OFF-SHELL BOUNDARY/CROSSCAP STATES AND ORIENTIFOLD PLANES

H. Itoyama,1 and S. Nakamura2

1 Department of Mathematics and Physics, Graduate School of Science
Osaka City University
3-3-138, Sugimoto, Sumiyoshi-ku, Osaka 558-8585, Japan
2 Theoretical Physics Laboratory
RIKEN (The Institute of Physical and Chemical Research)
2-1 Hirosawa, Wako, Saitama 351-0198, Japan

The authors’ recent works on off-shell boundary/crosscap states are reviewed.

1 Introduction

It is well-known that extending string theory off-shell is a hard problem. In the
first quantized string theory based on an action of a single string, the guiding
principle is a local Weyl invariance of string worldsheets. This immediately
translates into on-shellness of string scattering amplitudes. Off-shellness alone
does not suggest any particular symmetry and one has to consider insertions
of operators carrying dimensions of arbitrarily high degrees on the bulk of
the string worldsheets: this is a hard task to carry out. Situation can be
more manageable to handle if the local Weyl invariance is broken only on the
boundaries of the worldsheets.

To date there are two proposals for off-shell string theory through defor-
mation on the boundary of the string worldsheets. The one is the boundary
string field theory (BSFT) of Witten on a unit disc and the other is the
sigma model approach of open strings to the spacetime action $S$. (See for
example Ref. and the references therein.) To put simply,

$$Z^{\text{single}} \rightarrow S,$$

where $Z^{\text{single}}$ is a partition function for a single string. The former proposal
was successfully applied to the problem of open string tachyon condensation
[3]. It is well known that open strings alone do not serve as perturbatively

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stable system in flat spacetime [4]. A possible next step common to these two approaches will be to extend the idea to the other worldsheet geometry which shares the same Euler number as that of a disc, namely $RP^2$. This amounts to considering the string field theory of unoriented open and closed strings at least at the linearized level [3]. This was considered and developed in Refs. [6, 7].

The above thoughts have also suggested that the off-shell extension of boundary states [8] and that of crosscap states [6, 7] are an expedient tool and can be used in more general contexts. In particular, the computation of $g$ function is equivalent to the determination of the normalization of these states. (See Refs. [9, 10] and the references therein.)

2 BSFT and off-shell boundary state

BSFT [1] is defined on a disc worldsheet. Let us consider an unit disc $\Sigma$ with worldsheet action

$$I_{\text{disc}} = \frac{1}{2\pi \alpha'} \int_{\Sigma} d^2z \partial X^\mu \bar{\partial} X_\mu + \int_{\partial \Sigma} \frac{d\sigma}{2\pi} \mathcal{V}(\sigma) ,$$

(2)

where $\mathcal{V}$ is a generic scalar operator with ghost number 0. Let us consider the operator $O(\sigma) = c^\sigma(\sigma) \mathcal{V}(w, \bar{w})$ located on the boundary, where $c^\sigma(\sigma)$ is the tangent component of the ghost field along the boundary.

The defining differential equation for $S_{\text{disc}}$ is written as

$$\frac{\delta S_{\text{disc}}}{\delta \lambda_\alpha} \propto tr \int \int \frac{d\sigma}{2\pi} \frac{d\sigma'}{2\pi} \langle O^\alpha(\sigma) \{ Q_{\text{BRS}}, O \} (\sigma') \rangle_{\mathcal{V}, \text{disc}}^{\text{ghost}} ,$$

$$O(\sigma) = c^\sigma(\sigma) \mathcal{V}(\sigma) = \sum_\alpha \lambda_\alpha c^\sigma(\sigma) \mathcal{V}^\alpha(\sigma) = \sum_\alpha \lambda_\alpha \mathcal{O}^\alpha(\sigma) ,$$

