Heavy-to-light form factors of $B$ decays at large recoil

Jong-Phil Lee*
Department of Physics and IPAP, Yonsei University, Seoul, 120-749, Korea

Abstract

The form factors of $B \to \pi(\rho)$ decays are analyzed using the light-cone sum rules in the framework of the soft-collinear effective theory (SCET). We establish the sum rules for the leading and the next-to-leading order (NLO) nonperturbative functions of the SCET. Explicit calculation shows that (leading)+(NLO) $\sim 1/E^2 + 1/E^3$, where $E$ is the large recoiling energy. The results are compatible with the literatures. Also the validity of this hybrid formalism is discussed.

*e-mail: jplee@phya.yonsei.ac.kr
I. INTRODUCTION

One of the least known elements of the Cabibbo-Kobayashi-Maskawa (CKM) matrix $V_{ub}$ is the highlight of recent studies in particle physics. $B$ factories in KEK and SLAC are now producing copious $B$ mesons, reducing experimental errors considerably. In spite of remarkable developments in theory part during the last decade, theoretical uncertainties in determining $|V_{ub}|$ are still in the way of higher accuracy; very recent measurement by BABAR is $|V_{ub}| = (3.64 \pm 0.22 \pm 0.25^{+0.39}_{-0.56}) \times 10^{-3}$ from the rare decays $B \to \rho \ell \nu$ [1].

The extraction of $|V_{ub}|$ greatly depends on the heavy-to-light form factors. As for heavy-to-heavy transitions, the heavy quark effective theory (HQET) based on the heavy quark symmetry (HQS) works well to simplify the relations of the relevant form factors [2]. Semileptonic charmed decay modes $B \to D^{(*)} \ell \nu$ are good stuffs to apply the HQET and to extract $|V_{cb}|$ [2,3].

If the final state hadron is light, however, constraints from the HQET on the heavy-to-light form factors are less strong. In the kinematical regions where the light hadron carries a large energy $E \sim m_b$, other effective theories have been developed such as large energy effective theory (LEET) [4] or soft collinear effective theory (SCET) [5]. We adopt the SCET as a basic framework in this analysis.

In SCET, an energetic light particle defines the light-cone direction $n^\mu$ whose component of the momentum is the largest and scales as $E$. A small expansion parameter $\lambda$ is introduced to be the ratio of the transverse momentum $p_\perp$ to $E$. The smallest component of the momentum is the backward one which is of order $\lambda^2 E$. A systematic expansion of $\lambda$ is possible in SCET, and the subleading analysis was already done in [6].

The heavy-to-light currents of the full QCD are matched to the effective currents of SCET below the scale $\mu \lesssim E$. Effective weak currents are composed of the collinear quarks and the heavy quark fields, and a new set of Wilson coefficients is introduced during the matching.

It was already known that three independent functions suffice to describe heavy-to-light transition at large recoil [7]. The SCET analysis reproduced the same features in [5], and it was found that more functions are needed at subleading order of $\lambda$ [6].

The authors of [7] argued that the form factors of heavy-to-light decays scale as $1/E^2$, and cross-checked by the light-cone sum rule (LCSR) calculations. The argument for $1/E^2$ is the same as [8], where the end-point configuration for the "soft contribution" to $B \to \pi(\rho)$ yields $O(1/m_Q^2)$ behavior of the form factors, up to the normalization convention. Very similar scaling was first derived in [9]. In this paper, we give the HQET-LCSR calculations in the framework of SCET. LCSR is among the most reliable nonperturbative methods, especially for $B \to \rho$ decays [10]. In the traditional QCD sum rules nonperturbative nature is encoded in the vacuum condensates. But the concept of vacuum condensate may cause unphysical behavior of the distribution amplitudes at the end point region [13]. The LCSR is free from this nuisance because the low-energy physics is parametrized by the well-defined distribution amplitudes. Recently, the HQET-based LCSR analysis is given for $B \to \pi(\rho)$ in [11]. The usefulness of this hybrid formalism was discussed in [12], and the next-to-leading order of $1/m_Q^2$ calculations were given. The authors of [12] found that the large recoil limit of their results agrees with the LEET analysis of [7].

The fact that both SCET and LCSR are adequate to describe the heavy-to-light decays
at large recoil is the main motivation of this paper. We combine in this work the HQET LCSR [11] with subleading SCET [6] to examine the energy scalings of the form factors. The $1/E^2$ scaling of leading functions is checked and the energy dependence of the form factors are analyzed up to $\mathcal{O}(\lambda)$. In this work, we neglect the hard gluon-spectator effects which might occur for simplicity.

In Sec. II, $B \to \pi(\rho)$ transition matrix elements are parametrized in the context of SCET at order $\lambda$. Section III is devoted to evaluate the LCSR within the SCET framework. Relevant SCET functions are expressed in terms of distribution amplitudes for the light mesons. Our results and discussions appear in Sec. IV and summary is given in Sec. V.

