Entropy-corrected new agegraphic dark energy in Hořava-Lifshitz cosmology

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We study the entropy-corrected version of the new agegraphic dark energy (NADE) model and dark matter in a spatially non-flat Universe and in the framework of Hořava-Lifshitz cosmology. For the two cases containing noninteracting and interacting entropy-corrected NADE (ECNADE) models, we derive the exact differential equation that determines the evolution of the ECNADE density parameter. Also the deceleration parameter is obtained. Furthermore, using a parametrization of the equation of state parameter of the ECNADE model as $\omega_{\Lambda}(z) = \omega_0 + \omega_1 z$, we obtain both $\omega_0$ and $\omega_1$. We find that in the presence of interaction, the equation of state parameter $\omega_0$ of this model can cross the phantom divide line which is compatible with the observation.

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I. INTRODUCTION

Astronomical observations indicate that our Universe is in a phase of accelerated expansion (Perlmutter et al. 1999; Bennett et al. 2003; Tegmark et al. 2004; Allen et al. 2004). One explanation for the cosmic acceleration is the dark energy (DE), an exotic energy with negative pressure. The dynamical nature of DE, at least in an effective level, can originate from various fields, although a complete description requires a deeper understanding of the underlying theory of quantum gravity. Nevertheless, physicists can still make some attempts to probe the nature of DE according to some basic quantum gravitational principles. Two examples of such a paradigm are the holographic DE (HDE) and the agegraphic DE (ADE) models which have originated from quantum gravity and possess some of its significant features. The former, that arose a lot of enthusiasm recently (Cohen et al. 1999; Hsu 2004; Li 2004; Huang and Li 2004; Jamil et al. 2009a, 2009b; Jamil and Farooq 2010; Setare and Jamil 2010; Wang et al. 2005a, 2005b, 2008; Sheykhi

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is motivated from the holographic hypothesis (’t Hooft 1993; Susskind 1995) and has been tested and constrained by various astronomical observations (Feng et al. 2005; Zhang and Wu 2005, 2007). The latter originated from the uncertainty relation of quantum mechanics together with the gravitational effect of general relativity (GR). The ADE model assumes that the observed DE effect comes from spacetime and matter field fluctuations in the Universe (Sasakura 1999; Cai 2007; Wei and Cai 2008a, 2009). Following the line of quantum fluctuations of spacetime, Karolyhazy (Karolyhazy 1966; Karolyhazy et al. 1982, 1986) proposed that the distance \( t \) in Minkowski spacetime cannot be known to a better accuracy than \( \delta t = \varepsilon t_P^{2/3} t^{1/3} \), where \( \varepsilon \) is a dimensionless constant of order unity and \( t_P \) is the reduced Planck time. Based on the Karolyhazy relation, Maziashvili proposed that the energy density of metric fluctuations of Minkowski spacetime is given by (Maziashvili 2007a, 2007b)

\[
\rho_\Lambda \sim \frac{1}{t_P^2 t^2} \sim \frac{M_P^2}{t^2},
\]

where \( M_P \) is the reduced Planck mass \( M_P^2 = 8\pi G \). Since in the original ADE model the age of the Universe is chosen as the length measure, instead of the horizon distance, the causality problem in the HDE is avoided (Cai 2007). The original ADE model had some difficulties. In particular, it cannot justify the matter-dominated era (Cai 2007). This motivated Wei and Cai (2008a) to propose the new ADE model, while the time scale is chosen to be the conformal time instead of the age of the Universe. The NADE density is given by (Wei and Cai 2008a)

\[
\rho_\Lambda = \frac{3n^2 M_P^2}{\eta^2},
\]

where \( 3n^2 \) is the numerical factor and \( \eta \) is the conformal time and defined as

\[
\eta = \int \frac{dt}{a} = \int_0^{a} \frac{da}{Ha^2}.
\]

The ADE models have been examined and constrained by various astronomical observations (Kim et al. 2008a, 2008b; Wu et al. 2008; Zhang et al. 2008; Neupane 2009; Sheykhi 2009a, 2009b, 2010b; Karami et al. 2010; Karami and Abdolmaleki 2011; Wei and Cai 2008b). Besides, in the loop quantum gravity, the entropy-area relation can be modified due to the thermal equilibrium fluctuations and quantum fluctuations (Banerjee et al. 2008; Banerjee and Majhi 2008a, 2008b, 2009; Banerjee and Modak 2009; Majhi 2009, 2010; Modak 2009). The corrected entropy takes the form

\[
S = \frac{A}{4G} + \tilde{\alpha} \ln \frac{A}{4G} + \tilde{\beta},
\]
where $\tilde{\alpha}$ and $\tilde{\beta}$ are dimensionless constants of order unity. Taking the corrected entropy-area relation (4) into account, the NADE density will be modified as well. Motivated by the corrected entropy-area relation (4), the ECNADE density is given by (Wei 2009; Karami and Sorouri 2010; Karami et al. 2011)

$$\rho_\Lambda = \frac{3n^2 M_p^2}{\eta^2} + \frac{\alpha}{\eta^4} \ln (M_p^2 \eta^2) + \frac{\beta}{\eta^4},$$

(5)

where $\alpha$ and $\beta$ are dimensionless constants of order unity.

Recently a power-counting renormalizable UV complete theory of gravity was proposed by Hořava (Hořava 2009a, 2009b). This theory is not Lorentz invariant (except in the infrared limit), it is power-counting renormalizable and obeys anisotropic scaling or Lifshitz scaling. Than the time coordinate and the 3 spatial coordinates have to be treated separately, the theory is non-relativistic, the speed of light diverges in the ultraviolet limit, and test particles do not follow geodesics. In consequence causal structures are different from that in General Relativity (Greenwald 2011) and entropy cannot be defined. Quantum gravity models based on an anisotropic scaling of the space and time dimensions have recently attracted significant attention (Visser 2009; Pal 2009). In particular, Hořava-Lifshitz point gravity might not have desirable features, but in its original incarnation one is forced to accept a non-zero cosmological constant of the wrong sign to be compatible with observations (Nastase 2009). There are four different versions of this theory: with (or without) projectability condition and with (or without) detailed balance. At a first look it seems that this non-relativistic model for quantum gravity has a well defined IR limit and it reduces to GR. But as it was first indicated by Mukohyama (Mukohyama 2009, 2010; Saridakis 2010), Hořava-Lifshitz theory mimics GR plus dark matter (DM). This theory has a scale invariant power spectrum which describes inflation. For reviews on the scenario where the cosmological evolution is governed by Hořava-Lifshitz gravity see (Mukohyama 2009, 2010; Saridakis 2010; Wang and Wu 2009; Majhi and Samanta 2010; Wang 2011; Khatua et al. 2011).

Here, our aim is to investigate the ECNADE model in Hořava-Lifshitz cosmology. To do this in section 2, we first review the scenario of Hořava-Lifshitz gravity. In section 3, we consider a spatially non-flat Friedmann-Robertson-Walker (FRW) Universe containing ECNADE and DM in the framework of Hořava-Lifshitz gravity. We obtain the evolution of dimensionless energy density, deceleration parameter and equation of state of ECNADE with interaction/non-interaction. Section 4 is devoted to conclusions.
II. HOŘAVA-LIFSHITZ GRAVITY

Under the projectability condition, the full metric in the (3+1) dimensional Arnowitt-Deser-Misner formalism is written as (Calcagni 2009; Kiritsis and Kofinas 2009)

$$ds^2 = -N^2 dt^2 + g_{ij}(dx^i + N^i dt)(dx^j + N^j dt),$$  

(6)

where the dynamical variables $N$, $N^i$ and $g_{ij}$ are the lapse function, shift vector and 3-dimensional metric, respectively. Note that the projectability condition restricts the lapse function $N$ to be space-independent, while the shift vector $N^i$ and the 3-dimensional metric $g_{ij}$ still depend on both time and space.

