Novel Properties of Twisted-Photon Absorption

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We discuss novel features of twisted-photon absorption both by atoms and by micro-particles. First, we extend the treatment of atomic photoexcitation by twisted photons to include atomic recoil, derive generalized quantum selection rules and consider phenomena of forbidden atomic transitions. Second, we analyze the radiation pressure from twisted-photon beams on micro-sized particles and observe that for particular conditions the pressure is negative in a small area near the beam axis. A part of the beam therefore acts as a “tractor beam”.

1. Introduction

In 1992 Allen and collaborators [1] pointed out that Laguerre-Gaussian laser modes may carry large projections of angular momentum on the direction of beam propagation. It became a very active field of studies, with many applications of optical vortices in optical tweezers, encoding of information, and microscopy. Dedicated recent reviews can be found in Ref. 2, 3. A term “twisted photon” is used to describe such beams at a quantum level 4. Optical vortices were successfully demonstrated in a broad range of photon energies, from radio-waves 5 to X-rays 6. It was also suggested to use Compton backscattering 7, 8 to obtain high-energy photons that may be relevant for nuclear and particle physics research.

To analyze the unique properties of the twisted photons, we considered their absorption by an atom. This problem was previously addressed by several authors 9–12. Most of the above papers considered an atom placed in the center of an optical vortex. Atomic ionization in a more general case was addressed in Ref. 13.

In our recent paper on photoexcitation of atoms 14 we made a (well-justified) approximation that neglected atomic recoil at the cost of apparent non-conservation of total angular momentum. Here we derive quantum selection rules that describe atomic transitions caused by twisted photons, with effects of atomic recoil included.

In addition, this paper focuses on two novel phenomena that twisted photons can produce. In both cases we look at the low end of the angular momentum projection mγ, possibilities, namely mγ = 0 and mγ = −Λ, where Λ = ±1 is the usual helicity of the photon. These possibilities cannot occur for plane wave photons and are in the twisted photon realm.

Atomic reactions induced by mγ = 0 photons can produce final states with quantum numbers forbidden to plane wave photon reactions. For example, one can photoexcite ground state hydrogen by E1 transitions into P-state orbitals with magnetic quantum number mℓ equal zero (using the incoming wavefront’s propagation direction as the quantization axis). Plane wave photons produce mℓ = Λ = ±1. As a practical matter, there are factors associated with the way the twisted photon is composed that can suppress the rate, even though the E1 tradition itself is leading order. However, depending on circumstances, the rate could be large enough to be observable. Details, along with background material on twisted photons and the discussion of atomic recoil effects, are given in Sec. 2.

An examination of the Poynting vector for twisted photons shows regions, notably along the beam axis, where the component along the beam direction is directed back to the source. Hence radiation pressure will make it appear that objects in these regions are attracted to the source: there is a tractor beam. It can be interpreted as a “boomerang effect,” so that twisted photons localized near the beam axis appear to be propagating backwards. Tractor beam details are given in Sec. 3 using twisted photons with mγ = −Λ as a prime example that shows the largest tractor effect, and including a discussion of the size of the tractor region and the strength of the tractor pressure compared to the repulsive pressure in adjacent regions.

2. Atomic excitation by twisted photons

2.A. Atomic states

To establish notation, let the electron coordinate be r1 and the proton or nucleus coordinate be r2. The relative and center of mass (CM) coordinates are

\[ \vec{r} = \vec{r}_1 - \vec{r}_2, \]

\[ \vec{R} = \frac{1}{M} (m_1 \vec{r}_1 + m_2 \vec{r}_2), \]

(1)

for \( M = m_1 + m_2 = m_e + m_p \). Likewise,

\[ \vec{r}_1 = \vec{R} + \frac{m_2}{M} \vec{r}_2, \]

\[ \vec{r}_2 = \vec{R} - \frac{m_1}{M} \vec{r}_1. \]

(2)
Nonrelativistically, since the atomic binding potential depends only on $\vec{r}$, the wave function for the atomic state naturally separates into a product

$$
\Psi(\vec{r}, \vec{R}) = \psi(\vec{r}) \Psi(\vec{R}) .
$$

(3)

2.B. Twisted photon expansion

To define the twisted photons, we follow [7, 8], whose states may be viewed as extensions of the nondiffractive Bessel modes described in [13, 10]. More detail is given in [12]; we give a short and hopefully sufficient summary here.

