Optimisation of a radio-frequency atomic magnetometer: a Uniform Design approach

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Abstract: High-sensitivity operation of a radio-frequency atomic magnetometer (RF-AM) requires careful setting of the system parameters, including the lasers intensity and detuning, and the vapour cell temperature. The identification of the optimal operating parameters, which ensures high sensitivity, is typically performed empirically and is often a lengthy process, which is especially labour intensive if frequent retuning of the magnetometer is required to perform different tasks. This paper demonstrates an efficient approach to RF-AM performance optimisation which relies on an open-loop optimisation technique based on Uniform Design (UD). This paper specifically describes the optimisation of an unshielded RF-AM based on a 4-factor-12-level UD of the experimental parameters space. The proposed procedure is shown to lead to the efficient optimisation of the atomic magnetometer at different frequencies, and is applicable to both AC and DC sensitivity optimisation. The procedure does not require any detailed knowledge of the model underlying the operation of the RF-AM and is effective in reducing the number of experimental runs required for the optimisation. It is ideally suited to self-calibration of devices without human supervision.

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1. Introduction

Recent years have witnessed a rapid development in optically pumped atomic magnetometers (AM), which have now achieved sub-fT/Hz\(^{1/2}\) sensitivity [1]. A wide range of applications leverage AMs’ high sensitivity. Notable examples include fundamental physics, space technology [2], security screening, for example nuclear quadrupole resonance (NQR) detection of explosive materials [3,4] and through-barrier imaging [5–7], and biomedical imaging, validated by the recent demonstration of electromagnetic induction imaging (EMI) of low conductivity (\(\sigma \leq 1\)S/m) samples with RF-AMs [8].

The basic principle of optically pumped atomic magnetometers is the laser interrogation of magnetically sensitive sublevel transitions in a polarised atomic vapour. The magneto-optical rotation of a linearly polarised laser beam through the vapour is detected to characterize the magnetometers’ response. The performance of the magnetometer is determined by the combination of several parameters, related to the pump light, the probe light and the atomic density. In all cases, the search for optimum sensitivity is the starting point for the operation of any atomic magnetometer, independently of the specific configuration or the envisaged application. In particular, for an RF-AM, the operational frequency has in general to be tuned over a broad range so to suit different application contexts, which requires a separate optimisation of the operating parameters for the different frequencies. For example, EMI with RF-AMs will require tuning at few hundred Hz for high-conductivity metallic objects [9] and at a few MHz for low-conductive saline solutions [8]. Therefore, an efficient optimisation process is needed, so to make the most of a small number of experimental data.
The standard approach to optimisation relies on sweeping the experimental parameters separately while ignoring the interactions between them [10]. In practice, it is performed by adjusting the parameters’ values to obtain the combination of atomic magnetometer response and linewidth leading to maximum sensitivity. This process is time-consuming and requires many tests due to the exponential growth with the number of involved parameters, without considering repetitions required to reduce the measurement errors. Methods for the efficient optimisation of the magnetometers’ parameters which take the interactions into considerations while reducing the number of tests required to an acceptable level are still lacking.

In this paper, Uniform Design (UD), also known as space filling method, is introduced for the optimisation of the parameter settings of an RF-AM for maximum sensitivity. This method allows one to distribute experimental points uniformly on the experimental domain so to significantly reduce the number of experiments while rationally depicting the relationship between the selected contributing factors, i.e. the experimental parameters of the magnetometer, and the response variable, the sensitivity in this paper [11–13].

This paper is organized as follows. Section II details the construction of the UD which will be used throughout this work. In section III, the setup of our RF-AM and the measurement procedure of its sensitivity are described. Application of the UD procedure and analysis of the data obtained are presented in section IV. Conclusions are drawn in section V.

2. Uniform design methodology for optimisation of a RF-AM

The optimisation procedure of a RF-AM requires several experimental runs for different values of the system parameters so to evaluate the dependence of the AM’s performance on the parameter settings and identify the conditions for optimum performance. However, with an increase of the number of experimental parameters, the evaluation with all possible interactions between parameters becomes complex, and the number of experimental runs increases exponentially. UD is a statistical tool which allocates points uniformly in the parameter space so to significantly reduce the complexity of the experiment [11–15].

