Transient growth analysis of oblique shock-wave/boundary-layer interactions in a hypersonic flow

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We study the physical mechanisms responsible for transient growth in a hypersonic laminar boundary layer interacting with an oblique shock wave (SWBLI). The transient growth is computed using a power iteration approach that lets perturbations propagate both upstream and downstream. This allows us to capture the physics associated with the recirculation bubble and understand how it amplifies fluctuations in hypersonic SWBLIs. For a Mach 5.92 boundary layer with no oblique shock wave, we demonstrate that the transient response arises from the inviscid Orr mechanism, the Landahl lift-up effect, and first-mode instability. After we incorporate the oblique shock wave and generate a Mach 5.92 SWBLI, we observe an increase in transient growth by four orders of magnitude relative to that in the boundary layer without SWBLI. We also find that the presence of an oblique shock wave increases the spanwise wavenumber at which the largest transient growth takes place from $\beta = 0.6$ to $\beta = 2.6$. These changes are attributed to the sudden change in the streamline curvature in the upstream region of the flow field (where the wavepacket first reaches the recirculation bubble). Furthermore, the optimal initial condition for the SWBLI consists of elongated streaks in the upstream boundary layer. As this initial condition evolves to its final state, we observe the formation of streamwise streaks in the recirculation bubble (that are further amplified in the downstream boundary layer) along with a large perturbation that comes off of the bubble apex and convects downstream. Our results demonstrate large transient growth in a Mach 5.92 SWBLI and suggest that inevitable imperfections in a hypersonic wind tunnel would play an important role in the early stages of transition to turbulence.

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I. INTRODUCTION

A deeper understanding of modal and nonmodal stability mechanisms in hypersonic boundary layers can facilitate the development of better prediction tools for reducing heat loads and skin friction drag on a vehicle’s surface. Previous work has focused mainly on the exponential growth of perturbations corresponding to unstable eigenmodes for boundary layers [1–4] and shock-wave/boundary-layer interactions (SWBLIs) [5–9]. A large transient growth of perturbations in hypersonic boundary layers has been demonstrated recently [10–12]. In spite of the asymptotic decay of flow fluctuations, this transient growth can trigger nonlinear interactions that ultimately breakdown into turbulent flow, bypassing modal instabilities [13–16].

Nonmodal growth analyses first appeared in the low-speed or incompressible regime to study different transition mechanisms [17]. Butler and Farrell [18] demonstrated that streamwise elongated flow structures that oppose the mean shear flow grow rapidly and robustly in plane channel flows. These disturbances are often associated with the inviscid Orr mechanism [19,20]. The transient growth of initially small disturbances can induce bypass transition [14]. This has been seen in a Blasius boundary layer, where long streamwise streaks eventually breakdown and cause transition to turbulence [21,22]. Tempelmann et al. [23] showed that the physical mechanism driving nonmodal growth in three-dimensional boundary layers consists of both the Landahl lift-up effect [24,25] and the inviscid Orr mechanism. Using the temporal framework, Hanifi et al. [26] applied transient growth analysis to compressible boundary layers and showed that the optimal disturbances are similar to those in the incompressible regime. Several other studies have applied a spatial transient growth analysis [27], while focusing on the inclusion of nonparallel flow effects [28–30].

Optimal transient growth analyses have recently been applied to hypersonic flows. Bitter and Shepherd [10] investigated the importance of modal and nonmodal growth mechanisms in flat-plate hypersonic boundary layers. They also studied the effects of Mach number and wall cooling on these processes. Both spatial and temporal frameworks are utilized to show that the optimal disturbances consist of streamwise vortices, which develop into streaks of high velocity and temperature. Paredes et al. [12] used the parabolized stability equations (PSE) to study optimal transient growth in compressible zero-pressure-gradient boundary-layer flows at Mach numbers ranging from 3 to 10. They found that as the Mach number increases, the differences between the optimal gain computed about the time-averaged Navier-Stokes equations and a self-similar boundary-layer approximation becomes significant. Paredes et al. [31] performed an optimal transient
growth analysis about a laminar flow based upon the solution of the Navier-Stokes equations over a 7°
half-angle variable-bluntness cone at zero angle of attack. They concluded that disturbances initiated near
the juncture between the nosetip and the frustum exhibit greater transient amplification for larger values
of nosetip bluntness. Also, they suggested that wall roughness might be able to induce optimal initial
perturbations in this conical flow.

There have been many experimental and numerical investigations of shock waves interacting with both
laminar [32–36] and turbulent boundary layers [37–47]. In contrast, literature related to the application
of spatial or transient growth analysis to SWBLIs is relatively sparse. Sartor et al. [48] experimentally
and theoretically studied unsteadiness in transonic shock-wave/turbulent-boundary-layer interactions, and
showed that these types of interactions can lead to significant transient growth. Dwivedi et al. [49] mea-
sured the spatial growth of streamwise streaks in a Mach 5.92 SWBLI using an adjoint looping PSE-based
algorithm, highlighting the importance of centrifugal instability to the development of streamwise streaks.
In a follow-up study, Dwivedi et al. [50,51] employed input-output analysis to examine the spatial growth
of high-speed boundary layers and SWBLIs on a compression ramp. This study showed that the presence
of a recirculation bubble substantially increases spatial transient growth.

In our recent study of oblique shock-wave/laminar-boundary-layer interactions at Mach 5.92 [8], we utilized
global stability analysis to identify a mechanism that is not related to centrifugal instability. This modal
mechanism causes the flow field to experience an exponential growth, and it eventually leads to transition.
Yet, the presence of curved streamlines means that there is a significant potential for centrifugal instability
to amplify disturbances as they convect over the recirculation bubble. In the global framework, such spatio-
temporal behavior appears as nonmodal instability. This is because of the fact that although the flow
amplifies disturbances, such disturbances eventually convect out of the domain, returning the system to a
quiet state (so that in fact the flow is globally stable). In the global framework, convective amplification
is a consequence of the nonorthogonality of the underlying global eigenmodes [52,53]. This is analogous to
the concept of "convective instability" in the context of locally parallel flows [54–56]. For example, while
the first and second Mack modes [1,2] are instabilities that amplify disturbances spatially, they give rise to
convective instabilities in the spatio-temporal framework, as disturbances associated with them eventually
convect out of the computational domain in the absence of continuous upstream excitation. A challenge for
SWBLIs is that the base flow cannot be approximated as locally parallel and contains significant recirculation. Nevertheless, we develop a method in this paper to study spatio-temporal amplification of disturbances in complex hypersonic flows such as SWBLI. We use the term "convective instability" in the loose sense, to describe such nonmodal amplification even though the base flow is not parallel [57]. In this paper, we show that significant nonmodal growth is indeed possible, and that the mechanism responsible for such nonmodal growth selects a spanwise length scale that is different from the one predicted by global stability analysis.

We utilize a power iteration method [60] to compute the spatial structure of the optimal initial conditions and the resulting temporal growth envelopes. This approach is described in Section II D, and it lets perturbations propagate both upstream and downstream. Section III A verifies our power iteration method by comparing against the results of Hanifi et al. [26], which focused on understanding the transient response of several supersonic parallel boundary-layer profiles. In Section III B, we apply this iterative approach along with a local linear stability analysis to a Mach 5.92 spatially-developing hypersonic boundary layer and investigate the importance of convective instabilities (i.e., first and second modes). In Section III C, we apply the power iteration method to a SWBLI at the same operating conditions in order to understand how the formation of a recirculation bubble changes the spatio-temporal response of the linearized flow equations. We conclude this paper in Section IV with remarks and outlook for future research directions.

