Spin glasses and many-body localization (MBL) are prime examples of ergodicity breaking, yet their physical origin is quite different: the former phase arises due to rugged classical energy landscape, while the latter is a quantum-interference effect. Here we study quantum dynamics of an isolated 1d spin-glass under application of a transverse field. At high temperature, the system is ergodic, relaxing via resonance avalanche mechanism, that is also responsible for the destruction of MBL in non-glassy systems with power-law interactions. At low temperatures, the interaction-induced fields obtain a power-law soft gap, making the resonance avalanche mechanism inefficient. This leads to the preservation of the spin-glass order, as demonstrated by resonance analysis and by numerical studies. A small fraction of resonant spins forms a thermalizing system with long-range entanglement, making this regime distinct from the conventional MBL. The model considered can be realized in systems of trapped ions, opening the door to investigating interplay of glassiness and MBL.

### Introduction.

Spin glasses (SG) and many-body localization (MBL) are two broad classes of systems that break ergodicity. A SG is a system where frustration resulting from random interactions and fields cause spins to ‘freeze’ at low temperatures, leading to an ‘ordered’ phase without long-range order $^{13}$. Characteristic extremely slow dynamics in SG originates from a large number of metastable low-energy states separated by large energy barriers.

While SG models are essentially classical, the emergence of freezing in quantum systems is being actively studied theoretically and experimentally in the context of MBL $^{4,5}$. Although, as with SG, randomness is essential, the fundamental mechanism behind MBL is quantum interference, rather than frustration.

Another major discrepancy between SG and MBL immediately meets the eye: MBL exists in short-range interacting models in $d = 1$-dimensional systems $^{1,5}$; whereas SG require long-range interacting models in more than $d = 2$ dimensions. Moreover, analytical results for SG exist predominantly for models with infinite-range interactions $^{1}$. There are of course some exceptions, notably, a $d = 1$-dimensional long-range interacting spin glass model introduced by Kotliar, Anderson and Stein $^{6}$.

The question regarding similarities and differences in dynamics between spin glasses and many-body localized phases remains largely open. Recently, eigenstates $^{7,8}$ and dynamics $^{9,10}$ of infinite-range spin glasses have been studied. Such models are difficult to realize (see, however, Ref. $^{11}$); here, instead our focus will be on experimentally relevant systems with power-law decaying interactions.

In this paper, we propose to bridge the gap between spin glasses and MBL by studying the Kotliar-Anderson-Stein SG model $^{6}$ with a quantum transverse field. This model has the advantage of being realizable with current experimental capabilities. In particular, 1d disordered systems with long-range interactions have been recently studied with trapped ions $^{12}$. We investigate quench dynamics of this model at high and low (but nonzero) temperatures $T$, finding that the onset of glassiness dramatically modifies dynamics at low $T$. Throughout, we will focus on the properties of isolated systems; note that the dynamics of glasses in the presence of external bath has been investigated extensively $^{13}$.

Recent works $^{14-18}$ argued that MBL is impossible in the thermodynamic limit in 1d for sufficiently long-ranged power-law interactions, attributing numerical signatures of MBL reported in Refs. $^{19,20}$ to finite-size effects. We argue that the novel aspect – frustration and glassiness – of our model compared to those studied in Ref. $^{14-18}$ enables ergodicity breaking in the quantum model at low $T$. It is worth noting that Ref. $^{21}$ proposed that MBL may occur at low temperature in systems with long-range interactions via a very different mechanism of charge confinement.

### Model and setup.

The Hamiltonian of the long-range quantum spin glass model of interest is given by:

$$H = \sum_{ij} J_{ij} \hat{Z}_i \hat{Z}_j - h_x \sum_i \hat{X}_i$$

where $\hat{X}_i$, $\hat{Z}_i$ are the Pauli operators for the spin on site $i$. Following Ref. $^{6}$, $J_{ij}$ are chosen to be random, normal distributed with standard deviation 1. All energies and times are therefore dimensionless. Parameter $\alpha$ sets the power of long-ranged interactions, and lies in the range $\frac{1}{2} < \alpha < 1$ so that in the absence of a transverse field $h_x$ the system is SG at low temperatures $^{6}$. Monte Carlo simulations of the classical model with $h_x = 0$ demonstrated a SG phase with critical temperature $T_c = 0.6$ for $\alpha = 0.75$ $^{22}$.

