Quark - Resonance Model

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Abstract

We construct an effective Lagrangian for low energy hadronic interactions through an infinite expansion in inverse powers of the low energy cutoff $\Lambda_\chi$ of all possible chiral invariant non-renormalizable interactions between quarks and mesons degrees of freedom. We restrict our analysis to the leading terms in the $1/N_c$ expansion. The effective expansion is in $(\mu^2/\Lambda_\chi^2)^P \ln(\Lambda_\chi^2/\mu^2)^Q$. Concerning the next-to-leading order, we show that, while the pure $\mu^2/\Lambda_\chi^2$ corrections cannot be traced back to a finite number of non renormalizable interactions, those of order $(\mu^2/\Lambda_\chi^2) \ln(\Lambda_\chi^2/\mu^2)$ receive contributions from a finite set of $1/\Lambda_\chi^2$ terms. Their presence modifies the behaviour of observable quantities in the intermediate $Q^2$ region. We explicitly discuss their relevance for the two point vector currents Green’s function.

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1 Introduction.

Effective chiral Lagrangians have become a relatively powerful technique to describe hadronic interactions at low energy, i.e. below the chiral symmetry breaking scale \( \Lambda_\chi \approx 4\pi f_\pi \sim 1 \text{ GeV} \). Chiral perturbation theory (ChPt) \([1, 2]\) describes the low energy interactions among the pseudoscalar mesons \( \pi, K, \eta \), which are the lightest asymptotic states of the hadron spectrum and are identified with the Goldstone bosons of the broken chiral symmetry. The inclusion of resonance degrees of freedom in the model (vectors, axials, scalars, pseudoscalars and flavour singlets scalar and pseudoscalar) allows to describe the interactions of all the particles below \( \Lambda_\chi \) \([3, 4, 6, 7]\). This approach has a disadvantage connected with the non renormalizability of the effective low energy theory. The chiral expansion (i.e. the expansion in powers of derivatives of the low energy fundamental fields) results as an infinite sum over chiral invariant operators of increasing dimensionality. At each order in the chiral expansion the number of terms increases. The coefficient of each term is not fixed by chiral symmetry and the theory looses predictivity at higher orders. Many attempts have been done to reformulate the model in a more predictive fashion, both in the non anomalous \([6]\) and in the anomalous sector \([7]\) of the theory.

In particular, there have been attempts to derive the low energy effective theory from the more fundamental theory which describes the interactions of quarks and gluons. The first attempt to connect the low energy effective theory of pseudoscalar mesons and resonances with QCD has been proposed in \([8]\), where an application to strong interactions of the old and well known Nambu-Jona Lasinio (NJL) model \([9, 11]\) is made. The QCD Lagrangian is modified at long distances (i.e. below the cutoff \( \Lambda_\chi \)) by an effective 4-quarks interaction Lagrangian which remains chirally invariant.

The resonance and pseudoscalar mesons fields are introduced in the model through the bosonization of the fermions effective action.

The ENJL model proposed in \([8]\) includes only interaction terms which are leading in an expansion in inverse powers of the cutoff \( \Lambda_\chi \). This is a resonable approximation when we are interested in the behaviour of the effective theory for light mesons at a very low energy. Higher order terms bring powers of the derivative expansion term \( \partial/\Lambda_\chi \) which are indeed suppressed.

This is not the case in the intermediate and high energy region, i.e. throughout the resonance region, where non renormalizable power-like corrections arising from
higher order terms can be dominant. The ENJL is not the full answer in the intermediate $Q^2$ region, while it can be satisfactorily used to derive the effective Lagrangian of the pseudo-Goldstone bosons (pions) at $Q^2 = 0$, where the resonance degrees of freedom (effective at high $Q^2$) have been frozen out.

The paper is organized as follows. In section 2 we construct the quark-resonance model, which is the full effective model in the intermediate $Q^2$ region. In section 3 we derive explicitly all possible higher dimensional terms of order $1/\Lambda^2_\chi$. We explain how these new vertices enter in the derivation of the effective meson Lagrangian in section 4 by discussing in detail the case of the vector resonance Lagrangian. The presence of next-to-leading power - leading-log corrections (NPLL), i.e. of order $Q^2/\Lambda^2_\chi \ln \Lambda^2_\chi/Q^2$, is crucial in the running of the effective Lagrangian parameters at $Q^2 \neq 0$. The coefficients of the NPLL corrections arising in the QR model should be fixed by experimental data. In section 5 we concentrate on the case of the two-point vector correlation function, where we are able to extract significative informations on the $Q^2$ behaviour of the real part of the invariant functions from the existing data on the total $e^+e^-$ hadron cross section in the $I = 1$ channel. The results can be directly compared with the predictions obtained in the ENJL framework [12, 13]. The corrections improve the agreement with the experimental data significantly.

2 The model.

The effective quark models describing low energy strong interactions assume that the result of integrating over high frequency modes in the original QCD Lagrangian, defined above a given energy cutoff, can be expressed by additional non-renormalizable interactions.

For strong interactions the natural cutoff is the scale at which chiral symmetry spontaneously breaks: $\Lambda_\chi \simeq 1$ GeV. The cutoff sets the limit below which only the "low frequency modes" of the theory are excited.

The QCD Lagrangian for the low frequency modes is modified as follows:

$$\mathcal{L}_{QCD} \rightarrow \mathcal{L}_{QCD}^{\Lambda_\chi} + \mathcal{L}_{N.R}(n - \text{fermion}).$$

(1)

$\mathcal{L}_{QCD}^{\Lambda_\chi}$ is the standard QCD Lagrangian where only the low-frequency modes of quarks and gluons are present:

$$\mathcal{L}_{QCD}^{\Lambda_\chi} = \bar{q}(i\hat{D} - m_0)q.$$  

(2)
The second term in (1) is the most general non-renormalizable set of higher-di-
dimensional local n-fermion interactions which respect the symmetries of the original
theory and are suppressed at low energy by powers of \( Q^2/\Lambda^2 \).

Recently, the Nambu- Jona Lasinio (NJL) model has been reanalysed in a system-
atic way in the framework of hadronic low energy interactions [8]. Many applications
and reformulations can be found in [14].

The extended version of the NJL model (ENJL) includes in \( \mathcal{L}_{N.R.} \) all
the lowest dimension operators: 4-fermion local interactions which are leading in
the \( 1/N_c \) expansion (colour singlets) and respect all the symmetries of the original
theory (chiral symmetry, Lorentz invariance, P and C invariance). The form of the
effective Lagrangian is then uniquely determined:

\[
\mathcal{L}^{ENJL} = \mathcal{L}^{\Lambda} + \mathcal{L}^{SP} + \mathcal{L}^{VA},
\]

with

\[
\mathcal{L}^{SP} = \frac{8\pi^2 G_S(\Lambda)}{N_c \Lambda^2} \sum_{a,b} (\bar{q}_L^a q_R^b L)(\bar{q}_L^b q_R^a R),
\]

and

\[
\mathcal{L}^{VA} = \frac{8\pi^2 G_V(\Lambda)}{N_c \Lambda^2} \sum_{a,b} \left[ (\bar{q}_L^a \gamma_\mu q_R^b L)(\bar{q}_L^b \gamma_\mu q_R^a R) + L \rightarrow R \right].
\]

The current quarks \( q_{L,R} \) transform as \( q_{L,R} \rightarrow g_{L,R} q_{L,R} \) under the chiral flavour
group \( SU(3)_L \times SU(3)_R \), with elements \( g_{L,R} \). As pointed out in [8] the 4-quark
effective vertex can be thought as a remnant of a single "low frequency" gluon
exchange diagram (see fig. 1). The gluon propagator modified at high energy with
a cutoff

\[
\frac{1}{Q^2} \rightarrow \int_0^{\Lambda^2} d\tau e^{-\tau Q^2}
\]

leads to a local effective 4-quark interaction

\[
\frac{g_s^2}{\Lambda^2} \bar{q}^a \gamma_\mu \frac{\lambda^{(a)}}{2} q \bar{q}^b \gamma_\mu \frac{\lambda^{(b)}}{2} q.
\]

By means of the Fierz-identities one gets the \( S, P, V, A \) combinations of (4,5) with
the identification \( G_S = 4G_V \).
The non-renormalizable part of the fermion action $S_{NR}(q)$ admits an integral representation in terms of auxiliary boson fields:

$$e^{iS_{NR}[q]} = \int DB \ e^{iS[B,q]}.$$  \hfill (8)

The previous relation introduces the meson degrees of freedom into the effective quark Lagrangian. The following two identities hold:

$$\exp i \int d^4x \ L_{S,P}(x) = \int D H \ \exp i \int d^4x \left\{ -(\bar{q}_L H^\dagger q_R + \text{h.c.}) - \frac{N_c \Lambda^2}{8 \pi^2 G_S} \ tr(H H^\dagger) \right\}$$

$$\exp i \int d^4x \ L_{V,A}(x) = \int D L_\mu D R_\mu \ \exp i \int d^4x \left\{ \bar{q}_L \gamma_\mu L^\mu q_L + \frac{N_c \Lambda^2}{8 \pi^2 G_V} \ \frac{1}{4} tr(L^\mu L^\mu) + L \rightarrow R \right\},$$  \hfill (9)

where we have introduced three auxiliary fields: a scalar field $H(x)$ and the right-handed and left-handed fields $L_\mu$ and $R_\mu$. Under the chiral group they transform as:

$$H \rightarrow g_R H g_L^\dagger$$

$$L_\mu \rightarrow g_L L_\mu g_L^\dagger$$

$$R_\mu \rightarrow g_R R_\mu g_R^\dagger.$$  \hfill (10)

The field $H$ can be decomposed into the product of a new scalar field $M$ times a unitary field $U$:

$$H = MU = \xi \tilde{H} \xi,$$  \hfill (11)

where the field $\xi$ is the square root of the field $U$: $\xi^2 = U$. A connection with the physical fields is obtained by redefining the auxiliary fields as follows:

$$H = \xi \tilde{H} \xi$$

$$L_\mu^+ = \xi L_\mu \xi^\dagger + \xi^\dagger R_\mu \xi$$

$$L_\mu^- = \xi L_\mu \xi^\dagger - \xi^\dagger R_\mu \xi.$$  \hfill (12)

The new set of fields transforms homogeneously under chiral transformation:
\{ \bar{H}, W^+_\mu, W^-_\mu \} \rightarrow h \{ \bar{H}, W^+_\mu, W^-_\mu \} h^\dagger, \quad (13)

where \( h \) is a non-linear representation of the chiral group.

We redefine also the fermion fields by replacing the current quarks \( q_{L,R} \) with the constituent quarks:

\[
Q_L = \xi q_L \quad Q_R = \xi^\dagger q_R. \quad (14)
\]

They transform under the chiral group \( G = SU(3)_L \times SU(3)_R \) as:

\[
Q_L \rightarrow h(\Phi, g_L, g_R) Q_L \quad Q_R \rightarrow h(\Phi, g_L, g_R) Q_R, \quad (15)
\]

where the matrix \( h(\Phi, g_L, g_R) \) acts on the element \( \xi \) of the coset group \( G/SU(3)_V \):

\[
\xi(\Phi) \rightarrow g_R \xi(\Phi) h^\dagger = h \xi(\Phi) g^\dagger_L. \quad (16)
\]

The quark field \( Q \) is defined as \( Q = Q_L + Q_R \).