II. PROBLEM FORMULATION

A. Flow configuration

Figure 1 shows a schematic of a canonical SWBLI. High-speed freestream flow enters at the left boundary and flows over a flat plate situated along the bottom boundary. For this configuration, the leading edge of the flat plate is upstream of the left boundary and produces a bow shock that persists throughout the domain. At the inflow, the boundary layer has displacement thickness $\delta^*_m$ and slowly grows as it develops downstream (refer to Shrestha et al. [61] for details). Also, an oblique shock wave enters the domain through the left boundary well above both the bow shock and the boundary layer. Such an oblique shock wave might result from placing a turning wedge in the freestream a distance upstream. The incident oblique shock wave propagates at an angle $\theta$ until it impinges on the boundary layer. A recirculation bubble forms due to the
adverse pressure gradient of the impinging shock that causes the boundary layer to separate from the wall. In simulations of the hypersonic spatially-developing boundary layer, the incident shock and recirculation bubble do not appear, but the bow shock is still present. This bow shock creates an entropy layer nearby that can alter the spatial development of the boundary layer.

![Diagram of an oblique shock wave impinging on a Mach 5.92 boundary layer](image)

**FIG. 1.** Schematic of an oblique shock wave (red) impinging on a Mach 5.92 boundary layer adapted from Hildebrand et al. [8]. The adverse pressure gradient associated with the incident shock causes the boundary layer to separate from the wall, forming a recirculation bubble (blue).

We consider freestream flow conditions matching experiments performed in the ACE Hypersonic Wind Tunnel at Texas A&M University [62]. Upstream of the bow shock and the incident shock, the freestream Mach number, temperature, and pressure are $M_\infty = 5.92$, $T_\infty = 53.06$ K, and $p_\infty = 308.2$ Pa, respectively. A Reynolds number of $Re = \frac{\rho_\infty u_\infty \delta^*_m}{\mu_\infty} = 9660$ based on the undisturbed boundary-layer displacement thickness $\delta^*_m = 2.1$ mm at the left inlet is used in the present study. This corresponds to a unit Reynolds number of $4.6 \times 10^6$ m$^{-1}$. Here $\rho_\infty$, $u_\infty$, and $\mu_\infty$ denote the freestream density, velocity, and dynamic viscosity, respectively. When the incident oblique shock wave is present, it has an angle of $\theta = 13.0^\circ$. A Cartesian coordinate system is utilized hereafter with $x$, $y$, and $z$ denoting the streamwise, wall-normal, and spanwise directions, respectively.

**B. Governing equations**

The compressible Navier-Stokes equations are used to model the dynamics of a spatially-developing boundary layer and an oblique SWBLI at hypersonic speeds. These equations govern the evolution of the system state $\mathbf{q} = [p; \mathbf{u}^T; s]^T$, where $p$, $\mathbf{u}$, and $s$ are the non-dimensional fluid pressure, velocity, and entropy, respectively [63]. After nondimensionalization with respect to the displacement thickness $\delta^*_m$, freestream velocity
For an ideal fluid, the density \( \rho \) and temperature \( T \) are related to pressure \( p \) through the equation of state \( \gamma M_\infty^2 p = \rho T \). The freestream Mach number is defined as \( M_\infty = U_\infty / a_\infty \), where \( a_\infty = \sqrt{\gamma p_\infty / \rho_\infty} \) is the speed of sound in the freestream. Furthermore, \( \gamma = 1.4 \) is the assumed constant ratio of specific heats.

We define entropy as \( s = \ln(T)/[\gamma M_\infty^2] - \ln(p)/(\gamma M_\infty^2) \) so that \( s = 0 \) when \( p = 1 \) and \( T = 1 \). The viscous stress tensor \( \tau \) is written in terms of the identity matrix \( I \), velocity vector \( u \), and dynamic viscosity \( \mu \) to yield the following expression

\[
\tau = \mu \left[ \nabla u + (\nabla u)^T - \frac{2}{3} (\nabla \cdot u) I \right].
\]

The viscous dissipation is defined as \( \phi = (\tau : \nabla u)/\mu \), where the operator \( : \) represents a scalar product or a double dot product between two tensors. Note that the scalar product of two tensors is given by \( A : B = \text{trace}(A^T B) \). Furthermore, the second viscosity coefficient is set to \( \lambda = -2\mu/3 \). In order to compute the dynamic viscosity \( \mu \), Sutherland’s law is used with \( T_s = 110.3 \) K as follows:

\[
\mu(T) = T^{3/2} \frac{1 + T_s/T_\infty}{T + T_s/T_\infty}.
\]

The Prandtl number is set to a constant value \( Pr = \mu(T)/\kappa(T) = 0.72 \), where \( \kappa(T) \) is the coefficient of heat conductivity. Further, system (1) can be recast into the form \( \partial q/\partial t = F(q) \). Here, \( F \) represents the differential nonlinear Navier-Stokes operator.
C. Linearized model

To investigate the behavior of small fluctuations about various base flows, system (1) is linearized by decomposing the state variables $q = \bar{q} + q'$ into steady and fluctuating parts. By keeping only the first-order terms in $q'$, the linearized Navier-Stokes (LNS) equations are obtained

\[
\frac{\partial p'}{\partial t} + \bar{u} \cdot \nabla p' + u' \cdot \nabla \bar{p} + \bar{\rho} \bar{a}^2 \nabla \cdot u' + \gamma (\nabla \cdot \bar{u}) p' = \frac{1}{Re} \left\{ \frac{1}{M^2_{\infty} Pr} \nabla \cdot (\mu' \nabla T + \mu \nabla \bar{T}) \right. \\
\left. + (\gamma - 1) \left[ \tau : \nabla u' + \tau' : \nabla \bar{u} \right] \right\},
\]

\[
\frac{\partial u'}{\partial t} + \frac{1}{\bar{\rho}} \nabla p' - \frac{\rho'}{\rho^2} \nabla \bar{p} + \bar{u} \cdot \nabla u' + u' \cdot \nabla \bar{u} = \frac{1}{Re} \left\{ \frac{1}{\bar{\rho}} \nabla \cdot \tau' - \frac{\rho'}{\rho^2} \nabla \cdot \bar{\tau} \right\},
\]

\[
\frac{\partial s'}{\partial t} + \bar{u} \cdot \nabla s' + u' \cdot \nabla s = \frac{1}{Re \, \bar{\rho} T} \left\{ \frac{1}{(\gamma - 1) M^2_{\infty} Pr} \left[ \nabla \cdot (\mu' \nabla T + \mu \nabla \bar{T}) \right. \\
\left. - \frac{\rho'}{\rho} \nabla \cdot (\mu' \nabla \bar{T}) \right] \\
\right. + \bar{\tau} : \nabla u' + \tau' : \nabla \bar{u} - \frac{\rho'}{\rho} \bar{\tau} : \nabla \bar{u} \right\},
\]

Furthermore, the equation of state is linearized to obtain $\rho' / \bar{\rho} = p' / \bar{p} - T' / \bar{T}$. The expression $T' = (\gamma - 1) M^2_{\infty} (\bar{T} s' + p' / \bar{\rho})$ is derived by linearizing the definition of entropy and substituting in the equation of state. Further, the perturbed viscous stress tensor is determined by

\[
\tau' = \bar{\mu} [\nabla u' + (\nabla u')^T - \frac{2}{3} (\nabla \cdot u') I] + \mu' [\nabla u + (\nabla \bar{u})^T - \frac{2}{3} (\nabla \cdot \bar{u}) I],
\]

and the perturbed dynamic viscosity is $\mu' = (\partial \bar{\mu} / \partial \bar{T}) T'$. In [8], the accuracy of this formulation is verified by comparing our results to Malik’s stability analysis [4] of parallel high-speed boundary layers.

