Supersymmetric Janus solutions in four-dimensional $N = 3$ gauged supergravity

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Abstract

We construct supersymmetric Janus solutions using four-dimensional $N = 3$ gauged supergravity with $SO(3) \times SU(3)$ gauge group. The $N = 3$ supersymmetric $AdS_4$ vacuum with unbroken $SO(3) \times SU(3)$, identified with the compactification of eleven-dimensional supergravity on $AdS_4 \times N^{010}$, provides a gravity dual of supersymmetric $N = 3$ Chern-Simons-Matter theory in three dimensions with $SU(3)$ flavor symmetry. The Janus solutions accordingly describe supersymmetric conformal interfaces within this Chern-Simons-Matter theory via the AdS/CFT holography. We find two classes of Janus solutions preserving respectively $(2, 1)$ and $(0, 1)$ supersymmetry on the $(1 + 1)$-dimensional conformal defects. The solution with $(2, 1)$ supersymmetry preserves $SO(2) \times SO(2) \times SO(2) \subset SO(3) \times SU(3)$ symmetry while the $(0, 1)$ supersymmetric solution is invariant under a larger $SO(2) \times SU(2) \times SO(2)$ symmetry.
1 Introduction

Conformal field theories (CFTs) with a conformal defect are important in describing some properties of condensed matter and statistical physics systems \[1,2\]. The AdS/CFT correspondence \[3\] offers holographic duals to these theories via Janus solutions \[4\]. According to the usual relations in the AdS/CFT correspondence, these solutions would be useful in studying strongly coupled CFT with conformal interfaces. Along this line, holographic duals of conformal defects within \( N = 4 \) supersymmetric Yang-Mills (SYM) theory, one of the primary examples of the AdS/CFT duality, have been extensively studied in a number of previous works, see for example \[5,6,7,8\].

In general, Janus solutions can be obtained from AdS-sliced domain walls. A special case of flat domain walls with the AdS-slice replaced by the flat Minkowski space describes the usual holographic RG flows. Supersymmetric Janus solutions in five dimensions, dual to \( N = 4 \) SYM with conformal interfaces, have been obtained both from five-dimensional gauged supergravity and directly from ten-dimensional type IIB theory \[9,10,11,12\]. Janus solutions dual to defect conformal field theories (dCFTs) or interface conformal field theories (ICFTs) in two dimensions have also been studied in \[13,14,15,16,17\] including the multi-face Janus solution recently constructed in \[18\].

In four dimensions, a class of supersymmetric Janus solutions from eleven-dimensional M-theory have been classified in \[19\]. A number of supersymmetric Janus solutions in the maximal \( N = 8 \) gauged supergravity with various symmetries have been studied in \[20\]. The solution with \( SO(4) \times SO(4) \) symmetry can be uplifted to eleven dimensions and has been shown to be different from the solutions classified in \[19\]. All solutions considered in \[20\] have been obtained by truncating the \( E_7/\text{SU}(8) \) scalar manifold of the \( N = 8 \) supergravity to a single complex scalar living in \( SL(2, \mathbb{R})/SO(2) \) coset.

In this paper, we will give another example of supersymmetric Janus solutions within \( N = 3 \) gauged supergravity coupled to eight vector multiplets. This results in \( N = 3 \) gauged supergravity with \( SO(3) \times SU(3) \) gauge group which is expected to arise from a dimensional reduction of eleven-dimensional supergravity on a tri-sasakian manifold \( N^{010} \) \[21,22,23\]. Possible three-dimensional \( N = 3 \) SCFT dual to the supersymmetric \( AdS_4 \) critical point with \( SO(3) \times SU(3) \) symmetry has been given in \[24,25\]. Other \( AdS_4 \) critical points and holographic RG flows between \( AdS_4 \) critical points including flows to \( AdS_2 \) geometries have been extensively studied in \[26\].

We will consider supersymmetric Janus solutions in this \( N = 3 \) gauged supergravity with \( SO(2) \times SU(2) \times SO(2) \) and \( SO(2) \times SO(2) \times SO(2) \) symmetries. As we will see, the solutions preserve respectively \( N = (1,0) \) and \( N = (2,1) \) supersymmetries on the \((1+1)\)-dimensional interfaces. According to the AdS/CFT correspondence, these solutions should be dual to some conformal defects, breaking \( N = 3 \) supersymmetry and \( SO(3) \times SU(3) \) global symmetry, in the Chern-
Simons-Matter theory dual to the \(AdS_4 \times N^{010}\) background.

The paper is organized as follow. In section 2, we review the matter-coupled \(N = 3\) gauged supergravity in order to set up the notations and collect all the needed formulae. The analysis of BPS equations relevant to finding supersymmetric Janus solutions will also be given. Supersymmetric Janus solutions with \(N = (1, 0)\) supersymmetry and \(SO(2) \times SU(2) \times SO(2)\) symmetry is constructed in section 3 while the \(N = (2, 1)\) solution with \(SO(2) \times SO(2) \times SO(2)\) symmetry will be given in section 4. Conclusions and comments on the existence of supersymmetric Janus solution with \(SO(3) \times U(1)\) symmetry and the possibility of other solutions within non-compact gauge groups are given in section 5. In the appendix, we give a brief comments on the non-existence of supersymmetric Janus solution with \(SO(3) \times U(1)\) symmetry and the possibility of other solutions within non-compact gauge groups.

2 \(N = 3\) gauged supergravity

Before giving the solutions, we review the \(N = 3\) gauged supergravity in four dimensions and collect all relevant formulae which will be used in later sections. The reader is referred to [27, 28, 29] for the full construction. Apart from the mostly plus metric signature \((- + + +)\), all the notations are the same as in [27].

In \(N = 3\) supersymmetry, the supergravity multiplet consists of the graviton \(e^\mu\), three gravitini \(\psi_{\mu A}\), three vectors \(A_{\mu A}\) and one spinor field \(\chi\). Indices \(\mu, \nu, \ldots = 0, \ldots, 3\) and \(a, b, \ldots = 0, \ldots, 3\) are respectively space-time and tangent space indices. The \(SU(3)_R\) R-symmetry triplets are labeled by indices \(A, B, \ldots = 1, 2, 3\). Spinor indices will not be shown explicitly.

