A model of light pseudoscalar dark matter

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Abstract

The EW-ν_R model was constructed in order to provide a seesaw scenario operating at the Electroweak scale Λ_{EW} \sim 246 \text{GeV}, keeping the same SM gauge structure. In this model, right-handed neutrinos are non-sterile and have masses of the order Λ_{EW}. They can be searched for at the LHC along with heavy mirror quarks and leptons, the lightest of which has large decay lengths. The model also incorporates a rich scalar sector, consistent with various experimental constraints, predicts a \sim 125 \text{GeV} scalar with the SM Higgs characteristics satisfying the current LHC Higgs boson data. The seesaw mechanism requires the existence of a complex scalar which is singlet under the SM gauge group. The imaginary part of this complex scalar denoted by A_0^s is proposed to be the sub-MeV dark matter candidate in this manuscript. We find that the sub-MeV scalar can serve as a viable non-thermal feebly interacting massive particle (FIMP)-DM candidate. This A_0^s can be a naturally light sub-MeV DM candidate due to its nature as a pseudo-Nambu-Goldstone (PNG) boson in the model. We show that the well-studied freeze out mechanism falls short in this particular framework producing DM overabundance. We identify that the freeze in mechanism produce the correct order of relic density for the sub-MeV DM candidate satisfying all applicable constraints. We also discuss the allowed parameter space arising from the current indirect search bounds for this sub-MeV DM.

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I. INTRODUCTION AND FRAMEWORK

The various astronomical and cosmological evidences, like the galaxy rotation curves [1, 2], gravitational lensing [3], the bullet cluster [4] etc., vindicate the existence of the dark matter (DM). The Planck Collaboration [5] using the precise map of the cosmic microwave background (CMB) indicates that DM contributes almost 26% to the mass/energy budget of the Universe. These observations can be explained by postulating the existence of particles that interact either very weakly with ordinary particles or only through gravity or any other forces potentially not part of the Standard Model (SM). The limitation of the standard model of particle physics to account for a viable DM candidate opens up the pathway of various plausible scenarios beyond the standard model (BSM) to resolve the DM puzzle.

The most sought after candidate for the DM in any BSM scenario is the so-called weakly interacting massive particle (WIMP) with mass close to the Electroweak scale and weak couplings with the SM particles. In this scenario, once WIMPs are thermally produced at the very early Universe, they remain in thermal equilibrium with the SM bath up to a certain temperature depending on its mass and interaction strength. Eventually they decouple from the thermal bath at some temperature $T_f$, which is commonly called the Freeze-out temperature, where the DM interaction rate drops below the expansion rate of the Universe, governed by the Hubble parameter $H$. At that point, the DM freezes-out from the SM thermal bath providing the observed DM relic density, which is affected only by the expansion of the Universe. It is interesting to note that in the WIMP scenario, the theoretical prediction for the DM thermal relic density coincides with the observed DM relic abundance ($\Omega h^2 \sim 0.12$ [5]), popularly known as the WIMP-miracle. However, various null results coming from the WIMP searches at LHC [6–10], at the spin independent and dependent WIMP-nucleon scattering experiments at LUX [11], PANDAX-II [12], XENON-1T [13], PICO [14] etc. and from the indirect detection coming from FERMI-LAT [15] MAGIC [16] and PLANCK [5] experiments have already excluded a significant part of the parameter space in the WIMP-nucleon scattering cross-section vs WIMP mass plane, casting doubt on the simplest WIMP hypothesis. As the various WIMP searches continue to provide null results, one approach is to design and implement completely new search techniques for WIMP detection different from the aforementioned scattering experiments [17] or to deviate from the WIMP paradigm altogether towards an alter-
native scenario, where the DM particle has very feeble interactions with SM particles and never enter the thermal bath. In this scenario, the DM obtain their relic abundance very slowly through the decay and/or the annihilation of the bath particles, known as the Freeze-in process, providing a completely different explanation for the DM conundrum \cite{18, 19}. For such cases, the production of DM from the decays will contribute dominantly if the same couplings are involved in both decay and annihilation. Due to such feeble interactions, it is extremely challenging to detect the DM in the present direct search experiments. However, many new experimental techniques with low-threshold direct detection\cite{20–25} have been proposed having the capability to probe FIMPs in the near future. Particularly, for a MeV scale feebly interacting massive particle (FIMP) with interaction with electron could be tested by next generation experiments\cite{20, 21, 26} constraining the typical DM-electron cross sections. In general for a light DM, the number density have to be very large to satisfy the observed relic abundance enhancing their detection rate.

In this paper, we present a framework for a sub-MeV DM arising naturally from the theoretical construction of our model. It is to be noted that our detailed knowledge of nucleosynthesis puts a strong constraint on the thermal production of such sub-MeV DM. In particular, this kind of DM could be overabundant if produced by a thermal freeze-out mechanism and motivated by this fact, we have analyzed the non-thermal production of such MeV-scale DM. In building such a framework for the sub-MeV DM, it is legitimate to ask how natural is it to have such a light (sub-MeV) particle and the ways it could satisfy the aforementioned experimental constraints. This work is based on a well studied framework (the EW-\nu model) proposed by one of us \cite{27} for non-sterile right-handed neutrinos with Majorana masses being proportional to the Electroweak scale \( \Lambda_{EW} = 246.21 \) GeV and as a result, can be produced and hunted at the Large Hadron Collider (LHC). To see how the EW-\nu model generates a light DM particle, a brief summary of the seesaw mechanism of the model is in order. The model contains mirror fermions and, for the purpose of this introduction, it is sufficient to discuss one generation of SM leptons: \( \psi_L = (\nu_l, \ell)_L^T; \ell_R \), and mirror leptons: \( \psi^M_R = (\nu^M, \ell^M)_R^T; \ell^M_L \). The Majorana mass for right-handed neutrinos is obtained by coupling \( \ell^M_R \) to a complex Higgs triplet \( \tilde{\chi} \) as \( y_M \psi^{M,T}_R \sigma_2(\tau_2 \tilde{\chi}) \psi^M_R \). With \( \langle \tilde{\chi} \rangle = v_M \), one obtains \( M_R = \frac{y_M v_M}{\sqrt{2}} \). Right-handed neutrinos in this model, being non-sterile, have to be heavier than \( M_Z/2 \) constrained by the \( Z \)-width data, implying that \( v_M \propto \Lambda_{EW} \) severely affects the custodial symmetry which ensures that \( M_W = M_Z \cos \theta_W \) at tree-level. Custodial symmetry is restored by
the introduction of real triplet $\xi$ having the same VEV as $\tilde{\chi}$. (It turns out that this real triplet also provides a solution for a topologically stable, finite energy Electroweak monopole [28, 29].) The Dirac mass term in this seesaw mechanism comes from the coupling of a SM left-handed lepton doublet, a mirror right-handed lepton doublet with a complex singlet scalar $\Phi_s$: $\bar{\ell}_L \Phi_s \ell_R + h.c.$, giving $m_D = y_s v_s$ when $\langle \Phi_s \rangle = v_s$. As we show below, the imaginary part of this complex singlet $\Phi_s$ which is a pseudo-Nambu-Goldstone (PNG) boson, $A^0_s$, can be used as a light DM candidate, for the simple reason that $A^0_s$ would be massless when a global symmetry present in the EW-$\nu_R$ model is spontaneously broken. $A^0_s$ acquires a mass when there is an explicit breaking term in the scalar potential. As one has encountered similar situations in various other scenarios such as chiral symmetry breaking, the explicit breaking term is characterized by some mass scale which is usually assumed to be much smaller than the scale of spontaneous symmetry breaking (SSB). For example, hadronic $SU(2)_L \times SU(2)_R$ is an approximate symmetry with the SSB scale $\Lambda_{QCD} \approx 300$MeV and the scales of explicit breaking are the up and down quarks masses, 2.3 MeV and 4.8 MeV. As we shall see below, the SSB scale of the global symmetry in this framework is of the order of the Electroweak scale and the explicit breaking scale is the mass of $A^0_s$ which will be assumed to be in the sub-MeV region making this PNG boson $A^0_s$ an ideal sub-MeV DM candidate.

It will be shown that the production of $A^0_s$ through the freeze-out mechanism is not favored here since it will result in overabundance. We will further show that the Freeze-in mechanism is the most attractive alternate possibilities in this scenario. As $A^0_s$ interacts extremely feebly with other particles (FIMP), it can be non-thermally produced through the Freeze-in mechanism, yielding the correct relic density and satisfying current constraints from the indirect searches. The model contains a large parameter space exhibiting the aforementioned behavior of the DM candidate. From a particle physics point of view, this DM scenario has a very interesting implication concerning the seesaw mechanism of the EW-$\nu_R$ model: the symmetry breaking scales proportional to the Dirac and Majorana masses as described in the previous paragraphs are found to be comparable in sizes, avoiding the kind of hierarchy found in a generic seesaw mechanism.

The organization of the paper is as follows. In Section II, we present the framework for the rich scalar sector for the EW-$\nu_R$ model. Section III contains the influence of various theoretical and experimental constraints on the model parameters. In Section IV we present our selection of benchmark points and discuss the various LHC bounds examining the stability of this kind of light
dark matter candidate. Section V discusses how $A^0_s (\equiv \text{Im}(\Phi_s))$, a pseudo-Nambu Goldstone boson can successfully play the role of a potential sub-MeV dark matter in this framework and section VI includes the mechanism for the FIMP dark matter candidate $A^0_s$ producing the correct relic density. Section VII discusses the constraints pertaining to the indirect searches. The summary and implications are presented in Section VIII.

II. EXTENDED SCALAR SECTOR OF THE MODEL

The main idea of the EW-$\nu_R$ model [30] containing mirror fermions including Majorana masses for right-handed neutrinos proportional to the Electroweak scale with an extended scalar sector is very appealing. Unlike the Standard Model, the framework is not only left-right symmetric, but each left handed fermion multiplet is accompanied by new right handed fermion multiplet of opposite chirality. The framework contains a rich scalar sector incorporating four doublets (two belonging to the Two Higgs doublet model-THDM like $\Phi_1$, $\Phi_2$, two for mirror sector $\Phi_{1M}$, $\Phi_{2M}$), two triplets $\chi$, $\xi$ and one complex singlet $\Phi_s$ is represented by

$$
\begin{align*}
\Phi_1 &= \begin{pmatrix} 
\phi_{1}^{0*} & \phi_{1}^+ \\
\phi_{1}^- & \phi_{1}^0 
\end{pmatrix}, \\
\Phi_{1M} &= \begin{pmatrix} 
\phi_{1M}^{0*} & \phi_{1M}^+ \\
\phi_{1M}^- & \phi_{1M}^0 
\end{pmatrix}, \\
\Phi_2 &= \begin{pmatrix} 
\phi_{2}^{0*} & \phi_{2}^+ \\
\phi_{2}^- & \phi_{2}^0 
\end{pmatrix}, \\
\Phi_{2M} &= \begin{pmatrix} 
\phi_{2M}^{0*} & \phi_{2M}^+ \\
\phi_{2M}^- & \phi_{2M}^0 
\end{pmatrix}, \\
\tilde{\chi} &= \begin{pmatrix} 
\chi^+ / \sqrt{2} & \chi^{++} \\
\chi^0 & -\chi^+ / \sqrt{2} 
\end{pmatrix}, \\
\xi &= \begin{pmatrix} 
\xi^+ & \xi^0 & \xi^- 
\end{pmatrix}, \\
\Phi_s &= \begin{pmatrix} 
\phi_{s}^{0*} & \phi_{s}^+ \\
\phi_{s}^- & \phi_{s}^0 
\end{pmatrix}. 
\end{align*}
$$

The Higgs potential consisting of these scalars has a global $SU(2)_L \times SU(2)_R$ symmetry, under which the triplet and doublet scalars transform as $(3,3)$ and $(2,2)$ $^1$. The Electroweak symmetry is spontaneously broken by the VEVs of the neutral component of the doublet and triplet scalars. We denote VEVs of the scalar fields $\Phi_1, \Phi_2, \Phi_{1M}, \Phi_{2M}, \chi$ and $\Phi_s$ as $v_1, v_2, v_{1M}, v_{2M}, v_M$ and $v_s$ respectively, where, $\chi$ field is the combination of two triplet scalars $\tilde{\chi}$ and $\xi$ (see eqns.A1-A5). The standard model vacuum expectation value is given by, $v_{SM} \equiv \sqrt{\Delta v^2} \approx 246$ GeV. After the spontaneous Electroweak symmetry breaking, the VEVs are aligned in such a manner that there still remains an unbroken $SU(2)_D$ custodial symmetry, ie. $SU(2)_L \times SU(2)_R \rightarrow SU(2)_D$ and we get six $SU(2)_D$ singlet CP-even Higgs like scalars ($H_0^1, H_0^2, H_{1M}^0, H_{2M}^0, H_s^0, H_{1s}^0$).

