Quantitative Investigations on the Dimensional Stability of a CFRP Reflector Model against Temperature Variations*

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Carbon fiber reinforced plastic (CFRP) exhibits high specific elasticity and a low coefficient of thermal expansion. Therefore, CFRP is considered to be a suitable material for fabricating reflectors for high-precision, space-based astronomical observation systems. However, non-negligible out-of-plane thermal deformation on CFRP reflectors may occur due to errors in the fiber orientation of each layer. In addition, out-of-plane thermal deformation can be generated because of non-uniform temperature conditions even when no fiber orientation error exists. In this study, the out-of-plane thermal deformation of a CFRP reflector model due to normally distributed fiber orientation errors was probabilistically examined, and methods to mitigate the effect of fiber orientation errors were investigated. The Monte Carlo method was used for analysis. The results demonstrated that increasing the number of layers using thin prepregs was found to be an effective method to mitigate not only the effect of fiber orientation errors, but also out-of-plane thermal deformation modes caused by non-uniform temperature distributions. Furthermore, it was found that the stacking sequence should be appropriately determined to reduce the effect of fiber orientation errors.

Key Words: Reflector, Composite Materials, Thermal Deformation

1. Introduction

High-resolution astronomical observation systems are currently needed to achieve further advances in space science. These observation systems require highly precise, large reflectors (mirrors). In general, astronomical observations are performed in space to avoid the effect of the Earth’s atmosphere on the observations. Accordingly, observation systems are mounted on satellites and launched by rockets to perform in-space observations. Reducing the payload is of great importance when launching rockets, and even the weight of large observation systems needs to be reduced as much as possible. Reflectors account for a large proportion of the weight of observation systems; therefore, reducing the weight of the reflectors is important. In addition, observation systems are exposed to severe temperature variations in space; therefore, thermal deformation of the reflectors is an issue. Moreover, shape irregularity occurs because the gravity difference between the ground and space is non-negligible. Therefore, the materials of space-based reflectors need to have both large specific elasticity and a low coefficient of thermal expansion. Carbon fiber reinforced plastic (CFRP) exhibits the aforementioned properties and is therefore being considered as a next-generation material for space-based reflectors.

CFRP has already been utilized as the material of a 7.2-m-diameter ground-based optical reflector used for scientific research. Ozaki et al. developed high-precision CFRP pipes that were used for the optical telescope structure of the solar observation satellite Hinode. As work towards the realization of space-based CFRP reflectors, the effects of moisture absorption and microcracks in CFRP on its dimensional stability were investigated by Abusafieh et al., and they concluded that CFRP can be used for high-precision structures that require micron-orders of dimensional accuracy. Subsequently, Arao et al. assessed the long-term deformation of CFRP, including creep deformation, moisture absorption, self-shrinking, and residual stress relaxation. Yoon et al. studied the effect of thermal deformation and outgassing deformation of the composite structure of a telescope on its optical performance, and it was concluded that the thermal deformation degraded the optical performance of the telescope significantly more than outgassing deformation. This result indicated the necessity to research the thermal deformation of CFRP reflectors.

As previously mentioned, observation systems are exposed to severe temperature variations in space. Accordingly, the thermal deformation of CFRP needs to be investigated to discuss the feasibility of space-based CFRP reflectors. In a previous study, thermal deformation of a reflector model comprising a CFRP laminate was observed, and the extent of the out-of-plane thermal deformation was found to be non-negligible. In addition, a finite-element analysis was performed in a previous study, and the error in the fiber orientation angle when laminating prepregs was found to be a significant cause of the out-of-plane thermal deformation of CFRP reflectors. The authors of the previous study also investigated the suppression of a representative mode of the out-of-plane thermal deformation of a CFRP reflector model due to fiber orientation errors by applying ex-
ternal forces with linear actuators. However, the essence of the out-of-plane thermal deformation of CFRP reflectors due to fiber orientation errors, such as the magnitude of deformation and frequency of mode occurrence, was not discussed.

