Analysis of the mechanical coupling characteristics of a monolithic tri-axis MEMS gyroscope

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Abstract. The mechanical coupling characteristics of a monolithic tri- axis micro-electromechanical gyroscope have an important effect on the performance of the gyroscope, and the coupling is directly related to the processing error. This paper introduces the structure and principle of a monolithic tri-axis micro-electromechanical gyroscope, establishes a dynamic coupling model, and studies the mechanical coupling problem caused by beam width errors. The finite element analysis tool was used to simulate the harmonic response of the gyroscope structure with beam width error. The driving-to-sensing and sensing-to-sensing coupling characteristics, the equivalent coupling angular rate of the driving mode to each sensing mode, and the cross-coupling coefficients between the sensing modes are investigated. The coupling error under unit beam width error are obtained and can be used as a reference for the structural design of the single-chip tri-axis gyroscope and the precision requirements for processing.

1. Introduction
MEMS gyroscopes have been widely used in aerospace, automotive, consumer electronics, medical and other fields. In practical applications, it is necessary to measure the tri-axial angular rate simultaneously. Tri-axis gyroscopes are usually combined by multiple single-axis or dual-axis gyroscope packages, or multiple sensitive structures fabricated on a single chip [1-2]. Miniaturization is a trend in the development of micro-electromechanical gyroscopes. The single-chip tri-axis micro-electromechanical gyroscopes studied in this paper has a single-sensitive structure to realize the tri-axis gyroscope function, which can greatly reduce the volume, improve the integration level, and reduce the cost. However, the mechanical coupling problem of a single-structure tri-axis gyroscope is more complicated [3], and the coupling characteristics caused by processing errors in the structure need to be analysed.

With the advancement of the integration of micro-electromechanical gyroscopes, research on single-chip tri-axis gyroscopes have gradually increased. Beijing Institute of Aerospace Control Devices has proposed a monolithic integrated tri-axis micro-electromechanical gyroscope [4]. The bias stabilities were 53.4 (°) / h, 70.8 (°) / h and 18.4 (°) / h in x-axis, y-axis and the z-axis, respectively.

A micro-inertial monolithic tri-axis gyroscope with distributed mass is proposed by the Micro Inertial Instrument Laboratory of Southeast University [4]. The gyroscope has complex decoupling beams, anchors and separate sensing frames. The coupling stiffness is analysed, and the results show that the driving mode and the sensing mode are completely decoupled.

At present, most single-chip tri-axis gyroscopes adopt the four-mass structure [5-7]. Gyroscopes with different beam structures and anchors have different mechanical characteristics. There are also
tri-axis gyroscopes that use a single mass as a resonator, such as Fairchild Semiconductor’s FIS1100 [8].

Understanding and controlling modal coupling in micro-electromechanical devices is vital to the design of high-accuracy inertial sensors. Nanoscience Centre of University of Cambridge demonstrated that tuneable coupling stiffness and mode coupling can be achieved in a vacuum-sealed microelectromechanical silicon ring resonator through capacitive electrodes surrounding it [9]. Department of Mathematical Sciences of Montclair State University measured both the linear and nonlinear coupling between transverse modes in a nanomechanical resonator and found that as the modes are tuned through the degeneracy point, they remain linearly polarized, while their planes of vibration rotate by 90° [10]. Eyal Buks et al. induced mechanical coupling between 67 doubly clamped beam resonators by applying a controlled electrostatic interaction between them [11].

The single-chip tri-axis micro-electromechanical gyroscope studied in this paper uses a centre-supported four-mass structure [12]. The structure has complicated in-plane and out-of-plane vibration modes, including the driving mode and the 3 sensing modes. The natural frequencies of some vibration modes are close to each other and the vibration modes are liable to couple. Processing error always exists and makes the coupling more serious [13]. The mechanical coupling relationship between different modes is complex and has an important impact on the performance of the gyroscope. Therefore, it is necessary to investigate the coupling characteristics of the tri-axis gyroscope.

2. Monolithic tri-axis MEMS gyroscope structure

2.1. Structure

The single-chip tri-axis MEMS gyroscope studied in this paper is shown in Figure 1. The support frame and the four proof masses are connected by Y-shaped and N-shaped beams.

Combs are set on the masses to apply electrostatic forces and detect the motions in the plane of the structure. Plate electrodes are arranged above and below the masses to apply electrostatic force and detect the motions in the vertical direction of the structure plane. Two sinusoidal signals out of phase and with a DC offset are applied to the driving electrodes, and the four proof masses resonate at the driving mode frequency. For the convenience of description, the masses are numbered clockwise, and the comb electrodes are outlined. The positions of the driving electrodes and the sensing electrodes are marked.

