Radiative Muon Capture and Induced Pseudoscalar Coupling Constant in Nuclear Matter

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Abstract

The recent TRIUMF experiment for $\mu^-p \to n\nu_\mu\gamma$ gave a surprising result that the induced pseudoscalar coupling constant $g_P$ was larger than the value obtained from $\mu^-p \to n\nu_\mu$ experiment as much as 44 %. Reexamining contribution of the axial vector current in electromagnetic interaction, we found an additional term to the matrix element which was used to extract the $g_P$ value from the measured photon energy spectrum. This additional term, which plays a key role to restore the reliability of $g_P(-0.88m_\mu^2) = 6.77g_A(0)$, is shown to affect the $g_P$ quenching problems in nucleus.

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I. INTRODUCTION

The matrix element of vector and axial vector currents are generally given as

$$\langle N(p') | V_\mu^a(0) | N(p) \rangle = \bar{u}(p') [G_V(q^2)\gamma^\mu + \frac{G_S(q^2)}{2m} q^\mu + G_M(q^2)\sigma^{\mu\nu} q_\nu] \frac{\tau_a}{2} u(p)$$

$$\langle N(p') | A_\mu^a(0) | N(p) \rangle = \bar{u}(p') [G_A(q^2)\gamma^\mu + \frac{G_P(q^2)}{2m} q^\mu + G_T(q^2)\sigma^{\mu\nu} q_\nu] \gamma_5 \frac{\tau_a}{2} u(p) \, ,$$

(1)

where $G_A(0) = g_A(0)$, $G_M(0) = g_M(0)$, $G_V(0) = g_V(0)$ and $G_P(q^2) = (\frac{2m}{m_\mu}) g_P(q^2)$ with the nucleon and muon masses, $m$ and $m_\mu$. $\tau_a$ is the isospin operator. $G_S$ and $G_T$ belong to the second class current which has a different G-parity from the first class current, and they are assumed to be absent from the muon capture to be discussed in this paper. On the basis of PCAC (Partially Conserved Axial Current), the induced pseudoscalar coupling constant is calculated as

$$g_P(-0.88m_\mu^2) = \frac{2m}{m_\pi^2 + 0.88m_\mu^2} g_A(0) = 6.77 g_A(0) \, .$$

(2)

This value is confirmed by an experiment of the ordinary muon capture (OMC) on a proton, $\mu^- p \rightarrow n \nu_\mu \, [1]$. However, in order to obtain more precise data, the TRIUMF group measured recently the photon energy spectrum of the radiative muon capture (RMC) on a proton, $\mu^- p \rightarrow n \nu_\mu \gamma$ and extracted a surprising result $\, [2]$

$$\hat{g}_P \equiv g_P(-0.88m_\mu^2) / g_A(0) = 9.8 \pm 0.7 \pm 0.3 \, .$$

(3)

It exceeds the value obtained from OMC as much as 44%. This discrepancy is serious because the theoretical value of $g_P$ is predicted in a fundamental manner based on PCAC and agrees with the OMC value. As long as PCAC is assumed to be creditable, a doubt may be cast on the result of TRIUMF experiment. Recent calculations $\, [4,5]$ by chiral perturbation also says such a doubt. However, in order to solve this puzzle, one has to reexamine carefully the Beder-Fearing formula $\, [6,7]$, which is a phenomenological model, used to extract the $g_P$ value from the measured RMC spectrum.
In finite nuclei, through the theoretical analyses of OMC experimental results, it is already reported [3] that the $\hat{g}_P$ value is quenched in medium-heavy and heavy nuclei while it is enhanced in light nuclei. Since these analyses are carried out before the recent TRIUMF experiment one needs to reconsider those analyses from another viewpoint.

In this paper we present more successful analysis for the recent TRIUMF data and show some progressive results for $\hat{g}_P$ quenching problems in nucleus by applying our results on proton to nuclear matter.

