High-energy neutrino-induced cascade from the direction of the flaring radio blazar TXS 0506+056 observed by the Baikal Giganon Volume Detector in 2021

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ABSTRACT

The existence of high-energy astrophysical neutrinos has been unambiguously demonstrated, but their sources remain elusive. IceCube reported an association of a 290-TeV neutrino with a gamma-ray flare of TXS 0506+056, an active galactic nucleus with a compact radio jet pointing to us. Later, radio blazars were shown to be associated with IceCube neutrino events with high statistical significance. These associations remained unconfirmed with the data of independent experiments. Here we report on the detection of a rare neutrino event with the estimated energy of 224 ± 75 TeV from the direction of TXS 0506+056 by the new Baikal-GVD neutrino telescope in April 2021 followed by a radio flare observed by RATAN-600. This event is the highest-energy cascade detected so far by Baikal-GVD from a direction below horizon. The result supports previous suggestions that radio blazars in general, and TXS 0506+056 in particular, are the sources of high-energy neutrinos, and opens up the cascade channel for the neutrino astronomy.

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1. INTRODUCTION

Recently, the Baikal-GVD neutrino telescope collaboration reported (Avrorin et al. 2022; Dzhilkibaev 2022) on its first detection of astrophysical neutrinos with hundreds of TeV energies. This gives the first independent confirmation of the results of the IceCube telescope that has been observing high-energy neutrinos since 2008 (IceCube Collaboration 2013). Yet, astrophysical neutrino origins, as well as the details of their production mechanisms, still remain poorly understood, see e.g. Troitsky (2021) for a review.

Active galactic nuclei (AGNs) have been considered as potential neutrino sources since the very early days of multi-messenger astronomy. Recent statistical observational studies e.g. Plavin et al. (2020, 2021); Giommi et al. (2020); Hovatta et al. (2021); Buson et al. (2022) associate high-energy neutrino emission with blazars, a class of active galaxies with jets pointing towards the observer. Like photons, neutrinos experience relativistic boosting in these objects, facilitating their detection. There is evidence for neutrinos being preferentially produced during major flares in the central parsecs of blazars (IceCube Collaboration et al. 2018; Plavin et al. 2020; Hovatta et al. 2021). What are the specific regions where neutrinos are produced and what is the physical connection between neutrinos and electromagnetic flares, remain open questions.

The first association of an individual astrophysical object with a high-energy neutrino was found for the TXS 0506+056 blazar (IceCube Collaboration et al. 2018). On September 22, 2017, the IceCube observatory detected a probable 290-TeV neutrino from the direction of this blazar. TXS 0506+056 was experiencing a major flare across the entire electromagnetic spectrum, from radio to gamma rays. Since then, this object has been extensively studied using astronomical instruments operating at various electromagnetic wavelengths. This object is characterized by bright compact emission on parsec scales but appears to be completely ordinary otherwise (Kovalev et al. 2020). Further studies of neutrino emission of TXS 0506+056 can shed more light on the mechanisms of neutrino production, not only in this particular source but in the whole bright blazar population.

Observation of astrophysical high-energy neutrinos requires detectors of very large volume because neutrino interacts with matter very weakly. The Baikal Gigaton Volume Detector, or Baikal-GVD, is a Northern-hemisphere cubic-kilometer scale deep underwater Cherenkov detector aimed at the search for neutrinos with energies\(^1\) between several TeV and tens of PeV. The telescope is located in the Southern part of Lake Baikal (51°50’ N, 104°20’ E), at about 4 km from the shore. The lake depth at the facility site is 1366 m. The instrument has been operating since 2016 with the operational volume increasing every year in the course of the detector deployment.

Neutrinos are detected through the Cherenkov radiation emitted by secondary particles produced in neutrino interactions in the water or bedrock below the detector and observed by the detector light sensors, optical modules (OMs). Charged-current (CC) muon neutrino interactions yield long-lived muons that can pass several kilometers through the water, leading to a track signature crossing the detector, see e.g. Allakhverdyan et al. (2021). The accuracy of the angular reconstruction better than 1° is typical of high energy track-like events in neutrino telescopes. Tracks are therefore often used for the search of point sources of astrophysical neutrinos. However, the precision of the determination of the neutrino energy is poor in this case.

