Dark photon manifestation in the triplet-like QED processes

\[ \gamma + \ell_i \to \ell_j^+ \ell_j^- + \ell_i, \ i \neq j, \ i = e, \mu, \ j = e, \mu, \tau \]

G. I. Gakh\(^{\text{\textsection}}\) M.I. Konchatnij\(^{\text{\textsection}}\) and N.P. Merenkov\(^{\text{\textsection}}\)

National Science Centre, Kharkov Institute of Physics and Technology, Akademicheskaya 1, and V. N. Karazin Kharkov National University, Dept. of Physics and Technology, 31 Kurchatov, 61108 Kharkov, Ukraine

E. Tomasi–Gustafsson\(^{\text{\textsection}}\)

IRFU, CEA, Université Paris-Saclay, 91191 Gif-sur-Yvette, France

Abstract

The triplet-like QED processes \[ \gamma + \ell_i \to \ell_j^+ \ell_j^- + \ell_i \] with \( i \neq j \), and \( i = e, \mu, \ j = e, \mu, \tau \) has been investigated as the reactions where a dark photon, \( A' \), is produced as a virtual state with subsequent decay into a \( \ell_j^+ \ell_j^- \) pair. This effect arises due to the so-called kinetic mixing and is characterized by the small parameter \( \epsilon \) describing the coupling strength relative to the electric charge \( e \). The main advantage of searching \( A' \) in these processes is that the background to the \( A' \) signal is pure QED. Concerning \( A' \), we consider its contribution in the Compton-type diagrams only since, in this case, the virtual dark photon has time-like nature and its propagator has the Breit-Wigner form. So, near the resonance, \( A' \) can manifest itself. The contribution of \( A' \) in the Borsellino diagrams is negligible since, in this case, the virtual dark photon is space-like, the \( A' \) propagator does not peak and the effect is proportional, at least, to \( \epsilon^2 \). We calculate the distributions over the invariant mass of the produced \( \ell_j^+ \ell_j^- \) pair and search for the kinematical region where the Compton-type diagrams contribution is not suppressed with respect to the Borsellino ones. The value of the parameter \( \epsilon \) is estimated as a function of the dark photon mass for a given number of events.

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The concept of dark matter (DM) was introduced to explain the properties of large scale structures in the Universe. In 1933 it appeared that the masses of nebulae can not be determined only from the observations of luminosities and internal rotations, Ref. [1], but require a new source of mass, interacting by gravity and not emitting light.

Further cosmological observations such as the rotational speed of galaxies, fluctuations in the microwave background radiation, the motion of individual galaxies inside clusters can be explained either by additional gravity or a modification of the theory of gravity, the second option being less favored.

As the origins of DM are not known, a number of hypothesis and models exist. The understanding of dark matter implies the discovery of new particles and probably necessitates new interactions, belonging to the domain of 'physics beyond the Standard Model (SM)'. The search of new particles is going on at LHC, at what is called the 'high energy frontier', with scarce success up to now. Moreover, the activity in this field is growing and experiments are suggested or ongoing at various accelerators, and laboratories, with different probes, looking for hypothetical particles in a broad energy range (for recent reviews, see [2–4]). Other possibilities lying at 'the frontiers of precision', i.e., high precision experiments that could reveal very tiny signals, seem more and more appealing, demanding in general more limited resources. The results of these studies are expressed in terms of boundaries on the mass and coupling strength of the missing particle.

The simplest extension of SM is the introduction of the dark photon (DP), $A'$, a low mass gauge boson, a massive vector particle that can interact with a conventional photon through 'kinetic mixing' [5]. Its mass and interaction strength with the known charged particles, $\epsilon$ (in units of the electrical charge $e$), can not be predicted unambiguously by the theory since the mass of the DP can arise via different mechanisms. As the associated mixing coupling constant $\epsilon$ is not known, the non-observation of such particle at very high energies suggests that its mass could be in the GeV range, and, therefore, accessible to existing colliders or electron accelerators.

The electron-nucleus fixed-target scattering was considered at JLAB using the APEX test run data [6, 7] in the mass region $175 \text{ MeV} < M_{A'} < 250 \text{ MeV}$ and at MAMI (with a heavy nucleus) by the A1 Collaboration [8]. The JLAB DarkLight Collaboration [9] studied...
the possibility of detecting a light boson with mass \( \leq 100 \text{ MeV} \) in electron-proton scattering.

Different experimental setups can be used at electron-positron colliders to detect the \( A' \) signal. A peak in the dimuon mass peak was searched in the reaction \( e^+e^- \rightarrow \mu^+\mu^-\gamma \) with the radiative return method (RRM) by the KLOE and KLOE2 Collaborations at DAΦNE \[10\], setting constraints in a wide region of \( M_{A'} \), from 30 MeV up to 980 MeV. Further searches are ongoing at Frascati, by the PADME Collaboration, with a 550 MeV positron beam annihilating on electrons at rest in an active diamond target \[11\] \[12\]. A method based on missing mass measurement in the reaction \( e^+e^- \rightarrow A'\gamma \), suitable for the low mass region (5 MeV < \( M_{A'} < 20 \text{ MeV} \)), has been proposed at VEPP-3 \[13\]. Ref. \[14\] reviews the main dark sector searches performed at the Belle II experiment at the SuperKEKB energy-asymmetric \( e^+e^- \) collider. DP is searched for in the reaction \( e^+e^- \rightarrow \gamma A' \) by RRM, with subsequent decays of the DP to dark matter \( A' \rightarrow \chi\bar{\chi} \). Preliminary studies have been performed and the sensitivity to the kinetic mixing parameter strength is given. A competitive measurement is possible, in particular in the region \( M_{\chi} > \text{few tens of MeV} \), where the BABAR experiment starts dominating in terms of sensitivity \[15\].

Evidence of DP is also searched in the decay of the known particles. The authors of Ref. \[16\] have studied radiative pion decays \( \pi^+ \rightarrow e^+\nu\gamma \). The measurements were performed in the \( \pi E1 \) channel at the Paul Scherrer Institute (PSI), Switzerland. DP was also searched in the decay of \( \pi^0 \)-meson (\( \pi^0 \rightarrow \gamma A' \rightarrow \gamma e^+e^- \)) \[17\], produced in proton nuclei collisions at HIAF facility (China). The decay of \( \pi^0 \)-meson was also used for DP searches in the experiment WASA-at-COSY (Jülich, Germany) \[18\], where \( \pi^0 \)-mesons were produced in the reaction \( pp \rightarrow pp\pi^0 \) and also at CERN (where \( \pi^0 \) were formed through the decay of K-mesons, \( K^\pm \rightarrow \pi^\pm\pi^0 \)) \[19\].

The ability of reactor neutrino experiments TEXONO (Taiwan) and COGERENT (Tennessee, USA) to constrain the light-mass \( A' \) using the Compton-like process \( \gamma e^- \rightarrow A' e^- \) (TEXONO, \( M_{\chi} \approx 1 \text{ MeV} \)) and decay \( \pi^0 \rightarrow \gamma A' \) (COGERENT, \( M_{\chi} \) up to 65 MeV) was analysed assuming subsequent decay of \( A' \) into a pair of DM particles \[20\] \[21\].

From the theoretical side, DP formation in various reactions was investigated in a number of papers. Bjorken et al. \[22\] considered several possibilities for the detection of \( A' \) in the most probable range of masses from a few MeV to several GeV and confirmed that the experiments at a fixed target are suitable for the discovery of DP in this interval of masses. The production of DP in electron scattering on the proton or heavy nuclei has been
investigated in Ref. [23] for the experimental conditions of MAMI and JLAB experiments. The authors of Ref. [24] proposed to use rare leptonic decays of kaons and pions $K^+(\pi^+) \rightarrow \mu^+\nu_\mu e^+e^-$ to study the light DP (with mass about 10 MeV). The constraints on the very light DP in the 0.01 - 100 keV mass range are discussed in Ref. [25] (indirect constraints following from $A' \rightarrow 3\gamma$ decay are also revisited).

