Why the Parity Violation

G. Quznetsov
quznets@yahoo.com

June 29, 1999

Abstract

The sufficient and necessary conditions for a nonzero fermion mass without Higgs are considered. The Parity Violation is deduced from these conditions.

In this paper I consider a global gauge transformation. I use the following notation:

\[ 1_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad 0_2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad p_u = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \quad p_d = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \]

\[ \sigma_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad \sigma_y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}, \quad \sigma_z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \]

\[ \beta_1 = \begin{bmatrix} \sigma_x & 0_2 \\ 0_2 & -\sigma_x \end{bmatrix}, \quad \beta_2 = \begin{bmatrix} \sigma_y & 0_2 \\ 0_2 & -\sigma_y \end{bmatrix}, \quad \beta_3 = \begin{bmatrix} \sigma_z & 0_2 \\ 0_2 & -\sigma_z \end{bmatrix}, \]

\[ \beta_4 = i \cdot \begin{bmatrix} 0_2 & 1_2 \\ -1_2 & 0_2 \end{bmatrix}, \quad \beta_0 = \begin{bmatrix} 1_2 & 0_2 \\ 0_2 & 1_2 \end{bmatrix} = 1_4, \quad \gamma_0 = \begin{bmatrix} 0_2 & 1_2 \\ 1_2 & 0_2 \end{bmatrix}, \]

\[ \gamma_5 = \begin{bmatrix} 1_2 & 0_2 \\ 0_2 & -1_2 \end{bmatrix}, \]
$$1_{8} = \begin{bmatrix} \beta_0 & 0_4 \\ 0_4 & \beta_0 \end{bmatrix},$$

$$0_8 = 1_8 - 1_{8},$$

$$\gamma_1 = \gamma_0 \cdot \beta_1, \gamma_2 = \gamma_0 \cdot \beta_2, \gamma_3 = \gamma_0 \cdot \beta_3, \gamma_5 = i \cdot \gamma_0 \cdot \beta_4.$$ 

$$0_4 = \begin{bmatrix} 0_2 & 0_2 \\ 0_2 & 0_2 \end{bmatrix}, \gamma_0 = \begin{bmatrix} \gamma_0 & 0_4 \\ 0_4 & \gamma_0 \end{bmatrix}, \beta_n = \begin{bmatrix} \beta_n & 0_4 \\ 0_4 & \beta_n \end{bmatrix}.$$ 

## 1 Hints

1) Let us consider the free lepton Lagrangian [1]:

$$\mathcal{L} = 0.5 \cdot i \cdot \left( \overline{\psi} \cdot \gamma^\mu \cdot (\partial_\mu \psi) - (\partial_\mu \overline{\psi}) \cdot \gamma^\mu \cdot \psi \right) - m \cdot \overline{\psi} \cdot \psi.$$ 

Hence:

$$\mathcal{L} = 0.5 \cdot i \cdot \left( \psi^\dagger \cdot \beta^\mu \cdot (\partial_\mu \psi) - (\partial_\mu \psi^\dagger) \cdot \beta^\mu \cdot \psi \right) - m \cdot \psi^\dagger \cdot \gamma^0 \cdot \psi.$$ 

This Lagrangian contains four matrices from the Clifford pentad [2]

$$\{\gamma_0, \beta_1, \beta_2, \beta_3, \beta_4\},$$ but one does not contain \(\beta_4\).

2) Let us consider the lepton current:

$$j_\mu = \psi^\dagger \cdot \beta^\mu \cdot \psi.$$ 

for \(0 \leq \mu \leq 3\).