$^1$ The transformation $(SU(2)_L$ triplet, $SU(2)_R$ triplet)$\equiv (3,3)$ and the doublet transformation is denoted by $(2,2).$
forming a $6 \times 6$ matrix $\mathcal{M}_H^2$. We obtain the physical eigenstates $\tilde{H}^{''''}$, $\tilde{H}^{'''}$, $\tilde{H}''$, $\tilde{H}'$, $\tilde{H}$, $\tilde{H}_s$, in descending order of mass ($M_{\tilde{H}^{''''}} > M_{\tilde{H}^{'''}} > M_{\tilde{H}''} > M_{\tilde{H}'} > M_{\tilde{H}} > M_{\tilde{H}_s}$) after diagonalizing $\mathcal{M}_H^2$ by an orthogonal matrix $O_H$. Among these new physical scalars, $\tilde{H}$ behaves as the SM like Higgs boson with a mass of 125 GeV. This SM like Higgs boson state is expressed as a superposition of 6 weak eigenstates as: $\tilde{H} = O_{H1}^{51} H_1^0 + O_{H2}^{52} H_2^0 + O_{H3}^{53} H_{1M}^0 + O_{H4}^{54} H_{2M}^0 + O_{H5}^{55} H_s^0 + O_{H6}^{56} H_1^{0'}$. The mixing angles $O_{H1}^{51}$ and $O_{H2}^{52}$ control the $\tilde{H}$ couplings with various standard model particles and the SM like $SU(2)_L$ doublet scalars $H_1^0$ and $H_2^0$ respectively. The detailed analysis on the extended scalar sector of this present framework have been discussed in the Appendix A.

III. THEORETICAL AND EXPERIMENTAL CONSTRAINTS ON THE MODEL

The additional charged fermions and scalars in this model can lead to non-trivial implications on various existing experimental observations. Moreover, mathematical consistency of the model demands that various model parameters should satisfy certain theoretical conditions. In this section we discuss the various theoretical and experimental constraints crucial to the framework discussed in this paper.

- **Perturbativity**: The various quartic couplings in the Higgs potential are assumed to be perturbative: $|\lambda_i| < 4\pi$, where $\lambda_i$s are defined in the section II. The Yukawa couplings are taken to be $< \sqrt{4\pi}$.

- **Constraints from the Electroweak precision observables**: The additional mirror fermions and heavy $SU(2)$ doublet and triplet scalars contribute to the electroweak precision observables in this framework, namely the oblique S, T, U parameters [36]. It should be noted that the constraints arising from these parameters have been presented in the Ref. [37] in an earlier rendition of the model where only two Higgs doublets were incorporated (in addition to the two triplets and the singlet). As shown in Ref. [37], the positive contributions in the oblique (S and T) parameters stemming from the new mirror fermions can be offset by the negative contributions from the triplets present in the framework. Additional Higgs doublets will not alter this picture and we preserve the features of the earlier version of the model. Nevertheless, in our numerical analysis, we have applied the T-parameter constraints
on the mass difference for the doubly, singly charged and neutral scalars and the the mirror up-down fermions of the $SU(2)$ fermion doublets, $\Delta M_{5,ij} \equiv |M_i - M_j|$ (where $i \neq j$, $i,j = H_{5^\pm}, H_{5}^{0}$), $\Delta M_3 \equiv |M_{H_{5}^{\pm}} - M_{H_{5}^{0}}|$ and $\Delta M_f \equiv |M_{f_{MF}^{u,\nu R}} - M_{f_{MF}^{d,l}}|$ to be less than $\sim 50$ GeV with the assumption of a light 125 GeV SM like Higgs boson and 173.1 GeV top quark.

• **Constraints from the Lepton Flavor Violating processes:** The presence of mirror leptons would lead to additional contribution to $\mu \to e\gamma$ at the one loop level and $\mu \to 3e$ as well as $\tau \to 3\ell$ processes [38] at the tree level due to the charged lepton mixing through the Yukawa interaction $y_{s\ell} \tilde{\nu}_L^c M^M \Phi_s$. Among all the aforementioned LFV processes, the most stringent limit $\text{BR}(\mu^+ \to e^+\gamma) < 4.2 \times 10^{-13}$ at 90% C.L. [39, 40] comes from the MEG experiment at PSI on the $\mu \to e\gamma$ process. On the other hand for the $\mu$ to $e$ conversion in the nuclei, the strongest experimental upper limit on branching ratio is provided by the SINDRUM II experiment for titanium targets: $\text{BR}(\mu^- + \text{Ti} \to e^- + \text{Ti}) < 4.3 \times 10^{-12}$ at 90% C.L. [40, 41]. The current experimental upper limit on $\text{Br}(\mu \to e\gamma)$ sets the constraint of $y_{sl} \leq 10^{-4}$ for the mirror lepton mass $M_{f_{MF}} \sim 100-800$ GeV [34, 42, 43]. It is also important to note that according to previous studies performed by one us the most stringent constraints placed on the additional couplings $y_s \sim y_{sq} \sim y_{su} \sim y_{sd}$ are given by $y_s < 0.1 y_{sl}$ [33, 35] coming from the solution of the strong CP problem.

• **The Higgs signal strength:** It is to be noted that the experimentally measured properties of the 125-GeV scalar particle discovered at the LHC so far tend to favor the characteristics of the SM Higgs boson. Since we assume one of the CP-even scalars $\tilde{H}$ fulfilling the role of the SM like Higgs, it is necessary that we examine whether the production and decay of $\tilde{H}$ in our scenario is in agreement with the current experimental data. The agreement can be probed by estimating the Higgs signal in the $x$th decay of $\tilde{H}$ as $\mu_x = \frac{\sigma(pp\to \tilde{H})_{\text{BSM}}}{\sigma(pp\to h)_{\text{SM}}} \frac{\text{BR}(\tilde{H} \to x)_{\text{BSM}}}{\text{BR}(h \to x)_{\text{SM}}}$, where, $x = \gamma\gamma, WW, ZZ, b\bar{b}$ and $\tau^+\tau^-$. Theoretically obtained Higgs signal strengths depend very strongly on the choice of model parameters as shown in Table I. In the next section while describing the model parameters, we will elaborate the implication of the Higgs signal strength on various benchmark points.
TABLE I: Extended scalar sector parameters (scalar field VEV’s and quartic couplings $\chi$’s) and the corresponding scalar masses in GeV for five representative benchmark points.

IV. BENCHMARK POINTS IN THE MODEL

Here we discuss the significance of the choice of our model benchmark points as shown in Table I in the context of FIMP like dark matter phenomenology. The masses and phenomenology of the additional scalars and mirror fermions are strongly dependent on the vacuum expectation value (VEV) of the scalar fields (GeV) and scalar quartic couplings $\chi$’s ($\lambda_{1a}$, $\lambda_{1b}$, $\lambda_{2a}$, $\lambda_{2b}$, $\lambda_3$, $\lambda_4$, $\lambda_5$, $\lambda_8$ and $\lambda_s$). To analyze the dark matter phenomenology, we are interested in the region of parameter space allowed by various theoretical and experimental constraints discussed in the previous section.

In our analysis, we choose the model parameters (scalar quartic couplings, VEVs, etc.) to accommodate the second lightest component $\tilde{H}$ acting as the SM like Higgs boson with dominant contributions coming from the Higgs doublets $\Phi_1$ and $\Phi_2$. Thus, the VEVs $v_1$, $v_2$ and the scalar mixing elements $O_{H}^{31}$ and $O_{H}^{32}$ control the SM like Higgs boson couplings with the SM fermions and gauge bosons. The other scalars ($\Phi_{1M,2M}, \chi$) interact with the SM particles through the mixing with the doublet scalars and for the benchmark points shown in Table I, the mixing angles are

\footnote{We performed a scan on the parameter space randomizing the input parameters (vevs, quartic couplings etc) to obtain the Higgs-like scalar mass $M_{\tilde{H}} \approx 125$ GeV. Out of the one million data points used to scan with $M_{\tilde{H}} \approx 125$ GeV, we obtained a set of data points satisfying all the theoretical as well as experimental constraints as discussed in the text with most of the points being discarded by the LHC Higgs signal strength data.}
negligibly small. This leads to a scenario where these scalars have highly suppressed couplings with the SM particles. As a consequence of this, one can have heavy scalar with mass $\sim 300\text{ GeV}$ satisfying the aforementioned experimental constraints. In our parameter space scan, we have varied all the quartic couplings (parameters of the scalar potential) within their perturbative range.

In this framework, the Yukawa couplings of the SM like Higgs boson is analogous to the Type-II 2HDM\[44–46\] in both (SM and Mirror fermions) sectors. The first THDM like doublet $\Phi_1$ interacts with the SM charged leptons and down-type quarks whereas the second THDM like doublet $\Phi_2$ couples to the up-type quarks (see eqn. A19). Here, we assume that the Higgs-125 GeV scalar $\tilde{H}$ is mostly generated from the real part of the doublets $\Phi_1$ and $\Phi_2$. Hence, the contributions on the Higgs signal strength coming from the other scalar multiplets due to the mixings can be taken to be negligible. The decay rate of $\tilde{H}$ into two lightest CP-even scalar $\tilde{H}_s$ (and two dark matter $A^0_s$) fields can contribute to the Higgs invisible decay width depending on mixing, i.e., the value of the quartic coupling $\lambda_{4t}$ and the VEVs ($v_1, v_2, v_{1M}, v_{2M}, v_M$ and $v_s$) of the scalar fields. We exercise caution while choosing the values for these VEVs as they can alter the mixing and masses of all the model particles. In particular, the VEV $v_2$ can significantly modify the $\tilde{H}t\bar{t}$ yukawa coupling $Y_{\tilde{H}t\bar{t}}$, which in turn can alter the one loop effective Higgs to gluon-gluon ($\tilde{H}gg$) coupling away from its Standard Model value. Thus, the $Y_{\tilde{H}t\bar{t}}$ coupling modifier $\kappa_t$ can be parameterized as $\kappa_t = \frac{(Y_{\tilde{H}t\bar{t}})_{\text{New}}}{(Y_{\tilde{H}t\bar{t}})_{\text{SM}}} \equiv \frac{O_{\tilde{H}}^{52}}{v_2},$ the ratio of the top Yukawa coupling for $\tilde{H}$ in this model relative to the SM value. As a result of this, the production cross-section for the Higgs-125 GeV will be different in this framework in comparison to the SM and the deviation is approximately proportional to $\kappa_t^2$: $R_\sigma \equiv \frac{\sigma(pp \rightarrow \tilde{H})_{\text{BSM}}}{\sigma(pp \rightarrow h)_{\text{SM}}} \approx \frac{\sigma(gg \rightarrow \tilde{H})_{\text{BSM}}}{\sigma(gg \rightarrow h)_{\text{SM}}} \propto \kappa_t^2.$ For all the benchmark points shown in Table I, the mixing angle $O_{\tilde{H}}^{52} \approx 0.69$ and the minimization condition for the complete scalar potential imply that the individual values of both $v_1$ and $v_2$ must be smaller than $v_{\text{SM}}$. This results in the estimated value of $R_\sigma > 1$, leading to an enhanced production cross-section for the SM like $\tilde{H}$ in the gluon fusion channel compared to the SM. However, the Higgs signal strengths in the $\gamma\gamma, WW$ and $ZZ$ final states $\mu_{\gamma\gamma}$ and $\mu_{WW,ZZ}$ are measured very precisely at the LHC and provide stringent limits on the model parameters for any beyond the standard model scenario that contribute non-trivially to these final states \[47\]. Hence, to satisfy these constraints the enhancement in the Higgs production rate ($gg \rightarrow \tilde{H}$) must be compensated by the branching ratio $BR(\tilde{H} \rightarrow xx)_{\text{BSM}},$ where
\( x = \gamma, W, Z \). We find that for the benchmark points (see Table I) the branching ratio \( \text{Br}(\tilde{H} \to \gamma\gamma) \) is \((5 - 18\%) \) lower compared to the SM-value (see Table II) thus making the model predicted \( \mu_{\gamma\gamma} \) values consistent with the observed data \cite{47}. We also note that the contributions coming from the other heavy charged scalars at the one-loop level do not impact the \( \Gamma(\tilde{H} \to \gamma\gamma) \) due to the small quartic coupling \( \lambda_{\text{4f}} \). Also, due to the six custodial \( SU(2)_D \) CP-even scalar mixings, the \( \text{Br}(\tilde{H} \to WW, ZZ) \) is approximately \( 15 - 30\% \) lower than the corresponding branching ratios in the SM. As a consequence of this, the Higgs (\( \tilde{H} \)) signal strengths for \( \mu_{WW,ZZ} \) channels are consistent with the experimental values within 1\( \sigma \) range for our choice of benchmark points. While scanning the parameter space, we have found that the values of \( v_{1M} = v_{2M} \approx v_M \gtrsim 70 \text{ GeV} \) could violate the Higgs signal strength data for the following reasons. In this scenario, we have total four scalar doublets and two triplets, and their corresponding vacuum expectation values satisfy the constraint: 

\[ v_{\text{SM}} \equiv \sqrt{v_1^2 + v_{1M}^2 + v_2^2 + v_{2M}^2 + 8v_M^2} = 246.21 \text{ GeV}. \]

