Moduli Stabilization in Heterotic $M$-theory

Kiwoon Choi*, Hang Bae Kim†, and Hyungdo Kim‡

Department of Physics, Korea Advanced Institute of Science and Technology
Taejon 305-701, Korea

Abstract

We examine the stabilization of the two typical moduli, the length $\rho$ of the eleventh segment and the volume $V$ of the internal six manifold, in compactified heterotic $M$-theory. It is shown that, under certain conditions, the phenomenologically favored vacuum expectation values of $\rho$ and $V$ can be obtained by the combined effects of multi-gaugino condensations on the hidden wall and the membrane instantons wrapping the three cycle of the internal six manifold.

PACS Number(s):04.50.+h, 11.25.Mj, 04.65.+e
Recently Hořava and Witten proposed that the strong coupling limit of the $E_8 \times E_8$ heterotic string theory can be described by the $d = 11$ supergravity (SUGRA) on a manifold with boundaries. At energy scales below the $d = 11$ Planck scale $M_{11} = \kappa^{-2/9}$, the 11-dimensional bulk and 10-dimensional boundary actions allow a dimensional expansion in powers of $\kappa^{2/3}$ which may be interpreted as the inverse of the membrane tension. Phenomenological implications of this heterotic $M$-theory have been studied by compactifying the 11-dimensional SUGRA on a Calabi-Yau (CY) manifold times the eleventh segment. It has been noticed that the resulting effective theory can reconcile the observed $d = 4$ Planck scale $M_P \approx 2.4 \times 10^{18}$ GeV with the phenomenologically favored unification scale $M_{\text{GUT}} \approx 3 \times 10^{16}$ GeV in a natural manner, which was not possible in perturbative heterotic string theory. In addition to providing a natural framework for the unification of couplings, heterotic $M$-theory has other phenomenological virtues. For instance, gaugino masses in heterotic $M$-theory appear to be comparable to the squark masses even when the $d = 4$ supersymmetry is broken by hidden sector gaugino condensation. This is in contrast to the case of perturbative heterotic string theory in which hidden sector gaugino condensation leads to undesirably small gaugino masses compared to the squark masses. Another phenomenological virtue of $M$-theory is that there can be a QCD axion whose high energy axion potential is suppressed enough so that the strong CP problem can be solved by the axion mechanism.

Compactified heterotic $M$-theory involves the two geometric moduli, the length $\pi \rho$ of the eleventh segment $S^1/Z_2$ and the volume $V$ of the internal six manifold $X$ averaged over $S^1/Z_2$. The above-mentioned phenomenological virtues of heterotic $M$-theory have been discussed based on the assumption that $\pi \rho$ and $V$ are stabilized at the VEVs leading to the correct values of the $d = 4$ gauge and gravitational couplings together with $M_{\text{GUT}} \approx 3 \times 10^{16}$ GeV. In this paper, we wish to study the stabilization of $\rho$ and $V$ in the context of $d = 4$ effective action including various nonperturbative effects, e.g. gaugino condensations on the hidden wall, membrane and fivebrane instantons. Our analysis indicates that $\rho$ and $V$ can be stabilized at the phenomenologically favored VEVs by the combined effects of multi-
gaugino condensations and the membrane instantons wrapping the three cycle $C_3$ of $X$ if the hidden sector involves multi-gauge groups with appropriate hidden matter contents, and $X$ admits a complex structure for which the value of $(32\pi^2)^{1/2} |\int_{C_3} \Omega|/(i \int_X \Omega \wedge \bar{\Omega})^{1/2}$ is of order unity where $\Omega$ is the harmonic $(3,0)$ form on $X$.

