Precessing warped discs in close binary systems

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Abstract: We describe some recent nonlinear three dimensional hydrodynamic simulations of accretion discs in binary systems where the orbit is circular and not necessarily coplanar with the disc midplane. The calculations are relevant to a number of observed astrophysical phenomena, including the precession of jets associated with young stars, the high spectral index of some T Tauri stars, and the light curves of X-ray binaries such as Hercules X-1 which suggest the presence of precessing accretion discs.

1 Introduction

Protostellar discs appear to be common around young stars. Furthermore recent studies show that almost all young stars associated with low mass star forming regions are in multiple systems (Mathieu, 1994 and references therein). Typical orbital separations are around 30 astronomical units (Leinert et al. 1993) which is smaller than the characteristic disc size observed in these systems (Edwards et al. 1987). It is therefore expected that circumstellar discs will be subject to strong tidal effects due to the influence of binary companions.

The tidal effect of an orbiting body on a differentially rotating disc has been well studied in the context of planetary rings (Goldreich and Tremaine, 1981), planetary formation, and generally interacting binary stars (see Lin and Papaloizou, 1993 and references therein). In these studies, the disc and orbit are usually taken to be coplanar (see Artymowicz and Lubow, 1994). However, there are observational indications that discs and stellar orbits may not always be coplanar (see for example Corporon, Lagrange and Beust, 1996 and Bibo, The and Dawanas, 1992.)

In addition, reprocessing of radiation from the central star by a warped non coplanar disc has been suggested in order to account for the high spectral index of some T Tauri stars (Terquem and Bertout 1993, 1996).

A dynamical study of the tidal interactions of a non coplanar disc is of interest not only in the above contexts, but also in relation to the possible existence of precessing discs which may define the axes for observed jets which apparently precess (Bally and Devine, 1994).

Various studies of the evolution of warped discs have been undertaken assuming that the forces producing the warping were small so that linear perturbation theory could be used (Papaloizou and Pringle, 1983, Papaloizou and Lin, 1995)
and Papaloizou and Terquem, 1995). The results suggested that the disc would precess approximately as a rigid body if the sound crossing time was smaller than the differential precession frequency.

We describe here some recent non linear simulations of discs which are not coplanar with the binary orbits using a Smoothed Particle Hydrodynamics (SPH) code originally developed by Nelson and Papaloizou (1993, 1994). We study the conditions under which warped precessing discs may survive in close binary systems and the truncation of the disc size through tidal effects when the disc and binary orbit are not coplanar. The simulations indicate that the phenomenon of tidal truncation is only marginally affected by lack of coplanarity. Also our model discs were able to survive in a tidally truncated condition while warped and undergoing rigid body precession provided that the Mach number in the disc was not too large. The inclination of the disc was found to evolve on a long timescale likely to be the viscous timescale, as was indicated by the linear calculations of Papaloizou and Terquem (1995).

2 Basic equations

The equations of continuity and of motion applicable to a gaseous viscous disc may be written

\[
\frac{d\rho}{dt} + \rho \nabla \cdot \mathbf{v} = 0, \tag{1}
\]

\[
\frac{d\mathbf{v}}{dt} = -\frac{1}{\rho} \nabla P - \nabla \Psi + \mathbf{S}_{\text{visc}} \tag{2}
\]

where

\[
\frac{d}{dt} \equiv \frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla
\]

denotes the convective derivative, \( \rho \) is the density, \( \mathbf{v} \) the velocity and \( P \) the pressure. The gravitational potential is \( \Psi \), and \( \mathbf{S}_{\text{visc}} \) is the viscous force per unit mass.

For the work described here, we adopt the polytropic equation of state

\[
P = K \rho^\gamma
\]

where

\[
c_s^2 = K \gamma \rho^{\gamma - 1}
\]
gives the usual associated sound speed, \( c_s \). Here we take \( \gamma = 5/3 \), and \( K \) is the polytropic constant. This corresponds to adopting a fluid that remains isentropic throughout even though viscous dissipation may occur. This means that an efficient cooling mechanism is assumed.
3 Orbital Configuration

We consider a binary system in which the primary has a mass $M_p$ and the secondary has a mass $M_s$. The binary orbit is circular with separation $D$. The orbital angular velocity is $\omega$. We suppose that a disc orbits about the primary such that at time $t = 0$ it has a well defined mid-plane. We adopt a non rotating Cartesian coordinate system $(x, y, z)$ centred on the primary star and we denote the unit vectors in each of the coordinate directions by $\hat{i}$, $\hat{j}$ and $\hat{k}$ respectively. The $z$ axis is chosen to be normal to the initial disc mid-plane. We shall also use the associated cylindrical polar coordinates $(r, \varphi, z)$.