(3)

where $tr$ denotes the trace over the Chan-Paton space. The BRS charge $Q_{\text{BRS}}$ corresponds to a fermionic vector field $V$ which is a basic ingredient of the formalism of BSFT. Here $\langle \cdots \rangle_{\mathcal{V}, \text{disc}}^{\text{ghost}}$ is the unnormalized path integral with respect to the worldsheet action [4], and can be represented as a matrix element $\langle \cdots \rangle_{\mathcal{V}, \text{disc}} = \langle B | \cdots | 0 \rangle_{\mathcal{V}, \text{disc}}$. The bra vector $\langle B |$ obeys

$$\langle B | \left( \frac{1}{2\pi \alpha'} \left( z \frac{\partial}{\partial z} + \bar{z} \frac{\partial}{\partial \bar{z}} \right) X^\mu + \frac{1}{2\pi} \frac{\partial \mathcal{V}}{\partial X^\mu} \right) \bigg|_{z=e^{i\sigma}, \bar{z}=e^{-i\sigma}} = 0 .$$

(4)

We refer to $\langle B |$ as an off-shell boundary state (OBS) of the disc. The condition (4) is a consequence from the correspondence between the matrix element and
the end point of the path integral. The latter one obeys the boundary condition derived from the worldsheet action $I^{disc}$ on the disc. Eq. (4) tells us that, at the initial point of the coupling constant flow, the system obeys the Neumann boundary condition, and the end point of the flow is described by zero of $\frac{\partial V}{\partial X^\mu}$, namely the Dirichlet boundary condition.

3 Extension of BSFT on $RP^2$ Worldsheet Geometry

We now proceed to construct the $RP^2$ extension of BSFT. $RP^2$ is a non-orientable Riemann surface of Euler number one with no hole, no boundary and one crosscap. We construct the $RP^2$ worldsheet on a complex $z$-plane by using an involution where we identify $z$ and $-\frac{1}{\bar{z}}$. We choose the fundamental region $\Sigma'$ to be $\{z = re^{i\sigma}| 0 \leq r < 1, 0 \leq \sigma < 2\pi \} \cup \{z = re^{i\sigma}| r = 1, 0 \leq \sigma < \pi \}$. The crosscap $C$, the non-trivial closed loop of the $RP^2$ worldsheet, is represented as half of unit circle $\{z = re^{i\sigma}| r = 1, 0 \leq \sigma < \pi \}$ in this case.

Let us consider the $RP^2$ worldsheet with the following action:

$$I^{RP^2} = \frac{1}{2\pi\alpha'} \int_{\Sigma'-C} d^2z \partial X^\mu \bar{\partial} X_\mu + \int_C \frac{d\sigma}{2\pi} V'(\sigma).$$

Our first objective is to obtain the operator which corresponds to the fermionic vector field $V$ for the $RP^2$ case. Let us consider the following operator:

$$Q^{(\sigma)}_{BRS} \equiv \oint \frac{dz}{2\pi i} \left( j^{(g)}_z (z) + j^{(m)}_z (z) \right),$$

$$j^{(m)}_z (z) \equiv 2c_{even}^z (z) \left( -\frac{1}{\alpha'} \right) : (\partial_z X^\mu)_{even}(z)(\partial_z X_\mu)_{even}(z) : ,$$

$$j^{(g)}_z (z) \equiv : b_{zz} c_{even}^z (z) \partial_z c_{even}^z (z) : ,$$

where the subscript $even$ implies that the modes are restricted to the even ones.

We can show that $\delta_B c^\sigma (w) = i e^\sigma \partial_\sigma c^\sigma (w)$ and $\delta_B c^\sigma (w) = 0$, where $c^\sigma$ is the normal component of the ghost field along the loop. ($|w| = 1$ is understood.) We also obtain $\delta_B X^\mu = i e^\sigma \partial_\sigma X^\mu (w)$. Thus, the operator $Q^{(\sigma)}_{BRS}$ generates the BRS transformations $\delta_B$ associated with the diffeomorphisms in the $\sigma$ direction on $|w| = 1$.