II. MATRIX ELEMENTS IN THE SCET

The standard parametrization of $B \to \pi(\rho)$ decay matrix elements is

\begin{align}
\langle \pi(p)|V^\mu|B(p_B)\rangle & = f_+(q^2) \left[ p_B^\mu + p^\mu - \frac{m_B^2 - m_\pi^2}{q^2} q^\mu \right] + f_0(q^2) \frac{m_B^2 - m_\pi^2}{q^2} q^\mu, \\
\langle \rho(p, \epsilon^*)|V^\mu|B(p_B)\rangle & = \frac{2V(q^2)}{m_B + m_\rho} i\epsilon^{\mu\nu\alpha\beta} \epsilon^{*}_\nu p_\alpha (p_B)_\beta, \\
\langle \rho(p, \epsilon^*)|A^\mu|B(p_B)\rangle & = i(m_B + m_\rho) A_1(q^2) \epsilon^{*\mu} - i \frac{A_2(q^2)}{m_B + m_\rho} \epsilon^{*} \cdot p_B (p_B + p)^\mu - i \frac{A_3(q^2)}{m_B + m_\rho} \epsilon^{*} \cdot p_B q^\mu,
\end{align}

where $V^\mu(A^\mu) = \bar{u}\gamma^\mu b(\bar{u}\gamma^\mu\gamma_5 b)$, $q = p_B - p$, and $\epsilon^{*\mu}$ is the polarization vector of $\rho$. If the final-state mesons were heavy, then the HQS would allow to relate all the form factors, leaving only one independent Isgur-Wise function. For energetic light mesons, a similar reduction of the form factors can be obtained in the SCET systematically in powers of $\lambda$.

First we briefly review the structure of the SCET, following [5,6]. In the $B$-rest frame, the energetic light particle defines the light-cone direction $n^\mu = (1, 0, 0, 1)$. The quark momentum $p$ has large part $\bar{p}$ and small one $k$,

\begin{equation}
p = \bar{p} + k, \quad \bar{p} \equiv (\bar{n} \cdot p) \frac{n}{2} + p_\perp,
\end{equation}

where $\bar{n}^\mu = (1, 0, 0, -1)$ and $p^\mu = (p^+, p^-, p_\perp) = (n \cdot p, \bar{n} \cdot p, p_\perp)$. Extracting the large momentum $\bar{p}$ defines a new quark field $q_{n,p}$

\begin{equation}
q(x) = \sum_{\bar{p}} e^{-i\bar{p} \cdot x} q_{\bar{p}}(x),
\end{equation}

from which the collinear quark fields are introduced:

\begin{equation}
\xi_{n,p} = \frac{\bar{p} \cdot \bar{p}}{4} q_{\bar{p}}.
\end{equation}

The effective Lagrangian can be constructed at leading ($\mathcal{L}_0$) and subleading ($\mathcal{L}_1$) order of $\lambda$ as [6].
\[ L_0 = \bar{\xi}_n \left\{ n \cdot (iD - gA_n) + (\slashed{P}_\perp - gA_n^\perp)W \frac{1}{\slashed{P}} \right\} \frac{\slashed{n}}{2} \xi_n, \]
\[ L_1 = \bar{\xi}_n \left\{ i\slashed{D}_\perp W \frac{1}{\slashed{P}} W^\dagger (\slashed{P}_\perp - gA_n^\perp) + (\slashed{P}_\perp - gA_n^\perp)W \frac{1}{\slashed{P}} W^\dagger i\slashed{D}_\perp \right\} \frac{\slashed{n}}{2} \xi_n, \tag{5} \]

where
\[ W = \left[ \exp \left( \frac{1}{\slashed{P}} g\bar{n} \cdot A_n \right) \right], W^\dagger = \left[ \exp \left( g\bar{n} \cdot A_n^* \frac{1}{\slashed{P}} \right) \right], \tag{6} \]

and
\[ \gamma^\mu_\perp = \gamma^\mu - \frac{\slashed{n}}{2} \gamma^\mu_n - \frac{\slashed{n}}{2} n^\mu. \tag{7} \]

Here the gluon field is separated into the collinear and soft parts as \( A^\mu = A^\mu_c + A^\mu_s \), and the collinear gluon field \( A_n \) is given by
\[ A^\mu_c(x) = \sum_q e^{-i\bar{q} \cdot x} A^\mu_{n,q}(x). \tag{8} \]

The operator \( \slashed{P} \) acts on the collinear fields as
\[ f(\slashed{P})(\phi_{q_1}^\dagger \cdots \phi_{q_m}^\dagger \phi_{p_1} \cdots \phi_{p_n}) = f(\bar{n} \cdot p_1 + \cdots + \bar{n} \cdot p_n - \bar{n} \cdot q_1 - \cdots - \bar{n} \cdot q_m)(\phi_{q_1}^\dagger \cdots \phi_{q_m}^\dagger \phi_{p_1} \cdots \phi_{p_n}). \tag{9} \]

