An Approach of Fuzzy Logic $H_{\infty}$ Filter in Mobile Robot Navigation Considering Non-Gaussian Noise

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Abstract

This chapter has presented an analysis of $H_{\infty}$ filter-based mobile robot navigation with fuzzy logic to tolerate in non-Gaussian noise conditions. The technique exploits the information obtained through $H_{\infty}$ filter measurement innovation to reduce the noises or the uncertainties during mobile robot observations. The simulation results depicted that the proposed technique has improved the mobile robot estimation as well as any landmark being observed. Different aspects such as $\gamma$ values, noise parameters, intermittent measurement data lost and finite escape time issues are also analysed to investigate their effects in estimation. Different fuzzy logic design configurations were also studied to achieve better estimation results. As demonstrated in this work, fuzzy logic offers reliable estimation results compared to the conventional technique.

Keywords: navigation, mobile robot, $H_{\infty}$ filter, estimation, fuzzy logic

1. Introduction

Working in a hazardous area has always been an issue to human safety and health. This is a situation where robotics offers an alternate solution to perform any given task. In realization of this problem, an autonomous robot is providing a suitable approach with less human monitoring system. Since the term of ‘robot’ defined by Karel Capek in 1920s, robotics has experienced a lot of interesting advancements and evolutions.

In general, robotics is classified into several categories stated by the Robotics Institute of America such as the variable-sequence robot, playback robot, numerical control robot and...
intelligent robot. There are also a number of available robotic configurations such as manipulator robot, SCARA robot, spherical type robot, cartesian robot and cylindrical robot.

The robotic research, development and technology have been immense recently considering various fields and applications. In fact, the technology has spread and is widely used in home-based appliances such as the lawn mower robot, vacuum cleaner robot and floor-washing robot. These domestic robots are sometimes designed to work automatically and independently with their own pre-described algorithm or system. Considering these conditions, two main features must be made available for the robot to perform their task, which are navigation and autonomous capabilities.

Mobile robot navigation is the main concern being focussed in this chapter. In modelling the mobile robot navigation, three approaches are available [1, 2]. First, the mathematical modelling which develops the mathematical model of the robot and the environment through the robot dimensions, configurations and the environment conditions. Therefore, designer must understand clearly the mobile robot configurations as well as the environment conditions.

Sensor information is one of the influential elements during information processing in mobile robot navigation. The information enables better mobile robot confidence in the navigation processes. This is the second type of navigation modelling which uses the information extracted from sensors. Several types of sensors can be equipped in a mobile robot to assist the navigation. Compared to the mathematical model, this technique is less complex as sensors provide relative measurements between mobile robot and any observed landmarks.

The third type is focussing on how to tolerate the noises when the mobile robot navigates. Based on probabilistic rules, particularly the Bayes rule, the mobile robot attempts to identify its location and the landmarks observed with some uncertainties. This method consumed lower mathematical computation and does not rely heavily on the sensors information. Owing to these advantages, the third type has been famously applied nowadays. This chapter focusses on the probabilistic technique for mobile robot navigation.

On the aspect of making an autonomous mobile robot, a number of approaches have been proposed especially by using an artificial intelligence technique, for example [3–5]. Jaradat and Abdel-Hafez [4], for example, have successfully applied the neural network to process information obtained through the sensors, IMU and GPS, to navigate the vehicle. Vehicle-measured force and angular velocity are measured via IMU and for the vehicle location, GPS is employed to do the work. Two hidden layer with at least six neurons are designed to calculate and analyse the vehicle movement. Their results seem to provide a good accuracy of vehicle estimation.