It is now immediate to carry out the action of $Q^{(\sigma)}_{BRS}$ on a generic operator with ghost number one. This also brings us an operator which is "on-shell" with respect to $Q^{(\sigma)}_{BRS}$, that is, invariant under $\delta_B$. Let $\mathcal{O}$ be a scalar operator with ghost number one, $\mathcal{O}(w, \bar{w}) = c^\sigma (w) V' (X^\mu (w, \bar{w})) + c^\sigma (w) V' (X^\mu (w, \bar{w}))$,
where $V, V''$ are generic scalar relevant operators. We can show $\delta^2_{\mathcal{H}} \mathcal{O} = 0$ which is the condition we need in our formalism.

Following the disc case, we introduce a defining differential equation for $S_{\mathbb{R}P^2}$

$$\frac{\delta S_{\mathbb{R}P^2}}{\delta \lambda_\alpha} \propto \int_{\mathbb{C}} \int_{\mathbb{C}} \frac{d\sigma}{2\pi} \frac{d\sigma'}{2\pi} \langle \{O_\alpha(\sigma) \{Q_\text{BRS}, \mathcal{O}\}(\sigma')\}_{\text{ghost}} \rangle_{V', \mathbb{R}P^2}$$

$$= 2 \int_{\mathbb{C}} \int_{\mathbb{C}} \frac{d\sigma}{2\pi} \frac{d\sigma'}{2\pi} \sin(\sigma - \sigma') \langle c_1 c_0 c_{-1} \rangle_{\mathbb{R}P^2} f(\sigma, \sigma') , \quad (7)$$

where $f(\sigma, \sigma') \equiv \langle V^{\alpha}(\sigma) (\partial_{\sigma'} V'(\sigma') - i/2 \partial^2_{\partial X'\partial X'} V'(\sigma')) \rangle_{V', \mathbb{R}P^2}, V' \equiv \sum_\alpha \lambda_\alpha V^{\alpha}$.

Note that the allowed form of the nonvanishing ghost three point function has selected $V'$ alone and $V''$ has disappeared. Here the unnormalized path integral $\langle \cdot \cdot \cdot \rangle_{V'}$ is evaluated with respect to the action (5), and can be represented as a matrix element between the ket vector of the closed string vacuum and the bra vector of the off-shell crosscap state (OCS) $\langle C |$, namely $\langle \cdot \cdot \cdot \rangle_{V'} = \langle C | \cdot \cdot \cdot | 0 \rangle_{V'}$. Details of the OCS will be given in the next section.

4 $\mathbb{R}P^2$ worldsheet with background dilatons

In this section, we consider the $\mathbb{R}P^2$ worldsheet with background dilaton field $\Phi$ from the viewpoint of the sigma model approach [7]. The basic idea of the sigma model approach is that the spacetime action for string fields is essentially the renormalized partition function of the worldsheet with corresponding background string fields. In the case of the disc for bosonic strings, for example, we need corrections of the disc partition function in order to subtract the divergence from the Möbius infinity [11, 2]. However, the Möbius group of $\mathbb{R}P^2$ is $SO(3)$ whose volume is finite, and we have no Möbius infinity from the $\mathbb{R}P^2$ worldsheet. Therefore it is natural to assume that the partition function of the $\mathbb{R}P^2$ worldsheet with background fields itself is the exact loop correction term from the $\mathbb{R}P^2$ graph in the presence of the quadratic background fields.

To begin with, we set the metric inside the unit circle ($|z| \leq 1$) on the complex plane as $h_{zz} = h_{\bar{z}\bar{z}} = 0, h_{z\bar{z}} = h_{\bar{z}z} = 1/2$. The metric outside the unit circle ($|z'| \geq 1$) is obtained by the involution $z' = -\frac{1}{z}, \bar{z}' = -\frac{1}{\bar{z}}$ as $h_{z'\bar{z}'} = \frac{1}{r^2} h_{zz}$, where $r^2 = z'\bar{z}'$ for $r \geq 1$. The worldsheet curvature $R$ is then given by $\sqrt{g} R = 4 \{ \delta'(r - 1) r \ln r + (2 + \ln r) \delta(r - 1) \}$, and we obtain

$$\frac{1}{4\pi} \int_{\Sigma'} dr d\sigma \sqrt{g} R \Phi(r, \sigma) = \frac{1}{\pi} \int_0^\pi d\sigma \Phi(1, \sigma). \quad (8)$$
Here $g$ is the determinant of the worldsheet metric written in the polar coordinate $(r, \sigma)$. Note that Eq. (8) gives the correct Euler number of the $RP^2$ (which is one) if we set $\Phi = 1$. Therefore the contribution of the background dilaton concentrates on the crosscap with the above gauge choice.