We take the orbit of the secondary to be in a plane which has an initial inclination angle $\delta$ with respect to the $(x, y)$ plane. For a disc of negligible mass, the plane of the orbit is invariable and does not precess. We denote the position vector of the secondary star by $D$ with $D = |D|$. Adopting an orientation of coordinates and an origin of time such that the line of nodes coincides with, and the secondary is on, the $x$ axis at $t = 0$, the vector $D$ is given as a function of time by

$$D = D \cos \omega t \hat{i} + D \sin \omega t \cos \delta \hat{j} + D \sin \omega t \sin \delta \hat{k}. \quad (3)$$

The total gravitational potential $\Psi_{ext}$ due to the binary pair at a point with position vector $r$ is given by

$$\Psi_{ext} = -\frac{GM_p}{|r|} - \frac{GM_s}{|r - D|} + \frac{GM_s r \cdot D}{D^3}$$

where $G$ is the gravitational constant. The first dominant term is due to the primary, while the remainder, $\equiv \Psi'_{ext}$, gives perturbing terms due to the secondary. Of these, the last indirect term accounts for the acceleration of the origin of the coordinate system. We note that a disc perturbed by a secondary on an inclined orbit becomes tilted, precesses and so does not maintain a fixed plane. Our calculations presented below are referred to the Cartesian system defined above through the initial disc mid-plane. However, we shall also use a system defined relative to the fixed orbital plane for which the ‘$x$ axis’ is as in the previous system and the ‘$z$ axis’ is normal to the orbital plane. If the disc were a rigid body its angular momentum vector would precess uniformly about this normal, as indicated below.

3.1 Disc response

The form of the disc response to the perturbing gravitational potential due to the secondary is determined by the properties of the free modes of oscillation. These are divided into two classes according to whether the associated density perturbation has even or odd symmetry with respect to reflection in the unperturbed disc midplane. The modes with even symmetry are excited when the binary orbit and disc are coplanar and have been well studied in the context of angular momentum exchange between disc and binary leading to tidal truncation of and wave excitation in the disc (see Lin and Papaloizou, 1993 and...
references therein). They are also excited at a somewhat reduced level in the non coplanar case where they produce similar effects. Here we shall focus attention on the modes with odd symmetry and with azimuthal mode number $m = 1$. These are only excited in the non coplanar case. They are of interest because they are responsible for disc warping, twisting and precession.

3.2 Potential expansion

When the orbital separation is much larger then the outer disc radius such that for any disc particle, $r \ll D$, and $z \ll D$, we can expand $\Psi'_\text{ext}$ in powers of $r/D$ and $z/D$.

We are interested in bending modes which are excited by terms in the potential which are odd in $z$ and which have azimuthal mode number $m = 1$ when a Fourier analysis in $\varphi$ is carried out. The lowest order terms in the expansion of the potential which are of the required form are given by

$$
\Psi'_\text{ext} = -\frac{3}{4} \frac{GM}{D^3} r z \left[ (1 - \cos \delta) \sin \delta \sin (\varphi + 2\omega t) \\
- (1 + \cos \delta) \sin \delta \sin (\varphi - 2\omega t) \\
+ \sin 2\delta \sin (\varphi) \right]
$$

In linear perturbation theory, we can calculate the response of the disc to each of the three terms in $\Psi'_\text{ext}$ separately and superpose the results. The general problem is then to calculate the response due to a potential of the form

$$
\Psi'_\text{ext} = \text{const} \times rz \sin (\varphi - \Omega_P t)
$$
or in complex notation

$$
\Psi'_\text{ext} = \mathcal{R} \left( f_r z e^{im(\varphi - \Omega_P t)} \right)
$$

Here, the azimuthal mode number, $m = 1$, the pattern frequency $\Omega_P$ of the perturber is one of 0, $2\omega$ or $-2\omega$ and $f$ is the appropriate complex amplitude.

