Numerical Analyses on the Rockbolt Behaviour in Rock Wedges Reinforcement Using 2D-DDA

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Abstract. One major concern in the design and construction of caverns in jointed rock mass is the potential failure of rock wedges falling from the roof or sliding out from the sidewalls. The rockbolts are commonly used to improve the stability and to prevent failure in the caverns. In this paper, the two-dimensional discontinuous deformation analysis (2D-DDA) method was used to analyse the stability of the rock wedge supported by rockbolts on the cavern roof. The joint relaxation method was adopted to represent a deformable rock joint under stresses. Parametric studies were conducted to investigate the effects of the confining pressure, the spacing and the angle between rockbolt and direction of wedge falling on the stability of the rock wedge. The numerical simulation results indicate that the wedge is self-supporting after stress redistribution if the confining pressure is larger than the critical confining pressure and the semi-apical angle of the wedge is less than the friction angle of rock joint. The force in rockbolt slightly decreases with reduced rockbolts spacing but increase when the rockbolt is not along the falling direction of rock wedge.

Keywords: Rock cavern; Rock wedge; Rockbolt; 2D-DDA.

1. Introduction
For tunnels excavated in jointed rock mass, rock wedges formed by intersecting structural discontinuities may fall from the roof or slide out of the sidewalls as shown in Fig. 1a. As there might be no restraints from the boundary, the rock wedge in the tunnel roof may fall as soon as its base is fully exposed by the excavation of opening [10]. The falling of the roof wedge will reduce the restraints and interlocks in the surrounding rock mass. Unless proper supports are used to reinforce the loose wedges, the stability of the surrounding rock may deteriorate rapidly. The spot rockbolt and sparsely spaced pattern rockbolt are empirically recommended to be installed in jointed rock mass to stabilize the rock wedges. As the major driving force of the wedge falling is its deadweight, especially if the tunnel is excavated in hard jointed rock mass under the low field stress state, the rockbolts for wedge stabilization are usually designed based on the deadweight of the rock wedge [14,17].

The design parameters of the rockbolt include the bolt strength, length, diameter, spacing, patterns, and so on. The recommendations and suggestions of the design parameters of the rockbolt are summarized in Table 1. The relative location of the rockbolts to the major joints is a critical factor that needs to be considered in field. The embedded length (l0) defined as the length of rockbolt in the stable rock behind
the wedge is another critical parameter. As shown in Fig. 1b, the embedded length changes with the relative location of the rockbolt to the wedge boundaries. It should be long enough to ensure the anchorages in the stable interior rock. Otherwise, the rockbolt would be pulled out easily. For the fully grouted rockbolt, the required embedded length in field is usually about 1.2 m [15].

The angle between the rockbolt and the joint is also an important factor as it determines the internal forces that develop in the rockbolts, i.e. tension and/or shear loads, as shown in Fig. 1c. The shear components may reduce the load capacity of the rockbolt [3].

![Sketch of rock wedge falling from cavern roof](image)

**Fig. 1** Sketch of rock wedge falling from cavern roof (a) possible position of unstable rock wedges, (b) variables of embedded length and (c) loads generated along rockbolts.

The stable state of rock wedge changes with the stress re-adjustment in the surrounding rock mass during the excavation process [5]. The rock wedge may even be partially loaded by surface forces generated due to the influence from the residual internal pressure [30]. To estimate the magnitudes of the surface forces, the joint relaxation method is proposed by introducing the joint stiffness and wedge displacement caused by joint deformation [23,24]. The required support forces from rockbolts are calculated as the difference between the deadweight of rock wedge and the vertical deriving force [2]. Sensitivity studies are usually used to find out the influences of each significant parameter to the wedge displacement after joint relaxation. However, it will be more efficient if a numerical method could be used [24].

