D3 instantons in Calabi-Yau orientifolds with(out) fluxes

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Abstract

We investigate the instanton effects due to D3 branes wrapping a four-cycle in a Calabi-Yau orientifold with D7 branes. We study the condition for the nonzero superpotentials from the D3 instantons. For that matter we work out the zero mode structures of D3 branes wrapping a four-cycle both in the presence of the fluxes and in the absence of the fluxes. In the presence of the fluxes, the condition for the nonzero superpotential could be different from that without the fluxes. We explicitly work out a simple example of the orientifold of $K3 \times T^2/Z_2$ with a suitable flux to show such behavior. The effects of D3-D7 sectors are interesting and give further constraints for the nonzero superpotential. In a special configuration where D3 branes and D7 branes wrap the same four-cycle, multi-instanton calculus of D3 branes could be reduced to that of a suitable field theory. The structure of D5 instantons in Type I theory is briefly discussed.

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1 Introduction

Understanding the nonperturbative corrections to the superpotentials is an interesting topic in string theory. With the recent progress in understanding the flux compactification, it might be interesting to pursue this issue in this context. Especially, KKLT\cite{kklt} type scenario crucially needs such nonperturbative corrections. So it would be interesting to figure out the conditions where such corrections exist. One convenient starting point for the flux compactification is to consider the Type IIB orientifold on Calabi-Yau manifolds with three-form fluxes and D7-branes\cite{kal}. The nonperturbative corrections in this set up are due to D3-branes. In order to understand such instanton effects, we need the physical gauge approach as developed in \cite{phys1, phys2, phys3, phys4, phys5}. Furthermore the instanton effects due to D3 branes are not well understood in the Calabi-Yau compactification even without the fluxes.

Here we initiate the investigation of the D3 instanton effect in the Calabi-Yau orientifolds. In this paper we are mainly interested in the basic structure of D3 instantons and the conditions for the nonzero superpotentials. Much of the discussion is devoted to the fermion zero modes on D3 branes wrapping a four cycle in a Calabi-Yau orientifold, relegating the further discussion of the specific examples to future work. We find the conditions for the nonzero superpotentials in the presence of the fluxes and in the absence of the fluxes. The nonzero conditions in the presence of the fluxes are different from those in the absence of the fluxes. Related examples were studied in \cite{example1, example2} and the related M5 brane instanton effects were discussed in \cite{example3, example4, example5, example6}. We consider one simple orientifold of $K3 \times T^2/Z_2$ with D7 branes as an example where the modification of the nonzero conditions for the superpotentials occur due to the presence of the fluxes. We show that this orientifold can have nonzero superpotentials due to D3 instantons in the presence of the fluxes while without the fluxes there would be no superpotentials arising from D3 instantons. Along with this development we find many interesting facts as well. Especially the D3-D7 sectors are important in understanding the structure of the D3 instantons. When D3 branes and D7 brane wrapping the same four cycle, D3 instantons could be reduced to the usual field theory instantons. Then we can borrow the results of the multi-instanton calculus of the field theory to study the multi-instanton effects of D3 branes, which are not well understood so far. In other case of D3-D7 sectors, we find D1 string sector at the intersection of D3 brane and D7 brane. The effect of the D1 string is similar to that of the heterotic string instanton effect as considered in \cite{heterotic1, heterotic2} and this gives rise to further restrictions on the nonzero conditions for the superpotentials.

The content of the paper is as follows. In section 2, we consider the fermion zero modes on the D3 brane world volume to figure out the nonzero conditions for the superpotentials due to the D3 instantons wrapping a four-cycle in the Calabi-Yau orientifolds with D7 branes. In section 3, we consider the same problem in the presence of the fluxes. The nonzero conditions are
dependent on the fluxes and we are mainly interested in the simple orientifold of $K^3 \times T^2/Z_2$ with D7 branes. We show that there indeed nonzero superpotentials generated due to D3 instantons. In section 5, by the similar method we work out the condition where the contribution of the D5 brane instanton effects to the superpotential in Type I theory is nonzero. After this work has finished, we are aware of the work [15] which deals with the similar problem.

2 Zero mode analysis in the absence of the fluxes

We are mainly interested in the Calabi-Yau orientifold with D7 branes and (instantonic) D3 branes. We will consider two possibilities for D3-D7 systems. We assume that the normal directions to D7 branes are $x^8, x^9$ directions and Calabi-Yau manifold spans $x^4$ to $x^9$. In one case D3 brane world volume directions span $x^4, x^5, x^6, x^7$ directions so that within the Calabi-Yau manifold, the transverse directions of D3 and D7 branes coincide. This is T-dual to D5-D9 configurations if $x^8, x^9$ directions are compactified on $T^2$, which are small instanton ones. (In a nontrivial geometry of Calabi-Yau, the coordinates $x^4 \cdots x^9$ are local coordinates near the D3 brane.) Another possibility is that the D3 brane world volume directions contain $x^8, x^9$, which are the transverse directions of D7 brane. For example we can take the world volume directions of D3 are $x^6, x^7, x^8, x^9$. The D3-D7 sectors arising from this geometry are T-dual to D1-D9 string configurations if $x^8, x^9$ are coordinates of $T^2$, which is S-dual to the heterotic string. On the intersection of D7 worldvolume and D3 world volume is the worldsheet of D1 brane. This configuration is closely related to the D1 instanton of Type I string theory. We will see that these two configurations will arise for the IIB orientifold on $K^3 \times T^2/Z_2$, examples we consider later.

Let’s consider the first possibility. In the Type IIB theory, we have two chiral 10-d spinors $\epsilon_L, \epsilon_R$ satisfying $\Gamma_{11} \epsilon_L = \epsilon_L, \Gamma_{11} \epsilon_R = \epsilon_R$. In the presence of D7 branes, the unbroken supersymmetries are given by

$$\epsilon_L = \Gamma_0\Gamma_1 \cdots \Gamma_7 \epsilon_R = \Gamma_8 \Gamma_9 \epsilon_R$$

while in the presence of the D3 branes, we have further constraint

$$\epsilon_L = \Gamma_4 \Gamma_5 \Gamma_6 \Gamma_7 \epsilon_R.$$  \hspace{1cm} (2.2)

From these two equations, we obtain the condition

$$\epsilon_R = \Gamma_{456789} \epsilon_R.$$  \hspace{1cm} (2.3)