Uncertainties of the material properties of CFRP have been studied by means of probabilistic approaches since the 1980s. Chamis assessed the effect of uncertainties of CFRP material properties as a part of designing components of the space shuttle main engine. Mase et al. simulated the effect of fiber orientation errors on the material properties of a quasi-isotropic CFRP laminate using the Monte Carlo method. The Monte Carlo method was also adopted by Arao et al. to simulate hygrothermal deformation due to fiber orientation errors of a cross-ply CFRP laminate. In this study, the effect of normally distributed fiber orientation errors on the out-of-plane thermal deformation of a CFRP reflector model was quantitatively investigated using the Monte Carlo method, doing so in the same way as the previous studies mentioned above, to further understand their characteristics. In addition, the relation between the configurations of the CFRP, the number of layers and the stacking sequence, and the magnitude of the thermal deformation were discussed to find a method to mitigate the effect of fiber orientation errors. Further, thermal deformations due to non-uniform temperature distributions were assessed.

2. Thermal Deformation of the CFRP Reflector Model

In a previous study, the authors observed the out-of-plane thermal deformation of a reflector model composed of quasi-isotropic CFRP laminate (XN60/NM31, Sanki Composite Co.). The stacking sequence was [0/−45/90/45]s. The appearance and dimensions of the reflector model are described in Fig. 1. The reflector model had a φ30-mm jig fixed at the center of the bottom surface. The reflector model was placed inside a thermostatic chamber with a heat-resistance glass on the top surface, and the shape of the reflector model was observed using a laser displacement sensor before and after heating the reflector model. The center of the reflector model was attached to a jig by a screw. The out-of-plane thermal deformation of the CFRP reflector model was obtained by comparing the shape of the reflector model observed before heating with that observed after heating. The observed out-of-plane thermal deformation when the reflector model was heated approximately 40 K is shown in Fig. 2. The out-of-plane deformation was saddle-shaped and the maximum displacement was approximately 0.02 mm, which means the maximum displacement was approximately 0.5 μm/K. This deformation was unexpected because the stacking sequence was symmetric.

In another previous study, the thermal deformation of a CFRP reflector model under the same conditions as the experiment was simulated via finite-element analysis to find the cause of this unexpected out-of-plane thermal deformation. The result indicated that a mode of the out-of-plane thermal deformation similar to that observed in the experiment occurred when a fiber orientation error existed.

The same result was obtained using a different analysis model from that used in the previous research. The details of the thermal deformation analysis of the CFRP reflector model with a fiber orientation error are described below.

The engineering simulation software “ANSYS Mechanical APDL” was used for the analysis. An analysis model with the same dimensions as the CFRP reflector model used in the experiment, as shown in Fig. 1, was constructed. The analysis model comprised 4-noded shell elements (SHELL181), and the number of nodes and elements were 5580 and 5400, respectively. The thermal expansion of the reflector model in the thickness direction cannot be calculated with the shell elements. However, in this analysis, as with the previous study by Arao et al. in which the thermal deformation of CFRP plates were calculated based on the lamination theory, the displacement of the neutral plane...
of the reflector model in the out-of-plane direction was evaluated and the effect of the thermal expansion in the thickness direction was ignored. Accordingly, the shell elements were chosen. The global coordinate system was oriented so that the $Z$-axis corresponded to the normal of the surface on the center of the reflector model directed toward the center of the curvature. The $X$-axis corresponded to the $0^\circ$-direction of the lamination on the plane normal to the $Z$-axis. The nodes on the right, left, and upper edges of the $30$-mm hole in the center of the analysis model, which corresponded to the $30$-mm jig attaching the reflector model shown in Fig. 1, were fixed in the $X$-, $Y$-, and $Z$-directions, $Y$- and $Z$-directions, and $Z$-direction, respectively, to prevent rigid-body motion. Each CFRP layer was assumed to be a transversely isotropic material, and its material constants are listed in Table 1. These values, except $\nu_{TT}$, were obtained via tensile tests and theromechanical analysis of a specimen composed of the same material as the reflector model. The curve of the fiber orientation of each layer when its prepreg was laminated along the curvature of the reflector model was modeled by applying an individual stacking sequence on each element. The thickness of each of the eight layers was assumed to be $0.1075$ mm. In the analysis, a temperature variation of $+1$ K was applied for all of the nodes.