### 4.1 Off-shell crosscap conditions and OCS

Let us consider the $RP^2$ worldsheet $\Sigma'$ with the following action:

$$ I = \frac{1}{2\pi\alpha'} \int_{\Sigma' - C} d^2z \partial X^\mu \bar{\partial} X_\mu + \frac{1}{\pi} \int_C d\sigma \Phi(\sigma), $$

where $\Phi(\sigma) = a + \frac{1}{2\alpha'} \sum_{\mu=1}^{26} u_\mu (X^\mu(\sigma))^2$. Note that the worldsheet action is free in the “bulk” region $\{z = r e^{i\sigma} | 0 \leq r < 1, 0 \leq \sigma < 2\pi\}$ in this gauge choice.

The aim of this subsection is to extend the on-shell crosscap conditions into the case $u_\mu \neq 0$. We should find, in other words, the constraints on the closed-string modes in the neighborhood of $C$ in the presence of interaction $\Phi$. We call these constraints off-shell crosscap conditions. We assert that the off-shell crosscap conditions can be written as

$$ K(z, \bar{z}) |_{r \to 1} = 0, $$

$$ \{(z \partial_z + \bar{z} \partial_{\bar{z}}) K(z, \bar{z})\} |_{r \to 1} = 0, $$

where

$$ K(z, \bar{z}) \equiv \{(w \partial_w + \bar{w} \partial_{\bar{w}}) X^\mu(w, \bar{w}) + u_\mu X^\mu(w, \bar{w})\} |_{w = z, \bar{w} = \bar{z}} $$

$$ + \{(w \partial_w + \bar{w} \partial_{\bar{w}}) X^\mu(w, \bar{w}) + u_\mu X^\mu(w, \bar{w})\} |_{w = -1/z, \bar{w} = -1/\bar{z}}. $$

The right-hand side of Eq. (11) indicates the meaning of the off-shell crosscap conditions; Eq. (10) are the conditions so that the $X^\mu$ in the neighborhood of $C$, as well as its image by the involution, connects smoothly with the $X^\mu$ on $C$ which obeys the equation of motion $\{(z \partial_z + \bar{z} \partial_{\bar{z}} + u_\mu) X^\mu(z, \bar{z})\} |_C = 0$.

The off-shell crosscap conditions (10) are rewritten in terms of closed-string modes as

$$ K_1 \equiv -\{\alpha_n^\mu + (-1)^n \tilde{\alpha}_{-n}^\mu\} + \frac{u_\mu}{n} \{\alpha_n^\mu - (-1)^n \tilde{\alpha}_{-n}^\mu\} = 0, $$

$$ K_2 \equiv -i\alpha' p^\mu + u_\mu X_0^\mu = 0, $$

1 Although $RP^2$ has no boundary, $(z \partial_z + \bar{z} \partial_{\bar{z}}) X^\mu(z, \bar{z})$ which comes from the total derivative survives only on the crosscap due to the involution.
where we do not sum over $\mu$. We can easily check that the off-shell crosscap conditions interpolate between the usual on-shell crosscap conditions and their T-duals.

