Topological nodal line in superfluid $^3$He and the Anderson theorem

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Superconductivity and superfluidity with anisotropic pairing—such as $d$-wave in cuprates and $p$-wave in superfluid $^4$He—are strongly suppressed by impurities. Meanwhile, for applications, the robustness of Cooper pairs to disorder is highly desired. Recently, it has been suggested that unconventional systems become robust if the impurity scattering mixes quasiparticle states only within individual subsystems obeying the Anderson theorem that protects conventional superconductivity. Here, we experimentally verify this conjecture by measuring the temperature dependence of the energy gap in the polar phase of superfluid $^3$He. We show that oriented columnar non-magnetic defects do not essentially modify the energy spectrum, which has a Dirac nodal line. Although the scattering is strong, it preserves the momentum along the length of the columns and forms robust subsystems according to the conjecture. This finding may stimulate future experiments on the protection of topological superconductivity against disorder and on the nature of topological fermionic flat bands.

**INTRODUCTION**

Very soon after discovery of superconductivity, H. Kamerlingh Onnes noticed that addition of impurities to a superconducting metal does not change its critical temperature. Explanation of this counterintuitive effect had to wait until formulation of the Bardeen–Cooper–Schrieffer theory, based on which P.W. Anderson proved his famous theorem $^1$. The Anderson theorem states that non-magnetic impurities do not modify static properties of a superconductor with conventional $s$-wave pairing, including the critical temperature $T_c$ and value of the superconducting gap $\Delta(T)$. While origin of this robustness is closely linked to the time-reversal symmetry of the pairing state, a handwaving illustration is presented by the cartoon in Fig. 1. Impurity scattering mixes quasiparticle states with different momentum $\mathbf{p}$ directions, but if $\Delta(\mathbf{p}) = \text{const}$ then the “averaged” gap remains unchanged. For unconventional $d$-wave or $p$-wave systems the Anderson theorem is generally not applicable. Here the gap is usually anisotropic and often includes nodes, that is, points or lines in momentum space where $\Delta(\mathbf{p}) = 0$. As the cartoon in Fig. 1 suggests, scattering then suppresses the gap $\Delta(0)$, while the effect of disorder on the physics related to the energy nodes becomes a separate actively investigated problem $^{21-23}$.

For applications of unconventional and topological superconductors, strong suppression of $T_c$ is undesirable and mechanisms to improve robustness of Cooper pairs in such systems, in particular through extensions of the Anderson theorem, are now under intensive study $^{21,24}$. Here $p$-wave superfluid $^4$He provides an ideal platform to elucidate the effects of disorder: This system is naturally void of any impurities, while scattering centers, in the form of solid nanoparticles, can be immersed into the liquid under full experimentalists’ control. In fact, nanostructured confinement of $^3$He became a flagship tool to engineer novel topological phases of matter $^{14,15,22}$. Here we focus on the polar phase $^{22}$, where the confining matrix forms a set of nearly parallel strands, see Fig. 1. The polar phase is believed to have anisotropic gap with a Dirac nodal line in the plane perpendicular to strands, Fig. 1, although an experimental confirmation for the presence of the node has so far been missing.

A remarkable feature of the polar phase is that its $T_c$ is suppressed only marginally compared to the critical temperature $T_{\text{ch}}$ of clean bulk $^3$He (Fig. 2 $^1$), even when the distance between scattering strands is just a fraction of the coherence length $^{23,24}$. To explain this robustness, it has been suggested that the Anderson theorem can be extended to the polar phase provided impurities have the form of infinitely long non-magnetic strands, which are straight and parallel to each other and the scattering of quasiparticles is fully specular (see Fig. 1g). The reason is that the polar phase represents a set of independent two-dimensional (2D) superfluids with different $p_z$. (Here $^z$ is the direction along the strands.) For perfect columnar defects the scattering between different $p_z$ states (or 2D bands) is absent. Such 2D superfluids have the same properties as $s$-wave superconductors, including time-reversal symmetry, the Anderson theorem is applicable and impurities do not break the Cooper pairs. Conceptually similar approach was suggested for extension of the Anderson theorem to multiband unconventional superconductors $^{14,15,22}$.

Here we verify the applicability of the Anderson theorem to the polar phase with measurement of the temperature dependence of the gap $\Delta(T)$ at temperatures $T < 0.5 T_c$. We find that $\Delta(0) - \Delta(T) \propto T^3$, which is a signature of the Dirac nodal line. Moreover, the prefactor in this cubic dependence is close to the theoretical expectation based on the BCS theory in the clean limit. Generally, disorder in nodal systems is expected to affect both the power law of the temperature dependence of various properties and the absolute magnitude of the change, see, e.g., Ref. $^{25}$. Agreement of the gap measurements in the polar phase under strong columnar disorder with the clean-limit theory is a definite manifestation of the Cooper pair protection by the extended Anderson theorem. The small deviations from ideal clean-limit val-
ues in the magnitude of the gap change at low temperatures and in the critical temperature are found to have similar pressure dependence. These deviations probably originate from the real confinement providing channels for mixing between different $p_z$ subsystems, e.g., due to orientational disorder in confining strands.

**RESULTS**

**Suppression of $T_c$**

In the nafen-243 material used for confinement in this work, 94% of the volume is an open space between strands. This is a relatively low level of porosity: Impurity scattering is strong with $(\tau \Delta/h) \lesssim 1$ at all pressures and temperatures, see Supplementary Fig. 1. Here $\tau$ is the quasiparticle scattering time. When $^3$He is confined in silica aerogels of such porosity, superfluidity is suppressed completely and even in 98%-open silica aerogels the superfluid is suppressed at pressures below about 5 bar. The reason is that more randomly distributed impurities in silica aerogels do not allow extension of the Anderson theorem to that system. On the contrary, since the original measurement in the confinement with columnar defects, it is known that $T_c \gtrsim 0.9 T_{cb}$ in nafen-243. Our measurements confirm this result, Fig. 1.