Strictly speaking, we can impose the two conditions (2.2), (2.3) only if the positions of D3 branes and D7 branes along $x^8, x^9$ directions coincide. If the D7 branes are away from the
D3 branes, surviving supersymmetries on the D3 branes are given by (2.2). However in the evaluation of the superpotentials using the physical gauge approach, we use the Kaluza-Klein approximation so that compactified space is small. In this case, the zero modes of the spinors should satisfy (2.2), (2.3) simultaneously. This could be understood better if we consider $x^8, x^9$ directions are compactified on a torus $T^2$. This is T-dual to the D5-D9 configuration, where the D5 branes have 8 component surviving supersymmetries. The small radius limit of $T^2$ of the D3-D7 configurations corresponds to the large radius limit of D5-D9 configurations. With the conditions (2.2), (2.3) satisfied, the decomposition of the 10-d chiral spinor is given by

\begin{align}
S_{10}^+ &\equiv (S_6^+ \otimes S_4^+) \oplus (S_6^- \otimes S_4^-) \\
S_6^+ &\equiv (S_D^+ \otimes S_N^+) \oplus (S_D^- \otimes S_N^-)
\end{align}

(2.4)

where $S_6^+, S_7^+, S_N^+$ are a positive chirality spinor in the Calabi-Yau manifold, in the 4 cycle and in the normal direction to the 4-cycle within the Calabi-Yau manifold respectively while $S_6^-, S_D^-, S_N^-$ are negative chirality spinors defined on the corresponding spaces. Among 16 components of $S_{10}^+$ only 8 components $S_6^+ \otimes S_4^+$ are consistent with the above supersymmetry (2.2), (2.3). In order to proceed with the spacetime approach of D-brane instantons, one needs a $\kappa$ invariant action of D3 brane in the presence of D7 branes. This is not constructed yet. However we just need the quadratic part of the D3 brane action for that purpose and one important ingredient is the structure of worldvolume fermions. The $\kappa$ invariant action is written in terms of 10-d spacetime spinor. Upon the static gauge fixing, this turns into worldvolume spinors of D3 brane and the unbroken supersymmetries are given by (2.2) if the D3 brane is separated from the D7 branes along $x^8, x^9$ directions while those are given by (2.2) and (2.3) if the D3 brane is coincident with the D7 branes. Upon Kaluza-Klein reduction, the zero modes of the spinors satisfy (2.2) and (2.3) simultaneously. We assume that such $\kappa$ invariant action can be constructed. This is also consistent with the low energy supersymmetric theory obtained from the D3-D7 configurations, which is the theory of 8 supercharges with bifundamentals coming from D3-D7 sectors. Related example of $\kappa$ invariant action of membrane of M-theory with boundaries was constructed by [18] where in the bulk we have 32 supersymmetries before the static gauge fixing while in the boundaries we have 16 supersymmetries. Upon the double dimensional reduction we obtain the $\kappa$ invariant action of the heterotic string with 16 supersymmetries before the gauge fixing [6]. This is S-dual to D1-D9 configuration which is T-dual to D3-D7 configuration considered later. In this example, one can check explicitly that the above line of argument is correct. With the above arguments, we regard $S_6^+ \otimes S_4^+$ as D3 brane worldvolume spinors.

Using the identification between spinors and forms on a Kahler manifold [10], $S_D^+$ is decomposed as

$$S_D^+ \simeq K_D^{i/2} \otimes (\Omega^{(0,0)} \oplus \Omega^{(0,2)})$$
\[ S_D^{-} \simeq K_D^{1/2} \otimes \Omega^{(0,1)} \]  

where \( K_D \) is the canonical bundle of the four-cycle \( D \). Using the adjunction formula for the normal bundle \( N \) in a Calabi-Yau manifold, \( N = K_D \) we have

\[ S_N^+ \simeq K_D^{1/2}, \quad S_N^- \simeq K_D^{-1/2} \]  

so that

\[ S_D^+ \otimes S_N^+ \simeq (K_D^{1/2} \otimes (\Omega^{(0,0)} \oplus \Omega^{(0,2)})) \otimes K_D^{1/2} \]  
\[ S_D^- \otimes S_N^- \simeq (K_D^{1/2} \otimes \Omega^{(0,1)}) \otimes K_D^{-1/2} = \Omega^{(0,1)} \]  

here we denote the \( U(1) \) charge of the \( S_N^\pm \) as a subscript where \( U(1) \) is the rotation group of the normal direction to the 4-cycle. We have

\[ S_6^+ \simeq (K_D \otimes \Omega^{(0,0)}) \oplus (K_D \otimes \Omega^{(0,2)}) \oplus \Omega^{(0,1)} \]  
\[ S_6^- \simeq \Omega^{(0,0)} \oplus \Omega^{(0,2)} \oplus (K_D \otimes \Omega^{(0,1)}) \]  

We introduce the complex coordinates \( z^a, \bar{z}^a \) for the 4-cycle and \( z, \bar{z} \) for the normal direction to the 4-cycle in the Calabi-Yau manifold. The Dirac equation \( D : S_6^+ \rightarrow S_6^- \) is of the form

\[ (\gamma^a \nabla_a + \bar{\gamma}^\bar{a} \nabla_{\bar{a}})\theta = 0 \]  

We will use the standard arguments about spinors on Kahler manifolds to make the identification between spinors and forms explicit [13]. To this end, we let the gamma matrices act as

\[ \Gamma^{\bar{a}} = dz^\bar{a} \wedge, \quad \Gamma^a = g^{ab} i_b \]  

where \( i_b \) denotes a contraction on the differential forms and we define a Clifford vacuum as a state satisfying

\[ \Gamma^z | \Omega > = 0, \quad \Gamma^a | \Omega > = 0. \]  

In this formalism elements of \( S_6^+ \) can be written as

\[ (\phi_{\bar{z}} \Gamma^\bar{z} + \phi_{\bar{z}\bar{a}} \Gamma^{\bar{z}\bar{a}} + \phi_a \Gamma^a)| \Omega > \]  