Let us denote:

$$J_\gamma = \psi^\dagger \cdot \gamma^0 \cdot \psi$$ and $$J_4 = \psi^\dagger \cdot \beta^4 \cdot \psi.$$ 

In this case if
\( \rho = j_0 \)

then the average velocity vector is:

\[
\rho \cdot v_x = j_1, \quad \rho \cdot v_y = j_2, \quad \rho \cdot v_z = j_3.
\]

Let us denote:

\[
\rho \cdot V_\gamma = J_\gamma \quad \text{and} \quad \rho \cdot V_4 = J_4.
\]

In this case:

\[
v_x^2 + v_y^2 + v_z^2 + V_\gamma^2 + V_4^2 = 1.
\]

Hence of only all five elements of the Clifford pentad lends the entire kit of the velocity components.

3) In the Standard Model we have got the following entities:
the right electron field vector \( e_R \),
the left electron field vector \( e_L \),
the electron field vector \( e \) (\( e = \begin{bmatrix} e_L \\ e_R \end{bmatrix} \)),
the left neutrino fields vector \( \nu_L \),
the zero right neutrino fields vector \( \nu_R \),
the unitary \( 2 \times 2 \) \( SU(2) \) matrix \( U \) of the isospin transformation:

\[
U = \begin{bmatrix}
\cos(\varepsilon) + i \cdot n_3 \cdot \sin(\varepsilon) & (i \cdot n_1 + n_2) \cdot \sin(\varepsilon) \\
(i \cdot n_1 - n_2) \cdot \sin(\varepsilon) & \cos(\varepsilon) - i \cdot n_3 \cdot \sin(\varepsilon)
\end{bmatrix},
\]

\( \varepsilon, n_1, n_2, n_3 \) are real and:

\[
n_1^2 + n_2^2 + n_3^2 = 1.
\]

This matrix acts on the vectors of the kind: \( \begin{bmatrix} \nu_L \\ e_L \end{bmatrix} \).

Therefore, in this theory (the \((j,0)+(j,0)\) representation space: \( \exists \), \( \forall \), \( \mathbb{F} \)):

\[
U = \begin{bmatrix}
u_{1,1} & u_{1,2} \\
u_{2,1} & u_{2,2}
\end{bmatrix}
\]
then the matrix

\[
U = \begin{bmatrix}
  u_{1,1} \cdot 1_2 & 0 & u_{1,2} \cdot 1_2 & 0_2 \\
  0_2 & 1_2 & 0_2 & 0_2 \\
u_{2,1} \cdot 1_2 & 0_2 & u_{2,2} \cdot 1_2 & 0_2 \\
  0_2 & 0_2 & 0_2 & 1_2 \\
\end{bmatrix}
\]  

operates on the vector

\[
\begin{bmatrix}
  \nu_L \\
  \nu_R \\
  e_L \\
  e_R \\
\end{bmatrix}
\]

Because \( e_R, e_L, \nu_L, \nu_R \) are the two-component vectors then

\[
\begin{bmatrix}
  \nu_L \\
  \nu_R \\
  e_L \\
  e_R \\
\end{bmatrix}
is
\begin{bmatrix}
  \nu_{L1} \\
  \nu_{L2} \\
  \nu_{R1} \\
  \nu_{R1} \\
\end{bmatrix}
\]

\( U \) has got eight orthogonal normalized eigenvectors \( s_1, s_2, s_3, s_4, s_5, s_6, s_7, s_8 \):

\[
s_1 = \begin{bmatrix}
  0 \\
  1 \\
  0 \\
  0 \\
\end{bmatrix},
\quad
s_2 = \begin{bmatrix}
  0 \\
  0 \\
  1 \\
  0 \\
\end{bmatrix},
\quad
s_3 = \begin{bmatrix}
  a \\
  0 \\
  0 \\
  0 \\
\end{bmatrix},
\quad
s_4 = \begin{bmatrix}
  0 \\
  a \\
  0 \\
  0 \\
\end{bmatrix},
\]

\[
\begin{array}{cccc}
  0 & a \\
  a & 0 \\
  0 & 0 \\
  b + i \cdot c & 0 \\
\end{array}
\]

\[
\begin{array}{cccc}
  b + i \cdot c & 0 \\
  0 & 0 \\
  0 & 0 \\
  0 & 0 \\
\end{array}
\]
Let for all $k \leq 8$:

$$h_k = \gamma_0 \cdot s_k$$

and let:

$$M = \begin{bmatrix} h_1 & h_2 & h_3 & h_4 & h_5 & h_6 & h_7 & h_8 \end{bmatrix}.$$

