Training Neural Networks using SAT solvers

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Abstract

We propose an algorithm to explore the global optimization method, using SAT solvers, for training a neural net. Deep Neural Networks have achieved great feats in tasks like image recognition, speech recognition, etc. Much of their success can be attributed to the gradient-based optimisation methods, which scale well to huge datasets while still giving solutions, better than any other existing methods. However, though, there exist a chunk of learning problems like the parity function and the Fast Fourier Transform, where a neural network using gradient-based optimisation algorithm can’t capture the underlying structure of the learning task properly. Thus, exploring global optimisation methods is of utmost interest as the gradient-based methods get stuck in local optima. In the experiments, we demonstrate the effectiveness of our algorithm against the ADAM optimiser in certain tasks like parity learning. However, in the case of image classification on the MNIST Dataset, the performance of our algorithm was less than satisfactory. We further discuss the role of the size of the training dataset and the hyper-parameter settings in keeping things scalable for a SAT solver.

1 Introduction

Machine Learning, at its core, is an optimisation problem. With highly non-linear models like neural networks, which have ushered a revolution in many fields of machine learning over the past decade or so, optimisation is still a challenging and non-trivial task. The state of the art optimisers for Neural Networks such as Adam, Adagrad and RMSProp get stuck in spurious local optima. Finding a globally optimal solution is NP Hard. And to tackle this issue we leverage the prowess of SAT solvers which are highly engineered to find solutions to NP Hard Problems. The popular optimisation methods rely on a gradient based optimisation scheme and hence suffer from many drawbacks. For example, exploding and vanishing gradients tend to destabilise training and to mitigate this one has to resort to techniques like batch-normalisation. Including a suitable regularisation strategy, they have a large number of hyperparameters to tune and finding a good combination of which is resource hungry and often requires intuition and experience. Above all, these being greedy strategies we always end up with a sub-optimal solution. Thus we want need to move over the current trend of gradient based update methods and further strive to make neural network optimisation an elegant process with a minimal number of hyper-parameters to tune. In this paper we propose a non-greedy optimisation method to train a neural network featuring discrete weights. After bit-blasting the output of the neural network, we formulate the cost function as a Satisfiability problem a solution to which is found by using a SAT solver. As The weights and inputs being discretized we show how a modified version of relu called stepped-relu generalises better as an activation function. The number of hyper-parameters in our method is way lesser than the state of the art optimisers. Our method being non-greedy finds a global minima wrt to the mini-batch of training examples. To make our algorithm scalable to huge datasets, we propose a method to parallelise training across batches of training data. Lastly a novel method to decrease the solving time of the sat solvers has been discussed.

Preprint. Under review.
2 Related Works

The goal of this work is to design a non-greedy optimisation scheme for neural networks. To do this, the cost function of the neural network is mapped to a SAT encoding and further SAT solvers were used to find an assignment to the formula. Using such an encoding, Nina et al. [1] explored various properties of a Binarized Neural Network like robustness to adversarial perturbations. Furthermore, Huang et al. [2] proposed a general framework for automated verification of safety of classification decisions made by feed-forward deep neural networks which leverages SMT solvers and SAT encodings. Zahra et al. [3] proposed a framework that enables an untrusted server (the cloud) to provide a client with a short mathematical proof of the correctness of inference tasks that they perform on behalf of the client.

3 Reducing to Satisfiability

3.1 Satisfiability

The Boolean satisfiability problem (sometimes called propositional satisfiability problem and abbreviated SAT) is the problem of determining if there exists an interpretation that satisfies a given Boolean formula. In other words, it asks whether the variables of a given Boolean formula can be consistently replaced by the values TRUE or FALSE in such a way that the formula evaluates to TRUE. For example a neural network model with let’s say two variables only, \( l_1 \) and \( l_2 \), could look like:

\[
\Sigma = (l_1 \lor l_2 \lor t_0) \land (\bar{l}_1 \lor l_2 \lor \bar{t}_0)
\]

where \( t_0 \) is a temporary variable, created in the process of breaking the cost function into CNF.

