Detection of primordial non-Gaussianity ($f_{\text{NL}}$) in the WMAP 3-year data at above 99.5% confidence

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We present evidence for the detection of primordial non-Gaussianity of the local type ($f_{\text{NL}}$), using the temperature information of the Cosmic Microwave Background (CMB) from the WMAP 3-year data. We employ the bispectrum estimator of non-Gaussianity described in [1] which allows us to analyze the entirety of the WMAP data without an arbitrary cut-off in angular scale. Using the combined information from WMAP’s two main science channels up to $\ell_{\text{max}} = 750$ and the conservative kp0 foreground mask we find $27 < f_{\text{NL}} < 147$ at 95% C.L., with a central value of $f_{\text{NL}} = 87$. This corresponds to a rejection of $f_{\text{NL}} = 0$ at more than 99.5% significance. We find that this detection is robust to variations in $l_{\text{max}}$, frequency and masks, and that no known foreground, instrument systematic, or secondary anisotropy explains our signal while passing our suite of tests. We explore the impact of several analysis choices on the stated significance and find 2.5 $\sigma$ for the most conservative view. We conclude that the WMAP 3-year data disfavors canonical single field slow-roll inflation.

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It is now widely accepted that tests of primordial non-Gaussianity, parameterized by the non-linearity parameter $f_{\text{NL}}$, promise to be a unique probe of the early Universe [2] beyond the two-point statistics. Although the non-Gaussianity from the simplest inflation models is very small, $f_{\text{NL}} \sim 0.01 - 1$ [3,4,5], there is a very large class of more general models, e.g., models with multiple scalar fields, features in inflation potential, non-adiabatic fluctuations, non-canonical kinetic terms, deviations from the Bunch-Davies vacuum, among others, that predict substantially higher level of primordial non-Gaussianity (see [6] for a review and detailed references).

Recent calculations of the perturbations arising in the ekpyrotic or cyclic cosmological scenarios [7] have concluded that these scenarios can predict $f_{\text{NL}}$ much larger than single field slow-roll inflation [8]. Detailed calculations in these models are fraught with difficulties connected to matching the perturbations through the cosmological singularity at the bounce. However, the current calculations suggests that primordial non-Gaussianity of the $f_{\text{NL}}$ type could be a powerful discriminant between ekpyrotic models and standard slow-roll inflation. As such, the search for primordial non-Gaussianity is complementary to the search for the inflationary gravitational wave background. We will argue in this letter that the WMAP 3-year data already distinguishes $f_{\text{NL}} = 100$ from $f_{\text{NL}} \sim 0$ at a statistically significant level.

Primordial non-Gaussianity can be described in terms of the 3-point correlation function of Bardeen’s curvature perturbations, $\Phi(k)$, in Fourier space:

$$\langle \Phi(k_1)\Phi(k_2)\Phi(k_3) \rangle = (2\pi)^3 \delta^3 (k_1 + k_2 + k_3) F(k_1, k_2, k_3).$$

(1)

Depending on the shape of the 3-point function, i.e., $F(k_1, k_2, k_3)$, non-Gaussianity can be broadly classified into two classes [8]. First, the local, “squeezed,” non-Gaussianity where $F(k_1, k_2, k_3)$ is large for the configurations in which $k_1 \ll k_2, k_3$. Second, the non-local, “equilateral,” non-Gaussianity where $F(k_1, k_2, k_3)$ is large for the configuration when $k_1 \sim k_2 \sim k_3$.

The local form arises from a non-linear relation between inflaton and curvature perturbations [3,4], curvature models [10], or the ekpyrotic models [8]. The equilateral form arises from non-canonical kinetic terms such as the Dirac-Born-Infeld (DBI) action [11], the ghost condensation [12], or any other single-field models in which the scalar field acquires a low speed of sound [13]. While we focus on the local form in this letter, it is straightforward to repeat our analysis for the equilateral form.

