Microscopic origin of black hole entropy from tachyon condensation

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Abstract

We show generically that the dynamics of a probe particle near the event horizon of a non-extreme black hole is described by the tachyon effective action. The Hagedorn temperature in the action is always equal to the Hawking temperature of the background black hole. The fact suggests that the infalling particle should decay completely into gravitons or closed strings approaching the event horizon. The increased area in the black hole due to absorption of a particle should be interpreted as the entropy of degenerate states of the closed strings that the particle decays into. With the energy match condition between the infalling particle and the emitted closed strings on the event horizon, we examine this variational area-entropy relation and find that it matches in all cases if the closed string emission process from an unstable D0-brane obeys the first law.

1 Introduction

The variation of black holes satisfies a set of classical mechanic laws [1, 2, 3]. The first law of them tells how the area increases under the change of black hole quantities, like mass, charge and angular moment:

\[ \frac{1}{4} \delta A = \frac{2\pi}{\kappa} (\delta M - \Phi_H \delta Q - \Omega_H \delta J). \] (1)
The close resemblance of the laws to the thermodynamical ones lead people to realize that thermodynamical properties should be assigned to black holes, particularly after Hawking found that black holes could radiate [4]. The surface gravity $\kappa$ on the event horizon can be viewed as the Hawking temperature of radiation: $T_{\text{Haw}} = \kappa/2\pi$. The area of the black hole can be viewed as the entropy [2]:

$$S = \frac{1}{4}A.\tag{2}$$

A major task since then is to explain this area-entropy relation, as a means of exploration of quantum gravity. To date, many approaches have been proposed and adopted to reproduce the relation (e.g., see reviews [5, 6] for a list). But, in most of the approaches, the microscopic degrees of freedom responsible for the entropy still remain elusive. This is better understood for extremal black holes [7] whose near-horizon geometries are AdS space. It is also unknown how the information of infalling matter that forms the black hole is recoded in these degrees of freedom.

The unsettled puzzles, together with the information loss paradox, may imply that some key ingredient is missing in the present black hole theory. This ingredient is most possibly relevant to the event horizon, which is usually thought to be smooth for infalling material. In the fuzzball proposal [8], the “information-free” horizon is replaced by the boundary of a fuzzball filled with quantum fuzz. In an alternative proposal [9], it is suggested that infalling observer should encounter firewall at the horizon.

In previous works [10, 11], we found that extraordinary things could really happen naturally near the event horizon of a non-extreme black hole. In the near-horizon region whose geometry is generically Rindler space, the action of an infalling particle is the tachyon field action derived in string theory. The tachyon effective action for an unstable D0-brane is [12, 13, 14, 15, 16]

$$S_0(T) = -\int d\eta \frac{\tau_0}{\cosh(\beta T)} \sqrt{1 - (\partial_\eta T)^2},\tag{3}$$

where the mass of the D0-brane is $\tau_0 = 2\beta/g_s$. The constant $\beta = 1/(2l_s)$ for bosonic strings and $= 1/(\sqrt{2}l_s)$ for superstrings, with the string length $l_s = \sqrt{\alpha'}$. A probe particle in Rindler space behaves like an unstable D-particle, described by the action (3) at large field $T$. This means that the particle falling towards a non-extreme black hole will completely decay into closed strings approaching the horizon, in terms of the results of tachyon condensation [17].

If so, the energy of the infalling particle near the event horizon should be equal to that of the emitted closed strings. The increased area due to absorption of the particle, which
is given by the first law (1), should be accounted for by the entropy of the closed string states, which is calculated in the tachyon field theory (3): $\delta S = \delta A/4$. In this work, we verify this variational area-entropy relation by using the energy match condition on the event horizon for the three kinds of black holes in four dimensions: Schwarzschild, RN and Kerr. In contrast to our previous work [10], we make improvements in two aspects: (i) we show that the dynamics of the probe particle infalling along more general geodesics on more general black hole backgrounds can be described by the tachyon field action; (ii) we use the energy match condition to estimate the resulting entropy, with no need to consider the detailed dynamics of each probe particle, which makes a more universal approach. All through the paper, we adopt the conventions $G = \hbar = c = k_B = 1$.

