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Test of the Universality of the Three-Body Efimov Parameter at Narrow Feshbach Resonances

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We measure the critical scattering length for the appearance of the first three-body bound state, or Efimov three-body parameter, at seven different Feshbach resonances in ultracold 39K atoms. We study both intermediate and narrow resonances, where the three-body spectrum is expected to be determined by the nonuniversal coupling of two scattering channels. Instead, our observed ratio of the three-body parameter with the van der Waals radius is approximately the same universal ratio as for broader resonances. This unexpected observation suggests the presence of a new regime for three-body scattering at narrow resonances.

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The Efimov effect [1] was first described in the context of nuclear physics but is now explored also in atomic, molecular, and condensed-matter systems [2–5]. Recent experiments on ultracold atoms with Feshbach resonances [6–18] have opened up a new path to study the universal spectrum of three-body Efimov states. The resonant interaction is expected to give rise to a three-body potential scaling as 1/R2, where R is the hyperradius that parameterizes the moment of inertia of the system. This leads to an infinite series of trimer states with an universal geometrical scaling for the binding energies. For a finite, negative two-body scattering length a, the three-body potential has a long-range cutoff at R ≈ |a|, and only a finite number of bound states exist. The critical scattering length a− for the appearance of the first Efimov state at the three-body threshold, often called the three-body parameter, was expected to be the only parameter to be influenced by nonuniversal physics, i.e., by the microscopic details of two- or even three-body forces [1,3]. While clear evidence of the universal scaling of the Efimov spectrum is still missing, recent experiments on identical bosons suggested that also a− might be universal [19]. This surprising result has been interpreted in a recent series of theoretical studies [20–22]. The underlying idea is that the sharp drop in the two-body interaction potential at a distance of the order of the van der Waals radius Rdw results in an effective barrier in the three-body potential at a comparable distance [22]. This prevents the three particles from coming sufficiently close to explore nonuniversal features of the interactions at short distances and leads to a three-body parameter set by Rdw alone, a− ≈ −9.5Rdw [19,21,22].

However, this scenario is realized only for the broad Feshbach resonances studied so far in most experiments, which can be described in terms of a single scattering channel, the so-called open channel. For narrow resonances, one must instead take into account the coupling of the open and a second closed channel [23]. It has been shown that in this case a new length scale that depends on the details of the specific Feshbach resonance, the so-called intrinsic length R*, must be introduced to parameterize the two-body scattering. The three-body potentials are also modified, with an expected deviation from the Efimovian dependence into 1/(R*/R) for distances R < R* [24]. This tends to reduce the depth of the three-body potential and leads to the nonuniversal result a− = −12.90R* [24,25], which is much larger than that obtained for broad resonances. This prediction is valid only close to resonance, where |a| ≫ R*. It is still unclear how a− scales in the intermediate regime of |a| ≪ R* or generally for resonances of intermediate widths. Various general models have been proposed [26–31], but they either are not fully predictive or give contradicting results.

In this Letter, we address this problem by performing an experimental study of three-body collisions in ultracold bosonic 39K atoms, where we determine the three-body parameter a− at several Feshbach resonances of intermediate or narrow width. In particular, our measurements probe for the first time the regime of very small resonance strengths, sres = 0.956Rdw/R* ≈ 0.1, where R* might be expected to be the relevant length scale that determines a−. Surprisingly, we find values of a− that are around the same −9.5Rdw measured for broad resonances, suggesting the existence of a novel intermediate regime of three-body scattering.

The investigation of closed-channel-dominated Feshbach resonances is particularly favored in 39K, which has several resonances with moderate magnetic width Δ and relatively small background scattering length −abg ≈ (20–30)α0 [32]. These parameters, together with the difference of the magnetic moments of the closed and open channels, δμ,
determine the intrinsic length $R^* = \hbar^2/(m a_0 \Delta \delta \mu)$ [23], which can be related also to the on-resonance effective range (see the Supplemental Material [33]). In particular, we investigated seven different resonances with $s_{\text{res}}$ in the range 0.1–2.8 in the three magnetic sublevels of the hyperfine ground state $F = 1$ [32].

