THE SEARCH FOR ZOO-PERPARTICLES

JOSE-LUIS VÁZQUEZ-BELLO

Physics Department, Queen Mary and Westfield College,
Mile End Road, London E1 4NS,
UNITED KINGDOM.

ABSTRACT

This paper reviews the covariant formalism of N=1, D=10 classical superparticle models. It discusses the local invariances of a number of superparticle actions and highlights the problem of finding a covariant quantization scenario. Covariant quantization has proved problematic, but it has motivated in seeking alternative approaches that avoids those found in earlier models. It also shows new covariant superparticle theories formulated in extended spaces that preserve certain canonical form in phase-space, and easy to quantize by using the Batalin-Vilkovisky procedure, as the gauge algebra of their constraints only closes on-shell. The mechanics actions describe particles moving in a superspace consisting of the usual N = 1 superspace, together with an extra spinor or vector coordinate. A light-cone analysis shows that all these new superparticle models reproduce the physical spectrum of the N=1 super-Yang-Mills theory.

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1. Introduction.

All gauge theories, including super-Yang-Mills theories (SYM), are characterized by a common feature, namely, unphysical degrees of freedom. These unwanted degrees are contained within the fields and must be fixed by an appropriate mechanism (gauge fixing condition) when quantizing the model. In the lagrangian formulation the gauge fixing condition is implemented by a gauge breaking term which is added to the original gauge invariant classical action so as to make the quantum action nondegenerate [1]. However, in general, a gauge fixing term might spoil other global symmetries of the theory, like for example relativistic covariance. The importance of covariant quantization is that the global symmetries can be used to simplify calculations in the quantum theory, and therefore it is recommended to leave these global symmetries manifest. Although, there have been numerous contributions in this direction since the work by Faddeev and Popov (FP) [2], the Batalin and Vilkovisky (BV) formulation encompasses all previous developments [3]. Covariant quantization of superparticle and superstring theories typically requires the introduction of an infinite number of ghost fields (infinite reducible gauge theories) [4], and can be approached using the BV quantization procedure.

The Brink-Schwarz-Casalbouni (BSC) superparticle, which is a supersymmetric extension of the standard relativistic particle [5,6], has many properties in common with superstrings [4,7,8]. In particular, the D=10 superparticle describes the dynamics of the zero-modes of the D=10 superstring [8-10], and so it is often used as a toy model. The application of the BV procedure to the classical BSC superparticle action has lead the reproduction of an incorrect physical spectrum, which is known from a light-cone gauge (non-covariant) quantization method [11]. Although several modified superparticle actions have been proposed with the intention of solving this problem [12-15], a truly covariant quantization of the BSC model has not yet been found [7,16,17]. However, in recent works [18-24], new proposed superparticle models have the same physical spectrum as the original BSC superparticle.
The purpose of this manuscript is to review some aspects of the various super-particle models. It would take far too long to go into all the details, however, these can be found in the original papers [5,6,12,14,15,18,22,23].

2. Local Reparametrization Invariance.

Let us consider a dynamical system formulated in a phase-space with coordinates \((q_i, p^i)\) \((i = 1, \ldots, N)\) where \(q_i\) are the canonical coordinates and \(p^i\) are their corresponding conjugate momenta. The canonical Hamiltonian of the system is \(H_0\). Let us also suppose that there are \('m'\) first-class constraints \(G_a(q, p)\), and they satisfy \[\{G_a, G_b\} = f^{c}_{ab}G_c\] (2.1)

Here \(\{,\}\) denotes Poisson brackets. The constraints (2.1) must satisfy the following stability condition

\[\{H_0, G_b\} = V^a_{b}G_a, \quad (a, b = 1, \ldots, m).\] (2.2)

The canonical action is

\[S = \int d\tau \left[ p^i \dot{q}_i - H_0 - \lambda^a G_a \right]\] (2.3)

where \(\lambda^a(\tau)\) are Lagrange multipliers, and \(\tau\) parametrizes the world-line in the phase-space. The action (2.3) is made invariant under a local gauge transformation, generated by the constraints \(G_a(q, p)\), by choosing convenient transformation properties for the \(\lambda\)'s and appropriate boundary conditions for the infinitesimal

\(\Diamond\) The Poisson bracket of two functions \(f\) and \(g\) in the phase-space with \(N\) degrees of freedom is defined by \(\{f, g\} = \sum_{i=1}^{N} \left[ \frac{\partial f}{\partial q_i} \frac{\partial g}{\partial p^i} - \frac{\partial f}{\partial p_i} \frac{\partial g}{\partial q^i} \right] \).
gauge parameters of the transformation $\epsilon^a(\tau)$. The infinitesimal gauge transformations for (2.3) are given by [26,27]

$$\delta_\epsilon q_i = [q_i, G_a \epsilon^a], \quad \delta_\epsilon p^i = [p^i, G_a \epsilon^a],$$

(2.4)

and

$$\delta_\epsilon \lambda^a = \frac{\partial \epsilon^a}{\partial \tau} + [\epsilon^a, H_0 + \lambda^b G_b] - f^a_{bc} \epsilon^b \lambda^c - \epsilon^b V^a_b,$$

(2.5)

provided that (2.4) and (2.5) vanish at the endpoints of the world-line, $\tau_1$ and $\tau_2$. Precisely, the gauge parameters $\epsilon^a$ of the infinitesimal transformations (2.4) and (2.5) must satisfy the following boundary condition

$$\epsilon(\tau_1) = 0, \quad \epsilon(\tau_2) = 0.$$

(2.6)

The variation of the action (2.3) under the transformations (2.4) and (2.5) is proportional to the following boundary term

$$\left[ p^i \frac{\partial (\epsilon^a G_a)}{\partial p^i} - \epsilon^a G_a \right]_{\tau_2} - \left[ p^i \frac{\partial (\epsilon^a G_a)}{\partial p^i} - \epsilon^a G_a \right]_{\tau_1},$$

(2.7)

which cancels as a result of (2.6). It has become customary to call a system with constraints that are linear and homogeneous in the momenta systems with internal gauge symmetries; Yang-Mills systems are of this type [26]. For such systems

$$\left[ p^i \frac{\partial G_a}{\partial p^i} - G_a \right] = 0,$$

(2.8)

vanishes identically. ♣

We can choose a gauge in which $\lambda^a$ are constants, simply by integrating (2.5). However, this is not a simple task for local symmetries as one, in general, cannot integrate the first-order differential equation (2.5) with two simultaneous boundary conditions.

