Incentive-compatible mechanisms for continuous resource allocation in mobility-as-a-service: Pay-as-You-Go and Pay-as-a-Package

Haoning Xi\textsuperscript{a,b}, Wei Liu\textsuperscript{a,c}, David Rey\textsuperscript{a,*}, S.Travis Waller\textsuperscript{a}, Philip Kilby\textsuperscript{b}

\textsuperscript{a}Research Centre for Integrated Transport Innovation, School of Civil and Environmental Engineering, University of New South Wales, Sydney, NSW 2052, Australia
\textsuperscript{b}Data61, CSIRO, Canberra ACT 2601, Australia
\textsuperscript{c}School of Computer Science and Engineering, University of New South Wales, Sydney, NSW 2052, Australia

Abstract

Mobility as a Service (MaaS) has recently received a significant attention from researchers, industry stakeholders, and the public sector. The vast majority of existing MaaS paradigms are articulated based on the traditional segmentation of travel modes, e.g. private vehicle, public transportation (bus, metro, light rail) and shared mobility (car/bike/ride-sharing, ride-sourcing). In the context of ‘Everything-as-a-Service’ (XaaS), service providers have evolved from product-based models towards less segmented representations in which resources are priced in a continuous fashion. Yet, such continuous resource allocation formulations are inexistent for MaaS systems. This study attempts to address this gap in the literature by introducing innovative MaaS mechanisms that allocate mobility resources to users without any form of travel mode segmentation. We introduce an online auction framework where travelers have the possibility to bid for continuous mobility resources based on their requirements and willingness to pay (WTP). We propose two MaaS mechanisms, named Pay-as-You-Go (PAYG) and Pay-as-a-Package (PAAP), which allow travelers to either pay for the immediate use of mobility services or to subscribe to mobility service packages for a more protracted usage. Both MaaS mechanisms are based on mixed-integer or linear programming formulations designed to maximize social welfare in the transport system. We show that the proposed PAYG and PAAP mechanisms are incentive-compatible thus promoting truthful user bidding behavior. We develop efficient online primal-dual algorithms to implement the proposed MaaS mechanisms and derive theoretical bounds on the worst-case performance of these algorithms. Moreover, we design a rolling horizon framework to incorporate booking flexibility and improve the social welfare obtained by the proposed online algorithms. Numerical results on extensive problem instances generated from realistic mobility data highlight the benefits of the proposed MaaS mechanisms, and quantify the trade-offs among the proposed approaches.

Keywords: Mobility as a Service, mechanism design, incentive-compatibility, mixed-integer linear programming, primal-dual online algorithm, resources allocation

\textsuperscript{*}Corresponding author at: Research Centre for Integrated Transport Innovation, School of Civil and Environmental Engineering, University of New South Wales, Sydney, NSW 2052, Australia.

Email address: d.rey@unsw.edu.au (David Rey)

Preprint submitted to Elsevier September 16, 2020
1. Introduction

In recent years, the rapid evolution of the digital and sharing economy has brought significant changes in how various services are provided. Everything-as-a-Service concept (XaaS) – such as Computing-as-a-service, Platform-as-a-service, Software-as-a-Service – is revolutionizing traditional models, where people value experience over the possession of material commodity. The global economic transition from ‘commodity’ to ‘service’ to ‘experience’ has changed the way how services are delivered to travelers. In this context, the transport sector is experiencing a vast revolution brought by the Mobility as a Service (MaaS) concept.

The increasing number of transport services and the advances in information and automated vehicle technologies pave the way for the development of innovative MaaS paradigms. Yet, the definition of MaaS is is still a topic of discussion within the research community. The Dutch Ministry defined MaaS as the provision of multimodal, demand-driven mobility services, offering customised travel options to customers via a digital platform with real-time information, including payment and transaction processing in the white paper (MuConsult, 2017). Kamargianni et al. (2016) indicated that the term MaaS should focus on purchasing mobility services based on consumer needs, rather than purchasing travel across different transport modes. Hensher (2017) indicated that MaaS will shift the public transport contracts from the current output-based form (delivering kilometres on defined modes) to outcome-based models (delivering accessibility using any mode), thereby become mode-agnostic. Hensher and Mulley (2020) summarized that MaaS can be identified as offering users a multimodal package of mobility with a single payment platform.

Traditionally, a user travelling with different transport modes are required to make payments to multiple TSPs. In the MaaS paradigm, subscriptions or payments can be made through a single digital platform under two tariff models: Pay-as-You-Go (PAYG) and Pay-as-a-Package (PAAP). PAYG charges users according to the effective use of the service; whereas PAAP allows customers to subscribe a more protracted period usage of mobility packages, such as a weekly or monthly package through a single payment. Ho et al. (2018) discussed different payment mechanisms in MaaS and reported that PAAP was not more attractive due to its higher single payment. However, PAAP allows customers to pre-pay for the mobility services with lower marginal costs (Kamargianni et al., 2016), has the potential to attract more travelers to the shared modes and public transit (Matyas and Kamargianni, 2019a) and is beneficial for the sustainability.

We summarize the main characteristics of MaaS reported in the existing literature in Table 1.

| Features                          | Explanations                                                                 | Literature                                                                 |
|-----------------------------------|-------------------------------------------------------------------------------|---------------------------------------------------------------------------|
| Integration of modes              | Bundle different public and private transportation modes.                     | (Mulley et al., 2018; Utriainen and Pöllänen, 2018)                     |
|                                   |                                                                              | (Wong et al., 2020; Meurs et al., 2020)                                   |
| Customized subscription           | Tailor the service for a traveler based on preference and WTP                | (Polydoropoulou et al., 2020; Caiati et al., 2020)                       |
|                                   |                                                                              | (Mulley et al., 2018; Matyas and Kamargianni, 2019b)                     |
| Digital platform                  | Users can book or request services on a digital platform.                    | (Kamargianni and Matyas, 2017; Barreto et al., 2018)                     |
|                                   |                                                                              | (Merkert et al., 2020; Wang et al., 2016)                                 |
| On-demand                         | Offer demand-responsive services and tariff                                  | (Jitteaprom et al., 2017; Djavadian and Chow, 2017)                      |
|                                   |                                                                              | (Hensher, 2017; Mulley et al., 2018)                                     |
| Payment options                   | Pay-as-You-Go and Pay-as-a Package                                           | (Caiati et al., 2020; Ho et al., 2020)                                   |
|                                   |                                                                              | (Matyas and Kamargianni, 2019a)                                           |
| Operation mode                    | Free market operation and government-contracted operation                    | (Hensher et al., 2020; Hirschhorn et al., 2019)                          |
| Multi-actors                      | MaaS ecosystem focuses on multi-disciplinary collaborations                 | (Merkert et al., 2020; Pantelidis et al., 2020)                          |
|                                   |                                                                              | (Polydoropoulou et al., 2020; Beheshtian et al., 2020)                    |
Although MaaS appears to be attractive, Karlsson et al. (2020) have shown that most travellers are not willing to change their travel habits and register as a MaaS customer of the UbiGo pilot in Gothenburg. The authors concluded that whether travelers can perceive a ‘match’ between their mobility requirements and the solution offered by the service (in terms of cost, transport modes, etc.) or not, is the decisive factor to remove the barrier. This highlights that further research is needed to develop and to promote practical MaaS solutions.

We explore the potential of MaaS solutions through the lens of mechanism design, which focuses on the identification of economic incentives to achieve targeted objectives (Haeringer, 2018). Our goal is to develop online, incentive-compatible auction-based mechanisms for MaaS that aim to maximize social welfare. This is motivated by the rapid development of Internet of Things (IoT) which have made online auctions increasingly popular, notably in the context of Computing-as-a-Service (CaaS). CaaS allows users to locate applications and data on a third-party server, instead of investing in its own Information Technology (IT) infrastructures. The advent of cloud computing resources served by providers such as ‘Google cloud’ could efficiently allocate the multi-resources to end-users based on different online mechanisms. For instance, Zhang et al. (2013) proposed a truthful online cloud auction framework for users with heterogeneous requests. Shi et al. (2015) designed a truthful online combinatorial auction for cloud computing, which is computationally efficient and can guarantee a certain competitive ratio. Zhou et al. (2016) proposed an efficient online auction design to allocate the cloud resources by considering the completion deadlines and developed efficient approximation algorithms. More recently, Tafsiri and Yousefi (2018) proposed a combinational double auction-based mechanism for cloud computing and proved the efficiency and incentive compatibility of the mechanism.

Inspired by the literature and practice on CaaS, we propose a MaaS paradigm based on the premise that mobility services can be regarded as continuous quantities. For instance, distance, travel time, and price can be regarded as continuous features of a mobility service. Building on this framework, we propose MaaS online mechanisms to efficiently allocate mobility resources to user requests so as to maximize social welfare. The main contributions of this paper are summarized as follows (the main tasks in this paper are illustrated in Figure 1):

- We propose an innovative and alternative representation for MaaS in which continuous and unified mobility resources integrated from both public and private multi-modal transportation providers can be efficiently and effectively allocated to travelers based on their preference and willingness to pay (WTP) through a single payment on a digital platform.
- We design incentive-compatible PAYG and PAAP mechanisms based on the online auctions, and formulate mixed-integer and linear optimization formulations to allocate mobility resources to user requests so as to maximize social welfare.
- We develop PAYG and PAAP online algorithms for the proposed online formulations and derive guaranteed optimality bounds (competitive ratio) of the proposed online algorithms.
- We design a rolling horizon framework to consider booking flexibility and improve the solution quality of the MaaS online mechanisms by executing offline formulations in online fashion.
- We conduct numerical experiments to test the proposed online mechanisms, formulations, algorithms and illustrate their performance on a series of problems. The results also show that the proposed online algorithms are competitive and can be efficiently executed in large scale data sets.

The rest of this paper is organized as follows: Section 2 introduces the problem setting; Section 3 introduces MaaS online mechanisms; Section 4 (Section 5) gives PAYG (PAAP) online and
offline mobility resources allocation formulations, tailors PAYG (PAAP) primal-dual online algorithm, derives the competitive ratio of PAYG (PAAP) online algorithm and proves the incentive compatibility of the PAYG (PAAP) online mechanism. Section 6 designs a rolling horizon-based framework; Section 7 conducts numerical experiments for illustration; Section 8 concludes the paper, provides the remarks and discusses future research directions.

2. Problem Setting

2.1. Problem statement

In the proposed MaaS paradigm, assume that a traveler can request multiple mobility services in terms of different travel time (speed) for the given travelling distance, then the mobility resources integrated from different transportation service providers (TSPs) can be allocated to travelers in forms of a ‘bundle plan’, which combines different transport modes (e.g. taxi, ride-sharing, public transit, bicycle-sharing) in the most logic and efficient way. For each mode, the average speed and inconvenience degrees per time slot based on vehicle occupancy and transfers are assumed known (see examples in Figure 2); thus the average speed and inconvenience degree of a MaaS bundle can be determined based on the time spent in each mode. In analogy to a commodity-based transport paradigm, the quantity of a mobility service is described by the distance of a trip, and the quality is described by the speed, delay, and inconvenience degrees, etc.

In the PAYG mechanism, a traveler is assumed to bid for a mobility service for immediate use, and the traveler’s request should be either accepted or rejected, i.e. we represent this decision as binary allocation. In turn, under the PAAP mechanism, a traveler is assumed to subscribe a mobility package to be used over a pre-defined period of time, e.g. such as a monthly mobile phone subscription. In this context, travelers are assumed to bid for mobility package but may be allocated any fractional amount of their requested mobility services. The fraction of a bundle plan denotes the proportion of the requested distance and travel time, keeping the requested speed unchanged.

We depict two examples in Figure 2. Consider a traveller with the fixed inconvenience tolerance of 100 for each trip will purchase a 10 km-mobility service for immediate usage or subscribe a 70 km-mobility service for weekly usage. In the PAYG mechanism, the traveler can request for a ‘10 km-service’ by bidding for different speeds at different prices. For example, if the traveller submits
three bids such as (10 km, 12.2 km/h, $10), (10 km, 30 km/h, $50) and (10 km, 30 km/h, $20); then MaaS operator will arrange the most efficient combinations of transport modes for each bid from a practical standpoint of this trip, such as three bundle plans shown in Figure 2. Although any of the three bundle plans is feasible for the traveler, PAYG online mechanism allocates the traveler with the bundle plan that can maximize his/her utility, e.g. ‘Bundle plan C’.

In the PAAP mechanism, the traveller’s inconvenience tolerance is 700 for a weekly package plan, assume that the traveller submits three bids such as (70 km, 12.2 km/h), (70 km, 30 km/h, $350) and (70 km, 30 km/h), if MaaS operator arranges package bundle plan A, B and C for the traveler with a fraction such as 50%, 30% and 20%, then the weekly package plan can be denoted by (70 km, 18.23 km/h), which is the weighted sum of the corresponding distance, speed and inconvenience degrees in three package bundle plans (see in Figure 2). Note that the inconvenience degrees of any bundle plan can not exceed the traveler’s fixed inconvenience tolerance.

2.2. Preliminaries

We consider the problem of managing the allocation of mobility resources from the perspective of the MaaS operator with the aim to maximize social welfare in the transport system. In the proposed MaaS paradigm, users are assumed to be able to bid for mobility bundles to meet their travel needs. Hence, from the MaaS operator’s perspective, the decision problem at hand is to optimize the allocation of mobility resources while accounting for users’ (multi-)bids for MaaS bundles so as to maximize social welfare. To enable this MaaS paradigm to address the proposed optimization problem, we make the following assumptions on the context of the transport system.
A summary of the mathematical notations used throughout the paper is provided in Appendix A.

**Assumption 1: Autonomous transport system.** We consider autonomous vehicles and autonomous public transportation system providing on-demand public transit without fixed schedules and stops. Therefore, travel is assumed to be flexible with respect to time and space.

**Assumption 2: MaaS market under government contracting.** Wong et al. (2020) envisaged a government-contracted MaaS model that delivers on autonomous market freedoms while maintaining strong regulatory control, where the government can directly cooperate with a MaaS operator through a competitive tender. Similarly, this study considers the MaaS operator under government contracting who integrates mobility resources from various TSPs, such as taxi companies, ridesharing companies, metro operators, bus companies and bicycle-sharing companies (Figure 3). The aim of MaaS operator under government contracting is to optimize social welfare. Further, we assume that the MaaS operator and the TSPs are under reselling contracts, in which TSPs are paid by the MaaS operator to satisfy their reservation utility regardless of whether the provided mobility resources are utilized.

**Assumption 3: MaaS system with given capacity and without congestion.** We assume that the quantity of available mobility resources in the transport system is known and that the network is congestion-free. The summation of the number of vehicles times their speed represents the total distance traveled within one time slot, which is the movement capacity of the network itself used to derive the quantity of available mobility resources. Let $\mathcal{M}$ be the set of transport modes considered. To capture the difference among various transport modes, the distance corresponding to the TSP $m$ is weighted by the average speed of mode-$m$ vehicle denoted by $v_m$. We assume that the number of mode-$m$ vehicles $N_m$, the number of seats on a mode-$m$ vehicle $h_m$, and the average speed of mode-$m$ vehicle $v_m$ are fixed. Hence, the quantity of capacity-weighed mobility resources available at each time slot is denoted by $C$ (see in Eq. (1)):

$$C = \sum_{m \in \mathcal{M}} v_m \cdot N_m v_m h_m,$$

where $N_m v_m h_m$ represents the total distance provided by TSP $m$ per time slot.

---

1The reselling model has already been discussed by many in the literature for shared mobility, e.g., Zhang et al. (2020). One may refer to the literature for more detailed discussion regarding the reselling model.
Assumption 4: The average speed of different modes is not the “driving speed” but takes into potential service delays (e.g., commercial speed). In the proposed MaaS paradigm, we assume that the waiting time, pick-up time, drop-off time, transfer time, detour time, etc. are already incorporated into the average speed of each mode; thus the average speed is the commercial speed rather than the driving speed on the road. Different transport modes with varying vehicle occupancy are assumed to have different commercial speed, i.e., $v_4$ (public transit) < $v_3$ (ridesharing with 3 riders) < $v_2$ (ridesharing with 2 riders) < $v_1$ (taxi).

We next introduce MaaS online mechanisms to solve mobility resource allocation problems under the proposed MaaS paradigm.

3. MaaS online mechanisms

The proposed MaaS online mechanisms (PAYG and PAAP) are designed to efficiently match travellers’ requirements and WTP with the available mobility resources through an online auction process, where a bid is an expression of the traveller’s requirements and WTP, and the auction outcomes include the allocated weighted quantity of mobility resources and payment. We next present the online auction setting in Section 3.1, before introducing the auction process in Section 3.2. We then introduce time-varying payment rules in Section 3.3 and illustrate the proposed online auction in an example in Section 3.4.

