Maximum shear modulus evaluation based on continuous wavelet transform of bender element test

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Abstract. Maximum shear modulus ($G_{max}$) is a crucial parameter in the prediction of ground deformation and performance of seismic design. Practically, the $G_{max}$ parameter could be obtained from specific laboratory test or seismic field test. Bender element (BE) test is non-destructive laboratory test, which is simple yet reliable method in determining $G_{max}$ of soil. This study focuses on application; processes and analysis of $G_{max}$ values obtained from BE laboratory test using continuous wavelet transform (CWT) method. The $G_{max}$ values were validated with results from field seismic dilatometer test (sDMT) conducted at the same site of soil sampling for BE laboratory test. BE laboratory tests were conducted on undisturbed Auckland residual clay at various effective confining pressures (ECP) under fully saturated condition. The $G_{max}$ values analysis between CWT and field sDMT data were based on the overburden pressures values that matches with ECP applied in the BE laboratory test. In summary, the comparison between CWT analysis of laboratory and field sDMT data indicated good consistency of $G_{max}$ values. Thus, confirmed the reliability of CWT method in estimating $G_{max}$ parameter from BE laboratory test.

1. Introduction
Small strain shear modulus ($G_{max}$) is an important input parameter used in the wide ranges of dynamic analysis of geotechnical engineering applications including the identifying of the soil characterization, settlement computation and any soil dynamic problems such as vibration in the machine foundations, response on earth structure towards earthquake and also the analysing on traffic vibration. The value for the small strain shear modulus, ($G_{max}$) can be calculated by using the equation (1):

$$G_{max} = \rho \times v_s^2$$

where $G_{max}$ = shear modulus (MPa), $v_s$ = shear wave velocity (m/s) and $\rho$ = soil density (kg/m$^3$). While, the equation for shear wave velocity from bender element test is:

$$v_s = \frac{L}{t_s}$$

1. Introduction
where \(v_s\) = shear wave velocity (m/s), \(L_{tt}\) = distance between tip to tip of bender elements (m), and \(t_s\) = shear wave arrival time (s). Bender element test is a non-destructive dynamic testing that normally used for measuring the shear wave velocity \((v_s)\) of soil specimens in the laboratory. Since its development in the 1970’s, the BE test has gained reputation and significant improvement has taken place in the advance of integrated laboratory testing system [1, 2]. Although BE test is considered as a simple and reliable test to obtain the \(G_{max}\), there are several complications associated with bender element test and analysis including resonant frequency, electromagnetic coupling prevention and near field effects and detection of the first arrival of signal. The analysis of the BE data using manual (visual) method could also lead to the potential human errors in estimating the shear wave arrival time. This study focuses on shear wave arrival time estimation from bender element test results using continuous wavelet transform (CWT) method. The CWT method is able to replace the manual method due to its resistant to the noise in the signal. This method is selected to improve the estimation value of \(G_{max}\) in geotechnical engineering design application. A wavelet is a mathematical function which divides a given function into different frequency components and studies each component with a result that matches its scale. A wavelet transform is the representation of a function by wavelets. Continuous wavelet transforms method is a better method as compared to the traditional Fourier transforms method because it representing functions which can detect discontinuities and sharp peaks.

2. **Continuous wavelet transform method**

The wavelet transform has been used in a various aspects, such as image coding, singularity detection, digital filtering, speech processing and many more signal processing applications [3]. The wavelet transform is motivated by the possibility of finding singularity; it decomposes a signal into elementary building blocks that are well localized both in time and frequency [4]. This method is also known as time–scale domain method that consists of small waves which called as wavelets. Wavelet is a mathematical function that used to divide continuous time signal into different frequency and identify which frequency matches with the scale. The advantage of using wavelet transforms method is it represents functions that have discontinuity and sharp peaks, [5].The basic CWT is defined as the following function:

\[
CWT(x, t) = \frac{1}{\sqrt{a}} \int \psi\left(\frac{t-b}{a}\right) x(t) dt
\]