We remark that the magnitude of the tidal perturbation acting on the disc is measured by the dimensionless quantity $GM_s/(\Omega^2 D^3)$, $\Omega$ being the angular velocity in the disc and for comparable primary and secondary masses this is of order $\omega^2/\Omega^2$.

4 Free bending modes

Bending waves with $m = 1$ are naturally excited by a perturbing potential of the form given by (5). For a binary with large orbital radius, we may consider the pattern frequency $\Omega_P$ to be small compared to the rotation frequency in the disc, $\Omega$. Thus the forcing is at low frequency.
Bending waves in thin discs have been studied in the context of disc galaxies (Hunter and Toomre, 1969), planetary rings (Shu, 1984) and gaseous accretion discs (Papaloizou and Lin, 1995, Papaloizou and Terquem, 1995).

In a self-gravitating disc with no pressure and of small enough mass that the unperturbed disc is in a state of near Keplerian rotation, the local dispersion relation for bending waves with $m = 1$ is given by (Hunter and Toomre, 1969)

$$(\Omega_p - \Omega)^2 = \Omega^2 + 2\pi G \Sigma |k|,$$

(6)

where $\Sigma$ is the disc surface density. In the limit of small pattern speeds this takes the form

$$\Omega_p \Omega = -\pi G \Sigma |k|,$$

(7)

from which it follows that the waves propagate without dispersion with speed

$$c_g = \frac{\pi G \Sigma}{\Omega} = \langle c_s \rangle Q,$$

(8)

where the Toomre $Q = \Omega \langle c_s \rangle / (\pi G \Sigma)$, where the angled brackets denote an appropriate vertical mean of the sound speed. For stability to axisymmetric modes, we require $Q \geq 1$, which implies that bending waves in a stable disc propagate with a speed not exceeding the maximum sound speed at a particular radial location.

Papaloizou and Lin (1995) considered the case when pressure is included, giving the corresponding local dispersion relation for bending waves with $m = 1$ in the low frequency limit

$$\Omega_p \Omega = -\pi G \Sigma |k| - \frac{(1 - \Theta) \Omega \langle c_s \rangle^2 |k|^2}{4\Omega_p},$$

(9)

where $\Theta < 1$ is a dimensionless parameter which vanishes when self-gravity is unimportant. This gives a quadratic equation for $\Omega_p$ with the two roots

$$\Omega_p = -\frac{\langle c_s \rangle |k|}{2} \left( \frac{1}{Q^2} \pm \sqrt{\frac{1}{Q^2} + 1 - \Theta} \right).$$

(10)

In this case there are fast and slow waves which in the case of a stable disc propagate with speeds comparable to the sound speed. When self-gravity is unimportant there is a single sonic like wave. The excitation by the forcing potential (5), and angular momentum transport associated with these waves with non zero $\Omega_p$ has been considered by Papaloizou and Terquem (1995).

The secular term in the forcing potential (4) with zero pattern speed causes the disc to be subject to a precessional torque. The properties of the bending waves determine the form of the disc response. For the disc to precess approximately like a rigid body, the effects of the precessional torque which acts largely in the outer parts of the disc, must be communicated to the inner regions where it is weakest, within a precession period. This roughly corresponds to the condition that the disc sound crossing time be less than the precession period.
5 Precession Frequency

In order to calculate the precession frequency, we consider the time independent, or secular term in the perturbing potential (4) as it is only this term which produces a non zero net torque after performing a time average. This is given by

$$\Psi'_{ext0} = -\frac{3}{4} \frac{GM_s}{D^3} rz \sin 2\delta \sin(\varphi).$$  \hspace{1cm} (11)

For a conservative system, the Lagrangian displacement vector $\xi$ will satisfy an equation of the form (see Lynden-Bell and Ostriker 1967)

$$C(\xi) = -\nabla \Psi'_{ext0}. \hspace{1cm} (12)$$

Here $C$ is a linear operator, which needs to be inverted to give the response. When a barotropic equation of state applies, and the boundaries are free, $C$ is self-adjoint with weight $\rho$. This means that for two general displacement vectors $\xi(r)$ and $\eta(r)$ we have

$$\int_V \rho \eta^* \cdot C(\xi) d\tau = \left[ \int_V \rho \xi^* \cdot C(\eta) d\tau \right]^* \hspace{1cm} (13)$$

where $^*$ denotes complex conjugate and the integral is taken over the disc volume $V$.

