Study on the correlation model between the change of aircraft’s mass center and oil supply flow rates

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Abstract: The change of aircraft’s mass center affects the stability and control of flight, and the change of oil mass in the fuel tanks is one of the important factors that lead to the change of the aircraft’s mass center. In this paper, in order to ensure the stability and reliability of the oil supply for engine and minimize the deviation between the aircraft’s mass center and the ideal mass center, a correlation model between the change of aircraft’s mass center and the oil supply flow rates is presented. According to the requirements of the aircraft’s mass center, the optimal oil supply flow rates of each tank can be obtained quickly, and the deviation between the predicted mass center and the ideal mass center can be minimized during flight. The calculation examples show that the maximum deviation between the predicted mass center and the ideal mass center is 0.63μm when the attitude of the aircraft is always unchanged, and it's 13.6cm when the attitude of the aircraft is constantly adjusted, which can meet the practical requirements. Therefore, the model presented in this paper has a good applicability in solving the problem of oil supply for the mass center’s balance of aircraft.

1. Introduction

The stability of flight is the basic of aircraft navigation. As one of the important parameters of the aircraft, the position of mass center affects the stability and control of the aircraft [1,2]. Therefore, the mass center should be designed in the early stage of aircraft’s research and development[3,4], a reasonable fuel supply strategy should be formulated for each tank to ensure the stability and reliability of fuel supply for engine, and at the same time, the mass center of aircraft will not deviate greatly from the ideal mass center, which is of great significance for the control of the aircraft.

Adapting to the development of all-electric aircraft, digital intelligent integrated fuel management system has become the development trend of fuel system. At present, the researches on fuel supply strategy of aircraft can be roughly divided into two categories: one is that only one tank can supply fuel at the same time, and tanks are emptied one by one [5-7]; the other is that multiple tanks can supply fuel at the same time. The formulation of fuel supply strategy should take the selection of working tanks and the determination of fuel supply flow rates of tanks into consideration. Wei et al.[8] divided all tanks into two groups according to its distribution in the aircraft, the two tank groups work alternatively according to the actual mass center of the aircraft. Long[9] adopted the same grouping method and
integrated the calculation module of mass and mass center position to realize the automatic control of fuel supply. Guo et al. [10] used traversal method to try to calculate the deviation between predicted mass center and ideal mass center under different oil supply solutions to determine the optimal oil supply solution. The calculation efficiency was relatively low, and the oil supply strategy could not be adjusted in real time to meet the requirements of mass center position when the ideal mass center curve was temporarily adjusted.

Aiming at these shortcomings of existing researches in the fuel supply strategy, this paper proposes a correlation model between the change of aircraft’s mass center and fuel supply flow rates, which directly correlates the change of aircraft’s mass center with fuel supply flow rates of fuel tanks. Compared with traversal method, this model has a great advantage in calculation efficiency.

On the basis of this model, the fuel supply strategy is calculated step by step according to the idea of "greedy strategies", which can ensure the stable and reliable fuel supply for engine. In the meantime, the deviation between the predicted mass center of the aircraft and the ideal mass center can be minimized during the whole flight process.

2. The hypothesis of aircraft structure and flight conditions

In order to simplify the problem, the following assumptions are proposed for the structure and flight conditions of the aircraft:

(1) The mass of the aircraft components remains unchanged except for the oil mass.

(2) The engine’s fuel consumption flow rate, ideal mass center position and flight attitude are given in the flight process.

(3) The oil tanks are cuboid [8], and the length, width and height directions of oil tanks are parallel to the X, Y and Z axes respectively of the aircraft body coordinate system. The longitudinal center axis of the aircraft is X-axis, and the direction of the front is positive. Y axis is perpendicular to the longitudinal profile of the aircraft where X axis is located; The z-axis is determined by the right-hand rule [1].

(4) The fuel consumption flow rate of the engine and fuel supply flow rates of the oil tanks do not change in a very short time ∆t. therefore, they can be regarded as the superposition of several step functions in the whole time period.

(5) When the aircraft maintains a level flight state, the fuel bottom area remains unchanged, and the change of oil masses in the tanks is reflected by the change of liquid level height; When the attitude of the aircraft changes, the attitude of tanks relative to the ground will also change, and the fuel distribution of the tanks will change accordingly, making the mass center of the aircraft shift. The schematic of fuel shape changing with aircraft attitude is shown in Fig. 1. The left figure is the state of the fuel tank when the aircraft is in level flight, and the dotted line on the right figure represents the fuel level plane after attitude change of the aircraft.

