Current-induced motion of a domain wall in magnetic nanowires

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I. INTRODUCTION

The increased interest in the dynamics of domain walls in magnetic nanostructures is rooted mostly in perspective applications in novel spintronic devices. Several recent experiments demonstrated that the motion of domain walls can be effectively controlled by means of an external magnetic field or an electric current.$^{1,2,3,4,5}$

The theory of domain wall motion has been worked out long time ago.$^6,7,8$ Within this theory, the magnetic dynamics is usually described in the framework of a classical ferromagnet assuming separation of the magnetic and electronic degrees of freedom. In this way three- or two-dimensional ferromagnets are treated.

Present studies of the domain wall dynamics are devoted to the issue of the current-induced magnetic motion in nanowires and nanoconstrictions. In the presence of an electric current, the domain wall can move due to a spin torque exerted on the magnetic system by the spin-polarized electron gas. In addition, (linear) momentum can be transferred directly to the domain wall upon scattering of the charge carriers. The problem of calculating the spin torque is of a prime importance for the theory of the domain-wall motion, however only few recent works address this point specifically.$^9,10,11,12,13$

The domain-wall motion can be described by Landau-Lifshitz equations, which have a well-known static solution for the domain wall. Finding the corresponding dynamical solution is however a nontrivial task, particularly in the presence of an external force. The standard way is to describe the domain wall dynamics using an approximate scheme, which is based on physically reasonable arguments, but a strict mathematical justification is missing. The simplest approach is to use an approximate solution that assumes the moving wall to have exactly the same shape as deduced from the static solution. The rigorous conditions for the range of validity of this approximation are however unclear. To obtain reliable results numerical simulations are needed.$^{14}$

Most recently, new approaches have been put forward which address the calculation of the torque as well as the solution of the dynamical equations of motion of the domain wall. Some aspects of these works are developed further and partially revised in this paper.

Our treatment is concentrated on the spin torque and on the wall dynamics in a magnetic nanowire with a domain wall which is sharp on the length scale set by the wave length of the relevant charge carriers. The approach is thus more appropriate for the case of magnetic semiconductors with a small Fermi momentum of carriers (electrons or holes).$^{15}$

II. CURRENT-INDUCED SPIN TORQUE

Here we consider the spin torque transferred by the electric current from the spin polarized electron system to the domain wall. The main goal of our calculation is the demonstration of the presence of two components of the torque, which tend to rotate the magnetic moment in different directions.

We adopt a one-dimensional model for the charge carriers with a point-like interaction between the electron spin $\sigma$ and the magnetic moment $M(x)$ located at a point $x$ along the wire

$$H_{int} = g \sigma \cdot M(x),$$

where $g$ is the coupling constant. Here the one-dimensionality of the electronic system means that we consider the electrons within a wire with transversal dimensions smaller than the electron wavelength $\lambda$, so that only the lowest electron subband is relevant. Strictly speaking, the coupling of the moment to the electron spin depends on the coordinates $y, z$ that characterize
the location of the moment within the wire, \( g(y, z) \sim |\psi_0(y, z)|^2 \), where \( \psi_0(y, z) \) is the wave function of the transverse motion of electrons in the lowest subband. For simplicity, in the following we neglect this dependence, substituting it by an averaged coupling, \( g(y, z) = A^{-1} \int g(y, z) \, dy \, dz \), where \( A \) is the cross section of the wire.

Considering the scattering of electrons from a magnetic moment \( \mathbf{M}(x) \) we assume that the moment is frozen in the point \( x \) on the scale of the characteristic times of the electron motion. This assumption renders possible calculation of the torque as in the case of a static domain wall. The calculated torque is then utilized for the investigation of the domain wall dynamics. Our assumption relies on an adiabatic approximation insofar as we require the time scale for the motion of the magnetic subsystem to be slow as compared to that of the electrons.

To calculate the torque in the case of a sharp domain wall, we start from a model describing the one-dimensional scattering from a localized moment in nonmagnetic and magnetic wires. Then we use the results obtained for the simplified models, to calculate the torque acting locally on the moments within the domain wall.

A. Single magnetic moment in a nonmagnetic wire

Let us consider first scattering of electrons in a nonmagnetic one-dimensional system (nonmagnetic nanowire). The electrons are scattered from a single frozen magnetic moment \( \mathbf{M}_0 \) situated at the point \( x = 0 \), i.e., \( \mathbf{M}(x) = \mathbf{M}_0 \delta(x) \). Here we denote the coordinate along the wire as \( x \). In the absence of the spin-orbit interaction, it is more convenient to use a different coordinate system \((x', y', z')\) for the spin space. We calculate the total scattering amplitude (beyond the Born approximation) of an electron with an arbitrary spin polarization, coming from \( x = -\infty \), and elastically scattered into the states with different spin polarizations.

Assuming the quantization axis \( z' \) along the moment \( \mathbf{M}_0 \), we can write the spinor wave function of electrons as

\[
\psi(x) = \begin{cases} 
  e^{i k x} \begin{pmatrix} a \\ b \\ r a \\ t a \\ \bar{t} b \end{pmatrix}, & x < 0, \\
  e^{i k x} \begin{pmatrix} a \\ b \\ r a \\ t a \\ \bar{t} b \end{pmatrix}, & x > 0,
\end{cases}
\]

(2)

where the coefficients \( a \) and \( b \) correspond to an arbitrary spin polarization in the incident electron wave, \( r \) and \( t \) are, respectively, the reflection and the transmission coefficients in the spin up channel

\[
r = -\frac{\alpha}{1 + i\alpha}, \quad t = \frac{1}{1 + i\alpha},
\]

(3)

\[\alpha = g \mathbf{M}_0 m / k \hbar^2, \text{ and } \mathbf{M}_0 \text{ is the magnitude of the localized magnetic moment.}\]

Using Eqs. (2) and (3) we can calculate the spin density in the wire, associated with the wave function (2)

\[
S_\mu(x) = \psi^\dagger(x) \sigma_\mu \psi(x).
\]

(4)

\( S_\mu(x) \) stands for the oscillating spin density created by the electrons reflected backwards from the localized moment, \( x < 0 \). The spin density components are

\[
S_{x'}(x < 0) = s_{x'} \frac{1 + \alpha^2 + 2\alpha^4}{(1 + \alpha^2)^2} + s_{y'} \frac{2\alpha^3}{(1 + \alpha^2)^2} - 2 \cos(2kx) \left( s_{x'} \frac{\alpha^2}{1 + \alpha^2} + s_{y'} \frac{\alpha}{1 + \alpha^2} \right),
\]

(5)

\[
S_{y'}(x < 0) = s_{y'} \frac{1 + \alpha^2 + 2\alpha^4}{(1 + \alpha^2)^2} - s_{x'} \frac{2\alpha^3}{(1 + \alpha^2)^2} + 2 \cos(2kx) \left( -s_{y'} \frac{\alpha^2}{1 + \alpha^2} + s_{x'} \frac{\alpha}{1 + \alpha^2} \right),
\]

(6)

\[
S_{z'}(x < 0) = s_{z'} \left( \frac{1 + 2\alpha^2}{1 + \alpha^2} \right) - \cos(2kx) \left( \frac{2\alpha^2}{1 + \alpha^2} \right) - \sin(2kx) \frac{2\alpha}{1 + \alpha^2},
\]

(7)

where \( s_{y'} \) is the unit vector determining the spin polarization in the incident wave. Similarly, we obtain for \( x > 0 \)

\[
S_{x'}(x > 0) = s_{x'} \frac{1 - \alpha^2}{(1 + \alpha^2)^2} - s_{y'} \frac{2\alpha}{(1 + \alpha^2)^2},
\]

(8)

\[
S_{y'}(x > 0) = s_{y'} \frac{1 - \alpha^2}{(1 + \alpha^2)^2} + s_{x'} \frac{2\alpha}{(1 + \alpha^2)^2},
\]

(9)

\[
S_{z'}(x > 0) = s_{z'} \frac{1}{1 + \alpha^2}.
\]

(10)

As follows from Eqs. (5) to (10), the spin density induced by the spin-polarized wave incoming from \( x = -\infty \), oscillates with the period \( \pi/k \) at \( x < 0 \), and is constant for \( x > 0 \).

