PB TMD fits at NLO with dynamical resolution scale

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Outline

- Recap of Parton Branching method
- Fixed and Dynamical soft-gluon resolution scale $z_M$
- Fits with fixed $z_M$ at NLO
- Fits with dynamical $z_M$ at NLO

Merged talk:
- Z+b jet production in 4FL and 5FL
Recap of PB TMDs

TMD evolution in the PB formalism:

\[
\widetilde{A}_a(x, k_\perp, \mu^2) = \widetilde{A}_a(x, k_\perp, \mu^2) \Delta_a(\mu^2) + \sum_b \int \frac{d^2 \mu_\perp'}{\pi \mu'^2} \Theta(\mu^2 - \mu'^2) \Theta(\mu'^2 - \mu_0^2) \frac{\Delta_a(\mu'^2)}{\Delta_a(\mu^2)} \int_0^{z_M} dz P_{ab}^R(z, \alpha_s(q_\perp)) \times \widetilde{A}_b \left( \frac{x}{z}, k_\perp + (1 - z)\mu', \mu'^2 \right)
\]

- **\( z_M \): Resolution scale**:
  - Resolvable branching: \( z < z_M \)
  - Non-resolvable branching: \( z > z_M \)

- **Splitting functions: \( P_{ab}^R(z) \):** The real emission parts of the DGLAP splitting function:
  - Probability that a branching will happen

- **Sudakov form factor: \( \Delta_a = \exp(-\int_{\ln \mu_0^2}^{\ln \mu^2} d \ln \mu'^2) \sum_b \int_0^{z_M} dz z P_{ba}^R(\alpha_s, z) \)**
  - The probability of an evolution without any resolvable branching

[Hautmann et al., JHEP 01 (2018) 070, 1708.03279]
Recap of PB TMDs

Iterative form of the PB evolution equation:

\[
\widetilde{A}_a(x, k_\perp, \mu^2) = \widetilde{A}_a(x, k_\perp, \mu_0^2) \Delta_a(\mu^2) + \sum_b \int_{ln\mu_0^2}^{ln\mu^2} d\ln \mu_1^2 \frac{\Delta_a(\mu^2)}{\Delta_a(\mu_1^2)} \int_x^{z_M} dz \ P_{ab}^R(z, \alpha_s(k_\perp)) \Delta_b(\mu_1^2) \times \widetilde{A}_b \left( \frac{x}{z}, k_\perp + (1 - z)\mu_1, \mu_0^2 \right) + \ldots
\]

Solvable by MC iterative technique:

- generated \( \mu_1^2 \): if \( \mu_1^2 > \mu^2 \) stop, otherwise splitting,
- generated the next scale \( \mu_2^2 \): if \( \mu_2^2 > \mu^2 \) stop, otherwise splitting,
- \ldots
Angular Ordering:

**Color coherence phenomena:**

- Angular ordering of the soft gluon emissions
  \[
  \Theta_{i+1} > \Theta_i \\
  |q_{\perp,i}| = (1 - z_i) |E_i| \sin \Theta_i
  \]
  Associating \(|E_i| \sin \Theta_i\) with \(\mu'\)
  \[
  q_{\perp,i}^2 = (1 - z_i)^2 \mu_i'^2
  \]
- The **argument of \(\alpha_s\)** should be \(q_{\perp}^2\)
  \[
  \alpha_s(q_{\perp}^2) = \alpha_s((1 - z)^2 \mu'^2)
  \]
- resolvable & non-resolvable \(\rightarrow\) condition on \(\min q_{\perp,i}^2 \rightarrow z_M\)
  \[
  z_M = 1 - \left(\frac{q_0}{\mu'}\right)
  \]
Fixed and dynamical resolution scale

- **Fixed** $z_M$:  
  - $\mu$ independent  
  - $z_M = 1 - \epsilon$  
    where $\epsilon$ is small: $10^{-3}$, $10^{-4}$, $10^{-5}$,...

- **Dynamical Resolution scale in Angular Ordering:**  
  - $z_M = 1 - \left( \frac{q_0}{\mu'} \right)$  
    where $q_0$ is smallest emitted transverse momentum for resolvable partons

  - Sudakov form factor $\Delta_\alpha$: non-resolvable region
  - Splitting functions $P_{ab}^R$: resolvable region

[Hautmann, Keersmaekers, Lelek, van Kampen NuclPhysB (2019) 114795,1908.08524]
Dynamical resolution scale

The Condition on $q_0$ of

$$z_M = 1 - \left( \frac{q_0}{\mu'} \right)$$

- Scale of strong coupling:
  $$\alpha_s(q_{\perp}^2) = \alpha_s((1 - z)^2 \mu'^2)$$

