A New Look to Massive Neutron Cores

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February 9, 2022

Abstract

We reconsider the problem of modelling static spherically symmetric perfect fluid configurations with an equation of state from a point of view of that requires the use of the concept of principal transform of a 3-dimensional Riemannian metric. We discuss from this new point of view the meaning of those familiar quantities that we call density, pressure and geometry in a relativistic context. This is not simple semantics. To prove it we apply the new ideas to recalculate the maximum mass that a massive neutron core can have. This limit is found to be of the order of $3.8\, M_\odot$ substantially larger than the Oppenheimer and Volkoff limit.

Introduction

We review in Section 1 the basic equations of the models being considered as well as the concept of principal transform of a 3-dimensional Riemannian metric which is at the core of our new point of view to understand these models.

Section 2 is devoted to lay down the fundamental system of equations to be integrated. We use spherical space coordinates of the quo-harmonic class which allow to implement $C^1$ class smoothness across a sharp border when there is one.

In Section 3 we define the concept of proper mass $M_p$ to be used to define the binding energy $E_b$ of the models as the difference between the active gravitational mass $M_a$ and $M_p$. We define also the concept of proper mass density which is a fundamental hybrid concept related to $M_p$ and the principal transform of the quotient space metric. Its relation with the pressure $p$ characterizes the fluid source independently of the solution being considered.
Section 4 is devoted to establish that the binding energy $E_b$ can be obtained as the integral over all space of a localized energy density $\sigma$ which depends only on some of the gravitational potentials and its first radial derivatives.

In Section 5 we linearize the fundamental system of equations to obtain the linearized expression of the binding energy $E_b$, which coincides with the familiar Newtonian one, thus providing a partial justification to the definition of $E_b$ in the nonlinear regime. We obtain also the linearized expression of the energy density $\sigma$.

The last section contains our proposed application of the new point of view to the study of massive neutron cores. The equation of state is the usual one for a degenerate neutron gas, but both the density and the pressure are variants of those used by Oppenheimer and Volkoff \[1\]. Our main result is that the maximum limit mass that a neutron core can have is approximately $3.8\, M_\odot$ instead of $0.7\, M_\odot$ in \[1\].

1 Static Spherically Symmetric Models

We shall be interested in this paper on global spherically symmetric models, which we shall write using a time adapted coordinate and polar-like space coordinates:

$$ds^2 = -A^2(r)c^2dt^2 + ds^2$$

where:

$$ds^2 = B^2(r)dr^2 + B(r)C(r)r^2d\Omega^2, \quad d\Omega^2 = d\theta^2 + \sin^2\theta d\varphi^2,$$

solution of Einstein’s field equations:

$$S_{\alpha\beta} = -\chi T_{\alpha\beta}, \quad \chi = 8\pi G/c^4$$

where the r-h-s describes a compact fluid source, or with fast decreasing density, with two “flavours”: isotropic or a special kind of anisotropic pressure to be presented in a moment.

The quotient 3-dimensional metric \[2\] can be written using a variety of supplementary coordinate conditions belonging to two different types: algebraic or differential. The most often used is the curvature condition which uses a radial $\tilde{r}$ coordinate with the algebraic condition:

$$\tilde{C} = \tilde{B}^{-1}$$
We shall refer also to any other quantity which assumes the use of this radial coordinate with a tilde overhead.

We shall use here almost exclusively the quo-harmonic condition which restricts the $r$ coordinate with the differential condition:

$$C' = \frac{2}{r} (B - C)$$

(5)

where the prime means derivative with respect to $r$. The use of a differential condition makes possible the construction of global $C^1$ models with a sharp boundary with vacuum, something which is not possible when using the coordinate $\tilde{r}$.

Notice that if $B(r)$ and $C(r)$ are known the curvature coordinate $\tilde{r}$ is simply the following function of $r$:

$$\tilde{r} = \sqrt{\frac{B(r)C(r)}{r}}$$

(6)

while, on the contrary, to obtain explicitly the inverse function is in most cases of interest impossible or very cumbersome.

Despite the emphasis we put on the use of the quo-harmonic coordinate $r$ let us be clear from the beginning that the main conclusion of this paper will not owe anything to a particular choice of coordinates. It will owe instead all to a new concept: that of a principal transformation of a 3-dimensional Riemannian metric such as (2).

