Charge asymmetries in $e^+e^- \rightarrow \pi^+\pi^-\gamma$ at the $\phi$ resonance.

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We consider the forward-backward pion charge asymmetry for the $e^+e^- \rightarrow \pi^+\pi^-\gamma$ process. In addition to the Bremsstrahlung and the double resonance contributions, we use four different models to describe the final state radiation (Kaon Loop Model, Resonance Chiral Perturbation Theory, Unitarized Chiral Perturbation Theory and Linear Sigma Model), perform a Monte Carlo calculation and compare our results with experimental data. Our results indicate that none of these models yield an appropriate description of the asymmetry in the whole energy range reported by the KLOE collaboration and, contrary to expectations, the invariant mass region between 400 and 700 MeV may be the region of major interest since the models we consider are unable to reproduce the data in this region while the same models (except for Resonance Chiral Perturbation Theory) reproduce the overall characteristics of the data in the $f_0(980)$ region.

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I. INTRODUCTION

The nature of low mass scalar mesons nonet is a long-standing puzzle. The $\phi$ radiative decays are expected to provide information about the $f_0(980)$ and $a_0(980)$ scalar mesons. Unfortunately data reported by the KLOE collaboration on the $\phi$ decays to $f_0\gamma$ and $a_0\gamma$ with $\pi^0\pi^0\gamma$ and $\pi^0\eta\gamma$ final states respectively— together with results on the process $\phi \rightarrow \pi^+\pi^-\gamma$ including the $f_0\gamma$ as intermediate state—are not conclusive. In the latter work, results on the forward-backward asymmetry as a function of the $\pi^+\pi^-$ invariant mass are presented. The asymmetry is sensitive to the mechanisms involved in the final state radiation and has been proposed as an appropriate observable to test the importance of the different mechanisms at work in this reaction and the different proposals for their description. It also provides information on the pion form factor.

The asymmetry requires a non vanishing interference between initial (ISR) and final (FSR) state radiation, the latter being strongly model dependent. The invariant amplitude for the $e^+e^- \rightarrow \pi^+\pi^-\gamma$ process can be parameterized in terms of three independent Lorentz structures and thus the model dependence in FSR can be included in three scalar functions. The final state radiation has been calculated in different models. The simplest approximation has been named scalar QED ($sQED$) and it actually includes the $\rho$ contributions to the pion form factor. In that work the contributions of intermediate scalars ($f_0(980)$ and $\sigma$) are also considered using a point-like $\phi f_0\gamma$ interaction, in the so called "no-structure" model. Later on, final state radiation was calculated within Resonance Chiral Perturbation Theory ($R\chi PT$) at tree level. In particular, in sub-leading intermediate vector mesons contributions, like $e^+e^- \rightarrow \phi \rightarrow \rho^- \pi^+ \rightarrow \pi^+\pi^-\gamma$, named double resonance contributions, were incorporated.

The aim of this paper is to work out the predictions for $e^+e^- \rightarrow \pi^+\pi^-\gamma$ at the $\phi$ resonance at one loop level for four alternative models, namely $R\chi PT$, Unitarized Chiral Perturbation Theory ($U\chi PT$) (containing actually a resummation of loops), Linear Sigma Model ($LSM$) and the so-called "kaon-loop" model. In each case we add the $sQED + VMD$ model and the double resonance contribution, both proposed in $\phi$. We report the forward-backward pion charge asymmetry which is calculated using Monte Carlo methods and compare our results with KLOE data.

The paper is organized as follows: Section II includes the general formalism to describe the $e^+e^- \rightarrow \pi^+\pi^-\gamma$ process. In section III we derive the scalar functions that characterize the $R\chi PT$, $LSM$, $U\chi PT$ and the kaon loop models, including Bremsstrahlung process and the double resonance contribution. In section IV we present the numerical results after the Monte Carlo calculation and compare them with experimental results. Finally, conclusions are given in section V.
II. GENERAL FORMALISM

We are interested in the process

$$e^- (p_1) e^+ (p_2) \rightarrow \pi^+ (p_+), \pi^- (p_-), \gamma (k, \epsilon).$$  

(1)

