The GUT? Neutrino bi-large mixing and proton decay

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Abstract

In this talk, we introduce a new scenario of grand unified theory (GUT) with anomalous $U(1)_A$ gauge symmetry, which can explain doublet-triplet splitting, quark and lepton masses and mixing angles. In neutrino sector, the scenario realizes LMA solution for solar neutrino problem and large $U_{e3} = O(0.1)$. Moreover, the scenario predicts that the main decay mode of proton is from dimension 6 operators and the lifetime of proton must be near the present limit. The realization of gauge coupling unification requires that the cutoff scale of the scenario must be around the usual GUT scale $\Lambda_G \sim 2 \times 10^{16}$ GeV, which is smaller than the Planck scale. It may suggest the extra dimension in which gauge fields in visible sector do not propagate. This talk is based on the papers.\[1, 2, 3, 4, 5\]

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1 Introduction

The most of people regard the standard model as the real theory which describe our world but does not satisfy the model as the final theory, because there are a lot of things which are not explained by the model; unstability of the weak scale due to quadratic divergent loop correction to the Higgs mass term: the miracle anomaly cancellation between quark and leptons: the origin of hierarchies of gauge and Yukawa couplings: the origin of small mixings in quark sector and large mixings in lepton sector: the charge quantization: no gravity, etc. The idea of grand unified theories (GUT) not only explain the hierarchy of three gauge couplings in the standard model, anomaly cancellation and charge quantization, but also gives a natural unification of quark and leptons in a few multiplets in a simple gauge group. Since supersymmetry (SUSY) can stabilize the weak scale, SUSY GUT is one of the most promising model beyond the standard model. Unfortunately, it is not so easy to obtain the realistic SUSY GUT, because it is difficult to solve the doublet-triplet splitting problem with stable proton and to obtain realistic quark and lepton mass matrices. On the other hand, it is known that the hierarchy of Yukawa couplings in the supersymmetric standard model are realized by introducing anomalous $U(1)$ gauge symmetry, whose anomaly is cancelled by the Green-Schwarz mechanism. Of course, it is not so straightforward to extend the argument in the SUSY standard model into in the GUT scenario, especially with large neutrino mixing angles, but it is important to examine the GUT scenario with anomalous $U(1)_A$ gauge symmetry.

Recently, in a series of papers, the interesting GUT scenario with anomalous $U(1)_A$ gauge symmetry has been proposed with $SO(10)$ unified group and with $E_6$ unified group. In the scenario, the anomalous $U(1)_A$ gauge symmetry plays an important role not only in obtaining realistic quark and lepton mass matrices, including bi-large neutrino mixings (LMA for solar neutrino problem) but also in solving doublet-triplet splitting problem. Moreover, since generic interactions are allowed to be introduced, it predicts the mass spectrum of superheavy fields and (GUT) symmetry breaking scales once the symmetry of the theory is fixed. It is surprising that the success of coupling unification in the minimal SUSY standard model (MSSM) can be naturally explained in the scenario, though the mass spectrum of superheavy fields does not respect $SU(5)$ symmetry. It is shown that the gauge coupling unification requires the cutoff scale must be around the usual GUT scale $\Lambda_G = 2 \times 10^{16}$ GeV and the unification scale is just below the usual GUT scale if all the fields except those of MSSM have superheavy masses. It is interesting that this result is independent of the detail of Higgs sector. The GUT with anomalous $U(1)_A$ gauge symmetry with a simple unified gauge group predicts the above result. Therefore, proton decay via dimension 6 operators may be seen in near future experiments. Moreover, once SUSY breaking parameters are introduced, the $\mu$ problem is naturally solved.
and natural suppression of flavor changing neutral current (FCNC) is realized in $E_6$ GUT.\[4\]

As introduced in the above, the scenario predicts the proton decay via dimension 6 operators even though low energy SUSY is required. Moreover, bi-large neutrino mixings are obtained (especially, LMA solution for the solar neutrino problem is predicted). The scenario predicts large $U_{e3} \sim O(0.1)$ and small $\tan \beta$.

