Reconstruction of non-forward evolution kernels.

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Abstract

We develop a framework for the reconstruction of the non-forward kernels which govern the evolution of twist-two distribution amplitudes and off-forward parton distributions beyond leading order. It is based on the knowledge of the special conformal symmetry breaking part induced by the one-loop anomaly and conformal terms generated by forward next-to-leading order splitting functions, and thus avoids an explicit two-loop calculation. We demonstrate the formalism by applying it to the chiral odd and flavour singlet parity odd sectors.

Keywords: evolution equation, two-loop exclusive kernels, conformal constraints

PACS numbers: 11.10.Hi, 11.30.Ly, 12.38.Bx

\textsuperscript{1}Alexander von Humboldt Fellow.
1 Introduction

The possibility to access new characteristics of hadrons by means of the deeply virtual Compton scattering \[1, 2, 3\] and the hard diffractive hadron electroproduction \[3, 4\] processes has recently initiated a growing phenomenological interest in the underlying non-perturbative elements — the so-called off-forward parton distributions (OFPD) — which parametrize hadronic structure in these reaction making use of the QCD factorization theorems. The main feature of the processes is a non-zero skewedness, i.e. plus component, \(\Delta_+ = \eta\), of the \(t\)-channel momentum transfer \(\Delta\).

One of the central issues which has been addressed in this context is the description of the scaling violation phenomena in the cross section via the evolution of the off-forward parton distributions. Since the OFPD is defined as an expectation value of a non-local string operator, its \(Q^2\)-dependence is governed by the renormalization of this operator Fourier transformed to the momentum fraction space. Inasmuch as the generalized skewed kinematics can be unambiguously restored \[5\] from the conventional exclusive one, known as Efremov-Radyushkin-Brodsky-Lepage (ER-BL) region \(\eta = 1\), in what follows we deal formally with renormalization of the ordinary distribution amplitudes which obey the ER-BL equation \[6, 7\]

\[
\frac{d}{d \ln Q^2} \phi(x, Q) = V(x, y|\alpha_s(Q)) \otimes \phi(y, Q). \tag{1}
\]

Here we have introduced the exclusive convolution

\[
\otimes \equiv \int_0^1 dy,
\]

to distinguish it from the inclusive one used later. Here \(\phi = \left(\frac{Q}{c_\phi} \phi \right)\) is the two-dimensional vector and \(V(x, y|\alpha_s)\) is a \(2 \times 2\)-matrix of evolution kernels given by a series in the coupling.

Several methods have been offered so far to solve the off-forward evolution equation: numerical integration \[8\], expansion of OFPD w.r.t. an appropriate basis of polynomials \[9, 10\], mapping to the forward case \[12, 13\] and solution in the configuration space \[14, 15\]. The last three methods are based on the well-known fact that operators with definite conformal spin do not mix in the one-loop approximation. Beyond leading order the latter two methods can only be applied in the formal conformal limit of QCD where the \(\beta\)-function is set equal to zero and making use of the conformal subtraction scheme which removes the special conformal symmetry breaking anomaly appearing in the minimal subtraction scheme. Thus, we are only left with the former two methods which allows for a successive improvement of the perturbative approximations involved.

Up to now only orthogonal polynomial reconstruction method has allowed the analysis of the scaling violation in the singlet sector in two-loop approximation since only the anomalous

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\(^2\)This idea has earlier been applied directly to the kernels in \[11\].
dimensions required in the formalism were available so far [11, 16, 17]. This was sufficient to get a first insight into the NLO evolution corrections. However, in order to have an access to the whole kinematical region, especially for small $x$, $\eta$ and high precision handling of the $x \sim \eta$ domain, one should look for a more efficient numerical treatment. This can be achieved with the first method alluded to above. To do the direct numerical integration of the evolution equation one needs the corresponding evolution kernels whose Gegenbauer moments define the anomalous dimensions mentioned earlier. For the time being the former were available at LO order only. The flavour non-singlet ER-BL kernel ($\eta = 1$) was obtained in NLO by a cumbersome analytical calculation [18, 19, 20]. As we have mentioned above the continuation to $\eta \in [0, 1]$ is a unique procedure [5], so that one can obtain in a simple way the evolution kernels for OFPD. The goal of this paper is to outline a method that allows one to construct the singlet ER-BL kernels by applying conformal and supersymmetric constraints where the latter ones arise from the $\mathcal{N} = 1$ super Yang-Mills theory [21, 22]. In this way we can avoid the direct diagrammatical calculation which would be very difficult to handle otherwise since no appropriate technology has been developed yet.