Global modes of the linear system (4) take the form

\[
q'(x, y, z, t) = q(x, y) e^{i(\beta z - \omega t)},
\]
where $\beta$ is the nondimensional spanwise wavenumber and $\omega$ is the complex frequency. Substitution of (6) into (4) yields the eigenvalue problem

$$A\hat{q} = -i\omega\hat{q}. \quad (7)$$

We define the Jacobian operator as $A = \partial F/\partial q|_{\bar{q}}$, and it includes all of the terms in system (4) apart from the time derivative. In other words, the Jacobian operator gives the linear variation of the residual of the original nonlinear system (1) with respect to the state variables, taken about a base flow. As discussed below, our transient growth analysis requires the adjoint of the Jacobian operator, $A^H$, in addition to $A$. We compute $A^H$ using the continuous approach. This means applying integration by parts to system (4) and then discretizing the resulting equations. An advantage of the continuous approach over the discrete approach is that it allows for proper incorporation of the boundary conditions. For a discussion on adjoint operators and their properties see the comprehensive review by Luchini and Bottaro [65].

### D. Power iteration method

There are two common approaches to compute the transient energy growth. The first approach utilizes a singular value decomposition (SVD) of the state-transition matrix, and the second is a power iteration method that utilizes simulations of the direct and adjoint systems. A well-known problem with the SVD approach is that flow fields characterized by a strongly non-normal Jacobian operator [64] have difficulty reaching a converged transient growth envelope. This problem can even occur for 2D boundary layer flows [52]. Our earlier study [8] demonstrates that SWBLIs have significant non-normality caused by the strong convection present in the flow. Another problem associated with the SVD approach is that forming large matrices composed of hundreds (if not thousands) of eigenmodes is often computationally expensive.

In this study, we use an iterative approach that utilizes the Jacobian operator $A$ and its adjoint $A^H$ to compute the transient growth [60]. We define the disturbance energy as

$$E = \int \int \left[ \frac{\bar{m}'u'_i^*}{2} + \frac{M_x^2|p'|^2}{2} + \frac{(\gamma - 1)M_x^2|s'|^2}{2} \right] \, dx \, dy, \quad (8)$$

where $^*$ signifies the complex conjugate. This useful measure for compressible flow is derived in Hanifi et al.
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[26] by eliminating conservative compression work transfer terms. The expression above (8) is positive definite and monotone nonincreasing [66]. It also induces the following inner product \((q_1, q_2)_E = q_2^H W q_1\) for two system states \(q_1\) and \(q_2\). We define the weighting matrix as \(W = 1/2 \text{diag}[M^2, \bar{\rho}, \bar{\rho}, \bar{\rho}, \gamma(\gamma - 1)M^2] \Delta x \Delta y\) and \(G(t)\) to be the largest amplification at time \(t\), i.e.,

\[
G(t) = \max \frac{||q(t)||_E^2}{||q(0)||_E^2},
\]

where \(q(0)\) and \(q(t)\) represent the initial and final states. The power iteration method reformulates the problem of computing the largest transient growth \(G(t)\) in terms of a variational principle, and it is based on simulations of the direct and adjoint systems [16]. A standard power iteration can then be employed to arrive at the optimal initial and final states. The optimization process should respect the constraints given by the LNS equations (4). These constraints are enforced via Lagrange multipliers [67].

\[
q(0) = \frac{||q(0)||_E^2}{2||q(T)||_E^2} \tilde{q}(0)
\]

Update:

Direct:

\[
\frac{\partial q}{\partial t} = Aq
\]

Loop until convergence

Adjoint:

\[
\frac{\partial \bar{q}}{\partial t} = A^H \bar{q}
\]

Update:

\[
\tilde{q}(T) = \frac{2}{||q(0)||_E^2} q(T)
\]

FIG. 2. A detailed sketch of the power iteration method employed in this study, adapted from Schmid [16].

During one cycle of this iterative process, the LNS equations (4) are integrated forward in time using a given initial condition. After a fixed time interval \(t = T\), the output of this integration \(q(T)\) is converted into a terminal condition \(\tilde{q}(T)\) for the adjoint equations. Next, we solve the adjoint equations backward in time to produce a state \(\tilde{q}(0)\), from which a new initial condition \(q(0)\) for the direct problem is obtained. This procedure is repeated until convergence. Figure 2 shows a detailed sketch of the adjoint looping method.
including the updates between each time integration scheme. The final result is the unit energy initial condition that is amplified most over a time interval \( 0 \leq t \leq T \), from which the maximum transient growth \( G(t) \) is readily determined [16]. Similar power iteration techniques have been successfully implemented in various complex boundary layer flows [68,69].

E. Numerical methods

We solve the compressible Navier-Stokes equations in conservative form for the base flow calculations of the spatially-developing boundary layer and oblique SWBLI [61]. A stable low-dissipation scheme based upon the kinetic energy consistent (KEC) method developed by Subbareddy and Candler [70] is implemented for the inviscid flux computation. In this numerical method, the flux is split into a symmetric (or non-dissipative) portion and an upwind (or dissipative) portion. The inviscid flux is premultiplied by a shock-detecting switch, which ensures that dissipation occurs only around shocks [71]. A fourth-order centered KEC scheme is employed for the present study. Viscous fluxes are modeled with second-order central differences. Time integration is performed using an implicit second-order Euler method with point relaxation to maintain numerical stability [72]. The implicit system is also solved using the full matrix data-parallel line relaxation (DPLR) method, which has good parallel efficiency [73].

For the transient growth calculations, the LNS equations (4) are discretized by fourth-order centered finite differences applied on a stretched mesh. This results in a large sparse matrix [74]. Time integration is performed using an implicit first-order Euler method. The inversion step is computed by finding the lower-upper (LU) decomposition of the shifted sparse matrix with the massively parallel SuperLU package [75]. We use a Newton-Raphson method to converge the residual of each base flow in accordance with system (1) to machine zero. This mitigates any error introduced by going from a conservative formulation [72] to a nonconservative or characteristic formulation [63]. A numerical filter is used to add minor amounts of scale-selective artificial dissipation to damp spurious modes associated with the smallest wavelengths allowed by the mesh. We introduce the numerical filter by adding terms of the form \( \epsilon (\partial^4 q / \partial x^4) \) and \( \epsilon (\partial^4 q / \partial y^4) \). Sponge layers are employed at the top, left, and right boundaries of the spatially-developing boundary layer and oblique SWBLI flows to absorb outgoing information with minimal reflection [76].
III. TRANSIENT GROWTH ANALYSIS

A. Parallel boundary layers

To verify our implementation of the power iteration method in Figure 2, we consider a few locally parallel base flows that are described in Hanifi et al. [26]. The base flows consist of one-dimensional (1D) boundary-layer profiles that satisfy the Mangler-Levy-Lees transformation [77]. For these parallel flow computations, we use a Prandtl number $Pr = 0.7$ and a stagnation temperature $T_0 = 333$ K. We employ periodic boundary conditions at the left and right boundaries of the two-dimensional (2D) computational domains. A numerical sponge layer is placed along the top boundary, which enforces a freestream outlet. Finally, the bottom boundary is treated as an isothermal wall.

