Chiral perturbation theory for pentaquark baryons and its applications

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Abstract

We construct a chiral lagrangian for pentaquark baryons assuming that the recently found \( \Theta \) \((1540) \) state belongs to an antidecuplet of SU(3) flavor symmetry with \( J^P = \frac{3}{2}^- \). We derive the Gell-Mann-Okubo formulae for the antidecuplet baryon masses, and a possible mixing between the antidecuplet and the pentaquark octet. Then we calculate the cross sections for \( \pi^{-}p \to K^{-}\Theta^{+} \) and \( \gamma n \to K^{-}\Theta^{+} \) using our chiral lagrangian. The resulting amplitudes respect the underlying chiral symmetry of QCD correctly. We also describe how to include the light vector mesons in the chiral lagrangian.

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1. Introduction — Recently, five independent experiments reported observations of a new baryonic state \( \Theta^{+}(1540) \) with a very narrow width \( \approx 5 \text{ MeV} \) \cite{1,2,3,4,5}, which is likely to be a pentaquark state \( (uudds) \). Arguments based on quark models suggest that this state is a member of SU(3) antidecuplet with spin \( J = \frac{1}{2} \) or \( \frac{3}{2} \). The hadro/photo production cross section would depend on the spin \( J \) and parity \( P \) of the \( \Theta^{+} \), and it is important to have reliable predictions for these cross sections. The most proper way to address these issues will be chiral perturbation theory.

In this paper, we construct a chiral lagrangian for pentaquark baryons assuming that they are SU(3) antidecuplet with \( J = \frac{1}{2} \) and \( P = +1 \) or \(-1\). (The case for \( J = \frac{3}{2} \) can be discussed in a similar manner, except that antidecuplets are described by Rarita–Schwinger fields.) Then we calculate the mass spectra of antidecuplets, their possible mixings with pentaquark octets, the decay rates of antidecuplets, and cross sections for \( \pi^{-}p \to K^{-}\Theta^{+} \) and \( \gamma n \to K^{-}\Theta^{+} \). Finally we describe how to include light vector mesons in our framework, and how the low energy theorem is recovered in the soft pion limit.

2. Chiral lagrangian for a pentaquark baryon decuplet — Let us denote the Goldstone boson field by pion octet \( \pi \), baryon octet including nucleons by \( B \), and antidecuplet including \( \Theta^{+} \) by \( P \). Under chiral SU(3)_L x SU(3)_R \cite{6}, the Goldstone boson field \( \Sigma \equiv \exp(2i\pi f) \), where \( f \approx 93 \text{ MeV} \) is the pion decay constant, transforms as

\[
\Sigma(x) \to L\Sigma(x)R^{\dagger}.
\]

It is convenient to define another field \( \xi(x) \) by \( \Sigma(x) \equiv \xi^{3}(x) \), which transforms as

\[
\xi(x) \to L\xi(x)U^{\dagger}(x) \equiv U(x)\xi(x)R^{\dagger}.
\]

The \( 3 \times 3 \) matrix field \( U(x) \) depends on Goldstone fields \( \pi(x) \) as well as the SU(3) transformation matrices \( L \) and \( R \). It is convenient to define two vector fields with the following properties under chiral transformations:

\[
V_{\mu} = \frac{1}{2} \left( \xi^{\dagger} \partial_{\mu} \xi + \xi \partial_{\mu} \xi^{\dagger} \right), \quad V_{\mu} \to UV_{\mu}^{\dagger} + U\partial_{\mu}U^{\dagger},
\]

\[
A_{\mu} = \frac{i}{2} \left( \xi^{\dagger} \partial_{\mu} \xi - \xi \partial_{\mu} \xi^{\dagger} \right), \quad A_{\mu} \to UA_{\mu}U^{\dagger} + U\partial_{\mu}U^{\dagger}. \quad (1)
\]

Note that \( V_{\mu} \) transforms like a gauge field. The transformation of the baryon octet and pentaquark antidecuplet \( P \) including \( \Theta^{+} \) \((I = 0) \) can be chosen as

\[
B_{ij} \to U_{ia}^{\dagger}B_{ib}^{a}U_{jk}^{b}, \quad P_{ijk} \to P_{abc}U_{ia}^{a}U_{jb}^{b}U_{ck}^{c},
\]

where all the indices are for SU(3) flavor. The pentaquark baryons are related to \( P_{abc} = P_{(abc)} \) by, for example, \( P_{333} = \Theta^{+} \), \( P_{133} = \frac{1}{\sqrt{3}}\tilde{N}^{0} \), \( P_{113} = \frac{1}{\sqrt{3}}\tilde{\Sigma}^{-} \), and \( P_{112} = \frac{1}{\sqrt{3}}\tilde{\Sigma}^{-}/2 \). Then, one can define a covariant derivative \( D_{\mu} \), which transforms as

\[
D_{\mu}B = \partial_{\mu}B + [V_{\mu}, B].
\]

