Thermophysical properties of nanoparticles in carboxymethyl cellulose water mixture for heat enhancement applications

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Abstract
A successful drilling operation requires among others an effective drilling fluid which is also called drilling mud. Carboxymethyl cellulose (CMC) which is a derivative of cellulose is commonly used because of its high viscosity. Its performance as a drilling fluid is limited at high temperatures and pressures due to its low thermal conductivity. A colloidal mixture of nanoparticles in CMC enhances the performance of the fluid. In this work, a comparison of the thermophysical properties of three metal oxides namely Titanium Oxide (TiO2), Aluminium oxide (Al2O3) and Copper Oxide (CuO) dispensed in Carboxymethyl cellulose (CMC) water mixture in drilling operations were investigated at varying temperatures. Maximum of 0.4 volume fraction of the nanoparticles of each of the oxides in less or equal to 0.4 /100 of CMC concentration in water was considered. The governing equations obtained were simplified to a set of Ordinary Differential Equations which were solved numerically using Runge Kutta Scheme (order 4) along with shooting method. Results obtained showed that the metal oxides enhanced the heat transfer capability of the CMC water mixture. Besides, the conductivity enhancement is least with TiO2 in CMC water mixture and maximum with CuO in CMC water mixture. Viscosity and thermal conductivity increased with increasing volume fraction of the nanoparticles in the dispensing medium. These results were compared with existing literature and found to be in good agreement.

Keywords: Carboxymethyl cellulose, metal oxide, nanoparticles, Runge Kutta scheme, thermophysical properties, volume fraction
1 Introduction

The significant of fluids used in drilling operations especially in an environment with high pressure and high temperature condition cannot be overemphasized. These fluids are needed to maintain adequate flow properties of drilled materials and to provide cooling and lubrication of the string and bit of the drill pipe in an environment of such [1]. Carboxymethyl cellulose (CMC) a highly viscous derivative cellulose is a useful additive capable of maintaining adequate flow properties of drilled materials like clay suspensions in boreholes and rocks crushed into small fragments in telescopic holes of oil and gas [2].

A nanofluid, a potential coolant is a colloidal suspension of nanoparticles inside a base fluid, e.g. ethylene glycol, water and CMC solutions. The existence of nanoparticles of metal carbides, metal oxides, and so on in the convectional fluid enhances its thermal conductivity; thereby improve its heat transfer capability. Nanofluids also exhibit greater viscosity as the volume fraction of the nanoparticles is increased [3].

A number of works abound in literature on formulation of drilling fluids that is nano-based. In their preliminary test investigated the result of the thermophysical properties of iron oxide (Fe₂O₃) nanoparticles on bentonite fluids as base fluid, it was reported that when the volume fraction of iron oxide is raised, nanoparticles in the base fluid increased the viscosity of the base fluid [4]. Similar test conducted showed that the effect of increase in temperature on the properties of nanofluids tested is more significant than that of pressure [5].

Other works include that of Iqbal et al. [6] on the comparative investigation of Al₂O₃/H₂O, SiO₂/H₂O and ZrO₂/H₂O nanofluid for heat transfer applications, studies on thermophysical properties of silicon dioxide (SiO₂) in Ethyl Glycol/water mixture for proton exchange membrane fuel cell cooling application [7], and examination of the magnetohydrodynamic radiative flow of Casson fluid on a stretching area with heat source/sink [8].

Greater nanofluid viscosity and its improved capability to transfer heat relative to those of the base fluid like CMC solution ought to be examined for formulating effective drilling fluid with best possible properties within a choice of working conditions. This work therefore presents the comparison of effects of the thermophysical properties of TiO₂, Al₂O₃ and CuO nanoparticles under different temperature conditions. The properties include thermal conductivity and viscosity for drilling operations.
2. **Mathematical Formulation**

