Vaccination Strategies: a comparative study in an epidemic scenario

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Abstract. Epidemics are an extremely important matter of study within the Mathematical Modeling area and can be widely found in the literature. Some epidemiological models use differential equations, which are very sensible to parameters, to represent and describe the diseases mathematically. For this work, a variation of the SIR model is discussed and applied to a certain epidemic scenario, wherein vaccination is introduced through two different strategies: constant vaccination and vaccination in pulses. Other epidemiological and population aspects are also considered, such as mortality/natality and infection rates. The analysis and results are performed through numerical solutions of the model and a special attention is given to the discussion generated by the parameters variation.

1. Introduction

Mathematical models are essential tools that allow scientific and descriptive analysis, since their versatility admit the obtainment of several valuable results for a better comprehension of natural phenomena. Amongst a great number of matters studied in the mathematical modelling area, the epidemiology is one of big importance, since it may bring very useful results and is able to aid public health agencies to better deal with epidemics.

The SIR model (KERMACK, 1927), pioneer at the epidemiological modelling area, describes the spread of an infectious disease by proposing the division of a particular population in three individual categories: susceptible (S), infected (I) and recovered (R). A system of differential equations is used to run this model, which is fed with parameters representing population/epidemic intrinsic characteristics, such as the infection rate. More recent studies have developed mathematical models considering specific aspects for distinct epidemics, such as seasonal influence, cross-immunity and disease containment measures (STONE, 2000).

Attempts of epidemic control and containment in terms of vaccination programs are often seen and suggested. The governmental programs present, among their goals, the prevention of infant/older children diseases, a major health issue in any society. In 2012, the Global Vaccine Action Plan (GVAP) was approved in order to improve the access to vaccines around the world; thus, thousands of deaths can be avoided until 2020.

Several mathematical models allow the vaccination study in a certain population. Vaccination strategies are considered a measure that presents one of the best outcomes in preventing infectious diseases (ZHI-LONG HE, 2015). In (ANDERSON, 1995), an efficiency analysis is...
performed for the case when a certain parcel of the population is submitted to a vaccination campaign. The eradication condition of infectious diseases is achieved when the parcel of successfully vaccinated individuals is bigger than a specific critical value. The insertion of a vaccination campaign that could achieve this critical value of vaccinated individuals is often unreachable due to economical and time limitations.

In this work, it will be presented and analyzed a model that contains two vaccination strategies: a constant vaccination, wherein a certain parcel of the population that is born is immediately vaccinated; and vaccination in pulses, wherein a certain parcel of the population is vaccinated periodically. It is proposed the utilization of a function that illustrates the vaccination in pulses behavior. An implementation of the parameters variation study is performed. Finally, epidemiological scenarios wherein both strategies are alternately inserted in the same population and scenarios wherein only one of them is applied are numerically compared and discussed.

2. Model and Methods

The traditional SIR model was modified in order to be used with a time variant population considering the influence of the campaign of constant vaccination (STONE, 2000), where a certain parcel of the population \( \rho \) is vaccinated right after they are born, preventing its possibility of infection. Then, they are directly introduced in the group of recovered individuals. In order to use such model, it is necessary to describe some parameters that represent characteristics inherent of each disease. Thus, \( \beta, \alpha \) and \( \gamma \) represents infection rate, recovery rate and mortality and natality rates, respectively. Considering \( S(t) \) the class of susceptible individuals, \( I(t) \) the class of infected individuals and \( R(t) \) the class of recovered individuals, the equations that describe the SIR model in this specific vaccination situation are (SABETI, 2011):

\[
\begin{align*}
\frac{dS}{dt} &= (1 - \rho_1)\gamma - (\beta I + \gamma)S \\
\frac{dI}{dt} &= \beta IS - (\gamma + \alpha)I \\
\frac{dR}{dt} &= \alpha I + (\rho_1 - R)\gamma
\end{align*}
\]  

(1)

