Nuclear Pasta at Finite Temperature with the Time-Dependent Hartree-Fock Approach

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Abstract. We present simulations of neutron-rich matter at sub-nuclear densities, like supernova matter. With the time-dependent Hartree-Fock approximation we can study the evolution of the system at temperatures of several MeV employing a full Skyrme interaction in a periodic three-dimensional grid [1].

The initial state consists of α particles randomly distributed in space that have a Maxwell-Boltzmann distribution in momentum space. Adding a neutron background initialized with Fermi distributed plane waves the calculations reflect a reasonable approximation of astrophysical matter.

The matter evolves into spherical, rod-like, connected rod-like and slab-like shapes. Further we observe gyroid-like structures, discussed e.g. in [2], which are formed spontaneously choosing a certain value of the simulation box length. The ρ-T-map of pasta shapes is basically consistent with the phase diagrams obtained from QMD calculations [3]. By an improved topological analysis based on Minkowski functionals [4], all observed pasta shapes can be uniquely identified by only two valuations, namely the Euler characteristic and the integral mean curvature.

In addition we propose the variance in the cell-density distribution as a measure to distinguish pasta matter from uniform matter.

1. Introduction

We observe systems with mean densities of 10% of the nuclear density up to the nuclear density at finite temperatures of an order of 10 MeV. These conditions are present in supernova core collapses and also in neutron stars but with low temperatures. The nuclear matter consists under these conditions no longer of nearly spherical nuclei but rods, slabs and phases where liquid and gas phase are inverted called rod bubbles and spherical bubbles are built up (s. e.g. [5, 6, 7]).

Besides the liquid drop model, Thomas-Fermi and QMD approaches to nuclear pasta (s. e.g. [8, 9, 3]) the time-dependent Hartree-Fock (TDHF) approximation was used to describe matter at zero temperature [10, 11]. Here we describe matter at finite temperature with the TDHF equations with a Skyrme force. For a review see [12].

More details about the calculations of this work, especially about what is presented in the sections 2 and 3, can be found in [13].
2. Setup

The calculations are performed on a full 3D grid without any symmetry assumptions with 16 grid points in each direction with a grid spacing of 1 fm and periodic boundary conditions. To vary the density we initialize on this grid a number of $\alpha$ particles with wave functions which were calculated in a static HF calculation, adding plane-wave background neutron wave functions to get a realistic value for the proton fraction of 1/3. The $\alpha$ particles are distributed with the Maxwell Boltzmann distribution in momentum space and randomly in real space. The background neutron wave functions are distributed with the Fermi distribution using the same temperature as for the $\alpha$ particles. The time evolution is performed for 1500 fm/c.

The temperature which was used for the initialization does not stay constant during the fusion of the $\alpha$ particles and background neutrons which takes place in the first few hundred fm/c of the calculation. Therefore the real temperature is measured by observing the excitation energy from the ground state. For a detailed discussion see [13]. Note that even if the $\alpha$ particles are distributed with zero momentum and the background neutrons states are filled up to the Fermi energy, the resulting real temperature is about 7 MeV.

3. The map of pasta shapes

The left side of Fig. 1(a) shows the typical shapes obtained as final states at the lowest temperature. The surfaces of these states are in slight motion because of the high temperature but the topology of the shapes is stable. Additionally to the rod and slab shapes also connected rods, namely rod(2) and rod(3), were observed. The number denotes the connectivity of the rods in the shapes. Similar shapes have been discovered in [7, 10, 14].

On the right side of Fig 1(b) the distribution of the shapes in the density-temperature plane is shown. For the lowest value for the temperature the shapes are well ordered from spherical to rod, rod(2) and rod(3) mixed with the slab phase followed by the corresponding bubble shapes in the reverse order. Note that the slab and rod(3) shapes represent their own bubble shapes because liquid and gas phase are topologically identical. For higher temperature the phases are not ordered anymore and they are overlapping each other. For increasing density smaller temperatures already produce uniform matter.
Table 1. Signs of the integral mean curvature and Euler characteristic as a function of $\rho_{th}$.

| shape | sph | rod | rod(2) | rod(3) | slab | rod(2) | b | rod | b | sph | b |
|-------|-----|-----|--------|--------|------|--------|---|-----|---|-----|---|
| $W_2$ | > 0 | > 0 | > 0    | - to   | $\approx$| 0     | < 0| < 0 | < 0| < 0 | < 0|
| $W_3$ | > 0 | = 0 | < 0    | < 0    | = 0   | = 0    | > 0|

4. The gyroidal structure
In the mid-density range of the map of pasta shapes it emerges, that there are minimal surface-like shapes present. A minimal surface is characterized by a mean curvature equal to zero at each point of the surface. For the slab shape this holds trivially, but also the rod(3) shape shown in Fig. 1(a) left side is topologically equivalent to a minimal surface called primitive (p-) surface. The gyroid is closely connected to this surface via the Bonnet transformation [20] and was discussed as a candidate for pasta matter [21, 2]. In systems with an intrinsic length scale the gyroid should appear at higher box length than the p-surface does. Therefore we made additional calculations with a box length of 22 fm with a mean density of 0.06 fm$^{-3}$ and a temperature of 7 MeV. The initialization was prepared such that the $\alpha$ particles have as large as possible distances between each other but that it is still possible to have random distributions. The experience with the calculations with smaller box length showed that this avoids slab shapes.

Besides shapes which could not be assigned to a certain topological structure, we observed gyroidal shapes as final states of these calculations (s. Fig. 2). These structure were mostly strongly deformed but could be identified to be topologically identical to the gyroid by fitting a gyroid network into the calculated structure.

Although the gyroidal shape appeared only in about 30% of the calculations, the spontaneous ordering of a randomized scenario to a gyroidal structure shows that the gyroid may be at least a meta stable state for pasta matter.

5. Conclusion and Outlook
In this work matter at subnuclear densities was studied by initializing $\alpha$ particles and background neutrons with a certain amount of kinetic energy randomly over a grid with periodic boundary
Figure 2. Dividing surface of a gyroidal pasta structure. The simulated surface is repeated three times in \( x \) and \( y \) direction. Cyan corresponds to the side facing the liquid phase, magenta to the gas phase. A gyroid network was added (gray/red) to identify the gyroidal structure.

conditions. The time evolution was done with the TDHF method. After about 1000 \( \text{fm/c} \) the states evolve to topologically constant states. The obtained map of pasta shapes agrees qualitatively with the diagram calculated in QMD [3] calculations. The different shapes are uniquely characterized by the sign of the integral mean curvature and the Euler characteristic and the distinction from uniform matter can be realized by the cell-density distribution.

Additionally gyroidal structures can be identified which are formed from a random initial state at a box length of 22 \( \text{fm} \) in the mid-density range. This hints that the gyroid could be important at least as a meta stable ground state. This should be investigated further with calculations considering the ground state with varying box lengths and mean densities.

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6. References
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