A NEW PROBE OF DARK MATTER AND HIGH-ENERGY UNIVERSE USING MICROLENSING

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\section*{ABSTRACT}

We propose the existence of ultracompact minihalos as a new type of massive compact halo object (MACHO) and suggest an observational test to discover them. These new MACHOs are a powerful probe into the nature of dark matter and physics in the high-energy universe. Non-Gaussian energy-density fluctuations produced at phase transitions (e.g., QCD) or by features in the inflation potential can trigger primordial black hole (PBH) formation if their amplitudes are $\delta \gtrsim 30\%$. We show that a PBH accumulates over time a sufficiently massive and compact minihalo to be able to modify or dominate its microlensing magnification light curve. Perturbations of amplitude $0.03\% \lesssim \delta \lesssim 30\%$ are too small to form PBHs, but can nonetheless seed the growth of ultracompact minihalos. Thus, the likelihood of ultracompact minihalos as MACHOs is greater than that of PBHs. In addition, depending on their mass, they may be sites of formation of the first Population III stars. Ultracompact minihalos and PBHs produce a microlensing light curve that can be distinguished from that of a “point-like” object if high-quality photometric data are taken for a sufficiently long time after the peak of the magnification event. This enables them to be detected below the stellar-lensing “background” toward both the Magellanic Clouds and the Galactic bulge.

\textit{Key words:} dark matter – early universe – Galaxy: halo – gravitational lensing

\textit{Online-only material:} color figures

\section{1. INTRODUCTION}

The nature of dark matter is one of the fundamental unsolved questions in cosmology. Cosmic microwave background (CMB) anisotropy data indicate that about 20\% of the universe is composed of non-baryonic dark matter and 4\% is in baryons. The most popular dark matter candidates are weakly interactive massive particles (WIMPs) and a great effort is under way to detect them. Ongoing experiments using ground-based dark matter detectors are significantly restricting the parameter space for some favored supersymmetric models (e.g., Angle et al. 2008; Ahmed et al. 2009).

A less popular—but physically well-motivated—dark matter candidate is primordial black holes (PBHs). These are black holes predicted to form during the radiation-dominated era if the universe had regions with energy-density fluctuations of amplitude $\gtrsim 30\%$. PBHs can form with a range of masses that span many decades, from the Planck mass ($10^{-5}\text{ g}$) to thousands of solar masses.

A fully relativistic approach is necessary to model the formation of PBHs. However, the Newtonian approximation elucidates the basic concept of why PBHs can form before large-scale structures appeared in the universe. During the radiation era the cosmic Jeans mass approaches the Horizon mass, which is also of the same order of the mass of a black hole with density equal to the mean cosmic value. This means that before matter–radiation equality, relatively small perturbations on Horizon scales may become self-gravitating and they begin the collapse with an initial density that is already very close to the black hole regime. Thus, the mass of PBHs depends on the time of their formation because it is of the order of the mass of the particle Horizon at that time.

However, fluctuations of amplitude of about 30\%—required for PBH formation—are very large when compared to the rms amplitude of perturbations from inflation (about 0.001\%) that lead to the formation of normal galaxies. Thus, such large perturbations would be exceedingly rare, unless they can be produced by physical processes taking place during phase transitions (e.g., topological defects, bubble nucleation, softening of the cosmic equation of state, etc.) or by ad hoc features in the inflation potential. Due to theoretical uncertainties in the physics of the high-energy universe, we do not know the typical amplitude of perturbations created at phase transitions. Hence, theoretical modeling cannot tell us whether PBHs exist, whether they are the bulk of dark matter, or whether they are only an academic curiosity. We must rely on observations to probe them as dark matter candidates.

Even if PBHs are only a fraction (i.e., <10\%) of the dark matter, during matter domination, they can grow by up to 2 orders of magnitude in mass through the acquisition of large dark matter halos (Mack et al. 2007, hereafter MOR07). The dark halo is instrumental in increasing the ability of the PBHs to accrete gas, in boosting their X-ray emission, and thus in modifying the cosmic ionization history (Ricotti et al. 2008, hereafter ROM08).

In this paper, we consider whether the dark halo is sufficiently massive and compact to produce observable modifications of microlensing light curves of PBHs or even dominate the inferred mass of the lens. Most importantly, we propose a novel type of non-baryonic massive compact halo object (MACHO). The idea is a corollary of models of PBH formation: primordially laid perturbations that fail to form PBHs because their amplitude is lower than the required threshold of $\sim 30\%$ can nevertheless seed the growth of ultracompact minihalos that are also detectable as MACHOs using microlensing experiments. Due to the lower overdensity threshold required to form ultracompact minihalos, their existence is several orders of magnitude more likely than that of PBHs.

Observational evidence for (or constraints against) PBHs is often sought in conflicting claims for (MACHO: Alcock et al. 2000) or against (EROS: Tisserand et al. 2007; OGLE: Wyrzykowski et al. 2009) detection of MACHOs in microlensing experiments toward the Magellanic Clouds (MCs).
For example, Alcock et al. (2000) originally claimed that roughly 20% of Galactic dark matter was in the form of MACHOs of characteristic mass 0.4 $M_{\odot}$, which made PBHs a good candidate (since stars of this mass would easily be seen). However, ROM08 argued that such a population of PBHs would emit X-rays and produce distortion of the CMB spectrum that are incompatible with COBE FIRAS data.