In terms of the new variables the bosonized euclidean generating functional of the ENJL model reads:

\[
Z[v, a, s, p] = \int D\xi \ D\bar{H} \ D\mu \ D\nu \ e^{-\Gamma_{\text{eff}}[\xi, W^+_\mu, W^-_\mu; v, a, s, p]} \quad (17)
\]

where we have defined the total differential operator \( D_E \) as follows:

\[
D_E = \gamma_\mu \nabla_\mu - \frac{1}{2}(\Sigma - \gamma_5 \Delta) - \bar{H}(x), \quad (18)
\]

with the covariant derivative acting on the chiral quark field given by:

\[
\nabla_\mu = \partial_\mu + iG_\mu + \Gamma_\mu - \frac{i}{2}W^+_\mu - \frac{i}{2}\gamma_5(\xi_\mu - W^-_\mu). \quad (19)
\]

The field \( \Gamma_\mu \) acts like a vector field and is defined by:
\[
\Gamma_\mu = \frac{1}{2} \{ \xi^\dagger d_\mu \xi + \xi d_\mu \xi^\dagger \} = \frac{1}{2} \{ \xi^\dagger [\partial_\mu - i(v_\mu + a_\mu)]\xi + \xi [\partial_\mu - i(v_\mu - a_\mu)]\xi^\dagger \}. \tag{20}
\]

It transforms inhomogeneously under the local vector part of the chiral group

\[
\Gamma_\mu \rightarrow h \Gamma_\mu h^\dagger + h \partial_\mu h^\dagger \tag{21}
\]
and makes the derivative on the \( Q \) field invariant under local vector transformations. The field \( \xi_\mu \) is like an axial current and is defined by:

\[
\xi_\mu = i \{ \xi^\dagger d_\mu \xi - \xi d_\mu \xi^\dagger \} = i \{ \xi^\dagger [\partial_\mu - i(v_\mu + a_\mu)]\xi - \xi [\partial_\mu - i(v_\mu - a_\mu)]\xi^\dagger \} = \xi^\dagger_\mu. \tag{22}
\]

It transforms homogeneously under the chiral group \( G \):

\[
\xi_\mu \rightarrow h \xi_\mu h^\dagger. \tag{23}
\]

The fields \( \Sigma \) and \( \Delta \) are defined by:

\[
\Sigma = \xi^\dagger \mathcal{M} \xi^\dagger + \xi \mathcal{M} \xi \\
\Delta = \xi^\dagger \mathcal{M} \xi^\dagger - \xi \mathcal{M} \xi. \tag{24}
\]

They are both proportional to the quark mass matrix \( \mathcal{M} \) and vanish in the chiral limit. The field \( \tilde{H}(x) \) is the auxiliary scalar field of the bosonized action and can be parametrised as

\[
\tilde{H}(x) = M_Q 1 + \sigma(x), \tag{25}
\]

where we have split the \( \tilde{H} \) field into its vacuum expectation value and the fluctuation around it. The quantity \( M_Q \) is the value of the \( \tilde{H}(x) \) field (used in the so called mean field approximation of the ENJL model) which minimizes the effective action in absence of other external fields:

\[
\frac{\delta \Gamma_{eff}(\tilde{H}, \ldots)}{\delta \tilde{H}} \bigg|_{\xi=1, W_\mu^+ = W_\mu^-, v, a, s, p = 0, \tilde{H} = \langle \tilde{H} \rangle} = 0. \tag{26}
\]

\( M_Q \neq 0 \) corresponds to broken chiral symmetry \cite{15}. Its value is the solution of the mass gap equation generated by (26).
In the leading effective action (17) two constants appear: the scalar coupling \( G_S \) and the vector coupling \( G_V \). They are functions of the cutoff \( \Lambda_\chi \) and their estimate involves non-perturbative contributions.

The fundamental fields of the bosonized action are \( \xi_\mu, \Gamma_\mu, W^+_\mu, W^-_\mu, \tilde{H} \).

A full effective quark model à la NJL contains a priori an infinite tower of n-fermion operators with increasing dimensionality: the ENJL 4-fermion interactions are the leading terms both in \( 1/\Lambda \chi \) and \( 1/N_c \) expansions.

The QR model is the bosonization of the full effective quark model à la NJL. The resulting quark-resonance Lagrangian is a non-renormalizable Lagrangian which contains all possible interaction terms between quarks and resonances. The QCD euclidean generating functional of the correlation functions at low energy is modified as follows:

\[
Z[v, a, s, p] = e^{W[v, a, s, p]} = \int \mathcal{D}\Gamma \ e^{-\Gamma_{\text{eff}}[\Gamma; v, a, s, p]} e^{-f[\Gamma]},
\]

where \( \Gamma \) indicates the set of fields introduced in the bosonization of the QCD effective action and the effective low energy action \( \Gamma_{\text{eff}} \) is given by

\[
e^{-\Gamma_{\text{eff}}[\Gamma; v, a, s, p]} = \frac{1}{Z} \int DG \exp \left( - \int d^4x \frac{1}{4} C_{\mu \nu}^{(a)} G_{\mu \nu}^{(a)} \right) \int DQD\bar{Q} \exp \left[ \int d^4x \left( \bar{Q} \gamma^\mu (\partial_\mu + iG_\mu) Q + \sum_0^\infty \left( \frac{1}{\Lambda_\chi} \right)^n \bar{Q} \Gamma Q \right) \right].
\]

The functional \( f[\Gamma] \) in eq. (27) contains the mass terms of the auxiliary boson fields.

The second term in (28) is the non-renormalizable part of the action expressed as an infinite sum over all chiral invariant quark bilinears interacting with the low energy boson degrees of freedom. The most general structure of the \( \Gamma \) operator can be represented by:

\[
\Gamma = \beta(\Lambda_\chi) \times \{ \gamma_{\text{Dirac}} \} \times \{ \Gamma_\mu, \xi_\mu, W^+_\mu, W^-_\mu, \sigma \} \times d_\mu^n,
\]

where the coupling \( \beta(\Lambda_\chi) \) is not deducible from symmetry principles and not calculable in a perturbative way. \( d_\mu \) defines the covariant derivative acting on the meson fields or on the chiral quark fields and the set \( \{ \Gamma_\mu, \xi_\mu, W^+_\mu, W^-_\mu, \sigma \} \) contains all possible boson fields which can couple to the quark bilinears and which can be identified with the physical degrees of freedom of the low energy effective theory:
pseudoscalar mesons and resonances. As it is shown in detail in ref. [8], the integration over quark fields induces a mixing between the axial field $W_\mu$ and the pseudoscalar field $\xi_\mu$ and a diagonalization is required to define the true physical axial and pseudoscalar meson fields. Our Lagrangian at leading order in the $1/\Lambda_\chi$ expansion and in the $1/N_c$ expansion coincides with the bosonization of the ENJL model of eq. (17).

The additional quark-meson interaction terms originate from the bosonization of non-renormalizable n-quark vertices. These can be of two types:

(I) 4-quark $\times$ derivatives

(II) n (>4) - quark $\times$ derivatives

The higher dimension terms with n-quarks can be easily constructed using the chiral invariant building blocks of the ENJL model:

$$\bar{q} \hat{D}q = \bar{q}_L \hat{D}q_L + \bar{q}_R \hat{D}q_R$$

$$\bar{q}q - (\bar{q}\gamma_5 q)^2 = 4\bar{q}_L q_R \bar{q}_R q_L = S^2 - P^2$$

$$\bar{q} \gamma_\mu q)^2 + (\bar{q} \gamma_\mu \gamma_5 q)^2 = 2[(\bar{q}_L \gamma_\mu q_L)^2 + (\bar{q}_R \gamma_\mu q_R)^2] = V^2 + A^2$$

There is one chiral invariant term with four quarks, which does not appear in the ENJL model:

$$\mathcal{L}_\sigma = C(\bar{q}^a \sigma_{\mu\nu} q^b)(\bar{q}_b \sigma^{\mu\nu} q_a).$$

This term is a magnetic moment like term and contributes to the $g - 2$ of the muon, which however is next-to-leading (i.e. $O(1)$) in the $1/N_c$ expansion [10] and will be neglected. All possible higher order terms can be constructed as combinations of arbitrary powers of the basic building blocks of eq. (31) times the insertion of powers of the differential operator $\partial^2/\Lambda_\chi^2$.

The bosonization of the most general n-fermion action ends up in the most general quark-resonance action (28) constrained only by chiral symmetry.

The possible relevance of additional non-renormalizable terms in the scalar sector of the NJL model has been already pointed out in [11]. They modify the mass-gap equation and can be incorporated in a renormalization of the scalar coupling $G_S$. 


or alternatively of the expectation value of the scalar field $M_Q$ which minimizes the effective potential.

The effective meson theory is given by the integral over quarks and gluons of the Lagrangian (28). By neglecting the gluon corrections, which are inessential to our argument, the derivation of the low energy theory reduces to the integral over quarks of the quark-resonance effective Lagrangian:

$$\int\int \mathcal{D}Q\mathcal{D}\bar{Q} \exp \left[ \int d^4x \left( \bar{Q}\gamma^\mu (\partial_\mu + iG_\mu)Q + \sum_0^\infty \left( \frac{1}{\Lambda^2} \right)^n \bar{Q}\Gamma Q \right) \right] = \det \left[ \hat{D}_0 + \sum_0^\infty \left( \frac{1}{\Lambda^2} \right)^n \Gamma \right],$$

where $\hat{D}_0 = \gamma^\mu (\partial_\mu + iG_\mu)$ is the free fermion operator. The integral corresponds to the set of all one quark-loop diagrams which mediate the interactions among the meson fields as shown in fig. 2. Higher order terms contain factors of $\partial^2/\Lambda^2$. The derivative acts: a) on the quark in the loop, b) on the external boson legs. In both cases it produces powers of the external momenta in the diagram.

In order to clarify how these contributions enter in the effective meson Lagrangian, we consider the one quark-loop diagram constructed with the insertion of two leading vertices involving the vector field $\bar{Q}\gamma_\mu W_\mu Q$. This diagram gives the leading contribution to the vector wave function.

The next-to-leading diagram is given by the insertion of a next-to-leading vertex of the form $\frac{1}{\Lambda^2} \bar{Q}\gamma_\mu W_\mu Q$ and a leading vertex. This gives next-to-leading corrections to the vector wave function which contain powers of the ratio $Q^2/\Lambda^2$. The $Q^2$ dependence of next-to-leading terms affects the running in energy of the parameters of the effective meson Lagrangian.

In what follows we concentrate on the next-to-leading vertices (up to $\frac{1}{\Lambda^2}$) of the quark-resonance model and on their contributions to the parameters of the vector resonance Lagrangian which are leading in the chiral expansion.

The analysis shows two basic features:

- Next-to-leading corrections increase the number of independent parameters present at the leading order, so that higher order contributions cannot be reabsorbed in a redefinition of the leading parameters. This implies that relations among resonance parameters valid at zero energy (i.e. at the leading
order) are modified when the energy increases (i.e. including next-to-leading corrections).

- There are two types of next-to-leading corrections:
  
i) NPLL corrections = \( \frac{Q^2}{\Lambda^2} \chi \ln \frac{\Lambda^2}{Q^2} \)
  
ii) NTL power corrections without logarithms (NP) = \( \frac{Q^2}{\Lambda^2} \)

The first class receives contribution from a finite set of higher dimension operators (only \( \frac{1}{\Lambda^2} \) terms) and can be easily kept under control.

Chiral symmetry does not constrain the coefficients \( \beta(\Lambda, \chi) \) and one has to fix them from experimental data. We defer to section (5) a discussion of the particular case of the vector 2-point function.

