Laser-assisted nuclear photoeffect reexamined

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(Date textdate; Received textdate; Revised textdate; Accepted textdate; Published textdate)

Abstract

The S-matrix element and the cross section of the laser-assisted nuclear photoeffect are recalculated in a gauge invariant manner taking into account the effect of the Coulomb field of the remainder nucleus. The γ-photon energy dependence of the laser free cross section obtained in the plane wave and long-wavelength Coulomb-Volkov approximations are compared. Numerically the laser-assisted partial cross sections with laser photon energy 2 keV and some different polarization states of γ-photon of energy 3 MeV are investigated.

PACS numbers: 32.80.Wr, 25.20.-x, 42.55.Vc

Keywords: other multiphoton processes, photonuclear reactions, x-ray and γ ray lasers

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I. INTRODUCTION

The problem of requirement of gauge invariance of perturbation calculus in matter-field interactions in atomic physics and quantum optics was a central problem in the 60’s and 70’s of the last century, and it was finally satisfactorily clarified in the late 80’s [1], [2]. At this time the laser assisted x-ray photoeffect was widely investigated and the gauge invariant calculation of the cross section was also made [3]. In a recent paper [4] the effect of intense coherent electromagnetic field on the nuclear photoeffect was discussed. The laser-assisted nuclear photoeffect is a process, which is similar to the laser-assisted x-ray photo effect (x-ray absorption). Both processes are bound-free transitions of charged particles (protons and electrons, respectively) that are assisted by an intense electromagnetic field. In both cases the initial state is strongly bound so that its change due to the intense laser field may be neglected. Since the correspondence between the two processes is straightforward, one can use the results of [3] in calculating the gauge invariant cross section of laser assisted nuclear photoeffect making the necessary substitutions and modifications in the formulae.

Therefore we re-discuss the problem of the laser-assisted nuclear photoeffect satisfying the requirement of gauge invariance and taking into account the effect of the Coulomb field of the remaining nucleus. The recoil of the remainder nucleus and the initial momentum of the γ particle are neglected. The calculation is made in radiation (pA) gauge in long wavelength approximation (LWA) of the electromagnetic fields. In LWA the laser field has a vector potential \( \mathbf{A}_L(t) = A_0[\cos(\omega_0 t)\mathbf{e}_1 - \sin(\omega_0 t)\mathbf{e}_2] \), that corresponds to a circularly polarized monochromatic wave. For the unit vectors \( \mathbf{e}_1 \) and \( \mathbf{e}_2 \) the \( \mathbf{e}_1 \cdot \mathbf{e}_2 = 0 \) holds. The corresponding amplitude of the electric field of the laser is \( F_0 = \omega_0 A_0/c \). The unit vectors of the frame of reference used are \( \mathbf{e}_x = \mathbf{e}_1, \mathbf{e}_y = \mathbf{e}_2 \) and \( \mathbf{e}_z = \mathbf{e}_1 \times \mathbf{e}_2 \), with \( \mathbf{e}_z \) in the direction of propagation of the intense field. The vector potential describing the gamma radiation is \( \mathbf{A}_\gamma = \sqrt{2\pi \hbar/(V\omega_\gamma)} \mathbf{e} \exp(-i\omega_\gamma t) \) in the LWA. Here \( \hbar \omega_\gamma \) is the energy and \( \mathbf{e} \) is the unit vector of state of polarization of the \( \gamma \) photon, and \( V \) is the volume of normalization. The polar angles of \( \mathbf{e} \) are \( \Theta \) and \( \Phi \). The space dependent part of the initial nuclear (protonic) state \( \phi_0(\mathbf{r}) = (2\pi)^{-1}\beta^{3/2}e^{-\beta r}/(\beta r) \), with \( \beta = \nu \sqrt{2mE_b}/\hbar \), where \( m \) is the rest mass of the proton and \( -E_b \) is its initial energy. The actual values of \( E_b \) and \( \nu \) are \( E_b = 0.137 \text{ MeV} \) and \( \nu = 1.84 \) [4].
II. GAUGE INVARIANT S-MATRIX ELEMENT IN THE LWA COULOMB-VOLKOV MODEL