Drastic change of $T_c$ from zero to nearly $T_{cb}$ is the result of protection provided by the Anderson theorem. In ideal case of the theorem applicability, $T_c = T_{cb}$ independently of the defect density. It is interesting that two measurements in Fig. 1 performed on the samples of the same nominal density, demonstrate slightly different $T_c$ suppression. Thus, it is not the density of defects which directly determines suppression (in accordance with the understanding provided by the Anderson theorem), but deviations of the scattering properties from ideal $p_z$-preserving. A simple model accounting for this deviation is given in Ref. [26] $T_c = T_{cb} \frac{1 - n_s \xi_0 + \beta \xi_0^{-1} \sigma_\parallel}{1 - \frac{n_s \xi_0}{\beta}}$. Here $n_s = 9.55 \cdot 10^{14} \text{m}^{-2}$ is the density of nafen strands, $\xi_0$ is the pressure-dependent coherence length, $\beta$ is a parameter related to the strand diameter and $\sigma_\parallel$ is the scattering radius for a quasiparticle moving along the strand. (For ideal strand $\sigma_\parallel = 0$.) We fit two sets of data in Fig. 1 with the same $\beta$ and different $\sigma_\parallel$. The fit describes the observations reasonably well with $\beta = -0.1$ and $\sigma_\parallel = 1.7 \text{nm}$ for data from Ref. [22] and $\sigma_\parallel = 1.5 \text{nm}$ for our data. Note that $\sigma_\parallel \ll \langle d \rangle$. Thus the orientational disorder in strands is relatively weak. Below we demonstrate that also the low-temperature measurements in the polar phase are described well by the clean-limit theory, while deviations are relatively small.
FIG. 2: NMR measurements of the temperature dependence of the gap in the polar phase. a, NMR spectrum of $^3$He in the polar phase ($T < T_c$) demonstrates the frequency shift from the Larmor value $\omega_L$, where the spectrum in the normal state ($T > T_c$) is located. From the temperature dependence of the shift, the temperature dependence of the gap $\Delta(T)$ can be extracted. The spectra on the plot are normalized so that the total integral of absorption is unity. b, Temperature dependence of the frequency shift in the polar phase at the lowest temperatures at three pressures (symbols). Lines are fit of the data in the temperature range $0.3 < T/T_c < 0.5$ to Eq. (4) using $\alpha$ and $\omega(0)$ as fitting parameters. The fitted values are shown in Fig. 3a and 3b respectively. Absolute temperatures on the upper x-axis refer to 29.5 bar pressure.

### Measurement of the gap

The measurement of the gap utilizes spin-orbit interaction in Cooper pairs, which have total spin 1 and orbital momentum 1. As a result of this interaction, the precession frequency of spin in nuclear magnetic resonance (NMR) experiments $\omega$ deviates from the Larmor precession frequency $\omega_L = |\gamma|H$ in the normal phase. Here $\gamma \approx -204 \cdot 10^6 \text{rad s}^{-1} \text{T}^{-1}$ is the gyromagnetic ratio of $^3$He. In our measurements the magnetic field $H$ is oriented along $\hat{z}$ and the frequency shift is

$$\omega(T)^2 - \omega_L^2 = \lambda_D N_0 \frac{\gamma^2}{\chi} \Delta^2(T).$$

Here $\lambda_D \approx 10^{-6}$ describes interaction of two magnetic dipoles in a pair and is approximately constant, $N_0$ and $\chi$ are pressure-dependent but temperature-independent density of states and magnetic susceptibility of normal $^3$He, respectively, and $\Delta(T)$ is the maximum gap in the energy spectrum of Bogoliubov quasiparticles. Examples of the NMR spectra measured in normal helium and in the polar phase at $T \approx 0.2T_c$ at different pressures are shown in Fig. 2.

In the polar phase, the gap for arbitrary direction of momentum has the form $\Delta(T, p) = \Delta(0) \cos \mu$, where $\mu$ is the angle between $p$ and $\hat{z}$. The gap vanishes at $\mu = \pi/2$, which gives rise to the Dirac nodal line on the equator of the Fermi surface. This nodal line is topological since the Berry phase changes by $\pi$ on a closed path around an element of the line for each of the two spin projections of fermions, see Fig. 4a and Supplementary Note 1. Due to the nodal line the density of states in the polar phase as a function of energy $\epsilon$ is $N(\epsilon) \propto \epsilon$, which results in the cubic dependence of the free energy $F(T) - F(0) \propto T^3$ at low temperatures $T \ll T_c$. Such cubic dependence also extends to the gap amplitude:

$$1 - \frac{\Delta(T)}{\Delta(0)} = a \frac{T^3}{T_c^3}, \quad T \ll T_c,$$

where the dimensionless parameter $a$ in the BCS clean limit is

$$a \approx 8.5 \left[ \frac{k_B T_c}{\Delta(0)} \right]^3,$$

see Supplementary Note 2. With $\Delta(0) = 2.46 \mu\text{B}$, in the weak coupling approximation, the value is $a = 0.57$, see Supplementary Note 3. We remark that in the case of Weyl superfluids with point nodes (like the A phase of superfluid $^3$He), the expected temperature dependence of the gap amplitude is $(T/T_c)^4$.

From equations (1) and (2), we find that

$$\frac{\omega(0) - \omega(T)}{\omega(0) - \omega_L} = 1 - \frac{\Delta^2(T)}{\Delta^2(0)} = 2a \frac{T^3}{T_c^3}, \quad T \ll T_c,$$

where we assumed that $\omega - \omega_L \ll \omega_L$ and $\Delta(0) - \Delta(T) \ll \Delta(0)$. Thus the normalized frequency shift is a direct probe of the temperature dependence of the gap.