Some vaccination programs applied by health public agencies base themselves in periodic vaccination campaigns (AGUR, 1993), which happens to be an highly suggested method by the literature. In these cases, groups considered at risk are repeatedly vaccinated until the level of transmission and infection is reduced or stopped. So, defining \( \omega \) as the period of the pulses and \( \phi \) as their phase, the pulse modeling of this kind of vaccination is proposed by the insertion of a function \( \rho_1 \) given by:

\[
\rho_1 = \begin{cases} 
0 & \text{if } \cos(\omega t + \phi) \leq 0 \\
\rho(\cos(\omega t + \phi)) & \text{if } \cos(\omega t + \phi) > 0
\end{cases}
\]  

(2)

In this analysis, the set of parameters used is \( \beta = 1800, \gamma = 0.02, \alpha = 100 \) (STONE, 2000) and \( \phi = 0 \).

3. Results and Discussions

The constant vaccination model has two equilibrium points: \( P_1 = ((1 - \rho), 0) \), the infection free equilibrium, and \( P_2 = (\frac{\gamma + \alpha}{\beta}, \frac{\gamma(\beta - \gamma - \alpha - \beta \rho_1)}{\beta(\gamma + \alpha)}) \), the epidemic equilibrium. It is shown in Figure 1a) that the infected individuals behavior of a population not submitted to a vaccination campaign presents more infection peaks and tends to an equilibrium point with a bigger number of infected people, in comparison with the behavior of the same population submitted to either of the
vaccination strategies discussed here. It is also visible that, the bigger the parcel of the vaccinated population is, the bigger is the amplitude between infection peaks (and, therefore, the smaller is the number of these peaks in a same time interval).

Figure 1b) presents a phase plan of the susceptible and infected individuals. It is perceptible that the bigger the population parcel vaccinated is, the faster the epidemic stabilizes. When $\rho = 90\%$, for instance, the system tends fast towards the equilibrium point. This result is clearly explained by the Principle of the Mass Action, which basically says that the infection rate in an epidemic scenario is proportional to the number of infected and susceptible individuals currently present in that population.

(a) Graphics of infected population versus time submitted to a constant vaccination, where $\rho$ is assumed either 0% (no vaccination whatsoever), 70%, 80% or 90%.

(b) Phase plan of infected population versus susceptible population, where $\rho$ is assumed either 70%, 80% or 90%.

Figure 1. Constant vaccination results

(a) Infected individuals versus time for pulse vaccination campaigns with different periods.

(b) Susceptible individuals versus time for pulse vaccination campaigns with different periods.

Figure 2. Pulse vaccination results

Figure 2a) makes a comparison between three pulse vaccination strategies. The first strategy consists in vaccinating the population twice a year, whereas the second strategy vaccines the population once a year. Finally, the third strategy proposes a vaccination pulse happening in every two years. Such period change for the pulse is achieved by changing the value of $\omega$, responsible for the frequency of the pulse function. Figure 2b) shows the susceptible population
dynamics under the influence of these vaccination strategies. It is noticed that the bigger $\omega$ is, the more stable is their behavior.

In the Figure 3a) and 3b) the dynamics of the infected individuals are presented, in three situations: without the influence of the vaccination, with a constant vaccination technique and in a pulse vaccination scenario. It is seen that the population not submitted to a vaccination campaign tends to an equilibrium point with a higher number of infected individuals when compared to the vaccinated populations. The strategy of constant vaccination presents more infection peaks, but with smaller amplitudes: fewer susceptible individuals are infected in each infection peak.

![Figure 3. Comparison between different vaccination strategies](image)

4. Numerical comparison between vaccine insertion strategies

For this analysis, $\rho = 0.2$, $\omega = 4$ and $\phi = 0$ are chosen and the differential equation system is computationally simulated in three cases: when just one of the vaccinations strategies is applied in the population and when both of them are alternately applied in the population. For the last case, a algorithm would solve each time step for both of the strategies and then use the one it found to work more efficiently, based on a specific efficiency parameter. In the case here described, this parameter is the number of infected people (the lesser infected after a infinitesimal time, more efficient the algorithm considers the strategy to be for that specific time).