The calculated $Z$-directional displacement of the reflector model with a stacking sequence $[0/\pm45/90/45]$ is shown in Fig. 3. The reflector model appeared to warp with its axis in the direction of approximately $-20^\circ$ to the $X$-axis; however, the maximum displacement was $0.043$ $\mu$m/K, which is less than $1/10$ of that observed in the experiment. The mode of displacement was completely different. This result indicates that the cause of the out-of-plane thermal deformation observed in the experiment was not the curve of the fiber resulting from the curvature of the reflector model.

When fabricating CFRP by laminating prepregs, errors in the fiber orientation angle resulting from manufacturing errors are inevitable. Studies on the hygrothermal deformation of CFRP cross-ply plates by Arao et al. indicate that a fiber orientation error of $0.4^\circ$ in standard deviation is inevitable when prepregs are laminated by an expert. As an example, the thermal deformation of the CFRP reflector model calculated when a fiber orientation error of $1^\circ$ was applied on the uppermost surface of the laminate (i.e., the stacking sequence was assumed to be $[1/(\pm45/90/45)_s/0]$) is shown in Fig. 4. The positive and negative $Z$-directional displacements were distributed alternatingly along the edge of the reflector model. Although the mode of the positive and negative displacements was reversed, the distribution of the displacement was saddle-shaped and corresponded to that observed in the experiment. The maximum displacements in the positive and negative directions of $Z$-axis were $0.939$ $\mu$m/K and $-0.964$ $\mu$m/K, respectively. These displacements were twice larger than those of the experimental result; therefore, this result qualitatively corresponds to the experimental result.

This indicates that fiber orientation errors in the lamination can cause unexpected out-of-plane thermal deformation on CFRP reflectors. To construct highly precise space-based CFRP reflectors that need to exhibit high dimensional stability against temperature variations, the thermal deformation caused by fiber orientation errors needs to be suppressed.

### 3. Probabilistic Analysis of the Effect of Normally Distributed Fiber Orientation Errors

As mentioned above, fiber orientation errors are inevitable when laminating prepregs to fabricate CFRP reflectors. Therefore, the details of the thermal deformation of CFRP reflectors due to the existence of fiber orientation errors should be discussed. Accordingly, the out-of-plane thermal deformation of the same reflector model discussed in the previous section, caused by the existence of normally distributed fiber orientations,
orientation errors was probabilistically investigated using the Monte Carlo method.\textsuperscript{17}

In this analysis, the same analysis model as described in the last section was used. A random number was added as the fiber orientation error to the original fiber orientation angle of each layer. In accordance with the assessment by Arao et al.,\textsuperscript{15} in the analysis, the fiber orientation error was assumed to follow a normal distribution with a standard deviation of 0.4°. The number of stacking sequence sets was 10000; that is, 80000 random numbers were generated for the reflector model with eight layers. Next, the thermal deformations of the CFRP reflector model with each stacking sequence were calculated. As a parameter of the extent of out-of-plane thermal deformation, the root mean square (RMS) of dimensional error from the ideal shape of the reflector model was used. This parameter is important to assess the performance of reflectors. When the RMS exceeds 1/13 of the wavelength of target electromagnetic wave, reflector performance is significantly degraded.\textsuperscript{16} When this parameter approaches zero, reflector model dimensions approach the ideal shape. To calculate this parameter, the reflector model surface was divided into small domains so that each domain had only one node. Assuming that the dimensional error in each domain is equal to the dimensional error of the node included, the dimensional error RMS can be calculated using the dimensional error of the \(i\)-th node, \(\zeta_i\), and the area of the domain in which the \(i\)-th node exists, \(S_i\):

\[
\text{RMS} = \sqrt{\frac{\sum_{i} \zeta_i^2 S_i}{\sum_{i} S_i}}. \tag{1}
\]