![Figure 1. Schematic diagram of the tri-axis MEMS gyroscope.](image)

2.2. Vibration modes

The finite element analysis software ANSYS was used to simulate the vibration modes of the structure. The frequencies of the vibration modes are shown in Table 1, the working modes are shown in Figure 2, and several modes with frequencies close to the working modes are shown in Figure 3. The colours
in Figure 2 and Figure 3 represent the relative displacement, where blue is the smallest and red is the largest.

Table 1. Frequencies of vibration modes.

| No. | Frequency/Hz | Description | diagram number |
|-----|--------------|-------------|----------------|
| 1   | 4465.3       | Linear vibration along the z-axis | ——— |
| 2   | 4546.4       | The four masses move around the centre in plane, similar to an angular vibration | ——— |
| 3   | 4637.1       | Equivalent to the mode No. 4 rotated 90 degrees around the z-axis | ——— |
| 4   | 4637.9       | Mode No. 4 | Figure 3(a) |
| 5   | 4645.5       | x sensing mode | Figure 2(b) |
| 6   | 4647.7       | y sensing mode | Figure 2(c) |
| 7   | 4676.8       | Driving mode | Figure 2(a) |
| 8   | 4717.8       | Mode No. 8 | Figure 3(b) |
| 9   | 4721.6       | Mode No. 9 | Figure 3(c) |
| 10  | 4722.1       | Equivalent to the mode No. 9 rotated 90 degrees around the z-axis | ——— |
| 11  | 4774.5       | z sensing mode | ——— |
| 12  | 4909.8       | Mode No. 12 | Figure 3(d) |

Figure 2. Working modes: (a) driving mode, (b) x sensing mode, (c) y sensing mode, (d) z sensing mode.

It can be seen from the modal analysis results that the frequency differences between the driving and sensing modes are in the range of 29.1 ~ 97.7Hz, and there are other non-working modes between the driving mode and the z sensing mode.
2.3. **Principle and characteristics**

Referring to the modal shape diagram, in the driving mode, 4 proof masses move radially relative to the central anchor point, every two opposite masses vibrate in the opposite direction. When an angular rate is applied along x-axis direction, due to the Coriolis effect, the two masses in the x-direction (MASS1 and MASS3) remain stationary in z-direction, while the two masses in the y-direction (MASS2 and MASS4) vibrate reversely along the z-direction. If the gyro is subjected to a z-axis angular rate, MASS1 and MASS3 vibrate in reversely along the y-axis, and MASS2 and MASS4 vibrate reversely along the x-axis. Both x and y sensing modes are out-of-plane modes, and z sensing mode is an in-plane mode.

The structure of the monolithic tri-axis MEMS gyroscope has following advantages:
1) The single driving mode can reduce the number of driving comb required and save chip area;
2) Single anchor support can reduce the energy loss;
3) The fully symmetrical structure is beneficial to reduce errors through differential sensing method;
4) The simple beams contribute to reduce the process sensitivity.

2.4. **Coupling principle and dynamic model**

For a single-axis vibratory gyroscope, there is generally a coupling error from the driving mode to the sensing mode. This coupling error still exists in the monolithic tri-axis gyroscope. Moreover, a new coupling channel appears, that is the coupling between the sensing modes.

If there is structural asymmetry, the driving force will occur sensing displacement directly. The coupling principle between the sensing modes is displacement occurs on a sensing mode if there is a force (Coriolis or electrostatic force) applied to another sensing mode.

The simplified dynamic characteristics of the monolithic tri-axis MEMS gyroscope can be expressed as:

\[
\begin{bmatrix}
M_x \ddot{w} \\
M_y \ddot{x} \\
M_z \ddot{y} \\
M_z \ddot{z}
\end{bmatrix} + \mathbf{B} + \begin{bmatrix}
k_{ww} & k_{wx} & k_{wy} & k_{wz} \\
k_{ww} & k_{wx} & k_{wy} & k_{wz} \\
k_{ww} & k_{wx} & k_{wy} & k_{wz} \\
k_{ww} & k_{wx} & k_{wy} & k_{wz}
\end{bmatrix} \begin{bmatrix}
w \\
x \\
y \\
z
\end{bmatrix} = \begin{bmatrix}
f_{ew} \\
f_{ex} + f_{ea} \\
f_{ey} + f_{ey} \\
(f_{ez} + f_{ce})
\end{bmatrix}
\]  