3.1. Online auction setting

We propose a sealed-bid online auction, where a bidder submits multiple atomic bids to the auctioneer without knowing other bidders’ information. A bidding language (Nisan, 2006) defines the way that bidders are allowed to express their requirement and formulate their bids. In the PAYG mechanism, travelers use exclusive-OR (XOR) bidding language, which allows each traveler at most wins one of the submitted multiple bids. In the PAAP mechanism, travelers use additive-OR (OR) bidding language, which allows each bidder either gets all items contained in a bundle through the weighted sum of multiple bids or gets nothing. The participators of the online auction include:

- **Owner**: transportation service providers (TSPs) such as taxi companies, ridesharing companies, bus companies, metro operators and bicycle-sharing companies.
- **Customer**: A traveler can buy any quantity of mobility service with the required quality according to his or her preference and WTP.
- **Auctioneer**: MaaS operator is a third party to monitor and manage the auction process involving the TSPs and the travelers according to the auction rules.

The proposed online auction consists of three rules: i) winner determination rule, ii) payment rule and iii) mobility resources allocation rule. The MaaS operator first determines the number of travelers who can participate in the following auction process, and then determine the successful bidders based on the winner determination rule; finally allocates mobility resources to the successful bidders in terms of bundle plan and charges the travelers based on the payment rule. The above rules are assumed to be known to all the participants in the online auction process.
3.2. Online auction process

The proposed MaaS online mechanisms are executed at each time slot, and the main procedure of the auction process at time slot \( t \) is introduced in the following steps.

Step 1: **Submit the bidding language.** For each time slot \( t \), we consider a set of travelers \( \mathcal{I}(t) \) who submit their multi-bids to the MaaS operator with either PAYG or PAAP option.

In the PAYG mechanism, each bid submitted by Traveler \( i \) includes his or her origin and destination information (\( \zeta_i \)) which is converted into the shortest travelling distance (\( D_i \)) by MaaS operator, departure time (\( O_i \)), delay budget (\( \Phi_i \)), inconvenience tolerances (\( \Gamma_i \)). Further, let \( \mathcal{J}_i \) be the set of Traveler \( i \)'s atomic bids, for Traveler \( i \)'s bid \( j \): requested travel time (\( T_{ij} \)) and bidding price (\( b_{ij} \)) are provided, and the weighted quantity of mobility resources for Traveler \( i \)'s bid \( j \) is defined as \( Q_{ij} = D_i T_{ij} \). Overall Traveler \( i \)'s XOR bidding language is denoted by \( B_i = \{ D_i, O_i, \Phi_i, \Gamma_i, \{ Q_{ij}, b_{ij} : j \in \mathcal{J}_i \} \} \) or \( B_i = \{ O_i, \Phi_i, \Gamma_i, \{ Q_{ij}, b_{ij} : j \in \mathcal{J}_i \} \} \).

In the PAAP mechanism, a traveler directly requests for the distance (\( D_i \)) included in the mobility package, \( O_i \) represents Traveler \( i \)'s requested start date of the mobility package, Traveler \( i \)'s requested time period of the package (\( L_i \)) is also provided. Then the OR bidding language is written as \( B_i = \{ D_i, O_i, \Phi_i, \Gamma_i, L_i, \{ Q_{ij}, b_{ij} : j \in \mathcal{J}_i \} \} \) or \( B_i = \{ O_i, \Phi_i, \Gamma_i, L_i, \{ Q_{ij}, b_{ij} : j \in \mathcal{J}_i \} \} \).

Step 2: **Select the bidders.** Let Traveler \( i \)'s unit bidding price for bid \( j \in \mathcal{J}_i \) be the ratio of the bidding price \( b_{ij} \) to the weighted quantity of mobility resources \( Q_{ij} \). Let \( j_i^* \) be the bid corresponding to the maximum unit bidding price of user \( i \) in \( \mathcal{I}(t) \), defined as \( j_i^* = \arg \max_{j \in \mathcal{J}_i} \left\{ \frac{b_{ij}}{Q_{ij}} \right\} ; \forall i \in \mathcal{I}(t) \). Let \( A_t \) be the available mobility resources at time slot \( t \). If the mobility resources of the MaaS system are deficient, the MaaS operator sorts travelers by decreasing maximum unit bidding price, and determines the participants of the auction by identifying the critical index \( k \) such that \( \sum_{i=1}^{k-1} Q_{ij_i^*} \leq A_t < \sum_{i=1}^{k} Q_{ij_i^*} \). Travelers \( k, k + 1, \ldots, |\mathcal{I}(t)| \) are removed from the current auction, and the set \( \mathcal{I}(t) \) is updated to \( \mathcal{I}(t) = \{ 1, 2, \ldots, k - 1 \} \) (see in Figure 4).
Step 3: **Winner determination.** Let $p_t$ be the unit price determined by the MaaS operator at time slot $t$ based on the current available mobility resources $A_t$, which will be discussed later in Section 3.3. If all of Traveller $i$’s unit bidding prices are smaller than the unit price at time slot $t$, namely, if $b_{ij}/Q_{ij} \leq p_t, \forall j \in J_i$, then all of Traveler $i$’s bids will be rejected; otherwise, one of Traveller $i$ bids will be accepted. This is illustrated in Figure 4, where Travelers 1, 2, ..., $k-2$ succeed in the auction at time slot $t$, and Travelers $k-1, \cdots, |I(t)|$ are rejected at time slot $t$. The rejected travellers can participate in subsequent auctions at the following time slots with the same bidding price or submit new bids.

Step 4: **Resources allocation and payments.** In the PAYG mechanism, a successful bidder can get a bundle plan corresponding to the bid that can maximize his or her utility. In the PAAP mechanism, a successful bidder can get a package bundle plan based on the allocated fraction of each bid. Then the payment is charged according to the time-varying payment rules (see in Section 3.3). Traveler $i$’s utility obtained from bid $j$ is defined as Eq.(2):

$$u_{ij} = v_{ij} - p_{ij},$$

where $v_{ij}$ denotes Traveler $i$’s valuation (WTP) on bid $j$, $v_{ij} = b_{ij}$ holds in an incentive-compatible mechanism, $p_{ij}$ denotes Traveler $i$’s payment for bid $j$ defined in Eq.(3).

Step 5: **Update the occupied mobility resources.** After each auction, MaaS operator sums the arranged number of time slots during a trip for each accepted Traveler $i$ ($L_i$) and then update each traveler $i$’s quantity of occupied mobility resources within the range of his/her departure time slot ($O_i$) to arrival time slot ($O_i + L_i$), respectively.

### 3.3. Time-varying payment rules

The proposed time-varying pricing scheme aims to guarantee that when the mobility resources are deficient, the travellers who are willing to pay higher can secure the service, and is significantly different between peak and non-peak hours. This pricing scheme can be regarded as the posted pricing, in which the unit price ($p_t$) is posted at each time slot, and then the payment is determined by a traveler’s requested weighted quantity of mobility resources.

Traveler $i$’s payment for bid $j$ is calculated through Eq.(3):

$$p_{ij} = p_t \sum_{j \in J_i} Q_{ij} x_{ij}, \forall i \in I(t),$$

where $x_{ij}$ is a binary variable indicating the acceptance or rejection in the PAYG mechanism and is a continuous variable within $[0,1]$ indicating the allocated fraction in the PAAP mechanism. $Q_{ij}$ is the weighted quantity of mobility resources for Traveler $i$’s bid $j$, $p_t$ is the unit price at time slot $t$.

At time slot $t$, the range of unit price is within the minimum and maximum unit bidding price, $p_t \in [b_{\text{min}}, b_{\text{min}} + b_{\text{max}}]$, in which $b_{\text{min}} = \min_{i \in I(t), j \in J_i} \left\{ \frac{b_{ij}}{Q_{ij}} \right\}$, $b_{\text{max}} = \max_{i \in I(t), j \in J_i} \left\{ \frac{b_{ij}}{Q_{ij}} \right\}$. We consider three types of unit price functions: linear, quadratic and exponential. Recall that $C$ is the mobility resources capacity, let $z_t = C - A_t$ denote the quantity of allocated mobility resources at time slot $t$. The proposed unit price functions are discussed below.

- The linear unit price function is:

$$p_t = \frac{b_{\text{max}}}{C} z_t + b_{\text{min}}.$$  

Note that if $z_t = 0$, $p_t = b_{\text{min}}$; if $z_t = C$, $p_t = b_{\text{max}} + b_{\text{min}}$. 

• The quadratic unit price function is:

\[ p_t = \frac{1}{C^2} (z_t)^2 + \frac{b_{\max}}{C} z_t + b_{\min}. \]  

(5)

Since \( \frac{dp_t}{dz_t} \geq 0 \), Eq.(5) monotonically increases with respect to \( z_t \) within \([b_{\min}, b_{\min} + b_{\max}]\).

• The exponential unit price function in the PAYG mechanism is:

\[ p_t = \frac{b_{\max}}{\alpha_t - 1} \cdot \left( \alpha_t^{\frac{z_t}{t}} - 1 \right) + b_{\min}, \]  

(6)

where \( \alpha_t = (1 + R_t)^{\frac{1}{t}} \), \( R_t = \max_{i \in \mathcal{I}(t)} \{ \frac{Q_t}{\lambda_i} \} \), \( Q_t = \max_{j \in \mathcal{J}} \{ Q_{ij} \} \), \( \forall i \in \mathcal{I}(t) \). The unit price in Eq.(6) monotonically increases with respect to \( z_t \) within \([b_{\min}, b_{\min} + b_{\max}]\).

Similarly, the exponential unit price function in the PAAP mechanism is:

\[ p_t = \frac{b_{\max}}{\beta_t - 1} \cdot \left( \beta_t^{\frac{z_t}{t}} - 1 \right) + b_{\min}, \]  

(7)

where \( \beta_t = (1 + \varphi_t)^{\frac{1}{t}} \), \( \varphi_t = \min_{i \in \mathcal{I}(t)} \{ \frac{Q_t}{\lambda_i} \} \), \( Q_t = \min_{j \in \mathcal{J}} \{ Q_{ij} \} \), \( \forall i \in \mathcal{I}(t) \). Note that exponential unit price function will be used to design the iteration rule in Algorithm 1 (Line 16) and Algorithm 2 (Line 17).

3.4. Illustration of the proposed online auction mechanism

We next give an example to illustrate the online auction process in the PAYG mechanism shown in Figure 5. We assume that the capacity is \( C = 10 \) and we use a linear unit price function \( p_t = 0.2(C - A_t) + 1 \). \( A_t \) denotes the weighted quantity of available resources at time slot \( t \).

![Figure 5: An example to illustrate the online auction process](image_url)

**At time slot 1:**

Step 1: Both Traveler 1 and Traveler 1’ submit two bids to the MaaS operator:

\[ B_1 = \{(O_1 = 1, \Phi_1 = 1, \Gamma_1 = 20, (Q_{11} = 7, b_{11} = 7.7), (Q_{12} = 6, b_{12} = 7.2))\} \]

\[ B_1' = \{(O_1' = 1, \Phi_1' = 5, \Gamma_1' = 30, ((Q_{11}' = 7, b_{11}' = 6.3), (Q_{12}' = 6, b_{12}' = 3))\} \]

Step 2: The MaaS operator sorts the unit bidding price \( \frac{b_{12}}{Q_{12}} \geq \frac{b_{11}}{Q_{11}} \), since \( Q_{12} \leq C \leq Q_{12} + Q_{11}' \), only Traveler 1 can participate in the following online auction.
Step 3: Assume that all mobility resources are available at time slot 1, i.e. \( A_1 = C \), the unit price is \( p_1 = $1 \). Since \( p_1 \leq \frac{b_{11}}{Q_{11}} \) and \( p_1 \leq \frac{b_{12}}{Q_{12}} \), Traveler 1’s both bids may be accepted. To determine which bid to accept, the MaaS operator calculates Traveler 1’s utility for the two bids via Eq.(2):

\[
\begin{align*}
& u_{11} = b_{11} - p_1 Q_{11} = 7.7 - 7 = 0.7 , \quad u_{12} = b_{12} - p_1 Q_{12} = 7.2 - 6 = 1.2 .
\end{align*}
\]

Step 4: Since \( u_{11} < u_{12} \), the MaaS operator arranges a bundle plan for bid 2 and charges Traveler 1 the payment \( p_{12} = p_1 Q_{12} = $6 \).

Step 5: Since Traveler 1’s total travel time \( (L_1) \) is 3 time slots, the MaaS operator will update the quantity of occupied mobility resources within time slot 1 ∼ time slot 3 (Figure 5).

At time slot 2:

Step 1: Traveler 2 submits 2 bids to the MaaS operator.

\[
B_2 = \{ O_2 = 2, \Phi_2 = 5, \Gamma_2 = 35, \{ (Q_{21} = 3, b_{21} = $8), (Q_{22} = 6, b_{22} = $13) \} \}
\]

Step 2: \( Q_{21} \leq A_2 \), thus Traveler 2 can participate in the following online auction.

Step 3: The MaaS operator posts the unit price \( (p_2 = 0.2 \times 6 + 1 = $2.2) \). Since \( p_2 \leq \frac{b_{21}}{Q_{21}}, p_2 > \frac{b_{22}}{Q_{22}} \), Traveler 2’s bid 1 will be accepted.

Step 4: The MaaS operator will arrange a bundle plan for Traveler 2’s bid 1 and charge Traveler 2 the payment \( p_{21} = p_2 Q_{21} = $6.6 \).

Step 5: Since Traveler 2’s total travel time \( (L_2') \) is 9 time slots, the MaaS operator will update the quantity of occupied mobility resources within time slot 2 ∼ time slot 10 (Figure 5).

At time slot 3:

Step 1: Traveler 3 submits 2 bids to the MaaS operator.

\[
B_3 = \{ \{ O_3 = 3, \Phi_3 = 6, \Gamma_3 = 25, (Q_{31} = 1, b_{31} = $2), (Q_{32} = 0.5, b_{32} = $1.2) \} \}
\]

Step 2: \( Q_{31} = A_3 \), thus Traveler 3 can participate in the online auction.

Step 3: The MaaS operator posts the unit price \( (p_3 = 0.2 \times 9 + 1 = $2.8) \). Since \( p_3 \geq \frac{b_{31}}{Q_{31}}, p_3 \geq \frac{b_{32}}{Q_{32}}, \) Traveler 3 will be rejected and rolled over into the auction at time slot 4.

The main differences between the PAAP and PAYG mechanisms are in Step 2 and Step 5. In the PAAP mechanism, travellers can be allocated a fraction of their requested bids. E.g., if Traveler 1’s two bids are allocated \( x_{11} = 0.3 \) and \( x_{12} = 0.4 \), the total quantity of mobility resources allocated for Traveler 1 is \( x_{11} Q_{11} + x_{12} Q_{12} = 4.5 \), and Traveler 1’s payment is \( x_{11} Q_{11} p_1 + x_{12} Q_{12} p_1 = $4.5 \). Then MaaS operator will arrange Traveler 1 with a mobility package for one-week usage \( (L_1 = 7) \).

4. Pay-as-You-Go (PAYG) mechanism

We introduce the mobility resources allocation formulation for the PAYG mechanism in Section 4.1, the PAYG primal-dual online algorithm in Section 4.2 and the competitive ratio analysis in Section 4.3.
4.1. PAYG online social welfare optimization model

In PAYG mechanism, each time slot $t$ represents a small time interval within one day. Each traveler could transfer to another mode at the end of any time slot, e.g., a traveler served by a taxi in time slot $t$ can transfer to be served by a bus in time slot $t + 1$. In order to accurately describe travelers’ transfer among different modes and update the occupied and released mobility resources timely, $t$ should be set as a small-time interval, such as one minute. For a given time slot $t \in \Omega$, we formulate the online social welfare maximization problem for the PAYG mechanism (Model 1), as a mixed-integer linear program (MILP).