(6) The oil outlet of the oil tank is at the bottom of the oil tank, and the oil tank can supply oil smoothly no matter how much oil is left.

Fig. 1  The schematic of fuel shape changing with aircraft attitude

(7) Fuel sloshing and liquid viscosity are ignored, and the liquid level of fuel is simplified to a plane [10].
3. The proposal of the correlation model between aircraft’s mass center change and oil supply flow rates

When planning the fuel supply flow rates of tanks in a certain period of time, if the traversal method or the method based on genetic algorithm is used, in order to minimize the deviation between the predicted mass center and the ideal mass center of the aircraft, the fuel supply flow rates of tanks should be adjusted continuously and the mass center deviation should be calculated one by one to obtain the optimal solution. Although these methods can get the best fuel supply solution in theory, their computational efficiencies are very low. Therefore, a new model is proposed in this paper to relate the change of the mass center of aircraft with the fuel flow rates. As long as the requirements of the mass center change of aircraft in a certain period of time are known, the optimal selection of working tanks and fuel supply flow rate of each tank within that period of time can be quickly obtained.

Suppose the no-load mass of the aircraft is $M_0$, the mass center of the aircraft in the no-load state is $(x_0, y_0, z_0)$, fuel density is $\rho$. The aircraft has several fuel tanks, which can be divided into two types: the main tanks, which can supply fuel directly to the engine, and the vice tanks, which can only supply fuel to the other tanks. Suppose $i$ is the serial number of tanks. $j$ is the serial number of time. The interval from time $j$ to time $j+1$ is $\Delta t_j$. The remaining oil quantity of the tank $i$ at time $j$ is $m_{ij}$, the oil supply flow rate within the time $\Delta t_j$ is $m_{i\cdot j}$. Then the predicted aircraft’s mass center $c_j(x_{cj}, y_{cj}, z_{cj})$ at time $j$ can be calculated as:

$$
\begin{align}
    x_{cj} &= \frac{\sum_i (x_{ij} \cdot m_{ij}) \cdot \rho + M_0 \cdot x_0}{\sum_i m_{ij} \cdot \rho + M_0} \\
    y_{cj} &= \frac{\sum_i (y_{ij} \cdot m_{ij}) \cdot \rho + M_0 \cdot y_0}{\sum_i m_{ij} \cdot \rho + M_0} \\
    z_{cj} &= \frac{\sum_i (z_{ij} \cdot m_{ij}) \cdot \rho + M_0 \cdot z_0}{\sum_i m_{ij} \cdot \rho + M_0}
\end{align}
$$

(1)

In Equation (1), $(x_{ij}, y_{ij}, z_{ij})$ is the mass center of remaining oil in the tank $i$ at time $j$.

Since the aircraft fuel tanks are divided into the main fuel tanks and the vice fuel tanks, the changes of the aircraft’s mass center position are analysed when the main fuel tanks and the vice fuel tanks work separately and jointly. On this basis, the fuel supply flow solution of each tank within the time $\Delta t_j$ is carried out to minimize the deviation between the predicted mass center and the ideal mass center of the aircraft at the time $j+1$.

3.1 Only one of the main tanks works

If only one main tank supplies oil to the engine at the flow rate $m_{ij}$ within the time $\Delta t_j$, then the oil mass supplied in $\Delta t_j$ should be $m_{ij} \cdot \Delta t_j$. This part of fuel is supplied to the engine, burn into a gas emission, and provide thrust to vehicles. Before the interval $\Delta t_j$, suppose the mass center position of this part of the fuel in the aircraft is $(\hat{x}_{ij}, \hat{y}_{ij}, \hat{z}_{ij})$. Then the predicted mass center of aircraft $c_{j+1}(x_{c(j+1)}, y_{c(j+1)}, z_{c(j+1)})$ after this part of the fuel is expelled is:

$$
\begin{align}
    x_{c(j+1)} &= \frac{\sum_i (x_{ij} \cdot m_{ij}) \cdot \rho + M_0 x_0 - \hat{m}_{ij} \cdot \hat{x}_{ij} \cdot \Delta t_j}{\sum_i m_{ij} \cdot \rho + M_0 - \hat{m}_{ij} \cdot \hat{x}_{ij} \cdot \Delta t_j} \\
    y_{c(j+1)} &= \frac{\sum_i (y_{ij} \cdot m_{ij}) \cdot \rho + M_0 y_0 - \hat{m}_{ij} \cdot \hat{y}_{ij} \cdot \Delta t_j}{\sum_i m_{ij} \cdot \rho + M_0 - \hat{m}_{ij} \cdot \hat{y}_{ij} \cdot \Delta t_j} \\
    z_{c(j+1)} &= \frac{\sum_i (z_{ij} \cdot m_{ij}) \cdot \rho + M_0 z_0 - \hat{m}_{ij} \cdot \hat{z}_{ij} \cdot \Delta t_j}{\sum_i m_{ij} \cdot \rho + M_0 - \hat{m}_{ij} \cdot \hat{z}_{ij} \cdot \Delta t_j}
\end{align}
$$

