Single-electron current gain in a quantum dot with three leads

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Abstract
The conductance through a quantum dot (QD) between a source and a drain electrode is usually controlled electrostatically by a nearby gate electrode. A periodic modulation of the conductance versus gate voltage is observed, swapping between Coulomb blockade and single-electron tunneling. By controlling the Fermi level of a third (‘base’) lead attached to the QD, we were able to switch a single-electron current from source to drain, exceeding the single-electron current to or from the base lead. A simple model is presented revealing the role of ground- and excited states within the QD for this dynamic operation of a single-electron transistor.

Keywords: quantum dot excitations, coulomb blockade, electronic wavefunction, single-electron tunneling

(Some figures may appear in colour only in the online journal)

1. Introduction

Conventional n- and p-metal-oxide-semiconductor field-effect transistors (MOSFETs) act as almost perfect electronic switches as they are based on the concept of electrostatically tuning an energy barrier for the charge carriers from source to drain by changing the gate voltage \(V_G\). This yields the exponential current versus gate-voltage characteristics\(I \propto \exp(\alpha \Delta V_G/k_BT)\)\((\alpha \approx \pm 1)\) observed in the subthreshold regime of the transistor. It is this property and the complementary switching behavior of n- and p-type transistors which has enabled highly integrated digital circuits with billions of transistors on a single semiconductor die. In conventional bipolar npn or pnp transistors, the charge carrier flow from emitter to collector is also limited by an energy barrier, present in the base region of the transistor. However, in contrast to the field-effect transistor, this energy barrier is dynamically controlled, i.e. it requires a base current.

In the late 1980’s, the concept of a single-electron transistor was introduced [1]. As sketched in figure 1(a), such a transistor consists of a small region (‘island’) where conduction electrons are confined, weakly coupled by tunneling barriers to leads which are denoted as source and drain [2]. Due to the Coulomb interaction between electrons in the small island and their image charges induced in the surrounding electrodes, a single-electron charging energy is required to add an additional electron onto the island, and also to remove an electron. As indicated in figure 1(b), energy barriers for recharging the island exist, forming an energy gap for single-electron transport between source and drain. Due to electrostatics, this energy gap scales with the inverse of the linear spatial dimensions of the device. By using a gate electrode close-by, the electrostatic potential and therefore the position of the energy gap relative to the Fermi levels of the leads is tunable. Shifting the threshold level for charging the island with \(N+1\) electrons between the Fermi levels of source and drain, the number of electrons on the island can fluctuate between \(N\) and \(N+1\), i.e. single-electron tunneling takes place. Shifting this level below both Fermi levels, the electron transport is blocked, the electron number on the island is fixed to \(N+1\) (Coulomb-blockade regime). Shifting the electrostatic potential further, the behavior repeats, however...
for increased electron number on the island. Therefore, as a function of the applied gate voltage, a periodic modulation of the conductance is observable—often denoted as Coulomb-blockade oscillations. Such a single-electron transistor is an electrostatically controlled device, where with an appropriate device design even voltage gain is achievable [3], but hardly to obtain on nano-scaled or molecular devices [4].

2. Some basics

If the Fermi wavelength of the conduction electrons is comparable to the spatial dimensions of the island, a discrete single-particle spectrum is expected for the electrons confined on the island. Such an island is denoted as an ‘artificial atom’ or as a quantum dot (QD) [5]. QD systems have been considered as model systems to understand also the essence of electrical transport through single molecules. However, due to the mesoscopic size, the electron–electron interaction is usually the dominant energy scale so that single-electron tunneling processes have to be described in terms of transitions between many-electron states of $N$ and $N + 1$ electrons confined in the QD [9]. In this language, the threshold level, depicted in figure 1(b), indicates the energy difference $E(n + 1, 0) - E(n, 0)$ between the groundstate energies of $n$ and $n + 1$ electrons, and their energy separation usually varies with increasing $n$ [6]. This ladder of energy levels shifts linearly with voltage applied to adjacent electrodes (gates, source, drain, etc.). Transitions to excited states of either the $n$ or $n + 1$ electron system appear as additional energy levels in such energy schemes, however these energy levels are only usable for single-electron transport if at least this level and the respective threshold level lie between the Fermi levels of source and drain. Transitions to excited states can therefore only be accessed at larger source-drain voltage [7].

