Slow viscous flow in a microchannel with similar and different superhydrophobic walls

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Abstract. The paper continues our studies of Stokes flows over superhydrophobic surfaces using the Boundary Element Method. A parametric investigation of a pressure-driven viscous-fluid flow in a plane microchannel, whose walls are similarly or differently textured superhydrophobic surfaces, is performed. A general case is under consideration, when both walls of the channel have periodically located striped microcavities, which can only partially be filled with gas bubbles. The main aim of the paper is to estimate the reduction in the hydraulic drag of microchannels with different geometrical parameters of microcavities on the channel walls. The main new features of the problem formulation are the consideration of a narrow channel with microcavities located asymmetrically on the bottom and top walls and the textured walls with cavities completely filled with the fluid.

1. Introduction
Superhydrophobic surfaces (SHS) provide a noticeable friction reduction when a viscous fluid flows over such surfaces [1]. This effect on a SHS is achieved due to the heterogeneous nature of its surface on the microscale [2]. Such surfaces are produced via a combination of mechanical texturing (creation of microcavities of microns or tens of microns in size) and the chemical hydrophobicity of the surface. Inside the cavities, gas bubbles stably trapped by the surface tension may be present. Since the friction between the fluid and the gas is low, on the macroscale the average friction is noticeably reduced and the average slip velocity of the fluid on the surface is non-zero. The capacity of SHS to reduce friction in viscous fluid flows has made such surfaces an object of intense interdisciplinary studies. Along with the production of more and more advanced textured SHS, the current studies are aimed at the development of mathematical models of viscous fluid flows over SHS on the microscale (scale of the surface texture) and the calculation of the effective hydrodynamic characteristics of SHS (slip length), which can be used in the effective slip boundary condition of Navier type [3] for averaged description of flows near SHS. The main difficulty arising in modeling viscous fluid flows over SHS is the necessity to calculate the fluid motion equations in a domain with complex boundary conditions, containing one or several phase interfaces. This results in the need to develop efficient numerical methods for calculating the hydrodynamic characteristics of a textured surface, which then can be used in the design of advanced SHS.

In this paper, within the Stokes approximation we consider a pressure-driven flow in a microchannel with one or two superhydrophobic walls, the texture of which contain gas bubbles. The case when the gas bubbles are absent and the microcavities are completely filled with the fluid is also considered. On both walls, the striped microcavities are located periodically, hence the velocity field is assumed to
be periodic. However, in general case the geometrical parameters of the cavities and the periods are different for the bottom and the top wall; accordingly, the flow can be asymmetrical with respect to the central line of the channel. The bubble surfaces are assumed to have a static shape. The ends of the phase interfaces in the cavities may be either pinned at the cavity corners or shifted into the cavities. A numerical procedure based on the Boundary Element Method for Stokes flow on one period of the flow is developed. A parametric numerical study of the pressure drop reduction in the microchannel with similar and different types of the walls is performed and the flow patterns in a periodic flow domain with two phase interfaces are investigated. In our previous papers [4, 5], it was shown that the shift of the phase interface into the cavity of a SHS reduces the averaged velocity slip on such SHS. Below, we will estimate how the effect of slip on both superhydrophobic walls of the channel may increase the friction reduction effect and decrease the pressure drop in microfluidic devices with both SHS.

2. Pressure-driven flow in a microchannel with superhydrophobic walls

We consider a steady-state viscous fluid flow with a constant volume flow rate \(Q^*\) in a microchannel with both striped superhydrophobic surfaces. The flow field is assumed to be periodic, and hence we may consider a flow domain corresponding to one period of the flow (figure 1). It should be noted that such problems arise in microfluidic devices used for pumping a viscous fluid. It is anticipated that using both superhydrophobic walls may substantially reduce the pressure drop in the device. In a general case, the surfaces of the gas bubbles in the microcavities may either protrude into the bulk flow or be convex into the cavities.

![Figure 1](image_url)

**Figure 1.** Pressure-driven flow in a microchannel with superhydrophobic walls with rectangular microcavities partially filled with gas bubbles

Two particular cases are also considered, when only the bottom surface is superhydrophobic or both walls are textured but the bubbles are absent and the cavities are completely filled with the fluid.

