Ultracold Bose atoms in intense laser fields: intensity- and density-dependent effects

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Abstract

Starting from the first principles of nonrelativistic QED we have derived the system of Maxwell-Schrödinger equations, which can be used for theoretical description of atom optical phenomena at high densities of atoms and high intensities of the laser radiation. The role of multiple atomic transitions between ground and excited states in atom optics has been investigated. Nonlinear optical properties of interacting Bose gas are studied: formula for the refractive index has been derived and the polariton spectrum of a condensate interacting with an intense laser field has been investigated.

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1  Introduction

In last decades the problem of interaction of the laser radiation with ultracold atomic gases has attracted a lot of attention. With the aid of the laser radiation one can manipulate the center-of-mass motion of ultracold atoms and one can observe different wave phenomena with ultracold atomic beams. After the experimental realization of Bose-Einstein condensation, which allows to create rather dense atomic systems, the problem of interaction of photons with ultracold atoms has reached a new stage of development. Acting with the laser on a dense atomic sample one can induce nonlinearities in the behavior of the matter field caused by dynamical dipole-dipole interactions. This provides a possibility to create atomic solitons of different kinds \[1\], to change dramatically the effective scattering length of the condensate \[2\], to create nonlinear beam splitters \[3\], vortices \[4\], photonic band gaps and defect states in a condensate \[5\].

In recent years different approaches to the description of interaction of ultracold atoms in the field of optical radiation have been suggested and different aspects of the phenomenon have been considered. The properties of the laser radiation modified by atomic dipole-dipole interactions were investigated \[6, 7, 8\] and it was shown that they can be described by the refractive index which is governed in the linear case, when the light intensity is low enough, by the Clausius-Mossotti relation known from classical optics if we neglect quantum correlations. If the quantum statistical correlations are taken into account, the formula for the refractive index contains additional terms defined by a position dependent correlation function \[6, 7, 8\]. Nonlinear optical properties of noninteracting Bose gas were studied as well \[9\].

The modification of the properties of the laser radiation should have a back influence on the behavior of an ultracold atomic ensemble (for example, the motion of atomic beam). An attempt to consider this back influence was undertaken by several authors \[1, 3, 7, 10, 11, 12\]. In the first works on the subject the two-body interactions were modelled by the phenomenological contact potential \[10\]. Later on dynamical dipole-dipole interactions were taken into account within the framework of the nonrelativistic electrodynamics \[1, 3, 7, 11, 12, 13, 14, 15\]. However, in papers by W.Zhang and D.Walls \[3\] and G.Lenz et al.\[1\] the averaged polarization of ultracold atomic ensemble was computed as a function of the incident laser field, whereas it should be a function of the macroscopic or the local field which are different from the external laser field due to dynamical dipole-dipole interactions. Wallis \[12\] used the correct form of the equations for the electromagnetic field. However, his equation for the matter field was not consistent with the equation for the electromagnetic field \[13\]. This problem was considered also by Y.Castin and K.Mølmer \[11\] and J.Ruostekoski and J.Javanainen \[7\]. But the equations which are used in those papers are very complicated because they are written down in terms of the local field...
and dipole-dipole interactions are presented explicitly in the form of the sum over dipole fields. This makes the analysis very difficult and the equations, which govern the time evolution of the matter fields, remained unsolved.

In the papers [13, 14, 15] a self-consistent quantum theory of atom optical processes has been developed. Making use of the Lorentz-Lorenz relation, which allows to simplify the analysis, it was obtained the general system of Maxwell-Schrödinger equations for atomic creation and annihilation operators and the propagation equation for the laser field which can be used for the description of linear and nonlinear phenomena in atom optics of single- and multi-species condensates at high densities of the atomic system. However, the treatment in [13, 14, 15] was restricted by low light intensities.

In the present paper we shall continue the investigations started in [13, 14]. Having in mind mainly atom optical applications, we shall derive the system of Maxwell-Schrödinger equations for the case of high density of the atoms and high light intensity. Nonlinear optical properties of the interacting Bose gas will be also discussed.

2 Heisenberg equations of motion for the atomic operators

We consider a system of ultracold two-level atoms with masses \( m \), transition frequencies \( \omega_a \), and transition dipole moments \( d \). We shall describe such a system in terms of matter field operators. Let \( |g\rangle \) and \( |e\rangle \) are the vectors of the ground and excited states of the quantized atomic fields. Then the corresponding annihilation operators of the atoms in these internal states are \( \hat{\phi}_g \) and \( \hat{\phi}_e \). Matter-field operators are assumed to satisfy the bosonic equal time commutation relations.