Under the detailed balance condition, the gravitational action of Hořava-Lifshitz (HL) gravity is given by (Hořava 2009a, 2009b)

$$S_g = \int dt d^3x \sqrt{g} N \left[ \frac{2}{\kappa^2} (K_{ij} K^{ij} - \lambda K^2) + \frac{\kappa^2 \mu}{2 w^2} C^{ij} \frac{\epsilon^{ijk}}{\sqrt{g}} R^l_i \nabla_j R^l_k + \frac{\kappa^2 \mu^2}{8} R_{ij} R^{ij} + \frac{\kappa^2 \mu^2}{8(3\lambda - 1)} \left( 1 - \frac{4\lambda}{4} R^2 + \Lambda R - 3\Lambda^2 \right) \right],$$  

(7)

where

$$K_{ij} = \frac{1}{2N} \left( g_{ij} - \nabla_i N_j - \nabla_j N_i \right),$$  

(8)

is the extrinsic curvature and

$$C^{ij} = \frac{\epsilon^{ijk}}{\sqrt{g}} \nabla_k \left( R^l_i - \frac{1}{4} R \delta^l_i \right),$$  

(9)

is the Cotton tensor. Also $\epsilon^{ijk}$ is the totally antisymmetric unit tensor, $\lambda$ is a dimensionless constant and $\Lambda$ is a positive constant which as usual is related to the cosmological constant in the IR limit.

The variables $\kappa$, $w$ and $\mu$ are constants with mass dimensions $-1$, $0$ and $1$, respectively. Note that the detailed balance condition restricts the form of a general potential in a $(d+1)$-dimensional Lorentz action to a specific form that can be expressed in terms of a $d$-dimensional action of a relativistic theory with Euclidean signature, whereby the number of independent couplings is considerably limited.

In particular, Eq.(7) suffers several problems, including instability, inconsistency and strong coupling problems, for detail, see (Mukohyama,2010). To overcome these problems, one way is to
provokes the Vainshtein mechanism, as first done by Mukohyama for spherical spacetimes (Mukohyama, 2010) and by Wang and Wu in the cosmological setting (Wang & Wu, 2011). Such considerations were further carried out by using the so-called gradient expansion method (Izumi & Mukohyama, 2011; Gumrukcuoglu et al, 2011). Another very promising approach is to introduce an extra $U(1)$ symmetry, as first done by (Horava & Melby-Thompson, 2010) with $\lambda = 1$, and later generalized to the case with any $\lambda$ by (da Silva, 2011). These studies were further generalized to the case without the projectability condition (Zhu et al, 2011a; Zhu et al, 2011b). In both cases (with/without the projectability condition), due to the $U(1)$ symmetry, the spin-0 gravitons are eliminated, and all the problems related to them, such as instability, inconsistency and strong coupling problems, are resolved.

To include the matter component we add a cosmological stress-energy tensor to the gravitational field equations, by imposing a condition to recover the general relativistic formalism in the low-energy limit (Carloni et al. 2009; Leon and Saridakis 2009; Sotiriou et al. 2009; Chaichian et al. 2010; Dutta and Saridakis 2010). For this, the energy density $\rho_m$ and pressure $p_m$ satisfy the energy conservation equation

$$\dot{\rho}_m + 3H(\rho_m + p_m) = 0. \quad (10)$$

In the cosmological context, we use a FRW metric as

$$N = 1, \ g_{ij} = a^2(t)\gamma_{ij}, \ N^i = 0, \quad (11)$$

with

$$\gamma_{ij}dx^i dx^j = \frac{dt^2}{1-kt^2} + r^2d\Omega_2, \quad (12)$$

where $k = -1, 0, +1$ refer to spatially open, flat, and closed Universe respectively.

On varying the action with respect to the metric components $N$ and $g_{ij}$, one can obtain the modified Friedmann equations in the framework of HL gravity as

$$H^2 = \frac{\kappa^2}{6(3\lambda - 1)}\rho_m + \frac{\kappa^2}{8(3\lambda - 1)a^4} \left[ \frac{3\kappa^2 \mu^2 k^2}{8(3\lambda - 1)} + \frac{3\kappa^2 \mu^2 \Lambda^2}{8(3\lambda - 1)^2} \right] - \frac{\kappa^4 \mu^2 \Lambda k}{8(3\lambda - 1)^2 a^2}, \quad (13)$$

$$\dot{H} + \frac{3}{2}H^2 = -\frac{\kappa^2}{4(3\lambda - 1)}p_m - \frac{\kappa^2}{4(3\lambda - 1)}\left[ \frac{\kappa^2 \mu^2 k^2}{8(3\lambda - 1)a^4} - \frac{3\kappa^2 \mu^2 \Lambda^2}{8(3\lambda - 1)} \right] - \frac{\kappa^4 \mu^2 \Lambda k}{16(3\lambda - 1)^2 a^2}. \quad (14)$$
where \( H = \frac{\dot{a}}{a} \) is the Hubble parameter.

Noticing the form of the above Friedmann equations, we can define for DE

\[
\rho_\Lambda = \frac{3\kappa^2 \mu^2 k^2}{8(3\lambda - 1)a^4} + \frac{3\kappa^2 \mu^2 \Lambda^2}{8(3\lambda - 1)}, \tag{15}
\]

\[
p_\Lambda = \frac{\kappa^2 \mu^2 k^2}{8(3\lambda - 1)a^4} - \frac{3\kappa^2 \mu^2 \Lambda^2}{8(3\lambda - 1)}. \tag{16}
\]

The first term on the right hand side proportional to \( a^{-4} \) is effectively the “dark radiation term”, present in HL cosmology (Calcagni 2009; Kiritsis and Kofinas 2009), while the second term is referred as an explicit cosmological constant. Hence, in expressions (15) and (16) we defined the energy density and pressure for the effective DE, which include the aforementioned contributions. Finally, note that using (15) and (16) it is easy to show that these satisfy the following expression

\[
\dot{\rho}_\Lambda + 3H(\rho_\Lambda + p_\Lambda) = 0. \tag{17}
\]

Finally in order for these expressions to match with the standard Friedmann equations \((c = 1)\) we set (Calcagni 2009; Kiritsis and Kofinas 2009)

\[
G_c = \frac{\kappa^2}{16\pi(3\lambda - 1)}, \tag{18}
\]

\[
\frac{\kappa^4 \mu^2 \Lambda}{8(3\lambda - 1)^2} = 1, \tag{19}
\]

where \( G_c \) is the “cosmological” Newton’s constant. Note that in gravitational theories with the violation of Lorentz invariance (like HL gravity) the “gravitational” Newton’s constant \( G_g \), which is present in the gravitational action, differs with the “cosmological” Newton’s constant \( G_c \), which is present in Friedmann equations, unless Lorentz invariance is restored (Carroll and Lim 2004). For the sake of completeness we write

\[
G_g = \frac{\kappa^2}{32\pi}. \tag{20}
\]

Note that in the IR limit \((\lambda = 1)\), where Lorentz invariance is restored, \( G_c \) and \( G_g \) are the same.

