Interaction with Formal Mathematical Documents in Isabelle/PIDE

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Abstract. Isabelle/PIDE has emerged over more than 10 years as the standard Prover IDE for interactive theorem proving in Isabelle. The well-established Archive of Formal Proofs (AFP) testifies the success of such applications of formalized mathematics in Isabelle/HOL. More recently, the scope of PIDE has widened towards languages that are not connected to logic and proof in Isabelle, but taken from a broader repertoire of mathematics on the computer. The present paper provides a general overview of the PIDE project and its underlying document model, with built-in parallel evaluation and asynchronous interaction. There is also some discussion of original aims and approaches, successes and failures, later changes to the plan, and ideas for the future.

1 Introduction

Isabelle/PIDE means Prover IDE: its implementation relies on Isabelle/Scala, and the standard front-end is Isabelle/jEdit: so all these brand names can be used interchangeably at some level of abstraction. The presentation at Schloss Dagstuhl in October 2009 [18] provides an interesting historical view of the initial concepts of Isabelle/Scala and the preliminary implementation of Isabelle/jEdit. Work on that had already started one year earlier, so the Dagstuhl presentation in August 2018 could use the title “The Isabelle Prover IDE after 10 years of development” [27]. In the years between, there have been many papers about the project, notably [19, 16, 20, 22, 3, 25, 28, 29].

Considerable complexity of Isabelle/PIDE concepts and implementations has accumulated over time, and presenting a comprehensive overview in this paper poses a challenge. Subsequently, we start with two concrete application scenarios: standard Isabelle/jEdit (§1.1) and non-standard Isabelle/Naproche (§1.2). More systematic explanations of the PIDE document-model are given in §2. Discussion of aims and approaches of PIDE follows in §3: this provides a perspective on design decisions from the past, with projections into the future.

1.1 Isabelle/PIDE as IDE for Interactive Proof Documents

Isabelle is an interactive proof assistant (similar to Coq or HOL4), and PIDE is the Prover IDE framework for it. Isabelle/PIDE is implemented in Isabelle/ML (based on Poly/ML) and Isabelle/Scala (on the Java Virtual Machine). This
arrangement allows to use existing IDE front-ends from the Java ecosystem, e.g. the plain text editor jEdit (http://jedit.org). The combined Isabelle/jEdit [30] is presently the most sophisticated application of PIDE, and the default user-interface for Isabelle. There are other PIDE front-ends, e.g. Isabelle/VSCode and a headless server, but non-PIDE interaction has already been discontinued in October 2014. Consequently, the classic Proof General Emacs [1] does not work for Isabelle anymore: it was based on the TTY-loop that no longer exists.

Fig. 1. The all-inclusive Isabelle/PIDE/jEdit application

Users who download¹ and run the application bundle for their respective operating-system first encounter the Isabelle/jEdit desktop application, similar to fig. 1: it provides immediate access to documentation, examples, and libraries. All of these are Formal Mathematical Documents in Isabelle, which are organized as theories and sessions (i.e. collections of theories with optional document output). This includes the Isabelle manuals from the Documentation panel, which shows PDFs generated from sessions in the $ISABELLE_HOME/src/Doc directory.

By opening $ISABELLE_HOME/src/Doc/JEdit/JEdit.thy in Isabelle/jEdit, we see the theory sources of the Isabelle/jEdit manual in the Prover IDE. Its content mainly consists of traditional document structure: section headings and blocks of text. Such quotations of informal text may also contain formal items via antiquotations. The latter concept was introduced to allow prose text to talk

¹ https://isabelle.sketis.net/Isabelle_CICM2019
about logical terms and types, but the same mechanism is re-used to augment \LaTeX\ by formal elements: links to files or URLs, text styles with robust nesting (bold, emphasized, verbatim, footnote), item lists as in Markdown, citation management \wrt\ Bib\LaTeX\ databases etc. (see also [30, Chapter 4]).

Beyond self-application of Isabelle/PIDE/jEdit to its own documentation, the Isabelle distribution provides libraries and applications of formalized mathematics, mostly in Isabelle/HOL (see the directory $\$ISABELLE\_HOME/src/HOL). The material may be edited directly in the Isabelle/jEdit Prover IDE — except for the HOL session itself, which is preloaded as non-editable session image. Such spontaneous checking may require substantial hardware resources, though. E.g. $\$ISABELLE\_HOME/src/HOL/Analysis/Analysis.thy$ works best with 8 CPU cores and 16 GB main memory, and still requires several minutes to complete. Note that this is not just browsing, but \textit{semantic editing} of a live document: a checked state provides full access to the execution environment of the prover.

