Formation of primordial black holes from warm inflation

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Abstract. Primordial Black Holes (PBHs) serve as a unique probe to the physics of the early Universe, particularly inflation. In light of this, we study the formation of PBHs by the collapse of overdense perturbations generated during a model of warm inflation. For our model, we find that the primordial curvature power spectrum is red-tilted (spectral index $n_s < 1$) at the large scales (small $k$) and is consistent with the $n_s - r$ values allowed from the CMB observations. Along with that, it has a blue-tilt ($n_s > 1$) for the small PBH scales (large $k$), with a sufficiently large amplitude of the primordial curvature power spectrum required to form PBHs. These features originate because of the inflaton’s coupling with the other fields during warm inflation. We discuss the role of the inflaton dissipation to the enhancement in the primordial power spectrum at the PBH scales. We find that for some parameter range of our warm inflation model, PBHs with mass $\sim 10^3$ g can be formed with significant abundance. Such tiny mass PBHs have a short lifetime $\sim 10^{-19}$ s and would have evaporated into Hawking radiation in the early Universe. Further in this study, we discuss the evaporation constraints on the initial mass fraction of the generated PBHs and the possibility of Planck mass PBH relics to constitute the dark matter.

Keywords: primordial black holes, inflation, physics of the early universe

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1 Introduction

Primordial Black Holes (PBHs) [1–4] are the black holes that could have produced in the very early Universe. They can form in a number of ways - by the collapse of overdensities in the early inhomogeneous Universe [2, 3], from the collision of bubbles [5], by the collapse of strings [6, 7] or domain walls [8], etc. For a review on PBHs, see refs. [9–12].

It is crucial to study primordial black holes as they provide us a unique probe to the rich physics of the Universe at all epochs of its evolution. While the Cosmic Microwave Background (CMB) and Large Scale Structures (LSS) observations measure only the large scale modes ranging from $10^{-22} - 1$ Mpc$^{-1}$, PBHs span over a wide range of modes varying from $10^{-2} - 10^{23}$ Mpc$^{-1}$, and therefore provide a probe for a huge range of small scales. The mass of a PBH depends on the epoch when it is formed, with lighter mass PBHs forming earlier than the heavier PBHs. The lightest PBH mass can be as tiny as corresponding to the Planck mass, $M_P = 10^{-5}$ g [3].

The abundance of PBHs is constrained from various observations. These constraints can be classified for two mass ranges of PBHs, $M_{PBH} > 10^{15}$ g, which are obtained from the gravitational effects of PBHs, and $M_{PBH} \leq 10^{15}$ g, obtained from the effects of their evaporation. PBHs with mass $M_{PBH} < 10^{15}$ g have short lifetime compared to the present age of the Universe, and would have evaporated into Hawking radiation by the present time [13, 14]. Therefore, for PBHs with $M_{PBH} < 10^{15}$ g, the consequences of PBH evaporation on the nucleosynthesis [15–17], or the relic abundance of stable and long lived decaying particles produced from PBH evaporation [18–20] can provide constraints on their abundance (see ref. [21]). The upper limit on PBH abundance further give bounds on the amplitude of the primordial curvature power spectrum and hence various inflationary models. (For
details, see refs. [22–26]). In this way, PBHs can serve as a powerful and unique probe to the inflationary epoch.

Furthermore, PBHs with \(M_{\text{PBH}} \sim 10^{15}\) g would be evaporating into radiation at the present epoch and have interesting astrophysical consequences. Such PBHs can contribute to the diffuse gamma-ray background [27], or positrons and antiprotons in the cosmic rays [28] and therefore can provide useful information about the high energy physics of the PBH evaporation [21].

The primordial black holes with \(M_{\text{PBH}} > 10^{15}\) g have not evaporated till today. An interesting consequence of such PBHs is that they can contribute as some or all of the Dark Matter (DM) present in the Universe, and therefore their abundance should be less than the limits on the cold dark matter density at present. (For review, see refs. [29–31]). The constraints on the abundance of these PBHs can also be obtained from the different lensing experiments [32–34], or from the dynamical effects of PBHs on the astrophysical systems (For a review, see ref. [12]). Also, massive PBHs (few solar masses \(M_\odot\)) can accrete its surrounding gas and emit X-rays, which can change the ionization history of the Universe [35] and cause spectral distortions in the CMB radiation [36–39]. Therefore, the abundance of massive PBHs is constrained from the CMB anisotropy observations. The detection of gravitational wave signals from binary black hole merger by the LIGO collaboration revived the interest in PBHs as constituent of dark matter [40, 41]. There are many studies on the stochastic gravitational waves generated from the binary PBH mergers used to constraint the PBH abundance [42–46]. (For review, see refs. [12, 47].) Recent observations of microlensing of the stars in the Andromeda galaxy suggest that PBHs in the mass range \(10^{22} g < M_{\text{PBH}} < 10^{27}\) g can constitute only a small fraction < 0.1% of the dark matter energy density [48]. However, the possibility that PBHs comprise a significant fraction of DM still remains open for other unexplored mass ranges of PBHs, \(M_{\text{PBH}} \sim (10^{19} – 10^{22})\) g [29]. It is also argued that Planck mass (\(M_P = 10^{-5}\) g) remnants of the evaporating PBHs can comprise the dark matter [49, 50]. We will discuss it further in our study. For a comprehensive and recent review of the observational constraints on the abundance of various mass ranges of PBHs and their contribution as dark matter, see refs. [31, 51].