And the heavy quark fields \( h_v \) are accompanied in a HQET form,
\[ L_{\text{HQET}} = \bar{h}_v i\gamma^\mu D_h v. \tag{10} \]

The vector current \( V^\mu = \bar{q} \gamma^\mu b \) is then matched to the effective currents of the SCET up to \( O(\lambda) \) as [6]
\[ V^\mu \to \sum_i C_i(\mu) J^\mu_i + \sum_j B_j(\mu) O^\mu_j + \sum_k A_k(\mu) T^\mu_k, \tag{11} \]

where
\[ J^\mu_1 = \bar{\xi}_n W^\gamma^\mu h_v , \quad J^\mu_2 = \bar{\xi}_n W v^\mu h_v , \quad J^\mu_3 = \bar{\xi}_n W n^\mu h_v , \tag{12} \]
\[ O^\mu_1 = \bar{\xi}_n \frac{\slashed{n}}{2} (\slashed{P}_\perp - gA_\perp) W \frac{1}{\slashed{P}} \gamma^\mu h_v , \]
\[ O^\mu_2 = \bar{\xi}_n \frac{\slashed{n}}{2} (\slashed{P}_\perp - gA_\perp) W \frac{1}{\slashed{P}} v^\mu h_v , \]
\[ O^\mu_3 = \bar{\xi}_n \frac{\slashed{n}}{2} (\slashed{P}_\perp - gA_\perp) W \frac{1}{\slashed{P}} n^\mu h_v , \]
\[ O^\mu_4 = \bar{\xi}_n (\slashed{P}_\perp - gA_\perp) W \frac{1}{\slashed{P}} h_v. \tag{13} \]
perturbative functions

\[ T^\mu_k = i \int d^4 y T \{ J^\mu_k(0), L_1(y) \} \quad (k = 1, 2, 3). \]  

(14)

The Wilson coefficients \( C_i(\mu), B_i(\mu), \) and \( A_k(\mu) \) are summarized in the Appendix A.

Now the weak transition matrix elements of \( B \to \pi(\rho) \) are simplified, introducing non-perturbative functions \( \xi_{P,\perp,\bot} \) at leading order and \( a(b)_{V,1,V,2} \) at subleading order of \( \lambda \) [6]:

\[
\langle \pi(p)|V^\mu|B(p_B)\rangle = 2E_n^\mu \left\{ (C_1 + C_3)\xi_P + \frac{1}{2E}[-B_1 + B_3)v_P + (A_1 + A_3)b_P] \right\} \\
+ 2E\xi^\mu \left\{ C_2\xi_P + \frac{1}{2E}[(2B_1 + B_2)a_P + A_2b_P] \right\} , \tag{15a}
\]

\[
\langle \rho(p,\epsilon^*)|V^\mu|B(p_B)\rangle = 2Ee^{\mu\alpha\beta}\epsilon^\ast_{\alpha}v_{\beta}n_{\beta} \left\{ C_1\xi_\perp + \frac{1}{2E}(B_4a_{V1} + A_1b_{V1}) \right\} , \tag{15b}
\]

\[
\langle \rho(p,\epsilon^*)|A^\mu|B(p_B)\rangle = 2Ec^{\mu} \left\{ C_1\xi_\perp + \frac{1}{2E}(B_4a_{V1} + A_1b_{V1}) \right\} \\
+ 2E(\epsilon^* \cdot v)^\mu \left\{ C_2\xi_\parallel + \frac{1}{2E}[(2B_1 + B_2)a_{V2} + A_2b_{V2}] \right\} \\
- 2E(\epsilon^* \cdot v)n^\mu \left\{ C_1\xi_\perp - (C_1 + C_3)\xi_\parallel + \frac{1}{2E}[(B_1 - B_3)a_{V2} \\
+ B_4a_{V1} + A_1b_{V1} - (A_1 + A_3)b_{V2}] \right\} . \tag{15c}
\]

III. LIGHT CONE SUM RULES

Light cone sum rule begins with the two-point (2P) correlation function

\[
F_{B \to \pi}^\mu = i \int d^4 x e^{iqx} \langle \pi(p)|T\bar{u}(x)\gamma^\mu b(x)j^+_B(0)|0\rangle , \tag{16}
\]

where \( j^+_B(x) = \bar{b}(x)i\gamma_5 d(0) \). In the phenomenological representation, it simply becomes

\[
F_{B \to \pi}^\mu = \frac{\langle \pi(p)|\bar{u}\gamma^\mu b|B\rangle\langle B|j^+_B(0)|0\rangle}{m_B^2 - (p + q)^2} + \sum_{H \neq B} \frac{\langle \pi(p)|\bar{u}\gamma^\mu b|H\rangle\langle H|j^+_B(0)|0\rangle}{m_H^2 - (p + q)^2} . \tag{17}
\]

The second term of (17) corresponds to the resonance part. In the HQET and SCET, the matrix elements of the first term of (17) are parametrized as in (15a) and

\[
\langle 0|\bar{q}\Gamma h^{(b)}_\nu|B_\nu\rangle = \frac{F}{2}Tr[\Gamma M_\nu] , \tag{18}
\]

where

\[
M_\nu = \frac{1 + y}{2}(-\gamma_5) . \tag{19}
\]
Here $F$ is the $B$ meson decay constant in the effective theory.