The Heisenberg equations of motion for the atomic operators are derived from the Hamiltonian of the second quantized atomic field interacting with the photons. In the reference frame rotating with the frequency \( \omega_L = ck_L \) of the incident laser field \( E_m(r,t) \), which is assumed to be monochromatic, and in making use of the electric dipole approximation and the rotating-wave approximation we obtain the following dynamical equations for the matter-field operators [13, 14]

\[
\begin{align*}
    i\hbar \frac{\partial \hat{\phi}_g}{\partial t} &= \hat{H}_{cm} \hat{\phi}_g + \hat{H}_{ge} \hat{\phi}_e, \\
    i\hbar \frac{\partial \hat{\phi}_e}{\partial t} &= \hat{H}_{cm} \hat{\phi}_e - \hbar (\Delta + i\gamma/2) \hat{\phi}_e + \hat{H}_{eg} \hat{\phi}_g,
\end{align*}
\]
where $\hat{H}_{\text{cm}} = -\hbar^2 \nabla^2 / (2m)$, $\Delta = \omega_L - \omega_a - \delta$ is the detuning of the frequency of the laser wave from the frequency of the atomic transition, $\delta$ and $\gamma$ are the Lamb shift and the spontaneous emission rate of a single atom in free space, respectively. Here we have introduced the operators $\hat{H}_{eg} = -d \hat{E}_{\text{loc}}^+$ and $\hat{H}_{ge} = -d \hat{E}_{\text{loc}}^-$ which are responsible for the transitions $|g\rangle \rightarrow |e\rangle$ and $|e\rangle \rightarrow |g\rangle$, respectively. $\hat{H}_{eg}$ and $\hat{H}_{ge}$ are related to the operator of the local electric field $\hat{E}_{\text{loc}}^\pm(r, t)$. The positive-frequency part of this operator has the form

$$\hat{E}_{\text{loc}}^+(r, t) = E_{\text{in}}^+(r) + i \sum_{k\lambda} \frac{2\pi \hbar \omega_k}{v} e_{\lambda k\lambda}(0) \exp \left[ i\mathbf{k} \cdot \mathbf{r} - \mathbf{k} \cdot \mathbf{R} - \omega_k t \right] + \int d\mathbf{r}' \nabla \times \nabla \times \hat{\mathbf{P}}^+(\mathbf{r}', t - R/c) e^{ik\mathbf{R}},$$

where $\nabla \times$ refers to the point $\mathbf{r}$ and the operator $\hat{c}_{k\lambda}(0)$ corresponds to the free-space photon field (vacuum fluctuations). The polarization operator $\hat{\mathbf{P}}^+ = d\hat{\phi}_g^\dagger \hat{\phi}_e$. Note that in equation (3) a small volume around the observation point $\mathbf{r}$ is excluded from the integration. The last term in equation (3) describes dipole-dipole interaction. Due to the structure of the local field given by equation (3) one can distinguish two kinds of the photons: primary laser photons and secondary photons, re-radiated by the atoms.

Usually in atom optics of BEC one deals with equations for the ground state matter-field operator $\hat{\phi}_g$. Therefore, one has to eliminate the excited state matter-field operator $\hat{\phi}_e$ from the system of equations (1), (2). In the next section we shall develop a general procedure of the elimination of the excited state which is in contrast to previous works (see, for instance, [3]) valid for high densities of the atoms and high intensities of the laser radiation.

### 3 Elimination of the excited state

Let’s assume that initially there are no atoms in the excited state. Then we can rewrite equation (2) in the form

$$\hat{\phi}_e(t) = -i \int_0^t e^{i \left( \frac{\hat{H}_{\text{cm}}}{\hbar} - \hat{\Delta} \right) t'} \hat{H}_{eg}(t') e^{-i \frac{\hat{H}_{\text{cm}}}{\hbar} t'} \hat{\phi}_g(t') dt',$$

where $\hat{\Delta} = \Delta + i\gamma/2$. Making use of the identity

$$\int e^{ax} F(x) dx = \frac{e^{ax}}{a} \sum_{k=0}^{\infty} \frac{(-1)^k}{a^k} \frac{\partial^k}{\partial x^k} F(x),$$

