MODELING MAGNETOROTATIONAL TURBULENCE IN PROTOPLANETARY DISKS WITH DEAD ZONES

SATOSHI OKUZUMI 1 and SHIGENOBU HIROSE 2

1 Department of Physics, Nagoya University, Nagoya, Aichi 464-8602, Japan; okuzumi@nagoya-u.jp
2 Institute for Research on Earth Evolution, JAMSTEC, Yokohama, Kanagawa 236-0001, Japan

Received 2011 April 25; accepted 2011 August 23; published 2011 November 4

Abstract

Turbulence driven by magnetorotational instability (MRI) crucially affects the evolution of solid bodies in protoplanetary disks. On the other hand, small dust particles stabilize MRI by capturing ionized gas particles needed for the coupling of the gas and magnetic fields. To provide an empirical basis for modeling the coevolution of dust and MRI, we perform three-dimensional, ohmic-resistive MHD simulations of a vertically stratified shearing box with an MRI-inactive “dead zone” of various sizes and with a net vertical magnetic flux of various strengths. We find that the vertical structure of turbulence is well characterized by the vertical magnetic flux and three critical heights derived from the linear analysis of MRI in a stratified disk. In particular, the turbulent structure depends on the resistivity profile only through the critical heights and is insensitive to the details of the resistivity profile. We discover scaling relations between the amplitudes of various turbulent quantities (velocity dispersion, density fluctuation, vertical diffusion coefficient, and outflow mass flux) and vertically integrated accretion stresses. We also obtain empirical formulae for the integrated accretion stresses as a function of the vertical magnetic flux and the critical heights. These empirical relations allow us to predict the vertical turbulent structure of a protoplanetary disk for a given strength of the magnetic flux and a given resistivity profile.

Key words: dust, extinction – planets and satellites: formation – protoplanetary disks

Online-only material: color figures

1. INTRODUCTION

Plots are believed to form in protoplanetary gas disks. The standard scenario for planet formation consists of the following steps. Initially, submicron-sized dust grains grow into kilometer-sized planetesimals by collisional sticking and/or gravitational instability (Safronov 1969; Goldreich & Ward 1973; Weidenschilling & Cuzzi 1993). Planetesimals undergo further growth toward Moon-sized protoplanets through mutual collision assisted by gravitational interaction (Wetherill & Stewart 1989). Accretion of the disk gas onto protoplanets leads to the formation of gas giants (Mizuno 1980; Pollack et al. 1996), while terrestrial planets form through the giant impacts of protoplanets after the gas disk disperses by viscous accretion onto the central star and other effects (Chambers & Wetherill 1998).

Turbulence in protoplanetary disks plays a decisive role in planet formation as well as on disk dispersal. The impact of turbulence is particularly strong on the formation of planetesimals since the frictional coupling of gas and dust particles governs the process. Classically, planetesimal formation has been attributed to the collapse of a dust sedimentary layer by self-gravity (Safronov 1969; Goldreich & Ward 1973) and/or the collisional growth of dust grains (Weidenschilling & Cuzzi 1993). The presence of strong turbulence is necessary for dust growth when the dust particles are small enough that the Coulomb repulsion is effective (Okuzumi 2009; Okuzumi et al. 2011). However, strong turbulence affects the growth of macroscopic dust aggregates since it makes their collision disruptive (Weidenschilling 1984; Johansen et al. 2008). Furthermore, turbulence causes the diffusion of a dust sedimentary layer, making planetesimal formation via gravitational instability difficult as well (Weidenschilling 1984). Turbulence is also known to concentrate dust particles of particular sizes, but its relevance to planetesimal formation via gravitational instability is still under debate (Cuzzi et al. 2001, 2008, 2010; Pan et al. 2011). More recently, it has been suggested that two-fluid instability of gas and dust can produce dust clumps with density high enough for gravitational collapse, but successful dust coagulation to macroscopic sizes seems to be still required for this mechanism to become viable (Youdin & Goodman 2005; Johansen & Youdin 2007; Johansen et al. 2007; Bai & Stone 2010). Besides, turbulence also affects planetesimal growth as turbulent density fluctuations gravitationally interact with planetesimals and can raise their random velocities above the escape velocity (Ida et al. 2008; Nelson & Gressel 2010). The fluctuating gravitational field can even cause random orbital migration of protoplanets (Laughlin et al. 2004; Nelson & Papaloizou 2004). Thus, to understand the growth of solid bodies in various stages, it is essential to know the strength and spatial distribution of disk turbulence.

Interestingly, the evolution of solid bodies is not only affected by but also affects disk turbulence. The most viable mechanism for generating disk turbulence is the magnetorotational instability (MRI; Balbus & Hawley 1991). This instability has its origin in the interaction between the gas disk and magnetic fields, and therefore requires a sufficiently high ionization degree to operate. Importantly, whether the MRI operates or not in each location of the disk is strongly dependent on the amount of small dust grains because they efficiently capture ionized gas particles and thus the ionization degree (Sano et al. 2000; Ilgner & Nelson 2006; Okuzumi 2009). This implies that dust and MRI-driven turbulence affect each other and thus evolve simultaneously.

The purpose of this study is to present an empirical basis for studying the coevolution of solid particles and MRI-driven turbulence. It is computationally intensive to simulate the evolution of dust and MRI-driven turbulence simultaneously,
since the evolutionary timescale of solid bodies is generally much longer than the dynamical timescale of the turbulence. For example, turbulent eddies grow and decay on a timescale of one orbital period (e.g., Fromang & Papaloizou 2006), while dust particles grow to macroscopic sizes and settle to the midplane spending 100–1000 orbital periods (e.g., Nakagawa et al. 1981; Dullemen & Dominik 2005; Brauer et al. 2008). However, this also means that MRI-driven turbulence can be regarded as quasi-steady in each evolutionary stage of dust evolution. Motivated by this fact, we restrict ourselves to time-independent ohmic resistivity, but instead focus on how the quasi-steady structure of turbulence depends on the vertical profile of the resistivity.

To characterize the vertical structure of MRI-driven turbulence, we perform a number of three-dimensional MHD simulations of local stratified disks including resistivity and nonzero net vertical magnetic flux. Inclusion of a nonzero net vertical flux is important as it determines the saturation level of turbulence (Hawley et al. 1995; Sano et al. 2004; Suzuki & Inutsuka 2009; see also our Section 4). Similar simulations have been done in a number of previous studies (e.g., Miller & Stone 2000; Suzuki & Inutsuka 2009; Oishi & Mac Low 2009; Suzuki et al. 2010; Turner et al. 2010; Gressel et al. 2011; Simon et al. 2011; Hirose & Turner 2011). One important difference between our study and previous ones is that we focus on the general dependence of the saturated turbulent state on the model parameters such as the resistivity and net magnetic vertical flux.

Our modeling of MRI-driven turbulence consists of two steps. In the first step, we seek scaling laws giving the relations among turbulent quantities. We express the relations as a function of the vertically integrated accretion stress, which is the quantity that determines the rate at which turbulent energy is extracted from the differential rotation (Balbus & Papaloizou 1999). As we will see, excellent scaling relations are obtained if we divide the integrated stress into two components that characterize the contributions from different regions in the stratified disk (which we will call the “disk core” and “atmosphere”). In the second step, we find out empirical formulae that predict the vertically integrated stresses as a function of the resistivity profile and vertical magnetic flux.

The plan of this paper is as follows. In Section 2, we describe the setup and method used in our MHD simulations. In Section 3, we introduce “critical heights” derived from the linear analysis of MRI in stratified disks. As we will see later, these critical heights are useful to characterize the turbulent structure observed in our simulations. We present our simulation results in Section 4 and obtain scaling relations and predictor functions for the quasi-steady state of turbulence in Section 5. In Section 6, we simulate dust settling in a dead zone to model the diffusion of dust particles and X-rays and recombination (in the gas phase and on grain surfaces). Detailed structure of the resistivity profile depends on what processes dominate the ionization and recombination. However, a general tendency is that the ionization degree decreases toward the midplane of the disk, because the ionization rate is lower as the column depth is greater and because the recombination rate is higher as the gas density is higher (see, e.g., Sano et al. 2000). Based on this fact, we give the resistivity profile \( \eta(z) \) such that \( \eta \) increases as \( z \) decreases. To be more specific, we adopt the following resistivity profile:

\[
\eta = \eta_{\text{mid}} \exp\left(\frac{-z^2}{2h_\eta^2}\right),
\]

where \( \eta_{\text{mid}} \) is the resistivity at the midplane and \( h_\eta \) is the scale height of \( \eta \).

Equation (4) satisfies the important property of realistic resistivity profiles mentioned above. Furthermore, as shown in the Appendix, Equation (4) exactly reproduces the vertical resistivity profile of a disk in some limited cases. However, it will be useful to examine possible influences of limiting \( \eta \) to
Equation (4). In order to do that, we also consider a resistive profile used in Fleming & Stone (2003),

\[ \eta_{FS03} = \eta_0 \exp \left( -\frac{z^2}{4h^2} \right) \exp \left( \frac{\Sigma_0}{\Sigma_{CR}} \frac{1}{2\sqrt{2\pi}} \int_{|z|/h}^{\infty} e^{-\frac{z'^2}{2}} dz' \right), \]

which is characterized by two parameters \( \eta_0 \) and \( \Sigma_0/\Sigma_{CR} \) (see Equation (10) of Fleming & Stone 2003). Physically, Equation (5) corresponds to the resistivity profile when the ionization degree is determined by the balance between cosmic-ray ionization and gas-phase recombination (see also the Appendix).

