Location-Aware Transmission for Two-Cell Wireless Networks With Caching

BIN JIANG\textsuperscript{1}, (Member, IEEE), FU-CHUN ZHENG\textsuperscript{2}, (Senior Member, IEEE), TONG PENG\textsuperscript{3}, ALISTER G. BURR\textsuperscript{4}, (Senior Member, IEEE), AND MIKE FITCH\textsuperscript{5}, (Member, IEEE)

\textsuperscript{1}National Mobile Communications Research Laboratory, Southeast University, Nanjing 210096, China
\textsuperscript{2}School of Electronic and Information Engineering, Harbin Institute of Technology (Shenzhen), Shenzhen 518055, China
\textsuperscript{3}Department of Informatics Centre for Telecommunications Research, King’s College London, London WC2R 2LS, U.K.
\textsuperscript{4}Department of Electronic Engineering, University of York, York Y010 5DD, U.K.
\textsuperscript{5}5G Innovation Centre, University of Surrey, Guildford GU2 7XH, U.K.

Corresponding author: Bin Jiang (bjiang@seu.edu.cn)

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\section*{ABSTRACT} A novel downlink transmission selection scheme based on the locations of users for network coded cache-enabled, Multiple-Input Multiple-Output (MIMO) systems is proposed in this paper. Caches are available at both base stations (BSs) and mobile terminals (MTs) and network coding is employed in order to reduce power consumption. We compare the analysis of the proposed scheme with a simple direct transmission scheme in terms of achievable sum rate and outage probability. It is shown that the proposed scheme is preferred for high transmit signal-to-noise ratio (SNR) and/or high cross-link channel gain ratio scenarios, while the simple direct transmission scheme is better for low SNR and/or low cross-link channel gain ratio. Motivated by this analytical results, we present a selection algorithm for location-aware transmission scheme for two-cell networks. Numerical results confirm our theoretical analysis.

\section*{INDEX TERMS} Wireless caching, achievable sum rate, outage probability.

\section*{I. INTRODUCTION}
Caching at the edge of mobile networks (e.g., at base stations (BSs), access points (APs), or mobile terminals (MTs)) is a promising key technology for 5G and beyond wireless networks [1]. The benefits of caching popular content in the BSs or APs, such as backhaul load relief, low access latency and lower energy consumption, as well as creating more cooperative MIMO opportunities, have been studied in, e.g., [2]–[10]. The authors in [11] have proposed a novel coded caching scheme in which both local and global caching gains can be achieved by designing a particular pattern in cache placement at the receivers and exploiting coding in delivery. A cache-aided interference management (CAIM) scheme for general networks with caches at both transmitters and receivers was discussed in [12]. The authors have designed a scheme which loads the caches judiciously in order to maximize the opportunity not only for zero-forcing the ongoing interference from the transmitters but also for interference cancellation at the receivers due to the cached content at the receivers. In [13], the comparison between different caching strategies has been addressed.

Inspired by [12], we in this paper first design a network coded caching scheme ("Scheme I"). The popular contents are divided into different files and cached at BSs and MTs separately. When two MTs require the same or different contents, network coding is implemented between files at the same BS before transmission. The MTs will recover the desired content by using the original cached files and the network coded files received from different BSs. Our scheme differs from the work in [12] in that network coded files are transmitted in different time slots in order to reduce the energy consumption.

We then compare the proposed scheme with an alternative approach ("Scheme II") which to cache the popular contents at the BSs only (i.e., no need for MT caching). In this paper, we analyze and compare these two different schemes in terms of sum rate and outage probability and then propose a caching scheme selection algorithm based on the MT location information (or channel conditions).

The rest of the paper is organized as follows. In Section 2, an introduction to the system definition and the novel...
none of the MTs are located in the overlapped cell edge area. The MTs are located near the centers of different cells in Case (b), and in Case (c), both MTs are located near the center of the same cell. In order to obtain explicit expressions and clear boundaries between the different cases, we define three parameters $\omega$, $\kappa_0$ and $\kappa_1$, given by
\[
\omega = \left(\frac{r_{00}}{r_{11}}\right)^\alpha, \quad \kappa_0 = \left(\frac{r_{00}}{r_{01}}\right)^\alpha, \quad \kappa_1 = \left(\frac{r_{11}}{r_{10}}\right)^\alpha. \tag{2}
\]
The above three parameters are the ratios of the mean cross-link channel gains, which characterize the balances of the cross-links. Without loss of generality, we set cell 0 in Fig. 1 as the reference, and thus the values of $\omega$ and $\kappa_0$ are between 0 and 1. The expressions for different cases are given in Table 1. It is worth noting that Case (a) is a special case of Cases (b), (c) and (d) in which $\kappa = 1$, $\omega = 1$ and $\kappa = 0 = 1$, respectively. Under such a setup and assuming no channel state information (CSI) at the transmitter (i.e., BSs), two transmission schemes can be employed as described below.

