Extra dimensions and dark matter

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Abstract. In this paper we study the general scenario of an effective theory coming from the compactification of a higher dimensional theory in a string inspired setting. This leads to gauge coupling unification at an intermediate mass scale. After having computed all the threshold corrections (due to Kaluza–Klein modes) to the running of the couplings of the minimal supersymmetry extension of the standard model, we embark on a detailed phenomenological analysis of the model, based on the numerical package DarkSUSY, to find constraints on the scenario from dark matter data. The mass spectrum of the theory does not have tachyons. Moreover we find that the neutralino is still the lightest supersymmetric particle with a relic density compatible with the most recent experimental data. We find that with respect to the standard mSUGRA scenario the neutralino is higgsino-like in most of the parameter space. Our modifications to the DarkSUSY package will shortly be available upon request.

Keywords: dark matter, extra dimensions

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Contents

1. Introduction 2
2. Extra dimensions, renormalization and the effective potential 3
   2.1. An extra dimensions scenario .................................. 3
   2.2. Computations of the one-loop $\beta$-functions .................. 4
3. Running of the couplings 8
   3.1. Higgs $N = 1$ ................................... 9
      3.1.1. Couplings in the SUSY Lagrangian. .................. 9
      3.1.2. Soft terms in the minimal scenario. .................. 10
      3.1.3. The non-minimal scenario. ....................... 12
   3.2. Higgs $N = 2$ ................................... 13
      3.2.1. Soft terms in the minimal scenario. .................. 14
      3.2.2. Non-minimal scenario. ........................................ 16
4. Phenomenology 16
5. Conclusions 22
   Acknowledgments 24
   Appendix A. $\beta$-functions from the method of the effective potential 24
   Appendix B. Feynman diagram analysis of the soft terms 24
   References 27

1. Introduction

The idea of extra dimensions has brought about an explosion of activity in the field of high energy physics phenomenology. The unification of the couplings of the theory at an intermediate mass scale is in fact particularly appealing. Narrowing the gap between the energy scale that can be probed by current experimental techniques and the unification energy scale makes indirect testing of the latter more reliable.

The connection between the supersymmetry breaking energy scale and the size of the compactified extra dimensions first appeared in [1, 2]. At that time the result was considered a negative one: a breaking scale of the order of the TeV implied a compactification scale of $\approx 10^{18}$ GeV, a region in which perturbative computations in string theory are ruled out. This judgment was influenced by the fact that the fashionable string theory at the time was the heterotic string and that the other consistent string theories were not considered to be relevant from the phenomenological point of view. It was only later, with the discovery of dualities among different string theories [3], that this scenario became appealing [4]: all consistent string theories in fact gained equal status. Non-perturbative computations for a certain string theory became perturbative ones for its dual. Being in the perturbative regime is only one of the desirable features that the theory should have. Another major hurdle for a model building directly coming from string
Extra dimensions and dark matter

theory is given by the presence of moduli. These are scalar fields which appear after the compactification and give flat directions to the potential of the theory. How to fix this problem is not clear yet, even if encouraging progress has been made recently [5]–[9].

Other inputs on the relevance of the extra dimensions scenario for phenomenology could be obtained by a comparison with experimental data of the present and sensitivities of future experiments. The approach in which a low energy model is derived directly from a string theory is called a top-down one, while trying to prune models on the basis of low energy data is called a bottom-up approach. Eventually these two approaches will merge, but for the moment they are still distinct.

This paper provides an example of the bottom-up approach: as suggested in [10] a simple way to achieve an earlier unification is to modify the running of the couplings in an ‘exponential’ fashion. This can be accomplished by taking into account the effects of extra massive Kaluza–Klein (KK) states. The scenario here is quite generic. A detailed knowledge of the compactification mechanism is not needed. At the start only the gauge sector and the Higgs fields are allowed to have a tower of KK states. At a later time also part of the fermionic sector can have a tower.

In this paper we analyse the implications of this scenario for the problem of dark matter using the numerical package DarkSUSY [11] together with a modified version of the ISASUGRA [12] package. In order to do this we compute the one-loop running of all the couplings in the minimal SUSY extension of the standard model (MSSM) in the extra dimension scenario of [10]. To do this efficiently we use the method of the effective potential [14]. See the lecture notes in [15] for a nice summary. The computation of the running of the gauge and Yukawa couplings was performed already in [10], while the running of the soft parameters was first studied in [18] using the spurion formalism. Instead we used a diagrammatic analysis finding some discrepancies between our results and that of the previous reference.

This is the plan of the paper. In section 2 we outline the theoretical framework and the method of the effective potential. In section 3 we present the renormalization group equations in the presence of extra dimensions that we implemented in the numerical package ISASUGRA. In section 4 we show the results of our computations: the spectrum of the theory has no tachyons. The neutralino remains the LSP particle in almost all of the parameter space we have explored. If no fermions are allowed to have a KK tower then the neutralino is higgsino-like. Otherwise it is bino-like for \( \tan \beta < 30 \) and higgsino-like or mixed-like for \( \tan \beta > 30 \).

2. Extra dimensions, renormalization and the effective potential

In this section we deal with the theoretical issues which must be faced to adapt the DarkSusy package to the extra dimensions scenario of [10]. In section 2.1 we review the main aspects of the extra dimensions scenario of [10] while in section 2.2 we review the computation of the one-loop \( \beta \)-functions of the MSSM with the method of the effective potential [14,15].

2.1. An extra dimensions scenario

The MSSM emerges after the extra dimensions are compactified on circles of fixed radius \( R \). The energy scale \( R^{-1} \equiv \mu_0 \) exceeds experimentally attainable energy scales and
represents the threshold beyond which the effects of extra dimensions can be felt. Let $\delta$ be the number of extra dimensions. The effect of the compactification of a $(4+\delta)$-dimensional field along $\delta$ circles of radius $R$ is to allow for the presence of four-dimensional massive excitations, called KK modes, of mass $m_n^2 = m_0^2 + \vec{n} \cdot \vec{n}/R^2$. $\vec{n}$ is a $\delta$-dimensional vector with $n_i \in \mathbb{Z}$. $m_0$ is the mass of the zero-mode. If an additional $\mathbb{Z}_2$ is imposed on the compactified dimensions, the circles change into orbifolds. Fields sitting at the orbifold fixed points are not allowed to have KK modes. This is a convenient mechanism for selecting which fields have massive KK excitations. Let us now denote by $\eta$ the number of generations of MSSM chiral fermions. In the simplest possible scenario $\eta = 0$: no chiral MSSM fermions have KK excitations. We will discuss scenarios with both $\eta = 0$ and $\eta \neq 0$. Compactifications with $\delta = 1, 2$ will exhibit $N = 2$ SUSY. Higher values of $\delta$ will result in theories with an even larger number of supersymmetries. The $N = 1$ vector multiplet, and an additional chiral multiplet, $\Phi$, are such that their KK modes are organized into an $N = 2$ multiplet. Finally the two chiral fields, $Q, \tilde{Q}$, containing the two Higgs fields form an $N = 2$ hypermultiplet. For $\eta \geq 1$ also the chiral multiplet containing one of the chiral fermions and its mirror are organized into an $N = 2$ hypermultiplet. As we said earlier, this $N = 2$ SUSY for the KK modes is welcome since it allows the gauge couplings to be computed à la Wilson. To make the Lagrangian of the theory really $N = 2$ SUSY invariant we must add a term, $Q\Phi \tilde{Q}$, which couples the chiral field in the $N = 2$ multiplet with the fields in the hypermultiplet. This term is only there for the KK modes: zero-modes have, in fact, a $N = 1$ SUSY. To test how robust our scenario is against this term we have performed computations with and without this coupling: as we will see when we will discuss our results, there seem to be minor differences between the two cases which we will dub Higgs $N = 1$ and $N = 2$.

2.2. Computations of the one-loop $\beta$-functions

For energy scales larger than $\mu_0$, KK excitations become possible and their effects have to be incorporated in the computation of the $\beta$-functions. Given the large number of such functions to be computed in the MSSM we prefer to carry out this computation using the method of the effective potential [14,15]. In fact, in SUSY theories all divergences are found in the wavefunction renormalization of chiral multiplets and in the renormalization of gauge coupling constants (equivalent to wavefunction renormalization of vector multiplets). At the one-loop level, it is then sufficient to consider the renormalization of the scalar potential to determine all the renormalization constants of the theory. The one-loop divergent contributions to the scalar potential are given by

$$\delta V = \frac{\Lambda^2}{32\pi^2} \text{Str} \mathcal{M}^2(z) + \frac{1}{64\pi^2} \text{Str} \mathcal{M}^4(z) \ln \left( \frac{\mathcal{M}^2(z)}{\Lambda^2} \right) \quad (2.1)$$

where $\Lambda$ is a cut-off mass parameter. The first term contains the quadratic divergences. $\text{Str} \mathcal{M}^2(z)$ is considered as a function of the scalar fields $z^a$ and their vacuum expectation values. $\text{Str} \mathcal{M}^4(z)$ is the analogous supertrace for the fourth power of the mass matrices. The second term contains all logarithmic one-loop divergences.