Chiral symmetry is explicitly broken by non-vanishing current-quark masses and electromagnetic interactions. The former can be included by regarding the quark-mass matrix \( m = \text{diag}(m_{u}, m_{d}, m_{s}) \) as a spurion with transformation property \( m \to LmR^{\dagger} = RmL^{\dagger} \). It is more convenient to use \( \xi m\xi + \xi^{\dagger}m^{\dagger} \), which transforms as an SU(3) octet. Electromagnetic interactions can be included by introducing photon field \( A_{\mu} \) and its field strength tensor \( F_{\mu\nu} = \partial_{\mu}A_{\nu} - \partial_{\nu}A_{\mu} \):

\[
\partial_{\mu}\Sigma \to D_{\mu}\Sigma \equiv \partial_{\mu}\Sigma + ieA_{\mu}(Q, \Sigma), \quad (2a)
\]

\[
V_{\mu} \to V_{\mu} + \frac{e}{4}\Sigma A_{\mu}(\xi^{\dagger}Q\xi + \xi Q^{\dagger}e\xi) \equiv V_{\mu}^{\dagger} + d_{\mu}\xi^{\dagger}Q\xi, \quad (2b)
\]

\[
A_{\mu} \to A_{\mu} - \frac{e}{4}\Sigma A_{\mu}(\xi^{\dagger}Q\xi - \xi Q^{\dagger}e\xi), \quad (2c)
\]

where \( Q \equiv \text{diag}(2/3, -1/3, -1/3) \) is the electric-charge matrix for light quarks \((q = u, d, s)\). Now it is straightforward to construct a chiral lagrangian with lowest order in derivative expansion.
The parity and charge-conjugation symmetric chiral lagrangian is given by
\[ \mathcal{L} = \mathcal{L}_\Sigma + \mathcal{L}_B + \mathcal{L}_P, \] (3)
where
\[ \mathcal{L}_\Sigma = \frac{f_2^2}{4} \text{Tr} \left[ D_\mu \Sigma D^\mu \Sigma - 2 \mu m (\Sigma + \Sigma^\dagger) \right], \] (4a)
\[ \mathcal{L}_B = \text{Tr} \left[ \widehat{\mathcal{P}} (i \gamma^5 m_B) B + D \text{Tr} \left( \mathcal{A}, B \right) \right] + F \text{Tr} \left( \mathcal{B}_{\gamma 5} \left( \mathcal{A}, B \right) \right), \] (4b)
\[ \mathcal{L}_P = \left[ \widehat{\mathcal{P}} (i \gamma^5 m_P) P + \mathcal{C}_P \left( \mathcal{P} \Gamma_p \mathcal{A} B + \mathcal{P} \Gamma_p \mathcal{A} P \right) + \mathcal{H}_P \right] \mathcal{P}, \] (4c)
where \( P \) is the parity of \( \Theta^+ \), \( \Gamma^+ = \gamma_5 \), and \( \Gamma_- = 1 \), and \( m_P \) is the average of the pentaquark decuplet mass.

The Gell-Mann-Okubo formulae for pentaquark baryons will be obtained from
\[ \mathcal{L}_m = \alpha_m \left[ \left( \xi m_{\xi} + \xi^\dagger m_{\xi}^\dagger \right) \mathcal{P} \right]. \] (5)
Expanding this, we get the mass splittings \( \Delta m_i \equiv m_i - m_P \) within the antidecuplet:
\[ \Delta m_{\Theta^0} = 2 \alpha_m m_s, \] (6a)
\[ \Delta m_{\Xi^-} = \alpha_m (2 \tilde{m} + 4 m_s) / 3, \] (6b)
\[ \Delta m_{\Xi^0} = \alpha_m (4 \tilde{m} + 2 m_s) / 3, \] (6c)
\[ \Delta m_{\Xi^+} = 2 \alpha_m \tilde{m}, \] (6d)
where \( \tilde{m} = m_u + m_d \) ignoring small isospin-breaking effects. If the newly observed state at a mass 1862 ± 2 MeV is identified as \( \Xi_{3/2}^- \), we find
\[ m_{\Xi^-} = 1647 \text{ MeV}, \quad m_{\Xi^0} = 1755 \text{ MeV}. \] (7)

