SOLAR NEUTRINO DATA, SOLAR MODEL UNCERTAINTIES AND NEUTRINO OSCILLATIONS

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Abstract
We incorporate all existing solar neutrino flux measurements and take solar model flux uncertainties into account in deriving global fits to parameter space for the MSW and vacuum solutions of the solar neutrino problem.
The solar neutrino problem continues unabated after over 20 years and four experiments. In spite of the relatively high flux recently reported by the GALLEX [1] collaboration, a neutrino based solution of the solar neutrino problem still remains strongly favored [2], at least as long as the Homestake Cl [3] result remains unchanged. (Note that the 20 year Homestake weighted average used here and in most analyses is lower than an unweighted average, or an average over only the last 3 year’s data. If a higher Homestake value is used, this can affect the comparative range of various neutrino-based solutions.) In this paper, we investigate what range of neutrino masses and mixing angles (for 2 species mixing) remains consistent with a global fit of all the data, including the updated 2 year SAGE average of $58^{+17}_{-24} \pm 14$ SNU [4]. We stress the following: (1) The MSW range is not the only range of masses and mixing angles which is consistent with the observations. Indeed, while the original MSW parameter space allowed by the Homestake data was far larger than for vacuum oscillations, this difference is now much less significant, at least for the canonical small angle MSW region; (2) There is no justification for incorporating the new GALLEX result and not the SAGE result in model fits; (3) Solar model uncertainties noticeably increase the allowed range of neutrino parameter space which is consistent with the existing observations.

Neutrino based solutions of the solar neutrino problem fall into 2 major classes, related to suppression of the neutrino signal at the earth due to either oscillations between neutrino flavors or helicity states. Both involve at least one non-zero neutrino mass. The latter type involves potentially non-trivial time dependence of the solar neutrino signal over the solar cycle, but requires extremely large neutrino magnetic moments to remain viable, and has been recently investigated elsewhere [5]. The former falls into two subcategories: resonant neutrino oscillations inside the sun due to a changing electron density in the solar core (MSW oscillations [6]), and oscillations in vacuo between the earth and sun (vacuum oscillations). The latter of these
involves masses which are 2-3 orders of magnitude smaller, and consequently vacuum oscillation lengths which are up to 6 orders of magnitude larger than those appropriate for MSW matter oscillations.

Both MSW and vacuum oscillations result in an energy dependent suppression of the neutrino signal. The latter has a clear oscillation in energy [7]. In principle, this could be useful to distinguish between these possibilities once high statistics measurements of the solar neutrino spectrum are available. We find that the Kamiokande [8, 9] spectral measurements do not presently constrain regions of MSW parameter space beyond those which can be ruled out out by global fits to overall flux measurements. Vacuum oscillations also allow a possible seasonal variation in the observed signal, as the earth-sun distance changes, and different oscillation lengths are sampled. Time sequenced data is available for each of the experiments. However, there is a great deal of scatter in the data, and it is not clear at this point to what extent any systematic time variation can be extracted from the signal ( except that a systematic day-night variation which would occur for certain of the MSW parameter space is not observed [10]). Thus, in this paper, we merely compare the predicted time-averaged total signals in the Homestake, Kamiokande, SAGE and GALLEX experiments with the observed signals in order to derive, by statistical methods, the region of parameter space which remains viable, after solar model uncertainties have been taken into account. Additional tests based on spectral information, or time dependence can serve to further reduce this region, but probably only marginally so at present. Several recent analyses have examined these latter issues in more detail, especially for vacuum oscillations [11, 12].

The strategy for calculating the neutrino survival probability at the earth is different in the case of vacuum oscillations and MSW oscillations. In both cases, we consider here only possible oscillations to an active neutrino, as this remains at present the most likely possibility now that evidence for a 17 keV neutrino has diminished [13]. For the case of vacuum oscillations [4] the
details of the SSM production regions and \( N_e \) density profile are unimportant, the \( \nu_e \) survival probability \( P_{\text{vac}} \) at a distance \( L \) from the center of the Sun is simply [7, 14]

\[
P_{\text{vac}}(\nu_e \rightarrow \nu_e) = 1 - \sin^2 2\theta \sin^2 \left( \frac{\Delta m^2 L}{4E} \right)
\]  

(1)

We averaged \( P_{\text{vac}} \) over a varying Earth-Sun distance to take into account (in a simple minded way) the motion of the Earth around the Sun during each measurement. The average was taken (linearly) from \( L(1 - \epsilon) \) to \( L(1 + \epsilon) \) with \( \epsilon \) equal to the Earth’s orbital eccentricity [3].

