Resonant decay of parity odd bubbles in hot hadronic matter

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Abstract

We investigate the decay of metastable states with broken CP-symmetry which have recently been proposed by Kharzeev, Pisarski and Tytgat to form in hot hadronic matter. We consider the efficiency of the amplification of the $\eta'$-field via parametric resonance, taking the backreaction into account. For times of the order $t \approx 10 \text{ fm}$, we find a particle density of about $0.7 / \text{fm}^3$ and a correlation length of $\xi_{\text{max}} \approx 2.5 \text{ fm}$. The corresponding momentum spectra show a non-thermal behaviour.

Key words: hot hadronic matter, metastable CP-odd bubbles, parametric resonance
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1 Introduction

Recently, Kharzeev, Pisarski and Tytgat [1,2] presented the following idea, based on the nontrivial topological structure of the QCD-vacuum [3]: Metastable states which act like regions with a non-vanishing QCD vacuum angle $\theta$ may be excited when matter undergoes the deconfining phase transition, provided it is of second order. In these false vacua, parity (and also CP) is spontaneously broken, a quality that could lead to experimental signatures like parity odd correlations of the produced particles.

In the present work, we investigate the production of $\eta'$-particles during the decay of the CP-odd metastable states, concentrating on the amplification of the low momentum modes by parametric resonance when the zero mode rolls down from the false to the true vacuum. This mechanism plays an important role for particle production in inflationary cosmology [4–11] and has also been

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investigated for the formation of Disoriented Chiral Condensates [12–15] and for Axions [16]. Here we discuss the efficiency of the amplification mechanism for the production of $\eta$ from the decaying metastable bubble. As a main result we derive the corresponding momentum spectrum showing non-thermal behaviour. The integrated density of particles produced is estimated to be $0.7 \text{ fm}^{-3}$.

2 Metastable states in hot hadronic matter

The idea is based on the effective Lagrangian of the Witten-DiVecchia-Veneziano model [17–19]

$$\mathcal{L}_{\text{eff}} = \frac{f^2}{4} \left[ \text{tr}(\partial_\mu U \partial^\mu U^+) + \text{tr}(MU + MU^+ \right] = -\frac{a}{N_c} \left( \theta - \frac{i}{2} \text{tr}(\ln U - \ln U^+) \right)^2 , \tag{1}$$

which describes the low-energy dynamics of the pseudoscalar mesons in the large $N_c$-limit of QCD. In the following, we consider the "real world"-vacuum angle $\theta \equiv 0$. Formation and decay of a non-zero $\theta$-vacuum and its signatures are investigated in [20]. The $N_f \times N_f$-matrix $U$ in Eq. (1) describes the meson fields, and the term containing the mass matrix $M$ provides the explicit soft breaking of chiral symmetry due to the quark masses. $M$ can be written as $M_{ij} = \mu_i^2 \delta_{ij}$, where the diagonal elements are chosen as $\mu_1^2 \equiv \mu_2^2 \equiv m_\pi^2$ and $\mu_3^2 \equiv 2m_K^2 - m_\pi^2$. With the parametrization $U = \exp(i\phi/f_\pi)$, the matrix $\phi$ representing the singlet and the octet meson fields reads

$$\phi = \sqrt{2} \frac{\eta_1}{3} + \begin{pmatrix} \pi^0 + \frac{m_\pi}{\sqrt{3}} & \sqrt{2}\pi^+ & \sqrt{2}K^+ \\ \sqrt{2}\pi^- & -\pi^0 + \frac{m_\pi}{\sqrt{3}} & \sqrt{2}K^0 \\ \sqrt{2}K^- & \sqrt{2}K^0 & -2 \frac{2}{\sqrt{3}} \eta_8 \end{pmatrix} . \tag{2}$$

The last term in the effective Lagrangian reflects the $U_A(1)$-anomaly by giving a mass to the singlet, even in the chiral limit of vanishing quark masses. The parameter $a = 2N_f \lambda_{YM}/f_\pi^2$ represents the topological susceptibility.