2 Reissner-Nordström black hole

The metric of a Reissner-Nordström (RN) black hole can be expressed as

$$ds^2 = V(r)(-dt^2 + dR^2) + r^2 d\Omega^2, \quad V(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2}.$$ (4)

The constants $M$ and $Q$ are respectively the mass and charge of the black hole. The condition $V = 0$ determines the radii of the outer and inner horizons: $r_{\pm} = M \pm \sqrt{M^2 - Q^2}$. Here, we consider the non-extreme black hole with $|Q| < M$. The situation for Schwarzschild black hole is included as the special case $Q = 0$. We choose the relation between the radial coordinates $R$ and $r$ to be: $dR = -dr/V(r)$. This gives

$$R = -(r - r_+ - r_-) - \frac{1}{\epsilon} \left[ r_+ \ln \left( \frac{r}{r_+} - 1 \right) - r_- (1 - \epsilon) \ln \left( \frac{r}{r_-} - 1 \right) \right],$$ (5)

where $\epsilon = 1 - r_-/r_+$. Thus, $R \sim -r \rightarrow -\infty$ as $r \rightarrow \infty$, and $R \rightarrow \infty$ as $r \rightarrow r_+$. For Schwarzschild black hole, it reduces to be a shift of the negative tortoise coordinate $r_s$: $R = 2M - r_s$. The Hawking temperature for the black hole is

$$T_{\text{Haw}} = \frac{\epsilon}{4\pi r_+}.$$ (6)

2.1 Radial infall

We first consider the case of dropping a neutral particle with mass $m_0$ and arbitrary radial velocity at infinity. The action of the particle collapsing towards the black hole along a radial trajectory is given by

$$S_0 = -m_0 \int dt \sqrt{V(1 - R^2)},$$ (7)
where the dot represents the derivative with respect to the time coordinate $t$. The energy-momentum tensor from the action is

$$
T_{00} = \frac{m_0 \sqrt{V}}{\sqrt{1 - R^2}} = E, \quad T_{ij} = -\frac{m_0^2 V}{E} \delta_{ij}.
$$

The energy $E$ is constant.

At infinity with $r \to \infty$, $V \approx 1$ and the action describes a free particle moving in Minkowski spacetime. The energy of the particle is $E = \gamma m_0$, with $\gamma$ being the Lorentz factor.

In terms of the first law (1), the surface area of the black hole will increase when such a particle is eventually absorbed by the hole. Since the black hole mass increases by an amount: $\delta M = E$, the increased area is

$$
\frac{1}{4} \frac{\delta A}{T_{\text{Haw}}} = \frac{4 \pi r_+ E}{\epsilon}.
$$

Let us next focus on the dynamics near the event horizon with $r \to r_+$ or $R \to \infty$. In this region, the potential $V \approx \epsilon e^{-\epsilon R/r_+}$ and so the action of the particle is

$$
S_0 \approx -m_0 \sqrt{\epsilon} \int dt e^{-\epsilon R/(2 r_+)} \sqrt{1 - \dot{R}^2}.
$$

It is exactly the effective action (3) of a rolling tachyon with $T \to \infty$ by matching the parameters. This action arises because the near-horizon geometry is Rindler space [10, 11]. It is easy to find that the Hagedorn temperature in this tachyon action is equal to the Hawking temperature (6) of the black hole [10, 18]:

$$
T_{\text{Hag}} = T_{\text{Haw}}.
$$

From the energy-momentum tensor (8), we know that the effective theory will evolve into a pressureless “tachyon matter” state [19, 20] with $|\dot{R}| \to 1$ and $T_{ij} \to 0$ when approaching the event horizon.