For the MSW model [3, 14] there exist good analytic approximations to the \( \nu_e \) survival probability under the assumption that the electron number density \( (N_e) \) decreases exponentially \( N_e \sim \exp(-r/r_0) \) near the resonance. We used the approximations and best fit values of \( r_0 \) given in [15] who break the \( (\Delta m^2, \sin^2 2\theta) \) into several regions, giving approximations good to a few percent in each region. A natural distinction is between adiabatic and non-adiabatic transitions. If we define:

\[
4n_0 = r_0 \left( \frac{\Delta m^2}{2E} \right) \left( \frac{\sin^2 2\theta}{\cos 2\theta} \right)
\]  

(2)

then the non-adiabatic region is given by the condition \( 4n_0 \leq 1 \) while in the adiabatic region \( 4n_0 \gg 1 \) (a reasonable choice for the transition value is \( 4n_0 = 4 \) [11]). We then subdivide the non-adiabatic region into three parts, depending on the size of \( N_{e}^{\text{res}} = \Delta m^2 \cos 2\theta/2\sqrt{2G_F E} \) compared to \( N_e^{(1)} = N_e^{(0)} (1 + \tan 2\theta)^{-1} \) and \( N_e^{(2)} = N_e^{(0)} (1 - \tan 2\theta)^{-1} \), with \( N_e^{(0)} \) the electron density where the \( \nu \) is produced. Writing

\[
x = 2\pi \frac{r_0 \Delta m^2}{2E} \quad \text{(3)}
\]

\[
y = 2\pi n_0 (1 - \tan^2 \theta) \quad \text{(4)}
\]

\[
\cos 2\theta_m = (1 - \eta)/\sqrt{(1 - \eta)^2 + \tan^2 2\theta} \quad \eta \equiv N_e^{(0)}/N_e^{\text{res}} \quad \text{(5)}
\]
the analytic expressions for the $\nu_e$ survival probability we used were

\[ N_{\text{res}} (1) < N_e < N_{\text{res}} (2) : \]

\[ \bar{P}(\nu_e \rightarrow \nu_e) = \frac{1}{2} + \left( \frac{1 + e^{-x}}{1 - e^{-x}} \right) \left[ \frac{1}{2} - \frac{e^{-y}}{1 + e^{-x}} \right] \cos 2\theta_m \cos 2\theta, \] (6)

\[ N_{\text{res}} (1) \leq N_{\text{res}} \leq N_{\text{res}} (2) : \]

\[ \bar{P}(\nu_e \rightarrow \nu_e) = \frac{1}{2} \left[ 1 + \exp(-\pi n_0) \right]. \] (7)

\[ N_{\text{res}} > N_{\text{res}} (2) : \]

\[ \bar{P}(\nu_e \rightarrow \nu_e) = \frac{1}{2} + \frac{1}{2} \cos 2\theta_m \cos 2\theta. \] (8)

For neutrinos in the adiabatic region we used equation (6), which matches smoothly onto equation (8) in the limit $4n_0 \gg 1$. In addition we found that for the higher mass gaps including the possibility of double resonances [16, 17] noticeably altered the results. We assumed that for neutrinos produced at a radius $r_{\text{prod}}$ a fraction

\[ \frac{1}{2} \left( 1 - \sqrt{1 - \left( \frac{r_{\text{res}}}{r_{\text{prod}}} \right)^2} \right) \] (9)

of them passed through two resonances. This modifies the survival probability by $P_{\text{jump}} \rightarrow 2P_{\text{jump}}(1 - P_{\text{jump}})$ where $P_{\text{jump}}$ is the coefficient of the $\cos 2\theta_m \cos 2\theta$ term in (6-8).