II. BASIC FORMULAE

We start from the ordinary linear-$\sigma$ model;

$$L_0 = \overline{\Psi}[i\gamma^\mu \partial_\mu - g(\sigma + i\vec{\pi} \cdot \vec{\gamma}_5)]\Psi + \frac{1}{2}[(\partial_\mu \vec{\pi})^2 + (\partial_\mu \sigma)^2] + \frac{1}{2}\mu^2(\vec{\pi}^2 + \sigma^2) - \frac{\lambda^2}{4}(\vec{\pi}^2 + \sigma^2)^2 \quad (4)$$

, which gives the following axial current

$$A_\mu^a = \overline{\Psi}\gamma^\mu\gamma^5\frac{\tau^a}{2}\Psi + \pi^a_\mu \partial_\mu \sigma - \sigma \partial_\mu \pi^a \quad (5)$$

By the spontaneous breakdown of chiral symmetry, $\sigma$ field is shifted to $\sigma' = \sigma - \sigma_0$ with $\sigma_0 = f_\pi$. Consequently, the pion appears as Nambu-Goldstone boson. The PCAC can be satisfied by the additional inclusion of the explicit chiral symmetry breaking term as well known.

But the axial current

$$A_\mu^a = \overline{\Psi}\gamma^\mu\gamma^5\frac{\tau^a}{2}\Psi - f_\pi \partial_\mu \pi^a \quad (6)$$

gives $g_A = 1$ in the tree approximation. Following the recipe of Akhmedov [12] to cure this problem, we add chiral invariant lagrangian $L_1$ to $L_0$,

$$L_1 = C[\overline{\Psi}\gamma^\mu\frac{\vec{\pi}}{2}\Psi(\vec{\pi} \times \partial_\mu \vec{\pi}) + \overline{\Psi}\gamma^\mu\gamma^5\frac{\vec{\tau}}{2}\Psi(\vec{\pi} \partial_\mu \sigma - \sigma \partial_\mu \vec{\pi})] \quad (7)$$

where arbitrary parameter $C$ is determined so that the axial current pertinent to nucleons in $L = L_0 + L_1$
\[(N) A_\mu^a = \bar{\Psi} \gamma_\mu \gamma_5 \frac{\tau_a}{2} \Psi [1 + C^2 (\pi^2 + \sigma^2)] \]  

(8)

should satisfy \[(N) A_\mu^a = g_A \bar{\Psi} \gamma_\mu \gamma_5 \frac{\tau_a}{2} \Psi \] with \(g_A = 1.26\). The Goldberger-Treiman relation then is satisfied exactly. As a consequence, \((N) A_\mu\) includes the contribution not only from the nucleon but also from the \(\pi - N\) interactions.

Now, the axial vector current consists of the nucleon and pion sectors as

\[ A_\mu^a (x) = (N) A_\mu^a (x) + (\pi) A_\mu^a (x) \]

(9)

\[ = (N) A_\mu^a (x) + f_\pi \partial^\mu \phi_a (x) , \]

where \(f_\pi\) is the pion decay constant. \(\phi_a (x)\) is the pion field.

To describe RMC, we need a radiative axial current, which is used to obtain the transition amplitude of RMC by coupling to the weak current of lepton line. Three different methods are considered in order to construct a radiative axial current. The 1st method \[8\] is to start from the above lagrangians \(\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_1\) using covariant derivative, by which we introduce a photon field in U(1) gauge invariant way. The outcoming lagrangian gives a radiative axial current, which characteristic is its non-conservation through the explicit chiral symmetry breaking due to the electromagnetic interaction. The second method \[9\] is to use the extended Euler equation \[10\] for the lagrangian \(\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_1\). The third is to make it directly from the above axial currents, eq.(9), by exploiting it minimal coupling scheme to the momenta of relevant particles. Here we follow the third one. Of course the final radiative axial currents from these three different methods turned out to be equivalent.

Let us begin from the divergences of \((N) A_\mu^a (x)\) and \((\pi) A_\mu^a (x)\),

\[ \partial_\mu (N) A_\mu^a (x) = \partial_\mu [g_A \bar{\Psi} (x) \gamma_\mu \gamma_5 \frac{\tau_a}{2} \Psi (x)] \equiv f_\pi J^N_a , \]  

(10)

\[ \partial_\mu (\pi) A_\mu^a (x) = f_\pi \partial^\mu \phi_a (x) = - f_\pi [m_\pi^2 \phi_a (x) + J^N_a] , \]  

(11)

where we used pion field equation from the above lagrangians

\[ (\partial^2 + m_\pi^2) \phi_a = - J^N_a . \]  

(12)
The quantity $J_a^N$ denotes the pion source term. Therefore, the divergence of total axial currents is given in the following way

$$\partial_\mu A_\mu^a(x) = \partial_\mu^{(N)} A_\mu^a(x) + \partial_\mu^{(\pi)} A_\mu^a(x)$$

$$= -f_\pi [m_\pi^2 \phi_a(x) + J_a^N] + f_\pi J_a^N$$

$$= -f_\pi m_\pi^2 \phi_a(x).$$

This is PCAC. By eqs. (10) and (12), one can obtain

$$\phi_a(x) = -\frac{1}{f_\pi (\partial^2 + m_\pi^2)} \partial_\mu^{(N)} A_\mu^a(x).$$

