A GALAXY MODEL FROM TWO MICRON ALL SKY SURVEY
STAR COUNTS IN THE WHOLE SKY, INCLUDING THE PLANE

P. Polido, F. Jablonski, and J. R. D. Lépine

1 Divisão de Astrofísica, Instituto Nacional de Pesquisas Espaciais, Avenida dos Astronautas 1758, 12227-010 São José dos Campos SP, Brazil; pripolido@gmail.com
2 Instituto de Astronomia, Geofísica e Ciências Atmosféricas, Universidade de São Paulo, Rua do Matão 1226, 05508-900 São Paulo SP, Brazil

Received 2012 December 20; accepted 2013 September 10; published 2013 October 31

ABSTRACT

We use the star count model of Ortiz & Lépine to perform an unprecedented exploration of the most important Galactic parameters comparing the predicted counts with the Two Micron All Sky Survey observed star counts in the J, H, and K_s bands for a grid of positions covering the whole sky. The comparison is made using a grid of lines of sight given by the HEALPix pixelization scheme. The resulting best-fit values for the parameters are: 2120 ± 200 pc for the radial scale length and 205 ± 40 pc for the scale height of the thin disk, with a central hole of 2070-2000 pc for the same disk, 3050 ± 500 pc for the radial scale length and 640 ± 70 pc for the scale height of the thick disk, 400 ± 100 pc for the central dimension of the spheroid, 0.0082 ± 0.0030 for the spheroid to disk density ratio, and 0.57 ± 0.05 for the oblate spheroid parameter.

Key words: Galaxy: fundamental parameters – Galaxy: structure – infrared: stars – stars: statistics

1. INTRODUCTION

The star count method is an important tool for investigating Galactic structure and is based on the equation of stellar statistics (Binney & Merrifield 1998):

\[
N_s(m_1, m_2, l, b) d\Omega = \int_{m_1}^{m_2} dm \int_0^\infty r^2 dr \rho_s(r, M) \phi_s(M) d\Omega,
\]

(1)

which allows us to predict the number of objects for a certain line-of-sight defined by the Galactic longitude \( l \) and the Galactic latitude \( b \). \( N_s \) is the number of stars of type \( s \) with apparent magnitude between \( m_1 \) and \( m_2 \) in a solid angle \( d\Omega \) in the direction defined by \( (l, b) \); \( r \) is the heliocentric distance, \( \rho_s \) is the stellar density, and \( \phi_s \) is the luminosity function. We adopted the luminosity function as the number of stars per cubic parsec near the Sun with magnitudes in the range \((M, M + dM)\). It is convenient to handle the stellar densities normalized with respect to the values in the solar neighborhood.

The star count models can be classified into roughly two categories: those adopting an empirical luminosity function—based on observations of the stellar populations in the solar neighborhood and specific environments, like globular clusters—and those adopting a luminosity function derived from theoretical stellar evolutionary tracks and the associated distributions of stellar masses, ages, and metallicities. The pioneering investigation of Bahcall & Soneira (1980) fits into the first group, together with models like those by Wainscoat et al. (1992), Ortiz & Lépine (1993, hereafter OL93), Juric et al. (2008), and Chang et al. (2011). On the other hand, the works of Robin & Crézé (1986) and Girardi et al. (2005) belong to the group of models that adopt a stellar evolution approach to define the luminosity function.

The number of components used to represent the Galaxy varies from model to model, according to the focus of the work. Basically, two components are always present: a disk and a spheroid. After Gilmore & Reid (1983), many authors adopted the disk component consisting of a thin disk and a thick disk. Concerning the spheroid, despite this terminology being used in all models, it actually may indicate different Galactic structures, like the bulge, the halo, or even both (e.g., OL93). The densities of stars in the disks decrease exponentially with galactocentric radius and with vertical distance from the plane, whereas the density in the spheroid follows a decay similar to the de Vaucouleurs' law.

Despite strong evidence that our Galaxy has a spiral structure, the number of arms, as well as the parameters that describe their shapes and positions, vary from work to work (e.g., Russell 2003; Levine et al. 2006; Vallée 2008; Hou et al. 2009). The star count models, which attempt to take the spiral arms into account, are those due to OL93 and Wainscoat et al. (1992).

The existence of a bar in the central region of our Galaxy was proposed in the 1970s based on the large non-circular motions seen in the observations of H I and CO in the inner Galaxy (Peters 1975; Cohen & Few 1976; Liszt & Burton 1980), but only in the 1990s did the combination of evidence from near infrared (NIR) light distributions (Blitz & Spergel 1991; Weiland et al. 1994), star count asymmetries (Nakada et al. 1991; Stanek et al. 1997; Benjamin et al. 2005), gas kinematics (Binney et al. 1991; Englmaier & Gerhard 1999; Fux 1999) and large microlensing optical depth (Udalski et al. 1994; Zhao et al. 1995; Han & Gould 1995) become more persuasive.

Bahcall & Soneira (1980) examined the star counts in the BV bands and assumed that the Galaxy could be well represented by two components: a disk and a spheroid. Robin & Crézé (1986) examined the star counts in the UBV bands and used three components to model the Galaxy: a disk, a halo, and an exponential spheroid population with an intermediate age. Wainscoat et al. (1992) adopted five components (disk, halo, bulge, spiral arms, molecular ring) to model the Galaxy star counts as a foreground for their extragalactic source counts based on the Infrared Astronomical Satellite Survey (IRAS).^3^ OL93 attempted to model the star counts in RIJKHL, [12 μm],
### Table 1

| Flag                             | Catalog Code | Meaning                                                                 |
|----------------------------------|--------------|-------------------------------------------------------------------------|
| Photometric quality              | X, U\(^a\)   | Objects not detected in one of the bands or from which it was not possible to estimate the brightness |
| Read                             | 0, 9\(^b\)   | Objects not detected in one of the bands or, although detected, it was not possible to make useful estimates of brightness |
| Contamination and confusion      | p, c, d, s, b\(^c\) | Objects affected by the proximity of a bright source or image artifacts |
| Galactic contamination           | 1, 2\(^d\)   | Objects contaminated by the proximity of extended sources |

**Notes.**

\(^a\) Possible codes for this flag are X, U, F, E, A, B, C, and D.

\(^b\) Possible codes for this flag are 0, 1, 2, 3, 4, 6, and 9.

\(^c\) Possible codes for this flag are p, c, d, s, b, and 0.

\(^d\) Possible codes for this flag are 0, 1, and 2.

---

**Figure 1.** Updated luminosity functions for main sequence and giant stars in the J-band. The circles and squares indicate, respectively, the main sequence and giant luminosity functions according to OL93 (black), Wainscoat et al. (1992, red), Bochanski et al. (2010, blue), Reid & Gizis (1997, green), and Murray et al. (1997, cyan). The lines indicate the smooth fits for the main sequence (black) and giants (red).

and [25 μm] IRAS bands, using four components: a spheroid, two disks with scale heights 100 pc and 390 pc, and spiral arms. A version of the model with an intermediate disk and a bar is described on their Web site [4](http://www.astro.iag.usp.br/~jacques/pingas.html). The approach of Girardi et al. (2005) uses a stellar populations synthesis code as the input for the luminosity function to compare their four components model (thick disk, thin disk, halo, bulge) with optical and infrared data, including the Two Micron All Sky Survey (2MASS),[5](http://www.ipac.caltech.edu/2mass/) the Chandra Deep Field South, and Hipparcos.[6](http://www.rssd.esa.int/Hipparcos/) Jurić et al. (2008) compared the stars counts from the Sloan Digital Sky Survey[7](http://www.sdss.org/) with the predicted counts from their Galactic model with a disk and a halo. Chang et al. (2011) used a model with a spheroid and a disk and a power-law luminosity function for which the power index is a free parameter at each grid point on the sky for Galactic latitudes \(|b| > 30°\). The comparison was made for 8192 lines of sight in the 2MASS K\(_S\) band.

In this study, we use the homogeneous all-sky coverage of 2MASS to test the OL93 model in the JHK\(_S\) bands. We explore the parameters space of this model adopting an updated luminosity function to describe the stellar populations in the solar neighborhood.

In Sections 2 and 3, we describe the OL93 model and the method used to search for the best-fit parameters. The results and discussion are presented in Section 4. Our conclusions follow in the last section.

### 2. The Galactic Model

The OL93 model uses a young disk, an old disk, an intermediate disk composed of carbon stars, a spheroidal, spiral arms, and a bar, with the total number density of sources being the sum of all individual contributions. A detailed description of all components follows.