Inspired by the human nature of thinking and based on preceding literatures, the mobile robot can be guaranteed to move with less human monitoring and relies on the artificial intelligence to observe any environment. In this context, artificial intelligence is recognized as one of the possible techniques to provide efficient solutions. This research applies the fuzzy logic control to improve a probabilistic technique known as H∞ filter by analysing several issues such as finite escape time [6], operating in non-Gaussian Noise, intermittently on lost measurement data and parameter effects to the estimation.
2. Mobile robot navigation

The celebrated Kalman filter is the famously used approach for mobile robot navigation. The technique which is based on the minimum mean square error utilizes the prior information obtained by the sensor to update its current location. Even though Kalman filter offers reliable estimations, it is still incompetent in an environment that holds non-Gaussian noise characteristics. The noise can be considered as noise when it is not zero mean and holds a characteristic of not uniformly distributed noise. Therefore, a number of new methods are proposed such as the unscented Kalman filter, particle filter and ensemble Kalman filter, which generally use a lot of particles to infer the mobile robot position [2]. These techniques exhibit higher computational cost if the number of particles is increased. In fact, they require a fast processor to calculate and store the information during mobile robot observation. Thus, a simple and robust filter than the above-mentioned techniques is welcome to solve the navigation issues. Some of the Kalman filter limitations can be listed as follows:

- The mean and correlation of the process and measurement noise need to be known at each time instant.
- The covariances of the noises must be determined beforehand, since the Kalman filter uses these covariances as design parameters.
- The Kalman filter is the minimum variance estimator if the noise is Gaussian. However, if a different cost function (such as the worst-case estimation error) should be applied to the system, then the Kalman filter may not be suitable to accomplish the objectives.

$H_\infty$ filter is one of Kalman filter families. The filter is proven to work in non-Gaussian noise environment and it is assumed that the noises are bounded. By defining the tuning parameters known as $\gamma$, the filter can provide a better solution than Kalman filter as well as other techniques. In this chapter, the non-Gaussian noise is assumed to be bounded and acts as a random noise when mobile robot does its observations. It is also important to note that $H_\infty$ filter is facing one issue, that is the finite escape time where the estimation can be unbounded if the system is not satisfying the designed criteria [7]. Despite these shortcomings, the $H_\infty$ filter performance analysis is the main objective in this chapter to propose an alternative solution for mobile robot navigation. To ensure the finite escape time is not perceived during observation, fuzzy logic technique is applied for the proposed system. The system design consisting of these approaches is presented in the next section.

3. $H_\infty$ filter and fuzzy logic technique modelling

One of the main parts in navigation is the simultaneous localization and mapping (SLAM) problem. It states a situation where the mobile robot builds up an environment based on its sensor readings while at the same time localizes itself on the map [8, 9]. Typically, two models that simulate the system are referred to describe how the SLAM problem is being solved. The first model is known as the kinematic model and the latter is defined as measurement
model. Both models are essential to define the behaviour of the system with reference to the environmental conditions.

The kinematic model simply illustrates how the mobile robot movements are recorded. The following equation demonstrates the model.

\[ X_{k+1} = F_k X_k + B_k u_k + w_k \]  

(1)

where \( X \) is the augmented states of the mobile robot \((x, y)\) and landmark \(i (x_i, y_i)\). \(F\) and \(B\) are the state transition matrix and the control input matrix, respectively. \(u\) and \(w\) are the control input and the associated noise during mobile robot movements.

Mobile robot measures relative distance and angle to any observed landmarks location when moving throughout the environment. The measurement model holds the following equation.

\[ y_{k+1} = H_k X_k + v_k \]  

(2)

| The extended Kalman filter | The \( H_\infty \) filter |
|----------------------------|-------------------------|
| The system                 |                         |
| \( X_{k+1} = f(X_k, u_k, w_k, k) \) | \( X_{k+1} = f(X_k, u_k, w_k, k) \) |
| \( z_k = h(X_k, v_k, k) \)     | \( z_k = h(X_k, v_k, k) \) |
| \( y_k = D X_k \)              |                         |
| The prediction step         |                         |
| \( \dot{X}_{k+1} = f(\dot{X}_k, u_k, 0, k) \) | \( \dot{X}_{k+1} = f(\dot{X}_k, u_k, 0, k) \) |
| \( P_{k+1} = \nabla F_k P_{k+1} \nabla F_k^T + \nabla F_k Q_k \nabla F_k^T \) | \( \dot{X}_{k+1} = \nabla F_k \dot{X}_k + \nabla F_k K_k (z_k - \nabla H_k \dot{X}_k) + u_k \) |
| The update step             |                         |
| \( \dot{X}_{k+1} = \dot{X}_{k+1} + K_{k+1} \mu_{k+1} \) | \( P_{k+1} = \nabla F_k P_{k+1} (I - \gamma_k P_k + \nabla H_k R_k \nabla H_k P_k) \nabla F_k^T + Q_k \) |
| \( P_{k+1} = (I - K_{k+1} \nabla H_k) P_{k+1} \) | \( P_{k+1} = \nabla F_k P_{k+1} (I - \gamma_k P_k + \nabla H_k R_k \nabla H_k P_k) \nabla F_k^T + Q_k \) |
| \( \mu_{k+1} = z_{k+1} - h(\dot{X}_{k+1}, 0, k) \) | \( K_k = P_k (I - \gamma_k P_k + \nabla H_k R_k \nabla H_k P_k) \nabla H_k R_k \) |
| \( S_{k+1} = \nabla H_k P_{k+1} \nabla H_k^T + R_k K_{k+1} \) | \( S_{k+1} = \nabla H_k P_{k+1} \nabla H_k^T + R_k K_{k+1} \) |