3.2 Bit Blasting

The primary idea here is to decompose the weight variables into bits and using binary arithmetics for addition, multiplication and non-linear activations reduce the cost function to a 0-1 optimisation problem. The weights bear a signed binary representation. A weight \( w \) is represented as,

\[
w = w_0w_1\ldots w_{n-2}w_{n-1},
\]

where \( w_0 \) is the sign bit. The decimal value of \( w \) is then,

\[
w_{\text{decimal}} = -2^{n-1}w_0 + 2^{n-2}w_1\ldots 2^1w_{n-2} + 2^0w_{n-1}
\]

3.3 Signed and Unsigned Addition

Addition of 2 signed numbers is done by carry look ahead adder method as shown in algorithm [1] and setting the type to ‘signed’. As the sum has to be represented in the same number of bits as addends’, a constraint is generated which has to be satisfied for a valid addition. For example, in the case of signed addition, the constraint is \( \text{carryin} = \text{carryout} \) wrt to the signed bit. And for unsigned addition, the constraint is \( \text{carryout} \) from the signed bit = 0. These constraints are then converted to CNF formula. We do so by using z3 solver.

3.4 Signed Multiplication

First we convert the multiplicands into their magnitudes and then perform repeated unsigned addition by shift and add method. Having found the magnitude, we assign a positive sign to it if the multiplicands are of the same sign, else a negative sign. The algorithm [2] illustrates the steps for this.

The multiplicands are expressed in \( 2^{\text{num\_bits}} \) number of bits. The hyper parameter \( \text{product\_magnitude\_bits} \) sets a limit on the magnitude of the product. Ideally \( \text{product\_magnitude\_bits} = 2^{\text{num\_bits}} - 1 \) But, we can set it to a lower value which would generate constraints corresponding to product < \( 2^{\text{product\_magnitude\_bits}+1} \). This speeds up SAT solvers by limiting our search space to a much smaller domain, with a compromise in the accuracy.
Algorithm 1 Bitwise Addition: bitwiseAdd

Input: BitVectors: $a = a_0a_1 \ldots a_{n-2}a_{n-1}$, $b = b_0b_1 \ldots b_{n-2}b_{n-1}$, type $\in \{\text{signed, unsigned}\}$

Output: BitVector $y = y_0y_1 \ldots y_{n-2}y_{n-1}$, with $y_{n-1}$ and 1 constraint

1: $n \leftarrow \text{length}(a)$
2: $\text{carryPrev} \leftarrow 0$
3: $\text{carry} \leftarrow 0$
4: for $i \leftarrow n-1$ to 0 do
5:  $\text{carryPrev} \leftarrow \text{carry}$
6:  $G \leftarrow a_i \land b_i$
7:  $P \leftarrow a_i \lor b_i$
8:  $y_i \leftarrow G \oplus P \oplus \text{carryPrev}$
9:  $\text{carry} \leftarrow G \lor (P \land \text{carryPrev})$
10: end for
11: if type $=$ signed then
12:  $\text{constraint} \leftarrow \text{carry} == \text{carryPrev}$
13: else
14:  $\text{constraint} \leftarrow \text{carry} == 0$
15: return $y, \text{constraint}$
16: end if

3.5 Weighted Sum

After the $w_i * x_i$ operation, we need to add all such products, which then would be the input to one of the neurons in the next layer. Hence if the number of nodes in the previous layer is large, this raises a concern. Addition is done sequentially, this means there is an inherent upper bound on the partial sum. So, we introduce another hyper-parameter called slack_bits. Each term $w_i * x_i$ is sign extended from $2^{\text{num_bits}}$ to slack_bits number of bits. This mitigates the problem as now we can accommodate large temporary partial sums. Then this would mean that the input to the next layer would be in slack_bits number of bits. To prevent this, after doing the weighted sum, we get rid of the least significant bits, to reduce the number of bits from slack_bits to num_bits. In decimal, this corresponds to division with $2^{\text{slack_bits} - \text{num_bits}}$. Note, unlike other weights, the bias is represented in slack_bits number of bits.