(2)

Therefore, from time $j$ to time $j+1$, the change of aircraft’s predicted mass center in the X direction is
\[
\Delta x_{c_{j}} = x_{c_{j+1}} - x_{c_{j}} \\
= \frac{-m_{ij} \cdot \Delta t_{j} \cdot (\sum_{i} m_{ij} \cdot x_{ij})}{(\sum_{i} m_{ij} \cdot x_{ij}) \cdot (\sum_{i} m_{ij} \cdot \rho + M_{0})} + \frac{(\sum_{i} m_{ij} \cdot \rho + M_{0} - \bar{m}_{ij} \cdot \Delta t_{j}) \cdot \sum_{i} m_{ij} \cdot \rho + M_{0}}{\sum_{i} m_{ij} \cdot \rho + M_{0} - \bar{m}_{ij} \cdot \Delta t_{j}} \\
= \frac{\sum_{i} m_{ij} \cdot \rho + M_{0} - \bar{m}_{ij} \cdot \Delta t_{j} \cdot x_{ij}}{\sum_{i} m_{ij} \cdot \rho + M_{0} - \bar{m}_{ij} \cdot \Delta t_{j}} + \frac{\sum_{i} m_{ij} \cdot \rho + M_{0} - \bar{m}_{ij} \cdot \Delta t_{j}}{\sum_{i} m_{ij} \cdot \rho + M_{0} - \bar{m}_{ij} \cdot \Delta t_{j}} \\
= \frac{\sum_{i} m_{ij} \cdot \rho + M_{0} - \bar{m}_{ij} \cdot \Delta t_{j}}{\sum_{i} m_{ij} \cdot \rho + M_{0} - \bar{m}_{ij} \cdot \Delta t_{j}} \Delta x_{c_{j}}
\]

The derivation process of the mass center change in Y direction and Z direction is the same as above. Then the change of the predicted mass center \(\Delta c_{j}(\Delta x_{c_{j}}, \Delta y_{c_{j}}, \Delta z_{c_{j}})\) of aircraft is:

Equation (2) - Equation (1)

\[
= c_{j+1} - c_{j} \\
= \begin{cases} \\
\Delta x_{c_{j}} = \frac{m_{ij} \cdot \Delta t_{j} \cdot (x_{c_{j}} - \bar{x}_{ij})}{\sum_{i} m_{ij} \cdot \rho + M_{0} - \bar{m}_{ij} \cdot \Delta t_{j}} \\
\Delta y_{c_{j}} = \frac{m_{ij} \cdot \Delta t_{j} \cdot (y_{c_{j}} - \bar{y}_{ij})}{\sum_{i} m_{ij} \cdot \rho + M_{0} - \bar{m}_{ij} \cdot \Delta t_{j}} \\
\Delta z_{c_{j}} = \frac{m_{ij} \cdot \Delta t_{j} \cdot (z_{c_{j}} - \bar{z}_{ij})}{\sum_{i} m_{ij} \cdot \rho + M_{0} - \bar{m}_{ij} \cdot \Delta t_{j}} \\
\end{cases}
\]

(4)

If the time interval \(\Delta t_{j}\) is small enough, then \(\bar{m}_{ij} \Delta t_{j} \ll (\sum_{i} m_{ij} \cdot \rho + M_{0})\). Therefore, the mass of aircraft \((\sum_{i} m_{ij} \cdot \rho + M_{0} - \bar{m}_{ij} \cdot \Delta t_{j})\) can approximate to \((\sum_{i} m_{ij} \cdot \rho + M_{0})\) in \(\Delta t_{j}\).