Table 1. Kalman filter and \( H_\infty \) filter comparison [10].
Here, $y$ contains the information of relative distance and angle measurements and $H$ is showing the measurement matrix. $v$ is the associated noise occurred during measurements.

$\text{H}_\infty$ filter algorithm is almost similar to the Kalman filter. The only differences are notated on the state covariance and state update. For convenience, the comparison between $\text{H}_\infty$ filter and Kalman filter is presented in Table 1.

### 3.1. Fuzzy Logic-based navigation

In this chapter, the measurement innovation is referred as the main reference in designing the fuzzy logic. Different to Ref. [3] which utilizes the heading angle and relative distance range as their inputs to the system, fuzzy logic is designed in this research to process the angle and distance errors as its inputs. Our objective is to decrease those errors by configuring the fuzzy sets in producing smaller errors. By choosing the output appropriately, the effects or measurement error due to sensor inaccuracies can be minimized further.

There were also some researchers attempts to applied fuzzy Logic in navigation such as in Refs. [11–17]. Each of them demonstrated that fuzzy logic or artificial intelligence technique is capable to fuse the information obtained from the sensors for mobile robot estimation. Some of the researches are referred to evaluate the performance of this work.

Literature has stated that mobile robot has some confidence on its estimation especially when Kalman filter is applied for estimation [3]. For non-Gaussian noise characteristic, the sensors reading might be interfered and exhibits bigger error and hence results in bigger measurement noise covariance, $R$. If the gain $K$ is small at all times during observations, then it is possible to have smaller measurement errors. Inspired by this fact, fuzzy logic is proposed to find the best value of measurement innovation to pursue lower error. Kobayashi et al. [16] have selected the $P$, $Q$ and $R$ from fuzzy logic to gain smaller uncertainties. Work by Wang et al. [18] has recognized this as one of the ways to realize smaller measurement noise even when it was being applied to the other $\text{H}_\infty$ filter family, the Kalman filter. Mamdani technique is used for analysis purposes in determining the output of the system. The technique is proposed as it calculates the output by considering and utilizing the maximum information gained from measurement.

The general design is illustrated in Figures 1–3, that consists of the input and output and their respective fuzzy sets. The fuzzy sets are changed whenever the mobile robot moves in different motion and therefore, the values are not included. The following describes the rules of fuzzy logic that are used to define the output of the measurement innovation.

- IF angle error is negative and distance error is negative, THEN angle is negative.
- IF angle error is negative and distance error is normal, THEN angle is normal.
- IF angle error is negative and distance error is positive, THEN angle is negative and distance is normal.
• IF angle error is positive and distance error is normal, THEN angle is negative.
• IF angle error is positive and distance error is negative, THEN distance is normal.
• IF angle error is positive and distance error is positive, THEN angle is negative and distance is normal.

