The Quantum Field Theory Boundaries Applicability and Black Holes Thermodynamics

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Abstract
This paper presents a study of the applicability boundary of the well-known quantum field theory. Based on the results of black hole thermodynamics, it is shown that this boundary may be lying at a level of the energy scales much lower than the Planck. The direct inferences from these results are given, specifically for estimation of a cosmological term within the scope of the quantum field theory.

Keywords Quantum field theory · Black holes thermodynamics · Equivalence principle

1 Introduction
The local quantum field theory (QFT) is understood as a canonical Quantum Field Theory in flat space-time [1–3]. But in what follows it is demonstrated that a flat geometry of space-time in the processes of high energy physics is not ensured from the start, being based on validity of the Einstein’s Strong Equivalence Principle (EP). However, this principle has its applicability boundaries. The Planck scales present a natural (upper) bound for applicability because at these scales a natural geometry of space-time, determined locally by the particular metric $g_{\mu\nu}(x)$, disappears due to high fluctuations of this metric and is replaced by space-time (or quantum) foam.

In [4, 5] the author suggested a hypothesis that actually the real applicability boundary of EP lies in a domain of the energies $E$ considerably lower than the Planck energies. The principal objective of this paper is to demonstrate that the hypothesis is true for some, quite naturally arising, assumptions. Proceeding from the afore-said, this condition sets the applicability boundaries for the canonical QFT. Besides, direct inferences from the obtained result are considered. Hereinafter, EP is understood as a Strong Equivalence Principle.
The canonical quantum field theory (QFT) [1–3] is a local theory considered in continuous space-time with a plane geometry, i.e with the Minkowskian metric $\eta_{\mu\nu}(\mathbf{x})$. In reality, any interaction introduces some disturbances, introducing an additional local (little) curvature into the initially flat Minkowskian space $\mathcal{M}$. Then the metric $\eta_{\mu\nu}(\mathbf{x})$ is replaced by the metric $\eta_{\mu\nu}(\mathbf{x}) + o_{\mu\nu}(\mathbf{x})$, where the increment $o_{\mu\nu}(\mathbf{x})$ is small. But, when it is assumed that EP is valid, the increment $o_{\mu\nu}(\mathbf{x})$ in the local theory has no important role and, in a fairly small neighborhood of the point $\mathbf{x}$ in virtue EP.

The Einstein Equivalence Principle (EP) is a basic principle not only in the General Relativity (GR) [6–8], but also in the fundamental physics as a whole. In the standard formulation it is as follows: ([8],p.68):

"at every space-time point in an arbitrary gravitational field it is possible to choose a locally inertial coordinate system such that, within a sufficiently small region of the point in question, the laws of nature take the same form as in unaccelerated Cartesian coordinate systems in the absence of gravitation".

Then in ([8],p.68) "...There is also a question, how small is “sufficiently small”. Roughly speaking, we mean that the region must be small enough so that gravitational field in sensible constant throughout it...”.

However, the statement “sufficiently small” is associated with another problem. Indeed, let $\mathbf{x}$ be a certain point of the space-time manifold $\mathcal{M}$ (i.e. $\mathbf{x} \in \mathcal{M}$) with the geometry given by the metric $g_{\mu\nu}(\mathbf{x})$. Next, in accordance with EP, there is some sufficiently small region $\mathcal{V}$ of the point $\mathbf{x}$ so that, within $\mathcal{V}$ it is supposed that space-time has a flat geometry with the Minkowskian metric $\eta_{\mu\nu}(\mathbf{x})$.

In essence, sufficiently small $\mathcal{V}$ means that the region $\mathcal{V}'$, for which $\mathbf{x} \in \mathcal{V}' \subset \mathcal{V}$, satisfies this condition as well. In this way we can construct the sequence

$$(1) \quad \ldots \subset \mathcal{V}'' \subset \mathcal{V}' \subset \mathcal{V}. $$

The problem arises, is there any lower limit for the sequence in formula (1)?

