Research on system modeling and motion control simulation of automatic power trowel

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Abstract. Aiming at the problems that the existing control researches on the power trowel are limited to the analysis of the motion principle and the open-loop control of some mechanisms, taking a hydraulically-driven ride-on power trowel as the research object, the closed-loop control method of the point-to-point motion of the power trowel is studied. After analyzing the motion principle of the power trowel, based on the assumption of elastic deformation of concrete, the dynamic model of a single trowel is established, and the relationship between the driving force, driving moment and hydraulic moment, velocity, and angular velocity of the trowel is obtained. The whole machine motion equation of the power trowel is deduced, the point-to-point state feedback control algorithm of the power trowel is studied, and a simulation model is built to verify the accuracy of the system model of the power trowel and the effectiveness of the control algorithm. This research can provide reference for the control method design of other complex motions of the power trowel.

1. Introduction
The power trowel is a kind of construction machinery used for slurry lifting, smoothing and troweling in cement concrete surface construction [1]. It has been widely used in the construction of concrete surfaces such as high-standard workshops, warehouses, parking lots, squares, airports and frame buildings [2]. According to different structures, power trowels are mainly divided into two types: ride-on type and walk-behind type [2], both of which are constructed by manual operation, which has the problems of high labor intensity and low work efficiency. There is a big gap between domestic research on construction machinery automation and foreign countries. In recent years, with the continuous development of sensor technology, computer technology, artificial intelligence and other technologies, as well as the increase in labor costs, more and more construction units hope that the domestic automated power trowel can be quickly listed [3], so the research on the power trowel autonomous driving technology has urgent needs.

There have been researches on motion control of the power trowel at home and abroad. Dong Hun Shin et al. [4] studied the driving principle of the ride-on power trowel and deduced the relationship between the driving force and the control variable of the power trowel; Huang Zhihui et al. [2] considered the coupling effect between the motion velocity and friction of the power trowel, established the translational and rotational motion equations of the mechanically-driven ride-on power trowel and obtained the corresponding analytical solutions; Dong Hun Shin et al. [5] proposed an open-loop control method to control the velocity of the power trowel by controlling the inclination angle of a pair of trowels based on the derived relationship between the saturation velocity of the power trowel and the inclination angle of the trowel; Huang Zhihui et al. [3] modeled and simulated the hydraulic system of the power
trowel based on AMEsim, which provided a reference for the selection of system optimization design parameters and prototype experiments.

The existing control researches on the power trowel are limited to the analysis of the motion principle and the open-loop control of some mechanisms, and there is a lack of research on the closed-loop control of the entire power trowel system. This paper takes the hydraulically-driven ride-on power trowel as the research object, establishes its whole machine dynamic model and hydraulic circuit model, designs point-to-point state feedback control algorithm to control its point-to-point motion, and verifies the accuracy of the system model and the effectiveness of the control algorithm through simulation.

2. Motion principle of the power trowel

![Figure 1. Schematic diagram of the drive system of the power trowel](image)

The schematic diagram of the drive system of the hydraulically-driven ride-on power trowel is shown in figure 1. There are two trowels on the left and right, and each trowel is composed of four rectangular blades. In the construction process of the power trowel, the left and right trowels rotate at the same speed but turn opposite (the left trowel rotates clockwise and the right trowel rotates counterclockwise). The driving mode of the power trowel is friction drive, and its motion velocity is controlled by three hydro-cylinders. By controlling the hydraulic pressure of the left hydro-cylinder and the right-1 hydro-cylinder, the moment $M_{y1}$ and $M_{y2}$ along the $y_1$ and $y_2$ axes can be applied to the left and right trowel respectively, and the moment $M_{x2}$ along the $x_2$ axis can be applied to the right trowel by controlling the hydraulic pressure of the right-2 hydro-cylinder. Then the distribution of sliding friction force on the two trowels from the concrete ground is changed to realize the translation and rotation of the power trowel.