Further we can rewrite the modified Friedmann equations (13) and (14) in the usual form as

\[
H^2 + \frac{k}{a^2} = \frac{8\pi G_c}{3}(\rho_m + \rho_\Lambda), \tag{21}
\]

\[
\dot{H} + \frac{3}{2}H^2 + \frac{k}{2a^2} = -4\pi G_c(\rho_m + \rho_\Lambda). \tag{22}
\]
III. ECNADE IN HL COSMOLOGY

Here we would like to investigate the ECNADE in HL theory. To do this we consider a spatially non-flat FRW Universe containing the ECNADE and DM. Let us define the dimensionless energy densities as

\[ \Omega_m = \frac{\rho_m}{\rho_{cr}} = \frac{8\pi G_c}{3H^2} \rho_m, \]
\[ \Omega_\Lambda = \frac{\rho_\Lambda}{\rho_{cr}} = \frac{8\pi G_c}{3H^2} \rho_\Lambda, \]
\[ \Omega_k = -\frac{k}{a^2H^2}, \] (23)

then, the first Friedmann equation (13) yields

\[ 1 - \Omega_k = \Omega_\Lambda + \Omega_m. \] (24)

From definition \( \rho_\Lambda = \frac{3H^2}{8\pi G_c} \Omega_\Lambda \), we get

\[ \Omega_\Lambda = \frac{\eta^2}{H^2\eta^2} \gamma_n, \] (25)

where

\[ \gamma_n = \frac{G_c}{G_g} + \frac{8\pi G_c}{3n^2\eta^2} \left[ \alpha \ln (M_P^2\eta^2) + \beta \right], \] (26)

and \( M_P^2 = 8\pi G_g \). Taking time derivative of Eq. (24) and using \( \dot{\eta} = 1/a \), one can get

\[ \dot{\rho}_\Lambda = \left( \frac{2}{a\eta} \right) \left[ -2\rho_\Lambda + \frac{3n^2M_P^2}{\eta^2} + \alpha \right]. \] (27)

Taking time derivative of \( \Omega_\Lambda = \frac{8\pi G_c}{3H^2} \rho_\Lambda \) and using Eqs. (25), (27), \( \dot{\eta} = 1/a \) and \( \Omega_\Lambda' = \dot{\Omega}_\Lambda/H \), one can obtain the equation of motion for \( \Omega_\Lambda \) as

\[ \Omega_\Lambda' = -2\Omega_\Lambda \left[ \frac{\dot{H}}{H^2} + \frac{1}{na\gamma_n} \left( \frac{\Omega_\Lambda}{\gamma_n} \right)^{1/2} \left( 2\gamma_n - \frac{G_c}{G_g} \right) - \frac{8\pi G_c \alpha H^2}{3n^4 \gamma_n^2} \right]. \] (28)

Here, prime denotes the derivative with respect to \( x = \ln a \). Taking derivative of \( \Omega_k = -k/(a^2H^2) \) with respect to \( x = \ln a \), one gets

\[ \Omega_k' = -2\Omega_k \left( 1 + \frac{\dot{H}}{H^2} \right). \] (29)
A. Noninteracting case

Consider the FRW Universe filled with ECNADE and pressureless DM which evolves according to their conservation laws

$$\dot{\rho}_\Lambda + 3H(1 + \omega_{\Lambda})\rho_\Lambda = 0,$$

$$\dot{\rho}_m + 3H\rho_m = 0,$$

where $\omega_{\Lambda} = p_{\Lambda}/\rho_\Lambda$ is the equation of state (EoS) parameter of the ECNADE model.

Taking time derivative of the first Friedmann equation (13) and using Eqs. (23), (24), (25), (26), (27) and (31), one can get

$$\frac{\dot{H}}{H^2} = \frac{1}{2} \left[ \Omega_k - 3(1 - \Omega_{\Lambda}) \right] - \frac{1}{na} \left( \frac{\Omega_{\Lambda}}{\gamma_n} \right)^{3/2} \left( 2\gamma_n - \frac{G_c}{G_g} - \frac{8\pi G_c \alpha H^2 \Omega_{\Lambda}}{3n^4} \right).$$

(32)

Substituting this into Eq. (28), one obtains

$$\Omega'_{\Lambda} = \Omega_{\Lambda} \left[ 3(1 - \Omega_{\Lambda}) - \Omega_k + \frac{2}{na} \left( \frac{\Omega_{\Lambda}}{\gamma_n} \right)^{1/2} \times \left( \frac{\Omega_{\Lambda} - 1}{\gamma_n} \right) \left( 2\gamma_n - \frac{G_c}{G_g} - \frac{8\pi G_c \alpha H^2 \Omega_{\Lambda}}{3n^4} \right) \right].$$

(33)

Putting Eq. (32) into (29) reduces to

$$\Omega'_k = \Omega_k \left[ (1 - \Omega_k) - 3\Omega_{\Lambda} + \frac{2}{na} \left( \frac{\Omega_{\Lambda}}{\gamma_n} \right)^{3/2} \left( 2\gamma_n - \frac{G_c}{G_g} - \frac{8\pi G_c \alpha H^2 \Omega_{\Lambda}}{3n^4} \right) \right].$$

(34)

The deceleration parameter is given by

$$q = -\left( 1 + \frac{\dot{H}}{H^2} \right).$$

(35)

Replacing the term $\dot{H}/H^2$ from (32) into (35) yields

$$q = \frac{1}{2}(1 - \Omega_k - 3\Omega_{\Lambda}) + \frac{1}{na} \left( \frac{\Omega_{\Lambda}}{\gamma_n} \right)^{3/2} \left( 2\gamma_n - \frac{G_c}{G_g} - \frac{8\pi G_c \alpha H^2 \Omega_{\Lambda}}{3n^4} \right).$$

(36)

Note that if we set $\alpha = \beta = 0$ then from Eq. (26) $\gamma_n = G_c/G_g$. Therefore Eqs. (33) and (36) reduce to

$$\Omega'_{\Lambda} = -\Omega_{\Lambda} \left[ \Omega_k + (\Omega_{\Lambda} - 1) \left( 3 - \frac{2\Omega_{1/2}}{na} \left( \frac{G_g}{G_c} \right)^{1/2} \right) \right].$$

(37)
\[ q = \frac{1}{2} (1 - \Omega_k - 3 \Omega_\Lambda) + \frac{\Omega_\Lambda^{3/2}}{na \left( \frac{G_g}{G_c} \right)^{1/2}}, \]  

which are same as those results obtained for the NADE model in HL cosmology (Jamil and Saridakis 2010).