#### 2.1. Disks

We consider a thin (young) disk and a thick (old) disk. The density of each subcomponent \(i = Y\) (young), O (old), is given by a modified exponential (Lépine & Leroy 2000), which is an important change with respect to the original model:

\[
n_{d,i}(r, z, s) = n_{d,i}(R_0, 0, s) e^{-\frac{r}{\alpha_i}+\beta_i\left(\frac{z}{h_i}\right)^2}.
\]

Here, \(r\) is the distance to the Galactic center in the plane of the Galaxy, \(n_{d,i}(R_0, 0, s)\) is the star density of spectral type \(s\) in the solar neighborhood, \(\alpha_i\) is the radial scale length, which does not depend on the spectral type, \(h_i\) is the scale height, and \(\beta_i\) is the radius of a hole in the central part of the disk. The disk has a distribution of spectral types of giants similar to that found in globular clusters. All the stars of spectral types O and B, as well as the supergiants of all spectral types, were excluded from the composition of the thick disk. The scale height \(h_i\) is given by

\[
h_i(r) = z_\odot e^{-\frac{r}{R_0} + \frac{\alpha_i}{R_0}}.
\]
Figure 2. Illustration of the $N_{\text{side}} = 4$ (192 points) basic grid in green and a finer grid with 382 points drawn from the $N_{\text{side}} = 16$ HEALPix scheme (red).

Figure 3. Joint a posteriori probability densities for the parameters of our model after $10^5$ iterations of an MCMC, considering the $N_{\text{side}} = 4$ grid. The marginalized one-dimensional histograms for the parameters are displayed on the diagonal. We also included the correlation coefficients between each pair of parameters. The red cross indicates the mode of the two-dimensional distribution and the yellow contour shows the 95% confidence region.
where $z_i$ is the scale height in the solar neighborhood. The free parameters of the model related to the disks are $\alpha_Y, \alpha_O, \beta_Y, \beta_O, z_Y$ and $z_O$.

Recent studies (e.g., Sofue et al. 2009; Amore et al. 2009) suggest the existence of a minimum in the density of young objects in the solar neighborhood due to the effects of corotation of the spiral pattern with Galactic rotation. We experimented with this possibility by modeling the young disk density with

$$n_{d,Y}(r) = n_{d,Y}(R_0)[1 - 0.3e^{-(r-R_0)^2}], \quad (4)$$

where $r$ and $R_0$ are expressed in kpc. The results of this attempt are shown in Section 4.2.1.

The OLO3 model does not use warp components in the disks. Evidence for the presence of this contribution in 2MASS data are presented by López-Corredoira et al. (2002) and Reylé et al. (2009). Based on uncertainties related to the different space distributions of dust and stars (Freudenreich et al. 1994; Drimmel & Spergel 2001; Reylé et al. 2009) and differences in the distribution of giant and main sequence stars (e.g., López-Corredoira et al. 2002), we did not attempt to include this component in our modeling.

**2.2. Spheroid**

The spheroidal structure is subtle and when we observe it, especially close to the Galactic center, we see the superimposition of contributions from the young and old disks (which may or may not have central holes) and a Galactic bar. Furthermore, the innermost regions of the spheroid are subject to heavy reddening. A number of improvements to the description of this region as a whole have been suggested in the last decade. López-Corredoira et al. (2005) found that the best fit to 2MASS data is given by a structure that consists of a boxy...
bar associated with a tri-axial bulge. Vanhollebeke et al. (2009) also use a tri-axial figure that is truncated by a Gaussian decay and Robin et al. (2012) describe the 2MASS data in the central region of the Galaxy (|l| < 20°, |b| < 10°) with a tri-axial boxy bar/bulge plus a longer and thicker ellipsoid. Nataf et al. (2013), McWilliam & Zoccali (2010), and Saito et al. (2011) investigated the X-shaped component that is part of the bar/bulge structure.

The OL93 model uses an oblate spheroid in an attempt to describe both the inner region and the larger outer scales. The density in the spheroid is adapted from the mass density of Hernquist (1990) and is given by

\[ n_{sph}(R, s) = \frac{C_1}{\zeta (\zeta + a_H)^3}, \]

where \( R \) is the distance to the Galactic center, \( \zeta = \sqrt{(z/\kappa)^2 + R^2} \), where \( \kappa \) is the oblateness of the spheroid and \( z \) is the distance above the Galactic plane. \( a_H \) is a scale length, and \( C_1 = R_0(R_0 + a_H)^2(N_{sph}/N_D) \). The last term is the ratio of densities of spheroid and disk populations in the solar neighborhood. \( R_0 \) is the distance of the sun to the Galactic center, taken here as 8.0 kpc—very close to both the arithmetic and weighted means of the \( R_0 \) determinations obtained since 1992 (Malkin 2013; Morris et al. 2012; Reid 2012, 2013; Zhu & Shen 2013). OL93 used \( R_0 = 7.9 \) kpc, which was intermediate between the International Astronomical Union (IAU) recommended value of 8.5 kpc and the “short scale” value 7.5 kpc often used at the time (see, for instance, a discussion in Section 2 of Lépine et al. 2011). The stellar population of the spheroid is considered to be the same as that of the thick disk, as discussed below. \( a_H, \kappa, \) and \( N_{sph}/N_D \) are free parameters in the OL93 model.

2.3. Spiral Arms

One of the first representations of the spiral structure in the Milky Way, by Georgelin & Georgelin (1976), proposed a four armed pattern. Since then, many authors have studied this Galactic component and have obtained discrepant results for the number of arms and their locations. Recent results include Majaess et al. (2009), based on the space distribution of type II
Cepheids, which suggest deviations from what logarithmic spirals predict for the Sagittarius–Carina arm and the Local arm. Lépine et al. (2011) traced the carbon monosulfide (CS) molecular emission related to IRAS sources and concluded that they are not fit by logarithmic arms, but rather by a sequence of straight line segments. From 2MASS data and gas distributions, Francis & Anderson (2012) concluded that the Milky Way is a two-armed, grand-design spiral. Robitaille et al. (2012) found the existence of two dominant and two secondary arms in GLIMPSE, MIPSGAL, and IRAS data. In the outer parts of the Galaxy, the spiral arms are not very prominent. Quillen (2002) even suggested, based on 2MASS data, that the Galaxy is flocculent in that region. Comprehensive reviews of earlier works can be found in Vallée (1995, 2002, 2005).

The spiral pattern is the same as in the OL93 model, with four logarithmic arms, each described by

$$r_{\text{arm}} = q e^{(\theta - \theta_0)} \tan(i),$$

where $q$ is the Galactic radius where each arm begins, $\theta_0$ is the initial galactocentric angle, and $i$ is the pitch angle. The arms are confined to the range of Galactic radii $2 \text{kpc} < r < 15 \text{kpc}$ and consist of O5-B stars of all luminosity classes and supergiants of all spectral types. We adopted an improved version of the OL93 arm configuration, consistent with recent observations. Values for $q$, $i$, and $\theta_0$ were derived from the fit of Equation (6) to the representation seen in Churchwell et al. (2009), which is based on the model of Georgelin & Georgelin (1976) with some modifications: variations in positions according to the results of parallax distances to masers in regions of stellar formation (Xu et al. 2006), refinement to the tangential directions to the arms from CO surveys (Dame et al. 2001), revisions in the amplitudes of the arms due to GLIMPSE results and the work of Drimmel & Spergel (2001), and finally, the locations of the outer and distant arms using kinematical data (McClure-Griffiths et al. 2004).

The only parameter related to the spiral pattern that we attempted to derive from the 2MASS observations is the density contrast between the arms and the thin disk, $C_S$, since the arms
**Table 2**

| Parameter         | Symbol | Lower | Upper |
|-------------------|--------|-------|-------|
| Bar half length   | $d_b$  | 3.0   | 4.0   |
| Orientation angle of the bar (deg) | $\theta_{bar}$ | 30   | 30   |

**Table 3**

| Parameter                        | Symbol | Lower | Upper |
|----------------------------------|--------|-------|-------|
| Radial scale length of thin/thick disks (pc) | $d_y$ | 0.5   | 0.9   |
| Density contrast of the bar       | $C_{bar}$ | 0.0   | 0.6   |
| Orientation angle of the bar (deg) | $\theta_{bar}$ | 11   | 53   |

behave as enhancements of the disk density. The stellar density perpendicular to the arm’s length is described by a Gaussian function and its half-width at half-maximum (HWHM) is taken as $\sim 180$ pc. The tangential directions to the arms, which are also the directions where we expect to see the largest contribution of the arms to the star counts, are located at longitudes $32^\circ$, $49^\circ$, $284^\circ$, and $308^\circ$.

### 2.4 Bar

Prior information on the structural properties of the bar is somewhat sparse. It is generally accepted that our Galaxy has a bar, but its length, shape, and orientation cover wide ranges in the literature. The half length of the bar, for example, ranges from $0.67$ kpc in Cao et al. (2013) to $3.9$ kpc in López-Corredoira et al. (2007).

