Statistical simulation of radiation transfer in horizontally inhomogeneous stratus clouds

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Abstract. The aim of the paper is to analyze the influence of a cloud layer horizontal inhomogeneity on radiation fluxes in the atmosphere using the models based on observation data of the optical thickness for stratus clouds. To reproduce the correlation structure and one-dimensional distributions of the observed stratus optical thickness fields we have developed numerical stochastic cloud models based on the inverse distribution function method and autoregressive schemes. We used Monte Carlo methods to simulate the radiation fluxes for visible and infrared ranges of wavelength.

1. Introduction

The study of the influence of the cloud layers spatial inhomogeneity on radiation fluxes in the atmosphere is a significant task for developing climate models, weather forecast, and solving remote sensing problems. According to [1], [2], the spatial inhomogeneity increases the solar radiation transmittance of stratus and stratocumulus clouds by 5 – 15 % as compared with a homogeneous cloud layer of an average optical thickness.

Monte Carlo simulation for a cascade cloud model and reflection of light on the underlying surface has shown [3] that the absolute bias of albedo for homogeneous arctic stratus layers is smaller than of 0.02, the relative bias is less than 2 %. The absolute bias of transmittance is less than 0.05, while a relative bias can be larger than 10%.

Figure 1. The observed Arctic Stratus stochastic optical thickness field St-04, (see [5]).

In our research, we analyze the influence of spatial inhomogeneity of stratus clouds using
numerical models of random fields constructed by the inverse distribution function method and autoregressive schemes.

Statistical properties of the models correspond to the optical thickness of the Arctic Stratus observed during VERDI campaign [4], [5] in May 2012 around Inuvik, Canada. The values of optical thickness $\tau(x, y)$ are given on a regular grid with the intervals $h_x, h_y$ in a rectangular domain. Figure 1 demonstrates a part of the Arctic Stratus optical thickness field St-04 [5]. Figure 2 (a) shows an estimate of the one-dimensional distribution density for St-04 and its approximations by the lognormal distribution $lnN(\mu, \sigma^2)$ with the density

$$f_{lnN}(x) = \exp \left( -\frac{1}{2} \left[ \ln(x) - \mu \right]^2 / (x\sigma\sqrt{2\pi}) \right),$$

and the parameters $\mu = 2.69, \sigma = 0.0234$ and the gamma distribution $\Gamma(\nu, \theta)$ with the density

$$f_{\Gamma}(x) = \frac{x^{\nu-1}\exp(-x/\theta)}{\theta^{\nu}\Gamma(\nu)},$$

and the parameters $\nu = 40, \theta = 0.37$. Here $\Gamma$ denotes the gamma function. Estimates of (normalized) autocorrelation functions for the optical thickness field St-04 along the axes OX and OY, as well as their approximation by the exponential function $r(L) = \exp(-L/\rho_L)$, $\rho_L = 150$ m are presented in Figure 2 (b). According to [4], $\rho_L$ is the decorrelation length of the optical thickness field St-04.

2. A stochastic model of the Stratus optical thickness fields

We have constructed a model of a stratus cloud as a plane-parallel layer with a random optical thickness $\tau(x, y)$. To numerically simulate a random field $\tau(x, y)$ with a given one-dimensional distribution function $F(t)$ and an autocovariance function $r(x, y)$ we applied a nonlinear transformation of a Gaussian field [6], [7]. This simulation technique is often called the method of inverse distribution function. The simulation formula in this case has the form

$$\tau(ih, jh) = F^{-1}(\Phi(\xi_{i,j})).$$
Here $\Phi$ is the standard normal distribution function, $\xi_{i,j}$ is a Gaussian random field with zero mean, unit variance and the autocorrelation function $R_{i,j}$. The autocorrelation function $r_{i,j} = r(ih, jh)$ of the random field $\tau(ih, jh)$ has the form

$$r_{i,j} = W_F(R_{i,j}),$$

where

$$W_F(\rho) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} F^{-1}(\Phi(\xi))F^{-1}(\Phi(\eta))\varphi_{\rho}(\xi, \eta)d\xi d\eta,$$

