Two-nucleon momentum distribution and correlation in $A=6$ systems

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Abstract. The momentum distribution reflects the two-nucleon correlation in nuclei. The momentum distribution for the valence nucleons is calculated for both $^6$He and $^6$Li in a three-body model of $^{\alpha+N+N}$. The ground state solution for the three-body Hamiltonian is obtained accurately using correlated basis functions. The distribution depends on the type of the $N$-$N$ interaction. With use of a realistic potential, the $^6$He momentum distribution exhibits a dip around $2\text{ fm}^{-1}$ characteristic of $S$-wave motion. In contrast to this, the $^6$Li momentum distribution is similar to that of the deuteron; no dip appears because it is filled with the $D$-wave component arising from the tensor force.

1. Introduction

The correlation in nuclei plays an important role in binding a Borromean three-body system which has no pairwise bound states. An experiment using the technique of intensity interferometry [1] has been done in order to extract the spatial correlation function of the two neutrons in the halo nuclei such as $^6$He, $^{11}$Li and $^{14}$Be. In this experiment the momenta of the two neutrons and the core nucleus are measured after the dissociation of the halo nucleus. Another experiment concerning the two-nucleon correlation has been done by Piecetzky et al. [2]. The experiment measures the two-nucleon momentum distribution in two-nucleon knock out processes, $^{12}\text{C}(e,e'np)$, $^{12}\text{C}(e,e'np)$. R. Shiavilla et al. have calculated the two-nucleon momentum distributions of the ground states of nuclei with mass number $A \leq 8$ [3].

Recently, an experiment has been performed at RIKEN to probe the spatial correlation in $^6$He and $^6$Li from the relative momentum distribution of the valence nucleons [4]. The basic idea of this experiment is to utilize the well-established one-nucleon exchange process, $^6\text{He}(p, dn)\alpha$, $^6\text{Li}(p, dp)\alpha$. Since the reaction process is simple, the experiment expected to give information sensitive to the nucleon correlation.

The momentum distribution is one of the candidates which enables one to get information on the nucleon-nucleon correlation. A theoretical analysis of the momentum distribution in $^6$He and $^6$Li should be important to understand the physics involved in the experiment. In this report, we show the momentum distribution between the valence nucleons in $^6$He and $^6$Li and its comparison with use of an effective force and realistic one as $N$-$N$ potential [5].
2. Three-body calculation

The wave functions for $^6\text{He}$ and $^6\text{Li}$ are determined from variational calculations for the core($\alpha$ particle)+$N+N$ three-body system which is specified by the Hamiltonian

$$H = T_r + T_R + U_1 + U_2 + v_{12}.$$  

The subscripts of the kinetic energies $T$ stand for the relative distance vector $r$ between the two nucleons, and the relative distance vector $R$ from the $\alpha$ particle to the center of mass of the two nucleons. The potential $U_i$ is the $N-\alpha$ potential and $v_{12}$ is the $N-N$ potential. The $\alpha$ particle is treated as a structureless particle, but its compositeness is taken into account through the elimination of redundant states as explained below.

As the two-nucleon potential $v_{12}$, we use a realistic potential, G3RS potential [6], which contains central, spin-orbit and tensor terms. To show the importance of the correlation clearly, we employ an effective potential model to compare results of calculation with the realistic potential model. We use the Minnesota potential [7] (MN) which has only central terms and a mild short-ranged repulsion as the effective potential. This potential renormalizes the effect of the tensor force into the central force and reproduces the binding energy and the root mean square radius of the deuteron.

As for the $N-\alpha$ potential $U_i$, we adopt a parity-dependent phenomenological potential [8], abbreviated as KKNN, which contains the central and spin-orbit components, and reproduces very well the low-energy $N-\alpha$ scattering phase shifts of $S$ and $P$ waves. The Coulomb potential for $p-\alpha$ is taken into account. The KKNN potential is deep enough to have an $S$-wave bound state which should be removed because no bound states exist for $^5\text{He}$ and $^6\text{Li}$. The elimination is carried out by imposing that the trial wave function to be orthogonal to the $0s_{1/2}$ bound state of the KKNN potential [12].

