SUPERNOVAE CONSTRAINTS ON DGP MODEL AND COSMIC TOPOLOGY

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We study the constraints that the detection of a non-trivial spatial topology may place on the parameters of braneworld models by considering the Dvali-Gabadadze-Porrati (DGP) and the globally homogeneous Poincaré dodecahedral spatial (PDS) topology as a circles-in-the-sky observable topology. To this end we reanalyze the type Ia supernovae constraints on the parameters of the DGP model and show that PDS topology gives rise to strong and complementary constraints on the parameters of the DGP model.

1. Introduction

In the standard cosmology, the Universe is described by a space-time manifold $\mathcal{M}_4 = \mathbb{R} \times M_3$ endowed with a locally (spatially) homogeneous and isotropic metric

$$ds^2 = -dt^2 + a^2(t) \left[ \frac{dr^2}{1 - kr^2} + r^2(d\theta^2 + \sin^2 \theta d\phi^2) \right],$$

(1)

where, depending on the spatial curvature $k$, the geometry of the 3-space $M_3$ is either Euclidean ($k = 0$), spherical ($k = 1$), or hyperbolic ($k = -1$). The spatial section $M_3$ is usually taken to be one of the simply-connected spaces: Euclidean $\mathbb{R}^3$, spherical $S^3$, or hyperbolic $\mathbb{H}^3$. However, given that the connectedness of the spatial sections $M_3$ has not been determined by cosmological observations, and since geometry does not fix the topology, our 3-dimensional space may be one of the possible multiply connected quotient manifolds of the form $\mathbb{R}^3/\Gamma$, $S^3/\Gamma$, and $\mathbb{H}^3/\Gamma$, where $\Gamma$ is a fixed-point free group of isometries of the corresponding covering space. Thus, for example, for the Euclidean geometry ($k = 0$) besides $\mathbb{R}^3$ there are 6 classes of topologically distinct compact orientable spaces $M_3$.

The immediate observational consequence of a detectable nontrivial topology of $M_3$ is the existence of the circles-in-the-sky, i.e., pairs of matching circles will be imprinted on the CMBR anisotropy sky maps. Hence, to observationally probe a putative nontrivial topology of $M_3$, one should examine the full-sky CMBR maps in order to extract the pairs of correlated circles and determine the spatial topology.

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In the context of the 5D braneworld models the universe is described by a 5-dimensional metrical orbifold (bulk) $O_5$ that is mirror symmetric ($Z_2$) across the 4D brane (manifold) $M_4$. Thus, the bulk can be decomposed as $O_5 = M_4 \times E_1 = R \times M_3 \times E_1$, where $E_1$ is a $Z_2$ symmetric Euclidean space, and where $M_4$ is endowed with a Robertson–Walker metric $\mathbf{(1)}$, which is recovered when $w = 0$ for the extra non-compact spatial dimension. In this way, the multiplicity of possible inequivalent topologies of our 3-dimensional space, and the physical consequences of a non-trivial detectable topology of $M_3$ as the circles-in-the-sky are brought on the braneworld scenario.

Here we briefly study the constraints that a detection of a spatial topology may place on the parameters of a simple braneworld modified-gravity model that accounts for the accelerated expansion of the universe via infrared modifications to general relativity, namely the Dvali-Gabadadze-Porrati (DGP) model,$^4$ as generalized to cosmology by Deffayet.$^5$ To this end we reanalyze the type Ia supernovae constraints on the parameters of the DGP model and show that PDS topology gives rise to strong and complementary constraints on the parameters of the DGP model.

2. Constraints and Concluding Remarks

Using the first-year data the WMAP team reported a total density value$^6$ $\Omega_{\text{tot}} = 1.02 \pm 0.02$, while the three-year WMAP article$^7$ reports six different values for the $\Omega_{\text{tot}}$ ranging from a very nearly flat $\Omega_{\text{tot}} = 1.003^{+0.017}_{-0.013}$ to positively curved $\Omega_{\text{tot}} = 1.037^{+0.015}_{-0.021}$ depending on the combination of data set used to resolve the geometrical degeneracy.

