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Microscopic Investigation of Vortex-Vortex Interaction in Conventional Superconductors

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Abstract. Quasi-particle structures around a pair of vortices and its effect on the vortex-vortex interaction are investigated. For this purpose, a new numerical method is developed. This method uses the elliptic coordinate and (modified) Mathieu functions. Using this method and solving the Bogoliubov-de Gennes equation, we analyse how quasi-particle structures change with the vortex-vortex distance.

1. Introduction

Vortices in superconductors are interesting and important for application of superconductors, because the vortex pinning is essential for perfect conductivity. Phenomenologically, the vortex-vortex interaction is given as,

\[ f_{\nu} = -\frac{\Phi_0^2}{8\pi^2\lambda^3} K_1\left(\frac{|r|}{\lambda}\right) \hat{r} \]  

(1)

where \( \Phi_0 = hc/2e \) is a flux quantum, \( \lambda \) is the penetration depth, \( K_1(x) \) is the first order modified Bessel function and \( r \) is a vector from a vortex to another vortex\cite{1}. This is always a repulsive interaction, but previous our study shows that in a confined geometry, the vortex configuration becomes different from that is predicted from this repulsive interaction\cite{2}. In such case, the quasi-particle structure around the vortices affects the vortex-vortex interaction. Therefore the vortex-vortex interaction is not trivial, especially for short distance where the quasi-particle structure around one of vortices is affected by the other vortices.

There have been many studies about quasi-particle structures around a single vortex. Especially, the Bogoliubov-de Gennes equation was solved using the Fourier-Bessel expansion \cite{3-6}, where the phase singularity point of the order parameter, which is the center of vortex, is put at the origin. But when we consider vortex-vortex interaction, we must take into account of the two phase-singularity points. Therefore the Fourier-Bessel expansion method is not good for our problem. Therefore we develop new numerical method, which can treat two phase-phase singularity points. In this study, we
apply this method for two vortices state in s-wave superconductors and investigate how the quasi-particle structures change with the vortex-vortex distance.

2. Model
We start from the Bogoliubov-de Gennes equation,

\[
\begin{align*}
\left\{ \frac{1}{2m} \left( \frac{\hbar}{i} \nabla + \frac{e}{c} A \right)^2 - \mu \right\} u_n(r) + \Delta v_n(r) &= E_n u_n(r) \\
- \left\{ \frac{1}{2m} \left( \frac{\hbar}{i} \nabla - \frac{e}{c} A \right)^2 - \mu \right\} v_n(r) + \Delta^* u_n(r) &= E_n v_n(r)
\end{align*}
\] (2)

where \( E_n \) and \( u_n(r) \) and \( v_n(r) \) are n-th eigenenergy and electron and hole-like part of n-th quasi-particle wave function, respectively. Superconducting order parameter \( \Delta(r) \) is determined by the following equation,

\[
\Delta(r) = \sum_n u_n(r) v_n^*(r) \left( 1 - 2 f(E_n,T) \right)
\] (3)

where \( f(E_n,T) \) is the Fermi distribution function.

For two vortex states, we use elliptic coordinates, which are shown in figure 1. The elliptic coordinates are denoted by \((\xi, \eta)\), which are related the Cartesian coordinates as,

\[
x = h_0 \cosh \xi \cos \eta
\] (4)

\[
y = h_0 \sinh \xi \sin \eta
\] (5)

In figure 1, we represent the corves with constant \( \xi \) and \( \eta \) by broken and dotted lines, respectively. And points \((\pm h_0, 0)\) are two foci of the ellipse. We put two vortices at these foci.

![Figure 1. Elliptic coordinates. Two vortices are put at two foci.](image)

Using this elliptic coordinate system, the kinetic Hamiltonian becomes as,

\[
H_0 = -\frac{\hbar^2}{2m h_0^2 (\cosh 2\xi - \cos 2\eta)} \left( \frac{\partial^2}{\partial \xi^2} + \frac{\partial^2}{\partial \eta^2} \right)
\] (6)
Eigen wave functions of this Hamiltonian are given as
\[
H_0 \text{ce}_m(\xi, q_m) \text{ce}_m(\eta, q_m) = \frac{\hbar^2}{2m} \frac{4q_m}{h_0^2} \text{ce}_m(\xi, q_m) \text{ce}_m(\eta, q_m)
\]  
(7)