3. The idea

By adding a third lead (base) to the QD (figure 2(a)), the situation is modified: lifting the Fermi level $\varepsilon_f^B$ of the base lead above the threshold level for $N + 1$ electrons (figure 2(b)), the QD is charged from the base and in the next step discharged by one electron towards the source or drain lead. The QD undergoes a transition from the $N$-electron groundstate $|N, 0\rangle$ to the $(N + 1)$-electron groundstate $|N + 1, 0\rangle$ and vice versa (figure 2(c)). Under stationary bias condition, this charging/discharging cycle repeats and leads to a stationary current carried by single-electron tunneling from or to the respective lead. The left circles describe single-electron tunneling from the base to source/drain, the blue circle at the right side describes single-electron tunneling from source to drain, enabled by a single-electron charging process into the QD from the base lead.

![Figure 1](image1.png)

**Figure 1.** (a) Arrangement of a single-electron transistor. (b) Respective energy scheme for charging/discharging the island by a single electron. The energy levels for the island above the threshold level for $QD$ is charged from the base and in the next step discharged by island by a single electron. The energy levels for the island have already occupied the island. Situation of Coulomb blockade is depicted.

![Figure 2](image2.png)

**Figure 2.** (a) Arrangement of a QD with three leads—source, drain and base, and a gate electrode nearby. For the bias condition depicted in the energy scheme (b), the single-electron-tunneling dynamics between three quantum-dot states—the $N$-electron groundstate $|N, 0\rangle$, the $(N + 1)$-electron groundstate $|N + 1, 0\rangle$, the $N$-electron excited state $|N, +\rangle$—is sketched in (c). The half-circles with arrows indicate the direction for the QD-state change by single-electron tunneling from or to the respective lead. The left circles describe single-electron tunneling from the base to source/drain, the blue circle at the right side describes single-electron tunneling from source to drain, enabled by a single-electron charging process into the QD from the base lead.
For the bias condition depicted in the energy scheme (a), the single-electron-tunneling dynamics between three quantum-dot states—\(|N+1,0\rangle, |N,0\rangle, |N+1,\ast\rangle\)—is sketched in (b). The left circles describe single-electron-tunneling from source/drain to the base, the red circle at the right side single-electron tunneling from source to drain, enabled by a single-electron-discharging process of the QD to the base lead.

\(|N+1,0\rangle\) and \(|N,\ast\rangle\) is used several times for single-electron tunneling events (blue circle in figure 2(c)) before the groundstate \(|N,0\rangle\) of N electrons is reached, a single charging event from the base lead induces several single-electron tunneling events between source and drain. In such a case, the source-drain current exceeds the base current which has enabled the source-drain current to flow.

4. Modeling the dynamics

Modeling the dynamics of this single-electron-tunneling device by rate equations shows (see appendix A) that—besides a suppression of relaxation from the excited state—the tunneling rates between QD and leads have to fulfill certain requirements which can be summarized basically in the following way: firstly, discharging the QD into the excited state \(|N,\ast\rangle\) via drain (blue path from \(|N+1,0\rangle\) in figure 2(c)) has to have a higher rate than discharging the dot into the groundstate \(|N,0\rangle\) via source or drain (black paths from \(|N+1,0\rangle\) in figure 2(c)). Secondly, charging the dot from \(|N,\ast\rangle\) via the base lead has to have a lower probability than charging the dot via the source lead. Such conditions can only be expected for a QD with few electrons where the different tunnel couplings of individual quantum states with the leads are resolvable\(^3\).

The dynamics described with figure 2(c) is initiated by charging the QD from the base by an electron, and results in a source-drain current by leaving an excess energy in the QD after discharging which is used in the next single-electron charging process. Under other bias conditions, shown in figure 3(a), the QD might be excited with charging, instead of discharging by a single electron. The dynamics for this case, switching between the three quantum-dot states \(|N,0\rangle, |N+1,0\rangle\) and \(|N+1,\ast\rangle\), is sketched in figure 3(b). It is analogous to the one described with figure 2(c). Modeling the system by rate equations reveals also here that certain conditions for the transition rates allow for a source-drain current exceeding the single-electron-tunneling current to the base lead: firstly, charging the QD into the excited state \(|N+1,\ast\rangle\) via source (red path from \(|N,0\rangle\) in figure 3(b)) has to have a higher rate than charging the dot into the groundstate \(|N+1,0\rangle\) from source or drain (black paths from \(|N,0\rangle\) in figure 3(b)). Secondly, discharging the dot from \(|N+1,\ast\rangle\) to the base lead has to have a lower probability than discharging the dot via the drain lead.