By definition, in the particular case we are considering, the principal transform of (2) is a new 3-dimensional Riemannian metric

$$d\bar{s}^2 = \Phi^2(r)B^2(r)dr^2 + \Psi^2(r)B(r)C(r)r^2d\Omega^2$$

(7)

such that: i)

$$\bar{R}^i_{jkl} = 0, \quad i, j, k, l = 1, 2, 3$$

(8)

and: ii)

$$(\hat{\Gamma}^i_{jk} - \bar{\Gamma}^i_{jk})\hat{g}^{jk} = 0,$$

(9)

where, with otherwise obvious notations, the quantities with a hat overhead refer to the metric (2) and the quantities with a bar overhead refer to the metric (3). Notice that both conditions above being tensor conditions under any transformation of space coordinates the concept of Principal transformation is intrinsic to the Killing time congruence we are considering.

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1See for instance references [2] or [3]
One of the practical conveniences of using polar quo-harmonic space coordinates from the outset in the very process of model-building is that, when appropriate boundary conditions are taken into account, the principal transform of (4) is just:

\[ d\vec{s}^2 = dr^2 + r^2d\Omega^2 \]  \hspace{1cm} (10)

and:

\[ \Phi(r) = 1/B(r), \quad \Psi(r) = 1/\sqrt{B(r)C(r)} \]  \hspace{1cm} (11)

We shall consider two types of energy-momentum tensors: i) those describing standard perfect fluids with isotropic pressure:

\[ T_{00} = \rho A^2, \quad T_{ij} = p\hat{g}_{ij}, \quad (x^0 = ct) \]  \hspace{1cm} (12)

and a new type of fluid where the isotropy of the pressure is meant in the sense of the principal transform (7):

\[ T_{00} = \rho A^2, \quad T_{ij} = p\bar{g}_{ij} \]  \hspace{1cm} (13)

i.e. in the sense of (10) if polar quo-harmonic coordinates are used.

Notice that this second case, that we shall present in Section 3 as being the truly isotropic fluid, can be considered from the standard point of view as an anisotropic fluid of a particular type. Namely one for which the radial and tangential pressures are related to a single function \( p(r) \) as follows:

\[ p_r = pB^{-2}, \quad p_t = p(BC)^{-1} \]  \hspace{1cm} (14)

We can deal with both cases at once introducing in the field equations a two-valued flag \( f = 0 \) or \( f = 1 \) and the following expressions for \( p_r \) and \( p_t \):

\[ p_r = (fB^{-2} + 1 - f)p, \quad p_t = (f(BC)^{-1} + 1 - f)p \]  \hspace{1cm} (15)

thus \( f = 0 \) corresponds to the standard case and \( f = 1 \) corresponds to the new case.

As usual we shall make use of a compressibility equation:

\[ \rho = \rho(p) \text{ or } p = p(\rho) \]  \hspace{1cm} (16)

depending on convenience, to describe the physics of the source.

The boundary conditions at infinity will be:

\[ \lim_{r \to \infty} \Xi = 1, \quad \lim_{r \to \infty} r^3\Xi' = 0 \]  \hspace{1cm} (17)
where \( \Xi = A, B \) or \( C \); and the regularity conditions at the centre of symmetry of the configuration will be:

\[
\Xi'_0 = 0, \quad \Xi''_0 < \infty
\]  

(18)

Finally, in those cases where the model has a sharp boundary between an interior with \( \rho > 0 \) and vacuum we shall require the continuity of both \( X_i \) and \( X' \), so that the space-time metric will be of global \( C^1 \) class.

The system of units we shall use throughout will be such that:

\[
\bar{c} = 1, \quad \bar{G} = 1
\]  

(19)

where \( \bar{c} \) is the speed of light in vacuum and \( \bar{G} \) is Newton’s constant. A more specific system of units of this class will be chosen in Sect. 6.

2 Explicit field equations

Taking into account the coordinate condition (5) and also its derivative:

\[
C'' = 2 \frac{B'}{r} - \frac{6}{r^2} (B - C)
\]  

(20)

the field equation \( S_{00} = \rho A^2 \) can be written as follows:

\[
B'' = -3 \frac{B B'}{r r C} - \frac{B^2}{r^2 C} + \frac{5 B^2}{4 B} + \frac{B^3}{r^2 C^2} - 8 \pi \rho B^3
\]  

(21)