To proceed, let us estimate the phenomenologically favored values of $\langle \rho \rangle$ and $\langle V \rangle$. To be definite, we will use the compactification involving a smooth CY manifold. Including the corrections at $\mathcal{O}(\kappa^{2/3})$, the CY volume is given by

$$V_{CY}(x^{11}) = V - (4\pi \kappa^2)^{1/3} \left( x^{11} - \frac{1}{2} \right) \pi \rho \int \omega \wedge I_4,$$

where $x^{11}$ covers $S^1/Z_2 = [0,1]$ whose boundaries at $x^{11} = 0$ and $x^{11} = 1$ represent the visible sector wall and the hidden sector wall, respectively, $\pi \rho = \int x^{11} \sqrt{g_{11,11}}$ is the physical length of $S^1/Z_2$, $\omega$ is the CY Kähler form, and $I_4 = \frac{1}{8\pi^2} \text{tr}(F \wedge F - \frac{1}{2} R \wedge R)$. Here we use the downstairs $d = 11$ SUGRA coupling, i.e. $\kappa^2 \equiv \kappa_{\text{down}}^2 = \frac{1}{2} \kappa_{\text{up}}^2$, and also take into account the factor $2^{1/3}$ correction to the $d = 10$ YM coupling made in [8]. Obviously, $V$ corresponds to the CY volume averaged over $S^1/Z_2$. A simple dimensional reduction of the $d = 11$ bulk and $d = 10$ boundary actions leads to the $d = 4$ gauge coupling constant $\alpha_{\text{GUT}} = (4\pi)^{2/3} \kappa^{4/3} \langle V_{CY}(0) \rangle^{-1}$ at the unification scale $M_{\text{GUT}} = \langle V_{CY}(0) \rangle^{-1/6}$ and the $d = 4$ Planck scale $M_P = \kappa^{-1} \sqrt{\pi \rho V}$. Fitting the phenomenological values of $\alpha_{\text{GUT}}, M_P$ and $M_{\text{GUT}}$, one finds [4]

$$\langle \pi \rho \rangle \approx 15 \kappa^{2/9} \approx (4 \times 10^{15} \text{ GeV})^{-1},$$

$$\langle V \rangle \approx 80 \kappa^{4/3} \approx (3 \times 10^{16} \text{ GeV})^{-6}.$$  (2)

For the gravitino mass $m_{3/2} = \mathcal{O}(1) \text{ TeV}$, the Kaluza-Klein scales of compactified dimensions are much higher than the moduli masses which are presumed to be $\mathcal{O}(m_{3/2})$ and also than the dynamical scale of supersymmetry breaking, e.g. the hidden gaugino condensation scale which would be $\mathcal{O}(10^{13}) \text{ GeV}$. This justifies our approach studying the moduli stabilization in the context of the $d = 4$ effective SUGRA action.

In $d = 4$ effective SUGRA, $\rho$ and $V$ form the chiral superfields $S$ and $T$ together with the axions arising from the 3-form gauge field in $d = 11$ SUGRA. Since $\langle \pi \rho \rangle$ is larger than
by one order of magnitude, one can consider an intermediate $d = 5$ effective theory at length scales between $\langle V \rangle^{1/6}$ and $\langle \pi \rho \rangle$. In this scheme, at the leading order in $\kappa^{2/3}$, $S$ belongs the $d = 5$ hypermultiplet, while $T$ and $d = 4$ SUGRA multiplet belong to the $d = 5$ SUGRA multiplet. At any rate, $S$ and $T$ can be normalized by fixing the periodicity of their axion components:

$$\text{Im}(S) \equiv \text{Im}(S) + 1, \quad \text{Im}(T) \equiv \text{Im}(T) + 1. \quad (3)$$

