A COMPREHENSIVE COMPARISON OF THE SUN TO OTHER STARS: SEARCHING FOR SELF-SELECTION EFFECTS

José A. Robles,1 Charles H. Lineweaver,1 Daniel Grether,2 Chris Flynn,3 Chas A. Egan,2,4 Michael B. Pracy,4 Johan Holmberg,5 and Esko Gardner3

Received 2007 December 21; accepted 2008 May 10

ABSTRACT

If the origin of life and the evolution of observers on a planet is favored by atypical properties of a planet’s host star, we would expect our Sun to be atypical with respect to such properties. The Sun has been described by previous studies as both typical and atypical. In an effort to reduce this ambiguity and quantify how typical the Sun is, we identify 11 maximally independent properties that have plausible correlations with habitability and that have been observed by, or can be derived from, sufficiently large, currently available, and representative stellar surveys. By comparing solar values for the 11 properties to the resultant stellar distributions, we make the most comprehensive comparison of the Sun to other stars. The two most atypical properties of the Sun are its mass and orbit. The Sun is more massive than 95% ± 2% of nearby stars, and its orbit around the Galaxy is less eccentric than 93% ± 1% of FGK stars within 40 pc. Despite these apparently atypical properties, a χ2 analysis of the Sun’s values for 11 properties, taken together, yields a solar χ2 = 8.39 ± 0.96. If a star is chosen at random, the probability that it will have a lower value (i.e., be more typical) than the Sun, with respect to the 11 properties analyzed here, is only 29% ± 11%. These values quantify, and are consistent with, the idea that the Sun is a typical star. If we have sampled all reasonable properties associated with habitability, our result suggests that there are no special requirements for a star to host a planet with life.

Subject headings: stars: fundamental parameters — stars: statistics — Sun: fundamental parameters — Sun: general

1. INTRODUCTION

If the properties of the Sun are consistent with the idea that the Sun was randomly selected from all stars, this would indicate that life needs nothing special from its host star and would support the idea that life may be common in the universe. More particularly, if there is nothing special about the Sun, we have little reason to limit our life-hunting efforts to planets orbiting Sun-like stars. As an example of the type of anthropic reasoning we are using, consider the following situation. Suppose uranium (a low-abundance element in the solar system and in the universe) was central to the biochemistry of life on Earth. Further, suppose that a comparison of our Sun to other stars showed that the Sun had more uranium than any other star. How should we interpret this fact? The most reasonable way to proceed would be to try to evaluate the probability that such a coincidence happened by chance and to determine whether we are justified in reading some importance into it. Although a correlation does not necessarily imply cause, we think that a correlation between the Sun’s anomalous feature and life’s fundamental chemistry would give us important clues about the conditions necessary for life. Specifically, the search for life around other stars as envisioned by the NASA’s Terrestrial Planet Finder or ESA’s Darwin Project and as currently underway with SETI would change the strategy to focus on the most uranium-rich stars. Another example: Suppose the Sun had the highest [Fe/H] of all the stars that had ever been observed. Then high [Fe/H] would be strongly implicated as a precondition for our existence, possibly by playing a crucial role in terrestrial planet formation. These are exaggerated examples of the more subtle correlations that a detailed and comprehensive comparison of the Sun with other stars could reveal.

Whether the Sun is a typical or atypical star with respect to one or a few properties has been addressed in previous studies. Using an approach similar to ours (comparing solar to stellar properties from particular samples), some studies have suggested that the Sun is a typical star (Gustafsson 1998; Allende Prieto 2006), while other studies have suggested that the Sun is an atypical star (Gonzalez 1999a, 1999b; Gonzalez et al. 2001). This apparent disagreement arises from three problems:

1. The language used to describe whether the Sun is or is not typical is often confusingly qualitative. For example, reporting the Sun as “metal-rich” can mean that the Sun is significantly more metal-rich than other stars (e.g., more metal-rich than 80% of other stars) or it can mean that the Sun is insignificantly metal-rich (e.g., more metal-rich than 51% of other stars).
2. The stellar samples chosen for the comparison can be biased with respect to the property of interest.
3. The inclusion (or exclusion) of stellar properties for which it is suspected or known that the Sun is atypical will make the Sun appear more atypical (or typical).

In this paper we address problem 1 by using only quantitative measures when comparing the Sun’s properties to other stars. Our main interest is to move beyond the qualitative assessment of the Sun as either typical or atypical and obtain a more precise quantification of the degree of the Sun’s (a)typicality. In other words, we want to answer the question, “how typical is the Sun?” rather than “is the Sun typical or not?” There are at least two ways to quantify how typical the Sun is. This can be done for individual parameters by determining how many stars have values below or
above the solar value (Table 3). This can also be done by a joint analysis of multiple parameters (Table 2). If there are several subtle factors that have some influence over habitability, a quantitative joint analysis of the Sun’s properties may allow us to identify these factors without invoking largely speculative arguments.

With respect to problem 2, most previous analyses have compared the Sun to subsets of Sun-like stars selected to be Sun-like with respect to one or more parameters. In such analyses, the Sun will appear typical with respect to any parameter(s) correlated with respect to one or more parameters. In such analyses, the Sun is compared to subsets of Sun-like stars selected to be Sun-like linking specific properties to habitability.

The sample used to produce the stellar distributions plotted in Figures 1–10.

| Figure | Property | Range | Median $\mu_{1:2}$ | $\sigma_{15}$ | Solar Value | Number of Stars | Spectral type | $d_{\text{max}}$ (pc) | Source |
|--------|----------|-------|--------------------|-------------|-------------|----------------|--------------|----------------|--------|
| 1       | Mass ($M_\odot$) | 0.08–2 | 0.33 | 0.37 | 1 | 125 | A1–M7 | 7.1 | RECONS |
| 2       | Age (Gyr) | 0–15 | 5.4 | 3.25 | $4.0^{+0.3}_{-0.2}$ | 552 | F8–K2 | 200 | Rocha-Pinto et al. (2000b) |
| 3       | [Fe/H] | $-1.20$ to $+0.46$ | $-0.08$ | 0.20 | 0 | 453 | F7–K3 | 25 | Grether & Lineweaver (2007) |
| 4A      | [C/O] | $-0.22$ to $+0.32$ | 0.07 | 0.09 | 0 | 256 | FG | 150 | G99, R03, BF06 |
| 4B      | [Mg/Fe] | $-0.18$ to $+0.14$ | 0.01 | 0.04 | 0 | 231 | FG | 150 | R03, B05 |
| 5       | $v \sin i$ (km s$^{-1}$) | 0–36 | 2.51 | 1.27 | $1.28^e$ | 276 | F8–K2 | 80 | Valenti & Fischer (2005) |
| 6       | e | 0–1 | 0.10 | 0.05 | $0.036 \pm 0.002^g$ | 1987 | A5–K2 | 40 | Nordström et al. (2004) |
| 7       | $Z_{\text{max}}$ (kpc) | 0–0.60 | 0.14 | 0.10 | $0.104 \pm 0.06^g$ | 1987 | A5–K2 | 40 | Nordström et al. (2004) |
| 8       | $R_\text{gal}$ (kpc) | 0–30 | 4.9 | 5.03 | $7.62 \pm 0.32^d$ | ... | ... | ... | ... |
| 9       | $M_\text{gal}$ ($M_\odot$) | $10^4$–$10^{12}$ | $10^{10.2}$ | 0.47 | $10^{10.55\pm0.16}$ | ... | ... | $10^7$ | D94, CB99, L00, B01, J03 |
| 10      | $M_\text{group}$ ($M_\odot$) | $10^5$–$10^{13}$ | $10^{11.1}$ | 0.47 | $10^{10.91\pm0.07}$ | ... | ... | ... | ... |

a Characteristic width of distribution in the direction of the solar value.