Substitution of eq.(14) into eq.(9) yields

$$A_\mu^a(x) = (N)A_\mu^a(x) - \frac{i}{\partial^2 + m_\pi^2} (i \partial)^\mu [\partial_\mu^{(N)} A_\mu^a(x)].$$

In order to clarify a role of the axial current in description of the radiation process, we adopt the minimal coupling prescription, i.e. $\partial_\lambda \rightarrow \partial_\lambda - ieA_\lambda$ and $q^\mu \rightarrow q^\mu - eA^\mu$. This procedure leads to

$$A_\mu^a(x) = (N)A_\mu^a(x) - \frac{i}{\partial^2 + m_\pi^2} (i \partial)^\mu [\partial_\mu^{(N)} A_\mu^a(x)] + \frac{ie}{\partial^2 + m_\pi^2} \epsilon^\mu [\partial_\lambda^{(N)} A_\lambda^a(x)]$$

$$- \frac{e}{\partial^2 + m_\pi^2} (i \partial)^\mu \epsilon_\lambda^{(N)} A_\lambda^a(x) + \frac{e^2}{\partial^2 + m_\pi^2} \epsilon^\mu \partial_\lambda^{(N)} A_\lambda^a(x),$$

where the potential $A_\lambda$ is replaced by the photon polarization vector $\epsilon_\lambda$. Notice here that $\partial_\lambda^{(N)} A_\lambda^a(x) = g_A m \bar{\Psi}(x) i\gamma_5 \tau_a \Psi(x)$. The last term in eq.(16) can be neglected because it appears at $O(e^2)$ order. Thus we can express the axial current in the radiative processes in the following realistic form

$$A_\mu^a(x) = \bar{\Psi}(x) [g_A \gamma^\mu \gamma_5 + \frac{g_P (q^2)}{m_\mu} q^\mu \gamma_5 - \frac{eg_P (q^2)}{m_\mu} \epsilon^\mu \gamma_5 \frac{\tau_a}{2} \bar{\Psi}(x)$$

$$- \frac{eg_P (q^2)}{2mm_\mu} q^\mu [\bar{\Psi}(x) \epsilon_\alpha \gamma_5 \gamma_5 \frac{\tau_a}{2} \bar{\Psi}(x)].$$

The fourth term, which corresponds to “Seagull term”, is missing in the previous calculations. 

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Following the Fearing’s formulation and notation \[6\] for the diagrams given in ref. \[6\], one can evaluate the relativistic amplitude of RMC on a proton as

\[M_{fi} = M_a + M_b + M_c + M_d + M_e + \Delta M_e\] (18)

with

\[M_a = -\epsilon_a \bar{u}_n \gamma_5 \gamma_\delta (1 - \gamma_5) \frac{\not{k} - \not{k}}{-2k \cdot \mu} \gamma^\alpha u_\mu,\] (19)

\[M_b = \epsilon_a L_\delta \bar{u}_n \Gamma^\delta(K) \frac{\not{p} - \not{p}}{-2k \cdot p} + m_p \gamma^\alpha \gamma_\delta (1 - \gamma_5) u_\mu,\]

\[M_c = \epsilon_a L_\delta \bar{u}_n \left(-i\kappa_n \frac{\not{k} + m_n}{2k \cdot n} \Gamma^\delta(K) u_\mu,\right),\]

\[M_d = -\epsilon_a L_\delta \bar{u}_n \left(\frac{2Q^\alpha + k^\alpha g_p(K^2) K\delta \gamma_5)}{2 - m^2_p} m_\mu \right) u_\mu,\]

\[M_e = \epsilon_a L_\delta \bar{u}_n \left(\frac{g_{p}(K^2)}{2mm_\mu} Q^\delta \gamma_5 \gamma_\delta u_\mu,\right),\]

\[\Delta M_e = -\epsilon_a L_\delta \bar{u}_n \left(\frac{g_{p}(K^2)}{2mm_\mu} Q^\delta \gamma_5 \gamma_\delta u_\mu,\right),\]

