Octupole transitions in the $^{208}$Pb region

Zs Podolyák$^1$, C M Shand$^1$, E Wilson$^1$, B A Brown$^2$, H Grawe$^3$, C J Chiara$^{5,4}$, S Zhu$^4$, B Fornal$^6$, R V F. Janssens$^4$, M Bowry$^1$, M Bunce$^1$, M P Carpenter$^4$, N Cieplicka$^{6}$, A Y Deo$^{7,8}$, G D Dracoulis$^9$, C R Hoffman$^4$, R S Kempley$^1$, F G Kondev$^{10}$, G J Lane$^9$, T Lauritsen$^4$, G Lotay$^5$, M W Reed$^4$, P H Regan$^{1,11}$, C Rodríguez Triguero$^{12}$, D Seweryniak$^4$, B Szpak$^6$, P M Walker$^1$

$^1$Department of Physics, University of Surrey, Guildford, GU2 7XH, UK
$^2$Michigan State University, East Lansing, Michigan 48824-1321, USA
$^3$GSI Helmholtzzentrum für Schwerionenforschung GmbH, D-64291 Darmstadt, Germany
$^4$Physics Division, Argonne National Laboratory, Argonne, IL 60439, USA
$^5$Department of Chemistry and Biochemistry, University of Maryland, College Park, MD 20742, USA
$^6$H. Niewodniczański Institute of Nuclear Physics, PL-31342 Kraków, Poland
$^7$Department of Physics, University of Massachusetts Lowell, Lowell, MA 01854, USA
$^8$Department of Physics, Indian Institute of Technology Roorkee, Roorkee - 247667, India
$^9$Department of Nuclear Physics, Research School of Physics and Engineering, Australian National University, Canberra, ACT 0200, Australia
$^{10}$Nuclear Engineering Division, Argonne National Laboratory, Argonne, IL 60439, USA
$^{11}$National Physical Laboratory, Hampton Road, Teddington, Middlesex, TW11 0LW, UK
$^{12}$School of Computing, Engineering and Mathematics, University of Brighton, Brighton, BN2 4GL, UK

E-mail: z.podolyak@surrey.ac.uk

Abstract. The $^{208}$Pb region is characterised by the existence of collective octupole states. Here we populated such states in $^{208}$Pb + $^{208}$Pb deep-inelastic reactions. γ-ray angular distribution measurements were used to infer the octupole character of several E3 transitions. The octupole character of the 2318 keV $^{17-} \rightarrow 14^{+}$ in $^{208}$Pb, 2485 keV $^{19/2^{-}} \rightarrow 13/2^{+}$ in $^{207}$Pb, 2419 keV $^{15/2^{-}} \rightarrow 9/2^{+}$ in $^{209}$Pb and 2465 keV $^{17/2^{+}} \rightarrow 11/2^{-}$ in $^{207}$Tl transitions was demonstrated for the first time. In addition, shell model calculations were performed using two different sets of two-body matrix elements. Their predictions were compared with emphasis on collective octupole states.

1. Introduction

The even-even nucleus $^{208}$Pb is unusual as its first excited state, at 2615 keV, has spin-parity $3^{-}$. This state decays to the ground-state by a collective E3 octupole transition with a strength of $B(E3)=33.8(6)$ W.u. [1]. The $3^{-}$ collective state arises due to the large number of $\Delta l = \Delta j=3$ orbital combinations around Z=82 and N=126. Collective octupole transitions can be also found in the nearby nuclei, often built on single/multi particle/hole states. The excitations formed in this way are often yrast, therefore accessible experimentally [2]. For some transitions, their octupole character was determined directly or indirectly (from knowing the spin-parities of initial and final states). The character of other octupole transitions were proposed based purely
on theoretical considerations. The aim of this contribution is to demonstrate the octupole character of several such transitions, using angular distribution analysis. In terms of the shell model, octupole states have a very mixed wave function. We discuss the theoretical description of octupole states within the shell model.