3. System modeling of the power trowel

3.1. Inclination angle model of the trowel

![Figure 2. Force diagram of the left trowel](image)

Assuming that the trowel is a rigid body, the relationship between the inclination angle of the trowel and the hydraulic moment is analyzed. The force analysis of the left trowel is shown in figure 2. Where $x_1O_1y_1$ is the coordinate system of the left trowel, $\xi_1O_1\eta_1$ is the coordinate system of the combination...
blades, $M_{x1}$ and $M_{y1}$ are the hydraulic moment exerted by the $x_1$ and $y_1$ axes of the hydro-cylinders on the left trowel, $M_{\xi1}$ and $M_{\eta1}$ are the hydraulic moment after coordinate transformation, $R_1$ and $R_2$ are the shortest distance from the center of the trowel to the inside and outside of the blade, $a_m$ and $b_m$ are the length and width of the blade, and $\alpha$ is the angle between $\xi_1$ and $x_1$. The moment transformation relationship between the two coordinate systems is shown in equation (1).

$$
\begin{bmatrix}
M_{\xi1} \\
M_{\eta1}
\end{bmatrix} =
\begin{bmatrix}
\cos \alpha & \sin \alpha \\
-\sin \alpha & \cos \alpha
\end{bmatrix}
\begin{bmatrix}
M_{x1} \\
M_{y1}
\end{bmatrix}
$$

(1)

As the inclination angle of the trowel is very small in actual construction, the change of the trowel area caused by the angle is not considered. Assuming that the concrete ground is elastic deformation [2], the concrete deformation at the center of the two trowels is denoted as $\Delta_0$, then the concrete extrusion stress $\sigma$ at any point on the trowel can be calculated as follows:

$$
\sigma = k(\Delta_0 + d\theta)
$$

(2)

where $k$ is a constant, $d$ is the distance from the point to the center of the trowel, and $\theta$ is the inclination angle of the trowel.

By calculating the moment of the center $O_1$ of the trowel, equation (3) can be obtained from the force balance.

$$
M_{\xi1} = \frac{1}{6} \theta_{\xi1} [4kb_m(R_2^3 - R_1^3) + ka_m b_m^3] \quad M_{\eta1} = \frac{1}{6} \theta_{\eta1} [4kb_m(R_2^3 - R_1^3) + ka_m b_m^3]
$$

(3)

where $\theta_{\xi1}$ is the inclination angle of the trowel around the $\xi_1$ axis, and $\theta_{\eta1}$ is the inclination angle of the trowel around the $\eta_1$ axis.

3.2. Dynamic model of the trowel

Figure 3. Diagram of left trowel velocity and friction force

The whole weight of the power trowel is carried by the two trowels at the bottom, so the concrete deformation $\Delta_0$ at the center of the two trowels can be calculated by equation (4).

$$
\Delta_0 = mg(8ka_m b_m)^{-1}
$$

(4)

where $m$ is the mass of the power trowel and $g$ is the gravitational acceleration.

Analyze point $P$ on blade 1. According to equation (2), the concrete extrusion stress $\sigma_{p1}$ at this point is:

$$
\sigma_{p1} = k[\Delta_0 - \eta_1(\theta_{\xi1} - \gamma) + \xi_1 \theta_{\eta1}]
$$

(5)

where $\gamma$ is the inclination angle of the blade.
Similarly, the concrete stress $\sigma_{P2}$, $\sigma_{P3}$ and $\sigma_{P4}$ at any point on blades 2, 3 and 4 can be calculated by equation (6).

\[
\begin{align*}
\sigma_{P2} &= k[\Delta_0 - \eta_1 \theta_{\xi_1} + \xi_1(\theta_{\eta_1} - \gamma)] \\
\sigma_{P3} &= k[\Delta_0 - \eta_1 (\theta_{\xi_1} + \gamma) + \xi_1 \theta_{\eta_1}] \\
\sigma_{P4} &= k[\Delta_0 - \eta_1 \theta_{\xi_1} + \xi_1(\theta_{\eta_1} + \gamma)]
\end{align*}
\]

(6)

The absolute velocity of point $P$ is:

\[
v_P = (v_{01\xi_1} + \eta_1 \omega)i + (v_{01\eta_1} - \xi_1 \omega)j
\]

(7)

where $i$ and $j$ are the unit direction vectors of the $\xi_1$ and $\eta_1$ axes respectively, $v_{01\xi_1}$ and $v_{01\eta_1}$ are the components of the absolute velocity of the trowel center $O$ in the $\xi_1$ and $\eta_1$ axis, and $\omega$ is the angular velocity of the trowel.

