On Throughput Optimization and Bound Analysis in Cache-Enabled Fiber-Wireless Access Networks

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Abstract—The backhaul of fiber-wireless (FiWi) access networks is facing the bandwidth crunch due to the increasing demands for bandwidth-hungry applications. In this paper, we consider utilizing the caches in FiWi access networks to mitigate backhaul bottleneck and improve network throughput. As both the power budget in wireless access networks and the bandwidth of backhaul are constrained, it is challenging to allocate power for caching and wireless transmission properly to achieve superior system performance. Our goal is to maximize the downlink throughput by considering power allocation and caching strategy jointly. We first propose a volume-adjustable backhaul-constrained water-filling method (VABWF) to derive the optimal power allocation for wireless transmission. A dynamic programming algorithm is then designed by reformulating the problem as a multiple-choice knapsack problem (MCKP). To explore the potential of throughput in cache-enabled FiWi access networks, we derive a tractable expression of the ergodic capacity (EC) of wireless links and provide a theoretical upper bound of the throughput by utilizing the analytical properties of EC. We show that an appropriate allocation of transmission power and caching power is essential to achieve high throughput. Numerical and simulation results validate our theoretical analysis and demonstrate the effectiveness of our proposed optimization method.

Index Terms—Fiber-Wireless (FiWi) access networks, Backhaul, Throughput, Caching, Power Allocation

I. INTRODUCTION

Access networks are foreseen to suffer from a severe bandwidth crunch in the coming years [1], [2]. According to Cisco’s forecast [3], global Internet traffic will achieve a threefold increase over the next 5 years, and video applications will be approximately 82% of all Internet traffic by 2022, up from 75% in 2017. Evidently, supporting the increasing demand for bandwidth-hungry multimedia applications such as video on demand (VOD) services, access points (APs) are supposed to be densely deployed to improve network capacity in wireless front ends [7], thus putting substantial pressure on the fiber backhaul. Second, the recent IEEE 802.11ax standard specifies data rates of up to 10 Gbps for higher wireless access throughput [3]. Hence, fiber backhaul is facing bandwidth bottleneck that limits the network throughput. Some research efforts have been devoted to next-generation passive optical networks (NG-PONs) [9], [10], which aim at increasing the capacity of state-of-the-art PONs. However, existing optical network units (ONUs) and optical line terminals (OLTs) are not directly compatible with the technical specifications of NG-PONs [11], so ONUs and OLTs must be replaced or upgraded to support NG-PONs, which is a costly method at present. For example, according to [12], the average price of an NG-PON ONU is more than 20 times of that of a GPON ONU. Such a price premium is not cost-effective to alleviate the fiber backhaul bandwidth pressure.

At the same time, there have been growing recent researches towards data caching as an approach for alleviating bandwidth pressure in fiber networks. In [13], an architecture consisting of an ONU with an associated storage unit was proposed to save traffic in the feeder fiber and improve the system throughput as well as mean packet delays. In [14], a framework of software-defined PONs with caches was introduced to achieve a substantial increase in served video requests. In [15], a dynamic bandwidth algorithm based on local storage VOD delivery in PONs was proposed, and the achievable throughput levels have been improved when a local storage is used to assist VOD delivery. However, most of them mainly focus on providing storage capacity in PONs to improve the network throughput, so these methods cannot be directly applied to FiWi access networks.

On the other hand, power consumption has become one of the main concerns in modern communication networks due to the rapid growth in Internet traffic, especially when involving caches in optical networks, which introduces additional...
power consumption. The power efficiency of different caching hardware technology has been shown in [16]. As an example, the commonly used storage medium high-speed solid state disk (SSD) consumes about 5 W (Watt) of caching power per 100 GB. In [17], the power consumption brought about by the introduction of local cache is analyzed and the trade-off between enhanced Quality-of-Service performance versus the increased power requirement is examined. In [18], a peer-to-peer based caching scheme over passive optical networks is proposed to reduce traffic in the core network and decrease the overall network power consumption. In [19], the authors study the impact of deploying video cache servers on the power consumption over hybrid optical switching, and demonstrate a careful dimensioning of cache size to achieve high power efficiency.

In FiWi access networks, the functions of ONU and AP are integrated into a component called ONU-AP [20]. To mitigate the bandwidth crunch of fiber backhaul in FiWi access networks, we equip ONU-APs with caches. Note that in FiWi access networks, the wireless front end will also make an impact on the system throughput in addition to the fiber backhaul network [21]. Transmission power and channel conditions in the wireless front end should be taken into account, which is different from optical access networks with caches mentioned above. Particularly, transmission power of ONU-APs and channel conditions from user equipments (UEs) to ONU-APs are important factors determining the throughput in wireless access networks. When involving caches in FiWi access networks, it is challenging to properly leverage power for caching and that for wireless transmission to achieve optimal system performance. This is because if more power is used for wireless transmission, UEs can achieve higher wireless access throughput, whereas less power is used for caching, putting more pressure on fiber backhaul load. As both power budget in wireless access networks and bandwidth of fiber backhaul are constrained, joint power allocation and caching optimization are required to maximize the throughput in FiWi access networks.

In this paper, considering fiber backhaul bottleneck and channel conditions in wireless front end, as well as the sum power constraint of integrated ONU-APs, we formulate the joint power allocation and caching optimization problem as a Mixed Integer Programming (MIP) problem. To solve the joint optimization problem, firstly, we propose a volume adjustable backhaul constrained water-filling method (VABWF) using convex optimization to derive the expression of optimal wireless transmission power allocation. Then, we reformulate the problem as a multiple-choice knapsack problem (MCKP) by exploiting the highest-popularity-first property of files and design a dynamic programming algorithm based on the proposed VABWF method. To understand the potentials of involving caches in FiWi access networks, we derive a tractable expression of the ergodic capacity (EC) for wireless links and show that increasing the EC for wireless links does not necessarily improve the downlink throughput in cache-enabled FiWi access networks. To validate the effectiveness of the proposed algorithm, we also provide a theoretical upper bound of the downlink throughput by utilizing the analytical properties of EC. We prove that the theoretical downlink throughput upper bound is achieved where UEs follow a uniform spatial distribution. Simulation results show that the theoretical downlink throughput upper bound can be approached by using our proposed algorithm.

The rest of this paper is organized as follows. We first introduce the system model in Section II. In Section III, we derive the expression of optimal transmission power allocation and optimize the network throughput by joint consideration of power allocation and caching. We derive a tractable expression of EC for wireless links and give theoretical analysis on the downlink throughput upper bound by exploiting the analytical properties of EC in Section IV. Numerical and simulation results are shown in Section V and concluding remarks are provided in Section VI.

II. System Model

A. Network Model

The network model we propose is shown in Fig. 1. The FiWi access network is divided into a fiber backhaul network and a wireless front end network. We adopt Passive Optical Network (PON) as the fiber backhaul network. The optical line terminal (OLT) is located at the central office (CO), connecting to an optical splitter through the feeder fiber. The split ratio of the optical splitter is usually 1:32 or 1:64. In the FiWi access network, each ONU is collocated with an AP, and the integration of the ONU and AP is called integrated ONU-AP. The ONU-APs are denoted by an index set \( N = \{1, 2, \ldots, n, \ldots, N\} \). We assume that the feeder fiber connecting the splitter and the OLT has a capacity constraint of \( C \).

At the wireless front end of the FiWi access network, as shown on the right side of Fig. 1, ONU-APs (blue triangle) are assumed to be spatially deployed according to a hexagonal cell planning [22]. For easy analysis, we approximate the hexagonal cell shape as a circle with coverage radius \( D \) [23]. UEs (red circle) are distributed in the coverage of ONU-APs. The spatial distribution of the UEs is modeled as independent Poisson point process (PPP) [24], [25] where the average number of UEs in the whole network is \( \lambda \). We assume that each UE is associated with the closest ONU-AP, and use \( K = \{1, 2, \ldots, k, \ldots, K\} \) to denote the UE index set. Each UE is assumed to be associated with an ONU-AP in the fixed way, thus we denote the set of UEs associated with ONU-AP \( n \) by \( \Phi_n, n \in N \).