The spin current is defined as

\[
\mathbf{j}_\mu^s(x) = \frac{i \hbar}{2m} \left( i \nabla_x \psi^\dagger(x) \right) \sigma_\mu \psi(x) - \psi^\dagger(x) \sigma_\mu \nabla_x \psi(x),
\]

(11)

(\( \mu = x', y', z' \)) and it can be also calculated using Eqs. (2) and (3) for \( x < 0 \) and \( x > 0 \), respectively. Then we find that the spin current is constant for \( x < 0 \) and \( x > 0 \), with a jump of \( x' \) and \( y' \) components at \( x = 0 \).

We can calculate the spin torque acting on the moment \( \mathbf{M}_0 \) as the transferred spin current at the point \( x = 0 \)

\[
T_\mu = \mathbf{j}_\mu^s(-\delta) - \mathbf{j}_\mu^s(+\delta).
\]

(12)

Using Eqs. (2),(3),(11) and (12) we obtain

\[
T_{x'} = \frac{j_0}{e} \left[ s_{x'} \frac{4\alpha^2}{1 + \alpha^2} + s_{y'} \frac{2\alpha(1 - \alpha^2)}{1 + \alpha^2} \right],
\]

(13)
and $T_{x'} = 0$, where $e$ is the electron charge ($e < 0$), $j_0$ is the electric current

$$ j_0 = \frac{ie\hbar}{2m} \left[ (\nabla_x \psi(x)) \psi(x) - \psi(x) \nabla_x \psi(x) \right] = \frac{ev}{1 + \alpha^2} \tag{15} $$

and $\nu = \hbar k/m$ is the velocity. Note that we found the components of torque (13), (14) in the coordinate system related to the moment $M_0$, so that $x'$ and $y'$ axes are perpendicular to the vector $M_0$, and they are not related in any way to the direction of the current $j_0$.

Using Eqs. (5)-(10) one can show that exactly the same values for the torque components (13), (14) are obtained by utilizing the relation

$$ T_\mu = \frac{gM_0}{\hbar} \epsilon_{\mu\nu\lambda} n_\nu S_\lambda(0), \tag{16} $$

which follows from the equation of motion of the magnetic moment $M_0$, where $n$ is the unit vector along $M_0$, and $\epsilon_{\mu\nu\lambda}$ is the unit antisymmetric tensor.

One can also present the result for the torque (13), (14) in a form, which is more appropriate for an arbitrary coordinate system for the electron spin (not necessarily with the axis $z$ along $M_0$)

$$ T_\mu = \frac{j_0}{e} \left[ \eta (\delta_{\mu\nu} - n_\mu n_\nu) s_\nu + \zeta \epsilon_{\mu\nu\lambda} n_\nu s_\lambda \right], \tag{17} $$

where

$$ \eta = \frac{4\alpha^2}{1 + \alpha^2}, \quad \zeta = -\frac{2\alpha (1 - \alpha^2)}{1 + \alpha^2}. \tag{18} $$

As we see from Eqs. (17) and (18), there are two components of the torque – both transverse to the localized moment. Apart from this, one of them tends to align the moment along the direction of the spin polarization of the incoming electrons, whereas the other one is perpendicular to the spin polarization of the incident wave. Note that in the Born approximation for the scattering amplitude, which is valid for $\alpha \ll 1$, only the second term in (17) survives. It rotates the moment $M_0$ to the direction perpendicular to vector $s$ (and also to $n$).

Thus, the spin torque acting on a single magnetic moment can be found as a change of the spin current due to scattering from the localized moment. The same result is obtained from the calculation of the interaction of accumulated spin with the localized moment. In the following, to calculate the torque in the domain wall, we will use both methods but the second one (coupling to the accumulated spin) is more convenient in case of a sharp domain wall.

We assume that in the case of a smooth domain wall, it can be more convenient to solve the problem considering it as a propagation of a spin-polarized wave with subsequent scattering from different magnetic moments. Correspondingly, one can calculate the torque as a divergence of the spin current.

### B. Scattering from a single magnetic moment in a magnetic wire

Now we calculate the torque in the case of a magnetic wire with the magnetization $M$ being oriented along the axis $x$ for $x < 0$ (left to the wall) and in the opposite direction for $x > 0$ (right to the wall). Here we assume the spin coordinate system $(x', y', z')$ to coincide with the the $(x, y, z)$ one. Like in the previous problem, we introduce an additional frozen magnetic moment $M_0 = M_0(n_x, n_y, 0)$ located at the point $x = 0$. For definiteness, let the vector $M_0$ lie in the $x - y$ plane.

The relevant Hamiltonian has the form

$$ H = -\frac{\hbar^2}{2m} \frac{d^2}{dx^2} + gM \sigma_x \sigma(x) + gM_0 \mathbf{n} \cdot \sigma \delta(x), \tag{19} $$

First, we consider the torque created by a single spin-polarized wave (with the spin polarization along the axis $x$ labelled as "↑") coming from the left. We choose the quantization axis along the axis $z$. Then the wave function containing the reflected and transmitted waves of opposite polarization is

$$ \psi_\uparrow(x) = \begin{cases} e^{ik_\uparrow x} \left( \frac{1}{\sqrt{2}} \right) + e^{-ik_\uparrow x} \left( \frac{1}{1} \right), & \text{for } x < 0, \\ t_\uparrow e^{ik_\uparrow x} \left( \frac{1}{\sqrt{2}} \right) + t_\uparrow e^{-ik_\downarrow x} \left( \frac{1}{1} \right), & \text{for } x > 0, \end{cases} \tag{20} $$

where $k_{\uparrow, \downarrow} = [2m(\pm gM)]^{1/2}/\hbar$, and $\varepsilon$ is the energy. Note that the spin-up electrons are the spin-minority ones, while the spin-down electrons are the spin-majority ones.

Using the continuity of wave function at $x = 0$ and the condition resulting from the integration of Schrödinger equation in the vicinity of domain wall,

$$ -\frac{\hbar^2}{2m} \left( \frac{d\psi_\uparrow}{dx} \bigg|_{+\delta} - \frac{d\psi_\uparrow}{dx} \bigg|_{-\delta} \right) + gM_0 (n_x \sigma_x + n_y \sigma_y) \psi(0) = 0, \tag{21} $$

we find the transmission coefficients for the spin-up polarized wave

$$ t_\uparrow = 2k_{\uparrow} (k_{\uparrow} + k_{\downarrow} - ig_0 n_x), \quad \frac{(k_{\uparrow} + k_{\downarrow})^2 + g_0^2}{(k_{\uparrow} + k_{\downarrow})^2 + g_0^2} \tag{22} $$

and the reflection factors $r_\uparrow = t_\uparrow - 1$, $r_{\uparrow f} = t_{\uparrow f}$, where we denote $g_0 = 2gM_0/\hbar^2$.