\footnote{This chiral multiplet must be odd under the $\mathbb{Z}_2$, so that its zero-modes are missing, to recover the expected $N = 1$ MSSM.}
The scalar potential is
\[ V(z^a, z_a) = V_{\text{susy}}(z^a, z_a) + V_{\text{soft}}(z^a, z_a) \]  
(2.2)

where \( V_{\text{susy}} \) is the standard supersymmetric potential
\[ V_{\text{susy}}(z^a, z_a) = f_a f^a + \frac{1}{2} D^A D^A \]  
(2.3)

and where we used the standard notation \( f_a = df/dz^a, f^a = df/dz_a \). More subscript and superscript indices denote higher order derivatives. The \( D \)-terms read
\[ D^A = -g z_a T^A_{b} z^b \]  
(2.4)
in terms of the generators \( T^A \) of the gauge group for the representation of the chiral multiplets. The superpotential will be written as
\[ f = \frac{1}{2} \mu_{ab} z^a z^b + \frac{1}{6} f_{abc} z^a z^b z^c. \]  
(2.5)

Following the notation in [16], we set \( z^a = H_u, H_d, Q, L, \bar{u}, \bar{d}, \bar{e} \) for the chiral superfields of the theory. Since the top quark, bottom quark and tau lepton are the heaviest fermions in the Standard Model, we will use the approximation in which only the 33 component of the Yukawa matrices is important. Then, in terms of the components with respect to the weak \( SU(2) \), the relevant fields become
\[ Q_3 = (t, b), L_3 = (\nu_\tau, \tau), H_u = (H_u^+, H_u^0), H_d = (H_d^0, H_d^+) \], \( \bar{u}_3 = \bar{t}, \bar{d}_3 = \bar{b}, \bar{e}_3 = \bar{\tau} \) and the superpotential reads
\[ \int d^2 \theta f_a f^a \approx y_t(t\bar{t} H_u^0 - \bar{t}b H_u^+) - y_b(t\bar{b} H_d^- - \bar{b}b H_d^0) - y_\tau(\nu_\tau \bar{\tau} H_d^- - \bar{\tau}\tau H_d^0) + \mu(H_u^+ H_d^- - H_u^0 H_d^0). \]  
(2.6)

\( V_{\text{soft}}(z^a, z_a) \) takes into account soft breaking terms. These are gauge invariant terms which do not add quadratic divergences; that is \( \text{Str} \, M^2(z) \) does not receive any new field dependent contributions from these terms. Taking into account gauge invariance we only need to add scalar and fermionic terms
\[ V_{\text{soft}}(z^a, z_a) = (m^2)_a^b z^a z^b + \xi(z) + \bar{\xi}(\bar{z}) \]  
(2.7)

where \( \xi \) is an arbitrary gauge invariant cubic function of \( z_a \)
\[ \xi(z) = \xi(1)_a z^a + \xi(2)_{ab} z^a z^b + \xi(3)_{abc} z^a z^b z^c. \]  
(2.8)

\( V_{\text{soft}}(z^a, z_a) \) only adds constant terms to the squared mass matrix and (2.7) is the most general expression for scalar soft breaking terms. They contain scalar mass terms of the form \( z \bar{z}, z^2 + \bar{z}^2 \) and trilinear analytic interactions \( z^3 + \bar{z}^3 \). The spin 1/2 mass matrix
\[ -\frac{1}{2} \Delta^{AB} \chi^A \chi^B = -\frac{1}{2} \bar{\Delta}^{AB} \bar{\chi}^A \bar{\chi}^B \]  
(2.9)
instead will only give a constant contribution $\Delta^{AB} \Delta^{BA}$ to $\text{tr} \mathcal{M}_{1/2}^2$. This term is then a soft term too. All other additions to $\mathcal{M}_{1/2}^2$ are not soft in general. These soft breaking terms contain all we need to find an acceptable spectrum of SUSY particles. We have soft terms for raising the masses of gluinos and photinos. Using the same notation as above, denoting by $a_t, a_b, a_\tau$ the 33 component of the matrices $\xi (3)_{QH_u}, \xi (3)_{dQH_d}, \xi (3)_{eLH_d}$ and assuming that the squark and lepton matrices are flavour blind, we get

\begin{equation}
V_{\text{soft}} \approx \left[ -\frac{1}{2} (\Delta^{33} \tilde{g} \tilde{g} + \Delta^{22} \tilde{W} \tilde{W} + \Delta^{11} \tilde{B} \tilde{B}) + \text{c.c.} \right] - m_{Q_3}^2 \tilde{t}^* \tilde{t} - m_{\tilde{q}_3}^2 \tilde{b}^* \tilde{b} \\
- m_{\tilde{l}_3}^2 \tilde{\nu}_\tau \tilde{\nu}_\tau - m_{\tilde{t}_1}^2 \tilde{\tau}^* \tilde{\tau} - m_{\tilde{f}_1}^2 \tilde{\bar{f}}^* \tilde{\bar{f}} - m_{\tilde{e}_1}^2 \tilde{\bar{e}}^* \tilde{\bar{e}} \\
- a_t (t H_u^0 - \bar{b} H_d^+ - \bar{b} H_d^0) - a_b (b H_u^0 - \bar{t} H_d^- - \bar{t} H_d^0) - a_\tau (\nu_\tau \tilde{H}_d^- - \tilde{\tau} H_u^0) \\
+ \left[ b (H_u^+ H_d^- - H_u^0 H_d^0) + \text{c.c.} \right] - m_{H_u}^2 (H_u^+ H_u^- + (H_u^0)^* H_u^0) \\
- m_{\tilde{H}_u}^2 \left[ (H_u^-)^* H_u^- + (H_u^0)^* H_u^0 \right].
\end{equation}

$\tilde{g}, \tilde{W}, \tilde{B}$ are the gaugino, wino and bino. Let us now compute the supertrace of the quartic mass matrix for arbitrary values of the fields $z^a$. Let us start with the mass matrix for spin 1:

\begin{equation}
(M_1^2)^{AB} = D_a^A D_a^B + D_a^B D_a^A.
\end{equation}

The fermionic mass matrix has instead the form

\begin{equation}
\mathcal{M}_{1/2} = \begin{pmatrix} f_{ab} & i \sqrt{2} D_a^A \\ -i \sqrt{2} D_a^A & \Delta^{AB} \end{pmatrix}.
\end{equation}

From this matrix one obtains

\begin{equation}
-2 \text{tr} \mathcal{M}_{1/2}^4 = -2 f_{ab} f_{bc} f_{cd} f_{da} - 16 f_{ab} f_{ac} D_a^A D_{ab} - 16 (D_a^A D_{ab} (D^B_D D^B_a) \\
+ 8 f_{ab} D_a^A \Delta^{AB} D^B_B + f_{ab} D_a^A \Delta^{AB} D^B_B - 16 \Delta^{AC} \Delta^{CD} D_a^A D^B_B
\end{equation}

The mass matrix squared for the scalar fields is given, as usual, in terms of the second derivatives of the potential

\begin{equation}
\mathcal{M}_0^2 = \begin{pmatrix} V^a_b & V^{ac} \\ V^{ba} & V^c_d \end{pmatrix}
\end{equation}

where

\begin{equation}
V_{ab} = f_{a b c} f^c + D_a^A D_b^A + \eta_{ab}
\end{equation}

\begin{equation}
V^{ab} = f_{a b c} f_c + D^A_a D^A_b + \bar{\eta}^{ab}
\end{equation}

\begin{equation}
V^a_b = f^{a c} f_{b c} + D^A_a D^A_b + D^A_a D^A_b + (m^2)^a_b.
\end{equation}
Note that the mass matrix squared is field independent. The soft terms give
\[
\text{tr} \mathcal{M}_\text{soft}^2 = 2(m^2)_a - 2\Delta^{AB}\Delta^{BA}
\]
(2.18)
which is also field independent.

