Site-diluted three dimensional Ising Model
with long-range correlated disorder

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Abstract

We study two different versions of the site-diluted Ising model in three dimensions with long-range spatially correlated disorder by Monte Carlo means. We use finite-size scaling techniques to compute the critical exponents of these systems, taking into account the strong scaling-corrections. We find a $\nu$ value that is compatible with the analytical predictions.

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I. INTRODUCTION

Neutron and X-Ray critical scattering experiments in different systems \[1\] revealed an unexpected feature: the two length scales coexistence. In the theory of critical phenomena it is expected that the vector dependence of the scattering intensity corresponds to a Lorentzian function with a width proportional to the inverse of the correlation length. Nevertheless it was found \[1\] more reasonable to understand the experimental measures supposing the superposition of a broad Lorentzian (which width closely behaves as it is theoretically expected for these materials) and a sharper function, which behaviour is similar to a simple or a squared Lorentzian function. In Ref. \[2\] it is proposed that this new component of the scattering intensity has an defect related origin, in particular due to the presence of dislocations near the sample surface. So there is a crossover between the bulk critical behaviour (broad component) and the disorder critical behaviour. As the defects are not randomly placed points but randomly oriented lines, the quenched disorder is long-range correlated.

A Gaussian disorder with correlations decaying like a power law was studied in Ref. \[3\] for the vector spin models by analytical means, in particular using renormalization group expansion in $\epsilon = 4 - d$ and $\delta = 4 - a$, up to first order, where $a$ is the power of the potential decay of the spatial correlation function and $d$ is the ordinary spatial dimension. For example, straight dislocation lines with random orientation can be represented with $a = d - 1$. In addition to the Gaussian fixed point, the pure Ising point and the short-ranged disordered one it appears another fixed point. This new point has a pair of complex eigenvalues that lead to oscillating scaling corrections. The critical exponents of the long-range correlated disorder fixed point are

$$\nu = \frac{2}{a},$$
$$\eta = \mathcal{O}(\epsilon^2),$$

being, as usual, $\nu$ the thermal critical exponent, associated to the correlation length and $\eta$,
the anomalous dimension of the field. Through the scaling relations it is possible to obtain the other critical exponents of the system. The first of the relations in Eq. (1) should be valid at all order in perturbation theory and it should be an exact relation. The more interesting case of non-Gaussian disorder has never been studied in detail, although it is possible that the results for non-Gaussian disorder are the same of the Gaussian case. It should be noted that the previous relation is valid only if the disorder decays in a sufficiently slow manner.

A useful criterion due to Harris [4] tells us whether the disorder is an irrelevant perturbation in the pure system, in terms of the pure critical behaviour. The criterion states that the short-ranged disorder is not relevant when \( d\nu_{\text{pure}} - 2 = -\alpha_{\text{pure}} > 0 \). In Ref. [3] the Harris criterion is extended to the long-range correlated case. This kind of disorder is an irrelevant perturbation if the condition \( 2/a > \nu_{\text{pure}} \) holds, i.e. when the exponent \( \nu \) given by the relation (1) is larger than the exponent \( \nu_{\text{pure}} \) of the model without disorder.

A line of defects corresponds to a correlation decay with a \( a = 2 \) power, but with a non-Gaussian distribution of the disorder. In this case, using the data from the pure Ising case, \( 1 > \nu = 0.6294(5)(5) \) [5], the extended criterion tells us that the disorder is a relevant perturbation. Neglecting the non-Gaussian effects and applying the results of the Eq. (1) to this case, in Ref. [2], is found a \( \nu \) exponent close with the experimental data for different materials [1]. Furthermore, it was computed the local fluctuation on the critical temperature due to the line defects concentration finding also an acceptable agreement.

In Ref. [6] the same authors have studied the influence of the long-range correlated disorder in the line shape of the narrow component of the scattering intensity function, finding that it can be steeper than a Lorentzian one, also in agreement with the experimental data. They were assuming the Gaussian disordered model and the line defects one, where the disorder is non-Gaussian at large scale, do belong to the same Universality Class.

