Effects of dissipative hydrodynamics on elliptic flow and HBT interferometry in central collisions at RHIC

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Abstract. In this work, we study relativistic heavy ion collisions by using a hydrodynamic model for both ideal and viscous cases to investigate the influence of dissipative effects on the elliptic flow and HBT interferometry. We conclude that the bulk viscosity has small influence in hydrodynamic calculations. However, our results show that the shear effects are important for describing the experimental data in central collisions.

1. Introduction

Relativistic heavy ion collisions allow the study of the behavior of matter under extreme pressure and temperature conditions. Under these conditions it is possible to observe a transition from ordinary matter to a quark-gluon plasma (QGP) [1]. One possible tool for studying the system formed in these collisions is the hydrodynamic model [2, 3]. The application of this model is based on the assumption that the system reaches a state of local thermodynamic equilibrium and in the fact that the matter formed in these collisions shows a collective behavior.

Our aim in this work is to investigate the influence of shear and bulk viscosities on elliptic flow and HBT radii by using a hydrodynamic model with smooth initial conditions and an equation of state based on lattice QCD with a crossover transition between the QGP and the hadron gas. The calculations were performed for central Au+Au collisions (0-5%) with √s_{NN} = 200 GeV in 2+1 dimensions (boost invariance).

2. Hydrodynamic Model

In the hydrodynamic model each fluid element can be characterized by its energy-momentum tensor and other conserved numbers (baryonic number, strangeness, etc.). In this work, we assumed that all conserved numbers are zero, so the hydrodynamic equations can be written as

\[ \partial_\mu T^{\mu \nu} = \partial_\mu T^{\mu \nu} + \Gamma^\mu_{\sigma \mu} T^{\sigma \nu} + \Gamma^{\nu}_{\sigma \mu} T^{\mu \sigma} = 0, \]

where \( \Gamma^\mu_{\sigma \mu} \) are the Christoffel symbols and \( T^{\mu \nu} \) the energy-momentum tensor given by [4, 5]

\[ T^{\mu \nu} = \epsilon u^\mu u^\nu - (P + \Pi) \Delta^{\mu \nu} + \pi^{\mu \nu}. \]

Here \( \epsilon, u^\mu, P, \Pi, \pi^{\mu \nu} \) and \( \Delta^{\mu \nu} \) are, respectively, the energy density, the fluid four-velocity, the pressure, the bulk viscosity, the shear viscosity tensor and the orthongal projector to \( u^\mu \). We solved Eq. (1) in Milne coordinates [4].
The second order viscous hydrodynamic equations are calculated for both shear and bulk viscosities, using the Israel-Stewart framework [6]

\[ \Delta^{\mu\alpha} \Delta^{\nu\beta} u^\gamma \partial_{\gamma} \pi_{\alpha\beta} = -\frac{1}{\tau_\pi} [\pi^{\mu\nu} - \eta \sigma^{\mu\nu}] - \frac{4}{3} \pi^{\mu\nu} \partial_{\gamma} u^\gamma, \]

\[ u^\gamma \partial_{\gamma} \Pi = -\frac{1}{\tau_\Pi} [\Pi - \zeta \partial_{\gamma} u^\gamma] - \frac{4}{3} \Pi \partial_{\gamma} u^\gamma, \]

where \( \sigma^{\mu\nu} = \Delta^{\mu\lambda} \partial_{\lambda} u^\nu + \Delta^{\nu\lambda} \partial_{\lambda} u^\mu - \frac{2}{3} \Delta^{\mu\nu} \partial_{\lambda} u^\lambda \) and \( \eta, \zeta, \tau_\pi, \tau_\Pi \) are, respectively, the shear coefficient, the bulk coefficient, the shear relaxation time, the bulk relaxation time. The relaxation times depend on the viscous coefficients as \( \tau_\pi = \frac{3}{2} \frac{T}{s} \) and \( \tau_\Pi = \frac{6}{\pi^2} \frac{T}{s} \) [3], where \( T \) is the temperature and \( s \) the entropy density. The \( \eta/s \) and \( \zeta/s \) are parameters in our model. To perform the hydrodynamic evolution, we also have to provide the initial conditions (IC) and the equation of state (EoS).