**Table 4**

| Parameter              | MCMC | NS | MCMC+NS |
|------------------------|------|----|---------|
| $\alpha$ (pc)          | (1230$^{+190}_{-170}$) | (1200$^{+190}_{-170}$) | (1190$^{+170}_{-160}$) |
| $\alpha_O$ (pc)        | (4750$^{+920}_{-840}$) | (5150$^{+590}_{-560}$) | (4420$^{+850}_{-610}$) |
| $\beta_y$ (pc)         | (920$^{+2570}_{-570}$) | (2140$^{+2570}_{-520}$) | (2770$^{+1700}_{-1110}$) |
| $\beta_O$ (pc)         | (1740$^{+2570}_{-1450}$) | (100$^{+2570}_{-100}$) | (4760$^{+1240}_{-370}$) |
| $a_H$ (pc)             | (1940$^{+770}_{-570}$) | (1350$^{+570}_{-460}$) | (1000$^{+550}_{-410}$) |
| $a_H/N_D$              | (0.007$^{+0.004}_{-0.006}$) | (0.0058$^{+0.0064}_{-0.0038}$) | (0.0058$^{+0.0072}_{-0.0033}$) |
| $\kappa$               | (0.6$^{+0.10}_{-0.05}$) | (0.76$^{+0.10}_{-0.05}$) | (0.74$^{+0.09}_{-0.04}$) |
| $C_s$                  | $\alpha_Y^{-1}$ | $\alpha_Y^{-1}$ | $\alpha_Y^{-1}$ |
| $C_{bar}$              | (3.0$^{+0.6}_{-0.5}$) | (0.4$^{+0.5}_{-0.4}$) | (3.5$^{+0.3}_{-0.3}$) |
| $z_Y$ (pc)             | (170$^{+1560}_{-40}$) | (170$^{+40}_{-30}$) | (170$^{+40}_{-40}$) |
| $z_O$ (pc)             | (680$^{+1200}_{-20}$) | (710$^{+1200}_{-20}$) | (730$^{+130}_{-90}$) |
| $b_{bar}$ (pc)         | 2000 | 2000 | 2000 |

**Note.** * These parameters were kept fixed at the values indicated.

**Table 5**

| Parameter | $\sqrt{N}$ | 5-points Variance |
|-----------|-------------|-------------------|
| $\alpha$ (pc) | (1200$^{+220}_{-140}$) | (1330$^{+190}_{-140}$) |
| $\alpha_O$ (pc) | (5150$^{+250}_{-190}$) | (4970$^{+230}_{-170}$) |
| $z_Y$ (pc) | (170$^{+400}_{-30}$) | (180$^{+40}_{-30}$) |
| $z_O$ (pc) | (710$^{+1200}_{-20}$) | (770$^{+130}_{-90}$) |

In our modeling, the bar contribution to the stellar density is given by

$$r_{bar} = \left( \frac{x}{x_0} \right)^2 + \left( \frac{y}{y_0} \right)^2 + \left( \frac{z}{z_0} \right)^4$$

n_{bar} = (n_{d,y} + n_{d,o})C_{bar} e^{-\frac{r_{bar}^2}{2\sigma_{bar}^2}} \tag{7}

where $r_{bar}$ describes the shape ($x'$, $y'$, and $z'$ are along the three axes) and $C_{bar}$ gives the contribution of the bar relative to the sum of the disk densities (Dwek et al. 1995). We explored axes ratios in the ranges $\{x_0 : y_0 : z_0\} = [1.00 : (0.22 - 0.67) : (0.34 - 0.40)]$ and $x_0$ up to 4 kpc, a little more than the largest value found in the literature (López-Corredoira et al. 2007). The orientation of the bar with respect to the line Sun-Galactic center, $\theta_{bar}$, was allowed to be in the range $11^\circ$–$53^\circ$. Even though the geometrical parameters of the bar are not strongly constrained by the 2MASS data, we attempted to determine them, as well as the density contrast between the bar and the disks.

**2.5 Interstellar Extinction**

The interstellar extinction is an important ingredient of a Galactic model, especially in the Galactic plane. Due to the accumulation of dust in the Galactic plane, the extinction is larger in this region of the Galaxy. The model of Amôres & Lépine (2005) is based on the distribution of gas (H1 and CO) and interstellar dust (IRAS 100 $\mu$m), under the assumption that the dust is well mixed with the gas. In this model, the Galaxy has axial symmetry, the gas density varies radially in a smooth way, and the spiral arms produce no effects in the extinction. The interstellar extinction is calculated assuming
Table 6
Comparison with Results in the Literature

| Parameter | Our Result       | Value from Literature   | Source                        |
|-----------|------------------|-------------------------|-------------------------------|
| $\alpha_Y$ (pc) | $(2120 \pm 200)$ | $\left(2500^{+200}_{-200}\right)$ | Fux & Martinet (1994)         |
|           |                  |                         | Freudenreich (1998)          |
|           |                  | $2600$                  |                               |
|           |                  | $(2100 \pm 300)$        | Porcile et al. (1998)        |
|           |                  | $(3300 \pm 600)$        | Feast (2000)                 |
|           |                  | $1700$                  | Lépine & Leroy (2000)        |
|           |                  | $(2800 \pm 300)$        | Ojha (2001)                  |
|           |                  | $\left(2100^{+720}_{-720}\right)$ | López-Corredoira et al. (2002) |
|           |                  | $(3500 \pm 300)$        | Larsen & Humphreys (2003)    |
|           |                  | $2400$                  | Picaud & Robin (2004)        |
|           |                  | $(2600 \pm 520)$        | Juric et al. (2008)          |
|           |                  | $2200$                  | Reyé et al. (2009)           |
|           |                  | $(3700 \pm 1000)$       | Chang et al. (2011)          |
| $\alpha_O$ (pc) | $(3050 \pm 500)$ | $3500$                  | Reid & Majewski (1993)       |
|           |                  | $(2800 \pm 800)$        | Robin et al. (1996)          |
|           |                  | $(3000 \pm 1500)$       | Buser et al. (1999)          |
|           |                  | $2300$                  | Lépine & Leroy (2000)        |
|           |                  | $(3700 \pm 800)$        | Ojha (2001)                  |
|           |                  | $(4700 \pm 200)$        | Larsen & Humphreys (2003)    |
|           |                  | $(3600 \pm 720)$        | Juric et al. (2008)          |
|           |                  | $(5000 \pm 1000)$       | Chang et al. (2011)          |
| $\beta_Y$ (pc) | $(2070^{+2000}_{-800})$ | $3000$                  | Freudenreich (1998)         |
|           |                  | $2600$                  | Lépine & Leroy (2000)        |
|           |                  | $2000–4000$             | López-Corredoira et al. (2004) |
|           |                  | $(1310 \pm 1030)$       | Picaud & Robin (2004)        |
| $z_Y$ (pc) | $(205 \pm 40)$   | $325$                   | Reid & Majewski (1993)       |
|           |                  | $250–270$               | Robin et al. (1996)          |
|           |                  | $100$                   | Lépine & Leroy (2000)        |
|           |                  | $(310^{+60}_{-45})$     | López-Corredoira et al. (2002) |
|           |                  | $(245 \pm 49)$          | Juric et al. (2008)          |
|           |                  | $(360 \pm 10)$          | Chang et al. (2011)          |
| $z_O$ (pc) | $(640 \pm 70)$   | $1400–1600$             | Reid & Majewski (1993)       |
|           |                  | $760$                   | Robin et al. (1996)          |
|           |                  | $390$                   | Lépine & Leroy (2000)        |
|           |                  | $900$                   | Larsen & Humphreys (2003)    |
|           |                  | $(900 \pm 180)$         | Juric et al. (2008)          |
|           |                  | $(1020 \pm 30)$         | Chang et al. (2011)          |
| $a_H$ (pc) | $(400 \pm 100)$  | $3000$                  | Gilmore (1984)               |
|           |                  | $2670$                  | Reid & Majewski (1993)       |
|           |                  | $1900^{+a}$             | Binney et al. (1997)         |
|           |                  | $420$                   | Lépine & Leroy (2000)        |
|           |                  | $(4300 \pm 700)$        | Larsen & Humphreys (2003)    |
|           |                  | $(2500^{+1720}_{-160})$ | Vanhullebeke et al. (2009)   |
| $N_{sph}/N_D$ | $(0.0082 \pm 0.0030)$ | $0.00125$              | Bahcall & Soneira (1980)     |
|           |                  | $0.0083$                | Guglielmo (1990)             |
|           |                  | $0.00358$               | Ruelas-Mayorga (1991)        |
|           |                  | $(0.002–0.003)$         | Larsen & Humphreys (2003)    |
|           |                  | $0.0051$                | Juric et al. (2008)          |
|           |                  | $(0.002 \pm 0.001)$     | Chang et al. (2011)          |
| $\kappa$  | $(0.57 \pm 0.05)$| $(0.80 \pm 0.05)$       | Reid & Majewski (1993)       |
|           |                  | $0.8$                   | Lépine & Leroy (2000)        |
|           |                  | $0.6$                   | Robin et al. (2000)          |
|           |                  | $(0.55 \pm 0.06)$       | Chen et al. (2001)           |
|           |                  | $(0.65 \pm 0.05)$       | Girardi et al. (2005)        |
|           |                  | $(0.64 \pm 0.01)$       | Juric et al. (2008)          |
| $C_S$     | $(2.0^{+0.6}_{-0.4})$ | $1.32$                 | Drimmel & Spergel (2001)     |
|           |                  | $1.2–1.4$               | Grosbol et al. (2004)        |
|           |                  | $1.30$                  | Benjamin et al. (2005)       |
|           |                  | $1.3–1.5$               | Liu et al. (2012)            |
| $C_{bar}$ | $(3.4^{+1.0}_{-1.3})$ | $1610–2030$            | Dwek et al. (1995)           |
| $b_{bar}$ (pc) | $(1250^{+500}_{-200})$ | $900$                 | Stanek et al. (1997)         |
|           |                  | $<3128$                 | Freudenreich (1998)          |
|           |                  | $1750$                  | Bissantz & Gerhard (2002)    |
|           |                  | $3900$                  | López-Corredoira et al. (2007) |
|           |                  | $\sim1250$             | Gonzalez et al. (2011)       |
|           |                  | $\sim1460$             | Robin et al. (2012)          |
Table 6 (Continued)