It is interesting that all the above solutions are realized in non-trivial ways once only several anomalous $U(1)_A$ charges are determined. Actually, the input parameters are only 8 integer anomalous $U(1)_A$ charges ($+3$ for singlet Higgs) for the Higgs sector and 3(or 4) (half) integer charges for the matter sector in $E_6$ (or SO(10)) GUT. In this talk, we will explain some of them.

## 2 Doublet-triplet splitting

One of the most interesting feature of anomalous $U(1)_A$ gauge theory is that the vacuum expectation values (VEV) are determined by anomalous $U(1)_A$ charges as

\begin{align}
\langle Z^+ \rangle &= 0, \\
\langle Z^- \rangle &\sim \lambda^{-z^{-}},
\end{align}

where $Z^\pm$ are singlet operators with the charges $z^+ > 0$ and $z^- < 0$, and $\lambda = \langle \Theta \rangle / \Lambda$. Here $\Theta$ is a Froggatt-Nielsen field.\[18\] Through this paper, we use unit in which the cutoff $\Lambda = 1$ and denote all the superfields by uppercase letters and their anomalous $U(1)_A$ charges by the corresponding lowercase letters. Such VEVs do not change the order of the coefficients obtained by the Froggatt-Nielsen mechanism:

\begin{equation}
W = \left(\frac{\Theta}{\Lambda}\right)^{x+y+z} XYZ \to \lambda^{x+y+z} XYZ,
\end{equation}

if the total charge $x + y + z$ of the operator $XYZ$ is positive. Note that even if the operator $\frac{Z^-}{\Lambda}$ is used instead of $\left(\frac{\Theta}{\Lambda}\right)^{z^-}$ in the interactions, the order of the coefficients does not change. This feature is critically different from the naive expectation that the contribution from the higher dimensional operators is more suppressed. If the total charge $x + y + z$ is negative, such interaction is not allowed by the anomalous $U(1)_A$ gauge symmetry because only negatively charged fields have non-vanishing VEVs. This is called SUSY zero mechanism. Note that this mechanism leads to the finite number of non-renormalizable interactions, and therefore we can control the generic superpotential.

Actually, under the vacua (2.1), the generic superpotential to determine the VEVs of $Z^-$ can be written as

\begin{equation}
W = \sum_{i}^{n_+} W_{Z^+_i},
\end{equation}

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Table I. Typical values of anomalous $U(1)_A$ charges.

|  | non-vanishing VEV | vanishing VEV |
|---|---|---|
| 45 | $A(a = -1, -)$ | $A'(a' = 3, -)$ |
| 16 | $C(c = -4, +)$ | $C'(c' = 3, -)$ |
| $\bar{16}$ | $\bar{C}(\bar{c} = -1, +)$ | $\bar{C}'(\bar{c}' = 6, -)$ |
| 10 | $H(h = -3, +)$ | $H'(h' = 4, -)$ |
| 1 | $\Theta(\theta = -1, +), \bar{Z}(z = -2, -)$, $\bar{Z}(\bar{z} = -2, -)$ | $S(s = 5, +)$ |

where $W_X$ denotes the terms linear in the $X$ field. This is because the $F$-flatness conditions of negatively charged fields are automatically satisfied and the terms with more than two positively charged fields do not contribute in the $F$-flatness condition of positively charged fields.

Let us discuss an $SO(10)$ GUT model with anomalous $U(1)_A$ gauge symmetry in which doublet-triplet splitting is naturally realized. The Higgs content is listed in Table I. Here the symbols ± denote the $Z_2$ parity. The VEVs of the negatively charged Higgs fields are determined by the superpotential

$$W = W_{A'} + W_{C'} + W_{\bar{C}'} + W_{H'} + W_S.$$  (2.5)

We do not have spaces enough to explain the vacuum structure in detail, so we here point out only one good feature in realizing the doublet-triplet splitting.

