A reliability growth projection model based on similar failure mechanisms

Zhongsheng Li*, Jinwei Fan and Dongju Chen
Jidian building, room 304, pingleyuan 100, chaoyang district, 100124, Beijing, China
Beijing key laboratory of advanced manufacturing technology, Beijing University of Technology, Beijing, China

*13910909582@163.com

Abstract. Reliability growth test is a common method to evaluate complex electromechanical systems in mechanical engineering. Reliability growth models are usually used to predict the reliability index of systems after the reliability test. AMSAA (army materials systems analysis activity) projection-model-stein (APMS) model is widely used for delayed corrective strategy, since it only requires limited assumptions but covers all failure data. However, there is a controversy about whether the Stein factor introduced in this model is reasonable. To alleviate the problem, an APMS development model is presented in this paper, because different types of parts have different inherent failure mechanisms. The study showed the projection results of the proposed model are credible. Moreover the Stein shrinkage factors were more reasonable and acceptable, as they were derived from the components with similar inherent failure mechanisms. These provide a technical foundation for the wide application of the proposed model.

1. Introduction
The requirement of high reliability for the complex systems is obvious in mechanical engineering. Reliability test is a common method to evaluate complex electromechanical systems, which involves surfacing failure modes, analyzing them and implementing fixes to the observed modes. Reliability growth models are generally used to quantify realized reliability growth value at the end of the initial test phase and project expected reliability growth value based on assumed, fix effectiveness factors (FEFs) of corrective actions [1, 2]. AMSAA (army materials systems analysis activity) projection-model-stein (APMS) model is widely used for delayed-corrective-strategy testing, because it only requires limited assumptions but covers all failure data [3, 4]. Meanwhile the APMS model is statistically more robust against the effects of fix-effectiveness-factors variability than the AMSAA projection model [5]. Moreover, the Stein-based model can provide a more accurate reliability-projection value than other models, given that minimizing the expected squared error between the assessed and initial mode failure rates [6].

Since the seminal work of Stein [7], the ‘Stein Estimator’, has received enormous attention in the theoretical study and aroused great interest in the practical applications [8]. However, there is controversy about whether the stein factor introduced in this model is reasonable [9], due to inadmissible data involved. To alleviate this problem, an development model of APMS Model, in which the stein estimators are calculated individually by different failure mechanisms, is presented. It is known that different types of components have different failure mechanisms. For example, the failure mechanisms of mechanical products mainly include fatigue and corrosion. The failure
mechanisms of electronic products mainly include thermal effect, electrochemistry, electromagnetic compatibility, electro-static discharge, etc. The main objective of the model proposed is to be able to continuously assess the reliability growth of a system across multiple test phases, through dividing the failure data into several sections according to similar failure mechanisms.

2. APMS Model

APMS projection model, estimates the fraction of system failure rate, caused by the not yet occurring modes, after the test through stein shrinkage estimator, utilizing an optimality criterion to minimize the sum of squared errors, between the estimated initial failure ratios and the real rates. The base of the model is that below ratios of occurrence for separate failure modes are a case of a Gamma distribution random sample [3].

It is supposed the system has \( k > 1 \) potential failure modes that have initial failure rates \( \lambda_1, \ldots, \lambda_k \).

Let \( n_i \) denote the number of failures observed for mode \( i (i = 1, \ldots, k) \) that occur during the test. The maximumLikelihood estimate of \( \lambda_i \) is \( \hat{\lambda}_i = n_i / T \). After mitigation of the failure modes observed during the test period \([0, T]\), the realized system-failure-rate-intensity function is given as

\[
\rho_s(T) = \frac{n_A}{T} + \sum_{i \in \text{obs}(B)} (1 - d^*_i) \cdot \hat{\lambda}_i + \sum_{i \in \text{obs}(B)} \hat{\lambda}_i
\]

(1)

Where \( n_A \) is the observed number of Mode-A failures, \( \text{obs}(B) \) means the observed Mode-B failures, \( d^*_i \) is a realization of \( d_i \) (FEF). \( \hat{\lambda}_i \) represents the stein estimation of \( \lambda_i \), is defined by

\[
\hat{\lambda}_i \equiv \theta_s \cdot \hat{\lambda}_i + (1 - \theta_s) \cdot \frac{\sum \hat{\lambda}_i}{k_B}
\]

(2)

The shrinkage factor \( \theta_s \in [0, 1] \) in equation (2), that minimizes the expected sum of squares,

\[
E \left[ \sum_{i \in B} \left( \lambda_i - \lambda \right)^2 \right]
\]

can be expressed by

\[
\theta_s = \frac{\sum_{i \in B} (\lambda_i - \lambda)^2}{\frac{\lambda_B}{T} \cdot (1 - \frac{1}{k_B}) + \sum_{i \in B} (\hat{\lambda}_i - \lambda)^2}
\]

(3)

Where \( \lambda = \frac{1}{k_B} \sum_{i \in B} \hat{\lambda}_i \), and \( \lambda_B = n_B / T \), \( n_B \) is the observed number of Mode-B failures.

