In recent years significant advances have been achieved in the observations of stars gravitationally bound with the supermassive black hole SgrA* at the center of the Galaxy. Several so-called S0 stars, which move at very high velocities (\(>10^3\) km/s) in almost elliptic orbits around a very compact supermassive object, are observed in the infrared range [1, 2, 3, 4]. According to the measured parameters of the Kepler orbits of S0 stars, the mass of supermassive black hole SgrA* is \(M_{\text{BH}} = (4.1 \pm 0.4) \times 10^6 M_\odot\) [1, 2, 3, 4]. Independent and currently most accurate values of the mass \(M_{\text{BH}}\) and spin (Kerr parameter) \(a\) of the SgrA* black hole are determined from the observations of quasi-periodic oscillations with the average periods of 11.5 and 19 min [6, 7]. They are \(M_{\text{BH}} = (4.2 \pm 0.2) \times 10^6 M_\odot\) and \(a = 0.65 \pm 0.05\) [8, 9].

At the center of the Galaxy, in addition to the supermassive black hole SgrA*, there are invisible sources of mass such as compact gas clouds, dim stars and their remnants, and a distributed mass in the form of the diffuse dark matter. This additional mass would result in the deviation of the total Newtonian gravitational potential from the point mass potential of the black hole \(U = -GM_{\text{BH}}/r\). As a result, the orbits of S0 stars gravitationally bound with the black hole would be unclosed and precess. The openness of the orbit of the most studied S0-2 star will be measured in the next one or two years. Thus, the total mass of the dark matter within the orbit of this star with a characteristic radius of 0.005 pc will be determined.

The existence of fast S0 stars provides a unique possibility of reconstructing the gravitational potential and measuring the mass distribution at the center of the Galaxy by fitting their orbits. We discuss and develop a method for studying the distribution of the dark matter at the center of the Galaxy by measuring the precession angle of orbits of S0 stars. Nowadays it is known that the distributed mass within the orbit of the S0-2 star is no more than \(3 - 4\%\) of the mass of the supermassive black hole SgrA*. It is noteworthy that the expected measurement of the
nonrelativistic precession of the orbit of the S0-2 star will allow either improving the indicated bound on the distributed dark mass by two or three orders of magnitude or determining this dark mass.

The analytical formulas are derived general for the precession of S0 star orbits with a power law density profile of the dark matter distribution in the Galactic center. These formulas make it possible to easily determine the additional distributed mass from the measured precession angle. An additional independent method for determining the distribution of the dark matter is the search for a possible annihilation signal from the center of the Galaxy. We calculated [13] (see figure 1 and 2) the mass of the SUSY neutralino dark matter necessary for the explanation of the excess of gamma-radiation from the center of the Galaxy detected recently by the Fermi-LAT space gamma-telescope [10, 11].

In particular, we determined the dependence of the additional mass both on the profile of the central peak of the dark matter density and on the annihilation cross section of dark matter particles taking into account the Sommerfeld enhancement effect (see figure 1).

**Figure 1.** Shift angle of the apsis of the orbit of the star in one turn $\delta \phi$ versus the exponent of the power-law spectrum of the dark matter for realistic values of the mass fraction of the dark matter $\xi$ within the orbit of the S0-2 star. The indicated region is excluded by the constraints caused by the annihilation of dark matter particles if the dark matter makes the main contribution to the mass fraction of the dark matter $\xi$.

We consider the power-law density profile of matter responsible for the correction $\delta U$ to the potential of the black hole $\rho(r) = \rho_h (r/r_h)^{-\beta}$, where $\rho_h$, $r_h$, and $\beta$ are the parameters. The corresponding total mass of the dark matter within the sphere with the radius $r$ is

$$M_{DM}(r) = \frac{4\pi \rho_h r_h^\beta}{3 - \beta} \left[ r^{3-\beta} - R_{\text{min}}^{3-\beta} \right],$$

where $R_{\text{min}}$ is the minimum radius to which the density profile expands. We define the mass
Figure 2. Mass fraction of the dark matter $\xi$ versus the exponent $\beta$ in the density profile at the precession angle $\delta \phi = 0.01$. The indicated region is excluded by the constraints caused by the annihilation of dark matter particles.

fraction of the dark matter within the orbit of the S0 star $\xi = [M_{DM}(r_a) - M_{DM}(r_p)]/M_{BH}$, which is significant for the subsequent analysis.

In the presence of a small correction $\delta U$ to the Newtonian potential of the black hole, the precession angle of the orbit of a probe particle (S0-2 star) in one turn is (see [12], section 15, Problem 3)

$$\delta \phi = \frac{\partial}{\partial L} \left( \frac{2m}{L} \int_0^{\pi} r^2(\phi) \delta U d\phi \right).$$

(2)

Here, integration is performed with the trajectory of the particle in the form of an unperturbed elliptic orbit $r(\phi) = p(1 + e \cos \phi)^{-1}$, where $e$ is the eccentricity of the ellipse, $p = L^2/(GM_{BH}m) = a(1 - e^2)$ is the parameter of the orbit, $a$ is the major semiaxis, and $L$ is the conserved angular momentum of the star with the mass $m$. The observed parameters of the Kepler orbit of the S0-2 star: the eccentricity $e = 0.898 \pm 0.0034$, the radius of the pericenter $r_p = a(1 - e) = 0.585$ mpc, and the radius of the apocenter $r_a = a(1 + e) = 9.42$ mpc. We note that, in the case of relativistic precession, the orbit would rotate in the direction of the rotation of the star, but Newtonian precession (2) occurs in the opposite direction, i.e., $\delta \phi < 0$.

The calculation [13] of the precession angle of the orbit of the star in the time of one turn around the black hole $\delta \phi$ gives an expression with one hypergeometric function $\text{}_2F_1(a, b; c; z)$:

$$\delta \phi = -\frac{4\pi^2p_b r_b^3}{(1 - e)^{4-\beta}M_{BH}^2} \text{}_2F_1 \left( 4 - \beta, \frac{3}{2}; 3; -\frac{2e}{1-e} \right),$$

(3)

where $p = L^2/(GM_{BH}m) = a(1 - e^2)$ is the parameter of the elliptic orbit, $e$ is the eccentricity of the orbital ellipse of the S0-2 star, $a$ is the major semiaxis, and $L$ is the conserved angular
Figure 3. Mass fraction of the dark matter $\xi$ required by the observed excess of gamma radiation in the case of the annihilation of the dark matter particles with the cross section due to the Sommerfeld enhancement effect $\langle \sigma v \rangle \propto v^{-\eta}$ versus an index $\eta$.

momentum of the star with the mass $m$.

The currently existing observation accuracy is still insufficient for the measurement of the precession angle of fast S0 stars and the distributed invisible mass. However, there is a high probability of reaching in the near future the accuracy required either for the measurement of the precession angle or for the determination of a strong bound, which in turn will make it possible to impose stringent dynamic constraints on the additional dark mass.

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