Defense Model of Intercepting a High-Velocity Long Rod by Multiple Linear Shaped Charges

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Abstract. The interception of high-velocity long rods has always been a difficult problem in defense technology. The existing research lacks the comprehensive analysis on the relationship between the latency, interception position, intercepting velocity, aftereffect damage and other factors in the defense system. In this paper, we first established a time-space rendezvous model of the long rod intercepted by multiple Linear Shaped Charges (LSCs), and then analyzed the fracture model of the long rod, the yaw-ricochet model and the after-effect penetration model of the fractured rods. Finally, the optimal defense model was established. Based on the optimal defense model, the key parameters and their variation rules that affect the defense performance of the protection system were obtained and the key parameters were verified by numerical simulation. It was found that reducing the latency could shorten the length of the front-fractured rod and increasing the number of LSCs could also reduce the average length of the fractured rods. Shortened length of the fractured rods and various attack angles could greatly reduce the penetration depth, and then improve the defense performance. This article has an important significance for the design of the active protection system with LSCs intercepting long rods.

1. Introduction
The interception of high-velocity long rods has always been a difficult problem in defense technology. Owing to the delay of response system and the shortening interaction time, the existing active armor protection technology cannot effectively intercept and defend against high-velocity long rod projectiles [1]. Research on rapid response interception technology has been carried out in recent years. High-sensitive and fast-response sensors could detect high-velocity long rods, which were intercepted by moving plates [2-6], fly cross bars [7-8] or the explosive products of liner shaped charges (LSC). The explosive products of LSCs, including linear jets and linear slugs (i.e. linear explosively formed
projectiles, LEFPs), were the most efficient ways to defend high-velocity long rods. Deformation and fracture failure of a high-velocity long rod intercepted by LEFPs sequence was researched, and it was found that the length loss rate of the fractured rods is as high as 27% \cite{9}. Joo J et al. \cite{10} studied the defensive effectiveness of LEFP against high-velocity rods, and discussed the influence of LEFP’s intercept location and angle on the residual penetration depth. Bing Li et al. \cite{11}, \cite{12} studied the impact initiation characteristics of LEFP on shelled warheads. Yong Li et al. \cite{13} simulated the interaction between multiple LSCs and the high-velocity long rod by numerical simulation software.

It can be seen that the existing research results only reveal the interception effect of LSC on the long rod, but lack the comprehensive analysis on the relationship between the response time, interception position, intercepting velocity, aftereffect damage and other factors in the defense system. Therefore, it is necessary to establish a defense model to describe the relationship between various impacts. In this paper, we first established a time-space rendezvous model, and then analyzed the fracture model of the long rod after the interception of the multiple LSCs, the yaw ricochet model and the after-effect penetration model of the fractured rods. Finally, the optimal defense model of multi linear shaped charge intercepting long rod was established. Based on the optimal defense model, the influence of various factors on the defense performance was obtained and verified by simulation. This paper has significance for the design of the protection system using multi-linear shaped charge as the interception unit.

2. Time-space model of multiple LSCs intercepting long rod

2.1. The space coordinate system

The spatial position relationship of multiple LSCs and long rod intersection is shown in Figure 1. The rendezvous process includes the ground coordinate system \(OgXgYgZg\), the LSC coordinate system \(OsXsYsZs\), the long rod coordinate system \(OPXPYPZP\), the ballistic coordinate system \(OrXrYrZr\),etc. The long rod will deform and fracture after being impacted by the detonation products of the LSCs. The geometric relationship between the fractured rod coordinate system and the aftereffect target coordinate system is shown in Figure 2.

![Figure 1. The geometry diagram of defensive intersection.](image)
Figure 2. Geometric relation diagram of intersection of multi segment rod and target plate.

The coordinate systems of the penetration process of the fractured rods includes the coordinate system of the front-fractured rod ($O_{PF}X_{PF}Y_{PF}Z_{PF}$), the middle-fractured rod ($O_{PM}X_{PM}Y_{PM}Z_{PM}$), the rear-fractured rod ($O_{PR}X_{PR}Y_{PR}Z_{PR}$) and the aftereffect target ($O_{t}X_{t}Y_{t}Z_{t}$). The attack angle of the fractured rod is of great significance to the penetration depth and shape.