The field equation \( S_{22} = -p_r B C r^2 \) (or \( S_{33} = -p_r B C r^2 \sin^2 \theta \)) can be written using the preceding Eq. (21) and the second Eq. (15) as:

\[
A'' = \frac{1}{r C} \left( \frac{1}{2} A B' - B A' \right) + \frac{1}{2 r^2 C} \left( \frac{B}{C} - 1 \right) + \frac{1}{8} \frac{A B'^2}{B^2} + \frac{1}{2} \frac{A' B'}{B}
\]

\[
+ 4 \pi \rho B^2 A + 8 \pi p A B (f/C + B(1 - f))
\]  

(22)

The remaining equation we have to take care of is \( S_{11} = -p_r B^2 \), or taking into account (5):

\[
\frac{B}{r^2 C} - \frac{1}{4} \frac{B'^2}{B^2} - \frac{B^2}{r^2 C^2} - \frac{A' B'}{B A} - \frac{B'}{r C} - 2 \frac{B A'}{r C A} = -8 \pi p (f + (1 - f) B^2)
\]  

(23)

On the other hand from the conservation equation:

\[
\nabla_a S^a_1 = 0 \quad (x^1 = r)
\]  

(24)
or:

\[ S_1^{\prime} + \Gamma_1^{0}(S_1^1 - S_0^0) + \Gamma_2^{0}(S_1^1 - S_2^0) + \Gamma_3^{0}(S_1^1 - S_3^0) = 0 \]  

(25)

and Eqs. (13) we derive the equation:

\[ p' = -\frac{A'}{A} \left[ \rho(fB^2 + 1 - f) + p \right] + f \left[ 2B' \frac{B'}{B} - \left( \frac{B'}{B} + \frac{2B}{rC} \right) \left( 1 - \frac{B}{C} \right) \right] p \]  

(26)

Taking into account the regularity conditions (18) at the centre of symmetry of the configuration in the preceding equations it is easy to see that they imply:

\[ B_0 = C_0, \quad p_{\theta\theta} = p_{r0}, \quad S_{110} = -B_0^2 p_{\theta0} \]  

(27)

Therefore the remaining field equation \( S_{11} = -p_r B^2 \) is satisfied at the origin \( r = 0 \) and from from (21), (22) and (25) it follows that it is satisfied everywhere.

What follows is the summary of this section and the preceding one: The models we are considering will be fully described by the field variables \( A, B, C \) and the source variables \( \rho, p \); the latter being related by a compressibility equation of either type (16). This complete set of variables is constrained to satisfy the system of differential equations (1), (21), (22) and (26). Appropriate initial conditions will be \( A_0, B_0, C_0 = \rho_0 > 0 \) (or \( p_0 = 0 \)). The initial values of \( A_0 \) and \( B_0 \) have to be chosen such that the asymptotic conditions (17) are satisfied. The boundary of the source will be defined by the first zero \( r = R \) of the pressure \( p \), and beyond the vacuum field equations will be required. The continuity of \( A, B \) and \( C \) and its first derivatives is automatically implemented across the boundary of the source.

3 Physical and geometrical interpretations

This will be the more difficult section of this paper, although it does not contain any calculation, because it deals about the meaning of words of common use.

When we look at Eqs. (2) as equations to be solved we all refer to the r-h-s as the source term. But this is not quite correct because the energy-momentum tensor depends on the coefficients of the unknown metric. In the case we are considering in this paper the real source variables are the so-called density \( \rho \) and pressure \( p \) related by a compressibility equation (16). The meaning of these three ingredients deserve to be examined with some
detail. A density by definition is a mass per unit volume, and a pressure is a force per unit surface. Therefore to be clear about them we must tell of what mass are we talking about and to what geometry of space are we referring when using the words volume and surface.

As we all know the concept of mass is tricky because it comes in three flavours: inertial mass, passive gravitational mass and active gravitational mass. Newtonian theory assumes the proportionality of the three masses and General relativity assumes the proportionality of inertial and passive gravitational masses of test bodies. Beyond that we have a few decisions to be taken.

To decide what geometry of space to use is also a tricky problem in relativity theory because we have to decide whether this geometry has to be known before we solve the field equations or will be known only after they have been solved, in which case the meaning of \( \rho, p \) and the compressibility Eq. (16) will also be known only after the problem has been solved.