3.6 Activation Function

The hidden layer of the neural network features a non-linear activation function. We use Rectified Linear Unit (Relu) for this purpose.

$$\text{relu}(x) = \begin{cases} x, & \text{if } x > 0. \\ 0, & \text{otherwise.} \end{cases}$$

Let’s see how relu activation function transforms the input bits into the output bits. When the output $x < 0$, $x_{n-1}$ the sign bit is 1. The output of relu should be 0 (in decimal), i.e. the output bits are set to 0. This is achieved by and-ing rest of the bits with MSB. Notice how the output is the same as input when the input is a positive number i.e the sign bit is 0. So,

$$\text{relu}(x_{n-1}x_{n-2} \ldots x_1x_0) = 0 \overline{x_{n-1}} \land x_{n-2} \ldots \overline{x_{n-1}} \land x_1 \overline{x_{n-1}} \land x_0 \quad (3)$$

Note the input to the activation function is in slack_bits number of bits. Because the output of this node further would be multiplied with a weight which is in num_bits number of bits, we want the output to be also represented in num_bits number of bits. Also we don’t want the output to blow up with each forward pass. Thus the activation function itself should take care of this. So if we want relu as an activation function, we have to clip it’s maximum value at $2^{\text{num_bits} - 1}$. In this way the output can always be represented in num_bits number of bits with the MSB being the sign bit. The activation function is shown in figure and algorithm[3] describes the operation.

In the above plot we see that due to clipping at 7, the output can be represented in 4 bits. There is one problem: Because the $w_i$ and $x_i$ are discrete, the input to a hidden node is very much susceptible to a slight change in $x_i$ which would hamper the generalising capability of the neural network. Hence
Algorithm 2 Bitwise Multiplication: bitwiseMul

**Input:** BitVectors: \(a = a_0 a_1 \ldots a_{n-2} a_{n-1}, b = b_0 b_1 \ldots b_{n-2} b_{n-1}\)

**Output:** BitVector \(y = y_0 y_1 \ldots y_{n-2} y_{n-1}\), with \(y_{n-1}\) and constraints

1. \(n \leftarrow \text{length}(a)\)
2. \(a_{\text{sign}} \leftarrow a_0\)
3. \(b_{\text{sign}} \leftarrow b_0\)
4. Initialise product with slack_bits number of 0
5. Initialise \(a^{\text{mag}}, b^{\text{mag}}\) with num_bits number of 0
6. for \(i \leftarrow 0 \text{ to } n - 1\) do
7. \(a_i \leftarrow a_0 \oplus a_i\)
8. \(b_i \leftarrow b_0 \oplus b_i\)
9. end for
10. \(a_{n-1} \leftarrow a_{\text{sign}}\)
11. \(b_{n-1} \leftarrow b_{\text{sign}}\)
12. a, constraint \(\leftarrow\) bitwiseAdd\((a, a^{\text{mag}}, \text{signed})\)
13. constraints \(\leftarrow\) \{constraint\}
14. b, constraint \(\leftarrow\) bitwiseAdd\((b, b^{\text{mag}}, \text{signed})\)
15. constraints \(\leftarrow\) constraints \(\cup\) \{constraint\}
16. for \(i \leftarrow n - 1 \text{ to } 0\) do
17. Initialise \(b^{\text{new}}\) with slack_bits number of 0
18. for \(j \leftarrow 0 \text{ to } n - 1\) do
19. \(b^{\text{new}}_{\text{slack_bits} - 1 - j} \leftarrow b_{n-1 - j} \land a_i\)
20. end for
21. product, constraint \(\leftarrow\) bitwiseAdd\((\text{product}, b^{\text{new}}, \text{unsigned})\)
22. constraints \(\leftarrow\) constraints \(\cup\) \{constraint\}
23. end for
24. for \(i \leftarrow 0 \text{ to } \text{slack_bits} - \text{product_magnitude_bits}\) do
25. constraints \(\leftarrow\) constraints \(\cup\) \{product\_i == 0\}
26. end for
27. product\_sign \(\leftarrow a_{\text{sign}} \oplus b_{\text{sign}}\)
28. Initialise \(\text{product}^{\text{mag}}\) with slack_bits number of 0
29. \(\text{product}^{\text{mag}}_{\text{slack_bits} - 1} \leftarrow \text{product}_{\text{sign}}\)
30. for \(i \leftarrow \text{slack_bits} - 1 \text{ to } 0\) do
31. \(\text{product}_i \leftarrow \text{product}_{\text{sign}} \oplus \text{product}_i\)
32. end for
33. product, constraint \(\leftarrow\) bitwiseAdd\((\text{product}, \text{product}^{\text{mag}}, \text{signed})\)
34. constraints \(\leftarrow\) constraints \(\cup\) \{constraint\}
35. return \(y, \text{constraints}\)

To smoothen things out, we get rid of the last few least significant bits of the output, denoted as regret_bits, of the input and then apply the activation function. Getting rid of the last regret_bits bits has a effect of division with \(2^{\text{regret_bits}}\).