Because of the spherical symmetry of the unperturbed primary potential, the unperturbed system is invariant under applying a rigid tilt to the disc. This corresponds to the existence of rigid tilt mode solutions to (12) when there is no forcing ($\Psi'_{ext0} = 0$). For a rotation about the $x$ axis, the rigid tilt mode is of the form

$$\xi = \xi_T = (r \times \hat{\varphi}) \sin \varphi - \hat{\varphi} z \cos \varphi \hspace{1cm} (14)$$

where $\hat{\varphi}$ is the unit vector along the $\varphi$ direction. For time averaged forcing potentials $\propto \sin \varphi$, the existence of the solution (14) results in an integrability condition for (12). When $C$ is self-adjoint, this is

$$\int_V \xi_T \cdot \nabla \Psi_s \rho d\tau = \int_V (r \times \nabla \Psi_s) \rho d\tau = 0. \hspace{1cm} (15)$$

The above condition is equivalent to the requirement that the $x$ component of the external torque vanishes. This will clearly not be satisfied in the problem we consider.

To deal with this one may suppose that the disc angular momentum vector precesses about the orbital angular momentum vector with angular velocity $\omega_p$ (Papaloizou and Terquem 1995). This in turn is equivalent to supposing our coordinate system rotates with angular velocity $\omega_p$ about the orbital rotation axis. Treating the Coriolis force by perturbation theory produces an additional term on the right hand side of (12) equal to $-2r\Omega \hat{\omega}_p \times \hat{\varphi}$. Using the modified force in formulating (15) gives the integrability condition as
\[ \omega_p \sin \delta \int_V r^2 \Omega \rho d\tau = \int_V \hat{i} \cdot (\mathbf{r} \times \nabla \Psi_s) \rho d\tau, \quad (16) \]

with \( \omega_p = |\omega_p| \). Equation (16) gives a precession frequency for the disc that would apply if it were a rigid body. However, approximate rigid body precession is only expected to occur if the disc is able to communicate with itself, either through wave propagation or viscous diffusion, on a timescale less than the inverse precession frequency (see for example Papaloizou and Terquem 1995 and below). Otherwise, a thin disc configuration may be destroyed by strong warping and differential precession.

We comment that the situation described above in which the external perturbation produces a precessional torque in the \( x \) direction only is a consequence of the assumption of a conservative response for which the density and potential perturbations are in phase. However, if dissipative processes are included, there will be a phase shift between the perturbing potential and density response. This will result in a net torque in the \( y \) direction which can change the angle between the disc and orbital angular momentum vectors (see Papaloizou and Terquem, 1995) possibly leading to their alignment. Such a process is likely to occur on the long dissipative timescale. A torque in the \( y \) direction originating from a disc wind has been proposed by Schandl and Meyer (1994) in order to produce misalignment between the disc and binary orbit angular momentum vectors in HZ-Hercules.

6 Numerical Simulations

Three dimensional simulations of warped precessing discs in close binary systems have been carried out by solving the set of basic equations (1) and (2) numerically using an SPH code (Lucy 1977, Gingold and Monaghan 1977), developed by Nelson and Papaloizou (1994), which uses a conservative formulation of the method that employs variable smoothing lengths. A suite of test calculations illustrating the accurate energy conservation obtained with this method is described by Nelson and Papaloizou (1994), and additional tests and calculations are presented in Nelson (1994).

Larwood et al (1996) considered circumprimary discs in close binary systems with mass ratio of order unity and Larwood and Papaloizou (1997) have considered circumbinary discs in systems with a variety of mass ratios.

In order to stabilize the calculations in the presence of shocks, the artificial viscous pressure prescription of Monaghan and Gingold (1983) has been used in the simulations. This induces a shear viscosity which leads to angular momentum transport and the standard viscous evolution of an accretion disc (Lynden-Bell and Pringle 1974) in which disc expansion is produced by outward transport of angular momentum as mass flows inwards. In the studies presented here, the discs undergo angular momentum loss through tidal interaction with orbiting secondaries. Then disc expansion arising from outward transport of angular momentum is halted by tidal truncation. This effect, well known in the coplanar
case (see for example Lin and Papaloizou 1993), also occurs here when the disc and binary angular momenta are not aligned.