The only matter fields in \(N = 3\) supersymmetry are vector multiplets containing one vector field \(A_\mu\), four spinors \((\lambda_A, \lambda)\) which are a triplet and a singlet of \(SU(3)_R\), and three complex scalars \(z_A\). Each vector multiplet is labeled by indices \(i, j, \ldots = 1, \ldots, n\). Spinor fields are subject to the following chirality projections

\[
\begin{align*}
\psi_{\mu A} &= \gamma_5 \psi_{\mu A}, \quad \chi = \gamma_5 \chi, \quad \lambda_A = \gamma_5 \lambda_A, \quad \lambda = -\gamma_5 \lambda, \\
\psi_\mu^A &= -\gamma_5 \psi_\mu^A, \quad \lambda^A = -\gamma_5 \lambda^A.
\end{align*}
\] (1)

The \(N = 3\) supergravity coupled to \(n\) vector multiplets consists of \(3n\) complex, \(6n\) real, scalar fields \(z_A^i\) parametrized by the coset space \(SU(3, n)/SU(3) \times SU(n) \times U(1)\). The scalars can accordingly be parametrized by the coset representative \(L(z)^\Lambda_{\Sigma}\) which transforms under the global \(G = SU(3, n)\) and the local \(H = SU(3) \times SU(n) \times U(1)\) symmetries by left and right multiplications, respectively.

Indices \(\Lambda, \Sigma, \ldots = (A, i) = 1, \ldots, n + 3\) denote fundamental representation of \(SU(3, n)\). The coset representative can also be decomposed into \(L^\Lambda_{\Sigma} = (L^A_{\Lambda}, L^i_{\Lambda})\). The inverse of \(L^\Lambda_{\Sigma}\) is given in term of \(L^\Lambda_{\Sigma}\) via the relation

\[
(L^{-1})^\Lambda_{\Pi} = J_{\Lambda\Pi}^{\Sigma\Delta}(L^\Lambda_{\Delta})^*.
\] (2)
where $J_{\Lambda \Sigma}$ is an $SU(3, n)$ invariant tensor defined by

$$J_{\Lambda \Sigma} = J^{\Lambda \Sigma} = (\delta_{A B}, -\delta_{i j}).$$  
(3)

In the presence of $n$ vector multiplets, $(n + 3)$-dimensional subgroups of $SO(3, n) \subset SU(3, n)$ can be gauged provided that its structure constants defined by the gauge algebra

$$[T_\Lambda, T_\Sigma] = f_{\Lambda \Sigma \Gamma} T_\Gamma,$$  
(4)

satisfy the consistency condition

$$f_{\Lambda \Sigma \Gamma} = f_{\Lambda \Sigma \Gamma}^\prime J_{\Gamma \Gamma}^\prime = f_{[\Lambda \Sigma \Gamma]}.$$  
(5)

A number of compact and non-compact gauge groups of this “electric” type have been studied in [30]. In the present work, we mainly focus on the case of $n = 8$ vector multiplets and compact $SO(3) \times SU(3)$ gauge group with the corresponding structure constants given by

$$f_{\Sigma \Lambda \Gamma} = (g_1 \epsilon_{A B C}, g_2 f_{i+3,j+3,k+3}), \quad i, j = 1, \ldots, 8.$$  
(6)

In the above equation, $f_{ijk}$ are the usual $SU(3)$ structure constants. In what follows, we are interested only in supersymmetric Janus solutions with only the metric and scalars non-vanishing. All the other fields will accordingly be omitted from the following discussion.

The bosonic Lagrangian of the $N = 3$ gauged supergravity is given by

$$e^{-1} \mathcal{L} = \frac{1}{4} R - \frac{1}{2} P^A_{\mu} P^A_{\mu} - V.$$  
(7)

The vielbein $P^A_i$ of the $SU(3, n)/SU(3) \times SU(n) \times U(1)$ coset are given by the $(A, i)$-components of the Mourer-Cartan one-form

$$\Omega^\Pi_A = (L^{-1})^\Sigma_A d L_{\Sigma}^\Pi$$  
(8)

with $\Omega^A_i = (\Omega^A_i)^\ast$. The scalar potential is given in terms of the “boosted structure constants”

$$C^\Lambda_{\Pi \Gamma} = L^\Lambda_A (L^{-1})^\Pi_B (L^{-1})^\Gamma_C f_{B C}^A A^A$$  
and  $$C^\Lambda_{\Pi \Gamma} = J_{\Lambda \Lambda^\prime} J_{\Pi \Pi^\prime} J_{\Gamma \Gamma^\prime} (C^A_{\Pi \Gamma})^\ast$$
(9)

by the following relation

$$V = -2 S_{A C S} S_{C M} + \frac{2}{3} U_d U^A + \frac{1}{6} N_{i A} N^i A + \frac{1}{6} M^{i B}_{A} M^i_{A B}$$

$$= \frac{1}{8} |C_{i A}^B|^2 + \frac{1}{8} |C_{i}^P Q|^2 - \frac{1}{4} \left( |C_{A}^P Q|^2 - |C_{P}|^2 \right)$$  
(10)
where $C_P = -C_{PM}^M$. All tensors appearing in the above equations are defined by

$$S_{AB} = \frac{1}{4} \left( \epsilon_{BPQ} C_A^{PQ} + \epsilon_{ABC} C_M^{MC} \right),$$

$$\mathcal{U}^A = -\frac{1}{4} C_M^{MA}, \quad \mathcal{N}_{iA} = -\frac{1}{2} \epsilon_{APQ} C_i^{PQ},$$

$$\mathcal{M}_{iA}^B = \frac{1}{2}(\delta_i^B C_{iM}^M - 2C_{iA}^B).$$

(11)

The supersymmetry transformations of fermionic fields are given by

$$\delta \psi^\mu_A = D^\mu \epsilon_A + S_{AB} \gamma^\mu \epsilon_B,$$

$$\delta \chi = \mathcal{U}^A \epsilon_A,$$

$$\delta \lambda_i = -\mathcal{N}_{iA} \epsilon_A,$$

$$\delta \lambda^i_A = -\mathcal{M}_{iA}^B \epsilon_B.$$ (12-15)