To establish the light cone sum rule, one needs to evaluate (16) in terms of the pion distribution amplitudes (up to twist 4),

$$
\langle \pi(p)|\bar{u}(x)\gamma^\mu\gamma_5d(0)|0\rangle = -ip^\mu f_\pi \int_0^1 du \, e^{iup\cdot x}[\phi_\pi(u) + x^2g_1(u)]
$$

$$
+ f_\pi \left(x^\mu - \frac{x^2p^\mu}{x\cdot p}\right) \int_0^1 du \, e^{iup\cdot x}g_2(u)
$$

(20a)

$$
\langle \pi(p)|\bar{u}(x)i\gamma_5d(0)|0\rangle = f_\pi \mu \int_0^1 du \, e^{iup\cdot x}\phi_\pi(u)
$$

(20b)

$$
\langle \pi(p)|\bar{u}(x)\sigma^\mu\nu\gamma_5d(0)|0\rangle = \frac{if_\pi \mu}{6}(p^\mu x^\nu - p^\nu x^\mu) \int_0^1 du \, e^{iup\cdot x}\phi_\pi(u)
$$

(20c)

where $\mu_\pi \equiv m_\pi^2/(m_u + m_d)$.

After a proper Borel transformation ($T$ is the associated Borel parameter),

$$
iF e^{-2A/T} \left\{ 2En^\mu \left\{ (C_1 + C_3)\xi_P + \frac{1}{2E}[(B_1 - B_3)a_P + (A_1 + A_3)b_P] \right\}
+ 2Ev^\mu \left\{ C_2\xi_P + \frac{1}{2E}[(2B_1 + B_2)a_P + A_2b_P] \right\} \right\}
$$

$$
= \frac{1}{\pi} \int_0^{s_0} \text{Im} F_{B\to\pi}^\mu(E,s) e^{-s/T} ds
$$

(21)

The imaginary part of $F_{B\to\pi}^\mu$ is

$$
\frac{1}{\pi} F_{B\to\pi}^\mu(E,s) = f_\pi \Theta(\bar{u}_0)v^\mu \left[ \frac{\mu_\pi}{2E} \phi_\pi(\bar{u}_0) - \frac{g_2^\prime(\bar{u}_0)}{2E^2} + \frac{1}{2E} \frac{\mu_\pi}{6} \phi_\pi^\prime(\bar{u}_0) \right]
$$

$$
+ f_\pi \Theta(\bar{u}_0)p^\mu \left[ \frac{\phi_\pi(\bar{u}_0)}{2E} - \frac{1}{2E} \frac{\mu_\pi}{6} \phi_\pi^\prime(\bar{u}_0) + \frac{g_2^\prime(\bar{u}_0) - g_1^\prime(\bar{u}_0)}{2E^3} \right]
$$

(22)

where $\bar{u}_0 = 1 - s/2E$.

Comparing both sides of (21) gives the final results

$$
2E \left\{ C_2\xi_P + \frac{1}{2E}[(2B_1 + B_2)a_P + A_2b_P] \right\}
$$

$$
= -if_\pi N \left\{ \int_0^\theta du \left[ \mu_\pi \phi_\pi(\bar{u}) + \frac{2}{T}g_2(\bar{u}) - \frac{2E \mu_\pi}{T} \phi_\pi(\bar{u}) \right] e^{-2uE/T}
+ \left[ \frac{1}{E}g_2(\bar{\theta}) - \frac{\mu_\pi}{6} \phi_\pi(\bar{\theta}) \right] e^{-2\bar{\theta}E/T} \right\}
$$

(23a)

$$
2E \left\{ (C_1 + C_3)\xi_P + \frac{1}{2E}[(B_1 - B_3)a_P + (A_1 + A_3)b_P] \right\}
$$

$$
= -if_\pi N \left\{ \int_0^\theta du \left[ \phi_\pi(\bar{u}) + \frac{2 \mu_\pi}{T} \phi_\pi(\bar{u}) - \frac{2}{2ET}g_2(\bar{u}) - \frac{4}{T^2}g_1(\bar{u}) \right] e^{-2uE/T}
+ \left[ \frac{1}{E}g_2(\bar{\theta}) + \frac{1}{E^2}g_1(\bar{\theta}) \right] e^{-2\bar{\theta}E/T} \right\}
$$

(23b)
where $\bar{u} = 1 - u$, $\theta = \min(1, s_0/2E)$, $\bar{\theta} = 1 - \theta$, and $N = m_b e^{2\Lambda/T}/F$.

