Fluctuations of persistent current

A G Semenov and A D Zaikin

1 I E Tamm Department of Theoretical Physics, P N Lebedev Physics Institute, 119991 Moscow, Russia
2 Institute for Nanotechnology, Karlsruhe Institute of Technology (KIT), 76021 Karlsruhe, Germany

E-mail: semenov@lpi.ru

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Abstract

We theoretically analyze equilibrium fluctuations of the persistent current (PC) in nanorings. We demonstrate that these fluctuations persist down to zero temperature provided the current operator does not commute with the total Hamiltonian of the system. For a model of a quantum particle on a ring we explicitly evaluate PC noise power which has the form of sharp peaks at frequencies set by the corresponding interlevel distances. In rings with many conducting channels, a much smoother and broader PC noise spectrum is expected. A specific feature of PC noise is that its spectrum can be tuned by an external magnetic flux indicating the presence of quantum coherence in the system.

(Some figures in this article are in colour only in the electronic version)

1. Introduction

Meso- and nanorings formed by normal conductors and pierced by external magnetic flux develop persistent currents [1]. This fundamentally important equilibrium effect is a direct consequence of quantum coherence of electrons which—at low enough temperatures—may persist up to distances exceeding the perimeter of such rings.

Does the persistent current (PC) fluctuate? At first sight it might appear reasonable to assume that at least for $T \to 0$ no such fluctuations could occur. Indeed, while at non-zero $T$ thermal fluctuations of the PC should be present [2], in the zero temperature limit the system approaches its (non-degenerate) ground state and, hence, no PC fluctuations should be possible.

Below we will demonstrate that in many cases this is not so. Namely, no PC fluctuations are expected in the zero temperature limit only provided the current operator commutes with the total Hamiltonian of the system, otherwise fluctuations of the persistent current can occur even in the ground state exactly at $T = 0$. Theoretical and experimental investigations of such PC fluctuations can give important additional information about the ground state properties of meso- and nanorings not contained in the average value of the PC.

Note that fluctuations of the PC in the ground state may be induced provided the ring interacts with some quantum dissipative environment. In this case, such interaction is responsible for (i) phase-breaking effects implying suppression of both quantum coherence and PC and (ii) non-vanishing fluctuations of the PC down to $T \to 0$. For example, it was demonstrated [3] that interaction with the Caldeira–Leggett environment decreases the average value of the PC and simultaneously increases PC fluctuations which are directly related to fluctuations in the environment itself. Such effects are of importance, e.g., for PC qubits in which quantum states can be entangled with those of the environment [4]. Fluctuations of the PC down to $T \to 0$ could also occur provided the number of particles in a ring fluctuates due to its interaction with some reservoir [5].

The situation considered here is entirely different: we do not assume the presence of interaction or particle exchange with any environment at all. Accordingly, quantum coherence of the system is fully preserved and no PC suppression takes place. As will be demonstrated below, quantum coherence implies the possibility of tuning of PC fluctuations by an external magnetic flux applied to the system.

2. The model and general relations

For definiteness, let us consider a simple model of a quantum particle with mass $M$ on a 1d ring of radius $R$ pierced by magnetic flux $\Phi$, see, e.g., figure 1. The particle position on the ring is parametrized by the angle $\theta$ which will be the quantum mechanical variable of interest in our problem. The
Hamiltonian for this system reads
\[ \hat{H} = \frac{(\hat{\phi} + \phi_0)^2}{2MR^2} + U(\theta), \]  
where \( \hat{\phi} = -i\frac{\partial}{\partial \theta} \) is the (dimensionless) flux operator, \( U(\theta) \) defines the potential profile for our particle, \( \phi_0 = \Phi/\Phi_0 \) and \( \Phi_0 \) is the flux quantum. Using this model, PC was previously studied in a number of papers [6–11] in the presence of various dissipative environments. In addition, the model discussed here could be of interest for the problem of PC in superconducting nanorings in the presence of quantum phase slips [12].