(1)
Where \( w, x, y, \) and \( z \) represent the generalized displacements of the driving mode, the \( x, y \) and \( z \) sensing mode, respectively; 
\( M_w, M_x, M_y, \) and \( M_z \) represent the generalized masses of the driving mode, the \( x, y \), and \( z \) sensing modes, respectively; 
\( B \) is the damping coupling matrix, which is not discussed in this paper because the damping coupling is not as significant as the stiffness coupling for the structure studied. 
\( k_{ij}, i \in \{w, x, y, z\}, j \in \{w, x, y, z\}, \) are the elements of the stiffness matrix; 
\( f_{cw}, f_{cy}, f_{cz} \) are the Coriolis forces on the \( x, y, \) and \( z \) sensing modes, respectively; 
\( f_{cx}, f_{ey}, \) and \( f_{ez} \) are the feedback electrostatic forces applied on the \( x, y, \) and \( z \) sensing modes, respectively.

Taking the \( x \) sensing mode in the monolithic tri-axis gyroscope as an example, the coupling dynamics model is shown in Figure 4. Other sensing modes are similar to it. Due to the stiffness coupling, the electrostatic driving force, three Coriolis forces and three feedback electrostatic forces on the \( x, y, \) and \( z \) sensing modes will directly or indirectly affect the \( x \) sensing movement.

![Figure 4. Schematic diagram of coupling dynamic model.](image)

3. Simulation analysis of coupling characteristics caused by beam width error

3.1. Vibration mode frequency with a beam width error
In the study of the process error caused coupling characteristics of the gyroscope, a beam width error was artificially introduced as shown in Figure 5. Beam No. 2 has the narrowest width, and it can be
seen from Figure 2 that its deformation is relatively large in all working modes. Therefore, the width of beam No. 2 is selected as the main error source. A width error is applied to No. 2 beam to imitate the asymmetry of the beams caused during manufacturing. The finite element analysis software ANSYS is employed to conduct the simulation. The ideal width of beam No. 2 is 25 μm, we apply an +0.3 μm error in all simulations.

The natural frequencies of the vibration modes varied due to the beam width error. The natural frequencies of the 5 modes most relevant to the coupling characteristics are listed in Table 2.

| Mode                  | Without errors | With a +0.3μm width error in a single beam No.2 |
|-----------------------|----------------|-----------------------------------------------|
|                        | Natural frequency/Hz | Difference from driving frequency/Hz | Natural frequency/Hz | Difference from driving frequency/Hz |
| driving mode           | 4676.8          | 0                      | 4680.8          | 0                      |
| x sensing mode         | 4645.5          | 31.3                   | 4651.9          | 34.9                   |
| y sensing mode         | 4647.7          | 29.1                   | 4780.4          | 99.6                   |
| z sensing mode         | 4774.5          | 97.7                   | 4780.4          | 99.6                   |
| Mode No. 4             | 4637.9          | 38.9                   | 4644.4          | 36.4                   |

3.2. Driving-to-sensing coupling characteristics

Ideally, when the gyro is in the driving mode and there is no external input, each sensing mode should not be activated. However, due to the asymmetry of the beam, the supporting beam will transfer energy from the driving mode to sensing modes, causing coupling.

Figure 6. Amplitude-frequency characteristic curve of the driving-to-sensing coupling with 0.3 μm width error of a single beam.

In the study of driving-to-sensing coupling characteristics, the central anchor is set to be fixed and a sinusoidal force with an amplitude of 10^-7 N to each proof mass are applied to activate the driving mode, and the harmonic response of x, y, and z sensing modes are simulated. The amplitude-frequency characteristic of the harmonic response of each sensing modes in the range of ± 100Hz around the driving frequency is shown in Figure 6. The frequency-phase curves are omitted.

The coupling curves from driving axis to each sensing axis of the tri-axis gyroscope all have a peak at the driving frequency (4680.8Hz), and for the Y-axis and Z-axis, it is the highest peak. For the x-axis, the highest peak locates at 4644.4Hz, which is the natural frequency of mode No. 4.

Driving-to-sensing coupling causes displacement on the sensing modes, which can be regard as the equivalent angular rate on the sensing directions.
\[ \Omega = \frac{K_{sn} x_1 F_0}{4 \pi m u f_d F_1} \quad (2) \]

Where \( F_1 \) is the force applied to the mass during the simulation, \( F_0 \) is the driving forces in practice, \( x_1 \) is the simulated displacement, \( K_{sn} \) is the sensing mode stiffness, \( m, u, d, \) and \( f_d \) is the mass, displacement and frequency of the driving mode, respectively.

