Turbulence-driven thermal and kinetic energy in the atmospheres of hot Jupiters

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ABSTRACT

We have performed high resolution 3-dimensional compressible hydrodynamics simulations to investigate the effects of shocks and turbulence on energy transport into hot Jupiter atmospheres, under a variety of shear gradients. We focus on a local atmospheric region to accurately follow the small-scale structures of turbulence and shocks. We find that the effects of turbulence above and below a shear layer are different in scale and magnitude: Below the shear layer, the effects of turbulence on the vertical energy transfer are local, generally \( \lesssim 2 \times (\text{scale height}) \). However, turbulence can have a spatially and thermally-large influence on almost the entire region above the shear layer. We also find that shock formation is local and transient. Once the atmosphere becomes steady, the time-averaged heat energy flux at \( P \approx 1 \text{ bar} \) is insignificant, on the order of 0.001% of the incoming stellar flux with a shear motion at \( P \approx 1 \text{ mbar} \), and 0.1% with a deeper shear layer at \( P \approx 100 \text{ mbar} \). Accordingly, the diffusion coefficient is higher for the deeper shear layer. Therefore, our results suggest that turbulence near less dense regions does not cause a sufficiently deep and large penetration of thermal energy to account for radius inflation in hot Jupiters, regardless of how violent the turbulence is. However, as the shear layer gets deeper, heat energy transfer becomes more effective throughout the atmosphere (upwards and downwards) due to a larger kinetic energy budget. Therefore, it is more important how deep turbulence occurs in the atmosphere, than how unstable the atmosphere is for effective energy transfer.

Key words: Hot Jupiter – planetary systems : atmosphere – planetary systems : gaseous planet

1 INTRODUCTION

Hot Jupiters are a class of gas-giant exoplanets, characterized by short orbital periods (\( P \lesssim 50 \) days). Such close proximity to their parent stars leads to several interesting features, which include tidal synchronization, strong irradiation, and a generally large day-night temperature contrast. A number of hot Jupiters are observed to have radii larger than what predicted from standard cooling models (e.g. Showman & Guillot 2002; Guillot & Showman 2002; Howard et al. 2012; Wang et al. 2015). The origin of the radius inflation is still debated, and several ideas have been put forward to explain it.

Inflated radii imply that the bloated planets retain more internal entropy than expected. This could be produced by either injection/dissipation of heat, or less efficient energy loss, or a combination of both. Within this context, the mechanisms that have been put forward to explain the radius anomaly can be divided into two classes. The first category includes less efficient cooling due to enhanced opacity (Burrows et al. 2007). As the opacity increases, cooling becomes inefficient and the planets can naturally retain more internal heat. The second category invokes extra heat sources in the interior, such as the dissipation of heat via tidal forces (Bodenheimer et al. 2001; Jackson et al. 2008; Ibgui & Burrows 2009; Ibgui et al. 2011), conversion of the stellar flux into kinetic energy of the global atmospheric flow, driven by the large day-night temperature gradient (often called “hydrodynamic dissipation”); Showman & Guillot 2002; Guillot & Showman 2002), magnetic drag in ionized planetary winds, or “ohmic dissipation” (Batygin & Stevenson 2010; Perna et al. 2010a,b, 2012), and dissipation of energy induced by fluid instabilities (Li & Goodman 2010).

Among those, energy dissipation via turbulence (Li & Goodman 2010), likely accompanied by shocks (Perna et al. 2012; Dobbs-Dixon & Agol 2013; Heng 2012)\(^1\), could be a

\(^1\) Generally, fluid in a stably stratified atmosphere becomes unstable when the shear stress, or velocity gradient, is sufficiently
viable, or at least interesting mechanism to consider. This is because turbulence may be ubiquitous and present even in stably stratified atmospheres. It is hence natural to study its onset in globally circulating planetary atmospheres. Youdin & Mitchell (2010) proposed that forced turbulence can drive downward transport of heat in the outer radiative zone of stratified atmospheres. They call this the “mechanical greenhouse effect”\(^2\), and build an analytic model of the outer radiative zone, focusing on diffusion and dissipation by forced turbulence. They find that a heat flux generated by forced turbulence propagates downwards and can be deposited in deeper regions. Their analytic approach, undoubtedly necessary for understanding the underlying physics, is however more suitable for somewhat idealized cases. To account for a more realistic scenario, simulations with detailed modelling are essential. Recently, using the compressible shock-capturing code RAMSES, Fromang et al. (2016) developed a 3-dimensional model to examine the role of shear-driven instabilities and shocks in planetary atmospheres. They covered a large volume of the atmosphere to take into account global motions and included cooling via a Newtonian cooling method. Their simulations suggest that equatorial jets are subject to shear-driven instabilities, which can lead to a sufficiently large amount of downward kinetic energy flux and the formation of shocks at a few mbar pressure levels. Their results improve and deepen our understanding of the physics of turbulence and shocks. However, as they pointed out in their paper, it is possible that their spatial resolution may still be too large to capture processes occurring on small scales.

Motivated by those studies and in order to improve on some of their limitations, in this work we investigate the effect of shocks and turbulence on energy penetration in stable stratified atmospheres, using 3-dimensional compressible hydrodynamics simulations with high resolution with the code CASTRO (Almgren et al. 2010). We focus on a local atmospheric region to accurately capture the small-scale structures of the eddy motion. We estimate how much and how deep heat can be deposited in the atmosphere when shear motions are driven. Based on the measured heat flux, we further estimate the diffusion coefficient \(K_G\) (see Equation 19). Last, we discuss the formation, duration and distribution of shocks in the planetary atmospheres.

In our suites of simulations, we find that the effects of turbulence on the kinetic and heat energy transfer are local, generally confined to within a spatial range of \(z \sim 2H\) (where \(H\) is the scale height) below where eddies are created, but turbulence can make a spatially and thermally large-scale impact on the regions above it. We also find that shock formation is local and transient. The time-averaged heat energy flux at \(P \sim 1\) bar when the atmosphere becomes steady is on the order of 0.1 – 0.01% of the incoming stellar flux depending on the location of the shear layer (lower flux for an outer shear layer). Hence, our results suggest that turbulence near less dense regions \((P \gtrsim 1\) mbar\) does not lead to a sufficient amount of thermal energy burial in deeper regions to account for the inflated radii of hot Jupiters, regardless of how violent the turbulence is. On the other hand, thermal energy can be transferred more effectively throughout the atmosphere when turbulence is triggered at deeper regions \((P \gtrsim 100\) mbar\). Therefore, it is more important how deep turbulence occurs in the atmosphere, than how unstable the atmosphere is for effective transfer of energy.

This paper is organized as follows. In Section 2, we explain our numerical setup including the model description (Section 2.1) and the boundary conditions (Section 2.2), and describe our shear prescription (Section 2.3) and initial model parameters (Section 2.4). We present our results in Section 3. In Section 4, we first compare our results with two different numerical resolutions for the same set-up, and then we compare simulations at higher resolution but with different atmospheric depths for the shear layer. Finally, we conclude with a summary of our findings in Section 5.