| Parameter | Our Result | Value from Literature | Source |
|-----------|------------|-----------------------|--------|
| $\theta_{\text{bar}}$ (deg) | $(12^{\circ}1^{\circ}15^{\circ})$ | ~1490 | Wang et al. (2012) |
| | | ~680 | Cao et al. (2013) |
| | | $(20 \pm 10)$ | Dwek et al. (1995) |
| | | 20–30 | Stanek et al. (1997) |
| | | 12 | Freudenburg (1998) |
| | | 15–30 | López-Corredoira et al. (2002) |
| | | 20–35 | López-Corredoira et al. (2005) |
| | | 43 | López-Corredoira et al. (2007) |
| | | $(42.44 \pm 2.14)$ | Cabrera-Lavers et al. (2008) |
| | | $(15^{\circ}12^{\circ}7^{\circ}13^{\circ}3^{\circ})$ | Vanhollebeke et al. (2009) |
| | | ~30 | Gonzalez et al. (2011) |
| | | 25–27 | Nataf et al. (2013) |
| | | 13 | Robin et al. (2012) |
| | | 20 | Wang et al. (2012) |
| | | 29–32 | Cao et al. (2013) |

Notes.

* Bulge following a truncated power law.
* Axis ratio 1.0 : 0.6 : 0.4 and angle between the Sun-center line and the major axis of the bulge ~20°.
* Axis ratio 1.00 : 0.68$^{+0.19}_{-0.05}$ : 0.31$^{+0.04}_{-0.06}$ and angle between the Sun-center line and the major axis of the bulge $15^{\circ}4^{\circ}7^{\circ}12^{\circ}13^{\circ}3^{\circ}$.

Figure 7. Cumulative star count histograms for selected lines of sight in the $J$ band. The filled circles with error bars represent the observations while the steps correspond to the model. The three middle histograms refer to the Galactic plane or close to it, while the upper and lower histograms correspond to $|b| = 45°$. The dashed line indicates the magnitude limit for each line of sight.
that it is proportional to the column density of hydrogen, in both atomic ($N_{\text{H}\text{I}}$) and molecular ($N_{\text{H}_2}$) forms:

$$A_V = C_V(r)N_{\text{H}\text{I}}(R, z) + 2C_V(r)N_{\text{H}_2}(R, z),$$

(8)

Here, $C_V$ is a proportionality factor, with an average value of $5.3 \times 10^{-22}$ mag cm$^2$ (Bohlin et al. 1978), if $A_V = 3.1 E_{B-V}$, but it can change along the galactocentric radius due to the metallicity gradient. For galactocentric distances in the plane $r > 1.2$ kpc, the authors suggest a proportionality with $r^{-0.5}$. For $r < 1.2$ kpc, due to the highly uncertain metallicity of the region, a constant value is employed. Similar analytical expressions were adopted for both gas forms:

$$n_{\text{H}\text{I}, \text{H}_2} = ce^{-z^2/(2b^2)}$$

(9)

where, for $\text{H}\text{I}$, $a = 7$ kpc, $b = 1.9$ kpc, and $c = 0.7$ cm$^{-3}$. For $\text{H}_2$, $a = 1.2$ kpc, $b = 3.5$ kpc, and $c = 58$ cm$^{-3}$. Since there is a large concentration of $\text{H}_2$ in the Galactic center, for $r < 1.2$ kpc this was modeled separately with the function

$$n_{\text{H}_2} = de^{-f(r)}$$

(10)

with $f = 0.1$ kpc and $d = 240$ cm$^{-3}$.

The vertical distribution of hydrogen is given by a Gaussian function of $z$:

$$n_H(r, z) = n_H(r)e^{-z^2/[4(z_{1/2}/2)^2]}$$

(11)

where $z_{1/2}$ is the half width at half height of the scale height. Amôres & Lépine (2005) suggest for the scale height of $\text{H}_2$:

$$z_{1/2} = 45e^{0.1r} \text{pc},$$

(12)

while for $\text{H}\text{I}$ the same expression must be multiplied by a factor of 1.8.

### 2.6. Luminosity Function

The luminosity function used in this work follows OL93 and enters the code via a table containing up to 64 classes of objects among main sequence, giants, supergiants, and variable objects. The space densities were updated to be consistent with recent results. Figure 1 shows the luminosity function for main sequence and giant stars according to OL93, Wainscoat et al. (1992), Bochanski et al. (2010), Reid & Gizis (1997), and Murray et al. (1997). The luminosity functions for supergiants and variable objects are the same as in OL93.

### 3. METHODOLOGY

#### 3.1. The Data: Star Counts from the 2MASS Catalog

Since interstellar extinction is lower in the NIR, surveys in this region are efficient at investigating the structure of the Galaxy. 2MASS (Skrutskie et al. 2006; Cutri et al. 2003) covers 99.8% of
the sky in the NIR with 471 million point sources observed in $J$ (1.25 μm), $H$ (1.65 μm), and $K_S$ (2.17 μm). At a signal-to-noise ratio $S/N = 10$, the limit magnitudes for point objects are 15.8, 15.1, and 14.3 in the $J$, $H$, and $K_S$ bands, respectively. These limits refer to unconfused sources outside the Galactic plane ($|b| > 10°$) and far from areas where the interstellar extinction is large. The all-sky coverage of the 2MASS data allows us to make a comprehensive comparison of the observed counts with the predictions of the model by OL93.

The number of sources in a given line-of-sight through the Galaxy can be obtained from the 2MASS database in three forms: cone, box, and polygon search. Taking into account practical aspects such as the elapsed time for retrieving the data, we opted to use the cone search, with a cone area of one square degree. We use a grid of Galactic longitudes and latitudes as generated by the Hierarchical Equal Area Isolatitude Pixelization of a Sphere (HEALPix8; Górski et al. 2005) in order to obtain a uniform sampling in Galactic coordinates. The orientation of the grid is such that the “equator” of the HEALPix scheme coincides with the Galactic plane. Our basic grid contains 192 points (the $N_{\text{side}}$ parameter of HEALPix = 4). A $N_{\text{side}} = 8$ grid (768 points) was used to provide four neighbors to each point of the $N_{\text{side}} = 4$ scheme in order to estimate the variance of the star counts at each position of the basic grid.

8 http://healpix.jpl.nasa.gov/

This allows us to verify if the assumption of using a $\sqrt{N}$ law for the uncertainty in the counts, $N$, is reasonable. It turns out that the dispersion obtained from the five-point estimate is consistent with the Poisson uncertainties for most of the sky, but significant differences show up close to the Galactic plane. This simply reflects the structure of the Galaxy in a scale of a few degrees and the large gradients in star counts along the $z$-direction close to the Galactic plane. We investigated the results of using one or the other option for the uncertainties in the observed star counts and the results are shown in Section 4.

The output of a cone search is a list of $JHK_S$ magnitudes and catalog flags for each counted object. The flags include important indicators to eliminate low-quality counts. We examined the following flags: photometric quality, read, contamination and confusion, and Galactic contamination (extended objects). Table 1 summarizes the criteria for considering a source bad. Table 2 summarizes the percentage of rejections for a few lines of sight. One can see that even far from the Galactic plane, the $K_S$ band is more prone to being affected by rejections than the other bands are, while in regions near the Galactic plane the three bands have about 50% rejections. This indicates that comparisons in the Galactic plane should not be extended to magnitudes fainter than 11. Using an extreme field as an example ($l = 11°25', b = 0°0'$), in the 11th magnitude bin we find 74 rejections out of a total of 1062 counts for the $J$ band, 566 (out of a total of 4860) for the $H$ band, and 2076 (out of a total...
of 9990) for the $K_S$ band (7%, 11.6%, and 20.8%, respectively). The fractions of rejections are 25% to 30% due to photometric quality, 10%–15% from the read flag, and 65%–75% from the contamination/confusion flag.

