Wrapping corrections, reciprocity and BFKL beyond the $\mathfrak{sl}(2)$ subsector in $\mathcal{N} = 4$ SYM

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ABSTRACT: We consider $\mathcal{N} = 4$ SYM and a class of spin $N$, length-3, twist operators beyond the well studied $\mathfrak{sl}(2)$ subsector. They can be identified at one-loop with three gluon operators. At strong coupling, they are associated with spinning strings with two spins in $AdS_5$. We exploit the Y-system to compute the leading weak-coupling four loop wrapping correction to their anomalous dimension. The result is written in closed form as a function of the spin $N$. We combine the wrapping correction with the known four-loop asymptotic Bethe Ansatz contribution and analyze special limits in the spin $N$. In particular, at large $N$, we prove that a generalized Gribov-Lipatov reciprocity holds. At negative unphysical spin, we present a simple BFKL-like equation predicting the rightmost leading poles.
1. Introduction

The study of finite size corrections to states/operators in AdS/CFT correspondence can be addressed within the powerful and general approach of the mirror thermodynamic Bethe Ansatz (TBA) developed for the $AdS_5 \times S^5$ superstring in [1] and deeply tested in [2], mainly in $sl(2)$ closed subsector. The associated Y-system has been proposed in [3] based on symmetry arguments and educated guesses about the analyticity and asymptotic properties of the Y-functions. Further developments can be found in [4].

Very powerful explicit tests of wrapping corrections can be done in the $sl(2)$ sector of $\mathcal{N} = 4$ SYM. The relevant operators are represented by the insertion of $N$ covariant derivatives $D$ into the protected half-BPS state $\text{Tr} Z^L$ ($Z$ being one of the three complex scalars)

$$\mathcal{O}_{\mathcal{N},L}^N = \sum_{s_1, \ldots, s_L} c_{s_1, \ldots, s_L} \text{Tr} (D^{s_1} Z \cdots D^{s_L} Z), \quad \text{with} \quad N = s_1 + \cdots + s_L. \quad (1.1)$$
Their anomalous dimensions can be obtained from a non-compact, length-$L$ \text{sl}(2) spin chain with $N$ excitations underlying a factorized two-body scattering. The interaction range between scattering particles increases with orders of the coupling constant in perturbation theory. If it exceeds the length of the spin chain and wraps around it, the S-matrix picture fails, as no asymptotic region can be defined any longer. For twist $L$ operators this effect, delayed by superconformal invariance, starts at order $g^{2L+4}$.

The most advanced available calculations are at five-loop order for $L = 2$ \cite{5} and at six-loop order for $L = 3$ \cite{6}. The anomalous dimension of $O^A_{N,L}$ is presented in closed form as a function of $N$. This allows to study special limits which have physical significance and test the TBA framework. In particular, at large $N$, it is found that a generalized Gribov-Lipatov reciprocity (see \cite{7} and the recent review \cite{8}) holds predicting basically half of the expansion in terms of the other half. Also, it is possible to match the predictions of the BFKL equation \cite{9} governing the poles around unphysical negative values of $N$. Notice that both reciprocity and BFKL are not related to integrability and have a wider physical meaning. As such, we regard them as independent checks of integrability predictions.\footnote{See \cite{10} for a recent analysis of the interplay between reciprocity and BFKL poles.}

The tests in the \text{sl}(2) explore a relatively small part of the full \text{psu}(2,2|4) structure of $\mathcal{N} = 4$ SYM. For this reason, we believe that similar calculations in a larger sector would be very interesting. Of course, for general states, there is no reason to expect to be able to find similar closed formulae in any parameter and the proposal seems hopeless. A remarkable and rather peculiar exception are the twist operators studied at many loops in \cite{11}. We shall refer to these operators as 3-gluon operators $O^A_{N,3}$. The reason for this terminology is that at one-loop they have the same form as \text{sl}(2) operators, with the scalar $Z$ being replaced by a physical gauge field component. At strong coupling, these operators are part of a larger family studied in \cite{12} and dual to spinning string configurations with two spins in $AdS_5$. As discussed in \cite{11}, it is possible to derive the asymptotic (with no wrapping corrections) anomalous dimensions in closed forms as functions of $N$. Wrapping corrections are expected to appear at four loops. Up to this level, reciprocity holds for the asymptotic contribution. The BFKL poles have not been studied yet.

Thus, a quite reasonable plan is that of computing the four-loop wrapping corrections exploiting the structural features of the asymptotic contributions in order to determine a closed form for the wrapping as a function of the spin $N$. This is precisely what we shall describe in this paper. As a byproduct, we shall be able to test (positively) reciprocity as well as discuss the BFKL poles of the full four-loop result. Our analysis provides an extension of the well-known results for scalar twist-operators to the full \text{psu}(2,2|4) states. Notice indeed, that beyond one-loop the 3-gluon operators have a non-trivial mixing with other operators and do not belong to a closed subsector.

The plan of the paper is the following. In Sec. (2), we recall the details of the operators under consideration. In Sec. (3), we present their one-loop description and integrability properties. Sec. (4) is devoted to a brief summary of the relevant Y-system formulae. In Sec. (5), we present our results for the wrapping corrections. These are analyzed from the
point of view of reciprocity in Sec. (6), and in terms of a proposed BFKL resummation in Sec. (7).

2. Generalities on 3-gluon twist operators

The 3-gluon operators are single-trace maximal helicity quasi-partonic operators which in the light-cone gauge take the form

\[ O_{N,3}^A = \sum_{n_1+n_2+n_3=N} c_{n_1,n_2,n_3} \Tr [\partial_+^{n_1} A(0) \partial_+^{n_2} A(0) \partial_+^{n_3} A(0)], \]

(2.1)

where \( A \) is the holomorphic combination of the physical gauge degrees of freedom \( A_\perp^\mu \) and \( \partial_+ \) is the light-cone projected covariant derivative (in light-cone gauge the gauge links are absent). The coefficients \( \{c_{n_1,n_2,n_3}\} \) are such that \( O_{N,L}^A \) is an eigenvector of the dilatation operator. The total Lorentz spin is \( N \).

