PLASMA INDUCED FERMION SPIN-FLIP
CONVERSION \( f_L \to f_R + \gamma \)

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Abstract

The fermion spin-flip conversion \( f_L \to f_R + \gamma \) is considered, caused by the difference of the additional energies of the electroweak origin, acquired by left- and right-handed fermions (neutrino, electron) in medium. An accurate taking account of the fermion and photon dispersion in medium is performed. It is shown that the threshold arises in the process, caused by the photon (plasmon) effective mass. This threshold leaves no room for the so-called “spin light of neutrino” and “spin light of electron” in the real astrophysical situations.

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1 Introduction

The most important event in neutrino physics of the last decades was the solving of the Solar neutrino problem, made in the unique experiment on the heavy-water detector at the Sudbury Neutrino Observatory. [1] – [3] This experiment, together with the atmospheric and the reactor neutrino experiments, [4] – [7] has confirmed the key idea by B. Pontecorvo on neutrino oscillations. [8,9] The existence of non-zero neutrino mass and lepton mixing is thereby established. The Sun appeared in this case as a natural laboratory for investigations of neutrino properties.

There exists a number of natural laboratories, the supernova explosions, where gigantic neutrino fluxes define in fact the process energetics. It means that microscopic neutrino characteristics, such as the neutrino magnetic moment, the neutrino dispersion in an active medium, etc., would have a critical impact on macroscopic properties of these astrophysical events.

This is the reason for a growing interest to neutrino physics in an external active medium. In an astrophysical environment, the main medium influence on neutrino properties is defined by the additional energy $W$ acquired by a left-handed neutrino. [10] This additional energy gives, in part, an effective mass squared $m^2_L$ to the left-handed neutrino,

$$m^2_L = P^2 = (E + W)^2 - p^2,$$

where $P^\alpha$ is the neutrino four-momentum in medium, while $p^\alpha = (E, p)$ would form the neutrino four-momentum in vacuum, $E = \sqrt{p^2 + m^2_\nu}$.

Given a $\nu\nu\gamma$ interaction, the additional energy of left-handed neutrinos in medium opens new kinematical possibilities for the radiative neutrino transition:

$$\nu \rightarrow \nu + \gamma.$$

It should be self-evident, that the influence of the substance on the photon dispersion must be taken into account, $\omega = |\mathbf{k}|/n$, where $n \neq 1$ is the refractive index.

First, a possibility exists that the medium provides the condition $n > 1$ (the effective photon mass squared is negative, $m^2_\gamma \equiv q^2 < 0$) which corresponds to the well-known effect [11] – [13] of “neutrino Cherenkov radiation”. In this situation, the neutrino dispersion change under the medium influence is being usually neglected, because the neutrino dispersion is defined by the weak interaction while the photon dispersion is defined by the electromagnetic interaction.

One more possibility could be formally considered when the photon dispersion was absent, and the process of the radiative neutrino transition $\nu \rightarrow \nu + \gamma$ would be caused by the neutrino dispersion only. As the left-handed neutrino dispersion is only changed, transitions become possible caused by the $\nu\nu\gamma$ interaction with the neutrino chirality change, e.g. due to the neutrino magnetic dipole moment. Just this situation called the “spin light of neutrino” ($SL\nu$), was first proposed and investigated in detail in an extended series of papers. [14] – [24]

However, it is evident that in the analysis of this effect such an important phenomenon as plasma influence on the photon dispersion cannot be ignored. As will be shown below, this phenomenon closes the $SL\nu$ effect for all real astrophysical situations. [25] – [30]

In the recent papers [31] – [33] the authors have extended their approach to the so-called “spin light of electron” ($SLe$), $e_L \rightarrow e_R + \gamma$. It will be shown that just the same mistake of ignoring the photon dispersion in plasma was repeated in these papers.

It is interesting to note that it was not the first case when the plasma influence was taken into account for one participant of the physical process while it was not taken for other participant. The history is repeated.

As E. Braaten wrote in Ref. [34]:

“In Ref. [35], it was argued that their calculation for the emissivities from photon and plasmon decay would break down at temperatures large enough that $m_\gamma > 2 m_e$, since the decay $\gamma \rightarrow e^+e^−$ is then kinematically allowed. This statement, which has been repeated in subsequent papers, [36] – [39] is simply untrue. The plasma effects which generate the photon
mass $m_\gamma$ also generate corrections to the electron mass such that the decay $\gamma \rightarrow e^+e^-$ is always kinematically forbidden."

Thus, the authors [14] – [24], [31] – [33] made the same mistake when they considered the plasma-induced additional neutrino or electron energy and ignored the effective photon mass $m_\gamma$ arising by the same reason.

Consequently, the spin-flip processes $\nu_L \rightarrow \nu_R + \gamma$ and $e_L \rightarrow e_R + \gamma$ should be reanalysed with taking into account the photon dispersion in medium. Having in mind possible astrophysical applications, it is worthwhile to consider the astrophysical plasma as a medium, which transforms the photon into the plasmon, see e.g. Ref. [40] and the papers cited therein. Here, we perform a detailed analysis of the fermion spin-flip conversion $f_L \rightarrow f_R + \gamma$ with both the fermion dispersion and the photon dispersion in medium.