2 NUMERICAL SETUP

In this section we present our planetary atmospheric models. We describe the initial conditions of the model atmospheres and our shear prescription.

2.1 Model description

In order to follow the evolution of our model atmosphere, we solve the 3-dimensional hydrodynamic equations in a Cartesian coordinate system, using the code CASTRO (Almgren et al. 2010). CASTRO is an adaptive mesh, compressible radiation-hydrodynamics simulation code, based on an Eulerian grid. It supports a general equation of state, nuclear reaction networks, rotation, and full self-gravity. The fully compressible equations computed in the code CASTRO are as follows,

\[
\frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho \mathbf{u}),
\]

\[
\frac{\partial (\rho \mathbf{u})}{\partial t} = \nabla \cdot (\rho \mathbf{uu}) - \nabla \mathbf{P} + \rho \mathbf{g} + S_{\text{src}},
\]

\[
\frac{\partial (\rho E)}{\partial t} = -\nabla \cdot (\rho \mathbf{u}E + P \mathbf{u}) + \rho \mathbf{u} \cdot \mathbf{g},
\]

where \(\rho\), \(\mathbf{u}\), and \(P\) are the density, velocity vector, and pressure, respectively. \(E\) represents the total specific energy, given by the sum of the internal energy \(e\) and the kinetic energy, i.e., \(E = e + \mathbf{u} \cdot \mathbf{u}/2\). \(S_{\text{src}}\) is a user-specified momentum source term, which will be described in more detail in \$2.3. CASTRO is suitable for capturing small scale structures of turbulence, which is our primary goal in this study.

We consider a three-dimensional computational domain with the shape of a rectangular prism (the height is twice the width). In our simulations, we model a radiative region of strongly-irradiated planets assuming it is initially in hydrostatic equilibrium with a constant gravity \(g = 10^3\) cm s\(^{-2}\). We fill the domain with our model atmosphere, starting at \(P \approx 10\) bar at the bottom, towards the top such that the atmosphere at a level of \(P \gtrsim 1\) mbar occupies around 20% of the entire domain. We define \(P = 1\) mbar as the top of the atmospheres in this study. Then, we further extend the atmosphere until the density becomes smaller than

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2 See Izakov (2001) for the greenhouse effect in the atmosphere of Venus.
\[ \rho = 10^{-17} \text{ g cm}^{-3} \ (36\% \text{ of the domain}). \] We fill the rest of the domain with a constant density medium with \( \rho = 10^{-17} \text{ g cm}^{-3} \) and \( T = 10^{2} \text{ K} \). We refer to this region as a "buffer". We introduce this region to avoid possible spurious effects from an upper boundary condition (more details in the next section). The left panel in Figure 1 presents the \( \rho - z \) plot for two different temperatures (\( T = 1800 \text{ K} \) and 3000 K). The outer arrow outside the right vertical axis, annotated with "Right panel" in magenta, roughly shows the spatial scale of the schematic diagram of the atmosphere in our computation box shown in the right panel. The right panel in Figure 1 will be explained in detail in §2.3.

To ensure robustness of our results given the numerical accuracy, we run simulations for a given initial condition with two resolutions. In the lower resolution simulations, the number of cell is \( N_{\text{cell}} = (N_x, N_y, N_z) = (256, 256, 512) \), while in the higher resolution simulations, \( (N_x, N_y, N_z) = (512, 512, 1024) \). We choose the spatial scale of each single cell to be \( \sim 0.1 H \) (where \( H \) is the scale height) for the lower simulation case and \( \sim 0.05 H \) for the higher resolution. By filling the domain in this way, the total box size varies depending on what temperature we assume for the atmosphere. As will be explained in §2.1, we consider two different temperatures and the total box sizes of our models are given in Table 1.

Since our results are found to converge between the simulations with the two resolutions, in the result section we only focus on the atmosphere below \( P \approx 1 \text{ mbar} \) (corresponding to \( z \approx 10 H \)) for further analysis in the high resolution simulations. We also discuss the differences between the simulations with the two resolutions in §4.1.

### 2.2 Boundary condition

We consider a different boundary condition (BC) for each boundary. We use periodic BCs for the side boundaries. For the BC at the bottom, we employ a "hydrostatic" BC to provide the pressure support for the atmosphere against gravity. Here, the ghost cells outside the domain are initialized with no forced turbulence (with no forced vertical velocity \( v_z \)). Furthermore we use a reflective BC on the velocity (or the momentum). These hydrostatic boundary conditions are described in Zingale et al. (2002).

For the top BC, we employ an inflow boundary condition in which ghost cells are updated to be the same as the uppermost inner cells, except for momentum. We only allow for incoming flows (i.e. gas with a positive vertical velocity \( +v_z \)). However, if the gas at the boundary has a positive vertical velocity \( +v_z \), it is reset to be zero. In either case, the \( x \) and \( y \) components of velocities \( (v_x \text{ and } v_y) \) are always zero at the boundary. The top BC is not relatively well-defined compared to the BCs at the other sides. Hence we introduce a buffer region on top of the atmosphere to place the atmosphere of our interest sufficiently far away from the upper boundary of the domain. This may increase the computational cost as the volume of the entire domain (low \( P \) atmosphere-+buffer region) grows. However, this, along with the sponging applied in the buffer layer (see §2.3), ensures that the actual top BC does not matter and our results are robust against different choices for the upper BCs.

Employing these BCs, we have confirmed that our atmospheres stay in hydrostatic equilibriuim (with no forced turbulence) sufficiently longer \( (t > 1.5 \times 10^{5} \text{ s} \) for low resolution simulations and \( t > 3.0 \times 10^{5} \text{ s} \) for high resolution simulations), than the total physical times considered in this paper, namely, \( t < 5 \times 10^{4} \text{ s} \).
2.3 Shear prescription

It is found in numerical studies (e.g. Guillot & Showman 2002; Showman & Guillot 2002) that shear motions in the atmospheres of hot Jupiters can be caused by forcing due to the day-night temperature contrast. Furthermore, a typical Mach number at a level of $P = 1$ mbar is found to be around $M \approx 1 - 2$ (e.g. Showman & Guillot 2002; Fromang et al. 2016). Motivated by these studies, we give an initial shear velocity in our model atmospheres as follows.

We consider a bulk shear motion only in the $x$ direction. At $t = 0$ s, we give a constant shear velocity at a sound speed $c_s$ at $P_{sh} = P \lesssim 1$ mbar. Below this region (higher $P$), we place a shear layer with a positive velocity gradient, i.e. $v_x(z = z_{i+1}) - v_x(z = z_i) > 0$ (cell index $i$, increasing with $z$). We refer to this region as "shear layer" throughout the paper. The velocity at the top of the shear layer is set to be $c_s$ at $P = 1$ mbar, decreasing linearly down to $v_x = 0$ at the bottom of the shear layer. In other words, for two adjacent cells, $\Delta v_x/\Delta z = c_s/z_{sh} = \text{constant}$. Here, $z_{sh}$ is the height of the shear layer. Furthermore, we consider a small number of zero velocity corrugations in the $x$ and $y$ directions (similar to corrugations usually seen in billow clouds). This is to invoke more non-regular turbulent motions in every direction, albeit the corrugated pattern is regular. Velocity profiles (width, size and frequency of corrugation) inside the shear layer are unknown; future work specific to this will be necessary for more realistic modelling inside the shear layer. A schematic diagram illustrating the shear is shown in Figure 1. We emphasize that this forcing within the shear layer is only given at $t = 0$. Below the shear layer, starting typically from $P_{sh, bottom} \approx 2 - 3$ mbar ($P_{sh, bottom}$ refers to the pressure of the bottom of the shear layer), the atmosphere is assumed to be in static equilibrium with no initial velocity forcing.