The limiting magnitude varies from grid point to grid point due to differences in interstellar extinction and the existence of unresolved close sources. This is especially true for regions in the Galactic plane and close to the Galactic center. To set up a limiting magnitude for each grid point, we stepped in magnitude, counting the corresponding objects and monitoring the number of associated rejections. The limiting magnitude was set when the fraction of rejected objects reached 10%.

Objects not rejected are put into ascending order of magnitude in each band and the cumulative star counts are obtained from the sums up to a certain magnitude limit. The differential counts can be obtained from the latter.

### 3.2. Parameter Estimations

Models in which multiple parameters are to be evaluated are recognizably challenging in the sense that traditional search methods for an optimal solution, like those based on gradients in the figure of merit (e.g., the Downhill Simplex method of Nelder & Mead 1965), may lead to local maxima (or minima), which can be far from the best solution. A number of statistical methods have received attention in the last few decades because even though they are relatively slow, they produce reliable results. We chose to use the Markov Chain Monte Carlo (MCMC) method to have an overall view of the parameter space, including a rough localization but with a reliable measure of the spread of the parameters. We also use the Nested Sampling (NS) method to have a better estimate of the parameter values. We experimented with MCMC chains of up to $10^5$ iterations to have an overview of possible multiple modes in parameter space. Our conclusion is that one can unambiguously limit the region of interest for each parameter with typically $10^4$ iterations. The NS procedure subsequently progresses faster to the mode, in some cases with only a few hundred iterations.

#### 3.2.1. MCMC

MCMC (Gilks et al. 1996) is probably the preferred first approach for an overall view of the parameter space in a multi-parameter problem. It has the virtue of being very simple to code and provides a first assessment of the location and spread of the parameters. It is perfectly suited for the Bayesian context, where prior information may be relevant in parameter estimation or model selection. Briefly, if we have a set of data $D$ with individual points $d_1, d_2, \ldots, d_N$ and a model $M$ with a vector of parameters $\theta_1, \theta_2, \ldots, \theta_N$, for which we are able to calculate the likelihood $L$, the MCMC algorithm progresses as follows.

1. Consider an initial state $\theta$, randomly chosen, with associated likelihood $L$. 

---

**Figure 10.** Cumulative star count histograms for the Galactic poles in the $JHK_S$ bands. Symbols are the same as in Figure 7.
Figure 11. Cumulative star counts as a function of longitude for $b = 0^\circ$ in the $K_s$ band. The black/cyan filled circles with error bars represent the observations for the $N_{\text{obs}} = 4/16$ HEALPix grid. The blue and red steps correspond to the model with and without the correction of Equation (4) for thin disk density, respectively. The green steps indicate the counts calculated with the Besançon model.

Figure 12. Cumulative star counts along latitude for $l = 0^\circ$ in the $K_s$ band. The symbols are as in Figure 10, with green steps indicating the counts calculated with the Besançon model.

2. Draw a random candidate state $\theta^*$, statistically centered on $\theta$, for which the likelihood is $L^*$.

3. Calculate the ratio $\alpha = \frac{L^*}{\bar{L}}$.

4. If $\alpha > 1$ accept the new state else draw a random number $\beta$ in [0, 1] if $\beta < \alpha$ accept the new state else stay in the old state.

5. Record the chosen state and start again at step 2.

Since we considered flat priors for all parameters, the histograms of the chain for each parameter may be regarded as proportional to the a posteriori distribution of the parameter. Each histogram location gives us an estimate of the value of the parameter itself and the confidence region (or uncertainty) may be obtained by integrating the area (for example, the one corresponding to the 1σ quantile in a normal distribution) around the mode or the median of the histogram.

Step 2 in the scheme above is by far the most involved, since it implies defining a step size by which to statistically draw the proposal states $\theta^*$. Suppose a particular parameter $h$. For all other variables, we work on normalized quantities (here $h_n$) calculating $h_n = (h - h_{\text{min}})/(h_{\text{max}} - h_{\text{min}})$ and then draw the proposal $h_{n}^* \sim N(h_n, 0.289\Delta)$. Here, “$\sim$” means “distributed as.” The numerical factor 0.289 ensures that the step variance is similar to the variance generated by the step $\Delta$ in a uniform distribution. With this scheme, all normalized variables are subject to the same step size. We follow the recommendations in the literature (e.g., Skilling 2004) and tune $\Delta$ as the chain progresses to have an acceptance rate of $\sim 37\%$. The initial value of $\Delta$ is 0.25 and the adaptive scheme for changing its size is only started after $2^{N_p}$ steps of the chain to allow for exploration of distant regions in parameter space.

We investigated two forms for the likelihood $L$. In the first, we calculate for the whole grid of $n_{\text{pix}}$ lines of sight (in one band, say $H$):

$$\chi^2_H = \frac{1}{n_{\text{pix}}} \sum_{j=1}^{n_{\text{pix}}} \frac{m_{\text{obs},j} - m_{\text{model},j}}{C_{\text{obs},j} + C_{\text{model},j}}^2.$$  

Here, $C_{\text{obs},j}$ are the 2MASS observed counts in each line of sight, with $j$ running from the lower magnitude limit $m_{\text{lim}}$ to the upper magnitude limit $m_{\text{up}}$, and $C_{\text{model},j}$ are the corresponding model quantities. Each line of sight has a cumulative histogram $N_{\text{obs}} = \sum_{j=1}^{15}(m_{\text{up}} - m_{\text{lim}})/\Delta m_{\text{lim}} + 1$ magnitude bins. The other bands are treated similarly and we finally write for the likelihood

$$L \propto \exp \left( -\frac{1}{2} \chi^2_H \right).$$

where $\chi^2 = \chi^2_H + \chi^2_KS$.

Since Equation 14 assumes that the $C_{\text{obs}}$ terms in Equation (13) are statistically independent and normally distributed—and the latter is not the case for low counts—we also used the Poisson likelihood form prescribed in Bienaymé et al. (1987) and Robin et al. (1996):

$$\ln L = \sum_{j=1}^{n_{\text{pix}}} \sum_{i=m_{\text{lim}}}^{m_{\text{up}}} C_{\text{obs},j}(1 - \xi_j + \ln \xi_j),$$

where $\xi_j = (C_{\text{model},j}/C_{\text{obs},j})$.

The two likelihood forms allow us to perform a sanity check on the results since Equation (13) gives more weight to instances of large star counts while Equation (15) attributes equal weights to the counts in all magnitude bins. The largest difference in parameter values, however, is small, typically less than 5%. The median of the absolute value of the relative residuals $(C_{\text{obs},11} - C_{\text{model},11})/C_{\text{model},11}$, in a grid of 3072 lines of sight is typically less than 2%.

3.2.2. Nested Sampling

NS is an algorithm for optimization in multi-parameter problems invented by Skilling (2004). It relies on the idea that whatever the number of parameters in a model, we can always populate the parameter space with a number $N_{\text{ive}}$ of random samples and calculate their likelihoods. Instead of focusing on the best likelihoods, the algorithm works on the worst. Our implementation of NS is as follows.

1. Populate the parameter space with $N_{\text{ive}}$ samples chosen randomly. Note that a minimum acceptable value for $N_{\text{ive}}$ would be $2^{N_p}$, where $N_p$ is the number of parameters being sought. Calculate the associated $L_i$, $i = 1, ..., N_{\text{ive}}$. The associated $j = 1, ..., N_p$ parameters are stored in the vectors $\theta_{i,j}$.

2. Find the worst $L$ among the $N_{\text{ive}}L_i$ and call it $L^* = L_{i_{\text{ive}}}$.

3. Choose at random an index $k$ among $1, ..., N_{\text{ive}}$ such that $k \neq i_{\text{ive}}$. 

$\chi^2_H = \frac{1}{n_{\text{pix}}} \sum_{j=1}^{n_{\text{pix}}} \frac{m_{\text{obs},j} - m_{\text{model},j}}{C_{\text{obs},j} + C_{\text{model},j}}^2.$
Figure 13. (a) Observed cumulative star counts for $K_S < 11$ sampled according to the $N_{\text{side}} = 16$ HEALPix scheme (3072 grid points). Each grid point is the result of a cone search of one square degree area. The counts are color-coded in a logarithmic scale to facilitate visualization. (b) Predicted cumulative star counts from our model in the same band with the same count coding as in (a). (c) Relative differences $(C_{\text{obs},11} - C_{\text{M},11}) / C_{\text{M},11}$ of (a) and (b) color coded in a linear scale, to emphasize the details.