We construct 17 simulation models using Equations (4) and (5). Sixteen models are constructed from Equation (4) with various sets of the parameters (\( \eta_{mid}, h, \beta_{0} \)). Table 1 lists the parameters adopted in each run. The model “Ideal” assumes zero resistivity throughout the simulation box. Models X0–X3 are defined by the same value of \( \eta_{mid} \) but different values of \( h \). The difference between X, Y, and W models is in the value of \( \eta_{mid} \). The initial plasma beta is taken to be \( 3 \times 10^5 \) in all models except X1a–X1d. In addition, we construct one model from Equation (5) with \( \eta_{mid} = \eta_0 \exp(\Sigma_0/4\Sigma_{CR}) = 0.01 \) and \( \Sigma_0/\Sigma_{CR} = 34.7 \). This set of parameters corresponds to the “larger dead zone” model of Fleming & Stone (2003) defined by \( \text{Re}_{M,mid} = 100 \) and \( \text{Re}_{M,mid}=2\pi\sqrt{h} \approx 5.6 \times 10^6 \), where \( \text{Re}_{M} \equiv c_s h/\eta \) is the magnetic Reynolds number with the typical length and velocity set to be \( h \) and \( c_s \), respectively. We refer to the model with these parameters as model FS03L. The initial plasma beta in the FS03L model is taken to be \( 3 \times 10^5 \). Note that our FS03L run is not exactly the same as the larger dead zone run of Fleming & Stone (2003) since they assumed zero net vertical magnetic flux.

### 2.2. Method

We solve the equations of the isothermal resistive MHD using the ZEUS code (Stone & Norman 1992). The domain is a box of size \( 2\sqrt{2}h \times 8\sqrt{2}h \times 10\sqrt{2}h \) along the radial, azimuthal, and vertical directions, divided into 40 \( \times \) 80 \( \times \) 200 grid cells. For all runs, the radial and azimuthal boundary conditions are taken to be shearing-periodic and periodic, respectively. Therefore, the vertical magnetic flux is a conserved quantity of our simulations.

The vertical boundary condition for all runs except X1d is the standard ZEUS outflow condition, where the fields on the boundaries are computed from the extrapolated electromotive forces; for run X1d only, we assume for numerical stability that the magnetic fields are vertical on the top and bottom boundaries as done by Flaig et al. (2010). We have checked that the results hardly depend on the choice of the two types of the boundary conditions. Note that the radial and azimuthal components of the mean magnetic fields are not conserved due to the outflow boundary condition. We also added a small artificial resistivity near the boundaries for numerical stability (Hirose et al. 2009). A density floor of \( 10^{-5}\rho_0 \) is applied to prevent high Alfvén speeds from halting the calculations.

### 3. CHARACTERISTIC WAVELENGTHS AND CRITICAL HEIGHTS

As we will see in the following sections, it is useful to analyze the simulation results using the knowledge obtained from the linear analysis of MRI. In this subsection, we introduce several quantities that characterize the linear evolution of MRI.

#### 3.1. Characteristic Wavelengths of MRI

According to the local linear analysis of MRI including ohmic resistivity (Sano & Miyama 1999), the wavelength of the most unstable MRI mode can be approximately expressed as

\[ \lambda_{local} \approx \max \{ \lambda_{ideal}, \lambda_{res} \} \]

where

\[ \lambda_{ideal} \equiv 2\pi \frac{v_{Az}}{\Omega} \]

and

\[ \lambda_{res} \equiv 2\pi \frac{\eta}{v_{Az}} \]

are the characteristic wavelengths of MRI modes in the ideal and resistive MHD limits, respectively, and \( v_{Az} = B_z/\sqrt{4\pi\rho} \) is the vertical component of the Alfvén velocity. Equation (6) can be written as \( \lambda_{local} \approx \lambda_{ideal} \max \{ 1, \Lambda^{-1} \} \), where \( \Lambda \) is the Elsasser number defined by

\[ \Lambda \equiv \frac{v_{Az}^2}{\eta \Omega}. \]

The Elsasser number determines the growth rate of the MRI. If \( \Lambda > 1 \), ohmic diffusion does not affect the most unstable mode lying at \( \lambda = \lambda_{ideal} \), and the local instability occurs rapidly at a wavelength \( \lambda_{local} \approx \lambda_{ideal} \) and at a rate \( \approx \Omega \). If \( \Lambda < 1 \), ohmic diffusion stabilizes the most unstable mode, and the local instability occurs at a longer wavelength \( \lambda_{local} \approx \lambda_{res} = \Lambda^{-1}\lambda_{ideal} \) and at a slower rate \( \approx \Lambda^{-1}\Omega \). We will refer to the former case as the “ideal MRI” and to the latter case as the “resistive MRI.”

Figure 1 schematically illustrates how \( \lambda_{ideal} \) and \( \lambda_{res} \) vary with height \( |z| \). In general, \( \lambda_{ideal} \) grows toward higher \( |z| \) because the Alfvén speed \( v_{Az} \) increases as the density decreases. By contrast, \( \lambda_{res} \) grows toward lower \( |z| \) because \( \lambda_{res} \) is inversely proportional to \( v_{Az} \) and because \( \eta \) increases with decreasing \( |z| \) (see the discussion in Section 2.1.2).

The global instability of a stratified disk can be described in terms of the local analysis. As shown by Sano & Miyama (1999), the gas motion at height \( z \) is unstable if the local unstable wavelength \( \lambda_{local} \) is shorter than the scale height of the disk, i.e.,

\[ \max \{ \lambda_{ideal}, \lambda_{res} \} \lesssim h. \]
Ohmic diffusion allows the resistive MRI to operate at $h_{\text{res}} \lesssim \Lambda \lesssim h_{\Lambda}$. At $|z| \lesssim h_{\text{res}}$, ohmic diffusion stabilizes all the unstable MRI modes. Note that some previous studies (e.g., Gammie 1996; Sano et al. 2000) used the terminology “dead zone” for the region $|z| \leq h_{\text{res}}$ rather than $|z| \leq h_{\Lambda}$. In fact, as we will see in Section 4, the set of $h_{\Lambda}$ and $h_{\text{res}}$ best characterizes our dead zone. We regard $h_{\text{res}}$ as zero when $\lambda_{\text{res}}$ is less than $h$ at all heights. This is the case for $Y$ models.

The critical heights in the initial state ($h_{\text{ideal},0}$, $h_{\Lambda,0}$, and $h_{\text{res},0}$) are shown in Table 1 for all of our 17 simulations. Using Equations (1) and (4), one can analytically calculate the initial critical heights for models except FS03L as

\[ h_{\text{ideal},0} = \left( \frac{2 \ln \left( \frac{\beta z_0}{8 \pi^2} \right)}{1 + (h/h_{\eta})^2} \right)^{1/2} h, \] \[ h_{\Lambda,0} = \left( \frac{2 \ln \lambda_{\Lambda,0}^{-1} + \lambda_{\Lambda,0}^{-2}}{1 + (h/h_{\eta})^2} \right)^{1/2} h, \] \[ h_{\text{res},0} = \left( \frac{2 \ln \left( \frac{8 \pi^2 \beta z_0^{-1} \lambda_{\Lambda,0}^{-2}}{1 + (h/h_{\eta})^2} \right)}{1 + (h/h_{\eta})^2} \right)^{1/2} h, \]

where

\[ \lambda_{\Lambda,0} = \frac{2c_{\delta} h}{\eta_{\text{med}} \beta_0} \]

is the initial Elsasser number at the midplane.

Figure 2 shows the vertical profiles of the resistivity $\eta$ and the initial Elsasser number $\lambda_{\Lambda,0}$ for some of our models. The initial midplane Elsasser number $\Lambda_0$ and initial critical heights ($h_{\text{ideal},0}$, $h_{\Lambda,0}$, $h_{\text{res},0}$) are listed in Table 1 for all models. As one can see from Table 1 and the lower panel of Figure 2, models labeled by the same number are arranged so that they have similar values of $\lambda_{\Lambda,0}$.

For turbulent states, we evaluate $v_{Az}$ in the Elsasser number and the characteristic wavelengths as $(\overline{B^2}/4\pi \overline{\rho})^{1/2}$, where the overbars denote the horizontal averages.

### 4. SIMULATION RESULTS

#### 4.1. The Fiducial Model

We select model X1 as the fiducial model to describe in detail. Figure 3 shows how MRI-driven turbulence reaches a quasi-steady state in run X1. The upper and lower panels plot the horizontal averages of the Maxwell stress $\overline{w_M} = -\overline{\delta B_x \delta B_y}/4\pi$ and the density-weighted velocity dispersion $\overline{\rho \delta v^2}$, respectively, as a function of time $t$ and height $z$. The solid, dotted, and dashed lines are the loci of the critical heights $h_{\text{ideal}}$, $h_{\Lambda}$, and $h_{\text{res}}$, respectively. As seen in the figure, a quasi-steady state is reached within the first 40 orbits. The critical height $h_{\text{ideal}}$ measured in the quasi-steady state is slightly lower than that in the initial non-turbulent state. This is because the ideal MRI wavelength $\lambda_{\text{ideal}} \propto v_{Az}$ is increased by the fluctuation in the vertical magnetic field, $\delta B_z$. By contrast, $h_{\Lambda}$ and $h_{\text{res}}$ are almost unchanged, because the fluctuation of the magnetic field is suppressed in the dead zone.