The covariant derivative on the supersymmetry parameter $\epsilon_A$ is defined by

$$D\epsilon_A = d\epsilon_A + \frac{1}{4} \omega_{ab} \gamma_{ab} \epsilon_A + Q_A^B \epsilon_B + \frac{1}{2} n Q \epsilon_A$$

(16)

where the $SU(3) \times SU(8) \times U(1)$ composite connections $(Q_A^B, Q_i^j, Q)$ can be obtained from the $(A, B)$ and $(i, j)$ components of the Mourer-Cartan one-form via

$$\Omega_A^B = Q_A^B - n \delta_A^B Q, \quad \Omega_i^j = Q_i^j + 3 \delta_i^j Q.$$ (17)

Note that $(Q_A^B, Q_i^j)$ satisfy $Q_A^A = Q_i^i = 0$.

We can now construct the BPS equations for finding supersymmetric Janus solutions. The metric ansatz takes the form of $AdS_3$-sliced domain wall

$$ds^2 = e^{2A(r)} \left( e^{2\xi} dx_1^2 + dx_2^2 \right) + dr^2.$$ (18)

In the limit $\ell \to \infty$, this metric becomes a flat domain wall used in the study of holographic RG flows. The non-vanishing spin connections of the above metric can be computed to be

$$\omega^\xi = A' e^\xi, \quad \omega^{\xi_\ell} = \frac{1}{\ell} e^{-A} e^\mu, \quad \omega^{\mu_\ell} = A' e^\mu$$

(19)

where $'$ denotes the $r$-derivative. From now on, indices $\mu, \nu$ will take values 0, 1, and hatted indices are the tangent space, or flat, ones. In the above expressions, the vielbein components are given by

$$e^\mu = e^{A+\xi} dx^\mu, \quad e^{\xi} = e^A d\xi, \quad e^r = dr.$$ (20)
All of the scalar fields only depend on $r$. Therefore, only the $r$-component of $P^{iA}$ will be non-vanishing. The variations $\delta \lambda_i$ and $\delta \lambda_{iA}$ then require a $\gamma^r$ projection. Following [20], this projection takes the form of

$$\gamma^r \epsilon_A = e^{i\Lambda} \epsilon_A$$

(21)

where $\Lambda$ is a real phase. Using the Majorana representation with all gamma matrices real and $\gamma_5 = i\gamma_0\gamma_1\gamma_2\gamma_3$ purely imaginary, we have the relation $\epsilon^A = (\epsilon_A)^*$. This implies

$$\gamma^r \epsilon_A = e^{-i\Lambda} \epsilon_A.$$  

(22)

For the gravitino variations, we will denote the eigenvalues of $S_{AB}$ matrix corresponding to the unbroken supersymmetry by $-\frac{1}{2}W$. $W$ will play the role of the “superpotential”. With this and the above spin connections, the variation $\delta \psi_{A\tilde{A}} = 0$ gives

$$A'\gamma^r \epsilon_A + \frac{1}{\ell} e^{-A} \gamma_\xi \epsilon_A - W \epsilon_A = 0.$$  

(23)

As in [20], taking the complex conjugate and iterating the above equation lead to

$$A'^2 = W^2 - \frac{1}{\ell^2} e^{-2A}$$

(24)

where the “real superpotential” is defined by $W = |W|$. We now take the $\gamma_\xi$ projection to be

$$\gamma_\xi \epsilon_A = i\kappa e^{i\Lambda} \epsilon_A$$

(25)

with $\kappa^2 = 1$.

The equation coming from $\delta \psi_{A\tilde{A}} = 0$ gives

$$e^{-A} \partial_\xi \epsilon_A + \frac{1}{2} A' \gamma_\xi \epsilon_A - \frac{1}{2} W \gamma_\xi \epsilon_A = 0.$$  

(26)

Using equation (23), we find

$$\partial_\xi \epsilon_A = \frac{1}{2\ell} \epsilon_A$$

(27)

which implies $\epsilon_A = e^{\frac{\xi}{2\ell}} \tilde{\epsilon}_A$ for $\xi$-independent $\tilde{\epsilon}_A$. Following [20], we will denote the Killing spinor by

$$\epsilon_A = e^{\frac{\xi}{2\ell} + \frac{i\omega}{2} + \frac{i\Lambda}{2}} \epsilon_A^{(0)}$$

(28)

where the constant spinors $\epsilon_A^{(0)}$ satisfy

$$\gamma^r \epsilon_A^{(0)} = \epsilon^{(0)A} \quad \text{and} \quad \gamma_\xi \epsilon_A^{(0)} = i\kappa \epsilon^{(0)A}.$$  

(29)

In order to determine $e^{i\Lambda}$, we come back to equation (23) and take the real and imaginary parts

$$A' = \frac{1}{2} W (e^{i\omega + i\Lambda} + e^{-i\omega + i\Lambda}),$$

$$\frac{\kappa}{\ell} e^{-A} = \frac{i}{2} W (e^{i\omega - i\Lambda} - e^{-i\omega + i\Lambda})$$

(30)  

(31)
where we have written $\mathcal{W} = W e^{i\omega}$. There are two possibilities namely real and complex $\mathcal{W}$. For real $\mathcal{W}$, $\mathcal{W} = W$ or $\omega = 0$, we find

$$e^{i\Lambda} = \frac{A'}{W} + \frac{i\kappa}{\ell} e^{-A}.$$  

(32)

For complex $\mathcal{W}$, we simply have

$$e^{i\Lambda} = \frac{\mathcal{W}}{A' + \frac{\kappa}{\ell} e^{-A}}.$$  

(33)

Both cases occur in the solutions considered in subsequent sections.