### B. SCHEME I: CACHING AT BOTH BSs AND MTs

In Scheme I, as in [12], both the transmitters and receivers are equipped with caches. We define file $A$ and $B$ as two files stored in the library. Each BS and MT can cache at most a fraction $\mu_T$ and $\mu_R$ of the library content respectively, where $\mu_T, \mu_R \in [0, 1]$ is defined as the fractional cache size. For Scheme I, the BS and MT cache sizes are $\mu_T = 0.5$ and $\mu_R = 0.5$, respectively. In the prefetching phase, each file in the library is divided into 4 disjoint subfiles with the same size and then stored at the caches of the BSs and MTs. For example, file $A$ is divided into $A_{00}, A_{01}, A_{10}, A_{11}$, where $A_{ij}$ is stored at both the $j^{th}$ BS and the $i^{th}$ MT [12], $i, j \in \{0, 1\}$ (see Fig. 2(a)). File $B$ is partitioned and stored in the same way.

Without loss of generality, we assume that MT$_0$ and MT$_1$ requests file $A$ and $B$, respectively. Then the delivery phase is split into two time slots and in each delivery slot, one BS broadcasts a network coded signal while the other BS keeps silent. One example is shown in Fig. 2(b). In the first time slot, BS$_0$ transmits the content $A_{01} \oplus B_{00}$ and BS$_1$ keeps silent, where $\oplus$ stands the bit-wise XOR operation. In the second slot, BS$_0$ delivers the content $A_{11} \oplus B_{10}$ and BS$_1$ keeps silent. Other cases work in a similar fashion. The delivery scheme in [12] is illustrated in Fig. 2(c) where both BSs transmit contents at both time slots.

Let $x_k^{(i)}$ denote the transmitted signal from the $j^{th}$ BS at the $k^{th}$ time slot, then the received signal at the $i^{th}$ MT in the $k^{th}$
By substituting (2) into (5), we have

\[ \bar{\gamma}_i = \frac{\|g_i\|^2 E_s}{r_{ji}^{-\alpha} |g_j|^2 E_s + N_0} = \frac{\alpha^2 \gamma_0 |g_j|^2}{\mu_i |g_j|^2 + 1}, \quad j \neq i. \quad (8) \]

where \( \bar{\gamma}_i \) is the average value of \( \gamma_i \) given by

\[ \bar{\gamma}_i = \frac{\|g_i\|^2 E_s}{r_{ji}^{-\alpha} |g_j|^2 E_s + N_0}. \quad (5) \]

By substituting (2) into (5), we have

\[ \bar{\gamma}_i = \begin{cases} \psi(\gamma_0), & i = j, \\ \psi(\gamma_0) \gamma_i, & i \neq j, \end{cases} \quad (6) \]

where \( \gamma_0 \triangleq r_{00}^{-\alpha} E_s/N_0 \). Following [14], the normalized SNR is defined as \( \rho_{ji} = \gamma_{ji}/\bar{\gamma}_j = |g_j|^2 \). Since \( g_j \sim CN(0, 1) \), the probability density function (PDF) of \( \rho_{ji} \) is given by

\[ f_{\rho_{ji}}(x) = e^{-x}, \quad x \geq 0. \quad (7) \]

**C. SCHEME II: CACHING AT BSs**

In Scheme II, only BSs are equipped with caches. An MT is only served by its nearest BS, and each BS has cached all the files, (i.e., the BS and MT cache sizes are \( \mu_T = 1 \) and \( \mu_R = 0 \), respectively), which means that there is no cooperation between the BSs. With this assumption, the signal from the non-nearest BS will be treated as unknown interference [15]. We assume that the \( i^{th} \) MT is served only by the \( i^{th} \) BS. Then, the expression of the \( i^{th} \) MT’s received signal is the same as in (3) except that \( x_j \) and \( x_j' \) are both non-zero. The instantaneous received signal-to-interference-plus-noise ratio (SINR) for the \( i^{th} \) MT is given by

\[ \gamma_i = \frac{r_{ij}^{-\alpha} |g_i|^2 E_s}{r_{ji}^{-\alpha} |g_j|^2 E_s + N_0} = \frac{\alpha^2 \gamma_0 |g_j|^2}{\mu_i |g_j|^2 + 1}, \quad j \neq i. \quad (8) \]

where \( \mu_i \triangleq \omega^\gamma y_i \). Similarly, the normalized SINR is defined as \( \beta_i = \gamma_i/\bar{\gamma}_i \), where \( \bar{\gamma}_i \) is the mean value of \( \gamma_i \).

The PDF of \( \beta_i \) can be derived as

\[ q_i(x) = \frac{1 + \psi(\mu_i) x}{\psi(\mu_i) x} \exp\left( \frac{-\psi(\mu_i) x}{\mu_i} \right), \quad x > 0, \quad (9) \]

where

\[ \psi(x) \triangleq e^{1/x} E_1(1/x), \quad (10) \]

and \( E_1(x) \) is the exponential integral function [16].

**Proof:** Please refer to Appendix A.

It is worth noting that the cache and transmission strategies of Schemes I and II do not need channel state information at the transmitters (CSIT). The system performance can be improved by utilizing the instantaneous CSIT at very short time scales. However, in this work, the transceiver strategy is designed by using the location information of users at longer time scales [17], [18].

**III. PERFORMANCE ANALYSIS**

In this section, the achievable average sum rate and outage probability of Scheme I and II are given. The four cases illustrated in Fig. 1 have been considered in both schemes in order to address the effect of different locations of the MTs to the schemes.