3. The third one is $h_{\text{res}}$ defined by

\[ \lambda_{\text{res}}(z = h_{\text{res}}) = h. \]

![Figure 1](image-url) Schematic illustration showing the vertical structure of a stratified protoplanetary disk with vertical magnetic fields. The horizontal axis shows the distance $z$ from the midplane, while the vertical axis shows the characteristic wavelengths $\lambda_{\text{ideal}}$ (solid curve) and $\lambda_{\text{res}}$ (dashed curve) of the MRI at each $z$ as well as the gas scale height $h$ (dotted line). At $|z| > h_{\text{ideal}}$ ($\Lambda > 1$ and $\lambda_{\text{ideal}} > h$), MRI is stabilized due to the weak gas pressure compared to the magnetic tension. At $h_{\text{Lambda}} < |z| < h_{\text{res}}$ ($\Lambda > 1$ and $\lambda_{\text{ideal}} < h$; dark gray region), MRI operates without being affected by ohmic dissipation. At $h_{\text{res}} < |z| < h_{\Lambda}$ ($\Lambda < 1$ and $\lambda_{\text{ideal}} < h$; light gray region), ohmic dissipation is effective and MRI operates only weakly. At $|z| < h_{\text{res}}$ ($\Lambda < 1$ and $\lambda_{\text{res}} < h$), ohmic dissipation perfectly stabilizes MRI. We refer to the regions $|z| < h_{\text{ideal}}$ and $|z| > h_{\text{ideal}}$ as the “disk core” and “atmosphere,” respectively.

#### 3.2. Critical Heights

With the global instability criterion (Equation (10)) together with the vertical dependence of $\lambda_{\text{ideal}}$ and $\lambda_{\text{res}}$, we can define three different critical heights for a stratified disk.

1. The first one is $h_{\text{ideal}}$ defined by

\[ \lambda_{\text{ideal}}(z = h_{\text{ideal}}) = h, \]

or equivalently, $\beta_{c}(z = h_{\text{ideal}}) = 8\pi^2$, where $\beta_{c}(z) = 8\pi\rho(z)C_{s}^2/B_{z}^2(z)$. At $|z| \gtrsim h_{\text{ideal}}$, $\lambda_{\text{ideal}}(z) \lesssim 8\pi^2$, MRI does not operate because the wavelengths of the unstable modes exceed the disk thickness $\sim h$ (Sano & Miyama 1999). We refer to the region $|z| \gtrsim h_{\text{ideal}}$ as the “atmosphere” and to the region $|z| \lesssim h_{\text{ideal}}$ as the “disk core.”

2. The second one is $h_{\Lambda}$ defined by

\[ \lambda_{\Lambda}(z = h_{\Lambda}) = 1, \]

or equivalently, $\lambda_{\text{ideal}}(z = h_{\Lambda}) = \lambda_{\text{res}}(z = h_{\Lambda})$. The layer $h_{\Lambda} \leq |z| \leq h_{\text{ideal}}$ is the so-called active layer, where MRI operates without being affected by ohmic diffusion or gas stratification. The region $|z| \leq h_{\Lambda}$ is what we call the “dead zone,” where ohmic diffusion stabilizes the most unstable ideal MRI mode. For convenience, we regard $h_{\Lambda}$ as zero when a dead zone is absent. This is the case for model Ideal.
Figure 2. Vertical profiles of the ohmic resistivity $\eta$ (upper panel) and the initial Elsasser number $\Lambda_{t0}$ (lower panel) for models X0 (blue dot-dashed curve), X1 (blue dotted curve), X2 (blue dashed curve), X3 (blue solid curve), Y3 (red curve), W3 (green curve), and FS03L (black curve). (A color version of this figure is available in the online journal.)

Figure 4 shows the vertical structure of the disk averaged over a time interval $175 < \Omega t/(2\pi) < 350$. The dark and light gray bars in each panel indicate the heights where ideal and resistive MRIs operate, respectively (see also Figure 1). The brackets $\langle \cdots \rangle$ denote the averages over time and horizontal directions.

In Figure 4(a), we compare the averaged gas density $\langle \rho \rangle$ with the initial density given by Equation (1). We see that the density is almost unchanged in the disk core ($|z| < h_{\text{ideal}}$) but is considerably increased in the atmosphere ($|z| > h_{\text{ideal}}$). This is because the magnetic pressure is negligibly small in the disk core but dominates over the gas pressure in the atmosphere. Figure 4(a) also shows the amplitude of the density fluctuation, $\langle (\delta \rho)^2 \rangle^{1/2}$. As one can see, the density fluctuation is small ($\langle (\delta \rho)^2 \rangle^{1/2} \ll \langle \rho \rangle$) except at $|z| \gg h_{\text{ideal}}$.

The magnetic activity in the disk can be seen in Figure 4(b), where the vertical profiles of the magnetic energies ($\langle \delta B^2 \rangle/4\pi$, $\langle B_x^2 \rangle/4\pi$, and $\langle B_z^2 \rangle/4\pi$) and Maxwell stress $\langle w_M \rangle$ are plotted. One can see that these quantities peak near the outer boundaries of the active layers, $|z| \approx h_{\text{ideal}}$. This is because the largest channel flows develop at locations where $\lambda_{\text{ideal}} \approx h$ (see, e.g., Suzuki & Inutsuka 2009). In the dead zone, ohmic dissipation suppresses the fluctuation in the magnetic fields, $\langle \delta B^2 \rangle$, leaving the initial vertical field ($\langle B_z^2 \rangle \approx B_0^2$) and coherent toroidal fields ($\langle B_y^2 \rangle \approx \langle B_y \rangle^2$) generated by the differential rotation. In our simulations, ohmic resistivity is not high enough to remove shear-generated, coherent toroidal fields. In this sense, our dead zone is an “undead zone” in the terminology of Turner & Sano (2008).

Figure 4(c) shows the density-weighted velocity dispersions $\langle \rho \rangle \langle \delta v^2 \rangle$ and $\langle \rho \rangle \langle \delta v_z^2 \rangle$ and the Reynolds stress $\langle w_R \rangle = \langle \rho \delta v_x \delta v_y \rangle$. These quantities characterize the kinetic energy in the random motion of the gas. Comparing Figures 4(b) and (c), we find that the drop in these quantities in the disk core is not as significant as the drop in $\langle \delta B^2 \rangle$ and $\langle w_M \rangle$. This is an indication that shear-generated, coherent toroidal fields are not completely suppressed in the dead zone.

4 In our simulations, ohmic resistivity is not high enough to remove shear-generated, coherent toroidal fields. In this sense, our dead zone is an “undead zone” in the terminology of Turner & Sano (2008).

5 In the disk core ($|z| \lesssim h_{\text{ideal}}$), $\langle \rho \rangle \langle \delta v^2 \rangle$ is approximately equal to $\langle \rho \delta v_z^2 \rangle$ since the density fluctuation is small (see Figure 4(a)).
that sound waves generated in the active layers penetrate deep inside the dead zone (Fleming & Stone 2003). Furthermore, we find that \( \langle \rho \rangle \delta v^2 \) is approximately constant, i.e., the velocity dispersion \( \delta v^2 \) is inversely proportional to the mean density \( \rho \), in the disk core. This means that the kinetic energy density of fluctuation is nearly constant in the disk core. This is another indication of sound waves, because the amplitude of the velocity fluctuation \( \delta v \) is generally proportional to \( 1/\sqrt{\rho} \) for freely propagating sound waves. The root-mean-squared random velocity \( \langle \delta v^2 \rangle^{1/2} \) is shown in Figure 4(d). The random velocity is subsonic in the disk core and exceeds the sound speed only in the atmosphere.

An indication of freely sound waves can also be found in the density fluctuation. Shown in Figure 4(e) is the mean-squared density fluctuation \( \langle \delta \rho^2 \rangle \) divided by the mean density \( \langle \rho \rangle \). Since \( \langle \delta \rho^2 \rangle^{1/2} \ll \langle \rho \rangle \), the quantity \( \langle \delta \rho^2 \rangle / \langle \rho \rangle \approx \langle \delta \rho^2 / \rho \rangle \) is approximately proportional to the thermal energy density of fluctuation, \( \langle c_s^2 \delta \rho^2 / 2 \rho \rangle \). In the disk core, we see that \( \langle \delta \rho^2 / \rho \rangle \) is roughly constant along the vertical direction, meaning that the amplitude of the density fluctuation, \( \langle \delta \rho^2 \rangle^{1/2} \), is proportional to the square root of the mean density \( \rho \). The similarity between \( \langle \delta v \rangle \delta v^2 \) and \( \langle \delta \rho^2 / \rho \rangle / \rho \) is peculiar to sound waves, for which \( \delta v / c_s \sim \delta \rho / \rho \).

Figure 4(f) displays the profile of the vertical mass flux \( \langle \rho v z \rangle \). In the atmosphere, the vertical flux is outward, i.e., \( \langle \rho v z \rangle > 0 \) at \( z > h_{\text{ideal}} \) and \( \langle \rho v z \rangle < 0 \) at \( z < -h_{\text{ideal}} \). This outflow results from the breakup of large channel flows at the outer boundaries of the active layers, \( |z| \approx h_{\text{ideal}} \) (Suzuki & Inutsuka 2009; Suzuki et al. 2010). The vertical mass flux reaches a constant value at \( |z| > 5h \). This fact allows us to measure a well-defined outflow flux for each simulation (see Section 5.1.3).

### 4.2. Model Comparison

We now investigate how the vertical structure of turbulence depends on the resistivity profile and vertical magnetic flux.

Figure 5 displays the temporal and horizontal averages of the Maxwell stress \( \langle w_M \rangle \) and the density-weighted velocity dispersion \( \langle \rho \delta v^2 \rangle \) as a function of \( z \) for various \( \beta_0 = 3 \times 10^5 \) models. As in Figures 1 and 4, the dark and light gray bars in each panel indicate the heights where ideal and resistive MRIs operate, respectively.

