An $SU(2)_L \times U(1)_Y$ model with reflection symmetry in view of recent neutrino experimental result

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We demonstrate that an $SU(2)_L \times U(1)_Y$ model with the same particle content as Standard Model (SM) and discrete reflection symmetry between second and third generation of leptons gives rise to charged lepton and neutrino mass matrices which can accommodate the present solar, atmospheric, WMAP neutrino experimental results. The model predicts the value of $|U_{13}|$ which could be tested in neutrino factories and the value of effective Majorana neutrino mass $\langle m_{ee} \rangle$ comes out at the lower end of the present experimental limit. Neutrino masses are generated through $\text{dim}=5$ operators and the scale of which is constrained by the value of $\langle m_{ee} \rangle$. If, in future neutrino less double beta decay experiments namely, MOON, EXO, GENIUS etc. shifts the lower bound on $\langle m_{ee} \rangle$ by one order the present model will fail to accommodate the solar neutrino mixing angle due to LMA solution.

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I. INTRODUCTION

Super-Kamiokande (SK) atmospheric neutrino experiment [1] has strengthened the conjecture of neutrino flavor oscillation as well as non-zero neutrino mass through the measurement of magnitude and angular distribution of $\nu_\mu$ flux produced in the atmosphere due to cosmic ray interactions and the data provide the following values as $\Delta m^2 = 2.7 \times 10^{-3} \text{eV}^2$, $\sin^2 2\theta = 0.85$ at 99.73% c.l. Furthermore, the K2K reactor neutrino experiment [2] is also in concordance with the interpretation of atmospheric neutrino experimental result. Taking into account the K2K reactor neutrino experimental result and the SK neutrino experimental result, the best fitted value obtained as $\Delta m^2_{23} = (2.63 \pm 0.33) \times 10^{-3} \text{eV}^2$, $\sin^2 2\theta = 1.0$. Furthermore, it has been shown that global analysis of all solar neutrino experimental results, including SNO experiment [3], is in favor of Large Mixing Angle (LMA) MSW solution of solar $\nu_e$ deficit problem. This solution is also recently supported by the KamLAND reactor neutrino experimental result [4]. Combined analysis of KamLAND and all other solar neutrino experimental data restrict the parameter space of $\nu_e$ oscillation [5, 6] and there are slightly different ranges of $\Delta m^2_{23}$ and $\tan^2 \theta_{23}$ have been found in the literature [7], although the variation is not much. We consider for the present analysis the following ranges $\Delta m^2_{23} = (2.45 \pm 0.57) \times 10^{-3} \text{eV}^2$, $0.266 \leq \tan^2 \theta_{23} \leq 0.88$ at 3$\sigma$ level with the best-fitted value of $\Delta m^2_{23} = 7.17 \times 10^{-3} \text{eV}^2$, $\tan^2 \theta_{23} = 0.43$. Moreover, assuming $\theta_{12}$ as $\theta_{12}$ and $\theta_{atm}$ as $\theta_{23}$, the CHOOZ experiment [8] also has put an upper bound on the $U_{e3}$ element of the lepton mixing matrix (which is also known as MNSP matrix). It has been shown [9, 10] that combined three-neutrino oscillation analysis of solar, atmospheric and reactor data gives a lower bound on $U_{e3}$ as $|U_{e3}| \leq 0.22$ at 99.73% c.l. There are two other important experimental results, one of them, recently reported WMAP [11] result on cosmic microwave background anisotropies. Combining analysis of 2dF Galaxy Redshift survey, CBI and ACBAR [12], WMAP has determined the amount of critical density contributed by relativistic neutrinos which in turn gives an upper bound on the total neutrino mass as $\Sigma m_\nu \leq 0.71 \text{eV}$ at 95% c.l. [13]. Furthermore, neutrino-less double beta decay ($\beta \beta_{0v}$) experiment [14] has reported the bound on the effective Majorana neutrino mass (relaxing the uncertainty of the Nuclear Matrix elements up to $\pm 50\%$ and the contribution to this process due to particles other than Majorana neutrino is negligible [15]) as $\langle m_{ee} \rangle = (0.05 - 0.84) \text{eV}$ at 95% c.l. [16]. Keeping all these constraints in view, it is very much interesting to find out an appropriate texture of lepton mass matrix which satisfy all those experimental results with appropriate mixing. There is much literature investigating appropriate texture of neutrino mass matrix keeping charged lepton mass matrix diagonal. Another scenario, which considers both charged lepton and neutrino mass matrices are non-diagonal.

