Development of calculation method to study influence of end pieces and asphalt roadway impedance on total sound pressure measured at exhaust pipe mouth

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Abstract. The article contains a developed finite element model of the rear part of the vehicle with the mufflers made adjoining the sphere. Experimental research has been carried out in accordance with the rules of the ECE UN R51 amendment 03. Calculation procedure was made to study the influence of the reflecting surfaces on the total sound pressure by the example of the exhaust pipe mouth. The finite element model of the vehicle rear part has been developed. The results of calculations and experimental research of the sound pressures of the exhaust pipe mouth are given. The developed calculation procedure has been proved by the experiments.

1. Introduction
The total sound pressure measured in the reference point equals to the sum of the primary/direct field and the reflected fields. When performing the finite element modeling of propagation of the sound wave from the main noise sources of the vehicle, it is necessary to consider the acoustic characteristics of the reflecting surfaces of not only the body elements but also the ground surface [1-6]. Therefore, we shall consider the simplest case of monopole radiation over the surface shown in Figure 1.

Figure 1. Monopole radiation over the surface: A – wave front from the source and its imaginary component in the free field, B – real wave front, C – ray model (simplified).
The full field measured in point S is a superposition of the waves transferred by the PS linear segment and the PQ-QS incidence and reflection segment (Figure 1) from the surface that is equivalent to the presence of the \( P_1 \) imaginary source. In view of the above, the full field created by the monopole radiation will be written as follows in accordance with the theory of reflection:

\[
p(S) = A \frac{e^{-ikr}}{r} + A \cdot K \frac{e^{-ikr_1}}{r_1}
\]

where \( r = \sqrt{(y_0 - y)^2 + x^2} \) and \( r_1 = \sqrt{(y_0 + y)^2 + x^2} \)

\[
K = \frac{-1+2\cos\theta}{1+2\cos\theta} - \text{reflection coefficient;}
\]

\[
\cos\theta = \frac{y_0 + y}{\sqrt{(y_0 + y)^2 + x^2}}, \theta - \text{incidence/reflection angle;}
\]

\[
Z - \text{reflecting surface impedance;}
\]

As shown by Formula 1, the reflected field summand is multiplied by the reflection coefficient, which in turn depends on the reflecting surface impedance and incidence angle. In view of the above let us examine two border-line cases of reflecting surface impedance:

- When the reflecting surface impedance tends to infinity (absolutely rigid boundary/high-impedance contrast), then at any incidence angles, the summand \( \frac{\rho_g(\omega)K_g(\omega)}{\Omega} \) of the numerator and denominator will be much larger than the first summands of the numerator and denominator of the formula for determining the reflection coefficient, therefore the reflection coefficient is \( K = 1 \).

- When the reflecting surface impedance equals to zero (absolutely soft boundary/low-impedance contrast), the reflection coefficient will be \( K = -1 \) at any incidence angles.

In other cases, \( 0 < Z < \infty \), the reflection coefficient will depend on the angle.

The acoustic characteristics of the body panels can be considered as a rigid boundary, and based on the classical theory for determination of acoustic characteristics of the reflecting surfaces made of a pored uniform material, a lot of models describing ground impedance were developed, which are provided in this paper [7]. However, the results of experimental research [8] of the cut samples (in particular of asphalt and concrete) in the impedance tube have gained wide use. The developed model is defined by the following formula:

\[
Z = \sqrt[{|\rho_g(\omega)K_g(\omega)|}]{\Omega}
\]

where \( \rho_g(\omega) \) – effective density;
\( K_g(\omega) \) – effective elasticity;
\( \Omega = 0,15 \) – asphalt porosity.

Formula 3 in turn defines the effective density.

\[
\rho_g(\omega) = \rho_0 q^2 (1 + \frac{f_\Omega}{f})
\]

where \( \rho_0 = 1,204 \text{ kg/m}^3 \) – air density
\( q^2 = 3,3 \) – form parameter (asphalt waviness)
\( f_\Omega = \frac{R_s}{2\pi \rho_0 c_0 q^2} \), where \( R_s = 2000 \text{ N} \cdot \text{s/m}^3 \) – asphalt blowing resistance coefficient
\( c_0 = 343,1 \text{ m/s} \) – acoustic velocity in the air

And the effective elasticity is defined by Formula 4.

\[
K_g(\omega) = \frac{\gamma P_0}{1+(\gamma - 1)(1 - \frac{f}{f_y})}
\]

where \( P_0 = 2 \cdot 10^{-5} \text{ Pa} \) – atmospheric pressure
\( \gamma = 1,4 \) – heat capacity ratio for the air
\( f_y = \frac{R_s}{2\pi \rho_0 N_r} \), where \( N_r = 0,85 \) – Prandtl number, kinematic viscosity coefficient to asphalt thermal diffusivity coefficient ratio
At high frequencies, the radiation of the directional source (exhaust pipe end mouth of the exhaust system) becomes directional (torch-shaped), and its intensity depends on frequency.