The one-loop anomalous dimensions of the above operators can be found from the spectrum of a non-compact \( XXX_{-3/2} \) spin chain with 3 sites. At higher orders we are forced to abandon the quasipartonic detailed description and work in terms of superconformal multiplets. The identification of the \( \text{psu}(2,2|4) \) primary of the multiplet where such operators appear as descendant can be done thanks to the work of [13]. We decompose the symmetric triple tensor product \( (V_F \otimes V_F \otimes V_F)_S \) where \( V_F \) is the singleton infinite dimensional irreducible representation of \( \text{psu}(2,2|4) \)

\[ (V_F \otimes V_F \otimes V_F)_S = \bigoplus_{n=0}^{\infty} c_n \left[ V_{2k,n} + V_{2k+1,n+3} \right], \]

(2.2)

where \( c_n \) are suitable multiplicities and \( V_{n,m} \) well defined modules. For even \( N \) and \( m = 2 \), the one-loop lowest anomalous dimension in \( V_{N,F} \) is associated with an unpaired state and has been proposed to be [13]

\[ \gamma_{N,2} = \frac{\lambda}{8\pi^2} \left[ 2 S_1 \left( \frac{N}{2} + 1 \right) + 2 S_1 \left( \frac{N}{2} + 2 \right) + 4 \right] = \frac{\lambda}{8\pi^2} \left[ 2 S_1 \left( \frac{N}{2} + 1 \right) + \frac{4}{N+4} + 4 \right], \]

(2.3)

where \( g^2 = \lambda/(8\pi^2) = g_{YM}^2 N_c/(8\pi^2) \) is the scaled 't Hooft coupling, fixed in the planar \( N_c \to \infty \) limit. This result is in agreement with the analysis of maximal helicity 3 gluon operators in QCD [14]. The second term in (2.3) fully reveals the violation of the maximum transcendentality principle, as discussed (and exploited!) in [11].

These operators can also be presented as a special case of another family built with scalar fields [12]. They are the gauge theory dual of a minimal energy spinning string configuration with two spins, \( S_1 \) and \( S_2 \), in \( AdS_5 \) and charge \( J \) in \( S^5 \). The field content of these operators can be schematically represented by

\[ \Tr \left[ D^{n+m} \bar{D}^m \mathcal{L}^3 \right]. \]

(2.4)

The charges of the string are related to \( m \) and \( n \) through the identification

\[ S_1 = n + m - \frac{1}{2}, \quad S_2 = m - \frac{1}{2}, \quad J = L. \]

(2.5)
2.1 Asymptotic anomalous dimension

The asymptotic anomalous dimension has been studied at four loops in [11]. The three loop results is

\[ \gamma_1 = 4 S_1 + \frac{2}{n+1} + 4, \]  
\[ \gamma_2 = -2 S_3 - 4 S_1 S_2 - \frac{2 S_2}{n+1} - \frac{2 S_1}{(n+1)^2} - \frac{2}{(n+1)^3} + \]  
\[ -4 S_2 - \frac{2}{(n+1)^2} - 8, \]  
\[ \gamma_3 = +5 S_5 + 6 S_2 S_3 - 4 S_{2,3} + 4 S_{4,1} - 8 S_{3,1,1} \]
\[ + \left( 4 S_2^2 + 2 S_4 + 8 S_{3,1} \right) S_1 \]
\[ + \frac{-S_4 + 4 S_{2,2} + 4 S_{3,1}}{n+1} \]
\[ + \frac{4 S_1 S_2 + S_3}{(n+1)^2} + \frac{2 S_1^2 + 3 S_2}{(n+1)^3} \]
\[ + \frac{6 S_1}{(n+1)^4} + \frac{4}{(n+1)^5} \]
\[ -2 S_4 + 8 S_{2,2} + 8 S_{3,1,1} \]
\[ + \frac{4 S_2}{(n+1)^2} + \frac{4 S_1}{(n+1)^3} + \frac{6}{(n+1)^4} \]
\[ + 8 S_2 + 32, \]

where \( n = \frac{N}{2} + 1 \) and \( S_{a,b,...} = S_{a,b,...}(n) \) are nested harmonic sums. The four-loop result contains a contribution from the dressing phase and has a similar structure. Many interesting structural properties as well as a reciprocity proof have been discussed in [11]. Our working hypothesis will be that the same structure is also shared by the leading wrapping correction, precisely as it happens for genuinely \( sl(2) \) operators.

3. One-loop description of \( \mathcal{O}^{A}_{N,2} \) in \( sl(2) \) grading

We recall the one-loop description of the 3-gluon states under consideration. They are fully described in [11]. The 3-gluon states are associated with the following (higher) Dynkin diagram [15] assignments

\[ \begin{array}{ccccccc}
+1 & \times & \cdots & \times & \circ & \cdots & \times \\
N+3 & N+4 & N+2 & 1
\end{array} \]  

(3.1)

As explained in [11], after a chain of one-loop dualizations, the exact Bethe roots can be written in terms of those of a \( XXXX_{-3,2} \) spin chain. The explicit roots can be given in terms of the associated Baxter polynomials. Let us define the polynomial

\[ P_0(u) = \binom{-\frac{N}{2}}{\frac{N}{2}} + 4 \binom{\frac{1}{2} + i u}{\frac{1}{2} - i u} \binom{1}{1}. \]