The derivation is based on the fairly well established structure of the ER-BL kernel in NLO. Up to two-loop order we have

$$V(x, y|\alpha_s) = \frac{\alpha_s}{2\pi} V^{(0)}(x, y) + \left(\frac{\alpha_s}{2\pi}\right)^2 V^{(1)}(x, y) + \mathcal{O}(\alpha_s^3),$$

(2)

with the purely diagonal LO kernel $V^{(0)}$ in the basis of Gegenbauer polynomials and NLO one separated in two parts: $V^{(1)}(x, y) = V^{D(1)}(x, y) + V^{ND(1)}(x, y)$, with the diagonal part which is entirely determined by the well-known forward DGLAP splitting functions $P(z)$ [11]

$$ABV^D(x, y) = \int_0^1 dz \sum_{j=0}^\infty \frac{w(y|\nu)C^\nu(A)j(2x - 1)z^j}{N_j(\nu)} \left(\frac{\alpha_s}{2\pi}\right) \left[ g^e \otimes \left(V^{(0)}(x, y) + \mathcal{D}\right) - \mathcal{D} \mathcal{D}(x, y) \right],$$

(3)

where $N_j(\nu) = 2^{-4\nu+11\frac{3}{2}}\frac{\Gamma(\nu+1)}{\Gamma(\nu+1)^2}$ and $w(y|\nu) = (y^n)^{\nu-1/2}$ are the normalization and weight factors, respectively. The non-diagonal piece is fixed completely by the conformal constraints [17]

$$V^{ND(1)}(x, y) = -(\mathcal{I} - \mathcal{D}) \left\{ \dot{V}^e \otimes \left(V^{(0)}(x, y) + \mathcal{D}\right) - \left[ g^e \otimes V^{(0)}(x, y) \right] \right\},$$

(4)

in terms of

$$V^{(0)} = \begin{pmatrix} C_F^{QQ} V^{(0)} & 2T_F N_f C_F^{QG} V^{(0)} \\ C_F^{GQ} V^{(0)} & C_A^{GG} V^{(0)} \end{pmatrix}, \quad g = \begin{pmatrix} C_F^{QQ} g & 0 \\ C_F^{QG} g & C_A^{GG} g \end{pmatrix},$$

(5)

the ER-BL kernels at LO and the special conformal symmetry breaking matrix $g$. Here $\beta_0 = \frac{4}{3} T_F N_f - \frac{11}{2} C_A$ is the first expansion coefficient of the QCD $\beta$-function. In the parity odd sector the dotted kernel, $\dot{V}$, is simply given by a logarithmic modification of the $V^{(0)}$. Due to subtleties,
appearing in the parity even case \[11\], we deal here, for the sake of simplicity, only with the parity odd and transversity sectors.

The main problem is thus to restore the diagonal part of the NLO kernels. Since the use of Eq. (3) beyond LO is extremely complicated in practice, we are forced to look for other solutions. It turns out that the bulk of contributions in the ER-BL kernel can be deduced by going to the forward limit making use of the reduction

\[
P(z) = \text{LIM} V(x, y) \equiv \lim_{\tau \to 0} \frac{1}{|\tau|} \left( \begin{array}{cc}
QQV & \frac{1}{z} \frac{1}{2} \frac{QG}{z} V \\
\frac{1}{z} \frac{1}{2} \frac{GQ}{z} V & GGV
\end{array} \right)^{\text{ext}} \left( \frac{z}{\tau}, \frac{1}{\tau} \right).
\]

Then the difference

\[
P(z) - P^{\text{cross-ladder}}(z) - \text{LIM} V^{\text{ND}}(x, y)
\]
can be represented in terms of inclusive convolutions of simple splitting functions and the back transformation to the exclusive kinematics is trivial. The contributions of the purely diagonal cross-ladder diagrams \(V^{\text{cross-ladder}}(z)\) can be found from the known \(QQ\) sector \[18, 19, 20\] exploiting the \(\mathcal{N} = 1\) supersymmetric constraints.

The paper is organized as follows. In the next section we analyze the structure of the known flavour non-singlet ER-BL kernel and state the benchmarks of the formalism. The structure observed will give us a guideline for construction of all other kernels: quark chiral odd sector is considered in Section 3 and parity odd flavour singlet one is discussed in Section 4. Finally, we give our conclusions and an outlook.

