An improved engineering method for bearing capacity calculation of stiffened curved composite panels

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Abstract. The application of composite materials in the primary structures of large aircraft fuselage is a development trend in recent years. As an indispensable structure in design and structure selection, accurate and efficient evaluation of its performance is of great significance to reduce the research cost. In this paper, the bearing capacity of stiffened curved composite panel is calculated through an improved engineering calculation method, and the effectiveness of the method is verified by test, which provides technical support for the rapidly design and selection of stiffened curved composite panels.

1. Introduction

Composite materials have been widely used in structure of modern aerospace vehicles owing to their excellent weight reduction properties, unique material design-ability and good manufacturability. The weight of the composite material reaches 50% of the structure weight on Boeing 787. Airbus A350 adopts the fuselage structure of composite materials [1]. At present, the amount of composite materials in aircraft has become a mark of the aircraft's advanced nature.

The fuselage of modern aircraft usually adopts semi-rigid shell structure, which is composed of skin, stiffener and frame. For stiffened curved thin-walled shell structures, there are many failure modes, and each failure mode has different influence for structural failure [2]. The buckling forms that may lead to failure mainly include skin local buckling, panel buckling and global buckling. It is necessary to study the post-buckling behavior of the structure because the structure can still bear loads after skin buckling. The post- buckling research of composite structures is an urgent problem to solve. There are many design variables of stiffened cylindrical shell structure. It’s obviously unacceptable to work with FEM in preliminary design. Therefore, the research on the engineering calculation method of bearing capacity of stiffened curved composite panels can not only simplify the evaluation process of structural strength, reduce the waste of resources in the process of simulation analysis and improve the design efficiency, but also help to reduce the cost of design, manufacturing and test.

There were many experiments of stiffened curved panel under axial compression [3][4][5][6][7]. The buckling and postbuckling behaviors of stiffened curved panel is usually studied by numerical method especially by finite element method. Many researchers proposed new elements to simulated the
structures by using different modeling approaches and analytical procedures [8][9][10]. The buckling load bearing capacity of panel are examined by adopting the theoretical formula and the semi empirical formula by some scholars [11][12]. At present, the research on the bearing capacity of stiffened curved composite panels mostly focuses on the test and numerical analysis, and there is little experience that can be used for reference in engineering methods.

Therefore, this paper introduces the theoretical analysis method of engineering calculation of bearing capacity of stiffened curved composite panels, and its effectiveness and validity verification with the existing test data.

2. Theoretical analysis

2.1 Buckling load estimation of stiffened curved plate under axial compression load

The buckling of stiffened panels usually occurs first in the skin between stiffeners. Therefore, the buckling load of the skin between stiffeners is often regarded as the buckling load of stiffened panels. The stress diagram of skin between stiffeners of stiffened cylindrical shell under axial compression loads is shown in Figure 1. The definition of the coordinate system is shown in Figure 1, the x-axis is along the stiffener and the y-axis is along the circumferential direction of the skin. \(N_x\) is the load of unit length along the circumferential direction of the skin.

![Stress diagram of skin between stiffeners of stiffened curved composite panel under axial compression load](image)

According to the stress status of panel, assuming that the skin is simply supported, the generalized displacement function is set by Rayleigh Ritz method as

\[
\begin{align*}
    u_0 &= \sum_{m=1}^{\infty} u_{0m} \cos \frac{m\pi x}{a} \sin \frac{n\pi y}{b} \\
    v_0 &= \sum_{n=1}^{\infty} v_{0n} \sin \frac{m\pi x}{a} \cos \frac{n\pi y}{b} \\
    \gamma_x &= \sum_{m=1}^{\infty} \gamma_{xm} \cos \frac{m\pi x}{a} \sin \frac{n\pi y}{b} \\
    \gamma_y &= \sum_{n=1}^{\infty} \gamma_{yn} \sin \frac{m\pi x}{a} \cos \frac{n\pi y}{b} \\
    w &= \sum_{m=1}^{\infty} w_m \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b}
\end{align*}
\]

