Towards precise causal effect estimation from data with hidden variables

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Abstract

Causal effect estimation from observational data is a crucial but challenging task. Currently, only a limited number of data-driven causal effect estimation methods are available. These methods either only provide a bound estimation of the causal effect of a treatment on the outcome, or have impractical assumptions on the data or low efficiency although providing a unique estimation of the causal effect. In this paper, we identify a practical problem setting and propose an approach to achieving unique causal effect estimation from data with hidden variables under this setting. For the approach, we develop the theorems to support the discovery of the proper covariate sets for confounding adjustment (adjustment sets). Based on the theorems, two algorithms are presented for finding the proper adjustment sets from data with hidden variables to obtain unbiased and unique causal effect estimation. Experiments with benchmark Bayesian networks and real-world datasets have demonstrated the efficiency and effectiveness of the proposed algorithms, indicating the practicability of the identified problem setting and the potential of the approach in real-world applications.

1 Introduction

Causal inference [Imbens and Rubin, 2015; Pearl, 2009] has been widely studied to understand the underlying mechanisms of phenomena in economics, medicine, and social science, to name but a few. One major task for causal inference is causal effect estimation, e.g. estimating the effect of a drug on a disease or the effect of a policy on a certain population. Randomized Controlled Trials (RCTs) are usually used to estimate causal effects. However, RCTs are usually impossible to conduct due to ethical concern, cost and time constraints.

It is desirable to estimate causal effect from data since the collection of observational data is increasing rapidly and dramatically. Confounding bias is a major challenge for causal effect estimation from data. To reduce confounding bias, covariate adjustment is commonly used. However, it is challenging to determine which variables should be included in an adjustment set from data.

Graphical causal modelling provides a theoretical foundation for adjustment set selection [Pearl, 2009; Maathuis et al., 2015; Perković et al., 2017]. When a causal DAG (Directed Acyclic Graph) or MAG (Maximal Ancestral Graph) representing the causal mechanism is given, the back-door criterion or generalised back-door criterion can be used to determine an adjustment set [Pearl, 2009; Maathuis et al., 2015].

However, it is impossible to uniquely identify causal effects from data without additional constraints. In most real-world applications, the causal graphs are unknown, so they have to be learned from data. However, from data one cannot discover a unique causal graph. Instead, only an equivalence class of causal graphs encoding the same conditional independence relationships among variables can be learned from data. This results in the uncertainty in determining the proper adjustment sets and thus the uncertainty in the causal effects estimated using the found adjustment sets. This is why data-driven causal effect estimation methods often return a set of possible causal effects, i.e. a bound estimation, instead of a precise or unique estimation of the causal effect.

For example, a widely used data-driven causal effect estimation method, IDA (Intervention when the DAG is Absent) [Maathuis et al., 2009] provides a multiset of estimated causal effects based on data without hidden variables. For data with hidden variables, Hyttinen et al. proposed a method, CE-SAT, which uses logic representation and SAT-based inference to estimate causal effect from the data, but the method can only deal with very small datasets [Hyttinen et al., 2015]. Malinsky and Spirtes [Malinsky and Spirtes, 2017] have extended IDA to LV-IDA (Latent variable IDA), which is also a bound estimation method, to estimate causal effects from data with hidden variables.

The high uncertainty in the results returned by these bound estimation methods can seriously hinder the applicability of data-driven causal effect estimation. Hence researchers have imposed extra constraints on data to eliminate the uncertainty for unique causal effect estimation. Häggström [Häggström, 2018] developed the Bayesian network methods in conjunction with the Covariate Selection algorithms (CovSel for short) in [De Luna et al., 2011] to uniquely estimate the causal effect of a treatment on the outcome, with the assumptions that there are no hidden variables and all other
observed variables are pretreatment variables. With a more strong constraint, i.e. the data satisfies the unconfoundedness assumption, Shalit et al. [Shalit et al., 2017] have developed a deep learning based causal defect estimation method, CFR-Net. For data with hidden variables, Entner et al. [Entner et al., 2013] proposed EHS (for authors’ names, Entner, Hoyer, and Spirtes), a method based on conditional independence tests and with the pretreatment variable assumption too. EHS is very inefficient since it performs an exhaustive search.

