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Tensile Fracture Analysis of Fiber Reinforced Cement-Based Composites with Rebar Focusing on the Contribution of Bridging Forces

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Abstract

Tensile fracture of fiber reinforced cement-based composites (FRCC) with rebar was investigated via a mesoscale analysis using discretized short fibers. Herein, the effects of fiber volume fraction, steel reinforcement ratio, FRCC–rebar bond characteristics, and fiber distribution on tensile fracture behavior were investigated. In some cases, localized crack was observed in the post-yield range of rebar. The localization mechanism was numerically explained and then inhibited by focusing on the bridging forces of the fibers and rebar. The effectiveness of steel reinforcement in enhancing the strain capacity of strain-hardening cement-based composites was confirmed. This paper is based on an original paper (Ogura et al. 2016) written in Japanese.

1. Introduction

Various properties of cement-based materials, including resistance to shrinkage cracking, post-cracking strength, and toughness, are commonly improved by short-fiber reinforcement. In recent years, fiber reinforced cement-based composites (FRCC) with various performances have been applied to actual structures. In several cases, FRCCs can be applied alone because the tensile force after initial cracking can be transferred across the crack plane in the cement matrix through bridging fibers. However, in most of the real structures, FRCCs are combined with steel reinforcement, such as rebars and PC tendons. FRCC that exhibits strain softening is mainly reinforced using a rebar, which is similar to conventional reinforced concrete structures. Ultra-high performance fiber reinforced concrete is often combined with PC tendon (JSCE 2006). Strain-hardening cement-based composites (SHCC), which exhibit strain-hardening properties after initial cracking, are commonly combined with a rebar because the application without rebar is beyond the scope of a guideline (JSCE 2008).

To rationalize the performance of FRCCs in real structures, one must evaluate the mechanical performances of both the FRCC alone and its steel reinforced members. In particular, the risk of reducing the amount of rebar at the design phase must be evaluated by clarifying the load capacity, deformation performance, crack propagation behavior, and the ultimate behavior of the member, including the fracture mode.

Several studies have reported the crack propagation behavior of FRCC with a rebar. The behavior of strain-softening FRCC under uniaxial tensile stress was investigated by Deluce et al. (2012) and Deluce and Vecchio (2013). They found that with a decrease in load, the cracks generated in the elastic range of the rebar were localized to the region of rebar yielding (see Fig. 1). Noghabi (2000) has conducted experiments using six types of fibers and reported that the cracks localize after rebar yielding in some cases. Kunieda et al. (2010) demonstrated the effectiveness of a small amount of

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Fig. 1 Crack patterns after tensile tests (Deluce et al. 2012): (a) Concrete with rebar; (b) FRCC with rebar.
reinforcement to enhance the strain capacity and crack distribution of SHCC subjected to axial tension.

As crack localization might degrade the toughness and restorability of the member, it warrants a quantitative evaluation. For this purpose, the above-mentioned responses must be investigated in an integrated manner. However, a unified analysis is difficult because the responses are expected to differ not only between strain-softening and strain-hardening FRCCs but also among the various materials and conditions of members, such as steel reinforcement ratio, type of fiber, and fiber volume fraction, among others. Additionally, as FRCC resists tensile forces even at the cracking, its responses are difficult to evaluate by separating the contribution of the fibers and rebar. Consequently, the behaviors of FRCC members with rebar are less easily understood than those of ordinary reinforced concrete members.

On the other hand, numerical analyses can evaluate the fracture behavior for various parameter values, potentially enabling a unified picture of fracture behavior. A simulated mesoscale analysis using discretized short fibers has already confirmed the onset and propagation of cracks in FRCC (Kunieda et al. 2011; Ogura et al. 2013). Therefore, the analysis has potential that can separately evaluate the contribution of the rebar and fibers, which are difficult to measure in experiments.

This paper verifies the tensile fracture behavior of FRCCs with rebars in a numerical analysis. The behavior of localized cracks in the post-yield range of the rebar was simulated, and in addition, the localization mechanism and a proposal for a localization-avoidance strategy focusing on the bridging force were also investigated. Varying the fiber volume fraction and steel reinforcement ratio, the parameter combinations leading to localized and non-localizing cracks was identified. The fiber distribution of FRCC was also analyzed, and the influence of the accompanying nonuniformity of the material was discussed. The analysis targets are the materials of strain-softening and strain-hardening FRCCs. These materials, which were considered separately in past studies, are studied together for a comprehensive and unified understanding of their combined behaviors.

2. Analytical model

2.1 Rigid-Body-Spring Model (RBSM)

The cement matrix phase of the FRCC is assumed homogeneous and is modeled as a RBSM. The element stiffness matrix was formed from paired polyhedral cells (see Fig. 2) in the three-dimensional RBSM. Each cell is assumed as a rigid body with six degrees of freedom (defined at the nodal point from which the cell is constructed). Six springs connect the neighboring cells across their common facet; each spring set is composed of one facet-normal and two facet-tangential extensional springs and three rotational springs. The mechanical properties of the normal/tangential springs were defined by the cement matrix characteristics (such as tensile strength and elastic modulus). The mechanical property of the rotational spring was defined by the elastic modulus and facet geometry (Bolander and Berton 2004).

