The strange quark condensate in the nucleon in 2+1 flavor QCD

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Abstract

We calculate the “strange quark content of the nucleon”, \( \langle N | \bar{s}s | N \rangle \), which is important for interpreting the results of some dark matter detection experiments. The method is to evaluate quark-line disconnected correlations on the MILC lattice ensembles, which include the effects of dynamical light and strange quarks. After continuum and chiral extrapolations, the result is

\[
\langle N | \bar{s}s | N \rangle = 0.69(7)_{\text{statistical}}(9)_{\text{systematic}},
\]

in the modified minimal subtraction scheme (2 GeV), or for the renormalization scheme invariant form,

\[
m_s \frac{\partial M_N}{\partial m_s} = 59(6)(8) \text{ MeV}.
\]

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In addition to its relevance to nuclear structure, the strange quark condensate in the nucleon, $\langle N|\bar{s}s|N \rangle$, is important in understanding experimental searches for dark matter, since some of the leading candidates for dark matter couple most strongly to the nucleon through interactions with the strange quark loops. For example, the importance of this quantity is emphasized in Refs. [1, 2]. The strange and light quark condensates in the nucleon have been calculated through effective field theories of nucleons and mesons [3], and the heavy quark content can be studied perturbatively [4]. Previous lattice studies of the nucleon strange quark content have been done in the quenched approximation [5, 6], with two flavors of Wilson or overlap quarks [7, 8, 9] or an exploratory study with 2+1 flavor stout quarks [10]. A recent study uses 2+1 flavor baryon mass fits [11]. Some, though by no means all, of these studies have suggested that $\langle N|\bar{s}s|N \rangle$ might be much larger than found here.

In this work we use lattices with 2+1 flavors of dynamical quarks (two light, one strange) and nucleon correlators generated from them. These were generated by the MILC collaboration, except for one long ensemble from the UKQCD collaboration. These simulations use a Symanzik improved gauge action and an improved staggered quark action. Details of the action, the ensembles of gauge configurations, and the techniques for computing the nucleon correlators are in Ref. [12]. The simulations cover a range of light quark masses and a range of lattice spacings, which allows us to check the extrapolations to zero lattice spacing and to the physical light quark mass. The lattices have a spatial size of 2.4 fm or larger, with $m_\pi L$ ranging from 3.8 to 6, so that finite size effects will be small. Each ensemble, or set of gauge configurations with a given gauge coupling and quark masses, used here contains from 500 to 4500 equilibrated lattices, with a total of 25784 lattices used in the analysis.

Differentiation of a path integral expression for the nucleon mass with respect to the strange quark mass (the Feynman-Hellman theorem) relates the matrix element $\langle N|\bar{s}s|N \rangle$ to $\frac{\partial M_N}{\partial m_s}$. In particular,

$$\left\langle N \middle| \int d^3 x \bar{s}s \right| N \right\rangle - \left\langle 0 \middle| \int d^3 x \bar{s}s \right| 0 \right\rangle = \left. \frac{\partial M_N}{\partial m_s} \right|_{\alpha, m_l},$$

where the left hand side makes definite what we mean by $\langle N|\bar{s}s|N \rangle$. Note the vacuum subtraction and the integral over space.

Since we expect that the nucleon is made mostly from light quarks, it may seem strange to suppose that $M_N$ depends strongly on the mass of the strange quark, $m_s$. However, to equate $\langle N|\bar{s}s|N \rangle$ with $\frac{\partial M_N}{\partial m_s}$, differentiation of the path integral must be done with all other
parameters in the action held fixed. This change in $m_s$ would cause all dimensionful QCD quantities to change by roughly the same factor, with most of this change interpreted as a change in the physical lattice spacing. For example, if $f_\pi$ were used to determine the lattice spacing, both lattice quantities $aM_N$ and $af_\pi$ might change, with ratio $aM_N/af_\pi$ approximately constant. Thus it is not terribly surprising to find $\frac{\partial M_N}{\partial m_s}$ of order one.

The MILC ensembles contain runs with different values for $m_s$, so it is in principle possible to determine $\frac{\partial M_N}{\partial m_s}$ from a fit to $M_N$ on different ensembles. However, we find that correlations of $\bar{s}s$ with the nucleon correlator give a better signal. (In one case to be discussed later, we do have a useful check from mass fits.)