5. Experimental realization

To test our new concept of operating a single-electron transistor dynamically, we have fabricated quantum-dot devices with three leads. The samples are based on a GaAs/Al\(_{0.33}Ga_{0.67}\)As heterostructure\(^4\) containing a two-dimensional-electron system (2DES) at the heterojunction interface 50 nm below the surface (electron density \(n_s = (3.2 \pm 0.6) \times 10^{12} \text{ m}^{-2}\) and mobility \(\mu = (30 \pm 6) \text{ m}^2 \text{Vs}^{-1} \text{ at } T = 4.2 \text{ K}\)). A mesa structure was etched and metal contacts alloyed in certain regions of the mesa to 2DES. By using electron-beam lithography and a metal lift-off process, structured metal electrodes were deposited on the heterostructure surface. A scanning-electron-microscope

\(^3\)This differs to a resistive-coupled single-electron transistor (R-SET) which possesses a metallic island with a dense energy spectrum where relaxation is assumed to be fast.

\(^4\)Layer sequence starting from the semi-insulating GaAs substrate: 0.3 \(\mu\)m GaAs buffer, 0.64 \(\mu\)m GaAs/Al\(_{0.33}Ga_{0.67}\)As superlattice, 0.5 \(\mu\)m GaAs, 20 nm Al\(_{0.33}Ga_{0.67}\)As spacer, 20 nm Al\(_{0.33}Ga_{0.67}\)As homogeneously Si doped, 10 nm GaAs cap layer.
image of this split-gate structure is shown in figure 5(a). At temperatures below 0.1 K, negative voltages were applied to the gate electrodes dividing the 2DES into the QD, connected by tunneling barriers to three separate 2DES regions acting as leads—denoted in the figure as source, drain and base. Please note, (1) we have chosen a heterostructure with shallow lying 2DES to have rather steep electrostatic potential gradients defined by the split gates forming the QD and its tunneling barriers. (2) The size of the QD is rather small. (3) The split-gate design is intended to allow for an oval-shaped QD. (4) Source and drain contact are along a line via the QD while the base contact is lying aside.

6. Experimental results

Since in such quantum-dot systems the confining potential and the exact number of electrons is usually unknown, we have chosen a pragmatic approach: by systematically tuning the Fermi levels of source, drain and the base lead with respect to each other, tuning the electrostatic potential of the QD by a near-by gate electrode, and measuring the dc currents in all three leads under these bias conditions, we were able to well characterize our quantum-dot system. To probe the alignment of quantum-dot levels with the source or drain Fermi level, the differential conductance $dI_S/dV_D$ is measured: the voltage applied to the drain lead is modulated by $dV_D = 1 \mu V_{\text{rms}}$ while the modulation amplitude $dV_S$ of the source current is measured by lock-in technique (modulation frequency 13.5 Hz).