♣ Although (2.8) is true for the Yang-Mills field, it does not hold, for example, for the Freedman-Townsend model [29].
3. The Relativistic Scalar Particle.

The relativistic massive scalar particle is described by the evolution of a massive point particle in a D dimensional space, and its trajectory is represented by a world-line \((x^\mu(\tau))\) where \(\mu = 0, 1, \ldots, D - 1\) and \(\tau\) parametrizes the world-line. The action for the relativistic scalar particle is given by [27,28,30]

\[
S = -m \int d\tau \sqrt{-\dot{x}^\mu \dot{x}^\nu \eta_{\mu\nu}}, \quad (3.1)
\]

where \(\eta_{\mu\nu}\) is the D-dimensional Minkowski metric and \(\dot{x}^\mu = \frac{\partial x^\mu(\tau)}{\partial \tau}\). The action (3.1) can be interpreted as the action of \(D\) scalar fields in a 1-dimensional space-time, as it can be rewritten in the following form

\[
S_0 = \frac{1}{2} \int d\tau [e^{-1} \dot{x}^\mu \dot{x}^\nu \eta_{\mu\nu} - e m^2], \quad (3.2)
\]

where \(e\) is the \textit{einbein} of the world-line. The canonical conjugate momenta \(p^\mu\) is given by

\[
\frac{\partial L}{\partial \dot{x}^\mu} = p^\mu = m \frac{\dot{x}^\mu}{\sqrt{-\dot{x}^2}}, \quad (3.3)
\]

or

\[
p^\mu = e^{-1} \dot{x}^\mu \quad (3.4)
\]

where either (3.1) or (3.2) have been used, respectively, as the Lagrangian \(L\) in the definition of the canonical conjugate momenta \(p^\mu\). There is a first-class constraint

\[
G_a = p^2 - m^2. \quad (3.5)
\]

From the reparametrization invariance of the relativistic action (3.1), it follows
that the canonical Hamiltonian vanishes identically,

\[ H_0 = p_\mu \dot{x}^\mu - L = 0. \]  

(3.6)

The canonical action for the relativistic scalar particle is described in the phase-space \((x^\mu, p_\mu)\) by

\[ S_{\text{canonical}} = \int d\tau [p_\mu \dot{x}^\mu - \frac{1}{2} e (p^2 - m^2)], \]  

(3.7)

where \(e\) is a Lagrange multiplier. Therefore, the action (3.7) can be made invariant under a local gauge transformation generated by the constraint (3.5), by choosing convenient transformation properties for \(e\) and appropriate boundary conditions for the infinitesimal gauge parameters. Indeed, the action (3.7) is invariant under the following \(\tau\)-reparametrization generated by the \(G_a\) constraint

\[ \delta_\epsilon x^\mu = -2p^\mu \epsilon(\tau), \quad \delta_\epsilon e = \dot{\epsilon}(\tau), \quad \delta_\epsilon p^\mu = 0, \]  

(3.8)

where \(\epsilon(\tau)\) is the infinitesimal gauge parameter of the transformation with fixed endpoints. We also notice that there is no problem with the massless limit in either (3.2) or (3.7).

4. **BSC Superparticle.**

A superparticle is a particle moving in a superspace, and its evolution is represented by a world-line in a \(D\)-dimensional superspace \((x^\mu(\tau), \theta(\tau))\) where \(\mu = 0, 1, \ldots, D - 1; \theta\) is an anti-commuting Majorana-Weyl spinor and \(\tau\) parametrizes the world-line. The superparticle mechanics action proposed by Brink and Schwarz is given in first-order form by [5,6]

\[ S_{\text{BSC}} = \int d\tau [p_\mu \dot{x}^\mu - i \dot{\theta} \dot{\theta} - \frac{1}{2} e p^2], \]  

(4.1)

where \(\dot{\theta} = \frac{\partial \theta}{\partial \tau}\), \(p^\mu\) is the momenta of the superparticle and \(e\) is the *einbein* on
A variation of (4.1) with respect to \(e\) implies the massless Klein-Gordon equation \(p^2 = 0\). The remaining classical field equations are

\[
\dot{p}^\mu = 0, \quad \dot{p} \dot{\theta} = 0, \quad p^\mu = e^{-1}[\dot{x}^\mu - i\theta \gamma^\mu \dot{\theta}].
\]

The action (4.1) is invariant under \(\tau\)-reparametrization (generated by the \(G_a = p^2\) constraint) together with rigid space-time supersymmetry transformations

\[
\delta_e \theta = \epsilon, \quad \delta_e x^\mu = i\epsilon \gamma^\mu \theta, \quad \delta_e p^\mu = \delta_e e = 0,
\]

where the infinitesimal parameter \(\epsilon^A\) is a constant Grassmann-valued space-time anti-commuting spinor. The action (4.1) is also invariant under a local fermionic symmetry [31]

\[
\delta \theta = \delta \kappa, \quad \delta x^\mu = -i\kappa \gamma^\mu \theta, \quad \delta e = 4i\dot{\theta} \kappa, \quad \delta p^\mu = 0,
\]

where the infinitesimal gauge parameter \(\kappa = \kappa^A\) is a Majorana-Weyl space-time spinor. Although the superparticle action (4.1) is formulated in a superspace with coordinates \((x^\mu, \theta_A)\), it lacks the conjugate momenta associated with the supercoordinate \(\theta_A\). Consequently, the canonical structure of (2.3) is broken. In addition, the gauge algebra contains some extra symmetries when the equations of motion are not used (off-shell). In particular, the commutator of two fermionic symmetries gives a linear combination of world-line diffeomorphisms plus a new transformation of the form [32]

\[
\delta x^\mu = p^2 v^\mu, \quad \delta e = -2\dot{p}_\mu v^\mu,
\]

where \(v^\mu\) is a bosonic vector parameter. This is a local symmetry of the action (4.1) which is trivial, but it is needed to close the gauge algebra off-shell.