In this work, we study an inflationary scenario known as Warm Inflation [52–54] and discuss the formation of primordial black holes by the collapse of large inhomogeneities generated during it. Warm Inflation is a description of inflation in which the dissipation processes during the inflationary phase are taken in account. During the expansion, a thermal bath of particles (radiation) is simultaneously created from the inflaton dissipation and thus the Universe has a non-zero temperature during warm inflation. For a review, see refs. [55–58]. The primordial power spectrum for warm inflation is dominated by the contributions from the thermal fluctuations of the fields, unlike the quantum fluctuations in cold inflation, as will be explained in section 3. In this paper, we discuss the features in the primordial power spectrum of a model of warm inflation for various values of the dissipation parameter and then study the formation of PBHs from our model.

We find that for our warm inflation model, the primordial power spectrum is red-tilted (spectral index, \(n_s < 1\)) for the CMB scales, with an amplitude \(P_R(k_P) = 2.1 \times 10^{-9}\) (at the pivot scale \(k_P = 0.05\) Mpc\(^{-1}\)), and is consistent with the \(n_s\) and \(r\) (tensor-to-scalar ratio) values allowed from the CMB observations. Apart from this, it has a blue-tilt (\(n_s > 1\)) with a large amplitude of the primordial power spectrum for the PBH scales. In our analysis, we find that for some range of the model parameter, the amplitude of \(P_R(k)\) at the PBH scales is of \(O(10^{-2})\) which is required for the PBH formation, and therefore, a significant abundance
of PBHs can be produced. We first discuss the relevant range of model parameter and then calculate the initial mass fraction and the mass of the generated PBHs.

As mentioned previously, PBHs are a unique probe to inflation, as the observational bounds on the abundance of PBHs provide an upper limit on the amplitude of the primordial power spectrum. Therefore, a study of PBHs is crucial to test various inflationary models. Here we carry out the first study considering a warm inflation primordial power spectrum for the PBH formation. The importance of this work is that our warm inflation model is also consistent with the CMB bounds on $n_s - r$ and the theoretical prediction for the tensor-to-scalar ratio is within the sensitivity of the upcoming CMB polarisation experiments and hence can be tested in the near future.

The organization of this work is as follows. In section 2, we first discuss the formation of primordial black holes in terms of the mass of the generated PBH and its initial mass fraction. Then using the Press-Schechter formalism, we calculate the bounds on the primordial curvature power spectrum from the bounds on PBH abundance. After that, we discuss the fundamentals of warm inflation and the primordial power spectrum in section 3. Then we present our model of warm inflation in section 4 and analyze the obtained results in section 5. At last we conclude our study in section 6.

2 Formation of primordial black holes

As mentioned previously, PBHs can be generated through many mechanisms. Here we consider the PBH formation by the collapse of overdense perturbations generated during a model of warm inflation. These fluctuations leave the horizon during inflation and then re-enter at later epochs (we assume radiation dominated era), and collapse to form PBHs. In this section, we first calculate the mass of PBHs formed as a function of the fluctuation mode, $k$. We then define the initial mass fraction of PBH and discuss the constraints on it from observations. Further, we discuss the Press-Schechter formalism for PBH formation and using it, calculate the initial mass fraction of PBHs. After that, we show how the constraints on the abundance of PBHs can be used to obtain bounds on the amplitude of the primordial power spectrum.

2.1 Mass of the generated primordial black holes

Primordial black hole forms when an overdense fluctuation with a comoving wavenumber $k$ generated during inflation, re-enters the horizon at a later epoch (i.e. physical wavelength equals the horizon size, $H^{-1} = (k/a)^{-1}$) with an overdensity $\delta$ greater than a critical density $\delta_c$, and collapses through gravitational instability. The mass of the generated PBH, $M_{\text{PBH}}$ depends on the time of its formation and is taken to be a fixed fraction, $\gamma$, of the horizon mass at that epoch [24],

$$M_{\text{PBH}}(k) = \gamma \frac{4\pi}{3} \rho H^{-3} \bigg|_{k = aH}. \quad (2.1)$$

Here $H$ is the Hubble expansion rate and $\rho$ is the energy density of the Universe at the epoch of PBH formation. We consider that the PBH formation takes place during the radiation dominated era, and therefore $\rho$ is the energy density of the radiation, i.e. $\rho = \rho_r$. The fraction of the horizon mass collapsing into the PBHs is taken as $\gamma = 0.2$ [4].

In order to calculate the relation between the $k^{th}$ mode of perturbation, and the mass of the generated PBH, we write the parameters on the r.h.s. of eq. (2.1) explicitly as a function of $k$. This is carried out as follows.
The energy density of the radiation is given as \( \rho_r = \frac{\pi^2}{30} g_* T^4 \), where \( g_* \) is the number of relativistic degrees of freedom, and \( T \) represents the temperature of the Universe in the radiation dominated era. Following the conservation of entropy, we have \( S = g_* a^3 T^3 = \) constant, where \( S \) is the entropy, \( a \) is the scale factor of the Universe, and \( g_* \) represent the number of relativistic degrees of freedom contributing in the entropy. We assume that the number of relativistic degrees of freedom contributing to the radiation equals to the ones contributing in the entropy i.e. \( g_* \approx g_{*s} \) (also see ref. [59] for comments on massive neutrinos and relativistic degrees of freedom), and thus obtain \( \rho_r \propto g_*^{-1/3} a^{-4} \). With this relation, the radiation energy density at the initial time of PBH formation \( \rho_{ri} \) can be related to the present radiation energy density \( \rho_{r0} \) as

\[
\rho_{ri} = \left( \frac{g_{*i}}{g_{*0}} \right)^{-1/3} \left( \frac{a_i}{a_0} \right)^{-4} \rho_{r0}.
\]

