Quantum jump approach to the switching process of a Josephson junction coupled to a microscopic two-level system

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Abstract

With microwave irradiation, the switching current of a Josephson junction coupled to a microscopic two-level system jumps randomly between two discrete states. We modeled the switching process of the coupled system using the quantum jump approach that is well known from quantum optics. The parameters that affect the character of the quantum jumps between macroscopic quantum states are discussed. The results obtained from our theoretical analysis agree well with those of the experiments and provide a clear physical picture for macroscopic quantum jumps in Josephson junctions coupled with two-level systems. In addition, quantum jumps may serve as a useful tool to investigate the microscopic two-level structures in solid-state systems.

(Some figures in this article are in colour only in the electronic version)

1. Introduction

Recent progress on superconducting qubits based upon a Josephson junction (JJ) unambiguously demonstrate quantum behavior of the macroscopic variables [1–7]. Moreover, quantum jumps, an interesting quantum phenomenon previously studied in quantum optics [8–10], were recently experimentally demonstrated for the first time in a junction coupled with a microscopic two-level system (TLS) [11]. The JJ–TLS coupling system possesses a Λ-type energy level structure and microwave photons are used to generate transitions between quantum states. However, the state of the system is read out by detecting the macroscopic quantum tunneling process rather than by detecting photon emissions in quantum optics. Quantum jumps then manifest themselves in the form of random jumps between the upper branch and lower branch of the switching currents. In the language of quantum measurement theory, the switching currents in the upper branch or lower branch serve as a pointer from which the macroscopic quantum state of the JJ–TLS coupling system can be determined. In this situation the ensemble description of the dynamics of junctions based on the master equation method [12–14] fails in describing trajectories of a single quantum system. The quantum jump approach was developed in the 1980s and has been successfully used to describe fluorescence of single trapped ions [10]. In this paper we generalize the quantum jump approach to the switching process of the JJ–TLS coupling system and make a systematic study of the parameters that affect the process. The same method has also been used to investigate quantum jumps in Rabi oscillations of a JJ–TLS coupling system [15]. However, in that work the biased current of the junction is fixed at an appropriate value while here it keeps changing during the switching current measurement. Therefore, new mechanisms such as Landau–Zener transitions may be involved in the dynamics of the JJ–TLS coupling system.

This paper is organized as follows. In section 2, we describe the physics of the current-biased Josephson junction briefly and introduce the quantum jump approach for simulating the switching process of a current-biased junction.
In section 3 we generalize the quantum jump approach to the switching process of the JJ–TLS coupling system and discuss the parameters that have effects on the process. In section 4 we compare our theoretical results with experimental data and make a conclusion in section 5.

2. Quantum jump approach to the switching process of a current-biased Josephson junction

The Hamiltonian of a current-biased Josephson junction as shown in figure 1(a) is [16, 17]

\[ H_{JJ} = \frac{1}{2C} \dot{\delta}^2 - \frac{I_0 \Phi_0}{2\pi} \cos \delta - I \Phi_0 \delta, \]  

(1)

where \( I_0 \) is the critical current of the Josephson junction, \( I \) is the bias current, \( C \) is the junction capacitance, \( \Phi_0 = h/2e \) is the flux quantum, \( \dot{\delta} \) denotes the charge operator and \( \delta \) represents the gauge-invariant phase difference across the junction, which obeys the quantum commutation relation \([\delta, \dot{\delta}] = 2iei\). The states of the current-biased Josephson junction can be controlled through the bias current \( I(t) \) given by

\[ I(t) = I_{dc} + \Delta I(t) = I_{dc} + I_{\mu}\omega \cos \omega t, \]  

(2)

where the classical bias current is parameterized by a dc component \( I_{dc} \) and an ac component with the magnitude \( I_{\mu}\omega \) and frequency \( \omega \). For \( I_{dc} < I_0 \), the effective potential of the system (shown in figure 1(b)) has a series of metastable wells. At low temperature, the current-biased junction has quantized energy levels, with the two lowest energy states labeled as \( |0\rangle \) and \( |1\rangle \). Microwaves induce transitions between \( |0\rangle \) and \( |1\rangle \) at a frequency

\[ \omega_{10} = \frac{E_1 - E_0}{h} = \omega_p \left( 1 - \frac{5}{36} \frac{h\omega_p}{\Delta U} \right), \]  