3 The Lagrangian up to \( \frac{1}{\Lambda^2} \) order.

The invariant quark-resonance bilinears up to \( \frac{1}{\Lambda^2} \) order are all the possible quark-resonance bilinears which are chirally invariant and respect all the usual symmetries: Parity, Charge conjugation and Lorentz invariance. We report in Table 1 and Table 2 the P and C transformation properties of the quark bilinears and the fundamental fields in the \( \Gamma \) set respectively. If we are interested in producing the effective Lagrangian of meson resonances at leading order in the chiral expansion only two classes of quark-resonance bilinears give contribution:

\[
(I) \quad \left( \frac{1}{\Lambda^2} \right)^n \times 1 \times b \times n \times d \\
(II) \quad \left( \frac{1}{\Lambda^2} \right)^n \times 2 \times (n - 1) \times d,
\]

with \( b = \)boson field and \( d = \)covariant derivative. These are also the only possible terms that appear at order \( 1/\Lambda^2 \).

We work in the chiral limit and we set to zero all terms proportional to the mass fields \( \Sigma \) and \( \Delta \). As it is shown in detail in ref. [8] the integration over quarks induces a mixing between the pseudoscalar field \( \xi_{\mu} \) and the axial field \( W_{\mu}^- \) which is leading in the chiral expansion. The physical fields are obtained after a diagonalization of the quadratic matrix. In the ENJL model this corresponds to a rescaling of the
pseudoscalar field by the mixing parameter $g_A$, which the authors of ref. connect to the $g_A$ parameter of the effective quark-model by Georgi-Manohar. In the QR model the mixing parameter $g_A$ receives higher order corrections: the physical pseudoscalar field is given by the rescaling:

$$\xi_\mu \rightarrow g'_A \xi_\mu$$

with the new mixing parameter $g'_A$. Each insertion of a field $\xi_\mu$ brings an insertion of the mixing parameter $g'_A$.

The leading terms of the ENJL model have a logarithmic dependence upon the cutoff $\Lambda_\chi$. Terms without logarithms can receive contributions from all higher order terms. Indeed, besides the finite contributions of the leading renormalizable operators, higher dimensions non-renormalizable operators differing from the leading ones by powers of derivatives may develop divergences that, integrated up to the cutoff $\Lambda_\chi$, compensate the inverse powers of $\Lambda_\chi$ and contribute as constant terms. The same happens to the terms which are of order $1/\Lambda_\chi^2$ in the final low energy meson Lagrangian: those accompanied by logarithms can be traced back to terms of order $1/\Lambda_\chi$ and $1/\Lambda_\chi^2$ in the original quark-resonance Lagrangian while those without logarithms are determined by the whole tower of non-renormalizable interactions.

At the order $1/\Lambda_\chi$ we have the following terms:

$$\frac{1}{\Lambda_\chi} \times 1 \ b \times 1 \ d : \frac{1}{\Lambda_\chi} \times Q \left[ \sigma_{\mu\nu} W^{+\mu\nu} + \sigma_{\mu\nu}(W^{+\mu} d^\nu - W^{+\nu} d^\mu) \right] Q,$$

$$\frac{1}{\Lambda_\chi} \times Q \sigma_{\mu\nu} \Gamma^{\mu\nu} Q,$$

$$\frac{1}{\Lambda_\chi} \times 2 \ b : \frac{1}{\Lambda_\chi} \times Q \left[ W_{\lambda}^{+} W^{+\lambda} + \sigma_{\mu\nu}[W^{+\mu}, W^{+\nu}] \right] Q,$$

$$\frac{1}{\Lambda_\chi} \times Q \left[ \xi_\lambda \xi^\lambda + \sigma_{\mu\nu}[\xi_\mu, \xi_\nu] \right] Q,$$

where $\Gamma^{\mu\nu}$ is defined as $\Gamma_{\mu\nu} = \partial_\mu \Gamma_\nu - \partial_\nu \Gamma_\mu + [\Gamma_\mu, \Gamma_\nu]$. Terms containing the combination $(\Gamma^{\mu} d^\nu - \Gamma^{\nu} d^\mu)$ with derivatives acting on quark fields are forbidden by chiral invariance because of the inhomogeneous transformation of the $\Gamma_\mu$ field. The $\sigma_{\mu\nu}$ terms come from the bosonization of a 4-fermion vertex like the one in (3) with $\gamma_\mu$ replaced by the tensor $\sigma_{\mu\nu}$ and are negligible in a leading $N_c$ expansion.

All possible invariants at $1/\Lambda_\chi^2$ order are terms with one and two meson fields. For simplicity, we write explicitly only the terms containing the fields $W^{+\mu}_\mu$ (vector...
resonance) and \( \xi_\mu \) (the axial current of pseudoscalar mesons), which we will use in the next section. They are:

\[
\begin{align*}
\frac{1}{\Lambda_\chi^2} \times 1 b \times 2d : & \quad \frac{1}{\Lambda_\chi^2} \times Q \left[ \gamma_\mu d_\lambda W^{+\mu \lambda} + \gamma_\mu W^{+\mu \lambda} d_\lambda + \gamma_\mu (W^{+\mu} d^\lambda - W^{+\lambda} d^\mu) d_\lambda \right] Q, \\
\frac{1}{\Lambda_\chi^2} \times \bar{Q} \left[ \gamma_\mu \gamma_5 d_\lambda \xi^{\mu \lambda} + \gamma_\mu \gamma_5 \xi^{\mu \lambda} d_\lambda + \gamma_\mu \gamma_5 (\xi^{\mu} d^\lambda - \xi^\lambda d^\mu) d_\lambda \right] Q,
\end{align*}
\]

\[
\begin{align*}
\frac{1}{\Lambda_\chi^2} \times 2 b \times 1d : & \quad \frac{1}{\Lambda_\chi^2} \times Q \left[ \gamma_\mu [W^{+}_\nu, W^{+\mu \nu}] + \gamma_\mu [W^{+}_\nu, W^{+\mu} d^\nu - W^{+\nu} d^\mu] \right] Q, \\
\frac{1}{\Lambda_\chi^2} \times Q \left[ \gamma_\mu [\xi_\nu, \xi^{\mu \nu}] + \gamma_\mu [\xi_\nu, \xi^{\mu} d^\nu - \xi^\nu d^\mu] \right] Q, \\
\frac{1}{\Lambda_\chi^2} \times \bar{Q} \left[ \gamma_\mu \gamma_5 [\xi_\nu, W^{+\mu \nu}] + \gamma_\mu \gamma_5 [W^{+}_\nu, \xi^{\mu \nu}] + \gamma_\mu \gamma_5 [\xi_\nu, W^{+\mu} d^\nu - W^{+\nu} d^\mu] \right] \\
+ \gamma_\mu \gamma_5 [W^{+}_\nu, \xi^{\mu} d^\nu - \xi^\nu d^\mu] \right] Q.
\end{align*}
\]

(37)

To these terms we have to add all the analogous terms with the substitutions \( W^{+} \rightarrow \Gamma_{\mu \nu} \) and \( \xi_\mu \rightarrow A_\mu \), containing the axial resonance field \( A_\mu \) and the vector current of the pseudoscalar mesons \( \Gamma_{\mu \nu} \). In the next section we derive the next to leading corrections to the parameters of the leading chiral vector resonance Lagrangian.

### 4 The Vector Meson Lagrangian.

The leading non anomalous Lagrangian with one vector meson \( \mathcal{L}_V \) (i.e. of order \( p^3 \)) is the following:

\[
\mathcal{L}_V = -\frac{1}{4} < V_{\mu \nu} V^{\mu \nu} > + \frac{1}{2} M_V^2 < V_\mu V^\mu > - \frac{f_V}{2\sqrt{2}} < V_\mu f_+^{\mu \nu} > \\
- \frac{i g_V}{2\sqrt{2}} < V_{\mu \nu} [\xi^\mu, \xi^\nu] > + H_V < V_\mu [\xi_\nu, f_-^{\mu \nu}] > + i I_V < V_\mu [\xi_\mu, \chi_-] >.
\]

(38)

The form above corresponds to the so called Conventional Vector model \cite{3, 4} where the vector fields are introduced as ordinary fields. This is the natural form for the effective low energy theory after the bosonization of four-fermion interactions. In the chiral limit the \( I_V \) term is zero and the Lagrangian is parametrised by five constants: the vector resonance wave function \( Z_V \), the mass \( M_V \) and the coupling constants \( f_V, g_V \) and \( H_V \).
The ENJL estimate of the five parameters has been already derived in \cite{8, 18} using the heat kernel expansion technique in the calculation of the fermion determinant. We rederive both the leading and non-leading contributions using the loopwise expansion. If we write the fermion differential operator as a sum of a free part $D_0$ and of a perturbation $\delta$, the euclidean effective action at one loop is given by:

$$\Gamma_{\text{eff}}(\delta) = -Tr \ln [D_0 + \delta] + Tr \ln D_0 = -Tr D_0^{-1}\delta + \frac{1}{2} Tr D_0^{-1}\delta D_0^{-1}\delta + ....,$$  \hspace{1cm}(39)$$

where we have subtracted its value at $\delta = 0$.

The various terms on the r.h.s. are identified by the order $n$ in the series expansion of the logarithm. The term $Tr D_0^{-1}\delta$ ($n=1$) contains the tadpole graphs. The next term ($n=2$) contains the set of graphs with the insertion of two vertices in the loop. The contributions to the parameters of $L_V$ arise from the $n = 2$ and $n = 3$ insertions of vertices in the perturbative expansion, corresponding to the insertion of three meson fields at most.

At the leading order and in the chiral limit $\delta$ is given by:

$$\delta = \delta_0 = \gamma_{\mu}[\Gamma_{\mu} - \frac{i}{2} W^+_\mu - \frac{i}{2} \gamma_5(\xi_{\mu} - A_\mu)],$$  \hspace{1cm}(40)$$

and the free part $D_0$ is

$$D_0 = \gamma_{\mu}(\partial_\mu + iG_\mu - M_Q).$$  \hspace{1cm}(41)$$

The mass term $M_Q$ acts as an infrared cutoff in quark loop diagrams.

The complete operator $\delta$ is the sum of the leading part $\delta_0$ defined in (40) and the non leading contributions in the $1/\Lambda_\chi$ expansion:

$$\delta = \delta_0 + \sum_{n=1}^{\infty} \left(\frac{1}{\Lambda_\chi}\right)^n \Gamma.$$  \hspace{1cm}(42)$$

In Appendix A the single quark-loop diagrams for $n=2$ are explicitly calculated with the insertion of a generic form of the operator $\delta(x)$. Using the formulas reported there one can get the contribution to a given parameter of the vector Lagrangian with the substitution of the appropriate operator $\delta(x)$. The next order ($n=3$) is a straightforward generalization of the previous case and it will not enter in the calculation of the parameters that are analysed in detail in sections 4.3 and 5.

We distinguish two classes of terms in the $L_V$ Lagrangian: the kinetic term $Z_V$, the mass term $M_V$ and the interaction term $f_V$ belong to the first class and contain
two meson fields plus derivatives, while the $g_V$ and $H_V$ terms belong to the second class and contain three fields plus derivatives. The $H_V$ term is a three fields term because of the identity $f_{\mu\nu} = \xi_{\mu\nu} = d\mu\xi_{\nu} - d\nu\xi_{\mu}$, while the $g_V$ term belongs also to the two fields class because of the identity

$$\Gamma_{\mu\nu} = -\frac{i}{2} f^+_{\mu\nu} + \frac{1}{4}[\xi_{\mu}, \xi_{\nu}].$$

If the $g_V$ term receives contributions only through the combination $\Gamma_{\mu\nu}$, the relation $f_{\nu} = 2g_V$ (many times discussed in the literature [19, 7, 8]) remains valid. We summarize the two classes discussed above:

Class I : $Z_V, M_V, f_V, g_V = 2 b \times 2 d$
Class II : $H_V, I_V, g_V = 3 b \times 1 d$, (44)

where $g_V$ belongs to both classes by virtue of the identity (43). Class I receives contribution from a two operators diagram (n=2), which has the insertion of two boson fields at most, while the class II receives in general contribution from terms with n=2 and n=3 of the loop expansion.