The answer is positive. Currently, there is no doubt that at very high energies (on the order of Planck energies $E \approx E_p$, i.e. on Planck scales, $l \approx l_p$) quantum fluctuations of any metric $g_{\mu\nu}(\mathbf{x})$ are so high that in this case the geometry determined by $g_{\mu\nu}(\mathbf{x})$ is replaced by the “geometry” following from space-time foam that is defined by great quantum fluctuations of $g_{\mu\nu}(\mathbf{x})$, i.e. by the characteristic dimensions of the quantum-gravitational region (for example, [9–14]). The above-mentioned geometry is drastically differing from the locally smooth geometry of continuous space-time and EP in it is no longer valid [15–22].

From this it follows that the region $\mathcal{V}_{\mathbf{r},\mathbf{t}}$ with the characteristic spatial dimension $\mathbf{r} \approx l_p$ (and hence with the temporal dimension $\mathbf{t} \approx t_p$) is the lower (approximate) limit for the sequence in (1).

It is difficult to find the exact lower limit for the sequence in formula (1)–it seems to be dependent on the processes under study. Specifically, when the involved particles are considered to be point, their dimensions may be neglected in a definition of the EP applicability limit. When the characteristic spatial dimension of a particle is $\mathbf{r}$, the lower limit of the sequence from formula (1) seems to be given by the region $\mathcal{V}_{\mathbf{r}}$ containing the above-mentioned particle with the characteristic dimensions $\mathbf{r}' > \mathbf{r}$, i.e. the space EP applicability limit should always be greater than dimensions of the particles considered in this region. By the present time, it is known that spatial dimensions of gauge bosons, quarks, and leptons within the limiting accuracy of the conducted measurements < $10^{-18} m$. Because of this, the condition $\mathbf{r}' \geq 10^{-18} m$ must be fulfilled. In addition, the radius of interaction of
particles $r_{int}$ must be taken into account in quantum theory. And this fact also imposes a restriction on considering concrete processes in quantum theory. However, the interactions radii of all known processes lie in the energy scales $E \ll E_p$.

At the present time there is a series of the results demonstrating that EP may be violated at the energies $E$ considerably lower than $E_p$ (for example, the quantum phenomenon of neutrino oscillations in a gravitational background [23–26] and others [27–29]).

As QFT is a local theory applicable only to space-time with a flat geometry determined by the Minkowskian metric $\eta_{\mu\nu}(\vec{x})$, the applicability boundary EP may be considered the applicability boundary of QFT as well.

**Main Hypothesis** It is assumed that in the general case EP, and consequently, QFT is valid for the locally smooth space-time only if all the energies $E$ of the particles are satisfied the necessary condition

$$E \ll E_p, \quad (2)$$

In the following section this hypothesis is proven in the assumption that space-time foam consists of micro black holes (mbh) with the event horizon radius $r \approx l_p$ and mass $m \approx m_p$.

**Remark 2.1** Why in canonical QFT it is so important never forget about the fact that space-time has a flat geometry, or the same possesses the Minkowskian metric $\eta_{\mu\nu}(\vec{x})$? Simply, in the contrary case we should refuse from some fruitful methods and from the results obtained by these methods in canonical QFT, in particular from Wick rotation [3]. In fact, in this case the time variable is replaced by $t \mapsto it \equiv t_E$, and the Minkowskian metric $\eta_{\mu\nu}(\vec{x})$ is replaced by the four-dimensional Euclidean metric

$$ds^2 = dt_E^2 + dx^2 + dy^2 + dz^2. \quad (3)$$

Clearly, such replacement is possible only in the case when from the start space-time (locally) has a flat geometry, i.e. possesses the Minkowskian metric $\eta_{\mu\nu}(\vec{x})$. This is another argument supporting the key role of the EP applicability boundary. Otherwise, when we go beyond this boundary, Wick rotation becomes invalid. Naturally, some other methods of canonical QFT will lose their force too.