In this study we will compute by Monte Carlo means the critical exponents of the site-diluted Ising model with long-range correlated disorder, in the \( a = 2 \) case, in order to numerically check the analytical predictions that seem compatible with some experimental results. In order to do this we will use the finite-size scaling techniques that are recently
applied to the study of random site-diluted Ising systems \[7,8\]. We will study both Gaussian and non-Gaussian disorder and we will find similar results for the critical exponents supporting the correctness of the analysis of Ref. \[2,6\].

The layout of the paper is the following. In section 2 we will define the model we have simulated in the lattice and the two different ways used to introduce the long-range correlated disorder in the system. In section 3 we will show the finite-size techniques we used. The technical details of the simulation will be reported in section 4. The numerical results will be shown in section 5. Finally, in section 6, we will report the conclusions of this study.

II. THE MODELS AND THE OBSERVABLES

We have considered the following Hamiltonian defined in a cubic lattice of linear size \(L\) with periodic boundary conditions:

\[
H = -\beta \sum_{<i,j>} \epsilon_i \epsilon_j \sigma_i \sigma_j,
\]  

(2)

where the sum is extended to the nearest neighbors, \(\sigma\) are the usual \(Z_2\) spin variables and the \(\epsilon\)’s are quenched random variables, with long-range spatial correlation. An actual \(\epsilon_i\) set will be called a sample from now on. We have studied two different ways to introduce the correlation between the \(\epsilon_i\) variables.

The first one is to obtain a set of \(V = L^3\) correlated Gaussian random variables, \(\eta(\mathbf{x})\), where \(\mathbf{x}\) is the position vector in the lattice, with these properties:

\[
\langle \eta(\mathbf{x}) \rangle = 0,
\]

\[
\langle \eta(\mathbf{x}) \eta(\mathbf{y}) \rangle \propto \frac{1}{|\mathbf{x} - \mathbf{y}|^a} \quad (\equiv C(|\mathbf{x} - \mathbf{y}|)),
\]  

(3)

where \(d > a > 0\). In order to do this we have used the Fourier Filtering (FF) method \[9\].

Let \(\tilde{C}(\mathbf{p})\) be the Fourier transform of the function \(C(|\mathbf{x} - \mathbf{y}|)\) in momentum space. Let us define the set \(\tilde{\eta}(\mathbf{p})\) as
\[ \tilde{\eta}(\mathbf{p}) = \sqrt{\tilde{C}(\mathbf{p})} u(\mathbf{p}), \]  

where \( u(\mathbf{p}) \) is a Gaussian set of random numbers in the complex plane with the following properties:

\begin{align*}
\langle u(\mathbf{p}) \rangle &= 0, \\
\langle u(\mathbf{p})u(\mathbf{p}) \rangle &= 1, \\
\langle u(\mathbf{p})u(\mathbf{p}') \rangle &= 0, \text{ if } \mathbf{p} \neq \mathbf{p}'.
\end{align*}

At this point we construct \( \eta(\mathbf{x}) \) as the inverse Fourier transform of the \( \tilde{\eta}(\mathbf{p}) \) set. In order to assure that the \( \eta(\mathbf{x}) \) set is real we have to introduce the condition:

\[ \tilde{\eta}(\mathbf{-p}) = \tilde{\eta}^*(\mathbf{p}). \]

The zero mode divergence in a lattice treatment is eliminated by using the condition \( u(\mathbf{p} = 0) = 0 \). This choice agrees with the property \( \langle u(\mathbf{p}) \rangle = 0 \).

With these definitions \( \eta(\mathbf{x}) \) becomes a Gaussian random variable, as it is a sum of a large number of random variables, and it is easy to prove that the relations given by Eq. (3) are satisfied. Furthermore, it is also possible to calculate the variance of this Gaussian distribution, as the zero momentum inverse Fourier transform of \( \tilde{C}(\mathbf{p}) \).

With the \( \{\eta(\mathbf{x})\} \) set we proceed to choose, with a given probability \( p \), that we will call the mean concentration of the system, when each site is occupied \( (\epsilon_i = 1) \) or not \( (\epsilon_i = 0) \). We compute the area below the corresponding Gaussian distribution for the \( \eta \) random variable up to its actual value \( \eta(\mathbf{x}) \). We compare this quantity with \( p \) and we consider that the site is occupied if the area is smaller than \( p \).

