Electron gas oscillations in plasma.
Theory and applications.

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Abstract

We analyze the obtained solutions of the non-linear Shrödinger equation for spherically and axially symmetrical electrons density oscillations in plasma. The conditions of the oscillations process existence are examined. It is shown that in the center or on the axis of symmetry of the systems the static density of electrons enhances. This process results in the increasing of density and pressure of the ion gas. We suggest that this mechanism could occur in nature as rare phenomenon called the fireball and could be used in carrying out the research concerning controlled fusion. The description of the experiments, carried out for the purpose to generate long-lived spherical plasma structures, is presented.

1 Introduction

The studying of electron gas oscillations in plasma is interesting physical problem, mainly because it enables one to examine plasma properties from the theoretical point of view as well as it provides the basis for subsequent experiments in this area. Let us discuss the electric charge variation in electroneutral plasma. If a volume charge appears in such a system, i.e. electrons density increases or decreases in some finite area, then, after an external influence is over, the oscillating process consisting in periodical changes of the sign of the considered volume charge is known to appear. We can roughly neglect the motion of positively charged ions since their mass is several orders of magnitude greater then electron mass. The process of electron gas oscillations is schematically shown in Fig. 1. The electrons motion from the central area is presented in Fig. 1(a). Thus, the central region acquires excessive positive charge. Electrons moving to the central area are depicted in Fig. 1(b). In this case the central region becomes negatively charged. It is necessary

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Figure 1: the process of electron gas oscillations in plasma, with electrons moving from (a) and to (b) the central area.

to remind that the frequency of this process $\omega_p$, called plasma frequency, is related to free electrons density in plasma $n_0$ by the formula

$$\omega_p^2 = \frac{4\pi e^2 n_0}{m},$$

where $e$ and $m$ are the charge and the mass of the electron.

Numerous attempts were made to describe electron gas oscillations within the context of both classical and quantum mechanics (see, for example, Ref. [1]). The method of kinetic equation is one of the most commonly used approaches to this problem in frames of classical theory. The kinetic equation for each type of particles has the form:

$$\frac{\partial f_\alpha}{\partial t} + \mathbf{v} \frac{\partial f_\alpha}{\partial \mathbf{r}} + \mathbf{p} \frac{\partial f_\alpha}{\partial \mathbf{p}} = \text{St}_f_\alpha,$$

where $f_\alpha(\mathbf{r}, \mathbf{p}, t)$ is the distribution function depending on the coordinate $\mathbf{r}$, momentum $\mathbf{p}$ and time $t$ for either electron ($\alpha = e$) or ion ($\alpha = i$) component, $\text{St}_f_\alpha$ is the collision integral. The derivative $\dot{\mathbf{p}}$ is equal to $q \left( \mathbf{E} + \frac{1}{c} [\mathbf{v} \times \mathbf{B}] \right)$ for the case of electromagnetic interactions, $q$ is the particle charge.

The collision integral is negligible if the mean free path $l$ of a particle is much greater than the distance of the electromagnetic field (produced by charged particles in plasma) variation $L$: $l \gg L$. The Eq. (2) with $\text{St}_f_\alpha = 0$ for $\alpha = i, e$ should be supplied with the system of averaged Maxwell equations in order for the electric $\mathbf{E}$ and magnetic $\mathbf{B}$ fields to be specified. The electromagnetic field, determined in such a manner, is called the self-consistent field. The self-consistent field conception enables one to investigate electromagnetic properties of plasma and describe various plasma phenomena [1]. The kinetic equation with $\text{St}_f_\alpha \neq 0$ takes into account the collision processes between particles in plasma. There exist several types of collision integrals [1].
There is, however, another approach, developed in Refs. [2, 3] to describe the evolution of \( N \) interacting particles. It is known that the classical dynamics of \( N \) interacting particles can be presented by the system of differential equations of motion in configurational space of \( 3N \) dimensions or in \( 6N \)-dimensional phase space. It is also possible to describe the evolution of such a system by means of partial differential equations in three-dimensional physical space as the dynamics of microscopic singular material fields (see Ref. [2]). A particle in this approach was determined as \( \delta \) function distribution containing spatial coordinates and time independently. This method allows one to implement the mapping of trajectories dynamics in the fields dynamics and vice versa. To describe the particles dynamics on the macroscopic scale in this approach one should smooth the microscopic distributions. This procedure is well known in classical electrodynamics and hydrodynamics.

The transfer to the quantum-mechanical description is realized by replacing the dynamic functions with Hermitian operators. In this case the dimensionality of the configuration space is conserved and the state of the system is completely defined by the wave functions in \( 3N \)-dimensional space. However, physical characteristics of the \( N \) particles system are determined in three-dimensional physical space.