At $t > 0$ s, we consider a momentum source in the $x$ direction within 0.1 mbar < $P_{src} < 1$ mbar to continuously drive shear motions. This is to mimic the east-west stream found in many global circulation models, which is likely to last as long as the rotation of a planet is synchronized with the orbital motion. For the atmosphere within this pressure range, we add a certain amount of momentum to each cell equally at every time step such that horizontal average ve-
Table 1. Model parameters considered in this study. We categorize the parameters into two groups: model parameters that all models share, and those which differ among models. From left to right: The common parameters include the number of cells $N_{\text{cell}}$, the number of cells per scale height $H/\Delta l$ ($H$: scale height), the gravitational constant $g$ [cm s$^{-2}$], and the shear velocity in units of the Mach number $Ma$. In the category of the different parameters, we list the size of our computation box $L$ [10$^6$ km], the height $z_{\text{P1-mbar}}$ [10$^6$ km] at $P = 1$ mbar, the initial $T$ in the radiative zone $T_{\text{deep}}$ [K], $P$ at the radiative-convective boundary $P_{\text{RCH}}$ [bar], $P$ at the bottom of the domain $P_{P1-o}$ [bar], the Richardson number $Ri$, estimated at $t = 0$ and the sound speed $c_s$ [$\text{km s}^{-1}$].

| Model name       | same model parameters | different model parameters |
|------------------|-----------------------|----------------------------|
|                  | $N_{\text{cell}}$ = ($N_x$, $N_y$, $N_z$) | ($L_x$, $L_y$, $L_z$) | $z_{\text{P1-mbar}}$ | $T_{\text{deep}}$ | $P_{\text{RCH}}$ | $P_{P1-o}$ | $R(t = 0)$ | $c_s$ |
| $T3000 – R0.02$  | (512, 512, 1024)       | 20 | $10^3$ | 1               | 2.5 | 5.0 | 1.00 | 3000 | 268 | 12 | 0.1 | 3.86 |
| $T3000 – R0.1$   |                        |   |   |                | 0.25 |
| $T3000 – R0.25$  |                        |   |   |                | 0.02 |
| $T1800 – R0.02$  |                        |   |   |                | 0.1 | 2.99 |
| $T1800 – R0.1$   |                        |   |   |                | 0.25 |
| $T1800 – R0.25$  |                        |   |   |                | 0.25 |

The same six models above, but with a lower resolution $N_{\text{cell}}$ = (256, 256, 512)

The velocities in the $x$ direction $\nabla_x$ gradually approach $v_x = c_s$ over a certain time ($t_{\text{src}} = 1000$ s). While the atmosphere is dynamically evolving, it is possible that a mean motion of gas at any given time at some pressure happens to be supersonic in the $+x$ direction (i.e. $\nabla_x > c_s\hat{x}$, where $\hat{x}$ refers to the basis vector in the $x$ direction). Whenever that is the case, we do not apply this forcing to the gas at that pressure. This way, a bulk motion is gradually driven while small scales remain intact. We note that the lower pressure limit ($P = 0.1$ mbar) for the momentum input is arbitrarily chosen, but the atmosphere at $P \gg 1$ mbar is not sensitive to different choices of the lower limit. We can summarize this continuous forcing within 0.1 mbar $< P_{\text{rc}} < 1$ mbar as follows. For gas at a given pressure $P$ and time $t$ with a mean motion $\nabla_x$, an external momentum (i.e. $S_{\text{src}}\Delta t$ in Equation 2) is added to the momentum of the gas at each cell

$$S_{\text{src}} = \max(0, c_s - \nabla_x)_{t_{\text{src}}}$$

where “max( )” indicates the maximum of the two values in the parenthesis. To conserve the total energy of the gas, we take into account its additional energy accordingly.

As an example, Figure 2 shows different evolutions of gas motions in the $x$ direction and the temperature of the atmosphere with (“continuous”, left panel) and without (“one-time”, right panel) the additional momentum source. In the upper panel, we show $v_x$ throughout the atmosphere at different times and in the lower panel the relative temperature variations with respect to the initial temperature $T_0$ at different pressure levels. In all the plots, the shaded regions around the horizontal average values (solid lines) demarcate the ranges between the maximum and minimum values at a given pressure and time. The size of the shaded regions is relevant for our study, along with the average values. This is because it can serve as a good indicator for how chaotic the atmosphere is due to turbulence. We can see some clear differences in both the $v_x$ and the $T$ plots. Among those, the most noticeable difference is in the larger shaded regions with the extra momentum source. This means that the momentum source clearly contributes to amplifying the effects and lifetime of turbulent motions, as we expect. From now on, we will only consider the “continuous” shear case.

For the atmosphere even further above at $P < 0.01$ mbar, we damp the velocity of the gas to $v = 0$ in all directions. We use a damping scheme (or “sponge” damping) employed in the low Mach number code MAESTRO (Nonaka et al. 2010) and used in other studies (e.g. Zingale et al. 2009, 2011; Nonaka et al. 2012). This scheme was originally introduced to avoid a large growth in velocities in the low dense regions of a stellar surface due to intense heating. See Section 4.3.1 in Almgren et al. (2008) for the equations used for the damping scheme. In our case, an unphysical surge in velocities can occur in the buffer region. This scheme serves to damp the large velocity wakes which would otherwise propagate towards the atmosphere and significantly affect its stability.

To sum up, the atmosphere is modelled such that:

At $t = 0$ (from $z = 0$ to larger $z$),

(i) $P_{sh,\text{bottom}} \lesssim P$: hydrostatic equilibrium with $v = 0$ ($P_{sh,\text{bottom}}$ is determined by the size of the shear layer and it is around 2 – 3 mbar in our simulations. See §2.4 below),

(ii) $1$ mbar $\lesssim P \lesssim P_{sh,\text{bottom}}$: positive velocity gradient in the vertical direction,

(iii) $P \lesssim 1$ mbar: $v_x = c_s$ and $v_y = v_z = 0$.

At $t > 0$, the atmospheric region at $P \lesssim 1$ mbar is affected by:

(i) $0.1$ mbar $\lesssim P \lesssim 1$ mbar: momentum source ($v_x \rightarrow v_{sh} = c_s$);

(ii) $P \lesssim 0.01$ mbar: sponge damping ($v \rightarrow 0$).

2.4 Model parameters and initial condition

Our primary goal is to capture small-scale structures of turbulence and quantify the turbulent kinetic and heat energy flux which penetrates into the atmosphere. For this, we employ the analytic model in Youdin & Mitchell (2010) to determine the initial properties of our model atmospheres.

