Computer simulation of kinematics of parallel mechanisms

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Abstract. The paper deals with the kinematic analysis of a planar parallel mechanism. The use of parallel mechanisms makes the tasks of controlling the arms on the basis of these mechanisms difficult, since the solutions of the problems of kinematics and the dynamics of mechanisms are complicated. This leads to more complex control systems for such machines. Therefore, it becomes important to develop appropriate templates for solving certain problems of kinematics and dynamics for certain types of parallel mechanisms. In this paper, a planar parallel mechanism in the form of a tripod with three rotational pairs in each leg is considered.

1. Introduction

Among the modern problems of mechanical engineering, the problems of precise machining of complex surfaces, for example, for aviation and space, in disaster technology, in medicine, etc., are important [1–3]. There is a growing need for precision and high-speed processing of materials [4]. Therefore, the development of machines for performing precision machining of complex surfaces is an important task. Usually, flexible arms are used to solve these problems.

The disadvantage of sequential multilink arms is their lack of rigidity, which is not the case with arms built on the basis of parallel mechanisms. Parallel mechanisms are used in many other fields of mechanization and automation of technological processes, providing new opportunities for the use of industrial equipment and tools. The best known examples of parallel mechanisms are tripods and hexapods. For example, a hexapod known as the Stewart Platform, which consists of two plates connected by six prismatic pairs.

When the length of these pairs changes, the upper plate moves spatially relative to the lower one. There are many other projects dedicated to the development of robotic arms based on parallel mechanisms. Use of all the advantages of parallel mechanisms is difficult due to the complexity of analyzing the dynamics of such mechanisms and creating a corresponding management system. Well-known methods for solving these problems have been developed for parallel mechanisms [5–11].

At the same time, one of the ways to simplify and cheapen the process of developing such mechanisms is preliminary computer modeling of their work [12–15]. In this connection, the authors study the operation of mechanisms based on numerical and computer simulation [16–22].

2. Mathematical Model

The article describes the formation of a tripod trajectory with three rotational pairs – a planar parallel mechanism consisting of a mobile $C_1C_2C_3$ and fixed $A_1A_2A_3$ platforms (Fig. 1). The mobile platform of the mechanism is connected to the fixed base with three identical legs $A_1B_1C_1$, $A_2B_2C_2$ and $A_3B_3C_3$. All links of the mechanism are connected by rotational pairs.
That is, the rotational pairs are located at the points \(A_i, B_i,\) and \(C_i.\) The rotational pairs at points \(A_i\) connect the fixed platform \(A_1A_2A_3\) with the first link of the corresponding leg \(A_iB_i\) \((i = 1, 2, 3).\) Rotational pairs at points \(B_i\) connect the first link \(A_iB_i\) of each leg to the second link \(B_iC_i\) \((i = 1, 2, 3).\) And, finally, the rotational pairs at the \(C_i\) points connect the second \(B_iC_i\) link of each leg to the mobile platform \(C_1C_2C_3.\)

![Geometrical model of a tripod. In order not to complicate the drawing, only one leg of \(A_1B_1C_1\) is shown. The other two, \(A_2B_2C_2\) and \(A_3B_3C_3,\) are identical to the first.](image)

If we consider this mechanism as an arm with three degrees of freedom, then the position of the working body can be determined by the generalized coordinates of the mechanism or by the position and orientation of the last link of the arm. The direct kinematics problem is formulated as a definition based on given generalized coordinates of the actuated links of the arm position.

Solving the problems of kinematics of mechanisms with a parallel structure is possible in different ways: the screw method with using Plücker coordinates, the \(l\)-coordinate method, vector methods [20–22]. For the numerical solution, the authors used a matrix method based on homogeneous coordinates. The systems that were connected to the links of the mechanism by the Denavit–Hartenberg algorithm [23] were chosen as coordinate systems. The designations of the data from Table 1 have the meaning traditionally used in this method.

