GAMMA-RAY BURSTS FROM MAGNETIZED COLLISIONALLY HEATED JETS

INDREK VURM1,2,3, ANDREI M. BELOBORODOV4,5, AND JURI POUTANEN3

1 Racah Institute of Physics, Hebrew University of Jerusalem, Jerusalem 91904, Israel; indrek@phys.huji.ac.il
2 Tartu Observatory, Tõravere 61602, Tartumaa, Estonia
3 Astronomy Division, Department of Physics, P.O. Box 3000, 90014 University of Oulu, Finland; juri.poutanen@oulu.fi
4 Physics Department and Columbia Astrophysics Laboratory, Columbia University, 538 West 120th Street New York, NY 10027, USA; amb@phys.columbia.edu
5 Astro-Space Center of Lebedev Physical Institute, Profsojuznaja 84/32, Moscow 117810, Russia

Received 2011 March 30; accepted 2011 June 17; published 2011 August 12

ABSTRACT

Jets producing gamma-ray bursts (GRBs) are likely to carry a neutron component that drifts with respect to the proton component. The neutron–proton collisions strongly heat the jet and generate electron–positron pairs. We investigate radiation produced by this heating using a new numerical code. Our results confirm the recent claim that collisional heating generates the observed Band-type spectrum of GRBs. We extend the model to study the effects of magnetic fields on the emitted spectrum. We find that the spectrum peak remains near 1 MeV for the entire range of the magnetization parameter \( 0 < \epsilon_B < 2 \) that is explored in our simulations. The low-energy part of the spectrum softens with increasing \( \epsilon_B \), and a visible soft excess appears in the keV band. The high-energy part of the spectrum extends well above the GeV range and can contribute to the prompt emission observed by Fermi/LAT. Overall, the radiation spectrum created by the collisional mechanism appears to agree with observations, with no fine tuning of parameters.

Key words: gamma-ray burst: general – gamma rays: general – radiation mechanisms: non-thermal

Online-only material: color figures

1. INTRODUCTION

Gamma-ray bursts (GRBs) are produced by ultra-relativistic jets from short-lived and powerful energy sources, probably associated with black hole formation. Apart from the jet launching itself, a primary question of GRB theory concerns the emission mechanism of the burst: how does the jet emit the observed gamma-rays?

The jet must develop an ultra-relativistic speed if its initial thermal energy is much larger than its baryonic rest-mass energy. The thermal energy is dominated by radiation and can be released at the photospheric radius \( r_\star \), where the jet becomes transparent. For sufficiently clean (baryon-poor) thermal jets, transparency occurs when radiation still carries most of the energy. The thermal energy is dominated by radiation and can be released at the photospheric radius \( r_\star \), where the jet becomes transparent. For sufficiently clean (baryon-poor) thermal jets, transparency occurs when radiation still carries most of the energy. The thermal energy is dominated by radiation and can be released at the photospheric radius \( r_\star \), where the jet becomes transparent. For sufficiently clean (baryon-poor) thermal jets, transparency occurs when radiation still carries most of the energy. The thermal energy is dominated by radiation and can be released at the photospheric radius \( r_\star \), where the jet becomes transparent. For sufficiently clean (baryon-poor) thermal jets, transparency occurs when radiation still carries most of the energy. The thermal energy is dominated by radiation and can be released at the photospheric radius \( r_\star \), where the jet becomes transparent.

The observed emission indeed peaks near 1 MeV in most bursts, however its spectrum is nonthermal, with an extended tail of high-energy emission. This has two implications: (1) the GRB photosphere is rarely radiation-dominated, and (2) some form of dissipation operates in the GRB jets and greatly broadens their radiation spectra. The free energy available for dissipation may be kinetic or magnetic. In particular, the relatively small amount of energy in the rest of the jet can be dissipated (e.g., Rees & Meszaros 1994; Paczynski & Xu 1994). It leads to collisionless shocks as well as collisional dissipation.

Collisionless shocks are known as efficient accelerators of nonthermal particles in astrophysics. They offer a mechanism for generating nonthermal electrons in GRB jets, and their synchrotron emission may be associated with observed \( \gamma \)-rays. The idea became popular, however it faces difficulties. First, recent ab initio particle-in-cell simulations of collisionless shocks (e.g., Sironi & Spitkovsky 2009, 2011) do not support this scenario. Their results suggest that nonthermal electron acceleration by shocks is inefficient in GRB jets with expected transverse magnetic fields and the magnetization parameter \( \epsilon_B \approx 10^{-3} \). Second, the spectra of a large number of GRBs observed by the Compton Gamma Ray Observatory (CGRO)/BATSE and Fermi/GBM instruments are inconsistent with synchrotron emission, especially in the fast-cooling regime that is expected in the model (e.g., Preece et al. 2000). Some of the spectra show harder slopes than synchrotron emission can produce irrespective of the cooling regime. In addition, the synchrotron model does not predict the observed preferential position of the spectral peak, which is always in the MeV range for bright GRBs. A recent discussion of the internal-shock synchrotron model can be found in Daigne et al. (2011).

One can consider other phenomenological models of emission from the optically thin region of the jet at \( r > r_\star \). In particular, one can consider synchrotron self-Compton radiation from the jet (Stiem & Poutanen 2004; Vurm & Poutanen 2009). In this scenario, the optically thin plasma maintains a relativistic temperature, \( kT_e \gg m_e c^2 \), at which radiative cooling balances the heating, and keeps the jet expanding. While this model can produce hard spectra, it still does not explain the preferential position of the observed spectrum peaks. It also conflicts with the recently observed high-energy spectra of GRBs (e.g., Abdo et al. 2009b; Zhang et al. 2011).

A likely resolution of these problems is that the bulk of GRB luminosity is generated below the photosphere at \( r \lesssim r_\star \), rather than in the optically thick zone \( r > r_\star \). Then multiple Compton scattering participates in the spectrum formation and can naturally produce the observed spectra. Generally, any subphotospheric heating leads to Comptonization of the (initially thermal) photons, and then radiation released at the photosphere \( r_\star \) looks as nonthermal while the peak of its
2. PHYSICAL MODEL AND SIMULATION SETUP

The central engines of GRBs must be compact and hot objects, which are almost certainly neutron rich. The high density and temperature inevitably leads to $\beta$-equilibrium (Imshennik et al. 1967; Derishev et al. 1999; Beloborodov 2003). For all plausible parameters of the central engine, the equilibrium establishes a high fraction of free neutrons in the baryonic matter (this feature is seen, for example, in the accretion-disk model for GRBs, see Beloborodov 2008 for a review). Therefore, neutrons are generally expected in GRB jets.

The presence of neutrons creates perfect conditions for strong collisional heating. In any variable baryonic jet, the inter-penetrating neutron and proton components inevitably develop relative motions (see B10 and references therein). The two components acquire different Lorentz factors $\Gamma_n$ and $\Gamma_p$, and (rare) collisions between neutrons and protons dissipate enormous energy. The dissipation is strongest in those parts of the jet where $\Gamma/\Gamma_n$ is highest.

The variable jet consists of many causally disconnected shells. Different shells are accelerated to different Lorentz factors and emit different radiation, which is consistent with observed variability. In our simulations, we focus on the heating history and radiation of one shell. The shell is launched from the central source and accelerated at the expense of its internal energy until it enters the coasting matter-dominated phase of expansion. At radius $R_n$, the timescale for collisions between the neutron and proton components exceeds the expansion timescale of the jet, and neutrons start to migrate relative to protons, forming a compound flow with $\Gamma \neq 1$. At this point, strong collisional heating begins and the radiation spectrum starts deviating from a blackbody. Our simulation starts at $R_n$ and follows this evolution. We assume that $\Gamma$ and $\Gamma_n$ remain approximately constant until the end of the simulation (see B10 for discussion of this approximation). Note that migration of neutrons between different shells of a strongly variable jet can create compound flows with $\Gamma < \Gamma_n$.