The atmospheres are characterized by two different temperatures, $T_{\text{deep}}$ and $T_1$. Note that we use the same notation as in Youdin & Mitchell (2010), $T_{\text{deep}}$ is the temperature at the top of the atmosphere. Throughout this paper, we define the top of the atmosphere to be at $P = 1$ mbar. On the other hand, $T_1$ is the temperature that convective regions would have at $P = 1$ bar, and it measures the internal entropy of the...
where $N_{BV}$ represents the Brunt-Väisälä frequency,

$$N_{BV}^2 = \frac{\rho g}{P} [\nabla_{ad} - \nabla].$$

(7)

In the above, $\nabla$ refers to the lapse rate of the atmosphere, defined as,

$$\nabla = \left( \frac{d \ln T}{d \ln P} \right)$$

(8)

and the adiabatic lapse rate $\nabla_{ad} = 2/7$. From this expression, we can expect that if the vertical velocity gradient is chosen to be smaller, we start with a more unstable atmosphere. We choose the height of the shear layer to be small enough to give $Ri \lesssim 0.25$. In particular, we assume $Ri = 0.02, 0.1$ and $0.25$. These correspond to $N_{sh, z} = 6, 14$ and $24$ within the layer for the higher resolution (hence half the cell number for the lower resolution case).

Each of our models is integrated until the atmosphere reaches a steady state. We assume the atmosphere becomes steady when variables including $T$ and $v$ below $P = 3$ mbar do not change significantly. Typically, the atmospheres reach a steady state at $t \gtrsim 10^4$ s for $Ri = 0.02$, the time being shorter with larger $Ri$. Note that this is still sufficiently shorter than the time for our model atmosphere to remain in equilibrium when there is no initial shear motion.

The model parameters and their initial values are summarized in Table 1.

3 RESULTS

In this section, we analyze the evolution of the thermodynamic properties of our model atmospheres. In particular, we focus on how much and how deep heat and kinetic energy fluxes can penetrate into the atmospheres. Additionally, we discuss shock formation in the atmospheres.

3.1 Development of Eddies

In all of our models, turbulent motions are created first inside the shear layer. The unstable motions spread downwards over time, but they are limited within $z \sim (1 - 2) H$, as shown in Figure 4. Each panel shows a 2-dimensional snapshot of a $x-z$ plane in the middle of our 3-dimensional box, for $Ri = 0.02$ (top panel), $Ri = 0.1$ (middle panel) and $Ri = 0.25$ (bottom panel) at $t/t_{edd} \approx 0.6, 2, 7, 12$ and $25$ (from left to right). Here $t_{edd}$ is a characteristic time scale for eddy motions, which we define as follow,

$$t_{edd} \approx 2\pi \frac{H}{c_s} = \begin{cases} 1734 \text{ s} & \text{if } T_{deep} = 3000 \text{ K}, \\ 1343 \text{ s} & \text{if } T_{deep} = 1800 \text{ K}. \end{cases}$$

(9)

The top corresponds to a level of $P \approx 0.3$ mbar and the initial shear layer extends below from $P \approx 1$ mbar. The plots are color-coded according to the temperature from blue (lower $T$) to red (higher $T$). The vertical line with T-shaped heads in each panel indicates the $2H$ spatial scale. The temperatures in the atmospheres with lower $Ri$ are generally higher at a given time. Furthermore, eddies at larger scales break up into smaller eddies. These are typically expected in a standard picture of turbulence. It is noticeable that distinctive variations in $T$ are limited within a vertical range of

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1 - 2 H, i.e., P \lesssim 10 \text{ mbar}. In our models, P \approx 2.5 \text{ mbar} at z = z_1 \text{ mbar} - H, where z_1 \text{ mbar} is the height at P \approx 1 \text{ mbar}, and P \approx 6 \text{ mbar} at z = z_1 \text{ mbar} - 2H. Considering that turbulent heat energy is closely related with temperature fluctuations (see Equation 11), we can see already from the slice plots that the atmosphere below P \approx 10 \text{ mbar} is not significantly affected by the shear motions at the top of the atmosphere. This will be shown more clearly in the following sections.

### 3.2 Velocity and temperature variation

It has been suggested that turbulent mixing due to the non-linear Kelvin–Helmholtz instability plays an important role in the penetration and dissipation of kinetic energy. Vertical motions of gas are a primary factor to determine in which direction the kinetic and heat energy fluxes (along with temperature variations) propagate. Therefore, it is important to understand first how vertical velocities in our atmospheres...

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**Figure 4.** 2-dimensional snapshots for a x - z plane in the middle of our 3-dimensional domain for \( R_i = 0.02 \) (top panel), \( R_i = 0.1 \) (middle panel) and \( R_i = 0.25 \) (bottom panel), at \( t/\tau_{\text{edd}} \approx 0.6, 2, 7, 12 \) and \( 25 \) (from left to right). \( \tau_{\text{edd}} \) is a characteristic time scale for eddy motions, which is defined in Equation 9. The top of each panel corresponds to a level of \( P \approx 0.3 \text{ mbar} \) and the initial shear layer extends below from \( P \approx 1 \text{ mbar} \). The color coding indicates the temperature from lower (blue) to higher (red) \( T \). The vertical line with T-shaped head in each panel represents the 2H spatial scale.

**Figure 5.** The average vertical velocity \( v_z \) at different pressures for (from left to right) \( R_i = 0.02, 0.1 \) and 0.25. The boundaries of the shaded regions show the maximum and the minimum values around the average values (solid lines) at a given time and pressure.
evolve and how temperatures vary over time under the presence of the shear motion.

Figure 5 shows the average vertical velocity $\overline{v_y}$ at different pressures for (from left to right) $\text{Ri} = 0.02$, 0.1 and 0.25. Positive (negative) values indicate upward (downward) movements. Note that $P = 3$ mbar corresponds to the bottom of the shear layer for $\text{Ri} = 0.25$, whereas for lower $\text{Ri}$ the shear layers are positioned at $P \lesssim 3$ mbar. The shaded regions are bounded by the maximum and the minimum values around the average values (solid lines) at a given time and pressure. Shaded regions in other plots of this type below will have the same meaning. A general trend is that the magnitude of fluctuations in $v_y$ ($|v'_y| = |\overline{v_y} - v_y|$) increases up to $t/\tau_{\text{eddy}} \approx 5 - 10$, but gradually decreases afterwards. The velocity fields symmetrically fluctuate and $|v'_y|$ decreases by a factor of 3–5 from one pressure to the next higher pressure chosen in the plots. Meanwhile, average vertical motions remain almost zero at those pressure levels. This means that there is no dominant bulk motion in the vertical direction and eddies are confined within a small volume (a few mbar vertically). This is also shown in Figure 4. At $t/\tau_{\text{eddy}} \gtrsim 10$, $|v'_y|$ does not increase, but rather decays or stays constant. We next present the relative changes of $T$ with respect to the temperature at $t = 0$ (denoted by $T_0$) in Figure 6 for the same pressures as in Figure 5. Somewhat contrary to the symmetric changes in $\overline{v_y}$, as $\text{Ri}$ decreases (left and middle panel), turbulence leads to a more positive temperature fluctuation $T'$ at $P \approx 3$ mbar (corresponding to the bottom of, or slightly below, the shear layer). However this is not clearly seen for the models with $\text{Ri} = 0.25$ (right panel) and at $P \gtrsim 5$ mbar for $\text{Ri} = 0.02$ and 0.1.

