Equation of the gas oscillation frequencies in a closed-circuit Helmholtz resonator type combustion chamber

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Abstract. A solid fuel combustion chamber of the Helmholtz-resonator-type is investigated. In previous works, the air supply tube and the combustion product outlet tube were open at the outlet. In this paper, we consider a device in which the tubes are connected by a total volume. The linearization of the conservation equations in the combustion chamber and the interference chamber gives four boundary conditions for acoustic disturbances. From these conditions, an equation follows that allows us to calculate the frequencies of gas vibrations in the device of the considered type.

1. Introduction
The main problem of using pulsating combustion chambers in industry is a high noise level. One of the possible solutions is the use of combustion chambers with two resonant tubes of different lengths [1]. This difference can be such that the oscillations of the gas at the ends of the tubes have opposite phases. If the tubes have a common outlet, then the interference of sound waves will cause a significant reduction in the noise level [2, 3]. In [4, 5], the pulsating combustion of solid fuel in a combustion chamber of the Helmholtz-resonator-type with two resonance tubes open at the outlet was investigated (Figure 1). It is shown [6] that the gas oscillations at the outlet of the tubes have opposite phases if the air supply tube is approximately 3 times longer than the pipe for the outlet of combustion products.

Figure 1. Set the type of the Rijke tube: 1-combustion chamber, 2-grate holding fuel, 3-resonant tube, 4-air supply tube.
In this paper, in order to achieve the interference of the sound wave, it is proposed to connect the ends of the tube with a total volume of two holes (Fig. 2). The purpose of this work is to determine the equation of the frequencies of gas vibrations in the installation under consideration.

Figure 2. Double-circuit installation of the Helmholtz resonator type: 1-combustion chamber, 2-fuel grate, 3-air supply tube, 4-pipe for the exit of combustion products(chimney), 5-mixing chamber, 6-air supply hole, 7-hole for the exit of combustion products.

2. Frequency equation

The boundary conditions for the pulsations of velocity $u'$ and pressure $p'$ at the inlet and outlet of the combustion chamber are known [2] and have the form:

$$p'_{3,0} = p'_{4,0} = p'_{1}, \quad (1)$$

$$S'_3 u'_{3,0} + S'_4 u'_{4,0} = \frac{i \omega V'_3 p'_1}{\rho_3 \omega c_3^2}, \quad (2)$$

where $S$ is the cross-sectional area of the tube, $V$ is the volume of the chamber, $\omega$ is the cyclic frequency, $\rho_{3,0}$ is the average gas density, and $c$ is the speed of sound.

Linearization of the conservation equations and the equation of state for a gas in an interference chamber gives the following:

$$p'_{3,1} = p'_{4,1} = p'_{5} = p'_{6} = p'_{7}, \quad (3)$$

$$S'_3 u'_{3,1} + S'_4 u'_{4,1} = \frac{i \omega V'_3 p'_1}{\rho_3 \omega c_3^2} - S'_6 u'_{6} - S'_7 u'_{7}. \quad (4)$$

In an air supply tube, the speed of sound is constant. The pulsations of velocity and pressure are as follows:

$$u'_{3}(x_3, t) = C_3 \cos(\omega x_3/c_3 + \varphi_3)e^{i\omega t},$$

$$p'_{3}(x_3, t) = -i \rho_{3,0} c_3 C_3 \sin(\omega x_3/c_3 + \varphi_3)e^{i\omega t}.$$
\[
u'_4(x_4, t) = C_4 e^{i\omega t} \left( 1 - \frac{b_4 x_4}{a_4} \right)^{1/2} \cos(\varphi_4 - \psi_4), \quad \psi_4 = \frac{\omega \beta_4}{b_4} \ln \left( 1 - \frac{b_4 x_4}{a_4} \right),
\]

\[
p'_4(x_4, t) = -i \rho_{4,0}(x_4) c_4(x_4) C_4 e^{i\omega t} \left( 1 - \frac{b_4 x_4}{a_4} \right)^{1/2} \left[ \frac{b_4}{2\omega} \cos(\varphi_4 - \psi_4) + \beta_4 \sin(\varphi_4 - \psi_4) \right].
\]

Let's introduce acoustic impedances:

\[
Z_6 = p'_6/u'_6, \quad Z_7 = p'_7/u'_7,
\]

where \( Z = iY \) is the imaginary part of the round hole impedance.