**Model 1** (PAYG online mobility resources allocation formulation).

$$\text{max} \sum_{i \in I(t)} \sum_{j \in J_i} b_{ij} x_{ij}, \quad (8a)$$

subject to

$$\sum_{m \in M} \sum_{j \in J_i} v_m l_{ij}^m = \sum_{j \in J_i} D_i x_{ij}, \quad \forall i \in I(t), \quad (8b)$$

$$0 \leq \sum_{m \in M} l_{ij}^m - T_{ij} x_{ij} \leq \Phi_i, \quad \forall i \in I(t), j \in J_i, \quad (8c)$$

$$\sum_{m \in M} \sigma_m l_{ij}^m \leq \Gamma_i, \quad \forall i \in I(t), j \in J_i, \quad (8d)$$

$$x_{ij} (b_{ij} - p_{ij}) \geq 0, \quad \forall i \in I(t), j \in J_i, \quad (8e)$$

$$\sum_{i \in I(t)} \sum_{j \in J_i} Q_{ij} x_{ij} \leq A_t, \quad (8f)$$

$$\sum_{j \in J_i} x_{ij} \leq 1, \quad \forall i \in I(t), \quad (8g)$$

$$0 \leq l_{ij}^m \leq \ell_{ij}^m, \quad \forall i \in I(t), j \in J_i, m \in M, \quad (8h)$$

$$x_{ij} \in \{0, 1\}, \quad \forall i \in I(t), j \in J_i. \quad (8i)$$

The main decision variables of Model 1 are $l_{ij}^m$ a real variable denoting the travel time served by mode $m$ in the bundle plan arranged for Traveler $i$’s bid $j$ and $x_{ij}$ a binary variable indicating whether Traveler $i$’s bid $j$ is accepted or not. The social welfare in this paper is defined as the total utility of all travelers and TSPs. TSPs’ utility is the revenue obtained from travelers, $\sum_{i \in I(t)} \sum_{j \in J_i} p_{ij} x_{ij}$; travelers’ utility is the difference between their valuation and the actual payment, $\sum_{i \in I(t)} \sum_{j \in J_i} (b_{ij} - p_{ij}) x_{ij}$; thus the objective function (8a) can be written as $\sum_{i \in I(t)} \sum_{j \in J_i} (b_{ij} - p_{ij}) x_{ij} + \sum_{i \in I(t)} \sum_{j \in J_i} p_{ij} x_{ij}$. Constraint (8b) guarantees that the bundle plan arranged for Traveler $i$’s bid $j$ can satisfy the travelling distance requirement, constraint (8c) guarantees that the bundle plan arranged for Traveler $i$’s bid $j$ can satisfy the travel time and delay budget requirement, constraint (8d) guarantees that inconvenience degrees of a bundle plan brought by sharing space with others during the trip should not exceed Traveler $i$’s inconvenience tolerance ($\Gamma_i$). Constraint (8e) and constraint (8g) indicates that if a traveler’s bidding price for each bid is less than the actual payment, it will be rejected; otherwise, one of Traveler $i$’s bids will be accepted. Constraint (8f) restricts that the weighted quantity of requested mobility resources should not exceed the weighted quantity of available resources ($A_t$) at time slot $t$, in which $A_t$ is obtained by updating each traveler’s quantity of occupied mobility resources.
within his/her departure time slot \((O_i)\) and arriving time slot \((O_i + \mathcal{L}_i)\) at the end of time slot \(t\): \(A_{t+1} = A_t - \sum_{i \in \mathcal{I}(t)} \sum_{j \in \mathcal{J}_i} Q_{ij} x_{ij}\). Constraint \((8g)\) guarantees that at most one bid among a traveler’s multi-bids will be accepted.

Using Model 1, the proposed PAYG online mechanism determines whether to accept a traveler’s bid or not, and arranges a bundle plan for accepted bids. We next show that the proposed PAYG online mechanism is Incentive-Compatible (IC).

**Lemma 1.** Let \(S_i\) be the compact set of Traveler \(i\)’s bundle plans defined as \(S_i \triangleq \bigcup_{j \in \mathcal{J}_i} S_{ij}, \forall i \in \mathcal{I}(t)\),

\[
S_{ij} \triangleq \left\{ (x_{ij}, l_{ij}), l_{ij} = \lfloor \frac{m_{ij}}{e_{ij}} \rfloor \right\} \right| (8b) - (8d), b_{ij} \geq p_{ij} \right\}. \tag{9}
\]

Let \(\chi_{i,s}\) be a binary variable indicating whether the bundle plan \(s \in S_i\) is allocated to Traveler \(i\) or not. The compact integer program (IP) defined in Eq.\((10)\) is equivalent to Model 1.

\[
\begin{align*}
\max_{i \in \mathcal{I}(t)} \sum_{s \in S_i} b_{i,s} \chi_{i,s}, \tag{10a} \\
\text{subject to} \hspace{1cm}
\sum_{i \in \mathcal{I}(t)} \sum_{s \in S_i} Q_{i,s} \chi_{i,s} \leq A_t, \hspace{1cm} \tag{10b} \\
\sum_{s \in S_i} \chi_{i,s} \leq 1, \hspace{1cm} \forall i \in \mathcal{I}(t), \hspace{1cm} \tag{10c} \\
\chi_{i,s} \in \{0,1\}, \hspace{1cm} \forall i \in \mathcal{I}(t), s \in S_i. \hspace{1cm} \tag{10d}
\end{align*}
\]

**Proof.** We now show that MILP \((8)\) can be reformulated as IP \((10)\) using a compact packaging reformulation. According to the winner determination rule of the MaaS online mechanism (Step 4) and constraint \((8c)\) in Model 1, if \(b_{ij} \geq p_{ij}\), Traveler \(i\)’s bid \(j\) will be accepted, \(x_{ij} = 1\); if \(b_{ij} \leq p_{ij}\), Traveler \(i\)’s bid \(j\) will be rejected, \(x_{ij} = 0\). Let \(b_{i,s}\) and \(Q_{i,s}\) denote Traveler \(i\)’s bidding price and requested weighted quantity of mobility resources of the bundle plan \(s\) arranged for Traveler \(i\)’s bid \(j\), we have \(Q_{i,s} = Q_{ij}, \forall i \in \mathcal{I}(t), j \in \mathcal{J}_i, s \in S_{ij}\). Moreover, since the arranged bundle plan \(s\) meets all of the requirements on Traveler \(i\)’s bid \(j\), Traveler \(i\)’s bidding price for bid \(j\) is equal to that for bundle plan \(s\), \(b_{i,s} = b_{ij}, \forall i \in \mathcal{I}(t), j \in \mathcal{J}_i, s \in S_{ij}\). Thus Model 1 can be reformulated as a compact IP \((10)\) with packaging structure given in Lemma 1.

**Theorem 1.** The proposed PAYG online mechanism for MaaS is incentive-compatible (IC), in which bidding truthfully is a traveler’s weakly dominant strategy.

**Proof.** According to the payment rules of the MaaS online mechanism given in Eq.\((3)\), Traveler \(i\)’s payment for bid \(j\) is independent of Traveler \(i\)’s bidding price, and will be charged with the same payment, no matter how much the bidding price is. Traveler \(i\)’s utility for bid \(j\) is defined in Eq.\((2)\). If Traveler \(i\) bids truthfully, his/her bidding price on bid \(j\) is equal to his/her true valuation (WTP) on bid \(j\), \(b_{ij} = v_{ij}\); if bid \(j\) is accepted, \(x_{ij} = 1\), his/her utility is \(u_{ij} = v_{ij} - p_{ij}\); if bid \(j\) is rejected, \(x_{ij} = 0\), his/her utility is \(u_{ij} = 0\). If Traveler \(i\) lies about his bidding price, \(b_{ij} \neq v_{ij}\); if bid \(j\) is accepted, Traveler \(i\)’s utility is still \(u_{ij} = v_{ij} - p_{ij}\); if bid \(j\) is rejected, his/her utility is not greater than 0. We enumerate all possible cases to compare Traveler \(i\)’s utility for bid \(j\) with regards to truthful and non-truthful bidding behavior in Table 2.
Dual online
Minimize : \( Q \)
Subject to:
\( u \)
\( q \)

Lemma 2. Let \( O \) of primal variables, dual variables in a time loop. The iteration rule of \( R \) process. Line 8 and Line 9 define the parameters \( s \) bundle plan price function in Eq.(6), Line 16 and Line 17 guarantee that each traveler is allocated with the available resources at each time slot to capacity \( O \) of occupied mobility resources within time slot \( q \).\end{itemize}

\( \alpha \)
\( \beta \)

4.2. Primal-dual online algorithm for PAYG mechanism

In this section, we tailor an online algorithm for the proposed MaaS online mechanism by resorting to the primal-dual algorithm design technique (Borodin and El-Yaniv, 2005; Buchbinder and Naor, 2009).

The online compact IP (10) given in Lemma 1 can be converted into a compact LP by relaxing the binary variable \( \chi_{i,s} \in \{0,1\} \) to the continuous variable \( \chi_{i,s} \geq 0 \). The relaxed compact LP \( (OPT_1(t)) \) and its dual problem \( (OPT_2(t)) \) are summarized as follows:

\[
\begin{align*}
\text{Maximize:} & \quad \max \sum_{i \in I(t)} \sum_{s \in S_i} b_{i,s} \chi_{i,s}, \\
\text{Subject to:} & \quad \sum_{i \in I(t)} \sum_{s \in S_i} Q_{i,s} \chi_{i,s} \leq A_t, \\
& \quad \sum_{s \in S_i} \chi_{i,s} \leq 1, \forall i \in I(t), \\
& \quad \chi_{i,s} \geq 0, \forall i \in I(t), s \in S_i,
\end{align*}
\]

\[
\begin{align*}
\text{Minimize:} & \quad A_t q(t) + \sum_{i \in I(t)} (u_i), \\
\text{Subject to:} & \quad Q_{i,s} q(t) + u_i \geq b_{i,s}, \forall i \in I(t), s \in S_i, \\
& \quad q(t) \geq 0, \\
& \quad u_i \geq 0, \forall i \in I(t),
\end{align*}
\]

where the primal variable \( \chi_{i,s} \) corresponds to the dual constraint; and where \( q(t) \) and \( u_i \) are the dual variables corresponding to the two constraints of the primal problem \( (OPT_1(t)) \) at time slot \( t \). Observe that \( q(t) \) and \( u_i \) can be interpreted as the unit price at time slot \( t \) and Traveler \( i \)’s utility.

In Algorithm 1, \( \omega = |\Omega| \) denotes the last time slot, Line 1 initializes the weighted quantity of available resources at each time slot to capacity \( C \), Line 3 ~ Line 7 are based on the online auction process. Line 8 and Line 9 define the parameters \( R_t \) and \( \alpha_t \). Line 12 ~ Line 20 show the iterations of primal variables, dual variables in a time loop. The iteration rule of \( u_i \) (Line 15) is determined by the dual constraint, the iteration rule of \( q(t) \) (Line 16) is determined by the exponential unit price function in Eq.(6), Line 16 and Line 17 guarantee that each traveler is allocated with the bundle plan \( s^* \) that can maximize his utility. Line 21 and Line 22 update each traveler’s quantity of occupied mobility resources within time slot \( O_t \sim O_t + L_t \) at the end of time slot \( t \), and then provide the quantity of available mobility resources at time slot \( t + 1 \) in Line 10.

**Lemma 2.** Let \( S \) denotes the largest compact set of bundles across all travelers \( i \in I(t) \). The worst-case time complexity of Algorithm 1 is \( \mathcal{O}(|\Omega| |S| |I(t)|) \).
Lemma 3. In each time iteration of Algorithm 1 (Line 2), \( \Delta \mathcal{P}(i, s) = (1 - \frac{1}{\alpha_t}) \Delta \mathcal{D}(i, s) \) holds, in which \( \Delta \mathcal{D}(i, s) \) and \( \Delta \mathcal{P}(i, s) \) denote the change value in the objective functions of dual problem and primal problem, respectively.

Proof. In each iteration of a time loop, \( \Delta \mathcal{D}(i, s) \) can be written as Eq.(11), in which \( u_i \) and \( \Delta q(t) \) are obtained from Line 15 and Line 16 in Algorithm 1.

\[
\Delta \mathcal{D}(i, s) = A_t \Delta q(t) + u_i = A_t \left[ q(t) \frac{Q_i}{A_t} + \frac{b_{i,s}}{\alpha_t - 1} A_t \right] + u_i,
\]

\[
= Q_i q(t) + \frac{b_{i,s}}{\alpha_t - 1} + b_{i,s} - Q_i q(t),
\]

\[
= b_{i,s} \left( \frac{1}{\alpha_t - 1} + 1 \right).
\]

Since \( \chi_{i,s} = 1 \) (Line 18), the change value in objective function of primal problem is \( \Delta \mathcal{P}(i, s) = \)
\( b_{i,s} \), thus the relationship between \( \Delta D(i,s) \) and \( \Delta P(i,s) \) is written as Eq. (12).

\[
\Delta D(i,s) = \left( \frac{1}{\alpha_t} - 1 \right) \Delta P(i,s).
\]  

(12)

**Lemma 4.** Algorithm 1 constructs feasible solutions for both the primal and dual online problems.

**Proof.** We first prove that Algorithm 1 yields dual feasible solutions. Let \( q(t) \) denote the value of the dual variable at the end of each iteration in a time loop (Line 2). If \( q(t) \geq \frac{b_{i,s}}{Q_s} \), then the dual constraint always holds; else if \( q(t) \leq \frac{b_{i,s}}{Q_s} \), the dual variables will be increased until the dual constraints are satisfied. Since \( \alpha_t = b_{i,s} - Q_sq(t) \) (Line 15), the subsequent increase of \( q(t) \) is always feasible due to the winner determination rules: if Traveler \( i \)'s unit bidding price on bundle plan \( s \) is no less than the unit price at time slot \( t \), i.e \( q(t) \leq \frac{b_{i,s}}{Q_s} \), Traveler \( i \) will be allocated with the bundle plan \( s^* \) maximizing \( b_{i,s} - Q_sq(t) \); otherwise, if \( q(t) \geq \frac{b_{i,s}}{Q_s} \), Traveler \( i \) will be rejected \( (x_{i,s} = 0) \), and the dual variables will not be updated.

We next show that Algorithm 1 yields primal feasible solutions. The iteration rule of \( q(t) \) (Line 16) ensures that \( q(t) \) is bounded by the sum of a geometric sequence with the common ratio \((1 + \frac{Q_s}{A_t})\). Consider a geometric sequence produced by the iterations of \( q(t) \) for Traveler \( m \): the first item is \( \frac{b_{m,s}}{(\alpha_t - 1)A_t} \), in which the value of \( b_{m,s} \) is fixed in each iteration (Lines 13~18), and the common ratio is \( 1 + \frac{Q_s}{A_t} \). Based on the formula of the sum of geometric sequence, we obtain Eq.(13):

\[
q(t) \geq \frac{b_{m,s}}{Q_m} \cdot \frac{1}{\alpha_t - 1} \left[ \left( 1 + \frac{Q_m}{A_t} \right)^{\sum_{i \in I(t)} \sum_{s \in S} \chi_{i,s}} - 1 \right],
\]  

(13)

where the number of iterations in each time loop is larger than \( \sum_{i \in I(t)} \sum_{s \in S} \chi_{i,s} \). Eq.(13) can be rewritten as Eq.(14):

\[
q(t) \geq \frac{b_{m,s}}{Q_m} \cdot \frac{1}{\alpha_t - 1} \left[ \left( 1 + \frac{Q_m}{A_t} \right)^{A_t} \frac{\sum_{i \in I(t)} \sum_{s \in S} Q_s \chi_{i,s}}{A_t} - 1 \right].
\]  

(14)

Let \( R_t \) denote the maximum ratio of a traveler’s requested quantity of mobility resources to the available resources at time slot \( t \), \( R_t = \max_{i \in I(t)} \left\{ \frac{Q_m}{A_t} \right\} \). Since \( 0 \leq \frac{Q_i}{A_t} \leq \frac{Q_m}{A_t} \leq 1 \), we obtain Eq.(15):

\[
q(t) \geq \frac{b_{m,s}}{Q_m} \cdot \frac{1}{\alpha_t - 1} \left[ \left( 1 + \frac{Q_m}{A_t} \right)^{A_t} \frac{\sum_{i \in I(t)} \sum_{s \in S} Q_s \chi_{i,s}}{A_t} - 1 \right].
\]  

(15)

Since \( 0 \leq \frac{Q_i}{A_t} \leq R_t \leq 1 \), we have \( \frac{\ln(1 + \frac{Q_i}{A_t})}{R_t} \geq \frac{\ln(1 + R_t)}{R_t} \), namely, \( (1 + \frac{Q_i}{A_t})^{R_t} \geq (1 + R_t)^{\frac{Q_i}{A_t}} \), this yields Eq.(16):

\[
1 + \frac{Q_i}{A_t} \geq (1 + R_t)^{\frac{1}{R_t} \frac{Q_i}{A_t}}.
\]  

(16)
Since \( \alpha_t = (1 + R_t)^{1/t} \) and \( Q_i = \max_{s \in \mathcal{S}} \{ Q_{i,s} \} \), Eq. (17) can be obtained by substituting Eq. (16) into Eq. (15):

\[
q(t) \geq \frac{b_{m,s}}{Q_m} \cdot \frac{1}{\alpha_t - 1} \cdot \left( \frac{\sum_{i \in \mathcal{I}(t)} \sum_{s \in \mathcal{S}} Q_{i,s} \chi_{i,s}}{x_t} - 1 \right).
\]

According to Eq. (17), if \( \sum_{i \in \mathcal{I}(t)} \sum_{s \in \mathcal{S}} Q_{i,s} \chi_{i,s} \geq A_t \), then \( q(t) \geq \frac{b_{m,s}}{Q_m} \). Since Algorithm 1 does not update the primal optimal solution (Line 20) if \( q(t) \geq \frac{b_{m,s}}{Q_m} \) (Line 19), the primal optimal solution will not be updated any further once \( \sum_{i \in \mathcal{I}(t)} \sum_{s \in \mathcal{S}} Q_{i,s} \chi_{i,s} \geq A_t \).