For the trace of the fourth power of the mass matrix, after some calculations we finally obtain
\[
\text{Str} \mathcal{M}^4 = +2g^2[T(R) + 2C(R) - 3C(G)]D^AD^A - 8g^2C(R)f^af_a
\]
\[+ 4f^af_bD^A_bD^A + 2f^abcf^df_e - 2g^2C(R)[z^af_ab + z_a f_b f^{ab}]
\]
\[- 8g^2C(R)\Delta_A(f_a z^a + f^a z_a) - 16g^2C(R)\Delta^2_A z^a z_a - 2\Delta^4_A
\]
\[+ 2f^abcf_a\eta_{ab} + 2f^abcf^{\eta_{ab}} - 2g^2C(R)(\bar{\eta}^a z_a + \eta_a z^a) + 2\eta_{ab}\eta^{ab}
\]
\[+ 4f^abcf_a(m^2)_b + 4g^2C(R)(m^2)_b z_a z^b + 2(m^2)_b (m^2)_a
\]
\[+ 4(m^2)_b (D^A)_a D^A.
\]
(2.19)
From (2.19) we can compute the coefficients of the terms with different powers of the variables $z, \bar{z}$. The renormalization of the gauge coupling constant $g$ and the Yukawa couplings $f_{abc}$ is not affected by the soft terms which do not contribute to the dimension 4 terms $zz\bar{z}\bar{z}$. Then one defines the following renormalization constants:
\[
\hat{\delta}^a = (\delta^a_b - \frac{1}{2} \epsilon^a_b) z^b
\]
(2.20)
\[
\hat{g} = (1 + \rho)g
\]
(2.21)
\[
\hat{f}_{abc} = f_{abc} + \frac{1}{2} \epsilon^a_c f_a\theta_{bc} + \frac{1}{2} \epsilon^b_c f_{a\theta_{bc}} + \frac{1}{2} \epsilon^c_a f_{a\theta_{bc}} + f_{a\theta_{bc}} f_a\theta_{bc}
\]
(2.22)
\[
\hat{\mu}_{ab} = \mu_{ab} + \frac{1}{2} \epsilon^a_c \mu_{a\theta_{bc}} + \frac{1}{2} \epsilon^b_c \mu_{a\theta_{bc}} + \mu_{a\theta_{bc}} \mu_a\theta_{bc}
\]
(2.23)
with
\[
\rho = -2k g^2[T(R) - 3C(G)]
\]
(2.24)
\[
\epsilon^a_b = 4k g^2C(R)\delta^a_b - 2k X^a_b
\]
(2.25)
where $X^a_b = f^{acd}f_{bcd}$ and
\[
\delta^a_b = 2k g^2[C(A) + C(B)]\delta^a_b - 2k X^a_b
\]
(2.26)
\[
f^{a\theta_{bc}} = 2k g^2[C(A) + C(B) + C(C)]\delta^a_\theta\delta^b_\theta\delta^c_\theta.
\]
(2.27)
Furthermore
\[
k = \frac{1}{32\pi^2} \ln(\Lambda/Q)
\]
(2.28)
where $\Lambda$ is an ultraviolet cut-off and $Q$ is the energy scale at which the couplings are computed. As is well known in SUSY theories the couplings renormalize as

$$
\mu_{ab} = (Z_1^{1/2})_a^d (Z_1^{1/2})_b^d \mu_{d'd'}
$$

(2.29)

$$
f_{abc} = (Z_1^{1/2})_a^d (Z_1^{1/2})_b^d (Z_1^{1/2})_c^d f_{d'd'e'}
$$

(2.30)

From (2.30) we infer

$$(Z_1^{1/2})_b^a = \delta_b^a + 4k g^2 C(A) \delta_b^a - k X_b^a.
$$

(2.31)

It is now easy to compute the renormalizations of the couplings and masses of the theory and from this their runnings. In appendix A we collect, for the reader’s convenience, all these results following [15]. It is now easy to specify these formulae for the fields of the MSSM and recover the results of [16] for the running of the MSSM couplings and masses.

In the presence of KK modes the effect of the thresholds is to contribute to the running with power-like terms [10]. The general form of the running of a coupling $g$ will be of the type

$$
Q \frac{dg}{dQ} = (b - \tilde{b}) \ln \frac{\Lambda}{Q} + X_\delta \tilde{b} \left( \frac{\Lambda}{\mu_0} \right)^\delta
$$

(2.32)

$X_\delta$ is a constant depending on the number of compactified dimensions. $b$ is the standard one-loop value of the coefficient, while $\tilde{b}$ is the contribution of the thresholds.

### 3. Running of the couplings

In this section we list the runnings of the couplings of the theory. To find them we used the formulae of appendix A from which we extracted the contributions of the various KK towers. In certain cases we also checked these results against the appropriate Feynman diagrams which renormalize the theory. Appendix B contains some details of the computation which will be fully discussed in [17].

The runnings of the gauge and Yukawa couplings were first studied in [10]: in this case the coefficients $\tilde{b}_a$ take into account the Higgs and gauge boson KK tower and, separately, the contribution of $\eta$ families of chiral fermions. The one-loop coefficients (2.32) for the running of the gauge couplings are

$$
b_a = (\frac{33}{5}, 1, -3)$$

$$
\tilde{b}_a = (3/5, -3, -6) + \eta(4, 4, 4)
$$

(3.1)

where $a = 1, 2, 3$ labels the three gauge groups $U(1), SU(2), SU(3)$. The $U(1)$ generator is rescaled by a factor $3/5$.

As we said earlier, it is convenient to assemble the two $N = 1$ Higgs chiral fields into an $N = 2$ hypermultiplet. To be invariant under $N = 2$ SUSY, besides the kinetic terms for the vector multiplet and the hypermultiplet the Lagrangian must also contain
an interaction term of the following form:

$$\int d^2 \theta \left( \sqrt{2} H_u \Phi H_d + \mu H_u H_d \right) + h.c. \quad (3.2)$$

where $\mu$ plays the role of the usual $\mu$-term appearing in the MSSM while $\Phi$ is the chiral multiplet contained in the $N = 2$ vector multiplet. These are the only admissible terms for an $N = 2$ theory. The first term is a coupling between the hypermultiplet and the vector multiplet, and it is needed to enlarge the amount of supersymmetry. The one-loop diagram built from the interaction vertex associated with this term exactly cancels the one-loop diagram of figure 6(d) of [10] (in whose loop KK states can propagate) thus ensuring that the Higgs wavefunction is immune to the effects of the extra spacetime dimensions. The first term in (3.2) is not present at the level of zero-modes because $\Phi$ is odd under the action of the $Z_2$ of the orbifold. In this way even if the Higgs zero-modes are assembled into an $N = 2$ hypermultiplet they still have an anomalous dimension.

To check how robust the scenario is with respect to the presence of this term, we have studied two different cases which we call Higgs $N = 1$ and $N = 2$. In the former case we assume that the KK states behave as their zero-modes and that the first term in (3.2) is absent. In the latter case this term is switched on.

3.1. Higgs $N = 1$

3.1.1. Couplings in the SUSY Lagrangian. In this case the anomalous dimension receives a contribution from the diagrams in which particles from the KK tower of the gauge bosons circulate in the loop leading to a power law contribution. The wavefunction renormalization factor of the field $i$ is

$$Z_i = 1 - \frac{(\gamma_i(\mu_0) - \tilde{\gamma}_i(\mu_0))}{2\pi} \ln \frac{\Lambda}{\mu_0} - \frac{X_5}{\delta} \tilde{\gamma}_i(\mu_0) \left[ \left( \frac{\Lambda}{\mu_0} \right)^{\delta} - 1 \right] \quad (3.3)$$

leading, for the Higgs fields to

$$\tilde{\gamma}_{H_u} = -\frac{3}{10}\alpha_1 - \frac{3}{2}\alpha_2 \quad (3.4)$$

$$\tilde{\gamma}_{H_d} = -\frac{3}{10}\alpha_1 - \frac{3}{2}\alpha_2 \quad (3.5)$$

with $\alpha_i \equiv g_i^2/4\pi$. As far as the Yukawa couplings are concerned, we get

$$16\pi^2 \frac{d}{dt} y_t = y_t \left[ 3|y_t|^2 + X_5 \left( \frac{\Lambda}{\mu_0} \right)^{\delta} \left( 3|y_t|^2 + |y_b|^2 - \frac{16}{3} g_3^2 - 3g_2^2 - \frac{13}{15} g_1^2 \right) \right] \quad (3.6)$$

$$16\pi^2 \frac{d}{dt} y_b = y_b \left[ 3|y_b|^2 + |y_t|^2 + X_5 \left( \frac{\Lambda}{\mu_0} \right)^{\delta} \left( |y_t|^2 + 3|y_b|^2 - \frac{16}{3} g_3^2 - 3g_2^2 - \frac{7}{15} g_1^2 \right) \right] \quad (3.7)$$

$$16\pi^2 \frac{d}{dt} y_\tau = y_\tau \left[ |y_\tau|^2 + 3|y_b|^2 + X_5 \left( \frac{\Lambda}{\mu_0} \right)^{\delta} \left( 3|y_\tau|^2 - 3g_2^2 - \frac{9}{5} g_1^2 \right) \right]. \quad (3.8)$$
Finally we analyse the (supersymmetric) Higgs mass

\[ 16\pi^2 \frac{d}{dt} \mu = \mu \left[ \left( 3|y_t|^2 + 3|y_b|^2 + |y_\tau|^2 \right) + X_\delta \left( \frac{\Lambda}{\mu_0} \right)^\delta \left( -3g_2^2 - \frac{3}{5}g_1^4 \right) \right]. \] (3.9)

In the Higgs \( N = 2 \) case \( \mu \) will not be renormalized as a consequence of a non-renormalization theorem.