The wave function of a free proton in a repulsive Coulomb field of charge number \( Z \) has the form \( \varphi(\vec{r}) = e^{i\vec{Q} \cdot \vec{r}} \chi(\vec{Q}, \vec{r}) / \sqrt{V} \) [5]. Here \( \vec{Q} \) is the wave number vector of the proton. Applying the LWA in \( \chi(\vec{Q}, \vec{r}) \), i.e. taking \( |\chi(\vec{Q}, 0)| = \chi_c(Q) \),

\[
\varphi(\vec{r}) = e^{i\vec{Q} \cdot \vec{r}} \chi_c(Q) / \sqrt{V} \quad (1)
\]

with

\[
\chi_c(Q) = \left( \frac{2\pi Z \alpha_f}{\lambda_p Q} \right)^{1/2} \left[ \exp \left( \frac{2\pi Z \alpha_f}{\lambda_p Q} \right) - 1 \right]^{-1/2}. \quad (2)
\]

Here \( \alpha_f \) is the fine structure constant and \( \lambda_p = \frac{\hbar}{m \alpha_f} \) is the reduced Compton wavelength of the proton. The solution (1) is a LW A of the Coulomb-solution. This approximation leads to the Fermi correction factor of the decay rate of the \( \beta \) decay [6].

For the time dependent state of the proton in the intense field an approximate non-relativistic solution \( \psi \) of the time dependent Schrödinger equation of a particle in the laser plus Coulomb field is used, which is called Coulomb-Volkov solution [7], [8]. For \( \psi_{\vec{Q}}(\vec{r}, t) \) we use the LWA of nonrelativistic Coulomb-Volkov solution

\[
\psi_{\vec{Q}}(\vec{r}, t) = V^{-1/2} e^{i\vec{Q} \cdot \vec{r}} \chi_c(Q) \exp \left( -iEt/\hbar \right) f(t) \quad (3)
\]

with \( \vec{E} = \hbar^2 \vec{Q}^2 / (2m) + U_p \), that is the energy of the outgoing proton in the intense field, where \( U_p = e^2 F_0^2 / (2m \omega_0^2) \) is the ponderomotive energy. The polar angles of the wave number vector \( \vec{Q} \) of the outgoing proton are \( \vartheta \) and \( \eta_0 \). The function \( f(t) = \exp[i\alpha \sin(\omega_0 t + \eta_0)] \) where \( \alpha = \alpha_\vartheta \sin(\vartheta) \) with \( \alpha_\vartheta = eF_0 Q / (m \omega_0^2) \).

The gauge independent S-matrix element can be obtained with the aid of Eq.(27) of [3] as

\[
S_{fi} = -\frac{\chi_c(Q)}{\sqrt{V}} \int \exp[i\left(\vec{E} + E_b \right) t/\hbar] f^*(t) \frac{\partial}{\partial t} G[\vec{q} (t)] dt, \quad (4)
\]

where \( G(\vec{q}) = \int \phi_0(\vec{r}) e^{-i\vec{q} \cdot \vec{r}} d^3r \) is the Fourier transform of the initial stationary nuclear state, it is \( G(\vec{q}) = 2 \frac{2\pi \beta}{\sqrt{2}} (q^2 + \beta^2)^{-1} \) in our case, and \( \vec{q} (t) = \vec{Q} - \frac{e}{\hbar} \vec{A} \) with \( \vec{A} = \vec{A}_L \) \( + \vec{A}_\gamma \). Using the identity \( \partial_\vartheta G = \partial_\eta G \sum_{j=1}^3 \partial_\lambda_j q_j (\partial_\lambda_j A_j) \) identity \( \partial_\vartheta G = \left( \partial_\eta G \right) \frac{e}{\hbar} \left( \vec{Q} \cdot \vec{E} - \frac{e}{\hbar c} \vec{A} \cdot \vec{E} \right) \) with \( \vec{E} = -\frac{\partial}{\partial t} \vec{A} \), i.e. \( \vec{E} = \vec{E}_L \) \( + \vec{E}_\gamma \).