3 Janus solution with $SO(2) \times SU(2) \times SO(2)$ symmetry

We begin with the solution with $SO(2) \times SU(2) \times SO(2)$ symmetry. There are two singlet scalars invariant under this symmetry. They correspond to $SU(3,8)$ non-compact generators

$$\hat{Y}_1 = e_{3,11} + e_{11,3} \quad \text{and} \quad \hat{Y}_2 = -ie_{3,11} + ie_{11,3}$$  

(34)

where we have used the matrices $(e_{\Lambda \Sigma})_{\Gamma \Delta} = \delta_{\Lambda \Gamma} \delta_{\Sigma \Delta}$. These are non-compact generators of $SU(1,1) \subset SU(3,8)$ with the compact $U(1)$ subgroup generated by

$$J = 2i(e_{33} - e_{11,11}).$$  

(35)

We can parametrize this $SU(1,1)/U(1)$ coset in the form of

$$L = e^{\varphi J} e^{\varphi \hat{Y}_1}.$$  

(36)

The scalar potential can be computed to be

$$V = -\frac{1}{2} g_1^2 [1 + 2 \cosh(2\phi)]$$  

(37)

which admits only a critical point at $\phi = 0$ as already studied in [26].

We now consider the BPS equations. With the above coset representative and the $SO(3) \times SU(3)$ structure constants given previously, we find the $S_{AB}$ matrix

$$S_{AB} = -\frac{1}{2} \text{diag}(\mathcal{W}_1, \mathcal{W}_1, \mathcal{W}_2)$$  

(38)

where

$$\mathcal{W}_1 = g_1 \cosh(2\phi) \cosh(\phi), \quad \mathcal{W}_2 = g_1 e^{2i\phi} \cosh(\phi).$$  

(39)
As pointed out in the RG flows studied of [26], \( W_2 \) gives rise to supersymmetric solutions. Using \( W = |W_2| \), we can write the scalar potential as

\[
V = -\frac{1}{8} \sinh^2(2\phi) \frac{\partial W}{\partial \phi} \frac{\partial W}{\partial \phi} - \frac{1}{2} \frac{\partial W}{\partial \phi} \frac{\partial W}{\partial \phi} - \frac{3}{2} W^2. \tag{40}
\]

In order to solve all equations from the gravitini variations, we set \( \epsilon_{1,2} = 0 \), so the unbroken supersymmetry is associated with \( \epsilon_3 \). The preserved supersymmetry for the full solution will be \((1,0)\) or \((0,1)\) depending on the values of \( \kappa = 1, -1 \).

By computing \( P_{iA}' \) and using the projectors as described in the previous section, we obtain two different complex equations from \( \delta \lambda_i = 0 \) and \( \delta \lambda_{iA} = 0 \). It turns out that the latter only involves \( \epsilon_{1,2} \) and hence identically vanishes. Solving for \( \phi' \) and \( \varphi' \), the BPS equations from \( \delta \lambda_i = 0 \), involving only \( \epsilon_3 \), read

\[
\begin{align*}
\phi' &= \kappa \frac{e^{-A}}{4 \ell} \operatorname{sech}^2 \phi, \tag{41} \\
\varphi' &= -\tanh \phi A'. \tag{42}
\end{align*}
\]

Together with the equation

\[
A'^2 - g_1^2 \cosh^2 \phi + \frac{e^{-2A}}{\ell^2} = 0, \tag{43}
\]
we can solve for the supersymmetric solution. It can be verified that these equations also solve the second-order field equations. In the RG flow limit \( \ell \to \infty \), we find

\[
\begin{align*}
\phi' &= 0, \quad \varphi' = \mp g_1 \sinh \phi, \quad A' = \pm g_1 \cosh \phi \tag{44}
\end{align*}
\]

which are the flow equations studied in [26].

It is remarkable that equations (41), (42) and (43) turn out to be the same as those considered in [20]. The solution can be obtained similarly. By solving equation (42), we find

\[
A = C_1 - \ln \sinh \phi \tag{45}
\]

where \( C_1 \) is an integration constant.

Inserting the solution for \( A \) into equation (43), we find an equation for \( \phi \)

\[
\phi'^2 = g_1^2 \sinh^2 \phi - \frac{e^{-2C_1}}{\ell^2} \frac{\sinh^4 \phi}{\cosh^2 \phi}. \tag{46}
\]

Follow [20], we define a parameter

\[
a = g_1 \ell e^{C_1}. \tag{47}
\]

Accordingly, the solution for \( \phi \) can be found to be

\[
\sinh \phi = \zeta \frac{a}{\sqrt{1 - a^2} \cosh[g_1(r - r_0)]} \tag{48}
\]
for $a < 1$, and
\[
\sinh \phi = \zeta \frac{a}{\sqrt{a^2 - 1}} \frac{1}{\sinh [g_1(r - r_0)]}
\]  
for $a > 1$. In these solutions, the parameter $\zeta = \pm 1$ can be chosen to be $+1$ if we choose $\phi > 0$. Furthermore, the integration constant $r_0$ can be set to zero.

Using the solutions for $A$ and $\phi$ in equation (41), we obtain the solution for $\varphi$
\[
\tan(\varphi - \varphi_0) = -\kappa \zeta \sqrt{1 - a^2} \sinh [g_1(r - r_0)], \quad \text{for } a < 1,
\]
\[
\tan(\varphi - \varphi_0) = -\kappa \zeta \sqrt{a^2 - 1} \cosh [g_1(r - r_0)], \quad \text{for } a > 1.
\]

The metric warp factor can also be expressed as a function of $r$ as follow
\[
e^A = \zeta \frac{\sqrt{1 - a^2}}{g_1 \ell} \cosh [g_1(r - r_0)], \quad \text{for } a < 1,
\]
\[
e^A = \zeta \frac{\sqrt{a^2 - 1}}{g_1 \ell} \sinh [g_1(r - r_0)], \quad \text{for } a > 1.
\]

For further holographic study, it is useful to give an asymptotic expansion of the solution near the $AdS_4$ critical point.