For the aforementioned values of \( v_{1M}, v_{2M} \) and \( v_M \), the VEVs \( v_1 \) and \( v_2 \) are bound to be tiny to satisfy the condition \( v_{\text{SM}} = 246.21 \text{ GeV} \). Such low values for \( v_1 \) and \( v_2 \) would substantially increase the Yukawas \( Y_{u,d,\ell} \) which in turn will elevate the tree-level SM-like Higgs partial width in the \( b\bar{b}, \tau\tau \) final states. In addition to producing a surge in these tree-level decay widths, the loop induced \( \gamma\gamma \) and gluon-gluon partial widths could also rise as a consequence of this. The increase in these partial decay modes eventually add up in the total decay width of the SM-like Higgs boson. As a result of this inflated total decay width, all branching ratios for the SM-like Higgs boson are altered significantly, violating the experimentally measured Higgs signal strength data.

\[
\begin{array}{|c|c|c|c|c|c|c|}
\hline
\text{Benchmark Points} & \text{Br}(\tilde{H} \to b\bar{b}) & \text{Br}(\tilde{H} \to \tau\tau) & \text{Br}(\tilde{H} \to WW^*) & \text{Br}(\tilde{H} \to ZZ^*) & \text{Br}(\tilde{H} \to \gamma\gamma) & \text{Br}(\tilde{H} \to Other BSM) \\
\hline
\text{SM} & 5.66 \times 10^{-01} & 6.21 \times 10^{-02} & 2.26 \times 10^{-02} & 2.81 \times 10^{-02} & 2.88 \times 10^{-02} & - \\
\text{BP-1} & 6.01 \times 10^{-01} & 8.56 \times 10^{-02} & 1.98 \times 10^{-02} & 2.46 \times 10^{-02} & 1.96 \times 10^{-02} & < 1 \times 10^{-06} \\
\text{BP-2} & 6.98 \times 10^{-01} & 8.64 \times 10^{-02} & 1.92 \times 10^{-02} & 2.38 \times 10^{-02} & 1.95 \times 10^{-01} & < 1 \times 10^{-06} \\
\text{BP-3} & 6.71 \times 10^{-01} & 8.31 \times 10^{-02} & 2.12 \times 10^{-02} & 2.63 \times 10^{-02} & 2.11 \times 10^{-01} & < 1 \times 10^{-06} \\
\text{BP-4} & 7.25 \times 10^{-01} & 8.88 \times 10^{-02} & 1.63 \times 10^{-02} & 2.02 \times 10^{-02} & 1.94 \times 10^{-01} & < 1 \times 10^{-06} \\
\text{BP-5} & 7.23 \times 10^{-01} & 8.95 \times 10^{-02} & 1.64 \times 10^{-02} & 2.03 \times 10^{-02} & 1.93 \times 10^{-01} & < 1 \times 10^{-06} \\
\hline
\end{array}
\]

**TABLE II:** The Branching fraction of the SM Higgs and the 125 GeV \( \tilde{H} \) are shown for five benchmark points of Table I.

\(^3\) The Higgs quartic coupling \( \lambda_{4f} \leq 10^{-9} \) is required for a FIMP like dark matter production via freeze-in mechanism in this scenario.
| Signal Strength | Benchmark Points and Signal strength of SM like Higgs |
|-----------------|-----------------------------------------------------|
| μ_{Best−Fit}   | 2.51$^{+2.03}_{−2.01}$ | 1.05$^{+0.53}_{−0.37}$ | 1.35$^{+0.35}_{−0.21}$ | 1.22$^{+0.23}_{−0.21}$ | 1.16$^{+0.21}_{−0.18}$ |
| μ_{BP−1}       | 1.70  | 1.91  | 1.214 | 1.211 | 1.19  |
| μ_{BP−2}       | 1.81  | 2.03  | 1.239 | 1.236 | 1.25  |
| μ_{BP−3}       | 1.42  | 1.59  | 1.114 | 1.111 | 1.10  |
| μ_{BP−4}       | 1.85  | 2.06  | 1.03  | 1.029 | 1.23  |
| μ_{BP−5}       | 2.06  | 2.30  | 1.16  | 1.15  | 1.22  |

TABLE III: 125-GeV Higgs boson signal strengths for five benchmark points of Table I. The experimental best fit μ_{Best−Fit} are the CMS combined measurements of the Higgs boson couplings at 13 TeV run of the LHC with 35.9 fb\(^{-1}\) data [47]. The error bars shown on μ_{Best−Fit} are at the one sigma level.

In Table II we show different branching ratios (BR) for $\tilde{H}$ and also provide the corresponding SM higgs boson BR values for direct comparison. For our choice of benchmark points, the $\tilde{H}$ branching ratios in various SM two body final states are consistent with that of the current LHC Higgs boson data. It should also be noted that the invisible decay of $\tilde{H}$ is negligible small in this scenario. The various Higgs signal strengths calculated for our choice of benchmark points and the corresponding experimental best-fit values are shown in Table III. For all the chosen benchmark points, the Higgs signal strengths are consistent with the experimental bounds within 2σ [47].

V. THE SUB-MEV DARK MATTER CANDIDATE IN THE MODEL

Let us now turn our attention to the possibility of accommodating a sub-MeV dark matter candidate in this framework. In this scenario, it is noted that the imaginary part of the complex singlet scalar does not mix with the other scalars and as a result can serve as a viable dark matter candidate $A_0^s \equiv Im(\Phi_s)$. The transformation of the following fields can be expressed as $\Phi_{1,2} \rightarrow e^{-2i\alpha_{SM}} \Phi_{1,2}$, $\Phi_{1M,2M} \rightarrow e^{2i\alpha_{MF}} \Phi_{1M,2M}$, $\bar{\chi} \rightarrow e^{-2i\alpha_{MF}} \bar{\chi}$, $\xi \rightarrow \xi$ and $\Phi_s \rightarrow e^{-i(\alpha_{SM}+\alpha_{MF})} \Phi_s$. The $\lambda_{5,6}$’s terms, in the potential (see eqn. A6), break the $U(1)_{SM} \times U(1)_{MF}$ symmetries explicitly. Total three ‘massless’ Nambu-Goldstone bosons can be obtained after spontaneous breaking of $SU(2)_L \times U(1)_Y \rightarrow U(1)_{em}$ by imposing the condition $\lambda_{5a} = \lambda_{5b} = \lambda_{6a} = \lambda_{6b} = \lambda_{7a} = \lambda_{7b} =$

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\( \lambda_{7ab} = \lambda_{7Mab} = \lambda_{7aM} = \lambda_{7abM} = \lambda_5 \). Similarly the first line of \( \lambda_5 \) term in the potential (see eqn. A6) is \( U(1)_{SM} \times U(1)_{MF} \) conserving, and the second line explicitly violates these symmetries. All these terms help to get exact minimization of the scalar potential. In the absence of the \( \lambda_5 \) term, one can obtain an additional ‘massless’ neutral Nambu-Goldstone, i.e., the complex singlet type pseudoscalar (PSS) can remain massless. The \( U(1)_{SM} \times U(1)_{MF} \) breaking \( \lambda_5 \) terms help us to get non-zero sub-MeV mass for the singlet-type complex pseudoscalar field \( A_0^0 \). At the tree-level the mass of the complex singlet scalar is given by

\[
M_{A_0^0}^2 = 8 \lambda_5 (v_1 + v_2)(v_{1M} + v_{2M}). \tag{5.1}
\]

For the chosen BPs, the numerical values of \( (v_1 + v_2)(v_{1M} + v_{2M}) \) remain almost same and the dark matter mass only depends on the quartic coupling \( \lambda_5 \). The Higgs portal coupling \( (A_0^0 A_0^0 \bar{H}) \) is also proportional to the coupling \( \lambda_5, \lambda_4 \) and VEVs (see the eqns. 6.16). In the following sections, we discuss the various bounds on the model parameter space [27, 32–34] coming from terrestrial and laboratory-base experiments.

Motivated by the possibility of a viable dark matter candidate in this scenario, we investigate the mass ranges and the corresponding stability conditions for the \( A_0^0 \). If the dark matter \( A_0^0 \) is heavy, it can decay into two fermions at tree-level and the corresponding decay width is given by

\[
\Gamma(A_0^0 \rightarrow ff) = \frac{N_f^2M_{A_0^0}y_f^2}{8\pi}(1 - \frac{4m_f^2}{M_{A_0^0}^2})^{1/2}, \tag{5.2}
\]

where, \( y_f \approx \sqrt{2} \frac{y_f^2}{y_{s\ell}^2} \), \( i = u, d, \ell \) (see eqns A26 and A27) for \( v_M = v_{1M} = v_{2M} \). One can also find the decay life time in this case to be

\[
\tau_{A_0^0} = \frac{6.5821 \times 10^{-25}}{\Gamma_{A_0^0}^{Total} \text{[in GeV]}} \text{seconds} \tag{5.3}
\]

Here we use the natural unit conversion 1 GeV\(^{-1} = 6.5821 \times 10^{-25} \) seconds. If \( M_{A_0^0} > 2m_e \), it can decay only into two electrons. The stability of the DM demands that \( \tau_{A_0^0} > \tau_U \) where, \( \tau_U \approx 4.35 \times 10^{17} \) seconds is the lifetime of the Universe. For the chosen parameters, \( M_{A_0^0} = 1.023 \) MeV, \( \left( \frac{v_{2M}}{v_s} \right)^2 = 1.848 \times 10^{-5} \) and \( y_e^M \approx \sqrt{4\pi} \), we find the limit on the coupling \( y_{s\ell} \) to stabilize the dark matter to be

\[
y_{s\ell} < 5.14091 \times 10^{-11}. \tag{5.4}
\]

Furthermore, motivated from the theoretical framework present in the current scenario, we are
interested in the sub-MeV DM mass range. As the dark matter is assumed to be light (sub-MeV), it is unable to decay into two fermions at tree-level, however there is a possibility of it decaying to two photons through the SM and mirror charged particles (see Fig. 1). The decay width of the dark matter decaying into two photons is given by

$$\Gamma(A^0_s \rightarrow \gamma\gamma) = \Gamma(A^0_s \rightarrow \gamma\gamma)^{\text{SM fermions}} + \Gamma(A^0_s \rightarrow \gamma\gamma)^{\text{MF fermions}},$$

(5.5)

where,

$$\Gamma(A^0_s \rightarrow \gamma\gamma)^{\text{SM fermions}} = \frac{\alpha^2 M^3_{A^0_s}}{256\pi^3 v^2_{\text{SM}}} \left| \sum_f N^c_f Q^2_f y_f F_{1/2}(\chi_f) \right|^2,$$

(5.6)

$$\Gamma(A^0_s \rightarrow \gamma\gamma)^{\text{MF fermions}} = \frac{\alpha^2 M^3_{A^0_s}}{256\pi^3 v^2_{\text{SM}}} \left| \sum_f N^c_f Q^2_f y_f M F_{1/2}(\chi_f) \right|^2,$$

(5.7)

here, $\chi_i = M^2_{A^0_s}/4m^2_i$. $Q_f$ denotes electric charges of the corresponding particles. $N^c_f$ is the color factor. In the limit $y_{sd} \approx y_{sq} \approx y_{su} = y_s$, $y_f = -y^M_f \approx \sqrt{2} \frac{v_d \nu_s}{v^2_{\text{SM}}} \frac{y^M_s}{y^M_{e3}}$ (see eqns A26 and A27) denote $A^0_s$ couplings to $f_i\bar{f}_i$ and $f^M_i\bar{f}_i$ where, $i = u, d, \ell$. The loop function $F_{1/2}$ is defined as

$$F_{1/2}(\chi) = 2[\chi + (\chi - 1) f(\chi)]\chi^{-2},$$

(5.8)

where,

$$f(\chi) = \begin{cases} \frac{(\sin^{-1} \sqrt{\chi})^2}{4}, & \chi \leq 1 \\ \frac{1}{4}\left[\ln\frac{1+\sqrt{1-\chi^{-1}}}{1-\sqrt{1-\chi^{-1}}} - i\pi\right]^2, & \chi > 1 \end{cases},$$

(5.9)

As previously stated, to solve the strong CP problem in this framework the corresponding couplings are of the order $y_s < 0.1 y_{sd}$ [33, 35] as a result of which the decay $\Gamma(A^0_s \rightarrow \gamma\gamma)$ channel through SM-quarks are suppressed. Additionally, as the mass of the mirror-fermions are large ($\mathcal{O}(150) \text{ GeV}$),
the decays through the mirror-fermions are negligible compared to other channels. It is to be noted that the couplings of $A_0^s$ with two charged scalars or charged gauge bosons are absent (protected from the $U(1)_{\text{SM}} \times U(1)_{\text{MF}}$ symmetries) in this model. As a result, the decay $\Gamma(A_0^s \to \gamma\gamma)$ through the SM charged lepton loop becomes the dominant one and gives rise to the most stringent bound on the coupling $y_{s\ell}$. Hence we obtain

$$
\Gamma_{\text{tot}}(A_0^s \to \gamma\gamma) = \frac{\alpha^2 M_3^3 A_0^s}{256\pi^3 v_{\text{SM}}^2} \frac{y_{s\ell}^4}{(y_{\ell}^M)^2} \frac{v_s^2}{v_{2M}^2} \left| F_{1/2}(\chi_e) + F_{1/2}(\chi_\mu) + F_{1/2}(\chi_\tau) \right|^2. \quad (5.10)
$$