The unit direction vector $q$ in the opposite direction of the absolute velocity $v_P$ is:

\[
q = \frac{v_P}{|v_P|} \approx \frac{v_P}{|v_0|} = \frac{v_P}{\omega \sqrt{\eta_1^2 + \xi_1^2}} = \frac{- (v_{01\xi_1} + \eta_1 \omega)i + (v_{01\eta_1} - \xi_1 \omega)j}{\omega \sqrt{\eta_1^2 + \xi_1^2}}
\]

(8)

Since the trowel angular velocity $\omega$ is large, $v_0$ is much larger than the trowel center velocity $v_0$ [5], $q$ can be simplified as follows:

\[
q = \frac{v_P}{|v_P|} \approx \frac{v_P}{|v_0|} = \frac{- (v_{01\xi_1} + \eta_1 \omega)i + (v_{01\eta_1} - \xi_1 \omega)j}{\omega \sqrt{\eta_1^2 + \xi_1^2}}
\]

(9)

As the direction of friction force is opposite to the direction of absolute velocity, the friction force $f$ at any point on the trowel is:

\[
f = \mu \sigma_P q
\]

(10)

where $\mu$ is the sliding friction coefficient between the trowel and the concrete ground.

From the equations (5-10), the friction force $F_{\xi_1}$, $F_{\eta_1}$ and the friction moment $M_{O1}$ of the left trowel are:

\[
F_{\xi_1} = 4K_1 \mu k \theta_{\xi_1} - 8K_2 \frac{\mu k \Delta_0}{\omega} v_{01\xi_1},
F_{\eta_1} = 4K_1 \mu k \theta_{\eta_1} - 8K_2 \frac{\mu k \Delta_0}{\omega} v_{01\eta_1}
\]

(11)

\[
M_{O1} = \int_{0}^{b_m} \int_{R_1}^{R_2} r_{O1P} \cdot f dS = 8K_1 \mu k \Delta_0 - 4K_1 \frac{\mu k}{\omega} \theta_{\xi_1} v_{01\xi_1} - 4K_1 \frac{\mu k}{\omega} \theta_{\eta_1} v_{01\eta_1}
\]

(12)

where:

\[
K_1 = \int_{0}^{b_m} d\eta_1 \int_{R_1}^{R_2} \sqrt{\eta_1^2 + \xi_1^2} d\xi_1 ,
K_2 = \int_{0}^{b_m} d\eta_1 \int_{R_1}^{R_2} \frac{1}{\sqrt{\eta_1^2 + \xi_1^2}} d\xi_1
\]

(13)

The above simplify results of $F_{\xi_1}$, $F_{\eta_1}$ and $M_{O1}$ show that the inclination angle $\gamma$ of the blade is eliminated, so it has no influence on the dynamics process of the power trowel. Substituting equation (3) into equations (11) and (12), replacing the inclination angles of the trowel $\theta_{\xi_1}$, $\theta_{\eta_1}$ with the moments $M_{\xi_1}$, $M_{\eta_1}$ around the $\xi_1$ and $\eta_1$ axes, and eliminating the constant $k$, the results can be further simplified:

\[
F_{\xi_1} = \mu \left(K_3 M_{\xi_1} - K_4 v_{01\xi_1}\right),
F_{\eta_1} = \mu \left(K_3 M_{\eta_1} - K_4 v_{01\eta_1}\right)
\]

(14)
where:
\[
K_3 = K_1 \frac{24}{4b_m(R_2^3 - R_1^3) + a_m b_m}, \quad K_4 = K_2 \frac{mg}{\omega a_m b_m}, \quad K_5 = K_1 \frac{mg}{a_m b_m}
\]  