The equation (4) shows that the change of predicted mass center \(\Delta c_{j}\) is related to the position of predicted mass center \(c_{j}\) and mass center of the part of fuel \((\bar{x}_{ij}, \bar{y}_{ij}, \bar{z}_{ij})\) in time \(j\), which will be eliminated in the time interval \(\Delta t_{j}\). And \(\Delta c_{j}\) is proportional to the flow rate of oil emission, \(\Delta c_{j} \propto \bar{m}_{ij}\).

Therefore, to simplify the calculation reasonably, \(\Delta c_{j}\) can be approximately expressed as:

\[
\begin{cases} \\
\Delta x_{c_{j}} = \frac{\Delta t_{j} \cdot (x_{c_{j}} - \bar{x}_{ij})}{\sum_{i} m_{ij} \cdot \rho + M_{0}} \cdot \bar{m}_{ij} \\
\Delta y_{c_{j}} = \frac{\Delta t_{j} \cdot (y_{c_{j}} - \bar{y}_{ij})}{\sum_{i} m_{ij} \cdot \rho + M_{0}} \cdot \bar{m}_{ij} \\
\Delta z_{c_{j}} = \frac{\Delta t_{j} \cdot (z_{c_{j}} - \bar{z}_{ij})}{\sum_{i} m_{ij} \cdot \rho + M_{0}} \cdot \bar{m}_{ij} \\
\end{cases}
\]

(5)

Suppose \(S_{ij}(S_{xi_{j}}, S_{yi_{j}}, S_{zi_{j}})\) is the mass center change caused by per mass fuel supply by main tank \(i\) within time \(\Delta c_{j}\), referred to as the influence of the main tank \(i\) on the mass center of the aircraft at time \(j\), the expression is as follows:
\[
\begin{align*}
S_{xij} &= \frac{\Delta t_j \cdot (x_{cj} - \bar{x}_{ij})}{\sum_i m_{ij} \cdot \rho + M_0} \\
S_{yij} &= \frac{\Delta t_j \cdot (y_{cj} - \bar{y}_{ij})}{\sum_i m_{ij} \cdot \rho + M_0} \\
S_{zij} &= \frac{\Delta t_j \cdot (z_{cj} - \bar{z}_{ij})}{\sum_i m_{ij} \cdot \rho + M_0}
\end{align*}
\] (6)

Therefore, \( \Delta \mathbf{c}_j = S_{ij} \cdot \dot{m}_{ij} \) (7)

Equation (7) relates the change of aircraft's mass center \( \Delta \mathbf{c}_j \) to the oil supply flow rate \( \dot{m}_{ij} \). When the oil is only supplied by main tank \( i \) within the time \( \Delta t_j \), \( \Delta \mathbf{c}_j \propto \dot{m}_{ij} \), the proportional factor is \( S_{ij} \).

### 3.2. Only one of the vice tanks works

If only one vice oil tank \( i \) supplies oil to the remaining oil tanks during time interval \( \Delta t_j \), and the oil flow rate is \( \dot{m}_{ij} \), then the oil mass supplied by tank \( i \) in \( \Delta t_j \) is \( \dot{m}_{ij} \cdot \Delta t_j \). This part of fuel is transferred to other tanks within \( \Delta t_j \), and its mass center position changes. Suppose the mass center position of this part of the fuel in the aircraft is \( (\bar{x}_{ij}, \bar{y}_{ij}, \bar{z}_{ij}) \) and \( (\bar{x}_{(i+1)}, \bar{y}_{(i+1)}, \bar{z}_{(i+1)}) \) respectively before and after the transfer. Thus, the predicted mass center of aircraft \( \mathbf{c}_{j+1}(x_{c_{(j+1)}}, y_{c_{(j+1)}}, z_{c_{(j+1)}}) \) after the transfer of fuel should be expressed as:

\[
\begin{align*}
x_{c_{(j+1)}} &= \frac{\sum_i (x_{ij} \cdot m_{ij}) \cdot \rho + M_0 \cdot x_0 + \dot{m}_{ij} \cdot \Delta t_j \cdot (\bar{x}_{(i+1)} - \bar{x}_{ij})}{\sum_i m_{ij} \cdot \rho + M_0} \\
y_{c_{(j+1)}} &= \frac{\sum_i (y_{ij} \cdot m_{ij}) \cdot \rho + M_0 \cdot y_0 + \dot{m}_{ij} \cdot \Delta t_j \cdot (\bar{y}_{(i+1)} - \bar{y}_{ij})}{\sum_i m_{ij} \cdot \rho + M_0} \\
z_{c_{(j+1)}} &= \frac{\sum_i (z_{ij} \cdot m_{ij}) \cdot \rho + M_0 \cdot z_0 + \dot{m}_{ij} \cdot \Delta t_j \cdot (\bar{z}_{(i+1)} - \bar{z}_{ij})}{\sum_i m_{ij} \cdot \rho + M_0}
\end{align*}
\] (8)

