Isoscalar dipole mode in relativistic random phase approximation

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Abstract

The isoscalar giant dipole resonance structure in $^{208}$Pb is calculated in the framework of a fully consistent relativistic random phase approximation, based on effective mean-field Lagrangians with nonlinear meson self-interaction terms. The results are compared with recent experimental data and with calculations performed in the Hartree-Fock plus RPA framework. Two basic isoscalar dipole modes are identified from the analysis of the velocity distributions. The discrepancy between the calculated strength distributions and current experimental data is discussed, as well as the implications for the determination of the nuclear matter incompressibility.
The study of the isoscalar giant dipole resonance (IS GDR) might provide important information on the nuclear matter compression modulus $K_{nm}$. This, somewhat elusive, quantity defines basic properties of nuclei, supernovae explosions, neutron stars and heavy-ion collisions. The range of values of $K_{nm}$ has been deduced from the measured energies of the isoscalar giant monopole resonance (GMR) in spherical nuclei. The complete experimental data set on isoscalar GMR, however, does not limit the range of $K_{nm}$ to better than $200 - 300$ MeV. Also microscopic calculations of GMR excitation energies have not really restricted the range of allowed values for the nuclear matter compression modulus. On one hand, modern non-relativistic Hartree-Fock plus random phase approximation (RPA) calculations, using both Skyrme and Gogny effective interactions, indicate that the value of $K_{nm}$ should be in the range $210-220$ MeV [1, 2]. In relativistic mean-field models on the other hand, results of both time-dependent and constrained calculations suggest that empirical GMR energies are best reproduced by an effective force with $K_{nm} \approx 250 - 270$ MeV [3, 4, 5].

In principle, complementary information about the nuclear incompressibility, and therefore by extension about the nuclear matter compression modulus, could be obtained from the other compression mode: giant isoscalar dipole oscillations. In first order the isoscalar dipole mode corresponds to spurious center-of-mass motion. The IS GDR is a second order effect, built on $3\hbar\omega$, or higher configurations. It can be visualized as a compression wave traveling back and forth through the nucleus along a definite direction: the "squeezing mode" [6, 7]. There are very few data on IS GDR in nuclei (the current experimental status has been reviewed in Ref. [8]). In particular, recent results on IS GDR obtained by using inelastic scattering of $\alpha$ particles have been reported for $^{208}$Pb [9], and for $^{90}$Zr, $^{116}$Sn, $^{144}$Sm, and $^{208}$Pb [10]. As in the case of giant monopole resonances, data on heavy spherical nuclei are particularly significant for the determination of the nuclear matter compression modulus: for example $^{208}$Pb. However, recent experimental data on IS GDR excitation energies in this nucleus disagree: the centroid energy of the isoscalar dipole strength distribution is at $22.4 \pm 0.5$ MeV in Ref. [9], while the value $19.3 \pm 0.3$ MeV has been reported in Ref. [10]. In the analysis of Ref. [9], the "difference of spectra" technique was employed to separate the IS GDR from the high-energy octupole resonance (HEOR) in the $0^0 \rightarrow 2^0$ $\alpha$-scattering spectrum for $^{208}$Pb. On the other hand, in the experiment on $^{90}$Zr, $^{116}$Sn, $^{144}$Sm, and $^{208}$Pb of Ref. [10], the mixture of isoscalar $L = 1$ (IS GDR) and $L = 3$ (HEOR) multipole strength could not be separated by a peak fitting technique. Instead, the data were analyzed by a multipole analysis of 1 MeV slices of the data over the giant resonance structure, obtained by removing the underlying continuum.

In Ref. [8] it has been also pointed out that the experimental IS GDR centroid energies, and therefore the corresponding values of the nuclear in-
compressibility $K_A$, are not consistent with those derived from the measured energies of the isoscalar GMR in $^{208}$Pb. In the sum rule approach to the compression modes [11], two different models have been considered for the description of the collective motion: the hydrodynamical model and the generalized scaling model. The assumption of the scaling model leads to a difference of more than 40% between the values of the finite nucleus incompressibility $K_A$, when extracted from the experimental energies of the IS GDR and the GMR in $^{208}$Pb. A consistent value for $K_A$ can be derived from the experimental excitation energies, only if the two compression modes are described in the hydrodynamical model. The resulting value of $K_A \approx 220$ MeV, however, is much too high, and in fact it corresponds to the nuclear matter compression modulus $K_{nm}$, as derived from non-relativistic Hartree-Fock plus RPA calculations. This is not difficult to understand, since the expressions for $K_A$ in both models were derived in the limit of large systems and consequently do not account for surface effects [11]. Both models, however, are approximations to a full quantum description: the time-dependent Hartree-Fock or, equivalently, the RPA. Therefore, fully microscopic calculations might be necessary, in order to resolve the apparent discrepancy between the values of $K_A$ extracted from the IS GDR and the GMR in $^{208}$Pb.