Generally, Gaussian and triangular membership functions are considered in this chapter for evaluation purposes. Only three fuzzy sets are defined which are divided into three different categories: the negative, normal and positive regions. The scale of each of the fuzzy sets is selected based on the normal condition that has high errors. The value differs with each of the fuzzy sets, and it has been tuned several times to obtain the best estimation results. The tuning is taking into account the uncertainties behaviour throughout the simulation. Other than that, both angle and distance measurement characteristics are observed prior to the tuning process. Wang et al. [18] designed the membership function of the angle error to be positive at all times. However, random mobile robot’s movements may also show a negative angle especially when a global coordinate system is being considered. This aspect is one of the major differences between our approach and what they have investigated.
4. Results and discussions

This section describes the performance of the fuzzy logic-based \( H_\infty \) filter by considering several factors such as the finite escape time problem, localization and mapping problem and the effects of noise parameters. These three issues are important to be solved concurrently during the navigation. If not, then the estimation results will not be as expected. Analysis is based on the design parameters included in Table 2.

Figures 4 and 5 demonstrate the estimation results when \( \gamma = 0.7 \), which determines that fuzzy logic-based \( H_\infty \) filter outperforms the normal \( H_\infty \) filter about the mobile robot and landmarks estimations. Erroneous estimations are perceived for the mobile robot location estimations as well as the landmarks positions. The measurement details are presented in Figure 2 for each of the estimation error. Figure 6 illustrates the state covariance update performance. It can also be seen from this figure that the proposed method attempts to avoid the finite escape time from happening.

4.1. Effect of changing \( \gamma \)

The \( \gamma \) effect to the proposed technique has also been analysed to ensure that our proposed technique has consistent and reliable results. Figures 7 and 8 describe the performance of the state estimations for both system of normal \( H_\infty \) filter and fuzzy logic-based \( H_\infty \) filter, respectively, when \( \gamma = 0.23 \), which is smaller than the previous case. It is plotted clearly that normal \( H_\infty \) filter exhibits erroneous results compared to our proposed technique.

4.2. Effect of initial state covariance

Generally, in SLAM, mobile robot does not have any prior information about its location or the environment. Therefore, the initial state covariance is designed to pose high uncertainties. Owing to these conditions, the mobile robot can probably have lost its way and is unable to navigate effectively on the environment. The analysis covers this issue by simulating the results using both normal \( H_\infty \) filter and fuzzy logic-based \( H_\infty \) filter in the next few figures.

| Variables          | Parameter values         |
|--------------------|--------------------------|
| Process noise:     |                          |
| \( Q_{\text{min}} \), \( Q_{\text{max}} \) | \(-0.002, 0.001\)         |
| Measurement noise: |                          |
| \( R_{\text{theta-min}}, R_{\text{theta-max}} \) | \(-0.04, 0.01\)          |
| \( R_{\text{dist-min}}, R_{\text{dist-max}} \) | \(-0.15, 0.3\)           |
| Initial covariance:|                          |
| \( P_{\text{robot}}, P_{\text{landmark}} \) | \(0.001, 100\)           |
| Simulation time    | 1000 (s)                 |

Table 2. Simulation parameters.
Figure 4. The mobile robot movements through the environment. Lighter colour of round shape defines the actual positions while the triangle shape and darker colour round shape presents the $H_{\infty}$ Filter with Fuzzy Logic(FHF) and normal $H_{\infty}$ Filter (HF) estimation performance, respectively.

Figure 5. A performance comparison between FHF and normal HF estimations for both mobile robot (a) and landmarks (b) estimations about the errors.

Figure 6. The state covariance conditions between $H_{\infty}$ Filter (HF) and $H_{\infty}$ Filter with Fuzzy Logic(FHF). Normal HF exhibits frequent Finite Escape Time(FET) compared to the $H_{\infty}$ Filter with Fuzzy Logic(FHF).
Figure 9 demonstrates the results of estimation when mobile robot is in the above-mentioned condition. As expected, the normal H\textsubscript{\infty} filter did not deliver good estimation results. On the other hand, the fuzzy logic-based H\textsubscript{\infty} filter guarantees a good estimation that can still be preserved with a considerable estimation. In this analysis, the Gaussian membership is used for decision making.

Further inspection is done by using different fuzzy membership with the similar range of fuzzy sets in the case when the mobile robot attempts to localize itself in a given environment. Other associated and related simulation parameters remain unchanged to observe any significant improvement that fuzzy logic can offer. The initial state covariance for the landmarks is now known and mobile robot does not have any information of its location. Even other types
of membership functions are applied for decisions; the fuzzy logic-based $H_\infty$ filter surpassed the normal filter performance as depicted in Figure 10.