The tensile model of normal springs is assumed as linear-elastic up to the tensile strength. After cracking, the model behaves as a 1/4 model with a bilinear softening branch (JSCE 2010). The stress–strain behaviors of the constituent matrix models are presented in Fig. 3. The models account for the tensile fracture energy $G_f$. The unloading and reloading paths of the normal springs are directed through the origin.

In the tangential spring, the Mohr–Coulomb criterion is assumed as the failure criterion (Ueda et al. 1990). The shear stress decreases with increasing crack width; after a normal spring softened, stress of tangential spring is multiplied by the shear stress reduction factor, which is dependent on the tensile strain of normal spring, similar to Saito’s model (Saito and Hikosaka 1999).

2.2 Short-fiber model

(1) Outline of the modeling process

In this study, the cement matrix and each short fiber in a composite were modeled separately. The reinforcement effect of the short fiber was simulated by a discrete approach based on the bond stress–slip modeling of individual fibers. This is an approach that extended the numerical model proposed by Bolander and Yip (2003) to three-dimensional analysis and added a model that can quantitatively express the pulling behavior of the fiber (Kunieda et al. 2008; Ogura and Kunieda 2009). The capability of simulated SHCC tensile tests was demonstrated by Kunieda et al. (2011), who compared their
simulation results with those of experiments. The method is suitable for the current research by virtue of the following features.

1) A biased fiber distribution is easily constructed in this model. Therefore, one can investigate the effects of the material nonuniformity caused by the dispersed fiber distribution.

2) The fiber volume fraction is easily changed by increasing and decreasing the number of modeled fibers. Thereby, various material models with different characteristics, such as strain-softening and strain-hardening FRCC models, can be created.

3) As the method is based on RBSM (a discrete analysis method), the crack width is directly obtainable and the relation between the crack width and fiber-bridging force is easily verified.

In this model, short fibers with a given length are arranged individually in the specimen model until the predetermined fiber volume fraction was reached. As shown in Fig. 4, a zero-size spring is placed at each crossing point of a fiber and a facet common to two cells (crack plane), and the bond stress versus slip relations are integrated to obtain a fiber-bridging force that acts on the spring. The embedment length $l_e$ and angle from the facet-normal (orientation angle) $\phi$ are also calculated for each fiber. Note that the orientation angle can be determined as the inner product of the vector normal to the boundary surface of the element and the vector in the fiber-orientation direction.

(2) Modeling of fiber action

After cracking, the stress transfer by the fibers across the crack plane in the cement matrix is calculated from the embedment length $l_e$, the fiber–matrix bond characteristic, and the pullout displacement, as described in Ogura et al. (2012). The pullout displacement is calculated from the crack width of the cement matrix in each simulation step. This calculation is possible because the RBSM can directly calculate the crack width from the relative displacement between the adjacent elements.

Before the cement matrix cracks, the stress borne by the fibers is calculated using the shear lag theory of Cox (1952). Cox expressed the tensile stress distribution in the fibers within the matrix as

\[
\sigma_f = E_f \epsilon_m \left[1 - \frac{\cosh (\beta L_f / 2 - x)}{\cosh (\beta L_f / 2)} \right]
\]

where $\sigma_f$ is the fiber tensile stress (MPa), $E_f$ is the fiber elastic modulus (MPa), $\epsilon_m$ is the matrix strain, $L_f$ is the fiber length (mm), $x$ is the distance from the fiber end (mm), and $\beta$ is a factor-dependent constant (here set to 1.0).

2.3 Rebar and bond–slip model

(1) Modeling of the rebar and the matrix–rebar bond characteristic

The rebar model is formed as a series of regular beam elements that resist the axial and bending stress (see Fig. 5). In this model, the rebar can be freely positioned within the member, regardless of the mesh design of the concrete (Bolander and Saito 1998). Three translational and three rotational degrees of freedom are defined at each beam node. The stress–strain relation of the rebar (see Fig. 6) is determined by Shima et al.’s model (1987). In this study, the strain at the start of hardening $\epsilon_{sh}$ and the tensile strength $f_u$ are set to 1.5% and 1.2 $f_y$, respectively.

The rebar (beam element) is attached to the cement matrix particles by zero-size link elements, which provide a force-transfer mechanism between a cement matrix particle and a beam node (Saito and Hikosaka 1999). The bond interaction between the cement matrix and rebar strongly affects the crack propagation. Figure 7 shows the bond–slip relation between a cement matrix and a link element. Before and after the peak strength $\tau_{max}$, the strength is computed by Eq. (2) (Suga et al. 2001) and a relation proposed by Sawabe et al. (2006), respectively.