The expected positions of peaks in $dI_S/dV_D$ at $V_{DS} = 0$ mV as a function of the base voltage $V_B$ and gate voltage $V_G$ are sketched in figure 4: for $V_B = 0$ mV, the Fermi level $\epsilon_f^B$ of the base lead is equal to the source and drain Fermi level. Conductance is expected whenever a threshold level $E(N + 1, 0) - E(N, 0)$ for charging the QD aligns with the source and the drain Fermi level $\epsilon_f^S,D$—Coulomb-blockade oscillations appear as a function of gate voltage. Resonances to excited states cannot be used under these conditions as the QD is then in the Coulomb-blockade regime. Changing $V_B$, the electrostatic potential of the QD is affected, shifting the resonance condition of the Coulomb-blockade oscillations linearly in the gate voltage axis (black solid lines in figure 4). More importantly, shifting the Fermi level of the base lead above ($V_B < 0$) or below ($V_B > 0$) the charging-threshold level $E(N + 1, 0) - E(N, 0)$, single-electron tunneling is initiated between base and source/drain. As a function of gate voltage and base voltage, the well-known diamond-like charge-stability diagram is obtained. In the grey-shaded regions, the Coulomb blockade is overcome, single-electron tunneling between source/drain leads and base lead is possible, and excited states might become accessible. They are usable for single-electron tunneling between source and drain if the respective energy level aligns with the source/drain Fermi levels. Figure 4 shows the positions (red solid line) where the dynamics described with figure 3(b) can be expected: $E(N + 1, 0) - E(N, 0) = \epsilon_f^S,D$ and $\epsilon_f^S,D > E(N + 1, 0) - E(N, 0) \geq |\epsilon_f^B|$ (grey-shaded region at $V_B > 0$). The dynamics, given in figure 2(c), can be expected at positions marked by the blue solid line: $E(N + 1, 0) - E(N, 0) = \epsilon_f^S,D$ and $\epsilon_f^S,D < E(N + 1, 0) - E(N, 0) \leq |\epsilon_f^B|$ (grey-shaded region at $V_B > 0$).

Differential conductance measurements $dI_S/dV_D$ at $V_{DS} = 0$ mV on our quantum-dot system are shown in figure 5(b). Indeed, the expected conductance peaks are found, however usually starting at higher values $|\epsilon_f^B|$ than expected from the diamond-like charge-stability diagram. This indicates that the visible transitions are not starting from the respective groundstate. More than one excited state is involved, and transitions between excited states $|N, 0\rangle$ and $|N + 1, 0\rangle$ have to be included for the full description. This was experimentally confirmed by detailed transport-spectroscopy investigations on our QD in this regime.

7. Discussion

The important point is that current gain has been observed in these quantum-dot devices. Figure 5(c) shows an example: the source, drain and base dc currents are measured for fixed source-drain voltage ($V_S = -0.285$ mV, $V_D = +0.285$ mV) while the base voltage is tuned. A gate voltage is used to

$^5$ rms’ indicates that the modulation amplitude is given as ‘root mean square’.

$^6$ Within the constant-interaction model, the slopes of the borderlines between Coulomb blockade and single-electron tunneling are given by $\partial V_G/\partial V_B = -C_B/C_G$ and $\partial V_G/\partial V_B = (C_D - C_B)/C_D$, where $C_G$ denotes the total capacitance of the island, $C_B$ the capacitance to the base, $C_G$ to the gate.
compensate for the electrostatic shift caused by varying the base voltage. At zero base voltage, the QD is energetically in the Coulomb-blockade regime, and the observed current from source to drain can only be due to correlated electron tunneling, not single-electron tunneling. Lowering the base voltage to about $-0.4$ mV, the base current increases in an almost step-like manner, indicating that single-electron tunneling from the base via the QD to source and drain is possible. At the same time, the source and drain currents are increased (also visible at the same point marked by an arrow in figure 5(b)). The signs of the currents indicate that a net electron flow occurs from the source into the QD (and not vice versa) and from the QD to the drain lead,—a source-drain current is induced by the base current. Most importantly, the source current exceeds the base current by a factor of three. For even more negative base voltage, further excited states become involved, changing the single-electron tunneling dynamics. Similar behavior, although not that nicely expressed, is observed for another resonance (red line in figure 5(b), see appendix figure C1).

8. Conclusions

In summary, we have demonstrated a novel type of operation for a single-electron transistor where in the Coulomb-blockade regime a source-drain current is induced by single-electron recharging events to a third lead, the base lead. Experimentally we found, the induced source-drain current can exceed the base current. Current gain can only work if certain conditions for the tunneling rates using ground- and excited states are fulfilled. One option is that the spatial overlap and symmetry of the (single-electron) wavefunctions with the respective leads fulfill these conditions [13]. This can even be imposed by designing the QD’s confining potential,—for instance, with two local confining potential minima of different depth (asymmetric quantum-dot molecule). In literature it has been predicted that selection rules due to spin and/or correlation effect exist for relaxation from excited to groundstates and single-electron recharging of the QD [10]. This has been discussed in the context of negative differential conductance appearing in the single-electron tunneling regime of QDs with two leads [7]. We expect that indeed such selection rules are present governing the dynamics but are hard to control for the purpose of current gain. Our experimental data show that more than one excited state are involved. With each new state being accessed, transitions to other excited states become possible. The dynamics on the right side of figures 2(c) and 4(b) becomes more complex but might obviously be in favor that the QD does not end in the respective groundstate on the left side blocking transport from source and drain.