Now we deal with last factor of \( \partial_\vartheta G \). The \( \vec{Q} \cdot \vec{E}_L \) term can be neglected if the pure intense field induced proton stripping process is negligible since this term describes the process without the gamma photon. Furthermore, the ratio of the amplitudes of \( \vec{A}_\gamma \cdot \vec{E}_L \) and \( \vec{A}_L \cdot \vec{E}_\gamma \) equals
\( \omega_L/\omega_\gamma \ll 1 \). Therefore the \( \vec{Q} \cdot \vec{E} - \frac{\omega}{\hbar c} \vec{A} \cdot \vec{E} = \vec{Q} \cdot \vec{E}_\gamma - \frac{\omega}{\hbar c} \vec{A}_L \cdot \vec{E}_\gamma \) approximation is justified to use, where \( \vec{E}_\gamma = i \sqrt{2 \pi \hbar \omega_\gamma / \sqrt{\varepsilon}} \exp(-i \omega_\gamma t) \). The relative strength of the \( \vec{Q} \cdot \vec{E}_\gamma \) and \( \frac{\omega}{\hbar c} \vec{A}_L \cdot \vec{E}_\gamma \) terms is characterized by the parameter \( \delta = e A_0 / (\hbar c Q) \). In the laser free case \( Q = \sqrt{2m \left[ \hbar \omega_\gamma - E_b \right] / \hbar} \).

Numerical estimation shows that \( \delta \lesssim 0.004 \) if \( \hbar \omega_\gamma \geq 3 \text{ MeV} \) and \( \delta \) increases up to \( \delta \approx 0.05 \) if \( \hbar \omega_\gamma = 0.189 \text{ MeV} \) in the case of laser photon energy and intensity values discussed in \([4]\). Therefore the \( \vec{Q} \cdot \vec{E}_\gamma \) term is the leading one in the last factor of \( \partial_t G \) that after the substitution of the concrete form of \( \partial_q G \) results

\[
\frac{\partial}{\partial t} G = \left( \frac{\partial}{\partial q} G \right) \frac{\hbar}{\varepsilon} \frac{\vec{Q} \cdot \vec{E}_\gamma}{q} = - \frac{4 \sqrt{2\pi\beta}}{[q^2 + \beta^2]^2} \frac{\hbar}{\varepsilon} \frac{\vec{Q} \cdot \vec{E}_\gamma}{q}. \tag{5}
\]

As to the denominator of (5), the effect of \( \vec{A}_\gamma \) is negligible in \( \vec{q}(t) \) and thus \( \vec{q}(t) = \vec{Q} - \frac{\omega}{\hbar c} \vec{A}_L \). It was shown above that the amplitude of oscillation of \( \vec{q}(t) \) due to the intense field can be neglected. Moreover the amplitude of oscillation of \( \vec{q}(t) \) compared to \( \beta \) is less than 1.4%. Therefore \( q = Q \) can be used in the denominator of (5).