### 4.1 The Leading contributions

The leading contributions to the parameters in eq. (44) are obtained by the $\delta_0$ insertion for n=2 and n=3 of the perturbative expansion. The class I receives contribution from n=2, while the class II receives contribution from n=3. The diagrams are the following:

\[
\begin{align*}
  n = 2 & \Rightarrow 1 b \times 1 b \\
  n = 3 & \Rightarrow 1 b \times 1 b \times 1 b
\end{align*}
\]

(45)

The leading divergent contribution to the vector wave function $Z_V$ in a momentum cutoff regularization scheme is the following:

$$Z_V = \frac{N_c}{16\pi^2} \int_0^1 d\alpha\alpha(1 - \alpha) \ln \frac{\Lambda^2}{s(\alpha)}.$$  (46)

The values of the mass and the coupling constants are obtained after imposing the correct normalization of the kinetic term of the vector Lagrangian, which defines the physical vector field as:
\[ V_{\mu} = \sqrt{Z_V} W_{\mu}^+ \]  

(47)

The mass of the vector meson is given by the mass term of the ENJL action of eq. (17) rescaled by the wave function:

\[ M_V^2 = \frac{N_c}{16\pi^2} \left( \frac{\Lambda^2}{2G_V} \right) \frac{1}{Z_V}. \]  

(48)

The leading divergent contributions to the five parameters of the vector Lagrangian are given by:

\[
\begin{align*}
Z_V &= \frac{N_c}{16\pi^2} 2 \int_0^1 d\alpha \alpha (1 - \alpha) \ln \frac{\Lambda^2}{s(\alpha)} \\
M_V^2 &= \frac{N_c}{16\pi^2} \left( \frac{\Lambda^2}{2G_V} \right) \frac{1}{Z_V} \\
f_V &= \sqrt{2} \sqrt{Z_V} \\
g_V &= \frac{N_c}{16\pi^2} \sqrt{2} (1 - g_A^2) \frac{1}{\sqrt{Z_V}} \int_0^1 d\alpha \alpha (1 - \alpha) \ln \frac{\Lambda^2}{s(\alpha)} \\
H_V &= -i \frac{N_c}{16\pi^2} g_A^2 \frac{1}{\sqrt{Z_V}} \int_0^1 d\alpha \alpha (1 - \alpha) \ln \frac{\Lambda^2}{s(\alpha)}.
\end{align*}
\]  

(49)

The function \( s(\alpha) \) is equal to \( M_Q^2 + \alpha (1 - \alpha) l^2 \) and depends explicitly upon the external momentum \( l^2 \). If we set it to zero, we produce the low energy limit of the ENJL model derived in [8], where the values of the parameters are the following:

\[
\begin{align*}
Z_V &= \frac{N_c}{16\pi^2} \frac{1}{3} \ln \frac{\Lambda^2}{M_Q^2} \\
M_V^2 &= \frac{N_c}{16\pi^2} \left( \frac{\Lambda^2}{2G_V} \right) \frac{1}{Z_V} \\
f_V &= \sqrt{2Z_V} \\
g_V &= \frac{N_c}{16\pi^2} \frac{\sqrt{2}}{6} (1 - g_A^2) \frac{1}{\sqrt{Z_V}} \ln \frac{\Lambda^2}{M_Q^2} \\
H_V &= -i \frac{N_c}{16\pi^2} g_A^2 \frac{1}{6\sqrt{Z_V}} \ln \frac{\Lambda^2}{M_Q^2}.
\end{align*}
\]  

(50)

They coincide with the ones calculated in [8] in the proper time regularization scheme, where one has to use the expression of the incomplete Gamma function \( \Gamma(0, x = \frac{M_Q^2}{\Lambda^2}) = -\ln x - \gamma_E + \mathcal{O}(x) \) for small values of \( x \).
The leading contributions to the parameters of the vector meson Lagrangian are all logarithmic. Furthermore the five parameters are not all independent. They can be expressed in terms of three of the input parameters of the ENJL model:

\[ x = \frac{M_Q^2}{\Lambda^2}, \quad G_V, \quad g_A. \]  

(51)

As we will see in the next section this reduction of the number of independent parameters is no more valid at next-to-leading order.

### 4.2 The Next-to-Leading contributions

As already discussed, the insertion of higher dimension vertices in the \(1/\Lambda^2\) expansion generates two types of next-to-leading corrections to the parameters of the vector meson Lagrangian:

- leading log power corrections = \(\frac{Q^2}{\Lambda^2} \ln \frac{\Lambda^2}{Q^2}\)

- power corrections = \(\frac{Q^2}{\Lambda^2} \cdot 1\)

We restrict to the first type of corrections only which come from a finite set of diagrams constructed with the insertion of one \(\frac{1}{\Lambda^2}\) vertex.

Using the formulae in Appendix A one can derive the following results. The insertion of \(\delta(x)\) vertices with no derivatives on quark fields, i.e. contributing as internal quark loop momenta (Case 1. in Appendix A), contributes to the logarithms of the vector parameters as follows:

\[
\begin{align*}
1 & \Rightarrow \ln \frac{\Lambda^2}{Q^2} = \text{leading} \\
\frac{1}{\Lambda^2} & \Rightarrow \frac{Q^2}{\Lambda^2} \ln \frac{\Lambda^2}{Q^2} \\
\frac{1}{\Lambda^4} & \Rightarrow \left(\frac{Q^2}{\Lambda^2}\right)^2 \ln \frac{\Lambda^2}{Q^2} \\
\end{align*}
\]  

(52)

The insertion of \(\delta(x)\) vertices with one derivative on quark fields (Case 2. in Appendix A) contributes as follows:
\[
\begin{align*}
1 & \Rightarrow 0 \\
\frac{1}{\Lambda^2} & \Rightarrow \frac{Q^2}{\Lambda^2} \ln \frac{\Lambda^2}{Q^2} \\
\frac{1}{\Lambda^4} & \Rightarrow \left(\frac{Q^2}{\Lambda^2}\right)^2 \ln \frac{\Lambda^2}{Q^2} \\
\end{align*}
\]

(53)

The insertion of \(\delta(x)\) vertices with two derivatives on quark fields (Case 3. in Appendix A) contributes similarly to the previous case:

\[
\begin{align*}
1 & \Rightarrow 0 \\
\frac{1}{\Lambda^2} & \Rightarrow \frac{Q^2}{\Lambda^2} \ln \frac{\Lambda^2}{Q^2} \\
\frac{1}{\Lambda^4} & \Rightarrow \left(\frac{Q^2}{\Lambda^2}\right)^2 \ln \frac{\Lambda^2}{Q^2} \\
\frac{1}{\Lambda^6} & \Rightarrow \left(\frac{Q^2}{\Lambda^2}\right)^3 \ln \frac{\Lambda^2}{Q^2} \\
\end{align*}
\]

(54)

More than two derivatives produce terms at least of order \((Q^2/\Lambda^2)^2\ln(\Lambda^2/Q^2)\) and are beyond our NPLL corrections which get contribution only from vertices which are at most \(1/\Lambda^2\) and with at most two derivatives on quark fields.

In order to determine how many independent parameters we are left with after the inclusion of non-renormalizable interactions (NRI) in the quark-meson Lagrangian, we analyse the corresponding vertices that give contribution to the five parameters of the Lagrangian \(L_V\) at next-to-leading order.

For \(n=2\) the sets of pairs \((a, b)\) of vertices \(\{V_1^a \times V_1^b, V_2^a \times V_2^b\ldots\}\) contributing to each independent parameter are the following:

\[
\begin{align*}
Z_V & \Leftrightarrow \left\{-\frac{i}{2}\gamma^\mu W^{+\mu} \times \frac{1}{\Lambda^2} \left[\gamma_\mu W^{+\mu} + \gamma_\mu W^{+\mu} d_{\lambda} + \gamma_\mu (W^{+\mu} d^{\lambda} - W^{+\lambda} d^{\mu}) d_{\lambda}\right]\right\} \\
f_V & \Leftrightarrow \left\{\gamma_\mu \Gamma^\mu \times \frac{1}{\Lambda^2} \left[\gamma_\mu W^{+\mu} d_{\lambda} + \gamma_\mu W^{+\mu} d_{\lambda} + \gamma_\mu (W^{+\mu} d^{\lambda} - W^{+\lambda} d^{\mu}) d_{\lambda}\right], \right. \\
& \left. \frac{i}{2}\gamma_\mu W^{+\mu} \times \frac{1}{\Lambda^2} \left[\gamma_\mu d_{\lambda} \Gamma^{\mu\lambda} + \gamma_\mu \Gamma^{\mu\lambda} d_{\lambda}\right]\right\} \\
g_V & \Leftrightarrow \left\{\text{those of} f_V\right\}, \left\{\frac{i}{2}\gamma_\mu W^{+\mu} \times \frac{1}{\Lambda^2} \gamma_\mu [\xi^{\nu}, \xi^{\mu} d^{\nu} - \xi^{\nu} d^{\mu}], \right\}
\end{align*}
\]
\[
\frac{i}{2} \gamma_\mu \gamma_5 \xi^\mu \times \frac{1}{\Lambda^2_\chi} \left[ \gamma_\mu \gamma_5 [\xi_\nu, W^{+\mu\nu}] + \gamma_\mu \gamma_5 [\xi_\nu, W^{+\mu} d^\nu - W^{+\nu} d^\mu] \right] \\
+ \gamma_\mu \gamma_5 [W^{+\mu}_\nu \times \xi^\mu d^\nu - \xi^\nu d^\mu] \right] \\
H_V \Leftrightarrow \left\{ \frac{i}{2} \gamma_\mu W^{+\mu} \times \frac{1}{\Lambda^2_\chi} \left[ \gamma_\mu [\xi_\nu, \xi^{\mu\nu}] + \gamma_\mu [\xi_\nu, \xi^\mu d^\nu - \xi^\nu d^\mu] \right], \right. \\
\left. \frac{i}{2} \gamma_\mu \gamma_5 \xi^\mu \times \frac{1}{\Lambda^2_\chi} \left[ \gamma_\mu \gamma_5 [W^{+\mu}_\nu, \xi^{\mu\nu}] + \gamma_\mu \gamma_5 [\xi_\nu, W^{+\mu} d^\nu - W^{+\nu} d^\mu] \right] \\
+ \gamma_\mu \gamma_5 [W^{+\mu}_\nu, \xi^\mu d^\nu - \xi^\nu d^\mu] \right\}. \tag{55}
\]

The diagrams with \( n = 3 \) at \( 1/\Lambda^2_\chi \) order give contributions to \( g_V \) and \( H_V \) only:

\[
g_V \Leftrightarrow \left\{ \frac{i}{2} \gamma_\mu W^{+\mu} \times \frac{i}{2} \gamma_\mu \gamma_5 \xi^\mu \times \frac{1}{\Lambda^2_\chi} \gamma_\mu \gamma_5 (\xi^\mu d^\lambda - \xi^\lambda d^\mu) d_\lambda, \\
\left( \frac{i}{2} \gamma_\mu \gamma_5 \xi^\mu \right)^2 \times \frac{1}{\Lambda^2_\chi} \left[ \gamma_\mu W^{+\mu \lambda} d_\lambda + \gamma_\mu (W^{+\mu} d^\lambda - W^{+\lambda} d^\mu) d_\lambda \right] \right\} \\
H_V \Leftrightarrow \left\{ \frac{i}{2} \gamma_\mu W^{+\mu} \times \frac{i}{2} \gamma_\mu \gamma_5 \xi^\mu \times \frac{1}{\Lambda^2_\chi} \left[ \gamma_\mu \gamma_5 \xi^{\mu\lambda} d_\lambda + \gamma_\mu \gamma_5 (\xi^\mu d^\lambda - \xi^\lambda d^\mu) d_\lambda \right], \\
\left( \frac{i}{2} \gamma_\mu \gamma_5 \xi^\mu \right)^2 \times \frac{1}{\Lambda^2_\chi} \gamma_\mu (W^{+\mu} d^\lambda - W^{+\lambda} d^\mu) d_\lambda \right\}. \tag{56}
\]

