Method of Evaluation of Extreme Wave Probability

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Abstract. Method of evaluation of cumulative probability of appearance of wind waves with any given height is described. The method is based on direct modeling of wind wave fields and uses long-term dataset of significant wave height. The application of the method using wave forecast model data is presented. The disadvantages of the analyses of annually and seasonally average fields of extreme wave height probability obtained by the method are considered. Another application of evaluating of extreme wave height field based on the method is proposed.

1. Introduction

Highest risks of catastrophes for the human activities in the sea, i.e., sea traffic, fishery, sea mining etc., are mostly connected with strong storms. There are difficulties in predicting and avoiding emergency situations for long-term human economic activities in the ocean. The main reason for this is that the reliability of wave forecasts depends on reliability of wind forecast (or weather forecast in common). The period of the reliable forecast is no more than 10 days.

Possible method to reduce possible risks is to use climate dataset. It is obvious that it is impossible to keep numerical fields of sea surface elevation above zero level with enough time and spatial resolutions to make analyses of wave development and indentify cases of extreme waves. The integral characteristics of wave field are used. One of them is significant wave height $H_s$. This parameter are defined as a mean height of 1/3 of the highest waves in wave field (Ochi 2005). $H_s$ is evaluated as:

$$H_s = 4\int_0^\infty \int_0^\infty S(k_x,k_y)dk_x dk_y^{1/2}$$

(1)

in this formula $k_x$, $k_y$ are wave numbers in x and y direction, while $S(k_x,k_y)$ is wave spectrum.

The main problem of using this characteristic for extreme wave identifying is that $H_s$ doesn’t give information about maximum wave height in a given wave field. Extreme or freak waves which have the equal height can be observed in the wave fields with different values of $H_s$. For instance, a 15 meter waves can occur in a case of wave field with $H_s=10$ m and in a case of wave field with $H_s=7$ m. Thus, data of significant wave height aren’t enough to calculate the real wave height probability. Using any other integral characteristic of wave field (for example, period of peak) will lead to the same problem.
At present there are datasets consisting of the historical forecasts data on wind waves calculated by wave forecasts models and corrected by different methods, i.e., contact measurements with accelerometers or GPS-buoys; remote observation with using satellite altimetry or radars.

Estimations of freak wave’s probability nowadays are based on the analysis of dataset of $H_s$. In [1] long-term data are analyzed. It’s considered that an freak wave is a wave whose height is twice more than $H_s$. In [2] spectral method was used for analysis of long-term dataset, it was shown that the spectrum of modeled $H_s$ misses the energy for frequency more than $2.5 \times 10^5$ Hz (daily timescale and less). A correction method was formulated which allow to fill in the variability of the $H_s$ at high frequencies. In [3] estimating of the extreme wave heights from 30-year simulation was made by using the peaks-over-threshold method. In [4] extreme wave height (crest-mean level) was estimated by the methods of theory of extreme value. Methods of calculations of probability were discussed in many papers (for example [5, 6]).

2. Method description

In [7] method of calculating of cumulative probability of appearance of waves whose height above mean level exceeding a specific value ($P(h)$ and $h$ below) was developed.

The method was based on analysis of results of wave simulating made by 3-D model of potential flow. The numerical model used spectral representation of horizontal fields, finite differences for calculation of vertical derivatives. Fourth-order Runge–Kutta scheme was used for time integration. Fourier space resolution is 256X64 wave number, physical space resolution is 1024X256 (see [8] to get more detail). The calculations were done for 350 units of nondimensional time, i.e., for 70,000 time steps.

A series of 50 experiments was made by the model. Each experiments lasted 350 non-dimensional time units i.e., for 70,000 time steps. JONSWAP spectrum was used as initial conditions (more detail about experiments in [7]). Totally there are 3500000 wave fields.

The results were processed as follow: Each field of height of free surface above mean level ($h$) was normalized by the $H_s$ corresponding to this field $\tilde{H} = h / H_s$. Cumulative probability of nondimensional wave height $P(\tilde{H})$ was calculated. The distribution of $P(\tilde{H})$ was approximated by the function:

$$ P(\tilde{H}) = \exp(-3.97 \tilde{H} - 4.02 \tilde{H}^2), \quad 0 \leq \tilde{H} \leq 1.85. \quad (2) $$

The probability of wave which exceed value 1.85 (maximal value of normalized wave height in dataset) was considered as extremely low and neglected. Formula (2) can be considered as universal approximation for fields of wind waves where freak wave appearance is most likely.

Other wave types (for instance swells) have a low steepness and do not have impact on generation of extreme waves.

From the definition of conditional probability the probability of waves which exceeding specific height $h$ equals $P(h) = P(\tilde{H}) \cdot P(H_s)$, where $P(H_s)$ is distribution of $H_s$ probability. Then, $P(h)$ can be calculated as integral of $P(\tilde{H}) \cdot P(H_s)$ : over all possible value of $H_s$:

$$ P(h) = \int_0^{H_s_{max}} P(\tilde{H}) \cdot P(H_s) dH_s, \quad (3) $$

where $H_s_{max}$ is the maximum value of $H_s$ in the data. More information about method can be found in [7] and.

In [7] spatial distribution of the extreme wave probability was calculated by the method (2-3) from historical forecast data of $H_s$ [9]. This dataset was calculated by WAVEWATCH III wave forecast model for period from August 1999 to July 2015. The model used GFS-2 wind analysis [10] as dynamical forcing; the horizontal resolution of the fields is 0.5X0.5 degree. Validation of the model is made by using observations from wave buoys.
In [11] another method application (2-3) with using historical forecast data [9] was considered. The method was used for evaluation of height of waves which appear with cumulative probability $10^{-7}$. Average annual and seasonal fields obtained by the method were given. Areas where extreme waves can appear were shown.

3. Application of the method for engineering stationary structures

For design of stationary structure on the ocean, estimation of the highest wave which can strike the structure shall be taken into account. In [11] the cumulative probability $10^{-7}$ was considered as value which corresponds to appearance of extreme wave. Hereinafter the wave which appears with cumulative probability $10^{-7}$ will be considered as extreme wave.

Average annual value of height of extreme wave can be smoothed and consequently be small. It’s shown on figures 1-2.

![Figure 1. Extreme wave height averaged for January.](image1)

![Figure 2. Extreme wave height averaged annual averaged.](image2)
It can be seen, that in some area January average extreme wave height is more than the annual one (Northern parts of Atlantic and Pacific Ocean for example).

To take into account highest amount of possible extreme wave cases the new application of method (2-3) is proposed. The average monthly fields of cumulative probability distribution as function of wave height were calculated. Wave height which corresponds to the cumulative probability $10^{-7}$ was found for each month. Then the largest of wave height was chosen from among values of every month.

The result of this application is shown on fig.3. The field evaluated by this application can be considered as field of maximum annual wave height with cumulative probability $10^{-7}$ or maximum annual extreme wave height. The result of this field is shown on fig.3.

![Figure 3. Maximum annual extreme wave height.](image)

Field of maximum extreme wave height and field of annual averaged extreme wave height are similar in general. But there are differences: field of maximum annual extreme wave height is more detailed than annual average one. Values of maximum annual extreme wave height are larger than values annual averaged extreme wave height. There are some regions (thin curves) with large wave height on both figures which can be identified as trajectories of hurricanes. Number of these curves on fig.3 is considerably more than on fig.2. The reason of it is yearly averaging which smoothes field.

It could be said that the application proposed in this article allows take into account more cases of extreme waves that previously applications of method (2-3) can do.

## 4. References

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