2. Experiment
A $^{208}$Pb beam of 1446 MeV energy bombarded a 75 mg/cm$^2$ thick $^{208}$Pb target. Nuclei around $^{208}$Pb were populated by transferring few nucleons from target or beam. All reaction products were stopped in the thick target. The beam current was $\sim$0.25 particle-nA on average, and the experiment ran for $\sim$7 days. The $\gamma$ rays were detected with the Gammasphere array [3], consisting of 101 HPGe detectors and their BGO anti-Compton shields. Tantalum, cadmium and copper absorbers were used in front of the detectors in order to reduce the dominant Pb consisting of 101 HPGe detectors and their BGO anti-Compton shields. tantalum, cadmium

$\sim\gamma$ and copper absorbers were used in front of the detectors in order to reduce the dominant Pb $^{101}$HPGe detectors and their BGO anti-Compton shields. Tantalum, cadmium and copper absorbers were used in front of the detectors in order to reduce the dominant Pb X-ray yields. The trigger required three coincident $\gamma$-rays detected within 2 $\mu$s. Further details on the experimental conditions have been given in our previous conference papers [4, 5].

The data were sorted into $\gamma\gamma\gamma$ cubes with different time conditions. In addition, the data were sorted in two-dimensional $\gamma\gamma$ matrices for angular correlation and angular distribution studies.

3. Results
The high energy part of the recorded $\gamma$-ray spectrum is shown in figure 1. Several of the transitions visible in the 2.3-2.75 MeV region were previously identified/suggested to be E3 transitions [6]. Those discussed in the present paper are labelled.

In order to prove/establish the octupole character of the transitions angular distribution analysis was performed. As these transitions are strong, usually no gates on particular transitions were needed. The $\gamma$-ray intensities were fitted with the $W(\theta) = a_0 + a_2 P_2(cos\theta) = a_0[1 + a_2/a_0 P_2(cos\theta)]$ function. $P_2(cos\theta) = 0.5[3cos^2(\theta) - 1]$ is the second order Legendre polynomial and $\theta$ is the angle between the beam and the emitted $\gamma$ ray. The $a_2/a_0$ parameter is negative for stretched dipole transitions and positive for stretched quadrupole and octupole ones. $a_2/a_0$ is larger for octupole transitions. The exact value of $a_2/a_0$ depends on the amount of spin alignment generated in the reaction. Therefore we determined it for well known E3 and E2 transitions in $^{208}$Pb and other nuclei. Based on this, we expect an $a_2/a_0 \sim 0.36$ for E3 and $a_2/a_0 \sim 0.22$ for E2 transitions (see below).

The angular distribution results are shown in figure 2. Here we discuss the nuclei individually:

$^{208}$Pb: The E3 character of the 2615 keV transition with $B(E3; 3^- \to 0^+) =$33.8(6) W.u. is well known [1]. Similarly, the 4611 keV state with $I^z = 8^+$ is depopulated by a 1413 keV transition with a strength of $B(E3; 8^+ \to 5^-) =$12.6(20) W.u. We use these cases, together with the 583 keV $5^- \to 3^- E2$ transition, to calibrate our $a_2/a_0$ coefficients. We note that our angular correlations analysis, which can be directly compared with theory agree with the well known multipolarities of these transitions, as we have shown in [4]. In addition a (17$^+$) state at 9062 keV is known to decay into a (14$^-$) state by a 2318 keV transition [1, 7]. The spin-parity assignment was based purely on theoretical considerations. Our angular distribution proves the octupole character of this transition.

$^{206}$Pb: The 15$^-$ state at 6431 keV is depopulated by a stretched E3 2403 keV transition [8]. Therefore this transition is also used as a test case.

$^{207}$Pb: The 13/2$^+$ state at 1633 keV is based on the neutron-hole $\nu i_{13/2}$. According to the latest Nuclear Data Sheets evaluation from 2011 [9], this state is populated by a 2485 keV transition from the 4118 keV (15/2$^-$) $\nu j_{15/2}$ state. In contrast, this state has been suggested in 1992 as (19/2$^-$) with $\nu i_{13/2}^{-1} \times 3^-$ character by Schramm et al. [10]. The spin-parity assignment, and indeed the assignment of the transition to the $^{207}$Pb nucleus, was based on comparison with
particle-octupole vibration coupling calculations. Here, our measurement settles the controversy. The 2485 keV transition has an \(a_2/a_0=0.36(2)\) coefficient, therefore it has octupole character. The E1 character, suggested by the compilation, can be clearly disregarded.