### 3.7 Architectures

We propose two Neural Network Architectures namely-

- **Vanilla Neural Network:** The standard feedforward network with relu activation function, a single hidden layer with 10 hidden neurons and one single output node.

- **Kernelised Neural Network:** To take care of the problem as discussed in the discussion section, we want that the weights (in the first layer only) corresponding to the neighbouring pixels of the input image should not vary significantly. Hence we can have a square matrix and a sliding window of size window_size. And the weights of the neural network are the average of the elements in the sliding window with stride window_stride.
Algorithm 3 Activation Function: Relu

Input: BitVector $x = x_0x_1 \ldots x_{n-1}x_{n-1}$
Output: BitVector $y = y_0y_1 \ldots y_{n-1}y_{n-1}$

1: $n \leftarrow \text{length}(x)$
2: for $i \leftarrow 1$ to $n - 1$ do
3: \hspace{1em} $y_i \leftarrow x_i \land x_{n-1}$
4: end for
5: $y_0 \leftarrow 0$
6: return $y$

Algorithm 4 Activation Function: Relu_clipped

Input: BitVector $x = x_0x_1 \ldots x_{n-1}x_{n-1}$ Output: BitVector $y = y_0y_1 \ldots y_{n-1}y_{n-1}$ and a list of constraints.

1: $n \leftarrow \text{length}(x)$
2: temp $\leftarrow \text{Relu}(x)$
3: temp $\leftarrow \text{temp}_0 \text{temp}_1 \ldots \text{temp}_{n-1} - \text{regret_bits}$
4: Prepend temp with 0s to make its length n
5: for $i \leftarrow 0$ to $n - 1$ do
6: \hspace{1em} linearShiftDown_i $\leftarrow 1$
7: \hspace{1em} linearShiftUp_i $\leftarrow 0$
8: end for
9: for $i \leftarrow n - \text{num_bits}$ to $n - 2$ do
10: \hspace{1em} linearShiftDown_i $\leftarrow 0$
11: end for
12: for $i \leftarrow n - \text{num_bits}$ to $n - 1$ do
13: \hspace{1em} linearShiftUp_i $\leftarrow 1$
14: end for
15: temp, constraintDown $\leftarrow \text{bitwiseAdd}(\text{temp}, \text{linearShiftDown}, \text{signed})$
16: for $i \leftarrow 1$ to $n - 1$ do
17: \hspace{1em} temp_i $\leftarrow \text{temp}_i \land \text{temp}_0$
18: end for
19: temp, constraintUp $\leftarrow \text{bitwiseAdd}(\text{temp}, \text{linearShiftUp}, \text{signed})$
20: constraints $\leftarrow (\text{constraintDown}, \text{constraintUp})$
21: $y \leftarrow y_{\text{slack_bits}-1-n-\text{num_bits}} \ldots y_{\text{slack_bits}-1}$
22: return $y$, constraints

3.8 Cost Function

We propose a cost function for a binary classification problem. The final output layer has a single neuron. The cost function, where $y$ is the output:

- $y \geq +2^{\text{cost_bits}}$ if label = +ve
- $y \leq -2^{\text{cost_bits}}$ if label = -ve

Note that $y$ is represented in slack_bits number of bits. To implement the above cost function, we follow the algorithm.[5]

4 Implementation

We start with a small experiment to check whether we can classify linearly separable points by training a perceptron model using this approach. The input was a 4 dimensional vector with each number being 0 or 1. A labeling function $y_{\text{label}} = \text{sign}(2x_0 + 3x_1 - 4x_2 - 2x_3 + 1)$ was chosen and used to label the points. Then the perceptron model described by $y = w_0x_0 + w_1x_1 + w_2x_2 + w_3x_3 + b$ was declared and the weights $w_0 \ldots 3$ and the bias are expressed with num_bits = 4, product_magnitude_bits = 7, slack_bits = 8. Using the bitwiseAdd and bitwiseMul method described as above, we express $y$ in 8 bits. Setting the cost_bits = 0 we get SAT. The dataset and assignments are given in [?].
Algorithm 5 Cost Function: cost