**A. ACHIEVABLE SUM RATE ANALYSIS**

For Scheme I, the average achievable sum rate is defined as

\[ R_1 = \frac{1}{4} \sum_{i=0}^{1} \sum_{j=0}^{1} \int_0^\infty f_{ij}(x) \ln(1 + x\bar{\gamma}_j) dx, \quad (11) \]

where \( f_{ij}(x) \) is the PDF of \( \rho_{ij} \) and \( \bar{\gamma}_j \) is the mean value of \( \gamma_j \). Substituting (5) and (7) into (11) and using the equality \( \int_{-\infty}^{\infty} \exp(-x) \ln(1 + ax) dx = \psi(a) \) [16, 4.337.2] yields

\[ R_1 = \frac{\psi(\gamma_0) + \psi(\omega \gamma_0) + \psi(\kappa_0 \gamma_0) + \psi(\kappa_1 \omega \gamma_0)}{4}. \quad (12) \]

For Scheme II, the average achievable sum rate is defined as

\[ R_2 = \frac{1}{2} \sum_{i=0}^{1} \int_0^\infty q_i(x) \ln(1 + x\bar{\gamma}_i) dx, \quad (13) \]
where $q_i(x)$ is the PDF of $\beta_i$ and $\bar{y}_i$ is the mean value of $\gamma_i$. $R_2$ can be expressed as

$$R_2 = \frac{1}{2} \sum_i \left( 1 - \frac{\psi(\omega_i' \gamma_i) - \psi(\kappa_i \omega_i' \gamma_i)}{\omega_i' - \omega_i} \right), \quad \kappa_i \neq 1,$$

$$\frac{1}{2} \left( 1 - \frac{\psi(\omega_i' \gamma_i) - \psi(\omega_i' \gamma_i) - \psi(\kappa_i \omega_i' \gamma_i)}{\omega_i' - \omega_i} \right), \quad \kappa_i \neq 1, \quad i \neq j,$$

$$\frac{1}{2} \sum_i \left( 1 - \frac{\psi(\omega_i' \gamma_i)}{\omega_i' \gamma_i} \right), \quad \kappa_i = 1. \quad (14)$$

**Proof:** Please refer to Appendix B. \hfill $\blacksquare$

The function $\psi(x)$ is important to the average achievable sum rates of Scheme I and Scheme II derived in (12) and (14): it can be defined as the ergodic capacity of Rayleigh fading channels [19], [20]. $\psi(x)$ includes the non-elementary function $E_1(x)$, which makes it difficult to obtain much insight about the sum-rate. Estimations of $\psi(x)$ based on series expansions can be found in [19], [20], while polynomial and rational approximations of $\psi(x)$ can be found in [21]. For further discussions, we need to investigate the monotonicity of $\psi(x)$ and its related functions, and the main results are summarized in the following lemma.

**Lemma 1:** The function $\psi(x)$ is monotonically increasing in $x$. For the following $\psi(x)$ related functions

$$\theta_1(x) = \frac{\psi(x)}{x}, \quad x > 0, \quad (15)$$

$$\theta_2(x) = \psi(x) + \frac{\psi(x)}{x} - 1, \quad x > 0, \quad (16)$$

$$\theta_3(x) = \psi(ax) - \psi(bx), \quad x > 0, \quad a > 0, \quad b > 0, \quad (17)$$

$$\theta_4(x) = \frac{\psi(b) - \psi(bx)}{1 - x}, \quad x > 0, \quad x \neq 1, \quad b > 0, \quad (18)$$

$\theta_1(x)$ and $\theta_4(x)$ are monotonically decreasing in $x$, $\theta_2(x)$ is monotonically increasing in $x$, and $\theta_3(x)$ is monotonically increasing and decreasing in $x$ for $a > b$ and $a < b$, respectively.

**Proof:** Please refer to Appendix C. \hfill $\blacksquare$

With the general results of average achievable sum-rates for Scheme I and Scheme II as shown in (12) and (14), we give the sum-rate expressions for Cases (a) to (d) as follows.

**Case (a):** In this case, both $\omega$ and $\kappa_i$ are assumed to be constants. By substituting $\omega = 1$ and $\kappa_0 = \kappa_1 = 1$ into (12) and (14), we have

$$R_1^{(a)} = \psi(\gamma_0), \quad (19)$$

$$R_2^{(a)} = 1 - \psi(\gamma_0)/\gamma_0. \quad (20)$$

According to the monotonicity of $\psi(x)$ and $\theta_1(x)$ established in Lemma 1, both $R_1^{(a)}$ and $R_2^{(a)}$ are monotonically increasing with respect to $\gamma_0$.

**Case (b):** In this case, we assume that the MTs’ locations are symmetrical with respect to the cell edge. We also assume that $0 < \kappa < 1$. If $\kappa = 1$, Case (b) reduces to Case (a). By substituting $\omega = 1$ and $\kappa_0 = \kappa_1 = \kappa$ into (12) and (14), we have

$$R_1^{(b)} = \frac{\psi(\gamma_0) + \psi(\kappa \gamma_0)}{2}, \quad (21)$$

$$R_2^{(b)} = \frac{\psi(\gamma_0) - \psi(\kappa \gamma_0)}{1 - \kappa}. \quad (22)$$

According to the monotonicity of $\psi(x)$, $\theta_3(x)$ and $\theta_4(x)$ derived in Lemma 1, we conclude that $R_1^{(b)}$ is monotonically increasing in both $\gamma_0$ and $\kappa$, while $R_2^{(b)}$ is monotonically increasing in $\gamma_0$ and decreasing in $\kappa$.