![Fig. 4 Fiber location and zero-size spring. The variables $l_e$, $L_f$, and $\phi$ denote the fiber length, embedment length, and orientation angle between the facet and the normal, respectively.](image)

![Fig. 5 Beam and link elements.](image)

![Fig. 6 Stress–strain relationship in the rebar.](image)
\[
\tau = 0.4 \cdot 0.9 f'_c^{2/3} \left[ 1 - \exp \left( -40 \left( s / D \right)^{0.1} \right) \right]
\]

(2)

where \( f'_c \) is compressive strength of the cement matrix (MPa), \( s \) is the slip displacement (mm), and \( D \) is the rebar diameter (mm).

(2) Tensile behavior of cement matrix with rebar in numerical model

To confirm the simulation capability of the numerical approach, analytical results were compared with the experimental results of uniaxial tensile tests (Tamai et al. 1987). The specimens were concrete prisms with a cross section of \((200 \times 150) \text{ mm}^2\) and a rebar through the center. The model details are illustrated in Fig. 8. For simplicity and computational efficiency, Model (a) was constructed from rectangular cells with a \((200 \times 150) \text{ mm}^2\) cross section and 20 mm spacing. The cells were consecutively arranged in the longitudinal direction of the specimen. Model (b) was constructed from Voronoi cells with random geometries, as described by Bolander and Saito (1998). The average distance between the Voronoi cells was approximately 20 mm. The rebar was modeled as a beam element. Displacement-controlled loading was applied in the longitudinal direction. Table 1 shows the physical properties of Model (a), and Fig. 9 compares the load–average strain relations of both models with the experimental results. The average strain \( \varepsilon_{ave} \) was measured over the 2100-mm gage length. The simulated initial stiffness, tension-stiffening effect after cracking, and behavior in the post-yield range of the rebar were consistent with the experimental results.

Figure 10 compares the stress and strain distributions in the rebar obtained in the present analysis and previous experiments. As seen in that figure, the strain distributions and crack spacing derived from the numerical
model well agree with the experimental results, confirming the simulation capability of the numerical model for cement matrix with rebar. Moreover, the results are almost independent of element division. Figure 11 shows the crack patterns under average strain of 1.0%. As Models (a) and (b) yielded similar tensile failures and crack behaviors, Model (a) was employed in the following parametric study by virtue of its higher computational efficiency than Model (b).

3. Tensile fracture analysis of strain-softening FRCC

3.1 Outline of analysis

(1) Modeling of the cement matrix and rebar

Figure 12 shows the analysis model of strain-softening FRCC. The model is almost identical to the specimen in the previous section, but with four D19 rebars arranged at both end portions of the specimen. Therefore, the test section in this analysis was the 2100-mm length excluding the reinforcing sections (600 mm) at both ends. The matrix was divided into cells of rectangular cross section (200 ×150) mm² and regular spacing (20 mm), arranged in the longitudinal direction of the specimen, as described in the previous section. The loading was applied by displacement control of the beam element located at each end of the specimen.

The material parameters of the cement matrix and rebar were those in the previous section (see Table 1). However, the tensile strength of the cement matrix (normal spring of RBSM) was varied by a pseudo-random number. In a previous study (Kunieda et al. 2011), the tensile strength distribution in the FRCC matrix expressed the proper fracture behavior. Hence, the strength distribution was here assumed as a normal distribution with an average and standard deviation of 2.9 MPa and 0.3 MPa, respectively. At this stage, we lack sufficient quantitative data for optimizing these values, so data accumulation is desired to better discuss the distribution.

(2) Modeling of fibers

In this study, the fibers were randomly dispersed in the specimen. The centroid positions (x, y, z coordinates) and inclination angles of the fibers were determined by pseudo-random numbers, and the fibers were generated one by one until the predetermined fiber volume fraction \( V_f \) was reached. At that time, if any fibers were partially outside the specimen area, their inclination angles were reset by repeating the pseudo-random number until the whole fiber fitted within the specimen area.

Figure 12(b) shows the distribution of 47,746 fibers generated in the case wherein \( V_f = 0.5\% \). This model is the basic analysis model of the current research. Figure 12(c) shows the fiber distribution in a nonuniform dis-

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Fig. 11 Simulated crack patterns under 1.0% average strain.

Fig. 12 Discretized tensile specimen (\( V_f = 0.5\% \)): (a) cement matrix elements, (b) uniform fiber-distribution model and (c) nonuniform fiber-distribution model.
distribution model for the same volume fraction ($V_f = 0.5\%$). This case considers the nonhomogeneous FRCC-production conditions while considering varying fiber distribution during construction.

Figure 13 plots the number of fibers in the cross section of each model and fiber intersections at cross sections A, B and C. The fibers were counted in each of the 106 boundary surfaces between the consecutive adjacent elements of the test section. The number of fibers in the cross sections of the nonuniform distribution model ranged widely, from 147 to 352. Even in the uniform distribution model, where the fibers and their orientations were randomly generated by pseudo-random numbers, the number of fibers varied in the cross sections because of the discrete modeling approach. The averages and variation coefficients of the fiber numbers in the cross sections were 268.1 and 0.056, respectively, for the uniform distribution model, and 267.5 and 0.140 respectively, for the nonuniform distribution model.

