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Stationary Tetrolet Transform: an Improved Algorithm for Tetrolet Transform

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Abstract. In order to get an efficient image multi-scale geometrical representation, the basic principle of tetrolet transform is studied and an efficient stationary tetrolet transform algorithm based on Haar wavelet transform is proposed. Tetrolets are Haar-type wavelets whose supports are tetrominoes which are shapes made by connecting four equal-sized squares. Tetrolet transform is divide the image into 4 x 4 blocks in the horizontal and vertical direction, there is no overlap between the sub-blocks. Because there is no overlap between the images blocks, so the image decomposition coefficients in image processing is easy produce Gibbs phenomenon. Stationary tetrolet transform is a new adaptive Haar-type wavelet transforms which the block overlaps method in image decomposition process is inserted into the middle of the original coefficients in tetrolet transform. The corresponding filter bank algorithm is simple but enormously effective. Compare with standard two dimensional wavelet transform, stationary tetrolet transform is a novel tetrominoes based multi-scale geometrical transform tool, which can capture image anisotropic geometrical structures information efficiently by multi-direction selection. In this paper, the decomposition and reconstruction algorithms of the stationary tetrolet transform are described in detail, and the simulation and analysis of the decomposition of the image using the stationary tetrolet transform is carried out. Experimental results show that compared with traditional algorithm, the proposed algorithm can get better sparse representation and eliminate the blocking artifacts in image processing resulted from tetrolet transform algorithm.

1. Introduction
With the emergence of multi-scale geometric analysis algorithms, computational harmonic analysis and sparse approximation algorithms have been developed rapidly and have been widely used [1]. In the course of digital image processing study, achieve sparse representation of images and reconstruct them with the least number of coefficients has all been concerned hot by majority of researchers, also has been the most active areas of research subject in obtain the detail components of multi-scale image, which has a very important position in the practical application [2]. Wavelet transform is a multi-scale and multi-resolution analysis method, which has unique time frequency local analysis ability [3]. But it is found that wavelet transform has the characteristics of "isotropy", which is difficult to express higher dimensional geometric features, and can not express the edge direction of the structural characteristics of the image accurately. It is not the optimal method for two-dimensional images containing "line" or "surface". A lot of methods have been proposed to improve the treatment of orientated geometric image structures in the last years.
From the beginning of 1997, the idea of multiscale geometric analysis has been greatly developed [4]. Brushlets [5], Wedgelets [6], Ridgelets [7], Bandelets [8], Curvelets, Contourlet [9] and Directionlets [10-11] are wavelet systems with more directional sensitivity [12]. But the above multiscale geometric analysis method often involves over sampling; non separable convolution and complex filter design in the process of image decomposition. The computation is large, and the filter used is complex compare with wavelet transform. In 2009, Krommweh proposed a new adaptive Haar wavelet transform--Tetrolet transform [13].

Tetrolet transform has more advantages in image compression, image denoising, and image fusion and so on [14]. But the image blurred and Gibbs phenomenon is easy to appear in the image processing. In order to overcome the block effect defects in image fusion and image denoising, a stationary tetrolet transformation algorithm is proposed [15]. Stationary tetrolet transform not only preserves the multi-scale, multi-resolution, and anisotropy characteristics of the tetrolet transform, but also increases the redundancy of image decomposition process. It can effectively improve the effect of image processing, and can eliminate the block effect in the process of image processing by using tetrolet transform.

2. The Idea of Tetrolet Transform
Tetrolet transform is a basic four lattice board proposed by Jens Krommweh Harr wavelet transform concept, according to the local geometric features of images, adaptively select the corresponding four lattice plate of the square area of sparse representation. Compared with the traditional multi-scale transform Wavelet, Curvelet and Contourlet using the same number of transform coefficients reconstruction can get better image quality.

The tetrolet transform is designed with a simple structure of Haar filter. In the two-dimensional classical Haar case, the low-pass filter and the high-pass filters are just given by the averaging sum and the averaging differences of each four pixel values which are arranged in a 2 × 2 square. It can make the function system adapt to the local structure instead of selecting the prior basis or frame, but it can obtain the anisotropic decomposition with more directional selectivity. Tetrolet transform divide the image into 4×4 blocks, then we determine in each block a tetromino partition which is adapted to the image geometry in this block. Tetrominoes were introduced by Golomb in [16] are shapes made by connecting four equal-sized squares, each joined together with at least one other square along an edge. We can get more of the direction of image decomposition, image edge and texture can be extracted, and the optimal approximation of various geometric features of the image can be achieved.

Tetrolet transform is divide the image into 4 × 4 blocks in the horizontal and vertical direction, there is no overlap between the sub-blocks. Because there is no overlap between the images blocks, so the image decomposition coefficients in image processing is easy produce Gibbs phenomenon.