The aim of UD, given a parameter space $S$, the so-called search space in UD language, is to determine a set of $n$ points $P = \{x_1, x_2, \ldots, x_n\}$ which are uniformly scattered over $S$. This involves the minimization of a measure of non-uniformity of $P$. A widely accepted non-uniformity measure is the so-called discrepancy $D(S)$ (see Ref. [15] for its mathematical definition). Thus, the required points can be found by minimizing the discrepancy. For a given number $s$ of relevant parameters – called factors in the UD language – and $m$ experimental runs per factor, UD produces a $s \times m$ table $U_{m}(m^s)$ of the values to be used in the experiment for the different factors in the different runs. These uniformly map the search space.

Once the search space is uniformly mapped, regression techniques can be used to identify active effects and screen out unimportant factors within the search space. Thus, the procedure simultaneously eliminates insignificant variables and estimates the coefficients of significant variables. It does not require a prior knowledge of the underlying model, an important feature given that AM operating regimes and the dominant noise sources depend on the specific scheme used. UD thus allows to establish a relationship between the response variable and the contributing factors based on a reasonable number of measurements without reference to a specific model [15].

RF-AMs are widely tunable with bandwidth from several hundred Hz to a few MHz. While the fundamental sensitivity of an RF-AM is independent of its frequency, it does not always determine the actual sensitivity. The noise floor can be dominated by other factors, for example photon-shot noise or electronic noise, which is frequency dependent. RF-AMs operating at many different frequencies have been reported so far [8,16,17]. Here we consider as case studies the optimisation of an unshielded RF-AM at two different frequencies, 424 kHz and 3 MHz, which correspond to two different applications. The first application is NQR detection of nitrogen
rich materials, and specifically the detection of the 424 kHz resonance of ammonium nitrate, a common test bed for NQR detection. The other one is EMI of low-conductivity materials [8,17,18], which requires operation of the RF-AM at higher frequency, the frequency of 3 MHz considered here being beyond the range of frequencies previously considered in the context of EMI with RF-AMS [8].

The first step in the procedure is the identification of the contributing factors, that is, the parameters determining the performance of the RF-AM which should be included in the UD. In general, all experimental parameters can be considered as contributing factors, as the irrelevant ones would eventually be eliminated by the procedure.

Four parameters were selected as contributing factors: the probe laser frequency detuning $\Delta_{pr}$, the vapour cell temperature $T$, the pump ($P_{pu}$) and probe ($P_{pr}$) powers. This is also in agreement with previous works where a significant dependence of the sensitivity of AMs on these parameters was reported [19–22]. We notice that the frequency of the pump light was not included, as the pump laser is kept at resonance to avoid introducing a fictitious magnetic field along the bias field via the AC-stark effect [23]. As for the frequency of the probe, it is included as a contributing factor, however only two discrete values are considered as only two locking points for the laser were available experimentally.

A 4-factor 12-run UD is used in this work for the presented optimisation. This means that 12 different values, numbered as 1-12 in the table, for each factor, indicated by A-D, are explored in the experiment. The corresponding UD table is shown in Table 1. This is a $U_{12}(12^4)$ table of 0.2233 discrepancy derived from the standard uniform table $U_{12}(12^{10})$ [24]. We notice that UD optimisation requires 12 experimental runs, while a standard approach would require all the combination of parameters, i.e., $12^4$ runs (>20000), for the given range and interval between each parameter. The reduction in the complexity of the optimisation is significant, and even more so considering that one of the factors is the vapour temperature, which is inherently slow to vary.

The specific design and the corresponding results are presented in Section IV.

### Table 1. 4-factor-12-run $U_{12}(12^4)$ table

| Factor/No. | 1   | 2   | 3   | 4   | 5   | 6   | 7   | 8   | 9   | 10  | 11  | 12  |
|------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| A          | 1   | 2   | 3   | 4   | 5   | 6   | 7   | 8   | 9   | 10  | 11  | 12  |
| B          | 6   | 12  | 5   | 11  | 4   | 10  | 3   | 9   | 2   | 8   | 1   | 7   |
| C          | 8   | 3   | 11  | 6   | 1   | 9   | 4   | 12  | 7   | 2   | 10  | 5   |
| D          | 10  | 7   | 4   | 1   | 11  | 8   | 5   | 2   | 12  | 9   | 6   | 3   |