The decay through electron loop is the dominant one as $F_{1/2}(\chi_e) >> F_{1/2}(\chi_\mu) >> F_{1/2}(\chi_\tau)$ for $M_{A_0^s} \leq 1$ MeV. Hence

$$
\Gamma_{\text{tot}}(A_0^s \to \gamma\gamma) \approx \frac{\alpha^2 M_3^3 A_0^s}{256\pi^3 v_{\text{SM}}^2} \frac{y_{s\ell}^4}{(y_{\ell}^M)^2} \frac{v_s^2}{v_{2M}^2} \left| F_{1/2}(\chi_e) \right|^2 \quad (5.11)
$$

and using this we can obtain the life-time of the pseudoscalar given by

$$
\tau_{A_0^s} = \frac{6.582 \times 10^{-25}}{\Gamma_{\text{tot}}(A_0^s \to \gamma\gamma) \ [\text{in GeV}] \ [\text{seconds}}} \quad (5.12)
$$

The dark matter stability condition imposes an upper limit on the couplings. Taking $(\frac{v_{2M}}{v_s})^2 = 1.848 \times 10^{-5}$ and $(y_{\ell}^M)^2 \sim 4\pi$, the limit on $y_{s\ell}$ can be obtained as

$$
y_{s\ell} \lesssim \frac{6.582 \times 10^{-25}}{\tau_U \alpha^2 M_3^3 A_0^s} \frac{256\pi^3 v_{\text{SM}} (y_{\ell}^M)^2 \left( \frac{v_{2M}}{v_s} \right)^2}{\left| F_{1/2}(\chi_e) \right|^2} \quad (5.13)
$$

We take $|F_{1/2}(\chi_e)| = 1.922$ for $M_{A_0^s} = 1$ MeV and obtain the bound on $y_{s\ell}$

$$
y_{s\ell} < 9.305 \times 10^{-7}. \quad (5.14)
$$

Similarly taking $|F_{1/2}(\chi_e)| = 1.33$ for $M_{A_0^s} = 1$ keV, we get

$$
y_{s\ell} < 1.986 \times 10^{-4}. \quad (5.15)
$$

It is to be noted that the parameter space is constrained by $\mu \to e\gamma$ and $\mu - e$ conversion implying $y_{s\ell} \leq 10^{-4}$ [34]. In Fig. 2, we present the allowed parameter space in the $y_{s\ell} - M_{A_0^s}$ plane. The gray-region is excluded from the $\mu \to e\gamma$ decay and $\mu - e$ conversion constraints [34].
The indirect searches for the light dark matter can also place stringent constraints on the model parameter space which will be discussed in the later sections. It is to be noted that the direct detection limits [13] for such light dark matter masses may not be applicable in this framework. We will discuss the details of the relic density analysis (through the successful implementation of Freeze-in mechanism) in the upcoming section.

VI. FIMP-LIKE DARK MATTER DENSITY

From the discussion in the previous section, it is established that the pseudoscalar singlet (PSS) $A^0_s$ can be a viable candidate for the dark matter in this framework. However, a decaying MeV scale dark matter candidate poses a viable limitation due to the fact that one has to implement fine-tuning [48, 49] to stabilize the dark matter as the lifetime of the DM particles has to be at least larger than the age of the Universe [50, 51]. In this model, we observe a viable, stable dark matter candidate in the interesting mass range of $\mathcal{O}(< 1)$ MeV with the corresponding quartic coupling $15$. 

FIG. 2: Plot shows the exclusion region in $M_{A^0_s} - y_{s\ell}$ plane. The dark matter is stable in the region below the red-line given by, $\tau_{A^0_s} > \tau_U$ where the three red lines correspond to $y_{u}^M = 1$ (dashed), $\sqrt{4\pi}$ (solid) and $4\pi$ (dotted) respectively. The gray-region is excluded from the $\mu \rightarrow e\gamma$ constraints and $\mu - e$ conversion [34] implying $y_{s\ell} < 10^{-4}$. 

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related to the dark matter mass to be in the range $\lambda_{5c} < \mathcal{O}(10^{-12})$. It is possible that for the low dark matter mass $\mathcal{O}(< 1) \text{ MeV}$, the relic density at the right ballpark $\Omega h^2 = 0.1198 \pm 0.0026$ [5] is still produced through the well established Freeze-out mechanism with the choice of the large Higgs portal and other couplings, but it will violate the perturbative-unitarity limits. For example, very large Higgs portal couplings $\lambda's = \mathcal{O}(500)$ (here $\lambda's$ are $\lambda_{4a}$, $\lambda_{5c}$ and $\lambda_s$) for $M_{A^0} = 0.5 \text{ MeV}$ and $\lambda's = \mathcal{O}(1500)$ for $M_{A^0} = 100 \text{ keV}$ are needed to obtain the thermally averaged annihilation cross-section $\sim 2.0 \times 10^{-26} \text{ cm}^3/\text{s}$.

We observe that the non-thermally produced PSS can serve as a viable (depending on the parameters $\lambda_{5c}, \lambda_{4a}, \lambda_s, y_s, y_s^\ell$ and VEVs, see the Section II) $\mathcal{O}(< 1) \text{ MeV}$ dark matter candidate satisfying the dark matter relic density constraints. As the dark matter interacts with other particles very weakly (feebly), for such very weakly interacting particles, called feebly interacting massive particles or FIMPs, we can invoke the non-thermal, so-called Freeze-in mechanism. This mechanism needs feeble interactions which could be one of the reasons to have aforementioned tiny couplings existing in this framework. We examine the possibility of the dark matter sector getting populated through decay or annihilation of other heavy particles until the number density of the corresponding heavy particle species becomes Boltzmann-suppressed. To carry this out, we need to solve the Boltzmann equation that dictates the final relic abundance for the dark matter candidate $A^0_s$. The production of the dark matter resulting from the decay of any mother particle ($\bar{H}_i (i = 1, ..6), f_{MF}$) is in thermal equilibrium at early universe and is given by

$$\frac{\Gamma}{H} \geq 1,$$

(6.1)

where, $\Gamma$ is the relevant decay width and $H$ is the Hubble parameter given by $[18, 52, 53]$

$$H(T) = \left( g^* \frac{\pi^2}{90} \frac{T^4}{M_{Pl}^2} \right)^{1/2},$$

(6.2)

where, $M_{Pl} = 1.2 \times 10^{19} \text{ GeV}$ is the Planck mass and $T$ is the temperature ($1 \text{ GeV} = 1.16 \times 10^{13} \text{ Kelvin}$). If the production of the mother particles occur mainly from the annihilation of other particles in the thermal bath, $\Gamma$ will be replaced by $[18, 52, 53]$

$$\Gamma = n_{eq} < \sigma v >,$$

(6.3)
where $n_{eq}$ is their equilibrium number density and is given by [52]

$$n_{eq} = \begin{cases} 
  g^* \left( \frac{MT}{2\pi} \right)^{3/2} e^{-M/T}, & \text{for non-relativistic states } T << M \\
  \zeta_3 \pi^2 g^* T^3, & \text{for relativistic boson states } T >> M \\
  3 \pi^2 \frac{\xi_3}{4} g^* T^3, & \text{for relativistic fermion states } T >> M 
\end{cases}$$

(6.4)

where the Riemann zeta function has the value $\zeta_3 = 1.2$ and $g^* = 208.5$ (for $T >> M$) is the effective degrees of freedom in this framework and $M$ stands for the mass of the particle. Here $<\sigma v>$ is the thermally averaged annihilation cross-section for the particles in the thermal bath and can be written as [52, 54]

$$<\sigma_{xx} v> = \frac{2\pi^2 T \int_{4M^2}^{\infty} ds \sqrt{s (s - 4M^2)} K_1\left(\frac{\sqrt{s}}{T}\right) \sigma_{xx}}{(4\pi M^2 TK_2\left(\frac{M}{T}\right))^2},$$

(6.5)

where $\sigma_{xx}$ is the production annihilation cross-section of the mother particles ($x = \tilde{H}_{i=1,.6}, f_{MF}$) from other particles in the thermal bath (see the production annihilation diagrams in Fig. 3) and $K_{1,2}$ is the modified Bessel function of functions of order 1 and 2 respectively.

In this picture, we present various possible decay (see Fig. 4) and annihilation (see Fig. 5) diagrams for the production of the heavy Higgs and mirror fermions that facilitate thermal equilibrium in the early universe. The 2-body and 3-body decay widths for the heavy Higgs ($\tilde{H}_i$) are

FIG. 3: Annihilation-production diagrams for the dark matter from the Higgs, SM and mirror fermion.
FIG. 4: Decay diagrams contributing to the relic density. Decay-production diagrams for the heavy Higgs and mirror fermion help in thermal equilibrium in the early universe.

FIG. 5: Annihilation-production diagrams for the heavy Higgs and mirror fermions help in thermal equilibrium in the early universe. It is to be noted that there are many other similar diagrams that can contribute in the production of the heavy Higgs and mirror fermions.

given by

\[
\Gamma(\tilde{H}_i \to f_{MF} f_{MF}) = \frac{M_{\tilde{H}_i} y_M^2}{8\pi} \left(1 - 4 \frac{M_{MF}^2}{M_{\tilde{H}_i}^2}\right)^{3/2}
\]  \hspace{1cm} (6.6)

\[
\Gamma(\tilde{H}_i \to \tilde{H}_j \tilde{H}_k) = \frac{y_{\tilde{H}_i \tilde{H}_j \tilde{H}_k}^2}{16\pi M_{\tilde{H}_i}^3} \left((M_{\tilde{H}_i}^2 - M_{\tilde{H}_j}^2 - M_{\tilde{H}_k}^2)^2 - 4 M_{\tilde{H}_j}^2 M_{\tilde{H}_k}^2\right)^{1/2}
\]  \hspace{1cm} (6.7)
\[
\Gamma(\tilde{H} \to f_M f_{SM} A_s^0) = \frac{1}{2\pi^3} \frac{1}{32\pi^3} |\mathcal{M}|^2 dE_1 dE_3, \quad \text{where, } |\mathcal{M}|^2 \propto \frac{y_M^2 y_{st}^2}{M_{MF}^2}.
\]

(6.8)

Here \(E_{1,2,3}\) are the energies of the final state particle for three-body decay \([55]\). There are other processes and diagrams that can also contribute to the production of the heavy Higgs and mirror fermions which for simplicity, have been ignored in our calculation. We have closely followed the Ref. \([56]\) to calculate the three body decay widths and find the three-body decay rates to be always suppressed by additional propagator mass \(M_{MF}^2\) and the coupling \(y_{st}^2\). It is to be noted that it is not challenging to obtain \(\frac{\Gamma}{\mathcal{H}} \gg 1\) due to the large decay width of the heavy particles in the early universe, resulting the heavy fermions being in thermal equilibrium with the thermal bath particles. Also to note that the lighter CP-even Higgs could also be produced from the decay of the heavier CP-even Higgs with the corresponding decay width \(\mathcal{O}(10)\) GeV satisfying the condition \(\frac{\Gamma}{\mathcal{H}} \gg 1\) in this case too. Hence the lighter CP-even Higgs may also remain in thermal equilibrium with the thermal bath particles.

Similarly the scattering diagrams shown in Fig. 5 also satisfy the condition \(\frac{n_{eq} \sigma v}{\mathcal{H}} \gg 1\) in the model parameter space we are interested in. In this framework, the dark matter can be produced from the decay of the mirror fermions, heavy scalars and from the annihilation of the other particles. It has already been discussed in existing literatures \([18, 53, 57–66]\) that if same couplings are involved in both decays as well as scattering processes then the former has the dominant contribution to DM relic density over the latter one.