The quantum-mechanical description of the system of \( N \) charged particles with arbitrary masses was considered in Ref. [3]. Particles were taken to interact by Coulomb forces and with external classical electromagnetic field characterized by vector \( \mathbf{A}(r, t) \) and scalar \( \phi(r, t) \) potentials. It is worth mentioning that the evolution of \( N \) particles was described in three-dimensional physical space. The exact microscopic equations for the description of the quantum dynamics of the \( N \) particles system in physical space were obtained in that paper. The transfer to the macroscopic observable fields was implemented by means of smoothing procedure of microscopic functions. Note that the quantum description of the \( N \) particles system can be formally reduced to one particle Shr"odinger equation if we express all operators in terms of creation and annihilation operators. However, one particle wave functions are the operators in this approach. In Ref. [3] it was shown that the quantum dynamics of the \( N \) particles system can be described using non-operator wave function in three-dimensional physical space. This result is of great importance.

We applied the formalism developed in Ref. [3] to the description of the electron gas oscillations in plasma. The solutions of linearized Shr"odinger equation in the approximation of the self-consistent field were obtained in Ref. [4].

The main goal of this paper is to study electron gas oscillations in plasma using quantum mechanical approach. The quantum description is implemented in three-dimensional physical space. In Sec. 2 we analyze spherically and axially symmetrical solutions of the non-linear Shr"odinger equation. The conditions of the oscillations process existence are examined. It is found that in the center or on the axis of symmetry of the systems the static density of electrons enhances. This process leads to the increasing of density and pressure of the ion gas. Then, in Sec. 3 we discuss possible applications of the considered model. We suggest that this mechanism could occurs in nature as rare phenomenon called the fireball and could be used in carrying out the research concerning controlled fusion. The description of the experiments, carried out for the purpose to generate long-lived spherical
plasma structures, is presented. Finally, in Sec. 4 we discuss our results.

2 Quantum description of electron gas oscillations

We will assume electrons in plasma to be a quantum many body system. Such an assumption is due to the fact that, as it will be shown below, the obtained solutions have characteristic sizes of atomic order. The complex $\Psi$ function is introduced in three-dimensional space and has the form

$$\Psi (r, t) = \sqrt{n_e (r, t)} e^{i \sigma (r, t)},$$  

(3)

where $n_e (r, t) = |\Psi|^2$ is the density of electrons, $\sigma (r, t)$ is the phase of the function $\Psi$. The function $\Psi$ satisfies the partial differential equation:

$$i \hbar \frac{\partial \Psi}{\partial t} = \hat{H} \Psi.$$  

(4)

The Hamiltonian in the Eq. (4) is expressed in the following way

$$\hat{H} = \frac{1}{2m} \left( \frac{\hbar}{i} \nabla - \frac{e}{c} A (r, t) \right)^2 + e \varphi (r, t) + e^2 \int d^3 r' G (r - r') |\Psi|^2 + \theta (r, t),$$  

(5)

where $G (r - r') = \frac{1}{r - r'}$. In the Eq. (5) the first two terms are the components of single electron Hamiltonian in the external electromagnetic field. The third term presents the potential energy of the electron in the self-consistent electrostatic field created by the whole electrons system, with number density of particles being equal to $|\Psi|^2$. The function $\theta (r, t)$ describing exchange interactions between electrons has the form

$$\theta (r, t) = \frac{\hbar^2}{2m} \frac{\Delta \Psi}{|\Psi|^2} + \int_{r_0}^r \frac{dp_e (r, t)}{\Psi} + e^2 \int d^3 r' \int_{r_0}^r dG (r - r') \frac{q_2 (r, r', t)}{|\Psi|^2},$$  

(6)

where $p_e (r, t)$ is the pressure of the electron gas, $q_2 (r, r', t)$ is the correlation function.

Therefore, in order to resolve exactly the considered problem one should take into account all terms in the Eq. (5). In Ref. [4] we assumed that the contribution of exchange interactions to the dynamics of free electrons in plasma is much smaller than the contribution of self-consistent electrostatic field. This assumption is valid at low density of free electrons in plasma. Then, in describing the dynamics of the electron gas we neglected the function $\theta (r, t)$. As it will be seen from further speculations, these rough approximations allow us to get some characteristics of the oscillations process which are close enough to those obtained from the treatment of similar problem within the classical approach.