In this work we will study the case \( a = 2 \) corresponding to linear defects. With this value we have checked that the correlation obtained for the \( \{\eta(\mathbf{x})\} \) set with the FF method, performed in double precision, is in good agreement with the expected correlation function. Although it is evident that the \( \epsilon_i \) are not Gaussian variables, their connected correlation functions at all different points are equal to zero and therefore this model corresponds to
a Gaussian model at large scale (non-Gaussian effects are restricted at short scale and are likely irrelevant). This model is what we referred to as Gaussian distributed noise in the introduction.

A second way to obtain samples with long-range correlated disorder with a decay with the inverse of the square of the distance is to remove lines of a given configuration. We start with a filled cubic lattice and remove lines until we get the fixed concentration $p$. The last line considered in this procedure is removed or not with a given probability in order to get as the mean concentration the $p$ value. We also want that the probability of removal for all the lattice points to be the same, and the lattice symmetries to be preserved. We can do that by only removing lines along the axes. It is clear that the connected correlation functions with this method are definitely different from zero also at long distances. The noise is very far from being Gaussian and this model is referred to as the non-Gaussian distributed noise in the introduction.

In both cases, we will consider the quenched disorder, that is, we first calculate the average of a given observable on the $\{\sigma_i\}$ variables with the Boltzmann weight given by the Hamiltonian of the Eq. (2), the results on the different samples being later averaged. The quenched approximation is due to the fact that the defect dynamics is slower that the associated to the magnetic interaction. We will denote by brackets the thermal average and by overlines the sample average. The observables will be denoted with calligraphic letters, i.e. $\mathcal{O}$, and we will use the italics for the double average $\mathcal{O} = \langle \mathcal{O} \rangle$.

Thus, we can define the nearest-neighbours energy as

$$\mathcal{E} = \sum_{\langle i,j \rangle} \epsilon_i \sigma_i \epsilon_j \sigma_j .$$

This quantity is extensively used for extrapolating the results for a given observable, $O$, obtained at coupling $\beta$ to a nearby coupling $\beta'$ as well as for calculating $\beta$-derivatives through its connected correlation with the observable. For instance, one can define the specific-heat as

$$C = \partial_\beta \mathcal{E} = \frac{1}{V} \left( \langle \mathcal{E}^2 \rangle - \langle \mathcal{E} \rangle^2 \right) .$$
The order parameter of the phase transition is the usual normalized magnetization
\[ M = \frac{1}{V} \sum_i \epsilon_i \sigma_i . \] (9)

As in a finite lattice, its mean value, \( M \), is zero, we are restricted to work with even powers of the magnetization. The second power is related to the susceptibility of the system:
\[ \chi = V \langle M^2 \rangle . \] (10)

With the fourth power we can construct another interesting quantity, the cumulant \( g_4 \), defined as
\[ g_4 = \frac{3}{2} - \frac{1}{2} \frac{\langle M^4 \rangle}{\langle M^2 \rangle^2} . \] (11)

In the finite-size scaling method we use it is very convenient to have a well behaved estimate of the correlation length in a finite lattice. We have used the second-momentum definition, that reads \[ \xi = \left( \chi/F - \frac{1}{4 \sin^2(\pi/L)} \right)^{1/2} , \] (12)
where \( F \) is defined in terms of the Fourier transform of the spin distribution
\[ G(p) = \frac{1}{V} \sum_r e^{i p \cdot r} \epsilon_r \sigma_r , \] (13)
as
\[ F = \frac{V}{3} \langle |G(2\pi/L,0,0)|^2 + \text{permutations} \rangle . \] (14)

III. FINITE-SIZE SCALING TECHNIQUES

In the scaling region, the mean value of a given observable, \( O \), measured at a coupling \((\beta, p)\) pair can be written as
\[ O(L, \beta, p) = L^{x_O/\nu} \left( F_O(\xi(L, \beta, p)/L) + O(L^{-\omega}) \right) , \] (15)
where $x_O$ is the critical exponent of the operator $O$, $F_O$ is a smooth scaling function depending on the observable and $\omega$ corresponds to the eigenvalue of the first irrelevant operator of the theory from the Renormalization Group point of view.