3. Stationary Tetrolet Transform

3.1. Defects of Tetrolet Transform in Image Processing
From a signal processing point of view, the Gibbs phenomenon is the step response of a low-pass filter, and the oscillations are called ringing or ringing artifacts. The main cause of ringing artifacts is due to a signal being bandlimited (specifically, not having high frequencies) or passed through a low-pass filter; this is the frequency domain description.

The image multi-scale decomposition of tetrolet transform is transformed by 4 x 4 region block, which is equivalent to the rectangle window segmentation of the image. The Gibbs phenomenon is generated by the truncation approximation and the frequency hopping of the spectrum when the filter is performed. The rectangular window is suddenly truncated in the time domain, which makes the frequency spectrum of the rectangular window have more high frequency components, so the relative amplitude of the side lobe of the rectangular window is relatively large. So the block effect will be produced when the image is processed, the more the layers are decomposed, the stronger the block effect is produced.
3.2. The Basic Idea of Stationary Tetrolet Transform

In order to eliminate the defect of tetrolet transform in image processing, the concept of redundant data processing is proposed. The block overlaps method in image decomposition process is introduced in tetrolet transform. The new overlap block is inserted into the middle of the original coefficients. So that the size of the high frequency sub-image of each layer after the decomposition and the size of the original image have the same size. This is the basic idea of stationary tetrolet transform.

Stationary tetrolet transform of multiscale image decomposition is also divide the low-pass image into 4 × 4 blocks. In each block \( I_{i,j} = \{(2i-1:2i+2,j-1:2j+2)\}, \quad i = 1, 2, ..., M/2, \quad j = 1, 2, ..., N/2 \), \( M \) and \( N \) are the row and column size values of the decomposed images, respectively. Then the each piece of the image area is transformed using tetrolet base transform. For example, the image of the size of 8×8 by stationary tetrolet transform decomposition of the structure is shown in figure 1. In Figure 1, the selection of 4×4 region translates a block length pattern of one unit length in each horizontal and vertical direction, which is equivalent to inserting a new block into the tetrolet transform block.

![Figure 1. Multiscale Stationary Tetrolet Transform Construction](image)

3.3. Decomposition Algorithm Steps of Stationary Tetrolet Transform

We start with the input image \( a^0 = (a(i,j))_{i,j=0}^{N-1} \) with \( N = 2^J \), \( J \in \mathbb{N} \). In the \( r \)-th level, \( r = 1, 2, ..., J - 1 \), we apply the following computations.

1. Divide the low-pass image \( a^{r-1} \) into blocks \( Q_{i,j} = [(2i-1:2i+2,j-1:2j+2)] \) of \( i, j = 0, 1, ..., N/2 \).

2. Find in each block the sparsest tetrolet representation. In each block \( Q_{i,j} \), we compute the pixel averages for every admissible tetromino covering \( c = 1, 2, ..., 117 \), we can determine the lowpass part.

\[
a^{r,c} = \sum_{(m,n)\in I^{r,c}} a^{r-1}[m,n]
\]

as well as the three high-pass parts for \( l = 1, 2, 3 \)

\[
w^{r,c}_l = \left( \sum_{(m,n)\in I^{r,c}} a^{r-1}[m,n] \right)^3
\]
where the coefficients $c[l,m], l,m = 0,1,2,3$ are entries from the Haar wavelet transform matrix.

$$W = (c[l,m])_{l,m=0}^3 = \frac{1}{2} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & -1 & -1 \\ -1 & 1 & -1 & -1 \\ -1 & -1 & 1 & 1 \end{bmatrix}$$ (3)

Then we choose the covering $c^*$ such that the $l_1$-norm of the tetrolet coefficients becomes minimal.

$$c^* = \arg \min_c \sum_{l=1}^3 \sum_{s=0}^3 |w^r_{l,s}(c)|$$ (4)

Hence, for every block $Q_{i,j}$, we get optimal tetrolet decomposition $[a^r(e), w^r_1(e), w^r_2(e), w^r_3(e)]$. The best covering $c^*$ is a covering whose tetrominoes do not intersect an important structure like an edge in the image $a^{r-1}$. Because the tetrolet coefficients become as minimal as possible a sparse image representation will be obtained. For each block $Q_{i,j}$ we have to store the covering $c^*$ that has been chosen, since this information is necessary for reconstruction. If the optimal covering is not unique, then we take the tiling $c^*$ that has already been chosen most frequently in the previous blocks. Thus, the coding of the used coverings becomes cheaper.

3. In order to be able to apply further levels of the tetrolet decomposition algorithm, rearrange the low-pass and high-pass coefficients of each block into a $2\times2$ block.

$$a^r_{Q_{i,j}} = R(a^r(e)) = \begin{bmatrix} a^r(e) & 0 \\ a^r(e) & 2 \\ a^r(e) & 1 \\ a^r(e) & 3 \end{bmatrix}$$ (5)

and in the same way $w^r_{Q_{i,j}} = R(w^r(e))$ the degree of polarization is a determination as to how much of the captured light is polarized.