The effect of changing the size of the dead zone can be seen in Figure 5(a), where models X1, X2, and X3 are compared. These models are characterized by the same values of \( \beta_0 \) and \( \Lambda_0 \) but different values of \( h_\eta \). For all the models, \( \langle w_M \rangle \) sharply falls at \( z \sim h_{\text{res}} \), meaning that \( h_{\text{res}} \) predicts well where the resistivity shuts off the magnetic activity. By contrast, \( \langle \rho \delta v^2 \rangle \) exhibits a flat profile at \( |z| \lesssim h_{\text{ideal}} \) with no distinct change across \( z = h_{\text{res}} \) or \( z = h_\eta \). The only clear difference is the value of \( \langle \rho \delta v^2 \rangle \) in the disk core, i.e., the value is lower when the dead zone is wider. Note that \( \langle \rho \delta v^2 \rangle \) decreases more slowly than the column density of the active layers \( h_\eta < |z| < h_{\text{ideal}} \). For example, the active column density in model X1 is 20 times smaller than that in model X3. However, the midplane value of \( \langle \rho \delta v^2 \rangle \) in the former is only five times smaller than that in the latter. This suggests that even a very thin active layer can provide a large velocity dispersion near the midplane.

Interestingly, the vertical structure of turbulence depends on the critical heights (\( h_{\text{ideal}}, h_\eta \), and \( h_{\text{res}} \)) but are very insensitive to the details of the resistivity profile. This can be seen in Figure 5(b), where we compare runs with similar critical heights (runs X3, W3, and FS03L). We see that these models produce...
Figure 5. Vertical profiles of temporally and horizontally averaged Maxwell stress \(\langle w_M \rangle\) (red curves) and density-weighted velocity dispersion \(\langle \rho \rangle \langle \delta v^2 \rangle\) (black curves) normalized by the initial midplane gas pressure \(P_0 = \rho_0 c_s^2\) for \(\beta_0 = 3 \times 10^5\) models. The dark gray bars indicate where MRI operates without being affected by ohmic resistivity \((h_\Lambda < |z| < h_{\text{ideal}}\)) while the light gray bars show where MRI operates but is weakened by ohmic resistivity \((h_{\text{res}} < |z| < h_\Lambda)\).

(A color version of this figure is available in the online journal.)

very similar vertical profiles of \(\langle w_M \rangle\) and \(\langle \rho \rangle \langle \delta v^2 \rangle\) even though they assume quite different resistivity profiles (see the upper panel of Figure 2). This suggests that the vertical structure of turbulence is determined by the values of the critical heights.

The importance of distinguishing between \(h_\Lambda\) and \(h_{\text{res}}\) is illustrated in Figures 5(c) and (d). These panels compare five models (Y1–Y4 and Ideal) in which the resistive MRI is active at the midplane, i.e., \(h_{\text{res}} = 0\). Figure 5(c) shows models with \(h_\Lambda > 0.5h\). We see that the profiles of \(\langle \rho \rangle \langle \delta v^2 \rangle\) and \(\langle w_M \rangle\) are very similar for the three models. This implies that the vertical structure is determined by the value of \(h_{\text{res}}\) when \(h_\Lambda > 0.5h\). Model Ideal is clearly different from the other models. Model Y4 \((h_\Lambda = 0.4)\) is interesting because it exhibits both features. In the lower half of the disk \((z < 0)\), the Maxwell stress behaves as in the other Y models. In the upper half \((z > 0)\), however, the profile of \(\langle w_M \rangle\) is closer to that in model Ideal.

Next, we see how the saturation level of turbulence depends on the vertical magnetic flux. Figure 6 shows the vertical profiles of \(\langle w_M \rangle\) and \(\langle \rho \rangle \langle \delta v^2 \rangle\) for three runs with different values of \(\beta_0\) (X1b, X1, and X1d). We see that these values increase with decreasing \(\beta_0\). The peak value of \(\langle w_M \rangle\) is approximately \(10^{-4} P_0, 10^{-3} P_0, \) and \(10^{-2} P_0\) for runs X1b, X1, and X1d, respectively \((P_0 = \rho_0 c_s^2\) is the initial midplane gas pressure\). This indicates a linear scaling between the turbulent stress and \(\beta_0^{-1}\).

5. SCALING RELATIONS AND PREDICTOR FUNCTIONS

Now we seek how the amplitudes of turbulent quantities depend on the vertical magnetic flux and the resistivity profile. We do this in two steps. First, we derive relations between the amplitudes of turbulent quantities and the vertically integrated turbulent stress. We then obtain empirical formulae that predict the integrated stress as a function of the vertical magnetic flux and the resistivity profile.

5.1. Scaling Relations between Turbulent Quantities and Vertically Integrated Accretion Stresses

The ultimate source of the energy of turbulence is the shear motion of the background flow. The accretion stress \(w_{xy} = w_R + w_M\) determines the rate at which the free energy is extracted. Therefore, we expect that the accretion stress is
related to the amplitudes of turbulent quantities, such as the gas velocity dispersion and outflow mass flux.

To quantify the rate of the energy input in the simulation box, we introduce the effective $\alpha$ parameter

$$\alpha \equiv \frac{\int \langle w_{xy} \rangle \, dz}{\Sigma_c^2},$$

(18)

where $\Sigma = \int (\rho) \, dz$ is the gas surface density. In the classical one-dimensional viscous disk theory (Lynden-Bell & Pringle 1974), the parameter $\alpha$ is related to the turbulent viscosity $v_{\text{turb}}$ as $v_{\text{turb}} = (3/2) \alpha c_s^2 / \Omega$, where the prefactor $3/2$ comes from the slope of the Keplerian rotation. Thus, $\alpha$ also characterizes the vertically integrated mass accretion rate.

As we will see below, it is useful to decompose $\alpha$ as $\alpha = \alpha_{\text{core}} + \alpha_{\text{atm}}$, where

$$\alpha_{\text{core}} \equiv \frac{\int_{|z| < h_{\text{ideal}}} \langle w_{xy} \rangle \, dz}{\Sigma_c^2}$$

(19)

and

$$\alpha_{\text{atm}} \equiv \frac{\int_{|z| > h_{\text{ideal}}} \langle w_{xy} \rangle \, dz}{\Sigma_c^2}$$

(20)

are the contributions from the disk core ($|z| < h_{\text{ideal}}$) and atmosphere ($|z| > h_{\text{ideal}}$), respectively. Table 2 shows the values of $\alpha$, $\alpha_{\text{core}}$, and $\alpha_{\text{atm}}$ as well as the time-averaged critical heights ($h_{\text{ideal}}$, $h_A$, $h_{\text{res}}$) for all our simulations.

5.1.1. Velocity Dispersion

Random motion of the gas crucially affects the growth of dust particles as it enhances the collision velocity between the particles via friction forces (Weidenschilling 1984; Johansen et al. 2008). Here, we seek how the velocity dispersion $\langle \delta v^2 \rangle$ is related to the integrated accretion stress.

First, we focus on the velocity dispersion at the midplane, $\langle \delta v^2 \rangle_{\text{mid}}$. Figure 7(a) shows $\langle \delta v^2 \rangle_{\text{mid}}$ versus the total accretion stress $\alpha$ for all our runs. The value of $\langle \delta v^2 \rangle_{\text{mid}}$ for each run is listed in Table 2. One can see a rough linear correlation between the velocity dispersion and the accretion stress (for reference, a linear fit $\langle \delta v^2 \rangle_{\text{mid}} = 0.25 \alpha c_s^2$ is shown by the dashed line). However, detailed inspection shows that $\langle \delta v^2 \rangle_{\text{mid}}$ increases more rapidly than $\alpha$ as the dead zone increases in size.

As found from Table 2, this is because the contribution from the atmosphere, $\alpha_{\text{atm}}$, is insensitive to the size of the dead zone in

### Table 2

Time-averaged Properties of MHD Simulations

| Model  | $h_{\text{ideal}} / \Sigma$ | $h_A / \Sigma$ | $h_{\text{res}} / \Sigma$ | $\alpha_{\text{core}}$ | $\alpha_{\text{atm}}$ | $\langle \delta v^2 \rangle_{\text{mid}}$ | $\langle \delta \rho \rangle_{\text{mid}}$ | $\langle \delta \rho \rangle_{\text{mid}} / \langle \delta v^2 \rangle_{\text{mid}}$ | $w_{\text{d}}$ |
|--------|-----------------------------|----------------|---------------------------|------------------------|------------------------|-----------------------------|-----------------------------|---------------------------------|------------------|
| Ideal  | 2.4                         | 0.0            | 0.0                       | 18                     | 12                     | 6.0                         | 0.22                        | 6.4                             | 4.7               |
| X0     | 3.3                         | 3.0            | 2.8                       | 1.9                    | 0.30                   | 1.6                         | 0.13                        | 0.34                            | 0.13              |
| X1     | 3.2                         | 2.5            | 2.2                       | 2.0                    | 0.42                   | 1.5                         | 0.24                        | 0.32                            | 0.16              |
| X2     | 3.1                         | 1.7            | 2.3                       | 3.3                    | 0.68                   | 1.6                         | 0.45                        | 0.35                            | 0.27              |
| X3     | 3.0                         | 0.8            | 0.6                       | 3.3                    | 1.4                    | 1.9                         | 1.1                         | 0.32                            | 0.66              |
| Y1     | 2.9                         | 1.6            | 0.0                       | 4.3                    | 2.0                    | 2.2                         | 2.1                         | 0.26                            | 1.2               |
| Y2     | 3.0                         | 1.3            | 0.0                       | 3.6                    | 1.5                    | 2.1                         | 1.4                         | 0.31                            | 0.84              |
| Y3     | 3.0                         | 0.8            | 0.0                       | 3.5                    | 1.6                    | 1.9                         | 1.8                         | 0.23                            | 1.2               |
| Y4     | 2.8                         | 0.4            | 0.0                       | 7.0                    | 3.8                    | 3.3                         | 3.9                         | 0.26                            | 2.2               |
| W1     | 3.2                         | 2.6            | 2.4                       | 1.9                    | 0.37                   | 1.5                         | 0.24                        | 0.36                            | 0.16              |
| W2     | 3.2                         | 1.7            | 1.5                       | 2.1                    | 0.59                   | 1.5                         | 0.38                        | 0.37                            | 0.23              |
| W3     | 3.1                         | 0.9            | 0.7                       | 2.7                    | 1.1                    | 1.7                         | 1.0                         | 0.39                            | 0.51              |
| X1a    | 4.2                         | 3.2            | 2.7                       | 0.61                   | 0.014                  | 0.047                       | 0.0097                     | 0.17                            | 0.0083            |
| X1b    | 3.8                         | 2.9            | 2.5                       | 0.19                   | 0.056                  | 0.13                        | 0.031                      | 0.25                            | 0.023             |
| X1c    | 2.8                         | 2.2            | 1.9                       | 7.6                    | 1.4                    | 6.2                         | 0.93                       | 0.40                            | 0.50              |
| X1d    | 2.0                         | 1.6            | 1.4                       | 29.0                   | 8.0                    | 21                          | 7.0                        | 0.47                            | 1.8               |
| FS03L  | 3.0                         | 1.0            | 0.6                       | 3.4                    | 1.5                    | 2.0                         | 1.3                        | 0.35                            | 0.70              |