2.2. Transformation relationship between coordinate systems

If $M_a(b)$ is the rotation matrix of rotation angle $b$ around a axis, then the coordinate transformation matrices of rotation around X axis, Y axis and Z axis are

$$M_x(*) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(*) & \sin(*) \\ 0 & -\sin(*) & \cos(*) \end{bmatrix}$$  \hspace{1cm} (1)

$$M_y(*) = \begin{bmatrix} \cos(*) & 0 & -\sin(*) \\ 0 & 1 & 0 \\ \sin(*) & 0 & \cos(*) \end{bmatrix}$$  \hspace{1cm} (2)

$$M_z(*) = \begin{bmatrix} \cos(*) & \sin(*) & 0 \\ -\sin(*) & \cos(*) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$  \hspace{1cm} (3)

As you can see,

$$M^T_a(b) = M^{-1}_a(b) = M_a(-b)$$  \hspace{1cm} (4)

The transformation of each coordinate system in the process of interception and aftereffect penetration was shown in Table 1.

Table 1. The transformation of each coordinate system.

| Coordinate system                        | Rotation matrix | Angle interpretation                      |
|-----------------------------------------|-----------------|------------------------------------------|
| The long rod coordinate system $O_{PF}X_{PF}Y_{PF}Z_{PF}$ to the ballistic coordinate system $O_{s}X_{s}Y_{s}Z_{s}$ | $M_x(\varepsilon_i)M_y(\varepsilon_j)M_z(\varepsilon_k)$ | $\varepsilon_i, \varepsilon_j$ and $\varepsilon_k$ are the nutation angle of long rod on X, Y and Z axes in ballistic direction respectively. |
| The moving coordinate system of linear slug $O_{SV}X_{SV}Y_{SV}Z_{SV}$ to the ballistic coordinate system $O_{s}X_{s}Y_{s}Z_{s}$ | $M_y(\frac{\pi}{2} - \theta_j)M_z(\theta_k)$ | $\theta_j$ is the angle between the symmetry plane of the intercepting unit and the |
The ballastic coordinate system $O_1X_1Y_1Z_1$ to the coordinate system of the front broken-rod $O_{P1}X_{P1}Y_{P1}Z_{P1}$

$$M_x(\alpha_1)M_y(\alpha_1)M_z(\alpha_1)$$

$\alpha_1$, $\alpha_2$ and $\alpha_k$ are the yaw angle of the front broken-rod in X-axis, Y-axis and Z-axis respectively.

The ballastic coordinate system $O_2X_2Y_2Z_2$ to the coordinate system of the middle broken-rod $O_{PM}X_{PM}Y_{PM}Z_{PM}$

$$M_x(\varphi_1)M_y(\varphi_1)M_z(\varphi_k)$$

$\varphi_1$, $\varphi_2$ and $\varphi_k$ are the yaw angle of the middle broken-rod in X-axis, Y-axis and Z-axis respectively.

The ballastic coordinate system $O_3X_3Y_3Z_3$ to the coordinate system of the rear broken-rod $O_{P1}X_{P1}Y_{P1}Z_{P1}$

$$M_x(\gamma_1)M_y(\gamma_1)M_z(\gamma_k)$$

$\gamma_1$, $\gamma_1$ and $\gamma_k$ are the yaw angle of the rear broken-rod in X-axis, Y-axis and Z-axis respectively.

The coordinate system of the middle-fractured rod $O_{P1}X_{P1}Y_{P1}Z_{P1}$ to the coordinate system of aftereffect target $O_1X_1Y_1Z_1$

$$M_x(\alpha'_1)M_y(\alpha'_1)$$

$\alpha'_1$ and $\alpha'_k$ are the attack angle of the front broken-rod.

The coordinate system of the middle-fractured rod $O_{PM}X_{PM}Y_{PM}Z_{PM}$ the coordinate system of aftereffect target $O_2X_2Y_2Z_2$

$$M_x(\varphi'_1)M_y(\varphi'_k)$$

$\varphi'_1$ and $\varphi'_k$ are the attack angle of the front broken-rod.

The coordinate system of the rear-fractured rod $O_{P1}X_{P1}Y_{P1}Z_{P1}$ to the coordinate system of aftereffect target $O_3X_3Y_3Z_3$

$$M_x(\gamma'_1)M_y(\gamma'_k)$$

$\gamma'_1$ and $\gamma'_k$ are the attack angle of the front broken-rod.