Let us consider the electroneutral plasma formed by the singly ionized gas, with the energy of electrons being more than the ionization potential of this gas. We will suppose that plasma possesses a spherical symmetry for density and velocities distribution of the electron and the ion gases. Note that there is no electromagnetic radiation in this system.
since the magnetic field is absent throughout the volume. The magnetic field, and hence the radiation, can appear if plasma parameters deviate from the spherically symmetrical distribution. If such deviations are small, the system is likely to reconstruct its internal structure and becomes spherically symmetrical again. The problem of the system stability to such deviations is under investigation now.

Taking into account the small mobility of heavy ions in gas compared to the mobility of electrons we will suppose that the density of ions is the constant value \( n_i(r,t) = n_0 \). We will also consider that in our case there are no external electromagnetic fields except those of positively charged ions. The potential of self-consistent field created by the electron gas is represented by the formula (using spherical coordinates):

\[
U_e = e \int d^3r' G(r - r')|\Psi|^2 = 4\pi e \int_0^\infty \frac{dR}{R^2} \int_0^R x^2 |\Psi(x,t)|^2 dx. \tag{7}
\]

Similarly, for the potential \( \varphi \) of singly ionized gas with density of ions \( n_i(r,t) \) one has

\[
\varphi = -4\pi e \int_0^\infty \frac{dR}{R^2} x^2 n_i(x,t) dx. \tag{8}
\]

Thus, taking into account Eqs. (7) and (8), the Eq. (4) can be represented in the following way

\[
i\hbar \frac{\partial \Psi}{\partial t} + \frac{\hbar^2}{2m} \Delta \Psi - 4\pi e^2 \Psi F(|\Psi|^2 - n_0) = 0, \tag{9}
\]

where \( \Delta = \frac{\partial^2}{\partial r^2} + \frac{2}{r} \frac{\partial}{\partial r} \) is the Laplas operator in the spherical coordinate system and

\[
F(\ldots) = \int_0^\infty \frac{dR}{R^2} \int_0^R x^2 (\ldots) dx.
\]

Moreover, we demand that the system should be electroneutral as a whole, i.e. the condition must be satisfied:

\[
\lim_{R \to \infty} \frac{1}{R^3} \int_0^R x^2 |\Psi(x,t)|^2 dx = \frac{n_0}{3}. \tag{10}
\]

We will search for a solution of the Eq. (9) in the form:

\[
\Psi = \Psi_0 + \chi(r)e^{-i\omega t}. \tag{11}
\]

Here \( \Psi_0 \) is the solution in the case of unperturbed electroneutral plasma, \( |\Psi_0|^2 = n_0 \), \( \chi(r) \) is the complex function describing small perturbations on frequency \( \omega \), \( |\chi| \ll |\Psi_0| \). Using the Eq. (11), we get that \( |\Psi|^2 = n_0 + \Psi_0 f + |\chi|^2 \), where \( f = \chi e^{-i\omega t} + \chi^* e^{i\omega t} \). Taking into account that \( |\chi| \ll |\Psi_0| \), we obtain that \( |\Psi|^2 \approx n_0 + \Psi_0 f \).
Then, we substitute the function $\Psi$, given in the Eq. (11), and approximate expression for $|\Psi|^2$ in the Eq. (9). Having considered the complex-conjugate equation together with the obtained one, it was easy to get the following equation for the function $f$:

$$\hbar \omega f + \frac{\hbar^2}{2m} \Delta f - 4\pi e^2 \Psi_0 (2\Psi_0 + f) F(f) = 0.$$  \hspace{1cm} (12)

For the total linearization, it is necessary to suppose that $2\Psi_0 + f \approx 2\Psi_0$ in the third term of the Eq. (12). Then, we represent the function $\chi$ through its real and imaginary parts: $\chi = \chi_1 + i\chi_2$. Hence, $f = \chi_1 \cos \omega t + \chi_2 \sin \omega t$, and the Eq. (12) can be divided into two independent similar equations for $\chi_1$ and $\chi_2$:

$$\hbar \omega \chi_n + \frac{\hbar^2}{2m} \Delta \chi_n - 8\pi e^2 n_0 F(\chi_n) = 0, \quad n = 1, 2.$$  \hspace{1cm} (13)

One can find out that the functions $\chi_n = B_n \sin \gamma r$, $n = 1, 2$, where $B_n$ are real constants, are the solutions of the Eq. (13). The behavior of the functions $\chi_n$ is shown in Fig. 2. The parameter $\gamma$ satisfies the dispersion relation

$$\gamma^2 = \frac{\omega m}{\hbar} \left[ 1 \pm \left( 1 - \frac{4\omega_p^2}{\omega^2} \right)^{1/2} \right],$$  \hspace{1cm} (14)