2 Kinematical Analysis for “Spin Light of Neutrino”

To perform the kinematical analysis, it is necessary to evaluate the scales of the values of the left-handed neutrino additional energy $W$ and of the photon (plasmon) effective mass squared $m_\gamma^2$.

The expression for this additional energy of a left-handed neutrino with the flavor $i = e, \mu, \tau$ was obtained in the local limit of the weak interaction, [41] – [43] see also Ref. [44], and can be presented in the following form

$$W_i = \sqrt{2} G_F \left[ \left( \delta_{ie} - \frac{1}{2} + 2 \sin^2 \theta_W \right) (N_e - \bar{N}_e) + \left( \frac{1}{2} - 2 \sin^2 \theta_W \right) (N_p - \bar{N}_p) \right]$$

$$- \frac{1}{2} (N_n - \bar{N}_n) + \sum_{\ell = e, \mu, \tau} (1 + \delta_{i\ell}) (N_{\nu\ell} - \bar{N}_{\nu\ell})$$

where the functions $N_e, N_p, N_n, N_{\nu\ell}$ are the number densities of background electrons, protons, neutrons, and neutrinos, and $\bar{N}_e, \bar{N}_p, \bar{N}_n, \bar{N}_{\nu\ell}$ are the densities of the corresponding antiparticles.

To find the additional energy for antineutrinos, one should change the total sign in the right-hand side of Eq. (3).

For a typical astrophysical medium, except for the early Universe, using the notations $N_p \simeq N_e = Y_e N_B$, $N_n \simeq (1 - Y_e) N_B$, $N_{\nu\ell} = Y_{\nu\ell} N_B$, where $N_B$ is the barion density, one obtains from Eq. (3):

$$W_e = \frac{G_F N_B}{\sqrt{2}} \left( 3 Y_e + 4 Y_{\nu e} - 1 \right),$$

$$W_{\mu,\tau} = -\frac{G_F N_B}{\sqrt{2}} \left( 1 - Y_e - 2 Y_{\nu e} \right).$$

Here, $Y_{\nu e}$ is not zero in the supernova core only, and describes the fraction of trapped electron neutrinos.

In any medium except for the supernova core, the additional energy acquired by muon and tau left-handed neutrinos is always negative, because $Y_e < 1$. At the same time, the additional energy of electron left-handed neutrinos in these conditions becomes positive at $Y_e > 1/3$. And vice versa, the additional energy for electron antineutrinos is positive at $Y_e < 1/3$, while it is always positive for the muon and tauon antineutrinos. On the other hand, right-handed neutrinos and their antiparticles, left-handed antineutrinos, being sterile with respect to weak interactions, do not acquire an additional energy.

If $Y_{\nu e}$ can be neglected, one readily obtains from Eq. (4):

$$W \simeq 6 \text{ eV} \left( \frac{N_B}{10^{38} \text{ cm}^{-3}} \right) (3 Y_e - 1),$$

3
where the scale of the barion number density is taken, which is typical e.g. for the interior of a neutron star.

On the other hand, a plasmon acquires in medium an effective mass \( m_\gamma \) which is approximately constant at high energies. For the transversal plasmon, the value \( m_\gamma^2 \) is always positive, and is defined by the so-called plasmon frequency. In the non-relativistic classical plasma (i.e. for the solar interior) one has:

\[
m_\gamma \equiv \omega_{pl} = \sqrt{\frac{4\pi \alpha N_e}{m_e}} \simeq 4 \times 10^2 \text{ eV} \left( \frac{N_e}{10^{26} \text{ cm}^{-3}} \right)^{1/2}.
\]  

(7)

For the ultra-relativistic dense matter one has:

\[
m_\gamma^2 = \frac{2\alpha}{\pi} \left( \mu_e^2 + \frac{\pi^2}{3} T^2 \right),
\]

(8)

where \( \mu_e \) is the chemical potential of plasma electrons. For the case of cold degenerate plasma one obtains from Eq. (8):

\[
m_\gamma = \sqrt{\frac{3}{2}} \omega_{pl} = \left( \frac{2\alpha}{\pi} \right)^{1/2} \left( 3\pi^2 N_e \right)^{1/3} \simeq 10^7 \text{ eV} \left( \frac{N_e}{10^{37} \text{ cm}^{-3}} \right)^{1/3}.
\]

(9)

One more physical parameter, a great attention was paid to in the SL\( \nu \) analysis, [14] – [24] was the neutrino vacuum mass \( m_\nu \). As the scale of neutrino vacuum mass could not exceed essentially a few electron-volts, which is much less than typical plasmon mass scales for real astrophysical situations, see Eqs. (7)-(10), it is reasonable to neglect \( m_\nu \) in our analysis.

