Bulk Dynamics in Heavy Ion Collisions

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The features of heavy ion collisions that suggest the relevance of collective dynamics, as opposed to mere superpositions of nucleon-nucleon or even parton-parton collisions, are reviewed. The surprise of these studies is that bulk observables are far simpler than typical dynamical models of nucleus-nucleus collisions would imply. These features are shown to have a natural interpretation in terms of statistical-hydrodynamical models. The relevance of hydrodynamics to heavy ion collisions, coupled with the various similarities of the heavy ion data with that of more elementary collisions, raises very basic questions about its relevance to smaller systems.

1. Introduction

Recently, there has been dramatic progress in the understanding of the dynamics of heavy ion collisions. This is due to the availability of a large, high-quality data set spanning an enormous range in energy, rapidity and event centrality. With the turn-on of the RHIC facility in 2000, we now have information about the basic features of these collisions up to $\sqrt{s_{NN}} = 200$ GeV, where $\sqrt{s_{NN}}$ is the nucleon-nucleon center-of-mass energy. Although most of the analyzed data is from mid-rapidity (90 degrees in the center-of-mass system), all of the accelerator facilities have experiments dedicated to acquiring data over the full rapidity range (e.g. E895 at the AGS, NA49 at the SPS, and BRAHMS and PHOBOS at RHIC). Finally, the various heavy ion experiments are starting to converge on comparable centrality measures, making comparisons between the data sets as a function of the number of participating nucleons feasible.

During the summer of 2004, the four RHIC experiments (BRAHMS, PHENIX, PHOBOS, and STAR) produced draft “white papers”, summarizing their most important results and relevant interpretations [1, 2, 3, 4]. The result of these discussions indicates an ongoing paradigm shift both in the interpretation of the lattice results and the experimental data. The 20-25% deviation of the lattice energy density from the Stefan-Boltzmann Limit is no longer interpreted as approaching the weakly-interacting Quark Gluon Plasma. Rather, this deviation is presently understood (using N=4 SUSY QCD) as precisely the signature of a strongly-interacting plasma [5, 6]. Thus, rather than expecting a weakly-coupled gas of quarks and gluons to be liberated in high energy heavy ion collisions, one might expect to create a strongly-interacting state instead, with completely different properties than previously expected. More importantly, one would not expect systems with these properties to be generated by the two body scatterings of asymptotic hadrons. The energy density achieved in these collisions imply particle

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densities far too high for individual particle states to remain distinct.

If this is the case, then one should expect the system created in heavy ion collisions to show collective behavior, characteristic of a system with zero mean free path, i.e. the hydrodynamic limit. This treatment of the system stands in stark contrast to the usual dynamical transport approach to heavy ion collisions. In this sort of approach, one expects multiple, independent stages, characterized by different dynamical mechanisms (shadowed parton distributions, parton production and reinteractions, quark recombination and chemical freezeout, fragmentation functions, hadron rescattering, thermal freezeout and hadronic decays).

The independent combination of these various sources is encoded in models such as HIJING. There are also models that avoid the partonic stage altogether and proceed directly to the hadronic dynamics, such as RQMD. Interestingly, all of these models have difficulty reproducing the energy dependence of the particle density at mid-rapidity \((dN_{ch}/d\eta)|_{\eta<1}/\langle N_{part}/2\rangle\) as shown in Fig. 1, which compares predictions for Au+Au and p+p from HIJING and RQMD, and compares them with data. While the models are tuned appropriately on p+p, both miss the Au+Au data systematically as a function of beam energy. Variables such as this integrate over all of the possible dynamical processes outlined above, thus showing the utility of bulk observables to be sensitive to both hard and soft processes, and thus early and late stages. Of course, a single observable at a particular energy and centrality is of little use. Rather, it is the combined systematics of energy, centrality, and rapidity which provide a handle on the most important dynamical contributions.

The surprising conclusion, partially outlined in this proceedings, is that the systematics of bulk observables are far simpler than one might expect given the various mechanisms that could contribute to the final state. However, it seems that one must consider the features of particle production over the full phase space (and not just concentrate on
particular kinematic regions) to isolate the simple organizing features.