\[\♠\] It is often convenient to use a 16-component spinor notation to distinguish chirality, so that a right-handed Majorana-Weyl spinor \(\psi_A\) has lower spinor index \((A = 1, \ldots, 16)\). In this notation, the supercoordinates have components \(\theta_A, \theta_{\gamma^\mu} \dot{\theta} = \theta_A (\gamma^\mu)^{AB} \dot{\theta}_B, \dot{p}_{AB} = p_\mu (\gamma^\mu)^{AB}, (\gamma^\mu)_{AB} = (\gamma^\mu)_{BA}, \) and so on.
5. Siegel Superparticles.

An alternative action which restores the canonical form of (2.3), should include a conjugate momenta associated with the spinor coordinate $\theta_A$. An action of this type was proposed by Siegel [12] for a manifest space-time supersymmetry invariance by introducing a gauge field $\psi^A$ (referred as SSP1 superparticle or $AB$ system). The SSP1 action is [12,13]

$$S_{SSP1} = \int d\tau [p_\mu \dot{x}^\mu + i \dot{\theta} \dot{\theta} - \frac{1}{2} e p^2 + i \psi \hat{p} d]$$  \hspace{1cm} (5.1)

where $d^A$ is a fermionic space-time anticommuting spinor introduced so that the Grassmann coordinate $\theta_A$ has a conjugate momenta $\hat{\theta}^A = d^A - \hat{p}^{AB} \theta_B$. The superparticle action (4.1) is obtained from (5.1) by setting $d = 0$. A variation of (5.1) with respect to $e$ and $\psi^A$ implies, respectively, the following $G_a$ constraints

$$G_e = p^2, \quad G_\psi = \hat{p} d,$$  \hspace{1cm} (5.2)

so that the non-derivative terms in (5.1) are the product of Lagrange multipliers $\lambda^i$ with constraints $G_a$. The remaining classical equations of motion are

\[
\begin{align*}
\dot{p}^\mu &= 0, & \dot{\theta} &= 0, & \dot{d} &= 2 \dot{\theta}, \\
\dot{\hat{\theta}} &= \hat{p} \psi,
\end{align*}
\]

$$p^\mu = e^{-1}[\dot{x}^\mu - i \dot{\theta} \gamma^\mu \dot{\theta} + i \psi \gamma^\mu d].$$  \hspace{1cm} (5.3)

The algebra of constraints $G_a$ is

$$\{G_e, G_e\} = 0, \quad \{G_e, G_\psi\} = 0, \quad \{G_\psi, G_\psi\} = 2 \hat{p} G_e,$$  \hspace{1cm} (5.4)

subject to the reducibility of the constraints. Indeed, an important feature of this superparticle model is the reducibility of the constraints (5.2), as they are linearly dependent

$$G_e d - \hat{p} G_\psi = 0.$$  \hspace{1cm} (5.5)

Therefore, (5.5) expresses that the superparticle (5.1) is a system with reducible constraints. In addition, the $G_a$ are not only constraints, but the generators of a
number of local gauge symmetries by choosing convenient transformations properties for the Lagrange multipliers and appropriate boundary conditions for the infinitesimal gauge parameters of the transformations. The action (5.1) is then invariant under rigid Poincaré transformations together with the rigid space-time supersymmetry and a number of local symmetries [16,33]. It is convenient to combine the reparametrization invariance with a trivial symmetry ♠ to obtain a local bosonic $A$ symmetry

$$\delta x^\mu = p^\mu \xi, \quad \delta e = \dot{\xi},$$

(5.6)

The local fermionic symmetry (sometimes referred as $B$ symmetry) is given by

$$\delta \theta = \dot{\theta} \kappa, \quad \delta x^\mu = -i\kappa \dot{\gamma}^\mu \theta + i\dot{\gamma}^\mu \kappa, \quad \delta e = 4i\dot{\theta}\kappa, \quad \delta \psi = \dot{\kappa}, \quad \delta p^\mu = 0, \quad \delta d = 2p^2 \kappa,$$

(5.7)

where $\kappa^A$ is an anticommuting Majorana-Weyl spinor. The action (5.1) is also invariant under symmetries that act only on the gauge fields, a generalized local bosonic symmetry and global space-time supersymmetries. However, these further symmetries are not needed to close the gauge algebra of constraints and we shall ignore their status in the discussion given below, but its consequences are considered elsewhere [16].

There is another reformulation to the $SSP1$ superparticle action which includes a further gauge field $\chi$ so that one of the further global space-time symmetries of the $SSP1$ action is turned into a local symmetry [15]. The $SSP2$ superparticle action (sometimes referred as the $ABC$ system) is then an extension of the $SSP1$ action by the addition of a bilinear term in the field $d^A$, and it guarantees also the ♠ A trivial symmetry is one under which all fields transform into equations of motion. So that an action $S(\phi')$ dependent fields $\phi'$ will automatically be invariant under local transformations of the form $\delta \phi' = \lambda J^{ij}(\phi)\delta S/\delta \phi'$ (with local parameter $\lambda$) provided that $J^{ij}$ is (graded) anti-symmetric [16].
closure of the gauge algebra [8,15]. The SSP2 action is [15,33]

$$S_{SSP2} = \int d\tau [p_\mu \dot{x}^\mu + i\dot{\theta} - \frac{1}{2}e p^2 + i\dot{\psi}d + \frac{1}{2}d\chi d]$$ (5.8)

where $p^\mu$ is the momentum conjugate to the space-time coordinate $x^\mu$, while $\dot{\theta}^A = d^A - \dot{\theta}^{AB} \theta_B$ is conjugate to the Grassmann super-coordinate $\theta_A$, and $e$, $\psi^A$ and $\chi_{AB} = -\chi_{BA}$ are gauge fields which are also Lagrange multipliers imposing some classical constraints. The original $S_{BSC}$ superparticle action (4.1) is given by setting $d = 0$, while the SSP1 action is given by setting $\chi = 0$ in (5.8).