(3.2)
In terms of $P_6$, we can write the Baxter polynomials $Q_\ell(u) \sim \prod_{i=1}^{K_\ell} (u - u_{\ell,i})$ for the roots at $\ell$-th node with $\ell = 3, 4, 5$

$$Q_5(u) = (u + \frac{i}{2})^3 P_6 \left( u + \frac{i}{2} \right) - (u - \frac{i}{2})^3 P_6 \left( u - \frac{i}{2} \right), \quad (3.3)$$

$$Q_4(u) = \left( u + \frac{i}{2} \right)^3 Q_5 \left( u + \frac{i}{2} \right) - \left( u - \frac{i}{2} \right)^3 Q_5 \left( u - \frac{i}{2} \right), \quad (3.4)$$

$$Q_3(u) = Q_4 \left( u + \frac{i}{2} \right) - Q_4 \left( u - \frac{i}{2} \right). \quad (3.5)$$

The single root at node 6 is identically zero and $Q_6 \sim u$. The Bethe equations satisfied by these polynomials are

$$1 = \left. \frac{Q^-_4}{Q^+_4} \right|_{u_{6,p}} \quad , \quad (3.6)$$

$$- \left( \frac{u_{4,p} + \frac{i}{2}}{u_{4,p} - \frac{i}{2}} \right)^3 = \left. \frac{Q^-_4 Q^+_5 Q^-_3 Q^-_5}{Q^-_4 Q^-_3 Q^-_5} \right|_{u_{4,p}} \quad , \quad (3.7)$$

$$1 = \left. \frac{Q^-_6}{Q^-_6} \right|_{u_{4,p}} \quad , \quad (3.8)$$

$$-1 = \left. \frac{Q^-_6 Q^+_5}{Q^-_6 Q^+_5} \right|_{u_{4,p}} \quad . \quad (3.9)$$

For the following application, it will be convenient to dualize the Dynkin diagram and write it in $\mathfrak{sl}(2)$ grading. To this aim, we first dualize at node 3. From

$$Q^+_4 - Q^-_4 = Q_3 \tilde{Q}_3, \quad (3.10)$$

we deduce that $\tilde{Q}_3$ is a constant, i.e. we don’t have roots at node 3. We then dualize at node 5. From

$$Q^+_4 Q^-_6 - Q^-_4 Q^+_6 = Q_5 \tilde{Q}_5, \quad (3.11)$$

we deduce that the degree of $\tilde{Q}_5$ is 2. Explicitly, after a short computation, one finds

$$\tilde{Q}_5 = -\frac{1}{4} (N + 4)^2 - (N + 3)(N + 5) u^2. \quad (3.12)$$

3.1 Summary

After dualization (and omitting now the tilde), the one-loop Bethe equations in $\mathfrak{sl}(2)$ grading are associated with the diagram

$$\begin{array}{c}
\bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\
\circ & \circ & \circ & \circ & \circ & \circ \\
\circ & \circ & \circ & \circ & \circ & \circ \\
\circ & \circ & \circ & \circ & \circ & \circ \\
\circ & \circ & \circ & \circ & \circ & \circ \\
\circ & \circ & \circ & \circ & \circ & \circ \\
\circ & \circ & \circ & \circ & \circ & \circ \\
\circ & \circ & \circ & \circ & \circ & \circ \\
\circ & \circ & \circ & \circ & \circ & \circ \\
\circ & \circ & \circ & \circ & \circ & \circ \\
\end{array}$$

$$\begin{array}{c}
\bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\
\circ & \circ & \circ & \circ & \circ & \circ \\
\circ & \circ & \circ & \circ & \circ & \circ \\
\circ & \circ & \circ & \circ & \circ & \circ \\
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\circ & \circ & \circ & \circ & \circ & \circ \\
\circ & \circ & \circ & \circ & \circ & \circ \\
\circ & \circ & \circ & \circ & \circ & \circ \\
\end{array}$$

$$N + 4 \quad 2 \quad 1 \quad (3.13)$$

[Note: The image contains a figure that represents the Dynkin diagram with nodes and edges, but the text provides the equations and reasoning without the visual aid.]
with the auxiliary polynomials
\[
P_6(u) = 4 P_3 \left( -\frac{N}{2} + \frac{N}{2} + 4 \frac{i}{2} + iu \left[ \frac{1}{2} - iu \right] \right), \quad (3.14)
\]
\[
P_5(u) = (u + i)^3 P_6^+ - (u - i)^3 P_6^-, \quad (3.15)
\]
and the Baxter polynomials
\[
Q_4(u) = \left( u + \frac{i}{2} \right)^3 P_5^+ - \left( u - \frac{i}{2} \right)^3 P_5^-, \quad (3.16)
\]
\[
Q_5(u) = \frac{1}{4} (N + 4)^2 - (N + 3)(N + 5) u^2, \quad (3.17)
\]
\[
Q_6(u) = u. \quad (3.18)
\]
They obey the one-loop Bethe equations
\[
- \left( \frac{u_{4, p} + \frac{i}{2}}{u_{4, p} - \frac{i}{2}} \right)^3 = \frac{Q_4^- Q_5^+}{Q_4^+ Q_5^-} \bigg|_{u_{4, p}}, \quad (3.19)
\]
\[
1 = \frac{Q_4^+ Q_6^-}{Q_4^- Q_6^+} \bigg|_{u_{5, p}}, \quad -1 = \frac{Q_5^+ Q_6^-}{Q_5^- Q_6^+} \bigg|_{u_{6, p}}. \quad (3.20)
\]
This one-loop setup is enough to compute the leading wrapping correction as explained in the next Section.