Using Eqs. (11) and (20), we can calculate the components of the spin current induced by the incoming spin-up wave

$$ j^{\uparrow}_{x'}(x) = \begin{cases} v_\uparrow (1 - |r_{\uparrow f}|^2) + v_\downarrow |r_{\uparrow f}|^2, & x < 0, \\ v_\downarrow |t_{\uparrow f}|^2 - v_\uparrow |t_{\uparrow f}|^2, & x > 0, \end{cases} \tag{24} $$
\[ j^s_{ty}(x) = \begin{cases} \left( t_{tf} \text{Im} \left[ v_t (e^{ik_x x} - r^*_t e^{-ik_x x}) \right] + v_v (e^{-ik_x x} - r^*_t e^{ik_x x}) \right), & x < 0, \\ \left( -v_t t^* t e^{ik_x x} + v_v t^* t e^{-ik_x x} \right), & x > 0, \end{cases} \]

\[ j^s_{tz}(x) = \begin{cases} \left( t_{tf} \text{Re} \left[ v_t (e^{ik_x x} - r^*_t e^{-ik_x x}) \right] - v_v (e^{-ik_x x} - r^*_t e^{ik_x x}) \right), & x < 0, \\ \left( v_t t^* t e^{ik_x x} + v_v t^* t e^{-ik_x x} \right), & x > 0, \end{cases} \]

where \( k_{\pm} = k_{\uparrow} \pm k_{\downarrow} \) and \( v_{\uparrow, \downarrow} = \hbar k_{\uparrow, \downarrow}/m \).

Hence, the transverse components of the spin currents, \( j^s_{ty}(x) \) and \( j^s_{tz}(x) \), are nonzero for \( x < 0 \) and for \( x > 0 \). As we see from Eqs. (24)-(26), the transverse components of the spin current are oscillating functions of \( x \). The nonconservation of spin current in the magnetic wire is related to indirect magnetic interactions accompanying the inhomogeneous distribution of the spin density. In the nonmagnetic case, corresponding to the limit of \( k_x \to 0 \), it reduces to the conservation of spin current at \( x < 0 \) and \( x > 0 \), as we found in the previous section.

The calculation of the corresponding spin transfer (12) in this model gives us the following expressions for the components of the torque

\[ T_{tx} = 2v_t \text{Re} t_{tf} + (v_t + v_v) \left( |t_{tf}|^2 - |t_{tf}|^2 \right), \]

\[ T_{ty} = -2t_{tf} (v_t + v_v) \text{Im} t_{tf}, \]

\[ T_{tz} = 2t_{tf} [v_t - (v_t + v_v) \text{Re} t_{tf}]. \]

As in the case of a nonmagnetic wire with a localized moment, the torque \( \mathbf{T} \) can be calculated using Eq. (16) with the spin density \( \mathbf{S}_t(0) \), where the index "\( t \)" indicates the accumulated spin associated with the incident wave of spin-up polarization. Using (4) and (20) we can calculate the spin density \( \mathbf{S}_t(x) \) as in the previous section. The spin density oscillates for both \( x < 0 \) and \( x > 0 \), with the periods \( 2\pi/k_{\uparrow} \) and \( 2\pi/k_{\downarrow} \) for the spin current in Eqs. (24)-(26).

With our choice of the coordinate system, when the vector \( \mathbf{M}_0 \) lies in the \( x-y \) plane, the transverse components of the torque acting on the moment \( \mathbf{M}_0 \) are

\[ T_{tx} = -n_y T_{tx} + n_x T_{ty}, \]

rotating the moment in \( x-y \) plane, and \( T_{tz} \), rotating it in out-of-plane direction. The label \( \perp \) in (30) means the projection of torque on the direction perpendicular to the moment \( \mathbf{M}_0 \) in the \( x-y \) plane.

Similarly, we can consider the scattering of electron with the incident spin polarization opposite to the \( x \)-axis (labelled as \( \perp \)). The corresponding scattering state has the form of Eq. (20) with \( k_{\uparrow} \leftrightarrow k_{\downarrow} \) and interchanged spin states, and the expressions for transmission coefficients are

\[ t_{t\downarrow} = \frac{2k_{\downarrow} (k_{\uparrow} + k_{\downarrow} + ig_0n_x)}{(k_{\uparrow} + k_{\downarrow})^2 + g_0^2}, \]

\[ t_{t\downarrow} = \frac{2g_0n_y k_{\downarrow}}{(k_{\uparrow} + k_{\downarrow})^2 + g_0^2}, \]

where the change \( g_0 \to -g_0 \) is equivalent to the flip of momentum \( \mathbf{M}_0 \). The components of spin current \( j^s_{t\mu} \) have the following form

\[ j^s_{t\downarrow}(x) = \begin{cases} -v_t (1 - |r|^2) - v_v |r_{tf}|^2, & x < 0, \\ -v_t |t_{tf}|^2 + v_v |t_{tf}|^2, & x > 0, \end{cases} \]

\[ j^s_{t\downarrow}(x) = \begin{cases} -t_{tf} \text{Im} \left[ v_t (e^{ik_x x} - r^*_t e^{-ik_x x}) + v_v (e^{-ik_x x} + r^*_t e^{ik_x x}) \right], & x < 0, \\ t_{tf} \text{Im} \left[ v_t t^* t e^{ik_x x} - v_v t^* t e^{-ik_x x} \right], & x > 0, \end{cases} \]

\[ j^s_{t\downarrow}(x) = \begin{cases} t_{tf} \text{Re} \left[ v_t (e^{ik_x x} - r^*_t e^{-ik_x x}) - v_v (e^{-ik_x x} + r^*_t e^{ik_x x}) \right], & x < 0, \\ t_{tf} \text{Re} \left[ v_t t^* t e^{ik_x x} + v_v t^* t e^{-ik_x x} \right], & x > 0. \end{cases} \]

The torque components (27)-(29) can also be calculated from Eq. (16), taking into account the spin accumulation at \( x = 0 \),

\[ S_{t\downarrow x}(k\downarrow) = \frac{4k^2 (k_{\uparrow} + k_{\downarrow})^2 + g_0^2 (n_{\uparrow}^2 - n_{\downarrow}^2)}{(k_{\uparrow} + k_{\downarrow})^2 + g_0^2}, \]

\[ S_{t\downarrow y}(k\downarrow) = \frac{8g_0 k^2 n_{\uparrow} n_{\downarrow}}{(k_{\uparrow} + k_{\downarrow})^2 + g_0^2}, \]

\[ S_{t\downarrow z}(k\downarrow) = \frac{8g_0 k^2 (k_{\uparrow} + k_{\downarrow}) n_{\downarrow}}{(k_{\uparrow} + k_{\downarrow})^2 + g_0^2}. \]

The above formulas will be used later to calculate the torque exerted on a domain wall.

In the case of a 100% polarized electron gas, only the components of the spin current calculated according to Eqs. (33)-(35) corresponding to the majority electrons, are relevant. The transition to this case implies the substitutions \( k_{\uparrow} \to i k_{\uparrow} \).

In this section we calculated the wave functions of electrons in the magnetic wire with a sharp domain wall and a localized magnetic moment. The eigenfunctions of the Hamiltonian (19) correspond to the spin-polarized incoming waves (spin up and down). Obviously, an arbitrary-polarized incoming wave is not the eigenfunction of the Hamiltonian. Nevertheless, we can still consider the scattering of electron waves with different spin.
magnetization of minority and majority electrons, respectively), we can calculate the transmission of states with different incoming spin-polarized waves by means of an injection from the tip, and its lifetime \( \tau \) can be long enough on the scale of the characteristic time of the domain wall motion. In this case, a superposition of states with different incoming spin-polarized wave polarizations. For example, such a state can be created by means of an injection from the tip, and its lifetime \( \tau \) can be long enough on the scale of the characteristic time of the domain wall motion. In this case, a superposition of states with different incoming spin-polarized waves can be used to calculate the torque.

