Scalar Fields as Dark Matter in Spiral Galaxies

F. Siddhartha Guzmán* and Tonatiuh Matos†

Departamento de Física,
Centro de Investigación y de Estudios Avanzados del IPN,
AP 14-740, 07000 México D.F., MEXICO.

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We present a model for the dark matter in spiral galaxies, which is a result of a static and axial symmetric exact solution of the Einstein-Dilaton theory. We suppose that dark matter is a scalar field endowed with a scalar potential. We obtain that a) the effective energy density goes like \(1/(r^2 + r_c^2)\) and b) the resulting circular velocity profile of test particles is in good agreement with the observed one.

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One of the greatest puzzles of physics at the moment is without doubt the existence of dark matter in cosmos. The experimental fact that the galaxy masses measured with dynamical methods do not coincide with their luminous galaxy masses gives rise to the existence of a great amount of dark matter in galaxies, galaxy clusters and superclusters. At the present time, cosmological observations indicate that the universe is filled out with about 90 percent of dark matter, whose nature till now remains unexplained. Recently some authors have proposed the scalar field as a candidate for dark matter in cosmos, in some sense the inflationary cosmological model proposes the scalar field as cosmological dark matter as well. These models consider scalar-tensor theories of gravity where one is able to add mass terms to the total energy density of the space-time. All modern unifying field theories also contain scalar fields. For example, scalar fields are fundamental fields in Kaluza-Klein and Superstring theories, because such fields appear in a natural way after dimensional reduction. In both theories the scalar field could be endowed with an exponential scalar potential, in particular, when we deal with 5-dimensional Kaluza-Klein theories, the Lagrangian density reads \(L_5 = R_5 + \Lambda\) being \(\Lambda\) a 5-dimensional cosmological constant. After dimensional reduction and a conformal transformation one obtains the density \(L_4 = -R_4 + 2(\nabla \phi)^2 + e^{-2/\sqrt{3}\phi} \Lambda\), where \(\phi\) is the scalar field which actually states that an exponential potential appears in a natural way in this theory. An analogous procedure establishes that in the low energy limit of Superstring theory one gets a similar result. In general one obtains the Lagrangian from high-dimensional theories \(L_4 = -R_4 + 2(\nabla \Phi)^2 + e^{-2a\Phi} \Lambda\), therefore here we will restrict our selves to an exponential scalar potential. In this letter we show a possible model for the dark matter in spiral galaxies, supposing that such matter is of scalar nature.

There is a common approach to explain the rotation curves in spiral galaxies called Modified Newtonian Dynamics (MOND), which basically consists of modifying the Newton’s law of attraction for small accelerations by adding terms to the gravitational potential. In this way, by adjusting some free parameters for each galaxy, one can reproduce the asymptotic behavior of the rotation curves. However it appears to be artificial because it is nothing but a mere correction of Newton’s law, we are unable to know neither where the parameters and the correction terms come from, nor why nature behaves like that and therefore which is the Newton’s law at cosmological scale for instance.

A convincing phenomenological model for galactic dark matter is the called Isothermal Halo Model (IHM), which assumes the dark matter to be a self-gravitating ball of ideal gas (made of any kind of particles) at a uniform temperature \(kT = \frac{1}{2}m_{dm}v_c\), - being \(m_{dm}\) the mass of each particle and \(v_c\) its velocity - which eventually produces a dark matter distribution going as \(\sim 1/r^2\), implying in this way an increasing mass \(M(r) \sim r\). Then, by assuming that a galaxy is a system in equilibrium \((GM/r^2 = v_c^2/r)\) the velocity of particles surrounding the profile above should produce flat rotation curves into a region where the dark matter dominates, i.e. at large radii when one considers as exponential distribution of luminous matter as usual.

Observational data show that galaxies are composed by almost 90% of dark matter. This is so because the kinematics inside the dark matter dominated region

\*E-mail: siddh@fis.cinvestav.mx
\†E-mail: tmatus@fis.cinvestav.mx
is not consistent with the predictions of Newtonian theory, which explains well the dynamics of the luminous sector of the galaxy but predicts a keplerian falling off for the rotation curve. The region of the galaxy we are interested in is that in which the dark matter determines the kinematics of test particles. So we can suppose that luminous matter does not contribute in a very important way to the total energy density of the matter that determines the baryonic matter contribution to the total energy density for the explanation of assimptotic rotation curves.