Of the three types of mass, only the meaning of active gravitational mass was settled very early by Tolman [4] identifying it with the Newtonian mass at infinity and proving that it can be calculated as the following integral over the source:

\[
M_a = 4\pi \int_0^\infty \rho \tilde{r}^2 \, d\tilde{r}
\]  
(28)

where we remind that \( \tilde{r} \) is the curvature radial coordinate. Using (6) this formula can be written equivalently as:

\[
M_a = 4\pi \int_0^\infty \rho F(r)r^2 \, dr
\]  
(29)

where:

\[
F(r) = \frac{1}{2} \sqrt{BC} (B'C r + 2B^2)
\]  
(30)

Bonnor [5] proposed to eliminate the ambiguities that remain defining the passive gravitational mass as:

\[
M(B) = 4\pi \int_0^\infty (\rho + p) \tilde{B}\tilde{r}^2 \, d\tilde{r}
\]  
(31)

This means that \( \rho + p \) is interpreted as a density of inertial, or passive, gravitational mass and that the metric that gives a meaning to the word volume is (2). But this metric is known only once the problem has been solved and then it depends on the point of the body which is considered. This deprives the meaning of the variables \( \rho \) and \( p \) and the compressibility
equation (14) of any \textit{a priori} significance. Other difficulties that arise from this definition are discussed in Bonnor’s paper.

We do not believe that the consideration of a single body at rest, as we have been doing here, can say anything about its passive or inertial gravitational mass because this would require to know how it reacts to the presence of another comparable body. On the other hand we believe that we should be able to define its proper mass $M_p$ if we want to know what is the binding energy $E_b$ of any given configuration as defined by:

$$E_b = M_a - M_p$$  \hspace{1cm} (32)

More precisely, our point of view, along the lines of a long enduring effort to understand the concept of rigidity and establishing a theory of frames of reference in special and general relativity, consists in accepting the usual generalization of Schwarzschild’s “substantial mass” \footnote{An english translation of Schwarzschild’s paper has been provided by S. Antoci in arXiv:physics/9912033} as proper mass:

$$M_p = 4\pi \int_0^\infty \rho B^2 C r^2 \, dr, \quad \sqrt{\hat{g}} = B^2 C r^2 \sin \theta$$ \hspace{1cm} (33)

and defining at the same time a proper mass density $\rho_p$:

$$\rho_p = \rho B^2 C$$ \hspace{1cm} (34)

such that $M_p$ could be written:

$$M_p = 4\pi \int_0^\infty \rho_p r^2 \, dr, \quad \sqrt{\hat{g}} = r^2 \sin \theta$$ \hspace{1cm} (35)

This means then defining $\rho_p$ as a density of proper gravitational mass and interpreting the words volume and surface in the sense of the universal euclidian geometry \footnote{An english translation of Schwarzschild’s paper has been provided by S. Antoci in arXiv:physics/9912033} related to the quotient metric \footnote{An english translation of Schwarzschild’s paper has been provided by S. Antoci in arXiv:physics/9912033} by a principal transformation \footnote{An english translation of Schwarzschild’s paper has been provided by S. Antoci in arXiv:physics/9912033}. This guarantees the independence of the meanings of $\rho_p$, $p$ and the compressibility equation \footnote{An english translation of Schwarzschild’s paper has been provided by S. Antoci in arXiv:physics/9912033} independently of the location of the element of the fluid in the object and independently of the solution of the field equations that one is considering. This guarantees also that $M_p$ can be identified with an appropriate number of identical samples of a fluid as weighted with a balance at the “shop store” before being assembled into the body.

The interpretation we have just given of the metric \footnote{An english translation of Schwarzschild’s paper has been provided by S. Antoci in arXiv:physics/9912033} implies also as a corollary that, as above-mentioned, a fluid with isotropic pressure should be described by an energy-momentum tensor as written in \footnote{An english translation of Schwarzschild’s paper has been provided by S. Antoci in arXiv:physics/9912033}. And that for the compressibility equation to have a well defined \textit{a priori} meaning it should be given as a relationship between $\rho_p$ and $p$:
\[ \rho_p = \rho_p(p) \quad \text{or} \quad p = p(\rho_p) \quad (36) \]

4 Localized energy density

Following suit to the ideas of the preceding section we exhibit the quantity \( E_b \) as an integral extended over all space of an energy density function depending only on \( r, B, C \) and \( B', C' \). From (29) and (35) it follows that:

\[ E_b = 4\pi \int_0^\infty \rho B^3 Y r^2 \, dr \quad (37) \]

where:

\[ Y = \frac{F}{B^3} - \frac{C}{B} \quad (38) \]

while \( \rho B^3 \) can be obtained from (21) as:

\[ \rho B^3 = \frac{1}{8\pi} (X - B'') \quad (39) \]

with:

\[ X = \frac{B(-3rB' - B + B^2/C)}{r^2C} + \frac{5B^2}{4B} \quad (40) \]