A recent review of the phenomenology of DP in the mass range of a few MeV to GeV and of the constraints on the mass and decay of $A'$ from different experiments have been presented in [26, 27], where the $g - 2$ of muons, neutrino and electrons together with other precision QED data, as well as radiative decays of strange particles were analyzed.

The hypothesis of a hidden sector of particles that interact through a messenger with the known particles, has been advocated in other domains to explain a number of observed 'anomalies', i.e., the discrepancy of experiment and theory. Besides the models predicting DM which interacts with the leptons of the SM, there are models which predict new force carriers (vector or scalar) that couple preferentially to muons and decay predominantly to DM particles [28–30]. Some experiments are planned to search for these particles. The NA62 experiment (CERN) proposes to use the rare kaon decays, such as $K \rightarrow \mu\nu X$ (if $X$ decays to invisible DM particles) or $K \rightarrow 3\mu\nu$ decay (if $X$ decays to muons) [31]. LHC (using ATLAS calorimeter) can probe new force carriers that are coupled to muons [32]. The proposed analysis, based on muon samples from $W$ and $Z$ decays only, has a comparable reach to other proposals. The possibility to accelerate muons in a dedicated collider is discussed in Ref. [33].

The process of the triplet photoproduction on a free electron, $\gamma + e^- \rightarrow e^+ + e^- + e^-$, in which DP is formed as an intermediate state with subsequent decay into a $e^+e^-$ pair, was investigated in [34]. The sensitivity of the detection of the $A'$ particle over the QED background was calculated for different $A'$ masses. An original method of selecting events in which the invariant mass of one $e^+e^-$ pair remains fixed and the other pair is scanned, allowed to find constraints on the parameter $\epsilon$ depending on the DP mass and on the energy of the $\gamma e^-$ system.

In this work, we suggest a possible way to detect the DP signal through the reaction $\gamma + \ell_i \rightarrow \ell_j^+\ell_j^- + \ell_i$, where a few tens MeV photon collides with a high energy electron or muon beam. The reaction $\gamma + \mu^- \rightarrow \tau^+\tau^- + \mu^-$ can be used to probe of $A'$—signal if DP is coupled (at the tree level) predominantly to the second and third lepton generations.
FIG. 1. Feynmann diagrams for the process (1). Diagrams (a) are called the Borsellino diagrams, and diagrams (b) are the Compton-type diagrams.

(the so-called $L_{\mu} - L_{\tau}$ models [35, 36]). $A'$ would appear as a narrow resonance in the $\ell^+_j \ell^-_j$ spectrum, requiring the signal detection over a background. The advantage of these reactions is that the corresponding backgrounds in these cases are pure QED processes that can be calculated with the required accuracy. Moreover, no identical particles are present in the final state.

The diagrams describing these processes are calculated and estimations of the required luminosity for possible signatures in the invariant $\ell^+ \ell^-$ mass are given in terms of the parameter $\epsilon$ as a function of the DP mass.

II. FORMALISM

As it was mentioned in the introduction, DP can manifest itself as some intermediate state with the photon quantum numbers, that decays into a $\ell^+_j \ell^-_j$ pair. In such case, the processes

$$\gamma(k) + \ell^-_i(p) \rightarrow \ell^+_j(p_3) + \ell^-_j(p_1) + \ell^-_i(p_2),$$

in which a high energy electron or muon interacts with a few tens MeV photon, can be used, in principle, to probe the $A'$ signal in a wide range of $A'$ mass, from a few MeV up to ten GeV.

For this aim, we suggest to scan the differential cross section over the $\ell^+_j \ell^-_j$ invariant mass, $s_1 = (p_3 + p_1)^2$. The background due to pure QED exceeds essentially the DP effect and has to be calculated with a high accuracy. In the lowest approximation, the QED amplitude of the process (1) is given by the four diagrams shown in Fig. 1.

In this work, first we calculate the double differential cross section over the invariant variables $s_1$ and $u = (k - p_2)^2$. We find the kinematical regions, in terms of these variables, where the single-photon amplitudes of the Compton-type diagrams give a comparable or
larger contribution with respect to the double-photon amplitudes of the Borsellino diagrams. This region can be delimited by excluding large $|u|$ values with appropriate cuts. Then we perform integration over the variable $u$ in the restricted region and include the effect due to the DP contribution by modifying the photon propagator of the single-photon amplitudes.

A. Kinematics

The process (1) is of the type $2 \rightarrow 3$ particles, and one can use several sets of invariant variables to describe its kinematics [37]. Let us define the following five independent invariants:

\[
\begin{align*}
    s &= (k + p)^2 = (p_1 + p_2 + p_3)^2, \\
    s_1 &= (p_1 + p_3)^2 = (k + p - p_2)^2, \\
    s_2 &= (p_2 + p_3)^2 = (k + p - p_1)^2, \\
    t_1 &= (k - p_1)^2 = (p_2 + p_3 - p)^2, \\
    t_2 &= (p - p_2)^2 = (p_1 + p_3 - k)^2.
\end{align*}
\]

The scalar products of 4-momenta in the process and the variable $u$ are expressed, in terms of these invariants, as follows:

\[
\begin{align*}
    2(k p_2) &= s - s_1 + t_2 - M^2, \\
    2(k p) &= s - M^2, \\
    2(k p_1) &= m^2 - t_1, \\
    2(k p_3) &= s_1 + t_1 - t_2 - m^2, \\
    u &= 2M^2 - s + s_1 - t_2, \\
    2(pp_1) &= s - s_2 + t_1, \\
    2(pp_2) &= 2M^2 - t_2, \\
    2(pp_3) &= s_2 - t_1 + t_2 - M^2, \\
    2(p_1 p_3) &= s_1 - 2m^2, \\
    2(p_2 p_3) &= s_2 - M^2 - m^2, \\
    2(p_1 p_2) &= s - s_1 - s_2 + m^2,
\end{align*}
\]

where $M$ ($m$) is the mass of initial (created) lepton.

Taking into account azimuthal symmetry (we consider back-to-back events), the phase space of the final particles can be written as [37]:

\[
d R_3 = \frac{d^3 p_1}{2 E_1} \frac{d^3 p_2}{2 E_2} \frac{d^3 p_3}{2 E_3} \delta(k + p - p_1 - p_2 - p_3) = \frac{\pi}{16(s - M^2)} \frac{dt_1 dt_2 ds_1 ds_2}{\sqrt{-\Delta}},
\]

where $\Delta$ is the Gram determinant. In terms of the considered variables it reads

\[
\Delta = \frac{1}{16} \begin{vmatrix}
    2s_2 & M^2 + s_2 - t_1 & -m^2 + s + s_2 & m^2 - M^2 + s_2 \\
    M^2 + s_2 - t_1 & 2M^2 & M^2 + s & -M^2 + s_2 - t_1 + t_2 \\
    -m^2 + s + s_2 & M^2 + s & 2s & -m^2 - M^2 + s_1 + s_2 \\
    m^2 - M^2 + s_2 & s_2 - M^2 - t_1 + t_2 & s_1 + s_2 - m^2 - M^2 & 2m^2
\end{vmatrix}.
\]
In order to obtain the required distributions it is necessary to integrate the differential cross section over the variables $t_1$ and $s_2$. The limits of integration are defined by the condition of the positiveness of ($-\Delta$). Solving the equation $\Delta = 0$ relative to the variable $t_1$ one finds:

$$t_{1-} < t_1 < t_{1+}, \quad t_{1\pm} = \frac{A \pm 2\sqrt{B}}{(s-s_1)^2 - 2(s+s_1)M^2 + M^4},$$

where

$$A = \left\{ -M^4s_1 + s_1s_2(s_1 - t_2) + s^2t_2 + M^2 \left[ s_1^2 + s_1s_2 + s(s_1 - t_2) + s_2t_2 \right] - s[s_2t_2 + s_1(s_2 + t_2)] + m^2 \left[ M^4 + s(s-s_1) + (s+s_1)t_2 - M^2(2s + 3s_1 + t_2) \right] \right\},$$

$$B = \left[ s_2(t_2 - t_1) + M^2(s_1^2 - 2st_2 - s_1t_2) + M^4t_2 \right] \times \left[ m^2M^4 + m^4s_1 - m^2M^2(2s + s_1) + m^2(s^2 - ss_1 - 2ss_2) + M^2(s - s_2) + s_1s_2(-s + s_1 + s_2) \right].$$

The limits of the second integration over the variable $s_2$, $s_2- < s_2 < s_2+$, at fixed $s_1$ and $t_2$, are defined as the roots of the second factor in the expression for $B$, namely

$$s_{2\pm} = \frac{2m^2 + M^2 + s - s_1 \pm \lambda_1 \sqrt{1 - \frac{4m^2}{s_1^2}}}{2}, \quad \lambda_1 = \sqrt{(s - s_1)^2 - 2M^2(s + s_1) + M^4}.$$  

The roots of the first factor of the expression $B$ are defined in terms of $s_1$ and $t_2$. The corresponding range for $t_2$, $t_{2-} < t_2 < t_{2+}$ and for $s_1$, $4m^2 < s_1 < (\sqrt{s} - M)^2$ is limited by the curves

$$t_{2\pm} = \frac{1}{2s} \left[ C \pm \lambda_1(M^2 - s) \right], \quad C = s_1(s + M^2) - (s - M^2)^2.$$  

Below, the variable $u$ is preferred instead of $t_2$, because one of the Compton-type diagram has a pole behavior precisely in the $u$ channel. The kinematical region $(s_1, u)$ is shown in Fig. 2, where :

$$u_{\pm} = \frac{1}{2s} \left[ \tilde{C} \pm \lambda_1(s - M^2) \right], \quad \tilde{C} = M^4 + M^2(2s - s_1) - s(s_1).$$

**B. Calculation of the QED cross section**

In the case of unpolarized particles one has to average over (to sum over) the polarization states of the initial (final) particles. The differential cross section can be written in the form

$$d\sigma = \frac{1}{4} \frac{e^6}{4(kp)^5 (2\pi)^5} \sum_{pol} |M|^2 dR_3.$$


FIG. 2. Kinematical region for \((s_1, u)\). The quantity \(s_{10}\) is the solution of the equation \(u_0 = u_-\), where \(u_0\) is the negative cut parameter.

where \(M\) is the matrix element of the process \((1)\):

\[
\sum_{\text{pol}} |M|^2 = |M_b|^2 + |M_c|^2 + 2\Re(M_bM_c^*) ,
\]

and the index \(b\) (\(c\)) corresponds to the Borsellino diagrams (Compton-type diagrams).

The double differential cross section, as a function of the variables \(s_1\) and \(t_2\) (or the \((s_1, t_2)\)-distribution), is

\[
\frac{d\sigma}{ds_1 dt_2} = \frac{\alpha^3}{64\pi(s - M^2)^2} \int \int \frac{d s_2 d t_1}{\sqrt{-\Delta}} \sum_{\text{pol}} |M|^2 .
\]

The individual contributions in the matrix element are the contractions of the corresponding currents \(j_{\mu}\) with the photon polarization 4-vector \(A^\mu\)

\[
M_b = \frac{1}{t_2} A_{\mu} j_{\mu}^b , \quad M_c = \frac{1}{s_1} A_{\mu} j_{\mu}^c .
\]

The corresponding currents have the form

\[
j_{\mu}^b = \overline{u}(p_2) \gamma_{\lambda} u(p_1) \overline{u}(p_1) \tilde{Q}^{\mu\lambda} v(p_3) , \quad \tilde{Q}^{\mu\lambda} = \frac{1}{2d_1} \gamma^\mu \hat{k} \gamma^\lambda - \frac{1}{2d_3} \gamma^\lambda \hat{k} \gamma^\mu + e_{(31)}^{\mu} \gamma^\lambda ,
\]

\[
j_{\mu}^c = \overline{u}(p_1) \gamma_{\lambda} v(p_3) \overline{u}(p_2) \tilde{K}^{\mu\lambda} u(p) , \quad \tilde{K}^{\mu\lambda} = \frac{1}{2d_2} \gamma^\mu \hat{k} \gamma^\lambda + \frac{1}{2d} \gamma^\lambda \hat{k} \gamma^\mu + e_{(02)}^{\mu} \gamma^\lambda ,
\]

where \(d = (k p)\), \(d_i = (k p_i)\), and

\[
e_{\mu}^{(0i)} = \frac{p_{\mu}}{d} - \frac{p_{\mu}}{d_i} , \quad e_{\mu}^{(ij)} = \frac{p_{\mu}}{d_i} - \frac{p_{\mu}}{d_j} , \quad i, j = 1, 2, 3 .
\]
It is easy to verify that both currents in relations (12) satisfy the condition $j^\mu k_\mu = 0$. If we define

$$J^\mu = \frac{j^\mu}{t_2} + \frac{j^\mu}{s_1},$$  \hspace{1cm} (14)

then, in the unpolarized case, we can write

$$\sum_{\text{pol}} |M|^2 = -g_{\mu\nu} J^\mu J^{\nu*}. \hspace{1cm} (15)$$

The calculation of the matrix element squared gives:

$$|M_b|^2 = \frac{8}{t_2^2} \left\{ \frac{8d^2m^2}{d_3^2} + t_2^2 \left[ \left( \frac{1}{d_3^2} + \frac{1}{d_1^2} \right) m^2 - 2M^2 \frac{2d}{d_1d_3} \left( \frac{d}{d_1} + 1 \right) \right] + 8(pp_3)^2 \left[ \left( \frac{1}{d_3^2} + \frac{1}{d_1^2} \right) m^2 - \frac{t_2^2}{d_1d_3} \right] + 4(pp_3) \left[ t_2 \left( -m^2 \left( \frac{1}{d_3^2} + \frac{1}{d_1^2} \right) - \frac{t_2^2}{d_1d_3} \right) \right] + 4 \left( \frac{1}{d_3} + \frac{1}{d_1} \right) M^2 \right\},$$

$$|M_c|^2 = |M_b|^2 (p \leftrightarrow -p_3, p_1 \leftrightarrow p_2, m \leftrightarrow M),$$

$$M_b M_c^* + M_c M_b^* = \frac{8}{t_2s_1} \left\{ -\frac{16}{dd_1d_2d_3} (p p_3)^3 \right\} +$$

$$\frac{8}{dd_2} \left( \frac{1}{d_3^2} + \frac{1}{d_1^2} \right) [d_1^2 - d_3^2 + 2d(d_1 + d_3)] +$$

$$\left[ d \left( \frac{2}{d_3} + \frac{1}{d_1} \right) - 2 \left( \frac{1}{d_3} + \frac{2}{d_1} \right) \right] t_2^2 \left( pp_3 \right)^2 -$$

