Quantum randomness is considered to be an important resource for information processing. Quantum randomness stems from quantum coherence, whose information-theoretic characterization is currently under investigation. In this work, we link the quantum randomness with a coherence measure. The quantum randomness of a local measurement corresponds to the minimum amount of uncertainty about the outcome for a correlated party. Meanwhile, in another scenario where the correlated party performs a local measurement to gain classical information, the corresponding quantum randomness has also been shown to be a coherence measure, namely, coherence of formation. We show that the gap between the two quantum randomness, or coherence measures, corresponds to the quantum discord of the correlated state after the local measurement. The relation between local quantum randomness and correlation is discussed. We observe that the local quantum randomness on one party corresponds to the minimum amount of information required to be sent to the correlated party to unlock hidden data.

I. INTRODUCTION

According to the Born rule [2], the outcome of a quantum measurement is intrinsically uncertain. Given a quantum state \( |\alpha\rangle = \sum_i c_i |i\rangle \), where the \( \{c_i\} \) are complex coefficients, the result of a measurement represented by the operators \( \{|i\rangle \langle i|\} \) is not deterministic, having output \( i \) with probability \( p_i = |c_i|^2 \). Such randomness differs from the classical uncertainty due to imperfections of the measurement process. How to discriminate between quantum and classical contributions to the measurement randomness is a topic of interest in quantum information theory, as quantum randomness promises to be a potential resource for information processing tasks [3–8]. As it is immediately clear from the example above, quantum randomness is a consequence of breaking coherent superpositions of quantum states, a phenomenon nowadays routinely observed in the laboratory. Recently, several works studied the properties of coherent states as an information-theoretic resource [9, 10]. It turns out that the key notion to identify a resource, i.e., the one of free operations, is debatable in the case of coherence. Thus, several definitions of degree of coherent superposition are possible. The most intuitive way to quantify coherence is being the distance to the set of incoherent states \( I \) for a reference basis \( \{|i\rangle\} \), given an appropriate yet arbitrary (pseudo)-metric function: \( C_d(\rho) = \min_{\sigma \in I} d(\rho, \sigma) \). We label this notion of coherence as BCP coherence [11, 12]. A widely employed solution is to adopt the relative entropy of coherence \( C_R(\rho) = \min_{\sigma \in I} S(\rho||\sigma) \) as a measure, mainly because of its computability and the importance of the relative entropy in information theory [13]. While the parent notion of asymmetry has a clear-cut interpretation in a number of physical settings [9], and other significant advances have been reported [14], the computational power offered by the BCP coherence still needs to be clarified.

In this work, we provide an operational interpretation to the quantum contribution to a measurement randomness. We show that the quantum randomness of a measurement corresponds to the peculiar BCP coherence, as quantified by the relative entropy of coherence. The strategy we adopt is the following. In Section II, we pick the Shannon entropy \( H(p_i) = -\sum_i p_i \log p_i \) as the quantifier of the total randomness of a measurement with probability distribution \( \{p_i\} \) in a state \( \rho \). Such choice is not unique [15–19], yet inspired by several previous works. For example, entropic quantities have been employed to develop information-theoretic uncertainty relations, the first study on the topic being Everett’s thesis [20–31]. Our goal is to split the total randomness into quantum and classical contributions and provide an operational meaning to the quantum share. It is immediate to notice that for pure states the only kind of uncertainty is the truly quantum one, thus the Shannon entropy is itself a measure of quantum randomness. On the same hand, for incoherent states, the measurement randomness is purely classical, as it depends on the incomplete knowledge of the system state, i.e., its mixedness. We consider a scenario which is consistent with these two situations and allows to give an operational interpretation to the quantum randomness associated to a measurement in the more complex case of mixed coherent states. The idea implies to consider a bipartite extension of a system manipulated by an experimentalist Alice, say accessible to a pair Alice-Eve in the state \( \rho_{AE} \), and address the question of how much information Eve can access about Alice’s measurement outcome with probability distribution \( \{p_i\} \) and outputs being the elements of a reference basis \( \{|i\rangle_A\} \). We show that in the best case scenario Eve’s ignorance is quantified by the relative entropy of coherence with respect to the reference basis, which is a good quantifier of the quantum randomness affecting Alice’s
measurement: \( \min_{p \in \mathcal{P}} H(\{p_i\} | E)_{\rho_{AE}} = C_R(\rho_A) \).