3. Experimental setup

The experimental setup of the atomic magnetometer used in this work is shown in Fig. 1. It follows the standard design of a two-orthogonal-beam RF-AM [25], and is an upgraded version, both in terms of laser system and temperature control of the vapour cell, of the setup described in Refs. [26,27]. A bias field along the $z$-axis is applied by a pair of DC coils and locked to a set point, using the readout of a three-axis fluxgate (Bartington MAG690), and a feedback loop. The fluxgate is placed next to a (25 mm)$^3$ quartz vapour cell filled with isotopically enriched $^{87}$Rb and 40 Torr of N$_2$ as buffer gas. Additional sets of coils are used to passively compensate DC stray magnetic fields and gradients, so to allow operation in unshielded environment. The cell is heated by six heating pads, which are high-Tg printed circuit boards (PCBs) of Copper traces wired in pairs and powered by alternating currents of 51 kHz supplied by a full bridge power driver circuit [28]. A 3D-printed high temperature plastic enclosure is used to shelter the cell, the heating pads and the temperature sensor (Pt1000). The heater is turned off during the measurement so to avoid stray magnetic field noise. The RF field is provided by a pair of
Helmholtz coils along the y axis driven by a lock-in amplifier (LIA, Zurich Instruments HF2LI). The optical components are placed on a separate optical bench to reduce the magnetic fields generated by metallic components that could perturb the magnetometer.

Fig. 1. A sketch of the RF-AM. QWP refers to a $\lambda/4$ plate and HWP refers to a $\lambda/2$ plate.

The lasers are external cavity diode lasers (MOGLabs, CEL002). A $\sigma^{+}$ polarised pump beam light is locked to the $^{87}$Rb D1 line $F = 1 \rightarrow F' = 2$ via saturation absorption spectroscopy (SAS) and amplified by a tapered amplifier (TA) (MOGLabs, MOA). The pump beam passes through an 80 MHz acousto-optical modulator (AOM) for control of beam intensity. The linearly polarised probe beam was locked either to $^{85}$Rb D2 line $F = 2 \rightarrow F' = 3$, approximately 4.035 GHz blue detuned from the reference transition $^{87}$Rb D2 line $F = 2 \rightarrow F' = 3$ or to the $^{85}$Rb D2 line $F = 3 \rightarrow F' = 3/4$ crossover, which equates to a blue detuning of 1.35 GHz. Another 80 MHz AOM is also added in the probe light path. The pump beam is expanded to be a 10 mm$\times$15 mm ellipse and the probe beam is expanded to be a 12 mm$\times$9 mm ellipse. Both beams can fully cover the 8 mm diameter apertures of the enclosure covering the cell. The probe light after the cell is detected by a polarimeter, consisting of a polarizing beam splitter (PBS) and a balanced photodiode (Thorlabs, PDB210A). The output of the photodiode is interrogated by the LIA. The LIA simultaneously extracts four components: in-phase (absorptive, X), out-of-phase (dispersive, Y), radius ($R = X^2 + Y^2$) and phase ($\Phi = \text{atan2}(Y/X)$). All measurements are taken using an acquisition window of 1 ms with time domain filtering.

4. UD experimental results

The performance of an RF-AM is quantified by two different sensitivity figures, namely the DC field sensitivity and the AC field sensitivity. The optimisation of both is presented in the present paper, with for illustrative purposes the DC sensitivity optimised at 424 kHz and the RF one at 3 MHz. The DC sensitivity is the smallest detectable shift of the bias field. At a given operating frequency, the DC sensitivity is defined as

$$\delta B_{\text{DC}} = \hbar \frac{g_F \mu_B}{\Gamma \text{SNR}}.$$  \hspace{1cm} (1)$$

where $\mu_B$ is the Bohr magneton, $g_F$ is the Landé factor, $\hbar$ is the Planck’s constant, $\Gamma$ is the full width half maximum (FWHM) linewidth and SNR is the signal-to-noise ratio. The SNR is measured as the square root of the ratio between the maximum value of the power spectrum density (PSD) with the RF field on and the mean value of the noise power spectrum density with the RF field turned off, which is also referred to as the noise floor.
The AC sensitivity is the smallest detectable variation of a magnetic field oscillating at the Larmor frequency. The AC sensitivity is given by

$$\delta B_{AC} = \frac{B_{RF}}{\text{SNR}}$$ (2)

The amplitude of the RF calibration field driving the magnetometer is optimised by identifying ranges leading to optimum sensitivity. The calibration of $B_{RF}$ is realized by measuring the voltage across a non-inductive resistor connected in series to the RF coil driving the atomic precession.