$\varphi_{\rho}(\xi, \eta)$ is the probability density of the two-dimensional Gaussian random vector $(\xi, \eta)$ with zero mean, unit variance of the components and the correlation coefficient $\rho$ between the components,

$$\varphi_{\rho}(\xi, \eta) = \left[\frac{2\pi}{\sqrt{1-\rho^2}} \exp\left(\frac{\xi^2 + \eta^2 - 2\rho\xi\eta}{2(1-\rho^2)}\right)\right]^{-1}.$$

Thus, when constructing the field $\tau(x, y)$, it is necessary to calculate the autocorrelation function $R_{i,j} = W_F^{-1}(r_{i,j})$ of the homogeneous Gaussian field $\xi_{i,j}$, to simulate the field $\xi_{i,j}$, and to apply transformation (2).

We used an autoregressive scheme of the first order to simulate the homogeneous Gaussian field $\xi_{i,j}$ on the plane with zero mean and unit variance. This algorithm is fast, and it enables one to generate realizations of homogeneous Gaussian random fields with exponential correlation functions. The simulation algorithm for the two-dimensional homogeneous Gaussian field $\xi_{i,j}$ can be described as follows [8]:

**STEP 1.** Random processes along each axis are simulated according to the formula

$$\xi_{0,0} = \varepsilon_{0,0},$$

$$\xi_{i,0} = \hat{\rho} \xi_{i-1,0} + (1 - \hat{\rho}^2)^{1/2}\varepsilon_{i,0}, \quad i > 0,$$

$$\xi_{0,j} = \hat{\rho} \xi_{0,j-1} + (1 - \hat{\rho}^2)^{1/2}\varepsilon_{0,j}, \quad j > 0.$$  

Here and further $\varepsilon_{i,j}$ are independent Gaussian random variables.

**STEP 2.** The values $\xi_{i,j}$ of the two-dimensional field are sequentially simulated using the values from the previous step:

$$\xi_{i,j} = \hat{\rho} \xi_{i-1,j} + \hat{\rho} \xi_{i,j-1} + \hat{\rho}^2 \xi_{i-1,j-1} + (1 - \hat{\rho}^2)\varepsilon_{i,j}, \quad i, j > 0,$$

An autocorrelation function of the random field $\xi_{i,j}$ has the form

$$R_{i,j} = E\xi_{00}\xi_{i,j} = \hat{\rho}^{(i+j)} = \exp(-h(i + j)/\rho_L),$$

where $\hat{\rho} = \exp(-h/\rho_L)$.

We have constructed a model of a two-dimensional homogeneous random field, whose one-dimensional distribution and autocorrelation function approximate the corresponding statistical characteristics of the stratus optical thickness field St-04 from [4]. The one-dimensional distribution of this field is approximated by the lognormal one $lnN(m, \sigma^2)$ with the parameters $\mu = 2.69$, $\sigma = 0.0234$ or the gamma distribution $\Gamma(\nu, \theta)$ with the parameters $\nu = 40$, $\theta = 0.37$ (see Figure 2(a)). For the autoregressive scheme we took the parameter $\hat{\rho} = \exp(-h/\rho_L) = 0.9829$. Here $\rho_L = 150$ m and $h = 2.6$ m are the decorrelation length and the grid size of the field St-04.

The distortion $\delta_F$ of the autocorrelation function (3) for the nonlinear transformation (2) in the case of the lognormal distribution can be calculated by the formula [8].
\[ \delta_{lnN} = |W(R) - R| = \left| \frac{\exp(\sigma^2 R) - 1}{\exp(\sigma^2) - 1} - R \right|. \]