The fluid considered is electrically conducting, viscous and incompressible induced by buoyancy effect. The flow is two-dimensional and unsteady while the Cartesian coordinate is selected such that the x-axis is along which the fluid flows is perpendicular to the surface and y-axis along it. The fluid properties are taken to be constant and the temperature of the system is negligible. Carboxymethyl cellulose water (CMC) is the base fluid. Aluminium oxide (Al$_2$O$_3$), titanium dioxide (TiO$_2$) and copper oxide (CuO) are the nanoparticles. Their thermophysical properties used are as follows:

| Physical properties | Specific heat Capacity \( C'_p \) (Jkg$^{-1}$K$^{-1}$) | Density \( \rho \)kgm$^{-3}$ | Thermal Conductivity \( k \)Wm$^{-1}$K$^{-1}$ | Coefficient of thermal expansion \( \beta_T \times 10^5 \) (K$^{-1}$) |
|---------------------|--------------------------------|----------------|-------------------|-------------------|
| CMC-water (< 0.4%)   | 4179                          | 997.1          | 0.613             | 21               |
| Al$_2$O$_3$ nanoparticles | 765                     | 3970           | 40                | 0.85             |
| TiO$_2$ nanoparticles | 686.2                     | 4350           | 8.95              | 0.72             |
| CuO nanoparticles    | 531.8                      | 6320           | 76.5              | 1.80             |

Source: [3, 9, 10]

Based on the assumptions, the governing equations for the fluid flow can be modeled as follows;

**Continuity Equation**

\[
\frac{\partial \bar{u}}{\partial x} + \frac{\partial \bar{v}}{\partial y} = 0
\]

**Momentum Equation**

\[
\frac{u\frac{\partial \bar{u}}{\partial x}}{\rho} + \frac{v\frac{\partial \bar{u}}{\partial y}}{\rho} = \frac{1}{\rho_{nf}} \left( \frac{1}{1 + \frac{1}{\beta'}} \frac{\partial \mu_{nf}(T) \partial T}{\partial y} \frac{\partial \bar{u}}{\partial y} + \frac{\mu_{nf}(T)}{\rho_{nf}} \left( \frac{1}{1 + \frac{1}{\beta'}} \frac{\partial^2 \bar{u}}{\partial y^2} \right) \right) - \frac{\mu_{nf}(T)}{\rho_{nf}} \left( \frac{1}{1 + \frac{1}{\beta'}} \frac{u}{K} + g(\beta_T)_{nf} (T - T_w) - \frac{b^* u^2}{K} - \frac{\alpha B_0^2 u}{\rho_{nf}} \right)
\]
Energy Equation

\[
\frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = k_{\text{sf}}(T) \left( \frac{\partial^2 T}{\partial y^2} + \frac{1}{(\rho C_p)_{\text{sf}}} \frac{\partial k_{\text{sf}}(T)}{\partial T} \left( \frac{\partial T}{\partial y} \right)^2 + \frac{\mu_{\text{sf}}(T)}{(\rho C_p)_{\text{sf}}} \left( 1 + \frac{1}{\beta} \right) \left( \frac{\partial u}{\partial y} \right)^2 \right) + \tau \left[ D_B \frac{\partial C}{\partial y} \frac{\partial T}{\partial y} + \frac{D_f}{T_{\infty}} \left( \frac{\partial T}{\partial y} \right)^2 \right]
\]

Concentration Equation

\[
u \frac{\partial C}{\partial x} + \frac{\partial C}{\partial y} = D_B \frac{\partial^2 C}{\partial y^2} + \left( \frac{D_f}{T_{\infty}} \right) \frac{\partial^2 T}{\partial y^2} - k_f(C - C_{\infty})
\]

With the following boundary conditions:

\[
\begin{align*}
u &= Bx, & C &= C_w & \text{at} & y = 0 \\
u &\to 0 & T &\to T_{\infty} & C &\to C_{\infty} & \text{as} & y \to \infty
\end{align*}
\]

(3)

(4)

Using Rosseland approximation, the radiative heat flux, \( q_r \), as reported by [11] is given as:

\[
q_r = -\frac{4\sigma_{\text{sf}}^* \partial T^4}{3K^* \partial y}
\]

(7)

Applying Taylor series and neglecting higher terms

\[
q_r = -\frac{16\sigma_{\text{sf}}^* T_{\infty}^4 \partial T}{3K^* \partial y}
\]

(8)

Substituting (6) and (8) as well as their derivatives into the governing equations, it became:

Continuity equation

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0
\]

(9)