A twisted photon state moving in the $z$-direction, with helicity $m_\gamma$ and with symmetry axis passing through the origin, is in Hilbert space given by

$$
|\kappa m_\gamma k_z \Lambda\rangle = \sqrt{\frac{\kappa}{2\pi}} \int \frac{d\phi_k}{2\pi} (-i)^{m_\gamma} e^{im_\gamma \phi_k} |\vec{k}, \Lambda\rangle .
$$

(4)

The component states on the right are plane wave states, all with the same longitudinal momentum $k_z$, the same transverse momentum magnitude $\kappa = |\vec{k}_z|$, and same helicity $\Lambda$ (in the directions $\vec{k}$). The azimuthal angle $\phi_k$ varies, and with the phasing shown, $m_\gamma$ is the total angular momentum in the $z$ direction, with the possibility that $m_\gamma \gg h$. We also define a pitch angle $\theta_k = \arctan(\kappa/k_z)$, and $\omega = |\vec{k}|$.

The electromagnetic potential of the twisted photon in coordinate space, at the location of the electron is

$$
A_{\kappa m_\gamma k_z \Lambda}(t, \vec{r}_1) = \sqrt{\frac{\kappa}{2\pi}} e^{-i\omega t} e^{\frac{\Lambda}{\sqrt{2}} \sin \theta_k \mu} \int \frac{d\phi_k}{2\pi} (-i)^{m_\gamma} e^{im_\gamma \phi_k} \Psi^\mu_{\kappa \Lambda} e^{i\vec{k} \cdot \vec{r}_1} .
$$

(5)

The polarization vectors are

$$
e^\mu_{\kappa \Lambda} = e^{-i\Lambda \phi_k} e^{\frac{\theta_k}{2} \eta^{\mu}_+} + e^{i\Lambda \phi_k} e^{\frac{\theta_k}{2} \eta^{\mu}_-} + \frac{\Lambda}{\sqrt{2}} \sin \theta_k \mu
$$

(6)

with

$$
\eta^{\mu}_{\pm 1} = \frac{1}{\sqrt{2}} (0, \mp 1, -i, 0) , \quad \eta^{\mu}_0 = (0, 0, 0, 1) .
$$

(7)

It will be useful to expand the potential using cylindrical coordinates and the Jacobi-Anger formula, wherein

$$
e^{\vec{k}_\perp \cdot \vec{r}_\perp} = e^{i\vec{k}_\perp \cdot \vec{R}_\perp} e^{i\kappa (m_p/M) \vec{r}_\perp} = \sum_{n_1, n_2 = -\infty}^{\infty} e^{i(n_1 \phi_R - n_2 \phi_F)} e^{in_2 (\phi_F - \phi_R)}
$$

$$
\times J_{n_1}(\kappa R_\perp) J_{n_2}(\frac{m_p}{M} \kappa R_\perp) ,
$$

(8)

where $R_\perp = |\vec{R}_\perp|$, $r_\perp = |\vec{r}_\perp|$, and the azimuthal angles $\phi_R$ and $\phi_F$ are indicated in Fig. [1]. After some manip-
With two different momenta, proton and electron, and with each momentum becoming a derivative that acts on both CM and relative coordinates, one has four contributions to the above amplitude. We will explicitly work out the formulas for the electron case, which should be by far the largest contributions, and consider the derivatives on the CM and relative momenta serially. For the electron momentum,

\[
\vec{p}_e = \frac{m_e}{M} \vec{p} + \vec{p} = -i \frac{m_e}{M} \vec{p}_e R - i \vec{\nabla}_r \tag{12}
\]

Again because of the mass factor, the second term should be by far the larger, and we consider it first.

Also, we will put the initial atom in the ground state, \(\psi_i(\vec{r}) = \psi_{n_i,m_i}(\vec{r}) \rightarrow \psi_{100}(\vec{r}) = R_{10}(r)Y_{00}(\theta, \phi)\).