The distortions \( \delta_F \) for the autocorrelation function \( R \) for the lognormal \( lnN(2.69, 5.467e - 4) \) and the gamma distributions \( \Gamma(40, 0.37) \) are presented in Figure 3. We can see that for the lognormal distribution, the maximum distortion value \( \delta_{lnN} = 0.025 \) is achieved for \( R = -1 \). Since the autocorrelation function \( r_{i,j} \) of our model is positive, the distortion of the Gaussian autocorrelation function \( R_{i,j} \) does not exceed \( \delta_{lnN} \leq 0.5\% \) for the lognormal one-dimensional distribution. For the one-dimensional gamma distribution \( \Gamma(40, 0.37) \), the distortion of the Gaussian autocorrelation function \( R_{i,j} \) is not larger than 0.5\% if \( R_{i,j} \) is positive. Therefore, in our model, we can use the autocorrelation function \( r_{i,j} \) as the autocorrelation function \( R_{i,j} \) of the Gaussian field.

![Figure 3](image.png)

**Figure 3.** The distortions \( \delta_F \) of the autocorrelation function for the nonlinear transformations (2) for the distributions \( lnN(2.69, 5.5e-4) \) and \( \Gamma(40, 0.37) \).

3. The results of Monte Carlo simulation for stochastic cloud models

For Monte Carlo simulation of the radiation transfer in the water-drop stratus clouds with a spatially inhomogeneous structure, we applied numerical model (2), (5), (6) of the optical thickness field \( \tau(x, y) \) described in the previous section. We used optical models of Maritime Stratus (with mean droplet radius 8.18 \( \mu m \)) and Continental Stratus (with mean droplet radius 5.5 \( \mu m \)) from OPAC [9] for wavelengths 0.530 and 10.0 \( \mu m \). Single scattering albedo equals 1 with wavelength 0.53 \( \mu m \) and 0.6879 with wavelength 10.0 \( \mu m \). The mean values of the optical thickness are equal to 14.81 and 17.32 for wavelengths 0.530 \( \mu m \) and 10.0 \( \mu m \), respectively. The asymmetry factors of the phase functions for Maritime and Continental Stratus are equal to 0.868, 0.885 for the wavelength 0.530 \( \mu m \) and 0.921, 0.889 for the wavelength 10.0 \( \mu m \).

**Remark.** To study the radiation balance in the cloudy atmosphere in the IR wave range, it is necessary to take into account the radiation emission by the Earth’s surface, atmosphere and clouds. Here we present the results for a simple model in order to study the influence of the cloud structure randomness in the case of high radiation absorption in clouds. In our model, we assume that the IR radiation is emitted from a plane surface according to the Lambertian law and omit other sources of light as well as interaction of light with the atmosphere.

First, we have simulated the stratus cloud optical thickness field with the one-dimensional distribution (1) with the parameters \( \mu = 2.69, \sigma = 0.0234 \) and the exponential autocorrelation function, which approximates the one-dimensional distribution and the correlation function of the field St-04 (see Figure 2). Let us call this model ”field 1”. For this model, the parameter \( \hat{\rho} \) of the autoregressive scheme is equal to 0.9829 and the decorrelation length of the optical...
thickness field is equal to $\rho_L = 150$ m. In addition, we have simulated similar fields with $\hat{\rho}_2 = \hat{\rho}_1^2 = 0.9333$ ("field 2") and $\hat{\rho}_3 = \hat{\rho}_1^4 = 0.9957$ ("field 3"). "Field 2" has the decorrelation length 38 m, which is four times smaller than the decorrelation length of "field 1", while "field 3" has the decorrelation length 600 m, which is four times larger. Here let us mention that only the optical depth is a variable quantity within a cloud layer, while the single scattering albedo and the scattering phase function do not change from one pixel to another. Moreover, we have calculated the radiation fluxes for a non-random plane-parallel stratus cloud with the constant extinction and the scattering coefficients (model "constant").

In our study, we assumed that the height of the cloud layer is equal to 200 m, the number of cells in the horizontal plane is $4000 \times 500$ and the cell size is $2.6 \times 2.6$ m. The periodic boundary conditions were used for Monte Carlo simulation of photon trajectories at the lateral surfaces of the domain. In the numerical experiments, we assumed that the incident radiation is perpendicular to the scattering layer for the visible range of light and for IR radiation we considered a Lambertian plane source.

