High School Students' Mathematical Problem Solving Skills Based on Krulik and Rudnick Steps Reviewed from Thinking Style

A Rahmah¹*, Mardiyana¹, D R S Saputro¹
¹Postgraduate School of Mathematics Education, Faculty of Teacher Training and Education, Universitas Sebelas Maret, Indonesia

*Email: rahmah.adila@gmail.com

Abstract. This research explains the problem solving skills of students of Grade XII MIPA SMA MTA Surakarta based on Krulik and Rudnick's steps that are reviewed from the thinking style. This research is a qualitative study that produces descriptive data. The study began by determining the subject of the study by using a thought style questionnaire, given problem solving tasks and interviews. The results showed math problem solving skills; 1) concrete sequential student are systematically worked on and use an inductive mindset and the steps of reviewing and discussing are not implemented; 2) abstract sequential student are easy to present what is known in the question to mathematical symbol forms, collect data in detail, and manage the information obtained regularly and perform all of Krulik and Rudnick's steps; 3) concrete random student are tend to be consistent in using guessing or experimenting strategies and work without detail and the steps of reviewing and discussing are not implemented; 4) abstract random student are fundamental differences in processing the information received and planning problem solving in a way of their own accord and doing all the steps krulik and rudnick.

1. Introduction

The objectives of mathematical learning include developing skills: (1) mathematical communication, (2) mathematical reasoning, (3) mathematical problem solving, (4) mathematical connections, and (5) mathematical representation [1]. Problem solving activities of the learning process in the school math can be said to be core activities. Therefore, problem solving capabilities are indispensable to every human being. The problem-solving process teaches a thought process that is included in solving mathematical problems that teach the process of thinking mathematically. According to Rogers et al problem solving has become an important focus in the school's math curriculum starting from elementary school to high school level [2]. However, Indonesia's PISA results in 2018 were down compared to 2015. PISA 2018 results for the math category scored an average of 379, while the 2015 PISA results for the math category scored an average of 386[3]. The math PISA assessment focuses on measuring a student's capacity to formulate, use, and interpret mathematics in a variety of contexts. To complete the PISA test, students must be mathematically accurate and use mathematical concepts, procedures, facts, and tools to describe, explain, and predict phenomena [4].

Siswono asserts here are four factors that affect students' ability to solve problems. These four factors are early experience, mathematical background, problem structure, and motivation [5]. Math learning prioritizes process skills and strategy in solving problems. Troubleshooting requires new and different steps or strategies compared to steps or strategies in solving regular or ordinary questions. In line with
Tambychik et al opinion students face difficulties in solving math problems due to an inability to acquire many math skills and lacking in cognitive learning abilities [6]. Amri explained that the requirements of students in problem solving, (1) the prerequisites to tackle such problems; (2) the problem solving itself, (3) a matter of affordability, (4) students are willing and intend to solve the problem [7].

A common heuristic model of problem solving was first developed by Polya in his various papers. Heuristic Polya consists of 4 thought stages. While in development and practice, heuristic problem solving is developed in more detail by other experts[8]. One of them is Krulik and Rudnick's steps that explain in more detail the stages of problem solving than polya measures. Polya reveals there are four steps in solving the problem: (a) Understanding the problem; (b) Plan a settlement; (c) Implement the plan; (d) Re-examine the process and results [9]. Krulik and Rudnik also state there are 5 troubleshooting steps: (a) Reading and thinking; (b) Explore and plan; (c) Choose a strategy; (d) Find and answer; and (e) Review and discuss [10]. In this study, troubleshooting steps refer to krulik & rudnick troubleshooting steps.

Solving math problems also depends on the characteristics of students' thinking in finding and processing the learning outcomes they receive. A student's ability to solve problems is influenced by each student's thinking style, the style of thinking itself can be influenced by the student's habits when attending class as well as the habits of students studying at home. In the learning process, students who get varied learning (using various learning methods) with students who get learning with the same method each day will produce different student profile characteristics [11]. In line with the opinions of Zhang Thinking styles vary depending on both students’ and teachers’ characteristics [12] .

Sahatcija et al, students apply their skills in different ways. The application of skills in various ways is called a thinking style [13]. The style of thinking is a behavior dominated by the left brain (sequential) that is distinguished into two concrete sequences and an abstract sequence or a right brain (random) that is also distinguished into two random concrete and random abstracts in processing information until it obtains a solution [14]. This is in line with Gregorc's opinion suggesting that mind style is a behavior related to the dominance of brain work in processing information in a balanced manner, resulting in the most effective solution under a variety of different conditions [15]. The opinion suggests that the style of thinking is the activity of human brain nerve cells in processing and processing information in a balanced manner to produce a form of problem-solving measures. Tool and technology support is needed to support information processing in order to come up with effective solutions. The thinking style is divided into four groups: concrete sequential (SK), abstract sequential (SA), concrete random (AK), and random abstract (AA).

