Optimization of The Number of Layers for A Fixed Thickness of a Composite Material Subjected to Compression Loading

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Abstract. In this paper, the effect of varying the number of layers for a fixed total thickness is investigated. In order to conserve the integrity and the safety of a multilayers composite structure, the search of the optimal number of layers in the cross section, induced the low stress-strain field intensity, is strongly recommended. Meanwhile, this study aims to show how the mechanical response of a multilayers composite plate loaded on a compression in the axial direction, changes when the number of layers increases from for a fixed total thickness.

1. Introduction

Nowadays, composites materials are becoming an essential part in the world of industry, due at numerous advantages such as high fatigue strength, faster assembly, corrosion resistance, and high stiffness and strength to weight ratios. For example, Steel pipelines used for oil and gas transportation, are exposed to corrosion damage that can easily cause catastrophic failure of the pipeline [1-6], and composite materials could a perfect solution for the repairing and maintenance of the corroded pipeline [7,8]. Besides, the stacking sequence, the plies numbers and thickness, and the fibers orientations are considered as the primordial parameters that qualify a composite construction, and give trust to its integrity. Therefore, in order to tailor the material to specific structural needs, and attain the best results, the use of optimization techniques is an important step that should be taken in consideration. Meanwhile, the increase of the number of layers, which involves the thinning of layers, and the weakness of the intensity of the stress-strain field in the cross section, is required [9-21].

The field of the strains at a point M of the laminate of coordinates (x, y, z) is the sum of the membrane strains $\varepsilon_m$ and the bending strains $\varepsilon_f$ such that:

$$\varepsilon_m(M) = \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ Y_{xy}^0 \end{bmatrix} = \begin{bmatrix} \frac{\partial u_0}{\partial x} \\ \frac{\partial v_0}{\partial y} \\ \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} \end{bmatrix}$$ (1)
\[ \varepsilon_f(M) = \begin{bmatrix} \varepsilon_x^f \\ \varepsilon_y^f \\ \gamma_{xy}^f \end{bmatrix} = \begin{bmatrix} -z \frac{\partial^2 w_0}{\partial x^2} \\ -z \frac{\partial^2 w_0}{\partial y^2} \\ -2z \frac{\partial^2 w_0}{\partial x \partial y} \end{bmatrix} \] (2)

\[ \varepsilon_f(M) = zK(x,y) \] (3)

\[ K(x,y) = \begin{bmatrix} K_x \\ K_y \\ K_{xy} \end{bmatrix} = \begin{bmatrix} -\frac{\partial^2 w_0}{\partial x^2} \\ -\frac{\partial^2 w_0}{\partial y^2} \\ -2\frac{\partial^2 w_0}{\partial x \partial y} \end{bmatrix} \] (4)

\[ \varepsilon(M) = \varepsilon_m(x,y) + zK(x,y) \] (5)

According to the classical laminate theory, the expression of the membrane stress can be expressed by equation (6) [5].

\[ N(x,y) = A\varepsilon_m(x,y) + BK(x,y) \] (6)

\[ A_{ij} = \sum_{k=1}^{n} (Q_{ij})_k (h_k - h_{k-1}) \] (7)

\[ B_{ij} = \frac{1}{2} \sum_{k=1}^{n} (Q_{ij})_k (h_k^2 - h_{k-1}^2) \] (8)

A: Membrane matrix
B: Coupling matrix

The vector of bending and torsional moments can be expressed by equation (9):

\[ M_f(x,y) = B_{ij} \varepsilon_m(x,y) + D_{ij} K(x,y) \] (9)

\[ D_{ij} = \frac{1}{3} \sum_{k=1}^{n} (Q_{ij})_k (h_k^3 - h_{k-1}^3) \] (10)

2. Geometrical model

The geometric model is a square plate made with hybrid composite material, of dimension 100 mm × 100 mm, and the number of layers is increased from 2 to 40 for a fixed total thickness of 10 mm. The plate is composed of two plies with configuration -45°/45°, loaded in compression with a fixed intensity load of magnitude of -300N/mm. The induced stress – strain field assessment, is based on the classical laminate theory, for a progressive increase of layers configuration namely (-45°/45°)_1, (-45°/45°)_2, (-45°/45°)_4, (-45°/45°)_8, (-45°/45°)_12, (-45°/45°)_16, and (-45°/45°)_20. Figure 1, illustrates an example of a plate with a configuration of (-45°/45°)_4.
3. Results and discussions

In this section, the results are as follows expressed in terms of the ultimate lateral deflection with U magnitude, and failure load.

