Abstract

New string dynamics is formulated on the basis of the extended set of supergauge constraints including not only supergauge Virasoro conditions but also nilpotent supercurrent constraints. This approach arises from a natural generalization of the classical operator many-string vertices. The formulation of this model leads to three two-dimensional surfaces for description of hadron strings. It gives some dynamical generalization of Chan-Paton factor for string amplitudes in terms of operator vertices. Supersymmetry on the 2-d world surface is present but ten-dimensional supersymmetry is absent. In this approach two-dimensional fermion string fields make it possible to give a unified description of hadron and lepton degrees of freedom and of its dynamics. This model allows to solve the problem of elimination of the most part of parity twins in the baryon spectrum.

One-loop (and many-loops perhaps) amplitudes in this model are finite due to the extended set of supergauge constraints and to the significant excess of the total number of fermion two-dimensional fields over the number of boson 2-d fields.
1 Introduction

In spite of significant success in treatment of hadron interactions of high energy the quantum chromodynamics (QCD) up to now is unable to give a consistent quantitative description of most strong interactions up to 2-5 GeV, that is to say of soft hadron interactions. Chiral models \[1, 2\] are enough effective for small energies of light hadrons \(< 300 MeV\). However the chiral approach solely is incapable to describe the resonance region of hadron interactions in a consistent way.

Attempts to build string-based hadron amplitudes have been undertaken at dawn of the age of string theory (dual resonance models) \[3\]. They were initiated by the remarkable universal linearity of the Regge trajectories for spins of meson and baryon resonances \(J = \alpha(M^2) = \alpha(0) + \alpha' M^2\); \(\alpha' \approx 0, 85 GeV^{-2}\). Now we have these trajectories up to \(J = 5\). From this point of view the \(\rho\)-trajectory with \(\alpha(0) = 1/2\) turns out to be the leading hadron trajectory. Up to now there is no other satisfactory explanation of the striking linearity of hadron Regge trajectories than a stringlike form of hadrons. This hadron string approach was very encouraging with respect to the few particles amplitudes (\(\pi\pi \rightarrow \pi\pi\) for example) in good agreement with experimental data and the chiral limit. But many-particles amplitudes and loop corrections found then proved to break down unitarity. All further consistent string amplitudes for many particles have to include massless vector mesons in the resonance spectrum in order for unitarity requirements to be satisfied. They correspond to the \(\rho\)-trajectory with \(\alpha(0) = 1\).

It is in obvious contradiction with the hadron spectrum observed experimentally. Development of this string approach with the leading Regge trajectory \(\alpha(M^2) = J = 1 + \alpha'M^2\) has led to the usual superstring theory \[4\] for interactions of quarks, gluons and gravitons at Planck distances \((\alpha' \equiv \alpha'_P \sim 10^{-38} GeV^{-2})\) beyond reach of experiment.

The initial case with the \(\rho\)-trajectory \(\alpha(M^2) = J = 1/2 + \alpha'_H M^2\) was realized by Lovelace and Shapiro in the amplitude for \(\pi\pi \rightarrow \pi\pi\) \[5\]. It corresponds to the correct chiral limit and to the Adler-Weinberg condition \[6\]. A few years ago a consistent generalization of Lovelace-Shapiro amplitudes for arbitrary number of pions with the leading trajectory \(\alpha(M^2) = J = 1/2 + \alpha'_H M^2\) has been found \[7, 8\]. New many-pions amplitudes do not contradict unitarity.

Second difficulty of the string approach to hadron interactions was to de-
scribe hadron scattering at large angles for high energies $s$ and high transfer momenta $t$. It may appear that there is an exponential decrease for string amplitudes here in obvious contradiction with experimental data and the parton picture. However discovery of sister trajectories [9] has proved this exponential falloff to be reduced up to a power dependence owing to contributions of many particles states. Moreover as we shall see further both fundamental values of the slope $\alpha'$: the hadronic $\alpha'_H$ and the extreme small Planck $\alpha'_P$ for closed strings are possible in this model. Thus this obstacle was been overcome and this argument against string interpretation of hadrons has lost its strength too.

So the main problem for the string approach to hadrons is to combine requirements of unitarity and the intercept of the leading hadron trajectory to be equal one half (instead of the traditional value to be equal one in the case of classical superstring theories). The absence of ghosts (states with negative norm) in the spectrum of physical states in traditional string models is provided by the Virasoro supergauge constraints to be satisfied. In turn the fulfillment of the Virasoro conditions usually leads to the intercept of the leading Regge trajectory for mesons to be equal one and the presence of massless vector mesons in the resonance spectrum correspondingly. A possible solution of this problem on the basis of some natural generalization of many-string operator vertices ($N$-reggeon vertices [10]) was suggested in [8]. In this model the physical hadron spectrum does not consist massless vector states and the leading Regge trajectory for mesons has the intercept to be equal one half. It proves to be possible due to a new functional approach to string amplitudes. This construction includes some cyclic functional integration which generalizes traditional one and generates natural structure of the Harari-Rosner quark dual diagrams [1] for string amplitudes. Second new attribute of this model is nilpotent supercurrent gauge constraints apart from usual Virasoro supergauge ones. The previous formulation of this approach in [7] is not sufficiently advanced to deduce consistent supercurrent constraints and vertices which will satisfy requirements of unitarity. A new modification to be considered here allows to obtain necessary constraints and operator vertices. Furthermore two-dimensional fields of this model are capable of describing all hadron and lepton degrees of freedom and its interactions.
2 Fermion two-dimensional fields, functional formulation of the model and Virasoro supergenerators

Two-dimensional fields of the model are represented by two sets. First set includes usual fields of the Neveu-Schwarz model [12]: the string coordinate $X_\mu(\xi)$ and its fermion superpartner $H_\mu(\xi)$, $\mu = 0, \ldots, d - 1, d = 10; \xi$ are coordinates on two-dimensional world surface. Second set includes new two-dimensional fermion fields: $\Psi_\alpha(\xi)$ and $\Phi_a(\xi)$. New fields $\Psi_\alpha, \alpha = 1, \ldots 32$ are spinor fields in ten-dimensional target space (compare with the Bardakci-Halpern model [13, 14]). Other fields $\Phi_a$ are two-dimensional fermion fields which carry quantum numbers of all currents $J_a$ composed from $\Psi_\alpha$: $J_a^{(\psi)} = \bar{\Psi}\Gamma_a\Psi$. It is easy to find the number of independent matrices $\Gamma_a$ and of corresponding fields $\Phi_a$. We obtain this number to be equal $32 \times 31 / 2 = 496$ due to the anticommutation of $\Psi_\alpha$ fields. Actually this model includes two sets of new fields as we shall see below. New two-dimensional fermion fields provide a way of obtaining the following results:

1) unified description of all quark flavours and lepton degrees of freedom;
2) consistent from viewpoint of unitarity tree amplitudes for $N$ hadrons due to a new operator construction of these amplitudes;
3) finite one loop (and many loops perhaps) corrections for open strings due to the significant excess of the number of fermion two-dimensional fields over the number of boson fields (including BRST fields).

Let us remind that the divergence of an amplitude $A_1$ for one loop planar $N$-particle diagram in the sector of open strings is defined by integral over the variable $x = x_1x_2\ldots x_N$ of $f(x)$ near $x = 1$. For $x \to 1$ this function behaves in the following manner:

$$f(x) \to r \exp c(D - D')/(1 - x).$$

Here $D$ is the number of boson fields, $D'$ is the number of fermion two-dimensional fields; $x_i$ are the Chan variables; $c = \pi^2 / 6$; $r(x)$ is some power function for $x \to 1$. We have $D = D'$ for usual superstring approach and $D' - D = 2 \times 5 \times (2^5 + 1) / 2 = 512$ for the model under consideration.

It is worth attention that an nonzero value of the one loop amplitude to
be given by integral to be considered may be anomal small in this model due to the factor \( r \exp c(D - D')/(1 - x) \).

Now we define relations between the real quantum numbers and the components of the \( \Psi \) and \( \Phi \) fields. It is convenient to represent 32 components of \( \Psi \) as \( \Psi_{\alpha\beta\gamma\delta} \) components. Here \( \alpha = 1, 2, 3, 4 \) is the usual Dirac index; \( \beta = 1, 2; \gamma = 1, 2; \delta = 1, 2 \) are three isotopic indices which are corresponding to internal quantum numbers and give us eight isotopic components.