For completeness, in order to introduce our notation and conventions, in this section we include the basic equations used to describe the process. To this end, we follow the formalism developed in Ref. [7]. The invariant amplitude $M$ includes the initial state radiation $M_{ISR}$ and final state radiation $M_{FSR}$, i.e.

$$M_{ISR} = \frac{e}{q^2} L^{\mu \nu} \epsilon^\nu_\mu F_\pi (q^2),$$

(2)

$$M_{FSR} = \frac{e^2}{s} J_\mu M^\mu_{FSR} \epsilon_\nu,$$

(3)

where $F_\pi (q^2)$ denotes the pion electromagnetic form factor, $\epsilon_\nu$ is the photon polarization vector and the tensor $M^\mu_{FSR}$ describes the photon radiation from the final state. The lepton currents are given by

$$L^{\mu \nu} = e^2 \overline{u}_{s_2} (-p_2) \times \left[ \gamma^\mu \left( -p_2 + \overline{k} + m_e \right) t_2 + \gamma^\nu \left( p_1 - \overline{k} + m_e \right) t_1 \right] \times u_{s_1} (p_1),$$

(4)

$$J_\mu = e \overline{u}_{s_2} (-p_2) \gamma_\mu u_{s_1} (p_1).$$

(5)

The electron and positron spinors are $u_{s_1} (p_1)$ and $\overline{u}_{s_2} (-p_2)$ respectively. In terms of the external particles’ four-momenta, the following variables are introduced $Q = p_1 + p_2, q = p_+ + p_-, l = p_+ - p_-$ and five independent Lorentz scalars are defined

$$s \equiv Q^2 = 2p_1 \cdot p_2,$$  

$$t_1 \equiv (p_1 - k)^2 - m_e^2 = -2p_1 \cdot k,$$  

$$t_2 \equiv (p_2 - k)^2 = -2p_2 \cdot k,$$  

$$u_1 \equiv l \cdot p_1, u_2 \equiv l \cdot p_2,$$

(6)

where the electron mass, $m_e$, has been neglected. The differential cross section is

$$d\sigma = \frac{1}{2s(2\pi)^5} \int d^4 (p_1 + p_2 - p_+ - p_- - k) \times \frac{d^3 p_+}{2E_+} \frac{d^3 p_-}{2E_-} \frac{d^3 k}{2\omega} |M|^2,$$

(7)

with $p_+ = (E_+, p_+), p_- = (E_-, p_-), k = (\omega = |k|, k)$ and $|M|^2$ is the invariant amplitude squared, averaged over initial lepton polarizations [24]. The most general form of the FSR tensor $M^\mu_{FSR}$ is [7]

$$M_{FSR} = f_1 \tau_1^{\mu \nu} + f_2 \tau_2^{\mu \nu} + f_3 \tau_3^{\mu \nu},$$

(8)

where the $\tau_i^{\mu \nu}$ are three independent gauge invariant tensors which are dictated by parity, charge conjugation, crossing symmetry and gauge invariance

$$\tau_1^{\mu \nu} = k^\mu Q^\nu - g^{\mu \nu} k \cdot Q,$$

$$\tau_2^{\mu \nu} = k \cdot l (l^\mu Q^\nu - g^{\mu \nu} k \cdot l) + l^\nu (k^\mu k \cdot l - l^\mu k \cdot Q),$$

$$\tau_3^{\mu \nu} = Q^2 (g^{\mu \nu} k \cdot l - k^\mu l^\nu) + Q^\mu (l^\nu k \cdot Q - Q^\nu k \cdot l).$$

(9)

The scalar functions $f_i \equiv f_i (Q^2, k \cdot Q, k \cdot l)$ are either even ($f_1, f_2$) or odd ($f_3$) under the change of sign of the argument $k \cdot l$. Our first task will be to determine these scalar functions $f_1$ for $R\chi PT, U\chi PT, LSM$ and the $KL$ model including Bremsstrahlung process and the double resonance contribution [3]. Before we consider the models, we define the charge asymmetry. The pair of pions produced in (1) differ in charge conjugation, depending if the photon is emitted from the initial or from the final state, while the former is odd under charge conjugation the latter is even. So, any interference between the two amplitudes is odd under charge conjugation and gives rise to a charge asymmetry. The forward-backward charge asymmetry is defined as

$$A = \frac{N(\theta_{\pi^+} > 90^\circ) - N(\theta_{\pi^+} < 90^\circ)}{N(\theta_{\pi^+} > 90^\circ) + N(\theta_{\pi^+} < 90^\circ)},$$