Each diagram has one (or two) leading vertex and one NTL vertex. Each NTL vertex brings a new coefficient \( \beta(\Lambda_\chi) \). We conclude that at NTL order the five parameters of the vector Lagrangian are all independent. Each of them has a dependence upon \( Q^2 \) of the form:

\[
f_i = \left( 1 + \beta_i \frac{Q^2}{\Lambda^2_\chi} \right) \ln \frac{\Lambda^2_\chi}{Q^2}. \tag{57}
\]

### 4.3 The running of \( f_V^2 \) and \( M_V^2 \)

For a detailed evaluation of the NTL contributions we concentrate on two of the five parameters of the vector Lagrangian: the coupling \( f_V \) between the vector meson and the external vector current and the mass \( M_V \). In the next section we will use these two parameters in the calculation of the vector-vector correlation function. In the ENJL model (i.e. at the leading order in the logarithmic expansion) the two parameters are both expressed in terms of the wave function \( Z_V \) as follows:
\[ f_V = \sqrt{2Z_V} \quad M_{V}^2 = \frac{N_c}{16\pi^2} \left( \frac{\Lambda^2}{2G_V} \right) \frac{1}{Z_V}, \tag{58} \]

where \( Z_V \) is the leading logarithmic contribution to the wave-function

\[ Z_V = Z_V^l = 2 \frac{N_c}{16\pi^2} \int_0^1 d\alpha \alpha (1-\alpha) \ln \frac{\Lambda^2}{s(\alpha)}. \tag{59} \]

The product \( f_V^2 M_V^2 \) is scale invariant:

\[ f_V^2 M_V^2 = \frac{N_c}{16\pi^2} \frac{\Lambda^2}{G_V} \tag{60} \]

By adding the NPLL corrections, the \( f_V \) coupling receives contributions which are absent for the wave function \( Z_V \). The latter defines the renormalized vector mass \( M_V \), once the physical vector field has been defined through eq. (47).

A summary of the pairs of vertices entering the calculations of \( f_V \) and \( Z_V \) (in the same notation of eq. (55)), including the leading contributions, is given by:

\[ f_V \leftrightarrow \left\{ W^+_\mu \times \Gamma_\mu, \right. \]
\[ \left. W^+_\mu \times \frac{1}{\Lambda^2} \left( \beta^1 \lambda d^\lambda \Gamma_{\mu \lambda} + \beta^2 \Gamma_{\mu \lambda} d^\lambda \right) \right\}; \]
\[ \Gamma_\mu \times \frac{1}{\Lambda^2} \left( \beta^1 \lambda d^\lambda W^+_{\mu \lambda} + \beta^2 W^+_{\mu \lambda} d^\lambda + \beta^3 (d^\lambda W^+_{\mu \lambda} - d^\mu W^+_{\lambda \mu}) d^\lambda \right) \right\}, \tag{61} \]

where a \( \beta^i \) coefficient is explicitly written in front of each \( 1/\Lambda^2 \) vertex.

Using the formula in Appendix A one gets:

\[ f_V = \sqrt{2Z_V} + \frac{N_c}{16\pi^2} \frac{\sqrt{2}}{3} \frac{Q^2}{\Lambda^2} \sum_{i=1}^{2} \beta^i \int_0^1 d\alpha \frac{P_i(\alpha) \ln \frac{\Lambda^2}{s(\alpha)}}{Z_V} \]
\[ -\frac{1}{2} \sum_{i=1}^{3} \beta^i \int_0^1 d\alpha \frac{P_i(\alpha) \ln \frac{\Lambda^2}{s(\alpha)}}{Z_V} \]
\[ M_{V}^2 = \frac{N_c}{16\pi^2} \left( \frac{\Lambda^2}{2G_V} \right) \frac{1}{Z_V}. \tag{62} \]

where the wave function \( Z_V \) is given by:
\[ Z_V = \frac{N_c}{16\pi^2} \left\{ \frac{1}{3} \int_0^1 d\alpha \alpha(1-\alpha) \ln \frac{\Lambda^2}{s(\alpha)} + \sum_{i=1}^3 \beta^i_V \frac{Q^2}{\Lambda^2} \int_0^1 d\alpha P_i(\alpha) \ln \frac{\Lambda^2}{s(\alpha)} \right\} \]

\[ \equiv Z_V + \frac{N_c}{16\pi^2} \frac{1}{3} \sum_{i=1}^3 \beta^i_V \frac{Q^2}{\Lambda^2} \int_0^1 d\alpha P_i(\alpha) \ln \frac{\Lambda^2}{s(\alpha)}. \quad (63) \]

The \( \beta^i_{V,\Gamma} \) coefficients must be determined from experimental data. The function \( s(\alpha) \) is equal to \( M^2_Q + \alpha(1-\alpha)Q^2 \). The \( P_i(\alpha) \) are polynomials in the Feynman parameter \( \alpha \). Their explicit form can be derived by the formula in Appendix A and reads:

\[ P_1(\alpha) = \alpha(1-\alpha) \]
\[ P_2(\alpha) = \alpha^2(1-\alpha) \]
\[ P_3(\alpha) = \alpha^3(1-\alpha) - 3\alpha^2(1-\alpha)^2 \quad (64) \]

They correspond to the three possible classes of contributions we got: 1) one vertex with no derivatives on the quark fields 2) one vertex with one derivative 3) one vertex with two derivatives. The dependence upon \( Q^2 \) of the quantity \[ \int_0^1 d\alpha P_i(\alpha) \ln(\Lambda^2/s(\alpha)) \] for the different \( P_i \) is shown in fig. 3. The polynomials \( P_2 \) and \( P_3 \) lead up to a sign to the same \( Q^2 \) dependence. We will use this result for the construction of the vector-vector correlation function.

By including the NPLL corrections, the product (60) is given by:

\[ f^2_V M^2_V = \frac{N_c}{16\pi^2} \frac{\Lambda^2}{G_V} \left[ 1 + \frac{N_c}{16\pi^2} \frac{1}{3} \frac{1}{Z_V} \frac{Q^2}{\Lambda^2} \left( 2 \sum_{i=1}^2 \beta^i_V \int_0^1 d\alpha P_i(\alpha) \ln \frac{\Lambda^2}{s(\alpha)} \right) 
- \sum_{i=1}^3 \beta^i_V \int_0^1 d\alpha P_i(\alpha) \ln \frac{\Lambda^2}{s(\alpha)} \right]. \quad (65) \]

The presence of the new NTL terms with coefficients \( \beta^1_V \) and \( \beta^i_V \) breaks in general the scale invariance of the product in eq. (60).

5 Phenomenology of the Vector-Vector correlation function.

The finite set of \( 1/\Lambda^2 \) non-renormalizable quark-meson interactions generates the running of the parameters of the effective meson Lagrangian in the Quark-Resonance
To estimate the values of the coefficients which enter in the running of \( f_V \) and \( M_V^2 \) we will focus on the particular channel of the vector resonance sector, by studying the \( Q^2 \) behaviour of the vector-vector correlation function where we can compare our predictions with the experimental results. We closely follow the derivation of the 2-point vector function of ref. \[12\].

We define the 2-point vector function as:

\[
\Pi^{\mu \nu}_{V}(q^2) = i \int d^4 x \ e^{iqx} \langle 0|T(V^a_{\mu}(x)V^b_{\nu}(0)|0 \rangle, \tag{66}
\]

where \( V^a_{\mu}(x) \) is the flavoured vector quark current defined as:

\[
V^a_{\mu}(x) = \bar{q}(x)\gamma^\mu \frac{\lambda^a}{\sqrt{2}} q(x), \tag{67}
\]

with \( \lambda^a \) the Gell-Mann matrices normalised as \( \text{tr}(\lambda^a \lambda^b) = 2 \delta^{ab} \). The Lorentz covariance and \( SU(3) \) invariance imply for the \( \Pi^{\mu \nu}_{V} \) the following structure:

\[
\Pi^{\mu \nu}_{V}(q^2) = (q_{\mu}q_{\nu} - g_{\mu \nu} q^2) \Pi^1_{V}(Q^2)\delta^{ab} + q_{\mu}q_{\nu} \Pi^0_{V}(Q^2)\delta^{ab}, \tag{68}
\]

where \( Q^2 = -q^2 \), with \( q^2 \) euclidean. The \( SU(3)_L \times SU(3)_R \) ENJL model gives the low energy prediction for the invariant functions \( \Pi^1_{V}, \Pi^0_{V} \) in the chiral limit \( (\mathcal{M} \to 0) \) and without the inclusion of chiral loops \[12\]:

\[
\Pi^1_{V}(Q^2) = -4(2H_1 + L_{10}) + \mathcal{O}(Q^2) \\
\Pi^0_{V}(Q^2) = 0. \tag{69}
\]

The parameters \( H_1 \) and \( L_{10} \) are two of the twelve counterterms that appear in the non anomalous effective Lagrangian of pseudoscalar mesons at order \( p^4 \) in the chiral expansion:

\[
\mathcal{L}_4 = \ldots + L_{10} \text{tr}(U^\dagger R_{\mu \nu}U F_{L \mu \nu}^R) + H_1 \text{tr}(F_{\mu \nu R}^2 + F_{\mu \nu L}^2) \]
\[
= L_{10} \frac{1}{4}(f_{\mu \nu}^+ - f_{\mu \nu}^-) + H_1 \frac{1}{2}(f_{\mu \nu}^+ + f_{\mu \nu}^-), \tag{70}
\]

where \( f_{\mu \nu}^\pm \) are related to the external field-strenght tensors \( F_{\mu \nu}^{R,L} \) through the identity:

\[
f_{\mu \nu}^\pm = \xi F_{\mu \nu}^L \xi^\dagger \pm \xi^\dagger F_{\mu \nu}^R \xi \tag{71}\]
\[ F^L_{\mu\nu} = \partial_\mu l_\nu - \partial_\nu l_\mu - i[l_\mu, l_\nu] \]
\[ F^R_{\mu\nu} = \partial_\mu r_\nu - \partial_\nu r_\mu - i[r_\mu, r_\nu]. \quad (72) \]

The leading values of \( H_1 \) and \( L_{10} \) at \( Q^2 = 0 \) predicted by the QR model are:
\[ H_1 = -\frac{1}{12} \frac{N_c}{16\pi^2} (1 + g_A^2) \ln \frac{\Lambda^2}{M_Q^2} + \text{finite terms} \]
\[ L_{10} = -\frac{1}{6} \frac{N_c}{16\pi^2} (1 - g_A^2) \ln \frac{\Lambda^2}{M_Q^2} + \text{finite terms}. \quad (73) \]

The combination \( 2H_1 + L_{10} \) is free from finite contributions.