2.D. Relative momentum contribution

For this subsection we keep just the electron’s \(-i \vec{\nabla}_r\) contribution, and using

\[
-i \eta_\lambda \vec{\nabla}_r R_{10}(r)Y_{00} = -i \sqrt{\frac{1}{3}} Y_{1\lambda}(\theta,0)e^{i\lambda \phi} R'_{10}(r) \tag{13}
\]

for \(\lambda = \pm 1, 0\), we obtain after doing the d\(\phi\), integration,

\[
iM_{e,rel} = -\frac{e \Lambda}{m_e a_0} \sqrt{\frac{2\pi \kappa}{3}} G_{FI} \times \left\{ \cos^2 \frac{\theta_k}{2} g_{f\lambda} + \sqrt{\frac{i}{2}} \sin \theta_k g_{f0} - \sin^2 \frac{\theta_k}{2} g_{f,-\lambda} \right\}. \tag{14}
\]

Here,

\[
G_{FI} = \int d^3R \Psi^*_F(\vec{R}) \Psi_I(\vec{R}) \times e^{i(m_{\gamma'}-m_f)\phi_R} J_{m_{\gamma'}-m_f}(\kappa R \perp) e^{ik_z Z}. \tag{15}
\]

Further, the relative coordinate integrals over the atomic factors \(g_{f\lambda} = g_{n_f m_f \lambda}\) as in [13], up to some near unity factors \((m_p/M)_R\),

\[
g_{f\lambda} = -a_0 \int_0^\infty r^2dr R_{n_f m_f}(r)R'_{10}(r) \int_0^1 d(cos \theta_r) \times J_{m_{\gamma'}-\lambda}(\frac{m_p}{M} \kappa R \perp) Y_{1\lambda}(\theta_r,0) Y_{1\lambda}(\theta,0)e^{i(m_p/M)k_z z}. \tag{16}
\]

We consider two special cases for the CM wave function. If the CM wave function is well localized, we may center it in the \(Z = 0\) plane at a definite distance and azimuth angle from the photon axis, and approximate the unit normalized state by

\[
\Psi_I(\vec{R}) = \frac{1}{R} \delta^{1/2}(R-b)\delta^{1/2}(\cos \theta_R)\delta^{1/2}(\phi_R - \phi_b \pm \pi), \tag{17}
\]

where the square roots of the delta functions can be defined by a limiting procedure. If in addition we neglect recoil and use the same wave function for the final state, we obtain,

\[
iM_{e,rel} = -\frac{e \Lambda}{m_e a_0} \sqrt{\frac{2\pi \kappa}{3}} e^{i(m_{\gamma'}-m_f)\phi_R} J_{m_{\gamma'}-m_f}(\kappa R_b) \times \left\{ \cos^2 \frac{\theta_k}{2} g_{n_f m_f \lambda} + \frac{i}{\sqrt{2}} \sin \theta_k g_{n_f m_f 0} - \sin^2 \frac{\theta_k}{2} g_{n_f m_f,-\lambda} \right\}. \tag{18}
\]

in agreement with [14].

If the initial and final CM wave functions have definite eigenvalues of the angular momentum operator \(L_z\) [12], then we can consider transitions between CM states,

\[
\Psi_I(\vec{R}) = \frac{e^{im_R \phi}}{\sqrt{2\pi}} Y_I(R, \cos \theta_R),
\]

\[
\Psi_F(\vec{R}) = \frac{e^{im'_{\gamma} \phi}}{\sqrt{2\pi}} Y_{I'}(R, \cos \theta_R). \tag{19}
\]

This gives,

\[
iM_{e,rel} = \frac{e \Lambda}{m_e a_0} \sqrt{\frac{2\pi \kappa}{3}} \delta_{m_{\gamma'}-m_{\gamma},m_{\gamma'}-m_f} G_{FI} \times \left\{ \cos^2 \frac{\theta_k}{2} g_{f\lambda} + \frac{i}{\sqrt{2}} \sin \theta_k g_{f0} - \sin^2 \frac{\theta_k}{2} g_{f,-\lambda} \right\}, \tag{20}
\]

with,

\[
G_{FI} = \int_0^\infty R^2 dR \int_0^1 d(cos \theta_R) \times Y_{I'}^*(R, \cos \theta_R)Y_I(R, \cos \theta_R)J_{m_{\gamma'}-m_{\gamma}}(\kappa R \perp) e^{ik_z Z}. \tag{21}
\]

Even more particularly, if the \(Y\) wave functions are well localized in radius and angle, \(G_{FI} = J_{m_{\gamma'}-m_{\gamma}}(\kappa b)\), and the amplitude (20) has the same magnitude as (18).