Similar to the discussion in [20], the solution with $a < 1$ is smooth for $-\infty < r < \infty$ and approaches the $SO(3) \times SU(3)$ $AdS_4$ critical point for $r \to \pm \infty$. Using the new radial coordinate
\[
r = \pm \frac{1}{g_1} \ln \left[ \sqrt{1 - a^2} \frac{2a}{\rho} \right],
\]
we find that the limit $r \to \pm \infty$ corresponds to $\rho \to 0$. In this limit, the solution behaves as
\[
\phi \sim \rho + \frac{1}{12a^2}(a^2 - 3)\rho^3 + \ldots,
\]
\[
\varphi \sim \varphi_0 \mp \kappa \frac{\pi}{a} \pm \kappa \frac{1 + 3a^2}{12a^3} \rho^3 + \ldots,
\]
\[
A \sim -\ln \rho + \ln \frac{a}{g_1 \ell} + \frac{1 - a^2}{4a^2} \rho^2 + \ldots
\]

where we have set $\zeta = 1$ and $r_0 = 0$. The leading terms simply give
\[
A \sim \pm g_1 r
\]
and
\[
\phi \sim e^{\mp \frac{\pi}{a}}, \quad \varphi \sim e^{\mp \frac{\pi}{a}}
\]

where the $AdS_4$ radius is given by $L = \frac{1}{g_1}$. This indicates that $\phi$ and $\varphi$ are dual to relevant operators of dimensions $\Delta = 1, 2$ in the dual $N = 3$ SCFT arising
from the $AdS_4 \times N^{010}$ compactification.

As pointed out in [31], the values of $\Delta = 1, 2$ can lead to two different quantizations. Holographically, the two quantizations imply different identifications of the operator deformations and vacuum expectation values (vevs). In $N = 8$ ABJM theory, it has been shown in [32] that the correct holographic dictionary requires the “standard quantization” for scalars and the “alternative quantization” for pseudoscalars. In the standard quantization, non-normalizable modes are identified with deformations while normalizable modes describe vevs. The identification is reversed in the alternative quantization. Consequently, the operators dual to scalars and pseudoscalars are given respectively by bosonic and fermionic bilinears of dimensions one and two. It would be interesting to determine whether there exists such a unique dictionary in the case of $N = 3$ SCFTs considered here. This would make the holographic interpretation of supergravity solutions more transparent.

For $a > 1$, we still choose $\zeta = 1$ and $r_0 = 0$ but define the new radial coordinate by

$$r = \pm \frac{1}{g_1} \ln \left[ \frac{\sqrt{a^2 - 1}}{2a} \rho \right].$$

The behavior of $\phi$ and $\varphi$ near $r \to \pm \infty$ or $\rho \sim 0$ can be determined as in the previous case. The result is the same as in (55). Therefore, the solution approaches the $AdS_4$ similar to the $a < 1$ case.

However, the scalar $\phi$ and the metric function $A$ diverge at a finite value of $r = 0$. There are two possibilities for $r > 0$ and $r < 0$. For $r > 0$, we choose $\zeta = 1$, and for $r < 0$ we choose $\zeta = -1$ in order to make $e^A$ positive. We then find the expansion near $|r| \approx 0$

$$\phi \sim \mp \ln \left[ \frac{\sqrt{a^2 - 1}g_1|r|}{2a} \right] + \ldots,$$

$$\varphi \sim \mp \kappa \tan^{-1} \sqrt{a^2 - 1} + \ldots,$$

$$e^A \sim \sqrt{a^2 - 1} \frac{\ell}{|r|} + \ldots.$$ (59)

It can be readily seen that $\phi$ and the metric become singular at $r = 0$.

From the scalar potential (37), we see that $V(\phi \to \pm \infty) \to -\infty$. At least, by the criterion of [33], the singularity is acceptable. It would be interesting to investigate this singularity in eleven-dimensional context. The four-dimensional metric near this singularity is given by

$$ds^2 = \frac{a^2 - 1}{\ell^2} r^2 ds^2(AdS_3) + dr^2.$$ (60)

The Janus solution for $a > 1$ should accordingly correspond to an interface between $N = 3$ SCFT and a non-conformal field theory or a Coulomb phase. As pointed out in [34], this solution might also be useful in describing boundary
conformal field theories (BCFTs). In [34], it has been argued that the strength of the deformation determines whether the solution corresponds to an ICFT or a BCFT. In the present solution, the deformation is determined by the parameter $a$. The fact that the value of $a$ larger than a critical value gives the solution dual to a BCFT is in agreement with the discussion in [34].

4 Janus solution with $SO(2) \times SO(2) \times SO(2)$ symmetry

We now come to a more complicated solution with smaller residual symmetry. There are four scalars invariant under $SO(2) \times SO(2) \times SO(2)$ symmetry. They are given by the following $SU(3,8)$ non-compact generators

$$
\begin{align*}
\tilde{Y}_1 &= e_{3,11} + e_{11,3}, \\
\tilde{Y}_2 &= -ie_{3,11} + i e_{11,3}, \\
\tilde{Y}_3 &= e_{36} + e_{63}, \\
\tilde{Y}_4 &= -ie_{36} + i e_{63}.
\end{align*}
$$

These generators are also non-compact generators of $SU(2,1) \subset SU(3,8)$. The corresponding scalars are then coordinates of a submanifold $SU(2,1)/U(2)$. The resulting $SU(2,1)/SU(2) \times U(1)$ coset can be parametrized as in [26]

$$
L = e^{\phi_1 J_1} e^{\phi_2 J_2} e^{\phi_3 J_3} e^{\Phi \tilde{Y}_1}
$$

where the generators $J_i$ form the $SU(2)$ subgroup. Explicitly, they are given by

$$
\begin{align*}
J_1 &= -ie_{66} + ie_{11,11}, \\
J_2 &= e_{6,11} - e_{11,6}, \\
J_3 &= -ie_{6,11} - ie_{11,6}.
\end{align*}
$$

The scalar potential takes a simple form

$$
V = -\frac{1}{2} g_1^2 [1 + 2 \cosh(2\Phi)].
$$

In this case, although the potential turns out to be the same as in the previous case, the $S_{AB}$ matrix is given by

$$
S_{AB} = -\frac{1}{2} g_1 \cosh(\Phi) \delta_{AB}.
$$

We see that $S_{AB}$ has a three-fold degenerate real eigenvalue giving rise to a real superpotential.