The dimensional error RMS of the thermal deformation of the reflector model without any fiber orientation error, shown in Fig. 3, was 0.016 \(\mu\text{m}/\text{K}\). That of the reflector model with the fiber orientation error, shown in Fig. 4, was 0.395 \(\mu\text{m}/\text{K}\). On the other hand, that of the experimental result of the previous study was approximately 0.2 \(\mu\text{m}/\text{K}\). A histogram of the frequency of the dimensional error RMS for the CFRP reflector model with the original stacking sequence \([0/-45/90/45]_s\), is shown in Fig. 5. The mean RMS was 0.223 \(\mu\text{m}/\text{K}\). This result indicates that 90\% of the RMS was smaller than 0.42 \(\mu\text{m}/\text{K}\), and 99\% was smaller than 0.64 \(\mu\text{m}/\text{K}\). As described above, the RMS of the experimentally observed result was approximately 0.2 \(\mu\text{m}/\text{K}\). Therefore, the extent of the out-of-plane thermal deformation observed was coincidentally near the mean.

The angle between the direction of the node where the positive maximum Z-directional displacement was generated and the X-axis at the center of the reflector model is defined as the “maximum displacement direction,” which denotes the Z-directional displacement mode. The relation between the dimensional error RMS and the maximum displacement direction is shown in Fig. 6. The horizontal line indicates the mean RMS. This figure indicates that the magnitude of the RMS differs with respect to the maximum displacement direction. Relatively large thermal deformation occurs frequently when the maximum displacement direction is near ±45° and occurs less frequently when it is near 0° or ±90°. However, even if the maximum displacement direction is near 0° or ±90°, some points are still found above the line of the mean RMS. Therefore, an out-of-plane thermal deformation with an average extent can occur regardless of the displacement mode. As an example, Fig. 7 shows the distribution of the Z-directional displacement of the thermal deformation with its RMS near the mean. The stacking sequence was \([-0.437/44.676/90.222/44.925/45.256/89.946/45.551/-0.181]\). The maximum displacement direction was −24°, and the positive and negative maximum displacements were 0.494 \(\mu\text{m}/\text{K}\) and −0.562 \(\mu\text{m}/\text{K}\), respectively.

As shown above, some extent of out-of-plane thermal deformation is inevitable because of the existence of fiber orientation errors. Therefore, in this paper, methods to mitigate the effect of fiber orientation errors by changing the number of layers and the stacking sequence are discussed.

3.1 Effect of changing the number of layers

In this subsection, the effect of increasing the number of layers is investigated. The number of layers for the CFRP reflector model used in the experiment was eight. The RMS
distributions of the dimensional error of the thermal deformation caused by the existence of fiber orientation errors for two types of reflector models with the same dimensions and different numbers of layers were calculated. One model had 16 layers and the other had 32 layers. The original stacking sequences for the 16- and 32-layer models were $[0_2/-45_2/90_2/45_2]$, and $[0_4/-45_4/90_4/45_4]$, respectively. The total thickness was maintained at 0.86 mm; therefore, the thicknesses of each layer for the 16- and 32-layer models were 0.05375 mm and 0.026875 mm, respectively. Note that even though the original fiber orientation of a layer could be the same as that of the neighboring layers, the fiber orientation errors were applied to each layer individually.

The mean RMS and upper limits of the range in which 90% and 99% of the deformations are included in Table 2. It can be seen that these values decreased as the number of layers increased. As an example, Fig. 8 shows the histogram of the RMS frequency for the stacking sequence $[0_4/-45_4/90_4/45_4]$. Comparing this figure with Fig. 5, it can be seen that the distribution of the RMS frequency became narrow by increasing the number of layers. The mean RMS was 0.112 μm/K, which was 50% smaller than that of the 8-layer model. Therefore, the effect of fiber orientation errors on the extent of the thermal deformation of a CFRP reflector model can be mitigated by increasing the number of layers. This result means that, in the realistic fabrication of CFRP reflectors, the out-of-plane thermal deformation caused by the existence of fiber orientation errors can be mitigated using as thin a prepreg as possible while maintaining the same total thickness.