Let:

$$P_3 = \begin{bmatrix} 0_2 & 0_2 & 0_2 & 0_2 \\ 0_2 & p_u & 0_2 & 0_2 \\ 0_2 & 0_2 & 0_2 & 0_2 \end{bmatrix}, P_4 = \begin{bmatrix} 0_2 & 0_2 & 0_2 & 0_2 \\ 0_2 & p_d & 0_2 & 0_2 \\ 0_2 & 0_2 & 0_2 & 0_2 \end{bmatrix},$$

$$P_7 = \begin{bmatrix} 0_2 & 0_2 & 0_2 & 0_2 \\ 0_2 & 0_2 & 0_2 & 0_2 \\ 0_2 & 0_2 & 0_2 & p_u \\ 0_2 & 0_2 & p_u & 0_2 \end{bmatrix}, P_8 = \begin{bmatrix} 0_2 & 0_2 & 0_2 & 0_2 \\ 0_2 & 0_2 & 0_2 & 0_2 \\ 0_2 & 0_2 & 0_2 & p_d \end{bmatrix}.$$

In this case the projection matrices are:
\[ Y_1 = M \cdot P_3 \cdot M^\dagger, \]
\[ Y_2 = M \cdot P_4 \cdot M^\dagger, \]
\[ Y_3 = K \cdot P_3 \cdot K^\dagger, \]
\[ Y_4 = K \cdot P_4 \cdot K^\dagger, \]
\[ Y_5 = M \cdot P_7 \cdot M^\dagger, \]
\[ Y_6 = M \cdot P_8 \cdot M^\dagger, \]
\[ Y_7 = K \cdot P_7 \cdot K^\dagger, \]
\[ Y_8 = K \cdot P_8 \cdot K^\dagger. \]

The vectors:

\[
\begin{bmatrix}
0 \\
0 \\
0 \\
e_{L1} \\
e_{L2} \\
e_{R1} \\
e_{R2}
\end{bmatrix}
\begin{bmatrix}
0 \\
0 \\
0 \\
e_{R1} \\
e_{R2}
\end{bmatrix}
\begin{bmatrix}
0 \\
0 \\
0 \\
e_{L1} \\
e_{L2} \\
e_{L2}
\end{bmatrix}
\]

correspond to the vectors \( e, e_R \) and \( e_L \) resp.

Let:

\[
X_a = Y_1 + Y_2 + Y_3 + Y_4, \\
X_b = Y_5 + Y_6 + Y_7 + Y_8, \\
e_a = X_a \cdot e, \\
e_b = X_b \cdot e.
\]

In this case:

\[
X_a + X_b = 1_8, \\
X_a \cdot X_b = 0_8, \\
X_a \cdot X_a = X_a, \\
X_b \cdot X_b = X_b, \\
X_a^\dagger = X_a, \\
X_b^\dagger = X_b.
\]

Let:
\[\rho_a = e_a^\dagger \cdot e_a, \rho_b = e_b^\dagger \cdot e_b,\]
\[J_{\gamma,a} = e_a^\dagger \cdot \gamma_0 \cdot e_a, J_{\gamma,b} = e_b^\dagger \cdot \gamma_0 \cdot e_b,\]
\[J_{4,a} = e_a^\dagger \cdot \beta_4 \cdot e_a, J_{4,b} = e_b^\dagger \cdot \beta_4 \cdot e_b,\]
\[J_{\gamma,a} = \rho_a \cdot V_{\gamma,a}, J_{\gamma,b} = \rho_b \cdot V_{\gamma,b},\]
\[J_{4,a} = \rho_a \cdot V_{4,a}, J_{4,b} = \rho_b \cdot V_{4,b}.\]