Finite temperature is introduced into the model via the temperature dependence of the parameters: For $T = 0$, the value $a = m_\eta^2 + m_\eta^2 - 2m_K^2 \simeq 0.726 \text{ GeV}^2$ can be determined from experimental results [17–19], and $\mu^2 \equiv (\mu_1^2 + \mu_2^2 + \mu_3^2)/3 = (m_\pi^2 + 2m_K^2)/3 \simeq 0.171 \text{ GeV}^2$. The corresponding value of the pion decay constant is $f_\pi \simeq 93 \text{ MeV}$. 

2
When the temperature approaches the temperature $T_d$ of the phase transition, $a(T)$ goes to zero, indicating the effective restoration of the $U_A(1)$-symmetry for $T > T_d$ with enhanced $\eta$ - and $\eta'$ - production as a possible signature [21,22]. We use the temperature of the deconfinement phase transition, $T_d$, following the assumption of [1] that any other phase transition of this model also occurs at $T_d$. According to mean field estimates [1], $a(T) \propto (T_d - T)$ and $\mu^2(T) \propto (T_d - T)^{-1/2}$ near the phase transition.

In order to get an analytically tractable approximation that allows some insight into the physical processes, we consider only the singlet, which is the main component of the $\eta'$-meson. Using $N_c = 3$, the effective Lagrangian simplifies to

$$L_s = \frac{1}{2} (\partial_\mu \eta_1) (\partial^\mu \eta_1) + \frac{3}{2} f_\pi^2 \mu^2 \cos \left( \sqrt{\frac{2}{3}} \frac{\eta_1}{f_\pi} \right) - \frac{a}{2} \eta_1^2. \quad (3)$$

The singlet effective potential

$$V \left( \frac{\eta}{f} \right) \equiv \frac{V_s}{f^2 \mu^2} \left( \frac{\eta}{f} \right) = - \cos \frac{\eta}{f} + \frac{a}{2 \mu^2} \frac{\eta^2}{f^2}, \quad (4)$$

where we have dropped the index 1 in the notation of the singlet and introduced $f \equiv \sqrt{3/2} f_\pi$ for convenience, can be considered as a projection of the full effective potential. It takes three qualitatively different shapes, depending on the temperature, more specifically, on the value of $a(T)/\mu^2(T)$. For large values of $a/\mu^2$, corresponding to small $T$, the potential is a more or less deformed parabola (see Fig. 1a). For $a/\mu^2 = 0$, i.e. $T > T_d$, the potential is periodic with minima at $\eta/f = 0, 2\pi, 4\pi...$, which are all equivalent true vacua (see Fig. 1d). And for a small, nonzero value of $a/\mu^2$, corresponding to a temperature just below the phase transition, metastable states appear in the potential which are distinct from the true vacuum (see Fig. 1c). One recognizes that the false vacua are odd under parity transformation, but even under charge conjugation, and therefore odd under CP [1].

One can show that the effective potential exhibits metastable states also in the case of nonvanishing $\pi, \eta_8$ - and $K$-fields, provided that $m_\pi \neq 0$ [23]. But only the singlet is responsible for the formation of these states, in the sense that it is the only field which appears in the term of the effective potential which includes the topological susceptibility.

3 The decay of the metastable states
3.1 Dynamics of the model

Applying the model to the description of a heavy ion collision, we suppose that during the phase transition, the potential changes its shape from the parabola (Fig. 1a), corresponding to the low temperature phase, to that of the high temperature phase (Fig. 1d), and back to the low temperature phase, but this time including metastable states that have been created at a temperature just below the phase transition. After the phase transition, but still at high temperature, the $\eta'$—field may be trapped in a metastable state, forming a "CP-odd bubble inside the hadronic phase" in the language of [1]. Considering the effective potential as a function of temperature, one recognizes that there exists a value $a/\mu^2(T_{sp})$ for which the local minimum turns into a saddle point (see Fig. 1b). For the lowest false vacuum, this happens for $a/\mu^2(T_{sp}) = 0.217$ at the value $(\eta/f)_{sp} = 4.493$. Assuming the proportionality $a(T)/\mu^2(T) \propto (T_d - T)^{3/2}$ to hold even far from $T_d$, the temperature corresponding to this special case of the potential can be estimated to be $T_{sp} \approx 0.86 T_d$. Once the saddle point temperature is reached and the barrier has disappeared, the $\eta'$—field starts rolling down into the true vacuum and oscillating around it, while energy is transferred from the mean value of the field into its quantum fluctuations. The resulting growth of the occupation numbers of the quantum fluctuations is interpreted as particle production with respect to the true, CP even, vacuum, whereas the zero mode is damped. This process is investigated in more detail in the following.