We assume that ONU-APs adopt orthogonal frequency-division multiple access (OFDMA) to transmit data, that is to say, an ONU-AP allocates orthogonal subcarriers to UEs, and different transmission power levels are assigned to different UEs. Let \( P_{nk} \) denote the transmission power allocated to UE \( k \) by ONU-AP \( n \). We adopt a standard path loss propagation model [26] with path loss exponent \( \alpha > 2 \). In order to account for the effects of small-scale fading, we consider a Rayleigh fading channel, where the channel gain \( h \) follows an exponential distribution with mean 1, i.e., \( h \sim \exp(1) \). Then
the channel state information (CSI) from ONU-APₙ to UEₖ can be defined as

\[ g_{nk} = r_{nk}^{-\alpha} h_{nk}. \]  

(1)

In this case, the received transmission power at UEₖ is \( g_{nk} P_{nk} \). The received signal-to-interference-plus-noise ratio (SINR) at UEₖ associated with ONU-APₙ becomes

\[ \gamma_{nk} = \frac{g_{nk} P_{nk}}{\sum_{m \in N \backslash \{n\}} g_{mk} P_{mk} + \sigma^2}. \]  

(2)

where \( N \backslash \{n\} \) denotes all the ONU-APs except ONU-APₙ, and \( \sigma^2 \) characterizes the power level of background noise.

We assume that non-overlapping channels are assigned to adjacent ONU-APs, and ONU-APs that reuse the same spectrum are deployed far away from each other. Thus, the co-channel interference is negligible, namely, \( g_{nk} = 0, m \in N \backslash \{n\} \). The wireless transmission rate from ONU-APₙ to UEₖ is given by the Shannon’s theorem

\[ R_{nk} = B \log_2 \left( 1 + \frac{g_{nk} P_{nk}}{\sigma^2} \right), \]  

where \( B \) is the subchannel bandwidth for each UE.

### B. ONU-AP Caching Model

Each ONU-AP is cache-enabled, and the cache size of ONU-APₙ is denoted by \( Qₙ \). Requested files are denoted by an index set \( J = \{1, 2, \ldots, j, \ldots, J\} \). If ONU-APₙ already caches a requested file, it will respond to the request directly. Otherwise, the file should be fetched from the Internet via capacity-limited fiber backhaul. We assume that all the files have the same size, which is denoted by \( s \). The file popularity distribution of \( J \) files is modeled as Zipf distribution, so the probability of the \( j \)-th ranked file being requested by UEs is \( p_j = \frac{j^{-\delta}}{\sum_{i=1}^{J} i^{-\delta}} \), where \( \delta \) is a shape parameter that shapes the skewness of the popularity distribution. Let \( p_{hit}^n \) denote the probability that files requested by UEs are cached at ONU-APₙ (i.e., cache hit ratio), and \( p_{miss}^n \) denote the probability that files requested by UEs are not cached at ONU-APₙ (i.e., cache miss ratio), where \( p_{hit}^n + p_{miss}^n = 1 \).

Given the cache decision on the \( j \)-th file at ONU-APₙ as \( x_{nj}, x \in \{0, 1\} \), the cache hit ratio at ONU-APₙ can be written as

\[ p_{hit}^n = \sum_{j \in J} x_{nj} p_j = \frac{\sum_{j \in J} x_{nj} j^{-\delta}}{\sum_{j \in J} j^{-\delta}}. \]  

(4)

Note that files requested by UEs that are not cached at ONU-APs should be fetched from the Internet via fiber backhaul. According to the cache miss ratio \( p_{miss}^n \) at ONU-APₙ, the average fiber backhaul bandwidth occupied by UEₖ at ONU-APₙ can be written as \( p_{miss}^n R_{nk} \), and thus the backhaul constraint of all the \( K \) UEs should be satisfied as

\[ \sum_{n \in N} \sum_{k \in \Phi_n} p_{miss}^n R_{nk} \leq C. \]  

(5)

### C. Power Consumption Model

The total power consumed by an ONU-APₙ can be obtained by leveraging the typical power consumption model of wireless networks and including the power used for caching as

\[ P_{total}^n = \sum_{k \in \Phi_n} \rho P_{nk} + P_{ca}^n + P_{cc}^n, \]  

where \( P_{ca}^n \) denotes the power consumed at ONU-APₙ for caching, \( P_{cc}^n \) denotes the power consumed by circuits, which is a constant depending on the circuit design, and \( \rho \) is a coefficient that measures the impacts of power amplifier, power supply and cooling. We use an energy-proportional model for caching power consumption, which has been widely adopted in content-centric networking for efficient use of caching power. In this model, the consumption of caching power is proportional to the total number of bits cached at an ONU-AP, i.e., \( P_{ca}^n = \omega n \), where \( n \) denotes the total number...
of bits cached at ONU-AP, and ω is a power coefficient that is related to the caching hardware and reflects the caching power efficiency in watt/bit. In this work, we consider the common caching device, high-speed solid state disk (SSD), for caching files at ONU-APs. The value of ω for SSD is $6.25 \times 10^{-12}$ watt/bit.

Each ONU-AP can adjust the transmission power and the caching power to achieve higher throughput without violating the maximum power constraint $P_M$, namely, $P_{\text{total}} \leq P_M$. Here, for notational convenience, the circuit power $P_{\text{circ}}$ is omitted since it is a constant.

### III. Joint Power Allocation and Caching

In this section, we first formulate the joint power allocation and caching problem. Then, we propose an optimal transmission power allocation method. To perform the joint optimization of power allocation and caching, an efficient algorithm based on dynamic programming is also provided by converting the problem into a standard MCKP.

#### A. Problem Formulation

Our objective is maximizing the downlink wireless access throughput for UEs by optimizing power allocation and caching strategy at ONU-APs in the FiWi access network. This is formulated as follows.

**P1:** $\max_{x, P} \sum_{n \in \mathcal{N}} \sum_{k \in \Phi_n} B \log_2 \left(1 + \frac{g_{nk}P_{nk}}{\sigma^2} \right)$

subject to:

1. $\sum_{k \in \Phi_n} \rho P_{nk} + \omega \sum_{j \in \mathcal{J}} x_{nj}s \leq P_M, \forall n \in \mathcal{N}$ (7b)
2. $\sum_{n \in \mathcal{N}} \sum_{k \in \Phi_n} \sum_{j \in \mathcal{J}} p_j (1 - x_{nj}) R_{nk} \leq C$ (7c)
3. $s \sum_{j \in \mathcal{J}} x_{nj} \leq Q_n, \forall n \in \mathcal{N}$ (7d)
4. $P_{nk} \geq 0, \forall n \in \mathcal{N}, \forall k \in \mathcal{K}$ (7e)
5. $x_{nj} \in \{0, 1\}, \forall n \in \mathcal{N}, \forall k \in \mathcal{K}$ (7f)

Constraint (7b) makes sure that the sum of caching and transmission power does not exceed the maximum power constraint $P_M$. Constraint (7c) ensures that the backhaul occupancy by uncached files should satisfy the capacity constraint of fiber backhaul $C$. Constraint (7d) makes sure that the total size of the files cached at ONU-APs should not exceed its cache capacity $Q_n$. Constraint (7e) guarantees that the transmission power is non-negative. Constraint (7f) means that the cache decision on the $j$-th file at ONU-AP is a binary variable $x_{nj}$.

#### B. Optimal Transmission Power Allocation Method

Problem P1 is a typical Mixed Integer Programming (MIP) problem, which is non-linear and non-convex. However, the problem becomes a convex optimization problem for fixed $\{x_{nj}\}$. By deriving the Karush-Kuhn-Tucker (KKT) conditions [33], we can get the optimal solution to transmission power allocation.

The Lagrangian of P1 is given as

$$\mathcal{L}(P, \lambda, \mu, \varepsilon) = - \sum_{n \in \mathcal{N}} \sum_{k \in \Phi_n} R_{nk} + \varepsilon_{nk} P_{nk} + \lambda \left( \sum_{n \in \mathcal{N}} \sum_{k \in \Phi_n} \mu_{\text{miss}} R_{nk} - C \right)$$

$$+ \sum_{n \in \mathcal{N}} \mu_n \left( \sum_{k \in \Phi_n} \rho P_{nk} + \omega \sum_{j \in \mathcal{J}} x_{nj}s - P_M \right) - \sum_{n \in \mathcal{N}} \sum_{k \in \Phi_n} \varepsilon_{nk} P_{nk}$$

where $\lambda, \mu \in \mathbb{R}^N, \varepsilon \in \mathbb{R}^{N \times K}$ are Lagrangian multipliers.