| Parameters | Description | Recommendations and suggestions |
|------------|-------------|--------------------------------|
| Rockbolt length, \( L_b \) | For operation | \( L_b \leq 0.5h \) (Roof); \( L_b \leq 0.5B \) (Wall) |
| | For small depth failure zone | \( L_b \geq d_f + l_0 \) (m) |
| | For moderate depth failure zone | \( L_b = 1.40 + 0.184 B \) |
| | For highly fracture zone | \( L_b = 2 - 3 \) (m) |
| Spacing, \( s_b \) | In general case | \( s_b = 1.0 - 2.5 \) (m) |
| | For slightly jointed rock mass \((e = 0.3 - 1\text{m})\) | \( s_b = (3-4)e \) |
| | For moderately jointed zone \((e < 0.3)\) | \( s_b \leq 3e \) |
| | For highly fracture zone | \( s_b \leq L_b/2 \) |
| Pattern | For gravitational rock falls | Spot bolting |
| | For less deformable rock | Systematic bolting |
| | For squeezing rock | Yield support system |
| Other bolt parameters | Diameters: \( D_b = 16 - 20 \) (mm) | Mechanical / Frictional / others |
| | Bond stiffness | e.g., 339 kN for Dydidag\(^\circ\)R; 500 kN for Flexirope\(^\circ\)R; 110 kN for Split set\(^\circ\)R |
| | Ultimate strength | e.g., Ultimate axial strain, Strain hardening index, etc. |
| | Creep properties | |

Table 1. Parameters for rockbolt design (modified) [24,27].
Note

- $h$ – tunnel height; $B$ – tunnel width; $d_f$ – the depth of the failure zone;
- $l_0$ – Embedded length, empirically $l_0 = 1.2$ m; $e$ – mean joint spacing.

*In practice, the spacing of a systematic bolting design includes the in-row spacing and the spacing between rows. Here, both spacings are assumed equal.

The two-dimensional Discontinuous Deformation Analysis (2D-DDA) method developed by Shi [26] is capable to simulate the static and dynamic behaviors of discrete blocky systems based on block kinematics. Verification studies on the stability of rock blocks using 2D-DDA have been conducted by MacLaughlin and Doolin [19] and Jiao et al. [13]. They have concluded that the solutions of the 2D-DDA matched the limit equilibrium solution in high precision. The simulation results of 2D-DDA could be used to predict the interaction between rock wedge and rockbolt through the force-displacement relationships of rock blocks and compose ground reaction diagram [1, 21, 29].

In this study, the stability of the rock wedges reinforced by rockbolt in tunnel roof is analyzed using 2D-DDA. The stress state changes in the surrounding rock blocks after cavern excavation is represented using the joint relaxation method. Parameter studies are also carried out to investigate the effects of the horizontal pressure, the spacing between rockbolts and the installation angle on the reinforcement effort of the rockbolts for the rock wedge stabilization. A case study is conducted to illustrate the procedure of using the proposed method.

2. Wedge reinforcement in 2D-DDA

2.1. Governing equations

The 2D-DDA method adopts an incremental solution procedure. The dynamic equations of the block system are solved at each time step, while the incremental changes in energy are determined at the same time as the block system attempts to reach equilibrium [26]. The deformation of a single block has six basic variables in the first-order approximation. In each time step, the displacements of a point $(x, y)$ in a block are represented as:

$$\{u_i\} = [T_i]\{D_i\}$$

in which $[T_i]$ is expressed as follows,

$$[T_i] = \begin{bmatrix} 1 & 0 & -(y - y_0) & (x - x_0) & 0 & (y - y_0)/2 \\ 0 & 1 & (x - x_0) & 0 & (y - y_0) & (x - x_0)/2 \end{bmatrix}$$

where $\{D_i\} = (u_0, v_0, r_0, \varepsilon_x, \varepsilon_y, \gamma_{xy})^T$, $i$ is the block ID; $u_0$ and $v_0$ are the rigid body translation at a specific point $(x_0, y_0)$ within the block $i$; $r_0$ is the rotation angle of the block $i$ with respect to $(x_0, y_0)$; $\varepsilon_x$ and $\varepsilon_y$ are the normal strains in the $x$ and $y$ directions, respectively; and $\gamma_{xy}$ is the shear strain.

For a blocky system containing $n$ blocks, the global equilibrium equation in matrix form is shown as follows:

$$[K_y]\{D_i\} = \{F_i\}$$

The matrix $[K_y]$ ($i, j = 1, 2, \ldots, n$) is given as follows:

$$[K_y] = \frac{\partial^2 \Pi}{\partial d_i \partial d_j} \quad (r, s = 1, 2, \ldots, 6)$$

where $\Pi$ is the total potential energy of the whole system; $d_i$ is the displacement vector; $[K_y]$ is a $6 \times 6$ submatrix to represent the contacts between blocks $i$ and $j$ when $i \neq j$ ($i, j = 1, \ldots, n$), and $[K_j]$ is the local stiffness matrix when $i = j$.