\[
H_0 \text{se}_m(\xi, q_m) \text{se}_m(\eta, q_m) = \frac{\hbar^2}{2m} \frac{4q_m}{h_0^2} \text{se}_m(\xi, q_m) \text{se}_m(\eta, q_m)
\]  
(8)

where \(\{\text{ce}_m(\eta, q_m), \text{ce}_m(\xi, q_m)\}\) are cosine like Mathieu and modified Mathieu functions with characteristic number \(q_m\) and \(\{\text{se}_m(\eta, q_m), \text{se}_m(\xi, q_m)\}\) are sine like Mathieu and modified Mathieu functions with characteristic number \(q_m\).

Then the BdG equation becomes,
\[
\left(\frac{\hbar^2}{2m} \frac{4q_m}{h_0^2} - \mu\right) u_{vmr} + \sum_{v' \nu r m r_1} A^+_{v'v} (m r, m r_1) u_{v' m r_1} - \sum_{v' \nu r m r_1} \Delta_{v v_1} (m r, m r_1) v_{v' m r_1} = E_n u_{vmr} \quad (v = c, s) \]  
(11)

\[
-\left(\frac{\hbar^2}{2m} \frac{4q_m}{h_0^2} - \mu\right) v_{vmr} - \sum_{v' \nu r m r_1} A^-_{v v_1} (m r, m r_1) v_{v' m r_1} + \sum_{v' \nu r m r_1} \Delta_{v v_1}^* (m r, m r_1) u_{v' m r_1} = E_n v_{vmr} \quad (v = c, s) \]  
(12)

where \(q_m = q_m, q_m = \bar{q}_m\) and \(A^+_{v v_1} (m r, m r_1), A^-_{v v_1} (m r, m r_1)\)'s are the matrix elements of order parameter with the Mathieu function basis. And \(A^+_{v v_1} (m r, m r_1), A^-_{v v_1} (m r, m r_1)\)'s are given as,
\[
A^+_{v v_1} (m r, m r_1) = \frac{\hbar^2}{2m} (\alpha^2 h_0^2 A_{2v v_1} (m r, m r_1) + i2\alpha A_{v v_1} (m r, m r_1))
\]  
(13)

where \(\alpha = \pi H / \Phi_0\) and \(A_{v v_1} (m r, m r_1), A_{2v v_1} (m r, m r_1)\) are the matrix elements of the vector potential and square of the vector potential with the Mathieu function basis, respectively. We solve these equations and calculate the local density of states of quasi-particles.

3. Results
In order to investigate the quasi-particle structures around two vortices when distance between them is changed, we change the distance between two foci \(2h_0\). We set the length of the major axis as \(4.0\xi\), although the length of shorter axis depends on the value of \(h_0\). In figure 2, we show the local density of states (LDOS) at the lowest energy bound states for several vortex-vortex distance cases. In shorter distance, in figures 2(a) and (b), bound states around each vortex almost merge. In this case, two vortices may be considered to form a vortex molecule, where the electrons go around two vortices. With increasing the distance, the bound states around each vortex gradually separate but still interfere.
with each other (figure 2(c) and (d)). For further distance between vortices, the bound states exist almost independently, and only small interference peak in the LDOS appears around middle point of two foci (figure 2(e) and (f)). These results of interference of the quasi-particle bound states agree well with our previous study using the finite element method [2].

**Figure 2.** Local density of states of quasi-particles around two vortices, when distance between vortices are (a) $1.2\xi$, (b) $1.6\xi$, (c) $2.0\xi$, (d) $2.4\xi$, (e) $2.8\xi$, and (f) $3.2\xi$. The energies are (a) $E = -0.1\Delta_0$, (b) $E = -0.15\Delta_0$, (c) $E = -0.1\Delta_0$, (d) $E = -0.15\Delta_0$, (e) $E = -0.15\Delta_0$, and (f) $E = -0.15\Delta_0$.

4. Summary
We have analyzed the quasi-particle structures and the stable vortex configuration for two vortex states, using the Bogoliubov-de Gennes equation and the Mathieu functions with elliptic coordinates. We found that in shorter distance the bound states around two vortices merge and the vortices form a vortex molecule states. In such case the repulsive interaction between vortices may be overcome by this quasi-particle effect. Further investigation about the distance dependence of vortex-vortex interaction is a future problem.

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