The Vector-Vector correlation function allows to explore a sector of the QR model which is free from the effects of the axial-pseudoscalar mixing (i.e. the parameter \( g_A \)). Indeed, the \( g_A^2 \) dependence is introduced by the \( f^-_{\mu\nu} \) part of the invariant terms, which in turn depends on the \( \xi_\mu \) physical field because of the identity \( f^-_{\mu\nu} = \xi_{\mu\nu} \). The vector two-point function gets contribution only from the \( f^+_{\mu\nu} \) terms and therefore the parameters \( H_1 \) and \( L_{10} \) will only enter in a combination independent of \( g_A \). The combination that appears in front of the \( f^{+2}_{\mu\nu} \) term in the Lagrangian (70) is the following:
\[ \frac{1}{4} (2H_1 + L_{10}) = -\frac{N_c}{16\pi^2} \frac{1}{12} \ln \frac{\Lambda^2}{M_Q^2} \quad (74) \]
and contributes as in eq. (69) to the two-point vector correlation function. As was pointed out in [12], the vector resonance exchange also contributes to the \( Q^2 \) dependence of the \( \Pi_{1V}(Q^2) \) function. The diagram with one vector meson exchange in fig. 4 represents the vector resonance contribution to the \( \Pi_{1V}(Q^2) \). The total result is:
\[ \Pi_{1V}(Q^2) = -4(2H_1 + L_{10}) - 2\frac{f^2_V Q^2}{M_V^2 + Q^2}, \quad (75) \]
which includes the contribution at \( Q^2 = 0 \) from the genuine one quark-loop diagram (first term) and the contribution from the vector resonance exchange (second term). In this approximation the parameters \( f_V \) and \( M_V \) are the values at \( Q^2 = 0 \) predicted
by the ENJL model, i.e. they are generated by the single quark-loop diagrams with the insertion of leading vertices in the $1/\Lambda_\chi$-expansion (see fig. 5).

In the ENJL model \[8\] at $Q^2 = 0$ the following relation holds:

\[
(2H_1 + L_{10})(Q^2 = 0) = -\frac{f^2_V}{2}(Q^2 = 0)
\]  

(76)

so that the $\Pi^1_V(Q^2)$ function predicted by the ENJL model can be rewritten in a VMD way

\[
\Pi^1_V(Q^2) = 2 \frac{f^2_V M^2_V}{M^2_V + Q^2},
\]

(77)

where the parameters

\[
f^2_V = \frac{N_c}{16\pi^2} \frac{2}{3} \ln \frac{\Lambda^2_\chi}{M^2_Q}, \quad M^2_V = \frac{3}{2 G_V} \frac{\Lambda^2_\chi}{\ln \frac{\Lambda^2_\chi}{M^2_Q}}
\]

(78)

are the values at $Q^2 = 0$ predicted by the ENJL model.

The authors of \[12\] have resummed all quark-bubble diagrams in fig. 6 with the insertion of the leading 4-quark effective vertex with coupling $G_V$. In the VMD representation of eq. (77), the $Q^2$ dependent contributions coming from the n-loop diagrams can be reabsorbed in the running of the vector parameters $f_V(Q^2)$ and $M^2_V(Q^2)$, which are completely determined in terms of the ENJL parameters. The result quoted in \[12\] is the following:

\[
\Pi^1_V(Q^2) = 2 \frac{f^2_V(Q^2) M^2_V(Q^2)}{M^2_V(Q^2) + Q^2},
\]

(79)

with

\[
f^2_V(Q^2) = \frac{N_c}{16\pi^2} 4 \int_0^1 d\alpha \ \alpha(1-\alpha) \ln \frac{\Lambda^2_\chi}{M^2_Q + \alpha(1-\alpha)Q^2}
\]

\[
M^2_V(Q^2) = \frac{\Lambda^2_\chi}{4 G_V \int_0^1 d\alpha \ \alpha(1-\alpha) \ln \frac{\Lambda^2_\chi}{M^2_Q + \alpha(1-\alpha)Q^2}}.
\]

(80)

In the formula \[80\] we kept only the leading logarithmic contribution of the expansion of the incomplete Gamma function $\Gamma(0, x) \simeq -\ln x - \gamma_E + O(x)$ appearing in the calculation of ref. \[12\].

In this case the product $f^2_V(Q^2) M^2_V(Q^2)$ remains scale invariant.
5.1 $\Pi_V(Q^2)$ from the QR model

The full $Q^2$ dependence of the vector-vector function can be extracted from the bosonized generating functional. In this case pure fermion vertices are absent and in particular the 4-fermion vertex with coupling $G_V$ is replaced by the q-q-V vertex plus a vector mass term, as shown in fig. 7.

At the one quark-loop level the couplings $H_1, L_{10}, f_V$ and the mass $M_V$ get NTL logarithmic corrections as we have shown in paragraph 4.3. The combination $2H_1 + L_{10}$ gets NTL contributions from a subset of the NTL vertices $(1/\Lambda^2) \chi$ which give contribution to the coupling $f_V$. The pairs of vertices according to the notation of eq. (55) are:

\[
\begin{align*}
W^+ & \Leftrightarrow \{ W^+ \times \frac{1}{\Lambda^2} \left( \beta^1_V d^\lambda \Gamma_{\mu\lambda} + \beta^2_V \Gamma_{\mu\lambda} d^\lambda \right) \}, \\
\Gamma & \times \frac{1}{\Lambda^2} \left( \beta^1_V d^\lambda W^+_{\mu\lambda} + \beta^2_V W^+_{\mu\lambda} d^\lambda + \beta^3_V (d^\lambda W^+_{\mu} - d^\mu W^+_{\lambda}) d^\lambda \right) \\
2H_1 + L_{10} & \Leftrightarrow \{ \Gamma \times \frac{1}{\Lambda^2} \left( \beta^1_V d^\lambda \Gamma_{\mu\lambda} + \beta^2_V \Gamma_{\mu\lambda} d^\lambda \right) \}.
\end{align*}
\]

Because of the presence of independent unknown coupling constants the running of the two quantities $f^2_V/2$ and $2H_1 + L_{10}$ is not a priori the same. There are two possible solutions at $Q^2 \neq 0$:

- The running with $Q^2$ of the two parameters can be different, while their values at $Q^2 = 0$ are related through the identity (76). In this case the coefficients $\beta^i_V$ and $\beta^i_\Gamma$ of the NTL logarithmic corrections are not constrained.

- The relation (76) has to be scale invariant. This puts a constraint on the coefficients of the NTL logarithmic corrections to $f^2_V/2$ and $2H_1 + L_{10}$, $\beta^i_V$ and $\beta^i_\Gamma$.

The second solution appears to hold in resonance models and under the saturation hypothesis formulated in [6]. For kinematical reasons the CV model is the only vector model which does not generate the saturation of the $L_i, H_i$ counterterms of the $\mathcal{L}_4$ Lagrangian through vector resonance exchange. In the ENJL model the saturation is replaced by the direct contribution of one loop of quarks. Other vector models [3] saturate the relation (76) without the inclusion of quark-loops contribution. By construction the saturation by resonance exchange holds at the resonance
scale \((Q^2 = M_V^2)\). If we require a) the equivalence of the vector models (including the quark-loops contribution in the case of the CV model) and b) the validity of the saturation hypothesis, which in fact is experimentally well verified, we conclude that the relation (76) has to be scale invariant.

Let us see if this ansatz is satisfied by the coefficients \(\beta_{V,i}^{R}\).

The values of the two parameters of eq. (76), including the NPLL corrections, can be deduced by using the formulas in Appendix A and by inserting the list of vertices of eq. (81):

\[
\begin{align*}
\frac{f_V}{\sqrt{2}} &= \sqrt{Z_V} + \frac{N_c}{16\pi^2} \frac{1}{3} \sqrt{Z_V} \frac{Q^2}{\Lambda^2} \sum_{i=1}^{2} \beta_i \int_0^1 d\alpha \ P_i(\alpha) \ln \frac{\Lambda^2}{s(\alpha)} \\
&\quad - \frac{1}{2} \beta_V \int_0^1 d\alpha \ P_i(\alpha) \ln \frac{\Lambda^2}{s(\alpha)} \\
-(2H_1 + L_{10}) &= Z_V + \frac{N_c}{16\pi^2} \frac{2Q^2}{3} \sum_{i=1}^{2} \beta_i \int_0^1 d\alpha \ P_i(\alpha) \ln \frac{\Lambda^2}{s(\alpha)} \\
&\quad - \beta_V \int_0^1 d\alpha \ P_i(\alpha) \ln \frac{\Lambda^2}{s(\alpha)}.
\end{align*}
\]

where the wave function \(Z_V\) at one loop is given by:

\[
\begin{align*}
Z_V &= \frac{N_c}{16\pi^2} \frac{1}{3} \left[ 6 \int_0^1 d\alpha \ (1 - \alpha) \ln \frac{\Lambda^2}{s(\alpha)} + \sum_{i=1}^{3} \beta_i \int_0^1 d\alpha \ P_i(\alpha) \ln \frac{\Lambda^2}{s(\alpha)} \right] \\
&\equiv Z_V + \frac{N_c}{16\pi^2} \sum_{i=1}^{3} \beta_V Q^2 \int_0^1 d\alpha \ P_i(\alpha) \ln \frac{\Lambda^2}{s(\alpha)}.
\end{align*}
\]

If we compare the running of the two terms of the relation (76) up to the NPLL order, we have:

\[
\begin{align*}
\frac{f_V^2}{2} &= Z_V + \frac{N_c}{16\pi^2} \frac{1}{3} \sum_{i=1}^{2} \beta_i \int_0^1 d\alpha \ P_i(\alpha) \ln \frac{\Lambda^2}{s(\alpha)} \\
&\quad - \sum_{i=1}^{3} \beta_V \int_0^1 d\alpha \ P_i(\alpha) \ln \frac{\Lambda^2}{s(\alpha)} \\
-(2H_1 + L_{10}) &= Z_V + \frac{N_c}{16\pi^2} \frac{1}{3} \sum_{i=1}^{2} \beta_i \int_0^1 d\alpha \ P_i(\alpha) \ln \frac{\Lambda^2}{s(\alpha)}
\end{align*}
\]

25
They have the same running in $Q^2$ including the NPLL corrections.

$\Pi_V^1(Q^2)$ can be written as follows:

$$
\Pi_V^1(Q^2) = -4(2H_1 + L_{10})(Q^2) - \frac{2f_V^2(Q^2)Q^2}{M_V^2(Q^2) + Q^2}.
$$  

(85)

By using the property that the running of the two parameters in eq. (84) is the same (at least up to the NPLL order) the following expression holds:

$$
\Pi_V^1(Q^2) = \frac{2f_V^2(Q^2)M_V^2(Q^2)}{M_V^2(Q^2) + Q^2},
$$  

(86)

where the running of $f_V^2$ and $M_V^2$ is given by:

$$
f_V^2 = 2Z_V + \frac{N_c}{16\pi^2} \frac{2}{3} \frac{Q^2}{\Lambda^2} \left[ 2 \sum_{i=1}^2 \beta^i V \int_0^1 d\alpha \frac{P_i(\alpha)}{s(\alpha)} \ln \frac{\Lambda^2}{s(\alpha)} \right]
$$

$$
- \sum_{i=1}^3 \beta^i V \int_0^1 d\alpha \frac{P_i(\alpha)}{s(\alpha)} \ln \frac{\Lambda^2}{s(\alpha)}
$$

$$
M_V^2 = \frac{N_c}{16\pi^2} \left( \frac{\Lambda^2}{2G_V} \right) \frac{1}{Z_V}
$$

$$
Z_V = \frac{N_c}{16\pi^2} \frac{1}{3} \left[ 6 \int_0^1 d\alpha \frac{\alpha(1-\alpha)}{s(\alpha)} \ln \frac{\Lambda^2}{s(\alpha)} + \sum_{i=1}^3 \beta^i V \frac{Q^2}{\Lambda^2} \int_0^1 d\alpha \frac{P_i(\alpha)}{s(\alpha)} \ln \frac{\Lambda^2}{s(\alpha)} \right].
$$

(87)

The infinite resummation of quark bubbles considered in ref. [12] corresponds to replacing in the vector contribution the one quark-loop dressed propagator as shown in fig. 8.