2. Methods
This research is a descriptive qualitative study that aims to reveal problem solving skills based on Krulik and Rudnick's steps reviewed from a thinking style about third-dimensional subject matter. The selection of subjects began with the provision of a thinking style questionnaire as a guideline for grouping students. This thought style questionnaire developed Gregorc theory of concrete sequential, abstract sequential, random concrete, random abstract. The thinking style questionnaire consists of 40 questions describing the four styles and has been validated by three experts in their field. The selection of research subjects was conducted by providing a thinking style questionnaire to 75 students of grade XII MIPA SMA MTA Surakarta, Central Java, Indonesia. From the test results, students were grouped according to their thinking style and selected four students representing each style of thinking. Prospective subjects selected as research subjects, can be seen in Table 1 below.
Data collection techniques are performed with problem solving tests and interviews. The data validity technique performed in this study is a triangulation method. The data analysis techniques used are reducing data, presenting data, and drawing conclusions.

3. Result and Discussion

3.1. Result

The three-dimensional problem solving test of UNBK Matematika IPA 2018 is "Akbar room in the shape of a beam with a long size: width : height= 5:5:4. On the ceiling of the room there is a lamp located right at the center of the ceiling field. On one of the walls of the room is a switch that is located right in the middle of the wall. The distance of the switch to the lamp is...".

3.1.1 Subject SK. Exposure of research results, mathematics problems on the third dimension with SK subjects.

Table 2. Comparison of problem solving math test (TPMM) and interview analysis based on Krulik and Rudnick's steps.

| No | Step Krulik and Rudnick | TPMM | Interview                        |
|----|-------------------------|------|----------------------------------|
| 1  | read and think          | writing down what is known | to mention known elements and be asked |
| 2  | explore and plan        | plan to draw blocks        | mentioning the steps to be implemented |
| 3  | select a strategy       | sketching the distance of the switch to the lamp | explaining the sketch of the distance of the switch to the drawn lamp |
| 4  | find and answer         | applying the pythagorean theorem | applying the pythagorean theorem |
| 5  | reflect and extend      | not re-checking the answers | still concerned with the answers obtained |

3.1.2. Subject SA. Exposure of research results, mathematics problems on the third dimension with SA subjects. SA subjects applying Krulik and Rudnick steps from no 1-4 are still the same as SK Subject, which distinguishes the fifth step which is the reflect and extend step.
Table 3. Comparison of problem-solving math test (TPMM) and interview analysis based on Krulik and Rudnick's steps.

| No | Step Krulik and Rudnick | TPMM | Interview |
|----|--------------------------|------|-----------|
| 1  | read and think           | writing down what is known | to mention known elements and be asked |
| 2  | explore and plan         | plan to draw blocks         | mentioning the steps to be implemented |
| 3  | select a strategy        | sketching the distance of the switch to the lamp | explaining the sketch of the distance of the switch to the drawn lamp |
| 4  | find and answer          | applying the pythagorean theorem | applying the pythagorean theorem |
| 5  | reflect and extend       | Re-checking and re-write the conclusion | Mention the conclusion of the answer obtained |

3.1.3. Subject AK. Exposure of research results, mathematics problems on the third dimension with AK subjects. AK subject applies all steps Krulik and Rudnick exactly the same as SK Subject.

Table 4. Comparison of problem-solving math test (TPMM) and interview analysis based on Krulik and Rudnick's steps.

| No | Step Krulik and Rudnick | TPMM | Interview |
|----|--------------------------|------|-----------|
| 1  | read and think           | writing down what is known | to mention known elements and be asked |
| 2  | explore and plan         | plan to draw blocks         | mentioning the steps to be implemented |
| 3  | select a strategy        | sketching the distance of the switch to the lamp | explaining the sketch of the distance of the switch to the drawn lamp |
| 4  | find and answer          | applying the pythagorean theorem | applying the pythagorean theorem |
| 5  | reflect and extend       | not re-checking the answers | still concerned with the answers obtained |

3.1.4. Subject AA. Exposure of research results, mathematics problems on the third dimension with AA subjects. AA subject applies Krulik and Rudnick steps from no 1-4 still the same as SK subject, SA subject, and AK subject. Krulik and Rudnick’s fifth step: reflect and extend, the AA subject is different from the other three subjects.
Table 5. Comparison of problem-solving math test (TPMM) and interview analysis based on Krulik and Rudnick's steps.