Based on the classical laminate theory, the increase of the number of layers is independent from the extensional and bending reduced stiffness matrices. Only the coupling matrix is changed with the increase of the number of layers. Table 1 illustrates the impact of the number of layer on the variation of the coupling reduced stiffness matrix.

$$A = \begin{bmatrix} 280725 & 128566 & 0 \\ 128566 & 280407 & 0 \\ 0 & 0 & 158720 \end{bmatrix}$$

$$D = \begin{bmatrix} 2339371 & 1071379 & 0 \\ 1071379 & 2336723 & 0 \\ 0 & 0 & 1322666 \end{bmatrix}$$

Table 1. Element of coupling reduced stiffness matrix B

| n  | B13, B23, B31 et B32 |
|----|----------------------|
| 1  | 249604               |
| 2  | 124802               |
| 4  | 62401                |
| 8  | 31200                |
| 12 | 20800                |
| 16 | 15600                |
| 20 | 12480                |
3.1. Stress – strain field assessment

Figure 2 below illustrates the distribution of longitudinal, transverse and shear stresses across the thickness of the one layer of the laminate in a linear pattern.

The longitudinal stresses assessed to be 29 MPa are the most important ones in the direction of compression loading, as an external bornes. For this configuration, the transverse s and shear tresses are equivalent, and are estimated to be the halve of the maximum axial stresses.

It’s obvious that, a composite material composed of two layers present a low shrinking about 2%, when compared to monolayer composite. These is true in the axiale direction, but in the transverse one, the shrinkage bacame important about 32%. Figure 2, 3 illustrate the stress – strain field evolving in the composite material of configuration (-45°/45°), and Figure 4, and 4 illustrate the stress – strain for the configuration (-45°/45°)₄.

![Figure 2. Stress evolving in the composite material of configuration (-45°/45°).](image-url)
Figure 3. Strain evolving on the composite material of configuration (-45°/45°).

Figure 4. Stress-strain evolving on the composite material of configuration (-45°/45°).
3.2. Impact of the number of layers on the intensity of the stress – strain field

With the increase of the number of layers, the maximum stress in the cross section of the laminate is shrunk progressively with the progressive thinning of the composite material. If the number of layer is increased from 1 to 20, the ultimate axial stress is reduced about 20%, and the transverse stress is shrunk about 48%. Practically, there is no change in the shear stresses. In addition, the increase of the number of layers from 1 to 20, implies a shrinking in the ultimate strain range. As a result, the ultimate axial strain in the cross section is reduced about 20%, and about 59% in the transverse direction. Figure 6, illustrate the impact of the number of layer on the ultimate axial stress, and transverse strain. The decrease of ultimate stress and strain magnitude induced is multilayer composite material follow a quadratic evolution, and it stabilize after a number of layer equal to 20, corresponding a 40 plies oriented with -45°and 45°.

In fact, it’s very interesting to predict the effect of the number of layers, on the ultimate lateral displacement U. For all configurations (-45°/45°)ₙ with n varied from 1 to 20, the lateral deflection, of the plate subjected to a compressive loading is assessed by coupling the developed analytical model with finite element method, and the impact of the number of layers is investigated. The results of the displacement magnitude as a function of the normalized distance of the plate, for different number of layers, is illustrated in Figure 7.
Figure 6. Impact of the number of layers on the intensity of the stress-strain field.

Figure 7. Evolving of the lateral deflection as a function of the normalized distance.
As illustrated in Figure 7, the thinning or the increase of the number of layers for a fixed total thickness of the whole composite has a significant impact on the lateral displacement of the plate. At a low number of layers, it is practically constant, at the first quarter of the plate. After, the deviation is accentuated, and reach its maximum value at the corner of the plate. At a number of layers more than 32, the lateral deflection coincided, and became independent from the layers number. The tabulation of the results permits to extract the ultimate displacement, localized at the corner of the plate. Finally, a power law mechanical model is developed in order to predict the lateral displacement, as a function of the number of layers, with a determination coefficient equal to 99.86%, as expressed in equation (11).

\[ U_{\text{ult}} = 0.5465 \left( \frac{x}{L} \right)^{1.066} \]  

\[ R^2 = 99.86 \]

\[ U_{\text{ult}} = 0.5465(x/L)^{1.066} \]

Figure 8. Evolving of the ultimate lateral deflection as a function of the normalized distance.

4. Conclusion

The longitudinal stresses developed in the composites are the most important ones in the direction of compression loading. In the longitudinal direction, the composite material composed of two layers present a low shrinking about 2%, when compared to monolayer composite, and the shrinkage became important about 32%, in the transverse direction. Besides, with the increase of the number of layers, the maximum stress in the cross section of the laminate is shrinked progressively with the progressive thinning of the composite material. If the number of layer is increased from 1 to 20, the ultimate axial stress is reduced about 20%, and the transverse stress is shrinked about 48%, and the ultimate axial strain is reduced about 20%, and about 59% in the transverse direction. In addition, the increase of the number of layers for a fixed total thickness, has a significant impact on the lateral displacement of the plate, and a power law mechanical model is developed in order to predict the lateral displacement, as a function of the number of layers.
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