Trial wave functions for the ground states of $^6\text{He}$ and $^6\text{Li}$ are expressed, in $LS$ coupling scheme, as a combination of explicitly correlated Gaussians:

$$\Psi_{JM} = \sum_{i=1}^{K} C_i \Psi_{JM}(\Lambda_i, A_i, u_i),$$  

(2)

with the basis function

$$\Psi_{JM}(\Lambda=(LS), A, u, x) = (1 - P_{12}) \left\{ e^{-\frac{1}{2}A x^2} \left[ Y_L(\tilde{u} x) \chi_S(1, 2) \right]_{JM} \eta_{TM_r}(1, 2) \right\}.$$  

(3)

Here the permutation $P_{12}$ ensures the antisymmetry of the valence nucleons. We note that the basis function of Eq. (3) has a definite parity $(-1)^L$. As the ground states of $^6\text{He}$ and $^6\text{Li}$ have a positive parity, this basis function cannot be used for $L=1$. We need to extend the basis function to make it possible to include $L=1$ and a positive parity. This is made possible by replacing $Y_{LM_r}(\tilde{u} x)$ by $[Y_L(\tilde{u} x)Y_1(u' x)]_{LM_r}$ [11]. For the case of two nucleons with $L=1$, this replacement results in a new basis function

$$\Psi_{JM}(\Lambda=(1S), A, x) = (1 - P_{12}) \left\{ e^{-\frac{1}{2}A x^2} \left[ [Y_1(x_1)Y_1(x_2)]_1 \chi_S(1, 2) \right]_{JM} \eta_{TM_r}(1, 2) \right\}.$$  

(4)

The basis function is specified by a set of nonlinear parameters, the orbital and spin angular momenta $\Lambda=(LS)$, a $2 \times 2$ positive-definite, symmetric matrix $A$, and a $2 \times 1$ matrix $u$. The symbol $\tilde{\cdot}$ indicates the transpose of a matrix, and the square bracket $[\cdot\cdot]$ denotes the angular momentum coupling. The short-hand notation $\bar{x} A x$ stands for $A_{11}x_1^2 + 2A_{12}x_1x_2 + A_{22}x_2^2$, where the coordinates $x_1$ and $x_2$, are the distance vectors of the valence nucleons from the $\alpha$ particle. The cross term $A_{12}x_1x_2$ describes explicitly the two-nucleon correlation, which is vital to obtain a precise solution in a relatively small basis dimension [9]. The angular part of the basis function
is expressed by the solid spherical harmonics, $Y_{L,M_L}^{ll_1} (\vec{u} \vec{x}) = |\vec{u} \vec{x}|^L Y_{L,M_L}^{ll_1} (\vec{u} \vec{x})$, specified by a global vector $\vec{u} \vec{x} = u_1 \vec{x}_1 + u_2 \vec{x}_2$. The ratio of $u_1$ to $u_2$ characterizes the coordinate which is responsible for the rotation of the system [9, 10]. The isospin part of the system is expressed by $\eta T M_T$.

The set of $\Lambda = (LS)$ included in the present calculation are $(LS) = (00), (11)$ for $^6$He ($J^P = 0^+$) and $(LS) = (01), (10), (11), (21)$ for $^6$Li ($J^P = 1^+$). Here the basis function is given by Eq. (3) for even $L$ and by Eq. (4) for odd $L$, respectively.

To search for good basis functions, we use the stochastic variational method (SVM) [9]. The SVM increases the basis dimension one by one by testing a number of candidates that are chosen randomly. The candidates are actually generated by giving random numbers to the parameters chosen from physically important multi-dimensional parameter space. The SVM works efficiently to take care of both the short-ranged repulsion of the realistic force and the elimination of the redundant states.

To calculate the momentum distribution, we introduce the Wigner distribution function

$$W(r, k) = \frac{1}{(2\pi)^{2J+1}} \sum_M \left\langle \Psi_{JM} \left| r + \frac{s}{2} \vec{R} \right\rangle \left\langle r - \frac{s}{2} \vec{R} \right\rangle \Psi_{JM} \right\rangle_{ST} e^{ik \cdot s} dR dx. \quad (5)$$