The driving-to-sensing coupling is mainly investigated at the natural frequency of the driving mode as the gyroscope is driven at this frequency. Based on the above simulation, the equivalent angular rate per 0.1\( \mu \)m beam width error at the driving frequency is shown in Table 3.

**Table 3. Equivalent angular rate per 0.1\( \mu \)m beam width error at driving frequency.**

| axis | Amplitude | Real part | Imaginary part |
|------|-----------|-----------|----------------|
| x    | 3.70\( \times 10^{-9} \) | -1.31\( \times 10^{-9} \) | 3.5\( \times 10^{-9} \) |
| y    | 6.60\( \times 10^{-9} \) | -2.34\( \times 10^{-9} \) | 6.17\( \times 10^{-9} \) |
| z    | 8.37\( \times 10^{-9} \) | 7.97\( \times 10^{-9} \) | -2.54\( \times 10^{-9} \) |

The Coriolis force has the same phase as the velocity of the driving vibration, so in the amplitude-frequency characteristics of the driving force to sensing displacement coupling, the real part corresponds to the in-phase error, and the imaginary part is the quadrature error which becomes zero after demodulation in theory. Therefore, the effective equivalent angular rate of driving-to-sensing coupling should be the real part, which is 293° / s for x axis, 525° / s for y axis and 1892° / s for z axis. While the severity of the coupling can be roughly judged based just on the amplitude.

### 3.3. The calculation method of sensing-to-sensing coupling characteristics

The sensing-to-sensing coupling is mainly evaluated through cross-coupling coefficients. Taking the x-to-y coupling as an example, the cross-coupling coefficient of X to Y is:

\[ k_{x2y} = \frac{A_{x \rightarrow y}}{A_{y \rightarrow y}} \quad (3) \]

Where \( A_{x \rightarrow y} \) and \( A_{y \rightarrow y} \) are the y sensing mode amplitudes response to the forces applied to the x sensing and y sensing modes, respectively. The forces applied can be the Coriolis force or the electrostatic force.

In order to study the effect of angular rate frequency on the cross-coupling coefficient, the formula of the amplitude of the sensing axis when the input angular rate has a certain frequency is derived.

When the driving frequency is \( \omega_d \) and the driving amplitude is unit 1, the displacement of the driving mode can be expressed as \( \mu_d(t) = \sin \omega_d t \). The Coriolis force is obtained by applying a unit angular velocity \( \Omega(t) = \sin \lambda t \) with a frequency of \( \lambda \):

\[ f = m \omega_d \left[ \sin (\omega_d + \lambda) t - \sin (\omega_d - \lambda) t \right] \quad (4) \]
Therefore, the sensing displacement has two components with frequencies of $\omega_d + \lambda$ and $\omega_d - \lambda$. The displacement and phase at the two frequency points are:

\[
\begin{align*}
\omega_d - \lambda : A_1 &= A(\omega_d - \lambda), \varphi_1 = \varphi(\omega_d - \lambda) \\
\omega_d + \lambda : A_2 &= A(\omega_d + \lambda), \varphi_2 = \varphi(\omega_d + \lambda)
\end{align*}
\]  

(5)

The total response of the sensing mode before demodulation is:

\[
A(t) = A_1 \left[ \sin(\omega_d - \lambda)t + \varphi_1 \right] + A_2 \left[ \sin(\omega_d + \lambda)t + \varphi_2 \right]
\]  

(6)

Setting the demodulation phase to the value at the driving frequency on the sensing mode frequency-phase curve, the amplitude of the sensing vibration after demodulation is:

\[
A_{\text{sen}}(t) = \frac{A_1}{2} \cos(-\lambda t + \varphi_1) + \frac{A_2}{2} \cos(-\lambda t + \varphi_2 - \varphi_0)
\]  

(7)

**Figure 7.** The curves of the sensing-to-sensing cross-coupling coefficients when the input angular rate frequency is in the range of 0-115Hz: (a) y-to-x and z-to-x sensing coupling, (b) x-to-y and z-to-y sensing coupling, (c) x-to-z and y-to-z sensing coupling.
3.4. Simulation of sensing-to-sensing coupling characteristic

The sensing-to-sensing cross coupling characteristics are investigated in two steps. First, we apply sinusoidal force to one of the 3 sensing axes in the FEA model, which has a width error of 0.3 μm with the beam No.2, and probe each response of the 3 sensing axes. The frequency range is set to 115Hz above and below the driving frequency. Then, the amplitudes of 3 sensing movements are calculated according to Formula (7) in MATLAB. Finally, the curves of the sensing-to-sensing cross-coupling coefficients are obtained according to Formula (3), as shown in Figure 7.