The energy and momentum conservation law for the radiative spin-flip neutrino transition \( \nu_L \to \nu_R + \gamma \), with the additional left-handed neutrino energy included, has the form:

\[
E + W = E' + \omega, \quad \mathbf{p} = \mathbf{p}' + \mathbf{k},
\]

(11)

Thus, in accordance with (11), a simple condition for the kinematic opening of the process is:

\[
m_\gamma^2 \simeq 2EW > m_\gamma^2.
\]

(12)

This means that the process becomes kinematically opened when the neutrino energy exceeds the threshold value,

\[
E > E_0 = \frac{m_\gamma^2}{2W}.
\]

(13)

This threshold behavior of the process \( \nu_L \to \nu_R + \gamma \) in plasma becomes very clear if compared with the well-known process in vacuum. As is seen from Refs. [26,29], the authors believe that they have shown in Refs. [20,22] that “for the case of high-energy neutrinos the matter influence on the photon dispersion can be neglected” because “plasma is transparent for electromagnetic radiation on frequencies greater than the plasmon frequency”. It is rather naive consideration. Really, one can see that from the side of kinematics the discussed process \( \nu_L \to \nu_R + \gamma \) in plasma is identical to the process \( \bar{\nu}_e + e^- \to \tau^- + \bar{\nu}_\tau \), where the high-energy electron antineutrino scattered off the electron in rest, creates the \( \tau \) lepton. The 4-momentum conservation law can be written in the form

\[
p^\alpha + m_e u^\alpha = p'^\alpha + q^\alpha,
\]

(14)
where \( p = (E, p) \), \( p' = (E', p') \) are the initial and final neutrino 4-momenta, \( u \) is the 4-vector of the electron velocity, which in its rest frame is \( u^\alpha = (1, 0) \), \( q = (\omega, k) \) is the \( \tau \) lepton 4-momentum. In the lab system, which is the electron rest frame, the energy and momentum conservation takes the form:

\[
E + m_e = E' + \omega, \\
p = p' + k,
\]

which is just the same as Eq. (11), up to the notations. Neglecting the neutrino masses, let us write down the Mandelstam \( S \) variable in the lab system:

\[
S = (p + m_e u)^2 = 2m_e E + m_e^2. 
\]

(16)

On the other hand, in the center-of-mass frame one has:

\[
S = (\omega + E')^2 = (\sqrt{m_\tau^2 + p'^2} + p')^2 \geq m_\tau^2. 
\]

(17)

The threshold value for the initial neutrino energy arises from (16) and (17):

\[
E \geq E_0 = \frac{m_\tau^2 - m_e^2}{2m_e} \simeq \frac{m_\tau^2}{2m_e},
\]

(18)

to be compared with Eq. (13). The similarity is deliberately not accidental. Both inequalities are caused by the minimal value of the Mandelstam \( S \) variable which is equal to the mass squared of the heavy particle in the final state, \( m_\tau^2 \) (or \( m_\gamma^2 \) in our case). At the same time, the mass of the initial electron in rest in the process \( \bar{\nu}_e + e^- \rightarrow \tau^- + \bar{\nu}_\tau \) is kinematically identical to the additional left-handed neutrino energy \( W \) in the process \( \nu_L \rightarrow \nu_R + \gamma \) in plasma. Taking the approach of the authors, [14] – [24] one should forget about the threshold (18) and conclude that the process \( \bar{\nu}_e + e^- \rightarrow \tau^- + \bar{\nu}_\tau \) is always open if only the medium (which is vacuum in this case) is transparent for \( \tau \) leptons.

Let us evaluate the threshold neutrino energies (13) for different astrophysical situations.

In the classical plasma, the threshold neutrino energy does not depend on density, and do depend on the chemical composition only:

\[
E_0 \simeq \frac{Y_e}{3Y_e - 1} \frac{4 \sin^2 \theta_W m_W^2}{m_e}. 
\]

(19)

For the solar interior \( Y_e \simeq 0.6 \), and the threshold neutrino energy is

\[
E_0 \simeq 10^{10} \text{ MeV},
\]

(20)

to be compared with the upper bound \( \sim 20 \) MeV for the solar neutrino energies.

For the interior of a neutron star, where \( Y_e \ll 1 \), the additional energy for neutrinos (4), (5) is negative, and the process \( \nu_L \rightarrow \nu_R + \gamma \) is closed. On the other hand, there exists a possibility for opening the antineutrino decay. Taking for the estimation \( Y_e \simeq 0.1 \), one obtains from (6) and (8) the threshold value

\[
E_0 \simeq 10^7 \text{ MeV},
\]

(21)

to be compared with the typical energy \( \sim \text{ MeV} \) of neutrinos emitted via the URCA processes.

For the conditions of a supernova core, \( Y_e \sim 0.35, Y_{\nu_e} \sim 0.05 \), the additional energy of left-handed electron neutrinos obtained from Eq. (4) leads to:

\[
E_0 \simeq 10^7 \text{ MeV},
\]

(22)

to be compared with the averaged energy \( \sim 10^2 \) MeV of trapped neutrinos.
Thus, the analysis performed shows that the radiative spin-flip neutrino transition $\nu_L \rightarrow \nu_R + \gamma$ is possible at ultra-high neutrino energies only. However, it should be obvious in this case, that the local limit of the weak interaction does not describe comprehensively the additional neutrino energy in plasma, and the non-local weak contribution must be taken into account. The analysis of this contribution was first performed for the conditions of the early Universe. [41,44] In this case, the local weak contribution is suppressed, because plasma is almost charge symmetric.