Here $\langle \ldots \rangle_{ST}$ indicates that the integration is to be performed over the spin and isospin coordinates. The density and momentum distributions are expressed as follows

$$\rho(r) = \int W(r, k) dk, \quad \rho(k) = \int W(r, k) dr. \quad (6)$$

3. Results

Full calculations which couple all possible $\Lambda$ channels give the results listed in Table 1. The table shows the binding energies, the contribution of tensor force and the distance between the valence nucleons. The binding energy does not strongly depend on the potential models, but the constitutions from each term of the Hamiltonian is different. For the case of realistic force, the tensor component is quite large in $^6$He and deuteron. Large $N$-$N$ distance shows a halo structure in $^6$He and agrees with the $N$-$N$ distance estimated by the intensity interferometry experiment [1]. In $^6$Li, the distance between the valence nucleons is smaller than that of deuteron. Attraction from the core makes the deuteron in $^6$Li smaller than free deuteron. The common lack of the binding energy of the G3RS potential can be explained by at least three effects: One is the deficiency of the attraction in the $D$ and $F$ waves of the KKN potential [13, 14]. Next is the effect of three-body forces [15] and the third is the distortion of the $\alpha$ core [16].

| Force       | $^6$He   | $^6$Li   | $^6$He   | $^6$Li   | $d$ |
|-------------|----------|----------|----------|----------|-----|
| Energy[MeV] | $-0.421$ | $-0.460$ | $-3.91$  | $-3.31$  | $-2.20$  | $-2.28$  |
| Tensor[MeV] | 0.107    | -12.3    | -11.5    |          |      |
| $N$-$N$ distance [fm] | 5.05 | 4.86 | 3.48 | 3.58 | 3.90 | 3.96 |

Table 1. The ground state properties of $^6$He, $^6$Li and deuteron.

3.1. Density distribution

Figure 1 plots the density distributions $\rho(r)$ (normalized to unity) of the two-nucleon relative motion in $^6$He, $^6$Li and the deuteron. The densities calculated using the G3RS potential (right panel) show central dips due to the short-ranged repulsion, but beyond $r=1.5$ fm they are similar to those calculated with the MN potential (left panel). The density of $^6$He reaches furthest in the distance, and as its result the density around $r=1.5$ fm is considerably smaller than that of $^6$Li. Comparing the densities between $^6$Li and the deuteron, we see that the $np$ relative motion in $^6$Li shrinks compared to that of the deuteron (See also Table 1).
3.2. Momentum distribution

It is well-known that the momentum distribution of the \( np \) relative motion in the deuteron shows different behavior in the \( S \)- and \( D \)-wave contributions. As displayed in the right panel of Fig. 2, the \( S \)-wave contribution to the momentum distribution is peaked at lower momentum and has a dip at \( k \sim 2 \text{ fm}^{-1} \). The \( D \)-wave component of the deuteron, however, fills the dip in spite of the small \( D \) state probability, 4.8\% in the G3RS potential. This characteristics of the distribution is supported by experiment. In contrast to this, the momentum distribution (left panel) obtained with the MN potential does show a dip because it has no \( D \)-wave component, and in addition the momentum distribution decreases rapidly with increasing \( k \) because the short-ranged repulsion is not as strong as the G3RS potential. To compare with experiment at \( k \) higher than \( 2 \text{ fm}^{-1} \), however, it is important to include effect of the \( \Delta \) excitation.

The momentum distributions of \(^6\text{He}, \(^6\text{Li} \) and the deuteron are compared in Fig. 3 for the G3RS (right panel) and MN (left panel) potentials. The realistic potential of G3RS gives the momentum distributions characterized as follows: The momentum distribution of \(^6\text{Li} \) is very similar to that of the deuteron, but the one of \(^6\text{He} \) differs from them, showing a clear dip at \( k \sim 2 \text{ fm}^{-1} \). These features are understood from the difference in the partial wave contents of the \( N-N \) relative motion; \(^6\text{Li} \) contains the \( D \)-wave component as the deuteron does, whereas \(^6\text{He} \) is dominated by the \( S \)-wave component. The most distinctive difference between \(^6\text{He} \) and \(^6\text{Li} \) appears around \( k \sim 2 \text{ fm}^{-1} \). If the measurement of the momentum distribution is made in this region, one can learn the role of the tensor force acting between the valence nucleons, provided that the \( \Delta \) excitation is still not so important. R. Shiavilla et al. have calculated the two-nucleon momentum distributions of the ground states of nuclei [3]. They have considered the momentum distributions averaged over all the \( np \) or \( pp \) pairs in the nuclei, while we have calculated the momentum distribution for the valence nucleons in \(^6\text{He} \) and \(^6\text{Li} \). In spite of these differences, both calculations show similar results concerning the dominance of \( np \) distribution over \( nn \) (or \( pp \)) distribution, particularly in the region of \( k=2\text{ fm}^{-1} \), and the important role of the tensor force which lead to those characteristics.
3.3. Uncorrelated basis