The resulting Dirac equation is

\[ \partial^b \phi_{\bar{b}} = 0 \]  
\[ \partial_{[a} \phi_{b]} = 0 \]  
\[ \partial^A \phi_{\bar{z}} + 2 \partial^A \phi_{\bar{z}\bar{a}} = 0 \]
On forms which has $\bar{z}$ index, we use a covariant derivative $\partial^A \equiv \partial + A$ rather than the usual derivative. Without the presence of the flux, the forms appearing in the above expression are harmonic so the number of zero modes of the Dirac operator $D : S^+_6 \rightarrow S^-_6$ is given by the arithmetic genus $\chi_D \equiv h^{0,0} - h^{0,1} + h^{0,2}$ where the zero modes are counted as positive for the positive chirality spinor on the D3 brane and as negative for the negative chirality spinor. Since $S^+_4$ is of rank two, if the index $D$ is equal to one, we have two fermion zero modes, which is the condition required for the nonzero superpotential contribution. Note that this counting is related to $U(1)$ anomaly of the rotation in the transverse direction where $U(1)$ charge of $S^+_2$ is $1/2$ while that of $S^-_2$ is $-1/2$. The $U(1)$ anomaly is the same as $\chi_D$. The anomaly of the $U(1)$ symmetry plays the crucial role in Witten’s work on the nonperturbative superpotentials due to M5 branes\cite{16}. The above mode counting is done on the Calabi-Yau manifold before taking orientifolding action. If we consider the orientifold we should consider the further projection due to the orientifold to make it sure that the zero modes are surviving from the orientifold projections. We take the procedure to consider the cycles of the Calabi-Yau manifold before the orientifold action and consider the further restrictions coming from the orientifolding action. The simple case would be to consider D3 brane on four-cycles which are not fixed by the orientifold actions. It is argued in \cite{8}, if M5-brane wrapping a cycle $D \rightarrow S$ which are fibrations over $S$ with $P^1$ fiber, the arithmetic genus of $D$

$$\chi(D) = \sum_{i=0}^{3} (-1)^{i} h^{0,i}(D) = h^{0,0}(S) - h^{0,1}(S) + h^{0,2}(S)$$

(2.14)

using the Leray spectral sequence. Since we expect M5 brane wrapping on $P^1$ is mapped to D3 brane, M5-brane zero mode counting is consistent with that of D3 brane.\footnote{It’s known that M5 brane wrapping on $K3$ is dual to heterotic string or to Type I D1 string\cite{23}. If we consider the elliptic $K3$ and if we T-dualize fiberwise along the elliptic fiber, we obtain D3 brane in the dual side. For generic fiber, this holds. Bad fibers of elliptic fibration give rise to the D3-D7 sectors.}

The above zero mode analysis is carried out for a single D3 brane wrapping on a 4-cycle in a Calabi-Yau manifold. Here we make an important assumption that there are no massless D3-D7 sectors. However at the special point of moduli space we can have massless D3-D7 sectors. Once the Kaluza-Klein reduction is carried out, D7 branes and D3 branes are turning into D3 branes and D(-1) branes respectively from the 3+1 dimensional point of view. Thus D3-D7 brane configuration is reduced to the small instanton configuration along 0, 1, 2, 3 directions. Thus we have to consider a small instanton in $R^4$ with the assumption that the usual Kaluza-Klein reduction being valid. The additional moduli space we should integrate over is just the instanton moduli space of the super Yang-Mills theory obtained from the dimensional reduction of D7 branes wrapping the 4-cycle. Here the gauge coupling is related to the volume of the 4-cycle. In \cite{17}, the dimension of the instanton moduli space is counted as the independent of the number of hypermultiplets of D5 brane in the presence of D9 branes, which are T-dual to our
D3-D7 configurations. Let us denote the gauge group arising from D9 branes by $SO(N)$ and that from D5 branes by $Sp(k)$ then we have one hypermultiplet transforming $(N, 2k)$ arising from D5-D9 sectors while we have one hypermultiplet transforming as antisymmetric representation of $Sp(k)$ arising from D5-D5 sectors. When $k = 1$ this antisymmetric representation is nothing but the ordinary scalars representing the fluctuations of the D5-brane position. The dimension of the instanton moduli space is given by

$$4Nk + 4 \frac{2k(2k - 1)}{2} - 4 \frac{2k(2k + 1)}{2} = 4Nk - 8k = 4k(N - 2)$$

Related to the fermion zero mode counting, the important thing is that we have additional D-term constraints whose number is the same as the adjoint of the D5 brane gauge group. When we consider the case with D9 brane gauge group being $U(N)$ and D5 brane gauge group being $U(k)$ the dimension of the instanton moduli space is given by $4Nk + k^2 - 4k^2 = 4Nk$ where the first factor is the contribution from the D5-D9 sectors, the second factor is that from the D5-D5 sectors while the last one comes from the D-term constraints. When we consider the special case of D9-brane gauge group being $U(1)$, the dimension of the moduli space is $4k$ where the $4k$ can be regarded as the position of $k$ D5 branes transverse to its worldvolume but along the D9 brane world volume direction. Translated to D3-D7 configurations of our interest, these $4k$ hypermultiplets denote the position of D3 branes along 0, 1, 2, 3 directions.

Now we should consider the fermion zero modes. According to the field theory result, the fermion zero modes are $4Nk$ if we have $N = 2$ supersymmetry in the four-dimensions, while those are $2Nk$ if we have $N = 1$ supersymmetry for pure supersymmetric gauge theory without matter\[21\]. Especially for $U(1) \times U(1)$, i.e., the abelian D7-brane gauge group and a single D3 brane wrapping around a 4-cycle, we have 2 fermion zero modes, which is needed for a nonzero superpotential. Here we see that we can have the nontrivial superpotential from an abelian instanton configuration. Note that while two fermion zero modes follow from the previous analysis of D3-D3 sectors if D3-D7 sectors are massive, the counting of the zero modes with the massless D3-D7 sectors is the result of the D3-D3 sectors and D3-D7 sectors combined with the D-term constraints. Also note that in this configuration the analysis of the multiple D3 instantons is reduced to the multi instanton calculus in the field theory. This would be an interesting topic to pursue further. Note that if we have $SU(N)$ gauge group from the D7-branes and a single D3 brane the number of fermion zero mode is $2N$, which agrees with the number of fermion zero modes of one instanton for $SU(N)$ supersymmetric gauge theory.