The material nonuniformity in the discrete modeling approach mimics that in actual structures. Therefore, this modeling is convenient when such variations are important. These variations, which may not be considered in rebar reinforcement, are arguably necessary for understanding the fracture behavior of FRCC.

Table 2 Material properties of the short fiber.

| Fiber Length $L_f$ | 30mm |
|-------------------|------|
| Diameter $d_f$    | 0.6mm|
| Elastic modulus $E_f$ | 200 GPa |
| Strength $\sigma_{fu}$ | 1078MPa |
| Interface Frictional strength $\tau_i$ | 9.5MPa |
| Bond strength $\tau_s$ | 10.0MPa |
| Stiffness $G$ | 700MPa/mm |

Fig. 13 Number of fibers intersecting the cross sections along the model’s axis and fiber intersections at cross sections A, B and C.

Fig. 14 Tensile stress–crack width responses of FRCC ($V_f = 1.0\%$) obtained in the model (Num.) and in previous experiments [Exp., Uchida and Ozawa (2001)].

3.2 Analytical results

(1) Macroscopic response

Figure 15 shows the tensile load–average strain relations obtained at $V_f = 0.5\%$ and $V_f = 0$. In the case wherein $V_f = 0$, the stiffness declined after cracking, and gradually and asymptotically approached the bare bar response. In the case of $V_f = 0.5\%$ specimen, the curve did not descend to the bare bar response even after cracking, confirming that the yielding load was in-
increased in this specimen.

Figure 16 shows the crack patterns at the times of the ○ marks labeled B - D in Fig. 15. The deformation is magnified five times for clarity. At the time of rebar yielding (point B), six and nine cracks appeared in the $V_f = 0$ and $V_f = 0.5\%$ specimens, respectively, indicating that the fibers improved the crack distribution. The same tendencies before rebar yielding were observed in a previous experiment (Bischoff 2001).

After rebar yielding, the load in the load–average strain curve of the $V_f = 0.5\%$ specimen decreased from point B to point C. Whereas the opening displacement of only one crack increased in the $V_f = 0.5\%$ specimen, all crack planes opened in the $V_f = 0$ specimen, and the crack widths were approximately even. The load decrease and crack localization in the post-yield range of the rebar is consistent with the fracture behavior reported by Deluca et al. (2012).

The average stress–average strain responses of the rebar in the matrix are shown in Fig. 17(a). The average stress was obtained by averaging the stresses in the beam element of the test section. In the average stress–average strain curve of a rebar, it is known that 1) the yielding point is lower than the yielding point of the bare bar and 2) increasing the strain immediately increases the stress in the post-yield range of the rebar, as it occurs in strain-hardening regions of bare bar (Tamai et al. 1987). Both of these behaviors emerged in the numerical analysis ($V_f = 0$ and $V_f = 0.5\%$).

The average stress in the FRCC, plotted as a function of strain in Fig. 17(b), was obtained by subtracting the rebar contribution from the tensile load in Fig. 15 and dividing it by the cross-sectional area of the member. The rebar contribution was obtained by multiplying the average stress of rebar with the cross-sectional area of the rebar. The average stress in the FRCC was higher in the $V_f = 0.5\%$ specimen than in the $V_f = 0$ specimen. This can be explained by two effects: tension stiffening induced by bond action between the rebar and matrix, and the action of bridging forces of the fibers across the cracks. The same behaviors were observed in previous experimental studies (Shionaga et al. 2010).

(2) Stress distribution in the rebar and FRCC

Figure 18 shows the simulated crack patterns and fibers bridging the crack planes under the average strain of 0.57% (point C in Fig. 15). In the $V_f = 0$ specimen, the rebar strain was approximately similar in all crack planes and reached the strain at the start of hardening, $\varepsilon_{sh}$. The crack widths of the crack planes were also approximately similar. Conversely, in the case wherein $V_f = 0.5\%$, the rebar strain reached 1.8% at the surface of the localized crack, and was below $\varepsilon_{sh}$ at all other surfaces. In the crack plane of the localized cracks, half of the bridging fibers were pulled out.

Figure 19 plots the stress and strain distributions in the rebar and FRCC along the specimen axis at points A - C in Fig. 15. The local maxima of the distribution in Figs. 19(a) - (b) and 19(d) - (e) corresponds to the crack onset position. In the case wherein $V_f = 0$, the rebar strain at the local maxima increases at an almost similar rate. In contrast, in the case wherein $V_f = 0.5\%$, the rebar strain in one region increases rapidly at point C ($\varepsilon_{ave} = 0.57\%$).

In the FRCC stress distribution curve of the $V_f = 0.5\%$ specimen [Fig. 19(b)], the following features are noted:
1) The stress of the local minima (corresponding to the crack onset position) is never 0.
2) The stress at point C ($\varepsilon_{ave} = 0.57\%$) decreases remarkably in one region, and the stress difference is larger in this region than in other regions.