We define OCS $\langle C(u) \rangle$ as $\langle C(u) | K_1 = 0, \langle C(u) | K_2 = 0$. The explicit form of $\langle C(u) \rangle$ is given as

$$\langle C(u) \rangle = \langle 0 | \exp \left( -\frac{1}{2} X_0^{\mu} A_{\mu \nu} X_0^{\nu} \right) \exp \left( \sum_{m=1}^{\infty} \tilde{\alpha}_{\mu}^m C_{\mu \nu}^{(m)} \alpha_{\nu}^m \right) \rangle,$$

(13)

where $A_{\mu \nu} = \frac{1}{\alpha'} u_\mu \delta_{\mu \nu}$, and $C_{\mu \nu}^{(m)} = -\frac{(-1)^m}{m} \frac{m-u_\mu}{m+u_\mu} \delta_{\mu \nu}$. We can easily check that this OCS becomes (the T-dual of) the usual on-shell crosscap state if we take the limit $u^\mu \to 0$ ($u^\mu \to \infty$). Therefore the OCS naturally interpolates between the crosscap state for a higher-dimensional O-plane and that for a lower-dimensional O-plane. The coupling constant $u^\mu$, which is a parameter of the configuration of the background dilaton field, controls the dimension of the corresponding O-plane.

### 4.2 OCS and partition function

Next, we show that the OCS is a useful tool to evaluate the quantities on the $\mathbb{RP}^2$ worldsheet. For example, we can calculate the Green’s function and the partition function on the $\mathbb{RP}^2$ worldsheet in the presence of interaction $\Phi(\sigma)$ on the crosscap. Let us consider one-dimensional target space and omit the superscript $\mu$ of $X$ and $u$ for simplicity. The Green’s function for this case is given by

$$G(z, w) = \frac{\langle C(u) | X(z, \bar{z}) X(w, \bar{w}) | 0 \rangle}{\langle C(u) | 0 \rangle}$$

$$= -\frac{\alpha'}{2} \ln(|z - w|^2 |1 + z \bar{w}|^2) + \frac{\alpha'}{u} - \alpha' \sum_{k=1}^{\infty} \frac{(-z \bar{w})^k + (-\bar{z} w)^k}{k(k + u)}.$$

(14)

The expectation value of composite operator $X^2(\sigma)$ which is defined as

$$X^2(\sigma) \equiv \lim_{\epsilon \to 0} \left[ X(\sigma) X(\sigma + \epsilon) - \left\{ -\frac{\alpha'}{2} \ln |1 - e^{i \epsilon}|^2 + \text{(const.)} \right\} \right],$$

(15)

is obtained by substituting $z = e^{i \sigma}$ and $w = e^{i (\sigma + \epsilon)}$ into Eq. (14):

$$\langle X^2(\sigma) \rangle = -\alpha' \ln(2q) - \frac{\alpha'}{u} - 2\alpha' \left[ \Psi \left( \frac{u}{2} \right) - \Psi(u) \right],$$

(16)
where $\Psi(u) \equiv \frac{d}{du} \Gamma(u)$ and we have written the constant in Eq. (15) as $\alpha' \ln q$ by using a positive constant $q$. The value for $q$ is ambiguous at this stage and depends on the renormalization scheme. We will determine the value for $q$ later.

We can calculate the partition function of the $RP^2$ worldsheet by using the relationship $\frac{d}{du} \ln Z(u) = -\frac{1}{2\alpha'} \langle X^2(\sigma) \rangle$, and then we obtain

$$Z(u) = \frac{\sqrt{2\pi} e^u}{\Gamma(u)} \sqrt{\frac{u}{2}} \Gamma(u)$$

up to the overall normalization factor. In general, the partition function for 26-dimensional target space with $\Phi(\sigma) = a + \frac{1}{2\alpha'} \sum_{\mu=1}^{26} u_\mu (X^\mu(\sigma))^2$ on the crosscap can be written as

$$Z(a, u) \equiv e^{-a} \prod_{\mu=1}^{26} Z(u_\mu) = e^{-a} \prod_{\mu=1}^{26} \left( \frac{\sqrt{2\pi} e^u}{\Gamma(u)} \sqrt{\frac{u}{2}} \Gamma(u) \right),$$

up to the overall normalization factor.