In a general case, the non-local weak contribution into the additional neutrino energy in plasma, which is identical for both neutrinos and antineutrinos, can be presented in the form

$$\Delta^{(nloc)}W_i = -\frac{16 G_F E}{3\sqrt{2}} \left[ \langle E_{\nu_i} \rangle (N_{\nu_i} + \bar{N}_{\nu_i}) + \delta_{ie} \frac{\langle E_e \rangle}{m^2_W} (N_e + \bar{N}_e) \right], \quad (23)$$

where $E$ is the energy of a neutrino propagating through plasma, $\langle E_{\nu_i} \rangle$ and $\langle E_e \rangle$ are the averaged energies of plasma neutrinos and electrons correspondingly. In a particular case of a charge symmetric hot plasma, this expressions reproduces the result of Refs. [41,44]:

$$\Delta^{(nloc)}W_i = -\frac{7}{45} \sqrt{\frac{\pi^2 G_F T^4}{m^2_W}} \left( \frac{1}{m^2_Z} + 2 \frac{\delta_{ie}}{m^2_W} \right) E. \quad (24)$$

The minus sign in (24) unambiguously shows that in the early Universe, in contrast to the neutron star interior, the process of the radiative spin-flip transition is forbidden both for neutrinos and antineutrinos.

An analysis of the sum of the local and non-local weak contributions and shows that adding of the latter leads in general to the decreasing of the additional neutrino energy $W$ in plasma, i.e. to the increasing of the threshold energy. Strictly speaking, one has to perform an analysis of the kinematical inequality, which leads to the solving of the quadratic equation. As a result, there arises the window in the neutrino energies for the process to be kinematically opened, $E_0 < E < E_{\text{max}}$, where $E_0$ and $E_{\text{max}}$ are the lower and the upper limits connected with the roots of the above-mentioned quadratic equation, if they exist. For example, in the solar interior there is no window for the process with electron neutrinos at all, i.e. the transition $\nu e_L \rightarrow \nu e_R + \gamma$ is forbidden kinematically.

Thus, the above analysis shows that the nice effect of the “spin light of neutrino”, unfortunately, has no place in real astrophysical situations because of the photon dispersion. The sole possibility for the discussed process $\nu_L \rightarrow \nu_R + \gamma$ to have any significance could be connected only with the situation when an ultra-high energy neutrino threads a star.

### 3 Kinematical Analysis for “Spin Light of Electron”

Similarly to a neutrino, an electron acquires in medium the additional energy depending on its helicity, due to the parity non-conserving weak interaction. To find this energy, one should consider the electron self-energy operator in medium. For a real electron, with taking account of the renormalization of the chiral mass and the wave function, this self-energy operator can be presented in the form

$$\Sigma_r = \frac{1}{2} \hat{u}(V - A \gamma_5), \quad (25)$$

where $u^\alpha$ is the four-vector of plasma velocity which in its rest frame is $u^\alpha = (1, 0)$. The value $V$ is caused mainly by the electromagnetic interaction of an electron:

$$V \simeq V^{\text{em}} = \frac{\alpha}{\pi E} \left( \mu^2_e + \pi^2 T^2 \right). \quad (26)$$

Using (26), the value $V$ can be expressed via the plasmon mass:

$$V \simeq C \frac{m^2_e}{E}, \quad C = \frac{3}{2} \frac{\mu^2_e + \pi^2 T^2}{3 \mu^2_e + \pi^2 T^2}, \quad \frac{1}{2} \leq C \leq \frac{3}{2}, \quad (27)$$
where the lower and the upper limits for $C$ correspond to the cases of cold and hot plasma.

We do not present here the contribution into $V$ caused by the weak interaction, because in real astrophysical conditions the electromagnetic contribution $V^{\text{em}}$ always dominates, except for the unphysical case of a pure neutron medium considered in Refs. [31] - [33]. It should be noted that even in the conditions of a cold neutron star, the fraction of electrons and protons cannot be exactly zero, $Y_e \gtrsim 0.01$. [45] And even at such minimal value of $Y_e$ the electromagnetic contribution into $V$ dominates.

As about the value $A$, it is caused by the weak interaction only and has the form:

$$A = \frac{G_F}{\sqrt{2}} \left[ 2 \left(1 - 4 \sin^2 \theta_W \right) \left(N_e - \bar{N}_e\right) - (1 - 4 \sin^2 \theta_W) \left(N_p - \bar{N}_p\right) \right] + (N_n - \bar{N}_n) + 2 \sum_{\ell=e,\mu,\tau} (2 \delta_{\ell e} - 1) (N_{\ell \ell} - \bar{N}_{\ell \ell})$$

$$- \frac{8 G_F E}{3 \sqrt{2}} \left[ \frac{<E_e>}{m_Z^2} (N_e + \bar{N}_e) (1 - 4 \sin^2 \theta_W) + 4 \frac{<E_{e\nu}>}{m_W^2} (N_{e\nu} + \bar{N}_{e\nu}) \right]. \quad (28)$$