In the simulations reported here the shear viscosity, \( \nu \) operating in our disc models was well fitted by a constant value. To specify the magnitude of \( \nu \) we write

\[
\nu = \alpha c_s^2(R)/\Omega(R),
\]

where \( \alpha \) corresponds to the well known Shakura and Sunyaev (1973) \( \alpha \) parameterization and \( R \) denotes the outer radius of the disc. However, it is applicable only at the outside edge of the disc. The discs were here set up with a distribution of 17500 particles such that the surface density was independent of radius. Then the aspect ratio \( H/r \), with \( H \) being the semi-thickness was found to be approximately independent of radius. As a consequence of this the Mach number \( \mathcal{M} = (H/R)^{-1} \) is also approximately constant for a particular simulation and can be used to parameterize it. Also the radial dependence of the viscosity parameter is \( \alpha \propto r^{-1/2} \).

We comment that a characteristic value of \( \alpha = 0.03 \) that we have in the simulations is about two orders of magnitude larger than that expected to be associated with tidally induced inwardly propagating waves (Spruit 1987, Papaloizou and Terquem 1995). Accordingly, it is expected that tidal truncation will instead occur through strong nonlinear dissipation near the disc’s outer edge (Savonije, Papaloizou and Lin 1994). The large viscosity of the disc models considered here will damp inwardly propagating waves before they can propagate very far.

In order to deal with the central regions of the disc, the primary’s gravitational potential softened such that

\[
\psi_p = -\frac{GM_p}{\sqrt{r^2 + b^2}},
\]

where \( b \) is the softening length.

We adopt units such that the primary mass \( M_p = 1 \), the gravitational constant \( G = 1 \), and the outer disc radius \( R = 1 \). In these units the adopted softening parameter \( b = 0.2 \) and the time unit is \( \Omega(R)^{-1} \). The self-gravity of the disc material has been neglected in the simulations presented here.

## 7 Numerical Results

We have considered the evolution of disc models set up according to the procedure outlined above. The models were characterized only by the mean Mach number, \( \mathcal{M} \). After a relaxation period of about two rotation periods at the outer edge of the disc, the time was reset to zero and the secondary was introduced in an inclined circular orbit, moving in a direct sense, crossing the \( x \) axis at \( t = 0 \). In some cases the full secondary mass was included immediately corresponding to a sudden start. However, for strong initial tidal interactions such as those that occur when \( D/R = 3, M_s/M_p = 1 \), this can result in disruption of the outer edge of the disc with a small number of particles being ejected from the disc. It was found that this could be avoided by using a ‘slow start’ in which \( M_s \) was built up gradually (see Larwood et al., 1996). However, subsequent results were found to be independent of the initiation procedure.
In the above discussion of bending modes we indicated that the disc should be able to approximately precess as a rigid body if the sound crossing time was short compared to the characteristic precession period.

The general finding from the simulations was that a disc with an initial angular momentum vector inclined to that of the binary system tended to precess approximately as a rigid body, with a noticeable but small warp if $\mathcal{M}$ was not too large. In such cases only small changes in the inclination angle between the angular momentum vectors were found over the run time. This is consistent with the expectation from Papaloizou and Terquem (1995) that the timescale for evolution of the inclination in such cases should be comparable to the viscous evolution timescale of the disc, assuming outward disc expansion is prevented by tidal interaction.

We here describe simulations of three circumprimary disc models in a close binary system with $D/R = 3$ and $\delta = \pi/4$ initially. Models 1, 2 and 3 had $\mathcal{M}$ equal to 20, 25 and 30 respectively and the total run times for these models initiated with a slow start was 310, 217.7, and 397.9 units respectively. Note that the viscosity is larger in the models with smaller Mach number so that this aids disc communication in these cases also.

The calculations presented here use a coordinate system which is based on the initial disc mid-plane. However, as the disc precesses, the mid-plane changes location with time. It is then more convenient to use a coordinate system $(x, y_o, z_o)$ based on the fixed orbital plane, the $z_o$ axis coinciding with the orbital rotation axis. We shall refer to these as ‘orbital plane coordinates’. We locate the inclination angle $\iota$ (equal to $\delta$ at $t = 0$) between the disc and binary orbit angular momentum vectors through

$$\cos \iota = \frac{\mathbf{J}_D \cdot \mathbf{J}_O}{|\mathbf{J}_D||\mathbf{J}_O|}.$$ 

Here, $\mathbf{J}_O$ is the orbital angular momentum. The disc angular momentum is $\mathbf{J}_D = \sum_j \mathbf{J}_j$, where the sum is over all disc particle angular momenta $\mathbf{J}_j$.