The matrix $\{F_i\}$ ($i = 1, 2, \ldots, n$) is a $6 \times 1$ sub-matrix representing the loading on block $i$ distributed to the six deformation variables, and expressed as:

$$\{F_i\} = -\left(\frac{\partial \Pi}{\partial d_i}\right)_{d_i = 0} \quad (r = 1, 2, \ldots, 6)$$
More details of the matrix \([K_{ij}]\) and \(\{F_i\}\) could be found in Hatzor et al. [9] To simulate the reinforcement provided by rockbolts, the sub-matrix of point loads must be considered. Take a point load \((F_x, F_y)\) acting on a point \((x, y)\) for example, its contribution to the submatrix \(\{F_i\}\) is given as:

\[
\begin{bmatrix}
T_x
F_x
T_y
F_y
\end{bmatrix} \to \{F_i\}
\]  

(6)

2.2. Rockbolt model in 2D-DDA
A rockbolt model has been developed in 2D-DDA by Nie [20] to simulate the rockbolt reinforcement. An example of a fully grouted rockbolt installed through rock blocks A, B and C is shown in Fig. 2. The rockbolt is divided into three segments and each segment contains several rockbolt elements. The nodes of the installed rockbolt and rock nodes are designed in the same position. Once the rock block deforms, the relative movements between the nodes of rock and rockbolt are used to calculate their slip displacement. The force developed in the rockbolt elements are updated in each time step. The increase of axial load \(\Delta N_A\), shear load \(\Delta F_{sA}\) and bending moment \(M_A\) of a rockbolt element at a time-step can be derived as:

\[
\Delta N_A = \frac{E_b A_b}{l_e} \Delta U_s
\]  

(7)

\[
\Delta F_{sA} = K_s \Delta U_s
\]  

(8)

\[
\Delta M_A = F_{sA} l_e
\]  

(9)

where: \(E_b\) is the elastic modulus of rockbolt material; \(A_b\) is the cross-sectional area of rockbolt; \(l_e\) is the hinge length of the rockbolt; \(K_s\) is the shear stiffness of the rockbolt element; \(\Delta U_s\) is the shear displacement of rockbolt at joint, and \(\Delta U_i\) and \(\Delta U_b\) are the movements of rock and rockbolt, respectively.

The increase of axial stress \(\Delta \sigma_A\) induced by the axial load and the bending moment is give as:

\[
\Delta \sigma_A = \frac{\Delta N_A}{A} + \frac{\Delta M_A D_b}{2I_b}
\]  

(10)

where: \(D_b\) is the diameter of rockbolt and \(I_b\) is the inertia of rockbolt cross-section.

The Von Mises yield criterion is usually used to describe the yield of rockbolt element and shown as:

\[
\left(\frac{\sigma_A A_b}{N_y}\right)^2 + \left(\frac{F_{sA}}{F_{sA,max}}\right)^2 = 1
\]  

(11)

where \(N_y\) is the yield axial load of rockbolt element under pure tension and \(F_{sA,max} = N_y \sqrt{3}\).

Fig. 2 Element divisions of a rockbolt model installed through three rock blocks.
By assuming the shear stress and moment of rockbolt element do not increase with respect to the increase of shear displacement once the rockbolt element is at yielded stage, the axial stress versus axial strain curve is still linear with its strain-hardening modulus of $E_r$. The failure of the rockbolt element is determined by:

$$\left( \frac{\sigma_{s,y}}{N_f} \right)^2 + \left( \frac{F_{s,y}}{F_{s,max}} \right)^2 = 1$$

(12)

where $F_{s,y}$ is the shear load of a rockbolt element at yielding and $N_f$ is the ultimate axial load of a rockbolt element under pure tension.

Under the combined tension and shear loads, the point loads to the rock blocks induced by rockbolt restraints at a time-step could be calculated as:

$$\begin{bmatrix} F_x \\ F_y \end{bmatrix} = (-\Delta P) \begin{bmatrix} \cos \theta \\ \sin \theta \end{bmatrix} + (-\Delta F_{s,y}) \begin{bmatrix} \sin \theta \\ \cos \theta \end{bmatrix}$$

(13)

where: $\Delta P$ is the loading induced by bonding at a rockbolt node and expressed as,

$$\Delta P = kl_i \left( \Delta U_{b} - \Delta U_{r} \right)$$

(14)

where: $l_i$ is the length of a rockbolt element and $k = k_0 \pi D_b$, $k_0$ is the shear stiffness at the rockbolt and rock interface.