**Lemma 1.** Problem P1 becomes a convex optimization problem for a given solution to the caching strategy $\{x_{nj}\}$.

**Proof.** The objective function (7a) is a strictly concave and increasing function with respect to $P_{nk}$, the inequality constraints (7b) and (7c) are convex for a given caching solution $\{x_{nj}\}$. Therefore, problem P1 becomes a convex optimization problem for fixed $\{x_{nj}\}$. □

The KKT conditions can be expressed as

$$\frac{\partial \mathcal{L}}{\partial P_{nk}} = \frac{g_{nk}}{\sigma^2 + g_{nk}P_{nk} \ln 2} \left( -1 + \sum_{j \in \mathcal{J}} B p_j (1 - x_{nj}^0) \right)$$

$$+ \rho \mu_n - \varepsilon_{nk} P_{nk} = 0$$

$$\lambda \left( \sum_{n \in \mathcal{N}} \sum_{k \in \Phi_n} p_j (1 - x_{nj}^0) R_{nk} - C \right) = 0$$

$$\mu_n \left( \sum_{k \in \Phi_n} \rho P_{nk} + \omega \sum_{j \in \mathcal{J}} x_{nj}s - P_M \right) = 0$$

$$\varepsilon_{nk} P_{nk} = 0, \forall n \in \mathcal{N}, \forall k \in \mathcal{K}$$

$$\lambda, \mu_n, \varepsilon_{nk} \geq 0, \forall n \in \mathcal{N}, \forall k \in \mathcal{K}$$

where (9c) is a necessary condition for an optimal solution, (9b), (9c) and (9d) represent the complementary slackness, and (9e) represents the dual feasibility.

**Lemma 2.** The downlink throughput in the FiWi access network is maximized when the ONU-APs consume the maximum power, i.e., satisfy

$$\sum_{n \in \mathcal{N}} \rho P_{nk} + \omega \sum_{j \in \mathcal{J}} x_{nj}s = P_M, \forall n \in \mathcal{N}$$

**Proof.** Suppose that there exists $n \in \mathcal{N}$ satisfying $\sum_{k \in \Phi_n} \rho P_{nk} + \omega \sum_{j \in \mathcal{J}} x_{nj}s < P_M$, then ONU-AP can increase its caching power to $P_M - \sum_{k \in \Phi_n} \rho P_{nk}$ for a higher cache hit ratio without reducing the sum rate of UEs associated to ONU-APs. Through proof by contradiction, we get the conclusion that the downlink throughput of the FiWi access network is maximized when the ONU-APs consume the maximum level of power. □

By using Lemmas 1, 2 and the KKT conditions mentioned above, we can obtain an expression for the optimal transmission power $P_{nk}$ with respect to Lagrangian multipliers $\lambda, \mu_n$

$$P_{nk}^* = \left( \frac{B - \lambda B \sum_{j \in \mathcal{J}} p_j (1 - x_{nj}^0)}{\rho \mu_n \ln 2 - \sigma^2} \right)^+, \quad (11)$$

with $(\cdot)^+ = \max(\cdot, 0)$. 

Theorem 1. For problem P1, the optimal transmission power \( P_{nk}^* \) allocated to UE \( k \) by ONU-AP \( n \) can be expressed as

\[
P_{nk}^* = \left( \frac{B}{\rho \mu_k \ln 2} - \frac{\sigma^2}{g_{nk}} \right)^+ \tag{12}
\]

and the corresponding Lagrangian multiplier \( \mu_n \) is

\[
\mu_n = -\frac{|\Phi_n| B}{\left( P_M + \sum_{k \in \Phi_n} \frac{\sigma^2}{g_{nk}} - \omega \sum_{j \in \mathcal{J}} x_{nj}^0 \right) \ln 2} \tag{13}
\]

Proof. Eq. (12) holds if the backhaul constraint (7c) is neglected (i.e., \( \lambda = 0 \)). Substituting Eq. (11) into Eq. (10), we can obtain Eq. (13). What we need to prove is that the conclusion still holds with the backhaul constraint (7c). Suppose the optimal solution to problem P1 is \( \{ P_{nk}^*, x_{nj}^o \} \), then \( \sum_{k \in \Phi_n} \rho P_{nk}^* + \omega \sum_{j \in \mathcal{J}} x_{nj}^0 s = P_M \). Let \( \{ P_{nk}^o, x_{nj}^o \} \) denote the solution that satisfies Eq. (12), which indicates the maximum sum rate of UEs associated to ONU-AP \( n \) without the backhaul constraint and is expressed as

\[
\begin{align*}
\sum_{k \in \Phi_n} \log_2 \left( 1 + \frac{g_{nk} P_{nk}^o}{\sigma^2} \right) & \geq \sum_{k \in \Phi_n} \log_2 \left( 1 + \frac{g_{nk} P_{nk}^*}{\sigma^2} \right) \quad (14a) \\
\sum_{k \in \Phi_n} P_{nk}^o \leq \sum_{k \in \Phi_n} P_{nk}^* \quad (14b)
\end{align*}
\]

With Eqs. (10) and (14b), we obtain \( p_{hit}^h \geq p_{hit}^h* \), which means that \( \{ P_{nk}^o, x_{nj}^o \} \) gets a higher cache hit ratio without reducing the sum rate of UEs associated to ONU-AP \( n \). Therefore, Eq. (14a) also holds with the backhaul constraint, so \( \{ P_{nk}^o, x_{nj}^o \} \) yields the optimal solution that can be obtained by using Eq. (12). \(\square\)

In Eq. (12), \( \frac{\sigma^2}{g_{nk}} \) is the inverse of channel gain normalized by the noise variance \( \sigma^2 \). Eq. (12) complies with the form of water-filling method [34, Chapter 6]. We consider a vessel whose bottom is formed by plotting those values of \( \frac{\sigma^2}{g_{nk}} \) for each subchannel \( k \). Then, we flood the vessel with water to a depth \( \frac{B}{\rho \mu_k \ln 2} \). Note that the volume of water is not fixed and it depends on the caching solution \( \{ x_{nj}^o \} \). For a given caching solution, the total amount of water used is then \( P_M - (\omega \sum_{j \in \mathcal{J}} x_{nj}^0 s) \), and the depth of the water at each subchannel \( k \) is the optimal transmission power allocated to it. Theorem 1 guarantees that the optimal solution to transmission power allocation still holds with the backhaul constraint, so this method is called volume adjustable backhaul-constrained water-filling method (VABWF).

C. Problem Reformulation and Algorithms

In this subsection, we aim to design an optimization algorithm to maximize the downlink throughput of FiWi access networks.

Theorem 2. On the condition of total power constraint, files with higher popularity should be cached preferentially for maximizing the throughput in the FiWi access network.