Development of complex proof documents requires add-on tools: a theory library usually provides new logical content together with tools for specifications and proofs. Isabelle/HOL itself is an example for that, with many \textit{proof methods} to support (semi-)automated reasoning in Isabelle/Isar [17], and \textit{external provers} (ATP, SMT) for use with Sledgehammer [5]. Isabelle/PIDE orchestrates all tools within one a run-time environment of parallel functional programming. Results are exposed to the front-end via a stream of PIDE protocol messages. The editor can retrieve the resulting PIDE document markup in real-time (without waiting for the prover) and use conventional GUI elements to show it to the user: e.g. as text colours, squiggly underlines, icons, tooltips, popups. Output generated by the prover can have extra markup to make it \textit{active}: when the user clicks on it, edits will be applied to the text to continue its development, e.g. see [30, §3.9] for document-oriented interaction with Sledgehammer.

The example sessions of the Isabelle distribution are quite substantial, but most Isabelle/HOL formalizations are now maintained in \textit{AFP}, the Archive of Formal Proofs [7]. AFP is organized like a scientific journal, not a repository of “code”. Thus it is similar to the Mizar Mathematical Library\textsuperscript{2}, but with more flexibility and programmability of \textit{domain-specific formal languages} [28]. That continues the original LCF/ML approach [10, 9] towards active documents with full-scale Prover IDE support.

AFP version 28e97a6e4921 (April 2019) has 315 authors, 471 sessions, 4912 theories. In principle, it is possible to load everything into a single prover session for Isabelle/jEdit. But scaling is not for free, and doing that blindly requires two orders of magnitude more resources than for HOL-Analysis above. In practical development of large AFP entries, users still need some planning and manual arrangement of sessions, to restrict the focus to relevant parts of AFP.

A truly integrated development environment should do that automatically for the user, and treat Isabelle + AFP as one big mathematical document for editing (and browsing). Concrete ideas for further scaling of the PIDE technology are outlined in [26], but it will require some years to get there.

\textsuperscript{2} \url{http://mizar.org/library} and \url{http://mizar.org/fm}
1.2 Isabelle/Naproche for Automatic Proof-Checking of Ordinary Mathematical Texts

Naproche-SAD is a recent tool by Frerix and Köepke [8], based on the original System for Automated Deduction (SAD) by Paskevich and others [11]. It processes the Formal Theory Language (ForTheL), which is designed to look like mathematical text, but it is restricted to a small subset of natural language.

The tool is implemented in Haskell as a plain function from input text to output messages. A file is like a chapter of mathematical text, with a nested tree-structure of elements and sub-elements (for signatures, axiomatizations, statements, proofs). Output messages inform about the translation of mathematical text to problems of first-order logic, and indicate success or failure of external proof checking; the latter is delegated to the E Prover by Stephan Schulz and can take several seconds for each proof obligation.

To integrate Naproche-SAD into PIDE, Frerix and Wenzel have reworked the Haskell program over 2 months in 2018, to turn the command-line tool into a service for reactive checking of ForTheL texts. Isabelle integration was done via the new Isabelle/Haskell library and some glue code in Isabelle/Scala to register ForTheL as auxiliary file-format (extension .ftl).

![Fig. 2. Isabelle/Naproche with “ordinary mathematical text”](https://isabelle.sketis.net/Isabelle_Naproche_CICM2019)

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version of the text, which is sent in total to Naproche-SAD again. The back-end is sufficiently smart to avoid redundant checking of unchanged sub-elements: it keeps a global state with results of old versions: this is easy to implement as the program keeps running until shutdown of Isabelle/PIDE.

The general approach of using an external tool in Isabelle/PIDE like a function on input sources to output messages has been demonstrated before for \texttt{bibtex} databases [28, §5]. This technique depends on the following conditions:

– Input and output works via temporary files or anonymous streams for each invocation — no global state within the file-system.