Hence, the infalling particle near the horizon behaves like an unstable D0-brane in string theory. The collapsing process towards the event horizon of a non-extremal black hole should be actually a tachyon condensation process. The event horizon plays the role of the closed string vacuum. This implies that the infalling particle will decay into gravitons or closed strings completely before reaching the event horizon, via coupling to the gravitational or closed string modes of the background black hole.

We now estimate the entropy of the closed strings from the decay of the infalling particle based on the results obtained in [17] (see the Appendix). The results are derived
in the boundary conformal field theory (BCFT), which is an equivalent description to 
the tachyon effective theory (3). Therefore, to use the results, we need to first transform 
the action (10) into the standard “string-scale” form (3). With the redefinitions \( \tilde{R} = \epsilon R/(2\beta r_+) \) and \( \tilde{t} = ct/(2\beta r_+) \), we get

\[
S_0 \simeq -\frac{2\beta r_+ m_0}{\sqrt{\epsilon}} \int d\tilde{t} e^{-\beta \tilde{R}} \sqrt{1 - (\partial_{\tilde{R}}^2)^2}. \tag{12}
\]

The conserved energy of the infalling particle measured in the new time coordinate \( \tilde{t} \) 
is \( 2\beta r_+ E/\epsilon \) (the Hagedorn temperature measured in this time coordinate is now \( \beta/2\pi \), the one in string theory). This energy should be equal to the total energy \( E_c \) of the emitted closed strings. Inserting it in Eq. (A.6) in Appendix, we determine the entropy of degenerate states of the closed strings emitted from the infalling particle:

\[
\delta S = S_c \simeq \frac{4k\pi r_+ E}{\epsilon} = \frac{k}{4} \delta A. \tag{13}
\]

So it is comparable with the increased area (9), up to a numerical constant \( k \). When 
\( k = 1 \), i.e., the closed string emission process from an unstable D0-brane obeys the first 
law, the result is consistent with the expected identification (2).

### 2.2 Non-radial infall

For non-radial collapse, the action of the particle is

\[
S_0 = \int dt \mathcal{L}_0 = -m_0 \int dt \sqrt{V} \sqrt{1 - \dot{R}^2 - \frac{r^2 \dot{\Omega}^2}{V}}. \tag{14}
\]

The equation of motion for the angular part leads to

\[
-\frac{m_0^2 r^2 \dot{\Omega}}{\mathcal{L}_0} = L, \tag{15}
\]

where \( L \) is a constant. Solving the equation for \( \dot{\Omega} \) and inserting the solution into the 
action (14), we get

\[
\mathcal{L}_0 = -\frac{m_0^2 r}{\sqrt{L^2 + m_0^2 r^2}} \sqrt{V} \sqrt{1 - \dot{R}^2}. \tag{16}
\]

At infinity, the action is also that of a free particle moving in Minkowski spacetime. 
We have

\[
r \to \infty : \quad E = \gamma m_0, \quad L = \gamma m_0 r^2 \dot{\Omega}, \tag{17}
\]
where $\gamma = 1/\sqrt{1-r^2}$ with $R \sim -r$ towards infinity. So the constants $E$ and $L$ are respectively the conserved energy and angular momentum of the particle. Note that the linear velocity $r\dot{\Omega} \to 0$ as $r \to \infty$ from the latter equation.

Near the event horizon $R \to \infty$, the action (16) is

$$S_0 \simeq -\frac{\sqrt{\epsilon m_0^2 r_+}}{\sqrt{L^2 + m_0^2 r_+^2}} \int dt e^{-\epsilon R/(2\Omega_+)} \sqrt{1 - \dot{R}^2}. \quad (18)$$

The action indicates that the particle will decay completely into closed strings if it can reach the event horizon with appropriate $L$. With the same procedure, we can find that the relation between the increased area and entropy is the same as the previous case (13).