A precession angle $\beta_p$, measured in the orbital plane can be defined through

$$\cos \beta_p = -\frac{(\mathbf{J}_D \times \mathbf{J}_O) \cdot \mathbf{u}}{|\mathbf{J}_D \times \mathbf{J}_O||\mathbf{u}|},$$

where $\mathbf{u}$ may be taken to be any fixed reference vector in the orbital plane. We take this to point along the $y_o$ axis such that initially $\beta_p = \pi/2$ in all cases. For retrograde precession of $\mathbf{J}_D$ about $\mathbf{J}_O$ the angle $\beta_p$ should initially decrease as is found in practice.

For a disc with constant $\Sigma$ and radius $R$, the period of rigid body precession of a thin disc is $2\pi/\omega_p$ where, from equation (16) we obtain

$$\omega_p = -\left(\frac{3GM_s}{4D^3}\right) \cos \delta \int_0^R \frac{\Sigma r^3 dr}{\int_0^R \Sigma r^3 \Omega dr} = -\frac{15M_s R^3}{32M_p D^3 \Omega(R)} \cos \delta.$$ (17)
The condition for sound to propagate throughout the disc during a precession time is approximately that

\[ \frac{H}{R} > \frac{|\omega_p|}{\Omega(R)}. \]  \hspace{1cm} (18)

For models 1-3, equation (17) gives \( \omega_p/\Omega(R) = 0.012 \) corresponding to a precession period of 512 time units. Our results were consistent with the condition (18) to within a factor of two in that models 1 and 2 with \( M < 25 \), showed modest warps and approximate rigid body precession while Model 3 with \( M = 30 \) showed severe warping and a more complex precessional behaviour.

**Fig. 1.** Projection plots in orbital plane coordinates for model 1 at time \( t \simeq 0 \). The projections are in the \((x, y_o)\) plane (top left), \((x, z_o)\) plane (top right), \((y_o, z_o)\) plane (bottom left) and \((x, z_o)\) plane (bottom right).
Fig. 2. The precession angle $\beta_p$ (dashed line) and inclination angle $\iota$ (solid line) for model 1.

Fig. 3. Projection plots in orbital plane coordinates for model 1 at time $t = 297.8$.

Fig. 4. Projection plots in orbital plane coordinates for model 3 at time $t = 397.9$.

We now present particle projection plots for each of the Cartesian planes using orbital plane coordinates. In all such figures a fourth ‘sectional plot’ is also included in which only particles such that $-0.05 < y_o < 0.05$ are plotted.

7.1 Model 1

A projection plot for model 1 is shown in Fig. (1) near $t = 0$ when the disc is unperturbed. Note that the disc appears as edge on and inclined at 45 degrees in the $(y_o, z_o)$ plane. The time dependence of the angles $\iota$ and $\beta_p$ is plotted in Fig. (2). It may be seen that there is little change in $\iota$ during the whole run. On the other hand $\beta_p$ decreases approximately linearly, corresponding to uniform precession (note that $\beta_p$ is plotted as positive rather than negative for the latter section of this plot). The inferred precession period is around 600 units, in reasonable agreement with the value of 512 units obtained from equation (17). Fig. (3) shows a projection plot at $t = 297.8$, near the end of the run when the disc has precessed through about 180 degrees. This amount of precession is demonstrated by the fact that the disc appears almost edge on in the $(y_o, z_o)$ plane just as it did at time $t = 0$. However, its plane is inclined at about 90 degrees to the original disc plane. At this stage our results indicate that the disc has attained a quasi-steady configuration as viewed in a frame that precesses uniformly about the orbital rotation axis. The disc develops a warped structure that initially grows in magnitude but then levels off. The sectional plot in Fig. (3) indicates that the disc has developed a modest warp in this case.

7.2 Models 2, and 3

The behaviour of model 3 with $M = 30$ is considerably more complex than that of model 1. This disc develops a strong warp such that the inner and outer parts of the disc try to separate. A projection plot is shown at $t = 397.9$ in Fig. (4). The inner part of the disc seems to occupy a different plane from the outer part.
Fig. 5. The precession angle $\beta_p$ (solid and long-dashed lines) and inclination angle $\iota$ (short-dashed and dotted lines) for model 3. The solid and short-dashed lines correspond to the outer disc section with $r > 0.5$, and the long-dashed and dotted lines correspond to the inner disc section with $r < 0.3$.