The peaks of the cross-coupling coefficient curves and the coupling coefficients at 0 Hz are shown in Table 4.

| Table 4. Cross-coupling coefficient in the case of a single defected beam (0.3 μm width error). |
|----------------|----------------|----------------|----------------|----------------|
| axis           | Peak angular rate frequency | Peak mode       | Peak coupling coefficient | Coupling coefficient at 0Hz |
| x-axis to y-axis | 33.9 Hz        | x sensing mode  | 126%            | 0.98%          |
| x-axis to z-axis | 33.9 Hz        | x sensing mode  | 13.9%           | 1.70%          |
| y-axis to x-axis | 28.6 Hz        | y sensing mode  | 157%            | 1.18%          |
| y-axis to z-axis | 0 Hz           | driving mode    | 2.12%           | 2.12%          |
| z-axis to x-axis | 36.8 Hz        | Mode No. 4      | 2.04%           | 1.23%          |
| z-axis to y-axis | 99.7 Hz        | z sensing mode  | 3.74%           | 1.27%          |

The peaks of the x-to-y and x-to-z coupling coefficient curves locate at the x-sensing mode frequency. The y-to-x coupling coefficient curve has only one peak, which locates at the modal frequency of the y-sensing mode. The y-to-z coupling coefficient curve has two higher peaks, which locate at the frequency of driving and y-sensing modes. The two higher peaks in the z-to-x coupling coefficient curve are at the frequencies of mode 4 and the z-sensing modes. The maximum coupling coefficient of the z-to-y is at the frequency of z-sensing mode. In summary, the maximum cross-coupling of one axis to another generally locates at the frequency of the sensing mode of that axis.

It is worth mentioning that if there is a 0.3 μm error in a single beam, the maximum x-to-y and y-to-x coupling coefficients both exceed 100%. In Table 4, the cross-coupling coefficients at 0 Hz reflect the sensing-to-sensing coupling with constant angular rate, which is about 1~2%.

In order to study the relationship of the cross-coupling coefficients when there are errors in multiple beams, width errors are applied to the same side or diagonal No. 2 beams, respectively. Because the coupling error has a linear relationship with the beam width error in a relatively small range, the cross-coupling coefficient under the 0.3μm error is scaled to coefficients with a 0.1 μm error, and are shown in Table 5.

| Table 5. Peak sensing-to-sensing cross-coupling coefficients for every 0.1μm beam width error. |
|----------------|----------------|----------------|----------------|----------------|
| Error beam position | Single beam | Same side beam | Diagonal beam |
| x-axis to y-axis    | 42.0%        | 14.4%          | 58.3%          |
| x-axis to z-axis    | 4.6%         | 0.9%           | 1.5%           |
| y-axis to x-axis    | 52.3%        | 15.2%          | 48.7%          |
| y-axis to z-axis    | 0.7%         | 2.5%           | 1.5%           |
| z-axis to x-axis    | 0.7%         | 3.0%           | 2.0%           |
| z-axis to y-axis    | 1.3%         | 3.6%           | 3.7%           |

With the increase of the number of beams with width error, some of the cross-coupling coefficients increase. However, there is also some coefficients decreased. With errors in the two beams on the same side, the two most significant coupling error (x-to-y and y-to-x) are both reduced to one-third of the case that has an error in a single beam. The reason probably is although the number of error beams increases, the overall structure symmetry is improved.
4. Conclusions

According to the simulation analysis in this paper, with asymmetric beams, the energy in one vibration mode couples to other modes, and even small processing error can lead to large coupling error. Referring to Figure 4, in an open-loop state, for example, when a Coriolis force is applied to the y-sensing mode, it couples to the x-sensing mode, causing coupling error. However, if the gyroscope is operated in the force-rebalance mode, the Coriolis force can be cancelled by the electrostatic force in real time, and the force coupled from the y-axis to the x-axis becomes zero. In this way, the coupling error can be completely eliminated in ideal situation. Therefore, the three-axis force rebalance working mode can suppress the coupling error of the monolithic tri-axis micro-electromechanical gyroscope caused by processing errors significantly.

The performance requirements for gyroscopes vary in different applications. According to the analysis of this article, the beam width error range that meets the design requirements can be given and be referred to in the fabrication process.

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