The correlated motion of the valence nucleons reflects on the two-nucleon correlation function

$$\rho(x_1,x_2, \theta) = \frac{1}{2J+1} \sum_M \langle \Psi_{JM} | x_1 x_2 \rangle \langle x_1 x_2 | \Psi_{JM} \rangle_{ST}. \tag{7}$$

Figure 4 (left panel) displays the contour maps of $8\pi^2 x^4 \sin \theta \rho(x,x,\theta)$ for $^6$He calculated from the G3RS potential. The MN potential gives similar map. We clearly see asymmetric patterns with two distinct peaks.

To clarify the importance of the correlation, we examine the function $\rho(x_1,x_2, \theta)$ which is generated from an “uncorrelated” basis function $\Phi$ for $^6$He. For this purpose we take a combination of the two $p$-shell harmonic-oscillator functions, $\Phi = \sqrt{1-C^2} | S=0 \rangle + C | S=1 \rangle$. Here the shell model is extended to allow for different size parameters for both the components. The parameters are determined so as to maximize the overlap, $|\langle \Phi | \Psi_{00} \rangle|^2$, with the $^6$He ground-state wave function $\Psi_{00}$ obtained using the G3RS potential. The maximum value of $|\langle \Phi | \Psi_{00} \rangle|^2$ is 0.75. The simple wave function $\Phi$ has a surprisingly large overlap with the realistic wave function $\Psi_{00}$. Though the overlap is fairly large, $\Phi$ includes no correlated configurations and indeed the energy calculated with $\Phi$ is high (8.77 MeV). The two-nucleon correlation function $\rho(x_1,x_2, \theta)$ calculated from $\Phi$ becomes a function of $\cos^2 \theta$, so that function multiplied by $8\pi^2 x^4 \sin \theta$ is symmetric with respect to $\theta=90^\circ$. See Fig. 4. An asymmetry with respect to $\theta=90^\circ$ would indicate the presence of correlation in the $A=6$ nuclei. Comparing the left panel with right one in Fig. 4, we learn that the two-nucleon interaction enhances the asymmetric pattern.

Figure 5 compares the momentum distributions of $^6$He corresponding to the three different wave functions, those obtained with G3RS, MN and the uncorrelated one. Both the G3RS and MN distributions are similar up to the dip region. Beyond $k \sim 2$ fm$^{-1}$ the momentum distribution of G3RS surpasses that of MN, which is due to the difference in the short-range correlation. The uncorrelated wave function gives the momentum distribution which is quite different from those of the correlated wave functions even at $k \sim 1$ fm$^{-1}$.
4. Summary

To study the correlation and the momentum distribution of the two-nucleon relative motion in the ground states of $^6$He and $^6$Li, we have described these states in a three-body model of $\alpha+\text{N}+\text{N}$ where the $\alpha$ particle is assumed to be an inert core. We used a parity-dependent $\alpha$-N potential which reproduces the low-energy scattering phase shifts, and two different types of $N$-$N$ interactions for the two valence nucleons. One is a realistic potential which contains the tensor and spin-orbit forces and the other is an effective potential which includes no tensor component. These were used to compare how much the different $N$-$N$ potentials affect the correlation and the momentum distribution. We have obtained the solution of the three-body problem by approximating the $^6$He and $^6$Li ground state wave functions in terms of a combination of explicitly correlated Gaussian basis functions that provides us with a solution of high accuracy.

The momentum distributions of the $N$-$N$ relative motion have been compared between $^6$He and $^6$Li. The distributions obtained with the effective potential show the pattern characteristic of $S$-wave dominance and fall rapidly as the momentum increases. In the case of the realistic potential, the momentum distribution in $^6$Li is very similar to that of the deuteron. That is, both the $S$- and $D$-waves contribute to the momentum distribution which monotonically decreases with an increasing momentum. In contrast to this, the $^6$He momentum distribution is dominated by the $S$-wave, showing a clear dip at $k \sim 2\text{ fm}^{-1}$. The most prominent difference in their momentum distributions thus shows up around $k=2\text{ fm}^{-1}$. The difference between $^6$He and $^6$Li is primarily due to whether or not the tensor force plays an important role of mixing the $D$-state probability between the $N$-$N$ relative motion.

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