– Source positions in output uses precise text offsets — not just line numbers: this typically requires careful inspection of text encoding and line-endings.

– The tool starts quickly and runs only briefly. As a rule of thumb for IDE reactivity, 10 ms is very fast, 100 ms is fast enough, 500 ms is getting slow.

– A long-running tool needs to be interruptible via POSIX signals: C/C++ programs often require some corrections in this respect.

– A long-running tool should output messages incrementally on a stream to let the user see approximative PIDE markup early. E.g. syntax markup immediately after parsing, and the status of semantic checking later on.

The small and quick \texttt{bibtex} program can be started afresh for each version of input text, and messages returned as a single batch extracted from the log file. In contrast, the rather heavy \texttt{Naproche-SAD} executable can spend several seconds on small mathematical texts, so the continuously running server process with its internal cache is important for reactivity: it also avoids repeated startup of a big program. To achieve that, the Haskell program had to be changed significantly, but typed functional programming helps to keep an overview of global state and interactions with the world: this makes it easy to isolate the main function for concurrent invocations of multiple server requests.

In summary, we see that Isabelle is not just Isabelle/HOL: Isabelle/Naproche uses logic in its own way, independently of the Isabelle logical framework.

2 The PIDE Document Model

Abstractly, we can understand a PIDE document as large expression of embedded sub-languages that can be explored interactively in the editor, while the prover is processing the execution as a parallel functional program. Continued editing by the user may cancel ongoing executions by the prover, and replace no longer relevant parts of the document by new material to be checked eventually. The communication of the editor with the prover always works via document edits, e.g. to insert or remove segments of text to produce a new document version. Edits also determine the perspective on the text: it tells the prover which parts of the document are important for the user. E.g. scrolling an editor window will update the document and refocus the execution to include uncovered text.

Document content is represented as plain text within editor buffers, but it is also possible to include auxiliary files that could be external blobs in some
user-defined format (usually they are plain text as well). Both input sources and output messages are rendered in enhanced editor buffers to support text decorations and pretty-printing in the style of Oppen [13]: its formatting and line-breaking dynamically adjusts itself to font metrics and window sizes.

PIDE markup (XML) over input sources or within output pretty-trees is used for text colours, tooltips, popups etc. Thus the syntax-highlighting in jEdit is augmented by semantics-rendering in Isabelle/jEdit.

2.1 Document Structure and Organization: Theories and Sessions

PIDE documents follow the traditional structure of mathematical texts, which is a sequence of “definition–statement–proof” given in foundational order.

A definition is an atomic text element that extends the theory context: this could be a genuine definition of a constant in Isabelle/HOL, or the definition of a module in Isabelle/ML, or something else in a domain-specific formal language. A statement is a pending claim within the theory that requires justification by the subsequent proof: it could be a theorem statement in the logic or a derived definitional specification (e.g. a recursive function with termination proof). Definitions and statements can be arbitrary user-defined notions of the Isabelle application, but proofs always use the Isabelle/Isar proof language [17].

Foundational order means that the overall document elements can be seen as a well-founded sequence of elementary constructions: mutual recursion is restricted to a single definitional element, e.g. one that introduces multiple recursive functions. For better scalability, projects are usually built up as an acyclic directed graph of theory nodes: each theory imports existing theories and forms a canonical merge of the contexts before adding further material. Isabelle context management has a built-in notion of monotonicity to propagate results: after import, a well-formed term remains well-formed, a proven theorem remains proven etc. Entities from domain-specific languages need to conform to the same abstract principle of “monotonic reasoning”.

A session consists of a sub-graph of imported theories, together with auxiliary files for document-preparation in \LaTeX. This roughly corresponds to a “project” in conventional IDEs for programming languages. A session image is a “dumped world” of the underlying Isabelle/ML process of a session, to speed-up reloading of its state. Each session image may refer to a single parent, to form an overall tree of session images. Merging session images is not supported, but a session can have multiple session imports to load theories from other sessions. This is not a copy of the other theory, but an inclusion that requires re-checking for the new session (often this is faster than building a complex stack of session images).

In AFP, each entry usually consists of a single session for its formal theory content, together with the \LaTeX setup for presentation as a journal article [7].