| No | Step Krulik and Rudnick | TPMM | Interview |
|----|-------------------------|------|-----------|
| 1  | read and think          | writing down what is known | to mention known elements and be asked |
| 2  | explore and plan        | plan to draw blocks         | mentioning the steps to be implemented |
| 3  | select a strategy       | sketching the distance of the switch to the lamp | explaining the sketch of the distance of the switch to the drawn lamp |
| 4  | find and answer         | applying the pythagorean theorem | applying the pythagorean theorem |
| 5  | reflect and extend      | re-checking the answers and write the wrong conclusion | still concerned with the answers obtained |

3.2. Discussion

In the reading and thinking stage, students who have concrete sequential thinking styles (SK), concrete randoms (AK), and abstract randoms (AA) write down what is known on the three-dimensional question, students who have abstract sequential thinking styles (SA) write down what is known about the three dimensions and what is asked. Abstract sequential subjects (SA) easily present what is known in the question to the mathematical symbol form as Djadiri et al, Zhang & Sternberg SA students easily present facts in the form of mathematical symbols [16], [17]. Abstract random subjects (AA) students experience errors in understanding the problem, this is according to Djadir et al students who have abstract random thinking styles (AA) in solving math problems showing fundamental differences in processing information [16].

In the explore and plan stage, students who have sequential concrete (SK), sequential abstract (SA), random concrete (AK), and random abstract (AA) draw blocks according to the size of the comparison known by the problem. SA subjects collect data in detail, this is in accordance with Gregorc which says that abstract sequential subjects (SA) collect data before making conclusions [15]. AA subjects plan problem solving in a way of their own accord, this is in accordance with the opinions Zhang & Sternberg, Lestanti et al and Gregorc which says that abstract random subjects (AA) perform in their own way [17], [18], [15].

The stage selects a strategy, students who have sequential concrete (SK), sequential abstract (SA), random concrete (AK), and random abstract (AA) draw sketches of elbows connecting the switch to the lamp. SK subjects work systematically, step by step, this is in accordance with the opinions Sa’diah, Djadiri et al, Lestanti et al and Gregorc which says that the subject is sequential concrete (SK) process information step by step or in detail [14], [16], [18], [15]. AK subjects tend to consistently use guessing or trialling strategies, this is in line with opinions Djadiri et al, Zhang & Sternberg and Gregorc which say that concrete random subjects (AK) tend to consistently use guessing or trialling strategies [16], [17], [15]. Guess and check was the weakest and least effective strategy [19]. According to Kolovou et al this strategy does not entail high cognitive demand and it is widely used in a variety of mathematical and everyday situations [20].

In the stage of finding answers, students who use sequential concrete (SK), sequential abstract (SA), random concrete (AK), and random abstract (AA) calculate the distance of the switch to the lamp by applying the theorem of pythagoras. SK subjects use this inductive mindset in accordance with the opinions of Djadiri et al students with concrete sequential (SK) tend to use a basic inductive mindset [16]. SA subjects tend to like abstract thinking and can manage information obtained regularly, this is in accordance with the opinions Djadiri et al, Zhang & Sternberg and Gregorc which means that abstract
sequential subjects (SA) have a tendency to grow information abstractly and manage that information regularly [16], [17], [15]. AK subjects tend to work in no detail and work on the subject on their own accord, this is in the opinion of Sa’diah and Gregorc who says that random subjects are less structured in solving problems and doing things their own way [14], [15].

The stage of reviewing and discussing, students who have concrete sequential thinking style (SK) and concrete random (AK) do not double check the answers obtained and do not write the conclusions of the answers obtained, this is according to Kusdinar et al in all categories, find and answer indicators are more often performed by students when exploring and planning indicators are still rare [21]. Students who have an abstract sequential thinking style (SA) double-check the answers obtained and write the conclusions of the answers obtained, this is in accordance with the opinion Sa’diah SA students have re-examined the stages performed and corrected the answers obtained [14]. Students with abstract random thinking styles (AA) double-check the answers and write incorrect answers, this is in accordance with the opinions of Lestanti et al students with type AA do not check the results of work step by step in detail to correct the truth of the answer [18].

4. Conclusion
能力 to solve math problems based on Krulik and Rudnik's steps is reviewed from a concrete sequential thinking style that is systematically worked out and uses an inductive mindset and the steps of reviewing and discussing are not implemented. Abstract sequential thinking styles are easy to present what is known in the question to mathematical symbol forms, collect data in detail, and manage information obtained regularly and perform all krulik and rudnick steps. Concrete random thinking styles are likely to consistently use guessing or trialling strategies and working in no detail and the steps of reviewing and discussing are not implemented. Abstract random thinking styles is to show fundamental differences in processing information and planning problem solving in a way of his own accord as well as doing all the steps Krulik and Rudnick.

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**Acknowledgments**
The authors thank all participants who assisted in the research data collection phase as well as Universitas Sebelas Maret and SMA MTA Surakarta to enable this research and the team of assessors who helped improve this research.