A crucial assumption for this picture is that at least after the saddle point temperature has been reached, the effective potential changes slowly compared to the motion of the field, so that the oscillations can be considered as taking place in a static potential, frozen at $T = T_{sp}$. Accordingly, the ratio $a/\mu^2 = 0.217$ is kept fixed during the evolution in time, and the value of $f$ as well. The solutions we find (see below) show that the zero mode reaches the true vacuum for the first time after $1 - 2 \, fm$, whereas the relevant time scale to be considered for the particle production is of the order of $10 \, fm$.

3.2 Evolution equations

To study the dynamics of the decay process, we decompose the field into its expectation value $\varphi(t) \equiv \langle \eta(\vec{x}, t) \rangle$ and fluctuations $\chi$ about it,

$$\eta(\vec{x}, t) = \varphi(t) + \chi(\vec{x}, t)$$

with $\langle \chi(\vec{x}, t) \rangle = 0$, so that the "physical" fluctuations are given by the usual expression $\langle (\chi - \langle \chi \rangle)^2 \rangle = \langle \chi^2 \rangle$. With this decomposition, following the stan-
standard method as described in, e.g., [5,6,8,11], we obtain two evolution equations from the effective Lagrangian (3). The effect of the quantum fluctuations at lowest order is taken into account with a Hartree-type approximation [5,8,11] that consists of the factorization $\chi^3 \to 3\langle \chi^2 \rangle \chi$ and $\chi^2 \to \langle \chi^2 \rangle$ and the self-consistency condition represented by Eq. (7) given below; terms of higher order in $\chi$ are neglected. Taking the expectation value $\langle \rangle$, we arrive at the equation for the zero mode

$$\frac{d^2 \varphi(\tau)}{d\tau^2} \frac{f}{f} + \left(1 - \frac{\langle \chi^2 \rangle}{2 f^2}\right) \sin \frac{\varphi(\tau)}{f} + \frac{a}{\mu^2} \frac{\varphi(\tau)}{f} = 0,$$

written for the dimensionless mean field $\varphi/f$, where the dimensionless time variable $\tau \equiv \mu t$ is introduced. The expectation value of the fluctuations

$$\langle \chi^2 \rangle_{\tau} \equiv \int \frac{d^3 k}{(2\pi)^3} |\chi_k(\tau)|^2$$

is calculated from the mode functions $\chi_k(\tau)$ which are known from the Fourier expansion of the field. The integration is done over the interval of momenta for which the quantum fluctuations are enhanced.

We use the definition (7) for the back reaction, although it does not fulfill $\langle \chi^2 \rangle_0 = 0$, because it reflects the background of the calculation, the closed-time-path technique which is usually applied for the description of out-of-equilibrium dynamics in field theory. In this framework, the fluctuations are defined as the two-point correlation functions or equal-time Green’s functions [5]

$$\langle \chi^2 \rangle_{\tau} \propto \int \frac{d^3 k}{(2\pi)^3} G_k(\tau, \tau).$$

The mode functions satisfy the mode equation

$$\frac{d^2}{d\tau^2} \chi_\kappa(\tau) + \omega^2_\kappa(\tau) \chi_\kappa(\tau) = 0,$$

written for the dimensionless mode functions $\chi_\kappa \equiv \sqrt{\mu} \chi_k$, where

$$\omega^2_\kappa(\tau) = \kappa^2 + \left(1 - \frac{\langle \chi^2 \rangle}{2 f^2}\right) \cos \frac{\varphi(\tau)}{f} + \frac{a}{\mu^2}$$
is the time-dependent frequency squared with $\kappa \equiv k/\mu$. They are used for the calculation of the particle production: The induced particle number density

$$n(\tau) = \int \frac{d^3 k}{(2\pi)^3} n_k(\tau)$$  \hspace{1cm} (11)

is obtained from the spectral particle number density

$$n_\kappa(\tau) = \frac{\omega_\kappa}{2} \left( |\chi_\kappa(\tau)|^2 + \left| \frac{\dot{\chi}_\kappa(\tau)}{\omega_\kappa^2} \right|^2 \right) - \frac{1}{2},$$  \hspace{1cm} (12)