2 Structure of ER-BL kernel in non-singlet sector.

It is very instructive to demonstrate the machinery in the simplest case of non-singlet sector. Since the explicit two-loop calculation is available \[18, 19, 20\] the direct comparison can be made. The NLO \(QQ\)-kernel can be decomposed in colour structures as \[^3\]

\[
V(x, y|\alpha_s) = \frac{\alpha_s}{2\pi} C_F V^{(0)}(x, y)
+ \left( \frac{\alpha_s}{2\pi} \right)^2 C_F \left[ C_F V_F(x, y) - \frac{\beta_0}{2} \beta_0 V_\beta(x, y) - \left( C_F - \frac{C_A}{2} \right) V_G(x, y) \right] + \mathcal{O} \left( \alpha_s^3 \right).
\]

\[^3\text{Here } V^{\text{ND}} \text{ is understood without the } (I - D) \text{-projector.}\]

\[^4\text{We omit the superscript } QQ \text{ later in this section.}\]
with the LO kernel $V(0)(x, y) = [v(x, y)]_+$, where

$$v(x, y) = \theta(y - x)f(x, y) + \theta(x - y)\overline{f}(x, y), \quad \text{and} \quad f(x, y) = \frac{x}{y} \left(1 + \frac{1}{y - x}\right).$$

(8)

The shorthand notations $\bar{x} = 1 - x$ and $\overline{f} = f(\bar{x}, \bar{y})$ are used throughout the paper. The “+”-prescription is conventionally defined by

$$[V(x, y)]_+ = V(x, y) - \delta(x - y) \int_0^1 dz V(z, y).$$

Let us now recall a few properties of the kernel that are useful for the following considerations. Due to absence of the conformal symmetry breaking counterterms at leading order for the renormalization of the composite operators with total derivatives, one can use its consequences to fix the eigenfunctions which turn out to be the Gegenbauer polynomials $C^{3/2}_j(2x - 1)$ \[6, 7\]. Thus, the LO kernel is symmetric with respect to the weight function $x\bar{x}$: $y\bar{y}V(0)(x, y) = x\bar{x}V(0)(y, x)$. Its eigenvalues are given by the anomalous dimensions appeared in the analysis of deep inelastic scattering. Thus, it is not surprising that a simple limit already mentioned in Eq. (6) gives us the DGLAP kernel \[3\]:

$$P(z) = \text{LIM} V(x, y) \equiv \lim_{\tau \to 0} \frac{1}{|\tau|} V^\text{ext} \left(\frac{z}{\tau}, \frac{1}{\tau}\right).$$

(9)

To perform this limit, we have to extend at first the ER-BL kernel, originally defined in the domain $0 \leq x, y \leq 1$, to the whole region $x, y \in (-\infty, \infty)$ by a unique procedure which is given in practice by the replacement, e.g. at leading order, of the $\theta$-function by

$$\theta(y - x) \to \theta \left(1 - \frac{x}{y}\right) \theta \left(\frac{x}{y}\right) \text{sign}(y).$$

(10)

If a kernel is diagonal in the ER-BL representation, we can restore it from the known DGLAP kernel by the integral transformation (3). Because of branch cuts appearing in the convolutions of the NLO terms with the transformation kernel, it is highly nontrivial to handle the inverse reduction to the exclusive kinematics.

At NLO the kernel (4) contains besides a pure diagonal part with respect to the Gegenbauer polynomials also a non-diagonal part located in $V_F(x, y)$ and $V_\beta(x, y)$. These parts are predicted by conformal constraints (see QQ-entry of Eq. (4)) and are fixed by the one-loop special conformal anomaly kernels \[11, 16, 17\]:

$$\dot{v}(x, y) = \theta(y - x)f(x, y) \ln \frac{x}{y} + \left\{x \to \bar{x}, \quad y \to \bar{y}\right\}, \quad g(x, y) = -\theta(y - x) \frac{\ln \left(1 - \frac{x}{y}\right)}{y - x} + \left\{x \to \bar{x}, \quad y \to \bar{y}\right\}.$$  

(11)

Let us now analyze in detail the contributions to the NLO kernel from different colour structures. The expressions for $C^2_F$ terms arise from Feynman diagrams containing quark self-energy
insertions and ladder graphs. In order to subtract the ultraviolet (UV) divergences in subgraphs it requires the LO renormalization of the composite operator to which these lines are attached to. The explicit calculation gives \[18, 19, 20\]

\[
V_F(x, y) = \theta(y - x) \left\{ \left( \frac{4}{3} - 2\zeta(2) \right) f + 3 \frac{x}{y} - \left( \frac{3}{2} f - \frac{x}{2y} \right) \ln \frac{x}{y} - (f - f) \ln \frac{x}{y} \ln \left(1 - \frac{x}{y}\right) \right. \\
+ \left. \left( f + \frac{x}{2y} \right) \ln \frac{x}{y} - \frac{x}{2y} \ln x (1 + \ln x - 2 \ln \bar{x}) + \begin{cases} x \to \bar{x} \\ y \to \bar{y} \end{cases} \right\}.
\]