which is valid for a non-vanishing $a$, we get

$$\hat{\phi}_e(\mathbf{r}, t) = \frac{e^{-i \frac{\hat{H}_{\text{cm}}}{\hbar} t}}{h \hat{\Delta}} \sum_{k=0}^{\infty} \frac{(-i)^k \partial^k}{\Delta^k} \left[ e^{i \frac{\hat{H}_{\text{cm}}}{\hbar} \hat{H}_{eg}(\mathbf{r}, t)} \hat{\phi}_g(\mathbf{r}, t) \right].$$
If $\hat{H}_{eg}$ does not depend on time and the center-of-mass motion does not give a large contribution, we have

$$\hat{\phi}_e(r, t) = \frac{\hat{H}_{eg}(r)}{\hbar \Delta} \sum_{k=0}^{\infty} \frac{(-i)^k}{\Delta^k} \frac{\partial^k}{\partial t^k} \hat{\phi}_g(r, t).$$  \hspace{1cm} (7)

From the system of equations (1), (2), with the center-of-mass motion neglected one can derive the relation

$$\frac{\partial^n}{\partial t^n} \hat{\phi}_g(r, t) = -i^n \frac{\hat{H}_{ge}(r) \hat{H}_{eg}(r)}{\hbar^2 \Delta^{2-n}} \sum_{k=n-1}^{\infty} \frac{(-i)^k}{\Delta^k} \frac{\partial^k}{\partial t^k} \hat{\phi}_g(r, t), \quad n = 1, 2, \ldots$$  \hspace{1cm} (8)

Substituting this relation iteratively into equation (7), we obtain

$$\hat{\phi}_e(r, t) = \frac{\hat{H}_{eg}(r)}{\hbar \Delta} \sum_{m=0}^{\infty} (-1)^m a_m \left[ \frac{\hat{H}_{ge}(r) \hat{H}_{eg}(r)}{\hbar^2 \Delta^2} \right]^m \hat{\phi}_g(r, t).$$  \hspace{1cm} (9)

$$a_0 = 1, \quad a_{m+1} = \sum_{k_1=0}^{m} \sum_{k_2=0}^{m} \sum_{k_3=0}^{m} \cdots \sum_{k_m=0}^{m} = 2 \left( \frac{2m+1}{m!} \right)^2, \quad m = 0, 1, \ldots$$  \hspace{1cm} (10)

The zeroth order term ($m = 0$) in equation (9) corresponds to the transition $|g\rangle \rightarrow |e\rangle$. The next term ($m = 1$) corresponds to the transition $|g\rangle \rightarrow |e\rangle \rightarrow |g\rangle \rightarrow |e\rangle$ and so on. Therefore, different terms in equation (9) describe multiple transitions of the atoms between ground and excited states, caused by the influence of the photons. In a typical atom optical situation when the electromagnetic field has a form of a standing wave these terms correspond to the processes with the momentum transfer from the laser beam to the atoms $\pm 2\hbar K(m + 1)$, where $K$ is a wave vector of the laser wave in a medium.

We assume that $\hat{\epsilon} = \hat{H}_{ge} \hat{H}_{eg} / (\hbar^2 \Delta^2)$ acts only on states $|\psi\rangle$ for which the series in equation (9) converges, i.e., $|\psi\rangle$ can be decomposed into eigenstates of $\hat{\epsilon}$ whose eigenvalues satisfy the condition $|\epsilon_n| < 1/4$. Then the result of the summation is given by

$$\hat{\phi}_e(t) = \frac{\hat{H}_{eg} \sqrt{1 + 4\epsilon} - 1}{2\epsilon} \hat{\phi}_g(t),$$  \hspace{1cm} (11)

and the equation for the ground state (11) takes the form

$$i\hbar \frac{\hat{\phi}_g}{\partial t} = \hat{H}_{cm} \hat{\phi}_g + \frac{\hbar \Delta}{2} \left( \sqrt{1 + 4\epsilon} - 1 \right) \hat{\phi}_g(t).$$  \hspace{1cm} (12)

Although we do not see explicitly on the r.h.s. of equations (11), (12) $\hat{\phi}_e$, it is still there, because $\hat{E}_{loc}$ depends on $\hat{\phi}_e$ and this dependence, which is given by (9), is very complicated. Formally we could iteratively substitute equations (9) and (11) into (12). However, by doing
such a procedure we would get an equation for the ground state wave function in a form which would be impossible to use. Therefore, the algorithm described in this section does really eliminate the excited state only in the case when the dipole-dipole interactions are negligible, i.e., when $\hat{E}_{\text{loc}}^\pm \approx E_{\text{in}}^\pm$. In order to eliminate the excited state in the case when the dipole-dipole interactions play an important role it is useful to employ in addition the Lorentz-Lorenz relation, which will be briefly discussed in the next section.