b Wright et al. (2004); see footnote 8 of this paper.

c G99: Gustafsson et al. (1999); R03: Reddy et al. (2003); BF06: Bensby & Feltzing (2006).

d R03: Reddy et al. (2003); B05: Bensby et al. (2005).

e Solar rotational velocity corrected for random inclination (see § 2.5).

f Subset of stars within the mass range $0.9 M_\odot \leq M \leq 1.1 M_\odot$.

g Calculated using the solar galactic motion (Dehnen & Binney 1998) and the Galactic potential (see § 2.6).

h Subset of volume complete A5–K2 stars within 40 pc.

i Integrated solar orbit in the Galactic potential of Flynn et al. (1996; see § 2.6).

j Eisenhauer et al. (2005).

k BS80: Bahcall & Sonier (1980); G96: Gould et al. (1996); E05: Eisenhauer et al. (2005).

l Stellar mass, not total baryonic mass or total mass.
m D94: Driver et al. (1994); CB99: Courteau & van den Bergh (1999); L00: Loveday (2000); B01: Bell & de Jong (2001); J03: Jarrett et al. (2003).

---

2. STELLAR SAMPLES AND SOLAR VALUES

We are looking for a signal associated with a prerequisite for, or a property that favors, the origin and evolution of life (see Gustafsson [1998] for a brief discussion of this idea). If we indiscriminately include many properties with little or no plausible correlation with habitability, we run the risk of diluting any potential signal. If we choose only a few properties based on previous knowledge that the Sun is anomalous with respect to those properties, we are making a useful quantification, but we are unable to address problem 3. We choose a middle ground and try to identify as many properties as we can that have some plausible association with habitability. This strategy is most sensitive if a few unknown stellar properties (among the ones being tested) contribute to the habitability of a terrestrial planet in orbit around a star.

An optimal quantitative comparison of the Sun to other stars would require an unbiased, large representative stellar sample from which independent distributions for as many properties as
desired could be compared. Such a distribution for each property of interest would allow a straightforward analysis and outcome: the Sun is within \( n \% \) of stars around the centroid of the \( N \)-dimensional distribution. However, observational and sample selection effects prevent the assembly of such an ideal stellar sample.

In this study, we compare the Sun to other stars with respect to the following 11 basic physical properties: (1) mass, (2) age, (3) metallicity \([\text{Fe/H}]\), (4) carbon-to-oxygen ratio \([\text{C/O}]\), (5) magnesium-to-silicon ratio \([\text{Mg/Si}]\), (6) rotational velocity \( v \sin i \), (7) eccentricity of the star's galactic orbit \( e \), (8) maximum height to which the star rises above the galactic plane \( Z_{\text{max}} \), (9) mean galactocentric radius \( R_{\text{Gal}} \), (10) the mass of the star's host galaxy \( M_{\text{Gal}} \), and (11) the mass of the star's host group of galaxies \( M_{\text{group}} \). These 11 properties span a wide range of stellar and galactic factors that may be associated with habitability. We briefly discuss how each parameter might have a plausible correlation with habitability. For each property we have tried to assemble a large, representative sample of stars whose selection criteria is minimally biased with respect to that property. For each property the percentage of stars with values lower and higher than the solar value are computed. For properties 9, 10, and 11, the uncertainties in the percentages are determined from the uncertainties of the distributions. For the rest of the properties, nominal uncertainties \( \Delta \) on the percentages were calculated assuming a binomial distribution (e.g., Meyer 1975): \( \Delta = (n_{\text{low}} \times n_{\text{high}}/N_{\text{tot}})^{1/2} \), where \( n_{\text{low}} (n_{\text{high}}) \) is the fraction of stars with a lower (higher) value than the Sun and \( N_{\text{tot}} \) is the total number of stars in the sample. The solar value is indicated with the Sun symbol (\( \odot \)) in all figures.

We compare the Sun and its environment to other stars and their environments. The analysis of these larger environmental contexts provides information about properties that otherwise could not be directly measured. For example, suppose the metallicity of the Sun were normal with respect to stars in the solar neighborhood, but that these stars as a group had an anomalously high metallicity with respect to the average metallicity of stars in the universe. This fact would strongly suggest that habitability is associated with high metallicity, but our comparison with only local stars would not pick this up. In the absence of an \([\text{Fe/H}]\) distribution for all stars in the universe, we use galactic mass as a convenient proxy for any such property that correlates with galactic mass.

2.1. Mass

Mass is probably the single most important characteristic of a star. For a main-sequence star, mass determines luminosity, effective temperature, main-sequence lifetime, and the dimensions, UV insolation, and temporal stability of the circumstellar habitable zone (Kasting et al. 1993).

Low-mass stars are intrinsically dim. Thus, a complete sample of stars can only be obtained out to a distance of \(~7\) pc (\( \approx 23\) lt-yr). Figure 1 compares the mass of the Sun to the stellar mass distribution of the 125 nearest main-sequence stars within 7.1 pc, as compiled by the RECONS consortium.\(^7\) Overplotted is the stellar initial mass function (IMF; see Kroupa 2002, eqs. [4] and [5] and Table 1) normalized to 125 stars more massive than the brown dwarf limit of 0.08 \( M_{\odot} \). Since the IMF appears to be fairly universal (Kroupa & Weidner 2005), these nearby comparison stars are representative of a much larger sample of stars. There is good agreement between the histogram and the IMF; the Sun is more massive than 95\% ± 2\% of the nearest stars and more massive than 94\% ± 2\% of the stars in the Kroupa (2002) IMF. Fourteen brown dwarfs and nine white dwarfs within 7.1 pc were not included in this sample. Including them yields 94\%—the same result obtained from the IMF. Our 95\% ± 2\% result should be compared with the 91\% reported by Gonzalez (1999b). The Sun's mass is the most anomalous of the properties studied here.

2.2. Age

If the evolution of observers like ourselves takes on average many billions of years, we might expect the Sun to be anomalously old (Carter 1983). Accurate estimation of stellar ages is difficult. For large stellar surveys (more than a few hundred stars), the most commonly used age indicators are based on isochrone fitting and/or chromospheric activity (\( R'_{\text{HK}} \) index). Rocha-Pinto et al. (2000b) have estimated a star formation rate (SFR) or, equivalently, an age distribution for the local Galactic disk from chromospheric ages of 552 late-type (F8–K2) dwarf stars in the mass range \( 0.8 M_{\odot} \leq M \leq 1.4 M_{\odot} \) at distances \( d \leq 200 \) pc (Rocha-Pinto et al. 2000a). They applied scale-height corrections, stellar evolution corrections, and volume incompleteness corrections that converted the observed age distribution into the total number of stars born at any given time. Hernandez et al. (2000) and Bertelli & Nasi (2001) have made estimates of the star formation rate in the solar neighborhood and favor a smoother distribution (fewer bursts) than Rocha-Pinto et al. (2000b).

In Figure 2 we compare the chromospheric age of the Sun (\( \tau_{\odot} = 4.9 ± 3.0 \) Gyr) with that of the Sun (\( \tau_{\odot} = 4.57 ± 0.002 \) Gyr (Allegre et al. 1995)).

---

\(^{7}\) See RECONS database at http://www.recons.org/.
short list contains the dominant elements in the composition of terrestrial planets (O, Fe, Si, and Mg) and life (C, O, N, and S).