where \(u_0, v_0, \gamma_x, \gamma_y\) and \(w\) are generalized displacement functions; \(u_{0m}, v_{0n}, \gamma_{xm}, \gamma_{yn}\) and \(w_m\) are the partial coefficients of the generalized displacement functions; \(m\) and \(n\) are the half wave number of displacement function along x-axis and y-axis; \(a\) and \(b\) are the length of skin along x-axis and y-axis respectively.
The series expansion of load can be expressed as

\[ q_x = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} q_{xmn} \cos \left( \frac{m \pi x}{a} \right) \sin \left( \frac{n \pi y}{b} \right) \]

\[ q_y = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} q_{ymn} \sin \left( \frac{m \pi x}{a} \right) \cos \left( \frac{n \pi y}{b} \right) \]

\[ m_x = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} m_{xmn} \cos \left( \frac{m \pi x}{a} \right) \sin \left( \frac{n \pi y}{b} \right) \]

\[ m_y = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} m_{ymn} \sin \left( \frac{m \pi x}{a} \right) \cos \left( \frac{n \pi y}{b} \right) \]

\[ q_z = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} q_{znmn} \sin \left( \frac{m \pi x}{a} \right) \sin \left( \frac{n \pi y}{b} \right) \]

where \( q_x, q_y, m_x, m_y, \) and \( q_z \) are generalized load functions; \( q_{0x}, q_{0y}, m_{0x}, m_{0y}, \) and \( q_{zm} \) are partial coefficients of generalized load functions; subscripts \( m \) and \( n \) are positive integers.

Substituting Eq.(1) and (2) into the governing equation of the linear theory of composite shallow shells, Eq.(3) can be obtained as

\[
\begin{bmatrix}
T_{11} & T_{12} & T_{13} & T_{14} & T_{15} \\
T_{12} & T_{22} & T_{23} & T_{24} & T_{25} \\
T_{13} & T_{23} & T_{33} & T_{34} & T_{35} \\
T_{14} & T_{24} & T_{34} & T_{44} & T_{45} \\
T_{15} & T_{25} & T_{35} & T_{45} & T_{55} + N^0_1 \frac{m^2 \pi^2}{a^2} + N^0_2 \frac{n^2 \pi^2}{b^2} \\
q_{xmn} & q_{ymn} & m_{xmn} & m_{ymn} & w_{mn}
\end{bmatrix}
\begin{bmatrix}
u_{0mn} \\
v_{0mn} \\
\gamma_{0mn} \\
\gamma_{0mn} \\
w_{mn}
\end{bmatrix}
= \begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
0
\end{bmatrix}
\]

where \( T_{ij} \) represents coefficients related to buckling half wave number and stiffness coefficient.

Set \( q_x, q_y, m_x, m_y, \) and \( q_z \) be zero, then the condition of non-zero solutions for generalized displacement \( u_0, v_0, \gamma_x, \gamma_y, \) and \( w \) can be determined as

\[
N^0_1 \frac{m^2 \pi^2}{a^2} + N^0_2 \frac{n^2 \pi^2}{b^2} = 0
\]

Because the skin thickness is thin, and Eq.(4) can be simplified to Eq.(5) by Kirchhoff hypothesis.

\[
N^0_1 \frac{m^2 \pi^2}{a^2} + N^0_2 \frac{n^2 \pi^2}{b^2} = \frac{T_{11} T_{22} + T_{22} T_{15} + 2 T_{12} T_{15} T_{25} - T_{55}}{(T_{11} T_{22} - T_{12}^2) T_{25}}
\]

Under pure axial compression, the stress on the skin side of the stiffened curved panel can be expressed as

\[
N^0_y = 0
\]

Substituting Eq.(6) into (5), the axial force of skin under pure axial compression load can be obtained as

\[
N^0_x = \frac{T_{11} T_{22} + T_{22} T_{15} + 2 T_{12} T_{15} T_{25} - T_{55}}{(T_{11} T_{22} - T_{12}^2) T_{25}}
\]

For small curvature panels, set \( R \) be \( \infty \), and Eq.(7) can be reduced to the expression of axial compression buckling of orthotropic rectangular laminated plates. Let the function \( f(m, n) \) be Eq.(8), and solve Eq.(9). Buckling load can be obtained by substituting \( m \) and \( n \) into Eq.(7).

