Mode purification for ultrasonic guided waves under pseudo-pulse excitation

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Abstract. Under broadband excitation, the captured Lamb wave signals contain rich information of the structural properties. However, since multiple modes are highly dispersive and overlapped with each other, Lamb wave signals are especially complicated to be interpreted. To overcome this problem, a mode purification strategy is established under pseudo-pulse excitation. In this strategy, a pseudo-pulse excitation technique is introduced to obtain a high resolution for the inspection firstly. Dispersion compensation method is applied to remove the dispersion of the received signal subsequently, through which all the wave packets associated with the interferential mode propagating through different paths could be compressed into the same shape as the excitation. Benefiting from that, the energy of the wave packets corresponding to the interference mode could be concentrated in time domain as individual temporal pulses, which thereby could be eliminated by zero-amplitude rectangular time windows without affecting the desired mode much. After that, the inverse dispersion compensation is applied to the residual signals to restore the original waveform of the desired mode. Finally, experiments are introduced to validate the availability and robustness of the proposed strategy.

1. Introduction
Lamb wave techniques allow long-range inspections for large-scale plate-like waveguides (e.g., airplanes, pipelines and oil tanks) rapidly and nondestructively, because of its little energy loss and high sensitivity to both the surface and internal defects [1-4]. Actually, under broadband excitation, rich information of the structural properties could be obtained in the received Lamb wave signals [5]. Especially, the detection of incipient damage in the structure may be possible [6, 7]. However, for broadband excitations, the Lamb waves are especially complicated and thus cannot be directly interpreted in time domain, because multiple modes are highly dispersive and overlapped with each other.

One candidate way to solve this problem is the time-frequency representation (TFR), because it provides a clear illustration for the temporal variation modal energy stream in the time-frequency domain [8]. Unfortunately, with distinct dispersive effects and propagating through different distances, there are some frequency components corresponding to different modes arrive in phase at the receiver position, and thus these modes will interact at the neighborhood of those frequencies. The energy of those interacted modes, which covers the same time-frequency tiling in the time-frequency representation, will be processed together and cannot be separated by current methods [9]. Xu et al.

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developed a mode separation method on the basis of dispersion compensation method which could compress the dispersive signal into the initial shape of the excitation with prior i-knowledge of the dispersion curves [10]. However, the energy of the short spiky pulse excitation is the main limitation because guided wave measurements are frequently performed using low voltage excitations [11, 12].

In this paper, a pseudo-pulse excitation technique is introduced to obtain a high resolution for the inspection. The pseudo-pulse excitation lowers the probability of mode overlapping, while achieves a high signal-to-noise ratio. On this basis, a windowed function is employed to purify the Lamb modes in the received signals, and thus the wave-packets reflected from the structural features (defects or edges) would be much easier to be identified.

2. Methods

2.1. Pseudo-pulse excitation

Limited by the low generating voltage, when a spiky pulse acts as the excitation, it is hard to acquired response signal with high signal-to-noise ratio. Exchanging spiky pulse with the long duration excitation could significantly enhance the signal-to-noise ratio of the response signal. However, with the increase of duration, the probability of mode overlapping would rise, which complicates the interpretation of the response signal. Therefore, a pseudo-pulse excitation, with short duration and high signal-to-noise ratio, is introduced in this section.

Golay complementary code (GCC) is one type of the binary sequences, which contains two binary sequences [13-15], 8-bit for example:

\[ A[N] = [+1,+1,+1,-1,+1,-1,-1] \]
\[ B[N] = [+1,+1,+1,-1,-1,-1,+1] \] (1)

The auto-correlation functions of \( A[N] \) and \( B[N] \), \( \Psi_A[n] \) and \( \Psi_B[n] \), have side-lobes with equivalent magnitude but opposite sign. Therefore, the sum of the two auto-correlation functions has a main peak and zero side-lobes, which can be represented as

\[ \Psi_A[n] + \Psi_B[n] = \begin{cases} 16, & n = 0 \\ 0, & n \neq 0 \end{cases} \] (2)