Project the above results to the \(x\) and \(y\) axes, and from the coordinate conversion relationship, it can be obtained:
\[
\begin{bmatrix}
F_{x1} \\
F_{y1}
\end{bmatrix} = \mu \begin{bmatrix}
K_3 M_x - K_4 v_{0x1} \\
K_3 M_y - K_4 v_{0y1}
\end{bmatrix}, \quad M_{O1} = \mu \left( K_5 - K_3 M_x \frac{v_{0x1}}{\omega} - K_3 M_y \frac{v_{0y1}}{\omega} \right)
\]  

Based on the derivation process of the left trowel, the friction force \(F_{x2}, F_{y2}\) and the friction moment \(M_{O2}\) of the right trowel are calculated as follows:
\[
\begin{bmatrix}
F_{x2} \\
F_{y2}
\end{bmatrix} = \mu \begin{bmatrix}
-K_3 M_x - K_4 v_{0x2} \\
-K_3 M_y - K_4 v_{0y2}
\end{bmatrix}, \quad M_{O2} = \mu \left( -K_5 - K_3 M_x \frac{v_{0x2}}{\omega} - K_3 M_y \frac{v_{0y2}}{\omega} \right)
\]  

3.3. Whole machine dynamic model

Figure 4 shows the schematic diagram of whole machine motion and trowel motion, where \(x_1 O_1 y_1\) is the local coordinate system of the left trowel, \(x_2 O_2 y_2\) is the local coordinate system of the right trowel, \(x_0 O y_0\) is the local coordinate system of the power trowel, and \(x O y\) is the global coordinate system of the power trowel. Set the position of the power trowel as \((x, y, \phi)\), then the velocity of its center \(O\) is \((\dot{x}, \dot{y})\), and the angular velocity of the body around point \(O\) is \(\dot{\phi}\) (counterclockwise is positive). Equation (19) can be derived from the kinematic relationship.
\[
v_{01x1} = v_{0x0}, \quad v_{01y1} = v_{0y0} - \phi L, \quad v_{02x2} = v_{0x0}, \quad v_{02y2} = v_{0y0} + \phi L
\]  

where \(v_{0x0}, v_{0y0}\) are the projections of the velocity of the center \(O\) point of the power trowel on \(x_0\) and \(y_0\), and \(L\) is the distance between the center of the trowel and the center of the power trowel.

Since the rotation direction of the left trowel around the \(x_1\) axis is consolidated with the frame, there is no degree of freedom in this direction, so the following relationship can be obtained.
\[
M_{x1} = -M_{x2}
\]  

In the local coordinate system \(x_0 O y_0\) of the power trowel, the external force \(F_{x0}, F_{y0}\) and the external moment \(M_{O}\) of the whole machine are:
\[
F_{x0} = F_{x1} + F_{x2} = -2\mu K_3 M_x - 2\mu K_4 v_{0x0}
\]
\[
F_{y0} = F_{y1} + F_{y2} = \mu K_3 (M_y - M_{y2}) - 2\mu K_4 v_{0y0}
\]
\[ M_D = (F_{y2} - F_{y1})L + M_{D1} + M_{D2} = -\mu K_3 (M_{y1} + M_{y2})L - 2\mu K_4 L^2 \dot{\phi} \\
-\mu K_3 \omega^{-1} [v_{Oy0} - \dot{\phi}L] + M_{y2} (v_{Oy0} + \phi L) \tag{23} \]

In the global coordinate system \( xOy \), according to Newton's Second Law, the motion equation of the power trowel is established as follows:

\[ m \ddot{x} = -2\mu K_4 \dot{x} - 2\mu K_3 M_{x2} \cos \phi - \mu (K_3 M_{y1} - K_3 M_{y2}) \sin \phi \tag{24} \]

\[ m \ddot{y} = -2\mu K_4 \dot{y} - 2\mu K_3 M_{x2} \sin \phi + \mu (K_3 M_{y1} - K_3 M_{y2}) \cos \phi \tag{25} \]

\[ J \ddot{\phi} = -2\mu K_4 L^2 \dot{\phi} - \mu K_3 M_{y1} \omega^{-1} [-\dot{x} \sin \phi + \dot{y} \cos \phi + (\omega - \dot{\phi})L] \\
-\mu K_3 M_{y2} \omega^{-1} [-\dot{x} \sin \phi + \dot{y} \cos \phi + (\omega + \dot{\phi})L] \tag{26} \]