4. Explore the vicinity of point $\theta_k$ with a short MCMC and choose from its output a parameter set $\theta_{\text{copy}}$ for which $L > L^*$.  
5. Substitute $\theta_{\text{worst}}$ with $\theta_{\text{copy}}$ and jump to step 2.

A simple scheme for stopping the NS algorithm was set based on the size of the MCMC proposal step in step 4 above. When $\Delta < 0.0001$, we finish the iterations. Again, $\Delta$ is allowed to vary only after $2^{N_p}$ steps of the NS algorithm.

The limits we adopted for parameter searches are shown in Table 3. They were chosen to cover, with some slack, the range of values found in the literature.

3.3. Finer Search for Selected Parameters

The $N_{\text{side}} = 4$ HEALPix grid provides a good first assessment of the Galactic parameters but has the obvious limitation of being too coarse, especially in regions close to the Galactic center. To
circumvent that, we did several experiments with finer grids, for which only a few parameters were explored. For example, a grid of 382 lines of sight drawn from the $N_{\text{side}} = 16$ HEALPix scheme provides good coverage of the central region of the Galaxy and is not very expensive in terms of computing time. At $l = b = 0^\circ$, it samples every 5.6 in longitude and 2.4 in latitude. In order to also probe regions far from the Galactic center, the 382 points are drawn from the $N_{\text{side}} = 16$ scheme according to the density of star counts. Figure 2 illustrates the $N_{\text{side}} = 4$ basic grid and also the finer grid drawn from the $N_{\text{side}} = 16$ scheme.

The previously well determined parameters, $\alpha_Y$, $\alpha_O$, $z_Y$, and $z_O$, are minimally affected by the choice of a finer grid. However, $a_H$ and $N_{\text{sph}} / N_D$ are. This is due to the fact that the cusp in star counts clearly visible at $-10^\circ < l < +10^\circ$ is well sampled by the finer grid. The trend of the changes is in the sense that $a_H$ decreases and $N_{\text{sph}} / N_D$ increases.

4. RESULTS

4.1. Parameters

Figure 3 shows an overview of the joint distributions of the parameters from an MCMC run of $10^5$ iterations considering the $N_{\text{side}} = 4$ HEALPix grid. One can see that $\alpha_Y$, $\alpha_O$, $z_Y$, and $z_O$ are the best constrained parameters, followed by $N_{\text{sph}} / N_D$ and $\kappa$. Two factors contribute to making the estimates of $a_H$, $\beta_Y$, and $\beta_O$ more difficult: their much more subtle contribution to the overall behavior of the counts and the relatively poor resolution of the 192 point grid. One should recall that the separation of samples in longitude for $b = 0^\circ$ is 22.5 and that the first “parallels” are at $|b| \sim 9^\circ / 6$. The parameter related to the contrast of the spiral arms, $C_S$, and the parameters of the bar ($C_{\text{bar}}, \theta_{\text{bar}}, x_0, y_0, z_0$) were kept fixed at the best possible guesses when using the sparse grid. This means contrasts of the order of unity, $\theta_{\text{bar}} \sim 30^\circ$, $x_0 \sim 2000$ pc, and $(x_0 : y_0 : z_0) = (1.0 : 0.4 : 0.4)$.

Figure 4 illustrates our attempt to better constrain the parameters via the NS algorithm. For $\alpha_Y$, $\alpha_O$, $z_Y$, $z_O$, and $N_{\text{sph}} / N_D$, the best solution agrees well with the maximum likelihoods from the MCMC run. For $\beta_Y$, $\beta_O$, $a_H$, $\kappa$, $C_S$, and $C_{\text{bar}}$, the maxima tend to fall in regions that may be far from the correspondent in the MCMC distribution. This just reflects the “flat” nature of the likelihood landscape, with small differences between the discrete evaluations, which can lead to solutions along a wide range of values. We tried to mitigate this limitation by choosing 32 random likelihoods among the 5% best evaluated in an MCMC run of 25000 steps as the starting state for the NS algorithm. The result is shown in Figure 5. We see that the best determined parameters from the MCMC procedure converged to consistent values, while the rest tend—even though they converge to definite values—to show a substantial spread in parameter space as the NS algorithm progressed.

Table 4 summarizes the results shown in Figures 3, 4, and 5 in numerical form. The well constrained parameters, $\alpha_Y$, $\alpha_O$, $z_Y$, and $z_O$, are consistent in all three approaches and are probably more accurately determined than indicated by the MCMC procedure alone. For the rest of the parameters, the range indicated by the MCMC exploration is large and reflects the weak constraints imposed on them by the adopted sampling grid.

Table 5 shows the effects of adopting different uncertainties in the star counts as discussed in Section 3.1. To simplify the comparison, we chose to minimize only the well-determined parameters. As one can see, the different weighting schemes do not produce conflicting results. In the following, we discuss only results for which the $\sqrt{N}$ scheme was used.

The adoption of a finer grid (see Section 3.3) improves our ability to determine parameters that are related to spatially limited regions of the Galaxy. This is the case for the spheroid, for example. Figure 6 shows the result of an MCMC run on such a finer grid. Notice that we fixed ill-determined parameters like $\beta_O$, $C_S$, and $C_{\text{bar}}$ to the best possible guesses. A further NS run on the region constrained by the MCMC run of Figure 6 using the full resolution of a $N_{\text{side}} = 16$ (3072 points) grid provided us with what we consider to be the best estimate for the basic Galactic parameters from the 2MASS data. They are discussed in the following section.

4.2. Comparison with Results in the Literature

Table 6 shows the results of this work together with a compilation of corresponding values found in the literature, to facilitate a comparison and discussion of possible differences. The first result that catches our attention is $\alpha_Y$, for which we obtain values that are systematically smaller than those quoted in the literature, especially considering the results from the $N_{\text{side}} = 4$ coarse grid. We attribute at least part of the difference
to the interplay between $\alpha_\gamma$ and $\beta_\gamma$—the latter, in general, is not used in models by other authors. $\alpha_\gamma$ is comparable with the estimates of Larsen & Humphreys (2003) and Chang et al. (2011). Like in most cases seen in the literature, we found that $\alpha_\gamma$ is larger than $\alpha_\gamma$. López-Corredoira et al. (2002) present parameters for the Galactic disk also based on 2MASS data. They recovered a disk scale length of 2.1 kpc, which is in agreement with what we found for the thin disk. The value we obtain for $\beta_\gamma$ is consistent with the estimates by Freudenberg (1998), Lépine & Leroy (2000), López-Corredoira et al. (2004), and Picaud & Robin (2004).

The two scale heights, $z_\gamma$ and $z_\phi$, are also well determined parameters. $z_\gamma$ is in good agreement with the values obtained by Robin et al. (1996) and Juric et al. (2008) but slightly smaller than those found by López-Corredoira et al. (2002), Reid & Majewski (1993), and Chang et al. (2011). $z_\phi$ is definitively smaller than the values in the literature (Robin et al. 1996, Larsen & Humphreys 2003, Juric et al. (2008), and Chang et al. 2011)). López-Corredoira et al. (2002) determined the scale height of the sole disk in their model to be equal to 310 pc, which is intermediate between the values we found in our two-disk model. In this context, it is important to recall that $z_\gamma$ and $z_\phi$ follow Equation (3) in our model and, as consequence, any comparison with the literature should refer to the solar neighborhood values.

The scale length parameter of the spheroid, $a_{H}$, defines how well the “cusp” in star counts close to the Galactic center is fit. This cusp is conspicuous if we examine the $K_S$ longitudinal counts at $|b| \sim 2\textdegree$, the first “parallel” in the $N_{side} = 16$ HEALPix scheme. To be able to reproduce the cusp with the spheroidal population of Equation (5), $a_{H}$ necessarily has to be small and the normalization $N_{sph}/N_{D}$ has to be large. The largest values needed for $N_{sph}/N_{D}$, however, do not exceed 0.01. The oblateness of the spheroid, $\kappa$, estimated to be $\sim 0.57$, is consistent with the range of values in the literature, 0.55–0.8. We note that since there is a correlation between this parameter and $N_{sph}/N_{D}$, both parameters should always be optimized simultaneously. Larsen & Humphreys (2003) suggest that this parameter could vary with galactocentric radius. Carollo et al. (2007) concluded that the spheroid would be better described by two sub-populations, one related to an inner bulge, with oblateness $\sim 0.6$, and another related to an outer bulge, with oblateness $\sim 0.9$.