A variation of (5.8) with respect to $e$, $\psi$ and $\chi$ implies, respectively, the following classical constraints

$$G_e = p^2, \quad G_\psi = \psi d, \quad G_d = d d.$$

These constraints satisfy the algebra

$$\{G_\psi, G_\psi\} = 2\dot{\psi} G_e, \quad \{G_\psi, G_d\} = 2d G_e,$$

$$\{G_d, G_d\}_{ABCD} = 4(\dot{\psi})_A[C G_d D]|B - 4(\dot{\psi})_B[C G_d D]|A,$$

$$\{G_e, G_e\} = 0, \quad \{G_e, G_\psi\} = 0, \quad \{G_e, G_d\} = 0.$$ (5.10)

However, the right-hand sides of (5.10) are not unique due to certain linear relations among the constraints (5.9). These are given by

$$\dot{\psi} G_\psi - d G_e = 0, \quad \dot{\psi} G_d - d G_\psi = 0, \quad d G_d = 0,$$ (5.11)

and so on. Therefore, they imply that the SSP2 superparticle model (5.8) is also a reducible system. The remaining classical equations of motion are

$$\dot{\psi}^\mu = 0, \quad \dot{\theta} = 0, \quad \dot{d} = 2\dot{\psi} \dot{\theta},$$

$$\dot{\psi} = \dot{\psi}^\mu + i\chi d, \quad p_\mu = e^{-1}[\dot{x}^\mu - i\theta \gamma^\mu \dot{\theta} + i\psi \gamma^\mu d].$$ (5.12)

The SSP2 action has a large number of symmetries which generalize the ones found for the earlier superparticle models. It is invariant under rigid Poincaré
transformations together with the rigid space-time supersymmetry. The action (5.8) is also invariant under local gauge transformations generated by the constraints (5.9), which are referred as the \(\mathcal{A}, \mathcal{B}\) and \(\mathcal{C}\) symmetries [16,33]. Explicitly, the local \(\mathcal{A}\) symmetry is given by

\[
\delta x^\mu = p^\mu \xi, \quad \delta e = \dot{\xi},
\]

(5.13)

the \(\mathcal{B}\) symmetry is

\[
\delta \theta = \dot{\theta} \kappa, \quad \delta x^\mu = -i \kappa \dot{\psi} \gamma^\mu \theta + i d \gamma^\mu \kappa,
\]

\[
\delta e = 4i \dot{\theta} \kappa, \quad \delta \psi = \dot{\kappa}, \quad \delta p^\mu = 0, \quad \delta d = 2p^2 \kappa,
\]

(5.14)

and the \(\mathcal{C}\) symmetry is

\[
\delta \theta = -i \eta d, \quad \delta x^\mu = -i d \eta \gamma^\mu \theta, \quad \delta d = i \dot{\eta} d,
\]

\[
\delta e = -i \psi \eta d, \quad \delta \chi = \dot{\eta} + i (\chi \dot{\eta} - \eta \dot{\chi}),
\]

(5.15)

where \(\kappa^A\) is a fermionic spinor parameter, while \(\eta_{AB} = -\eta_{BA}\) is a bosonic bispinor parameter associated with the gauge field \(\chi_{AB}\). There are also a number of local symmetries that act only on the gauge fields and their presence reflects ambiguities in the definition of the \(\mathcal{A}, \mathcal{B}\) and \(\mathcal{C}\) symmetries and their relations among the constraints.

6. Gauge Fixing and Quantization.

We now consider both the counting of the fields and the choice of gauge conditions that will fix the gauge invariances for the above superparticle models. For the classical relativistic scalar particle (3.7), there are 10 degrees of freedom corresponding to the ten components of \(x^\mu\). The degree of freedom corresponding to the field \(e\) is gauged away, while the momentum \(p^\mu\) is an auxiliary field and so can be eliminated by its equation of motion. In the quantum theory, the net number
of degrees of freedom is given by a graded total count. The momentum $p^\mu$ is still an auxiliary field, while the gauge invariance is fixed. There is now one negative degree of freedom corresponding to the ghost of diffeomorphisms giving a graded total of $10 + 1 - 1 = 0$ (off-shell).

For the classical BSC superparticle (4.1), the counting of degrees of freedom for the bosonic sector is the same, as before. For the fermionic sector, however, there are 16 degrees of freedom corresponding to the components of the Majorana-Weyl spinor $\theta_A$. The fermionic local symmetry allows the choice of a non-covariant (physical) gauge in which half of the 16-components of the fermionic spinor coordinate $\theta_A$ are gauged away, while the eight surviving components of $\theta_A$ are self-conjugate, leaving a total net number of 8 fermionic degrees of freedom. In the quantum theory, the graded counting of degrees of freedom for the bosonic sector still remains the same. For the fermionic sector, there is a sequence of 16-component spinors $(\theta, \kappa_1, \kappa_2, \ldots, \kappa_n, \ldots)$ corresponding to the fermionic spinor $\theta_A$ and the corresponding ghosts degrees of freedom for the fermionic local symmetry $\kappa$, giving a graded total of $16 \times (1 - 1 + 1 - 1 + \cdots)$ degrees of freedom (the alternating sign corresponds to alternating statistics in the fields). This ill-defined series is regularized to give a total of $16 \times \frac{1}{2} = 8$ degrees of freedom, which is in agreement with the classical counting [16,32].