Momentum Equation

\[
\frac{\partial u}{\partial x} + v \frac{\partial v}{\partial y} = -\frac{a\mu_{\text{sf}}^*}{\rho_{\text{sf}}} \left( 1 + \frac{1}{\beta} \right) \frac{\partial T}{\partial y} \frac{\partial u}{\partial y} + \frac{\mu_{\text{sf}}^*}{\rho_{\text{sf}}} \left[ 1 + a(T_w - T) \right] \left( 1 + \frac{1}{\beta} \right) \frac{\partial^2 u}{\partial y^2}
\]

\[
- \frac{\mu_{\text{sf}}^*}{\rho_{\text{sf}}} \left[ 1 + a(T_w - T) \right] \left( 1 + \frac{1}{\beta} \right) \frac{u^2}{K} + g(\beta_f)_{\text{sf}} (T - T_{\infty}) - \frac{b^* u^2}{K} - \frac{\sigma B_{\text{sf}} u}{\rho_{\text{sf}}}
\]

(10)
Energy Equation
\[
\begin{align*}
\frac{\partial T}{\partial t} + v \frac{\partial T}{\partial y} &= \frac{k_m(T)}{(\rho C_p)_T} \left[ 1 + b(T - T_w) \right] \frac{\partial^2 T}{\partial y^2} + \frac{b k_m}{(\rho C_p)_T} \left( \frac{\partial T}{\partial y} \right)^2 \\
&+ \frac{\mu_T}{(\rho C_p)_T} \left[ 1 + a(T_w - T) \right] \left( 1 + \frac{1}{\beta} \right) \left( \frac{\partial T}{\partial y} \right)^2 - \frac{16 \sigma T^3}{3 K (\rho C_p)_T} \frac{\partial^2 T}{\partial y^2} + \tau \left[ D_b \frac{\partial C}{\partial y} + D_f \left( \frac{\partial T}{\partial y} \right)^2 \right]
\end{align*}
\]  

(11)

Concentration Equation
\[
\begin{align*}
\frac{\partial C}{\partial t} + v \frac{\partial C}{\partial y} &= D_b \frac{\partial^2 C}{\partial y^2} + \left( \frac{D_x}{T_w} \right) \frac{\partial^2 T}{\partial y^2} - k_r (C - C_{\infty})
\end{align*}
\]  

(12)

Subject to the following boundary conditions
\[
\begin{align*}
u &= Bx & v &= -Vx & T &= T_w & C &= C_w & \text{at} & & y = 0 \\
u &\to 0 & T &\to T_{\infty} & C &\to C_{\infty} & \text{as} & & y \to \infty
\end{align*}
\]  

(13)

Introducing the following relations
\[
\begin{align*}
u &= Bxf' (\eta), & v &= -\sqrt{BV_f} f (\eta), & \eta &= \sqrt{\frac{B}{V_f}} y, & \theta (\eta) &= \frac{T - T_{\infty}}{T_w - T_{\infty}}, & \phi &= \frac{C - C_{\infty}}{C_w - C_{\infty}}
\end{align*}
\]  

(14)

Thus, \[ u = Bxf' (\eta) \Rightarrow \frac{\partial u}{\partial x} = Bf' \quad \text{and} \quad \frac{\partial u}{\partial \eta} = \frac{\partial u}{\partial y} \cdot \frac{\partial y}{\partial \eta} = \frac{x \sqrt{B}}{\sqrt{f'}} f'^*
\]  

(15)

Substituting equations (15) – (18) into equations (10) – (12) are re-arranging yielded:
Using the following boundary conditions:

\[ f'(\eta) = 1 \quad f(\eta) = S \quad \theta = 1 \quad \phi = 1 \quad \text{at} \quad \eta = 0 \]
\[ f'(\eta) \to \infty \quad \theta \to \infty \quad \phi \to \infty \quad \text{as} \quad \eta \to \infty \quad (22) \]

Where,

- \( \beta \) is the base fluid parameter
- \( \xi = a(T_u - T_w) \) is the temperature-dependent viscosity parameter
- \( \varepsilon = b(T_u - T_w) \) is the temperature-dependent thermal conductivity parameter

\( F_s = \frac{b^*}{x} \) is the Forchheimer parameter \quad \( D_a = \frac{K}{x^2} \) is the Darcy number