We choose to model parallel boundary layers with 2D computational domains because our stability code is two dimensional and to be consistent, due to the fact that all of the other flow configurations in this study are two dimensional. A total of $n_x = 101$ and $n_y = 301$ grid points are used to resolve the 2D domains in the streamwise and wall-normal directions, respectively. In viscous units [77], the first grid point above the wall is positioned at $y^+ = 0.6$. The mesh spacing gradually increases as one moves away from the bottom wall. In the streamwise direction, the mesh spacing is uniform. We nondimensionalize by the Blasius length scale $\ell = \sqrt{\nu_\infty x/u_\infty}$, where $\nu_\infty$ is the kinematic viscosity in the freestream.

FIG. 3. Nondimensional streamwise velocity (solid blue) and temperature (dashed red) profiles of a locally-parallel Mach 2.5 boundary layer with $Re_\ell = 3000$. 
Figure 3 shows the boundary-layer profiles for $M_\infty = 2.5$ and $Re_\ell = 3000$. To generate a 2D parallel flow, we impose these boundary-layer profiles at every streamwise location along the wall-normal direction. We first consider the case with a nondimensional streamwise wavenumber $\alpha = 0.06$ and a nondimensional spanwise wavenumber $\beta = 0.1$. To calculate the streamwise domain length, we use the relation $L_x = 2\pi/\alpha$. The wall-normal domain length is set to $y = 30\ell$ for every locally-parallel base flow. We compute the energy growth of this parallel boundary-layer flow with the power iteration method described in Section II D. We start the iterative process displayed in Figure 2 using a randomly generated initial condition of unit energy. To make sure the randomly generated initial condition only contains $\alpha = 0.06$, we perform fast Fourier transforms (FFTs) in the streamwise direction and only keep the coefficients corresponding to the specified streamwise wavenumber. After this computation is done, we transform back to physical space.

Figure 4 compares the transient growth $G(t)$ that results from the power iteration method to the results of Hanifi et al. [26]. Note that $G(t)$ is an envelope of all possible optimal responses, since the optimum initial condition $q(0)$ depends on the finite time interval $[0, T]$ considered. The iterative procedure in Figure 2 takes approximately 5-6 iterations to converge (see Appendix B) and obtain an energy growth $G(t)$ at time $t$. Therefore, to trace out the entire envelope, we have to repeat this procedure for many different time intervals (represented by blue dots in Figure 4). We see good agreement between the two transient growth
envelopes in Figure 4. Hanifi et al. [26] calculated the transient growth $G(t)$ in one spatial dimension; therefore, exactly one streamwise wavenumber is being resolved at all times. Regardless, both methods converge to almost the exact same transient growth envelope with a temporal instability starting to take over at $t = 1000$. Temporal instability is a necessary condition for convective instability [56]. We know that there is a temporal instability present in the flow field because the transient growth envelope tends toward infinity as time advances (long-time exponential growth).

![Graph](image-url)

**FIG. 5.** Transient growth envelope of a locally-parallel Mach 2.5 boundary layer at $Re_\ell = 300$ computed with an SVD (solid black) from Hanifi et al. [26] and a power iteration method (solid/dots blue). Here, $\alpha = 0$ and $\beta = 0.1$ are the streamwise and spanwise wavenumbers, respectively.

We repeat the transient growth analysis for a parallel boundary layer with $M_\infty = 2.5$, $Re_\ell = 300$, $\alpha = 0$, and $\beta = 0.1$. Figure 5 shows the transient growth envelope for this case. Again we compare to the results of Hanifi et al. [26], and we see that there is good agreement between the power iteration method and the singular value decomposition. We observe that the energy peaks at $t = 1750$ in Figure 5, after which the transient growth $G(t)$ decreases with increasing time $t$. This means that the parallel boundary-layer flow is temporally stable at these conditions. We plot the optimal initial and final states corresponding to the peak at $t = 1750$ in Figure 6. The optimal initial condition consists mostly of wall-normal and spanwise velocities, while the streamwise velocity is negligible. This type of initial state has been seen in many incompressible [14,18] and compressible [27,28] flows. The disturbances in Figure 6 have been scaled to have a maximum
of one, but the final state has actually grown by about two orders of magnitude relative to the initial state [10]. In the final state, the streamwise velocity is very large compared to the other velocity components. The physical interpretation of this amplification is the Landahl lift-up effect [24,25]. This is where initial streamwise vortices that have relatively small $u'$ and $T'$ decay in time, while elongated streamwise structures grow rapidly and robustly [10]. Figure 6 illustrates that it is most effective to excite the wall-normal and spanwise velocity components. Also, the most energy is carried by the streamwise velocity component.

![Graph](image)

**FIG. 6.** Optimal (a) initial and (b) final states corresponding to the peak at $t = 1750$ in Figure 5. The absolute value of the streamwise (solid blue), wall-normal (dashed red), and spanwise (dash-dot green) velocity components are plotted for each state. Both states are scaled to have a maximum of one.

### B. Spatially-developing boundary layer

Since the transient growth of parallel boundary layers has been studied before [10], and we have successfully verified our power iteration method against these studies [26], we now focus our efforts on investigating the transient growth of nonparallel boundary layers. Bitter [11] recently examined the propagation of localized instability wave packets in spatially-developing boundary layers. This comprehensive study included a conventional stability analysis involving fixed-frequency disturbances, examining the development of Gaussian-shaped second-mode wave packets that are placed inside the boundary layer, and analyzing the response of planar acoustic wave packets in the freestream.
In order to conduct an optimal transient growth analysis of a nonparallel boundary layer, we first need to compute a steady two-dimensional base flow. The domain we consider extends $235\delta^*_m$ in the streamwise direction and $36\delta^*_m$ in the wall-normal direction. We use a Cartesian mesh to discretize this domain. The mesh is stretched in the wall-normal direction with $y^+ = 0.6$ and uniformly spaced in the streamwise direction. A total of $n_x = 500$ and $n_y = 420$ grid points are used to resolve this domain in the streamwise and wall-normal directions, respectively. The base flow simulations are run for approximately 60 flow-through times with the US3D hypersonic flow solver [78] until the residual is on the order of machine zero. Here a flow-through time is defined as the time it takes for a fluid particle to traverse the entire streamwise length of the domain, traveling with the freestream at Mach 5.92.

![Base flow contours](image)

**FIG. 7.** Base flow contours of nondimensional streamwise velocity and density for a Mach 5.92 spatially-developing boundary layer with $Re_{\delta^*_m} = 9660$.

Figure 7 shows base flow contours of the spatially-developing boundary layer. This corresponds to the flow conditions of the SWBLI, but without an incident oblique shock wave. We nondimensionalize $x$, $y$, and $z$ by the displacement thickness $\delta^*_m$. At the left inlet, we apply boundary-layer profiles computed from an earlier study [61]. These profiles also produce a bow shock and an entropy layer in close vicinity of one another. The lower boundary is modeled as an adiabatic wall. We enforce a hypersonic freestream inlet along the top edge of the domain. Furthermore, we impose a characteristic-based supersonic-outlet boundary condition along the right edge of the domain. Notice in Figure 7 that the boundary layer, and more specifically the boundary layer displacement thickness, grows substantially in the streamwise direction.