Making use of the known non-diagonal part \[4\], \(V_F\) can be represented up to a pure diagonal term, denoted as \(D_F(x, y)\), by the convolution

\[
V_F(x, y) = -\left( \dot{v} \otimes v + g \otimes v - v \otimes g \right) (x, y) + D_F(x, y).
\]

To find an appropriate representation of this missing diagonal element we first take the forward limit. Since the forward limit of the convolution is \[6\]

\[
\text{LIM} \left\{ [\dot{v}]_+ \otimes [v]_+ \right\} = \left\{ \text{LIM} [\dot{v}]_+ \right\} \otimes \left\{ \text{LIM} [v]_+ \right\},
\]

where we have introduced the inclusive convolution

\[
P_1(z) \otimes P_2(z) \equiv \int_0^1 dx \int_0^1 dy \delta(z - xy) P_1(x) P_2(y),
\]

and the commutator \(g \otimes V^{(0)} - V^{(0)} \otimes g\) drops out in the forward limit, we obtain

\[
\text{LIM} V_F(x, y) = -\dot{p} \otimes p + \text{LIM} D_F(x, y),
\]

where \(\dot{p} = \text{LIM} \dot{v} = p(z) \ln z + 1 - z \) and \(p(z) = \text{LIM} v(x, y) = (1 + z^2)/(1 - z)\). The comparison of \(\text{LIM} V_F(x, y)\) with the corresponding part of the DGLAP kernel \[23\]

\[
P_F(z) = \left\{ \frac{4}{3} - 2\zeta(2) - \frac{3}{2} \ln z + \ln^2 z - 2 \ln z \ln (1 - z) \right\} p(z)
+ \left. 1 - z + \frac{1 - 3z}{2} \ln z - \frac{1 + z}{2} \ln^2 z, \right\}
\]

yields the result in which all double log terms are contained in the convolution \(\dot{p} \otimes p\) and, therefore, only single logs survive in \(D_F(z) = \text{LIM} D_F(x, y)\):

\[
D_F(z) = P_F + \dot{p} \otimes p \\
= -\frac{1}{2} p^a(-z) \ln z - p^a(z) \left\{ \ln z - 2 \ln (1 - z) - \frac{1}{2} \right\} - \frac{5}{12} p(z).
\]

\[5\] For simplicity we imply the diagrams in the light-cone gauge \[18, 19\].

\[6\] We remind as well that \([A]_+ \otimes [B]_+ = [C]_+.\]
Here we have introduced for convenience the kernel $p^a(z) = 1 - z$. The next important point is that
the remaining log terms can be represented as convolutions of $p^a$ and $p$. Thus, we have finally
\[ D_F(z) = \frac{1}{2} p^a \otimes \{ 2 p + p^a \} (z) + \frac{1}{12} p(z) + \frac{5}{2} p^a(z). \] (18)

Since $D_F(x, y)$ is by definition diagonal, the extension of $D_F(z)$ towards the ER-BL kinematics is trivial:
\[ D_F(z) \rightarrow D_F(x, y) = \frac{1}{2} v^a \otimes (2 v + v^a) (x, y) + \frac{1}{12} v(x, y) + \frac{5}{2} v^a(x, y), \] (19)
where a new diagonal element is $v^a(x, y) = \theta(y - x) \frac{x}{y} + \theta(x - y) \frac{y}{x}$. Evaluating the convolutions one can establish the equivalence of our prediction with Eq. (12).

Next, the Feynman diagrams containing vertex and self-energy corrections provide $V \beta$ proportional to $\beta_0$. Its off-diagonal part is induced by the renormalization of the coupling and is contained in the dotted kernel \[ \text{(11)} \]
\[ V \beta(x, y) = \dot{v}(x, y) + D \beta(x, y). \] (20)
The remaining diagonal piece, $D \beta$, is deduced from the known NLO DGLAP kernel \[ \text{(23)} \]
\[ P \beta(z) = \frac{5}{3} p(z) + p^a(z) + \dot{p}(z) \] (21)
by going to the forward kinematics and restoring then the missed contributions from it. Thus,
\[ D \beta(x, y) = \frac{5}{3} v(x, y) + v^a(x, y). \] (22)
Indeed, the final result coincides with \[ \text{(18, 19, 20)} \].