\( P_p = \frac{V_f}{K_B} \) is the porosity parameter \quad \( J_{GR} = \frac{g(\beta_f)_{f} T_w}{axB^2} \), \quad \( H_a = \frac{\sigma B_0^2}{B \rho_f} \)

\( Pr = \frac{\mu_f (C_p)_f}{k_f} \) is the Prandtl number \quad \( E_c = \frac{x^2 B^2}{(C_p)_f (T_u - T_w)} \) is the Eckert number

\( N_t = \frac{\tau D_j (T_u - T_w)}{V_f T_w} \) is the thermophoresis parameter \quad \( N_b = \frac{\tau D_j (C_w - C_u)}{V_f} \)

\[ A_1 = \frac{\mu_f}{\mu_{nf}} \quad A_2 = A_1 \left[ 1 - \psi + \psi \left( \frac{\rho_k}{\rho_f} \right) \right] \quad A_3 = A_1 \left[ 1 - \psi + \psi \left( \frac{(\rho \beta_f)_k}{(\rho \beta_f)_f} \right) \right] \]
A_4 = A_1 \left[ \frac{k_0^w}{k_f} \right] = A_1 \left[ \frac{k_s + 2k_f - 2\psi(k_f - k_s)}{k_s + 2k_f + \psi(k_f - k_s)} \right]

A_5 = A_1 \left[ 1 - \psi + \psi \left( \frac{\rho C_p}_s}{(\rho C_p)_f} \right) \right] = A_1 \left[ 1 - \psi + \psi \left( \frac{(\rho C_p)_s}{(\rho C_p)_f} \right) \right]

The numerical representation of the solutions was displayed graphically for the velocity and temperature distributions of the fluid flow. A table was also used to illustrate the computational values of the thermophysical properties of the nanofluid in order to enhance the discussion. When \( \psi = 0 \), the results are reduced to that of the base fluid problem without any nanofluid property. The following values of the dimensionless numbers were used except otherwise to investigate the effects of the fluid parameter \( \beta \), and the volume fraction of the nanoparticles, \( \psi \).

\[ \beta = 1.0, P_r = 0.7, k_r = 0.2, S_C = 0.22, F_s = 1.0, H_a = 0.1, P_\rho = 0.3, D_a = 1.0, E_C = 0.01 \]

3. Results and Discussion

In Table 2, it is noticed that the effective thermophysical properties of the CMC water mixture and the nanoparticles improved with increase in the volume fraction of the nanoparticles across the temperature. Aluminium oxide has the least density and copper oxide (CuO) has the highest thermal conductivity. The effective viscosity decreased with temperature while the thermal conductivity increased with temperature.

Figure 1 illustrates the velocity distribution of the flow for different values of the base fluid parameter (\( \beta \)) at negligible temperature. It is observed that the velocity of the fluid decreased with increase in \( \beta \) with and without nanoparticles as reported in the work of [12]. This is because increase in \( \beta \) implies decrease in yield stress of the base fluid and the corresponding increase in the dynamic viscosity of the fluid which created internal resistance to the fluid flow. It is also revealed that the flow of the fluid with nanoparticles is faster than without nanoparticles as a result of the enhanced thermal conductivity of the former. The fluid with aluminium oxide (Al\(_2\)O\(_3\)) nanoparticles is the fastest because it has the least density.

Figure 2 depicts the velocity distribution of the flow for different values of the base fluid parameter (\( \beta \)) at higher temperature. As \( \beta \) increases, the velocity profile of the fluid also increased particularly towards the strain function. This is because the temperature of the fluid is
high enough to overpower the resistance of the fluid flow. The fluid with titanium dioxide (TiO$_2$) nanoparticles has the maximum velocity.

Figure 3 shows the temperature distribution of the fluid flow for different values of the base fluid parameter ($\beta$) at negligible temperature. As the $\beta$ increases, the temperature of the fluid also increased. This is due to the fact that heat is absorbed by the system which implies a rise in the temperature of the fluid with and without nanoparticles. The rise is slower with the fluid with nanoparticles because its heat capacity has been enhanced. The rise in temperature is slowest in the aluminium oxide (Al$_2$O$_3$) based drilling fluid because Al$_2$O$_3$ is a good insulator.