Therefore we have:

\[ E_b = \frac{1}{2} \int_0^\infty XY r^2 \, dr - \frac{1}{2} \int_0^\infty B'' Y r^2 \, dr \quad (41) \]

Integrating by parts the second integral we obtain:

\[ E_b = 4\pi \int_0^\infty \sigma(r) r^2 \, dr - 4\pi \lim_{r \to \infty} \sqrt{\frac{C}{B}} B' r^2 \left( \frac{1}{4} \frac{B'C}{B^2} + 1 - \sqrt{\frac{C}{B}} \right) \quad (42) \]

where:

\[ \sigma(r) = \frac{1}{8\pi} (XY + Z) \quad (43) \]

with:

\[ Z = \frac{B'}{8rB^3 \sqrt{BC}} (-5r^2C^2 B'^2 + 2rBC(4\sqrt{BC} + C)B' - 8B^3(2\sqrt{BC} - B + C)) \quad (44) \]
depends only on $r, B, C$ and $B'$. From the asymptotic conditions (17) it follows that:

$$\lim_{r \to \infty} B/C = 1 \text{ and } \lim_{r \to \infty} r^3 B' = 0$$

and therefore, the limit in (42) being zero, the final result is:

$$E_b = 4\pi \int_0^\infty \sigma(r) r^2 dr$$

5 Linear approximation

We consider here the linear approximation of the models that we have been considering, to take a closer look to two of the def concepts that we have implemented in Sect. 3. Namely: the mass defect, or binding energy $E_b$ and the proper mass density $\rho_p$.

We assume that $A, B$ and $C$ can be written as:

$$A = 1 + A_1, \quad B = 1 + B_1, \quad C = 1 + C_1$$

where $A_1, B_1$ and $C_1$ are small quantities, of order $\epsilon$ say. We assume also that $\rho$ is also of order $\epsilon$ and that $p$ is of order $\epsilon^2$ and can be ignored, as well as any other quantity of the same order or smaller, in the field equations.

The coordinate condition (5) and the field equations (21), (22) become:

$$C_1' = \frac{2}{r} (B_1 - C_1)$$

$$B_1'' = -3 \frac{B_1'}{r} + \frac{1}{r^2} (B_1 - C_1) - 8\pi \rho$$

$$A_1'' = \frac{1}{2r} (B_1' - 2A_1') + \frac{1}{2r^2} (B_1 - C_1) + 4\pi \rho$$

As our purpose is purely illustrative here we consider below the simplest case where the source is a spherical body of finite radius $R$ and constant $\rho$.

The interior solution satisfying the regularity conditions (18) at the centre is:

$$A_1 = \frac{2}{3} \pi \rho r^2 + a_0, \quad B_1 = -\frac{16}{15} \pi \rho r^2 + b_0 \quad C_1 = -\frac{8}{15} \pi \rho r^2 + b_0$$

where $a_0$ and $b_0$ are two allowed constants of integration; the exterior solution satisfying the asymptotic conditions (17) at infinity is:
\[ A_1 = -\frac{b_1}{r}, \quad B_1 = \frac{b_1}{r} + \frac{b_3}{r^3}, \quad C_1 = \frac{2b_1}{r} - \frac{2b_3}{r^3} \]  
(52)

where \(a_1\) and \(b_3\) are two new constants of integration. Demanding the continuity of \(A_1\), \(B_1\) and \(C_1\) and their first derivatives across the border \(r = R\) fixes \(a_0\) and all the \(b_\)'s as follows:

\[ a_0 = -2\pi \rho R^2, \quad b_0 = \frac{8}{3} \pi \rho R^2, \quad b_1 = \frac{4}{3} \pi \rho R^3, \quad b_3 = \frac{4}{15} \rho R^5 \]  
(53)

From the preceding results we can calculate the leading approximation, which is of order \(\epsilon^2\), of the localized energy density (43). For \(r < R\) the result is using an arbitrary system of units:

\[ \sigma = -\frac{3}{20} \frac{G^2 M^2 r^2}{\pi R^6} \]  
(54)

where at this approximation \(M\) is either \(M_a\) or \(M_p\). For \(r > R\) the result is:

\[ \sigma = -\frac{3}{160} \frac{G^2 M^2 (5r^4 + 6R^2 r^2 - 3R^4)}{\pi r^8} \]  
(55)