\(^{209}\text{Pb}\): The 1423 keV 15/2\(^-\) state has a configuration \(\nu j_{15/2}\) [11] and decays by an E3 transition to the ground-state. This case is also a calibration point for the \(a_2/a_0\) coefficient. Rejmund et al. [6] proposed a 2419 keV transition populating the 15/2\(^-\) state. Based on theoretical expectations, they assigned (21/2\(^+\)) to this new 3842 keV state. The angular distribution of the 2419 keV \(\gamma\) ray, as measured in the present experiment, confirms its octupole character.

\(^{207}\text{Tl}\): The 11/2\(^-\) \(\pi h_{11/2}\) state at 1348 keV is isomeric. According to the 2011 Nuclear Data Sheets evaluations [9] it is populated by a 2465 keV transition from a state at 3813 keV. The compilation does not suggest any spin-parity assignment for this state. In contrast, the authors who observed this state in 2000 interpreted it as 17/2\(^+\) [6] based on the particle-vibration coupling model. Our angular distribution measurement of the 2465 keV \(\gamma\) ray, as measured in the present experiment, confirms its octupole character.

\(^{209}\text{Bi}\): The ground state is 9/2\(^-\) based on the \(\pi h_{9/2}\) orbital. The 15/2\(^+\) state at 2741 keV has \(\pi h_{9/2} \times 3\)\(^-\) character, and it decays by an E3 transition with strength of 25(4) W.u. [11]. Therefore this case is also a calibration point for the \(a_2/a_0\) coefficient.

![Figure 1](image-url). The high-energy region of the full projection of the \(\gamma\gamma\gamma\) cube. The octupole transitions discussed in the paper are labelled.

The description of collective states is difficult in the shell model. Consequently the usual approach is to consider the octupole as a vibrational phonon. Particle octupole-phonon coupling calculations proved to be very powerful in predicting/explaining the energies of such excitations [6]. Here we performed shell model calculations using two different sets of interactions (and model spaces). The OXBASH code [12] was employed. The single particle/hole energies relative to \(^{208}\text{Pb}\) were taken from experiments. Calculations were performed for \(^{208}\text{Pb}\) and for the four neighbouring single particle/hole nuclei \(^{207}\text{Tl}\), \(^{209}\text{Bi}\), \(^{207,209}\text{Pb}\). The details of the two shell model calculations are given below:

\(KHH7B\): The model space considered consisted of proton orbitals \(d_{5/2}, h_{11/2}, d_{3/2}, s_{1/2}\) below \(Z=82\) and \(h_{9/2}, f_{7/2}, i_{13/2}\) above, and neutron orbitals \(i_{13/2}, p_{3/2}, f_{5/2}, p_{1/2}\) below \(N=126\) and \(g_{9/2}, i_{11/2}, j_{15/2}\) above. These are in total 14 orbitals, with four \(\Delta j=\Delta l=3\) pairs across
Figure 2. Angular distributions of octupole transitions in the $^{208}$Pb region. For comparison the angular distribution of the 583 keV E2 transition is also shown.

Z=82 and N=126. The cross shell two-body interaction matrix elements (TBMEs) are based on the H7B interaction [13], while the neutron-proton ones on the Kuo-Herling interaction [14]. Mixing between proton and neutron particle-hole excitations were not considered. We note that these calculations describe accurately valence particle excitations (when no core-breaking is needed). They were used extensively on nuclei above Z=82 [15], and below Z=82 along the N=126 line [16, 17, 18, 19], as well as on both in the N>126 [20] and N<126 [19] regions.