Assessment on different mobile robot movements during its observations with the same simulation parameters is also conducted to evaluate the consistency of estimation. The results are not disappointing and show a reliable estimation when comparing it to the normal $H_\infty$ filter as illustrated in Figure 11.

Figure 8. The state update conditions of FHF do not encounter high uncertainties or errors during mobile robot navigation.
4.3. Effect of non-Gaussian measurement noise

$H_\infty$ filter is known to be more robust to the extended Kalman filter (EKF) especially whenever non-Gaussian noise is available. The measurement noise is increased to observe whether the $H_\infty$ filter with fuzzy logic still able to preserve a good estimation. Figure 12 presents the results that still define that the $H_\infty$ filter with fuzzy logic (triangular memberships) maintains better performance than the normal $H_\infty$ filter. Similar performance is also observed with different fuzzy memberships even though the results are not included in this chapter.

Figure 9. The normal $H_\infty$ Filter(dotted line) shows erroneous results compared to the true mobile robot path(lighter line) and Fuzzy Logic based $H_\infty$ Filter(darker line). Landmarks estimation of normal $H_\infty$ Filter(round shape) is also results in erroneous estimation.

Figure 10. The normal $H_\infty$ Filter (dotted line) cannot localize itself as well as the available landmarks even though the landmarks information is given compared to the Fuzzy Logic based $H_\infty$ Filter (darker line) with reference to the truth locations (lighter line). Triangle shows the Fuzzy Logic based $H_\infty$ Filter and round shape showing the normal $H_\infty$ Filter for landmarks estimation.
The performance analysis between fuzzy logic-based $H_\infty$ filter and extended Kalman filter is also considered. A condition where mobile robot loses its measurement data at random time is referred in this case. Interesting results are obtained in Figure 13 showing that the $H_\infty$ filter with fuzzy logic inference is still better compared to the normal EKF estimation in non-Gaussian noise. The landmarks estimation for $H_\infty$ filter with fuzzy logic outperforms the EKF. Therefore, based on the figure, $H_\infty$ filter with fuzzy logic technique offers better solutions when non-Gaussian noise as well as when measurement data is lost unexpectedly during mobile robot observations.

There are few remarks to be considered in designing the fuzzy logic control for mobile robot navigation. The measurement innovation characteristics must be first examined prior to the estimation to ensure the results achieve the desired conditions. Besides measurement

![Figure 11](image1.png)

*Figure 11.* Normal $H_\infty$ Filter (dotted line) produces erroneous results compared to the proposed technique (FHF in darker line) for different mobile robot movements. Triangle shows the Fuzzy Logic based $H_\infty$ Filter and round shape showing the normal $H_\infty$ Filter for landmarks estimation.

![Figure 12](image2.png)

*Figure 12.* Normal $H_\infty$ Filter (dotted line) performance is still low compared to the Fuzzy Logic based $H_\infty$ Filter (darker line) with reference to the truth positions(lighter line) for bigger measurement noise. Triangle shows the Fuzzy Logic based $H_\infty$ Filter and round shape showing the normal $H_\infty$ Filter for landmarks estimation.
innovation, the noise characteristics can also influence the estimation performance. Therefore, the designer should carefully understand and model the noise according to the environment to be observed. $H_\infty$ filter is also sensitive to some parameters as stated by Bolzern and Maroni [7] and those parameters must be studied before conducting further analysis on the proposed technique.

5. Concluding remarks

This research has presented the analysis and study of $H_\infty$ filter for mobile robot navigation using fuzzy logic control. The investigation was mainly focusing on the development of the fuzzy logic control to analyse the relative angle and distance measurement as its input to produce smaller error of navigation. Besides, fuzzy logic control was also found to be a possible technique to avoid finite escape time problem in $H_\infty$ filter. A number of tests have been conducted for the proposed technique which includes the effects of having different $\gamma$ value, different noise parameters and intermittently data lost. Preliminary results describe that the fuzzy logic-based $H_\infty$ filter is able to tolerate the problem by using only few number of rules and fuzzy sets. Thus, the technique can be one of the alternative solutions for navigation.

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