For our problem the current operator in the standard Schrödinger representation is defined as
\[ \hat{I} = \frac{e}{2\pi} \hat{\theta} = \frac{ie}{2\pi} [\hat{H}, \hat{\theta}] = \frac{e(\hat{\phi} + \phi_0)}{2\pi MR^2}. \]
Switching to the Matsubara representation
\[ \hat{I}(\tau) = e^{i\omega \tau} \hat{I} e^{-i\omega \tau}, \]
we define the current–current correlation function
\[ \Pi(\tau) = \langle \langle \hat{I}(\tau) \hat{I}(0) \rangle \rangle = T + \sum_{n=-\infty}^{\infty} \Pi_{n0} e^{-\beta \epsilon_n \tau}, \]
which describes equilibrium current noise. Here \( T \) is the time-ordering operator, \( \epsilon_n = 2\pi n T \) are Matsubara frequencies, \( \langle \cdot , \cdot \rangle \equiv \text{tr}(\hat{\rho} \cdot \cdot \cdot) \) denotes averaging with the equilibrium density matrix \( \hat{\rho} = e^{-\beta \hat{H}} / Z \), where \( Z = \text{tr} e^{-\beta \hat{H}} \) is the grand partition function and \( \beta = 1/T \). The symbol \( \langle \cdot , \cdot \rangle \) stands for irreducible correlators (cumulants), e.g., \( \langle \langle \hat{I}(\tau) \hat{I}(0) \rangle \rangle = \langle \hat{I}(\tau) \hat{I}(0) \rangle - \langle \hat{I}(\tau) \rangle \langle \hat{I}(0) \rangle \).

Employing the full set of eigenstates \( \hat{H} |m\rangle = \epsilon_m(\phi_x) |m\rangle \), after a straightforward calculation we obtain
\[ \Pi(\tau) = \mathcal{P} + \tilde{\Pi}(\tau), \]
where \( \mathcal{P} \) does not depend on imaginary time and reads
\[ \mathcal{P} = \frac{1}{Z} \sum_m |\langle m | \hat{I} | n \rangle| e^{-\beta \epsilon_m} - |\langle \hat{I} \rangle|^2, \]
while the Fourier components of \( \Pi(\tau) \) are defined as
\[ \tilde{\Pi}_{n0} = \frac{1}{Z} \sum_{m \neq n} |\langle m | \hat{I} | n \rangle|^2 e^{-\beta \epsilon_n - \beta \epsilon_n} \langle 0 | \epsilon_m - \epsilon_n | 0 \rangle. \]

In order to establish the relation between the correlator (4) and the current noise power we also define the Heisenberg operators \( \hat{I}(\tau) = e^{i\hat{H} \tau} e^{-i\hat{H}} \) and the Keldysh Green function
\[ S(t) = \langle \langle \hat{I}(\hat{I}(0) + \hat{I}(0) \hat{I}(\tau) \rangle \rangle = \int \frac{d\omega}{2\pi} S_\omega e^{-i\omega t}. \]
As before, decomposing the result for \( S(t) \) into time-independent and time-dependent contributions we find
\[ S(t) = 2\mathcal{P} + \tilde{S}(t), \]
where \( \mathcal{P} \) is again defined in equation (6) and the Fourier components of \( \tilde{S}(t) \) take the form
\[ \tilde{S}_\omega = \frac{2\pi}{Z} \sum_{m \neq n} |\langle m | \hat{I} | n \rangle|^2 (e^{-\beta \epsilon_m} + e^{-\beta \epsilon_n}) \delta(\omega + \epsilon_m - \epsilon_n). \]
Comparing equations (7) and (10) we arrive at the fluctuation–dissipation relation
\[ \tilde{S}_\omega = 2 \coth \frac{\beta \epsilon_0}{2} \text{Im} \tilde{\Pi}_{\omega+i0}. \] which allows us to immediately recover the current noise power from the correlator (5)–(7). Note that this relation links together the quantities \( \tilde{\Pi} \) and \( \tilde{S} \) which—according to equations (5) and (9)—differ from the correlators \( \Pi \) and \( S \) by the constant in time terms, respectively \( \mathcal{P} \) and \( 2\mathcal{P} \), which produce singularities in the frequency domain.

We would like to point out that our formalism also allows us to analyze the linear current response to the time-dependent flux inside the ring and to formally define the ac conductance of the system. According to the Kubo formula this ac conductance is expressed in terms of the commutator of the current operators, unlike the noise spectrum defined by the anticommutator of these operators (8). Below we restrict our attention only to time-independent values \( \phi_x \) and do not address the behavior of the ac conductance which we are not interested in here.