Also for \( G_c \rightarrow G_g \), Eqs. (33) and (36) recover the results obtained for the ECNADE model in the standard FRW cosmology (Karami and Sorouri 2010) as

\[ \Omega'_\Lambda = \Omega_\Lambda \left[ 3(1 - \Omega_\Lambda) - \Omega_k + \frac{2}{na} \left( \frac{\Omega_\Lambda}{\gamma_n} \right)^{1/2} \right. \]
\[ \times \left( \frac{\Omega_\Lambda - 1}{\gamma_n} \right) \left( 2 \gamma_n - 1 - \frac{\alpha H^2}{3 M_p^2 n^4 \gamma_n} \right), \]

\[ q = \frac{1}{2} (1 - \Omega_k - 3 \Omega_\Lambda) + \frac{1}{na} \left( \frac{\Omega_\Lambda}{\gamma_n} \right)^{3/2} \left( 2 \gamma_n - 1 - \frac{\alpha H^2}{3 M_p^2 n^4 \gamma_n} \right). \]

The EoS parameter of the ECNADE model can be parameterized as (Huterer and Turner 1999, 2001; Weller and Albrecht 2001)

\[ \omega_\Lambda(z) = \omega_0 + \omega_1 z. \]

Using (30) and (41), the ECNADE density evolves as (Huterer and Turner 1999, 2001; Weller and Albrecht 2001; Copeland et al. 2006)

\[ \frac{\rho_\Lambda}{\rho_{\Lambda_0}} = a^{-3(1 + \omega_0 - \omega_1)} e^{3 \omega_1 z}. \]

The Taylor expansion of the DE density around \( a_0 = 1 \) at the present time yields

\[ \ln \rho_\Lambda = \ln \rho_{\Lambda_0} + \frac{d \ln \rho_\Lambda}{d \ln a} \bigg|_0 \ln a + \frac{d^2 \ln \rho_\Lambda}{2 d \ln a^2} \bigg|_0 (\ln a)^2 + \cdots, \]

where the index 0 denotes the value of a quantity at present. Using \( \ln a = -\ln(1 + z) \simeq -z + \frac{z^2}{2} \) for small redshifts, Eqs. (32) and (33), respectively, reduce to

\[ \frac{\ln (\rho_\Lambda/\rho_{\Lambda_0})}{\ln a} = -3(1 + \omega_0) - \frac{3}{2} \omega_1 z, \]

\[ \frac{\ln (\rho_\Lambda/\rho_{\Lambda_0})}{\ln a} = \frac{d \ln \rho_\Lambda}{d \ln a} \bigg|_0 - \frac{d^2 \ln \rho_\Lambda}{2 d \ln a^2} \bigg|_0 z. \]

Comparing Eqs. (44) and (45), one can obtain the parameters \( \omega_0 \) and \( \omega_1 \) as

\[ \omega_0 = -\frac{1}{3} \frac{d \ln \rho_\Lambda}{d \ln a} \bigg|_0 - 1, \]
\[ \omega_1 = \frac{1}{3} \frac{d^2 \ln \rho_\Lambda}{d (\ln a)^2} \big|_0. \] (47)

From Eq. (31), the energy density of DM evolves as \( \rho_m = \rho_{m0} a^{-3} \). Now using (24) one can get

\[ \rho_\Lambda = \frac{\rho_{m0} a^{-3}}{(1 - \Omega_k - \Omega_\Lambda)} \Omega_\Lambda. \] (48)

Substituting the above relation into (46) yields

\[ \omega_0 = -\frac{1}{3} \left[ \frac{\Omega''_\Lambda}{\Omega_\Lambda} + \frac{\Omega'_\Lambda + \Omega'_k}{(1 - \Omega_k - \Omega_\Lambda)} \right]_0. \] (49)

Adding Eqs. (33) and (50), we get

\[ \Omega'_\Lambda + \Omega'_k = (1 - \Omega_k - \Omega_\Lambda) \left[ \Omega_k + 3 \Omega_\Lambda - \frac{2}{n a} \left( \frac{\Omega_\Lambda}{\gamma_n} \right)^{3/2} \right. \]
\[ \times \left. \left( 2 \gamma_n - \frac{8 \pi G_c \alpha H^2 \Omega_\Lambda}{3 n^4 \gamma_n} \right) \right]. \] (50)

Substituting Eqs. (33) and (50) into (49) yields

\[ \omega_0 = -1 + \frac{2}{3 n \gamma_n} \left( \frac{\Omega_\Lambda}{\gamma_n} \right)^{1/2} \left( 2 \gamma_n - \frac{G_c}{G_g} \right) \]
\[ - \frac{8 \pi G_c \alpha H^2_0 \Omega_\Lambda}{3 n^4 \gamma_n}. \] (51)

Note that in the absence of correction terms, i.e. \( \alpha = \beta = 0 \) and \( \gamma_n = G_c/G_g \), Eq. (51) yields

\[ \omega_0 = -1 + \frac{2 \Omega_\Lambda^{1/2}}{3 n} \left( \frac{G_g}{G_c} \right)^{1/2}, \] (52)

which is the EoS parameter of the NADE model in HL cosmology (Jamil and Saridakis 2010).

Also in the limit of \( G_c \to G_g \), Eq. (51) recovers the EoS parameter of the ECNADE model in the standard FRW cosmology (Karami and Sorouri 2010) as

\[ \omega_0 = -1 + \frac{2 \Omega_\Lambda^{1/2}}{3 n} \left( \frac{G_g}{G_c} \right)^{1/2} \]
\[ \times \left( 2 \gamma_n - 1 - \frac{\alpha H^2_0 \Omega_\Lambda}{3 M_p^4 n^4 \gamma_n} \right). \] (53)

Using Eqs. (47) and (48), one can get

\[ \omega_1 = \frac{1}{3} \left[ \frac{\Omega''_\Lambda}{\Omega_\Lambda} - \frac{\Omega'^2_\Lambda}{\Omega_\Lambda} + \frac{\Omega'_\Lambda + \Omega''_k}{(1 - \Omega_k - \Omega_\Lambda)} \right. \]
\[ + \frac{(\Omega'_\Lambda + \Omega'_k)^2}{(1 - \Omega_k - \Omega_\Lambda)^2} \big|_0. \] (54)
Taking derivative of Eqs. (33) and (34) with respect to $x = \ln a$, one can obtain

\[
\frac{d^2}{dx^2}\Omega \Lambda = -6\Omega \Lambda \Omega' \Lambda + 3\Omega' \Lambda (1 - 2\Omega \Lambda) + (\Omega \Lambda - \Omega' \Lambda - 1) \left[ \frac{G_c}{G_g} \frac{2n^2}{aH^3\eta^3} - \frac{4\Omega \Lambda}{aH\eta} + \frac{16\pi G_c \alpha}{3aH^3\eta^3} \right] \\
+ (1 - \Omega \Lambda) \left[ \left( \frac{4\Omega \Lambda}{aH\eta} - \frac{G_c}{G_g} \frac{6n^2}{aH^3\eta^3} \right) \left( \frac{H}{H^2} + \frac{\dot{H}}{H \eta} \right) - \frac{4\Omega' \Lambda}{aH\eta} \right] - \frac{16\pi G_c \alpha}{3aH^3\eta^3} \left( 3\frac{\dot{H}}{H^2} + \frac{5\dot{\eta}}{H \eta} \right),
\]

and

\[
\frac{d^2}{dx^2}\Omega = \Omega' (1 - 2\Omega - 3(\Omega \Lambda \Omega' \Lambda + \Omega \Lambda \Omega' \Lambda) + (\Omega \Lambda) \Omega' \Lambda) \left[ \frac{G_c}{G_g} \frac{2n^2}{aH^3\eta^3} - \frac{4\Omega \Lambda}{aH\eta} + \frac{16\pi G_c \alpha}{3aH^3\eta^3} \right] \\
- \Omega \Lambda \left[ \left( \frac{4\Omega \Lambda}{aH\eta} - \frac{G_c}{G_g} \frac{6n^2}{aH^3\eta^3} \right) \left( \frac{H}{H^2} + \frac{\dot{H}}{H \eta} \right) - \frac{4\Omega' \Lambda}{aH\eta} \right] - \frac{16\pi G_c \alpha}{3aH^3\eta^3} \left( 3\frac{\dot{H}}{H^2} + \frac{5\dot{\eta}}{H \eta} \right). \tag{55}
\]