Considering all these discussion, we take into account that the dark matter candidate is stable and can be produced only from the decay of the mirror fermions and heavy Higgses in this framework. The Boltzmann equation for the dark matter can be written as \([18, 52, 53]\)

\[
\frac{dn}{dt} + 3 H n = - \sum_i S(X_{\text{Heavy},i} \to A_s^0 A_s^0, f_{SM} A_s^0),
\]

(6.9)

where \(X_{\text{Heavy},i} = \tilde{H}_{j=1,6} \quad (\tilde{H}_{j=1,6} \text{ stand for the six physical CP-even mass eigenstates } \tilde{H}^{m}, \tilde{H}^{m'}\), \(\tilde{H}^{m''}, \tilde{H}', \tilde{H}, \text{ and } \tilde{H}_s\), \(f_{MF}\). We have summed over all these heavy particle contributions. The notation \(f_{SM}\) stands for SM fermions. Here the decay-based source term \(S\) can be written as

\[
S = \Gamma(X_{\text{Heavy},i} \to A_s^0 A_s^0, f_{SM} A_s^0) \frac{K_1(m_{X_{\text{Heavy},i}}/Y)}{Y} n_{eq_{\text{Heavy},i}}
\]

(6.10)

where \(K_{1,2}\) is the modified Bessel function of the first and second kind. For \(x = \frac{m_{X_{\text{Heavy},i}}}{Y}\) and \(Y = \frac{m}{T}\),

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the eqn. 6.9 now reads [52]
\[
dY(x) = \frac{\sum_i g_{X_{\text{Heavy},i}} \Gamma(X_{\text{Heavy},i} \rightarrow A^0_s A^0_s; f_{SM} A^0_s)}{2\pi^2 H(x \approx 1)} x^3 K_1(x),
\]
where \(g_{X_{\text{Heavy},i}}\) is the degrees of freedom of the heavy particle. We can integrate the dark matter production over the entire thermal history and find the final yield \(Y(x_0)\) with the help of the appropriate integral [18, 52, 53]
\[
Y(x_0) = \frac{45 M_{Pl}}{6.64 \pi^4 g^S \sqrt{g^\rho}} \sum_i g_{X_{\text{Heavy},i}} \Gamma(X_{\text{Heavy},i} \rightarrow A^0_s A^0_s; f_{SM} A^0_s) \int_0^\infty x^3 K_1(x) dx
\]
\(g^S\) and \(g^\rho\) are the effective relativistic of degrees of freedom for the entropy density and energy density respectively. The relic density now can be written as [18, 52, 53]
\[
\Omega h^2 = \frac{h^2}{3 H_0^2 M_{Pl}^2} \frac{M_{A^0_s}^2}{28 T_0^S} Y(x_0)
\approx 1.09 \times 10^{27} M_{A^0_s} \sum_i g_{X_{\text{Heavy},i}} \Gamma(X_{\text{Heavy},i} \rightarrow A^0_s A^0_s; f_{SM} A^0_s) \frac{M_{X_{\text{Heavy},i}}^2}{M_{A^0_s}^2}
\]

FIG. 6: Dark matter production diagrams from the decay of the heavy particles contributing to the relic density.

We now use eqn. 6.13 to calculate the relic density in this scenario. The main DM production diagrams from the decay widths of the heavy particles are shown in Fig. 6. The partial decays of the heavy Higgs into the pair of dark matter and the mirror fermion decaying into a SM fermion and single dark matter are given by
\[
\Gamma(\tilde{H}_i \rightarrow A^0_s A^0_s) = \frac{g_{\tilde{H}_i A^0_s A^0_s}^2}{32\pi M_{\tilde{H}_i}} \left(1 - \frac{M_{A^0_s}^2}{M_{\tilde{H}_i}^2}\right)^{\frac{1}{2}}
\]
\[ \Gamma(f_{MF} \to f_{SM} A_s^0) = \frac{M_{MF} y_{f_{MF}}^2}{8\pi} f_{SM} A_s^0. \] (6.15)

with the corresponding coupling strengths given in terms of the mixing angles in the scalar sector (see eqns. A6-A15)

\[
y H_s A_{2}A_{2} = O_1^{11} y H_1^0 A_{2}^0 A_{2}^0 + O_1^{21} y H_2^0 A_{2}^0 A_{2}^0 + O_1^{31} y H_{1M}^0 A_{2}^0 A_{2}^0 + O_1^{41} y H_{2M}^0 A_{2}^0 A_{2}^0 + O_1^{51} y H_{1M}^0 A_{2}^0 A_{2}^0 + O_1^{61} y H_{2M}^0 A_{2}^0 A_{2}^0, \\
y H_{A_{2}A_{2}} = O_1^{12} y H_1^0 A_{2}^0 A_{2}^0 + O_1^{22} y H_2^0 A_{2}^0 A_{2}^0 + O_1^{32} y H_{1M}^0 A_{2}^0 A_{2}^0 + O_1^{42} y H_{2M}^0 A_{2}^0 A_{2}^0 + O_1^{52} y H_{1M}^0 A_{2}^0 A_{2}^0 + O_1^{62} y H_{2M}^0 A_{2}^0 A_{2}^0, \\
y H_{A_{2}A_{2}} = O_1^{13} y H_1^0 A_{2}^0 A_{2}^0 + O_1^{23} y H_2^0 A_{2}^0 A_{2}^0 + O_1^{33} y H_{1M}^0 A_{2}^0 A_{2}^0 + O_1^{43} y H_{2M}^0 A_{2}^0 A_{2}^0 + O_1^{53} y H_{1M}^0 A_{2}^0 A_{2}^0 + O_1^{63} y H_{2M}^0 A_{2}^0 A_{2}^0, \\
y H_{A_{2}A_{2}} = O_1^{14} y H_1^0 A_{2}^0 A_{2}^0 + O_1^{24} y H_2^0 A_{2}^0 A_{2}^0 + O_1^{34} y H_{1M}^0 A_{2}^0 A_{2}^0 + O_1^{44} y H_{2M}^0 A_{2}^0 A_{2}^0 + O_1^{54} y H_{1M}^0 A_{2}^0 A_{2}^0 + O_1^{64} y H_{2M}^0 A_{2}^0 A_{2}^0, \\
y H_{A_{2}A_{2}} = O_1^{15} y H_1^0 A_{2}^0 A_{2}^0 + O_1^{25} y H_2^0 A_{2}^0 A_{2}^0 + O_1^{35} y H_{1M}^0 A_{2}^0 A_{2}^0 + O_1^{45} y H_{2M}^0 A_{2}^0 A_{2}^0 + O_1^{55} y H_{1M}^0 A_{2}^0 A_{2}^0 + O_1^{65} y H_{2M}^0 A_{2}^0 A_{2}^0, \]

where,

\[
y H_1^0 A_{2}^0 A_{2}^0 = \lambda_{4a} v_1 + 2\lambda_{5c} v_{1M} + 2\lambda_{5c} v_{2M}, \\
y H_2^0 A_{2}^0 A_{2}^0 = 2\lambda_{5c} v_{1M} + \lambda_{4a} v_2 + 2\lambda_{5c} v_{2M}, \\
y H_{1M}^0 A_{2}^0 A_{2}^0 = 2\lambda_{5c} v_1 + \lambda_{4a} v_{1M} + 2\lambda_{5c} v_2, \\
y H_{2M}^0 A_{2}^0 A_{2}^0 = 2\lambda_{5c} v_1 + 2\lambda_{5c} v_2 + \lambda_{4a} v_{2M}, \\
y H_{s}^0 A_{2}^0 A_{2}^0 = 2\lambda_{s} v_s, \quad y_{H_{s}^0 A_{2}^0 A_{2}^0} = 0. \]

where \( H_1^0, H_2^0, H_{1M}^0, H_{2M}^0, H_s^0 \) and \( H'_1^0 \) is the unphysical scalar fields (before mixing, see the Section II for details). The CP-even scalar incorporating both the triplet scalars \( H_1'^0 = \sqrt{\frac{2}{3}} \lambda^0 r + \sqrt{\frac{1}{3}} \xi^0 \) does not have the direct coupling to PSS dark matter, i.e., \( y H_{s}^0 A_{2}^0 A_{2}^0 = 0 \). The other scalars are unable to decay to PSS dark matter due to conservation of the \( U(1)_{SM} \times U(1)_{MF} \) symmetry, charge, etc. Hence the initial DM density coming from the decay scenario mainly depends on the decay of these CP-even scalars and mirror fermions. One can see from these eqns. 6.16 and 6.17 that the decay of these heavy (physical) scalar fields and mirror fermions can be controlled by the \( \lambda_{5c}, \lambda_{4a}, \lambda_s, y_s, y_{s\ell} \) and VEVs with the mass of the dark matter mainly depending on the \( \lambda_{5c} \) and VEVs (see eqn. 5.1).

As an example, let us first neglect the contribution from the decay of the mirror fermions \( f_{MF} \to f_{SM} A_s^0 \) (we consider \( y_{s\ell} < < 10^{-9} \)) and consider the benchmark point BP-3 (see Table I).
FIG. 7: Plots show the variation of parameters $\lambda_{5c}$ and $\lambda_{4a}$ in left panel and similarly show the dark matter mass against $\lambda_{4a}$ variation in right panel. These plots are generated for BP-3 as in Table I. In both plots the red solid line represents $\Omega h^2 = 0.1198$ and the dashed red lines correspond to the $3\sigma$ variation in $\Omega h^2$. The lighter region corresponds to higher values of $\Omega h^2$. For both these plots the contribution from the mirror fermion decay $f_{MF} \rightarrow f_{SM} A_s^0$ is sub-dominant and has been neglected.

We choose $\lambda_s = 10^{-15}$, hence the lightest CP-even scalar field will not be able to decay to the dark matter ($M_{\tilde{H}_s} < 2M_{A_0^0}$). The other CP-even states including the 125 GeV scalar field could decay into the dark matter which increases the abundance of the dark matter. Using BP-3 and $\lambda_{5c} = 3.2 \times 10^{-12}$ and $\lambda_{4a} = 7.377 \times 10^{-9}$, we obtain the dark matter mass as $M_{A_0^0} = 0.808$ MeV and find the numerical values of the coupling strengths $y_{\tilde{H}_s} A_0^0 A_0^0 = 1.630 \times 10^{-11}$, $y_{\tilde{H}} A_0^0 A_0^0 = -1.021 \times 10^{-6}$, $y_{\tilde{H'}} A_0^0 A_0^0 = 3.390 \times 10^{-6}$, $y_{\tilde{H}''} A_0^0 A_0^0 = 1.059 \times 10^{-21}$, $y_{\tilde{H}'''} A_0^0 A_0^0 = -9.424 \times 10^{-6}$ and $y_{\tilde{H}''''} A_0^0 A_0^0 = 3.669 \times 10^{-6}$. Finally, we obtain the relic density to be $\Omega h^2 = 0.1198$. We show the variation of the parameters $\lambda_{5c}$ and $\lambda_{4a}$ in Fig. 7(left) and similarly show the dark matter mass against $\lambda_{4a}$ variation in Fig. 7(right). In both plots the red solid line represents $\Omega h^2 = 0.1198$ and the dashed red lines correspond to the $3\sigma$ variation in $\Omega h^2$ with the darker region corresponding to the lower values of $\Omega h^2$.

We find that if we neglect the contribution coming from the scalar fields, the dark matter abundance could increase from the decay of the mirror fermions $f_{MF} \rightarrow f_{SM} A_s^0$. In this case, the
contribution from the mirror quarks will also be negligibly small; as the corresponding Yukawa couplings are very small \((y_s \sim y_{sq} \sim y_{su} \sim y_{sd} \sim 0.1 y_{s\ell} [33, 35])\). We obtain the relic density in the right ballpark for \(y_{s\ell} \sim 4.428 \times 10^{-9}\) for \(M_{A^0} = 10\ \text{keV}\) and \(y_{s\ell} \sim 4.429 \times 10^{-10}\) with \(M_{A^0} = 1\ \text{MeV}\). The variation of dark matter mass against \(y_{s\ell}\) is shown in Fig. 8. The blue dashed line indicates the 3\(\sigma\) relic density \(\Omega h^2 = 0.1198 \pm 0.0026\) band [5]. The line above this blue line will overclose the Universe. The indirect detection bounds form HEAO and INTEGRAL experiments will be discussed in the section VII.

![Graph showing the variation of dark matter mass against \(y_{s\ell}\)](image)

**FIG. 8:** The same \(y_{s\ell} - M_{A^0}\) plot as in Fig. 2 showing the relic density constraint and including the indirect detection bounds applicable in the parameter space. The blue dashed line indicates the relic density 3\(\sigma\) band \(\Omega h^2 = 0.1198 \pm 0.0026\). The region above the purple and black shaded lines are ruled out from the HEAO-1 [67] and INTEGRAL [68] indirect detection experiments.

We now consider all these aforementioned contributions in the relic density calculation. Increasing \(\lambda_{4a}\) increases the contributions coming from the heavy Higgs decays whereas large values of \(y_{s\ell}\) increases the contributions from the mirror fermions. We present such variations for two different dark matter masses \(M_{A^0} = 10\ \text{keV}\) and \(0.5\ \text{MeV}\) respectively in Fig. 9. The contribution is almost equal for \(\lambda_{4a} \sim 4.90 \times 10^{-8}\) and \(y_{s\ell} \sim 2.98 \times 10^{-9}\) with dark matter mass \(M_{A^0} = 10\ \text{keV}\) and similarly for \(\lambda_{4a} \sim 7.447 \times 10^{-9}\) and \(y_{s\ell} \sim 3.78 \times 10^{-10}\) with dark matter mass \(M_{A^0} = 0.5\ \text{MeV}\). The dark matter abundance increases as a result of the decay of the mirror fermions \(f_{MF} \rightarrow f_{SM} A^0_s\). In this
FIG. 9: Plots show the variation of parameters $\lambda_{4a}$ and $y_{s\ell}$ for two different dark matter masses $M_{A_0^s} = 10$ keV (left) and 0.5 MeV (right). These plots are also generated for BP-3 as in Table I. In both plots the red solid line represents $\Omega h^2 = 0.1198$ and the dashed red lines correspond to the $3\sigma$ variation in $\Omega h^2$. The lighter region corresponds to higher values of $\Omega h^2$.