(2.2)

The subscript ‘0’ and ‘i’ to any quantity represent its value at the present epoch and at the initial time when PBH formed, respectively. The scale factor at present \( a_0 = 1 \). The present radiation density can be written as \( \rho_{r0} = \rho_{\text{crit}} \Omega_{r0} \), where \( \rho_{\text{crit}} = 3H_0^2/8\pi G_N = 1.054 \times 10^{-5} h^2 \) GeV cm\(^{-3} \) is the critical energy density today, \( G_N \) is the Newton’s gravitational constant, \( H_0 = 100 \ h \) is the present Hubble expansion rate with \( h = 0.678 \), and \( \Omega_{r0} \approx 5.38 \times 10^{-5} \) is the radiation density parameter today [60]. Thus we can write eq. (2.2) as

\[
\rho_{ri} = \left( \frac{g_{*i}}{g_{*0}} \right)^{-1/3} a_i^{-4} \rho_{\text{crit}} \Omega_{r0},
\]

(2.3)

and substitute it in eq. (2.1) to obtain the mass of the generated PBH as

\[
M_{\text{PBH}}(k) = \gamma \frac{4\pi}{3} \left( \frac{g_{*i}}{g_{*0}} \right)^{-1/3} a_i^{-4} \rho_{\text{crit}} \Omega_{r0} (\frac{k}{a_i})^{-3},
\]

(2.4)

Now we will determine the scale factor \( a_i \) at the time of PBH formation, as shown in ref. [44]. The Hubble rate of expansion at the time of PBH formation, when \( k^{th} \) fluctuation mode re-enters the horizon can be written as,

\[
H^2 = \left( \frac{k}{a_i} \right)^2 = \frac{8\pi G_N}{3} \rho_{ri}.
\]

(2.5)

By substituting the expression for the initial radiation energy density from eq. (2.3) into this, we get

\[
\left( \frac{k}{a_i} \right)^2 = \frac{8\pi G_N}{3} \rho_{\text{crit}} \left( \frac{g_{*i}}{g_{*0}} \right)^{-1/3} a_i^{-4} \Omega_{r0}
\]

(2.6)

which gives

\[
a_i^{-1} = \left( k^2 H_0^{-2} \left( \frac{g_{*i}}{g_{*0}} \right)^{1/3} \Omega_{r0}^{-1} \right)^{1/2}.
\]

(2.7)
Finally, we substitute eq. (2.7) in eq. (2.4) and obtain

\[
M_{\text{PBH}}(k) = \frac{4\pi}{3} \left( \frac{g_{*i}}{g_{*0}} \right)^{-1/3} k H_0^{-1} \left( \frac{g_{*i}}{g_{*0}} \right)^{1/6} \Omega_{r0}^{-1/2} \rho_{\text{crit}} \Omega_{r0} k^{-3}
\]

\[
= \frac{4\pi}{3} \rho_{\text{crit}} \left( \frac{g_{*i}}{g_{*0}} \right)^{-1/6} \Omega_{r0}^{1/2} k^{-2} H_0^{-1}. \tag{2.8}
\]

This relation implies that the mass of the generated PBH is proportional to the inverse square of the \(k^{th}\) mode of fluctuation that creates it, \(M_{\text{PBH}} \propto k^{-2}\). Therefore, more massive PBHs form when small \(k\) modes re-enter the horizon, whereas the lighter PBHs form when large \(k\) modes re-enter the horizon, with the amplitude of power spectrum large enough to generate them. As large \(k\) mode leaves the horizon late during inflation and re-enters in the horizon first, this implies that lighter PBHs form early in the radiation era, and the small \(k\) modes corresponding to the more massive PBHs enter late and form later in the radiation era.

Further, we can express eq. (2.8) in terms of the present horizon mass which is given as

\[
M_0 = \frac{4\pi}{3} \rho_{\text{crit}} H_0^{-3} \approx 4.62 \times 10^{22} M_\odot.
\]

This is as follows

\[
M_{\text{PBH}}(k) = \gamma M_0 \left( \frac{g_{*0}}{g_{*i}} \right)^{1/6} \Omega_{r0}^{1/2} \left( \frac{H_0}{k} \right)^2 \tag{2.9}
\]

\[
\approx 5 \times 10^{15} g \left( \frac{g_{*0}}{g_{*i}} \right)^{1/6} \left( \frac{10^{15} \text{Mpc}^{-1}}{k} \right)^2. \tag{2.10}
\]

This relation implies that for an overdense fluctuation mode with \(k \sim 10^{15} \text{Mpc}^{-1}\), PBHs of mass \(M_{\text{PBH}} \sim 5 \times 10^{15} \text{g}\) are formed.

### 2.2 Initial mass fraction of primordial black holes

The present abundances of PBHs are constrained from various observations, which gives upper limits on their initial mass fraction defined as

\[
\beta(M_{\text{PBH}}) \equiv \frac{\rho_{\text{PBH},i}}{\rho_{\text{total},i}}. \tag{2.11}
\]

For any PBH of mass \(M_{\text{PBH}}\), the initial mass fraction is the ratio of its energy density at the time of its formation, \(\rho_{\text{PBH},i}\) to the total energy density of the Universe at that epoch, \(\rho_{\text{total},i}\). The initial mass fraction of PBHs is a mass dependent quantity.