(10)

where $\theta_{\pi^+}$ is the $\pi^+$ polar angle, which is measured with respect to the incident electron momentum. It should be clear that the asymmetry depends strongly on the experimental conditions, in particular on the cutoff polar angle and the minimal photon energy that can be measured.
III. FSR MODELS

A. Bremsstrahlung process and double resonance contribution

Before discussing the $\phi$ decay models, we first consider the Bremsstrahlung and the double resonance contribution. The Bremsstrahlung process was calculated in [9] using the $sQED\ast VMD$ model and $R\chi PT$. The functions $f_i$ for this contribution are given by equations (11) to (20) in [9]. The double resonance contribution $e^+e^- \rightarrow \phi \rightarrow p^\mp\pi^\pm \rightarrow \pi^+\pi^-\gamma$, is formulated in the context of VMD [14] and the $f_i$ terms can be found in equations (26) to (28) in [9]. We used these expressions in our numerical calculation. We now consider specific models.

B. The $R\chi PT$ and $U\chi PT$ models

The amplitude for the $e^+e^- \rightarrow \pi^+\pi^-\gamma$ process at the $\phi$ peak involves the $\gamma\phi\pi\pi$ vertex function with all particles on-shell. On the other hand, within the analysis of $e^+e^- \rightarrow \phi\pi\pi$ reaction, this vertex function was calculated in the context of $R\chi PT$ and $U\chi PT$ [15] for an off-shell photon. The starting point was the one loop calculation of the $\gamma\phi\pi\pi$ vertex function [10] which turns out to be finite. The diagrams relevant to our analysis are shown in Fig. (1). These diagrams include kaons in the loops, thus they involve the off-shell $KK-\pi\pi$ amplitude. It was shown in [15] that, to leading order in the chiral expansion, the contribution of diagrams $d, e, f, g$ cancels the off-shell contributions of the $KK-\pi\pi$ amplitude, entirely contained in diagrams $a, b, c, h$, so that the calculation reduces to evaluate diagrams $a, b, c, h$ with the $KK-\pi\pi$ amplitude on-shell. This procedure yields the $R\chi PT$ result for the $\gamma\phi\pi\pi$ vertex function and we would expect it to reproduce experimental results at low dipion invariant mass. However, due to the appearance of the widely discussed light scalar resonance (the $\sigma$ meson), this expansion brakes down in the scalar channel even at the dipion threshold.

The scalar poles can be generated unitarizing the leading order meson-meson scattering amplitudes for definite isospin. Following [16], the unitarized $K^+K^--\pi^+\pi^-$ scattering amplitude is calculated projecting onto the zero spin and isospin channel the leading order on-shell meson-meson amplitude and performing a coupled channel analysis involving iterations of all intermediate states in the $s$ channel. As far as the calculation of the $e^+e^- \rightarrow \pi^+\pi^-\gamma$ amplitude is concerned, the scalar poles are incorporated replacing the leading order on-shell $K^+K^- - \pi^+\pi^-$ amplitude by the $K^+K^- - \pi^+\pi^-$ unitarized amplitude. For details of the calculation we refer the interested reader to [16], here we just quote the result for $\phi \rightarrow \pi^+\pi^-\gamma$. The resulting amplitude for $\phi(Q, \eta^{\nu}) \rightarrow \pi^+ (p_+) \pi^- (p_-) \gamma (k, \epsilon^\mu)$ in $U\chi PT$ is
\[-iM = \frac{2}{\sqrt{3}} \frac{e}{2\sqrt{2}\pi^2 m_K^2 f^2} t_{K\pi}^{0} \left[ G_V \left( I_P^b(W, k g_{\mu\nu} - Q_\mu k_\nu) \right) Q_\alpha \right. \right. \]
\[- \left. - \left( G_V - \frac{F_V}{2} \right) \frac{m_K^2}{4} g_K(q^2) g_{\mu\nu} k_\alpha \right] \eta^{\alpha\nu} \epsilon^\mu, \]