For the vector meson production, we calculate the $2\mathrm{P}$ function
\begin{equation}
F_{\rho \to \rho}^\mu = i \int d^4x \, e^{-iPB \cdot x} \langle \rho(p, \epsilon^*) | T \bar{u}(0)\gamma^\mu(1 - \gamma_5)b(0)j_B^+(x)|0 \rangle
= \frac{\langle \rho(p, \epsilon^*)|\bar{u}\gamma^\mu(1 - \gamma_5)b|B \rangle \langle B| j_B^+|0 \rangle}{m_B^2 - (p + q)^2} + \sum_{H \neq B} \frac{\langle \rho(p, \epsilon^*)|\bar{u}\gamma^\mu(1 - \gamma_5)b|H \rangle \langle H| j_B^+|0 \rangle}{m_H^2 - (p + q)^2}, \tag{24}
\end{equation}
where the second line is the phenomenological description.

Using (15), the sum rule has the form of
\begin{equation}
\frac{iFe^{-2\Lambda/T}}{m_b} \left[ 2E\epsilon^\mu \left\{ C_1\xi_\perp + \frac{1}{2E}(B_4a_{V1} + A_1b_{V1}) \right\} \\
+ 2E(\epsilon^* \cdot v)\epsilon^\mu \left\{ C_2\xi_\parallel + \frac{1}{2E}[(2B_1 + B_2)a_{V2} + A_2b_{V2}] \right\} \\
- 2E(\epsilon^* \cdot v)n^\mu \left\{ C_1\xi_\perp - (C_1 + C_3)\xi_\parallel + \frac{1}{2E}((B_1 - B_3)a_{V2} + B_4a_{V1} + A_1b_{V1} - (A_1 + A_3)b_{V2}) \right\} \right] = \frac{1}{\pi} \int_0^{s_0} \text{Im} F_{\rho \to \rho}^\mu(E, s)e^{-s/T} ds, \tag{25}
\end{equation}
Introducing the distribution amplitudes of $\rho(p, \epsilon^*)$ (up to twist 3)
\begin{align}
\langle \rho(p, \epsilon^*)|\bar{u}(0)\sigma^{\mu\nu}d(x)|0 \rangle &= -if_\rho^\pm(\epsilon^\mu p^\nu - \epsilon^\nu p^\mu) \int_0^1 du \, e^{i\epsilon^\nu \cdot x}\phi_\perp(u), \tag{26a} \\
\langle \rho(p, \epsilon^*)|\bar{u}(0)\gamma^\mu d(x)|0 \rangle &= f_\rho m_\rho p^\mu \frac{\epsilon^\nu \cdot x}{p \cdot x} \int_0^1 du \, e^{i\epsilon^\nu \cdot x}\phi_\parallel(u) \\
&\quad+ f_\rho m_\rho \left( \frac{\epsilon^\nu - p^\nu \epsilon^\nu \cdot x}{p \cdot x} \right) \int_0^1 du \, e^{i\epsilon^\nu \cdot x} g_\perp^{(v)}(u), \tag{26b} \\
\langle \rho(p, \epsilon^*)|\bar{u}(0)\gamma^\mu\gamma_5d(x)|0 \rangle &= \frac{1}{4} f_\rho m_\rho \epsilon^{\mu\nu\alpha\beta} \epsilon^*_{\nu\rho} x_{\beta} \int_0^1 du \, e^{i\epsilon^\nu \cdot x} g_\perp^{(a)}(u), \tag{26c}
\end{align}
the imaginary part of $2\mathrm{P}$ function becomes
\begin{equation}
\frac{1}{\pi} \text{Im} F_{\rho \to \rho}^\mu(E, s) = \frac{\Theta(\bar{u}_0)}{2iE} \left[ -\frac{1}{4} f_\rho m_\rho \epsilon^{\mu\nu\alpha\beta} \epsilon^*_{\rho\alpha} v_{\beta} \frac{i}{E} g_\perp^{(a)y}(u_0) + if_\rho^\pm \epsilon^{\mu\nu\alpha\beta} \epsilon^*_{\nu\rho} a_{\beta}\phi_\perp(u_0) \\
- f_\rho m_\rho p^\mu \epsilon^* \cdot v \phi_\parallel(u_0) + f_\rho m_\rho \left( \epsilon^\mu - p^\mu \epsilon^\nu \cdot v \right) \frac{g_\perp^{(v)}(u_0)}{E} \\
+ v_{\nu} f_\rho^\pm (\epsilon^\nu p^\nu - \epsilon^* p^\mu) \phi_\perp(u_0) \right], \tag{27}
\end{equation}
where $u_0 \equiv s/2E$. Plugging (27) into (25), we arrive at
\begin{equation}
2E \left\{ C_1\xi_\perp + \frac{1}{2E}(B_4a_{V1} + A_1b_{V1}) \right\} = -\frac{N}{2E} \int_0^\infty ds \, e^{-s/T} \left[ \frac{1}{4} f_\rho m_\rho g_\perp^{(a)}(u_0) - Ef_\rho^\perp \phi_\perp(u_0) \right], \tag{28a}
\end{equation}
\[ 2E \left\{ C_1 \xi_\perp + \frac{1}{2E} (B_4 a_{V1} + A_1 b_{V1}) \right\} = \frac{N}{2E} \int_0^\kappa ds \ e^{-s/T} \left[ f_\rho m_\rho g_{\perp}^{(v)}(u_0) + Ef_\rho^\perp \phi_{\perp}(u_0) \right], \quad (28b) \]