The point load matrix as shown in Eqs. (13) and (14) will be added to the submatrix of $\{ F_i \}$ in Eq. (6) in the multi-step procedure to simulate the reinforcements of rockbolt onto the rock blocks.

3. Numerical model

For 2D problems, the geometry of rock wedge is usually assumed to be unchanged in the direction perpendicular to the rock discontinuity [30]. Therefore, it is assumed in this study that the forces acting on the wedge only allow the block movement parallel to the discontinuity. The rock wedge is only loaded by its own weight, surface forces, and the support load generated by rockbolts. The hydraulic water pressure is not considered in this study.
3.1. 2D-DDA model

A symmetric rock wedge with height of 2.0 m and semi-apical angle of 30° is hanging on the bottom of rock mass as shown in Fig. 3a. The surrounding rock mass are simulated using two blocks with their height of 6.0 m and width of 5.0 m. The joints between the wedge and two surrounding rock blocks are purely frictional. The three rock blocks have same engineering properties, such as density $\rho = 2.6 \times 10^3$ kg/m$^3$, elastic modulus $E = 30$ GPa and Poisson’s ratio $\nu = 0.1$. Two rigid blocks are added at the two sides of the rock mass to supply the horizontal confining pressure. Roller supports are applied at the bottom of the rock blocks to restrain their vertical displacements.

The numerical analysis is conducted in three steps. The first step is to progressively apply lateral confining pressure $p$ onto the two lateral rigid blocks to achieve the initial equilibrium stress state. The second step is to remove the roller at the bottom of rock wedge to simulate the excavation process. During this process, the deformation of the joint defining the rock wedge is taken account. As shown in Fig. 3b, the contact forces at points A to D between the rock wedge and the surrounding rock blocks are recorded. It should be noted that points A and B are used to separate the contacts of the top vertex to the right and left joints, respectively. The third step is to add two symmetrical rockbolts into the model right after the second step to support the rock wedge. As shown in Fig. 3c, the length, embedded length,
inclined angle and spacing of rockbolt are denoted as $L_b$, $l_0$, $\theta$ and $s_b$, respectively. To provide enough anchorage force, the embedded length of a rockbolt is designed as $l_0 = 1.2 \, \text{m}$ as suggested by empirical method given by Li [14]. The length of the rockbolt element in the rock is set as $0.1 \, \text{m}$, while that near the rock joint is set as $2l_e$. The parameter setting is listed in Table 2.

### 3.2. Model verification

To verify the accuracy of the wedge model with no rockbolt, the simulation result is compared with that from analytical solution given by Brandy and Brown [2]. Before relaxation progress as shown in Fig. 4a, the force equilibrium in the vertical direction gives the vertical deriving load $P$ of the rock wedge as:

$$ P = 2N_0 \sec \phi \sin (\phi - \alpha) $$

where: $P$ is the vertical deriving load; $N_0$ is the surface forces normal to the joint, assume $N_0 = N_{0,1} = N_{0,2}$; $S_0$ is the surface forces tangent to the joint; and $S_0 = S_{0,1} = S_{0,2}$; $\alpha$ is the wedge semi-apical angle of joint, $\alpha = \alpha_1 = \alpha_2$, and $\phi$ is the friction angle of joint, $\phi = \phi_1 = \phi_2$.

### Table 2. Parameters used in 2D-DDA model

| Item                  | Parameter | Values | Item                  | Parameter | Values |
|-----------------------|-----------|--------|-----------------------|-----------|--------|
| Rock                  | Elastic modulus $E_r$, GPa: | 30     | Diameter $D_b$, mm    | 20        |
| Poisson ratio, $v$    | 0.1       |        | Elastic modulus $E_b$, GPa | 210     |
| Unit weight $\gamma_r$, $\times 10^3 \, \text{kN/m}^3$ | 26.0      |        | Yield axial strength, kN | 180     |
| 2D-DDA calculation    | Step max. displacement ratio | 0.0004 | Ultimate axial strength, kN | 217     |
|                       | Upper limit of time interval | 0.0002 | Hinge length $l_e$, mm | 40       |
|                       | SOR factor | 1.4    | Shear stiffness $K_s$, MN/m | 67.3     |

By introducing the joint stiffness $k_u$ and $k_s$ as shown in Fig. 4b, the limiting vertical load $P_l$ is calculated as:

$$ P_l = 2 M \tan \alpha_0 / D $$

where: $H_0$ is the horizontal force before joint relaxation, $H_0 = N_0^2 + S_0^2$ and $D$ and $M$ are constants which are calculated as $D = k_u \cos \alpha \cos \phi + k_s \sin \alpha \sin \phi$ and $M = (k_s \sin^2 \alpha + k_u \sin^2 \phi)$.