We consider first the optimisation of the AC sensitivity at 3MHz. In the optimisation process, we used the SNR to examine the AC sensitivity as $B_{RF}$ was fixed for each operating frequency. The RF level was optimised before the UD optimisation procedure was carried out, with the results shown in Fig. 2.

**Fig. 2.** Optimisation of the RF level at a frequency of 3 MHz and a cell temperature of 85 °C, for a pump beam power of 55 mW, a probe beam power of 2.714 mW and a probe frequency detuned by 4.035 GHz from the $^{87}$Rb D2 line $F = 2 \rightarrow F' = 3$ transition. ‘Signal’ of the y-axis in panel (a) represents the signal amplitude of $|F = 2, m_F = 2 \leftrightarrow |F = 2, m_{F-1} = 1 \rangle$ transition of the X response extracted at the resonance frequency. In panel (b), the RF level corresponding to the minimum ratio between linewidth and signal magnitude was chosen as the work point. For 3 MHz, the RF level was set to 92.7 nT.

$U_{12}(12^3 \times 2)$ experiments at 3 MHz, a frequency of interest for EMI in the biomedical domain, were performed within the range of probe detuning ($\Delta_{pr}$) 1.35 GHz and 4.035 GHz, temperature (T) 42~119°C, pump power ($P_{pu}$) 0.7 mW~143.7 mW and probe power ($P_{pr}$) 0.15 mW~8.4 mW. 12 values of each factors (apart from the 2 values of the probe detuning) were generated linearly within the ranges and then allocated to each test of $U_{12}(12^3 \times 2)$ according to Table 1. Results are given in rows 1-12 of Table 2. Figure 3 (a) shows a resonance measured at 3 MHz where nonlinear Zeeman splitting occurs and thus each sublevel can be resolved in the frequency response. The strongest RF transition, which corresponds to $|F = 2, m_F = 2 \leftrightarrow |F = 2, m_{F-1} = 1 \rangle$, was used for the measurements at 3 MHz.

The optimisation procedure is structured as follows: A multiple quadratic regression model is derived from the UD data. Here quadratic terms $\Delta_{pr}^2$, $\Delta_{pr}T$, etc. were used as independent variables along with $\Delta_{pr}$, T, $P_{pu}$, and $P_{pr}$ in a linear regression. More specifically, backward stepwise multifactor linear regression was applied using the software Statistical Product and Service Solutions (SPSS) [15].

It is a stepwise regression approach that begins with a saturated model and at each step gradually eliminates variables from the regression model in search for a reduced model that best describes the input data. This method can reduce the number of predictors, reducing the multi-collinearity problem and the overfitting issue. F-test is adopted as the stepping method criteria. To let in full independent variables at the beginning of the regression, alpha-to-entry pertaining to the level of significance is set as $P_{IN} = 0.1$ and alpha-to-remove as $P_{OUT} = 0.15$. 
Fig. 3. Resonances (a) and spectra (b) under the settings corresponding to the highest SNR measured at 3 MHz (row 8 in Table 2): 98 °C, a pump power of 143.7 mW, a probe power of 0.9 mW and a probe frequency detuning of 4.035 GHz. In panel (a), four evenly separated resonance peaks can be observed due to the non-linear Zeeman effect where the sublevels split unequally. Two-photon RF transitions between sublevels are visible in this case as the RF field is close to its saturation point [29]. The rightmost peak was used for the measurements. In panel (b), the blue trace was measured with RF excitations; the red trace was measured without RF excitations; the yellow trace was measured without RF excitations and pump light; the purple trace was measured without RF excitations and probe light; the green trace was measured without RF excitations and with bias field shifted.