Eq. (20) contains a \(\delta\)-function showing conservation of angular momentum along the \(z\)-direction. Whatever part of the photon’s helicity that does not go into exciting the atomic wave function goes into making the atom as a whole revolve about the photon’s symmetry axis.

If the initial CM wave function is centered in the vicinity of some definite azimuthal point, as in Fig. [1] then the initial wave function could be expanded as a sum of \(L_z\) eigenfunctions over a range of \(m_R\). We can work, as we do here, with the individual terms in such a sum, noting that if the initial wave function is not revolving about the origin, then the expectation value \(\langle m_R \rangle = 0\). In general,

\[
\langle m'_R \rangle = \langle m_R \rangle + m_\gamma - m_f. \tag{22}
\]

Jáuregui has obtained similar results [12]. The methods used here lead to results that appear qualitatively simpler and more compact, and without lingering summations in the final results, for the CM wave functions displayed. No dipole approximation has been used, so that excitation of highly excited atomic states can be accurately calculated.
2.E. CM derivative contribution

Continuing to work out the case where the electron absorbs the photon, we now consider the \( \nabla_R \) term in \( p_1 = -i(m_e/M)\nabla_R - i\nabla_\tau \). We have

\[
i \mathcal{M}_{e,R} = \frac{ie_1}{M} \int d^3r \, R_{\alpha I}(r) Y_{\alpha m_f} (\theta, \phi, 0) e^{-im_f \phi - i} \times R_{10}(r) Y_{\alpha 0}(0, \bar{r}_1) \nabla_R \Psi_I(\bar{R}). \tag{23}\]

We again expand the CM wave functions as, for the initial state, \( \Psi_I(\bar{R}) = e^{i(m_f \phi + \Theta_\tau)\phi} \partial_{R \lambda} \bar{Y}_I(R_\perp, Z) \), and let

\[
\hat{e}_\lambda \nabla_R \Psi_I(\bar{R}) = e^{i(\lambda + m_f \phi)\phi} \partial_{R \lambda} \bar{Y}_I(R_\perp, Z), \tag{24}\]

where

\[
\partial_{R \lambda} \bar{Y}_I = \left\{ \begin{array}{cl}
\frac{1}{\sqrt{2}} \left( -\Lambda_\lambda \frac{\partial}{\partial R_\perp} + \frac{m_f}{R_\perp} \right) \bar{Y}_I, & \lambda = \Lambda = \pm 1, \\
\frac{\partial}{\partial Z} \bar{Y}_I, & \lambda = 0.
\end{array} \right. \tag{25}\]

After performing the \( d\phi_R \) and \( d\phi_\tau \) integrals, one obtains

\[
i \mathcal{M}_{e,R} = \frac{2\pi e\Lambda}{M a_0} \sqrt{\frac{k}{2}} \delta_{m_R - m_f, m_\tau} g_{f_i} \cos \theta_k G_{fi,\Lambda} + \frac{i}{\sqrt{2}} \sin \theta_k G_{fi,0} - \sin^2 \frac{\theta_k}{2} G_{fi, -\Lambda}\right\}, \tag{26}\]

This time,

\[
g_{f_i} = \int_0^\infty r^2 dr \, R_{a f}(r) R_{10}(r) \int_0^1 d(\cos \theta_r) \times J_{m_f} \left( \frac{m_p}{M} \kappa r_\perp \right) Y_{a m_f}(\theta_r, 0) e^{-i(m_p/M)k_r} \tag{27}\]

and

\[
G_{fi,\Lambda} = -a_0 \int_0^\infty R_\perp dR_\perp \int_0^{\infty} dZ \times \bar{Y}_I^*(R_\perp, Z) J_{m_R - m_f - \Lambda} (\kappa R_\perp) e^{ik_r Z} \partial_{R \lambda} \bar{Y}_I(R_\perp, Z). \tag{28}\]

In general, unless the CM state is extremely localized, so that the derivatives of the \( \bar{Y}_I \) are large, the contributions coming from the derivative acting on the CM coordinate, \( \mathcal{M}_{e,R} \), are small compared to the term where the derivative acts on the relative coordinate \( \mathcal{M}_{e,rel} \).