2. Features of charged-particle multiplicities in $A + A$ collisions

The most striking feature of bulk particle production in heavy ion collisions is the approximate linearity of the total number of charged particles ($N_{ch}$) with the number of participating nucleons ($N_{part}$). Results on $p + A$ collisions in the 1970’s and 80’s \[8\], found that $N_{ch}$ scaled linearly as $N_{part} \times N_{pp}^{ch}$. Recent PHOBOS data on $d + Au$ collisions, shown in Fig. 2 \[9\] shows that this phenomenon persists at RHIC energies of $\sqrt{s_{NN}} = 200$ GeV. This simple behavior occurs despite non trivial changes to the particle density over the whole pseudorapidity range, shown in Fig. 3. What was not expected was that the same scaling would be present in $Au + Au$ collisions from $\sqrt{s_{NN}} = 20$ to 200 GeV. This is a non-trivial result, in that the variation of $dN_{ch}/d\eta$ with centrality, shown in Figs. 4 still maintains an overall constancy of $\langle N_{ch} \rangle / \langle N_{part}/2 \rangle$. It should also be pointed out that while the $d + Au$ data connects simply to the $p + p$ data, the $A + A$ data does not seem to extrapolate smoothly to the $p + p$ limit.

Another striking global feature is “limiting fragmentation”, the energy independence of the particle yields with energy when considering a system with a fixed collision geometry in a frame where one of the projectiles is at rest \[10\]. This feature was expected to be found in very limited regions of pseudorapidity, characteristic of the fragmentation of each projectile \[11\]. Instead, limiting fragmentation is observed over a relatively wide range in pseudorapidity, as shown in Fig. 5. Moreover, one finds that the shape of the “limiting curve” depends only on the impact parameter, and once this is determined,
data from different energies lies on the same curve. Since the data peels off of the limiting curve at approximately the same distance from mid-rapidity, this phenomenon also seems to strongly constrain the mid-rapidity yields as well. Thus, the energy dependence of particle yields seems to be controlled by a global constraint that is obeyed at all available beam energies.

The third striking feature of bulk particle production is the apparent universality of the total multiplicity for $A + A$, $p + p$ and $e^+e^-$ reactions at high energy, shown in Fig. 6[12, 13]. Of course, this is only observed if one accounts for the “leading particle effect” in $p + p$ collisions by working with an “effective energy” $\sqrt{s_{\text{eff}}} = \sqrt{s}/2$. The agreement of $A + A$ and $e^+e^-$ without rescaling suggests on the contrary that there is no similar leading particle effect in heavy ion collisions. This is presumably connected to the fact that a typical participant is struck multiple times in $A + A$ (unlike $p + p$ and $p + A$ collisions). The deviation of $A + A$ and $e^+e^-$ at low energies can be understood as a consequence of the presence of a substantial baryochemical potential [13], which tends to suppress the overall entropy (via the relationship $S = (E - pV - \mu_B N_B)/T$.

In conclusion, there seem to be three essential features of particle multiplicities in $A + A$ collisions: 1) $N_{\text{part}}$-scaling of the total multiplicity, 2) A universal value of $\langle N_{\text{ch}} \rangle / \langle N_{\text{part}}/2 \rangle$ in $A + A$, $p + p$ and $e^+e^-$ reactions, and 3) “limiting fragmentation”, which seems to constrain the global angular distributions and thus even the mid-rapidity density. Such simple behavior would not naturally be expected to arise out of a dynamical approach, where each stage is in-principle distinct from the others. For instance, it would seem fortuitous for nuclear shadowing, energy loss, and fragmentation functions to conspire to arrive at $N_{\text{part}}$-scaling from processes where semi-hard physics is controlled by the number of binary collisions, $N_{\text{coll}}$. It seems that any relevant physics scenario (such as the Color Glass Condensate described by L. McLerran [14]) must capture these essential features.
3. Hydrodynamic Descriptions of High Energy Collisions

As discussed in the introduction, one of the major surprises from the RHIC data has been the relevance of relativistic hydrodynamics in the overall understanding of the experimental data. In some sense, this should not be so surprising. It is well-known that any system in which the mean free path approaches zero should show hydrodynamic behavior, even an apparently disparate system such as cold \(^6\text{Li}\) atoms [13]. This system can be initialized with a strongly asymmetric geometry and the subsequent pressure gradients lead to spatial deformations that have been successfully modeled with hydrodynamics.