### 3 The Strong Equivalence Principle, Black Holes, and QFT

It is supposed that a large (i.e., classical) four-dimensional Schwarzschild black hole is existent with the metric

$$ds^2 = \left(1 - \frac{2MG}{r}\right) dt^2 - \left(1 - \frac{2M}{r}\right)^{-1} dr^2 - r^2 d\Omega^2, \quad (4)$$

where $M$ is the mass of this black hole, and the Schwarzschild horizon radius $r_{BH}$ is defined by

$$r_{BH} = 2MG. \quad (5)$$

As shown in [30–32], EP is violated for an observer distant from the black hole event horizon. Considering our objective, it seems expedient to give in brief the main results from [30–32].
In view of the Unruh effect, an accelerating observer does detect thermal radiation (so-called Unruh radiation) with the Unruh temperature given by [34]

$$T_{\text{Unruh}} = \frac{\hbar a}{2\pi},$$

where $a = |a|$ is a corresponding acceleration.

When an observer is at the fixed distance, $r > r_{BH}$, from a Schwarzschild black hole of mass $M$ and event horizon radius $r_{BH} = 2GM$, then, due to the existence of Hawking radiation [35], the observer will measure radiation with thermal spectrum and a temperature given by formula [30, 31]

$$T_{H,r} = \frac{\hbar}{8\pi GM \sqrt{1 - \frac{r_{BH}}{r}}},$$

where $r > r_{BH}$.

In the foregoing formulae and in what follows, we use the normalization $c = k_B = 1$.

Besides, in [30, 31] it is shown that an observer, positioned at the fixed distance $r > r_{BH}$ from the above-mentioned black holes and measuring Hawking temperature with the value $T_{H,r}$, experiences the local acceleration

$$a_{BH,r} = \frac{1}{\sqrt{1 - \frac{r_{BH}}{r}}} \left( \frac{r_{BH}}{2r^2} \right).$$

Another observer in the Einstein elevator, moving with acceleration through Minkowskian space-time, will measure the same acceleration toward the floor of the elevator, thermal radiation with the Unruh temperature given by formula (6). As shown in [30, 31], $a_{BH}$ is coincident with the quantity $a$ from the formula in (6). Then substituting the acceleration $a = a_{BH}$ from formula (8) into formula (6), we can obtain a formula for $T_{\text{Unruh},r}$ in this case [32]:

$$T_{\text{Unruh},r} = \frac{\hbar}{2\pi \sqrt{1 - \frac{r_{BH}}{r}}} \left( \frac{r_{BH}}{2r^2} \right).$$

If EP is valid, the quantities $T_{\text{Unruh},r}$ from formula (9) and $T_{H,r}$ in (7) should be coincident for $r > r_{BH}$ to a high degree of accuracy. However, we see that this is not true. In [32], e.g. for $r = 4GM = 2r_{BH}$, we have $T_{H,r} = 4T_{\text{Unruh},r}$.

So, far from the event horizon, EP is not the case. Moreover, violation of EP is the greater the farther it is from the black hole event horizon. Indeed, for an observer at the distance $r > r_{BH}$ we can write $r = \alpha(r)GM = \frac{1}{2}\alpha(r)r_{BH}$, $\alpha(r) > 2$. Then

$$T_{\text{Unruh},r} = \frac{\hbar}{2\pi \alpha^2(r)GM \sqrt{1 - \frac{2}{\alpha(r)}}}.$$

In this way $T_{H}/T_{\text{Unruh},r} = \alpha^2(r)/4$. And the ratio is the greater, the higher $\alpha(r)$, i.e. the farther from horizon the observer is. Next, for compactness, we denote $T_{\text{Unruh},r}$ in terms of $T_{U,r}$. Of course, in this case we bear in mind only an observer at a sufficiently great but finite distance from a black hole, i.e. only when a gravitational field is thought significant and must be taken into consideration. So, in the general case $r_{BH} < r \ll \infty$, whereas in the case of a distant observer we have

$$r_{BH} \ll r \ll \infty.$$

Obviously, this case of violated EP is not directly associated with the Main Hypothesis concerning the boundaries of EP validity (formula (2)) from the previous section, because
in [30–32] consideration is given to a large black hole with the event horizon radius $r_{BH}$ much greater than Planck length $r_{BH} \gg l_p$ at sufficiently low energies.