The above exact relations fully determine the PC correlators in terms of the system eigenstates. These relations allow us to observe that as long as the current operator \( \hat{I} \) commutes with the Hamiltonian of the system the Fourier components (7) and (10) vanish identically together with the matrix elements \( |m | \hat{I} | n \rangle \) with \( m \neq n \), while the time-independent term \( \mathcal{P} \) (6) tends to zero only in the zero temperature limit. Thus, in this case no PC fluctuations can occur at \( T \to 0 \) and at non-zero temperatures PC noise does not vanish only in the zero frequency limit, \( S_\omega = 2\mathcal{P} \delta(\omega) \).

If, however, the operators \( \hat{I} \) and \( \hat{H} \) do not commute with each other the situation becomes entirely different. In that case the matrix elements \( |m | \hat{I} | n \rangle \) in general remain non-zero for any pair of eigenstates and, hence, PC fluctuations may persist down to \( T = 0 \).
For the simple model of figure 1, the PC correlators can be evaluated directly from equations (4)–(10). In more complicated situations, however, the above general relations employing the representation of eigenstates could become less convenient for practical calculations. For this reason, below we will develop alternative approaches which can also be useful for the analysis of PC fluctuations.

3. Free energy and current noise

It turns out that in both limits of zero Matsubara frequency and zero imaginary time the current–current correlator can be conveniently related to the free energy of the system \( F = -T \ln Z \). Making use of the expression for the PC

\[
\langle \hat{I} \rangle = \frac{1}{2\pi} \frac{\partial F}{\partial \phi_0}
\]

(12)

together with the identity

\[
\frac{\partial (e^{-\beta H})}{\partial \phi_0} = -\int_0^\beta \frac{\partial}{\partial \phi_0} e^{-\beta(-\tau)\hat{H}} \frac{\partial \hat{H}}{\partial \phi_0} e^{-\beta \hat{H}},
\]

(13)

for the second derivative of the free energy with respect to the flux, we obtain

\[
e^2 \frac{\partial^2 F}{\partial \phi_0^2} = -e^2 \frac{\partial^2 F}{\partial \phi_0^2} - \int_0^\beta \frac{\partial}{\partial \phi_0} e^{-\beta(-\tau)\hat{H}} \frac{\partial \hat{H}}{\partial \phi_0} e^{-\beta \hat{H}}.
\]

(14)

From this equation one readily finds

\[
\int_0^\beta d\tau \Pi(\tau) \equiv \Pi_0 = \frac{e^2}{4\pi^2 M R^2} - \frac{e^2}{4\pi^2} \frac{\partial^2 F}{\partial \phi_0^2}.
\]

(15)

On the other hand, employing the identity

\[
\frac{1}{R^2} \frac{\partial}{\partial M} \frac{\partial}{\partial M} e^{-\beta \hat{H}} = \frac{2\pi^2}{e^2} \frac{\partial^2 F}{\partial \phi_0^2} e^{-\beta \hat{H}},
\]

(16)

we get

\[
\langle \hat{I}^2 \rangle = -\frac{e^2}{2\pi^2 R^2} \frac{\partial^2 F}{\partial M^2}
\]

(17)

and, hence,

\[
\Pi(0) = -\frac{e^2}{2\pi^2 R^2} \frac{\partial^2 F}{\partial M^2} - \frac{e^2}{4\pi^2} \left( \frac{\partial F}{\partial \phi_0} \right)^2.
\]

(18)

As in many cases, the free energy of the system can be readily evaluated and equations (15) and (18) provide a great deal of information about PC noise. Further simplifications may appear in the zero temperature limit since in this case the free energy reduces to the ground state energy \( F(T \to 0) = \epsilon_0(\phi_0) \). For example, for a free particle on a ring (i.e. for \( U(\theta) = 0 \)) one has \( \epsilon_0(\phi_0) = \phi^2/(2MR^2) \) and, hence, in this case in the limit \( T \to 0 \) from equations (15) and (18) one trivially finds

\[
\Pi_0 = \Pi(0) = 0.
\]

(19)

In agreement with our general analysis, in the absence of an external potential and at \( T = 0 \) the correlator (4) vanishes identically for all values of \( \tau \) implying that no fluctuations of the PC occur in this case. This is because for \( U(\theta) = 0 \) the current operator commutes with the Hamiltonian.