3. Model of the long rod fracture and penetration

3.1. Fracture model of long rod after intercepting by multiple LSCs

When the detonation products of LSC intercept on the long rod, the long rod and its contact part displace along the impact direction. The long rod was sheared and deformed, forming a plugging failure around the impact area.

In the process of plug failure, the width of the sheared plug is similar to that of the detonation products. Based on the law of conservation of energy and the theory of impact dynamics, the failure mechanism of a high-velocity long rod impacted by detonation products of LSC was studied. In the process of interaction, the long rod absorbs the energy of detonation products, and the detonation products also work against the shear stress of long rod material.

The process that the broken part of the long rod was pushed away from the long rod by the detonation products of LSCs can be regarded as a kinetic energy conversion problem. The liner mass and length of LSC are $M$ and $L$ respectively. Then the mass of the part contacting with the long rod is $M_0 = \frac{M}{L_0} \cdot D_P$. The mass of broken part of the long rod (plug) is $m_k$. Assuming that there is no mass loss during impact fracture, the energy conservation equation can be written as:

$$\frac{1}{2}M_0V_{S_0}^2 = \frac{1}{2}(M_0 + m_k)V_{S_t}^2 + W_P + W_f$$  (5)
Where $W_p$ is the sum of work, heat loss and elastic-plastic deformation energy of the detonation products of LSC to overcome the shear stress of long rod material; $W_f$ is the consumed energy when the both parts form a common velocity $V_0$ during impact.

When ignoring the impulse acquired by the material around the fractured rod, the momentum conservation equation can be expressed as

$$M_0 V_{S_0} = (M_0 + m_k)V_{SP}$$  \hspace{1cm} (6)

Then,

$$V_{SP} = \frac{M_0}{M_0 + m_k} V_{S_0}$$  \hspace{1cm} (7)

At the beginning of the collision, the kinetic energy consumed to reach the common velocity is

$$W_f = \frac{1}{2} M_0 V_{S_0}^2 - \frac{1}{2} (M_0 + m_k) V_{SP}^2 = \frac{1}{2} (M_0 + m_k) V_{l}^2 + W_p$$  \hspace{1cm} (8)

Then, the energy equation can be rewritten as

$$\frac{1}{2} M_0 V_{S_0}^2 - \frac{1}{2} (M_0 + m_k) V_{SP}^2 = \frac{1}{2} (M_0 + m_k) V_{l}^2 + W_p$$ \hspace{1cm} (9)

$$\frac{1}{2} (M_0 + m_k) V_{SP}^2 = \frac{1}{2} (M_0 + m_k) V_{l}^2 + W_p$$ \hspace{1cm} (10)

Assuming that the plug is just pushed out of the long rod, it can be obtained according to the definition of ballistic limit velocity

$$W_p = \frac{1}{2} (\frac{M_0}{M_0 + m_k}) V_{c}^2$$ \hspace{1cm} (11)

Where $V_{c}$ is the limit velocity of LSC detonation products. Thus, the expression of residual velocity can be deduced

$$V_{S_r} = \frac{M_0}{M_0 + m_k} \sqrt{V_{S_0}^2 - V_{c}^2}$$ \hspace{1cm} (12)

Where, $\lambda = m_k/M_0$. Assuming that the shear stress ($\tau$) acting on the surface of the plug is constant, then the work done by LSC detonation products to push plug is

$$W_p = \pi H_j D_p^2 \tau$$ \hspace{1cm} (13)

Where, the width and diameter of the fractured rod are $H_j$ and $D_p$ respectively; $\tau = Y_t/\sqrt{3}$, $Y_t$ is the flow stress of long rod material. If the kinetic energy of LSC detonation products at the limit velocity is equal to the plastic work required to cut off the long rod, then

$$\frac{1}{2} M_0 V_{c}^2 = \pi H_j D_p^2 \tau$$ \hspace{1cm} (14)

The ultimate fracture projectile velocity is obtained from equation (14)

$$V_{c} = D_p \sqrt{\frac{2\pi H_j \tau}{M_0}}$$ \hspace{1cm} (15)