We then compare the result with the scenario in which Eve gains information about Alice’s measurement “classically” by performing a measurement on her part (Section III A). A previous work proved that, as in the former setting, Eve’s uncertainty on Alice’s outcome is a full-fledged measure of BCP coherence, namely, coherence of formation [1]. Such a measure yet differs from the relative entropy of coherence, being instead obtained by a convex roof construction. We show that the gap between the two quantities corresponds to the quantum discord of the Alice-Eve’s system after Alice’s measurement (Section III B), a kind of quantum correlation which is more general than entanglement [33–35]. We show that the discrepancy between the two quantities, which characterizes the irreversibility of coherence resource theory [32], corresponds to the quantum discord of the state. In Section III C, we further highlight that the result is in agreement with findings identifying discord as the resource for quantum data locking [36, 37], i.e., information which is made accessible through quantum correlations, yet kept secure against attacks, by sharing a relatively small supplemental amount of data (a key, in the cryptography parlance). The convex roof quantum randomness and the relative entropy of coherence correspond to the amount of bits that Alice has to send to a colleague Eve to reconstruct the message respectively before and after sharing the key. In Section IV we draw our conclusions.

II. QUANTIFYING QUANTUM RANDOMNESS

Let us consider a \( d \)-dimensional Hilbert space and a reference basis \( I := \{|i\} = \{|1\}, |2\}, \ldots, |d\} \). Suppose a projective measurement \( \{|i\} \langle i| \) is performed on a given quantum state \( \rho_A \) accessed by an experimentalist Alice. The measurement outcome has a probability distribution \( \{p_i\}, \sum_{i=1}^{d} p_i = 1, p_i = \text{Tr}[\rho_A |i \rangle \langle i|] \geq 0, \forall i \). In quantum information theory, a practical quantifier of the total randomness associated to the measurement is given the Shannon entropy \( H(\{p_i\}) = -\sum_{i} p_i \log(p_i) \). However, the randomness of the measurement is intrinsically twofold: a classical uncertainty due to Alice’s ignorance about the system state; and a quantum one due to the coherence of the state in the reference basis. For a mixture of incoherent states \( \rho_X = \sum_i q_i |i \rangle \langle i| \), the measurement randomness is given by the state mixedness, i.e., a classical source of uncertainty, quantified by the von Neumann entropy: \( H(\{q_i\}) = H(\rho_X) \). On the other hand, for pure states, \( \rho_X = |\psi \rangle \langle \psi| \), the randomness is due to the genuinely quantum overlap between the state and the basis elements: \( H(\{p_i\})_{|\psi \rangle} = H(|\langle i| \psi \rangle|^2) \).

Here we present an operational characterization of the quantum randomness for arbitrary coherent mixed states. To be a good measure of quantum uncertainty, a quantity should satisfy the following properties:

1. Being nonnegative;
2. Vanishing if and only if the measurement uncertainty is only due to the state mixedness;
3. Representing the total uncertainty for pure states;
4. Being convex [11, 12, 16–18].

We consider the worse case scenario depicted in Fig.1, where Alice and Eve share a bipartite system in state \( \rho_{AE} \). Alice makes a measurement and obtains outcomes following a probability distribution \( \{p_i\}, p_i = \text{Tr}[\rho_A |i \rangle \langle i|] \). The total randomness associated to the measurement is \( H(\{p_i\}) \). The uncertainty of Eve about Alice’s measurement outcome is quantified by the conditional entropy \( H(\{p_i\} | E)_{\rho_{AE}} \).