C. Magnetic wire with a thin domain wall

In the case of a thin metallic wire, when \( k_{F\uparrow,\downarrow}d \ll 1 \) (\( d \) is the wire diameter and \( k_{F\uparrow,\downarrow} \) are the Fermi momenta of minority and majority electrons, respectively), we can assume that only one quantization level is filled with electrons.

Let us consider a magnetic wire with a single domain wall, with the magnetization \( \mathbf{M} \) being directed along the \( x \) axis for \( x < -w \) and along the opposite direction for \( x > w \). Here \( 2w \) is the domain wall width, and we chose the spin coordinate system like in the previous section, with the \( x \) axis along the wire.

Upon applying a small voltage an electric current flows in the wire. For definiteness, we assume the current to flow in the direction opposite to the \( x \) axis direction (i.e., the electron flux is oriented along \( x \)). If the only imperfection in the wire is the presence of the domain wall, one can assume a jump \( \Delta \phi \) in the potential at the wall, and both the charge and the spin currents can be calculated as integrals over energies in the interval between \( \varepsilon_{FR} \) and \( \varepsilon_{FL} = \varepsilon_{FR} + \Delta \phi \), where \( \varepsilon_{FL} \) and \( \varepsilon_{FR} \) are the Fermi levels on the left and right sides. In the limit of small voltage, \( |e\Delta \phi| \ll \varepsilon_F \), the transport is linear and is associated with electrons at the Fermi level.

We assume the electrons approaching the domain wall from the left are spin-polarized according to the magnetization direction in the left part of the wire. The incoming electrons are scattered from a large number of magnetic moments in the domain wall. We consider this scattering using the point interaction of electron with each of the localized moments. This corresponds to the picture with an array of well-separated magnetic moments like in the case of magnetic semiconductor doped with magnetic impurities. We assume that there is a large number of magnetic atoms with different orientation of moments within the domain wall. Accordingly, the electron transmitted through the wall is multiply scattered from many magnetic moments.

Thus, to calculate the transmission of electrons through the domain wall, we should take the perturbation created by the total magnetic moment \( \mathbf{M}(x) = \sum \mathbf{M}_i \delta(x-x_i) \), where \( \mathbf{M}_i \) is the localized moment at the point \( x = x_i \), and all of the moments \( \mathbf{M}_i \) are located within a region of the domain wall width, \( |x_i| < w \), which in turn is assumed to be small as compared to the wavelength of electrons, \( k_{F\uparrow,\downarrow}w \ll 1 \).

The scattering from the total moment \( \tilde{\mathbf{M}}(x) \) located within a region much smaller than the electron wavelength can be described using the model of a spin-dependent delta-function potential. Then, in the limit of small voltage, we obtain the current

\[
\tilde{j}_0 \approx \frac{e^2 \Delta \phi}{2\pi \hbar} \left( |\tilde{t}_{\uparrow f}\rangle \langle \tilde{t}_{\uparrow f}| + |\tilde{t}_{\downarrow f}\rangle \langle \tilde{t}_{\downarrow f}| + |\tilde{t}_{\uparrow d}\rangle \langle \tilde{t}_{\uparrow d}| - |\tilde{t}_{\downarrow d}\rangle \langle \tilde{t}_{\downarrow d}| \right),
\]

(39)

where the tilde means the transmission coefficients for the scattering of electrons from an effective moment \( \tilde{\mathbf{M}} \).

\( M_{eff} \) is the localized moment at \( x \) is the Fermi level at the Fermi surface, \( k_{F\uparrow,\downarrow} \equiv k_{F\uparrow,\downarrow} \) is the Fermi level at the Fermi surface, \( k_{F\uparrow,\downarrow} \equiv k_{F\uparrow,\downarrow} \) is the Fermi level at the Fermi surface.

In the case the domain wall possesses a magnetization profile as depicted in Fig. 1, the effective moment \( M_{eff} \) is oriented along the \( y \) axis. The transmission coefficients \( \tilde{t}_{\uparrow,\downarrow} \), \( \tilde{t}_{\uparrow f} \) and \( \tilde{t}_{\uparrow d} \) have the form of Eqs. (22), (23) and (31), (32), respectively, with \( n_x = 0, n_y = 1 \), and the substitution \( g_0 \rightarrow \tilde{g}_0 = 2mg_{eff}/\hbar^2 \). We can relate the magnitude of \( M_{eff} \) to the continuous magnetic profile within the wall, \( M_{eff} \approx \int_{-w}^{w} M_y dx \).

The spin current can be also calculated in linear response approximation using the scattering states. It includes the sum of partial spin currents

\[
\tilde{j}^s(x) = \frac{e}{2\pi \hbar} \left( \tilde{j}^s_{\uparrow}(x) + \tilde{j}^s_{\downarrow}(x) \right),
\]

(40)

where the components of \( \tilde{j}^s_{\uparrow,\downarrow} \) can be found using Eqs. (24)-(26) and (33)-(35) with the substitution \( t_{\uparrow,\downarrow} \rightarrow \tilde{t}_{\uparrow,\downarrow} \), \( t_{\uparrow f} \rightarrow \tilde{t}_{\uparrow f} \). The appearance of \( v_{\uparrow} \) and \( v_{\downarrow} \) in the denominators of Eq. (40) is related to the 1D density of states for spin up and down electrons. The components of the spin current perpendicular to the \( x \) axis, are...
oscillating functions. As we see from Eqs. (24)-(26) and (33)-(35), the wavelength of oscillations is determined by the inverse momentum at the Fermi level. Hence, the wavelength of oscillation of the transverse component of the spin current is much larger than the domain wall width.

It is worth noting that in three-dimensional systems, the transverse component of the spin current decays due to the integration over momentum in the DW plane. In metallic ferromagnets, the decay is very fast due to the large Fermi momentum of electrons. However, there is a nonvanishing spin transfer for the transverse component in the 3D case, too.

We can also calculate the accumulated spin density induced by the external current $j_0$. It can be found as the expectation value of the spin $\sigma_\mu$ in the scattering state of the incoming electrons, integrated over all energies between $\varepsilon_F$ and $\varepsilon_F + e\Delta\phi$, similar to the calculation of the charge and spin currents. Accordingly, we find

$$S(0) = \frac{e\Delta\phi}{2\pi\hbar} \left( \frac{\tilde{S}_\uparrow(0)}{v_\uparrow} + \frac{\tilde{S}_\downarrow(0)}{v_\downarrow} \right),$$

(41)

where $\tilde{S}_{\uparrow,\downarrow}(0)$ can be found using Eqs. (36)-(38) with $n_x = 0$ and the substitution $g_0 \rightarrow \tilde{g}_0$, which corresponds to the scattering from the effective magnetic moment $M_{eff}$.