### 3.3 Kinetic energy and heat flux transport due to turbulence

Figures 7 and 8 show the time-averaged turbulence vertical kinetic energy flux $\bar{F}_{\text{KE}}$ and the turbulence vertical heat flux $\bar{F}_H$, respectively. In the flux figures, the three horizontal lines near the bottom indicate 1% ($\approx -10^5$ W m$^{-2}$, solid horizontal line), 10% ($\approx -10^6$ W m$^{-2}$, dot-dashed line) and 100% ($\approx -10^7$ W m$^{-2}$, dotted line) of the incoming stellar flux $F_\star$ at $T = 3000$ K. We first define an instantaneous kinetic energy flux $\tilde{F}_{\text{KE}}$ (Hamnoum et al. 1988), while $\tilde{F}_H$ defines the heat flux as $\rho c_p v_z T'$ at $t$ and $P$ as follows,

$$\tilde{F}_{\text{KE}}(t, P) = \overline{\rho \overline{v'_y} v'_y}, \quad (10)$$

$$\tilde{F}_H(t, P) = \overline{\rho c_p v_z T'}, \quad (11)$$

where $\rho$ is the specific mass density at constant pressure, $c_p = \rho c_{p\text{ad}}$. $\tilde{F}_{\text{KE}}$ is the turbulence specific kinetic energy,

$$\tilde{F}_{\text{KE}} = \frac{1}{2}(v'_y^2 + v'_x^2 + v'_z^2). \quad (12)$$

Using then the instantaneous fluxes extracted from the output data at specific time intervals ($\Delta t/\tau_{\text{eddy}} \approx 0.06$), we calculate a time average of the fluxes at a given $t$ and $P$ as,

$$\bar{F}(t, P) = \frac{1}{\Delta t} \sum_{t'} \tilde{F}(t', P) \Delta t. \quad (13)$$

The turbulent kinetic energy flux $\bar{F}_{\text{KE}}$ in Figure 7 at $\text{Ri} = 0.25$ (right panel) is nearly zero throughout the atmosphere below the shear layer. On the other hand, for the lower $\text{Ri}$ (left and middle panels), the magnitudes of the fluxes $|\bar{F}_{\text{KE}}|$ are smaller at higher $P$. The fluxes at $P \approx 3$ mbar for both $\text{Ri}$’s are the largest and remain constant, but they are at most $0.01 – 0.001\%$ of $F_\star$. This means that the continuous shear forcing at the top keeps exciting eddy motions, but confined at $P \lesssim 10$ mbar, then the turbulence kinetic energy rapidly dissipates into heat. Next, we quantify the turbulence heat energy flux $\bar{F}_H$.

Unlike the kinetic energy flux, the turbulence heat flux $\bar{F}_H$ in Figure 8 propagates downwards at all pressures with its magnitude smaller for higher $Ri$. However, this flux is not significant, roughly $\lesssim 0.01\%$ of the incoming flux. Furthermore, $\bar{F}_H$ for $Ri = 0.25$ at all pressures gradually converges to zero. Only for $Ri = 0.01$, $\tilde{F}_H$ at $P \approx 3$ mbar maintains a

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4 Hurlburt et al. 1984 defines the heat flux such that positive $\tilde{F}_H$ is directed downward. Notice the negative sign in their definition.

5 To maximize the code speed, the current default set-up of the code allows to print out the main state variables (e.g. $\rho$, $T$ and etc.) and some derived variables (e.g., entropy and etc.). We post-process the data for more model-specific variables, such as $F_\star$.

6 We find that $\tilde{F}_{\text{KE}}$ in some deeper regions ($5 \leq P \leq 10$ mbar) becomes positive. We believe that this is mostly due to small random fluctuation of $\tilde{F}_{\text{KE}}$ around zero.
0.1% level. We can roughly estimate the order of magnitude for the inferred $\tilde{f}_H$ as follows. The fact that $\tilde{f}_H$ remains constant at later times means an unvarying inflow rate of the instantaneous flux $\tilde{f}_H$ over the unit time (consider Equation 13 with constant $\tilde{f}_H$). Next using the ideal gas equation of state and expanding the term $v_z T^2$,

$$\tilde{f}_H = \frac{P}{RT} c_p (v_z T - v_T) = \frac{\gamma}{\gamma - 1} \frac{P}{T} (v_z T - v_T).$$

(14)

Here we make a crude approximation for mathematical simplicity such that i) we consider $\tilde{P}$ independently, i.e. $\tilde{\rho} v_z T^2 = \tilde{P} v_z T^2$ and ii) $\tilde{\rho} = \frac{\tilde{P}}{RT} = \tilde{\rho}$.

(15)

which can be interpreted as the normal horizontal average of $\rho$ being very close to the temperature-weighted average of $\tilde{\rho}$. This may not be valid for an unstable atmosphere where $\rho$ and $\tilde{\rho}$ significantly oscillate. In our case the error from the approximation would be roughly $\sim R \sigma(\rho) \sigma(T)/T$, where $\sigma$ refers to the root mean square (RMS) of a variable, and it turns out that this is quite small even in the shear layer. Using this approximation and for $\gamma = 7/5$, we find that,

$$\tilde{f}_H \sim \frac{7}{2} \frac{\tilde{P}}{T} (v_z T - v_T) \sim \frac{7}{2} \frac{\tilde{P} \sigma(v_z) \sigma(T)}{T}.$$  

(17)

Now we can see that the order of magnitude of the flux is largely determined by the difference between the normal average and the temperature-weighted average of the vertical velocity at a given pressure. Let’s consider the inside ($P = 1 \text{ mbar}$) and the outside ($P = 1 \text{ bar}$) of the shear layer, separately. With the values for $\sigma(v_z)$ and $\sigma(T)$ from the simulations, e.g. for $Ri \approx 0.01$, we finally find the following relations,

$$\tilde{f}_H \approx \begin{cases} 3 \times 10^3 \text{ W m}^{-2} & \left( \frac{\tilde{P}}{\text{bar}} \right) \left( \frac{\sigma(v_z) / \sigma(T)}{10^{12}} \right) \frac{\sigma(T)}{T} \text{ near top,} \\ 3 \times 10^3 \text{ W m}^{-2} & \left( \frac{\tilde{P}}{\text{bar}} \right) \left( \frac{\sigma(v_z) / \sigma(T)}{10^{12}} \right) \frac{\sigma(T)}{T} \text{ near bottom,} \end{cases}$$

(18)

which is in good agreement with $\tilde{f}_H$ shown in Figure 8. Plots for $\sigma(v_z)$ and $\sigma(T)$ for the two $T_{\text{deep}}$ are presented in Figure 13 and a plot showing a correlation between $\sigma(v_z)$ and $\sigma(T)$ at different times is given in Figure A1 in Appendix A.