Where, $M_0 = \rho_L V_0 = \rho_L D_p H_j H_l$, $\rho_L$ and HL and $H_l$ is the liner density and thickness of LSC respectively. From equation (15),

$$V_{c} = \sqrt{\frac{2\pi D_p \tau}{\rho_L H_L}}$$ \hspace{1cm} (16)

The residual velocity $V_{u}$ can be obtained by solving equations (16) and (12). When the velocity of LSC detonation products exceeded the ultimate velocity $V_{c}$, the long rod would be fractured.
If the shear force on the long rod is \( F \), the force on the tail of the front fractured rod is \( F/2 \). The front fractured rod deflects around the mass point, and the deflection angle is \( \alpha_i \), then the work done by the shear force on the front fractured rod is

\[
dA = \frac{F}{2} \frac{L_x}{2} \sin \theta_j d\alpha_i
\]  

(17)

According to the conservation of energy,

\[
F \frac{D_p}{2} = W_p
\]

(18)

\[
\int_0^{\alpha_i} dA = W_p
\]

(19)

From equation (17), equation (18) and equation (19),

\[
\alpha_i = 2 \frac{D_p}{b_y} \sin \theta_j
\]

(20)

Where, \( \alpha_i \) is in radians.

Similarly, the deflection angle of the back fractured rod can be calculated,

\[
\gamma_i = 2 \frac{D_p}{(L-L_\gamma)} \sin \theta_j
\]

(21)

3.2. Penetration model of the fractured rods

The penetration depth of the fractured rod should be obtained to verify the defensive performance of the protection system. We discussed the situation where the fractured rods penetrate the semi-infinite target in this paper. After the long rod fractured, the moving direction of the fractured rods changed differently. This section first discussed the fractured rod horizontal penetration model, and then discussed the fractured rod penetration model with attack angle and the fractured rod ricochet model.

3.2.1. The fractured rod horizontal penetration model

The penetration motion of the fractured rods can be regarded as a quasi-steady motion. The parameters in the penetration process change linearly with time. The pressure on the head of the rod came from the cavity expansion pressure. The target plate material is a hard elastoplastic material, and the long rod is an ideal rigid plastic material. It is assumed that the material of neither the target plate nor the fractured rod is compressible during the penetration process.

When the fractured rod hits the target plate, the target plate will apply static resistance force \( R_t \) to the rod head. The resistance force is the same as the Brinell hardness \( H_b \) of the target plate. According to the theoretical model of fluid dynamics, assuming that the depth \( x \) of the hole is equal to the diameter \( D_p \) of the long rod, the following results are obtained:

\[
R_t = C_1 e^{\frac{D_p}{b_y}} + C_2
\]

(22)

Where, \( x \)—Penetration depth, \( D_p \)—Long rod diameter.

Let the boundary condition be:

\[
\begin{align*}
\text{When } x &= 0, R_t = H_b \\
\text{When } x &= D_p, R_t = ps
\end{align*}
\]

Substitute equation (22) , then

\[
\begin{align*}
C_1 &= \frac{p_d^+ - H_b}{e^{-1}} \\
C_2 &= \frac{e^{H_b - p_d^+}}{e^{-1}}
\end{align*}
\]

(23)

(24)

So,
\[
R_t = \frac{1}{e^{-1}} \left[ (p^s - H_b)e^{\beta p} + (eH_b - p^s) \right] \tag{25}
\]

\[
p^s = \frac{1}{3} \sigma_0 \left( 1 + \ln \frac{2E}{3\sigma_0} \right) + \frac{2}{27} \pi^2 E_t \tag{26}
\]

According to the classical Alekseevskii-Tate penetration model \textsuperscript{[14-15]}, we can get

\[
U_p = \sqrt{\frac{4V_p^2 - (8 - \rho_p^2 \alpha \rho_p^2) [2V_p^2 \frac{V_p - H_b}{\rho_p \rho_p^2}]}{4 - 3\rho_p^2 \alpha \rho_p^2}} \tag{27}
\]

Finally, the horizontal penetration depth of the fractured rod can be obtained. For example, the penetration depth of the front-fractured rod is as follows:

\[
x_F = L_F - \frac{U_p}{V_p - U_p} \tag{28}
\]

3.2.2. Penetration model of the fractured rods with the attack angle

Through the transformation from ballistic coordinate system to the fractured rod coordinate system and from the fractured rod coordinate system to the aftereffect plate coordinate system, the penetration velocity and the attack angle of the fractured rod in the aftereffect plate coordinate system were obtained. According to the penetration velocity and the attack angle, the penetration depth of the fractured rods on the aftereffect target can be calculated.

