The Average Kinetic Energy of the Heavy Quark in $\Lambda_b$

in the Bethe-Salpeter Equation Approach

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Abstract

In the previous paper, based on the $SU(2)_f \times SU(2)_s$ heavy quark symmetries of the QCD Lagrangian in the heavy quark limit, the Bethe-Salpeter equation for the heavy baryon $\Lambda_b$ was established with the picture that $\Lambda_b$ is composed of a heavy quark and a scalar light diquark. In the present work, we apply this model to calculate $\mu_\pi^2$ for $\Lambda_b$, the average kinetic energy of the heavy quark inside $\Lambda_b$. This quantity is particularly interesting since it can be measured in experiments and since it contributes to the inclusive semileptonic decays of $\Lambda_b$ when contributions from higher order terms in $1/M_b$ expansions are taken into account and consequently influences the determination of the Cabibbo-Kobayashi-Maskawa matrix elements $V_{ub}$ and $V_{cb}$. We find that $\mu_\pi^2$ for $\Lambda_b$ is $0.25 GeV^2 \sim 0.95 GeV^2$, depending on the parameters in the model including the light diquark mass and the interaction strength between the heavy quark and the light diquark in the kernel of the BS equation. We also find that this result is consistent with the value of $\mu_\pi^2$ for $\Lambda_b$ which is derived from the experimental value of $\mu_\pi^2$ for the $B$ meson with the aid of the heavy quark effective theory.

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I. Introduction

The physics of heavy quarks has attracted intense interests in recent years, partly because of the discovery of the flavor and spin symmetries in QCD, $SU(2)_f \times SU(2)_s$, in the heavy quark limit and the establishment of the heavy quark effective theory (HQET) [1]. Compared with the research on heavy mesons, heavy baryons have been studied less both theoretically and experimentally. However, more and more experimental data for heavy baryons have been and will be obtained. This will help to test theoretical predictions for heavy baryons. For example, the lifetime of $\Lambda_b$ has been measured in several experiments [2]. The measurement of the nonleptonic decay of $\Lambda_b$, $\Lambda_b \rightarrow \Lambda J/\psi$, has been done [3]. There have also been the measurements of the semileptonic decays of $\Lambda_b$, $\Lambda_b \rightarrow \Lambda_c l^- \bar{\nu}_l$anything [4] and $\Lambda_b \rightarrow \Lambda_c l^- \bar{\nu}_l$ [5]. On the other hand, since heavy baryons are composed of three quarks instead of two, theoretical studies for heavy baryons become more complicated. In order to understand the hadronic structure of heavy baryons, more theoretical and experimental studies are needed.

HQET can simplify the physical processes involving heavy quarks. $\Lambda_b$ is composed of a heavy $b$ quark and two light quarks, $u$ and $d$. When the heavy quark mass is very large compared with the QCD scale $\Lambda_{QCD}$, the light degrees of freedom (the light quark system) in a heavy baryon becomes blind to the flavor and spin quantum numbers of the heavy quark because of the $SU(2)_f \times SU(2)_s$ symmetries. Therefore, the angular momentum and flavor quantum numbers of the light degrees of freedom become good quantum numbers. Hence it is natural to regard the heavy baryon $\Lambda_b$ to be composed of a heavy quark and a light scalar diquark, $[ud]_0$, with $[ud]$ flavor quantum number and zero spin and isospin.

The Bethe-Salpeter (BS) equation is a formally exact equation to describe the relativistic bound state [6, 7, 8]. In the heavy quark limit the BS equation can be simplified to a great extent and has been applied to give many theoretical results.
concerning heavy mesons and heavy baryons [9, 10, 11, 12, 13, 14]. With the model
for the composition of $\Lambda_b$ which is described above the heavy baryon $\Lambda_b$ is reduced
from a three-body system to a two-body system. In this picture the BS equation
for $\Lambda_b$ was established [10, 11]. A scalar confinement and a one gluon exchange
term compose the kernel of the BS equation in this model. Furthermore, this model
was generalized to the heavy baryons $\Sigma_b^{(*)}, \Xi_b^{(*)},$ and $\Omega_b^{(*)}$ which are regarded to be
composed of a heavy quark and an axial-vector diquark [12].

