Full characterization of Gaussian bipartite entangled states by a single homodyne detector

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We present the first complete experimental reconstruction of Gaussian entangled states generated by a type–II optical parametric oscillator (OPO) below threshold. Our scheme provides the entire covariance matrix using a single homodyne detector and allows for the complete characterization of bipartite Gaussian states, including the evaluation of purity, entanglement and nonclassical photon correlations, without a priori assumptions on the state under investigation. Our results show that single homodyne schemes are convenient and robust setups for the full characterization of OPO signals and represent a tool for quantum technology based on continuous variable entanglement.

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Introduction—In this letter we address the complete experimental characterization of bipartite Gaussian states. In our experiment continuous-wave (CW) entangled light beams are generated by a single type–II optical parametric oscillator (OPO) below threshold, and then their covariance matrix (CM) is fully reconstructed using a novel scheme [1] that involves a single homodyne detector [2]. To our knowledge this is the first complete characterization of OPO signals without a priori assumptions and paves the way to a deeper investigation of continuous variable entanglement without experimental loopholes.

Light beams endowed with nonclassical correlations [3] are crucial resources for quantum technology and find applications in quantum communication [4], imaging [5] and precision measurement [6,7]. Their full characterization has a fundamental interest in its own and represents a tool for the design of quantum information processing protocols in realistic conditions. Remarkably, entangled states produced by OPOs are Gaussian states [8,9] and thus may be fully characterized by the first two statistical moments of the field modes. In turn, the CM contains the complete information about entanglement [10,11], i.e. about their performances as a resource for quantum technology.

Bipartite entangled states may be generated by mixing at a beam splitter (BS) two squeezed beams obtained by a degenerate OPO below threshold [12]. The beams exiting the BS are entangled [13] and a partial reconstruction of the corresponding CM has been obtained [14]. The complete reconstruction of a CM has been obtained for different entangled states, by varying single-mode squeezing [15]. In this configuration two OPOs are used and the amount of entanglement critically depends on the symmetry between the two squeezed beams. From the experimental point of view this requires an accurate setting on the two squeezers and a strict control on the relative phase. The measured CM presents some unexpected deviations from a proper form so leaving a question open on the reliability of double homodyne schemes due to technical difficulties [14,16]. A more direct way to generate quadrature entanglement is to use a single non-degenerate OPO [17], which represents a robust and reliable source of EPR-type correlation either below [18,19] or above threshold [20,21,22]. Partial reconstructions of the CM in the pulsed regime have been achieved for the spectrally degenerate but spatially non-degenerate twin beams at the output of a type–I parametric amplifier [23], whereas in the CW regime cross polarized beams emitted by a self-locked type–II OPO have been examined at frequency degeneracy [16]. Although the correlation properties of OPO signals have been widely investigated, no proper CM reconstruction has been performed so far. In turn, in previous proposals and experiments non-physical CM entries [12,14,16,23] or deviations from a proper CM [15] appeared, thus requiring a priori hypothesis on the measured state to understand the experimental results.

In this letter we report the first complete measurement of the CM for the output of a single non-degenerate OPO. The two entangled beams are emitted with orthogonal polarization and degenerate frequency by a CW type–II OPO below threshold. In order to reconstruct the ten independent elements of the CM, the beams are optically combined into six auxiliary modes, whose quadratures are measured using a single homodyne detector [11]. The first two moments of the relevant quadratures are obtained by tomographic reconstruction using the whole homodyne data set, the CM is, then, fully reconstructed after assessing the Gaussian character of the signal and compared with a general model describing a realistic OPO. Entanglement is demonstrated using the partial transpose method [10], the Duan inequality [11] and the stricter EPR criterion [24], and quantified upon evaluating the logarithmic negativity and the entanglement of formation (EoF).

