Cooling of Color Superconducting Compact Stars

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We review the status of research on the cooling of compact stars, with emphasis on the influence of color superconducting quark matter phases. Although a consistent microscopic approach is not yet available, severe constraints on the phase structure of matter at high densities come from recent mass and cooling observations of compact stars.

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1. Introduction

The fundamental questions for the origin of the mass of hadrons (chiral symmetry breaking), or the mechanism for confinement of quarks and gluons require answers from QCD as the fundamental quantum field theory of strong interactions in its non-perturbative domain. Despite progress with lattice QCD simulations and systematic approaches such as Dyson-Schwinger equations and renormalization-group equations, effective models for the strong interaction will remain indispensable for bridging the gap to phenomenological results accumulated in nuclear and particle physics as well as in astrophysics.

A major challenge of experimental programs at large-scale facilities such as CERN Geneva, BNL Brookhaven or GSI Darmstadt is to produce hadronic matter under extreme conditions of temperature and density in ultrarelativistic heavy-ion collisions in order to investigate the phase transition to a state of matter where the chiral symmetry and asymptotic freedom (deconfinement) of the QCD Lagrangian are restored. Quite opposite to the exploration of these phase transitions under terrestrial laboratory conditions, the strongly interacting matter in the heaven, namely in compact stars, is not plagued by the limitations of small volumes, short timescales and strong nonequilibrium and provides a unique view into the phase diagram at low temperatures and high densities, see Fig. 1. In this region quark matter, being a cold and dense Fermi system with attractive interactions, is expected to appear in a color-superconducting state due to the instability against Cooper-pairing and formation of diquark Bose condensates. The present contribution reviews recent investigations of the question whether observations of cooling compact stars can provide constraints on the development of microscopic approaches to phase diagram, equation of state (EoS) and transport properties of dense QCD matter, performed along the lines of the scheme

![Figure 1: The phase diagram for strongly interacting matter is under exploration in Lattice gauge theory simulations, heavy-ion collisions and astrophysics of compact stars.](image-url)
given in Fig. 2. Starting from a hadronic baseline for the EoS and cooling regulators we introduce

![Diagram](image)

**Figure 2:** Scheme for the interplay between different aspects of compact star phenomenology and microscopic theory of dense QCD matter. In the present contribution the focus is on aspects of the quark-hadron phase transition and effects of color superconductivity.

the new, stringent observational constraints on the masses, cooling curves and population of compact stars. We present state-of-the-art phase diagrams of three-flavor QCD matter under compact star constraints for chiral quark models of the Nambu–Jona-Lasinio (NJL) type and examine possible hybrid star configurations regarding the questions (i) do strange quark matter phases occur? and (ii) which patterns of color superconductivity are admissible?

2. **EoS and Structure of Hadronic Stars**

Hadronic matter in compact stars can reach densities above $1 \text{ fm}^{-3}$, so that systematic investigations shall be based on relativistic quantum field-theoretical approaches as reviewed, e.g. in [1, 2]. Comparative studies often employ the representation of the energy per nucleon

$$E(n, \beta) = E_0(n) + \beta^2 E_\beta(n),$$

where $n$ is the baryon number density, $\beta = 1 - 2x$ the asymmetry parameter depending on the proton fraction $x = n_p/n$; $E_0(n)$ is the energy per nucleon in symmetric nuclear matter to which in the case of pure neutron matter the asymmetry energy $E_\beta(n)$ has to be added. In a recent study of constraints on the high-density behavior of the nuclear EoS [3], a set of relativistic mean-field (RMF) EoS of the Walecka type (NL$\rho$, NL$\rho\delta$) with density dependent coupling constants and masses (DD, D$^3$C, DD-F, KVR, KVOR) together with that of the Dirac Brueckner Hartree-Fock (DBHF) approach based on the Bonn-A nucleon-nucleon potential has been considered to which we will refer here as a hadronic baseline, see Fig. 3. Note that all these EoS describe the properties of symmetric nuclear matter at the saturation density $n_0 \sim 0.16 \text{ fm}^{-3}$ but differ considerably in the high-density behavior. An astrophysical constraint on this behavior comes from the hydrostatic equilibrium configurations of spherical stars obtained by solving the Tolman-Oppenheimer-Volkoff
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Figure 3: Energy per nucleon in symmetric nuclear matter (left), symmetry energy (middle) and energy per nucleon in beta-equilibrated neutron star matter for the set of relativistic EoS investigated in [3].

The maximum attainable mass is an important feature of the EoS to be tested against observational constraints. Recently the mass of the pulsar in the double system PSR J0751+1807 has been determined to $2.1 \pm 0.2 \, M_\odot$ [4], see Fig. 5. From the analysis of RXTE data on quasi-periodic oscillations in the low-mass X-ray binary 4U 1636-536 a measurement of the innermost stable circular orbit frequency $v_{ISCO}$ corresponding to a mass of $1.9 - 2.1 \, M_\odot$ [5] has been reported [6], see Fig. 4. These constraints favor a stiff high-density behavior of the EoS as displayed for DD, $D^3C$ and DBHF. Note in Fig. 5 that for stiff EoS the maximum masses are at values above $2.3 \, M_\odot$ but at a lower central density than for soft EoS which allows larger star radii of $R \sim 12 - 13$ km, see Fig. 4. The mass-radius constraint obtained from the observation of the thermal emission of RX J1856

Figure 4: Mass-radius constraints from compact stars compared to results for a set of relativistic EoS [3].

