A Test of the AdS/CFT Duality on the Coulomb Branch

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Abstract

We consider the $N = 4$ SU($N$) Super Yang Mills theory on the Coulomb branch with gauge symmetry broken to $SU(N_1) \times U(N_2)$. By integrating the W particles, the effective action near the IR $SU(N_i)$ conformal fixed points is seen to be a deformation of the Super Yang Mills theory by a non-renormalized, irrelevant, dimension 8 operator. The correction to the two-point function of the dilaton field dual operator near the IR is related to a three-point function of chiral primary operators at the conformal fixed points and agrees with the classical gravity prediction, including the numerical factor.

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1 Introduction

Recently there has been a considerable effort in studying the AdS/CFT duality [1, 2, 3] when conformal invariance is broken (see [4] and references therein). The general philosophy is that conformal invariance can be broken by considering a non-conformal vacuum of the conformal theory, or otherwise by considering a deformation of its Lagrangian. In both cases the dual gravitational background will have an AdS boundary but it will differ from this space in the inside. Identifying the radial coordinate with the field theory energy scale the geometry will describe the RG flow starting from the UV conformal theory at the AdS boundary.

In this note we shall consider the case of a non-conformal vacuum of \( SU(N) \) \( \mathcal{N} = 4 \) Super Yang-Mills (SYM) theory. The moduli space of this theory is given by \( \mathbb{R}^{6(N-1)}/S_N \), parameterizing the relative position of \( N \) identical D3-branes in the transverse space \( E^6 \). Separating the D3-branes in two stacks of \( N_1 \) and \( N_2 \) branes by a distance \( 2\Delta \) is equivalent to Higgs the theory by giving an expectation value to the scalar fields according to

\[
\langle \vec{Y} \rangle \equiv \langle \vec{\phi} \rangle = \frac{\Delta}{2\pi\alpha'} \left( \begin{array}{cc} I_1 & 0 \\ 0 & -I_2 \end{array} \right),
\]

where \( I_i \) is the \( N_i \times N_i \) unit matrix. The gauge symmetry is broken to \( S(U(N_1) \times U(N_2)) \) and the theory is left with 16 conserved supercharges because conformal symmetry is broken. The symmetry breaking process gives rise to massive W particles with mass \( m_W = \Delta/(\pi\alpha') \). This construction provides an example of maximally supersymmetric RG flow: starting from the UV \( SU(N) \) conformal theory we flow to the IR conformal fixed points with \( SU(N_i) \) gauge symmetry. Related aspects of the AdS/CFT duality can be found in references [5-23].

To derive the dual gravitational background we start with the double-centered D3-brane geometry with metric

\[
ds^2 = H^{-1/2} d\mathcal{M}^4 + H^{1/2} d\mathcal{E}^6,
\]

where the harmonic function \( H \) reads

\[
H = 1 + \frac{R_1^4}{|\vec{y} - \vec{\Delta}|^4} + \frac{R_2^4}{|\vec{y} + \vec{\Delta}|^4},
\]

with \( R_i^4 = 4\pi g_s\alpha'^2 N_i \) and \( \vec{y} \) the coordinates in the transverse space \( E^6 \). Next, we take the decoupling limit of [4] which corresponds to drop the factor 1 in the harmonic function \( H \) and to consider energy scales in the brane theory much smaller than the string scale, i.e. \( \epsilon, m_W \ll 1/\sqrt{\alpha'} \). We further require the string coupling \( g_s \) to be small and \( g_s N_i \) to be large for the supergravity approximation to hold.
In a recent paper [21] we studied the classical absorption for a minimally coupled scalar, e.g. the dilaton field, in the double-centered D3-brane geometry. The analysis was valid for low energies such that $\epsilon \ll m_g \ll m_W$, where $m_g = m_W/\sqrt{gN}$ is the gravity mass gap. In the above decoupling limit the cross-section for absorption by the $i$-th hole was seen to be

$$\sigma_i = \frac{\pi^4 \omega^3 R_i^8}{8} \left[ 1 - \frac{(\omega R_i)^4}{12} \left( \frac{R_j}{2\Delta} \right)^4 \log \left( \frac{\omega R_i^2}{\Delta} \right) + \cdots \right], \quad (j \neq i). \tag{4}$$

In the field theory side, the total absorption cross section (i.e. $\sigma = \sigma_1 + \sigma_2$) for a scalar field $\phi$ is related to the two-point function of the field theory dual operator $O_\phi$

$$\Pi(x) = \langle O_\phi(x) O_\phi(0) \rangle, \tag{5}$$
calculated in the non-conformal vacuum described above. In the case of the dilaton field the exact relation is [21]

$$\sigma = \frac{2\kappa^2}{2i\omega} \text{Disc} \Pi(s)\bigg|_{p_0=\omega, \ p=0}, \tag{6}$$

where $\kappa$ is the gravitational coupling, $s = -p^2$ and $\Pi(s)$ is the Fourier transform of $\Pi(x)$.