The single-frequency sinusoidal signal with the frequency \( f=150 \text{ kHz} \) and the cycle \( k=1 \) is used for GCC modulation. \( a(t) \) and \( b(t) \) are the \( A[N] \) and \( B[N] \) modulation, respectively, shown in figure 1(a) and 1(b). \( \psi_A(t) \) and \( \psi_B(t) \) are the auto-correlation of \( a(t) \) and \( b(t) \), respectively. As shown in figure 1(c) and 1(d), \( \psi_A(t) \) and \( \psi_B(t) \) have side-lobes with equivalent magnitude but opposite sign. After summing \( \psi_A(t) \) and \( \psi_B(t) \), as shown in figure 1(e), the sum, \( s(t) \), has a main peak and zero side-lobes, which is the pseudo-pulse signal.
Consider that two transducers are mounted on a plate, acting as the transmitter and receiver, respectively. The entire system, consisting of the instrumentations, transducers and structures, can be considered as a linear system [16]. If an excitation signal $f(t)$ is generated by the transmitter, the response can be denoted as

$$r(t) = \int F(\omega)H(\omega)\exp(-i\omega t)d\omega$$  \hspace{1cm} (3)$$

where $F(\omega)$ is the Fourier transform of $f(t)$, $H(\omega)$ is the transfer function of the system.

To gain the pulse compression signal $c(t)$, the response signal $g(t)$ is cross-correlated with the excitation signal $f(t)$. The pulse compression signal is represented as

$$c(t) = \int F(\omega)H(\omega)F(\omega)^* \exp(-i\omega t)d\omega$$

$$= \int |F(\omega)|^2 H(\omega)\exp(-i\omega t)d\omega.$$  \hspace{1cm} (4)$$

where the superscript $*$ represents the complex conjugate, $|F(\omega)|^2$ represents the Fourier transform of the auto-correlation of the excitation signal $f(t)$. Equation (4) means that the pulse compression of the response is equal to adopting this auto-correlation curve as the new excitation signal [17-19].

When $a(t)$ and $b(t)$ are acted as excitation, cross-correlating the response signals with $a(t)$ and $b(t)$, respectively, the sum of the cross-correlated signals is the response signal of pseudo-pulse excitation $s(t)$.

### 2.2. Dispersion compensation algorithm

The dispersion phenomenon in Lamb wave propagation has been acknowledged. Considering that the transducers generate/receive only a Lamb wave mode and the propagation distance is $d_j$, the amplitude attenuation is $A_j(\omega)$, the received signal can be predicted as [20]

$$g(t) = \sum_{j=-\infty}^{\infty} A_j(\omega)S(\omega)\exp[-ik(\omega)d_j]\exp(i\omega t)d\omega$$  \hspace{1cm} (5)$$

where $k(\omega)$ is the wavenumber of the guided wave mode, which is a function of angular frequency $\omega$, and $S(\omega)$ is the Fourier transform of excitation $s(t)$.

In equation (5), because the wavenumber $k(\omega)$ is a non-linear function with respect to the angular frequency $\omega$, the non-linear phase adjustment term $\exp[-ik(\omega)d_j]$ will change the time delay of
harmonic of $F(\omega)$, according to the time shift property of Fourier transform [21]. It causes the received signal $g(t)$ spreading out in time domain and deforming comparing with the original excitation, which is the so-called dispersion. Therefore, when $s(t)$ is used as excitation, the received signal will not still maintain pulse-shape.