To probe the dynamical properties of the model $^{1}$, we will focus on a quantum quench protocol, in which the system is initially prepared in a product state $|\psi_0\rangle$, with spins pointing along $z$-direction, $Z_i = \pm 1$. The quantity
of interest is the decay of the initial magnetization pattern under unitary evolution with the Hamiltonian \[ H = \sum_{ij} J_{ij} \sigma_i \sigma_j \] and trapped-ion \([12]\) experiments to probe ergodicity breaking via MBL.

To access dynamics at high and low temperatures, we will consider two kinds of initial product states. First, choosing \( Z_i = \pm 1 \) at random yields an initial state that corresponds, on average, to zero energy density and therefore infinite temperature, \( T = \infty \). Second, we will construct initial low-temperature states stable to single spin flips (see below).

**Effective fields.** In an initial product state \( \ket{\psi_0} \), each spin is subject to a random \( z \)-field \( \phi_i \) arising from the interaction term in Eq. \( [1] \),

\[
\sum_{ij} \frac{J_{ij}}{|i-j|^\alpha} Z_i Z_j = \sum_i Z_i \phi_i, \quad \phi_i = \sum_j \frac{J_{ij}}{|i-j|^\alpha} Z_j. \tag{2}
\]

For \( h_x \ll 1 \), a typical spin will have \( h_x \ll |\phi|_{\text{typ}} \), and such spin will at first precess around \( z \)-axis, maintaining the memory of its initial state. To relax and “forget” its magnetization, the spin should either be involved in a higher-order multi-spin resonant process, or, alternatively, its on-site field \( \phi_i \) should become smaller than \( h_x \), as a result of time evolution of other spins. Clearly, both kinds of processes are very sensitive to the distribution of on-site fields. We will first study this distribution, finding a markedly different behavior at high and low \( T \).

At infinite temperature, and for random, uncorrelated \( Z_i = \pm 1 \), the effective fields are normal distributed with standard deviation \( \sigma_{T=\infty} = \sqrt{2\zeta(2n)} \), which diverges as \( \alpha \) approaches 0.5 as \( 1/[\alpha-1/2] \). Fields experienced by different spins are uncorrelated.

In contrast, if we restrict ourselves to low temperatures, the number of small fields gets suppressed. To prepare initial low-temperature states, for \( \alpha = 0.7 \), and various sizes \( L \), averaged over 10,000 (for \( L = 3000 \)) to 1,000,000 (for \( L = 200 \)) disorder realizations. The distributions are extrapolated to \( L = \infty \) (black solid line) using 1/\( L \) scaling. The \( L = \infty \) distribution is fitted for \( |\phi| \ll 1 \) by a power-law \( \rho(\phi) \sim |\phi|^\tau \). Inset: The power \( x \) depends on the interaction range parameter \( \alpha \). The dashed line represents \( x \sim 1 \), the threshold for stability against resonance avalanches.

We computed the power \( x \) for the single-flip states as a function of \( \alpha \), and the result is shown in Fig. [1]. That the field distribution follows a power-law at small fields is reminiscent of the soft gap in Coulomb glasses \([26, 27]\). However, in our model no analogue of the strong Efros-Shklovskii bounds exists because the random interactions \( J_{ij} \) can have either sign, as opposed to the positive-definite Coulomb repulsion. As discussed below, the power \( x \) is crucial for the stability of the spin glass phase under a transverse field.

**Resonance avalanches.** Equipped with the distribution of onsite fields, we are now prepared to discuss quantum dynamics generated by transverse field. We will focus on the analysis of resonant processes arising under the application of a (generally small) transverse field \( h_x \ll 1 \). Although the analysis has many parallels with the arguments developed by Burin and others \([8, 14, 15]\), the glassiness of the underlying classical problem significantly modifies resonant processes.

When the system is prepared in an initial product state, only the spins with \( |\phi_i| < h_x \) can flip over the time \( t \sim 1/h_x \). We will call such spins resonant. The number of resonant spins \( N_r(L) \) depends on system size \( L \), on the temperature (or energy density) and on the transverse field \( h_x \). We denote the typical distance between resonant spins by \( d(h_x) = L/N_r(L) \).

Turning to dynamics, the resonant spins will oscillate with frequency \( \omega \sim h_x \). This, in turn, will affect the effective fields \( \phi_i \) felt by other non-resonant spins. The key question is whether these changes can drive new spins to become resonant, and whether such a resonance avalanche can eventually include the whole system.
For any (initially) non-resonant spins, the expected change in effective field is $\Delta \phi_j \sim \sum_{i=1}^{N_r(L)} \frac{1}{d_{i,j}}$ where $d_{i,j}$ is the distance from the $i$th resonant spin to the site $j$. Assuming that there are no correlations in the positions of resonant spins, the typical change of field becomes $|\Delta \phi|_{typ} \sim d^{-2\alpha}$.