$R_n$ may be defined as the radius where the neutron flow becomes “optically thin” to collisions with protons. The corresponding “optical depth” for this process at radius $r$ is given by

$$\tau_n = \frac{n_n \sigma_n r}{\Gamma_n} = \frac{L_n \sigma_n}{4\pi m_n c^3 \Gamma_n^3} = \frac{R_n}{r}.$$  

where $n_n$ is the comoving number density of the neutron flow, $L_n = 4\pi n_n c^3 r^2 \Gamma_n^2 \sigma_n$ is its kinetic luminosity (isotropic equivalent), and $\sigma_n \sim 3 \times 10^{-26} \text{cm}^2$ is the effective cross-section for nuclear collisions. Radius $R_n$ is defined so that $\tau_n(R_n) = 1$,

$$R_n = \frac{L_n \sigma_n}{4\pi m_n c^3 \Gamma_n^3}.$$  

We aim to calculate the evolution of electron/positron and photon distribution functions in the heated jet. The photon spectrum is controlled by electrons (and positrons) via Compton scattering and synchrotron emission. Therefore, the key ingredient of any GRB model is how energy is injected into electrons. For the model studied in this paper, the electron heating is unambiguously determined by the collisional processes as described below (see B10 for detailed discussion).

The electron heating comes from baryons, in two forms.

1. Nuclear $n$–$p$ collisions generate $e^+$ pairs whose energy in the comoving frame of the jet is comparable to the pion rest
mass, \( y m_e c^2 \sim m_\pi c^2 \approx 140 \text{ MeV} \). The injection rate of these energetic particles is proportional to the rate of nuclear collisions, \( \dot{n}_{\text{coll}} = c_0 \dot{n}_e n_e \Gamma \), where \( \Gamma \), the Lorentz factor and kinetic luminosity, is the relative Lorentz factor of collisions. The \( e^\pm \) injection rate is given by \( B10 \)

\[
\dot{n}_{\text{ej}}^\pm \approx \frac{1}{4} \Gamma_{\text{rel}} m_\pi c \dot{n}_{\text{coll}} \approx \frac{3}{8} \Gamma \tau_0 n \frac{t}{t_{\text{exp}}}.
\]

Here \( n \) is the proton number density in the plasma rest frame and \( t_{\text{exp}} \) is the expansion timescale of the plasma, measured in its rest frame. The \( e^\pm \) injection peaks near the Lorentz factor \( \gamma_0 \approx m_\pi / m_e \approx 300 \). We will approximate it by a Gaussian distribution centered at \( \gamma_0 = 300 \). These energetic particles experience rapid radiative cooling and join the thermal \( e^\pm \) population.

2. The \( e^\pm \) population receives energy from protons via Coulomb collisions. Note that protons have at least mildly relativistic random velocities—they are stirred by nuclear collisions. The dominant majority of \( e^\pm \) pairs are kept at a much lower temperature, because of Compton cooling. Coulomb collisions drain the energy from protons to \( e^\pm \) with the following rate (e.g., Ginzburg & Syrovatskii 1964):

\[
\dot{Q}_{\text{inj}} = \frac{3}{2} \ln \Lambda \frac{\sigma_T m_e n_e}{\beta_p} \approx 0.02 \tau \frac{n m_p c^2}{t_{\text{exp}}},
\]

where \( \ln \Lambda \) is the Coulomb logarithm, \( \beta_p \sim 1 \) is the random velocity of the protons stirred by nuclear collisions, \( n_e = n_- + n_+ \) is the number density of electrons and positrons, and \( \tau_T \) is a characteristic Thomson optical depth of the plasma, as seen by photons at radius \( r \). The pair density \( n_p \) is controlled by the process of pair injection (Equation 3), the subsequent pair-photon cascade in the radiation field, and \( e^\pm \) annihilation, all of which must be self-consistently calculated.

Equations (3) and (5) determine the heating history of the expanding plasma shell and eventually the radiation spectrum emitted by this shell. The shell itself is a part of the (variable) jet. It has the following parameters.

1. The Lorentz factor \( \Gamma \) and kinetic luminosity \( (\Gamma L) \) for the plasma component and \( \Gamma_n \) and \( L_n \) for the neutron component. Note that the characteristic radius \( R_n \) for a given shell is determined by its \( \Gamma_n \) and \( L_n \) according to Equation (2).

2. The temperature of the thermal radiation at \( R_n \), prior to the onset of collisional heating, \( T_{\text{BB}}(R_n) \). The corresponding luminosity is \( L_{\text{BB}} = (4/3) c a T_{\text{BB}}^4 \Gamma^4 \Gamma \approx 10^{44} \text{ erg s}^{-1} \) (here \( a \) is the radiation density constant).

3. Magnetization \( \varepsilon_B \). It is defined as the ratio of the magnetic energy density \( U_B = B^2 / 8\pi \) to the proper energy density of the plasma flow \( U = L / 4\pi r^2 \Gamma^2 \) (which includes rest-mass energy). Both \( U_B \) and \( U \) are measured in the comoving frame of the plasma. We assume that the magnetic field is advected from the central engine. Then the field must be transverse to the (radial) velocity and scale with radius as \( B \propto r^{-1} \) (assuming \( \Gamma \approx \text{const} \)). In this case \( \varepsilon_B = U_B / U \approx \text{const} \), i.e., it does not change with its radius.

3. METHOD OF CALCULATIONS

For our calculations we use a new version of the numerical code developed by VP09. The code is designed to model the coupled evolution of the heated outflow and the radiation it carries. The evolution is tracked by solving the time-dependent kinetic equations for the particles and photons. The included interactions are Compton scattering, cyclotron synchrotron emission and absorption, photon–photon pair production and annihilation, and Coulomb collisions. We use the exact cross-sections or rates for all these processes.

The original version of the code had one significant limitation: it used the “one-zone” or “leaking-box” approximation. This approximation pictures a uniform plasma cloud of an optical depth \( \tau_T \) with isotropic populations of particles and photons. It treats the loss of photons from the cloud using an escape probability instead of accurate calculations of radiation diffusion through the cloud (VP09).

The leaking-box picture is not good for GRB jets. In contrast to static sources, where radiation escapes the source on timescale \( \sim \tau_T R / c \), GRB radiation remains embedded in the relativistic jet at all radii of interest. It evolves with radius according to the radiative transfer equation, which is in serious conflict with the one-zone approximation. For example, the true angular distribution of radiation in the jet comoving frame is far from being isotropic, even in the subphotospheric region (B11). The isotropic approximation of the one-zone model becomes invalid when the optical depth \( \tau_T \) decreases below \( \sim 10 \). Furthermore, the large free path of photons near the photosphere leads to mixing of radiation emitted by different parts of the photospheric region with different Doppler shifts. This mixing has a significant effect on the local photon spectrum and the spectrum received by a distant observer.

For these reasons the simplified one-zone treatment has to be abandoned in favor of proper radiative transfer calculations. The kinetic equation for photons in VP09 is replaced by the transfer equation as described below.

3.1. Radiative Transfer

The equation of radiative transfer in an ultra-relativistic, matter-dominated outflow reads (B11)

\[
\frac{\partial I_v}{\partial \ln r} = (1 - \mu) \left( \frac{\partial I_v}{\partial \ln v} - 3 I_v \right) - \left( 1 - \mu^2 \right) \frac{\partial I_v}{\partial \mu} + \frac{r (j_\nu - k_\nu I_v)}{\Gamma (1 + \mu)},
\]

Here \( I_v \) is the specific intensity, \( v \) is the photon frequency, \( \mu = \cos \theta \) and \( \theta \) is the angle relative to the radial direction, and \( r \) is the distance from the central source. The emission and absorption terms \( j_\nu \) and \( k_\nu \) are defined in the usual fashion. The term \( j_\nu \) represents the total emission coming from the source, \( k_\nu = k_\nu + 2 \delta \) is the total absorption including the Doppler shift to the target frequency \( \nu_0 \) (in the frame of the target medium).

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absorption coefficients \( j_\nu \) and \( \kappa_\nu \) represent all the interactions of radiation with plasma as well as with radiation itself. All quantities (except \( r \) and \( \Gamma \)) are measured in the rest frame of the outflow.

Equation (7) and the formulation of the ultra-relativistic transfer problem are discussed in detail in B11. The problem simplifies because essentially all photons flow toward a larger \( r \), and hence only inner boundary conditions need to be specified. The radiative transfer then takes the form of an initial-value problem. One can think of it as the evolution of \( I_\nu \) with \( r \) or, equivalently, with the jet comoving time \( t \). Equation (7) is similar to the kinetic equation for the evolution of the photon distribution function (VP09), with additional terms due to the angular dependence.

The inner boundary condition for \( I_\nu \) is the blackbody radiation of a given temperature. We track the evolution of radiation with \( r \) consistently with the plasma evolution, which is described in Section 3.2. In the case of a passively cooling jet (no heating), the transfer equation reproduces the adiabatic cooling of photons in the subphotospheric region. Heating changes the state of the plasma and greatly affects the source function \( S_\nu \), which becomes more significant at high photon energies, comparable to \( m_\epsilon c^2 \) in the plasma frame. This approximation allows us to use the numerical tools for \( j_\nu \) and \( \kappa_\nu \) developed in VP09. A detailed discussion of the angle-averaged \( j_\nu \) and \( \kappa_\nu \) and their numerical treatment is found in that work.

