Tachyon Vortex *

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Abstract

The property and gravitational field of global string of tachyon matter are investigated in a four dimensional approximately cylindrically-symmetric spacetime with a deficit angle. Especially, we give an exact solution of the tachyon field in the flat spacetime background and we also find the solution of the metric in the linearized approximation of gravity.

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Various topological defects such as domain wall, string(vortex) and monopole could be formed by the symmetry-breaking phase transitions in the early universe and their existence has important implications in cosmology\[1, 2\]. The symmetry breaking model of ordinary scalar field\[3\] can be prototypically written as

\[
L = \frac{1}{2} \partial_\mu \phi^a \partial^\mu \phi^a - V(f)
\]

where \( \phi^a \) is a set of scalar fields, \( a = 1, ..., N, \) \( f = (\phi^a \phi^a)^{\frac{1}{2}} \). The model has O(N) symmetry and admits domain wall, string and monopole solutions for \( N = 1, 2 \) and \( 3 \), respectively. Usually, the potential \( V(f) \) has a minimum at a finite non-zero value of \( f \). On the other hand, in Ref.\[4, 5\], Cho and Vilenkin investigated the defects in models where \( V(f) \) has a local maximum at \( f = 0 \) but no minima; instead, it monotonically decrease to zero at \( f \to \infty \). And they called this kind of defects ”vacuumless defects”. The main gravitational property of all the defects mentioned above is the divergent mass (or mass density), which leads to an effect of deficit angle and negative mass (or mass density)\[6, 7\].

Recently, pioneered by Sen\[8\], the study of non-BPS objects such as non-BPS branes, brane-antibrane configurations or space-like branes\[9\] has been attracting physical interests in string theory. Sen showed that classical decay of unstable D-brane in string theories produces pressureless gas with non-zero energy density. The basic idea is that the usual open string vacuum is unstable but there exists a stable vacuum with zero energy density. There is evidence that this state is associated with the condensation of electric flux tubes of closed string\[10\]. These flux tubes described successfully using an effective Born-Infeld action\[12\]. The tachyon rolling towards its minimum at infinity as a dark matter candidate was also proposed by Sen\[10\]. Gibbons took into account coupling to gravitational field by adding an Einstein-Hilbert term to the effective action of the tachyon on a brane, and initiated a study of ”tachyon cosmology”\[11\]. Several authors have investigated the process of rolling of the tachyon in the cosmological background\[13, 14\].

Especially, in the case of merging \( D-\bar{D} \) branes, the tachyon field is complex and the potential has a phase symmetry \( V(T) = V(T e^{i\alpha}) \), the cosmic strings would be produced after the annihilation of the branes\[15\].

It is therefore of importance to investigate the property and the gravity of the topological defects of tachyon field in the stationary spacetime background. In this paper, we first present the rolling tachyon dynamics in the stationary spacetime, and then discuss the
property of global string of the tachyon field in the flat spacetime and in the curved spacetime respectively.

A general static, cylindrically-symmetric metric can be represented as

\[ ds^2 = B(r)(dt^2 - dz^2) - dr^2 - r^2 A(r)d\theta^2. \]  

(2)

And the Lagrangian density of rolling tachyon with potential \( V(\phi) \), which couples to the Einstein gravity, can be written as the following Born-Infeld form:

\[ L = L_R + L_T = \sqrt{-g} \left[ \frac{R}{2\kappa} - V(|T|) \sqrt{1 - g_{\mu\nu} \partial_\mu T \partial_\nu T^\dagger} \right] \]  

(3)

where \( T \) is a complex tachyon field, and \( g_{\mu\nu} \) is the metric. The field configuration describing a string(vortex) is given by

\[ T = f(r)e^{i\theta}, \quad T^\dagger = f(r)e^{-i\theta}. \]  

(4)

Using Eqs.(2)-(4), we can obtain the following Euler-Lagrange equation:

\[ \frac{1}{V} \frac{dV}{df} + f' \left( \frac{B'}{B} + \frac{A'}{2A} + \frac{r}{r} \right) = f'' + \frac{f'f''f' - f^2A'}{1 + f'^2 + \frac{f^2}{Ar^2}} \]  

(5)

where the prime denotes the derivative with respect to \( r \) and the Einstein equations reads:

\[ -\frac{1B''}{2B} - \frac{B'}{2rB} - \frac{B'A'}{4BA} - \frac{A'}{rA} + \frac{B'^2}{4B^2} - \frac{A''}{2A} = \kappa T_{0}^0 \]  

