Algorithm for Harmonic Tidal Analysis along T/P Tracks: Taking Differences of Observed Sea Surface Heights at Adjacent Points as Observations

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Abstract  An algorithm (differential mode) is presented for the improvement of harmonic tidal analysis along T/P tracks, in which the differences between the observed sea surface heights at adjacent points are taken as observations. Also, the observation equations are constrained with the results of the crossover analysis; the parameter estimations are performed at 0.1° latitude intervals by the least squares. Cycle 10 to 330 T/P altimeter data covering the China Sea and the Northwest Pacific Ocean (2°-50°N,105°-150°E) are adopted for a refined along-track harmonic tidal analysis, and harmonic constants of 12 constituents in 8474 points are obtained, which indicates that the algorithm can efficiently remove non-tidal effects in the altimeter observations, and improve the precision of tide parameters. Moreover, parameters along altimetry tracks represent a smoother distribution than those obtained by traditional algorithms. The root mean squares of the fitting errors between the tidal height model and the observations reduce from 11 cm to 1.3 cm.

Keywords  satellite altimetry; T/P; harmonic tidal analysis; differential mode

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Introduction

The satellites, which perform exact repeat missions, can survey sea level variations along a track at a high resolution. The satellite altimeter observations can be processed for the tidal constituents by the harmonic or response analysis. The TOPEX/POSEIDON (T/P) satellite has collected over 10 years worth of data, which can overcome the aliasing constraints and estimate the tidal constituents along a track[1]. The reports of Tieney[2] and Bao Jingyang[3] show that precise tidal parameters can be obtained by the along-track harmonic analysis not only in deep oceans but also in shallow seas. For crossover points, the double data and complex sampling rule can estimate more precise tidal parameters.

The principle of the tidal analysis is that the observed sea surface heights are taken to fit the tidal mode by a certain rule (the least squares rule), and the tidal parameters in the mode are estimated. The magnitude of the fitting errors reflects the contribution of the non-tidal effects (including orbit errors, environmental correction errors, etc), and affects the precision.
sion and reliability of the estimated parameters.

1 Equations of along-track harmonic tidal analysis and the differential mode

1.1 Expression of the tidal height and analysis

The tidal height at a given location is written as:

\[ h(t) = \bar{h} + \sum_{i=1}^{m} f_i H_i \cos[\chi_i(t) + u_i - g_i] + \Delta = \bar{h} + \sum_{i=1}^{m} f_i \cos[\chi_i(t) + u_i] U_i + \sum_{i=1}^{n} f_i \sin[\chi_i(t) + u_i] V_i + \Delta \]

where \( t \) is surveying time; \( h \) is observed sea surface height; \( \bar{h} \) is the mean sea level; \( f_i \) and \( u_i \) are nodal factors; \( \chi_i \) is the phase of the constituent; \( m \) is the number of the constituents in the expression; \( H_i \) and \( G_i \) are harmonic constants; \( U_i \) and \( V_i \) are in-phase part and quadrature part of the constants; \( \Delta \) is the total of the survey errors, the non-tidal effects and the contribution of the minor constituents not included in Eq.(1). When the constituents in Eq.(1) can express the tidal component of the sea level, the non-tidal water level is the dominant component of the \( \Delta \).

\( n \) cycles of sea surface height will form observation equations. Then, tidal parameters can be estimated by the least squares method.

1.2 Differential mode

The above content is the basic principle of the along-track tide analysis at a given location. Considering the correlation of the sea surface heights and tidal parameters among the adjacent points along track, we take the difference of the sea surface heights of the adjacent points as observations, and the observation equations are constrained with the results of the crossover analysis. We name the algorithm differential mode.