Let us denote the dimensional thickness of the microchannel as \(H\), the length of the chosen flow period as \(L\), and the widths of the microcavities on the bottom and top walls as \(d_1, d_2\). As \(\delta_1, \delta_2\), we denote the dimensional shifts of the menisci into the microcavities on the bottom and top SHS. Here, the subscripts 1 and 2 correspond to the bottom and top wall, respectively. As the velocity scale \(U\), we choose \(Q^*/H\), and the pressure scale is \(\mu U/L\), where \(\mu\) is the dynamic viscosity of the fluid. The shape of the gas bubble surface is assumed to be static and can be approximated with a good accuracy by a circle arc [6], with the
arc radius being determined by the static angle of wettability. The static angles of wettability \( \theta_1 = -90^\circ \) and \( \theta_2 = 90^\circ \) correspond to the gas bubbles fully protruding into the cavities. The angles \( \theta_1 = 90^\circ \) and \( \theta_2 = -90^\circ \) correspond to the bubbles fully protruding into the bulk flow in the microchannel. This notation is similar to that introduced in [7]. In our calculations, we assume that the shape of the phase interfaces is prescribed. After the nondimensionalization of the problem, we obtain the following set of similarity parameters: \( H/L, d_{1,2}/L, s = \delta_{1,2}/L, \theta_{1,2} \) and \( x_{1,2} \). Here, \( x_{1,2} \) denote the scaled coordinates of the centers of the cavities on the microchannel walls, measured from the middle point of the chosen periodic flow domain. The set of the governing parameters includes all possible cases of geometry of a pressure-driven flow in a microchannel with SHS which can arise in practice. For convenience, the semi-widths of the cavities \( c_{1,2} = d_{1,2}/2L \) are introduced. Using these parameters, we can write the geometric relation of the bubble radius with the static angle of wettability as \( c_{1,2} = R_{1,2}/|\cos(\theta_{1,2} \mp 90^\circ)| \). The case \( \theta_{1,2} = 0^\circ \) corresponds to a flat phase interface, being either pinned at the cavity corners or shifted into the cavity. We note that when the phase interface is shifted into the cavities it is assumed that the bubble surface protrudes into the cavity.

3. Stokes problem of a pressure-driven viscous fluid flow in the selected domain

In dimensionless form, the equations of fluid motion in a chosen periodic 2D domain containing microcavities on the bottom and top walls (filled with either gas bubbles or viscous fluid) can be written as follows:

\[
\Delta \mathbf{u} = \nabla p, \quad \nabla \cdot \mathbf{u} = 0
\]

Here, \( \mathbf{u} \) is the fluid velocity vector and \( p \) is the pressure in the fluid. We specify the following boundary conditions: on the solid walls, the no-slip condition for the fluid velocity is valid \( \mathbf{u} = 0 \); and on the bubble surfaces, the kinematic (no-flow) condition \( \mathbf{u} \cdot \mathbf{n} = 0 \) and the dynamic condition of zero shear stress \( \sigma_{ij} \mathbf{n}_i \mathbf{t}_j = 0 \) are satisfied. Here, \( \mathbf{n}_i \) and \( \mathbf{t}_j \) are the components of the unit normal and the unit tangent to the flow domain boundary. At the inlet and outlet of the calculation domain \( x = \pm 0.5 \), we specify the identical Poiseuille velocity profiles: \( u(x = \pm 0.5) = -y^2/2 + yH/2L \), \( v(x = \pm 0.5) = 0 \).

Such velocity profiles satisfy the periodicity condition for small values of the gas fraction \( d_{1,2}/L \) (see [5, 6], where a similar problem was considered for a fluid flow over bottom SHS). In the general case, when the gas fraction \( d_{1,2}/L \) on both channel walls tends to unity and/or the cavities are located asymmetrically, to ensure the smoothness of the periodic velocity field, it is necessary to assume the equality of the partial derivatives of the fluid velocity components with respect to \( x \) at the inlet and outlet boundaries of the flow domain. When \( d_{1,2}/L \) tend to unity or the cavities on different walls are noticeably shifted relative each other, the real periodic velocity profile will deviate from the standard Poiseuille profile [5, 6]. Below, we present only the results for the case of sufficiently small gas fractions. We will analyze the pressure drop reduction in a channel with different types of channel walls containing fairly narrow cavities, occupied by either the gas bubbles or the fluid.