### Cubic law

We measured the NMR frequency shift in the polar phase at several pressures between 0.1 and 29.5 bar. The temperature dependence of the shift at three pressures is shown in Fig. 2. The data are fit very well with the cubic dependence of equation (1). The data for all pressures are presented in Supplementary Figs. 2 and 3 and the details of the fitting procedure are discussed in Supplementary Note 4. Combining uncertainties of the fit with uncertainties of the temperature calibration (see Methods) we determine the confidence interval for the
exponent in the temperature dependence of the gap as \(2.9 - 3.2\). Within this uncertainty, the power law is in full agreement with the theoretical temperature dependence of the gap in the clean limit, which comes from the Dirac nodal line.

The prefactor \(a\) in the gap temperature dependence varies between 0.7 at low pressures and 0.3 at high pressures, see Fig. 3. It is comparable to its theoretical clean-limit value, although deviations from the weak-coupling value of 0.57 and systematic pressure dependence are clearly seen. Since \(a \propto (T_c/\Delta(0))^3\), one contribution to this change is the increase of the \(\Delta(0)/T_c\) ratio due to strong coupling effects, which in \(^3\)He becomes more important with increasing pressure\(^{29,31}\). Another possible contribution is a weak violation of the Anderson theorem due to non-ideal scattering at the strands. That is, the same effect as responsible for \(T_c\) suppression. We discuss these two contributions in more details below.

We stress that beautiful agreement of the measurements with the clean-limit theory is achieved despite the fact that in our sample the impurity scattering is strong with \(\tau \ll h/\Delta\) in the most parts of the phase diagram. Nevertheless, as seen in Supplementary Fig. 1, at the lowest temperatures and at higher pressures \(\tau \Delta/h\) can reach or even exceed unity. That opens a possibility of phase transitions to different superfluid phases, not protected by the Anderson theorem. We believe that the upward deviation of the points from the fit line for 29.5 bar data at the lowest temperatures in Fig. 3 is an indication of such transition. The new phase is probably the polar-distorted A phase as observed in the less dense nafen-243 used in this work. Triangles are the 30 bar data from Ref. 38 scaled with the strong-coupling corrections, see the text for details. On the right \(y\) axis: Suppression of \(T_c\) from Fig. 1 (squares). The scale of the axis is selected to stress similar dependence of \(a\) and \(T_c/T_{cb}\) on pressure.

**DISCUSSION**

Our demonstration of the extension of the Anderson theorem to the \(p\)-wave system is applicable for non-magnetic scattering from impurities. This condition is fulfilled in our experiments by preplating the nafen strands with a \(^3\)He layer. With insufficient \(^4\)He cov-
The density of states is a parameter in the vicinity of strands. The low-temperature predictions for the polar phase are the following: (i) $T^3$ law in the gap dependence is preserved at least until $T = 0.5T_c$; (ii) coefficient $\alpha$ increases compared to the clean limit, see diamonds in Fig. 3b; (iii) coefficient $\alpha$ decreases with pressure as $T_c$ suppression decreases. These predictions agree with our observations, at least qualitatively. The measured decrease of $\alpha$ is much stronger than found in this model, but indeed has a similar pressure dependence as $T_c$ suppression, see squares in Fig. 3b.

An explanation for the enhanced pressure dependence of $\alpha$ may come from the strong-coupling effects in superfluid $^3$He. These effects result in the increase of $\Delta(0)/T_c$ with pressure and renormalization of $\alpha$ to smaller values. The correction is well known in the B phase, but its value cannot explain our observations fully, see the solid line and the filled triangle in Fig. 3b. In principle, it is not excluded that presence of confining strands between $^3$He atoms interacting in a Cooper pair (since $D \lesssim \xi_0$) may affect the strong-coupling effects. We can find $\Delta(0)/T_c$ from the zero-temperature frequency shift, Fig. 4 using Eq. (1), see Supplementary Fig. 4. With this correction applied with respect to 0.1 bar data, the observed overall pressure change in $\alpha$ is reproduced, see stars and an empty triangle in Fig. 3b, although detailed pressure dependence somewhat differs from the observations. Despite the agreement, one should be careful, though, that the assumption of $\lambda_D = \text{const}$ used in this analysis, may be an oversimplification.

Overall, it is likely that orientational disorder in the strands and the strong coupling effects together explain the observed deviations in the gap temperature dependence from the prediction of the clean-limit weak-coupling theory. The detailed accounting for these effects as well as for the contribution of the strand surface-bound states which may change the temperature dependence further at the lowest temperatures, remains a task for future. Nevertheless, these effects provide just relatively small deviations from the clean-limit expectations, as confirmed by the experiment, while in the absence of protection by the Anderson theorem, the system behavior would be completely revamped.

Non-trivial symmetry and topology of the polar phase of superfluid $^3$He has been used to experimentally demonstrate the analog of the cosmological Alice string (the half-quantum vortex)\textsuperscript{10}. A realization of the Kibble–Lazarides–Shafi cosmic wall bounded by strings\textsuperscript{39} has also been observed: These walls are formed\textsuperscript{39} after the phase transition from the polar phase to the other phases of superfluid $^3$He. He found with the less dense confining matrix, where the scattering time is increased to $\tau \sim$...
To a linear dependence of the critical temperature on the two-dimensional surface of zero energy states (a flat band) in bulk material generates the two-dimensional surface of zero energy states (a flat band) on the boundary. In metals and semimetals, the presence of the topological flat band at the surface normal to the direction of the confining strands, see Supplementary Fig. 5b. As a result of the topological flat band at the surface normal to the direction of the confining strands, see Supplementary Fig. 5b. The robustness of the BFS to disorder remains an interesting question for future research, since in the presence of the superflow the Anderson theorem is not automatically applicable. The non-thermal quasiparticles located in the pockets covered by BFS reduce the average gap value. This suppression can be observed, for example, by attaching a piece of nafen with confined polar phase to a vibrating object immersed in helium or by rotating the entire sample and detecting the corresponding frequency shift in NMR, as a direct extension of the method used in the present work.