Feature 1) can be explained as follows. In the case
wherein $V_f = 0$, only the rebar resists the total tensile force in the crack plane. In contrast, in the case wherein $V_f = 0.5\%$, the fibers across the cracks also transfer the stress. The stress of the local minima corresponds to the bridging forces of the fibers. As these bridging forces vary with the crack width, the local minimum values

![Deformation magnification: ×5](image)

![Fracture localization](image)

Fig. 18 Simulated crack patterns and fibers bridging the crack planes under 0.57\% average strain: (a) $V_f = 0$; (b) $V_f = 0.5\%$. Fibers bearing the tensile force and the pulled out fibers (pullout displacement = embedment length) are shown in blue and green, respectively.

![Strain of rebar (%)](image)

![Stress of rebar (MPa)](image)

![Stress of concrete (MPa)](image)

![Stress of FRCC (MPa)](image)

Fig. 19 Stress and strain distributions in the rebar and FRCC along the specimen axis at Points A, B, and C in Fig. 15: (a - c) $V_f = 0$; (d - f) $V_f = 0.5\%$. 
increase or decrease with increasing average strain. The stress difference at each crack plane (local minima) from the initial loading stage (point A) is attributable to the varying numbers of fibers across the crack plane, as shown in the previous section. Meanwhile, feature 2) can be best explained as follows.

1) Once the bridging force of the fibers begins decreasing at a crack plane, the rebar stress increases to compensate the loss.
2) Increasing the rebar stress promotes the rebar slippage at the crack plane, further increasing the crack width.
3) Increasing of the crack width further reduces the bridging force of the fiber.

Thus, once the bridging force of the fibers begins diminishing at the crack plane, the rebar stress and the crack width at that position rapidly increase, and localized crack occurs.

(3) Mechanism of crack localization

Figure 20(a) illustrates the relation between the crack width and the bridging force $\sigma_{bf}$. This relation, called the fiber-bridging constitutive law, quantifies the contribution of the fibers. The tension softening curve of plain concrete (also shown in the figure) relates the fictitious crack width to the cohesive stress in the fracture process zone of the cement matrix. Meanwhile, the fiber-bridging constitutive law assumes that the bridging forces are contributed only by the fibers across cracks. In the tension softening curve of the cement matrix, the stress decreases with broadening crack width $w$. In the fiber-bridging constitutive law, the stress increases up to $w_{peak}$ and then declines with further broadening.

Moreover, in the rebar, the crack-bridging performance can also be represented by plotting crack width versus the rebar force at the crack plane. This curve [see Fig. 20(b)] assumes that the bridging force (referred to as rebar-bridging force hereafter) is contributed only by the rebar. In this study, the rebar-bridging force $P_{bs}$ was obtained by multiplying the rebar stress $\sigma_s$ at the crack location by the rebar cross-sectional area $A_r$. This enables a direct comparison with the fiber-bridging force $P_{bf}$ obtained by multiplying the fiber bridge stress $\sigma_{bf}$ by the FRCC cross-sectional area $A_c$.

Li and Leung (1992) found that SHCC materials can be quantitatively designed from the tension softening curve and the fiber-bridging constitutive law. However, in FRCC with a rebar, the sum of these two curves and the rebar-bridging-force curve can be regarded as the total bridging force versus crack width curve, as shown in Fig. 20(c).

Figure 21 shows the calculated bridging-force curve at the crack plane indicated by the arrow in Fig. 18. In the case wherein $V_f = 0$, cracking reduces the cement matrix stress. At a crack width of 0.17 mm, the cement matrix makes no contribution, and the rebar bears the entire tensile force. Once the member yields, the rebar at the crack location reaches the strain-hardening zone. As the crack opens, the bridging force of the rebar gradually increases. Even as the crack widens, the overlapped bridging-force curve (relation between total bridge force and crack width) maintains a positive gradient.
Conversely, in the case wherein $V_f = 0.5\%$, the bridging force of the fibers remains even when the cement matrix stress vanishes. Although the bridging force gradually decreases beyond $w_{\text{peak}}$ (0.9 mm), it never vanishes even when the crack width reaches 6 mm. The total bridging force versus crack width curve was determined by summing the bridging forces of the fibers and rebar. In the present FRCC analysis, the negative gradient of the fiber-bridging-force curve exceeded the positive gradient of the rebar-bridging-force curve [see Fig. 21(b)]. Therefore, the total bridging-force curve in the post-yield range of the rebar possessed a negative gradient.

In the softening zone of the total bridging-force curve (if such a zone exists), the crack plane can bear no further load, and the crack width rapidly increases while the load decreases. In contrast, the other crack planes are in an unloaded state and their crack widths decrease. Consequently, the fractures were concentrated at the crack planes on which the crack widths first reach $w_{\text{peak}}$.

### 3.3 Parametric study

This subchapter investigates the influence of fiber volume fraction, steel reinforcement ratio, and fiber distribution on the tensile fracture behavior. Focusing on the bridging-force curve, it then discusses preventative measures against fracture localization. The other analysis conditions were unchanged from the previous subchapter.