When the mean free path becomes of the order of the system size, the hydrodynamic model is no longer valid and the particles decouple by traveling in straight line to the detector. Here, we employ the freeze-out framework [7] to calculate the decoupling by using the Cooper-Frye formula [8]

\[ \frac{1}{2\pi} \frac{dN}{p_T dp_Y dy d\phi} = \int d\Sigma_\mu p^\mu f(p_\mu u^\mu), \]

where \( N \) is the number of particles, \( p_T \) the transverse momentum, \( y \) the rapidity, \( \phi \) the azimuthal angle, \( p^\mu \) the four-momenta and \( d\Sigma_\mu \) is the normal vector to the freeze-out surface. The distribution function is given by \( f = f_0 + \delta f_\pi + \delta f_\Pi \), where \( f_0 \) is equilibrium distribution function [7]; \( \delta f_\pi \) and \( \delta f_\Pi \) are the corrections for shear and bulk viscosities, respectively [3].

3. Numerical Results

We used IC generated by TRENTo [9]. The hydrodynamic equations were solved by using vHLLE [4] with the s95p EoS [10], and the decoupling calculated by using THERMINATOR2 [7]. We perform simulations for ideal fluids and different scenarios of viscous fluids. In viscous simulations we consider \( \eta/s \) constant in all the hydrodynamical evolution and \( \zeta/s \) constant in the hadron gas phase and zero in the QGP phase [3].

3.1. Elliptic Flow

The collective behavior of the system formed in heavy ion collisions is described by the anisotropic flow [11], where the initial spacial anisotropy is reflected in the final state of particles. In order to calculate the anisotropic flow, we rewrite the azimuthal part in equation (5) using a Fourier tranform as

\[ \frac{dN}{d\phi} = 1 + 2 \sum_{n=0}^{\infty} v_n \cos (\phi - \psi_R), \]

where \( v_n \) are the Fourier components and \( \psi_R \) is the reaction plane angle. Here, we calculated the second Fourier component \( (v_2) \), called elliptic flow, using the event plane method [12]. Figure 1 shows the \( p_T \) dependence of \( v_2 \) compared with data. The result for ideal hydrodynamics is shown in red line, the blue line is for viscous hydrodynamics with \( \eta/s = 0.08 \) and \( \zeta/s = 0.04 \), magenta line is for \( \eta/s = \zeta/s = 0.08 \), green line is for \( \eta/s = 0.16 \) and \( \zeta/s = 0.04 \), and black squares are experimental data. We see that the ideal hydrodynamics can describe the data for \( p_T < 1 \) GeV. However, for describing the data for higher values of \( p_T \) one has to include shear effects. Also, we can see that the bulk viscosity has no significant effects on \( v_2 \).
3.2. HBT Interferometry

The HBT effect [15] allows measuring the space-time dimensions of the interacting region, at freeze-out, in a high energy collision. To calculate the so-called HBT radii, we used the correlation function defined, for a pair of pions with momentum $p_1$ and $p_2$, as

$$C(p_1, p_2) = \frac{P_2(p_1, p_2)}{P_1(p_1)P_1(p_2)},$$

where $P_2$ is the probability of detection of both particles simultaneously and $P_1$ is the probability of detection of each particle individually. We calculated the equation (7) using the Monte Carlo approach implemented in THERMINATOR2 by using the Berstch-Pratt coordinates [14]. The long coordinate is defined along the beam direction; the out coordinate is parallel to the average transverse momentum of the pair ($\vec{K}_T = (\vec{p}_1 + \vec{p}_2)/2$) and the side coordinate is orthogonal to both long and out. We fitted the correlation function using a Gaussian function

$$C = 1 + \lambda \exp \left\{ -R_{out}^2q_{out}^2 - R_{side}^2q_{side}^2 - R_{long}^2q_{long}^2 \right\},$$