Table 1 provides a summary of the above discussed data-driven methods.

In this paper, we aim to develop an efficient data-driven method towards precise causal effect estimation from data with hidden variables, and make the following main contributions:

- We have identified a practical problem setting where causal effects can be estimated uniquely from data with hidden variables. We have developed the theorems to support causal effect estimation under the problem setting to ensure the soundness of the proposed algorithms.
- We have developed two algorithms for precise causal effect estimation. The algorithms are efficient and provide more accurate causal effect estimation than existing methods dealing with hidden variables.

2 Preliminaries and background

2.1 Basic definitions and assumptions

A graph \( G = (V, E) \) consists of a set of nodes \( V = \{V_1, \ldots, V_n\} \) and a set of edges \( E \subseteq V \times V \). An edge can be directed, bi-directed, non-directed or partially directed. We use “\( \rightarrow \)” to denote an arbitrary edge mark.

A mixed graph may contain directed and bi-directed edges. A partial mixed graph may contain any type of directed edges. A path is a sequence of distinct adjacent vertices.

A causal path from \( V_i \) to \( V_j \) is a path on which all edges are directed towards \( V_j \). A path from \( V_i \) to \( V_j \) is a possibly causal path if it contains no arrowhead pointing towards \( V_j \). If a path \( \pi \) contains \( V_k \) if \( V_k \rightarrow V_j \) is a collider on \( \pi \). A collider path is a path on which every non-endpoint node is a collider. A path of length one is a trivial collider path. If \( V_i \rightarrow V_j \), \( V_i \) is a parent of \( V_j \). If \( V_i \rightarrow V_j \), \( V_i \) is a spouse of \( V_j \). If there is a direct edge (possibly directed) path from \( V_i \) to \( V_j \), \( V_i \) is an ancestor (possible ancestor) of \( V_j \), and \( V_j \) is a descendant (possible descendant) of \( V_i \). \( Pa(V_i) \), \( Sp(V_i) \), \( An(V_i) \), \( De(V_i) \), \( PossAn(V_i) \) and \( PossDe(V_i) \) denote the sets of all parents, spouses, ancestors, descendants, possible ancestors and possible descendants of \( V_i \) respectively. There is a directed cycle between \( V_i \) and \( V_j \) if \( V_i \rightarrow V_j \) and \( V_j \in An(V_i) \). There is an almost directed cycle between \( V_i \) and \( V_j \) if \( V_i \leftrightarrow V_j \) and \( V_j \in An(V_i) \). An ancestral graph [Richardson et al., 2002] is a mixed graph without directed and almost directed cycles.

When a DAG possesses the following defined Markovian property and faithfulness, we can read dependency/independency of data distribution from the DAG.

Definition 1 (Markovian property [Pearl, 2009]). A probability distribution \( P \) over variables \( V \) is Markov relative to a given DAG \( G \) if and only if for every independence in \( G \), the joint distribution of \( V \) is factorised as \( \text{prob}(V) = \prod_i \text{prob}(V_i|Pa(V_i)) \) based on the Markovian property.

Definition 2 (Faithfulness [Spirtes et al., 2000]). A graph \( G \) is faithful to a joint distribution \( P \) over a set of variables \( V \) if and only if every independence present in \( P \) is entailed by \( G \) and satisfies the Markovian property. A joint distribution \( P \) is faithful to a graph \( G \) if and only if there exists a graph \( G \) which is faithful to the joint distribution \( P \).

Definition 3 (Causal sufficiency [Spirtes et al., 2000]). A given dataset satisfies causal sufficiency if for every pair of observed variables, all their common causes are observed.

In data, the assumption of causal sufficiency is often unwarranted. Ancestral graphs are used to represent data generating processes which may involve hidden variables.