\[
f(m, n) = \frac{a^2}{m^2 \pi^2} \left( \frac{T_{11} T_{22} + T_{22} T_{15} + 2 T_{12} T_{15} T_{25} - T_{55}}{(T_{11} T_{22} - T_{12}^2) T_{25}} \right)
\]
2.2 Bearing capacity estimation of stiffened curved panel under axial compression load

2.2.1 Column analogy method
If the curvature of the curved panel is small, the estimation of the bearing capacity for stiffened curved panel under axial compression can be appropriately simplified to the estimation of stiffened panel.

The three different failure models which are named the short column failure, the long column failure, the medium-long column failure are discussed on the basis of \( \lambda = L'/\rho \), as shown in Figure 2.

The failure mode in short column is stiffener crippling. The failure mode in long column is global buckling, which can be calculated according to Euler formula. The failure mode in medium-long column is collapses caused by local buckling occurred on skin or stiffener. The failure stress of stiffened panel can be calculated as

\[
\sigma_{cc} = \left[ 1 - \left( 1 - \frac{\sigma_{cr}}{\sigma_{cc}} \right) \frac{\sigma_{cr}}{\sigma_{r}} \right] \sigma_{cc} \tag{10}
\]

where \( \sigma_{co} \) is average failure stress of stiffened panel; \( \sigma_{cr} \) is local buckling stress of skin; \( \sigma_{cc} \) is the results of short column panel; \( \sigma_{r} \) is global buckling load without considering stiffness degradation; \( \sigma_{cr} \) is global buckling stress of the stiffened panel calculated according to the Euler formula without considering the influence of the stiffness drop after local buckling of the skin or stiffener.

The crippling stress of stiffened panel can be replaced approximately by the minimum local buckling stress of each plate element of stiffener when lack of crippling test data. At the same time, it is proposed that failure load is obtained as the smaller one between medium-long column crippling load and panel Euler load.

In the preliminary estimation, set \( \sigma_{cr} \approx \sigma_{co}/2 \). The global buckling stress by Euler formula can be calculated as

\[
\sigma_{r} = \frac{\pi^2 \bar{E}_x}{(L'/\rho)^2} \approx \frac{c \pi^2 \bar{E}_x}{A} \frac{1}{L^2} \tag{11}
\]

Eq.(10) can be simplified as

\[
\sigma_{co} = \sigma_{cc} - (\sigma_{cc})^2 (L'/\rho^2)/(4\pi^2 \bar{E}_x) \tag{12}
\]

where \( \bar{E}_x \) is equivalent axial elastic modulus of stiffened panel; \( \rho \) is least radius of gyration(see Eq.(13)); \( L' \) is effective column length of stiffened panel(see Eq.(14)); \( I \) is moment of inertia to centroidal axis of panels; \( A \) is area of stiffened panel; \( L \) is length of panels; \( c \) is end support coefficient of stiffened panel; \( A_i \) is sectional area of the ith plate element; \( E_{xi} \) is axial elastic modulus of the ith plate element(see Eq.(15)); \( t_i \) is thickness of the ith plate element.

\[
\rho = \sqrt{I/A} \tag{13}
\]

\[
L' = L/\sqrt{c} \tag{14}
\]

\[
E_{xi} = \left( A_{11} - A_{11}^2/A_{22} \right)/t_i \tag{15}
\]
2.2.2 Improved column analogy method

According to the test data, the parabolic formula of medium-long column by column analogy method is modified to the linear formula (see Figure 2). A simple method for estimating the average crippling failure stress of stiffened panel is recommended. The failure stress of medium-long column is calculated as

$$\sigma_{\infty} = \frac{\sigma_{cr} - \sigma_{cc}}{\lambda_{cr} - \lambda_{cc}} \lambda + \sigma_{cr} - \frac{\sigma_{cr} - \sigma_{cc}}{\lambda_{cr} - \lambda_{cc}} \lambda_{cr}$$ \hspace{1cm} (16)

where \( \lambda_{cr} \) is effective slenderness ratio of stiffened panel when Euler buckling and local buckling occur simultaneously.