We suppose that six additional dimensions form a flat six-dimensional compact space with enough small sizes and with global asymmetry which corresponds to the real world.

It is suitable to build 32 \( \otimes \) 32 matrices in this model in accordance with the \( \Psi_{\alpha\beta\gamma\delta} \) components as direct product of the 4 \( \otimes \) 4 Dirac matrices in the Majorana representation and the 8 \( \otimes \) 8 isotopic matrices.

For instance
\[
\Gamma_{\mu} = \gamma_{\mu} \otimes I \otimes I \otimes I;
\]
\[
T_3^{(1)} = \frac{1}{2} I \otimes \tau_3 \otimes I \otimes I; T_3^{(2)} = \frac{1}{2} I \otimes I \otimes \tau_3 \otimes I; T_3^{(3)} = \frac{1}{2} I \otimes I \otimes I \otimes \tau_3.
\]

Four \( \alpha \) components of \( \Psi \) carry usual spin 1/2, and eight \( \beta, \gamma, \delta \) components correspond to the quark flavours and the lepton numbers according to the Table 1.

|          | \( T_3^{(1)} \) | \( T_3^{(2)} \) | \( T_3^{(3)} \) |
|----------|-----------------|-----------------|-----------------|
| \( \nu \) | +1/2            | +1/2            | +1/2            |
| \( e^- \) | -1/2            | -1/2            | -1/2            |
| \( u \)  | +1/2            | -1/2            | -1/2            |
| \( d \)  | -1/2            | +1/2            | +1/2            |
| \( c \)  | -1/2            | +1/2            | -1/2            |
| \( s \)  | +1/2            | -1/2            | +1/2            |
| \( t \)  | -1/2            | -1/2            | +1/2            |
| \( b \)  | +1/2            | +1/2            | -1/2            |

Table I. Quantum numbers of the \( \Psi \) components

Absence of transitions between quark and lepton degrees of freedom and the differences in the dynamics of its interactions at the experiment one can to present as consequence of some global asymmetry of the compact target.
space and fields on it. This is an asymmetry in reference to $T_3^{(1)}, T_3^{(2)}, T_3^{(3)}$ between "symmetrical" lepton components and "nonsymmetrical" quark ones.

Here we shall give the functional definition of the tree amplitudes of this approach. This representation generalizes the usual functional approach and proves to be the direct consequence of generalization of many-string operator vertices in papers [4, 5].

At first let us introduce some continual generalization of usual multiplication of matrices.

Let $F_{12}(\Delta(\xi^1); \Theta(\chi^2))$ be a functional of functions $\Delta(\xi)$ and $\Theta(\chi)$ on two-dimensional world surfaces $\xi$ and $\chi$ which depends on two fixed points $\xi^1$ and $\chi^2$.

Now we consider the functional integral

$$W(\tilde{\Delta}(\xi^1); \Delta(\xi^2)) = \int D\Theta \exp iS(\Theta)F_{12}(\tilde{\Delta}(\xi^1); \Theta(\chi^2))F_{23}(\Theta(\xi^2); \Delta(\xi^3))$$

(Where $S(\Theta)$ is an action for string fields $\Theta(\xi)$) as a continual generalization of multiplication of two matrices $W_{n1n3} = \sum_{n2} F_{n1n2}F_{n2n3}$.

Correspondingly we consider the following multiple functional integral

$$CTrF_{12}F_{23}F_{34} \ldots F_{N1} = \int D\Psi_1 \exp iS(\Psi_1) \int D\Psi_2 \exp iS(\Psi_2) \ldots \int D\Psi_N \exp iS(\Psi_N)F_{12}(\Psi_1; \Psi_2)F_{23}(\Psi_2; \Psi_3)F_{34}(\Psi_3; \Psi_4) \ldots F_{N1}(\Psi_N; \Psi_1)$$

as a continual generalization of the trace of the product of $N$ matrices.

N-particles string amplitudes for [4] are presented by the usual functional integral $\int DgDXDH \exp iS(X, H, g)$ and a continual trace $CTr$ of the product of vertices $V_{i,i+1}$:

$$A_N = \int d^2\xi^1 \int d^2\xi^2 \ldots \int d^2\xi^N U(\xi^1, \xi^2, \ldots, \xi^N) \int DgDXDH \exp iS(X, H, g)CTr \prod_i V_{i,i+1}$$

$$CTr \prod_i V_{i,i+1} = \int D\Psi_1 D\Phi_1 \exp iS(\Psi_1, \Phi_1)$$

6
\[ \int D\Psi_2 D\Phi_2 \exp iS(\Psi_2, \Phi_2) \ldots \int D\Psi_N D\Phi_N \exp iS(\Psi_N, \Phi_N) \]

\[ V_{12}(\Psi_1(\xi^1), \Phi_1(\xi^1); X(\xi^1), H(\xi^1); \Psi_2(\xi^1), \Phi_2(\xi^1)) \]

\[ V_{23}(\Psi_2(\xi^2), \Phi_2(\xi^2); X(\xi^2), H(\xi^2); \Psi_3(\xi^2), \Phi_3(\xi^2)) \ldots \]

\[ V_{N1}(\Psi_N(\xi^N), \Phi_N(\xi^N); X(\xi^N), H(\xi^N); \Psi_1(\xi^N), \Phi_1(\xi^N)) \] (4)

Vertices of our model are choosing to have a simple symmetry:

\[ V_{ij}(\Psi_i, \Phi_i; X, H; \Psi_j, \Phi_j) = V_{ij}(\Psi_j, \Phi_j; X, H; \Psi_i, \Phi_i) \] (5)

From here on we shall consider tree string amplitudes in the framework of the operator approach which allow us to obtain the necessary consistent description of superconformal constraints and corresponding vertex operators which satisfy these constraints. As usually tree amplitudes of interaction of N strings are represented by some integrals of vacuum expectation values of the product of corresponding operator vertices:

\[ A_N = \int \prod_{i=1}^{N} dz_i Tr \langle 0|V_{12}V_{23}...V_{N1}|0 \rangle \] (6)

For superstring approach operator vertices have the form of commutators with Virasoro supergenerators:

\[ V_{i,i+1}(z_i) = z_i^{-L_0} [G_r, W_{i,i+1}(1) \exp ip_{i,i+1}X(1)] z_i^{L_0} \]

\[ \exp ip_{i,i+1}X(1) = \exp ip_{i,i+1}X^{(+)}(1) \exp ik_i Y_{i0} \]

\[ \exp iq_{i,i+1}X_0 \exp (-ik_{i+1} Y_{i+1,0}) \exp ip_{i,i+1}X^{(-)}(1) \] (7)

The momentum of the physical state \( p_{i,i+1} \) is separated into two parts: the momentum \( k_i - k_{i+1} \) which corresponds to the new set of two-dimensional fields and corresponding surfaces and the momentum \( q_{i,i+1} \). The last moment corresponds to usual two-dimensional NS-surface. In the sector of open strings:

\[ p_{i,i+1} = (k_i - k_{i+1}) + q_{i,i+1}; \]

\[ q_{i,i+1} = \beta (k_i + k_{i+1}) \]

\[ k_i^2 = k_{i+1}^2 = \nu^2; \sum_{i=1}^{N} k_i = 0 \] (8)
\[ \beta^2 \sim \frac{\alpha_H'}{\alpha_H} \]  

We have for arbitrary channels in tree amplitudes:

\[
p_{i,j} = \sum_{l=i}^{j} p_{l,l+1} = (k_i - k_j) + \sum_{l=i}^{j} q_{l,l+1};
\]

In our model we consider factorized vertices in correspondence with (3,4).

\[
W_{i,i+1}(1) = \tilde{F}_i(\Psi; \Phi)\Pi^{(SF)}F_{i+1}(\Psi; \Phi); i = 2, 3, ... N - 1; i \neq 1, N
\]

\[
W_{1,2}(1) = \tilde{F}_1(\Psi'; \Phi')F_2(\Psi; \Phi); W_{N,1}(1) = \tilde{F}_N(\Psi; \Phi)F_{1}(\Psi'; \Phi')
\]

\[\Pi^{(SF)} = |0(\Phi\Phi)\rangle \langle 0(\Phi\Phi)|\] is the projector onto zero occupation numbers of modes of \(\Psi\) and \(\Phi\) fields. It is evidently that namely the projector \(\Pi^{(SF)}\) and separation of \(\Psi'; \Phi'\) and \(\Psi; \Phi\)-modes bring amplitudes (5) with vertices (6,7,8) in correspondence with (3,4). Now we consider the construction of the Virasoro generators of the superconformal algebra for this model.