#### (1) Effect of fiber volume fraction

The fiber volume fraction $V_f$ was varied as 0.3, 1.0, and 1.5%, maintaining the steel reinforcement ratio $p$ as 2.2%. The fibers were introduced to the specimen one by one until their numbers satisfied the predetermined fiber volume fraction. The number of fibers in the 0.3, 1.0 and 1.5% specimens were 28 647, 95 492, and 143 239, respectively.

Figures 22(a) and 22(b) plot the tensile loads and maximum crack widths as functions of average strain in the specimens with different fiber volume fractions. The load capacity tended to increase with increasing of $V_f$ and the maximum crack width tended to reduce as $V_f$ increased up to rebar yielding. In the post-yield range of the rebar, the $V_f = 0.3\%$ specimen maintained its load increment up to 1.5% average strain, owing to the fiber reinforcement. In contrast, the load in the $V_f = 1.0\%$ specimen decreased at average strains of 0.6% and higher, accompanied by a sharp increase in the maximum crack width. The behavior notably differed between this case and the other cases. In the case wherein $V_f = 1.5\%$, the load gradually increased in the post-yield range of the rebar, and the maximum crack width increased more moderately than in the other cases.

Figure 23 shows the simulated crack patterns of the specimens with different fiber volume fractions under average strain of 1.2%. Increasing the fiber volume fraction narrowed the crack spacing and increased the number of cracks. In the $V_f = 0.5$ and 1.0% specimens, the crack width was larger in one locality than in the remainder of the specimen. This localization behavior disappeared in the $V_f = 0.3\%$ specimen.
Figure 24 plots the total number of cracks as a function of fiber volume fraction under average strain of 1.2%. The number of “cracks in the loading state” are also plotted, where the current crack width has exceeded the maximum crack width experience in the past. In the $V_f = 0.5$ and $V_f = 1.0$ specimens, two and one crack planes developed in the loading state, respectively (corresponding to the localized cracks in Fig. 23). In contrast, all crack planes in the $V_f = 0$ and 1.5% specimens were in the loading state. Under the imposed conditions, we found a range of $V_f$ in which localized cracks developed in the post-yield range of the rebar.

(2) Effect of rebar reinforcement
To investigate the effect of steel reinforcement ratio on the fracture behavior, we varied the main rebar material as D13, D19, D29 and D41 (obtaining steel reinforcement ratios of 0.4, 1.0, 2.2 and 4.5% respectively). The fiber volume fraction $V_f$ was maintained at 0.5%.

Figure 25 shows the tensile load versus average strain responses in the specimens with different steel reinforcement ratios. In the $p = 0.4$% specimen, the load decreased in the post-yield range of the rebar, and the response asymptotically approached the bare bar response. Loss of post-yielding load was better prevented in specimens with higher steel reinforcement ratio, and in the $p = 4.5$% specimen, the load was maintained even when the average strain reached 2%.

Figure 26 shows the simulated crack patterns under average strain of 2.0% in the specimens with different steel reinforcement ratios. All specimens except for $p = 4.5$% case developed localized cracks in the post-yield range of the rebar. No noticeable fracture localization was observed at $p = 4.5$%, and the crack widths gradually increased in the post-yield range of the rebar. Increasing crack numbers at higher steel reinforcement ratios have also been observed in ordinary reinforced concrete members (Tamai et al. 1987).

(3) Effect of fiber distribution
The influence of fiber dispersion on the fracture behavior was evaluated in the nonuniform distribution model shown in Fig. 12(c). Figure 27 shows the simulated crack patterns under average strain of 2.0%. In the $p = 3.2$% specimen, the width of each crack was more variable than in the reference (uniform distribution) model (see Fig. 26). In the other specimens, the crack patterns closely resembled those of the reference model. In addition, the load–average strain relations did not significantly differ between the nonuniform distribution model and the reference model.

The fiber distribution little affected the fracture behavior under the analytical conditions. However, in the $p = 3.2$% specimen, the crack width increased at locations containing relatively few bridging fibers. This
suggests that further nonuniformity of the fiber distribution would promote fracture localization.

4) Parameters affecting crack propagation behavior
The effect of each parameter on the crack propagation behavior (within the scope of the study) is summarized in Table 3.

4. Tensile fracture analysis of SHCC

4.1 Outline of the analysis
The previous section focused on strain-softening FRCCs. This section analyzes strain-hardening FRCCs that exhibit quasi-ductile behavior with a strain capacity of up to several percent, resulting from the formation of multiple fine cracks prior to reaching the tensile strength of the composite.

To simulate an FRCC material with strain hardening and multiple fine cracks, the fiber volume fraction \(V_f\) was set at 1.5%, and the other analysis conditions were same as those in Section 3. The steel reinforcement ratio \(p\) was varied as 0, 0.4, 1.0, and 2.2%. As described in the previous chapter, the tensile strength of the matrix (normal spring of the RBSM) was set using normal random numbers, assuming normally distributed tensile strengths with an average of 2.9 MPa and a standard deviation of 0.3 MPa.

4.2 Analytical results
(1) Macroscopic responses
Figure 28 shows the tensile stress as a function of average strain response in the rebar-less specimen. Under the imposed parameter settings, the model replicated the behavior of a material with strain hardening and multiple fine cracks. The new cracks were continuously generated after initial cracking.

