Optical properties of hybrid quasiperiodic Fibonacci photonic crystals

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Abstract. In this paper, we exploit new optical properties of hybrid quasiperiodic Fibonacci photonic crystals by the transfer matrix method. The quasiperiodic photonic crystals are consist of various generation orders of photonic crystal series that follow the Fibonacci sequence. By regarding the building blocks like $S_3$, $S_4$, $S_5$ as a unit cell ($S_3S_4S_5$) of photonic crystals, we get multiple bandgaps in its spectrum. Furthermore, the bandgaps of the Fibonacci hybrid photonic crystals are tunable by merely altering the refractive index of the dielectric materials. The proposed configuration is a promising candidate as a multichannel filter and a refractive index sensor in the field of integrated photonic circuits.

1. Introduction

Photonic crystals (PCs), known as a milestone in the progress of photonics and modern science, have been investigated extensively during the past thirty years [1]. Similar to crystals showing electron bandgaps in solid physics, this type of multilayer structures composed by alternative dielectric materials exhibit photonic bandgaps for some frequency ranges of photons. A photon with eigenfrequency inside the bandgap of PCs decays exponentially inside the structure. Thus, the photon cannot propagate through the whole geometry, forming a photonic bandgap [2]. With the development of photonic crystals, quasiperiodic photonic crystal, as a branch of PCs, is proposed to achieve more complicated functionalities [3]. With research going further, the field of quasiperiodic photonic crystals (QPCs) thrives and flourishes. Traditionally, QPCs follow some substitution rules, such as the Fibonacci sequence, Thue-Morse sequence, and Cantor sequence. QPCs that follow substitution sequences exhibit complicated and irregular properties in their related transmittance and reflectance spectra. Furthermore, substitution sequences formed QPCs always show self-similarity in their spectra, which is induced by the disorder or quasi-order in the sequence orders [4]. Self-similarity, as the name suggests, behaves as the similarity in the various spectra of different generation orders of a specific substitution sequence. And the tendency is altogether unchanged with the increment of the generation order. To exploit more useful applications in diverse fields, researchers have made an abundant of efforts in trying different configurations of quasiperiodic photonic crystals. One of the most accepted approaches is to regard the various QPCs as building blocks, by taking the combination of building blocks as a unit cell, we can construct a new type of aperiodic PCs, which is also called hybrid photonic crystals (HPCs). For example, by simply composing two periodic PCs with different period
and dielectric constants (treat two periodic PCs as new building blocks), we shall get resonance transmission modes in its spectrum. Moreover, the number of resonant modes is adjustable by changing the period number of the structure [5]. Also, the other combination configuration such as combining aperiodic structures and quasi-periodic structures are proposed. Till now, a large number of devices are investigated, such as omnidirectional reflectors composed by Fibonacci sequence and Cantor sequence, and the broadband filters made up of period and Fibonacci sequence [4].

In this paper, we exploit new transmittance and reflectance properties of hybrid quasiperiodic Fibonacci photonic crystals by the transfer matrix method. The quasiperiodic photonic crystals consist of various generation orders of photonic crystal series that follow the Fibonacci sequence. By regarding the building blocks like $S_3S_4S_5$ as a unit cell of photonic crystals, we shall get multiple bandgaps in its spectrum. Furthermore, the bandgaps of the Fibonacci hybrid photonic crystals are tunable by merely altering the refractive index of the dielectric materials. The proposed configuration is a promising candidate as a multichannel filter and a refractive index sensor in the field of integrated photonic circuits.