The operators \(G_r\) are presented by the sum

\[G_r = G_r^{NS} + G_r^{SF} + G_r^{SF'}\]

where

\[G_r^{NS} = \frac{1}{2\pi} \int_0^{2\pi} d\tau (H^\mu \frac{d}{d\tau} X_\mu + \dot{P}_\nu H^\nu) e^{-i\tau r}\]

is the supergenerator to be formed in the ordinary way with help of the old Neveu-Schwarz fields \(X_\mu(\tau)\) and \(H_\mu(\tau)\)

\[\dot{P}_\nu = \sum_i \omega_{i\nu} \Gamma_i\]

for \(\nu = 0, 1, 2, 3\)

\[\dot{P}_\nu = \Gamma^{(\alpha')} P_{\alpha'}\]

for \(\nu = 4, 5, 6, 7, 8, 9\)

\[\dot{P}_\nu = (\frac{R_{\nu}}{n_{\nu}} + \sum_i \tilde{\omega}_{i\nu} \Gamma_{cm})\]

\(n_{\nu} = 0; \pm 1; \pm 2; \pm 3...\) and

\[G_r^{SF} = G_r^{(0)SF} + \Delta_r^{SF}\]

is the supergenerator to be formed with help of the new fermion fields \(\Psi\) and \(\Phi\),

\[G_r^{(0)SF} = \frac{1}{2\pi} \int_0^{2\pi} d\tau \eta (\frac{1}{4} \tilde{\Psi} \Gamma_a \Phi^a \Psi + \sum_{a,b,c} \Phi_a \Phi_b \Phi_c) e^{-i\tau r}\]
\[ \eta = \frac{1}{2i\sqrt{3}}; \]  
\[ \Delta_{r}^{SF} = \frac{1}{2\pi} \int_{0}^{2\pi} d\tau \rho^{cd} \tilde{\Gamma}_{e} \Phi_{d} e^{-i\tau}; \]  
\[ \tilde{\Psi} = \Psi T_{0}; \]  
\[ T_{0} = \gamma_{0} \otimes \tau_{2} \otimes \tau_{2} \otimes \tau_{2} \]

The fields \( \Phi_{a}, \Phi_{b}, \Phi_{c} \) in second sum \( \sum_{a,b,c} \Phi_{a} \Phi_{b} \Phi_{c} \) correspond to currents \( \tilde{\Psi} \Gamma_{a} \Psi, \tilde{\Psi} \Gamma_{b} \Psi \) and \( \tilde{\Psi} \Gamma_{c} \Psi \) in the sum \( \tilde{\Psi} \Gamma_{a} \Phi^{a} \Psi \). These matrices \( \Gamma_{a}, \Gamma_{b} \) and \( \Gamma_{c} \) obey the following equations:

\[ \Gamma_{a} \Gamma_{a} = \Gamma_{b} \Gamma_{b} = \Gamma_{c} \Gamma_{c} = I \]

\[ [\Gamma_{a}, \Gamma_{b}] = i\Gamma_{c} \]

The supergenerator \( G_{r}^{SF'} \) is precisely the same as \( G_{r}^{SF} \) with substitution of \( \Psi' \) and \( \Phi' \) fields for \( \Psi \) and \( \Phi \) ones.

The important distinguishing feature of this approach from previous works \[7, 8\] is an inclusion of special \( 32 \otimes 32 \) matrices \( \Gamma \) in the linear parts \( \Delta_{r}^{SF} \) of \( G_{r}^{SF} \) and in the linear part of \( G_{NS}^{r} \). It reminds introduction of \( \gamma_{5} \) and \( \gamma_{\mu} \) in the supergenerators \( F_{n} \) of the Ramond model \[15\]. So vertex operators become matrix operators and then corresponding amplitudes are able to reproduce the structure of the dual quark Harari-Rosner diagrams and the Chan-Paton factor \[16\] in the natural way. The matrix \( \Gamma^{(a')} \) provides a separation of hadron and nonhadron Regge trajectories in their slopes \( a' \). To be more precise we suppose some universal slope for hadron trajectories and some very small slope for nonhadron Regge trajectories in the Born approximation.

The field \( \Phi_{d} \) corresponds to the matrix \( \Gamma_{d} \) which is present in the term \( \tilde{\Psi} \Gamma_{d} \Phi^{d} \Psi \) of the expression for \( G_{r}^{SF} \). In its turn \( \Phi_{d} \) defines components of the current \( J^{d}_{n} \):

\[ J^{d}_{n} = [G_{r}^{SF}, \Phi^{d}_{n-r}]. \]

For fields of the model under consideration we have the usual commutation relations corresponding to free two-dimensional fields. The commutators for the \( X_{\mu} \) components \( X_{\mu} = \sum_{n \neq 0} \frac{1}{m} a_{n \mu} e^{-inv} \) are

\[ [a_{n\mu}, a_{m\nu}] = -ng_{\mu\nu} \delta_{n,-m}; \]

\[ [P_{\nu}, X_{0\mu}] = -\frac{1}{i} g_{\mu\nu} ; \]

\[ a^{+}_{n \mu} = a_{-n \mu}; \]
The anticommutators for the others $H_\mu$, $\Psi_\alpha\beta\gamma\delta$, $\Phi_a$ are following ones:

$$
H_\mu = \sum_r b_{r\mu} e^{-ir\tau}; \quad \Phi_a = \sum_r \phi_{a,r} e^{-ir\tau};
$$

$$
\Psi_\alpha = \sum_r \psi_{\alpha,r} e^{-ir\tau}; \quad r = \pm 1/2; \pm 3/2; \pm 5/2... ;
$$

$$
\{b_{r\mu}, b_{s\nu}\} = -g_{\mu\nu} \delta_{r,-s}; \quad \{\phi_{a,r}, \phi_{b,s}\} = \delta_{a,b} \delta_{r,-s}; \quad \{\bar{\psi}_{\alpha,r}, \psi_{\beta,s}\} = \delta_{\alpha,\beta} \delta_{r,-s}
$$

As usually $b^+_r = b_{-r}$; $\phi^+_{a,r} = \phi_{a,-r}$; $\psi^+_{\alpha,r} = \psi_{\alpha,-r}$

$$
b_r|0> = \phi_{a,r}|0> = \psi_{\alpha,r}|0> = \phi_{a,-r} = \psi_{\alpha,-r} = 0 ; \quad r > 0
$$

The anticommutators for the $\Psi'_{\alpha\beta\gamma\delta}$, $\Phi'_a$ are the same as for $\Psi_{\alpha\beta\gamma\delta}$, $\Phi_a$.

We suppose $[\tilde{\Gamma}_c, \tilde{\Gamma}'_c] = 0; \quad [\tilde{J}_0^c, \tilde{J}'_0^c] = 0;
$ $[\hat{\Gamma}_d, \hat{P}_\nu] = 0; \quad [\hat{P}_{-\mu}, \hat{P}_{-\nu}] = 0;$

Then we get the canonical superconformal algebra of generators

$$
G_r = G_r^{NS} + G_r^{SF} + G_r^{SF'}:
$$

$$
\{G_r, G_s\} = 2L_{r+s} + \frac{c}{3}(r^2 - 1/4)\delta_{r,-s}
$$

$$
[L_n, L_m] = (n - m)L_{n+m} + \frac{c}{12}n(n^2 - 1)\delta_{n,-m}
$$

$$
[L_n, G_r] = (n/2 - r)G_{n+r}
$$

Here $c = 3d/2 + d'(d' + 1)/4;$

$d = 10; d' = 2d/2 = 32$

Due to these commutation relations additional matrix components $\tilde{\Gamma}^2_d$ arise in the operator $L_0$ from linear parts $\tilde{\Gamma}_d\Phi^d$ of operators $G^SF$. They do not vanish on vacuum states and hence will break down the crossing symmetry and duality of string amplitudes. We regain these symmetries by way of a natural generalization of the matrix parts of operators $G_r$. We shall modify somewhat matrix terms of $G_r$ in order for the operator $L_0$ to be vanished while acting on the vacuum state. With this aim in mind we introduce operators to be similar to zeroth modes of the fields $\Psi$. Here we do not consider relation between these modes and a symmetry of the compact space of six-dimensional torus. We hope to discuss this point in future.
Namely we introduce usual spinor operators with quantum numbers of $\Psi$-fields:

$$\lambda^{(\pm)}; \mu^{(\pm)};$$

These operators obey simple equations:

$$\{\lambda^{(-)}, \lambda^{(+)}\} = \delta_{\alpha,\beta}; \lambda = \lambda T_0;$$

$$\langle 0 | \lambda^{(+)} = 0; \lambda^{(-)} | 0 \rangle = 0;$$

And we have precisely the same equations for $\mu$ - operators.