### 2.3 Charged particle

The discussion can be extended to the case for a charged particle, even though we do not know the detailed dynamics. We can do so only by using the energy match condition on the event horizon.

When the RN black hole absorbs a particle with charge $q$, its charge increases with $\delta Q = q$. Thus, in terms of the first law (1), we only need to replace $E$ in Eq. (9) by $E - q\Phi_H$, where the surface electric potential $\Phi_H = Q/r_+$.

In the field theory description, it can be found that the energy of the charged particle will become $E - q\Phi_H$ when it reaches the event horizon because work will be done on the charge as it falls in the electric field of the black hole (the sign of the work depends on the relative sign between $Q$ and $q$). Approaching the horizon, the charged particle should also decay completely into closed strings. In terms of the energy match condition on the event horizon, the energy of the radiated closed strings is therefore: $E_c = 2\beta r_+(E - q\Phi_H)/\epsilon$. Thus, we get the same variational area-entropy relation: $\delta S \simeq k\delta A/4$.

### 3 Kerr black hole

The metric for a Kerr black hole with mass $M$ and angular momentum $J$ is

$$ds^2 = -\frac{\rho^2 \Delta}{\Sigma} dt^2 + \frac{\Sigma \sin^2 \theta}{\rho^2} (d\phi - \omega dt)^2 + \frac{\rho^2}{\Delta} dr^2 + \rho^2 d\theta^2, \quad (19)$$

where $\rho^2 = r^2 + a^2 \cos^2 \theta$, $\Delta = r^2 - 2Mr + a^2$, $\Sigma = (r^2 + a^2)^2 - a^2 \Delta \sin^2 \theta$ and $\omega = 2Mar/\Sigma$ with the spin parameter $a = J/M$. The radii of the horizons are: $r_\pm = M \pm \sqrt{M^2 - a^2}$. On the event horizon, the angular velocity of the hole is $\Omega_H = \omega(r_+) = a/(2Mr_+)$. The
Hawking temperature for the non-extreme Kerr black hole is

\[
T_{\text{Haw}} = \frac{\epsilon}{8\pi M},
\]

where \( \epsilon = 1 - r_- / r_+ \).

The Lagrangian of a probe particle moving in the spacetime is also given by \( \mathcal{L}_0 = -m_0\sqrt{g_{\mu\nu}x^\mu x^\nu} \). The equation of motion about \( \dot{\phi} \) leads to

\[
-\frac{m_0^2 \Sigma \sin^2 \theta (\dot{\phi} - \omega)}{\rho^2 \mathcal{L}_0} = L,
\]

where \( L \) is a constant. The case \( L = 0 \) corresponds to the trajectory of the zero angular momentum observer (ZAMO). With the solution to the equation, the Lagrangian becomes

\[
\mathcal{L}_0 = -\frac{m_0^2 \rho \sin \theta \sqrt{\Delta}}{\sqrt{\rho^2 L^2 + m_0^2 \Sigma \sin^2 \theta}} \sqrt{1 - \frac{\Sigma}{\Delta^2} \left( \dot{\rho}^2 + \Delta \dot{\theta}^2 \right)}.
\]

Towards the event horizon with \( \Delta = 0 \), the angular motion along the \( \theta \) direction is suppressed. We only consider the special case of geodesics with constant angle \( \theta \) in what follows. This is allowed at least for geodesics on the equatorial plane in terms of the Carter constant of motion [21].