\[ 2E \left\{ C_2 \xi_\parallel + \frac{1}{2E} \left[ (2B_1 + B_2) a_{V2} + A_2 b_{V2} \right] \right\} = 0, \quad (28c) \]

\[ 2E \left\{ C_1 \xi_\perp - (C_1 + C_3) \xi_\parallel + \frac{1}{2E} \left[ (B_1 - B_3) a_{V2} + B_4 a_{V1} + A_1 b_{V1} - (A_1 + A_3) b_{V2} \right] \right\} \]

\[ \quad = -\frac{N}{2E} \int_0^\kappa ds \ e^{-s/T} \left[ f_\rho m_\rho \left\{ \phi_{\parallel}(u_0) - g_{\perp}^{(v)}(u_0) \right\} - Ef_\rho^\perp \phi_{\perp}(u_0) \right]. \quad (28d) \]

IV. ENERGY SCALINGS OF THE FORM FACTORS

At leading order of \( \lambda \), SCET introduces three independent functions \( \xi_P, \xi_\perp \), and \( \xi_\parallel \) to describe \( B \to \pi(\rho) \) transition. At the NLO, additional six subleading functions \( a_i \) and \( b_i \) \((i = P, V1, V2)\) are needed. We can now evaluate these functions from the LCSR established in (23) and (28).

Let us see first Eq. (23). For sufficiently large \( E \) and \( m_b \) (at leading order of \( \alpha_s \))

\[ 2a_P = -if_\pi N \left\{ \mu_\pi (\tilde{B}_2 + \tilde{B}_4 - 6\tilde{C}_2 - 15\tilde{C}_4) \frac{T}{2E} + \left[ \frac{20\delta^2}{3T} - \mu_\pi (6\tilde{B}_2 + 20\tilde{B}_4) \right] \frac{T^2}{4E^2} \right\}, \quad (29a) \]

\[ 2E \left\{ \xi_P + \frac{-a_P + b_P}{2E} \right\} \]

\[ = -if_\pi N \frac{1}{4E} \left\{ 6(1 + 6a_2 + 15a_4)T^2 + 2\mu_\pi (1 + 6\tilde{C}_2 + 15\tilde{C}_4)T - 4e\delta^2 \right\} - \frac{40\delta^2}{3} \frac{T}{2E} \right\}. \quad (29b) \]

From (29b), the energy scaling of leading and subleading functions are

\[ \xi_P + \frac{-a_P + b_P}{E} \sim \frac{1}{E^2} + \frac{1}{E^3}. \quad (30) \]

Such a behavior is quite reasonable and very compatible with the previously known \( \xi_P \sim 1/E^2 \). Referring to (30), Eq. (29a) implies

\[ \tilde{B}_2 + \tilde{B}_4 - 6\tilde{C}_2 - 15\tilde{C}_4 = 0, \quad (31) \]

which corresponds to (see (B1b) and (B1c))

\[ \phi_\rho(1) + \frac{1}{6} \phi'_\rho(1) = 0, \quad (32) \]

of [7]. Numerically,

\[ \tilde{B}_2 + \tilde{B}_4 - 6\tilde{C}_2 - 15\tilde{C}_4 = 0.025, \quad (33) \]

where the values of (B3) are used.

From the sum rules for \( B \to \rho \) decay, we have, by (28a) and (28b),
\[ \int_0^\theta ds e^{-s/T} \left[ g_\perp^{(v)}(u_0) + \frac{1}{4} g_\perp^{(a)^\mu}(u_0) \right] = 0 . \]  

(34)

It should be understood that the relation holds near the end-point region. At \( u = 1 \), the relation is exact from (B2b) and (B2c),

\[ g_\perp^{(v)}(u) + \frac{1}{4} g_\perp^{(a)^\mu}(u) = \int_u^1 dv \frac{\phi\parallel(v)}{v} . \]

(35)

In the vicinity of \( u \sim 1 \), we use

\[ g_\perp^{(v)}(u) \approx \frac{3}{2}(1 + a_2\parallel), \]

\[ g_\perp^{(a)}(u) \approx 6(1 + a_2\parallel)\bar{u} , \]