However, if the wavenumber $k(\omega)$ is in a linear relation with respect to the angular frequency $\omega$, $k(\omega)$ will not cause phase distortions, and there will be no dispersion in a propagated waveform [22]. On the basis of this concept, the dispersion compensation algorithm maps the dispersion relation $k(\omega)$ to the linear $k_{\text{lin}}(\omega)$ to remove the dispersion effect [23]. The linear wavenumber is approximated by using the Taylor expansion of the wavenumber $k(\omega)$ at the excitation central frequency $\omega_c$ up to the first order as

$$k(\omega) \approx k_{\text{lin}}(\omega) = k(\omega_c) + k'(\omega_c)(\omega - \omega_c) = k(\omega_c) + \frac{(\omega - \omega_c)}{c_{gr}(\omega_c)}$$  \hspace{1cm} (6)

where $k'(\omega_c)$ is the first-order derivative of $k(\omega)$ at $\omega_c$, and $c_{gr}$ is the group velocity of the guided wave mode.

![Graph](image)

**Figure 2.** The dispersion relation and its Taylor expansion

The procedure of dispersion compensation can be summarized as follows:

(i) Apply Fourier transform to the received signal $g(t)$ to get $G(\omega)$. Since $k$ is a function of $\omega$, $G(\omega)$ can be rewritten as $G(k)$;

(ii) Use equation (6) to calculate the linear wavenumber $k_{\text{lin}}(\omega)$;

(iii) Interpolate $[k, G(k)]$ pair at $k_{\text{lin}}(\omega)$ to get $G[k_{\text{lin}}(\omega)]$;

(iv) Apply inverse Fourier transform to $G[k_{\text{lin}}(\omega)]$ to obtain a wave with the dispersion removed.

After dispersion compensation, all the wave packets associated with the compensated mode propagating through different paths could be compressed into the same shape as the excitation.

### 2.3. Mode elimination

When $s(t)$ is generated by transmitter, assuming that the received modes (A0 and S0, for instance) are overlapped with each other in time domain, the received signal can be represented as

$$G(\omega) = S(\omega)H_{A0}(\omega) + S(\omega)H_{S0}(\omega) = G_{A0}(\omega) + G_{S0}(\omega)$$  \hspace{1cm} (7)

where $S(\omega)$ is the Fourier transform of the excitation, $H_{A0}(\omega)$ and $H_{S0}(\omega)$ are the transfer function of A0 and S0, respectively, $G_{A0}(\omega)$ and $G_{S0}(\omega)$ are the Fourier transform of the receive signals of the A0 and S0, respectively.
After applying dispersion compensation to the received signal with knowledge of the dispersion characteristics of A0, the processed signal can be denoted as

\[ Q_{dc}(\omega) = G_{A0}(k) \circ K_{A0,\text{lin}}(\omega) + G_{S0}(k) \circ K_{A0,\text{lin}}(\omega) \]

\[ = S(\omega) H_{A0}(k) \circ K_{A0,\text{lin}}(\omega) + G_{S0}(k) \circ K_{A0,\text{lin}}(\omega) \]

\[ = S_{dc}(\omega) + G_{S0}(k) \circ K_{A0,\text{lin}}(\omega) \]  \hspace{1cm} (8)

where \( \circ \) is composition operator, and \( H_{A0}(k) \) and \( H_{S0}(k) \) are implicit functions of \( k \). \( K_{A0,\text{lin}}(\omega) \) is the Taylor expansion of the wavenumber \( k(\omega) \) of A0 mode. \( Q_{dc}(\omega) \) is the Fourier transform of the compensated signal and \( S_{dc}(\omega) \) is the compensated A0 mode.

In equation (8), \( S_{dc}(\omega) \) shows that the energy of the wave packets corresponding to the compensated mode would be concentrated in time domain as individual temporal pulses. \( G_{S0}(k) \circ K_{A0,\text{lin}}(\omega) \) means that the energy of the other mode (S0 mode) will spread out further in time domain. Therefore, the main lobe of the compensated pulses could be eliminated by zero-amplitude rectangular time windows without affecting other modes much, because of short duration of the excitation.

\[ Q_{dc}(\omega) - S_{dc}(\omega) = S_{dc}(\omega) - S_{dc}(\omega) + G_{S0}(k) \circ K_{A0,\text{lin}}(\omega) \approx G_{S0}(k) \circ K_{A0,\text{lin}}(\omega) \]  \hspace{1cm} (9)

where \( S_{dc}(\omega) \) is the main lobe of \( s(t) \), and \( S_{dc}(\omega) \) contains most energy of the compensated mode \( S_{dc}(\omega) \).