Here, however, we adopt a very different orientation. First, we reopen the possibility that PBHs can be identified with microlenses detected toward the MCs by noting that the PBH contribution to the microlens mass is small compared to that of accreted dark matter. Hence the X-ray signature is likewise small. Second, we argue that these PBH-seeded compact objects can have most of their mass outside the lensing Einstein radius, meaning that the objects might be cosmologically very important even if naive interpretations of the lensing results constrain their density to be <10% of Galactic dark matter (which would be consistent with both experiments claiming upper limits). Third, and most important, we argue that because they are embedded in accreted dark matter, PBHs generate a lensing signal that is qualitatively different from that due to stars and other “point-like” objects. Hence, PBHs would be detectable (and distinguishable from stars) in microlensing experiments even if their lensing rate was equal to or below that of stars. Moreover, even if the amplitude of early-universe perturbations is too small to create PBHs, it may still be large enough to generate compact minihalos that would give rise to lensing signatures. Like the PBH-minihalo signatures, these could also be robustly distinguished from garden-variety microlensing due to stars. Thus, we propose an observational test for PBHs and other early-universe perturbations that is far more sensitive and robust than any previously contemplated.

The plan of the paper is as follows. In Section 2, we discuss the formation and derive the density profile of ultracompact minihalos. In Section 3, we calculate the magnification light curve of microlensing events due to PBHs embedded in their minihalos and ultracompact minihalos without PBHs. In Section 4, we discuss the prospects of detecting PBHs and ultracompact minihalos below the stellar-lensing “background” toward both the MCs and the Galactic bulge. Finally, in Section 5, we present the discussion and conclusions.

2. DENSITY PROFILE OF ULTRACOMPACT MINIHALOS

It is generally accepted that the mass of PBHs does not increase significantly after their formation (Zel’Dovich & Novikov 1967; Carr & Hawking 1974). However, any locally overdense region in an expanding universe seeds the formation of dark matter structures. PBHs are local overdensities in the dark matter distribution (made of either WIMPs or smaller mass PBHs); hence, they seed the growth of spherical halos.

The theory of spherical gravitational collapse in an expanding universe (i.e., assuming radial infall), also known as secondary infall theory (Bertschinger 1985), predicts that during the matter-dominated era, the dark halo grows as $t^{2/3} \propto (1 + z)^{-1}$. During the radiation-dominated era the halo growth is of order unity, thus the halo mass grows with redshift as

$$M_h(z) = M_{\text{pbh}} \left( \frac{1 + z}{1 + z_{\text{eq}}} \right)^{-1},$$

where $z_{\text{eq}} \approx 3500$ is the redshift of matter–radiation equality (MOR07). After $z \sim 30$, the growth of the dark minihalo depends on the environment. If the PBH evolues in isolation it can continue to grow; otherwise, it will either stop growing or will lose mass due to tidal interactions as it is incorporated into a larger galactic halo.

2.1. Revisiting Secondary Infall Seeded by PBHs

In previous work on halo growth seeded by PBHs, MOR07 have assumed a point-like overdensity (i.e., the PBH) accreting gas and dark matter from a uniform density expanding universe with $\rho = \Omega_m \rho_c(z)$ (the growth of the halo mass during radiation epoch is negligible). However, this assumption is correct only if the mass of the perturbation that originates the PBH is equal to the PBH mass. This is typically incorrect. We expect $M_{\text{pbh}} \approx M_{\text{Hor}}$ only if the perturbation that creates the PBH has amplitude much larger than the critical amplitude for collapse.

One-dimensional fully relativistic simulations of gravitation collapse of perturbations on horizon scales show that the critical overdensity needed to trigger PBH collapse is $\approx w$, where $P = w \rho c^2$ is the cosmic equation of state (Carr 1975; Green et al. 2004). During the radiation era $w = 1/3 \sim 30\%$. The mass of PBHs is typically smaller than the mass within the particle horizon at the redshift of its formation, $z_f$: $M_{\text{pbh}} = f_{\text{Hor}} M_{\text{Hor}}(z_f)$, where $f_{\text{Hor}} < 1$ depends on the amplitude of the perturbation relative to the critical value $w$. Perturbations with amplitude significantly larger than $w$ produce $f_{\text{Hor}} \sim 1$, while perturbation that are near the critical value produce PBHs with masses much smaller than $M_{\text{Hor}}$. In addition, the total mass of the perturbation can be $\delta m > M_{\text{Hor}}(z_f)$ if it has wings of subcritical amplitude extending outside of the horizon at $z_f$. Partially for this reason, it has been suggested by several authors (e.g., Dokuchaev et al. 2004; Carr 2005; Chisholm 2006) that PBHs have high probability of forming clusters or binaries.

According to the theory of secondary infall any mass excess $\delta m > 0$ within a sphere of radius $R$ seeds the growth of a minihalo. Here $\delta m = M(R) - 4\pi \Omega_m \rho_c R^3$ includes the mass of the PBH and any overdensity surrounding it. We ran several simulations (see Section 2.1.1) in which the PBH is located in an overdense region confirming that the mass of the minihalo is proportional to the mass of the overdense region $\delta m$:

$$M_h(z) = \delta m \left( \frac{1 + z}{1 + z_{\text{eq}}} \right)^{-1}.$$

If the accretion has spherical symmetry (i.e., radial infall), the dark halo develops a self-similar power-law density profile $\rho \propto R^{-\nu}$ with $\nu = 2.25$ (Bertschinger 1985) truncated at a halo radius

$$R_h = 0.019 \text{ pc} \left( \frac{M_h(z)}{1 M_\odot} \right)^{1/3} \left( \frac{1 + z}{1000} \right)^{-1},$$

where $R_h$ is about one-third of the turnaround radius (Ricotti 2007; ROM08). Hence, the cumulative mass of the halo is