The cases where the proton absorbs the photon can be similarly written down, and will be numerically small because of the mass factors.

All the photoproduction amplitudes display a \( \delta \)-function that explicitly shows conservation of angular momentum along the \( z \)-direction, assuring us that the initial helicity either goes into excitation of the atomic state, or into revolution of the entire atom about the origin defined by the twisted photon’s symmetry axis.

2.F. 1S → 2P atomic photoexcitation

For ordinary plane wave photons with helicity \( \Lambda \), a photoexcited 2P state necessarily has \( m_f = \Lambda \), if we start from the ground state. For a twisted photon initial state, other final quantum numbers (QN) are possible. In particular, for a twisted photon with \( m_\gamma = 0 \), the zero angular momentum projection final state may be produced with sufficient amplitude to be observable.

We will take the approximation that amplitude \( \mathcal{M}_{e,rel} \) is most of the amplitude and use the notation \( \mathcal{M}_{e,rel}^{(m_\gamma)} \), to keep track of the transitions from and to different QN states. We calculate amplitudes from Eq. (18) or from the localized version of the definite \( L_Z \) states mentioned after Eq. (21).

We take \( \Lambda = 1 \) for definiteness. For \( m_\gamma = 0 \), examine Eq. (18), we can produce only the \( m_f = 0 \) final state if the target atom is on the photon axis, and progressively produce more of the \( m_f = 1 \) final state when the target atom is off the axis. The largest of the atomic factors for the \( m_f = 0 \) is the one labeled \( g_{2100} \), and similarly for \( m_f = 1 \) the largest atomic factor is \( g_{2111} \). In the dipole approximation, \( k \ll a_0 \), these are the same, either by the Wigner-Eckart theorem or by direct examination,

\[
g_{21\lambda \lambda} = \frac{1}{2\pi} \int_0^{\infty} r^2 dr R_{21}(r) R_{10}(r) = \frac{1}{\pi} \left( \frac{2}{3} \right)^{7/2}. \tag{29}\]

Hence the main difference between the peak values of the amplitudes for \( m_f = 0 \) and \( m_f = 1 \) final states produced from the special \( m_\gamma = 0 \) twisted photon comes because the pitch angle \( \theta_k \) factors are different in the different terms of the overall amplitude, Eq. (18).

Fig. 2 plots the absolute values of the amplitudes for the 1S → 2P transitions for \( m_\gamma = 0 \) and the different \( m_f \) as a function of the transverse distance of the target from the photon axis (measured in photon wavelengths), for a pitch angle \( \theta_k = 0.2 \) radians.

![Fig. 2. 1S → 2P photexcitation amplitudes for \( m_\gamma = 0 \) twisted photons, with pitch angle \( \theta_k = 0.2 \) radians.](image-url)
Fig. 3 for the $m_\gamma = 1$ twisted photon, which has essentially the same peak amplitudes as the plane wave case. The magnitudes are not significantly larger than the peaks in the $m_\gamma = 0$ case.

![Graph](image)

Fig. 3. 1S → 2P photexcitation amplitudes for $m_\gamma = 1$ twisted photons, with pitch angle $\theta_k = 0.2$ radians.

### 3. Tractors Property of Twisted-Photon Beams

In this section, rather than considering interactions with single atoms, we will consider the effect of a twisted photon beam upon small objects, dust grains perhaps, that the beam shines upon. We will notice that for certain choices of the quantum numbers of the twisted photon beam, the longitudinal component of the Poynting vector along the beam axis points back to the source, i.e., a well aimed and well produced twisted photon becomes a tractor beam. We will also give an example to see how strong the tractor beam is and how large the tractor region can be.