where $q = p_1 - p_2$ is the relative four-momentum of the pair in out, side and long directions and $\lambda$ is the chaoticity parameter. Figure 2 shows $K_T$ dependence of $R_{out}$, $R_{side}$, $R_{long}$ and $R_{out}/R_{side}$ for hydrodynamic simulations compared with data. The red-dashed line is the result for ideal hydrodynamics; the blue-dashed line is for viscous hydrodynamics with $\eta/s = 0.08$ and $\zeta/s = 0.04$; the magenta-dashed line is for $\eta/s = \zeta/s = 0.08$; the green-dashed line from $\eta/s = 0.16$ and $\zeta/s = 0.04$; black squares are experimental data. Although our results fail to reproduce the experimental data for $R_{out}$, $R_{side}$, and $R_{long}$, we can see no influence of bulk viscosity in HBT radii, analogously to $v_2$ results. We also have a better agreement with data for $R_{out}/R_{side}$ when we increase the shear coefficient $\eta/s$.

4. Conclusions and Perspectives

In summary, we have used the hydrodynamic model in 2+1 dimensions for both ideal and viscous cases to calculate $v_2$ and HBT radii. We have shown that the effect of bulk viscosity is to small for both observables in central collisions. However, the inclusion of shear viscosity brings a better agreement with data for $v_2$ in higher $p_T$ and for the ratio $R_{out}/R_{side}$. A more detailed study about the influence of dissipative effects can be found in [17]. Some improvements to this work can be made in the future: it includes hydrodynamic simulations in 3+1 dimensions, event-by-event initial conditions, viscous dependence on temperature.
Figure 2. $K_T$ dependence of HBT radii for charged pions in the Bertsch-Pratt system $R_{out}$ (top left), $R_{side}$ (top right), $R_{long}$ (bottom left) and the ratio $R_{out}/R_{side}$ (bottom right) for central Au+Au collisions at 200 GeV. Data from STAR collaboration [16].

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References

[1] Ludlam T and Aronson S 2005 Hunting the quark gluon plasma BNL-73847-2005.
[2] Hama Y, Kodama T and Socolowski Jr O 2005 Braz. J. Phys. 35 24; Andrade R P G, Grassi F, Hama Y and Qian W L 2011 Nucl. Phys. A 854 81; Noronha-Hostler J, Denicol G S, Noronha J, Andrade R P G and Grassi F 2013 Phys. Rev. C 88 044916.
[3] Bozek P 2012 Phys. Rev. C 85 034901.
[4] Karpenko Iu, Huovinen P and Bleicher M 2014 Comp. Phys. Commun. 185 3016.
[5] Landau L D and Lifshitz E M 1987 Fluid Mechanics Butterworth-Heinemann Ltd.
[6] Israel W 1976 Annals Phys. 100 310; Israel W and Stewart J M 1979 Annals Phys. 118 341.
[7] Chojnacki M, Kisel A, Florkowski W andBroniowski W 2012 Comput. Phys. Commun. 183 746.
[8] Cooper F and Frye G 1974 Phys. Rev. D 10 186;
[9] Moreland J S, Bernhard J E and Bass S A 2015 Phys. Rev. C 92 011901.
[10] Huovinen P and Petreczky P 2010 Nucl. Phys. A 837 26.
[11] Ollitrault J Y 1992 Phys. Rev. D 46 229.
[12] Poskanzer A M and Voloshin S A 1998 Phys. Rev. C 58 1671.
[13] STAR Collaboration 2005 Phys. Rev. C 72 014904.
[14] Bertsch G F 1989 Nucl. Phys. A 498 173C.
[15] Hanbury Brown R and Twiss R Q 1958 Nature 178 4541; Padula Sandra S 2005 Braz. J. Phys. 35 25;
[16] STAR Collaboration 2005 Phys. Rev. C 71 044906.
[17] Lemos D S 2017 Efeitos da Hidrodinâmica Dissipativa e da Equação de Estado sobre o Fluxo Elíptico e a Interferometria HBT Masters dissertation FURG.