Under the $SO(2)_R \subset SO(3)_R$ identified with the first $SO(2)$ in $SO(2) \times SO(2) \times SO(2)$, the supersymmetry transformation parameters $\epsilon_A$ transform as $2 + 1$. The singlet corresponds to $\epsilon_3$. Similar to the previous cases, $\delta \lambda_i = 0$ equations only involve $\epsilon_3$ while $\delta \lambda_{iA} = 0$ only have non-vanishing components along $\epsilon_{1,2}$.

As pointed out in [20], different representations of $\epsilon_A$ under the residual
symmetry can be assigned different \( e^{i\Lambda} \) phases. In the following, we will choose the \( \gamma_r \) projections to be

\[
\gamma_r \epsilon_{1,2} = e^{i\Lambda} \epsilon_{1,2} \quad \text{and} \quad \gamma_r \epsilon_3 = e^{-i\Lambda} \epsilon^3.
\]

For \( \gamma_\xi \) projection, we choose

\[
\gamma_\xi \epsilon_{1,2} = i\kappa e^{i\Lambda} \epsilon_{1,2} \quad \text{and} \quad \gamma_\xi \epsilon_3 = -i\kappa e^{-i\Lambda} \epsilon^3.
\]

This implies the opposite chirality of \( \epsilon_{1,2} \) and \( \epsilon_3 \) on the \((1 + 1)\)-dimensional interface. With these projectors and the expression for \( e^{i\Lambda} \) given in (32), \( \delta \lambda_i = 0 \) and \( \delta \lambda_{iA} = 0 \) variations reduce to the same set of equations

\[
4e^{2\Phi} \left[ 2g_1 \sinh \Phi + ie^{i\Lambda} \sin(2\varphi_3) \sin(2\Phi) \varphi_2 + 2e^{i\Lambda} \varphi_3' \right] \\
+ 2i(e^{4\Phi} - 1) e^{i\Lambda} \cos(2\varphi_2) \cos(2\varphi_3) \varphi_1' = 0,
\]

\[
8e^{2\Phi} \left[ \sinh \Phi \left( g_1 + ie^{i\Lambda} \cosh \Phi \sin(2\varphi_3) \varphi_2' + e^{i\Lambda} \Phi' \right) \right] \\
+ 2i(e^{4\Phi} - 1) e^{i\Lambda} \cos(2\varphi_2) \cos(2\varphi_3) \varphi_1' = 0.
\]

Note that choosing different projectors as shown above means that equations from \( \delta \psi_{A\mu} \) variations involving \( \epsilon_{1,2} \) and \( \epsilon_3 \) are complex conjugate of each other. This is possible by the fact that \( \mathcal{W} \) is real. Since in this case \( e^{i\Lambda} \) and \( e^{-i\Lambda} \) differ effectively by a sign change in \( \kappa \) as can be seen from equation [32], the solution to these equations then preserves \( \mathcal{N} = (2, 1) \) supersymmetry on the interface.

Solving all of these equations results in the following BPS equations

\[
\varphi_1' = \frac{2\kappa \cos \varphi_3 \sec \varphi_2 e^{-A}}{\ell \cosh^2 \Phi}, \quad \varphi_2' = \frac{2\kappa \sin \varphi_3 e^{-A}}{\ell \cosh^2 \Phi}, \\
\varphi_3' = \frac{-2\kappa \cos \varphi_3 \tan \varphi_2 e^{-A}}{\ell \cosh^2 \Phi}, \quad \Phi' = -A' \tanh \Phi, \\
0 = A'^2 + \frac{e^{-2A}}{\ell^2} - g_1^2 \cosh^2 \Phi.
\]

These equations can readily be verified to satisfy the corresponding field equations. It should also be noted that in the limit \( \ell \to \infty \), we recover the BPS equations for holographic RG flows studied in [26]

\[
\varphi_1' = \varphi_2' = \varphi_3' = 0, \quad \Phi' = \mp g_1 \sinh \Phi, \quad A' = \pm g_1 \cosh \Phi.
\]

From the above equations, \( \Phi' \) and \( A' \) equations form a close set since they do not couple to all of the \( \varphi_i \). These two equations can be solved by the same solutions as in the previous case. We will not give their explicit form here to avoid repetitions.

With \( \Phi \) and \( A \) solutions as given in the previous section, we can solve for
\( \varphi_i, i = 1, 2, 3, \) as follow

\[
\cos \varphi_3 = \zeta C_2 \sec \varphi_2, \\
\tan(\varphi_1 - \tilde{\varphi}_0) = \frac{2C_2 \sin \varphi_2}{\sqrt{2 - 4C_2^2 + 2 \cos(2\varphi_2)}}, \\
\sinh[g_1(r - r_0)] = -\frac{\kappa}{\sqrt{1 - a^2}} \tan \left[ \frac{1}{2} \tan^{-1} \frac{\tan(\varphi_1 - \tilde{\varphi}_0)}{C_2} \right], a < 1, \\
\cosh[g_1(r - r_0)] = -\frac{\kappa}{\sqrt{a^2 - 1}} \tan \left[ \frac{1}{2} \tan^{-1} \frac{\tan(\varphi_1 - \tilde{\varphi}_0)}{C_2} \right], a > 1.
\]

(72) (73) (74) (75)

Note that for \( \varphi_2 = \varphi_3 = 0, \) we recover the solutions of previous section provided that the identifications \( C_2 = \zeta \) and \( \varphi_1 = 2\varphi \) are made. Therefore, turning on \( \varphi_2 \) and \( \varphi_3 \) further breaks the \( SO(2) \times SU(2) \times SO(2) \) symmetry to \( SO(2) \times SO(2) \times SO(2), \) but enhanced supersymmetry from \( N = (1, 0) \) to \( N = (2, 1). \)