(36)

to calculate (28a)

\[ \frac{(2E)^2}{N} \left\{ \xi\parallel + \frac{b v_1}{2E} \right\} = \left[ f_\rho m_\rho \frac{3}{2}(1 + a_2\parallel)T + f_\rho \frac{1}{2}(3 + 18a_2\parallel)T^2 \right] - \left[ f_\rho m_\rho (3 + 18a_2\parallel)T^2 + f_\rho \frac{1}{2}(6 + 216a_2\parallel)T^3 \right] \frac{1}{2E} . \]

(37)

Thus we have

\[ \xi\parallel + \frac{b v_1}{E} \sim \frac{1}{E^2} + \frac{1}{E^3} . \]

(38)

Eq. (28c) tells that \( a_{V2} = 0 \). The absence of \( v^\mu \) component in the sum rules is due to the twist expansion in terms of the \( \rho \) meson’s distribution amplitudes (26). Vanishing \( a_{V2} \) does not mean that it is suppressed by \( \lambda^\mu \). Instead, at higher twist, there are terms proportional to \( x^\mu \) (which turns out to be proportional to \( v^\mu \) because the heavy quark propagator is \( \langle T h_\mu(0)\bar{h}_\nu(x) \rangle = \int_0^\infty dt \delta^4(-x - vt)(1 + \gamma)/2 \) in (26) with new distribution functions [13].

Another nontrivial relation comes out from (28b)—(28d):

\[ \frac{(2E)^2}{N} \left\{ \xi\parallel + \frac{-a_{V2} + b v_2}{2E} \right\} = f_\rho m_\rho \int_0^\theta ds e^{-s/T} \phi\parallel(u_0) \]

\[ = f_\rho m_\rho \left[ \left( 6 + 36a_2\parallel \right) \frac{T^2}{2E} - \left( 6 + 216a_2\parallel \right) \frac{T^3}{4E^2} + \cdots \right] . \]

(39)

Considering (28c), the above equation means that

\[ \xi\parallel + \frac{b v_2}{E} \sim \frac{1}{E^3} + \frac{1}{E^4} . \]

(40)

The longitudinal component is, therefore, 1/\( E \)-suppressed compared to the transverse one of (38).

Some remarks are in order. First, the LCSR calculation is done at leading order of \( \alpha_s \). In [14,15], leading radiative corrections to the twist-2,3 contributions of \( B \to \pi \) are calculated.
Corrections to the twist-2 are rather large ($\approx 30\%$), but they are almost canceled by large radiative corrections to the decay constant $f_B$, leaving the form factor $f_+$ free from large corrections [14]. Authors of [15] found that the sizes of the radiative corrections to the twist-3 distribution amplitudes $\phi_p$ and $\phi_\sigma$ are of the same size as that of the twist-2 while the sum of both is quite small. Hence it would be very interesting to check whether that kind of cancellation might occur at higher twist and in $B \to \rho$ decays. One more point to be noticed is that the radiative corrections preserve the factorized scheme of the LCSR [15],

$$\langle 2P \text{ functions} \rangle \sim \sum_{\text{twists}} T_H \otimes \phi , \tag{41}$$

where $T_H$ is the process-dependent hard kernel and $\phi$ is the nonperturbative distribution amplitudes, up to twist-3. Such a factorization in (41) is desirable to establish sum rules between the LCSR and the SCET. The role of $T_H$ is very similar to that of the Wilson coefficients in SCET which separate the long- and short-distance physics. Thus the study of short-distance structure will check the validity of the sum rule, or provide more information on both sides.

Second, we have neglected the hard contributions. Hard contributions arise when an energetic light quark from the weak vertex exchanges the hard gluon with the soft degrees of freedom. But it was argued that the effects are suppressed by $\alpha_s(\sqrt{m_b \Lambda_{\overline{\text{QCD}}}})$ [16]. Also, explicit calculations in [14,15] show that the soft contributions which are responsible for the case where the recoiling quark carries almost all the momentum are dominant.  

V. SUMMARY

In this paper we established the light-cone sum rules for the $B \to \pi(\rho)$ form factors in the context of SCET. Both LCSR and SCET are useful tools to describe the heavy-to-light transitions at large recoil, and favor the asymmetric configuration of quark and antiquark to form the energetic meson through the ”soft contributions”. From the sum rules, leading and subleading nonperturbative functions at $O(\lambda)$ of SCET are found to be $\sim 1/E^2 + 1/E^3$, which is compatible with other literatures. The fact that radiative corrections to the LCSR preserve the separation between long- and short-distance physics is encouraging to study the hybrid scheme of SCET and LCSR. More refinements in every direction such as radiative corrections, hard spectator effects, etc. will provide better understanding of $B \to \pi(\rho)$ as well as $|V_{ub}|$.