In this normalization,

$$\text{Re}(S) = (4\pi)^{-2/3}\kappa^{-4/3}V,$$

$$\text{Re}(T) = (4\pi \sum_{IJK} C_{IJK}/6)^{-1/3}\kappa^{-2/3}\pi \rho V^{1/3}, \quad (4)$$

where $C_{IJK} = \int \omega_I \wedge \omega_J \wedge \omega_K$ are the intersection numbers for $\omega_I$ ($I = 1, \ldots, h_{1,1}$) which form the basis of the integer $(1, 1)$ cohomology. To derive the second relation, we have used $\kappa^{-2/3}\pi \rho \omega = (4\pi)^{1/3}\text{Re}(T) \sum_I \omega_I$ and $V = \frac{1}{3} \int \omega \wedge \omega \wedge \omega$ for the CY Kähler form $\omega$. Then the moduli VEVs of Eq. (2) correspond to

$$\langle \text{Re}(S) \rangle \equiv \langle S_R \rangle = \mathcal{O}(\alpha_{\text{GUT}}^{-1}),$$

$$\langle \text{Re}(T) \rangle \equiv \langle T_R \rangle = \mathcal{O}(\alpha_{\text{GUT}}^{-1}), \quad (5)$$

and our problem becomes to stabilize both $S_R$ and $T_R$ at the VEVs of $\mathcal{O}(\alpha_{\text{GUT}}^{-1})$. To make a comparison, let us note that the dilaton and the overall Kähler modulus in perturbative heterotic string theory are stabilized at $\langle S_R \rangle = \mathcal{O}(\alpha_{\text{GUT}}^{-1})$ and $\langle T_R \rangle = \mathcal{O}(1)$ when $S$ and $T$ are normalized as (3) [10].

The moduli effective potential would be determined by the Kähler potential $(K)$ and the superpotential $(W)$ via the standard SUGRA formula: $V_{\text{eff}} = e^K [K \hat{\partial}(D_i W)(D_j W)^* - 3 |W|^2]$. Since we are interested in the possibility for $\langle S_R \rangle \approx \langle T_R \rangle = \mathcal{O}(\alpha_{\text{GUT}}^{-1})$, let us consider the behavior of $K$ and $W$ in the limit $S_R \gg 1$ and $T_R \gg 1$. In this limit, $K$ can be divided into two pieces, $K = K_p + K_{np}$, where $K_p$ is the part which allows an asymptotic expansion in powers of $1/S_R$ and $1/T_R$ and $K_{np}$ stands for the rest which originates from nonperturbative effects.
As was noted in [11], $K_p$ can be determined either in the context of the $M$-theory large radius expansion which is available in the $M$-theory limit with $\kappa^{-2/3}(\pi \rho)^3 \approx \pi^2 T_R^3/S_R \gg 1$ or in the context of the string loop and $\sigma$-model expansion which is available in the perturbative string limit $e^{2\phi}/(2\pi)^5 \approx \pi^2 T_R^3/S_R \ll 1$. (Here $\phi$ denotes the heterotic string dilaton.) Note that the asymptotic expansion of $K_p$ in powers of $1/S_R$ and $1/T_R$ is valid in both the $M$-theory limit and the perturbative string limit as long as both $S_R$ and $T_R$ are large enough [12]. One key observation made in [11] is that, when expanded in powers of $1/\pi S_R$ and $1/\pi T_R$, the nonvanishing expansion coefficients are generically of order unity. We then have

$$K = -\ln(S + \bar{S}) - 3\ln(T + \bar{T}) + \delta K_p + K_{np},$$

(6)

where the leading logarithmic terms are determined at the leading order in the $M$-theory large radius expansion [13] or in the string loop and $\sigma$-model expansion, and the perturbative corrections are given by $\delta K_p = \sum_{n,m} C_{(n,m)}/(\pi S_R)^n(\pi T_R)^m$ with the coefficients $C_{(n,m)}$ which are essentially of order unity. For the compactification on a CY space with the minimal embedding, one finds $C_{(0,1)} = C_{(0,2)} = 0$, $C_{(0,3)} = 3\zeta(3)\chi/16 \sum_{IJK} C_{IJK}$, and $C_{(1,0)} = \chi/288$ [11], showing that $C_{(n,m)}$ are indeed of order unity (or less) for reasonable values of the Euler number $\chi$ and the intersection numbers $C_{IJK}$. In fact, the most important corrections with the coefficients $C_{(1,0)}$ and $C_{(0,1)}$ can always be absorbed into the leading logarithmic terms. After absorbing these corrections, we have

$$\delta K_p = O(1/(\pi S_R)^n(\pi T_R)^m) = O((\alpha_{\text{GUT}}/\pi)^2),$$

(7)

for $n + m \geq 2$ and the moduli VEVs of (5). It is hard to imagine that such a small $\delta K_p$ can play a significant role for the moduli stabilization, and so we will ignore it in the subsequent discussions.