If $-3a \leq a' < -5a$, the superpotential $W_{A'}$ is in general written

$$W_{A'} = \lambda a'^+ \alpha A'A + \lambda a'^+3a(\beta(A'A)_1(A^2)_1 + \gamma(A'A)_{54}(A^2)_{54}),$$  (2.6)

where the suffices 1 and 54 indicate the representation of the composite operators under the $SO(10)$ gauge symmetry, and $\alpha$, $\beta$ and $\gamma$ are parameters of order 1. Here we assume $a + a' + c + \bar{c} < 0$ to forbid the term $CA'AC$, which destabilizes the DW form of the VEV $\langle A \rangle$. The $D$-flatness condition requires the VEV $\langle A \rangle = i\tau_2 \times \text{diag}(x_1, x_2, x_3, x_4, x_5)$, and the $F$-flatness conditions of the $A'$ field requires $x_i(-\alpha \lambda^{-2a} + (2\beta - \frac{4}{3})(\sum_j x_j^2) + \gamma x_i^2) = 0$. This allows only two solutions, $x_i^2 = 0$ and $x_i^2 = \frac{\alpha}{(1-\frac{4}{3})\gamma + 2N\beta}\lambda^{-2a} = v^2$. Here $N = 0 - 5$ is the number of solutions $x_i = v$. When $N = 3$, the vacuum becomes $\langle A(45) \rangle_{B-L} = \tau_2 \times \text{diag}(v, v, v, 0, 0)$, which breaks $SO(10)$ into $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ at the scale $\Lambda_A \equiv \langle A \rangle \sim \lambda^{-a}$. This Dimopoulos-Wilczek form of the VEV plays an important role in solving the DT splitting problem. Actually through the interaction $W = H'AH$, the DW type of the VEV gives superheavy masses only to the triplet Higgs, and therefore the doublet Higgs remains massless. Taking account of the mass term $H'^2$, only one pair of Higgs doublets becomes massless.

Note that the higher terms $A'A^{2L+1}$ ($L > 1$) are forbidden by the SUSY zero mechanism. If they were allowed, the number of possible VEVs other than the
DW form would become larger, and thus it would become less natural to obtain the DW form. This is a critical point of this mechanism, and the anomalous $U(1)_A$ gauge symmetry plays an essential role in forbidding the undesired terms. The spinor Higgs fields $C$ and $\bar{C}$ break $SU(2)_R \times U(1)_{B-L}$ into $U(1)_Y$ by developing $\langle C \rangle = \langle \bar{C} \rangle \equiv \Lambda_C \sim \lambda^{-(c+\bar{c})/2}$. Then this model becomes MSSM at a low energy scale.

3 Quark and lepton mass matrices

One of the most attractive features of grand unified theory is to unify the quark and lepton into fewer multiplets. For example, in $SO(10)$ GUT scenario, a $16$ representation field contains one family quark and lepton fields including right-handed neutrino field. However, this attractive feature directly leads to unrealistic Yukawa relations. For example, if we introduce $3$ right-handed neutrino field, this attractive feature directly leads to unrealistic Yukawa relations. We have to pick up the VEV $\langle C \rangle$ in the Yukawa matrices to avoid the former unrealistic relation $Y_u = Y_d$, and the VEV $\langle A \rangle$ to avoid the latter unrealistic relation $Y_d = Y_e$. In our scenario, we introduce an additional matter field $T(10)$. Then after breaking the GUT gauge group into the standard model gauge group, one pair of vector-like fields $5$ and $\bar{5}$ of $SU(5)$ becomes massive. The mass matrix is obtained from the interaction

$$ W = Y_{ij} \Psi_i \Psi_j H $$

(3.1)
as

$$ 5_T (\lambda^t \psi_1 + (c-\bar{c})/2, \lambda^t \psi_2 + (c-\bar{c})/2, \lambda^t \psi_3 + (c-\bar{c})/2, \lambda^{2t}) \left( \begin{array}{c} \bar{5}_{\psi_1} \\ \bar{5}_{\psi_2} \\ \bar{5}_{\psi_3} \\ \bar{5}_T \end{array} \right) \right),