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Appendix A. Dynamics described by a rate-equation ansatz

To describe the dynamics visualized in figure 2 we choose here a rate-equation ansatz [11]. For simplicity, we assume that the confined electron system of the QD can only change between the groundstates $|N,0\rangle$ and $|N+1,0\rangle$ of $N$ and $N+1$ confined electrons in the QD, and an excited state $|N,\ast\rangle$ of the confined N-electron system. It means the probabilities of finding the electron system in one of these states add up to unity,

$$P_{N,0} + P_{N,\ast} + P_{N+1,0} = 1. \quad (A.1)$$

The rate for the transition from the state $|n,l\rangle$ to the state $|n',l'\rangle$—derived from Fermi’s golden rule—is denoted by

$$\Gamma_{[n,l] \rightarrow [n',l']}(X).$$

The label $X \in \{S,D,B,rel\}$ indicates from/to where an electron has entered/left the QD—source (S), drain (D) or base (B), or a relaxation process (rel) at fixed number of electrons in the QD has happened. Intrinsic (thermal) excitations in the leads and in the QD are excluded, for simplicity.

The dynamics of the quantum-dot system for the bias conditions depicted in figure 2 is described by the following set of equations:

$$\frac{d}{dt} P_{N,0} = + \Gamma_{[N+1,0] \rightarrow [N,0]} P_{N+1,0} + \Gamma_{[N,1] \rightarrow [N,0]} P_{N+1,0} - \Gamma_{[N,0] \rightarrow [N+1,0]} P_{N,0},$$

where

$$\frac{d}{dt} P_{N,\ast} = + \Gamma_{[N+1,\ast] \rightarrow [N,\ast]} P_{N+1,\ast} - \Gamma_{[N,\ast] \rightarrow [N+1,\ast]} P_{N,\ast},$$

and

$$\frac{d}{dt} P_{N+1,0} = + \Gamma_{[N+1,0] \rightarrow [N+1,0]} P_{N+1,0} - \Gamma_{[N+1,0] \rightarrow [N,0]} P_{N+1,0}.$$

fulfilling relation (A.1). Under stationary conditions we have $dP_{n,l}/dt = 0$ for all states, which allows us to calculate probability ratios from (A.2)

$$\frac{P_{N,\ast}}{P_{N+1,0}} = \frac{\Gamma_{[N+1,\ast] \rightarrow [N,\ast]} + \Gamma_{[N,\ast] \rightarrow [N+1,\ast]} + \Gamma_{rel}}{\Gamma_{[N+1,0] \rightarrow [N,0]} + \Gamma_{[N,0] \rightarrow [N+1,0]}},$$

$$\frac{P_{N,0}}{P_{N+1,0}} = \frac{\Gamma_{[N,0] \rightarrow [N+1,0]} + \Gamma_{[N+1,0] \rightarrow [N+1,0]}}{\Gamma_{[N+1,0] \rightarrow [N+1,0]}} + \frac{\Gamma_{rel}}{\Gamma_{[N+1,0] \rightarrow [N,0]}} P_{N,\ast},$$

and

$$\frac{P_{N+1,0}}{P_{N,0}} = \frac{\Gamma_{[N+1,0] \rightarrow [N,0]} + \Gamma_{[N,0] \rightarrow [N+1,0]}}{\Gamma_{[N+1,0] \rightarrow [N+1,0]}} + \frac{\Gamma_{rel}}{\Gamma_{[N+1,0] \rightarrow [N,0]}} P_{N,\ast}. \quad (A.3)$$