Finally, we come to the contribution which mainly originates from the crossed ladder diagram proportional to $(C_F - C_A/2)$:
\[ V_G(x, y) = 2 v^a(x, y) + \frac{4}{3} v(x, y) + \left( G(x, y) + \left\{ \begin{array}{c} x \rightarrow \bar{x} \\ y \rightarrow \bar{y} \end{array} \right\} \right). \] (23)
Since this diagram has no UV divergent subgraph and thus requires no subtraction, its contribution has to be diagonal w.r.t. the Gegenbauer polynomials. This is obvious for the first two terms appearing in Eq. (24). The function $G(x, y)$ contains in the unusual $\theta(y - \bar{x})$-structure the mixing between quarks and antiquarks
\[ G(x, y) = \theta(y - x) H(x, y) + \theta(y - \bar{x}) \bar{H}(x, y), \] (24)
\[ \text{We have slightly changed the original definition given in} \text{[18, 19]} \text{by} \text{G(x, y) + 2} \theta(y - x) \bar{f} \ln y \ln \bar{x} \rightarrow G(x, y). \text{[24]}. \]
with
\[
H(x, y) = 2 \left[ \mathcal{F} \left( \text{Li}_2(x) + \ln y \ln x - f \text{Li}_2(y) \right) \right],
\]
\[
\overline{H}(x, y) = 2 \left[ (f - \mathcal{F}) \left( \text{Li}_2 \left( 1 - \frac{x}{y} \right) + \frac{1}{2} \ln^2 y \right) + f \left( \text{Li}_2(y) - \text{Li}_2(x) - \ln y \ln x \right) \right],
\]
where \( \text{Li}_2 \) is the dilogarithm. It can be easily checked that the \( G \)-contribution (not the terms \( H \) and \( \overline{H} \) separately) is symmetrical w.r.t. the weight \( x \). Performing the limit (9) we obtain the following correspondence with the non-singlet DGLAP kernel [1]:
\[
G(z) \equiv \text{LIM} G(x, y) = \theta(z) \theta(1 - z) H(z) + \theta(-z) \theta(1 + z) \overline{H}(z),
\]
where
\[
H(z) \equiv \text{LIM} H(x, y) = p(z) \left( \ln^2 z - 2 \zeta(2) \right) + T(z),
\]
\[
\overline{H}(z) \equiv \text{LIM} \overline{H}(x, y) = 2 p(z) S_2(-z) + T(-z).
\]
Here \( S_2(z) = \int_{z/(1+z)}^{1/(1+z)} \frac{dx}{x} \ln \frac{1-x}{x} \) and \( T(z) = 2(1 + z) \ln z + 4(1 - z) \). We should emphasize that this structure of \( G \) is the most general, especially, it is the only contribution that contains Spence functions. In the forward limit we obtain therefore a typical combinations given in (28) and (29), which can be found in all other channels as well. This observation provides us with a hint for the construction of all singlet \( G \) kernels in the ER-BL representation. For completeness, we give the corresponding part of the DGLAP kernel [23]
\[
P_G(z) = \text{LIM} V_G(x, y) = 2p^o(z) + \frac{4}{3} p(z) + G(z),
\]
and \( G(z) \) defined above in Eqs. (27)-(29).

Recapitulating the results obtained in this section, we have observed a rather simple structure of the non-singlet NLO kernel in the \( QQ \)-channel. Up to the diagonal \( G \)-function, which is in fact the only new element in the two-loop approximation, we can represent all other terms by a simple convolution of LO kernels already known. It is not accidental but a mere consequence of the topology of contributing Feynman graphs at \( \mathcal{O}(\alpha_s^3) \). Thus, we anticipate the same feature to appear in all other channels as well.

### 3 Quark kernel in chiral odd sector.

After we have outlined and tested in the preceding section our formalism, we can apply it to the previously unknown transversity two-loop ER-BL kernel. We decompose the transversity kernel
analogous to the chiral even case (7). We also use the same decomposition for the DGLAP kernels (24). The leading order kernel is

$$V^{(0)T}(x, y) = \left[v^b(x, y)\right]_+ - \frac{1}{2} \delta(x - y), \quad (31)$$

with

$$v^b(x, y) = \theta(y - x)f^b(x, y) + \theta(x - y)f^b(x, y), \quad \text{and} \quad f^b(x, y) = \frac{x}{y} \frac{1}{y - x}. \quad (32)$$