Figure 2: the coefficient $Y = \chi_n/(\gamma B_n)$ versus the parameter $X = \gamma r$. 
Figure 3: the coefficient $y = \gamma [(2\omega_p m)/\hbar]^{-1/2}$ versus the parameter $x = \omega/\omega_p$. If $\omega = 2\omega_p$, then $\gamma_{1,2} = \pm [(2m\omega_p)/\hbar]^{1/2}$. If $\omega \gg 2\omega_p$, for the upper branch we have $\gamma_{1,2} \sim \pm [(2m\omega)/\hbar]^{1/2}$, and for the lower branch $\gamma_{3,4} \sim \pm [(2m\omega^2)/\hbar\omega]^{1/2}$.

where $\omega_p$ is determined in the Eq. (1). The positive part of the Eq. (14) as a function of frequency $\omega$ is presented in Fig. 3. It is worth noticing that if the values of $B_n$ are limited, the expressions for $\chi_n$ satisfy the condition of system electroneutrality Eq. (10).

Now we present the estimate of some characteristics of the oscillations process. For the density $n_0 = 2.7 \times 10^{19}\text{cm}^{-3}$, i.e. for completely singly ionized gas under the atmospheric pressure and when $\omega = 2\omega_p$, the frequency of electron oscillations is:

$$\nu = \frac{\omega_p}{\pi} = 2 \left(\frac{e^2 n_0}{\pi m}\right)^{1/2} \approx 9 \times 10^{13}\text{Hz}.$$

This frequency corresponds to the electromagnetic radiation in the infrared range with the wavelength $\lambda = c/\nu \approx 3 \times 10^{-4}\text{cm}$. In this case $\gamma_{1,2} = \pm [(2m\omega_p)/\hbar]^{1/2} \approx \pm 2.3 \times 10^7\text{cm}^{-1}$. The size of the central region $\delta$, where the most intensive oscillations of the electron gas are observed, is equal to $\pi/\gamma \approx 1.4 \times 10^{-7}\text{cm}$. The obtained value for $\delta$ is of atomic order. That is why we have adopted the quantum approach to the electron gas oscillations problem.

From the Eq. (14) one can see that the frequency $\omega = 2\omega_p$ is the critical value, since for frequencies less than $2\omega_p$, $\gamma$ becomes a complex value and under these circumstances the oscillations of the electron gas are damped. However, there is no contradiction between this result and the well known fact that in using the classical approach to similar problem one gets the value $\omega = \omega_p$ for the critical frequency. The plasma frequency $\omega_p$ is the critical value for a frequency in a sense that oscillations of electron gas can appear only if $\omega \geq \omega_p$. According to our result $\omega$ should be greater (or equal) than $2\omega_p$. However, under such
condition the inequality \( \omega \geq \omega_p \) is sure to be satisfied. The obtained constraint for possible oscillations frequencies \( \omega \geq 2\omega_p \) is unlikely to be the feature of geometry of the system in question. As it will be shown below, there is the same condition for axially-symmetrical electron gas oscillations. We suspect that the deviation from the result of classical theory \( (\omega \geq \omega_p) \) results from the quantum approach used in our paper.

Now let us discuss the origin of each of the branches in the dispersion relation. At high oscillations frequencies the parameter \( \gamma \) is great for the upper branch. Therefore, we can define the considered branch as the "high-energy" one. Moreover, it is possible to treat the upper branch as the "classical" one. If \( \omega \gg 2\omega_p \), the dispersion relation for this branch becomes similar to the relation between energy \( \bar{\hbar}\omega \) and "momentum" \( \bar{\hbar}\gamma \) for a classical particle:

\[
\bar{\hbar}\omega = \frac{(\bar{\hbar}\gamma)^2}{2m}.
\]

Note that the definition "high-energy" is consistent with "classical" owing to the correspondence principle between classical and quantum mechanics (see, for instance, Ref. [5]).

The lower branch is the antithesis to the upper branch in all respects. To begin with the behavior of the parameter \( \gamma \) at high frequencies. Contrary to the upper branch the lower one vanishes as \( \omega^{-1/2} \). This fact means that we can define considered branch as the "low-energy" one. Thus, according to the correspondence principle again we can also treat it as the "quantum" branch. We suspect that such substantially quantum phenomenon as superconductivity of electron gas may take place if the dispersion relation is realized as the lower branch. This problem is also discussed in Sec. 3 devoted to possible applications of the described model.