**Case (c):** In this case, we assume $0 < \omega < 1$. If $\omega = 1$, it transforms to Case (a). Substituting $\kappa_0 = \omega$ and $\kappa_1 = \omega^{-1}$ into (12) and (14) yields

$$R_1^{(c)} = \frac{\psi(\gamma_0) + \psi(\omega \gamma_0)}{2}, \quad (23)$$

$$R_2^{(c)} = \frac{\psi(\gamma_0) - \psi(\omega \gamma_0)}{1 - \omega}. \quad (24)$$

Interestingly, the only difference between the sum-rate expression in Case (c) and that in Case (b) is the use of the parameters $\omega$ or $\kappa$.

**Case (d):** In this case, we assume $0 < \omega < 1$ and $0 < \kappa_0 < 1$. Case (a) is obtained if $\omega = \kappa_0 = 1$. Then by substituting $\kappa_1 = 1$ into (12) and (14) gives

$$R_1^{(d)} = \frac{\psi(\gamma_0) + 2\psi(\omega \gamma_0) + \psi(\kappa \gamma_0)}{4}, \quad (25)$$

$$R_2^{(d)} = 1 - \frac{\psi(\omega \gamma_0)}{2\omega \gamma_0} + \frac{\psi(\gamma_0) - \psi(\kappa_0 \gamma_0)}{2(1 - \kappa_0)}. \quad (26)$$

According to Lemma 1, $R_1^{(d)}$ is monotonically increasing in $\gamma_0$, $\kappa_0$ and $\omega$, while $R_2^{(d)}$ is monotonically increasing in both $\gamma_0$ and $\omega$ and decreasing in $\kappa_0$.

**B. OUTAGE PROBABILITY ANALYSIS**

The outage probability is another important performance indicator in order to evaluate communication over fading channels; it indicates the probability that the instantaneous SNR or SINR falls below a certain threshold $\gamma_{th}$ [14], [22]. The average outage probabilities of Schemes I and II are given by

$$P_1 = \frac{1}{4} \sum_{i=0}^1 \sum_{j=0}^1 \int_0^{\gamma_{th}} f_{\beta_i}(x)dx$$

$$= 1 - \frac{1}{4} \sum_{i=0}^1 \left( \exp \left( - \frac{\gamma_{th}}{\omega_i' \gamma_i} \right) + \exp \left( - \frac{\gamma_{th}}{\kappa_i \omega_i' \gamma_i} \right) \right). \quad (27)$$

$$P_2 = \frac{1}{2} \sum_{i=0}^1 \int_0^{\gamma_{th}} q_i(x)dx$$

$$= 1 - \frac{1}{2} \sum_{i=0}^1 \frac{1}{1 + \kappa_i \gamma_{th}} \exp \left( - \frac{\gamma_{th}}{\omega_i' \gamma_i} \right). \quad (28)$$

In general, $P_1$ is monotonically decreasing in $\gamma_0$, $\kappa_i$ and $\omega$, while $P_2$ is monotonically decreasing in $\gamma_0$ and $\omega$ and increasing in $\kappa_i$. With Table 1, we can obtain the expressions of the outage probabilities for Cases (a)-(d). We omit here for brevity.
IV. LOCATION-AWARE DELIVERY SCHEME SELECTION ALGORITHM

Based on the analysis in the previous section, clearly, a key question is: which caching/transmission scheme should be used under which conditions? As such, this section will discuss the conditions under which Scheme I or II should be selected.

The caching/transmission scheme selection criteria are designed based on the comparison of sum rates and outage probabilities of Schemes I and II. It is worth pointing out that the conditions are determined by the user locations (rt, or equivalently ω and κ, i, j ∈ {0, 1}) and the transmit SNR.

Proposition 1: According to the locations of MTs described in Cases (a) to (d), the conditions that Scheme I achieves a higher sum rate than Scheme II are given by

- R(1) < R(2) for all γ0 > 0.
- R(1) ≥ R(2) if and only if the condition that
  \( (3 - \kappa)\psi(\kappa \gamma_0) - (1 + \kappa)\psi(\gamma_0) \geq 0 \) \tag{29}

is satisfied.
- R(1) ≥ R(2) if and only if the condition (29) (κ replaced by ω) is satisfied.
- R(1) ≥ R(2) if and only if the condition that
  \( (3 - \kappa_0)\psi(\kappa_0 \gamma_0) - (1 + \kappa_0)\psi(\gamma_0) \geq 0 \)
  \( + 2\theta_2(0) \gamma_0 \kappa \) \geq 0 \tag{30}

is satisfied, and R(1) ≥ R(2) if the condition (29) (κ replaced by κ0) is satisfied.

Proof: For Case (a), the difference between R(1) and R(2) can be expressed as \( \theta_2(0) \gamma_0 \kappa \). Since \( \lim_{\gamma_0 \to 0} \psi(\gamma_0) = 0 \), \( \lim_{\gamma_0 \to 0} \psi(\kappa_0 \gamma_0) = 1 \), and \( \theta_2(0) \) is monotonically increasing in \( \gamma_0 \), we have \( \theta_2(0) > \theta_2(0) = 0 \), i.e., R(1) > R(2) for all \( \gamma_0 > 0 \). For Cases (b)-(d), the necessary and sufficient conditions are obtained by directly simplifying the corresponding inequality R1 ≥ R2. The sufficient condition for Case (d) is obtained by using the fact that the term \( \theta_2(0) \gamma_0 \) in (30) is always greater than zero for all \( \gamma_0 \) and \( \omega \).