In HQET the strong interaction of a heavy quark with four-velocity $v$ can be
described by the following Lagrangian density [1, 15]:

$$\mathcal{L} = \bar{h}_v i v \cdot D h_v + \frac{1}{2M_Q} \bar{h}_v [(iD_\perp)^2] h_v + \frac{g_s}{4M_Q} \bar{h}_v \sigma_{\mu\nu} G^{\mu\nu} h_v, \quad (1)$$

where $h_v$ denotes the field of the heavy quark, $M_Q$ is the mass of the heavy quark,
$D^\mu = \partial^\mu - ig_s A^\mu$ is the covariant derivative, $D_\perp = D^\mu - \nu^\mu v \cdot D$, and $G^{\mu\nu}$ is the
gluon field tensor. The second operator in Eq. (1) is related to the average kinetic
energy of the heavy quark due to the residual motion of the heavy quark inside the
heavy hadron and the third one corresponds to the spin energy of the heavy quark.
The kinetic energy and the spin energy of the heavy quark can be described by the
following two local matrix elements respectively:

$$\mu_\pi^2 = -\frac{\langle H_Q | \bar{h}_v (iD_\perp)^2 h_v | H_Q \rangle}{2M}, \quad (2)$$

and

$$\mu_G^2 = \frac{\langle H_Q | g_s \bar{h}_v \sigma^{\mu\nu} G_{\mu\nu} h_v | H_Q \rangle}{4M}, \quad (3)$$

where $H_Q (Q = b$ or $c)$ denotes a heavy baryon containing a heavy quark $Q$ and $M$
is the mass of $H_Q$.

The parameters $\mu_\pi^2$ and $\mu_G^2$ are of particular interests since they contribute to
the inclusive semileptonic decays of heavy hadrons when contributions from higher
order terms in $1/M_Q$ expansions are taken into account and, therefore, influence the
determination of the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements $V_{ub}$ and
V_{cb}. Therefore, it is very interesting to calculate these nonperturbative quantities theoretically.

There have been extensive studies in literature on inclusive semileptonic decays of bottom hadrons, $H_b \rightarrow X e\bar{\nu}_e$, especially since the establishment of HQET [16, 17, 18, 19, 20, 21, 22]. These studies include corrections to the leading order results both from perturbative QCD ($\alpha_s(M_b)$) terms and from nonperturbative terms which are suppressed by powers of $M_b$. It has been pointed out that there is no $1/M_b$ corrections to the leading order result in $1/M_b$ for the differential decay width of semileptonic decays of bottom hadrons, $d\Gamma/dq^2 dE_e$, where $q$ is total momentum of the electron and the neutrino and $E_e$ is the electron energy [16]. Then Bigi et al. studied $1/M_b^2$ corrections to the decay width $d\Gamma/dE_e$ [21]. Manohar and Wise analyzed extensively $1/M_b^2$ corrections to $d\Gamma/dq^2 dE_e$ for unpolarized bottom hadron $H_b$ and for polarized $\Lambda_b$ [22]. In recent years, theoretical calculations for the inclusive semileptonic decay widths and for the moments of inclusive observables have been carried out to order $1/M_b^3$ and $\alpha_s^2\beta_0$ ($\beta_0 = 11 - 2n_f/3$) [23, 24, 25, 26, 27]. It was found that the $1/M_b^2$ corrections are characterized by the two parameters $\mu_\pi^2$ and $\mu_G^2$, which can be extracted from experimental data and theoretically should been determined in a nonperturbative way.

In the case of the $B$ meson, the parameter $\mu_G^2$ can be extracted from the data for the hyperfine splitting between $B$ and $B^*$ mesons. The other parameter $\mu_\pi^2$ has been extracted from the experimental data for the inclusive semileptonic $B$ meson decays $B \rightarrow X_c l\bar{\nu}$ and $B \rightarrow X_s \gamma$ [28, 29, 30, 31, 32, 33]. In Ref. [33], using the theoretical formulae provided in Refs. [24, 25], Buchmüller and Flächer obtained the most recent result for $\mu_\pi^2$, $\mu_\pi^2 = 0.401 \pm 0.040 GeV^2$, from a combined fit to the moments of the hadronic mass distribution and the moments of the leptonic energy spectrum in $B \rightarrow X_c l\bar{\nu}$ and the moments of the photon energy spectrum in $B \rightarrow X_s \gamma$ which are measured in the BABAR, Belle, CDF, CLEO, and DELPHI experiments [32, 34, 35, 36, 37]. Theoretically, $\mu_\pi^2$ has been calculated in various phenomenological
models such as QCD sum rules and the BS equation [14, 38, 39, 40, 41] and by lattice QCD [42]. The theoretical results of $\mu^2_\pi$ for the $B$ meson depend on models strongly. Some of them are consistent with the experimental value, $\mu^2_\pi = 0.401 \pm 0.040 GeV^2$, while some of them, including that from the BS approach for the $B$ meson, are not. This needs further and more careful investigations.