Under stationary conditions, the electron currents flowing in the respective leads towards the QD are obtained from (with electron charge $-e$)
Figure B1. (a) Schematics of the experimental setup. The voltages $V_G$, $V_D$, $V_S = -V_D$ are controlled, the currents $I_D$, $I_S$, and $I_B$ are measured. (b) Measured dc currents $I_S$, $I_B$, and $I_D$ in colorscale as a function of $V_{DS}$ ($V_S = -V_D$) and $V_G$ while $V_B = 0$ (with about $\pm 5$ $\mu$V uncertainty). Colored in red, the current flows towards the QD, in blue away from the QD (note, the electrons flow opposite to the technically defined current direction). All currents $I_S$, $I_B$, $I_D$ vary with $V_G$, $V_{DS}$ and $V_B$ but add in any constellation altogether to zero. In these measurements the Fermi levels $\epsilon_S^f$, $\epsilon_D^f$ are shifted symmetrically around $\epsilon_B^f$. (c) Differential conductance $dI_S/dV_D$ and $dI_B/dV_B$ measured in parallel with (b) enhancing the step-like changes in the current-bias voltage characteristics. The differential conductances $dI_S/dV_D$ and $dI_B/dV_B$ are measured with two lock-in amplifiers in parallel by modulation $V_D$ (at 13.5 Hz) and $V_B$ (at 21.5 Hz), both with $1 \mu V_{rms}$. (d) As (b) but for lowered Fermi level $\epsilon_B^f$ ($V_B = (374 \pm 5) \mu$V). Current flow becomes visible where in (b) a Coulomb-blockade region has been identified (see area of dashed triangle). (e) Differential conductance $dI_S/dV_D$ and $dI_B/dV_B$ measured in parallel with (d).
Changes in the single-electron current flow are expected whenever certain resonance conditions are fulfilled. For a QD with $|N, 0⟩$, $|N + 1, 0⟩$, $|N, +⟩$ and $|N + 1, +⟩$ as possible states with the respective total energy $E(n, i; V_G, V_DS)$ the resonance conditions for changing and discharging the QD to the respective leads are schematically plotted in the $(V_G, V_DS)$ plane. We assume $V_B > 0$, i.e. a lowered Fermi level of base. Lines reflecting resonance conditions with the Fermi level of source ($e_{fS}$) and drain ($e_{fD}$) have a finite, positive or negative slope, whereas lines reflecting resonance conditions with the Fermi level of base ($e_{fB}$) are flat. For certain points in the $(V_G, V_DS)$ plane, the respective energy schemes for the quantum-dot system are sketched. In grey-shaded regions we expect single-electron tunneling through the QD being possible, in the dark grey the single-electron tunneling dynamics is only enabled by discharge events to the base. In the dashed region, single-electron discharging to the base lead is not possible, however as shown in the energy scheme (7), inelastic cotunneling through the QD being possible, in the dark grey the single-electron tunneling dynamics is only enabled by discharge events to the base. Such a region seems to be visible in figure B1(d).

$$\frac{I_S}{e} = + \Gamma_{[N, +]}^{(S)}(|N + 1, +⟩ → |N⟩) \cdot P_{[N, +]} \cdot \Gamma_{N + 1, 0}^{(S)}(|N⟩ → |N + 1, 0⟩)$$

$$\frac{I_D}{e} = - \Gamma_{N + 1, 0}^{(D)}(|N, 0⟩ → |N + 1, 0⟩) \cdot P_{N + 1, 0} \cdot \Gamma_{[N, +]}^{(D)}(|N + 1, +⟩ → |N, +⟩)$$

$$\frac{I_B}{e} = + \Gamma_{[N, 0]}^{(B)}(|N + 1, 0⟩ → |N, 0⟩) \cdot P_{N + 1, 0} + \Gamma_{[N, +]}^{(B)}(|N + 1, +⟩ → |N, +⟩) \cdot P_{N, +}$$

Current conservation requires $I_S + I_D + I_B = 0$.

Due to (A.4), the ratio $|I_S/I_B|$ exceeds the factor $g > 1$ (case of 'current gain') if

$$g^{-1} \left( \frac{\Gamma_{N + 1, 0}^{(D)}(|N, 0⟩ → |N + 1, 0⟩) \cdot P_{N, +}}{\Gamma_{[N, 0]}^{(B)}(|N + 1, 0⟩ → |N + 1, +⟩) \cdot P_{N, +}} \right) > \frac{\Gamma_{[N, 0]}^{(B)}(|N + 1, 0⟩ → |N, 0⟩) \cdot P_{N, +}}{\Gamma_{[N, 0]}^{(B)}(|N + 1, +⟩ → |N, +⟩) \cdot P_{N, +}}$$