Firstly, the flight velocity of the front fractured rod \(V_{PF}(V_{PF_1}, V_{PF_2}, V_{PF_3})\) was obtained through the transformation from the trajectory coordinate system to the front broken link coordinate system:

\[
v_{PF_1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos (\alpha_i) & \sin (\alpha_i) \\ 0 & -\sin (\alpha_i) & \cos (\alpha_i) \end{bmatrix} v_{P_1} \tag{29}
\]

\[
v_{PF_2} = \begin{bmatrix} \cos (\alpha_j) & 0 & -\sin (\alpha_j) \\ 0 & 1 & 0 \\ \sin (\alpha_j) & 0 & \cos (\alpha_j) \end{bmatrix} v_{P_2} \tag{30}
\]

\[
v_{PF_3} = \begin{bmatrix} \cos (\alpha_k) & \sin (\alpha_k) & 0 \\ -\sin (\alpha_k) & \cos (\alpha_k) & 0 \\ 0 & 0 & 1 \end{bmatrix} v_{P_3} \tag{31}
\]

Where, \(\alpha_i\), \(\alpha_j\) and \(\alpha_k\) are the yaw angle of the front-fractured rod in x-axis, Y-axis and z-axis respectively.

Then, the coordinate system of the front-fractured rod was transformed into the coordinate system of the aftereffect target, and the velocity of the front-fractured rod \(v_{PF} (v'_{PF_1}, v'_{PF_2}, v'_{PF_3})\) impacting the target plate was obtained.

\[
v'_{PF_1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos (\alpha_i') & \sin (\alpha_i') \\ 0 & -\sin (\alpha_i') & \cos (\alpha_i') \end{bmatrix} v_{PF_1} \tag{32}
\]

\[
v'_{PF} = v'_{PF_1} \tag{33}
\]
\[ \dot{v}_{P_{F_k}} = \begin{bmatrix} \cos(\alpha_{k}') & \sin(\alpha_{k}') & 0 \\ -\sin(\alpha_{k}') & \cos(\alpha_{k}') & 0 \\ 0 & 0 & 1 \end{bmatrix} v_{P_{F_k}} \] (34)

Where, \( \alpha_{i}' \) and \( \alpha_{k}' \) are the yaw angle and deflection angle of the front-fractured rod in the coordinate system of the aftereffect target respectively. The attack angle of the fractured rod \( \delta_F \) is the angle between the axis of the rod and the velocity vector of the mass center, so the attack angle is as follows:

\[ \tan^2\delta_F = \tan^2\alpha_{i}' + \tan^2\alpha_{k}' \] (35)

Finally, according to Rosenberg’s penetration model\(^{[16]}\), the attack angle \( \delta_{F_c} \) and the penetration length of the fractured rod \( L_{eff} \) can be calculated as follows:

\[ \sin\delta_{F_c} = \frac{D_c-D_p}{2L_1} \] (36)

\[ L_{eff} = N_{eff} \cdot 1 \] (37)

Where, \( D_c \) is the diameter of the long side of the crater on the target plate, \( N_{eff} \) is the maximum number of elements that the fractured rod does not hit the pit wall.

3.2.3. Ricochet model of the fractured rod

When the fractured rods hit the aftereffect target plate at a certain attack angle, the phenomenon of ricochet may occur. The damage to the aftereffect target will be greatly reduced because of the fractured rods ricochet off. According to Rosenberg ricochet model\(^{[17]}\), the minimum value of the critical ricochet angle \( \beta_{c\min} \) can be obtained,

\[ \tan^2\beta_{c\min} = \frac{\rho_p v_p^2}{R_t} = 2 \left( 1 - \frac{v_p}{R_t} \right), \quad \dot{v}_p = v_c \] (38)

Where, \( Y_p \) is the strength of the long rod and \( R_t \) is the resistance of the target material. According to the AT model, the critical impact velocity \( v_c \) can be obtained,

\[ v_c = \sqrt{\frac{2(R_t-Y_p)}{\rho_p}} \] (39)

When the attack angle of the fractured rod \( \delta_{F_c} \) is greater than \( \beta_{c\min} \) and the velocity of the fractured rod \( \dot{v}_{P_{F_i}} \) is less than the critical impact velocity \( v_c \), the fractured rod will ricochet and occur penetration failure.