We also reconstruct the joint photon number distribution and demonstrate nonclassical photon correlations by evaluating the noise reduction factor. In the following, after defining no-
Reconstruction method—Upon introducing the vector \( R = (x_1, y_1, x_2, y_2) \) of canonical operators, in terms of the mode operators \( a_k, x_k = (a_k^+ + a_k)/\sqrt{2}, y_k = (i(a_k^+ - a_k))/\sqrt{2}, \) \( k = 1, 2, \) the CM \( \sigma \) of a bipartite state \( \rho \) is defined as the block matrix \( \sigma = \begin{pmatrix} A & C \\ \frac{1}{B} & D \end{pmatrix} \), \( \sigma_{kk} = \frac{1}{2}\{R_k, R_k\} - \{R_k\}^2 \)

where \( A, B \) and \( C \) are \( 2 \times 2 \) real matrices, \( \{O\} = \text{Tr}(\rho \rho^\dagger) \) and \( \{f, g\} = fg + gf \). In the following we will use the notation \( a \equiv a_1 \) and \( b \equiv a_2 \) and also consider the four additional auxiliary modes \( c = (a+b)/\sqrt{2}, d = (a-b)/\sqrt{2}, e = (ia+b)/\sqrt{2}, \) and \( f = (ia-b)/\sqrt{2} \) obtained by the action of polarizing beam splitters (PBS) and phase-shifters on modes \( a \) and \( b \). Positivity of the density matrix for physical states is written in terms of the uncertainty relation for the minimum symplectic eigenvalue \( \nu \) of the CM, i.e., \( \nu \geq 1/2 \). For Gaussian states, the state purity is given by \( \mu(\sigma) = (4 \sqrt{\det(\sigma)})^{-1} \)

where separability corresponds to positivity of the partially transverse (PPT) density matrix. A bipartite Gaussian state is separable iff \( \nu \geq 1/2 \), whereas \( \nu \) is the minimum symplectic eigenvalue of \( \Delta^\sigma \Delta. \) \( \Delta = \text{Ding}[1, 1, 1, -1] \).

A convenient measure of entanglement is thus given by the logarithmic negativity \( E_N(\sigma) = \max(0, -\ln 2\nu) \) and the EoF \( E_{\rho}(\sigma) \) can be evaluated following Ref. \[25\]. In addition to \( E_N(\sigma) \) the Duan criterion gives a necessary condition for non-separability in terms of the noise properties of \( c \) and \( d \)[11] whereas a stricter condition \[24\], referred to as EPR criterion, involves the conditional variances on \( x_a \) and \( y_a \) obtained from a measurement of \( \sigma_{ab} \) for the explicit expressions in terms of the noise on modes \( a, b, c \) and \( d \).

In our experiment, the block \( A \) of the CM is retrieved by measuring the single-mode quadratures of mode \( a \): the variances of \( x_a \) and \( y_a \) give the diagonal elements, while the off diagonal ones are obtained from the additional quadratures \( z_a = (x_a + y_a)/\sqrt{2} \) and \( t_a = (x_a - y_a)/\sqrt{2} \) as \( \sigma_{12} = \frac{1}{2}\{(z_a^2) - (t_a^2)\} - \{x_a\}^2 \).

The block \( B \) is reconstructed in the same way from the quadratures of \( b \), whereas the elements of the block \( C \) are obtained from the quadratures of the auxiliary modes \( c, d, e \) and \( f \) as follows \( \sigma_{13} = \frac{1}{2}\{(x_c^2) - (x_d^2)\} - \{x_a\} \{x_b\}, \sigma_{14} = \frac{1}{2}\{(y_c^2) - (y_d^2)\} - \{x_a\} \{y_b\}, \sigma_{23} = \frac{1}{2}\{(x_e^2) - (x_f^2)\} - \{y_a\} \{x_b\}, \sigma_{24} = \frac{1}{2}\{(y_e^2) - (y_f^2)\} - \{y_a\} \{y_b\}. \)

Notice that the measurement of the \( f \)-quadratures is not mandatory, since \( \{x_f^2\} = \{x_c^2\} + \{y_a\}^2 - \{x_d^2\} \) and \( \{y_f^2\} = \{x_c^2\} + \{y_a\}^2 - \{x_d^2\} \). Analogous expressions hold for \( \{x_e^2\} \) and \( \{y_e^2\}. \)