In the lattice simulations, the nucleon mass $M_N$ is obtained by a fit to the nucleon correlator $P(t) = \langle O_N(0)O'_N(t) \rangle$, where $O_N$ and $O'_N$ are lattice operators with the best practicable overlap with the nucleon. As such, it is just a complicated function of the correlator at different times:

$$M_N = f(P(t_1), P(t_2), P(t_3) \ldots ) .$$  \hspace{1cm} (2)

Using the chain rule to rewrite the derivative:

$$\frac{\partial M_N}{\partial m_s} = \sum_i \frac{\partial M_N}{\partial P(t_i)} \frac{\partial P(t_i)}{\partial m_s} .$$  \hspace{1cm} (3)

The partial derivatives $\frac{\partial P(t_i)}{\partial m_s}$ can be evaluated by using the Feynman-Hellman theorem in reverse to relate them to $\langle P(t_i) \int d^4x \bar{s}s \rangle - \langle P(t_i) \rangle \langle \int d^4x \bar{s}s \rangle$. Then $\frac{\partial M_N}{\partial m_s}$ is evaluated by adding and subtracting a small multiple of $\frac{\partial P(t)}{\partial m_s}$ to the nucleon correlator $P(t)$ and examining the change in the fit result. It may seem that this second use of the Feynman-Hellman theorem has just reversed the original calculation relating $\langle N|\bar{s}s|N \rangle$ to $\frac{\partial M_N}{\partial m_s}$. If the source and sink for the lattice nucleon correlator $P(t)$ created nothing but a normalized nucleon state, this would be the case. But in practice the lattice correlator contains opposite parity particles with almost the same amplitude as the nucleon but with higher mass, and excited states of both parities. The fitting procedure implicit in Eq. (2) is designed to determine a nucleon mass from this complicated correlator, by explicitly including opposite parity (alternating in $t$) contributions and by ignoring the correlator at short separations, so that the excited state contributions are suppressed.

Nucleon correlators have been computed on most of the MILC ensembles. Typically these are averaged over eight Coulomb gauge wall sources in each lattice, so most of the lattice
volume is involved in computing this correlator. Also, the MILC code does a stochastic estimate of \( \int d^4x \bar{s}s \) using a random source (covering the entire lattice) when the lattice is generated or read in for a measurement. Thus, we have \( \bar{s}s \) measurements from several random sources on each lattice, and can compute the correlation between the nucleon correlator and \( \bar{s}s \). (The number of random sources ranged from two to fifteen, depending on the ensemble, with an average of ten.)

We fit the nucleon correlators to a form including the nucleon and an opposite parity state, using distance range \( D_{\text{min}} \) to \( D_{\text{max}} \)

\[
P(t) = Ae^{-M_N t} + A'(-1)^t e^{-M' t}.
\]

Since the fractional statistical errors on the nucleon correlator increase quickly with minimum distance, it is advantageous to use as small a minimum distance in the fits as possible. Since we have a quark-line disconnected correlation function, statistical errors are much larger than in simple hadron mass calculations. Thus, in fitting the perturbed nucleon correlators, we have chosen smaller minimum distances than in our fits to the nucleon masses themselves. In particular, we have chosen \( D_{\text{min}} = 5, 7 \) and \( 10 \) for the \( a = 0.12 \) fm, \( a = 0.09 \) fm and \( a = 0.06 \) fm ensembles respectively, or a consistent physical distance of about \( 0.6 \) fm. Since the nucleon mass, \( M_N \), computed from these same correlators can be determined with a statistical error of order one percent, its dependence on minimum distance can be used to estimate the resulting systematic error. Fits to \( M_N \) with these minimum distances differ by between 1% and 5% from the \( M_N \) fit with larger minimum distances. Alternatively, from looking at the values of \( \frac{\partial M_N}{\partial m_s} \) for various minimum distances, it appears that there could be errors as large as 10% from the choice of fit range. (The choice of \( D_{\text{max}} \) has negligible effects.)

Figure 4 shows the nucleon correlator and its derivative with respect to the strange quark mass for a sample ensemble. The second panel of the figure shows \( \frac{\partial M_N}{\partial m_s} \) (unrenormalized) for three of the \( a \approx 0.09 \) fm ensembles versus the minimum distance included in the fit, while the third panel shows the nucleon mass itself as a function of \( D_{\text{min}} \).

Since the quantity we are computing is a complicated, and implicitly defined, function of the averages measured on the lattice, we use a jackknife analysis to estimate statistical errors. Since consecutive lattices are correlated, we eliminated blocks of ten consecutive lattices, or 50 to 60 simulation time units, in the jackknife analysis. Using larger blocks made only a small difference.
FIG. 1: The nucleon correlator and the derivative of this correlator with respect to $m_s$ for the ensemble with $am_l = 0.0093$ and $am_s = 0.031$ (first panel). For the derivative, the squares are points where the derivative is negative, and crosses are points where it is positive. The vertical lines show the range used in fitting the correlator. The second panel shows $\partial M_N/\partial m_s$ for three ensembles with $a \approx 0.9$fm as a function of the minimum distance used in the fitting, and the third panel shows the fitted nucleon mass itself versus $D_{min}$. The error bars labelled “10%” in the second and third panels show the size of the ten percent systematic error estimate from excited state contamination.