Finally to compute the survival probability in the SSM we averaged (6-8) over the $\nu$ production regions in the sun [14].

For each model and each $(\Delta m^2, \sin^2 2\theta)$ the survival probabilities, along with the SSM $\nu_e$ spectra [14] were used to compute $\nu_e$ and $\nu_\mu$ spectra at the Earth. These spectra were then convolved with simple detector efficiencies and cross sections [2, 3, 14] to obtain predicted rates. In addition we generated 100,000 “flux variations” using a Monte-Carlo procedure (as outlined in
[2], based on earlier solar model Monte-Carlos [14, 16]) to estimate the SSM errors in the predictions.

The results of this stage then are a set of predicted average event rates in the detectors (Homestake, Kamiokande, SAGE and GALLEX) with errors coming from the SSM uncertainties. These predicted rates \( p(i) \pm \sigma_p(i) \) were then compared with the experimental rates \( e(i) \pm \sigma_e(i) \) by calculating the ratio

\[
\frac{p(i) \pm \sigma_p(i)}{e(i) \pm \sigma_e(i)}
\]

(combining errors in quadrature as usual) and computing \( \chi^2 \) for a fit of the ratios to unity. The averaged rates [1, 4, 3, 8] used for the fits are shown in table 1.

Finally, in order to estimate the effect of solar model uncertainties and our fitting procedure, we compared the statistical fits described above, after incorporating the results of our Monte Carlo over flux uncertainties with those obtained before these uncertainties were incorporated. Note that the fits for the data without standard solar model uncertainties were done using a standard \( \chi^2 \) fitting procedure. Thus the incorporation of solar model uncertainties also changes the fitting procedure. Our results are then displayed in figures 1-3. As expected, the inclusion of the GALLEX result is to reduce the allowed MSW phase space compared to that allowed for SAGE alone. The inclusion of solar model uncertainties on the other hand increases the region allowed at the 90% confidence level for both MSW and vacuum oscillations (this result should be contrasted to that obtained in [18]).

As indicated above, spectral measurements and a search for possible day-night variations can in principle further constrain the allowed regions displayed here. Present spectral constraints from Kamiokande [9] do not yet provide further constraints on the MSW parameter space, but constraints from day-night variations do. We display in the figures those regions of the otherwise allowed MSW parameter space which are ruled out by these con-
To conclude, the progress on the resolution of the solar neutrino problem has been uneven. While the data does favor a neutrino based solution, unfortunately at present the results of the Gallium experiments are sufficiently inconclusive so as to delay a definitive statement in this regard. Nevertheless, when all the existing time-averaged data is incorporated, along with solar model uncertainties, a wide range of possible neutrino masses and mixing angles, involving either matter or vacuum neutrino oscillations, remains viable. We may have to await a combination of high statistics experiments and experiments with enhanced spectral sensitivity, such as SNO [19], super-Kamiokande [20], and perhaps Borexino [21], before a specific solution can be unambiguously determined.

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| Experiment | Rate       |
|------------|------------|
| Homestake  | 0.28 ± 0.04|
| Kamiokande | 0.49 ± 0.08|
| SAGE       | 0.44 ± 0.21|
| GALLEX     | 0.63 ± 0.16|

Table 1: The experimental rates, normalized to the standard solar model predictions, used in the fits.

**Figure Captions**

Figure 1: Region of mass-mixing angle space for neutrino oscillation solutions allowed at the 90% confidence level for the combined Homestake-Kamiokande-SAGE data, (a) without, and (b) including solar model uncertainties. Within this region the portion excluded by the day-night effect in Kamiokande has been removed.

Figure 2: Region of mass-mixing angle space for neutrino oscillation solutions allowed at the 90% confidence level for the combined Homestake-Kamiokande-GALLEX data, (a) without, and (b) including solar model uncertainties. Within this region the portion excluded by the day-night effect in Kamiokande has been removed.

Figure 3: Region of mass-mixing angle space for neutrino oscillation solutions allowed at the 90% confidence level for the combined Homestake-Kamiokande-SAGE-GALLEX data, (a) without, and (b) including solar model uncertainties. Within this region the portion excluded by the day-night effect in Kamiokande has been removed.
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