Finally, we find the torque acting on a single localized moment in the domain wall. For this purpose we use Eq. (16) with $S(0)$ from (41), describing the spin accumulation created by scattering from the domain wall as a whole. In its turn, $\tilde{S}_{\uparrow,\downarrow}(0)$ is calculated as explained after Eq. (41) using (36)-(38). The result can be presented in the general form

$$T = \frac{j_0}{e} [\eta \mathbf{n} \times (\mathbf{n} \times \mathbf{s}) + \zeta \mathbf{n} \times \mathbf{s}].$$

(42)

where

$$\eta = \frac{g_0 \tilde{g}_0 (k_\uparrow^2 - k_\downarrow^2)}{2k_\uparrow k_\downarrow (k_\uparrow + k_\downarrow)^2 + \tilde{g}_0^2 (k_\uparrow^2 + k_\downarrow^2)},$$

(43)

$$\zeta = -\frac{g_0 (k_\uparrow + k_\downarrow)^2 [(k_\uparrow + k_\downarrow)^2 - \tilde{g}_0^2]}{2k_\uparrow k_\downarrow (k_\uparrow + k_\downarrow)^2 + \tilde{g}_0^2 (k_\uparrow^2 + k_\downarrow^2)},$$

(44)

and $\mathbf{s}$ is the unit vector along the spin polarization corresponding to magnetization $\mathbf{M}$ at $x < -w$. The dependence of the coefficients $\eta$ and $\zeta$ on the parameters $\tilde{g}_0$ and on the electron gas polarization $P = (k_\downarrow - k_\uparrow)/(k_\uparrow + k_\downarrow)$ is presented in Figs. 2 and 3. As we see, both coefficients strongly depend on the parameters of the ferromagnet and on the parameters of the wall. In the case of small coupling $\tilde{g}_0$, we obtain $\zeta \gg \eta$, i.e., the torque is mostly related to the second component in Eq. (42). In contrast, if $\tilde{g}_0$ is larger, the first term in (42) dominates.

FIG. 2: Dependence of the factor $\eta$ on the effective coupling $\tilde{g}_0$ for different values of the electron polarization $P$.

FIG. 3: Coefficient $\zeta$ vs. coupling constant $\tilde{g}_0$ for different values of $P$. 
D. Spin torque in p-type magnetic semiconductors

The conductivity of magnetic semiconductors like GaMnAs is usually of the p-type. The valence band of these semiconductors can be described by a matrix Hamiltonian, which includes the spin-orbit interaction. Thus, the calculation of the transmission of holes through the domain wall in GaMnAs semiconductors needs a different model, which takes into account the complex band structure.

Here we use the Luttinger model for the energy spectrum of holes with the momentum $J = 3/2$. We also neglect the anisotropy of the energy spectrum. To simplify the calculations, in this section we take the quantization axis $z$ along the wire. Then in the quasi-one-dimensional case with the domain wall in the $y-z$ plane, the Hamiltonian of holes is

$$H = \frac{\hbar^2}{2m_0} \left( \gamma_1 + \frac{5\gamma_2}{2} \right) \frac{d^2}{dz^2} - \frac{\hbar^2 \gamma_2}{m_0} J^2 \frac{d^2}{dz^2} - g \left[ J_y M_y(z) + J_z M_z(z) \right] ,$$

where $m_0$ is the mass of free electron, $\gamma_1$ and $\gamma_2$ are the Luttinger parameters, and $J_y$ are the matrices of the total momentum $3/2$. Note that we are using Hamiltonian (45) to describe the holes like unfilled electron states in the valence band. Then the correct statistics of holes corresponds to the reversed energy axis of holes as compared to that of electrons.

Similar to the previous consideration, we take the magnetization $\mathbf{M}$ along the axis $z$ for $z < -w$ and in the opposite direction for $z > w$, whereas in the region of $-w < z < w$ the moment changes its orientation rotating in $y-z$ plane. For $z < -w$ the hole can be described by the energy spectrum consisting of four parabolic bands of the particles labelled by the momentum projection $J_z$

$$E_{\pm3/2}(k) = -\frac{\hbar^2 k^2}{2m_t} + \frac{3gM}{2} ,$$

$$E_{\pm1/2}(k) = -\frac{\hbar^2 k^2}{2m_t} + \frac{gM}{2} ,$$

where $m_t = m_0/(\gamma_1 - 2\gamma_2)$ and $m_t = m_0/(\gamma_1 + 2\gamma_2)$ are the masses of heavy and light holes, respectively. In accordance with Eqs. (46) and (47), the energy band of the heavy holes with the moment projection $J_z = -3/2$ is above all other bands. In the region of $z > 0$ the spectrum is the same but with the opposite signs of $J_z$.

We assume that the holes are fully polarized so that the hole density is rather small. Correspondingly, we assume that the chemical potential $\mu$ is located in the interval of energies $gM/2 < \mu < 3gM/2$.

The scattering state of holes corresponding to the wave $J_z = -3/2$ incoming from $z = -\infty$ is

$$\psi^\dagger(z) = \left( r_3 e^{-ikz} , r_2 e^{ikz} , r_1 e^{\kappa_1 z} , e^{-ikz} + r^* e^{ikz} \right) ,$$

$$z < -w ,$$

where $r, r_1, ... r_3$ and $t, t_1, ... t_3$ are the reflection and the transmission coefficients, respectively. The momentum of heavy hole $k$ is taken at the Fermi surface, $-\hbar^2 k^2/2m_t + 3gM/2 = \varepsilon$. The other momenta $\kappa_i$ correspond to the decaying components of the wave function, $\kappa_1 = [2m_t(\varepsilon - gM/2)/\hbar^2]^{1/2}$, $\kappa_2 = [2m_t(\varepsilon + gM/2)/\hbar^2]^{1/2}$, and $\kappa_3 = [2m_t(\varepsilon + 3gM/2)/\hbar^2]^{1/2}$. Note that the transmission coefficient $t$ in this notation corresponds to the transmission from the state with moment $J_z = -3/2$ to the state $J_z = 3/2$.

In the limit of $w \to 0$, the matching condition can be presented in the matrix form

$$\text{diag} \left( m_t^{-1} , m_t^{-1} , m_t^{-1} , m_t^{-1} \right) \left( \frac{d\psi}{dz} - \frac{d\psi}{dz} \right) = -\lambda_0 J_y \psi(0) = 0 ,$$

where $\lambda_0 = 2gM_{\text{eff}}/\hbar^2$.

Using Eq. (50) and the continuity of the wave function at $z = 0$ we can calculate eight reflection and transmission coefficients. The accumulated spin density $\mathbf{S}(0)$ induced by the current flowing along $z$ can be calculated like in the previous section, but with the opposite sign because the accumulation of polarized holes means a loss of real particles (electrons). Then we find

$$S_x(0) = -\frac{e \Delta \phi}{2\pi \hbar v_t} \text{Im} \left( \sqrt{3} t_1^* t_1 + 2 t_2^* t_2 + 3 t_3^* t_3 \right) ,$$

$$S_y(0) = -\frac{e \Delta \phi}{2\pi \hbar v_t} \text{Re} \left( \sqrt{3} t_1^* t_1 + 2 t_2^* t_2 + 3 t_3^* t_3 \right) ,$$

$$S_z(0) = -\frac{e \Delta \phi}{4\pi \hbar v_t} \left( |t_1|^2 + |t_1|^2 - |t_2|^2 - 3 |t_3|^2 \right) ,$$

where $v_t = \hbar k/m_t$ is the velocity of heavy holes at the Fermi level, $e \Delta \phi = \varepsilon_{\text{FR}} - \varepsilon_{\text{FL}} > 0$, and $\varepsilon_{\text{FL}}$ and $\varepsilon_{\text{FR}}$ are the Fermi levels at $z < -w$ and $z > w$, respectively.

Using Eqs. (16) and (42), we find the parameters $\eta$ and $\zeta$ determining the torque acting on a single magnetic moment $M_0$

$$\eta = \frac{e gM_0}{j_0 \hbar} S_x(0) ,$$

$$\zeta = -\frac{e gM_0}{j_0 \hbar} S_x(0) ,$$

where $j_0 = -e^2 \Delta \phi |t|^2 / 2\pi \hbar$, and the "-" sign in current is due to the opposite charge of holes.

The dependence of $\eta$ and $\zeta$ on the magnitude of magnetic splitting $gM$ for different bulk hole densities $p$ is presented in Figs. 4 and 5. We take the cross section...
A = 1 nm², and the momentum of heavy holes \( k = \pi p_{1D} \), where \( p_{1D} \) is the linear density of holes.