Overall, our results suggest that turbulence due to the shear motion near the top cannot lead to deep penetration of the energy flux, which remains confined in a vertical spatial scale of $\sim 2H$. The atmosphere below $P = 10 \text{ mbar}$ is barely affected by the turbulent motion at $P \approx 1 - 3 \text{ mbar}$.

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Figure 7. Time evolution of the turbulence kinetic energy flux $\tilde{f}_{\text{KE}}$ for $Ri = 0.02$ (left panel), $Ri = 0.1$ (middle panel) and $Ri = 0.25$ (right panel). The horizontal lines near bottom indicate 1% ($\sim 10^6 \text{ W m}^{-2}$, solid horizontal line), 10% ($\sim 10^5 \text{ W m}^{-2}$, dot-dashed line) and 100% ($\sim 10^4 \text{ W m}^{-2}$, dotted line) of an incoming stellar flux at $T = 3000 \text{ K}$.

Figure 8. Time evolution of the turbulence heat flux $\tilde{f}_H$ calculated using Equation 11 for $Ri = 0.02$ (left panel), $Ri = 0.1$ (middle panel) and $Ri = 0.25$ (right panel).
3.4 Eddy diffusion coefficient

Even though heat energy transport due to the forced shear does not render large-scale effects, it is still worth quantifying the eddy diffusion coefficient $K_{zz}$ for negative $F_{H}$. We estimate $K_{zz}$ by combining equation (20) in Youdin & Mitchell (2010),

\[ F_{H} = -K_{zz} \rho T \frac{dS}{dz} \]

with the time-averaged heat flux $\bar{F}_{H}$ (Equations 11 and 13). These two equations give the following expression for $K_{zz}$:

\[ K_{zz} \approx \frac{\sum_{t=0}^{t=T} \rho \bar{v}_{z} T (\Delta t)}{\rho T \Delta z} \]

where $\frac{dS}{dz}$ is estimated as follows,

\[ \frac{\Delta S}{\Delta z} (z = h) = \frac{S(z = h + \Delta z) - S(z = h - \Delta z)}{2 \Delta z} \]

Based on the time-averaged heat fluxes found in our models, we find (see Figure 9) that for $Ri = 0.01$ (upper panel), $K_{zz} \approx 10^8 - 10^{11}$ cm$^2$ s$^{-1}$ at a few Pmbar pressures, decreasing down to $K_{zz} \approx 10^7$ cm$^2$ s$^{-1}$ at $P = 1$ bar. There is no significant difference between $Ri = 0.1$ (middle panel) and $Ri = 0.25$ (bottom panel), except for $P = 3$ mbar. These values are reasonably consistent with Spiegel et al. (2009), where they estimate a $K_{zz} \sim 10^7 - 10^{11}$ cm$^2$ s$^{-1}$ to be necessary to maintain sufficient TiO in the upper atmospheres ($P \approx 1$ mbar) for thermal inversion.

As seen in Figure 9, $K_{zz}$ decreases as $P$ increases. One may notice that the lines in Figure 9 look very similar to those in Figure 8, only flipped. This is probably because $K_{zz}$ still has the same dependence on the term $\sigma(v_{z}) \sigma(T) / \langle T \rangle$. Assuming $\Delta T \ll T$ within a characteristic height and the ideal gas law, we find that,

\[ \frac{\Delta S}{\Delta z} \approx -\frac{R}{\Delta z} \log \left( 1 + \frac{\Delta T}{H} \right) \]

which gives,

\[ K_{zz} \approx \frac{2}{\langle T \rangle} \frac{\sigma(v_{z}) \sigma(T)}{\Delta z} \log(1 + \frac{\Delta T}{H}) \]

In particular, for $\Delta z = H$, we estimate $K_{zz} \approx 10^{11}$ cm$^2$ s$^{-1}$ at $P \approx 1$ mbar and $10^5$ cm$^2$ s$^{-1}$ at $P \approx 1$ bar. For this, we
have used the same values for $\sigma(v_2)$ and $\sigma(T)/\bar{F}$ given in Equation 18.

These estimates, however, do not clearly inform us on how the coefficient $K_{zz}$ varies with pressure. To find the dependence of $K_{zz}$ on $P$, we show $K_{zz}$ as a function of $P$ for our atmospheric models with $T_{\text{deep}} = 3000$ K (solid lines) and $T_{\text{deep}} = 1800$ K (dot-dashed line) in Figure 10. We see that we have different $P$ dependences at $P \gtrsim 3 - 4$ mbar and $P \lesssim 3 - 4$ mbar for all the models. $K_{zz}$ at $P \lesssim 3 - 4$ mbar dramatically declines, followed by a relatively mild decrease in the deeper atmosphere. Such steepness near the top is clearly due to chaotic eddy motions. Interestingly, $K_{zz}$ for $Ri = 0.02$ (black lines) and $Ri = 0.25$ (blue lines) throughout the atmosphere have almost the same dependence on $P$, only differing by a factor of $\sim 5$ in magnitude. On the other hand, $K_{zz}$ for $Ri = 0.1$ (red lines) shows a transitional behavior between that for $Ri = 0.02$ (black lines) and $Ri = 0.25$ (blue lines): the lines for $Ri = 0.1$ are very close to those for $Ri = 0.02$ at $P \lesssim 3 - 4$ mbar whereas they are still lingering near the lines for $Ri = 0.02$ at $P \gtrsim 3 - 4$ mbar. From this, we may be able to conjecture the following: 1) $Ri = 0.1$ could be a characteristic value below which the heat flux starts effectively penetrating into the deeper region and 2) inflow of a heat flux in the inner region may occur via episodic jumps, rather than by a gradual growth. However, it is important to emphasize that our conjectures are made only based on our models, which cover a subset of the whole parameter space. In order to find more general trends of $K_{zz}$ (e.g., how $K_{zz}$ would increase in atmospheres for $Ri < 0.01$, in particular whether it would gradually increase or whether there would be another lingering phase like the one we find for $Ri = 0.25 - 0.02$), we need to explore a larger parameter space with different initial conditions, which we will leave for future work.

Assuming $K_{zz}$ follows a power law of $P$ such that $K_{zz} \propto P^{-\alpha}$, the following provides a fit for $K_{zz}$:

$$K_{zz} \approx \begin{cases} 5 \times 10^8 \beta \left( \frac{P}{P_c} \right)^{-6} \text{cm}^2 \text{s}^{-1} & P < P_c, \\ 5 \times 10^8 \beta \left( \frac{P}{P_c} \right)^{-1.2} \text{cm}^2 \text{s}^{-1} & P \geq P_c, \end{cases}$$  

(24)

where $\beta$ is a normalization factor, possibly depending on $T_{\text{deep}}$, the velocity gradient due to shear ($Ri$) and so on. As mentioned above, at least for $Ri = 0.02$ and $Ri = 0.25$, $\beta$ is a constant, differing by around 5. $P_c$ can be approximately found to be $P_c \approx P$ at $z = z_1$ mbar - 1.5$H$.