Using the Jacobi-Anger formula in the Fourier series expansion of \( f^*(t) \) \([9]\) the S-matrix element can be written as

\[
S_{fi} = \sum_{n=n_0}^{\infty} \frac{2\pi i}{V} \delta [\omega_n(Q)] \chi_C(Q) \left( \frac{\partial G}{\partial q} \right)_{q=Q} \frac{\sqrt{2\pi\hbar \omega_\gamma}}{\hbar} M_n(\xi, \alpha)
\]

with \( M_n(\xi, \alpha) = \xi J_n(\alpha) e^{-i \eta \omega_\gamma} \), where \( \xi = \vec{Q} \cdot \varepsilon / Q \), \( J_n(\alpha) \) is a Bessel function of the first kind, and

\[
\omega_n(Q) = \frac{\hbar Q^2}{2m} + \frac{U_p + E_b}{\hbar} - \omega_\gamma - n \omega_0.
\]

### III. GAUGE INVARIANT CROSS SECTION OF LASER-ASSISTED NUCLEAR PHOTOEFFECT

The cross section has the form

\[
\sigma = \sum_{n=n_0}^{\infty} \sigma_n,
\]

where the partial cross section

\[
\sigma_n = \sigma_{n0}(Q_n) |\mu_n|^2
\]

with

\[
\sigma_{n0}(Q_n) = \chi_C^2(Q_n) \alpha_f \frac{Q_n k_f}{2\pi \lambda_p} \left[ \frac{\partial}{\partial q} G(Q_n) \right]_{q=Q_n}^2.
\]
In our case

$$\sigma_{n0}(Q_n) = 16\alpha \frac{k \beta}{\alpha_p} \chi^2_c(Q_n) \frac{Q_n^3}{[Q_n^2 + \beta^2]^3}.$$  \hspace{1cm} (11)

Here $Q_n = \frac{1}{\hbar} \sqrt{2m \left[ \frac{\hbar}{\omega} + n \omega_0 - U_p - E_b \right]}$, $k = \omega \gamma/c$. The cases $n < 0$ and $n > 0$ correspond to laser photon emission and absorption, respectively. $n_0(<0)$ is the smallest possible value of $n$, it just fulfills the $\frac{\hbar}{\omega} + n \omega_0 - U_p - E_b > 0$ condition ($|n_0| \simeq \left( \frac{\hbar \omega - U_p - E_b}{\hbar \omega_0} \right)$).

$$|\mu_n|^2 = \int_0^{2\pi} \int_0^{\pi} \xi^2 J_n^2(\alpha_n) \sin \theta \, d\theta \, d\eta,$$  \hspace{1cm} (12)

where $\alpha_n = \alpha_n \sin \theta$ with $\alpha_n = eF_0Q_n/\left( m\omega_0^2 \right)$.

The partial cross section is proportional to the Coulomb factor $\chi^2_c(Q_n)$. The Coulomb factor and therefore also the partial cross section rapidly decrease with the decrease of $Q_n$, i.e. near the threshold ($\hbar \omega \rightarrow E_b$), and it is the fact in the laser free case too. Therefore near the threshold the cross section is unobservable.

If the direction of $\vec{x}$ in the gamma flux is random then $|\mu_n|^2$ must be averaged as $\left< |\mu_n|^2 \right> = \int |\mu_n|^2 \sin \Theta d\Theta d\Phi / (4\pi)$. Applying the spherical harmonics addition theorem, the orthonormal properties and the sum rule of the spherical harmonics \[10\] one can obtain $\left< \xi^2 \right> = 1/3$ and

$$\left< |\mu_n|^2 \right> = \int_0^{2\pi} \int_0^{\pi} \frac{1}{3} J_n^2(\alpha_n \sin \theta) \sin \theta \, d\theta \, d\eta_0.$$  \hspace{1cm} (13)

Than the average of the partial cross section $\sigma_n$ reads $\left< \sigma_n \right> = \sigma_{n0}(Q_n) \left< |\mu_n|^2 \right>.$

It can be seen from (13) that the averaged differential partial cross section can be written as

$$\frac{d \left< \sigma_n \right>}{d\Omega_\theta} = \sigma_{n0} \frac{1}{3} J_n^2(\alpha_n \sin \theta),$$  \hspace{1cm} (14)

where $d\Omega_\theta = \sin \theta \, d\theta \, d\eta$.