where

\[\Gamma^\delta(q) = g_V \gamma^\delta + \frac{ig_M}{2m} \sigma^\delta \gamma_3 + g_A \gamma^\delta \gamma_5 + \frac{g_{p}(q^2)}{m_\mu} q^\delta \gamma_5,\] (20)

\[L_\delta = \bar{u}_\nu \gamma_\delta (1 - \gamma_5) u_\mu, K = n - p + k \text{ and } Q = n - p \text{ with momenta of neutron, proton and photon, } n, p \text{ and } k, \text{ respectively. And } m \sim m_p \sim m_n.\] Other constants are taken as \(g_V = 1.0, g_A = -1.25, g_M = 3.71, \kappa_p = 1.79 \text{ and } \kappa_n = -1.91\). \(M_e\) term is originated from the third term in eq.(17) and \(\Delta M_e\) term comes from the fourth term. But the latter, \(\Delta M_e\), is missing in the paper by Fearing \[3, 7\]. Accordingly, this term was not included in the previous procedure of extracting \(g_P\) value from the experimental RMC photon energy spectrum \[2\].

The above transition amplitude can be also understood in terms of pseudo vector (PV) coupling scheme between nucleons and virtual pion, through which external axial current interacts with nucleons. Moreover in nuclear matter, this PV coupling type is preferred rather than PS coupling type because the former is consistent with PCAC while the latter contradicts to PCAC in nuclear matter \[12\].
The RMC transition rate is given by

\[ \frac{d\Gamma_{RMC}}{dk} = \frac{\alpha G^2 |\phi_\mu|^2 m_N}{(2\pi)^2} \int_{-1}^{1} dy \frac{k E_{\nu}^2}{W_0 - k(1 - y)} \frac{1}{4} \sum_{\text{spins}} |M_{fi}|^2 , \]  

(21)

where \( \alpha \) is the fine structure constant, \( G \) is the standard weak coupling constant, \( y = \hat{k} \cdot \hat{\nu}, \ k_{\text{max}} = (W_0^2 - m_n^2)/2W_0, \ E_\nu = W_0(k_{\text{max}} - k)/[W_0 - k(1 - \nu)], \ W_0 = m_p + m_n - \) (muon binding energy) and \( |\phi_\mu|^2 \) is the absolute square of muon wave function averaged over the proton which is taken as a point Coulomb. In order to compare to the experimental results, we take the following steps. For liquid hydrogen target, muon capture is dominated through the ortho and para \( p\mu p \) molecular states \[2,13\]. Since these molecular states can be attributed to the combinations of hyperfine states of \( \mu p \) atomic states \[13\] i.e. single and triplet states, we decompose the statistical spin mixture \( \frac{1}{4} \sum_{\text{spins}} |M_{fi}|^2 \) into such hyperfine states by reducing \( 4 \times 4 \) matrix elements to \( 2 \times 2 \) spin matrix elements. At this step, we confirmed that when the \( \Delta M_e \) term was not included, eq.(21) reproduced the curves given in ref. \[4\].

For the description of RMC in nuclear matter, we follow the Fearing’s paper \[14\], i.e. we adopted the relativistic mean field theory \[15\] where the nucleons are treated as free Dirac particles with effective mass due to the scalar and vector potential. Then the nucleus are the Fermi gas. Our RMC capture rate in nuclear matter is given in the following way

\[ \Gamma_{RMC}^{NM} = \frac{\alpha G^2 m_p m_N}{4\pi^2 4\pi k_F^4} \int_{k_{\text{min}}}^{k_{\text{max}}} dk E_{\nu} n p^2 k \int_{(\cos \theta_k)_{\text{min}}}^{(\cos \theta_k)_{\text{max}}} d\cos \theta_k \int_{p_{\text{min}}}^{p_{\text{max}}} dp \int_{k_F}^{m_{\text{max}}} dn \sum_{\text{spins}} |M_{fi}|^2 , \]  

(22)

where the integration intervals of nucleon momenta, which comes from the Pauli blocking in nuclear matter, are calculated in detailed kinematics and \( k_F \) is the Fermi momentum. For finite nuclei, we need to know the corresponding muon wave functions and \( k_F \) values, but which depends on the model. We already suggested a model \[9\] for this purpose, but will be skipped here and concentrate on the case of nuclear matter.
III. RESULTS AND DISCUSSIONS

Our results for RMC on proton are shown in Fig.1. The solid curve is the spectrum obtained in ref. 2, i.e. the result without $\Delta M_e$ term for $\hat{g}_P = 9.8$. On the other hand, the dotted curve is calculated without $\Delta M_e$ term for $\hat{g}_P = 6.77$. This curve is obviously much lower than the solid curve. When $\Delta M_e$ term is taken into account for $\hat{g}_P = 6.77$, we obtain the dashed curve which is very close to the solid curve for the energy spectrum on $k \geq 60 MeV$. The minor discrepancy may be due to the neglect of higher order contributions and other degree of freedom such as $\Delta$. Our result shows that $\Delta M_e$ term restores the credit of $\hat{g}_P = 6.77$.