In the ideal case the OPO output is in a twin-beam state \( \mathcal{S}(\xi) = \exp\{\xi \sigma_{ab} - \xi \sigma_{ba}\} \) being the entangling two-mode squeezing operator: the corresponding CM has diagonal blocks \( A, B, C \) with the two diagonal elements of each block equal in absolute value. In realistic OPOs, cavity and crystal losses lead to a mixed state, \( i.e., \) to an effective thermal contribution. In addition, spurious nonlinear processes, not perfectly suppressed by the phase matching, may combine to the down conversion, contributing with local squeezing. Finally, due to small misalignments of the nonlinear crystal, a residual component of the field polarized along \( a \) may project onto the orthogonal polarization (say along \( b \)), thus leading to a mixing among the modes [21]. Overall, the state at the output is expected to be a zero amplitude Gaussian entangled state, whose general form may be written as \( \rho_g = U(\beta)\mathcal{S}(\zeta)\mathcal{L}_S(\xi_1, \xi_2)\mathcal{L}_S^\dagger(\xi_1, \xi_2)\mathcal{S}(\zeta) U^\dagger(\beta), \)

where \( \mathcal{L} = \mathcal{T}_1 \otimes \mathcal{T}_2, \) with \( \mathcal{T}_k = (1 + \bar{n}_k)^{-1}\{\bar{n}_k/(1 + \bar{n}_k)\}a^k a^k \) denotes a two-mode thermal state with \( \bar{n}_k \) average photons per mode, \( \mathcal{L}_S(\xi_1, \xi_2) = S(\xi_1) \otimes S(\xi_2), \) \( S(\xi) = \exp\{\frac{1}{2}\{\xi \sigma_{ab} - \xi \sigma_{ba}\}\} \) denotes local squeezing and \( \mathcal{U}(\beta) = \exp\{\beta a^b - \beta^* a^b\} \)

a mixing operator, \( \zeta, \xi_1, \xi_2 \) and \( \beta \) being complex numbers. For our configuration, besides a thermal contribution due to internal and coupling losses, we expect a relevant entangling contribution with a small residual local squeezing and, as mentioned above, a possible mixing among the modes. The CM matrix corresponding to \( \rho_g \) has diagonal blocks \( A, B, \) and \( C \) with possible asymmetries among the diagonal elements.

Experimental setup—The experimental setup shown in Fig. 1 relies on a CW internally frequency doubled Nd:YAG laser pumping (@532 nm) a non degenerate OPO based on a periodically poled \( \alpha \)-cut KTP (PKPPT) crystal (Raincol Crystals Ltd. on custom design) [28]. The use of the \( \alpha \)-cut PKPPT allows implementing a type-II phase matching with cross polarized signal \( (a) \) and idler \( (b) \) waves, frequency degenerate @1064 nm for a crystal temperature of \( \approx 53^oC. \) The OPO cavity is locked to the pump beam by Pound-Drever technique [29] and adjusted to work in triple resonance by finely tuning its geometrical properties [27]. The cavity output coupling @1064 nm is \( \approx 0.73, \) corresponding to an experimental line-width of 16 MHz @1064 nm. The measured oscillation threshold is \( P_{th} \approx 50 \text{ mW}; \) during the acquisition the system has been operated below threshold at 60% of the threshold power.

In order to select mode \( a \) and \( b \) or their combinations \( c \) and \( d, \) the OPO beams are sent to a half-wave plate and a PBS. Modes \( e \) and \( f \) are obtained by inserting an additional quarter-wave plate [11]. The PBS output goes to a homodyne detector, described in details in [1,30], exploiting the laser output @1064 nm as local oscillator (LO). The overall homodyne detection efficiency is \( \eta = 0.88 \pm 0.02. \) The LO reflects
on a piezo-mounted mirror (PZT), which allows varying its phase $\theta$. In order to avoid the laser low frequency noise, data sampling is moved away from the optical carrier frequency by mixing the homodyne current with sinusoidal signal of frequency $\Omega = 3$ MHz [30]. The resulting current is low–pass filtered ($B = 300$ kHz) and sampled by a PCI acquisition board (Gage 14100, 1M–points per run, 14 bits resolution). The total electronic noise power has been measured to be $16$ dBm below the shot–noise level, corresponding to a signal to noise ratio of about 40.