$$M_h(R) = M_h(z) (R/R_h(z))^{3/4}.$$

In addition, the density profile of the dark halo enveloping the PBH may become steeper than $\nu < 2.25$ in the vicinity of the central PBH. Simulations by Quinlan et al. (1995) of dark matter halos with a central black hole of mass $M_{\text{bh}}$, find a steepening of the slope of the density profile from $\nu \sim 2$ in the outer parts of the halo to $\nu \sim 2.5$ in the region around the black hole that contains a mass $M(R) < 2M_{\text{bh}}$. For the purpose of our study, this small steepening of the density profile can be important for the cases in which the mass of the lens is comparable to the mass of the PBH (see Section 2.1.2).
In Section 3, we show that if \( M_\perp(R_E) \gtrsim 30\% M_{\text{pbh}} \) the difference of the light curve of the lensing event from that of a point mass with \( M_{\text{lens}} = M_{\text{pbh}} \) is measurable. Thus, if \( \delta m = M_{\text{pbh}} \) the effect of the minihalo in modifying the magnification light curve of the PBH is hard to detect assuming realistic measurement errors. However, as we have noted before, we generally expect that \( \delta m \approx M_{\text{Hor}} > M_{\text{pbh}} \). If \( \delta m / M_{\text{pbh}} > 9 \) (or \( f_{\text{Hor}} < 0.11 \) assuming \( \delta m = M_{\text{Hor}} \)) the signature of the minihalo is observable in the lensing light curve.

In deriving Equations (4) and (5), we have neglected the steepening of the halo density profile expected in the vicinity of the PBH (Quinlan et al. 1995). As we have mentioned in the previous paragraph, this effect is important when the contribution to the lens mass from the halo is smaller or comparable to the PBH mass. If we take this effect into account, the slope of the density profile becomes \( \nu = 2.5 \) and Equation (5) becomes

\[
\frac{M_\perp(R_E)}{M_{\text{pbh}}} \approx 20\% \left( \frac{\delta m}{1 M_\odot} \right)^{1/2} \left( \frac{M_{\text{pbh}}}{1 M_\odot} \right)^{-3/4}.
\]

Thus, even for the most unfavorable and unlikely case in which \( \delta m \approx M_{\text{pbh}} \), it may be possible to detect the deviation of the lensing light curve from that of a point mass.

In the limit of \( M_\perp(R_E) \gg M_{\text{pbh}} \), then \( M_{\text{lens}} \approx M_\perp(R_E) \) and we find

\[
M_{\text{lens}} = 1 M_\odot \left( \frac{\delta m}{44.5 M_\odot} \right)^{6/5}.
\]

In this case, the mass of the lens is independent of the PBH mass. In the following section, we consider the case in which the PBH does not form: \( M_{\text{pbh}} = 0 \). This case is particularly interesting because the formation of ultracompact minihalos is statistically more likely than the formation of PBHs.

### 2.2. Ultracompact Minihalos Without PBHs

Dark matter density fluctuations that seed the formation of large-scale structure and galaxies have an amplitude \( \delta \approx 10^{-5} \) when they enter the Horizon, roughly independently of their mass (for scale-invariant perturbations). Here we focus on small mass fluctuations that enter the Horizon during the radiation era. The relativistic component of the (adiabatic)
perturbations stops growing rather quickly after it enters the Horizon because its mass becomes smaller than the Jeans mass \(M_\text{J}(z) \sim M_\text{Hor}(z)\) for \(z > z_\text{eq}\). The dark matter component grows only logarithmically because of the Meszaros effect (Meszaros 1974). Overall, small mass linear perturbations grow by 2–3 orders of magnitude from the time they enter the Horizon to matter–radiation equality, depending on their mass. A relatively rapid initial growth when the perturbation enters the Horizon is followed by a slow logarithmic growth. For the mass range of interest to us, the Horizon mass increases by 10 or more orders of magnitude from the time the perturbation enters the Horizon to the redshift of equality \(M_\text{Hor}(z) \sim 10^5 M_\odot\). For example, scale-invariant density fluctuations with mass typical of dwarf galaxies \(10^9 - 10^6 M_\odot\) grow to an amplitude \(\delta \sim 10^{-2}\) by redshift of matter–radiation equality. They collapse when \(\delta \sim 1.68\) at \(z_\text{vir} \sim 15\) following further growth by a factor \(z_\text{eq}/z_\text{vir} \sim 200\) during matter-domination era.

Dark matter perturbations that enter the Horizon with amplitude \(\gtrsim 0.1\%\)—i.e., \(\sim 100\) times larger than the approximately scale-invariant perturbations from inflation—collapse before \(z = 10^1\). These perturbations are too small to form PBHs but can seed the growth of ultracompact minihalos that accrete from a nearly uniform universe, before the epoch of formation of the first galaxies at \(z \sim 30\). Thus, the angular momentum of the accreted dark matter is small and the infall quasi-radial.

The aforementioned arguments lead us to conclude that the formation of ultracompact minihalos is more likely than the formation of PBHs. The threshold overdensity required for their formation at \(z \gtrsim 10^3\) is \(\sim 0.1\%\), compared to \(\approx 30\%\) required for PBH formation.

Similar to the case for PBHs, observational constraints or the discovery of ultracompact minihalos is a powerful probe of high-energy physics in the early universe. Microlensing experiments can help constrain the amplitude of perturbations produced during two recent phase transitions: the QCD (quark-hadron) phase transition with \(M_\text{Hor} \sim 1 M_\odot\) and the \(e^+e^-\) annihilation epoch with \(M_\text{Hor} \sim 10^2 M_\odot\). Perturbations with masses in this range can be constrained by microlensing experiments because the mass of the lens, given by Equation (7), is in the range probed by current microlensing experiments.