Over the last few decades, much effort has gone into determining abundances in nearby stars for a wide range of elements. Stellar elemental abundances for element X are usually normalized to the solar abundance of the same element using a logarithmic abundance scale: \( \log (X/H) \equiv \log (X/H)_\odot - \log (X/H)_\odot \). Hence, all solar elemental abundances \( [X/H]_\odot \) are defined as zero. Spectroscopic abundance analyses are usually made differential relative to the Sun by analyzing the solar spectrum (reflected by the Moon, asteroids, or the telescope dome) in the same way as the spectrum of other stars. In this approach, biases introduced by the assumption of local thermodynamic equilibrium (LTE) largely cancel out for Sun-like stars (Edvardsson et al. 1993b).

A comparison between solar and stellar iron abundances is a common feature of most abundance surveys, and most have concluded that the Sun is metal-rich compared to other stars (Gustafsson 1998; Gonzalez 1999a, 1999b). However, for our purposes, the appropriateness of these comparisons depends on the selection criteria of the stellar sample to which the Sun has been compared. Stellar metallicity analyses such as Edvardsson et al. (1993a), Reddy et al. (2003), Nordström et al. (2004), and Valenti & Fischer (2005) have stellar samples selected with different purposes in mind. For example, Edvardsson et al. (1993a) aimed to constrain the chemical evolution of the Galaxy, and their sample is biased toward low metallicity (average \( [\text{Fe/H}] = -0.25 \)). The sample of Valenti & Fischer 2005 (average \( [\text{Fe/H}] = -0.01 \)) was selected as a planet candidate list and contains some bias toward high metallicity (see Grether & Lineweaver 2007). To assess how typical the Sun is, Gustafsson (1998) limited the sample of Edvardsson et al. (1993a) to stars with galactocentric radii within 0.5 kpc of the solar galactocentric radius, and to ages between 4 and 6 Gyr. The distribution of stars given by this criteria has an average \( [\text{Fe/H}] = -0.09 \).

Grether & Lineweaver (2006, 2007) compiled a sample of 453 Sun-like stars within 25 pc. These stars were selected from the \textit{Hipparcos} catalog, which is essentially complete to 25 pc for stars within the spectral type range F7–K3 and absolute magnitude of \( M_F = 8.5 \) (Reid 2002). Metallicities for this sample were assembled from a wide range of spectroscopic and photometric surveys. In Figure 3 we compare the Sun to the Grether & Lineweaver (2007) sample, which has a median \( [\text{Fe/H}] = -0.08 \). To our knowledge this is the most complete and least-biased stellar spectroscopic metallicity distribution. The Sun is more metal-rich than 65% ± 2% of these stars.

This result should be compared with that of Favata et al. (1997), who constructed a volume-limited \( (d_{\text{max}} = 25 \) pc) sample of 91 G and K dwarfs ranging in color index \((B-V)\) between 0.5 and 0.8 (Favata et al. 1996). Their distribution has a median \( [\text{Fe/H}] = -0.05 \), and compared to this sample, the Sun is more metal-rich than 56% ± 5% of the stars. Fuhrmann (2008) compared the Sun to a volume-complete \( (d_{\text{max}} = 25 \) pc) sample of about 185 thin-disk mid-F-type to early K-type stars down to \( M_F = 6.0 \). He finds a mean \( [\text{Fe/H}] = -0.02 \pm 0.18 \). This mean \( [\text{Fe/H}] \) is lowered by 0.01 dex if the 43 double-lined spectroscopic binaries in his sample are included. His results are consistent with ours.

\subsection{2.4. Elemental Ratios \([\text{C/O}]\) and \([\text{Mg/Si}]\)}

The elemental abundance ratios of a host star have a major impact on its protoplanetary disk chemistry and the chemical compositions of its planets. Oxygen and carbon make up ~62% of the solar system’s non-hydrogen–non-helium mass content \((Z = 0.0122; \text{Asplund et al. 2005})\). Carbon and oxygen abundances are among the hardest to determine. This is due to high temperature

![Figure 2](image-url)
sensitivity and non-LTE effects in their permitted lines (e.g., C i \( \lambda 6588 \)), and to the presence of blends in the forbidden lines \((\text{[C i]} \lambda 8727, [\text{O i}] \lambda 6300)\). See Allende Prieto et al. (2001) and Bensby & Feltzing (2006) for details on C and O abundance derivations.

Carbon pairs up with oxygen to form carbon monoxide. In stars with a C/O ratio larger than 1, most of the oxygen condenses into CO, which is largely driven out of the incipient circumstellar habitable zone by the stellar wind. In this oxygen-depleted scenario, planets formed within the snow line are formed in reducing environments and are mostly composed of carbon compounds, for example, silicon carbide (Kuchner & Seager 2005). Thus, the C/O ratio could be strongly associated with habitability.

As most heavy element abundances relative to hydrogen (e.g., [O/H], [C/H], [N/H]) are correlated with [Fe/H], they were not included in our analysis. After the overall level of metallicity (represented by [Fe/H]), and after the ratio of the two most abundant metals, [C/O], the magnesium to silicon ratio [Mg/Si] is the most important ratio of the next most abundant elements (excluding the noble gas Ne). For example, [Mg/Si] sets the ratio of olivine to pyroxene, which determines the ability of a silicate mantle to retain water (H. O’Neill 2007, private communication).

Stellar rotational velocities are related to the specific angular momentum of a protoplanetary disk and possibly to the magnetic field strength of the star during planet formation, and to protoplanetary disk turbulence and mixing. An unusually low stellar rotational velocity may be associated with the presence of planets (Soderblom 1983). One or several of these factors could be related to habitability.

There is a known correlation between mass and \( v \sin i \) at higher stellar masses (e.g., see Gray 2005, Fig. 18.21). In order to minimize the effect of this correlation (and maximize independence between parameters), we assembled a sample containing 276 stars within the mass range 0.9—1.1 M\(_{\odot}\) (F8—K2) from Valenti & Fischer (2005). The selection criteria of the Valenti & Fischer (2005) stars introduces some bias against more active stars. We compared the high \( v \sin i \) tail of our Valenti & Fischer (2005) sample with the high \( v \sin i \) tail of a subsample from Nordström et al. (2004). We estimate that for our Valenti & Fischer (2005) sample, the bias introduced by the selection criteria is lower than \( \sim 5\% \). The \( v \sin i \) values in Valenti & Fischer (2005) are obtained by fixing the macroturbulence for the stars of a given color without modeling the stars individually. If the macroturbulence value was underestimated for \( T > 5800 \) K, the resulting \( v \sin i \) values (especially when \( v \sin i \) is near zero) would be overestimated (Valenti & Fischer 2005, § 4).

The inclination of the stellar rotational axis to the line of sight is usually unknown, so the observable is \( v \sin i \). Using the solar spectrum reflected by the asteroid Vesta, Valenti & Fischer (2005) derived a solar \( v \sin i = 1.63 \) km s\(^{-1}\). For the purposes of this analysis we use the mean value that would be derived for the Sun when viewed from a random inclination: \( v \sin i_0 = 1.63(\pi/4) \) km s\(^{-1}\) \( \approx 1.28 \) km s\(^{-1}\).

The Sun rotates more slowly than 83% ± 7% of the stars in our Valenti & Fischer (2005) sample (Fig. 5). This is in agreement with Soderblom (1983, 1985), who reported that the Sun is within 1 standard deviation of stars of its mass and age.