5 Applications of OBS and OCS

5.1 $g$-function

OBS’s are useful tools for analyses of two-dimensional systems with boundaries. In the case the boundary interaction is quadratic, the proper form of the Green’s function and the $g$-function has been given by using OBS [8, 10]. In the determination of the $g$-function which is defined as $\langle 0 \mid e^{-H} \mid B \rangle$, the overall normalization of the OBS is crucial. The normalization can be fixed by computing the partition function of the cylinder worldsheet, $Z^{\text{cylinder}}_{B,B'}$, and demanding $Z^{\text{cylinder}}_{B,B'} = \langle B' \mid e^{-iH} \mid B \rangle$, without taking the long cylinder limit [10]. Here $H$ is the Hamiltonian on the cylinder of length $l$, and $B$ and $B'$ indicate the boundaries. The left-hand side is calculated via path integral with $\zeta$-function regularization. For example, the $g$-function for bosonic theory (4) with the quadratic boundary interaction $V(\sigma) = uX^2$ is given by

$$g(u) = (\alpha'/2)^{1/4} (2\pi u)^{-1/2} \Gamma(u + 1) (e/u)^u.$$  

(19)

For supersymmetric extension with boundary mass, the fermionic sector gives

$$g_+ = \frac{\sqrt{2\pi} e^u}{\Gamma(u + 1/2)} (\frac{u}{e})^u \quad \text{(for NS sector)}, \quad g_- = 2^{1/4} \sqrt{2\pi u} \frac{1}{\Gamma(u + 1)} \quad \text{(for R sector)},$$

(20)

which agree with the results of Refs. [12].
5.2 Descent relation among O-plane tensions

The descent relation among O-plane tensions can be also derived by using the result of Sec. \[7\].

Let us define quantity $S_p$ as $Z(a, u) \rightarrow S_p$, where the limit is taken as $u^1, \ldots, u^{p+1} \rightarrow 0$ and $u^{p+2}, \ldots, u^{26} \rightarrow \infty$. We do not touch the parameter $a$ here. According to the argument in Sec. \[7\], $S_p$ is equal to $V_p \times T_p$ where $V_p$ and $T_p$ are the volume and the tension of an $Op$-plane. Here, the dimension of the O-plane is $p + 1$ and is defined as the number of parameters $u^\mu$ which are taken to zero. Then we can write

$$\frac{S_{25}}{S_{24}} = \frac{Z(u_{25})|_{u_{25} \rightarrow 0}}{Z(u_{25})|_{u_{25} \rightarrow \infty}} = \frac{\int dx^{25}V_{24}T_{25}}{V_{24}T_{24}} = \frac{\int dx^{25}T_{25}}{T_{24}}. \quad (21)$$

We find

$$\lim_{u_{25} \rightarrow 0} Z(u_{25}) = \int dx^{25} \frac{4}{\sqrt{2\pi\alpha'}} \quad \lim_{u_{25} \rightarrow \infty} Z(u_{25}) = 4\sqrt{\frac{\pi}{2}}, \quad (22)$$

where $x^{25}$ is the zero mode of $X^{25}$ and we have correctly extracted the integral of the zero-mode part of $Z(u_{25})$. Here we have assigned the value 2 to $q$ because we can obtain a finite and non-zero value of $Z(u)$ in the limit $u \rightarrow \infty$ if and only if $q = 2$. In other words, we have chosen the renormalization scheme in Eq. (15) so that we obtain a finite and non-zero value of $Z(u)$ in the limit $u \rightarrow \infty$. We therefore obtain

$$\frac{T_{24}}{T_{25}} = \frac{\sqrt{2\pi\alpha'}}{4} \sqrt{\frac{\pi}{2}} = \pi \sqrt{\alpha'}. \quad (23)$$

This is precisely the ratio of the tension of an $O24$-plane and that of an $O25$-plane. In general, we can show in a similar manner that $T_p/T_q = (\pi \sqrt{\alpha'})^{q-p}$.

6 Conclusions

Off-shell crosscap states as well as off-shell boundary states are useful tools both for formulating off-shell theory (worldsheet string field theory or off-shell non-linear sigma model) of unoriented open and closed strings, and for performing computation.

In configuration space of strings, interpolations of O-planes of various dimensions are certainly possible as are in D-branes, and the dilaton background is responsible for this. One important physical issue is how these interpolations are materialized as physical processes in systems with moderate instabilities.
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