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$^1$Recent work of [17] includes the hard contribution to extend the SCET.
APPENDIX A: WILSON COEFFICIENTS

In this Appendix, Wilson coefficients for the SCET weak currents are given [6]:
\[
C_1(\mu) = 1 - \frac{\alpha_s C_F}{4\pi} \left[ 2 \ln^2 \left( \frac{m_b}{\mu} \right) - 5 \ln \frac{m_b}{\mu} + \frac{3x - 2}{1 - x} \ln x + 2 Li_2(1 - x) + \frac{\pi^2}{12} + 6 \right]
\]
\[
C_2(\mu) = \frac{\alpha_s C_F}{4\pi} \left[ \frac{2}{1 - x} + \frac{2x}{(1 - x)^2} \ln x \right]
\]
\[
C_3(\mu) = \frac{\alpha_s C_F}{4\pi} \left[ - \frac{x}{1 - x} + \frac{x(1 - 2x)}{(1 - x)^2} \ln x \right], \tag{A1}
\]
where \( x = 2E/m_b \), \( C_F \) is the color factor, and \( Li_2(x) = -\int_0^x \frac{dt}{t} \ln(1-t) \) is the dilogarithmic function. Other coefficients are related to \( C_i \) as
\[
B_i(\mu) = C_i(\mu) \ (i = 1, 2, 3), \quad B_4(\mu) = 2C_3(\mu),
\]
\[
A_i(\mu) = C_i(\mu). \tag{A2}
\]

APPENDIX B: DISTRIBUTION AMPLITUDES

We summarize the distribution amplitudes for \( \pi(p) \) and \( \rho(p, \epsilon^*) \):
\[
\phi_\pi(u, \mu) = 6u\bar{u}\left\{ 1 + a_2(\mu) \frac{3}{2} [5(u - \bar{u})^2 - 1] + a_4(\mu) \frac{15}{8} [21(u - \bar{u})^4 - 14(u - \bar{u})^2 + 1] \right\}, \tag{B1a}
\]
\[
\phi_\rho(u, \mu) = 1 + \tilde{B}_2(\mu) \frac{1}{2} [3(u - \bar{u})^2 - 1] + \tilde{B}_4(\mu) \frac{1}{8} [35(u - \bar{u})^4 - 30(u - \bar{u})^2 + 3], \tag{B1b}
\]
\[
\phi_\sigma(u, \mu) = 6u\bar{u}\left\{ 1 + \tilde{C}_2(\mu) \frac{3}{2} [5(u - \bar{u})^2 - 1] + \tilde{C}_4(\mu) \frac{15}{8} [21(u - \bar{u})^4 - 14(u - \bar{u})^2 + 1] \right\}, \tag{B1c}
\]
\[
g_1(u, \mu) = \frac{5}{2} \delta^2(\mu)u^2\bar{u}^2 + \frac{1}{2} \epsilon(\mu)\delta^2(\mu) \left\{ u\bar{u}(2 + 13u\bar{u}) + 10u^3 \ln u \left[ 2 - 3u + \frac{6}{5}u^2 \right] \right\}
+ 10u^3 \ln \bar{u} \left[ 2 - 3\bar{u} + \frac{6}{5}\bar{u}^2 \right], \tag{B1d}
\]
\[
g_2(u, \mu) = \frac{10}{3} \delta^2(\mu)u\bar{u}(u - \bar{u}), \tag{B1e}
\]
for the pion, and
\[
\phi_{\perp(\parallel)}(u, \perp) = 6u\bar{u}\left\{ 1 + a_2^{\perp(\parallel)} \frac{3}{2} [5(u - \bar{u})^2 - 1] \right\}, \tag{B2a}
\]
\[
g_\perp^{(v)}(u, \mu) = \frac{1}{2} \left[ \int_u^1 dv \frac{\phi_{\perp (v, \mu)}}{1 - v} + \int_u^1 \frac{\phi_{\parallel (v, \mu)}}{v} \right], \tag{B2b}
\]
\[
g_\perp^{(o)}(u, \mu) = 2 \left[ (1 - u) \int_u^1 dv \frac{\phi_{\perp (v, \mu)}}{1 - v} + u \int_u^1 \frac{\phi_{\parallel (v, \mu)}}{v} \right], \tag{B2c}
\]
for \( \rho \) meson. Here \( \mu \) is the renormalization scale. The values of the coefficients at \( \mu = 1 \) GeV are
\[ a_2 = 0.44 \, , \quad a_4 = 0.25 \, , \quad \tilde{B}_2 = 0.48 \, , \quad \tilde{B}_4 = 1.15 \, , \]
\[ \tilde{C}_2 = 0.10 \, , \quad \tilde{C}_4 = 0.067 \, , \quad \delta^2 = 0.2 \text{ GeV}^2 \, , \quad \epsilon = 0.5 \, , \]
\[ a^\perp_2 = 0.2 \, , \quad a^\parallel_2 = 0.18 \, . \]  

(B3)
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