where \(\psi(t)\) is a mother wavelet function and acts as a window function to localize the integration. The \(\psi\left(\frac{t-b}{a}\right)\) is a dilated and shifted version of the mother wavelet function where \(t\) is time parameter \(a\) is the dilation (scaling) factor and \(b\) is the translation factor. The scaling factor is an inherently positive quantity and shifting factor in wavelet analysis means delaying or advancing its onset. The condition for zero mean of wavelet function \(\psi_{b,a}(t)\) is given as equation:

\[
\int_{-\infty}^{+\infty} \psi_{b,a}(t) dt = 0
\]

Basically the term mother wavelet is defined as a prototype for generating the other window functions. Figure 1 shows several common mother wavelets belonging to different wavelet families are Haar wavelet, Gaussian wavelet, Daubechies wavelet and Morlet wavelet [6]. Selection of a particular wavelet function \(\psi(t)\) is crucial in continuous wavelet transform for reliable analysis. Based on the transmitted and received raw signals obtained from bender elements test, the most suitable mother wavelet for this particular CWT analysis is Daubechies wavelet.

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Figure 1. Common mother wavelets used for wavelet transform analysis (after Baker, 2007).
(a) Haar wavelet (b) Gaussian wavelet (c) Daubechies (dB) wavelet (d) Morlet wavelet.

3. Bender element test
Bender elements test is a reliable and cost effective alternative to undertaking locally instrumented stress path triaxial tests and can be readily performed on unconfined samples in the laboratory [7]. Figure 2 shows a schematic diagram of bender elements system and the pulse excitation that applied in the bender elements test [8].

Figure 2. Bender elements system installed in the small strain triaxial apparatus.
Determination of $G_{\text{max}}$ from bender element test is relatively straightforward as the bulk density of the specimen and the $L_d$ used in the calculation are accurately pre-determined before the test is performed. The main issue that could compromise the value of $G_{\text{max}}$ is the reliability of $t_s$ obtained from the test. Potential errors related to $t_s$ measurement in BE test could be from the boundary limitation, signal deflection and environmental noise interference contribute toward the severity of near field effect. In general, the method used to determine the $t_s$ could be divided into two types; namely, time and frequency domain. The time domain method is more frequently used by researchers as compared to the latter due to its simplicity in the test procedures and data analysis itself. Figure 3 shows the $t_s$ estimation from bender element test result.

![Figure 3. Shear wave arrival time estimation from transmitted and received signals.](image)

### 3.1. Shear wave arrival time estimation based on continuous wavelet transform method

Estimation of shear wave arrival time of bender elements test data using continuous wavelet transform method was performed using MATLAB program. Continuous wavelets transform (CWT) is used to display the quality of the transmitted and received signal's match in red green blue (RGB) contour plots as shown in figure 4 and figure 5. In this method, the brighter spot indicates where all of the stretched and shifted mother wavelet is parallel and similar to the signals, while the darker spots represent only some of the wavelets are align with the signals received. While the highest number of index at 240 and RGB equal to one represents the maximum peak of the signal. The difference between maximum peak of the transmit signal and the received signal are refer to the number of pixels in the x-axis. For this analysis, one pixel represents 0.001ms refereeing to the time scale used in oscilloscope. The numbers of pixel between transmit and received signal are multiplied with 0.001ms and the value represents travel time between both signals. Figure 4 and figure 5 show the relationship between the shear wave arrival time between transmitted and received signals in RGB contour plots compared with signals from oscilloscope. The brighter spot with maximum index of 240 and RGB value equal to 1 correspond to the peak of transmitted and received signals from bender elements test. After the shear wave arrival time delay has been obtained by using CWT method, the value of $v_s$ and $G_{\text{max}}$ were determined respectively.
Figure 4. Peak of transmitted signal is determined at index 240 where RGB value is one.