As the PBH formation is assumed to take place in the radiation dominated era, the total energy density at that epoch is in the radiation, i.e. \(\rho_{\text{total},i} = \rho_{ri}\), given in eq. (2.3), while the energy density of PBHs evolve as \(\rho_{\text{PBH},i} = \rho_{\text{PBH,0}} a_i^{-3}\). Thus we can write eq. (2.11) as

\[
\beta(M_{\text{PBH}}) = \frac{\Omega_{\text{PBH0}}(M_{\text{PBH}})}{\Omega_{r0}} \left( \frac{g_{*i}}{g_{*0}} \right)^{1/3} a_i, \tag{2.12}
\]

where \(\Omega_{\text{PBH0}}(M_{\text{PBH}}) = \rho_{\text{PBH0}}/\rho_{\text{crit}}\) is the density parameter for PBH of mass \(M_{\text{PBH}}\). On substituting eq. (2.7) in eq. (2.12) and then using eq. (2.10), we obtain

\[
\beta(M_{\text{PBH}}) = \frac{\Omega_{\text{PBH0}}(M_{\text{PBH}})}{\Omega_{r0}^{3/4}} \left( \frac{g_{*i}}{g_{*0}} \right)^{1/4} \left( \frac{M_{\text{PBH}}}{M_0} \right)^{1/2} \gamma^{-1/2}. \tag{2.13}
\]
With this relation, the observational constraints on $\Omega_{\text{PBH}}$ for a PBH of mass $M_{\text{PBH}}$, can be used to calculate the upper bound on its initial mass fraction $\beta(M_{\text{PBH}})$ (see refs. [21, 24, 25]). The number of relativistic degrees of freedom (considering all 3 neutrinos as massless) at the present, $g_\text{0} = 3.36$, and $g_{\ast i}$ denotes at the time of PBH formation in the radiation dominated era. For our supersymmetric model of warm inflation discussed in section 4, we take $g_{\ast i} \sim 200$. The bounds on $\beta(M_{\text{PBH}})$ can then be further used to constraint the amplitude of primordial power spectrum, as will be shown in the next subsection.

2.3 PBH formation using the Press-Schechter formalism

Now we discuss the Press-Schechter theory for the formation of a primordial black hole and then show how the primordial power spectrum can be constrained by the PBH abundance calculated using this formalism. We consider that the PBH forms by the collapse of a density perturbation generated during inflation. We assume that the initial gaussian seeds of density perturbations re-enter the horizon during the radiation dominated epoch and the PBH formation takes place in the regions with overdensity above a critical value, $\delta > \delta_c$ where $\delta_c \sim \mathcal{O}(1)$ [4] (see refs. [61, 62] for more details). On smoothening the density perturbations using a Gaussian window function, the probability distribution for a smoothed density contrast over a radius $R = (aH)^{-1}$ is given as [63],

$$p(\delta(R)) = \frac{1}{\sqrt{2\pi}\sigma(R)} \exp \left( -\frac{\delta^2(R)}{2\sigma^2(R)} \right).$$  \hspace{1cm} (2.14)

Here $\sigma(R)$ is the mass variance evaluated at the horizon crossing, and is defined as,

$$\sigma^2(R) = \int_{0}^{\infty} \tilde{W}^2(kR)P_\delta(k)\frac{dk}{k}$$  \hspace{1cm} (2.15)

where $P_\delta(k)$ is the matter power spectrum, and $\tilde{W}(kR)$ is the Fourier transform of the window function

$$\tilde{W}(kR) = \exp(-k^2R^2/2).$$  \hspace{1cm} (2.16)

The primordial curvature power spectrum $P_R(k)$ for the fluctuations generated during the inflation can be related to the density power spectrum $P_\delta(k)$ as [64],

$$P_\delta(k) = \frac{4(1+w)^2}{(5+3w)^2} \left( \frac{k}{aH} \right)^4 P_R(k),$$  \hspace{1cm} (2.17)

where $w$ is the equation of state of the fluid and is equal to 1/3 for a radiation dominated era.

Using the Press-Schechter theory, the initial mass fraction of a PBH with mass $M_{\text{PBH}}$ is obtained as [65],

$$\beta(M_{\text{PBH}}) = 2 \int_{\delta_c}^{1} p(\delta(R))d\delta(R) = \frac{2}{\sqrt{2\pi}\sigma(R)} \int_{\delta_c}^{1} \exp \left( -\frac{\delta^2(R)}{2\sigma^2(R)} \right) d\delta(R)$$  \hspace{1cm} (2.18)

$$= \text{erfc} \left( \frac{\delta_c}{\sqrt{2}\sigma(R)} \right)$$  \hspace{1cm} (2.19)

where erfc is the complimentary error function, and we consider $\delta_c = 0.5$ in this study. For any parameterization of the primordial power spectrum, we carry out the integration in eq. (2.15) by using eq. (2.17), and then substitute the obtained mass variance into eq. (2.19). The
expression thus obtained for $\beta(M_{\text{PBH}})$ using Press-Schechter theory is equated to eq. (2.13), and constrained using the observational bounds on $\Omega_{\text{PBH}}(M_{\text{PBH}})$, as argued in the previous subsection. In this way, the primordial power spectrum, and hence inflationary models are constrained from the bounds on abundance of PBHs [22–25]. For the various mass of PBHs, the upper bound on the amplitude of the primordial power spectrum is obtained to be, $P_R(k_{\text{PBH}}) \sim \mathcal{O}(10^{-2} - 10^{-1})$ [11, 12].