A detailed description of the experimental setup is given elsewhere [34]. The three-body parameter was determined by finding the maximum of the three-body loss coefficient $K_3$ in the region of negative $a$ at each Feshbach resonance, as in previous experiments [6–18]. In the presence of three-body losses, both the atom number $N$ and temperature $T$ evolve according to $dN/dt = -K_3 \langle n^2 \rangle N$ and $dT/dt = (K_3/3) \langle n^2 \rangle T$, where $\langle n^2 \rangle = (1/N) \int n(x)^2 d^3x$ is the mean square density [35]. The temperature increase is due to the preferential removal of atoms in the high-density region around the trap center. The typical starting condition was a noncondensed sample with $3–80 \times 10^4$ atoms in a temperature range of 20–400 nK, depending on the spin channel and Feshbach resonance (see the Supplemental Material [33]). The atoms were held in a purely optical trap (or with an additional magnetic confinement, depending on the specific resonance) at sufficiently low density to have a negligible mean-field interaction energy. Care was taken to have a trap depth sufficiently large to avoid evaporation associated to the heating. The samples were initially prepared at small negative $a$ in proximity of the Feshbach resonances; the measurements started 10 ms after $a$ was ramped to the final value in about 2 ms.

Figure 1 shows a typical evolution of $N$ and $T$, as measured by absorption imaging after a free expansion. They were simultaneously fitted with

$$N(t) = N_0 \left(1 + \frac{3 \beta^2 N_0^2}{\sqrt{27} T_0 K_3 t}\right)^{1/3}.$$  \hspace{1cm} (1)

$$T(t) = T_0 \left(1 + \frac{3 \beta^2 N_0^2}{\sqrt{27} T_0 K_3 t}\right)^{1/9}. \hspace{1cm} (2)$$

Here $N_0$ and $T_0$ are the initial atom number and temperature, respectively, and $\beta = (m \bar{\omega}^2 / 2 \pi k_B)^{1/2}$, with $\bar{\omega}$ the mean trap frequency. In such a fit, one-body losses were neglected, since they occur on a much longer time scale.

Crucial ingredients for a reliable measurement of the $K_3$ dependence on the scattering length were an accurate calibration of the magnetic field $B$ and the use of a high-quality coupled-channel (CC) model for $a(B)$, based on a large number of experimental observations for the positions and widths of the Feshbach resonances [32,33]. The centers and widths of the Feshbach resonances were redetermined in the present work, finding a good agreement with the theoretical ones. An additional confirmation of the CC model was derived from a direct measurement of the dimer binding energy at the two narrowest resonances by radio-frequency spectroscopy.

We observed for all Feshbach resonances a clear peak in $K_3$ in the region of $|a| = (600–1000) a_0$, as shown in Figs. 2 and 3. We compared the observations to the known

![FIG. 1 (color online). Example of the time evolution of the atom number (circles) and temperature (triangles), fitted to Eq. (1) (solid line) and Eq. (2) (dashed line) to determine the three-body loss coefficient $K_3$.](image1.png)

![FIG. 2 (color online). Three-body loss rate measured in the proximity of five Feshbach resonances of intermediate strength (see Table I for the assignment of the spin state). The experimental data (squares) are fitted to Eq. (3) (solid line).](image2.png)
with an effective rate of the form
\[ \left( \text{dashed line} \right) \] to determine state. The experimental data (squares) are fitted with a Gaussian (solid line). The dash-dotted lines provide a comparison to a
\[ \left( \text{solid line} \right) \] for identical bosons at zero collision energy and
\[ \left( \text{solid line} \right) \] temperature of the experiment, there is a limitation in
\[ \left( \text{solid line} \right) \] deeply bound molecular states [\ref{7}], and also compared with Eq. (3), using \[ \eta_+ = 0.1 \] (solid line). The dash-dotted lines provide a comparison to a
\[ |a|^{7/2} \] behavior for low \[ |a| \].

relation for identical bosons at zero collision energy and in the zero-range approximation, for \( a < 0 \):
\[ K_3(a) = \frac{3\hbar a^4}{m} \frac{\sinh(2\eta_-)}{\sin^2(s_0 \ln(a/a_-)) + \sin^2\eta_-}. \]