At non-zero external potentials \( U(\theta) \neq 0 \), however, these two operators do not commute anymore and, hence, fluctuations of the PC in general do not vanish even at very low \( T \). This conclusion can be reached, e.g., from equation (15) without any additional calculation. Indeed, for \( U(\theta) \neq 0 \) the ground state energy \( \epsilon_0(\phi_0) \) deviates from \( \phi^2/(2MR^2) \) and, hence, \( \Pi_0 \neq 0 \) down to \( T = 0 \).

4. Generating functional

Let us now formulate a general technique that will allow us to fully describe current fluctuations of the PC within our model. For this purpose we define the generating functional

\[
Z[\eta] = \int D\phi D\theta \hat{e}^{\beta \int \phi \theta - \frac{1}{4\pi^2} \epsilon_0(\phi) + U(\theta) - \eta \theta}.
\]

(20)

where \( \eta(\tau) \) is the source field for the flux variable \( \phi \). Performing integration over \( \phi \) we obtain

\[
Z[\eta] \sim \int D\phi e^{-\frac{1}{2} \int \frac{1}{2} \epsilon_0(\phi) + \phi_0 \theta + U(\theta)}.
\]

(21)

Taking the variational derivative of \( Z[\eta] = -T \ln Z[\eta] \) over the source field \( \eta(\tau) \) and setting this field equal to zero afterwards, we derive the relation between the expectation values for the current and the particle ‘velocity’ \( \dot{\theta} \):

\[
\langle I(\tau) \rangle = \frac{i e}{2\pi} \langle \dot{\theta}(\tau) \rangle.
\]

(22)

Similarly, the second derivative of \( Z[\eta] \) with respect to \( \eta(\tau) \) yields the second current cumulant:

\[
\langle \hat{I}(\tau_1) \hat{I}(\tau_2) \rangle = \frac{e^2}{4\pi^2 M R^2} \delta(\tau_1 - \tau_2) - \frac{e^2}{4\pi^2} \langle \hat{\theta}(\tau_1) \hat{\theta}(\tau_2) \rangle.
\]

(23)

Analogously, one can establish the relations between higher current and velocity cumulants. Up to some unimportant \( \delta \)-functions at coinciding times (which cancel out in the final result as we will see below) the latter cumulants, in turn, are evaluated from the relation

\[
\langle \hat{\theta}(\tau_1) \cdot \hat{\theta}(\tau_N) \rangle = (-i)^N \frac{\delta^N}{\delta \zeta(\tau_1) \cdots \delta \zeta(\tau_N)} \bigg|_{\zeta=0}.
\]

(24)

where \( Z[\zeta] \) is the generating functional

\[
Z[\zeta] = \int_0^{2\pi} D\theta_0 \sum_{m=-\infty}^{\infty} e^{2\pi i m \theta_0} \langle \hat{\theta}(\tau_1) \hat{\theta}(\tau_2) \rangle.
\]

(25)

The above general expressions allow for straightforward evaluation of all current cumulants, thus establishing ‘full-counting statistics’ of the PC in our problem.
5. Current–current correlator

Below we will focus our attention on the current–current correlation function (4) which will be evaluated in the specific limiting case [11]

\[ U(\theta) = U_0(1 - \cos(\kappa \theta)), \quad U_0 \gg \kappa^2/(MR^2). \]  

In other words, we will assume that the particle confined to the 1D ring is moving in a periodic potential with the distance \(2\pi/\kappa\) between adjacent minima. For \(\kappa = 1\) our model reduces to that derived for ultra-thin superconducting rings in the presence of quantum phase slips [12]. As indicated in equation (26), the potential barriers between these minima are high, in which case the particle moves around the ring due to hopping from one minimum to another. Semiclassically these hops are described by multi-instanton trajectories [11]

\[ \Theta(\tau) = \sum_j \nu_j \theta(\tau - \tau_j), \quad \nu_j = \pm 1, \]  

which dominate the path integral (25). Here \(\dot{\theta}(\tau) = 4\arctan(e^{2\beta})/\kappa\) is the well known kink solution, describing the particle tunneling with the amplitude

\[ \Delta/2 = 4(\Omega U_0/\pi)^{1/2} e^{-\pi^2/\Omega}, \]  