### 2.3. Angular Momentum of Accreted Dark Matter

The angular momentum of accreted dark matter determines the density profile of the dark halo enveloping a PBH and whether the mass of a PBH can grow substantially by accreting a fraction of its enveloping dark halo. ROM09 found that angular momentum of dark matter accumulated by PBHs with masses \(M_\text{pbh} > 1000\) is so small that a fraction of the dark matter is directly accreted by the PBH. But for smaller mass PBHs and ultracompact minihalos the angular momentum is larger and the accretion can become non radial in the inner parts of the halo. The \(\nu = 2.25\) power-law profile of the density of minihalos results from the assumption of radial infall. If the accreted material has some angular momentum the density profile will be shallower near the center and will contain less mass. Here we check whether the assumption of radial infall is justified in the inner parts of the halo profile enclosing a mass comparable to the lensing mass (i.e., within the Einstein radius).

The angular momentum of the dark matter accreting onto PBHs can be estimated from Equation (19) in ROM09, which gives the mean values of the velocity dispersion within a comoving volume of radius equal to the halo turnaround radius,

\[
\sigma_{dm} \approx \sigma_{dm,0} \left(\frac{1 + z}{1000}\right)^{-\frac{2}{3}} \left(\frac{M_h}{1 M_\odot}\right)^{0.28}, \tag{8}
\]

with \(\sigma_{dm,0} \approx 1.4 \times 10^{-4} \text{ km s}^{-1}\).

Applying conservation of angular momentum, we find that the rotational (i.e., tangential) velocity of the gas at a distance \(r\) from the black hole is \(v(r) = \sigma_{dm} R_h\), where \(R_h\) is the halo radius. If the velocity is smaller than the Keplerian velocity in the proximity of the Einstein radius, then the accretion is quasi-spherical; if vice versa, a shallower core can form. The Keplerian velocity at radius \(R\) is \((GM(R)/R)^{1/2}\). Thus, the accretion is quasi-spherical within the Einstein radius if \(\sigma_{dm} R_h < (GM_{\text{lens}} R_E)^{1/2}\). This inequality is satisfied for

\[
\delta m < 1350 M_\odot \left(\frac{1 + z}{1000}\right)^{3.46} \left(\frac{M_{\text{lens}}}{1 M_\odot}\right)^{1.22} \tag{9}
\]

In Equation (9), there is a steep dependence on redshift. A value of redshift \(z \sim 1000\) is most appropriate for probing the inner parts of the density profile because the inner mass is dominated by material accreted at high redshift. Bertschinger (1985) has shown that the growth of the halo is self-similar because the material accreted at late times from outer shells does not change the density profile in the inner parts (despite the fact that these carry most of the halo mass). This is because the outer shells have high speed when they reach the halo center and spend little time there. Thus, the mass profile near the center of the minihalo—within the Einstein radius—is built by material accreted at redshifts \(z \gtrsim 1000\). Let us now rewrite Equation (9) for two special cases.

1. If \(M_{\text{lens}} \approx M_{\text{pbh}}\), assuming \(\delta m = M_{\text{pbh}} / f_{\text{Hor}}\), we find that the infall is quasi-radial at \(R_E\) if

\[
7.4 \times 10^{-4} \left(\frac{1 + z}{1000}\right)^{-3.46} \left(\frac{M_{\text{pbh}}}{1 M_\odot}\right)^{-0.22} < f_{\text{Hor}} < 1. \tag{10}
\]

2. If \(M_{\text{lens}} \approx M_L\) from Equation (7), we find that the infall is quasi-radial at \(R_E\) if

\[
\delta m > 2.9 \times 10^{-2} M_\odot \left(\frac{1 + z}{1000}\right)^{-7.4}. \tag{11}
\]

### 2.4. Profile Steepening from Adiabatic Compression and Population III Star Formation

In this section, we discuss the role of baryons on shaping the density profile of ultracompact minihalos and whether the first Population III stars can form in their centers. The discussion here will be only qualitative as the simulations required to obtain quantitative results are beyond the scope of this paper.

Due to gas cooling and the concentrated dark matter profile of ultracompact minihalos, baryons may sink dissipatively by several orders of magnitude in radius toward the halo center. The infall perturbs the underlying dark matter distribution, pulling it inward and creating an even smaller and denser core than would have evolved without baryon condensation. For the case of ultracompact minihalos, it is most appropriate to assume perfectly radial orbits (but the same equations are valid assuming only circular orbits). Since \(M_h(R)\) varies in a self-similar fashion, the quantity \(R_{\max} M_h(R_{\max})\), where \(R_{\max}\) is the maximum radius of the radial orbit, is conserved during the “slow” contraction of the gas component. Defining the fraction
of baryonic mass $F \equiv M_b/M_h$, the invariant of the dark matter particle orbits implies