Input: BitVector \( x = x_0 x_1 \ldots x_{n-1}x_{n-1}, \text{label} \in \{-1, +1\} \)

Output: constraints

1: \( n \leftarrow \text{length}(x) \)
2: if \( y \) is +1 then
3: \( \text{constraints} \leftarrow \{ x_0 = 0, \bigvee_{i=1}^{n-\text{cost\_bits}-1} x_i = 1 \} \)
4: else
5: \( \text{constraints} \leftarrow \{ x_0 = 1, \bigwedge_{i=1}^{n-\text{cost\_bits}-1} x_i = 0 \} \)
6: end if
7: return \( \text{constraints} \)

4.1 Dataset and Pre-processing

4.1.1 MNIST

We run our experiments on MNIST Dataset. We want to learn a 3-recogniser. For this we first separate 3 and non-3 images. Then create a train and test set of sizes 63 and 10139 with same number of both types. We follow 2 different pre-processing schemes for the vanilla and Kernelised Neural Network. The reasoning and details follow:

- Vanilla Neural Network: Downsampled the 28*28 image to 14*14 image.
- Kernelised Neural Network: Borders with zero pixel values were removed while preserving the square structure of the image. Then downsampled to 10*10 image.

After pre-processing, the images were reshaped to a single dimensional form. Individual pixels (floating point number varying between 0 and 1) were discretized and represented in \( \text{num\_bits} \) number of bits. The following transformation gives the decimal representation of each pixel with value \( \text{pixel\_val} \):

\[
K = \text{int}(\text{pixel\_val} \times (2^\alpha - 1))
\]

where \( \alpha = \text{num\_bits} - \text{preprocess\_psub} \) and \( 1 < \alpha < \text{num\_bits} \). Then \( K \) represented in \( \text{num\_bits} \) number of bits, is the discretized value for that pixel. In our experiments we set \( \alpha = 2 \). It was observed that, with \( \alpha > 2 \), the SAT solver couldn’t find a solution to the cnf.

4.1.2 Parity Learning

Nye et al. [4] show that gradient based optimisers like [5], cannot be used to train a deep neural network to learn the parity function. So, to test our algorithm we create two datasets of binary input vectors with \( \text{dimensionality} \in \{8, 16\} \). To label every training example, we first randomly choose bit positions, and then the label for that particular data becomes the xor of the bits present in those positions. For example, for the dataset with dimensionality = 8, the positions chosen were 0, 1, 3, and 5. So, the \( i^{th} \) example \( x_i \) has the label \( y_i \) as,

\[
y_i = x_i[0] \oplus x_i[1] \oplus x_i[3] \oplus x_i[5]
\]

Similarly, for \( \text{dimensionality} = 16 \), the positions were 0, 1, 2, 3, 4, 6, 11, and 14.

4.2 Generating CNF

We use z3 solver to declare the weights of the neural network and its’ libraries to do the binary arithmetic. The forward pass of an image generates constraints (boolean equalities) because of addition, multiplication, activation functions and cost functions. The constraints for all the images in a given batch are fed to z3 solver, which then breaks down the complex expressions into cnf format. Let’s denote the generated clauses by \( \Sigma_i \) for \( i^{th} \) batch.
4.3 Training

From the train set of 63 images, we randomly sample 20 batches with \textit{batch\_size} 30. The training is divided into 2 phases-

- We generate \( \Sigma_i \) for \( i: 1 \rightarrow \text{num\_batch} \). Then Using the following principle we collect implications from every batch.

\[ \Sigma_i \implies x_j \lor x_k \lor x_l \iff \Sigma_i \land x_j \land x_k \land x_l \text{ is UNSAT} \]

From above we conclude if \( x_j, x_k \) and \( x_l \) were assigned 1, 0 and 1 respectively in \( \Sigma_i \) and \( \Sigma_i \) becomes UNSAT, then \( x_j \lor x_k \lor x_l \) is a learned clause. Every implied clause carries information on how the weight variables are related among themselves to correctly classify that particular batch. Let \( \lambda_i \) be defined such as:

\[ \lambda_i = \{ l \mid \Sigma_i \implies l \} \]

\( \lambda_i \) is generated in the following manner. Multiple batches are run parallely across different machines. One thing to note is time taken to find \( \Sigma \land \text{literals} \) grows as \( \text{var\_ChunkSize} \) decreases. Because we don’t want to get stuck at one such initialisation, we choose to halt the process for a given assignment of \( \text{literals} \) if we don’t find UNSAT in time = 180 seconds of run time. This number was chosen empirically.