$$4 \left[ \frac{d(3d_1 + d_3) - 2d(d_1 + 3d_3)}{2dd_1d_2d_3} t_2^2 + \right]$$

$$\frac{7d(d - d_2)^2 + 2(d + d_2)d_3^2 + (m^2 + M^2)(d_1 + d_3)^2 + d_2(d_2^2 - d_2^2) + 4d(d_2 - 2d)d_3^2t_2}{dd_1d_2d_3} +$$

$$\frac{(d_1 + d_3)(9d_3^2 + 14d_2d_1^2 + 13d_3d_1^2 + 6d_2^2d_1 + 5d_3^2d_1 + 12d_3d_1d_3 + d_3^2 + 2d_3d_3^2 + 2d_2^2d_3)}{dd_1d_2d_3} +$$

$$\frac{(d_1 + d_3)[(d_3^2 + 6d_3d_1 + d_3^2)M^2 + m^2(d + d_2)^2]}{dd_1d_2d_3} \right\}(pp_3) +$$

$$\left( \frac{1}{d_2d_3} - \frac{1}{d_1} \right) t_2^2 + 2 \left[ \frac{(m^2 + M^2)(dd_1 - 2d_2d_3)}{dd_1d_2d_3} + \right]$$
\[
\frac{4dd_1^2 - (d_1 - d_2)(d_1 + d_2)d_1 + d_3(d_1 + d_3)d_1 + (d_1 - d_2)d_3(d_2 + d_3)}{dd_1d_2d_3} t_2^2 + \\
\frac{4M^2}{dd_2d_3} (d_1^3 + d_2d_1^2 + 5d_3d_1^2 + 7d_3^2d_1 + 2d_2d_3d_1 + 3d_3^3 + 5d_2^2d_1^2) + \\
\frac{4}{dd_1} (d_1 + d_3) [2(d_1 + d_2)^2 + (d_1 + d_3)^2 + 2d_2d_3] + \\
2 \left[ \frac{7d_1^3}{dd_2d_3} + \frac{13d_2^3}{dd_3} + \frac{15(d_1 + d_2)d_1}{dd_2} + \frac{11d_3d_1}{dd_2} + \frac{8d_2d_1}{dd_3} + \frac{3d_3^3}{dd_3} + \frac{5d_3}{dd_3} + \frac{2d_2^2}{dd_3} - \\
\frac{d_3^3 + 2(d_1 - d_2)d_3}{dd_1} - \frac{2d_2d_3}{dd_1} + \frac{M^2(d_1 + d_3) [3dd_1 + (2d_1 - d_2)d_3]}{dd_2d_5d_1} + \\
\frac{m^2(d_1 + d_3) [3dd_1 + 2d_2(d_1 + d_2) + d_2d_3]}{dd_2d_3d_1} \right] t_2 + \frac{4m^2(d + d_2)^2}{dd_2d_3} \right\}.
\]

Introducing the short notation:

\[
\frac{\pi}{64 (s - M^2)} \int_{s_2}^{s_2+} d s_2 \int_{t_1}^{t_1+} d t_1 \frac{W}{\sqrt{-\Delta}} \equiv \vec{W},
\]

and, applying the relation \[(3)\] between the variables \(t_2\) and \(u\), we have:

\[
|\vec{M}_c|^2 = \frac{2\pi^2}{3(M^2 - s)^2 s_1^3 (M^2 - u)} \times \left\{ -4M^6 + (s - s_1 + u)(5M^4 - 2s_1^2 + su) - M^2 \left[ s^2 - 4s_1^2 + s_1u + u^2 + s(s_1 + 6u) \right] \right\},
\]

\[
|\vec{M}_b|^2 = \frac{4\pi^2}{(M^2 - s)(2M^2 - s + s_1 - u)^2} \times \left\{ \log \left( \frac{s_1 - 4m^2 + \sqrt{s_1}}{2m} \right) \left[ 4s_1^2 s_1^2 - 2m^2(2M^2 - 3s_1) - \frac{(M^2 - s)^2}{(-2M^2 + s + u)^4} \right] - \right. \\
\left. \frac{4(M^2 - s)[2m^2 (2M^2 - s - s_1) + s_1(-5M^2 + 5s + 3s_1)] + s_1^2 (-2M^2 + 2s + s_1)}{(2M^2 - s - u)^3} \right\} - \frac{4m^4 + m^2(-8M^2 + 8s + 2s_1) + (M^2 - s)^2 + s_1(-M^2 + 3s + 2s_1)}{-2M^2 + s + u} + \\
\frac{1}{(-2M^2 + s + u)^2} \left[ 2m^2 (4M^2 - s)^2 + s_1(10s + s_1 - 8M^2) - 2m^2(2s + s_1) \right] + \\
s_1 (3(M^2 - s)^2 + s_1(4s + s_1 - 2M^2))] + 2M^2 + s + 3s_1 - u \right] - \\
\sqrt{1 - \frac{4m^2}{s_1}} \left[ -4(M^2 - s)[m^2 s_1(3M^2 - s - s_1) + 2s_1^2(-2M^2 - 2s - s_1)] \right] + \\
\frac{m^2 s_1(3M^2 - s - s_1) + 2s_1^2(-2M^2 - 2s - s_1)}{(-2M^2 + s + u)^3}.
\]
\[
\frac{4s_1^2(M^2 - s)^2(m^2 + 2s_1)}{(-2M^2 + s + u)^4} + \\
\frac{s_1[2m^2(2s + s_1) + 9(M^2 - s)^2 + 2s_1(-6M^2 + 8s + s_1)]}{(-2M^2 + s + u)^2} - \\
\frac{s_1(2m^2 - 5M^2 + 9s + 4s_1) + (M^2 - s)^2}{-2M^2 + s + u} + \frac{1}{2}(2M^2 + s + 5s_1 - u) \}
\] 

The interference of the Compton-type and Borsellino diagrams vanishes after integration over \( t_1 \) and \( s_2 \), as a consequence of the Furry theorem [38].

Following Eq. (11) and the definition of the quantity \( \hat{W} \), the double differential cross section can be written as

\[
\frac{d\sigma}{ds_1 \, du} = \frac{\alpha^3}{\pi^2(s - M^2)} \sum_{\text{pol}} |M|^2,
\]

where \( \sum_{\text{pol}} |M|^2 \) is defined by Eq. (10). To measure the differential cross section \( d\sigma/(ds_1 \, du) \) it is sufficient to detect the final muon 4-momenta. The analytic form of this double differential cross section is given in the Appendix.