(6)

\[ -\frac{B'}{rB} - \frac{A'B'}{2AB} - \frac{B'^2}{4B^2} = \kappa T_{1}^1 \]  

(7)

\[ \frac{B'^2}{4B^2} - \frac{B''}{B} = \kappa T_{2}^2 \]  

(8)

where the energy-momentum tensor \( T_{\mu}^{\nu} \) of the system are given by

\[ T_{0}^0 = T_{3}^3 = V(f)\sqrt{1 + f'^2 + \frac{f^2}{Ar^2}} \]  

(9)
and the rest is zero. Obviously, the configuration of the system depends on the tachyon potential $V(T)$. According to Sen [10], the potential should have an unstable maximum at $T = 0$ and decay exponentially to zero when $T \to \infty$. There are lots of functional forms that satisfy the above two requirements. In the following sections, we choose the tachyon potential as follows:

$$V(f) = M^4(1 + 4\lambda f^4)^{1/4} \exp(-\lambda f^4)$$

(12)

where $M$ and $\lambda$ are two constants and both are greater than zero. It is not difficult to find that the potential satisfies the two requirement proposed by Sen.

In the flat spacetime, the Euler-Lagrange equation (5) can be reduced to the following equation:

$$\frac{1}{V} \frac{dV}{df} + \frac{f}{r^2} = f'' + \frac{f'}{r} - \frac{f' f'' + \frac{f'}{r^2}(f' - \frac{f}{r})}{1 + f'^2 + \frac{f^2}{r^2}},$$

(13)

and the energy density of the system $\varepsilon$ is given by

$$T_0^0 = V(f) \sqrt{1 + f'^2 + \frac{f^2}{r^2}}.$$  

(14)

For the potential (12), the equation (13) has a simple exact solution

$$f(r) = \lambda^{-\frac{1}{4}} \left( \frac{r}{\delta} \right)^{-1},$$

(15)

where $\delta = (4\lambda)^{-1/4}$ is the size of the string core, and the corresponding energy density can be written as

$$T_0^0 = M^4 \left[ 1 + 4 \left( \frac{\delta}{r} \right)^4 \right]^{3/4} \exp \left[ - \left( \frac{\delta}{r} \right)^4 \right].$$

(16)

The energy per unit length of string at $r \gg \delta$ is

$$\mu(R) = 2\pi \int_0^R T_0^0 r dr \sim \pi M^4 (R^2 - \frac{1}{2\lambda R^2}).$$

(17)
where the cutoff radius $R$ has the meaning of a distance to the nearest string (or the loop radius in the case of a closed loop). We find easily from Eq. (17) that tachyon strings are very diffuse objects with most of the energy distributed at large distances from the string core, and we expect their spacetime to be substantially distinct from the ordinary case. They are much more diffuse than ordinary global strings which have $\mu(R) \propto \ln R$, so that most of the energy is concentrated near the core. Furthermore, they are similar to the vacuumless strings which have $\mu(R) \sim (R/\delta)^{4/(n+2)}$, so that most of the energy is also diffused at large distance [4]. By analogy with vacuumless strings and ordinary strings, one can anticipate that tachyon strings will eventually reach a scaling regime in which the typical distance between the strings is comparable to the horizon, $R \sim t [4]$. Let us now briefly discuss the cosmological evolution of tachyon strings. The mass per unit length of string is given by Eq. (17) where $t$ is substituted for $R$. The relative contribution of strings to the energy density of the universe is given by

$$\rho_s/\rho \sim \mu(t)/M_p^2 \sim \pi(M^2/M_p)^2(t^2 - 1/2\lambda t^2)$$

where $M_p$ is the Planck mass. It is easy to find the fraction of energy in strings monotonically grows with time, and the universe becomes dominated by the strings when $f \sim \pi M^4/2M_p^4$. The observed isotropy of the cosmic microwave background implies $\mu(t_0)/M_p^2 \lesssim 10^{-5}$, where $\mu(t_0)$ is the present value of energy per unit length. Therefore, we have a corresponding constraint on $M$ is $M \lesssim 1MeV$ as the explanation of structure formation. The characteristic scale of the observed large scale structure crossed the horizon at $t \sim t_{eq} \sim 10^{-6}t_0$. The density fluctuations due to tachyon strings on that scale are of the order $\delta \rho_s/\rho_s \sim \mu(t_{eq})/M^2 \lesssim 10^{-17}$.