In the literature, there are a lot of studies which discuss the PBH production from the collapse of large inhomogeneities generated from various inflationary scenarios. Some examples are the hybrid inflation models [66–68], running-mass inflation models [69–72], hilltop inflation model [73], inflating curvaton model [74], axion curvaton inflation model [75, 76], double inflation model [77, 78], thermal inflation [79], single field inflation with a broken scale invariance [80], or by introducing a inflection point (plateau) in the potential [30, 81], running of the spectral index [82, 83], etc. It is shown in ref. [84] that for a power-law form of the primordial power spectrum, $P_R(k) = A_s(k/k_P)^{n_s-1}$, the spectral index has to be blue-tilted ($n_s > 1$) at the small scales for a significant formation of PBHs. But from the CMB observations, the spectral index of the power spectrum is precisely measured to be red-tilted ($n_s < 1$) at the large scales. If the running of the spectral index, $\alpha_s$, and running of the running, $\beta_s$, are also considered in the primordial power spectrum, $P_R(k) = A_s(k/k_P)^{n_s(k)-1}$, then the amplitude of the power spectrum can become $P_R(k) \sim \mathcal{O}(10^{-2})$ for some values of $n_s$, $\alpha_s$, $\beta_s$ allowed from the CMB observations. However, ref. [85] shows that such a Taylor series expansion of $n_s(k)$ in the parameterization of $P_R(k)$ can lead to large errors in the amplitude of primordial power spectrum at the small scales and hence PBH formation.

In this study, we consider a primordial power spectrum in the context of warm inflation, which is dominantly sourced by the thermal fluctuations of the fields. It is characterized in terms of the inflaton self-coupling, and a dissipation parameter, which is a measure of inflaton couplings to the other fields. In the next section, we first describe the fundamentals of warm inflation and the primordial power spectrum, and afterwards discuss the PBHs generated from our warm inflation model.

3 Theory of warm inflation

In the warm inflation description, one accounts for the inflaton couplings and dissipation to the other fields during inflation, unlike in cold inflation where they are neglected. As a result, the Universe constitutes a thermal bath of particles and has a temperature throughout the inflation. The equation of motion of the inflaton field $\phi$ rolling down a potential $V(\phi)$ has an additional dissipation term $\Upsilon \dot{\phi}$ arising because of the inflaton coupling to other fields and is given as

$$\ddot{\phi} + (3H + \Upsilon)\dot{\phi} = -V'(\phi).$$

(3.1)

Here $H$ is the Hubble expansion rate and overdot and $'$ denote the derivative w.r.t. time and $\phi$, respectively. $\Upsilon$ is called the dissipation coefficient, and it depends on the inflaton field $\phi$ and temperature of the Universe $T$. Due to the inflaton dissipation, there is an energy transfer from the inflaton into the radiation as given by

$$\dot{\rho}_r + 4H \rho_r = \Upsilon \dot{\phi}^2.$$

(3.2)

We define a dissipation parameter,

$$Q \equiv \frac{\Upsilon}{3H}.$$
which is the ratio of the relative strength of inflaton dissipation, compared to the Hubble expansion rate, and rewrite eq. (3.1) as

$$
\ddot{\phi} + 3H(1 + Q)\dot{\phi} + V'(\phi) = 0.
$$

(3.3)

When the dissipation parameter $Q \ll 1$, it is the weak dissipative regime of warm inflation, and when $Q \gg 1$, it is the strong dissipation regime. For our model of warm inflation, the dissipation parameter evolves during inflation. In our study, we consider that at the pivot scale ($k_P = 0.05$ Mpc$^{-1}$), it is the weak dissipation regime of warm inflation. But as inflation proceeds, $Q$ increases and we obtain the strong dissipative regime of warm inflation by the end of inflation.

The microphysics description of warm inflation is described by non-equilibrium field theory [86], based on which a field theoretical model was constructed in ref. [87] for studying the strong dissipative regime of warm inflation. However, it was indicated in [87] and also pointed out in [88] that it is difficult to obtain a successful strong dissipative regime of warm inflation. The problem was that in the high temperature limit taken in ref. [87], the thermal corrections to the effective potential become large, due to which the shape of the potential no longer remains flat and thus inflation ends quickly without achieving a sufficient number of e-folds of expansion [88]. Therefore, subsequent studies considered new models, such as supersymmetric distributed mass model in the context of string theory [54], or a two-stage decay mechanism of inflaton, where the inflaton couples to a heavy intermediate catalyst field which then further couple to the light radiation fields [89–92], or recently discrete interchange symmetry in the warm little inflaton model [93, 94] to control these corrections [95], and attain a strong dissipation regime of warm inflation.

In warm inflation, the primordial power spectrum has dominant contributions from the thermal fluctuations of the fields and is given in refs. [96–99] as

$$
P_R(k) = \left( \frac{H_k^2}{2\pi\phi_k} \right)^2 \left[ 1 + 2n_k + \left( \frac{T_k}{H_k} \right) \frac{2\sqrt{3\pi}Q_k}{\sqrt{3 + 4\pi Q_k}} \right] G(Q_k),
$$