We now briefly look at asymptotic behaviors of the solution. In this case, the expansion is more complicated. Therefore, we will only give the asymptotic behavior for \( r \to \infty. \) The expansion for \( r \to -\infty \) can be obtained similarly. By defining the coordinate \( \rho \) as in (53) and setting \( \zeta = 1 \) and \( r_0 = 0, \) we find

\[
\varphi_1 \sim \tilde{\varphi}_0 - \tan^{-1} \left[ C_2 \tan \frac{\kappa \pi}{12a^3} \right] + \frac{4\kappa C_2}{a \left[ 1 + C_2^2 + (1 - C_2^2) \cos \frac{\kappa \pi}{6a^3} \right]} \rho + \frac{8C_2(C_2^2 - 1) \sin \frac{\kappa \pi}{6a^3}}{a^2 \left[ 1 + C_2^2 + (1 - C_2^2) \cos \frac{\kappa \pi}{6a^3} \right]} \rho^2 + \ldots, \\
\varphi_2 \sim \alpha_0 + \alpha_1 \rho + \alpha_2 \rho^2 + \ldots, \\
\varphi_3 \sim \beta_0 + \beta_1 \rho + \beta_2 \rho^2 + \ldots
\]

(76)

with the expansions for \( \Phi \) and \( A \) given by the \( \phi \) and \( A \) expansions in (55). \( \alpha_i \) and \( \beta_i \) are constants depending on \( C_2 \) and \( a. \) Explicitly, \( \alpha_i \) are given by

\[
\alpha_0 = -\frac{\sqrt{1 - C_2^2}}{12C_2^2} \left[ 5C_2^2 - 2 + (2 + 7C_2^2) \cos \frac{\kappa \pi}{6a^3} \right] \sec^2 \frac{\kappa \pi}{12a^3} \tan \frac{\kappa \pi}{12a^3}, \\
\alpha_1 = \kappa \frac{\sqrt{1 - C_2^2}}{2aC_2^3} \sec^4 \frac{\kappa \pi}{12a^3} \left[ C_2^2 - 2 + (2 + C_2^2) \cos \frac{\kappa \pi}{6a^3} \right] \sec \frac{\kappa \pi}{12a^3} \tan \frac{\kappa \pi}{12a^3}, \\
\alpha_2 = \frac{\sqrt{1 - C_2^2}}{a^2C_2^2} \left[ 6 + C_2^2 - (2 + 3C_2^2) \cos \frac{\kappa \pi}{6a^3} \right] \sec^4 \frac{\kappa \pi}{12a^3} \tan \frac{\kappa \pi}{12a^3}.
\]

(77)

The explicit form of \( \beta_i \) is much more complicated, so we refrain from giving them here. It should be noted that the above expansion reduces to the \( SO(2) \times SU(2) \times SO(2) \) solution for \( C_2 = \zeta = \pm 1 \) with \( \varphi_1 \sim 2\varphi \) and \( \varphi_2 \sim \varphi_3 \sim 0. \)

The solution for \( a > 1 \) is singular as in the previous case. The asymptotic
expansion near $r \sim 0$ can be obtained analogously. Near the singularity $r \sim 0$, the metric and $\Phi$ are singular while all of the $\varphi_i$ remain finite. In this case, the singularity also satisfies the criterion of [33]. We then expect the solution to be dual to a BCFT.

5 Conclusions

We have found supersymmetric Janus solutions within $N = 3$ gauged supergravity for $SO(3) \times SU(3)$ gauge group. The solutions have $SO(2) \times SU(2) \times SO(2)$ and $SO(2) \times SO(2) \times SO(2)$ symmetries with unbroken $N = (1,0)$ and $(2,1)$ supersymmetry on the $(1+1)$-dimensional interface. The solutions provide a holographic dual of conformal interfaces in the $N = 3$ Chern-Simons-Matter theory with $SU(3)$ flavor symmetry in three dimensions and might be interesting in applications to condensed matter physics systems along the line of [35]. Similar to the maximal $N = 8$ theory, Janus solutions require non-vanishing pseudoscalars as opposed to the RG flow solutions. In the case of $SO(3) \times U(1)$ symmetry in which the BPS equations require constant pseudoscalars, there does not exist supersymmetric Janus solutions. This will be shown in the appendix.

It would be very interesting to identify precisely the interface SCFTs dual to the gravity solutions given here. Since the solutions given here are all analytic, they could be useful in a holographic study of the correlation functions in the dual $N = 3$ dCFT by the method introduced in [36]. It would be interesting to further study the solutions that become singular in the IR, at a finite value of the $AdS_4$ radial coordinate and give a precise interpretation in the dual field theory. According to [34], these solutions should be interpreted as gravity dual of boundary conformal field theories (BCFTs). The analytic solutions should also be of particular interest in this context as well as in computations of the entanglement entropy.

Another interesting direction is obviously to look for possible uplift of these solutions to eleven dimensions and identify the corresponding M-brane configurations. The full embedding of $N = 3$ gauged supergravity considered in this paper to eleven dimensions is currently not known. This is due to the lack of a complete reduction ansatz on $N^{010}$ although an embedding keeping only $SU(3)$ singlets has been constructed in [37]. Since the solutions found in the present paper involve scalars that transform non-trivially under $SU(3)$, a more general truncation is needed.

On the other hand, the full embedding might be possibly first in $N = 4$ gauged supergravity. As has been pointed out in some previous works, see for example [25, 37, 38], the $N = 3$ $AdS_4$ vacuum can be realized as a supersymmetry breaking $AdS_4$ vacuum of $N = 4$ gauged supergravity. It would be desirable to explicitly construct this truncation and study the eleven-dimensional uplift of the Janus solutions found here and the holographic RG flows in [26] similar to the
solutions of the maximal gauged supergravity recently studied in [39].

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A Supersymmetric solutions with $SO(3) \times U(1)$ symmetry

In this appendix, we consider supersymmetric solutions with $SO(3) \times U(1)$ symmetry and look for possible Janus solutions. It turns out that, in this case, there is no supersymmetric Janus solution. We now present some details of the analysis.

It has been found in [26] that apart from the $SO(3) \times SU(3)$ symmetric $AdS_4$ critical point, there is another $N=3$ critical point with $SO(3) \times U(1)$ symmetry. The unbroken $SO(3)$ is a diagonal subgroup of $SO(3) \times SO(3)$ with the second factor being $SO(3) \sim SU(2) \subset SU(3) \times U(1) \subset SU(3)$. The $U(1)$ is an explicit $U(1)$ factor in $SU(2) \times U(1)$. The uplift of this critical point to eleven dimensions is presently unknown.