where \(\Omega = \kappa\sqrt{U_0/(MR^2)}\). Substituting the trajectories (27) into (25) and performing Gaussian integration we get

\[ Z[\xi] = \sum_{n=0}^{\infty} \sum_{\nu_j = \pm 1} \left( \frac{\Delta}{2} \right)^n \times \int_0^\beta \int_{\tau_1}^\beta \int_{\tau_2}^\beta \cdots \int_{\tau_{n-1}}^\beta \sum_{m=-\infty}^{\infty} e^{2\pi i m \nu_j}, \]

where the terms

\[ Z_0[\xi] = e^{\frac{2\pi i}{\kappa} \int \xi(\tau) G(\tau, \tau') \frac{d\tau}{d\tau'}} \]  

are set by Gaussian fluctuations around \(n\)-instanton trajectories \(\Theta(\tau)\) (27). The correlator \(G(\tau, \tau') = (\delta(\dot{\theta}(\tau) \delta(\dot{\theta}(\tau'))\) can easily be evaluated for a dilute instanton gas provided both times \(\tau\) and \(\tau'\) are outside the instanton cores, i.e. \(|\tau - \tau_j|, |\tau' - \tau_j| \gg \Omega^{-1}\) for every \(j\). In this case \(Z_0[\xi]\) reduces to the generating functional for a harmonic oscillator \(Z_0[\xi]\) defined by equation (30) with

\[ G(\tau, \tau') \approx \frac{\Omega^2}{2k^2 U_0} e^{-\Omega|\tau - \tau'|} + \frac{1}{2MR^2} \delta(\tau - \tau'), \]  

where the last expression remains valid for \(\beta \Omega \gg 1\) and \(|\tau - \tau'| \ll \beta\). Proceeding analogously to [11] and employing Poisson’s resummation formula we obtain

\[ Z[\xi] = Z_0[\xi] \sum_{k=1}^{\infty} e^{\frac{2\pi i}{\kappa} \int \xi(\tau) \dot{\theta}(\tau - \tau_j) d\tau_1}. \]

In the limit \(\xi \to 0\), equation (32) reduces to the partition function [11] and the average value of the PC is obtained from equations (32) and (24) with \(N = 1:\)

\[ I = \frac{e^{\frac{\Delta}{\kappa}} \sum_{k=1}^{\infty} \sin \left( \frac{2\pi (\phi_k - \kappa)}{\kappa} \right)}{\sum_{k=1}^{\infty} e^{\frac{\Delta}{\kappa} \cos \left( \frac{2\pi (\phi_k - \kappa)}{\kappa} \right)}}. \]

At low temperatures \(T \ll \Delta/\kappa^2\), equation (33) reduces to a simple formula

\[ I = \frac{e^{\Delta}}{\kappa} \sin \left( \frac{2\pi \phi_k}{\kappa} \right), \quad -1/2 < \phi_k < 1/2. \]

Figure 2. Persistent current \(I\) (measured in units \(\Delta/2\pi\)) as a function of the magnetic flux \(\phi_k\) for \(\kappa = 3\) at different temperatures: \(T = 0, 0.125\Delta, 0.5\Delta, 2\Delta\).

The same equations for \(N = 2\) yield the second current cumulant

\[ \Pi(\tau) = \Pi + \Pi_{osc}(\tau) + \Pi_{osc}(\beta - \tau) + \tilde{\Pi}(f(\tau) + f(\beta - \tau)), \]

where for \(\Omega^{-1} \leq \beta \leq 1 - \Omega^{-1}\) we find

\[ \Pi = \frac{e^{2\Delta^2}}{k^2} \sum_{k=1}^{\infty} \sin \left( \frac{2\pi (\phi_k - \kappa)}{\kappa} \right) e^{\frac{\Delta}{\kappa} \cos \left( \frac{2\pi (\phi_k - \kappa)}{\kappa} \right)} \]

and

\[ \tilde{\Pi} = -\frac{e^{2\Delta^2}}{k^2} \sum_{k=1}^{\infty} e^{\frac{\Delta}{\kappa} \cos \left( \frac{2\pi (\phi_k - \kappa)}{\kappa} \right)} \]