Neutral-current (NC) interactions, as well as most of CC interactions of electron and tau neutrinos, yield hadronic and electromagnetic showers (cascades). The showers are quasi point-like, highly anisotropic sources of Cherenkov radiation. The energy of the cascade progenitor neutrino is determined with a good accuracy; however, the angular reconstruction is worse for cascades than for tracks. The cascade channel is thus complementary to the track one. Previously, cascades were used mostly in searches for the diffuse neutrino flux and in studies of the astrophysical neutrino spectrum.

Optical properties of Baikal deep water are characterized by the light absorption length \(L_a \approx (21 - 23) \text{ m}\) and the scattering length \(L_s \approx (60 - 80) \text{ m}\) at \(\lambda = (480-500) \text{ nm}\) wavelength. Compared to the ice used as the target medium in the IceCube neutrino telescope, this low-scattering medium makes it possible to reconstruct the arrival directions of cascade progenitor neu-

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\(^1\) 1 TeV= \(10^{12}\) eV; 1 PeV= \(10^{15}\) eV.
Table 1. Reconstructed parameters of the highest-energy upgoing cascade event: arrival time (Modified Julian Day, MJD), reconstructed energy $E_{sh}$, number of hits $N_{hit}$, zenith angle $\theta$, right ascension (RA) and declination (Dec), Galactic longitude $l$ and latitude $b$.

| Event ID       | MJD       | $E_{sh}$ (TeV) | $N_{hit}$ | $\theta$ (deg) | RA (deg) | Dec (deg) | l (deg) | b (deg) |
|---------------|-----------|---------------|----------|----------------|----------|-----------|---------|---------|
| GVD210418CA   | 59322.94855324 | 224±75       | 24       | 115            | 82.4     | 7.1       | 163.2   | -14.6   |

A distinctive feature of the Baikal-GVD design is its modular structure, which allows performing physical studies even at early stages of setup deployment. The telescope is composed of several sub-arrays, clusters of optical modules. Each cluster is an independent array comprising a total of 288 OMs and is connected to the shore station by its own electro-optical cable. The current rate of array deployment is about two clusters per year (see for details Appendix A).

In this work, we use the approach to the search for high-energy neutrinos of astrophysical origin based on the selection of events from showers generated in the sensitive volume of the Baikal-GVD neutrino telescope. The procedures of shower-like event selection and cascade reconstruction by a cluster of Baikal-GVD are presented elsewhere (e.g. Avrorin et al. 2022; Dzhilkibaev et al. 2022). The precision of the energy reconstruction is ~ (10–30)% and depends substantially on the energy of the cascade, its position and orientation relative to the cluster. The precision of reconstruction of the progenitor neutrino direction depends strongly on OM hit multiplicity and is about 2°–4° (median value). In this analysis, we use the Baikal-GVD data collected between April, 2018, and March, 2022, which corresponds to 13.5 years of equivalent live time for one cluster. The sample of 3.49×10^{10} events was collected by the basic trigger of the telescope. After noise-hit suppression, cascade reconstruction and application of cuts on the reconstruction quality parameters, a sample of 14328 cascades with reconstructed energy $E_{sh}>10$ TeV and OM hit multiplicity $N_{hit}>11$ was selected. Most of these events have atmospheric origin and constitute the background for the astrophysical neutrino search.

Additional, drastic background suppression is achieved by selecting only upward moving cascades, as it was described in Dzhilkibaev (2022). Cascade-like events with $E_{sh}>15$ TeV and reconstructed zenith angle $\theta$ satisfying $\cos\theta<-0.25$, were selected as astrophysical neutrino candidates. A total of 11 such events have been found in the data sample, while on average, 2.7 events are expected from atmospheric neutrinos and 0.5 events from mis-reconstructed atmospheric muons.

The highest-energy cascade detected so far by Baikal-GVD from a direction below the horizon arrived on April 18, 2021. The basic parameters of this event are presented in Table 1. The pattern of this event in the Baikal-GVD telescope is show in Figure 1.
For the given $E_{\text{sh}}$, $N_{\text{hit}}$, and shower direction, the probability that this event comes from the atmospheric background can be estimated from Monte-Carlo simulations to be as low as $P_{\text{atm}} = 0.0033$. Assuming $E^{-2.36}$ spectrum of astrophysical neutrinos (Abasi et al. 2021), we determine the “signalness” parameter (Aartsen et al. 2017), which is a measure of the likelihood of the event to have the astrophysical origin relative to the background. The signalness of the GVD210418CA cascade is 97.1%. For comparison, the signalness of the track event IceCube-170922A, associated with TXS 0506+056, is only 56.5% (IceCube Collaboration et al. 2018). The Monte-Carlo simulations of this event demonstrate that in 90% of cases, the reconstructed neutrino arrival direction is within the opening angle of 6.0° from the assumed one. This 90%-containment circle contains the direction to the TXS 0506+056 blazar, as illustrated in Figure 2. Below we discuss the possibility that the highest-energy Baikal-GVD upgoing cascade, GVD210418CA, is indeed associated with this well-known blazar.