Really, the resulting distribution of the particles emitted by a black hole has the form (last formula on p.122 in [36])

$$n_E = \Gamma_{gb} \left[ \exp \left( \frac{E}{T_H} \right) - 1 \right]^{-1},$$

where $n_E$ is the number of particles with the energy $E$ and $\Gamma_{gb} < 1$ is the so-called greybody factor. As the black hole mass $M$ is large, the temperature $T_H$ is low, and then from the last formula it follows that arbitrary large values of $n_E$ will be given only by particles with the energies $E$ close to a small value of $T_H$.

The principal result from the remarkable papers [30–32] may be summarized as follows:

Comment 3.1 In any point of space-time that is in a field of a large classical Schwarzschild black hole, and in the cases when this field must be taken into consideration, it is impossible to remove this field in the vicinity of the point even locally, i.e. to consider space-time as flat.

Comment 3.2 It is important to refine some formulations from [30–32]. Specifically, if $r \to r_{BH}$, then $T_{H,r} \to \infty$, $T_{Unruh,r} \to \infty$ in formulae (7) and (9), respectively. Note that for $r \to r_{BH}$ these temperatures become infinite $T_{H,r} = \infty$, $T_{Unruh,r} = \infty$. Based on this fact, in [30–32] it is inferred “that the equivalence principle is restored on the horizon”. But this statement is not correct. Restoration of EP is not following from the fact that the above temperatures take infinite values. We can only state that temperature on the BH horizon and in its vicinity cannot be the parameter detecting a deviation from EP. In the opposite case one can arrive at violation: on the black hole event horizon, where a gravitational field is very large in value, EP holds, whereas far from the event horizon, where a gravitational field is much weaker, this principle is violated.

Let us return to high energy physics and to the subject of the previous section. One of the preferable models for space-time foam is the model based on the assumption that its unit cells are $mbh$, with radius and mass on the order of the Planck (for example, [14, 19, 20]. Of great importance for $mbh$ are the quantum-gravitational effects and the corresponding quantum corrections of black hole thermodynamics at Planck scale (for example, [37]).

Then, in line with formula (20) in [37], we have minimal values for radius and mass of a black hole

$$r_{min} = \sqrt{\frac{\pi}{2}} \alpha' l_p, \quad m_{min} \doteq m_0 = \frac{\alpha' \sqrt{\frac{\pi}{2}} m_p}{2},$$

(13)

where the number $\alpha'$ is on the order of 1, and in [37] we take the normalization $\hbar = c = k_B = 1$ in which $l_p = m^{-1} = T^{-1} = \sqrt{G}$.

From (13) it directly follows that the formula for the event horizon radius $r = 2MG$, valid for large classical black holes, will be valid in the case when we include the quantum-gravitational effects for $mbh$ because $r_{min} = 2m_0G$. Such a black hole of a minimal size is associated with a maximal temperature (formula (24) in [37]):

$$T_{H}^{\max} = \frac{T_p}{2\pi \sqrt{2\alpha'}}.$$

(14)

A black hole satisfying the formulae (13), (14) is termed as minimal (or Planck).
Without loss of generality, it is assumed that for event horizon radii and masses of $\text{mbh}$ the following is valid:

$$r_{\text{mbh}} \approx r_{\text{min,mbh}}, m_{\text{mbh}} \approx m_0,$$

(15)
i.e. $\text{mbh}$ are Planck black holes.

For the energies $E$ somewhat lower than the Planck energies (i.e., $E \ll E_p$) involved in the condition (2), a semiclassical approximation is valid. This means that, on substitution of $\text{mbh}$ with the mass $m_{\text{mbh}}$ for a large (classical) black hole with the mass $M$, in the case under study the results, substantiated when an observer uses the standard Unruh-Dewitt detector in radiation measurement for coupled to a massless scalar field [38, 39], are valid with the corresponding quantum corrections [33].