2. Theory model

For multilayer structures, we usually use the transfer matrix method (TMM) when calculating the transmittance and reflectance spectra. The characteristic matrix $M_i (i = A, B)$ for $A/B$ dielectric layer is calculated by

$$M_i = \begin{pmatrix}
\cos(k_i d_i) & -i q_i \sin(k_i d_i) \\
-i q_i \sin(k_i d_i) & \cos(k_i d_i)
\end{pmatrix} \tag{1}
$$

for TE-polarization wave, where $k_i = k_0 n_i (i = A, B)$, $q_i = \sqrt{\epsilon_i \mu_i}$ and $d_i$ is the thickness of layer $A$ or $B$. $k_0$, $\epsilon_i$, and $\mu_i$ is the wave vectors in the vacuum, the permittivity and the permeability of corresponding dielectric layer, respectively. As for the TM-polarized waves, we shall change $q_i = \sqrt{\mu_i \epsilon_i}$. Thus, the characteristic matrix of the entire structure can be given by

$$M = \prod_i M_i = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} \tag{2}
$$

where $j$ is the generation order and $i = A, B$ corresponds to particular dielectric layer materials of specific generation sequence. For example, the one-dimensional Fibonacci-sequence-based structure is expressed as $ABAABABA$ for generation number $j = 5$, and the transfer matrix for this corresponding Fibonacci photonic crystal with generation order $S_5$ can be presented by $M_5 = M_A M_B M_A M_3 M_B M_A M_4 M_A$. The reflection and transmission coefficient of the structure is

$$r = \frac{(M_{11} + M_{12} p_0) p_0 - (M_{21} + M_{22} p_0)}{(M_{11} + M_{12} p_0) p_0 + (M_{21} + M_{22} p_0)} \tag{3}
$$

and

$$t = \frac{2 p_0}{(M_{11} + M_{12} p_0) p_0 + (M_{21} + M_{22} p_0)} \tag{4}
$$

where $p_0$ and $p_L$ are the external ambient parameters for the incident and exit area. In this article, we mainly focus on the TE mode, while the result for the TM case can also be derived by the same method.

3. Result and discussion

Regarding the QPCs generated by the Fibonacci sequence as building blocks, we can construct a unit cell by combining the building blocks. In Fig. 1, we show a unit cell of the HPC composed by different generation order of Fibonacci sequences. The HPC is composed of Fibonacci sequence $S_3$, $S_4$, and $S_5$ with alternating dielectric material $A$ and $B$. The thickness of two dielectric layers is noted as $d_A$ and $d_B$ below, and the refractive index is $n_A$ and $n_B$. In this paper, dielectric layer is chosen as traditional optical material TiO$_2$ and SiO$_2$, with refractive index $n_A = 2.33$ (TiO$_2$) and $n_B = 1.45$.
(SiO₂), respectively. The thickness of the dielectric layers is set as $d_A = 75$ nm and $d_B = 100$ nm. It is worth to note that to get a complete bandgap in the spectrum, the total length of the photonic crystal structure must be long enough. Thus, the unit cell of the Fibonacci photonic crystal needs to be periodically repeated. Besides, to keep the structure length roughly the same, the period number $N$ is adjusted in the calculation.

**Figure 1.** Schematic illustration of the hybrid photonic crystal composed by $S_3$, $S_4$, and $S_5$ of Fibonacci sequences.

**Figure 2.** (a) Spectrum of periodic photonic crystal with $N = 45$ (dashed black line) and the FQPC structure with a unit cell of $S_3$ (solid blue line), $N_3 = 30$. (b) The FQPC structure with unit cell of $S_4$, $S_5$, $S_6$ with $N_4 = 18$, $N_5 = 11$ and $N_6 = 7$, respectively.