In a light-cone gauge (non-covariant), the reparametrization invariance is used to set the gauge field $e$ to be a constant and the fermionic symmetry is used to eliminate half of the 16-components of $\theta_A$. An $SO(9,1)$ vector $x^\mu (\mu = 0, 1, \ldots, 9)$ decomposes into an $SO(8)$ vector $x^i (i = 1, \ldots, 8)$ and two singlets $x^+, x^- (x^\pm = x^0 \pm x^9)$, so that if $x^+$ is set equal to the solution of its equation of motion, then $x^-$ is determined by solving $p^2 = 0$. A 16-component spinor of $SO(9,1)$ decomposes into two 8-component $SO(8)$ spinors, so that $\gamma^+ \theta = 0$ gauges away exactly eight of the 16-components of $\theta_A$. The remaining eight bosonic $x^i$ and eight fermionic $\theta^a (a = 1, \ldots, 8)$ variables transform as a vector and spinor, respectively, of the little group $SO(8)$ [4,16,34]. One can further break the Lorentz covariance of the superparticle from $SO(8)$ to $U(4)$, since the eight surviving fermionic components
are selfconjugate. So that in a canonical approach they can be decomposed into four coordinates $\theta^a_c$ ($a = 1, \ldots, 4$) and four momenta $\pi^a_c$. The physical content of the superparticle is then given by the fields in the component expansion of a complex superfield $\Phi(p^\mu, \theta^a_c)$ which satisfies the constraint $p^2 \Phi(p^\mu, \theta^a_c) = 0$ and a reality condition. The component expansion gives $1 + 6 + 1 = 8$ real bosonic components, and $4 + 4 = 8$ real fermionic components. Such physical content of the BSC superparticle fit together to give the spectrum for the $N = 1$ super-Yang-Mills Theory [35,36].

For the classical SSP1 superparticle (5.1), there are 10 degrees of freedom corresponding to $x^\mu$, the field $e$ is gauged away and $p^\mu$ is eliminated by its equation of motion. There are also 16 degrees of freedom corresponding to $\theta_A$. However, the fermionic symmetry can be used to gauge away eight of the 16 components of $\theta$ which is in accord with the expected physical spectrum of the theory. In a **covariant quantization** it is necessary to find a covariant gauge choice for both the reparametrization and fermionic symmetries. The reparametrization invariance can be fixed by imposing a constraint on the einbein $e = \text{constant}$, while the fermionic invariance can be fixed by imposing a gauge condition on the fermionic gauge field, $\psi = 0$. However, a light-cone gauge quantization of the SSP1 reveals that its corresponding spectrum contains $2^8$ states and it is therefore not equivalent to the $N = 1$ BSC superparticle action (4.1) which has a spectrum of $2^4$ states [16,37]. It was emphasized previously that the SSP1 model is invariant under a further global space-time supersymmetry so that the corresponding superalgebra of the generators of these global supersymmetries have a **twisted** nature. Therefore, the $N = 1$ SSP1 superparticle is on-shell equivalent to a *twisted* version of the $N = 2$ SSP0 superparticle that has negative norm states [16].

The SSP2 superparticle (5.8) is an extension of the SSP1 action by the addi-

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♣ If the supercharges generating the global space-time supersymmetries are denoted by $Q_1$ and $Q_2$, respectively, the corresponding superalgebra have the twisted algebra $\{Q_1^A, Q_2^B\} = -\{Q_2^A, Q_1^B\} = 2(\gamma^\mu)^{AB} p^\mu$, and $\{Q_1^A, Q_2^B\} = 0$. The relative minus sign means a $N = 2$ twisted supersymmetry which has automorphism group $SO(1,1)$, and it also leads the presence of negative norm states [16].
tion of a bosonic bi-spinor $\chi_{AB}$ which acts as a Lagrange multiplier imposing a constraint that is intended to remove those unwanted negative norm states of the SSP1. The extra term breaks the further global space-time supersymmetry of the SSP1, so that the SSP2 action is left invariant under a single global space-time supersymmetry. In a light-cone gauge analysis of the SSP2, it is found that there are again eight bosonic degrees of freedom and eight $\theta$'s plus eight independent $d$'s, but also there is a residual $SO(8)$ symmetry which is used to gauge $d$ to a constant, so that the physical spectrum of the SSP2 consists of eight bosons and eight fermions, corresponding to the spectrum for the $N = 1$ super-Yang-Mills theory [16,36,38]. However, covariant quantization of the SSP2 superparticle yields unsatisfactory results, because its BRST operator does not give the correct BRST cohomology classes [17].

To summarize, two reformulations of the original superparticle action BSC (5.1) have been proposed which include a gauge field for the local fermionic symmetry. Neither of these reformulated models give satisfactory results, but their quantization illustrates a number of interesting features. Covariant quantization of any of these superparticles or further modifications require an infinite number of ghost fields which reflects the ambiguities on the infinite reducibility of the constraints.

7. Quadratic Superparticles.

Covariant quantization of the first-class $\mathcal{ABC}$ system [17], either by BRST or BV methods has failed, because the BRST operator does not give the correct cohomology [22]. In Ref. [22], it was shown that the BRST quantization of any set of constraints forming a compact gauge algebra should contain a singlet of the gauge group. The above problem for the $\mathcal{ABC}$ system can be avoided by the appropriate modification of the constraints. In Ref. [22], two formulations were also considered for solving the difficulties of the $\mathcal{ABC}$ system (referred as first or second-ilk superparticles). The first-ilk or $\mathcal{ABCD}$ superparticle is defined by introducing a new fermionic variable $\Gamma_a$ (which satisfies $\{\Gamma_a, \Gamma_b\} = 2\eta_{ab}$) and a
new constraint $\mathcal{D}$. The full set of constraints for this system is given by [19,20]

$$
\mathcal{A} = p^2, \quad \mathcal{B} = \hat{p}d, \quad \mathcal{D} = p \cdot \Gamma,
$$

$$
C_{\alpha\beta} = \frac{1}{2}(\gamma^{abc})_{\alpha\beta} C_{abc} = d_{[\alpha}d_{\beta]} + \frac{1}{2}(\gamma^{abc})_{\alpha\beta} p_a \Gamma_b \Gamma_c. \tag{7.1}
$$