In equation (35) for \(\tau \gg \Omega^{-1}\) we also defined

\[ f(\tau) = \frac{k^2}{4\pi^2} \int_{-\infty}^{\infty} \dot{\theta}(\tau_1 - \tau) \dot{\theta}(\tau_1) d\tau_1 \approx \frac{4\Omega^2}{\pi^2} e^{-\Omega \tau}. \]

As one could expect from our general analysis in terms of the exact eigenstates, the result (35) indeed consists of two different—time-independent and time-dependent—contributions. The meaning of each of these terms can be
identified with the aid of equations (5)–(7). As we already discussed, exactly at $T = 0$ the time-independent part $\mathcal{P}$ should vanish, $\mathcal{P} = 0$. This fact is indeed directly observed from our result (36) in the limit $T \to 0$.

We also note that the expression for $\mathcal{P}$, (36), can be established within the effective tight-binding model in which case the particle successively hops between $\kappa$ nodes on the ring. This observation demonstrates that the term $\mathcal{P}$ is universal, meaning that it depends only on the tunneling amplitude $\Delta$ but not on the profile of the periodic potential. Within the tight-binding model the current operator commutes with the total Hamiltonian, the current–current correlator does not depend on $\tau$ and vanishes in the limit $T \to 0$ in accordance with our general considerations.

At non-zero temperatures, however, the term $\mathcal{P}$ does not vanish. At $\Delta/\kappa \ll T \ll \Omega$ from equation (36) we get

$$\mathcal{P} = \frac{e^2\Delta^2}{2\kappa} (I_0(\beta \Delta) - I_2(\beta \Delta)) - \frac{e^2(\kappa - 1)\Delta^e}{2^{\frac{5}{2}}\kappa!T^{\frac{1}{2}}\tau} \cos(2\pi \phi_e),$$

where $I_n(x)$ are the Bessel functions.

Let us now turn to the time-dependent contribution to $\Pi(\tau)$. With the aid of equations (5), (7) and (35) we identify

$$\mathcal{P}_{osc} + \bar{P} f(\tau) = \frac{1}{Z} \sum_{m > n} \langle |m| I |n| \rangle^2 e^{-\beta \epsilon_n} e^{-\epsilon_n - \epsilon_m}.$$  \hfill (41)

The form of this—non-universal—contribution cannot be recovered within the tight-binding model as it explicitly depends on the instanton solution and, hence, on the particular shape of the periodic potential.

In order to proceed let us observe that there exist $\kappa$ low-lying quantum levels with energies $\Omega/2 - \Delta \cos(2\pi (\phi_e - k)/\kappa)$ in our problem. These states originate from tunneling depletion of the ground state energy level $\Omega/2$ in each of the $\kappa$ potential wells. Below we will label these states as $|0k\rangle$ with $k = 1, \ldots, \kappa$. Due to the rotation symmetry of our model all matrix elements between these states vanish and, hence, do not contribute to the current–current correlation function.

The next $\kappa$ energy levels $|1l\rangle$ with $l = 1, \ldots, \kappa$ occur due to depletion of the first excited state $3\Omega/2$. These states are characterized by the energies $3\Omega/2 + \Delta \cos(2\pi (\phi_e - k)/\kappa)$, where the parameter $\Delta$ is to be defined below. With the aid of the symmetry arguments one can again demonstrate that the matrix elements of the current operator between the states $|1l\rangle$ with different $l$ vanish while the matrix elements between the states $|0k\rangle$ and $|1l\rangle$ remain non-zero provided $k = l$. In order to evaluate these matrix elements it suffices to consider the instanton contribution small by setting $(\Delta + \Delta \tau)\tau < 1$ in equation (41) and to expand the right-hand side of this equation in powers of $\Delta \tau$ and $\Delta \tau$. Comparing the first two terms of this expansion with $\mathcal{P}_{osc}$ and $\bar{P} f$ (37)–(39), in the limit $U_0 \gg \Omega$ considered here we identify

$$\langle 0k | I | 1l \rangle^2 \approx \delta_{kl} e^{2\Omega^3/8\pi \kappa^2 U_0} \Delta \approx \frac{32 U_0}{\Omega}.$$ \hfill (42)

Note that exactly the same expressions can also be recovered from the WKB analysis of the Schrödinger equation for the cosine potential.