(3.3)$$

where actually the VEV $\langle C \rangle = \langle \bar{C} \rangle \sim \lambda^{-(c+\bar{c})/2}$ appear in the mass matrix. Since $\psi_3 < \psi_2 < \psi_1$, the massive mode $5_M$, the partner of $5_T$, must be either $5_{\psi_3}(\Delta \equiv 2t + (t + \psi_3 + (c-\bar{c})/2) > 0)$ or $5_T(\Delta < 0)$. The former case is interesting, and in this case, the three massless modes $(\bar{5}_{\psi_1} + \lambda^{\psi_1-\psi_2} 5_{\psi_3}, 5_T + \lambda^\Delta 5_{\psi_3}, 5_{\psi_2} + \lambda^{\psi_2-\psi_3} 5_{\psi_3})$ can be written. If we adopt their charges $(\psi_1, \psi_2, \psi_3, t) = (9/2, 7/2, 3/2, 5/2)$ in addition to the charges of Higgs fields, then we can estimate quark and lepton mass matrices as

$$ M_u = \left( \begin{array}{ccc} \lambda^6 & \lambda^5 & \lambda^3 \\ \lambda^5 & \lambda^4 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{array} \right) \langle H_u \rangle, \ M_d = M_e = \lambda^2 \left( \begin{array}{ccc} \lambda^4 & \lambda^{3.5} & \lambda^3 \\ \lambda^3 & \lambda^{2.5} & \lambda^2 \\ \lambda^1 & \lambda^{0.5} & 1 \end{array} \right) \langle H_d \rangle.$$

(3.4)
And for the neutrino sector, we take into account the interaction
\[ \lambda \psi_i \psi_j + 2 \bar{\psi}_i \bar{\psi}_j \bar{C} C, \]
which lead to the right-handed neutrino masses
\[ M_R = \lambda \psi_i \psi_j + 2 \bar{\psi}_i \bar{\psi}_j \bar{C} C \]
\[ = \lambda^{2n+\epsilon-c} \left( \begin{array}{ccc} \lambda^6 & \lambda^5 & \lambda^3 \\ \lambda^5 & \lambda^4 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{array} \right). \]
(3.6)

Since the Dirac neutrino mass is given by
\[ M_{\nu D} = \lambda^{4/3.5} \left( \begin{array}{ccc} \lambda^{4} & \lambda^{3} & \lambda \\ \lambda^{3.5} & \lambda^{2.5} & \lambda^{0.5} \\ \lambda^{2} & \lambda & 1 \end{array} \right) \langle H_u \rangle \eta, \]
(3.7)
the neutrino mass matrix is obtained by the seesaw mechanism as
\[ M_{\nu} = M_{\nu D} M_{R}^{-1} M_{\nu D}^T = \lambda^{4-2n+\epsilon-c} \left( \begin{array}{ccc} \lambda^2 & \lambda^{1.5} & \lambda \\ \lambda^{1.5} & \lambda & \lambda^{0.5} \\ \lambda & \lambda^{0.5} & 1 \end{array} \right) \langle H_u \rangle^2 \eta^2. \]
(3.8)

Note that the ratio \( \frac{m_{\nu e}}{m_{\nu \tau}} \sim \lambda \) is realized, that predicts LMA solution for the solar neutrino problem. It is interesting that we obtain the small mixing angles for the Cabibbo-Kobayashi-Maskawa matrix
\[ U_{\text{CKM}} = \left( \begin{array}{ccc} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{array} \right), \]
(3.9)
and the large mixing angles for the Maki-Nakagawa-Sakata matrix
\[ U_{\text{MNS}} = \left( \begin{array}{ccc} 1 & \lambda^{0.5} & \lambda \\ \lambda^{0.5} & 1 & \lambda^{0.5} \\ \lambda & \lambda^{0.5} & 1 \end{array} \right). \]
(3.10)