Figure 5. Peak of received signal is determined at index 240 where RGB value is one.
4. Results discussion and analysis

BE tests were carried out with different ECP applied (20kPa, 40 kPa, 60kPa, 100kPa, 150kPa, 220kPa, 500kPa and 800kPa). The frequencies used in this study were in the range of 3 kHz – 10 kHz. Based on the pre-analysis of BE data, the average of $G_{\text{max}}$ is calculated from the data between 9 kHz to 13 kHz frequency ranges for good consistency of shear wave arrival time results. The results of average $G_{\text{max}}$ values plotted against various effective confining pressure shows that the $G_{\text{max}}$ value increases with the increment of applied effective confining pressure. Table 1 presents the data analysis of $G_{\text{max}}$ for continuous wavelet transform at various effective confining pressures (ECP). The analysis shows consistent results for CWT methods. Figure 6 shows the relationship between the effective confining (ECP) and $G_{\text{max}}$ for CWT data analysis. The results from bender element tests indicate that shear modulus is dependent on the effective stress with the relationship given by equation (5):

$$G_{\text{max}} = 9 \left( \sigma' \right)^{0.45}$$

where, $G_{\text{max}}$ and $\sigma'$ are in MPa.

| ECP (kPa) | $t_s$ (µs) | $\nu_s$ (m/s) | $G_{\text{max}}$ (Mpa) |
|----------|------------|---------------|-----------------------|
| 20       | 868        | 152           | 42                    |
| 40       | 820        | 161           | 47                    |
| 60       | 763        | 173           | 54                    |
| 100      | 702        | 188           | 63                    |
| 150      | 629        | 210           | 78                    |
| 220      | 491        | 269           | 130                   |
| 250      | 473        | 279           | 140                   |
| 500      | 437        | 302           | 163                   |
| 800      | 310        | 426           | 171                   |

Figure 6. $G_{\text{max}}$ values obtained using cwt methods for various ECP.
5. Verification of bender elements test with seismic dilatometer test (sDMT) field data
Reliability and consistency of the bender elements laboratory test result is verified based on field seismic dilatometer test (sDMT) conducted at the vicinity of the sampling pit. The sDMT test is conducted at four different points until 9.5 metre depth. Tests were carried out at 500 mm depth intervals with the first reading taken at 1 m depth from the ground surface. The comparison between the $G_{\text{max}}$ values from the bender element test results, with data from field sDMT are shown in figure 7. The estimated $G_{\text{max}}$ values (orange dotted line) shows excellent agreement with the field sDMT data started from 3m to 9 m depth. The discrepancies that occurred at the top 3 m of soil layer could be explained based on the difference between site and laboratory test conditions. Bender elements tests were performed under fully saturated condition, whereas the actual condition at the top 3 m is partially saturated. Under unsaturated condition, matric suction plays an important role by providing higher effective stress to the soil. Thus the $G_{\text{max}}$ of soil under unsaturated soil is expected to be much higher than the soil under fully saturated condition with similar applied pressure due to the extra effective stress induced by matric suction. A study on modulus-suction-moisture relationship conducted by [9] found that shear wave velocity of the soil specimen increased as the matric suction increased and the volumetric water content decreased (i.e., as the specimen was drying along soil water characteristic curve).

![Figure 7. Comparison of $G_{\text{max}}$ values between continuous wavelet transforms and sDMT with respect to the depth.](image-url)
6. Conclusion

Based on the research works undertaken in developing the bender elements testing system, incorporating it in the small strain triaxial apparatus and various analysis methods of bender elements results, the following conclusions could be drawn. In general the results obtained from laboratory BE test is reliable in estimating $G_{\text{max}}$ values provided that all precautions and procedures were followed accordingly. The verification of $G_{\text{max}}$ values calculated based on continuous wavelet transform method from BE test data show excellent agreement with field sDMT data, thus confirmed the reliability of the method in BE data analysis. Moreover, the good agreement with field data verification using auto-analysis CWT system based on MATLAB software could provide good alternative to the popular manual method of visual picking.

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