Floquet Hamiltonian engineering of an isolated many-body spin system

Sebastian Geier1, Nithiaware Thaicharon1,2,†, Clément Hainaut1,†, Titus Franz3, Andre Salzinger1, Annika Tebben1, David Grimshandl1, Gerhard Zürn1, Matthias Weidemüller1,*

Controlling interactions is the key element for the quantum engineering of many-body systems. Using time-periodic driving, a naturally given many-body Hamiltonian of a closed quantum system can be transformed into an effective target Hamiltonian that exhibits vastly different dynamics. We demonstrate such Floquet engineering with a system of spins represented by Rydberg states in an ultracold atomic gas. By applying a sequence of spin manipulations, we change the symmetry properties of the effective Heisenberg XYZ Hamiltonian. As a consequence, the relaxation behavior of the total spin is drastically modified. The observed dynamics can be qualitatively captured by a semiclassical simulation. Engineering a wide range of Hamiltonians opens vast opportunities for implementing quantum simulation of nonequilibrium dynamics in a single experimental setting.

According to Floquet’s theorem (1), the stroboscopic dynamics of periodically driven quantum systems is effectively described by a time-independent Hamiltonian that can be engineered by controlling the properties of the drive (2, 3). This type of Floquet Hamiltonian engineering (FHE) leads to the observation of unconventional phases of matter (4, 5), dynamical phase transitions (6), lattice gauge theories (7) and Floquet-topological matter (8, 9). A paradigmatic example of FHE involves a sequence of spin echo sequences (10, 11) applied to a system of spins embedded in an inhomogeneous environment, as encountered, for example, in nuclear magnetic resonance (NMR). The pulse sequence inverts the spins multiple times such that they finally return to their initial states and appear to be effectively decoupled from the environment. In a Hamiltonian description, this corresponds to the emergence of a vanishing Hamiltonian, that is, the effective time-independent Hamiltonian over the spin echo sequence becomes zero. Although this type of sequence is suitable to decouple the dynamics of single spins, more sophisticated sequences have been used to decouple two-body interactions (12) and to engineer Hamiltonians for studying many-body localization of mixed quantum states (13).

The application of multipulse protocols, as, for example, introduced by (14, 15), to closed quantum systems facilitates the quantum engineering of many-body spin systems. Owing to the genuine decoupling from the environment and the high degree of control down to the single-particle level, the system’s universality classes, symmetries, and type of interactions become tunable. In this study, we demonstrate the realization of such programmable many-body spin Hamiltonians and observe the modification of the out-of-equilibrium dynamics as a function of the target Hamiltonian’s parameters. We use an ultracold atomic Rydberg gas which can be well decoupled from the environment and has already been established as an excellent platform for quantum simulation of closed systems (16, 17) enabling, for example, the preparation of pure many-body quantum states in random (18) and controllable spatial geometries (19, 20). The unitary dynamics of this system are captured by spin models (21), which serve as prototypical templates for studying quantum magnetism emerging from different classes of Hamiltonians (22).

The key idea of this work is to transform the naturally given spin Hamiltonian $H_{\text{nat}}$ into a target Floquet Hamiltonian $H_{\text{Floq}}$ using a sequence of periodically applied control pulses (Fig. 1A). To demonstrate programming of the resulting effective Hamiltonian, we engineer a tunable Heisenberg XYZ Hamiltonian from a naturally given XX Hamiltonian. We benchmark FHE by comparing the emerging dynamics of the driven system with the one expected from an effective time-independent Hamiltonian. As a demonstration of tunability, we modify the symmetries in an effective XYZ Heisenberg model and analyze the resulting change of the out-of-equilibrium relaxation dynamics.

More specifically, we consider a Heisenberg XX Hamiltonian with an external driving field

$$H(t) = H_{\text{XX}} + H_{\text{drive}}(t)$$

where $H_{\text{XX}} = \sum_{i<j} J_{ij} \hat{S}_i^x \hat{S}_j^x + \sum_{i<j} J_{ij} \hat{S}_i^y \hat{S}_j^y$ and $H_{\text{drive}}(t) = \sum_i \Omega(t) \cos(t) S_i^x \hat{S}_j^y + \sin(t) S_i^y \hat{S}_j^x \hat{S}_j^x$, with $\Omega(t)$ being the time-dependent Rabi frequency of the drive, $\phi(t)$ is the phase, and $J_{ij}$ is the interaction coefficient between spins $i$ and $j$.

As schematically depicted in Fig. 1B, we use the drive $H_{\text{drive}}(t)$, which consists of a periodic sequence of four global $\pi/2$ pulses realizing different spin operators ($\hat{S}_x^z, -\hat{S}_y^z, \hat{S}_y^z, -\hat{S}_x^z$); the pulses are separated by delay times $t_1 = t(1 - 2v + 2w), t_2 = t(1 + 2v - 2w)$, and $t_3 = t(1 - 2u + 2v)$, where $u$, $v$, and $w$ are dimensionless parameters. Under the periodic driving $H_{\text{drive}}(t)$ in the ultracold gas of $^{87}$Rb atoms, the two states are well decoupled from the environment.

In our experimental implementation, the spin is represented by the two Rydberg states $|4S_{1/2}, m_J = 1/2\rangle = |0\rangle$ and $|4S_{1/2}, m_J = 3/2\rangle = |1\rangle$ in an ultracold gas of $^{87}$Rb atoms. The two states couple through a dipole-dipole interaction, yielding coefficients $J_{ij} = 2 C_0(\theta_{ij}) / r^3_{ij}$ in Eq. 1, where $C_0(\theta_{ij})$ is the angle-dependent dipole coupling parameter and $r_{ij}$ represents the spatial separation between atoms $i$ and $j$.