At infinity, the conserved energy and angular momentum of the particle are respectively

\[
r \rightarrow \infty : \quad E = \gamma m_0, \quad L = \gamma m_0 r^2 \sin^2 \theta \dot{\phi}.
\]

When the particle is absorbed, the mass and angular momentum of the black hole increase respectively by: \( \delta M = E \) and \( \delta J = L \). In this case, the first law (1) reads

\[
\frac{1}{4} \delta A = \frac{8\pi M}{\epsilon} \left( E - \Omega_H L \right).
\]

Near the event horizon \( r \rightarrow r_+ \), the Lagrangian (22) with \( \dot{\theta} = 0 \) is approximately

\[
\mathcal{L}_0 \approx -\frac{m_0^2 r_+ \rho_+ \sin \theta \sqrt{\epsilon}}{\sqrt{\rho_+^2 L^2 + 4m_0^2 M^2 \sin^2 \theta}} e^{-\epsilon R/4M} \sqrt{1 - R},
\]

where \( \rho_+^2 = 1 + (a/r_+)^2 \cos^2 \theta \). The \( R \) coordinate is defined by \( dR = -\sqrt{\Sigma} dr / \Delta \). We can also see that the Hagedorn temperature \( T_{\text{Hag}} \) in the action is equal to the Hawking temperature (20). The tachyon action suggests that the particle should decay into closed strings completely if it can reach the event horizon. Its energy should be equal to the total energy of the emitted closed strings on the event horizon.
Since the black hole spacetime is rotating, we need to choose a fiducial frame to make the energy match. The fiducial frame can be chosen as that of a ZAMO [22]. At infinity, the observer is static. The measured conserved energy and angular momentum are respectively $p_t = -E$ and $p_\phi = L$, as given in Eq. (23). On the event horizon, the observer is co-rotating with the horizon at $\Omega_H$, so is the infalling particle, i.e., $\dot{\phi} \to \Omega_H$ regardless of the value of $L$ as $r \to r_+$ in Eq. (21). Thus, it is natural to think that the emitted closed strings near the horizon should also be co-rotating with the horizon (in the co-rotating frame, the particle falls radially, like the previous case). Generally for an observer rotating with $\Omega_H$, the measured conserved energy becomes $-p \cdot \xi = E - \Omega_H L$, where $\xi^\mu = \partial_t + \Omega_H \partial_\phi$ is the co-rotation Killing vector that is normal to the horizon. This is the energy to match with the total energy of the emitted closed strings on the horizon.

Similarly, to make the match on the same scale, we need to transform the action (25) to the string-scale form (3) with the redefinitions $\tilde{R} = \epsilon R/(4\beta M)$ and $\tilde{t} = ct/(4\beta M)$. Under the transformations, the quantity $\Omega_H L$ transforms as $E$. In the redefined coordinates $(\tilde{t}, \tilde{R})$, the energy match condition on the event horizon is thus: $E_c = 4\beta M(E - \Omega_H L)/\epsilon$. Inserting it into Eq. (A.6), we get the entropy of the closed string states:

$$\delta S \simeq \frac{8k\pi M}{\epsilon}(E - \Omega_H L) = \frac{k}{4}\delta A.$$  \hspace{1cm} (26)

4 Discussion

We show that infalling matter trying to enter into a non-extreme black hole will encounter a tachyon condensation process near the event horizon, with the Hagedorn temperature equal to the Hawking temperature. It will decay into closed strings completely before reaching the event horizon, via coupling to the background gravitational or closed string modes. So this provides a description of the infall process beyond the semiclassical approximation and is obscure in the usual geodesic equation approach.

The revealed fact implies that the black hole area or entropy should be interpreted as the degeneracy of discrete states of the closed strings that the infalling matter decays into. All the information of the matter is recorded in the emitted closed strings. The information of each infalling particle is mainly recorded at the closed string state at some cut-off level which is relevant to energy of the particle on the horizon.

This is a picture for an exterior observer. For an observer co-moving with the infalling matter, the matter still possibly enters into the black hole, with nothing special happening near the horizon, if it encounters no firewall [9]. The co-moving observer sees Minkowski spacetime there, if the equivalence principle still holds. The states on the infalling mater
are not excited and the coupling to gravitational modes becomes trivial.