A recent study by Bitter and Shepherd [79] investigated the stability of hypervelocity boundary layers by using a locally-parallel linear stability framework. They found that the presence of supersonic modes...
cause the flow to become unstable over a wide band of frequencies. To examine the importance of convective instabilities (i.e., first and second modes) in a spatially-developing boundary layer, we perform a local linear stability analysis about 2D base flows that are created with 1D profiles extracted from Figure 7. We make the assumption that these boundary-layer profiles are locally parallel. Again we choose to model 1D parallel boundary layers with 2D computational domains to be consistent with the previous section and because our stability code is 2D. The steady base flows are similar to those reported in Section III A due to the fact that they have a streamwise length of $L_x = 2\pi/\alpha$. A total of $n_x = 101$ grid points are used to resolve the 2D domains in the streamwise direction. Periodicity is enforced at the left and right boundaries of the computational domain instead of applying numerical sponge layers.

FIG. 8. Growth rate contours of the first and second modes in the wavenumber plane resulting from a temporal stability analysis applied to a boundary-layer profile extracted from Figure 7 at $x = 235$.

The real and imaginary parts of the complex frequency $\omega = \omega_r + i\omega_i$ denote the temporal frequency and growth rate, respectively. Figure 8 displays the growth rate contours of the first and second modes resulting from a local linear stability analysis applied to a boundary-layer profile extracted from Figure 7 at $x = 235$. We extract a profile far downstream to minimize the strong impact of the bow shock and the entropy layer. In the wavenumber plane, the second mode has larger growth rates than the first mode. A positive growth rate corresponds to temporal instability, which is a necessary condition for convective instability [56]. Notice that the second mode becomes unstable at larger streamwise wavenumbers than the first mode, which agrees...
TRANSIENT GROWTH ANALYSIS OF OBLIQUE ...

with the results of Hanifi et al. [26] and Bitter [11]. Outside of the confined regions of instability shown in Figure 8 the first and second modes have negative growth rates. Similar results are obtained for boundary-layer profiles extracted from Figure 7 at streamwise positions before $x = 235$ (not shown). Since the first and second modes are unstable over a wide range of streamwise and spanwise wavenumbers, it is important to examine their contribution to the optimal transient growth of the spatially-developing boundary layer and oblique shock-wave/laminar-boundary-layer interaction.

**FIG. 9.** Contour plots of the transient growth $G(\beta, t)$ of a Mach 5.92 spatially-developing boundary layer with $Re_\delta^* = 9660$ for streamwise lengths of the domain equal to (a) $59\delta^*_{in}$, (b) $118\delta^*_{in}$, (c) $176\delta^*_{in}$, and (d) $235\delta^*_{in}$. The streamwise lengths correspond to the factors 1/4, 1/2, 3/4, and 1 of the original length. These plots illustrate convective instability of the spatially-developing boundary layer.

To further determine the role of convective instabilities, we compute the optimal transient growth of the spatially-developing boundary layer. Instead of enforcing periodicity at the streamwise boundaries, we set all perturbations to zero with the help of two artificial sponge layers [76]. The iterative process is initialized using a randomly generated flow field with unit energy. Figure 9 shows the transient growth over a range of
time intervals $t$, nondimensional spanwise wavenumbers $\beta$, and streamwise domain lengths $L_x$. We observe that the transient growth becomes larger as the streamwise extent of the domain increases. This indicates that the spatially-developing boundary layer we consider is convectively unstable. In other words, even though the system is globally stable, localized perturbations introduced upstream will grow substantially as they convect downstream with the flow [80,81].

Figure 10 displays the maximum transient energy growth and its dependence on the spanwise wavenumber for five different streamwise domain lengths. We generate the curves in Figure 10 by taking the maximum of $G(\beta, t)$ in time $t$ from Figure 9 and keeping only the variation with spanwise wavenumber $\beta$. The maximum transient growth occurs for the longest domain and is equal to $5.63 \times 10^3$ at $\beta = 0.38$, $t = 350$, and a streamwise domain length of $470s_{in}$. Taking the square root of this energy gain yields an amplification of $A_G = \sqrt{G_{\text{max}}} = 7.50 \times 10^1$ for the spatially-developing boundary layer without SWBLI.

While the transient growth increases with larger streamwise domain lengths, the spanwise wavenumber corresponding to the maximum growth decreases. In fact, Figure 11 shows that the spanwise wavelength (as measured by the inverse spanwise wavenumber) scales with the boundary layer thickness at the right outlet of the domain. Together, Figures 10 and 11 imply that a significant portion of the transient growth happens
near the streamwise end of the domain, and that this region selects the spanwise wavelength supporting the largest growth. This is a natural consequence of the square root growth of the boundary layer in the presence of convective instabilities: the boundary layer remains tuned to instabilities for longer extents downstream because it grows at a slower rate [11]. While the transient growth in the spatially-developing boundary layer is controlled by convective instability, and thus domain length, we will show that the separation bubble caused by SWBLI introduces a streamwise length scale and an additional mechanism of transient growth that does not depend on the streamwise domain length. We quantify this additional transient growth by considering a fixed domain length ($L_x = 235$) and then comparing the transient responses of the flow with and without SWBLI relative to one another.

In Figure 11, we illustrate the optimal initial and final states of the spatially-developing boundary layer colored by the real part of the normalized streamwise velocity perturbation for $L_x = 235$ with $\beta = 0.6$. The initial state is comprised of elongated streamwise structures near the left inlet that are tilted against the mean shear of the boundary layer. Notice that these streamwise streaks in Figure 12(a) have a shallow angle due to the high-speed nature of the flow. These tilted streamwise streaks start to align themselves with the mean shear as time passes causing substantial growth. This is commonly referred to as the inviscid
Orr mechanism [19,20]. Butler and Farrell [18] showed that these types of elongated streamwise structures grow rapidly and robustly. The final state in Figure 12(b) is also comprised of tilted streamwise streaks that extend to the right outlet. We obtain similar optimal initial and final states for different streamwise domain lengths (not shown), which confirms that they do not depend on the streamwise length of the domain.

FIG. 12. Optimal (a) initial and (b) final states of a Mach 5.92 spatially-developing boundary layer at $\text{Re}_\delta^* = 9660$ with $\beta = 0.6$. These states correspond to the peak in Figure 10 and are colored by the real part of the normalized streamwise velocity perturbation. The white lines indicate where we extracted the wall-normal profiles in Figure 13.

To examine whether the Landhal lift-up effect [24,25] contributes to the transient response of the spatially-developing boundary layer, we extract wall-normal profiles of the streamwise, wall-normal, and spanwise velocity components of the optimal initial and final disturbances at $x = 35$ and $x = 185$, respectively. This is shown in Figure 13, where we normalized the disturbances to have a unit maxima. Similar to Figure 6, we see that the optimal initial condition consists mostly of wall-normal and spanwise velocities, while the streamwise velocity is negligible. In the final state, however, the streamwise velocity is much larger than the other velocity components. This means that the lift-up effect does indeed contribute to the optimal transient growth of the spatially-developing boundary layer. In other words, initial streamwise vortices decay in time, while elongated streamwise structures grow rapidly and robustly.
FIG. 13. Normalized wall-normal cross sections of the optimal (a) initial and (b) final states of a Mach 5.92 spatially-developing boundary layer. The absolute value of the streamwise (solid blue), wall-normal (dashed red), and spanwise (dash-dot green) velocity components are plotted for each state. Here, $Re_{\delta_i} = 9660$ and $\beta = 0.6$. The initial state is extracted at $x = 35$ and the final state is extracted at $x = 185$.