The integration of the double differential cross section with respect to the variable \( u \) in the limits defined above, gives

\[
\frac{d\sigma_c}{ds_1} = \frac{\alpha^3}{3\pi s_1^3(s - M^2)^3} \left\{ \frac{\lambda_1[M^6 - M^4(s + s_1) + M^2s(15s + 2s_1) + s^2(s + 7s_1)]}{2s^2} - \right. \\
\left. [3M^4 + M^2(6s - 2s_1) - s^2 + 2(s - s_1)s_1\ln \left( \frac{(M^2 + s - s_1 + \lambda_1)^2}{4M^2s} \right) \right\},
\]

\[
\frac{d\sigma_b}{ds_1} = \frac{2\alpha^3}{\pi(s - M^2)} \left\{ \frac{2\lambda_1\left[1 - \frac{4m^2}{s_1}\right]}{3s_1^3(M^2 - s)^2} \\
\left[ m^2[17(M^2 - s)^2 + 2s_1(4M^2 - 2s + s_1)] + s_1[7(M^2 - s)^2 + s_1(4M^2 - 2s + s_1)] \right] + \\
\left. \frac{1}{s_1^4(s - M^2)} \left[ s_1\sqrt{1 - \frac{4m^2}{s_1}}[2m^2[2(M^2 - s)^2 + s_1(6M^2 - 2s + s_1)] \\
+ s_1[(M^2 - s)^2 + s_1(5M^2 - s))]] + 2[4m^4[2(M^2 - s)^2 + s_1(6M^2 - 2s + s_1)] - \\
- 2m^2s_1[2(M^2 - s)^2 + s_1(6M^2 - 2s + s_1)] - \\
s_1^2[(M^2 - s)^2 + s_1(3M^2 - s)]\ln \left( \frac{\sqrt{s_1 - 4m^2} + \sqrt{s_1}}{2m} \right) \right] \times \\
\ln \left( \frac{(\lambda_1 - M^2 + s + s_1)^2}{4ss_1} \right) \right\}
\]
\[
\frac{2\lambda_1}{3s_1^4(M^2 - s)^2} \left[ -4m^4(17(M^2 - s)^2 + 2s_1(4M^2 - 2s + s_1)) + 6m^2s_1(M^2 - s)(7M^2 - 7s + 2s_1) + s_1^2[8(M^2 - s)^2 + s_1(5M^2 - s + 2s_1)] \right] \times \ln \left( \frac{\sqrt{s_1 - 4m^2} + \sqrt{s_1}}{2m} \right) + \\
\frac{1}{2s_1^4(s - M^2)} \left[ s_1 \sqrt{1 - \frac{4m^2}{s_1}} \left[ 4m^2[2(M^2 - s)^2 + s_1(6M^2 - 2s + s_1)] + s_1[2(M^2 - s)^2 + s_1(10M^2 - 2s + s_1)] \right] + 2(8m^4 - 4m^2s_1 - s_1^2)[2(M^2 - s)^2 + s_1(6M^2 - 2s + s_1)] \times \ln \left( \frac{\sqrt{s_1 - 4m^2} + \sqrt{s_1}}{2m} \right) \ln \left[ \frac{[M^4 + \lambda_1(M^2 - s) - M^2(2s + s_1) + s(s - s_1)]^2}{4M^2s_1^2} \right] \right].
\]

In the limiting case \( s \gg (s_1, M^2) \gg m^2 \), as for electron-positron pair production, these expressions are essentially simplified, namely
\[
\frac{d\sigma_c}{ds_1} = \frac{\alpha^3}{3 \pi s s_1} \left[ \frac{1}{2} + \frac{17M^2 + 2s_1}{2s} + \left( 1 - \frac{3M^2 + 2s_1}{s} \right) \ln \frac{s}{M^2} \right],
\]
\[
\frac{d\sigma_b}{ds_1} = \frac{2\alpha^3}{\pi s_1^2} \left\{ \ln \frac{s_1}{m^2} \ln \frac{s^2}{s_1 M^2} - \frac{8}{3} \ln \frac{s_1}{m^2} - \ln \frac{s^2}{s_1 M^2} + \frac{14}{3} + \frac{1}{s} - s_1 \ln \frac{s_1}{m^2} \ln \frac{s^2}{s_1 M^2} + (s_1 - 2M^2) \ln \frac{s_1}{m^2} + s_1 \ln \frac{s^2}{s_1 M^2} - 4s_1 + 2M^2 \right\}.
\]

Eqs. (18) and (19) hold for particles with arbitrary masses. They can be applied to the reactions:
\[
\gamma + e^- \to \mu^+\mu^- + e^- , \quad \gamma + e^- \to \tau^+\tau^- + e^- , \quad \gamma + \mu^- \to \tau^+\tau^- + \mu^- ,
\]
and the asymptotic formulae, in the limit \( s \gg (s_1, m^2) \gg M^2 \), that hold for muon pair production, become:
\[
\frac{d\sigma_c}{ds_1} = \frac{\alpha^3}{6\pi s s_1} (2 + y) \sqrt{1 - y} \left[ \frac{1}{2} + x_1 + (1 - 2x_1) \ln \frac{s}{M^2} \right], \quad x_1 = \frac{s_1}{s} , \quad y = \frac{4m^2}{s_1} ,
\]
\[
\frac{d\sigma_b}{ds_1} = \frac{2\alpha^3}{\pi s_1^2} \left\{ \left[ (2 + 2y - y^2)L - (1 + y) \sqrt{1 - y} \right] (1 - x_1) \ln \frac{s^2}{M^2s_1} + \frac{17}{6} y^2 - 7y - \frac{16}{3} + x_1 \left( 2 + 5y - \frac{3}{2}y^2 \right) \right\} L + \sqrt{1 - y} \left[ \frac{14}{3} + \frac{17}{6} y - x_1 \left( 4 + \frac{3}{2}y \right) \right].
\]

\[ L = \ln \frac{\sqrt{s_1} + \sqrt{s_1 - 4m^2}}{2m}. \]

In the case of restricted phase space, the analytical form of the \( d\sigma/ds_1 \) is more complicated and is given as the difference of two quantities (see Appendix).
C. Dark photon contribution

The DP effective interaction Lagrangian with the SM electromagnetic current \[5\] can be written as

\[ \mathcal{L} = i \epsilon e \bar{\psi}(x) \gamma^\mu \psi(x) A'_\mu(x), \]

where \( A'_\mu \) is the 4-potential of the \( A' \) field and the small parameter \( \epsilon \) characterizes the coupling strength relative to the electric charge \( e \). In this approach, DP manifests itself as an intermediate state in the Compton-type diagrams with the ordinary Breit-Wigner propagator for spin-one particles:

\[ V^{\mu\nu}(q) = \left( -g^{\mu\nu} + \frac{q^\mu q^\nu}{M_A'^2} \right) P_{BW}(q^2), \quad P_{BW}(q^2) = \frac{1}{q^2 - M_A'^2 + iM_A' \Gamma}, \]

where \( M_A'(\Gamma) \) is the DP mass (total decay width).

The width of the DP decay to the SM lepton pair is

\[ \Gamma(\gamma' \rightarrow \ell^+ \ell^-) = \epsilon^2 \frac{\alpha}{3M_A'^2} (M_A'^2 + 2m_\ell^2) \sqrt{M_A'^2 - 4m_\ell^2} \Theta(M_A' - 2m_\ell) = \epsilon^2 \Gamma_0, \quad (20) \]

where \( m_\ell \) is the SM lepton mass and \( \Theta(x) \) is the Heaviside Theta function. In our numerical calculations, when \( e^+e^- \) or \( \mu^+\mu^- \) pairs are created we restrict ourselves to the analysis of a light DP signal and suppose that its mass \( M_A' < 1 \, \text{GeV} \). In this condition, the decay \( A' \rightarrow \tau^+ \tau^- \) is forbidden and, therefore, \( m_\ell \) in Eq. (20) is the electron or the muon mass.

The effect of the DP contribution is to modify the matrix element as

\[ M_c \rightarrow M_c R(s_1), \quad R(s) = 1 + s \epsilon^2 P_{BW}(s), \quad (21) \]

leading to the enhancement of the cross section in the resonance region near \( s_1 \approx M_A'^2 \). On the top of the resonance, the parameter \( \epsilon \) vanishes in the modification factor \( R \) because the decay width \( \Gamma(\gamma' \rightarrow \ell^+ \ell^-) \) is proportional to \( \epsilon^2 \):

\[ R(s = M_A'^2) = 1 - i \frac{M_A'}{\Gamma_0}. \]

Taking into account the DP contribution, the modified matrix element squared can be written as

\[ |M|^2 = |M_b|^2 + |M_c|^2 |R(s_1)|^2 + 2 \text{Re}\{M_b M_c^* R^*(s_1)\}, \]

where

\[ |R(s)|^2 = 1 + \frac{se^2}{D(s)} [2(s - M^2) + se^2], \quad D(s) = (s - M^2)^2 + M^2 \Gamma^2, \]

and the modified interference term does not contribute, of course, only when the final muon 4-momentum is measured (for double \( (s_1, u) \) distribution).
FIG. 3. (a) Differential cross section of the process $\gamma + \mu^- \rightarrow e^+e^- + \mu^-$, as a function of the $e^+e^-$ invariant mass squared, calculated with Eqs. (18) and (19); (b) Ratio of the contributions to the cross section of the Compton-type diagrams to the Borsellino ones.