Proof. Suppose that \( \{ P_{nk}^o, x_{nj}^o \} \) is the optimal solution to problem P1 with the corresponding downlink throughput \( P_{dl}^o \), and \( \{ x_{nj}^o \} \) violates the popularity based caching policy mentioned in Theorem 2. Let \( \{ P_{nk}, x_{nj}^o \} \) be another set and \( \{ x_{nj}^o \} \) satisfies

\[
\{ x_{nj}^o \} : x_{nj}^o \geq x_{nj}^o(j+1), \sum_{j \in \mathcal{J}} x_{nj}^o = \sum_{j \in \mathcal{J}} x_{nj}^*, \forall n \in N, \forall j \in \mathcal{J}. \tag{15}
\]

Namely, \( \{ x_{nj}^o \} \) is the caching solutions that obey Theorem 2. As \( p_j \) is monotonically decreasing with \( j \), the cache miss ratio satisfies

\[
\sum_{j \in \mathcal{J}} p_j (1 - x_{nj}^o) < \sum_{j \in \mathcal{J}} p_j (1 - x_{nj}^*). \tag{16}
\]

Note that \( \{ P_{nk}, x_{nj}^o \} \) satisfies \( \sum_{j \in \mathcal{J}} x_{nj}^o = \sum_{j \in \mathcal{J}} x_{nj}^* \). With constraint (10), we can obtain a set of transmission power solutions that satisfy \( P_{nk}^o = P_{nk}^o \). With (16), \( \{ P_{nk}^o, x_{nj}^o \} \) is ensured to satisfy the constraint (7c), and thus \( \{ P_{nk}^o, x_{nj}^o \} \) is a solution to problem P1, with the corresponding downlink throughput \( P_{dl}^o \). By the optimality of \( \{ P_{nk}, x_{nj}^o \} \) to problem P1, we have

\[
P_{dl}^o - P_{dl}^* \geq 0. \tag{17}
\]

Since we have \( P_{nk}^o = P_{nk}^o \), the equality sign in (17) must hold, which concludes that \( \{ P_{nk}^o, x_{nj}^o \} \) yields the optimal solution to problem P1. \(\square\)

By using Theorems 1 and 2 and the VABWF method, we can reduce the solution space of problem P1 greatly. If we denote the solution space of problem P1 as \( A \), then

\[
A = \left\{ \{ P_{nk}, x_{nj}^o \} \mid P_{nk} = \left( \frac{B}{\rho \mu_k \ln 2} - \frac{\sigma^2}{g_{nk}} \right)^+ ; \right. \tag{18}
\]

\[
x_{nj}^o \geq x_{nj}^o(j+1), j \in \mathcal{J} \right\}
\]

An element of \( A \), denoted as \( A_{nj} = \{ P_{nk}^o, x_{nj}^o \} \in A \), should satisfy

\[
\sum_{j \in \mathcal{J}} x_{nj}^o = j. \tag{19}
\]

Let \( \mu(A_{nj}) \) denote whether \( A_{nj} \) is chosen to be the solution, i.e.,

\[
\mu(A_{nj}) = \begin{cases} 1, & A_{nj} \text{ is chosen to be the solution} \\ 0, & \text{otherwise} \end{cases} \tag{20}
\]

Define \( \omega(A_{nj}) \) to be the backhaul bandwidth occupied by ONU-AP \( n \) with respect to solution \( A_{nj} \)

\[
\omega(A_{nj}) = \begin{cases} \sum_{k \in \Phi_n} R_{nk}^o | P_{nk}, x_{nj}^o = A_{nj}^o |, \mu(A_{nj}) = 1 \\ 0, \mu(A_{nj}) = 0 \end{cases} \tag{21}
\]

Likewise, define \( \nu(A_{nj}) \) to be the sum rate of UEs associated to ONU-AP \( n \) with respect to solution \( A_{nj} \)

\[
\nu(A_{nj}) = \begin{cases} \sum_{k \in \Phi_n} R_{nk}^o | P_{nk}, x_{nj}^o = A_{nj}^o |, \mu(A_{nj}) = 1 \\ 0, \mu(A_{nj}) = 0 \end{cases} \tag{22}
\]

It should be noted that Eq. (12) requires an iterative approach to be solved in the simulations or in actual FiWi network.
Algorithm 1: Iterative Transmission Power Allocation based on VABWF method

Input: \( N, K, J, B, P_M, Q_n, g_{nk}, \omega, s, \rho, \sigma, \delta \)

Output: Backhaul bandwidth and sum rate of UEs with respect to solution \( A_n \);

1: for all \( A_n \in A \), \( \forall n \in N, \forall j \in J \) do
2: Calculate \( \mu_n \) according to Eq. (13);
3: \( P_{nk} \leftarrow \frac{\mu_n}{g_{nk}} - s^2 \);
4: while \( \exists P_{nk} < 0, \forall k \in \Phi_n \) do
5: \( P_{nk} \leftarrow (P_{nk})^+ \);
6: Recalculate \( \mu_n \) and reallocate \( P_{nk} \) to those \( P_{nk} > 0 \);
7: end while
8: Calculate \( \omega(A_n), \nu(A_n) \) according to Eqs. (21) and (22);
9: end for
10: return \( \omega(A_n), \nu(A_n) \);

Algorithm 2: Dynamic Programming Algorithm for P2

Input: \( C, \omega(A_n), \nu(A_n) \);

Output: \( \{P^*_n, x^*_n\} \), and the downlink throughput \( R_{dl} \);

1: Calculate \( R(1, c), c = 0, 1, 2, \cdots, C \);
2: for \( n = 2; n \leq N; n + + \) do
3: for all \( c \in \{0, 1, 2, \cdots, C\} \) do
4: if \( c - \min \{\omega(A_n) | \forall j \in J\} < 0 \) then
5: \( R(n, c) \leftarrow R(n - 1, c) \);
6: else
7: Update \( R(n, c) \) according to Eq. (24);
8: end if
9: if \( R(n, c) \neq R(n - 1, c) \) then
10: \( \pi(n, c) \leftarrow \arg \max_j \{\{R(n - 1, c) \cup \{R(n - 1, c - \omega(A_n)) + \nu(A_n)\}\}\};
11: end if
12: end for
13: end for
14: for \( n = N; n \geq 1; n - - \) do
15: if \( R(n, c) \neq R(n - 1, c) \) then
16: The optimal item index for the \( n \)-th class in the case of backhaul bandwidth \( c \) is obtained by \( \pi(n, c) \), then let \( c \leftarrow c - \omega(A_n) \);
17: end if
18: end for
19: Find the solution \( A_n \) according to \( \pi(n, c) \) and Eq. (18) in the reverse order of \( n \);
20: return \( \{P^*_n, x^*_n\}, R_{dl} \);

scenarios. This is because the value of \( \mu_n \) that affects the required water-filling level cannot be directly determined, even though it can be calculated from Eq. (13). Specifically, the number of UEs \( |\Phi_n| \) served by an ONU-AP in Eq. (13) may vary due to the differences in channel conditions of UEs. For example, the calculated value of \( P_{nk} \) may be less than zero when the channel condition of a particular UE \( n \) is rather poor. Since the transmission power cannot be negative, it is set to zero according to Eq. (12), which means that the algorithm tends to allocate more transmission power to UEs with better channel conditions to maximize the system throughput at the sacrifice of not being able to serve certain UEs with poor channel conditions. This is reasonable from the perspective of improving power efficiency because allocating transmission power to UEs with rather poor channel conditions is a waste of limited resources. In this case, the number of UEs served by an ONU-AP has been changed, so the value of \( \mu_n \) should be recalculated according to Eq. (13). This procedure is repeated until there is no negative transmission power allocated to a UE. The detailed algorithm is outlined in Algorithm 1.

Then problem P1 can be converted into the problem of determining the value of \( \mu(A_n) \) with the aim to maximizing the downlink wireless access throughput of the FiWi access network as follows,

\[
P_2: \max_{\mu(A_n)} \sum_{n \in N} \sum_{j \in J} \nu(A_n) \tag{23a}
\]

s.t.

\[
\sum_{n \in N} \sum_{j \in J} \omega(A_n) \leq C \tag{23b}
\]

\[
\sum_{j \in J} \mu(A_n) \leq 1, \quad \forall n \in N, \forall j \in J \tag{23c}
\]

\[
\mu(A_n) \in \{0, 1\}, \quad \forall n \in N, \forall j \in J \tag{23d}
\]

Problem P2 is in the form of a multiple-choice knapsack problem (MCKP) [35, Chapter 11]. The problem is to choose no more than one item from each class such that the profit sum is maximized without exceeding the capacity \( C \) in the corresponding backhaul limitation. Under the assumption that the values of \( \omega(A_n) \) and \( \nu(A_n) \) are integers, we design an efficient algorithm through dynamic programming based on VABWF method as outlined in Algorithm 2. The reason we adopt dynamic programming is that problem P2 has the property of an optimal substructure as described in [36, Section 15.3].