In both plots the red solid line represents $\Omega h^2 = 0.1198$ and the dashed red lines correspond to the $3\sigma$ variation in $\Omega h^2$. The lighter region corresponds to higher values of $\Omega h^2$.

In this case, the contributions from the mirror quarks are negligibly small as $y_s \sim y_{sq} \sim y_{su} < y_{s\ell}$ \cite{33, 35}. We have also shown the evolution of the dark matter with the temperature of the Universe in Fig. 10 for the following parameters: $\lambda_s = 10^{-15}$, $\lambda_{4a} \sim 4.90 \times 10^{-8}$ and $y_{s\ell} \sim 2.98 \times 10^{-9}$ and $M_{A_0^s} = 10$ keV. The plot clearly represents the significance of the Freeze-in mechanism in this framework, i.e., the initial DM density being zero and increasing during the cooling of the Universe. After a certain temperature ($T \sim \mathcal{O}(100$ GeV) as shown in Fig. 10) the dark matter density becomes constant.

We have also looked into the bounds coming from the free streaming length $l_{fs}$ which will denote whether the dark matter will behave as hot, warm or cold. The $l_{fs} > 2$ Mpc region stands for hot dark matter region and can create challenges for the structure formation \cite{69}. We avoid these regions (dark matter mass $<< 1$ keV) in this analysis and calculate the free streaming length \cite{70} for the dark matter with mass range $10^{-5} - 1$ MeV and find it to be consistently less than 10 kpc in the parameter space referred to in the Fig. 8. Hence we conclude that in this scenario $A_0^s$ behaves as a cold dark matter candidate \cite{69}.

In summary, in this section we have discussed the Freeze-in scenario for the dark matter can-
FIG. 10: Plot shows the variation of the yield $Y(x)$ against $x$ for all the contribution coming from the heavy Higgs and mirror fermion decays. The Freeze-in mechanism effect can be clearly seen as the initial DM density is zero and increases during the cooling of the Universe and attaining a constant value after a certain temperature. We scale the mirror fermion decay contribution (black line) by a factor $1/10$ to distinguish from the others and used $M_X = 150$ GeV mass scale factor.

didate possible in this framework and have discussed the stability bounds on the dark matter in the previous section. Now as we recognize that the galactic and extra-galactic diffuse $X−ray$ or $γ−ray$ may come from the decay or annihilation of a dark matter through the loop placing stringent constraints on the model parameter space of any dark matter model [71], in the upcoming section we devote a discussion on the possible additional bounds coming from the available data from various indirect dark matter search experiments.

VII. INDIRECT DETECTION OF DARK MATTER

Let us now divert our attention towards the various dark matter detection prospects plausible in this framework. One interesting possibility is to look for the excess of photon or charged particles from different directions of our sky coming from the annihilation or the decay of dark matter particles. In this scenario, the dark matter decay is less constrained from the early Universe cosmology and the galactic and extra-galactic diffuse $X−ray$ or $γ−ray$ background puts the most stringent bounds on the model parameters for the decaying DM [71].
The observations of the Fermi Large Area Telescope (Fermi-LAT) put limits on the weak-scale DM (mass $\mathcal{O}(100) \text{ MeV}$), having decay lifetime of $\tau_{A_0} > 10^{26} \text{ s}$, many orders of magnitude larger than the age of the Universe [72, 73]. Although the usual gamma-ray constraints from the Fermi-LAT do not apply for the $< \mathcal{O}(100) \text{ MeV}$ masses, several other satellite-based experiments such as HEAO-1 [67], INTEGRAL [68], COMPTEL [74], EGRET [75] are sensitive to photons with energies well below $\mathcal{O}(100) \text{ MeV}$. In this particular framework with dark matter mass less than $\mathcal{O}(1) \text{ MeV}$, only HEAO-1 [67] and INTEGRAL [68] can put stringent constraints on the dark matter parameters. In Fig. 8, we show 4 the limits coming from HEAO-1 [67] and INTEGRAL [68] in $y_{st} - v s - M_{A_0}$ plane which puts a strong upper limit on the value of $y_{st} \leq 10^{-6} - 10^{-7}$. However, as we are considering the non-thermal production of dark matter, the relic density bound can still be satisfied with smaller $y_{st}$ as shown in the blue line in Fig. 8.

VIII. COLLIDER ANALYSIS

We now investigate the Collider signals as the mirror fermion interactions with the SM particles make it feasible to probe DM via the Collider sector. We would like to point out that the mirror fermion masses are assumed to be less than 1 TeV and these mass range is currently valid evading the Collider bounds under the current assumptions applicable in this framework. We focus on the existing literature (see Ref. [76]) outlining some possible ways to explore the DM signals. The Ref. [76] has primarily investigated the FIMP dark matter scenario in the context of 14 TeV LHC data with high integrated luminosity at the MATHUSLA surface detector in the near future. In this framework, there is a possibility of charged track formations due to the mirror fermion decays into SM fermion and dark matter fields at the Colliders. The decay width of the charged mirror fermion is given by eqn. 6.15. The decay width ($\Gamma(f_{MF} \rightarrow f_{SM} A_0)$ for the mirror fermions is proportional to $y_{st}^2$ and the mirror fermion masses. As discussed previously, an estimated value of the coupling $y_{st} \lesssim 10^{-8}$ is needed to obtain the correct dark matter density through the Freeze-in process. We find the decay length for these charged fermions to be $\mathcal{O}(1)$ meter for this choice of $y_{st}$. We are interested to see if we can get sufficient number of events from this charged tracks for

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4 We extract these data from Ref. [71] where the authors have put bounds on the lifetime of a scalar dark matter decaying to two photons. We translate it into the interaction strength ($y_{st}$), i.e. the interaction between dark matter, SM and mirror leptons, to constrain our model parameter space.
the DM detection. It mainly depends on the production cross-section $\sigma_{\text{LHC}}^{\sqrt{s}}$ of the mother particle and luminosity $\mathcal{L}$ at the detector. The number of events at the LHC is calculated in Ref. [76] and is given by

$$N_{\text{events}} = \sigma_{\text{LHC}}^{\sqrt{s}} \mathcal{L} \int P_{\text{Decay}}^{\text{MATH}}, \quad \text{with} \quad P_{\text{Decay}}^{\text{MATH}} = 0.05(e^{-\frac{e^{-L_{a}}}{\beta_{c} M_{F}}} - e^{-\frac{e^{-L_{b}}}{\beta_{c} M_{F}}}).$$  

(8.1)

It has been reported in Ref. [76] that the number of events is $N_{\text{events}} \geq 3$ for $\sqrt{s} = 13$ TeV with an integrated luminosity $\mathcal{L} = 3000$ fb$^{-1}$ using a specific model parameter space, showing that the MATHUSLA100/200 detector could detect these mother particles up to 1 TeV mass with the dominant production of the mother particles coming from the Drell-Yan processes. Following this reference, in this framework, as the production of the mother particles (mirror fermions) are hugely suppressed due to the small mixing (see the BPs) between the mirror and standard model particles, we find the production cross-section to be less than $\mathcal{O}(10^{-10})$ fb. Hence for this scenario, larger luminosity ($>> 1$ ab$^{-1}$) and energy is needed to get any significant events at the current MATHUSLA surface detector.

**IX. CONCLUSION**

In this work, we have presented a model incorporating a sub-MeV DM based on the exploration of the scalar sector of the Electroweak-scale Right-handed neutrino model. The idea of EW-$\nu_{R}$ model with additional GeV scale mirror fermions with large displaced vertices containing long lived particles (LLP) signatures is already highly appealing from the LHC perspective and has been extensively studied before [32, 77, 78]. The rich scalar sector of EW-$\nu_{R}$ includes doublets, triplets and an additional complex-singlet scalar $\Phi_{s}$ and the imaginary part of this complex singlet (pseudo-Nambu Goldstone (PNG) boson), $A_{0}^{s}$ is investigated to be a plausible DM candidate in the present context. The dark matter $A_{0}^{s}$ acquires a sub-MeV mass from the explicit breaking term in the scalar potential; this explicit breaking term is characterized by some mass scale assumed to be much smaller than the scale of spontaneous symmetry breaking (SSB). The various model parameters present in the scalar sector of this framework are investigated to generate possible benchmark points in the context of a sub-MeV dark matter, satisfying the current 125 GeV Higgs branching ratio and signal strength constraints from the LHC. In this work, we have focused on the limitations of the well established Freeze out mechanism, for which the observed abundance
is set almost exclusively by the annihilation cross-section and is largely insensitive to unknown
details of early Universe and to the mass, producing overabundance for the sub-MeV DM particle
$A_0^s$ we are interested to study. Null results at direct detection experiments have currently put
tight constraints on the WIMP paradigm and alternative possibilities like ALP, axions, SIMPs,
FIMPS have become relevant in this context. We have implemented the Freeze-in mechanism to
obtain the correct order of relic density for the chosen dark matter masses $< 1$ MeV. For such
feeble interacting massive particles or FIMPs, we can invoke the non-thermal Freeze-in mechanism
that necessitates feeble interactions making it one of the reasons to have such a tiny fine-tuned
coupling present in this EW-$\nu_R$ model. We find that the non-thermally produced PSS (pseudo-
scalar singlet) can serve as a viable $\leq 1$ MeV dark matter depending on the parameters $\lambda_{5c}, \lambda_{4a}, \lambda_s,$ $y_s, y_{s\ell}$ and VEVs satisfying the dark matter relic density.

Using the Freeze in mechanism to investigate the scalar sector of the EW-$\nu_R$, we obtain a significant
parameter space of $y_{s\ell} - M_{A_0^s}$ for the sub-MeV dark matter mass satisfying the correct relic density
and successfully put bounds on the coupling strength $y_{s\ell}$ vs $M_{A_0^s}$ exclusion region from the stability
(lifetime of DM $> lifetime of the Universe) of the dark matter, the rare processes ($\mu \to e\gamma$, and
$\mu - e$ conversion), several indirect detection experiments constraining this particular mass region
(HEAO-1 and INTEGRAL) etc. We also found that indirect detection experiments, such as Fermi-
LAT data are currently unable to successfully constrain the parameter space of $y_{s\ell} - M_{A_0^s}$ for the
mass range of $< 1$ MeV. Also, due to such feeble interactions, it is challenging to get handle on the
signatures of this light dark matter from the direct-detection experiments through nucleon-dark
matter scattering as well as dark matter-electron interactions via the magnon excitation.

Through our investigation, we found that $y_{s\ell} \sim 10^{-8}$ is needed for the correct relic abundance
and have pointed out the parameter space available for sub-MeV FIMP dark matter ready to be
explored by the future experiments. We have discussed in detail the possible future implications of
this scenario in the Collider searches, in specific the MATHUSLA detector. From a particle physics
point of view, this scenario is highly interesting as the model framework has already been successful
incorporating the non sterile right handed neutrinos with Electroweak scale Majorana masses.

Having a substantial parameter space available to explore after implementing relevant constraints
for a natural sub-MeV FIMP Dark matter particle in the current and future experiments makes
this scenario even more relevant and exciting. The current framework casts light on the feeble
explored sub-MeV dark sector frontier, and offers many opportunities for exciting and profound discoveries in the future.

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Appendix A: Model

The model framework and motivations including the gauge structure, particle content [30] has already been introduced previously in this work. Here we include a summary of the details on the extended scalar sector of the Electro-weak Right handed neutrino model that is vital for studying the dark matter portion of this framework. The present framework includes a rich scalar sector incorporating four doublets (two for the THDM like, two for mirror sector), two triplets and a singlet given by

\[
\Phi_1 = \begin{pmatrix} \phi^0_1 & \phi^+_1 \\ \phi^-_1 & \phi^0_1 \end{pmatrix}, \quad \Phi_{1M} = \begin{pmatrix} \phi^0_{1M} & \phi^+_1 \\ \phi^-_{1M} & \phi^0_{1M} \end{pmatrix}, \quad \Phi_2 = \begin{pmatrix} \phi^0_2 & \phi^+_2 \\ \phi^-_2 & \phi^0_2 \end{pmatrix}, \quad \Phi_{2M} = \begin{pmatrix} \phi^0_{2M} & \phi^+_2 \\ \phi^-_{2M} & \phi^0_{2M} \end{pmatrix},
\]

\[
\tilde{\chi} = \begin{pmatrix} \chi^+ / \sqrt{2} \\ \chi^0 \\ -\chi^+/\sqrt{2} \end{pmatrix}, \quad \xi = (\xi^+, \xi^0, \xi^-), \quad \text{Complex singlet scalar} = \Phi_s \quad (A1)
\]