Here, both the local and the non-local weak contributions are included. $E$ is the energy of an electron propagating through plasma, $<E_e>$ and $<E_{e\nu}>$ are the averaged energies of plasma electrons and neutrinos correspondingly. For typical astrophysical medium, $\bar{N}_e \simeq \bar{N}_p \simeq \bar{N}_n \simeq \bar{N}_{\ell \ell} \simeq 0$, $N_p \simeq N_e = Y_e N_B$, $N_{e\nu} \simeq Y_{e\nu} N_B$, the expression (28) for $A$ can be simplified:

$$A = \frac{G_F}{\sqrt{2}} N_B \left[ 1 - 4 \sin^2 \theta_W \right] Y_e \left[ 1 + 2 Y_e - \frac{8}{3} \frac{<E_{e\nu}>}{m_W^2} \right] \left(1 - 4 \sin^2 \theta_W\right) - \frac{32}{3} \frac{E <E_{e\nu}>}{m_W^2} Y_e \left(1 - 4 \sin^2 \theta_W\right). \quad (29)$$

The self-energy operator (25) defines the additional electron energy which can be written in the plasma rest frame as:

$$\Delta E = \frac{1}{2} (V - A \lambda v), \quad (30)$$

where $\lambda = -1$ for the left helicity of the electron, and $\lambda = +1$ for the right helicity, $v$ is the electron velocity.

By this means electrons with left and right helicities acquire different additional energies in plasma. Similarly to the previous analysis for neutrinos, one can guess that the process $e_L \rightarrow e_R + \gamma$ is possible for ultra-high energy electrons only, due to the relative smallness of the helicity-depending energy shift $A$. The energy $E_\lambda$ of the ultra-relativistic electron takes the form:

$$E_{\pm 1} \simeq p + \frac{\tilde{m}_e^2}{2p} \pm \frac{A}{2}, \quad (31)$$

where the effective electron mass in plasma is introduced, which is defined by:

$$\tilde{m}_e^2 = m_e^2 + C m_\gamma^2. \quad (32)$$

Given Eq. (31), the energy and momentum conservation law for the process $e_L \rightarrow e_R + \gamma$ can be written in the form:

$$p + \frac{\tilde{m}_e^2}{2p} + \frac{A(p)}{2} = p' + \frac{\tilde{m}_e^2}{2p'} - \frac{A(p')}{2} + k + \frac{m_\gamma^2}{2k}, \quad (33)$$

$$p = p' + k. \quad (34)$$

For the first step, let us neglect the non-local weak contribution into $A$ in (25). It is justified when the energies are not very high. In this case, it is easy to obtain the kinematic condition for the process to be opened:

$$E > E_0 = \frac{m_e^2 + 2 \tilde{m}_e m_\gamma}{2 A}. \quad (35)$$
Similarly to the neutrino case, the numerical analysis gives for the threshold energies:

i) for the solar interior:
\[ E_0 \simeq 10^{13} \text{ MeV} ; \]  
(36)

ii) for the interior of a neutron star:
\[ E_0 \simeq 10^7 \text{ MeV} . \]  
(37)

Evidently, the inclusion of the non-local weak contribution is necessary. As the analysis shows, for the solar interior the process \( e_L \rightarrow e_R + \gamma \) is totally forbidden exactly as the process \( \nu_e L \rightarrow \nu_e R + \gamma \). As for the neutron star interior, there exists a window approximately from \( 10^7 \) MeV to \( 10^{10} \) Mev for the process to be opened.

Hence the process \( e_L \rightarrow e_R + \gamma \) could be realized in the same exotic case when an ultra-high energy electron tries to thread a neutron star.

4 Solution of the Dirac Equation for a Fermion in Medium

For definiteness, we consider the spin-flip process \( f_L \rightarrow f_R + \gamma \) with the plasmon creation, where \( f \) could be both neutrino and electron. The amplitude of the process contains the bispinor amplitudes with the definite helicities, \( u_L \) and \( u'_R \), of the initial left-handed and the final right-handed fermions. We should remind that helicity states do not coincide in general with the chirality states, and tend to them in the ultra-relativistic limit only. To define the bispinor amplitudes, one should write down the modified Dirac equation for a fermion in plasma. In the momentum space, the plasma influence is described by the self-energy operator \( \Sigma_r \)

\[
\left( \hat{p} - m - \Sigma_r \right) u^{(\lambda)} = 0 ,
\]  
(38)

where \( m \) is the fermion mass. The operator \( \Sigma_r \) defined in Eq. (25) provides the additional fermion energy \( \Delta E \) defined in Eq. (30). It is well known that the additional energy in plasma modifies the phase of the de Broglie wave of the fermion:

\[
\psi(x) \sim e^{-i(Px)} , \quad p^\alpha = \left( E + \frac{1}{2} (V - A \lambda v) , p \right) .
\]  
(39)

Substituting (39) into (38), one can rewrite the Dirac equation in the plasma rest frame as follows:

\[
\left( \hat{p} - m - \frac{A}{2} \gamma_0 (\lambda v - \gamma_5) \right) u^{(\lambda)} = 0 .
\]  
(40)

It is worthwhile to note that the relatively large value \( V \) is exactly cancelled here and it does not influence the bispinor amplitude \( u^{(\lambda)} \).

The solution of Eq. (40) obtained in the natural approximation \( A \ll E \), is

\[
u^{(\lambda)} \simeq \left( 1 - \frac{mA}{4E^2 \gamma_0 \gamma_5} \right) u_0^{(\lambda)} . \]  
(41)

where \( u_0^{(\lambda)} \) is the solution with definite helicity of the Dirac equation in vacuum, \( (\hat{p} - m) u_0^{(\lambda)} = 0 \).