2.2. RATAN-600 monitoring of TXS 0506+056

The 600-meter ring radio telescope RATAN-600 of the Special Astrophysical Observatory (Korolkov & Pariskii 1979), the Russian Academy of Sciences, performs long-term observations of continuum spectra at centimeter wavelengths of a large sample of extragalactic radio sources selected with very long baseline interferometry (VLBI) technique (e.g. Kovalev et al. 1999, 2002). One of them is the blazar TXS 0506+056.

In order to characterize the flaring activity of the blazar in radio waves, we decomposed the two highest-frequency light curves into three flares, see Appendix C, Table 2, and Figure 3. The neutrino event IceCube-170922A occurred on September 22, 2017, during a slow rising of the strongest radio flare of this source over the past 25 years (Kovalev et al. 2020) and a large gamma-ray flare (Figure 3). The radio flare started in 2015-2016 after a long low-activity state and reached its maximum at 22 GHz at the end of 2019, while lower frequency emission was delayed, as expected from synchrotron opacity. The Baikal neutrino event detected on April 18, 2021, may coincide with the beginning of a new radio flare peaking at 2022.0 (see Figure 3 and Table 2).

2.3. Fermi LAT light curve

We constructed the gamma-ray photon flux light curve of the source 4FGL J0509.4+0542, positionally associated with the quasar 0506+056, based on the Fermi LAT data taken within the energy range 0.1–300 GeV since 2008 August 4 through 2022 May 11 (Figure 3). We applied the adaptive binning technique (Lott et al. 2012) with a constant target flux uncertainty of 20%, and assuming that the source energy spectrum follows a power-law function with photon index $\Gamma = 2.079$ (Abdollahi et al. 2020; Ballet et al. 2020). The resulting light curve comprises 281 bins with a median bin duration of 17 days, a median Test Statistics (Mattox et al. 1996) $TS = 64$, and a median photon flux $F_p = 6.8 \times 10^{-8} \text{ ph cm}^{-2} \text{s}^{-1}$. The source experienced activity periods centered on 2011 and mid-2017, but at the time of the Baikal-GVD cascade neutrino event, the source was in the low-state, with the photon flux $3.2 \times 10^{-8} \text{ ph cm}^{-2} \text{s}^{-1}$, twice as low as the median level.

3. DISCUSSION

We have found that an exceptional Baikal-GVD event, GVD210418CA, having a 99.67% probability to be of astrophysical origin, is associated with an exceptional source singled out by previous studies. Indeed, not only the energetic, $E = 290^{+2010}_{-75} \text{ TeV}$ (see IceCube Collaboration et al. 2018, Figure S2), neutrino candidate event IC170922A was associated with a gamma-ray flare of TXS 0506+056, but it was the only source claimed by IceCube Collaboration at estimated neutrino energies $E > 200 \text{ TeV}$. A significant correlation between the neutrino arrival and the gamma-ray activity of a source was found for this case only. It is interesting to estimate the probability that the exceptional Baikal-GVD event coincides with the unique source by chance. All extremely high-energy events with published energies, used to establish TXS 0506+056 as a neutrino source,
have $E > 200$ TeV, and this threshold was previously chosen for various other analyses (e.g. by Aartsen et al. 2016; Plavin et al. 2020). In what follows, we also adopt this energy threshold for the present analysis. Note that lowering the threshold down to $\sim 100$ TeV or increasing it up to the energy of the exceptional event, we consider here would not change our numerical estimates considerably.