The non-diagonal part has been analyzed in Ref. [17] and is completely analogous to the chiral even case discussed above. Thus,

$$V^T_F(x, y) = -\left\{\left[\dot{v}^b\right]_+ \otimes V^{(0)T} + [g]_+ \otimes V^{(0)T} - V^{(0)T} \otimes [g]_+\right\}(x, y) + D^T_F(x, y), \quad (33)$$

where $\dot{v}^b$ is obtained from Eq. (11) by replacing $f$ by $f^b$ and the $g$ kernel is the same as in Eq. (11). Taking the forward limit of $V^T_F(x, y)$ and comparing it with the known result for the DGLAP kernel in Ref. [24], we find the following trivial representation of the remaining diagonal part

$$D^T_F(x, y) = -\frac{2}{3} \left[v^b(x, y)\right]_+ - \frac{19}{24} \delta(x - y). \quad (34)$$

There is essentially no extra work required to find the contribution proportional to the $\beta_0$-function, since it can be easily traced from the DGLAP kernel [24] to be

$$V^{T\beta}(x, y) = \frac{5}{3} \left[v^b(x, y)\right]_+ + \left[\dot{v}^b(x, y)\right]_+ - \frac{13}{12} \delta(x - y). \quad (35)$$

The case of the $G$ function is easy to handle as well. If we replace $f$ by $f^b$ in the definition (24), we obtain the diagonal $G^T(x, y)$ kernel. Taking the forward limit and comparing it with the DGLAP kernel, we immediately find the remaining $\delta$-function contribution, so that the whole result reads

$$V^{T\beta}(x, y) = \left[G^T(x, y) + \left\{x \rightarrow \bar{x}, y \rightarrow \bar{y}\right\}\right]_+ - \frac{19}{6} \delta(x - y). \quad (36)$$

This completes the discussion of the quark chiral-odd channel.

### 4 Flavour singlet parity odd sector.

Let us now address the flavour singlet parity odd sector responsible for the evolution of axial-vector distribution amplitudes. For even parity there are few subtleties, which will be discussed elsewhere. Here we would only like to note that in the latter case a direct leading order calculation provides a result that suffers for the mixed channel from off-diagonal matrix elements in the unphysical
sector. Although the improved result has been found in Ref. [11], it still remains a difficult task
to find an appropriate representation for the dotted kernels and the two-loop \( G \) functions.

Making use of the known non-diagonal part of the ER-BL kernel [11], the whole NLO result in
the axial-vector case reads

\[
V^{(1)A} = -V^A \otimes \left( V^{(0)A} + \frac{\beta_0}{2} \mathbb{1} \right) - g \otimes V^{(0)A} + V^{(0)A} \otimes g + D^A + G^A,
\]

where the kernels \( D^A(x, y) \) and \( G^A(x, y) \) are purely diagonal. Here the matrix of the LO kernels
is given in a compact form by

\[
V^{(0)A}(x, y) = \begin{pmatrix}
C_F \left[ QQ \nu(x, y) \right]_+ & -2T_F N_f QG \nu^a(x, y) \\
C_F QG \nu^a(x, y) & C_A \left[ GG \nu^A(x, y) \right]_+ - \frac{\beta_0}{2} \delta(x - y)
\end{pmatrix},
\]

where \( QQ \nu \equiv QQ \nu^a + QQ \nu^b \) and \( GG \nu^A \equiv 2 GG \nu^a + GG \nu^b \). The general structure of the functions \( \nu^i \) is

\[
\nu^i(x, y) = \theta(y - x) \nu f^i(x, y) \pm \begin{cases} x \to \bar{x} \\ y \to \bar{y} \end{cases} \text{ for } \begin{cases} A = B \\ A \neq B \end{cases},
\]

with

\[
\begin{aligned}
\nu f^a & = x^\nu(a) - 1/2 \\
\nu f^b & = y^\nu(b) - 1/2 \\
\end{aligned}
\]

The index \( \nu(A) \) coincides with the index of Gegenbauer polynomials in the corresponding channel,
i.e. \( \nu(Q) = 3/2 \) and \( \nu(G) = 5/2 \). The dotted kernels involved in the definition [37] can simply
be obtained by differentiating LO results w.r.t. the index \( \nu \) which gives rise to the additional
\( \ln(x/y) \)-multiplier in front of the former

\[
\dot{V}^{(0)A}(x, y) = \begin{pmatrix}
C_F \left[ QQ \nu(x, y) \right]_+ & -2T_F N_f QG \nu^a(x, y) \\
C_F QG \nu^a(x, y) & C_A \left[ GG \nu^A(x, y) \right]_+ 
\end{pmatrix},
\]

with the matrix elements

\[
\nu(x, y) = \theta(y - x) \nu f(x, y) \ln \frac{x}{y} \pm \begin{cases} x \to \bar{x} \\ y \to \bar{y} \end{cases} \text{ for } \begin{cases} A = B \\ A \neq B \end{cases},
\]