We substitute the expressions

$$\sum_{c,d} \rho_{cd} \tilde{\Gamma}_c \Phi^d$$

by

$$\sum_{c,d} \rho_{cd} (\tilde{\lambda}^{(+)} \tilde{\Gamma}_c \lambda^{(-)}) \Phi^d$$

and the expressions

$$\sum_{c,d} \rho_{cd} \tilde{\Gamma}_c \Phi^d$$

in $G^{SF}_r$ by

$$\sum_{c,d} \rho_{cd} (\tilde{\mu}^{(+)} \tilde{\Gamma}_c \mu^{(-)}) \Phi^d$$

Besides we substitute other matrix terms in $G_r$. Namely we substitute the expression $\hat{P}_\nu = \sum_i \omega_{i\nu} \Gamma_i$ by the sum

$$\hat{P}_\nu = \tilde{\lambda}^{(+)} \Gamma^{(\alpha')} \lambda^{(-)} \frac{1}{i} \frac{\partial}{\partial Y_0} + \tilde{\mu}^{(+)} \Gamma^{(\alpha')} \mu^{(-)} \frac{1}{i} \frac{\partial}{\partial Y_0} + \frac{1}{i} \frac{\partial}{\partial Y_0}$$

(21)

for $\nu = 0, 1, 2, 3$

$$\Gamma^{(\alpha')} = b_q \Pi_q + b_l \Pi_l$$

(22)

and by the sum

$$\frac{n_\nu}{R_\nu} + \tilde{\lambda}^{(+)} \Gamma^{cm}_\nu \lambda^{(-)} + \tilde{\mu}^{(+)} \Gamma^{cm}_\nu \mu^{(-)} + J^{cm}_\nu + J^{'cm}_\nu;$$

$$n_\nu = 0, \pm 1, \pm 2, \pm 3, ...$$

for $\nu = 4, 5, 6, 7, 8, 9$.

$$J^{cm}_\nu = \frac{2}{\eta} \left[ G^{SF}_r, (\Phi^{cm})_{-r} \right]$$

(23)

$$J^{'cm}_\nu = \frac{2}{\eta} \left[ G^{SF'}_r, (\Phi^{cm})_{-r} \right]$$

(24)
The field $\Phi_{\nu}^{cm}$ corresponds to the matrix $\tilde{\Gamma}_{\nu}^{cm}$. Here

$$\Pi_\nu = I \otimes \left( \frac{3}{4} I \otimes I \otimes I - \frac{1}{4} (\tau_3 \otimes \tau_3 \otimes I + \tau_3 \otimes I \otimes \tau_3 + I \otimes \tau_3 \otimes \tau_3) \right)$$

$\Pi_\nu$ is the projector on quark components

$$\Pi_l = I \otimes \frac{1}{4} (I \otimes I \otimes I + \tau_3 \otimes \tau_3 \otimes I + \tau_3 \otimes I \otimes \tau_3 + I \otimes \tau_3 \otimes \tau_3)$$

$\Pi_l$ is the projector on lepton components.

The currents $J_{\nu}^{cm}, \lambda^{(+)} \Gamma_{\nu}^{cm} \lambda^{(-)}; \nu = 4, 5, 6, 7, 8, 9$ enter along with compact components of momenta $P_\nu$ and related to quantum numbers of physical states.

This modification does not change the superconformal algebra of operators $G_r$ considered above and the conditions of nilpotency of the BRST charge but provides for vanishing of $L_0$ on vacuum states, crossing symmetry and permits to bring string amplitudes of our model in accordance with the quark dual diagrams.

It is worth to note that the operator $L_0$ may be represented as the following sum:

$$L_0 = R - \frac{\hat{D}^2}{2} + \Delta^{(L_0)}$$

$$\Delta^{(L_0)} = \left\{ G_r^{(0)SF}, \Delta_{SF}^{SF} \right\} + \left\{ G_r^{(0)SF'}, \Delta_{SF'}^{SF'} \right\}$$

in correspondence with equations (15)-(20).

Let us introduce some local multiplication $\lambda-$ and $\mu-$ currents:

$$\tilde{\lambda}^{(+)} \Gamma_i \lambda^{(-)} \ast \tilde{\lambda}^{(+)} \Gamma_j \lambda^{(-)} \equiv \tilde{\lambda}^{(+)} \Gamma_i \Gamma_j \lambda^{(-)}$$

$$\tilde{\mu}^{(+)} \Gamma_i \mu^{(-)} \ast \tilde{\mu}^{(+)} \Gamma_j \mu^{(-)} \equiv \tilde{\mu}^{(+)} \Gamma_i \Gamma_j \mu^{(-)}$$

These results can be obtained as corresponding operator products:

$$\tilde{\lambda}^{(+)} \Gamma_i \Pi^{(\lambda)} \lambda^{(-)} \tilde{\lambda}^{(+)} \Gamma_j \Pi^{(\lambda)} \lambda^{(-)} \equiv \tilde{\lambda}^{(+)} \Gamma_i \Gamma_j \Pi^{(\lambda)} \lambda^{(-)}$$

where

$$\Pi^{(\lambda)} \equiv |0^{(\lambda)} \rangle \langle 0^{(\lambda)}|$$
is the projector onto zero occupation numbers of the \( \lambda \) modes. And we can write down similar expressions for \( \mu \)-modes.

Now it is possible to formulate our meson amplitudes \( A_N \) by entering \( \lambda, \mu \)-modes in operator vertices (7,8,13,14) and replacing the operator \( \Pi^{(SF)} \) by the product \( \Pi^{(SF)}\Pi^{(\lambda)}\Pi^{(\mu)} \) in the expression (13). The cyclic operator trace for \( \lambda \) and \( \mu \)-modes generalizes the Chan-Paton factor in a natural way.

For example let us consider the following expression:

\[
T_N = \langle 0 | \mu^{(-)} W_1 \lambda^{(-)} \bar{\lambda}^{(+)} W_2 \lambda^{(-)} \bar{\lambda}^{(+)} W_3 \lambda^{(-)} \ldots \\
\bar{\lambda}^{(+)} W_{N-1} \lambda^{(-)} \bar{\lambda}^{(+)} W_N \mu^{(+)} | 0 \rangle
\]

This product turns into the traditional Chan-Paton factor in the simplest case of matrices \( \Gamma_i \Pi^{(\lambda)} \) instead of \( W_i \):

\[
\langle 0 | \mu^{(-)} \Gamma_1 \lambda^{(-)} \bar{\lambda}^{(+)} \Gamma_2 \Pi^{(\lambda)} \lambda^{(-)} \bar{\lambda}^{(+)} \Gamma_3 \Pi^{(\lambda)} \lambda^{(-)} \ldots \\
\bar{\lambda}^{(+)} \Gamma_{N-1} \Pi^{(\lambda)} \lambda^{(-)} \bar{\lambda}^{(+)} \Gamma_N \mu^{(+)} | 0 \rangle \equiv Tr(\Gamma_1 \Gamma_2 \Gamma_3 \ldots \Gamma_N)
\]

3 Nilpotency of the BRST-charge operator and the supercurrent conditions

So we have the additional central charge \( c = d'(d' + 1)/2 \) in the Virasoro superalgebra of \( G_r \) and \( L_n \) operators owing to the new fields \( \Psi \) and \( \Phi \). There is a need to have new additional gauge symmetries in order that one can compensate this new central charge by the corresponding central charge of BRST-ghosts and obtain the nilpotency of the BRST-charge operator.