In this paper, the spacetime of tachyon strings will be investigated using the linearized gravity approximation. We shall first consider the Newtonian approximation. The Newtonian potential $\Phi$ can be found for the equation

$$\nabla^2 \Phi = \frac{\kappa}{2}(T^0_0 - T^i_i).$$

For the tachyon string, $f(r)$ is given by Eq. (15) and

$$T^0_0 - T^i_i \simeq 2M^4$$

(20)
at \( r >> \delta \). The solution of Eq. (19) is then

\[
\Phi(r) \simeq - \frac{M^4}{8 \lambda M^2 f^2}.
\]

(21)

The linearized approximation applies as long as \(|\Phi(r)| << 1\), which is equivalent to \( f >> \frac{M^2}{\sqrt{8 \lambda M_p}} \). Therefore, we should take the parameters \( \lambda \) and \( M \) satisfy

\[
(4 \lambda)^{-1/4} << R << \frac{2 M_p}{M^2}
\]

(22)

where \( R \) is the distance of string separation (or the loop radius in the case of closed loop). Next, we express the metric functions \( A(r) \) and \( B(r) \) as

\[
A(r) = 1 + \alpha(r), \quad B(r) = 1 + \beta(r).
\]

(23)

Linearizing in \( \alpha(r) \) and \( \beta(r) \), and using the flat space expression (15) for \( f(r) \), Eqs. (6)-(8) can be written as follows

\[
\alpha'' + \frac{2 \alpha'}{r} = -\frac{\kappa M^4}{4(r/\delta)^4} \left[ 1 + 4 \left( \frac{\delta}{r} \right)^4 \right]^{-1/4} \exp \left[ - \left( \frac{\delta}{r} \right)^4 \right],
\]

(24)

and

\[
\beta'' + \frac{\beta'}{r} = -2\kappa M^4 \left[ 1 + 2 \left( \frac{\delta}{r} \right)^4 \right] \left[ 1 + 4 \left( \frac{\delta}{r} \right)^4 \right]^{-1/4} \exp \left[ - \left( \frac{\delta}{r} \right)^4 \right].
\]

(25)

The solution of external metric is easily found

\[
ds^2 = (1 - \frac{\kappa M^4}{2} r^2)(dt^2 - dz^2) - dr^2 - r^2 (1 - \frac{\kappa M^4}{2} \lambda r^2) d\theta^2.
\]

(26)

The metric (26) can be expressed by the form of Newtonian potential

\[
ds^2 = (1 + 2\Phi)(dt^2 - dz^2) - dr^2 - r^2 \left[ 1 + \left( \frac{2\delta}{r} \right)^4 \Phi \right] d\theta^2.
\]

(27)

Here, we note two qualitative differences between the metrics of tachyon and ordinary gauge string: (i) for a tachyon string, the gravitational field is strongly repulsive and the spacetime becomes singular at a finite distance from the string core; (ii) the effective deficit angle for a tachyon string

\[
\Delta(r) \simeq \pi \kappa M^4 \left( r^2 - \frac{1}{2\lambda r^2} \right) + \frac{\pi}{8} \kappa^2 M^8 r^2 \left( r^2 - \frac{\kappa M^4}{\lambda} \right).
\]

(28)
increases with distance from the core, while the ordinary gauge string deficit angle remains constant.

In this paper, we have studied the property and gravitational field of global string of tachyon matter (tachyon vortex) in a four dimensional approximately cylindrically-symmetric spacetime. We give an exact solution of the tachyon field in the flat spacetime background and in particular, using the linearized approximation of gravity, we find a solution of the metric, which denotes a spacetime with an increasing deficit angle. Contrast to the ordinary string, the tachyon strings are very diffuse objects with most of the energy distributed at large distances from the string core, and their spacetime is substantially distinct from that of the ordinary string. In this respect, they are more similar to the vacuumless strings [4, 5].

In the $D-\bar{D}$ branes theory, tachyon field is complex and the potential has a phase symmetry $V(T) = V(e^{i\alpha}T)$ [15]. In this model one would expect formation of the cosmic strings after annihilation of brane. It is worth noting that although we used the potential [12] in our discussion, all the conclusions will be preserved qualitatively if we use the potentials which are discussed by Sen [10].

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