2.6. Galactic Orbital Parameters

The Galactic velocity components of a star \((U, V, W)\) with respect to the local standard of rest (LSR) may be used to compute a star’s orbit in the Galaxy. How typical or atypical is the solar orbit compared to the orbits of other nearby stars in the Galaxy? The orbit may be related to habitability because more eccentric orbits bring a star closer to the Galactic center where there is a larger danger to life from supernova explosions, cosmic...
gamma and X-ray radiation, and any factors associated with higher stellar densities (Gonzalez et al. 2001; Lineweaver et al. 2004).

For a standard model of the Galactic potential, Nordström et al. (2004) computed orbital parameters for the Sun and for a large sample (~16,700) of A5–K2 stars. Their adopted components of the solar velocity relative to the local standard of rest were $(U, V, W) = (10.0 \pm 0.4, 5.25 \pm 0.62, 7.17 \pm 0.38)$ km s$^{-1}$ (Dehnen & Binney 1998).

For each of the 1987 stars within 40 pc in the Nordström et al. (2004) catalog, inner and outer radii $R_{\text{min}}$ and $R_{\text{max}}$ were computed. This yielded the orbital eccentricity $e = (R_{\text{max}} - R_{\text{min}})/(R_{\text{min}} + R_{\text{max}})$. The solar eccentricity was computed using the components of the solar motion (Dehnen & Binney 1998) relative to the local standard of rest in the Galactic potential of Flynn et al. (1996). The bottom panel of Figure 6 shows the correlation between Galactic orbital eccentricity $e$ and the magnitude of the galactic orbital velocities with respect to the local standard.

![Figure 4](image-url)
of rest: \( r_{\text{LSR}} \equiv (U^2 + V^2 + W^2)^{1/2} \). Eccentricity \( e \) and \( r_{\text{LSR}} \) are strongly correlated. We include \( e \), not \( r_{\text{LSR}} \), in the analysis since \( e \) is less correlated with the maximum height above the Galactic plane \( Z_{\text{max}} \) than is \( r_{\text{LSR}} \). This is shown in Figure 16 in Appendix A.

The Sun's eccentricity was determined with the same relation as the stellar eccentricities. The uncertainty in our estimate of solar eccentricity came from propagating the uncertainty in the adopted solar motion. We find \( e_{\odot} = 0.036 \pm 0.002 \) (consistent with the \( e_{\odot} = 0.043 \pm 0.016 \) found by Metzger et al. 1998). The Sun has a more circular orbit than \( 93\% \pm 1\% \) of the A5–K2 stars within 40 pc (with median eccentricity \( \mu_{1/2} = 0.1 \)). This is the second most anomalous of the 11 solar properties we consider here.

The frequency of the passage of a star through the thin disk could be associated with Galactic gravitational tidal perturbations of Oort cloud objects that might increase the impact rate on potentially habitable planets. This is correlated with the maximum height, \( Z_{\text{max}} \), to which the stars rise above the Galactic plane. Figure 7 shows the stellar distribution of \( Z_{\text{max}} \) for the stars shown in Figure 6. We find that \( 59\% \pm 3\% \) of the A5–K2 stars within 40 pc of the Sun reach higher above the Galactic plane than the Sun does (\( Z_{\text{max,}\odot} = 0.104 \pm 0.006 \) kpc). The solar \( Z_{\text{max,}\odot} \) was derived by integrating the solar orbit in the Galactic potential. The uncertainty on \( W \) produces the uncertainty on \( Z_{\text{max}} \) and hence the \( \pm 3\% \) uncertainty on \( 59\% \). Our results for eccentricity and \( Z_{\text{max}} \) are consistent with those obtained using Hogg et al. (2005) LSR values: \( (U, V, W) = (10.1 \pm 0.5, 4.0 \pm 0.8, 6.7 \pm 0.2) \). Using the Hogg et al. LSR values, \( 92\% \pm 1\% \) of A5–K2 stars within 40 pc have higher eccentricities than the Sun and \( 62\% \pm 4\% \) of A5–K2 stars within 40 pc have larger \( Z_{\text{max}} \) values.

How does the Sun’s distance from the center of the Milky Way compare to the distances of other stars from the center of the Milky Way? In Figure 8 we show the distribution of the mean radial distances of stars from the Galactic center, based on the star count model of Bahcall & Soneira (1980). To represent the entire Galactic stellar population we include the disk (thin + thick) and spheroidal (bulge + halo) components. Using the current solar distance from the center \( (R_0 = 7.62 \pm 0.32 \) kpc; Eisenhauer et al. 2005) and a disk scale length \( h = 3.0 \pm 0.4 \) kpc (Gould et al. 1996), we estimate that the Sun lies farther from the Galactic center than \( 72\% \pm 5\% \) of the stars in the Galaxy. The uncertainty on
the result comes from the 68% bounds of the total distribution, which come from the scale length uncertainty (±0.4 kpc).

2.7. Host Galaxy Mass

The mass of a star's host galaxy may be correlated with parameters that have an influence on habitability. For example, galaxy mass affects the overall metallicity distribution that a star would find around itself—an effect that would not show up in Figure 3, which only shows the local metallicity distribution.

The Milky Way is more massive than 99% of all galaxies; the precise fraction depends on the lower mass limit chosen for an object to be classified as a galaxy and the behavior of the low-mass end of the galaxy mass function (Silk 2007). We are referring here to the stellar mass, not the total baryonic mass or the total mass. Despite the Milky Way's large mass compared to other galaxies, if most stars in the universe resided in even more massive galaxies, the Milky Way would be a rather low mass galaxy for a star to belong to. To estimate the fraction of all stars in galaxies of a given mass, we first estimate the distribution of galaxy masses by taking the $K$-band luminosity function of Loveday (2000; the $K$-band most closely reflects stellar mass since it is less sensitive than other bands to differences in stellar populations) and weighting it by luminosity. We convert this to stellar mass assuming a constant stellar mass-to-light ratio of 0.5 (Bell & de Jong 2001). This function, plotted in Figure 9, shows the amount of stellar mass contributed by galaxies of a given mass. Approximately 77±11% of stars live in galaxies less massive than ours. The cross-hatched band shows the 1σ uncertainty associated with the uncertainty in the two Schechter function parameters, $\alpha$ and $L^*$ (Loveday 2000; Schechter 1976). The dashed line shows the unweighted luminosity function (the number of galaxies per luminosity interval $dN_{\text{gal}}/dM$) according to which the Milky Way is more massive than ~99% of galaxies.

Fig. 7.—Distribution of maximum heights above the Galactic plane for the Nordström et al. (2004) sample. 59% ± 3% of nearby A5–K2 stars ($d_{\text{max}} = 40$ pc) reach higher above the Galactic plane than the Sun reaches. There are 22 stars evenly distributed over $Z_{\text{max}}$ between 1.5 and 9.6 kpc. Their exclusion from the comparison reduces the 59% result by less than 1%.