Figure 4 represents the temperature distribution of the fluid flow for different values of the base fluid parameter ($\beta$) at higher temperature. As $\beta$ increases, the temperature of the fluid decreased, which is in agreement with earlier findings [13]. This is because heat is released which implies reduction in the temperature of the fluid. Heat is released faster in the fluid with nanoparticles because of its enhanced thermal conductivity. It is fastest with copper oxide (CuO) nanoparticles which has the highest thermal conductivity across the temperature.

**Table 2.** Thermophysical properties of the three nanofluids at various volume fractions of the three nanoparticles

| Fluid            | Volume Fraction of Nanoparticles ($\psi$) | Density ($\rho_{nf}$) | Heat Capacitance ($\rho C_p_{nf}$) | Viscosity ($\mu_{nf}$) | Thermal Conductivity ($k_{nf}$) |
|------------------|------------------------------------------|-----------------------|-----------------------------------|------------------------|-------------------------------|
|                  |                                          |                       | $\xi = 0$                         | $\xi = 3.5$            | $\varepsilon = 0$             | $\varepsilon = 3.5$           |
| CMC/Water        | 0                                        | 997.1                 | 4166880.90                        | 0.00890000             | 0.00323635                    | 0.613                         | 1.68575034                    |
|                  |                                          |                       |                                   |                        |                               |                               |                               |
| CMC/Water + CuO  | 0.1                                      | 1529.36               | 4086290.41                        | 0.01158200             | 0.00421163                    | 0.8191776814                  | 2.8419218849                  |
|                  | 0.2                                      | 2061.68               | 4005699.92                        | 0.01554766             | 0.00565369                    | 1.0592328827                  | 3.7073148949                  |
|                  | 0.3                                      | 2593.97               | 3925109.43                        | 0.02170924             | 0.00789427                    | 1.3747709989                  | 4.8116984948                  |
|                  | 0.4                                      | 3126.26               | 3844518.94                        | 0.03191626             | 0.01160591                    | 1.7914053856                  | 6.2699188496                  |
| CMC/Water + TiO$_2$ | 0.1                                      | 1332.29               | 4048689.80                        | 0.00890000             | 0.00421163                    | 0.7711101120                  | 2.719885392                   |
|                  | 0.2                                      | 1667.68               | 3930498.72                        | 0.01158200             | 0.00565369                    | 0.9733822721                  | 3.4068379521                  |
|                  | 0.3                                      | 2002.97               | 3812307.63                        | 0.01554766             | 0.00789427                    | 1.2122941797                  | 4.2430296290                  |
|                  | 0.4                                      | 2338.36               | 3694116.54                        | 0.02170924             | 0.01160591                    | 1.5094358884                  | 5.2830260940                  |
| CMC/Water + Al$_2$O$_3$ | 0.1                                      | 1294.39               | 4053897.80                        | 0.00890000             | 0.00421163                    | 0.8072556661                  | 2.8253948314                  |
|                  | 0.2                                      | 1591.68               | 3940914.72                        | 0.01158200             | 0.00565369                    | 1.0473972041                  | 3.6658902143                  |
|                  | 0.3                                      | 1888.97               | 3827931.63                        | 0.01554766             | 0.00789427                    | 1.3518603123                  | 4.7315110931                  |
|                  | 0.4                                      | 2186.26               | 3714948.54                        | 0.02170924             | 0.01160591                    | 1.7504837935                  | 6.126932773                   |
Figure 1. Velocity profile for different values of β when ξ=0 and ε=0

Figure 2. Velocity profile for different values of β when ξ=3.5 and ε=3.5
Figure 3. Temperature profile for different values of $\beta$ when $\xi=0$ and $\varepsilon=0$

Figure 4. Temperature profile for different values of $\beta$ when $\xi=3.5$ and $\varepsilon=3.5$
4 Conclusion

According to the study, it is established that the presence of nanoparticles in CMC solution enhances its viscosity and thermal conductivity across the temperature. Moreover, the flow properties of the fluid with nanoparticles improved as the temperature increased which can bring about reduction in the pumping cost of the fluid. Also, the volume fraction of the nanoparticles improves the heat transfer capability of the drilling fluid while CuO nanoparticles offered the optimal increased cooling effect for the drilling pipe at low and high temperature with highest thermal conductivity and density.

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