Since the strange quark mass is renormalization scheme and scale dependent, so is the derivative $\partial M_N/\partial m_s$. For a useful result, we wish to express our answer in a renormalization scheme useful for computations of cross sections. The relation between the strange quark mass in the Asqtad regularization and in the $\overline{\text{MS}}$ scheme is known to two loop order in perturbation theory \cite{13}.

$$\frac{\partial M_N}{\partial m_s(\overline{\text{MS}}, 2 \text{ GeV})} = \frac{u_0}{Z_m} \frac{\partial M_N}{\partial m_s(\text{Asqtad}, 1/a)}$$  \(5\)

where the factor of $u_0$ converts the lattice definition of the quark mass used here to the definition used in Ref. \cite{13}, and $Z_m$ can be found in Ref. \cite{13}. Since in the subsequent steps in this analysis we will be combining results at different lattice spacings $a$, it is most consistent to make the conversion to the $\overline{\text{MS}}(2 \text{ GeV})$ scheme before making chiral and continuum extrapolations.

The strange quark masses, $m_s$, used in the MILC simulations were of necessity estimated before the simulations were done, and the correct strange quark masses were only known after the pseudoscalar masses were analyzed. These differ significantly from the $m_s$ used in
the simulations. For lattice spacings 0.12 fm, 0.09 fm and 0.06 fm the values of \(am_s\) used in most of the simulations were \(am_s = 0.050, 0.031\) and 0.018, while the corrected values are 0.036, 0.026 and 0.019, respectively. (A few ensembles were run with a lighter strange quark mass 0.6 times the above mass.) To adjust to the correct \(m_s\), we use the fact that light quark \(\bar{\psi}\psi\) was also evaluated on all of these lattices. In some of the largest ensembles the actual light quark mass used was still fairly large — 0.2, 0.4 or even 0.6 times the simulation strange quark mass, outside the chiral regime and with qualitative behavior similar to heavy quarks. For example, one of the ensembles with \(a \approx 0.12\) fm was generated with light and strange quark masses \(am_l = 0.03\) and \(am_s = 0.05\), where the correct strange quark mass determined later was about 0.036. On these ensembles we use the difference between \(\frac{\partial M_N}{\partial m_s}\) and \(\frac{\partial M_N}{\partial m_l}\) (\(m_l\) is the light quark mass used in the simulation) to calculate the derivative of \(\frac{\partial M_N}{\partial m_s}\) with respect to \(m_s\), and use this to adjust the results to the correct strange quark mass. Since \(\frac{\partial M_N}{\partial m_s}\) and \(\frac{\partial M_N}{\partial m_l}\) are measured on the same lattices and with the same nucleon propagators, albeit with different random sources, they are highly correlated and the error on their difference is greatly reduced. With the additional assumption that this slope in physical units is the same for all ensembles, a correction factor can be estimated. In particular, using five long ensembles with \(m_l \geq 0.2m_s\), we find \(\frac{\partial}{\partial r_1m_s}\left(\frac{\partial M_N}{\partial m_s}\right) = -2.2(3)\). Here \(r_1\) is a hadronic length scale determined from the heavy quark potential, and is approximately 0.31 fm\([12, 14, 15]\).

Figure 2 shows \(\frac{\partial M_N}{\partial m_s}\) on all of the ensembles used, where the results have been adjusted to the correct strange quark mass, and the quark mass converted to the \(\overline{\text{MS}}(2\text{ GeV})\) regularization. In this plot we see that the best results are in the \(a = 0.12\) fm ensembles, mainly because of the larger numbers of lattices.

Finally, it is necessary to extrapolate the result to the physical light quark mass and to the continuum \((a = 0)\) limit. To do this, we fit the results to the form\([16]\)

\[
\frac{\partial M_N}{\partial m_s} = A + Bm_lr_1 + C(a/r_1)^2
\]

(6)

Since the results from the \(a = 0.06\) and 0.09 fm ensembles have much larger statistical errors than the 0.12 fm results, the term linear in \(a^2\) is very poorly determined. However, we can use experience with other quantities to estimate the likely size of lattice corrections. In particular, the masses of the \(\rho\), nucleon, and \(\Omega^-\) at \(a = 0.12\) fm differ by about 4%, 10% and 9% respectively from their continuum extrapolation. Therefore we constrain \(C\) to be small by using a (Gaussian) Bayesian prior with a one standard deviation width corresponding to
FIG. 2: The derivative $\frac{\partial M_N}{\partial m_s}$ on the various ensembles. As discussed above, the data have been adjusted to the correct strange quark mass, and the quark mass converted to the $\overline{\text{MS}}(2\,\text{GeV})$ regularization. In the horizontal axis, $r_1$ is a hadronic length scale, approximately 0.31 fm. In these plots the symbol size is proportional to the number of lattices in the ensemble, with the largest symbol corresponding to about 4500 lattices. In each panel, the cross at $m_l r_1 \approx 0.05$ ($m_l \approx 0.4 m_s$) also shows the value of the nearby point before adjusting the strange quark mass. The line on each panel is the continuum and chiral fit in Eq. 6 evaluated at the corresponding lattice spacing, and the error bar at the left is the error on the combined fit to all the data.