#### 3.A. Poynting vector for twisted photons

The coordinate space expression for the vector potential of a twisted photon moving in the z-direction, following the normalization and other conventions of Serbo and Jentschura [7, 8], is in general

\[
\mathcal{A}_{\kappa m_\gamma, k\Lambda}^\mu(x) = -i\alpha e^{-i(\omega t - k_z z)} \sqrt{\frac{\kappa}{2\pi}} \\
\times \left\{ e^{i(m_\gamma - \Lambda)\phi} \cos^2 \frac{\theta_k}{2} J_{m_\gamma - \Lambda}(\kappa \rho) \eta_\Lambda^\mu \\
- e^{i(m_\gamma + \Lambda)\phi} \sin^2 \frac{\theta_k}{2} J_{m_\gamma + \Lambda}(\kappa \rho) \eta_{-\Lambda}^\mu \\
+ \frac{i}{\sqrt{2}} e^{im_\gamma \phi} \sin \theta_k J_{m_\gamma}(\kappa \rho) \eta_0^\mu \right\}. \tag{30}
\]

In cylindrical coordinates this is

\[
\mathcal{A}_{\kappa m_\gamma, k\Lambda}(x) = e^{-i(\omega t - k_z z - m_\gamma \phi)} \sqrt{\frac{\kappa}{4\pi}} \\
\times \left\{ i\hat{\rho} \left[ \cos^2 \frac{\theta_k}{2} J_{m_\gamma - \Lambda}(\kappa \rho) + \sin^2 \frac{\theta_k}{2} J_{m_\gamma + \Lambda}(\kappa \rho) \right] \\
- \Lambda \hat{\phi} \left[ \cos^2 \frac{\theta_k}{2} J_{m_\gamma - \Lambda}(\kappa \rho) - \sin^2 \frac{\theta_k}{2} J_{m_\gamma + \Lambda}(\kappa \rho) \right] \\
+ \Lambda \hat{z} \sin \theta_k J_{m_\gamma}(\kappa \rho) \right\}. \tag{31}
\]

where $\hat{\rho}$, $\hat{\phi}$, and $\hat{z}$ are unit vectors in the appropriate directions.

The electric field is given by $\vec{E} = -\partial \vec{A}/\partial t = i\omega \vec{A}$ and may be read directly from Eq. (31). After some manipulation, the magnetic field is found to be just $90^\circ$ out of phase with the electric field, and is given by

\[
\vec{B} = -i\Lambda \vec{E}, \tag{32}
\]

generalizing Eq. (12) of [14]. Using the physical electric and magnetic fields, one finds the Poynting vector $\vec{S} = \vec{E} \times \vec{B}$ is

\[
S_\rho = 0, \quad S_\phi = \frac{\kappa \omega^2}{4\pi} \sin \theta_k J_{m_\gamma}(\kappa \rho) \\
\times \left[ \cos^2 \frac{\theta_k}{2} J_{m_\gamma - \Lambda}(\kappa \rho) + \sin^2 \frac{\theta_k}{2} J_{m_\gamma + \Lambda}(\kappa \rho) \right],
\]

\[
S_z = \frac{\kappa \omega^2}{4\pi} \left[ \cos^4 \frac{\theta_k}{2} J_{m_\gamma - \Lambda}(\kappa \rho) - \sin^4 \frac{\theta_k}{2} J_{m_\gamma + \Lambda}(\kappa \rho) \right]. \tag{33}
\]

Note that the preceding equation is not time averaged: the Poynting vector for these states is naturally time independent, albeit position dependent.

### 3.B. The tractor beam

A light beam exerts radiation pressure on absorptive or reflective objects in its path. Regarding pressures along the beam direction, the longitudinal component of the Poynting vector, Eq. (33), shows that for $m_\gamma = -\Lambda$ and close to the axis one has a tractor beam, i.e., a radiation pressure attracting objects toward the radiation source. In the case of $m_\gamma = -\Lambda$ the second term for the expression of $S_z$ Eq. (33) is given by a zeroth-order Bessel function, thereby maximizing the tractor effect for a fixed pitch angle $\theta_k$. More generally, if $|m_\gamma - \Lambda| < |m_\gamma + \Lambda|$, we obtain a region near the beam axis with a negative z-component of the Poynting vector. This condition is equivalent to having opposite signs of $m_\gamma$ and $\Lambda$.