- After all the processes are complete in the previous step, we combine get

\[ \lambda^{all} = \bigwedge_{i=1}^{\text{num\_batches}} \lambda_i \]

To find an assignment we run SAT solvers on all \( \Sigma_i^{all} = \Sigma_i \land \lambda^{all} \) parallely across multiple machines.

\[ \textbf{Algorithm 6 Clause Sharing: ImpliedClauses} \]

\textbf{Input:} Clauses \( \Sigma \)  
\textbf{Output:} Implied Clauses \( \lambda \)

1: \( n \leftarrow \text{length}(\text{vars}) \)
2: \( \text{var\_ChunkSize} \leftarrow n \)
3: Initialise \( \lambda = \{ \} \)
4: while getting learned clauses do
5: \( s \leftarrow \lceil \frac{n}{\text{var\_ChunkSize}} \rceil \)
6: \( \text{count} \leftarrow 0 \)
7: while \( \text{count} < 100 \) do
8: \( \text{for } i \leftarrow 1 \text{ to } s \text{ do} \)
9: \( \text{literals} \leftarrow \text{randomly sample var\_ChunkSize no. of elements from vars} \)
10: \( \text{randomly set each element in literals to either 0 or 1} \)
11: if \( \Sigma \land \text{literals} \) is UNSAT then
12: \( \lambda \leftarrow \lambda \cup \{ \bigvee_{j=1}^{\text{var\_ChunkSize}} \overline{x_j} \forall x_j \in \text{literals} \} \)
13: end if
14: end for
15: \( \text{count} \leftarrow \text{count+1} \)
16: end while
17: \( \text{var\_ChunkSize} \leftarrow \max(\text{var\_ChunkSize} - 0.05 \times \text{var\_ChunkSize}, 50) \)
18: end while
19: return \( \lambda \)

4.4 Speeding Up computations in SAT Solver

With the given choice of \textit{batch\_size} and architecture of the Network Network, the SAT solver could not find a solution to the \( \Sigma^{all}_i \) in 48 hours of running the code. However things are boosted significantly by randomly setting a chunk of the model variables to either 0s or 1s and then running
the solver. We start by assigning some 90% model variables randomly to 0s and 1s and follow the curriculum is shown in [7]. This also empowers us to find multiple solutions which are much different from each other. As \( var\text{ChunkSize} \) decreases, the likelihood to find a solution and solving time both increase.

**Algorithm 7 Solving: AssumptionSolving**

**Input:** Clauses \( \Sigma \), model variables \( vars \)

**Output:** Solutions to the Clauses \( \lambda \)

1: \( n \leftarrow \text{length}(vars) \)
2: Initialise \( sols=\{\} \)
3: \( \text{solFound} \leftarrow 0 \)
4: \( \text{while } \text{solFound} < \text{numSols} \text{ do} \)
5: \( \text{count} \leftarrow 0 \)
6: \( \text{while } \text{count} < 100 \text{ do} \)
7: \( \text{for } i \leftarrow 0 \text{ to } s \text{ do} \)
8: \( \text{literals} \leftarrow \text{randomly sample } var\text{ChunkSize} \text{ no. of elements from vars} \)
9: \( \text{randomly set each element in literals to either } 0 \text{ or } 1 \)
10: \( \text{if } \Sigma \land \text{literals} \text{ is SAT then} \)
11: \( \text{assignment} \leftarrow \text{solution to } \Sigma \land \text{literals} \)
12: \( \text{sols} \leftarrow \text{sols} \cup \{\text{assignment}\} \)
13: \( \text{solFound} \leftarrow \text{solFound}+1 \)
14: \( \text{if } \text{size}(\text{sols}) = \text{numSols} \text{ then} \)
15: \( \text{return } \text{sols} \)
16: \( \text{end if} \)
17: \( \text{end if} \)
18: \( \text{end for} \)
19: \( \text{count} \leftarrow \text{count}+1 \)
20: \( \text{end while} \)
21: \( \text{var}\text{ChunkSize} \leftarrow \max(\text{var}\text{ChunkSize} - 0.05*\text{var}\text{ChunkSize}, 50) \)
22: \( \text{end while} \)
23: \( \text{return } \text{sols} \)

