Hydraulic and gas-dynamic models of a steam-water mixture flow in a granular bed

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Abstract. The paper presents a comparative analysis of the hydraulic and gas-dynamic models of a steam-water mixture flow through a granular bed. The analysis is carried out using experimental data obtained at an inlet pressure of 0.6 - 15.5 MPa, flow quality of 0.002 - 0.3, and mass velocity through a cross-section of a packed bed from 44.5 to 410 kg/(m²s). In the hydraulic model, with a view to improving the accuracy, the algebraic calculation of an average pressure is replaced with numerical integration over the bed height. In the gas-dynamic model, which allows analytical integration, refined constitutive relations are proposed. An analysis of the obtained results indicates that the gas-dynamic model provides a good description of experimental data, whereas the hydraulic model leads to a considerable error in calculation in the case of large relative pressure drops.

1. Introduction

Recently, the steam-water mixture flow in packed beds of spherical particles, which is used in the energy industry, chemical industry and other areas of industrial production, has been actively studied. One of the main objectives of this study is to establish a relationship between the flow rate and pressure drop. Out of a large number of works devoted to the experimental study of two-phase flows in porous media [1–4], the works [1, 2], where the authors investigate the forced flow of steam-water mixture, are of particular interest. The results of the experiments were summarized using different models. In the model by Avdeev et al. [5] (denoted by 1), the packed bed resistance is calculated in the same way as for a component of a hydraulic circuit. The model by Tairov et al. [6] (denoted by 2) is based on the Goldshtik gas dynamics equations [7]. In both models, the transition to a two-phase mixture involves finding empirical dependences that make it possible to express the density of the steam-water mixture through pressure and flow quality. In model 1, algebraic in form, the density of the steam-water mixture and the hydraulic resistance coefficient are calculated from the average pressure in the bed. In model 2, the polytropic law approximates the intermediate states of the mixture between the bed inlet and outlet.

In this paper, a comparative analysis of the hydraulic and gas-dynamic models is carried out using two independent sets of experimental data. The first data set was obtained by Avdeev et al. [1], the second data set was obtained by a research team from Melentiev Energy Systems Institute (ESI). The similarities and differences in the construction of hydraulic and gas-dynamic models are highlighted.
2. Mathematical models

2.1. Hydraulic model

The hydraulic model (model 1) [5] is based on the quadratic dependence between the mass velocity of the two-phase mixture in the full cross-section of channel, $\rho w_0$, and pressure gradient.

$$\frac{dP}{dy} = -\lambda \left( \frac{\rho w_0}{2 \rho d} \right)^2, \quad (1)$$

where $\rho$ is a density of the two-phase mixture expressed through void fraction $\varphi$ as follows

$$\rho = \rho' (1 - \varphi) + \rho'' \varphi \quad (2)$$

Hereinafter, one stroke refers to saturated liquid, and two strokes refer to saturated vapor. Model 1 employs the following interpolation expression to relate void fraction with flow quality $x$:

$$\varphi = \begin{cases} 
0.83 \beta, & \beta < 0.8; \\
\left[1 + 6.25(1 - \beta)\right]^{-0.5}, & \beta \geq 0.8,
\end{cases} \quad (3)$$

where $\beta = \left[1 + (\rho''/\rho')(1 - x)/x\right]^{-1}$.

Expression (1) indicates an increase in pressure gradient along the packed bed with a decrease in the mixture density, $\rho$, and a decrease in the diameter of the packed bed particles $d$. The coefficient of hydraulic resistance $\lambda$ is expressed by the formula experimentally obtained for one-phase liquid flow:

$$\lambda = \begin{cases} 
3.56 \ m^{-1.8} \ Re_{0}^{0.2}, & Re_{0} < 6500; \\
0.615 \ m^{-1.8}, & Re_{0} \geq 6500,
\end{cases} \quad (4)$$

where $Re = \rho w_0 d / \mu$ is calculated for the liquid flow with a mass velocity equal to a mass velocity of the mixture through the full cross-section of the channel, and $m$ is the average porosity of the packed bed.

Pressure drop in the packed bed is an integral of expression (1) with respect to $y$ from 0 to the height of the packed bed $H$. In [5], the data with relatively small pressure drops were analyzed using the linear approximation of the pressure profile which was expressed by a simplified formula

$$P_1 - P_2 = \Delta P = \frac{1}{2} \lambda \left[ \frac{\rho w_0}{\rho' + \rho''} \right]^{2} \frac{H}{d}, \quad (5)$$

where $P_0 = (P_1 + P_2)/2$ is an average pressure. Further on, subscripts 1 and 2 refer to the packed bed inlet and outlet, respectively. It is worthwhile to note that in the case where the flow rate and the inlet pressure are known and it is required to know their values at the outlet, the average pressure here is also unknown, consequently equation (5) is solved iteratively.