A comparison of our results for the spheroid with those of Robin et al. (2012) would be interesting since this is a recent result also based on 2MASS data. Unfortunately, it is not straightforward. Those authors use additional components in their boxy bar/bulge + thicker ellipsoid structure. Furthermore, they express their results as maps of residuals given by $(C_M - C_{obs})/C_{obs}$ for a very fine grid with 15 arcmin separation in the region $|l| < 20\textdegree$ and $|b| < 10\textdegree$. One can see from their Figure 3 that only the central region, a few square degrees in size, presents residuals well in excess of 20%. However, their best fit model (case S + E in that paper) involves 15 parameters, while our description uses 7 parameters for the spheroid that are optimized simultaneously with the parameters that describe the rest of the Galaxy. Considering the same $20\times10^\textdegree$ region, our model presents a smooth trend of overestimating the star counts for $|l| \lesssim 10^\textdegree$, with typical values of the residuals of $\sim 20\%$. The largest residual in the $20\times10^\textdegree$ region is 0.46.

In models where the bulge follows a truncated power law (Binney et al. 1997; Vanhollebeke et al. 2009), the scale length of the bulge plays a different role with respect to our $\alpha_\gamma$—it indicates, roughly, the truncation radius of the bar. For this reason, we cannot quantitatively compare these scales. Similar difficulties are found in the case of the boxy-bulge of López-Corredoira et al. (2005). The HWHM of a Gaussian density profile would be 425 pc, close to our length scale, but the contribution of the boxy-bulge structure extends to larger galactocentric radii.

The parameter describing the contrast of the spiral arms, $C_S$, is very loosely constrained by the 2MASS data. In fact, even with the $N_{side} = 16$ grid, which gives a separation of 5/6 at $|b| = 0\textdegree$, the enhancement of the counts in the tangential directions discussed at the end of Section 2.3 are hardly seen. Clearly, higher resolution is needed to characterize this component. A similar conclusion was reached by Quillen (2002). We find indications that $C_S$ is of the order of unity, roughly consistent with previous estimates from Drimmel & Spergel (2001), Grosbol et al. (2004), Benjamin et al. (2005), and Liu et al. (2012).

The inclusion of a bar in the modeling of the data sampled with the $N_{side} = 16$ HEALPix scheme definitely improves the quality of the resulting fits, even though the limit of $K_S = 11$ and the spacing of $\Delta l = 5/6$ and $\Delta b = 2/4$ are not the best to constrain that feature. We find that the bar has a half length of $\sim 1.25$ kpc, with axis ratios 1.00 : 0.22 : 0.39. The angle $\theta_{bar}$ is close to the lower limit of the values found in the literature, $\sim 12^\circ$ (López-Corredoira et al. 2000). The contrast of the bar with the ambient disks, $C_{bar}$, is $\sim 3$. We did not find previous estimates of the latter quantity in the literature.

4.2.1. Comparison of Selected Lines of Sight

Figures 7, 8, and 9 show cumulative histograms of star counts in nine sets of $(l, b)$ in $J$, $H$, and $K_S$, respectively. We see that the largest differences occur close to the Galactic plane, especially for $l \sim 300\textdegree$ and $l \sim 60\textdegree$. One can easily see the effects of the proximity of the Small Magellanic Cloud in the line of sight corresponding to $(l, b) = (300, -45)$ for magnitudes fainter than $\sim 13$. Figure 10 shows similar histograms for $|b| = 90\textdegree$. Here, we see that the limiting magnitudes extend all the way to the depth of the 2MASS catalog.

Counts along the Galactic plane, or intercepting the plane, used to be a tour de force for star count models. Figure 11 shows the comparison between observations and models for 64 lines of sight in the Galactic plane, with and without the adoption of the density profile for the young disk given by Equation (4). The limit magnitude is 11 in the $K_S$ band. For comparison, we also show the predicted counts from the Besançon model (Robin & Crézé 1986; Robin et al. 2003), obtained from the online form. Since the only parameter that can be varied by the user in the online form is the extinction, we show the fit that provides the best description close to the Galactic center. Figure 12 shows the observed and predicted counts along Galactic latitude for $l = 0\textdegree$. The largest differences are confined to the region close to the Galactic plane. Again, for comparison, we show the predicted counts from the Besançon model. Comparing our counts with those from the Besançon model, one can see that our fit is slightly better.

An all sky map of the relative differences between the observed and predicted counts produces a comprehensive view of the merits and limitations of a model. Figure 13(c) shows as a figure of merit the ratio $(C_{obs} - C_M)/C_M$ obtained from the cumulative histograms up to magnitude 11. This is not very

---

10 http://model.obs-besancon.fr/
different from but a little more intuitive than the figure of merit $C_M/C_{obs}$ used by Chang et al. (2011). For reference, both the observed and model counts are presented in Figures 13(a) and (b), respectively. As we can see in Figure 13(c), the largest differences between the observed and predicted counts are concentrated in the Galactic plane. This is a limitation shared by all models in the literature. Our Figure 13(c) resembles Figure 2 of Reylé et al. (2009), as both present the differences between the observations and the model predictions. Although the model of Reylé et al. (2009) includes a description of a warp, not present in our model, the $O - C$ differences are smaller in our case. The main differences between our map and that from Reylé et al. (2009) are the locations where the Galactic model does not agree with observations. The excess of observed stars counts that they attribute to the warp is present in the outer Galaxy, while the largest excesses observed by us are close to $l \sim 300^\circ$ and $l \sim 60^\circ$.

Figure 14 shows an all sky map of $C_M/C_{obs}$ for a more direct comparison with the result obtained by Chang et al. (2011). Our description is better both at the center and anti-center regions.

5. CONCLUSIONS

We have used 2MASS data to test a modified version of the model from OL93 in the JHK$_s$ bands. We emphasize that this study is the first attempt to determine the main parameters describing the Milky Way using the observed star counts over the entire sky, including the Galactic plane.

A survey of the parameter space was done using the MCMC method, followed by attempts to better constrain the parameters via the NS algorithm. A grid with 192 points generated by the HEALPix scheme was adopted as a baseline for sampling and via the NS algorithm. A grid with 192 points generated by the HEALPix scheme was adopted as a baseline for sampling and via the NS algorithm.

The largest excesses observed by us are close to $l \sim 300^\circ$ and $l \sim 60^\circ$.

$\alpha_Y$ HEALPix scheme was adopted as a baseline for sampling and via the NS algorithm. A grid with 192 points generated by the HEALPix scheme was adopted as a baseline for sampling and via the NS algorithm.

A survey of the parameter space was done using the MCMC method, followed by attempts to better constrain the parameters via the NS algorithm. A grid with 192 points generated by the HEALPix scheme was adopted as a baseline for sampling and via the NS algorithm.

Our model describes the star counts in 80% of the sky with an accuracy better than 10%. In the remaining area, a few dozen lines of sight (out of 3072) show absolute residuals in excess of 20%. They are concentrated close to the plane ($|b| < 8^\circ$), especially around $l \sim 300^\circ$, $l \sim 60^\circ$ and surrounding the Galactic center.

An overall view of the Galaxy according to our work suggests that it can be described by two disks with radial scales slightly smaller than those found in the literature. Only the thick disk needs a “hole” in its inner part. The conspicuous concentration of sources in the direction of the Galactic center can be described by a combination of contributions from the disks, from a spherical with scale length 400 pc, and from a bar that has an aspect ratio 1.00:0.22:0.39, which is seen at an angle of 12°.

P.F.P. received financial support for this work from Coordenação de Aperfeiçoamento de Pessoal de Nível Superior. This publication makes use of data products from the Two Micron All Sky Survey, which is a joint project of the University of Massachusetts and the Infrared Processing and Analysis Center/California Institute of Technology, funded by the National Aeronautics and Space Administration and the National Science Foundation.

