Fuzzy multi objective transportation problem – evolutionary algorithm approach

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Abstract: This paper deals with fuzzy multi objective transportation problem. An fuzzy optimal compromise solution is obtained by using Fuzzy Genetic Algorithm. A numerical example is provided to illustrate the methodology.

1. Introduction:
Fuzzy Transportation problem is a fuzzy optimization problem deals with transporting commodities from various sources to various destinations in such a way so that the total fuzzy transportation cost is minimum. When a fuzzy transportation problem involves more than one objective function the task of finding one or more fuzzy optimal solution is known as fuzzy multi objective transportation problem. For multiple conflicting fuzzy objectives, there cannot be a single fuzzy optimum solution which simultaneously optimizes all the fuzzy objectives. The resulting outcome is a set of fuzzy optimal solutions with varying degree of objective values. Hence it is better to compute the fuzzy compromise solution between two or more conflicting fuzzy objectives. In this article, we propose a fuzzy genetic algorithm approach for the solution of fuzzy multi objective transportation problems.

In real life situations, supply, demand and unit transportation cost are uncertain. Hence idea of fuzzy sets was introduced by Zadeh [2] in 1965. Zimmerman [9] applied the fuzzy programming techniques to solve multi objective linear programming problems. C. Vijayalakshmi [3] solved the bi objective transportation problem using genetic algorithm and represented it by bipartite graphs. Waiel F. Abd El- Wahed [8] applied fuzzy programming approach to determine the optimal compromise solution of a crisp multi objective transportation problem. For the balanced fuzzy multi objective transportation problem[7] T. Leelavathy and et.al applied weighted sum of the objectives method and obtained the compromise solution by decision maker’s preference.

The rest of the paper is organized as follows: In section 2, we have discussed the basic concepts of triangular fuzzy number and their arithmetic operations. In section 3, we introduce the fuzzy multi objective transportation problem with cost coefficients, supplies and demands as triangular fuzzy numbers. In section 4, we define the basic concepts of fuzzy genetic algorithm. In section 5, a numerical example is provided to illustrate the efficiency of the proposed methodology.

2. PRELIMINARIES
**Definition 2.1:** A fuzzy set $\tilde{A}$ defined on the set of real numbers $\mathbb{R}$ is said to be a fuzzy number, if its membership function $\mu_\tilde{A} : \mathbb{R} \to [0,1]$ has the following characteristics:

(i) $\mu_\tilde{A}$ is convex.

(ii) $\mu_\tilde{A}$ is normal.

(iii) $\tilde{A}$ is upper semi–continuous

(iv) $\text{sup}(\tilde{A})$ is bounded in $\mathbb{R}$.

**Definition 2.2:** A fuzzy number $\tilde{A}$ is a triangular fuzzy number denoted by $\tilde{A} = (a_1, a_2, a_3)$ where $a_1, a_2, a_3$ are real numbers and its membership function $\mu_\tilde{A}(x)$ is given below and

$$
\mu_\tilde{A}(x) = \begin{cases} 
\frac{x-a_1}{a_2-a_1}, & a_1 \leq x \leq a_2 \\
\frac{a_3-x}{a_3-a_2}, & a_2 \leq x \leq a_3 \\
0, & \text{otherwise}
\end{cases}
$$

**Definition 2.3:** A triangular fuzzy number $\tilde{A} = (a_1, a_2, a_3) \in \mathbb{F}(\mathbb{R})$ can also be represented as a pair $\tilde{A} = \left( \left[ a_1 \right], \left[ a_3 \right] \right)$ of function of $\underline{a}(r), \overline{a}(r)$ for $0 \leq r \leq 1$ which satisfies the following requirements:

(i) $\underline{a}(r)$ is a bounded monotonic increasing left continuous function.

(ii) $\overline{a}(r)$ is a bounded monotonic decreasing left continuous function.

(iii) $\underline{a}(r) \leq \overline{a}(r), 0 \leq r \leq 1.$

2.1 **Ranking of Triangular Fuzzy Numbers**

For every $\tilde{A} = (a_1, a_2, a_3) \in \mathbb{F}(\mathbb{R})$, the ranking function $R : \mathbb{F}(\mathbb{R}) \to \mathbb{R}$ by graded mean is defined by

$$R(\tilde{A}) = \frac{a_1 + 2a_2 + a_3}{4}.$$ For any two triangular fuzzy number $\tilde{A} = (a_1, a_2, a_3)$ and $\tilde{B} = (b_1, b_2, b_3)$ in $\mathbb{F}(\mathbb{R})$. We have the following comparison:

(i) $\tilde{A} \succ \tilde{B}$ if and only if $R(\tilde{A}) > R(\tilde{B})$.

(ii) $\tilde{A} \prec \tilde{B}$ if and only if $R(\tilde{A}) < R(\tilde{B})$.

(iii) $\tilde{A} \approx \tilde{B}$ if and only if $R(\tilde{A}) = R(\tilde{B})$.

(iv) $\tilde{A} - \tilde{B} = 0$ if and only if $R(\tilde{A}) - R(\tilde{B}) = 0$. 