If we consider a simple case of a rigid 4-cycle with $h^{(1,0)} = 0$ within the Calabi-Yau, then we just have 4 translational bosonic zero modes along $x^0$ to $x^3$ directions and 2 fermion zero modes, which are the Goldstone fermion zero modes associated with the breaking of the supersymmetry
in the presence of the D3 instanton. The superpotential expression is given by

\[ W = \exp(-A + i \int_D C') \frac{\text{Pf} \frac{D_F}{\sqrt{\det D_B}}}{\sqrt{\det D_B}} \] (2.16)

if the D3-D7 sectors are massive while the integration of the instanton moduli space should be considered if D3-D7 sectors are massless. Here \( A \) denotes the volume of the 4-cycle \( D \) and \( C \) is a RR 4-form potential, the prime means that we are considering the determinant factors for nonzero modes and \( \sqrt{\det D_B} \) comes from the one-loop determinant of the bosonic modes and \( \text{Pf} \frac{D_F}{\sqrt{\det D_B}} \) comes from that of the fermionic modes of the D3 branes. The exponential terms come from the classical instanton action, while the other factors represent the one-loop integral over the quantum fluctuations around the classical instanton solutions. The superpotential generated by the instanton, apart from the usual exponential terms, is independent of the Kahler class of the Calabi-Yau manifold \( M \) and so can be computed by scaling up the metric of \( M \). The one-loop determinant is invariant under the scaling while higher loop corrections to the worldvolume computation would be proportional to the inverse power of the Kahler class and so vanish by holomorphy. The one-loop approximation to the superpotential is exact [16].

Let’s now turn to the second possibility. We assume that D7 brane worldvolume direction spans \( x^0 \) to \( x^7 \) while D3 brane worldvolume spans \( x^4, x^5, x^8, x^9 \) directions. From D7 branes we have the unbroken supersymmetry

\[ \epsilon_L = \Gamma_8 \Gamma_9 \epsilon_R \] (2.17)

while from the D3 brane we have the condition

\[ \epsilon_L = \Gamma_{0123} \Gamma_{67} \epsilon_R = \Gamma_{4589} \epsilon_R. \] (2.18)

And from these we obtain \( \epsilon_L = \Gamma_{45} \epsilon_L \). Note that along the intersection of D7 and D3 we have a two-dimensional worldsheet and the supersymmetry above represents the surviving supersymmetry on this two-dimensional worldsheet. Note that this represents a chiral theory in two dimensions. This is consistent with the fact that upon the T-dualities the D7-D3 configurations are mapped to D1-D9 configurations, which is D1-string configuration in Type I theory, S-dual to the heterotic string. Since the surviving supersymmetry is different from the previous one, the zero mode analysis is also different. Again we decompose the spinors \( S^+_{10} = (S^+_6 \otimes S^+_4) \oplus (S^-_6 \otimes S^-_4) \).

Now let \( N \) be the normal direction of D3 brane so that

\[ S^+_6 \simeq (S^+_N \otimes S^+_D) \oplus (S^-_N \otimes S^-_D) \]
\[ S^-_6 \simeq (S^+_N \otimes S^-_D) \oplus (S^-_N \otimes S^+_D) \] (2.19)

Now decompose the spinors on the D3 brane world-volume along the two-dimensional worldsheet and its normal direction

\[ S^+_D \simeq (S^+_2 \otimes S^+_N) \oplus (S^-_2 \otimes S^-_N) \]
\[ S^-_D \simeq (S^+_2 \otimes S^-_N) \oplus (S^-_2 \otimes S^+_N) \] (2.20)

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where we define the spinors $S_{N*}$ associated with the holomorphic conormal bundle to the two-dimensional worldsheet $C$ so that $S_{N*}^+ \sim (N^*)^{1\over 2}$, $S_{N*}^- \sim (N^*)^{-1\over 2}$ and $S_2$ be the spinors associated with the canonical bundle on the two-dimensional worldsheet so that $S_2^+ \sim K_2^{1\over 2}$, $S_2^- \sim K_2^{-1\over 2}$. This convention is consistent with the identification of the spinors with the antiholomorphic forms tensored with the square root of the canonical bundle, eq. (2.23). Note that

$$S_2^+ \oplus S_2^- \simeq K_2^{1\over 2} \oplus K_2^{-1\over 2} \simeq K_2^{1\over 2} \otimes (\Omega^{(0,0)}(C) \oplus \Omega^{(0,1)}(C))$$

(2.21)

and similarly

$$S_D \simeq (S_2^+ \oplus S_2^-) \otimes (S_{N*}^+ \oplus S_{N*}^-)$$

$$\simeq (K_2^{1\over 2} \oplus K_2^{-1\over 2}) \otimes (\tilde{N}^{1\over 2} \oplus \tilde{N}^{-1\over 2})$$

$$\simeq K_2^{1\over 2} \otimes \tilde{N}^{1\over 2} \otimes (1 \oplus K_2^{-1} \oplus \tilde{N}^{-1} \otimes (K_2 \otimes \tilde{N})^{-1})$$

$$\simeq K_2^{1\over 2} \otimes (\Omega^{(0,0)}(D) \oplus \Omega^{(0,1)}(D) \oplus \Omega^{(0,2)}(D))$$

(2.22)

If we decompose the 16 component spinors $S_{10}^+$ in terms of $S_2, S_{N*}, S_N$ the surviving 8 components are given by

$$S_N^+ \otimes S_2^+ \otimes S_{N*}^+ \otimes S_4^+ \subset S_6^+ \otimes S_4^+$$

$$S_N^- \otimes S_2^+ \otimes S_{N*}^- \otimes S_4^+ \subset S_6^+ \otimes S_4^-$$

$$S_N^+ \otimes S_2^- \otimes S_{N*}^+ \otimes S_4^- \subset S_6^- \otimes S_4^-$$

$$S_N^- \otimes S_2^- \otimes S_{N*}^+ \otimes S_4^- \subset S_6^- \otimes S_4^-$$

(2.23)

Depending on the embedded geometry of the 2d worldsheet in the Calabi-Yau one can have different number of zero modes. If we choose the 2d worldsheet to be a rigid $P^1$, then the normal bundle over $P^1$ is described by $O(-1) \oplus O(-1)$ bundles over $P^1$, where $O(n)$ is a holomorphic line bundle whose sections are functions homogeneous of degree $n$ in the homogeneous coordinates of $P^1$. Then $S_2^+ \simeq O(-1), S_2^- \simeq O(1), S_{N*}^+ \simeq O(1/2), S_{N*}^- \simeq O(-1/2)$ and $S_N^+ \simeq O(-1/2), S_N^- \simeq O(1/2)^3$ one can see that from eq. (2.23) only

$$S_N^- \otimes S_2^+ \otimes S_{N*}^+ \otimes S_4^- \simeq O \oplus O$$

(2.24)

contributes to the zero modes and $S_N^- \otimes S_2^+ \otimes S_{N*}^+ \simeq \Omega^{(0,0)}(D)$ if we use the identification eq. (2.23).