On the other hand, the exact symmetry of the dark halo stills unknown, but it is reasonable to suppose that it is symmetric with respect to the rotation axis of the galaxy. In this letter we let the symmetry of the halo as being affected by the rotation of the galaxy and we can consider a time reversal symmetry of the space-time. So, the motion of test particles around the galaxy, dragging the baryonic matter contribution to the tests particles traveling around the galaxy. In this letter we let the symmetry of the halo as being symmetric with respect to the rotation axis of the galaxy. In this letter we let the symmetry of the halo as being symmetric with respect to the rotation axis of the galaxy. Hence, in the region of our interest we can suppose the space-time to be static, given that the circular velocity of stars (like the sun) of about 230 Km/s seems not to affect the tests particles (stars) traveling around the galaxy.

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For the above equations \[ \lambda = \frac{1}{2} \ln(M) + \ln(f_0) \]

\[ \Phi = \Phi_0 + \frac{1}{2} \sqrt{\frac{1}{\kappa_0 M}} \ln(M) \]

\[ V = \frac{4f_0}{\kappa_0 M} \]

\[ k = \frac{1}{2} \left( \ln(M) + \ln(M) \right) \]

where \( f_0 \) and \( \Phi_0 \) are integration constants and \( W = M \) is a function restricted only by the condition
whose solutions are $M = Z(z)Z(\bar{z})$, where $Z$ is an arbitrary function. The reader can check that (\ref{eq:9}) is a solution of the field equations substituting the set (\ref{eq:9}) into (\ref{eq:2}) using the metric (\ref{eq:2}).

In what follows we study the circular trajectories of a test particle on the equatorial plane taking the space-time (\ref{eq:2}) as the background. The motion equation of a test particle in the space-time (\ref{eq:2}) can be derived from the Lagrangian

$$L = \frac{1}{f} e^{2k} \left[ \left( \frac{d \rho}{d \tau} \right)^2 + \left( \frac{d \zeta}{d \tau} \right)^2 + W^2 \left( \frac{d \phi}{d \tau} \right)^2 \right] - f c^2 \left( \frac{d t}{d \tau} \right)^2.$$  \hspace{1cm} (11)

This Lagrangian contains two constants of motion, the angular momentum per unit of mass

$$\frac{W^2}{f} \frac{d \phi}{d \tau} = B,$$  \hspace{1cm} (12)

and the total energy per unit of mass of the test particle

$$f c^2 \frac{d t}{d \tau} = A,$$  \hspace{1cm} (13)

where $\tau$ is the proper time of the test particle. An observer falling freely into the galaxy, with coordinates $(\rho, \zeta, \phi, t)$, will have a line element given by

$$ds^2 = \left\{ \frac{1}{c^2} \left[ e^{2k}(\rho^2 + \zeta^2) + W^2 \dot{\phi}^2 \right] - f \right\} c^2 d\tau^2$$

$$= \left( \frac{v^2}{c^2} - f \right) c^2 d\tau^2$$

$$= -c^2 d\tau^2.$$ \hspace{1cm} (14)

The velocity $v^a = (\dot{\rho}, \dot{\zeta}, \dot{\phi})$, is the three-velocity of the test particle, where a dot means derivative with respect to $t$, the time measured by the free falling observer. The squared velocity $v^2$ is then

$$v^2 = g_{ab} v^a v^b = \frac{e^{2k}}{f} (\dot{\rho}^2 + \dot{\zeta}^2) + \frac{W^2}{f} \dot{\phi}^2,$$ \hspace{1cm} (15)

where $a, b = 1, 2, 3$. Using (\ref{eq:14}) into (\ref{eq:13}) we obtain an expression for the squared energy

$$A^2 = \frac{f^4 f^2}{f - \rho^2}.$$ \hspace{1cm} (16)