$$R[M_h(R) + M_b(R)] = R_i M_i(r_i) = \frac{R_i M_i(R)}{1 - F},$$

which can be solved iteratively for the final dark matter mass distribution $M_b(R)$, given the initial total mass distribution $M_{h,i}(R)$ and the final baryon mass distribution $M_b(R)$ (Blumenthal et al. 1986). The equality $M_{h,i} = (1 - F) M_i(R_i)$ further assumes that initially $F(R_i) = \text{const}$. In Figure 2, we show the compression of the dark matter profile due to baryonic contraction obtained by solving Equation (12) iteratively. The initial minihalo mass profile is a power law $M_{h,i} = M_i(z) (R/R_i)^{3.75}$ with $M_i(z)$ from Equation (2) assuming $\delta m = 1 M_\odot$ and $z = 10$ (dotted line). The final dark matter profiles are shown by the solid and dashed lines for different configurations of the final baryon distribution. We assume that a fraction of the baryons $f_{\text{baryon}}(\text{core}) = 0.01, 0.001$ settles to a constant density core with radii $R_{\text{core}} = 0.5 \text{ AU}$ and $8 \text{ AU}$. The value of $R_{\text{core}}$ is not important as long as $R_{\text{core}} < R_E$. The mass of the lens is given by the intersection of the final dark matter mass profile with the long dashed line, which traces the lens equation $M_{\text{lens}}/(1 M_\odot) \approx (R_E/8 \text{ AU})^2$. The mass of the lens increases only if the gas mass within $R_E$ for the initial mass profile is comparable or dominates initial dark matter mass. If later, the baryons are removed from the center of the halo on a short time scale when compared to the orbital period of the dark matter particles (for instance due to reionization, SN explosions, etc.) the cuspy dark matter profile may become less steep, but it will not go back to the original profile before the baryon compression (Gnedin et al. 2004; Sellwood & McGaugh 2005).

The next step consists in determining whether a fraction of the baryons in the minihalo can become sufficiently concentrated to dominate the mass within $R_E$ at some point during the minihalo evolution. We will consider separately the case of PBHs embedded in minihalos and ultracompact minihalos without PBH.

1. For PBHs accreting at high $z$ from the intergalactic medium, the gas mass within the Einstein radius of the lens is proportional to the dimensionless gas accretion rate $\tilde{m} = M_{\text{g}}/M_{\text{Ed}}$, where $M_{\text{Ed}}$ is the Eddington rate. For PBHs with mass $\lesssim 500 M_\odot$, the gas accretion is quasi-radial. Thus, assuming $M_{\text{lens}} \sim M_{\text{pbh}}$, the gas approaches free-fall velocity $v \sim (GM_{\text{lens}}/R)^{1/2}$. From mass conservation $4\pi\rho(R) R^2 \tilde{m} = M_{\text{Ed}}$, we find that

$$\frac{M_{\text{g}}(R_E)}{M_{\text{lens}}} \sim 5.4 \times 10^{-8} \tilde{m} \left( \frac{M_{\text{lens}}}{1 M_\odot} \right)^{1/4}.$$  \hspace{1cm} (13)

Thus, in this regime, the gas mass never dominates the potential within $R_E$. However, Equation (13) does not apply in the following cases: (1) PBHs more massive than $M_{\text{pbh}} > 10^5 M_\odot$, for which the gas is self-gravitating (Ricotti 2007); and (2) PBHs with mass $> 500 - 1000 M_\odot$ accreting gas at $z < 100$, for which the gas has sufficient angular momentum to form an accretion disk. Thermal feedback effects, which are important at $z < 100$, will tend to increase the value of the PBH critical mass for disk formation (ROM08). An accretion disk may also form around PBHs that are able to accrete gas efficiently from the interstellar medium (ISM) in galaxies. However, to sustain efficient gas accretion the relative velocity of PBHs with respect to the ISM must be subsonic. Only in this latter case, if the accretion rate is near Eddington, the accretion disk may dominate the mass profile within $R_E$. We conclude that compression of dark minihalos around PBHs due to baryon dissipative collapse is in most cases negligible.

2. The first stars of mass $\sim 10^5 M_\odot$ at $z > 30$ are thought to form in rare $\sigma$ perturbations (Tegmark et al. 1997). Ultracompact minihalos, because they are already in place at $z \sim 1000$, may form the first stars in smaller mass halos of $10^4 M_\odot$ at redshifts $200 \lesssim z \lesssim 1000$. The simplest and naive condition for star formation is given by the requirement that the minihalos must have $T_{\text{vir}} > T_{\text{CMB}}$: the gas would not be able to condense if it were initially colder than the temperature of the CMB (see Figure 6 in Tegmark et al. 1997). Thus, ultracompact minihalos with $\delta m > 100 M_\odot$ are able to host the formation of Population III stars, and the baryon dissipation can increase the concentration of the dark matter halo.

However, the naive estimate is likely too conservative because ultracompact minihalos have a power-law density profile with $v = 2.25$, and thus are much more dense than normal galaxies (e.g., assuming an NFW profile). Their steep mass profile leads to significantly larger gas densities in their cores. Assuming an isothermal equation of state with $T_E = T_{\text{CMB}}$ (a good approximation at $z > 100$ due to Compton cooling/heating), the gas density profile in hydrostatic equilibrium in the potential of an ultracompact minihalo is

$$\rho_b(r) = \frac{\tilde{\rho}_b}{4 r_b} \exp \left[ \alpha \left( \frac{R}{R_h} \right)^{−1/4} - 1 \right],$$

where $\alpha = 4 r_b/r_h$ (Ricotti 2009). For $T_E = T_{\text{CMB}}$, we get $\alpha = (M_h/240 M_\odot)^{2/3} \approx 0.054 \delta m^{2/3} (1 + z/1000)^{−2/3}$. If we estimate $\rho_b(R)$ at $R = R_E$, where

$$\frac{R_E}{R_h} = 1.6 \times 10^{-5} \left( \frac{\delta m}{1 M_\odot} \right)^{13/15} \left( \frac{1 + z}{1000} \right)^{4/3},$$

$$\rho_b(R) \lesssim \frac{\rho_b}{4 r_b} \exp \left[ \alpha \left( \frac{R}{R_h} \right)^{−1/4} - 1 \right].$$