It is seen from Eq. (41) that in the ultra-relativistic limit the deviation of the solution in plasma from the vacuum one contains an additional suppressing factor \( m/E \ll 1 \). Thus, one can use the vacuum solution with a great accuracy.

We have to note that in Refs. [19,20,24], [31] – [33] the authors claimed to obtain the exact solution of the modified Dirac equation for a fermion in plasma. However, it can be easily seen that at least in one case their solution is simply incorrect. Really, for the interior of a neutron
star, the additional neutrino energy \( W \sim 10\,\text{eV} \) can exceed the momenta of soft neutrinos, \( p \sim 1\,\text{eV} \). In this case the bispinor (5) in Ref. [24] for the right-handed massless neutrino is not the solution of the modified Dirac equation (4) there. Moreover, according to Eq. (6) of Ref. [24] such right-handed neutrinos acquire an additional energy in plasma, in spite of being sterile.

5 Ultra-High Energy Neutrino Threads a Star

Here, we consider the above-mentioned possibility of the radiative spin-flip ultra-high energy neutrino transition \( \nu_L \to \nu_R + \gamma \) when the neutrino threads a star. Obviously it could have only a methodical meaning. For these purposes, we present a correct calculation of the process width, [28] with some details. [30]

A neutrino having a magnetic moment \( \mu_\nu \) interacts with photons, and the Lagrangian of this interaction is

\[
\mathcal{L} = -\frac{i}{2} \mu_\nu (\bar{\nu} \sigma_{\alpha \beta} \nu) F^{\alpha \beta},
\]

(42)

where \( \sigma_{\alpha \beta} = (1/2)(\gamma_\alpha \gamma_\beta - \gamma_\beta \gamma_\alpha) \), and \( F^{\alpha \beta} \) is the tensor of the photon electromagnetic field.

With the Lagrangian (42), one readily obtains the amplitude of the process \( \nu_L \to \nu_R + \gamma \) with a creation of the plasmon with the four-momentum \( q^\alpha = (\omega, k) \) and the polarization \( \xi \):

\[
\mathcal{M}(\xi) = \mu_\nu \left( \bar{u}_R \gamma \xi \nu_L \right),
\]

(43)

where \( u_L \) and \( u'_R \) are the bispinor amplitudes for the initial left-handed and the final right-handed neutrinos.

The amplitude (43) squared

\[
|\mathcal{M}(\xi)|^2 = \mu_\nu^2 \text{Tr} \left[ \rho_L(p) \bar{\xi}(\xi) \rho_R(p') \bar{\xi}(\xi) \right],
\]

(44)

with the neutrino density matrices substituted,

\[
\rho_L(p) = u_L \bar{u}_L = \frac{1}{2} \hat{p} (1 + \gamma_5), \quad \rho_R(p') = u'_R \bar{u}_R = \frac{1}{2} \hat{p}' (1 - \gamma_5),
\]

(45)

is:

\[
|\mathcal{M}(\xi)|^2 = 2 \mu_\nu^2 \left[ 2(pq) (p'q) - q^2 (pp') - 2q^2 \left( \bar{p} \xi(\xi) \right) \left( p' \bar{\xi}(\xi) \right) \right].
\]

(46)

Using the energy-momentum conservation law and keeping in mind that \( E > E_0 \gg W \), one obtains:

\[
(pq) = W (E - \omega) + \frac{m_\gamma^2}{2}, \quad (p'q) = W E - \frac{m_\gamma^2}{2}, \quad (pp') = W \omega - \frac{m_\gamma^2}{2}.
\]

(47)

Substituting Eqs. (47) into Eq. (46) and summarizing over the transversal plasmon polarizations:

\[
\sum_\xi \left( \bar{p} \xi(\xi) \right) \left( p' \bar{\xi}(\xi) \right) = E^2 \sin^2 \theta, \quad \sum_\xi |\mathcal{M}(\xi)|^2 = |\mathcal{M}|^2,
\]

(48)

where \( \theta \) is the angle between the initial neutrino momentum \( \mathbf{p} \) and the plasmon momentum \( \mathbf{k} \), one readily obtains the amplitude squared in the form:

\[
|\mathcal{M}|^2 = 4 \mu_\nu^2 E^2 \left[ 2 W^2 \left( 1 - \frac{\omega}{E} \right) - m_\gamma^2 \sin^2 \theta \right].
\]

(49)

We should note that this expression presented in our Ref. [28] came under criticism in Ref. [29] where it was declared not to be positively-defined. One can see that the plasmon mass \( m_\gamma \) in
the second negative term is much greater than the additional neutrino energy \( W \). However, one should wonder what is the restriction on the \( \theta \) angle. Really, from the energy-momentum conservation law, the \( \theta \) angle can be expressed in terms of the energy \( \omega \) of the plasmon to be considered relativistic with a good accuracy, as follows:

\[
\sin^2 \theta \simeq \theta^2 \simeq \frac{2W(\omega - E_0)}{\omega^2} \left(1 - \frac{\omega}{E}\right),
\]

and the positivity of the amplitude squared becomes manifest:

\[
|\mathcal{M}|^2 = \frac{8 \mu^2_\nu E^2 W^2}{\omega^2} \left(1 - \frac{\omega}{E}\right) [(\omega - E_0)^2 + E_0^2].
\]

It should be stressed that discussing ultra-high energy neutrinos, and consequently the high plasmon energies, one can consider with a good accuracy the plasmon mass \( \gamma \), which is the dynamical parameter, see Eq. (1).