(3)

where \( \omega_p(I_{dc}) = 2^{1/4}(2\pi I_0/\Phi_0 C)^{1/2}(1 - I_{dc}/I_0)^{1/4} \) is the small oscillation frequency at the bottom of the washboard potential and \( \Delta U(I_{dc}) = (2\sqrt{2}I_0\Phi_0/3\pi)(1 - I_{dc}/I_0)^{3/2} \) is the barrier height. It is apparent from equation (3) that the energy spacing \( \omega_{10} \) is a function of the bias current \( I_{dc} \). Therefore, if we ramp \( I_{dc} \) from 0 to \( I_0 \), the barrier \( \Delta U \) is decreasing. At a certain \( I_{dc} \), called the switching current, the system will tunnel out of the potential well. In addition, a microwave with frequency matching the energy level spacing will generate a transition between \( |0\rangle \) and \( |1\rangle \). As shown in the top panel of figure 2(b), the main peak of the switching current distribution corresponds to tunneling from the ground state \( |0\rangle \), and the resonant peak corresponds to tunneling from the first excited state \( |1\rangle \). By plotting the frequency of microwaves versus the parameter of the resonant peak we can obtain the energy spectrum of the junction.

To simulate the switching process of a current-biased junction, we firstly write the Hamiltonian of the junction in subspace \(|0\rangle, |1\rangle\):

\[ H_{JJ} = \hbar \begin{pmatrix} 0 & \Omega_m \cos \omega t \omega_{10}(I_{dc}) \\ \Omega_m \cos \omega t \omega_{10}(I_{dc}) & 0 \end{pmatrix}, \]  

(4)

where \( \Omega_m = I_{\mu}\sqrt{1/(2\hbar\Omega_1 C)} \) is the Rabi frequency. Considering the dissipative effect of the environment, the time evolution of the system can be described by the non-Hermitian effective Hamiltonian:

\[ H_{eff} = H_{qb} - \frac{i\hbar}{2}(\gamma_{10} + \Gamma_1)|1\rangle\langle 1| - \frac{i\hbar}{2}\Gamma_0|0\rangle\langle 0|, \]  

(5)

where \( \gamma_{10} \) is the energy relaxation rate from \( |1\rangle \) to \( |0\rangle \) and \( \Gamma_1 \) is the tunneling rate from the state \(|i\rangle \) \((i = 0, 1)\) out of the potential (figure 1(b)). It is noticed that both \( \gamma_{10} \) and \( \Gamma_1 \) are functions of the bias current \( I_{dc} \). At temperature \( T \), the relaxation rate \( \gamma_{10} \) is given by [13]

\[ \gamma_{10} = \frac{\omega_{10} R_Q}{2\pi R} \left[ 1 + \coth \left( \frac{\hbar \omega_{10}}{2k_B T} \right) \right] \times \langle |0\rangle|1\rangle^2, \]  

(6)

where \( R_Q \equiv h/4e^2 \approx 6.45 \text{k}\Omega \) is the natural quantum unit of resistance and \( R \) is the shunting resistance in the RCSJ model (figure 1(a)). The tunneling rate \( \Gamma_1 \), from the state \(|i\rangle \) can be obtained with the WKB method:

\[ \Gamma_i = \frac{1}{T(E_i)} \exp \left( -\frac{2S_j(E_i)}{h} \right), \]  

(7)

where \( T(E_i) \) is the classical period of motion and \( S_j(E_i) \) is the action across the classically forbidden region.

Then the quantum jump approach for simulating the switching process of junctions can be summarized as follows:

(i) At \( t = 0 \), initialize the junction in the ground state: \(|\Psi(t = 0)\rangle = |0\rangle\).

(ii) For \( I_{dc}(t + \Delta t) = I_{dc}(t) + (dI_{dc}/dt)\Delta t \), calculate the corresponding energy spacing and various transition rates according to equations (3)–(7).

(iii) Determine whether the system evolves according to the Schrödinger equation, or makes a "jump" [15].

(a) If a quantum tunneling escape happens, register the switching current \( I_s = I_{dc}(t) \), and then turn to step (v).

(b) If a relaxation event happens, then the system jumps to the ground state \(|0\rangle\).