The set of diagrams corresponding to the full two-point vector correlation function predicted by the QR model is shown in fig. 9.

5.2 Determination of $\Pi_V^1(Q^2)$ at NTL order

The real part of the invariant $\Pi$ function is related to its imaginary part through a standard dispersion relation

$$
Re\Pi_V^1(Q^2) = \int_0^\infty ds \frac{1}{s} \frac{1}{s + Q^2} Im\Pi_V^1(s).
$$  

(88)
For a review on QCD spectral Sum rules and the calculation of QCD two-point Green’s functions see [22].

For our analysis we choose the channel of the hadronic current with the ρ meson quantum numbers \((I = 1, J = 1)\) \(J_\rho^\mu = 1/\sqrt{2}(\bar{u}\gamma^\mu u - \bar{d}\gamma^\mu d)\). The imaginary part of \(\Pi_1^V\) is experimentally known in terms of the total hadronic ratio of the \(e^+e^-\) annihilation in the isovector channel defined as follows:

\[
R_I^{1=1}(s) = \frac{\sigma^{1=1}(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}
\]  

(89)

The following dispersion relation holds [20, 21]:

\[
\text{Re}\Pi_1^V(Q^2) = \frac{2}{12\pi^2} \int_0^\infty ds R_I^{1=1}(s) \frac{s}{s+Q^2}
\]  

(90)

We have performed a comparison between the QR model parametrization of the vector 2-point function in the isovector channel, valid in the energy region \(0 < Q^2 < \Lambda^2\), and the prediction obtained from a modelization of the experimental data on \(e^+e^- \to \text{hadrons}\) [24]. For a determination of the function \(\Pi_1^V(Q^2)\) in the high \(Q^2\) region (i.e. beyond the cutoff \(\Lambda\)) see [23].

We adopted the following parametrization of the experimental hadronic isovector ratio:

\[
R_I^{1=1}(s) = \frac{9}{4\alpha^2} \frac{\Gamma_{ee}\Gamma_\rho}{(\sqrt{s} - m_\rho)^2 + \Gamma_\rho^2} + \frac{3}{4} \left(1 + \frac{\alpha_s(s)}{\pi}\right) \theta(s - s_0).
\]  

(91)

This is a generalization of the one proposed in ref. [20], where the rho meson width corrections have not been included. \(\Gamma_{ee} = 6.7 \pm 0.4\) KeV is the \(\rho \to e^+e^-\) width and \(\Gamma_\rho = 150.9 \pm 3.0\) is the total width of the neutral ρ [25]. We used the leading logarithmic approximation for \(\alpha_s(s)\):

\[
\alpha_s(s) = \frac{12\pi}{33 - 2n_f} \frac{1}{\log(s/\Lambda_{QCD}^2)}.
\]  

(92)

The modelization [21] includes a dependence of the ρ channel upon the ρ width and the contribution from the continuum starting at a threshold \(s_0 = 1.5\) GeV \(^2\) [20]. For the running of \(\alpha_s\) we used a value of 260 MeV for \(\Lambda_{QCD}\), according to the average experimental value \(\Lambda_{QCD}^{(4)} = 260_{-46}^{+54}\) MeV [25] and with \(n_f = 4\) flavours.

The results are practically insensitive to the \(\alpha_s\) running corrections and our leading log approximation turns out to be adequate.
The Vector Green’s function in the QR model has been parametrized in eqs. (86, 87). To extract information on the $\beta_i^V, \beta_i^\Gamma$ coefficients of the NTL logarithmic corrections we made a best fit of the first derivative of the 2-point function coming from the modelization (91) of the experimental data:

$$\Pi'(Q^2)_{\text{exp}} = -\frac{2}{12\pi^2} \int_0^\infty ds \frac{R^{I=1}(s)}{(s + Q^2)^2},$$

(93)

where the derivative of the VV function in the QR model is given by:

$$\Pi'(Q^2)_{QR} = \left[ \frac{2f^2V}{M_V^2}(1 + \frac{Q^2}{M_V^2}) - 2f^2V(1 - \frac{Q^2M_V^2}{M_V^2}) \right] \frac{1}{\left(1 + \frac{Q^2}{M_V^2}\right)^2}.$$

(94)

We have used $M_Q = 265$ MeV for the IR cutoff and $\Lambda_\chi = 1.165$ GeV for the UV cutoff, determined by a global fit in ref. [8].

The fit has been done in the region: $0.5 < Q < 0.9$ GeV. At lower momenta the NPLL corrections $Q^2/\Lambda_\chi^2 \ln(\Lambda_\chi^2/Q^2)$ we are considering compete with the otherwise suppressed logarithmic corrections $M_Q^2/\Lambda_\chi^2 \ln(\Lambda_\chi^2/Q^2)$ proportional to $M_Q^2$. At higher momenta we are sufficiently near to the cutoff to require the inclusion of higher order contributions.

The full set of NPLL coefficients consists of the three coefficients $\beta_i^V$ and the two coefficients $\beta_i^\Gamma$. As we observed in section 4.3 the polynomials $P_2(\alpha)$ and $P_3(\alpha)$ give rise to the same $Q^2$ dependence. We reduced correspondingly the number of free coefficients in the fit.

We have performed the best fit with four free parameters: $\beta_1^V, \beta_2^V, \beta_1^\Gamma, \beta_2^\Gamma$.

In fig. 10 we show the $Q^2$ behaviour of the derivative of the experimental 2-point function, the data from the best fit, and the derivative of the ENJL prediction with quark-bubbles resummation of eqs. (79, 80) and including only the logarithmic contributions to the $Q^2$ running of the parameters. The values of the coefficients are:

$$\frac{N_c}{16\pi^2} \beta_1^V = -0.045 \pm 0.001 \quad \frac{N_c}{16\pi^2} \beta_1^\Gamma = -0.072 \pm 0.009 \quad \frac{N_c}{16\pi^2} \beta_2^V = -0.034 \pm 0.002 \quad \frac{N_c}{16\pi^2} \beta_2^\Gamma = -0.035 \pm 0.01.$$

(95)

The terms with polynomials $P_1$ and $P_2$ give almost the same contribution. The $\chi^2$ of the fit has been defined as $\sum_i (\Pi_i' - \Pi_i'^{\text{exp}})^2/\sigma_i^2$, where we defined the $\sigma_i$ assuming...
a 10% of uncertainty on the experimental data: $\sigma_i = 0.1 \Pi_i^{\exp}(Q^2)$. A $\chi^2/n.d.f. = 5 \cdot 10^{-2}$ has been obtained. At energies lower than 0.4 GeV the derivative starts to be sensitive to corrections proportional to the infrared cutoff $M_Q = 265$ MeV, while at energies higher than 0.9 GeV becomes sensitive to higher order corrections. The ENJL prediction differs by roughly a 40% from the experimental curve at 0.8 GeV.

The invariant function $\Pi_1^V(Q^2)$ has been obtained by requiring a matching with the ENJL function at $Q = M_Q$:

$$\Pi_1^V(Q^2) = \Pi_{1}^{ENJL}(Q^2) \theta(M_Q^2 - Q^2) + \int_{M_Q^2}^{Q^2} \frac{d\Pi_{F}^{fit}}{dQ^2} dQ^2 \theta(Q^2 - M_Q^2).$$

The $\Pi_1^V(Q^2)$ function obtained with the values (95) and with the matching of eq. (96) is plotted in fig. 11 and compared with the ENJL prediction of eq. (79) (i.e. including the resummation of linear chains of quark bubbles and including only logarithmic corrections). The difference between the two curves reaches a 30% at 0.7 GeV.

The inclusion of gluons in the ENJL model makes worse the agreement with the experimental data.

The modelization of (91) does not include the higher $I = 1, J = 1$ resonance states with $\rho$ quantum numbers $\rho(1450), \rho(1700)$. There is no measurement at present of their leptonic width. The addition of more resonance states increases the difference between the two curves. The sensitivity to the continuum threshold value $s_0$ of $R_I=1(s)$ is contained inside a 10% of variation in the range $s_0 = 1.5 \div 4 \text{ GeV}^2$.

The practical insensitivity to large variations of the $\Lambda_{QCD}$ parameter, due to the smallness of the contributions involving $\alpha_s$, has been also verified.

A last point concerns the numerical values used for the IR cutoff $M_Q = 265$ MeV and the UV cutoff $\Lambda_{\chi} = 1.165$ GeV. They come from a global fit as explained in detail in ref. [8]. The data used as inputs are not truly $Q^2 = 0$ quantities, while the fitted parameters are the truly $Q^2 = 0$ values predicted by the ENJL model. Higher order corrections in $1/\Lambda_{\chi}^2$, in the scalar sector, can in fact induce corrections to the mass-gap equation and as a consequence to the numerical value of the IR cutoff $M_Q$. A change in the scalar parameters can modify the prediction for the mass of the scalar resonance given by the ENJL model. We are currently investigating on this point.

The analysis of the vecto-vector Green’s function shows how a sizable magnitude of NP LL corrections can be estimated from the data. Correlations in other chan-
nels which are experimentally less accessible could be estimated by QCD lattice simulations which could be used to fix the parameters of the effective Lagrangian.

6 Conclusions.

Effective quark models inspired to the old Nambu-Jona Lasinio model [9] have proven to be a promising tool to describe low energy hadronic interactions. In this type of models the hadron fields are introduced through the bosonization of the effective quark action. The effective meson Lagrangian comes from the integration over the quarks and gluons degrees of freedom.

The simplest model one can construct is the so called ENJL model [8], where only the lowest dimension effective quark operators are included, leading in the $1/\Lambda_\chi$ and $1/N_c$ expansions.

As we have shown in detail, the ENJL model correctly predicts the value of the parameters of the effective meson Lagrangian in the zero energy limit. In this limit the model is noticeably more predictive with respect to the usual effective meson Lagrangian approach [1, 2, 3, 4]. As an example, the twelve counterterms of the effective pseudoscalar meson Lagrangian at order $p^4$ in the chiral expansion together with the parameters of the chiral leading effective resonance Lagrangian are all expressed in terms of only three input parameters of the NJL model: $G_S, G_V$ and $\Lambda_\chi$. Adding gluon corrections to order $\alpha_s N_c$ introduces ten more unknown constants which can be estimated in terms of a single unknown parameter $g$ [8].

Nevertheless, the ENJL model is not able to describe the behaviour of the low energy hadronic observables at $Q^2 \neq 0$.

We indicate a systematic way to get predictions on the behaviour of the hadronic observables in the whole low energy range of $Q^2$ (i.e. $0 < Q^2 < \Lambda_\chi^2$) which could provide a bridge between the non-asymptotic and the asymptoptic regime of QCD.

The Quark-Resonance model formulated in this work is based on the observation that higher dimension n-quark effective interactions give relevant contributions to the values of the low energy hadronic observables at $Q^2 \neq 0$.

We have shown that higher dimension operators produce next-to-leading power - leading log corrections of the type $(Q^2/\Lambda^2_\chi) \ln(\Lambda^2_\chi/Q^2)$ to the parameters of the effective meson Lagrangian and corrections without logarithms of order $(Q^2/\Lambda^2_\chi)$. 30
The former are produced by a finite set of $1/\Lambda^2_\chi$ terms, while the latter arise from an infinite tower of higher dimension operators.

We have focused our attention on the first class of contributions, which are assumed to be dominant for values of $Q^2$ above the IR cutoff $M_Q^2$ and below the UV cutoff $\Lambda^2_\chi$.