As $M$-theoretic nonperturbative effects which may contribute to $K_{np}$, one can consider the following types of instantons: $I_1 =$ membrane instantons wrapping the CY 3-cycle ($C_3$), $I_2 =$ membrane instantons which wrap the CY 2-cycle ($C_2$) and are stretched along the 11-th
segment, $I_3 =$ fivebrane instantons wrapping the entire CY volume. These instantons have been discussed in [7] in the context of type IIA $M$-theory, however it is rather straightforward to extend the discussion to the heterotic $M$-theory. A complete computation of the effects of these instantons would require the full nonperturbative formulation of $M$-theory, which is not available at this moment. However one can still compute the most important semiclassical factor $e^{-A(I)}$ where $A(I)$ is the Euclidean action of the instanton $I$. A simple computation using the membrane tension $T_2 = (2\pi^2)^{1/3} \kappa^{-2/3}$ and the fivebrane tension $T_5 = (\pi/2)^{1/3} \kappa^{-4/3}$ yields [7]

$$A(I_1) = b\sqrt{S_R}, \quad A(I_2) = 2\pi kT, \quad A(I_3) = 2\pi S,$$

(8)

where $k = |\sum_I \int \omega_I|$ is a positive integer and

$$b = (32\pi^2)^{1/2} \frac{|\int C_3 \Omega|}{(i \int \Omega \wedge \Omega)^{1/2}}$$

(9)

for the harmonic $(3, 0)$ form $\Omega$ on CY. Note that generically $b$ is a function of the complex structure moduli. Since $A(I_2)$ and $A(I_3)$ are holomorphic, $I_2$ and $I_3$ can affect not only the Kähler potential, but also the holomorphic gauge kinetic functions and superpotential. However $A(I_1)$ depends only on $S_R$, and thus the effects of $I_1$ can be encoded only through the Kähler potential.

In fact, since their locations on $S^1/Z_2$ are not specified, $I_1$ and $I_3$ induce 5-dimensional local interactions which are suppressed by $e^{-A(I)}$ where $A(I_1) = b\sqrt{\text{Re}(S)}$ and $A(I_3) = 2\pi S$ for the $d = 5$ field $S$ in the universal hypermultiplet which is normalized as $\text{Re}(S) = (4\pi)^{-2/3} \kappa^{-4/3} V_{CY}$. For the CY volume $V_{CY}$ depending upon $x^{11}$ as Eq. (1), we have $\text{Re}(S) = S_R - n(1 - \frac{1}{2}) T_R$ where the integer $n = \sum_I \int \omega_I \wedge I_4$. After the reduction to $d = 4$, the 5-dimensional interactions induced by $I_1$ and $I_3$ are average over $x^{11}$, yielding the 4-dimensional local interactions suppressed by $e^{-A(I_1, 3)}$ with $A(I_{1,3})$ given in (8).