As listed in Table 3, the rock wedge is potentially stable when $P_l > W$ where $W$ is the dead weight of the wedge if the allowable displacement is not considered. When $P_l < W$, the rock wedge is not stable anymore. The required support force from rockbolt is $R = W - P_l$. The critical state for stable analysis of the rock wedge is $P_l = W$. By assuming the horizontal force before joint relaxation at this critical state as $H_0 = p_c H$, the critical horizontal confining pressure $p_c$ is calculated as:

$$ p_c \geq \frac{W D}{2 M H \sin (\phi - \alpha)} $$

### Table 3. Roof wedge stability analysis based on the analytical solutions.

| Case | Semi-apical angle, $\alpha$ | Internal horizontal force, $H_0$ | $P_l$ | Support force, $R$ | Displacement, $u_c$ |
|------|-----------------------------|----------------------------------|------|-------------------|---------------------|
| A    | $\phi$ close to $\alpha$ and $\alpha \leq \phi$ | Low | $0 < P_l \leq W$ | Any decrease of $\phi$ and decrease of $H_0$ will result in the increase of required support forces. | Potentially stable if sufficiently support provided. |
\[
\begin{array}{|c|c|c|c|c|}
\hline
B & \alpha \leq \varphi & \text{Low} & P_l > W & \text{No or less support required (for safety).} \\
\hline
C & \alpha \leq \varphi & \text{High} & P_l > W & \text{No support required if the displacement is allowable; Require reinforcement to increase the joint stiffness if the joint displacement is large.} \\
\hline
\end{array}
\]

Note:
- A larger \( P_l \) indicates a more stable condition.
- \( u_t \) affected by \( H_0 \) and the joint stiffness, and
- \( H_0 \) creates confining on the rock wedge

Assume the joint has an area of \( A \), the normal reaction forces due to the joint deformation could be calculated as:

\[
R_n = \bar{k}_n A d_n
\]

(18)

where: \( d_n \) is the normal deformation of a joint; \( R_n \) is the normal reaction forces, and \( \bar{k}_n \) is the joint stiffness at the normal directions.

Similarly, we have the expression of the shear reaction force and shown as:

\[
|R_s| = \bar{k}_s A |d_s|
\]

(19)

where: \( d_s \) is the shear deformation of a joint; \( R_s \) is the shear reaction forces, and \( \bar{k}_s \) is the joint stiffness at the shear directions.

The critical horizontal pressure, \( p_{cr} \), versus friction angle, \( \varphi \), curves are calculated using Eq. (17) and plotted in Fig. 5. It shows for a certain rock wedge, the critical horizontal confining pressure \( p_{cr} \) is varied. The ratio of joint stiffness on the critical horizontal confining pressure could also be considered as:

\[
\frac{\bar{k}_n}{\bar{k}_s} = \frac{R_n/d_n}{R_s/d_s} = \frac{k_n}{k_s}
\]

(20)

Fig. 4. Free-body diagrams of a 2D roof wedge in (a) rigid joint condition and (b) relaxed joint condition (modified) [2].
Fig. 5. Comparisons of the critical horizontal pressures obtained from numerical and analytical solutions.