Therefore, from time \( j \) to time \( j+1 \), the change of predicted mass center of aircraft \( (\Delta x_{cj}, \Delta y_{cj}, \Delta z_{cj}) \) is

\[
\Delta \mathbf{c}_j = \text{equation (8) } - \text{equation (1)} = \mathbf{c}_{j+1} - \mathbf{c}_j
\]

\[
\begin{align*}
\Delta x_{cj} &= \frac{\dot{m}_{ij} \cdot \Delta t_j \cdot (\bar{x}_{(i+1)} - \bar{x}_{ij})}{\sum_i m_{ij} \cdot \rho + M_0} \\
\Delta y_{cj} &= \frac{\dot{m}_{ij} \cdot \Delta t_j \cdot (\bar{y}_{(i+1)} - \bar{y}_{ij})}{\sum_i m_{ij} \cdot \rho + M_0} \\
\Delta z_{cj} &= \frac{\dot{m}_{ij} \cdot \Delta t_j \cdot (\bar{z}_{(i+1)} - \bar{z}_{ij})}{\sum_i m_{ij} \cdot \rho + M_0}
\end{align*}
\] (9)

According to the equation (9), the change of predicted mass center \( \Delta \mathbf{c}_j \) is related to the position of the mass center of this part of fuel in time \( j \) and time \( j+1 \), which will be transferred in the time interval \( \Delta t_j(\bar{x}_{ij}, \bar{y}_{ij}, \bar{z}_{ij}) \), which can be expressed as \( (\bar{x}_{ij}, \bar{y}_{ij}, \bar{z}_{ij}) \) and \( (\bar{x}_{(i+1)}, \bar{y}_{(i+1)}, \bar{z}_{(i+1)}) \) respectively. And \( \Delta \mathbf{c}_j \) is proportional to the flow rate of oil transfer, \( \Delta \mathbf{c}_j \propto \dot{m}_{ij} \).

Then \( \Delta \mathbf{c}_j \) can also be expressed as:
\[
\begin{align*}
\Delta x_{cj} &= \frac{\Delta t_j \cdot (\ddot{x}_{i(j+1)} - \ddot{x}_{ij})}{\sum_i m_{ij} \cdot \rho + M_0} \cdot \dot{m}_{ij} \\
\Delta y_{cj} &= \frac{\Delta t_j \cdot (\ddot{y}_{i(j+1)} - \ddot{y}_{ij})}{\sum_i m_{ij} \cdot \rho + M_0} \cdot \dot{m}_{ij} \\
\Delta z_{cj} &= \frac{\Delta t_j \cdot (\ddot{z}_{i(j+1)} - \ddot{z}_{ij})}{\sum_i m_{ij} \cdot \rho + M_0} \cdot \dot{m}_{ij}
\end{align*}
\] (10)

Set \( S_{ij} (S_{xij}, S_{yij}, S_{zij}) \) as the mass center change caused by fuel supply per mass by vice tank \( i \) within time interval \( \Delta t_j \), referred to as the influence of vice tank \( i \) on the mass center of aircraft at time \( j \), the expression is as follows:

\[
\begin{align*}
S_{xij} &= \frac{\Delta t_j \cdot (\ddot{x}_{i(j+1)} - \ddot{x}_{ij})}{\sum_i m_{ij} \cdot \rho + M_0} \\
S_{yij} &= \frac{\Delta t_j \cdot (\ddot{y}_{i(j+1)} - \ddot{y}_{ij})}{\sum_i m_{ij} \cdot \rho + M_0} \\
S_{zij} &= \frac{\Delta t_j \cdot (\ddot{z}_{i(j+1)} - \ddot{z}_{ij})}{\sum_i m_{ij} \cdot \rho + M_0}
\end{align*}
\] (11)

Then \( \Delta c_j = S_{ij} \cdot \dot{m}_{ij} \) (12)

Equation (12) relates the change of aircraft’s mass center \( \Delta c_j \) to the oil supply flow rate \( \dot{m}_{ij} \). When the oil is only supplied by vice tank \( i \) within the time \( \Delta t_j \), \( \Delta c_j \propto \dot{m}_{ij} \), the proportional factor is \( S_{ij} \).