![Fig. 1. Quantum randomness. In a bipartite Alice-Eve system described by a pure state \( \psi_{AE} \), the quantum randomness of a measurement performed by Alice on the system in the mixed state \( \rho_A \) is given by the amount of uncertainty Eve has on the measurement outcome. Such quantum uncertainty is quantified by the relative entropy of coherence \( R^Q_I(\rho_A) \).](image)

After the Alice’s measurement, define the global state by \( \rho'_{AE} \) and Alice’s resulted state becomes \( \rho'_{A}^{\text{diag}} := \sum_i p_i |i \rangle \langle i| \). Hence, in the best case scenario for Eve, her uncertainty is given by the von Neumann conditional entropy

\[
R^Q_I(\rho_A) := \min_{\rho_E} H(\{p_i\} | E)_{\rho_{AE}} = \min_{\rho_E} S(A|E)_{\rho'_{AE}} = S(\rho'_{AE}) - S(\rho_E).
\]

where the optimization runs over all the possible Eve’s states such that \( \text{Tr}[E(\rho_{AE})] = \rho_A \), and the conditional entropy is given by \( S(A|E)_{\rho'_{AE}} = S(\rho'_{AE}) - S(\rho_E) \). It is not hard to see that the best case scenario for Eve is to hold a purification of Alice, \( |\psi \rangle_{AE} \). In fact, one can always extend Eve’s part to hold a purification of a mixed state \( \rho_{AE} \), which will not increase her uncertainty about Alice’s measurement outcome.

When \( \rho_A \) is a pure state, then \( |\psi \rangle_{AE} \) and hence \( \rho'_{AE} = \rho'_{A}^{\text{diag}} \otimes \rho_E \) are both product states. It is easy to verify that Eve’s uncertainty corresponds to the total randomness of Alice’s measurement:

\[
R^Q_I(\rho_A) = S(\rho_A^{\text{diag}}) = H(\{p_i\})_{\rho_A}.
\]

When \( \rho_A \) is not a pure state, after Alice’s measurement, the state is changed to \( \rho_{AE}' = \sum_i p_i |i \rangle_A \langle i| \otimes \rho_E \), where \( \rho_E = \sum_i p_i \rho_E^i \). In fact, \( \rho_A = A \langle i| (\langle \psi \rangle_{AE} |\psi \rangle_{AE}) |i \rangle_A / p_i \) is a pure state. The conditional entropy of the post
measurement state is given by \( S(A|E)_{\rho_{AE}} = S(\rho_{AE}) - S(\rho_{E}) \). Using the equality \( S(\sum_i p_i |i\rangle \langle i| \otimes \rho_i) = H(p_i) + \sum_i p_i S(\rho_i) \), the conditional entropy is then \( S(A|E)_{\rho'_{AE}} = H(p_i) + \sum_i p_i S(\rho_{E}^i) - S(\rho_{E}) \). Since \( H(p_i) = S(\rho_A^{(i)}), S(\rho_{E}) = S(\rho_{A}), \) and \( S(\rho_{E}^i) = 0, \forall i, \) we have

\[
R_I^Q(\rho_A) = S(\rho_A^{\text{diag}}) - S(\rho_A). \tag{3}
\]

It is immediate to observe that the Eve’s uncertainty is equal to the relative entropy of coherence

\[
R_I^Q(\rho_A) = S(\rho_A||\rho_A^{\text{diag}}) = C_R(\rho_A), \tag{4}
\]

thus satisfying all the requirements for a consistent measure of quantum randomness as well as being a measure of BCP coherence [11].

Note that, when considering a tripartite pure state \( |\psi_{ABE}\rangle \) and a projective measurement \( \{|i\rangle \langle i|\} \) on system \( A \), it is shown [31] that the quantum randomness of the measurement outcome conditioned on system \( E \) corresponds the distance between state \( \rho_{AB} = \text{tr}_E[|\psi_{ABE}\rangle \langle \psi_{ABE}|] \) and state \( \rho'_{AB} \) after the measurement. Furthermore, by regarding system \( B \) as a trivial system, the analysis in Ref. [31] also applies to our scenario.