Figure 2. Dark matter mass profile of minihalos before (dotted line) and after (solid and dashed lines) baryonic compression. The initial mass profile is for a minihalo at $z = 10$ with $\delta m = 1 M_\odot$. We show the mass profile for a set of final baryon configurations. For simplicity here we assume that the baryons have a constant density core with radius $R_{\text{core}} = 0.5 \text{ AU}$ (solid lines) and $8 \text{ AU}$ (dashed lines) and with gas mass that is a small fraction $f_{\text{baryon}}(\text{core}) = 0.01, 0.001$ of $M_h$ (see labels).
we find that the core gas density is
\[ \rho_b(R_E) \sim \rho_b \exp \left[ \alpha \left( \frac{R}{R_h} \right)^{-1/4} \right] \sim \rho_b 10^{0.37 \left( \frac{R}{R_h} \right)^{1/4} \left( \frac{R_h}{R_{10}} \right)^{-1}}. \]

The gas density in the core becomes increasingly large with decreasing redshift. Equation (16) is a good estimate of the gas density in the core as long as the gas temperature is close to or lower than the CMB temperature (i.e., \( z > 10^{-30} \)) and \( \rho_b(R_E) < \rho_{d_{10}}(R_E) \).

If the gas in the core is able to form H₂ and cools in less than a Hubble time, it may collapse and form the first Population III stars. The large initial density in the core increases the rate of formation of H₂ with respect to the estimates by Tagmark et al. (1997) for normal galaxies (although the reduced fractional ionization and H⁻ abundance in the high-density core have the opposite effect, these do not dominate). Thus, ultracompact minihalos of mass significantly smaller than \( 10^4 M_\odot \) at \( z \approx 30 \) may be able to form dense gas cores or Population III stars. It is therefore possible that minihalos with 5 \( M_\odot \leq 1 \ M_\odot \), as the one shown in Figure 2, may be compressed by baryon collapse and produce lenses with much larger masses than given by Equation (7). Quantitative calculations of the gas collapse require the integration of a set of time-dependent equations of the chemical reaction network for H₂ in the evolving gravitational potential of the minihalo. These calculations are beyond the scope of this paper.

3. MICROLENSING LIGHT CURVE

To calculate the magnification of minihalo MACHOs (initially without PBHs at their center), we consider density profiles of the form \( \rho = C r^{-\nu} \), where C is set so that the projected mass \( M_L \) within a radius \( R_E \) is just equal to the mass required to form an Einstein ring at \( R_E \). That is,
\[ M_L(R) = \int_0^R 2 \pi db \int_{-\infty}^{\infty} dz C (b^2 + z^2)^{-\nu/2}, \]

where \( D_L \) is the distance from Earth to the minihalo lens and \( D_S \) is the distance to the microlensed source. Note that \( M_L(R) \) is related to the mass inside a sphere of radius \( R \) by
\[ M_L(R) = \frac{(1/2)! \nu^{(v-3)/2}! [v-2]/2!}{v(v-3)/2!}, \]

which is 1.350 for \( v = 2.25 \). Hence, \( M_L(R) \propto R^{3-v} \).

We then use Newton’s method to solve the lens equation, which gives the image position \( \theta_i \) as an implicit function of the source position \( \theta_s \),
\[ \theta_i - \theta_s = \frac{4GM_L(D_L \theta_i)}{\theta_i c^2} \left( \frac{1}{D_L} - \frac{1}{D_S} \right). \]

Newton’s method automatically returns the radial derivative of \( M_L \), and so allows calculation of the magnification,
\[ A = A_+ + A_-; \quad A_\pm = \left| \frac{\partial \theta_i}{\partial \theta_s} \frac{\partial \theta_i}{\partial \theta_s} \right|. \]

For the case that the minihalo is seeded by a PBH, we simply add a point mass to the center of the halo profile and repeat the above procedure.

We construct simulated light curves (with errors proportional to the square root of the magnification) and proceed to fit the simulated flux \( F \) to a standard five-parameter point-lens microlensing model of the form
\[ F = f_x A(t; t_0, u_0, t_b) + f_0, \]

where \( t_E \) is the Einstein radius crossing time, \( u_0 \) is the impact parameter in units of the Einstein radius, \( t_0 \) is the time of maximum (closest approach), \( f_x \) is the model source flux, and \( f_0 \) is background light that lies in the photometric aperture but does not participate in the event. We adopt the conservative approach of allowing negative blending (which would be unphysical) for two reasons. First, the great majority of microlensing events are blended because they are in crowded fields. This blended light typically absorbs any negative blending from a spurious fit, rendering the latter undetectable. Second, even truly isolated sources can suffer negative blending because of poor determination of the sky.

The two examples in Figure 3 show deviations from standard microlensing of somewhat different degree. Whether either of these would be detectable in practice would depend on the quality of the experiment. However, the crucial point is that they are detectable just based on their structure. When microlensing dark matter searches were proposed by Paczynski (1986), their major selling point was that if the dark matter were composed of MACHOs, then microlensing events would be 25 times more common than if it were not. Hence the event rate itself would unambiguously settle the question. Then, when MACHO reported an event rate that was the rms of the prediction for MACHOs and the prediction for stars, the result became completely ambiguous. Recent work by EROS and OGLE has seemed to show that the event rate is compatible with that expected from stars. If confirmed, this would have seemed to be the end-game for microlensing dark-matter searches: even if there were dark-matter objects, they would be too few to detect as an excess event rate (without enormously larger experiments) and, in any event, would be cosmologically uninteresting because their density was so far below the dark-matter density.