Another striking consequence of the nodal line is the presence of the topological flat band at the surface, normal to the direction of the confining strands, see Supplementary Fig. 5b. As a result of the topological phenomenon of bulk-boundary correspondence, the one-dimensional nodal line in bulk material generates the two-dimensional surface of zero energy states (a flat band) on the boundary. In metals and semimetals, the singular density of electronic states in the flat band leads to a linear dependence of the critical temperature $T_c$ of the superconducting transition on the strength of the pairing interaction. Thus, in a flat-band system $T_c$ can be essentially higher than in conventional superconductors, where $T_c$ is exponentially suppressed as a function of the pairing interaction. A particular example is provided by the superconductivity in the twisted graphene layers, while some other experiments with graphite materials demonstrate signatures of superconductivity even at room temperature. In the polar phase of $^3$He, the surface flat band may give rise to a new superfluid phase on the surface with additional symmetry breaking.

This work provides an experimental evidence for the existence of the nodal line in the polar phase and demonstration of the extension of the Anderson theorem to an unconventional p-wave system. Quite likely, by varying the geometry of the confinement, protection of the Anderson theorem can be extended to other superfluid phases with p-wave pairing, like the chiral A phase in the planar geometry. The extension of the Anderson theorem has also been considered for unconventional and multi-band superconductors. Although the superconducting model systems discussed in Refs. 11,12 differ significantly from the polar phase with columnar defects, the mechanism of the robustness towards disorder is the same: impurity scattering between different bands should be properly suppressed. There are also suggested scenarios in which the disorder leads to enhancement of the transition temperature due to inelastic scattering and to even larger enhancement of superfluidity due to multifractality of the fermion wave functions in the special arrangements of strands. At the moment we did not see any hints of the enhancement of $T_c$, but different classes of disorder can be experimentally constructed in future by varying synthesis techniques of the confinement material and its post-processing or by nanofabrication: with different arrangements of strands, with different densities, with different fractal distributions, with different shapes of strands, with different bound states etc. Our work opens the road to future experiments on the protection of topological superconductivity against disorder, on Bogoliubov Fermi surfaces, on fermionic topological flat bands, and on the effective metric which allows a transition to antispacetime.

**METHODS**

**Sample**

The $^3$He sample is confined in a $4 \times 4 \times 4$ mm$^3$ cubic container made from Styacstock 1266 epoxy. The container volume is filled with a nanostructured material called nafen of 0.243 g/cm$^3$ density. The nafen consists of nearly parallel $Al_2O_3$ strands (Fig. 1) and provides about 94% open space within the structure. The nafen was produced by AFN Technology Ltd in Estonia. To avoid formation of paramagnetic solid $^3$He layer on the strands, which breaks the requirement of non-magnetic specular scattering, all surfaces are preplated by about 2.5 monolayers of $^4$He. The $^3$He pressure is regulated between 0.1 and 29.5 bar through a filling line from a room-temperature gas handling system. The container with confined sample is connected to a larger volume of bulk $^3$He which in turn is attached to a copper nuclear demagnetisation cooling stage through a heat exchanger made from sintered silver powder. The temperature is regulated by changing current in the solenoid creating the demagnetisation field. Depending on pressure, the lowest temperatures reached vary between 0.19 and 0.21 $T_c$, or between 200 and 450 $\mu$K. The lowest temperature is limited by the residual heat leak to the sample (about 25 $\mu$W) and the thermal resistance of the sinter enhanced by the $^4$He layer.

The SEM photograph of the nafen material in Fig. 1 is acquired with Jeol JSM-7100F microscope using acceleration voltage of 5 kV and working distance of 9.3 mm. The imaged sample is from the same production batch as the one used in confining superfluid $^3$He, but is a physically different piece.
Nuclear magnetic resonance measurements

NMR spectrometer includes pick-up coils made from copper wire, which enclose the sample. The same pair of coils with the axis transverse to the static NMR field $H$ is used both to excite and detect the nuclear magnetisation precession. The coils are part of a tuned tank circuit, with a $Q$-value of 150. Signal from the pick-up coils is preamplified with a cold amplifier thermalized to a 4K plate and then fed for further amplification and detection using a lock-in amplifier located at room temperature.

In the measurements, the excitation is continuously applied at frequency $\omega_\text{rf}/2\pi = 363\text{kHz}$, which is fixed to the resonance frequency of the tank circuit, while the magnitude of the magnetic field $H$ is swept around the value of $H_L = \omega_\text{rf}/|\gamma| \approx 11\text{mT}$ to record the NMR spectrum. Here the Larmor field $H_L$ corresponds to location of the NMR response in normal $^4\text{He}$. In the polar phase, the NMR response shifts to lower fields $H < H_L$. For analysis, we convert the field shift to the equivalent frequency shift as $\omega - \omega_\text{rf} = \omega_\text{rf}(1 - H/H_L)$, which is applicable since the observed shifts are sufficiently small. We ensure that the magnetic field $H$ is oriented along the nafen strands by rotating the field using 2-axis vector magnet and checking for the maximum value of the frequency shift.

The inhomogeneity of the applied magnetic field is $\Delta H/H \approx 10^{-4}$. This value determines the width of the NMR spectrum in the normal phase. In the polar phase when the temperature decreases the spectrum becomes wider due to inhomogeneity of the nafen, see Fig. 2a for spectra examples. To determine the frequency shift of the spectrum from the normal state to the lowest probed temperature, the first moment of absorption spectra are compared (the spectra are normalized to unit area). The spectrum shift with respect to the one measured at the lowest temperature is found by matching the whole absorption and dispersion profiles recorded at two temperatures using the relative shift as a fitting parameter. To accommodate the changing spectrum shape, the shift is determined relatively, within groups of 10 consecutively measured spectra, where the line shape does not visually change. To verify that the error in accumulation of the relative shifts is not significant, we in parallel determine the NMR peak position by fitting a parabola close to the absorption maximum. Both methods produce the same overall dependence, but the peak-finding method results in significantly larger scatter, since it uses much fewer data from the each recorded spectrum.

For pressures of 0.1, 4, 7, 11 and 15 bars one temperature sweep with increasing temperature and one with decreasing temperature are measured, while for 23 and 29.5 bars two pairs of increasing/decreasing temperature sweeps are recorded. All sweeps at the same pressure are averaged for the analysis.