Radiation that will be received by a distant observer is determined by \( I_\nu \) at a large enough radius, where photons stream almost freely at \( \Gamma(1+\mu) \) (note that this radius is much larger than \( R_\ast \)). Using \( 1-\Gamma^{-2} \approx 1 \), we define the Doppler factor as \( \mathcal{D} = \Gamma(1+\mu) \). The spectral distribution of the observed (isotropic equivalent) luminosity \( L_\gamma \) is given by the Lorentz transformation of \( I_\nu \) d\( \mu \) to the observer frame and integration over angles,

\[
L_\gamma(E) \equiv \frac{dL_\gamma}{dE} = \frac{8\pi^2}{\hbar} R^2 \int_1^{-1} \mathcal{D}(\mu) I_\nu(\mu) \, d\mu, \tag{8}
\]

where \( E = \mathcal{D} h \nu \) is the observed photon energy and \( \hbar \) is the Planck constant. We emphasize that the simple relation (8) is valid only at large radii, after all transfer effects on \( I_\nu \) have already been calculated according to Equation (7).

Finally, we note that Equation (7) was formally derived for steady outflows, however it is also applicable to strongly variable, non-uniform jets (see B11 and the Appendix). The basic reason for this is that radiation moves together with the ultra-relativistic plasma flow. The photon diffusion in the plasma rest frame is limited to scales \( \delta \sim ct \sim r/\Gamma \), which are much smaller than \( r \). When viewed in the fixed observer frame, the scale of diffusion in the radial direction is additionally compressed by the factor of \( \Gamma^{-1} \). The transfer occurs in a thin shell (or “pancake”) of the jet as if it were part of a steady outflow.

### 3.2. Kinetic Equation for Plasma

The comoving-frame kinetic equation for electrons and positrons can be written as

\[
\frac{\partial n_\pm(p)}{\partial t} = -\frac{\partial}{\partial p} \left[ \frac{\partial n_\pm(p)}{\partial \nu} \frac{1}{2} \frac{\partial}{\partial \nu} (Dn_\pm(p)) \right] + j_\pm - \kappa_\pm n_\pm(p) - \frac{2}{\Gamma} n_\pm(p). \tag{9}
\]

Here \( p = \sqrt{\gamma^2 - 1} \) is the electron/positron momentum in units of \( m_e c \), \( n_\pm(p) \) are the distribution functions of positrons and electrons, and \( t = r/c \Gamma \) is the proper time of the flow. When calculating \( \dot{p} \) we take into account Coulomb heating by protons (Equation 5) and adiabatic cooling (Equation 10 below). The term \( \dot{p} \) also includes the usual contributions from Compton scattering (in the Thomson regime), synchrotron emission and self-absorption, and Coulomb collisions between \( e^\pm \), as described in VP09. The diffusion coefficient \( D \) governs particle thermalization at low energies due to synchrotron processes and Coulomb collisions between \( e^\pm \) (VP09). The source \( j_\pm \) includes the injection of high-energy pairs by inelastic nuclear collisions as described in Section 2. The source and sink terms \( j_\pm \) and \( \kappa_\pm \) also contain contributions from photon–photon pair production, pair annihilation, and Compton scattering (in the Klein–Nishina regime). Our code calculates all these processes using the exact cross-sections, as described in detail in VP09. Finally, the last term in Equation (9) takes into account the dilution of particle densities due to the two-dimensional (sideways) expansion of the coasting flow.

The contribution of adiabatic cooling to \( \dot{p} \) is given by

\[
\dot{p}_{ad} = -\frac{2}{3} \frac{p}{\Gamma}. \tag{10}
\]

This is easy to verify by considering an adiabatic expansion of \( N_e \) monoenergetic electrons occupying a volume \( V \). Their pressure \( P_e = (N_e/V)(p^2/3\gamma)m_e c^2 \) determines the adiabatic cooling rate \( \dot{N}_e m_e c^2 d\gamma = -P_e dV \). The coasting outflow expands in two dimensions, \( V \propto r^2 \propto c^2 t^2 \). Using \( dp = (\gamma/p) d\gamma \), one gets Equation (10).

In our numerical simulations, the electron/positron kinetic equation (9) is discretized on a grid of particle momenta and solved simultaneously with the radiative transfer equation (7). The grid of dimensionless momentum \( p \) extends from \( 10^{-4} \) to \( 10^4 \), with 25 points per decade. The photon-energy grid spans 15 decades from \( h\nu/m_e c^2 = 10^{-11} \) to \( 10^9 \) (comoving frame), with approximately 13 grid points per decade. The lower boundary is set so to properly simulate the effects of synchrotron self-absorption in mildly relativistic pairs in a weak magnetic field. The photon angular grid is uniform in \( \theta \) and has 40 grid points.

### 4. Non-Magnetized Outflows

We begin with the non-magnetized model that can be directly compared with the results in B10. The model has the following parameters: proton flow luminosity \( L = 10^{52} \text{ erg s}^{-1} \), neutron flow luminosity \( L_n = 2 \times 10^{51} \text{ erg s}^{-1} \), Lorentz factor of the proton flow (baryon loading) \( \Gamma = 600 \), Lorentz factor of the neutron flow \( \Gamma_n = 100 \), and initial radius of the flow \( r_0 = 10^9 \text{ cm} \). The starting radius of the simulations is determined by Equation (2), which gives \( R_\ast = 10^{11} \text{ cm} \). The comoving temperature of the blackbody radiation field at \( R_\ast \) is found from the passively cooling outflow model at \( r < R_\ast \), which gives \( kT_{BB}(R_\ast) \approx 0.5 \text{ keV} \).
distribution that would be obtained if the injected
is normalized so that it shows the optical depth
τ
and were Compton-cooled in the Thomson regime.

4.1. Electron/Positron Distribution

At the radius \( R_n \), where the dissipation begins, the injected high-energy pairs start to upscatter the thermal photons to energies up to \( \sim 100 m_e c^2 \) in the rest frame of the jet. The optical depth for photon–photon pair production seen by the high-energy photons is initially small, as the target photons are confined to the blackbody component that is below the threshold for the reaction. However, this soon changes as more and more photons are upscattered from the thermal distribution. Then the jet becomes optically thick to pair production and a cascade develops. Soon the annihilation balance is established, in which the cooled \( e^\pm \) pairs annihilate at the same rate as the new pairs are created by the cascade.

The continual cascade quickly reaches a steady state, on a timescale shorter than the jet expansion timescale. Then a quasi-steady \( e^\pm \) distribution is maintained, which gradually changes with radius (Figure 1). The distribution has two parts, thermal and nonthermal (cf. B10). The nonthermal tail consists of continually injected, fast-cooling \( e^\pm \) pairs. If the injected \( e^\pm \) generated no cascade and cooled in Thomson regime at all \( p \), the nonthermal tail would have a power-law form \( n_\pm \propto p^{-2} \).

The actual distribution is affected by the Klein–Nishina effects in Compton cooling (which are significant at \( p_\gamma > 1 \)). The Thomson energy injection has the same \( p^{-1} \) dependence. Thus, the ratio of thermal and nonthermal heating rates remains nearly constant along the flow. This ratio is close to unity. Most of the kinetic energy of the flow is dissipated (and re-dissipated) by collisional heating. One should keep in mind though that the heat given to plasma and radiation in the subphotospheric region is continually degraded by adiabatic cooling. As a result, the actual luminosity released at the photosphere is smaller by a factor of 1/2 compared with the total heat deposited in the flow (see Section 4.6 in B10).

4.2. Radiation Spectrum

The spectrum that should be received by a distant observer from the collisionally heated jet is shown in Figure 2. For comparison, we also show the result of the Monte Carlo simulation in B10. The two spectra are very similar.

The emitted spectrum peaks near 1 MeV. Radiation below the peak is made of blackbody photons advected from the central source and released at the photosphere. Note that the low-energy photon index \( \alpha \approx 0.4–0.5 \) is significantly different from the Rayleigh–Jeans \( \alpha = 1 \). This softening is a result of the superposition of emissions from different parts of the photosphere having different Doppler factors, i.e., effectively a multi-temperature spectrum is observed below the peak (B10). The spectrum between 1 and 20 MeV is mainly shaped by thermal Comptonization by Coulomb-heated pairs. The thermal pairs upscatter photons to energies up to \( E \sim 2E_k T_e \sim 20 \) MeV in the observer frame, where \( E_k T_e \sim 15 \) keV is the electron temperature in the comoving frame.