Under $SO(3)_{\text{diag}} \times U(1)$, there are two singlet scalars corresponding to the non-compact generators

\begin{align*}
\hat{Y}_1 &= e_{14} + e_{41} + e_{25} + e_{52} + e_{36} + e_{63}, \\
\hat{Y}_2 &= -ie_{14} + ie_{41} - ie_{25} + ie_{52} - ie_{36} + ie_{63}.
\end{align*}

(78)

The coset representative can be parametrized by

\begin{equation}
L = e^{\varphi J} e^{\Phi \hat{Y}_1} e^{-\varphi J}.
\end{equation}

(79)

where

\begin{equation}
J = \text{diag}(2i\delta^{i+3,j+3} , 0, 0, 0, 0, 0), \quad i, j = 1, 2, 3.
\end{equation}

(80)

The $J$ generator, corresponding to $U(1) \sim SO(2)$, together with $\hat{Y}_1$ and $\hat{Y}_2$ form a non-compact group $SU(1,1) \subset SU(3,8)$. The $SO(3) \times U(1)$ critical point is given by

\begin{align*}
\varphi &= 0, \\
\Phi &= \frac{1}{2} \ln \left[ \frac{g_2 - g_1}{g_2 + g_1} \right], \\
V_0 &= -\frac{3g_1^2 g_2^2}{2(g_2^2 - g_1^2)}.
\end{align*}

(81)

We now consider the BPS equations for supersymmetric solutions. The scalar matrix $S_{AB}$ takes the form of

\begin{equation}
S_{AB} = -\frac{1}{2} W 0_{AB}.
\end{equation}

(82)
where
\[ W = \frac{1}{8} e^{-3\Phi} \left[ (1 + e^{2\Phi})^3 g_1 + (e^{2\Phi} - 1)^3 g_2 \right] \cos(2\varphi) \\
- i \left[ (1 + e^{2\Phi})^3 g_1 - (e^{2\Phi} - 1)^3 g_2 \right] \sin(2\varphi) \]. \quad (83)

The variations \( \delta \lambda_i \) and \( \delta \lambda_{iA} \) give the following equations
\[ \Phi' - \frac{1}{3} e^{-i\Lambda} \frac{\partial W}{\partial \Phi} \pm ie^{-2\Phi}(e^{4\Phi} - 1) \varphi' = 0 \] \quad (84)

which implies \( \varphi' = 0 \) or \( \varphi = \varphi_0 \). We will set the constant \( \varphi_0 = 0 \) in order to recover the two AdS\(_4\) critical points at \( r \to \pm \infty \). Using the expression for \( e^{i\Lambda} \) in term of \( W \), equation (33), we find that the above equation requires
\[ \Phi = 0 \text{ or } \Phi = \frac{1}{2} \ln \left[ \frac{g_2 - g_1}{g_2 + g_1} \right]. \quad (85) \]

This means the scalars are fixed at the critical points.

The metric function \( A(r) \), for \( \Phi = 0 \), can be determined from the gravitini variations
\[ A^2 + \frac{e^{-A}}{\ell^2} - g_1^2 = 0 \] \quad (86)

whose solutions are
\[ A = \ln \left[ \frac{e^{-g_1r} (e^{g_1r} + g_1^2 \ell^2)^2}{4g_1^4 \ell^4} \right] \quad (87) \]

or
\[ A = \ln \left[ \frac{e^{-g_1r} (e^{g_1r} g_1^2 \ell^2 + 1)^2}{4g_1^4 \ell^4} \right]. \quad (88) \]

For \( \Phi = \frac{1}{2} \ln \left[ \frac{g_2 - g_1}{g_2 + g_1} \right] \), we find
\[ e^A \ell^2 (g_2^2 - g_1^2) A^2 = g_1^2 (1 + e^A g_2^2 \ell^2) - g_2^2. \quad (89) \]

With a suitable integration constant, the solutions can be written as
\[ e^A = \frac{e^{-\frac{91 g_2^2}{16g_1^2 g_2^2 \ell^4}}}{16g_1^2 g_2^2 \ell^4} \left[ e^{-\frac{91 g_2^2}{16g_1^2 g_2^2 \ell^4}} \ell^2 (g_2^2 - g_1^2)^2 + 4 \ell^2 (g_2^2 - g_1^2) \right]^2 \] \quad (90)

or
\[ e^A = \frac{e^{-\frac{91 g_2^2}{16g_1^2 g_2^2 \ell^4}}}{16g_1^2 g_2^2 \ell^4} \left[ 4 \ell^2 (g_2^2 - g_1^2) e^{\frac{91 g_2^2}{16g_1^2 g_2^2 \ell^4}} + 1 \right]^2. \quad (91) \]

These solutions are nothing but AdS\(_4\) backgrounds in the AdS\(_3\)-sliced parametrization. Therefore, there are no supersymmetric Janus solutions connecting the \( SO(3) \times U(1) \) critical point identified in [26].
In addition, we have also looked for Janus solutions in the case of non-compact gauge groups. It has been pointed out in [30] that among the “electric” gaugings, \(SO(3, 1)\) and \(SL(3, \mathbb{R})\) gauge groups admit supersymmetric \(AdS_4\) vacua. In the case of \(SO(3, 1)\), there is no supersymmetric Janus solution with \(SO(3)\) symmetry. For \(SL(3, \mathbb{R})\) gauge group, there are no scalars which are singlets under \(SO(3) \subset SL(3, \mathbb{R})\). Another possibility would be to consider solutions with \(SO(2) \subset SO(3)\) symmetry.

In both gauge groups, there are six \(SO(2)\) singlets parametrized by the coset manifold \(SU(1, 1)/U(1) \times SU(1, 1)/U(1) \times SU(1, 1)/U(1)\) which is a submanifold of \(SU(3, 3)/SU(3) \times U(3)\) and \(SU(3, 5)/SU(3) \times U(5)\), respectively. The analysis turns out to be highly complicated. In addition, there does not seem to be any simple truncation to fewer scalars that can give rise to supersymmetric Janus solutions. It would be useful to carry out the full analysis and definitely determine whether \(N = 3\) gauged supergravity with \(SO(3, 1)\) and \(SL(3, \mathbb{R})\) gauge groups admits supersymmetric Janus solutions.

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