**Thermometry**

Near the superfluid transition temperature, the NMR frequency shift of the bulk $^3\text{He}$ (in the B or A phase, depending on pressure) is used as a thermometer. Small contribution to the NMR spectra from the unconfined helium is visible due to the geometry of the pick-up coils. At temperatures below about $0.7T_c$ the bulk signal is however too wide and too small to be distinguished from the noise. The experiment is equipped with a quartz tuning fork of nominal 32768 Hz frequency, which is immersed in bulk $^3\text{He}$. The width of the resonance $\omega_b$ of such fork is a sensitive probe of the temperature-dependent density of quasiparticles and it is widely used as a thermometer.
in superfluid $^3$He measurements. In our experiment, presence of the $^4$He film on all surfaces modifies the fork response, in particular with appearance of a pressure-dependent resonance-like feature seen in Fig. 5. Thus, independent calibration of the fork becomes unreliable and challenging. Nevertheless, the fork keeps its sensitivity in the whole studied temperature range, and we developed a procedure which allowed to recover the temperature assuming that the fork and NMR responses are reproducible for the same temperature.

The temperature sweep in the experiment is performed with the sweep of the current $I(t)$ in the demagnetization solenoid. For ideal cooling in the absence of losses, heat leaks and thermal resistances, the resulting temperature is $T^*(t) = T_{cb}[I(t)/I_{cb}]$. Here $I_{cb}$ is the demagnetization current at the moment of crossing $T_{cb}$ temperature as determined from the maximum resonance width of the tuning fork immersed in bulk liquid. To calculate the actual temperature of the sample $T(t)$ we use a thermal model which includes the nuclear stage and the sample cell with separate temperatures and the thermal resistance between them. The heat capacity of the stage and of the cell are assumed to be known (for the copper stage – from the design, for the cell – theoretically calculated from the known volume). The thermal resistance is taken as $R_T \exp(\Delta/k_B T)$ where $\Delta$ is the value of the superfluid gap in the B phase, which is in the contact with the sinter, calculated with trivial strong-coupling corrections and $R_T$ is an adjustable parameter. Other two parameters of the model are the heat leaks to the stage $q_{as}$ and to the sample $q_{sam}$. We measure the NMR spectra and the fork resonance while sweeping the temperature up and down in the range of the interest with different rates. The model parameters are then selected in such a way, that at the same calculated temperature of the cell the measured shift of the NMR spectrum and the width of the fork resonance coincide independently of the direction and rate of the temperature sweep. An example of such temperature calibration is shown in Fig. 5. Same values of $q_{as} = 2 \text{nW}$ and $q_{sam} = 25 \text{pW}$ are used at all pressures. The value of $R_T$ we characterize with the minimum attainable temperature of the sample $T_{min}$ as $R_T = T_{min} q_{sam}^{-1} \exp(-\Delta/k_B T_{min})$ and use $T_{min} = 0.17 T_{cb}$ at all pressures. These values agree with independent measurements of the heat leak to the nuclear stage and with expectations based on the previous experiments which had better thermal control of bulk $^3$He sample.

To estimate influence of possible temperature calibration error on the determination of the gap temperature dependence, we reanalyzed measurements at 7 bars with about 30 sets of the thermal model parameters distributed in the range of 0.5 to 2 times the values quoted above for the heat leaks and from 0.1 to 10 times for the thermal resistance. This range exceeds possible uncertainty in the parameters, as the fork and NMR responses become clearly hysteretic at the boundaries of the distribution. For all these sets, the fitted values of the gap power-law exponent are between 2.95 and 3.2 while the values of the parameter $a$ are between 0.47 and 0.58 when the fit is performed in the $(0.3–0.5)/T$ temperature range.

Data availability

The data generated in this study have been deposited in the Zenodo database under accession code https://doi.org/10.5281/zenodo.8055891.

Code availability

The code for the thermal model used in the temperature calibration is available from the corresponding author upon request.

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Authors contributions

The experiments were conducted by T.K. J.R., and M.M.V. The data were analyzed by J.R. and V.B.E. Theoretical work was carried out by G.E.V. Writing was done by V.B.E. and G.E.V. with contributions from all the authors. V.B.E. supervised the project.

Competing interests

The authors declare no competing interests.

Supplementary Note 1: Dirac nodal line and its topology

Polar phase belongs to the superfluid states with the so-called equal spin pairing. This means that it can be represented as an equal mixture of two superfluids with the projections of the Cooper pair spin $S_z = +1$ and $S_z = -1$. In superfluid $^3$He the spin-orbit interaction is very small compared to the superfluid gap and can be neglected. That is why for each of the two spin projections one has the following Bogoliubov-de Gennes Hamiltonian:

$$ H = v_F (p - p_F) \tau^3 + \Delta \frac{p_z}{p_F} \tau^1, \quad (5) $$

where $\Delta$ is the gap amplitude, and $\tau^a$ are the Pauli matrices in the Bogoliubov-Nambu particle-hole space. The Hamiltonian is nullified on the line $p_z = 0$, $p = p_F$. This is the Dirac nodal line, which stability is supported by topology and symmetry. The corresponding topological invariant can be written in terms of the Hamiltonian:

$$ N = -\frac{1}{4\pi} \oint_C dl \tau^2 H^{-1} \nabla_l H. \quad (6) $$

Here the contour $C$ of integration is around the element of the nodal line, see Fig. 1a in the main text. This integral is invariant under deformations preserving the time reversal symmetry, due to which the Hamiltonian must anti-commute with $\tau^2$, i.e. $\{H, \tau^2\} = 0$. For each spin projection the invariant has the value $N = 1$, which also means that the Berry phase changes by $\pi$ along $C$. 