5 Results

Experiments were done to see how various parameters and model architectures influence the quality of a solution found by the Sat Solver. For experiments in [1][2][3][4] each instance of the solver was run on a 8 core cpu with hyper-threading, 16 GB RAM and 4 plingeling solver threads. The experiments in [3] and [4] were run on a 24 core machine with hyperthreading, 32GB RAM and 12 plingeling threads as finding a solution was much harder in these cases. For all the experiments \( \text{num}\text{_bits} = 4, \text{product}\_\text{magnitude}\_\text{bits} = 7 \) and a neural network with a single hidden layer with 10 nodes were used unless otherwise stated.

In table [1] we set \( \text{slack}\_\text{bits} = 8, \text{cost}\_\text{bits} = 3 \), and observe that increasing \( \text{regret}\_\text{bits} \) improves generalising capacity of the neural network. \( \text{slack}\_\text{bits} \) bottle-neck the weighted sums. For example, if the input node is of 196 dimensions, hidden nodes in the subsequent layer receive the weighted sum of 196 numbers. With \( \text{slack}\_\text{Bits} \) set to 8, we impose a constraint such which enforces partial sums to be small enough to be stored as a 8 bit fixed-point number. To study the effect of \( \text{slack}\_\text{Bits} \), in table [2] we set \( \text{cost}\_\text{bits} = 4, \) we find that increasing \( \text{slack}\_\text{bits} \) doesn’t improve accuracy. Because \( \text{regret}\_\text{bits} \leq \text{slack}\_\text{bits} - \text{num}\_\text{bits} \), with more \( \text{slack}\_\text{bits} \) we could vary the \( \text{regret}\_\text{bits} \) too. And we see that increasing \( \text{regret}\_\text{bits} \) doesn’t make accuracy any better.

In table [3] with \( \text{regret}\_\text{bits} = 4, \text{slack}\_\text{bits} = 8 \), we realise even with stronger separations between \( y^+\text{pred} \) and \( y^-\text{pred} \), the accuracy doesn’t go up. In table [4] we study if increasing the complexity of the neural network improves the accuracy. We set \( \text{regret}\_\text{bits} = 4, \text{cost}\_\text{bits} = 4, \text{slack}\_\text{bits} = 8 \). The accuracy still doesn’t improve.
Table 1: Variation with $\text{regret\_bits}$

| $\text{regret\_bits}$ | Min     | Median   | Max     |
|------------------------|---------|----------|---------|
| 2                      | 47.98%  | 60.93%   | 74.62%  |
| 3                      | 51.79%  | 66.36%   | 77.15%  |
| 4                      | 41.48%  | 67.18%   | 82.04%  |

Table 2: Variation with $\text{slack\_bits}$

| $\text{slack\_bits}$ | $\text{regret\_bits}$ | Min     | Median   | Max     |
|-----------------------|------------------------|---------|----------|---------|
| 8                     | 4                      | 53.46%  | 71.08%   | 82.81%  |
| 9                     | 5                      | 58.62%  | 74.37%   | 80.11%  |
| 10                    | 6                      | 61.00%  | 71.87%   | 80.51%  |

Table 3: $\text{cost\_bits}$

| Type | Min     | Median   | Max     |
|------|---------|----------|---------|
| 3    | 41.48%  | 67.18%   | 82.04%  |
| 4    | 53.46%  | 71.08%   | 82.81%  |

Table 4: Variation with model architecture

| Layers | Hidden nodes | Min     | Median   | Max     |
|--------|--------------|---------|----------|---------|
| 1      | 10           | 53.46%  | 71.08%   | 82.81%  |
| 1      | 20           | 56.56%  | 68.06%   | 78.14%  |
| 2      | 5, 5         | 61.75%  | 72.52%   | 78.75%  |

In table 5, $\text{regret\_bits} = 4$, $\text{cost\_bits} = 4$, $\text{slack\_bits} = 8$, by looking at the min and median scores we conclude that the clause sharing across batches is not effective at all. Also seeing twice as more data doesn’t improve the max accuracy.