It is rather obvious that $I_2$ and $I_3$ are irrelevant for the moduli stabilization at $\langle S_R \rangle \approx \langle T_R \rangle = \mathcal{O}(\alpha_{\text{GUT}}^{-1})$ since their effects are suppressed by the extremely small $e^{-A(I)} = \mathcal{O}(e^{-2\pi/\alpha_{\text{GUT}}})$. As was noticed in [7], $I_1$ can generate a four-dilatino operator,
thereby modifying the Riemann-Kähler curvature tensor. This would result in the nonperturbative correction $K_{np} \propto e^{-A(I_1)}$. A key feature distinguishing $I_1$ from $I_2$ and $I_3$ is that it is possible that $A(I_1) = \mathcal{O}(1)$ even for $S_R = \mathcal{O}(\alpha_{GUT}^{-1})$, which would be the case if $b = \mathcal{O}(1)$ for some values of the complex structure moduli. In this case, $I_1$ can give a sizable contribution to $K_{np}$ and thus be relevant for the moduli stabilization. However we stress that a rather particular form of the complex structure is required to have $b = \mathcal{O}(1)$. For instance, $b = 3^{-3/4}(32\pi^2)^{1/2} \approx 7.8$ for the simple $Z_3$ orbifold, yielding $A(I_1) \approx 35$ for $S_R \approx 20$. In this case, the effects of $I_1$ would be too small to be relevant for the moduli stabilization.

At any rate, the discussions in the previous paragraph suggest the following form of the nonperturbative Kähler potential:

$$K_{np} = d \left( \frac{S_R}{4\pi} \right)^{p/2} e^{-b\sqrt{S_R}} \left[ 1 + \mathcal{O}\left( \frac{1}{\pi S_R}, \frac{1}{\pi T_R} \right) \right] + \mathcal{O}(e^{-2\pi S}, e^{-2\pi T}),$$

where $d$ and the integer $p$ are introduced to parameterize the unknown parts of the $I_1$-induced Kähler potential. We note that this membrane instanton-induced Kähler potential corresponds to the $M$-theory realization of the stringy nonperturbative effects which has been discussed by Shenker [14] and later applied to the dilaton stabilization in perturbative heterotic string vacua [15].

Let us now turn to the effective superpotential ($W$) of $S$ and $T$. Since $I_1$ does not affect $W$ and also the effects of $I_2$ and $I_3$ are suppressed by $e^{-2\pi T}$ and $e^{-2\pi S}$ respectively, in the limit $S_R \gg 1$ and $T_R \gg 1$, $W$ is expected to be dominated by the field-theoretic gaugino condensations. More explicitly [10],

$$W = \sum_a C_a e^{-\lambda_a f_a} + \mathcal{O}(e^{-2\pi S}, e^{-2\pi T}),$$

where $f_a$ denotes the gauge kinetic function of the $a$-th hidden gauge group $G_a$, $\lambda_a$ is determined by the one-loop beta function of $G_a$, and $C_a$ is also determined by $G_a$ and the hidden matter contents. For instance, if $G_a = SU(N)$ and there are $M$ quarks in $(N + \bar{N})$ representation, we have $C_a = -(N - M/3)(32\pi^2) \left( \frac{M}{N-M/3} \right)^{\frac{N-M/3}{M}} (M/3)^{\frac{M}{N-M/3}}$ and $\lambda_a = 8\pi^2/(N - M/3)$.

The gauge kinetic functions of the compactified heterotic $M$-theory are given by [16]
where \( n_a \) are model-dependent quantized coefficients. For compactifications on a smooth CY times \( S^1/Z_2 \), \( n_a = \frac{1}{8\pi^2} \sum_I \omega_I \wedge [\text{tr}(F^2 - \frac{1}{2} R^2)] \) = integer, however for compactifications involving a singular six manifold, e.g. orbifold, \( n_a \) are generically rational numbers depending upon the orbifold twists and also the instanton numbers of the gauge bundle \([16]\).