Using Eq. (51), the above expressions for $\Omega'' \Lambda$ and $\Omega''$ can be rewritten as

\[
\frac{d^2}{dx^2}\Omega \Lambda = -6\Omega \Lambda \Omega' \Lambda + 3\Omega' \Lambda (1 - 2\Omega \Lambda) + (\Omega \Lambda - \Omega' \Lambda - 1) \omega_0 \\
+ (1 - \Omega \Lambda) \left[ 3(\Omega \Lambda + \Omega' \Lambda) + A \right], \tag{57}
\]

and

\[
\Omega'' = \Omega' (1 - 2\Omega) - 3\Omega \Lambda (\Omega \Lambda - \Omega' \Lambda) - 1) \omega_0 \\
- \Omega \Lambda \left[ 3(\Omega \Lambda + \Omega' \Lambda) + A \right], \tag{58}
\]

where

\[
A = \left( \frac{4\Omega \Lambda}{aH\eta} - \frac{G_c}{G_g} \frac{6n^2}{aH^3\eta^3} \right) \left( \frac{H}{H^2} + \frac{\dot{H}}{H \eta} \right) - \frac{4\Omega' \Lambda}{aH\eta} - \frac{16\pi G_c \alpha}{3aH^3\eta^3} \left( 3\frac{\dot{H}}{H^2} + \frac{5\dot{\eta}}{H \eta} \right). \tag{59}
\]

Using Eqs. (25), (35) and $\dot{\eta} = 1/a$, one can rewrite (59) as

\[
A = -9\Omega \Lambda (1 + q)(1 + \omega_0) \\
+ \frac{\Omega \Lambda}{na \left( \frac{\Omega \Lambda}{\gamma_n} \right)^{1/2}} \left[ 17 + 4(\Omega_k + 2q) \right] + 3(7 - 4\Omega \Lambda) \omega_0 - \frac{8}{na \left( \frac{\Omega \Lambda}{\gamma_n} \right)^{1/2}} \\
- \frac{32\pi G_c \alpha H^2 \left( \frac{\Omega \Lambda}{\gamma_n} \right)^{5/2}}{3an^5 \Omega \Lambda}. \tag{60}
\]
Adding Eqs. (57) and (58), we get

\[ \Omega'' + \Omega_k'' = -(\Omega \Lambda_k' + \Omega_k' \Omega) + \Omega_k'(1 - 2\Omega) - 3(\Omega_k - \Omega_k' + \Omega - \Omega' - 1)\Omega_\Lambda \omega_0 + (1 - \Omega - \Omega_k) \left[ 3(\Omega + \Omega' + A) \right]. \]  

(61)

Substituting Eqs. (33), (50), (57) and (61) into (54) yields

\[ \omega_1 = (1 + \omega_0) \left[ 1 - \Omega_{k\Lambda} - 3(1 - \Omega_\Lambda_0)\omega_0 \right] + \frac{A_0}{3\Omega_\Lambda_0}, \]

(62)

which can be rewritten by the help of Eq. (60) as

\[ \omega_1 = (1 + \omega_0) \left[ 1 - \Omega_{k\Lambda} - 3(1 - \Omega_\Lambda_0)\omega_0 \right] - 3(1 + q_0)(1 + \omega_0) + \frac{1}{3n} \left( \frac{\Omega_\Lambda_0}{\gamma_{n_0}} \right)^{1/2} \times \left[ 17 + 4(\Omega_{k\Lambda} + 2q_0) + 3(7 - 4\Omega_\Lambda_0)\omega_0 - \frac{8}{n} \left( \frac{\Omega_\Lambda_0}{\gamma_{n_0}} \right)^{1/2} - \frac{32\pi Gc_\alpha H_0^2}{3n^2\Omega_\Lambda_0} \left( \frac{\Omega_\Lambda_0}{\gamma_{n_0}} \right)^{5/2} \right]. \]

(63)

**B. Interacting case**

Here, we consider a case in which the ECNADE and DM interact with each other. This causes the energy conservation law for each dark component not to be held separately, i.e.

\[ \dot{\rho}_\Lambda + 3H(1 + \omega_\Lambda)\rho_\Lambda = -Q, \]

(64)

\[ \dot{\rho}_m + 3H\rho_m = Q, \]

(65)

where \( Q = 3b^2H\rho_\Lambda \) (Pavón and Zimdahl 2005) stands for the interaction term with coupling constant \( b^2 \). For \( Q > 0 \), there is an energy transfer from ECNADE to DM. The recent observational evidence provided by the galaxy cluster Abell A586 supports the interaction between DE and DM (Bertolami et al. 2007, 2009; Abdalla et al. 2009).

Taking time derivative of the first Friedmann Eq. (13) and using (23), (24), (25), (26), (27) and (65) yields

\[ \frac{\dot{H}}{H^2} = \frac{1}{2} \left[ \Omega_k - 3(1 - \Omega_\Lambda) + 3b^2\Omega_\Lambda \right] - \frac{1}{n\alpha} \left( \frac{\Omega_\Lambda}{\gamma_n} \right)^{3/2} \times \left( 2\gamma_n - \frac{Gc_\alpha H^2 \Omega_\Lambda}{Gg^3} \right). \]

(66)
Substituting this into Eqs. (28) and (29) one can get

\[
\Omega'_\Lambda = \Omega_{\Lambda} \left[3(1 - \Omega_{\Lambda}) - 3b^2 \Omega_{\Lambda} - \Omega_k + \frac{2}{na} \left(\frac{\Omega\Lambda}{\gamma_n}\right)^{1/2} \times \left(\frac{\Omega_{\Lambda} - 1}{\gamma_n}\right) \left(2\gamma_n - \frac{8\pi G_c c H^2 \Omega_{\Lambda}}{3n^4 \gamma_n}\right)\right],
\] (67)

and

\[
\Omega'_k = \Omega_k \left[(1 - \Omega_k) - 3\Omega_{\Lambda} - 3\Omega_{\Lambda} + \frac{2}{na} \left(\frac{\Omega_{\Lambda}}{\gamma_n}\right)^{3/2} \times \left(2\gamma_n - \frac{G_c}{G_g} - \frac{8\pi G_c c H^2 \Omega_{\Lambda}}{3n^4 \gamma_n}\right)\right].
\] (68)

Replacing the term $\dot{H}/H^2$ from (66) into (35) yields

\[
q = \frac{1}{2}(1 - \Omega_k - 3\Omega_{\Lambda}) + \frac{1}{na} \left(\frac{\Omega_{\Lambda}}{\gamma_n}\right)^{3/2} \left(2\gamma_n - \frac{G_c}{G_g} - \frac{8\pi G_c c H^2 \Omega_{\Lambda}}{3n^4 \gamma_n}\right) - \frac{3}{2} b^2 \Omega_{\Lambda}.
\] (69)