(c) If no jumps happen, the system evolves under the influence of the non-Hermitian form [15].
Figure 2. Simulated switching currents (lower panel) of a junction obtained by a quantum jump approach (a) without microwaves and (b) with microwaves, respectively. The parameters used for simulations are: $I_0 = 35.9 \, \mu\text{A}$, $C = 4 \, \text{pF}$, $\omega/2\pi = 9.02 \, \text{GHz}$, $\Omega_m = 10 \, \text{MHz}$ and $\gamma_0 = 0.6 \, \mu\text{s}^{-1}$. The ramping rate is $\frac{dI_0}{dt} = 4.5 \times 10^{-5} \, \mu\text{A} \cdot \text{s}^{-1}$. By making a histogram of the switching currents we obtained the switching current distribution, shown as the symbols in the top panel. The solid lines are ensemble results obtained using the master equations.

(iv) For cases (b) and (c), repeat from step (ii).
(v) Repeat to obtain the switching current $I_s$.
(vi) Average the switching current $I_s$ over many simulation runs.

The numerical results obtained with the quantum jump approach are shown in figure 2. The parameters we used in the simulation are from experiments [11]. In addition, we calculate the switching current distribution with the master equation method. The agreement between the quantum jump approach and the master equation indicates that the quantum jump approach is valid to model the switching process of the junction. Furthermore, as discussed in section 3, the quantum jump approach is more powerful than the master equation method when the stochastic characteristics of a single quantum system play an important role.

3. Quantum jump approach to the switching process of a JJ–TLS coupling system

Firstly we give a brief description of the physics of the JJ–TLS coupling system. TLSs have recently been extensively observed in the superconducting phase [18–20], charge [21] and flux [22] qubits. The mechanism of TLS is still an open question [18–20]. For simplicity, a TLS is usually understood to be a particle or a small group of particles that tunnels between two lattice configurations, with different wavefunctions $|L\rangle$ and $|R\rangle$, respectively (figure 3(a)).
The interaction Hamiltonian between the junction and TLS can be written as [18]

$$H_{\text{int}} = -\frac{\Phi_0 I_{0k}}{2\pi} \cos \delta \otimes |R\rangle \langle R| - \frac{\Phi_0 I_{0L}}{2\pi} \cos \delta \otimes |L\rangle \langle L|.$$  \hspace{1cm} (8)

For convenience, we transfer to the energy eigenstate basis of TLS with $|g\rangle$ and $|e\rangle$ being the ground state and the excited state, respectively. Then the total Hamiltonian of the JJ–TLS in the basis $\{|0g\rangle, |1g\rangle, |0e\rangle, |1e\rangle\}$ is given by [15, 23]

$$H = \hbar \begin{pmatrix}
0 & \Omega_m \cos \omega t & 0 & 0 \\
\Omega_m \cos \omega t & \Omega_c & 0 & 0 \\
0 & \Omega_c & \omega_{\text{TLS}} & \Omega_m \cos \omega t \\
0 & 0 & \Omega_m \cos \omega t & \omega_{\text{TLS}} + \omega_{\text{TLS}}
\end{pmatrix},$$  \hspace{1cm} (9)

where $\omega_{\text{TLS}}$ is the energy frequency of the TLS and $\Omega_c$ is the coupling strength between the Josephson junction and the TLS. In experiments, the coupling strength $\Omega_c$ can be characterized in spectroscopic measurements and usually lies from 20 to 200 MHz [11, 18–22]. The time evolution of the JJ–TLS coupling system under the dissipative effect of the environment can be described by the effective Hamiltonian

$$H_{\text{eff}} = H - \frac{i\hbar}{2} \Gamma_0 |0g\rangle \langle 0g| - \frac{i\hbar}{2} (\gamma_{10} + \Gamma_{1g}) |1g\rangle \langle 1g|$$
$$- \frac{i\hbar}{2} \Gamma_0 |0e\rangle \langle 0e| - \frac{i\hbar}{2} (\gamma_{10} + \Gamma_{1e}) |1e\rangle \langle 1e|,$$  \hspace{1cm} (10)

where $\Gamma_i$ is the tunneling rate from state $|i\rangle$. We emphasize that in the asymmetric double-well model of TLS, the energy basis of TLS is approximated to the position basis. In this approximation, states $|g\rangle$ and $|e\rangle$ correspond to different critical currents. Therefore, the tunneling rates from different states are different. With no loss of generality, suppose the state $|e\rangle$ corresponds to the smaller critical current. Then the procedure for simulating the switching process of the JJ–TLS coupling system can be summarized as follows:

(i) Initializing the system in state $|0g\rangle$ for flag $= 0$, or in state $|0e\rangle$ for flag $= 1$, where ‘flag’ is a marker.