Set the hydraulic pressure corresponding to the three moments of \( M_{x2} \), \( M_{y1} \), and \( M_{y2} \) to be \( F_1 \), \( F_2 \), \( F_3 \), and the force arms are respectively \( l_1(l_{02}g_2) \), \( l_2(l_{01}A_1) \), \( l_3(l_{02}A_2) \), where \( l_2 = l_3 \) (pressing down is positive), then equation (27) can be obtained from figure 1.

\[ M_{x2} = F_1 l_1, M_{y1} = -F_2 l_2, M_{y2} = F_3 l_2 \tag{27} \]

Substituting equation (27) into equations (24-26), the motion equation of the power trowel can be simplified as follows:

\[ m \ddot{x} = -2\mu K_4 \dot{x} - 2\mu K_3 l_1 F_1 \cos \phi + \mu K_3 l_2 (F_2 + F_3) \sin \phi \tag{28} \]

\[ m \ddot{y} = -2\mu K_4 \dot{y} - 2\mu K_3 l_1 F_1 \sin \phi - \mu K_3 l_2 (F_2 + F_3) \cos \phi \tag{29} \]

\[ J \ddot{\phi} = -2\mu K_4 L^2 \dot{\phi} + \mu K_3 l_2 F_2 \omega^{-1} [-\dot{x} \sin \phi + \dot{y} \cos \phi + (\omega - \dot{\phi})L] \\
-\mu K_3 M_{y2} \omega^{-1} [-\dot{x} \sin \phi + \dot{y} \cos \phi + (\omega + \dot{\phi})L] \tag{30} \]

Taking \( x, y, \phi, \dot{x}, \dot{y}, \dot{\phi} \) as state variables and \( F_1, F_2, F_3 \) as input variables:

\[ x = [x \ y \ \phi \ \dot{x} \ \dot{y} \ \dot{\phi}]^T, u = [F_1 \ F_2 \ F_3]^T \tag{31} \]

Written in the form of a state space equation [6]:

\[ \dot{x} = Ax + Bu \tag{32} \]

where:

\[ A = \begin{bmatrix} O_{3 \times 3} & E_3 \\ O_{3 \times 3} & a \end{bmatrix}, B = \begin{bmatrix} O_{3 \times 3} \\ b \end{bmatrix}, a = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & m L^2 \end{bmatrix}, \]

\[ b = \begin{bmatrix} \mu K_3 m \\ -2l_1 \cos \phi \\ -2l_1 \sin \phi \\ l_2 \sin \phi \\ -l_2 \cos \phi \\ 0 \end{bmatrix}, \begin{bmatrix} l_2 \sin \phi \\ -l_2 \cos \phi \\ 0 \end{bmatrix}, \begin{bmatrix} l_2 \sin \phi \\ -l_2 \cos \phi \\ 0 \end{bmatrix} \tag{33} \]

3.4. Hydraulic circuit model

Figure 5 shows the principle diagram of the hydraulic circuit of the power trowel. The hydraulic pressure of the three hydro-cylinders are controlled by six electro-hydraulic proportional pressure reducing valves. Assuming that the acting area of the hydro-cylinder with the piston rod end is \( S_1 \), and the acting area of the non-piston rod end is \( S_2 \), the equation (34) can be obtained from figure 5.

\[ \begin{aligned} F_1 &= p_4 S_2 - p_5 S_1 \\
F_2 &= p_2 S_2 - p_5 S_1 \\
F_3 &= p_6 S_2 - p_5 S_1 \end{aligned} \tag{34} \]
where $p_1$, $p_2$, $p_3$, $p_4$, $p_5$, and $p_6$ are the hydraulic pressures of the output ports of each electro-hydraulic proportional valve.