- Lowest scale in $\alpha_s$ corresponds to minimal $q_{\perp}$

- $q_{\perp,\text{min}} = q_0 \& q_0 > \Lambda_{\text{QCD}} \Rightarrow$ we stay in the weak coupling region!
The Past PB TMD fits at NLO calculation using angular ordering : fixed $z_M$

"NLO DIS Matrix Element (ME) and NLO evolution kernel"

- Associating the evolution scale with some physical interpretation:
  - Set 1: $\alpha_s(\mu'^2)$
  - Set 2: $\alpha_s(q_{1\perp}^2) = \alpha_s((1 - z)^2 \mu'^2)$

- The resulting TMD parton densities, PB-NLO-2018-set1 and PB-NLO-2018-set2 are available in TMDLIB2: *The European Physical Journal C* 81.8 (2021): 1-10

- Data set: HERA 1+2 inclusive DIS data

\[
\begin{array}{c|c|c|c}
\text{PB NLO Set 1} & \alpha_s(\mu^2) & \chi^2 & \text{d.o.f} \\
\hline
\mu^2_0 = 1.9 \text{ GeV}^2 & 1363.37 & 1131 & 1.21 \\
\end{array}
\]

\[
\begin{array}{c|c|c|c}
\text{PB NLO Set 2} & \alpha_s(q_{1\perp}^2) & \chi^2 & \text{d.o.f} \\
\hline
\mu^2_0 = 1.4 \text{ GeV}^2 & 1369.80 & 1131 & 1.21 \\
\end{array}
\]

Measurement of the inclusive DIS cross section obtained at HERA compared to predictions using Set 1 and Set 2
PB TMD fits at NLO with *dynamical* $z_{\text{max}}$

**New study**

From *fixed* resolution scale to *dynamical* resolution scale
PB TMD fits at NLO with dynamical zmax: $z_M = 1 - \left(\frac{q_0}{\mu^2}\right)$

New fits with dynamical zmax at LO and NLO with HERA 1 + 2 Data set: Using JFitter arXiv:1709.01151v1

✓ Performing different fits, each time by varying $Q_{\text{min}}^2$ and on top of that with different $q_0$ values

- At LO, for small $Q_{\text{min}}^2$ and $0.9 \text{ GeV} < q_0 < 1.2 \text{ GeV} \quad \rightarrow \quad 2.2 < \frac{\chi^2}{\text{dof}} < 3$
- AT NLO, for small $Q_{\text{min}}^2$ and all values of $q_0$, we have better fits with good $\frac{\chi^2}{\text{dof}}$!
The difference between LO and NLO

- Does the difference between LO and NLO come from the kernels? or ME?!..

For $q_0 = 1.0$ GeV

4 states for this purpose:
1. Fitting with NLO kernel & NLO ME
2. Fitting with NLO kernel & LO ME
3. Fitting with LO kernel & LO ME
4. Fitting with LO kernel & NLO ME

The difference is dominated by the kernel not ME!..!
The difference between LO and NLO

- Which part of the kernel is responsible?

\[ P_{ab}(z, \mu^2) \text{? or } \alpha_s? \]

4 states for this purpose:
1. Fitting with NLO \( P_{ab} \) & NLO \( \alpha_s \)
2. Fitting with NLO \( P_{ab} \) & LO \( \alpha_s \)
3. Fitting with LO \( P_{ab} \) & LO \( \alpha_s \)
4. Fitting with LO \( P_{ab} \) & NLO \( \alpha_s \)

The difference is dominated by the splitting functions not \( \alpha_s \)!!
Which part of the splitting functions is responsible for the difference between LO and NLO?

- For high values of $q_0$ (e.g., [1.0 Gev, 1.2 Gev]) or low values of $z_M = 1 - \left(\frac{q_0}{\mu'}\right)$, LO and NLO have different behavior.

  The first piece for checking is $\frac{1}{z}$

  - In the NLO, all the splitting functions have pieces with $(1/z)$ term:
    
    $P_{ab}(z, \mu^2) \sim P_{qq}(1/z, \mu^2), P_{qg}(1/z, \mu^2), P_{gg}(1/z, \mu^2), P_{gq}(1/z, \mu^2)$

  - In the LO, just the splitting functions with “gluon” in the final state have $(1/z)$ piece:
    
    $P_{gg}(z, \mu^2) = \frac{1}{1-z} + \frac{1}{z} - 2 + z(1 - z)$,
    
    $P_{gq}(z, \mu^2) = \frac{1+(1-z)^2}{z}$

  - And the splitting functions with “quark” in the final state don’t have $(1/z)$ piece:
    
    $P_{qq}(z, \mu^2) = \frac{2}{1-z} - 1 - z$,
    
    $P_{qg}(z, \mu^2) = z^2 + (1 - z)^2$

- Is the lack of $(1/z)$ piece in LO splitting function with quark in the final state responsible for this difference?