2.2 **Arithmetic Operations:** In particular for any two fuzzy numbers \( \tilde{A} = (a_1, a_2, a_3) \) and \( \tilde{B} = (b_1, b_2, b_3) \), we define:

- **Addition**: \( \tilde{A} + \tilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3) \)
- **Subtraction**: \( \tilde{A} - \tilde{B} = (a_1 - b_1, a_2 - b_2, a_3 - b_3) \)
- **Multiplication**: \( \tilde{A} \times \tilde{B} = (\min(a_1b_1, a_1b_2, a_1b_3, a_2b_1, a_2b_2, a_2b_3, a_3b_1, a_3b_2, a_3b_3), \max(a_1b_1, a_1b_2, a_1b_3, a_2b_1, a_2b_2, a_2b_3, a_3b_1, a_3b_2, a_3b_3)) \)
- **Division**: \( \tilde{A} / \tilde{B} = (\min(a_1 / b_1, a_1 / b_2, a_1 / b_3, a_2 / b_1, a_2 / b_2, a_2 / b_3, a_3 / b_1, a_3 / b_2, a_3 / b_3), \max(a_1 / b_1, a_1 / b_2, a_1 / b_3, a_2 / b_1, a_2 / b_2, a_2 / b_3, a_3 / b_1, a_3 / b_2, a_3 / b_3)) \)

3. **FUZZY MULTI OBJECTIVE TRANSPORTATION PROBLEM**

3.1 **Mathematical formulation of Fuzzy Transportation Problem**
Consider a fuzzy multi objective transportation problem with \( m \) sources and \( n \) destinations. Let \( \tilde{a}_i \) \( (\tilde{a}_i \geq 0) \) be the fuzzy availability at source \( i \) and \( \tilde{b}_j \) \( (\tilde{b}_j \geq 0) \) be the fuzzy requirement at destination \( j \).

Let \( \tilde{c}_{ij} \) be the fuzzy unit transportation cost from source \( i \) to destination \( j \). Let \( \tilde{x}_{ij} \) denote the number of fuzzy units to be transported from source \( i \) to destination \( j \). Now the problem is to determine a feasible way of transporting which minimizes the total fuzzy transportation cost.

\[
\text{Minimize } z^k(x) = \sum_{i=1}^{m} \sum_{j=1}^{n} c^k_{ij} \tilde{x}_{ij}
\]

subject to \( \sum_{j=1}^{n} \tilde{x}_{ij} = \tilde{a}_i, i = 1, 2, ..., m \)

\( \sum_{i=1}^{m} \tilde{x}_{ij} = \tilde{b}_j, j = 1, 2, ..., n \)

and \( \tilde{x}_{ij} \geq 0, \text{for all } i, j \).

where \( z^k(x) = \{z^1(x), z^2(x), ..., z^k(x)\} \) is a vector of \( k \) fuzzy objective functions and the superscript on both \( z^k(x) \) and \( c^k_{ij} \) are used to indicate the number of fuzzy objective functions. Without loss of generality, it is assumed in the paper that \( \tilde{a}_i > 0, \tilde{b}_j > 0, \forall i, \forall j, c^k_{ij} \geq 0, \forall i, \forall j \) and \( \sum_{i} \tilde{a}_i = \sum_{j} \tilde{b}_j \).

**Definition 3.1:** If the fuzzy objective functions are said to be conflicting, then there exists a fuzzy pareto optimal solution.

**Definition 3.2:** A fuzzy solution is called fuzzy non dominated, fuzzy pareto optimal, fuzzy pareto efficient or non inferior, if none of the fuzzy objective functions can be improved in value without degrading some of the other fuzzy objective values.

**Definition 3.3:** Fuzzy Pareto efficiency or fuzzy pareto optimality is a state of allocation of resources in which it is impossible to make anyone individual better off without making atleast one individual worse off.
Definition 3.4: If the fuzzy compromise solution satisfies the decision maker's preferences, then the solution is called the fuzzy preferred compromise solution.

4. Fuzzy Genetic Algorithm

Fuzzy Genetic Algorithm consists of mainly three steps:

1. Fuzzy Selection
2. Fuzzy Crossover
3. Fuzzy Mutation

Fuzzy Selection: Of the three methods, Fuzzy North West corner rule, fuzzy least cost method, fuzzy Vogel's approximation method we select FVAM to obtain the initial fuzzy basic feasible solution.

Fuzzy Crossover: There are different types of fuzzy crossover namely

1. **Fuzzy single point Crossover** - One fuzzy crossover point is selected, fuzzy allocation from the beginning to the fuzzy crossover point is copied from the first fuzzy parent solution, the rest is copied from the other fuzzy parent solution.

2. **Fuzzy Two point Crossover** - Two fuzzy crossover points are selected, fuzzy allocation from the beginning of the first fuzzy crossover point is copied from the first fuzzy parent, the part from the first to the second fuzzy crossover point is copied from the other fuzzy parent and the rest is copied from the first fuzzy parent again.

3. **Fuzzy Uniform Crossover** – Fuzzy allocations are randomly copied from the fuzzy first or from the fuzzy second parent.