The relevant geometry in the early stages of a heavy ion collision (or even a \(p + p\) collision) is one characterized by the nuclear radius in the transverse direction, and a large contraction in the longitudinal direction. Already one sees that the relevant time scales for longitudinal and transverse dynamics are very different, of order \(\tau_T \sim R/c\) (approximately several fm/c) in the transverse direction and \(\tau_L \sim mR/(c\sqrt{s})\) (\(\ll 1\) fm/c) in the longitudinal direction.

3.1. Landau Hydrodynamics

Already in the early 1950’s, Fermi and Landau considered a system with the geometry just described, replacing the potential complexities of a high energy nuclear collision with a slab of area \(\pi R_A^2\) and length \(\Delta = R_A m/\sqrt{s}\) (and thus volume \(V \propto R^3/\sqrt{s}\)) in which all of the energy of the incoming projectiles is assumed to thermalize. This leads to an initial energy density of \(\epsilon \propto s_{NN} \sim 4\) TeV/fm\(^3\) at RHIC energies, a value typically seen as unphysically large. And yet, when one uses the massless blackbody
equation of state \( p = \frac{\epsilon}{3} \) and converts this to a relation for the entropy density, \( \sigma \propto \epsilon^{3/4} \), the total entropy in the collision volume is \( S = \sigma V \propto s^{3/4}/\sqrt{s} = s^{1/4} \), which is itself proportional to the total multiplicity \([16,17,18]\). It may not be immediately clear from these definitions, but in a nuclear collision, \( S \propto V \propto N_{\text{part}} \), leading naturally to the \( N_{\text{part}} \)-scaling of total multiplicity. Landau extended this statistical model by using the equations of relativistic hydrodynamics \([17,18]\). It turned out that the strong compression along one axis leads to highly anisotropic distributions, with Gaussian rapidity distributions of width \( \sigma = \ln \sqrt{s/m^2} \). It is the application of the Fermi-Landau initial conditions to the generally-accepted formalism of 3D relativistic hydrodynamics that is known as the “Landau hydrodynamical model”, as advanced by Cooper et al. \([19]\), Carruthers et al. \([20]\) and Shuryak \([21]\), among others. What is striking is just how much existing data is broadly consistent with Landau’s original predictions from the 1950’s. As shown in Fig. 6, the Landau multiplicity formula, tuned on the low-energy data \( N_{\text{ch}} = 2.2s^{1/4} \) does a reasonable job on describing the trend of \( e^+e^- \) over a wide range of \( \sqrt{s} \), and thus the \( p+p \) data at \( \sqrt{s}/2 \) and the \( A+A \) data above \( \sqrt{s_{NN}} = 20 \text{ GeV} \) \([13]\). Of course it should not be overlooked that pQCD \([22]\) can describe the \( e^+e^- \) data just as well, if not better, than the Landau formula. However, it is not clear why the two approaches agree to better than 10% over the range for which data exists, as shown in Fig. 6.

The BRAHMS rapidity distributions of charged pions are distinctly Gaussian (shown in Fig. 7) and have a width which deviates less than 10% from Landau’s parameter-free predictions (as seen in the inset of Fig. 7) \([23]\). It is less well-known that Landau’s formulas (\( N_{\text{ch}} \propto s^{1/4} \) and \( \sigma = \ln \sqrt{s/m^2} \)) actually predict limiting fragmentation when plotted as a function of \( y' = y - y_{\text{beam}} \), as shown in Fig. 8 \([13]\). It appears that Landau’s physical picture is already consistent with the essential features of particle multiplicities in heavy ion collisions. Moreover, in contrast to models that explain the rapidity distributions as the consequence of scatterings between the partons in the initial-state nucleon and nuclear wave functions \([24]\), the Landau model starts with a static initial state and rapidly generates the rapidity distribution by means of hydrodynamics. Thus, \( dN_{\text{ch}}/d\eta \) is not a static “initial-state” effect, but rather the result of a dynamical process in the very first stages of the collision.