Because we are mainly interested in atom optical problems and want to study the coherent evolution of the center-of-mass motion of the gas, we shall neglect spontaneous emission. This is valid for situations where the absolute values of the detunings are much bigger than the spontaneous emission rates and Rabi frequencies $|\Delta| \gg \gamma$, $|\text{d}E_{\text{loc}}^\pm/\hbar|$. In order to do this approximation self-consistently we drop in the following the vacuum fluctuations and the spontaneous emission rates $\gamma$ from our equations. Then we may replace all the operators by macroscopic functions.

4 Local-field correction

As it was mentioned in the previous section, the solution of equations (3), (11), (12) is a rather complicated mathematical problem because these equations contain explicitly the dipole-dipole interactions. In many particular situations such a detailed microscopic description of matter is not necessary and it is more convenient to consider optical properties of the medium on a macroscopic level. This can be done by introducing the macroscopic field $E_{\text{mac}}(r, t)$, instead of the local field $E_{\text{loc}}(r, t)$ in the equations for the matter fields.

As in Ref. [16] we can introduce the macroscopic field by imposing the requirement that it is a solution of the macroscopic Maxwell equations for a charge-free and current-free polarization medium. Using Maxwell equations and the definition of the local field (3), we get the following relation

$$E_{\text{loc}}^\pm(r, t) = E_{\text{mac}}^\pm(r, t) + \frac{4\pi}{3}P^\pm(r, t).$$

This equation is often called in the literature the Lorentz-Lorenz relation. It constitutes the basis of the local-field effects in classical, quantum and nonlinear optics.

5 Nonlinear matter equation

Let’s consider first a special case of the low light intensity. In this case we can keep only the first term ($m = 0$) in equation (3), which is linear with respect to the electromagnetic field
strength:
\[ \phi_e(r, t) = -\frac{dE^+(r)}{\hbar \Delta} \phi_g(r, t). \]  
(14)

From equations (13), (14) we obtain
\[ \phi_e(r, t) = -\frac{\Omega(r) \phi_g(r, t)}{2\Delta} \left[ 1 - \frac{4\pi}{3} \alpha |\phi_g(r, t)|^2 \right]^2 \phi_g(r, t), \]  
(15)

where the position dependent Rabi frequency \( \Omega(r) = 2dE^+_{mac}(r)/\hbar \) is related to the macroscopic electric field, \( \alpha = -d^2/\hbar \Delta \) is the atomic polarizability. The series in (15) converges, provided that \( \frac{4\pi}{3} \alpha |\phi_g|^2 < 1 \). In this case we get
\[ \phi_e(r, t) = -\frac{\Omega(r) \phi_g(r, t)}{2\Delta} \left[ 1 - \frac{4\pi}{3} \alpha |\phi_g(r, t)|^2 \right]. \]  
(16)

This adiabatic solution has been obtained in Ref. [13] using slightly different technique. Note that a singularity occurs in equation (16) under the condition \( \frac{4\pi}{3} \alpha |\phi_g|^2 = 1 \). However, as it follows from our present derivation, we never encounter this singularity, because in the region \( \frac{4\pi}{3} \alpha |\phi_g|^2 \geq 1 \) equation (16) is not valid. This was not clear from the derivation given in Ref. [13].

Then substituting (16) in (1), we obtain as the result an equation for the ground-state matter field \( \phi_g \) [13, 14]
\[ i\hbar \frac{\partial \phi_g(r, t)}{\partial t} = \hat{H}_{cm} \phi_g + \frac{\hbar |\Omega(r)|^2}{4\Delta} \left[ 1 + \frac{8\pi}{3} \alpha |\phi_g|^2 + 3 \left( \frac{4\pi}{3} \alpha |\phi_g|^2 \right)^2 \right] |\phi_g|^2 \phi_g. \]  
(17)

Now we substitute the Lorentz-Lorenz relation (13) into equations (11), (12) and keep the terms up to the order \( 1/\Delta^3 \). As a result we get the following equation for the ground-state wave function
\[ i\hbar \frac{\partial \phi_g}{\partial t} = \hat{H}_{cm} \phi_g + \frac{\hbar |\Omega(r)|^2}{4\Delta} \left[ 1 + \frac{32}{3} \sqrt{a^3/3} |\phi_g|^2 \right] |\phi_g|^2 \phi_g. \]  
(18)

Varying the density of atoms \( |\phi_g|^2 \), the light intensity, which is proportional to \( |\Omega|^2 \), and the magnitude and the sign of the detuning \( \Delta \), one can change the effective potential for the ground-state matter field.