From

\[
\rho_{\Lambda} = \frac{\rho_m}{\Omega_m} \Omega_{\Lambda} = \frac{\rho_m}{(1 - \Omega_k - \Omega_{\Lambda})} \Omega_{\Lambda},
\] (70)

one can obtain

\[
\frac{d \ln \rho_{\Lambda}}{d \ln a} = \frac{\rho'_m}{\rho_m} - \frac{\Omega'_m}{\Omega_m} + \frac{\Omega'_\Lambda}{\Omega_{\Lambda}}.
\] (71)

From Eq. (64) and using (41), the interacting ECNADE density evolves as

\[
\frac{\rho_{\Lambda}}{\rho_{\Lambda_0}} = a^{-3(1 + \omega_0 - \omega_1 + b^2)} e^{3\omega_1 z}.
\] (72)

Using $\ln a = -\ln(1 + z) \simeq -z + \frac{z^2}{2}$ for small redshifts, Eq. (72) reduces to

\[
\ln \left(\frac{\rho_{\Lambda}/\rho_{\Lambda_0}}{\ln a}\right) = -3(1 + \omega_0 + b^2) - \frac{3}{2} \omega_1 z.
\] (73)

Comparing Eqs. (45) and (73), one can obtain the parameters $\omega_0$ and $\omega_1$ for the interacting case as

\[
\omega_0 = -\frac{1}{3} \left.\frac{d \ln \rho_{\Lambda}}{d \ln a}\right|_0 - 1 - b^2,
\] (74)

\[
\omega_1 = \frac{1}{3} \left.\frac{d^2 \ln \rho_{\Lambda}}{d (\ln a)^2}\right|_0.
\] (75)
Substituting Eq. (71) into (74) and using (65) one can get
\[
\omega_0 = -\frac{1}{3}\frac{\Omega_A' + \Omega_k'}{\Omega_A} + \frac{\Omega_A' + \Omega_k'}{(1 - \Omega_k - \Omega_A)}_0 - b^2\left(\frac{1 - \Omega_k}{1 - \Omega_k - \Omega_A}\right)_0.
\] (76)

Using Eqs. (70) and (75) one can obtain
\[
\omega_1 = \frac{1}{3}\left[\frac{3b^2\Omega_A'}{(1 - \Omega_k - \Omega_A)} + \frac{3b^2\Omega_A(\Omega_A' + \Omega_k')}{(1 - \Omega_k - \Omega_A)^2} + \frac{\Omega_A''}{\Omega_A} - \frac{\Omega_A' + \Omega_k''}{(1 - \Omega_k - \Omega_A)} + \frac{(\Omega_A' + \Omega_k')^2}{(1 - \Omega_k - \Omega_A)^2}\right]_0.
\] (77)

Adding Eqs. (67) and (68) gives
\[
\Omega_A' + \Omega_k' = (1 - \Omega_k - \Omega_A)\left[\Omega_k + 3\Omega_A - \frac{2}{n\alpha}(\frac{\Omega_A}{\gamma_n})^{3/2} \times \left(2\gamma_n - \frac{G_c}{G_g} - \frac{8\pi G_c \alpha H^2 \Omega_A}{3n^4 \gamma_n}\right) - 3b^2\Omega_A(\Omega_k + \Omega_A)\right].
\] (78)

Substituting Eqs. (67) and (78) into (76) yields
\[
\omega_0 = -1 + \frac{2}{3n\gamma_n \gamma_n} \left(\frac{\Omega_A}{\gamma_n}\right)^{1/2} \times \left(2\gamma_n - \frac{G_c}{G_g} - \frac{8\pi G_c \alpha H^2 \Omega_A}{3n^4 \gamma_n}\right) - b^2,
\] (79)

which in the absence of interaction, i.e. \(b^2 = 0\), reduces to Eq. (51). The EoS parameter \(\omega_0\) versus the interacting coupling parameter \(b^2\) is plotted in Fig. 1. Figure shows that in the absence of interaction, \(b^2 = 0\), \(\omega_0 = -0.85\) and cannot cross the phantom divide line. However, in the presence of interaction, \(b^2 \neq 0\), the phantom EoS \(\omega_0 < -1\) can be obtained when \(b^2 \geq 0.15\) for the coupling between ECNADE and DM. This value for coupling constant \(b^2\) is consistent with recent observations (Wang et al. 2005a, 2005b). Also the phantom divide crossing is compatible with the observation (Komatsu et al. 2011).

Taking derivative of Eqs. (67) and (68) with respect to \(x = \ln a\) and using (79), one can get
\[
\Omega_A'' = -(\Omega_A\Omega_k' + \Omega_A'\Omega_k) - 3\Omega_A(\Omega_A - \Omega_A' - 1)(\omega_0 + b^2) + (1 - \Omega_A)\left[3(\Omega_A + \Omega_A') + A\right] - 6b^2\Omega_A\Omega_A,
\] (80)
and

\[
\Omega_k'' = \Omega_k'(1 - 2\Omega_k) - 3\Omega_A(\Omega_k - \Omega_k')(\omega_0 + b^2) \\
- \Omega_k \left[ 3(\Omega_A + \Omega_A') + A \right] \\
- 3b^2(\Omega_k'\Omega_A + \Omega_k\Omega_A'),
\]

(81)

where \( A \) is given by Eq. (59). Using Eqs. (25), (35), (66), (79) and \( \dot{\eta} = 1/a \), one can rewrite (59) for the interacting case as

\[
A = -9\Omega_A(1 + q)(1 + \omega_0 + b^2) + \frac{\Omega_A}{na} \left( \frac{\Omega_A}{\gamma_n} \right)^{1/2} \\
\times \left[ 17 + 4(\Omega_k + 2q) + 3(7 - 4\Omega_A)\omega_0 + 21b^2 \\
- \frac{8}{na} \left( \frac{\Omega_A}{\gamma_n} \right)^{1/2} - \frac{32\pi G_c\alpha H^2}{3an^5\Omega_A} \left( \frac{\Omega_A}{\gamma_n} \right)^{5/2} \right].
\]

(82)

Adding Eqs. (80) and (81) gives

\[
\Omega_A'' + \Omega_k'' = -(\Omega_A\Omega_k' + \Omega_A'\Omega_k) + \Omega_k'(1 - 2\Omega_k) \\
- 3\Omega_A(\omega_0 + b^2)(\Omega_k - \Omega_k' + \Omega_A - \Omega_A' - 1) \\
+ (1 - \Omega_A - \Omega_k) \left[ 3(\Omega_A + \Omega_A') + A \right] \\
- 3b^2(\Omega_k'\Omega_A + \Omega_k\Omega_A') - 6b^2\Omega_A'\Omega_A.
\]

(83)