During the simulation, the boundary of joint will be pushed back as no-penetration and no-tension are allowed between blocks in 2D-DDA. The shear contact stiffness $k_s$ is assumed as $0.4k_n$. Three normal contact spring stiffness $k_n$ are adopted in the numerical simulations to analyse the critical horizontal confining pressure after joint relaxation, i.e., $k_n = 3 \times 10^9$, $30 \times 10^9$ and $300 \times 10^9$ N/m, or $1.3E_r L$, $13E_r L$ and $130E_r L$, respectively, where $E_r$ is the Young’s modulus of the rock block and $L$ is the length of the line across which the contact springs are attached. The dimension of the joint out-of-plane is assumed as 1 unit. The results of numerical analysis agree reasonably well with those obtained from analytical solutions, especially for the simulation using normal contact stiffness $k_n = 3 \times 10^9$ N/m. The mismatch between the two results is because the unsymmetrical contacts forces are generated during the simulation. This could be further explained using the numerical models with friction angle $\phi = 40^\circ$ and three contacts. The magnitudes of the displacements in the 2nd step decrease with the increase of contact stiffness as shown in Fig. 6. The model with a normal contact stiffness $k_n = 3 \times 10^9$ N/m shows progressive displacement before the new balance achieves. The normal contact force versus the time step curves at the contact points A to D are plotted in Fig. 7a. Even the load is supposed to be applied linearly at the 1st time step, the obtained curves are not purely linear. The contact forces are symmetrically distributed. The differences between upper contact points (A and B) and lower contact points (C and D) are limited. For the case of $k_n = 30 \times 10^9$ N/m (see Fig. 7b), the normal contact forces increase linearly with the increase of time step during loading time. In both stages, the normal contact forces are symmetric. However, the contact forces of the upper contacts (A and B) are less than those of the bottom contacts (C and D). In case of $k_n = 300 \times 10^9$ N/m (see Fig. 7c), the differences between the normal forces at upper contact points (A and B) and those at bottom contact points (C and D) become larger. The forces at the upper contact points are only 70% of those of the bottom contacts. It also shows that the contact forces have oscillations in the simulations leading to unsymmetrical loadings at two sides of rock wedge. This might be the reason that a block model with soft contacts (i.e., $k_n = (3 ~ 30) \times 10^9$ N/m or $(1.3 ~ 13) \times E_r L$) are resulted in a close agreement with the analytical solutions.
Fig. 6 Effects of contact stiffness on the vertical displacements versus time curves.

Fig. 7 Effects of the normal contact stiffness on the normal contact forces versus time step ($\phi = 40^\circ$) (a) $k_n = 3 \times 10^9$ N/m, (b) $k_n = 30 \times 10^9$ N/m and (c) $k_n = 300 \times 10^9$ N/m.

3.3. Parametric study

Parametric studies are carried out to investigate the key parameters that might influence the reinforced effects of the rockbolts when they are used to stabilize the rock wedge. The 2D-DDA model with the contact stiffness $k_n = 3 \times 10^9$ N/m and $k_s = 0.4k_n$ is adopted to assess the stability of rock wedge. The friction angle of the joints between the wedge and two surrounding rock blocks are assumed as 40°. The variables are the initial horizontal pressure ($p$), the spacing between two rockbolts ($s_b$) and the incline angles of the rockbolt with respect to the direction of rock falling ($\theta$).

The confining pressure $p = 20, 40, 60$ and 80 kPa, and constant $s_b = 2.0$ m and $\theta = 0^\circ$ are used to investigate their effects on the critical confining pressure. As shown in Fig. 8a, the rockbolts are slightly loaded due to the joint displacement if the horizontal pressure $p_p > p_{cr}$, which means the rock wedge could be self-supported after stress redistribution. However, for $p \leq p_{cr}$, the rock wedge is unstable. The support force provided by each rockbolt is about half of the wedge weight (30 kN). The horizontal stress in the wedge after stress re-balance is about 6.2 kPa. Three spacing between the two rockbolts i.e., $s_b = 0.5$, 1.0 and 2.0 m with $p = 80$ kPa ($p < p_{cr} = 93$ kPa) and $\theta = 0^\circ$ are adopted to analyze the effects of the spacing. The variations of rockbolt force from the simulation results are shown in Fig. 8b. The axial load at the rockbolt element is decreasing with the decrease of spacing ($s_b$). The support force of each rockbolt when $s_b = 0.5$ m is about 77% of that when $s_b = 2.0$ m. The horizontal stress in the wedge after stress re-balance in the former case is about 27% of that of the latter case. This could be explained as the rockbolts with narrow spacing induce less deformation in the deformable wedge. Two cases are analyzed to investigate the effects of incline angles of the rockbolt with respect to the direction of rock falling.
falling. For $p = 80$ kPa ($< p_{cr} = 93$ kPa) and $s_b = 1.0$ m, the required support force when $\theta = 45^\circ$ will increase with a magnifying factor of $\sec(\theta)$ from that when $\theta = 0^\circ$, as shown in Fig. 8c.