We first compare the transmittance of the HPCs consisting of only one Fibonacci series with that of traditional periodic photonic crystals, as shown in Fig. 2. The corresponding reflectivity spectrum can be derived by $1-T$ as the structure is reciprocal. In Fig. 2(a), we plot the transmittance of the periodic photonic crystal structure (black dashed line) and the Fibonacci quasiperiodic photonic crystal (FQPC) structure with a unit cell of $S_3$ (solid blue line). Furthermore, in Fig. 2(b) we depict the transmission spectrum of the Fibonacci quasiperiodic photonic crystal structure with various unit cells. The unit cells are $S_4$, $S_5$, $S_6$ corresponds to the solid dark line, dash red line, and dash-dotted blue line, respectively. To make sure the total length of these structures roughly equal, the period number of the periodic PC is set as $N = 45$, and the period number for the unit cell of $S_3$, $S_4$, $S_5$, and $S_6$ are $N_3 = 30$, $N_4 = 18$, $N_5 = 11$ and $N_6 = 7$, correspondingly. In calculation, the structure is assumed put in the air with the outside ambient $n_0 = 1$ and placed on a silica substrate. The other structural parameters are identical to the above. For the sake of illustration, we take the logarithm operation for the transmittance. In Fig. 2(a), for period PC, only one photonic bandgap is observed, which is located at $\lambda_0 = 640$ nm. Following the Bragg condition for periodic photonic crystals $2(n_A d_A + n_B d_B) = m\lambda$, where $m$ is an integer. The wavelength of the bandgap for periodic PC is the same as $\lambda_0$ in the plot. While for FQPCs with unit cells $S_3$, two bandgaps emerge at wavelength $\lambda_{31} = 495$ nm and $\lambda_{32} = 988$ nm. In Fig. 2(b), the spectrum of $S_4$, $S_5$, and $S_6$ also show two bandgaps locating at similar positions. The center wavelengths of the bandgaps are $\lambda_{41} = 540$ nm, $\lambda_{42} = 825$ nm for FQPC with unit cell $S_4$, $\lambda_{51} = 524$ nm, $\lambda_{52} = 875$ nm for FQPC with unit cell $S_5$, and $\lambda_{61} = 520$ nm, $\lambda_{62} = 860$ nm for FQPC with unit cell $S_6$, respectively. Comparing the spectrum of the period PC and the Fibonacci quasiperiodic photonic crystals, we see that one more bandgap is generated near the original bandgap of the period PC. Thus, the idea of changing the building blocks of periodic PC would introduce more
bandgaps in its spectrum. Further, with the increment of the Fibonacci sequence generation order \( S_j \), more narrow bandgaps are induced (the small dips grow with the increment of generation order). However, the locations of the two basic bandgaps of FQPCs change little, which can be referred to the self-similar property caused by the Fibonacci sequence. Though the spectrum and the bandgaps alter with the change of the generation order, the main basic property in the spectrum remains still.

Furthermore, we calculated the spectra of multiple HPCs that are composed of various combinations of the Fibonacci series. As shown in Fig. 3(a), the transmittance of Fibonacci quasiperiodic photonic crystal with different building blocks are computed. The unit cells are assembled by \( S_3S_4S_5 \), \( S_4S_5S_6 \), and \( S_5S_6S_7 \) correspond to the solid black line, dash red line, and blue dash-dotted line, respectively. Apparently, the total layer number and the total length for unit cells are unequal. To offset the length differences, we choose different period number \( N \) as illustrated above. For series \( S_3S_4S_5 \), we set \( N_{345} = 5 \), and for \( S_4S_5S_6 \), \( N_{456} = 3 \) and for \( S_5S_6S_7 \), \( N_{567} = 2 \), correspondingly. The first bandgap of the three composited structures locate at 523 nm, 529 nm, and 528 nm, and the second bandgap lies in 877 nm, 862 nm and 865 nm, respectively.