\[ \begin{align*}
\text{Left hydro-cylinder} & \quad \text{Right-2 hydro-cylinder} & \quad \text{Right-1 hydro-cylinder} \\
\multicolumn{3}{c}{p_1 \quad p_2} \\
\multicolumn{3}{c}{p_3 \quad p_4} \\
\multicolumn{3}{c}{p_5 \quad p_6}
\end{align*} \]

Figure 5. Principle diagram of hydraulic circuit

4. Controller design

4.1. Design objectives of the controller

The control algorithm of the power trowel is to quickly calculate the current that needs to be input to the six electro-hydraulic proportional valves based on the current state value and the expected value of the path planning. Realizing the point-to-point motion control of the power trowel is the basis for realizing any motion path control of it. The design objectives of the point-to-point motion controller are as follows:

- When the power trowel is working, the translational velocity is limited to $0.6 \text{m/s}$, the attitude angular velocity is limited to $0.2 \text{rad/s}$, and short-term overshoot is allowed.
- The power trowel should run as close to the maximum velocity as possible to reach the target position.
- When reaching the target position, it needs to decelerate to less than $0.1 \text{m/s}$ to stop the machine.

The state space model equations (31) and (32) have been obtained through the dynamic modeling of the whole machine, and there are coupling terms in matrix $b$:

\[
\begin{bmatrix}
-\ddot{x}\sin\phi + \dot{y}\cos\phi + (\omega - \dot{\phi})L \\
0
\end{bmatrix}
\begin{bmatrix}
-2l_1 \cos\phi \\
-2l_1 \sin\phi
\end{bmatrix}
\begin{bmatrix}
l_2 \sin\phi \\
l_2 \cos\phi
\end{bmatrix}
\begin{bmatrix}
l_2 \sin\phi \\
-l_2 \cos\phi
\end{bmatrix}
\begin{bmatrix}
l_2 \sin\phi \\
l_2 \cos\phi
\end{bmatrix}
\begin{bmatrix}
0 \\
ml_2 \frac{L}{J}
\end{bmatrix}
\]

In order to simplify the control algorithm, the coupling terms need to be simplified reasonably. When the power trowel is working, $\dot{x}, \dot{y}, \dot{\phi}$ is small ($\dot{x} \leq 0.3 \text{m/s}, \dot{y} \leq 0.3 \text{m/s}, \dot{\phi} \leq 0.2 \text{rad/s}$). Consult its design materials and work manuals, the distance between the center of the trowel and the center of the power trowel is $L = 0.5025 \text{m}$, and the rotation speed of the trowel is $\omega = 5\pi \text{rad/s}$. Analyze the coupling terms:

\[
\frac{-\ddot{x}\sin\phi + \dot{y}\cos\phi + (\omega - \dot{\phi})L}{\omega} \leq \left| \frac{-\ddot{x}\sin\phi + \dot{y}\cos\phi}{L} \right| + \left| \dot{\phi} \right| \leq \frac{\sqrt{\dot{x}^2 + \dot{y}^2}}{L} + \left| \dot{\phi} \right| < 0.067
\]

It can be seen that the coupling terms is quite small compared to the rotation speed $\omega$ of the trowel, so the matrix $b$ can be simplified as:

\[
b = \frac{\mu K_3}{m} \begin{bmatrix}
-2l_1 \cos\phi & l_2 \sin\phi & l_2 \sin\phi \\
-2l_1 \sin\phi & -l_2 \cos\phi & -l_2 \cos\phi \\
0 & ml_2 \frac{L}{J} & -ml_2 \frac{L}{J}
\end{bmatrix}
\]
4.2. Point-to-point state feedback control algorithm

Assume that the state space equation of the controlled system is [6]:

$$\begin{align*}
\dot{x} &= Ax + Bu \\
y &= Cx
\end{align*} \quad (38)$$

where $x$ is an $n$-dimensional state variable, $u$ is an $r$-dimensional input variable, $y$ is an $m$-dimensional output variable, $A$ is an $n \times n$-dimensional system matrix, $B$ is an $n \times r$-dimensional input matrix, and $C$ is an $m \times n$-dimensional output matrix.