The differential width of the process \( \nu_L \to \nu_R + \gamma \) is defined as:

\[
d\Gamma = \frac{|\mathcal{M}|^2}{8 E (2\pi)^2} \delta(E + W - E' - \omega) \delta^{(3)}(p - p' - k) \frac{d^3p'}{E' \omega},
\]

where the plasmon energy \( \omega \) cannot be taken the vacuum one (\( \omega = |k| \)), as it was done in the \( SL\nu \) analysis, [14] – [24] but it is defined by the dispersion in plasma, \( \omega = \sqrt{k^2 + m_\gamma^2} \).

Performing a partial integration in Eq. (52), one obtains for the photon (i.e. transversal plasmon) energy spectrum

\[
d\Gamma = \frac{\alpha}{4} \left(\frac{\mu_\nu}{\mu_B}\right)^2 \frac{m_\gamma^2 W}{m_e^2} \frac{1 - x}{\epsilon x^2} \left[(\epsilon x - 1)^2 + 1\right] dx \left(\frac{1}{\epsilon} \leq x \leq 1\right),
\]

where \( \mu_B = e/(2m_e) \) is the Bohr magneton, and the notations are used \( x = \omega/E \), and \( \epsilon = E/E_0 \). Recall that \( E_0 = m_\gamma^2/(2W) \) is the threshold neutrino energy for the process to be opened.

Instead of the photon energy spectrum (53), one can obtain also the spatial distribution of final photons. As the analysis of Eq. (50) shows, all the photons are created inside the narrow cone with the opening angle \( \theta_0 \),

\[
\theta < \theta_0 \simeq \frac{\epsilon - 1}{\epsilon} \frac{W}{m_\gamma} \ll 1.
\]

The distribution of final photons over the \( \theta \) angle can be presented in the form:

\[
d\Gamma = \frac{\alpha}{8} \left(\frac{\mu_\nu}{\mu_B}\right)^2 \frac{m_\gamma^2 W}{m_e^2} \epsilon (\epsilon - 1) \frac{dy}{\sqrt{1 - y}}
\]

\[
\times \left\{ \frac{1}{\epsilon + 1 - (\epsilon - 1)\sqrt{1 - y}} \left[ 1 - \frac{2}{\epsilon + 1 - (\epsilon - 1)\sqrt{1 - y}} - \frac{(\epsilon - 1)^2 y}{2\epsilon^2} \right] \right. \]

\[
\left. + \frac{1}{\epsilon + 1 + (\epsilon - 1)\sqrt{1 - y}} \left[ 1 - \frac{2}{\epsilon + 1 + (\epsilon - 1)\sqrt{1 - y}} - \frac{(\epsilon - 1)^2 y}{2\epsilon^2} \right] \right\},
\]

where \( y = \theta^2/\theta_0^2 \), \( 0 \leq y \leq 1 \).

Performing the final integration in Eq. (53) over \( x \), as well as in Eq. (55) over \( y \), one obtains the total width of the process

\[
\Gamma = \frac{\alpha}{8} \left(\frac{\mu_\nu}{\mu_B}\right)^2 \frac{m_\gamma^2 W}{m_e^2} F(\epsilon) \quad (\epsilon \geq 1),
\]

\[
F(\epsilon) = \frac{1}{\epsilon} [(\epsilon - 1)(\epsilon + 7) - 4(\epsilon + 1) \ln \epsilon].
\]
In Figs. 1 and 2 the functions \( f(x, \varepsilon) \) and \( \Phi(y, \varepsilon) \) are shown, describing the normalized energy spectrum and the normalized spatial distribution of final plasmons for some values of the ratio \( \varepsilon = E/E_0 \) of the neutrino energy to the threshold neutrino energy:

\[
f(x, \varepsilon) = \frac{1}{\Gamma} \frac{d\Gamma}{dx}, \quad \Phi(y, \varepsilon) = \frac{1}{\Gamma} \frac{d\Gamma}{dy},
\]

\[ (57) \]

Figure 1: The function \( f(x, \varepsilon) \) defining the normalized energy spectrum of plasmons from the left-handed neutrino decay for different values of the ratio \( \varepsilon, \varepsilon = 30 \) (solid line), \( \varepsilon = 5 \) (dotted line), and \( \varepsilon = 2 \) (dashed line).

The well-known singularity is seen in Fig. 2 at \( \theta = \theta_0 \left( y = 1 \right) \) which is typical for the angular distribution of a final massive particle in the two-particle decay on flight.

It should be noted that in the situation when \( W < 0 \), and the transition \( \nu_L \rightarrow \nu_R + \gamma \) is forbidden, the crossed channel \( \nu_R \rightarrow \nu_L + \gamma \) becomes kinematically opened. As the analysis shows, the plasmon spectrum and the total decay width are described in this case by the same Eqs. (53) and (56), with the only substitution \( W \rightarrow |W| \).