Now let us discuss the ion gas density distribution. We supposed that number density of the ion gas was constant throughout the volume because of the small mobility of heavy ions. However, it leads to incorrect results. It is necessary to remind that the exact expression for the density of the electron gas in searching for the solution in the form of the Eq. (11) is presented as \(|\Psi|^2 = n_0 + \Psi_0 f + |\chi|^2\). Let us define in this expression \(\Psi_0 f\) and \(\bar{n}_e = n_0 + |\chi|^2\) as the dynamic and static components of the electron gas density. In deriving the approximate linearized Eq. (13) the function \(|\chi|^2\) was supposed to be small and thus neglected. This procedure is not correct because the integral operator \(F(|\chi|^2)\) becomes divergent for the function \(\chi = (B/r)\sin \gamma r\). The divergence is caused by the time independent quantity \(|\chi|^2\), which means the excessive static component of the electron gas density. We assumed that the density of the ion gas was constant. The average of the frequently oscillating dynamic component over time is zero, but \(|\chi|^2\) does not depend on time. It is naturally to expect that under some conditions the negative volume charge, described by the function \(|\chi|^2\), will be compensated (or neutralized) by the removing of positive ions. This process will result in local changing of density and pressure of the ion gas.

Indeed, it can be shown that for our case the condition of static stability of the ion gas
is:

\[
kT \frac{\partial n_i}{\partial r} = \frac{4\pi e^2}{r^2} n_i \int_0^r (n_i - \bar{n}_e) x^2 \, dx \tag{15}
\]

where \( k \) is Boltzmann constant, \( T \) is the ion gas temperature, \( n_i \) is the ions density.

For the neutralization process to occur we demand that the ions density \( n_i \) should be equal to the static electrons density \( \bar{n}_e = n_0 + |\chi|^2 \) with high level of accuracy. Then, from Eq. (15) we get the following inequality:

\[
\frac{kT r^2}{4\pi e^2 n_i} \left| \frac{\partial n_i}{\partial r} \right| \ll 1.
\]

Having substituted the expression for \( n_i \) in the last formula, we obtained the condition of the neutralization: \((kTB^2\gamma)/(4\pi e^2 n_0) \ll 1\). Taking into account that ions density in the center of the system should be equal to \( n_c = B^2 \gamma^2 \), this condition can be rewritten in the following way:

\[
n_c \ll \frac{4\pi e^2 \gamma}{kT} n_0. \tag{16}
\]

For instance, for \( T = 10^3 \) K and the value \( \gamma = 2.3 \times 10^7 \) cm\(^{-1}\), which was obtained above, we get that \( n_c \ll 500n_0 \). We supposed that the perturbations described by the function \( \chi \) were small, i.e. \( n_c \ll n_0 \). Hence, the condition Eq. (16) is satisfied and the excessive static electron charge is undoubtedly compensated by the ion charge and the divergence in the integral \( F(|\Psi|^2 - n_i) \) can be eliminated. Thus, the approximate, linearized theory trends to describe the pressure enhancement of the ion gas in the center of symmetrically oscillating electron gas.

The ion density enhancement, described in our paper, and the Debye shielding are the completely different phenomena since the latter is the static effect and the former is the dynamic phenomenon. It should be also noted that the electron gas density enhancement is the essentially quantum effect and cannot be obtained within the classical approach with the use, for example, of the kinetic equation method. Indeed, in the classical description of the electron gas oscillations in plasma we get that electron distribution function in the Eq. (2) has the form:

\[
f_e(r,p,t) = f_0(p) + \delta f(r,p,t),
\]

where \( f_0(p) \) is the unperturbed stationary as well as spatial coordinates independent distribution function, \( \delta f(r,p,t) \) is the perturbation of the distribution function. The electrons number density is expressed in the following way

\[
n_e(r,t) = \int d^3p f_e(r,p,t) = n_0 + \delta n(r,t),
\]

where

\[
\delta n(r,t) = \int d^3p \delta f(r,p,t). \tag{17}
\]

The function \( \delta f(r,p,t) \) is usually taken to be proportional to \( \exp\left[i(kr - \omega t)\right] \). Therefore, the average of \( \delta n(r,t) \) over time in the Eq. (17) is zero. Thus, it is impossible to obtain the electron gas density enhancement within the context of the classical theory.
Basing on the obtained approximate solutions of the non-linear Shrödinger equation, we suggest the following dynamics of the spherically symmetrical electron gas oscillations with finite amplitude:

1. The appearance of the excessive negative volume charge in the center of the system (quantum mechanical effect in electron gas oscillations).
2. The neutralization of the small negative volume charge as a result of the ions motion towards the center.
3. The enhancement of the ion gas density and pressure.
4. The Hamiltonian in the Eq. (5) is changed allowing for the new electric charges distributions. The approximate solution of the new equation provides further pressure enhancement of the ion gas.