In addition to session documents, Isabelle2019 supports session exports: tools in Isabelle/ML may publish arbitrary blobs for the session database (with optional XZ compression). For example, the AFP entry Buchi_Complementation generates a functional program from its specification and exports the compiled
executable. The session export mechanism avoids uncontrolled write-access to the global file-system: since PIDE document processing operates concurrently on multiple versions, writing out physical files would be ill-defined.

In batch mode (e.g. isabelle build), session sources are read from the file-system, and session exports are added to the SQLite database for this session build. Command-line tools like isabelle export or isabelle build -e can retrieve exports later on: here the session database acts like a zip archive.

In interactive mode (e.g. isabelle jedit), session sources are managed by the editor (with backing by the file-system), and session exports are stored in Isabelle/Scala data structures. Isabelle/jEdit provides virtual file-systems with URL prefixes isabelle-session: and isabelle-export: to access this information interactively, e.g. see the jEdit File Browser, menu Favorites, the last two items. An example is session Tools/Haskell, theory Haskell, command export_generated_files at the bottom: it exports generated sources of the Isabelle/Haskell library for re-use in other projects, e.g. Naproche-SAD (§1.2).

2.2 Common Syntax for Embedded Languages: Cartouches

The outer syntax of Isabelle theories is the starting point for user-defined language elements: the header syntax theory A imports B₁ ... Bₙ is hardwired, but everything else is a defined command within the theory body. The Isabelle bootstrap provides the initial ML command: Isabelle/Pure and Isabelle/HOL are using that to define a rich collection of commands that users often understand as the Isabelle theory language, but it is merely a library.

A command definition requires a keywords declaration in the theory header, and a command parser with semantic command transaction in the theory body. All command parsers operate on the token language of Isabelle/Isar: it provides identifiers, numerals, quoted strings, embedded source etc.

Quoted string tokens are similar to string literals in ML. Some decades ago, there was no outer syntax and everything embedded into ML like that. Today we still see embedded types, terms, and propositions in that historic notation. Nested quotations do work, but require awkward backslash-escapes for quotes: the number of backslashes is exponential in the depth of nesting, so only one or two levels are seen in practice.

Embedded source tokens use the cartouche notation of Isabelle, which was introduced a few years ago to facilitate arbitrary nesting. The quotes are directed (like open/close parentheses) and chosen carefully to remain outside of usual application languages. The Isabelle symbols \<open> and \<close> are used for that; they are rendered in the front-end as French single-quotes: e.g. ⟨source⟩. Thus a command that takes a cartouche as outer syntax remains free to use its own lexical conventions in the nested source.

For example, the subsequent ML snippet defines a term t within the ML environment of the theory; the term-antiquotation inside ML uses regular term notation to construct the corresponding ML datatype value; there are further nested cartouches for comments inside these domain-specific languages.
ML (val t = term ⟨λx. x ≤ y + z — comment in term⟩ — comment in ML)

This approach of nesting languages resembles s-expressions in LISP, but the cartouche delimiters are visually less intrusive than parentheses, and the sub-expressions can be arbitrary sub-languages with their own concrete syntax. The Prover IDE helps users to understand complex nesting of languages, e.g. via text colors and popups (see fig. 3). Isabelle/ML helps language implementors with operations for common concepts, e.g. embedded comments seen here.

2.3 Auxiliary Files with Implicit Theory Context

The outer syntax of Isabelle supports a special class of theory load commands: there is a single argument that refers to a file, relative to the directory where the theory file is located. Isabelle/PIDE manages the content of that file in a stateless manner: the command implementation gets its source as attachment to outer syntax tokens — there is no direct access to the file-system. In general, the editor could have unsaved buffers with changed content: the prover needs to process that intermediate state, not an old saved copy.

Embedded source via auxiliary files is more scalable than inlined cartouches. For example, consider the commands ML vs. ML_file: both incorporate Isabelle/ML definitions into the current theory context, and both are ubiquitous in the construction of Isabelle/Pure and Isabelle/HOL. ML is preferred for small snippets, up to one page of source text. ML_file is better suited for big modules: Isabelle/jEdit provides a mode for the corresponding .ML files, with static syntax tables and a SideKick parser to generate tree views.