### III. QUANTUM RANDOMNESS AND QUANTUM DISCORD

#### A. Coherence of formation

In Section II we showed that the quantum randomness of a local measurement can be quantified by the best case uncertainty of a correlated party Eve. Such uncertainty has been quantified by the quantum conditional entropy. We compare the result with an alternative measure of quantum randomness reported in Ref. [1]. The setting is for the sake of clarity depicted in Fig. 2. The difference is that Eve performs a measurement with probability distribution \( \{q_i^E\}, q_i^E = \text{Tr}\rho_{E} |e_i^E\rangle \langle e_i^E| \) on her own system to predict Alice’s measurement outcome. The best case uncertainty is then given by the classical conditional entropy:

\[
R_I^C(\rho_A) = \min_{\rho_{E}, \{q_i^E\}} H(\{p_i\} | q_i^E)_{\psi_{AE}}, \tag{5}
\]

where the minimization runs over all the possible Eve’s states and measurements. When Alice’s system is in a pure state \( |\psi\rangle_A = \sum_i \sqrt{p_i} |i\rangle \), the probability distributions of \( A \) and \( E \) are uncorrelated as the the global system is in a tensor product state. Hence, we have

\[
R_I^C(\rho_A) = H(\{p_i\} | q_i^E)_{\psi_{AE}} = H(\{p_i\}) \quad \text{for any Eve’s strategy.}
\]

The quantity corresponds to the total randomness as expected. For an arbitrary mixed state \( \rho_A \), it turns out that the Eve’s uncertainty on Alice’s measurement is given by

\[
R_I^C(\rho_A) = \min_{\{p_i, |\psi_i\rangle\}_A} \sum_i p_i R_I^C(\psi_i)_A, \tag{6}
\]

where the minimization is over all possible decompositions of \( \rho_A \). We briefly review the proof here. Given the spectral decomposition \( \rho_A = \sum_i \lambda_i |a_i\rangle \langle a_i| \), then a purification of \( \rho_A \) is \( |\psi\rangle_{AE} = \sum_i \sqrt{\lambda_i} |a_i\rangle \otimes |e_i\rangle_E \). Here \( \{|e_i\rangle_E\} \) is an orthogonal basis of Eve’s system. Eve performs a projective measurement \( \{|e_i\rangle_E\} \) on her local system, then based on her measurement outcome \( |\psi_i\rangle_E \), the Alice’s state is

\[
|\psi_i\rangle_A = \frac{1}{\sqrt{p_i}} \sum_j \sqrt{\lambda_j} |\langle e_i | e_j| \rangle |a_i\rangle_A, \tag{7}
\]

where \( p_i = \sum_j \lambda_j |\langle e_i | e_j| \rangle|^2 \). As the state of Alice is pure for each outcome of Eve, the averaged quantum randomness is \( \sum_i p_i R_I^C(\psi_i)_A \). On the other hand, Eve can choose an arbitrary measurement basis, which determines a decomposition of \( \rho_A \), to maximize his prediction success probability. Therefore, the quantum randomness measure should be optimized over all the possible decompositions of \( \rho_A \). When Eve performs a general measurement (POVM), we can always enlarge the system of Eve and consider a projective measurement, then the proof follows accordingly q.e.d.

**FIG. 2.** Alternative definition of Quantum randomness. In a bipartite Alice-Eve system described by a pure state \( |\psi_{AE}\rangle \), the quantum randomness of a measurement performed by Alice on the system in the mixed state \( \rho_A \) is given by the minimum amount of uncertainty Eve has on the measurement outcome after performing a measurement on her own systems. Such quantum uncertainty is quantified by the convex roof measure \( R_I^Q(\rho_A) \).

The quantum randomness measure obtained by convex roof extension of the pure state randomness is a measure of BCP coherence as well [1]. Let us compare the two quantities \( R_I^C(\rho_A), R_I^Q(\rho_A) \) in a simple example about a qubit system. In the Bloch sphere representation, \( \rho_A = (I + \vec{n} \cdot \vec{\sigma})/2 \), where \( \vec{n} = (n_x, n_y, n_z) \) and \( \vec{\sigma} = (\sigma_x, \sigma_y, \sigma_z) \) are the Pauli matrices. Suppose that the measurement basis is the \( \sigma_2 \) eigenbasis, which is denoted by \( \{|0\}, |1\} \), then we obtain

\[
R_I^C(\rho_A) = H\left( \frac{1 + \sqrt{1 - n_x^2 - n_y^2}}{2} \right), \tag{8}
\]

\[
R_I^Q(\rho_A) = H\left( \frac{n_x + 1}{2} \right) - H\left( \frac{|n| + 1}{2} \right),
\]
where \(|n| = \sqrt{n_x^2 + n_y^2 + n_z^2}\) and \(H\) is the binary entropy.