The state feedback is used to control the above-mentioned controlled system. The state feedback closed-loop system is shown in figure 6 [7], and the state feedback control law is:

$$u = v - Rx \quad (39)$$

where $v$ is the $r$-dimensional reference input variable, and $R$ is the $r \times n$-dimensional state feedback matrix.

The purpose of adjusting the pole of the system can be achieved by adjusting the value of the state feedback matrix $R$. One method of tuning $R$ is the linear quadratic regulator algorithm (LQR). The goal of the LQR algorithm is to minimize the energy function $J$ in equation (40) [8-10]:

$$J = \frac{1}{2} \int_0^\infty x^T(t)Q_{xx}x(t)dt + \frac{1}{2} \int_0^\infty u^T(t)Q_{uu}u(t)dt \quad (40)$$

where $Q_{xx}$ is an $n \times n$-dimensional state variable weight matrix, and $Q_{uu}$ is an $r \times r$-dimensional input variable weight matrix.

When the energy function $J$ takes the minimum value, the state feedback matrix $R$ needs to satisfy equation (41), where $P$ is the positive definite solution of equation (42) [11].

$$R = -Q_{uu}^{-1}B^TP \quad (41)$$

$$PBQ_{uu}^{-1}B^TP - PA - A^TP - Q_{xx} = 0 \quad (42)$$

The above is the most general state feedback control method, and the reference input variable $v$ needs to be calculated separately. Therefore, the control block diagram with practical significance is shown in figure 7, where $w$ is the expected value of the $m$-dimensional output variable $y$, $u^*$ is the expected value of the $r$-dimensional input variable $u$, $x^*$ is the expected value of the $n$-dimensional state variable, $M_x$ is the $n \times r$-dimensional matrix, $M_u$ is the $r \times r$-dimensional matrix, and $M_x$ and $M_u$ are calculated by equation (43).
\[ \begin{bmatrix} M_x \\ M_u \end{bmatrix} = \begin{bmatrix} A & B \\ C & O \end{bmatrix}^{-1} \begin{bmatrix} O \\ E_r \end{bmatrix} \]  

The control law of the closed-loop system in figure 7 is:

\[ u = M_u w + R(M_x w - x) = -Rx + (M_u + RM_x)w \]  

In order to avoid the overload of the controller and the large difference between the current state variable and the expected state variable caused by the large amplitude of the step input, as shown in figure 8, Saturation Module-1 and Saturation Module-2 need to be added. In figure 8, the Precompute Function is to calculate \( x^* \) and \( u^* \) using the expected value \( w \), the Saturation Module-1 limits the control signal \( u \) to the actuator, and the Saturation Module-2 limits the difference between the current state variable \( x \) and the expected state variable \( x^* \).

The control objective of the point-to-point control algorithm is the position and attitude of the power trowel when it is finally shutdown. Therefore, taking the output variable \( y \) as \( [x \ y \ \phi]^T \), the output matrix \( C \) is shown in equation (45). \( M_u \) is \( 3 \times 3 \)-dimensional matrix, \( M_x \) is \( 6 \times 3 \)-dimensional matrix, and \( R \) is \( 3 \times 6 \)-dimensional feedback matrix.

\[ C = [E_3 \ O_{3 \times 3}] \]  