Above 20 MeV, the observed spectrum is generated by inverse Compton scattering by the nonthermal pairs. Note that there is no dip between the thermal and nonthermal Comptonization components. Two reasons contribute to this remarkable smoothness of the spectral shape. First, the energy budgets of thermal and
nonthermal Comptonization are comparable, and the two components have comparable luminosities at 10–20 MeV. Second, most of the photons above 20 MeV undergo several scattering events on thermal pairs before escaping. They experience significant Compton downscattering, which has an overall smoothing effect on the spectrum. The only distinct feature in the predicted high-energy spectrum is a broad annihilation line on top of the smooth continuum just below 1 GeV. The line is produced by the cooled $e^\pm$ pairs (photon energy $E \sim 1$ GeV in the observer frame approximately corresponds to $h\nu \sim m_e c^2$ in the jet frame).

The observed spectrum at energies $E > 1$ GeV is affected by absorption due to photon–photon ($\gamma-\gamma$) pair production. The jet is optically thick to GeV photons close to $R_0$, where most of the heating occurs, and all high-energy upscattered photons are quickly absorbed, creating secondary $e^\pm$ pairs. Thus, the developing pair-photon cascade is initially in the saturated regime.

The photon emissivity $j_\nu$ produced by the saturated cascade may be estimated analytically (Svensson 1987). An important parameter of the cascade is $z_T = (2/3)\gamma_0\lambda_0$, where $\gamma_0$ is the injection Lorentz factor of the primary nonthermal pairs and $\lambda_0 = h\nu_0/m_e c^2$ is the typical energy of soft photons. The quantity $z_T$ determines whether the cascade takes place in the Thomson or Klein–Nishina regime ($z_T \lesssim 0.5$ or $> 0.5$, respectively), the number of generations of secondary pairs etc. In our case $x_0 \sim 1/200$ and $\gamma_0 = 300$, so $z_T \sim 1$, i.e., the cascade starts in the Klein–Nishina regime at high energies and proceeds in the Thomson regime at lower energies. There are several generations of secondary pairs and photons, leading to a smooth overall spectrum. For the cascade with $z_T/r \sim 1$ the analytic solution predicts a flat emissivity of high-energy photons, $\nu j_\nu \propto \text{const}$ (Svensson 1987). The radiation intensity inside an opaque source is given by $I_\nu = j_\nu/\kappa_{\nu,\gamma\gamma}$. Here the absorption coefficient $\kappa_{\nu,\gamma\gamma} \propto \nu^{-\beta-1}$ and $\beta$ is the photon index of the target radiation (typically photons of multi-MeV energy in the observer frame). Thus, the high-energy spectrum $I_\nu \propto \nu^\beta$ is maintained inside the opaque jet.

The escaping multi-GeV emission may be estimated by considering the evolution of $\gamma-\gamma$ opacity with radius (B10). The rate of nuclear collisions fades at large radii, but it still extends far beyond 1 GeV, up to 100 GeV.

Finally, note that our spectrum deviates from that of B10 more significantly in the GeV range. This is likely caused by our angle-averaged approximation for $j_\nu$ and $\kappa_{\nu}$ (Section 3.1). Anisotropy effects are particularly significant at the high-energy end of the spectrum that forms at large radii. At $r \gg R_0$, the soft photon field is strongly collimated, and the scattering of nearly radial photons by relativistic pairs is preferentially backward. The backward-scattered photons see a higher $\gamma-\gamma$ optical depth and less of them can get out. The anisotropy of $\gamma-\gamma$ opacity is further enhanced by the very same collimation of the soft radiation field. These effects create particularly strong angular dependence of $j_\nu$ and $\kappa_{\nu}$ at high energies, which is missed by our angle-averaged approximation.

5. MAGNETIZED OUTFLOWS

Magnetization of the jet is described by the parameter

$$\varepsilon_B = \frac{L_B}{L} = \frac{4\pi r^2 c T^2 U_B}{L},$$

where $U_B = B^2/8\pi$ is the magnetic energy measured in the rest frame of the outflowing plasma. The quantity $\varepsilon_B$ is the ratio of the Poynting flux $L_B$ (measured in the static frame) to the kinetic luminosity of the outflow $L$. We discuss below the GRB model with the same parameters as in Section 4 and investigate how the results change with increasing $\varepsilon_B$. We have calculated a set of models with $\varepsilon_B = 10^{-3}$, $10^{-2}$, 0.1, 0.5, and 2. The corresponding ratios of (comoving) magnetic and radiation energy densities at the start of simulations are $U_B(R_0)/U_{r}(R_0) = 0.006, 0.06, 0.6, 3, \text{and 12}$. The results of our calculations are summarized in Table 1 and described in detail below.

### 5.1. Electron/Positron Distribution

Strong magnetic fields imply significant synchrotron cooling of the high-energy $e^\pm$ pairs injected by nuclear collisions, which can compete with Compton cooling. Since the synchrotron photons do not create secondary pairs, the magnetic field has a suppressing effect on the $e^\pm$ cascade. The multiplicity of each subsequent pair generation in a saturated cascade is approximately proportional to $\varepsilon_{rad}/(\varepsilon_{rad} + \varepsilon_B)$, where $\varepsilon_{rad} = L_{r}/L$ is the fraction of the jet energy carried by radiation. For magnetic fields approaching equipartition with radiation (i.e., $\varepsilon_B L/L_{r} \sim 1$), only two generations of secondary pairs can make a significant contribution to the total pair multiplicity.

As a result, the total pair yield decreases from $Y \approx 0.2$ to about 0.005 as $\varepsilon_B$ increases from 0 to 2 (Table 1). The pair

### Table 1: Results of Simulations

| $\varepsilon_B$ | $E_{peak}$ (MeV) | $\alpha^2$ | $\varepsilon_{rad}^2$ | $R_\gamma/R_\varepsilon$ | $Y$ | $\varepsilon_B/L_{r}$ (keV) |
|----------------|------------------|------------|-----------------------|------------------------|-----|--------------------------|
| 0              | 2.9              | -0.6       | 0.46                  | 16.5                   | 0.19| 0.94                     | 15.0 |
| 10^{-3}        | 2.5              | -0.8       | 0.46                  | 16.1                   | 0.18| 0.92                     | 14.7 |
| 10^{-2}        | 1.7              | -1.2       | 0.45                  | 14.2                   | 0.14| 0.79                     | 13.8 |
| 0.1            | 1.2              | -1.4       | 0.46                  | 8.8                    | 0.058| 0.32                     | 13.1 |
| 0.5            | 1.3              | -1.4       | 0.52                  | 2.3                    | 0.014| 0.10                     | 13.1 |
| 2.0            | 1.4              | -1.3       | 0.55                  | 0.005                  | 0.05| 14.4                     |

Notes.
1. Magnetization defined in Equation (11).
2. Peak energy of the $E_{peak}$ spectrum.
3. Photon index in the 100–500 keV range.
4. Radiative efficiency $\varepsilon_{rad} = L_r/L$.
5. Radius of the Thomson photosphere $R_\gamma$ relative to radius $R_\varepsilon = 10^{11}$ cm where the dissipation starts. In the case of $\varepsilon_B = 2$, the jet remains optically thin to scattering throughout the collisionally heated region ($R_\gamma < R_\varepsilon$).
6. Pair yield $Y = M/\gamma_0$, where $M$ is the secondary pair multiplicity and $\gamma_0 = 300$ is the Lorentz factor of injected electrons.
7. Ratio of thermal and nonthermal heating rates at $R_r$ for cases with $\varepsilon_B < 1$. The line is produced by $\nu j_\nu/\kappa_{\nu,\gamma\gamma}$.
8. Pair temperature at $R_r$ for $\varepsilon_B < 1$, and at maximum $\varepsilon_B$.
yield is defined as \( Y = M / \rho_0 \), where \( M \) is the multiplicity of secondary pairs. In the model with \( \varepsilon_B = 2 \), the pair yield is not far from the minimum value \( Y = 1 / \rho_0 \), which corresponds to \( M = 1 \) (no secondary pairs). The strong field completely suppresses the cascade.