The low energy expansion of the classical cross section (4) corresponds to consider the dual field theory near the $SU(N_i)$ IR fixed point. The purpose of this letter is to determine the exact form of the effective action near the IR fixed points that results from integrating out the W particles and therefore to reproduce exactly the logarithmic correction to the cross section using the dual field theory. We shall see that the effective action is given by a deformation of the $\mathcal{N} = 4$ SYM theory by a non-renormalized, irrelevant, dimension 8 operator as anticipated in [25, 21]. Then, the correction to the cross section will be related to a three-point function of chiral primary operators calculated at the IR fixed points [26]. To obtain exact agreement with the classical cross section calculation it was essential to use the symmetrized trace of the Yang Mills fields in the deformed Lagrangian and to keep only the planar diagrams. This result is further evidence for a non-renormalization theorem for three-point functions of chiral primary operators [27-32] and provides a test of the AdS/CFT duality on the Coulomb branch.

## 2 Effective Action in the Infrared

We start by writing the Lagrangian for the bosonic sector of $\mathcal{N} = 4$ $SU(N)$ SYM theory in the following form

$$\mathcal{L}_0 = -\frac{1}{4} \text{tr} \left[ F_{AB} F^{AB} \right], \tag{7}$$
where $A, B, \cdots$ are ten-dimensional indices and $F_{AB}$ is short for $F_{\mu\nu} = D_{\mu} A_{\nu} - D_{\nu} A_{\mu}$, $F_{\mu m} = D_{\mu} \phi_m$ with $D_{\mu} \equiv \partial_{\mu} + ig_{YM}[A_{\mu}, \cdot]$ and $F_{mn} = ig_{YM}[\phi_m, \phi_n]$. We removed the gauge coupling from the front of the action by rescaling the fields as $(A_\mu, \varphi^m) \rightarrow g_{YM}(A_\mu, \varphi^m)$, which also rescales the expectation value for the scalars in equation (1).

We want to integrate the $W$’s in order to obtain an effective action for the light $SU(N_1)$ and $SU(N_2)$ coloured fields at energy scales $\epsilon \ll m_W$ as explained in [21]. This is similar to the probe calculations extensively considered in the literature [33-36], where $N_1 = N$ and $N_2 = 1$. In the latter case the first non-vanishing one-loop diagram involves 4 $SU(N)$ coloured legs, with the following contribution to the effective bosonic Lagrangian [35, 36]

$$L_1 = -\frac{\pi^2 g_{YM}^4}{(2\Delta/\alpha')^4} \text{Str} \left[F_{AB} F^{BC} F_{CD} F^{DA} - \frac{1}{4} \left(F_{AB} F^{AB}\right)^2\right].$$

Notice that we are using the symmetrized trace as argued in [36]. This fact will be essential to obtain agreement with the dual classical calculation of the absorption cross-section. Also, these $F^4$ terms are protected [37], i.e. they are not renormalized by higher loop diagrams and therefore comparison with the strongly coupled supergravity regime is allowed.

Now we generalize the above result to arbitrary large $N_1$ and $N_2$. The only difference is that the term in the effective action involving 4 $SU(N_i)$ fields will be multiplied by a factor $N_j$ ($i \neq j$) because we have a $SU(N_j)$ colour index to sum over around the loop (due to the $W$’s exchange). Also, for large $N$ graphs with both $SU(N_1)$ and $SU(N_2)$ coloured external legs are subleading since they are associated with non-planar graphs. Hence, in the IR and for large $N$ there are no terms in the effective action of the type $\text{tr}_1(F^2)\text{tr}_2(F^2)$, where the subscript in $\text{tr}_i$ means that the trace is over $SU(N_i)$ fields. We conclude that the $F^4$ terms in the effective action for the IR $SU(N_i)$ theory read

$$L_1 = -\frac{\pi^2 g_{YM}^4 N_j}{(2\Delta/\alpha')^4} \text{Str}_i \left[F_{AB} F^{BC} F_{CD} F^{DA} - \frac{1}{4} \left(F_{AB} F^{AB}\right)^2\right], \quad (j \neq i).$$

