Comparative Research of Redundant Strapdown Inertial Navigation System Based on Different Configuration Schemes

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Abstract Aiming at various configuration scheme and inertial measurement units of Strapdown Inertial Navigation System, selected tetrahedron skew configuration and coaxial orthogonal configuration by nine low cost IMU to build system. Calculation and simulation the performance index, reliability and fault diagnosis ability of the navigation system. Analysis shows that the reliability and reconfiguration scheme of skew configuration is superior to the orthogonal configuration scheme, while the performance index and fault diagnosis ability of the system are similar. The work in this paper provides a strong reference for the selection of engineering applications.

Keywords configuration scheme; performance index; reliability; fault diagnosis ability

I. introduction

Strapdown Inertial Navigation System based on MEMS-IMU play a more and more important role in the field of navigation system for its advantages such as simple structure, small size, light weight, low cost, high reliability and so on\textsuperscript{[1,2]}. In order to improve reliability and accuracy of the navigation system, hardware redundancy technology is currently the mainstream way. That is, using redundant components with lower reliability to improve the overall reliability of the system.

Domestic and foreign researches on the redundant system of Inertial Navigation System mainly focus on the research of redundant configuration and the management of the residual sensor \textsuperscript{[3,4]}. Most of the subjects investigated are MEMS-IMU devices with high precision and high price in large institutions such as Aeronautics and Astronautics. This paper selects two configurations to study their differences in reliability, performance index and fault diagnosis ability based on the low cost MEMS-IMU devices.

II. Configuration scheme selection

Redundant configuration is an important research direction of strapdown inertial navigation system. According to the measurement axis, there are many available configurations such as orthogonal configuration scheme, non orthogonal configuration scheme. In this paper, orthogonal configuration and skew configuration scheme are adopted to build the system. Each system contains three sets of IMU, each containing three inertial sensors (gyro or accelerometer).

2.1 skew configuration scheme

The skew configuration of tetrahedral is showed in figure (1). In this scheme, IMU-1 is installed in the bottom of the tetrahedron. The center of the IMU-1 bottom surface coincides with the center of the tetrahedron, OX is perpendicular to BC, OY is parallel to BC. The X, Y, and Z axes of IMU-1 coincides with the X, Y, and Z axes of the navigation carrier coordinate system. The IMU-2 and IMU-3 are installed on the side ABC and ABD respectively. The center of the bottom surface coincides with the
center of the tetrahedron side, the Z axis points to the outside side of the tetrahedron and the Y axis is perpendicular to the bottom edge.

2.2 orthogonal configuration scheme

The orthogonal configuration is showed in figure (2). In this scheme, three sets of the same components are used common support and installed on the same direction. Each IMU is installed according to the three orthogonal routine which can be considered that there are 3 identical sensors (AxAyAz, BxByBz and CxCyCz) on the 3 orthogonal axes (X axis, Y axis and Z axis).

![Fig 1 skew configuration scheme](image1.png)  ![Fig 2 orthogonal configuration scheme](image2.png)

### III Performance comparison of different configuration schemes

#### 3.1 performance index comparison

The equation of system performance index is shown (1).

$$F_p = \left( H^T H \right)^{1/2}$$

In equation (1), $H$ is the measurement matrix; $H^T$ is the transposed matrix of measurement matrix.

For the skew configuration scheme shown in figure (1), the inertial sensor measurement matrix is:

$$H_1 = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

For the orthogonal configuration scheme shown in figure (2), its inertial sensor measurement matrix is:

$$H_2 = \begin{bmatrix} 1 & 0 & 0 & 1 & -0.3333 & 0.9428 & 0.866 & 0.1667 & -0.4714 \\ 0 & 1 & 0 & -1 & 0 & 0 & 0.5 & -0.2887 & 0.8165 \\ 0 & 0 & 1 & 0 & -0.9428 & -0.33331 & 0 & -0.9428 & -0.3333 \end{bmatrix}$$

From the formula (1), the FP values of two different configurations are calculated by Matlab software, $FP_1=FP_2=0.1925$ is calculated. Result shows that the performance indexes of low cost MEMS-IMU devices are equivalent in numerical operation under two different configurations.

#### 3.2 reliability comparison

Since each sensor obeys the same statistical function, system reliability $R_n(t)$ is the permutation and combination of single sensors. The expression is shown (2).

$$R_n(t) = \sum_{i=0}^{n} C_n^i [R(t)]^i [1 - R(t)]^{n-i}$$

Reliability of single sensor obeys: $R(t) = e^{-\lambda t}$. 
Define system $T_{MTBF}$:

$$T_{MTBF} = \int_0^\infty R_n(t)dt$$

(3)

For 9 inertial sensors in the redundant Strapdown Inertial Navigation System, system can run normally as long as there are no failures in any three Inertial sensor (non-coplanar). The system of skew configuration has 455 reliable reconfiguration methods.

$$C_o^3 + C_o^4 + C_o^5 + C_o^6 + C_o^7 + C_o^8 + C_o^9 - 11 = 455$$

The system of Orthogonal configuration has 346 reliable reconfiguration methods.

$$C_o^9 + C_o^8 + C_o^7 + (C_o^6 - 3) + (C_o^5 - 18) + (C_o^4 - 45) + (C_o^3 - 54) = 346$$

The reliability and MTBF calculation results of the redundant Strapless Inertial Navigation System (9 inertial sensors) are shown in table 1.