Figure 3 shows the \( e^\pm \) distribution function at \( r = 3R_a \). The overall shape of the distribution is similar to that found in non-magnetized jets. It has thermal and nonthermal parts. With increasing \( \varepsilon_B \), the normalization of the distribution significantly decreases—the number of \( e^\pm \) is reduced as the cascade is suppressed. At \( \varepsilon_B \gtrsim 0.1 \) the synchrotron cooling becomes dominant and the \( e^\pm \) distribution at high energies is described by the power law \( n_e(p) \propto p^{-2} \). A new feature appears when the magnetization is high—the bump in the electron spectrum at \( p < 10 \). This bump is a result of synchrotron self-absorption that tends to “thermalize” the high-energy pairs before they join the thermal population at low energies and thermalize via Coulomb collisions. A similar effect is seen in the spectral simulations of accreting black holes in X-ray binaries and active galaxies (Poutanen & Vurm 2009; Veledina et al. 2011).

The reduction in pair production implies a smaller Thomson optical depth \( \tau_T \propto Y^{1/2} \), see Equation (23) in B10). As a consequence, the photospheric radius \( R_c \) decreases with increasing \( \varepsilon_B \) (Table 1). Another significant implication is the reduction in the thermal heating rate \( \dot{Q}_{\text{th}} \), which is proportional to \( \tau_T \) (see Equation 5). Since the nonthermal injection rate \( \dot{Q}_{\text{nth}} \) remains unchanged, the ratio \( \dot{Q}_{\text{th}} / \dot{Q}_{\text{nth}} \) is reduced in magnetized jets.

The temperature of thermalized pairs remains remarkably stable as \( \varepsilon_B \) is increased (see Figure 4). This fact may be understood by noticing that the thermal heating rate per particle does not decrease with increasing magnetization (the volume heating does). Note also that the thermal pairs are unable to cool via synchrotron emission because of strong self-absorption. As a result their equilibrium temperature is weakly affected by the magnetic field.

5.2. Radiation Spectrum

The spectrum that should be observed from the magnetized jet is shown in Figure 5. Magnetization significantly changes the radiative properties of the jet. It suppresses the pair cascade, reduces the photospheric radius, and changes the \( e^\pm \) distribution function. It also implies a new emission component—synchrotron emission from the nonthermal \( e^\pm \) pairs.

Our detailed calculations confirm that the synchrotron radiation from strongly magnetized collisionally heated jets peaks at energies comparable to 1 MeV in the observer frame, as estimated in previous works (Koers & Giannios 2007; B10). The position of the synchrotron peak varies with magnetization as \( \varepsilon_B^{-1/2} \). Its amplitude increases linearly with \( \varepsilon_B \) for weak magnetizations \((\varepsilon_B \lesssim 0.01)\) and approaches a constant value when synchrotron emission becomes the dominant energy loss mechanism for high-energy pairs. The synchrotron peak remains practically buried under the Comptonized thermal spectrum in all models shown in Figure 5. Thus, the emerging spectrum preserves the Band-type shape with the MeV peak.

The presence of synchrotron radiation significantly affects the low-energy slope of the Band peak—it makes the spectrum softer, i.e., \( \alpha \) is significantly reduced compared with the non-magnetized model. The obtained spectral shape below 1 MeV may be described as follows. The mixture of synchrotron and Comptonized thermal photons create a nearly power-law spectrum between 100 and 500 keV (with a slope \( \alpha \) that depends on \( \varepsilon_B \)). Below this energy range, the spectrum is dominated by synchrotron radiation that has a smaller, softer slope. The resulting curvature of the spectrum may be described by observers as a soft excess above the power law. At energies \( E < 10 \) keV the spectral slope changes again, as synchrotron self-absorption becomes important (see Section 5.2.2).

5.2.1. High-energy Emission

Three mechanisms contribute to emission at energies \( E \geq 1 \) MeV: thermal Comptonization by \( e^\pm \) pairs with temperature

\[
\frac{d^2\tau}{d\ln p} \propto n_e(p) \propto \frac{1}{p^2}
\]

\[
\frac{d^2\tau}{d\ln p} \propto n_e(p) \propto \frac{1}{p^2}
\]
curves correspond to magnetizations \( \epsilon \) of 0.1, 0.10, 0.1, 0.01, and 0.01, respectively. The straight dotted line shows a power-law spectrum with \( \alpha = -1.2 \). The spectrum \( L_E \) is multiplied by photon energy \( E \) to make the differences between the models more visible in the figure.

(A color version of this figure is available in the online journal.)

\(~15\) keV in the jet frame, nonthermal Comptonization by the \( e^\pm \) cascade, and synchrotron emission (which extends to tens of MeV when \( \epsilon_B \) is large).

Thermal Comptonization dominates at \( E \gtrsim 1 \) MeV, near the spectral peak. The magnetized jets have approximately the same electron temperature \( T_e \) as non-magnetized jets, and their optical depth \( \tau_e \) is smaller. As a result, the Kompaneets’ parameter \( \gamma = 4\pi k T_e / m e^2 \) is reduced with increasing \( \epsilon_B \). This leads to a steeper slope of the thermally Comptonized spectrum above the peak. Similar to the non-magnetized model, the thermally Comptonized power law declines at \( \epsilon \sim 15 \) keV in the jet frame, nonthermal Comptonization by the \( e^\pm \) cascade, and synchrotron emission (which extends to tens of MeV when \( \epsilon_B \) is large).

Overall, the suppression of the pair cascade by synchrotron cooling destroys the simple power-law shape of the high-energy spectrum. Instead, a distinct hard component (nonthermal inverse Compton) appears above \( 50–100 \) MeV.

### 5.2.2. Low-energy Emission

The low-energy end of the predicted spectrum is dominated by synchrotron emission, even when \( \epsilon_B \) is small (Figure 5). The spectrum at energies \( E \lesssim 1–10 \) keV is affected by self-absorption. It can be derived analytically as follows.

In the rest frame of the plasma, the angle-averaged synchrotron emissivity and the absorption coefficient are given by (see, e.g., Ghisellini & Svensson 1991)

\[
j_s(v) = \int j_s(v, p)n_e(p)dp, \tag{12}
\]

\[
\kappa_s(v) = -\frac{1}{2m_e v^2} \int j_s(v, p) \gamma p \frac{d}{dp} \left[ \frac{n_e(p)}{p^3} \right] dp. \tag{13}
\]

Here all quantities are measured in the plasma rest frame; \( n_e(p) = n_e(p) + n_{\ldots}(p) \) is the distribution function of \( e^\pm \) pairs, and \( j_s(v, p) \) is the angle-averaged synchrotron emissivity per electron. For analytical estimates we will use the delta-function approximation for the emissivity

\[
4\pi j_s(v, p) = \frac{4}{3} c \sigma_T U_B p^2 \delta (v - \gamma^2 v_B), \tag{14}
\]

where \( v_B = eB/2\pi mc^2 \) is the Larmor frequency. The synchrotron emission is produced by relativistic \( e^\pm \) particles with \( \gamma \approx p \). Then Equations (12)–(14) give

\[
j_s(v) = \frac{\alpha_f}{9} h v_B p n_e(p), \tag{15}
\]

\[
\frac{j_s(v)}{\kappa_s(v)} = \frac{2}{2 + \delta} m e v_B^2 p^5, \tag{16}
\]

where \( \alpha_f = e^2/\hbar c = 1/137 \), \( \delta = -d \ln n_e(p)/d \ln p \) is the local slope of the \( \epsilon^\pm \) distribution function, and

\[
p \approx \gamma = \left( \frac{v}{v_B} \right)^{1/2}. \tag{17}
\]

The distribution function \( n_e(p) \) in Equation (15) can be determined by assuming a quasi-steady flow of \( e^\pm \) particles in the momentum space and writing

\[
\dot{p} n_e(p) = \dot{n}^{\text{inj}} p M(p), \tag{18}
\]

where \( \dot{n}^{\text{inj}} \) is the rate of particle injection at the highest energy \( \gamma_0 \approx 300 \) (Equation 3) and \( M(p) \) is the multiplicity of secondary \( e^\pm \) pairs created with momenta above \( p \). The synchrotron energy losses for particles emitting in the optically thin regime are given by

\[
\gamma^2 m_e e^2 \frac{\epsilon_B}{\epsilon_{\text{rad}} + \epsilon_B} = \frac{4}{3} c \sigma_T U_B p^2. \tag{19}
\]

Using Equations (18) and (19) (with \( p \approx \gamma \)) the synchrotron emissivity (15) becomes

\[
j_s(v) = \frac{m_e e^2 \dot{n}^{\text{inj}} B |\gamma|}{8\pi v_B (\epsilon_{\text{rad}} + \epsilon_B) \gamma}. \tag{20}
\]