Special cases for Case (b): (i) If the value of \( \gamma_0 \) is large, then the condition (29) will always hold for all \( \kappa > 0 \), which means that Scheme I achieves higher sum rate than Scheme II when the SNR is high. (ii) If \( \kappa \) approaches 1, then the condition (29) will always hold, which means that Scheme I achieves higher sum rate than Scheme II when the cross-link channel gain is strong enough. (iii) If \( \kappa \) approaches 0, then the condition (29) will not hold, which means that Scheme II is preferred when the cross-link channel gain is weak.

The scheme selection criteria according to outage probability for Cases (a) to (d) are contained in the following proposition.

Proposition 2: Considering the locations of MTs in Section II-A, the conditions that Scheme I achieves lower outage probabilities than Scheme II in Cases (a) to (d) are given by

- \( P_1^{(a)} < P_2^{(a)} \) for all \( \gamma_0 > 0 \).
- \( P_1^{(b)} \leq P_2^{(b)} \) if and only if the condition that
  \[ \gamma_0 \geq \frac{(1 - \kappa)\gamma_0}{\kappa(1 + \kappa)\gamma_0 - (1 - \kappa)\gamma_0)} \tag{31} \]

is satisfied.
- \( P_1^{(c)} < P_2^{(c)} \) for all \( \gamma_0 > 0 \) and \( 0 < \omega < 1 \).
- \( P_1^{(d)} \leq P_2^{(d)} \) if and only if the following condition
  \[ 2\gamma_0 e^{-\frac{\gamma_0}{\kappa}} \geq \frac{1 - \kappa_0 \gamma_0 e^{-\frac{\gamma_0}{\kappa}}}{1 + \gamma_0} \] \tag{32}

is satisfied.

Proof: For Cases (a) and (c), by respectively substituting \( \omega = \kappa_0 = \kappa = 1 \) and \( \omega = \kappa_0 = \kappa^{-1} \) into (27) and (28), we have

\[ P_1^{(a)} - P_2^{(a)} = -\frac{\gamma_0 e^{-\gamma_0 \kappa}}{1 + \gamma_0}, \tag{33} \]

\[ P_1^{(c)} - P_2^{(c)} = -\frac{1}{2} \left( \frac{\gamma_0 e^{-\gamma_0 \kappa} + \gamma_0 e^{-\gamma_0 \kappa}}{1 + \gamma_0 \kappa} \right). \tag{34} \]

Thus, \( P_1^{(a)} < P_2^{(a)} \) for all \( \gamma_0 > 0 \) and \( P_1^{(c)} < P_2^{(c)} \) for all \( \gamma_0 > 0 \) and \( \omega > 0 \). For Cases (b) and (d), the necessary and sufficient conditions are obtained by simplifying the corresponding inequality \( P_1 \leq P_2 \).

Proposition 2 indicates that the outage probability of Scheme I is always lower than that of Scheme II for grouped users (Cases (a) and (c)). If the locations of users are located in a cell, we do not discuss some extreme cases for the condition as shown in (31). (i) The condition (31) will always hold if \( \gamma_0 \) approaches infinity, while it will not hold if \( \gamma_0 \) approaches zero. This property means that Scheme I and Scheme II should be selected to achieve the lower outage probability in high and low SNR regions, respectively. (ii) The condition (31) will always hold if \( \kappa \) approaches 1, while it will not hold if \( \kappa \) approaches 0. Hence, Scheme I should be selected for strong cross-links, while Scheme II should be selected for weak cross-links. (iii) For Case (d), if \( \omega \) approaches 0, then the condition in (32) reduces to the condition in (31). In addition, (32) will always hold if \( \omega \) approaches 1 and \( \gamma_0 \rightarrow \infty \).

V. NUMERICAL RESULTS

A. ACHIEVABLE SUM RATE

Fig. 3 shows the achievable sum rates for Case (b).\(^1\) It is worth pointing out that Case (a) is a special case of Case (b) with \( \kappa = 1 \). Fig. 3(a) indicates the average sum rate as a function of \( \gamma_0 \) with different \( \kappa \), while Fig. 3(b) shows the average sum rate as a function of \( \kappa \) with different \( \gamma_0 \). The curves with and without markers depict the Monte-Carlo and analytical results, respectively. Clearly, our theoretical analysis matches the numerical results very well. According to Fig. 3(a), there is a switching point of \( \gamma_0 \) for a given value of \( \kappa \). For example, if \( \kappa = -12 \) dB, then the average sum rate of Scheme I will

\(^1\)If we replace the parameter \( \kappa \) in Fig. 3 with \( \omega \), then this figure becomes the achievable sum rates for Case (c).
outperform that of Scheme II when $\gamma_0 \geq 18$ dB. Similarly, Fig. 3(b) illustrates a switching point of $\kappa$ for a given value of $\gamma_0$. For example, if $\gamma_0 = 15$ dB, then the average sum rate of Scheme I will outperform that of Scheme II when $\kappa \geq -10$ dB.