### Table 2. The mean RMS and upper limits of the range of RMS where 90% and 99% of the deformations are included.

| Original stacking sequence | RMS [μm/K] |
|----------------------------|------------|
|                            | Mean | 90% | 99% |
| 8 ply $[0/−45/90/45]$     | 0.223 | 0.42 | 0.64 |
| 16 ply $[0/−45/90/45]$     | 0.158 | 0.30 | 0.46 |
| 16 ply $[0/−45/90/45]$     | 0.129 | 0.23 | 0.34 |
| 32 ply $[0_4/−45_4/90_4/45_4]$ | 0.112 | 0.22 | 0.33 |
| 32 ply $[0/−45/90/45]$     | 0.0865 | 0.15 | 0.21 |

Fig. 7. An example of the calculated Z-directional displacement distribution of thermal deformation of a CFRP reflector model with a normally distributed fiber orientation error (8 layers, original stacking sequence: $[0/−45/90/45]$).

Fig. 8. A histogram of the frequency of calculated dimensional error RMS of thermal deformation of a CFRP reflector model with a normally distributed fiber orientation error (32 layers, original stacking sequence: $[0/−45/90/45]$).

Fig. 9. A histogram of the frequency of calculated dimensional error RMS of thermal deformation of a CFRP reflector model with a normally distributed fiber orientation error (32 layers, original stacking sequence: $[0/−45/90/45]$).

3.2. Effect of changing the stacking sequence

In this subsection, the effect of changing both the stacking sequence and number of layers is investigated. The number of layers examined were 16 and 32. The original stacking sequences for the 16- and 32-layer models were $[0_2/−45_2/90_2/45_2]$ and $[0_4/−45_4/90_4/45_4]$, respectively. The total thickness of the laminate was maintained at 0.86 mm. The mean RMS and upper limits of the range in which 90% and 99% of the deformations included in Table 2. It can be seen that these values decreased as the number of layers increased, the same with those of the results mentioned in the last subsection. However, these values were much smaller than those of the previous results. As an example, Fig. 9 shows the histogram of the RMS frequency for the stacking sequence $[0/−45/90/45]$. Comparing this figure with Fig. 8, it can be seen that the distribution of the RMS frequency became much narrower by changing the stacking sequence, although the number of layers is the same. The
mean RMS was 0.0865 μm/K, which is 61% smaller than that of the 8-layer model, and 23% smaller than that with the original stacking sequence [0ș/−45ș/90ș/45ș]. Therefore, the effect of fiber orientation errors can be mitigated not only by increasing the number of layers, but also by applying appropriate stacking sequences.

3.3. Discussion on the results

The results showed that the out-of-plane thermal deformation of CFRP reflectors due to fiber orientation errors can be mitigated by increasing the number of layers of the laminate. It is believed that this is because, by increasing the number of layers, the total effect of fiber orientation errors on all layers is weakened, that is, the possibility of the existence of fiber orientation error which brought about the deformation mode that suppress the opposite deformation mode which was brought about by the fiber orientation errors in other layers increased. Thus, it can be expected that if the number of layers is increased further, the effect of fiber orientation errors would decrease further.