---

**Figure 7**

*Distribution of maximum heights above the Galactic plane for the Nordström et al. (2004) sample. 59% ± 3% of nearby A5–K2 stars ($d_{\text{max}} = 40$ pc) reach higher above the Galactic plane than the Sun reaches. There are 22 stars evenly distributed over $Z_{\text{max}}$ between 1.5 and 9.6 kpc. Their exclusion from the comparison reduces the 59% result by less than 1%.*

**Figure 8**

*Mean stellar galactocentric radius distribution $dN_{\ast}/dR_{\text{Gal}}$. The solid curve represents the sum of the disk (dashed curve) and spheroidal (dotted curve) stellar components. The 68% uncertainty of the total distribution is shown by the cross-hatched area. The Sun is farther from the Galactic center than 72±5% of the stars in the Galaxy.*

**Figure 9**

*Fraction of all stars that live in galaxies of a given mass, $dN_{\ast}/dM$ (solid curve). The mass of the Sun’s galaxy is indicated by the Sun symbol (○). This distribution represents the amount of stellar mass contributed by galaxies of a given mass. Approximately 77±11% of stars live in galaxies less massive than ours. The cross-hatched band shows the 1σ uncertainty associated with the uncertainty in the two Schechter function parameters, $\alpha$ and $L^*$ (Loveday 2000; Schechter 1976). The dashed line shows the unweighted luminosity function (the number of galaxies per luminosity interval $dN_{\text{gal}}/dM$) according to which the Milky Way is more massive than ~99% of galaxies.*
spiral galaxy from the 2 MASS Large Galaxy Atlas (Jarrett et al. 2003) and applying the color conversion from Driver et al. (1994). We then convert this to stellar mass using the same stellar mass-to-light ratio used above, i.e., 0.5. In this way we estimate the stellar mass content of the Milky Way to be $10^{10.55 \pm 0.16} = 3.6^{+1.5}_{-1.1} \times 10^{10} M_\odot$ (see also Flynn et al. 2006). Comparing this to the stellar masses of other galaxies (Fig. 9), we find that $77^{+19}_{-14}\%$ of stars reside in galaxies less massive than the Milky Way.

2.8. Host Group Mass

The mass of a star’s host galactic group or galactic cluster may be correlated with parameters that have an influence on habitability. For example, group mass is correlated with the density of the galactic environment (number of galaxies per Mpc$^3$) that could, like galactocentric radius, be associated with the dangers of high stellar densities: “The presence of a giant elliptical at a distance of 50 kpc would have disrupted the Milky Way Galaxy, so that human beings (and hence astronomers) probably would not have come into existence” (van den Bergh 2000). Our Local Group of galaxies seems rather typical (van den Bergh 2000), but we would like to quantify this. Proceeding similarly to our analysis of galaxy mass in § 2.7, we ask, what fraction of stars live in galactic groups more massive than our Local Group?

Figure 10 shows the luminosity-weighted (i.e., stellar mass-weighted) number density of galactic groups. The number distribution and luminosity distribution of galactic groups is taken from the Two-degree Field Galaxy Redshift Survey Percolation-Inferred Galaxy Group (2PIGG) catalog (Eke et al. 2004). It spans the range from weak groups to rich galaxy clusters.

We estimated the stellar masses of the 2PIGG groups and Local Group galaxies (Courteau & van den Bergh 1999) by converting from the B band assuming a constant stellar mass-to-light ratio of 1.5 (Bell & de Jong 2001). This gives an estimated stellar mass of the local group of $10^{11.09 \pm 0.07} = 8.1^{+1.4}_{-1.2} \times 10^{10} M_\odot$.

Figure 10 indicates that our Local Group is a typical galactic grouping for a star to be part of. Approximately $58\% \pm 5\%$ of stars live in galactic groups more massive than our Local Group. With respect to the mass of its galaxy and the mass of its galactic group, the Sun is a fairly typical star in the universe.

3. Joint Analysis of 11 Solar Properties

3.1. Solar $\chi^2$ Analysis

We would like to know whether the solar properties, taken as a group, are consistent with noise, i.e., are they consistent with the values of a star selected at random from our stellar distributions. We take a $\chi^2$ approach to answering this question. First we estimate the solar $\chi^2$, by adding in quadrature, for all 11 properties, the differences between the solar values and the median stellar values. We find

$$\chi^2 = \sum_{i=1}^{11} \left( \frac{x_{\odot,i} - \mu_{1/2,i}}{\sigma_{68,i}} \right)^2 = 7.88^{+0.68}_{-0.30},$$

where $i$ is the property index, $N = 11$ is the number of properties we are considering, $\mu_{1/2,i}$ is the median of the $i^{th}$ stellar distribution, and $\sigma_{68,i}$ is the difference between the median and the upper or lower 68% zone, depending on whether the solar value $x_{\odot,i}$ is above or below the median. The uncertainty on $\chi^2_\odot$ is obtained using the uncertainties of $x_{\odot,i}$.

Equation (1) can be improved on by taking into account (1) the non-Gaussian shapes of the stellar distributions and (2) the larger uncertainties of the medians of smaller samples (our smallest sample is ~100 stars).

We employ a bootstrap analysis (Efron 1979) to randomly resample data (with replacement) and derive a more accurate estimate of $\chi^2_\odot$. Because the bootstrap is a nonparametric method, the distributions need not be Gaussian.

We obtain $\chi^2_\odot = 8.39 \pm 0.96$. Figure 11 shows the resulting solar $\chi^2$ distribution. The median of this distribution is our adopted solar $\chi^2$ value. Dividing our adopted solar $\chi^2$ by the number of degrees of freedom gives our adopted reduced solar $\chi^2$ value:

$$\chi^2_\odot/11 = 0.76 \pm 0.09.$$ (2)

The standard conversion of this into a probability of finding a lower $\chi^2$ value (assuming normally distributed independent variables) yields

$$P(<\chi^2_\odot) = 8.39/N = 11) = 0.32 \pm 0.09.$$ (3)

3.2. Estimate of $P(<\chi^2_\odot)$

To quantify how typical the Sun is with respect to our 11 properties, we compare the solar $\chi^2$($=8.39$) to the distribution of $\chi^2$ values obtained from the other stars in the samples.

We perform a Monte Carlo simulation (Metropolis & Ulam 1949) to calculate an estimate of each star’s $\chi^2$ value ($\chi^2_i$). The histogram shown in Figure 12 is the resulting Monte Carlo stellar $\chi^2$ distribution. Three standard $\chi^2$ distributions have been over-plotted for comparison ($N = 10, 11, 12$). The probability of finding a star with $\chi^2$ lower than or equal to solar is

$$P_{\text{MC}}(\leq \chi^2_\odot = 8.39/N = 11) = 0.29 \pm 0.11.$$ (4)

The Monte Carlo $\chi^2$ distribution has a similar shape to the standard $\chi^2$ distribution function for $N = 11$, and thus both yield...
almost all other stars would have lower solar:

from 0 similar probabilities: \( P_{\text{MC}}(x^2 < \chi^2) = 0.29 \) to the 11 properties analyzed here) is only 29% typical (i.e., have a lower solar.

However, this preliminary low value of 0.29 indicates that if we have good reason to suspect that the Sun is not a typical star.

Table 2 summarizes our analysis for the solar \( \chi^2 \) values and the probabilities \( P(<\chi^2) \). Our simple \( \chi^2 = 7.88 \) estimate increased to 8.39, and the uncertainty increased by a factor of \( \sim 3 \) after non-Gaussian and sample size effects were included as additional sources of uncertainty. Our improved analysis yields \( P_{\text{MC}}(x^2 < \chi^2) \), with a longer tail and brings the probability down from 0.32 ± 0.09 to 0.29 ± 0.11. If this value were close to 1, almost all other stars would have lower \( \chi^2 \) values, and we would have good reason to suspect that the Sun is not a typical star.

However, this preliminary low value of 0.29 indicates that if a star is chosen at random, the probability that it will be more typical (i.e., have a lower \( \chi^2 \) value) than the Sun (with respect to the 11 properties analyzed here) is only 29% ± 11%. The details of our improved estimates of \( \chi^2 \) and \( P(<\chi^2) \) can be found in Appendix B.

4. RESULTS

Figure 13 shows four different representations of our results.