3.3. The main tanks and vice tanks work at the same time

Equations (6) and (11) establish the relationship between the change of mass center position of aircraft and oil supply flow rates. It should be noted that the calculation processes of the influence \( S_{ij} \) of the main tanks and vice tanks on the mass center are different.

If multiple tanks supply oil at the same time in \( \Delta t_j \), the predicted mass center change of the aircraft can be approximated to the linear superposition of the mass center change when each tank supplies oil separately, and the predicted mass center change of the aircraft is shown in Equation (13):

\[
\begin{align*}
\sum_i (S_{xij} \cdot \dot{m}_{ij}) &= \Delta x_{cj} \\
\sum_i (S_{yij} \cdot \dot{m}_{ij}) &= \Delta y_{cj} \\
\sum_i (S_{zij} \cdot \dot{m}_{ij}) &= \Delta z_{cj}
\end{align*}
\] (13)

3.4. Planning of oil supply flow rate of each tank within time step \( \Delta t_j \)

Based on this model, when planning the oil supply flow rates of tanks with a time step, it is not necessary to use traversal method to repeatedly adjust the oil supply flow rates and calculate the corresponding deviation of the mass centers. Instead, the optimal oil supply flow rates can be directly solved through the mathematical relationship. The method is as follows:

It can be seen from equation (13) that the optimal fuel supply flow rates \( \dot{m}_{ij} \) can be obtained by solving the linear equations if the requirements of the aircraft’s mass center change \( \Delta c_j \) from time \( j \) to time \( j+1 \) and the selection of working tanks within \( \Delta t_j \) are given.

Suppose the matrix formed by the influences of working tanks on the aircraft’s mass center \( S_j \) is \( S_{ij} \), and the vector composed of the oil supply flow rates of working tanks is \( \dot{m}_j \), and then Equation (13) can also be expressed as

\[ S_j \cdot \dot{m}_j = \Delta c_j \] (14)

Restricted by the structure limit of aircraft, most three tanks can work at the same time [10,11]. Thus the equations (14) contain three equations in the X, Y and Z directions and one to three unknowns,
namely the oil supply flow rates of each tank \( m_{ij} \). If three oil tanks work at the same time in a certain time step, the optimal oil supply flow rate of each tank can be obtained by solving the equation (14) accurately. If only 1 or 2 oil tanks work within a certain time step, the least square solution of the equation (14) can be obtained. The solution method is as follows:

\[
m_j = (S_j^*S_j)^{-1}S_j^*\Delta c_j
\]  

(15)

The advantage of the correlation model between the change of the aircraft's mass center and the fuel supply flow rates is as follows:

1. The relation between the change of the mass center position and the fuel supply flow rates is constructed. When there is only one fuel tank working, the change of the aircraft's mass center is approximately proportional to the fuel supply flow rate.

2. When multiple oil tanks work at the same time, the change of mass center position is approximated to the linear superposition of the mass center change when each tank works separately, which dramatically reduces the scale of computing.

Therefore, as long as the requirements of mass center change from one moment to the next and the selection of working tanks are known, the equations can be solved directly to obtain the optimal oil supply flow rate of each tank. Compared with the traversal method and genetic algorithm, this model has a great efficiency advantage.

4. Steps for formulating oil supply strategy

4.1. Formulation of oil supply strategy within the time of \( \Delta t_j \)

1. According to the requirements of mass center-time curve, the requirement of mass center change \( \Delta c_j \) of the aircraft from time \( j \) to time \( j + 1 \) is calculated. The influence \( S_j \) of each tank on the mass center at time \( j \) is calculated by using the correlation model of the change of aircraft’s mass center and the fuel supply flow rates.

2. List all possible working tank combinations.

3. In the formula, \( M_{i0} \) is the fuel quality when the tank \( i \) is fully loaded.

   \[
   M_{i0} - \sum_j (m_{ij} \cdot \Delta t_j) > 0?
   \]

   If this formula is not valid, it means that the fuel in the fuel tank \( i \) has been used up, so the fuel tank \( i \) can not work, and all the working tank combinations including this tank should be given up;

   If this formula is valid, proceed to the next step.

4. Solve the above equations to obtain the optimal oil supply rate of each tank \( m_{ij} \) in every tank combination.

If three oil tanks supply oil at the same time \( \Delta t_j \), then the exact solution of the equations is solved to obtain the oil supply flow rate of each tank. If only 1 or 2 oil tanks work in the time \( \Delta t_j \), then the least square solution of the equations is solved to obtain the oil supply flow rate of each tank.