The expressions (33) are normalized following Refs. [7, 8]. If instead, we normalize to have beam luminosity $L_0$ within a circular cross sectional disc of area $A$ or radius $R = \sqrt{A/\pi}$, centered on the beam axis, then the
appropriate normalization yields for the pressure on an absorptive object,

\[ p_\rho (\rho) = \frac{S_\omega}{c} = \frac{\kappa L_0}{2 \pi \rho \cos \theta_k} \times \left( \cos \frac{\theta_k}{2} J^{2}_{m_\gamma, \Lambda} (\kappa \rho) - \sin \frac{\theta_k}{2} J^{2}_{m_\gamma, \Lambda} (\kappa \rho) \right). \tag{34} \]

The normalization is for \( R \gg \lambda \) and can be checked using the identity

\[ \lim_{R \to \infty} \int_0^R \rho \, d\rho \, J^2_{m_\gamma} (\kappa \rho) = \frac{R}{\pi \kappa}, \tag{35} \]

for any index.

The pressure toward or away from the source is plotted in Fig. 4 for the case \( m_\gamma = 1 = -\Lambda \), for a luminosity of 1 Watt upon the innermost 1 cm\(^2\) of the wavefront, and with a pitch angle \( \theta_k = 0.2 \) radian. Both panels of the Figure show the longitudinal radiation pressure \( p_\rho \) \( v_s \) the transverse distance from the twisted photon axis measured in wavelengths of light. The scale of the axes is different. The tractor part of the beam is visible close to the axis, although the pressure associated with it is small compared to the radiation pressure farther out. On the other hand, the pressure peaks a few wavelengths from the beam axis are large compared to the radiation pressure averaged over the full 1 cm\(^2\) innermost circular area of the wavefront, which is about 0.03 mPa.

There is also an azimuthal component of the Poynting vector, which will spin objects around the beam axis. For the sake of comparison, plots of the azimuthal pressure, \( p_\phi (\rho) = S_\phi / c \) are shown in Fig. 5.

To observe the tractor part of the beam requires that the beam be fairly purely the desired \( m_\gamma \), as other \( m_\gamma \) can have large positive radiation pressures on axis, so that contamination will obscure the effect.

The most appropriate particles to observe the tractor effect would be absorptive or reflective, since they tend to drift toward the axis of the twisted-photon beam. The dimensions of the particles should not exceed the dimensions of the negative radiation pressure area, as seen in Fig. 4.

Tractor beams have been previously discussed, for examples see [17]–[18]. The previous discussions of tractor beams depend upon field gradient forces rather than upon radiation pressure (for some explanation, see [19]–[20]). The gradient forces depend on interference between different beams, and give alternating regions of attraction and repulsion along the beam axis. By manipulating the relative phases of the beams, the attractive wells and objects trapped in them can be moved toward or away from the source of the beams. In the present case, there is no field gradient along the beam direction and no gradient forces. The tractor forces in this paper depend only on radiation pressure, and to the best of our knowledge, represent a qualitatively new way to make a tractor beam.

4. Summary and Discussion

We have discussed twisted photons, or photons whose total angular momentum projected along the direction of wavefront propagation can be very large. These photons, unlike plane wave photons, have with a Poynting vector that is not zero in the transverse plane and swirls about a symmetry axis. We have, in particular, focused on two applications, photoexcitation of atomic states and situations where the Poynting vector along the propagation direction is reversed, \( i.e. \), tractor beams.

Regarding atomic photoexcitation, twisted photons offers unique opportunities to produce final states with arbitrary angular momentum projections, using the direction of motion of the incoming wavefront as a quantization axis. We have extended our previous photoexcitation analysis [14] to show how in our formalism photon angular momentum divides conservatively between rotating the CM state and angular momentum of the excited state (see also [12]). For a specific example, instead of a very large orbital angular momentum, we considered projected angular momentum \( m_\gamma = 0 \), a value also not possible with plane wave photons. This allows leading order, in the atomic matrix elements, \( 1S \to 2P \) tran-
The rates are suppressed by factors dependent upon the transitions with final magnetic quantum number $m_f = 0$. The rates are suppressed by factors dependent upon the pitch angle (the angle from the twisted photon propagation direction to its component plane wave states), but could be large enough to notice if the pitch angle is not small.