The principal feature of Eq. (15) is that all the quantities are measurable in a finite lattice. In order to obtain the critical exponents we need to remove the unknown scaling function $F_O$. Let us define the quotient of a given observable $O$ at two different lattice sizes and at the same coupling pair as

$$Q_O = O(sL, \beta, p)/O(L, \beta, p),$$

and let us compute this quotient at the coupling where the correlation length in units of the lattice size is the same for both lattices. Thus we get:

$$Q_O|_{Q_\xi = s} = s^{x_o/\nu} + A_p^O L^{-\omega} + \cdots,$$

where $A_p^O$ is a constant which depends on the observable and the spin concentration $p$ and the dots stand for higher-order scaling corrections. From this equation we can extract the critical exponent associated to a given observable.

The observables used to obtain the different critical exponents are: the $\beta$-derivative of the correlation length in order to calculate $\nu$ ($x_{\partial_\beta \xi} = \nu + 1$) and the susceptibility, $\chi$, to get the magnetic $\eta$ exponent, ($x_\chi = \nu(2 - \eta)$).

In order to compute the infinite volume critical coupling we will use the crossing points of the observables with $x_O = 0$, as $g_4$ or $\xi/L$, when measured at two different lattice sizes, $L$ and $sL$. The shift of these points from the critical coupling behaves as:

$$\Delta \beta_c^L \propto \frac{1 - s^{-\omega}}{s^{1/\nu} - 1} L^{-\omega - 1/\nu}.$$  

IV. NUMERICAL METHODS

The best update method for an Ising model simulation is a cluster algorithm [13]. In particular, the most efficient one in the pure case is the single-cluster Wolff method [14].
Nevertheless, in a diluted system small groups of isolated spins appear, which are scarcely visited with this algorithm. Furthermore in the non-Gaussian case also appear isolated occupied lines. In order to update all-sized spin clusters, after a fixed number of single-cluster updates we perform a Swendsen-Wang sweep. We call this ensemble our MC Step (MCS). We have discarded 100 MCS for thermalization and then we have measured the different observables for every single MCS. We have checked the correct thermalization of the system by starting from hot and cold configurations. We have chosen the single-cluster updates number in such a way that the autocorrelation times for all the observables are nearly one MCS. The simulations are carried out in the RTNN machine at Zaragoza University.

Other interesting parameters are the number of measures to perform in a given disorder realization, $N_I$, using the Ising Hamiltonian, Eq. (2), and the number of different samples, $N_S$. We refer to [7,8] for a discussion of the optimal choice of these parameters. In our case, we have performed $N_I = 100$ measures in $N_S = 20000$ different samples for $L \leq 64$ and in $N_S = 10000$ samples for $L = 128$.

We have used in this study the usual $\beta$-extrapolation [10]. Thus we restrict to not too strong dilutions.

In order to work with large dilutions it is convenient to perform a $p$ extrapolation as we will see later from the phase diagram of the systems. In the random site-diluted Ising model case this is possible because the density distribution probability of the actual configurations (a binomial one) is known. In the Gaussian model, due to the correlation between the different sites, this distribution it is not known. Neither in the non-Gaussian case, as it is presented here. Nevertheless it is possible to perform a slight variation of this latter model allowing to perform a dilution extrapolation. It is enough to choose with a given probability when a line it is empty or filled, but this variation will be not considered in this study.

We recall [7,8] that a bias of order $2\tau/N_I$ is present in the $\beta$-extrapolation, where $\tau$ is the correlation time between the energy and the observable we consider. This fact is not relevant in the usual MC calculations, because the statistical errors are of order $1/\sqrt{N_I}$. But in diluted system investigations, when $\sqrt{N_S} \sim N_I$, this bias could be not negligible. We
have performed a proper extrapolation procedure \[7,8\] in order to obtain unbiased estimates of the $\beta$-derivatives and the values of the different observables in the neighbourhood of the simulated couplings. The MCS is chosen in such a way that $\tau$ is nearly one measure. For the largest lattice we have considered, $L = 128$, in the non-Gaussian case, the single cluster update number for every Swendsen-Wang sweep is 1200 and for the Gaussian case it is 400. For the statistical error computation we have used the jack-knife method with 50 blocks, that allows us to obtain a 10% of accuracy in the error bars.

V. NUMERICAL RESULTS

We have studied the Gaussian case at two different dilutions, $p = 0.8$ and $p = 0.65$, performing simulations in lattice sizes $L = 8, 16, 32, 64$ and 128. In the non-Gaussian case we have only considered $p = 0.8$ in the same lattice sizes.