6. PC noise power

Finally, let us evaluate the real time current noise power $S_\omega$ defined in equations (9) and (10). Employing the above results at $T \ll \Omega$ we obtain

$$S_\omega = 4\pi P \delta(\omega) + \epsilon_2\Omega^3 \sum_{k=1}^{\kappa} e^{i\Delta \cos(\frac{2\pi (\phi_e - k)}{\kappa})}$$

$$\times (\delta(\omega - \Omega - \epsilon_k) + \delta(\omega + \Omega + \epsilon_k)) \quad \hfill (43)$$

where we defined

$$\epsilon_k = \frac{32 U_0 \Delta}{\Omega} \cos\left(\frac{2\pi (\phi_e - k)}{\kappa}\right). \quad \hfill (44)$$

We observe that—in agreement with our general analysis—PC noise power has the form of peaks at frequencies equal to the distance between the energy levels with non-zero matrix elements of the current operator plus an additional peak at zero frequency. In the zero temperature limit $T \to 0$, the amplitude of this peak tends to zero along with the terms related to transitions to higher energy levels and equation (43) reduces to

$$S_\omega = e^{-\Omega^3/4\pi \kappa^2 U_0} (\delta(\omega - \epsilon_0) + \delta(\omega + \epsilon_0)). \quad \hfill (45)$$

where $\epsilon_0(\phi_e) = \max_e \epsilon_e(\phi_e)$. This result demonstrates again that PC fluctuations indeed persist down to $T = 0$ in which case peaks of PC noise power $S_\omega$ occur at frequencies corresponding to transitions between the two lowest energy levels for which the matrix elements of the current operator differ from zero. We also note that $S_\omega$ differs from zero even at zero external flux $\phi_e = 0$ when the average PC value is zero. In the presence of dissipation due to interaction of the particle with other (quantum) degrees of freedom the energy levels acquire a finite width, the peaks broaden and the noise power should differ from zero, also in a wider range of frequencies.

Similarly, broadening of such peaks inevitably occurs in ensembles of rings or individual rings with many conducting channels. Within our model this broadening can be illustrated by considering an ensemble of rings with the parameter $U_0$ uniformly distributed within some energy interval, e.g., as is indicated in the caption to figure 3. In this case the total PC noise produced by the system is given by the sum of a large number of very close peaks (45) effectively resulting in a much smoother and broader noise spectrum, as is shown in figure 3.

A specific feature of PC noise is the dependence of $S_\omega$ on the external magnetic flux $\phi_e$. This dependence occurs due to the presence of quantum coherence in the system and disappears if this coherence gets destroyed. Hence, such sensitivity of the PC noise spectrum to the flux can be used as a measure of quantum coherence in our system. Taking the derivative of $S_\omega$ with respect to the flux, for the model considered here we obtain

$$\partial S_\omega / \partial \phi_e \propto \sin(2\pi \phi_e / \kappa). \quad \hfill (46)$$

A typical dependence of $\partial S_\omega / \partial \phi_e$ on $\omega$ is illustrated in figure 3. We believe that the main qualitative features of our
results displayed in figure 3 should also survive in other models and can be detected in experiments with nanorings.

It is also interesting to point out a direct physical analogy between our results and those of [13–15] where equilibrium supercurrent noise in point contacts between superconductors was investigated. In that case, the noise power spectrum also has the form of peaks which occur both at zero frequency and at frequencies equal to the distance between Andreev levels inside the contact. At \( T \to 0 \) the zero frequency peak disappears while the other peaks do not vanish except in the limit of fully transparent barriers. In the case of many channel diffusive contacts, the supercurrent noise spectrum broadens [15] in a qualitatively similar way to the result displayed in figure 3. In addition, the noise spectrum [13–15] turns out to depend on the phase difference across the superconducting weak link. This dependence has the same physical origin as the flux dependence of PC noise considered here.

7. Conclusion

In summary, we investigated equilibrium fluctuations of the persistent current in nanorings and demonstrated that these fluctuations do not vanish even at \( T = 0 \) provided the current operator does not commute with the total Hamiltonian of the problem. A specific feature of PC noise is its quantum coherent nature, implying that the noise spectrum can be tuned by an external magnetic flux inside the ring. We believe that the key features captured by our analysis will also survive in other models and can be verified in future experiments. Our further analysis will be devoted to the effect of dissipation on PC fluctuations in systems with many degrees of freedom.

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