HARD AND SOFT COLOUR SINGLET EXCHANGE IN THE
SEMICALSSICAL APPROACH

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In the present talk diffraction in deep inelastic scattering is discussed in the framework of the semiclassical approach. The main emphasis is on the possibility of a consistent semiclassical description of both hard and soft colour singlet exchange processes. This approach allows the comparison of hard and soft colour neutralization in diffractive electroproduction of high-$p_{\perp}$ jets or heavy quarks.

1 Introduction

One of the most interesting theoretical problems of diffraction in deep inelastic scattering is the precise nature of the $t$-channel colour singlet exchange, which is the main characteristic of this type of processes. On the one hand, the presence of the hard scale $Q^2$, provided by the virtual photon, and possibly additional hard scales, such as high-$p_{\perp}$ jets or heavy quarks in the final state, suggest the applicability of perturbation theory. From this point of view the natural mechanism would be perturbative two-gluon exchange in the $t$-channel. On the other hand, it is well known that at high energy the wave function of a highly virtual photon develops a soft component which has a large, hadronic cross section and contributes to inclusive deep inelastic scattering at leading twist. This clearly suggests a corresponding leading twist contribution to diffraction in which the hard scales from photon and diffractive final state do not render the colour singlet exchange perturbative.

The above two mechanisms for colour singlet exchange result in different dependences of cross sections on $Q^2$, $x$ and possible additional parameters like $p_{\perp}$ or heavy quark masses in the diffractive final state. However, as will be discussed below, the relative normalization of both contributions remains unknown.

In the present talk, which is based on, it will be shown how a unified description of both hard and soft colour singlet exchange naturally arises within the semiclassical framework. This observation can be taken as a starting point for the phenomenological analysis of high-$p_{\perp}$ jets and charm in diffraction. A more detailed discussion of the semiclassical approach to the above two diffractive processes and the resulting phenomenological predictions can be found in.
2 Hard colour singlet exchange and squared gluon density

Before focusing on the description of hard colour singlet exchange recall the general picture of leading order diffraction in the semiclassical approach. Working in the proton rest frame and modelling the proton by a classical colour field the simplest process is the creation of a colour neutral $q\bar{q}$-pair by the virtual photon (see Fig. 1). For transverse photon polarization and for one massless quark generation with one unit of electric charge the corresponding cross section reads:

$$\frac{d\sigma_T}{dt} \bigg|_{t=0} = \frac{\alpha_{em}}{6(2\pi)^6} \int d\alpha dp_{\perp}^2 (\alpha^2 + (1 - \alpha)^2) \times$$

$$\left| \int d_{x_{\perp},y_{\perp},p_{\perp}} e^{iy_{\perp}(p_{\perp}-p'_{\perp})} \text{tr} W^{F}_{x_{\perp}}(y_{\perp}) \frac{p_{\perp}}{\alpha(1 - \alpha)Q^2 + p'_{\perp}^2} \right|^2,$$

where $t = (q - p' - l')^2$ is the momentum transfer to the proton and $\alpha = p'_{0}/q_{0}$.

![Figure 1: Production of a $q\bar{q}$-pair in the semiclassical approach.](image)

The proton colour field is described by the quantity

$$W^{F}_{x_{\perp}}(y_{\perp}) = U^\dagger(x_{\perp} + y_{\perp}) U(x_{\perp}) - 1$$

which is built from the non-Abelian eikonal factors $U$ and $U^\dagger$ of the quark and antiquark whose light-like paths penetrate the colour field of the proton at transverse positions $x_{\perp}$ and $x_{\perp} + y_{\perp}$, respectively. The superscript $F$ is used because the quarks are in the fundamental representation of the gauge group.