The $Q^2$ behaviour of the low energy observables can be well reproduced. We have shown explicitly how the next-to-leading power - leading log corrections enter the calculation of the two-point vector Green’s function. In the $I = 1, J = 1$ channel we were able to fix the four coefficients of these corrections through a fit to the experimental data on the $e^+e^- \rightarrow \text{hadrons}$ cross section. The comparison with the ENJL prediction of ref. [12] provides evidence for a quantitative relevance of the next-to-leading terms in the $1/\Lambda_\chi$ expansion in the $Q^2$ dependence of the hadronic observables throughout the intermediate $Q^2$ region.

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A Effective Potential calculation: n=2

The formula to calculate a generic contribution for n=2 in Euclidean space is the following:

$$\frac{1}{2} \int \int dx \, dy \, Tr \left( \frac{d^4 k}{(2\pi)^4} e^{ik(x-y)} \frac{1}{i k + M_Q} \delta(y) \right) \int \frac{d^4 q}{(2\pi)^4} e^{iq(y-x)} \frac{1}{i q + M_Q} \delta(x),$$

(97)

where $Tr$ is the trace over Dirac, colour and flavour indices.

It corresponds to a one quark-loop diagram with two insertions of the operator $\delta(x)$ as defined in eqs. (42 and 40).
Defining $l \equiv k - q$ and introducing the Feynman parameter $\alpha$, the formula reduces to:

$$-\frac{1}{2} \int dx \ dy \int \frac{d^4l}{(2\pi)^4} e^{il(x-y)} \int_0^1 d\alpha \int \frac{d^4l'}{(2\pi)^4} \frac{1}{[k'^2 + \alpha(1-\alpha)l'^2 + M_Q^2]^2} \cdot$$

$$Tr \left\{ [i(\hat{k}' + \alpha \hat{l}) - M_Q]\delta(y) [i(\hat{k}' - (1-\alpha) \hat{l}) - M_Q] \delta(x) \right\}. \quad (98)$$

We give here the final formula for the contributions diverging logarithmically with the cutoff $\Lambda_\chi$ obtained with the insertion of three different forms of the local operator $\delta(x)$. These are the only calculations needed to obtain the corrections to the parameters of the vector meson Lagrangian generated by the insertion of one next-to-leading vertex $1/\Lambda_\chi^2$ and one leading vertex.

- Case 1. $\delta(y) = \gamma_\mu(\gamma_5)\delta^\mu(y)$  \quad $\delta(x) = \gamma_\mu(\gamma_5)\delta^\mu(x)$

$$\Gamma_{log} = \frac{1}{2} \frac{N_c \pi^2}{(2\pi)^8} \int dx \ dy \int d^4l \ e^{il(x-y)} (l_\mu l_\nu - g_{\mu\nu}l^2)$$

$$tr[\delta^\mu(y)\delta^\nu(x)]8 \int_0^1 d\alpha \ \alpha(1-\alpha) \ln \frac{\Lambda^2}{s(\alpha)}, \quad (99)$$

where $tr$ is the trace over the flavour indices of the $\delta(x)$ matrices and $s(\alpha) = M_Q^2 + \alpha(1-\alpha)l^2$.

Expression (99) can be simplified to:

$$\Gamma_{log} = \frac{1}{2} \frac{N_c \pi^2}{(2\pi)^4} \int dz dy \int d^4z \ [\partial^\mu \partial^\nu - g^{\mu\nu} \partial^2]$$

$$tr[\delta^\mu(y)\delta^\nu(z+y)]8 \int_0^1 d\alpha \ \alpha(1-\alpha) \ln \frac{\Lambda^2}{s(\alpha)}$$

$$= \frac{1}{2} \frac{N_c \pi^2}{(2\pi)^4} \int dy \ tr[\delta^\mu(y)\partial^\nu - g_{\mu\nu} \partial^2] \delta^\nu(y)$$

$$8 \int_0^1 d\alpha \ \alpha(1-\alpha) \ln \frac{\Lambda^2}{s(\alpha)} \quad (100)$$

- Case 2. $\delta(y) = \gamma_\mu(\gamma_5)\delta^\mu(y)$  \quad $\delta(x) = \gamma_\mu(\gamma_5)\delta^\mu\lambda(x) d_\lambda$

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\[ \Gamma_{\text{log}} = -\frac{1}{2} \frac{N_c \pi^2}{(2\pi)^8} \int dx \, dy \int d^4l \, e^{il(x-y)} i l_\lambda (l_\mu l_\nu - g_{\mu
u} l^2) \]

\[
= -\frac{1}{2} \frac{N_c \pi^2}{(2\pi)^4} \int dy \, tr \left[ \delta^\mu(y) \delta^\nu(x) \right] 8 \int_0^1 d\alpha \, \alpha^2 (1-\alpha) \ln \frac{\Lambda^2}{s(\alpha)}.
\]

\[
\Gamma_{\text{log}} = 1 \frac{N_c \pi^2}{(2\pi)^8} \int dx \, dy \int d^4l \, e^{il(x-y)} tr \left[ \delta^\mu(y) \delta^\nu(x) \right] \\
4 \int_0^1 d\alpha \, \ln \frac{\Lambda^2}{s(\alpha)} \left( \left[ \alpha^3(1-\alpha) - 3\alpha^2(1-\alpha)^2 \right] l^2 (l_\mu l_\nu - g_{\mu\nu} l^2) \\
+ \frac{3}{2} \alpha^2(1-\alpha)^2 - \alpha^3(1-\alpha) \right) g_{\mu\nu} l^4. \tag{101}
\]

- Case 3.

\[ \delta(y) = \gamma_\mu (\gamma_5) \delta^\mu(y) \quad \delta(x) = \gamma_\mu (\gamma_5) (\delta^\mu(x) d^\lambda - \delta^\lambda(x) d^\mu) d_\lambda \]

\[
\Gamma_{\text{log}} = 1 \frac{N_c \pi^2}{(2\pi)^8} \int dx \, dy \int d^4l \, e^{il(x-y)} tr \left[ \delta^\mu(y) \delta^\nu(x) \right] \\
4 \int_0^1 d\alpha \ln \frac{\Lambda^2}{s(\alpha)} \left( \left[ \alpha^3(1-\alpha) - 3\alpha^2(1-\alpha)^2 \right] l^2 (l_\mu l_\nu - g_{\mu\nu} l^2) \\
+ \frac{3}{2} \alpha^2(1-\alpha)^2 - \alpha^3(1-\alpha) \right) g_{\mu\nu} l^4. \tag{102}
\]

The last term proportional to \( g_{\mu\nu} l^4 \) in eq. (102) does not contribute to the logarithmically divergent part of the integral and one obtains:

\[
\Gamma_{\text{log}} = 1 \frac{N_c \pi^2}{(2\pi)^8} \int dx \, dy \int d^4l \, e^{il(x-y)} tr \left[ \delta^\mu(y) \delta^\nu(x) \right] \\
4 \int_0^1 d\alpha \left( \left[ \alpha^3(1-\alpha) - 3\alpha^2(1-\alpha)^2 \right] \ln \frac{\Lambda^2}{s(\alpha)} \right). \tag{103}
\]

We have not included logarithmic terms proportional to the IR cutoff mass \( M_Q \).

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TABLE CAPTIONS

1) Parity and Charge Conjugation transformation properties of the quark bilinears.

2) Parity and Charge Conjugation transformation properties of the fundamental fields of the effective meson theory.
FIGURE CAPTIONS

1) The QCD diagram with one gluon exchange generates an effective 4-quark interaction vertex.

2) A quark-loop diagram with at least one meson field as external leg. The integration over quarks (and gluons) produces the vertices of the effective meson Lagrangian. Double lines are resonances, dotted lines are pions and wavy lines are the external currents.

3) The integrals \( \int_0^1 d\alpha P_1(\alpha) \ln(\Lambda^2 \chi/s(\alpha)) \) which occur in the NTL logarithmic corrections to the effective meson Lagrangian are shown as a function of \( \sqrt{Q^2} \). The three polinomials correspond to the three cases of Appendix A.

4) The vector meson exchange at the tree level gives the \( Q^2 \) dependent term of the vector-vector correlation function in a vector resonance model. The ENJL model gives the prediction \([51]\) for the parameters of the vector meson Lagrangian.

5) The running of \( f_V \) with \( Q^2 \) generated by the QR model: the full circle indicates the insertion of a leading \( (O(1)) \) or a next-to-leading \( (O(1/\Lambda^2 \chi)) \) vertex in the one quark-loop diagram.

6) The resummation of n-quark bubble diagrams which gives the full \( Q^2 \) dependence of the vector-vector correlation function in the ENJL model of ref. \([12]\). They contain the insertion of the leading 4-quark vector vertex with coupling \( G_V \).

7) The 4-quark vector vertex of the fermion action with coupling \( G_V \) is replaced by the sum of the q-q-vector vertex and the mass term of the vector field in the bosonized action.

8) The ”dressed” vector meson propagator is given by the resummation of n quark-loop diagrams which are leading in the \( 1/N_c \) expansion.

9) The full vector two-point function as predicted by the QR model which we remind is developed at the leading order in the \( 1/N_c \) expansion. The vector meson propagator of the second term is defined in fig. 8.
10) The derivative of the experimental vector-vector function $-d\Pi_1^V(Q^2)/dQ^2$ (solid line), the fitted curve of the QR model (dashed line) and the prediction of the ENJL model including quark-bubble resummation and the logarithmic contributions in the incomplete Gamma functions $\Gamma(0, x)$ [12] (dot-dashed line) are shown as a function of $\sqrt{Q^2}$. The fit has been performed in the region $0.5 \geq \sqrt{Q^2} \leq 0.9$ GeV.

11) The invariant function $\Pi_1^V(Q^2)$ (dashed line) is obtained from the fitted derivative of fig. 10 by imposing the matching with the ENJL function at $Q = M_Q$. The ENJL prediction of eq. (79) (full line) is also shown. Gluon contributions have not been included.
|       | P          | C          |
|-------|------------|------------|
| $V_\mu$ | $\epsilon(\mu)$ | $-V_\mu^T$ |
| $A_\mu$ | $-\epsilon(\mu)$ | $A_\mu^T$ |
| $\sigma$ | $\sigma$ | $\sigma^T$ |
| $\Gamma_\mu$ | $\epsilon(\mu)$ | $-\Gamma_\mu^T$ |
| $\xi_\mu$ | $-\epsilon(\mu)$ | $\xi_\mu^T$ |
| $f_{\mu\nu}^\pm$ | $\pm\epsilon(\mu)\epsilon(\nu)$ | $\mp f_{\mu\nu}^{\pm T}$ |
| $\chi^\pm$ | $\pm \chi^\pm$ | $\chi^T_\pm$ |

Table 1.
| | P | C |
|---|---|---|
| $\bar{Q}Q$ | + | + |
| $\bar{Q}\gamma_5 Q$ | − | + |
| $\bar{Q}\gamma_\mu \gamma_5 Q$ | $-\epsilon(\mu)$ | $(\bar{Q}\gamma_\mu \gamma_5 Q)^T$ |
| $\bar{Q}\gamma_\mu Q$ | $\epsilon(\mu)$ | $-(\bar{Q}\gamma_\mu Q)^T$ |
| $\bar{Q}\sigma_{\mu\nu} Q$ | $\epsilon(\mu)\epsilon(\nu)$ | $-(\bar{Q}\sigma_{\mu\nu} Q)^T$ |
| $\bar{Q}\sigma_{\mu\nu}\gamma_5 Q$ | $\sim \epsilon^{\mu\nu\alpha\beta} V_{\alpha\beta}$ | |

Table 2.
This figure "fig1-1.png" is available in "png" format from:

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