Let us now evaluate the range of Lorentz factors \( \gamma > \gamma_5 \) for particles that emit synchrotron radiation in the optically thin regime, as a function of radius \( r \). The synchrotron photosphere can be found from the approximate condition

\[
\frac{r \kappa_s(v)}{\Gamma} = 1. \tag{21}
\]

Using Equations (16) and (20), together with the relations (3) and (11), we find from Equation (21)

\[
\gamma_5^0 \sim \frac{3(2 + \delta) \epsilon_B}{27(\epsilon_{\text{rad}} + \epsilon_B)} \frac{c^3}{v_B^2} \frac{\Gamma}{n} \Gamma \Gamma n \tau_n M(\gamma_5). \tag{22}
\]
With \((2 + \delta)^{1/6} \approx 21/2\), this equation gives

\[ \gamma_v \approx \frac{(m_e/m_p)^{1/3}}{2^{2/3} \epsilon_{\text{BL}}^{1/12} \gamma_0^{1/3}} \left[ \frac{\pi M(\gamma_0) \sigma_T L_{\text{rad}}}{\epsilon_{\text{BL}} + \epsilon_{\text{syn}}} \right]^{1/6} \left[ \frac{L_{\gamma}}{m_e c^3} \right]^{1/12}, \tag{23} \]

where \(r_{\text{ce}} = e^2/m_e c^2 \approx 2.82 \times 10^{-13} \text{ cm}\) is the classical electron radius. This gives \(\gamma_v \approx 10\) for the parameters adopted in our models. Note that \(\gamma_v\) does not depend on \(r\) and weakly depends on the parameters of the jet.

Synchrotron radiation at a given radius \(r\) is self-absorbed at energies \(E < E_s\) where

\[ E_s(r) \sim \gamma_v^2 \epsilon_{\text{BL}} \frac{\hbar \nu}{c} \left( \frac{2 \gamma_v L_{\gamma}}{c} \right)^{1/2}. \tag{24} \]

The entire heating region \(r > R_{\text{f}}\) is transparent to synchrotron absorption for photons above \(E_s(R_{\text{f}})\). For the jet models calculated in this paper \(E_s(R_{\text{f}}) \sim 1\)–10 keV. Observed radiation at lower energies \(E\) comes mainly from the corresponding synchrotron photosphere \(R_{\gamma}(E)\) that can be found by expressing \(r\) from Equation (24).

Neglecting the self-absorbed radiation from \(r < R_{\gamma}(E)\), we estimate the observed spectral luminosity \(L_{\gamma}(E)\) by integrating the synchrotron emissivity over the optically thin region \(r > R_{\gamma}(E)\),

\[ L_{\gamma}(E) \approx \int_{R_{\gamma}(E)}^{\infty} \frac{4\pi j_{\gamma}(v)}{h} 4\pi r^2 dr. \tag{25} \]

Here we approximated \(h \nu \approx E/\Gamma\) and used the fact that the angle-integrated emissivity \(4\pi j_{\gamma}\) is the same in the static and comoving frames with a factor \(C \approx 1.8\). Using the slope of \(e^\pm\) distribution \(\delta \approx 2\), one can show that \(j_{\gamma}(v) r^2\) scales with radius as \(r^{-1/2})/r = r^{-3/2}\) and its integral peaks near \(R_{\gamma}(E)\). Substituting \(j_{\gamma}(v)\) from Equation (20), using the relations \(\gamma_\nu^2 \sim E/\Gamma \epsilon_{\text{BL}}\), \(\Gamma^2 B^2 r^2 \sim 2 \epsilon_{\text{BL}} L_{\gamma}/c\) and Equation (3), we obtain

\[ \frac{L_{\gamma}(E)}{L} \approx \frac{\alpha_i}{m_e c^2} \gamma_0^5 \epsilon_{\text{BL}}, \quad E < E_s(R_{\text{f}}), \tag{26} \]

where \(\gamma_0\) is given by Equation (23). The emission at any photon energy \(E < E_s(R_{\text{f}})\) peaks near \(R_{\gamma}(E)\) and is produced by \(e^\pm\) particles with the same \(\gamma_\nu \sim 10\). Note that \(L_{\gamma}(E) = \text{const}\) (flat spectrum), which corresponds to the photon index \(\alpha = -1\). This fact is a consequence of \(B \propto r^{-1}\) and \(n_e(p) \propto r^{-2}\).

For a similar reason, a flat spectrum was derived for opaque radio jets in active galactic nuclei (Blandford & Königl 1979).

Equation (26) is in approximate agreement with our numerical results (Figure 5).

5.3. Radiative Efficiency

The radiative efficiency of the jet \(\epsilon_{\text{rad}}\) is defined as the ratio of the photon luminosity \(L_{\gamma}\) to the kinetic luminosity of the plasma outflow \(L\). Note that \(L(r) \approx \text{const}\) for the matter-dominated jet and \(L_{\gamma}\) evolves with radius. This evolution is shown in Figure 6. The final efficiency is given by the asymptotic value of \(\epsilon_{\text{rad}} = L_{\gamma}/L\) at large radii.

6. DISCUSSION

The photon luminosity prior to the onset of dissipation \((r = R_{\text{f}})\) is found from the passively cooling jet model. One can see from Figure 6 that \(L_{\gamma}\) is greatly increased by the intense collisional heating at \(r > R_{\text{f}}\). The net energy given to radiation by collisional heating outside a given radius is obtained by integrating Equations (3) and (5) over volume; the result is proportional to \(r^{-1}\). Thus, heating peaks near \(R_{\text{f}}\) and continues with a smaller rate at larger radii. On the other hand, \(L_{\gamma}\) is reduced by adiabatic cooling, in particular in the opaque zone \(r < R_{\gamma}\). The competition between collisional heating and adiabatic cooling shapes \(L_{\gamma}(r)\). In weakly magnetized jets, the photospheric radius is large, \(R_{\gamma} \sim 20R_{\text{f}}\), and adiabatic cooling is more efficient. It begins to win over collisional heating at \(r \sim 5R_{\text{f}}\) and somewhat reduces \(L_{\gamma}\). In strongly magnetized jets, the photospheric radius \(R_{\gamma}\) is smaller because the magnetic field suppresses the production of \(e^\pm\) pairs (Section 5.1). In this case, adiabatic cooling is less efficient.

In all cases shown in Figure 6 the final \(\epsilon_{\text{rad}}\) is close to 50%. We conclude that the radiative efficiency of collisionally heated jets remains high for the entire range of \(\epsilon_{\text{B}}\) considered in this paper.
and the central engine.\footnote{The situation may be compared with X-ray binaries, where Comptonization is thought to occur in a corona of the accretion disk or the inner hot flow, and the main puzzle is how the plasma is heated.} Progress in this direction was hampered for many years by the complexity of collisionless processes that were usually invoked in GRB production.

GRB spectra can be shaped by three possible heating mechanisms: (1) collisionless shocks, (2) magnetic dissipation, and (3) collisional dissipation. Least understood is magnetic dissipation, and it is usually modeled by introducing phenomenological parameters. A similar phenomenological approach was used for internal shocks in the jet until recent numerical simulations began to provide insights into shock physics. Collisional dissipation is the most straightforward mechanism of these three. It can be calculated exactly, from first principles. B10 recently showed that collisional heating possesses the key features of a successful GRB mechanism: efficient electron heating, high radiative efficiency and, most importantly, it generates the observed Band-type spectrum.

In this paper, we calculated the emission from collisionally heated jets by solving the time-dependent, coupled kinetic equations for the particle and photon distributions inside the jet. The advantage of our numerical code is the accurate modeling of all relevant kinetic and radiative processes including Coulomb collisions and synchrotron self-absorption. It allowed us to systematically explore the effects of jet magnetization on the emerging spectrum. We calculated the emerging radiation using the equation of radiative transfer (B11) instead of the Monte Carlo method used by B10. The only disadvantage of our code is that it uses angle-averaged coefficients of emission and absorption in the transfer equation. Comparison with exact transfer models of B10 and B11 shows that this approximation is reasonable, in particular at photon energies below GeV. Note also that this is the first implementation of radiative transfer calculations in a kinetic GRB code. The previously developed kinetic codes assumed isotropy of radiation in the rest frame of the plasma (Pe’er & Waxman 2005; VP09), which is invalid in the main region of interest $\tau < 10$.