To check this result consider the action for a probe of $N_i$ D3-branes in the AdS near-horizon geometry of $N_j$ D3-branes, i.e. we assume that $N_j \gg N_i$. The probe dynamics is determined by the non-abelian DBI action [38] which describes the effect of integrating all the massive strings stretching between the probe and the branes. The AdS background is described by the metric [2] with the harmonic function $H \equiv f = (R_j/r)^4$. If the center of mass for the $N_i$ probes is placed at $r$ we have

$$S_{\text{probe}} = -T_3 \int d^4 x f^{-1} \text{Str}_i \left[\sqrt{-\det \left(\eta_{\alpha\beta} + f \partial_\alpha Y^m \partial_\beta Y_m + 2\pi \alpha' \sqrt{f} F_{\alpha\beta}\right)} - I_i\right],$$

where $T_3 = ((2\pi)^3 g_{YM} \alpha'^2)^{-1}$ is the D3-brane tension. Next we give an expectation value to the scalars $\langle \bar{Y} \rangle = 2\Delta I_i$, which means that $r = 2\Delta + \delta r$ where $\delta r$ is the center of mass fluctuating
field in the radial direction. For large \( N_i \) we can discard the fields in the center of the gauge group and consider only \( SU(N_i) \) fields. Thus, setting \( f = (R_j/2\Delta)^4 \), expanding the DBI action and rescaling the fields according to \( (A_\alpha, \phi_m) \rightarrow g_{YM}(A_\alpha, \phi_m) \) we obtain the probe Lagrangian

\[
\mathcal{L}_{\text{probe}} = -\frac{1}{4} \text{tr} \left[ F_{AB} F^{AB} \right] - \frac{\pi^2 g_{YM}^4 N_i}{(2\Delta/\alpha')^4} \text{Str} \left[ F_{AB} F^{BC} F_{CD} F^{DA} - \frac{1}{4} (F_{AB} F^{AB})^2 \right],
\]

(11)

which is in agreement with the previous result as expected from the non-renormalization of the \( F^4 \) terms.

In the context of the asymptotically flat D3-brane geometry, the form of the above deformation of the SYM Lagrangian was conjectured based on the DBI corrections to the SYM theory [26], or alternatively on the basis of \( PSU(2|2,4) \) representation theory [39]. For a geometry with harmonic function

\[
H = h + \left( \frac{R}{r} \right)^4,
\]

(12)

the dual field theory was conjectured to be [39, 25]

\[
\mathcal{L} = \mathcal{L}_0 - \frac{h}{8T_3} \mathcal{O}_8,
\]

(13)

where \( \mathcal{O}_8 \) is an irrelevant, dimension 8 operator. This deformation is irrelevant in the IR which agrees with the fact that for \( r \rightarrow 0 \) the constant \( h \) becomes irrelevant in the harmonic function \( H \). What remains an open question is if the Lagrangian (13) describes the dual gravity theory as we flow from the IR. If we assume that the DBI action is dual to the full D3-brane geometry \( (h = 1) \), then we can regard (13) as the first correction to the SYM theory and determine \( \mathcal{O}_8 \) to be exactly \( \mathcal{O}_8 = \text{Str}[F^4 - \frac{1}{4}(F^2)^2] \). However, even in this case agreement between the gravity and the field theory calculations of the cross section is not found [26]. Fortunately the case here studied is entirely under control. While in the asymptotic flat space case the DBI action arises from integrating the massive string states that are dropped out in the decoupling limit, in our case the deformation of the IR Lagrangian arises from integrating the W’s that do survive the decoupling limit. In fact, in the decoupling limit of the double-centered D3-brane geometry the harmonic function \( H \) is given down the \( i \)-th throat by

\[
H = \left( \frac{R_j}{2\Delta} \right)^4 + \left( \frac{R_i}{r} \right)^4.
\]

(14)

Then the deformation of the SYM action in the IR \( SU(N_i) \) conformal fixed point is indeed given by (13) with \( h = (R_j/2\Delta)^4 \) and \( \mathcal{O}_8 = \text{Str}[F^4 - \frac{1}{4}(F^2)^2] \) [23, 21]. This (non-renormalized) deformation was computed exactly as a result of integrating the W’s. Hence,
if we believe the AdS/CFT correspondence to hold on the Coulomb branch, the gravity and perturbative field theory calculation of protected quantities in the IR using the Lagrangian (13) must give exactly the same answer.