Fig. 4 displays the achievable sum rates for Case (d) in terms of $\gamma_0$, $\kappa_0$, and $\omega$. For each of these three figures, one of these variables changes over the whole range, while the remaining two variables take two typical and fixed values. From Fig. 4(a), we have the following observations: (i) Scheme I is always better than Scheme II when $\kappa_0 = 0$ dB. (ii) If $\kappa_0 = -10$ dB, then Scheme I will be better than Scheme II when $\gamma_0 > 1$ dB and $\gamma_0 > 11$ dB for $\omega = 0$ dB and $\omega = -10$ dB, respectively. Fig. 4(b) shows that: (i) Scheme I is always better than Scheme II when $\gamma_0 = 30$ dB. (ii) If $\gamma_0 = 15$ dB, then Scheme I will be always better than Scheme II for $\omega = 0$ dB, and is conditionally better than Scheme II for $\omega = -10$ dB when $\kappa_0 > -14$ dB.
Fig. 5 shows the outage probabilities of Case (b). The SNR threshold $\gamma_0$ is set to 0 dB. We observe that:

(i) Schemes I and II will be always better for all $\gamma_0 > 0$ when $\kappa$ is big enough (e.g., 0 dB) and small enough (e.g., $\gamma_0 = -24$ dB), respectively. However, for moderate values of $\kappa$ (e.g., $\gamma_0 = -12$ dB), there is a switching point ($\gamma_0 = 21$ dB) between Schemes I and II. (ii) If $\gamma_0 = 15$ dB, then Scheme I is better than Scheme II only when $\kappa > -9$ dB. (iii) The switching point of $\kappa$ will decrease when the SNR $\gamma_0$ increases.

Fig. 6 shows the outage probabilities of Case (d) with respect to $\gamma_0$, $\kappa_0$ and $\omega$. We observe from Fig. 6(a) that: (i) Scheme I is always better than Scheme II when the SNR $\gamma_0$ is high enough, or $\kappa_0 = 0$ dB or $\omega = 0$ dB. (ii) In case of $\kappa_0 = \omega = -10$ dB, the switching point of Scheme I and Scheme II is $\gamma_0 = 10$ dB. Fig. 6(b): (i) Scheme I is always better than Scheme II when the $\kappa_0$ is big enough, or $\gamma_0 = 30$ dB or $\omega = 0$ dB. (ii) In case of $\gamma_0 = 15$ dB and $\omega = -10$ dB, the switching point between Schemes I and II is $\kappa_0 = -16$ dB. Fig. 6(c): (i) Scheme I is always better

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2For Cases (a) and (c), both the monotonicity and the optimality of Scheme I have been analytically discussed in Sec. III, so we omit their numerical results for brevity.
SNR and/or high cross channel gain ratio, the better strategy and the cross-link channel gain ratio depends on both the average transmitted SNR \(\gamma_0\). In Cases (b), (c) and (d), respectively. Note that the optimal variable \(\omega\) implies that Scheme II is better. In both figures, the Y-axis strategy; while the region where the inequality is reversed mark out the regions where inequality in Proposition 1 or \(\kappa\), \(\alpha\), \(\gamma_0\), and \(\kappa_0\), we obtain the results for \(\gamma_i\) is Scheme I, while for low SNR and/or low cross channel gain ratio, Scheme II is the better strategy.

C. SWITCHING REGION BETWEEN SCHEMES I AND II

FIGURE 7. Optimal regions of Schemes I and II.

than Scheme II when the \(\omega\) is big enough, or \(\gamma_0 = 30\) dB or \(\kappa_0 = 0\) dB. (ii) In case of \(\gamma_0 = 15\) dB and \(\omega = -10\) dB, the switching point between Schemes I and II is \(\omega = -18\) dB.

The switching regions between Schemes I and II based on achievable sum rate and outage probability are plotted in Fig. 7(a) and Fig. 7(b), respectively. The figures clearly mark out the regions where inequality in Proposition 1 or Proposition 2 is satisfied, meaning that Scheme I is the better strategy; while the region where the inequality is reversed implies that Scheme II is better. In both figures, the Y-axis variable \(x\) stands for the ratio of cross-link channel gain. By replacing \(x\) with \(\kappa\), \(\omega\) and \(\kappa_0\), we obtain the results for Cases (b), (c) and (d), respectively. Note that the optimal strategy depends on both the average transmitted SNR \(\gamma_0\) and the cross-link channel gain ratio \(x\). In general, for high SNR and/or high cross channel gain ratio, the better strategy.

VI. CONCLUSION

We have proposed a novel network coded caching scheme for two-cell downlink networks. In order to investigate its performance, the achievable sum-rate and outage probability have been derived in closed-form. A comparison between the proposed scheme and a simple direct transmission scheme has also been conducted and a location-based cache/ transmit scheme selection algorithm has been explored. The proposed selection scheme allows BSs to select cache and transmit scheme for different scenarios based on different locations of MTs in order to achieve higher sum rate or lower outage probability.