III. ANALYSIS OF THE DARK PHOTON SIGNAL

First, we estimate the QED background and find the kinematical conditions when the cross section $d\sigma_c/ds_1$ exceeds $d\sigma_b/ds_1$, because the DP signal in our searches is connected with a modification of the Compton-type diagrams. The calculations is done here for $e^+e^-$ pair creation.

It is well known that at photon energies larger than 10 MeV, the main contribution to the cross section of the triplet-like processes arises from the Borsellino diagrams due to the events at small values of $t_2$ [39].

In Fig. 3 we show the $s_1$—distribution differential cross section that is the sum of (18) and (19) and the ratio of the Compton-type diagrams contribution to the Borsellino ones,

$$R_b^c = \frac{d\sigma_c}{d\sigma_b},$$

at different colliding energies: $s=6, 30, 60$ GeV$^2$ provided that the whole kinematical region ($s_1, u$) is allowed. We see that in a wide, physically interesting range of variable $s_1$, the quantity $R_b^c(s_1)$ is rather small (does not exceed $2\cdot10^{-2}$). Therefore, for full phase space, the Borsellino contribution leads to a very large QED background for the search of a small DP signal, that is present only in the Compton-type contribution.

To find the kinematical region where the signal over background ratio is maximized, we analyse the double ($s_1, u$) distribution separately for the Compton and Borsellino contributions, using relations (16) and (17), and the results are presented in Fig. 4 where we plot...
FIG. 4. Double differential cross section for the reaction $\gamma + \mu^- \rightarrow e^+ e^- + \mu^-$ - in the first row as a function of $x_1 = s_1/s$ for fixed values of the variable $x_2 = |u|/s$: for Compton-like diagrams $x_2 = 1/30$ (solid line), $x_2 = 1/6$ (dotted line), for Borsellino diagrams $x_2 = 1/30$ (dash-dotted line), $x_2 = 1/6$ (dashed line); and for different values of the total energy squared $s$: (a) $s = 6$ GeV$^2$; (b) $s = 30$ GeV$^2$; (c) $s = 60$ GeV$^2$; - in the second row as a function of $x_2$ for fixed values of the variable $x_1$: for Compton-like diagrams $x_1 = 1/30$ (solid line), $x_1 = 1/6$ (dotted line), for Borsellino diagrams $x_1 = 1/30$ (dash-dotted line), $x_1 = 1/6$ (dashed line), and for different values of the total energy squared $s$: (a) $s = 6$ GeV$^2$; (b) $s = 30$ GeV$^2$; (c) $s = 60$ GeV$^2$.

The corresponding double differential cross sections and from which one can easily determine the regions where the Compton contribution exceeds the Borsellino one. We see that the contribution due to the Compton type diagrams increases with decrease of both, $s_1$ and $|u|$, whereas the contribution of the Borsellino diagrams indicates just opposite behaviour. Since we have to scan the $s_1$—distribution, we can restrict the $(s_1, u)$ region by cutting the large values of $|u|$ to reach our goal.

As one can see from the curves in Fig. 4, the region of the variables $s_1$ and $u$ where the Compton contribution exceeds the Borsellino one is wide. The measurements should be preferentially performed in this region to detect the DP signal in form of a resonance in the single photon intermediate state. The corresponding regions for the different reactions are
FIG. 5. Kinematical region where the Compton contribution to the double differential cross section exceeds the Borsellino contribution, at \( s = 30 \text{ GeV}^2 \): (a) for the process \( \gamma + \mu^- \rightarrow e^+ e^- + \mu^- \), (b) for the process \( \gamma + e^- \rightarrow \mu^+ \mu^- + e^- \), and (c) for the processes \( \gamma + e^- (\mu^-) \rightarrow \tau^+ \tau^- + e^- (\mu^-) \). shown in Fig. 5.

To reduce the Borsellino contribution, we suggest to remove the events with small values of the variable \(|t_2|\) (or with large values of \(|u|\)) by a kinematical cut:

\[
u > u_0,
\]

where \( u_0 \) is a negative parameter that takes different values in our numerical calculations. We can perform the integration over the variable \( u \) and derive an analytical result for the \( s_1 \)-distribution also in the restricted phase space given by (22). The region of the variables \( u \) and \( s_1 \), in this case, is shown in Fig. 2 where the quantity \( s_{10} \) is the solution of the equality \( u_0 = u_- \) and reads

\[
s_{10} = \frac{(M^4 - su_0)(2M^2 - s - u_0)}{(M^2 - u_0)(s - M^2)}, \quad u_0 < \bar{u}, \quad \bar{u} = M \left( \frac{M^2}{\sqrt{s}} + M - \sqrt{s} \right).
\]

Note that the solution (23) is the same for the equality \( u_0 = u_+ \), but in this case

\[
u_0 > \bar{u}, \quad [4m^2 < s_1 < s_{10}, \quad u_0 < u < u_+].
\]

Two subregions can be delimited:

\[
[4m^2 < s_1 < s_{10}, \quad u_0 < u < u_+] \lor [s_{10} < s_1 < (\sqrt{s} - M)^2, \quad u_- < u < u_+].
\]

The event selection, following the constraints (22) (with limited phase space), decreases essentially the Borsellino contribution, whereas the Compton-type contribution decreases
FIG. 6. Differential cross section of the process $\gamma + \mu^- \to e^+e^- + \mu^-$, as a function of the $e^+e^-$ invariant mass squared, for the sum of the Compton-type [18] and the Borsellino [19] contributions, with the constraint [22], is shown for $s = 6$ GeV$^2$ (a); $s = 30$ GeV$^2$ (b); $s = 60$ GeV$^2$ (c). The corresponding values of the parameter $u_0$ for different values of $s$ are given. The corresponding ratio of the Compton-type [18] and the Borsellino contributions is shown in the inserts (d),(e), and (f).

little. Their ratio

$$\tilde{R}_b^c = \frac{d\sigma_c(u > u_0)}{d\sigma_b(u > u_0)}$$

for the limited phase space (to be compared to the ratio $R_b^c$), is shown in Fig. 6.

In Figs. 6a,b,c the differential cross section of the process $\gamma + \mu^- \to e^+e^- + \mu^-$ is illustrated, taking into account all the contributions in the matrix element squared [10] and the constraint [22]. This cross section is plotted as a function of the dimensionless variable $x_1 = s_1/s$, for different values of $u_0$. Fig. 6 shows a steep decrease of this cross section, when $s$ and $u_0$ increase. The ratio of the Borsellino and Compton-type contributions for the corresponding kinematics is shown in Figs. 6d,e,f.