If the sea surface height expressions of two adjacent points \( A \) and \( B \) are expressed as Eq.(1), their difference is:

\[ \delta h_{AB} (t_A, t_B) = \delta \bar{h}_{AB} + \sum_{i=1}^{n} f_i \cos[\chi_i(t_B) + u_i] U_{B_i} - f_i \cos[\chi_i(t_A) + u_i] U_{A_i} + \delta U_{AB} \]  

\[ = f_i \cos[\chi_i(t_B) + u_i] U_{B_i} - f_i \cos[\chi_i(t_A) + u_i] U_{A_i} + \delta U_{AB} \]  

(2)

The satellite moves along a track very fast, so the time interval between the two adjacent points is very small and the sea surface height variation arising from meteorological and oceanographic effects can be considered equal approximately. Thus, the magnitude of \( \delta U_{AB} \) is smaller than that of \( \Delta_A \) and \( \Delta_B \) by far. Taking 15°-30°N part of Pass 177 Cycle 11 as an example, we calculate \( \Delta_A \) and \( \Delta_B \) by the result of the along-track harmonic analysis from Eq.(1), and \( \delta U_{AB} \) from Eq.(2). Fig.1 is the comparison. The solid line is \( \delta U_{AB} \), and the dashed one is \( \Delta \).

Fig.1 shows that the magnitude of \( \delta U_{AB} \) is smaller than that of \( \Delta \), and varies around zero. The other data comes to the similar conclusion that the non-tidal effects of the adjacent points are equal approximately. In Eq.(2), \( \delta U_{AB} \) is the residual of the mode. If the magnitude of \( \delta U_{AB} \) reduces, the RMS of the residual and the variance of the parameters reduces. It means that the differential mode can improve the precision and reliability of the parameters.

Given that a track section consists of \( N \) normal points and that there are observed sea surface heights in every cycle, the tidal parameters vector of \( N \) normal points can be expressed as an integrative vector:

\[ Y = [X_1^T X_2^T \cdots X_N^T]^T \]  

(3)

The differences between the observed sea surface heights at adjacent points are taken as observations. In cycle \( j \), \( N - 1 \) observation equations are written as:

\[ \delta L_j = B Y + \delta \Delta \]  

(4)
where $\delta L_j = \begin{bmatrix} \delta h_{12} \\ \delta h_{23} \\ \vdots \\ \delta h_{N-1,N} \end{bmatrix}$, $\delta \Delta_j = \begin{bmatrix} \delta \Delta_{12} \\ \delta \Delta_{23} \\ \vdots \\ \delta \Delta_{N-1,N} \end{bmatrix}$

$$B_j = \begin{bmatrix} -b_1 & b_2 \\ -b_2 & b_3 \\ \vdots \\ -b_{N-1} & b_N \end{bmatrix}$$  \hspace{1cm} (5)

The row vector $b$ represents the projection relation between the sea surface heights with the tidal parameters for every normal point. The subscript is the serial number of the normal point. For cycle $j$, the weight matrix of $\delta L_j$ is:

$$P_j = \begin{bmatrix} 2 & -1 & & \\ -1 & 2 & -1 & \\ & & \ddots & \\ & & & 2 & -1 \\ & & & & 2 \end{bmatrix}$$  \hspace{1cm} (6)

The integrative weight matrix is:

$$P = \begin{bmatrix} P_1 \\ P_2 \\ \vdots \\ P_n \end{bmatrix}$$  \hspace{1cm} (7)

The integrative observation equations consist of equations as Eq.(4) for every cycle. Therefore, the tidal parameters of $N$ normal points can be estimated by the least squares method. This is the basic principle of the differential mode.

### 1.3 Constraints

In Eq.(4), the observation is the difference of the sea surface heights of the adjacent two normal points. The normal equations are rank-deficient. So the constraints (one at least) are needed.