The tractor beams follow from the observation that there are regions where the Poynting vector points back toward the light source, and hence radiation pressure will push small objects located in these regions back to the source. Any region where the first Bessel function in the equation for $S_z$ (Eq. (33)) is zero or near zero will serve as an example. The strength of the tractor beam is suppressed by several powers of the pitch angle, so will not be found with a paraxial approximation, nor will it be found if polarization is neglected. We have shown a numerical example for the near-axis case with $m_{\gamma} = 1$ and component photon helicity $\Lambda = -1$ for which the tractor effect is maximized. Interpreting the result in terms of the photon flux, we notice that the tractor property is due to the twisted photons localized near the beam axis that propagate toward the light source. For the physical beams, the likely origin of these photons is a front of the wave packet.

The tractor beam shown here, again to the best of our knowledge, is qualitatively different from tractor beams previously presented in the literature, for example in [17][18]. These depend on interference effects to produce gradient forces along the axis, and obtain the tractor effect by varying phases to move toward the source the attractive centers of the gradient forces. Here, with no field gradients along the axis, there is only the radiation force. It is an open question if there are other configurations or situations where the strength and size of the tractor regions could be larger.

Appendix A: Comparison of twisted photon expansions

For a given $\{\kappa, m_{\gamma}, k_z\}$, we have a pair of independent states with $\Lambda = 0$. For the same $\{\kappa, m_{\gamma}, k_z\}$, Jäuregui [12] also has a pair of states, labeled TE and TM. The two pairs of states are equivalent, as we shall show.

Using cylindrical coordinates $\vec{r} = (r, \phi, z)$ and

$$\psi_m = \psi_m(r, \phi, \kappa) = J_m(k r) e^{i m \phi}, \quad \text{(A1)}$$

the TE and TM modes from [12] are

$$\vec{A}_{k m, k z}^{(\text{TE})}(t, \vec{r}) = \frac{i E_0}{k_z \sqrt{2}} e^{i(k_z z - \omega t)} \left( \psi_{m, -1} \hat{n}_- - \psi_{m, +1} \hat{n}_+ \right),$$

$$\vec{A}_{k m, k z}^{(\text{TM})}(t, \vec{r}) = -\frac{E_0}{\omega \sqrt{2}} e^{i(k_z z - \omega t)} \times \left( \psi_{m, -1} \hat{n}_- + \psi_{m, +1} \hat{n}_+ + i \sqrt{2} \tan \frac{\theta_k}{\kappa} \psi_m \hat{n}_0 \right). \quad \text{(A2)}$$

With $\Lambda = \pm 1$, the states we use are

$$\vec{A}_{k m, k z, 1} = \frac{1}{i} \sqrt{\frac{\kappa}{2 \pi}} e^{i(k_z z - \omega t)} \left\{ \psi_{m, -1} \cos^2 \frac{\theta_k}{\kappa} \hat{n}_- - \psi_{m, +1} \sin^2 \frac{\theta_k}{\kappa} \hat{n}_+ + i \sqrt{2} \psi_m \hat{n}_0 \right\},$$

$$\vec{A}_{k m, k z, -1} = \frac{1}{i} \sqrt{\frac{\kappa}{2 \pi}} e^{i(k_z z - \omega t)} \left\{ \psi_{m, -1} \sin^2 \frac{\theta_k}{\kappa} \hat{n}_+ - \psi_{m, +1} \cos^2 \frac{\theta_k}{\kappa} \hat{n}_- - i \sqrt{2} \psi_m \hat{n}_0 \right\}. \quad \text{(A3)}$$

Hence,

$$\vec{A}_{k m, k z}^{(\text{TE})} = \frac{E_0}{k_z} \sqrt{\frac{\pi}{\kappa}} \left( \vec{A}_{k m, k z, 1} + \vec{A}_{k m, k z, -1} \right),$$

$$\vec{A}_{k m, k z}^{(\text{TM})} = -i \frac{E_0}{k_z} \sqrt{\frac{\pi}{\kappa}} \left( \vec{A}_{k m, k z, 1} - \vec{A}_{k m, k z, -1} \right). \quad \text{(A4)}$$

The relations are clearly invertible, so the expansions are equivalent.

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