3.2. Bjorken Hydrodynamics

Although Landau hydrodynamics appears to be relevant for the physics in the very early stages \( (\tau \ll 1\text{ fm}/c) \), it is still a model only understood semi-analytically with some particularly drastic approximations. By contrast, hydrodynamic calculations which assume boost-invariance in the initial conditions have been used for quantitative comparisons with experimental observables that are sensitive to early-time pressure gradients \([24]\). These models are based on the pioneering work of Bjorken \([27]\), who postulated the imposition of boost-invariance \([26]\) as a guiding principle for high energy reactions. Of course, since the calculations are initialized at time scales on the order of 1 fm/c, they are unable to calculate the initial-state entropy. However, given this single piece of experimental data, and an assumed equation of state (usually a hybrid of the Landau EOS and a hadronic EOS, with a mixed phase), they are able to successfully calculate the effects of transverse pressure on particle spectra (radial flow) the mapping from the initial-state geometry to anisotropies in the final-state transverse momentum.
Several clear signatures of radial flow are present in the experimental data even without recourse to particular models. At low transverse momentum ($p_T < m$) it is observed that the particle spectra harden with increasing particle mass. This is characteristic of a collective flow velocity field that gives heavier particles a larger momentum kick ($p = \gamma \beta m$). This immediately breaks the $m_T$-scaling seen in $p + p$ data, which is often thought to result from emission from a thermalized source. Models which include a more detailed handling of chemical freezeout are able to reproduce these trends\cite{28}.

Elliptic flow has now been comprehensively described by calculations using boost-invariant hydrodynamics, both as a function of centrality and as a function of $p_T$ and particle mass for a fixed centrality. An example is shown in Fig. 9 which shows the characteristic “fine structure”, or mass-splitting of the asymmetry parameter $v_2$ as a function of the particle’s transverse momentum \cite{29}. These are non-trivial relationships between various species that are not typically described well in the dynamical approach described above. Nor can typical parton transport models reproduce the magnitude of the asymmetries \cite{30}. However, given the obvious discrepancy between the assumption of boost-invariance used in these calculations and the manifestly boost non-invariant particle distributions shown by the BRAHMS data, it is not surprising to find that it is difficult to reproduce the dependence of $v_2$ on $\eta$ as measured by PHOBOS, shown in Fig. 10 \cite{31}. This is true even for truly 3D hydrodynamic calculations, showing the need to understand the initial conditions in some detail. The current state of the art calculations \cite{32} use gluon structure functions calculated in the CGC framework, which explicitly avoids the longitudinal dynamics of the Landau approach.

In conclusion, we have seen that the hydro approach appears to warranted by a wide range of data, although no existing model or code can describe every detail correctly. This is especially true when considering longitudinal dynamics, which has not yet been fully
incorporated into models that describe many features of the transverse dynamics.

4. Relation to Elementary Systems

Typically, the success of hydrodynamic models in heavy ion collisions is attributed to their being larger than hadronic length scales, which are thought to control the scattering processes by which the system equilibrates. And yet, it has already been shown that certain features of elementary collisions, such as the total entropy density, are manifestly similar to heavy ion collisions. Thus, it is not ruled out a priori to seriously consider the relevance of hydrodynamics to smaller systems, such as $p + p$ and $e^+e^−$.

The previous discussions have suggested that rapidity distributions are not merely the consequence of the nucleon or nuclear wave functions, but may be dynamically generated by Landau initial conditions. The overall similarity between the pseudorapidity distributions in $A + A$ and $e^+e^−$ collisions at the same $\sqrt{s} = 200$ GeV (shown in Fig. 11 in the limiting fragmentation frame) certainly complicate separate explanations of these systems. It is also observed that the particle density per participant pair in $A + A$ is similar to the density relative to the thrust axis in $e^+e^− \rightarrow \text{hadrons}$. Given these similarities, the observation of limiting fragmentation in $p + p$ and $e^+e^−$ (also shown in Fig. 11 although less precise than that observed in $A + A$) is not surprising.