Normally, theory load commands occur within a particular theory body to augment its content. In contrast, an implicit theory context helps when the file is considered stand-alone: it refers to an imported context for its language definition, but the results of checking are restricted to PIDE markup shown to the user. The Isabelle/Scala programming interface allows to define a file-format (according to the file extension): thus PIDE knows which theory template needs to be generated internally for such auxiliary files. Example file-formats are those for bibtex and Naproche-SAD (§1.2).

Still missing is support for simultaneous loading of files by a single command, e.g. a whole sub-project in an external language. That would require the

Fig. 3. PIDE exploration of nested sub-languages within this paper
Isabelle/Scala interface to understand the syntax of the load command, beyond a single file argument. Even more ambitious would be transitive exploration of included files, to refer to a complete graph via a few root entries in the text.

2.4 Shallow Presentation of Document Sources

PIDE interaction is about creating documents, and this is taken literally for the final outcome: a traditional PDF produced via \LaTeX. An example is the present paper itself: document sources are edited in Isabelle/jEdit (see fig. 3) and the batch-mode tool isabelle build -o document=pdf produces the typeset document for publication. This works according to a rather shallow presentation scheme going back to the early days of Isabelle/Isar (20 years ago), with a few later additions. The idea is that the source language is sufficiently close to a proper mathematical document, such that simple pretty-printing is sufficient:

- Isabelle symbols like \langle alpha\rangle are blindly replaced by \LaTeX macros: the Isabelle style files provide a meaning for that to typeset α.
- Document markup commands like “section (source)” or “text (source)” are turned into corresponding \LaTeX macros for sections, paragraphs etc.
- Document markdown items with Isabelle symbols \langle item\rangle, \langle enum\rangle, \langle descr\rangle are turned into corresponding \LaTeX environments itemize, enumerate, description.
- Embedded comments like “— (source)” are turned into suitable \LaTeX macros from the Isabelle style files.
- Document antiquotations are evaluated and inlined: the user-defined implementation in ML generates document output within the formal context, e.g. to pretty-print a term using its logical notation.

Note that genuine PIDE markup is not yet used for document output: it is only available in Isabelle/Scala, but document preparation works in Isabelle/ML. We can see that omission in the example of §2.2: the typeset version of fig. 3 does not treat the ML keyword val specifically in \LaTeX, it looks like a regular identifier. Full semantic document preparation is an important area of future work: presently there are only some experiments with HTML preview in Isabelle/jEdit.

PIDE editor presentation of document sources works differently: while the prover is processing the sources, the online document model accumulates XML markup over the original sources. This can be used for painting the editor view in real-time, using whatever is available at a particular point in time; formally this is a snapshot of the PIDE document state. The jEdit editor is a bit limited in its visual rendering capabilities, though: a single font with small variations on style, and uniform font-size and line-height. To make the best out of that, Isabelle/jEdit uses custom Unicode fonts derived from the DejaVu collection, with mathematical symbols taken from \TeX fonts. Isabelle2019 includes a standard set of font families: Sans Mono (default for text buffers), Sans (default for GUI elements), and Serif (default for help texts). To emphasize the “ordinary
mathematical text” format of Naproche-SAD (§1.2), the screenshot in fig. 2 has actually used the proportional Sans instead of the (almost) fixed Sans Mono.

There are additional tricks in Isabelle/jEdit rendering to support subscript, superscript, and bold-face of the subsequent Isabelle symbol, but without nesting of font styles. Furthermore, there are some icons in the font to render special control symbols nicely, e.g. \item for Isabelle Markdown as a square bullet, \file as a sheet of paper, \file as a folder, \url as a W3C globe.

It is interesting to see how far the jEdit text editor can be stretched, with the help of an open-ended collection of Isabelle symbols and specifically generated application fonts. Compared to that, the modest HTML5/CSS3 styling in Isabelle/VSCode is still lagging behind: the makers of VSCode are taking away most of the rendering power of the underlying Chromium browser, because they want to deliver an editor only for “code”, not documents.

In the near future, there should be better convergence of offline PDF presentation and online editor rendering of PIDE documents. In particular, the Isabelle \LaTeX toolchain needs to be integrated into the IDE, to avoid several seconds of wait time to produce PDFs. Further ideas for renovation of Isabelle document preparation (mostly for HTML) are sketched in [26, §3.3].