APPENDIX A

PDF OF THE NORMALIZED SINR \(\beta_j\)

We first derive the PDF of the instantaneous SINR for the \(i^{th}\) MT. From (8), we see that \(\gamma_i\) is a ratio of two independent random variables \(X_i\) and \(Y_i\), both of which are the linear transform of chi-squared variables with 2 degrees of freedom, and their distributions are respectively given by

\[
U_{X_i}(x) = \frac{1}{r_{ii}^{\alpha} \gamma_0} \exp\left(-\frac{x}{r_{ii}^{\alpha} \gamma_0}\right), \quad x > 0, \quad (35)
\]

\[
U_{Y_i}(y) = \frac{1}{r_{ij}^{\alpha} \gamma_0} \exp\left(-\frac{y - 1}{r_{ij}^{\alpha} \gamma_0}\right), \quad y > 1. \quad (36)
\]

Then, the ratio distribution can be obtained from integration of the following form [23]

\[
U_{\gamma_i}(z) = \int_{-\infty}^{\infty} |y|U_{X_i}(yz)U_{Y_i}(y)dy. \quad (37)
\]

By substituting (35) and (36) into (37), we have

\[
U_{\gamma_i}(z) = \frac{1 + \alpha^i \kappa_i \gamma_0 + \kappa_i z}{\alpha^i \gamma_0(1 + \kappa_i z)^2} \exp\left(-\frac{z}{\alpha^i \gamma_0}\right). \quad (38)
\]

The mean of the \(\gamma_i\) is

\[
\bar{\gamma}_i = \int_{0}^{\infty} zU_{\gamma_i}(z)dz = \frac{\psi(\alpha^i \gamma_0 \gamma_0)}{\kappa_i}. \quad (39)
\]

Thus, the PDF of normalized SINR \(\beta_i\) can be expressed as

\[
q_i(x) = \bar{\gamma}_i U_{\gamma_i}(\bar{\gamma}_i x). \quad (40)
\]

Substituting (38) and (39) into (40) yields (9). This completes the proof.

APPENDIX B

DERIVATION OF (14)

From (9), the integration of \(q_i(x)\) can be written in the following closed-form

\[
\int q_i(x)dx = -\frac{1}{1 + \psi(\mu_i)x} \exp\left(-\frac{\psi(\mu_i)}{\mu_i} x\right). \quad (41)
\]
Let $y = \frac{\psi(x)}{\mu_i}$, then the $i^{th}$ summation term of (13) can be calculated as

$$R_{2,i} = \int_0^\infty \ln \left(1 + \omega y \theta_i\right) d \left(\frac{\exp(-y)}{1 + \mu_i y}\right),$$

$$= \int_0^\infty \omega y \exp(-y) \left(1 + \kappa_i \omega y \theta_i\right) dy.$$ (43)

The integration of (43) can be calculated by using the identities [16, 3.252.4] and [16, 3.353.3] for the conditions that $\kappa_i \neq 1$ and $\kappa_i = 1$, respectively, which yields (14). This completes the proof.

**APPENDIX C PROOF OF LEMMA 1**

According to the definition of $\psi(x)$ as in (10), we have

$$\psi(1/x) = e^x E_1(x).$$ (44)

The right-hand side (RHS) of (44) is a well studied function in the literature [21, 24, 25]. By simple transformation of the inequalities of [21, 5.1.19, 5.1.20], we have the following upper and lower bounds of $\psi(x)$:

$$\frac{x}{1 + x} < \psi(x) < x,$$ (45)

$$\frac{1}{2} \ln(1 + 2x) < \psi(x) < \ln(1 + x).$$ (46)

With (46), we can easily obtain the limits of $\psi(x)$ and its related functions, as $x$ approaches 0 or $\infty$. However, we omit the derived results here for brevity.

Now we begin to prove the monotonicity and convexity of $\psi(x)$, which is useful for proving the monotonicity of functions related to $\psi(x)$. The first and second order derivatives of $\psi(x)$ are respectively given by

$$\dot{\psi}(x) = \frac{x - \psi(x)}{x^2},$$ (47)

$$\ddot{\psi}(x) = \frac{(1 + 2x)\psi(x) - (x + x^2)}{x^4}. $$ (48)

Using the upper bound of $\psi(x)$ in (45), we see that $\dot{\psi}(x) > 0$ for $x > 0$. Thus, $\psi(x)$ is monotonically increasing in $x$.

In order to determine the sign of $\dot{\psi}(x)$, we first define an auxiliary function

$$\phi(x) = e^{-1/x} \left(\psi(x) - \frac{x + x^2}{1 + 2x}\right).$$ (49)

The first order derivative of $\phi(x)$ can be expressed as

$$\dot{\phi}(x) = \frac{2x^2 e^{-1/x}}{(1 + 2x)^2} < 0.$$ (50)

This shows that $\phi(x)$ is monotonically decreased with respect to $x$, so $\dot{\psi}(x) < \dot{\psi}(0) = 0$, or equivalently

$$\dot{\psi}(x) < \frac{x + x^2}{1 + 2x}. $$ (51)

Substituting (51) into (48) yields $\ddot{\psi}(x) < 0$, which means that $\psi(x)$ is a concave function with respect to $x$. It is worth pointing out that (51) is a new upper bound of $\psi(x)$, which is tighter than that of (45) for all $x > 0$ and is tighter than that of (46) only for small positive $x$.