Since we use a rule that \( \lambda^{0.5} + \lambda^{0.5} \sim \lambda^{0.5} \) in calculating the MNS matrix and \( \lambda^{0.5} \sim 0.5 \), this model gives large mixing angles \([19, 20]\) for the atmospheric neutrino problem and for the solar neutrino problem. And \( U_{e3} \sim \lambda \) is predicted, which is around the present upper limit given by CHOOZ.\([21]\) At this stage, the unrealistic GUT relation \( Y_d = Y_e \) still remains. However, in our scenario, the same amount of the Yukawa couplings are given by the higher dimensional interactions
\[ W = \lambda \psi_i \psi_j + na + h \bar{\psi}_i A^n \bar{\psi}_j H \]
(3.11)
by developing the VEV \( \langle A \rangle \sim \lambda^{-a} \). It is critical that the Yukawa couplings from the higher dimensional interactions have not kept the unrealistic GUT relation. Usually, the corrections from such higher dimensional interactions are suppressed by the factor \( \frac{\langle A \rangle}{\lambda} \). But in our scenario, the suppression factor \( \frac{\langle A \rangle}{\lambda} \) is just cancelled by the enhancement factor \( \lambda^a \) in the coefficients, and therefore we can obtain the same order coefficients as from the tree interaction. This is an attractive feature in our scenario, and the realistic mass matrices are naturally obtained.
4 Gauge coupling unification

First, we show the relation of the determinants of the mass spectrum of superheavy fields in terms of their anomalous $U(1)_A$ charges. If we use the notation of the fields $Q(3,2)_{\frac{1}{6}}, U^c(\bar{3},1)_{\frac{1}{3}}, D^c(\bar{3},1)_{\frac{1}{3}}, L(1,2)_{\frac{1}{2}}, E^c(1,1)_1, N^c(1,1)_0, X(3,2)_{\frac{5}{6}}$ and their conjugate fields, and $G(8,1)_0$ and $W(1,3)_0$ with the standard gauge symmetry, under $SO(10) \supset SU(5) \supset SU(3)_C \times SU(2)_L \times U(1)_Y$, the spinor $16$, vector $10$ and the adjoint $45$ of $SO(10)$ are divided as

$$16 \rightarrow [Q + U^c + E^c] + [D^c + L] + \frac{N^c}{1}, \quad (4.1)$$

$$10 \rightarrow [D^c + L] + [\bar{D}^c + \bar{L}], \quad (4.2)$$

$$45 \rightarrow [G + W + X + \bar{X} + N^c] + [Q + U^c + E^c] + [\bar{Q} + \bar{U}^c + \bar{E}^c] + \frac{N^c}{1}, \quad (4.3)$$

Then the determinants of the mass matrices $M_I$ of superheavy fields $I = Q,U^c,E^c,D^c,L,G,W,X$ are estimated as

$$\det M_I = \lambda \sum_i c_i, \quad (4.4)$$

where $c_i$ are anomalous $U(1)_A$ charges of superheavy fields.

Secondly, the conditions of the gauge coupling unification in using one loop renormalization group equations

$$\alpha_3(\Lambda_A) = \alpha_2(\Lambda_A) = \frac{5}{3} \alpha_Y(\Lambda_A) \equiv \alpha_1(\Lambda_A), \quad (4.5)$$

where $\alpha_i^{-1}(\mu > \Lambda_C) \equiv \frac{3}{\beta_0} \alpha_{R_i}^{-1}(\mu > \Lambda_C) + \frac{3}{5} \alpha_{B_i}^{-1}(\mu > \Lambda_C)$, are rewritten by the determinants of the mass matrices of the superheavy fields. Here $\alpha_X = \frac{23}{47}$, and the parameters $g_X(X = 3, 2, R, B - L, Y)$ are the gauge couplings of $SU(3)_C$, $SU(2)_L$, $SU(2)_R$, $U(1)_{B - L}$ and $U(1)_Y$, respectively.