Here \( s_0 \approx 1.00624 \) is an universal constant, and \( \eta_- \) is the decay parameter which sets the width of the Efimov resonance and incorporates short-range inelastic transitions to deeply bound molecular states [\ref{3}]. At the finite temperature of the experiment, there is a limitation in the maximum observable \( K_3 \) set by unitarity at \( K_3^{\text{max}} = 36\sqrt{3}\pi^2\hbar^2/(k_BT)^2m^3 \) [\ref{36,37}]. Therefore, we fitted the data with an effective rate of the form \( [1/K_3(a) + 1/K_3^{\text{max}}]^{-1} \) [\ref{7,13,38}], with \( a_- \), \( \eta_- \), and \( K_3^{\text{max}} \) as fitting parameters. The experimental \( K_3(a) \) for the five broadest resonances, shown in Fig. 2, is in good agreement with Eq. (3), besides a multiplicative factor of the order of 3 that can be justified with the experimental uncertainty in the determination of the density (see the Supplemental Material [\ref{33}]).

Also, the two narrowest Feshbach resonances feature a maximum in \( K_3 \) around \(-1000a_0 \), as shown in Fig. 3. There is, however, a slower background variation of \( K_3 \) with \( a \), not reproduced by Eq. (3). It was shown that for narrow resonances one should expect a slower evolution in the regime \( |a| < R^* \), with \( K_3 \propto |a|^{7/2} \) [\ref{24}], but also this does not seem to reproduce the data at small \( |a| \). In the absence of a better model and in analogy with the broad resonances, we determined the position of the measured maximum in \( K_3 \) with a Gaussian fit, as shown in Fig. 3, and we interpreted it as the \( \alpha_- \) parameter. As uncertainty, we conservatively took the \( 1/e^2 \) half-width of the Gaussian.

For all the resonances in excited spin states, there is in principle also a contribution of two-body losses, which have a slower dependence on \( a \) [\ref{23}]. While it was not possible to distinguish in a reliable way two- from three-body losses in the experiment, we have verified that the calculated two-body losses from the CC models are typically negligible, besides some large-\( a \) regions close to the two narrow resonances (see the Supplemental Material [\ref{33}]).

A summary of our analysis is reported in Table I. For the calculation of \( a(B) \), we used the experimentally determined Feshbach resonance centers \( B_0^{\text{exp}} \) and the resonance widths and the background scattering lengths from the CC model. The uncertainties in \( B_0^{\text{exp}} \) include those in the calibration of \( B \) and in the determination of \( B_0 \) from the loss resonances. Particular care was put in the determination of \( B_0 \) for the two narrowest resonances, where we found a rather good agreement between independent measurements of the atom losses and of the binding energy (see the Supplemental Material [\ref{33}]). The uncertainties in \( a_- \) include the statistical uncertainties from the fit of the \( K_3 \) data and from the determination of \( a(B) \). For the two narrowest resonances, the dominant source of uncertainty comes from the determination of \( B_0 \). These two resonances are coupled, and \( a(B) \) can be represented only over an

| \( m_F \) | \( B_0^{\text{exp}} \) (G) | \( R^* \) (\( a_0 \)) | \( s_{\text{res}} \) | \(- a_- \) (\( a_0 \)) | \( \eta_- \) | \( T \) (nK) |
|---|---|---|---|---|---|---|
| 0 | 471.0 (4) | 22 | 2.8 | 640 (100) | 0.065 (11) | 50 (5) |
| +1 | 402.6 (2) | 22 | 2.8 | 690 (40) | 0.145 (12) | 90 (6) |
| -1 | 33.64 (15) | 23 | 2.6 | 830 (140) | 0.204 (10) | 120 (10) |
| -1 | 560.72 (20) | 24 | 2.5 | 640 (90) | 0.22 (2) | 20 (7) |
| -1 | 162.35 (18) | 59 | 1.1 | 730 (120) | 0.26 (5) | 40 (5) |
| 0 | 65.67 (5) | 456 | 0.14 | 950 (250) | 330 (30) |
| 0 | 58.92 (3) | 556 | 0.11 | 950 (150) | 400 (80) |
extended range of magnetic fields in terms of a two-pole expression containing two widths and a single $d_{bg}$ [32].