Compared with the case of the $B$ meson, $\Lambda_b$ has been studied less both experimentally and theoretically. Since $\Lambda_b$ is composed of a heavy quark and a light scalar diquark, the parameter $\mu_G^2$ is zero for $\Lambda_b$. Although there has been no direct experimental measurement of $\mu^2_\pi$ for $\Lambda_b$, one can expect it to be measured in the future since more and more data on $\Lambda_b$ will be collected. Furthermore, with the aid of HQET, $\mu^2_\pi$ for $\Lambda_b$ can be related to $\mu^2_\pi$ for the $B$ meson [22]. Hence, one can derive the value of $\mu^2_\pi$ for $\Lambda_b$ from the experimental value of $\mu^2_\pi$ for the $B$ meson. Therefore, it is important to give results for $\mu^2_\pi$ for $\Lambda_b$ from theoretical calculations.

The aim of the present work is to calculate the average kinetic energy of the $b$ quark in the heavy baryon $\Lambda_b$ with the BS equation model for $\Lambda_b$ [10, 11]. We will give the numerical result for this parameter, discuss its dependence on the parameters in the model, and compare our result with the value of $\mu^2_\pi$ for $\Lambda_b$ derived from the experimental value of $\mu^2_\pi$ for the $B$ meson through HQET.

The reminder of this paper is organized as the following. In Section II we review the basic formalism for the BS equation for $\Lambda_b$. In Section III we give numerical solutions for the BS wave function and then apply the BS equation to calculate $\mu^2_\pi$ numerically. We also discuss the dependence of our result on the parameters in the model and compare this result with the value of $\mu^2_\pi$ for $\Lambda_b$ derived from the experimental value of $\mu^2_\pi$ for the $B$ meson with the aid of HQET. Finally we give a summary and discussion in Section IV.

II. Formalism for the BS equation for $\Lambda_b$

As discussed in Introduction, $\Lambda_Q$ is regarded as the bound state of a heavy quark
and a light diquark. Based on this picture the BS wave function of $\Lambda_Q$ is defined as follows:

$$\chi(x_1, x_2, P) = \langle 0| T\psi(x_1)\phi(x_2)|\Lambda_Q\rangle,$$

(4)

where $\psi(x_1)$ and $\phi(x_2)$ are field operators of the heavy quark and the diquark, respectively, and $P$ is the momentum of $\Lambda_Q$. The BS wave function in the momentum space, $\chi_P(p)$, is related to $\chi(x_1, x_2, P)$ through the following equation:

$$\chi(x_1, x_2, P) = e^{iPX} \int \frac{d^4p}{(2\pi)^4} \chi_P(p) e^{ipx},$$

(5)

where $p$ and $x(= x_1 - x_2)$ are the relative momentum and the relative coordinate of the heavy quark and the light scalar diquark, respectively, and $X$ is the center of mass coordinate which is defined as $X = \lambda_1 x_1 + \lambda_2 x_2$, where $\lambda_1 = \frac{M_Q}{M_Q + M_D}$, $\lambda_2 = \frac{M_D}{M_Q + M_D}$, with $M_D$ being the mass of the diquark. The momentum of the heavy quark is $p_1 = \lambda_1 P + p$ and that of the diquark is $p_2 = -\lambda_2 P + p$.

The mass of the heavy baryon, $M$, satisfies the following relation:

$$M = M_Q + M_D + E_0 + O(\frac{1}{M_Q}),$$

(6)

where $E_0$ is the binding energy in the leading order of $1/M_Q$ expansion.

The BS equation in the momentum space can be written as follows [10]:

$$\chi_P(p) = S_F(\lambda_1 P + p) \int \frac{d^4q}{(2\pi)^4} G(P, p, q) \chi_P(q) S_D(-\lambda_2 P + p),$$

(7)

where $G(P, p, q)$ is the kernel which is defined as the sum of all the two particle irreducible diagrams with respect to the heavy quark and the light diquark. $S_F$ and $S_D$ in Eq. (7) are propagators of the heavy quark and the light scalar diquark, respectively.

The kernel $G(P, p, q)$ includes two terms in the model: a scalar confinement term $V_1$ and a one gluon exchange term $V_2$ [9, 10, 12],

$$-iG = I \otimes IV_1 + v_\mu \otimes (p_2 + p_2')^\mu V_2,$$

(8)
where \( p_2 \) and \( p'_2 \) are the momenta of the light diquark attached to the gluon. The vertex of the gluon with the diquark depends on the structure of the diquark. This is taken into account by introducing a form factor \( F(Q^2) = \frac{\alpha_{\text{eff}}^2 Q_0^2}{Q^2 + Q_0^2} \) [43], where \( Q_0^2 \) is a parameter which freezes \( F(Q^2) \) when \( Q^2 = (p_t - q_t)^2 \) is very small.