Recently Jaffe and Wilczek suggested there could be an ideal mixing between pentaquark antidecuplet \( \mathcal{P}_{abc} \) and pentaquark octet \( \mathcal{O}_{b}^c \) with the same parities \( [8] \). This idea has been generalized further by other groups \( [10] \). In chiral lagrangian approach, such a general mixing arises from
\[ \beta_m \left[ \mathcal{P} (\xi m_{\xi} + \xi^\dagger m_{\xi}^\dagger) \mathcal{O} + \mathcal{O} (\xi m_{\xi} + \xi^\dagger m_{\xi}^\dagger) \mathcal{P} \right]. \] (8)
Expanding this leads to
\[ \mathcal{L} = \mathcal{B}_m \left[ \mathcal{P} \nabla^\pm \mathcal{P} \nabla^0 + \nabla^+ \nabla^0 - \nabla^\pm \nabla^0 + \nabla^+ \nabla^0 + \text{H.c.} \right], \] (9)
where \( \mathcal{B}_m = 2 \beta_m (m_u + m_d) / \sqrt{3} \) and we borrowed baryon-octet notation for pentaquark octet states in Eq. \( [10] \). Note that the relative sign between \( \nabla^0 \mathcal{P} \mathcal{P} \mathcal{O} \) and \( \mathcal{P} \mathcal{O} \mathcal{P} \mathcal{P} \mathcal{O} \) is negative, unlike the case in Ref. \( [10] \). This is due to the \( [10] \) nature of the \( \mathcal{P} \). One could write down the same mixing between pentaquark antidecuplet \( \mathcal{P}_{abc} \) and the ordinary baryon octet \( B \), but such terms will be highly suppressed compared to the above term, since it is a mixing between \( qqq \) and \( qqqq \).

Finally the baryon decuplet can only couple to pentaquark octet \( \mathcal{O} \), but not to pentaquark antidecuplet \( \mathcal{P} \), since \( 10 \otimes 8 \otimes 10 \) does not contain \( SU(3) \) singlet. This implies that \( N(1440) \) or \( N(1710) \) cannot be pure pentaquark antidecuplets, because they have substantial branching ratios into \( \Delta \pi \) final states. They could be mixed states of pentaquark octet and pentaquark antidecuplet, and their productions and decays will be more complicated than pure antidecuplet case. Since the current data on baryon sectors are not enough to study such mixings in details, we do not pursue the mixing further in the following.

Parameters in the above lagrangian are taken to have the following numerical values: \( m_B \approx 940 \text{ MeV} \) is the nucleon mass, \( D \approx -0.81 \) and \( F \approx -0.47 \) at tree level, and we assume \( \tilde{m} = 0 \) and \( m^2_n = (4/3) m^2_K \).

The coupling \( \mathcal{C}_{PN} \) is determined from the decay width \( \Gamma_\Theta \) of the \( \Theta^+ \), which is dominated by \( K^+ n \) and \( K^0 p \) modes as \( \Gamma_\Theta/2 = \Gamma_{\Theta^+ \rightarrow K^+ n} = \Gamma_{\Theta^+ \rightarrow K^0 p} \).
\[ \Gamma_\Theta = \frac{C^2_{PN} |p^*|}{8 \pi f^2 m^*_\Theta} \left( (m_{\Theta^0} \pm m_B)^2 - m_K^2 \right), \] (10)
where \( p^* \) is the kaon momentum in the \( \Theta^+ \) rest frame and the signs are for \( P(\Theta^+) = \pm 1 \), respectively. Then the \( \mathcal{C}_{PN} \) is determined as
\[ C^2_{PN} (P = +, -) = (2.7, 0.90) \times \Gamma_\Theta/\text{GeV}. \]

Cahn and Trilling \( [12] \) argues that \( \Gamma_\Theta = (0.9 \pm 0.3) \text{ MeV} \) using the DIANA results \( [2] \). We present our results proportional to the \( C^2_{PN} \) rescaled by the factor 1 MeV/\( \Gamma_\Theta \). Such a small \( \mathcal{C}_{PN} \) can be understood as following: this coupling is related to the matrix element of hadronic axial vector current operator (with zero baryon number) between a pentaquark baryon and an ordinary baryon. Since two states have different number of valence quarks, this matrix element should be highly suppressed compared to the ordinary axial vector coupling \( (D + F) \) or \( \mathcal{H}_{PN} \).

The coupling \( \mathcal{H}_{PN} \) is also unknown, and determines transition rates between pentaquark antidecuplets with pion or kaon emission. Unfortunately, such decays are all kinematically forbidden, and cannot be used to fix \( \mathcal{H}_{PN} \). However, we expect that \( \mathcal{H}_{PN} = O(1) \), without any suppression as in \( \mathcal{C}_{PN} \). With this remark in mind, we will assume \( \mathcal{H}_{PN} \) can vary between −4 and 4 in the numerical analysis to be as general as possible.