In figure 1 we show the phase diagram for the Gaussian model. The percolation critical point, $p_c \approx 0.25$, was obtained by studying the behaviour of the $g_4$ function in a $L = 128$ lattice. In the thermodynamical limit, $g_4 = 0$ in the disordered phase and $g_4 = 1$ in a ferromagnetic ordered one. The corresponding phase diagram for the non-Gaussian case is qualitatively the same, with a ferromagnetic ordered phase for large $\beta$, when $p$ is larger that the percolation threshold for this case.

A. Thermal exponent

In table 1 we present the results for the $\nu$ exponent in the two cases considered, the Gaussian and the non-Gaussian disorder. It was computed by applying the Eq. (17) to $\partial_\beta \xi$ using $s = 2$.

As we see, there are visible scaling corrections in all the cases. Nevertheless the values we obtain for the $\nu$ exponent are very different from those of the pure Ising model, $\nu = 0.6294(5)(5)$ \[5\] and from those of the three-dimensional random site-diluted case, $\nu = 0.6837(24)(29)$ \[7\]. In order to obtain the critical exponent, we have to perform an infinite
volume extrapolation procedure. It is possible that corrections-to-scaling that we observe are complicated by the presence of oscillatory terms, as is suggested by the first order of the $\epsilon$-expansion, but we are unable to confirm or discard this possibility. Strong corrections to simple scaling are present as we will see, so it is rather difficult (although we study lattices ranging from $8^3$ to $128^3$) to get conclusive statements on the nature of finite volume corrections.

We can try to parameterize the scaling corrections as in Eq. (17) only with the first term. We have used the data from the two different dilutions of the Gaussian case performing a joint fit assuming a single value for $\omega$ and $\nu$ exponents, following the picture of a single Universality Class along the critical line. Using $L \geq 8$ data and the full covariance matrix to compute the statistical function $\chi^2$, we find a very large value of $\chi^2/d.o.f. = 13.9/4$. Nevertheless, discarding the data from the $L = 8, 16$ pair, we find $\chi^2/d.o.f. = 1.20/2$. The value obtained for the thermal exponent, $\nu = 1.012(16)$, is compatible with the analytic predictions. We also find $\omega = 1.01(13)$.

We can control the presence of the higher-order corrections in a simple and na"ive way. We could perform a quadratic fit for each dilution with $L \geq 8$ data, assuming the $\omega = 1$ value compatible with our results and only using the diagonal part of the covariance matrix. If we do so, we obtain for $p = 0.8$, $\chi^2/d.o.f. = 0.76/1$ with $\nu = 1.012(10)$ and for $p = 0.65$, $\chi^2/d.o.f. = 0.73/1$ and $\nu = 1.005(14)$. So, the presence of second-order corrections for the thermal exponent data seems reasonable. Furthermore we have found that the $\nu$ value is not affected by the presence of these terms.

In the non-Gaussian case large finite volume corrections are also present. Nevertheless we find that the estimates from the two biggest lattice pairs for the $\nu$ exponent are compatible with the analytical calculations.
B. Magnetic exponent

In table [I] we present the estimates of the magnetic exponent $\eta$ using the Eq. (17), from the susceptibility $\chi$ measured at the point where $Q_\xi = 2$ for all the concentrations considered.

As we can see from the table, there are strong scaling effects in all the cases, specially in the non-Gaussian case. An infinite volume extrapolation procedure is therefore needed in order to get an $\eta$ estimate.

Assuming only the presence of first-order corrections with our previously calculated $\omega$ value we do not find reasonable fits to our data. So, we could consider higher-order correction terms. As we have found $\omega \simeq 1$, the second-order terms and the analytical corrections are of the same order, so we can try a quadratic joint fit using the $\omega = 1$ value. With the $L \geq 16$ data for all the concentrations studied, using only the diagonal part of the correlation matrix, we get $\chi^2/d.o.f. = 1.63/2$ and $\eta = 0.043(4)$. So we have found compatible results with the picture of a single $\eta$ value, scaling corrections parameterized by $\omega \simeq 1$, but with non-negligible higher-order correction effects. Nevertheless, this estimate has two different sources of systematic error: the first one is due to the possible evolution of the $\eta$ value with the minimum lattice size considered in the fits, and the second, to the uncertainty on the fitted functional form.