The constraints $\mathcal{A}, \mathcal{B}, \mathcal{C}$ and $\mathcal{D}$ satisfy the algebra [19,20]

$$
\{\mathcal{B}, \mathcal{B}\} = 2\hat{p}\mathcal{A}, \quad \{\mathcal{C}, \mathcal{B}\}_{\alpha\beta} = -4\delta_{[\alpha}^{\gamma}d_{\beta]}\mathcal{A}, \quad \{\mathcal{D}, \mathcal{D}\} = 2\mathcal{A},
$$

$$
\{\mathcal{C}, \mathcal{C}\}_{\alpha\beta\gamma\delta} = 4(\hat{p})_{\alpha[\gamma} C_{\delta]\beta] - 4(\hat{p})_{\beta[\gamma} C_{\delta]\alpha}
$$

$$
- \frac{1}{2}(\gamma^{a}_{cd})_{\alpha\beta}(\gamma^{bcd})_{\gamma\delta}\frac{1}{2} \Gamma_{[a} \Gamma_{b]} \mathcal{A} + p_{[a} \Gamma_{b]} \mathcal{D}. \tag{7.2}
$$

However, the right-hand sides of (7.2) are not unique due to the reducibility of the constraints $\mathcal{A}, \mathcal{B}, \mathcal{C}$ and $\mathcal{D}$. The system $\mathcal{ABCD}$ is infinitely reducible in the sense of Batalin and Vilkovisky [3].

On the other hand, the second-ilk superparticle consists of an infinite number of constraints and an infinite number of spinorial coordinates and momenta. The full set of constraints for this model is given by [22]

$$
\mathcal{A} = p^2, \quad \mathcal{B} = \hat{p}q_0, \quad \mathcal{C}_n = d_{2n} + q_{2n+2}, \tag{7.3}
$$

where $d_n = -i\dot{\theta}_n + \hat{p}\theta_n$ and $q_n = -i\dot{\theta}_n - \hat{p}\theta_n$ ($n = 0, 1, \ldots$) are fermionic space-time anticommuting spinors, $\hat{\theta}_n$ are conjugate momenta to the Grassmann supercoordinates $\theta_n$ ($n = 0, 1, \ldots$), and satisfy $\{\hat{\theta}_m, \theta_n\} = i\delta_{mn}$. The constraints (7.3) are also infinitely reducible.

In Ref. [22], a new classical action was found based upon the constraints (7.3). This second-ilk superparticle describes and infinite sequence of SSP1 superparticles plus a term that breaks down all the twisted supersymmetries of the infinite sequences of SSP1 superparticles. This model is formulated in a ten-dimensional superspace with coordinates $(x^\mu, \theta_0, \ldots, \theta_{2n}, \ldots)$, where $(\theta_0, \ldots, \theta_{2n}, \ldots)$ are anti-
commuting spinors. The action is \[21,22\]

\[
S_{\text{sec-ilk}} = \int d\tau \left[ p_\mu \dot{x}^\mu - \frac{1}{2} ep^2 + \sum_{n=0}^{+\infty} \dot{\theta}_{2n} \dot{\theta}_{2n} - \psi_1 \dot{p}q_0 \right. \\
\left. - \sum_{n=0}^{+\infty} \lambda_{2n+1}(d_{2n} + q_{2n+2}) \right],
\]

(7.4)

where \(\dot{\theta}_{2n} = d_{2n} - \dot{p}\theta_{2n}\) are the conjugate momenta to the supercoordinates \(\theta_{2n}\), and \(e, \psi_1\) and \(\lambda_{2n+1}\) are gauge fields which are also Lagrange multipliers imposing the infinite set of classical constraints \(A, B\) and \(C_n\).

The remaining classical field equations of motion are \[21]\n
\[
p^\mu = 0, \quad \dot{p}\dot{\theta}_0 - p^2 \psi_1 = 0, \quad \dot{\theta}_{2n} \dot{p} - i \lambda_{2n-1} \dot{p} = 0, \\
p^\mu = (2e)^{-1}[\dot{x}^\mu - i\psi_1\gamma^\mu d_0 + 4i\psi_1\gamma^\mu \theta_0 - i \sum_{n=0}^{+\infty} \dot{\theta}_{2n} \gamma^\mu \theta_{2n} \\
+ 2 \sum_{n=0}^{+\infty} \lambda_{2n+1} \gamma^\mu \theta_{2n+2}].
\]

(7.5)

The second-ilk superparticle (7.4) is invariant under a number of local gauge symmetries generated by the infinite set of constraints \(A, B\) and \(C_n\) \[21\]. It is also invariant under local gauge symmetries that act only on the gauge fields and their presence reflects ambiguities on the infinite reducibility of the constraints \(A, B\) and \(C_n\).

There is another similar superparticle action of the second-ilk type. It was found when a special combination of a certain \(N = 2\) supersymmetry was promoted to a local symmetry, and whose presence is crucial for obtaining the correct BRST operator \[14,18\]. It starts by considering a SSP1 action and introducing an infinite tower of anti-commuting fermionic variables, which are identified with the zero ghost number sector of the pyramid of ghosts appearing in the covariant
quantization of the BSC superparticle. A further term that breaks down the infinite tower of twisted $N = 2$ supersymmetries is added and as a result the final action is invariant under a global $N = 1$ supersymmetry. The action is [18]

$$S_{sec-ilk} = \int d\tau \left\{ p_\mu \dot{x}^\mu - \frac{1}{2} e p^2 + \sum_{n=0}^{+\infty} \lambda_n (\dot{\theta}_n - \dot{\psi}_n) - \sum_{n=0}^{+\infty} [\left( \lambda_n + ip\theta_n \right) + \left( \lambda_{n+1} - ip\theta_{n+1} \right)] \chi_n \right\}, \quad (7.6)$$

where $\psi_n$ is the gauge field for the fermionic symmetry at the $n’th$ level, while $\lambda_n$ is the conjugate momenta to the supercoordinate $\theta_n$, and $e$ and $\chi_n$ are gauge fields which are also Lagrange multipliers imposing the following infinite set of classical constraints

$$A = p^2, \quad B'_n = \dot{\psi}_n, \quad C'_n = (\lambda_n + ip\theta_n) + (\lambda_{n+1} - ip\theta_{n+1}). \quad (7.7)$$

The superparticle action (7.6) is invariant under the usual $A$ symmetry, together with an infinite sequence of fermionic symmetries with infinitesimal anticommuting spinor parameters $\kappa_n$ and $\xi_n$, respectively [18]. There are also symmetries of the second-type which act only on the gauge fields and reflect the infinite reducibility of the system.♠

♠ A complete cohomology analysis of the BRST operator for the first-ilk superparticle (7.1) was given in Refs. [19,20], while similar cohomology analysis for the second-ilk superparticles (7.4) and (7.6) were given in Refs. [22] and [14,18], respectively. These analysis showed that the cohomology reproduces the desired spectrum of the ten dimensional super-Yang-Mills theory.
8. Superparticles in Extended Spaces.