In the $n \neq 0$ channels the proton emission vanishes in the direction parallel (\( \theta = 0 \) or $\theta = \pi$) with the laser beam since $\lim_{x \to 0} J_n(x) = 0$. In the $n = 0$ case in this direction $\frac{d \left< \sigma_n \right>}{d\Omega_\theta} = \sigma_{00} \frac{1}{3}$ since $\lim_{x \to 0} J_0(x) = 1$.

In calculating $\left< \sigma_n \right>$ one can use the $J_{-n}(x) = J_n(-x) = (-1)^n J_n(x)$ relations to change the $\int_0^\pi d\theta$ to $2 \int_0^{\pi/2} d\theta$, and one can apply the

$$\int_0^{\pi/2} J_n^2(\alpha_n \sin \theta) \sin \theta \, d\theta = \frac{1}{\alpha_n} \sum_{k=0}^\infty J_{2n+2k+1}(2\alpha_n \eta)$$  \hspace{1cm} (15)

and the

$$2 \sum_{k=0}^\infty J_{2n+2k+1}(2\alpha_n \eta) = \int_{0}^{2\alpha_n \eta} J_{2n}(x') \, dx'$$  \hspace{1cm} (16)
identities. Using the $x' = 2\alpha_{n0}x$ and $dx' = 2\alpha_{n0}dx$ change of variable

$$\langle \sigma_n \rangle = \frac{4\pi}{3}\sigma_{n0} \int_0^1 J_{2n} (2\alpha_{n0}x) \, dx. \tag{17}$$

In the weak field limit ($\alpha_{n0} \to 0$) the $\int_0^1 J_{2n} (2\alpha_{n0}x) \, dx \to \int_0^1 J_{2n} (0) \, dx = 0$ because $\lim_{x \to 0} J_{2n} (x) = 0$ in the case of $n \neq 0$ and the $\int_0^1 J_{2n} (2\alpha_{n0}x) \, dx \to \int_0^1 J_0 (0) \, dx = 1$ because $\lim_{x \to 0} J_0 (x) = 1$ in the $n = 0$ case. Therefore the averaged total cross section $\langle \sigma \rangle \to \frac{4\pi}{3} \sigma_{00}$. On the other hand $\langle \sigma \rangle \to \sigma_T$, that is the total cross section of the laser free case and with random polarization in the $\gamma$-flux therefore $\sigma_{00} = \frac{3}{4\pi} \sigma_T$.

If we are far from the threshold, i.e. if $\hbar \omega_{\gamma} \gg E_b$, then the $Q_n = Q_0$ approximation holds for the intensity and photon energy parameter pairs ($I = 10^{15} \text{ W/cm}^2$ with $\hbar \omega_{\gamma} = 2 \text{ eV}$; $I = 6.25 \times 10^{21} \text{ W/cm}^2$ with $\hbar \omega_{\gamma} = 100$ and 200 eV; $I = 4.0 \times 10^{21}$, $1. \times 10^{23}$ and $2.5 \times 10^{24} \text{ W/cm}^2$ with $\hbar \omega_{\gamma} = 2 \text{ keV}$) discussed in [4] and one can use the $\sigma_{n0} = \sigma_{00} = \frac{3}{4\pi} \sigma_T$ substitution in all the above formulae of the partial cross section. Using this approximation in (14) and applying $\sum_{n=-\infty}^{n=\infty} J_n^2 (x) = 1$, since $n_0$ is a negative integer of large magnitude, $\frac{d\langle \sigma \rangle}{d\omega_{\gamma}} \approx \frac{1}{4\pi} \sigma_T$ and $\langle \sigma \rangle \approx \sigma_T$ at the intensities and laser photon energies discussed.