From the Kähler potential (6) and (10), the moduli effective potential is calculated to be

\[
V_{\text{eff}} = e^{K_{\text{np}}^{\nu/p}} \left\{ \frac{|2S_R W_S - (1 - \Delta)W|^2}{1 + \Delta'} + \frac{1}{3} |2T_R W_T - 3W|^2 - 3|W|^2 \right\},
\]

(13)

where \( W_S = \partial W / \partial S \), \( W_T = \partial W / \partial T \) and \( \Delta = \frac{1}{2}(p - b S_R^{1/2})K_{\text{np}}, \Delta' = \frac{1}{4}(p(p - 2) - (2p - 1)b S_R^{1/2} + b^2 S_R)K_{\text{np}}. \) When there is an appropriate minimum of the potential, we can calculate the gravitino mass and also the auxiliary \( F \)-components of moduli which are given by

\[
m^2_{3/2} = e^{K_{\text{np}}^{\nu/p}} |W|^2 / 16 S_R T_R^2, \quad F_S = 2m_{3/2} S_R [2S_R W_S - (1 - \Delta)W] / (1 + \Delta') W, \quad \text{and} \quad F^T = 2m_{3/2} T_R [2T_R W_T - 3W] / 3W.
\]

We wish to examine whether the potential (13) given by the superpotential (11) can achieve a (local) minimum with the moduli VEVs of (5) and supersymmetry breaking with the weak scale gravitino mass, for a reasonable choice of the hidden sector gauge group and matter fields, and also of the values of \( d, p, \) and \( b \) describing the membrane instanton-induced Kähler potential. We will require that the moduli Kähler metrics are positive-definite over a sizable domain around \( S_R \approx T_R = O(\alpha^{-1/2}_{\text{GUT}}) \), however \( \text{not} \) require the moduli potential to vanish at the minimum since it does not correspond to the fully renormalized vacuum energy density. In this paper, we present some of the results of our analysis to show the existence of the desired \( M \)-theory minimum, and the full details will be presented elsewhere \([17]\).

Let us first consider the case with single gaugino condensation yielding \( W = C_1 e^{-\lambda_1 f_1} \) for the hidden sector gauge kinetic function \( 4\pi f_1 = S - \frac{n}{2} T \). The value of \( \langle S_R \rangle \) can be set to \( O(\alpha^{-1}_{\text{GUT}}) \) by a reasonable choice of hidden sector gauge group and also of the membrane-instanton-induced Kähler potential \( K_{\text{np}} \). However, we have \( \langle T_R \rangle = 12\pi / n \lambda_1 \) which can \( \text{not} \) be \( O(\alpha^{-1}_{\text{GUT}}) \) for any reasonable values of \( \lambda_1 \) and \( n \), particularly for the values which
give rise to the weak scale gravitino mass. In fact, the minimum found in this case has $\langle S_R \rangle = O(\alpha^{-1}_{GUT})$ and $\langle T_R \rangle = O(1)$, and thus corresponds to the perturbative heterotic string vacuum discussed in [11,15], not the M-theory vacuum that we are looking for. We therefore conclude that even in the presence of a sizable $K_{np}$, single gaugino condensation does not lead to the phenomenologically favored M-theory vacua with the moduli VEVs (5) and the weak scale gravitino mass.

When there are two gaugino condensates with $W = C_1 e^{-\lambda_1 f_1} + C_2 e^{-\lambda_2 f_2}$ and also a sizable $K_{np}$, it turns out that the potential can have the desired M-theory minimum. One may first consider a superpotential implementing the simplest form of the T-duality, $T \rightarrow 1/T$: $W = \eta^{-6}(T)(C_1 e^{-\lambda_1 S/4\pi} + C_2 e^{-\lambda_2 S/4\pi})$. However this type of superpotential always leads to $\langle T_R \rangle = O(1)$ and thus the perturbative heterotic string vacuum, not the M-theory vacuum.