It has been shown that in the leading order of \( 1/M_Q \) expansion we only need one scalar function, \( \phi_P(p) \), to describe the BS wave function [10]. \( \phi_P(p) \) is related to \( \chi_P(p) \) as the following:

\[
\chi_P(p) = \phi_P(p) u_{\Lambda_Q}(v),
\]  

where \( v \) is the velocity of the heavy baryon and \( u_{\Lambda_Q}(v) \) is the spinor of the heavy baryon.

Define the longitudinal and transverse momenta with respect to \( v \):

\[
p_l = v \cdot p - \lambda_2 M, \quad p_t = p - (v \cdot p)v.
\]

Using the covariant instantaneous approximation, \( p_l = q_l \), at the vertex of the heavy quark and the gluon, we have the BS equation in the leading order of \( 1/M_Q \) expansion,

\[
\phi_P(p) = \frac{-i}{(p_t + E_0 + M_D + i\epsilon)(p_t^2 - W_p^2 + i\epsilon)} \int \frac{d^3 q_t}{(2\pi)^4} (\tilde{V}_1 + 2p_l \tilde{V}_2) \tilde{\phi}_P(q_t), \tag{10}
\]

where \( \tilde{\phi}_P(p_l) \equiv \int (dp_l/2\pi) \phi_P(p) \) and \( \tilde{V} \) stands for \( V \) in the covariant instantaneous approximation \( p_l = q_l \).

Integrating Eq. (10) by \( \int dp_l/2\pi \) and applying the residue theorem we obtain the equation for the BS wave function, \( \tilde{\phi}_P(p_l) \),

\[
\tilde{\phi}_P(p_l) = -\frac{1}{2W_p(-W_p + E_0 + M_D)} \int \frac{d^3 q_t}{(2\pi)^3} (\tilde{V}_1 - W_p 2 \tilde{V}_2) \tilde{\phi}_P(q_t). \tag{11}
\]

The kernel \( \tilde{V}_1 \) and \( \tilde{V}_2 \) have the following expression in the case of the heavy baryon [10, 12]:

\[
\tilde{V}_1 = \frac{8\pi\kappa}{[(p_t - q_t)^2 + u^2]^2} - (2\pi)^3 \delta^3(p_t - q_t) \int \frac{d^3 k}{(2\pi)^3} \frac{8\pi\kappa}{(k^2 + u^2)^2}, \tag{12}
\]

\[
\tilde{V}_2 = -\frac{16\pi}{3} \left[ \frac{(\alpha_{\text{eff}}^2) Q_0^2}{[(p_t - q_t)^2 + u^2][(p_t - q_t)^2 + Q_0^2]} \right]. \tag{13}
\]
where \( \kappa \) and \( \alpha_{\text{eff}}^s \) are coupling parameters related to the scalar confinement and the one gluon exchange diagram, respectively. The second term in Eq. (12) is the counter term which removes the infra-red divergence in the integral equation. The parameter \( u \) is introduced to avoid the infra-red divergence in numerical calculations. The limit \( u \to 0 \) is taken in the end.

Substituting \( \tilde{V}_1 \) and \( \tilde{V}_2 \) into Eq. (11) we have

\[
(E_0 + M_D - W_p) \tilde{\phi}_P(p_t) = -\frac{1}{2W_p} \left\{ \int \frac{q_t^2 dq_t}{4\pi^2} \left( \frac{16\pi \kappa}{(p_t^2 + q_t^2 + u^2)^2} - \frac{32\pi (\alpha_{\text{eff}}^s)^2 Q_0^2 W_p}{3(Q_0 - u^2)} \int \frac{q_t^2 dq_t}{4\pi^2} \frac{1}{2p_t q_t} \left[ \ln \frac{(p_t + q_t)^2 + u^2}{(p_t - q_t)^2 + u^2} - \ln \frac{(p_t + q_t)^2 + Q_0^2}{(p_t - q_t)^2 + Q_0^2} \right] \tilde{\phi}_P(q_t) \right] + \frac{1}{2W_p} \int \frac{q_t^2 dq_t}{4\pi^2} \frac{16\pi \kappa}{(p_t^2 + q_t^2 + u^2)^2} - \frac{4p_t^2 q_t^2}{4p_t^2 q_t^2} \tilde{\phi}_P(p_t) \right\} 
\]

\[
+ \frac{16\pi \kappa}{(p_t^2 + q_t^2 + u^2)^2} - \frac{4p_t^2 q_t^2}{4p_t^2 q_t^2} \tilde{\phi}_P(p_t). \tag{14}
\]

III. Calculation of the average kinetic energy of the \textit{b} quark in \( \Lambda_b \)

In this section we solve the BS equation numerically and then apply the results to calculate the average kinetic energy of the \textit{b} quark inside the heavy baryon \( \Lambda_b \), \( \mu_{\pi}^2 \), which is defined in Eq. (2). The BS wave function for \( \Lambda_b \), \( \tilde{\phi}_P(p_t) \) in Eq. (14), can be solved numerically by discretizing the integration region \((0, \infty)\) into \( n \) pieces \((n \) is chosen to be sufficiently large). We use the \( n \)-point Gauss quadrature rule to evaluate the integral. Then Eq. (14) becomes an eigenvalue equation. The numerical results for \( \tilde{\phi}_P(p_t) \) are obtained by solving this eigenvalue equation. Eq. (14) is a homogeneous equation which leaves the normalization of \( \tilde{\phi}_P(p_t) \) undetermined.