Definition 4 (m-separation [Richardson et al., 2002]). In an ancestral graph \( G \), a path \( \pi \) between \( V_i \) and \( V_j \) is said to be m-separated by a set of nodes \( Z \) (possibly \( \emptyset \)) if \( \pi \) does not contain any collider which is in \( Z \), or \( \emptyset \) for any collider \( V_i \) on the path \( \pi \), \( V_i \notin Z \) and no descendant of \( V_i \) is in \( Z \). Two nodes \( V_i \) and \( V_j \) are said to be m-connected by \( Z \) in \( G \) if \( V_i \) and \( V_j \) are not m-separated by the \( Z \).

Definition 5 (MAG [Richardson et al., 2002]). An ancestral graph \( G \) is referred to as a maximal ancestral graph (MAG) when every pair of non-adjacent nodes \( V_i \) and \( V_j \) can be m-separated by a set \( Z \) of \( \{V_i, V_j\} \).

A MAG encodes conditional independence relationships by m-separation. When two MAGs represent the same conditional independence relationships, they are Markov equivalent. Markov equivalent MAGs can be represented uniquely by a partial ancestral graph (PAG).

Definition 6 ([Zhang, 2008]). Let \( [M] \) be the Markov equivalence class of a MAG \( M \). The PAG \( G \) for \( [M] \) is a partial mixed graph such that \((I)\) \( G \) has the same adjacencies as \( M \) does; \((II)\) a mark of arrowhead (same to the mark of tail) is in \( G \) if and only if it is shared by all MAGs in \( [M] \).

2.2 Confounding adjustment

Let \( G = (V, E) \) be a causal graph, and \( V = \{W, Y\} \cup X \), where \( W \) is the treatment variable of interest, \( Y \) the outcome variable, and \( X \) the set of all other observed variables. We are interested in estimating the average total causal effect of \( W \) on \( Y \), as defined below.

Definition 7 (Average total causal effect). The total Average Causal Effect of \( W \) on \( Y \) is defined as \( ACE(W, Y) = E(Y|do(W = 1)) - E(Y|do(W = 0)) \), where \( do(W = w) \)
is the do-operator indicating the manipulation of \( W \) by setting it to the value \( w \) [Pearl, 2009].

Given a proper adjustment set \( Z \subseteq X \), \( ACE(W, Y) \) can be estimated unbiasedly and consistently as follows.

\[
ACE(W, Y) = E(Y|w, Z = z) - E(Y|w', Z = z)
\]

where \( w \) and \( w' \) denote \( W = 1 \) and \( W = 0 \), respectively. When the causal DAG is known, the back-door criterion [Pearl, 2009] can be used to identify an adjustment set. However, when a dataset contains hidden variables, i.e. the causal graph is a MAG (or a PAG), we will need the generalised adjustment criterion introduced below.

**Definition 8 (Visibility [Zhang, 2008]).** Given a PAG or MAG \( G \), a directed edge \( V_i \rightarrow V_j \) is visible if there is a node \( V_k \) not adjacent to \( V_j \), such that either there is an edge between \( V_k \) and \( V_i \), or there is a collider path between \( V_k \) and \( V_i \) that is into \( V_i \), or every node on this path is a parent of \( V_j \). Otherwise, \( V_i \rightarrow V_j \) is said to be invisible.

To introduce the generalised adjustment criterion, we need define amenability [van der Zander et al., 2014].

**Definition 9 (Amenability).** Given a PAG or MAG \( G \) with \( (W, Y) \), \( G \) is adjustment amenable w.r.t. \( (W, Y) \) if each proper possibly directed path from \( W \) to \( Y \) in \( G \) starts with a visible edge out of \( W \).

The forbidden set in \( G \) includes the set of variables that cannot in an adjustment set when we estimate \( ACE(W, Y) \).