Motivated by the results in CY cases, here we consider an alternative simple case that the two hidden gauge groups have the same gauge kinetic function: $4\pi f_1 = 4\pi f_2 = S - \frac{1}{2}T$, while the visible sector gauge kinetic function is $4\pi f_v = S + \frac{1}{2}T$. In this case, the two gaugino condensations fix the VEV of $\text{Im}(S - \frac{1}{2}T)$ to be $4\pi^2 l/(\lambda_1 - \lambda_2)$ where $l$ is an odd integer and also can stabilize $S_R - \frac{1}{2}T_R$ by the conventional race-track mechanism [10]. In the absence of a sizable $K_{np}$, the moduli potential still has a run-away behavior along $S_R + \frac{1}{2}T_R$. However with a proper membrane instanton-induced $K_{np}$, a minimum with the desired moduli VEVs (5) can be formed. This minimum is located in the valley of the potential which is formed because the two curves of $F^S = 0$ and $F^T = 0$ come close. We could actually find several examples which give rise to the desired moduli VEVs (5) for the reasonable values of parameters, which are shown in Table 1. Note that the solution to $F^S = F^T = 0$ is always an extremum point, however in our example it turns out to be a saddle point. Without the hidden matter fields, the minimum is located at near $S_R - \frac{1}{2}T_R = 0$, which would result in a too large gravitino mass. If appropriate hidden matter fields are assumed, we can get the minimum with the desired moduli VEVs (5) and the weak scale gravitino mass. For the examples depicted in Table 1, supersymmetry breaking is characterized by $F^S \approx F^T$ which may lead to an interesting pattern of soft terms [17].
In conclusion, we have examined the stabilization of the two typical $M$-theory moduli, the length $\rho$ of the 11-th segment $S^1/Z_2$ and the volume $V$ of the internal six manifold $X$ averaged over $S^1/Z_2$. A particular attention was paid for the possibility that these moduli are stabilized at the VEVs which give rise to the correct values of the 4-dimensional gauge and gravitational coupling constants together with $M_{\text{GUT}} \approx 3 \times 10^{16}$ GeV. Such moduli VEVs could be obtained by the combined effects of multi-gaugino condensations and the membrane instantons wrapping the three cycle $C_3$ of $X$ if the hidden sector involves multi-gauge groups with appropriate hidden matter contents, and $X$ admits a complex structure for which the value of $(32\pi^2)^{1/2}|\int_{C_3} \Omega|/(i\int_X \Omega \wedge \bar{\Omega})^{1/2}$ is of order unity where $\Omega$ is the harmonic $(3,0)$ form on $X$.

**ACKNOWLEDGMENTS**

This work is supported in part by KOSEF, through CTP of Seoul National University, KRF under the Distinguished Scholar Exchange Program, and Basic Science Institute Program BSRI-97-2434.
REFERENCES

[1] P. Horava and E. Witten, Nucl. Phys. B460, 506 (1996); Phys. Rev. D54, 7561 (1996).

[2] Note that all other dimensionful parameters, e.g. the bulk SUGRA coupling (κ^2), the boundary YM coupling (κ^4/3), and also the fivebrane tension (κ^−4/3), appear as some powers of κ^2/3. As a result, counterterms induced by the loops involving the SUGRA and/or YM couplings and also those induced by fivebranes can be properly taken into account in the expansion in powers of κ^2/3.

[3] E. Witten, Nucl. Phys. B471, 135 (1996).

[4] T. Banks and M. Dine, Nucl. Phys. B479, 173 (1996).

[5] H. P. Nilles, M. Olechowski and M. Yamaguchi, Phys. Lett. 415B, 24 (1997); H. P. Nilles, M. Olechowski and M. Yamaguchi, hep-th/9801030.

[6] K. Choi, Phys. Rev. D56, 6588 (1997).

[7] K. Becker, M. Becker and A. Strominger, Nucl. Phys. B456, 130 (1995)

[8] J. O. Conrad, hep-th/9708031; T. Harmark, Phys. Lett. 431B, 295 (1998).

[9] A. Lukas, B. A. Ovrut, K. S. Stelle and D. Waldram, hep-th/9803235,9806051; J. Ellis, Z. Lalak, S. Pokorski and W. Pokorski, hep-ph/9805377.

[10] V. Krasnikov, Phys. Lett. 193B, 37 (1987); J. A. Casas, Z. Lalak, C. Munoz and G. G. Ross, Nucl. Phys. B347, 243 (1990); T. Taylor, Phys. Lett. 252B, 59 (1990); B. de Carlos, J. A. Casas and C. Munoz, Nucl. Phys. B399, 623 (1993).