Let us revert to the formulae from [30–32]. In particular, to formula (8) for the real acceleration measured by an observer who is positioned at the fixed distance $r \gg r_{\text{BH}}$ in the Schwarzschild space-time, given by (formula (14) in [30, 31], formula (9.170) in [51])

$$a_{BH,r} = a_S = \frac{\sqrt{\nabla^\mu V \nabla^\nu V}}{V} = \frac{MG}{r^2 \sqrt{1 - 2MG/r}} = \frac{MG}{r^2 V},$$

(16)

where it is supposed that a static observer at the radius $r$ moves along orbits of the time-like Killing vector $K = \partial_t$ and $V = \sqrt{-K^\mu K^\nu} = \sqrt{1 - 2MG/r}$ is the red-shift factor for the Schwarzschild space-time (p.413 in [51]).

How changes formula (16) on going to $\text{mbh}$? It is clear that the condition $r \gg r_{\text{BH}}$ is replaced by the condition $r \gg r_{\text{mbh}}$ (corresponding to the condition $E \ll E_p$ and semiclassical approximation), $M$ is replaced by $m_{\text{mbh}}$, the red-shift factor $V$ should be replaced by $V_q$, where $V_q$ is the quantum deformation of $V$ with regard to quantum corrections in the field $\text{mbh}$. Then, for $\text{mbh}$, formula (16) is of the form

$$a_{S,q} = \frac{m_{\text{mbh}} G}{r^2 V_q},$$

(17)

where $a_{S,q}$ is the real acceleration with regard to the quantum corrections measured by a distant observer in the field $\text{mbh}$.

Clearly, formula (7) for $T_{H,r}$, due to formulae for the red-shift factor $V$, in the general case may be given as

$$T_{H,r} = \frac{\hbar}{8\pi GMV}.$$

(18)

Then its quantum analog, i.e. the corresponding formula for temperature in the field $\text{mbh}$, for $r \gg r_{\text{mbh}}$ is as follows:

$$T_{H,r,q} = \frac{\hbar}{8\pi Gm_{\text{mbh}}V_q}.$$

(19)

In virtue of formula (16), formula (9) takes the form

$$T_{U,r} = \frac{\hbar}{2\pi \sqrt{1 - \frac{r_{BH}}{r}}} \left( \frac{r_{BH}}{2r^2} \right) = \frac{\hbar}{2\pi V} \left( \frac{r_{BH}}{2r^2} \right).$$

(20)

For $r > r_{BH}$ and due to formula (10), we have

$$T_{U,r} = \frac{\hbar}{2\pi \alpha^2(r) GM \sqrt{1 - \frac{2}{\alpha(r)}}} = \frac{\hbar}{2\pi \alpha^2(r) GMV}.$$

(21)

As indicated above, for $r \gg r_{BH}$ we have $\alpha(r) \gg 1$.

What are the changes on going to $\text{mbh}$?
Considering the case $r \gg r_{mbh}$ and semiclassical picture, we again come to $\alpha(r) \gg 1$, whereas formula (21) is replaced by formula

$$T_{U,r,q} = \frac{\hbar}{2\pi \alpha^2(r) G m_{mbh} V_q}. \quad (22)$$

Proceeding from the above, we have

$$\frac{T_{U,r,q}}{T_{U,r}} = \frac{T_{H,r}}{T_{U,r}} = \frac{\alpha^2(r)}{4}. \quad (23)$$

Formula (23) points to the fact that, within the scope of a semiclassical approximation, relations of a black hole temperature to the Unruh temperature for a distant observer in the case of a large (classical) black hole and $m_{bh}$ are coincident because these quantities are dependent on the same factors:

first, on $1/MV$ and, second, on $1/m_{mbh}V_q$.