Additionally, the extent of the out-of-plane thermal deformation of the CFRP reflector models with stacking sequences [0ș/−45ș/90ș/45ș]ș (n = 2, 4) was smaller than that with stacking sequences [0ș/−45ș/90ș/45ș]ș for the same number of n. It is believed that this difference was brought about by the differences in the bending stiffness distribution of CFRP laminates. Each layer of CFRP laminates is anisotropic, and its bending stiffness is high in the fiber direction and low in the transverse direction. Because of this anisotropy, the bending stiffness of CFRP laminates differ depending on the direction, even for quasi-isotropic laminates. The bending stiffness distribution of CFRP laminates can be controlled by arranging the stacking sequence appropriately. In a laminate, the outer layers affect a large proportion of the bending stiffness distribution. For specific n (≥ 2), when comparing the stacking sequence [0ș/−45ș/90ș/45ș]ș with [0ș/−45ș/90ș/45ș]ș, the bending stiffness distribution is more uniform for [0ș/−45ș/90ș/45ș]ș because there are more layers with different fiber directions in the outer layers. The extent of out-of-plane thermal deformation is affected by the difference in bending stiffness distribution. For example, Figs. 10 and 11 show the relations between the dimensional error RMS and maximum displacement direction with stacking sequences [0ș/−45ș/90ș/45ș]ș and [0ș/−45ș/90ș/45ș]ș, respectively. Additionally, that with stacking sequence [0ș/−45ș/90ș/45ș], is shown in Fig. 6. Comparing Fig. 6 with Fig. 10, it can be seen that the RMS distribution became lower for [0ș/−45ș/90ș/45ș]ș because of the effect of increasing the number of layers described in the last paragraph. However, both distributions have their high peaks near the directions of ±45°. On the other hand, comparing Fig. 10 with Fig. 11, it can be seen that the peaks of the distribution near the directions of ±45° became lower. Additionally, the distributions near the directions of 0° and 90° became higher for [0ș/−45ș/90ș/45ș]ș, which means that the distribution became closer to uniform. As a result, the overall extent of thermal deformation became smaller for [0ș/−45ș/90ș/45ș]ș. This is because relatively large deformations having peaks near the directions of ±45° were suppressed because of more uniform bending stiffness distribution, even though the deformations with peaks near the directions of 0° and 90° became large.

Since the extent of thermal deformation differed with different stacking sequences having the same number of layers, it is implied that the thermal deformation caused by fiber orientation errors can be minimized by optimizing the stacking sequence while maintaining the same number of layers.

4. Thermal Deformation Due to Non-Uniform Temperature Distributions

Out-of-plane thermal deformation can occur on CFRP reflectors with curvature when exposed to non-uniform temperature distribution, even when there is no fiber orientation error. Non-uniform temperature distribution can be generated when heat is conducted between the satellite structure and observation system or when radiation from the Sun is not uniform. In this section, the out-of-plane thermal deformations of a CFRP reflector model without fiber orientation error for two types of non-uniform temperature distributions...
The number of layers was increased. Furthermore, the displacement maximum displacement and RMS decreased when the number of layers increased. Similarly, the displacement mode rotated clockwise; in addition, the out-of-plane thermal deformations of CFRP reflector models with stacking sequences arranged to mitigate the effect of fiber orientation errors, as discussed in the previous section (i.e., \([0/-45/90/45]_{12s}\) and \([0/-45/90/45]_{14s}\), were examined for the same temperature distributions.

### 4.1. Concentric circular temperature distribution

First, the out-of-plane thermal deformation generated on CFRP reflector models with three different stacking sequences under the condition of concentric circular temperature variation assuming heat conduction through the jig, as shown in Fig. 12, was investigated. The temperature rise was 0 K in the center and 1 K at the edge of the reflector models.

The negative maximum displacement and dimensional error RMS of the results calculated for the three stacking sequences are listed in Table 3. Note that only the negative maximum displacement is listed because nearly no positive Z-directional displacement was generated. Figures 13 and 14 show Z-directional displacement distributions of the thermal deformation for stacking sequences \([0/-45/90/45]_{12s}\) and \([0/-45/90/45]_{14s}\), respectively, as an example. From Fig. 13, it is understood that both the negative maximum displacement and dimensional error RMS were larger than those of the model with a uniform temperature distribution, shown in Fig. 3, even though the total rise in temperature was smaller.

It can be seen from these results that both the negative maximum displacement and RMS decreased when the number of layers was increased. Furthermore, the displacement mode became close to concentric circular. It is believed that this was because the bending stiffness distribution became close to uniform, the same as that discussed in the last section.

### 4.2. Uniform gradient temperature distribution

Second, out-of-plane thermal deformations generated by uniform gradient temperature variation assuming non-uniform radiation from the Sun, as shown in Fig. 15, were investigated. The temperature rise was 0 K on the bottom edge and 1 K on the upper edge of the reflector models.

The negative maximum displacement and dimensional error RMS of the results calculated for the three stacking sequences are listed in Table 4. Figures 16 and 17 show Z-directional displacement distributions of the thermal deformation for stacking sequences \([0/-45/90/45]_{12s}\) and \([0/-45/90/45]_{14s}\), respectively, as an example. From Fig. 16, it can be seen that the displacement mode was different from that of the model with a uniform temperature rise, as shown in Fig. 3.

