Origami-kirigami approach to materials structures modelling

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Abstract. Additive manufacturing and origami/kirigami principles fit together like a coffee cup and saucer. Recently derived translational surface with an elliptic curve (not an ellipse), as the governing one, led to constructing origami-like object. Set of eight surfaces, each with a square-shaped orthogonal projection, turned out to be enchantingly similar to heaven-and-hell-origami. In presented work engaged starting translational surface (STS) is originally subjected to the selected linear and nonlinear transformations, in two ways. The first one is dedicated to spacial placement of STS replicas. Shape STS variations represent the second approach. Square-shape of STS orthogonal projection is preserved during all the mappings, it does not change. On the other hand, especially in this point, the similarity with origami/kirigami structures is broken; generally, square-shaped sheet of paper changes via folding. In our paper, preserved orthogonal projection offers suitable opportunities to illustrate a structure of the hypothetical material. Explicit and parametric equations of geometrical elements, transformational matrices and MATLAB application MuPAD serve as the useful mathematical and computational tools for formal and graphical representation of the modeled structures.

1. Introduction
Grey eminence of what follows is an elliptic curve (EC). Not an ellipse. Nor a part of an ellipse. But an elliptic curve. Its significant and typical field of application is ECC – elliptic curve cryptography. Not a materials structures modelling.

In the last years, new technological alternatives have initialized an opening the door to the advent of novelty materials. On the one hand novelty from the viewpoint of structure/geometry. And simultaneously from physical properties perspective. References [1-13] represent multi-taste spectrum.

![Figure 1. 3D geometrical structure of hypothetical material. An elliptic curve is a governing key curve of translational surface element.](image-url)
We start from findings in [14] and [15] and their specific relation to origami. We offer an original, simple approach to modelling of hypothetical materials structures as can be seen in Figure 1. There is a future challenge to look into the role of geometrical properties of EC in physical aspects of material structures described herein. By then: brief creating-material-architecture-tour.

The aims of a next section are twofold: a) to sketch a key translational surface (TS) element with EC as a governing one [14-15] and b) to give a mathematically formal and a graphical illustration of a structure with solely linear mappings of TS as well as with TS-shape conservation. The changes of the TS-form as a consequence of nonlinear transformations are discussed in section 3. Selected nonlinear mappings do not modify squared orthogonal projection of TS; thank to this, elements arranged in a row have continuous projection, nevertheless transformed TS elements are of 3D shapes, more or less isolated and similar to kirigami structures. Replicas of transformed TS are arranged to layer, eventually layers are posed on top of each other.

2. Elliptic curve in a starting translational surface element and preserving-the-object-shape linear transformations

Hoping in clarity of interpretation 😉 we will continue by naming verbal, formal and graphical (FGV) representations of the objects. Here they are, one after the other:

- implicit equations of the elliptic curve, parameters $a, b$ are fixed,
- explicit equations of the elliptic curve, part for a positive Cartesian coordinate $y$,
- parametric equations of translational surface; EC is engaged to the position of each of both governing curves; dividing by 2 is for to obtain less slope of the curve;

$$y^2 = x^3 + ax + b, \ (a, b) = (1, 0) \quad (1)$$
$$y(x) = \sqrt{x^3 + x} \quad (2)$$
$$r(t, u) = (x(t, u), y(t, u), z(t, u)) = \left( t, u, \left( \sqrt{t^3 + t} + \sqrt{u^3 + u} \right)/2 \right) \quad (3)$$

where $t, u$ vary from 0 to 1.

Translational surface (Figure 2) serves as the basic element in the geometrical sets. In the following, firstly symmetry with respect to a horizontal plane is done. Plane intersects starting TS in the middle of its height. Duet of starting TS and a planarly symmetrical one are shown in Figure 3 from various viewpoint. TS duets are spatially arranged to the layer (Figure 4).

**Figure 2.** There are step by step a) 2D elliptic curve (EC), b) 3D translational surface: look at the borders of the surface – these are ECs, c) quartet of TSs, d) octet of TSs, which is similar to heaven-and-hell-origami-like object [14-15].
Figure 3. Duet of translational surface and its planarly symmetrical one in a three different positions. Figure 3c) represents a view from the above.

Figure 4. Three different views on the doublet-layer.

3. Nonlinear transformations
Sine-function was chosen to the experiments with nonlinear transformations of starting TS. In parametrical equations, \( x \) and \( y \) coordinates are without changes. Nonlinearity is placed to \( z \)-coordinate in two different ways:

- original \( z \)-coordinate is multiplied by sine,
- \( z \)-coordinate becomes an argument of a sine.

Here are the corresponding analytical expressions:

\[
r^*(t, u) = (x(t, u), y(t, u), z^*(t, u)) = \left( t, u, f^{-1}(t, u)\left(\sqrt{t^3 + t^3 + u^3 + u}\right)/2 \right)
\] (4)
\[ f(t, u) = A \sin(Btu). \quad (A, B) = (1/10, 24) \tag{5} \]
\[ r^{**}(t, u) = (x(t, u), y(t, u), z^{**}(t, u)) = (x(t, u), y(t, u), \sin(\frac{Bz(t,u)}{2})) \]
\[ z^{**}(t, u) = \sin(Bz(t,u)), \quad (A, B) = (1/6, 16), \tag{7} \]

where \( t, u \) vary from 0 to 1, in the same way as in the case of the previous linear mappings. Finally, graphical representation is shown in Figure 5 and Figure 6.

**Figure 5.** Two different views on the nonlinearily transformed TS and the series of its replicas.

**Figure 6.** The second nonlinear transformation. One shape-changed TS element (a) and the set of layers from these nonlinearily transformed surfaces (b).

### 4. Conclusion and a future work

It is a question of a future work to develop existing knowledge at these promising directions:

a) variations in the very beginning – at the heart of the matter: engaging the **elliptic curves** of the different \((a, b)\) parameters and consequently of the various differential-geometry-features,

b) **geometry** variations in a shape and a position of the STS, i. e. involvement other choices from infinity menu of linear and nonlinear transformations,
c) research of physical properties of simulated or real structures,
d) investigation/control in b) and c) issues with respect to a possible exploiting of a functional gradient, taking into account existing EC analytical representation via explicit, implicit and/or parametric functions.

Appendix
An incredible coincidence from the article preparation phase – snapshot from the copy center: kirigami-like pattern on the lady dress (Figure 7). This structure is similar to the one on the Figure 8 [2]; original figure is here rotated ninety degrees.

Figure 7. Pattern on the fabric.  Figure 8. Microscale kirigami pattern [2].

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