Figure 13a compares the solar values to each stellar distribution’s median and 68% and 95% zones. The Sun lies beyond the 68% zone for three properties: mass (95%), eccentricity (93%), and rotational velocity (88%). No solar property lies beyond the 95% zone. The histogram in Figure 13b is the distribution of solar values in units of standard deviations:

\[
z_i = \frac{x_{\odot,i} - \mu_{\text{1/2},i}}{\sigma_{68,i}}.
\]

For each stellar property \( i \), the Sun has a larger value than \( n_i \% \) of the stars. If the Sun were a randomly selected star, we would expect the percentages \( n_i \% \) to be scattered roughly evenly between 0% and 100%. When the \( n_i \% \) values are lined up in decreasing order (Fig. 13c), we expect them to be near the line given by

\[
n_i \text{ expected}\% = \left[ 1 - \left( \frac{i - 1/2}{N} \right) \right] \times 100\%
\]

and plotted in Figure 13c. Any anomalies would show up as Sun symbols significantly distant from the line.

Figure 13d compares the percentages \( n_i \% \) of stars having subsolar values (shown in Fig. 13c) with the solar values expressed in units of standard deviations from each distribution’s median (shown in Fig. 13b). If the stellar distributions were perfect Gaussians, the translation from \( z_i \) to \( n_i \) would be given by the cumulative Gaussian distribution (Fig. 13d, solid curve). That the points lie along this line demonstrates that the approximation of our distributions as Gaussians is reasonable.

Table 3 lists percentages \( n_i \% \) of stars for each property (as shown in Fig. 13). In the lower half of the table we list properties not included in this analysis because of correlations with properties that are included.

Individual stellar uncertainties make the observed characteristic widths (\( \sigma_{68} \), Table 1, col. [5]) larger than the widths of the intrinsic distributions. This broadening effect makes the Sun appear more typical than it really is when \( \sigma_{68} \) and the individual stellar uncertainties (\( \sigma_i \)) are of similar size and the individual stellar uncertainties are much larger than the solar uncertainty (\( \sigma_{\odot} \)). We estimate that our results are not significantly affected by this broadening effect.

Our resulting probability of finding a star with a \( \chi^2 \) lower or equal to the solar value of 29% ± 11% (eq. [4]) is consistent with the probability we would obtain if stellar multiplicity were included in our study. Using the volume-limited sample used for stellar mass in § 2.1 (125 A1–M7 stars within 7.1 pc) the
Table 2
Summary of $\chi^2$ and $P(\chi^2)$ Results

| Analysis       | $\chi^2_{\odot}$ | $\chi^2_{\odot}/11$ | $P(\chi^2_{\odot}|N=11)$ | $P_{MC}(\chi^2_{\odot}|N=11)$ |
|----------------|-------------------|----------------------|-----------------------------|---------------------------------|
| Simple         | 7.88 $^{+0.06}_{-0.08}$ (eq. [1]) | 0.72 $^{+0.03}_{-0.00}$ | 0.28 $^{+0.00}_{-0.00}$ (Eq. [B1]) | ...                             |
| Improved       | 8.39 $\pm$ 0.96    | 0.76 $\pm$ 0.09 (eq. [2]) | 0.32 $\pm$ 0.09 (eq. [3]) | 0.29 $\pm$ 0.11 (eq. [4])     |

Fig. 13.—Various representations of our main results. (A) Solar values of 11 properties compared to the distribution for each property. Each distribution’s median value is indicated by a small filled circle. The dark and light gray scale represent the 68% and 95% zones, respectively. (B) Histogram of the number of properties as a function of the number of standard deviations the solar value is from the median of that property. The gray curve is a Gaussian probability distribution normalized to 11 parameters. (C) Percentage $n_i\%$ of stars with subsolar values as a function of property. The average signal expected from a random star is shown by the solid line (see § 4). (D) Percentage $n_i\%$ of stars with subsolar values as a function of the number of standard deviations the solar value is from the median of that property. The solid curve is a cumulative Gaussian distribution—if every sample were a Gaussian distribution, every solar dot would sit exactly on the line. Just as in (C), the dashed lines encompass the 68% and 95% zones. Similar to the results from Fig. 12, these four panels indicate that the Sun is a typical star.
probability that a randomly selected star will be single is 52.8% ± 4.5%, which means that about half of stars are single while the other half have one or more companions. Including this in our bootstrap analysis and Monte Carlo simulations (see Appendix B, § B1) marginally increases the probability in equation (4) to 33% ± 11%. If the multiplicity data for 246 G dwarfs from Duquennoy & Mayor (1991) is used instead—the probability that a randomly selected G dwarf will be single is 37.8% ± 2.9%—then the probability in equation (4) would increase to 34% ± 11%. The inclusion of stellar multiplicity marginally increases our reported probability.

In Figures 6 and 7 of Radick et al. (1998) the Sun’s short-term variability as a function of average chromospheric activity appears ~1 σ low, compared to a distribution of 35 F3 – K7 Sun-like stars (Lockwood et al. 1997). Lockwood et al. (2007) suggest that the Sun’s small total irradiance variation compared to stars with similar mean chromospheric activity may be due to their limited sample and the lack of solar observations out of the Sun’s equatorial plane. We do not include short- or long-term variability (chromospheric or photometric) in Table 3 because of the small size of the Lockwood et al. (2007) sample. We also do not include the chromospheric index RV (see Table 3, bottom panel) as one of our 11 properties because of its correlation with the chromospheric ages of our sample.

5. DISCUSSION AND INTERPRETATION

The probability $P_{MC}(\chi^2 \leq \chi^2_{\text{MC}}) = 0.29 \pm 0.11$ classifies the Sun as a typical star. How robust is this result? The probability of finding a star with a $\chi^2$ lower than or equal to $\chi^2_{\text{MC}}$ depends on the properties selected for the analysis (see problem 3 in § 1). For example, if we had chosen to consider only mass and eccentricity data, this analysis would yield $P_{MC}(\chi^2 \leq \chi^2_{\text{MC}}) = 0.94 \pm 0.4$; i.e., the Sun would appear mildly (~2 σ) anomalous. If on the other hand, we had chosen to remove mass and eccentricity from the analysis, we would obtain $P_{MC}(\chi^2 \leq \chi^2_{\text{MC}}) = 0.07 \pm 0.04$, which is anomalously low. The most common cause of such a result is the overestimation of error bars. The next most common cause is the preselection of properties known to have $n\% \sim 50$.

Gustafsson (1998) discussed the atypically large solar mass and proposed an anthropic explanation—the Sun’s high mass is probably related to our own existence. He suggested that the solar mass could hardly have been greater than ~1.3 $M_\odot$ since the main-sequence lifetime of a 1.3 $M_\odot$ star is ~5 billion years (Clayton 1983). He also discussed how the dependence of the width of the circumstellar habitable zone on the host star’s mass probably favors host stars within the mass range 0.8 – 1.3 $M_\odot$.

Our property selection criteria is to have the largest number of maximally independent properties that have a plausible correlation with habitability and ones for which a representative stellar sample could be assembled. Our joint analysis does not weight any parameter more heavily than any other. If the only properties associated with habitability are mass and eccentricity, then we have diluted a ~2 σ signal that would be consistent with Gustafsson’s proposed anthropic explanation.