There are some indications of existence of such symmetries in appearance of a great many currents to be conserving on the mass shell in the Bardakci-Halpern string model [14]. Moreover there are evidences for these symmetries in the very approach under consideration. Indeed it turns out that the operators \( L_n^{SF} \) in this model may be written in Sugavara form [17] as \( L_n^{SF} =: \sum_a J_a^\lambda \bar{J}_a^\mu : \) where \( J_a^\mu = \{ G_{l-r}, \phi_r^a \} \). In general case the Sugavara form to be a normal product of current constituents contains nonsingular four-fermion components after its expansion in normal products in relation to fermion constituents. However here as in [7, 8] all these four-fermion components are cancelled and the operators \( L_n^{SF} \) acquire usual form of the Neveu-Schwarz model operators. The scalar product \( \sum_a J_a^\mu(\tau)J_a^\mu(\tau) \) has a
symmetry in relation to rotations of vectors $J$. These rotations are generated by some linear combination $\hat{J}$ of the very currents. By virtue of the commutation relations:

$$[J_a^l, J_b^l] = f_{abc} J_c^{l+n} + l\delta_{l-n} \delta_{a,b},$$

invariance of the operator expression $\sum_a J^a(\tau) J^a(\tau)$ with respect to these rotations will take place in the case that second anomalous term in the commutation relations will be absent for this combination $\hat{J}$ i.e.

$$[\hat{J}^s_n, \hat{J}^s_{-n}] = 0.$$

This situation reminds two-dimensional models with four-fermion interaction \[18\]. These models may obey similar symmetries and in this case ones have the spectrum of physical states of a free model. It corresponds to our commutation relations which are valid for free fields. On other hand these new fields $\Psi$ and $\Phi$ lead to new states of negative norm (”ghosts”). So the additional symmetries are necessary for new ”ghost” states to be vanished as unitarity requires. Due to the superconformal symmetry of our model new current gauge conditions to be generated by the operators $\hat{J}$ must be accompanied by fermion conditions to be generated by fields $\hat{\Phi}$ which are superpartners of $\hat{J}$: $-l\hat{\Phi}_r = \{G_{r-l}, \hat{J}_l\}$.

It is easy to find the number $N$ of necessary additional supercurrent constraints from the nilpotency BRST-charge $\Omega$: $\Omega^2 = 0$. We have previous BRST-ghost fields of the dimensionality $j$ to be equal 2 and 3/2 for $L_n$ and $G_r$ correspondingly. Now new BRST-ghost fields of the dimensionality $j$ to be equal 1 and 1/2 for $N$ new $\hat{J}^{(i)}_n$ and $\hat{\Phi}^{(i)}_r (i = 1 \ldots, N)$ constraints have to be added. The contribution of the BRST-ghost field of the dimensionality $j$ to central charge is equal to $\pm (3(2j - 1)^2 - 1)$. The sign is determined by the statistics of the BRST-ghost field. Let us denote the number of boson matter string fields by $D$ and the number of fermion fields by $D'$. Then we obtain first condition of the nilpotency BRST-charge from the requirement of the total central charge to be vanished: $D + \frac{D'}{2} - 26 + 11 - 2N - N = 0$.

For $D = 10, D' = 10 + 32 + 32 \ast 31/2$ we get $N=88$. In the case of the arbitrary even dimensionality of $D$ we have

$$D' = D + 2^{D/2}(2^{D/2} + 1)/2$$

here $2^{D/2}$ is the number of spinor components of $\Psi_\alpha$; $2^{D/2}(2^{D/2} - 1)/2$ is the number of currents from these spinor components. This condition gives possible values of $D$: $D = 6(mod(4)); D = 6, 10, 14, 18, 22, 26, \ldots$. 

Second condition of the nilpotency $\Omega^2 = 0$ requires $L_0 = 1/2$ on physical states. Due to the additional terms $(\sum_{c,d} \rho_{cd}(\lambda(+)\Gamma_c\lambda(-))^2$ and $(\sum_{c,d} \rho_{cd}(\bar{\mu}^+(+)\bar{\Gamma}_c\mu(-))^2$ in $L_0$ this condition does not lead now to $J = 1 + \alpha M^2$ for leading hadron Regge trajectory and does not require $M_\rho^2 = 0$ correspondingly.

Third nontrivial condition from $\Omega^2 = 0$ results in norms of the $\hat{\Phi}$ to be vanished. And as consequence the norms of the currents $\hat{J}$ are to be vanished too. In its turn it brings to the requirement of the vanishing corresponding Killing metrics $g_{ij}$ for group space of $\hat{J}$:

$$\left[ \hat{J}_i^s, \hat{J}_{-i}^s \right] = 0, \{ \hat{\Phi}_i^s, \hat{\Phi}_{-i}^s \} = 0$$

$$g_{ij} = \sum_{kl} f_{ikl} f_{jlk} = 0 ; \left[ \hat{j}_i^j, \hat{j}_n^k \right] = f_{ij} f_{kn}$$

Now we shall define current operators $\hat{J}^{(s)}$ for our nilpotent supercurrent conditions on the basis of general requirements found above. In order to explain the proposed choice of $\hat{J}$ we shall consider more simple case \cite{19}: $d = 6$; the field $\Psi_{\alpha\beta}$ is an eight-component spinor, $\alpha = 1, 2, 3, 4$ is an usual Dirac index, $\beta = 1, 2$ is an isotopic index.

We take the field $\Phi_k$ to be corresponding to the matrix:

$$\Gamma^k = \gamma_\mu k^\mu/\sqrt{k^2} \otimes I = \gamma_0 \otimes I$$

And then we can choose the required fields $\hat{\Phi}$ for supercurrent conditions as the fields to be determined by the matrices $\Gamma^k_i$:

$$\Gamma^k_i = (I + \Gamma^k)\Gamma_i$$

here $\Gamma_i$ are matrices which anticommute with $\Gamma^k$:

$$\Gamma_i = \gamma_5 \otimes \tau_i; \ i = 1, 2, 3; \quad \Gamma_j = \gamma_j \otimes I ; \ j = 1, 2, 3 ;$$

$$\{ \Gamma_{ij}, \Gamma^k \} = 0 \text{ We suppose here } \gamma_\mu k^\mu/\sqrt{k^2} = \gamma_0.$$

Matrices $\Gamma^k \Gamma_{ij}$ anticommute with $\Gamma^P$ too. But they do not bring to new matrices $\Gamma^k_i$.

Corresponding to $\Gamma^k_i$ the currents $J^k_i$ transfer the $\Psi$ components with eigenvalue of operator $\Gamma^k$ to be equal -1 into other $\Psi$ components with the same eigenvalue of parity. Components of $\Psi = \Psi T_0 \equiv \Psi \gamma_0 \otimes \tau_2$ have eigenvalues of $\Gamma^k$ which are opposite to eigenvalues of $\Psi$ components by virtue of antisymmetry of the $\Gamma^k$ matrix. ( We use in our approach matrices $\gamma_{\mu}$ in the
Majorana representation.) Namely from $\Gamma^k \Psi = \lambda \Psi$ we have $\bar{\Psi} \Gamma^k = -\lambda \bar{\Psi}$. In this simple case the number of corresponding nondiagonal transitions is equal to $N = 4 \times 3/2 = 6$. Since $\Gamma_i^k \Gamma_j^k = 0$ commutators of operators $\hat{\Phi}$ and $\hat{J}$ are vanished and we have $\hat{J}_n^i = \{G_{n-r}, \hat{\Phi}_r^i\}, i = 1, 2, 3, \ldots 6$; $[\hat{J}_n, \hat{J}_{-n}] = 0$

We note that full current operators $\hat{J}_n^i$ defined above contain the $\Phi^j \Phi^k f_{ijk}$ terms besides $\bar{\Psi} \Gamma_i \Psi$ components.