While considering non-linear Shrödinger equation (9), it can be seen that along with the components on frequency $\omega$, the terms which do not depend on time as well as on frequencies $2\omega$, $3\omega$ and etc appear. One can make sure of this representing the solution of the Eq. (9) in the form:

$$\Psi(r, t) = \Psi_0 + \sum_{k=1}^{\infty} \Psi_k(r, t),$$

where $\Psi_1 = (B_1/r)e^{-i\omega t}\sin\gamma r$.

It is worth mentioning that along with spherically symmetrical solution of the Eqs. (4)-(6), there is at least one axially-symmetrical solution which has the form:

$$\Psi(r, t) = n_0^{1/2} + B J_0(\gamma r)e^{-i\omega t},$$

where $J_0$ is the zero-order Bessel function, $B$ is a real constant. In this case the dispersion relation takes the same form as the Eq. (11). All consequences obtained for spherically-symmetrical oscillations are valid for this case as well.

### 3 Applications

In this section we present the data of the fireball appearance in nature and various theoretical models for the description of this phenomenon. We also analyze the possibility for a fireball to be accounted for in frames of our model of quantum electron gas oscillations in plasma.

The fireball is a very rare natural phenomenon. However, numerous observations of fireballs have been collected (see, for example, Ref [6]). It should be noted that most of the fireball observers were not professional researchers and, thus, their descriptions of this phenomenon may appear to be quite subjective. However, the substantial number of these observations reveal some regularity. We present below the key aspects of the fireball observations.
• **The fireball appearance.** A fireball more often appears beside the lightning stroke. There are, however, a considerable amount of observations when a fireball appeared without a lightning. Different sharpened objects such as metallic pieces, antennae, wires etc are reported to favor the fireball appearance. The fireball observers usually mention that there can be the high electric field strength, for instance, corona discharge on the metallic objects.

• **The fireball disappearance.** A fireball often disappears smoothly attenuating. However, rather frequently a fireball disappears with explosion, releasing of great amount of heat energy and melting metal and sand.

• **Stationary state of a fireball** The usual average size of a fireball is about 30÷40cm. The lifetime is in the range from 1 to 200s. The color of the fireball glow is various: red, yellow, white and blue. The minimal number of observed fireballs had green hues. Neither high brightness of a fireball nor the effects which can be regarded as the evidence of thermal radiation are the characteristics of a fireball. Its light looks like a glow discharge. Even transparent fireballs are reported to appear. Sometimes the observers mention that a fireball has more bright core.

• **The fireball form.** A fireball often has elliptical or spherical form. The form, which can be identified with toroidal one, is rarely observed. The fireball surface is reported to consist of needles. The fireball sparking is the frequently mentioned feature.

• **The fireball motion and location.** A fireball is regularly located near objects on the ground or in the atmosphere (for example, near airplanes). It moves, as a rule, along these objects on a certain distance. A fireball can be elastically reflected by the objects or disappears in touch with them. A fireball moves towards the air current as well as in the reversed direction. A fireball can penetrate indoors through a chimney, window glass and holes, with their diameter being less than the visible one of a fireball. It is worth noticing that window glass is not destroyed sometimes when a fireball travels through it. The fireball interaction with ferromagnetic materials is not observed.

• **The biological action of a fireball.** The biological effects of a fireball are the legs and arms paralysis as well as the tan.

The fireball models can be divided into several groups. According to one of them a fireball appears if the electromagnetic radiation with the wavelength of several centimeters or shorter is closed on itself forming a stable wave package. This process occurs either in the atmosphere or on the ground in a certain form a lightning discharge. Other fireball models are based on the step leader. A fireball is either identified with the step leader or is its extension under certain conditions (“unfinished” lightning, combination of currents values etc).

According to the model of quantum electron gas oscillations in plasma, developed in this paper, we found that if one presents the solution of the non-linear Shrödinger equation
in the form of the Eq. (11), then time independent excessive component of the electron gas density always appears. It leads to the ion gas density enhancement in the center of the system. This property is unlikely to be the feature of the adopted Hamiltonian.

In order to maintain plasma in ionized state and realize the process described above, the energy ingress from the outside or the energy release within the system is wanted. Nuclear fusion reactions can, in principle, serve as similar source of energy. The reactions will proceed if pressure and density of the ion gas attain appropriate values.