4.3. Competitive ratio analysis

To evaluate the performance of the proposed online primal-dual algorithm (Algorithm 1), we formulate an offline optimization model for PAYG mechanism which takes as input the entire travel demand over the multi-time period under consideration summarized in Model 2.

Model 2 (PAYG offline mobility resources allocation formulation).

\[
\begin{align*}
\text{max} & \quad \sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}_i} \sum_{t \in \Omega} b_{ij} x_{ij}^t, \\
\text{subject to} & \quad \sum_{m \in \mathcal{M}} v_{m,ij}^{mt} = D_{ij} x_{ij}^t, \quad \forall i \in \mathcal{I}, j \in \mathcal{J}_i, t \in \Omega : t \leq O_i, \\
& \quad 0 \leq \sum_{m \in \mathcal{M}} m_{ij}^{mt} - T_{ij} x_{ij}^t \leq \Phi_t, \quad \forall i \in \mathcal{I}, j \in \mathcal{J}_i, t \in \Omega : t \leq O_i, \\
& \quad \sum_{m \in \mathcal{M}} a_{m,ij}^{mt} \leq \Gamma_t, \quad \forall i \in \mathcal{I}, j \in \mathcal{J}_i, t \in \Omega : t \leq O_i, \\
& \quad x_{ij}^t (b_{ij} - p_{ij}^t) \geq 0, \quad \forall i \in \mathcal{I}, j \in \mathcal{J}_i, t \in \Omega : t \leq O_i, \\
& \quad p_t = \frac{b_{\text{max}}}{C} \left( \sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}_i} Q_{ij} x_{ij}^t \right) + b_{\min}, \quad \forall t \in \Omega, \\
& \quad p_{ij}^t = Q_{ij} p_t, \quad \forall i \in \mathcal{I}, j \in \mathcal{J}_i, t \in \Omega : t \leq O_i, \\
& \quad \sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}_i} Q_{ij} x_{ij}^t \leq A_t, \quad \forall t \in \Omega, \\
& \quad \sum_{j \in \mathcal{J}_i, t \in \Omega} x_{ij}^t \leq 1, \quad \forall i \in \mathcal{I}, \\
& \quad 0 \leq m_{ij}^{mt} \leq l_{ij}^{mt}, \quad \forall i \in \mathcal{I}, j \in \mathcal{J}_i, m \in \mathcal{M}, t \in \Omega : t \leq O_i, \\
& \quad b_{\min} \leq p_t \leq b_{\max} + b_{\min}, \quad \forall t \in \Omega, \\
& \quad p_{ij}^t \geq 0, \quad \forall i \in \mathcal{I}, j \in \mathcal{J}_i, t \in \Omega : t \leq O_i, \\
& \quad x_{ij}^t \in \{0, 1\}, \quad \forall i \in \mathcal{I}, j \in \mathcal{J}_i, t \in \Omega : t \leq O_i.
\end{align*}
\]

Model 2 can be viewed as a time-extended version of Model 1 where variable \( x_{ij}^t \) indicates whether Traveler \( i \)'s bid \( j \) will be allocated at time slot \( t \) or not, variable \( l_{ij}^{mt} \) represents the number
of time slots served by mode $m$ in the bundle plan arranged for Traveler $i$’s bid $j$, $p_t$ denotes the unit price at time slot $t$, and $p^t_{ij}$ denotes Traveler $i$’s actual payment for bid $j$ at time slot $t$. The objective function (18a) aims to maximize social welfare overall all time slots. Constraints (18b), (18c) and (18d) guarantee that the bundle plan can satisfy the traveler’s requested distance requirement, travel time requirement and inconvenience tolerance, respectively. Constraint (18e) illustrates that when the bidding price is smaller than the actual payment, the bid will be rejected. Constraint (18f) illustrates that the unit price will grow with the increasing quantity of allocated mobility resources at different time slots, and vary within $[b_{\min}, b_{\min} + b_{\max}]$. Constraint (18g) illustrates that the actual payment at time $t$ is determined by the unit price at time slot $t$ and the requested weighted quantity of mobility resources. Constraint (18h) restricts the weighted quantity of requested mobility resources at time slot $t$ can not exceed the weighted quantity of available mobility resources at time slot $t$. Constraint (18i) guarantees at most one of Traveler $i$’s multi-bids can be accepted at time slot $t$. Observe that if $|\Omega| = 1$, Model 2 is equivalent to Model 1.

Similar to the online formulation of the PAYG mechanism, Model 2 can be reformulated as a compact IP which can be relaxed into a compact LP.

**Lemma 5.** Let $S^t_i$ be the compact set of Traveler $i$’s bundle plans defined as $S^t_i \triangleq \bigcup_{j \in J_i} S^t_{ij}, \forall i \in I(t), \forall t \in \Omega$, with

$$S^t_{ij} \triangleq \{(x^t_{ij}, t^t_{ij}), l^t_{ij} = \lfloor l^m_{ij} \rfloor \mid (18b) - (18g), b_{ij} \geq p^t_{ij}\},$$

Let $\chi^t_{i,s}$ be a binary variable indicating whether bundle plan $s \in S^t_i$ is allocated to Traveler $i$ at time slot $t \in \Omega$ or not. The compact IP (20) defined below is equivalent to Model 2.

$$\begin{align*}
\max \sum_{i \in I} \sum_{s \in S^t_i} \sum_{t \in \Omega} b_{i,s} \chi^t_{i,s}, \\
\text{subject to } \\
\sum_{s \in S^t_i} \sum_{t \in \Omega} Q_{i,s} \chi^t_{i,s} \leq A_t, & \quad \forall t \in \Omega, \\
\sum_{s \in S^t_i} \sum_{t \in \Omega \leq O_{i}} \chi^t_{i,s} \leq 1, & \quad \forall i \in I, \\
\chi^t_{i,s} \in \{0, 1\}, & \quad \forall i \in I, t \in \Omega : t \leq O_{i}, s \in S^t_i.
\end{align*}
$$

*Proof.* The proof follows from that of Lemma 1. \qed

The compact IP (20) given in Lemma 5 can be converted into a compact LP by relaxing the binary variable $\chi^t_{i,s} \in \{0, 1\}$, leading to the following primal ($OPT_3$) and dual ($OPT_4$) offline formulations:
The competitive ratio of Algorithm 1 is \( \Theta \) (Borodin and El-Yaniv, 2005).

**Theorem 2.** The competitive ratio of Algorithm 1 is \( \Theta = (1 - R_{\max}) \left( 1 - \frac{1}{\alpha} \right) \), where \( \alpha = \min_{t \in T} \alpha_t \), \( R_{\max} = \max_{t \in T} R_t \), \( \alpha_t = (1 + R_t)^{\frac{1}{\alpha}} \), \( R_t = \max_{i \in I(t)} \{ Q_i / A_t \} \), \( \forall t \in T \), \( Q_i = \max_{s \in S_i} \{ Q_{i,s} \} \), \( \forall i \in I(t) \).

**Proof.** Let \( k \) be the critical index determined at Line 7 of Algorithm 1. Let \( q(t) \text{end} \) and \( q(t) \text{start} \) denote the value of \( q(t) \) before and after each iteration in the loop of \( i = k \) in Algorithm 1, respectively. Substituting \( q(t) \text{end} \) and \( q(t) \text{start} \) into Line 16 of Algorithm 1, yields Eq.(21):

\[
q(t) \text{end} = q(t) \text{start} \left( 1 + \frac{Q_k}{A_t} \right) + \frac{b_{k,s}}{(\alpha_t - 1)A_t}. \tag{21}
\]

According to Eq.(17) in Lemma 5, before examining Traveler \( k \)'s bids, the value of \( q(t) \text{start} \) is bounded as:

\[
q(t) \text{start} \geq \frac{b_{k,s}}{Q_k} \cdot \frac{1}{\alpha_t - 1} \cdot \left( \frac{\sum_{i \in I(t) \setminus \{k\}} \sum_{s \in S_i} Q_i \chi_{i,s} t \chi_i}{A_t} - 1 \right). \tag{22}
\]

Substituting Eq.(22) into Eq.(21), and simplifying yields:

\[
q(t) \text{end} \geq \frac{b_{k,s}}{Q_k} \cdot \frac{1}{\alpha_t - 1} \cdot \left( \frac{\sum_{i \in I(t) \setminus \{k\}} \sum_{s \in S_i} Q_i \chi_{i,s} t \chi_i}{A_t} - 1 \right) \cdot \left( 1 + \frac{Q_k}{A_t} \right) + \frac{b_{k,s}}{(\alpha_t - 1)A_t}. \tag{23}
\]

According to Eq.(16), we have \( 1 + \frac{Q_k}{A_t} \geq (1 + R_t)^{\frac{1}{\alpha}} \frac{Q_k}{A_t} \), thus Eq.(24) can be rewritten as:

\[
q(t) \text{end} \geq \frac{b_{k,s}}{Q_k} \cdot \frac{1}{\alpha_t - 1} \cdot \left[ \frac{\sum_{i \in I(t) \setminus \{k\}} \sum_{s \in S_i} Q_i \chi_{i,s} t \chi_i}{A_t} \cdot (1 + R_t)^{\frac{1}{\alpha}} \frac{Q_k}{A_t} - 1 \right]. \tag{25}
\]

Assume that Traveler \( k \) is accepted at time slot \( t \), then \( \sum_{i \in I(t) \setminus \{k\}} \sum_{s \in S_i} Q_i \chi_{i,s} t \chi_i + Q_k \chi_{k,s} = \)
Given the input sequence $\tau$, mobility resources at time slot $t$ $R_q$, and since $\alpha_t = (1 + R_t)\frac{1}{\alpha_t}$, Eq.(25) can be written as:

$$q(t)^{\text{end}} \geq \frac{b_{k,s}}{Q_k} \cdot \frac{1}{\alpha_t - 1} \cdot \left( \frac{\sum_{i \in \mathcal{I}(t)} \sum_{s \in S_i} Q_{i,s}}{A_t} \cdot \frac{Q_k}{\alpha_t} - 1 \right),$$

$$= \frac{b_{k,s}}{Q_k} \cdot \frac{1}{\alpha_t - 1} \cdot \left( \frac{\sum_{i \in \mathcal{I}(t)} \sum_{s \in S_i} Q_{i,s}}{A_t} - 1 \right).$$

Eq.(27) illustrates that if the requested quantity of mobility resources exceeds the available mobility resources at time slot $t$, $\sum_{i \in \mathcal{I}(t)} \sum_{s \in S_i} Q_{i,s} > A_t$, then Traveler $k$ will not be allocated any resources $q^\text{end} \geq \frac{b_{k,s}}{Q_k}$; thus $\sum_{i \in \mathcal{I}(t)} \sum_{s \in S_i} Q_{i,s} \geq A_t - \max_{i \in \mathcal{I}(t)} \{Q_i\}, \forall t \in \Omega$. Since $R_t = \max_{i \in \mathcal{I}(t)} \{Q_i\}, \forall t \in \Omega$, the social welfare obtained by Algorithm 1 at time slot $t$ is at least:

$$\sum_{i \in \mathcal{I}(t)} \sum_{s \in S_i} b_{i,s} Q_{i,s} (1 - R_t) = \sum_{i \in \mathcal{I}(t)} \sum_{s \in S_i} b_{i,s} \chi_{i,s} \cdot (1 - R_t).$$

According to Lemma 4, $\Delta \mathcal{D}(i, s)$ and $\Delta \mathcal{P}(i, s)$ denote the change value in the objective functions of dual problem ($OPT_2(t)$) and primal problem ($OPT_1(t)$), respectively, at each iteration of Algorithm 1. Let $\mathcal{P}(t)$ and $\mathcal{D}(t)$ denote the objective value of the dual and primal problems obtained by Algorithm 1. At iteration $t$, we have $\mathcal{D}(t) = \sum_{i \in \mathcal{I}(t)} \sum_{s \in S_i} \Delta \mathcal{D}(i, s)$, $\mathcal{P}(t) = \sum_{i \in \mathcal{I}(t)} \sum_{s \in S_i} \Delta \mathcal{P}(i, s)$. Based on Lemma 3, the relationship between $\mathcal{D}(t)$ and $\mathcal{P}(t)$ can be described as Eq.(29):

$$\mathcal{P}(t) = \left(1 - \frac{1}{\alpha_t}\right) \mathcal{D}(t).$$

Given the input time sequence $\tau = [1, 2, \cdots, t - 1, t]$, let $\mathcal{P}(\tau)$ and $\mathcal{D}(\tau)$ denote the objective value of the primal problem ($OPT_3$) and the dual problem ($OPT_2$) obtained by Algorithm 1. Since $\alpha = \min_{t \in \tau} \alpha_t$, Eq.(29) implies:

$$\mathcal{P}(\tau) \geq \left(1 - \frac{1}{\alpha}\right) \mathcal{D}(\tau).$$

Let $W_{\text{Alg1}}(\tau)$ denote the social welfare obtained by Algorithm 1 corresponding to the sequence $\tau$. Since $R_{\text{max}} = \max_{t \in \tau} R_t$, according to Eq.(28), we have $W_{\text{Alg1}}(\tau) \geq (1 - R_{\text{max}}) \mathcal{P}(\tau)$ and:

$$W_{\text{Alg1}}(\tau) \geq (1 - R_{\text{max}}) \left(1 - \frac{1}{\alpha_t}\right) \mathcal{D}(\tau).$$

Given the input sequence $\tau$, Let $Z^{IP}(\tau)$ denote the optimal value of the offline compact IP (20) and $Z^*(\tau)$ denote the optimal value of the offline compact LP ($OPT_3$). According to weak duality, we have $\mathcal{D}(\tau) \geq Z^*(\tau)$, and Eq.(31) can be rewritten as follows:

$$W_{\text{Alg1}}(\tau) \geq (1 - R_{\text{max}}) \left(1 - \frac{1}{\alpha_t}\right) Z^*(\tau),$$

$$\geq (1 - R_{\text{max}}) \left(1 - \frac{1}{\alpha}\right) Z^{IP}(\tau).$$
Hence the competitive ratio of Algorithm 1 is \( \Theta = (1 - R_{\text{max}}) \left(1 - \frac{1}{\alpha}\right) \).

5. Pay-as-a-Package (PAAP) mechanism

In this section, we present the PAAP mechanism which targets mobility packages that can be purchased in fractional amounts. We introduce the mobility resources allocation formulation for the PAAP online mechanism in Section 5.1, the PAAP primal-dual online algorithm in Section 5.2 and the competitive ratio analysis in Section 5.3.

5.1. PAAP online social welfare optimization model

In the proposed PAAP mechanism, time slots are intended to represent longer time periods compared to the PAYG mechanism, e.g. one day vs one minute. For a given time slot \( t \in \Omega \), we formulate the online social welfare maximization problem for the PAAP mechanism as a linear program (LP) (Model 3).

**Model 3** (PAAP online mobility resources allocation formulation).

\[
\begin{align*}
\max & \sum_{i \in I(t)} \sum_{j \in J_i} b_{ij} x_{ij}, \\
\text{subject to} & \quad (8b), (8h), \\
0 \leq & \ x_{ij} \leq 1, \quad \forall i \in I(t), j \in J_i.
\end{align*}
\]

Observe that Model 3 is the LP-relaxation of Model 1 wherein \( x_{ij} \) is a continuous variable representing the fraction of mobility resources allocated to Traveler i’s bid \( j \). Analogously to the PAYG mechanism, we next show that the proposed PAAP online mechanism is IC.

**Lemma 6.** Let \( S_i \) be the compact set of Traveler i’s bundle plans defined as \( S_i \triangleq \bigcup_{j \in J_i} S_{ij}, \forall i \in I \),

\[
S_{ij} \triangleq \{ (r_{ij}, l_{ij}) \mid l_{ij} = \lfloor l_m^{ij} \rfloor | (8b) - (8d), b_{ij} \geq p_{ij} \}
\]

Let \( r_{i,s} \) be a continuous variable indicating the fraction of package bundle plan \( s \in S_i \) allocated to Traveler i. The online compact linear program (LP) defined in Eq. (36) is equivalent to Model 3.

\[
\begin{align*}
\max & \sum_{i \in I(t)} \sum_{s \in S_i} b_{i,s} r_{i,s}, \\
\text{subject to} & \quad \sum_{s \in S_i} Q_{i,s} r_{i,s} \leq A_t, \\
\sum_{s \in S_i} & \ r_{i,s} \leq 1, \quad \forall i \in I(t), \\
\ r_{i,s} & \geq 0, \quad \forall i \in I(t), s \in S_i.
\end{align*}
\]

**Proof.** Proof of Lemma 6 is provided in Appendix B.1.

\[\square\]
Theorem 3. The PAAP MaaS online mechanism is IC, in which bidding truthfully is a traveler’s weakly dominant strategy.