FIG. 14. Optimal final state in 3D of a Mach 5.92 spatially-developing boundary layer with $Re_{\delta_i} = 9660$ and $\beta = 0.38$. The streamwise domain length is set to $2L_x = 470$. Isosurface contours represent the normalized streamwise velocity perturbation, where the red and blue contours indicate positive and negative velocities, respectively.

Our results demonstrate the importance of both the inviscid Orr mechanism \cite{19,20} and the Landahl lift-up effect \cite{24,25} in the optimal transient response. This is similar to the nonmodal results obtained in \cite{23} for incompressible three-dimensional boundary layers. According to Figure 8, the first and second modes are unstable over a wide range of streamwise and spanwise wavenumbers. Figures 9 and 10 also suggest the presence of convective instability because the transient growth becomes larger as the streamwise extent of the
domain increases. We plot the three-dimensional optimal final state of the Mach 5.92 spatially-developing boundary layer with $Re_{\delta^*} = 9660$, $\beta = 0.38$, and a streamwise domain length of $2L_x = 470$ in Figure 14. Notice that the final state consists of long oblique streamwise structures that are reminiscent of first-mode instability. The curved and tilted nature of these streamwise streaks suggest that there could be a numerical component impacting the three-dimensional structure. We plot wavepacket response in the following section to corroborate our claim that there is first-mode instability present in the optimal transient response of the spatially-developing boundary layer (refer to Figure 20). Thus, the optimal transient response of the spatially-developing boundary layer has contributions from the inviscid Orr mechanism, the Landahl lift-up effect, and first-mode instability.

C. Shock-wave/boundary-layer interaction

As shown in the previous section, a flat-plate boundary layer at Mach 5.92 supports significant transient growth. In this section, we consider an oblique SWBLI at the same conditions to examine its effect on transient growth. We show that the recirculation bubble created by SWBLI alters the physical mechanisms and enhances transient growth in a globally stable flow.

![FIG. 15. Contour plots of nondimensional streamwise velocity and density for a Mach 5.92 SWBLI with an incident shock angle of $\theta = 13^\circ$ at $Re_{\delta^*} = 9660$. Here, S and R denote the separation and reattachment points, respectively. The white contours indicate streamlines inside the recirculation bubble.](image)

We compute a steady two-dimensional base flow of an oblique shock-wave/laminar-boundary-layer interaction at Mach 5.92 with the US3D hypersonic flow solver [78]. The incident oblique shock wave is introduced
by modifying the inlet boundary layer profile so that the Rankine-Hugoniot jump conditions are satisfied at the point it enters the domain. We select this point so that the oblique shock impinges upon the wall at a distance of $119\delta_*^e$ from the leading edge. For this study, we are only interested in an oblique shock angle of $\theta = 13^\circ$. The other boundary conditions are explained in Section III B because they are used for the spatially-developing boundary layer with no incident oblique shock wave. We consider a domain that extends $235\delta_*^e$ and $36\delta_*^e$ in the streamwise and wall-normal directions, respectively. Our domain is discretized by a Cartesian mesh that is nonuniformly spaced in the wall-normal direction with $y^+ = 0.6$ and uniformly spaced in the streamwise direction. A total of $n_x = 998$ and $n_y = 450$ grid points are used to resolve this domain in the streamwise and wall-normal directions, respectively. Furthermore, the base flow simulations are run for approximately 60 flow-through times until the residual is on the order of machine zero.

![Image](image.png)

FIG. 16. Optimal transient growth versus spanwise wavenumber of a Mach 5.92 SWBLI with an incident shock angle of $\theta = 13^\circ$ at $Re\delta_*^e = 9660$ and $L_x = 235$.

Figure 15 depicts the SWBLI base flow with color contours of nondimensional streamwise velocity and density. We nondimensionalize $x$, $y$, and $z$ by the displacement thickness $\delta_*^e$. The incident oblique shock wave causes the boundary layer to separate from the wall at $x \approx 50$. Around this location, a separation shock wave forms. The separated boundary layer also causes a recirculation bubble of nearly constant density to develop. At the apex of the recirculation bubble, an expansion fan forms and extends up into the freestream. Moreover, at $x \approx 155$, the flow reattaches to the wall and compression waves coalesce to form a second
reflected shock. Figure 15 also shows a bow shock that enters the computational domain through the left inlet. This bow shock is created by the flat-plate leading edge and does not interact with the bubble.

Dwivedi et al. [49] measured the spatial growth of elongated streamwise structures that form after the reattachment point of this oblique shock-wave/laminar-boundary-layer interaction using a PSE-based power iteration. Because the parabolized stability equations require a slowly-varying base flow [82], this approach cannot be applied within the recirculation bubble. However, the power iteration method described in Section II D requires no such assumption. Therefore, it can be applied to the entire 2D steady base flow in Figure 15.

We compute the optimal transient growth of the SWBLI in Figure 15 with our power iteration method using a randomly generated initial condition that has unit energy. Similar to Figure 10, we plot the maximum transient energy growth versus the spanwise wavenumber in Figure 16.

Figure 17. Transient growth envelope (solid black) of a Mach 5.92 SWBLI with $\theta = 13^\circ$, $Re_{\delta_m} = 9660$, and $\beta = 2.6$. Initial conditions pertaining to the fixed time intervals $t = 150$, $t = 214$, and $t = 260$, where the optimal transient energy growth is computed using our power iteration method, are marched forward in time to yield the other three growth curves (dashed blue, dashed red, and dashed green). These transient growth curves will touch the envelope at exactly one point. A solid circle indicates where each curve is tangent to the transient growth envelope.

It is worth noting that the peak transient growth of $1.36 \times 10^7$ at $\beta = 2.6$ in Figure 16 is roughly four orders of magnitude larger than the transient growth seen in Figure 10 for $L_x = 235$. The spanwise
wavenumber also increases from $\beta = 0.6$ for a nonparallel boundary layer to $\beta = 2.6$ for an SWBLI. As the spanwise wavenumber increases away from the maximum at $\beta = 2.6$, the transient growth decreases. We also see in Figure 16 that as the spanwise wavenumber decreases to zero, the transient growth drops several orders of magnitude. Moreover, the corresponding optimal spanwise wavelength for the SWBLI is $\lambda_z = 2\pi/\beta = 2.41 \approx 2$ (or $2\delta^*$ in dimensional units). This agrees well with SWBLI experiments [58,59] that often find the spanwise wavelength associated with the streamwise streaks near the reattachment location to be on the order of twice the boundary layer thickness.

FIG. 18. Evolution of the (a) optimal initial condition to its (d) final state at $t=214$ for a Mach 5.92 SWBLI with $\theta = 13^\circ$, $Re_{\delta^*} = 9660$, and $\beta = 2.6$. The two intermediary states occur at (b) $t = 150$ and (c) $t = 182$. Contour levels, representing the real part of the normalized spanwise velocity perturbation, are identical in each frame. The black dashes indicate the start of three different sponge layers.

The transient growth envelope of the SWBLI in Figure 15 with $\beta = 2.6$ is displayed in Figure 17. Initial conditions pertaining to the fixed time intervals $t = 150$, $t = 214$, and $t = 260$, where the optimal transient energy growth is computed using our power iteration method, are marched forward in time to yield the other three growth curves. Notice in Figure 17 that each of these growth curves are tangent to the transient...
growth envelope at exactly one point. The optimal transient energy growth of this SWBLI occurs at $t = 214$. Since the transient growth approaches zero as time increases beyond the maximum at $t = 214$ to infinity, we know the SWBLI in Figure 15 is globally stable at $\beta = 2.6$. We showed this in an earlier study using a linear global stability analysis [8].