Define \( R(n, c) \) to be the maximum downlink throughput of the FiWi access network where there exist only the first \( n \) classes with backhaul limitation \( c \). Then we can consider an additional class to calculate the corresponding maximum throughput and the following recursive formula describe how the iteratively method is performed

\[
R(n, c) = \max \left\{ \left\{ R(n - 1, c - \omega(A_n)) + \nu(A_n) \right\} \cup \left\{ R(n - 1, c) \right\} \right\} \geq 0, \forall j \in J \right\}. \tag{24}
\]

Note that the constraint (23c) is satisfied by placing the recursive formula in the innermost loop. In each iteration, we choose the optimum solution to the given number of classes \( n \) and bandwidth limitation \( c \). The running time of Algorithm 2 is dominated by the \( c \) iterations of the second for-loop, each of which contains at most \( J \) iterations to compute a new solution of a subproblem. The \( N \) ONU-APs in the FiWi access network leads to \( N \) subproblems, and thus the overall time complexity is \( O(NCJ) \).
IV. ERGODIC CAPACITY & THEORETICAL THROUGHPUT 
UPPER BOUND ANALYSIS

In this section, we aim at obtaining the downlink throughput upper bound of FiWi access networks, which can be used to validate the effectiveness of the proposed algorithms in the previous section.

A. Ergodic Capacity of Wireless Links

In this subsection, we derive the expression of the downlink ergodic capacity for wireless links, which can eventually give the expression of the throughput upper bound and some guidance for improving the system performance.

The distance $r$ between an AP and a typical UE has a strong influence on the downlink rate. Since each UE is associated with the closest AP, the probability density function (pdf) of $r$ can be given by \( f_r(r) = \frac{2r}{D^2}. \) (25)

According to the Shannon formula, the wireless capacity is related to its assigned transmission power, the distance to the associated AP, and the fading parameter. Under assumption of knowledge of statistical CSI, the average ergodic rate of a typical UE is the expectation with respect to the distance distribution and the random variable $h$, which can be written as \[ \tau(P_n) = \mathbb{E}_r \mathbb{E}_{SNR} [\log_2 (1 + SNR(r))] \]

\[ = \int_{r>0}^D \mathbb{E}_h [\log_2 (1 + SNR(r))] f_r(r) dr \]

\[ = \int_{r>0}^D \mathbb{E}_h [\log_2 (1 + SNR(r))] \frac{2r}{D^2} dr \]

\[ \approx \int_{r>0}^D \int_{t>0}^\infty \mathbb{P} \left[ \log_2 (1 + \frac{P_{nk} h r^{\alpha}}{\sigma^2}) > t \right] dt \frac{2r}{D^2} dr, \] (26)

where (a) follows from Eq. (25) and (b) holds because the SNR is a strictly positive random variable.

Eq. (26) is the average ergodic wireless link capacity of a randomly chosen user $k$ associated to ONU-AP$_n$. The transmission power $P_{nk}$ is the variable we need to optimize, and it is normally difficult to give an expression of $P_{nk}$. Therefore, in most traditional PPP-based analysis of network performance, an equal transmission power allocation scheme is adopted \(27\)–\(29\). However, the equal transmission power allocation scheme fails to consider the throughput gain due to the allocation of transmission power to different users. By contrast, we take into account the impact of transmission power allocation on the network throughput. Specifically, the transmission power of a typical user is not a fixed value, but varies with large-scale and small-scale fading. The main advantage of introducing transmission power allocation to users in our analysis is that we can assign the optimal transmission power levels to the users based on the analysis in the previous section, which affects the average ergodic rate and the upper bound of the throughput in the FiWi network.

According to Eq. (12), the optimal transmission power $P_{nk}^*$ allocated to UE$_k$ depends on the distance $r$ and the fading parameter. Substituting Eq. (13) into Eq. (12) and using the fact that the average number of UEs associated with ONU-AP$_n$ is $\lambda$, we have

\[ P_{nk}^* = \frac{r^{-\alpha} h_{nk} P_{nk}^* N + \lambda \sigma^2}{\rho \lambda r^{-\alpha} h_{nk} \ln 2} - \frac{\sigma^2}{r^{-\alpha} h_{nk}}, \] (27)

where $P_{nk}^*$ denotes the total transmission power consumed by ONU-AP$_n$. Eq. (27) shows the linear relationship between $P_{nk}^*$ and $P_{nk}^T$, which can help us obtain the expression of ergodic wireless link capacity with respect to $P_{nk}^T$.

Theorem 3. The expression of the ergodic capacity for wireless links is

\[ \tau(P_{nk}^T, \lambda, D) = 2D^{-2} \mathcal{N}^{-\frac{\lambda \sigma^2}{\rho^2}} \int_{r>0}^D \int_{t>0}^\infty r e^{-\frac{t^2 + \lambda \rho \sigma^2 \ln 2}{N P_{nk}^T}} dt dr. \] (28)

Proof. By substituting $P_{nk}$ from Eq. (27) into Eq. (26), and using the fact that $h \sim \exp(1)$, the ergodic capacity can be expressed as

\[ \tau(P_{nk}^T, \lambda, D) = \int_{r>0}^D \int_{t>0}^\infty \mathbb{P} \left[ h > \frac{2t^2 + \lambda \rho \sigma^2 \ln 2 - \lambda \sigma^2}{N P_{nk}^T} \right] dt \frac{2r}{D^2} dr \]

\[ = \int_{r>0}^D \int_{t>0}^\infty \mathbb{E}_h \left[ \frac{2t^2 + \lambda \rho \sigma^2 \ln 2 - \lambda \sigma^2}{N P_{nk}^T} \right] e^{-\frac{t^2 + \lambda \rho \sigma^2 \ln 2 - \lambda \sigma^2}{N P_{nk}^T}} 2r \frac{1}{D^2} dr dt. \] (29)

Rearranging (29) gives the desired result. \(\square\)

Although Eq. (28) is not in a closed-form, it can be efficiently computed numerically as opposed to the usual Monte Carlo methods that rely on repeated random sampling to compute the results. Another advantage of this expression is that the variable $P_{nk}^T$ does not refer to the transmission power assigned to a typical user, but to the total transmission power consumed by ONU-AP$_n$. If the total transmission power consumption is determined, according to Lemma 2, the caching power consumption can be calculated. Then, the cache hit ratio can be obtained, thereby obtaining the backhaul bandwidth occupied by the UEs associated with ONU-AP$_n$, and this provides the theoretical guidance for trading the power allocated to caching for that assigned to transmission.

B. Theoretical Downlink Throughput Upper Bound

In this section, we obtain a theoretical upper bound of the throughput in FiWi access networks by exploiting several analytical properties related to the ergodic capacity derived above.

We first define the sum rate of UEs associated with ONU-AP$_n$ as

\[ R_n \triangleq k_n \tau(P_{nk}^T, \lambda, D), \] (30)

where $k_n$ denotes the number of UEs associated with ONU-AP$_n$. Then, the backhaul bandwidth occupied by these $k_n$ UEs can be written as

\[ C_n = p_n^{miss} R_n = (1 - p_n^{hit}) R_n, \] (31)
where $p_{hit}^{n}$ denotes the cache hit ratio at ONU-AP$_n$ and can be calculated as

$$p_{hit}^{n} = \frac{\sum_{j=1}^{P_{\text{hit}}^{n}} p_j}{\sum_{j=1}^{P_{\text{hit}}^{n}} p_j}.$$  

(32)

Note that $P_{\text{hit}}^{n}$ is a concave function of $P_T^{n}$. This is due to the fact that $p_j < p_{j+1}$ according to the Zipf distribution of file popularity, and files with higher popularity are cached preferentially according to Theorem 2. Therefore, the cache hit ratio will decrease faster as the transmission power increases (i.e., the caching power decreases).

**Lemma 3.** $\tau(P_T^{n}, \lambda, D)$ is a monotonically increasing function of $P_T^{n}$, and $C_n$ is a monotonically increasing function of $P_T^{n}$.