Figure 29(a) plots the tensile loads as functions of average strain response in the simulated specimens with different steel reinforcement ratios. The triangle symbol in the figure is the time point at maximum load, namely, the ultimate point. In the specimens with rebars, the strain at maximum load (hereafter referred to as ultimate strain \(\varepsilon_{\text{cu}}\)) was 1.8 to 2.6 times higher than in the rebar-less specimen. Beyond the ultimate strain, the load-strain responses gradually approached the behavior of the bare bar. Whereas the steel reinforcement ratio largely affected the macroscopic behavior of the strain-softening FRCC in the previous section, it little influenced the macroscopic behavior of the strain-hardening FRCC. That is, the load was maintained after rebar yielding at all steel reinforcement ratios, indicating an effective contribution of the rebar even at low steel reinforcement ratios.

(2) Crack propagation behavior
Figure 29(b) shows the relations between the average strain and number of cracks in the strain-hardening FRCC specimens. All cracks with opening widths of 0.01 mm or higher were counted. Unlike the strain-softening FRCC, the strain-hardening FRCC continuously developed new cracks in the post-yield range of the rebar, as evidenced by the increasing crack count in this range.

Figure 29(c) shows the relation between the average strain and maximum crack width in the strain-hardening FRCC specimens. In the specimens with rebars, the maximum crack width did not sharply increase in the post-yield range of the rebar when the deformation was relatively small. This behavior differs from that of the strain-softening FRCC analyzed in the previous section. When the average strain exceeded 1.0% and the crack width reached \(w_{\text{peak}}\), the fracture was concentrated on one crack with a rapidly increasing maximum crack width.

| Parameter                              | Crack localization after rebar yielding                      |
|----------------------------------------|--------------------------------------------------------------|
| Fiber volume fraction \(V_f\)          | Prevented by lowering the value                              |
| Steel reinforcement ratio \(p\)         | Prevented by increasing the value                            |
| Bond interaction between rebar and FRCC| Prevented by improving the interaction                      |
| Fiber distribution                      | Possibly prevented by reducing nonuniformity                 |

Fig. 27 Simulated crack patterns under 2.0% average strain in the nonuniform distribution model.

Fig. 28 Tensile stress–average strain responses in the rebar-less specimen \(p = 0\).
width. The crack localization reduced the load at this point, which corresponded to the maximum load point. The number of cracks at the maximum load point was higher in the rebar specimens than in the rebar-less specimen \( (p = 0) \), and increased with increasing steel reinforcement ratio.

Figure 30 shows the cracking behavior at the maximum load and the time points marked \( \bullet \) in Fig. 29(a). In all specimens, localized cracks appeared at the post-peak region. The rebar-less specimen was characterized by nonuniform crack spacing, but in the \( p = 0.4 \) and \( 2.2\% \) specimens, the cracks were dispersed across the specimen at more-or-less even intervals. Uniform crack intervals in SHCC with rebar reinforcement have also been observed in previous experiments (Kunieda et al. 2010).

(3) Stress distributions in the rebar and FRCC

Figure 31 plots the strain and stress distributions in the rebar and FRCC along the specimen axis of the \( p = 2.2\% \) specimen. The results are plotted at points A to C (marked by open circles) in Fig. 29(a). The rebar strain at the crack locations (points of local maxima) reached the strain at the start of hardening \( \varepsilon_{sh} \). The number of cracks (number of convexities) increased even in the post-yield range of the rebar (after point B). This behavior was absent in the strain-softening FRCC cases.

In the FRCC stress (lower panel of Fig. 31), the stress of local maxima exceeded the fiber-bridging stress at the crack locations, due to bond action with the rebar. Consequently, even when the average stress was below the tensile strength, the stress in the point of local maxima reached the tensile strength, and new cracks appeared. Thus, the crack dispersion improved and the cracks were uniformly distributed, increasing the ultimate strain.

4.3 Effect of fiber distribution

The influence of fiber distribution on the fracture behavior was evaluated in the nonuniform distribution model shown in Fig. 12(e). The \( p \) and \( V_f \) were unchanged from the previous subchapter.

Figure 32 plots the load–average strain responses in specimens with different steel reinforcement ratios. The number of cracks and the ultimate strain are given in Table 4. The ultimate strain of the rebar-less specimen in the nonuniform distribution model was 0.13%, indicating that dispersing the fiber distribution lowered the performance of the FRCC alone. Even in the specimens with rebars, ultimate strain was lower in the nonuniform than in the uniform distribution model. Varying the fiber distribution exerted no clear effect on the tensile frac-

| \( p \) | \( \Delta \) (Maximum load) | \( \bullet \) (Post peak) |
|------|----------------------------|---------------------|
| 0    | ![Image](image1.png)       | ![Image](image2.png) |
| 0.4% | ![Image](image3.png)       | ![Image](image4.png) |
| 2.2% | ![Image](image5.png)       | ![Image](image6.png) |

Fig. 30 Simulated crack patterns in specimens with different steel reinforcement ratios \( (V_f = 1.5\%) \).
ture behavior of the strain-softening FRCCs discussed in the previous subchapter but certainly affected that of the strain-hardening material. As the number of cracks increased, new cracks were more likely to be formed at positions of few bridging fibers. This explains the considerable influence of material nonuniformity in the strain-hardening FRCC specimens, which developed more cracks than the strain-softening specimens. In fact, the localized crack plane in the nonuniform distribution model were located where the number of bridging fibers was small.