(ii) For $I_{dc}(t + \Delta t) = I_{dc}(t) + (dI/dt)\Delta t$, calculate the corresponding energy spacing $\omega_{10}(I_{dc})$ and various transition rates.

(iii) Determine whether the system evolves according to the Schrödinger equation, or makes a ‘jump’.

(a) If a quantum tunneling event happens, register the switching current $I_s = I_{dc}(t)$. Furthermore, if the system tunnels from $|0g\rangle$ or $|1g\rangle$, set flag $= 0$; else if the system tunnels from $|0e\rangle$ or $|1e\rangle$, set flag $= 1$; and then turn to step (v).

(b) If a relaxation event happens, then the system jumps to the corresponding ground state $|0g\rangle$ or $|0e\rangle$, i.e. the system jumps from $|1g\rangle$ to $|0g\rangle$, or from $|1e\rangle$ to $|0e\rangle$.

(c) If no jumps happen, the system evolves under the influence of the non-Hermitian form.

(iv) For cases (b) and (c), repeat from step (iii).

(v) Repeat to obtain the switching current $I_s$.

The simulation results are shown in Figure 4. It is apparent that the switching current jumps between the upper branch and
In addition, the switching current in both branches becomes higher for the larger ramping rate. As the ramping rate increasing, the jumps between the upper branch and lower branch become less frequent, resulting in a longer lifetime in each branch. In addition, the switching current in both branches becomes higher for the larger ramping rate.

lower branch randomly, which is the major characteristic of macroscopic quantum jumps observed in experiments [11]. In addition, it is found that the jumps become more frequent with the increasing microwave amplitude (figure 4). The underlying physics can be understood as follows. The jumps between the upper branch and lower branch of the switching current are fulfilled through the coupling between state |1g⟩ and |0e⟩. On increasing the microwave amplitude, the system initialized in |0g⟩ has a larger transition rate to |1g⟩ in the expression

$$\Gamma = \frac{\Omega_m^2 \gamma}{2(\Delta^2 + \gamma^2)}.$$  \hfill (11)

where \(\gamma = (\gamma_{10} + \Gamma_g + \Gamma_{1g})/2\) and \(\Delta = \omega_{10} - \omega\). Therefore, it is much easier for the system to jump to state |0e⟩, i.e. jump from the upper branch to the lower branch, and vice versa. It is easier to understand for the extreme case \(\Omega_m = 0\). Then the system has no probability to occupy state |1g⟩ and thus no probability to transfer to |0e⟩.

Furthermore, it is noticed that the transition process from |1g⟩ to |0e⟩ is actually a Landau–Zener transition as illustrated in figure 3(b). Disregarding all decay terms, the asymptotic probability of a Landau–Zener transition is given by [24, 25]

$$P_{LZ} = \exp \left(-2\pi \frac{\hbar \Omega_c^2}{\nu} \right),$$  \hfill (12)

where \(2\hbar \Omega_c\) is the magnitude of the energy splitting and \(\nu \equiv d\epsilon/dt\) denotes the variation rate of the energy spacing for noninteracting levels. Notice that \(\nu \equiv (d\epsilon/dI_{dc})(dI_{dc}/dt), \) where \(d\epsilon/dI_{dc}\) is determined by the intrinsic parameters of the junction and \(dI_{dc}/dt\) is determined by the ramping rate. It can be easily inferred from equation (12) that, as the ramping rate increased, the transition rate between |1g⟩ and |0e⟩ becomes smaller. Therefore, the jumps between the upper branch and lower branch of the switching currents become less frequent and the lifetime for each branch is longer. To support this argument, we simulate the trajectories of the switching currents for different ramping rates. As shown in figure 5, with the ramping rate increasing, the jumps between different branches become less frequent, as expected from our theoretical analysis.

4. Experiments

We have compared the results of our theoretical analysis with the experimental data. The sample used in our experiments was a 10 \(\mu\)m \(\times\) 10 \(\mu\)m Nb/AlOx/Nb Josephson junction. The junction parameters are \(I_0 \approx 36 \mu\)A and \(C \approx 4\) pF, respectively. The device was thermally anchored to the mixing chamber of a dilution refrigerator with a base temperature of about 18 mK. The additional three-layer mu-metal surrounding the dewar was used to shield the magnetic field. All electrical leads that connect the junction to room temperature electronics were carefully filtered by resistor–capacitor (RC) filters and copper powder filters. The center conductor of an open-ended coaxial cable was placed above the junction for application of microwaves. This arrangement resulted in >110 dB attenuation between the end of the coaxial cable.