Non-relativistic self-consistent Hartree-Fock plus RPA calculations of dipole compression modes in nuclei were reported in the work of Van Giai and Sagawa [12], and more recently in Refs. [13] and [14]. A number of different Skyrme parameterizations were used in these calculations, and the result is that all of them systematically overestimate the experimental values of the IS GDR centroid energies, not only for $^{208}$Pb, but also for lighter nuclei. In particular, those interactions that reproduce the experimental excitation energies of the GMR (SGII and SKM$^*$), predict centroid energies of the IS GDR in $^{208}$Pb that are $4 - 5$ MeV higher than those extracted from small angle $\alpha$-scattering spectra. In Ref. [14] effects that go beyond the mean-field approximation have been considered: the inclusion of the continuum and $2p - 2h$ coupling. It has been shown that the coupling of RPA states to $2p - 2h$ configurations, although it reproduces the total width, results in a downward shift of the resonance energy of less than 1 MeV with respect to the RPA value. It appears, therefore, that the presently available data on excitation energies of the compression modes in nuclei: the GMR and the IS GDR, cannot be consistently reproduced by theoretical models.

In Ref. [3] we have performed time-dependent and constrained relativistic mean-field calculations for the monopole giant resonances in a number of spherical closed shell nuclei, from $^{16}$O to $^{208}$Pb. It has been shown that, in the framework of relativistic mean field theory, the nuclear matter compression modulus $K_{nm} \approx 250 - 270$ MeV is in reasonable agreement with the available data on spherical nuclei. This value is approximately 20% larger than the values deduced from non-relativistic density dependent Hartree-
Fock calculations with Skyrme or Gogny forces. In particular, among the presently available effective Lagrangian parameterizations, the NL3 effective force \[ K_{nm} = 271.8 \text{ MeV}, \] provides the best description of the mass dependence of the GMR excitation energies. Preliminary calculations with the time-dependent relativistic mean-field model \[ \text{[16]}, \] indicate that the NL3 effective interaction, which reproduces exactly the excitation energy of the GMR in $^{208}$Pb (14.1 MeV), overestimates the reported centroid energy of the IS GDR by at least 4 MeV. However, due to complications arising from the spurious center-of-mass motion, the time-dependent relativistic mean-field model computer code develops a numerical instability which prevents the precise determination of the IS GDR excitation energy. In the present analysis, therefore, we apply the relativistic random phase approximation (RRPA) to the description of the isoscalar dipole oscillations in $^{208}$Pb.

The RRPA represents the small amplitude limit of the time-dependent relativistic mean-field theory. Self-consistency will therefore ensure that the same correlations which define the ground-state properties, also determine the behavior of small deviations from the equilibrium. The same effective Lagrangian generates the Dirac-Hartree single-particle spectrum and the residual particle-hole interaction. Some of the earliest applications of the RRPA to finite nuclei include the description of low-lying negative parity excitations in $^{16}$O \[ \text{[17]}, \] and studies of isoscalar giant resonances in light and medium nuclei \[ \text{[18]}. \] These RRPA calculations, however, were based on the most simple, linear $\sigma - \omega$ relativistic mean field model. It is well known that for a quantitative description of ground- and excited states in finite nuclei, density dependent interactions have to be included in the effective Lagrangian through the meson non-linear self interaction terms. The RRPA response functions with nonlinear meson terms have been derived in Refs. \[ \text{[19, 20]}, \] and applied in studies of isoscalar and isovector giant resonances. However, the calculated excitation energies did not reproduce the values obtained with the time-dependent relativistic mean-field model \[ \text{[3, 16]}. \] The reason was that the RRPA configuration spaces used in Refs. \[ \text{[19, 20]}, \] did not include the negative energy Dirac states. In Ref. \[ \text{[21]}, \] it has been shown that an RRPA calculation, consistent with the mean-field model in the no – sea approximation, necessitates configuration spaces that include both particle-hole pairs and pairs formed from occupied states and negative-energy states. The contributions from configurations built from occupied positive-energy states and negative-energy states are essential for current conservation and the decoupling of the spurious state. In addition, configurations which include negative-energy states give an important contribution to the collectivity of excited states. In a recent study \[ \text{[22]}, \] we have shown that, in order to reproduce results of time-dependent relativistic mean-field calculations for giant resonances, the RRPA configuration space must contain negative-energy Dirac states, and the two-body matrix elements must include contributions from the spatial
components of the vector meson fields. The effects of the Dirac sea on the excitation energy of the giant monopole states have been also recently studied in an analytic way within the $\sigma - \omega$ model [23].