**Table 1.** Parameters of connected systems, specifying the transformation of the coordinates of a point from one coordinate system to another.

|        | \(A_1B_1C_1\) | \(A_2B_2C_2\) | \(A_3B_3C_3\) |
|--------|-------------|-------------|-------------|
| \(d\)  | \(a\)       | \(\theta\)  | \(a\)       | \(d\)       | \(a\)       | \(\theta\)  | \(a\)       |
| \(A_iB_i\) | 0          | \(a_{1,1}\) | \(\theta_{1,1}\)| 0         | 0         | \(a_{2,1}\) | \(\theta_{2,1}\)| 0         | 0         | \(a_{3,1}\) | \(\theta_{3,1}\)| 0         |
| \(B_iC_i\) | 0          | \(a_{1,2}\) | \(\theta_{1,2}\)| 0         | 0         | \(a_{2,2}\) | \(\theta_{2,2}\)| 0         | 0         | \(a_{3,2}\) | \(\theta_{3,2}\)| 0         |
| \(C_iK\) | 0          | \(a_{1,3}\) | \(\theta_{1,3}\)| 0         | 0         | \(a_{2,3}\) | \(\theta_{2,3}\)| 0         | 0         | \(a_{3,3}\) | \(\theta_{3,3}\)| 0         |

Vectors \(\rho_{i,0}\) and \(\rho_{i,3},\) specified in the fixed and moving coordinate systems, respectively, are connected by a matrix equation of the form

\[
\rho_{i,0} = \rho_{i,1}A_{i,1}A_{i,2}^T \rho_{i,3}, i = 1, 2, 3,
\]

where \(A_{i,0} = \rho_{i,1}A_{i,1}A_{i,2}^T \rho_{i,3}, i = 1, 2, 3\),

\[
\text{Figure 1. Geometrical model of a tripod. In order not to complicate the drawing, only one leg of } A_1B_1C_1 \text{ is shown. The other two, } A_2B_2C_2 \text{ and } A_3B_3C_3, \text{ are identical to the first.}
\]
\[ T_{i,0}^3 = \begin{pmatrix} c_{i,1} & -s_{i,1} & 0 & a_{i,1}c_{i,1} \\ s_{i,1} & c_{i,1} & 0 & a_{i,1}s_{i,1} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} c_{i,2} & -s_{i,2} & 0 & a_{i,2}c_{i,2} \\ s_{i,2} & c_{i,2} & 0 & a_{i,2}s_{i,2} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} c_{i,3} & -s_{i,3} & 0 & a_{i,3}c_{i,3} \\ s_{i,3} & c_{i,3} & 0 & a_{i,3}s_{i,3} \end{pmatrix}. \]

or, after multiplying the matrices

\[ T_{i,0}^3 = \begin{pmatrix} c_{i,1}c_{i,2}s_{i,3} - c_{i,1}s_{i,2}s_{i,3} - s_{i,1}s_{i,2}s_{i,3} - c_{i,1}s_{i,2}s_{i,3} + c_{i,1}s_{i,3}c_{i,2}s_{i,3} \\ -s_{i,1}s_{i,2}s_{i,3} + c_{i,1}s_{i,2}s_{i,3} - s_{i,1}s_{i,2}s_{i,3} + c_{i,1}s_{i,3}s_{i,2}s_{i,3} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} a_{i,1}c_{i,1} \left( c_{i,2}c_{i,3} - s_{i,2}s_{i,3} \right) - a_{i,1}s_{i,1} \left( s_{i,2}c_{i,3} + c_{i,2}s_{i,3} \right) + a_{i,1} \left( c_{i,1}c_{i,2} - s_{i,1}s_{i,2} \right) + a_{i,1}c_{i,1} \\ a_{i,1}s_{i,1} \left( c_{i,2}c_{i,3} - s_{i,2}s_{i,3} \right) + a_{i,1}c_{i,1} \left( s_{i,2}c_{i,3} + c_{i,2}s_{i,3} \right) + a_{i,1} \left( s_{i,1}c_{i,2} + c_{i,1}s_{i,2} \right) + a_{i,1}s_{i,1} \end{pmatrix}. \]