4. Y-system for the AdS$_5 \times$ S$^5$ superstring

4.1 Generalities

The Y-system is a set of functional equations for the functions $Y_{a,i}(u)$ defined on the fat-hook of psu(2,2|4) [3]. These equations are (their boundary conditions are discussed in [16])
\[
\frac{Y_{a,i}^+ Y_{a,i}^-}{Y_{a+1,i} Y_{a-1,i}} = \frac{(1 + Y_{a,i+1})(1 + Y_{a,i-1})}{(1 + Y_{a+1,i})(1 + Y_{a-1,i})}. \quad (4.1)
\]
The anomalous dimension of a generic state is given by the TBA formula
\[
E = \sum_i \epsilon_i(u_{4,i}) + \sum_{a \geq 1} \int_{\mathbb{R}} \frac{du}{2\pi i} \frac{\partial \epsilon_a^+}{\partial u} \log(1 + Y_{a,0}^+(u)). \quad (4.2)
\]
In this formula, the dispersion relation is
\[
\epsilon_a(u) = a + \frac{2ig}{x^a} - \frac{2ig}{x^{-a}}, \quad (4.3)
\]
and the star means evaluation in the mirror kinematics $^2$. The Bethe roots $\{u_{4,i}\}$ are fixed by the exact Bethe equations (in physical kinematics)
\[
Y_{1,0}(u_4) = -1. \quad (4.8)
\]

$^2$We recall that the physical and mirror branches of the Zhukovsky relation
\[
x + \frac{1}{x} = \frac{u}{g}, \quad (4.4)
\]
Any solution of the Y-system can be written as
\[ Y_{a,s} = \frac{T_{a,s+1}T_{a,s-1}}{T_{a+1,s}T_{a-1,s}}, \]
(4.9)
in terms of the solution \( T_{a,s} \) of the Hirota integrable discrete equation
\[ T_{a,s}^+ T_{a,s}^- = T_{a+1,s}T_{a-1,s} + T_{a,s+1}T_{a,s-1}. \]
(4.10)
This equation is gauge invariant (i.e. leads to the same \( Y \)) under
\[ T_{a,s} \rightarrow g_1^{a+s} g_2^{b-s-1} g_3^{-a+s} g_4^{-a-s} T_{a,s}, \]
(4.11)
The crucial assumption in the identification of relevant solutions to the Y-system is
\[ Y_{a \geq 1,0} \sim \left( \frac{x^{-a}}{x^{+a}} \right)^L, \]
(4.12)
for large \( L \), or large \( u \) (or small \( g \)). In this limit, it can be shown that the Hirota equation splits in two \( su(2|2)_{L,R} \) wings. One can have a simultaneous finite large \( L \) limit on both wings after a suitable gauge transformation. The solution is then
\[ Y_{a,0}(u) \sim \left( \frac{x^{-a}}{x^{+a}} \right)^L \phi^{-a} \phi^{+a} T_{a,1}^L T_{a,1}^R, \]
(4.13)
where \( \phi \) is an arbitrary function and \( T_{a,1}^{L,R} \) are transfer matrices of the antisymmetric rectangular representations of \( su(2|2)_{L,R} \). They are given explicitly by the generating functional
\[ \sum_{a=0}^\infty (-1)^a T_{a,1}^{1-a} \phi^a = \left( 1 - \frac{Q_3^+}{Q_3} \right)^{-1} \left( 1 - \frac{Q_3^-}{Q_3} \right)^{-1} \left( 1 \right) \left( 1 - \frac{Q_2^++Q_1^-}{Q_2 Q_1} \right)^{-1} \left( 1 \right) \left( 1 - \frac{Q_2^-+Q_1^+}{Q_2 Q_1} \right)^{-1}, \]
(4.14)
where \( \phi = e^{-i\partial_u} \) and
\[ R^{(\pm)} = \prod_{i=1}^{K_x} \frac{x(u) - x_{\pm i}^\mp}{(x_{\pm i}^\mp)^{1/2}}, \quad B^{(\pm)} = \prod_{i=1}^{K_x} \frac{1}{(x_{\pm i}^\mp)^{1/2}}. \]
(4.15)

\( x_{ph}(u) = \frac{1}{2} \left( \frac{u}{g} + \sqrt{\frac{u}{g} - 2\sqrt{\frac{u}{g} + 2}} \right), \quad x_{mix}(u) = \frac{1}{2} \left( \frac{u}{g} + i \sqrt{4 - \frac{u^2}{g^2}} \right). \]
(4.5)

Shifted quantities are defined as
\[ F^\pm a(u) = F^{[\pm a]}(u) = F \left( u \pm \frac{a}{2} \right). \]
(4.6)

For real \( u \) and \( a > 0 \), we have the relations
\[ x_{ph}^{[a]} = x_{ph}^{-a}, \quad x_{ph}^{-a} = \frac{1}{x_{ph}^{[a]}}, \]
(4.7)
The function $\phi$ can be fixed as explained in [3] and reads
\[
\frac{\phi^-}{\phi^+} = \sigma^2 \frac{B^{(+)}}{B^{(-)}} \frac{R^{(-)}}{R^{(+)}} \frac{B_{3L}^- B_{3L}^+}{B_{1L}^- B_{1L}^+} \frac{B_{3R}^- B_{3R}^+}{B_{1R}^- B_{1R}^+},
\] (4.16)
where $\sigma$ is the dressing phase\(^3\). At weak coupling, evaluating the various terms at leading order in the mirror dynamics, the wrapping correction (second term in the r.h.s. of (4.2)) is simply given by the expression
\[
\Delta E = -\frac{1}{\pi} \sum_{a=1}^{\infty} \int_{R} du \ Y_{a,0}^*.
\] (4.17)

### 4.2 Explicit formulae for the efficient computation of $Y_{a,0}^*$

In the following, we shall need a compact efficient formula for the evaluation of $Y_{a,0}^*$. According to (4.13), we need the contribution from the dispersion (ratio of $x^\pm$), the fusion of scalar factors ($\phi$ terms), and the $\text{su}(2|2)$ transfer matrices. The transfer matrices can be written in terms of the two spin dependent polynomials $Q_{4.5}$. After a straightforward computation we obtain:

**Dispersion**

This is the universal factor
\[
\left( \frac{4g^2}{a^2 + 4u^2} \right)^3.
\] (4.18)