Supplementary Note 2: $T^3$ dependence of the superfluid gap in the nodal-line polar phase

The gap equation for the polar phase:

$$
\frac{1}{g} = \int_0^1 dx' \int_0^{E_{uv}} d\xi \left( \frac{1}{\sqrt{\xi^2 + \Delta^2(0)x'^2}} - \frac{1}{\sqrt{\xi^2 + \Delta^2(T)x'^2}} \right) - 2 \int_0^1 dx' \int_0^{\infty} d\xi \frac{d\xi}{\sqrt{\xi^2 + \Delta^2(T)x'^2}} \exp \left( \frac{1}{T} \frac{\Delta(0)}{\sqrt{\xi^2 + \Delta^2(T)x'^2}} \right) + 1
$$

(7)

where in Eq. (8) the cut-off dropped out because of cancellation of logarithmic terms. Here $g$ is the normalized coupling, which is not important for us, since it drops out from further equations; $\Delta(T)$ is the gap amplitude; $x = \cos \mu$ shows the dependence of the gap function on the polar angle $\mu$; $E_{uv}$ is the ultraviolet cut-off of the logarithmically divergent integrals, which also drops out from the further equations, where the logarithmic terms cancel each other; the $x^2 = \cos^2 \mu$ in the integral over $x$ comes from the $\cos \mu$-dependence of the $p$-wave interaction potential $V_{k,k'} \propto k_x k'_x$ and the gap in the polar phase. We have from Eq. (7):

$$
\int_0^1 dx'^2 \int_0^{\infty} d\xi \left( \frac{1}{\sqrt{\xi^2 + \Delta^2(0)x'^2}} - \frac{1}{\sqrt{\xi^2 + \Delta^2(T)x'^2}} \right) =
$$

(8)

$$
-2 \int_0^1 dx'^2 \int_0^{\infty} d\xi \frac{d\xi}{\sqrt{\xi^2 + \Delta^2(T)x'^2}} \exp \left( \frac{1}{T} \frac{\Delta(0)}{\sqrt{\xi^2 + \Delta^2(T)x'^2}} \right) + 1
$$

(9)

where in Eq. (8) the cut-off dropped out because of cancellation of logarithmic terms. At low $T$ this Eq. (8) is proportional to $\Delta^2(T) - \Delta^2(0)$, while in the Eq. (9) one can take the $T = 0$ limit:

$$
\int_0^1 dx'^2 \int_0^{\infty} d\xi \left( \frac{1}{\sqrt{\xi^2 + \Delta^2(0)x'^2}} - \frac{1}{\sqrt{\xi^2 + \Delta^2(T)x'^2}} \right) =
$$

(10)

$$
2 \int_0^1 dx'^2 \int_0^{\infty} d\xi \frac{d\xi}{\sqrt{\xi^2 + \Delta^2(0)x'^2}} \exp \left( \frac{1}{T} \frac{\Delta(0)}{\sqrt{\xi^2 + \Delta^2(T)x'^2}} \right) + 1
$$

(11)

Expansion in $\Delta^2(0) - \Delta^2(T)$ gives

$$
\frac{1}{2} (\Delta^2(0) - \Delta^2(T)) \int_0^1 dx'^4 \int_0^{\infty} d\xi (\xi^2 + \Delta^2(T)x'^2)^{-3/2} =
$$

(12)

$$
2 \int_0^1 dx'^2 \int_0^{\infty} d\xi \frac{d\xi}{\sqrt{\xi^2 + \Delta^2(T)x'^2}} \exp \left( \frac{1}{T} \frac{\Delta(0)}{\sqrt{\xi^2 + \Delta^2(T)x'^2}} \right) + 1
$$

(13)

or

$$
\frac{1}{2} \left( 1 - \frac{\Delta^2(T)}{\Delta^2(0)} \right) \int_0^1 dx'^4 \int_0^{\infty} d\xi (\xi^2 + x'^2)^{-3/2} =
$$

(14)

$$
2 \int_0^{\infty} dx'^2 \int_0^{\infty} d\xi \frac{d\xi}{\sqrt{\xi^2 + x'^2}} \exp \left( \frac{\Delta(0)}{T} \frac{1}{\sqrt{\xi^2 + x'^2}} \right) + 1
$$

(15)

In Eq. (15) the integration over $x$ has been extended to $\infty$, because in this equation $x^2 + \xi^2 \sim T^2/\Delta(0)^2 \ll 1$. Introducing cylindrical coordinates $x = r \cos \phi$, $\xi = r \sin \phi$ in Eq. (15), one obtains:

$$
\frac{1}{6} \left( 1 - \frac{\Delta^2(T)}{\Delta^2(0)} \right) \int_0^{\infty} d\xi (\xi^2 + 1)^{-3/2} =
$$

(16)

$$
= \frac{\pi}{2} \int_0^{\infty} r^2 dr \frac{1}{\exp \left( \frac{r \Delta(0)}{T} \right) + 1}
$$

(17)

$$
= \frac{\pi}{2} \frac{T^3}{\Delta^3(0)} \int_0^{\infty} r^2 dr \frac{1}{e^r + 1}
$$

(18)
The integrals in Eq. (16) and in Eq. (18) are:

\[ \int_0^\infty d\xi (\xi^2 + 1)^{-3/2} = 1, \]

\[ \int_0^\infty \frac{r^2 dr}{e^r + 1} = \frac{3}{2} \zeta(3), \]

and we obtain the universal temperature dependence of the gap at low \( T \)

\[ 2 \left( 1 - \frac{\Delta(T)}{\Delta(0)} \right) = \left( 1 - \frac{\Delta^2(T)}{\Delta^2(0)} \right) = \frac{9\pi}{2} \zeta(3) \frac{T^3}{\Delta^3(0)}, \]

or

\[ 1 - \frac{\Delta(T)}{\Delta(0)} = \frac{9\pi}{4} \zeta(3) \frac{T^3}{\Delta^3(0)} = a \frac{T^3}{T_c^3}, \]

where

\[ a = \frac{9\pi}{4} \zeta(3) \left[ \frac{T_c}{\Delta(0)} \right]^3 \approx 8.5 \left[ \frac{T_c}{\Delta(0)} \right]^3. \]

Using the result \( \Delta(0) = 2.46T_c \) from the Supplementary Note 3 we find \( a = 0.57 \). We stress that this value is obtained for the pure polar phase without impurities. Its agreement with the measurements presented in the main text supports extension of the Anderson theorem to the polar phase with the columnar non-magnetic defects.