The local form of non-Gaussianity may be parametrized in real space as [2,4,14]:

$$\Phi(r) = \Phi_L(r) + f_{\text{NL}} (\Phi_L^2(r) - \langle \Phi_L^2(r) \rangle)$$

(2)

where $f_{\text{NL}}$ characterizes the amplitude of primordial non-Gaussianity. Note that the Newtonian potential has the opposite sign of Bardeen’s curvature perturbation, $\Phi$.

The first fast bispectrum based $f_{\text{NL}}$ estimator using temperature anisotropies alone was introduced in [18] (the KSW estimator). The idea of adding a linear term to reduce excess variance due to noise inhomogeneity was introduced in [19]. Applied to a combination of the Q, V and W channels of the WMAP 3-year data up to $\ell_{\text{max}} \sim 400$ this estimator has yielded the tightest constraint on $f_{\text{NL}}$ so far: $-36 < f_{\text{NL}} < 100$ (2$\sigma$) [16]. This estimator was generalized to utilize both the temperature and E-polarization information in [1], where we pointed out that the linear term had been incorrectly implemented in Eq. 30 of [1]. The corrected estimator enables us to analyze the entire WMAP data without suffering from a blow-up in the variance at high $\ell$.

Our analysis. We assume a standard Lambda CDM cosmology with following cosmological parameters: $\Omega_b = \ldots$
The analysis is done for 4 combinations of the frequency channels: coadded Q+V+W, coadded V+W, V, and W.

We performed Monte Carlo simulations to assess the statistical significance and errors of our \( f_{NL} \) estimates. For example for the Q+V+W coadded simulated map, we first simulated 8 Gaussian maps using the noise and foreground contamination biases \( f_{NL} \) negatively by similar amounts in both the data and the model.

![FIG. 1: We show the measured value of the non-linear coupling parameter \( f_{NL} \) using WMAP 3-year maps, and the corresponding 95% error bars derived from the Gaussian simulations. For this analysis the WMAP Kp0 mask was used. The analysis is done for 4 combinations of the frequency channels: coadded Q+V+W, coadded V+W, V, and W.](image-url)

| \( \ell_{max} \) | VW | Q | QVW |
|----------------|----|---|-----|
| Kp0 | Kp2 | Kp0 | Kp0+ |
| Kp0 | Kp12 | Kp2 | Kp0+ |
| 350 | -1290 | -27 | 35 | 19 | 1 | -2384 | -75 | 25 | 8 |
| 450 | -1425 | -16 | 68 | 65 | -6 | -2792 | -80 | 55 | 65 |
| 550 | -1510 | -13 | 80 | 84 | -11 | -3136 | -94 | 66 | 80 |
| 650 | -1560 | -22 | 79 | 81 | -14 | -3307 | -94 | 63 | 77 |
| 750 | -1575 | -23 | 87 | 87 | -20 | -3368 | -108 | 65 | 78 |
| 750* | -1105 | ±0.10 | -42±0.5 | -6±0.4 | -0.3±0.4 | -13±0.5 | 1±0.6 |

TABLE I: Non-linear coupling parameter \( f_{NL} \) using the V+W, Q, and Q+V+W WMAP 3-year raw maps, as a function of maximum multipole used in the analysis \( \ell_{max} \) and mask Kp12, Kp2, Kp0, and Kp0+ (corresponding \( f_{sky} \) is stated in the text and the masks are shown in Fig 2). The last row (750*) shows the mean \( f_{NL} \) estimated from Gaussian simulations including the WMAP foreground model. Foreground contamination biases \( f_{NL} \) negatively by similar amounts in both the data and the model.

This rules out the null hypothesis of Gaussian primordial perturbations at 99.5% significance.