We are interested in test particles (stars) moving on the equatorial plane $\dot{\zeta} = 0$ and the equation of motion derived from the geodesics of metric (\ref{eq:2}) reads

$$\frac{1}{f} e^{2k} \left( \frac{d \rho}{d \tau} \right)^2 + \frac{B^2}{W^2} - \frac{A^2}{c^2 f} = -c^2.$$ \hspace{1cm} (17)

where we have used the conservation equations (\ref{eq:12}) and (\ref{eq:13}). Equation (\ref{eq:17}) determines the trajectory of a test particle around the equator of the galaxy, in this trajectory $A$ and $B$ remain constant. If we change of test particle, we could have another constants of motion $A$ and $B$ determining the trajectory of the new particle. A spiral galaxy is practically a disc of stars traveling around the equatorial plane of the galaxy in circular trajectories in the period of observation, although it had to be formed from enormous clouds of gas going around a symmetry axis with average values of $A$ and $B$. Thus for a circular trajectory $\dot{\rho} = 0$, the equation of motion transforms into

$$\frac{B^2}{W^2} - \frac{A^2}{c^2 f} = -c^2.$$ \hspace{1cm} (18)

This last equation determines the circular trajectories of test particles travelling on the equator of the galaxy. Using (\ref{eq:18}) and (\ref{eq:19}) we find an expression for $B$ in terms of $v^2$,

$$B^2 = \frac{v^2 W^2}{f - \rho^2}$$

$$\sim v^2 W^2 f^2,$$ \hspace{1cm} (19)

since $v^2 \ll c^2$. Now using (\ref{eq:14}) one concludes that for our solution (\ref{eq:19}) $v^2 = f_0^2 B^2$, i.e.

$$v_{DM} = f_0 B,$$ \hspace{1cm} (20)

where we call $v \to v_{DM}$ the contribution of our dark matter to the circular velocity of a star.

When $Z = z$ our solution in Boyer-Lindquist coordinates $\rho = \sqrt{r^2 - 2ar + b^2} \sin \theta$, $\zeta = (r - a) \cos \theta$ reads

$$ds^2 = \left( 1 - \frac{a^2}{r} \right) \left( 1 - \frac{a^2}{r} + \frac{b^2}{r^2} \right) + \frac{dr^2}{r^2} + r^2 d\theta^2$$

$$+ \frac{K^2 \sin^2 \theta}{\rho_0} d\phi^2$$

$$- f_0 c^2 \frac{(r - a)^2 + K^2 \sin^2 \theta}{\rho_0} dt^2$$ \hspace{1cm} (21)

where $K = b^2 - a^2$ and $\rho_0$ only scales. The effective energy density $\mu_{DM}$ of (\ref{eq:19}) is given by the expression

$$\mu_{DM} = \frac{1}{2} V(\Phi) = \frac{2 f \rho_0}{\kappa_0((r - a)^2 + K^2 \sin^2 \theta)}$$ \hspace{1cm} (22)
and plays the role of our dark matter density profile.

Keeping in mind that this is only the contribution of dark matter to the energy density we are in conditions to compare these results with those given by measurements. In order to do so we recall the paper by Begeman et al. \[6\] where an energy density profile of the IHM \(\mu(r) = \rho_0 r_0^2 / (r^2 + r_0^2)\) for dark matter is used, being \(r_c\) a core radius. It is evident that this profile is a particular case of the expression we present here, namely, for matter localized on the equator of the galaxy. So, we can fit some of the free parameters of metric \((21)\) comparing these two profiles. We set \(b = r_c\), \(a = 0\) and \(\frac{\rho_0 r_0^2}{\kappa_0} = \rho_0 r_c^2\).

Let us model the circular velocity profile due to the luminous matter of the disc in a spiral galaxy by the function

\[
v_L^2 = v^2(R_{opt}) \beta \frac{1.97 x^{1.22}}{(x^2 + 0.78^2)^{1.43}}
\]

(23)

which is the approximated model for the Universal Rotation Curves (URC) as was propossed by Persic et al. \[8\] for an exponential thin disc, valid for a sample of 967 galaxies; in this expression \(x = r / R_{opt}\), the parameter \(\beta = v_L(R_{opt}) / v(R_{opt})\) being \(R_{opt}\) the radius into which it is contained the 83% of the observable mass of the galaxy and \(v\) the observed circular velocity.