The D3-D7 sector represents the chiral current algebra of $SO(32)$ and can be represented as left-moving fermions. The action is given by

$$S_L = \int C \, d^2 \sigma \sqrt{g} \bar{\Psi}^a \gamma^i (D_i \delta^{ab} - A_i^{ab}) \Psi^b$$

(2.25)

$^3$Since we are considering the tensor products of the spinors, all of the relevant expressions are well defined
where \( a, b \) denote \( SO(32) \) indices and \( A_i = A_\mu \frac{\partial X^\mu}{\partial \sigma^i} \) is a pullback of the spacetime field \( A_\mu \). If we consider the D1-D9 configurations, \( A_\mu \) are simply \( SO(32) \) bundle configuration. In the case of D3-D7 configurations \( A_8, A_9 \) are position moduli of D7 branes while the other components represent bundle configurations. Upon pullback to the worldsheet, the combination of position moduli and the bundle moduli induces a background gauge field on the world-sheet. Let \( V \) denote such induced field. Let us denote the left-handed spin bundles of the 2d-worldsheet \( C \) by \( S_- \). In a suitable complex structure the kinetic operator for a left moving fermion is a \( \bar{\partial} \) operator. The left moving fermions are a section of \( S_- \otimes V \). The superpotential expression for the D3 brane wrapping a rigid cycle is

\[
W = \exp(-A + i \int_D C) \frac{\text{Pf}f^D F}{\sqrt{\text{det} D B}} \text{Pf}f(\bar{\partial} S_- \otimes V) \tag{2.26}
\]

If we consider the the 2-d worldsheet of D3-D7 sectors is \( P^1 \), it is analyzed that the fermion determinant is nonzero only if the bundle restricted on \( P^1 \) is trivial. Thus in addition to the usual zero mode analysis concerning on the D3-D3 sectors, we should check D3-D7 sectors give rise to nontrivial fermion determinant. This could be a severe restriction for the generic \( SO(32) \) bundle configurations. Such nontrivial examples were worked out in the heterotic M-theory setting in [25]. In a simple example of \( K3 \times T^2 \), the D9 brane configurations are simply \( SO(32) \) instanton bundles along \( K3 \) and Wilson lines along \( T^2 \). T-dualizing to D7 branes wrapping on \( K3 \), we have a collection of D7 branes located at various points on \( T^2 \) with instanton bundles along \( K3 \). For each D7 brane group located at a separate point on \( T^2 \) we have the pullback of bundles of \( K3 \) into the two-dimensional worldsheet. The Pfaffian on the worldsheet is the product of all such contributions. This Pfaffian factor gives rise to the additional dependence of the superpotential on the vector bundle moduli in the Type I theory[25]. Translated to our case, this implies the dependence of the superpotential on the position and shape moduli of D7 branes as well as the bundle moduli on the D7 branes. It would be interesting to find nontrivial examples where explicit dependences could be exhibited.

## 3 D3 instanton effects in the presence of the flux

Now we consider the D3 instanton effects for a Calabi-Yau orientifold in the presence of the fluxes. In [11], fermion mass term is derived in the presence of the flux using the D3-brane action with \( \kappa \) symmetry and taking the static gauge consistent with our cases. This is needed in the evaluation of the one loop determinant appearing in the superpotential expression. D-brane actions in a general bosonic backgrounds in component form are considered by [12], for example. The result is that for the D3 brane wrapping a four-cycle, the quadratic action \( S_2 = S_k + S_{\text{mass}} \)
with

\[ S_k = -\mu_3 \int d^4x \sqrt{|g|} \left( e^{-\phi} \tilde{\Theta} \Gamma_k D^k \Theta \right) \]

\[ S_{\text{mass}} = -\mu_3 \int d^4x \sqrt{|g|} \Theta \left( e^{-\phi} \frac{1}{48} \Gamma^{mn} H_{mn} - \frac{1}{16} e^{-\phi} \Gamma_{pq} H^{pq} \right. \]

\[ \left. - \frac{1}{32} \epsilon^{|ijk|} \Gamma_{ij} F_{kklp} \right) \Theta \]  

(3.1)

Here we assume that the fermion mass terms arise due to the 3-form fluxes and there are no \( F - B \) terms in the D3 brane worldvolume. And \( H \) denotes NS-NS 3-form, \( F' \) denotes RR 3-form and \( i,j,k,l \) are along the worldvolume while \( m,n,p \) takes 0 to 9 in spacetime. It’s easy to see that if \( H, F' \) have two legs along the brane and one leg along the normal direction then

\[ S_{\text{mass}} = \mu_3 \int d^4x \sqrt{|g|} \Theta \left( -\frac{1}{16} e^{-\phi} \Gamma_{ij} H^{ij} - \frac{1}{32} \epsilon^{|ijkl|} \Gamma_{ij} F_{kklp} \right) \Theta \]

(3.2)

where the Hodge star is defined for the D3 worldvolume. The terms appearing in eq. (3.2) breaks the U(1) symmetry in the normal direction allowing in particular two fermions with opposite sign charge to pair up and get heavy, which suggests that the zero mode analysis could be different in the presence of the fluxes. We define \( G \equiv F' - ie^\phi H \) for later purposes.

As an example of the Calabi-Yau orientifold with the fluxes, we consider a simple example, IIB orientifold on \( K3 \times T^2/Z_2 \). According to Sen\cite{19}, F-theory on \( K3 \) is equivalent to IIB orientifold on \( T^2/Z_2 \) with the orientifold action \( \Omega R_{89}(-1)^F \). Thus IIB orientifold on \( K3 \times T^2/Z_2 \) is dual to F-theory on \( K3 \times K3 \). Closely related theory, the M5 brane instanton on M theory on \( K3 \times K3 \) was discussed in \cite{13} \cite{5}. Here we consider a simple orientifold which is dual to F-theory on \( K3 \times K3 \) with flux \( G_4 = \Omega_1 \wedge \Omega_2 + \Omega_2 \wedge \Omega_3 \) where \( \Omega_1, \Omega_2 \) are holomorphic two forms of \( K3 \)s. This is known to have N=1 supersymmetry\cite{8} \cite{20}. The relation between the 4-form in F-theory is given by

\[ G_4 = -\frac{1}{\phi - \bar{\phi}} G_3 \wedge dz' + \frac{1}{\phi - \bar{\phi}} \bar{G}_3 \wedge d\bar{z}' \]  

(3.3)

where \( z' \) is the elliptic fiber direction of the F-theory. Thus \( G_3 \) on the orientifold \( K3 \times T^2/Z_2 \) is given by

\[ \Omega \wedge d\bar{z} \]  