Specifically, for the state \(\rho_A(v) = v \ket{+} \langle + | + \frac{v}{2} \mathbb{I}, \text{ where } \ket{+} = (|0\rangle + |1\rangle)/\sqrt{2}, v \in [0, 1], n(v) = (v, 0, 0)\), we have

\[
R^C_x(\rho_A) = H\left(\frac{1 + \sqrt{1 - v^2}}{2}\right),
\]

\[
R^Q_x(\rho_A) = 1 - H\left(\frac{v + 1}{2}\right).
\]

In Fig. 3, we plot the quantum randomness versus the mixing parameter \(v\). As expected, the quantum randomness measure \(R^Q_x\) obtained through the fully quantum picture is smaller than \(R^C_x\), which is derived by the measurement-based method, while they both vanish when the state is incoherent, and are equal to the Shannon entropy in the pure state case.

![Comparison of the measures of quantum randomness](image)

**FIG. 3.** Comparison of the measures of quantum randomness \(R^Q_x\) (red dotted line) and \(R^C_x\) (blue dot-dashed line) in the qubit state \(\rho_A(v) = v \ket{+} \langle + | + \frac{v}{2} \mathbb{I}\) versus the mixing parameter \(v\).

### B. Quantum Discord as Quantum Randomness gap

We observed that, in general, there is a non-zero gap between the two quantum randomness measures: \(R^C_x(\rho_A) \geq R^Q_x(\rho_A)\), \(\forall \rho_A\). As the difference between the two frameworks in Figs. 1, 2 is brought about by making a measurement on Eve’s party, it is intuitive to think that the gap is related to the local measurement disturbance. Indeed, we show that such a gap is associated to the quantum discord of the bipartite state \(\rho_{AE} = \sum_i p_i \ket{i} \bra{i} \otimes \rho^L_E\) of the system after Alice carried out her measurement. Discord (we omit the quantum label from now on) is a kind of quantum correlation which equals entanglement for pure states but also shows up in all but a null measure set of separable states. It can be interpreted as the minimum disturbance induced on a bipartite system by a local measurement \[35\]. Its peculiarity is its asymmetry, as a measurement on one party has in general a different effect than performed on a different subsystem. For a state \(\rho_{AB}\), the discord defined as

\[
D_B(\rho_{AB}) = \min_{\{q^B_i\}} S(A|\{q^B_i\})_{\rho_{AB}} - S(A,B)_{\rho_{AB}} + S(B)_{\rho_{AB}}
\]

measures the least possible disturbance of a measurement with probability distribution \(\{q^B_i\}\) on the B party. Simple algebra steps show that \(\min S(A|\{q^B_i\})_{\rho_{AE}} = \min H((\{p_i\}|\{q^E_i\})_{\psi_{AE}}\) Hence, we obtain that the gap between the two quantum randomness measures is given by the discord of the state, i.e. the least possible state change induced by an Eve’s measurement:

\[
R^C_x(\rho_A) - R^Q_x(\rho_A) = D_E(\rho_{AE}).
\]

To interpret the result, we consider a standard communication task between correlated parties.

### C. Local Quantum Randomness and non-local Information access

We refer to literature for a detailed treatment of the quantum locking of classical correlations \[36, 37\], and the role played therein by discord \[38–42\], which we here summarize. An experimentalist Alice encodes a classical message represented by a random variable \(X\) taking values \(\{i\}\) with probability \(\{p_i\}\) in a quantum system and sends it to a friend Eve. The goal is to maintain the secrecy of Alice’s information against to an eavesdropper, yet making it accessible to Eve by sending her a key (another random variable). The information which is accessible to Eve is given by the mutual information between the variable \(X\) and the outcome of the most informative Eve’s measurement. Classically, i.e. if only zero discord states are allowed, the size of the key has to be approximately equal to the size of the message. Conversely, discord allows to disclose the necessary bits of information to decode the message, as measured by the X-Eve mutual information \(H((\{p_i\}) - H((\{p_i\})|E)\), by sharing a comparatively small key. Thus, it gives an advantage in maintaining secrecy against adversaries who tries to access such information without knowing the key. In the paradigmatic case of a classical-quantum state