3.1.2. Soft terms in the minimal scenario. While the effective potential method is powerful in determining the runnings for the parameters in the SUSY Lagrangian, it is difficult to apply it to the computation of the soft terms in the presence of the KK states. This is why we have decided to also carry out an analysis in terms of Feynman diagrams. In appendix A we then collect the general forms of the runnings of the couplings and masses in (2.10). As a check, the interested reader can sum (B.1)–(B.4) to recover the result of (A.5) when specialized to the parameter \( b \) in (2.10). The computation of the effect of extra dimensions is now straightforward: each time that in a Feynman diagram KK states are allowed to circulate in the loop (external states can only be zero-modes), the contribution of the diagram must be added to the \( \tilde{\mu} \) coefficient in (2.32). We finally remind the reader that in an orbifold compactification the KK momentum is conserved only in the bulk. The wavefunction in the compactified dimension must, in fact, be expanded in a basis which is invariant under the discrete group acting on the compactified dimension. Furthermore the orbifold fixed points break translation invariance along the extra dimensions.

Following the previous prescription we obtain the \( \beta \)-functions for the trilinear couplings

\[ 16\pi^2 \frac{d}{dt} a_t = 9a_t|y_t|^2 + X_\delta \left( \frac{\Lambda}{Q_0} \right)^\delta \left\{ a_t \left[ 9|y_t|^2 + |y_b|^2 - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{13}{15}g_1^4 \right] \right\} \]

\[ + 2a_b y_b^* y_t + y_t \left[ \frac{32}{3}g_3^2 M_3 + 6g_2^2 M_2 + \frac{26}{15}g_1^2 M_1 \right] \] (3.10)

\[ 16\pi^2 \frac{d}{dt} a_b = 9a_b|y_b|^2 + a_b|y_\tau|^2 + 2a_\tau y_\tau^* y_b + X_\delta \left( \frac{\Lambda}{Q_0} \right)^\delta \left\{ a_b \left[ 9|y_b|^2 + |y_\tau|^2 \right] \right\} \]

\[ - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{7}{15}g_1^4 \right]+ 2a_\tau y_\tau^* y_b + y_\tau \left[ \frac{32}{3}g_3^2 M_3 + 6g_2^2 M_2 + \frac{14}{15}g_1^2 M_1 \right] \} \] (3.11)

\[ 16\pi^2 \frac{d}{dt} a_\tau = 3a_\tau|y_\tau|^2 + 3a_\tau|y_\tau|^2 + X_\delta \left( \frac{\Lambda}{Q_0} \right)^\delta \left\{ a_\tau \left[ 9|y_\tau|^2 \right] \right\} \]

\[ - 3g_2^2 - \frac{8}{3}g_1^4 \right]+ 6a_b y_b^* y_\tau + y_\tau \left[ 6g_2^2 M_2 + \frac{18}{5}g_1^2 M_1 \right] \} \). \] (3.12)

The \( \beta \)-function for the \( b \) parameter (crucial for the electroweak symmetry breaking
mechanism) is given by
\[
16\pi^2 \frac{d}{dt} b = b \left( 3|y_t|^2 + 3|y_b|^2 + |y_\tau|^2 \right) + \mu \left[ 6a_t y_t + 6a_b y_b + 2a_\tau y_\tau \right] \\
+ X_\delta \left( \frac{\Lambda}{Q_0} \right) \delta \left\{ b \left( -3g_2^2 - \frac{3}{5}g_1^2 \right) + \mu \left( +6g_2^2 M_2 + \frac{6}{5}g_1^2 M_1 \right) \right\}. \tag{3.13}
\]

Now we can show the $\beta$-function for all the soft squared masses. For the two Higgs mass parameters we have
\[
16\pi^2 \frac{d}{dt} m_{H_u}^2 = 6|y_t|^2 \left( m_{H_u}^2 + m_t^2 + m_{\tilde{t}}^2 \right) + 6|a_t|^2 + \frac{3}{5}g_1^2 \text{tr}'(Y m^2) \\
+ X_\delta \left( \frac{\Lambda}{Q_0} \right) \delta \left\{ -6g_2^2 |M_2|^2 - \frac{6}{5}g_1^2 |M_1|^2 + \frac{3}{5}g_1^2 \left( m_{H_u}^2 - m_{H_d}^2 \right) \right\} \tag{3.14}
\]
\[
16\pi^2 \frac{d}{dt} m_{H_d}^2 = 6|y_b|^2 \left( m_{H_d}^2 + m_t^2 + m_{\tilde{t}}^2 \right) + 6|a_b|^2 + 2|a_\tau|^2 \\
+ 2|y_\tau|^2 \left( m_{H_d}^2 + m_t^2 + m_{\tilde{t}}^2 \right) - \frac{3}{5}g_1^2 \text{tr}'(Y m^2) \\
+ X_\delta \left( \frac{\Lambda}{Q_0} \right) \delta \left\{ -6g_2^2 |M_2|^2 - \frac{6}{5}g_1^2 |M_1|^2 - \frac{3}{5}g_1^2 \left( m_{H_u}^2 - m_{H_d}^2 \right) \right\} \tag{3.15}
\]

while the equations for the squark mass terms read
\[
16\pi^2 \frac{d}{dt} m_{\tilde{t}}^2 = 2|y_t|^2 m_{\tilde{t}}^2 + 2|y_b|^2 m_{\tilde{b}}^2 + \frac{1}{5}g_1^2 \text{tr}'(Y m^2) \\
+ X_\delta \left( \frac{\Lambda}{Q_0} \right) \delta \left\{ -\frac{32}{3}g_3^2 |M_3|^2 - \frac{32}{3}g_2^2 |M_2|^2 - \frac{2}{15}g_1^2 |M_1|^2 + 2|a_t|^2 + 2|a_b|^2 \\
+ 2|y_\tau|^2 \left( m_{H_u}^2 + m_{\tilde{t}}^2 \right) + 2|y_\tau|^2 \left( m_{H_d}^2 + m_{\tilde{t}}^2 \right) + \frac{1}{5}g_1^2 \left( m_{H_u}^2 - m_{H_d}^2 \right) \right\} \tag{3.16}
\]
\[
16\pi^2 \frac{d}{dt} m_{\tilde{b}}^2 = 4|y_b|^2 m_{\tilde{b}}^2 - \frac{4}{5}g_1^2 \text{tr}'(Y m^2) \\
+ X_\delta \left( \frac{\Lambda}{Q_0} \right) \delta \left\{ -\frac{32}{3}g_3^2 |M_3|^2 - \frac{32}{3}g_2^2 |M_2|^2 + 4|a_t|^2 \\
+ 4|y_t|^2 \left( m_{H_u}^2 + m_{\tilde{t}}^2 \right) - \frac{4}{5}g_1^2 \left( m_{H_u}^2 - m_{H_d}^2 \right) \right\} \tag{3.17}
\]
\[
16\pi^2 \frac{d}{dt} m_{\tilde{\tau}}^2 = 4|y_\tau|^2 m_{\tilde{\tau}}^2 + \frac{2}{5}g_1^2 \text{tr}'(Y m^2) \\
+ X_\delta \left( \frac{\Lambda}{Q_0} \right) \delta \left\{ -\frac{32}{3}g_3^2 |M_3|^2 - \frac{8}{15}g_1^2 |M_1|^2 + 4|a_b|^2 \\
+ 4|y_b|^2 \left( m_{H_d}^2 + m_{\tilde{b}}^2 \right) + \frac{2}{5}g_1^2 \left( m_{H_u}^2 - m_{H_d}^2 \right) \right\}. \tag{3.18}
\]
Finally, for the slepton masses we have
\[
16\pi^2 \frac{d}{dt} m^2_L = 2|y_\tau|^2 m^2_\tau - \frac{3}{5} g_1^2 \text{tr}'(Ym^2)
\]
\[
+ X_\delta \left( \frac{\Lambda}{Q_0} \right)^{\delta} \left\{ -6g_2^2 |M_2|^2 - \frac{6}{5} g_1^2 |M_1|^2 + 2|a_\tau|^2
\right.
\]
\[
+ 2|y_\tau|^2 (m^2_{H_d} + m^2_{\tilde{t}}) - \frac{3}{5} g_1^2 (m^2_{H_u} - m^2_{H_d}) \right\}
\]
\[
16\pi^2 \frac{d}{dt} m^2_\tilde{e} = 4|y_\tau|^2 m^2_L + \frac{6}{5} g_1^2 \text{tr}'(Ym^2)
\]
\[
+ X_\delta \left( \frac{\Lambda}{Q_0} \right)^{\delta} \left\{ -\frac{24}{5} g_1^2 |M_1|^2 + 4|a_\tau|^2
\right.
\]
\[
+ 4|y_\tau|^2 (m^2_{H_d} + m^2_{\tilde{t}}) + \frac{6}{5} g_1^2 (m^2_{H_u} - m^2_{H_d}) \right\}
\]
where, in order to avoid cumbersome expressions, we have introduced the quantity
\[
\text{tr}'(Ym^2) \equiv \text{tr}(Ym^2) - m^2_{H_u} + m^2_{H_d}
\]
(3.19)
where the usual trace over the hypercharge, defined in the MSSM, is given by
\[
\text{tr}(Ym^2) = m^2_{H_u} - m^2_{H_d} + m^2_Q - 2 \left( m^2_{\tilde{u}} + m^2_{\tilde{c}} + m^2_{\tilde{b}} \right)
\]
\[
+ m^2_{\tilde{d}} + m^2_{\tilde{s}} + m^2_{\tilde{t}} - m^2_{\tilde{L}} + m^2_{\tilde{\mu}} + m^2_{\tilde{\tau}}.
\]
(3.20)
In our computations we have assumed the first two families to be degenerate in mass, such that
\[
m^2_Q = 2m^2_{Q_1} + m^2_{Q_3}
\]
\[
m^2_L = 2m^2_{L_1} + m^2_{L_3}
\]
\[
m^2_\tilde{e} = m^2_{\tilde{\mu}}
\]
\[
m^2_\tilde{\mu} = m^2_{\tilde{\tau}}.
\]
(3.21)