(3.4)

where \( \Omega \) is the holomorphic two-form on \( K3 \) and \( z \) is a holomorphic coordinate of \( T^2 \). In a local coordinates \( G_3 \) has a nontrivial component \( G_{\text{z\bar{z}}} \) and \( G_{\text{z\bar{z}}} \) where \( z \) is a local holomorphic coordinate on \( T^2 \). For simplicity we consider the case with the constant dilaton. This occurs if the tadpole cancellation occurs locally. The resulting gauge group is \( SO(8)^4 \). If we consider the D3 instantons, there are two types of four-cycles we can consider. The fist one is the D3-brane wrapping on \( K3 \). And the second case is D3-brane wrapping on \( P^1 \times T^2/Z_2 \) where \( P^1 \) is a holomorphic curve in \( K3 \).
3.1 First case: D3 brane wrapping on K3

We introduce the complex coordinates $z^a, \bar{z}^{\bar{a}}$ for the K3 and $z, \bar{z}$ for the normal directions to K3 in the Calabi-Yau manifold. The Dirac equation $D : S_6^+ \to S_6^-$ is of the form

$$(\gamma^a \nabla_a + \gamma^{\bar{a}} \nabla_{\bar{a}} + G_{ab\bar{z}} \Gamma^{ab\bar{z}} + G_{\bar{a}b\bar{z}} \Gamma^{\bar{a}b\bar{z}}) \theta = 0 \quad (3.5)$$

after absorbing constants into the definition of $G$ in eq.(3.2). With the same identification between spinors and forms, the resulting Dirac equation is

$$\partial_{\bar{b}} \phi_{\bar{b}} = 0 \quad (3.6)$$
$$\partial_{[a} \phi_{\bar{b}]} + G_{[a\bar{b}]z} \phi^{\bar{z}} = 0 \quad (3.7)$$
$$\partial_{\bar{a}} \phi_{z} + 2 \partial^{[A} \phi_{z\bar{b}a]} = 0 \quad (3.8)$$

Without the presence of the flux, the index $D$ on the D3 brane wrapping on K3 is given by $h^{0,0} - h^{0,1} + h^{0,2} = 2$, which would not contribute to the superpotential. However the presence of the flux changes the zero mode analysis. Here we can use the similar trick to [13]. We introduce the projector $H$ onto harmonic forms so that $H(\omega)$ is a harmonic form (possibly zero) for any form $\omega$. The projector gives zero on any exact or co-exact form

$$H(\bar{\partial} \omega) = 0, \quad H(\bar{\partial}^\dagger \omega) = 0 \quad \forall \omega. \quad (3.9)$$

By acting $H$ on eq. (3.7) we have

$$H(G_{\bar{a}b\bar{z}} \phi^{\bar{z}} dz^{\bar{a}} \wedge dz^b) = 0. \quad (3.10)$$

These are $h^{2,0}$ equations for $h^{2,0}$ variables. One expects that generically $\phi^{\bar{z}}$ satisfying this condition is trivial. For a nonzero mode, one uses the formula $1 - H = \Delta G$ where $\Delta$ is the Laplacian and $G$ is the Green function of the corresponding Laplacian

$$(1 - H)(\omega) = (\bar{\partial} \bar{\partial}^\dagger + \bar{\partial}^\dagger \bar{\partial}) G \omega \quad \forall \omega \quad (3.11)$$

to obtain from the eq. (3.7)

$$(1 - H)G_{\bar{a}b\bar{z}} \phi^{\bar{z}} dz^{\bar{a}} \wedge dz^b = G_{\bar{a}b\bar{z}} \phi^{\bar{z}} dz^{\bar{a}} \wedge dz^b \quad (3.12)$$
$$= (\bar{\partial} \bar{\partial}^\dagger + \bar{\partial}^\dagger \bar{\partial}) GG_{\bar{a}b\bar{z}} \phi^{\bar{z}} dz^{\bar{a}} \wedge dz^b \quad (3.13)$$
$$= -\partial_{\bar{a}} \phi_{\bar{b}} dz^{\bar{a}} \wedge dz^b \quad (3.14)$$

From this we obtain one special solution

$$\phi_{\bar{b}0} = -2g^{\bar{a}a} \partial_{\bar{a}}(GG_{\bar{a}b\bar{z}} \phi^{\bar{z}}). \quad (3.15)$$
Now one can add $h^{(1,0)}$ zero modes to this special solution. Since the equation governing $h^{(0,0)}$ is not changed, the number of zero modes are given by $h^{(0,0)} - h^{(1,0)} + n$ with $n$ being the dimension of the solution space satisfying (3.10). In particular, if $G = \Omega \wedge d\bar{z}$, since $\phi^z = g^{z\bar{z}} \phi_{\bar{z}}$ is an element of $K \otimes \Omega^{(0,0)} \equiv \Omega^{(2,0)}$ this is just an element of 1-dimensional vector space. Multiplied by $\Omega$, $G_{\bar{a}b\bar{z}} \phi^{\bar{a}} dz^\bar{b} \wedge dz^\bar{\bar{b}}$ is proportional to the contraction of $\Omega$ and $\bar{\Omega}$. Thus $H(G_{\bar{a}b\bar{z}} \phi^{\bar{a}} dz^\bar{b} \wedge dz^\bar{\bar{b}})$ is nonzero unless $\phi^z$ is nonzero. Hence the number of zero modes $h^{(0,0)} - h^{(1,0)} + n = 1$ so that this could contribute to the superpotential. However we should make it sure that additional D3-D7 sector contribution does not give the null result. In order for a single D3-brane to contribute to the superpotential, we should have either massive D3-D7 sectors or there should be an abelian configuration of D7. In the spacetime approach of the evaluation of the superpotential, the D3 brane configuration also satisfies the classical equation of motion[4]. In the presence of the flux, D3 brane feels the same potential as the D7 brane along the Calabi-Yau manifold. Thus the D3 brane could be located only at the minimum of the potential. It looks rather difficult to analyze the abelian D7 brane configuration in the presence of the flux since this generally have the dilaton gradient. Anyway if such configuration exists, we can put a single D3 brane at the single D7 brane. This would give rise to the nontrivial superpotential. It would be worthwhile to look for such configurations. However in the case at hand, we know for a field theory point of view that there are nontrivial superpotentials. In the constant dilaton configuration with $SO(8)^4$ gauge group, the gauge theory on the four groups of D7 branes are just $N=1$ supersymmetric gauge theory without additional supersymmetric matter. In this case we know there are superpotential due to the gaugino condensation.  