Figure 7. Gas velocity dispersion at the midplane, $\langle \delta v^2 \rangle_{\text{mid}}$, for all runs presented in this study. Panels (a) and (b) plot the data vs. $\alpha$ and $\alpha_{\text{core}}$, respectively. The symbols correspond to models Ideal (circle), X0–X3 (open squares), Y1–Y4 (triangles), W1–W3 (crosses), X1a–X1d (tiled squares), and FS03L (plus sign). The lines show the best linear fits (Equation (21)) for panel (b).

(A color version of this figure is available in the online journal.)
predicted profiles are plotted only at and dotted curves correspond to runs Ideal, X1, and X1a, respectively. The predicted profiles are plotted only at $|z| < h_{\text{ideal}}$, where Equation (22) is valid.

the disk core. In Figure 7(b), we replot the data by replacing $\alpha$ with the accretion stress in the disk core, $\alpha_{\text{core}}$. Comparison between Figures 7(a) and (b) shows that $\langle \delta v^2 \rangle_{\text{mid}}$ more tightly correlates with $\alpha_{\text{core}}$ than with $\alpha$. We find that the data can be well fit by a simple linear relation

$$\langle \delta v^2 \rangle_{\text{mid}} = 0.78 \alpha_{\text{core}} c_s^2, \quad \text{(21)}$$

which is shown by the solid line in Figure 7(b). This result indicates that the accretion stress in the atmosphere does not contribute to the velocity fluctuation near the midplane.

Once $\langle \delta v^2 \rangle_{\text{mid}}$ is known, it is also possible to reproduce the vertical profile of the velocity dispersion. For the disk core ($|z| < h_{\text{ideal}}$), we already know that $\langle \delta v^2 \rangle$ is inversely proportional to the mean gas density $\langle \rho \rangle$ and that $\langle \rho \rangle$ hardly deviates from the initial Gaussian profile. From these facts, we can predict the vertical variation of $\langle \delta v^2 \rangle$ as

$$\langle \delta v^2 \rangle \approx \langle \delta v^2 \rangle_{\text{mid}} \frac{\langle \rho \rangle_{\text{mid}}}{\langle \rho \rangle} \approx \langle \delta v^2 \rangle_{\text{mid}} \exp \left( \frac{z^2}{2h^2} \right) \approx 0.78 \alpha_{\text{core}} c_s^2 \exp \left( \frac{z^2}{2h^2} \right), \quad \text{(22)}$$

where Equation (21) has been used in the final equality. In Figure 8, we compare the vertical profiles of the random velocity $\langle \delta v^2 \rangle_{1/2}$ directly obtained from runs Ideal, X1, and X1a with the predictions from Equation (22), where the values of $\alpha_{\text{core}}$ are taken from Table 2. We see that Equation (22) successfully reproduces the vertical profiles of $\langle \delta v^2 \rangle_{1/2}$ in the disk core. We remark that Equation (22) greatly overestimates the velocity dispersion at $|z| \gg h_{\text{ideal}}$, where the gas density can no longer be approximated by the initial Gaussian profile (see Figure 4(a)).

### 5.1.2. Density Fluctuation

Density fluctuations generated by MRI-driven turbulence gravitationally interact with planetesimals and larger solid bodies, affecting their collisional and orbital evolutions in protoplanetary disks (Laughlin et al. 2004; Nelson & Papaloizou 2004; Nelson & Gressel 2010; Gressel et al. 2011). Here, we examine how the amplitude of the density fluctuations is determined the vertically integrated accretion stress.

As in Section 5.1.1, we begin with the analysis of the density fluctuations at the midplane, $\langle \delta \rho^2 \rangle_{\text{mid}}^{1/2}$. We find from Table 2 that $\langle \delta \rho^2 \rangle_{\text{mid}}$ correlates more tightly with $\alpha_{\text{core}}$ than with $\alpha$. Figure 9 shows $\langle \delta \rho^2 \rangle_{\text{mid}}$ versus $\alpha_{\text{core}}$ for all runs. The best linear fit is given by

$$\langle \delta \rho^2 \rangle_{\text{mid}} = 0.47 \alpha_{\text{core}} \langle \rho \rangle_{\text{mid}}^2, \quad \text{(23)}$$

which is shown by the solid line in Figure 9. If we use this equation with Equation (21), we can also obtain the relation between the velocity dispersion and density fluctuation,

$$\langle \delta v^2 \rangle_{\text{mid}} / c_s^2 = 1.7 \langle \delta \rho^2 \rangle_{\text{mid}} / \langle \rho \rangle_{\text{mid}}^2. \quad \text{(24)}$$

This is consistent with the idea that the fluctuations near the midplane are created by sound waves, for which $\delta v / c_s \sim \delta \rho / \rho$ (see also Section 4.1).

As shown in Section 4.1, $\langle \delta \rho^2 \rangle$ is roughly proportional to $\langle \rho \rangle$ along the vertical direction in the disk core. Hence, if $\langle \delta \rho^2 \rangle_{\text{mid}}$ is given, one can reconstruct the vertical profile of the density fluctuation in the disk core according to

$$\langle \delta \rho^2 \rangle \approx \langle \delta \rho^2 \rangle_{\text{mid}} \langle \rho \rangle_{\text{mid}} \approx \langle \delta \rho^2 \rangle_{\text{mid}} \exp \left( -\frac{z^2}{2h^2} \right) \approx 0.47 \alpha_{\text{core}} \langle \rho \rangle_{\text{mid}}^2 \exp \left( -\frac{z^2}{2h^2} \right), \quad \text{(24)}$$

where we have used $\langle \rho \rangle \approx \langle \rho \rangle_{\text{mid}} \exp(-z^2/2h^2)$ and Equation (23) in the second and third equalities, respectively.

#### 5.1.3. Outflow Flux

We have seen in Section 4.1 and Figure 4(f) that MRI drives outgoing gas flow at the outer boundaries of the active layers. The MRI-driven outflow has been first observed by Suzuki & Inutsuka (2009) in shearing-box simulations and been recently demonstrated by Flock et al. (2011) in global simulations. Suzuki & Inutsuka (2009) and Suzuki et al. (2010) point out that this outflow might contribute to the dispersal of protoplanetary disks, although it is still unclear whether the outflow can really escape from the disks (see below). Meanwhile, MRI also contributes to the accretion of the gas in the radial direction. For consistent modeling of these two effects, we seek how the accretion stress and outflow flux are correlated with each other.
Although outflow from the simulation box is a general phenomenon in our simulations, it is unclear whether the outflow leaves or returns to the disk. In fact, the outflow velocity observed in our simulations does not exceed the sound speed even at the vertical boundaries. Since the escape velocity is higher than the sound speed, this means that the outflow does not have an outward velocity sufficient to escape out of the disk. Acceleration of the outflow beyond the escape velocity has not been directly demonstrated by previous simulations as well (Suzuki et al. 2010; Flock et al. 2011). However, Suzuki et al. (2010) point out the possibility that magnetocentrifugal forces and/or stellar winds could accelerate the outflow to the escape velocity. If the escape of the outflow is confirmed in the future, our scaling formula for $\dot{m}_w$ will certainly become a useful tool for discussing the dispersal of protoplanetary disks.

5.2. Saturation Predictors for the Accretion Stresses

In the previous subsection, we have shown that the amplitudes of various turbulent quantities scale with the vertically integrated stresses $\alpha_{\text{core}}$ and $\alpha_{\text{atm}}$. The next step is to find out how to predict $\alpha_{\text{core}}$ and $\alpha_{\text{atm}}$ in the saturated state from the vertical magnetic flux $B_0$ (or equivalently $\beta_0$) and the resistivity profile $\eta$. As shown in Section 4.2, the turbulent state of a disk depends on the resistivity only through the critical heights of the dead zone, $h_\Lambda$ and $h_{\text{res}}$. Furthermore, the values of $h_\Lambda$ and $h_{\text{res}}$ are only weakly affected by the nonlinear evolution of MRI since the fluctuations in $B_z$ and $\rho$ are small inside the dead zone (see Section 4.1). Therefore, we expect that the effect of the resistivity can be well predicted by the values of $h_\Lambda$ and $h_{\text{res}}$ in the initial state, i.e., $h_{\Lambda,0}$ and $h_{\text{res},0}$. With this expectation, we try to derive saturation predictors for $\alpha_{\text{core}}$ and $\alpha_{\text{atm}}$ as a function of $\beta_{0,0}$, $h_{\Lambda,0}$, and $h_{\text{res},0}$.