The first bandgap of the three composited structures locate at 523 nm, 529 nm, and 528 nm, and the second bandgap lies in 877 nm, 862 nm and 865 nm, respect to Fibonacci series \( S_3 \), \( S_4 \), and \( S_5 \). The positions of the two bandgaps are almost the same, which can be explained for the self-similarity caused by the Fibonacci sequence. We tried another configuration of series as \( S_3S_4S_5 \), \( S_4S_5S_6 \), and \( S_5S_6S_7 \), and the results are shown in Fig. 3(b). For this configuration, we choose \( N_{343} = 8 \), \( N_{454} = 5 \), and \( N_{565} = 3 \) when calculating the transmittance, respect to series \( S_3S_4S_5 \), \( S_4S_5S_6 \), and \( S_5S_6S_7 \). We can read from the picture that the main properties of the Fibonacci quasiperiodic photonic crystals keep unchanged, with little change in the location of bandgaps. The first bandgap in Fig. 3(b) locates at 514 nm, 531 nm, and 528 nm, and the second bandgap lies in 905 nm, 853 nm, and 877 nm, concerning series \( S_3S_4S_5 \), \( S_4S_5S_6 \), and \( S_5S_6S_7 \). The positions of the two bandgaps stay roughly unchanged, which is due to the self-similarity induced by the Fibonacci sequence following the substitution rule. The small blue shift is the result of unequal optical length as we only keep the structural length roughly the same. Comparing Fig. 3(a) and (b), we can see that the self-similarity induced by the quasiperiodic sequence is strong to hold the external changes. In short, the proposed configuration of the Fibonacci series is a promising candidate as a multichannel filter in the field of integrated photonic circuits.

**Figure 3.** Transmittance spectrum of Fibonacci quasiperiodic photonic crystal with different unit cells. (a) The series of the unit cells are \( S_3S_4S_5 \) (solid black line), \( S_4S_5S_6 \) (dashed red line), and \( S_5S_6S_7 \) (dash-dotted blue line), relate to \( N_{345} = 5 \), \( N_{456} = 3 \) and \( N_{567} = 2 \), respectively. (b) The other series of the unit cells are \( S_3S_4S_5 \) (solid black line), \( S_4S_5S_6 \) (dashed red line), and \( S_5S_6S_7 \) (dash-dotted blue line), relate to \( N_{343} = 8 \), \( N_{454} = 5 \) and \( N_{565} = 3 \), respectively.

Finally, we calculate the transmission spectrum of Fibonacci sequence \( S_1 \) for period number \( N = 11 \) as a function of various refractive index \( n_A \) and wavelength. The refractive index for \( A \) dielectric is set as \( n_A = 1.8 \) (solid black line), \( n_A = 2.0 \) (dash red line) and \( n_A = 2.2 \) (dash-dotted blue line). As shown in Fig. 4, the first bandgap of the hybrid structures locate at 445 nm, 473 nm, and 507 nm, and the second bandgap lies in 738 nm, 786 nm, and 842 nm, respect to \( n_A \) equals to 1.8, 2.0 and 2.2. Obviously, with the increment of the dielectric index, the two bandgaps of Fibonacci quasiperiodic...
photonic crystal both undergo a prominent redshift in its spectrum. The reason for the redshift can be attributed to the growth of the optical length of the structure. That is to say, the increment of the optical length induces the redshift of the bandgap. The trend is the same as the periodic PC structures obeying Bragg condition. This result means that the multichannel filter is tunable by adjusting the index of the dielectric materials.

![Figure 4](image)

**Figure 4.** Transmission spectrum of Fibonacci quasiperiodic photonic crystal with unit cells $S_5$ for $N_5 = 11$ as a function of various refractive index $n_A$. The refractive index for $A$ dielectric is set as $n_A = 1.8$ (solid black line), $n_A = 2.0$ (dash red line) and $n_A = 2.2$ (dash-dotted blue line).

4. **Conclusion**

In this paper, we exploit new spectrum properties by constituting various generation orders of PCs that follow the Fibonacci sequence. By treating series like $S_3 S_4 S_5$ and other compositions as building blocks, we get multiple bandgaps in its spectrum. Thanks to the self-similarity induced by quasiperiodic sequence, the bandgaps hold still for various Fibonacci series. Furthermore, the bandgaps of the Fibonacci hybrid PCs are tunable by merely switching the refractive index of the dielectric materials. For a variety of hybrid structures, the optical properties are worthy of further researches. Moreover, the proposed configuration is a promising candidate as a multichannel filter and a refractive index sensor in the field of integrated photonic circuits.

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