However, the issue of whether or not these systems are truly equilibrated is contingent on whether or not collective behavior can be discerned. It is well-known that the particle yields in $e^+e^−$ and $p + p$ collisions are described by statistical models at the same chemical freezeout temperature as $A + A$ at RHIC energies [33, 34]. However, it is a subject of debate whether or not this is simply “phase space dominance” as opposed to a true equilibration process [35]. Thus, the small systems are not regarded as truly collective. And yet, recent preliminary STAR data on identified spectra in $p + p$ collisions, shown in Fig. 12 [36], shows a qualitatively similar rise in the mean $p_T$ as a function of particle...
5. Conclusions

It is argued in this proceedings that soft physics in strongly interacting systems may well be simpler than typical dynamical models would generally suggest. In fact, hydrodynamics may provide a unified conceptual framework from the first moment of the collision all the way to freezeout. More than that, it may provide a basis for understanding elementary systems as well, perhaps complementing the approach of understanding these systems with perturbative QCD. Of course, QCD will be fundamental in understanding the basic degrees of freedom of these strongly-interacting systems and how they thermalize so quickly, as implied by the Landau approach. And yet, basic issues purely related to hydrodynamics, such as the integration of longitudinal and transverse dynamics will have to be tackled for real progress to be made. Finally, while the focus of RHIC physics has been at mid-rapidity, detailed understanding of the global features of the event across the whole rapidity range will be necessary to fully understand the collective nature of these collisions.
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REFERENCES

1. I. Arsene [BRAHMS Collaboration], arXiv:nucl-ex/0410020.
2. B. B. Back [PHOBOS Collaboration], arXiv:nucl-ex/0410022.
3. K. Adcox [PHENIX Collaboration], arXiv:nucl-ex/0410003.
4. The STAR White Paper can be found at http://www.star.bnl.gov/STAR/public/STAR-QGP-White.
5. S. S. Gubser, I. R. Klebanov and A. W. Peet, Phys. Rev. D 54, 3915 (1996).
6. E. V. Shuryak, Phys. Rept. 391, 381 (2004).
7. V. Topor Pop et al., Phys. Rev. C 68, 054902 (2003).
8. J. E. Elias et al., Phys. Rev. Lett. 41, 285 (1978).
9. B. B. Back et al., arXiv:nucl-ex/0409021.
10. B. B. Back et al., Phys. Rev. Lett. 91, 052303 (2003).
11. J. Benecke, T. T. Chou, C. N. Yang and E. Yen, Phys. Rev. 188, 2159 (1969).
12. B. B. Back et al., arXiv:nucl-ex/0301017.
13. P. Steinberg, arXiv:nucl-ex/0405022.
14. L. McLerran, these proceedings.
15. K.M. O’Hara et al., Science 298, 2179 (2002).
16. E. Fermi, Prog. Theor. Phys. 5, 570 (1950).
17. L. D. Landau, Izv. Akad. Nauk Ser. Fiz. 17, 51 (1953).
18. S. Z. Belenkij and L. D. Landau, Nuovo Cim. Suppl. 3S10, 15 (1956) [Usp. Fiz. Nauk 56, 309 (1955)].
19. F. Cooper, G. Frye and E. Schonberg, Phys. Rev. Lett. 32, 862 (1974).
20. P. Carruthers, Annals N.Y.Acad.Sci. 229, 91 (1974).
21. O. V. Zhirov and E. V. Shuryak, Yad. Fiz. 21, 861 (1975).
22. A. H. Mueller, Nucl. Phys. B 213, 85 (1983).
23. I. G. Bearden et al., arXiv:nucl-ex/0403050.
24. D. Kharzeev and E. Levin, Phys. Lett. B 523, 79 (2001).
25. P. F. Kolb and U. Heinz, arXiv:nucl-th/0305084.
26. R. P. Feynman, Phys. Rev. Lett. 23, 1415 (1969).
27. J. D. Bjorken, Phys. Rev. D 27, 140 (1983).
28. P. F. Kolb and R. Rapp, Phys. Rev. C 67, 044903 (2003).
29. R. Snellings, , arXiv:nucl-ex/0407010.
30. D. Molnar, arXiv:nucl-th/0406066.
31. T. Hirano, Phys. Rev. C 65, 011901 (2002).
32. T. Hirano and Y. Nara, Nucl. Phys. A 743, 305 (2004).
33. J. Cleymans, Pramana 60, 787 (2003).
34. P. Braun-Munzinger, K. Redlich and J. Stachel, arXiv:nucl-th/0304013.
35. V. Koch, Nucl. Phys. A 715, 108 (2003).
36. R. Witt, arXiv:nucl-ex/0403021.
37. T. D. Gutierrez, arXiv:nucl-ex/0403012.