Proof. The dual problem of the compact LP (36) given in Lemma 6 is:

\[
\begin{align*}
\min \ A_t q(t) + \sum_{i \in I(t)} u_i, \\
\text{subject to} \\
Q_{i,s} q(t) + u_i &\geq b_{i,s}, & \forall i \in I(t), s \in S_i, \\
q(t) &\geq 0, \\
u_i &\geq 0, & \forall i \in I(t).
\end{align*}
\]

(37a)

(37b)

(37c)

(37d)

where \(q(t)\) and \(u_i\) are the dual variables corresponding to constraints (36b) and (36c), respectively, and (37b) is the dual constraint corresponding to the primal variable \(r_{i,s}\) in the compact LP (36).

According to primal-dual complementary slackness, if \(u_i > 0\), constraint (36c) is binding, \(\sum_{s \in S_i} r_{i,s} = 1, \forall i \in I(t)\), namely, Traveler \(i\) will be allocated with the requested quantity of resources; else if \(u_i = 0\), then \(\sum_{i \in I(t)} \sum_{s \in S_i} r_{i,s} \leq 1\), namely, Traveler \(i\) will be fractionally allocated with the requested quantity of resources. Moreover, if \(r_{i,s} > 0\), constraint (37b) is binding, \(u_i = \sum_{i \in I(t)} \sum_{s \in S_i} b_{i,s} - Q_i q(t)\); if \(r_{i,s} = 0\), \(u_i = \sum_{i \in I(t)} \sum_{s \in S_i} b_{i,s} - Q_i q(t)\). Hence:

\[
u_i^* = \max \{0, \{b_{i,s} - Q_i q(t)\}\}, \forall i \in I(t), s \in S_i.
\]

(38)

where \(u_i^*\) and \(q^*(t)\) denote the optimal solutions of the dual problem (36). If we interpret \(q(t)\) as the unit price at time slot \(t\), then \(Q_{i,s} q(t)\) can be interpreted as Traveler \(i\)’s payment for package plan \(s\) and \(u_i\) can be interpreted as Traveler \(i\)’s utility obtained from package bundle plan \(s\). According to Eq.(38), if \(u_i \geq 0\), the proposed PAAP MaaS online mechanism maximizes Traveler \(i\)’s utility.

Since we have proved that the maximum utility of each traveler holds at the optimal solution of the primal problem (36) and dual problem (37), we conduct a sensitivity analysis on Traveler \(i\)’s bidding price \((b_{i,s})\) through the simplex tableau method to show IC. Without any loss of generality, we consider the case in which two travelers \(i\) and \(i + 1\) bid for bundle plan \((s)\) with bidding price \(b_{i,s}\) and \(b_{i+1,s}\). Assume that Traveler \(i\) is non-truthful i.e., bids higher or lower than his valuation on bundle plan \(s\) \((v_{i,s})\). The bid of Traveller \(i\) can be written as \(\hat{b}_{i,s} = b_{i,s} + \Delta b\), with \(\hat{b}_{i,s} = v_{i,s}\). In turn, assume that Traveler \(i + 1\) bids truthfully, i.e. \(b_{i+1,s} = v_{i+1,s}\). The standardized formulation of LP (36) can be formulated as LP (39):

\[
\begin{align*}
\max \ b_{i,s} r_{i,s} + b_{i+1,s} r_{i+1,s}, \\
\text{subject to} \\
Q_{i,s} r_{i,s} + Q_{i+1,s} r_{i+1,s} + y_1 &= A_t, \\
r_{i,s} + y_2 &= 1, \\
r_{i+1,s} + y_3 &= 1, \\
r_{i,s}, r_{i+1,s}, y_1, y_2, y_3 &\geq 0.
\end{align*}
\]

(39a)

(39b)

(39c)

(39d)

(39e)

where \(y_1, y_2\) and \(y_3\) are the slack variables corresponding to constraints (39b),(39c) and (39d), respectively. The initial simplex tableau (Table 3.1) obtained from the standard formulation LP (39) can be converted into the final simplex tableau (Table 3.2) via standard methods.

In Table 4, we conduct sensitivity analysis on \(b_{i,s}\) by substituting \(b_{i,s}\) in Table 3.2 with \(\hat{b}_{i,s} = v_{i,s}\)
Since the reduced cost of $\hat{y}_1$ and $\hat{y}_2$ always hold non-positive values, $\sigma(\hat{y}_1) \leq 0$, $\sigma(\hat{y}_2) \leq 0$, the optimal solutions do not change with $\Delta b$. Then we investigate how the dual variables $\hat{u}_i$ and $\hat{q}(t)$ change with $\Delta b$. The reduced cost coefficients ($\sigma$) corresponding to the slack variables in the primal final tableau give the opposite value of the reduced cost coefficient of the slack variable associated with the $i$th primal constraint (Arora, 2004). Since the slack variable $\hat{y}_1$ in Table 4 corresponds to the dual variable $\hat{q}(t)$, we have $\hat{q}(t) = -\sigma(\hat{y}_1) = \frac{b_{i+1,s}}{Q_{i+1,s}}$. In Table 3.2 we have $q^*(t) = -\sigma(y_1^*) = \frac{b_{i+1,s}}{Q_{i+1,s}}$, the change value of $q(t)$ is written as Eq.(40):

$$\Delta q(t) = \hat{q}(t) - q^*(t) = 0.$$  \hspace{1cm} (40)

Since $\hat{q}(t)$ can be interpreted as the unit price at time slot $t$, Traveler $i$’s utility is $\hat{u}_i = v_{i,s} - Q_{i,s}\hat{q}(t)$, the change in Traveler $i$’s utility ($\Delta u_i$) is 0. Moreover, the slack variable $y_2$ corresponding to the dual variable $u_i$, and thus we have: $\hat{u}_i = \sigma(y_2) = \frac{(b_i + \Delta b)Q_{i+1,s} + b_{i+1}Q_i}{Q_{i+1,s}}$, $u_i^* = \sigma(y_2^*) = \frac{b_iQ_{i+1,s} + b_{i+1}Q_i}{Q_{i+1,s}}$. Note that Traveler $i$’s true valuation on package plan $s$ is $b_{i,s}$, instead of $b_{i,s} + \Delta b$, thus Traveler $i$’s utility is $\hat{u}_i - \Delta b$, and the change in Traveler $i$’s utility ($\Delta u_i$) is written as (41):

$$\Delta u_i = \hat{u}_i - \Delta b - u_i^* = 0.$$  \hspace{1cm} (41)

We have shown that if Traveler $i$ bids higher than his/her true valuation ($\Delta b > 0$), his/her utility remains unchanged, whereas if he/she bids lower than this true valuation ($\Delta b < 0$), his/her utility may remain the same or be reduced. Hence bidding truthfully is a traveler’s weakly dominant strategy under proposed the PAAP online mechanism.

5.2. PAAP primal-dual online algorithm

We tailor a primal-dual online algorithm for the PAAP mechanism summarized in Algorithm 2. Although Algorithm 2 has a similar structure to that of Algorithm 1, the definition of parameters

| $\Delta q(t)$ | $\hat{q}(t) - q^*(t)$ | $0$ |
| $\Delta u_i$ | $\hat{u}_i - \Delta b - u_i^*$ | $0$ |
\( q_i, \varphi_t, \beta_t \), critical index selection (it is not needed to select in the PAAP mechanism) and iteration rules of variables \( r_{i,s} \) and \( q(t) \) are different.

**Algorithm 2**: PAAP primal-dual online algorithm

1. initialize \( A[1, 2, \ldots, \omega] \leftarrow C 
2. for \( t \in \Omega \) do
   3. each traveler submits a travel request \( B_i = \{O_i, \Phi_i, \Gamma_i, L_i, \{Q_{i,s}, b_{i,s} : s \in S_i\}\} \)
   4. \( Q_i \leftarrow \min_{s \in S_i} \{Q_{i,s} : \forall i \in I(t)\} \)
   5. \( \varsigma^*_i \leftarrow \arg \max_{s \in S_i} \left\{ \frac{b_{i,s}}{Q_{i,s}} \right\}, \forall i \in I(t) \)
   6. sort the unit bidding price: \( b_{1,s}^* \geq b_{2,s}^* \geq \ldots \geq b_{k,s}^* \geq \ldots \geq b_{I,s}^* \)
   7. \( \varphi_t \leftarrow \min_{i \in I(t)} \left\{ \frac{Q_i}{A_t} \right\} \)
   8. \( \beta_t \leftarrow (1 + \varphi_t)^{\frac{1}{\beta_t}} \)
   9. \( A_t \leftarrow A[t] \)
   10. initialize \( q(t) \leftarrow 0 \)
   11. for \( i \in I(t) \) do
       12. for \( s \in S_i \) do
           13. if \( q(t) \leq \frac{b_{i,s}}{Q_{i,s}} \) then
               14. \( r_{i,s} \leftarrow \frac{b_{i,s}}{Q_{i,s}} \sum_{s \in S_i} Q_{i,s} \)
               15. \( u_i \leftarrow b_{i,s} - Q_i q \)
               16. \( q(t) \leftarrow q(t)(1 + \frac{Q_i}{A_t}) + \frac{b_{i,s}r_{i,s}}{(\beta - 1)A_t} - \frac{(1-r_{i,s})b_{i,s}}{A_t} \)
           else if \( q(t) > \frac{b_{i,s}}{Q_{i,s}} \) then
               17. \( r_{i,s} \leftarrow 0 \)
           18. update \( A[O_i : O_i + L_i - 1] \leftarrow A[O_i : O_i + L_i - 1] - \sum_{s \in S_i} Q_{i,s} r_{i,s} \)
   20. return \( r \) and \( q \)

We next give a series of theoretical results for the PAAP mechanism in Lemmas 7-9 which are equivalent to those obtained in the PAYG mechanism, however, there are some difference in the proof procedure. For conciseness, the proofs are given in Appendix B.

**Lemma 7.** The worst-case time complexity of Algorithm 2 is \( O(\|\Omega\|\|S\|\|I(t)\|) \).

**Lemma 8.** In each iteration of Algorithm 2, \( \Delta P(i, s) = \left( 1 - \frac{1}{\beta} \right) \Delta D(i, s) \) holds, in which \( \Delta D(i, s) \) and \( \Delta P(i, s) \) denote the change value in objective function of dual problem and primal problem after each iteration in a time loop, respectively.

**Lemma 9.** Algorithm 2 constructs feasible solutions for both the primal and dual online problems.

5.3. Competitive ratio analysis

To analyse the performance of Algorithm 2, we formulate the offline social welfare maximization optimization problem for the PAAP mechanism as summarized in Model 4:
Model 4 (PAAP offline mobility resources allocation formulation).

\[
\begin{align*}
\text{max} \quad & \sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}_i} \sum_{t \in \mathcal{O}_i \mid t \leq O_i} b_{ij} x_{ij}^t, \\
\text{subject to} \quad & (18b)-(18l), \\
& 0 \leq x_{ij}^t \leq 1, \quad \forall i \in \mathcal{I}, j \in \mathcal{J}_i, t \in \mathcal{O} : t \leq O_i.
\end{align*}
\]

Model 4 is analogous to Model 2, and if $|\mathcal{O}|=1$, Model 4 is equivalent to Model 3.

Lemma 10. Let $S^t_i$ be the compact set of Traveler $i$’s package bundle plans defined as $S^t_i \triangleq \{(x_{ij}^t, l_{ij}^t) \mid (18b)-(18g), b_{ij} \geq p_{ij}\}$.

Let $r_{i,s}$ be a continuous variable representing the fraction of package bundle plan $s \in S^t_i$ allocated to Traveler $i$. The compact LP defined in Eq. (44) is equivalent to Model 4.

\[
\begin{align*}
\text{max} \quad & \sum_{i \in \mathcal{I}} \sum_{s \in S^t_i} \sum_{t \in \mathcal{O}_i \mid t \leq O_i} b_{i,s} r_{i,s}^t, \\
\text{subject to} \quad & \sum_{s \in S^t_i} \sum_{t \in \mathcal{O}_i \mid t \leq O_i} Q_i r_{i,s}^t \leq A_t, \quad \forall t \in \mathcal{O} : t \leq O_i, \\
& \sum_{s \in S^t_i} \sum_{t \in \mathcal{O}_i \mid t \leq O_i} r_{i,s}^t \leq 1, \quad \forall i \in \mathcal{I}, \\
& r_{i,s}^t \geq 0, \quad \forall i \in \mathcal{I}, s \in S^t_i, t \in \mathcal{O} : t \leq O.
\end{align*}
\]

Proof. The proof of Lemma 10 is provided in Appendix B.5.

Theorem 4. The competitive ratio ($\Theta$) of Algorithm 2 is $1 - \frac{1}{\beta}$, in which $\beta = \min_{t \in \mathcal{T}} \beta_t$, $\beta_t = (1 + \varphi_t) \frac{1}{\mathcal{A}_t}$, $\varphi_t = \min_{i \in \mathcal{I}(t)} \left\{ \frac{Q_i}{A_t} \right\}, \forall t \in \mathcal{O}$, $Q_i = \min_{s \in S^t_i} \{ Q_{i,s} \}, \forall i \in \mathcal{I}(t)$.

Proof. The proof of Theorem 4 is provided in Appendix C.

Corollary 1. When the ratio of a traveler’s requested weighted quantity of mobility resources to the available weighted quantity of mobility resources in a time slot approaches zero, the competitive ratio of PAYG primal-dual online algorithm (Algorithm 1) is equivalent to the competitive ratio of the PAAP primal-dual online algorithm (Algorithm 2). Namely, when $\frac{Q_i}{A_t} \to 0$, we have $\Theta = (1 - \frac{1}{\alpha}) \left(1 - \frac{1}{\beta}\right) = 1 - \frac{1}{e}$, in which $e$ denotes Euler’s number.

6. Rolling horizon framework

In the proposed MaaS online mechanisms, the travel demand data are input into the model as a ‘stream’ and the online mechanisms are executed periodically based on the current input
data, without knowledge of future demand. To implement the proposed MaaS online mechanisms, we use a rolling horizon algorithm (RHA) where $\Delta t$ denotes the time frequency at which the optimization problems are solved, and $T$ denotes the length of the time horizon considered at the current iteration. The time horizon length reflects travelers’ booking flexibility: at the iteration at time $t$, only travelers with a requested departure time in the time window $[t, t + T]$ are considered in the MaaS online mechanisms.

Figure 6 shows that the proposed rolling horizon framework rolls forward per $\Delta t$ time slots and solve the corresponding mobility resource allocation problem within the current time horizon $T$. Let $\omega$ denote the last time slot, the number of rolling processes ($n$) is written as $\omega/\Delta t$. The detailed procedure of the rolling horizon framework is introduced in Algorithm 3.

Algorithm 3: Rolling horizon framework

| Line | Description |
|------|-------------|
| 1    | initialize time horizon lengths $T$ and time step $\Delta t$ |
| 2    | for $n \in \{0, \Delta t, 2\Delta t, \cdots, \omega/\Delta t\}$ do |
| 3    | for $i \in \mathcal{I}(n)$ do |
| 4    | if $\Delta t = 1$ then |
| 5    | if MaaS mechanism=PAYG then |
| 6    | $[\chi_{i,s}(t), q(t)] \leftarrow \text{Algorithm 1} (B_i) \{\text{execute PAYG online algorithm}\}$ |
| 7    | $p_{t,s} \leftarrow \max_{s \in S_i} Q_{t,s} \chi_{i,s}$ |
| 8    | else if MaaS mechanism=PAAP then |
| 9    | $[r_{i,s}(t), q(t)] \leftarrow \text{Algorithm 2} (B_i) \{\text{execute PAAP online algorithm}\}$ |
| 10   | $p_{t,s} \leftarrow \max_{s \in S_i} Q_{t,s} r_{i,s}$ |
| 11   | else if $\Delta t > 1$ then |
| 12   | if MaaS mechanism=PAYG then |
| 13   | $[\chi_{t,s}^t, p_t, p_t^t] \leftarrow \text{Model 2} (B_i) \{\text{execute PAYG offline formulation}\}$ |
| 14   | else if MaaS mechanism=PAAP then |
| 15   | $[r_t, p_t, p_t^t] \leftarrow \text{Model 4} (B_i) \{\text{execute PAAP offline formulation}\}$ |
| 16   | return $\chi, p$ and $r$ |

We consider four RHA configurations with different time step ($\Delta t$), time horizon lengths ($T$) and optimization methods. All configurations can be executed with both the PAYG and the PAAP mechanisms. In each case, the corresponding mobility resources allocation problem can be solved either exactly (using Model 2/Model 4) or heuristically (using Algorithm 1/Algorithm 2).