**Proof.** Taking the derivative of $\tau(P_{nk})$ we obtain

$$\frac{d\tau(P_{nk})}{dP_{nk}} = \int_{r>0} \int_{r>0} e^{-\frac{r^\alpha}{r_{nk}^\alpha} - \frac{r^\alpha}{\sum_j r_j^\alpha}} r^\alpha \sigma^2 (2^t - 1) P_{nk}^{-2} \frac{2r}{D^2} dr dt.$$  

(33)

With $t > 0$, $\tau > 0$, we have $\frac{d\tau(P_{nk})}{dP_{nk}} > 0$, and thus $\tau(P_{nk})$ is monotonically increasing with $P_{nk}$. Based on Eq. (27), we have $\frac{dP_{nk}}{dP_T^{n}} > 0$, which means $P_{nk}$ is monotonically increasing in $P_T^{n}$. According to the chain rule of composition function, we know that $\tau(P_T^{n}, \lambda, D)$ is a monotonically increasing function of $P_T^{n}$.

With Eq. (32), we can get that $\frac{dp_{\text{miss}}^{n}}{dP_T^{n}} > 0$. Since both positive-valued function $p_{\text{miss}}^{n}$ and $\tau(P_T^{n}, \lambda, D)$ increase with $P_T^{n}$, we can prove that $C_n$ is a monotonically increasing function of $P_T^{n}$.

**Remark 1:** In order to achieve a higher average downlink rate, the transmission power allocated to UEs needs to be increased, so it is intuitive that the average downlink rate monotonously increases with the total transmission power. On the other hand, the increase of transmission power results in the decrease of caching power, which will cause more fiber backhaul bandwidth occupancy. This gives us the insight that it does not make sense to increase the transmission power when the backhaul bottleneck becomes a bottleneck of the network throughput, so increasing the transmission power does not necessarily increase the network throughput.

Next, we would like to explore the relationship between the sum rate of $k_n$ UEs associated with ONU-AP$_n$ and the backhaul bottleneck occupied by these UEs. So $R_n$ is regarded as a function of $C_n$, i.e., $R_n = f(C_n)$, then the concavity property of $f(C_n)$ can be found.

**Lemma 4.** For a given coverage radius $D$ of ONU-APs and average number of UEs $\lambda$ in the FiWi network, $R_n$ is a concave function of $C_n$.

**Proof.** Please refer to Appendix A

**Remark 2:** The significance of the concave function is that it implies the non-linear relationship between the fiber backhaul bottleneck occupancy and the downlink throughput. Namely, the increase of the fiber backhaul bottleneck occupancy results in a slower increase of the wireless downlink throughput. Considering that all the ONU-APs share the same throughput-limited fiber backhaul, we get an insight that each ONU-AP should not occupy much more backhaul bandwidth than others. This is because a much higher backhaul bottleneck occupancy does not lead to a much higher increase of wireless downlink throughput. Thus, an intuitive approach would be to make all the ONU-APs occupy equal fiber backhaul bandwidth such that the wireless downlink throughput is maximized. It turns out that this approach is valid when the UEs follow a uniform spatial distribution, and this particular spatial distribution also serves as a necessary and sufficient condition to ensure that the actual throughput can reach the theoretical upper bound derived below.

**Theorem 4.** The downlink throughput of the FiWi access network is upper-bounded by

$$R^+ = \min(\lambda \tau(P_T^{n}, \lambda, D), C + p_{hit}^{n}\lambda \tau(P_T^{n}, \lambda, D)).$$  

(34)

**Proof.** Please refer to Appendix B

Theorem 4 provides the upper bound of the system downlink throughput in cache-aided FiWi networks considering the fiber backhaul bottleneck. It can be observed that each AP consumes the same amount of power $P_T^{n}$ for transmission when the throughput reaches the upper bound. In other words, the upper bound of the throughput is given by Eq. (34) as long as the transmission power $P_T^{n}$ for each AP is determined. However, the total transmission power is not given in our problem formulation and it only needs to satisfy constraint (7b), i.e., not exceeding the maximum power $P_M$. Therefore, it is necessary to determine the amount of transmission power to maximize the downlink throughput. Based on Theorem 4, we then have the following corollary.

**Corollary 1.** $R^+$ is a concave function of $P_T^{n}$, and the downlink throughput upper bound under the maximum power constraint is the supremum of $R^+$ with respect to $P_T^{n} \in [0, P_M]$, i.e.,

$$R^+(P_{nk}, x_n) \leq \sup_{P_T^{n} \in [0, P_M]} \{R^+(P_T^{n}, P_M)\}.$$  

(35)

**Proof.** Please refer to Appendix C

**Remark 3:** It should be noted that the throughput upper bound is achievable in the case of a uniform spatial distribution of UEs. However, due to the assumption of a more realistic scenario where UEs are modeled as independent poisson point process (PPP), UEs will be located closely together in certain areas and far away in others, and thus the overall spatial distribution of UEs is not uniform in most of the PPP-based realizations. Considering this fact, we can see that the condition that the equality sign holds in (41) (in Appendix B) cannot be satisfied, which might make the actual downlink throughput be lower than the theoretical upper bound.

Since the throughput upper bound is obtained, we can calculate the corresponding cache capacity utilization, which by definition can be expressed as

$$\eta = \frac{1}{\omega Q} \left( P_M - \max_{P_T^{n} \in [0, P_M]} \{R^+(P_T^{n})\} \right).$$  

(36)
This gives an analytical result on the appropriate average cache capacity utilization when the downlink throughput reaches the theoretical upper bound, and we will also compare it with the simulation results in the next section.

V. NUMERICAL AND SIMULATION RESULTS

In this section, we present both numerical and Monte Carlo simulation results to validate the theoretical analysis and the performance of our proposed caching and power allocation optimization algorithm under different FiWi access network scenarios. The performance is averaged over 1000 network deployments in the Monte Carlo simulations, where UEs are scattered based on independent PPPs. The simulation parameters are summarized in Table I. To better assess the performance of our proposed algorithm, which we refer to as volume adjustable backhaul constrained wafer filling-dynamic programming (VABWF-DP) below, we compare it with three benchmark power allocation and caching algorithms as follows:

- **Water Filling-Full Caching** (WF-FC): Each ONU-AP fully uses the cache capacity to store the most popular files, and the classical water-filling method is used to allocate transmission power to the UEs.
- **Equal Power-Popularity First** (EP-PF): Transmission power is equally allocated to the UEs. Meanwhile, each ONU-AP chooses the most popular files to cache, but does not necessarily use the cache capacity in full.
- **Water Filling-Random Caching** (WF-RC): ONU-APs randomly choose files to cache and transmission power is also equally allocated to the UEs.

### A. Validation of the Theoretical Analysis and Discussion

We first validate the analysis of the throughput upper bound as well as the cache capacity utilization. Fig. 2(a) shows the performance of downlink throughput for different fiber backhaul bandwidths. It can be seen that the trend of throughput upper bound of the theoretical analysis is consistent with that of the simulation results. Specifically, when the fiber backhaul bandwidth is relatively low, the downlink throughput increases linearly with the fiber backhaul bandwidth, whereas it increases slowly with a relatively high backhaul bandwidth. We also observe that the downlink throughput is higher when ONU-APs have smaller coverage radius $D$. Quantitatively, the simulation results of the throughput have reached an average of 94.8% of the theoretical upper bound. For a typical GPON backhaul bandwidth of 2.488 Gbps, the corresponding downlink throughput is about 2.9 Gbps, 3.5 Gbps and 4.3 Gbps under three different AP coverage radii, respectively. This corresponds to a gain of about 16.6%, 40.7%, and 72.8% with respect to a FiWi access network without caches, respectively. The downlink throughput increases more significantly for a typical EPON backhaul bandwidth of 1.25 Gbps. For example, it increases by about 3.2 times with respect to the backhaul bandwidth when the ONU-AP coverage radius is 50 m. Fig. 2(b) shows the performance of the cache capacity utilization determined by Eq. (36) and the simulation results for different fiber backhaul bandwidths. As can be seen, the two sets of results match each other very well for all the considered backhaul bandwidth $C$ and coverage radius $D$. Full cache capacity is used in both theoretical analysis and simulations when the fiber backhaul bandwidth is relatively low, and the cache capacity utilization decreases as the fiber backhaul bandwidth increases. Besides, less cache capacity is used for a larger ONU-AP coverage radius.