But Figure 3 shows that minihalo MACHOs have light curve signatures that are different from those of stars and therefore can be detected even if they are only as common as stars, or indeed, even if they are far less common. Moreover, as we have discussed in Section 1, minihalo MACHOs could make up all the dark matter and yet have a microlensing optical depth that is one or several orders of magnitude below that expected for point-lens dark matter.

We note that there are several observed microlensing effects that produce deviations from the standard microlensing form given by Equation (23) such as parallax effects due to the Earth’s acceleration, xallarap effects due to source acceleration from an orbiting companion, light curve caustics due to binary or planetary companions, and finally finite-source effects that occur if the lens transits the source. However, except for the last, all of these are generically asymmetric, whereas the light curves shown in Figure 3 are symmetric. Moreover, although
finite-source effects are symmetric, they have a very specific form which is unlike the form due to minihalo MACHOs. Furthermore, they are almost completely restricted to extremely high-magnification events, which are rare.

Now it is true that the other effects could, in particular cases, be very symmetric but this is highly improbable, and furthermore these effects, even in their more typical asymmetric form, are not all that common. Hence, the level of contamination in minihalo-MACHO searches would be extremely low.

4. MACHO SEARCHES TOWARD THE BULGE

Originally, Paczynski (1986) proposed MACHO searches toward the LMC not because this line of sight had the highest expected number of MACHO microlensing events, but because it had very little contamination from non-MACHO events. However, if the MACHO and non-MACHO events can be distinguished based on their light curves, one should really focus MACHO searches on the line with the highest rate of dark-halo events and the highest-quality light curves (needed to decisively characterize the deviations from point-lens microlensing). This is the Galactic bulge, where currently about 800 events are discovered per year and where the sources are six times closer, and so 36 times brighter (for similar luminosities). It is true that the fraction of these due to MACHOs is only 0.6% (f/0.1), where f is the ratio of MACHO optical depth toward the LMC to that expected for a full point-lens MACHO halo (Gould 2005). However, these ∼5(f/0.1) per year events are to compared to the ∼1 per year currently detected toward the MCs.

Of course, the problem of background rejection is much more severe because there are ∼10^3 times more stellar lensing events. But the quality of the light curves is also higher, not only because the sources are closer but also because the bulge fields are much more aggressively monitored than the MCs, due to the possibility of finding extra-solar planets. This will be even more true in the future, as next generation microlensing planet-search experiments (already funded) begin to come on line.

And further in the future, there are proposals to put a wide-field imager in space, which could obtain the exquisite photometry that would enable one to distinguish even extremely subtle differences between point-lens and compact-minihalo microlensing events. Such a satellite would already create a synergy between dark-energy and extra-solar planet studies (Gould 2009), and its additional power to probe for compact minihalos (and so the early-universe phase transitions that generate them) would add both dark-matter searches and early-universe physics to the mix.

5. CONCLUSIONS AND DISCUSSION

We propose a new type of dark matter MACHO that may form copiously in the early universe. Upper limits on, or the discovery of these MACHOs is a powerful new tool to study cosmic phase transitions and the nature of dark matter. The new MACHOs are ultracompact minihalos that may form from cosmological perturbations in the universe that are only 10–100 times larger than scale-invariant perturbations from inflation that seeded the formation of large-scale structures and galaxies. These objects are more likely to form than PBHs due to the much larger amplitude of perturbations that are required to form PBHs. Ultracompact minihalos may be sites of formation of the first Population III stars. We show that the magnification light curve of ultracompact minihalos is similar to the one of a point mass but can be easily distinguished from it by sampling the light curve for a sufficiently long time after the peak of the magnification event.

Similarly, if PBHs exist and are only a fraction of the dark matter, their light curves would differ from those due to point masses because PBHs seed the growth of an enveloping minihalo that is sufficiently massive and concentrated to modify the microlensing magnification curve. This modification is substan-
tial whenever the mass of the minihalo within the lens Einstein radius is \( \sim 30\% \) of the PBH mass. However, the PBH mass can be much smaller than the minihalo and MACHO masses.

For the mass profile of the minihalo-enveloping PBHs as predicted by MOR07, the mass within the Einstein radius is only 3\% of the PBH mass; thus, the PBH magnification curve does not show measurable modifications. However, MOR07 assumed secondary infall from an expanding uniform universe seeded by a point-mass PBH. The later assumption in most cases is incorrect: PBHs form from collapse of linear fluctuations on Horizon scales, but their mass is typically smaller than the Horizon mass: \( M_{\text{PBH}} = f_{\text{Hor}} M_{\text{Hor}} \), with \( f_{\text{Hor}} < 1 \). Thus, the mass that seeds the secondary infall is larger than the PBH mass. This leads to more massive and more compact minihalos enveloping PBHs, which have observable signatures on the microlensing light curves.

The mass of the lens produced by compact minihalos scales with the mass of the perturbation as \( \delta m^{6/5} \). Thus, statistical fluctuations of the mass of the perturbations \( \delta m \) produce a mass range of the gravitational lenses (i.e., a range of MACHO masses). This is also the case for minihalos surrounding PBHs, if they exist. In this second case, we expect variations of the ratio of MACHO masses to PBH masses.