3.1.3. The non-minimal scenario. In the non-minimal scenario we introduce KK towers also for the matter fields. We must distinguish among three different cases: \( \eta = 1 \) in which only the third family have a KK tower, \( \eta = 2 \) in which only the first two families have a KK tower and \( \eta = 3 \) in which all the families have KK tower. The choice for the \( \eta = 2 \) case is dictated by the usual third-family approximation and by a constraint imposed by the ISASUGRA code concerning the mass degeneracy of the first two families.

One of the consequences of having KK towers for the matter fields is that they no longer live at the orbifold fixed points and they are allowed to be in the bulk.

The anomalous dimensions of the Higgs fields are the same in the \( \eta = 0 \) and \( \eta = 2 \) cases. For \( \eta = 1, 3 \) we find
\[
\tilde{\gamma}_{H_u} = \gamma_{H_u},
\]
(3.24)
\[
\tilde{\gamma}_{H_d} = \gamma_{H_d}.
\]
(3.25)
For the chiral fields $t$, $b$, $\tau$ we have a similar situation and the coefficients of the $\beta$-function have only a power law contribution.

Let us now discuss the soft terms: for the trilinear couplings and the $b$ parameter, we must distinguish between the $\eta = 2$ and $\eta = 1, 3$ cases. In the former case the running is the same of the minimal case due to the third-family approximation. In the latter case, once again, the $\beta$-function coefficients have only a power law behaviour.

Finally the running of soft scalar masses in the non-minimal cases is given by a logarithmic term multiplied by

$$\text{tr}'(Ym^2)$$

and a power law term given by

$$\text{tr}(Ym^2) - \text{tr}'(Ym^2).$$

In the $\eta = 1$ case the primed trace is defined as

$$\text{tr}'_{\eta=1}(Ym^2) = 2m_Q^2 - 2(m_0^2 + m_e^2) + m_d^2 + m_s^2 - 2m_{L_1}^2 + 2m_{\bar{e}}^2$$

while for $\eta = 3$

$$\text{tr}'_{\eta=3}(Ym^2) = 0$$

because all the families have KK tower and there is no logarithmic contribution. Finally for the case $\eta = 2$ we have

$$\text{tr}'_{\eta=2}(Ym^2) = m_Q^2 - 2(m_t^2 + m_b^2 + m_{L_3}^2 + m_{\bar{e}}^2).$$

### 3.2. Higgs $\mathcal{N} = 2$

With respect to the previous section, in this case the changes are given by the presence of the term (3.2). Following [10] the wavefunction renormalizations of the Higgs fields are given by

$$\tilde{\gamma}_{H_u} = 0$$

$$\tilde{\gamma}_{H_d} = 0.$$  

The one-loop beta functions for the Yukawa couplings are then

$$16\pi^2 \frac{d}{dt} y_t = y_t \left[ 3|y_t|^2 - \frac{3}{10} g_1^2 - \frac{3}{2} g_2^2 + X_\delta \left( \frac{\Lambda}{Q_0} \right)^\delta \left( 3|y_t|^2 + |y_b|^2 - \frac{16}{3} g_3^2 - \frac{3}{2} g_2^2 - \frac{17}{30} g_1^2 \right) \right]$$

$$16\pi^2 \frac{d}{dt} y_b = y_b \left[ 3|y_b|^2 + |y_t|^2 - \frac{3}{10} g_1^2 - \frac{3}{2} g_2^2 + X_\delta \left( \frac{\Lambda}{Q_0} \right)^\delta (|y_t|^2 + 3|y_b|^2) - \frac{16}{3} g_3^2 - \frac{3}{2} g_2^2 - \frac{17}{30} g_1^2 \right]$$

$$16\pi^2 \frac{d}{dt} y_\tau = y_\tau \left[ |y_\tau|^2 + 3|y_b|^2 - \frac{3}{10} g_1^2 - \frac{3}{2} g_2^2 + X_\delta \left( \frac{\Lambda}{Q_0} \right)^\delta \left( 3|y_\tau|^2 - \frac{3}{2} g_2^2 - \frac{3}{2} g_1^2 \right) \right].$$
For the (supersymmetric) Higgs mass, by virtue of the non-renormalization theorem, we have
\[
\frac{d}{dt}\mu = 0.
\] (3.36)

This equation holds for any \( \eta \). The \( \beta \)-functions for the \( b \) parameter read
\[
16\pi^2 \frac{d}{dt} b = b \left( 3|y_t|^2 + 3|y_b|^2 + |y_\tau|^2 \right) + \mu \left( 6a_t y_t^* + 6a_b y_b^* + 2a_\tau y_\tau^* \right)
+ b \left( -3g_2^2 - \frac{3}{3} g_1^2 \right) + \mu \left( +6g_2^2 M_2 + \frac{6}{3} g_1^2 M_1 \right). 
\] (3.37)

3.2.1. Soft terms in the minimal scenario. We begin with the trilinear terms, which look as follows:
\[
16\pi^2 \frac{d}{dt} a_t = a_t \left( 9|y_t|^2 - \frac{3}{10} g_1^2 - \frac{3}{2} g_2^2 \right) + X_\delta \left( \frac{\Lambda}{Q_0} \right)^{\delta} \left\{ a_t \left[ \frac{16}{3} g_3^2 - \frac{3}{2} g_2^2 - \frac{17}{30} g_1^2 \right] 
+ a_t \left( 9|y_t|^2 + |y_b|^2 \right) + 2a_b y_b^* y_t 
+ y_t \left[ \frac{32}{3} g_3^2 M_3 + 6g_2^2 M_2 + \frac{26}{15} g_1^2 M_1 \right] \right\} 
\] (3.38)
\[
16\pi^2 \frac{d}{dt} a_b = a_b \left( 9|y_b|^2 + |y_\tau|^2 - \frac{3}{10} g_1^2 - \frac{3}{2} g_2^2 \right) + 2a_\tau y_\tau^* y_b
+ X_\delta \left( \frac{\Lambda}{Q_0} \right)^{\delta} \left\{ a_b \left[ 9|y_b|^2 + |y_t|^2 \right] - \frac{16}{3} g_3^2 - \frac{3}{2} g_2^2 - \frac{1}{6} g_1^2 \right] + 2a_t y_t^* y_b 
+ y_b \left[ \frac{32}{3} g_3^2 M_3 + 6g_2^2 M_2 + \frac{14}{15} g_1^2 M_1 \right] \right\} 
\] (3.39)
\[
16\pi^2 \frac{d}{dt} a_\tau = a_\tau \left( 3|y_\tau|^2 + 3|y_\tau|^2 - \frac{3}{10} g_1^2 - \frac{3}{2} g_2^2 \right) + 6a_b y_b^* y_\tau 
+ X_\delta \left( \frac{\Lambda}{Q_0} \right)^{\delta} \left\{ a_\tau \left[ 9|y_\tau|^2 - \frac{3}{2} g_2^2 - \frac{3}{2} g_1^2 \right] 
+ y_\tau \left[ 6g_2^2 M_2 + \frac{18}{5} g_1^2 M_1 \right] \right\}. 
\] (3.40)