We show the 2D optimal initial and final states of a Mach 5.92 SWBLI with $\theta = 13°$ and $Re_{\delta^*} = 9660$ colored by the normalized spanwise velocity perturbation in Figure 18. There are also two intermediary states plotted, one at $t = 150$ and the other at $t = 182$. This evolution of the optimal initial condition corresponds to the transient growth curve with red dashes in Figure 17. Notice that by $t = 150$ there is still barely any transient growth in the system. After $t = 150$, there is a significant increase in the transient growth that persists until $t = 214$. During the same time interval, we see that in Figure 18 there is a large spanwise velocity perturbation that forms after the recirculation bubble and convects downstream until it hits the right sponge layer. When this large perturbation starts to decrease as it moves through the right sponge layer, we observe that the transient growth curve in Figure 17 also starts to decrease. Thus, if the domain had a longer streamwise length the optimal transient growth could be even larger. This indicates the presence of convective instability.

FIG. 19. Optimal initial and final states of a Mach 5.92 SWBLI with $\theta = 13°$, $Re_{\delta^*} = 9660$, and $\beta = 2.6$. Isosurface contours, which represent the normalized streamwise velocity perturbation, are identical in each frame. The red and blue contours indicate positive and negative velocities, respectively.
Figure 19 depicts the optimal initial and final states of a Mach 5.92 SWBLI in 3D. The initial state consists of elongated streamwise structures that start near the inflow and extend past the separation point at $x \approx 50$. These three-dimensional structures are made up of spanwise periodic streamwise vortices (see Figure 18). High and low momentum zones, that reside within the incoming boundary layer in the initial state, cause the recirculation bubble to become three dimensional such that the separation line is corrugated. The SWBLI responds by producing streamwise streaks, which form in the recirculation bubble and are amplified further downstream. The final state, which includes these long streamwise streaks, is shown in Figure 19. This optimal transient growth mechanism for an oblique SWBLI is similar to the physical mechanism found in an SWBLI that forms on a compression ramp using input-output analysis [50,51].

Figures 12, 18, and 19 suggest that the optimal spatio-temporal responses of both the spatially-developing boundary layer and the oblique SWBLI consist of wavepackets that amplify as they propagate downstream. Wavepackets occur in a variety of convectively-unstable flows including low-speed boundary layers [52] and high-speed jets [81], and have been linked to transient growth when viewed in the global framework [56]. In order to better understand the physical mechanisms present in the oblique shock-wave/laminar-boundary-layer interaction and determine the role of convective instabilities, we plot the wavepacket responses of both the spatially-developing boundary layer and SWBLI in Figure 20. We compute the disturbance amplitude at each streamwise station by integrating the Chu energy norm [66] in the wall-normal direction and then taking a square root. Each curve in Figure 20 represents a different snapshot in time, which we then plot as a function of group velocity $v_g = (x - x_0)/t$ to track the wavepacket as it propagates. Figure 20 shows that for certain group velocities the wavepacket amplifies, indicating convective instability.

The plots in Figure 20 allow us to employ the procedure described in Gallaire and Chomaz [83] to estimate the growth rate and wavelength of the instability waves that comprise a wavepacket. We define the temporal growth rate $\sigma$ observed while traveling at group velocity $v_g$ as

$$\sigma(v_g) = \frac{\ln(A_G(x_0 + v_g t_2)/A_G(x_0 + v_g t_1))}{t_2 - t_1} + \frac{\ln(t_2/t_1)}{2(t_2 - t_1)}.$$

where $x_0$ is the initial position of the wavepacket. Here, the times $t_1$ and $t_2$ are far enough apart (we use $t_2 = 1.5t_1$) to ensure convergence on the least stable mode and circumvent low-frequency oscillations [84].
In the fixed frame, the growth rate and streamwise wavenumber are determined by

\[
\omega_i = \sigma(v_g) - v_g \frac{\sigma(v_g + \delta x/t_2) - \sigma(v_g)}{\delta x/t_2}, 
\tag{11}
\]

\[
\alpha(v_g) = \frac{\phi(x_o + v_g t_2 + \delta x, t_2) - \phi(x_o + v_g t_2, t_2)}{\delta x}, 
\tag{12}
\]

where \(\phi\) denotes the phase angle of the resulting wavepacket. This angle is computed by taking the inverse tangent of the Hilbert transform of the real nondimensional pressure perturbation. Here, \(\delta x\) represents a small change in the streamwise position, and the quantities \(\omega_i\) and \(\alpha\) can be directly compared to those shown in Figure 8 of the previous section.

![Wavepacket responses of the (a) spatially-developing boundary layer at \(\beta = 0.6\) and (b) oblique SWBLI at \(\beta = 2.6\).](image)

FIG. 20. Wavepacket responses of the (a) spatially-developing boundary layer at \(\beta = 0.6\) and (b) oblique SWBLI at \(\beta = 2.6\). The dashed lines indicate the component of the wavepacket that resides within a sponge layer. For the oblique SWBLI in (b), the red contours represent the times when the main peak of the wavepacket has crossed the streamwise position of the separation point.

By evaluating equations (10), (11), and (12) using the data in Figure 19(a), we obtain \(\omega_i = 0.0076\) and \(\alpha = 0.110\) for the spatially-developing boundary layer at \(\beta = 0.6\). These values agree relatively well with those of the first mode in Figure 8. This means that convective instability, or more specifically first-mode instability, plays a vital role in the optimal transient response of the spatially-developing boundary layer. Similarly, evaluating the same equations for the first peak in Figure 19(b) colored with blue contours (times
before the wavepacket has crossed the separation point) results in a growth rate $\omega_i = 0.0098$ and streamwise wavenumber $\alpha = 0.109$. Therefore, first-mode instability also contributes to the transient response in the upstream boundary layer of the oblique SWBLI at $\beta = 2.6$. The second largest peak in Figure 19(b) colored with red contours (times after the wavepacket has crossed the separation point) yields a growth rate $\omega_i = 0.0768$ that is an order of magnitude larger than the values reported in Figure 8. Furthermore, the streamwise wavenumber $\alpha = 0.029$ is also a lot smaller than the other two values computed with equation (12), which is a consequence of the long streamwise streaks that form downstream of the recirculation bubble.

Since $\omega_i$ and $\alpha$ for the oblique SWBLI at $\beta = 2.6$ after separation are significantly different than the values before separation and the values obtained for the spatially-developing boundary layer at $\beta = 0.6$, the convective instability mechanism is no longer associated with the first or second mode. We attribute this change to centrifugal instability that arises from streamline curvature in the upstream region of the flow field where the wavepacket first reaches the recirculation bubble. In addition to the larger growth that occurs after the wavepacket moves past the separation point, the wavepacket is convectively unstable at lower group velocities too, as compared to Figure 19(a). This is because of the recirculation that exists within the bubble. Notice also in Figure 19(b) that the sponge layer starts to damp the wavepacket at its largest peak. In summary, our spatio-temporal analysis demonstrates that the formation of a recirculation bubble increases the optimal transient growth in a spatially-developing boundary layer at least in part by modifying the mechanism responsible for convective instability.