where \( q^2 = (p_+ + p_-)^2 \) and \( t_{K\pi}^{0} \) denotes the unitarized isoscalar scalar \( K\pi - \pi\pi \) amplitude. The factor \( 2/\sqrt{3} \) is required to single out the \( \pi^+\pi^- \) contribution in the isoscalar \( \pi\pi \) channel. The function \( g_K(q^2) \) is given by

\[ g_K(q^2) = -1 + \log \frac{m_K^2}{\mu^2} + \left[ \frac{\sigma(q^2) \log \frac{\sigma(q^2) + 1}{\sigma(q^2) - 1}}{\sigma(q^2) - 1} \right], \]

with \( \sigma(q^2) = \sqrt{1 - \frac{4m_K^2}{q^2}} \). Note that the particular form of this function involves a regularization scheme as well as a subtraction point. We use dimensional regularization and the value \( \mu = 1.2 \text{ GeV} \) which reproduces the \( f_0 \) peak at \( 980 \text{ MeV} \) in the squared meson-meson amplitudes in the scalar channel \([16] \). The loop integral is given by

\[ \tilde{I}_P^b = \frac{1}{2(a - b)} - \frac{2}{(a - b)^2} \left[ f \left( \frac{1}{b} \right) - f \left( \frac{1}{a} \right) \right] + \frac{a}{(a - b)^2} \left[ g \left( \frac{1}{b} \right) - g \left( \frac{1}{a} \right) \right], \]

\[ f(z) = \begin{cases} \arcsin \left( \frac{1}{\sqrt{2}z} \right)^2 & z > \frac{1}{4}, \\ \frac{1}{2} \ln \left( \frac{n_+}{n_-} \right) - i\pi & z < \frac{1}{4}, \end{cases} \]

\[ g(z) = \begin{cases} \sqrt{4z - 1} \arcsin \left( \frac{1}{\sqrt{2}z} \right) & z > \frac{1}{4}, \\ \frac{1}{2} \sqrt{1 - 4z} \ln \left( \frac{n_+}{n_-} \right) - i\pi & z < \frac{1}{4}, \end{cases} \]

\[ a = \frac{Q^2}{m_K^2}, \quad b = \frac{q^2}{m_K^2}, \quad n = \frac{1}{2} \left[ 1 \pm \sqrt{1 - 4z} \right]. \]

Results for \( R\chi PT \) are obtained from Eq. (13) by replacing \( t_{K\pi}^{0} \) by the leading order on-shell interaction \( V_{K\pi} = -\sqrt{3}q^2/4f^2 \). Using the propagator for a vector meson in the tensor formalism we obtain the amplitude for \( \gamma^*(Q, \mu) \rightarrow \pi\pi\gamma(k, \nu) \) via the exchange of the vector meson \( \phi \) as

\[-iM = \frac{ie^2 F_V \sqrt{2}}{3 Q^2 - M_{\phi}^2 + i\Gamma_{\phi} M_{\phi}} \left[ (Q^2 + J) (k_\mu Q_\nu - Q_\mu k_\nu) \right] \epsilon^\nu, \]

where

\[ I = -\frac{G_V}{\sqrt{6}\pi^2 m_K^2 f^2} \tilde{I}_P^b, \]

\[ J = \frac{1}{\sqrt{6}\pi^2 m_K^2 f^2} \frac{t_{K\pi}^{0}}{\sqrt{3}} \left( G_V - \frac{F_V}{2} \right) \frac{m_K^2}{4} g_K(q^2). \]

In terms of this vertex function we can identify the final state radiation invariant tensor \( M_{\mu\nu}^{\mu\nu} \) (see Eq. 5)

\[ M_{\mu\nu}^{\mu\nu} = -ie^2 (f_1 \tau_1^{\mu\nu} + f_2 \tau_2^{\mu\nu} + f_3 \tau_3^{\mu\nu}) = -ie^2 M_{\mu\nu}^{\mu\nu} \]

with

\[ f_1 = -\frac{1}{\sqrt{3}} \frac{F_V}{3 f^2 Q^2 - M_{\phi}^2 + i\Gamma_{\phi} M_{\phi} / 2}, \quad e_0 \frac{t_{K\pi}^{0}}{\sqrt{3}} \left( \frac{Q^2}{m_K^2} G_V \tilde{I}_P^b - \frac{1}{4} \left( G_V - \frac{F_V}{2} \right) g_K(q^2) \right), \]

\[ f_2 = 0, \]

\[ f_3 = 0. \]