The highest-purity sample of astrophysical neutrino candidates in the Baikal-GVD analysis contains upgoing cascade events only, with zenith angles satisfying $\cos \theta < -0.25$. Only one Baikal-GVD cascade with $E > 200$ TeV was detected from this $0.75 \cdot 2\pi$ sr of the sky, the one which we discuss here. The exposure of the detector is not uniform and depends on $\theta$: the Earth is not fully transparent to neutrinos at the highest energies. We use Monte-Carlo simulations of of isotropically distributed incoming astrophysical neutrinos of the observed energy, 224 TeV, to take this effect into account. We obtain the chance probability of coincidence, within the 90% confidence level angular-resolution cone of $6.0^\circ$, of the arrival direction of a single event with the position of TXS 0506+056 in the sky, as $p_{\text{TXS}} = 0.0074$.

The Baikal-GVD detection of the second $E > 200$ TeV event from TXS 0506+056, in addition to the IceCube event of 2017, significantly changes the estimate of high-energy neutrino flux from this particular source. Indeed, while it was possible to obtain an order-of-magnitude estimate of the flux with a single IceCube event, it might be misleading both because of the Eddington bias (Strotjohann et al. 2019) and due to the lack of knowledge of the typical duration of a flare. Combining the IceCube and Baikal-GVD measurements, the flux can be estimated with an improved uncertainty, and some information about the duration of the flare can be obtained.

At the time of the Baikal-GVD event from TXS 0506+056 in 2021, eight clusters of GVD were in operation, with the effective area for cascades several times smaller than that of IceCube for the track channel. The observation of one event in GVD and zero events in larger IceCube are reconcilable with the Poisson fluctuations provided the expected number of the observed events is small. This holds only under the assumption of a flaring source, with the duration of flares about a year or shorter. Otherwise, non-observation of events by IceCube would become incompatible with the assumed flux due to its large cumulative exposure.

Assume that the source emits neutrinos in flares with the flux $F$ (all flavors, energy > 200 TeV) and duration $T \sim 1$ yr, as it is motivated by the radio lightcurve studies (Plavin et al. 2020; Hovatta et al. 2021). The 2017 IceCube event was reported as an extremely high-energy (EHE) muon-track alert; the corresponding energy-dependent effective area was presented in Figure S1 of IceCube Collaboration et al. (2018) for the declination bin containing the source of interest. Since 2018, all EHE alerts are included, among others, in the GOLD alert channel (Blaufuss et al. 2020), and no such event was reported from the direction of TXS 0506+056. The
2021 Baikal event was observed in the cascade mode; the average all-sky effective area for the 7-cluster GVD configuration was published by Avrorin et al. (2022). However, the GVD effective area is a growing function of time in the construction period; here we use the time-dependent exposure estimated for the declination of TXS 0506+056. Starting from the number of the observed events, 1 by IceCube in the 2017 flare, 0 by IceCube, and 1 by Baikal-GVD in the 2021 flare, we estimate the $E > 200$ TeV fluence of such a flare as $5.6^{+7.2}_{-1.8} \times 10^{-14}$ cm$^{-2}$s$^{-1}$ assuming the power-law spectrum with index 2.0. For the assumption of the index 2.5, the estimated fluence is $9.7_{-3.0}^{+12.7} \times 10^{-14}$ cm$^{-2}$s$^{-1}$.

The probability to observe one or more Baikal events during the 2021 flare while not more than one is observed by IceCube during the 2017 and 2021 flares is about 11% (15%) for the assumed spectral indices 2.0 (2.5), respectively.

The estimated neutrino flux corresponds to $E^2 F(E) \sim 10^{-11}$ TeV cm$^{-2}$s$^{-1}$, consistent by the order of magnitude with the IceCube estimates of 2017 and with the gamma-ray flux in the Fermi LAT band during the high state of TXS 0506+056 IceCube Collaboration et al. (2018). However, the Fermi LAT flux remains low in the period of detection of the Baikal-GVD event. Together with the earlier observation of the lack of gamma-ray activity during the neutrino flare of the same source in 2014 (Figure 3), this suggests that simple one-zone models are disfavored as an explanation of both gamma-ray and neutrino data. This is in line with strongly constraining results of the population studies, e.g. Neronov et al. (2017); Murase et al. (2018), for a recent discussion see Kun et al. (2021).