**KHM3Y:** The model space consisted of proton orbitals \( g_{7/2}, d_{5/2}, h_{11/2}, d_{3/2}, s_{1/2} \) below Z=82 and \( h_{9/2}, f_{7/2}, i_{13/2}, f_{5/2}, p_{3/2}, p_{1/2}, h_{9/2}, f_{7/2} \) below N=126 and \( g_{9/2}, i_{11/2}, f_{15/2}, g_{7/2}, d_{5/2}, d_{3/2}, s_{1/2} \) above. The additional orbitals, compared to the KHH7B calculations, are shown in bold. There are in total 24 orbitals, with
### Conclusions

The octupole character of several transitions in the $^{208}$Pb region was demonstrated using angular distribution analysis. The octupole character of the $2318$ keV $17^- \rightarrow 14^+$ in $^{208}$Pb, $2485$ keV $19/2^- \rightarrow 13/2^+$ in $^{207}$Pb, $2419$ keV $21/2^+ \rightarrow 15/2^-$ in $^{209}$Pb, and $2465$ keV $17/2^+ \rightarrow 11/2^-$ in $^{207}$Tl transitions was shown for the first time. Two sets of shell model calculations, labelled KHH7B and KHM3Y, were performed to describe these transitions. The main characteristics of the two calculations are:

1. Both sets of calculations give a good description of the non-collective core-excited states in $^{208}$Pb.
2. The energies of the collective octupole states are overestimated by the KHH7B interaction and underestimated by the KHM3Y interaction.
3. The ordering of the three-particle states in $^{208}$Pb, $^{207}$Tl, $^{209}$Bi are similar in the two calculations, with the KHH7B interaction predicting slightly higher excitation energies.
4. The two calculations are compared with the experimental level energies for $^{208}$Pb and $^{207}$Tl [24] in figure 3.

8. $\Delta j = \Delta l=3$ pairs across $Z=82$ and $N=126$. The cross shell two-body matrix elements are based on the M3Y interaction [21], while the neutron-proton interactions are based on the Kuo-Herling interaction as given in [22]. Such calculations gave a good description of single and double octupole states in $^{208}$Pb [23]. In the present work the calculations were extended to the neighbouring nuclei.

The main characteristics of the two calculations are: (i) both sets of calculations give a good description of the non-collective core-excited states in $^{208}$Pb; (ii) the energies of the collective octupole states are overestimated by the KHH7B interaction and underestimated by the KHM3Y one; (iii) the ordering of the three-particle states in $^{207,208}$Pb, $^{207}$Tl, $^{209}$Bi are similar in the two calculations, with the KHH7B interaction predicting slightly higher excitation energies.

The two calculations are compared with the experimental level energies for $^{208}$Pb and $^{207}$Tl [24] in figure 3.

The biggest differences between the two calculations are regarding the octupole states. In order to understand the underlying reasons we looked into the wave functions. For example in $^{207}$Tl the KHM3Y interaction for the $17/2^+$ state, interpreted as $\pi h^{-1}_{11/2} \times 3^-$ [6, 24], gives a very fragmented wave function. The highest contributions are from $\pi h^{-1}_{11/2} \nu^0_{3/2}g_9/2$ (15.5%), $\pi h^{-1}_{11/2} \nu^0_5/2g_9/2$ (9.0%), $\pi h^{-1}_{11/2} s^{-1}_{1/2}f_{7/2}$ (8.9%), $\pi h^{-1}_{11/2} \nu^0_{13/2}j_{15/2}$ (7.5%). The $\Delta j = \Delta l=3$ contribution is of the order $5\%$. In contrast the KHH7B interaction gives a predominantly $\nu^1_{11/2} \nu^0_{1/2}g_9/2$ (80.8%) character. The $\Delta j = \Delta l=3$ contribution is of the order of $5\%$. The introduction of 2p-2h states in these calculations are expected to increase the mixing considerably.

Table I summarises the results on the discussed octupole states and transitions. The energies of collective octupole states are overpredicted and slightly underpredicted by the KHH7B and KHM3Y interaction, respectively. We note the good description of the $8^+$ and $5^-$ states in $^{208}$Pb, suggesting limited collectivity in them. This is in line with the measured $B(E3)=12.6(20)$ W.u., which is $\sim 2.5$ times smaller than the collective $B(E3; 3^- \rightarrow 0^+)=33.8(6)$ W.u. [1] in the same nucleus.