3. \( \pi^+ p \rightarrow K^- \Theta^+ \). — Let us first consider \( \pi^+ p \rightarrow K^- \Theta^+ \) as applications of our chiral lagrangian for pentaquark baryons. In Figs. \( [10] \) (a) and (b), we show the relevant Feynman diagrams. Note that only Fig. \( [10] \) (a) was considered in the literature. However, there is an \( s \)-channel \( N^0(1647) \) exchange diagram [ Fig. \( [11] \) (b) ] in our chiral lagrangian, since \( \Theta^+ \) is not an \( SU(3) \) singlet, but belongs to the antidecuplet. Therefore one has to
is sensitive to $H$ the results differ from the previous results where only will have constructive (destructive) interference. Thus very important. If $H$ be distinguished from the cross section for the parity-odd keeping in mind that our results get unreliable as the order, we did not include model-dependent form factors, following the spirit of chiral perturbation theory at lowest production of $\Theta^+$ magnetic dipole interaction terms. For the nucleon octet, varied $\leq H_4$ in the proton rest frame: (a) $P(\Theta) = +1$ and (b) $P(\Theta) = +1$.

We show the total cross section, rescaled by the factor $\Gamma_\Theta/\text{MeV}$, as functions of pion energy $E_\pi$ at the proton rest frame in Fig. 2 depending on the $\Theta^+$ parity, and varied $-4 \leq H_{PN} \leq 4$. Note that the sign of $H_{PN}$ is very important. If $H_{PN} > 0 (<0)$, two contributions will have constructive (destructive) interference. Thus our results differ from the previous results where only the $n$ contribution was included. Also the cross section is sensitive to $H_{PN}$, and may be useful to fix $H_{PN}$. Following the spirit of chiral perturbation theory at lowest order, we did not include model-dependent form factors, keeping in mind that our results get unreliable as the $E_\pi$ gets as large as $\sim 2.5$ GeV.

We expect that $|\kappa_\Theta| \approx |\kappa_F|$. On the other hand, a calculation in soliton picture predicts that $\kappa_\Theta \approx 0.3$, which is rather small. We vary $\kappa_\Theta$ between $-1$ and $1$. We ignore transition magnetic moments between nucleon octet and pentaquark antidecuplet, since this transition involves $qqq$ and $qqqq$. The relevant Feynman diagrams for $\gamma n \to K^- \Theta^+$ are shown in Fig. 3. One salient feature of our approach based on chiral perturbation theory is the existence of a contact term for $\gamma K^- n \Theta^+$ vertex [Fig. 3 (d)] that arises from the $C_{PN}$ term in Eq. (4c) with Eq. (2c), which is necessary to recover $U(1)_{em}$ gauge invariance within spontaneously broken global chiral symmetries. The resulting amplitude is

$$\mathcal{M} = \frac{\epsilon C_{PN}}{\sqrt{2f}} \epsilon_\mu \bar{u}_\Theta F^\mu u_n, \quad (12a)$$
\[ F^\mu = -\Gamma_{\rho k} \frac{\kappa_n}{4m_B} \frac{1}{\hat{p}_n + p_n - m_B} [\gamma^\mu, \hat{p}_\gamma] + \Gamma_{\rho} \gamma^\mu + \left( \gamma^\mu - \frac{\kappa_\Theta}{4m_\Theta} \right) \frac{1}{\hat{p}_n - \hat{p}_K - m_\Theta} \Gamma_{\rho} \hat{p}_K - \frac{(2p_k - p_\tau)^\mu}{(p_K - p_\tau)^2 - m_K^2} \Gamma_{\rho} (\hat{p}_K - \hat{p}_\gamma). \] (12b)

The cross sections and the angular distributions in the center of momentum frame are shown in Fig. 4 (a) and (b). Note that the parity-even case has a sharp rise near the threshold. The angular distribution shows that the forward/backward scattering is suppressed in the negative parity case, whereas the forward peak is present in the positive parity case. Therefore the angular distribution could be another useful tool to determine the parity of \( \Theta^+ \). Once the coupling \( C_{\rho N} \) is determined from the decay width of \( \Theta^+ \), then the parity and other couplings \( H_{\rho N} \) and \( \kappa_\Theta \) could be determined from the hadro/photo-production cross sections for \( \Theta^+ \). It is straightforward to apply our approach to other related processes such as \( \gamma p \to K^0\Theta^+ \) or \( K^+p \to \pi^+\Theta^+ \), or other pentaquark baryons. Finally we have outlined how to incorporate the vector meson degrees of freedom in our scheme, the details of which will be discussed in the separate publication [12].

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