The gauge couplings at the scale $\Lambda_A$ are roughly given by

$$\alpha_1^{-1}(\Lambda_A) = \alpha_1^{-1}(M_{SB}) + \frac{1}{2\pi} \left( b_1 \ln \left( \frac{M_{SB}}{\Lambda_A} \right) + \sum_i \Delta b_{1i} \ln \left( \frac{m_i}{\Lambda_A} \right) - \frac{12}{5} \ln \left( \frac{\Lambda_C}{\Lambda_A} \right) \right), \quad (4.6)$$

$$\alpha_2^{-1}(\Lambda_A) = \alpha_2^{-1}(M_{SB}) + \frac{1}{2\pi} \left( b_2 \ln \left( \frac{M_{SB}}{\Lambda_A} \right) + \sum_i \Delta b_{2i} \ln \left( \frac{m_i}{\Lambda_A} \right) \right), \quad (4.7)$$

$$\alpha_3^{-1}(\Lambda_A) = \alpha_3^{-1}(M_{SB}) + \frac{1}{2\pi} \left( b_3 \ln \left( \frac{M_{SB}}{\Lambda_A} \right) + \sum_i \Delta b_{3i} \ln \left( \frac{m_i}{\Lambda_A} \right) \right), \quad (4.8)$$

where $M_{SB}$ is a SUSY breaking scale, $(b_1, b_2, b_3) = (33/5, 1, -3)$ are the renormalization group coefficients for the minimal SUSY standard model (MSSM),
and $\Delta b_{ai}$ ($a = 1, 2, 3$) are the corrections to the coefficients from the massive fields with mass $m_i$. The last term in Eq. (4.4) is from the breaking $SU(2)_R \times U(1)_{B-L} \to U(1)_Y$ caused by the VEV $\langle C \rangle$. Since the gauge couplings at the SUSY breaking scale $M_{SB}$ are given by

$$
\alpha_i^{-1}(M_{SB}) = \alpha_G^{-1}(\Lambda_G) + \frac{1}{2\pi} \left( b_i \ln \left( \frac{\Lambda_G}{M_{SB}} \right) \right), \quad (i = 1, 2, 3) \quad (4.9)
$$

where $\alpha_G^{-1}(\Lambda_G) \sim 25$ and $\Lambda_G \sim 2 \times 10^{16}$ GeV, the unification conditions $\alpha_1(\Lambda_A) = \alpha_2(\Lambda_A), \alpha_1(\Lambda_A) = \alpha_3(\Lambda_A)$ and $\alpha_2(\Lambda_A) = \alpha_3(\Lambda_A)$ can be rewritten

$$
\left( \frac{\Lambda_A}{\Lambda_G} \right)^4 \frac{\Delta G}{\Lambda_A} \frac{(\det M_L)}{(\det M_{Dc})} \left( \frac{(\det M_Q)}{(\det M_U)} \right)^4 \left( \frac{(\det M_{Qc})}{(\det M_E)} \right)^2 \left( \frac{(\det M_W)}{(\det M_X)} \right)^5 = \Lambda_A^{-\bar{r}_D + \bar{r}_L - 4\bar{r}_U - 3\bar{r}_E - 7\bar{r}_Q - 5\bar{r}_X + 5\bar{r}_Y}, \quad (4.10)
$$

$$
\left( \frac{\Lambda_A}{\Lambda_G} \right)^6 \frac{\Delta G}{\Lambda_A} \frac{(\det M_{Dc})}{(\det M_L)} \left( \frac{(\det M_Q)}{(\det M_U)} \right)^4 \left( \frac{(\det M_{Qc})}{(\det M_E)} \right)^2 \left( \frac{(\det M_W)}{(\det M_X)} \right)^5 = \Lambda_A^{-\bar{r}_D + \bar{r}_L - \bar{r}_U - 4\bar{r}_E + 3\bar{r}_Q - 5\bar{r}_X + 5\bar{r}_Y}, \quad (4.11)
$$

$$
\left( \frac{\Lambda_A}{\Lambda_G} \right)^4 \frac{(\det M_{Dc})}{(\det M_L)} \left( \frac{(\det M_U)}{(\det M_Q)} \right)^2 \left( \frac{(\det M_G)}{(\det M_W)} \right)^2 \left( \frac{(\det M_G)}{(\det M_X)} \right)^5 = \Lambda_A^{-\bar{r}_L + \bar{r}_D - \bar{r}_U + 2\bar{r}_W - 3\bar{r}_X + 3\bar{r}_Y}. \quad (4.12)
$$