Here we have introduced the notation \( c_{ij} = \cos \theta_{ij} \) and \( s_{ij} = \sin \theta_{ij} \) and each column of the matrix \( T_{i,0}^3 \) is marked with curly brackets. For the second and third legs, the equations of motion are obtained similarly. Note that, in view of the consideration of the planar mechanism, this 4×4 matrix can be reduced to a 3×3 matrix. However, this is not done by the authors purposefully, in order to be able to move from a planar mechanism to a spatial one.

By differentiating equation (1) on time, one obtains the equations of velocity and acceleration for the corresponding leg.

By setting different laws for changing the generalized coordinates of a tripod or an arm position, one obtains various systems of equations of the form (1). The system of equations obtained in this way is solved analytically or numerically with the help of specialized packages of application programs and obtains the unknown quantities. After that, the obtained solutions are compared with computer modeling data, which will be described in the next section of this article.
3. Computer Model

For computer simulation, a model was created using block diagrams of the SimMechanics library, which is an extension of the MatLab package. Fig. 2 shows a fragment of the $A_1B_1C_1$ leg of the tripod model under consideration.

Figure 2. Fragment of the block diagram of the model of a planar parallel mechanism with three identical legs, which simulates the leg $A_1B_1C_1$. The blocks correspond to the following elements of the model: $A_1$ to the point $A_1$ of the fixed platform, which is the base of the first leg, $A_1B_1$ and $B_1C_1$ to the links $A_1B_1$ and $B_1C_1$, $R_{11}$, $R_{12}$, $R_{13}$ to the rotational pairs connecting platforms and links.

To set the motion kinematics, the corresponding blocks of the Simulink libraries were used. One of such cases is shown in Fig. 3. In this case, the movement of the mobile platform, which is set using the appropriate blocks, is initial. Here the mobile platform performs reciprocate-translatory plane motion, in which the coordinates of the platform points are changed according to harmonic laws. The parameters of these harmonics are set in the appropriate blocks of the block diagram fragment shown in Fig. 3.

Figure 3. Fragment of the block diagram of the model of a planar parallel mechanism with three identical legs, simulating the movement of the center of gravity to a mobile platform for the case when the generalized coordinates of the center of gravity change according to a harmonic law.
The results of computer simulation for this case of movement of a mobile platform are shown in Fig. 4. This figure shows the laws of generalized coordinate variation and their time derivatives for the first, second and third legs of the tripod, respectively, from top to bottom.

![Graph showing coordinate variations](image)

Figure 4. Results of computer simulation of the tripod for the first six seconds of movement.

This method of computer simulation allows us to consider not only the direct problem of the kinematics of the mechanism. It is useful in determining the working space of a parallel mechanism, when considering its specific positions. This method is also convenient for solving problems of dynamic analysis of the mechanism.

4. Conclusions
This study of the kinetics of a planar tripod makes it possible to obtain an analytical and / or numerical solution of problems associated with the analysis of kinematics and the dynamics of parallel mechanisms. These solutions can be compared with the results of computer modeling for their verification. In particular, this solution algorithm will be useful and easy to use when determining the working space of a planar tripod. The developed algorithm of kinematic analysis, due to its generality, is easily transferred to spatial parallel mechanisms.

It should be noted that in the general case the definition of the working space of the parallel mechanism will be more difficult task. At the same time, numerical solution and computer modeling (in this case using the Denavit–Hartenberg method and Simulink libraries and SimMechanics libraries) simplifies the work of designers and engineers related to the synthesis and analysis of parallel mechanisms of this particular type and track the accuracy of their solutions.

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