Table 2. UD at 3 MHz

| No. | Probe detuning, $\Delta_{p}$/GHz | Temperature, $T$/ °C | Pump power, $P_{pu}$/mW | Probe power, $P_{pu, max}$/mW | HWHM (Hz) | SNR DC | Sensitivity DC (fT/Hz$^{1/2}$) | AC Sensitivity DC (fT/Hz$^{1/2}$) |
|-----|---------------------------------|----------------------|-------------------------|-----------------------------|-----------|--------|-----------------|-----------------|
| 1   | 1.35                            | 77.00                | 91.70                   | 6.9                         | 1169      | 5.70E+04 | 932             | 1150            |
| 2   | 1.35                            | 119.00               | 26.70                   | 4.65                        | 281       | 2.63E+01 | 486000         | 2490000         |
| 3   | 1.35                            | 70.00                | 130.70                  | 2.4                         | 636       | 5.56E+04 | 520             | 1180            |
| 4   | 1.35                            | 112.00               | 65.70                   | 0.15                        | 281       | 2.89E+01 | 443000         | 2270000         |
| 5   | 1.35                            | 63.00                | 0.70                    | 7.65                        | 441       | 1.45E+03 | 13800           | 45500           |
| 6   | 1.35                            | 105.00               | 104.70                  | 5.4                         | 245       | 8.56E+01 | 130000         | 765000          |
| 7   | 4.035                           | 56.00                | 39.70                   | 3.15                        | 239       | 3.14E+04 | 346             | 2080            |
| 8   | 4.035                           | 98.00                | 143.70                  | 0.9                         | 362       | 1.11E+05 | 149             | 593             |
| 9   | 4.035                           | 49.00                | 78.70                   | 8.4                         | 451       | 1.86E+04 | 1100            | 3520            |
| 10  | 4.035                           | 91.00                | 13.70                   | 6.15                        | 399       | 6.21E+04 | 292             | 1060            |
| 11  | 4.035                           | 42.00                | 117.70                  | 3.9                         | 358       | 1.58E+04 | 1030            | 4150            |
| 12  | 4.035                           | 84.00                | 52.70                   | 1.65                        | 210       | 9.31E+04 | 102             | 704             |
|     | 4.035                           | 105.69               | 77.6108                 | 5.1276                      | 353       | 1.07E+05 | 150             | 613             |

The regression function is then used to find the conditions within the experimental search space for its local optimum value, and finally additional experiments are carried out to verify the suggested settings. Once the regression model is defined, a local optimum can be determined. If it is within the search space, it will be adopted as working point. Otherwise, if outside the search space, the point in the UD showing best performance will be selected as the starting point for further optimisation.

In our first optimisation, the SNR was used as the dependent variable at 3 MHz. A regression function was determined with an adjusted $R^2$ of 0.807. Table 3 lists the coefficient of each factor
of the regression function, together with the partial correlations $\rho$ between each factor and the dependent variable.

### Table 3. Coefficients and partial correlations of UD regression model of SNR at 3 MHz

| Independent variable | Constant | T | $\Delta_{pr}^{2}$ | $\Delta_{pu} P_{pu}$ | $\Delta_{pu} P_{pu}$ |
|----------------------|----------|---|------------------|---------------------|---------------------|
| Coefficient          | -259303  | 3396.6 | 19159.25         | -569.95            | -10925.45           |
| Partial correlation   | 0.922    | 0.969 | -0.955           | -0.954             |                     |
|独立变量              |          |       |                  |                     |                     |
| Coefficient          | -24.83   | -286.59 | 20.77           | 173.87             | 4425.12             |
| Partial correlation   | -0.896   | -0.907 | 0.946            | 0.877              | 0.948               |

The model accuracy can be evaluated by several factors such as F-statistic, significance and most importantly the experimental validation of the predicted optimised sensitivity. The significance of the given model is 0.149. Possible optimised points are calculated from the minimum among the stationary points within the experimental range since extrapolation is not considered reliable in UD [30]. Based on the regression function, a stationary point was determined, with settings given in the last row of Table 2. In the same row the experimentally measured DC and AC sensitivities are reported. We note that the reported sensitivities are about the same as the trial point at row 8. Thus, in this case the procedure did not reveal any point with better sensitivity than the one considered as trial point in the table.