  Let’s check it!
Does the difference come from 1/z piece of NLO splitting function?

For better understanding: “We added to the LO splitting functions($P_{qq}, P_{qg}$) the 1/z pieces of NLO”

✓ $P_{qq} (z, \mu^2) = \frac{2}{1-z} - 1 - z + \left(\frac{1}{z}\right)$ pieces of $P_{qq}$ NLO

✓ $P_{qg} (z, \mu^2) = z^2 + (1 - z)^2 + \left(\frac{1}{z}\right)$ pieces of $P_{qg}$ NLO

For $q_0=1.0$ GeV

✓ In NLO we have an extra (1/z) pieces in the quark channels compared with LO which is responsible for this difference!

✓ With this piece we are describing data well! Amount of $\chi^2/dof$ is reasonably good!

** For PB-TMD fit with dynamical zmax we obtain a reasonably good $\chi^2/dof$ at NLO! **
How does dynamical $z_{\text{max}}$ affect the fitted TMD (iTMD)?

**Set 2: fixed $z_{\text{max}}$ & $\alpha_s(q_T^2) = \alpha_s((1 - z)^2 \mu^2)$**

- The dynamical $z_{\text{max}}$ fit implies an effect not only in the $k_T$ dependence but also in the $x$ dependence!
The predictions in dynamical zmax frame

Predictions with ME generated by MCatNLO combined with obtained TMDs.
The merged talk: Z+b jet production in 4FL and 5FL

5 FLNS
- Full coupled evolution with all flavours & $\alpha(M_Z^{nf=5}) = 0.118$
- HERAPDF parametrization form
- Using full HERAI+II inclusive DIS data
- $\chi^2/\text{dof} = 1.21$

[Phys. Rev. D 99 (2019) no. 7, 074008]

4 FLNS
- The same functional form and data as 5FL-parameters re-fitted
- $M_b \rightarrow \infty$ & $\alpha(M_Z^{nf=4}) = 0.1128$
- $\chi^2/\text{dof} = 1.25$

[arXiv:2106.09791]

Matrix elements from MC@NLO (HERWIG6 subtraction)
- 5FLVNS: Z + one parton process
- 4FLVNS: Z + bb process

PDFs: TMDs (4FL & 5FL)
- 5FLVNS: b-quark is treated as a light quark
- 4FLVNS: no b-quark in the parton density

Parton shower following TMDs for intial state
- 5FL & 4FL PB-TMDs included in the Cascade3

Eur. Phys. J. C 81 (2021) 425
Differential cross section for $Z + b\bar{b}$ as a function of $p_t(Z)$ as measured by CMS collaboration

the full prediction + the result of using only the LHE files are shown.

[18]

Eur. Phys. J. C 77 (2017) 751
$Z + b\bar{b}$ as a function of $\Delta \phi$ ($b\bar{b}$)

Differential cross section for $Z + b\bar{b}$ as a function of $\Delta \phi$ ($b\bar{b}$) as measured by CMS collaboration.

5 FLNS

4 FLNS

4FL : both $b$ partons are already produced at the ME level
5FL : $b\bar{b}$ must be simulated in the parton shower.
Breakdown of the different contributions to quantify their roles

4FL: weakly depends on PB-TMD/parton shower
5FL: significant contribution from TMD parton shower
Summary

- PB TMD fits at NLO with dynamical zmax for the first time!
- For PB-TMD fit to HERA data with dynamical zmax, we obtain a reasonably good $\frac{\chi^2}{dof}$ at NLO!
- The difference between LO and NLO fits is mostly due to (1/z) pieces in quark channel in NLO splitting functions!
- The dynamical zmax impacts both the $k_T$ dependence and the x dependence of the fitted parton distribution!
- The next step: Using the PB TMD with dynamical zmax in phenomenology of LHC and lower energy colliders!
- The 4FL and 5FL PB-TMD distributions used to calculate Z + bb production
  - Good agreement with measurements obtained by the CMS collaboration
  - The evolution of the PB-TMD parton densities as well as in the PB-TMD parton shower is checked.

Thank you …
BACK UP ...
Back up...
PB TMD fits at NLO with dynamical $z_{\text{max}}$:

NLO fits with dynamical $z_{\text{max}}$

$Q^2_{\text{min}} = 2, 3.5, \ldots, 100 \text{ GeV}^2$