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In a first series of experiments, we choose the pulse sequence such that \( \mathbf{u} = \mathbf{v} = \mathbf{w} \), which is formally equivalent to the so-called WAHUHA sequence (12) widely used in NMR to realize dynamical decoupling and to suppress spin-spin interactions. For the isolated quantum system considered here, this particular sequence implies \( \delta_x = \delta_y = \delta_z \), resulting in the engineering of a symmetric Floquet Hamiltonian \( \mathcal{H}_{\text{tot}} = \mathcal{H}_{XXX} \) where the total spin constitutes a conserved quantity (23).

The experimental protocol, as shown in Fig. 1B, starts with the excitation of the \( | \downarrow \rangle^N \) Rydberg state, followed by a \( \pi/2 \) pulse, thus initializing the spins in the product state \( | \downarrow \rangle^N \) to \( | \uparrow \rangle^N \) (total magnetization pointing along the \( x \) direction). The spin system then evolves under the Hamiltonian given by Eq. 1. After time \( t_c \), the total magnetization is measured.

Without periodic driving, the dynamics is governed by \( \mathcal{H}_{XXX} \), resulting in a fast relaxation toward a demagnetized state within \( \approx 10 \text{ ms} \) (red points in Fig. 1C). In stark contrast, we observe a pronounced slowing down of the relaxation dynamics when we apply the WAHUHA sequence with \( \mathcal{H}_{XXX} \) as the target Hamiltonian (blue points in Fig. 1C). To confirm that the stalling of the dynamics is independent of the initial state, we let the system first evolve under \( \mathcal{H}_{XXX} \) for \( 2 \mu s \) to create an entangled state. We then apply the WAHUHA sequence and find a complete freezing out of the magnetization dynamics (see Fig. 1D), as indeed expected for the symmetric \( \mathcal{H}_{XXX} \).

To understand why the evolution in Fig. 1C is not entirely frozen, we have to consider the disordered nature of the system. The distance between a pair of Rydberg atoms has a lower bound \( \eta_a \), determined by the so-called Rydberg...
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Fig. 4. Consecutive symmetry breaking for a dual-component initial state. (A) Representation of the state initialized with a \( \pi/4 \) pulse on the Bloch sphere. (B) Dynamics of the magnetization components for a XXZ Hamiltonian. (C) Dynamics of the magnetization component for a fully anisotropic XYZ Hamiltonian. In (B) and (C), parameters are \( t_c = 0.5 \) μs, \( t_{x/2} = 10.7 \) ns, and \( J_m/2\pi = 0.4 \) MHz, and the shaded areas correspond to the density uncertainty of dTWA simulations of \( H(t) \) (dashed lines) (23).

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Blockade induced by the laser excitation process (24), resulting in a maximum interaction strength of \( J_{\text{max}}/2\pi = 2 \) Gz/(2πτ_p^2) = 18 MHz. For the closest pairs, this turns out to be large compared to the inverse cycling time \( 1/\tau_c = 2 \) MHz, thus violating the condition required by Eq. 3 for the validity of an effective time-independent Hamiltonian description. The result is a slow remnant relaxation of the total magnetization. In Fig. 1D, however, the strongest interacting pairs of spins have demagnetized before the drive is applied and thus no longer contribute to the total magnetization. This qualitative interpretation is confirmed by a semiclassical simulation based on the discrete truncated Wigner approximation method (dTWA) (25) for the full time-dependent Hamiltonian given by Eq. 1, which quantitatively reproduces all essential features of the magnetization dynamics.
scales with $\delta_c - \delta_e = 0.9$, whereas the dynamics of the latter scales with $\delta_c - \delta_e = 0.45$ [for more details, see (23)].

FHE of isolated spin systems enables quantum simulation of many-body systems beyond equilibrium. The ability to synthesize a variety of many-body Hamiltonians allows the experimental exploration of important concepts such as spin transport (27), the generalization of fluctuation dissipation relations (28) to out-of-equilibrium systems, and the nature of thermalization mechanisms (29). For disordered systems, the FHE technique can be combined with the realization of time-reversal operations (13), with the aim of measuring out-of-time-order correlators; this will facilitate experimental investigation of the role of entanglement and information scrambling in out-of-equilibrium systems (30, 31) as well as of nonergodic many-body localized states (32) in various spin Hamiltonians. The approach can be directly combined with current developments, for example, in preparing Rydberg atom arrays (17); this synthesis of efficient engineering of the Hamiltonian with precise control over the spatial arrangement provides a viable scenario for the realization of fully programmable quantum spin simulators.

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ACKNOWLEDGMENTS
We acknowledge insightful discussions with M. Gärttner, S. Whitlock, P. Cappellaro, and K. X. Wei. Funding: This work has been supported by the DFG under Germany’s Excellence Strategy EXC 228/1 - 390900948 (the Heidelberg STRUCTURES Excellence Cluster) and the European Commission FET flagship project PASQuanS (grant no. 817482). It is funded by the DFG (German Research Foundation) – Project-ID 278311115 – SFB 1225 ISOQUANT, and the DFG Priority Program 1929 “GiRyd” (WE2661/12-2). Support by the Heidelberg Center for Quantum Dynamics is acknowledged. N.T. received funding from the European Union’s Horizon 2020 program under Marie Skłodowska-Curie grant agreement no. 798402. C.H. is supported by the Alexander von Humboldt Foundation. T.F. received a graduate scholarship from the Heidelberg University (LGFG).

The authors declare no competing interests. Data and materials availability: The experimental data presented in the paper (33), the numerical dTWA code (34), and the analysis code (35) are available at Zenodo.

SUPPLEMENTARY MATERIALS
accessed 15 October 2021

11 August 2020; accepted 15 October 2021