The considered mathematical problem of slow fluid motion in a domain containing curved phase interfaces or cavities filled with the fluid is solved by the Boundary Element Method (BEM). According to the BEM, the original Stokes equations are replaced by an equivalent system of boundary integral equations [8]. An original numerical procedure based on the BEM and a collocation method for solving the boundary integral equations corresponding to similar problems with complex boundary conditions were developed in our previous publications (see [4–6]). After the solution of the boundary integral equations, we calculate the fluid velocity on the bubble surfaces and inside the flow domain.

As the main result of our numerical study, we present the calculated values of the pressure drop in microchannels with different types of the walls. To calculate the pressure drop, we use the equation for the pressure in the longitudinal direction. It takes the following form:

\[
\frac{\partial p}{\partial x} = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}, \quad p\left(-0.5, \frac{H}{2L}\right) = 1
\]
By integrating this equation from the inlet $x = -0.5$ to $x = 0.5$, we obtain the value of the pressure at the outlet and then the pressure drop $\Delta p$ corresponding to the geometry and type of the channel walls. From the parametric numerical study, it was found that for small $d_{1,2}/L$ and $x_{1,2} = 0$ the value of the pressure at the outlet of the channel is constant across the channel thickness. We checked the coincidence of the partial derivatives of the fluid velocity components with respect to $x$ at the inlet and outlet of the computational domain for the considered case of the flow geometry and the prescribed Poiseuille profiles on the flow boundaries. To check the accuracy of the pressure drop calculation, along with the solution of the equation for the pressure, we also calculated the pressure drop from the integral momentum conservation equation. We note that when the phase interface protrudes into the cavities it is possible to use both approaches to find the value of the pressure drop. However, when the gas bubbles protrude into the channel, it is simpler to solve differential equation for the pressure.

4. Results

The results of our calculations show that for a fairly narrow channel the mutual influence of its superhydrophobic walls may result in a substantial reduction of the hydraulic drag and the corresponding pressure drop in microchannels with both similar and different textures of the walls.

![Figure 2](image-url)

Figure 2. Pressure drop $\Delta p$ dependence on: (a) shape of the phase interface and equivalent shape of the cavity $\theta_{1,2} = 0$, $d_{1,2}/L = 0.4$, $H/L = 0.6$, $x_{1,2} = 0$; (b) shift of the phase interface into the cavity and equivalent depth of the cavity $s_{1,2} = s$, plane phase interface and bottom of the rectangular cavity; $1$ – one cavity, $2$ – two cavities; cavities filled with a fluid (---), cavities containing gas babbles (——)

Figure 2a shows the pressure drop $\Delta p$ in a channel as a function of the shape of the phase interface ($\theta_{1,2}$) pinned at the cavity corners and the shape of the cavity fully filled with the fluid. The centers of the cavities on both channel walls are located at the middle point of the calculation domain $x_{1,2} = 0$. In figure 2a, we present the results for the case of the channel with both SHS or only bottom SHS and the results for the case when gas bubbles are absent and the cavities have the shape similar to the phase interface in the first case. We consider the situation when the phase interfaces and the cavities on the walls are geometrically equal ($R_1 = R_2$). As is clear, a pressure drop reduction in a macrochannel can be achieved using different ways. When the shapes of the phase interface and the cavity coincide with a circle arc of the smallest curvature radius ($R_{1,2} = c$), the pressure drop reduction attains the same values. Superhydrophobicity of the channel walls results in a friction reduction and reduces the pressure drop. In figure 2a, it is clear that the presence of two superhydrophobic walls of the channel results in a noticeable decrease in the pressure drop, as compared to a channel with a single SHS. From our
calculations, it follows that there exists a threshold value of the protrusion angle of the bubble surfaces into the bulk flow above which the pressure drop starts to increase.

In figure 2b, we present the pressure drop versus the shift of the phase interface into the cavity or the depth of the rectangular cavities filled with the fluid. The geometry of the flow domain is as in figure 2a. It is clear that, as in the case with a single SHS [5], in a channel with two superhydrophobic walls, there also exists a critical shift $s$, when the pressure drop reaches a limiting value. The limiting value of the pressure drop is the same both for walls containing cavities partially filled with gas bubbles and for walls with cavities completely filled with the fluid.

5. Conclusion
Our calculations demonstrate how the mutual influence of superhydrophobic walls with different textures in a fairly narrow microchannel can result in a substantial reduction in the pressure drop. It is also shown that, even in the case of the absence of gas bubbles in the microcavities, the presence of textured channel walls can be a promising way to reduce the pressure drop in microfluidic devices.

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