To validate our approach, we investigated the eventual existence of settings leading to better sensitivities. Two sets of measurements were performed to this purpose.

First, both beam powers were swept separately around the stationary points at 106 °C. The SNR plateaued when the pump power reached 100 mW and the probe beam power reached 10 mW. The highest SNR obtained from sweeping around the stationary point was $9.69 \times 10^4$, for an AC sensitivity of $692 \text{ fT/Hz}^{1/2}$, thus worse than the one obtained at the stationary point. This validates our approach.

Second, in addition to the results obtained at the stationary point, we also swept the beam powers separately around the settings at row 8 at 98 °C, which generated a slightly higher SNR among the UD tests. The SNR plateaued when the pump power reached 140 mW and the probe beam power reached 5 mW. We obtained an SNR of $1.035 \times 10^5$ from beam power sweepings at 98°C. These settings lead to a slightly worse performance with respect to the optimal settings (the last row) previously found, thus further validating the approach put forward in this work.

In summary, the highest SNR obtained via the UD optimisation procedure at 3 MHz was around $1.1 \times 10^5$ and sweeping the beam powers either around the best point within the UD table or around the stationary point of the regression model failed to further increase the SNR.

An additional optimisation was performed at 424 kHz, a frequency of interest for NQR. For illustrative purposes, we consider here the DC sensitivity, so to complement the optimisation procedure at 3 MHz for the AC sensitivity. We firstly optimised the RF level at 424 kHz, as we did at 3 MHz, before carrying out UD at this higher operating frequency. Fixing the RF level to 70.3 nT, we performed $U_{12}(12^3 \times 2)$ experiments within the range of probe detuning $1.35\text{GHz}$ and $4.035 \text{GHz}$, temperature $45{-}100\text{°C}$, pump power $1.38\text{mW}$ to $55\text{mw}$ and probe power $0.15\text{mW}$ to $8.4\text{mW}$. Results are given in rows 1-12 of Table 4.

A regression function of DC sensitivity as the dependent variable was generated with an adjusted $R^2$ of 0.801 and significance of 0.03. The coefficients and the partial correlations are shown in Table 5.

In the present case, the calculated stationary points were outside the considered search space. However, the partial correlation analysis suggested the sequence of influence as $P_{pr}^2 > P_{pu} P_{pr} > P_{pu} P_{pu} > T P_{pr}$. As discussed in Section II, UD can function as an effective screening
Table 4. UD measurements at 424 kHz

| No. | Probe detuning, $\Delta_{pr}$/GHz | Temperature, T/°C | Pump power, $\Delta P_{pu}$/mW | Probe power, $P_{pr}$/mW | HWHM (Hz) | SNR | DC Sensitivity (fT/(Hz$^{1/2}$)) | AC Sensitivity (fT/(Hz$^{1/2}$)) |
|-----|----------------------------------|-------------------|-------------------------------|--------------------------|----------|-----|---------------------------------|---------------------------------|
| 1   | 1.35                             | 70                | 35.5                          | 6.9                      | 1740     | 4.67E+04 | 1690                           | 1060                            |
| 2   | 1.35                             | 100               | 11.12                         | 4.65                     | 338      | 1.21E+04 | 1270                           | 4100                            |
| 3   | 1.35                             | 65                | 50.12                         | 2.4                      | 710      | 1.09E+05 | 297                            | 456                             |
| 4   | 1.35                             | 95                | 25.75                         | 0.15                     | 252      | 1.83E+03 | 6260                           | 27200                           |
| 5   | 1.35                             | 60                | 1.37                          | 7.65                     | 778      | 1.81E+03 | 19500                          | 27500                           |
| 6   | 1.35                             | 90                | 40.37                         | 5.4                      | 699      | 1.90E+05 | 168                            | 262                             |
| 7   | 4.035                            | 55                | 16                            | 3.15                     | 315      | 9.27E+04 | 155                            | 536                             |
| 8   | 4.035                            | 85                | 55                            | 0.9                      | 279      | 4.88E+05 | 26                             | 102                             |
| 9   | 4.035                            | 50                | 30.62                         | 8.4                      | 470      | 2.50E+04 | 856                            | 1990                            |
| 10  | 4.035                            | 80                | 1.375                         | 6.15                     | 383      | 3.80E+04 | 459                            | 1310                            |
| 11  | 4.035                            | 45                | 45.25                         | 3.9                      | 339      | 3.69E+04 | 418                            | 1350                            |
| 12  | 4.035                            | 75                | 20.87                         | 1.65                     | 270      | 3.52E+05 | 34.9                           | 141                             |
| 13  | 4.035                            | 85                | 55                            | 2.714                    | 314      | 6.04E+05 | 23.7                           | 82.3                            |