$$\lambda_{cr} = \frac{bm}{\sqrt{\frac{AE_2}{c(\frac{E}{I})D_{22}}} \left[ \sqrt{(D_{12} + 2D_{66})^2 - D_{22} \left( D_{11} - \frac{c(\frac{E}{I})t^2}{m^2A} \right)} - D_{12} - 2D_{66} \right]}$$ \hspace{1cm} (17)

where \( b \) is width of skin between stiffeners; \( D_{ij} \) is bending stiffness coefficient of skin.

3. Verification

3.1 Specimen

The configuration of stiffened curved composite panel specimen is shown in Figure 3. The specimen has a length of 3100mm and a width of 2150mm (curve length), including 5 frames and 8 stiffeners. The stiffener span is 210mm and the frame span is 620mm. The skin, stiffeners and reinforced plates were made of M21E/IMA. Material Properties are listed in Table 1. The thickness of a monolayer is 0.186 mm. Stacking sequences of the specimen are given in Table 2.
Figure 3 The structure configuration of stiffened curved composite panel

Table 1 The mechanical property parameters of M21E/IMA.

| Property | Value/MPa |
|----------|-----------|
| $E_1$    | 154000    |
| $E_2$    | 8500      |
| $G_{12}$ | 4200      |
| $\nu_{12}$ | 0.35   |
| $X_T$    | 2610      |
| $X_C$    | 1450      |
| $Y_T$    | 55        |
| $Y_C$    | 285       |
| $S$      | 105       |

Table 2 The geometry dimension of stiffened panel

| Component | Thickness/mm | Layer number | Layer sequence |
|-----------|--------------|--------------|----------------|
| Skin      | 2.21         | 12           | [45/-45/-45/90/45/0]s |
| Stiffener | 1.66         | 9            | [45/0/0/-45/90/-45/0/45/0] |
| Frame     | 2.58         | 14           | [45/-45/0/90/45/-45/0]s |

3.2 Test results
The failure load is 2166.5kN. The failure mode is the skin wrinkled between two middle frames, and the skin is debonded from the stiffeners, as is shown in Figure 4.

3.3 Calculation
When calculating the local buckling load of the skin, three methods according to the skin width (see Figure 5) and two boundary conditions (simply supported or clamped) are taken into calculation.

In this paper, it is recommended that the skin width is taken as the distance $b_2$ between adjacent flange centerlines, where $b_2 = (b_1 + b_3) / 2$. And the boundary condition is taken as simply supported on four sides.
According to the calculation method, the local buckling stress of skin is 86.67 MPa. The buckling stress of each plate element of stringer is calculated, and the local buckling stress of the hat stiffener top is 2008.78 MPa and the local buckling stress of the hat stiffener waist is 695.18 MPa. Therefore, the minimum buckling stress is 695.18 MPa.

The strength of each plate element of stringer is calculated by Tsai-Wu criterion. The compressive ultimate strength is 911.76 MPa. The global buckling stress of the stiffened panel is 278.21 MPa. And the corresponding global buckling load is 2221.79 kN. The failure load can be calculated by substituting the above calculation results into Eq.(15). The failure stress of the stiffened panel is 505.61 MPa, and the corresponding failure load is 4037.84 kN.

The effective slenderness ratio is 53.969. Therefore, it is necessary to calculate global buckling failure load. Since the global buckling load is less than the failure load calculated by column analogy method, the failure load of the stiffened panel should be modified as 2221.786 kN. The failure load of test is 2166.5 kN, and the calculation error is 2.6%.

4. Conclusions
In this paper, an engineering method for bearing capacity of stiffened curved composite panels is presented. It’s proposed that the crippling stress of stiffened panel can be replaced approximately by the minimum local buckling stress of each plate element when the crippling test data cannot be obtained. It is proposed that failure load is obtained as the smaller one between medium-long column crippling load and panel Euler load. The method established in this paper can effectively predict the bearing capacity of stiffened curved composite panels, and the prediction accuracy is less than 10%.

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