3. APMS development model

It is supposed there are \( j (j = 1, \ldots, q) \) sections of the complex system, in term of the similar inherent failure mechanism and correction strategy. That is, the stein estimate values are calculated individually as per identified sections. As a result, the stein projection of the system–failure-rate intensity developed by identified sections is given as:

\[
\rho_{S,j}(T) = \frac{n_A}{T} + \sum_{j=1}^{q} \sum_{i \in \text{obs}(B)} (1 - d^*_j) \cdot \hat{\lambda}_j + \sum_{i \in \text{obs}(B)} \hat{\lambda}_j
\]

(4)

Where \( d^*_j \) is a realized \( d_j \) of the \( j \)th section. \( \hat{\lambda}_j \) represents the stein estimation of \( \lambda_i \) for the \( j \)th divided section, are defined by

\[
\hat{\lambda}_j = \theta_{S,j} \cdot \hat{\lambda}_j + (1 - \theta_{S,j}) \cdot \frac{\sum \hat{\lambda}_j}{k_{B,j}}
\]

(5)
Similarly,  

\[ \theta_{s,j} = \sum_{i \in B(j)} \left( (\lambda_{ij} - \overline{\lambda})^2 \right) \]

(6)

\[ \frac{\hat{\lambda}_{B,j}}{T} \left( 1 - \frac{1}{k_{B,j}} \right) = \sum_{i \in B(j)} (\lambda_{ij} - \overline{\lambda})^2 \]

Where \( \overline{\lambda} = \frac{1}{k_{B,j}} \sum_{i \in B(j)} \lambda_{ij} \) and \( \hat{\lambda}_{B,j} = n_{B,j} / T \).

Meanwhile

\[ \sum_{i \in B} \lambda_{ij} \approx \frac{\hat{\lambda}_B}{1 + \beta_{S,B} \cdot T} \]

With \( \hat{\lambda}_B \) denoting \( n_B / T \) and \( 0 < \beta_{S,B} \leq \lambda_B \), where \( \beta_{S,B} \) and \( \lambda_B \) are respectively equal to \( \frac{1}{\lambda_B} \sum_{i \in B} \lambda_{i}^2 \) and \( \sum_{i \in B} \lambda_i \).

The first term in equation (4) is the constant failure intensity contribution due to the Mode-A failures. The second term in equation (4) is the expected failure intensity contribution due to Mode-B failures that have occurred, and completed corrective actions at the end of the test, as per distinguished sections. The last term is the unconditional expected failure intensity contribution due to the Mode-B failures that have not been surfaced by time \( T \).

4. Case Study

A reliability-engineering test of NC machines with three testing phases was conducted in sequence. All corrective actions of surfaced B-modes were implemented at the end of the current test phase, prior to starting a follow-on test phase. The current test phase will be referred to as phase I, and the follow-on test phase as phase II. The failure data are shown in table 1, table 2, and table 3 below. The failure data are divided into five sections according to different failure mechanisms: Base, Feed system, Electronic-control system, Hydraulic and Lubrication system, Pneumatic system.

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|c|c|c|}
\hline
Sections & First-time (h) & Mode & Freq. & \( d_i \) & Sections & First-time (h) & Mode & Freq. & \( d_i \) \\
\hline
Electronic Control & 52.6 & B1 & 1 & 0.63 & Electronic Control & 48.0 & A & 1 & --- \\
& 80.3 & A & 1 & --- & & 98.7 & B8 & 1 & 0.59 \\
& 158.3 & B2 & 1 & 0.72 & Lubrication & 113.0 & B9 & 1 & 0.63 \\
& 57.9 & B3 & 1 & 0.77 & & & & & \\
& 73.1 & B4 & 1 & 0.92 & & & & & \\
& 105.9 & B5 & 2 & 0.75 & & & & & \\
& 129.4 & A & 1 & --- & Pneumatic & 100.9 & B12 & 1 & 0.84 \\
& 25.2 & B6 & 1 & 0.86 & & & & & \\
& 102.6 & B7 & 2 & 0.91 & & & & & \\
\hline
\end{tabular}
\caption{Failure data of NC machines in phase I.}
\end{table}

Regarding phase I, the system is tested for \( T_1 = 200 \) hours. As shown in table 1, there is a total number of 18 failures and all corrective actions will be incorporated at the end of the 200-hour test. There are 3 Mode-A failures and 15 Mode-B failures during the test. There are 13 unique Type-B failure modes seen, which means there are 13 distinct corrective actions incorporated into the system at the end of test.