First, we focus on $\alpha_{\text{core}}$. Figure 11(a) plots $\alpha_{\text{core}}$ versus $\beta_{0,0}^{-1}$ for all our simulations. We see that $\alpha_{\text{core}}$ scales roughly linearly with $\beta_{0,0}^{-1}$. The deviation from the linear scaling is expected to come from the difference in the dead zone size, i.e., $h_{\Lambda,0}$ and $h_{\text{res},0}$. In Figure 12, we plot the product $\alpha_{\text{core}} \beta_{0,0}$ as a function of $h_{\text{res},0}$ for models except for Ideal and Y4. We find that $\alpha_{\text{core}} \beta_{0,0}$ is well predicted by a simple formula

$$\alpha_{\text{core}} \beta_{0,0} = 510 \exp(-0.54 h_{\text{res},0}/h).$$

Figure 11(b) replots the data in Figure 11(a) by replacing $\beta_{0,0}$ with $\exp(-0.54 h_{\text{res},0}/h) \beta_{0,0}^{-1}$. For models Ideal and Y4, Equation (27) underestimates $\alpha_{\text{core}}$. As explained in Section 4.2, these models exhibit higher magnetic activity near the midplane than the other models because of no or a thin dead zone ($2h_\Lambda < h$). We expect that the higher magnetic activity gives additional contribution to $\alpha_{\text{core}}$. Taking into account this effect, we arrive at the final predictor function,

$$\alpha_{\text{core}} = 510 \exp(-0.54 h_{\text{res},0}/h) \beta_{0,0}^{-1} + 0.011 \exp(-3.6 h_{\Lambda,0}/h).$$

Here, the numerical factors 0.011 and 3.6 appearing in the second term have been chosen to reproduce the results of runs Ideal and Y4, respectively. Figure 11(c) compares the final fitting formula with the numerical data. It can be seen that Equation (28) predicts well $\alpha_{\text{core}}$ for all our models. Note that the second term of the predictor function is assumed to have no explicit linear dependence on $\beta_{0,0}^{-1}$ unlike the first term. In fact, it is possible to reproduce our data by multiplying the second term by a prefactor ($3 \times 10^3 / \beta_{0,0}$). However, as we will see below, the absence of the prefactor makes the predictor function consistent with the results of ideal MHD simulations in the literature.
Disk core accretion stress $\alpha_{\text{core}}$ for all our simulations. (a) Versus the inverse initial plasma beta $\beta_{z0}^{-1}$. (b) Versus $\beta_{z0}^{-1}$ multiplied by $\exp(-0.54 h_{\text{res},0}/h)$ (see also Figure 12). (c) Versus the final predictor function, Equation (28) (solid line). The symbols correspond to models Ideal (circle), X0–X3 (open squares), Y1–Y4 (triangles), W1–W3 (crosses), X1a–X1d (filled squares), and FS03L (plus sign). The dashed lines in panels (a) and (b) are linear fits, shown only for reference.

(A color version of this figure is available in the online journal.)

The predictor function for $\alpha_{\text{atm}}$ can be obtained in a similar way. Figure 13(a) shows $\alpha_{\text{atm}}$ versus $\beta_{z0}^{-1}$ for all our runs. We find that a simple linear relation $\alpha_{\text{atm}} = 530\beta_{z0}^{-1}$ fit the data well except for models Ideal and Y4. This means that $\alpha_{\text{atm}}$ is characterized only by $\beta_{z0}^{-1}$ as long as the dead zone is thick ($2h_\Lambda > h$). To take into account the cases of thin dead zones, we add a term proportional to $\exp(-3.6 h_{\text{atm},0}/h)$ as has been done for $\alpha_{\text{core}}$, and obtain

$$\alpha_{\text{atm}} = 530\beta_{z0}^{-1} + 0.0043 \exp(-3.6 h_{\text{atm},0}/h),$$

(29)

where the prefactor 0.0043 for the second term has been determined to fit the result of run Ideal. As seen in Figure 13(b), Equation (29) predicts the value of $\alpha_{\text{atm}}$ well for all our runs.

(A color version of this figure is available in the online journal.)

Our predictor functions indicate that the vertically integrated accretion stress is inversely proportional to $\beta_{z0}$ when a large dead zone is present ($h_\Lambda > h$). As we show below, this dependence originates from the magnitude of the accretion stress at the outer boundaries of the active layers, $|z| \approx h_{\text{ideal}}$. When a dead zone exists, the dominant contribution to $\alpha$ comes from the accretion stress at that location (see Figures 5 and 6). As shown in Figure 14, our simulations suggest that the accretion stress at
of the active layer ($z = h_{\text{ideal}}$) for all runs. The symbols correspond to models Ideal (circle), X0–X3 (open squares), Y1–Y4 (triangles), W1–Y3 (crosses), X1a–X1d (filled squares), and FS03L (plus sign). The solid line shows a linear fit (Equation (30)).

(A color version of this figure is available in the online journal.)

Figure 14. Accretion stress ($w_{xy}$) vs. gas density ($\rho$) at the upper boundary of the active layer ($z = h_{\text{ideal}}$) for all runs. The symbols correspond to models Ideal (circle), X0–X3 (open squares), Y1–Y4 (triangles), W1–Y3 (crosses), X1a–X1d (filled squares), and FS03L (plus sign). The solid line shows a linear fit (Equation (30)).

This implies that

$$
|z| = h_{\text{ideal}} \text{ obeys a simple relation}
$$

$$
\langle w_{xy} \rangle(h_{\text{ideal}}) \approx 0.18(\rho)(h_{\text{ideal}}) c_s^2.
$$

(30)

This means that the averaged accretion stress at $|z| = h_{\text{ideal}}$ is 18% of the averaged gas pressure $\langle \rho c_s^2 \rangle$ at the same height. For the definition of $h_{\text{ideal}}$, the gas density at $|z| = h_{\text{ideal}}$ is related to $\langle B \rangle$ at the same height as $\langle \rho \rangle(h_{\text{ideal}}) = (2\pi)^2 (\langle B \rangle)^2(h_{\text{ideal}})/4\pi c_s^2$.

Since our simulations suggest $\langle B \rangle(h_{\text{ideal}}) \approx 10 B_{\text{001}}$, the relation means $\langle \rho \rangle(h_{\text{ideal}}) \approx 10(2\pi)^2 B_{\text{001}}^2/4\pi c_s^2 \approx 3^4 \rho_{\text{001}}^{-1} \rho_0$. Using this fact, Equation (30) can be rewritten into the linear relation between $\langle w_{xy} \rangle(h_{\text{ideal}})$ and $\rho_0$:

$$
\langle w_{xy} \rangle(h_{\text{ideal}}) \approx 100 \beta_{\text{z0}}^{-1} \rho_0 c_s^2.
$$

(31)

When a dead zone is present, the level of $\alpha$ is determined by $\langle w_{xy} \rangle(h_{\text{ideal}})$ (see above), so we have $\alpha \propto \beta_{\text{z0}}^{-1}$.

We remark that the vertically integrated stress does not scale linearly with the column density of active layers. This is shown in Figure 15, where we compare $\alpha$ with the column density $\Sigma_{\text{active}}$ of the active region $h_\text{mid} < |z| < h_{\text{ideal}}$. We see that $\alpha$ decreases much more slowly than $\Sigma_{\text{active}}$ when $\Sigma_{\text{active}}$ is less than 10% of the total gas surface density. This reflects the fact that the dominant contribution to $\alpha$ comes from the outer boundaries of the active zones, $|z| \approx h_{\text{ideal}}$.

It is useful to see how the predictor functions work when a dead zone is absent. If $h_{\Lambda,0} = h_{\text{res,0}} = 0$, Equations (28) and (29) predict the total accretion stress $\alpha = 1.0 \times 10^3 \beta_{\text{z0}}^{-1} + 0.015$. This implies that $\alpha$ is constant ($\alpha \approx 10^{-2}$) for $\beta_{\text{z0}} \gtrsim 10^5$ and increases linearly with $\beta_{\text{z0}}^{-1}$ ($\alpha \approx 10^{-2}(10^5/\beta_{\text{z0}})$) for $\beta_{\text{z0}} \lesssim 10^5$. Strikingly, this prediction is consistent with the finding of Suzuki et al. (2010, see their Figure 2). The existence of the floor value $\alpha \approx 10^{-2}$ at low net vertical magnetic fluxes (i.e., at high $\beta_{\text{z0}}$) is also supported by recent stratified MHD simulations with zero net flux (Davis et al. 2010). These facts suggest that our predictor functions are applicable even when a dead zone is absent.

6. VERTICAL DIFFUSION COEFFICIENT

As seen in the previous section, sound waves excited in the upper layers create fluctuations in the gas velocity near the midplane. It has been well known that fully developed MRI-driven turbulence causes the diffusion of small dust particles (Joehansen & Klahr 2005; Turner et al. 2010). However, it has not been fully understood how the sound waves inside a dead zone affect the dynamics of dust particles there. For example, Suzuki & Inutsuka (2009) speculated that the sound waves might promote dust sedimentation by transferring the downward momentum to dust particles. On the other hand, Turner et al. (2010) reported that the waves excite vertical oscillation of dust particles deep inside the dead zone and thus prevent the formation of a thin dust layer. Since dust sedimentation is crucial to planetesimal formation via gravitational instability, it is worth addressing here how it is affected by the velocity dispersion created by sound waves.