We take the tidal parameters of the crossovers estimated by Eq.(1) as the constraints, and the section limited by two crossovers as a calculative section. Thus, two constraints are added for reliable estimation. Based on Eq.(4) and Eq.(7), the observation equations can be expressed as:

$$\delta L = BY + \delta \Delta, \hspace{0.5cm} P$$  \hspace{1cm} (8)

The constraint equations are:

$$X_1 = E_1Y = X_1^o$$  \hspace{1cm} (9)

$$X_N = E_NY = X_N^o$$

If just one crossover is usable (for example: the other one is on land), the corresponding constraint (just one) is added. Thus, the constraint equations can be given as:

$$EY = X^o$$  \hspace{1cm} (10)

Therefore, the normal equations are:

$$\begin{bmatrix} B^TPB & E^T \\ E & 0 \end{bmatrix} \begin{bmatrix} Y \\ K \end{bmatrix} = \begin{bmatrix} B^TP\delta L \\ X^o \end{bmatrix}$$  \hspace{1cm} (11)

where $k$ is correlate.

### 2 Data processing and results analysis

#### 2.1 Data processing

All discussions are based on that the ground tracks of the satellite pass the same points in different cycles. However, the sampled points of different cycle are actually not same. Therefore, we should generate normal points along a track at intervals ($0.1^\circ$ latitude). The sea surface height of each normal point is obtained by a polynomial which is set up by the points in $10^\circ$ around. The sea surface height of the crossovers is interpolated by the same polynomial approach. Then the tidal parameters in the crossovers are estimated by along-track harmonic tidal analyses (Eq.(1)). The results of crossover analyses are the constraints of the differential mode.

#### 2.2 Results analysis

The area on research is the China seas and the northwest Pacific ($0^\circ$-$38^\circ$N, $109^\circ$-$150^\circ$E). We use the MGDR-B from AVISO of Cycle 11 to Cycle 330. Harmonic constants of 12 constituents of 8 474 normal points are estimated by the differential mode.

The ocean tide propagates as a long wave, so there is strong correlation between two adjacent points along a track. It means that the variation of the tidal parameters along a track should be smooth. The differential mode takes the difference of the sea surface heights of two adjacent normal points as observation.
It takes the strong correlation between two adjacent points into account. Therefore, the variation of the tidal parameters obtained by the differential mode should be smoother than that obtained by the conventional along-track harmonic analysis. Fig.2 is the comparison between the results of the differential mode and the conventional along-track harmonic analysis. In Fig.2, the horizontal axis is the track of the satellite, and the vertical axis is orthogonal to the horizontal axis, and the scale is different for different constituents. From Fig.2, it is apparent that compared with the results of the conventional along-track harmonic analysis, the results of the differential mode is consistent in trend, but smoother.

The root mean squares of the fitting errors are about 11 cm\(^1\), since the conventional along-track harmonic analysis cannot remove non-tidal effects in the altimeter observations. The radial orbit errors have a long wavelength, and the environmental correction errors have middle wavelength. The latitude interval of two adjacent normal points is 0.1°, which means that the longest distance is about 11 km when they locate near the equator. Therefore, the radial orbit errors and the environmental correction errors can be considered approximately equal, and the difference of the observations can remove these errors. The satellite moves along the track very fast, so the time interval between two adjacent points is very small and the errors arising from meteorological and oceanographic effects can be removed too. Thus, the non-tidal errors are removed in the differential mode, and the residual errors are the errors of the altimeter, about 1.3 cm calculated by the T/P data.

3 Conclusions

The differential mode can efficiently remove non-tidal effects in the altimeter observations, and improve the precision of tide parameters. The differential mode takes the difference of the sea surface heights of two adjacent normal points as observations. It takes the strong correlation between two adjacent points into account. Therefore the variation of the tidal parameters obtained by the differential mode is smoother than that obtained by the conventional along-track harmonic analysis. There are many medium or large-scale non-tidal effects in the northwest Pacific, such as the north equatorial current, north equatorial countercurrent, subtropical countercurrent, Kuroshio and medium scale eddies. The differential mode has practical meaning for estimating high precise tidal parameters from altimetry data.

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