We can compare this result with those from the random site-diluted Ising model, $\eta = 0.0374(36)(9)$ [7], and with those from the Ising case, $\eta = 0.0374(6)(6)$ [5], finding that they are similar to our estimate for $\eta$.

C. Critical couplings

In table [II] we show the crossing points of $g_4$ and $\xi/L$ from $(L,2L)$ lattice pairs for the Gaussian case. As we can see in the table, there is a non monotonic $L$ behaviour for $g_4$ crossing points in both concentrations, so it is expected the presence of high-order scaling
corrections. A way to extract the infinite volume critical coupling is to perform a fit to the functional form of the Eq. (18). In order to find a proper extrapolated value, we have to check that we are in the linear regime and that we can control the higher-order corrections. As the crossing points for $g_4$ show a minimum value around the $L = 32, 64$ pair the former condition is not satisfied.

We could fit the $\xi/L$ crossing points to Eq. (18). Nevertheless, for both concentrations, using $L \geq 8$ data, we have found a large value of $\chi^2/\text{d.o.f.}$, being d.o.f. $= 1$. So we have also to assume the presence of higher-order scaling corrections.

In order to control the finite volume effects, assuming our estimate for $\omega + 1/\nu = 2.00(13)$, we can discard the $L = 8$ data and perform a linear fit for the $\xi/L$ data. In the $p = 0.65$ case we find a reasonable $\chi^2/\text{d.o.f.} = 1.28/1$ in the central value, and we get $\beta_c(\infty) = 0.332929(13)(12)$, being the second error bar due to the uncertainty in $\omega + 1/\nu$. Nevertheless, for $p = 0.8$ we do not find a reasonable fit, showing that the higher-order corrections are important also in the $L = 16$ lattice. We can check this latter picture performing a fit with $1/L^2$ and $1/L^3$ terms, only using the diagonal part of the covariance matrix for $L \geq 8$ data. Then we obtain $\chi^2/\text{d.o.f.} = 1.69/1$ and $\beta_c(\infty) = 0.272715(10)$. So the picture of second-order scaling corrections is compatible with our data in this case.

A similar analysis can be done by studying the $g_4$ and $\xi/L$ crossing points measured with a $(L_1, L_2)$ pair but fixing the $L_1$ value.

In the $p = 0.65$ case performing a linear fit for $\xi/L$ with $L \geq 16$, and $\omega + 1/\nu = 2.00(13)$ we get $\chi^2/\text{d.o.f.} = 1.43/1$ for the central value of this interval and $\beta_c(\infty) = 0.332927(13)(15)$, where the second error bar is due to the uncertainty on the critical exponents.

In the $p = 0.8$ case a diagonal fit for $\xi/L$ with $1/L^2$ and $1/L^3$ terms using $L \geq 8$ data, we get $\chi^2/\text{d.o.f.} = 0.52/1$ and $\beta_c(\infty) = 0.272722(10)$. So we find a similar behaviour with our previous analysis, finding reasonable fits and compatible estimates for the infinite volume critical couplings.

In table IV we show the crossing points of $g_4$ and $\xi/L$ measured at $L$ and $2L$ lattice sizes for the non-Gaussian case with mean concentration $p = 0.8$. 13
Also in this case we see that the $g_4$ crossing point is not a monotonic function of $L$. In the $\xi/L$ case we find that assuming our previous $\omega$ estimate, a linear fit for $L \geq 16$ is not reasonable, so we have to conclude that also in this case the higher-order terms are present. In order to check in a simple way this assumption, we perform a fit with $1/L^2$ and $1/L^3$ terms, using $L \geq 8$ and using only the diagonal part of the covariance matrix. Thus we obtain $\chi^2/\text{d.o.f.} = 0.34/1$ and $\beta_c(\infty) = 0.257126(14)$, so this picture is compatible with our data.

In order to compute the value of the scaling functions $g_4$ and $\xi/L$ at the critical coupling in the thermodynamical limit, we have measured the values of these quantities at the crossing points of $g_4$ and $\xi/L$ respectively. In the $g_4$ case, the finite volume corrections are large, finding values for this observable in the range 0.58-0.64. In the $\xi/L$ case, we have also found that an infinite volume extrapolation procedure is needed. Performing a $1/L$ extrapolation we quote for this quantity the value $0.36(2)$.