Notice that Eq. (18) contains a term with the combination \( G_V - \frac{F_V}{2} \). This combination is small and it vanishes in the context of Vector Meson Dominance \([23] \). For the sake of completeness we will keep this term and study its effect on the charge asymmetry.
C. The phenomenological $KL$ model

In this model the process under consideration proceeds through the chain

$$e^- (p_1) e^+ (p_2) \rightarrow \phi \rightarrow f_0 (q) \gamma (k, \epsilon) \rightarrow \pi^+ (p_+) \pi^- (p-) \gamma (k, \epsilon).$$

The corresponding amplitude is

$$M_\phi = \frac{-ie^2}{s} Ac\Theta (p_2) \gamma_{\mu u} (p_1) (Q^\mu k^\mu - Q \cdot k g^{\mu \nu}) \epsilon_\nu,$$

where we have defined

$$A = \frac{g_s g_\phi g_f}{f_\phi} \frac{g_f}{2\pi^2 m_{K^+}} \tilde{I}^b_{\phi} F_\phi (s) P_f (q^2) e^{i\delta_B},$$

with

$$F_\phi (s) = \frac{m_{\phi}^2}{s - m_{\phi}^2 + i\sqrt{s} \delta_{\phi}}, \quad P_f (q^2) = \frac{1}{q^2 - m_f^2 + im_f \Gamma_f},$$

and $g_s, g_\phi, g_f$ and $f_\phi$ stand for the $f_0 \pi^+ \pi^-, \phi K^+ K^-, f_0 K^+ K^-$ and $\phi - \gamma$ couplings respectively (for details concerning the precise definition of these quantities we refer the reader to the appendix of Ref. [13]). The angle $\delta_B$ is the phase of elastic background and it must be included with the kaon loop factor [17]. This phase becomes relevant for the interference between ISR and FSR amplitudes and thus can be measured for charged pions [6], in this case $\delta_B = b \sqrt{q^2 - 4m_\pi^2}$ with $b = 75^\circ/\text{GeV}$ [17, 18]. The kaon loop function $\tilde{I}^b_{\phi}$ is given in [13]. Finally, the scalar functions for this model are obtained by comparing (22) with (8) and (3), in this way we get

$$f_1 = A, \quad f_2 = 0, \quad f_3 = 0.$$  

D. The Linear Sigma Model

The calculation in this approach is similar to the $KL$ model, the difference arising from the treatment of the scalars. For the neutral pion case, the amplitude has already been derived in [13] using the improved chiral loop approach. Thus we can obtain the amplitude we are interested in just by making the following replacement in the $KL$ amplitude

$$g_s g_f P_f (q^2) \rightarrow A (K^+ K^- \rightarrow \pi^+ \pi^-)_{s,M} = \sqrt{2} A (K^+ K^- \rightarrow \pi^0 \pi^0)_{s,M},$$

where the amplitude for the meson scattering is given by

$$A (K^+ K^- \rightarrow \pi^0 \pi^0)_{s,M} = \frac{m_{\pi}^2 - q^2/2}{2f_\pi f_K} + \frac{q^2 - m_\pi^2}{2f_\pi f_K} \left[ \frac{m_K^2 - m_{\phi S}^2}{D_S (q^2)} c_\phi S - \sqrt{2} s_\phi S \right] + \frac{m_K^2 - m_{\phi S}^2}{D_{\phi S} (q^2)} (s_\phi S + \sqrt{2} c_\phi S),$$

with $D_S (q^2) = q^2 - m_S^2 + im_S \Gamma_S$, $f_K = 1.22 f_\pi$, $(c_\phi S, s_\phi S) \equiv (\cos \phi S, \sin \phi S)$ and $\phi S$ is the scalar mixing angle in the strange-non-strange basis [12]. Therefore, the scalar functions for this model can be obtained from (25) by making the replacement indicated in the previous paragraph.