Note, this is only possible if $P_{N + 1, 0} > 0$ which requires $\Gamma_{N, 0}^{(B)}(|N, 0⟩ → |N + 1, 0⟩) > 0$. Replacing the state probabilities by using (A.3), the constrain for the transition rates reads as

$$g^{-1} > 1 + \frac{\Gamma_{N + 1, 0}^{(D)}(|N, 0⟩ → |N + 1, 0⟩)}{\Gamma_{[N, 0]}^{(B)}(|N + 1, 0⟩ → |N + 1, +⟩)} + \frac{\Gamma_{N + 1, +}^{(D)}(|N + 1, 0⟩ → |N + 1, +⟩)}{\Gamma_{[N, +]}^{(B)}(|N + 1, 0⟩ → |N + 1, +⟩)}$$

which leads for $g^{-1} \ll 1$ to
is shifted above the Fermi level of source, a larger window opens of the base lead $\Gamma$ the differential conductance plane, the measured currents $I_D$, $I_S$, $I_G$ for $V_{DS} = 0.571$ mV are plotted in (b). The step height in the drain current exceeds the step height in the base current, although not that dramatic as in case of figure 5. Left: relative positions of energy levels. Along the red height in the base current, although not that dramatic as in case source and drain.

$\Gamma^{(S)}_{|N,+\rangle \rightarrow |N,0\rangle} > g \cdot \Gamma^{(B)}_{|N,+\rangle \rightarrow |N,0\rangle}$, i.e. charging of the QD being in the state $|N, +\rangle$ has to happen mostly from the source and not from the base lead, and

$\Gamma^{(S)}_{|N,+\rangle \rightarrow |N+1,0\rangle} > g \cdot \Gamma^{(B)}_{|N,+\rangle \rightarrow |N+1,0\rangle}$, i.e. direct relaxation from the excited state $|N, +\rangle$ to the groundstate $|N, 0\rangle$ has to be suppressed.

Under these circumstances, one can expect that, starting from the groundstate $|N, 0\rangle$, the QD is charged from the base and the blue circle in figure 2(c) is taken—in average—$g$ times, before the QD reaches again the groundstate $|N, 0\rangle$.

**Appendix B. Current and differential conductance versus $V_G$ and $V_{DS}$ for different $V_B$**

With shifting up or down the Fermi level $\epsilon^B_f$ of the base lead relatively to the Fermi levels $\epsilon^S_f$ and $\epsilon^D_f$, a larger window opens in gate voltage $V_G$ allowing for single-electron tunneling which is not possible for $V_B = 0$ (see figure B1). Within the single-electron tunneling regime, peaks in the differential conductance indicate excited states becoming available for single-electron tunneling. The shift of the differential conductance peak in the $(V_G, V_{DS})$ parameter space tells whether the resonance condition is with the Fermi level of source, drain or base, and an excited state is reached either by charging ($|N + 1, +\rangle$) or by discharging ($|N, +\rangle$) the QD [7]. This can be nicely seen by following the changes in the energy scheme in the $(V_G, V_{DS})$ parameter space for a QD with possible states $|N, 0\rangle$, $|N + 1, 0\rangle$, $|N, +\rangle$, and $|N + 1, +\rangle$ shown in figure B2. The essential features visible in the experimental data shown in figures B1(d) and (e) can be identified.

**Appendix C. Current gain for positive base voltage**

In figure 5(c), current gain is shown for negative base voltage $V_B$, i.e. $\epsilon^B_f$ is shifted above the Fermi level of source and drain and an excited state of the confined $N$-electron system is reached by discharging the QD towards source or drain (see figure 2). In figure C1, data are presented, showing current gain for positive $V_B$, i.e. an excited state of the confined $(N + 1)$-electron system is used for single-electron tunneling between source and drain (see figure 4, right side).
Appendix D. DC currents versus $V_G$ and $V_B$ in the regime of figure 5

To supplement the data of figure 5, the dc currents $I_S$, $I_D$, and $I_B$ measured for bias $V_{DS} = 0.190$ mV over a part of $(V_B, V_G)$ parameter space are presented in figure D1. Despite the different $V_{DS}$ bias, current gain is observed in the same $(V_B, V_G)$ regime.

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