**Fusion of scalar factors**

After some manipulations, one finds the formula
\[
\Phi_a^* = \left[ (Q_4^+ (0))^2 \right] \frac{Q_4^{[1-a]} Q_5^{[a-1]} Q_4^{[a+1]} Q_5^{[-a]}}{Q_4^{[1-a]} Q_5^{[a-1]} Q_4^{[a+1]} Q_5^{[-a]}}.
\] (4.19)

**Left $\text{su}(2|2)$ wing**

The left wing has no excitations. It depends only on $Q_4$ and an efficient expression has been already given in [17]
\[
T_{L,0}^a = \left. i \ c \ g^2 \left( -1 \right)^{a+1} \sum_{p=-a}^{a} \frac{Q_4^{[-1-p]} - Q_4^{[1-p]}}{u - \frac{p}{\pi} \Delta_p} \right|_{Q_4^{[a-1]} Q_4^{[a+1]} \to 0},
\] (4.20)

where
\[
c = \sum_j \frac{1}{u_{4,j} + \frac{1}{2} \ u_{4,j} - \frac{1}{2}} = \left. i (\log Q_4) \right|_{u \to +i/2}.
\] (4.21)

\(^3\)By a suitable analytic continuation, it is possible to work in a strip of the mirror kinematics where the dressing phase coincides with the physical one and gives higher order contributions that does not affect our computation.
is proportional to the one-loop anomalous dimension.

**Right su(2|2) wing**

The right wing depends on $Q_4$ and $Q_5$. It also contains $Q_6$ which is simply $u$. An efficient expression is

\[ T_{a,0}^{R} = i (-1)^{a+1} \frac{Q_6^{[b]}}{Q_4^{[-(a-1)]}} \sum_{p=-\infty}^{a-1} \frac{Q_4^{[p]}}{u + \frac{1}{2} p} \left( \frac{1}{Q_5^{[p+1]}} - \frac{1}{Q_5^{[p-1]}} \right). \]  

(4.22)

**5. Evaluation and results for the wrapping corrections**

Let us inspect the first values of $Y_{a,0}^*$ in order to identify the structure of the result. For all even $N$, we find empirically that it is a rational function of $a$ and $u$. The expressions have a complexity which grows rapidly with $N$, but are very suitable for symbolic manipulations. For instance, at $N = 2$ the explicit expression is

\[ Y_{a,0}^* = -2^{28} \cdot 5 \cdot 7 \frac{a^2 g^8}{(a^2 + 4u^2)^2} \frac{N_1(a, u) N_2(a, u)}{D(a, u) D(-a, u)}, \]  

(5.1)

where

\[ N_1(a, u) = \left(35a^2 - 36\right)\left(a^4 + 44a^2 - 288\right) - 4 \left(35a^4 - 3440a^2 + 5328\right)u^2 \]
\[ -80 \left(35a^2 - 272\right)u^4 - 6720u^6, \]  

(5.2)

\[ N_2(a, u) = \left(7a^6 + 120a^4 + 944a^2 - 1152\right) - 4 \left(63a^4 + 240a^2 - 944\right)u^2 \]
\[ -80 \left(7a^2 + 72\right)u^4 + 2240u^6, \]  

(5.3)

\[ D(a, u) = \left(35a^6 + 210a^5 + 920a^4 + 2280a^3 + 3536a^2 + 3072a + 1152\right)^2 + \]
\[ +8 \left(3675a^{10} + 36750a^9 + 206850a^8 + 777200a^7 + 2155360a^6 + 4582560a^5 + \right. \]
\[ +7220320a^4 + 8052480a^3 + 6144256a^2 + 2982912a + 645120\right)u^2 + \]
\[ +16 \left(18375a^8 + 147000a^7 + 569800a^6 + 1360800a^5 + 2603200a^4 + 4084800a^3 + \right. \]
\[ +3847360a^2 + 1585920a + 614656\right)u^4 + 1280 \left(1225a^6 + 7350a^5 + 15610a^4 + \right. \]
\[ +13440a^3 + 24016a^2 + 40752a - 4912\right)u^6 + 3840 \left(1225a^4 + 4900a^3 + \right. \]
\[ +1820a^2 - 6160a + 9088\right)u^8 + 358400 \left(21a^2 + 42a - 58\right)u^{10} + 5017600u^{12}. \]  

The wrapping correction is evaluated as usual at leading order as

\[ W_{N=2} = -\frac{1}{\pi} \sum_{a \geq 1} \int_{\mathbb{R}} Y_{a,0}^*(u) du. \]  

(5.5)
We checked that this can be obtained by summing the residues in \( u = \frac{i\pi}{2} \). Notice also that the residue is a rational function of \( a \) of the form

\[
\mathcal{R}(a) = \text{Res}_{u=ia} a^2 Y_\alpha^0(u) = \frac{P(a)}{a^5 \left[ Q_4^+ (ia) Q_4^- (ia) \right]^4},
\]

(5.6)

where \( P \) is a polynomial. The sum over \( a \) can be done exploiting the following important properties. First, the singular part of \( \mathcal{R}(a) \) at \( a = 0 \) is a combination of two poles \( \sim a^{-3} \) and \( a^{-5} \). These contributions can be summed giving terms proportional to \( \zeta_3 \) and \( \zeta_5 \). Then, the regularized rational function

\[
\mathcal{R}(a) = \mathcal{R}(a) - \text{"pole part at } a = 0", \quad (5.7)
\]

obeys the remarkable property

\[
\sum_{a=1}^{p} \mathcal{R}(a) = \frac{\bar{P}(p)}{[Q_4^+ (ip)]^4}, \quad (5.8)
\]

where \( \bar{P} \) is another polynomial with the same degree of the denominator. It can be determined easily by polynomial interpolation. Then, the infinite sum is just the trivial \( p \to \infty \) limit.