Our analysis provides the most information to date on the primordial non-Gaussianity of the local type. For the sake of comparison with the previous best result in the literature \((-36 < f_{NL} < 100\), for the coadded Q+V+W map at the 2\(\sigma \) level for \( \ell_{max} \approx 400 \)) [15, 16, 17], our constraints using the coadded Q+V+W map truncated at \( \ell_{max} = 400 \) are: \(-20.84 < f_{NL} < 83.4 \) (at 95% C.L.). We may conclude that the additional information uncovered by the Yadav et al. estimator [1] at \( \ell > 400 \) is
important for our result. As calculated by Creminelli et al.\textsuperscript{20} and verified in simulation by \textsuperscript{21}, there is a contribution to the estimator variance due to non-zero \( f_{\text{NL}} \). This widens the confidence interval of the estimator by 3\%. It does not however modify the significance of our rejection of the Gaussian null hypothesis.

**Interpretation.** A detection of non-Gaussianity has profound implications on our understanding of the early Universe. We will now argue based on an extensive suite of null tests and theoretical modeling that our results are not due to any known systematic error, foregrounds or secondary anisotropy.

Since our estimator is based on three-point correlations, any mis-specification of the WMAP noise model would not bias our estimator, since Gaussian instrument noise has a vanishing three-point function. Similarly, if the CMB were Gaussian, asymmetric beams cannot create non-Gaussianity. Beam far-side lobes can produce a small level of smooth foreground contamination at high galactic latitude \([22]\) at \( \ell < 10 \). This effect has been corrected in the 3-year maps\,[23]. Since our signal is not frequency dependent this is clearly not a dominant effect. Even so, we checked for this or any other large scale anomaly (such as the axis of evil) by deleting modes with \( \ell < 20 \) from our analysis. We find that our estimate increases to \( f_{\text{NL}} = 135 \pm 96 \) (95 \% C.L.), leaving the statistical significance of our signal at a similar level.

To test for non-Gaussian time-domain systematics or non-Gaussian noise, we take difference between the pairs of yearly WMAP data. This creates 3 jackknife realizations of WMAP noise maps for each detector including real instrument systematics. Applying our estimator to these maps gave negligible \( f_{\text{NL}} \) (\( \sim 1 \)) for all three pairs of years, leading us to a conservative bound on the systematic error arising from such effects of \( \pm 2 \).

Seeing the same behavior in all channel combinations suggests that even if the detection of non-Gaussianity were not primordial, its source would not be frequency dependent and not associated with the main galactic foregrounds near the galactic plane. This is confirmed by repeating our analyses with several foreground masks (WMAP Kp12 mask with \( f_{\text{sky}} = 94.2 \% \), WMAP Kp2 mask with \( f_{\text{sky}} = 84.7 \% \), WMAP Kp0 mask with \( f_{\text{sky}} = 76.8 \% \), and a larger mask (Kp0+) with \( f_{\text{sky}} = 64.3 \% \), which was obtained by smoothing and thresholding the Kp0 mask) for computing the value of \( f_{\text{NL}} \) using the Q, Q+V+W and V+W maps. The results for these analyses are shown in Table I. It is clear from these analyses that the Kp12 and Kp2 masks do not exclude galactic foregrounds at the required level. The Q+V+W combination is foreground contaminated even for Kp0 mask, as is also clear from Q band analysis. However, for V+W, increasing the mask beyond Kp0 does not change the results significantly.

To quantify the level at which foreground contamination might be expected to affect our results, we perform a null test (see last row of Table I). We apply our analysis to simulated Gaussian CMB skies, expecting to measure \( f_{\text{NL}} \) consistent with 0. We model the expected diffuse synchrotron, dust and free-free emission the same way the WMAP team did to produce their foreground cleaned maps. We generate simulated WMAP Q, V and W maps including this foreground model, Gaussian CMB and WMAP noise, and we found negligible \( f_{\text{NL}} \) (\( -6 \pm 4 \) standard error in the mean for 70 simulations) with \( \ell_{\text{max}} = 750 \) and Kp0 mask. The same analysis run on simulated Q+V+W data gives an \( f_{\text{NL}} \sim -13 \pm 5 \). \( f_{\text{NL}} \) becomes increasingly negative for smaller masks, following the same pattern seen in the real data, and is negligible for the Kp0+ mask.