To illustrate the extreme weakness of the effect considered, let us evaluate numerically the mean free path of an ultra-high energy neutrino with respect to the radiative decay, when the neutrino propagates through a neutron star. For the typical neutron star parameters, \( N_B \simeq 10^{38} \text{ cm}^{-3}, Y_e \simeq 0.05 \), we obtain

\[
L \simeq 10^{19} \text{ cm} \times \left( \frac{10^{-12} \mu_B}{\mu_\nu} \right)^2 \left[ F \left( \frac{E}{10 \text{ TeV}} \right) \right]^{-1},
\]

\[ (58) \]

where the neutrino energy \( E > E_0, E_0 \simeq 10 \text{ TeV} \) is the threshold energy for such conditions. The mean free path (58) should be compared with the neutron star radius \( \sim 10^6 \text{ cm} \).

6 Ultra-High Energy Electron Spin-Flip Process \( e_L \rightarrow e_R + \gamma \) in Neutron Star

As was already mentioned, in the papers [31] – [33] the authors have extended their approach to the so-called “spin light of electron” (SLe), \( e_L \rightarrow e_R + \gamma \). However, the same mistake of ignoring the photon dispersion in plasma was repeated there.
Figure 2: The function $\Phi(y, \varepsilon)$ defining the normalized angular distribution of plasmons from the left-handed neutrino decay for different values of the ratio $\varepsilon$, $\varepsilon = 30$ (solid line), $\varepsilon = 10$ (dotted line), and $\varepsilon = 5$ (dashed line).

According to the analysis performed in Sec. 3, the $SLe$ effect has no place in real astrophysical conditions. Here, we consider for methodical purposes the sole possibility when an ultra-high energy electron threads a star.

The process amplitude is caused by the electromagnetic interaction of an electron with a photon, and has the form:

$$M(\xi) = e\left(\varepsilon_R^{(\xi)} \varepsilon_L^{(\xi)} e_L\right).$$

(59)

After standard but rather cumbersome calculations, one obtains for the amplitude squared summarized over the transversal plasmon polarizations $\xi$:

$$|M|^2 \equiv \sum_{\xi} \left|M(\xi)\right|^2 \simeq 4\pi\alpha m_e^2 \left(\frac{E'}{E} + \frac{E}{E'} - 2\right),$$

(60)

where $E$ and $E'$ are the energies of the initial and final electrons. Here, $m_e$ is the chiral electron mass which differs from the effective mass $\bar{m}_e$, see Eq. (32).

We note that the amplitude squared of the spin-flip process $e_L \rightarrow e_R + \gamma$ would be zero if the chiral electron mass $m_e$ is equal to zero, because of the chirality conservation in the electromagnetic interaction.
The process probability is calculated by the standard way:

\[
\Gamma = \frac{\alpha m^2}{2 E^2} \int_{E_1}^{E_2} \frac{dE'}{E} \left( \frac{E'}{E} + \frac{E}{E'} - 2 \right),
\]

\[
\Gamma \approx \frac{\alpha m^2}{2 E} \left[ \ln \frac{E_2}{E_1} - \frac{E_2 - E_1}{E} \left( 2 - \frac{E_2 + E_1}{2E} \right) \right],
\]

\[
E_{1,2} = \frac{E}{2} \left( E + E_0 \frac{C}{2\sqrt{C} + 1} \right)^{-1} \left[ E + E_0 \frac{2\sqrt{C} - 1}{2\sqrt{C} + 1} \right],
\]

\[
E_0 = \frac{m_e^2}{2A} \left( 2\sqrt{C} + 1 \right),
\]

It is seen from Eqs. (61) - (64), that \( E_0 \) is the threshold energy. The value \( C \) is defined in Eq. (27).

In the limit \( E \gg E_0 \approx 10^7 \text{ MeV} \), the expression for the process width is simplified essentially:

\[
\Gamma \approx \frac{\alpha m^2}{2 E} \left( \ln \frac{2EA}{m_e^2} - \frac{3}{2} \right),
\]

where \( m_e \) is the electron effective mass, defined in Eq. (32). We remind that the probability is not zero only inside the window \( E_0 < E < E_{\text{max}} \). The formulas (61) - (64) were obtained within the approximation \( E \ll E_{\text{max}} \approx 10^{10} \text{ MeV} \).

In contrast to the neutrino spin-flip radiative transition, where the mean free path appeared to be extremely large, Eq. (65) gives the scale of the mean free path for electrons of the order of tens of centimeters. However, it is evident that the process considered can not compete with the standard electromagnetic processes of the electron scattering in dense plasma.

7 Conclusion

In conclusion, we have shown that the effects of the “spin light of neutrino” and “spin light of electron” have no place in real astrophysical situations for neutrinos and electrons “living” in plasma, because of the photon dispersion. The photon (plasmon) effective mass causes the threshold which leaves no room for both processes. For a pure theoretical situation when an ultra-high energy neutrino (or electron) threads a star, the total probabilities of the processes \( \nu_L \rightarrow \nu_R + \gamma \) and \( e_L \rightarrow e_R + \gamma \) are calculated with correct taking account of the fermion and photon dispersion in plasma.

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