Here, we focus on the dynamics of small dust particles, and model the swarm of the particles as a passive scalar as was previously done by Johansen & Klahr (2005) and Turner et al. (2010). We assume that dust particles are very small and their stopping time $\tau_s$ is much shorter than the turnover time of turbulence ($\sim \Omega^{-1}$). We also assume that the dust density is lower than the gas density and hence the dust has no effect on the gas motion. Under these assumptions, the velocity of dust particles relative to the gas can be approximated by the terminal velocity $V_T = -\Omega^2 \tau_s z \hat{z}$, where $\hat{z}$ is the unit vector for the $z$-direction. Then, the equation of continuity for dust is given by

$$
\frac{\partial \rho_d}{\partial t} + \nabla \cdot [\rho_d (v + V_T)] = 0,
$$

(32)

where $\rho_d$ is the dust density. Equation (32) has an advantage that the time step can be taken longer than $\tau_s$ in numerical calculation.

We have solved Equation (32) for four models (Ideal, X1, X3, and Y1) with the initial condition that the dust-to-gas mass ratio $f = \rho_d/\rho$ is constant throughout the simulation box. To extract the effect of the quasi-stationary turbulence, we insert the dust 100 (for models X1, X3, and Y1) or 250 (for model Ideal) orbits after the beginning of the MHD calculations. The stopping time $\tau_s$ is set to $\tau_s = 0.01 \Omega^{-1}$ for model Ideal and $\tau_s = 0.001 \Omega^{-1}$ for the other models. The longer $\tau_s$ has been adopted for model Ideal to allow the dust to settle appreciably in the stronger turbulence. In reality, the stopping time of a dust particle depends on the gas density and hence on $z$, but we ignore this dependency for simplicity.

Figure 16 shows the temporal evolution of the dust density at the midplane, $\rho_d, \text{mid}$, for the four MHD runs. The solid curves
Figure 16. Temporal evolution of the dust density at the midplane in various MHD runs. The four panels show the results for models Ideal (upper left), X1 (upper right), X3 (lower left), and Y1 (lower right). The solid curves show the horizontally averaged dust density $\rho_{d,\text{mid}}$ observed in the MHD runs, while the dotted curves show the evolution of $\rho_{d,\text{mid}}$ in a laminar disk. The dashed curves are the predictions from the one-dimensional advection-diffusion equation (Equation (33)). The diffusion coefficient $D_z$ in Equation (33) is assumed to be proportional to $\langle \delta v_z^2 \rangle / \Omega$, and the proportionality factor has been chosen to best reproduce the evolution of $\rho_{d, \text{mid}}$ in the MHD runs.

Figure 17. Snapshots of the vertical distribution of the dust-to-gas mass ratio $f = \rho_d / \rho$ at $t = 100$, 250, 400, and 550 orbits for model X1. The solid curves show the horizontally averaged $f$ observed in run X1. The dotted curves show the solutions to the one-dimensional advection-diffusion equation (Equation (33)) with $D_z(z) = 0.5 \langle \delta v_z^2 \rangle / \Omega$. The dashed curves show the solution to Equation (33) with a constant diffusion coefficient $D_z = 0.5 \langle \delta v_z^2 \rangle / \Omega$.

We examine here whether Equations (33) and (34) work well even when a dead zone is present. We solve Equation (33) with $D_z = b \langle \delta v_z^2 \rangle / \Omega$, where the vertical distribution of $\langle \delta v_z^2 \rangle$ is taken from temporally and horizontally averaged MHD data and $b$ is a dimensionless fitting parameter. The dashed curves in Figure 16 show the predictions by the advection-diffusion model, where $b$ is set to be 1.0, 0.5, 0.9, and 1.0 for runs Ideal, X1, X3, and Y1, respectively. It can be seen that the advection-diffusion model with $b \sim 1$ successfully reproduces the long-term evolution of the observed $\rho_{d, \text{mid}}$ for all the models. It is striking that a constant $b$ reproduces well the evolution of the dust density at all heights, as is shown in Figure 17. In this figure, the solid and dashed curves show the vertical distribution of the dust-to-gas mass ratio $f = \rho_d / \rho$ observed in run X1.
and predicted by the advection-diffusion model with $b = 0.5$, respectively. In this run, the boundaries between the active and dead zones are located at $|z| = h_\Lambda \approx 2.5h$ (see Table 2). However, Equation (33) successfully predicts the evolution of $f$ even if we do not change the value of $b$ across the boundaries. This fact supports the idea that sound waves propagating across a dead zone contribute to the diffusion of dust particles just as turbulence does in active zones.

It is worth mentioning here that the diffusion coefficient $D_z$ increases with $|z|$ as has been pointed out by Turner et al. (2006) and Fromang & Nelson (2009). This effect is particularly significant at high altitudes where the gas density is much lower than that at the midplane, because $D_z \propto \langle \delta v^2_z \rangle$ is roughly proportional to the inverse of the gas density.6 The dotted curves in Figure (17) show how Equation (33) would fail to predict dust evolution if one assumed a constant diffusion coefficient $D_z = 0.5 \langle \delta v^2_z \rangle_{\text{mid}}/\Omega$. We see that the constant diffusion coefficient model significantly underestimates the dust density at $|z| \gg h$. This fact will merit consideration when modeling the chemical evolution of protoplanetary disks, in which the vertical mixing of molecules is of importance (Heinzeller et al. 2011).

Finally, we give a simple analytic recipe for the vertical distribution of $D_z$. It is useful to rewrite Equation (34) in terms of $\langle \delta v^2_z \rangle$, for which the scaling relation (Equation (22)) and predictor function (Equation (28)) are available. Table 2 lists the ratio of $\langle \delta v^2_z \rangle_{\text{mid}}/\langle \delta v^2_z \rangle_{\text{atm}}$ for all our simulations. It can be seen that $\langle \delta v^2_z \rangle_{\text{mid}} \approx (32\% \pm 15\%) \times \langle \delta v^2_z \rangle_{\text{atm}}$, indicating that $\langle \delta v^2_z \rangle_{\text{mid}}$ is roughly equal to a third of $\langle \delta v^2_z \rangle_{\text{atm}}$. Furthermore, the ratio $\langle \delta v^2_z \rangle_{\text{mid}}/\langle \delta v^2_z \rangle_{\text{atm}}$ is approximately constant in the disk core, as is illustrated in Figure 4(c). Based on these facts, we approximate $\langle \delta v^2_z \rangle_{\text{mid}}$ as $\langle \delta v^2_z \rangle/3$ in the disk core. Using this approximation together with the scaling relation for $\langle \delta v^2_z \rangle$ (Equation (22)), we rewrite Equation (34) as

$$D_z \approx \frac{1}{3} \langle \delta v^2_z \rangle/\Omega \approx 0.3 \frac{\alpha_{\text{core}} c_s^2}{\Omega} \exp\left(\frac{z^2}{2h^2}\right).$$

If one uses this equation together with the predictor function for $\alpha_{\text{core}}$ (Equation (28)), one can calculate the vertical distribution of $D_z$ in the disk core for given $\beta_{\text{eq}}$ and $\eta$.

7. DISCUSSION: EFFECTS OF NUMERICAL RESOLUTION

All MHD simulations presented in the previous sections were performed with the numerical resolution of $40 \times 80 \times 200$ grid cells for the simulation box of size $2\sqrt{2} h \times 8\sqrt{2} h \times 10\sqrt{2} h$. Here, we examine how the numerical resolution saturated the state of turbulence.

We carry out X1 simulations changing the numerical resolution to $20 \times 40 \times 100$ cells and $80 \times 160 \times 400$ cells. Figure 18 compares the saturated values of various quantities obtained from the two runs with the values from the original X1 run (Table 2). Here, the horizontal axis shows the number of grid cells per length $\sqrt{2} h$ in the vertical direction, $n_z$ (see footnote 3). The value $n_z = 20$ corresponds to our original resolution. We see that the change of the resolution hardly affects the integrated accretion stresses $\alpha$ and $\alpha_{\text{core}}$ and outflow flux $m_w$, suggesting that the resolution of $n_z = 20$ is sufficient for these quantities to converge well. By contrast, the velocity and density dispersions $\langle \delta v^2_z \rangle_{\text{mid}}$ and $\langle \delta \rho^2 \rangle_{\text{mid}}$ increase with improving the resolution. Since the energy input rate to turbulence should be the same if the integrated accretion stress is unchanged, the resolution dependence of the velocity and density dispersions is expected to mainly come from the artificial dissipation of sound waves in the simulation box. However, we also see that this effect becomes less significant as the resolution is improved. Detailed inspection shows that the fractional increase in $\langle \delta v^2_z \rangle_{\text{mid}}$ is 81% when going from $n_z = 10$ to $n_z = 20$ but is 40% when going from $n_z = 20$ to $n_z = 40$. This suggests that the amplitudes of the velocity and density fluctuations should converge to finite values in the limit of high resolutions ($n_z \to \infty$). This is to be expected, since sound waves in a stratified disk physically dissipate through, e.g., shock formation, particularly at high altitudes where the gas density is low and therefore the amplitudes of the waves become large. We find that the data for $\langle \delta v^2_z \rangle_{\text{mid}}/c_s^2$ shown in Figure 18 lie on a curve $\langle \delta v^2_z \rangle_{\text{mid}}/c_s^2 = 0.012 - 0.0015 n_z^{-0.15}$. This implies a converged value of $\langle \delta v^2_z \rangle_{\text{mid}}/c_s^2 \approx 0.012$, which is five times higher than that obtained in our $n_z = 20$ simulation ($\langle \delta v^2_z \rangle_{\text{mid}}/c_s^2 \approx 0.0024$). From this estimate, we see that $\langle \delta v^2 \rangle$ and $\langle \delta \rho^2 \rangle$ could be underestimated by a factor of several in the simulations presented in this study.