5. \( m_{ij} \leq \sum_i m_{ij} \leq 1.2 m_j \) (tank \( i \) can only be taken from main tanks)

In this equation, \( m_{ij} \) is the prescribed oil consumption rate for the engine within \( \Delta t_j \). The total oil supply flow rate of main tanks should not be less than or more than 1.2 times as much as the engine oil consumption flow rates (to conserve fuel).

If the condition is not met, the working tank combination should be given up.

6. \( 0 \leq m_{ij} \leq m_{i\max} \) (\( m_{i\max} \) is the maximum oil supply rate of tank \( i \))

If the condition is not met, the solution should be given up.

7. \( 0 \leq M_{i0} - \sum_j (m_{ij} \cdot \Delta t_j) \leq M_{i0} \) (Remaining oil mass requirement)

If the above conditions are not met, then the solution should also be given up.

8. Judge whether there is a feasible solution after the above selections, and whether the solution is unique?

   If a there is only 1 feasible solution, this solution is chosen.
If there is more than 1 feasible solution, the solution with the lowest total fuel consumption flow rate of the main tanks is selected.

If all solutions fail to meet the conditions after selections, then modify the solution to force it to meet the constraints, and the solution that can minimize the deviation of aircraft predicted mass center from ideal mass center is selected.

Through the above steps, the final oil supply solution within $\Delta t_j$ is obtained.

4.2. The Formulation of fuel supply strategy during the whole flight process

According to the idea of greedy strategies [12], the steps in 4.1 are iterated within the time range to obtain the fuel supply solution within each time step, and the fuel supply strategy in the whole flight process is finally obtained, so as to minimize the deviation between the predicted mass center of aircraft and the ideal mass center in the whole process.

5. Example analyses

5.1. Parameters of a certain type of aircraft and related fuel supply restrictions

| The tank number | 1   | 2   | 3   | 4   | 5   | 6   |
|-----------------|-----|-----|-----|-----|-----|-----|
| The position of tank center (m) |     |     |     |     |     |     |
| x               | 8.91304 | 6.91304 | -1.68696 | 3.11304 | -5.28696 | -2.08696 |
| y               | 1.20652 | -1.39348 | 1.20652 | 0.60652 | -0.29348 | -1.49348 |
| z               | 0.61669 | 0.21669 | -0.28331 | -0.18331 | 0.41669 | 0.21669 |
| The size of tank (m) |     |     |     |     |     |     |
| length          | 1.5 | 2.2 | 2.4 | 1.7 | 2.4 | 2.4 |
| width           | 0.9 | 0.8 | 1.1 | 1.3 | 1.2 | 1   |
| Height          | 0.3 | 1.1 | 0.9 | 1.2 | 1   | 0.5 |
| The initial oil mass (m$^3$) |     |     |     |     |     |     |
| 0.3 | 1.5 | 2.1 | 1.9 | 2.6 | 0.8 |
| The maximum fuel flow rate of the tank (kg/s) | 1.1 | 1.8 | 1.7 | 1.5 | 1.6 | 1.1 |

(1) The no-load mass $M_0$ of the aircraft is 3000kg, and the fuel density is 850 kg/m$^3$.
(2) There are six oil tanks in the aircraft. The position of center point, size, initial oil volume and upper limit of oil supply flow rate of each tank are shown in Table 1.
(3) Main tanks 2, 3, 4 and 5 can supply fuel directly to the engine, while vice tanks 1 and 6 can only supply fuel to the main tank 2 and 5 respectively, but can not supply fuel directly to the engine. The oil supply diagram of the aircraft is shown in Figure 2.
(4) Restricted by the structure of the aircraft, no more than 2 main oil tanks can supply fuel to the engine at the same time, and no more than 3 oil tanks can supply fuel at the same time [11].

Fig. 3 shows the fuel consumption flow rate curve of the aircraft engine during a mission, and Fig. 4 shows the ideal mass center position curve of the aircraft [11]. In order to minimize the deviation between the predicted mass center of the aircraft and the ideal mass center during the flight, the oil supply flow rate of each tank is required to be reasonably planned.

5.2. Example verification

5.2.1. The example verification under the condition that the aircraft always keeps level flight
Assuming that the aircraft is always in the state of level flight, the fuel supply solution within each time step is calculated on the basis of the correlation model between the change of aircraft’s mass center and the fuel supply flow rates. According to the idea of "greedy strategies", through repeated iteration in the time range, the deviation between the predicted mass center and the ideal mass center of the aircraft in the whole flight process is finally minimized.