2.3 $M_\odot$ but at a lower central density than for soft EoS which allows larger star radii of $R \sim 12 - 13$ km, see Fig. 4. The mass-radius constraint obtained from the observation of the thermal emission
of the isolated neutron star RX J1856.5-3754 seems to favor the stiff high-density behavior too \cite{7}, see Fig. 4.

![Graph showing mass constraints](image)

**Figure 5:** Mass constraint from PSR J0751+1807 with 1σ and 2σ error ranges \[4\] compared to results for a set of relativistic EoS \[3\]. Crosses denote maximum mass configurations and dots indicate the threshold for the direct Urca process. The mass distribution of nearby neutron stars obtained from a population synthesis model binned over eight intervals, Ref. \[8\] defines the range of “typical neutron stars”.

### 3. Cooling of Hadronic Stars

A detailed recent analysis of the regulators of the compact star cooling and their in-medium effects (such as neutrino emissivities, specific heats and thermal conductivities of different components) has revealed the decisive role of the direct Urca (DU) process \cite{9}. Once this process gets initiated at a proton fraction of \( \sim 14\% \) (including muons), the cooling gets dramatically enhanced in disagreement with observational data for surface temperature versus age. In Fig. 6 the sensitivity to minor changes in the neutron star mass is demonstrated in such a case. This problem cannot be solved by a suppression of cooling regulators due to superfluid pairing gaps since in this case very efficient pair breaking and pair formation emissivities will lead to strongly enhanced cooling, again in disagreement with the data \cite{10}. This observation leads to the formulation of the DU constraint: Hadronic DU processes should not occur for star masses below or within the region of typical neutron stars \( 1.0 \, M_\odot \leq M_{\text{typ}} \leq 1.5 \, M_\odot \) following from a population synthesis, see Fig. 5, where the DU thresholds are marked by fat dots. A detailed discussion of the modern astrophysical constraints on the behavior of the EoS at high densities \cite{3} shows that none of the purely hadronic EoS from the set presented above could fulfill all constraints simultaneously. This result motivates the application of this test scheme to hybrid EoS with a deconfinement phase transition to quark matter. First results for a DBHF-NJL hybrid EoS indicate that indeed the problems with the DU and flow constraints can be solved this way \cite{11}.
Figure 6: Demonstration of the rather unlikely cooling evolution when the DU threshold would lie within the interval of typical neutron star masses: all cooling neutron stars should have the same masses within less than 1% above the DU threshold. Results for standard cooling of a nonlinear Walecka model (courtesy: H. Grigorian), see also [9, 10].

4. QCD Phase Diagram and Stability of Hybrid Stars

For the discussion of the QCD phase diagram in the nonperturbative low-temperature/ high-density domain (see Fig. 1) it is customary to use effective models of the NJL type [12] since asymptotic QCD approaches [13] are limited to the region of $\mu > 500$ MeV and Lattice QCD studies have principal problems with the sector $T < \mu$. DSE studies were still bound to rather schematic interactions [14, 15], more realistic forms of the interaction are at present under consideration and will become very interesting when the covariant momentum dependence of Lattice QCD studies of the quark propagator [16] could be reproduced within, e.g., a nonlocal separable quark model [17, 18]. Here we base our report on state-of-the-art results within a NJL model for three-flavor quark matter including a selfconsistent determination of the strange quark mass [19], see Fig. 7. Similar results have been obtained by [20, 21]. The striking result of these investigations is a sequential melting of the light and strange quark condensates due to the large difference in the dynamically generated light and strange constituent quark masses which entails rather different critical chemical potentials for the chiral transition in the light and the strange quark sector at low temperatures and a dominance of two-flavor superconductivity (2SC phase) in the vicinity of the deconfinement transition. Three-flavor phases, such as color-flavor-locking (CFL), occur only at rather large chemical potentials only when the strange quarks appear and become light enough to fulfill the approximate SU(3) flavor symmetry required for the CFL phase, see Fig. 7. We demonstrate in that Figure that increasing the diquark coupling shifts the critical chemical potentials for the onset of color superconducting quark matter phases to lower values. This leads to an early onset of quark core formation in compact star configurations, i.e. to stable hybrid stars already for the typical star masses, see Fig. 8. Note, however, that once a CFL phase occurs the corresponding hybrid star configurations turn out to be unstable against collapse [11, 12]. Introducing a finite neutrino chemical potential (for the discussion of neutrino trapping in the early, hot stages of pro-
Figure 7: Phase diagrams for three-flavor quark matter within the NJL model for intermediate (left) and strong (right) diquark coupling [19].