We can then prove the monotonicities of $\dot{\psi}(x)$ related functions. The first order derivatives of $\dot{\theta}_1(x)$, $\dot{\theta}_2(x)$ and $\dot{\theta}_3(x)$ are respectively given by

$$\dot{\theta}_1(x) = x - (1 + x)\psi(x) < 0,$$ (52)

$$\dot{\theta}_2(x) = \frac{(x + x^2)(1 + 2x)}{x^3} \psi(x) > 0,$$ (53)

$$\dot{\theta}_3(x) = \frac{\dot{\theta}_1(bx) - \dot{\theta}_2(ax)}{x}, $$ (54)

where the signs of (52) and (53) are determined by using (45) and (51), respectively. Thus, $\dot{\theta}_1(x)$ and $\dot{\theta}_2(x)$ are monotonically decreasing and increasing in $x$, respectively. Since $\dot{\theta}_3(x)$ is monotonically decreasing in $x$, the sign of (54) is the same as that of $a - b$. Hence, $\dot{\theta}_3(x)$ is monotonically increasing and decreasing in $x$ for $a > b$ and $a < b$, respectively.

The function $\dot{\theta}_4(x)$ defined in (18) is proportional to the slope of the line segment between fixed point $b$ and changeable point $(bx, \psi(bx))$. From the property of concave function, $\dot{\theta}_4(x)$ is monotonically decreasing in $x$ if and only if $\psi(x)$ is concave. This completes the proof of Lemma 1.

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Research Fellow with British Telecom, U.K. His current research interests include signal processing for communications, multiple antenna systems, green communications, and ultra-dense networks. He has been an active IEEE member since 1995. From 2001 to 2004, he was an Editor of the IEEE Transactions on Wireless Communications. In 2006, he has served as the General Chair of the IEEE VTC 2006-S. Melbourne, VIC, Australia—the first ever VTC held in the southern hemisphere in VTC’s history of six decades. More recently, he was the Executive TPC Chair for VTC 2016-S, Nanjing, China (the first ever VTC held in mainland China).

TONG PENG received the B.Eng. degree in electronic engineering from Liaocheng University, Shandong, China, in 2009, and the M.Sc. degree in communications engineering and Ph.D. degree in electronic engineering from the University of York, U.K., in 2010 and 2014, respectively. From 2014 to 2015, he was with the Centre for Studies in Telecommunications, Pontifical Catholic University, Rio de Janeiro, as a Research Associate. From 2015 to 2017, he was with the Communications Group, Department of Electronic Engineering, University of York, working on the network coded modulation for next generation wireless access networks project. He joined the Centre for Telecommunications Research, Department of Informatics, King’s College London, in 2017, and was a Research Associate on the Internet of Silicon Retina Project. His research interests include space-time coding, network coding, resource allocation, and multimedia communications in the 5G IoT.

ALISTER G. BURR (Senior Member, IEEE) was born in London, U.K., in 1957. He received the B.Sc. degree in electronic engineering from the University of Southampton, U.K., in 1979, and the Ph.D. degree from the University of Bristol, U.K., in 1984. From 1975 to 1985, he worked with Thorn-EMI Central Research Laboratories, London. In 1985, he joined the Department of Electronics (now Electronic Engineering) with the University of York, U.K., where he has been a Professor of communications since 2000. His research interests are in wireless communication systems, especially MIMO, cooperative systems, physical layer network coding, and iterative detection and decoding techniques. He has published around 250 papers in refereed international conferences and journals, and is the author of Modulation and Coding for Wireless Communications (Prentice-Hall/PHEI), and coauthor of Wireless Physical-Layer Network Coding (Cambridge University Press, 2018). In 1999, he was awarded a Senior Research Fellowship by the U.K. Royal Society, and in 2002, he received the J. Langham Thompson Premium from the Institution of Electrical Engineers. He has also given more than 15 invited presentations, including three keynote presentations. He was the Chair, working group 2, of a series of European COST programmes, including IC1004 Cooperative Radio Communications for Green Smart Environments, and has also served as an Associate Editor for the IEEE Communications Letters, Workshops Chair for the IEEE ICC 2016, and the TPC Co-Chair for PIMRC 2018 and 2020.

FIG. 1

**FU-CHUN ZHENG** (Senior Member, IEEE) received the B.Eng. and M.Eng. degrees in radio engineering from the Harbin Institute of Technology, China, in 1985 and 1988, respectively, and the Ph.D. degree in electrical engineering from the University of Edinburgh, U.K., in 1992. From 1992 to 1995, he was a Postdoctoral Research Associate with the University of York, U.K. His research interests include signal processing and wireless communications for multiple-input–multiple-output systems. He received the Science and Technology Progress awards of the State Education Ministry of China and Jiangsu province in 2009 and 2014, respectively.

**BIN JIANG** (Member, IEEE) received the B.S. and Ph.D. degrees in electrical engineering from Southeast University, Nanjing, China, in 2002 and 2011, respectively. Since 2011, he has been with the National Mobile Communications Research Laboratory, Southeast University. From January to December 2017, he was a Postdoctoral Research Associate with the University of York, U.K. His research interests include signal processing and wireless communications for multiple-input–multiple-output systems. He received the Science and Technology Progress awards of the State Education Ministry of China and Jiangsu province in 2009 and 2014, respectively.

**MIKE FITCH** (Member, IEEE) received the bachelor’s degree in maths and physics and the Ph.D. degree in satellite communications. His previous experience is with modeling, trials and deployments of Satellite, WiMAX, 5G, and LTE systems. He is currently with BT Research and Innovation, providing technical leadership to a small research team specializing in physical and systems aspects of wireless communications. He is also involved in a number of projects on emerging wireless technologies, such as small cells, radio resource management, and 5G. In addition, he provides engineering consultancy to other parts of BT on LTE, WiFi, and other wireless topics. He is a member of IET.