The Monte Carlo estimates of transmittance ($T$) and albedo ($A$) with wavelength 530 nm for the described optical thickness field models are presented in Table 1. Similarly, Table 2 demonstrates the values of transmittance and albedo for the cloud layers with wavelength of 10 $\mu$m. To compute the estimates, we have simulated $10^{10}$ trajectories of photons. The mean square deviations of the estimates are less than $2 \times 10^{-6}$. Moreover, in Table 2 the mean square deviations of the transmittance estimates are smaller than $2 \times 10^{-7}$.

| Phase function | "field 1" | "field 2" | "field 3" | "const." |
|----------------|-----------|-----------|-----------|-----------|
| St (cont.)     | A         | T         | A         | T         | A         | T         |
|                | 0.5159    | 0.4841    | 0.5169    | 0.4831    | 0.5120    | 0.4880    | 0.5194    | 0.4806    |

| Phase function | "field 1" | "field 2" | "field 3" | "const." |
|----------------|-----------|-----------|-----------|-----------|
| St (mari.)     | A         | T         | A         | T         | A         | T         |
|                | 0.5002    | 0.4998    | 0.5012    | 0.4988    | 0.4963    | 0.5037    | 0.5037    | 0.4963    |

| Phase function | "field 1" | "field 2" | "field 3" | "const." |
|----------------|-----------|-----------|-----------|-----------|
| St (mari.)     | A         | T         | A         | T         | A         | T         |
|                | 0.0272    | 0.00059   | 0.0272    | 0.00051   | 0.0273    | 0.00081   | 0.0273    | 0.00038   |

One can see that for the visible radiation, characterized by the absence of absorption in water-drop clouds, the difference in transmittance and albedo is insignificant between the non-random ("constant") cloud layer and random layers with different values of the horizontal decorrelation length. When the single scattering albedo is noticeably less than one, transmittance by the random cloud layers is larger than transmittance by the non-random layer. Also, in this case, the decorrelation length value affects the radiation transmittance: for a larger decorrelation length, the transmittance is larger. For the case under study, the transmittance for random cloud layers is 1.3 - 2.2 times larger than that for the "constant" model.
Figure 4 shows the numerical realization of the optical thickness random field $\tau(x, y)$ of size $1.3 \text{ km} \times 10.4 \text{ km}$ with the decorrelation length $150 \text{ m}$, and the transmitted and reflected irradiance fields with the wavelength $530 \text{ nm}$ for this realization of $\tau(x, y)$.

![Figure 4](image)

**Figure 4.** Numerical realization of the optical thickness random field of $1.3 \text{ km} \times 10.4 \text{ km}$ with the decorrelation length $150 \text{ m}$ (a), the transmitted (b) and reflected (c) irradiance fields with the wavelength $530 \text{ nm}$ for this realization.

The estimates of the autocorrelation functions for the fields of the optical thickness, the transmitted and reflected irradiance are presented in Figure 5 for the three values of the decorrelation length of $\tau(x, y)$.

**Conclusion**

We have constructed a stochastic model of the atmospheric cloud layers, which enables one to reproduce the decorrelation length and one-dimensional distribution of the optical thickness field. The model is based on the first order 2D Gaussian autoregressive scheme and nonlinear transformation of the Gaussian field. The model allows the fast numerical implementation for a variety of one-dimensional distributions and autocorrelation functions close to exponential. In our paper, we have applied this model for the simulation of the Arctic Stratus taking into account the observations in natural clouds. By Monte Carlo method, we analyzed the influence of the cloud stochastic structure on the radiation transfer. The numerical experiments have
shown that for the visible radiation, when the absorption in a cloud layer is close to zero and the asymmetry factor of the phase function is rather high, a relative difference in albedo and transmittance between random and non-random models is insignificant. For the IR radiation, when the absorption in clouds is essential, the transmittance for random models is significantly larger in comparison with the transmittance for the corresponding non-random model.

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