\[
\rho_{AE} = \sum_i p_i \ket{i}_A \bra{i} \otimes \rho^L_E,
\]

it has been proved that in the asymptotic limit of many state copies the discord does equal the quantum advantage in locking data \[40\]. The maximum information Eve can extract from a measurement is given by \(H((\{p_i\}) - \min_{\rho_E} H((\{p_i\})|E)_{\rho_{AE}}\). In particular, Eve decodes completely the message \(X\) only if Alice sends her at least \(\min_{\rho_E} H((\{p_i\})|E)_{\rho_{AE}} = R^Q_x(\rho_A)\) bits of information per state copy. On the other hand, if Eve performs
her measurement with probability distribution \( \{ q_i^E \} \) before receiving the key. Alice has to send Eve at least \( \min H(\{ p_i \}| \{ q_i^E \}) \rho_{AE} = R_I^E(\rho_A) \) bits. Thus, the quantum randomness, and therefore the BCP coherence, has an operational interpretation as the amounts of bits Alice must share in order to allow complete decoding by a correlated party Eve. Indeed, \( R_I^E \) is the minimum uncertainty of Eve on Alice’s measurement outcome if she waits for the key before measuring on her system, while \( R_I^F \) is the best case uncertainty if Eve measures before receiving the key.

As a simple example, we consider the cryptographic scenario of the BB84 protocol [43]. Alice processes two bits information representing eigenbasis and polarization of a quantum state \( \rho_A \). If the basis bit is 0 (1), she prepares the state in the \( X \) (\( Z \)) basis, while if the polarization bit is 0 (1), the state has polarization up (down) in the chosen eigenbasis. To set the notation, if the two bits are 00, 01, 10, 11, Alice prepares \( |0\rangle , |1\rangle , |+\rangle , |−\rangle \), respectively. Let us suppose the probability of choosing each state is equal, and that Alice sends the quantum state to Eve, who tries to guess the state. In the best case scenario, Alice and Eve’s system is initially in the pure state

\[
|\psi\rangle_{AE} = \frac{1}{2}(|00\rangle|0\rangle + |01\rangle|1\rangle + |10\rangle|+\rangle + |11\rangle|−\rangle),
\]

where the Alice’s marginal state is given by

\[
\rho_A = \frac{1}{4}(|0\rangle\langle0| + \frac{1}{\sqrt{2}}(|10\rangle + |11\rangle))(|0\rangle\langle0| + \frac{1}{\sqrt{2}}(|10\rangle + |11\rangle)) + \frac{1}{\sqrt{2}}(|01\rangle\langle01| + |11\rangle\langle11|))(|0\rangle\langle0| + \frac{1}{\sqrt{2}}(|10\rangle + |11\rangle)).
\]

After Alice’s choice, represented by a measurement in the \( I = \{ |00\rangle , |01\rangle , |10\rangle , |11\rangle \} \) basis, one has

\[
\rho'_{AE} = \frac{1}{4}(|00\rangle\langle00| \otimes |0\rangle\langle0| + |01\rangle\langle01| \otimes |1\rangle\langle1| + |10\rangle\langle10| \otimes |+\rangle\langle+| + |11\rangle\langle11| \otimes |−\rangle\langle−|).
\]

The convex roof quantum randomness about Alice’s measurement is \( R_I^F(\rho_A) = 3/2 \), while if she shares a key with Eve, the minimum number of bits to recover the message is instead \( R_I^E(\rho_A) = 1 \). As expected, the discord of the state in Eq. (12), which is obtained by a numerical algorithm, is 1/2. Indeed, Alice can just send the basis information to Eve, who can then get the polarisation as well.