The binding energy can be calculated using (46), (54) and (55), or (32), (29) and (35) at the appropriate approximation. The result is, using arbitrary units, the familiar Newtonian amount:

\[ E_b = -\frac{3}{5} \frac{GM^2}{c^2 R}, \]  
(56)

a result that can be obtained using a variety of other approaches \(^3\)

### 6 Massive Neutron Cores

Any particular model will be characterized by an equation of state and the value of its central density, or central pressure, or both in the important case in which one assumes that the density is constant. Taking into account the regularity conditions the initial conditions of the gravitational potentials \(A, B\) and \(C\) have to be chosen such that:

\[ B_0 = C_0, \quad A'_0 = B'_0 = C'_0. \]  
(57)

And taking into account the asymptotic conditions the values of \(A_0\) and \(B_0\) have to be chosen such that:

\(^3\)See for instance reference \([6]\)
\[
\lim_{r \to \infty} A = \lim_{r \to \infty} B = 1
\]
the condition:
\[
\lim_{r \to \infty} C = 1
\]
as, well as the remaining asymptotic conditions, being then automatically satisfied because the solution behaves as the exterior Schwarzschild one at infinity.

The numerical integration of the system of equations (5), (21), (22) and (26) where:
\[
\rho = \rho_p (f(B^2C)^{-1} + 1 - f)
\]
and \(\rho_p\) is a known function of \(p\), is a trial and error procedure. Arbitrary values of \(A_0, B_0\) and \(p_0 > 0\) have to be chosen; the integration has to proceed until \(p = 0\); then the equation of state has to be abandoned and \(\rho = p = 0\) has to be required; the integration has then to proceed to sufficiently large values of \(r\) to check the asymptotic conditions (58). If the check is not satisfactory the whole process has to be started again with new values of \(A_0, B_0\) and \(p_0 > 0\).

As an important example we consider the equation of state of a degenerate neutron gas as it suits to a model of massive neutron cores. This was considered in a famous paper by Oppenheimer and Volkoff from the standard point of view which consists in putting \(f = 0\). Our point of view consist in using instead the value \(f = 1\).

The equation of state can be written in parametric form, including both points of view, as:
\[
\rho = K (\sinh u - u)(f(B^2C)^{-1} + 1 - f)
\]
\[
p = \frac{1}{3}K (\sinh u - 8 \sinh \frac{1}{2} u + 3u)
\]
where, using arbitrary units:
\[
K = \frac{m^4c^5}{4h^3}
\]
m being the mass of a neutron.

We recall below the values of \(M_a\) obtained in [1] for several values of the initial values of \(u_0\), and include the values of \(M_p\) as calculated from (33):
The system of units that has been used is that satisfying (19) completed with the supplementary condition:

$$K = \frac{1}{4\pi}$$

which is the choice made in [1] to define a unit of mass. The most notorious result is existence of a maximum mass corresponding approximately to $u_0 = 3$ whose value is $M_a = 0.078$ which corresponds to $M_a = 0.71 M_\odot$. Oppenheimer and Volkoff concluded also from a very crude non relativistic argument that above $u_0 = 3$ the equilibrium configurations were not stable.

For $f = 1$ the results that we have obtained are:

| $u_0$ | $M_a$ | $M_p$ | $E_b$  |
|-------|-------|-------|--------|
| 1     | 0.033 | 0.033 | -0.000 |
| 2     | 0.066 | 0.071 | -0.005 |
| 3     | 0.077 | 0.088 | -0.011 |
| 4     | 0.070 | 0.085 | -0.015 |
| 5     | 0.060 | 0.074 | -0.014 |
| 6     | 0.049 | 0.062 | -0.013 |
| 7     | 0.042 | 0.054 | -0.012 |

Here also we obtain that there is a maximum mass $M_a = 0.41$ which corresponds to $M_a = 3.8 M_\odot$ but it is substantially larger than the value in [1] as well as the mass of some models with anisotropic pressure considered by Corchero in [7]. It is even somewhat larger than the limit value, $M_a = 3.2 M_\odot$ obtained by Rhoades and Ruffini in [8] from very general considerations complying with the conventional point of view.

## Acknowledgments

I gratefully acknowledge the help provided by J. M. Aguirregabiria checking parts of this manuscript, the patience of A. Chamorro for a careful reading of
it, and their comments as well as those of E. S. Corchero and J. Martín. I also gratefully acknowledge the position of visiting professor to the UPV/EHU that I have been holding while this paper was being prepared.

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