As we can see from Figs. 4 and 5, the factor \( \zeta \) is negligibly small as compared to \( \eta \). In our model, the density of holes and the spin splitting are independent parameters. Thus, the magnitude of torque \( \eta \) increases with the decreasing hole density \( p \) at a fixed value of \( gM \). However, in real magnetic semiconductors these values are not independent, and the magnetic splitting increases with the increasing hole density.¹⁹

It should be noted that the used condition of \( w \to 0 \) implies that not only the wavelength of holes with \( J_z = 3/2 \) is large as compared to the domain wall width, \( kw \ll 1 \), but also the conditions \( \kappa_i w \ll 1 \) for all \( \kappa_i \) should be fulfilled. This condition is restrictive for the magnitude of the magnetic splitting, \( (gMm_t)^{1/2}w/\bar{h} \ll 1 \).

III. MOTION OF THE DOMAIN WALL

A. Hamiltonian and equations of motion

Now we consider the Hamiltonian \( \mathcal{H}_0 \) describing a quasi-one-dimensional magnetic system with a domain wall. We adopt a model including the magnetic exchange interaction and two different anisotropy constants \( \lambda_1 \) and \( \lambda_2 \) in the \( z \) and \( y \) directions, respectively (see Fig. 1)

\[
\mathcal{H}_0 = \frac{a}{2} \left( \frac{\partial \mathbf{n}}{\partial x} \right)^2 + \frac{\lambda_1}{2} n_z^2 + \frac{\lambda_2}{2} n_y^2,
\]

where \( a \) is the constant of exchange interaction, and \( \mathbf{n}(x) \) is the unit vector of magnetization. Using this Hamiltonian we are going to describe the magnetic nanowire like presented in Fig. 1. Correspondingly, we assume that the magnetization vector field \( \mathbf{n} \) depends only on the coordinate \( x \) and time \( t \). The Hamiltonian \( \mathcal{H}_0 \) is the energy density of the magnetic system in the absence of the spin torque.

In the following we restrict ourselves by considering the above-calculated spin torque as a driving force acting on the domain wall. Hence, we neglect the direct transfer of momentum from electrons reflected from the domain wall. As we show in Appendix, this effect is much smaller than the above-calculated spin torque. On the other hand, our consideration has a general character without specifying the mechanism determining the values of the factors \( \eta \) and \( \zeta \).

Using the spherical angles \( \theta(x, t) \) and \( \varphi(x, t) \), we can rewrite the Hamiltonian \( \mathcal{H}_0 \) as

\[
\mathcal{H}_0 = \frac{a}{2} \left( \frac{\partial \theta}{\partial x} \right)^2 + \frac{a}{2} \left( \frac{\partial \varphi}{\partial x} \right)^2 \sin^2 \theta + \frac{\lambda_1}{2} \cos^2 \theta + \frac{\lambda_2}{2} \sin^2 \theta \sin^2 \varphi.
\]

The Landau-Lifshitz-Gilbert equation of motion includes a damping term and two possible components of
the current-induced torque, as discussed in the previous section,
\[
\frac{1}{\gamma} \frac{\partial \mathbf{n}}{\partial t} = -\mathbf{n} \times \left( \frac{\delta \mathcal{H}_0}{\delta \mathbf{n}} - \frac{\partial}{\partial x} \delta \frac{\mathcal{H}_0}{\partial (\mathbf{n}/\partial x)} \right) - \alpha \mathbf{n} \times \frac{\partial \mathbf{n}}{\partial t} \\
+ J_0 \zeta \mathbf{n} \times \mathbf{s} + J_0 \eta \mathbf{n} \times (\mathbf{n} \times \mathbf{s}),
\]
where \(\alpha\) is the damping constant, \(\gamma = g \mu_B / h M\) is the gyromagnetic ratio divided by \(M\), \(J_0 = j_0 h / e g \Omega_0\), and \(\Omega_0\) is a volume per magnetic moment. In Eq. (58) the spin torque is expressed in terms of transferred moment per unit volume, and it enters directly the equation of motion. The corresponding spin-torque terms in the magnetic Hamiltonian can be presented as
\[
\mathcal{H}_{int} = J_0 \zeta \mathbf{n} \cdot \mathbf{s} + J_0 \eta \int_0^1 d\tau \mathbf{n} \cdot \left( \frac{\partial \mathbf{n}}{\partial \tau} \times \mathbf{s} \right),
\]
where \(\mathbf{n}(\tau = 0) = 0\) and \(\mathbf{n}(\tau = 1) = \mathbf{n}\).

The Lagrangian of the magnetic system contains a term with a time derivative as follows:
\[
\mathcal{L} = A \int dx \left[ \frac{1}{\gamma} \frac{\partial \varphi}{\partial t} (\cos \theta - 1) - \mathcal{H} \right].
\]

Neglecting the damping term, the Landau-Lifshitz equations of the magnetic dynamics take the form
\[
\frac{1}{\gamma} \frac{\partial \theta}{\partial t} = -a \frac{\partial^2 \varphi}{\partial x^2} \sin \theta + \lambda_2 \sin \theta \sin \varphi \cos \varphi \\
- J_0 \eta \cos \theta \cos \varphi, -J_0 \zeta \sin \varphi,
\]
\[
\frac{\sin \theta}{\gamma} \frac{\partial \varphi}{\partial t} = a \frac{\partial^2 \theta}{\partial x^2} - a \left( \frac{\partial \varphi}{\partial x} \right)^2 \sin \theta \cos \theta \\
+ \lambda_1 \cos \theta \sin \theta - \lambda_2 \sin \theta \cos \theta \sin^2 \varphi \\
+ J_0 \eta \sin \varphi + J_0 \zeta \cos \theta \cos \varphi.
\]

In the absence of current, \(j_0 = 0\), they have the well-known kink-like static solution \(\varphi_0(x) = \arccos [\tanh(\beta_0 x)]\) and \(\theta_0 = \pi/2\), where \(\beta_0 = (\lambda_2 / a)^{1/2}\) is the inverse width of the static domain wall. From now on we assume for definiteness that \(\lambda_1 > \lambda_2\), so that the static domain wall with the magnetization in the \(x - y\) plane is energetically more favorable.

In a general case, the solution of the nonlinear dynamical Eqs. (61)-(62), describing the moving domain wall, is a difficult problem. Therefore, we assume in the following that one of the anisotropy constants is large, \(\lambda_1 \gg \lambda_2\).

### B. Strong easy-plane anisotropy

We consider the case of a large easy-plane anisotropy, and, accordingly, assume that for the moving domain wall (subjected to the torque) the deviation of magnetization vector \(\mathbf{M}\) from the \(x - y\) plane is small. Then we can write \(\theta(x, t) = \pi/2 + \chi(x, t)\) and take \(|\chi(x, t)| \ll 1\). Up to the second order in \(\chi(x, t)\) field, the Lagrangian \(\mathcal{L}\) is
\[
\mathcal{L} = A \int dx \left[ -\frac{1}{\gamma} \frac{\partial \varphi}{\partial t} (\chi + 1) - a \frac{\partial \varphi}{\partial x} \right]^2 - a \frac{\partial \varphi}{\partial x}^2 \\
\times (1 - \chi^2) - \lambda_1 \chi^2 - \frac{\lambda_2}{2} \sin^2 \varphi \left(1 - \chi^2\right) + J_0 \eta \sin \varphi \\
+ J_0 \zeta \cos \varphi.
\]

Since we restricted the treatment to quadratic terms in \(\chi\) in the Lagrangian, the integral over \(\chi\) is Gaussian, and we can integrate out the \(\chi\) fields to obtain
\[
\mathcal{L} = A \int dx \left[ \frac{1}{2} \int dx' G(x, x') \left( \frac{1}{\gamma} \frac{\partial \varphi(x)}{\partial t} - J_0 \eta \sin \varphi(x) \right) \\
\times \left( \frac{1}{\gamma} \frac{\partial \varphi(x)}{\partial t} - J_0 \eta \sin \varphi(x) \right) - a \frac{\partial \varphi}{\partial x}^2 - \lambda_1 - \lambda_2 \sin^2 \varphi \\
+ J_0 \zeta \cos \varphi, \right]
\]
where the Green function \(G(x, x')\) obeys the following equation
\[
\frac{\partial^2 G(x, x')}{\partial x^2} - a \left( \frac{\partial \varphi}{\partial x} \right)^2 + \lambda_1 - \lambda_2 \sin^2 \varphi \right) = \delta(x - x').
\]

Note that \(\varphi\)-fields are taken at the same time \(t\) in Eq. (65). It follows from the equation for Green function \(G(x, t; x', t') \sim \delta(t - t')\) describing the propagation in time.