1) RHA ($\Delta t = 1, T = 1$): rolls forward per time unit slot, and travelers placing an order at time slot $t$ can request a service departing at time slot $t$ ($O_i = t$).
2) **RHA** \((\Delta t = 1, \mathcal{T} = 240)\): rolls forward per unit time slot, and travelers placing an order at time slot \(t\) can book a service departing within the time window \([t, t + \mathcal{T}]\).

3) **RHA** \((\Delta t > 1, \mathcal{T} = 240)\): rolls forward every \(\Delta t\) time slots to solve a small scale mobility resources allocation problem, and the travelers placing an order at time \(t\) can book a service departing within the time window \([t, t + \mathcal{T}]\).

4) **SHA** \((\Delta t = \mathcal{T}, \mathcal{T} = 240)\): this is a single horizon configuration aiming to provide a benchmark to evaluate alternative configurations. In this configuration, the offline mobility resources allocation problem is solved with the entire travel demand data for the time period under consideration.

Since the first three configurations run in an online fashion, they are also referred to as online configurations. Instead, SHA is referred to as the offline configuration and is used as an oracle to benchmark the performance of the online RHA configurations.

7. **Numerical experiments**

We conduct a series of numerical experiments to evaluate the performance of the proposed MaaS online mechanisms. Specifically, we discuss the impacts of several parameters, i.e. the number of bids, the ratio of the maximum unit bidding price to the minimum unit bidding price, travel demand in a time slot, speed change ratio, and capacity. We also examine the impact of different types of unit price functions on social welfare for both PAYG and PAAP mechanisms, and numerically quantify the competitive ratios of the proposed online algorithms. All numerical experiments are conducted using Python 3.7.4 and CPLEX Python API on a Windows 10 machine with, Intel(R) Core i7-8700 CPU @ 3.20 GHz, 3192 Mhz, 6 Core(s) and with 64 GB of RAM.

7.1. Input data and parameter settings

Travel demand/requests information is an input to the proposed mechanisms. To determine travelers’ request data, i.e. bidding language sets, we conduct stochastic simulations under different auction settings, in which the parameters are set as follows. We consider five types of travel modes with different vehicle occupancy and speed. The average speed (km/min) and inconvenience (degrees/min) of different modes are given in Table 5.

| Modes       | m = 1 | m = 2 | m = 3 | m = 4 | m = 5 |
|-------------|-------|-------|-------|-------|-------|
| Taxi        | \(v_1\) | \(v_2\) | \(v_3\) | \(v_4\) | \(v_5\) |
| Ride sharing with 2 riders | 0.5 | 0.3 | 0.25 | 0.18 | 0.1 |
| Ride sharing with 3 riders | | | | | |
| Public transit | | | | | |
| Bicycle-sharing | | | | | | |

| Inconvenience | \(\delta_1\) | \(\delta_2\) | \(\delta_3\) | \(\delta_4\) | \(\delta_5\) |
|---------------|-------------|-------------|-------------|-------------|-------------|
| degrees/min   | 0           | 0.5         | 1           | 2           | 6           |

In the PAYG simulations, Traveller \(i\)'s bidding language is \(\{D_i, O_i, \Phi_i, \Gamma_i, \{T_{ij}, b_{ij} : j \in \mathcal{J}_i\}\}\). Traveler \(i\)'s requested distance \((D_i)\) for each trip is randomly generated within \([1\text{km}, 18\text{km}]\). Traveler \(i\)'s requested departure time is generated in different ways under different rolling horizon configurations as discussed in Section 6. Both delay budget (\(\Phi_i\)) and inconvenience tolerance (\(\Gamma_i\)) are Traveler \(i\)'s own characteristic and have a reverse relationship with his/her bidding price, \(\Phi_i\) is randomly generated within \([0, 100/b_{ij}]\), and \(\Gamma_i\) is randomly generated within \([0, 100D_i/b_{ij}]\). To generate a realistic MaaS system, the requested travel time for each trip should be within the range of
travel time taken by the fastest mode (taxi) and the slowest mode (bicycle-sharing), namely, the requested travel time of bid \( j \) of user \( i \) \((T_{ij})\) is randomly generated within the range of \([\frac{D_i}{v_5}, \frac{D_i}{v_1}]\). Given the travelling distance, the bidding price \((b_{ij})\) is set based on the tariff of current transport system in Sydney including: Uber, metro, bus, Tram and Lime. At each time slot, the minimum unit bidding price \((b_{min})\) is set based on the price of public transit in Sydney\(^2\), and the maximum unit bidding price \((b_{max})\) is set based on the price of UberX in Sydney, which varies over the time during one day\(^3\). Accordingly, Traveler \( i \)'s bidding price \((b_{ij})\) is randomly generated within \([b_{min} Q_{ij}, b_{max} Q_{ij}]\).

The operation time of the MaaS system is set as 20 hours every day (6:00am-01:00am). We consider time slot of 1 min, hence there are 1200 time slots per day, and the capacity of mobility resources in each time slot is set to 500. The number of travellers placing an order at time slot \( t \) \((\lambda_t)\) satisfies the normal distribution, where the mean value and standard deviation are set as different values between peak-hours and non-peak hours as indicated in Table 6.

| Time       | 6:00 - 7:00 | 8:00 - 9:00 | 10:00 - 17:00 | 18:00 - 19:00 | 20:00 - 01:00 |
|------------|-------------|-------------|---------------|---------------|---------------|
| Time slot  | [1, 2, \ldots, 120] | [121, 122, \ldots, 240] | [241, 182, \ldots, 720] | [721, 722, \ldots, 840] | [841, 842, \ldots, 1200] |
| \lambda_t | \sim \mathcal{N}(2, 1^2) | \sim \mathcal{N}(8, 2^2) | \sim \mathcal{N}(2, 1^2) | \sim \mathcal{N}(8, 2^2) | \sim \mathcal{N}(2, 1^2) |

In the PAAP simulations, the bidding language is \(\{D_i, O_i, \Phi_i, \Gamma_i, L_i, \{T_{ij}, b_{ij} : j \in J_i\}\}\). Traveler \( i \)'s requested distance \((D_i)\) for a mobility package is randomly generated within [1km, 300km], Traveler \( i \)'s requested time period of mobility package \((L_i)\) is randomly generated within [5, 14]. The other parameters are generated in the same way as in the PAYG simulations. In the PAAP simulations, each time slot represents one day and we consider 100 time slots (days) for a time cycle, and the capacity of mobility resources at each time slot is set to 10000. The number of travellers placing an order at time slot \( t \) \((\kappa_t)\) satisfies the normal distribution, where the mean value \((\mu)\) is 50 and the standard deviation \((\sigma)\) is 10, \(\kappa_t \sim \mathcal{N}(50, 10^2)\).

7.2. Numerical Results

We conduct sensitivity analysis on a series of parameters in Section 7.2.1, compare the PAYG and PAAP mechanisms in Section 7.2.2, validate the derived competitive ratios in Section 7.2.3 and discuss the booking flexibility under different rolling horizon lengths in Section 7.2.4; Finally compare the social welfare and computation time under different online and offline configurations in Section 7.2.5.

7.2.1. Sensitivity analysis on the MaaS online mechanisms

In this subsection, we execute Algorithm 1 to evaluate the performance of the PAYG online mechanism by conducting a sensitivity analysis on the bid range ratio \(b_{max}/b_{min}\), unit price functions, the speed change ratio and capacity in terms of acceptance ratio and social welfare.

We define the acceptance ratio as the ratio of the number of accepted travelers to the number of travelers participating the online auction at time slot \( t \), which is an important index for evaluating user satisfaction. For clarity, we only show the hourly average acceptance ratio, i.e. the average

\(^2\)Opal (2020) Trip Planner can be used to estimate the fare of different public transport modes in NSW, Australia.
\(^3\)Uber (2020) Real-time Estimator provides real-time fare estimates on each trip.
acceptance ratio for each bin of 60 time slots. Figure 7a, Figure 7b and Figure 7c show the acceptance ratio under linear unit price function (Eq.(4)), quadratic unit price function (Eq.(5)) and exponential unit price function (Eq.(6)) for varying bid range ratios, respectively.

Figure 7 shows that the acceptance ratio under exponential unit price function is higher than that under other types of functions, this is the reason why the exponential unit price function (Eq.(6)) is used as the iteration rule in Line 16 of Algorithm 1. Then we report the variation of the hourly average social welfare in Figure 8. We find that if $b_{\text{max}}/b_{\text{min}} = 5$, the value of social welfare is higher than its counterparts under all three types of unit price functions. Moreover, the social welfare in terms of time slot exhibits a similar pattern under three types of unit price functions.

To observe the influence of mobility resources capacity $C$ and of the travel mode speeds onto social welfare we set the value of $b_{\text{max}}/b_{\text{min}}$ to 5 and conduct sensitivity analysis on these parameters reported in Figure 9. Figure 9a shows that social welfare grows with the increase of capacity over all time slots, and that the social welfare remains unchanged beyond a capacity of 500. then apply
a speed factor (−75%, −50%, −25%, +25%, +50%) on the speed of each travel mode $v_m$ and report the results of social welfare. Figure 9b shows that the speed change ratio has very limited influence on social welfare over all time slots, and thus illustrate the reliability of the proposed MaaS system.

![Figure 8: Sensitivity analysis on different types of pricing functions under different $b_{max}/b_{min}$](image)

7.2.2. Comparison of the PAYG and PAAP mechanisms

In this subsection, we compare the daily social welfare achieved using both the PAYG and PAAP mechanisms, denoted $S_{PAYG}$ and $W_{PAAP}$, respectively. The travel demand at time slot $t$ in the PAYG mechanism ($\lambda_t$) and in the PAAP mechanism ($\kappa_t$) are given in Table 6 and Table 7, respectively. Since each time slot represents one day in the PAAP mechanism and represents one minute in the PAYG mechanism, travel demand at time slot $t$ in PAAP mechanism ($\kappa_t$) is set as the summation of $\lambda_t$ over 1200 time slots on workdays, ($\kappa_t = \sum_{t=1}^{1200} \lambda_t$), and is set as $\kappa_t = h \sum_{t=1}^{1200} \lambda_t$ on weekends, where $h$ is randomly set within [40%,80%]. The time period ($L_t$) is set as 1, 5, 6 and 7 in different weeks, then the bidding language sets in PAAP mechanism are obtained based on that in the PAYG mechanism by multiplying the parameters ($T_{ij}, b_{ij}, \Phi_i, \Gamma_i, Q_i$) with $L_t$. Let

![Figure 9: Sensitivity analysis on capacity (C) and speed change ratio ($\Delta v$)](image)
\(W_{PAYG}\) denote the social welfare of one time slot (min) in the PAYG mechanism. If \(L_i = 1\), \(W_{PAAP}\) corresponds to the summation of \(W_{PAYG}\) over 1200 time slots, \(S_{PAYG} = \sum_{t=1}^{1200} W_{PAYG}\).

### Table 7: Parameters setting in the online PAAP mechanism

| Day     | Week 1       | Week 2       | Week 4       | Week 3 (6,7,8) |
|---------|--------------|--------------|--------------|----------------|
|         | weekday      | weekend      | weekday      | weekday        |
|         | \([-1, 5]\)  | \([6, 7]\)   | \([8, 12]\)  | \([13, 14]\)   |
| \(L_i\)| 1            | 1            | 7            | 6              |

To compare both PAYG and PAAP mechanisms, we execute Algorithms 1 and 2, and report results in Figures 10 and 11. Figure 10a shows that the social welfare during morning peak hours (time slot 120 – 240) and evening peak hours (time slot 720 – 840) is considerably higher than that in off-peak hours using the PAYG mechanism.

![Figure 10](image1.png)

(a) \(t - W_{PAYG}\)

![Figure 11](image2.png)

(b) \(t - p_t\) and \(t - A_t\)

Figure 10: Social welfare \((W_{PAYG})\), unit price \((p_t)\) and available mobility resources \((A_t)\) in the PAYG mechanism

![Figure 11](image3.png)

(a) \(t - W_{PAAP}\)

(b) \(t - p_t\) and \(t - A_t\)

Figure 11: Social welfare \((W_{PAAP})\), unit price \((p_t)\) and available mobility resources \((A_t)\) in the PAAP mechanism
Comparing the social welfare achieved over time period $L_i$ using the PAYG mechanism, $L_i \cdot S_{PAYG}$, with that achieved using the PAAP mechanism, $W_{PAAP}$ (e.g., comparing $S_{PAYG}$ ($183398$) with $W_{PAAP}$ in week 1 ($L_i = 1$) and $7S_{PAYG}$ ($1283789$) with $W_{PAAP}$ in week 2 ($L_i = 7$)), we find that the social welfare of the PAAP mechanism is higher than that using the PAYG mechanism (Figure 11a). Moreover, Figure 10b and Figure 11b show that the unit price at time slot $t$ exhibits an opposite pattern against the available resources at time slot $t$ due to the proposed time-varying pricing scheme, and the range of unit price ($p_t$) in the PAAP mechanism ($$1 \sim 9$$) is lower than the unit price ($p_t$) in the PAYG mechanism ($$2 \sim 12$$).

### 7.2.3. Competitive ratio analysis

We analyse the competitive ratio ($\Theta$) given in Theorem 2/Theorem 4 by comparing with the social welfare ratio ($R$) defined as the ratio of social welfare obtained by Algorithm 1/Algorithm 2 to the corresponding offline formulation (Model 2/Model 4).

In the PAYG mechanism, the number of bids ($|J|$) is assumed to be comprised between $1 \sim 10$, and the travel demand at each time slot is given in Table 7. Figure 12a shows the relationship among the value of the competitive ratio ($\Theta$), the number of bids ($|J|$) and the number of time slots ($N_t$). Figure 12b shows that if $j = 1$, the value of $\Theta$ is within $0.54511 \sim 0.62145$; if $j = 2$, the value of $\Theta$ is within $0.60109 \sim 0.63128$; whereas if $j = 3, 4, \cdots, 10$, the value of $\Theta$ is within
that competitive ratio ($\Theta$) is independent on the number of time slots ($N_t$).

Figure 13a shows the relationship among social welfare ratio ($R$), the number of bids ($|J|$) and the number of time slots ($N_t$) in the PAYG mechanism. Figure 13b shows that if $j = 1$, the value of $R$ is within $0.74451 \sim 0.84882$, if $j = 2$, the value of $R$ is within $0.74125 \sim 0.86158$, whereas if $j = 3, 4, \cdots, 10$, the value of $R$ is within $0.78451 \sim 0.90882$. Figure 13c shows that the value of social welfare ratio ($R$) is independent on the number of time slots ($N_t$). Figure 12 and Figure 13 show that the competitive ratio ($\Theta$) given in Theorem 2 can always provide a lower bound for the social welfare ratio ($R$) under different size of input time sequence, which validates the derived competitive ratio ($\Theta$) in the PAYG mechanism. The gap between $R$ and $\Theta$ is within $0.19614 \sim 0.27754$. In the PAAP mechanism, Figure 14a shows the relationship of $R$, $|J|$ and $N_t$.

![Figure 14: Competitive ratio ($\Theta$), the number of bids ($|J|$) and the number of time slots ($N_t$) in PAAP mechanism](image)

Figure 14b shows that if $j = 1$, the value of $\Theta$ is within $0.632117 \sim 0.632120$; if $j = 2, 3, \cdots, 10$, the value of $\Theta$ is within $0.632116 \sim 0.632120$. Figure 14b and Figure 14c show that the value of $\Theta$ is independent on $|J|$ and $N_t$. Figure 15b shows that if $j = 1$, the value of $R$ is within $0.71451 \sim 0.80123$; if $j = 2$, the value of $R$ is within $0.73412 \sim 0.80123$; whereas if $j = 3, 4, \cdots, 10$, the value of $R$ is within $0.78321 \sim 0.82742$. Figure 15c shows that the value of $R$ is independent on $N_t$. Figure 14 and Figure 15 verify the derived competitive ratio ($\Theta$) given in Theorem 4. The gap between $R$ and $\Theta$ is within $0.082394 \sim 0.1953$. 