Note that the inverse problem is also of practical value: to find flow rate at a given pressure drop. To this end, formula (5) is converted to

$$\rho w_0 = \frac{2 \Delta P \ d \left[ \rho / H \lambda \right]_{P_1=P_2}^{1/2}}{H} \quad (6)$$

Equation (6) is also solved iteratively because although the initial data of the inverse problem already contain the outlet and average pressure, $\lambda$ depends not only on the average pressure but on the mass velocity as well.

The use of the two-phase flow properties averaged over the bed height in (5) gives a considerable error when the pressure drops are large. In some cases, the calculation using the average velocity
produces an outlet pressure below its physically acceptable value. Therefore, in this research, instead of formulas (5) and (6), we additionally used the numerical integration of equation (1) to find the outlet pressure and the secant method to solve the inverse problem of finding the flow rate at a known pressure drop.

2.2. Gas-dynamic model
The gas-dynamic model (model 2) also relies on the quadratic dependence of form (1) between the mass velocity of the mixture and the pressure drop and on the calculation of the mixture density by finding the void fraction (2). However, unlike model 1, model 2 uses a differing approach to calculating the hydraulic resistance coefficient and the mixture density. The hydraulic resistance coefficient obtained theoretically by Goldshlik for the jet flow of a gas around the particles with Reynolds numbers of above several hundred is used [7]:

\[ \lambda = \frac{3(1-m)}{\psi m}, \]  

(7)

where \( \psi = 0.508 - 0.56(1-m) \) is the relative minimum flow section of the packed bed. In the considered range 0<Re<7000, the difference in the values of coefficients \( \lambda \), calculated by the formulas (4) and (7), accounts on average for 15 percent.

In model 2, void fraction \( \varphi \) is expressed through slip ratio \( s \):

\[ \varphi = \left[ 1 + s (\rho^* / \rho)(1-x)/x \right]^{-1}, \]  

(8)

\[ s = 1 + 11.4 \gamma (1- \gamma) / \rho^{0.408}, \]  

(9)

where \( \gamma = \left[ 1 + 0.1(1-x)/x \right]^{-1}. \)

The constitutive relation (9) was obtained using a combined set of data from [1] and from ESI. Figure 1 presents a comparison of void fraction values obtained by formulas (3) and (8) at different pressures. As is seen, these values are generally close but for small flow qualities, the calculation using formula (3) gives a higher value of void fraction and, consequently, a lower mixture density.

\[ \rho = \rho_1 \left( \frac{P}{P_1} \right)^{1/\alpha}, \]  

(10)

Figure 1. Comparison of void fraction in models 1 and 2 versus flow quality and pressure.
where $n$ is a polytropic coefficient. The mixture density was calculated at several intermediate values of pressure between inlet pressure and atmospherics pressure for all experimental conditions by using (2), (8), (9) and conventional tabular data for $\rho'$ and $\rho''$. The obtained pressure-density data was fitted with the power function. It was found that exponent $(1/n)$ in formula (10) depends mostly on the inlet flow quality, giving the following expression for $n$:

$$n = 0.55 + 0.45(1 - \exp(-x_1/0.237)) .$$  \hspace{1cm} (11)

Thus, the following expression is obtained for the outlet pressure:

$$p_2 = p_1 \left[ 1 - \frac{\lambda}{2} \left( \frac{\rho_1}{\rho} \right)^\frac{2}{n+1} \right]^{\frac{n+1}{n}} .$$  \hspace{1cm} (12)

An analytical expression for the outlet pressure (12) can only be obtained for $\lambda$ that does not depend on pressure, which is true for the gas-dynamic model.

The inverse problem is also solved analytically.

$$\rho_2 w_0 = \left[ 2 \frac{d}{H} \left( \frac{n}{n+1} \right) p_1 \frac{\rho_1}{p} \left( \frac{p}{p_1} \right)^{\left(\frac{n+1}{n}\right)} \right]^{1/2} .$$  \hspace{1cm} (13)

Formula (13) allows finding the maximum mass velocity corresponding to the maximum possible pressure drop. The phenomenon of choking, limiting the available pressure drop and mass velocity, is considered in [8-11]. As it is shown in [8], according to the data of experiments on the flow to the atmosphere, although the maximum (critical) mass velocity is achieved under an outlet pressure greater than the atmospheric pressure, substitution of the atmospheric pressure for an outlet one in formula (13) allows predicting the critical mass velocity with an error not exceeding 3%.

3. Results and discussion

The data from [1] contain 78 experiments involving a packed bed 200 mm high of the particles with a diameter of $d=2.123$ mm, the porosity of the packed bed is 0.392. The ranges of other values are presented in Table 1.

The ESI data contain the results of 485 experiments collected in 54 series, each representing the relationship between the outlet pressure and the mass velocity at fixed values $d$, $H$, $P_1$, $x_1$. In this case, we used spherical particles, 2 and 4 mm in diameter, with a packed bed porosity of 0.37 and 0.396, respectively, combined in various ways with the bed heights of 50, 100, 250 and 355 mm. The inlet pressure is 0.6, 0.9 and 1.2 MPa.