Recently, an investigation in this path has been done in Ref.[17], through the introduction of a discrete $Z_3$ symmetry within the framework of an SU(5) model to generate appropriate mixing in both quark and lepton sector. Neutrino masses are generated in this model through seesaw mechanism and the model gives rise to appropriate mixing which reconciles with the neutrino experimental results. In the present work, we consider a different texture of charged lepton and neutrino mass matrices within the framework of an $SU(2)_L \times U(1)_Y$ model with a reflection symmetry and keeping the particle content same as Standard Model (SM). Neutrino masses are generated through $\text{dim}=5$ operators due to explicit violation of Lepton number. In addition, we consider a reflection symmetry between second and third generation of leptons. We have considered both charged lepton and neutrino mass matrices non-diagonal. Apart from the phase factor, the

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structure of neutrino mass matrix is same as the texture of neutrino mass matrix presented in Ref. [18]. The parameter space of the model admits solar, atmospheric and WMAP experimental results and predicts the value of $\theta_{13}$ well below the CHOOZ experimental result, the value of which may be possible to check in future neutrino factories [19]. The present model also predicts the value of effective Majorana neutrino mass $m_{ee}$, which may be possible to check in future neutrino factories. Neutrino phenomenology is discussed in Section III. Section IV contains summary of the present work.

II. PROPOSED MODEL

We consider an $SU(2)_L \times U(1)_Y$ model with particle content same as Standard Model. We consider the following reflection symmetry in the lepton sector

$$l_{2L} \leftrightarrow l_{3L}, \mu_R \leftrightarrow \tau_R. \quad (2.1)$$

The most general discrete symmetry invariant lepton-Higgs Yukawa interaction in the present model which gives rise to charged lepton and neutrino mass is given by

$$L_Y^L = [g_{1L} l_{1L} e_R + g_{2L} l_{2L} (\mu_R + \tau_R) + g_{21}(l_{2L} + l_{3L})e_R + g_{22}(l_{2L}\mu_R + l_{3L}\tau_R) + g_{23}(l_{2L}\tau_R + l_{3L}\mu_R)]\phi + h.c. \quad (2.2)$$

$$L_Y^\nu = [f_1 l_{1L} l_{1L} + f_2 l_{1L} (l_{2L} + l_{3L}) + f_3 (l_{2L} l_{2L} + l_{3L} l_{3L})]\phi/\phi/M \quad (2.3)$$

where $M$ is an additional scale of the theory apart from the electroweak symmetry breaking scale. Substituting VEV of the Higgs field $\phi$, $\langle \phi \rangle = v$, we obtain the charged lepton and neutrino mass matrices as

$$M_E = \begin{pmatrix} a & b & b \\ c & d & e \\ c & e & d \end{pmatrix} \quad (2.4)$$

where

$$a = g_{11}v, b = g_{12}v, c = g_{21}v,$$

$$d = g_{22}v, e = g_{23}v \quad (2.5)$$

$$M_\nu = \begin{pmatrix} p & q & q \\ q & r & q \\ q & r & r \end{pmatrix} \quad (2.6)$$

where

$$p = f_1\lambda, \quad q = f_2\lambda, \quad r = f_3\lambda, \quad \lambda = v^2/M \quad (2.7)$$

To diagonalize the charged lepton and neutrino mass matrix, we consider an orthogonal matrix, $O_\alpha, (\alpha = e, \nu)$ as

$$O_\alpha = \begin{pmatrix} c_{12}^\alpha & c_{31}^\alpha & s_{31}^\alpha \\ -s_{12}^\alpha c_{23}^\alpha & c_{12}^\alpha & -s_{23}^\alpha c_{31}^\alpha \\ s_{12}^\alpha c_{23}^\alpha & c_{23}^\alpha & s_{31}^\alpha e^{\phi_0/2} \end{pmatrix} \quad (2.8)$$

where $O_e$ and $O_\nu$ are utilised to diagonal $M_E$ and $M_\nu$, respectively.

By considering $M_E$ is real, we diagonalize the charged lepton mass matrix as

$$O_e^T M_E M_E^T O_e = \text{diag}(m_1^2, m_2^2, m_3^2) \quad (2.9)$$

and we obtain the three eigenvalues as

$$m_1^2 = \frac{(X + W + Z) - \sqrt{(X - W - Z)^2 + 8Y^2}}{2} \quad (2.10)$$

$$m_2^2 = \frac{(X + W + Z) + \sqrt{(X - W - Z)^2 + 8Y^2}}{2}$$

$$m_3^2 = Z - W \quad (2.10)$$

and the three mixing angles are

$$\theta_{13}^e = 0, \theta_{23}^e = -\pi/4,$$

$$\tan^2 \theta_{12}^e = \frac{(X - m_1^2)}{(m_2^2 - X)} \quad (2.11)$$

where

$$X = a^2 + 2b^2, Y = ac + bd + be, Z = c^2 + d^2 + e^2,$$

$$W = c^2 + 2de \quad (2.12)$$

The neutrino mass matrix $M_\nu$ is diagonalized as

$$(P_\nu^o O_\nu)^\dagger D_\nu (P_\nu O_\nu) = M_\nu \quad (2.13)$$
where

\[ D_\nu = \text{diag}(m_1^\nu, m_2^\nu, m_3^\nu) \quad (2.14) \]