Table 5. Coefficients and partial correlations of regression model of DC sensitivity at 424 kHz

| Independent variable | Constant | $\Delta_{pr}P_{pr}$ | $TP_{pr}$ | $P_{pr}P_{pr}$ | $P_{pr}^2$ |
|----------------------|----------|----------------------|-----------|-----------------|------------|
| Coefficient          | 6727.95  | -418.80              | -19.54    | -36.27          | 396.02     |
| partial correlation  | -0.85    | -0.721               | -0.843    | 0.911           |            |

Procedure for identification of active effects from the experiment. By choosing row 8 in Table 4, which gave the best DC sensitivity, as the starting point, we swept the probe power at 1.35 GHz and 4.035 GHz separately, since we only have access to two different values of probe detuning. Figure 4 shows clearly that a detuning of 4.035 GHz generated narrower linewidth and higher signal level at same probe powers than 1.35 GHz did.

Then keeping the probe power at 2.714 mW, 4.035 GHz detuned, we swept the pump power, with results shown in Fig. 5, and found its optimum value at 55 mW.
Fig. 5. HWHM (a) and signal amplitude (b) as a function of pump power at a vapour temperature of 85°C, probe laser detuning of 4.035 GHz and probe beam power of 2.714 mW.

The result of the optimisation for 424 kHz is given in the last row in Table 4, whose resonance and the signal and noise spectra at the optimised settings are shown in the Fig. 6, along with typical noise levels measured with no pump beam, no probe beam, and the bias field shifted.

Fig. 6. Resonances (a) and spectra (b) of RF-AM at 424 kHz. Measurements were taken at the stationary point of UD (the last row in Table 4): 85°C, a pump power of 55 mW, a probe laser detuning of 4.035 GHz and a probe beam power of 2.714 mW. In panel (b), the blue trace was measured with RF excitations; the red trace was measured without RF excitations; the yellow trace was measured without RF excitations and pump light; the purple trace was measured without RF excitations and probe light; the green trace was measured without RF excitations and with bias field shifted.

In general, optimisation at a given frequency, 3 MHz, does not guarantee optimal performance at another frequency, 424 kHz in the present case. This depends on the sources of noise in the system. We discuss this further in detail, with the spectra at 3MHz presented in Fig. 3 (b) in comparison with those at 424 kHz shown in Fig. 6 (b). The ‘RF off’ trace at 3 MHz was less separated from the ‘pump off, RF off’ trace. This demonstrates that the noise was more dominated by the probe noise at 3 MHz and the probe noise was imprinted with frequency-dependent noise sources such as the magnetic noise [31]. It is probably because of the pumping effect of the circularly-polarized component of the probe beam generated by the cell or the imperfection of the optical elements [32]. Thus, an increase of the incident probe light intensity would generate a larger response to the stray magnetic field. Also, the bias field fluctuations could be larger at a higher operating frequency.
5. Conclusion

In conclusion, we have demonstrated the design and implementation of UD optimisation for the sensitivity of an unshielded RF-AM at 424 kHz and 3 MHz. The procedure was shown to apply to both AC and DC optimisation. Our approach significantly reduces the number of experimental runs required for the optimisation by using an optimal mapping of the search space. The method improves the efficiency of sensitivity optimisation by suggesting a local optimum within the search space and indicating the degree of influence of contributing factors. It is very useful when frequent retuning of the magnetometer is necessary, and when the optimisation involve varying the cell temperature or other time intensive parameters. It is also a step towards self-optimisation of RF-AMs, without the direct intervention of an operator. The UD technique can also be applied to other types of optically pumped atomic magnetometers, and more generally to many physics experiments.

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Data availability. Data underlying the results presented in this paper are not publicly available at this time but may be obtained from the authors upon reasonable request.

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