(3.4)

where the subscript $k$ represents the time when the $k^{th}$ mode of cosmological perturbations leaves the horizon during inflation. By the fluctuation-dissipation theorem, the dissipation in the system sources the thermal fluctuations, which modifies the Langevin equation for the inflaton fluctuations. As a result, the primordial power spectrum gets augmented with an additional term (third in the square bracket) due to the thermal contributions. Furthermore, the thermal bath of radiation also excites the inflaton from its vacuum state to a Bose-Einstein distribution $n_k$, given by

$$
n_k = \frac{1}{\exp\left(\frac{k/a_k}{T_k}\right) - 1},
$$

which alters the primordial power spectrum. Additionally, as the dissipation coefficient $\Upsilon$ depends on the temperature, therefore the perturbations in the radiation can also couple to the inflaton perturbations and lead to a growth in the primordial power spectrum [97]. This growth factor $G(Q_k)$ depends on the form of $\Upsilon$ and is obtained numerically, as shown in ref. [97]. For a dissipation coefficient with a cubic dependence on temperature $\Upsilon \propto T^3$, the growth factor [100]

$$
G(Q_k)_{\text{cubic}} = 1 + 4.981 Q_k^{1.946} + 0.127 Q_k^{4.330}.
$$

(3.5)

In the weak dissipation regime for small $Q$, the growth factor does not enhance the power spectrum much. But for $Q \gg 1$ in the strong dissipation regime, the power spectrum is
hugely enhanced due to the growth factor. Further, if the produced radiation is an imperfect fluid, then there is also another damping factor in the power spectrum due to the viscosity in the fluid, as calculated in ref. [101]. For a weak self-coupling of the radiation, the shear damping can even counter balance the growth in the power spectrum, and only for a large self-coupling in the radiation, there is a net growth in the power spectrum. However, in this study, we do not consider any viscosity in the radiation and the associated damping factor.

4 Warm inflation model and formation of PBHs

Now we discuss the formation of PBHs from our model of warm inflation. We consider a supersymmetric model of warm inflation with a two stage decay mechanism, where the inflaton couples to intermediate $X$ superfields, which decay into $Y$ superfields, which thermalise and form a radiation bath. In this study, we consider a quartic potential of warm inflation ($V(\phi) = \lambda \phi^4$) with a dissipation coefficient given by [102],

$$\Upsilon(\phi, T) = C_\phi \frac{T^3}{\phi^2}. \tag{4.1}$$

This kind of dissipation term emerges in the low temperature regime of warm inflation, when the inflaton couples to the heavy ($m_X \gg T$) bosonic components of the $X$ fields [102–104]. The constant $C_\phi$ depends on the couplings and the multiplicities of $X$ and $Y$ as $C_\phi = \frac{1}{4} \alpha N_X$, where $\alpha = \frac{\hbar^2 N_Y}{4\pi} < 1$, $\hbar$ is the coupling between $X$ and $Y$, and $N_{X,Y}$ are the multiplicities of the $X$ and $Y$ fields [99, 104].

The motivation for considering this warm inflationary model is that it is the simplest renormalizable potential which is consistent with the CMB observations for some parameter space of the model parameters. Also, the tensor-to-scalar ratio prediction for this model is within the sensitivity of the next generation CMB polarisation experiments and therefore can be tested in the near future. Furthermore, we shall see that the primordial power spectrum for this model of warm inflation has features required for PBH generation. These features arise due to intrinsic properties of the inflaton-radiation system and therefore are interesting to study.

In our earlier work in ref. [105], we parameterized the primordial power spectrum for this model in terms of two model parameters - the inflaton self coupling, $\lambda$, and the dissipation parameter at the pivot scale, $Q_P$, and estimated them using large scale CMB observations. By using the same parameterization, we study the formation of small scale PBHs for our warm inflationary model considering different values of $Q_P$ in this work. For each $Q_P$ value, we consider $\lambda$ such that the primordial power spectrum is normalised at the pivot scale as $P_R(k_P) = 2.1 \times 10^{-9}$.

5 Analysis and discussion

In this section, we analyze the results of our study about the formation of PBHs from warm inflation model. Firstly, we plot the primordial power spectrum for our warm inflation model as a function of $k$, following the parameterization in our previous study [105]. In order to study the role of the dissipation parameter, $Q_P$ on the formation of PBHs, we consider different values of $Q_P$ to plot the primordial power spectrum. Then we discuss the range of $Q_P$ values allowed from the $n_s$ observations. Further, we discuss the effects of higher dissipation on the initial mass fraction and the mass of the generated PBHs. Finally, we
remark on the implications of the calculated initial mass fraction for tiny PBHs formed from our warm inflation model, and the possibility of PBH relics as dark matter.

5.1 Primordial power spectrum for our warm inflation model

We first plot the primordial curvature power spectrum as a function of comoving wavenumber $k$ in figure 1. We consider that the number of e-folds when the pivot scale ($k_P = 0.05$ Mpc$^{-1}$) leaves the horizon, $N_P = 60$. (In our notation, at the end of inflation, $N_e = 0$.) As already mentioned, PBHs provide a probe for a huge range of small scale modes. Here we consider only those $k$ modes that leave the horizon near the end of inflation, and can form PBHs when they re-enter in the radiation era.

In order to produce a significant number of PBHs, the amplitude of $P_{\mathcal{R}}(k)$ needs to be $O(10^{-2})$. We consider various cases of inflation with different values of the dissipation parameter at the pivot scale, $Q_P = 10^{-1.0}, 10^{-1.5}, 10^{-2.0},$ and $10^{-2.5}$ (weak dissipation regime when the pivot scale exits the horizon) to plot figure 1. As can be seen, for some $Q_P$ values, a large amplitude of $P_{\mathcal{R}}(k)$ is achieved near the end of inflation at $k \sim 10^{21}$ Mpc$^{-1}$. These small scale modes leave the horizon when inflation is near its end, and then re-enter in the horizon during radiation dominated era. When they re-enter, these overdense perturbations collapse to form the primordial black holes, as discussed in section 2. For the strong dissipation regime as well, the amplitude of $P_{\mathcal{R}}(k)$ would be $O(10^{-2})$ and higher, but those cases are not of interest, for the reason we discuss later.