The \( \beta \)-functions for the soft squared masses of the Higgs fields are
\[
16\pi^2 \frac{d}{dt} m_{H_u}^2 = 6|y_t|^2 \left( m_{H_u}^2 + m_t^2 + m_\tau^2 \right) + 6|a_t|^2 + \frac{3}{5} g_1^2 \text{tr}(Y m^2)
- 6g_2^2 |M_2|^2 - \frac{6}{3} g_1^2 |M_1|^2 
\] (3.41)
\[
16\pi^2 \frac{d}{dt} m_{H_d}^2 = 6|y_b|^2 \left( m_{H_d}^2 + m_t^2 + m_\tau^2 \right) + 6|a_b|^2 + 2|a_\tau|^2
+ 2|y_\tau|^2 \left( m_{H_d}^2 + m_t^2 + m_\tau^2 \right) - \frac{3}{5} g_1^2 \text{tr}(Y m^2)
- 6g_2^2 |M_2|^2 - \frac{6}{3} g_1^2 |M_1|^2 
\] (3.42)

while for the squarks we have

\[
16 \pi^2 \frac{d}{dt} m_t^2 = 2 |y_t|^2 m_t^2 + 2 |y_b|^2 m_b^2 + \frac{1}{5} g_1^2 \text{tr}'(Y m^2) \\
+ X_\delta \left( \frac{\Lambda}{Q_0} \right)^\delta \left\{ -\frac{32}{3} g_2^2 |M_3|^2 - 6 g_2^2 |M_2|^2 - \frac{2}{15} g_1^2 |M_1|^2 + 2 |a_t|^2 + 2 |a_b|^2 \\
+ 2 |y_t|^2 (m_{H_u}^2 + m_t^2) + 2 |y_b|^2 (m_{H_d}^2 + m_b^2) + \frac{1}{5} g_1^2 (m_{H_u}^2 - m_{H_d}^2) \right\} \tag{3.43}
\]

\[
16 \pi^2 \frac{d}{dt} m_t^2 = 4 |y_t|^2 m_t^2 - \frac{4}{5} g_1^2 \text{tr}'(Y m^2) \\
+ X_\delta \left( \frac{\Lambda}{Q_0} \right)^\delta \left\{ -\frac{32}{3} g_2^2 |M_3|^2 - \frac{32}{15} g_1^2 |M_1|^2 + 4 |a_t|^2 \\
+ 4 |y_t|^2 (m_{H_u}^2 + m_t^2) - \frac{4}{5} g_1^2 (m_{H_u}^2 - m_{H_d}^2) \right\} \tag{3.44}
\]

\[
16 \pi^2 \frac{d}{dt} m_b^2 = 4 |y_b|^2 m_b^2 + \frac{2}{5} g_1^2 \text{tr}'(Y m^2) \\
+ X_\delta \left( \frac{\Lambda}{Q_0} \right)^\delta \left\{ -\frac{32}{3} g_2^2 |M_3|^2 - \frac{8}{15} g_1^2 |M_1|^2 + 4 |a_b|^2 \\
+ 4 |y_b|^2 (m_{H_d}^2 + m_b^2) + \frac{2}{5} g_1^2 (m_{H_u}^2 - m_{H_d}^2) \right\} \tag{3.45}
\]

Finally the equations for the leptons read

\[
16 \pi^2 \frac{d}{dt} m_L^2 = 2 |y_\tau|^2 m_L^2 - \frac{3}{5} g_1^2 \text{tr}'(Y m^2) \\
+ X_\delta \left( \frac{\Lambda}{Q_0} \right)^\delta \left\{ -6 g_2^2 |M_2|^2 - \frac{6}{5} g_1^2 |M_1|^2 + 2 |a_\tau|^2 \\
+ 2 |y_\tau|^2 (m_{H_d}^2 + m_L^2) - \frac{3}{5} g_1^2 (m_{H_u}^2 - m_{H_d}^2) \right\} \tag{3.46}
\]

\[
16 \pi^2 \frac{d}{dt} m_\tau^2 = 4 |y_\tau|^2 m_\tau^2 + \frac{6}{5} g_1^2 \text{tr}'(Y m^2) \\
+ X_\delta \left( \frac{\Lambda}{Q_0} \right)^\delta \left\{ -\frac{24}{5} g_1^2 |M_1|^2 + 4 |a_\tau|^2 \\
+ 4 |y_\tau|^2 (m_{H_d}^2 + m_\tau^2) + \frac{6}{5} g_1^2 (m_{H_u}^2 - m_{H_d}^2) \right\}. \tag{3.47}
\]

Our results (3.43)–(3.47) for the masses of the squarks and sleptons differ from those reported in [18] in two respects: on the one hand the terms proportional to \( Y \) and \( m_{H_u}^2 - m_{H_d}^2 \), due to the \( U(1) \) gauge factor, are absent in [18]. On the other hand in (3.43)–(3.47) there are terms coming from the diagrams (B.13) which do not get contributions from the KK states and therefore do not have a power law type of running.

\(^2\) For an explicit evaluation of this term, given by the diagram (B.14), see for example (4.12) in [19].
3.2.2. Non-minimal scenario. The results we found in section 3.1.3 for \( N = 1 \) are still valid. The only difference concerns the soft mass terms: for the Higgs masses, \( m_{H^u}^2, m_{H^d}^2 \), no diagrams will contribute to the power law. The equations are then the same as in the MSSM. On the other hand the squark masses have only power law contributions. For the other soft terms the equations are the same as for the minimal scenario, regardless of the value of \( \eta \).

4. Phenomenology

In this section we present the results for the low energy phenomenology of our scenario.

To compute the weak scale parameters we used the publicly available codes ISASUGRA [12] and DarkSUSY [11]. We modified the ISAJET routines in order to take into account the power law running of the gauge couplings, of the Yukawa couplings and of all the soft terms following the procedure outlined in the previous sections. We solved the differential equations given by the running of the couplings and masses, imposing boundary conditions as in the usual mSUGRA scenario [13]. In this way we have, at the new unification scale \( M'_\text{GUT} \), a universal scalar mass \( m_0 \), a universal gaugino mass \( m_{1/2} \) and a common trilinear coupling \( A_0 \). The only other parameters which need to be specified are \( \tan \beta \) and \( \text{sgn}(\mu) \).

For a fixed choice of the extra dimensional parameters \( \mu_0, \delta, \eta \) we performed a detailed scan of the \((m_0, m_{1/2})\) parameter space keeping \( \tan \beta, A_0 \) and \( \text{sgn}(\mu) \) fixed. We computed for every model the thermal relic density

\[
\Omega_\chi h^2 = \frac{m_\chi n_\chi}{\rho_c} \quad (4.1)
\]

solving the Boltzmann equation for the neutralino number density \( n_\chi(t) \)

\[
\frac{dn_\chi}{dt} + 3Hn_\chi = -\langle \sigma_A v \rangle \left[ (n)^2 - (n_{\text{eq}})^2 \right] \quad (4.2)
\]

where \( H = \dot{a}/a \) is the Hubble expansion parameter (with the dot denoting the time derivative), \( a = a(t) \) is the scale factor of the Universe, \( \langle \sigma_A v \rangle \) is the thermal averaged annihilation cross section and \( n_{\text{eq}} \) is the equilibrium number density. We considered only tree-level annihilation processes in the computation of \( \langle \sigma_A v \rangle \). This implies that we do not have to worry about the presence of the KK states. In fact KK modes are too heavy to be produced in the intermediate tree-level annihilation channels. This is guaranteed by the fact that the neutralino mass (i.e. the mass of the lightest supersymmetric particle) is at most of the order of 1 TeV for every model we considered. For the relic density computation we have taken into account all the possible coannihilations with other sparticles. Because the mass of the first KK excited level is of order \( \mu_0 \) there is no possible coannihilation between the neutralino (a zero-mode particle) and the KK particles. In this way the standard treatment of the coannihilations is still valid. A possible difference from the standard scenario may arise from the Hubble expansion rate \( H \). In the standard framework, this term is responsible for the density \( n_\chi \propto a^{-3} \), in the absence of interactions. Unfortunately in the literature there is not a general expression for the Hubble expansion rate \( H \) for an arbitrary number of extra dimensions \( \delta \). There are some interesting issues.

\[\text{See [24]–[26] for general reviews on the subject.}\]
Table 1. Values of the $D = 5$ Planck mass $M_5$ and of $\rho_0$ for different values of the compactification scale $\mu_0$.

| $\mu_0 \equiv 1/R$ (GeV) | $M_5 = (\mu_0 M_5^2)^{1/3}$ (GeV) | $\rho_0 = 96\pi G M_5^6$ (GeV cm$^{-3}$) |
|-------------------------|------------------------------|---------------------|
| $10^5$                  | $2.15 \times 10^{14}$       | $10^8$              |
| $10^6$                  | $4.64 \times 10^{14}$       | $10^{10}$           |
| $10^8$                  | $2.15 \times 10^{15}$       | $10^{14}$           |
| $10^{10}$               | $1 \times 10^{16}$         | $10^{18}$           |
| $10^{12}$               | $4.64 \times 10^{16}$       | $10^{22}$           |

concerning the $(D = 6)$-dimensional case in [27, 28] and also in [29, 30]. We considered explicitly only the case $D = 5$ [31] and we argued that the conclusion still holds in the higher dimensional case\(^4\). Let us consider the case $\delta = 1$. Following [20] and [32] we have

$$H^2 = \frac{8\pi G}{3} \rho \left(1 + \frac{\rho}{\rho_0}\right) + \frac{C}{a^4}$$

where

$$\rho_0 = 96\pi G M_5^6$$

with $M_5$ being the five-dimensional Planck mass. The third term, where $C$ is an integration constant, is referred to as the dark radiation term. The second term proportional to $\rho^2$ and the dark radiation term are the new ingredients in the brane world cosmology and they lead to a non-standard expansion law. The second term is the dominant one when $\rho \gg \rho_0$. In order to satisfy this condition we must have $M_5 \lesssim 10^4$ TeV.