a 10% effect at $a = 0.12$ fm. This gives $\frac{\partial M_N}{\partial m_s} = 0.69 \pm 0.07$\text{statistical} in the continuum limit, with $\chi^2/D = 17.0/17$.

There are also a number of systematic errors. As discussed above, we include a 10% systematic error for the effects of excited states in the nucleon correlator. The extrapolation to the physical light quark mass contains higher order terms in chiral perturbation theory than the linear form used here. To estimate the likely size of these terms, we note that if the nucleon mass over this range of quark masses is fit to constant plus linear, the result at the physical point is seven percent different from the result including two more orders in the pion mass. We therefore take seven percent as an estimate of the effect of higher order terms in chiral perturbation theory. In one case where we have two spatial volumes, the nucleon mass on the volume used here was different by about one percent from the mass in the larger volume. It is possible that disconnected contributions are more sensitive to the volume, so we take three percent as an estimate of this systematic error. Finally, Ref. \[13\] estimates an
error of four percent in \( Z_m \). The combined systematic error estimate from excited states, finite volume, higher order \( \chi PT \) and \( Z_m \) is 0.09.

Evaluating the fit in the continuum limit at the physical light quark mass, we find \( \frac{\partial M_N}{\partial m_s} = 0.69 \pm 0.07 \)\(^\text{statistical} \pm 0.09 \)\(^\text{systematic} \), where \( m_s \) is in the \( \overline{\text{MS}} \) regularization at 2 GeV. It is also common to quote the renormalization scheme invariant quantity \( m_s \frac{\partial M_N}{\partial m_s} \). Using a similar chiral and continuum fit to the one used for \( \frac{\partial M_N}{\partial m_s} \), we find \( m_s \frac{\partial M_N}{\partial m_s} = 59(6)(8) \) MeV. The systematic error here does not include error in \( Z_m \), which cancels, but does include a lattice systematics error of almost the same amount, coming from uncertainty in the lattice strange quark mass and an overall two percent error in scale setting.

In general the MILC ensembles were run at different lattice coupling for each quark mass, which makes it complicated to extract \( \frac{\partial M_N}{\partial m_q} \) from fits to the table of nucleon masses. However, in one case there is an accidental check. Through an error, two ensembles were run with the same coupling constant \( 10/g^2 \) and tadpole factor \( u_0 \). These ensembles had sea quark masses \( m_l/m_s = 0.0062/0.0186 \) and \( 0.0093/0.031 \) respectively. By computing a partially quenched nucleon mass on the latter ensemble and examining its difference from the nucleon mass on the former, we can make a check on a particular combination of \( \frac{\partial M_N}{\partial m_s} \) and \( \frac{\partial M_N}{\partial m_l} \), \( (0.031 - 0.0186)\frac{\partial M_N}{\partial m_s} + 2(0.0093 - 0.0062)\frac{\partial M_N}{\partial m_l} \). Here \( \frac{\partial M_N}{\partial m_q} \) is evaluated at the midpoint of the sea quark masses on these two ensembles, and the factor of two comes from the two light flavors. These nucleon masses are computed in the usual way by a fit to the nucleon correlator. The resulting difference in masses was \( 0.016(3)_{\text{stat}}(2)_{\text{fit range}} = 0.016(4) \). The fit to \( \frac{\partial M_N}{\partial m_s} \) above, converted back into lattice units, together with a similar fit to \( \frac{\partial M_N}{\partial m_l} \), gives \( 0.020(3) \), in reasonable agreement.

Our result for \( \langle N|\bar{s}s|N \rangle \) is smaller than the results of the quenched calculations in Refs. \[3, 6\]. However, our result is reasonably consistent with the small value of \( y \) recently found in the two flavor overlap calculation in Ref. \[9\], where combining their result \( y < 0.05 \) with the value of \( \sigma_{\pi N} = 53 \) MeV found in the same fit gives \( m_s < \bar{s}s >= 36 \) MeV. Similarly, our result is marginally consistent with the result from fits to baryon masses in Ref. \[11\], who find \( m_s < N|\bar{s}s|N >= 31(15) \) MeV, although there may be differences in how the derivative with respect to \( m_s \) is taken.

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