In this language the hardness of the process is characterized by the relevant transverse distances $|y_{\perp}|$. Large $Q^2$ in itself is not sufficient to make the colour singlet exchange hard since it can be compensated by small $\alpha$ or $1 - \alpha$. However, the requirement of large $p'_{\perp}$ leads to the dominance of large $p_{\perp}$ and, eventually, to the dominance of small distances in the $y_{\perp}$-integration in Eq. (2). The same effect can also be achieved by giving the quarks a large mass, in which case the process remains hard for all $p'_{\perp}$. As a result, the leading order cross section
is only sensitive to the first term in the Taylor expansion of \( \text{tr} W^F \),

\[
\int_{x_\perp} \text{tr} W^F_{x_\perp} (y_\perp) = \text{const.} \times y^2_\perp + \mathcal{O}(y^4_\perp). \tag{3}
\]

In the leading-log approximation the coefficient of the above \( y^2_\perp \)-term can be related to the inclusive gluon density. To see this observe that, within the semiclassical approach, the colour dipole cross section \( \sigma(\rho) \) is given by

\[
\sigma(\rho) = -\frac{2}{3} \int_{x_\perp} \text{tr} W^F_{x_\perp} (\rho_\perp). \tag{4}
\]

Using the well-known relation of the short distance behaviour of \( \sigma(\rho) \) and the gluon density \( \rho \)

\[
\sigma(\rho) = \frac{\pi^2}{3} \alpha_s[xg(x)] \rho^2 + \mathcal{O}(\rho^4), \tag{5}
\]

the constant in Eq. (3) and hence the high-\( p_\perp \) cross section of Eq. (1) can be determined. This normalization of the \( y^2_\perp \)-term of \( \text{tr} W^F \) by the gluon density can also be derived by calculating the Compton amplitude of Fig. 2 and relating it to inclusive deep inelastic scattering via the optical theorem.

Figure 2: The Compton scattering amplitude within the semiclassical approach.

Combining the above formulae the following expression for the diffractive production of two high-\( p_\perp \) jets can be derived

\[
\left. \frac{d\sigma_T}{dtdyd^2p^2_\perp} \right|_{t=0} = \frac{2\pi^2 \alpha_e \alpha_s^2 [xg(x)]^2}{3} \frac{[\alpha^2 + (1 - \alpha)^2] [\alpha(1 - \alpha)]^2 Q^4 p'^2}{[\alpha(1 - \alpha) Q^2 + p'^2]^6}. \tag{6}
\]

The identification of the squared gluon density in hard diffractive processes has been demonstrated in \( ^1 \) in the framework of vector meson production. As expected, Eq. (6) is in agreement with the corresponding two-gluon exchange calculations for diffractive jet production (see e.g. \( ^4 \)).
3 Soft colour singlet exchange and diffractive gluon density

The physics of the colour neutralization changes radically if, in addition to the two high-\(p_\perp\) quark jets, a gluon is present in the diffractive final state. As has been shown in\(^6\) now the dominant contribution comes from the phase space region where this gluon is relatively soft, i.e. it has small transverse momentum and carries only a small fraction of the longitudinal momentum of the photon. This means that the gluon develops a large transverse separation from the \(q\bar{q}\)-pair, thus testing the proton field non-perturbatively (see Fig. 3).

![Figure 3: Diffractive jet production with an additional soft gluon in the final state.](image)

The process can be reinterpreted as high-\(p_\perp\) jet production in boson-gluon fusion\(^1\) and the cross section

\[
\frac{d\sigma}{d\xi dp_\perp^2} = \int_x \frac{d\hat{\sigma}_{T}^{\gamma^* g \rightarrow q\bar{q}}(y, p'_\perp)}{dy \, dp_\perp^2} \, dg(y, \xi) \, d\xi \tag{7}
\]

involves a diffractive gluon density

\[
\frac{dg(y, \xi)}{d\xi} = \frac{1}{8\xi^2} \left( \frac{b}{1-b} \right) \int d^2k'_{\perp} \left( \kappa_2^2 \right)^2 \int_{x_\perp} \left| \int d^2k_{\perp} \, \text{tr}\left[ \hat{W}_{x_\perp}^A (k'_{\perp} - k_{\perp}) t^{ij} \right] \right|^2, \tag{8}
\]

\[
t^{ij} = \delta^{ij} + \frac{2k^j_1 k^i_2}{k^2_\perp} \left( \frac{1-b}{b} \right). \tag{9}
\]

Here \(\xi = x(Q^2 + M^2)/Q^2\) for a final state with diffractive mass \(M\), the momentum fraction of the proton carried by the incoming gluon is denoted by \(y\), and \(b = y/\xi\). The function \(W_{x_\perp}^A\) is the Fourier transform of \(W_{x_\perp}^A\) which is defined as in Eq. (2) but with the \(U\)-matrices in the adjoint representation. Since Eq. (8) does not involve the hard scales of the process the function \(W_{x_\perp}^A(y_\perp)\) is tested in the whole range of \(y_\perp\). This is in contrast to Eq. (4) where only the perturbative small-\(y_\perp\) region matters in the high-\(p_\perp\) limit.