As a result, from the relations (1) – (4) we have:

\[
\begin{align*}
\varepsilon_3 C_3 \cos \varphi_3 + C_4 \cos \varphi_4 &= \sigma_0 C_3 \sin \varphi_3, \\
\theta_0 C_3 \sin \varphi_3 &= C_4 \left( \frac{b_4}{2\omega} \cos(\varphi_4) + \beta_4 \sin(\varphi_4) \right), \\
\varepsilon_3 C_3 \left[ \cos(\varphi_4 - \psi_{4,l}) + \lambda_4 C_4 \cos(\varphi_4 - \psi_{4,l}) \right] &= \sigma_1 C_3 \sin(\varphi_3 + \psi_{3,l}), \\
\sigma_3 C_3 \sin(\varphi_3 + \psi_{3,l}) &= C_4 \left[ \frac{b_4}{2\omega} \cos(\varphi_4 - \psi_{4,l}) + \beta_4 \sin(\varphi_4 - \psi_{4,l}) \right]
\end{align*}
\]

where \( \varepsilon_3 = \varepsilon_{3,4}, \quad \varepsilon_6 = \varepsilon_{6,7}, \quad \varepsilon_7 = \varepsilon_{7,4}, \quad b_3 = 0, \quad \beta_3 = 1, \quad \frac{\omega I_3}{c_3} = -\psi_{3,l}, \)

\[
\lambda_4 = \sqrt{1 - \frac{b_4}{a_4}}, \quad \lambda_3 = 1, \quad \sigma_0 = -\frac{\omega V_4}{c_3 s_4}, \quad \sigma_1 = \frac{\omega V_5}{c_3 s_4} + \rho_{3,0} c_3 \left( \frac{e_6}{y_6} + \frac{e_7}{y_7} \right).
\]

From (5) we have

\[
C_4 \cos \varphi_4 = C_3 (\sigma_0 \sin \varphi_3 - \varepsilon_3 \cos \varphi_3).
\]

Taking into account (5), from (6) it follows:

\[
C_4 \cos \varphi_4 = C_3 (N_1 \cos \varphi_3 + N_2 \sin \varphi_3).
\]

Substitution of (9), (10) into equation (7) gives:

\[
\tan \varphi_4 = \frac{N_1}{N_4}.
\]

From (9) and (10) we obtain

\[
\begin{align*}
\tan \varphi_4 &= \frac{N_1 + N_2 \tan \varphi_3}{\sigma_0 \tan \varphi_3 - \varepsilon_3}.
\end{align*}
\]

As a result, from equation (8) we have:

\[
\sigma_{3,4} M_3 M_2 - M_4 = 0.
\]
where $M_1 = \theta_0 \sin \varphi_3$, $M_2 = \frac{b_4}{2\omega} \cos \varphi_4 + \beta_4 \sin \varphi_4$.

$M_3 = \sin(\varphi_3 + \psi_{3,1})$, $M_4 = \frac{b_4}{2\omega} \cos(\varphi_4 + \psi_{4,1}) + \beta_4 \sin(\varphi_4 - \psi_{4,1})$.

The angles $\varphi_3$, $\varphi_4$ are determined by the relations (11), (12). Then equation (13) contains one unknown quantity – the frequency of gas oscillations.

Consider a particular case: $V_5 \to \infty$, i.e. the tubes are in communication with the atmosphere and do not interact. After a series of transformations, equation (13) takes the form

$$
\frac{b_4}{2\omega} + \beta_4 \tan \varphi_4 F^{-1} = 0, \tag{14}
$$

where $F = \left(\omega V_1 \left(\frac{c_3}{c_{3,4}} - \frac{\varepsilon_3}{\gamma,0} \frac{c_3}{c_{4,0}}\right) \right) \tilde{Y}_{3,0} = -\tan \varphi_3 = \text{Im} \left(\frac{p_{3,0}/w_{3,0}}{\rho_{3,0} c_3} \right)$.

The resulting equation, after the corresponding replacement of the indices, coincides with the known equation for installations of the Helmholtz-resonator-type in the case when the ends of the resonant tubes are free, i.e. they do not communicate.

3. Conclusion

So, in this paper, an equation is obtained that allows us to calculate the frequencies of gas vibrations in a combustion chamber of the Helmholtz-resonator-type of a closed circuit. Knowledge of the vibration frequencies of the gas and other acoustic characteristics of the pulsating combustion chamber allows further research to find the relationships that determine the conditions of excitation and the amplitude of the gas vibrations.

References

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