Fig. 12. The concentric circular temperature distribution applied in the analysis.

Table 3. The maximum and minimum Z-directional displacements and RMS for concentric circular temperature distribution.

| Stacking sequence | Negative maximum displacement [\(\mu m/K\)] | RMS [\(\mu m/K\)] |
|-------------------|------------------------------------------|------------------|
| 8 ply \([0/-45/90/45]_{12s}\) | -0.068 | 0.0253 |
| 16 ply \([0/-45/90/45]_{14s}\) | -0.050 | 0.0223 |
| 32 ply \([0/-45/90/45]_{16s}\) | -0.043 | 0.0217 |

Fig. 13. Calculated Z-directional displacement of the thermal deformation of a CFRP reflector model with a concentric circular temperature rise (8 layers, stacking sequence: \([0/-45/90/45]_{12s}\)).

Fig. 14. Calculated Z-directional displacement of the thermal deformation of a CFRP reflector model with a concentric circular temperature rise (32 layers, stacking sequence: \([0/-45/90/45]_{16s}\)).
other words, it approached the Y-directional gradient. It is believed that this was because the bending stiffness distribution became close to uniform, again the same as that discussed in the last section.

The results described above indicate that the out-of-plane thermal deformation of CFRP reflector models generated by the non-uniform temperature distribution discussed in this section decreased when the stacking sequence was arranged to mitigate the effect of fiber orientation errors. Therefore, increasing the number of layers would not cause an increase in the out-of-plane thermal deformation nor degradation in the dimensional stability of CFRP reflectors when considering the temperature conditions assumed in this study.

5. Conclusions

In this study, the out-of-plane thermal deformation due to fiber orientation errors in the lamination of a CFRP reflector model was probabilistically investigated. The results indicate that the out-of-plane thermal deformation observed in a previous study had the same order of magnitude with the average magnitude of the out-of-plane thermal deformation due to fiber orientation errors of 0.4° in standard deviation. In addition, the out-of-plane thermal deformation of CFRP reflector models with different numbers of layers and stacking sequences were investigated. It was demonstrated that the effect of fiber orientation errors can be suppressed by increasing the number of layers (i.e., by utilizing as thin a prepreg as possible) and by arranging the stacking sequence appropriately. Moreover, it was concluded that this method would not degrade the dimensional stability of CFRP reflectors against the non-uniform temperature conditions discussed in this study.

Because out-of-plane thermal deformations can be suppressed by altering the stacking sequence, it is likely that further suppression can be accomplished by adopting other stacking sequences not discussed in this paper. There are a large number of studies optimizing stacking sequences of CFRP laminates to maximize specific characteristics. However, the number of studies on optimization considering fiber orientation error is few. Therefore, in future work, the stacking sequence needs to be optimized to minimize out-of-plane thermal deformation due to the existence of fiber orientation error.

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Table 4. The maximum and minimum Z-directional displacements and RMS on uniform gradient temperature distribution.

| Stacking sequence | Negative maximum displacement $[\mu m/K]$ | RMS $[\mu m/K]$ |
|-------------------|------------------------------------------|-----------------|
| 8 ply $[0/-45/90/45]_8$ | $-0.034$ | $0.0090$ |
| 16 ply $[0/-45/90/45]_{16}$ | $-0.027$ | $0.0083$ |
| 32 ply $[0/-45/90/45]_{32}$ | $-0.024$ | $0.0081$ |

Fig. 15. The Y-directional uniform gradient temperature distribution applied in the analysis.

Fig. 16. Calculated Z-directional displacement of the thermal deformation of a CFRP reflector model with a Y-directional temperature rise (8 layers, stacking sequence: $[0/-45/90/45]_8$).

Fig. 17. Calculated Z-directional displacement of the thermal deformation of a CFRP reflector model with a Y-directional temperature rise (32 layers, stacking sequence: $[0/-45/90/45]_{32}$).
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