To get a better understanding of things hampering the performance, look at the fig 1

As we observe, the problem is not the architecture of the network but the kind of weights that are being learnt. Above we see that the weights in the first layer learnt by the sat solver are quit arbit as compared to the ones learnt by adam optimiser. Probably this is the reason we are not able to increase the accuracy. This is the main motivation for Kernelised Neural Network. Now that we have weights that are moving window averages, the neighbouring weights do not vary much. We set $\text{slack\_bits} = 8$, $\text{kernel\_stride} = 2$, $\text{kernel\_rb} = 1$ and run the experiments in 6.

Figure 1: Plot of the weights in the 1st layer, from the input to a hidden node.
Table 5: Clause sharing vs no sharing vs all

| Type                | Min    | Median | Max    |
|---------------------|--------|--------|--------|
| Clause Sharing      | 52.46% | 69.73% | 81.75% |
| No Sharing          | 53.46% | 71.08% | 82.81% |
| Entire dataset      | 70.19% | 75.24% | 80.64% |

Figure 2: Distribution of the weights in the 1st layer.

Table 6: Variation with Kernel params

| kernel_size | cost_bits | Min    | Median | Max    |
|-------------|-----------|--------|--------|--------|
| 3           | 3         | 73.96% | 76.89% | 80.07% |
| 3           | 4         | 77.37% | 77.57% | 78.02% |
| 4           | 3         | 61.91% | 78.02% | 81.78% |
| 4           | 4         | -      | -      | -      |

As stated earlier, we make random assumptions to the weight bits and feed the cnf to the sat solver. This might be the reason we learn bits that are really arbit. But then again it is equally likely for a Sat solver to find any assignment as long as the cnf is SAT. So not making any prior assignment would not guarantee a solution where the learnt weights are not co-related among neighbouring pixels.

In case of Parity Learning, we see our algorithm outperforms gradient descent by a significant margin. With gradient descent the training accuracy remains around 50%. CITE This being a binary classification task, we could infer that gradient descent doesn’t make any useful updates to the neural network. But, our algorithm not only achieves a 100% train accuracy but also 100% train accuracy for the 8 bit and 57.50% accuracy in case of the 16 bit dataset.

Table 7: Binarized Neural Network

| batch_size | cost_bits | Min    | Median | Max    |
|------------|-----------|--------|--------|--------|
| 20         | 1         | 46.81% | 59.20% | 69.38% |
| 20         | 2         | 45.55% | 59.29% | 69.34% |
| 20         | 3         | 55.76% | 64.91% | 76.21% |
| 20         | 4         | 67.16% | 71.59% | 77.56% |
| 30         | 1         | 64.80% | 68.65% | 77.31% |
| 30         | 2         | 66.13% | 76.07% | 83.10% |
Table 8: XOR (clause sharing)

| Bits | Min       | Median    | Max   |
|------|-----------|-----------|-------|
| 8    | 62.50%    | 87.50%    | 100.00%|
| 16   | 40.00%    | 50.00%    | 57.50% |

6 Discussion

In this work, we have presented a non-greedy optimisation scheme to train neural networks. Despite being non-greedy, we find in tasks like image classification gradient descent based optimisers outperform our algorithm. This can be attributed to the fact that our algorithm doesn’t scale to the point where we see the entire dataset. Due to the very small amount of data we see, our algorithm overfits easily. For our future work, we would like to explore more methods to parallelise learning across batches so that the net training error improves.

References

[1] Nina Narodytska, Shiva Kasiviswanathan, Leonid Ryzhyk, Mooly Sagiv, and Toby Walsh. Verifying properties of binarized deep neural networks. In Proceedings of the AAAI Conference on Artificial Intelligence, volume 32, 2018.

[2] Xiaowei Huang, Marta Kwiatkowska, Sen Wang, and Min Wu. Safety verification of deep neural networks. In International conference on computer aided verification, pages 3–29. Springer, 2017.

[3] Zahra Ghodsi, Tianyu Gu, and Siddharth Garg. Safetynets: Verifiable execution of deep neural networks on an untrusted cloud. Advances in Neural Information Processing Systems, 30, 2017.

[4] Maxwell Nye and Andrew Saxe. Are efficient deep representations learnable? arXiv preprint arXiv:1807.06399, 2018.

[5] Diederik P Kingma and Jimmy Ba. Adam: A method for stochastic optimization. arXiv preprint arXiv:1412.6980, 2014.