Let us choose operators $\hat{J}$ in our model in a similar way. From point of view of necessary exclusion of ”ghost” components and quantum numbers of observed states the most appropriate operator which gives eigenvalues for transitions corresponding to currents $\hat{J}$ is the parity operator $J_P$. The operator $\Phi_P$ for the current $J_P$ corresponds to the matrix $\Gamma_P$:

$$\Gamma_P = \frac{\hat{P}}{\mu} \otimes I \otimes I \otimes I$$  

(27)

Here $\mu = \sqrt{P^2}$.

Now we shall choose 66 current operators $\hat{J}$ by using eigen values of $\Psi$ quark components in relation to the operator $J_P = \frac{2}{\eta} \{G_{-r}, \hat{\Phi}_P^r\}$ for corresponding transitions. The field $\Phi_P$ corresponds to the matrix $\Gamma_P$. The spinor field $\Psi$ has 24 components of upper and lower quarks. From them 12 components have the eigenvalues of $J_P$ to be equal +1 and the other 12 components have the opposite eigenvalues to be equal -1. By taking transitions between the latter components we obtain $12 \times 11/2 = 66$ transitions and 66 corresponding operators $\hat{J}$ for quarks. The matrices for these transitions are analogous to the above ones

$$\Gamma_{qP}^i = (I + \Gamma_P^{(q)}) \Pi_q \Gamma_{ij}^q;$$  

(28)

$$\{\Gamma_{ij}^q, \Gamma_{ij}^{(q)}\} = 0$$  

(29)

$$\Gamma_P^{(q)} = \Gamma_P \Pi_q$$  

(30)

In the same way 8 lepton components of $\Psi$ can be divided in relation to the eigenvalues of $J^{(P)}$ into 4 components with the eigen value of $J^{(P)}$ to
be equal +1 and 4 ones with the eigen value to be equal -1. Hence we have here 4 \times 3/2 = 6 operators to be corresponded transitions from components \( \Psi \) with the eigen value of \( J^{(l)}_P \) to be equal -1 into ones with the same eigen value and 6 currents \( \hat{J} \).

\[
\Gamma^i_{lp} = (I + \Gamma^{(l)}_P)\Pi_l \Gamma^{il};
\]

(31)
\[
\{\Gamma^{il}, \Gamma^{(l)}_P\} = 0
\]

(32)
\[
\Gamma^{(l)}_P = \Gamma_p \Pi_l
\]

(33)

Now we shall choose the remaining 16 operators \( \hat{J} \) as corresponding components of \( \Psi \):

\[
\tilde{\Psi}(I + \Gamma_P) = \tilde{\Psi}(I + \frac{\hat{P}}{\mu} \otimes I \otimes I \otimes I) = -(I - \Gamma_P)\Psi
\]

(34)

In all, we have chosen 66+6+16=88 operators \( \hat{J} \) required for the supercurrent constraints.

It is easy to see that all commutators of the 88 operators of three sets are vanished and give the vanishing corresponding Killing metrics \( g_{(ij)} \). All these operators have vanishing norm. Thus all requirements of the nilpotency of the BRST charge are satisfied for this choice of supercurrent constraints. This choice is quite definite and natural but it can not pretend to the uniqueness. It is interesting to note that the number of operators \( \hat{J} \) is equal to 88 only for given division of 32 \( \Psi \) components into 24 quark components and 8 lepton ones. Other divisions lead to other values of the number of operators \( \hat{J} \).

Similarly we choose \( \hat{J}_n' \)- and \( \hat{\Phi}_r' \)-constraints replacing \( \Psi, \Phi, \lambda \)-operators by \( \Psi', \Phi', \mu \)-operators in the expressions considered above.

As discussed above we are guided by properties of the real physical states and the requirements of the generalized algebra of gauge constraints i.e. the Virasoro superalgebra with the nilpotent supercurrent conditions.

A wave function of physical state is determined by the expression:

\[
|\text{Phys}\rangle = G_{\frac{1}{2}}|W_{\text{phys}}\rangle
\]

(35)
So our constraints on the wave function of physical state are expressed by the following equations:

\[
(G^N_S + G^S_F + G^S_F')|W_{phys}\rangle = 0; \quad (NS) \\
(L^N_S + L^S_F + L^S_F')|W_{phys}\rangle = 0; \quad r, n > 0 \quad (36)
\]

\[
\hat{\Phi}_r^i|W_{phys}\rangle = 0; \quad \hat{\Phi}_r^{ij}|W_{phys}\rangle = 0; \quad i = 1, 2, 3, ... 72; \quad r > 0 \quad (37)
\]

\[
(L^0_N S + L^0_S F + L^0_S F')|W_{phys}\rangle = 1/2|W_{phys}\rangle; \quad (41)
\]

It is worth noting that the evidence of the conditions mentioned above for physical states in tree amplitudes can be obtained with help of the expansion of unity for given channel as it was made in previous version \cite{7} with the inclusion into consideration of fields operators to be superfluous for given channel. In doing so it is necessary at first to separate states generated by nilpotent supercurrent operators, then states generated by Virasoro supergenerators with \(r, n > 1/2\), then states generated by superfluous for given channel operators and at last by Virasoro supergenerators with \(r=1/2\).

4 Fermion states and elimination of the most part of parity twins from the baryon spectrum

The supercurrent conditions (39) confine essentially choice of fermion physical states and eliminate the most part of parity twins from the fermion spectrum of physical states due to the projector \((I - \Gamma_P)\) in the operator (34).

It is necessary to find the formulation of proper operator vertices which satisfy these supercurrent constraints in fermion channels. The direct inclusion of the projector on eigen values +1 of the current \(J_P\) is not compatible
with analyticy owing to presence in $J_P$ the singular function $\mu = \sqrt{P^2}$. We can to avoid this obstacle by replacing $\mu = \sqrt{P^2}$ with an equivalent nonsingular operator expression which does not break down the Virasoro conditions. For this purpose we introduce an additional dependence of some angle $\phi_i$ in operator vertices $V_i(z_i)$ and corresponding integration over these angles from 0 to $2\pi$. This construction is similar to the operator vertex for emission of an usual closed string state \[21\]:

$$V_{i,i+1}(z_i; \phi_i) = \phi_i^{R - \tilde{R}} z_i^{-L_0} [G_r, W_{i,i+1}(1) \exp i\tilde{p}_{i,i+1}X(1)] z_i^{L_0} \phi_i^{-R + \tilde{R}}; \quad (42)$$

Here (see (25),(26))

$$\bar{R} = L_0 + \frac{\tilde{p}^2}{2} - \Delta^{(L_0)}$$

$$\tilde{R} = 2J_{\mu\nu}^{SF} J_{NS}^{\mu\nu} + \frac{1}{4}(1 + (-1)^{G_{NS}}) + 1; \quad \mu, \nu = 0, 1, 2, 3$$

$$G_{NS} = \sum_r (b_{\mu})_r (b^\mu)_r$$

The operators $\frac{1}{2}(1 \pm (-1)^{G_{NS}})$ are the projectors onto states which have the positive or negative $G_{NS}$-parity.

$J_{NS}^{\mu\nu}$ is the part of the angular momentum in the Minkowski space which determines by the NS fields i.e. $X$; $H$-fields;

$J_{SF}^{\mu\nu}$ is the operator of the angular momentum for the $\Psi$; $\Phi$; $\Psi'$; $\Phi'$-fields only.

Let us note that the operators $J_{\mu\nu}^{\mu\nu}$ and $J_{\mu\nu}^{SF}$ are commuting with the Lorentz scalar supergenerators $G_r, L_n$. The operators included into vertices do not break down the supergauge constraints (36)-(41).

These operators $R$ and $\tilde{R}$ have half-integer and integer eigen values, the operator $R - \tilde{R}$ has integer eigen values only.