![Graphs showing the influence of confining pressure, bolt spacing, and angle on rockbolt performance](image)

**Fig. 8** Parameter studies of the influences of (a) the horizontal confining pressure $p$, (b) the bolt spacing $s_b$, and (c) the angle $\theta$ on the rockbolt performances.

### 4. Conclusions

Unstable wedges might be exposed on the roof of the underground excavation. 2D-DDA is used to evaluate the rockbolt design to stabilize roof wedges in underground opening. The joint relaxation method is adopted to represent a deformable rock mass under stresses. The numerical model is calibrated...
using the analytical solution. Parametric studies are carried out to investigate the key parameters that might influence the effects of rockbolts when they are used to stabilize the rock wedge. The numerical results show 2D-DDA could be used to find the critical horizontal pressure to sustain the rock wedges using the contact stiffness \( k_s = (3 \sim 30) \times 10^6 \) N/m or \((1.3 \sim 13)\times E\times L\) and \(k_n = 0.4k_s\) where \(E\) is the Young’s modulus of the rock block and \(L\) is the length of the line across which the contact springs are attached. The wedge could be self-supported after stress redistribution if the horizontal pressure \(p > p_{cr}\) and \(\alpha \leq \varphi\). The rockbolts are slightly loaded due to joint displacement. It also shows the bolt force slightly decreases when the spacing between two rockbolts decreases. However, the bolt force increases if there is shear components during the wedge deformation.

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Reference

[1] Aydin A, Ozbek A, Acar A (2014) Geomechanical characterization, 3-D optical monitoring and numerical modeling in Kirkcedge-1 tunnel, Turkey. Engineering Geology 181: 38-47. https://doi.org/10.1016/j.enggeo.2014.08.010.

[2] Brady BGH, Brown ET (2006) Rock mechanics for underground mining. 3rd edition. Springer Netherlands, Dordrecht, pp 242-270. https://doi.org/10.1007/978-1-4020-2116-9.

[3] Chen Y, Li CC (2015) Performance of fully encapsulated rebar bolts and D-Bolts under combined pull-and-shear loading. Tunnelling and Underground Space Technology 45: 99-106. http://dx.doi.org/10.1016/j.tust.2014.09.008.

[4] Christaras B (2003) Support capacity of wedges along tunnels of Egnatia highway. Engineering Geology 68: 361-372. https://doi.org/10.1016/S0013-7952(02)00240-5.

[5] Dwivedi RD, Singh M, Viladkar MN, Goel RK (2014) Estimation of support pressure during tunnelling through squeezing grounds. Engineering Geology 168: 9-22. https://doi.org/10.1016/j.enggeo.2013.10.020.

[6] Gerdeen JC, Snyder VW, Viegelman GLUO (1981) Design criteria for roof bolting plans using fully resin-grouted nontensioned bolts to reinforce bedded mine roof. Volume 5. Synthesis and design criteria: 46(5)-80. 22 July 1977. 129P. International Journal of Rock Mechanics and Mining Sciences & Geomechanics Abstracts 18: 38. https://doi.org/10.1016/0148-962(81)90937-2.

[7] Goodman RE, Shi GH (1985) Block theory and its application to rock engineering. Englewood Cliffs, N.J.: Prentice-Hall.

[8] Grasselli G (2005) 3D Behaviour of bolted rock joints: experimental and numerical study. International Journal of Rock Mechanics and Mining Sciences 42: 13-24. http://dx.doi.org/10.1016/j.ijrmms.2004.06.003.

[9] Hatzor Y, Ma G, Shi G (2017). Discontinuous deformation analysis in rock mechanics practice. Leiden, The Netherlands, CRC Press/Balkema.

[10] Hoek E, Kaiser PK, Bawden WF (1995) Support of underground excavations in hard rock. Rotterdam, Netherlands; Brookfield, VT, USA: A.A. Balkema.

[11] Itasca Consulting Group (2011) UDEC Ver3.0, Special features. Mineapolis: Itasca Consulting Group, Inc.

[12] Jalalifar H, Aziz N, Hadi M (2006) The effect of surface profile, rock strength and pretension load on bending behaviour of fully grouted bolts. Geotechnical and Geological Engineering 24: 1203-1227. http://doi.org/10.1007/s10706-005-1340-6.