REFERENCES

Amôres, E. B., & Lépine, J. R. D. 2005, AJ, 130, 659
Amôres, E. B., Lépine, J. R. D., & Mishurov, Y. N. 2009, MNRAS, 400, 1768
Bahcall, J. N., & Soneira, R. M. 1980, ApJS, 44, 73
Benjamin, R. A., Churchwell, E., Babler, B. L., et al. 2005, ApJL, 630, L149
Bienaymé, O., Robin, A. C., & Crézé, M. 1987, A&A, 180, 94
Binney, J., Gerhard, O., & Spiegel, D. 1997, MNRAS, 288, 365
Binney, J., & Merrifield, M. 1998, Galactic Astronomy (Princeton, NJ: Princeton Univ. Press)
Binney, J. J., Gerhard, O. E., Stark, A. A., Bally, J., & Uchida, K. I. 1991, MNRAS, 252, 210
Bissantz, N., & Gerhard, O. 2002, MNRAS, 330, 591
Blitz, L., & Spergel, D. 1991, ApJ, 379, 631B
Bochanski, J. J., Hawley, S. L., Covey, K. R., et al. 2010, AJ, 139, 2679
Bohlin, R. C., Savage, B. D., & Drake, J. F. 1978, ApJ, 224, 132
Buser, R., Rong, J., & Karaali, S. 1999, A&A, 348, 98
Cabrera-Lavers, A., González-Fernández, C., Garzón, F., Hammersley, P. L., & López-Corredoira, M. 2008, A&A, 491, 781
Cao, L., Mao, S., Natalf, D., Rattenbury, N. J., & Gould, A. 2013, MNRAS, 434, 595
Carollo, D., Beers, T. C., Lee, Y. S., et al. 2007, Nat, 450, 1020
Chang, C.-K., Ko, C.-M., & Peng, T.-H. 2011, ApJL, 740, 34
Chen, B., Stoughton, C., Smith, J. A., et al. 2001, ApJL, 553, 184
Churchwell, E., Babler, B. L., Meade, M. R., et al. 2009, PASP, 121, 213
Cohen, R. J., & Few, R. W. 1976, MNRAS, 176, 495
Cutri, R. M., Skrutskie, M. F., van Dyk, S., et al. 2003, yCat, 2246, 0
Dame, T. M., Hartmann, D., & Thaddeus, P. 2001, ApJ, 547, 792
Drimmel, R., & Spiegel, D. N. 2001, ApJ, 556, 181
Dwek, E., Arendt, R. G., Hauser, M. G., et al. 1995, ApJ, 445, 716
Englmaier, P., & Gerhard, O. 1999, MNRAS, 304, 512
Feast, M. 2000, MNRAS, 313, 596
Francis, C., & Anderson, E. 2012, MNRAS, 422, 1283
Freudenreich, H. T. 1998, ApJ, 492, 495
Freudenreich, H. T., Berjamin, G. B., Dwek, E., et al. 1994, ApJL, 429, L69
Fux, R. 1999, A&A, 345, 787
Fux, R., & Martinet, L. 1994, A&A, 287, L21
Georgelin, Y. M., & Georgelin, Y. P. 1982, A&A, 118, 1
Gilmore, G., & Reid, N. 1983, MNRAS, 202, 1025
Girardi, L., Groenewegen, M. A. T., Mathis, M. K., et al. 2005, A&A, 436, 895
Gonzalez, O. A., Rejkuba, M., Minniti, D., et al. 2011, A&A, 534, L14
Görski, K. M., Hivon, E., Banday, A. J., et al. 2005, Ap, 622, 759
Grosholz, P., Patis, F. A., & Pompé, E. 2004, A&A, 432, 549
Guglielmo, F., Le Bertre, T., & Epchtein, N. 1998, A&A, 334, 609
Han, C., & Gould, A. 1995, ApJ, 449, 521
Herquish, L. 1990, ApJ, 356, 359
Hou, L. G., Han, J. L., & Shi, W. B. 2009, A&A, 499, 473
Jurić, M., Ivezić, ˇZ., Brooks, A., et al. 2008, ApJ, 673, 864
Larsen, J. A., & Humphreys, R. M. 2003, AJ, 125, 1958
Lépine, J. R. D., & Leroy, P. 2000, MNRAS, 313, 263
Lépine, J. R. D., Roman-Lopez, A., Abrahm, Z., Junqueira, T. C., & Mishurov, Y. 2011, MNRAS, 414, 1607
Levine, E. S., Blitz, L., & Heiles, C. 2006, Sci, 312, 1773
Liszt, H. S., & Burton, W. B. 1980, ApJ, 236, 779
Liu, C., Xue, X., Fang, M., et al. 2012, ApJL, 753, L24
López-Corredoira, M., Cabrera-Lavers, A., Garzón, F., & Hammersley, P. L. 2002, A&A, 394, 883
López-Corredoira, M., Cabrera-Lavers, A., & Gerhard, O. E. 2005, A&A, 439, 107
López-Corredoira, M., Cabrera-Lavers, A., Gerhard, O. E., & Garzón, F. 2004, A&A, 421, 953
López-Corredoira, M., Cabrera-Lavers, A., Mahoney, T. J., et al. 2007, AJ, 133, 154
López-Corredoira, M., Hammersley, P. L., Garzón, F., Simonneau, E., & Mahoney, T. J. 2000, MNRAS, 313, 392
Majuess, D. J., Turner, D. G., & Lane, D. J. 2009, MNRAS, 398, 263
Malkin, Z. M. 2013, ARep, 57, 128
McClure-Griffiths, N. M., Dickey, J. M., Gaensler, B. M., & Green, A. J. 2004, ApJ, 607, L127
McWilliam, A., & Zoccali, M. 2010, ApJ, 724, 1491
Morrison, M. R., Meyer, L., & Ghez, A. M. 2012, RAa, 12, 995
Murray, C. A., Penston, M. J., Binney, J. J., & Houk, N. 1997, in Proc. ESA Symp on Hipparcos - Venice ’97, ed. M. A. C. Perryman & P. L. Bernacca (ESA SP-402; Noordwijk: ESA), 485
Nakada, Y., Onaka, T., Yamamura, I., et al. 1991, Natur, 353, 140
Nataf, D. M., Gould, A., Fouqué, P., et al. 2013, ApJ, 769, 88
Nelder, J. A., & Mead, R. 1965, CompJ, 7, 308
Ojha, D. K. 2001, MNRAS, 322, 426
Ortiz, R., & Lépine, J. R. D. 1993, A&A, 279, 90
Peters, W. L. 1975, ApJ, 195, 617
Picaud, S., & Roblin, A. C. 2004, A&A, 428, 891
Porcel, C., Garzón, F., Jimenez-Vicente, J., & Battaner, E. 1998, A&A, 330, 136
Quillen, A. C. 2002, AJ, 124, 924
Reid, I. N., & Gizis, J. E. 1997, AJ, 113, 2246
Reid, M. J. 2012, in IAU Symp. 287, Cosmic Masers—from OH to He, ed. R. S. Booth, W. H. F. Vlemmings, & E. M. L. Humphreys (Cambridge: Cambridge Univ. Press), 359
Reid, M. J. 2013, in IAU Symp. 289, Advancing the Physics of Cosmic Distances, ed. R. de Grijs (Cambridge: Cambridge Univ. Press), 188
Reid, N., & Majewski, S. R. 1993, ApJ, 409, 635
Reylé, C., Marshall, D. J., Robin, A. C., & Schultheis, M. 2009, A&A, 495, 819
Robin, A., & Crézé, M. 1986, A&A, 157, 71
Robin, A. C., Haywood, M., Crézé, M., Ojha, D. K., & Bienayme, O. 1996, A&A, 305, 125
Robin, A. C., Marshall, D. J., Schultheis, M., & Reylé, C. 2012, A&A, 538, A106
Robin, A. C., Reylé, C., & Crézé, M. 2000, A&A, 359, 103
Robin, A. C., Reylé, C., Derrière, S., & Picaud, S. 2003, A&A, 409, 523
Robitaille, T. P., Churchwell, E., Benjamin, R. A., et al. 2012, A&A, 545, A39
Ruelas-Mayorga, R. A. 1991, RevMexAA, 22, 27
Russel, D. 2003, A&A, 397, 133
Saito, R. K., Zoccali, M., McWilliam, A., et al. 2011, AJ, 142, 76
Sivia, D., & Skilling, J. 2006, Data Analysis: a Bayesian Tutorial (2nd ed.; Oxford: Oxford Univ. Press)
Skilling, J. 2004, in AIP Conf. Proc. 735, Nested Sampling, ed. R. Fischer, R. Preuss, & U. V. Toussaint (Melville, NY: AIP), 395
Skrutskie, M. F., Cutri, R. M., Stiening, R., et al. 2006, AJ, 131, 1163
Sofue, Y., Honma, M., & Omodaka, T. 2009, PASJ, 61, 227
Stanek, K. Z., Mateo, M., Udalski, A., et al. 1997, ApJ, 477, 163
Udalski, A., Szymanski, M., Stanek, K. Z., et al. 1994, AcA, 44, 165
Vallée, J. P. 1995, ApJ, 454, 119
Vallée, J. P. 2002, ApJ, 566, 261
Vallée, J. P. 2005, AJ, 130, 569
Vallée, J. P. 2008, AJ, 135, 1301
Vanhollebeke, E., Groenewegen, M. A. T., & Girardi, L. 2009, A&A, 498, 95
Wainscoat, R. J., Cohen, M., Volk, K., Walker, H. J., & Schwartz, D. E. 1992, ApJS, 83, 111
Wang, Y., Zhao, H., Mao, S., & Rich, R. M. 2012, MNRAS, 427, 1429
Weiland, J. L., Arendt, R. G., Berriman, G. B., et al. 1994, ApJL, 425, L81
Xu, Y., Reid, M. J., Zheng, X. W., & Menten, K. M. 2006, Sci, 311, 54
Zhao, H., Spergel, D. N., & Rich, R. M. 1995, ApJL, 440, L13
Zhu, Z., & Shen, M. 2013, in IAU Symp. 289, Advancing the Physics of Cosmic Distances, ed. R. de Grijs (Cambridge: Cambridge Univ. Press), 444