Fig. 3 The engine fuel consumption flow curve [11]

Fig. 4 The ideal mass center position curve of the aircraft [11]
5.2.2. The example verification under the condition that the attitude of aircraft is continuously adjusted

With the other conditions unchanged, if the attitude of the aircraft is constantly adjusted over time (only the change of pitch angle is considered in this paper), the curve of pitch angle is shown in Fig. 5. The fuel supply strategy is re-planned according to the steps in 4.2.1. It's worth noting that the fuel shape change caused by the attitude change of the aircraft should also be considered in the calculation, and the fuel mass center calculation method is adjusted according to the fuel shape.

5.3. Results analyses

5.3.1. Analysis of the example results under the condition that the aircraft always keeps level flight

Python programming is used to obtain the fuel supply strategy under the condition that the aircraft always keeps level flight. It only takes 47.3s to calculate the oil supply strategy within 7200s, and 0.0066s to calculate the solution within a second. Therefore, the oil supply strategy can be adjusted in real time according to the adjustment of the ideal mass center curve.

As can be seen from Fig.6, the maximum deviation between the predicted aircraft’s mass center and the ideal mass center is only 6.3e-7m, that is, 0.63μm, which is in good agreement with each other.
Fig. 7 Oil supply flow rate curve of tanks under the condition that the aircraft always keeps level flight

From the perspective of time history, the deviation between the predicted mass center of the aircraft and the ideal mass center generally shows a fluctuating upward tendency. It is speculated that the reason is as follows: with the increase of time, some tanks run out of fuel, and there are fewer options for the selection of working tanks. Even if a combination selection is the best solution that can meet the requirements of the mass center, it must be forced to give up because of the fuel exhaustion of a certain tank in the combination selection, and only the remaining combinations can be chosen. In this case, there is a certain deviation between the predicted mass center and the ideal mass center. As time goes on, the deviation fluctuates upward.

As can be seen from the fuel supply flow rate curve of each tank in Figure 7, the fuel supply flow rate of each tank changes smoothly, which effectively reduces the mechanical damage to the tanks caused by frequent switching.

5.3.2. Analysis of example results under the condition that the aircraft attitude is constantly adjusted

Fig. 8 The deviation curve of the aircraft's predicted mass center from the ideal mass center under the condition that the aircraft attitude is constantly adjusted
The fuel supply strategy under the condition that the aircraft’s pitch angle is constantly adjusted is calculated. It only takes 52.2s to calculate the oil supply strategy within 7200s, and 0.0073s to calculate the solution within a second. Therefore, the oil supply strategy can be adjusted in real time according to the adjustment of the ideal mass center curve.

As can be seen from Figure 8, the maximum deviation between the predicted aircraft’s mass center and the ideal mass center is 13.6cm, which is in relatively good agreement with each other.

![Fig. 9 Oil supply flow rate curve of tanks under the condition that the aircraft attitude is constantly adjusted](image)

As can be seen from the fuel supply flow rate curve of each tank in Fig. 9, each tank is switched on and off frequently. It is speculated that the reason is: when the pitch angle of the aircraft is constantly adjusted, the mass center shifts rapidly in a short time, which makes the fuel supply strategy need to change rapidly constantly so as to meet the requirements of mass center position. Due to the rapid adjustment of the pitch angle, the oil supply solution is inevitably hysteretic, so the maximum deviation of the mass centers obtained is larger than that in the level flight state, but it is still only 13.6cm, which can meet the practical needs.

6. Conclusion

(1) A correlation model between the change of the aircraft’s mass center and the fuel supply flow rates is proposed, which associates the change of the mass center of the aircraft with the fuel supply flow rates. When there is only one fuel tank works, the change of aircraft’s mass center is approximately proportional to the fuel flow rate.

(2) If multiple oil tanks supply oil at the same time, the change of mass center position is approximated to the linear superposition of mass center change when each tank works separately, which greatly simplifies the calculation.

(3) The maximum deviation between the predicted mass center and ideal mass center is only 0.63μm and 13.6cm under the condition of level flight and constant attitude adjustment.

(4) The optimization program consumes less time and has higher computational efficiency than traversal method and other methods. The fuel supply strategy can be adjusted in real time according to the ideal mass center curve to meet the requirements of the aircraft’s mass center position. Therefore, the model presented in this paper has a good applicability in solving the problem of fuel supply for mass center balance of aircraft.
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