In a previous study [8], we applied global stability analysis to the same oblique shock-wave/laminar-boundary-layer interaction at $\theta = 13^\circ$ and found an unstable stationary mode that causes the flow to bifurcate from its original laminar state. We know that the maximum growth rate of this global mode is $\omega_i = 0.0025$ at $\beta = 0.25$ [8]. The time horizon at which the largest transient growth occurs, however, is $t = 214$. During this time, the global mode would amplify only $e^{\omega_i t} = 1.71$, while the transient growth produces an amplification of $A_G = \sqrt{1.36 \times 10^7} = 3.69 \times 10^3$ (or an N-factor of 8) in the same time period. If the environment upstream introduces disturbances with amplitudes greater than $2.71 \times 10^{-4}$ compared to the elongated streaks downstream, one would see the streaks associated with transient growth on shorter time horizons in SWBLI experiments. These disturbances, if they are strong enough, can also trigger nonlinear interactions leading to transition to turbulence bypassing the modal route [85]. Transient growth calculations
provide an upper bound on the amplification possible, which might be important to designers of SWBLI experiments in quiet wind tunnels as it gives an estimate of tolerable noise levels. These calculations also provide a means by which to distinguish streaks produced by nonmodal mechanisms versus those produced by other ways (e.g., modal instability).

IV. CONCLUSIONS

We used a power iteration approach to study the dynamics of flow fluctuations in parallel and nonparallel boundary layers as well as in an oblique shock-wave/laminar-boundary-layer interaction by computing the optimal transient growth. This iterative method converges quickly (see Appendix B) allowing for several runs needed to trace out transient growth envelopes and investigate the impact of different streamwise and spanwise wavenumbers. A verification of our power iteration method against test cases reported by Hanifi et al. [26] resulted in good agreement.

In this study, we considered a spatially-developing boundary layer as an intermediate step between parallel boundary layers and transitional SWBLIs. A local stability analysis was applied to boundary layer profiles extracted from the nonparallel base flow. We found that both the first and second mode are spatially unstable at many different streamwise and spanwise wavenumbers. The optimal transient growth for a Mach 5.92 spatially-developing boundary layer with \( L_x = 235 \) and \( Re_{\delta_m} = 9660 \) is \( 1.69 \times 10^3 \) at \( \beta = 0.6 \). This corresponds to an amplification of \( A_G = \sqrt{1.69 \times 10^3} = 4.11 \times 10^1 \). The inviscid Orr mechanism [19,20], the Landahl lift-up effect [24,25], and first-mode instability all contribute to the optimal transient response. We also found that there is an inversely proportional relationship between the boundary layer thickness and the nondimensional spanwise wavenumber corresponding to the maximum transient growth.

Next, we compute the transient growth of an SWBLI at the same conditions as the spatially-developing boundary layer. The presence of an oblique shock wave changes the optimal transient response such that \( G(\beta) = 1.36 \times 10^7 \) at \( \beta = 2.6 \). It is worth noting that the transient growth in an SWBLI is four orders of magnitude larger than the transient growth in a spatially-developing boundary layer with \( L_x = 235 \). The spanwise wavenumber of the optimal transient response also increases from \( \beta = 0.6 \) to \( \beta = 2.6 \). Moreover, the corresponding optimal spanwise wavelength for the SWBLI is roughly twice the boundary layer thickness, agreeing with SWBLI experiments [58,59]. The optimal initial condition consists of spanwise periodic...
streamwise vortices that reside within the incoming boundary layer and cause the recirculation bubble to become three dimensional such that the separation line is corrugated. The final state consists of elongated streamwise streaks that form in the recirculation bubble and are amplified further downstream.

From the spatio-temporal analysis of different wavepacket responses, we observe that the presence of a recirculation bubble in SWBLI modifies the convective instability present in the spatially-developing boundary layer such that it is no longer associated with the first mode. This causes the temporal growth rate and streamwise wavenumber to increase and decrease by an order of magnitude, respectively. The resulting change is due to centrifugal instability caused by streamline curvature once the wavepacket collides with the recirculation bubble. By comparing the results from a previous global stability analysis [8] to the transient growth results of this paper, we observe a significant impact due to the nonmodal mechanisms in a Mach 5.92 SWBLI. This indicates that inevitable imperfections in wind tunnels [15] would play an important role in the early stages of transition in hypersonic flows.

In future work, we plan to compare our transient growth results with the predictions resulting from input-output analysis [50,51]. Both of these nonmodal analyses obtain a spanwise wavelength corresponding to the maximum spatio-temporal transient growth in SWBLIs on the order of twice the boundary layer thickness, even though their numerical methodologies are completely different. We also plan to investigate the effects of Mach number and wall temperature on the optimal transient growth similar to the studies performed by Bitter and Shepherd [10,79]. It is already known that the growth rates of both the first and second mode are quite sensitive to these parameters [2,86,87]. For example, wall cooling stabilizes the first mode, but destabilizes the second and higher modes. Their effects on transient growth in compressible boundary layers, however, are not so simple (refer to [23,27,28,88]). Therefore, a systematic study on the effects of Mach number and wall temperature is needed to further clarify the role of these parameters in the transient growth of spatially-developing boundary layers and SWBLIs.

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APPENDIX A: VERIFICATION OF OUR INITIAL/FINAL STATES

We apply transient growth analysis to a parallel boundary layer with $M_\infty = 2.5$, $Re_\ell = 300$, $\alpha = 0.0$, and $\beta = 0.25$. This case corresponds to the optimal gain reported in Hanifi et al. [26] for $M_\infty = 2.5$ and $Re_\ell = 300$. We show the optimal initial and final states computed with two different approaches in Figure 21. Comparing the temporal states obtained with the power iteration method to the SVD approach used in Hanifi et al. [26] results in good agreement. Notice that the final state has grown by three orders of magnitude relative to the initial state in Figure 21. We see along with the streamwise velocity, that the temperature fluctuations are a vital component to the Landahl lift-up effect [24,25] for an adiabatic wall. Due to the fact that we get relatively good agreement by comparing the transient growth envelopes as well as the optimal initial and final states from our power iteration method to the results of Hanifi et al. [26], we conclude that our approach accurately models the transient response of high-speed boundary layers.

![Figure 21](image)

**FIG. 21.** Optimal (a) initial and (b) final states for a Mach 2.5 boundary layer at $Re_\ell = 300$ computed with an SVD (black) from Hanifi et al. [26] and a power iteration method (blue/red). The absolute value of the streamwise velocity (solid) and temperature (dashed) perturbations are plotted for each state. Here, $\alpha = 0.0$ and $\beta = 0.25$ are the streamwise and spanwise wavenumbers, respectively.

APPENDIX B: CONVERGENCE OF OUR POWER ITERATION METHOD

Since the power iteration method in Figure 2 only solves for one point on a transient growth envelope at a time, it takes several iterations of this approach to trace out a full envelope (see Figures 4 and 5).
In order to include variations with respect to the streamwise or spanwise wavenumber (see Figure 9), this power iteration method has to be repeated hundreds of times. We show the convergence history of this iterative approach for the Mach 5.92 spatially-developing boundary layer and SWBLI with $Re_{\delta_m} = 9660$ and $L_x = 235$ in Figure 22. Notice that it takes roughly 5-6 iterations to converge to the optimal initial and final states. We know that the convergence depends on the given initial condition. For this study, we used a randomly-generated flow field with unit energy norm, but a Gaussian pulse or another type of initial disturbance could take much longer to converge. Further, optimization techniques (for example, conjugate gradient algorithms) could speed up convergence even more.

**FIG. 22.** Convergence of the power iteration method in Figure 2 for the Mach 5.92 (a) spatially-developing boundary layer and (b) oblique shock-wave/laminar-boundary-layer interaction with $Re_{\delta_m} = 9660$ and $L_x = 235$. Both plots correspond to the optimal transient growth in each case.

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