**Definition 10 (Forbidden set; \( Forb(W, Y, G) \)).** Given a PAG or MAG \( G \) with \( (W, Y) \), the forbidden set w.r.t. \( (W, Y) \) is \( Forb(W, Y, G) = \{X \in V : X \in PossDe(W, G) \text{ which lies on a possible causal path from } W \text{ to } Y \text{ in } G\} \).

Following [Colombo et al., 2012], for developing a small number of candidate adjustment sets, we need to define Possible-D-SEP (W,Y,G), pds(W,Y,G) for short.

**Definition 11 (pds(W,Y,G) [Colombo et al., 2012]).** Let \( X \in pds(W,Y,G) \) if and only if there is a path \( \pi \) between \( W \) and \( Y \) in \( G \) such that for every subpath \( <X_1, X_2, X_3> \) on \( \pi \) either \( X_2 \) is a collider on the subpath in \( G \) or \( <X_1, X_2, X_3> \) is a triangle in \( G \), i.e. each pair of nodes in the triple are adjacent.

Now, we introduce the criterion for testing a proper adjustment set in a MAG or PAG.

**Definition 12 (Generalised back-door path [Maathuis et al., 2015]).** Given a MAG or PAG \( G \) with \( (W, Y) \), a back-door path between \( W \) and \( Y \) is a directed path between \( W \) and \( Y \) that does not have a visible edge out of \( W \).

**Definition 13 (Generalised Adjustment Criterion (GAC) [Perkovic et al., 2017]).** Given a MAG or PAG \( G \) with \( (W, Y) \), a set of nodes in \( G \), denoted as \( Z \), satisfies the GAC relative to \( (W, Y) \) in \( G \), i.e. \( Z \) is a proper adjustment set for unbiased estimation of \( ACE(W, Y) \) if (I), \( G \) is adjustment amenable relative to \( (W, Y) \), (II), \( Z \cap Forb(W, Y, G) = \emptyset \), and (III), all generalised back-door paths between \( W \) and \( Y \) are blocked (i.e. m-separated) by \( Z \) in \( G \).

Note in this paper, we aim to identify minimal adjustment sets, i.e. a set is a minimal adjustment set with none of its subset is an adjustment set. GAC is a sufficient and complete criterion for identifying an adjustment set from a MAG or PAG [Perkovic et al., 2017].

### 3 Causal effect estimation from data with hidden variables

In this section, we develop the theory towards precise causal effect estimation from data with hidden variables, including the problem setting, i.e. the identified case where the causal effect can be uniquely estimated, and the theorems presenting the conditions for finding the proper adjustment sets from data with hidden variables.

#### 3.1 Problem setting

As mentioned previously, from observational data what we can learn is an equivalence class of causal graphs, hence the problem of causal effect estimation does not have a unique solution in general. To obtain a fixed valued estimation of causal effect, assumptions are needed. In this paper, we have identified a practical situation where the causal effect can be uniquely estimated from data with hidden variables. Specifically, we assume that in the equivalence class of the MAGs learned (represented by a PAG), there exists a variable which is a Cause Or Spouse of the treatment Only (COSO) variable, as defined below.

**Definition 14 (COSO).** Let \( G \) be the PAG learned from a dataset with treatment \( W \), outcome \( Y \) and the set of other observed variables \( X \). A variable \( Q \in X \), is a COSO variable if \( Q \in (Pa(W) \cup Sp(W)) \) and \( Q \notin (Pa(Y) \cup Sp(Y)) \) in \( G \).

A COSO variable can be easily found in many applications. For example, when studying the effect of smoking on lung cancer [Spirtes et al., 2000], family influence causes smoking but does not cause lung cancer directly. Family influence is a COSO w.r.t. (smoking, lung cancer); in the study of the effect of job training on income, marriage impacts job training but does not directly affect income [LaLonde, 1986]. Marriage is a COSO for (job training, income). Compared to the existing data-driven methods towards precise causal effect estimation [Entner et al., 2013], which all require that \( X \) contains only pretreatment variables, our COSO variable assumption is more practical and enables broader real-world applications of the proposed solution presented in the next section.