Prior to this work, microlensing searches for PBHs were reaching a dead end. Although the MACHO experiment (Alcock et al. 2000) had initially found a microlensing signature toward the MCs that was too large to be produced by known populations of stars, and indeed was difficult to explain by any object other than PBHs, subsequent results reported by EROS (Tisserand et al. 2007) and OGLE (Wyrzykowski et al. 2009) have tended to cast doubt on MACHO’s claim. But even if these are ultimately verified, ROM08 have argued that the density of PBHs required to explain the MACHO results would give rise to other signatures that have not been observed. This seemingly led to limits of order 1\% on PBHs, far below the reported MACHO value, but also (and more importantly), well below the background level expected from Galactic and MC stellar microlenses. Hence, it had appeared as though PBHs were inaccessible to microlensing experiments and, in any event, cosmologically unimportant (even if perhaps interesting).

However, we have shown that PBH-seeded minihalos remain viable candidates for the origin of the signal reported by MACHO, and further, that even if this reported signal proves spurious, it is still possible to search for PBHs in microlensing data at lower levels (well below the previously conceived “floor” of the stellar background). Both these realizations derive from the fact that PBHs make up only a small part of the mass of PBH-seeded minihalos. Hence, the cosmological density of PBHs can be low while the halos they seed can generate substantial microlensing signal, and the fact that minihalo+PBHs are not point-like leads to a qualitatively different microlensing signal.

Moreover, we have shown that the same type of early-universe perturbations that give rise to PBHs, will (at lower amplitude) give rise to minihalos that lack PBHs. Such minihalos are both more likely (because they are produced by a larger range of initial conditions) and easier to detect (because they deviate more strongly from point-lens profiles). Therefore, microlensing experiments (past and future) are a far more powerful probe of early-universe physics than previously understood.

Probing the clumpiness of dark matter at mass scales accessible to microlensing experiments has several implications to understand the nature of dark matter and the high-energy universe.

1. If ultracompact minihalos are found, they would provide tight constraints on the nature of dark matter particles. The masses of thermal or non-thermal relics and their free-streaming length could be constrained more tightly than in studies that rely on measurements of the power spectrum at small scales using the Ly\( \alpha \) forest (Viel et al. 2008) and the number of satellites in the Milky Way (E. Polisensky & M. Ricotti 2010, in preparation).

2. Upper limits on the abundance of ultracompact minihalos and PBHs can improve our understanding of the physics of the high-energy universe by constraining the amplitude of inhomogeneities created at phase transitions due to bubble nucleation, topological defects formation, and other non-Gaussian processes.

3. The clumpiness of dark matter in the Milky Way halo affects the flux of dark matter particles on Earth—important for detections by ground-based experiments. Unless a clump happens to intersect the Earth, the average dark matter density on Earth would be reduced with respect to the standard value \( \rho_\odot \sim 0.008 M_\odot \text{pc}^{-3} \). Assuming that the bulk of the dark matter is in ultracompact minihalos, the probability that the Earth is located inside one of these is roughly given by the volume filling factor of the clumps: \( f_{\text{vol}} \sim \rho_\odot (\langle \rho_h \rangle) \sim 10^{-3} [(1+z)/100]^{-3} \), where \( \langle \rho_h (z) \rangle \) is the mean density of ultracompact minihalos and \( z \sim 10–100 \) is the typical redshift at which the growth of the minihalos stops because they have run out of available dark matter to accrete.

4. If the bulk of the dark matter consists of self-annihilating particles, ultracompact minihalos should be powerful emitters of gamma-rays. Soon after the submission of this paper Scott & Sivertsson (2009) have followed up on our proposal to detect ultracompact minihalos (that they refer to as PLUMs: primordially laid ultracompact minihalos) from their gamma-ray emission. They consider PLUMs of different masses: the most massive ones formed during the \( e^+−e^- \) annihilation epochs, medium ones formed during the QCD phase transition, and smaller ones formed during the electroweak phase transition. They find that PLUMs produced during the \( e^+−e^- \) epoch should be easily detectable today, either by the Fermi satellite, current Air Cerenkov telescopes, or even in archival data from EGRET. However, medium mass PLUMs formed during the QCD phase transition have similar predicted fluxes to the dwarf spheroidal galaxies (e.g., Diemand et al. 2007) targeted by current indirect searches for dark matter. These might be detectable by future instruments. PLUMs arising from the electroweak phase transition will not be detectable in gamma-rays for the foreseeable future. Thus, we conclude that combining data from microlensing searches toward the Galactic bulge with data from Fermi or EGRET catalogs appears as a promising avenue for setting tight constraints on the existence of PLUMs and thus on the amplitude of perturbations generated at phase transitions.

5. Finally, ultracompact minihalos may have been the sites of formation of the first stars formed in our universe. However, because of their steep density profile, the accreted primordial gas may be able to cool more efficiently than in conventional minihalos. As a result, the mass of the Pop III stars formed in ultracompact minihalos may be significantly smaller than \( 100 M_\odot \) (the typical mass of regular Pop III stars). Thus, this process may allow for the formation of zero-metallicity stars that have a lifetime longer than the
age of the universe (due to their small mass) and that could survive to the present day. Low-mass Pop III stars could therefore be found in surveys of low-metallicity stars in the Galactic halo (Christlieb et al. 2002) or could be found at the center of ultracompact minihalos identified using microlensing searches.

A positive identification of PBHs as MACHOs would motivate future space missions to detect signatures of the energy injection by PBHs in the early universe (ROM08). The best measurements of the CMB spectrum to date are by FIRAS onboard COBE, now 15 years old. A follow-up mission would be of great scientific value and easily justified if evidence is found of the existence of non-baryonic MACHOs. Early energy injection also modifies the cosmic recombination history and thus the spectrum of anisotropies of the CMB. Future data from the Planck mission can be used to improve the upper limits on the abundance of PBHs or may detect signatures of PBHs existence.

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