3.3. Optimal defense model of multiple LSCs intercepting long rod

The intersect distance between the detonation products of LSC and the long rod is

\[ d_n = (n-1)s \cdot \tan\theta + d \] (40)

Where, \( d = y_{r_s} - y_{r_1} \), \( s \) is the axis spacing of adjacent LSCs, as shown in Figure 3.
**Figure 3.** Schematic diagram of intersection of multiple side-by-side LSCs and long rod.

Assuming that multiple LSCs are detonated at the same time, the action positions are

\[
L_n = [T + \frac{d + (n-1)s\tan\theta_j}{V_{S_j}}]V_{P_i}(\theta_j < 90^\circ) \quad (41)
\]

\[
L_n = [T + \frac{d}{V_{S_j}}]V_{P_i} + (n - 1)s(\theta_j = 90^\circ) \quad (42)
\]

The length of the fractured rod from the beginning to the end is as follows: \( L_{n-1}, L_{n-2} - L_{n-1}, \ldots, L_1 - L_2, L - L_{n} \),

\[
L_n - L_{n-1} = \frac{v_{P_i}}{V_{S_j}}s, \quad (\theta_j < 90^\circ) \quad (43)
\]

\[
L_n - L_{n-1} = s, \quad (\theta_j = 90^\circ) \quad (44)
\]

Therefore, the penetration depth of the fractured rod to the aftereffect steel plate is as follows:

\[
x_F = (T + \frac{y_{r_s} - y_{r_1}}{V_{S_j}})V_{P_i}(0.65 + 1.06\ln_{P_i}^{\prime} - 0.055\ln_{D_{eff}}^{\prime}) \quad (45)
\]

\[
x_M = \frac{v_{P_i}}{V_{S_j}}(0.65 + 1.06\ln_{P_i}^{\prime} - 0.055\ln_{D_{eff}}^{\prime})\tan\theta_j, \quad (\theta_j < 90^\circ) \quad (46)
\]

\[
x_M = \frac{v_{P_i}}{V_{S_j}}(0.65 + 1.06\ln_{P_i}^{\prime} - 0.055\ln_{D_{eff}}^{\prime})s, \quad (\theta_j = 90^\circ) \quad (47)
\]

\[
x_r = [L - \left(T + \frac{d + (n-1)s\tan\theta_j}{V_{S_j}}\right)V_{P_i}][0.65 + 1.06\ln_{P_i}^{\prime} - 0.055\ln_{D_{eff}}^{\prime}], \quad (\theta_j < 90^\circ) \quad (48)
\]

\[
x_r = [L - \left(T + \frac{d}{V_{S_j}}\right)V_{P_i} - (n - 1)s][0.65 + 1.06\ln_{P_i}^{\prime} - 0.055\ln_{D_{eff}}^{\prime}], \quad (\theta_j = 90^\circ) \quad (49)
\]

The attack angle model of the front-fractured rod \( \delta_F \) is as follows:

\[
\delta_F = \arctan \sqrt{\tan^2\alpha_i^{\prime} + \tan^2\alpha_k^{\prime}} \quad (50)
\]

\[
\alpha_i^{\prime} = \frac{\pi}{2} \pm \alpha_i - \psi \pm \varepsilon_i \quad (51)
\]

\[
\alpha_i = 2 \frac{D_P V_{S_j} \sin\theta_j}{(TV_{S_j} + y_{r_s} - y_{r_1})V_{P_i}} \quad (52)
\]

\[
\alpha_k^{\prime} = \alpha_k \pm \varepsilon_k \quad (53)
\]

\[
\alpha_k = 2 \frac{D_P V_{S_j} \sin\theta_k}{(TV_{S_j} + y_{r_s} - y_{r_1})V_{P_i}} \quad (54)
\]