The integration over $\phi_i$ leads to the operator

$$\sin \frac{\pi}{\pi}(R - \tilde{R})$$

and hence to the condition:

$$R = \tilde{R}; \quad L_0 = 2J_{\mu\nu}^{SF} J_{NS}^{\mu\nu} + \frac{1}{4}(1 + (-1)^{G_{NS}}) - \frac{\tilde{p}^2}{2} + \Delta^{(L_0)} + 1. \quad (43)$$
Now we are able to introduce the necessary parity projector for fermion states in the supergauge invariant manner as the operator $\Pi^{(\Psi)}_P$ and to substitute our vertices by $\Pi^{(\Psi)}_P V_{i,i+1}(z_i; \phi_i) \Pi^{(\Psi)}_P$:

$$\Pi^{(\Psi)}_P = \tilde{J}_P - (-1)^{G_{NS}} \hat{M};$$  \hspace{1cm} (44)

$$\tilde{J}_P = \frac{2}{\eta} \{ G_{-r}, \tilde{\Phi}^P_r \};$$  \hspace{1cm} (45)

$$\frac{\hat{M}^2}{2} = 2 J^{SF}_{\mu\nu} J^{NS}_{\mu\nu} + \frac{1}{4} (1 + (-1)^{G_{NS}}) + \Delta^{(L_0)}$$  \hspace{1cm} (46)

The operator $\tilde{\Phi}^P$ corresponds to the matrix $\tilde{\Gamma}_P$:

$$\tilde{\Gamma}_P = \tilde{P} \otimes I \otimes I \otimes I$$  \hspace{1cm} (47)

These modified operator vertices satisfy the conditions (39) and give some possible solution of the problem of the parity twins. Here the most of the parity twins appear to be spurious gauge states excluded from the spectrum of the physical states.

5 Linear terms of the supergauge conditions and fundamental interactions

Now we shall define more precisely linear in fields terms in the operator $G_r^{NS}$ and in the operator $G_r^{SF} = G_r^{(0)SF} + \Delta_r^{SF}$. Their detailed form determines vertex operators for amplitudes of interaction of particles and our approach in description of fundamental interactions to a great extent. Let us note that our treatment of gauge supercurrent nilpotent conditions and formulation of linear terms here differ essentially from previous versions [7, 20, 22].

We have rigorous commutation conditions for these linear terms noted above in second section. These requirements of commutations give the following equations for corresponding matrices:

$$[\tilde{\Gamma}_c, \tilde{\Gamma}'_c] = 0; [\tilde{\Gamma}_d, \Gamma^{(\alpha')}] = 0; [\tilde{\Gamma}_\nu^{cm}, \tilde{\Gamma}'_\nu^{cm}] = 0; [\tilde{\Gamma}_d, \Gamma^{cm}] = 0; [\Gamma^{cm}_\nu, \Gamma^{(\alpha')}] = 0.$$

These commutation relations are necessary for conservation of the form of commutators of $G_r$. These constraints and the Lorentz covariance restrict significantly a choice of possible matrices $\tilde{\Gamma}_c, \Gamma^{cm}_\nu$ and fields $\Phi^d$. We shall
determine fields $\Phi^d$ by matrices $\Gamma_d$ which enter in the part $\Psi \Gamma_d \Phi^d \Psi$ of the operator $G^{SF}_{r}$ with $\Phi^d$. We take the operators $\tilde{J}^d$ which appear in $L_0$ to be commuting with the operator $J_P$ so these fields $\tilde{\Phi}^d$ will be consistent with the new supercurrent constraints.

This choice of suitable matrices $\Gamma^e_{\nu}$ provides a possibility of electromagnetic and weak fundamental interactions due to vector mesons arising in nonplanar one-loop amplitudes (closed string sector) in addition to usual tensor particles (gravitons). Their generalized ten-dimensional masses are vanished but usual four-dimensional masses are nonzero for $W_\pm$ and $Z$-bosons. Let us remind that the momentum $P = \sum_i p_{i,i+1}$ is separated into two parts:

$$\sum_i p_{i,i+1} = \sum_i (k_i - k_{i+1}) + \sum_i q_{i,i+1}; \quad (48)$$

$$k^2_i = k^2_{i+1} = \nu^2; \quad q_{i,i+1} = \beta (k_i + k_{i+1}); \quad \beta^2 \sim \frac{\alpha'}{\alpha_H^2}.$$  

The momentum $\sum_i (k_i - k_{i+1})$ corresponds to the new sets of two-dimensional fields $(\Psi; \Phi)$ on new two-dimensional surfaces. The momentum $\sum_i q_{i,i+1}$ corresponds to the usual two-dimensional NS-surface. In the closed string sector $\sum_i (k_i - k_{i+1})$ is vanished and there are here the momentum $P = \sum_i q_{i,i+1} = \beta \sum_i k_i$ only. Hence the Regge trajectory for the closed string sector takes the following form:

$$\tilde{\alpha}(0) + \frac{1}{4} \beta^2 K^2; \quad K = \sum_i k_i \quad (49)$$

instead of

$$\alpha(0) + \frac{1}{2} P^2; \quad P \approx \sum_i (k_i - k_{i+1}) \quad (50)$$

for the open hadron string sector. And the Regge slope $\alpha'$ for the closed string sector is equal to $\frac{1}{2} \beta^2 \alpha'_H \sim \frac{1}{2} \alpha'_P \sim (10^{19} Gev)^{-2}$ in this approach.

Let us take corresponding expressions for the operator $\Delta_r$ and the matrices $\tilde{\Gamma}_c$ and $\Gamma^e_{\nu}$.

$$\Delta^{SF}_r = (\alpha_q \tilde{\lambda}^{(+)} \Pi_q \lambda^{(-)} + \frac{1}{4\alpha_q} \tilde{J}_q^+ \Phi^q_+); \quad \Delta^{SF'}_r = (\alpha_q \tilde{\mu}^{(+)} \Pi_q \mu^{(-)} + \frac{1}{4\alpha_q} \tilde{J}_q^+ \Phi^q_{+q}).$$  

21
\[ \Pi_q = I \otimes \left( \frac{3}{4} I \otimes I \otimes I - \frac{1}{4} (\tau_3 \otimes \tau_3 \otimes I + \tau_3 \otimes I \otimes \tau_3 + I \otimes \tau_3 \otimes \tau_3) \right) \]

The fields \( \Phi^{qs}, \Phi'^{qs} \) are corresponding to the matrices \( \Gamma_q \).

\[ \Gamma_q = I \otimes \left( \frac{3}{4} \tau_3 \otimes \tau_3 \otimes \tau_3 - \frac{1}{4} (\tau_3 \otimes I \otimes I + I \otimes I \otimes \tau_3 + I \otimes \tau_3 \otimes I) \right) \]

The currents \( \tilde{J}_q, \tilde{J}'_q \) are defined by the fields \( \tilde{\Phi}^{qs}, \tilde{\Phi}'^{qs} \):

\[ \tilde{J}_q = \frac{2}{\eta} \left[ G^{SF}_r, \tilde{\Phi}^{qs}_r \right] \]

\[ \tilde{J}'_q = \frac{2}{\eta} \left[ G^{SF}_r, \tilde{\Phi}'^{qs}_r \right] \]

The fields \( \tilde{\Phi}^{qs}, \tilde{\Phi}'^{qs} \) are corresponding to the matrices \( \tilde{\Gamma}_q \):

\[ \tilde{\Gamma}_q = \frac{\hat{P}}{M} \otimes \left( \frac{3}{4} I \otimes I \otimes I - \frac{1}{4} (\tau_3 \otimes \tau_3 \otimes I + \tau_3 \otimes I \otimes \tau_3 + I \otimes \tau_3 \otimes \tau_3) \right) \]

The value of coefficient \( \alpha_q \) provides the necessary shift of the intercept of the leading \( \rho \)-trajectory to the value equal to \( \frac{1}{2} \). It is noteworthy that the value of \( \alpha^2_q > \frac{1}{2} \) bring to unacceptable negative norms of states i.e. to ghosts in the spectrum of physical states.

Now we consider linear terms for operators to be accompanying compact components of momentum. In order to obtain necessary number of lepton and quark types we extend our set of the \( \lambda \) and \( \mu \) - modes correspondingly and introduce a triple set of \( \lambda^a; \mu^a; a = 1, 2, 3 \).