This process seems to support the existence of a fireball. Water is known to contain deuterium in the amount of \( \approx 1/5000 \) under normal conditions. If water vapors are present in atmosphere, the running the nuclear fusion reactions will release energy, which supports the oscillations of the electron gas and prevents the recombination of plasma. Possible exothermic nuclear fusion reactions are:

\[
\begin{align*}
2^1\text{D} + 2^1\text{D} & \rightarrow 3^2\text{He} + ^1n_0, \\
2^1\text{D} + 2^1\text{D} & \rightarrow 3^3\text{T} + ^1p_1, \\
2^1\text{D} + 3^3\text{T} & \rightarrow 4^4\text{He} + ^1n_0,
\end{align*}
\]

where \( 2^1\text{D} \) and \( 3^3\text{T} \) are the heavy isotopes of hydrogen (deuterium and tritium), \( 3^2\text{He} \) and \( 4^4\text{He} \) are the helium isotopes, \( ^1n_0 \) and \( ^1p_1 \) are the neutron and the proton. Taking into account small sizes of the central (active) region and small amount of deuterium in atmosphere, it is possible to use the term ‘microdose’ nuclear fusion reactions for the process in question. Axially-symmetrical oscillations of the electron gas are likely to appear as very seldom observed type of a fireball in the form of shining, sometimes closed cord (see also the list of the observed fireball properties in the beginning of this section). Uncomplicated calculation shows that energy released in deuterium nuclei fusion in \( 1\text{dm}^3 \) of water vapors (the average size of a fireball) has the value of about 1MJ, that corresponds to energy evaluations of some observed fireballs [6, 7].

However, along with high-energy ‘fireballs’ there often appear low-energy ones. The energy estimate of such ‘fireballs’ (see Ref. [6]) indicates that the nuclear fusion reactions are unlikely to support their existence. Low-energy fireball may be presented in frames of our model as the solution with the dispersion relation described by the lower branch (see Fig. 3). The superconductivity might be the mechanism preventing possible attenuation of the electron gas oscillations caused by various dissipation processes.

Moreover, the facts, indirectly verifying that quantum electron gas oscillations in plasma are the possible model of a fireball, are (see also the list of the observed fireball properties in the beginning of this section):

1. Numerous fireball observers mention the elevated level of the atmospheric gas ionization when a fireball appears. This fact points out that a fireball generates very strong electromagnetic radiation with the frequency in the ultraviolet range (or even higher). Although the systems with spherically-symmetrical electron gas oscillations do not reveal radiation, the ultraviolet electromagnetic radiation could be generated in the outer layers of such systems. The origin of this radiation would be various secondary effects, for example, small deviations from spherically-symmetrical state.
2. Persons who were beside a fireball sometimes revealed the tan. The explanation of this fact is the same as in the item 1.

3. A fireball often disappears with heavy explosion. This point indicates that there is the elevated pressure region within a fireball. This region should have small size. If the size had been great, the weight of a fireball would have exceeded the weight of air and a fireball would have descended to the ground. However, it confronts the observations that a fireball freely moves in the atmosphere. The pressure and density of the ion gas are inevitably increased if the electron gas oscillations in plasma are described in frames of our model.

4. A fireball burns small holes, from several millimeters to 2 ÷ 3 cm across, when it travels through dielectric materials (glass, plastic) as well as through thin metallic sheets. If a fireball is described on the basis of our model, the nuclear fusion reactions may take place in its central region. Thus, high temperature, required to melt, for example, glass, can be achieved.

5. The fireball lifetime is anomalously high. This fact can be accounted for by none of the models based on the classical electrodynamics or hydrodynamics.

3.1 Experimental studying of electron gas oscillations in plasma

Groups and separate researchers developing the problem of the controlled fusion are suggested to pay attention to self-consistent, radially and axially oscillating electron ‘plasmoids’ as a base models to self-supported nuclear fusion reactions. The authors of this article have certain experience in generating ball-like plasma structures. We present below the description of the experiments carried out in the N. N. Andreev Institute of Acoustics in 1995-1999.

The plant, used to generate spherical plasma structures, is shown in Fig. 4(a). The reservoir capacitor $C_0 \approx 10^{-7}$ F was charged up to the voltage $U_0 \approx 2 \times 10^5$ V. The electrostatic field energy of the capacitor was $W_{el} = \frac{C_0 U_0^2}{2} \approx 2 \times 10^3$ J. After the complete capacitor discharge, the point $Q$ formed very strong corona charging about 5 cm across. The corona was visible even in diffuse sunlight. The point $Q$ was the sharpened quenched steel wire. The space near the point was enriched with heavy water ($D_2O$). When a high-voltage pulse was applied to the electrode $S$, the break-down of the spark gap occurred and the capacitor $C_0$ was short-circuited. The process of the capacitor discharge is schematically presented in Fig. 4(b). The voltage fell from 200 kV to zero during the time $\tau \approx 10^{-8}$ s. Glowing dark-red spheres about 3 cm across separated from the point $Q$ at the moment of the break-down. The lifetime of these structures was in the interval 1 ÷ 5 s.