Here $\bar{r}_I$ are the ranks of the mass matrices of superheavy fields $\bar{M}_I$. Note that the above conditions are dependent only on the ratio of the determinants of mass matrices which are included in the same multiplet of $SU(5)$ and on the symmetry breaking scales $\Lambda_A, \Lambda_G$. If all the component fields in a multiplet have been superheavy, the above ratios would be of order one, because the determinants are given by det $\bar{M} = \lambda \Sigma c_i$. However, since part of the component fields (massless Higgs doublets or Nambu-Goldstone modes) do not appear in the mass matrices, the above ratios are dependent only on the charges of these massless modes. If all the other fields than in MSSM become superheavy, the above ratios are easily estimated as

$$
\frac{\det M_L}{\det M_{Dc}} \sim \lambda^{-2h}, \quad (4.13)
$$

$$
\frac{\det M_Q}{\det M_{Uc}} \sim \lambda^{\alpha + \tilde{\alpha} - 2a}, \quad (4.14)
$$

$$
\frac{\det M_G}{\det M_{Wc}} \sim \lambda^{-2a}. \quad (4.15)
$$

Then the conditions for the coupling unification becomes

$$
\Lambda \sim \lambda^{\frac{4}{3}} \Lambda_G, \quad \Lambda \sim \lambda^{-\frac{4}{3}} \Lambda_G, \quad \Lambda \sim \lambda^{-\frac{4}{3}} \Lambda_G. \quad (4.16)
$$
So the unification conditions become $h \sim 0$, and thus the cutoff scale must be taken as $\Lambda \sim \Lambda_G$. It is obvious that if the cutoff scale have been another scale (for example, the Planck scale), in MSSM three gauge couplings would meet at the scale. This means that in this scenario it is not accidental that three gauge couplings meet at a scale in MSSM, even though the unification scale in our scenario is different from the usual unification scale. Note that the above results are independent of the detail of the Higgs sector, because the requirement that all the other fields than those in MSSM become superheavy determines the field content of the massless fields, whose charges are important to examine whether gauge couplings meet at the unification scale $\Lambda_A$ or not. The above argument can be applied also to the scenario of $E_6$ unification, though instead of usual doublet Higgs charge $h$ we have to use effective Higgs charges $h_{\text{eff}} \equiv h + \frac{1}{4}(\phi - \bar{\phi})$, where $E_6$ is broken into $SO(10)$ by the VEV $|\langle \Phi \rangle| = |\langle \bar{\Phi} \rangle| \sim \lambda^{-\frac{1}{2}}(\phi + \bar{\phi})$.

Note that the condition $h \sim 0$ does not mean $h = 0$, because there is an ambiguity involving order 1 coefficients and we have used only one loop RGEs. However, the above analysis is fairly useful to provide a rough picture of the behavior.

5 Proton decay

The proton decay via dimension 5 operators\[22\] is suppressed in our scenario, because effective colored Higgs mass is given by $m_H^c \sim \lambda^{2h}$ and the dimension 5 operators are suppressed due to anomalous $U(1)_A$ gauge symmetry. On the other hand, proton decay via dimension 6 operators is reachable, because the cutoff scale must be around the usual GUT scale $\Lambda_G \sim 2 \times 10^{16}$ GeV and the unification scale is given by $\lambda^{-a}$. If we adopt $a = -1$, then the lifetime of the proton is roughly estimated as

$$\tau_p(p \to e\pi^0) \sim 3 \times 10^{33} \left(\frac{\Lambda_A}{5 \times 10^{15} \text{ GeV}}\right)^4 \left(\frac{0.015(\text{GeV})^3}{\alpha}\right)^2 \text{years,} \quad (5.1)$$

which is near the present experimental lower bound\[23\]. Here $\alpha$ is the hadron matrix element parameter. Here we use the formula\[24\] and the value $\alpha$ given by lattice calculation. \[25\] Though the prediction is strongly dependent on the actual unification scale which is dependent on the order one coefficients, this rough estimation gives a strong motivation for the future experiments for proton decay search.

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