Contrary, Figure 3 reveals an important similarity between the IceCube (2017) and Baikal-GVD (2021) events in terms of the radio observations: both neutrinos arrived at the beginning of prominent radio flares. This gives additional support for the statistical association between high-energy neutrinos and compact radio-loud structures in blazars proposed by Plavin et al. (2020), which may give important clues to ultimate understanding of the mechanisms of neutrino production. For instance, both the radio flare and the enhanced rate of neutrino emission may be caused by an increase in the continuous flow of matter falling from the accretion disk onto the supermassive black hole located in the center of the blazar. Part of this matter is thrown away and, after being accelerated to very high energies, forms a relativistic jet. The observed radio emission is caused by the synchrotron radiation of these accelerated particles, and the flare is accompanied by an increase in the density of radiation and matter, providing targets for neutrino production in interactions of accelerated protons.

VLBI monitoring (e.g. the currently ongoing 15 GHz MOJAVE program at the VLBA, Lister et al. 2021) will help to understand what is happening at parsec scales and check if there is a connection between neutrino production, core brightening, a birth, and prorogation of new features through the jet base. See the first results of VLBI monitoring observations after the 2017 high energy IceCube event in Ros et al. (2020).

Previous population studies suggested that the number of sources of high-energy neutrinos is large (Neronov & Semikoz 2020; Capel et al. 2020). This is true in particular for radio blazars (Plavin et al. 2021): statistically significant results were obtained for their large samples only. One does not expect many high-energy neutrino events coming from a single typical source, and this is what was observed. The accumulation of statistics and the contribution from a new experiment, Baikal-GVD, will make these conclusions more refined: multiple high-energy neutrinos start to be recorded from directions to particular blazars, see also Plavin et al. (2022). Some of these sources, TXS 0506+056 among them, may be singled out by prolonged periods of higher activity or just demonstrate lucky fluctuations; future higher-exposure studies will be necessary to make the choice.

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APPENDIX

A. DETAILS OF BAIKAL-GVD OBSERVATIONS

The Baikal-GVD neutrino telescope has a modular configuration with a basic structure named cluster. Each cluster is operated as an independent detector, taking data upon its deployment (Allakhverdyan et al. 2022). A standard cluster with eight strings (one central and seven peripheral strings) of 288 OMs in total covers a cylindrical instrumental volume of 525 m in height and 120 m in diameter. A basic element of the cluster is an OM equipped with a 25 cm photo-multiplier tube and related electronics. Inter-cluster distances measured between the central strings vary between 250 m and 300 m. The choice of the geometrical parameters follows the results of a Monte Carlo study optimizing the high-energy neutrino detection rate at trigger level. It implies a balance between two factors. On the one hand, large distances between photo detectors are needed to obtain a larger effective volume. On the other hand, sufficient density of OMs is required to accurately reconstruct the neutrino interaction vertex, energy, and direction. The operational configurations of Baikal-GVD in 2021 (8 clusters, 64 strings, 2304 OMs) and 2022 (10 clusters, 80 strings, 2880 OMs) are shown schematically in Figure 4.

Figure 4. Scheme of Baikal-GVD. Left: string structure (side view). Right: cluster arrangement scheme (top view).
Luminescence of organic particles suspended in water is a natural background for the detection of Cherenkov photons from charged particles by OMs. The luminescence results in typical OM illumination at the level of one photo-electron (ph.el.). The cut on the lowest allowed charge of OM hits $Q > 1.5$ ph.el. allows suppressing the number of noise pulses from water luminescence substantially. At the time of recording the event from the direction of TXS 0506+056, the level of noise rate of photons from water luminescence was low, as usually happens in April every year. The monitoring of the water optical properties in April 2021 indicates an absorption length of about 22 m — very close to the long-term average — and a relatively high value of the scattering length, more than 60 m. Thus the event we discuss in this paper was observed under favorable operational conditions.

The Cherenkov radiation from electromagnetic and hadronic showers is formed by photons emitted by charged particles of the shower (mainly by electrons and positrons) and is determined by their spatial, angular, and temporal distributions. Cascades are reconstructed as point-like sources of light. The coordinates, energy, and orientation of showers are reconstructed in two stages. At the first stage, the shower coordinates are reconstructed using temporal information from the telescope’s triggered OMs (in the approximation of a point source). The reconstruction procedure consists in determining of the shower coordinates that correspond to the minimum of the $\chi^2$ function. At the second stage, the shower energy and direction are reconstructed by the maximum likelihood method using the shower coordinates reconstructed at the first stage (Avrorin et al. 2009). The precision of the reconstruction of the arrival direction of the neutrino, which caused the GVD210418CA cascade, was estimated by the reconstruction of Monte-Carlo generated cascades with energies and directions equal to reconstructed values of the event. The distribution of the mismatch angle between generated and reconstructed directions is shown in Figure 5.