We also plot the power law power spectrum parameterization, considered in cold inflation (without running of $n_s$) (black line) in figure 1 for comparison. It can be seen that for a power law power spectrum, the amplitude of $P_{\mathcal{R}}(k)$ can never reach the large value $O(10^{-2})$, as the spectrum is red-tilted ($n_s < 1$), and therefore no PBHs can be produced for such a form of $P_{\mathcal{R}}(k)$. But for our model of warm inflation, we find that the power spectrum changes to blue-tilted ($n_s > 1$) at the PBH scales and therefore PBH formation can take place for some range of model parameters.

Now we discuss the effects of the inflaton dissipation during warm inflation to the primordial power spectrum. It can be seen from figure 1 that at the PBH scales (large $k$),
Figure 2. Plot of the spectral index at the pivot scale, $n_s$ versus $\log_{10} Q_P$. The colored band signifies the results of CMB observations from Planck 2018 for the allowed range of $n_s$ within 68% and 95% C.L.

for a large dissipation parameter $Q_P$, the amplitude of the primordial power spectrum $P_R(k)$ is larger as compared to the small dissipation case. This implies that for a large $Q_P$, the amplitude of $P_R(k)$ is enhanced to $\mathcal{O}(10^{-2})$ at a comparatively smaller $k$, and all the larger $k$ modes leaving the horizon further are sufficiently overdense to form PBHs. From the plot, it is also seen that for $Q_P < 10^{-2}$, the amplitude of primordial power spectrum is not sufficient to generate PBHs. Therefore, we limit our study of PBH formation till $Q_P = 10^{-2}$.

5.2 Relevant range of the dissipation parameter consistent with CMB

Next, we plot the spectral index of the primordial power spectrum at the pivot scale, $n_s$ for the different values of the dissipation parameter, $Q_P$ in figure 2. For reference, we also plot the allowed range of $n_s$ values in the 68% and 95% C.L. from the recent Planck 2018 results for TT,TE,EE + lowE dataset as given in ref. [106],

\[
  n_s = 0.9649 \pm 0.0044 \quad (68\% \, \text{C.L.}), \quad \text{and} \quad \quad n_s = 0.9649 \pm 0.0044 \quad (95\% \, \text{C.L.}).
\]  

(5.1)

From figure 2, it can be seen that only a small range of $Q_P$ values in the weak dissipation regime are consistent with the CMB observations. Therefore, we do not consider the cases with $Q_P > 10^{-1.7}$, despite the fact that the amplitude of the primordial power spectrum at PBH scales, $P_R(k_{PBH}) \sim \mathcal{O}(10^{-2})$, which is sufficient to form PBHs.

5.3 Initial mass fraction and mass of the PBHs formed

Now, we are equipped with the knowledge that for a certain range of $Q_P$ values, the amplitude of the primordial power spectrum at the PBH scales, $P_R(k_{PBH}) \sim \mathcal{O}(10^{-2})$, is sufficient enough to generate PBHs. For each scenario of inflation, represented by the different values of $Q_P$, we first calculate the mass variance by substituting the warm inflation power spectrum from eqs. (3.4), and (2.17) in eq. (2.15). The mass variance, $\sigma^2(R)$ is obtained as a function of $R = (aH)^{-1}k^{-1}$ at the horizon crossing, which can be then related to $M_{PBH}$ through eq. (2.10). After that, we substitute the obtained mass variance in eq. (2.19) and numerically calculate the initial mass fraction for the PBHs as a function of its mass, $\beta(M)$. 


Finally, we plot the obtained initial mass fraction $\beta(M)$ of the generated PBH versus the mass of the PBH in figure 3 for the cases when $Q_P = 10^{-1.7}, 10^{-1.8}, 10^{-1.9},$ and $10^{-2.0}$. As shown in figure 1, for $Q_P < 10^{-2.0}$, the amplitude of $P_R(k)$ is not sufficient to form PBHs, and hence is not considered in this study.

From figure 3, we can see that the mass of PBHs generated from our warm inflation model is of the order $M_{PBH} \sim 10^3$ g. From the plots, we infer that a large dissipation during inflation leads to a comparatively more massive PBH formation, whereas small dissipation produces small mass PBHs. The reason for this is that, as shown in figure 1, for a larger dissipation, the desired amplitude of the primordial power spectrum $P_R(k_{PBH}) \sim \mathcal{O}(10^{-2})$, is achieved at a comparatively smaller $k$. As the mass of the generated PBH is proportional to $k^{-2}$, this implies that PBHs formed by small $k$ overdense modes are more massive, compared to the large $k$ overdense modes.

5.4 Constraints on the abundance of PBHs formed

As the range of $Q_P$ values relevant for PBH study is very small, there is a very narrow range of $k$ modes available for the PBH generation, and consequently a narrow range of mass of the generated PBHs. The order of mass of the PBH formed from our warm inflation model, $M_{PBH} \sim 10^3$ g. Such tiny mass PBHs have an extremely short lifetime $\sim 10^{-19}$ sec and would have evaporated by now into Hawking radiation [13, 14]. Interestingly, there are other studies also which discuss the formalism to stabilize the light extremal PBHs against evaporation so that it comprises the present dark matter density [107]. The abundance of tiny mass PBHs is not strictly bounded, however, depend on the physics beyond the Standard Model of particle physics.