The transformation of these scalar multiplet under \(U(1)_{SM} \times U(1)_{MF}\) symmetry are as follows: \(\Phi_{1,2} \rightarrow e^{-2i\alpha_{SM}} \Phi_{1,2}, \Phi_{1M,2M} \rightarrow e^{2i\alpha_{MF}} \Phi_{1M,2M}, \tilde{\chi} \rightarrow e^{-2i\alpha_{MF}} \tilde{\chi}, \xi \rightarrow \xi \) and \(\Phi_s \rightarrow e^{-i(\alpha_{SM} + \alpha_{MF})} \Phi_s\). Additionally, the Higgs potential has a global \(SU(2)_L \times SU(2)_R\) symmetry. The triplet and doublet scalars transform as \((3,3)\) and \((2,2)\) under that global symmetry. The combination of these triplet scalars can be written as [30]

\[
\chi = \begin{pmatrix} \chi^0 & \xi^+ & \chi^{++} \\ \chi^- & \xi^0 & \chi^+ \\ \chi^- & \xi^- & \chi^{0*} \end{pmatrix}, \quad (A2)
\]
Proper vacuum alignment gives

\begin{align}
< \Phi_1 > &= \begin{pmatrix} v_1 / \sqrt{2} & 0 \\ 0 & v_1 / \sqrt{2} \end{pmatrix}, \quad < \Phi_{1M} > = \begin{pmatrix} v_{1M} / \sqrt{2} & 0 \\ 0 & v_{1M} / \sqrt{2} \end{pmatrix}, \\
< \Phi_2 > &= \begin{pmatrix} v_2 / \sqrt{2} & 0 \\ 0 & v_2 / \sqrt{2} \end{pmatrix}, \quad < \Phi_{2M} > = \begin{pmatrix} v_{2M} / \sqrt{2} & 0 \\ 0 & v_{2M} / \sqrt{2} \end{pmatrix}, \\
< \chi > &= \begin{pmatrix} v_M & 0 & 0 \\ 0 & v_M & 0 \\ 0 & 0 & v_M \end{pmatrix}, \quad < \Phi_s > = v_s \tag{\text{A5}}
\end{align}

The generic scalar potential for these scalars can now be written as

\[ V = \lambda_{1a} \left[ Tr\Phi_1^\dagger \Phi_1 - v_1^2 \right] + \lambda_{2a} \left[ Tr\Phi_{1M}^\dagger \Phi_{1M} - v_{1M}^2 \right] + \lambda_{1b} \left[ Tr\Phi_2^\dagger \Phi_2 
- \frac{\tau_2^2}{2} + \lambda_{2b} \left[ Tr\Phi_2^\dagger \Phi_{2M} - v_{2M}^2 \right] + \lambda_3 \left[ Tr\chi^\dagger \chi - 3v_M^2 \right]^2 
+ \lambda_s \left[ \Phi_s^\dagger \Phi_s - v_s^2 \right] + \lambda_4 \left[ Tr\Phi_1^\dagger \Phi_1 - v_1^2 + Tr\Phi_{1M}^\dagger \Phi_{1M} - v_{1M}^2 + Tr\Phi_2^\dagger \Phi_2 - v_2^2 
+ Tr\Phi_{2M}^\dagger \Phi_{2M} - v_{2M}^2 + Tr\chi^\dagger \chi - 3v_M^2 \right] \left[ \Phi_s^\dagger \Phi_s - v_s^2 \right] 
+ \lambda_5a \left[ Tr\Phi_1^\dagger \Phi_1 \left( Tr\chi^\dagger \chi \right) - 2 \left( Tr\Phi_1^\dagger \frac{\tau_a^a}{2} \Phi_1 \frac{\tau_b}{2} \right) \left( Tr\chi^\dagger T^a T^b \right) \right] 
+ \lambda_6a \left[ Tr\Phi_{1M}^\dagger \Phi_{1M} \left( Tr\chi^\dagger \chi \right) - 2 \left( Tr\Phi_{1M}^\dagger \frac{\tau_a^a}{2} \Phi_{1M} \frac{\tau_b}{2} \right) \left( Tr\chi^\dagger T^a T^b \right) \right] 
+ \lambda_5b \left[ Tr\Phi_2^\dagger \Phi_2 \left( Tr\chi^\dagger \chi \right) - 2 \left( Tr\Phi_2^\dagger \frac{\tau_a^a}{2} \Phi_2 \frac{\tau_b}{2} \right) \left( Tr\chi^\dagger T^a T^b \right) \right] 
+ \lambda_6b \left[ Tr\Phi_{2M}^\dagger \Phi_{2M} \left( Tr\chi^\dagger \chi \right) - 2 \left( Tr\Phi_{2M}^\dagger \frac{\tau_a^a}{2} \Phi_{2M} \frac{\tau_b}{2} \right) \left( Tr\chi^\dagger T^a T^b \right) \right] 
+ \lambda_5c \left\{ \Phi_s^\dagger \Phi_s \left( Tr\Phi_1^\dagger \Phi_{1M} + Tr\Phi_1^\dagger \Phi_2 + Tr\Phi_2^\dagger \Phi_{1M} + Tr\Phi_2^\dagger \Phi_{2M} \right) + h.c. \right\} 
- 2 \Phi_s^\dagger \Phi_s \left[ Tr\Phi_1^\dagger \Phi_1 + Tr\Phi_1^\dagger \Phi_2 + Tr\Phi_2^\dagger \Phi_{1M} + Tr\Phi_2^\dagger \Phi_{2M} \right] 
+ \lambda_7a \left[ Tr\Phi_1^\dagger \Phi_1 \left( Tr\Phi_{1M}^\dagger \Phi_{1M} \right) - \left( Tr\Phi_{1M}^\dagger \Phi_{1M} \right) \left( Tr\Phi_1^\dagger \Phi_1 \right) \right] 
+ \lambda_7b \left[ Tr\Phi_2^\dagger \Phi_2 \left( Tr\Phi_{2M}^\dagger \Phi_{2M} \right) - \left( Tr\Phi_{2M}^\dagger \Phi_{2M} \right) \left( Tr\Phi_2^\dagger \Phi_2 \right) \right] 
+ \lambda_7ab \left[ Tr\Phi_1^\dagger \Phi_1 \left( Tr\Phi_{2M}^\dagger \Phi_{2M} \right) - \left( Tr\Phi_{2M}^\dagger \Phi_{2M} \right) \left( Tr\Phi_1^\dagger \Phi_1 \right) \right] \tag{\text{A6}}
\]
convenient notation:

\[ + \lambda_{7ab} \left[ (Tr \Phi_{1M}^\dagger \Phi_{1M})(Tr \Phi_{2M}^\dagger \Phi_{2M}) - (Tr \Phi_{1M}^\dagger \Phi_{2M})(Tr \Phi_{2M}^\dagger \Phi_{1M}) \right] \]
\[ + \lambda_{7aMb} \left[ (Tr \Phi_{1M}^\dagger \Phi_{1})(Tr \Phi_{2M}^\dagger \Phi_{2M}) - (Tr \Phi_{1M}^\dagger \Phi_{2})(Tr \Phi_{2M}^\dagger \Phi_{1}) \right] \]
\[ + \lambda_{7abM} \left[ (Tr \Phi_{2M}^\dagger \Phi_{2})(Tr \Phi_{1M}^\dagger \Phi_{1M}) - (Tr \Phi_{2M}^\dagger \Phi_{1})(Tr \Phi_{1M}^\dagger \Phi_{2}) \right] \]
\[ + \lambda_8 \left[ Tr \chi \chi^\dagger \chi - (Tr \chi \chi)^2 \right] , \]

where \( a, b = 1, 2, 3 \) and from [31]

\[
T^1 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} ; \quad T^2 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix} ; \quad T^3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix} ; \quad (A7)
\]

\[
\tau^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} ; \quad \tau^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} ; \quad \tau^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} ; \quad (A8)
\]

Please note the transformation \( \Phi_{1,2} \to e^{-2i\alpha_{SM}} \Phi_{1,2}, \Phi_{1M,2M} \to e^{2i\alpha_{MF}} \Phi_{1M,2M}, \tilde{\chi} \to e^{-2i\alpha_{MF}} \tilde{\chi} \), \( \xi \to \xi \) and \( \Phi_s \to e^{-i(\alpha_{SM} + \alpha_{MF})} \Phi_s \). Hence the \( \lambda_{5,6,8} \)'s terms break explicitly the \( U(1)_{SM} \times U(1)_{MF} \) symmetries. The three ‘massless’ Nambu-Goldstone bosons can be obtained after spontaneous breaking of \( SU(2)_L \times U(1)_Y \to U(1)_{em} \), with the condition \( \lambda_{5a} = \lambda_{5b} = \lambda_{6a} = \lambda_{6b} = \lambda_{7a} = \lambda_{7b} = \lambda_{7ab} = \lambda_{7aMb} = \lambda_{7abM} = \lambda_5 \) imposed on the potential above. The first line of \( \lambda_{5c} \) term is \( U(1)_{SM} \times U(1)_{MF} \) conserving, and the second line explicitly violates these symmetries. Both of them will help us to get exact minimization of the scalar potential and non-zero mass for the singlet-type complex scalar field. There are eighteen physical scalars grouped into \( 5 + 3 + 3 + 3 + 3 + 1 \) of the custodial \( SU(2)_D \) with 6 real singlets. Here, we would like to mention that the dedicated study of vacuum stability condition for a multi-Higgs model like ours is extremely complicated and is beyond the scope of this paper.

To express the Nambu-Goldstone bosons and the physical scalars let us adopt the following convenient notation:

\[
v_{SM} = \sqrt{v_1^2 + v_{1M}^2 + v_2^2 + v_{2M}^2 + 8v_M^2} \approx 246 \text{ GeV}
\]
\[
s_1 = \frac{v_1}{v_{SM}}, \quad c_1 = \frac{\sqrt{v_{1M}^2 + v_2^2 + v_{2M}^2 + 8v_M^2}}{v_{SM}},
\]
\[
s_2 = \frac{v_2}{v_{SM}}, \quad c_2 = \frac{\sqrt{v_{1M}^2 + v_1^2 + v_{2M}^2 + 8v_M^2}}{v_{SM}},
\]

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Thus, $s_1^2 + c_1^2 = s_2^2 + c_2^2 = s_{1m}^2 + c_{1m}^2 = s_{2m}^2 + c_{2m}^2 = s_m^2 + c_m^2 = 1$. We also defined

$$\phi_1^0 = \frac{1}{\sqrt{2}} (\phi_1^{0r} + v_1 + i\phi_1^{0i}), \quad \phi_2^0 = \frac{1}{\sqrt{2}} (\phi_2^{0r} + v_2 + i\phi_2^{0i}),$$

$$\phi_{1M}^0 = \frac{1}{\sqrt{2}} (\phi_{1M}^{0r} + v_{1M} + i\phi_{1M}^{0i}), \quad \phi_{2M}^0 = \frac{1}{\sqrt{2}} (\phi_{2M}^{0r} + v_{2M} + i\phi_{2M}^{0i}),$$

$$\chi^0 = \frac{1}{\sqrt{2}} (\chi^{0r} + v_M + i\chi^{0i}), \quad \zeta^0 = (\zeta^0 + v_M), \quad \Phi_s = \phi_s^{0r} + v_s + i\phi_s^{0i}$$

and

$$\psi^\pm = \frac{1}{\sqrt{2}} (\psi^\pm + \bar{\psi}^\pm), \quad \zeta^\pm = \frac{1}{\sqrt{2}} (\zeta^\pm - \bar{\zeta}^\pm).$$

for the complex neutral and charged fields respectively. With these fields the Nambu-Goldstone bosons are given by

$$y_1^\pm = s_1 \phi_1^\pm + s_2 \phi_2^\pm + s_{1M} \phi_{1M}^\pm + s_{2M} \phi_{2M}^\pm + s_M \psi^\pm,$$

$$y_1^0 = -i(s_1 \phi_1^{0i} + s_2 \phi_2^{0i} + s_{1M} \phi_{1M}^{0i} + s_{2M} \phi_{2M}^{0i}) + is_M \chi^{0i}. \quad (A10)$$