Note that in this consideration there’s no need to have an explicit formula for $V_q$ as this quantity is not involved in the key expression (23). Specifically, to derive an explicit expression for $V_q$, we can use the results from [52] on quantum deformation of the Schwarzschild solution due to spherically symmetric quantum fluctuations of the metric and the matter fields. In this case the Schwarzschild singularity at $r = 0$ is shifted to the finite radius $r_{min} \approx r_{mbh} \propto \ell_p$, where the scalar curvature is finite. In this way the results from [52] correlate well with the results from [37].

Quantum corrections at Planck scales were obtained in [37] proceeding from validity of the Generalized Uncertainty Principle (GUP) [40–43]. But the results presented in this work are independent of this aspect. Actually, during studies of black hole thermodynamics at Planck scales with the use of other methods [44, 45] (differing from those in [37]), in particular, Loop Quantum Gravity (LQG) [45], the obtained results were similar to [37]. Because of this, for $m_{bh}$ with all the thermodynamic characteristics (mass, radius, temperature, ...) on the order of the corresponding Planck quantities, all the calculations in this section are valid.

As noted above, far from horizon of $m_{bh}$, i.e. at the energies $E \ll E_p$ (2), the results from [30–32] remain valid in this case as well.

Next, similar to [14], we assume that in every cell of space-time foam a micro black hole ($m_{bh}$) with a typical gravitational radius of $r_{min} \propto \ell_p$ may be present. Then, in according with the results in [30–32] and in virtue of the formula (23) we come to violation of the strong EP for distance $r$, satisfying the condition

$$\ell_p \ll r \ll \infty, \quad (24)$$

that is equivalent to $\tilde{E}_r \ll E_p$ for the energies $\tilde{E}_r$ associated with the scale of $r$.

In the last formula it is implicitly (and purely conditionally) assumed that a minimum length is equal to $\ell_p$ and to $r_{min}$. But, as noted above, in the general case we have $r_{min} \propto \ell_p$, i.e., the order is similar to that of $\ell_p$. Specifically, in [46] by natural assumptions it has been demonstrated that the minimum length may be twice and more as great as the Planck length. It is obvious that all the above calculations and derivations of the present work are independent of the specific value of $r_{min}$.

In this way, if the quantum foam structure is determined by $m_{bh}$, the applicability of QFT is limited to the energies $E \leq \tilde{E}_r \ll E_p$ and the formula (2) is the case. This supports the Main Hypothesis from Section 2 within the assumption concerning the quantum foam structure made in this section.
Comment 3.3 In the case of mbh Comment 3.2 is absolutely clear. In fact, at a horizon of mbh, i.e., for \( r = r_{\text{min}}, T_{H,r} = T_{\text{Unruh},r} = \infty \) similar to large black holes but, naturally, without any restoration of EP as the domain \( r = r_{\text{mbh}} \approx r_{\text{min}} \propto l_p \) is the region of Planck energies or of quantum foam, where EP in its canonical formulation becomes invalid. It is obvious that at the event horizon \( r = r_{\text{mbh}} \) of mbh and in its vicinity a gravitational field becomes very strong due to quantum effects and nothing could destroy it.

4 Some Immediate Consequences

4.1 Quantum Field Theory at Low and High Energies

Based on the above results, all the energies \( E \) we can classify into 3 groups:

a) low energies \( 0 < E \leq \tilde{E} \ll E_p \) – energies, for which the Strong Equivalence Principle is valid in virtue of formula (2), and hence this energy interval sets the QFT applicability boundaries. a1) Since \( \tilde{E} \ll E_p \), it is natural to assume that \( \tilde{E} \approx 10^{-N} E_p \), where \( N \geq 2 \). Obtaining of more accurate estimates for \( N \) is a separate problem;

b) intermediate energies \( \tilde{E} < E < E_p \) – energies, for which the Strong Equivalence Principle and, consequently QFT, becomes invalid but the corresponding scales are greater than the Planck. It can be assumed that QFT in this energy range will be a theory in a gravitational field that could not be destroyed even locally. In the case under study it is assumed that this field is created by mbh. Impossibility of destroying this field even locally is associated with large quantum corrections for the corresponding quantities which should be taken into consideration at these energies \([37, 44, 45]\). Let us call the energy scale \( \tilde{E} < E < E_p \) as prequantum gravity phase;

c) high (essentially maximal) energies \( E \approx E_p \) or \( E > E_p \). This interval is the region of quantum gravity energies.