**VI. CONCLUSIONS**

We have studied by Monte Carlo means the three-dimensional site-diluted Ising model, with long-range spatially correlated disorder. We have considered Gaussian and non-Gaussian disorder to study the influence of this fact in the critical behaviour of the system. We have used finite-size scaling techniques for the computation of the critical exponents.

We have found strong scaling-corrections for the $\nu$ exponent. In the Gaussian case, we succeed to parameterize them only with the first corrections-to-scaling term, finding an infinite volume $\nu$ value that is compatible with the analytical prediction in this model. In the non-Gaussian case, the value we obtain for the two largest lattice-size pairs is also compatible with this calculation.

For $\eta$ exponent, large finite volume effects are also present. Our data for $L \geq 16$ are compatible with the picture of a single $\eta$ value independent of the kind of disorder and the concentration considered, but with non-negligible second-order scaling-correction terms.
So we have obtained a consistent picture of the existence of a single fixed point (single \( \eta, \nu \) and \( \omega \) values) using Gaussian and non-Gaussian correlated disorder, but with non-negligible second-order scaling-corrections. This fact introduces systematic errors in our analysis making it very difficult to measure them, in order to obtain solid estimates for the critical exponents.

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FIG. 1. Phase diagram of the site-diluted Ising model with long-range correlated Gaussian disorder, in the inverse temperature–dilution plane. The dots correspond to the simulated points, while the arrow points to the percolation limit ($\beta = \infty$).
### TABLE I. Critical exponent $\nu$ computed using $\partial_3 \xi$ when measured in $(L, 2L)$ lattice pairs at the couplings where $Q_\xi = 2$ for both models at the different concentrations considered.

| $L$ | Gaussian model | Non-Gaussian |
|-----|----------------|--------------|
|     | $p = 0.8$      | $p = 0.65$   | $p = 0.8$     |
| 8   | 0.7626(19)     | 0.871(3)     | 0.8335(24)    |
| 16  | 0.833(3)       | 0.942(6)     | 0.934(4)      |
| 32  | 0.907(4)       | 0.969(7)     | 1.009(9)      |
| 64  | 0.964(9)       | 0.996(11)    | 1.009(13)     |

### TABLE II. Magnetic exponent $\eta$ computed from $\chi$, using lattice $(L, 2L)$ pairs at the couplings where $Q_\xi = 2$ for both Gaussian and non-Gaussian cases at the different concentrations simulated.

| $L$ | Gaussian model | Non-Gaussian |
|-----|----------------|--------------|
|     | $p = 0.8$      | $p = 0.65$   | $p = 0.8$     |
| 8   | 0.0085(11)     | 0.0256(14)   | -0.0513(14)   |
| 16  | 0.0082(14)     | 0.0274(16)   | -0.0532(12)   |
| 32  | 0.0137(15)     | 0.0384(18)   | -0.0259(18)   |
| 64  | 0.0259(19)     | 0.038(3)     | 0.0052(24)    |
TABLE III. Crossing points from \((L, 2L)\) pairs of \(g_4\) and \(\xi/L\) for the Gaussian case at the different concentrations simulated.

| \(L\) | \(p = 0.8\) | \(p = 0.65\) |
|-------|-------------|-------------|
| \(\xi/L\) | \(g_4\) | \(\xi/L\) | \(g_4\) |
| 8    | 0.274535(34) | 0.273760(52) | 0.335269(76) | 0.33358(12) |
| 16   | 0.273545(15) | 0.272862(22) | 0.333709(41) | 0.332617(72) |
| 32   | 0.2729883(96) | 0.272604(14) | 0.333099(19) | 0.332682(28) |
| 64   | 0.2727805(70) | 0.272624(11) | 0.332989(15) | 0.332872(25) |

TABLE IV. Crossing points of \(g_4\) and \(\xi/L\) from \((L, 2L)\) pairs for \(p = 0.8\) in the non-Gaussian disorder.

| \(L\) | \(\xi/L\) | \(g_4\) |
|-------|-------------|-------------|
| 8    | 0.25926(4)  | 0.25803(5)  |
| 16   | 0.257935(22) | 0.25706(3)  |
| 32   | 0.257375(13) | 0.25708(21) |
| 64   | 0.257188(9)  | 0.257110(13) |