### B. DETAILS OF RATAN-600 MONITORING

The RATAN-600 measurements of TXS 0506+056 discussed in this paper were carried out at frequencies of 2.3, 4.7, 8.2, 11.2, and 22.3 GHz in 2010-2022. The earlier RATAN-600 instantaneous continuum spectra since 1997 are published in Kovalev et al. (2020). The observations were made in the RATAN transit mode, and the multi-frequency broad-band radio spectra were obtained at the five above-mentioned frequencies within 3-5 min. The radio telescope parameters are described in Parijskij (1993); Kovalev et al. (1999, 2000); Sotnikova (2020). The measurements were processed using the Flexible Astronomical Data Processing System (FADPS) standard package modules (Verkhodanov 1997) for the RATAN continuum radiometers and the automated data reduction systems (Tsybulev 2011; Tsybulev et al. 2018; Udvovitskii et al. 2016; Kovalev et al. 1999). We used the following flux density secondary calibrators: 3C48, 3C138, 3C147, 3C161, 3C286, and NGC 7027. The calibrator measurements were corrected for the angular size and linear polarization, according to the data from Ott et al. (1994) and Tabara & Inoue (1980). The flux densities of the amplitude calibrators were used within the scale by Baars et al. (1977), their temporal evolution was considered following Ott et al. (1994) and Perley & Butler (2013, 2017).
Figure 6. RATAN-600 light curves of TXS 0506+056 measured at 2.3-22.3 GHz in 2010.0–2022.5. The blue vertical lines mark different IceCube and Baikal-GVD neutrino events which are labeled in the plot.

C. FLARE DECOMPOSITION OF RADIO LIGHT CURVES

It is often assumed that radio flux density variations in blazars are due to the features forming and flaring in the jet and a contribution from the underlying jet (e.g. Valtaoja et al. 1999). We decomposed our radio light curves into a few exponential flares and a constant quiescent level, $\Delta S_0$. A growth and decay of each flare was modeled in the following form (e.g. Hovatta et al. 2009):

$$\Delta S(t) = \begin{cases} \Delta S_{\text{max}} e^{(t-t_{\text{max}})/\tau}, & t < t_{\text{max}} \\ \Delta S_{\text{max}} e^{(t_{\text{max}}-t)/1.3\tau}, & t > t_{\text{max}} \end{cases}$$ \hspace{1cm} (C1)

where $\Delta S_{\text{max}}$ is the maximum flux density of the flare in [Jy] calculated on the top of the quiescent level $\Delta S_0$ and observed at the time $t_{\text{max}}$; $\tau$ is the rise time of the flare. This approach is commonly used for radio data at high frequencies since flux density varies on typically smaller timescales due to opacity. Thus, we performed a flare decomposition at 11 and 22 GHz. To find the number of flares that best describes our data, we applied the Akaike information criterion (e.g. Akaike 1974). It showed that the source light curve is best described by three flares, see the red curve in Figure 3 for the 11 GHz data. The fitted parameters of each flare and quiescent flux densities are given in Table 2.
Table 2. Main parameters of the exponential flares. First row for each frequency indicates the quiescent flux density $\Delta S_0$. The other rows show corresponding parameters of each flare.

| Frequency (GHz) | Flux density, $\Delta S$ (Jy) | Peak time, $t_{\text{max}}$ (yr) | Rise time, $\tau$ (yr) |
|----------------|--------------------------------|---------------------------------|------------------------|
| 11             | 0.36 ± 0.04                    | 2020.13 ± 0.02                  | 1.05 ± 0.05            |
|                | 1.78 ± 0.05                    | 2020.87 ± 0.01                  | 0.23 ± 0.04            |
|                | 0.67 ± 0.05                    | 2022.01 ± 0.02                  | 0.23 ± 0.05            |
| 22             | 0.44 ± 0.05                    | 2019.90 ± 0.02                  | 0.67 ± 0.05            |
|                | 1.78 ± 0.07                    | 2020.92 ± 0.01                  | 0.19 ± 0.03            |
|                | 0.65 ± 0.05                    | 2021.99 ± 0.02                  | 0.12 ± 0.04            |