Repeating the calculation for higher values of \( N \), we find the following list (omitting \( g^8 \))

\[
\begin{align*}
W_{N=2} &= \frac{5348840}{2187} + \frac{528640}{81} \zeta_3 - \frac{89600}{9} \zeta_5 \\
W_{N=4} &= \frac{1216307603}{331776} + \frac{235081}{24} \zeta_3 - 14910 \zeta_5 \\
W_{N=6} &= \frac{4018092206843}{810000000} + \frac{24863234}{1875} \zeta_3 - \frac{100848}{5} \zeta_5 \\
W_{N=8} &= \frac{27624795456401}{4374000000} + \frac{56992078}{3375} \zeta_3 - \frac{231088}{9} \zeta_5 \\
W_{N=10} &= \frac{206037054943950841}{26682793200000} + \frac{3718258258}{180075} \zeta_3 - \frac{1538160}{49} \zeta_5 \\
W_{N=12} &= \frac{143200464784761259303}{15613248496000000} + \frac{80764059527}{3292800} \zeta_3 - \frac{521985}{14} \zeta_5 \\
W_{N=14} &= \frac{7678365193173099146293}{72027074544768000000} + \frac{32079436339187}{112521150} \zeta_3 - \frac{24571256}{567} \zeta_5 \\
W_{N=16} &= \frac{185723391274028370487}{1524382530048000000} + \frac{77595407257}{2381400} \zeta_3 - \frac{148580}{3} \zeta_5 \\
W_{N=18} &= \frac{10202293030345127436784638469}{742647575811450355200000} + \frac{640511693446157}{17433038700} \zeta_3 - \frac{20271464}{363} \zeta_5.
\end{align*}
\]

We extended the list up to \( N = 70 \). The general form is always

\[
W_N = (r_{0,N} + r_{3,N} \zeta_3 + r_{5,N} \zeta_5) g^8, \quad (5.9)
\]

with rational coefficients \( r_{0,N} \), \( r_{3,N} \), and \( r_{5,N} \).
5.1 Closed formulae

Given our long list of explicit values \( \{W_N\} \), we can look for a closed formula based on the structure of the asymptotic anomalous dimensions. Notice also that from the expression of \( T^L \), we see that the wrapping correction is proportional to the one-loop anomalous dimension, i.e. the combination

\[
4 S_1(n) + \frac{2}{n + 1} + 4, \quad n = \frac{N}{2} + 1. \tag{5.10}
\]

**Coefficient of \( \zeta_5 \)**

After some trial and error we find

\[
r_{5,N} = 80 \left( 4 S_1 + \frac{2}{n + 1} + 4 \right) \left( -4(n + 1) + \frac{1}{n + 1} \right). \tag{5.11}
\]

**Coefficient of \( \zeta_3 \)**

Inspired by the structure of the previous coefficient we find

\[
r_{3,N} = 16 \left( 4 S_1 + \frac{2}{n + 1} + 4 \right) \times \left[ 8(n + 1) S_2 + 8 + \frac{2}{n + 1} (2 - S_2) - \frac{2}{(n + 1)^3} - \frac{1}{(n + 1)^3} \right]. \tag{5.12}
\]

**Purely rational part**

With major effort, we obtain

\[
r_{0,N} = 2 \left( 4 S_1 + \frac{2}{n + 1} + 4 \right) \times \left[ 16(n + 1)(2 S_{2,3} - S_6) + 32 S_3 + \frac{4}{n + 1}(S_5 - 2S_{2,3} + 4S_3) + \frac{8}{(n + 1)^3}(-S_3 + 2) + \frac{4}{(n + 1)^3}(-S_3 + 4) - \frac{4}{(n + 1)^5} - \frac{1}{(n + 1)^6} \right]. \tag{5.13}
\]

6. Large \( N \) expansion and generalized Gribov-Lipatov reciprocity

Let us factor out the one-loop anomalous dimension and write \((k = 0, 3, 5)\)

\[
r_{k,N} = \left( 4 S_1 + \frac{2}{n + 1} + 4 \right) \bar{r}_{k,N}. \tag{6.1}
\]

By standard methods, we worked out the large \( N \) of \( \bar{r}_{k,N} \). It reads

\[
\bar{r}_{5,N} = -320 n - 320 + \frac{80}{n} - \frac{80}{n^2} + \frac{80}{n^3} - \frac{80}{n^4} + \frac{80}{n^5} + \ldots,
\]

\[
\bar{r}_{3,N} = 128 \zeta_2 n + 128 \zeta_2 - \frac{32 \zeta_3}{n} + \frac{16 (6 \zeta_2 - 4)}{3 n^2} - \frac{16 (6 \zeta_2 - 8)}{3 n^3} +
\]
As discussed in [11], the asymptotic anomalous dimensions can be expanded at large

\[ \eta_{0,N} = \left(320 \zeta_5 - 128 \zeta_2 \zeta_3\right) n + \left(32 \zeta_2 \zeta_3 - 80 \zeta_5\right) + \frac{32 \zeta_2 \zeta_3 - 80 \zeta_5}{n} + \frac{16 \left(6 \zeta_2 \zeta_3 + 2 \zeta_3 - 15 \zeta_5 - 2\right)}{3n^2} + \frac{16 \left(6 \zeta_2 \zeta_3 + 4 \zeta_3 - 15 \zeta_5 - 4\right)}{3n^3} + \frac{8 \left(60 \zeta_2 \zeta_3 + 51 \zeta_3 - 150 \zeta_5 - 36\right)}{15n^4} + \frac{16 \left(30 \zeta_2 \zeta_3 + 22 \zeta_3 - 75 \zeta_5 + 8\right)}{15n^5} + \ldots. \]  

(6.4)

Remarkably, the leading \( \mathcal{O}(n) \) and next-to-leading \( \mathcal{O}(n^0) \) contributions cancel. The resulting total wrapping contribution is thus (including the one-loop factor, and writing the expansion in the \( N \) variable)

\[ \frac{1}{g^2} W_N = -\frac{512}{3} (3 \zeta_3 - 1) \frac{\log N + 1}{N^2} + \frac{2048}{3} (3 \zeta_3 - 1) \frac{2 \log N + 1}{N^3} + \frac{1536}{5} (77 \zeta_3 - 24) \frac{\log N}{N^4} - \frac{512}{45} (264 \zeta_3 - 43) \frac{1}{N^5} + \frac{8192}{15} (213 \zeta_3 - 56) \frac{\log N}{N^5} - \frac{2048}{45} (543 \zeta_3 - 316) \frac{1}{N^6} + \ldots. \]  

(6.5)

where \( \bar{N} = \frac{N}{2} e^{y_1} \). Notice that all terms containing \( \zeta_2 = \pi^2/6 \) as well as \( \zeta_5 \) cancel.