We use the following normalization condition to fix the amplitude of the BS wave function\(^3\):

\[
\langle \Lambda_b | \bar{h}_v h_v | \Lambda_b \rangle = 2M. \tag{15}
\]

\(^3\)One can also use the expression \( \mu_{\pi}^2 = -\frac{\langle H_Q | \bar{h}_v (iD^2 | h_v | H_Q) \rangle^2}{\langle H_Q | h_v h_v | H_Q \rangle} \) to calculate \( \mu_{\pi}^2 \). This expression is independent of how the BS wave function is normalized.
In the model we have several parameters, i.e. $\alpha_{s}^{\text{eff}}, \kappa, Q_{0}^{2}, M_{D}$ and $E_{0}$. The parameter $Q_{0}^{2}$ is taken as $Q_{0}^{2} = 3.2\text{GeV}^{2}$ [10, 12, 43]. The parameters $\alpha_{s}^{\text{eff}}$ and $\kappa$ are related to each other when we solve the eigenvalue equation with a fixed eigenvalue [10]. The parameter $\kappa$ varies in the region between $0.02\text{GeV}^{3}$ and $0.1\text{GeV}^{3}$ [10]. From Eq. (6) the parameters $M_{D}$ and $E_{0}$ are constrained by the relation $M_{D} + E_{0} = M - M_{b}$ for $\Lambda_{b}$ in the leading order of $1/M_{b}$ expansion. In our numerical calculations we use $M_{b} = 5.02\text{GeV}$ which leads to consistent predictions with experiments from the BS equation in the meson case [9]. Consequently we have $M_{D} + E_{0} = 0.62\text{GeV}$ for $\Lambda_{b}$ (where we have neglected $1/M_{b}$ corrections). The parameter $M_{D}$ can not be determined and hence we let it vary within some reasonable range. For $\Lambda_{b}$, we choose $M_{D}$ to be in the range $0.65\text{GeV} \sim 0.80\text{GeV}$. With this choice for $M_{D}$, the binding energy $E_{0}$ is negative and varies from around $-30\text{MeV}$ to $-180\text{MeV}$.

The numerical results for $\alpha_{s}^{\text{eff}}$ corresponding to various values of $\kappa$ are given in Tables 1, 2, and 3. Then the numerical results for the BS wave function depend on two parameters, $\kappa$ and $M_{D}$. In Fig. 1 we show the solutions for the BS wave function for some typical values of $\kappa$ and $M_{D}$.

Since $\mu^{2}_{\pi}$ is a Lorentz scalar [22] we are free to choose a special frame for the calculation of this parameter. For simplicity we choose the rest frame of $\Lambda_{b}$ in which Eq. (2) becomes

$$\mu^{2}_{\pi} = \frac{\langle \Lambda_{Q}|\bar{h}_{v}(i\slashed{D})^{2}h_{v}|\Lambda_{Q}\rangle}{2M}.\tag{16}$$

The diagram for calculating the average kinetic energy of the $b$ quark inside $\Lambda_{b}$ is shown in Fig. 2. Assuming the light diquark acts as a spectator, we obtain the following expression for $\mu^{2}_{\pi}$ which is related to the BS wave function of $\Lambda_{b}$:

$$\mu^{2}_{\pi} = \frac{1}{2M} \int \frac{d^{4}p}{(2\pi)^{4}} \bar{\chi}_{P}(p)\bar{P}^{2}\chi_{P}(p)S_{D}^{-1}(-\lambda_{2}P + p).\tag{17}$$

Substituting Eq. (9) and the relation between $\phi_{P}(p)$ and $\bar{\phi}_{P}(p_{t})$, Eq. (10), into Eq. (17) and integrating the $p_{t}$ component by selecting the proper contour we have

$$\mu^{2}_{\pi} = \frac{1}{2M} \int \frac{d^{3}p_{t}}{(2\pi)^{3}} p_{t}^{2}(2W_{p})\bar{\phi}_{P}^{2}(p_{t}).\tag{18}$$
The three-dimensional integral in Eq. (18) can be simplified to one-dimensional integral. This leads to

$$\mu_\pi^2 = \frac{1}{2M} \int \frac{p_t^2 dp_t}{2\pi^2} p_t^2 W_p \tilde{\phi}_P(p_t).$$