It is interesting to compare the nonlinear terms in equation (18) with the terms in the equation for the condensate wave function, which takes into account the effects of quantum fluctuations [17]
\[ i\hbar \frac{\partial \phi_g}{\partial t} = \hat{H}_{cm} \phi_g + \frac{4\pi \hbar^2 a}{m} \left[ 1 + \frac{32}{3} \sqrt{a^3/3} |\phi_g|^2 \right] |\phi_g|^2 \phi_g, \]  
(19)
where \( a \) is a scattering length. The leading nonlinear terms in equations (18), (19) are defined by the quantities

\[
U_E = -\frac{\Omega^+ |2\pi| d^2}{\Delta^2}, \quad U_C = \frac{4\pi \hbar^2 a}{m},
\]

respectively. The typical orders of magnitude of the parameters for alkali metal atoms are (in CGS system of units) \( d \sim 10^{-18} \) esu, \( |a| \sim 10^{-7} \) cm, \( m \sim 10^{-23} \) g, and we get the estimates

\[
|U_E| \sim \frac{|\Omega^+|^2}{\Delta^2} 10^{-36} \text{ erg cm}^{-3}, \quad |U_C| \sim 10^{-37} \text{ erg cm}^{-3}.
\]

Therefore, we see that at \( |\Omega^+|^2 / \Delta^2 \sim 0.1 \) \( U_E \) and \( U_C \) are of the same order of magnitude.

Higher order nonlinear terms in equations (18) and (19) are defined by

\[
V_E = \frac{\hbar |\Omega^+|^2}{4\Delta^3} 3 \left( \frac{4\pi}{3 \hbar \Delta} \right)^2 |\phi_g|^2, \quad V_C = \frac{4\pi \hbar^2 a}{m} 32 \sqrt{\frac{a^3}{\pi}} |\phi_g|,
\]

respectively. For the densities \( |\phi_g|^2 \sim 10^{14} \) cm\(^{-3}\) and detunings \( |\Delta| \sim 10^8 \) Hz, we have

\[
|V_E| \sim \frac{|\Omega^+|^2}{\Delta^2} 10^{-38} \text{ erg cm}^{-3}, \quad |V_C| \sim 10^{-39} \text{ erg cm}^{-3}.
\]

Thus, we see that the nonlinear terms in equations (18) and (19) have the same order of magnitude.

### 6 Optical properties of the ultracold gas

#### 6.1 Refractive index

We substitute equation (11) into the definition of the polarization field. This gives us a general nonlinear relation between the polarization and the local field:

\[
P^\pm = \alpha \sqrt{1 + \frac{4\epsilon}{2\epsilon} - 1} |\phi_g|^2 E^\pm_{\text{loc}}.
\]

In the special case of the low light intensity we can use the adiabatic solution (16). Then the expression for the polarization takes the form

\[
P^\pm = \alpha |\phi_g|^2 E^\pm_{\text{loc}} = \chi E^\pm_{\text{mac}},
\]

where dielectric susceptibility is given by

\[
\chi = \frac{\alpha |\phi_g|^2}{1 - \frac{4\epsilon}{3\alpha} |\phi_g|^2}.
\]
This expression for the dielectric susceptibility $\chi$ leads to the Clausius-Mossotti formula for the refractive index. As it follows from our physical interpretation of the expansion (9), the Clausius-Mossotti formula corresponds to the quantum transition of the type $|g\rangle \rightarrow |e\rangle$. Therefore, it takes into account pair interactions between the atoms when one atom emits a photon, then this photon is absorbed by another atom and so on.