IV. NUMERICAL RESULTS

In order to obtain the numerical results we implemented a Monte Carlo code where the experimental conditions of the KLOE collaboration are included. Thus, for the $\pi^+$ polar angle - defined respect to the electron beam - we considered the range $45^\circ < \theta_{\pi^+} < 135^\circ$. As far as the photon is concerned we took $45^\circ < \theta_\gamma < 135^\circ$ and used a cut at 10 MeV, i.e. we assumed $E_\gamma > 10$ MeV [3]. In order to compare with the experimental data for the forward-backward
FIG. 2: Comparison of models predictions with KLOE data in region I (400-700 MeV). A scalar mixing angle $\phi_S = -5^\circ$ is used in the LSM calculations. The predictions of the four models are in strong disagreement with data.

Asymmetry it is convenient to consider three complementary regions in the dipion invariant mass. We observe that in the 400-700 MeV the data decreases slowly while in the 700-900 MeV range the asymmetry grows up. Above 900 MeV, at 930 MeV, the asymmetry decreases until it reaches 950 MeV where it starts growing again and reaches the maximum in the $f_0(980)$ region. Hereafter we will refer to these as regions I, II and III respectively. In the following paragraphs we compare our numerical calculation with the experimental data.

Calculations in $U\chi PT$ and $R\chi PT$ involve parameters that have already been fixed from meson phenomenology. We use the following values in the numerics: $G_V = 53$ MeV, $F_V = 154$ MeV, $f_\pi = 93$ MeV and $\mu = 1.2$ GeV [16]. As to the KL model, a summary of the values we use for the parameters of this model is given in Table I, including the $g_s$ and $g_f$ values reported in [20] and the Particle Data Group [21]. The complete definitions of these constants are described in [13].

$$
\begin{array}{|c|c|c|}
\hline
\text{Parameter} & \text{Value} & \text{Reference} \\
\hline
m_f (\text{MeV}) & 980 & [21] \\
\Gamma_f (\text{MeV}) & 70 & [21] \text{ (Averaged)} \\
g_s^2 (\text{GeV}^2) & 3.61 & [20] \\
g_f^2 (\text{GeV}^2) & 19.56 & [13, 21] \\
g_s^2 (\text{GeV}^2) & 7.78 & [20] \\
f_s^2 & 179.14 & [13, 21] \\
g_s^2 & 35.95 & [13, 21] \\
f_f^2 & 24.66 & [13, 21] \\
\hline
\end{array}
$$

TABLE I: Constants appearing in the $KL$ model and the numerical values used in this work.

Besides the intrinsic parameters of the scalar mesons, the $LSM$ involves the scalar mixing angle. We use the following values: $m_f = 980$ MeV, $\Gamma_f = 70$ MeV, while for the the sigma meson we took the values reported in [22] $m_\sigma = 528$ MeV and $\Gamma_\sigma = 414$ MeV and for the scalar mixing angle we consider three values: $\phi_S = -3^\circ$ (LSM3), $-5^\circ$ (LSM5) and $-7^\circ$ (LSM7).

In the following lines we briefly describe comparison of KLOE data with the model predictions for the forward
backward asymmetry (Regions: I 400 - 700 MeV, II 700 - 900 MeV and III above 900 MeV):

- Fig. (2) contains our results for the four models in region I. We observe that in this region the results of all four schemes are in strong disagreement with the experimental data.

- Figure (3) presents the $U\chi PT$ and $R\chi PT$ results for regions II and III. The difference between $U\chi PT$ and $U\chi PTw$ results is that the latter exclude contributions from the $G_V - \frac{F_V}{2}$ term in Eq. (18). We observe that in region II both models provide a satisfactory description of the behavior of the asymmetry. In region III $U\chi PT$ predictions are in very good agreement with data. The effect of the $G_V - \frac{F_V}{2}$ term in Eq. (18) is small but necessary to achieve a good description of data. In this region $R\chi PT$ does not reproduce the data.

- Kaon loop model predictions for regions II and III are shown in figure (4). In region II the results are systematically above the data, while in region III the predictions of this model follow the trend of the data.