If $|\Delta \phi|_{typ} > h_x$, spins that we originally did not count as being resonant can become resonant. It is expected (and verified below) that if this condition is met, flipping one resonant spin will generally cause other spins to become resonant. In this case, the system will exhibit characteristic relaxation dynamics: as resonant spins are flipping, an avalanche of new resonances will lead to a complete loss of spin polarization. We note that such resonance avalanches are reminiscent of the phenomenon of spectral diffusion (see e.g. in Ref. [17]) in the context of non-glassy models.

The frozen spins can therefore only remain frozen if the resonances do not cause such an avalanche, which requires

$$h_x > |\Delta \phi|_{typ} \sim d^{-\alpha} \tag{4}$$

where $d$ is the typical distance between resonant spins.

At infinite temperature $T = \infty$ the distribution of fields is Gaussian, which implies that the average distance between resonant spins is $d \sim h_x^{-1}$. The condition for the stability of spin freezing, Eq. (4), becomes $h_x > c h_x^{\alpha}$ with $c$ some $h_x$-independent constant. Because $\alpha < 1$, at small $h_x$ this inequality is violated. Therefore, at infinite temperature, we will always have an avalanche of resonances. We explicitly demonstrate it numerically below.

However, at low temperatures we found a qualitatively different distribution of fields, $\rho(\phi) \sim |\phi|^2$. Now the distance between resonant spins scales as $d \sim h_x^{(x+1)}$. Therefore the condition for stability now becomes $h_x > c h_x^{\alpha(x+1)}$, with $c$ an $h_x$-independent constant. This criterion is satisfied for small $h_x$ as long as $\alpha(x+1) > 1$. Therefore, there will be no avalanche of resonances as long as

$$x > 1/\alpha - 1. \tag{5}$$

As shown in Fig. 1 this relation is satisfied for the SFS states in our model for any $1/2 < \alpha < 1$, although values of $x$ and $1/\alpha - 1$ are close to each other. We therefore expect that the majority of spins, the exception being the resonant spins, remain frozen at low temperatures, and that SG order will survive a small $h_x$. Note that SG order is even more stable at temperatures lower than that corresponding to the SFS states.

Next, we test the existence of resonance avalanches in a classical numerical simulation. Given an initial spin configuration (at either high or low $T$), we compute the onsite fields $\phi_i$ and identify all resonant spins that satisfy $|\phi_i| < h_x$. We then flip one of the resonant spins, chosen at random, re-compute the distribution of fields $\phi_i$, and check how many new spins become resonant. This is iterated many times. The results, illustrated in Fig. 2, show that at low $T$ the number of resonant spins quickly saturates within this recursive scheme, suggesting the stability of spin-glass order. At high $T$, in contrast, we see an avalanche of resonances: the number of spins affected by the avalanche grows approximately as a square root of the number of iterations.

**Two-spin-flip stability.** At the single spin-flip level our analysis shows that for a small field $h_x$ and low temperatures most spins remain frozen. However, one can imagine processes involving two spin-flips, where each individual spin flip is not resonant but their combination is. The amplitude of a second-order perturbative correction corresponding to flipping spins 1 and 2 equals

$$A_{12-12} = \frac{h_x^2}{-\phi_1 - \phi_2 + 2 \frac{h_x}{T_{12}} \left( \frac{1}{-\phi_1} + \frac{1}{-\phi_2} \right)} \tag{6}$$

If $|A_{12-12}| > 1$, we will call this process resonant. Now if either 1 or 2 are already single spin-flip resonant sites with $\phi_{1,2} < h_x$, the process is naturally accounted for by the resonance avalanches considered above. Our question is thus: how many genuine two-spin-flip resonances will exist in this system?

The number of genuine two-spin-flip resonances at $T = \infty$ as a function of system size $L$ can be estimated[28], yielding $N_{2res}(L) \sim L^{2-\alpha}$. We have verified that this approximately holds numerically for systems of size $L \leq 4000$. The case of low-temperature states is more intricate, because of correlations between different spins.
Quantum dynamics and (non)ergodicity. Next, we discuss the implications of the resonance avalanches (or their absence) for quench dynamics and eigenstate properties. Since experiments are conducted for finite systems (with tens - hundreds of spins), we will in particular be interested in the effect of finite $L$. Additionally, finite-$L$ behavior of quantum model will be tested below using exact diagonalization (ED), confirming the expectations from resonance considerations.