(19)

As shown in Fig. 1, the numerical results for the BS wave function $\tilde{\phi}_P(p_t)$ depend on the parameters $\kappa$ (or $\alpha_s^{\text{eff}}$) and $M_D$. Therefore, the results for $\mu_\pi^2$ also depend on these parameters. For example, taking $M_D = 0.7 GeV$ and $\kappa = 0.04 GeV^3$, we get $\mu_\pi^2 = 0.47 GeV^2$. In Tables 1, 2, and 3 we list the numerical results for $\mu_\pi^2$ for various values of the parameters $\kappa$ and $M_D$. 

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Fig. 1

Fig. 2
It can be seen from these tables that the value of $\mu^2_{\pi}$ changes from $0.25 GeV^2$ to $0.95 GeV^2$ in the variation ranges of the model parameters $M_D$ and $\kappa$. The dependence of the average kinetic energy of the $b$ quark inside $\Lambda_b$ on the model parameters is quite strong. Furthermore, the dependence on $\kappa$ is stronger than that on $M_D$. For instance, for $M_D = 0.7 GeV$, when $\kappa$ varies from $0.02 GeV^3$ to $0.1 GeV^3$, the change of $\mu^2_{\pi}$ is about $0.44 GeV^2$; for $\kappa = 0.04 GeV^3$, when $M_D$ varies from $0.65 GeV$ to $0.80 GeV$, the change of $\mu^2_{\pi}$ is about $0.27 GeV^2$.

In principle, the parameters in the model can be determined through the comparison between theoretical predictions and experimental measurements about some physical processes of $\Lambda_b$ if the data are accurate enough. In Refs. [10, 11, 13] some phenomenological predictions for $\Lambda_b$ such as semileptonic and nonleptonic decay widths of $\Lambda_b$ are given in the BS equation approach. Since the heavy quark mass is not infinite in reality, in order to give more exact phenomenological predictions $1/M_Q$ corrections to the BS equation for $\Lambda_Q$ are analyzed in Ref. [11] based on the assumption that $\Lambda_Q$ is composed of a heavy quark and a scalar light diquark. Including both the $1/M_Q$ corrections and the QCD corrections to the weak decay form factors [44] the prediction for the decay rate for $\Lambda_b \to \Lambda_c l \bar{\nu}$ can be obtained as $2.70 \sim 4.07 \times 10^{10} s^{-1}$ in the variation ranges of $M_D$ and $\kappa$ (where $V_{cb}$ is taken as $0.042$ [45]). The uncertainty of this prediction is mostly from the uncertainty from $\kappa$. The experimental data for the decay rate for $\Lambda_b \to \Lambda_c l \bar{\nu}$, which is in the range $2.3 \sim 6.7 \times 10^{10} s^{-1}$ [5], is consistent with the prediction. Therefore, we can not determine the parameters in the BS equation model from this process at present due to the large error in the experimental data. With more and more data available in the future one can constrain the parameters in the BS model much better. Furthermore, the experimental data for the nonleptonic decay widths for $\Lambda_b \to \Lambda_c$ plus a pseudoscalar or a vector meson (the predictions for them have been given in Ref. [11]) can also be used to determine the parameters in the model.

Theoretically, there have been some phenomenological calculations for the di-
quark mass from the BS equation for the diquark [46] and from the relativistic potential model for the diquark [47], respectively. The masses for the \([ud]_0\) diquark obtained in these two approaches depend on the model parameters and are consistent with what are used in our BS model.

As mentioned in Introduction, although there has been no direct experimental measurement of \(\mu_\pi^2(B)\) for \(\Lambda_b\), one can relate this quantity to \(\mu_\pi^2(B)\) for the \(B\) meson with the aid of HQET [22]. In this way, one can derive the value of \(\mu_\pi^2(B)\) for \(\Lambda_b\) from the experimental value of \(\mu_\pi^2(B)\). It was shown that when the masses of heavy hadrons are expanded to order \(1/M_Q^2\) one has the following relation:

\[
\mu_\pi^2(\Lambda_b) - \mu_\pi^2(B) = \frac{2M(B)M(D)}{M(B) - M(D)}\left\{[M(\Lambda_c) - M(D)_{\text{avg}}] - [M(\Lambda_b) - M(B)_{\text{avg}}]\right\}, \tag{20}
\]