Next note that, as all the experimentally involved energies \( E \) are low, they satisfy condition a) or b). Specifically, for LHC, maximal energies are \( \approx 10^{4} GeV \approx 10^{19} GeV \), that is by 15 orders of magnitude lower than the Planck energy \( \approx 10^{19} GeV \). Moreover, the characteristic energy scales of all fundamental interactions also satisfy condition a). Indeed, in the case of strong interactions this scale is \( \Lambda_{QCD} \sim 200 MeV \); for electroweak interactions this scale is determined by the vacuum average of a Higgs boson and equals \( \nu \approx 246 GeV \); finally, the scale of the (Grand Unification Theory (GUT)) \( M_{GUT} \) lies in the range of \( \sim 10^{14} GeV - 10^{16} GeV \).

It should be noted, however, that on validity of assumption a1) the energy scale \( M_{GUT} \) lies within the applicability region of the energy group a) and hence of QFT. Provided the EP applicability boundaries are lying at considerably lower energies, a study of GUT necessitates a theory with (even locally) unremovable curvature.

At the same time, it is clear that the requirement of the Lorentz-invariant QFT, due to the action of Lorentz boost (or same hyperbolic rotations) (for example formula (3) in \([7]\)), results in however high momenta and energies. But it has been demonstrated that unlimited growth of the momenta and energies is impossible because in this case we fall within the energy region, where the conventional quantum field theory \([1–3]\) is invalid.

Note that at the present time there are experimental indications that Lorentz-invariance is violated in QFT on passage to higher energies (for example, \([47]\)). Besides, one should note important recent works associated with EP applicability boundaries and violation in nuclei...
and atoms at low energies (for example [27]). Proceeding from the above, the requirement for Lorentz-invariance and EP is possible only within the scope of the condition (2).

4.2 The Ultraviolet Divergences Absence in Local QFT

Proceeding from the above results, it is inferred that the well-known QFT [1–3], from the start, is a ultraviolet-finite theory with the natural cutoff parameter \( l_E \propto \hbar / \tilde{E} \). Note that the quantum-gravitational parameter \( r_{mbh} \propto l_p \) is beyond the applicability limits of QFT.

4.3 Asymptotic Safety Problem

In the present approach it is of interest to study the problem of asymptotic safety introduced by Steven Weinberg in [48]. We use the definition of this notion given in ([49],p.67): “A theory is said to be asymptotically safe if all essential coupling parameters \( g_j \) (these are the ones that are invariant under field redefinitions) approach, for energies \( k \to \infty \), a fixed point where at least one of them does not vanish.” If initially it has been assumed that \( r_{min} \) is considered within the scope of (GUP) [40-43], this definition necessitates certain refinements. In particular, if (GUP) supposes the existence of a maximum momentum \( p_{\text{max}} \), as in [42] or (Section V in [43]), it is clear that the condition \( k \to \infty \) can not be fulfilled. So, the condition \( k \to \infty \) should be replaced by the condition \( k \to p_{\text{max}} \). Most often, it is assumed that the momentum \( p_{\text{max}} \) is on the order of the Planck momentum, i.e., we have \( p_{\text{max}} \propto p_{pl} \). However, in the most general case the quantity \( p_{\text{max}} \) may be even trans-Planck.

When \( p_{\text{max}} \) is inexistent (i.e. \( p_{\text{max}} = \infty \)), still by this approach of asymptotic safety the problem should be reformulated in accordance with the fact (shown above) that, beginning with the energies \( E, E > \tilde{E} \), a theory must be considered as QFT in curved space-time in a field created by \( mbh \). In his further works the author is planning to study this problem within the scope of this approach in greater detail.