Note that for \( A = B \) the dotted kernels are defined with the “+”-prescription. The \( g \) function is
given by

\[
g(x, y) = \theta(y - x) \begin{pmatrix}
- C_F \left[ \frac{\ln(1 - \frac{\bar{x}}{\bar{y}})}{y - x} \right]_+ \\
C_F \frac{\bar{x}}{y} & - C_A \left[ \frac{\ln(1 - \frac{\bar{x}}{\bar{y}})}{y - x} \right]_+ \end{pmatrix} \pm \begin{cases} x \to \bar{x} \\ y \to \bar{y} \end{cases},
\]

with \( - \) + sign corresponding to (non-) diagonal elements. Note, that we have used the property
\( (I - D) \ln(1 - \frac{\bar{x}}{\bar{y}}) = -(I - D) \frac{\bar{x}}{\bar{y}} \) for the element of \( GQ \)-channel to make contact with the results of Ref. [17].
Next we construct the diagonal $G(x, y)$ kernel. At first glance one would naively expect that one can obtain these kernels by only inserting appropriate $AB$ functions in the definition (24), so that the symmetry properties of the $f$ functions w.r.t. the weight induce then the desired symmetry of the $ABG$ functions. Unfortunately, the symmetry is not sufficient for the diagonal form of the $G(x, y)$ kernel. To ensure the diagonality, we have to add terms containing single logs and rational functions. Let us define the matrix

$$G^A(x, y) = -\frac{1}{2} \begin{pmatrix} 2C_F \left( C_F - C_A \right) \left[ QQ \ G^A(x, y) \right]_+ & 2C_A T_F N_f \left[ QG \ G^A(x, y) \right]_+ \end{pmatrix},$$

with the following general structure

$$ABG^A(x, y) = \theta(y - x) \left( ABH^A + \Delta ABH^A \right)(x, y) + \theta(y - x) \left( ABT^A + \Delta ABT^A \right)(x, y).$$

Here analogous to the non-singlet case we set

$$ABH^A(x, y) = 2 \left[ \pm ABf^A \left( \text{Li}_2(\bar{x}) + \ln y \ln x \right) - ABf^A \text{Li}_2(\bar{y}) \right],$$

$$ABT^A(x, y) = 2 \left[ \left( ABf^A + \Delta ABf^A \right) \left( \text{Li}_2 \left( 1 - \frac{x}{y} \right) + \frac{1}{2} \ln^2 y \right) + ABf^A \left( \text{Li}_2(\bar{y}) - \text{Li}_2(x) - \ln y \ln x \right) \right],$$

where the upper (lower) sign corresponds to the $A = B$ ($A \neq B$) channels. An explicit use of the reduction $P \to V^D$ procedure (3) to restore the $\Delta H$ contributions is rather involved due to complexity of the integrand function. Rather we have succeeded to deduce them using different arguments. Since the crossed ladder diagrams have no UV divergent subgraphs the kernels $ABG$ in different channels are related in a scheme independent way by supersymmetry and conformal covariance of $\mathcal{N} = 1$ super Yang-Mills theory [22]. Employing these symmetries we restored all necessary terms in a straightforward manner to be

$$\Delta QQ H^A(x, y) = \Delta QQ T^A(x, y) = 0,$$

$$\Delta QG H^A(x, y) = 2 \frac{\bar{x}}{yy} \ln \bar{x} - 2 \frac{x}{yy} \ln y, \quad \Delta QG T^A(x, y) = 2 \frac{x}{yy} \ln x - 2 \frac{\bar{x}}{yy} \ln y,$$

$$\Delta GG H^A(x, y) = 2 \frac{\bar{x}}{y} \ln \bar{x} - 2 \frac{x}{y} \ln y, \quad \Delta GG T^A(x, y) = -2 \frac{\bar{x}}{y} \ln x + 2 \frac{x}{y} \ln y,$$

$$\Delta TT^A(x, y) = \frac{x^2}{y^2} - \frac{1 + (x - y)^2}{y^2} - 2 \frac{\bar{x}}{y} \ln \bar{x} + 2 \frac{x}{yy} \ln x - 2 \frac{(\bar{x} - x)}{yy} \ln \bar{x} - 2 \frac{\bar{x} - x}{yy} \ln y,$$

$$\Delta TT^A(x, y) = 2 \frac{x}{y^2} - \frac{x}{y^2} - 2 \frac{1 - x\bar{x}}{yy^2} + 2 \frac{x}{y^2} \ln \bar{x} + 2 \frac{(x + y)\bar{x}}{yy^2} \ln x - 2 \frac{1 - x\bar{x}}{yy} \ln x + 6 \frac{\bar{x}}{yy} \ln y.$$