The $D = 5$ Planck mass is related to the $D = 4$ Planck mass through the relation

$$(M_5)^3 R = M_5^2.$$ 

where $R \equiv 1/\mu_0$ is the compactification scale. In our model we obtain the values collected in table 1. This result holds in general in a model with flat extra spacetime dimensions. Typical values of the halo density $\rho$ are of order of 1 GeV cm$^{-3}$ so the $\rho^2$-term is always subdominant. Hence the usual Friedmann equation holds.

Let us fix the notation defining the lightest neutralino as the linear combination

$$\tilde{\chi}_1^0 = N_{11} \tilde{B} + N_{12} \tilde{W} + N_{13} \tilde{H}_u + N_{14} \tilde{H}_d$$

where $\tilde{B}$ and $\tilde{W}$ are the bino and wino fields while $\tilde{H}_u$ and $\tilde{H}_d$ are the two higgsinos. We also define the gaugino fraction as

$$Z_g = |N_{11}|^2 + |N_{12}|^2.$$ 

We say that a neutralino is gaugino-like (in particular in our case bino-like) if $Z_g > 0.9$ while is higgsino-like when $Z_g < 0.1$. In all the intermediate cases we denote the neutralino as mixed-like.

As explained in section 3 we considered two different scenarios: one in which the two zero-mode Higgs fields are chiral $N = 1$ superfields as in the MSSM and the other in which they form an $N = 2$ matter hypermultiplet.

\(^4\) E Kiritsis told us he performed unpublished computations that support this assumption.
We first analyse the $N = 1$ Higgs case in the minimal scenario $\eta = 0$ in which the matter fermions do not have KK towers. We performed a detailed scan in the parameter space $(m_0, m_{1/2})$ fixing all the other parameters. In figure 1 we show the regions already excluded for either theoretical or experimental reasons for a given model ($\mu_0 = 10^5$ GeV, $\delta = 1$ and $\eta = 0$). The dark grey region is excluded because the $\tilde{\tau}$ is the LSP rather than the neutralino, the red region is excluded because the models do not achieve electroweak symmetry breaking (EWSB), while the blue region is excluded because the models do not satisfy the current accelerator bounds (limits on the chargino masses, $b \to s\gamma$, etc). In almost all the parameter space the neutralino is still the LSP. One of the main results is that, unlike in the standard mSUGRA case, the neutralino is no longer bino-like but tends to be a very pure higgsino (see figure 2). This conclusion strongly depends on the value of the compactification scale $\mu_0$. In fact the regions in which the neutralino is higgsino-like get smaller for higher values of $\mu_0$ (see the left panel of figure 2 for the case $\mu_0 = 10^5$ GeV and figure 4 for the two cases $\mu_0 = 10^8$ GeV and $\mu_0 = 10^{10}$ GeV). For higher values of $\mu_0$ the higgsino-like region approaches the region excluded due to an incorrect EWSB. The previous result is only slightly dependent on the number of extra dimensions $\delta$ as can be seen by comparing the two panels of figure 2. The only difference is a change in the shape of the excluded regions. In other words the crucial property of the $\beta$-functions is the power law behaviour rather than the effective power law index, i.e. $\delta$.

In every contour plot for the gaugino fraction we have shown the cosmologically allowed regions. The red regions are those for which the relic density $\Omega h^2$ satisfies the WMAP constraints $[33, 34]$

$$0.09 \leq \Omega h^2 \leq 0.13$$

(4.8)
Figure 2. Contour plots of the gaugino fraction in the plane \((m_0, m_{1/2})\) for different values of \(\delta\).

while the green regions denote the pre-WMAP constraints

\[
0.13 < \Omega h^2 \leq 0.30. \tag{4.9}
\]

The cosmologically allowed regions have a huge overlap with the pure higgsino-like region, especially for low values of \(\mu_0\), i.e. \(\mu_0 \lesssim 10^8\) GeV. For higher values of \(\mu_0\) the cosmologically allowed regions tend in general to overlap with a mixed-like neutralino region.

We also present in figure 3 the isomass contour plots for the neutralino for increasing \(\mu_0\) and for fixed \(\delta\). As in the case of the gaugino fraction we show the cosmologically allowed regions. For higher values of \(\mu_0\) high neutralino mass contours (around 1 TeV) are shifted upwards so that models that possess the right relic density have \(m_\chi \lesssim 500\) GeV. Increasing the value of \(A_0\), for example to \(A_0 = 2500\), implies a shift of the cosmologically allowed regions towards higher values of the neutralino mass of about \(m_\chi \gtrsim 800\) GeV and a wider region excluded by the accelerator bounds. Moreover, in this case, the neutralino is a very pure higgsino \(Z_g < 0.1\) throughout the allowed parameter space.

For increasing values of \(\tan \beta\) we obtain the same shift of the cosmologically allowed regions as in the case of high \(A_0\). Moreover the excluded region in which the neutralino is not the LSP grows towards higher values of \(m_0\).

Let us now analyse the non-minimal models with \(\eta > 0\). As explained in section 3.1.3, we considered three different cases: \(\eta = 1\) in which only the third family have a KK tower, \(\eta = 2\) in which only the first two families have a KK tower and \(\eta = 3\) in which all the families have a KK tower.

For \(\eta = 2\) we must have \(\mu_0 \geq 10^8\) GeV in order to avoid non-perturbative gauge couplings at the unification scale (see for example figure 4 of [10]). In this case regions with \(m_0 \gtrsim 3000\) GeV are excluded due to an incorrect EWSB while for \(\tan \beta \lesssim 30\) there is no region in which the neutralino is not the LSP. For higher \(\tan \beta\) the parameter space develops a region, for small \(m_0\), excluded by EWSB constraints and by the stau being the
LSP. In almost all of the allowed region the neutralino turns out to be a very pure bino (as in mSUGRA) though there is still a significant overlap between the cosmologically favoured region and the region in which the neutralino is higgsino or mixed-like (see figure 5). The possible values of the neutralino mass are lower than in the minimal scenario, with typically $m_\chi \lesssim 500 \text{ GeV}$.

In the case $\eta = 1$ in which only the third family has a KK tower almost all of the parameter space is allowed for low $\tan \beta$. There are only two small regions excluded by the accelerator bounds and by the EWSB constraints. The neutralino turns out to always be a bino. The cosmologically allowed region is very small and in correspondence with a low neutralino mass region, i.e. $m_\chi \lesssim 50 \text{ GeV}$. For higher $\tan \beta$ we have the same behaviour.

Figure 3. Contour plots of the neutralino mass (in GeV) in the plane ($m_0, m_{1/2}$) for increasing values of $\mu_0$. 

Journal of Cosmology and Astroparticle Physics 06 (2006) 002 (stacks.iop.org/JCAP/2006/i=06/a=002) 20
Figure 4. Contour plots of the gaugino fraction in the plane \((m_0, m_{1/2})\) for increasing values of \(\mu_0\).

Figure 5. Gaugino fraction in the plane \((m_0, m_{1/2})\). Left panel: non-minimal scenario \(\eta = 1\). Right panel: non-minimal scenario \(\eta = 2\).

except for the presence of the two excluded regions (EWSB constraints and the stau being the LSP) for small \(m_0\).

Finally we consider the \(\eta = 3\) case in which all the families possess a KK tower. In these models the region for which \(m_{1/2} \lesssim 750\) GeV is always excluded for any value of \(\tan \beta\). For \(\tan \beta \lesssim 30\) there is only a very small region in which the neutralino is not the LSP, namely for \(m_0 \approx 0\) GeV and \(m_{1/2} \approx 0\) GeV. For higher values of \(\tan \beta\) there is another region in which the neutralino is not the LSP together with a region excluded
by the EWSB on the left of the parameter space. The neutralino mass for such models is lower, $m_\chi \lesssim 200$ GeV, with respect to the minimal case (left panel of figure 6). The neutralino composition is essentially that of a pure bino except for a region close to the excluded regions in which $Z_g \leq 0.1$ (right panel of figure 6). Once again the relic density is in the right range in the regions where the neutralino is higgsino-like or mixed-like.

In general non-minimal models are disfavoured from the point of view of the cosmological relic density because the allowed regions are very small.