![Figure 5](image_url)
Figure 6. Experimental trajectories of the switching current of the JJ–TLS coupling system under different microwave powers with (a) $-7.2$ dBm and (b) $-5.8$ dBm. The right panel of each plot shows the histogram of switching events, i.e. the switching distribution $P(I)$. The ramping rate is $dI_{dc}/dt = 4.5 \times 10^3 \mu A s^{-1}$. On increasing the microwave power, the jumps become much more frequent.

and the junction. A saw-tooth bias current was applied with a repetition rate of 30–300 Hz [11, 26]. The junction voltage was amplified by a differential amplifier and the switching current was recorded when a voltage greater than the threshold was first detected during every ramp.

In the spectroscopy measurement of the junction, an avoided crossing caused by the coupling between junction and TLS was observed at $\omega/2\pi = 8.7$ GHz with an energy splitting $2\Omega_c/2\pi = 400$ MHz [11]. When a microwave field with $\omega/2\pi = 9.02$ GHz was applied, the coupling between junction and TLS was turned on. In this case macroscopic quantum jumps between the upper branch and lower branch of the switching current were observed. To investigate the effect of microwave power as discussed in section 3, we fixed the microwave frequency at $\omega = 9.02$ GHz and the ramping rate at $dI_{dc}/dt = 4.5 \times 10^3 \mu A s^{-1}$. The microwave power was adjusted from $-20$ to $-3$ dBm. As shown in figure 6, with the microwave power increasing, quantum jumps between different branches become much more frequent. Similarly, to investigate the effect of ramping rate, we adjusted the ramping rate $dI_{dc}/dt$ from $2.0 \times 10^3$ to $16.0 \times 10^3 \mu A s^{-1}$ while keeping other parameters fixed. As expected, the increasing ramping rate results in less frequent jumps between the upper branch and lower branch of the switching current (figure 7). The agreement between our simulated results and the experimental data confirmed the validity of the quantum jump approach.

It has been recognized that JJ–TLS coupling is harmful to quantum coherence and qubit control. Therefore, a correct understanding of TLS in Josephson junctions is crucial for realizing superconducting qubits. The quantum jump approach developed here may serve as a helpful tool to investigate the TLS in a solid-state system. Unfortunately, we are unable to determine the mechanism of the JJ–TLS coupling with available data. Previous works have proposed several mechanisms, including two widely used models: coupling through the critical current dependence on the TLS position [18] or the direct dipole coupling to the junction charge $\hat{Q}$ [20]. However, the effective interaction Hamiltonian of both mechanisms has the same form except for the expression of the coefficient. Further experiments on quantum jumps with different parameters may help to clarify this issue.

It is worth mentioning that the ensemble characteristics of the simulations (figures 4 and 5) are not in accordance with those of the experiments (figures 6 and 7) very well. For example, the histograms show two peaks in figure 4 while three peaks in figure 6. Actually, the average switching currents of three states are determined by the microwave frequency and the critical current we used in the simulation [11]. The parameters we selected here resulted in that two lower current peaks overlap each other. Nevertheless, we can still resolve the three branches from trajectory plots. A quantitative comparison between the theory and experiments could be done by better tuning the parameters.

5. Conclusion

We have used the quantum jump approach to simulate the switching process of the JJ–TLS coupling system. The mechanism that dominates the quantum jump phenomenon was discussed. In addition, we investigated the parameters that have an effect on the behavior of quantum jumps. It is found
that a higher microwave power or a smaller ramping rate can make the quantum jumps happen more frequently, which has significance in controlling the state of TLS. Furthermore, our theoretical results agree with the experimental data, indicating the validity of our approach. The model and method we used here can be easily generalized to other solid-state systems such as flux and charge qubits, quantum dots, trapped ions and so on.

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Figure 7. Experimental trajectories of the switching current of the JJ–TLS coupling system under different ramping frequencies. The microwave power is –6.6 dBm. The right panel of each plot shows the histogram of switching events, i.e. the switching distribution $P(I)$. The ramping rates are (a) $dI_d/dt = 4.5 \times 10^3 \mu A s^{-1}$ and (b) $dI_d/dt = 8.0 \times 10^3 \mu A s^{-1}$, respectively. With the ramping rate increasing, the jumps between different branches become less frequent.
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