Let us estimate the limits for the parameter $\epsilon$ following Refs. [22] [24], and introduce the definition of standard deviation

$$\sigma = \frac{S}{\sqrt{B}}.$$
where $S(B)$ is the number of signal (background) events ($\sigma = 2$ corresponds to $\approx 95\%$ confidence limit). The event number of any process $i$ is the product of the corresponding cross section and the integrated luminosity of the experimental apparatus

$$N_i = d\sigma_i \cdot L \cdot T,$$

where $L$ is the luminosity, $T$ is the time necessary to the registration of the total number of events. All differentials in $d\sigma_i$ are dimensionless.

The following relation holds:

$$\frac{S}{B} = \frac{d\sigma_{A'}}{d\sigma_Q}, \quad (26)$$

where $d\sigma_{A'}$ is the calculated differential distribution due to the DP mechanism

$$d\sigma_{A'}(\epsilon, M^2_{A'}) = \frac{\epsilon^2 s_1[2(s_1 - M^2_{A'}) + \epsilon^2 s_1]}{D(s_1)} d\sigma_c,$$

and $d\sigma_Q$ is the pure QED contribution. Eqs. (25) and (26) imply:

$$\sigma d\sigma_Q = \sqrt{N} d\sigma_{A'}(\epsilon, M^2_{A'}), \quad (27)$$

where $N$ is the number of detected events for a definite experimental setting. The condition for the event selection is that the invariant mass $\sqrt{s_1}$ of the detected $e^+e^-$ pair falls in the energy region

$$M_{A'} - \delta m/2 < \sqrt{s_1} < M_{A'} + \delta m/2,$$

where $\delta m$ is the experimental invariant mass resolution, i.e., the bin width containing the events corresponding to the possible signal. Assuming $\Gamma \ll \delta m \ll M_{A'}$ we can rewrite the quantity $D(s_1)$ (see Eq. (21) and the text below) in the approximate form

$$[D(s_1)]^{-1} = \frac{\pi}{M_{A'} \epsilon^2 \Gamma_0} \delta(s_1 - M^2_{A'}).$$

Integrating both sides of Eq. (27) over the variable $s_1$, within the bin interval $\delta m$ we obtain

$$\epsilon^2 = \frac{2\sigma \delta m \Gamma_0 d\sigma_Q(M^2_{A'})}{\pi \sqrt{N} M^2_{A'} d\sigma_c(M^2_{A'})}. \quad (28)$$

The last relation defines the constraints on the parameters $\epsilon^2$, $M^2_{A'}$, and the number of the detected events $N$ for a given standard deviation $\sigma$. We illustrate these constraints for $\sigma = 2$ and the energy bin value $\delta m = 1$ MeV. In our calculations we assume that three channels are open for the decay of the dark photon into SM leptons: $A' \rightarrow e^+e^-, \mu^+\mu^-, \tau^+\tau^-$. 

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FIG. 7. Constraints on the DP parameters in terms of $\epsilon^2$ as a function of $M_{A'}$ for the conditions: $N = 10^4$, $\sigma = 2$, and $\delta m = 1$ MeV, in the process $\gamma + \mu^- \rightarrow e^+ e^- + \mu^-$, for $M_{A'} < 2M$, where only one channel of DP decay is allowed, for: (a) $s = 1$ GeV$^2$; (b) $s = 6$ GeV$^2$; (c) for $s = 30$ GeV$^2$. Different curves correspond to different values of $u_0$, i.e., different kinematical cuts and give the lower limit on DP parameters, in case of no DP event detected. Insert (d) shows the ($\epsilon^2, M_{A'}$) dependence for $s = 1$ GeV$^2$ (solid line); $s = 6$ GeV$^2$ (dashed line) without kinematical cuts.

The plots of $\epsilon^2$ versus $M_{A'}$ at the above mentioned conditions, for the reaction $\gamma + \mu^- \rightarrow e^+ e^- + \mu^-$ are shown in Figs. 7, 8 for $M_{A'} < 2M$ and $M_{A'} > 2M$, correspondingly. In the case of $e^+ e^-$ pair production the value of the DP mass is restricted to $M_{A'} < 2m_\tau$ ($m_\tau$ is the $\tau$ lepton mass) and, therefore, we take into account the DP decays into $e^+ e^-$ and $\mu^+\mu^-$ when calculating quantity $\Gamma_0$. The plots for $\gamma + \mu^- \rightarrow \tau^+ \tau^- + \mu^-$ process are shown in Fig. 9. In this case all three channels of DP decays contribute into $\Gamma_0$.

We can conclude that setting kinematical limits to reduce the contribution of the Borsellino diagrams to the cross section, increase essentially the sensitivity to the DP
signal and should be implemented in the experimental analysis for the event selection.

The corresponding results of calculations for the reactions $\gamma + e^- \rightarrow \mu^+ \mu^- + e^-$ and $\gamma + e^- \rightarrow \tau^+ \tau^- + e^-$ are shown in Figs. 10, 11. Again, in the case of $\mu^+\mu^-$ production we use the restriction $M_{A'} < 2m_\tau$.

The points in the $(\epsilon^2, M_{A'})$ region (at a given values of $s$) below the curves, correspond to $\sigma < 2$ and above the curves, to $\sigma > 2$. If the real $A'$ signal corresponds, at least, to three (or more) standard deviations, then the quantity $\epsilon^2$ (at fixed $M_{A'}$) where this signal can be recorded increase at least by a factor 1.5 with respect to the corresponding points on the curves in Figs. 7, 11.

It is easy to see from Eq. (28) that an increase of the bin value $\delta m$ decreases the sensitivity of the detection of the $A'$ signal in the process $\gamma + e^- \rightarrow \mu^+ \mu^- + e^-$: the QED background increases as it is proportional to $\delta m$, whereas the events corresponding to the DP signal within the narrow $A'$ resonance, remain unchanged.

The dependence of this sensitivity on the DP mass $M_{A'}$ is defined by the interplay of
FIG. 9. $e^2$ versus $M_{A'}$ for the reaction $\gamma + \mu^- \rightarrow \tau^+\tau^- + \mu^-$, for $s = 30 \text{ GeV}^2$ (a); $s = 60 \text{ GeV}^2$ (b). Different curves correspond to different values of $u_0$, i.e., different kinematical cuts.

FIG. 10. The same as Fig. 8a,b,c for the process $\gamma + e^- \rightarrow \mu^+\mu^- + e^-$. The $M_{A'}$-dependences of $\Gamma_0$, $d\sigma_Q$ and $d\sigma_e$ entering Eq. (28). This remains true also if we use the exact form of $D(s_1)$ in the integration of both sides of Eq. (27). Accounting for the kinematical restriction (22) increases essentially the sensitivity, due to the suppression of the QED background. To illustrate the corresponding effect, $e^2(M_{A'})$ is also plotted for $\gamma + \mu^- \rightarrow e^+e^- + \mu^-$ without the kinematical cuts.

The number of events $N$ in the denominator of the right hand side of Eq. (28), under the described event selection, can be written with a good approximation as

$$N = \frac{2 \delta m M_{A'} d\sigma_Q}{s} \left( x_1 = \frac{M_{A'}^2}{s} \right) L \cdot T.$$  

(29)

Using this formula it is easy to estimate the integrated luminosity that is necessary to accumulate $10^4$ events. Taking values in the ranges: $10^{-31} \leq d\sigma_Q \leq 10^{-34} \text{ cm}^{-2}$ (as it follows from the curves in Fig. 6) and $10^{-3} \leq 2 \delta m M/s \leq 10^{-1}$, one finds the interval $10^{29} \leq L \cdot T \leq 10^{31} \text{ cm}^{-2}$ for the considered values of $s$ in the range 1 GeV$^2$ and 60 GeV$^2$. The larger energies require the larger integrated luminosity and vice versa. Note that radiative
FIG. 11. The same as in Fig. 9 for the process $\gamma + e^- \rightarrow \tau^+\tau^- + e^-$. Corrections to the QED contribution cancel in the ratio $\sigma_Q/\sigma_c$ entering Eq. (28) and can not change essentially the curves in Figs. [7,11].