\[ P_\nu = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\delta_2} & 0 \\ 0 & 0 & e^{i\delta_3} \end{pmatrix} \quad (2.15) \]

and the three neutrino masses come out as

\[ m_1^\nu = \frac{(p + 2r) - \sqrt{p^2 + 8q^2}}{2} \]
\[ m_2^\nu = \frac{(p + 2r) + \sqrt{p^2 + 8q^2}}{2} \]
\[ m_3^\nu = 0 \quad (2.16) \]

with the three mixing angles are given by

\[ \tan^2 \theta^\nu_{12} = (p - m_1)/(m_2 - p) \quad (2.17) \]

where \( m_1, m_2 \) are the charged lepton masses given in eqn.(2.10). It is to be noted that, the symmetry proposed in the model given in eqn.(2.1) constrains the relationship between \( \delta^\nu_3 \) and \( \delta^\nu_2 \) as \( \delta^\nu_2 = -\delta^\nu_3 \). More precisely, this can be understood if we consider \( l_2L \rightarrow e^{\delta^\nu_1}l_1L, l_3L \rightarrow e^{i\delta^\nu_1}l_2L \) which necessarily leads \( \delta^\nu_2 = -\delta^\nu_3 \). However, such transformation also leads charged lepton mass matrix complex. Further demanding the symmetry \( \mu_R \rightarrow e^{i\alpha_1} \tau_R, \tau_R \rightarrow e^{i\alpha_2} \mu_R \) (and as previous \( \alpha_1 = -\alpha_2 \)) will give rise to a phase matrix \( P_\nu \) which is similar in structure to \( P_\ell \). Both \( P_\ell \) and \( P_\nu \) will appear in \( U^{\text{MNSP}} \) matrix (Maki - Nakagawa - Sakata - Pontecorvo) and the product \( P_\ell^T P_\nu = P = \text{diag}(1, e^{i\delta_2}, e^{i\delta_3}) \) has the same structure as \( P_\ell \) or \( P_\nu \) with the constraint relation \( \delta_2 = -\delta_3 \). Furthermore if we consider Dirac phase in \( M_\ell \) the structure of \( U^{\text{MNSP}} \) will not change. Thus the MNSP mixing matrix appear in the charged current interaction as

\[ U^{\text{MNSP}} = O^T_\ell P_\nu \]

\[ = \begin{pmatrix} e^{\nu} e^{\nu} - \rho s^e s^\nu & e^{\nu} s^e - \rho s^e c^\nu & -s^e c^\nu \\ s^e e^{\nu} - \rho c^e s^\nu & s^e s^\nu + \rho c^e c^\nu & s^e c^\nu \\ -s^\nu c^e & \sigma & \rho \end{pmatrix} \quad (2.18) \]

where

\[ c^{\nu, \nu} = c^{\nu}_{12}, \ s^{\nu, \nu} = s^{\nu}_{12}, \]
\[ \rho = (e^{i\delta_2} + e^{i\delta_3})/2, \sigma = (e^{i\delta_3} - e^{i\delta_2})/2 \quad (2.19) \]

III. NEUTRINO PHENOMENOLOGY

Let us discuss neutrino phenomenology of the present model. First of all, we have fix the value of \( m_2^\nu \) by the required mass-squared difference needed to explain atmospheric neutrino problem as

\[ \Delta m^2_{\text{atm}} = \Delta m^2_{\text{23}} = (m_2^\nu)^2 = 2.7 \times 10^{-3} \text{eV}^2 \quad (3.1) \]

where we have used the best-fitted value of \( \Delta m^2_{\text{atm}} \) and \( m_2^\nu \) comes out as \( m_2^\nu = 0.0519 \text{ eV} \). Next, we fix the value of \( m_1^\nu \) with the solar neutrino experimental result as \( \Delta m^2_{\odot} = \Delta m^2_{\text{31}} = 7.1 \times 10^{-5} \text{eV}^2 \) and the value of \( m_1^\nu \) becomes \( m_1^\nu = 0.0512 \text{ eV} \). The three mixing angles required to explain the solar, atmospheric and CHOOZ neutrino experimental results are given by

\[ \sin^2 2\theta_{\text{atm}} = 4|U_{23}|^2 |U_{33}|^2 \]
\[ = 4|\rho|^2 |\sigma|^2 c_e^2 \]
\[ = (m_2^\nu - X)/(m_2^\nu - m_1^\nu)^2 \sin^2(\delta_2 - \delta_3) \quad (3.2) \]

where \( m_1, m_2 \) are the masses of charged leptons given in eqn.(2.10). Similarly,