![Figure 15: Social welfare ratio ($R$), the number of bids ($|J|$) and the number of time slots ($N_t$) in PAAP mechanism](image)
7.2.4. Impact of booking flexibility

In this subsection, we investigate the impact of booking flexibility by comparing two types of rolling horizon configurations with same time step and different time horizon lengths in the context of the PAYG mechanism: RHA ($\Delta t = 1, \mathcal{T} = 1$) and RHA ($\Delta t = 1, \mathcal{T} = 240$). The input data is the same for both configurations. Considering the booking feasibility in RHA ($\Delta t = 1, \mathcal{T} = 240$), we simulate travelers’ booking behavior and set each traveler’s requested departure time ($O_i$) as a random number satisfying a triangular distribution within $[t, t + 240]$.

Figure 16a shows the relationship among social welfare ($W$), the number of bids ($|J|$) and time slot ($t$) under RHA ($\Delta t = 1, \mathcal{T} = 1$) and $|J|$ has little influence on $W$. Figure 16b and Figure 16c are projection plots of $t - |J| - W$ figure under RHA ($\Delta t = 1, \mathcal{T} = 1$) and RHA ($\Delta t = 1, \mathcal{T} = 240$), respectively. Compared with Figure 16b, Figure 16c show that the social welfare in non-peak hours starts to increase 240 time slots beforehand RHA ($\Delta t = 1, \mathcal{T} = 240$). Since the total travel demand remains unchanged, the social welfare increases during off-peak hours and decreases during peak hours. Namely, Setting a longer time horizon lengths ($\mathcal{T}$) can improve travelers’ booking flexibility and balance the social welfare across peak hours and off-peak hours.

![Figure 16](image_url)

(a) $t - |J| - W$ with RHA ($\mathcal{T} = 1$)  
(b) $t - W$ with RHA ($\mathcal{T} = 1$)  
(c) $t - W$ with RHA ($\mathcal{T} = 240$)

Figure 16: RHA ($\Delta t=1,\mathcal{T}=1$ ) and RHA ($\Delta t=1,\mathcal{T}=240$)

7.2.5. Comparison of rolling horizon algorithm configurations

In this section, we conduct 100 Monte Carlo simulations to compare the mean value of computation time and social welfare obtained by the proposed online algorithms, online formulations and offline formulations under four types of rolling horizon configurations introduced in Section 6.

The results for the PAYG mechanism are reported in Figure 17 and based on the following four configurations:
1) Solve online algorithm (Algorithm 1) with Python based on RHA ($\Delta t = 1, \mathcal{T} = 240$);
2) Solve online MILP (Model 1) with CPLEX based on RHA ($\Delta t = 1, \mathcal{T} = 240$);
3) Solve offline MILP (Model 2) with CPLEX based on RHA ($\Delta t = 10, \mathcal{T} = 240$);
4) Solve offline MILP (Model 2) with CPLEX based on SHA ($\Delta t = 240, \mathcal{T} = 240$).

Figure 17a and Figure 17b show that the computation time of online and offline algorithms increase with the increasing number of time slots ($N_t$). The computational runtime of the offline algorithm increases exponentially with the number of time slots.
Analogously, the results for the PAAP mechanism are reported in Figure 18 based on the following four configurations:

1) Solve online algorithm (Algorithm 2) with Python based on RHA ($\Delta t = 1, T = 100$);
2) Solve online MILP (Model 3) with CPLEX based on RHA ($\Delta t = 1, T = 100$);
3) Solve offline MILP (Model 4) with CPLEX based on RHA ($\Delta t = 10, T = 100$);
4) Solve offline MILP (Model 4) with CPLEX based on SHA ($\Delta t = 100, T = 100$).

Figure 17: Comparing the computation time and social welfare obtained by different configures in PAYG mechanism

Figure 18: Comparing the computation time and social welfare obtained by different configures in PAAP mechanism

Figure 17a and Figure 18a show that the offline configuration runs in exponential time and takes significantly more time than online configurations. Figure 17c and Figure 18c show that the social welfare obtained by the offline configurations is considerably larger than that obtained by the online configurations, among which ‘RHA: offline MILP’ can obtain the maximum social welfare. We observe that the social welfare obtained by Algorithm 1 (resp. Algorithm 2) is only marginally lower than that obtained by the online MILP Model 1 (resp. Model 3) when solved to optimality. In turn, Algorithm 1 and Algorithm 2 are faster compared to exact optimization methods. Overall, the rolling horizon configuration ($\Delta t = 10, T = 240$) is shown to reduce the gap in social welfare observed between the online and offline algorithms, while retaining substantial computational efficiency.

We conducted a sensitivity analysis on the time step ($\Delta t$) of the RHA using the PAYG mechanism (the results in reported in Figure D.19 in the Appendix). Our findings show that when
\( \Delta t \) under SHA increases from 10\,\sim\,200, in terms of social welfare we have ‘SHA: offline MILP’ > ‘RHA: offline MILP’ > ‘RHA: online MILP’ > ‘RHA: online algorithm’, which is consistent with the results shown in Figure 17.

8. Conclusion and remarks

We first summarize the main contributions of this work before outlining the differences between the proposed MaaS paradigm and traditional approaches and discussing future research directions.

8.1. Conclusion

This paper introduces an innovative MaaS paradigm under two types of payment options (mechanisms): Pay-as-You-Go and Pay-as-a-Package. Two MaaS online mechanisms, which can effectively and efficiently match mobility resources with travelers’ requirements and WTP, are proposed based on a dynamic auction setting in which travelers have the possibility to place multiple bids for MaaS bundles. The PAYG online mechanism is formulated as a mixed-integer linear program whereas the PAAP online mechanism is formulated as a linear program. Both formulations are shown to be incentive-compatible. We propose polynomial-time, online primal-dual algorithms for both PAYG and PAAP mechanisms and derive the competitive ratio of these online algorithms. Specifically, we show that the PAYG online algorithm (Algorithm 1) has a competitive ratio of \((1 - \frac{1}{\alpha})(1 - R_{\text{max}})\) (Theorem 2), and that the PAAP online algorithm (Algorithm 2) has a competitive ratio of \((1 - \frac{1}{\beta})\) (Theorem 4). Further, if \(Q_{\Delta t} \to 0\), the competitive ratio of Algorithm 1 tends to that of Algorithm 2.

The proposed MaaS online mechanisms are tested through extensive numerical experiments which highlight the performance of the proposed online primal-dual algorithms, as well as the benefits which can be obtained through the proposed rolling horizon algorithm framework. The results indicate that MaaS online mechanisms are feasible with regard to the parameters (e.g., the number of bids, the ratio of the maximum unit bidding price to the minimum unit bidding price, travel demand in a time slot, speed change ratio, and capacity) and different types of unit price functions. Comparing the PAYG and the PAAP mechanisms, we find that the PAAP mechanism can improve social welfare and reduce the daily unit price of mobility resources. However, as noted by Ho et al. (2018), in the PAAP mechanism, travelers have to make a large payment compared to the PAYG mechanism, which may negatively affect the practicality of this mechanism. The competitive ratio \((\Theta)\) of Algorithm 1 (resp. Algorithm 2) can provide a lower bound for the social welfare ratio \((R)\) in the PAYG (resp. PAAP) mechanism. Further, we find that rolling horizon configurations with larger time horizons can improve users’ booking flexibility. We also observe that the rolling horizon configuration \((\Delta t = 10, T = 240)\) provides an efficient compromise between solution quality and computational scalability.

8.2. Remarks and future research

The global economic transition and state-of-the-art technologies are driving the transformation of the transport sector from an infrastructure/manufacturing focused industry to a service/experience focused industry (Hong, 2018). In line with the transition from a focus on ‘products’ to ‘service’ to ‘experience’, we proposed an innovative MaaS paradigm emphasizing the nature of service nature and user experience. In this paradigm, travelers can bid for mobility resources in a continuous fashion and expect multi-modal mobility services tailored to their willingness to pay and travel requirements.
The proposed MaaS paradigm is travel mode-agnostic and delivers transport accessibility by allocating continuous quantities of mobility resources to travelers based on their preferences, in contrast to more conventional discrete mode choice models. In this MaaS paradigm, travel distance is represented as a weighted combination of travel modes and travel speed, thus allowing travelers to purchase any quantity of mobility resources based on their preferences. The proposed MaaS online mechanisms are incentive-compatible, in which each traveler is allocated mobility services that maximize his or her utility, thus providing a sustainable transportation model. Further, the multi-bid online auction setting offers travelers the possibility to consider multiple MaaS bundles. Moreover, the proposed MaaS online mechanisms can guarantee that travelers with a higher WTP can preferentially obtain a mobility service with higher quality.

Sochor et al. (2018) proposed a four-level taxonomy to divide different MaaS schemes, current MaaS schemes such as UbiGo have not reached Level 3 (integration of the service offer). In comparison, the proposed MaaS paradigm aims to reach the highest level (integration of societal goals). Although the proposed paradigm may provide several advantages as discussed above, autonomous transport systems are the premise for this MaaS paradigm (Chen et al., 2020). Many more practical issues have to be addressed before such a MaaS paradigm can be implemented, which are discussed as follows.

At the micro level, travelers’ habits and attitudes are recognized as essential factors. In the proposed MaaS paradigm, travelers need to quantify the abstract characters (e.g., inconvenience tolerance and delay budget) and report them to MaaS operator, which might be difficult for non-sensitive travelers or mode-specific travelers. Karlsson et al. (2020) showed that it is difficult to change people’s travel behaviour due to the established habits and the individual’s perceived ‘action space’; thus we have to consider travelers’ adoption: are travelers willing to change their habits and substitute private cars with mobility service? Can travelers adopt the new way of being served?

At the micro level, Merkert et al. (2020) identified the importance of system integration and the elimination of the influence of boundary effects on different modes in a system, and indicated that the public and private combined operation of MaaS systems may increase the pressure upon TSPs to provide a multi-modal and seamless service. Thus the collaboration and responsibilities of different stakeholders in the MaaS ecosystem need to be considered: will different stakeholders form coalitions? How to improve the benefits of multiple stakeholders?

This paper has taken a first step towards designing an experience-relevant MaaS paradigm based on a continuous representation of mobility service. However, market dynamics and other complex factors in MaaS systems are not accounted for in the proposed approach motivating future research in this direction. In particular, two-sided economic deregulated MaaS markets, or hierarchical configurations such as Stackelberg competition between TSPs and travelers may provide more realistic configurations for the development of emerging MaaS ecosystems.

Acknowledgments

The authors wish to express their thanks to Dr Haris Aziz from the University of New South Wales and Prof. David Hensher from the University of Sydney for their useful suggestions. This research was partially supported by the Australian Government through the Australian Research Council’s Discovery Projects funding scheme (DP190102873). Dr Wei Liu acknowledges the support from Australian Research Council (DE200101793).
Appendix A. Mathematical Notation

| Notation | Specification |
|----------|---------------|
| $i$      | Set of travelers (1, 2, ..., $I$) requesting for a mobility service overall time slots. |
| $I(t)$   | Set of travelers (1, 2, ..., $I$) requesting for a mobility service at time slot $t$. |
| $I(n)$   | Set of travelers in the $n$-th rolling process in the rolling horizon framework. |
| $J_i$    | Set of bids submitted by traveler $i$. |
\( \mathcal{M} \) Set of transportation modes \( \mathcal{M} = \{1, 2, 3, 4, 5\} \)

\( U(n) \) Set of travelers in the \( n \)-th rolling process, who request a service in the previous time horizon.

\( S_i \) Set of Traveler \( i \)'s bundle plans in online formulation.

\( S_i^o \) Set of Traveler \( i \)'s bundle plans in offline formulation.

\( \Omega \) Set of time slots in the PAYG mechanism.

Appendix B. Proof of Lemmas and Theorem for the PAAP mechanism

Appendix B.1. Proof of Lemma 6

Proof. We now show that Model 3 can be reformulated as compact packaging reformulation. Let \( d_{i,s} \) denote the total distance in the bundle plan \( s \),

\[
d_{i,s} = \sum_{m \in \mathcal{M}} v_m l_{mij}, \quad \forall i \in \mathcal{I}(t), \, j \in \mathcal{J}_i, \, s \in S_{ij}.\]

According to Traveler \( i \)'s distance requirement in constraint (8b) of Model 3, we obtain Eq.(B.1):

\[
\sum_{s \in S_i} d_{i,s} = \sum_{j \in \mathcal{J}_i} \sum_{s \in S_{ij}} d_{i,s} = \sum_{m \in \mathcal{M}} \sum_{j \in \mathcal{J}_i} v_m l_{mij} = \sum_{j \in \mathcal{J}_i} D_i x_{ij}, \quad \forall i \in \mathcal{I}(t). \tag{B.1}
\]

Assume that package plan \( s \) is arranged for Traveler \( i \)'s bid \( j \), let \( b_{i,s} \) and \( Q_{i,s} \) denote Traveler \( i \)'s bidding price and weighted quantity of mobility resources in the package plan \( s \), then we have

\[
b_{i,s} = b_{ij}, \quad Q_{i,s} = Q_{ij}, \quad \forall i \in \mathcal{I}(t), \, j \in \mathcal{J}_i, \, s \in S_{ij}.
\]

Summing \( i \in \mathcal{I}(t) \) over both sides of \( \sum_{s \in S_i} d_{i,s} = \sum_{j \in \mathcal{J}_i} D_i x_{ij} \) yields:

\[
\sum_{i \in \mathcal{I}(t)} \sum_{s \in S_i} d_{i,s} = \sum_{i \in \mathcal{I}(t)} \sum_{j \in \mathcal{J}_i} D_i x_{ij}. \tag{B.2}
\]

Based on Eq.(B.2), the objective function of Model 3, \( \sum_{i \in \mathcal{I}(t)} \sum_{j \in \mathcal{J}_i} b_{i,s} x_{ij} \), can be rewritten as Eq.(B.3c). Then constraint (8f) in Model 3 can be rewritten as: \( \sum_{s \in S_i} Q_{i,s} \leq A_t \). Thus Model 3 can be reformulated as a compact LP (B.3) with packaging structure given in 6.

\[
\text{max} \sum_{i \in \mathcal{I}(t)} \sum_{s \in S_i} b_{i,s} d_{i,s}, \tag{B.3a}
\]

subject to

\[
\sum_{i \in \mathcal{I}(t)} \sum_{s \in S_i} Q_{i,s} \leq A_t, \tag{B.3b}
\]

\[
\sum_{s \in S_i} \frac{d_{i,s}}{D_i} \leq 1, \quad \forall i \in \mathcal{I}(t), \tag{B.3c}
\]

\[
d_{i,s} \geq 0, \quad \forall i \in \mathcal{I}(t), \, s \in S_i, \tag{B.3d}
\]

where constraint (B.3b) and (B.3c) corresponds to constraint (8f) and (8g). If we replace \( d_{i,s} \) with \( D_i r_{i,s} \), then compact-LP (B.3) can be reformulated as a compact-LP (36) given in Lemma 6.

Appendix B.2. Proof of Lemma 7

Proof. The proof follows from that of Lemma 2.