which can be calculated from the mode functions. Since the unstable solutions of the mode equations contribute to the resonant particle production, the integration in (11) is carried out over the momenta corresponding to these amplified solutions, as in Eq. (7). For practical purposes, we use $\tilde{\omega}_\kappa^2 \equiv \kappa^2 + a/\mu^2$ in definition (12), because it keeps the particle number positive definite for all values of $\tau$ and minimizes the fluctuations in the particle number. Accordingly, the initial values of the mode functions are chosen as $\chi_\kappa(0) = 1/\sqrt{2\tilde{\omega}_\kappa}$ and $\dot{\chi}_\kappa(0) = -i\sqrt{\tilde{\omega}_\kappa}/2$ such that $n_\kappa(0) = 0$. However, the results depend only weakly on the choice of $\tilde{\omega}_\kappa$ both in the initial conditions and in the definition of the particle number. The choice for the initial conditions of the zero mode is suggested by the model: $\phi$ is ”released” right below the saddle point $\varphi_{sp}/f = 4.493$ with zero velocity, i.e. $\varphi(0) = 0$. For the numerical results shown below, we used the starting value $\varphi(0)/f = 4.2$.

3.3 \textit{Solutions and results}

Considering the coupled system (6) and (9) with (10), one recognizes that the expectation value transfers energy to the mode functions via a time dependent frequency, and the mode functions in turn modify the equation of motion for the zero mode. A natural first step in the solution of this system is to neglect the fluctuation terms in both equations, obtaining the ”classical” evolution equations with $\langle \chi^2 \rangle_\tau = 0$ where the mode functions do not react back on the zero mode. If the variable frequency

$$\omega_{\kappa,cl}^2 = \kappa^2 + \cos \frac{\varphi(\tau)}{f} + \frac{a}{\mu^2},$$  \hspace{1cm} (13)

in the mode equation depends \textit{periodically} on time, according to Floquet’s theorem [24] it has quasiperiodic solutions of the form

$$\chi_{\kappa}^{cl}(\tau) = \exp(\mu_\kappa \tau) P(\tau),$$  \hspace{1cm} (14)
where the characteristic exponent $\mu_\kappa$ depends on $\omega_{\kappa,cl}$, and $P(\tau)$ is a periodic function of the same period as $\omega_{\kappa,cl}$, usually normalized to have unit amplitude. For certain values of the parameter $\omega_{\kappa,cl}$, there exist so called resonance bands that contain the exponentially growing (or resonant) solutions with a real characteristic exponent $\mu_\kappa$. These resonance bands are known, for example, for the solutions of the Mathieu- or the Lamé-equation.

An analytic solution of the classical system can be found with a Sine-Gordon-equation [7], which consists essentially of setting $a/\mu^2 \approx 0$ in our case. The corresponding approximate zero mode equation is given by

$$\ddot{\varphi}_{cl}(\tau) + \rho \sin \left( \frac{\varphi_{cl}(\tau)}{\rho f} \right) = 0,$$

(15)

where $\rho = 1.43$ represents the radius of the potential and is needed to adjust it to the original potential. It has the solution

$$\frac{\varphi_{cl}(\tau)}{f} = 2 \rho \arctan \left( \sqrt{\frac{\epsilon}{2 - \epsilon}} \cn(\tau, m) \right),$$

(16)

with $\epsilon \equiv 1 - \cos(\varphi(0)/\rho f)$, and $m = \epsilon/2$ is the square of the modulus of the Jacobian elliptic function $\cn$. The resulting mode equation is a Lamé-equation with the resonance band [7] $0 \leq \kappa^2 \leq \epsilon/2$. The resulting $\mu_\kappa$ is shown in Fig. 2 in comparison with the result of the numerical calculation of $\mu_\kappa$ for the solution of the classical mode equation. In the range of the single resonance band of the Lamé-equation, $0 \leq \kappa^2 \leq 0.97$, the numerical calculation yields three resonance bands, and at least two for $\kappa^2 > 1$. (This structure reminds of the Mathieu-equation which has more than one resonance band.) The maximum value $\mu_{\kappa,\text{max}} \simeq 0.42$ is found around $\kappa^2 \approx 0.1$, and the maximum value for the Sine-Gordon-solution, $\mu_{\kappa,\text{max}}^{sg} \simeq 0.5$, yields an acceptable approximation.