\[ |U_{13}|^2 = (X - m_1^\nu)/(2(m_2^\nu - m_1^\nu)) \quad (3.3) \]

and

\[ \sin^2 2\theta_{\text{CHOOZ}} = 4|U_{13}|^2 (1 - |U_{13}|^2) \quad (3.4) \]

\[ \sin^2 2\theta_{\odot} = 4|U_{11}|^2 |U_{12}|^2 \]
\[ = 4[(c_e c_{\nu})^2 + s_e s_{\nu}^2 + \sqrt{2} c_e c_{\nu} s_e s_{\nu} \cos (\delta_2 + \delta_3)] \times \]
\[ [(c_e c_{\nu})^2 + s_e s_{\nu}^2 - \sqrt{2} c_e c_{\nu} s_e s_{\nu} \cos (\delta_2 + \delta_3)] \quad (3.5) \]

Using the value of \( m_1, m_2 \) as \( m_1 = 0.4868 \text{ MeV}, m_2 = 105.7513 \text{ MeV} \), we now fix the value of \( X \) from eqn.(3.2). It is obvious from eqn.(3.2) that \( \delta_2 \neq \delta_3 \) and \( X \neq m_2^\nu \) to get non-zero atmospheric mixing angle. However, if \( X = m_2^\nu \) and \( \delta_2 = -\delta_3 = \pi/4 \) the atmospheric mixing angle becomes \( \sin^2 2\theta_{\text{atm}} = 1.0 \) and in that case \( |U_{13}|^2 = 0 \) and the solar neutrino mixing angle also shoots up to maximal value which is an ideal case for bi-maximal neutrino mixing. To find out a realistic parameter space, the rest of the work we consider \( \delta_2 = -\delta_3 = \pi/4 \) and from numerical estimation we find the value of \( X \) as \( X = m_1 m_2/100 \) and with such value of \( X \) we get

\[ \sin^2 2\theta_{\text{atm}} = 0.99 \]
\[ \sin^2 2\theta_{13} = 4.99 \times 10^{-5}, |U_{13}|^2 \simeq 1.25 \times 10^{-5} \]  
(3.6)

Such small value of \( \theta_{13} \) or \( U_{13} \) may be probed in future neutrino factories with superbeam facilities [14]. The solar neutrino mixing angle comes out as \( \sin^2 2\theta_{13} \approx 0.85 \) for the choice of \( p = 0.0516 \) eV. The parameter space is very sensitive to the parameter \( p \) and \( p \) can be varied upto \( p < 0.0515 \) eV so that \( \sin^2 2\theta_{13} < 1 \).

The parameter \( p \) also determines the value of effective Majorana neutrino mass \( \langle m_{\nu_e} \rangle = p \) and the value obtained marginally satisfies the bound obtained from recent \( \beta\beta_{0v} \) decay. Thus, if the lower bound of \( \langle m_{\nu_e} \rangle \) shifts by one order, the present model will fail to accommodate the solar neutrino mixing angle. Furthermore, the value of \( p \) also determines the scale \( M \) of the present model by considering perturbative unitarity bound on the Yukawa coupling \( f_1 \) as \( f_1^2/4\pi \leq 1 \). Thus, the additional scale \( M \) of the theory can be written as

\[ M = \frac{f_1 \phi^2}{p} \leq \frac{2\sqrt{p} \phi^2}{p} \]  
(3.6)

and for \( \langle \phi \rangle \simeq 200 \text{ GeV} \), we get \( M \leq 10^{13} \text{ GeV} \). Finally, in the present model, sum of the three neutrino mass comes out as \( \Sigma m_i = 0.1031 \text{ eV} \) which is also far below than the bound obtained from WMAP experimental result.

**IV. SUMMARY**

We demonstrate that an \( SU(2)_L \times U(1)_Y \) model with particle content same as SM and with a discrete reflection symmetry between second and third generation of leptons give rise to an appropriate texture of charged lepton and neutrino mass matrices which can accommodate the present atmospheric, solar, CHOOZ, WMAP and \( \beta\beta_{0v} \) decay experimental results. Neutrino masses are generated through explicit lepton number violation due to dim 5 mass terms and the charged lepton masses are generated according to SM. Apart from electroweak symmetry breaking scale, the present model contains an additional scale \( M \), which is constrained as \( M \leq 10^{13} \text{ GeV} \) due to the \( \beta\beta_{0v} \) decay experimental result and if the present lower bound of \( \beta\beta_{0v} \) decay result shifts by one order in future \( \beta\beta_{0v} \) experiments, such as, MOON, EXO, GENIUS, the present model will fail to accommodate the solar neutrino mixing angle due to LMA solution recently favoured by the SNO and KamLAND experimental results. It is worthwhile to investigate application of such symmetry in the context of other models where neutrino masses are generated through see-saw mechanism or by radiative way.

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