39
Appendix B.3. Proof of Lemma 8

Proof. In each iteration of a time loop, $\Delta D(i, s)$ can be written as Eq. (B.4), in which $u_i$ and $\Delta q(t)$ is obtained from Line 15 and Line 16 in Algorithm 2.

$$
\Delta D(i, s) = A_t \Delta q(t) + u_i = A_t \left( q(t) \frac{Q_i}{A_t} + \frac{b_{i,s} r_{i,s}}{(\beta_t - 1) A_t} - \frac{(1 - r_{i,s}) b_{i,s}}{A_t} \right) + u_i, \tag{B.4}
$$

In each iteration of a time loop, the change in the objective value of primal problem is:

$$
\Delta P(i, s) = \sum_{s' \in S_i} b_{i,s'} r_{i,s'} - \sum_{s' \in S_i \setminus \{s\}} b_{i,s'} r_{i,s'} = b_{i,s} r_{i,s}. \tag{B.5}
$$

According to Eq. (B.4) and Eq. (B.5), we have Eq. (B.6):

$$
\left(1 + \frac{1}{\beta_t - 1}\right) \Delta P(i, s) = \Delta D(i, s). \tag{B.6}
$$

Appendix B.4. Proof of Lemma 9

Proof. We first prove that Algorithm 2 yields dual feasible solutions. If $q(t) \geq \frac{b_{i,s}}{\varphi_t}$, then the dual constraint always holds. If $q(t) \leq \frac{b_{i,s}}{\varphi_t}$, the dual variable $q(t)$ will be increased until the dual constraint are satisfied. else if $q(t) \leq \frac{b_{i,s}}{\varphi_t}$, the dual variables will be increased until the dual constraint are satisfied. Since $u_i(t) = b_{i,s} - Q_i q(t)$ (Line 15), the subsequent increase of $q(t)$ can always guarantee the feasible of solutions. We next show that Algorithm 2 yields primal feasible solutions. The iteration rule of $q(t)$ (Line 16) ensures that $q(t)$ is bounded by the sum of a geometric sequence with the common ratio $(1 + \frac{Q_i}{A_t})$. Consider a geometric sequence produced by the iterations of $q(t)$ for Traveler $m$: the first item is at most $\frac{b_{m,s}}{(\beta_t - 1) A_t}$, in which the value of $b_{m,s}$ is fixed in each iteration (Lines 12-16), and the common ratio is $1 + \frac{Q_i}{A_t}$. Based on the formula of the sum of geometric sequence, we obtain Eq. (B.7), where the number of iterations of Algorithm 2 in each time loop is smaller than $\sum_{i \in I(t)} \sum_{s \in S_i} r_{i,s}$. Then Eq. (B.7) can be rewritten as Eq. (B.8).

$$
q(t) \leq \frac{b_{m,s}}{Q_m} \cdot \frac{1}{\beta_t - 1}, \left(1 + \frac{Q_m}{A_t}\right) \sum_{i \in I(t)} \sum_{s \in S_i} r_{i,s} - 1, \tag{B.7}
$$

$$
\leq \frac{b_{m,s}}{Q_m} \cdot \frac{1}{\beta_t - 1}, \left(1 + \frac{Q_m}{A_t}\right) \sum_{i \in I(t)} \sum_{s \in S_i} Q_i r_{i,s} - 1, \tag{B.8}
$$

Since $\varphi_t = \min_{i \in I(t)} \left\{ \frac{Q_i}{A_t} \right\}$. Since $0 \leq \frac{D_m}{A_t} \leq \frac{Q_i}{A_t} \leq 1$, Eq. (B.8) can be rewritten as Eq. (B.9):

$$
q(t) \leq \frac{b_{m,s}}{Q_m} \cdot \frac{1}{\beta_t - 1}, \left(1 + \frac{Q_m}{A_t}\right) \sum_{i \in I(t)} \sum_{s \in S_i} Q_i r_{i,s} - 1. \tag{B.9}
$$
Since 0 ≤ ϕ_t ≤ Q_i \frac{A_t}{A_i} ≤ 1, we have \( \frac{\ln(1 + Q_i)}{\frac{A_t}{A_i}} \leq \frac{\ln(1 + \varphi_t)}{\varphi_t} \), namely, \( (1 + \varphi_t) \frac{Q_i}{A_i} \leq (1 + Q_i) R_{\text{min}} \leq (1 + \varphi_t) \frac{Q_i}{A_i} \), yields:

\[
1 + \frac{Q_i}{A_t} \leq (1 + \varphi_t) \frac{Q_i}{A_t}, \tag{B.10}
\]

Since \( \beta_t = (1 + \varphi_t) \frac{1}{\varphi_t} \), \( Q_i = \min_{s \in S_i} \{ Q_{i,s} \} \), Eq.(B.9) is written as Eq.(B.11) based on Eq.(B.10):

\[
q(t) \leq b_{m,s} D_m \cdot \frac{1}{\beta_t - 1} \left( \beta_t \sum_{i \in I(t)} \sum_{s \in S_i} Q_{i,s} r_{i,s} \left( A_t - 1 \right) \right), \tag{B.11}
\]

According to Eq.(B.11), if \( \sum_{i \in I(t)} \sum_{s \in S_i} Q_{i,s} r_{i,s} \leq A_t \), then \( q(t) \leq \frac{b_{m,s}}{D_m} \); Since Algorithm 2 only updates the primal solutions if \( q(t) \leq \frac{b_{m,s}}{D_m} \) (Line 13), the primal optimal solutions will be updated when \( \sum_{i \in I(t)} \sum_{s \in S_i} Q_{i,s} r_{i,s} \leq A_t \).

**Appendix B.5. Proof of Lemma 10**

**Proof.** The proof follows from that of Lemma 6.

**Appendix C. Proof of Theorem 4**

**Proof.** Let \( k \) be the critical index determined at Line 7 of Algorithm 1). Let \( q(t)^{\text{end}} \) and \( q(t)^{\text{start}} \) denote the value of \( q(t) \) before and after each iteration in the loop of \( i = k \) in Algorithm 1, respectively. Substituting \( q(t)^{\text{end}} \) and \( q(t)^{\text{start}} \) into Line 16 of Algorithm 1, yields Eq.(C.1).

\[
q(t)^{\text{end}} \leq q(t)^{\text{start}} \left( 1 + \frac{Q_k}{A_t} \right) + \frac{b_{k,s}}{(\beta_t - 1) A_t}. \tag{C.1}
\]

According to Eq.(B.9), we have Eq.(C.2):

\[
q(t)^{\text{start}} \leq b_{k,s} Q_k \cdot \frac{1}{\beta_t - 1} \left( \beta_t \sum_{i \in I(t) \setminus \{k\}} \sum_{s \in S_i} Q_{i,s} r_{i,s} \left( A_t - 1 \right) \right). \tag{C.2}
\]

According to Eq.(C.1) and Eq.(C.2), we obtain Eq.(C.3):

\[
q(t)^{\text{end}} \leq b_{k,s} Q_k \cdot \frac{1}{\beta_t - 1} \left[ \beta_t \sum_{i \in I(t) \setminus \{k\}} \sum_{s \in S_i} Q_{i,s} r_{i,s} \left( 1 + \frac{Q_k}{A_t} \right) - 1 \right]. \tag{C.3}
\]

According to Eq.(B.10), we have \( 1 + \frac{Q_k}{A_t} \leq (1 + \varphi_t) \frac{1}{\varphi_t} \), Eq.(C.3) can be rewritten as Eq.(C.4).

\[
q(t)^{\text{end}} \leq b_{k,s} Q_k \cdot \frac{1}{\beta_t - 1} \left[ \beta_t \sum_{i \in I(t) \setminus \{k\}} \sum_{s \in S_i} Q_{i,s} r_{i,s} \left( 1 + \frac{Q_k}{A_t} \right) - 1 \right]. \tag{C.4}
\]
Since $\beta_t = (1 + \varphi_t) r_t^{-\frac{1}{\gamma_t}}$, Eq.(C.4) can be written as Eq.(C.5):

$$q(t)^{\text{end}} \leq \frac{b_{k,s}}{Q_k} \cdot \frac{1}{\beta_t - 1} \cdot \left( \frac{\sum_{i \in \mathcal{I}(t)} \sum_{s \in \mathcal{S}_i} Q_i r_{i,s}}{A_t} - 1 \right).$$  \hspace{1cm} (C.5)

According to Eq.(C.5), the primal variable $r_{i,s}$ will be updated if $\sum_{i \in \mathcal{I}(t)} \sum_{s \in \mathcal{S}_i} Q_i r_{i,s} \leq A_t$, elseif $\sum_{i \in \mathcal{I}(t)} \sum_{s \in \mathcal{S}_i} Q_i r_{i,s} > A_t$ , MaaS operator will stop allocating mobility resources at time slot $t$, and there is no allocation with the remaining resources at time slot $t$, $r_{k,s}(t) = A_t - \sum_{i \in \mathcal{I}(t)} \sum_{s \in \mathcal{S}_i} Q_i r_{i,s}(t)$. Since all of the mobility resources at time slot $t$ will be used up due to the fractional allocation, namely, $\sum_{i \in \mathcal{I}(t)} \sum_{s \in \mathcal{S}_i} Q_i r_{i,s} = A_t$, the total social welfare obtained by Algorithm 2 extracted from time slot $t$ is at least $\sum_{i \in \mathcal{I}(t)} \sum_{s \in \mathcal{S}_i} b_{i,s} \chi_{i,s} \sum_{i \in \mathcal{I}(t)} \sum_{s \in \mathcal{S}_i} Q_i r_{i,s}$. Let $P(t)$ and $D(t)$ denote the objective value of primal problem (Eq.(36)) and its dual problem obtained by Algorithm 2 in time $t$ loop, respectively. Based on Lemma 8, we have $P(t) = (1 - \frac{1}{\beta_t}) D(t)$.

Given the input time sequence $\tau = [1, 2, \cdots, t - 1, t]$, let $P(\tau)$ and $D(\tau)$ denote the objective value of the primal problem (Eq.(44)) and its dual problem obtained by Algorithm 1. Let $W_{\text{Alg2}}(\tau)$ denote the social welfare obtained by Algorithm 2, $Z^*(\tau)$ denote the optimal value of the offline primal problem Eq.(44). Since $\beta = \min_{t \in \tau} \beta_t$, Eq.(29) implies $P(\tau) \geq (1 - \frac{1}{\beta}) D(\tau)$. Moreover, we have $W_{\text{Alg2}}(\tau) \geq (1 - \frac{1}{\beta}) D(\tau)$. According to the weak duality, we have $D(\tau) \geq Z^*(\tau)$, yields:

$$W_{\text{Alg2}}(\tau) \geq (1 - \frac{1}{\beta}) Z^*(\tau).$$  \hspace{1cm} (C.6)

Hence the competitive ratio of Algorithm 2 is $\Theta = 1 - \frac{1}{\beta}$.  \hfill \Box

**Appendix D. Sensitivity analysis on the time step**

![Graphs showing social welfare vs. the number of rolling processes for different time steps](image_url)
Figure D.19: Sensitivity analysis on the time step (\(\Delta t\)) in terms of social welfare in the PAYG mechanism.

References

Arora, J.S., 2004. Introduction to optimum design. Elsevier.
Barreto, L., Amaral, A., Baltazar, S., 2018. Urban mobility digitalization: towards mobility as a service (maas), in: 2018 International Conference on Intelligent Systems (IS). IEEE, pp. 850–855.
Beheshtian, A., Geddes, R.R., Rouhani, O.M., Kockelman, K.M., Ockenfels, A., Cramton, P., Do, W., 2020. Bringing the efficiency of electricity market mechanisms to multimodal mobility across congested transportation systems. Transportation Research Part A: Policy and Practice 130, 178–191.
Borodin, A., El-Yaniv, R., 2005. Online computation and competitive analysis. Cambridge University Press.
Buchbinder, N., Naor, J., 2009. Online primal-dual algorithms for covering and packing. Mathematics of Operations Research 34, 270–286.
Caiati, V., Rasouli, S., Timmermans, H., 2020. Bundling, pricing schemes and extra features preferences for mobility as a service: Sequential portfolio choice experiment. Transportation Research Part A: Policy and Practice 131, 123–148.
Chen, Z., Lin, X., Yin, Y., Li, M., 2020. Path controlling of automated vehicles for system optimum on transportation networks with heterogeneous traffic stream. Transportation Research Part C: Emerging Technologies 110, 312–329.
Djavadian, S., Chow, J.Y., 2017. An agent-based day-to-day adjustment process for modeling ‘mobility as a service’ with a two-sided flexible transport market. Transportation Research Part B: methodological 104, 36–57.
Haeringer, G., 2018. Market design: auctions and matching. MIT Press.
Hensher, D.A., 2017. Future bus transport contracts under a mobility as a service (maas) regime in the digital age: Are they likely to change? Transportation Research Part A: Policy and Practice 98, 86–96.
Hensher, D.A., Mulley, C., 2020. Special issue on developments in mobility as a service (maas) and intelligent mobility. Transportation Research Part A: Policy and Practice 130, 1–4.
Hensher, D.A., Mulley, C., Ho, C., Smith, G., Wong, Y., Nelson, J.D., 2020. Understanding Mobility as a Service (MaaS): Past, Present and Future. Elsevier.
Hirschhorn, F., Paulsson, A., Sørensen, C.H., Veeneman, W., 2019. Public transport regimes and mobility as a service: Governance approaches in amsterdam, birmingham, and helsinki. Transportation Research Part A: Policy and Practice 130, 178–191.
Ho, C.Q., Hensher, D.A., Mulley, C., Wong, Y.Z., 2018. Potential uptake and willingness-to-pay for mobility as a service (maas): A stated choice study. Transportation Research Part A: Policy and Practice 117, 302–318.

Ho, C.Q., Mulley, C., Hensher, D.A., 2020. Public preferences for mobility as a service: Insights from stated preference surveys. Transportation Research Part A: Policy and Practice 131, 70–90.

Hong, A., 2018. Mobility as a service (maas): What it is and where it is headed. [EB/OL]. https://www.andyhong.org/single-post/2018/03/21/Mobility-as-a-Service-definition Accessed March 22, 2018.

Jittaprirom, P., Caiati, V., Feneri, A.M., Ebrahimigharehbaghi, S., Alonso González, M., Narayan, J., 2017. Mobility as a service: A critical review of definitions, assessments of schemes, and key challenges. Urban Planning 2, 13.

Kamargianni, M., Li, W., Matyas, M., 2016. A comprehensive review of mobility as a service systems. The National Academies of Sciences, Engineering, and Medicine.

Kamargianni, M., Matyas, M., 2017. The business ecosystem of mobility-as-a-service, in: Proceedings of the 96th Transportation Research Board Annual Meeting, Transportation Research Board, pp. 157–164.

Karlsson, I., Mukhtar-Landgren, D., Smith, G., Koglin, T., Kronsell, A., Lund, E., Sarasini, S., Sochor, J., 2020. Development and implementation of mobility-as-a-service—a qualitative study of barriers and enabling factors. Transportation Research Part A: Policy and Practice 131, 283–295.

Matyas, M., Kamargianni, M., 2019a. The potential of mobility as a service bundles as a mobility management tool. Transportation 46, 1951–1968.

Matyas, M., Kamargianni, M., 2019b. Survey design for exploring demand for mobility as a service plans. Transportation 46, 1525–1558.

Merkert, R., Bushell, J., Beck, M.J., 2020. Collaboration as a service (caas) to fully integrate public transportation—lessons from long distance travel to reimage mobility as a service. Transportation Research Part A: Policy and Practice 131, 267–282.

Meurs, H., Sharmeen, F., Marchau, V., van der Heijden, R., 2020. Organizing integrated services in mobility-as-a-service systems: Principles of alliance formation applied to a maas-pilot in the Netherlands. Transportation Research Part A: Policy and Practice 131, 178–195.

MuConsult, 2017. Mobility as a Service.

Mulley, C., Nelson, J.D., Wright, S., 2018. Community transport meets mobility as a service: On the road to a new a flexible future. Research in Transportation Economics 69, 583–591.

Nisan, N., 2006. Bidding languages. Combinatorial Auctions, 400–420.

Opal, 2020. Opal trip planner. [EB/OL]. https://transportnsw.info/trip#/trip.

Pantelidis, T.P., Chow, J.Y., Rasulkhani, S., 2020. A many-to-many assignment game and stable outcome algorithm to evaluate collaborative mobility-as-a-service platforms. Transportation Research Part B: Methodological 140, 79–100.

Polydoropoulou, A., Pagoni, I., Tsirimpa, A., Roumboutsos, A., Kamargianni, M., Tsouros, I., 2020. Prototype business models for mobility-as-a-service. Transportation Research Part A: Policy and Practice 131, 149–162.

Shi, W., Zhang, L., Wu, C., Li, Z., Lau, F.C., 2015. An online auction framework for dynamic resource provisioning in cloud computing. IEEE/ACM Transactions on Networking 24, 2060–2073.

Sochor, J., Arby, H., Karlsson, I.M., Sarasini, S., 2018. A topological approach to mobility as a service: A proposed tool for understanding requirements and effects, and for aiding the integration of societal goals. Research in Transportation Business & Management 27, 3–14.

Tafsiri, S.A., Yousefi, S., 2018. Combinatorial double auction-based resource allocation mechanism in cloud computing market. Journal of Systems and Software 137, 322–334.

Uber, 2020. Uber estimator: Real-time uber estimator. [EB/OL]. https://uberestimator.com.

Utriainen, R., Pöllänen, M., 2018. Review on mobility as a service in scientific publications. Research in Transportation Business & Management 27, 15–23.

Wang, X., He, F., Yang, H., Gao, H.O., 2016. Pricing strategies for a taxi-hailing platform. Transportation Research Part E: Logistics and Transportation Review 93, 212–231.

Wong, Y.Z., Hensher, D.A., Mulley, C., 2020. Mobility as a service (maas): Charting a future context. Transportation Research Part A: Policy and Practice 131, 5–19.

Zhang, F., Liu, W., Wang, X., Yang, H., 2020. Parking sharing problem with spatially distributed parking supplies. Transportation Research Part C: Emerging Technologies 117, 102676.

Zhang, Y., Li, B., Jiang, H., Liu, F., Vasilakis, A.V., Liu, J., 2013. A framework for truthful online auctions in cloud computing with heterogeneous user demands, in: Proceedings - IEEE INFOCOM, pp. 1060–1073.

Zhou, R., Li, Z., Wu, C., Huang, Z., 2016. An efficient cloud market mechanism for computing jobs with soft deadlines. IEEE/ACM Transactions on Networking 25, 793–805.