The attack angle model of the middle-fractured rod \( \delta_M \) is as follows:
\[ \delta_M = \arctan \sqrt{\tan^2 \varphi_i' + \tan^2 \varphi_k'} \] (55)

\[ \varphi_i = \frac{\pi}{2} \pm \varphi_i - \psi \pm \varepsilon_i \] (56)

\[ \varphi_i = 2 \frac{D_P}{L_2} \sin \theta_j - 2 \frac{D_P}{L_2} \sin \theta_j' \] (57)

\[ \varphi_k = \varphi_k \pm \varepsilon_k \] (58)

\[ \varphi_k = 2 \frac{D_P}{L_2} \sin \theta_k - 2 \frac{D_P}{L_2} \sin \theta_k' \] (59)

Where, \( \theta_j' \) is the angle between the symmetry plane of the adjacent LSC cover and the trajectory; \( \theta_k' \) is the inclination angle.

The attack angle model of the rear-fractured rod \( \delta_M \) is as follows:

\[ \tan^2 \delta_r = \tan^2 r_i' + \tan^2 r_k' \] (60)

\[ r_i' = \frac{\pi}{2} \pm r_i - \psi \pm \varepsilon_i \] (61)

\[ \gamma_i = 2 \frac{D_P}{(L-L_n)} \sin \theta_j' \] (62)

\[ r_k' = r_k \pm \varepsilon_k \] (63)

\[ r_k = 2 \frac{D_P}{(L-L_n)} \sin \theta_k' \] (64)

4. Factors affecting defense performance

According to the best defense model in Section 3.4, the influences of latency, intercepting velocity, intercept angle and number of LSCs on the defense performance were discussed, and numerical simulation was used to verify it.

4.1. Influence of latency on penetration depth

From equation (45), the relationship curve between the penetration depth of the fractured rods and different variables can be obtained, as shown in Figure 4. The scattered points in the figure are the numerical simulation results. It can be seen from the figure that as the delay increases, the penetration depth \( x \) of the front-fractured rod increases. At the same time, the higher the projectile velocity is, the greater the influence of latency on penetration depth is. The numerical simulation results are consistent with the changing trend of the relationship curve.
Figure 4. The relationship curve between the penetration depth of the front-fractured rod and the latency at different projectile velocities (d = 0.1m, V = 1.5km/s).

When the velocity of the long rod is 1.5km/s and the intercepting velocity of LSCs are 1.5km/s, 2km/s and 2.5km/s respectively, the relationship curves between the latency T and the penetration depth of the front-fractured rod are shown in Figure 5. It can be seen from the figure that under the same latency, the higher the intercepting velocity is, the smaller penetration depth of the front-fractured rod is.

Figure 5. The relationship curve between the penetration depth of the front-fractured rod and the latency at different intercepting velocities (d = 0.1m).

When the latency is 100μs, the intercepting distance is 10cm, and the intercepting velocity are 1.5km/s, 2km/s and 2.5km/s respectively, the relation curves between the projectile velocity and the penetration depth of the front-fractured rod are shown in Figure 6. The relationship between projectile velocity and penetration depth is nonlinear. The penetration effect enhances with the increase of the projectile velocity, while the penetration depth decreases with the increase of the intercepting velocity.
Figure 6. The relationship curve between the penetration depth of the front-fractured rod and the projectile velocity at different intercepting velocities. When the intercepting velocity is 2km/s, the latency is 100μs, and the projectile velocity is 1.3km/s, 1.5km/s and 1.7km/s respectively, the relationship curves between the intercepting distance and the penetration depth of the front-fractured rod are shown in Figure 7. It can be seen that the penetration depth of the front-fractured rod increases with the increase of intercepting distance. The results also show that the slope of the curve grows with the increase of the projectile velocity. The intercepting distance has different effects on residual penetration depth at different projectile velocity; the greater the projectile velocity is, the more serious the impact of intercepting distance on penetration depth is.

Figure 7. The relationship curve between the penetration depth and the intercepting distance of the front-fractured rod at different projectile velocities.