This choice gives an appropriate equivalent of colour quark degrees of freedom and a possibility for three generations of leptons. Let us to write out corresponding linear terms:

\[ \sum_{\gamma=4,5,6,7,8,9} \sum_{a=1,2,3} \tilde{\lambda}^{a(+)} \Gamma^a_{\gamma} \lambda^{a(-)} + \sum_{a=1,2,3} \tilde{\mu}^{a(+)} \Gamma^a_{\gamma} \mu^{a(-)} + J^\text{cm}_\gamma + J'^\text{cm}_\gamma) H^\gamma \]  (52)

\[ \sum_{a=1,2,3} \tilde{\lambda}^{a(+)} \Gamma^a_4 \lambda^{a(-)} = \sum_{a=1,2,3} \tilde{\lambda}^{a(+)} \tilde{\Gamma}_e \lambda^{a(-)} \]  (53)
\[ \Gamma_e = \frac{e}{2} (I \otimes \tau_3 \otimes \tau_3 \otimes \tau_3 + \frac{\hat{P}}{M} \otimes I \otimes I \otimes I) \]  
(54)

\[ \tilde{\Gamma}_e = \frac{\hat{c}}{2\nu} \otimes \tau_3 \otimes \tau_3 \otimes \tau_3 \]  
(55)

\[ J_{e}^{cm} = \frac{2}{\eta} \left[ G_{r}, \Phi_{-r}^{4} \right] \]  
(56)

The field \( \Phi_4 \) corresponds to the matrix \( \Gamma_e \).

This choice suggests the electric charges to be equal to \( \pm \frac{1}{2} \) for \( \lambda,\mu \)-components and to \( \pm 1, 0 \) for \( \Psi \)-components.

\[ \sum_{a=1,2,3} \tilde{\lambda}_R^{a(+)} \Gamma_5^a \lambda_L^{a(-)} = \sum_{a=1,2,3} \tilde{g}_W \tilde{\lambda}_R^{a(+)} \hat{\kappa}_5 \otimes \tau_1 \otimes \tau_1 \otimes \tau_1 \lambda_L^{a(-)} \]  
(57)

\[ J_{5}^{cm} = \frac{2}{\eta} \left[ G_{r}, \Phi_{-r}^{5} \right] \]  
(58)

The operator \( J_{6}^{cm} \) is absent.

The operators \( \lambda_L^{a(-)} \) have the left chirality; the operators \( \tilde{\lambda}_R^{a(+)} \) have the right chirality:

\[ \gamma_5 \lambda_L^{a(-)} = \lambda_L^{a(-)} \]  
(59)

\[ \tilde{\lambda}_R^{a(+)} \gamma_5 = -\tilde{\lambda}_R^{a(+)} \]  
(60)

\[ \sum_{a=1,2,3} \tilde{\lambda}_R^{a(+)} \Gamma_6^a \lambda_L^{a(-)} = \sum_{a=1,2,3} g_Z \tilde{\lambda}_R^{a(+)} (a\Pi_q + b\Pi_l) \lambda_L^{a(-)} \]  
(61)

\[ J_{6}^{cm} = g_Z \left[ G_{r}, \Phi_{-r}^{6} \right] \]  
(62)

The field \( \Phi_6 \) corresponds to the matrix \( (\tilde{a}\Pi_q + \tilde{b}\Pi_l) \).

\[ \hat{P}_T = \alpha_T \left( \sum_{a=1,2,3} \tilde{\lambda}_R^{a(+)} \Gamma_7^a \lambda_L^{a(-)} + J_{7}^{cm} + \sum_{a=1,2,3} \tilde{\mu}_R^{a(+)} \Gamma_7^a \mu_L^{a(-)} + J_{7}^{cm} \right) \]  
(63)
\[ J_{7}^{em} = \frac{2}{\eta} \left[ G_{SF}^{r}, \Phi_{7}^{r} \right] \] (64)

The field \( \Phi_{7} \) corresponds to the matrix \( \tilde{\Gamma}_{7} \):

\[
\Gamma_{7} = \frac{1}{2} \Pi_{3}\left( \frac{\hat{k}}{v} \otimes \tau_{3} \otimes \tau_{3} \otimes \tau_{3}(M_{t} - M_{b}) + M_{t} \otimes I \right) + \nonumber \]
\[ f_{12}\Pi_{12} + f_{23}\Pi_{23} + f_{13}\Pi_{13} \] (65)

\[
\tilde{\Gamma}_{7} = \frac{1}{2} \Pi_{3}(\Gamma_{P}(M_{t} - M_{b}) + M_{t} \otimes \tau_{3} \otimes \tau_{3} \otimes \tau_{3}) + \nonumber \]
\[ f_{12}\Pi_{12} + f_{23}\Pi_{23} + f_{13}\Pi_{13} \] (66)

\[
\Pi_{3} = \frac{1}{4}[I(\otimes I \otimes I \otimes I + \tau_{3} \otimes \tau_{3} \otimes I) - (\tau_{3} \otimes I \otimes I + I \otimes \tau_{3} \otimes \tau_{3})] \tag{67}
\]

\[
\Pi_{12} = I \otimes (\tau_{+} \otimes \tau_{-} \otimes I + \tau_{-} \otimes \tau_{+} \otimes I) \tag{68}
\]

\[
\Pi_{23} = I \otimes (I \otimes \tau_{+} \otimes \tau_{-} + I \otimes \tau_{-} \otimes \tau_{+}) \tag{69}
\]

\[
\Pi_{13} = I \otimes (\tau_{+} \otimes I \otimes \tau_{-} + \tau_{-} \otimes I \otimes \tau_{+}) \tag{70}
\]

\[
\sum_{a=1,2,3} \tilde{\lambda}^{a(+)} \Gamma_{8}^{a} \chi^{a(-)} = \sum_{a=1,2,3} \tilde{\lambda}^{a(+)}(\rho_{ae}\Pi_{e} + \rho_{au}\Pi_{u})\chi^{a(-)} \tag{71}
\]

The operator \( J_{7}^{em} \) is absent.

\[
\Pi_{e} = \frac{1}{2}\Pi_{l} \otimes (I \otimes I \otimes I - \tau_{3} \otimes \tau_{3} \otimes \tau_{3}) \tag{72}
\]

\[
\Pi_{u} = \frac{1}{2}\Pi_{l} \otimes (I \otimes I \otimes I + \tau_{3} \otimes \tau_{3} \otimes \tau_{3}) \tag{73}
\]

The coefficients \( M_{i}; \rho_{i} \) and \( f_{i} \) can give necessary mass shifts of quark flavours, lepton masses and coefficients in the Cabibbo-Kobayashi-Maskawa mixing (CKM) matrix.
6 Wave functions of underlying physical states in this model

Let us define some wave functions in the approach suggested here:

a) for $\pi$-meson

$$\langle 0 \mid \exp ikX_0(\hat{\mu}_i(-)\lambda(-))G_{\frac{1}{2}} \rangle$$

$$\Gamma_{\pi^+} = \gamma_5 \otimes \tau^+ \otimes \tau^- \otimes \tau^-; \Gamma_{\pi^-} = \gamma_5 \otimes \tau^- \otimes \tau^+ \otimes \tau^+; \Gamma_{\pi^0} = \Gamma^{(q1)} \gamma_5 \otimes \tau_3 \otimes \tau_3 \otimes \tau_3$$

b) for K-meson

$$\langle 0 \mid \exp ikX_0(\hat{\mu}_i(-)\lambda(-))G_{\frac{1}{2}} \rangle$$

$$\Gamma_{K^+(+)} = I \otimes (I - \tau_3) \otimes (I + \tau_3) \otimes \tau_+ / 4$$

$$\Gamma_{K^-(−)} = I \otimes (I - \tau_3) \otimes (I + \tau_3) \otimes \tau_- / 4$$

c) for nucleons

$$\langle 0 \mid \exp ikX_0(\hat{\mu}_i(-)\lambda(-))\tilde{U}(1 + \hat{b}^{(q1)}\bar{\Psi})G_{\frac{1}{2}} \rangle$$

Here $\hat{b} = \hat{b} - \frac{(bP)P}{M^2}$. d) for leptons of $i$-th generation

$$\langle 0 \mid \exp ikX_0(\hat{\mu}_i(-)\lambda(-))\tilde{U} \Pi_i \bar{\Psi} G_{\frac{1}{2}} \rangle$$

The operator $\Gamma^{(q1)}$ is the projector on the quark components of first generation:

$$\Gamma^{(q1)} = \frac{1}{4}I \otimes ((I \otimes I \otimes I + I \otimes \tau_3 \otimes \tau_3) - (\tau_3 \otimes \tau_3 \otimes I + \tau_3 \otimes I \otimes \tau_3))$$

More detailed analysis of physical states and corresponding amplitudes would be carried in following publications.

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