#### 3.2 The theorems

The following theorem presents the condition for searching for an adjustment set in data with hidden variables, under our problem setting.

**Theorem 1.** Given a PAG \( G \) which is adjustment amenable relative to \( (W, Y) \) and contains a COSO variable \( Q \in X \), \( Z \subseteq X \) is an adjustment set if and only if \( Z \subseteq X \setminus \{Q\} \cup Forb(W, Y, G) \) and \( Z \perp Y | Z \cup \{W\} \).

**Proof.** As \( Q \) is a COSO variable, \( G \) must contain an edge \( Q \rightarrow W \) and has no direct edge from \( Q \) into \( Y \). As nodes on any causal path from \( W \) to \( Y \) should not be included in any adjustment set according to Definition 13 of GAC. Hence, \( Z \subseteq X \setminus \{Q\} \cup Forb(W, Y, G) \).
Theorem 1. Given a PAG $\mathcal{G}$ and contains a COSO variable $Q \in \mathbf{X}$. $Z \subset \mathbf{X}$ is an adjustment set if and only if $Z \subset \mathbf{X}$ is an adjustment set and contains a COSO variable $Q \in \mathbf{X}$. $Z \subset \mathbf{X}$ is an adjustment set if and only if $Z \subset \mathbf{X}$ is an adjustment set and contains a COSO variable $Q \in \mathbf{X}$.

Proof. $pds(W, Y, \mathbf{G})$ is a subset of $\mathbf{X}$ and from [Malinsky and Spirtes, 2017] $pds(W, Y, \mathbf{G})$ contains all minimal adjustment sets, we only need to search for a minimal adjustment set in $pds(W, Y, \mathbf{G}) \setminus (\{Q\} \cup Forb(W, Y, \mathbf{G}))$ instead of in $\mathbf{X} \setminus (\{Q\} \cup Forb(W, Y, \mathbf{G}))$. The proof follows from the proof of Theorem 1.

The following example illustrates Theorems 1 and 2.

Example 1. Refer to the PAG in Figure 1, denoted as $\mathcal{G}$, which is adjustment amenable relative to $(W, Y)$ and has $X_1$ as a COSO variable. From $\mathcal{G}$, $Forb(W, Y, \mathcal{G}) = \{X_3, Y\}$ and $pds(W, Y, \mathcal{G}) = \{X_1, X_2, X_3, Y\}$.

Considering Theorem 1, as $X_1 \setminus (\{Q\} \cup Forb(W, Y, \mathcal{G})) = \{X_2, X_4\}$ and $Z$ can be a subset of $\{X_2, X_4\}$, i.e. $\{X_2\}$ or $\{X_4\}$ and $X_1 \perp Y|\{X_2, X_4\}$, so following Theorem 1, $\{X_2\}$ and $\{X_2, X_4\}$ are adjustment sets for unbiased estimation of $ACE(W, Y)$. According to $\mathcal{G}$, we actually can see that $X_2$ blocks all the four generalised back-door paths from $W$ to $Y$: $W \leftarrow X_1 \rightarrow X_2 \rightarrow Y$; $W \leftarrow X_1 \rightarrow X_2 \rightarrow X_3 \rightarrow Y$; $W \leftarrow X_2 \rightarrow X_3 \rightarrow Y$; and $W \leftarrow X_2 \rightarrow X_3 \rightarrow Y$.

Considering Theorem 2, as $pds(W, Y, \mathcal{G}) \setminus (\{Q\} \cup Forb(W, Y, \mathcal{G})) = \{X_2\}$, the search space for $Z$ reduces to $\{X_2\}$, comparing to $\{X_2, X_4\}$ when following Theorem 1.

Theorem 3. Given a PAG $\mathcal{G}$ which is adjustment amenable relative to $(W, Y)$ and contains a COSO variable $Q \in \mathbf{X}$, let $Z$ be the set of all adjustment sets found following Theorem 1 or Theorem 2, the estimated value of $ACE(W, Y)$ for adjusting $Z$ is fixed $\forall Z \in Z$.