3 Aims and Approaches of Isabelle/PIDE

What has Isabelle/PIDE tried to achieve over the past 10 years, what worked out and what failed? The subsequent overview of important aims and approaches summarizes success, failure, changes in the plan, and ideas for future work.

3.1 Isabelle/ML vs. Isabelle/Scala: “Mathematics” vs. “Physics”

At the bottom, Isabelle is an LCF-style proof assistant [10] that is freely programmable in Isabelle/ML. That is based on Poly/ML, which is well-tuned towards applications of symbolic logic on multicore hardware.

To complement the ultra-pure ML environment by tools and libraries from the “real” world, Isabelle/PIDE has been based on Scala/JVM from the very beginning in 2008. The JVM gives access to GUI frameworks, HTTP servers, database engines (SQLite, PostgreSQL) etc. The programming style of Isabelle/Scala follows that of Isabelle/ML to a large extent, and there are many basic libraries that are provided on both sides.

Success: The clean and efficient functional style of ML has been transferred to Scala. There are many modules on both sides that follow the typical Isabelle mindset of minimality and purity. It is feasible to move the language boundary of tool implementations, according to technical side-conditions of ML vs. Scala.

Failure: Isabelle users often find Isabelle/ML as tool implementation language already too difficult. The additional Isabelle/Scala for tool integration is beyond the multilingual capabilities of most people. This could be partly caused by
common misunderstandings about both sides: Isabelle/ML is not just Standard ML, and Isabelle/Scala not just Scala — both are “Isabelle” with an idiomatic style that deviates from customs seen elsewhere.

**Changes:** The original conception of Isabelle/Scala as add-on library for system integration turned out insufficient. Instead, Isabelle/Scala and Isabelle/ML have become equal partners in forming the Isabelle infrastructure. Consequently, Isabelle/Pure now contains many ML and Scala modules side-by-side, sometimes with equivalent functionality (e.g. portable file and process operations), and sometimes complementary (e.g. for the PIDE protocol).

**Future:** Isabelle/Scala still needs proper IDE integration: its development model resembles that of Isabelle/ML in earlier decades. It is a funny paradox that the Prover IDE infrastructure is developed with a plain text editor and command-line build process. Either Scala could be integrated into PIDE as another back-end, or a regular Scala IDE could be used (e.g. IntelliJ IDEA).

### 3.2 Private PIDE protocol (untyped) vs. public APIs (typed)

Isabelle/PIDE resides both in Isabelle/ML and Isabelle/Scala, with typed functional programming interfaces. The PIDE implementation uses a custom-made protocol that fits tightly to the requirements of the interactive document model. Over the years, there have been frequent changes and adjustments of the protocol. The communication works over a pure byte-channel, with low overhead for structured messages and ML-like datatype values. The paper [21] explains the PIDE protocol for demonstration purposes with a back-end in Coq 8.4 (2013).

**Success:** Efficient and robust implementation of the bi-lingual PIDE framework in Isabelle/ML/Scala works. Easy maintenance of corresponding modules in the same directory location is feasible.

**Failure:** It is cumbersome to develop and maintain different PIDE implementations for different provers: the Coq/PIDE project [14] did not reach end-users and is now lagging behind years of further PIDE development.

**Changes:** The initial conception of the PIDE protocol was quite basic, but it acquired complexity and sophistication over time.

**Future:** Back-end protocol: the old idea to retarget PIDE for other provers (like Coq) could be re-opened eventually, but it requires significant personal dedication and resources to do that properly. Front-end protocol: there is already a simplified public PIDE protocol for headless interaction with the document-model. That could eventually become a client-server protocol for web applications.

### 3.3 Pervasive Parallelism on Multicore Hardware

Both Isabelle/ML (“pure mathematics”) and Isabelle/Scala (“real physics”) support parallel programming with shared memory and immutable values. User-space tools do not have to care much about it, as long as standard Isabelle
programming idioms and libraries are used. Scaling of parallel programs is always a challenge: Isabelle/ML performs well into the range of 8–16 cores (on a single CPU node). Isabelle/Scala rarely uses more than 2–4 cores.

**Success:** Parallel Isabelle/ML became routinely available in 2008 [15], and has been refined many times [12, 23]. That proved so successful that an initial motivation for PIDE was to make an IDE that can properly connect to a parallel proof engine: for the user front-end this added the aspect of asynchronous interaction, which is central to PIDE [3, 24].