### Table 1  Reliability and MTBF of the system under different configurations

| Configuration mode | Reliability | MTBF | ratio |
|--------------------|-------------|------|-------|
| Single IMU orthogonal configuration | $e^{-3\lambda t}$ | $1/3\lambda$ | 1 |
| Skew configuration of three IMU | $25e^{-9.4\lambda t} - 170e^{-8.2\lambda t} + 490e^{-7.4\lambda t} - 770e^{-6.6\lambda t} + 70e^{-5.8\lambda t} - 355e^{-4.6\lambda t} + 80e^{-3.4\lambda t}$ | $413/315\lambda$ | 3.93 |
| Orthogonal configuration of three IMU | $25e^{-9.4\lambda t} - 210e^{-8.2\lambda t} + 340e^{-7.4\lambda t} - 530e^{-6.6\lambda t} + 43e^{-5.8\lambda t} - 300e^{-4.6\lambda t} + 58e^{-3.4\lambda t}$ | $257/415\lambda$ | 1.86 |

Reliability and MTBF of the two configurations are calculated respectively, results are shown in table 1. Reliability of the multi IMU redundant system is higher than single IMU system; Reconfiguration modes and the reliability of skew configuration are better than orthogonal configuration at the same number of IMU.

### 3.3 Comparison of fault diagnosis ability

#### 3.3.1 Principle of fault-detection

The method of fault detection and diagnosis mainly uses generalized likelihood ratio method, mean test method, local estimation method. This paper choose the most commonly used generalized likelihood ratio method[5]. First, we construct parity vectors and use fault parity vectors to construct fault decision function, System failure depends on whether the function value exceeds the pre-determined threshold of fault detection.

The measurement equation of the multi-IMU Redundant Strapdown Inertial navigation system is as follows:

$$Z = H \cdot X + \xi$$

(4)

In equation(4), $Z$ is the measured value of sensor; $H$ is the Installation matrix; $\xi$ is the noise malfunction, the odd and even equation is as below(5):

$$P = V \cdot Z = V(H \cdot X + \xi)$$

(5)
Parity vector \( P \) is only related to noise or possible failures, the design of \( V \) matrix should be satisfied: \( V^H = 0 \).

Define detection function:

\[
FD_{GLT} = \frac{1}{\delta^2} \left[ P^T (V^T V)^{-1} P \right]
\]  

(6)

3.3.2 Simulation experiment of skew configuration scheme

According to Potter algorithm\(^{(6)}\), the \( V \) matrix is calculated by equation \( V^H_1 = 0 \).

\[
V_1 = \begin{bmatrix}
0.8165 & 0 & 0 & 0.1361 & -0.3849 & -0.3546 & -0.0681 & 0.1925 \\
0 & 0.8165 & 0 & 0.2041 & 0 & 0 & -0.1021 & 0.0589 & -0.1667 \\
0 & 0 & 0.8165 & 0 & 0.1925 & 0.068 & 0 & -0.1925 & 0.068 \\
0 & 0 & 0 & 0.7906 & 0 & 0 & 0.1186 & -0.0685 & -0.1936 \\
0 & 0 & 0 & 0.7817 & 0.0251 & 0.0923 & -0.1954 & -0.1256 \\
0 & 0 & 0 & 0 & 0.7164 & -0.2865 & -0.1337 & 0.1282 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]

Step fault 5 degree /S and non-faults are added to the second sensors, simulation results by Matlab software are as follows:

![Fig 3 fault detection simulation without failure](image)

![Fig 4 fault detection simulation with step fault](image)

3.3.3 Simulation experiment of orthogonal configuration scheme

According to Potter algorithm, the \( V \) matrix is calculated by equation \( V^H_2 = 0 \).

\[
V_2 = \begin{bmatrix}
0.8165 & 0 & 0 & -0.4082 & 0 & 0 & -0.4082 & 0 & 0 \\
0 & 0.8165 & 0 & 0 & -0.4082 & 0 & 0 & -0.4082 & 0 \\
0 & 0 & 0.8165 & 0 & 0 & -0.4082 & 0 & 0 & -0.4082 \\
0 & 0 & 0 & 0.7071 & 0 & 0 & -0.7071 & 0 & 0 \\
0 & 0 & 0 & 0.7071 & 0 & 0 & -0.7071 & 0 & 0 \\
0 & 0 & 0 & 0 & 0.7071 & 0 & 0 & -0.7071 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.7071 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]

Step fault 5 degree /S and non-faults are added to the second sensors, simulation results by Matlab software are as follows:
From the figure 3 and figure 5, the FD value has no step change in the case of no fault add to the sensors. It exceeded the threshold at some point, but it quickly disappears. The FD values in the figure 4 and figure 6 are all increased after the step fault add to the second sensor in the two different configuration schemes. Simulation results shows that the fault judgment ability of low cost IMU devices are equivalent in the numerical operation of two different configurations.

IV Conclusion

System performance index, reliability and fault judgment ability at the different configuration scheme are analysed in this paper. Analysis shows that: reliability and reconstruction method of skew configuration scheme are better than orthogonal configuration, the system performance index and fault judgement are similar. this work for the engineering application of choice provides a reference:

1. In the field of precision instruments such as large equipment, Aeronautics and Astronautics, high reliability and more reconfiguration modes of skew configuration should be selected;

2. In the areas of low cost, easy installation and low reliability, the orthogonal configuration of low cost IMU devices can be selected.

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