There are certain bounds from the PBH evaporation leading to the generation of stable massive particles (Supersymmetric LSP) [18] or long lived decaying particles (eg. gravitino, modulii) [19, 20], and their relic abundance, which can be used to put constraints on PBH initial abundance [11, 21, 25]. Stable supersymmetric particles may contribute as the present dark matter, therefore, their abundance should be constrained so that they do not overclose the Universe. For a PBH with mass $10^3$ g evaporating into LSP of mass $m_{LSP} = 100$ GeV, the upper bound on the abundance $\beta(M_{PBH}) < 10^{-14}$ [21, 25]. For various scenarios (different
of our warm inflation model, we find that the calculated initial abundance of PBHs is in accordance with this limit for $Q_P = 10^{-1.8}, 10^{-1.9}, \text{and } 10^{-2.0}$. But for the case with $Q_P = 10^{-1.7}$, the theoretical estimate of the initial mass fraction is higher than the above mentioned constraint, which implies that this case is inconsistent with the PBH bounds, and should be ruled out.

Additionally, PBH evaporation can lead to a large abundance of gravitinos and modulii which has important cosmological consequences. These quasi stable massive particles decay into energetic particles, which destroy the light elements nuclei created in the era of nucleosynthesis. Thus, the abundance of PBHs evaporating into such species should be controlled so that these problems can be avoided. To an order of magnitude, for $M_{\text{PBH}} \sim 10^3$ g, the upper bound on $\beta(M_{\text{PBH}}) < 10^{-16}$ \cite{21, 25}. This is a more stronger bound than from the LSP. We find that for our warm inflation model, the calculated initial abundance of PBHs is in accordance with the observational limit for $Q_P = 10^{-1.9}$ and $10^{-2.0}$. But for the cases with $Q_P = 10^{-1.7}$ and $10^{-1.8}$, the theoretical estimate is higher than the upper limit, which implies that these values of the dissipation parameter should be ruled out for the model of warm inflation studied here.

### 5.5 PBH relics as constituent of dark matter

It is argued that Primordial Black Hole evaporation could leave a stable relic of Planck mass, which can contribute to the dark matter \cite{49, 50}. In order that the Planck mass relics do not overclose the Universe today, the present density of Planck mass relics should be less than the present cold dark matter density. To an order of magnitude, the constraints on the initial mass fraction of PBH of mass $M_{\text{PBH}} \sim 10^3$ g, as calculated in ref. \cite{108} is given to be $\beta(10^3 g) < 10^{-16}$. (Also see, refs. \cite{9, 21, 25}.) We find that in our warm inflation models with $Q_P = 10^{-1.9}$, and $Q_P = 10^{-2.0}$, the initial mass fraction lies within the above estimated limits, and therefore the possibility that PBH remnants form DM, remains valid for these cases. But as the Planck mass relics are very small, it is extremely difficult or nearly impossible to observationally detect them non-gravitationally. However, in ref. \cite{109}, it is argued that if such relics carry electric charge, then they can be possibly explored in the direct detection experiments in the near future. A detection of such events would be crucial to deepen our understanding about the dark matter and black holes physics.

### 6 Summary

Primordial Black Holes are a remarkable probe to the physics of the early Universe. They provide us an opportunity to investigate a huge range of small scale perturbations generated during the inflationary phase. In this study, we consider one model of warm inflation scenario, and discuss the PBH formation by the collapse of large inhomogeneties generated during it. Warm inflation is a description of inflation in which the radiation production takes place concurrent to the inflationary phase. The inflaton couples and dissipates into other fields during inflation, which creates a thermal bath of particles during inflation. The primordial power spectrum for warm inflation depends on by the inflaton dissipation parameter $Q_P$, which grows to large values near the end of inflation, and thus enhances the amplitude of the primordial power spectrum at the small scales to $\mathcal{O}(10^{-2})$, required to generate PBHs.

We find that for some parameter range of our model, PBHs can be generated with a significant abundance. We consider those cases with values of the dissipation parameter at the pivot scale as, $Q_P = 10^{-1.7}, 10^{-1.8}, 10^{-1.9}, 10^{-2.0}$. We calculate the initial mass fraction
and the mass of the generated PBHs for these values of the dissipation parameter. We obtain that our model of warm inflation can produce a significant abundance of PBHs with mass, $M_{\text{PBH}} \sim 10^3$ g. Such tiny mass PBHs have a very short lifetime of $10^{-19}$ sec, and would have evaporated into Hawking radiation. Our analysis shows that for the cases with $Q_P = 10^{-1.9}, 10^{-2.0}$, the obtained initial mass fraction is in accordance with the upper limit obtained from the abundance of stable and long lived decaying particles produced by evaporating PBHs. But the cases with $Q_P = 10^{-1.7}, 10^{-1.8}$ overproduce PBHs, which is inconsistent with the upper bounds on $\beta$, and hence should be ruled out.

Furthermore, it is also argued that PBH evaporation ceases when PBH mass gets close to the Planck mass, and such Planck mass relics can thus constitute the present dark matter. The present density of the Planck mass relics should be less than the cold dark matter density, so that it does not overclose the Universe today. This gives a rough bound on the PBH initial mass fraction for a PBH of mass $10^3$ g of an order, $\beta(10^3 g) < 10^{-16}$. For our warm inflation models with $Q_P = 10^{-1.9}$, and $Q_P = 10^{-2.0}$, we find that the calculated initial mass fraction lies within the limits, and hence the possibility to form DM remains valid. The Planck mass relics are extremely tiny and almost impossible to detect by non-gravitational measures. But if they carry electric charge, then they may be possibly detected in the dark matter direct detection experiments, which will have a lot of implications for the black hole physics and dark matter.

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