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In a recent paper on mixed-effects models for confirmatory analysis, Barr et al. (2013) offered the following guideline for testing interactions: “one should have by-unit [subject or item] random slopes for any interactions where all factors comprising the interaction are within-unit; if any one factor involved in the interaction is between-unit, then the random slope associated with that interaction cannot be estimated, and is not needed” (p. 275). Although this guideline is technically correct, it is inadequate for many situations, including mixed factorial designs. The following new guideline is therefore proposed: models testing interactions in designs with replications should include random slopes for the highest-order combination of within-unit factors subsumed by each interaction. Designs with replications are designs where there are multiple observations per sampling unit per cell. Psychological experiments typically involve replicated observations, because multiple stimulus items are usually presented to the same subjects within a single condition. If observations are not replicated (i.e., there is only a single observation per unit per cell), random slope variance cannot be distinguished from random error variance and thus random slopes need not be included.

This new guideline implies that a model testing $AB$ in a $2 \times 2$ design where $A$ is between and $B$ within should include a random slope for $B$. Likewise, a model testing all two- and three-way interactions in a $2 \times 2 \times 2$ design where $A$ is between and $B, C$ are within should include random slopes for $B, C$, and $BC$.

The justification for the guideline comes from the logic of mixed-model ANOVA. In an ANOVA analysis of the $2 \times 2$ design described above, the appropriate error term for the test of $AB$ is $MS_{UB}$, the mean squares for the unit-by-$B$ interaction (e.g., the subjects-by-$B$ or items-by-$B$ interaction). For the $2 \times 2 \times 2$ design, the appropriate error term for $ABC$ and $BC$ is $MS_{UBC}$, the unit-by-$BC$ interaction; for $AB$, it is $MS_{UB}$; and for $AC$, it is $MS_{UC}$.

To what extent is this ANOVA logic applicable to tests of interactions in mixed-effects models? To address this question, Monte Carlo simulations were performed using R (R Core Team, 2013). Models were estimated using the $\text{lmer()}$ function of $\text{lme4}$ (Bates et al., 2013), with $p$-values derived from model comparison ($\alpha = 0.05$). The performance of mixed-effects models (in terms of Type I error and power) was assessed over two sets of simulations, one for each of two different mixed factorial designs. The first set focused on the test of the $AB$ interaction in a $2 \times 2$ design with $A$ between and $B$ within; the second focused on the test of the $ABC$ interaction in a $2 \times 2 \times 2$ design with $A$ between and $B, C$ within. For simplicity all datasets included only a single source of random effect variance (e.g., by-subject but not by-item variance). The number of replications per cell was 4, 8, or 16. Predictors were coded using deviation ($-0.5, 0.5$) coding; identical results were obtained using treatment coding. In the rare case ($\approx 2\%$) that a model did not converge, it was removed from the analysis. Power was reported with and without adjustment for Type I error rate, using the adjustment method reported in Barr et al. (2013).

For each set of simulations at each of the three replication levels, 10,000 datasets were randomly generated, each with 24 sampled units (e.g., subjects). The dependent variable was continuous and normally distributed, with all data-generating parameters drawn from uniform distributions. Fixed effects were either between $-2$ and $-1$ or between 1 and 2 (with equal probability). The error variance was fixed at 6, and the random effects variance/covariance matrix had variances ranging from 0 to 3 and covariances corresponding to correlations ranging from $-0.9$ to 0.9.

For the $2 \times 2$ design, mixed-effects models with two different random effects structures were fit to the data: (1) by-unit random intercept but no random slope for $B$ (“RI”), and (2) a maximal model including a slope for $B$ in addition to the random intercept (“Max”). For comparison purposes, a test of the interaction using mixed-model ANOVA (“AOV”) was performed using R’s $\text{aov()}$ function.

Results for the test of the $AB$ interaction in the $2 \times 2$ design are in Tables 1 and 2. As expected, the Type I error rate for ANOVA and maximal models were very close to the stated $\alpha$-level of 0.05. In contrast, models lacking the random slope for $B$ (“RI”) showed unacceptably high Type I error rates, increasing with the number of replications. Adjusted power was comparable for all three types of analyses (Table 2), albeit with a slight overall advantage for RI.