Note that in terms of geometric engineering, this provides an interesting test bed. Since the effect of the flux is to give the mass to the adjoint matter this system is closely related example as analyzed in[24]. Turning things around, the field theory analysis gives the expression for the superpotential for the flux where the adjoint mass is proportional to the flux strength. Since the measures of the multi instanton moduli spaces for supersymmetric gauge theories were worked out[21][22], this is a good starting point to work out the multi-instanton effect of D3 branes. Here the multi instantons are represented by nonabelian configurations of D3 branes. It would be interesting to explicitly work out these.

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\(^4\)If four-dimensional theory is compactified on a circle $S^1$, the superpotential is due to the magnetic monopoles. The superpotential does not depend on the radius of the circle and we can take the decompactification limit. If we realize the four-dimensional theory by D3 brane, the magnetic monopole in three-dimension is represented as D0 brane stretching between the D2 branes which are separated on a dual circle $\tilde{S}^1$ upon T-dualizing the D3 branes with Wilson lines.
3.2 Second case: D3 brane wrapping on $P^1 \times T^2/Z_2$

In this case massless modes of D7-D3 sectors always exist. Without the flux, D3 brane wrapping on $P^1 \times T^2/Z_2$ is T-dual to D1 brane wrapping on $P^1$ and $T^2$ is transverse to D1 brane. The transverse geometry to $P^1$ of D1 worldvolume is $O \oplus O(-2)$ in the large volume limit where $O$ denotes the $T^2$ directions. Without the flux this geometry has additional zero modes than is needed for nonzero superpotential. The same is true of D3 brane and one should be careful in dealing with $Z_2$ action acting on $T^2$, which is not the usual geometric action but the combined action of the orientifold action and the geometric action. Since $P^1 \times T^2$ is mapped to itself under the orientifold action, the gauge group of U(N) of D3 branes are reduced to $SO(N)$.

The worldsheet degrees of freedom remain the same so the zero mode structure is the same as before the orientifolding. Here we have

$$S^+_N \otimes S^+_2 \otimes S^-_{N*} \otimes S^-_4 \simeq O \oplus O \subset S^+_6 \otimes S^+_4$$

$$S^-_N \otimes S^+_2 \otimes S^+_N \otimes S^-_4 \simeq O \oplus O \subset S^-_6 \otimes S^-_4$$

(3.16)

contribute to the zero modes. Without the presence of fluxes, this is more than needed for the nonzero superpotential. However in the presence of the flux some of the zero modes are removed so that it can contribute to the superpotential. Since $S^+_N \otimes S^+_2 \otimes S^-_{N*} \subset S^+_6$ and is an element of $\Omega^{(0,1)}(D)$ from the analysis of the Dirac equation $D: S^+_6 \rightarrow S^-_6$ one will see that $h^{(0,1)}$ is removed in the presence of the flux. If we denote the normal direction by a complex coordinate $y$ then the components of $G_3$ and $\bar{G}_3$ in the local coordinates can be written as $G_{yb^c}$ and $\bar{G}_{yb^c}$. Here $y, b$ span the K3 directions while $b, c$ span the D3 worldvolume and $c$ denotes the $T^2/Z_2$ direction. As in the previous case, we represent the spinor by forms. The Dirac equation for the D3 worldvolume fermion in the presence of the flux is given by

$$\partial^A \phi_b = 0 \quad (3.17)$$

$$\partial^A \phi_y + 2\partial^A (\phi_{yb^a}) + \bar{G}_{yb^c} \phi^{bc} = 0 \quad (3.18)$$

$$\partial_a \phi_b + G_{yc^a} \phi^{yc} - (a \leftrightarrow b) = 0 \quad (3.19)$$

Again adopt the Harmonic projector, then we have

$$H(\bar{G}_{yb^a} \phi^{c} d\bar{z} \wedge d\bar{a}) = 0 \quad (3.20)$$

and

$$H(G_{yc^a} \phi^{yc^c}) - (a \leftrightarrow b) = 0 \quad (3.21)$$

The first equation (3.20) is the map from $h^{(0,1)}$ to $h^{(0,1)}$ so generically remove all modes of $h^{(0,1)}$. The second equation is a map from $\Omega^{(0,0)}$ to $\Omega^{(0,2)}$. In the case of our interest, since $h^{(0,2)} = 0$. 

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this is satisfied automatically. For the first equation $\phi^c = g^{c\bar{c}}\phi_{\bar{c}}$ is an element of one dimensional vector space $\Omega^{(0,1)}$ and $\hat{G}_{\bar{g}\bar{a}} = \Omega_{g\bar{a}} \wedge dz^c$ is proportional to a nontrivial element of $\Omega^{(1,0)}$. Thus $\hat{G}_{\bar{g}\bar{a}}\phi^c$ is proportional to the contraction of the nontrivial elements of $\Omega^{(1,0)}$ and $\Omega^{(0,1)}$ if $\phi^c$ is nontrivial. Thus $\phi^c$ should vanish in order for the eq. (3.20) to be satisfied.

On the other hand one can check that $S_N^{-} \otimes S_2^+ \otimes S_8^+ \subset S_6^-$ is an element of $\Omega^{(0,0)}$ and by the similar analysis of the Dirac equation $D : S_6^- \to S_6^+$ one sees that $h^{(0,0)}$ is retained in the presence of the flux. Thus we are are left with the two fermion zero modes so that it can contribute to the superpotential. For $SO(8)^4$ configuration, for each group of D7 branes there are no nontrivial bundles so that the pullback is also trivial. Hence the Pfaffian factor gives the nonzero contribution. According to [14], the $K3$ manifold we consider is rather special one, so called attractive $K3$. The Picard number of this $K3$ is 20 so that it has 20 holomorphically embedded $P^1$s. In our orientifold we have 21 Kahler moduli. Since we also have 21 types of D3 instantons available and all of them are nonzero, we expect that all of the Kahler moduli would be fixed by the instanton effects as happened in M theory on $K3 \times K3$ [14]. One final remark should be added. The above zero mode analysis is carried out assuming a specific complex structures for $K3 \times T^2$. However in the presence of the fluxes, the actual complex structure is not necessarily the same complex structure as we assume. However the number of zero modes remains the same as we deform the complex structures. In the actual evaluation of the nonzero superpotential, different complex structures give different nonzero values.