Figure 4a) presents the behavior of the three classes in the simulation of a traditional SIR model (wherein no vaccination is applied). The analysis of Figure 4b) shows that the susceptible population in the alternated simulation tends to be smaller than the others. This is expected, since the vaccinated population is automatically moved from susceptible to removed class. Thus, the smaller number of susceptible individuals does not mean more infected individuals, but shows the effectiveness of the vaccination campaign.

Figure 4c) shows that the smaller number of infected individuals in the alternated simulation in fact translates how the strategy of using the algorithm for determining the alternated vaccination works. Finally, Figure 4d) shows that the alternated vaccination strategy makes the removed population tend to a number larger than those reached by the pulse vaccination and constant vaccination strategies alone. Again, this exemplifies the strategy effectiveness by vaccinating a larger number of vaccinated population.
(a) Behavior of the three individual classes through time

(b) Susceptible population behavior in different simulations

(c) Infected population behavior in different simulations

(d) Removed population behavior in different simulations

Figure 4. Simulation results for $\beta = 3.0, \gamma = 0.5, \alpha = 1.0, S_0 = 8.95$ e $I_0 = 1.05$.

5. Conclusion

The modified model has shown itself as an efficient tool for describing and foreseeing the epidemic behavior of a population under vaccination strategies. The proposal of the function $\rho$ for the pulse vaccination strategy seems reasonable for governmental vaccination campaigns and worked well, accordingly to previous works in literature (STONE & SABETI, 2011). In the case of this work, the pulse vaccination has shown better and faster results and stability, which does not mean that will occur every time. However, the numerical analysis performed with an alternated vaccination strategy indicates that a better outcome could be reached if an algorithm like the one used for this work decided the best sequence of vaccinations to apply in each time interval.

Since it is a perk of mathematical models its versatility and how they could be used to analyze different epidemics and sets of parameters, the methods and model here presented could be used in order to aid prevention studies and containment measures by health agencies, and even generate cost-benefit and time-sensitive campaigns.

References
[1] Stone, L., Shulgin, B. and Agur, Z., (2000) Theoretical examination of the pulse vaccination policy in the SIR epidemic model Mathematical and computer modelling, v. 31, n. 4, p. 207-215.
[2] Sabeti, M., Castilho, A. R. (2011). Modelo Epidêmico Discreto SIR com estrutura etária e aplicaçãode vacinação em pulsos e constante, Tese de Doutorado, UFPE.
[3] Agur, Z., Cojocaru, L., Mazor, G., Anderson, R. M., Danon, Y. L. (1993). Pulse mass measles vaccination across age cohorts. Proceedings of the National Academy of Sciences, v. 90, n. 24, p. 11698-11702.
[4] Barros, A. M. R. (2007) Modelos Matemáticos de equações diferenciais ordinárias aplicados à epidemiologia UFLA.
[5] Jardim, C. L. T. F., Prates, D. B., Silva, J. M., Ferreira, L. A. F., and Kritz, M. V. (2015). Simulations of a epidemic model with parameters variation analysis for the dengue fever. In Journal of Physics: Conference Series (Vol. 633, No. 1, p. 012008). IOP Publishing.
[6] Kermack, W.; McKendrick, A. (1927) Contributions to the mathematical theory of diseases. Proceedings of the Royal Society, 141 1933.
[7] R. Anderson and R. May, (1995) Infectious Disease of Humans, Dynamics and Control, Oxford University Press, New York, NY, USA.

[8] Prates, Dérek B., Silva, Jaqueline M., Gomes, Jessica L. and Kritz, Maurcio V., An epidemiological model with vaccination strategies, AIP Conference Proceedings, 1738, 480035, 2016.