Figure 33 shows the fiber-bridging stress–versus crack width curves of the cracked plane obtained from the analysis case of $p = 2.2\%$. The red line corresponds to the localized cracked plane in this analysis, and it is the crack that actually reached the softening zone. The triangle symbol in the figure corresponds to the instant at which maximum stress was reached. The black line represents the crack that was unloaded before reaching the softening zone in this analysis. To compare the red and black lines’ results, a separate analysis was performed to calculate the black lines. Figure 33 confirms that the crack localizes on the weakest plane in the generated cracks, and the difference in the fiber-bridging stress curve (material nonuniformity) of each crack plane is larger in the nonuniform distribution model. This result suggests that the weakest plane affects the macroscopic response.

Also notable in Table 4, the rebar largely improved the ultimate strain in the nonuniform model, in which the material performance was degraded by the nonuniformity of fiber distribution. The ultimate strain was 5 and 7 times higher, respectively, in the $p = 0.4\%$ and $2.2\%$ specimens than in the rebar-less specimen. Therefore, rebar reinforcement can effectively reverse the decline in ultimate strain caused by the material nonuniformity.

Next, five pseudo-random numbers were selected to generate five different fiber-distribution models, and the ultimate strain in each model was investigated. Changing the pseudo-random number changes the fiber distribution by altering the position and angle of each generated fiber. The fiber distributions were varied in both the uniform distribution model [see Fig. 12(b)] and the model with dispersed fiber distribution [see Fig. 12(c)], and all models were analyzed by varying the steel reinforcement ratio.

Figure 34 plots the ultimate tensile strain as functions of steel reinforcement ratio in the uniform and nonuniform models. The error bars in show the data section. In Table 4 Number of cracks and ultimate strain at maximum load.

| Uniform distribution | Nonuniform distribution |
|----------------------|------------------------|
| $p = 0$              | $p = 0$                |
| Number of cracks     | 13 (0.52)              |
| $€_{cu}$ (%)         | 4 (0.13)               |
| $p = 0.4\%$          | $p = 0.4\%$           |
| Number of cracks     | 32 (0.95)              |
| $€_{cu}$ (%)         | 21 (0.62)              |
| $p = 1.0\%$          | $p = 1.0\%$           |
| Number of cracks     | 41 (1.15)              |
| $€_{cu}$ (%)         | 30 (0.67)              |
| $p = 2.2\%$          | $p = 2.2\%$           |
| Number of cracks     | 49 (1.35)              |
| $€_{cu}$ (%)         | 38 (0.91)              |

Fig. 31 Strain and stress distributions in the rebar and FRCC along the specimen axis ($p = 2.2\%$).
all cases, the ultimate strain was improved by the rebar reinforcement. The average improvement effect (relative to the rebar-less case) was 1.5 times or more in the uniform distribution model, and 2.6 times or more in the nonuniform distribution model. In addition, increasing the steel reinforcement ratio increased the ultimate strain of the composites. In this analysis, the ultimate strains in the five fiber-distribution models were quite variable, giving rise to large error bars. To better discuss the fiber distribution, the material nonuniformity of FRCC should be quantified by accumulating more data in both experimental and actual construction materials.

5. Conclusions

In this study, FRCC with a rebar were investigated in a mesoscale tensile fracture analysis with discretized short fibers. The effects of fiber volume fraction, steel reinforcement ratio, FRCC-rebar bond characteristics, and fiber distribution on tensile fracture were numerically investigated. The study conclusions are summarized below:

1) The proposed analytical method expressed the crack localization behavior in the post-yield range of the rebar. This behavior was consistent with previously reported behaviors.
2) By studying the overlap of the bridging force versus crack-width curves in the fiber and rebar, the mechanism of crack localization in the post-yield range of the rebar was explained.
3) By varying the various parameters and studying the fracture behavior, crack localization can be mitigated by reducing the fiber volume fraction and increasing the steel reinforcement ratio.

An analysis of strain-hardening FRCCs yielded the following additional conclusions:
4) At any steel reinforcement ratio, the rebar reinforcement effectively improved the ultimate strain, and the cracks did not localize immediately in the post-yield range of the rebar. Under the present numerical conditions, the ultimate strain was improved even at low steel reinforcement ratios (0.4%).
5) In samples with nonuniform fiber distribution, the rebar improved the ultimate strain by 5–7 times (for steel reinforcement ratios of 0.4–2.2%) relative to the rebar-less case. This result suggests that rebar reinforcement prevents the degradation of the ultimate strain caused by material nonuniformity.

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