Let us consider further modifications of the superparticle that lead to free BRST invariant quantum actions. These new superparticle theories are obtained by the addition of extra coordinates to the superspace \((x^\mu, \theta_A)\), and their physical states are described either by a superspace spinor or vector wave function satisfying some linear or quadratic constraints [36]. The wave function for these new models is a superfield whose physical components are those of the super-Yang-Mills theory [23,36]. In this section, we present a summary of these new superparticle theories. A more complete treatment is presented in Ref. [23].

We first seek superparticle theories formulated in an extended superspace with coordinates \((x^\mu, \theta_A, \phi^A)\), where \(\theta_A\) and \(\phi^A\) are anti-commuting Majorana-Weyl spinors. Here \((x^\mu, \theta_A)\) are the usual coordinates of the ten-dimensional \(N = 1\) superspace and \(\phi^A\) is a new spinor coordinate. In [24] an extra spinor coordinate \(\phi^A\) was introduced, together with its conjugate momentum \(\hat{\phi}^A\) and a momentum \(\hat{\theta}^A\) conjugate to \(\theta_A\). It is convenient to define \(\hat{\theta}^A = d^A - \hat{p}^{AB} \theta_B\). The action is the sum of a free action

\[
S_0 = \int d\tau \left[ p_\mu \dot{x}^\mu + i\hat{\theta} \dot{\theta} + i\phi \dot{\phi} \right], \tag{8.1}
\]

plus the term

\[
S_1 = \int d\tau \left[ -\frac{1}{2} ep^2 + i\psi \hat{p} d + i\Lambda \phi \hat{\phi} + \frac{1}{2} \Gamma \dot{\phi} \hat{\phi} + \frac{1}{2} d\chi d + 2\hat{\phi} \Gamma^\mu \chi \Gamma_\mu \phi \hat{\phi} \right], \tag{8.2}
\]

where \(e, \psi^A, \chi_{AB} = -\chi_{BA}, \Lambda_A\) and \(\Gamma^{AB} = -\Gamma^{BA}\) are gauge fields which are also Lagrange multipliers imposing some classical constraints. A variation of (8.1) and

\begin{footnote}{We suppress spinor indices and use a notation, so that \(d\phi = d^A \delta A\), \(\theta \Gamma^\mu \phi = \theta_A \Gamma^\mu \phi^A\), \(d\chi d = d^A \chi_{AB} d\), \(\phi \Gamma^\mu \chi \Gamma_\mu \phi \phi = \phi_A \Gamma_{A}^{\mu} \chi_{BC} \Gamma_\mu \phi^C \phi^D \phi^E\), etc.}\end{footnote}
(8.2) with respect to $e, \psi, \Lambda, \Upsilon$ and $\chi$ implies the following classical constraints

\[
\begin{align*}
\mathcal{A} &= p^2, & \mathcal{B} &= \dot{p}d, & \mathcal{D} &= \dot{\phi}^\dagger, \\
\mathcal{G} &= \hat{\phi}_A \hat{\phi}_B, & \mathcal{C} &= d^A d^B - 8(\hat{\phi} \Gamma^\mu)^A(\Gamma_{\mu} \hat{\phi}) B].
\end{align*}
\]  

(8.3)

This new superparticle action formulated in an extended space has a large number of local gauge symmetries which generalize the ones found for earlier models. The covariant quantization of this superparticle model was discussed in [24] by choosing the gauge $e = 1$, with the other gauge fields set to zero. Covariant quantization uses the BV procedure, as the gauge algebra only closes on-shell, and requires an infinite number of ghosts fields.

There is another reformulation which leads to a spinor wave function satisfying certain linear constraint [36]. The superparticle action is [23]

\[
S_{\text{spinor}} = \int d\tau \left[ p_\mu \dot{x}^\mu + i\dot{\theta} \dot{\theta} + i\dot{\phi} \dot{\phi} \right],
\]  

(8.4)

plus the term

\[
S_{\text{extra}} = \int d\tau \left[ -\frac{1}{2} e p^2 + i\psi \dot{p} d + i\phi \dot{\phi} + i\Lambda_{\mu \rho \sigma} \Gamma^\mu \Gamma^\rho \Gamma^\rho \dot{\phi} \right. \\
\left. - i\beta(\phi \dot{\phi} - 1) + \frac{1}{2} \dot{\phi} \omega \dot{\phi} \right],
\]  

(8.5)

where, as usual, $p_\mu$ is the momentum conjugate to the space-time coordinate $x^\mu$, $d^A$ is a spinor introduced so that the Grassmann coordinate $\theta$ has a conjugate momentum $\dot{\theta}^A = d^A - \dot{\theta}^A \theta_B$, $\phi^A$ is also a new spinor coordinate and $\dot{\phi}_A$ its conjugate momentum. The fields $e, \psi^A, \varphi_A, \Lambda_{\mu \rho \sigma}, \beta$ and $\omega^{AB}$ are all gauge fields which are also Lagrange multipliers imposing some finite set of classical constraints. These are given by

\[
\begin{align*}
\mathcal{A} &= p^2, & \mathcal{B} &= \dot{p}d, & \mathcal{D} &= \dot{\phi}^\dagger = 0, \\
\mathcal{G} &= \hat{\phi}_A \hat{\phi}_B, & \mathcal{H} &= \phi^A \hat{\phi}_A - 1, & \mathcal{C} &= d^A (\Gamma^\mu \Gamma^\rho) B_A \hat{\phi}_B.
\end{align*}
\]  