In the scenario in which the Higgs particles form an $N=2$ matter hypermultiplet the results as regards the neutralino composition and the corresponding relic density remain essentially unchanged. However there are some differences in shape of the allowed and excluded regions in the parameter space. For example in the minimal scenario $\eta=0$ (see figure 7(a)) the regions excluded due to EWSB and to the accelerator bounds are smaller than in the $N=1$ case. This holds for all $\tan \beta$ and $A_0$. Keeping all the other parameters fixed there are in general allowed regions in the right part of the parameter space with $m_0 > 2500$ GeV. The cosmologically allowed regions get shifted in order to ‘follow’ the regions in which $Z_g \lesssim 0.1$. In the non-minimal cases $\eta > 0$ the behaviour is the opposite and the parameter space is in general more constrained, although there are cases in which the region excluded by the neutralino not being the LSP is absent or at least very small (see figures 7(c) and (d)).

5. Conclusions

In this paper we presented a general scenario that involves the presence of large extra spacetime dimensions. We computed all the threshold corrections (due to KK modes) to the running of the couplings of the MSSM, including all the soft terms. Then we performed
a detailed phenomenological analysis of the model. We found that the neutralino is still the LSP with a relic density compatible with the most recent WMAP data. The main difference between our model and the well studied mSUGRA scenario is that the neutralino is higgsino-like in most of the parameter space. Moreover the mass spectrum of the theory does not contain tachyons. These results remain essentially the same in the two cases we considered, \( N = 1 \) and \( N = 2 \) Higgs.

**Figure 7.** Contour plots for \( N = 2 \) Higgs models.
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Appendix A. $\beta$-functions from the method of the effective potential

In this appendix we collect the full set of renormalization group equations following the notation of [15]:

\[ Q \frac{d}{dQ} f_{abc} = -\frac{1}{32\pi^2} [4g^2(C(A) + C(B) + C(C))] f_{abc} \]
- \( X_a f_{a'bc} + X_b f_{a'bc'} + X_c f_{a'bc'} \) \hspace{1cm} (A.1)

\[ Q \frac{d}{dQ} \mu_{ab} = -\frac{1}{32\pi^2} [4g^2(C(A) + C(B))] \mu_{ab} - (X_a \mu_{a'b} + X_b \mu_{ab'}) \] \hspace{1cm} (A.2)

\[ Q \frac{d}{dQ} (m^2)_b = -\frac{1}{32\pi^2} [-X^a_c (m^2)_b - X^a_c (m^2)_d - 4f^{ade} f_{bec} (m^2)_d] \]
- \( 8\xi_{(3)abc} \xi_{(3)d} + 8g^2 [C(A)\Delta_2^A + C(B)\Delta^2_2] \delta^a_b \)
- \( 4g^2 \text{tr}(T^A m^2) (T^A)_g \) \hspace{1cm} (A.3)

\[ Q \frac{d}{dQ} \xi_{(3)abc} = -\frac{1}{32\pi^2} [4g^2(C(A) + C(B) + C(C))] \xi_{(3)abc} \]
- \( 2(f_{abc} f_{(b'c')d} \xi_{(3)a'b'c'}) + f_{acd} f^{ad'} f_{(a'b'c')d} \xi_{(3)a'b'c'} + f_{abd} f^{ad'} f_{(a'b'c')d} \xi_{(3)a'b'c'} \)
- \( X^a_b \xi_{(3)abc} - X^b_c \xi_{(3)abc} - X^c_a \xi_{(3)abc} \)
+ \( 4g^2 [C(A)\Delta_2^A + C(B)\Delta_2^B + C(C)\Delta_2^C] f_{abc} \) \hspace{1cm} (A.4)

\[ Q \frac{d}{dQ} \xi_{(2)ab} = -\frac{1}{32\pi^2} [4g^2(C(A) + C(B))] \xi_{(2)ab} - [X^a_b \xi_{(2)a'b} + X^b_c \xi_{(2)abc}] \]
- \( 2f_{abc} f^{cde} \xi_{(2)cde} - 4f^{cde} [\xi_{(3)abcd} \mu_{ab} + \xi_{(3)abcd} \mu_{ab}] \)
+ \( 8g^2 [C(A)\Delta_2 + C(B)\Delta_2] \mu_{ab} \) \hspace{1cm} (A.5)

\[ Q \frac{d}{dQ} \xi_{(1)a} = -\frac{1}{32\pi^2} [4g^2 C(A) \xi_{(1)a} - X^b_a \xi_{(1)b} - 2f^{abc} \mu_{ab} \xi_{(2)cd}] \]
- \( 4\xi_{(3)abc} \xi_{(2)c} - 4f_{abc} \mu_{cd} (m^2)_d \). \hspace{1cm} (A.6)

The generalization of these equations to non-simple gauge groups is very simple. Terms like \( g^2 [C(A) + C(B)] \) are to be replaced by a sum over the gauge groups (all simple components and \( U(1) \) factors). There is a gauge coupling constant and a group invariant \( C(R) \) for each group factor.

Appendix B. Feynman diagram analysis of the soft terms

In this appendix we collect all the Feynman diagrams which renormalize the soft terms in (2.10). Next to each diagram we give its contribution to the coefficient of the respective $\beta$-function. As one can easily check, in the absence of KK states, every diagram is
logarithmically divergent in four dimensions. In presence of extra dimensions, those diagrams in which KK states can circulate contribute to the power law piece in (2.32). This is the recipe we have adopted to obtain (3.10)–(3.20) and (3.38)–(3.47).

- Renormalization of the coefficient $b$ in (2.10):

\begin{equation}
6y_t^2 b (3y_t^2 + 3y_b^2 + y_\tau^2) \tag{B.1}
\end{equation}

\begin{equation}
2\mu (3a_t y_t + 3a_\tilde{b} y_b + a_\tau y_\tau) \tag{B.2}
\end{equation}

\begin{equation}
b (-3g_2^2 - \frac{3}{2} g_1^2) \tag{B.3}
\end{equation}

\begin{equation}
-\mu (6g_2^2 \Delta^{22} + \frac{6}{5} g_1^2 \Delta^{11}) \tag{B.4}
\end{equation}

- Renormalization of the coefficient $m_{H_u}^2$, $m_{H_d}^2$ in (2.10):

\begin{equation}
6y_t^2 m_{H_u}^2 \tag{B.5}
\end{equation}

\begin{equation}
6a_t^2 \tag{B.6}
\end{equation}

\begin{equation}
-\frac{6}{5} g_1^2 (\Delta^{11})^2 - 6g_2^2 (\Delta^{22})^2 \tag{B.7}
\end{equation}

\begin{equation}
6y_t^2 (m_t^2 + m_\tilde{t}^2) \tag{B.8}
\end{equation}

\begin{equation}
\frac{2}{5} g_1^2 \text{tr}(Y m^2). \tag{B.9}
\end{equation}

The last diagram is present only for the $U(1)$ gauge group.
• Renormalization of the coefficient $m_t^2$ in (2.10): 

\[ (2g_t^2 + 2g_b^2)m_t^2 \]  
(B.10) 

\[ 2(a_t^2 + a_b^2) \]  
(B.11) 

\[ -8g^2\Delta^2a_AC(t) \]  
(B.12) 

\[ 2y_t^2(m_{H_u}^2 + m_t^2) + 2y_b^2(m_{H_d}^2 + m_b^2) \]  
(B.13) 

\[ \frac{1}{3}g_t^2 \text{tr}(Ym^2). \]  
(B.14) 

Similar diagrams hold for the $\tilde{b}$ squark and the $\tilde{\tau}$ slepton.

• Renormalization of the coefficient $a_t, a_b, a_\tau$ in (2.10): 

\[ a_t(-\frac{3}{10}g_1^2 - \frac{3}{2}g_2^2) \]  
(B.15) 

\[ a_t(-\frac{17}{30}g_1^2 - \frac{3}{2}g_2^2 - \frac{16}{3}g_3^2) \]  
(B.16) 

\[ a_t(3y_t^2 + y_b^2) \]  
(B.17) 

\[ 3a_t y_t^2 \]  
(B.18) 

\[ 6a_t y_t^2 \]  
(B.19)
Extra dimensions and dark matter

\[ 6a_t y_t^2 + 2a_b y_b y_b \]  
\[ - y_t \left( \frac{16}{3} g_3^2 \Delta_{33} + 3 g_2^2 \Delta_{22} + \frac{13}{15} g_1^2 \Delta_{11} \right) \]  
\[ - y_t \left( \frac{16}{3} g_3^2 \Delta_{33} + 3 g_2^2 \Delta_{22} + \frac{13}{15} g_1^2 \Delta_{11} \right) . \]

Similar diagrams hold for the other two trilinear couplings, \( a_b, a_{\tau} \), the one for the squark \( \tilde{b} \) and the one for the slepton \( \tilde{\tau} \).

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