4 D5 instanton in Type I theory

If we consider D5 brane wrapping on a Calabi-Yau, we expect the instanton effect $e^{-V+\cdots}$ where $V$ is the volume modulus of Calabi-Yau. We can proceed the analysis in the similar way to that of D3 instanton. For Type I, the unbroken supersymmetry is given by

$$\epsilon_L = \epsilon_R$$

and in the presence of D5 branes along Calabi-Yau we have

$$\epsilon_L = \Gamma_{456789}\epsilon_R.$$  (4.2)

Thus we have

$$\epsilon_L = \Gamma_{456789}\epsilon_L.$$  (4.3)

On the D5 brane world volume are there 8 component of spinors $S_6^+ \otimes S_4^+$. Since $S_6^+$ is the positive chirality spinor on the Calabi-Yau, we can identify them as

$$S_6^+ \simeq \Omega^{0,0} \oplus \Omega^{0,2}$$  (4.4)
since the canonical bundle of the Calabi-Yau is trivial. The number of zero modes of the Dirac operator $D: S^+_6 \to S^-_6$ are given by $h^{0,0} + h^{0,2}$. For a generic Calabi-Yau $h^{(0,0)} = 1$, $h^{(0,2)} = 0$ and combined with the two degrees of freedom $S^+_4$, we have two fermion zero modes from the D5-D5 sectors of a single D5-brane. Again we should consider the effect of the D5-D9 sectors. Upon the Kaluza-Klein reduction, this is again reduced to the small instanton configuration in the 4d theory. Following the same logic in the D3 brane analysis, if we have the $SO(32)$ bundle configuration which breaks $SO(32)$ into $U(1)$ so that we are left with an abelian gauge group, the number of zero modes coming from D5-D5 and D5-D9 sectors with D-term constraints are precisely two so that we can have non-zero superpotential. But since the Calabi-Yau volume plays the role of the inverse gauge coupling of D9 sectors compactified on a Calabi-Yau for a nonabelian gauge group we can have the superpotential for the Calabi-Yau volume moduli, which makes the gaugino condensation possible. This depends on the matter content of the four-dimensional theory realized in D9 brane configuration compactified on the Calabi-Yau. For a generic Calabi-Yau manifold, $h^{(0,1)} = h^{(0,2)} = 0$. D9 brane gives rise to pure supersymmetric gauge theory.

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References

[1] S. Kachru, R. Kallosh, A. Linde and S. Trivedi, “De Ditter vacua in string theory,” Phys. Rev. D68 (2003) 046005, [hep-th/0301240]

[2] S. Giddings, S. Kachru and J. Polchinski, “Hierarchies from fluxes in string compactifications,” Phys. Rev. D66 (2002) 106006, [hep-th/0105097]

[3] J. Harvey and G. Moore, “Superpotentials and membrane instantons,” [hep-th/9907026]

[4] E. Witten, “Worldsheet instanton corrections via D-instantons,” [hep-th/9907041]

[5] G. Moore and G. Peradze and N. Saulina, “Instabilities in heterotic M theory induced by open membrane instantons,” Nucl. Phys. B607 (2001) 117, [hep-th/0012104]

[6] E. Lima, B. Ovrut, J. Park and R. Reinbacher, “Nonperturbative superpotentials from membrane instantons in heterotic M theory,” Nucl. Phys. B614 (2001) 117, [hep-th/0101049]
[7] E. Lima, B. Ovrut and J. Park, “Five-brane superpotentials in heterotic M theory,” Nucl. Phys. B626 (2002) 113, hep-th/0102046

[8] L. Gorlich, S. Kachru, P. Tripathy and S. Trivedi, “Gaugino condensation and nonperturbative superpotentials in flux compactifications,” hep-th/0407130

[9] P. Berglund and P. Mayr, “Nonperturbative superpotentials in F-theory and string duality,” hep-th/0504058

[10] N. Saulina, “Topological constraints on stabilized flux vacua,” hep-th/0503125

[11] P. Tripathy and S. Trivedi, “D3 brane action and fermion zero modes in presence of background flux,” hep-th/0503072

[12] D. Marolf, L. Martucci and P. Silva, “Actions and fermionic symmetries for D-branes in bosonic backgrounds,” JHEP 0307 (2003) 019, hep-th/0306066. D. Marolf, L. Martucci and P. Silva, “Fermions, T duality and effective actions for D-branes in bosonic backgrounds,” JHEP 0304 (2003) 051, hep-th/0303209

[13] R. Kallosh and A. Kashani-Poor and A. Tomasiello, “Counting fermion zero modes on M5 with fluxes,” hep-th/0503138

[14] P. Aspinwall and R. Kallosh, “Fixing all moduli for M-theory on K3×K3,” hep-th/0506014

[15] E. Bergshoeff, R. Kallosh, A. Kashani-Poor, D. Sorokin and A. Tomasiello, “An index for the Dirac operator on D3 brane with background fluxes,” hep-th/0507069.

[16] E. Witten, “Non-perturbative superpotentials in string theory,” Nucl. Phys. B474 (1996) 343, hep-th/9604030.

[17] E. Witten, “Small instantons in string theory,” Nucl. Phys. B460 (1996) 541, hep-th/9511030.

[18] M. Cederwall, “Boundaries of eleven-dimensional membranes,” Mod. Phys. Lett. A12 (1997) 2641, hep-th/9704161

[19] A. Sen, “F theory and orientifolds,” Nucl. Phys. B475 (1996) 562, hep-th/9605150.

[20] P. Tripathy and S. Trivedi, “Compactification with flux on K3 and tori,” JHEP 0303 (2003) 028, hep-th/0301139.

[21] N. Dorey, T. Hollowood, V. Khoze and M. Mattis, “Supersymmetry and multi-instanton measure 2. From N=4 to N=0,” Nucl. Phys. B519 (1998) 470, hep-th/9709072.
[22] N. Dorey, T. Hollowood and V. Khoze, “The calculus of many instantons,” Phys. Rept. 371 (2002) 231, hep-th/0206063.

[23] S. Cherkis and J. Schwarz, “Wrapping the M theory five-brane on K3,” Phys. Lett. B403 (1997) 225, hep-th/9703062.

[24] N. Dorey, T. Hollowood and S. Kumar, “An exact elliptic superpotential for $N = 1$ deformations of finite $N=2$ gauge theories,” Nucl. Phys. B624 (2002) 95, hep-th/0108221.

[25] E. Buchbinder, R. Donagi and B. Ovrut, “Superpotentials for vector bundle moduli,” Nucl. Phys. B653 (2003) 400, hep-th/0205190. E. Buchbinder, R. Donagi and B. Ovrut, “Vector bundle moduli superpotentials in heterotic superstrings and M theory,” JHEP 0207 (2002) 066, hep-th/0206203.