where \(\mu_\pi^2(\Lambda_b)\) (\(\mu_\pi^2(B)\)) is \(\mu_\pi^2\) for \(\Lambda_b\) (\(B\)), \(M(B)\) (\(M(D)\)) is the mass of \(B\) (\(D\)), and \(M(D)_{\text{avg}}\) (\(M(B)_{\text{avg}}\)) is defined as the spin averaged mass of \(D\) (\(B\)) mesons (for instance, \(M(D)_{\text{avg}} = [M(D) + 3M(D^*)]/4\) for \(D\) mesons). The masses of \(D, B,\) and \(\Lambda_c\) have been measured accurately and the largest uncertainty of the right hand side of Eq. (20) comes from the mass of \(\Lambda_b\) [45]. Using \(M(\Lambda_b) = 5624 \pm 9\text{MeV}\) and the masses of \(D, B,\) and \(\Lambda_c\) provided in Ref. [45], the right hand side of Eq. (20) is \(0.025 \pm 0.052\text{GeV}^2\) where the error comes mostly from the error of the mass of \(\Lambda_b\) (the errors of the masses of \(D, B,\) and \(\Lambda_c\) contribute little). Consequently we obtain \(\mu_\pi^2(\Lambda_b)\) from \(\mu_\pi^2(B) = 0.401 \pm 0.040\text{GeV}^2\) (which was obtained by fitting the data in the so-called kinetic scheme [33]) as follows:

\[
\mu_\pi^2(\Lambda_b) = 0.426 \pm 0.066\text{GeV}^2, \tag{21}
\]

where the error includes those from both \(\mu_\pi^2(B)\) and the mass of \(\Lambda_b\).

Besides the uncertainty in Eq. (21), the \(1/M_Q^2\) terms in the expansion for the masses of heavy hadrons may also cause some uncertainty to \(\mu_\pi^2(\Lambda_b)\). Two parameters, \(\rho_D^3\) and \(\rho^3\), appear in the \(1/M_Q^2\) terms in the masses of \(\Lambda_b, \Lambda_c,\) and the spin
averaged masses of $D$ and $B$ mesons\textsuperscript{4} [40, 48]. The parameter $\rho^3_D$ has been extracted from the fit in Ref. [33] while $\rho^3$, which is a nonlocal correlator of the two operators $\bar{h}_v(\vec{\sigma} \cdot \vec{D})^2 h_v$, has not been determined. $\rho^3_D$ is of order $\tilde{\Lambda}^3$ ($\tilde{\Lambda}$ is defined as the difference between the mass of a heavy hadron and the mass of the heavy quark inside the hadron in the heavy quark limit) [33]. $\rho^3$ is also expected to be of order $\tilde{\Lambda}^3$. Although there may be some cancellation between the parameters $\rho^3_D$ and $\rho^3$ for the heavy baryons and those for the heavy mesons in the mass difference $[M(\Lambda_c) - M(D)_{\text{avg}}] - [M(\Lambda_b) - M(B)_{\text{avg}}]$ on the right hand side of Eq. (20), we assume that the $1/M_Q^2$ terms in this mass difference is of order $\tilde{\Lambda}^3/M_Q^2$ to make a conservative estimate on the influence of the $1/M_Q^2$ terms on $\mu^2_\pi(\Lambda_b)$. The $\tilde{\Lambda}^3/M_Q^2$ terms give the main contribution to $\mu^2_\pi(\Lambda_b)$ in Eq. (20), which is about $0.09 GeV^2$ if we take $\tilde{\Lambda}$ to be $0.6 GeV$ [33].

Taking into account all the uncertainties from $\mu^2_\pi(B)$, the mass of $\Lambda_b$, and the $\tilde{\Lambda}^3/M_Q^2$ terms in the masses of heavy hadrons, one may expect $\mu^2_\pi(\Lambda_b)$ to be roughly in the range $0.27 GeV^2 \sim 0.58 GeV^2$. This is consistent with our result in the BS model, $0.25 GeV^2 \sim 0.95 GeV^2$. Conversely, one may give a rough constraint on the ranges of the parameters in the BS model from the range of $\mu^2_\pi(\Lambda_b)$, $0.27 GeV^2 \sim 0.58 GeV^2$. For instance, when $M_D$ is $0.65 GeV$, $\kappa$ is roughly in the range $0.02 GeV^3 \sim 0.08 GeV^3$ from Table 1, while when $M_D$ are $0.7 GeV$ and $0.8 GeV$, $\kappa$ are roughly in the ranges $0.02 GeV^3 \sim 0.06 GeV^3$ and $0.02 GeV^3 \sim 0.04 GeV^3$ from Tables 2 and 3, respectively.

IV. Summary and Discussion

The average kinetic energy of the $b$ quark inside $\Lambda_b$, $\mu^2_\pi$, is an interesting quantity both theoretically and experimentally. It contributes to the inclusive semileptonic decays of $\Lambda_b$ when contributions from higher order terms in $1/M_b$ expansions are taken into account and influences the determination of the CKM matrix elements.