Now it is supposed that the $\gamma$-photon is polarized. If the polarization vector $\vec{\varepsilon} = \vec{e}_2$, i.e. the polarization vector $\vec{\varepsilon}$ lies in the plane of polarization of the circularly polarized laser beam (case $\text{pol} \, 1$) then $\xi = \vec{Q} \cdot \vec{e}_2 / Q = \sin \theta \sin \eta_0$. If the polarization vector $\vec{\varepsilon}$ of the $\gamma$-photon is $\vec{\varepsilon} = \vec{e}_1 \times \vec{e}_2$ (case $\text{pol} \, 2$) then $\xi = \cos \theta$. Using (12) and carrying out the integration over $\eta_0$

$$|\mu_{n, \text{pol}, 1}|^2 = \pi \int_0^\pi J_n^2 (\alpha_{00} \sin \theta) \sin^3 \theta d\theta, \tag{18}$$

$$|\mu_{n, \text{pol}, 2}|^2 = \int_0^{2\pi} \int_0^\pi J_n^2 (\alpha_{00} \sin \theta) \cos^2 \theta \sin \theta d\theta d\eta_0, \tag{19}$$

and $\sigma_n$ has the form

$$\sigma_{n, \text{pol}, j} = \frac{3}{4\pi} \sigma_T \frac{\mu_{n, \text{pol}, j}^2}{\mu_{n, \text{pol}, j}^2}. \tag{20}$$

The total cross sections $\sigma_{\text{pol}, 1} = \sum_{n_0}^{\infty} \sigma_{n, \text{pol}, 1}$ can be obtained applying $\sum_{n_0}^{\infty} J_n^2 (x) \approx 1$ again resulting $\sigma_{\text{pol}, 1} = \sigma_{\text{pol}, 2} = \sigma_T$ valid in the cases of the laser intensities and photon energies discussed.

**IV. NUMERICAL RESULTS**

First the $E_p = \hbar \omega_{\gamma} - E_b$ dependence of the laser free averaged total cross section is investigated. $E_p$ is the kinetic energy of the outgoing proton, $\hbar \omega_{\gamma}$ is the energy of the $\gamma$ photon and $E_b = 0.137$
FIG. 1: The full line shows the $E_p = \hbar \omega - E_b$ dependence of the laser free averaged total cross section given by $\sigma_T = \frac{4\pi}{3} \sigma_{00}$ (for $\sigma_{00}$ see (11) with $Q_0$). $E_p$ is the kinetic energy of the outgoing proton, $\hbar \omega$ is the energy of the $\gamma$ photon and $E_b = 0.137$ MeV is the binding energy of the proton initially bound in $^8B$. For comparison the $\frac{4\pi}{3} \sigma_{00}/\chi^2_C(Q_0)$ is also plotted as a dotted line.

MeV is the binding energy of the proton initially bound in $^8B$ [4], [12]. The charge number of the final nucleus ($^7Be$) is $Z = 4$. The full line in Fig. 1. shows $\sigma_T = \frac{4\pi}{3} \sigma_{00}$, which is the laser free, averaged cross section of our model (for $\sigma_{00}$ see (11) with $Q_0$). For comparison the $\frac{4\pi}{3} \sigma_{00}/\chi^2_C(Q_0)$ is also plotted as a dotted line. That is the result of the gauge independent laser-free calculation in the plane wave approximation, i.e. without the Coulomb correction. On the base of Fig. 1. one can conclude that the Coulomb correction becomes more essential with decreasing $\gamma$ photon energy.

Next the averaged partial cross sections $\langle \sigma_n \rangle$ (applying the $\sigma_{n0} = \frac{4\pi}{3} \sigma_T$ substitution in (11)) are investigated numerically with $\hbar \omega_0 = 2 keV$, $\hbar \omega = 3 MeV$ and the intensities discussed in [4], i.e. at $I = 4.0 \times 10^{21}, 1. \times 10^{23}$ and $2.5 \times 10^{24}$ W/cm$^2$ (Fig. 2). Figs. 3 and 4 show the partial cross sections $\sigma_{n,pol,j}$ in the two cases of polarization of the $\gamma$ photon discussed. Fig. 3 is devoted to the case of $\vec{\gamma} = \vec{e}_2$ (case pol, 1) and Fig. 4 shows the partial cross sections in the case of $\vec{\gamma} = \vec{e}_1 \times \vec{e}_2$ (case pol, 2).