6.1. Spectra from Non-magnetized and Magnetized Jets

First, we calculated the emission from the fiducial baryonic jet model of B10 with zero magnetic field ($\varepsilon_B = 0$). Our results agree with B10: the spectrum peaks at 1 MeV, has the low-energy photon index $\alpha \approx 0.4$ and the high-energy photon index $\beta \approx -2.5$. We conclude that the Band-type spectrum is a robust prediction of the collisional-heating model. Similar to B10, we stress the ab initio character of our calculations: the dissipation process and generated radiation are derived from first principles, without introducing any phenomenological parameters for the heating mechanism.

The collisional heating peaks below the photosphere, where the $\gamma-\gamma$ opacity is large for multi-GeV photons emitted in the $e^\pm$ cascade. The nuclear collisions also occur at much larger radii (although with a smaller rate) where the $\gamma-\gamma$ optical depth is reduced and the high-energy photons can escape. As a result, instead of a cutoff at a few GeV, the predicted spectrum exhibits a moderate steepening at $\sim 5$ GeV and extends up to $\sim 100$ GeV (Figure 2). A similar behavior is seen in the Monte Carlo results of B10, with somewhat smaller normalization of the multi-Gev emission. This difference must be caused by the angle-averaged approximation for $j_\gamma$ and $\kappa_\gamma$ that was adopted in our calculations of radiative transfer. As discussed above, the accuracy of this approximation is reduced at reduced $E > 1$ GeV.

Then we calculated the emission from magnetized jets. We considered the range of magnetization parameters $0 < \varepsilon_B < 2$, so the magnetic energy was at most comparable to the energy of the baryonic component. We did not consider magnetically dominated jets with $\varepsilon_B \gg 1$ for two reasons. First, the strong magnetic field can change the jet dynamics. Then the cooling approximation (which is reasonable for a matter-dominated jet) would be invalid. Second, the strongly magnetized case is computationally more difficult. The generated spectra in this regime are left for a future study.

Our calculations show that magnetization has a small effect on the position of the spectral peak $E_{\text{peak}}$—it is only slightly shifted to lower energies. The spectrum still has the Band-type shape around the peak, with $\alpha$ and $\beta$ depending on $\varepsilon_B$ (Figure 5 and Table 1). Magnetization significantly affects the shape of the spectrum at high ($E \gg E_{\text{peak}}$) and low ($E \ll E_{\text{peak}}$) energies.

The high-energy emission is reduced with increasing $\varepsilon_B$ (Figure 5), because the nonthermal $e^\pm$ generated by nuclear collisions emit less via inverse Compton scattering and more at low energies via the synchrotron mechanism. This change is significant if $\varepsilon_B \gg 10^{-2}$. Synchrotron cooling of the injected high-energy $e^\pm$ has a throttling effect on the pair cascades. As a result, magnetized jets have smaller densities of $e^\pm$ pairs and smaller photospheric radii $R_\gamma$.

The low-energy emission is increased with increasing $\varepsilon_B$. An important effect of magnetization is the softening of the spectral slope below $E_{\text{peak}}$. This is the result of synchrotron emission from nonthermal pairs. At energies below $\sim 30$ keV a clear soft excess is predicted, even for small $\varepsilon_B \sim 10^{-3}$. Our model also predicts that the spectrum below a few keV should have the photon index $-1$. This spectrum extends down to the optical band.

Note that photon spectra in this paper are not corrected for the cosmological redshift $z$. The actually observed energy of all photons is smaller by the factor of $(1+z)^{-1}$. In particular, the soft excess is redshifted to $\sim 10$ keV for a typical $z$ of GRBs.

6.2. Comparison with Other Models of Subphotospheric Comptonization

Thompson (1994) proposed that GRB photons are Comptonized in the subphotospheric region by turbulent bulk motions of the plasma in the jet. His model assumed that the jet is dominated by magnetic field and pictured Alfvénic turbulence generated by some instabilities (e.g., by reconnection). Building a detailed theory of turbulence from first principles would be a formidable task, and Thompson (1994) made estimates assuming that the plasma motions are limited by radiation drag. Then the bulk Comptonization occurs in the unsaturated regime and is controlled by the efficiency of turbulence generation against the drag. The predicted GRB spectrum is sensitive to this unknown efficiency, which may vary with radius. The mechanism is very different from collisional heating. Direct comparison of the model predictions is difficult, because Thompson (1994) used the theory of Comptonization in static sources, which is not valid for GRBs (see discussion in Section 3). Like in the expanding universe, the number of scatterings in a GRB jet scales as $\sim \tau_r$ (not as $\tau_r^2$) and is controlled by the decrease in density, not the escape of photons from the plasma. The expected spectrum can only be found by solving radiative transfer in the
expanding jet, and we emphasize the importance of accurate transfer calculations for GRB models.\textsuperscript{10} Mészáros & Rees (2000b) and Rees & Meszaros (2005) discussed subphotospheric Comptonization in general terms and highlighted its importance for GRBs. They argued that Comptonization helps explain observations, although they did not consider any concrete model. More detailed calculations were performed by Pe’er et al. (2006). They still did not focus on any concrete physical scenario and experimented with various phenomenological models, using free parameters to describe the heating process: the amount of dissipated energy and its fraction that is given to electrons, the shape of the electron distribution, and the optical depth at which the dissipation occurs. Various combinations of these parameters led to various spectra. In contrast, we investigated the concrete physical model of collisional dissipation, which predicts the thermal and nonthermal heating rates, both scaling as $r^{-1}$. It is therefore hard to compare their results with ours. We can only compare the technical tools. Pe’er et al. (2006) used the kinetic code developed by Pe’er & Waxman (2005). In many respects, their code is similar to ours. Their code assumes, however, that radiation is isotropic in the comoving frame of the jet. We gave up this approximation because it is violated even below the photosphere (see Section 3) and instead solved the radiative transfer equation. Note also that Pe’er et al. (2006) argued that GRB jets never become dominated by $e^\pm$ pairs. In contrast, we find that jets with magnetization $e_B < 1$ have a large pair loading factor $n_e/n \sim 20–40$, which changes the photospheric radius $R_e$ by the factor of 20–40.

Giannios (2006) investigated Comptonization in jets that carry alternating magnetic fields and are heated by magnetic reconnection (Drenkhahn & Spruit 2002). This model has two phenomenological parameters—the reconnection rate and the fraction of dissipated energy that is deposited into electrons. The effect of magnetic dissipation on the electron distribution is unknown. Giannios (2006) assumed a thermal electron distribution and studied the radiation produced by this model. He used a Monte Carlo code and an iterative technique to simulate the radiative transfer in the jet.\textsuperscript{11} We note that the jet model in Giannios (2006) is significantly different from B10 and our work. In particular, the jet continues to accelerate as $\Gamma \propto r^{1/3}$ in the heated subphotospheric region while we focus on matter-dominated jets with $\Gamma \approx$ const. The electron temperature scales with radius approximately as $T_e \propto r^{5/3}$ while in our model we find that $T_e$ scales only by a factor of $\sim 2$ for three decades in radius (Figure 4). Note also that Giannios (2006) assumed pure thermal heating, leading to negligible pair creation. In contrast, our work and B10 predict a nonthermal $e^\pm$ cascade, which has significant effects on the observed spectrum at high energies (above 10 MeV) and at low energies where synchrotron emission from the cascade becomes dominant.

Lazzati & Begelman (2010) considered a simplified model for Comptonization following an impulsive heating event below the photosphere. In their model, all electrons in a slab of optical depth $\tau_T \sim 2$ are suddenly given a Lorentz factor $\gamma$ and then cooled by blackbody radiation. This setup can produce a variety of photon spectra depending on photon-to-electron ratio $n_p/n_e$, $\tau_T$, and $\gamma$. It has, however, a problem. The sudden heating of the slab (faster than Compton cooling) can hardly describe any physical dissipation mechanism, as different parts of the slab are causally disconnected. The causal contact (light-crossing) time for a region with $\tau_T \sim 2$ is orders of magnitude longer than Compton cooling time.

6.3. Comparison with Observed GRB Spectra

The main part of observed GRB radiation is emitted with a Band-type spectrum—a smoothly broken power law that peaks near 0.3 MeV. The spectral slopes below and above the peak ($\alpha$ and $\beta$) vary (Preece et al. 2000). The most frequently measured low-energy slope $\alpha$ is close to $-1$, but in some cases it reaches $1$ (Rayleigh–Jeans slope of the Planck spectrum). The most frequently measured high-energy slope $\beta$ is near $-2.5$; it also significantly varies from burst to burst, roughly between $-2$ and $-3$.