The number of RMC photons observed for $k \geq 60 MeV$ is $279 \pm 26$ and the number of those from the solid curve is 299, while our result obtained by integrating the dotted curve spectrum is 273. Since the contribution of $\Delta$ degree of freedom is known to be a few percent [7], it is not included in the present calculation. Vector mesons such as $\rho$ and $\omega$ are also turned out to have very small contributions in this calculation. Higher order terms are pointed out to be insignificant [3].

It is confirmed that the matrix elements upto the $M_e$ term in eq.(19) satisfy gauge invariance. However, it is broken when the $\Delta M_e$ term and $\Delta$ degree of freedom are included. As far as diagram method is adopted, the gauge invariance is more or less broken because a series of diagrams will be cut somewhere. In order to estimate the rate of gauge invariance broken by the $\Delta M_e$ term, we evaluated the spectrum with $\Delta M_e$ alone. The result is shown by a dot-dashed curve in Fig.1. The gauge invariance breaking is not so large as expected. However, in the limit $m_\mu \to 0$, the gauge invariance is restored.

In spite of such a burden of gauge invariance, our present calculation shows that $\hat{g}_P = 6.77$ is reasonable for both OMC and RMC on a proton.

Figure 2 shows the photon energy spectrum in nuclear matter, which is presented as the ratio of RMC to OMC in order to reduce the uncertainty from the nuclear structure. We compared our amplitude to Fearing’s analysis, which is PS coupling and has been used to
extract the $\hat{g}_P$ from the experimental data. At the same $\hat{g}_P$, our ratio R is higher than the PS coupling scheme. It means that $\hat{g}_P$ value to fit some experimental data becomes lower in our PV scheme. Therefore $\hat{g}_P$ quenching rate is smaller than the usual PS coupling scheme. These behaviour are nearly independent of effective nucleon mass in nuclear matter (see the solid, dotted and dashed lines).

Figure 3 shows another interesting results for $\hat{g}_P$ quenching in finite nuclei. The larger $k_F$, which may mean the heavier nuclei, the smaller becomes the ratio R. As a result, $\hat{g}_P$ quenching may be larger in heavier nuclei. But in relatively lower $k_F$ region just reversed results are shown. Consequently $\hat{g}_P$ may be enhanced in lower $k_F$ region. Since we have to integrate the nucleon’s Fermi motion from the possible lowest momentum up to the Fermi momentum, the mechanism in lower $k_F$ region could play a role of compensating $\hat{g}_P$ quenching in larger $k_F$ region. Therefore this could be an indication for the $\hat{g}_P$ enhancement in light nuclei.

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Figure Captions

Figure 1. Photon energy spectrum for triplet states in RMC on proton. The solid curve is deduced without $\Delta M_e$ term for $\hat{g}_P = 9.8$, whose result corresponds to the experimental results in ref.2. The dotted curve is obtained without $\Delta M_e$ term for $\hat{g}_P = 6.77$. The dashed curve is with $\Delta M_e$ for $\hat{g}_P = 6.77$. The dot-dashed curve is calculated with $\Delta M_e$ term alone for $\hat{g}_P = 6.77$.

Figure 2. The photon energy spectrum for the ratio of RMC and OMC in nuclear matter. The thick curves are results from our transition amplitudes, but thin curves are Fearing’s amplitudes. The solid curves are for $M^* = M$, the dotted curves are for $M^* = 0.57M$ and the dashed curves are for $M^* = 0.7M$.

Figure 3. The ratio of RMC and OMC versus fermi momentum $k_F$. The dot-dashed curve ($k_\gamma = 60$ MeV) and dotted curve ($k_\gamma = 80$ MeV) come from our amplitude, while the solid and long dashed curves are from Fearing’s, respectively from $k_\gamma = 60$ and 80 MeV.
\[
\frac{d\Gamma}{dk} \left( s^{-1} \text{MeV}^{-1} \right)
\]

vs.

\[
E_\gamma \text{(MeV)}
\]