The laser free, averaged total cross section $\sigma_T = 1.02 mb$ at $\hbar \omega = 3 MeV$. At this $\gamma$ energy and in all cases we obtained $\langle \sigma \rangle = \sigma_{pol,1} = \sigma_{pol,2} = \sigma_T = 1.02 mb$ contrary to $\sigma = \sum_n \sigma_n \approx 63.4 mb$ obtained by [4]. Also at this $\gamma$ energy the gauge independent laser-free result in the plane wave approximation gives $\sigma_T/\chi_C^2(Q_0) = 4.11 mb$. Thus one can conclude that the 62.2 times larger
total cross section of [4] compared to our result may be originated partly from the gauge invariant calculation (a factor of 15.4) and partly from ignoring the Coulomb repulsion between the final particles by [4] (a factor of 4.03). In the $E_p \rightarrow 0 \left( \hbar \omega_\gamma \rightarrow 0.137 MeV \right)$ limit this type of difference is more enhanced.

V. SUMMARY

The problem of laser-assisted nuclear photo-effect was discussed in a gauge invariant manner and taking into account the effect of the Coulomb repulsion between the ejected proton and the remainder nucleus. The investigation is mainly concerned with those $\gamma$ photon energies that are far from the threshold. In our model the calculation of the total cross section leads to 0.0161 times smaller result than that of [4] at the $\gamma$ photon energy $\hbar \omega_\gamma = 3 MeV$ and in all cases of state of polarization of the $\gamma$ photon discussed. It was found that the hindering effect of the Coulomb repulsion in the final state, that is manifested in the appearance of the Coulomb factor in the cross section is huge for small kinetic energies of the outgoing proton and it must also be taken into account in the $MeV$ energy range. At $\hbar \omega_\gamma = 3 MeV$ it causes about a factor of $1/4$ decrease of the total cross section, which is incorporated in the factor 0.0161, compared to the result of the plane wave approximation. The further 0.065 times decrease of the total cross section may be originated from its gauge invariant calculation. The numerical investigation, similar to [4], was made at $\hbar \omega_\gamma = 3 MeV$, and it shows, that the main effect of the presence of the laser field is that the total cross section is distributed between the partial cross sections of the channels absorbing or emitting different numbers of laser photons.

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FIG. 2: The averaged partial cross sections $\langle \sigma_n \rangle$ for some intensity parameters discussed in [4]; the intensities are (a) $I = 4.0 \times 10^{21}$, (b) $1.0 \times 10^{23}$ and (c) $2.5 \times 10^{24}$ W/cm$^2$ with photon energy $\hbar \omega_0 = 2$ keV. The polarization vector of the $\gamma$ photon of energy $3$ MeV is random.
FIG. 3: The partial cross sections $\sigma_{n,pol,1}$ in the case of the state of polarization of the $\gamma$ photon $\vec{e} = \vec{e}_2$.

The intensities are (a) $I = 4.0 \times 10^{21}$ (b) $1.1 \times 10^{23}$ and (c) $2.5 \times 10^{24}$ W/cm$^2$ with photon energy $\hbar \omega_0 = 2$ keV.
FIG. 4: The partial cross sections $\sigma_{n,\text{pol},2}$ in the case of the state of polarization of the $\gamma$ photon $\vec{e} = \vec{e}_1 \times \vec{e}_2$.

The intensities are (a) $I = 4.0 \times 10^{21}$ (b) $1. \times 10^{23}$ and (c) $2.5 \times 10^{24}$ W/cm$^2$ with photon energy $\hbar \omega_0 = 2$ keV.