(8.6)

The constraints (8.6) are also infinitely reducible in the sense of Batalin and Vilkovisky [3].
In the remainder of this section, we are concerned with superparticle theories which lead to a vector wavefunction satisfying either a linear or a quadratic constraint [36]. We first seek a superparticle action formulated in an extended superspace with coordinates \((x^\mu, \theta_A, \phi^\mu)\) where \(\theta_A\) is an anti-commuting Majorana-Weyl spinor, and \(\phi^\mu\) is a vector field. Here \((x^\mu, \theta_A)\) are the coordinates of the usual 10-dimensional \(N=1\) superspace and \(\phi^\mu\) is a new vector coordinate. The superparticle action (SSP-vector) is given by [23]

\[
S_0 = \int d\tau \left[ p_\mu \dot{x}^\mu + i \dot{\theta} \theta + \hat{\phi}_\mu \dot{\phi}^\mu \right],
\]

plus the term

\[
S_1 = \int d\tau \left[ -\frac{1}{2} \epsilon \mathcal{P}^2 + i \psi \mathcal{P} d + \frac{1}{2} d \chi d + 2i \hat{\phi}_\mu \chi \hat{\mathcal{P}}^\mu \nu \phi^\nu 
- \omega p^\mu \hat{\phi}_\mu + \frac{1}{2} \hat{\phi}_\mu \epsilon^{\mu \nu \rho} \phi^\rho \right],
\]

where \(p_\mu\) is the momentum conjugate to the space-time coordinate \(x^\mu\), \(d^A\) is a spinor introduced so that the Grassmann coordinate \(\theta\) has a conjugate momentum \(\hat{\theta}^A = d^A - \hat{p}^A B \theta_B\). Here, \(\phi^\mu\) is a new vector coordinate and \(\hat{\phi}_\mu\) its conjugate momentum. The fields \(e, \psi^A, \chi_{AB} = -\chi_{BA}, \rho_{\mu \nu} = \rho_{\nu \mu}\) and \(\omega\) are all Lagrange multipliers imposing the following finite set of classical constraints

\[
\mathcal{A} = p^2, \quad \mathcal{B} = \mathcal{P} d, \quad \mathcal{D} = p^\mu \hat{\phi}_\mu, 
\mathcal{G} = \hat{\phi}_\mu \hat{\phi}_\nu, \quad \mathcal{C} = d^A d^B + 4 \hat{\phi}_\mu (\hat{p}^{\mu \nu})_{AB} \phi^\nu.
\]

The \(\mathcal{A}, \mathcal{B}, \mathcal{C}, \mathcal{D}\) and \(\mathcal{G}\) constraints are the generators of a number of local gauge symmetries. There are also symmetries of the second-kind which reflect the infinite reducibility of the constraints \(\mathcal{A}, \mathcal{B}, \mathcal{C}, \mathcal{D}\) and \(\mathcal{G}\).

We seek now a superparticle theory which lead to a vector wave function satisfying a linear constraint [36]. The superparticle action is formulated in an extended superspace with coordinates \((x^\mu, \theta_A, \phi^\mu)\) where \(\theta_A\) is an anti-commuting Majorana-Weyl spinor. Here, \((x^\mu, \theta_A)\) are the coordinates of the usual ten dimensional \(N = 1\)
superspace and $\phi^\mu$ is a new vector coordinate. The new superparticle action (SSP-vector) is given by [23]

$$S_{\text{vector}} = \int d\tau \left[ p_\mu \dot{x}^\mu + i \dot{\theta} \theta + \dot{\phi}_\mu \phi^\mu \right],$$

(8.10)

plus the term

$$S_{\text{extra}} = \int d\tau \left[ -\frac{1}{2} e p^2 + i \phi \dot{p} d + \lambda p^\mu \dot{\phi}_\mu + \frac{1}{2} \phi_\mu \omega^{\mu\nu} \phi_\nu 
+ \beta (\phi^\mu \phi_\mu - 1) + \Upsilon^\mu A C_\mu^A \right],$$

(8.11)

where $C_\mu^A = \phi_\mu d^A - \frac{1}{2} \phi_\nu (\Gamma_\mu^\nu)^A_B d^B$, as usual, $p^\mu$ is the momentum conjugate to the space-time coordinate $x^\mu$, $d^A$ is a spinor introduced so that the Grassmann coordinate $\theta_A$ has a conjugate momentum $\dot{\theta}_A$, and $\phi^\mu$ is a new vector coordinate together with its conjugate momentum $\dot{\phi}_\mu$. The fields $e, \psi^A, \lambda, \omega^{\mu\nu} = \omega^{\nu\mu}, \beta$ and $\Upsilon^\mu A$ are all Lagrange multipliers imposing some classical constraints. A variation of (8.10) and (8.11) with respect to $e, \phi, \lambda, \omega, \beta$ and $\Upsilon$ implies the following set of classical constraints

$$\mathcal{A} = p^2, \quad \mathcal{B} = \phi d, \quad \mathcal{D} = p^\mu \phi_\mu, 
\mathcal{G} = \phi_\mu \hat{\phi}_\nu, \quad \mathcal{C} = C_\mu^A, \quad \mathcal{H} = \phi^\mu \phi_\mu - 1,$$

(8.12)

which are also the generators of a number of local gauge symmetries, together with symmetries of the second-kind due to the reducibility of the constraints $\mathcal{A}, \mathcal{B}, \mathcal{C}, \mathcal{D}, \mathcal{G}$ and $\mathcal{H}$.

Covariant quantization of any of the previous superparticle extended models require the use of the Batalin and Vilkovisky procedure, as the gauge algebra of their constraints only closes on-shell, and calls for an infinite number of ghost fields.

Finally, there are still other models for the superparticle as the Sokatchev’s harmonic superparticle [39], or actions with light cone directions chosen in a covariant way using dynamical variables [40]. However, the structure of these actions is different from those we are considering here and shall not be reviewed, but further details can be found elsewhere [7,39,40,41].
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