Let:
\[e_a' = U \cdot e_a, e_b' = U \cdot e_b,\]
\[\rho_a' = e_a'^\dagger \cdot e_a', \rho_b' = e_b'^\dagger \cdot e_b',\]
\[J_{\gamma,a}' = e_a'^\dagger \cdot \gamma_0 \cdot e_a', J_{\gamma,b}' = e_b'^\dagger \cdot \gamma_0 \cdot e_b',\]
\[J_{4,a}' = e_a'^\dagger \cdot \beta_4 \cdot e_a', J_{4,b}' = e_b'^\dagger \cdot \beta_4 \cdot e_b',\]
\[J_{\gamma,a}' = \rho_a' \cdot V_{\gamma,a}, J_{\gamma,b}' = \rho_b' \cdot V_{\gamma,b},\]
\[J_{4,a}' = \rho_a' \cdot V_{4,a}, J_{4,b}' = \rho_b' \cdot V_{4,b}.\]

In this case:
\[V_{\gamma,a} = V_{\gamma,b}, V_{4,a} = V_{4,b}\]

but:
\[V_{\gamma,a}' = V_{\gamma,a} \cos(\lambda) - V_{4,a} \sin(\lambda),\]
\[V_{4,a}' = V_{4,a} \cos(\lambda) + V_{\gamma,a} \sin(\lambda),\]
\[V_{\gamma,b}' = V_{\gamma,b} \cos(\lambda) + V_{4,b} \sin(\lambda),\]
\[V_{4,b}' = V_{4,b} \cos(\lambda) - V_{\gamma,b} \sin(\lambda).\]

Hence, every isospin transformation \(U\) divides a electron on two components which scatter on the angle \(2 \cdot \lambda\) in the space of \((J_\gamma, J_4)\).

Hence \(\beta^4\) must be inserted into Lagrangian.

\section{Sufficient Conditions}

Let \(\psi\) be any field of the following type:
\[
\psi = \begin{bmatrix}
0 \\
0 \\
0 \\
\psi_{L_1} \\
\psi_{L_2} \\
\psi_{R_1} \\
\psi_{R_2}
\end{bmatrix}.
\]
The value of the form

$$
\left( (\psi^\dagger \cdot \gamma^0 \cdot X_a \cdot \psi)^2 + (\psi^\dagger \cdot \beta^4 \cdot X_a \cdot \psi)^2 \right)^{0.5} +
$$

$$
+ \left( (\psi^\dagger \cdot \gamma^0 \cdot X_b \cdot \psi)^2 + (\psi^\dagger \cdot \beta^4 \cdot X_b \cdot \psi)^2 \right)^{0.5}
$$

(2)

does not depend from the choice of the \(SU(2)\) matrix \(U\) and the Lagrangian:

$$
\mathcal{L}_\psi = 0.5 \cdot i \cdot \left( (\psi^\dagger \cdot \beta^\mu \cdot (\partial_\mu \psi) - (\partial_\mu \psi)^\dagger \cdot \beta^\mu \cdot \psi \right) -
$$

$$
- m_\psi \cdot \left( \left( (\psi^\dagger \cdot \gamma^0 \cdot X_a \cdot \psi)^2 + (\psi^\dagger \cdot \beta^4 \cdot X_a \cdot \psi)^2 \right)^{0.5} +
$$

$$
+ \left( (\psi^\dagger \cdot \gamma^0 \cdot X_b \cdot \psi)^2 + (\psi^\dagger \cdot \beta^4 \cdot X_b \cdot \psi)^2 \right)^{0.5} \right)
$$

is invariant for this \(SU(2)\) transformation.