The convergence of $K_{zz}$ at the bottom ($P \approx 10 - 30$ bar), along with the sharp decrease, in all our models is probably due to the boundary condition. This drop has been found in Youdin & Mitchell (2010) for negative $\alpha$ (see their Figure 7), but near quite large pressures at which $\nabla = \nabla_{\text{adj}}/2$. In our models, the pressure which satisfies the condition corresponds to $P \approx 270$ bar (45 bar) for $T_{\text{deep}} = 3000$ K (1800 K), which is much higher than $P$ at the bottom.

### 3.5 Shock formation

Here we consider the formation of shocks in our atmosphere. We use a basic multi-dimensional shock detection algorithm (Colella & Woodward 1984; Colella 1990) embedded in the code CASTRO to trace shocks. Overall, we find that shocks form, but they are sporadic (in space) and transient (in time).

Shocks form within a range of $P \lesssim 2.5$ mbar and the fraction of the areas where shocks are detected, defined as the ratio of the number of cells identified with shocks to the total number of cells at a given pressure, remains below the $10^{-3} - 10^{-4}$ % level even in the most unstable atmosphere. We show horizontal slice plots for $M$ in the atmosphere with $T_{\text{deep}} = 3000$ K in Figure 11 at $P = 1$ mbar (left panel) and $P = 2$ mbar (right panel) at $t = 7 \tau_{\text{edd}}$. This figure is made for the model with $Ri = 0.02$, but the other models look very similar. The color indicates the magnitude of $M$ as given in the color bar. We also mark where shocks form using magenta dotted circles. They are local and scattered.

The shocks last longer in the atmosphere starting with smaller $Ri$, but they are no longer detected at $t \lesssim 12 \tau_{\text{edd}}$ for $Ri = 0.02$. This is visualized in Figure 12. This figure shows the largest pressure at which shocks form ($P_{\text{max}}$) as a function of time in our models with $T_{\text{deep}} = 3000$ K (solid.

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**Figure 11.** Horizontal slice plots for $M$ in the atmosphere with $T_{\text{deep}} = 3000$ K at $P = 1$ mbar (left panel) and $P = 2$ mbar (right panel) at $t = 7 \tau_{\text{edd}}$ when the initial $Ri = 0.02$. The arrow at the top-right corner indicates the direction of continuous shear motion. The magenta circles indicate where shocks are detected. The plots are color-coded based on the magnitude of $M$.**
lines) and $T_{\text{deep}} = 1800$ K (dotted line). The times when the lines cross the $P = 1$ mbar level correspond to moments when there are no shocks in the atmospheres. As clearly shown in the figure, the shocks cannot penetrate deeper than $P \simeq 2$ mbar and disappear rather quickly.

From the above, we conclude that shock formation is insignificant; therefore, shocks are not expected to affect the evolution of the atmospheres. Interestingly, even though Fromang et al. (2016) investigated shocks based on different atmosphere models and criteria for shock formation, both studies suggest similar conditions for shock formation. Shocks are not found in their low resolution simulations. However, at resolution high enough to resolve finer structures of jets, they find instabilities which cause velocity fluctuations, ultimately transforming into weak shocks at $P \simeq 2$ mbar. This trend (i.e. resolution dependence and weak shocks confined to lower pressures) is in good agreement with the findings from our simulations.

4 DISCUSSION

4.1 Comparison: different resolutions and the role of $T_{\text{deep}}$

As mentioned in §2.4, to test the numerical reliability of our results, we performed every set of our simulations with two resolutions ($N_t = 1024$ and 512). Generally, we find that the lower and higher resolution runs show reasonably consistent outcomes, which adds more robustness to our results. However, we also find a few differences. In this section, we discuss these differences in more detail.

(i) Resolution

We find converging results between the high and low-resolution simulations. More specifically: In the low-resolution simulations it is found that fluctuations in all the variables are generally larger at the initial times and inside the shear layer; however, when the atmosphere becomes steady, the final values of the variables are in a quite good agreement with the higher resolution simulations. One difference worth noting is the maximum depth at which shocks form, or $P_{\text{max}}$. In the low resolution simulations, the depth extends to $P \simeq 5$ mbar and the shocks last longer, roughly by a factor of 2 than the higher resolution runs, and more shocks are detected (the fraction of atmospheric area con-
and the heat fluxes for the two models in Figure 13. For a MNRAS (high resolution) is exactly the same. The number of cells per scale height for \( N \) is 9. Note that \( \Delta \) (with respect to \( H \)) for the models with different \( T_{\text{deep}} \) and their initial \( T = P \) profiles (such as \( P(z = 0) \)) are not exactly the same. The number of cells per scale height for \( N_L = 1024 \) (high resolution) is 21.8 and 20.4 for \( T_{\text{deep}} = 3000 \, \text{K} \) and \( T_{\text{deep}} = 1800 \, \text{K} \), respectively.

(ii) \( T_{\text{deep}} \)

The evolution of all the relevant variables mentioned so far, namely \( T \), \( v_T \) and the energy fluxes, are almost identical. Among those variables, we show the RMS \( \sigma \) of \( v_T \) and \( T \) and the heat fluxes for the two models in Figure 13. For a self-similar comparison, we normalize each value by a characteristic variable in the same dimension, such as \( T_0 \), \( c_s \) and \( F_* \). The black horizontal line in the bottom panel indicates 1\% of \( F_* \). \( T \) and \( v_T \) are found to be very comparable whereas \( T_{\text{H}}/F_* \) shows some discrepancies. Indeed, it is because the magnitudes of \( T_{\text{H}} \), not \( T_{\text{H}}/F_* \), for the two models are very similar. As we explained in §3.3 (see Equation 17 and 18), for a given \( P \), the same \( \sigma(v_T) \) and \( \sigma(T)/T \) likely lead to the same \( T_{\text{H}} \). This may mean that, as long as the Mach number of the horizontal shear motion at top is the same, the amount of heat flux reaching a certain pressure level is independent of \( T_{\text{deep}} \). This argument will need to be further explored in future work.

4.2 Turbulence in the deep regions

So far, we have focused on turbulence initially created at \( P_{sh} \approx 1 \, \text{mbar} \). However, we cannot rule out the possibility that turbulence is generated more deeply. In stable stratified atmospheres, turbulence can be caused by a breakdown of internal buoyancy waves, like in the atmospheres of the Earth, Mars and Venus (Izakov 2001, 2002). Furthermore, in many global model simulations for hot Jupiters, it has been found that transonic zonal winds extend vertically down to \( P \approx 1 \, \text{bar} \) (e.g. Showman et al. 2009; Rauscher & Menou 2010, 2012; Fromang et al. 2016). For example, Showman et al. (2009) find from their global 3-dimensional numerical simulations peak zonal wind speeds of 3.5 km s\(^{-1} \) at \( P \approx 10 \, \text{–} \, 100 \, \text{mbar} \) (corresponding to \( M > 1 \) assuming \( T = 1200 \, \text{K} \)) and Fromang et al. (2016) find \( R_i \) at \( P \approx 1 \, \text{bar} \) can be as low as 0.1 – 0.25. This means that atmospheres at those pressure levels may also be subject to shear instabilities.