[11] K. Choi, H. B. Kim and C. Muñoz, Phys. Rev. D57, 7521 (1998).

[12] Although not phenomenologically interesting, M-theory moduli space includes a domain on which κ^2/9 ≪ πρ ≪ V^1/6. On this domain, starting from the d = 11 bulk and d = 10 boundary actions expanded in powers of κ^2/3, one can systematically compute the d = 10 action which is valid at the energy scales below (πρ)^−1 but still above V^−1/6. In this
$d = 10$ action, the original $\kappa^{2/3}$-expansion is splitted into the double expansion in powers of $\kappa^{4/3}$ and $\kappa^{2/3}/\pi \rho$, which corresponds to the string loop expansion in $e^{-2\phi} (\alpha')^3 \sim \kappa^{4/3}$ and the $\sigma$-model expansion in $\alpha' \sim \kappa^{2/3}/\pi \rho$ when this action is extrapolated to the perturbative string vacua. If one goes further down to the energy scales below $V^{-1/6}$, the $d = 4$ action appears to allow the double expansion in powers of $1/S_R \sim \kappa^{4/3}/V$ and $1/T_R \sim \kappa^{2/3}/\pi \rho V^{1/3}$. Note that the $M$ and string theory duality implies that the string loop expansion in the $d = 10$ action corresponds to a dimensional expansion involving $(\alpha')^3$. This leads to for instance the nonrenormalization of $R^{(1+3k)}$ by the string $l(\geq k + 1)$-loop effects where $R$ denotes the $d = 10$ Riemann curvature tensor.

[13] T. Li, J. L. Lopez, and D. V. Nanopoulos, Phys. Rev. D56, 2602 (1997); E. Dudas and C. Grojean, Nucl. Phys. B507, 553 (1997); A. Lukas, B. A. Ovrut and D. Waldram, hep-th/9710208.

[14] S. H. Shenker, Proceedings of the Cargese School on Random Surfaces, Quantum Gravity and Strings, Cargese (France) 1990.

[15] T. Banks and M. Dine, Phys. Rev. D50, 7454 (1994); J. A. Casas, Phys. Lett. 384B, 103 (1966); P. Binetruy, M. K. Gaillard and Y. Wu, Nucl. Phys. B481, 109 (1996); hep-th/9611149,9702105; T. Barreiro, B. de Carlos and E. J. Copeland, Phys. Rev. D57, 7354 (1998).

[16] S. Stieberger, hep-th/9807124.

[17] K. Choi, H. B. Kim and H. Kim, in preparation.
TABLE I. Moduli VEVs for the hidden gauge group $SU(N_1) \times SU(N_2)$ with the hidden matters $M_1(N_1,1) + M_2(1,N_2) + \text{c.c.}$ and $4\pi f_1 = 4\pi f_2 = S - \frac{1}{2}T$.

| $N_1$ | $M_1$ | $N_2$ | $M_2$ | $d$ | $p$ | $b$ | $\langle S_R \rangle$ | $\langle T_R \rangle$ | $m_{3/2}$(GeV) |
|-------|-------|-------|-------|-----|-----|-----|------------------|------------------|------------------|
| 3     | 0     | 4     | 8     | 8   | 8   | 0.5 | 19               | 16               | $3.9 \times 10^2$ |
| 3     | 0     | 4     | 8     | 2   | 12  | 1   | 19               | 17               | $3.9 \times 10^2$ |
| 3     | 0     | 4     | 8     | 8   | 16  | 1.5 | 19               | 16               | $4.3 \times 10^2$ |
| 3     | 1     | 4     | 10    | 2.3 | 12  | 1   | 19               | 17               | $9.6 \times 10^2$ |
| 3     | 2     | 4     | 11    | 2.5 | 12  | 1   | 18               | 17               | $7.1 \times 10^2$ |