IV. DISCUSSION AND CONCLUSION

Given the twofold uncertainty of a quantum measurement, we provided an operational interpretation to the genuinely quantum randomness about a measurement performed by an observer Alice, which we quantify with the relative entropy of coherence, as the minimum uncertainty about the outcome which can be reached by a quantum correlated party Eve. We then compared the result to an alternative strategy to quantify quantum randomness by a convex roof extension of the Shannon entropy. The gap between the two strategies is equal to the discord of the bipartite state shared by Alice and Eve. The result provides an operational interpretation to the relative entropy of coherence and a link between single system quantumness and quantum correlations, following previous studies on the trade-off between local and global quantum properties [44–46]. We observe that, in the resource theory of quantum coherence, the coherence of formation and the relative entropy of coherence measure the coherence cost and the distillable coherence in the asymptotic limit, respectively [32]. Thus, the coherence cost and the distillable coherence equal the quantum uncertainty conditioned on Eve’s classical [1] and quantum strategies here discussed. The scenario is similar to what happens in the entanglement resource theory [47], where there is a nonzero gap between the entanglement cost and the distillable entanglement (Table 1). In particular, some entangled states have zero distillable entanglement, a phenomenon called bound entanglement. However, a key difference is that there is no coherent states with zero coherence of distillation [32].

Operationally, our work highlights the role of coherence in the secret communication problem called quantum data locking: the local quantum randomness equals the amount of supplemental information to be sent to a correlated party for decoding a message. To the best of our knowledge, this is the first quantitative study relating coherence measures and practical communication schemes. We hope that this work will stimulate further research on the role of quantum randomness and other coherence effects in quantum information and computation protocols.

ACKNOWLEDGEMENTS

This work was supported by the National Basic Research Program of China Grants No. 2011CBA00300 and No. 2011CBA00301, the 1000 Youth Fellowship program in China, the EPSRC (Grant No. EP/L01405X/1), and the Wolfson College, University of Oxford.

[1] X. Yuan, H. Zhou, Z. Cao, and X. Ma, Phys. Rev. A 92, 022124 (2015).
[2] M. Born, Zeitschrift für Physik 37, 863 (1926).
TABLE I. Comparison between coherence and entanglement: cost and distillation measures

| Properties                      | Coherence/randomness | Entanglement          |
|---------------------------------|---------------------|-----------------------|
| Cost                            | Coherence of formation/quantum randomness $R_C^f$, Eq. (5) | Entanglement of formation |
| Distillation                    | Relative entropy of coherence/quantum randomness $R_C^R$, Eq. (1) | Distillable Entanglement |
| Gap                             | Discord of the state after Alice’s measurement, Eq. (10) | Bound Entanglement [48] |