As an important remark, we emphasize that the leading wrapping correction is sub-leading at large \( N \) compared with the asymptotic contribution and does not change the scaling function already computed for these operators in [11] and in more general cases in [12].

6.1 Generalized Gribov-Lipatov reciprocity

As discussed in [11], the asymptotic anomalous dimensions can be expanded at large \( J \) where \( J^2 = n(n + 2) \) and turn out to admit an expansion in even powers of \( 1/J \). The absence of odd powers is the so-called reciprocity property. For instance, the one-loop factor has the expansion

\[ 4 S_1(n) + \frac{2}{n + 1} + 4 = (2 \log J^2 + 4) + \frac{5}{3 J^2} - \frac{19}{30 J^4} + \frac{79}{315 J^6} - \frac{1}{420 J^8} + \ldots. \]  

(6.6)

where \( J = Je^{y_1} \).

We now look at the reciprocity property of the wrapping correction associated with the \( \bar{f} \) factors. From the previous expansions we find

\[ \bar{f}_{5,N} = J \left( -320 - \frac{80}{J^2} + \frac{10}{J^3} - \frac{25}{2 J^4} + \frac{105}{8 J^6} + \ldots \right), \]  

(6.7)

\[ \bar{f}_{3,N} = 128 \zeta_2 J + \frac{32 \zeta_2}{J} - \frac{64}{3 J^2} + \frac{464}{15 J^4} - \frac{4 \zeta_2}{J^5} - \frac{4688}{105 J^6} + \frac{5 \zeta_2}{J^7} + \frac{7088}{105 J^8} + \ldots, \]  

(6.8)

\[ \bar{f}_{0,N} = (320 \zeta_5 - 128 \zeta_2 \zeta_3) \frac{J}{J} + \frac{80 \zeta_5 - 32 \zeta_2 \zeta_3}{J} - \frac{32 (\zeta_3 - 1)}{J^2} + \frac{22 \zeta_3}{15 J^4} - \frac{36 \zeta_2}{15 J^5} + \frac{4 \zeta_2 \zeta_3 - 10 \zeta_5}{J^6} + \ldots \]

This means that the large \( N \) expansion can be organized in a part with even powers of \( 1/N \) and an induced part with odd powers of \( 1/N \) completely determined by the former.
Notice that none of the three terms is separately reciprocity respecting. However, the combination appearing in the wrapping correction reads

\[
\zeta_5 \mathcal{F}_{5,N} + \zeta_3 \mathcal{F}_{3,N} + \mathcal{F}_{0,N} = \frac{32}{3} - 32 \zeta_3 + \frac{2344}{3} \frac{\zeta_3}{3} + \frac{3544}{3} - \frac{83966}{945} \frac{\zeta_3}{j^7} + \frac{18772901_3}{38045} - \frac{8793002}{45365} + \frac{540472}{45365} + \frac{4917304}{38045} - \frac{5747755528}{883275} + \ldots.
\]

All the odd powers of $1/j$ cancel proving that the reciprocity property does hold.

### 7. BFKL analytic continuation

In the sl(2) sector, and for length $L = 2$, it is possible to explore the analytic continuation of the anomalous dimensions at negative spin. The leading and next-to-leading poles are captured by the BFKL equation thus providing a strong cross check of the calculation. For the 3-gluon states, we shall now compute the BFKL poles and show that a very simple and natural modification of the twist-2 BFKL equation predicts the correct pole structure.

#### 7.1 Continuation at $n = -1$

The four loop anomalous dimension is

\[
\gamma(g) = g^2 \gamma_{1}^{ABA} + g^4 \gamma_{2}^{ABA} + g^6 \gamma_{3}^{ABA} + g^8 \left( \gamma_{4}^{ABA} + \zeta_3 \gamma_{4}^{\text{Dressing}} + W \right) + \ldots.
\]

Since we have the explicit closed form of all terms as functions of $N$, we analytically continue in the variable $n = \frac{N}{2} + 1$ around $n = -1$. Setting $n = -1 + \omega$, we find

\[
\begin{align*}
\gamma_{1}^{ABA} &= -\frac{4}{\omega} + \ldots, \\
\gamma_{2}^{ABA} &= \frac{8}{\omega^2} + \frac{4 \pi^2}{3 \omega} + \ldots, \\
\gamma_{3}^{ABA} &= \frac{0}{\omega^3} - \frac{16 \left( -3 \zeta_3 + \pi^2 + 12 \right)}{3 \omega^2} + \ldots, \\
\gamma_{4}^{ABA} &= -\frac{4}{\omega^5} + \frac{24}{\omega^6} + \frac{4 \left( \pi^2 - 24 \right)}{3 \omega^4} - \frac{8 \left( 3 \zeta_3 + 2 \pi^2 + 36 \right)}{3 \omega^4} + \ldots, \\
\gamma_{4}^{\text{Dressing}} &= -\frac{32}{\omega^4} + \ldots.
\end{align*}
\]

The expansion of the wrapping contribution is

\[
W = \frac{4}{\omega^7} - \frac{24}{\omega^6} + \frac{32}{\omega^5} - \frac{4 \pi^2}{3 \omega^5} - \frac{24 \zeta_3 + 16 \pi^2 + 64}{\omega^4} + \ldots.
\]

---

5This section has been prepared together with Nikolay Gromov. We thank him for his kind help and insight.