Our analysis points in another direction. If mass and eccentricity were the only properties associated with habitability, then the solar values for the remaining nine properties would be consistent with noise. However, a joint analysis of just the remaining nine properties produces a $\chi^2_{\text{MC}} = 3.6 \pm 0.4$ and the anomalously low probability $P(\chi^2_{\text{MC}}) = 0.07 \pm 0.04$, which suggests that the nine properties are unlikely to be the properties of a star selected at random with respect to these properties.
The χ² fit of the 11 points in Figure 13c to the diagonal line yields a fit that is substantially better then the fit of the remaining nine properties to equation (B1) with N = 9. In other words, the joint analysis suggests that although mass and eccentricity are the most anomalous solar properties, it is unlikely that they are associated with habitability, because without them it is unlikely that the remaining solar properties are just noise. Thus, the Sun, despite its mildly (~2σ) anomalous mass and eccentricity, can be considered a typical, randomly selected star.

There may be stellar properties crucial for life that were not tested here. If we have left out the most important properties, with respect to which the Sun is atypical, then our Sun-is-typical conclusion will not be valid. If we have sampled all properties associated with habitability, our Sun-is-typical result suggests that there are no special requirements on a star for it to be able to host a planet with life.

6. CONCLUSIONS

We have compared the Sun to representative stellar samples for 11 properties. Our main results are as follows:

1. Stellar mass and Galactic orbital eccentricity are the most anomalous properties. The Sun is more massive than 95% ± 2% of nearby stars and has a Galactic orbital eccentricity lower than 93% ± 1% FGK stars within 40 pc.
2. Our joint bootstrap analysis yields a solar χ² of χ² = 8.39 ± 0.96 and a solar reduced χ² of χ²/11 = 0.76 ± 0.09. The probability of finding a star with a χ² lower than or equal to solar $P_{MC}(\leq \chi_0^2) = 8.39 \pm 0.96) = 0.29 \pm 0.11$.

To our knowledge, this is the most comprehensive and quantitative comparison of the Sun with other stars. We find that taking all 11 properties together, the Sun is a typical star. This finding is largely in agreement with Gustafsson (1998); however, our results undermine the proposition that an anthropic explanation is needed for the comparatively large mass of the Sun.

Further work could encompass the inclusion of other properties potentially associated with habitability. Another improvement would come when larger stellar samples become available for which all properties could be derived, instead of using different samples for different properties as was done here. In addition, research in the molecular evolution that led to the origin of life may, in the future, be able to provide more clues as to which stellar properties might be associated with our existence on Earth, orbiting the Sun.

We would like to thank Charles Jenkins for clarifying discussions of statistics, particularly on how to include stellar multiplicity, and Martin Asplund and Jorge Meléndez for discussions of elemental abundances. J. A. R. acknowledges an RSAA Ph.D. research scholarship. M. P. acknowledges the financial support of the Australian Research Council. E. G. acknowledges the financial support of the Finnish Cultural Foundation.

APPENDIX A

PROPERTY CORRELATIONS

The χ² formalism and the use of the χ² distribution to obtain $P(<\chi_0^2|N)$—improved using Monte Carlo simulations in § 3.2 to obtain $P_{MC}(\leq \chi_0^2)$—assumes that each parameter is independent of the others. In selecting our 11 properties we have selected properties that are maximally independent based on plotting property 1 versus property 2 for the same stars. We show seven such plots in this appendix.

If there are correlations between the analyzed properties, then the number of degrees of freedom N could drop from 11 to ~10.5 (see Fig. 12). Some properties have been excluded from the analysis due to a correlation with another property in the analysis.

A1. ELEMENTAL RATIOS

In Figure 14 we show the distribution for carbon to oxygen ratio [C/O] versus the magnesium to silicon ratio [Mg/Si] of 176 FG stars.

A2. MASS, AGE, AND ROTATIONAL VELOCITY

In Figure 15 we show four correlation plots for mass, chromospheric age, rotational velocity, and $v \sin i$. We use the stars common to both Wright et al. (2004) and Valenti & Fischer (2005) for which these observables are available.

A3. GALACTIC ORBITAL PARAMETERS

In Figure 16 the Galactic orbital eccentricity (e) and the magnitude of the galactic orbital velocities with respect to the local standard of rest ($v_{LSR}$) are strongly correlated (see Fig. 6). We selected e instead of $v_{LSR}$ because of its near independence of the maximum height above the galactic plane ($Z_{max}$).

APPENDIX B

IMPROVED ESTIMATES OF $\chi_0^2$ AND $P(<\chi_0^2)$

In § 3.2, with 11 degrees of freedom, the reduced χ² from equation (4) is $\chi_0^2/11 = 0.72^{+0.01}_{-0.03}$. Since $\chi_0^2/11 < 1$, the Sun’s properties are consistent with the Sun being a randomly selected star.

To improve on this preliminary analysis (but with a similar conclusion), as mentioned in § 3.2, we employ a bootstrap analysis (Efron 1979) to randomly resample data (with replacement) and derive a more accurate estimate of $\chi_0^2$. Because the bootstrap is a nonparametric method, the distributions need not be Gaussian.
For every iteration, each parameter's stellar distribution is randomly resampled and a $\chi^2$ value is calculated using equation (1). The uncertainties $\sigma_{\odot,i}$ of the solar values $x_{\odot,i}$ are also included in the bootstrap method; for every iteration, the solar value for each parameter is replaced in equation (1) by a randomly selected value from a normal distribution with median $\mu_{1/2,i} = x_{\odot,i}$ and standard deviation $\sigma_{\odot,i}$. The process was iterated 100,000 times, although the resulting distribution varies very little once the number of iterations reaches $\sim 10,000$.

The median of this distribution and the error on the median yields our improved value for the reduced $\chi^2$ (Fig. 11). The uncertainty of the median of each resampled distribution varies inversely proportionally to the square root of the number of stars in the distribution, $\Delta \mu_{1/2,i} \propto 1/(N_i)^{1/2}$. In other words, median values are less certain for smaller samples, and this uncertainty is included in our improved estimate of $\chi^2$ and its uncertainty.

We find the probability of finding a star with a $\chi^2$ value lower than the solar $\chi^2$ for $N = 11$ degrees of freedom in the standard way (Press et al. 1992) and obtain

$$P(<\chi^2_{\odot}) = 7.88^{+0.08}_{-0.06}(11) = 0.28^{+0.01}_{-0.03}. \quad (B1)$$

To improve our estimate of the probability of finding a star with lower $\chi^2$ value than the Sun, we perform a Monte Carlo simulation (Metropolis & Ulam 1949) to calculate an estimate of each star's $\chi^2$ value ($\chi^2_i$). For every iteration, we randomly select a star from each stellar distribution. We then calculate its $\chi^2_i$ value by replacing the solar value $x_{\odot,i}$ with that star's value $x_{*,i}$ in equation (1). This process was repeated 100,000 times to create our Monte Carlo stellar $\chi^2$ distribution. Stars were randomly selected with replacement; thus, the simulated $\chi^2$ distribution accounts for small number statistics and non-Gaussian distributions. The probability of finding a star with $\chi^2$ lower than or equal to solar is $P_{MC} = 0.29 \pm 0.11$.

The results of our analysis for the solar $\chi^2_{\odot}$ values and the probabilities $P(<\chi^2_{\odot})$ are summarized in Table 2.