**Reconstruction and experimental results**—Acquisition is triggered by a linear ramp applied to the PZT and adjusted to obtain a $2\pi$ variation in 200 ms. Upon spanning the LO phase $\theta$, the quadratures $x(\theta) = x \cos \theta + y \sin \theta$ are measured. Calibration with respect to the noise of the vacuum state is obtained by acquiring a set of data with the output from the OPO obscured. All the expectation values needed to reconstruct $\sigma$ are obtained by quantum tomography [31], which allows to compensate nonunit quantum efficiency and to reconstruct any expectation value, including those of specific quadratures and their variances, by averaging special pattern functions over the whole data set. As a preliminary check of the procedure, we verified that the CM of the vacuum state is consistent with $\sigma_0 = \frac{1}{2} I$ within the experimental errors.

![FIG. 2: (Left): experimental homodyne traces and reconstructed CM; (Right): reconstructed Wigner functions. Top plots are for modes $c$ and bottom ones for $d$. Arrows on the homodyne plots show the positions of the maximum and minimum variances. $\theta$ is the relative phase between the signal and the LO.](image)

We start our analysis by checking the Gaussian character of the OPO signals, i.e. upon evaluating the Kurtosis of homodyne distribution at fixed phase of the LO [30]. Besides, we checked that the mean values of all the involved quadratures are negligible, in agreement with the description of OPO output as a zero amplitude state. Then, we have measured the quadratures of the six modes $a$–$f$. We found the modes $a$ and $b$ excited in a thermal state, thus confirming the absence of relevant local squeezing. Their combinations $c$, $d$, $e$ and $f$ are squeezed thermal states with squeezing appearing on $y_c$, $x_d$, $t_e$ and $z_f$, respectively. In Fig. 2 we show the experimental homodyne traces for modes $c$ and $d$ as well as the corresponding Wigner functions, obtained by reconstructing the single-mode CM. As it is apparent from the plots both modes are squeezed with quadratures noise reduction, corrected for nonunit efficiency, of about 2.5 dB. An analogue behavior has been observed for modes $e$ and $f$. The CM of Fig. 2 indeed reproduce that of an entangled thermal state with small corrections due to local squeezing and mixing. The relevant parameters to characterize the corresponding density matrix $\rho_0$ are the mean number of thermal photons $\bar{n}_i \approx 0.67$, $\bar{n}_e \approx 0.18$ and entangling photons $\bar{n}_e \approx 2 \sinh^2 |\chi| \approx 0.87$ [32]. The errors on the CM elements for the blocks $A$ and $B$ are of the order $\delta \sigma_{jk} \approx 0.004$ and have been obtained by propagating the tomographic errors. In this case phase fluctuations are irrelevant, since the two modes are both excited in a thermal state. On the other hand, in evaluating the errors on the elements of the block $C$ the phase-dependent noise properties of the involved modes have to be taken into account, and the tomographic error has to be compared with the error due to the finite accuracy in setting the LO phase $\theta$. The elements $\sigma_{13}$ and $\sigma_{24}$ are obtained as combinations of squeezed/anti-squeezed variances, which are quite insensitive to fluctuations of $\theta$. As a consequence the errors on these elements are given by the overall tomographic error $\delta \sigma_{jk} \approx 0.004$. On the other hand, the elements $\sigma_{14}$ and $\sigma_{23}$ depend on the determination of $x_{e,f}^2$ and $y_{e,f}^2$, which are sensible to phase fluctuations. In order to take into account this effect we evaluate errors as the fluctuations in the tomographically reconstructed quadratures induced by a $\delta \theta \approx 20$ mrad variation in the LO phase, corresponding to the experimental phase stability of the homodyne detection. The resulting errors are about $\delta \sigma_{14} = \delta \sigma_{23} \approx 0.03$ for both CM elements. The off-diagonal elements of the three matrices $A$, $B$ and $C$ are thus zero within their statistical errors, in agreement with the expectation for an entangled thermal state. As mentioned above, the experimental procedure may be somehow simplified exploiting the relationships among modes, and expressing mode $e$ or $f$ in terms of the others: only five modes are then needed. Upon rewriting the off-diagonal terms of $C$ in terms of the five modes we arrive at $\sigma_{14} = 0.02 \pm 0.03$ and $\sigma_{23} = 0.04 \pm 0.03$ when eliminating the mode $f$ and $\sigma_{14} = 0.06 \pm 0.03$ and $\sigma_{23} = 0.06 \pm 0.03$ when eliminating the mode $e$. Both procedures provide results in agreement with those obtained by using the complete set of homodyne data for the six modes.