- LSM predictions for regions II and III are shown in figure (5). In region II good agreement with data is obtained for the three values of the mixing angle. The asymmetry in region III is highly sensitive to the value of the scalar mixing angle and a very good description of data is obtained for $\phi_S = -5^\circ$. This value is close to the one reported in [19] where the $f_0$ propagator is approximated by a Breit-Wigner.

- Figure (6) contains a summary of the best predictions of the four models we consider in this work.

### A. Phase dependence of the asymmetry.

The strong disagreement between the models and the KLOE data for low dipion invariant mass motivated us to consider the effect of a relative phase. We explore the effect of a constant relative phase between $sQED * VMD$ plus double resonance model contributions and the models considered here. We included a constant phase $e^{i\alpha}$ in the $KL, LSM$ and $U\chi PT$ amplitudes. The calculation was performed for different values of $\alpha$ and the results are shown in figures (7) and (8). Results are labeled with the corresponding phase, e.g. $LSM45$ corresponds to the Linear Sigma Model with a phase $\alpha = 45^\circ$. An strong dependence on the relative phase is observed. In the case of $U\chi PT$ a
FIG. 4: Comparison of KLOE data with the results for the forward-backward asymmetry in the kaon loop model. Region II (left) and region III (right).

relative phase close to $\alpha = 135^\circ$ yields a reasonable description of region I. However, such a phase destroys the good description of the data in Region III. The inclusion of a phase in the $KL$ and $LSM$ does not improve the description of data. We conclude that a constant relative phase does not provide a good description of data in the whole invariant mass range reported by the KLOE collaboration for the models considered here.

FIG. 5: Comparison of KLOE data with results for the forward-backward asymmetry as predicted by the $LSM$ using the scalar mixing angles $\phi_2 = -3^\circ$ (LSM3), $-5^\circ$ (LSM5) and $-7^\circ$ (LSM7). Region II (left) and region III (right).
V. SUMMARY AND CONCLUSIONS

We used Unitary Chiral Perturbation Theory, Resonance Chiral Perturbation Theory, Linear Sigma Model and the Kaon Loop model to describe the final state radiation of the $e^+e^- \rightarrow \pi^+\pi^-\gamma$ process at the loop level. We implemented the models in a Monte Carlo code in order to obtain the forward-backward asymmetry from these models and the conventional tree level mechanisms ($sQED + VMD$ plus double resonance model). Detailed comparison with
FIG. 8: Phase effect on the forward-backward asymmetry for $U_\chi PT$.

 experimental data is reported. The following conclusions can be drawn from the comparison of the models with the KLOE data:

• Predictions of the four models for the asymmetry in the low invariant mass region (400 - 700 MeV) are in strong disagreement with the KLOE data.

• In region II (between 700 and 900 MeV) the four models yield an appropriate description of KLOE results.

• The trend of the data in region III (above 900 MeV) is well reproduced by all models except for $R_\chi PT$.

• The asymmetry in region III is highly sensitive to the scalar mixing angle in the Linear Sigma Model and a value $\phi_S = -5^\circ$ is favored by data.

• None of the models studied here yields an appropriate description of the asymmetry in the whole range of energy reported by the KLOE collaboration.

• The inclusion of a relative phase between tree level contributions and the loop level contributions studied here is not helpful. In the $U_\chi PT$ case a phase close to $\alpha = 135^\circ$ improves the prediction in the low invariant mass but this phase destroys the good description of data in the $f_0(980)$ region.

• Contrary to expectations, the invariant mass region between 400 and 700 MeV may be the region of major interest. In fact, models are unable to reproduce the data in this region while the same models correctly reproduce the overall characteristics of the data in the $f_0(980)$ region. A possible interpretation is that the $f_0(980)$ pole is correctly described in these models while the joint description of poles in different regions requires a more careful analysis.

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Ref. [7] uses $\pi^\nu(p) u_\nu(p) = -\pi_\nu(-p) u_\nu(-p) = 2m_\nu \delta_{\mu\nu}$ and $\sum_{\text{polar.}} e_\mu \epsilon_\sigma = -g_{\mu\sigma}$. 

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