**Supplementary Note 3: The gap amplitude at \( T = 0 \) vs \( T_c \)**

From the gap equation at \( T = 0 \)

\[ \frac{1}{g} = \int_0^1 dxx^2 \int_0^{E_{iv}} d\xi \frac{\xi}{\sqrt{\xi^2 + \Delta^2(T)x^2}} \tanh \left( \frac{\sqrt{\xi^2 + \Delta^2(T)x^2}}{2T} \right) = \int_0^1 dxx^2 \int_0^{E_{iv}} d\xi \frac{\xi}{\sqrt{\xi^2 + \Delta^2(0)x^2}} \frac{\xi}{2T_c} \]

we obtain

\[ 0 = \int_0^1 dxx^2 \int_0^\infty d\xi \left( \frac{1}{\xi} \tanh \frac{\xi}{2T_c} - \frac{1}{\sqrt{\xi^2 + \Delta^2(0)x^2}} \right) = \int_0^1 dxx^2 \int_0^\infty d\xi \left( \frac{1}{\xi} \tanh \frac{\xi}{2T_c} - \frac{1}{\sqrt{\xi^2 + x^2}} \right) \]

Introducing the parameter \( \alpha = \Delta(0)/2T_c \) one finds the value of \( \alpha \) at which the function \( f(\alpha) \)

\[ f(\alpha) = \int_0^1 dxx^2 \int_0^\infty d\xi \left( \frac{\tanh \alpha \xi}{\xi} - \frac{1}{\sqrt{\xi^2 + x^2}} \right) \]

crosses zero. Solving the equation numerically, we find \( \alpha = 1.23 \).

**Supplementary Note 4: Temperature dependence of the frequency shift**

The interpretation of the frequency shift measurements is complicated by the fact that theoretical models refer to the change of the frequency shift from its zero-temperature value, which is unknown experimentally. The zero-temperature shift should be determined by extrapolation of the measured data. Extrapolation requires some model for the data upfront. This circular dependence of data and interpretation results in the uncertainty in the zero-temperature value of the shift being the main source of uncertainty in the values of the exponent and of the coefficient in the temperature dependence of the frequency shift and thus of the gap.
A reasonable general model for the frequency shift for a superfluid with gap nodes is \( \omega(T) - \omega_L = b_1 + b_2(T/T_c)^{b_3} \) with \( b_1, b_2 \) and \( b_3 \) being the free parameters. The fit of the measured data to this model is shown in Supplementary Fig. 2. Observations at 0.1, 4 and 7 bar pressure demonstrate nearly ideal cubic temperature behavior with \( b_3 \) lying in the interval 2.9–3.1 and the coefficient \( a = -b_2/(2b_1) \) being very close to the clean-limit weak-coupling value of 0.57 for 4 and 7 bars and slightly larger for 0.1 bar in accordance with stronger \( T_c \) suppression at the lowest pressure. The fitted values are also robust with respect to the range of temperatures included in the fit. The upper limit of 0.5\( T_c \) should be safe to determine the low-temperature behavior according to the calculations in Fig. 4 of Ref. 38. In principle, the measurements at these three pressures are sufficient to prove the main messages of the paper on the presence of the nodal line and extension of the Anderson theorem in the polar phase, as well as to demonstrate the link between \( T_c \) suppression and the \( a \) coefficient value due to disorder breaking the requirements of the theorem.

The behavior at higher pressures is nevertheless interesting and provides more data for future analysis of the interplay between topology and disorder. The fit in the whole available temperature range gives values of \( b_3 \) up to 4 at 15 bar pressure. In principle, the exponent 4 is expected in the clean-limit case for the point nodes, which for confined \(^3\text{He}\) in this work means the polar-distorted A (PdA) phase. We note, however, that for the PdA phase the extension of the Anderson theorem is not supposed to work and thus the exponent to expect is not known. Also the value of \( b_3 \) changes depending on the temperature range included in the fit. Thus we interpret the observation as an indication of a phase transition from the polar phase at higher temperatures to a different phase at lower temperatures with a maximum extent of the different phase, up to about 0.3\( T_c \), at 15 bars and the range decreasing both at lower and higher pressures, recovering the polar-phase behavior in almost whole temperature range at 29.5 bars. (Note that in an inhomogenous confined sample, different phases can coexist even in the case of the second-order phase transition and the transition becomes extended in temperature).