4. After finding a sparse representation in every block $Q_{i,j}$, we store the low-pass matrix $a^r$ and the high-pass matrices $w^r_{l,s}$, replacing the low-pass image $a^{r-1}$ by the matrix

$$a^{r-1} = \begin{bmatrix} a^r & w^r_1 \\ w^r_1 & w^r_2 \end{bmatrix}$$ (6)

5. Apply step 1 to 4 to the low-pass image we can get multi-scale and multi-direction tetrolet decompose.

We divide the image index set into $4 \times 4$ blocks. Then instead of using the classical Haar wavelet transform, which corresponds to a partition of the $4 \times 4$ block into squares, we compute the 'optimal' partition of the block into four tetrominoes according to the geometry of the image.

4. Experiment and Analysis of Stationary Tetrolet Transform Decomposition

4.1. Image Decomposition Based on Stationary Tetrolet Transform

In order to verify the validity of the decomposition and reconstruction algorithm of the stationary tetrolet transform, $256 \times 256$ Lena image is used to decompose and restructure. At the same time, the decomposition of stationary tetrolet transform is compared with tetrolet transform decomposition, wavelet decomposition and stationary wavelet decomposition. The decomposition level is 3 layers, and the wavelet decomposition is based on the biorthogonal 6.8 filter bank.

Figure 2(a) is the 3 layer decomposition coefficient image decomposed by wavelet transform. Figure 2(b) is the 3 layer decomposition coefficients with Tetrolet transform.
Figure 3(a), (b) are second layer stationary tetrolet transform to the third layers coefficients of image decomposition.

Experiments show that the stationary tetrolet transform can effectively realize the image multiscale decomposition and retain more image details, the actual experiments show that the multiscale decomposition for large scale image can also achieve good image decomposition.

![Figure 2](image2.png)

**Figure 2.** The three layer decomposition coefficient using wavelet transform and Tetrolet transform respectively

![Figure 3](image3.png)

**Figure 3.** The 3 layer decomposition coefficients using stationary tetrolet transform

4.2. Sparsity Experiment of Stationary Tetrolet Transform

In order to verify the sparsity of the coefficient using the stationary tetrolet transform, 256 x 256 "Lena" image decomposition is used to verify. The wavelet transform decomposition basis function adopts the biorthogonal6.8 filter bank. After the 3 layers decomposition of wavelet transform (WT) and stationary wavelet transform (SWT), the diagonal subband coefficients of third layer are compared with the corresponding tetrolet transform (TT) and stationary tetrolet transform (STT).

Figure 4(a), (b), (c), (d) are normalized histogram distribution of third layer diagonal subband coefficient using four kinds of methods decomposition.

It can be seen from the graph that the stationary tetrolet transform can obtain good coefficient sparsity, and more coefficients are concentrated near 0.
4.3. Image Fusion Experiment Based on Stationary Tetrolet Transform

In order to test our image fusion algorithm, in this section, the methods proposed above were tested for image fusion of the different focus clock image. Meanwhile, in order to test stationary tetrolet transform is effective, we firstly fusion image using tetrolet transform [17-19].

Figure 5 are fused results of the different focus clock image with the local energy method using different levels of decomposition based on tetrolet transform. Figure 5(a) is the 5 layer decomposition fusion results; Figure 5(b) is the 7 layer decomposition fusion results.

(a) 5 layer decomposition  (b) 7 layer decomposition

Figure 5. Fused results with the local energy method using different levels of decomposition based on tetrolet transform.
Figure 6. Fused results with the local energy method using different levels of decomposition based on stationary tetrolet transform.

Figure 6 are fused results using different levels of decomposition based on stationary tetrolet transform. Figure 6(a) is the 5 layer decomposition fusion result; Figure 6(b) is the 7 layer decomposition fusion results.

5. Conclusion
The stationary tetrolet transform has the characteristics of simple filter design, multi-direction matching, multi-scale decomposition, multi feature expression and so on. The edge and contour of the image can be sparse and approximate expression and retain more image details using stationary tetrolet transform. It has better accuracy and simplicity compared with other multiscale geometric transformations. Since the decomposition function is adapted to the local structure rather than to choose a priori basis or frame. In the Haar filter bank, the low-pass filter and the high-pass filters are just given by the averaging sum and the averaging differences of each four pixel values which are arranged in a $2 \times 2$ square.

Stationary tetrolet transform is the optimal processing of the tetrolet transform using overlapping algorithm of block window. It effectively eliminate Gibbs phenomenon of the tetrolet transform in image processing equivalent to the conventional tetrolet transform decomposition coefficient interpolation operation. The proposed algorithm improves the image decomposition redundancy and provides a new algorithm for the image processing process. Experiments results also verify the effectiveness of the algorithm.

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7. References
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