Replacing Eqs. (67), (78), (80) and (83) into (77) gives

\[
\omega_1 = \frac{b^2\Omega_{A_0}}{(1 - \Omega_{A_0} - \Omega_{k_0})} \left[ -3(\omega_0 + b^2) \\
+ 6\Omega_{A_0}\omega_0 - 2\Omega_{k_0} \right] \\
+ (3\Omega_{A_0}\omega_0 - \Omega_{k_0}) \left[ \frac{\Omega_{k_0}}{3} - \Omega_{A_0}\omega_0 \right] \\
- \frac{b^2\Omega_{A_0}}{(1 - \Omega_{A_0} - \Omega_{k_0})} \right] \\
+ \left[ \frac{b^2\Omega_{A_0}}{(1 - \Omega_{A_0} - \Omega_{k_0})} - \Omega_{A_0}\omega_0 + \frac{\Omega_{k_0}}{3} - 1 \right] \\
\times \left[ 3\omega_0(1 - \Omega_{A_0}) + 3b^2 + \Omega_{k_0} \right] \\
+ (\omega_0 + b^2) \left[ 3(\omega_0 + b^2)(2\Omega_{A_0} - 1) \\
+ 1 - 2\Omega_{k_0} - 6b^2\Omega_{A_0} \right] + \frac{A_0}{3\Omega_{A_0}} + 1,
\]

(84)

which in the absence of interaction, i.e. \( b^2 = 0 \), yields Eq. (63). The EoS parameter \( \omega_1 \) versus the interacting coupling parameter \( b^2 \) is plotted in Fig. 2. Figure shows that in the absence of
interaction \((b^2 = 0)\), \(\omega_1 = 0.07\) and the EoS parameter \(\omega_1\) increases when the interacting coupling parameter \(b^2\) increases. This is in agreement with the result obtained by Jamil and Saridakis (2010).

IV. CONCLUSIONS

Here, we investigated the ECNADE scenario in a FRW Universe in the framework of HL gravity. We considered an arbitrary spatial local curvature for the background geometry and allowed for an interaction between the ECNADE and DM. In both the regular and interacting case we obtained the deceleration parameter as well as the differential equations which determine the evolution of the ECNADE density parameter. Finally, using a low redshift expansion of the EoS parameter of DE as \(\omega_A(z) = \omega_0 + \omega_1 z\), we calculated \(\omega_0\) and \(\omega_1\) as functions of the DE and curvature density parameters, \(\Omega_{\Lambda_0}\) and \(\Omega_k_0\) respectively, of the running parameter \(\lambda = (2 \frac{\rho}{\rho_c} + 1)/3\) of HL gravity, of the parameter \(n\) of ECNADE, of the interaction coupling \(b^2\), and of the coefficients of correction terms \(\alpha\) and \(\beta\).

Interestingly enough we found that in the presence of interaction between ECNADE and DM, the EoS parameter \(\omega_0\) of ECNADE in HL gravity, can cross the phantom divide line which is compatible with the recent observations. The interaction between ECNADE and DM can be detected in the formation of large scale structures. It was suggested that the dynamical equilibrium of collapsed structures such as galaxy clusters would be modified due to the coupling between DE and DM (Bertolami et al. 2007, 2009; Abdalla et al. 2009). The idea is that the virial theorem is modified by the energy exchange between the dark sectors leading to a bias in the estimation of the virial masses of clusters when the usual virial conditions are employed. This provides a probe in the near Universe of the dark coupling. The other observational signatures on the dark sectors’ mutual interaction can be found in the probes of the cosmic expansion history by using the type Ia supernovae (SNeIa), baryonic acoustic oscillation (BAO) and cosmic microwave background (CMB) shift data (Guo and Ohta 2007; Feng et al. 2008; He et al. 2009; Honorez et al. 2010).

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FIG. 1: The EoS parameter $\omega_0$ of the ECNADE in HL gravity, Eq. (79), versus the interacting coupling parameter $b^2$. Auxiliary parameters are: $n = 2.716$ (Wei and Cai 2008b), $\alpha = -7.5$, $\gamma_{n_0} = 15$ (Karami and Abdolmaleki 2010c), $\Omega_{\Lambda_0} = 0.728$, $\Omega_{k_0} = -0.013$ (Komatsu et al. 2011), $\lambda = 1.02$ (Dutta and Saridakis 2010), $G_c/G_g = 2/(3\lambda - 1)$, $H_0 = 74.2$ Km s$^{-1}$ Mpc$^{-1}$ (Riess et al. 2009) and $M_p^{-2} = 8\pi G_g = 1$.

FIG. 2: The EoS parameter $\omega_1$ of the ECNADE in HL gravity, Eq. (84), versus the interacting coupling parameter $b^2$. Auxiliary parameters as in Fig. 1.