We check this interpretation in Supplementary Fig. 3. Here the frequency shift at temperatures 0.3\( T_c < T < 0.5T_c \) is fit with the cubic power law as established for the polar phase to find the zero-temperature extrapolated shift. After subtracting the zero-temperature shift, the residual shift is fit with the power law in the same temperature range. The fit exponent \( b'_3 \) comes very close to 3, even at the intermediate pressures, which confirms that in this temperature range the polar phase dominates also at these pressures. At lower temperatures the normalized frequency shift goes above the fit line for pressures 11 bar and above, with the maximum deviation observed at 15 bar. Qualitatively, it is the direction of deviation expected on transition to the PdA phase as seen in the lower-density nafen-90\textsuperscript{22}. Proper identification, though, requires more measurements in particular versus magnetic field direction and tipping angle of magnetization. At 0.1 bar one can see deviation of the data below the fit line at the lowest temperatures. In principle, this is the direction expected on the transition from the polar to the polar-distorted B (PdB) phase\textsuperscript{41}. The deviation, however, is small and may be caused by inaccuracies in the temperature calibration. Finally, we note that at temperatures about and above 0.6\( T_c \) one finds the deviation of data above the fit line. This qualitatively agrees with calculations for the polar phase in Ref. 38.
Supplementary Figure 1. Scattering in confined superfluid $^3$He as a function of pressure. Calculated product of the quasiparticle scattering time $\tau(\mu)$ and the superfluid gap $\Delta(\mu)$ averaged over direction of the quasiparticle momentum $\mathbf{p}$ determined by the angle $\mu$ between $\mathbf{p}$ and the strands of the confining matrix. Solid line represents confining material used in this work, nafen with density 243 mg/cm$^3$. The dashed line is for the less dense nafen with density 90 mg/cm$^3$, for which phase diagram of superfluid $^3$He states is also known$^{22,41}$. Here $\tau(\mu)$ is determined from $[\tau(\mu)v_F]^{-1} = l_{\perp}^{-1}\sin\mu + l_{\parallel}^{-1}\cos\mu$, where $v_F$ is the Fermi velocity and $l_{\perp}$ and $l_{\parallel}$ are quasiparticle mean free path in the direction perpendicular and parallel to the strands, respectively. For the gap, the zero-temperature value in the weak-coupling BCS theory $\Delta(\mu) = 1.23 \cdot 2k_B T_c \cos\mu$ is taken (see Supplementary Note 3). Thus, plotted value is an upper bound of $\tau\Delta$ as a function of temperature. For nafen-243, $\tau < h/\Delta$ (except the lowest temperatures and elevated pressures), and in the major part of the phase diagram all superfluid phases are suppressed except the polar phase, which is robust due to extension of the Anderson theorem. For nafen-90, the scattering is less prominent and the polar phase is stable near $T_c$, while other phases replace it at lower temperatures. Values of $l_{\perp}$ and $l_{\parallel}$ are extracted from spin-diffusion measurements in the normal phase$^{63}$. For nafen-243 $l_{\perp} = 70$ nm and $l_{\parallel} = 570$ nm while for nafen-90 $l_{\perp} = 290$ nm and $l_{\parallel} = 960$ nm.
Supplementary Figure 2. Three-parameter fit of the frequency shift. The measured frequency shift of the NMR response is fitted as $\omega(T) - \omega_L = b_1 + b_2(T/T_c)^{b_3}$ with $b_1$, $b_2$ and $b_3$ being the fit parameters. **a**, The normalized shift, $1 - [\omega(T) - \omega_L]/b_1$ as a function of temperature at all measured pressures (symbols). The solid lines are fits performed to data at temperature $T < 0.5T_c$. Dash-dotted line corresponds to $T^3$ and dashed line to $T^{2.14}$. **b**, Zero-temperature shift $b_1 = \omega(0) - \omega_L$ as a function of pressure from fits to data at $T < 0.5T_c$ (circles) and at $0.35T_c < T < 0.5T_c$ (squares). Error bars show statistical uncertainties from the fit ($\pm1\sigma$). **c**, The same plot as **b** for the exponent $b_3$. Dashed line shows clean-limit weak-coupling value 3 expected for the polar phase. **d**, The same plot as **b** for the coefficient $a = -b_2/(2b_1)$. Dashed line shows clean-limit weak-coupling value 0.57 expected for the polar phase. For discussion, see Supplementary Note 4.
Supplementary Figure 3. Two-parameter fit of the frequency shift. The measured frequency shift of the NMR response is fitted at temperatures $0.3T_c < T < 0.5T_c$ as $\omega(T) - \omega_L = b_1 + b_2(T/T_c)^3$ with $b_1$ and $b_2$ being the fit parameters. This is the same fit as shown in Figs. 2b and 3a in the main text. 

**a.** The normalized shift, $1 - \frac{\omega(T) - \omega_L}{b_1}$ as a function of temperature at all measured pressures (symbols). The solid lines are fits of the normalized shift as $b'_2(T/T_c)^3$ for data in the range $0.3T_c < T < 0.5T_c$. Dash-dotted line corresponds to $T^3$ and dashed line to $T^2$. 

**b.** Zero-temperature shift $b_1 = \omega(0) - \omega_L$ as a function of pressure. Error bars show statistical uncertainties from the fit ($\pm 1\sigma$). 

**c.** The same plot as **b** for the exponent $b'_3$. Dashed line shows clean-limit weak-coupling value 3 expected for the polar phase. 

**d.** The same plot as **b** for the coefficient $a = -b'_2/(2b_1)$. Within error bars, the values are indistinguishable from $-b_2/(2b_1)$ plotted in Fig. 3b in the main text. Dashed line shows clean-limit weak-coupling value 0.57 expected for the polar phase. For discussion, see Supplementary Note 4.
Supplementary Figure 4. Determination of the strong coupling corrections in the polar phase of $^3$He from the frequency shift. Ratio $[\Delta(T = 0, P)/T_c(P)]/[\Delta(T = 0, P = 0.1\text{ bar})/T_c(P = 0.1\text{ bar})]$ as a function of pressure $P$ as determined from the zero-temperature frequency shift, Fig. 4 in the main text and Supplementary Fig. 3, using Eq. (1) in the main text (circles). The line shows accepted value of this ratio in the B phase. Statistical error from the fit of the temperature dependence of the frequency shift is smaller than the symbol size. Fits on Supplementary Figs. 2 and 3 give identical results within the symbol size. Uncertainties from the temperature calibration determined as described in Methods and from the finite width of the NMR spectra are both about 1%. We note that this analysis assumes $\lambda_D$ being independent of pressure, which might be an oversimplification.

Supplementary Figure 5. Consequences of the nodal line in the polar phase. a, Illustration of the Bogoliubov Fermi surface in the polar phase in the presence of superflow $v_s$. Under superflow the Dirac nodal line transforms to two Fermi pockets, which touch each other at two points (pseudo-Weyl points$^{32}$). In the zero-temperature limit, this Fermi surface should lead to the cubic suppression of the gap amplitude as a function of $v_s$: $1 - \Delta(v_s)/\Delta(0) \propto (v_s/v_c)^3$, where $v_c = \Delta(0)/p_F$. b, Owing to the surface-bulk correspondence in topological matter with Dirac lines$^{48,50}$ the fermionic flat band appears on the surface normal to the direction of the confining strands.