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|c|c|c|}
\hline
Sections & First-time (h) & Mode & Freq. & \( d_i \) & Sections & First-time (h) & Mode & Freq. & \( d_i \) \\
\hline
Electronic Control & 104.5 & A & 1 & --- & Electronic Control & 111.6 & B6 & 1 & 0.69 \\
& 165.4 & B1 & 2 & 0.68 & Hydraulic & 196.5 & A & 1 & --- \\
& 83.1 & B2 & 2 & 0.79 & Lubrication & 339.8 & B7 & 2 & 0.75 \\
& 125.3 & B3 & 1 & 0.82 & & & & & \\
& 384.2 & A & 2 & --- & Pneumatic & 170.6 & B9 & 2 & 0.91 \\
& & & & & & 308.7 & B10 & 2 & 0.82 \\
\hline
\end{tabular}
\caption{Failure data of NC machines in phase II.}
\end{table}
For the data in Phase II, the system is tested for $T_2 = 400$ hours. As shown in table 2, there is a total number of 23 failures and all corrective actions will be incorporated at the end of the 400-hour test. There are 4 Mode-A failures and 19 Mode-B failures during the test. There are 10 unique Type-B failure modes seen, which means there are 10 distinct corrective actions incorporated into the system at the end of test.

**Table 3.** Failure data of NC machines in phase III.

| Sections       | First-time (h) | Mode | Freq. $d_i$ | Sections       | First-time (h) | Mode | Freq. $d_i$ |
|----------------|----------------|------|-------------|----------------|----------------|------|-------------|
| Electronic     | 96.8           | A    | 1           | 149.5          | B7             | 3    | 0.70        |
| Control        | 366.2          | B1   | 2           | 553.1          | B8             | 1    | 0.76        |
|                | 433.6          | B2   | 2           |                |                |      |             |
|                | 86.0           | B3   | 1           |                |                |      |             |
| Feed           | 158.7          | A    | 2           |                |                |      |             |
|                | 300.4          | B4   | 3           | 218.4          | B9             | 3    | 0.82        |
|                | 389.7          | B5   | 2           |                | 298.6          | B10  | 1           | 0.86    |
|                | 187.6          | B6   | 2           |                | 582.3          | B11  | 2           | 0.75    |
| Base           |                |      |             |                |                |      |             |

For the data in Phase III, the system is tested for $T_3 = 800$ hours. As shown in table 3, there is a total number of 25 failures and all corrective actions will be incorporated at the end of the 800-hour test. There are 3 Mode-A failures and 22 Mode-B failures during the test. There are 11 unique Type-B failure modes seen, which means there are 11 distinct corrective actions incorporated into the system at the end of test. The predicted results of three phase data are shown in table 4.

**Table 4.** Calculation result of three phase data.

|                      | Phase I       | Phase II      | Phase III     |
|----------------------|---------------|---------------|---------------|
| $\rho_s (T)$         | 0.064306      | 0.034449      | 0.018313      |
| $\rho_{s,s} (T)$     | 0.063729      | 0.0342        | 0.018353      |
| Observed Value       | 0.09          | 0.0575        | 0.03125       |

The projection performance are plotted in Figure 1. The dashed-dotted lines show the projection value of failure-intensity-function value of APMS projection model. The dashed lines indicate the projection value of the failure-intensity-function value of APMS development-projection model. Finally, the solid lines are the observed value during the test.

**Figure 1.** Projection performance of two phases.

As shown in figure 1, the prediction results of the proposed APMS development model are more accurate than APMS Model, by comparing the predicted intensity rate of Phase I with the observed intensity rate of Phase II, similarly Phase II with Phase III. Furthermore, the observed intensity rate at a certain stage is less than the predicted intensity rate during the test. This is probably because the conservative FEFs are used instead of the realized FEFs, for example, the FEF of mode B8 is only 0.59 in table 1. Additionally, the stein shrinkage factors of the proposed model, which derive from five
partitioned sections, have the similar failure mechanism. That is, the proposed model is easier to understand and more acceptable in engineering than the APMS Model.

5. Conclusions
With the purpose of evaluating systems reliability under development, the paper presents an APMS development-projection model based on partitioned sections by failure data reconstitution. Then systems failure-intensity function is derived on the basis of reorganized data. Based on the study, the following results are obtained:

1) The stein shrinkage factors of partitioned sections constructed in term of similar inherent failure mechanisms, are easier to understand and accept in engineering field. This provides a technical foundation for the wide application of this proposed model.

2) As the stein factors are solved separately, the presented model seems more plausible than the APMS model for reliability prediction of complex systems, which contains many aspects, such as mechanical, hydraulic, pneumatic, electrical, software and integrated systems.

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