4.4 Correction of Estimates Some Quantities in QFT

It is possible to correct the estimates obtained within the scope of the known QFT by means of the condition (2). Let us consider a typical example. In his well-known lectures [50] at the Cornell University Steven Weinberg considered an example of calculating, within the scope of QFT, the expected value for the vacuum energy density \( <\rho> \) that is proportional to the cosmological term \( \lambda \). To this end, zero-point energies of all normal modes of some field with the mass \( m \) are summed up to the wave number cutoff \( \Lambda \gg m \) for the selected normalization \( \hbar = c = 1 \) (formula (3.5) in [50]):

\[
<\rho> = \int_0^\Lambda \frac{4\pi k^2 dk}{(2\pi)^3} \frac{1}{2} \sqrt{k^2 + m^2} \approx \frac{\Lambda^4}{16\pi^2}. \tag{25}
\]

Assuming, similar to [50], that GR is valid at all the energy scales up to the Planck, we have the cutoff \( \Lambda \approx (8\pi G)^{-1/2} \) and hence (formula (3.6) in [50]) leads to the following result:

\[
<\rho> \approx 2 \cdot 10^{71} \text{GeV}^4, \tag{26}
\]

that by \( 10^{118} \) orders of magnitude differs from the well-known experimental value for the vacuum energy density

\[
<\rho_{\text{exp}}> \lesssim 10^{-29} \text{g/cm}^3 \approx 10^{-47} \text{GeV}^4. \tag{27}
\]

Here \( G \) is a gravitational constant.
It is clear that in this case the condition (2) is not fulfilled and this leads to such a monstrous discrepancy with $<\rho_{\text{exp}}>$. Based on the afore-said, the following estimate for $<\rho>$ is more correct:

$$<\rho_{\tilde{E}}>=\int_{0}^{\Lambda_{\tilde{E}}} \frac{4\pi k^2 dk}{(2\pi)^3} \left[ \frac{1}{2} \sqrt{k^2+m^2} \right] \simeq \frac{\Lambda_{\tilde{E}}^4}{16\pi^2},$$

(28)

where $\Lambda(\tilde{E})$—cut-off parameter of the corresponding energy $\tilde{E}$ from point a) in 4.1. Of course, the main contribution into the integral in the right side of (25) is made by high energies $\tilde{E}<E<E_p$ from point b) in 4.1, which are not involved in formula (28). Consequently, it seems possible that $<\rho_{\tilde{E}}><<<\rho>$ and hence $<\rho_{\tilde{E}}>$ may be much closer to $<\rho_{\text{exp}}>\text{ than }<\rho>$. In the whole, the methods of a quantum field theory may be effectively used to obtain the vacuum energy density. In his very interesting work [53] the author, based on the quantum hoop conjecture, has obtained a natural cutoff for the vacuum energy of a scalar field, also giving an estimate for $<\rho>$ much closer to $<\rho_{\text{exp}}>$ than in formulae (25), (26). It is interesting to know how close are the estimates for the vacuum energy density in [53] and in formula (28). To this end, we first need a sufficiently accurate estimate of the quantity $\tilde{E}$ in line with the QFT boundaries applicability. Besides, it is important to find whether the methods of QFT are enough to obtain $<\rho_{\text{exp}}>\text{ or some additional assumptions will be required, specifically, the Holographic Principle applying to the Universe [54–61].}

5 Conclusion

Thus, within the scope of natural assumptions, in this paper it is demonstrated that the applicability boundary of the well-known QFT is lying in the region of energies considerably lower than the Planck energies, i.e. the canonical QFT [1, 3] is an ultraviolet-finite theory. In this paradigm it is important to understand the way to transform the well-known results for ultraviolet regularization, renormalization, and so on from QFT within the scope of the applicability boundary $\tilde{E}$ of QFT indicated in point a) of the preceding section. Possibly, this boundary will be dependent on a nature of the processes under study in high energy physics.

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Declarations

Conflict of Interests The author declares that there is no conflict of interests regarding the publication of this paper.

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