6Notice a factor 2 of difference in the definition of $g^2$ in this paper and in $[1]$. 
Summing all terms for the $g^6$ contribution, we get

$$\gamma_4 = \frac{0}{\omega^4} + \frac{0}{\omega^6} + \frac{0}{\omega^5} + \frac{-64\zeta_3}{\omega^4} + \frac{160\zeta_3}{\omega^3} + \ldots \quad (7.8)$$

The cancellation of the three leading poles (present in $\gamma_4^{ABA}$ and canceling against $W$) is remarkable. We can reproduce the leading poles from a BFKL-like equation as in [18, 19] as follows. In the Konishi case, one defines

$$\chi_a(z) = \psi(-z) + \psi(z + 1 + a) - 2\psi(1), \quad (7.9)$$

and solves perturbatively the equation

$$\frac{\omega}{g^2} = \chi_0 \left( \frac{\gamma}{2} \right), \quad (7.10)$$

where $\gamma$ is written as a power series in $g^2$. To extend this equation to our case, we notice that the BFKL kernel can be written as

$$\chi_0(z) = h(z) + h(-z - 1), \quad (7.11)$$

where $h(z) = \psi(z + 1) - \psi(1) \equiv S_1(z)$ is the analytic continuation of the basic harmonic sum, proportional to the one-loop anomalous dimension. In our case, the one-loop anomalous dimension (2.6) can be written in the form

$$\gamma_1(n) = 2 S_1(n) + 2 S_1(n + 1) + 4. \quad (7.12)$$

We thus led to replace in (7.10)

$$\chi_0(z) \rightarrow \chi_0(z) + \chi_0(z + 1) \equiv 2\chi_1(z). \quad (7.13)$$

Thus, we have to solve

$$\frac{\omega}{g^2} = 2 \chi_1 \left( \frac{\gamma}{2} \right). \quad (7.14)$$

Expanding at weak coupling, we indeed obtain

$$\gamma = \left( -\frac{4}{\omega} + \ldots \right) g^2 + \left( \frac{8}{\omega^2} + \ldots \right) g^4 + \left( \frac{\theta}{\omega^3} + \ldots \right) g^6 + \ldots \quad (7.15)$$

$$+ \left( -\frac{32 (1 + 2 \zeta_3)}{\omega^4} + \ldots \right) g^8 + \left( \frac{512 \zeta_3}{\omega^5} + \ldots \right) g^{10} + \ldots. \quad (7.16)$$

This is in full agreement with our four loop results. Notice that the agreement of the four loop term is achieved thanks to the non-trivial wrapping contribution. The higher order poles are a prediction.

One can also attempt to write down a NLO BFKL equation of the form

$$\frac{\omega}{g^2} = 2 \chi_1 \left( \frac{\gamma}{2} \right) - g^2 \delta(\gamma), \quad (7.17)$$

and fix the expansion of $\delta(\gamma)$ from the coefficients of the next to leading poles. At the four loop order one finds the remarkably simple result

$$\delta(\gamma) = \frac{2}{\gamma} \left[ 4 \zeta_2 + 2 (4 + \zeta_2 - \zeta_3) \gamma - (8 + \zeta_2 - 7 \zeta_3) \gamma^2 + \ldots \right] \quad (7.18)$$

with all integer coefficients.
7.2 Continuation at $n = -2$

We can also compute the singular expansion around $n = -2$. We do this with the aim of comparing with what happens in the Konishi case [18, 19]. Setting $n = -2 + \omega$, and considering only the leading poles, the result is

$$\gamma_1^{ABA} = -\frac{8}{\omega} + \ldots,$$  \hspace{1cm} (7.19)

$$\gamma_2^{ABA} = -\frac{8}{\omega^3} + \ldots,$$  \hspace{1cm} (7.20)

$$\gamma_3^{ABA} = -\frac{8}{\omega^5} + \ldots,$$  \hspace{1cm} (7.21)

$$\gamma_4^{ABA} = -\frac{8}{\omega^7} + \ldots,$$  \hspace{1cm} (7.22)

$$\gamma_4^{Dressing} = -\frac{128}{\omega^4} + \ldots$$  \hspace{1cm} (7.23)

The expansion of the wrapping contribution is

$$W = \frac{96}{\omega^6} + \ldots$$  \hspace{1cm} (7.24)

We see the same pattern as in the Konishi case. In particular, the ABA leading poles are of the same order as in Konishi, i.e. $\sim g^{2n} / \omega^{2n-1}$, and wrapping correction does not change them.

8. Conclusions

In this paper we have applied the Y-system formalism proposed in [3] to the computation of the wrapping correction for a class of twist operators reducing at one-loop to 3-gluon maximal helicity quasipartonic operators. We provided the leading four-loop contribution as a closed function of the operator spin $N$ by a generalized transcendentality Ansatz already found in the asymptotic contribution. The result can be checked by means of two important physical constraints: A generalized reciprocity of the large $N$ expansion, and a BFKL-like resummation of the leading pomeron poles. Both tests are passed extending similar conclusions holding for the simpler $\mathfrak{sl}(2)$ twist operators.

Our analysis is a novel test of the Y-system for the $AdS_5 \times S^5$ superstring involving a larger part of the $psu(2, 2|4)$ Dynkin diagram and leading to exact prediction for a new set of short operators. It would be very interesting to extend the analysis to the strong coupling regime by solving the full (numerical or possibly semiclassical) TBA equations. In this perspective, it would be very nice to assess the validity and consequences of the reciprocity and BFKL constraints for the string duals of the considered operators.

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