Finite element methods application for calculation of defective composite leaf spring

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Abstract. One of the most demonstrative examples of fibrous composites application in the vehicle power structures is a spring. Fiberglass springs are not uncommon, the existing analytical design methods allow calculating its overall dimensions under the conditions of strength and stiffness, but the behavior of leaf springs with defects under load is the point of interest. At the same time, the enormously approach is extremely complicated, but modern calculation methods allow to model defects, their growth and the process of destruction, taking into account the specifics of fibrous composites. In the paper considered the process of modeling the progressive destruction of fiber reinforcement plastic (FRP) leaf spring. The here presented calculation procedure is for one-leaf spring. The features of the calculation using MSC Patran are described.

1. Introduction
Despite the long history of springs, they continue to be used to this very day. The first applications of springs probably were in the carriages, where elliptic or semi-elliptic multi-leaf springs were used. Semi-elliptic multi-leaf springs have remained unchanged to this day. Traditional spring is made of high strength steel and it applies in suspension of heavy trucks. The field of our interest is appliance of composite materials for heavily loaded elastic elements.

Glass-fiber reinforced plastic (GFRP) or fiberglass (one of the most inexpensive FRP) due to the low elasticity modulus are the most effective for the elastic elements, because the elastic energy is stored at a predetermined force in reverse proportional to elasticity modulus. Unidirectional fiberglass composite leaf-springs are applied in some modern passenger and sport cars and SUV. The aim of this article is description of defective GFRP leaf spring calculation by FEM. Also the analytical method of profiled leaf spring calculation is shortly mentioned.

2. Analytical method of leaf spring calculation
The rational design of the console profiled beam is shown in figure 1. Section dimensions, width \( w(x) \) and thickness \( t(x) \) vary according to power laws:

\[
w(x) = w(0) \left(1 - \frac{x}{l}\right) ^{\alpha}, \quad t(x) = t(0) \left(1 - \frac{x}{l}\right) ^{\beta}
\]

(1)
Choosing the laws of variation in width and thickness along the length, one can achieve strength uniformity- the same stresses in each section $\sigma_{\text{max}}(x) = \text{const}$: which implies a linear variation of the bending moment along the length of the spring.

When designing bending elastic elements, it is necessary (knowing the material properties: elasticity modulus $E(x, y, z)$ and flexing strength $[\sigma]$) to find the dimensions of the root section $w(0)$ and $t(0)$ and the laws of their change ($\alpha - \beta$) for simultaneous fulfillment of the conflicting requirements:

- the requirement for stiffness:

$$e = \frac{P}{v}$$  \hspace{1cm} (2)

- the requirement for strength:

$$\sigma_{\text{max}} \leq [\sigma]$$  \hspace{1cm} (3)

Strength uniformity condition:

$$\sigma_{\text{max}}(x) = \frac{6P(l-x)}{w(x)t(x)^2} = \frac{6P}{w(0)t(0)}$$  \hspace{1cm} (1)

is resulting from (1) to a linear function:

$$\alpha + 2\beta = 1$$  \hspace{1cm} (4)

The sag $v$ of shaped in accordance with (1) console beam with the load $P$ is determined by the traditional method using the equation for the accumulated elastic energy $\frac{1}{2}Pv$:

$$v(l) = \int_0^l \frac{P(l-x)}{E(x)I(x)} dx = \frac{Pl^3}{3EI(0)} \frac{1}{1 - \frac{\alpha}{3} - \beta}$$  \hspace{1cm} (5)

where the moment of inertia is

$$I(x) = \frac{w(x)t(x)^3}{12} = \frac{w(0)t(0)^3}{12} \left(1 - \frac{x}{l}\right)^{\alpha + \beta}$$  \hspace{1cm} (6)

In (6), for simplicity, the elasticity modulus is assumed to be constant. But it, of course, changes due to a change in the misorientation angle of the fibers.

The shape factor for a sag (6):
\[ \delta_0 = \frac{v(l)}{v_0(l)} = \frac{1}{1 - \alpha^2/3 - \beta} \]

It equals to the ratio of the maximum sag \( v(l) \) of the profiled beam to sag \( v_0(l) \) of a rectangular beam with the same dimensions of the root section.