Since the minimum symplectic eigenvalue of $\sigma$ is $\nu_- = 0.68 \pm 0.02 \geq 0.5$, the CM corresponds to a physical state. State purity is $\mu(\sigma) = 0.31 \pm 0.01$. The minimum symplectic eigenvalue for the partial transpose is $\tilde{\nu}_- = 0.24 \pm 0.02$, which corresponds to a logarithmic negativity $E_N(\sigma) = 0.73 \pm 0.02$. 
the state is entangled, with $E_x(\sigma) = 1.46 \pm 0.02$. In turn, it satisfies the Duan inequality with the results $0.29 \pm 0.01 < 1/2$ and the EPR criterion with $0.21 \pm 0.01 < 1/4$.

Entangled Gaussian states as $g_y$ may be endowed with non-classical photon number correlations, i.e. squeezing in the difference photon number. This may be checked upon evaluating the noise reduction factor $R = \text{Var}(D_{ab})/(\langle N_a \rangle + \langle N_b \rangle)$ where $\text{Var}(D_{ab})$ denotes the variance of the difference photocurrent $D_{ab} = N_a - N_b$, $N = a^{\dagger}a$ being the number operator, and $\langle N_k \rangle$ the average photon number. A value $R < 1$ is a marker of nonclassical correlations between the two modes. We obtained $R = 0.50 \pm 0.02$, in agreement with the theoretical description \cite{33} for the values of thermal and entangling photons reported above. Starting from the CM one can reconstruct the full joint photon distribution $p(n, m)$ of the modes $a$ and $b$: the result is shown in Fig. 3 where the correlations between the two modes are clearly seen. We have also evaluated the single-mode photon distributions (either from data or from the single-mode CM) for modes $a, d$. Results are reported in Fig. 3 distributions of $a$ and $b$ are thermal, whereas the statistics of modes $c$ and $d$ correctly reproduces the even-odd oscillations expected for squeezed thermal states.

**Conclusion**—We have presented the complete reconstruction of the CM for the output of a CW type II non-degenerate OPO, below threshold and frequency degenerate. The CM elements have been retrieved as combinations of expectations and variances of suitable mode quadratures, obtained by combining the entangled modes by linear optics. The quantities of interest have been obtained tomographically, processing the whole data set and thus reducing statistical fluctuations. Upon exploiting a general model allowing local squeezing and polarization cross-talking inside the crystal, we have very precisely described the experimental CM with the theory underlying parametric downconversion, thus providing a full explanation of experimental findings. The reconstructed state is a Gaussian entangled state close to a two-mode squeezed thermal state, the corresponding entanglement and nonclassical photon number correlations have been demonstrated. We conclude that single homodyne schemes are convenient and robust setups for the full characterization of OPO signals and, in turn, represent a relevant tool for quantum technology based on CV entanglement, e.g., the full characterization of CV Gaussian channels by input-output signals’ characterization. Finally, making use of a single OPO and a single homodyne detector, our setup represents also a compact and robust tool for entanglement generation and characterization.

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