### Figure 3.

Energy difference between experimental [1, 24] and shell-model excitation energies for core excited states in $^{208}$Pb and $^{207}$Tl. For details of the two shell model calculations, labelled KHH7B and KHM3Y, see the text.
Table 1. Summary of the E3 transitions investigated in the present work. The experimental excitation energies are compared with two sets of calculations. All energies are in keV.

| Nucleus | $I_i^π \rightarrow I_f^π$ | $E_γ$ | $a_2/a_0$ | $E_i$ | $E_f$ | $E_i$ | $E_f$ |
|---------|----------------|-------|----------|-------|-------|-------|-------|
| 208Pb   | $3^- \rightarrow 0^+$ | 2614  | 0.37(2)  | 2614  | 0     | 3483  | 0     |
| 208Pb   | $8^+ \rightarrow 5^-$ | 1413  | 0.35(3)  | 4861  | 3198  | 4864  | 3415  |
| 208Pb   | $17^+ \rightarrow 14^-$ | 2318  | 0.40(2)  | 9061  | 6743  | -     | 6746  |
| 207Pb   | $19/2^- \rightarrow 13/2^+$ | 2485  | 0.36(2)  | 4118  | 1633  | 5003  | 1633  |
| 208Pb   | $15/2^- \rightarrow 9/2^+$ | 1423  | 0.37(2)  | 1423  | 0     | 1423  | 0     |
| 207Pb   | $21/2^+ \rightarrow 15/2^-$ | 2419  | 0.39(2)  | 4783  | 1423  | 3748  | 1423  |
| 207Tl   | $17/2^+ \rightarrow 11/2^-$ | 2465  | 0.35(2)  | 3813  | 1348  | 3681  | 1349  |
| 209Bi   | $15/2^+ \rightarrow 9/2^-$ | 2741  | 0.36(5)  | 2741  | 0     | 3653  | 0     |

both collective octupole states and less collective core breaking states, were performed and compared. In the future, the possible use of modern effective interactions, e.g. [25], will shed more light on the role of interactions in describing collective octupole states.

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References

[1] Martin M J 2007 Nucl. Data Sheets 108 1583
[2] Broda R et al. 2004 Eur. Phys. J. A 20 145
[3] Lee I Y 1990 Nucl. Phys. A 520 641c
[4] Wilson E et al. 2013 Acta Phys. Pol. B 44 381
[5] Wilson E et al. 2014 EPJ: Web of Conferences 66 02110
[6] Rejmund M et al. 2000 Eur. Phys. J. A 8 161
[7] Wrzesinski J et al. 2001 Eur. Phys. J. A 10 259
[8] Kondev F G 2008 Nucl. Data Sheets 109 1527
[9] Kondev F G and Lalkovski S 2011 Nucl. Data Sheets 112 707
[10] Schramm M et al. 1992 Z. Phys. 344 121
[11] Martin M J 1991 Nucl. Data Sheets 63 723
[12] Brown B A et al. 2004 OXBASH for Windows, MSU-NSCL report 1289
[13] Hosaka A, Kubo K I, Toki H 1985 Nucl. Phys. A 444 76
[14] Kuo T T S and Herling G H 1971 US Naval Research Laboratory Report 2258 unpublished
[15] Cieplicka N et al. 2012 Phys. Rev. C 86 054322
[16] Podolyák Zs et al. 2009 Eur. Phys. J. A 42 489
[17] Podolyák Zs et al. 2009 Phys. Lett. B 672 116
[18] Steer S J et al. 2008 Phys.Rev. C 78 061302(R)
[19] Steer S J et al. 2011 Phys.Rev. C 84 044313
[20] Al-Dahan N et al. 2009 Phys. Rev. C 80 061302
[21] Bertsch G et al. 1977 Nucl. Phys. A 284 399
[22] Warburton E K, Brown B A 1991 Phys. Rev. C 43 602
[23] Brown B A 2000 Phys. Rev. Lett. 85 5300
[24] Wilson E et al. 2014 to be published
[25] Coraggio L, Covello A, Gargano A, Itaco N 2009 Phys. Rev. C 80 021395(R)