Proof. Any adjustment set $Z$ found by following Theorem 1 or Theorem 2 is a proper adjustment set for all MAGs enumerated from the given PAG since all back-door paths from $W$ to $Y$ in all the MAGs are blocked by $Z$. Hence, the causal effect estimated by adjusting any $Z$ is unbiased. In other words, the estimated causal effect is always the same when adjusting any $Z$ found with Theorem 1 or Theorem 2.

Theorem 3 shows that given a dataset under our problem setting, the estimated causal effect is unique when adjusting any of the adjustment sets found with Theorem 1 or Theorem 2. This is a significant advantage of our proposed method over most existing data-driven methods, such as LV-IDA [Malinsky and Spirtes, 2017], which provides a bound or a multiset of estimated causal effects.

3.3 The proposed algorithms

In this section, based on Theorem 2, we propose two data-driven algorithms for causal effect estimation from data with hidden variables, DAVS-Q (Algorithm 1) and DAVS (Algorithm 2). Both algorithms take as input a dataset and a PAG learned from the dataset using a structure learning algorithm at the choice of users.

DAVS-Q requires a given COSO variable, while DAVS finds COSO variables from data. In many applications, a COSO variable is known based on domain knowledge, then DAVS-Q is the better choice. When a COSO variable is unknown, users can employ DAVS, which finds the set of candidate COSO variables, $Q$. For each variable $Q \in Q$, DAVS calls DAVS-Q to estimate $ACE(W, Y)$.

So the two algorithms take the same search strategy, a level wise strategy to search for an adjustment set based on Theorem 2, and stop the search process once an adjustment set is found, and estimate the causal effect using this found adjustment set (see lines 7-12 of (Algorithm 1). The correctness of the search strategy is guaranteed by Theorem 3, i.e. the uniqueness of the estimated causal effect using the adjustment sets found based on Theorem 2. Furthermore, the algorithms use a bottom-up search (from single variables to multiple variables) for efficient utilisation of data in conditional independence tests. Note that, empty set is also a legitimate adjustment set if it satisfies Theorem 2. This situation is checked and dealt with in lines 2-3 of Algorithm 1.

In our implementation, the input PAG $\mathcal{G}$ is learned using rFClI [Colombo et al., 2012] implemented in the R package pcalg [Kalisch et al., 2012]. The conditional independence test tool is implemented by GaussianCtest and binCtest in pcalg for Gaussian and binary datasets respectively. Eq. (1), the calculation of $ACE(W, Y)$ is implemented by im in R package stats [Maathuis et al., 2009; Malinsky and Spirtes, 2017] and stGlm in R package stReg [Witte and Didelez, 2018] for continuous and binary outcome $Y$ respectively.

The time complexity of DAVS-Q and DAVS is largely determined by the time taken by the rFClI algorithm for learning PAG $\mathcal{G}$ from $D$. Lines 7 - 12 in DAVS-Q (Algorithm 1) can be time-consuming, but the size of $Z$ is often small in practice, usually 2-5. When the size of $Z$ is large, a very large dataset is needed for conditional independence tests. Hence, the size of $Z$ is normally kept small for reliable tests. In DAVS, the
Algorithm 1 Data-driven Adjustment Variable Selection for estimating $ACE(W, Y)$ given $Q$ (DAVS-Q)

Input: Dataset $D$ with $W, Y$ and $X$, given $Q$ and PAG $\mathcal{G}$ learned from $D$.

Output: $ACE(W, Y)$.