**Failure:** The predictions of CPU manufacturers in 2005 about consumer machines with many cores (32–128) have not become true, because mainstream applications cannot use so much parallelism. Instead we have seen a trend towards light-weight mobile devices (with 2–8 cores). This can confuse new Isabelle users: they think that big applications from AFP should work on e.g. 2 CPU cores and 4 GB RAM, but reality demands to double or quadruple these resources. When we see server-class machines with a lot of cores, there is often an internal division into separate CPU nodes (NUMA) with significant penalty for a shared-memory application, e.g. a machine with 64 cores might turn out as 8 nodes of 8 core CPUs with delay factor 1.6–3.2 to access data on a distant node.

**Future:** The Isabelle/PIDE front-end (on a small mobile device) and the back-end (on a big server) could be separated, to follow a general trend towards “cloud computing”. This could be done without degrading the IDE into a mere web-browser application. Instead, the existing Isabelle/jEdit or Isabelle/VSCode desktop applications could connect to a remote version of Isabelle/PIDE, see also [26, §3.3]. Headless PIDE functionality has already been implemented and used elsewhere, e.g. to export formal content of AFP entries in Isabelle/MMT [6, §3.1]. A proper client-server environment across the Web would require substantial work, e.g. robust management of lost connections to the remote back-end.

### 3.4 Desktop Application Bundles on Linux, Windows, macOS

Isabelle/PIDE is available as a single download that can be unpacked and run without further ado. There is no requirement for self-assembly by end-users Isabelle does not need packaging by OS providers (e.g. Debian): it is a genuine end-user application; there can be different versions side-by-side without conflict.

**Success:** The majority of users is happy with the all-inclusive Isabelle application bundle (2019: 300 MB download size, 1 GB unpacked size). It works almost as smoothly as major Open Source products (e.g. Firefox, LibreOffice).

**Failure:** Full equality of all three platforms families has still not been achieved. Java GUI rendering on Linux is worse than on Windows and macOS; exotic Linux/X11 window managers can cause problems. Add-on tools are sometimes not as portable (and robust) on Windows: Isabelle often refers to Cygwin as auxiliary POSIX platform, but that may cause its own problems. Even for native Windows tools (e.g. via MinGW) the Unix-style orchestration of multiple
processes can have timing problems (e.g. Sledgehammer provers) or fail due to antivirus software. Moreover, the recent tendency of Windows and macOS towards “application stores” makes it harder to run downloaded Isabelle bundles on the spot: users need to bypass extra vendor checks.

**Changes:** The initial approach was more optimistic about availability of certain “standard” components on the OS platform, notably Java and Scala. Later it became clear that almost everything needs to be bundled with Isabelle, except for the most basic system components (e.g. libc, libc++, curl, perl). Note that other projects have come to a similar conclusion, e.g. SageMath with very thorough all-inclusive bundling.

**Future:** The “download–unpack–run” experience of Isabelle/PIDE needs fine-tuning for first-time users. In particular, AFP needs to be included in this, to avoid manual intervention with session ROOTS and ROOT files. There could be more support for alternative applications based on the Isabelle/PIDE platform, to suppress unused components of Isabelle/HOL, e.g. see Isabelle/Naproche (§1.2).

## 4 Conclusion

Isabelle/PIDE is a long-term effort to support live editing of complex document structures with “active” content. Its cultural background is that of interactive theorem proving, which has high demands for execution management: interrupts, real-time requirements, parallel threads, external processes. So this provides a generous upper bound of technology for less ambitious applications of PIDE. We have seen the example of Isabelle/Naproche.

Ultimately, we may understand PIDE as a continuation and elaboration of the following approaches:

- The LCF/ML approach to interactive theorem proving by Milner et-al [10, 9].
- The Isar approach to human-readable proof documents by Wenzel [17].
- Parallel ML and future proofs by Matthews and Wenzel [15, 12, 23].
- Early prover interfaces by Aspinall [2], Bertot [4] and others.

All of that taken together amounts to decades of research: PIDE attempts to form a limit over that, to reach a new stage of semantic editing that can be taken for granted. The present paper has illustrated some applications and sketched the overall construction. It is to be hoped that builders of other mathematical tools are encouraged to re-use PIDE for their own projects.

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