For a direct comparison with the exclusive two-jet production of Eq. (1) the differential cross section, Eq. (7), has to be rewritten in terms of \(d\alpha\). This
is easily done introducing the invariant mass of the two quark jet system, \( M_j^2 = p_{\perp}^2 / \alpha (1 - \alpha) \), and using the relation \( y Q^2 = x (Q^2 + M_j^2) \). For more phenomenological details see [5].

4 The relative normalization of hard and soft contribution

So far it has been discussed how high-\( p_{\perp} \) jets in diffractive electroproduction can be generated by either hard or soft colour singlet exchange mechanisms. The relative normalization of these two mechanisms is not easily predicted from first principles and should be determined from experiment.

Consider first the case of hard colour singlet exchange described by Eq. (1) and, more explicitly, by Eq. (6). Although the cross section at \( t \approx 0 \) (or, more precisely, at the very small value \( -t = m^2 p^{\xi^2} \)) can be expressed in terms of the squared gluon density this is not the case for the full cross section of diffractive jet production. The reason is that the \( t \)-integration introduces the unknown constant

\[
C = \left( \int \frac{d\sigma}{dt} dt \right) / \left( \frac{d\sigma}{dt}_{t \approx 0} \right) \sim \Lambda^2 ,
\]

where \( \Lambda \) is a typical hadronic scale.

The normalization of the contribution with soft colour neutralization is also not calculable. As can be seen from Eqs. (7) and (8) the production of the hard jets is governed by the standard cross section for boson-gluon fusion. In contrast, the radiation of the soft gluon and the overall normalization is described by the non-perturbative quantity \( dq(y, \xi)/d\xi \).

In spite of the fact that the normalization of the cross section is not known in both of the above cases some more general statements about the ratio of soft and hard contributions can be made. First of all, it is well known that the contribution with additional gluon is dominant in the region of large diffractive masses. It is, however, also interesting to consider the region of medium \( M^2 \) in more detail.

Setting \( M^2 = Q^2 \) (which corresponds to \( \beta = 1/2 \)) and integrating over all \( p_{\perp}^2 > p_{\perp, \text{min}}^2 \) the following dependences on \( Q^2 \) and \( p_{\perp, \text{min}}^2 \) are obtained for the exclusive two-jet and the two-jet plus gluon contribution respectively,

\[
\left( \frac{d\sigma}{d\xi} \right)_{qq} \sim \frac{1}{Q^2} \frac{\Lambda^2}{p_{\perp, \text{min}}^2} , \quad \left( \frac{d\sigma}{d\xi} \right)_{qg} \sim \frac{1}{Q^2} \ln Q^2 / p_{\perp, \text{min}}^2 .
\]

These rough estimates neglect additional dependences on the parameters introduced via the scales of factors of \( \alpha_s \). Nevertheless, Eq. (11) clearly shows that for sufficiently large \( p_{\perp, \text{min}}^2 \) the \( qg \) final state, and hence the soft colour neutralization mechanism, is dominant even in the region of medium \( M^2 \).
5 Conclusions

In diffractive electroproduction of high-$p_{\perp}$ jets contributions from pure $q\bar{q}$ and $q\bar{q}g$ final states rely on different mechanisms of colour neutralization. While in the first case the colour singlet exchange is hard, corresponding to perturbative two-gluon exchange, in the second case the soft gluon implies a non-perturbative mechanism of colour neutralization. The semiclassical approach to diffraction provides a consistent framework for the treatment of both contributions. It can therefore serve as a starting point for the phenomenological analysis of the experimental cross section of diffractive jet production.

For both the soft and the hard colour neutralization mechanisms the normalization of the jet cross sections can not be predicted from first principles. However, for sufficiently large transverse momenta of the produced jets the contribution with additional gluon and soft colour neutralization is expected to dominate. This is true even in the region of moderate diffractive masses $M$.

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