1: Let $ACE(W, Y) = NULL$
2: if $Q \perp Y | W$ then
3: Calculate $ACE(W, Y)$ via Eq.(1) given $Z = \emptyset$
4: else
5: Obtain $pds(W, Y, \mathcal{G})$ and $Forb(W, Y, \mathcal{G})$ from $\mathcal{G}$.
6: $\mathcal{Y} = pds(W, Y, \mathcal{G}) \setminus \{(Q \cup Forb(W, Y, \mathcal{G}))
7: for each subset $Z \subseteq \mathcal{Y}$ (level wise test) do
8: if $Q \perp Y | Z \cup W$ then
9: Calculate $ACE(W, Y)$ via Eq.(1) given $Z$
10: break for loop
11: end if
12: end for
13: end if
14: return $ACE(W, Y)$.

Algorithm 2 Data-driven Adjustment Variable Selection for estimating $ACE(W, Y)$ without $Q$ (DAVS)

Input: Dataset $D$ with $W, Y$ and $X$, PAG $\mathcal{G}$ learned from $D$.

Output: $ACE(W, Y)$.

1: Let $ACE(W, Y) = NULL$
2: Find $Pa(W), Sp(W), Pa(Y)$ and $Sp(Y)$ from $\mathcal{G}$.
3: $Q \leftarrow Pa(Y) \cup Sp(Y)$
4: for each $Q \in Q$ do
5: $ACE(W, Y) = DAVS-Q(D, Q, \mathcal{G})$
6: if $ACE(W, Y) \neq NULL$ then
7: break for loop
8: end if
9: end for
10: return $ACE(W, Y)$

The implementation of EHS [Entner et al., 2013] is from https://sites.google.com/site/dorisentner/publications/CovariateSelection, and the implementation of CFRNet is from https://github.com/clinicalml/cfrnet. The classic PSM [Rubin, 1974] is implemented by glm in package stats and Matching in R package Matching [Ho et al., 2007].

The same parameter settings are used for DAVS-Q, DAVS and LV-IDA, and the significance level ($\alpha$) is set to 0.05. Since IDA and LV-IDA each return a multiset of causal effects, the average is considered as the most probable estimation. For EHS, we constraint that the size of conditional set to 6, otherwise it cannot work within two hours for the IHDP and Twins datasets.

4.1 Experiments with synthetic data

We use 5 benchmark Bayesian networks (BNs) from the repository\footnote{http://www.bnlearn.com/bnrepository/}: CHILD, INSURANCE, MILDEW, ALARM and BARLEY to generate synthetic datasets. For each BN, we choose a variable with multiple incoming edges as the outcome variable $Y$, and select one of $Y$'s parents as the treatment variable $W$. We generate 5 synthetic datasets from the 5 BNs with 10,000 samples each by using the R package bnlearn [Scutari, 2009]. Then we hide 5% variables which are on back-door paths of $W$ to $Y$ from each synthetic dataset.

As the pretreatment variable assumption does not hold in these datasets, only methods without assuming pretreatment variables, i.e. IDA and LV-IDA are used in the comparison.

The causal effects calculated using Eq.(1) and the adjustment set identified based on back-door criterion [Pearl, 2009] on DAGs of the complete BNs are the ground truth causal effects. Bias(%) is absolute estimated error rate w.r.t. the ground truth causal effects.

From the results in Table 2, DAVS-Q and DAVS have significantly smaller biases than IDA and LV-IDA. The result has demonstrated that our algorithm not only can provide unique causal effect estimation, and the estimated causal effects are more accurate.

4.2 Experiments on two real-world datasets.

We evaluate the performance of DAVS-Q and DAVS on two real-world datasets, IHDP [Hill, 2011] and Twins [Almond et al., 2005; Louizos et al., 2017].

IHDP is the Infant Health and Development Program (IHDP) dataset from a collection based on a randomized controlled experiment studying high-quality intensive care provided to low-birth-weight and premature infants. There are 24 pretreatment variables (excluding race) and 747 infants, including 139 treated and 608 control. The simulated outcome variable was generated with the true $ACE(W, Y) = 4.36$ from the R packagenpci [Hill, 2011].