The described discharge scheme seems to have implemented the excitation of the spherically symmetrical electron gas oscillations. The electron gas was formed by the high-voltage corona discharge on the point $Q$.

The disadvantages of the considered plant are:
Figure 4: (a) the circuit for the generation of spherical plasma structures, (b) the process of the capacitor $C_0$ discharge.

- high power of the charging devise caused by strong leakage current in displaying a corona discharge;
- high level of noise in short-circuiting of the capacitor $C_0$;
- big overall dimensions of the plant.

The listed disadvantages were partially removed in the plant which is shown in Fig. 5(a). In this circuit, $C_1$ denotes the reservoir capacitor. When high voltage $U_0$ was applied to the point 2 of the circuit, the potential of the point $Q$ was always equal to zero because of the shunting resistor $R$. After the break-down of the spark gap, the potential of the capacitor $C_0$ fell to zero and the potential of the point $Q$ became equal to $U_0$. This process is schematically depicted in Fig. 5(b). The parameters of this circuit were: $C_0 \approx 10^{-10}$F, $C_1 \approx 10^{-9}$F, $R \approx 10^9$Ω, $U_0 \approx 1.5 \times 10^5$V. Leakage currents were almost eliminated since all reservoir elements had been placed in transformer oil. The principal elements of this circuit have passed preliminary testing.

At the end of this section we briefly discuss the results of the experiments, carried out for the purpose to study the interaction of laser radiation with the tantalum targets, which are presented in Ref. [8]. The authors of this paper report that long-lived structures were produced in the experiments. These objects are reported to emit hard gamma-ray radiations (with energy of quanta $E \geq 0.1$MeV) and abandon specific tracks on the metallic walls of the vacuum chamber. These facts imply that, as it was also mentioned by the authors of Ref. [8], the density of matter within the generated structures is very high. Such stable, long-lived structures cannot be described be means of classical electrodynamics. This point was also noticed in Ref. [8]. We suppose that the authors of this article have
Figure 5: (a) the modified circuit for the generation of spherical plasma structures, (b) the potential of the point $Q$.

exited the electron gas oscillations which may be described in frames of our model. Thus, for example, the elevated pressure in these objects can be successfully accounted for. The suggestion that the superconductive electric current might circulate within the produced structures was put forward in Ref. [8]. This hypothesis agrees with our anticipation that superconductivity might occur in the outer layers of spherically and axially symmetrical electron gas oscillations in plasma.

4 Conclusion

The quantum mechanical description of the electron gas oscillations in plasma has been presented in this paper. We have analyzed spherically and axially symmetrical solutions of the non-linear Shrödinger equation. The conditions of the oscillations process existence have been examined. It has been found that in the center or on the axis of symmetry of the systems the static density of electrons enhanced. This process led to the increasing of density and pressure of the ion gas. We have also discussed possible applications of the obtained solutions. It has been suggested that this mechanism could occurs in nature as rare phenomenon called the fireball and could be used in carrying out the research concerning controlled fusion. The description of the experiments, carried out for the purpose to generate long-lived spherical plasma structures, has been presented.

New approaches to the technical development of the controlled fusion problem are requested nowadays. The collected experience of plasma physics allows one to operate with very complicated plasma configurations. The problem of the self-consistent plasma confinement in power-generating plants can probably be imposed. In this situation a fireball
is of particular interest because of its high energy and long lifetime. A great number of the observations concerning a fireball appearance was collected. Plenty of the fireball models has been constructed. The advances in the studying of plasma interactions with rf radiation and matter point out that the stable plasma structures, supplied by the energy of nuclear fusion reactions, can, in principle, be implemented.

Taking into account limited amount of the exhaustible resources of natural energy carriers, the mankind will inevitably face energy catastrophe in 21st century. The consequence will be the complete destruction of the modern industrial civilization. The alternative energy sources, such as geothermal, tidal, wind-power, solar and water-power stations, will be unable to prevent this catastrophe. The only possibility for the mankind is to acquire the energy released in the controlled fusion, using for this purpose almost inexhaustible deuterium resource in the oceanic water. However, the numerous attempts to implement the stable controlled fusion, using the technique of electrical heating of plasma with its subsequent squeezing by the external magnetic field, are likely to fail. Now it is possible to claim that this technique is a dead-end for the outlined problem. In our opinion, the self-consistent plasma structure - a fireball, is one of the possible ways to acquire almost inexhaustible energy resource.

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