We can suppose that luminous matter near the center of a galaxy behaves like in Newtonian mechanics. Thus with the luminous velocity \((23)\) it is now easy to calculate the angular momentum (per unity of mass) of the test particle in the luminous matter dominated region

\[
B = v_L D,
\]

(24)

where \(D\) is the distance between the center of the galaxy and the test particle. For our metric, \(D = \int ds\), keeping \(\theta\), \(\phi\) and \(t\) constant, we obtain \(D = \sqrt{(r^2 - 2ar + b^2) / f_0 r_0}\). Observe that after we have determined the dark matter energy density \(\mu_{DM}\), \(B\) is uniquely determined by \(v_L\) via \((24)\); it is easy to show that \(B\) in \((24)\) equals that of equations \((17-20)\) by including a luminous newtonian component into the radial geodesic equation \[(13)\]. Therefore \((20)\) and \((24)\) imply the total circular velocity

\[
v_C = \sqrt{v_L^2 + v_{DM}^2} = v_L \sqrt{1 + f_0 (r^2 - 2ar + b^2)}
\]

(25)

expression that should fit the observed rotation curves. In order to do so, we present in Fig. 1 the plots containing the fittings of four spiral galaxies and into Table \[1\] the values of the parameters \(f_0\) and \(b\) keeping \(a = 0\) and the scale \(r_0 = 1\). From it we see that the agreement of the resulting circular velocity profiles given by the scalar field as dark matter and the observed is very good not only far away from the center of the galaxy but inside the part of the galaxy dominated by luminous matter as well.

The criterion used to choose the sample was the ratio of the dark to luminous mass inside \(r_{25}\), which was selected to be \(\sim 1\) in order to test our model by using "dark enough galaxies". The plots shown in Fig. 1 would not be enough to state that our model works, it is necessary to be consistent with the phenomenological URC approach, i.e. the contribution of our dark matter should be the same as that propossed into the URC frame which is strongly luminosity dependent. A formula consistent with \((23)\) is given by \[8\]

\[
v_{25}^2 = v^2(R_{opt})(1 - \beta)(1 + \gamma^2) \frac{x^2}{x^2 + \gamma^2}
\]

(26)

being \(\beta = 0.72 + 0.44 \log L / L_*\) the same parameter as in \((23)\) and \(\gamma = 1.5(L / L_*)^{1/5}\).

According to \(20\) and \(24\) the contribution of our dark matter is

\[
\nu_{DM}^2 = f_0 (r^2 + b^2) v^2(R_{opt}) \beta \frac{1.97 x^{1.22}}{(x^2 + 0.78^2)^{1.43}}
\]

(27)

which after using the fitting parameters of Table \[1\] the comparisson of both approaches are compared in Fig. 2 from which it can be concluded that our dark matter model respects the luminous matter model we have used.
FIG. 2. The contribution of dark matter to the circular velocity of test particles is shown. Solid lines emerge from the model described in this letter, dashed ones correspond to the URC approach. The discrepancies are of 8.3, 11, 23.3 and 5.2 percent respectively.

Some remarks can be drawn. The energy density (22) coincides with that required for a galaxy to explain the rotation curves of test particles in its halo, but in our model, this energy density is product of the scalar field and the scalar field potential, that is, this dark matter is produced by a $\Phi$ particle. So we have shown that there is an exact solution which describes the rotation curves of particles in a spiral galaxy. The crucial point for having the circular velocity $v_{DM}=f_0B$ is that $f_0 \sim W$ in the solution (9). But this fact remains unaltered after conformal transformations in the metric $\hat{d}s^2 = A(\Phi)ds^2$, so that the circular velocity $v_{DM}$ remains the same for all theories and frames related with metric (2) by conformal transformations.

How does our model looks like into the cosmological context? When a density profile for galactic dark matter goes as the inverse of $r^2$ and it is supposed that the halo of a galaxy ends in the region where those of neighboring galaxies start, the integrated amount of galactic dark matter is close to that needed for the Universe to be flat for the observed average distance between them, flatness inferred from the cosmic background radiation and thus permitting our model to be inside the bounds. In fact, we have developed a cosmological model that considers the same theory as here with the same scalar potential which has been able to explain the redshifts of Type Ia supernovae, and all the parameters for structure formation lie into the ranges imposed by observations which make us to put forward the model presented in this letter.

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| Galaxy    | $f_0$ (Kpc$^{-1}$) | $b$ (Kpc) | $R_{opt}$ (Kpc) | $\beta$  |
|-----------|------------------|---------|----------------|---------|
| NGC1560   | 0.0726           | 2.119   | 4.6            | 0.344   |
| NGC2403   | 0.0171           | 5.399   | 6.7            | 0.546   |
| NGC3198   | 0.0038           | 12.88   | 11             | 0.547   |
| NGC6503   | 0.0290           | 3.035   | 3.8            | 0.702   |

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