Comparing Fig. 2(a) with Fig. 2(b), we find that the throughput is lower when full cache capacity is used. This reflects the impact of cache capacity on downlink throughput. Due to the limited cache capacity on ONU-APs, even though full cache capacity is used, the backhaul is still unable to carry all the traffic of cache-missed files. Because of the insufficient fiber backhaul bandwidth, it remains the bottleneck of the system throughput. We find that the fiber backhaul is less likely to become a bottleneck of the system throughput when the coverage radius $D$ of ONU-APs is large. This is because the average distance between ONU-APs and UEs is farther, resulting in smaller ergodic capacity of wireless links, and thus there is less demand for fiber backhaul bandwidth. On the contrary, the ergodic capacity of wireless links becomes large when the coverage radius $D$ is small. Considering the limited cache capacity, the fiber backhaul is more likely to become the bottleneck of the throughput. As long as the fiber backhaul bandwidth increases to a certain value, the above problem will not occur. It can be found that the cache capacity utilization is reduced when the backhaul bandwidth is higher. This is because there is no need to use full cache capacity at this time, but rather to use more power for wireless transmission so that the wireless link capacity is increased, thereby achieving higher downlink throughput. In addition, although the UEs are spatially randomly distributed in the simulations, our proposed algorithm can approach the theoretical throughput upper bound since we have proposed an optimal transmission power allocation scheme and a dynamic programming algorithm. As will be shown later, the proposed algorithm outperforms existing algorithms in terms of the system throughput.

In Fig. 3, we plot the numerical results of the throughput as

### TABLE I

| Parameter                          | Value                              |
|------------------------------------|------------------------------------|
| Number of ONU-APs                  | 32                                 |
| System bandwidth                   | 20MHz                              |
| Subchannel bandwidth               | 500kHz                             |
| Thermal noise density              | $-17$ dBM/Hz                       |
| Number of files                    | 1000                               |
| Cache size of each ONU-AP          | 40GB                               |
| Caching power efficiency           | $6.25 	imes 10^{-12}$ Watt/abit    |
| Circuit power at each ONU-AP       | 2W                                 |
| Transmission power coefficient     | 1.2                                |
| Path loss exponent                 | 4                                  |
| Coverage radius of ONU-AP          | 100m (unless stated otherwise)      |
| Fiber backhaul capacity            | 2.488 Gbps (unless stated otherwise)|
| Maximum total power for each ONU-AP| 7W (unless stated otherwise)        |
| Zipf parameter                     | 0.8 (unless stated otherwise)       |
a function of the cache capacity utilization. It is not difficult to find that we normally do not use full cache capacity when the maximum downlink throughput is achieved, because using too much or too little cache capacity will result in a decrease in the downlink throughput. It should be noted that the cache capacity utilization here is related to the transmission power $P_T$ in Eq. (34). For example, if the cache capacity utilization is 0, the transmission power $P_T$ equals the maximum power $P_M$, in which case all the files need to be acquired through the backhaul, so the downlink throughput cannot be higher than the fiber backhaul bandwidth. Conversely, if the cache capacity utilization is 100%, then the transmission power is minimized. In this case, it is not the backhaul bandwidth, but the wireless link capacity becomes the bottleneck of the throughput due to the limited transmission power. The goal of the Eq. (35) is to find the value of transmission power $P_T$ corresponding to the optimal cache capacity utilization, such that the downlink throughput is maximized. In addition, the downlink throughput is higher for smaller coverage radius $D$ of ONU-APs, and the corresponding cache capacity utilization is increased, which is consistent with the simulation results in Fig. 2(b). The reason for this phenomenon is that the ergodic wireless link capacity is higher when $D$ has a smaller value. This makes the average wireless transmission rate reach a higher value, so more caches are used to alleviate the backhaul bandwidth pressure.

B. Comparison between the Proposed Algorithm and Existing Algorithms

The average transmission power of an ONU-AP with different algorithms are demonstrated in Fig. 4. It can be seen that as the number of UEs increases or the coverage radius of ONU-APs decreases, our proposed algorithm VABWF-DP tends to use less transmission power $P_T$. This is reasonable because the backhaul bandwidth pressure increases with more UEs and the ergodic capacity of wireless links increases with smaller ONU-AP coverage radius. Hence, our proposed algorithm uses more power for caching files to alleviate the backhaul bandwidth bottleneck, which leads to a reduction in the transmission power. In comparison, EP-PF takes the opposite tack to increase the transmission power when the number of UEs increases, while WF-FC uses fixed transmission power. As discussed later in this subsection, the proposed algorithm outperforms the benchmarks in terms of system throughput.

Fig. 5 shows the caching probability of files under different algorithms. In the simulation, a total number of 400 files can be cached if we use full cache capacity. By utilizing the highest-popularity-first caching strategy proposed in Theorem 2, files indexed after 400 will not be cached. It can be seen that VABWF-DP adjusts the caching probability of each file according to the network parameters (e.g., ONU-AP coverage radius). For instance, when the coverage radius of ONU-APs is large, most files will be cached with a lower probability, which means that the average cache capability utilization is lower. This is because the cache hit ratio does not need to be high with a relatively low wireless link capacity, and thus more
power is used for transmission to further enhance the system throughput.

Next, we compare the proposed algorithm with the other three algorithms in terms of the downlink throughput. Fig. 6 shows the performance of downlink throughput as the ONU-AP coverage radius $D$ varies. It is observed that we achieve higher throughput by using VABWF-DP at different ONU-AP coverage radii, which can be explained by Figs. 4 and 5. As the coverage radius increases, the proposed algorithm gradually increases the transmission power and reduces the number of cached files, which can provide more flexibility than the other algorithms that cannot adapt to different network conditions. The performance of the downlink throughput under different numbers of UEs is shown in Fig. 7. The throughput increases approximately linearly with the number of UEs, and VABWF-DP achieves higher throughput than the other algorithms. Combining with Fig. 4 we can conclude that when the number of UEs increases, more power should be used for caching instead of transmission to achieve higher power efficiency.

In Fig. 8 the performance of the downlink throughput under different algorithms is shown, with the maximum total power $P_M$ of an ONU-AP ranging from 5 W to 15 W. It can be seen that VABWF-DP outperforms the other algorithms under different maximum total powers, and the gap between VABWF-DP and the upper bound of the theoretical analysis is small. This is because the proposed algorithm can adaptively adjust the transmission power and caching power of different ONU-APs. Under the premise that the backhaul bandwidth occupied by cache-missed files not exceeding the maximum backhaul bandwidth $C$, the proposed algorithm maximizes the transmission power to achieve higher system throughput. It can be seen that the gap between VABWF-DP and WF-FC is large when the maximum total power $P_M$ is low, but the gap gradually decreases with the increase of $P_M$. This reflects the fact that VABWF-DP and WF-FC use a more similar strategy as the maximum total power increases. In other words, by adopting VABWF-DP, more caching power is used with the purpose of mitigating backhaul bandwidth pressure with the increase of maximum total power. It is also observed that EP-PF fails to achieve higher throughput, because it does not take into account the impact of either the wireless channel conditions or the non-uniform spatial distri-
distribution of UEs. In contrast, the proposed algorithm allocates more transmission power to UEs with better channel conditions to improve the total throughput, and dynamically determines the total transmission power to cope with the throughput degradation due to the non-uniform spatial distribution of UEs. Therefore, the throughput performance of the proposed algorithm is close to the theoretical upper bound. The worst performance occurs when WF-RC is adopted. This is due to the fact that the highest-popularity-first caching strategy in Theorem 2 is not implemented, and thus the cache capacity cannot be fully utilized, which results in a decrease in cache hit ratio and limits the system throughput.

In Fig. 8, we evaluate the performance of different algorithms as fiber backhaul capacity $C$ varies. We can observe that the throughput from the proposed algorithm is still the closest to the theoretical upper bound. It is worth noting that in the case of low fiber backhaul capacity, the throughput achieved by using WF-FC is very close to that of the proposed algorithm, which is consistent with the simulation results shown in Fig. 2(b). This observation suggests that increasing the cache capacity utilization is an effective way to achieve higher throughput when the backhaul bandwidth is low. However, WF-FC cannot keep increasing the throughput as the backhaul bandwidth increases. The reason is that the wireless link capacity is limited by the transmission power regardless of the increase in backhaul bandwidth. In comparison, our proposed algorithm uses more power for wireless transmission instead of caching, thereby further increasing the capacity of wireless links and obtaining throughput gain.