The physical scalars can be grouped, as stated in the previous section, based on their transformation properties under $SU(2)_D$ as follows:

five-plet (quintet) → $H_5^{\pm\pm}, H_5^\pm, \text{and } H_5^0$

triplet → $H_3^\pm, \text{ and } H_3^0$

triplet → $H_3'^\pm, \text{ and } H_3'^0$

triplet → $H_M^\pm, \text{ and } H_M^0$ \hspace{1cm} (A11)

triplet → $H_M'^\pm, \text{ and } H_M'^0$

Real singlet → $H_1^0, H_2^0, H_{1M}^0, H_{2M}^0, H_1'^0, \text{ and } H_1^0$

(A12)

where,

$$H_5^{\pm\pm} = \chi^{\pm\pm}, \quad H_5^\pm = \zeta^\pm, \quad H_5^0 = \frac{1}{\sqrt{6}} (2\zeta^0 - \sqrt{2} \chi^{0r}).$$
where, $H^{--} = (H^{++})^*$, $H_{\text{All}}^{-} = -(H_{\text{All}}^{+})^*$, $H_{\text{All}}^{0} = -(H_{\text{All}}^{0})^*$. The masses of these physical scalars can easily be obtained from (A6). Since, the potential preserves the $SU(2)_{D}$ custodial symmetry, members of the physical scalar multiplets have degenerate masses. These masses are

\[
m_5^2 = 3 \left( \lambda_5 \left( v_1^2 + v_{1M}^2 + v_2^2 + v_{2M}^2 \right) + 8 \lambda_8 v_{3M}^2 \right) \equiv 3 \left( \lambda_5 c_m^2 + 8 \lambda_8 s_m^2 \right) v_{\text{SM}}^2,
\]

\[
m_{3,H^+,H^0}^2 = \lambda_5 \left( v_1^2 + v_{1M}^2 + v_2^2 + v_{2M}^2 + 8 v_{3M}^2 \right) \equiv \lambda_5 v_{\text{SM}}^2
\]

\[
m_{3,\text{All others}}^2 = 2 \lambda_5 \left( v_1^2 + v_{1M}^2 + v_2^2 + v_{2M}^2 + 4 v_{3M}^2 \right) \equiv \lambda_5 (1 + c_m^2) v_{\text{SM}}^2.
\]
In general, $H_1^0$, $H_2^0$, $H_{1M}^0$, $H_{2M}^0$, $H_s^0$ and $H_1^0$ can mix according to the mass-squared matrix

$$M_H^2 = v_{SM}^2 \begin{pmatrix}
8(\lambda_{1a} + \lambda_4)s_1^2 & 8\lambda_{14}s_1s_2 & 8\lambda_{41}s_1s_m & 8\lambda_{42}s_2s_m & 8\lambda_{4a} & 2\sqrt{6}\lambda_{41}s_m
8\lambda_{14}s_1s_2 & 8(\lambda_{1b} + \lambda_4)s_2^2 & 8\lambda_{41}s_2s_m & 8\lambda_{42}s_1s_m & 8\lambda_{4a} & 2\sqrt{6}\lambda_{42}s_m
8\lambda_{41}s_1s_m & 8\lambda_{42}s_2s_m & 8(\lambda_{2a} + \lambda_4)s_m^2 & 8\lambda_{4s} & 8\lambda_{4a} & 2\sqrt{6}\lambda_{4s}s_m
8\lambda_{4a} & 8\lambda_{4s} & 8(\lambda_{2b} + \lambda_4)s_2m & 8\lambda_{4a} & 2\sqrt{6}\lambda_{4a}s_2m
2\sqrt{6}\lambda_{41}s_m & 2\sqrt{6}\lambda_{42}s_m & 2\sqrt{6}\lambda_{4s}^2 & 2\sqrt{6}\lambda_{4s}s_2m & 2\sqrt{6}\lambda_{4s} & 3(\lambda_3 + \lambda_4)s_m^2
\end{pmatrix}$$ \hspace{1cm} (A15)

We denote the mass eigenstates by $\tilde{H}^{'''}$, $\tilde{H}^{'''}$, $\tilde{H}''$, $\tilde{H}'$, $\tilde{H}$, $\tilde{H}_s$. We adopt a convention of denoting the lightest of the six by $\tilde{H}_s$. The next heavier one by $\tilde{H}$, the next one is $\tilde{H}'$ and so on. The heaviest state is $\tilde{H}^{'''}$. The descending order of mass of the physical eigenstates is $(M_{\tilde{H}^{'''}} > M_{\tilde{H}'''} > M_{\tilde{H}''} > M_{\tilde{H}'} > M_{\tilde{H}} > M_{\tilde{H}_s})$. The diagonalizing $6 \times 6$ orthogonal matrix element is denoted by $O_{ij}^H (i, j = 1...6)$. The 125-GeV Higgs-like scalar component can be written as

$$\tilde{H} = O_{51}^H H_1^0 + O_{52}^{H2} H_2^0 + O_{53}^{H1M} H_{1M}^0 + O_{54}^{H2M} H_{2M}^0 + O_{55}^{Hs} H_s^0 + O_{56}^{H1'} H_1'$$ \hspace{1cm} (A16)

In this analysis, multiple scalar fields alongside the Standard Model (SM) Higgs boson raise concerns regarding the experimentally measured SM Higgs boson mass under radiative corrections. The scalar sector of our model is also very similar to various versions of 2HDMs and Triplet scalar models, and we expect that the lightness of the SM Higgs boson mass will be ensured here, too. The SM fermion sector in this EW-$\nu_R$ model are given by [27, 32–34]

$$\psi_L = \begin{pmatrix}
\nu_l \\
\ell
\end{pmatrix}_L, \quad \text{and} \quad \ell_R, \quad Q_L = \begin{pmatrix}
u \\
\ell
\end{pmatrix}_L, \quad u_R \quad \text{and} \quad d_R, \quad (A17)$$

where $\ell = e, \mu, \tau$ and $u$ stands for the up - type quarks $(u, c, t)$ and $d$ denotes the down - type quarks $(d, s, b)$. $L$ indicates the left-chirality and $R$ is right-chirality. In the EW-$\nu_R$ model, right-handed neutrinos are parts of $SU(2)$ doublets along with their charged partners (the mirror charged leptons). Anomaly freedom dictates the existence of doublets of right-handed and singlets left-handed mirror quarks [27, 32–34]

$$\psi_R^M = \begin{pmatrix}
\nu^M \\
\ell^M
\end{pmatrix}_R, \quad \text{and} \quad \ell^M_R, \quad Q^M_L = \begin{pmatrix}
u^M \\
d^M
\end{pmatrix}_R, \quad u_L^M \quad \text{and} \quad d_L^M \quad (A18)$$

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It is noted that the left-handed SM fermions and right-handed Mirror fermions are doublet under $SU(2)_L$ whereas right-handed SM fermions and left-handed Mirror fermions are singlet under $SU(2)_L$ transformations. Again $SU(2)$-singlet right-handed SM fermions, left-handed mirror fermions are taken to be singlet under $U(1)_{SM} \times U(1)_{MF}$ transformation in Ref. [27, 32]. To solve the strong CP problem, we follow different transformation as in Ref. [33]. The SM $SU(2)$ singlet fermions transform as $(\ell_R, u_R, d_R) \to e^{i\alpha_{SM}}(\ell_R, u_R, d_R)$ whereas $SU(2)$ singlet fermions go as $(\ell^M_R, u^M_R, d^M_R) \to e^{i\alpha_{MF}}(\ell^M_R, u^M_R, d^M_R)$.

The Higgs doublet $\Phi_2$ only couples to SM up – quarks while another doublet $\Phi_1$ couples to down – quarks and leptons. It behaves like type-II two Higgs doublet model. Similar interactions are also there in the mirror sector with $\Phi_{1M}$ and $\Phi_{2M}$ scalar doublets. The $\tilde{\chi}$ in the Higgs triplet fields with hypercharge $Y = 2$ and $\chi$ is a real Higgs triplet with $Y = 0$. $\Phi_s$ is a complex singlet scalar which helps to generate the neutrino observables and strong CP problems. The total Yukawa part of the Lagrangian is given by [27, 32–34]

$$
\mathcal{L}_y = y_L \overline{\psi}_L \Phi_1 \ell_R + y_{Ls} \overline{\psi}_L \psi_M^\dagger iC \Phi_1 \ell_M + y_{Rc} \overline{\psi}_R \psi_M \Phi_s + y_\ell \overline{\psi}_R \chi \psi_M^\dagger \sigma_2 \tilde{\chi} \psi_M^\dagger
$$

$$
+ y_d \overline{Q}_L \Phi_1 d_R + y_d \overline{Q}_L \Phi_{1M} d^M_R - y_d \overline{Q}_L \sigma_2 \Phi_2 u_R - y_{u} \overline{Q}_L \sigma_2 \Phi_{2M} u^M_R
$$

$$
+ y_u \overline{d}_R d^M_L \Phi_s + y_{sq} \overline{Q}_L Q^M_R \Phi_s + y_{su} \overline{u}_R u^M_R \Phi_s + h.c
$$

(A19)

where $C$ is the charge conjugation operator, $\sigma_2$ being the second Pauli’s spin matrix.

Now we will calculate the various mass-mixing matrix after electroweak symmetry breaking, physical mass eigenstates of the fermions in this model. The charged-lepton mixing matrix can be found as [27]

$$
\mathcal{M}_l = \begin{pmatrix} m_\ell & m_\nu^D \\ m_\nu^D & m_\ell^M \end{pmatrix},
$$

(A20)

where, $m_\ell = y_\ell v_1/\sqrt{2}$, $m_\ell^M = y_\ell v_{1M}/\sqrt{2}$ and $m_\nu^D = y_{s\ell} v_s$. The $l$ and $\ell^M$ stand for flavor eigenstates whereas $\tilde{l}$ and $\tilde{l}^M$ stand for the mass eigenstates. The mixing angle between $\ell$ and $\ell^M$ is $\theta_\ell$, hence $\tan 2\theta_\ell = \frac{2m_\nu^D}{m_\ell^M - m_\ell}$. The mixing matrix is $R_\ell = \{\cos \theta_\ell, \sin \theta_\ell\}, \{-\sin \theta_\ell, \cos \theta_\ell\}$. For $m_\ell^M \gg m_\ell, m_\nu^D$, one can write $\tan \theta_\ell \approx \sin \theta_\ell \approx \theta_\ell \approx \frac{m_\nu^D}{m_\ell} = \frac{\sqrt{2y_{s\ell} v_s}}{y_\ell v_{1M}}$. The mass eigenstates can be written as

$$
\tilde{l} = \ell \cos \theta_\ell + \ell^M \sin \theta_\ell
$$

35
\[ \ell^M = -\ell \sin \theta_\ell + \ell^M \cos \theta_\ell \quad (A21) \]

As there is no singlet right-handed neutrino in this model, there is no such mixing in the neutrino sector and hence the pseudoscalar $A^0_s$ could not decay into two light neutrinos. The up and down sector mixing matrix are given by \[33, 35\]

\[ \mathcal{M}_u = \begin{pmatrix} m_u & m_{sq} \\ m_{su} & m^M_u \end{pmatrix}, \quad \text{and} \quad \mathcal{M}_d = \begin{pmatrix} m_d & m_{sq} \\ m_{sd} & m^M_d \end{pmatrix}, \quad (A22) \]

where, $m_d = y_d v_1/\sqrt{2}$, $m^M_d = y_d v_1 M/\sqrt{2}$, $m_u = y_u v_2/\sqrt{2}$, $m^M_u = y_u v_2 M/\sqrt{2}$, $m_{sq} \approx y_u v_s$, $m_{su} \approx y_s v_u$, and $m_{su} \approx y_s v_u$. The mixing matrix are $R_{u,d} = \{\{\cos \theta_{u,d}, \sin \theta_{u,d}\}, \{-\sin \theta_{u,d}, \cos \theta_{u,d}\}\}$, where

\[ \sin \theta_u = \sqrt{\frac{\left( m^M_u - m_u \right) \left( m^M_u - m_u - \sqrt{\left( m^M_u - m_u \right)^2 + 4 m_{sq} m_{su}} \right) + 2 m_{sq} m_{su} + 2 m^2_{su}}{2 \left( \left( m^M_u - m_u \right)^2 + 2 m_{sq} m_{su} + m^2_{sq} + m^2_{su} \right)}}. \]  

\[ \sin \theta_d \approx 5 \sqrt{\theta_u \theta_d} \quad (A23) \]

We can also get the similar analytical form of $\sin \theta_d$ by replacing $u$ to $d$ in eqn. A23. Let $u, d$ and $u^M, d^M$ stand for flavor eigenstates and $\tilde{u}, \tilde{d}$ and $\tilde{u}^M, \tilde{d}^M$ stand for the mass eigenstates. Thus one can write

\[ \tilde{u} = u \cos \theta_u + u^M \sin \theta_u \]
\[ \tilde{u}^M = -u \sin \theta_u + u^M \cos \theta_u \]
\[ \tilde{d} = d \cos \theta_d + d^M \sin \theta_d \]
\[ \tilde{d}^M = -d \sin \theta_d + d^M \cos \theta_d \]

The terms $(y_{sd} \tilde{\psi}_L^M \psi_R^M + y_{sq} \tilde{d}_R d^M_L + y_{sq} \tilde{Q}_L^M Q_R + y_{su} \tilde{\pi}_R u^M_R) \Phi_s$ in eqn A19 can help us to get the $A^0_s \tilde{f}_1 \tilde{f}_1$ coupling strengths. $A^0_s$ to two leptons (both the SM and MF charged and neutral leptons) can be written as

\[ y^f_f = y_{sf} \sin \theta_\ell \cos \theta_\ell \gamma_5 \approx y_{sf} \sin \theta_\ell \gamma_5 \approx \sqrt{2} \frac{y_{s\ell}^2 v_s}{y^M_{1M} v_{1M}} \gamma_5. \]  

\[ (A26) \]

For $y_{sq} \sim y_{su} \sim y_{sd} = y_s$, $v_{1M} = v_{2M}$ and $y_{d}^M = y_{u}^M$, we can write the mixing angle $\theta_u = \theta_d \approx \frac{\sqrt{2} y_{s\ell} v_s}{y_{s\ell}^M v_{1M}}$. Similarly $A^0_s$ to two quarks (both the SM and MF up and down quarks) coupling strengths are given by

\[ y^q_q = y_s \sin \theta_u \cos \theta_u \gamma_5 \approx y_s \sin \theta_u \gamma_5 \approx \sqrt{2} \frac{y^2_{s\ell} v_s}{y_{s\ell}^M v_{1M}} \gamma_5. \]  

\[ (A27) \]
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