[1] Abdalla, E., et al.: Phys. Lett. B 673, 107 (2009)
[2] Allen, S.W., et al.: Mon. Not. Roy. Astron. Soc. 353, 457 (2004)
[3] Banerjee, R., Majhi, B.R., Samanta, S.: Phys. Rev. D 77, 24035 (2008)
[4] Banerjee, R., Majhi, B.R.: Phys. Lett. B 662, 62 (2008a)
[5] Banerjee, R., Majhi, B.R.: JHEP 06, 095 (2008b)
[6] Banerjee, R., Majhi, B.R.: Phys. Lett. B 674, 218 (2009)
[7] Banerjee, R., Modak, S.K.: JHEP 05, 063 (2009)
[8] Bennett, C.L., et al.: Astrophys. J. Suppl. 148, 1 (2003)
[9] Bertolami, O., Gil Pedro, F., Le Delliou, M.: Phys. Lett. B 654, 165 (2007)
[10] Bertolami, O., Gil Pedro, F., Le Delliou, M.: Gen. Relativ. Gravit. 41, 2839 (2009)
[11] Cai, R.G.: Phys. Lett. B 657, 228 (2007)
[12] Calcagni, G.: JHEP 09, 112 (2009)
[13] Carloni, S., Elizalde, E., Silva, P.J.: Preprint (2009). arXiv:0909.2219
[14] Zhu T., Wu Q., Wang A., & Shu F.-W., Phys. Rev. D 84, 101502 (R) (2011)
[15] Zhu T., Shu F.-W., Wu Q., & Wang A., (2011) arXiv:1110.5106
[16] Carroll, S.M., Lim, E.A.: Phys. Rev. D 70, 123525 (2004)
[17] Chaichian, M., et al.: Class. Quantum Grav. 27, 185021 (2010)
[18] Cohen, A., Kaplan, A., Nelson, A.: Phys. Rev. Lett. 82, 4971 (1999)
[19] Copeland, E.J., Sami, M., Tsujikawa, S.: Int. J. Mod. Phys. D 15, 1753 (2006)
[20] Dutta, S., Saridakis, E.N.: JCAP 05, 013 (2010)
[21] Feng, B., Wang, X., Zhang, X.: Phys. Lett. B 607, 35 (2005)
[22] Feng, C., et al.: Phys. Lett. B 665, 111 (2008)
[23] Guo, Z.K., Ohta, N., Tsujikawa, S.: Phys. Rev. D 76, 023508 (2007)
[24] He, J.H., Wang, B., Zhang, P.: Phys. Rev. D 80, 063530 (2009)
[25] Honorez, L.L., et al.: J. Cosmol. Astropart. Phys. 09, 029 (2010)
[26] Hořava, P.: JHEP 03, 020 (2009a)
[27] Hořava, P.: Phys. Rev. D 79, 084008 (2009b)
[28] Greenwald J., Lenells J., Lu J.X., Satheeshkumar V.H., Wang A.: Phys. Rev. D 84, 084040 (2011).
[29] Hsu, S.D.H.: Phys. Lett. B 594, 13 (2004)
[30] Huang, Q.C., Li, M.: JCAP 08, 013 (2004)
[31] Huterer, D., Turner, M.S.: Phys. Rev. D 60, 081301 (1999)
[32] Huterer, D., Turner, M.S.: Phys. Rev. D 64, 123527 (2001)
[33] Jamil, M., Farooq, M.U., Rashid, M.A.: Eur. Phys. J. C 61, 471 (2009a)
[34] Jamil, M., Saridakis, E.N., Setare, M.R.: Phys. Lett. B 679,172 (2009b)
[35] Jamil, M., Farooq, M.U.: Int. J. Theor. Phys. 49, 42 (2010)
[36] Jamil, M., Saridakis, E.N.: JCAP 07, 028 (2010)
[37] Karami, K.: JCAP 01, 015 (2010a)
[38] Karami, K.: Preprint (2010b). arXiv:1002.0431
[39] Karami, K., Abdolmaleki, A.: Astrophys. Space Sci. 330, 133 (2010a)
[40] Karami, K., Abdolmaleki, A.: Phys. Scr. 81, 055901 (2010b)
[41] Karami, K., Abdolmaleki, A.: Preprint (2010c). arXiv:1009.2459
[42] Karami, K., Abdolmaleki, A.: Int. J. Theor. Phys. 50, 1656 (2011)
[43] Karami, K., Fehri, J.: Int. J. Theor. Phys. 49, 1118 (2010a)
[44] Karami, K., Fehri, J.: Phys. Lett. B 684, 61 (2010b)
[45] Karami, K., Khaledian, M.S., Felegary, F., Azarmi, Z.: Phys. Lett. B 686, 216 (2010)
[46] Karami, K., Sorouri, A.: Phys. Scr. 82, 025901 (2010)
[47] Karami, K., et al.: Gen. Relativ. Gravit. 43, 27 (2011)
[48] Karolyhazy, F., Nuovo. Cim. A 42, 390 (1966)
[49] Karolyhazy, F., Frenkel, A., Lukacs, B.: in Physics as natural Philosophy, edited by Shimony, A., Feschbach, H., MIT Press, Cambridge, MA, (1982)
[50] Karolyhazy, F., Frenkel, A., Lukacs, B.: in Quantum Concepts in Space and Time, edited by Penrose, R., Isham, C.J., Clarendon Press, Oxford, (1986)
[51] Khatua, P.B., Chakraborty, S., Debnath, U.: Preprint (2011). arXiv:1105.3393
[52] Kim, K.Y., Lee, H.W., Myung, Y.S.: Phys. Lett. B 660, 118 (2008a)
[53] Kim, Y.W., et al.: Mod. Phys. Lett. A 23, 3049 (2008b)
[54] Kiritsis, E., Kofinas, G.: Nucl. Phys. B 821, 467 (2009)
[55] Mukohyama, S.: Class. Quantum Grav. 27, 223101 (2010)
[56] Wang, A., Wu, Q.: Phys. Rev. D 83, 044025 (2011)
[57] Izumi K., Mukohyama S.: Phys. Rev. D 84, 064025 (2011)
[58] Gumrukcuoglu, A.E., Mukohyama, S., Wang, A.: (2011) arXiv:1109.2609
[59] Horava P., & Melby-Thompson.: Phys. Rev. D 82, 064027 (2010)
[60] da Silva, A.M.: Class. Quantum Grav. 28, 055011 (2011)
[61] Komatsu, E., et al.: Astrophys. J. Suppl. 192, 18 (2011)
[62] Leon, G., Saridakis, E. N.: JCAP 11, 006 (2009)
[63] Li, M.: Phys. Lett. B 603, 1 (2004)
[64] Majhi, B.R.: Phys. Rev. D 79, 044005 (2009)
[65] Majhi, B.R.: Phys. Lett. B 686, 49 (2010)
[66] Majhi, B.R., Samanta, S.: Annals of Physics 325, 2410 (2010)
[67] Maziashvili, M.: Int. J. Mod. Phys. D 16, 1531 (2007a)
[68] Maziashvili, M.: Phys. Lett. B 652, 165 (2007b)
[69] Modak, S.K.: Phys. Lett. B 671, 167 (2009)
[70] Mukohyama, S.: Phys. Rev. D 80, 064006 (2009)
[71] Mukohyama, S.: Class. Quantum Grav. 27, 223101 (2010)
[72] Nastase, H.: Preprint (2009). arXiv:0904.3604
[73] Neupane, I.P.: Phys. Lett. B 673, 111 (2009)
[74] Pal, S.: Preprint (2009). arXiv:0901.0599
[75] Pavón, D., Zimdahl, W.: Phys. Lett. B 628, 206 (2005)
[76] Perlmutter, S., et al.: Astrophys. J. 517, 565 (1999)
[77] Riess, A.G., et al., Astrophys. J. 699, 539 (2009)
[78] Saridakis, E.N.: Eur. Phys. J. C 67, 229 (2010)
[79] Sasakura, N.: Prog. Theor. Phys. 102, 169 (1999)
[80] Setare, M.R., Jamil, M.: JCAP 02, 010 (2010)
[81] Sheykhi, A.: Int. J. Mod. Phys. D 18, 2023 (2009a)
[82] Sheykhi, A.: Phys. Lett. B 680, 113 (2009b)
[83] Sheykhi, A.: Class. Quantum Grav. 27, 025007 (2010a)
[84] Sheykhi, A.: Phys. Rev. D 81, 023525 (2010b)
[85] Sotiriou, T.P., Visser, M., Weinfurtner, S.: JHEP 10, 033 (2009)
[86] Susskind, L.: Math. Phys. 36, 6377 (1995)
[87] Tegmark, M., et al.: Phys. Rev. D 69, 103501 (2004)
[88] 't Hooft, C.: Preprint (1993) [gr-qc/9310026]
[89] Visser, M.: Phys. Rev. D 80, 025011 (2009)
[90] Wang, B., Gong, Y., Abdalla, E.: Phys. Lett. B 624, 141 (2005a)
[91] Wang, B., Lin, C.Y., Abdalla, E.: Phys. Lett. B 637, 357 (2005b)
[92] Wang, B., et al.: Phys. Lett. B 662, 1 (2008)
[93] Wang, A., Wu, Y.: JCAP 07, 012 (2009)
[94] Wang, A.: Mod. Phys. Lett. A 26, 387 (2011)
[95] Wei, H., Cai, R.G.: Phys. Lett. B 660, 113 (2008a)
[96] Wei, H., Cai, R.G.: Phys. Lett. B 663, 1 (2008b)
[97] Wei, H.: Commun. Theor. Phys. 52, 743 (2009)
[98] Wei, H., Cai, R.G.: Eur. Phys. J. C 59, 99 (2009)
[99] Weller, J., Albrecht, A.: Phys. Rev. Lett. 86, 1939 (2001)
[100] Wu, J.P., Ma, D.Z., Ling, Y.: Phys. Lett. B 663, 152 (2008)
[101] Zhang, X., Wu, F.Q.: Phys. Rev. D 72, 043524 (2005)
[102] Zhang, X., Wu, F.Q.: Phys. Rev. D 76, 023502 (2007)
[103] Zhang, J., Zhang, X., Liu, H.: Eur. Phys. J. C 54, 303 (2008)