Fig. 10 shows the performance of downlink throughput as the Zipf distribution parameter $\delta$ varies, which reflects the performance under various file popularities. It can be seen that the downlink throughput of the proposed algorithm increases with parameter $\delta$. This is reasonable because more user requests centralize on a smaller number of files with a larger $\delta$, while the rest of the files are rarely requested, which results in the increase of cache hit ratio and the reduction of backhaul bandwidth occupancy. Therefore, our proposed algorithm tends to cache fewer files, saving more caching power to increase transmission power and achieving higher throughput. In contrast, WF-FC fails to achieve higher throughput despite the change of Zipf parameter $\delta$. This indicates that using full-cache algorithm is rather wasteful because the files that are rarely requested are also cached in ONU-APs. Thus, no more power can be used for wireless transmission, limiting the system throughput. We also notice that EP-PF is not even comparable to WF-FC. This indicates the importance of transmission power allocation based on wireless channel conditions of UEs, which has a great impact on the system throughput. Therefore, the proposed VABWF-DP for power allocation is more efficient in improving the throughput than EP-PF in the case of sum power constraints of ONU-APs.

VI. CONCLUSION

In this paper, to cope with fiber backhaul bottleneck in FiWi access networks, we equipped ONU-APs with caches to improve the system throughput. We illustrated the necessity of jointly considering power allocation and caching to enhance the network performance. We derived a closed-form expression of optimal wireless transmission power allocation using the
proposed VABWF method. To perform the joint optimization of power allocation and caching, we converted the problem to a standard MCKP and designed a dynamic programming algorithm to maximize the downlink throughput. To understand the potentials of involving caches in FiWi access networks, we then theoretically analyzed the ergodic capacity for wireless links and obtained an upper bound of the downlink throughput. We proved that the theoretical throughput upper bound is achievable in the case of a uniform spatial distribution of UEs. Numerical and simulation results showed that our proposed algorithm significantly outperforms existing algorithms in terms of system throughput and is a good approximation of the derived throughput upper bound under different FiWi access network scenarios. The results conclude that an appropriate allocation of transmission power and caching power is needed to achieve higher throughput in FiWi access networks.

**APPENDIX A**
**PROOF OF LEMMA 4**

For all $P_{n(1)}^T < P_{n(2)}^T$ that satisfies $P_{n(1)} < P_{n(2)}$, choose any $P_{n(1)}^T, P_{n(2)}^T$ that satisfies $P_{n(1)}^T < P_{n(2)}^T$. Since $C_n$ is a monotonically increasing function of $P^T$, we have $C_n(P_{n(2)}^T) = C_n(P_{n(1)}^T)$ and $p_{miss}(P_{n(1)}^T) < p_{miss}(P_{n(2)}^T)$

\[
\frac{R_n^2 - R_n}{C_n^2 - C_n} = \frac{R_n^2 - R_n}{p_{miss}(P_{n(2)}^T) R_n^2 - p_{miss}(P_{n(1)}^T) R_n^2} < \frac{R_n^2 - R_n}{p_{miss}(P_{n(1)}^T) R_n^2 - p_{miss}(P_{n(2)}^T) R_n^2} = \frac{1}{p_{miss}(P_{n(1)}^T)} = f'(C_n). \tag{37}
\]

By rearranging the terms in Eq. (37), we obtain

\[
R_n^2 < R_n^2 + f'(C_n)(C_n^2 - C_n), \tag{38}
\]

which is the first-order Taylor approximation of $f(C_n)$. As it is always a global upper estimator of $f(C_n)$, we can conclude that $f(C_n)$ is a concave function.

**APPENDIX B**
**PROOF OF THEOREM 4**

The downlink throughput can be calculated by adding up the rates of all the UEs, which can be expressed as

\[
R = \sum_{n \in \mathcal{N}} R_n = \sum_{n \in \mathcal{N}} f(C_n). \tag{39}
\]

By applying Jensen’s inequality, we obtain $R \leq N f\left(\frac{\sum_{n \in \mathcal{N}} C_n}{N}\right)$, where the equality sign holds if and only if all the ONU-APs occupy equal fiber backhaul bandwidth denoted by $\bar{C}$, i.e.,

\[
C_n = \bar{C} = p_{miss} k_n \tau(P_n^T, \lambda, D), n \in \mathcal{N}. \tag{40}
\]

By substituting Eq. (40) into Eq. (39), and using the fact that $[p_{miss}(P_n^T)]^{-1}$ is concave, we obtain

\[
R = \bar{C} \sum_{n \in \mathcal{N}} [p_{miss}(P_n^T)]^{-1} \leq \bar{C} N \left[p_{miss}\left(\frac{\sum_{n \in \mathcal{N}} P_n^T}{N}\right)\right]^{-1}, \tag{41}
\]

where the equality sign holds if and only if all the ONU-APs consume the same amount of transmission power, i.e.,

\[
P_n^T = P^T, n \in \mathcal{N}. \tag{42}
\]

Combining Eqs. (42) and (40), we can obtain that $k_n = \frac{\lambda}{N}, n \in \mathcal{N}$, which indicates that each ONU-AP serves the same number of UEs. Plugging Eq. (40) into Eq. (41) and substituting $k_n$ with $\frac{\lambda}{N}$, we can obtain

\[
R \leq \lambda \tau(P^T, \lambda, D). \tag{43}
\]

Considering the constraint of fiber backhaul capacity, if $p_{miss}(P^T)\lambda \tau(P^T, \lambda, N) \geq C$, we have

\[
R \leq C + p_{hit}(P^T)\lambda \tau(P^T, \lambda, D). \tag{44}
\]

Taking the minimum of Eqs. (43) and (44) gives the desired result.

**APPENDIX C**
**PROOF OF COROLLARY 1**

To prove the concavity property of $R^+$, we first show that $p_{hit} \tau(P^T, \lambda, D)$ is a concave function of $P^T$. According to the analysis in Subsection III-B, $p_{hit}$ is a concave function of $P^T$, and $\tau(P^T, \lambda, D)$ is a concave function of $P^T$. What we need to prove is that the product of these two functions is still a concave function of $P^T$. Taking the second order derivative of $p_{hit} \tau(P^T, \lambda, D)$ with respect to $P^T$, we have

\[
\frac{\partial^2 (p_{hit} \tau(P^T, \lambda, D))}{\partial (P^T)^2} = \frac{\partial^2 (p_{hit})}{\partial (P^T)^2} \tau(P^T, \lambda, D) + 2 \frac{\partial p_{hit}}{\partial P^T} \frac{\partial \tau(P^T, \lambda, D)}{\partial P^T} + \frac{\partial^2 \tau(P^T, \lambda, D)}{\partial (P^T)^2} p_{hit}. \tag{45}
\]

As $\frac{\partial p_{hit}}{\partial (P^T)^2} < 0$, $\frac{\partial \tau(P^T, \lambda, D)}{\partial (P^T)^2} > 0$, and $\frac{\partial^2 \tau(P^T, \lambda, D)}{\partial (P^T)^2} < 0$, due to the concavity properties, we can conclude that $\frac{\partial^2 (p_{hit} \tau(P^T, \lambda, D))}{\partial (P^T)^2} > 0$, so $p_{hit} \tau(P^T, \lambda, D)$ is a concave function of $P^T$. Let $\Phi_1(P^T) = \lambda \tau(P^T, \lambda, D)$, $\Phi_2(P^T) = C + p_{hit} \lambda \tau(P^T, \lambda, D)$, since both $\Phi_1$ and $\Phi_2$ are concave functions, then their pointwise minimum $R^+$, defined by

\[
R^+(P^T) = \min (\Phi_1(P^T), \Phi_2(P^T)), \tag{46}
\]

is also concave due to the pointwise minimum operations that preserve concavity of functions as described in [33]. In this case, there exists a unique supremum with respect to $P^T \in [0, P_M]$, which is the downlink throughput upper bound under the maximum power constraint.
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