\textsuperscript{4}$\rho^3$ contain four terms, $\rho^3_\pi\pi$, $\rho^3_{\pi\rho}$, $\rho^3_\delta$, $\rho^3_A$, while only $\rho^3_\pi\pi$ and $\rho^3_\delta$ contribute to the masses of $\Lambda_b$ and $\Lambda_c$ and the spin averaged masses of $D$ and $B$ mesons [40, 48].
$V_{ub}$ and $V_{cb}$. By comparing the experimental data with the theoretical predictions for such decays one can extract the value of $\mu_\pi^2$.

Based on the BS equation model for the heavy baryon $\Lambda_b$, which is regarded as composed of the heavy $b$ quark and a light diquark, we have calculated the average kinetic energy of the $b$ quark inside $\Lambda_b$. The kernel of the BS equation consists of a one gluon exchange term and a scalar confinement term. Since $\mu_\pi^2$ is expressed as the overlap integral of the BS wave function of $\Lambda_b$, we first solved out this BS wave function numerically by transferring the integral equation for the BS wave function into an eigenvalue equation. We have found that the value of $\mu_\pi^2$ varies in the region between $0.25 GeV^2$ and $0.95 GeV^2$ depending on the parameters in the model. The dependence of $\mu_\pi^2$ on the parameters in the model was discussed in some detail. We have compared our result with the value of $\mu_\pi^2$ for $\Lambda_b$ which is derived from the experimental value of $\mu_\pi^2$ for the $B$ meson with the aid of HQET and found that they are consistent. Conversely, the latter may also be used to give a rough constraint on the parameters in the BS model.

Compared with the meson case, heavy baryons are much more complicated since there are three quarks in a baryon. Even though we have simplified the bound state equation for a heavy baryon with the diquark picture, large uncertainties are still introduced in the BS equation for the heavy baryon. This is reflected in the large ranges of the parameters in the model, i.e. $\kappa$ and $M_D$. This leads to a much larger range for the phenomenological prediction for the average kinetic energy of the $b$ quark inside $\Lambda_b$. Fortunately much more data will be available in the future experiments, e.g. LHCb. This provides an opportunity to constrain the model parameters more accurately by comparing the experimental data with the BS model predictions for the physical processes, say semileptonic and nonleptonic decays of $\Lambda_b$. 

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Figure captions

Fig. 1 Numerical results for the BS wave function $\tilde{\phi}_P(p_t)$. The solid (dashed) line corresponds to $M_D = 0.7 GeV$ and $\kappa = 0.02 (0.1) GeV^3$. The dotted (dot-dashed) line corresponds to $\kappa = 0.04 GeV^3$ and $M_D = 0.65 (0.8) GeV$.

Fig. 2 The diagram for calculating the average kinetic energy of the $b$ quark inside $\Lambda_b$. The black dot represents the operator $\bar{h}_v(i\not{D})^2 h_v$.

| $\kappa (GeV^3)$ | 0.02 | 0.04 | 0.06 | 0.08 | 0.10 |
|------------------|-----|-----|-----|-----|-----|
| $\alpha_s^{\text{eff}}$ | 0.62 | 0.67 | 0.70 | 0.72 | 0.75 |
| $\mu_\pi^2 (GeV^2)$ | 0.25 | 0.39 | 0.51 | 0.62 | 0.72 |

Table 1: The values of $\kappa$, $\alpha_s^{\text{eff}}$, and the corresponding $\mu_\pi^2$ for $M_D = 0.65 GeV$.

| $\kappa (GeV^3)$ | 0.02 | 0.04 | 0.06 | 0.08 | 0.10 |
|------------------|-----|-----|-----|-----|-----|
| $\alpha_s^{\text{eff}}$ | 0.67 | 0.71 | 0.73 | 0.75 | 0.77 |
| $\mu_\pi^2 (GeV^2)$ | 0.34 | 0.47 | 0.58 | 0.69 | 0.78 |

Table 2: The values of $\kappa$, $\alpha_s^{\text{eff}}$, and the corresponding $\mu_\pi^2$ for $M_D = 0.7 GeV$.

| $\kappa (GeV^3)$ | 0.02 | 0.04 | 0.06 | 0.08 | 0.10 |
|------------------|-----|-----|-----|-----|-----|
| $\alpha_s^{\text{eff}}$ | 0.76 | 0.78 | 0.80 | 0.81 | 0.82 |
| $\mu_\pi^2 (GeV^2)$ | 0.55 | 0.66 | 0.76 | 0.86 | 0.95 |

Table 3: The values of $\kappa$, $\alpha_s^{\text{eff}}$, and the corresponding $\mu_\pi^2$ for $M_D = 0.8 GeV$. 

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