In summary, we find that the outflow mass flux and vertically integrated accretion stress converge well within our numerical resolution. This suggests that the predictor functions for $\alpha_{\text{core}}$ and $\alpha_{\text{atm}}$ (Equations (28) and (29)) and the scaling relation between $m_w$ and $\alpha_{\text{atm}}$ are hardly affected by the resolution. On the other hand, the amplitudes of sound waves could be underestimated by a factor of several because of the finite grid size. Future high-resolution simulations will enable us to better quantify the scaling relations between $\langle \delta v^2 \rangle$ and $\alpha_{\text{core}}$ and between $\langle \delta \rho^2 \rangle$ and $\alpha_{\text{core}}$ (Equations (22) and (24)).

8. SUMMARY

Good knowledge about the turbulent structure of protoplanetary disks is essential for understanding planet formation. To provide an empirical basis for modeling the coevolution of dust and MRI, we performed MHD simulations of a vertically strat-
ified shearing box with an MRI-inactive “dead zone” of various sizes and with a vertical magnetic flux of various strengths. Our findings are summarized as follows.

1. We have introduced the critical heights \( h_{\text{ideal}}, h_\Lambda, \) and \( h_{\text{res}} \) that characterize the MRI in a stratified disk (Section 3). We have found that the vertical structure of MRI-driven turbulence depends on the resistivity profile only through the critical heights for the dead zone \( (h_\Lambda \text{ and } h_{\text{res}}) \) and is insensitive to the detail of the resistivity profile (Section 4.2).

2. In the “disk core” \( (|z| < h_{\text{ideal}}) \), the density-weighted velocity dispersion \( \langle \delta v_z^2 \rangle / \langle \rho \rangle \) is nearly constant along the vertical direction (Section 4.1). This means that the velocity dispersion is approximately inversely proportional to the gas density. Weak dependence on \( z \) is also found for \( \langle \delta \rho^2 \rangle / \langle \rho \rangle \), meaning that the density fluctuation \( \langle \delta \rho^2 \rangle^{1/2} \) is proportional to the square root of the averaged density.

3. The accretion stresses in the disk core and “atmosphere” \( (|z| > h_{\text{ideal}}) \) contribute differently to the turbulent structure of a disk (Section 5.1). The velocity dispersion \( \langle \delta v_z^2 \rangle \) and density fluctuation \( \langle \delta \rho^2 \rangle \) in the disk core depend linearly on the accretion stress integrated over the core, \( \alpha_{\text{core}} \) (Equations (21) and (23)). By contrast, the outflow mass flux \( m_w \) depends linearly on the stress integrated over the atmosphere, \( \alpha_{\text{atm}} \) (Equation (26)).

4. We have obtained simple empirical formulae that predict the vertically integrated stresses \( \alpha_{\text{core}} \) and \( \alpha_{\text{atm}} \) in the saturated state (Section 5.2; Equations (28) and (29)). These are written as a function of the strength of the vertical magnetic flux \( \beta_\alpha \) and the critical heights of the dead zone measured in the nonturbulent state \( (h_{\text{res}},0 \text{ and } h_\Lambda,0) \). These predictor functions together with the saturation relations described above allow us to calculate various turbulent quantities for a given resistivity profile and a net vertical flux.

5. We have confirmed that the vertical diffusion coefficient \( D_z \) of contaminants is given by \( D_z \approx \langle \delta v_z^2 \rangle / \Omega \) both inside and outside a dead zone (Section 6). This implies that sound waves propagating across a dead zone contribute to the diffusion of dust particles just as turbulence does in active zones. We have obtained a simple analytic recipe for the vertical distribution of \( D_z \) as a function of \( \alpha_{\text{core}} \) on the basis of our MHD simulation data (Equation (35)).

The empirical formulae obtained in this study enable us to predict the amplitudes of various turbulent quantities in a protoplanetary disk with a dead zone. The steps to be performed are as follows.

1. Prepare the vertical profile of the ohmic resistivity \( \eta \), and find \( h_\Lambda \) and \( h_{\text{res}} \) from Equations (12) and (13). A realistic profile of \( \eta \) in the presence of dust particles can be obtained by solving the ionization state of the gas and the charge state of the dust simultaneously (e.g., Sano et al. 2000; Ilgner & Nelson 2006; Okuzumi 2009).

2. Calculate \( \alpha_{\text{core}} \) and \( \alpha_{\text{atm}} \) using the predictor functions, Equations (28) and (29).

3. One can now calculate the turbulent viscosity of the disk as \( \nu_{\text{turb}} = (3/2)(\alpha_{\text{core}} + \alpha_{\text{atm}} h_\Lambda^2 / \Omega) \) (see Equations (18)–(20)). The vertical distribution of the gas velocity dispersion and density fluctuation in the disk core \( (|z| < h_{\text{ideal}}) \) can be calculated from Equations (22) and (24), respectively. The outflow mass flux can be evaluated from Equation (26). For the diffusion coefficient in the disk core, one can use Equation (35).

When using our empirical formulae, it should be kept in mind that our scaling relations for velocity and density fluctuations (Equations (22) and (24)) could underestimate their mean-squared amplitudes relative to the integrated accretion stress by a factor of several because of the numerical dissipation of sound waves (Section 7). Future high-resolution simulations will allow to better quantify the saturation level of sound wave amplitudes.

We are grateful to Neal Turner and Takayoshi Sano for providing us with their MHD simulation data that have motivated us to start this study. We also thank Shu-ichiro Inutsuka, Takeru Suzuki, Taku Takeuchi, Hidekazu Tanaka, Takayuki Tanigawa, and Mordecai-Mark Mac Low for useful discussion and fruitful comments. Calculations were made on the Cray XT4 at the CICA, National Astronomical Observatory of Japan, and SR16000 at the Yukawa Institute for Theoretical Physics, Kyoto University. S.O. is supported by a Grant-in-Aid for JSPS Fellows (22 - 7006) from the MEXT of Japan.

APPENDIX

IONIZATION DEGREE AND OHMIC RESISTIVITY IN PROTOPLANETARY DISKS

In this appendix, we explain how the resistivity profile adopted in this study (Equation (4)) is related to realistic resistivity profiles in protoplanetary disks. Since the resistivity is inversely proportional to the ionization degree (more precisely, the electron abundance; see Blaes & Balbus 1994), we will see how the ionization degree depends on the height \( z \) above the midplane.

Recombination occurs in the gas phase and on dust surfaces. The gas-phase recombination dominates if the total surface area of dust particles is negligibly small. In this case, the equation for the ionization-recombination equilibrium is given by

\[
\xi n_n = \gamma_{ie} n_i n_e = \gamma_{ie} n_e^2,
\]

where \( \xi \) is the ionization rate (the probability per unit time at which a molecule is ionized), \( n_n \), \( n_i \), and \( n_e \) are the number densities of neutrals, ions, and electrons, respectively, and \( \gamma_{ie} \) is the gas-phase recombination rate coefficient. The second equality in the above equation assumes the charge neutrality in the gas phase, \( n_i = n_e \). Equation (A1) leads to the electron abundance

\[
x_e = \frac{n_e}{n_n} = \sqrt{\frac{\xi}{\gamma_{ie} n_n}} \propto \sqrt{\xi} \exp\left(\frac{z^2}{4h^2}\right),
\]

which means that the resistivity is proportional to \( \xi^{-1/2} \exp(-z^2/4h^2) \). Thus, if the vertical dependence of \( \xi \) can be neglected, the resistivity profile is given by Equation (4) with \( h_\eta = \sqrt{2}h \). Note that Equation (5) is derived instead of Equation (4) if cosmic-ray ionization is assumed and the attenuation of cosmic rays toward the midplane is taken into account (Fleming & Stone 2003).

If the total surface area of dust particles is large, recombination occurs mainly on dust surfaces. In this case, the equation for the ionization-recombination equilibrium is given by

\[
\xi n_n = \gamma_{de} n_d n_e,
\]

where \( \gamma_{de} \) is the sticking rate coefficient for dust–electron collision and \( n_d \) is the number density of dust particles. The
sticking rate coefficient depends on the charge of the dust particles, and ultimately on $n_e$ via the charge neutrality (see Okuzumi 2009), but we will ignore this dependence in the following. From Equation (A3), we have

$$x_e = \frac{\zeta}{\gamma_d n_d} \propto \zeta \exp \left( \frac{z^2}{2h_d^2} \right),$$  \hspace{1cm} (A4)

where we have assumed that $n_d \propto \exp(-z^2/h_d^2)$ with $h_d$ being the scale height of the dust particles (in fact, one can show using Equation (33) that $n_d$ obeys a Gaussian distribution in sedimentation–diffusion equilibrium if $D_z \propto \tau_s \propto n^{-1} \propto \exp(-z^2/h_d^2)$; the condition $\tau_s \propto n^{-1} \propto n^{-1}_z$ is satisfied if the size of the dust particles is smaller than the mean free path of the gas). Thus, ignoring the dependence of $\zeta$ on $z$, the resistivity $\eta \propto x_e^{-1}$ is given by Equation (4) with $h_\eta = h_d$.

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