The field theory description here can not predict the fate and the final state of the emitted closed strings. They may cross the horizon and enter into the black hole. In this sense, the black hole looks like a condensate of gravitons [23, 24, 25]. But we prefer a membrane paradigm [22]: the information of an infalling particle will be left on the membrane above the event horizon once the particle disappears from our world. The surrogates of the particle on the membrane are the emitted closed strings. Hawking radiation of a particle can be viewed as the inverse process of tachyon condensation from a collective of closed strings on the membrane. Hence, for an exterior observer, the information of the infalling matter is reformulated in random ways in Hawking radiation, but is retrievable.

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Appendix

A Entropy of tachyon condensation

The closed string emission from unstable D-branes is studied in the BCFT in [26, 17]. The tachyon effective theory (3) is an equivalent description to this conformal theory [27, 17, 28]. Thus, the results should be common for the two equivalent theories. In this Appendix, we summarize the results for the decay from an unstable D0-brane in bosonic string theory and estimate the entropy of degenerate states of the emitted closed strings, mainly based on [17, 28].

The tachyon effective action (3) or the BCFT action coupling to the closed string modes will lead to closed string emission from the rolling tachyon. The total energy of the emitted closed strings, up to some numerical factor, is given by:

$$E_c \sim \sum_n d_n \int \frac{d^{25} p_+}{(2\pi)^{23}} e^{-\frac{\pi}{2} E^{(n)}}. \quad (A.1)$$

The number of states of the emitted closed strings with left-right symmetry is the same as that of free open strings for large $n$:

$$d_n \sim n^{-\frac{22}{23}} e^{4\pi\sqrt{n}}. \quad (A.2)$$
The energy $E^{(n)}$ at large level $n$ can be approximated to be

$$E^{(n)} = \sqrt{M_n^2 + p_\perp^2} \simeq M_n + \frac{p_\perp^2}{2M_n} + \cdots,$$

where $M_n = 4\beta\sqrt{n-1} \simeq 4\beta n^{1/2}$ for bosonic string theory and $p_\perp$ is the transverse momentum. Note that the most important contribution of $p_\perp$ to the integral (A.1) occurs around $|p_\perp/\beta| \sim n^{1/4} \ll n^{1/2}$ for large $n$, therefore justifying the above approximation in (A.3).

Evaluating the integral in (A.1) gives

$$E_c \simeq \frac{\beta}{k} \sum_n \frac{1}{\sqrt{n}},$$

where $k$ is an undetermined numerical constant [28]. The sum is divergent. Since the energy of D0-brane is finite, this divergence cannot be true and is due to our lack of consideration of the backreaction of the closed string emission process during the rolling of the tachyon. So a natural cut-off should be chosen and this should be on the order of $1/g_s$, due to the D0-brane energy of order $1/g_s$. So this natural cut-off can be large for a weak string coupling. Denote the cut-off level as $N \sim 1/g_s^2$. For large $n$ (in this case, we can write the sum as an integral approximately), we have

$$E_c \simeq \frac{2\beta}{k} \sqrt{N} \simeq \frac{1}{2k} M_N.$$  \hspace{1cm} (A.5)

Most of the energy $E_c$ is stored in closed strings at level $N \sim 1/g_s^2$ since any large cut-off $N \ll 1/g_s^2$ will give an energy $E_c \sim \beta\sqrt{N} \ll 1/g_s$, much less than the D0-brane energy. The number of states at this level is also the largest. Thus, the entropy of the emitted closed strings can be estimated to be

$$S_c \simeq \ln d_N \simeq \frac{2k\pi}{\beta} E_c.$$ \hspace{1cm} (A.6)

Obviously, this tachyon condensation process obeys the first law for $k = 1$. When most energy is converted to the highest level $N$, the emission process tends to attain the maximum entropy.

For an unstable D0-brane, the energy $E_c$ is the scale of its mass $\tau_0$. For our case here, it is completely sourced by the energy of the infalling particle near the event horizon since the infalling particle decays completely into close strings there, as indicated by the near-horizon tachyon effective actions.
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