To explore the role of turbulence in the deeper regions, we additionally perform two simulations for the hotter atmosphere (\( T_{\text{deep}} = 3000 \, \text{K} \)) with a shear layer at \( P_{sh} \approx 100 \, \text{mbar} \), instead of \( P_{sh} \approx 1 \, \text{mbar} \). In this experiment, we only consider \( R_i = 0.25 \) and \( R_i = 0.02 \). All other model parameters, except for \( P_{sh} \), are identical to our fiducial models with \( P_{sh} \approx 1 \, \text{mbar} \), including the continuous momentum input at \( P = 1 \, \text{bar} \). We present slice plots in Figure 14 for the two models with \( R_i = 0.02 \) (upper panel) and \( R_i = 0.25 \) (lower panel) at the same times as in Figure 4, i.e., \( t/\tau_{\text{edd}} = 0.6, 2, 7, 12 \) and 25. Figures 4 and 14 share the same color-coding scheme.

Similarly to the models with \( P_{sh} = 1 \, \text{mbar} \), the temperature of gas near the shear layer becomes hotter as the kinetic energy of the gas dissipates into heat energy. Then the heat energy spreads out towards regions with relatively low \( T \) from the shear layer. Finally, the atmosphere becomes steady. The propagation of the heat energy can be visualized from how \( K_{zz} \) at each pressure level evolves over time. This is shown in the left panel of Figure 15 for \( R_i = 0.02 \). This panel shows \( K_{zz} \) every 0.1 \( t/\tau_{\text{edd}} \) for \( 0 \leq t/\tau_{\text{edd}} \leq 1 \) (the dotted lines indicate \( t/\tau_{\text{edd}} = 0 \) and 1, while the solid lines mark the intermediate times).

There are two points worth noting: 1) One outcome
which has not been seen in the fiducial models, but it is seen in this experiment, is that the regions above the shear layer go through larger increases in \( T \) (see high temperatures at \( P < 100 \text{ mbar} \) in Figure 14 compared to those at \( P > 100 \text{ mbar} \)). This is because a relatively small amount of heat energy is necessary to increase the temperature in a less dense region. 2) Unlike our fiducial models with \( P_{sh} = 1 \text{ mbar} \), we find that the values of \( K_{zz} \) for \( R_i = 0.02 \) and \( R_i = 0.25 \) outside the shear layer are comparable when a deeper shear layer is considered. This can be explained from trade-offs between efficiency of heat energy conversion via turbulence and the total kinetic energy budget which can dissipate into heat energy: according to our shear prescription, the total initial momentum (kinetic energy) of the shear layer increases as \( R_i \). Since we do not consider the continuous momentum input near the shear layer in these simulations, the total kinetic energy budget for \( R_i = 0.02 \) which can dissipate to heat energy is (five times) smaller than that for \( R_i = 0.25 \). Therefore, even though the heat energy can be converted via turbulence more efficiently in a more unstable atmosphere with \( R_i = 0.02 \), however it is limited by the smaller kinetic energy budget contained in the shear layer. On the other hand, for \( R_i = 0.25 \), the conversion efficiency is lower, but the shear layer has a larger reservoir of kinetic energy.

Overall, a larger heat flux can reach deeper regions when eddy motions are created at larger pressures. In this additional experiment with \( P_{sh} = 100 \text{ mbar} \), \( \dot{F}_H \) at \( P \approx 1 \text{ bar} \) (10 bar) becomes comparable to \( \sim 0.1\% \) (0.01%) of \( F_s \) for both \( R_i \)’s; these values are larger than those with \( P_{sh} = 1 \text{ mbar} \) by roughly two orders of magnitude. As a result, as shown in the right panel of Figure 15, \( K_{zz} \) with higher \( P_{sh} \) (dotted lines) is higher than that with \( P_{sh} = 1 \text{ mbar} \) (solid lines) by several orders of magnitude throughout the atmosphere, except near \( P_{sh} = 1 \text{ mbar} \). However, no significant difference in \( \dot{F}_H \) at \( P \approx 1 \text{ bar} \) is found between the two \( R_i \)’s.

These additional results strengthen and broaden our argument that the effect of turbulence on the atmosphere below where eddies form is local, whereas it can cause a spatially large impact on the thermal evolution in the regions above it. Therefore, what is more important for effective heat energy transfer into deeper regions via turbulence is probably where an atmosphere becomes unstable, rather than how unstable it is. Our results further add another aspect, which is that deep shear instabilities can significantly affect the atmosphere above where eddies are created.

5 SUMMARY AND FUTURE DIRECTION

We have performed 3-dimensional hydrodynamics simulations to investigate the effects of shock and turbulence on energy penetration into hot Jupiter atmospheres, under a variety of shear gradients. We find that the effects of turbulence on the kinetic and heat energy transfer are local, generally within a spatial range of \( \geq 2H \), below the shear layer. However, turbulence can drive a spatially and thermally great influence on in the regions above it. The temperature increases most significantly near the shear layer due to turbulence, which can further enhance the temperature inversion, in addition to the other effects already discussed in the literature (Showman et al. 2008; Rauscher & Menou 2010). We also find that shock formation is insignificant. The time-averaged heat energy flux at \( P \approx 1 \text{ bar} \) when the atmosphere becomes steady is on the order of 0.001% of \( F_s \) with a shear motion at the top of the atmosphere \( (P_{sh} \approx 1 \text{ mbar}) \) and 0.1% with a deeper shear layer at \( P_{sh} = 100 \text{ mbar} \). Accordingly, \( K_{zz} \) is higher for the deeper shear layer. Therefore, our results suggest that turbulence near less dense regions \( (P \geq 1 \text{ mbar}) \) does not lead to transport of heat energy deep enough to explain the inflated radii of hot Jupiters, regardless of how violent the turbulence is. On the other hand, as eddy motions occur at deeper regions \( (P \geq 100 \text{ mbar}) \), it is more likely that the heat energy is transferred more effectively throughout the atmosphere (upwards and downwards) due to relatively large kinetic energy budgets. Therefore, it is more important how deep turbulence occurs in the atmosphere (or, \( P_{sh} \)), than how unstable the atmosphere is (or, \( R_i \)) for effective transfer of energy.

Understanding the role of turbulence itself is a crucial step prior to modelling global-scale atmospheres. Future
work will aim to model global circulation of hot Jupiters including radiation.

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APPENDIX A: TIME EVOLUTION OF $\sigma(V_z)$ AND $\sigma(T)$

We present a plot showing a correlation between $\sigma(V_z)$ and $\sigma(T)$ at different time ranges.

This paper has been typeset from a TeX/LaTeX file prepared by the author.
Figure A1. Correlations between $\sigma(v_z)$ and $\sigma(T)$ at different times for the models for $Ri = 0.25$ with $T_{\text{deep}} = 3000$ K (blue dots) and $T_{\text{deep}} = 1800$ K (red dots). The solid black line diagonally running through the plane indicates for $\sigma(T)/T_0 = \sigma(v_z)/c_s$. 