Note that in the cases that we expect to be affected by foregrounds (e.g. small masks), \( f_{\text{NL}} \) is biased negatively both in the real data and in simulations. It is therefore plausible that any bias due to residual foreground contamination would falsely reduce the significance of our detection. We assign a systematic error of \( \pm 5 \) to diffuse foreground effects in V+W, kpo and \( \ell_{\text{max}} = 750 \).

To get an additional handle on foregrounds, we have also analyzed the WMAP foreground-reduced maps. Irrespective of the mask used we always find that foreground subtraction increases the \( f_{\text{NL}} \) estimate. However, since we cannot guarantee that cleaning the foregrounds does not oversubtract foregrounds we conservatively quote the results from the raw maps.

For both the Kp0 and the extended masks, increasing \( \ell_{\text{max}} \) increases the central value of \( f_{\text{NL}} \). This effect may also be seen in Fig. 1. This change is somewhat larger than what we measure in Monte Carlo simulations, but attaching a significance to this observation is complicated by its \textit{a posteriori} nature. In any case, foregrounds are likely not responsible since most models of the high latitude galactic foreground emission predict more rapid decay of spatial structure in these foregrounds compared.
to the CMB on the relevant scales. Note that the same arguments also apply to a spinning dust component.

Our added statistical power relies on information at relatively small angular scales, where unresolved point source emission may contaminate the map. There are several reasons why it is unlikely to be responsible for our results. First, jointly fitting for both the primordial and the point source bispectrum contributions has shown that neglecting the point source contribution does not bias $f_{\text{NL}}$ on the relevant range of scales $\sim 2, 11, 24$. Second, since bright point sources are highly clustered, one could hypothesize that there may be a contribution due to clustered point sources that are not explicitly masked by the Kp0 mask but that are near the point source exclusion regions. A re-analysis of the data using a mask with larger point source exclusion regions showed no significant shift. Galactic point sources with a gradient in galactic latitude cannot be responsible because our estimate is insensitive to extending the mask.

All studies of the dominant secondary anisotropies conclude that they are negligible for the analysis of the WMAP data for $l_{\text{max}} < 800 \ [2, 24]$. The largest expected biases arise from the ISW-lensing and SZ-lensing contributions. These partially cancel due to their opposite signs and are expected to contribute a net bias of $\sim 2$, and a much smaller effect on the error bar. Masking the Cold Spot found by $[24]$ we find the central value of $f_{\text{NL}}$ increases to $\sim 94$, but we ignore this enhancement of our signal as an a posteriori effect.

Finally we study the dependence of $f_{\text{NL}}$ on cosmological parameters. We find that within the $2\sigma$ allowed range of cosmological parameters, the central value and the error bar varies by than 10% with the largest effect due to variations in $n_s$. Setting $n_s = 0.95$ reduces all k polluted estimates by $5 - 20\%$ while simultaneously reducing the variance by a similar amount. For $n_s = 0.95$, $l_{\text{max}} = 750$ has a significantly smaller variance than those for lower $l_{\text{max}}$ and gives $f_{\text{NL}} = 83.5 \pm 27$, increasing the statistical significance of the detection to more than $3\sigma$.

We conclude that the WMAP 3 year data contains evidence that allows us to reject the null-hypothesis of primordial Gaussianity at the 99.5% significance level. Including our systematic error estimates, our result differs from $f_{\text{NL}} = 0$ at the $2.5\sigma$ level. If our result holds up under scrutiny and the statistical weight of future data, it will have profound implications on our understanding of the physics of the early Universe. As it stands, the data disfavors canonical single field slow-roll inflation.

In addition to repeating our analysis on future data, further tests on currently available data using different higher order moments and additional bispectrum configurations may provide additional clues. The implications of our result for other probes of the cosmological density field, such as the mass function and large scale structure data should also be considered. This detection demonstrates the promise of targeted searches for primordial non-Gaussianity as a probe of the early Universe.

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