After eliminating the A0 mode, the inverse dispersion compensation is applied to the residual signals to restore the original waveform of the S0 mode.

\[ \left[ Q_{dc}(\omega) - S_{dc}(\omega) \right] \circ K_{A0,\text{lin}}^{-1}(\omega) \approx G_{S0}(\omega) \circ K_{A0,\text{lin}}(\omega) \circ K_{A0,\text{lin}}^{-1}(\omega) \approx G_{S0}(\omega) \]  \hspace{1cm} (7)

where \( \circ K_{A0,\text{lin}}^{-1}(\omega) \) is the inverse dispersion compensation.

3. Experimental Example
The experiment was carried out on an aluminium plate with dimensions 2000 mm×1000 mm×2 mm. Two piezoelectric ceramic discs with a diameter of 8 mm and 0.5 mm in thickness were mounted on the plate, acting as the transmitter and receiver, respectively, with 340 mm between them. An artificial damage is 170 mm away from the receiver and the two PZTs are 660 mm away from the right boundary, shown in figure 3.

Figure 3. Experimental setup for measuring reflections of Lamb waves in an aluminium plate
In this experiment, \( a(t) \) and \( b(t) \) (shown in figure 1(a) and 1(b)) are generated by an Agilent 33220A function/arbitrary waveform generator, respectively, whose the centre frequency is 150 kHz. After being amplified with the peak-to-peak voltage of 5 V by a Piezo Systems EPA-104 voltage amplifier, they are applied to the actuator. The Lamb wave signals pass through the monitoring area and are captured by the receivers. After being amplified by the AVANT NI-2000 conditioning amplifiers, they are acquired at a sampling rate of 5 MHz by a NI PXIe-1082 data acquisition.

The received signals are cross-correlated with the corresponding excitations and the sum of the cross-correlated signals is the equivalent received signal, shown in figure 4(a). In this experiment, only the two fundamental A0 and S0 modes exist due to the frequency range is with [0-300] kHz. As it can be seen, A0 mode is highly dispersive and the direct arrival of A0 last a long duration, which may cover some information of other modes.

After applying dispersion compensation based on A0 mode dispersion relation, the A0 mode wave packets are compressed into pseudo-pulses and the S0 mode packets are spread out further, shown in figure 4(b).

The compensated A0 mode packets are pulse-like and can be eliminated by short zero-amplitude rectangular time windows, shown in figure 4(c). After elimination, the residual signal is restored by the inverse dispersion compensation, with the reflection of S0 mode from the damage being obvious, which could be applied to evaluate the health of the structure, as shown in figure 4(d).
4. Conclusion
In this study, a mode purification strategy is established to solve mode overlapping. Some conclusions are obtained as follows.

(i) The pseudo-pulse excitation could be generated easily by PZTs with a short duration and a high signal-to-noise ratio.

(ii) Since different modes are of distinct dispersion characteristics, when the compensated mode is compressed with the application of dispersion compensation, the energy of the other modes may further spread out.

(iii) Through mode purification strategy, the energy of interferential modes could be obviously alleviated and thus wave packets corresponding to the interest mode would be much easy to be identified.