### B1. ADDITION OF A DISCRETE PARAMETER

In § 4 we discuss the addition of stellar multiplicity to our analysis. Since stellar multiplicity cannot easily be approximated by a one-sided Gaussian (particularly because the Sun is on the edge of the distribution, i.e., it is of multiplicity one), we modified our Monte Carlo procedure to include this discrete parameter. The likelihood of observing a particular $\chi^2$ for the 11 parameters is

$$\exp \left( -\frac{1}{2} \sum_{i=1}^{11} \chi^2_i \right). \quad \text{(B2)}$$

We take the probability $p(1)$ of a star being a single star to be $53.8\% \pm 4.5\%$, obtained from our sample of nearby stars (§ 2.1). The likelihood $L$ of observing a particular $\chi^2$ and $p(1)$ is the product

$$L = p(1) \exp \left( -\frac{1}{2} \sum_{i=1}^{11} \chi^2_i \right). \quad \text{(B3)}$$
Taking logarithms we can then compute the distribution of the statistic $S$, where

$$S = \ln p(1) - \frac{1}{2} \sum_{i=1}^{11} \chi_i^2.$$  \hfill (B4)

The distribution of $S$ allows us to obtain the results for the multiplicity reported at the end of § 4.
Allende Prieto, C., Lambert, D. L., & Asplund, M. 2001, ApJ, 556, L63
Asplund, M., Grevesse, N., & Sauval, A. J. 2005, in ASP Conf. Ser. 336, Cosmic Abundances as Records of Stellar Evolution and Nucleosynthesis, ed. T. G. Barnes III & F. N. Bash (San Francisco: ASP), 25
Bahcall, J. N., & Soneira, R. M. 1980, ApJS, 44, 73
Bell, E. F., & de Jong, R. S. 2001, ApJ, 550, 212
Bensby, T., & Feltzing, S. 2006, MNRAS, 367, 1181
Bertelli, G., & Nasi, E. 2001, AJ, 121, 1013
Carter, B. 1983, Philos. Trans. R. Soc. London A, 310, 347
Clayton, D. D. 1983, Principles of Stellar Evolution and Nucleosynthesis (Chicago: Univ. Chicago Press)
Courteau, S., & van den Bergh, S. 1999, AJ, 118, 337
Dehnen, W., & Binney, J. J. 1998, MNRAS, 298, 387
Driver, S. P., Phillipps, S., Davies, J. I., Morgan, I., & Disney, M. J. 1994, MNRAS, 268, 393
Duquennoy, A., & Mayor, M. 1991, A&A, 248, 485
Edvardsson, B., Andersen, J., Gustafsson, B., Lambert, D. L., Nissen, P. E., & Tomkin, J. 1993a, A&A, 275, 101
———. 1993b, A&AS, 102, 603
Efron, B. 1979, Ann. Stat., 7, 1
Eisenhauer, F., et al. 2004, ApJ, 628, 246
Eke, V. R., et al. 2004, MNRAS, 355, 769
Favata, F., Micela, G., & Sciortino, S. 1996, A&A, 311, 951
———. 1997, A&A, 323, 809
Feltzing, S., Holmberg, J., & Hurley, J. R. 2001, A&A, 377, 911
Flynn, C., Holmberg, J., Portinari, L., Fuchs, B., & Jahreiβ, H. 2006, MNRAS, 372, 1149
Flynn, C., Sommer-Larsen, J., & Christensen, P. R. 1996, MNRAS, 281, 1027
Fuhrmann, K. 2008, MNRAS, 384, 173
Gonzalez, G. 1999a, MNRAS, 308, 447
———. 1999b, Astron. Geophys., 40, 25
Gonzalez, G., Brownlee, D., & Ward, P. 2001, Icarus, 152, 185
Gould, A., Bahcall, J. N., & Flynn, C. 1996, ApJ, 465, 759
Gray, D. F. 2005, The Observation and Analysis of Stellar Photospheres (Cambridge: Cambridge Univ. Press)
Grether, D., & Lineweaver, C. H. 2006, ApJ, 640, 1051
———. 2007, ApJ, 669, 1220
Gustafsson, B. 1998, Space Sci. Rev., 85, 419
Gustafsson, B., Carlsson, M., Olsorn, E., Edvardsson, B., & Ryde, N. 1999, A&A, 342, 426
Hernandez, X., Valls-Gabaud, D., & Gilmore, G. 2000, MNRAS, 316, 605
Hogg, D. W., Blanton, M. R., Roweis, S. T., & Johnston, K. V. 2005, ApJ, 629, 268
Hopkins, A. M., & Beacom, J. F. 2006, ApJ, 651, 142
Jarrett, T. H., Chester, T., Cutri, R., Schneider, S. E., & Huchra, J. P. 2003, AJ, 125, 525
Kasting, J. F., Whitmire, D. P., & Reynolds, R. T. 1993, Icarus, 101, 108
Kroupa, P. 2002, Science, 295, 82
Kroupa, P., & Weidner, C. 2005, in The Initial Mass Function 50 Years Later, ed. E. Corbelli, F. Palla, & H. Zinnecker (ASL Vol. 327; Dordrecht: Springer), 175
Kuchner, M. J., & Seager, S. 2005, preprint (astro-ph/0504214)
Lineweaver, C. H., Fenner, Y., & Gibson, B. K. 2004, Science, 303, 59
Lockwood, G. W., Skiff, B. A., Henry, G. W., Henry, S., Radick, R. R., Balunias, S. L., Donahue, R. A., & Soon, W. 2007, ApJS, 171, 260
Lockwood, G. W., Skiff, B. A., & Radick, R. R. 1997, ApJ, 485, 789
Loveday, J. 2000, MNRAS, 312, 557
Metropolis, N., & Ulam, S. 1949, J. Am. Stat. Assoc., 44, 335
Meyers, S. I. 1975, Data Analysis for Scientists and Engineers (New York: Wiley)
Nordström, B., et al. 2004, A&A, 418, 989
Press, W. H., Teukolsky, S. A., Vetterling, W. T., & Flannery, B. P. 1992, Numerical Recipes in FORTRAN: The Art of Scientific Computing (2nd ed.; Cambridge: Cambridge Univ. Press)
Radick, R. R., Lockwood, G. W., Skiff, B. A., & Balunias, S. L. 1998, ApJS, 118, 239
Ramírez, I., Allende Prieto, C., & Lambert, D. L. 2007, A&A, 465, 271
Reddy, B. E., Tomkin, J., Lambert, D. L., & Allende Prieto, C. 2003, MNRAS, 340, 304
Reid, I. N. 2002, PASP, 114, 306
Reid, I. N., Turner, E. L., Turnball, M. C., Mountain, M., & Valenti, J. A. 2007, ApJ, 665, 767
Rocha-Pinto, H. J., Maciel, W. J., Scalo, J., & Flynn, C. 2000a, A&AS, 135, 850
Rocha-Pinto, H. J., Scalo, J., Maciel, W. J., & Flynn, C. 2000b, ApJ, 531, L115
Schechter, P. 1976, ApJ, 203, 297
Silk, J. 2007, Astron. Geophys., 48, 30
Soderblom, D. R. 1983, ApJ, 263, 525
van den Bergh, S. 2000, The Galaxies of the Local Group (Cambridge: Cambridge Univ. Press)
Wright, J. T., Marcy, G. W., Butler, R. P., & Vogt, S. S. 2004, ApJ, 152, 261

Fig. 16.—Left: Galactic orbital eccentricity $e$ vs. $Z_{\text{max}}$ for 1987 FGK stars within 40 pc (Nordström et al. 2004). The orbital eccentricity is not correlated with $Z_{\text{max}}$. Right: $v_{\text{LSR}}$ vs. $Z_{\text{max}}$ for the same stars. Because $v_{\text{LSR}}$ is more strongly correlated with $Z_{\text{max}}$ than eccentricity, eccentricity has been selected for the joint analysis instead of $v_{\text{LSR}}$. As in Fig. 4, the contours correspond to 38%, 68%, 82%, and 95%.