3 Field Theory Calculation of Cross-Section

We proceed by calculating the cross section for absorption of the dilaton field using the field theory approach. This calculation was done in [26] using a $U(1)$ model. We refer the reader to that reference for the details and will present here only the relevant steps necessary to obtain the correct answer. For world-volume on-shell processes that involve the coupling of the dilaton to the brane only the gauge field is relevant. In the IR $SU(N_i)$ theory the operator $O_\phi$ dual to the dilaton field reads

$$S_{int} = \int d^4x \phi \left( \frac{1}{4} \text{tr}_i [F_{\mu\nu} F^{\mu\nu}] + \frac{h}{T_3} \text{Str}_i \left[ F_{\mu\nu} F_{\eta\lambda} F^\lambda\mu - \frac{1}{4} (F_{\mu\nu} F^{\mu\nu})^2 \right] \right),$$

$$\equiv \int d^4x \phi O_\phi \equiv \int d^4x \phi \left[ \mathcal{O}_4 + \frac{h}{T_3} \mathcal{O}_8 \right].$$

Then the two-point function for the operator $O_\phi$ is

$$\Pi(x) = \left\langle O_\phi(x) O_\phi(0) \right\rangle_{h=(R_j/2\Delta)^4} = \int \mathcal{D}A_\mu e^{-\int d^4z \left[ \frac{1}{4} \mathcal{O}_4 + \frac{h}{8T_3} \mathcal{O}_8 \right] O_\phi(x) O_\phi(0)}$$

$$= \left\langle O_\phi(x) O_\phi(0) \left( 1 - \left( \frac{R_j}{2\Delta} \right)^4 \frac{1}{8T_3} \int d^4z \mathcal{O}_8(z) \right) \right\rangle_{h=0}$$

$$= \frac{1}{2^4} \left\langle \mathcal{O}_4(x) \mathcal{O}_4(0) \right\rangle_{h=0} - \left( \frac{R_j}{2\Delta} \right)^4 \frac{1}{2^7T_3} \int d^4z \left\langle \mathcal{O}_4(x) \mathcal{O}_8(z) \mathcal{O}_4(0) \right\rangle_{h=0}$$

$$\equiv \Pi_0(x) + \Pi_1(x),$$

where we are just keeping the leading terms that will give the logarithmic corrections in the classical gravity result for the cross section. We see that the correction to the two-point function is related to a three-point function of chiral primary operators calculated at a IR conformal fixed point.

We start by writing the Euclidean space propagator for the field strength $(F_{\mu\nu})^{ab}$ in the conformal theory

$$(F_{\mu\nu})^{ab}(x)(F_{\alpha\beta})^{cd}(0) \equiv \left\langle (F_{\mu\nu})^{ab}(x) (F_{\alpha\beta})^{cd}(0) \right\rangle = \frac{\delta^{ad} \delta^{bc}}{\pi^2 x^4} \left[ \delta_{\mu\alpha} \delta_{\nu\beta} - \delta_{\mu\beta} \delta_{\nu\alpha} - \frac{2}{x^2} \left( \delta_{\nu\beta} x_\mu x_\alpha + \delta_{\mu\alpha} x_\nu x_\beta - \delta_{\nu\alpha} x_\mu x_\beta - \delta_{\mu\beta} x_\nu x_\alpha \right) \right],$$

(17)
where \( a, b, \cdots \) are \( SU(N_i) \) colour indices. In what follows it is convenient to write first the following contraction of the field strength propagators

\[
\left( (F_{\alpha\beta})^{ab}(F_{\alpha\beta})^{ba} \right)(x) \left( (F_{\mu\nu})^{cd}(F_{\eta\lambda})^{ef} \right)(0) = \frac{2\delta^{de}\delta^{cf}}{\pi^4 x^8} \left( \delta_{\mu\eta}\delta_{\nu\lambda} - \delta_{\mu\lambda}\delta_{\nu\eta} \right). \tag{18}
\]