IV. CONCLUSION

We studied the possibility of direct detection of the dark photon, $A'$, one of the new particles that may possibly shed light on the nature and on the interaction of dark matter. DP is mixed with the ordinary photon due to the effect of the kinetic mixing and it can, therefore, interact with the SM leptons. It is characterized by a mass $M_{A'}$ and a small parameter $\epsilon$ describing the coupling strength relative to the electric charge $e$.

We analysed a possible way to detect the DP signal when its mass lies in the range between few MeV and few GeV presently accessible at the existing accelerators. The idea is to scan the distribution of the invariant mass squared distribution of the $\ell_j^+\ell_j^-$ system, $s_1$, in the reactions $\gamma + \ell_i \rightarrow \ell_j^+\ell_j^- + \ell_i$ with $i \neq j$ and $i = e, \mu; j = e, \mu, \tau$, where few tens MeV photons collide with high energy electron or muon beams. Due to the interaction with SM leptons, DP appears as a narrow resonance in $\ell_j^+\ell_j^-$ system over a background, and modifies the Compton-type diagrams by the Breit-Wigner term. Choosing processes with $i \neq j$ we avoid the problems with interpretation of the measurements arising due to final particles identity.

First we calculated the double differential cross section with respect to the variables $s_1$, and $u$ and then derived the $s_1$-distribution after integration over the variable $u$. Note that to measure such double differential cross section it is sufficient to measure the four
momentum of the final lepton $\ell_i$.

The advantage of this reaction is that the background is of pure QED origin and can be calculated exactly with the necessary precision.

If all the kinematically allowed region for the variable $u$ is taken into account, a large QED background arises due to the contribution of the Borsellino diagrams, as illustrated in Fig. 3. To reduce this background we analysed the $(s_1, u)$ distribution and identified the kinematical regions where the contribution of the Compton-type diagrams exceed the Borsellino ones (see Fig. 4). The background contribution increases when the variables $s_1$ and $u$ decrease, whereas the DP signal has just opposite behaviour. Thus, we applied different cuts ($u > u_0$) to exclude the range of large values of $|u|$ (see Figs. 5-6).

Selecting the restricted $(s_1, u)$ region, we estimated the constraints on the possible values of the parameters $\epsilon^2$ and $M_{A'}$ for a given number of the detected events, $N = 10^4$, and the standard deviation $\sigma = 2$ for all considered reactions (see Figs. 7-11). Our results suggest that the convenient bin width containing all the events of the possible DP signal near $s_1 = M_{A'}$ is $\delta m = 1$ MeV. Eq. (28) defines the relation between $\epsilon^2$ and $M_{A'}$ as a function of the parameters $N$, $\sigma$ and $\delta m$. Estimates of the integrated luminosity necessary to obtain $10^4$ events in the considered experimental conditions, show that such experiment is indeed presently feasible.

V. ACKNOWLEDGMENTS

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VI. APPENDIX

The analytic expression of the double differential cross section for a restricted phase space is given here.

\[
\frac{d\sigma_b}{ds_1du} = \frac{\alpha^3}{\pi (s - M^2)} \left\{ \frac{\sqrt{s_1 - 4m^2}}{\sqrt{s_1}} \right\} \left[ \frac{2(4m^2 + s_1)M^2}{(M^2 - s)(-2M^2 + s + u - s_1)} + \frac{8(M^2 - s)(m^2 + 2s_1)}{3(-2M^2 + s + u)^3} \right. \\
+ \frac{2}{(M^2 - s)(-2M^2 + s + u)s_1^2} \left[ 2\left(2M^4 - 4(s - s_1)M^2 + 2s^2 + s_1^2 - 2ss_1\right)M^2 + s_1[M^4 - 2(s - 2s_1)M^2 + s^2 + 2s_1^2] \right] + \log \left( \frac{\sqrt{s_1} + \sqrt{s_1 - 4m^2}}{2m} \right) \times \\
\left[ \frac{4(-8m^4 + 4s_1m^2 + s_1^2)M^2}{(M^2 - s)(-2M^2 + s + u - s_1)s_1^2} + \frac{8(M^2 - s)(-4m^4 + 6s_1m^2 + s_1^2)}{3(-2M^2 + s + u)^3s_1} \right. \\
+ \frac{4}{(M^2 - s)(-2M^2 + s + u)s_1^2} \left[ s_1^2[(M^2 - s)^2 + s_1(2M^2 + s_1)] + 2m^2(2m^2 - s_1)[2M^4 - 4(s - s_1)M^2 + 2s^2 + s_1^2 - 2ss_1] \right] \\
+ \frac{2\log(-2M^2 + s + u)}{(M^2 - s)s_1^4} \left[ -\sqrt{s_1}\sqrt{s_1 - 4m^2} \times \\
\left[ 2\left[2M^4 + (6s_1 - 4s)M^2 + 2s^2 \\
+ s_1^2 - 2ss_1\right]m^2 + s_1[M^4 + (5s_1 - 2s)M^2 + (s - s_1)] \right] - \\
\right. \\
2\log \left( \frac{\sqrt{s_1} + \sqrt{s_1 - 4m^2}}{2m} \right) \times \left[ 2m^2(2m^2 - s_1)[2M^4 + (6s_1 - 4s)M^2 + 2s^2 + s_1^2 - 2ss_1] \\
+ s_1^2[M^4 + (3s_1 - 2s)M^2 + s(s - s_1)] \right] \right. \\
\left. \log(-2M^2 + s + u - s_1) \right\} \left[ 2\log \left( \frac{\sqrt{s_1} + \sqrt{s_1 - 4m^2}}{2m} \right) \left( 8m^4 - 4s_1m^2 - s_1^2 \right) \times \\
[2M^4 + (6s_1 - 4s)M^2 + 2s^2 + s_1^2 - 2ss_1] + \sqrt{s_1}\sqrt{s_1 - 4m^2} \times \\
\left[ 4\left[2M^4 + (6s_1 - 4s)M^2 + 2s^2 + s_1^2 - 2ss_1\right]m^2 + \\
s_1[2M^4 + (10s_1 - 4s)M^2 + 2s^2 + s_1^2 - 2ss_1] \right] \right\} \right. \\
\right. \\
\left. \frac{d\sigma_c}{ds_1du} = \frac{-\alpha^3\sqrt{s_1 - 4m^2}(2m^2 + s_1)}{3\pi s_1^{5/2}(M^2 - s)^4} \left[ \right. \\
- \frac{1}{2}u^2(M^2 - s) \times \left[ \frac{2M^2(2M^2 + s_1)(M^2 - s)^2}{M^2 - u} \right. \\
\left. + \frac{[3M^4 + M^2(6s_1 - 2s) - s^2 - 2s_1^2 + 2ss_1](M^2 - s)\log(u - M^2) + u(M^4 - 5M^2s - 2ss_1)}{24} \right] \right. \\
\right. \\
\right. \left(30\right)
\]
The analytical form of the $s_1$—distribution for the restricted phase space, defined by Eq. (22), is given as

$$
\frac{d\sigma_i}{ds_1}(s, s_1, u_0) = \frac{d\sigma_i}{ds_1 du}(s, s_1, u = u_+) - \frac{d\sigma_i}{ds_1 du}(s, s_1, u = u_0), \quad i = b, c.
$$

This expression is valid if $s_1 < s_{10}$, where $s_{10}$ is the solution of the equation $u_- = u_0$ (see Fig. 2.)

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