Initial basic fuzzy feasible solutions are considered as the fuzzy parent solutions. By using Fuzzy Crossover operator we generate a second generation population of Fuzzy solutions from those Fuzzy parent solutions and we obtain Fuzzy child 1 and Fuzzy child 2.

Fuzzy Mutation: Fuzzy Mutation alters one or more gene values in a chromosome from its initial state. In mutation, the fuzzy solution may change entirely from the previous fuzzy solution. Hence fuzzy GA can come to better fuzzy solution by using mutation.

5. Numerical Example

Consider the balanced fuzzy multi objective transportation problem [7]

\[
\tilde{C}^1 = \begin{bmatrix}
(0,1,2) & (1,2,3) & (6,7,8) & (6,7,8) \\
(0,1,2) & (8,9,10) & (2,3,4) & (3,4,5) \\
(7,8,9) & (8,9,10) & (3,4,5) & (5,6,7)
\end{bmatrix} \quad \tilde{C}^2 = \begin{bmatrix}
(2,3,4) & (3,4,5) & (2,3,4) & (3,4,5) \\
(4,5,6) & (7,8,9) & (8,9,10) & (9,10,11) \\
(5,6,7) & (1,2,3) & (4,5,6) & (0,1,2)
\end{bmatrix}
\]

Fuzzy supplies: \(\tilde{a}_1 = (0,3,5), \tilde{a}_2 = (4,6,9), \tilde{a}_3 = (4,6,7)\)

Fuzzy demands: \(\tilde{b}_1 = (2,4,5), \tilde{b}_2 = (0,1,2), \tilde{b}_3 = (2,5,7), \tilde{b}_4 = (4,5,7)\)

Initial Allocation
\[ \tilde{C}_1 = \begin{bmatrix} (0,2,3) & (0,1,2) & (0,0,0) & (0,0,0) \\ (0,1,2) & (0,0,0) & (0,0,0) & (4,5,7) \\ (2,1,0) & (0,0,0) & (2,5,7) & (0,0,0) \end{bmatrix} ; \tilde{C}_2 = \begin{bmatrix} (0,0,0) & (0,0,0) & (0,3,5) & (0,0,0) \\ (2,4,5) & (0,1,2) & (2,1,2) & (0,0,0) \\ (0,0,0) & (0,0,0) & (0,1,0) & (4,5,7) \end{bmatrix} \]

Fuzzy Parent 1: \( \tilde{Z}_1 = (18, 53, 104), \tilde{Z}_2 = (44, 91, 167) \)

Fuzzy Parent 2: \( \tilde{Z}_1 = (24, 71, 127), \tilde{Z}_2 = (24, 56, 102) \)

**Fuzzy Single Point Crossover**

\[ \tilde{C}_1 = \begin{bmatrix} (0,2,3) & (0,1,2) & (0,0,0) & (0,0,0) \\ (0,2,2) & (0,0,0) & (2,4,7) & (0,0,0) \\ (0,0,0) & (0,0,0) & (0,1,0) & (4,5,7) \end{bmatrix} ; \tilde{C}_2 = \begin{bmatrix} (0,0,0) & (0,0,0) & (0,0,0) & (0,3,5) \\ (0,3,5) & (0,1,2) & (0,0,0) & (4,2,2) \\ (2,1,0) & (0,0,0) & (2,5,7) & (0,1,2) \end{bmatrix} \]

Fuzzy child 1: \( \tilde{Z}_1 = (24, 52, 93) , \tilde{Z}_2 = (44, 91, 153) \)

Fuzzy child 2: \( \tilde{Z}_1 = (12, 69, 143), \tilde{Z}_2 = (26, 78, 173) \)

**Fuzzy Mutation**

\[ \tilde{C}_1 = \begin{bmatrix} (0,2,3) & (0,1,2) & (0,0,0) & (0,0,0) \\ (2,2,2) & (0,0,0) & (0,0,0) & (2,4,7) \\ (0,0,0) & (0,0,0) & (2,5,7) & (2,1,0) \end{bmatrix} ; \tilde{C}_2 = \begin{bmatrix} (0,0,0) & (0,1,2) & (0,0,0) & (0,2,3) \\ (0,3,5) & (0,0,0) & (0,0,0) & (4,3,4) \\ (2,1,0) & (0,0,0) & (2,5,7) & (0,0,0) \end{bmatrix} \]

Fuzzy child 3: \( \tilde{Z}_1 = (12, 48, 86), \tilde{Z}_2 = (30, 84, 155) \)

Fuzzy child 4: \( \tilde{Z}_1 = (18, 51, 101), \tilde{Z}_2 = (44, 85, 155) \)

After Fuzzy Mutation, we have the better fuzzy optimal solution: \( \tilde{Z}_1 = (12, 48, 86), \tilde{Z}_2 = (30, 84, 155) \)

**Conclusion:**

For the Bi objective fuzzy transportation problem solved by fuzzy Genetic Algorithm, the fuzzy compromised solution obtained is \( \tilde{Z}_1 = (12, 48, 86), \tilde{Z}_2 = (30, 84, 155) \)

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