Fig. 6. The precession angle $\beta_p$ (solid and long-dashed lines) and inclination angle $\iota$ (short-dashed and dotted lines) for model 2. The solid and short-dashed lines correspond to the outer disc section with $r > 0.5$, and the long-dashed and dotted lines correspond to the inner disc section with $r < 0.3$.

The outer part was found to precess like a rigid body at the expected rate, and it tended to drag the inner section behind it. This is indicated in Fig. (5) where we plot the angle $\beta_p$ calculated using the outer disc section with $r > 0.5$ only and also the same angle calculated using only the inner section with $r < 0.3$. The angle associated with the inner segment progresses at a variable rate indicating coupling to the outer section. The angle $\iota$ associated with the two sections is also plotted. This becomes significantly smaller for the inner segment consistent with the existence of a large amplitude warp. We note that the relatively larger inclination associated with the outer segment enables a closer matching of the precession frequencies associated with the two segments and aids coupling, due to the presence of large pressure gradients induced by the strong warping. Each segment of this model remained thin throughout the run.

Fig. (6) shows the evolution of $\beta_p$ and $\iota$ for model 2, which has $\mathcal{M} = 25$ and is intermediate between model 1 and model 3. There is an indication of differential precession initially. But the inner part of the disc couples to the outer part in such a way that the precession becomes uniform after about 150 time units. It appears that the inner part is able to adjust its precession frequency to the outer part by changing slightly its relative inclination. In this way the dependence of the precession frequency on inclination is exploited to remove the differential precession.

8 Discussion

We have described nonlinear simulations of an accretion disc in a close binary system when the disc midplane is not necessarily coplanar with the plane of the binary orbit. For our constant viscosity SPH models we found the tidal truncation phenomenon to be only marginally affected by non coplanarity. We found that modestly warped and thin discs undergoing near rigid body precession may survive in close binary systems. However, extremely thin discs may be severely disrupted by differential precession depending on the magnitude of the characteristic Mach number, $\mathcal{M}$. The crossover between obtaining a warped, but coherent disc structure, and disc disruption occurs for a value of the Mach number $\mathcal{M} \sim 30$. We also found that the inclination evolved on a long timescale, likely
to be the viscous timescale, as indicated by the linear calculations of Papaloizou and Terquem (1995).

A class of models formulated to explain the generation of jets in young stellar objects assumes that a wind flows outwards from the disc surface. This is then accelerated and collimated by the action of a magnetic field (see Königl and Ruden, 1993 and references therein). It is reasonable to assume that a precessing disc may lead to the excitation of a precessing jet. The precession period obtained from our calculations with a mass ratio of unity is about 500 units. When scaled to a disc of radius 50 AU, surrounding a star of 1M⊙, the unit of time is \( \Omega(R)^{-1} \approx 56 \) yr, leading to a precession period of \( 3 \times 10^4 \) yr.

Bally and Devine (1994) suggest that the jet which seems to be excited by the young stellar object HH34* in the L1641 molecular cloud in Orion precesses with a period of approximately \( 10^4 \) yr. This period is consistent with the source being a binary with parameters similar to those we have used in our simulations with a separation on the order of a few hundred astronomical units.

Some of the results presented here demonstrate how a warped disc can present a large surface area for intercepting the primary star’s radiation. The effect that the consequent reprocessing of the stellar radiation field can have on the emitted spectral energy distribution has been investigated by Terquem and Bertout (1993, 1996). They find that it may account for the high spectral index of some T Tauri stars or even for the spectral energy distribution of some class 0/I sources. The Model 3 simulation indicates that the required strongly warped disc could be physically realisable.

Finally, there is evidence from the light curves of X–ray binaries such as Hercules X–1 and SS433, that their associated accretion discs may be precessing in the tidal field of the binary companion (Schandl and Meyer, 1994). Larwood et al (1996) have demonstrated that the disc precession periods seen in simulations are in reasonable agreement with those that are inferred observationally (Pettersson, 1975, Gerend and Boynton, 1976, Margon, 1984).

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