In Fig. 1 we display the IS GDR strength distributions in $^{208}\text{Pb}$:

$$B^{T=0}(E1, 1_i \to 0_f) = \frac{1}{3} |\langle 0_f | \hat{Q}^{T=0}_{1\mu} | 1_i \rangle|^2,$$

where the isocalar dipole operator is

$$\hat{Q}^{T=0}_{1\mu} = e \sum_{i=1}^{A} \gamma_0 (r^3 - \eta r) Y_{1\mu}(\theta_i, \varphi_i),$$

and

$$\eta = \frac{5}{3} < r^2 >_0.$$  

The calculations have been performed within the framework of the self-consistent Dirac-Hartree plus relativistic RPA. The effective mean-field Lagrangian contains nonlinear meson self-interaction terms, and the configuration space includes both particle-hole pairs, and pairs formed from hole states and negative-energy states. The choice of the dipole operator (2), with the parameter $\eta$ determined by the condition of translational invariance, ensures that the IS GDR strength distribution does not contain spurious components that correspond to the center-of-mass motion [12]. The strength distributions in Fig. 1 have been calculated with the NL1 ($K_{nm} = 211.7$ MeV) [24], NL3 [15] ($K_{nm} = 271.8$ MeV), and NL-SH ($K_{nm} = 355.0$ MeV) [25] effective interactions. These three forces, in order of increasing values of the nuclear matter compressibility modulus, have been extensively used in the description of a variety of properties of finite nuclei, not only those along the valley of $\beta$-stability, but also of exotic nuclei close to the particle drip lines. In particular, in Ref. [3] it has been shown that the NL3 ($K_{nm} = 271.8$ MeV) effective interaction provides the best description of experimental data on isoscalar giant monopole resonances.

The calculated strength distributions are similar to those obtained within the non-relativistic Hartree-Fock plus RPA framework, using Skyrme effective forces [12, 14]. In disagreement with reported experimental results, all theoretical models predict a substantial amount of isoscalar dipole strength in the $8 - 14$ MeV region. The centroid energies of the distributions in the high-energy region between 20 and 30 MeV, are $4 - 5$ MeV higher than those extracted from the experimental spectra. It also appears that the centroid energies of the low-energy distribution do not depend on the nuclear matter incompressibility of the effective interactions. On the other hand, the IS GDR strength distributions in the low-energy region display the expected mass dependence. We have also performed calculations for a number of lighter spherical nuclei, and verified that with increasing mass the centroid is indeed shifted to lower energy. When comparing with experimental
data, it should be pointed out that the usable excitation energy bite in the experiment reported in Ref. [9] was $14 - 29$ MeV, and therefore a low-energy isoscalar dipole strength could not be observed. In this respect, somewhat more useful are the data from the experiment reported in Ref. [10], where spectra in the energy range $4 < E_x < 60$ MeV have been observed. The results of an DWBA analysis of the experimental spectra, however, attribute the isoscalar strength in the $10 - 15$ MeV region exclusively to the giant monopole (GMR) and giant quadrupole (GQR) resonances. It should be emphasized that a possible excitation of isoscalar dipole strength in this energy region and its interference with the GQR cannot be excluded [6].

In the high energy region the calculated dipole strength exhibits the expected dependence on the nuclear matter compressibility modulus of the effective interactions (NL1, NL3, NL-SH). The centroid of the strength distribution is shifted to higher energy with increasing values of $K_{nm}$. These energies, however, are considerably higher than the corresponding experimental IS GDR centroids [3,10]. Though, in order to precisely determine the IS GDR excitation energy from the experimental spectrum, the dipole strength has to be separated from the high-energy octupole resonance (HEOR), and this is not always possible [10]. Using the NL3 effective interaction, we have calculated the octupole strength distribution. The centroid of the HEOR is found at $\approx 22$ MeV, well below the IS GDR main peak, but more than 2 MeV above the experimental value for the HEOR centroid [11]. Incidentally, our calculated HEOR peak approximately coincides with the experimental value of the IS GDR centroid [9].