The simultaneous fulfillment of the conditions (2) and (3) allows from (4) and (6) to find the rational sizes of root section of the profiled beam:

\[ t(0) = \frac{2c\sigma l^2}{3EP_{\text{max}}} = t_0 \delta_0, \quad w(0) = \frac{27E^2P_{\text{max}}^3}{2c^2[\sigma]^3l^4\delta_0^2} = \frac{w_0}{\delta_0} \]

where \( t_0, w_0 \) are cross-sectional dimensions of the rectangular beam which satisfy the conditions (2) and (3).

The cross-sectional dimensions of the end section \( w(a), t(a) \) and its lengths \( a \) (figure 1) are found from criterion of shear stresses fracture for GFRP:

\[ w(l-a) \times t(l-a) = t(0) \times w(0) \left( \frac{a}{l} \right)^{\alpha+\beta} = \frac{3P_{\text{max}}}{2\tau^*}. \]

3. Design method of leaf spring progressive fracture
The step-by-step modeling method of analysis the general and local strength is used for the modeling and calculating the progressive fracture of a composite spring. The modeling algorithm is shown in figure 2.

![Figure 2. Calculating algorithm of the progressive fracture of a composite leaf spring.](image)

Description of the method presented in figure 2 provided below.

4. Step 1 - calculation of spring designs
First of all, it is necessary to model the geometry of the spring and build a FEM model. Depending on the required detail, different FEM elements (2D or 3D) can be used. Features of model are to be set according to anisotropic and orthotropic composite materials. It is possibly to set individual characteristics in each material layer.

The choice of fracture criterion depends on operating conditions and the types of loading. Criteria for the destruction of Hill, Hoffman, Tsai-Wu, Hashin, maximum stresses, maximum strains, and others can be given.

The specification of boundary conditions and loads depends on the type of problem - this can be either static or dynamic calculations.

To determine the stress-strain state a computational MSC Patran finite-element model of the leaf spring was created see figure 3.

The obtained results of the stress-strain state of construction are presented on figures from 3 to 6, the results are used for determination of the most probable destruction areas. The local coordinate system is used for calculation: the X axis is directed along the spring, the Y axis is perpendicular to the plane of the final element, the Z axis is across the spring.
The stress concentration areas are located near the spring eye (highlighted with ellipses in figures 5 and 6). Fracture probability in this area is high and we pay more attention to the eye area.

5. Step 2 – leaf spring progressive fracture calculation
The defect that was "laid" - the delamination or absence of glue. Geometric dimensions are set at the start of modeling, after which a finite element model of the area with the defect is created (figure 7).

The layers connection was modeled using the adhesive interface. The process of progressive fracture is modeled by the disruption of the links between the nodes of elements. Adhesive layer fracture criteria are set in the form of maximum permissible stresses in polymer matrix and criteria for the monolayers fracture. For other areas (in which the delamination is not considered) ordinary finite elements were used instead of an adhesive layer.

Before calculation the obtained model is integrated into the initial finite element model by replacing the corresponding array of finite elements.

In total, the results were obtained for 30 load steps (Figure 8). The illustrations of crack progress are presented in figure 9.

6. Step 3 - analysis of results
The process of progressive fracture of the leaf spring is divided into characteristic stages:

Stage of damage accumulation – during the loading steps 1 and 2 the spring deformed as a whole, the values of stresses and deformations in the local area near the eye have not reached critical values yet, the stress patterns have only insignificant differences.

With a further increase in load (at loading step 3, figure 9), the stress values reach critical values, which causes local destruction of the binder material between the layers. This effect is modeled by the break between the nodes of the elements in the corresponding area. There is an abrupt increase in the size of the damage, the values of displacements increase sharply.

Then, when the load is increased (steps 4-5, figure 9), the deformation of the spring is accompanied by a simultaneous increase of the area of the defect, as well as the magnitude and distance between the banks of the break.

At the loading step 6, the growth of the delamination is suspended, the spring is deformed as whole again. It is connected with the effects of increasing the spring thickness, the delamination will continue after the values of the stresses again become equal to the critical ones.

7. Conclusion
The proposed analytical calculation and the finite element model of the defect make it possible to reproduce quite accurately the process of progressive delamination of the composite leaf spring.

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