Twins is a benchmark dataset about twin births and deaths in the USA from 1989-1991 [Almond et al., 2005]. We only choose the same-sex twins with weights less than 2000g from the original data and each twin-pair contains 40 pre-treatment variables relate to the parents, the pregnancy and the birth [Louizos et al., 2017]. We eliminate all records with missing values, so 4821 twin-pairs remain. For each pair,
Table 2: Estimating $\text{ACE}$ on synthetic datasets. (a, b) next to a dataset name denotes the number of nodes (a) and arcs (b).

Table 3: Estimating $\text{ACE}$ on IHDP and Twins.

we observe both the treated ($W=1$, heavier twin) and control ($W=0$, lighter twin). The mortality after one year is the true outcome for each twin such that the true $\text{ACE}(W; Y)$ is -0.02489. For simulating an observational study, we follow [Louizos et al., 2017] to randomly hide one of the two twins. We use the setting: $W_i|x_i \sim \text{Bern}(\text{sigmoid}(\beta^T x + \varepsilon))$, where $\beta^T \sim \mathcal{U}((-0.1, 0.1)^{40 \times 1})$ and $\varepsilon \sim \mathcal{N}(0, 0.1)$.

Both datasets have pretreatment variables and hence we can include all methods in Table 1 (excluding CE-SAT) in the comparison. We use the default settings of the methods with the pretreatment assumption.

From the results in Table 3, when data is sufficient for learning the PAG, such as the Twins dataset, DAVS performs the best. When data is insufficient, such as the IHDP dataset, algorithms relying on the learned causal structure (i.e. IDA and LV-IDA) perform worse than other methods.

4.3 Efficiency evaluation
All computations were conducted on a PC with 2.6GHz Intel Core i7 and 16GB of memory. The runtime of the algorithms on all datasets (synthetic and real-world data) is shown in Figure 2. Note that there is no runtime recorded for EHS on the synthetic datasets as they do not met the pretreatment variable assumption. From Figure 2, IDA is the fastest since learning DAGs from data is faster than learning MAGs. EHS is the slowest since it uses exhaustive search. LV-IDA and our proposed algorithms have very similar time efficiency.

5 Related work
The major work on data-driven causal effect estimation has been discussed in the Introduction, so here we discuss the work related to the test condition in our theorems. The condition $Q \perp \perp Y | Z \cup \{W\}$ is the third condition (of three) in Pearl’s Genuine cause [Pearl, 2009] and the second condition (of two) of the infer rule to determine an adjustment set proposed in [Entner et al., 2013]. Only after we have introduced COSO variables, the rule can be used alone and has a property to support unbiased causal effect estimation. Furthermore, Pearl has not presented an algorithm for adjustment set identification based on the rules. Although Entner et al. presented an algorithm for searching for adjustment sets, the algorithm assumes pretreatment variables and it is very inefficient.

Adapting machine learning algorithms for causal inference has gained great attention recently [Athey and Imbens, 2016; Künzel et al., 2019]. Athey et al. [Athey and Imbens, 2016] and Wager et al. [Wager and Athey, 2018] adapted tree-based algorithms to estimate conditional average treatment effect (CATE) from data. In [Louizos et al., 2017], it was proposed to utilise Variational Autoencoder to estimate the unknown latent space of confounders and CATE simultaneously. [Hasanpour and Greiner, 2019] proposed a context aware importance sampling re-weighing scheme to address distributional shift due to selection bias. [Künzel et al., 2019] introduced a new meta-learner, the X-learner which uses the observed outcomes to estimate the unobserved individual treatment effects. Our work differs from all the work in that we estimate the average total causal effect from data with hidden variables, by covariate adjustment.

6 Conclusion & Future work
In this paper, we have studied the data-driven approach towards precise causal effect estimation. We have developed the theorems for determining an adjustment set from data with hidden variables. The theorems support the development of two data-driven algorithms to obtain unique and more accurate causal effect estimation from data with hidden variables. The experimental results have shown the good performance of the algorithms.
Selection bias is another major factor affecting causal effect estimation. In future, we will extend the theorems in this paper for causal effect estimation with selection bias.

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