The IS GDR transition densities for $^{208}$Pb are shown in Fig. 2. The transition densities correspond to the NL3 strength distribution in Fig. 1. Since it appears that none of the effective interactions reproduces the experimental position of the IS GDR, the remainder of the present analysis will be only qualitative, and we choose to display only results obtained with the NL3 set of Lagrangian parameters. On the qualitative level, the other two effective interactions produce similar results. In Fig. 2, we plot proton (dot-dashed), neutron (dashed), and total (solid) transition densities for two representative peaks from Fig. 1: 10.35 MeV (a) is the central peak in the low-energy region, and 26.01 MeV (b) is the energy of the main peak in the region above 20 MeV. The transition densities for both peaks exhibit a radial dependence characteristic for the isoscalar dipole mode, and they can be compared with the corresponding transition densities in the scaling model, or with those which result from constrained calculations [11]. While for the high-energy peak the proton and neutron transition densities display an almost identical radial dependence, the pattern is more complicated for the peak at 10.35 MeV.

RPA calculations, therefore, predict the fragmentation of the isoscalar dipole strength distribution into two broad structures: one in the energy
window between 8 – 14 MeV, and the other in the high-energy region around $\approx 25$ MeV. The position of the low-energy structure does not depend on the compressibility modulus, i.e. it does not correspond to a compression mode. Additional information on the underlying collective dynamics can be obtained through a study of transition currents. In Fig. 3 we plot the velocity fields for the two peaks at 10.35 MeV (a) and 26.01 MeV (b). The velocity distributions are derived from the corresponding transition densities, following the procedure described in Ref. [26]. The ”squeezing” compression mode is identified from the flow pattern which corresponds to the high-energy peak at 26.01 MeV. The flow lines concentrate in the two ”poles” on the symmetry axis at $z \approx \pm 2.5$ fm. The velocity field corresponds to a density distribution which is being compressed in the lower half plane, and expands in the upper half plane. The centers of compression and expansion are located on the symmetry axis, at approximately half the distance between the center and the surface of the nucleus. It is obvious that the excitation energy of this mode will strongly depend on the compressibility modulus. The flow pattern for the lower peak at 10.35 MeV is very different. The flow lines describe a kind of toroidal motion, which is caused by the surface effect of the finite nucleus. The density wave travels through the nucleus along the symmetry axis. The reflection of the wave on the surface, however, induces radial components in the velocity field. Although it corresponds to dipole oscillations, this is not a compression mode. We have verified that also other dipole states in this energy region display similar velocity fields.

In conclusion, the isoscalar giant dipole resonance in $^{208}$Pb has been calculated in the framework of the relativistic RPA, based on effective mean-field Lagrangians with meson self-interaction terms. The results have been compared with recent experimental data and with calculations performed in the Hartree-Fock plus RPA framework. While the results of the present RRPA study are consistent with previous theoretical analyses, they strongly disagree with reported experimental data on the position of the IS GDR centroid energy in $^{208}$Pb. This is a serious problem, not only because the disagreement between theory and experiment is an order of magnitude larger than for other giant resonances, but also because the present data on IS GDR are not consistent with the value of the nuclear incompressibility $K_A$ derived from the measured excitation energy of the isoscalar GMR. This inconsistency could, perhaps, be explained by a possible excitation of isoscalar dipole strength in the low-energy window between 8 MeV and 14 MeV. Although predicted by all theoretical models, the low-lying IS GDR strength is not observed in the experimental spectra. From the analysis of the velocity fields, we have identified two basic isoscalar dipole modes. The ”squeezing” compression mode is found in the high-energy region at $\approx 26$ MeV. The low-energy dipole mode does not correspond to a compression mode, and its dynamics is determined by surface effects.
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Figure Captions

- **Fig. 1** IS GDR strength distributions in $^{208}$Pb calculated with the NL1 (dashed), NL3 (solid), and NL-SH (dot-dashed) effective interactions.

- **Fig. 2** IS GDR transition densities for $^{208}$Pb calculated with the NL3 parameter set. Proton (dot-dashed), neutron (dashed), and total (solid) transition densities are displayed for the peaks at 10.35 MeV (a) and 26.01 MeV (b).

- **Fig. 3** Velocity distributions for the two isoscalar dipole modes in $^{208}$Pb calculated with the NL3 effective interaction. The velocity fields correspond to the two peaks at 10.35 MeV (a) and 26.01 MeV (b).