If we keep the terms up to the order $1/\Delta^3$, the expression for the polarization takes the form

$$P^\pm = \alpha |\phi_g|^2 \left[ 1 - \frac{|dE_{loc}^\pm|^2}{\hbar^2 \Delta^2} \right] E_{loc}^\pm = \chi E_{mac}^\pm,$$

where

$$\chi = \alpha |\phi_g|^2 \left[ 1 + \frac{4\pi}{3} \alpha |\phi_g|^2 + \left( \frac{4\pi}{3} \alpha |\phi_g|^2 \right)^2 - \frac{|dE_{mac}^+|^2}{\hbar^2 \Delta^2} \right].$$

Dielectric susceptibility is a rather important parameter, because it describes the propagation of the laser radiation inside a medium. In most of the practical situations the electromagnetic processes are much faster than the center-of-mass motion of the atoms. Therefore, $\chi$ can be considered as a time-independent quantity. Let us assume in addition that the spatial variations of the atomic density are not very large, such that $\nabla \chi \rightarrow 0$. Then $\text{div} E_{mac}^\pm \approx 0$, and we have the following Helmholtz equation for the macroscopic electric field

$$\nabla^2 E_{mac}^\pm + k_L n^2 E_{mac}^\pm = 0,$$

with the refractive index $n$ given by

$$n = \sqrt{1 + 4\pi \chi} = 1 + 2\pi \alpha |\phi_g|^2 \left[ 1 + \frac{\pi}{3} \alpha |\phi_g|^2 + \frac{5}{8} \left( \frac{4\pi}{3} \alpha |\phi_g|^2 \right)^2 - \frac{|dE_{mac}^+|^2}{\hbar^2 \Delta^2} \right],$$

which corresponds to the Kerr-type optical nonlinearity.

Equations (18), (25), (26) can be considered as an atom optical analogue of the system of Maxwell-Schrödinger equations used in quantum and nonlinear optics. In general they have to be solved in a self-consistent way.

### 6.2 Polariton band gap

In this section we investigate the polariton spectrum of a condensate interacting with an intense laser field. Let’s consider a special case of dilute atomic gas. In this case one can neglect the local-field correction in equation (13) and we have $E_{loc}^\pm(r, t) \approx E_{mac}^\pm(r, t)$. In order to simplify the analytical analysis we assume that the intensity of the laser radiation is not too high. Then
from Maxwell equations and equation (24) we get the following expression for the refractive index

\[ n^2(\omega) = 1 - \frac{R(\omega - \omega_0)}{(\omega - \omega_0)^2 + |\Omega^+|^2/4}, \quad R = \frac{4\pi d^2 |\phi_g|^2}{\hbar}, \] (27)

which contains saturation effects, owing to the intense electric field.

The polariton spectrum \( \omega(K) \) can be obtained from the equation

\[ K = \frac{\omega}{c} n(\omega). \] (28)

As it follows from equation (27) \( n(\omega) \) is imaginary for the frequencies \( \omega \) within the range

\[ \omega_0 + \frac{R - \sqrt{R^2 - |\Omega^+|^2}}{2} < \omega < \omega_0 + \frac{R + \sqrt{R^2 - |\Omega^+|^2}}{2}, \quad R > |\Omega^+|. \] (29)

In this case equation (28) has no real solutions. This means that light can not propagate in a condensate, i.e., we get a polariton band gap with the width \( \Delta_G = \sqrt{R^2 - |\Omega^+|^2} \). The gap width \( \Delta_G \) increases with the density of the atoms in the ground state \( |\phi_g|^2 \) and decreases with the laser intensity. In the case of the low light intensity \( (|\Omega^+| \ll R) \) we get a well-known result \( \Delta_G = R \) [18, 19]. If \( R \leq |\Omega^+| \), equation (28) has only real solutions. This means that in this case the polariton gap disappears and the medium is always transparent.

Note that our analysis does not take into account effects caused by the influence of the spontaneous emission. As it was discussed in Ref. [19], the spontaneous emission smears out the polariton spectrum, providing accessible polariton states inside the gap.

### 7 Conclusion

Starting from the microscopic model and making use of the multipolar formulation of QED, we have derived the general system of Maxwell-Schrödinger equations for atomic creation and annihilation operators and the propagation equation for the laser field. It describes the modification of the properties of the external off-resonant laser radiation in a medium due to dipole-dipole interactions and the influence of this modification on the center-of-mass motion of the ultracold atoms as a single dynamical process. The system can be used, for instance, for the self-consistent analysis of linear and nonlinear phenomena in atom optics at high densities of the atomic system and high intensities of the laser radiation.

A general procedure of the elimination of the excited state has been developed. The annihilation and creation operators of the excited state for large detuning are represented in the form of a series expansion in powers of the inverse detuning, which corresponds to multiple transitions between the ground and excited electronic states of the atoms.
Optical properties of an interacting ultracold Bose gas are studied: formula for the intensity-dependent refractive index is derived and the polariton spectrum of a condensate interacting with an intense laser field is investigated.

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