Finally, we have to extract the remaining diagonal piece $D^A$ of $V^A$ in the forward limit (3) from the known DGLAP kernel $P^A$ [23]. We take into account the underlying symmetry of the

\footnote{The details will be presented elsewhere.}
singlet parton distributions to map the antiparticle contribution, i.e. \( z < 0 \), into the region \( z > 0 \). As expected we find from

\[
D^A(z) = P^A(z) - \text{LIM} \left\{ - \dot{V} \otimes \left( V^{(0)A} + \frac{\beta_0}{2} \mathbb{1} \right) - g \otimes V^{(0)A} + V^{(0)A} \otimes g + G^A \right\}
\]  

(52)
a simple convolution-type representation for ER-BL kernels which can be immediately deduced from the forward results for singlet \( QQ \)-channel

\[
QQ D^A = C_F^2 [D_F]_+ - C_F \frac{\beta_0}{2} [D_\beta]_+ - C_F \left( C_F - \frac{C_A}{2} \right) \left[ \frac{4}{3} QQ v + 2 QQ v^a \right]_+ - 6 C_F T_F N_f QQ v^a,
\]

(53)
where \( D_F, D_\beta \) are given by Eqs. (19) and (22), respectively. The rest of channels is expressed as

\[
QQ D^A = 3 C_F T_F N_f \left\{ QQ v^a \otimes QQ v^a - \frac{1}{2} QQ v^a \right\}
\]

(54)
\[
- 2 C_A T_F N_f \left\{ 3 QQ v^a \otimes QQ v^a + [1 + 2 \zeta(2)] QQ v^a \right\},
\]

\[
GQ D^A = C_F^2 \left\{ \frac{1}{2} \left[ GG v^A \right]_+ \otimes QQ v^a - \frac{3}{2} QQ v^a \right\} - C_F \frac{\beta_0}{2} \left\{ QQ v^a \otimes [QQ v]_+ - \frac{1}{6} QQ v^a \right\},
\]

(55)
\[
- C_F C_A \left\{ \frac{3}{2} \left[ GG v^A \right]_+ \otimes QQ v^c + \frac{1}{2} \left[ GG v^A \right]_+ \otimes QQ v^a - \frac{3}{2} \left[ GG v^A \right]_+ \otimes QQ v^a - \frac{7}{3} - 2 \zeta(2) \right\} QQ v^a \right\},
\]

\[
GG D^A = C_A^2 \left\{ \left[ GG v^A \right]_+ + \frac{1}{2} GG v^a \right\} \otimes GG v^a + \frac{2}{3} \left[ GG v^A \right]_+ + \frac{1}{4} GG v^a \right\} - 2 \delta(x - y) \}
\]

(56)
\[
- C_A \frac{\beta_0}{2} \left\{ - \frac{1}{2} GG v^a \otimes GG v^a + \frac{5}{3} \left[ GG v^A \right]_+ + GG v^a + 2 \delta(x - y) \}
\]

\[
- C_F T_F N_f \left\{ GG v^a \otimes GG v^a - GG v^a + \delta(x - y) \right\},
\]

where we have introduced a new kernel

\[
GG v^c(x, y) = \theta(y - x) \frac{x^2}{y} (2 \bar{x} y - \bar{y}) - \left\{ \frac{x \to \bar{x}}{y \to \bar{y}} \right\}.
\]

(57)
These results provide us with the explicit parity odd singlet evolution kernels.

5 Conclusions.

In this paper, we have presented a simple method for construction of the exclusive evolution kernels in NLO from the knowledge of the conformal anomalies and the available two-loop splitting functions. The main task was, of course, the reconstruction of the diagonal part of the kernel in the basis of Gegenbauer polynomials.

In the course of study we have established convolution-type formulae for the bulk of contributing two-loop graphs with an exception of cross-ladder diagrams. The complications which arise in the restoration of the latter from the known forward kernels has been overcome making use of
The former feature suggests that by disentangling the topology of corresponding diagrams, it might allow for an effective and facilitated way of explicit calculation. One may expect that this property persists for a subset of diagrams at higher orders and can be used, e.g. for diagrammatical derivation of NNLO splitting functions.

The details of the present formalism together with the flavour singlet parity even case, where new subtleties appear, will be discussed elsewhere.

A.B. was supported by the Alexander von Humboldt Foundation.

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