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6. References
[1] Prosser W H, Seale M D and Smith B T 1999 Time-frequency analysis of the dispersion of Lamb modes J. Acoust. Soc. Am. 105 2669-76
[2] Wilcox P D, Lowe M J S and Cawley P 2001 The effect of dispersion on long-range inspection using ultrasonic guided waves NDT & E Int. 34 1-9
[3] Alleyne D N, Pialucha T P and Cawley P 1993 A signal regeneration technique for long-range propagation of dispersive Lamb waves Ultrasonics 31 201-4
[4] Zeng L, Lin J, Lei Y and Xie H 2013 Waveform design for high-resolution damage detection using Lamb waves IEEE. Trans. Ultrason. Ferroelectr. Freq. Control. 60 1025-9
[5] Michaels J E, Lee S J, Croxford A J and Wilcox P D 2013Chirp excitation of ultrasonic guided waves Ultrasonics 53 265-70
[6] Pierce S G and Culshaw B 1998 Laser generation of ultrasonic Lamb waves using low power optical sources IEE Proc-21Meas. Technol 145 5 244-49
[7] Fasel T R, Olson C C and Todd M D. 2008 Optimized guided wave excitations for health monitoring of a bolted joint The 15th International Symposium on: Smart Structures and Materials & Nondestructive Evaluation and Health Monitoring. International Society for Optics and Photonics 69351N-69351N-8
[8] Xu K, Ta D and Wang W 2010 Multiridge-based analysis for separating individual modes from multimodal guided wave signals in long bones IEEE Trans. Ultrason. Ferroelectr. Freq. Control 57102480-90

Figure 4. Experiment results of mode purification: (a) received signal; (b) compensated signal; (c) elimination of A0 mode; (d) restored signal
[9] Zhao M, Zeng L, Lin J and Wu W T 2014 Mode identification and extraction of broadband ultrasonic guided waves Meas. Sci. Technol. 25 (11): 11505
[10] Xu K, Ta D, Moilanen P and Wang W Q 2012 Mode separation of Lamb waves based on dispersion compensation method J. Acoust. Soc. Am. 131 (4) 2714-22
[11] Pertsch A, Kin J, Wang Y and Jacobs L 2011 An intelligent stand-alone ultrasonic device for monitoring local structural damage: implementation and preliminary experiments Smart Mater. Struct. 20 015022
[12] Marchi L D, Perelli A and Marzani A 2013 A signal processing approach to exploit chirp excitation in Lamb wave defect detection and localization procedures Mech. Syst. Signal Process. 39 (1-2) 20-31
[13] Shen C C and Shi T Y 2011 Golay-encoded excitation for dual-frequency harmonic detection of ultrasonic contrast agents IEEE Trans. UFFC 58 349-56
[14] Garcia-Rodriguez M, Yanez Y, Garcia-Hernandez M J, Salazar J, Turo A and Chavez J A 2010 Application of Golay codes to improve the dynamic range in ultrasonic Lamb waves air-coupled systems NDT & E Int. 43 677-86
[15] Veres I A, Cleary A, Thursby G, Mckee C, Armstrong I, Pierce G and Culshaw B 2013 Golay code modulation in low-power laser-ultrasonic Ultrasonics 53 122-9
[16] Michaels J E, Lee S J, Croxford A J and Wilcox P D 2013 Chirp excitation of ultrasonic guided waves Ultrasonics 53 265-70
[17] Gan T H, Hutchins D A, Billson D R and Schindel D W 2001 The use of broadband acoustic transducers and pulse-compression techniques for air-coupled ultrasonic imaging Ultrasonics 39 181-94
[18] Toiyama K 2004 High S/N ratio guided wave inspection of pipe using chirp pulse compression In. Proc. ASME/JSME Press, PVP 484 41-5
[19] Toiyama K and Hayashi T 2008 Pulse compression technique considering velocity dispersion of guided wave AIP Conf. Proc. 975 587-93
[20] Wilcox P D 2003 A rapid signal processing technique to remove the effect of dispersion from guided wave signals IEEE. Trans. Ultrason. Ferroelectr. Freq. Control. 50 419-27
[21] Liu L and Yuan F G 2010 A linear mapping technique for dispersion removal of Lamb waves Struct. Health Monit. 9 75-86
[22] Zeng L and Lin J 2014 Chirp-based dispersion pre-compensation for high resolution Lamb wave inspection NDT & E Int. 61 35-44
[23] Xu B, Yu L and Giurgiutiu V 2009 Lamb wave dispersion compensation in piezoelectric wafer active sensor phased-array application Proc. SPIE, Health Monit. Struct. Biol. Syst. 7295 729516