Figure 8: Mass-central density relations for hybrid star configurations with a phase transition between hadronic phase (HHJ: Heiselberg–Hjorth-Jensen fit to Akmal-Pandharipande-Ravenhall EoS) and quark matter (SM: separable model). The occurrence of a two-flavor color superconducting phase lowers the onset density of the phase transition, see Refs. [23] and [22].

toneutron star evolution) enlarges the domain of the 2SC in the phase diagram [24, 11]. Gapless modes in the quark dispersion relations occur when the asymmetry in the chemical potentials of the quark species forming the pair exceeds the size of the gap, \( \delta \mu_{ij} = |\mu_i - \mu_j| > \Delta_{ij} \) [25]. Their appearance is therefore tied to the lines of the critical temperatures for the vanishing of color superconducting phases in the \( T - \mu \) plane where gaps become small enough to fulfill this condition. They occur here only at high temperatures, not relevant for the discussion of (late) compact star evolution, in contrast to [26].
5. Hybrid Star Cooling

For the 2SC phase stable hybrid star configurations with masses even below 1.3 $M_\odot$ have been obtained [22]. This phase has one unpaired color of quarks (say blue) for which the very effective quark DU process works and leads to a too fast cooling of the hybrid star in disagreement with the data. For details of the cooling calculation see, e.g., Refs. [27, 28]. We have suggested to assume a weak pairing channel which could lead to a small residual pairing of the hitherto unpaired blue quarks [23, 29]. For the resulting gap $\Delta_X$ a density dependence following the ansatz

$$\Delta_X(\mu) = \Delta_c \exp\left[-\alpha(\mu - \mu_c)/\mu_c\right]$$

(5.1)

has been assumed with $\mu_c = 330$ MeV being the critical chemical potential. The choice of parameters $\alpha = 25$ and $\Delta_c = 5$ MeV has been found to give an excellent cooling phenomenology, see left panel of Fig. 9, fulfilling a new set of additional constraints [8]:

(i) the brightness constraint [30] given by the upper barred region in that figure;
(ii) the expected number of objects within a mass bin from the population synthesis (displayed in Fig. 9 by the darkness of the strip in the T-t plane, cf. Fig. 5) is in accordance with actual number of observed coolers.
(iii) the Log N - Log S test [31], see right panel of Fig. 10.
(iv) young coolers like Vela are explained within the mass region of typical stars, $M < 1.5 M_\odot$.

The physical origin of the X-gap remains to be identified, one possible hypothesis is the condensation of color neutral quark sextett complexes [32]. Such calculations have not yet been performed using chiral quark models.

For sufficiently small diquark coupling, the 2SC pairing may be inhibited at all [33]. In this case, due to the absence of this competing spin-0 phase with large gaps, one may invoke a spin-1 pairing channel in order to avoid the DU problem. In particular the color-spin-locking (CSL) phase [34] may be in accordance with cooling phenomenology as all quark species are paired and the smallest gap channel may have a behavior similar to Eq. (5.1), see [35]. A consistent cooling calculation for this phase, however, still requires the evaluation of neutrino emissivities and transport coefficients. Progress in this direction has recently been obtained [36, 37].

6. Conclusions

We have shown that the maximum mass and DU constraints allow for selective tests of EoS for QCD matter at high densities. Strange quark matter phases in compact stars are not supported by present selfconsistent microscopic approaches due to the instability of corresponding configurations.

Clearly, an improvement of the approaches towards QCD is desired. One possible path uses the Dyson-Schwinger equation (DSE) approach to the QCD partition function [15], from which covariant nonlocal models can be derived [17, 18] with interaction form factors to be fitted to Lattice QCD. Our understanding of the confinement/deconfinement mechanism at finite temperatures and densities needs to be developed [18]. A promising direction within the QCD DSE approach is a generalization of the Kugo-Ojima criterion [39]. A less ambitious, but effective approach augments the chiral quark dynamics with the Polyakov loop potential fitted to Lattice data [40].
Figure 9: Left: Hybrid star cooling curves for model IV of Ref. [8]. Different lines correspond to compact star mass values given in the legend in units of $M_\odot$. Right: LogN-LogS distribution for the same model

A possible microscopic explanation of the yet unknown X-gap for the 2SC+X pairing pattern could come from condensation of bosonic multiquark correlations, e.g. quark sextett states in analogy to the quartetting effect in nuclear matter [43]. Once baryonic states are consistently included into the description it will be clarified whether the diquark coupling (yet considered as a free parameter, then fixed by the description of the baryon mass spectrum) will be strong enough to allow pairing in the scalar diquark channel. The mismatch between up and down quark Fermi levels under $\beta$ equilibrium conditions in neutron stars could easily destroy the 2SC phase. In this case the spin-1 pairing channels with small gaps, such as the CSL phase would turn out as a viable alternative. A prerequisite for them to play a role in the explanation of compact star cooling would be, however, the absence of the otherwise dominant scalar diquark channel(s). For further reading on the fascinating topic of strong matter in the heaven we recommend [44, 45, 46, 47].

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