The reported values of $R^*$ are determined on resonance, from a comparison of the predictions of our CC calculation to a generalization of the quantum-defect model of Ref. [39] to the case of coupled resonances. The coupling causes a dependence of $R^*$ on $B$, which is, however, limited to about 20% in the experimental range [see the Supplemental Material [33]].

We observe a whole range of values of $\eta_-$ for the different Efimov resonances; this is probably a consequence of the different measurement temperatures but possibly also of the nonuniversal nature of $\eta_-$ [3,40].

A comparison of the results in Table I leads to the striking conclusion that the three-body parameter $a_-$ stays around values of the order of $-10R_{vdW}$ for all the Feshbach resonances explored in $^{39}$K, including the ones with $R^*$ as large as $\sim 600a_0$, hence much larger than $R_{vdW}$. We note that in the earlier measurement at the $m_F = 1$ resonance, we found two $K_3$ resonances at $|a| \approx 700a_0$ and $|a| \approx 1500a_0$, which we tentatively identified as a four- and a three-body resonance, respectively [7]. We now think that the previous resonance around 1500$a_0$, which we no longer observe, was an artifact of the analysis of the limited time-dependent data, and we reassign the one around 700$a_0$ as the three-body resonance (see the Supplemental Material [33]).

Figure 4 shows the measured $|a_-|/R_{vdW}$ as a function of $s_{res}$. We observe just a moderate deviation of our data from the mean value 9.73(3) measured for open-channel-dominated resonances [17,19,21,41] and also for other intermediate resonances [9–11,18,19,41]. This observation is far from the already mentioned prediction for narrow resonances [24,25], which indicates that the Efimov resonances should appear at scattering lengths that are multiples of $a_- = -12.9R^*$ by a factor exp$(\pi/s_0) = 22.7$. One might note that this result is expected to be valid only in the limit of a scattering length larger than any other length scale, $|a| \gg R^* \gg |d_{bg}|$, where the three-body potential at large hyperradii $R > R^*$ has an Efimovian character [24]. The present experiment does not access this extreme limit but is in an intermediate regime also for the two narrowest resonances, which show indeed $R^* \approx |a_-|$. Other models for the three-body physics at Feshbach resonances of intermediate strength have been proposed [26–31]. The specific problem of connecting the results for the three-body parameter in the open-channel-dominated regime, where $a_-$ is determined by $R_{vdW}$, and the closed-channel limit, where it is $R^*$ which sets the scale for $a_-$, has been addressed recently [26,31], finding, however, considerably different results. In particular, the model of Ref. [26] predicts that a crossover between the two regimes of broad and narrow resonances would take place around $s_{res} \approx 1$, as shown in Fig. 4. Additionally, the regime of $a_- = -12.9R^*$ should be reached only for excited Efimov states, while the first one has a slightly smaller $a_- = -10.3R^*$. Although an increase of $|a_-|$ with decreasing $s_{res}$ might be present in the experimental data, there is a clear disagreement with such predictions. Experiments on $^7$Li and $^{133}$Cs have also measured similar values for $a_-$ at three intermediate resonances with $s_{res} = 0.5–1$ [10,11,18,19], indicating that this behavior might not be peculiar of $^{39}$K. Also, a system without Feshbach resonances like metastable $^4$He might be consistent with these results [42].

We note that for the two narrowest resonances $|a_-|$ is only a factor of 2 larger than $R^*$. This observation seems to indicate that the three-body potential can support a bound state that resides only in the region with hyperradius $R \approx 2R^*$. This is a regime that was not accessible in previous one-channel models, and a multichannel approach [43], possibly comprising the coupled-resonance aspect, will presumably be necessary to model the observations.

In conclusion, our study showed an apparent universal behavior of the three-body parameter on several different Feshbach resonances of the same atomic species, down to a resonance strength $s_{res} \approx 0.1$. This gives important information on the three-body physics in this narrow-resonance regime, where an interplay of the open and the closed molecular channels is expected.

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