**Table 1.** Limits of experimental parameters values.

| Parameter                            | Data from [1] | Data from ESI |
|--------------------------------------|---------------|---------------|
| Mass velocity, $\rho w_0$, kg/m²s    | min 99, max 268 | min 45, max 410 |
| Inlet pressure, $P_1$, MPa           | 1.61, 15.3    | 0.6, 1.2      |
| Relative pressure drop, $\Delta P/P_1$, % | 0.2%, 12%    | 7.7%, 85%     |
| Inlet flow quality, $x_1$            | 0.002, 0.300  | 0.011, 0.178  |
| Reynolds number, $Re_0$              | 1969, 6719    | 511, 6244     |

Figure 2 demonstrates the pressure profiles in the packed bed, calculated with models 1 and 2 in comparison with the experimentally known conditions at the packed bed inlet and outlet. The calculation with model 1 was performed using the average pressure with formula (5) proposed in [5], and with numerical integration of equation (1), while the calculation with model 2 employed polytropic approximation using formula (12). The comparison shows that for the data of [1] (Fig. 2a),
both models give results close to the experimental ones, whereas, for the data of ESI (Fig. 2b), model 1 overestimates the pressure gradient and, as a result, underestimates the outlet pressure. The use of numerical integration of equation (1) leads to a smaller error compared to the calculation involving the average pressure. Therefore, further, calculations with model 1 were done using the numerical integration of equation (1). The gas-dynamic model allows predicting the outlet pressure with higher accuracy.

**Figure 2.** The calculated pressure profiles versus the experimental values of the inlet and outlet pressure. a) Data 1, $P_1=4.48$ MPa, $x_1=0.061$, $d=2.123$ mm, $H=200$ mm, $\rho w_0=138$ kg/(m$^2$s) b) Data 2, $P_1=0.6$ MPa, $x_1=0.011$, $d=2$ mm, $H=250$ mm, $\rho w_0=151$ kg/(m$^2$s). Lines: 1 – calculation based on equation (5), 2 – model 1 with numerical integration, 3 – calculation by equation (12).

The attainment of maximum flow rate with an increase in pressure drop at a constant inlet pressure is shown in Figure 3. Here the points correspond to the experimental data of ESI, and the lines indicate the calculation with models 1 and 2. The calculation using model 1 at low pressures leads to an underestimation of the theoretical curve relative to experimental points. This is especially evident in the area of low void fraction and large pressure drops across the packed bed height. Model 2 shows satisfactory accuracy over the entire range of void fractions considered.

**Figure 3.** The attainment of maximum flow rate according to the ESI data and calculation with the models: dashed lines - model 1, solid lines - model 2, points - experiment. a) $P_1=0.6$ MPa, $d=2$ mm, $H=250$ mm. b) $P_1=0.9$ MPa, $d=4$ mm, $H=355$ mm.
Figure 4. Comparison of calculated and experimental values of mass velocity. (a) Model 1, data from [1], (b) Model 2, data from [1], (c) Model 1, data from ESI, (d) Model 2, data from ESI.

In Figure 4, the calculated values of mass velocity are compared with the experimental values for both models. It can be seen that the data from [1] are well described by both models, whereas for the data from ESI, model 1 shows an underestimated mass velocity. Here $R^2$ is the determination coefficient.

Conclusions
The study focuses on the hydraulic and gas-dynamic models of a steam-water mixture flow through a packed bed of spherical particles. The calculation results are compared with the experimental data. A common feature of both models is the proportionality of the pressure gradient to the square of mass velocity. However, the hydraulic model uses the coefficient of hydraulic resistance obtained experimentally for the flow of a single-phase liquid, whereas in the gas-dynamic model, the packed bed resistance is derived analytically based on the model of gas jet flow. The constitutive relations for the void fraction in both models are obtained from the experiment on the flow of a steam-water mixture through a packed bed. In the hydraulic model, the void fraction is expressed through the flow quality, and in the gas dynamic model, it is presented in terms of a slip ratio. The expression for the slip ratio in model 2 was refined using a combined set of data from [1] and from ESI.

The pressure profiles were calculated in two ways: 1) using the hydraulic model with numerical integration and 2) using the simplified formula. Their comparison has indicated that the calculation of the mixture parameters by the average pressure gives a considerable error in the case of nonlinear pressure profile. The two models were used to solve both the direct problem of calculating the outlet pressure at a given mass velocity and the inverse problem of calculating the mass velocity at a given outlet pressure.
Testing the models on two sets of experimental data demonstrated that the gas-dynamic model describes all experiments with good accuracy, while the use of the hydraulic model [5] is limited by the conditions of the experiment [1].

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