Let us start with the case of infinite temperature. If $L$ is small such that there are no resonant spins at all (this occurs if $L \lesssim h_\tau^{-1}$), the system will exhibit usual MBL-like properties, in particular, the initial magnetization pattern will fail to relax even at $t \to \infty$, and the system will appear non-ergodic. The eigenstates are also expected to appear MBL-like: in particular, the level statistics is expected to be Poisson.

Once $L$ is increased such that there are at least a few resonances for a typical initial state, the avalanche will be effective and lead to the decay of initial magnetization. A typical spin will decay after time $t_d \sim 1/h_\tau^2$, but a broad distribution of relaxation times is expected, because spins are gradually included into the avalanche. This provides a direct experimental signature of the resonance avalanche. In this regime, eigenstates at $T = \infty$ are expected to become ergodic, and level statistics will obey Wigner-Dyson distribution.

At low $T$ the absence of the resonance avalanche at small $h_\tau$ will lead to a qualitatively different dynamics and eigenstate properties. The initially non-resonant spins stay non-resonant and will thus retain the memory of their initial magnetization even at very long, and possibly infinite times. Experimentally, this provides a direct signature of ergodicity breaking.

An interesting question concerns the effect of resonant spins on the 'frozen' ones. One possibility is that the resonant spins form a thermal bath which mediates the relaxation of initially non-resonant spins and erasure SG order. As reported below, we have studied the dynamics of the most resonant spins for SFS states using ED, finding indications that this scenario is not realized and SG order remains stable. Thus, resonant spins form a "bad bath" which is inefficient in relaxing other spins. The resonant-spin systems are expected however to exhibit non-trivial dynamics (e.g. of entanglement growth), which will be analyzed in a future work.

We note that SG order may also potentially be destroyed by higher-order, multi-spin resonances, which can e.g. couple different SFS states. In that case, ergodicity may be restored. Given the extreme sparsity of two-spin resonances, we believe this possibility to be unlikely, and this is confirmed by ED studies below.

Exact diagonalization. We will now study the model of Eq. (1) using exact diagonalization (ED), which has been used to diagnose MBL phases (see Refs. [3] for a review). We also note that the experiments with trapped ions were conducted with similar system sizes $L \approx 10 \sim 20$, so results below have direct experimental implications.

A common tool to distinguish between MBL and ergodic behavior, employed below, is to characterize level statistics via the ratio of adjacent eigenvalue gaps, $r = \min(\delta_n, \delta_{n+1})/\max(\delta_n, \delta_{n+1})$ where $\delta_n = E_n - E_{n-1}$ is the gap between two neighboring eigenvalues. This $r$-value approaches 0.53 in the ergodic phase, and $r = 0.39$ for Poisson level statistics in the localized phase. A second measure we will use is the non-equilibrium response after a quench starting from an initial product state at different energy densities. We measure the infinite time remnant imbalance, defined as $I_\infty = \lim_{t \to \infty} \frac{1}{T} \sum_i \langle Z_i(t) \rangle / \langle Z_i(0) \rangle$. The remnant imbalance is nonzero in the localized/SG ordered phase, and zero in the ergodic phase.

The two diagnostics, $\langle r \rangle$ and $I_\infty$ as a function of energy density $E/L$ and $h_\tau$ for system size $L = 12$ averaged over 2000 disorder realizations, are illustrated in Fig. 3. The range of interaction is fixed at $\alpha = 0.7$.

Notably, at very small $h_\tau \lesssim 0.1$ states at all temperatures appear localized (and $I_\infty$ is close to 1 signalling strongest non-ergodicity). This is consistent with systems being too small to have resonances. For $h_\tau \gtrsim 0.2$ where resonance avalanche becomes effective, in the middle of the spectrum ($T = \infty$) the system is clearly ergodic, as the $r$-value approached 0.53, and the rem-
nant imbalance vanishes. At lower temperatures (near the ends of the spectrum) the system stays localized up to larger values of $h_x$. Additionally, we have studied the Edwards-Anderson order parameter \[ \langle \sigma \rangle \] including finite size scaling up to $L = 16$ which yielded consistent results.