It is trivial to check that \( \Pi_0(x) \equiv \frac{1}{2h} \langle O_4(x)O_4(0) \rangle_{h=0} = (3N_i^2)(\pi^4 x^8) \tag{24} \). To calculate \( \Pi_1(x) \) first we expand the symmetrized trace of \( O_8 \) in terms of simple traces as

\[
O_8(z) = \text{Str}_i \left[ F^4 - \frac{1}{4}(F^2)^2 \right] = \frac{2}{3} \text{tr}_i \left[ F_{\mu\nu}F^{\mu\eta}F^{\nu\lambda}F_{\lambda\eta} + \frac{1}{2} F_{\mu\nu}F^{\nu\eta}F_{\eta\lambda}F^{\lambda\mu} \right. \\
- \frac{1}{4} F_{\mu\nu}F^{\mu\nu}F_{\eta\lambda}F^{\eta\lambda} - \frac{1}{8} F_{\mu\nu}F_{\eta\lambda}F^{\mu\nu}F^{\eta\lambda} \right]. \tag{19}
\]

Then using standard perturbative field theory techniques we have (see also \[13\])

\[
\langle O_4(x)O_8(z)O_4(0) \rangle_{h=0} = \frac{(3 \times 2^8)N_i^3}{\pi^8(x-z)^8 z^8}, \tag{20}
\]

and therefore

\[
\Pi_1(x) = - \left( \frac{R_j}{2\Delta} \right)^4 \frac{1}{2^7 T_3} \int d^4 z \frac{(3 \times 2^8)N_i^3}{\pi^8(x-z)^8 z^8}. \tag{21}
\]

Notice that we have to be careful in applying Wick’s theorem to obtain this result. The reason is that for large \( N_i \) only contractions between the fields in \( O_4 \) and fields that are consecutive in the trace expansion \([13]\) of \( O_8 \) will contribute. The other contractions correspond to non-planar graphs and are subleading in the large \( N_i \) limit \([13]\) (see figure). A consequence of this fact is that it was essential that we used the symmetrized trace in the effective action, otherwise we would obtain a different answer. Using the result

\[
\int d^4 u \frac{e^{ip \cdot u}}{u^8} = \frac{\pi^2}{3 \times 2^6} p^4 \log \left( \frac{p^2/\Lambda^2}{L^2} \right), \tag{22}
\]

\(^{2}\)I thank Igor Klebanov for bringing this point to my attention.
a simple calculation gives the Fourier transform of $\Pi_1(x)$

$$
\Pi_1(p) = -\left(\frac{R_j}{2\Delta}\right)^4 \frac{N_i^3}{(3 \times 2^{11})\pi^4} p^8 \left(\log \left(\frac{p^2}{\Lambda^2}\right)\right)^2,
$$

(23)

where $\Lambda$ is a cut-off scale. Then the absorption cross section is related by equation (6) to the momentum space two-point function $\Pi(p)$. The result is

$$
\sigma_i = \frac{\kappa^2 \omega^3 N_i^2}{32\pi} \left[1 - \left(\frac{R_j}{2\Delta}\right)^4 \frac{N_i \omega^4}{(3 \times 2^{3})T_3\pi^2} \log \left(\frac{\omega}{\Lambda}\right)\right],
$$

(24)

which agrees exactly with the classical gravity prediction, including the numerical factors.

We remark that in the perturbative field analysis it seems natural to set the cut-off scale to $\Lambda = m_W$, while the strong coupled gravity calculation suggests that $\Lambda = \Delta/R_i^2 \sim m_W/\sqrt{gN}$. This fact may be related to the existence of colour singlet condensates of $W$ particles at strong coupling with a large binding energy [18, 40]. However, a better understanding of the cut-off scale would require the extension of this calculation to the next order [26].

In resume, we tested the AdS/CFT duality on the Coulomb branch by finding agreement between the gravity and field theory absorption cross sections for the dilaton field near the $SU(N_i)$ IR conformal fixed points. This was done by determining the large $N$ (non-renormalized) $F^4$ terms in the field theory IR effective action that arise from integrating the massive $W$ particles. Then the correction to the absorption cross section is related to a three-point function of chiral primary operators. To obtain the correct result required large $N$ considerations as well as the use of the symmetrized trace in the effective action. This result provides further evidence for a non-renormalization theorem for three-point functions of chiral primary operators [27-32].

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