The Poincaré dodecahedral space (PDS), $D = S^3/I^*$, explains both the suppression of power of the low multipoles and this observed total density. We note, however, that other topologies as $\mathcal{O} = S^3/O^*$ also remain viable.$^9$ Attempts to find antipodal or nearly-antipodal circles-in-the-sky in the WMAP data have failed.$^{10}$

There is, however, claim of hints of matching circles$^{11}$ in ILC WMAP maps, which a second group has confirmed$^{12}$ but have also shown that the circle detection lies below the false positive threshold.$^{12}$ On the other hand, even if one embraces the result that pairs of antipodal (or nearly antipodal) circles of radius $\gamma \geq 5^\circ$ are undetectable in the current CMBR maps,$^{12}$ the question arises as to whether the circles are not there or are merely hidden by various sources of contamination (Doppler, integrated Sachs-Wolfe, e.g.), or even due to the angular resolution of the current CMBR maps, as suggested in Ref. 14. The answer to these questions requires great care, among other things, because the level of contamination depends on both the choice of the cosmological models (parameters) and on the topology.$^{15}$ Results so far remain non-conclusive, i.e., one group finds their negative outcome to be robust for globally homogeneous topologies, including the dodecahedral space, in spite of contamination,$^{12}$ while another group finds the contamination strong enough to hide the possible correlated circles in the current CMBR maps.$^{13,14}$

In $D$ space the pairs of matching circles are necessarily antipodal as shown in
Fig. 1. A schematic illustration of two antipodal matching circles in the LSS.

Clearly the distance between the centers of each pair of the correlated circles is twice the injectivity radius of the smallest sphere inscribable $D$. A straightforward use of trigonometric relations for the right-angled spherical triangle shown in Fig. 1 yields

$$\chi_{lss} = \frac{d_{lss}}{a_0} = \sqrt{|\Omega_k|} \int_1^{1+z_{lss}} \frac{H_0}{H(x)} \, dx = \tan^{-1} \left[ \frac{\tan r_{inj}}{\cos \alpha} \right],$$

(2)

where $d_{lss}$ is the radius of the LSS, $x = 1 + z$ is an integration variable, $H$ is the Hubble parameter, $\Omega_k = 1 - \Omega_{tot}$, $r_{inj}$ is a topological invariant (equals to $\pi/10$ for $D$), the distance $\chi_{lss}$ is measured in units of the curvature radius, $a_0 = a(t_0) = (H_0 \sqrt{|1-\Omega_{tot}|})^{-1}$, and $z_{lss} = 1089.6$.

Equation (2) makes apparent that $\chi_{lss}$ depends on the cosmological scenario. For the DGP model one has

$$\left( \frac{H}{H_0} \right)^2 = \Omega_k (1+z)^2 + \left( \sqrt{\Omega_m (1+z)^3 + \Omega_c} + \sqrt{\Omega_c} \right)^2,$$

(3)

where $r_c$ is a length scale beyond which gravity starts to leak out into the bulk. Equations (2) and (3) give the relation between the angular radius $\alpha$ and the parameters of the DGP model, and thus can be used to set constraints on these parameters.

To illustrate the role of the cosmic topology in constraining the DGP parameter we consider the $D$ spatial topology, and assume the angular radius $\alpha = 50^\circ$ and uncertainty $\delta \alpha \simeq 6^\circ$. Figure 2 shows the results of our joint SNe Ia plus cosmic topology analysis, where the gold sample of 157 SNe Ia, as compiled by Riess et al., was used. There we display the confidence regions in the parametric plane $\Omega_k - \Omega_m$ and also the regions from the conventional analysis with no such a topology assumption. The comparison between these regions makes clear that the effect of the $D$ topology is to reduce considerably the area corresponding to the confidence intervals in the parametric plane as well as to break degeneracies arising from the current SNe Ia measurements. The best-fit parameters for this joint analysis are $\Omega_m = 0.232$ and $\Omega_k = -0.018$. 

Fig. 2. Confidence contours (68.3%, 95.4% and 99.7%) in the $\Omega_m - \Omega_k$ plane for DGP model obtained with the SNe Ia gold sample assuming a $\mathcal{D}$ space topology with $\gamma = 50^\circ \pm 6^\circ$. Also shown are the contours obtained assuming no topological data (dash-dotted lines) and the ones corresponding to topology only (dotted lines).

For a detailed analysis of topological constraints in the context of the DGP and other models, including braneworld inspired models, see Refs. 16 and Refs. 17. Finally, we note that in Refs. 19 constraints are placed on the DGP models using supernova and other data but with no such a topological constraint.

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