4.2. Influence of intercepting velocity on penetration depth

When the projectile velocity is 1.5km/s, the latency is 100μs, and the intercepting distance is 7cm, 10cm, 13cm and 16cm, the relationship between intercepting velocity and penetration depth is shown in Figure 8. It can be seen that the penetration depth of the front-fractured rod decreases with the increase of the intercepting velocity. At the same intercepting velocity, the penetration effect enhances with the increase of the intercepting distance. The relationship curve between the intercepting velocity and the penetration depth is a variable slope curve, and the greater the intercepting velocity is, the smaller the reduction of the penetration depth is.
4.3. The effect of intercepting angle on the attack angle

According to formula (50) and formula (60), the relationship between the intercept angle of LSC and the attack angle of the fractured rod can be obtained. The attack angle of the fractured rod is the key factor that determined the penetration or ricochet of the fractured rod, so it is of great significance to discuss the influence of the LSC intercept angle on the attack angle of the fractured rod.

When the projectile velocity is 1.5 km/s, the intercept distance is 10cm, the latency is 100μs, and the intercepting velocity is 1.5 km/s, 2.0 km/s and 2.5 km/s respectively, the relationship curves between the attack angle of the front and rear fractured rods and the intercept angle of LSC are shown in Figure 9, and the scattered points in the figure are the simulation values. It can be seen that with the increase of LSC intercept angle, the attack angle of the fractured rod increases; at the same time, the higher the intercepting velocity is, the larger the attack angle is. When the latency is 100μs, the attack angle of the front-fractured rod is obviously larger than that of the rear-fractured rod.
When the projectile velocity is 1.5 km/s, the intercept distance is 10 cm, the intercepting velocity is 2.5 km/s, and the latency is 70 μs, 120 μs, 170 μs respectively, the relationship curves between the attack angle of the front and rear fractured rods and the intercept angle of LSC are shown in Figure 10, and the scattered points in the figure are the simulation values. It can be seen from the figure that the shorter the latency is, the more obvious the change of attack angle of the front-fractured rod is; and the effect of the latency on the rear-fractured rod is lower than that of the front-fractured rod.

When length-to-diameter ratio of the projectile is different and the intercept position is different, the relationship curve between the intercept angle of LSC and the attack angle of the front and rear-fractured rods is shown in Figure 11. It can be seen that as the intercept angle increases, the attack angle of the fractured rods also increases. When the length-to-diameter ratio of the long rod increases at the same intercept angle of LSCs, the attack angle decreases; when the interception position is 1/3 of the length of the long rod, the attack angle of the front-fractured rod is smaller than when that of the 1/4 intercept position; The change of the attack angle of the rear-fractured rod is opposite to the front-fractured rod.
4.4. The effect of LSC’s number on the length of the rear-fractured rod

In the defense model of multiple LSCs, the number and spacing of LSCs determined the number of fractured rods. The length of the front-fractured rod was mainly determined by the latency, projectile velocity, intercept distance, intercepting velocity and intercept angle, regardless of the number of LSCs. The number of LSCs affected the number of fractured rods and the length of the rear-fractured rod. From equations (40) to (49), the relationship curve between the length of the rear-fractured rod and the LSCs’ spacing can be obtained, as shown in Figure 12. It can be seen from the figure that the length of the rear-fractured rod decreases as the number of LSCs increases and the spacing of the LSCs increases. Reasonable matching some parameters, such as the LSCs’ spacing, the number of LSCs and the intercept angle, can make the length and attack angle of the fractured rods meet the conditions of ricochet, and then achieve the best defense effect.

![Figure 12. The relationship between the LSC lateral spacing and the length of the rear-fractured rod at the different of LSCs.](image)

5. Conclusion

In this paper, the optimal defense model of multiple LSCs intercepting high-speed long rod was established, and the key parameters and their variation rules that affect the defense performance of the protection system were obtained. The key parameters in the defense model were verified by numerical simulation. According to the defense model, the following conclusions can be made:

(1) Reducing the latency can shorten the length of the front-fractured rod; Shortening the length of the front-fractured rod can reduce the penetration depth of the aftereffect and improve the defense performance.

(2) Increasing the number of LSCs can reduce the average length of the fractured rods; at the same time, various attack angles and shortened length of the fractured rods can greatly reduce the penetration depth, and then improve the defense performance.

(3) According to the defense model, the intercept velocity and intercept angle are reasonably matched to increase the attack angle of the fractured rods; when the attack angle is greater than the minimum ricochet angle, the middle-fractured rods and the rear-fractured rod will ricochet at the moment of impacting the target, so as to improve the defense performance.

This article has an important significance for the design of the active protection system with LSCs intercepting long rods.
6. References

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