Equation (64) contains \(\varphi(x, t)\), which should be the saddle point solution of the Lagrangian, i.e., the self-consistency should be preserved.

Neglecting the first term in Eq. (65) we can find an approximate formula for the Green function proportional to \(\delta(x - x')\)
\[
G(x, x') = \delta(x - x') \left[ -a \left( \frac{\partial \varphi}{\partial x} \right)^2 + \lambda_1 - \lambda_2 \sin^2 \varphi \right]^{-1}.
\]

This form of \(G(x, x')\) leads to the point interaction of the \(\varphi\)-fields in the first term of Eq. (64). Physically, neglecting the first term with derivatives in Eq. (65), we substitute the finite-range interaction by the \(\delta\)-like one.

One can estimate at which conditions the use the Green function in the form of Eq. (66) is justified. The exact solution of Eq. (65) can be presented as
\[
G(x, x') = \sum_n \frac{\phi_n(x) \phi_n^*(x')}{\varepsilon_n + \lambda_1},
\]
where \(\phi_n(x)\) and \(\varepsilon_n\) are the eigenfunctions and corresponding eigenvalues of the equation
\[
\left[ -a \frac{\partial^2}{\partial x^2} - a \left( \frac{\partial \varphi}{\partial x} \right)^2 - \lambda_2 \sin^2 \varphi - \varepsilon_n \right] \phi_n(x) = 0.
\]
We expect that the function $\varphi(x)$ in Eqs. (65) and (68) is similar to the form of the static solution $\varphi_0(x)$. Thus, Eq. (68) corresponds to the Schrödinger equation for a particle of mass $m = \hbar^2/2a$ in the potential well $V(x)$ of width $L_0 \sim (a/\lambda_2)^{1/2}$. The energy spectrum of this problem consists of a level in the well $\varepsilon_0 \simeq -\lambda_2$ and a continuous spectrum for all positive energies.

For $\varphi(x) = \varphi_0(x)$, the potential has the form of $V(x) = -2\lambda_2 \cosh^2(\beta_0 x)$, and the discrete energy spectrum has one level $\varepsilon_0 = -4\lambda_2$. The eigenfunctions $\phi_n(x)$ corresponding to the continuous spectrum, are oscillatory functions, so that their contribution to Eq. (67) can be neglected as compared to the short-range interaction. Using the condition of strong easy-plane anisotropy, $\lambda_1 \gg \lambda_2$, we can simplify Eq. (66) essentially and obtain the expression

$$G(x, x') \simeq \frac{\delta(x - x')}{\lambda_1}. \quad (69)$$

In this approximation, the Lagrangian (64) acquires the following form

$$L = A \int dx \left[ \frac{1}{2\lambda_1} \left( \frac{\partial \varphi}{\partial t} - J_0 \eta \sin \varphi \right)^2 - a \left( \frac{\partial \varphi}{\partial x} \right)^2 \right] - \frac{\lambda_2}{2} \sin^2 \varphi + J_0 \zeta \cos \varphi. \quad (70)$$

The corresponding saddle-point equation is

$$- \frac{1}{\gamma^2 \lambda_1} \frac{\partial^2 \varphi}{\partial t^2} + \frac{J_0^2 \eta^2}{\lambda_1} \sin \varphi \cos \varphi + a \frac{\partial^2 \varphi}{\partial x^2} - \lambda_2 \sin \varphi \cos \varphi - J_0 \zeta \sin \varphi = 0. \quad (71)$$

Now we can study the problem of the domain wall dynamics in terms of a single $\varphi(x, t)$ field.

### C. Solution for $\zeta = 0$

Let us consider the possibility of the kink-like solution moving with an arbitrary constant velocity $v$, $\varphi(x, t) \equiv \varphi(x - vt) \equiv \varphi(x - vt)$. We can find such solutions in the case of $\zeta = 0$, trying a function which obeys the equality $\partial \varphi(x)/\partial x = \beta \sin \varphi(x)$. This function differs from the static solution only by a different choice of $\beta$ instead of $\beta_0 = (\lambda_2/a)^{1/2}$. Substituting it into (71), we obtain the equation that relates the values of $\beta$ and $v$ as

$$\beta^2 \left( a - \frac{v^2}{\gamma^2 \lambda_1} \right) - \lambda_2 + \frac{J_0^2 \eta^2}{\lambda_1} = 0. \quad (72)$$

The dependence $\beta(\tilde{v})$ is presented in Fig. 6 where we denoted $\tilde{v} = v/\gamma \sqrt{\lambda_1 a}$ and $\tilde{j}_0 = J_0 \eta/\sqrt{\lambda_1 \lambda_2}$.

When $j_0 = 0$, we find from (72) that $\beta^2 = \beta_0^2/(1 - v^2/\gamma^2 \lambda_1 a)$. It means, that in the absence of the current, the solution for a moving domain wall is more sharp as compared to the static wall.

For $j_0 \neq 0$ and $v \to 0$, the value of $\beta$ depends on the current as $\beta^2 = \tilde{\beta}_0^2 (1 - J_0^2 \eta^2 a/\lambda_1 \lambda_2)$, i.e., the current makes the thickness of the static domain wall larger.

Now we can use the velocity $v$ as a variational parameter to minimize the Lagrangian

$$L = \frac{AF(v)}{2\beta(v)} \int \sin^2 \varphi(x) \, d(\beta x), \quad (73)$$

where

$$F(v) = \frac{1}{\lambda_1} \left[ \frac{v \beta(v)}{\gamma} + J_0 \eta \right]^2 - a \beta^2 - \lambda_2, \quad (74)$$

the function $\beta(v)$ is defined by Eq. (72), and the integral in (73) does not depend on $v$.

Using Eqs. (72) to (74) we find that for $j_0 = 0$, the quantity $F(v) = -\lambda_2$ for any $v$. Thus, the minimum of $L$ corresponds to $\beta = \beta_0$, which is the minimum value of the dependence $\beta(v)$ for $j_0 = 0$. In the limit of a small velocity, $v^2 \ll \gamma^2 \lambda_1 a$, and using the relation $\int \sin^2 \varphi(x) \, dx = 2$, we find the kinetic energy of the moving domain wall in the form of $E_{kin} = m^* v^2/2$, where $m^* = \lambda_1 \sqrt{\lambda_2 / \gamma^2 \lambda_1} \sqrt{\pi}$ is the effective mass of the domain wall. This is in agreement with the definition from Ref. [4] for $\lambda_1 = 2\pi M^2$.