The test of the $ABC$ interaction in the $2 \times 2 \times 2$ design was evaluated under four different random effects structures, all including a random intercept but varying in which random slopes were included. The models were: (1) random intercept only (“RI”); (2) slopes for $B$ and $C$ but not for $BC$ (“nBC”); (3) slope for $BC$ but not for $B$ nor $C$ (“BC”); and (4) maximal (slopes for $B, C$, and $BC$; “Max”).

For the test of the $ABC$ interaction, ANOVA and maximal models both
just as important to attend to this guide-
sumed by each interaction of interest. It is
bination of within-subject factors sub-
it is critical to include the random slope
	ions simultaneously with replication,
(power was comparable across all analyses
random-intercept-only model. Adjusted
dom slope may be even worse than a
ceptably high Type I error rates. Indeed,
in the test of an interaction can yield unac-
isolate when one seeks to simplify a non-
verging model as when one is decid-
ing on what structure to fit in the first
order interaction, the model
should include a random slope for the con-
intercepts-only model, even though such
random slopes except for the single criti-
a model is nearly maximal. Finally, note
including only the critical random
slope in the model was sufficient to
obtain acceptable performance, as illus-
trated by the “BC” model in the $2 \times 2 \times 2$
design.

Table 1 | Type I error rate for the test of $AB$ in the $2 \times 2$ design.

| Reps | RI       | Max       | AOV       |
|------|----------|-----------|-----------|
| 4    | 0.170    | 0.063     | 0.050     |
| 8    | 0.267    | 0.064     | 0.052     |
| 16   | 0.395    | 0.063     | 0.049     |

Table 2 | Power for the test of $AB$ in the $2 \times 2$ design, Adjusted (Raw) $p$-values.

| Reps | RI       | Max       | AOV       |
|------|----------|-----------|-----------|
| 4    | 0.495 (0.704) | 0.469 (0.507) | 0.471 (0.471) |
| 8    | 0.594 (0.847) | 0.558 (0.604) | 0.558 (0.565) |
| 16   | 0.649 (0.922) | 0.619 (0.657) | 0.619 (0.619) |

Table 3 | Type I error rate for test of $ABC$ in $2 \times 2 \times 2$ design.

| Reps | RI | nBC | BC | Max | AOV |
|------|----|-----|----|-----|-----|
| 4    | 0.069 | 0.102 | 0.050 | 0.046 | 0.046 |
| 8    | 0.124 | 0.159 | 0.059 | 0.057 | 0.051 |
| 16   | 0.197 | 0.241 | 0.063 | 0.062 | 0.052 |

Table 4 | Power for test of $ABC$ in $2 \times 2 \times 2$ design, Adjusted (Raw) $p$-values.

| Reps | RI       | nBC       | BC     | Max     | AOV     |
|------|----------|-----------|--------|---------|---------|
| 4    | 0.422 (0.478) | 0.418 (0.546) | 0.396 (0.397) | 0.412 (0.412) | 0.405 (0.405) |
| 8    | 0.562 (0.711) | 0.567 (0.753) | 0.552 (0.575) | 0.564 (0.578) | 0.554 (0.557) |
| 16   | 0.649 (0.866) | 0.651 (0.889) | 0.653 (0.690) | 0.657 (0.687) | 0.656 (0.661) |

Although the current simulations only
considered interactions between categori-
variables, the guideline applies when-
ever there are replicated observations,
regardless of what types of variables are
involved in an interaction (e.g., continu-
ous only, or a mix of categorical and con-
tinuous). For example, consider a design
with two independent groups of subjects,
where there are observations at multiple
time points for each subject. When testing
the time-by-group interaction, the model
should include a random slope for the con-
tinuous variable of time; if time is mod-
eled using multiple terms of a polynomial,
then there should be a slope for each of
the terms in the polynomial that interact
with group. For instance, if the effect of
time is modeled as $Y = \beta_0 + \beta_1 t + \beta_2 t^2$
and the interest is in whether the $\beta_0$ and
$\beta_1$ parameters vary across group, then the
random effects structure should include
slopes for both the group-by-$t$ and group-
by-$t^2$ interactions.

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