2. Calculated analysis

In order to factor in the abovementioned peculiarities, a finite element model of the rear part of the vehicle with the mufflers was made adjoining the sphere having the 6 meters diameter, which allows simulating the free field, with the purpose of visualizing the sound pressure distribution and installing the microphones in the points of interest of the space. The type of elements used in the model is the second-order tetrahedral ones as shown in Figure 2.

**Figure 2.** Finite element model of the vehicle rear part in section.

Figure 3 shows boundary conditions, the sphere surface is simulated by infinite PLM elements (complete attenuation with no acoustic wave reflection). The asphalt roadway impedance was set under the vehicle bottom according to Formula 2, the rest of the surfaces had an absolutely rigid boundary.

**Figure 3.** Boundary conditions.
The channel modes, which were located at the end face of the muffler cylindrical pipe «input», were selected as a source of sound waves. As it is known, any acoustic wave, which propagates inside a channel, is divided into the incident and reflecting waves.

\[ p = p^+ + p^- \]

The peculiarity of channel modes is as follows: only a plane wave propagates at low frequencies inside the channel, and at higher frequencies, the acoustic field may be more complex, but in any case, the pressure may be considered a mathematical superposition of channel modes, the formulas of which are shown below:

\[ p^+ = \sum \alpha_i^+ \psi_i^+ \]
\[ p^- = \sum \alpha_i^- \psi_i^- \]

where \( \alpha \) is a contribution factor, \( \psi \) is a mode form.

The boundary conditions for cylindrical surface reflecting walls in the muffler itself are given as an example (see Fig. 4 and Formulas 5, 6).

\[ \frac{\partial p}{\partial x} \bigg|_{x=0} = \frac{\partial p}{\partial x} \bigg|_{x=r} = 0 \] (5)
\[ \frac{\partial p}{\partial y} \bigg|_{y=0} = \frac{\partial p}{\partial y} \bigg|_{y=r} = 0 \] (6)

Figure 4. Boundary conditions: reflecting wall of the muffler model cylindrical surface

Solution of the Helmholtz equation for round channel modes.

\[ p(x, y, z) = A \cos(k_x x)\cos(k_y y)e^{ik_z z} \] (7)

is true as long as the following ratio is maintained:

\[ k_x^2 + k_y^2 + k_z^2 = k^2 \] (8)

The following conditions shall be met for defining the boundary conditions of the channel modes with reflecting walls:

\[ k_x = m \frac{\pi}{r}, k_y = n \frac{\pi}{r} \] (9)

From Equation 8 taking into account Equation 9, we define

\[ k_z = \sqrt{k^2 - \frac{\pi^2}{r^2} m^2 - \frac{\pi^2}{r^2} n^2} \] (10)
In case of excitation by a channel mode, the pressure profile in the cross-section is defined by Bessel functions and represents a set of waves. M and n are wave numbers showing how many half-waves fit along the radius of the cross-section. Thus, the sound pressures were calculated in the reference point which were located at the exhaust pipe mouth – in the horizontal plane at 14 cm distance at 45° angle.

The calculation result for sound pressures of two modifications with and without the end piece are given in Figure 5.

When installing the end pieces into the exhaust pipe, the increase of sound pressure is observed within the entire frequency range up to 3 dB.

In Figure 6A, the spherical radiation is observed from the exhaust pipe end piece outlet, thus the sound pressure does not increase in the space between the internal side of the bumper and the external side of the muffler. In Figure 6B, the sound pressure increases in the space between the internal side of the bumper and the external side of the muffler, and the equidirectional radiation falls at the lower part of the bumper.

In order to check the adequacy of the calculated studies, the experimental research was performed. Test mode: idling. The testing was performed with two configurations:
1. With the guiding end pieces and inserts in the bumper;
2. Without the guiding end pieces, but with the inserts in the bumper.

In figure 7, the test object is shown with the end pieces at the exhaust pipe mouth and the experimental research results.

Indeed, the frequency response functions of the measured noise of the exhaust system differ from the frequency response functions of the calculated study, therefore, in order to define the adequacy of the calculated studies, the differences between the sound pressures with and without the end pieces were compared in the bandwidth up to 200 Hz.

The average values of the difference of the experimental research with and without the end piece amounted to ~3 dB and of the calculated study amounted to ~3 dB, which confirms the adequacy of the developed procedure.
3. Conclusion
The developed procedure enables to consider and take into account the influence of the end pieces, as well as the reflecting surface of the asphalt roadway that influences significantly the total sound pressure measured at the exhaust pipe mouth.

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