[3] R. Colbeck, *Quantum And Relativistic Protocols For Secure Multi-Party Computation*, University of Cambridge, PhD Thesis (2009).
[4] V. Scarani, H. Bechmann-Pasquinucci, N. J. Cerf, M. Dusek, N. Lutkenhaus, and M. Peev, *Rev. Mod. Phys.* **81**, 1301 (2009).
[5] S. Pironio, *et al.*, *Nature* **464**, 1021 (2010).
[6] X. Ma, X. Yuan, Z. Cao, B. Qi, and Z. Zhang, arXiv:1510.08957.
[7] Z. Cao, H. Zhou, X. Yuan, and X. Ma, *Phys. Rev. X* **6**, 011020 (2015).
[8] A. Acín, S. Massar, and S. Pironio, *Phys. Rev. Lett.* **92**, 100402 (2012).
[9] I. Marvian and R. W. Spekkens, arXiv:1602.08049, and references therein.
[10] J. Aberc. *Phys. Rev. Lett.* **113**, 150402 (2014).
[11] T. Baumgratz, M. Cramer, M. B. and Plenio, *Phys. Rev. Lett.* **113**, 140401 (2014).
[12] F. Herbut, *J. of Phys. A* **38**, 2959 (2005).
[13] V. Vedral, *Rev. Mod. Phys.* **74**, 197 (2002).
[14] B. Yadin, J. Ma, D. Girolami, M. Gu, and V. Vedral, arXiv:1512.02085.
[15] M. J. W. Hall, *Phys. Rev. Lett.* **74**, 3307 (1995).
[16] S. Luo, *Phys. Rev. Lett.* **91**, 180403 (2003).
[17] S. Luo, *Theor. Math. Phys.* **143**, 681 (2005).
[18] S. Luo, S. Fu, and C. H. Oh, *Phys. Rev. A* **85**, 032117 (2012).
[19] D. Girolami, *Phys. Rev. Lett.* **113** 170401 (2014).
[20] H. Everett III, *The Theory of the Universal Wave-Function*, pp 3-140, in *The Many-Worlds Interpretation of Quantum Mechanics*, Princeton Series in Physics, Princeton University Press (1973).
[21] I. I. Hirschmann, *Am. J. of Math.* **79**, 152 (1957).
[22] W. Beckner, *Ann. of Math.* **102**, 159 (1975).
[23] D. Deutsch, *Phys. Rev. Lett.* **50**, 631 (1983).
[24] I. Bialynicki-Birula, *Phys. Lett. A* **103**, 25 (1984).
[25] H. Maassen, and J. Uffink, *Phys. Rev. Lett.* **60**, 1103 (1988).
[26] J. M. Renes and J.-C. Boileau, *Phys. Rev. Lett.* **103**, 020402 (2009).
[27] M. Berta, M. Christandl, R. Colbeck, J. M. Renes, and R. Renner, *Nature Phys.* **6**, 659 (2010).
[28] P. J. Coles, L. Yu, V. Gheorghiu, and R. B. Griffiths, *Phys. Rev. A* **83**, 062338 (2011).
[29] P. J. Coles, R. Colbeck, L. Yu, and M. Zwolak, *Phys. Rev. Lett.* **108**, 210405 (2012).
[30] K. Korzekwa, M. Lostaglio, D. Jennings, and T. Rudolph, *Phys. Rev. A* **89**, 042122 (2014).
[31] Coles, Patrick J., *Phys. Rev. A* **85**, 042103 (2012).
[32] A. Winter and D. Yang, *Phys. Rev. Lett.* **116**, 120404 (2016).
[33] D. Ollivier and W. H. Zurek, *Phys. Rev. Lett.* **88**, 017901 (2001).
[34] L. Henderson and V. Vedral, *J. Phys. A* **54**, 6899 (2001).
[35] K. Modi, A. Brodutch, H. Cable, T. Paterek, and V. Vedral, *Rev. Mod. Phys.* **84**, 1655 (2012).
[36] D. P. Di Vincenzo, M. Horodecki, D. W. Leung, J. A. Smolin, and B. M. Terhal, *Phys. Rev. Lett.* **92**, 067902 (2004).
[37] S. Guha, P. Hayden, H. Krovi, S. Lloyd, C. Lupo, J. H. Shapiro, M. Takeoka, and M. M. Wilde *Phys. Rev. X* **4**, 011016 (2014).
[38] A. Datta and S. Gharibian, *Phys. Rev. A* **79**, 042325 (2009).
[39] S. Wu, U. V. Poulsen, and K. Molmer, *Phys. Rev. A* **80**, 032319 (2009).
[40] S. Boixo, L. Aolita, D. Cavalcanti, K. Modi, and A. Winter, *Int. J. of Quant. Inf.* **9**, 1643 (2011).
[41] M. Piani, V. Narasimhachar, and J. Calsamiglia, *New J. Phys.* **16**, 113001 (2014).
[42] S. Pirandola, *Sci. Rep.* **4**, 6956 (2014).
[43] C. H. Bennett, and G. Brassard, *Proceedings of the IEEE International Conference on Computers, Systems and Signal Processing*, IEEE Press, 175 (1984).
[44] D. Girolami, T. Tufarelli, and G. Adesso, *Phys. Rev. Lett.* **110**, 240402 (2013).
[45] A. Streitsoov, U. Singh, H. S. Dhar, M. N. Bera, and G. Adesso *Phys. Rev. Lett.* **115**, 020403 (2015).
[46] J. Ma, B. Yadin, D. Girolami, V. Vedral, and M. Gu, *Phys. Rev. Lett.* **116**, 160407 (2016).
[47] R. Horodecki, P. Horodecki, M. Horodecki and K. Horodecki, *Rev. Mod. Phys.* **81**, 865 (2009).
[48] G. Jaeger, *Quantum Information*, Springer-Verlag, New York (2007).