The momentum spectrum of the produced particles reflects the structure of the resonance bands. When the general expression for the mode functions (14) is inserted into Eq. (12), it is obvious that the particle production is essentially determined by $\mu_\kappa$:

$$\ln n_\kappa(\tau) - \ln \frac{\omega_\kappa(\tau)}{2} \simeq 2\mu_\kappa\tau.$$

(17)

In the classical approximation discussed above, the back reaction of the fluctuations on the zero mode (and also on the fluctuations themselves) is ignored. This implies that the approximation is not energy conserving, which can immediately be seen from the zero mode solution which is not damped although it is supposed to transfer its energy to the mode functions, which indeed grow
exponentially. Since the mode functions are not decoupled from the zero mode, at least the other direction of energy transfer is reflected by the approximation: The larger the initial value $\varphi(0)$, the more energy can be transferred, and the larger the mode functions and the particle number grow.

The evolution equations including the back reaction are solved numerically. For the evaluation, we use $f(T_{sp}) \simeq f(0)$ and $\mu(T_{sp}) \simeq 676 \text{ MeV}$. Because the value of $\mu$ in our approximation is only fixed by the value of $T_{sp}$, which depends on our restriction to the singlet case and may not correspond to the real physical situation, we also try $\mu(T = 0) = 412 \text{ MeV}$ as the other limit of the range of possible values for $\mu(T)$.

Although the inclusion of the back reaction destroys the parametric resonance, it does not suppress the particle production completely. The particle number (Fig. 3) reaches its asymptotic value for $t_{\text{end}} \approx 10 \text{ fm}$. At about the same time, the value of the back reaction term $\langle \chi^2 \rangle / 2f^2$ grows larger than 1, indicating that the approximations made become inappropriate for the description of the dynamics for larger times (cf. Eq. (6)). One recognizes from Fig. 3 that the choice of $\mu(T)$ has some influence on $t_{\text{end}}$; the asymptotic number density of the produced particles lies between $0.7 \text{ fm}^{-3}$ and $0.9 \text{ fm}^{-3}$. The absolute number of particles can be calculated from this density if the size of the CP-odd bubbles is known. For example, for a domain with radius $5 \text{ fm}$, as suggested in [25], we estimate a yield of 90-100 particles from our model.

A measure for the size of correlated domains of particles is the correlation length $\xi$, which can be calculated from the two-point correlation function

$$D(x, \tau) \propto \int \frac{d\kappa \kappa^2}{2\pi^2} j_0(\kappa x)|\chi_\kappa(\tau)|^2,$$

where $x = \mu r$ denotes the dimensionless length. For large $x$,

$$D(x, \tau) \propto \exp\left(\frac{-x^2}{\xi^2(\tau)}\right),$$

from which we estimate the correlation length for $t_{\text{end}}$. As a lower limit for the domain radius, we obtain $\xi \simeq 2.4 \text{ fm}$ for $\mu(T = 0)$ and $\xi \simeq 1.3 \text{ fm}$ for $\mu(T_{sp})$.

The momentum spectrum of the produced particles at $t_{\text{end}} \approx 10 \text{ fm}$ is shown in Fig. 4. Its maximum is located at about the value of $\kappa$ as the maximum of $\mu_\kappa$ in Fig. 2, which had been calculated for the system without back reaction. The spectrum deviates significantly from a thermal Bose-Einstein distribution.
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Figure Captions

Fig. 1. Form of the singlet effective potential for different temperatures: (a) $T = 0$, (b) $T = T_{sp}$, (c) $T_{sp} \leq T \leq T_d$, (d) $T_d < T$.

Fig. 2. The characteristic exponent $\mu_\kappa$ for the solutions of the classical evolution equations (solid line) and for the analytical solution of the Sine-Gordon-approximation (dotted line).

Fig. 3. Particle number density per $fm^3$ as a function of time for $\mu(T = 0) = 412 \text{ MeV}$ (dashed line) and $\mu(T_{sp}) \simeq 676 \text{ MeV}$ (full line).

Fig. 4. Momentum spectrum of produced particles at $t_{end} \simeq 10 \text{ fm}$, including back reaction and using $\mu(T = 0) = 412 \text{ MeV}$.
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