[13] Jiao YY, Zhang HQ, Tang HM, Zhang XL, Adoko AC, Tian HN (2014) Simulating the process of reservoir-impoundment-induced landslide using the extended DDA method. Engineering Geology 182: 37-48. https://doi.org/10.1016/j.enggeo.2014.08.016.

[14] Li CC (2017) Principles of rockbolting design. Journal of Rock Mechanics and Geotechnical Engineering 9: 396-414. https://doi.org/10.1016/j.jrmge.2017.04.002.

[15] Li CC, Kristjansson G, Høien AH (2016) Critical embedment length and bond strength of fully encapsulated rebar rockbolts. Tunnelling and Underground Space Technology 59: 16-23. http://dx.doi.org/10.1016/j.tust.2016.06.007.
[16] Li X, Nemcik J, Mirzaghorbanali A, Aziz N, Rasekh H (2015) Analytical model of shear behaviour of a fully grouted cable bolt subjected to shearing. International Journal of Rock Mechanics and Mining Sciences 80: 31-39. http://dx.doi.org/10.1016/j.ijrmms.2015.09.005.

[17] Low BK, Einstein HH (2013) Reliability analysis of roof wedges and rockbolt forces in tunnels. Tunnelling and Underground Space Technology 38: 1-10. https://doi.org/10.1016/j.tust.2013.04.006.

[18] Ma S, Zhao Z, Nie W, Zhu X (2017) An Analytical Model for Fully Grouted Rockbolts with Consideration of the Pre- and Post-yielding Behavior. Rock Mechanics and Rock Engineering 50: 3019-3028. http://doi.org/10.1007/s00603-017-1272-5.

[19] MacLaughlin MM, Doolin DM (2006) Review of validation of the discontinuous deformation analysis (DDA) method. International Journal for Numerical and Analytical Methods in Geomechanics 30: 271-305. https://doi.org/10.1002/nag.427.

[20] Nie W (2019) Reinforcement mechanism of rockbolt system for underground excavation. Ph.D., Nanyang Technological University, Singapore. https://www.doi.org/10.32657/10220/49777.

[21] Nie W, Zhao ZY, Ma SQ, Guo W (2018) Effects of joints on the reinforced rock units of fully-grouted rockbolts. Tunnelling and Underground Space Technology 71: 15-26. https://doi.org/10.1016/j.tust.2017.07.005.

[22] Nie W, Zhao ZY, Ning YJ, Guo W (2014) Numerical studies on rockbolts mechanism using 2D discontinuous deformation analysis. Tunnelling and Underground Space Technology 41: 223-233. http://dx.doi.org/10.1016/j.tust.2014.01.001.

[23] Nomikos PP, Sofianos AI, Tsoutrelis CE (2002) Structural response of vertically multi-jointed roof rock beams. International Journal of Rock Mechanics and Mining Sciences 39: 79-94. http://dx.doi.org/10.1016/S1365-1609(02)00019-9.

[24] Nomikos PP, Yiouta-Mitra PV, Sofianos AI (2006). Stability of asymmetric roof wedge under non-symmetric loading. Rock Mechanics and Rock Engineering 39: 121-129. https://doi.org/10.1007/s00603-005-0058-3.

[25] Pellet F, Egger P (1996) Analytical model for the mechanical behaviour of bolted rock joints subjected to shearing. Rock Mechanics and Rock Engineering 29: 73-97. https://doi.org/10.1016/0048-6505(96)00072-0.

[26] Shi GH (1988) Discontinuous deformation analysis: A new numerical model for the statics and dynamics of block systems. Ph.D., University of California, Berkeley.

[27] Stillborg B (1986) Professional users handbook for rock bolting. Trans Tech Publications.

[28] Tsesarsky M, Hatzor YH (2006) Tunnel roof deflection in blocky rock masses as a function of joint spacing and friction – A parametric study using discontinuous deformation analysis (DDA). Tunnelling and Underground Space Technology 21: 29-45. https://doi.org/10.1016/j.tust.2005.05.001.

[29] Windsor CR (1997) Rock reinforcement systems. International Journal of Rock Mechanics and Mining Sciences 34: 919-951. http://dx.doi.org/10.1016/S1365-1609(97)80004-4.

[30] Wittke W (2014) Stability of Rock Wedges and Excavation Surfaces. In: Wittke, W. (ed.) Rock Mechanics Based on an Anisotropic Jointed Rock Model (AJRM), pp 288-329.