Let us denote:

$$
\frac{\psi^\dagger \cdot \gamma^0 \cdot X_a \cdot \psi}{\sqrt{(\psi^\dagger \cdot \gamma^0 \cdot X_a \cdot \psi)^2 + (\psi^\dagger \cdot \beta^4 \cdot X_a \cdot \psi)^2}} = \cos (\alpha_a),
$$

$$
\frac{\psi^\dagger \cdot \beta^4 \cdot X_a \cdot \psi}{\sqrt{(\psi^\dagger \cdot \gamma^0 \cdot X_a \cdot \psi)^2 + (\psi^\dagger \cdot \beta^4 \cdot X_a \cdot \psi)^2}} = \sin (\alpha_a),
$$

$$
\frac{\psi^\dagger \cdot \gamma^0 \cdot X_b \cdot \psi}{\sqrt{(\psi^\dagger \cdot \gamma^0 \cdot X_b \cdot \psi)^2 + (\psi^\dagger \cdot \beta^4 \cdot X_b \cdot \psi)^2}} = \cos (\alpha_b),
$$

$$
\frac{\psi^\dagger \cdot \beta^4 \cdot X_b \cdot \psi}{\sqrt{(\psi^\dagger \cdot \gamma^0 \cdot X_b \cdot \psi)^2 + (\psi^\dagger \cdot \beta^4 \cdot X_b \cdot \psi)^2}} = \sin (\alpha_b)
$$

Let:

$$
\gamma = (\cos (\alpha_a) \cdot \gamma^0 + \sin (\alpha_a) \cdot \beta^4) \cdot X_a + (\cos (\alpha_b) \cdot \gamma^0 + \sin (\alpha_b) \cdot \beta^4) \cdot X_b.
$$

In this case:
\[ \gamma \cdot \gamma = 1_8 \]

and if \( 1 \leq k \leq 3 \) then
\[ \gamma \cdot \beta^k = -\beta^k \cdot \gamma \]

and the Euler-Lagrange equation for \( L_\psi \) is the following:
\[ \left( i \cdot \beta^\mu \cdot \partial_\mu - m_\psi \cdot \gamma \right) \cdot \psi = 0. \]

Since
\[ \alpha_a = \alpha_b \]

then
\[ \gamma = \cos (\alpha_a) \cdot \gamma^0 + \sin (\alpha_a) \cdot \beta^4. \]

Let \( \psi \) be a plane wave electron spinor \[ \begin{array}{l} \psi \\ \end{array} \] with a positive energy:
\[ \psi = \left( \begin{array}{c}
 a_1 \cdot \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\
 \frac{p_\mu}{E+m_e} \\
 \frac{p_\mu^2+p_y}{E+m_e}
\end{bmatrix} + a_2 \cdot \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\
 \frac{p_e+p_y}{E+m_e} \\
 \frac{-p_e}{E+m_e}
\end{bmatrix} \right) \cdot \exp (-i \cdot p \cdot x), \]

here:
\( a_1, a_2 \) are complex, and \( E = \sqrt{p^2 + m_e^2}. \)

In this case:
\[ \cos (\alpha_a) = 1. \]

Hence
\[ \gamma = \gamma^0 \]

and the Euler-Lagrange equation is the following:
\[ \left( i \cdot \gamma^\mu \cdot \partial_\mu - m_\psi \right) \cdot \psi = 0. \]
3 Necessary Conditions

Let $U$ be any $8 \times 8$ complex matrix for which the Lagrangian

$$L_0 = 0.5 \cdot i \cdot \left( \psi^\dagger \cdot \beta^\mu \cdot (\partial_\mu \psi) - (\partial_\mu \psi)^\dagger \cdot \beta^\mu \cdot \psi \right)$$

is invariant. Hence

$$U^\dagger \cdot U = 1_8$$

and for all $\mu$ ($1 \leq \mu \leq 3$):

$$U \cdot \beta^\mu = \beta^\mu \cdot U.$$

$U$ must be of the following type from this commutativity:

$$U = \begin{bmatrix}
  z_{1,1} & 0 & 0 & 0 & z_{1,5} & 0 & 0 & 0 \\
  0 & z_{1,1} & 0 & 0 & 0 & z_{1,5} & 0 & 0 \\
  0 & 0 & z_{3,3} & 0 & 0 & 0 & z_{3,7} & 0 \\
  0 & 0 & 0 & z_{3,3} & 0 & 0 & 0 & z_{3,7} \\
  z_{5,1} & 0 & 0 & 0 & z_{5,5} & 0 & 0 & 0 \\
  0 & z_{5,1} & 0 & 0 & 0 & z_{5,5} & 0 & 0 \\
  0 & 0 & z_{7,3} & 0 & 0 & 0 & z_{7,7} & 0 \\
  0 & 0 & 0 & z_{7,3} & 0 & 0 & 0 & z_{7,7}
\end{bmatrix}$$

(here $z_{j,k}$ are a complex) and from the unitarity if

$$z_{j,k} = x_{j,k} + i \cdot y_{j,k}$$

then

$$1 - x_{1,5}^2 - y_{1,5}^2 - y_{5,5}^2 \geq 0,$$

$$x_{1,1} = \sqrt{1 - x_{1,5}^2 - y_{1,5}^2 - y_{5,5}^2},$$

$$x_{5,5} = x_{1,1},$$

$$x_{5,1} = -x_{1,5},$$

$$y_{1,1} = -y_{5,5},$$

$$y_{5,1} = y_{1,5}$$

and
1 - x_{3,7}^2 - y_{3,7}^2 - y_{7,7}^2 \geq 0, \\
x_{3,3} = \sqrt{1 - x_{3,7}^2 - y_{3,7}^2 - y_{7,7}^2}, \\
x_{7,7} = x_{3,3}, \\
x_{7,3} = -x_{3,7}, \\
y_{3,3} = -y_{7,7}, \\
y_{7,3} = y_{3,7}.

If

\begin{align*}
x_{3,7} &= 0, \\
y_{3,7} &= 0, \\
y_{7,7} &= 0
\end{align*}

then

z_{3,3} = 1

and \( U \) is the matrix of type (1). In this case the mass form (2) is invariant for \( U \) and a right-handed particles do not interact by this transformation.

Therefore if an electron has got a nonzero mass, provided with the mass form (2), then all neutrinos must be left-handed.

Like this, for \( z_{1,1} = 1 \), all antineutrinos must be right-handed to an positron has got a nonzero mass.

If \( z_{1,1} \neq 1 \) and \( z_{3,3} \neq 1 \) then a mass form, invariant for \( U \), does not exist.

4 Acknowledgment

Thanks very much to Prof. V. V. Dvoeglazov for his papers which he had sent me kindly in 1998.

References

[1] for example, from L. H. Ryder, Quantum Field Theory . (Cambridge University Press, 1985)

[2] for instance, E.Madelung, Die Mathematischen Hilfsmittel des Physikers. (Springer Verlag, 1957)
[3] V. V. Dvoeglazov, Additional Equations Derived from the Ryder Postulates in the (1/2,0)+(0,1/2) Representation of the Lorentz Group. [hep-th/9906083]. Int. J. Theor. Phys. 37 (1998) 1909. Helv. Phys. Acta 70 (1997) 677. Fizika B 6 (1997) 75; Int. J. Theor. Phys. 34 (1995) 2467. Nuovo Cimento 108 A (1995) 1467. Nuovo Cimento 111 B (1996) 483. Int. J. Theor. Phys. 36 (1997) 635.

[4] D. V. Ahluwalia, (j,0)+(0,j) Covariant spinors and causal propagators based on Weinberg formalism. [nucl-th/9905047]. Int. J. Mod. Phys. A 11 (1996) 1855.

[5] D. V. Ahluwalia et al., Phys. Lett. B 316 (1993) 102.