**Large systems: dynamics of resonant spins.** Finally, we further investigate the stability of SG at low $T$ by studying the dynamics of most resonant spins in large systems. Here, we make the assumption that spins with fields $|\phi| \gg h_x$ stay frozen, and are not destabilized by the collection of resonant spins with $|\phi| \lesssim h_x$. To check the self-consistency of this assumption, we exactly solved the dynamics of the $L_s = 6 - 14$ most resonant spins using ED. Extrapolating this dynamics we estimate the remnant long-time magnetization of the 6 most resonant spins. We find that this magnetization remains sizeable (see Fig. 2), slowly decaying to zero as $L \to \infty$, thus establishing that there is no run-away effect of incorporating more and more spins into the exact dynamics of the most resonant spins.

Even though the resonant spins do not incite a loss of polarization in the non-resonant spins, amongst themselves they will eventually form an ergodic system. This can be understood by looking at the Hamiltonian for just the resonant spins (labeled by $I, J$): $H_{\text{res}} = \sum I \hat{h}_I \cdot \vec{\sigma}_I + \sum_{I, J} J_{IJ} \frac{\vec{\sigma}_I \cdot \vec{\sigma}_J}{\sqrt{|I|}}$. The random field $\hat{h}_I = (-h_x, 0, \phi_I)$ has a norm of order $h_x$. The interactions $J_{IJ}$, on the other hand, are now much reduced in strength. Given that the typical spacing between resonant spins scales as $d_{\text{res}} \sim h_x^{-(1+x)}$, the interaction between neighboring resonant spins is of the order $|J_{IJ}| \sim h_x^{\alpha(x+1)}$. As we saw above, $\alpha(x+1) > 1$. Thus, the interactions between resonant spins are weaker than the on-site fields, and their long-range nature is expected to lead to eventual thermalization of the resonant-spin subsystem. A complete thermalization, however, requires extremely large system sizes, as incomplete decay of polarization of resonant spins in Fig. 2 (right) suggests.

**Discussion.** In summary, we proposed to study the interplay of glassiness and MBL – two generic mechanisms of ergodicity breaking – in a power-law interacting model in 1d. We hope that our work will stimulate experiments with trapped ions, where long-range interactions with a tunable exponent have been demonstrated.

The onset of glassy behavior leads to an unconventional regime of quantum dynamics: in contrast to high $T$ where the system behaves as ergodic, forgetting its initial magnetization pattern under time evolution via resonance-avalanche mechanism, at low $T$ the memory is retained. In contrast to MBL systems, a set of resonant spins forms a thermalizing system. At the same time, the eigenstates are expected to be strongly non-ergodic, and violate the eigenstate thermalization hypothesis.

In the future work, it will be interesting to study if they exhibit clustering similar to that found in infinite-range models \[^4]\).

We emphasize that the non-ergodic low-$T$ regime, signalled by the persistence of SG order, revealed here is a unique consequence of glassiness: indeed, previous works \[^7]\) that studied dynamics of long-range interacting systems with power $d < \alpha < 2d$, found eventual ergodic behavior (accompanied by diffusive dynamics) even at low $T$.

We finally note that in the future it will be interesting to study the high-order tunneling processes between low-energy states separated by large energy barriers – a problem which has central importance to the performance of the adiabatic quantum algorithm for difficult optimization problems \[^30]\).

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[30] V. Bapst, L. Foini, F. Krzakala, G. Semerjian, and F. Zamponi, Physics Reports 523, 127 (2013) the Quantum Adiabatic Algorithm Applied to Random Optimization Problems: The Quantum Spin Glass Perspective.
Appendix: Large-scale dynamics

To obtain the remnant polarization of the most resonant spins in a large system, we find a spin configuration that is stable against single spin flips for each new disorder configuration. Once such a random low-energy state has been found we identify the "most resonant spins" as the sites with the lowest absolute value of their onsite field. We then restrict ourselves to the $L_s$ most resonant spins and perform a quench with a $h_x = 0.05$ transverse field using exact diagonalization. The expectation value of each spin in the $z$-direction is measured as a function of time, relative to its initial value, $P_t(L_s, n, L) = \langle Z_n(t) \rangle / \langle Z_n(t = 0) \rangle$

Given the exact time evolution, we estimate the long-time remnant spin polarization, $P_\infty$, as a function of $L_s$ (the number of resonant spins included) and $L$ (the total size of the system), by averaging over the quench values between $500 < t < 1000$. Then we compute the $L_s \to L$ limit of this remnant polarization for each spin, by performing an $1/L_s$ scaling. This extrapolation allows us to estimate the correct value of $P_\infty(n, L)$, as if we had an exact quench dynamics of the whole system. The resulting long-time remnant spin polarization $P_{\text{inf}}(n, L)$ as a function of system size $L$, for the six most resonant spins is shown in Fig. 2 right.