For $j_0 \neq 0$, we can present the dependence of the factor $F$ on both parameters $v$ and $j_0$ as

$$F(v) = -\lambda_2 \left[ 1 - 2\tilde{j}_0^2 - 2j_0 \tilde{v} \left( \frac{1 - \tilde{j}_0^2}{1 - \tilde{v}^2} \right)^{1/2} \right]. \quad (75)$$
In the limit of \( v \rightarrow 0 \), the factor \( F \) changes its sign for \( j_0 > j_{0,cr} \), where

\[
j_{0,cr} = \frac{e \Omega_0 \sqrt{\lambda_1 \lambda_2}}{\sqrt{2} \hbar \eta}
\]

is the critical current. Thus, if \( j_0 > j_{0,cr} \), the solution with moving domain wall is energetically favorable.

We can interpret the effect of the current as leading to an effective reduction of the effective mass of the domain wall. For \( j_0 > j_{0,cr} \) the current induces an instability towards a spontaneous motion of the wall.

**D. Case of \( \zeta \neq 0 \)**

In the case of \( \zeta \neq 0 \), there are no solutions of Eq. (71) corresponding to the motion of the domain wall with a constant velocity because the last term in this equation acts as a force accelerating the domain wall. Indeed, if we assume a probe solution in the form of \( \varphi(x, t) \equiv \varphi(x - x_0(t)) \) we find

\[
m^* \ddot{x}_0(t) + J_0 A \zeta = 0,
\]

where \( m^* \) is a constant in the limit of a small velocity. In other words, Eq. (77) describes the acceleration of the domain wall just after we apply some voltage. Hence, our model can describe the steady state if we include a viscosity (friction) into the equation of motion. We can use the damping term from Eq. (58). Writing the corresponding additional term in Eq. (71) as \( F_d = -\alpha \partial \varphi / \partial t \), we find the following equation that determines the velocity of the moving domain wall as follows

\[
v \beta(v) \simeq \frac{J_0 \zeta}{\alpha}.
\]

This equation indicates a linear dependence of the velocity on the current in the limit of a small velocity, when \( \beta \) is constant. As we see from Eq. (78), it corresponds to a large damping.

Effective friction may as well stem from the pinning by impurities. This case can be described phenomenologically leading to another mechanism for the critical current.

**IV. CONCLUSIONS**

We calculated the components of the spin torque acting on a thin domain wall in a magnetic nanowire subject to an electric current. These components can induce a rotation of magnetic moments in different directions.

We also considered the dynamics of a domain wall in the presence of the current. We demonstrated that a moving magnetic kink, similar to the static domain wall, can be a solution of the equations for the magnetic dynamics only at some special conditions. We found these conditions in the case of a large ratio of the magnetic anisotropy constants. In the limit of small velocities, the solution does look like a kink but its width decreases with increasing velocity. In the limit of a small velocity, the domain wall moves like a particle of a mass determined by the exchange interaction and anisotropies. One of the spin torque components \( \zeta \), dominating at the small coupling, acts as a moving force on the domain wall, accelerating the wall, provided that there is no pinning to impurities.

Recent direct observations of the domain-wall configurations show that the spin structure of the wall changes with the current, and the structure depends on the velocity of the domain wall motion.

We performed the calculation of torque in the limit of thin domain wall, \( w \ll \lambda_F \). It allows to simplify essentially the problem and to do all the calculations analytically. In reality, this inequality may be not well satisfied even in the magnetic semiconductors. Here we present the estimations for the case when the above-mentioned inequality is obeyed.

Let us take the cross section of the wire \( A = 1 \ \text{nm}^2 \) and the bulk carrier density \( n_{3D} = 10^{19} \ \text{cm}^{-3} \), corresponding to the linear density \( n_{1D} = n_{3D} A = 10^5 \ \text{cm}^{-1} \). It gives us \( k_F = \pi n_{1D} \simeq 3 \times 10^5 \ \text{cm}^{-1} \), and the carrier wavelength \( \lambda_F = 2\pi/k_F \simeq 100 \ \text{nm} \).

To estimate the domain wall width, we assume the magnitude of magnetization \( M = 100 \ \text{Oe} \), the demagnetizing factor along the \( y \) axis \( n^{(y)} = 0.3 \), and calculate the anisotropy constant as \( \lambda_2 \simeq 8\pi n^{(y)} M^2 \simeq 10^5 \ \text{erg/cm}^3 \).

For the energy of magnetic interaction \( E_{\text{int}} \simeq 10 \ \text{meV} \) at a distance between magnetic ions of \( c_0 = 1 \ \text{nm} \), the exchange parameter \( a = E_{\text{int}} c_0 / A \simeq 10^{-8} \ \text{erg/cm} \).

Then the domain wall width has a reasonable value of \( w = (a / \lambda_2)^{1/2} \simeq 10 \ \text{nm} \). Comparing these estimations, we see that the main inequality of \( w \ll \lambda_F \) is satisfied.

At a larger carrier density, both \( w \) and \( \lambda \) can be of the same order of magnitude or the inequality is reversed like in magnetic metals. In this case, the constants \( \zeta \) and \( \eta \) should be calculated numerically.

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**APPENDIX A: SPIN TORQUE DUE TO THE MOMENTUM TRANSFER**

The reflection of electrons from the domain wall is accompanied by the transfer of momentum from the elec-
tron system to the domain wall. In the presence of electric current transmitted through the magnetic wire, it creates an additional force acting on the wall. Here we estimate the magnitude of this effect in the case of a thin DW, $k_F w \ll 1$.

The force $F$ is determined by the total transferred momentum in unit time. Taking into account the contribution of spin up and down scattering states corresponding to the waves incoming from $-\infty$ in the energy range between $\varepsilon_F$ and $\varepsilon + e \Delta \phi$, we find

$$F = \frac{e \Delta \phi}{2\pi} \left[ k_t \left( 1 + |\tau|^2 - |t_{\tau f}|^2 \right) + k_t' \left( 1 + |\tau'|^2 - |t_{\tau f}'|^2 \right) \right].$$

This force tends to shift the domain wall along the $x$ direction. For a local moment within the wall it is equivalent to the appearance of a torque. To estimate the magnitude of this mechanical torque acting on a single moment we use a simplified model.

We describe the domain wall by the $\varphi(x)$ field, which is the angle in $x-y$ plane determining the direction of moment $M(x)$ as shown in Fig. 1. We assume that the shift along $x$ is related to the following interaction

$$H_{int} = \lambda \varphi(x) v(x), \quad (A2)$$

where $\lambda$ is a constant, $v(x) = -d\varphi_0/dx$, and $\varphi_0(x)$ is the static solution for the domain wall. The potential $v(x)$ has the form of a potential well in the vicinity of the domain wall, and it forces (makes energetically favorable) a correction to the $\varphi(x)$ field of the same form, $\delta \varphi(x) \sim \delta \varphi_0/x_0$. On the other hand, the correction $\delta \varphi(x) = (d\varphi_0/dx) \delta x_0$ is the shift along the axis $x$ by $\delta x_0$. Thus, the interaction term in form of (A2) in the equation of motion for the $\varphi(x)$ acts as a shifting force.

The constant $\lambda$ should be determined by the condition that the energy $dE$ associated with the shift, gives the force $F$:

$$F = -\frac{dE}{dx} = \lambda A \int \left( \frac{d\varphi_0}{dx} \right)^2 dx. \quad (A3)$$

Using the known solution, $d\varphi_0/dx = \beta \sin \varphi_0(x)$, we find

$$\lambda = F/2\beta A. \quad (A4)$$

where $\Omega$ is the volume of an elementary cell. We find that the relative contribution of the momentum-induced torque with respect to the spin transfer is

$$T_{mt}/T_{st} \approx k_F \Omega/A \ll 1. \quad (A5)$$

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