The spectra predicted by our model appear to agree with observations. The predicted spectral peak is close to $\text{MeV}/(1+z)$. The average observed values of $\alpha \sim -1$ and $\beta \sim -2.5$ are consistent with the collisionally heated jet with magnetization $e_B \sim 10^{-3}$ to $10^{-2}$. A lower magnetization $e_B < 10^{-3}$ leads to harder $\alpha$ up to 0.4. Larger slopes are not expected from the matter-dominated jets. The slope $\alpha = 1$ is predicted only in the radiation-dominated regime—in this case the jet emits a Planckian spectrum (B11).

It should be noted that our calculations give practically instantaneous observed spectra. The main part of the prompt emission (photon energies from soft X-rays to a few GeV) is dominated by the photosphere in our model. The emission is produced by a sequence of short-lived (in observer time) independent emitters passing through $R_e$—the sequence of shells of thickness $\delta r \lesssim R_e/\Gamma^2$, which corresponds to duration $\delta t_{\text{abs}} \sim 10^{-4}$ s for typical parameters of GRB jets. The spectra obtained by modern detectors do not provide this high temporal resolution, because of insufficient photon statistics. The observed spectra must be a mixture of emissions from different shells, possibly with significant variations in $L$ and $E_{\text{peak}}$. The superposition of different instantaneous spectra generally tends to reduce the observed slope $\alpha$ and can give $\alpha \sim -1$ even for non-magnetized jets (R. Mochkovitch 2011, in preparation).

An interesting feature of the magnetized model of collisionally heated jet is the soft excess that appears at energies $\sim 30(1+z)^{-1}$ keV. It may provide an explanation for the X-ray excesses that have been observed in several bursts (Preece et al. 1996), including the recent example of GRB 090902B (Abdo et al. 2009a).

Prompt optical emission is expected from collisionally heated jets, with a slope $L_\gamma(E) \equiv \text{const}$ (photon index $-1$). This emission peaks at the self-absorption radius $R_{\text{abs}}(E) \gg R_e$ and should be observed with a time lag $\sim R_e/\Gamma^2 \sim 1$ s with respect to the MeV radiation that is released at $r \sim R_e$. The prompt optical emission has been detected in some GRBs, but the data are sparse, as prompt optical observations are difficult. In some cases (e.g., GRB 080319B) the optical luminosity appears to exceed the predictions of our model, suggesting an additional source of emission (e.g., shocks or neutron decay, see B10).

The high-energy index $\beta$ predicted by the collisional model can easily be steeper than $-2.5$, because of a large $e_B$ (Figure 5) or less efficient heating (B10). The model can also accommodate

\textsuperscript{10} When heating occurs in a static source, the emerging spectrum can have two peaks, resembling the Comptonized spectra of accretion-disk corona. In contrast, the spectrum emitted by the GRB jet has one peak.

\textsuperscript{11} The numerical methods used by Giannios (2006) and B10 are similar, except that the iterations of the electron temperature in Giannios (2006) are simplified: the profile $T_e(r)$ is searched in the power-law form, with two parameters to iterate—the normalization and the slope of the power law.
very hard slopes $\beta > -2$ that are observed in some GRBs (see, e.g., Figure 7 in B10).

Our model predicts significant multi-GeV emission, especially when $e_\gamma$ is small. It implies that the prompt GRB emission can contribute to the radiation observed by LAT instrument of the Fermi telescope. The prompt high-energy photons should mix together with radiation from the other, long-lived, high-energy source that was identified by LAT in a fraction of GRBs and whose origin is debated.

Overall, our results support the view that the main peak of GRB spectra is dominated by photospheric radiation of the ultra-relativistic jet from the central engine. Note that photospheric emission is often erroneously pictured as a Planckian component in GRB spectra. As shown in B11, the photospheric spectrum has a Planckian shape only if the jet energy is strongly dominated in GRB spectra. In GRB spectra is dominated by photospheric radiation of the ultra-relativistic jet and whose origin is debated.

Mix together with radiation from the other, long-lived, high-energy source that was identified by LAT in a fraction of GRBs and whose origin is debated.

Here the spatial derivative $\partial/\partial r$ is taken along the worldline (i.e., at $s = $ const). Note that the flow worldlines do not cross (i.e., there are no caustics) as long as the bracket in Equation (A4) is non-zero.

If the flow varies on scales $dr \sim r/\Gamma^2$, the two parts of the operator (A4) are comparable. The problem simplifies (becomes formally equivalent to the steady problem) if the flow varies on scales much larger than $r/\Gamma^2$. Then the operator $\hat{L}_s$ is dominated by the first term that is proportional to $\partial/\partial r$. The left-hand side of Equation (A1) simplifies to $\hat{L}_s = \Gamma(\mu + \beta) \partial I_v/\partial r$. Similarly, the right-hand side of Equation (A1) simplifies and the transfer equation becomes

$$
\frac{1}{r^2} \frac{\partial}{\partial \ln r} \left( (\mu + \beta) r^2 I_v \right) = \frac{r}{\Gamma} \left( j_v - \kappa_v I_v \right) + \beta(1 - \mu^2) \frac{\partial I_v}{\partial \ln v} - \frac{\partial}{\partial \mu} \left[ (1 - \mu^2)(1 + \beta \mu) I_v \right],
$$

which becomes Equation (7) for outflows with $\beta \approx 1$. Formally, this is achieved simply by replacing $\partial/\partial r|_s$ by the derivative along the flow worldline $\partial/\partial r|_s$. Equation (A5) can also be written using the flow proper time as the independent variable, $t = r/c\beta \Gamma$.

ACKNOWLEDGMENTS

This work was supported by the Whuri foundation and ERC Advanced Research Grant 227634 (I.V.), NSF grant AST-1008334, and NASA grant NNX10AO58G (A.M.B.), and the Academy of Finland grant 127512 (J.P.).

**APPENDIX**

**RADIATIVE TRANSFER**

Equation (7) is valid for relativistic outflows in the limit $\Gamma \rightarrow \infty$. For finite $\Gamma \gg 1$, the equation accurately describes the transfer in a steady jet. Its accuracy is reduced if the jet is variable on scales $\delta r \lesssim r/\Gamma^2$, as discussed in B11. Here we give a more formal discussion of transfer in variable jets and the applicability of the steady Equation (7). The general equation of radiative transfer in a spherically symmetric outflow with any Lorentz factor $\Gamma(t, r)$ can be written as (Mihalas 1980)

$$
\hat{L}_s \Gamma \left[ \frac{\Gamma \beta}{r} (1 - \mu^2) + \frac{\mu}{\beta \Gamma} \hat{\Gamma} \right] v^3 \frac{\partial}{\partial \ln v} \left( \frac{l_v}{v^3} \right) - (1 - \mu^2) \left[ \frac{\Gamma}{r} (1 + \beta \mu) - \frac{1}{\beta \Gamma} \hat{\Gamma} \right] \frac{\partial l_v}{\partial \mu} + j_v - \kappa_v I_v,
$$

(A1)

where $\hat{L}$ is the differential operator

$$
\hat{L} = \Gamma(1 + \beta \mu) \frac{\partial}{\partial c t} + \Gamma(\mu + \beta) \frac{\partial}{\partial c r}.
$$

(A2)

Here $\beta = (1 - \Gamma^{-2})^{1/2}$ is the flow velocity in units of $c$, $t$ and $r$ are time and radial coordinate measured in the static frame, whereas the cosine $\mu$ of the photon angle relative to the radial direction, the specific intensity $I_v$, and the emission and absorption coefficients $j_v$ and $\kappa_v$ are all measured in the local comoving frame.

Let us define a coordinate $s$ by the following implicit relation:

$$
s = ct - \frac{r}{\beta(s)}.
$$

(A3)

For a matter-dominated (coasting) flow, $\Gamma(s) \approx$ const and $s$ can serve as a Lagrangian coordinate that labels different shells of the flow, as $s = $ const along the worldlines of the shells. Changing variables $r, t \rightarrow r, s$, we rewrite the operator (A2) as

$$
\hat{L} = \Gamma(\mu + \beta) \frac{\partial}{\partial r} - \frac{\mu}{\beta \Gamma} \left[ 1 - \frac{r \, d \ln \Gamma}{d s} \right]^{-1} \frac{\partial}{\partial s}.
$$

(A4)

Here the spatial derivative $\partial/\partial r$ is taken along the worldline (i.e., at $s = $ const). Note that the flow worldlines do not cross (i.e., there are no caustics) as long as the bracket in Equation (A4) is non-zero.

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