5. Simulation and result analysis

The simulation model of automatic power trowel is shown in figure 9, which consists of five parts: System Model, Controller, Position Feedback Model, Expected Position Module and Simulation Termination Module. The actual positioning system of the power trowel can measure its position value \( (x, y, \phi) \), which is discrete with time delay and noise. However, the position value obtained by the simulation system model is ideal. Therefore, a Position Feedback Model is designed to simulate the actual positioning system of the power trowel, and the actual position value \( \hat{x}, \hat{y}, \hat{\phi} \) of the power trowel are fed back to the Controller. After the test and analysis of the actual positioning system, the time delay and adjacent discrete time interval is 25ms, and the noise is white noise with mean value of 0 and standard deviation of 0.05. The Expected Position Module is responsible for providing the expected final position to the Controller. The function of the Simulation Termination Module is to simulate the stop of the power trowel when it reaches the target position and attitude. Since the six values of position and velocity \( (x, y, \phi, \dot{x}, \dot{y}, \dot{\phi}) \) of the power trowel in practical engineering are almost impossible to reach the expected value at the same time, the termination condition equation (46) is set, and the simulation is completed when the termination condition is satisfied.

\[ \begin{cases} |x| \leq 0.05m, |y| \leq 0.05m, |\phi| \leq 0.05rad \\
|\dot{x}| \leq 0.01m/s, |\dot{y}| \leq 0.01m/s, |\dot{\phi}| \leq 0.5rad/s \end{cases} \]  

Figure 9. Structure of simulation model
Table 1. The parameters of the power trowel and concrete

| Parameter | Value |
|-----------|-------|
| \( m \) (kg) | 360 |
| \( f \) (kg \( \cdot \) m\(^2\)) | 80 |
| \( a_m \) (m) | 0.35 |
| \( b_m \) (m) | 0.185 |
| \( R_1 \) (m) | 0.0973 |
| \( R_2 \) (m) | 0.4473 |
| \( \omega \) (rad/s) | 5\( \pi \) |
| \( L \) (m) | 0.5025 |
| \( S_1 \) (m\(^2\)) | 9.4\times10^{-4} |
| \( S_2 \) (m\(^2\)) | 1.3\times10^{-5} |
| \( L_1, L_2, L_3 \) (m) | 0.285, 0.26, 0.26 |
| \( k \) (N \( \cdot \) m\(^{-1}\)) | 365559 |
| \( \mu \) | 0.35 |

In order to verify the effectiveness of the point-to-point state feedback control algorithm, the starting position of the power trowel is set as \((0,0,0)\), the expected termination position is set as \((4m, 6m, \pi/4)\), and the actual operation of the power trowel is simulated. The parameter configuration of the power trowel and concrete is shown in table 1. The simulation results are shown in figure 10.

From figure 10, it can be seen that there is a large acceleration at the beginning of the simulation, which makes each velocity increase rapidly to the set maximum value, and then each position value increases at nearly uniform speed, reaching the expected value at 14s; When the position value reaches near their respective expected value, the corresponding velocity will gradually decrease to zero; Because the position feedback contains noise, the velocity fluctuates to a certain extent, but within an acceptable range; The velocity \( \dot{x} \) overshoot is large, but due to increasing the weight in the corresponding LQR algorithm in case of overspeed, the velocity quickly decreases to the allowable range. The simulation results are in line with expectations, the power trowel reaches the target expected value stably, and individual velocities have overshoots but the duration is short, which meets the design objectives of the controller.

6. Conclusions

Through the research on the composition and working principle of the drive system of the hydraulically-driven ride-on power trowel, it is concluded that the driving mode of the power trowel is friction drive. Its motion principle is that by controlling the hydraulic pressure of the three hydro-cylinders, the friction distribution from the concrete ground of the left and right trowels is changed to realize the motion control of the power trowel.

Based on the assumption of elastic deformation of concrete, the dynamic model of the left trowel is established, the relationship between the driving force, driving moment and hydraulic moment, velocity and angular velocity of single trowel is obtained, and the whole machine motion equation of the power trowel is deduced.

In view of the most basic point-to-point motion of the power trowel, a point-to-point state feedback control algorithm is designed with the expected position of the power trowel obtained by path planning as the input. The data characteristics collected by the actual positioning system of the power trowel are analyzed and the system simulation model is built accordingly.

The simulation results are in line with the design objectives of the controller, and the power trowel reaches the expected termination position stably. In the subsequent research, the control algorithm needs
to be verified by experiments, and the experimental results need to be compared with the simulation results.

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