Study on the design method of interference fit between gear and shaft of automobile transmission

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Abstract. The interference fit of gear and shaft in automobile transmission has an important influence on the performance of gear transmission. In order to design reasonably and calculate accurately, the theoretical calculation model of the interference fit between gear and shaft is established according to the principles of elastic-plastic mechanics. A new method of calculating the amount of interference is designed based on the Lame equation. Three limiting conditions are taken in the calculation. They are load transfer capacity of gear, no plastic deformation of fitting surface and control of gear tooth deformation. The theoretical calculation and finite element simulation analysis are carried out in an application case. The results of calculation and analysis show that the interference fit of gear and shaft has an important influence on the gear tooth deformation. The amount of interference calculation method in this paper can reduce the deformation of gear tooth and make the gear have higher accuracy.

1. Introduction
With the rapid progress of automobile technology and the increasing development of transportation, automobile has become the most important means of transportation. The requirements on the performance of automobile transmission have become higher and higher [1]. Gear is one of the core parts of automobile transmission. It has simple structure, smooth transmission and plays the role of transmitting engine torque. There are many kinds of connections between gear and shaft, among which involute spline connection has been widely used. The alignment modes of spline connection mainly include tooth side alignment and major diameter alignment [2]. Because there is a clearance at the tooth tip or tooth side, the accuracy and transmission performance will be affected after the alignment contact surface is worn during working. The interference fit connection of gear and shaft is also widely used to improve the alignment accuracy. Interference fit has many advantages such as simple structure, large bearing capacity, good alignment and strong impact resistance [3]. At present, many researches on the interference fit of gears and shafts have been carried out which mainly focuses on the bearing capacity of gear, elastic-plastic deformation and stress distribution at the fitting contact surface [4,5,6]. However, the essence of interference fit is the elastic deformation of the connector at the fitting surface that can not only affect the fitting surface of the inner hole, but also lead to the deformation of tooth profile of gears. For the small module gears in the automotive transmission the deformation of tooth profile is more obvious, which will reduce the accuracy of the gear, affect the backlash, meshing line, contact stress of the gear pair, and reduce the fatigue life of transmission ultimately [7]. Therefore, it is necessary to research the influence on the deformation of tooth profile when designing the interference fit of gear and shaft, so as to ensure the gear accuracy and the performance of automobile transmission.

2. Theoretical of interference fit Between gear and shaft

2.1 Geometric model
The geometric model of interference fit between gear and shaft is shown in Fig. 1. The elastic deformation occurs near the fitting contact surface of the hole and shaft after the interference assembly, which will cause
contact pressure. The fitting contact surface is not smooth and has static friction, so that the gear can transmit torque.

![Fig. 1. Geometric model of gear and shaft.](image)

2.2 Theoretical calculation model

Gear and shaft are made of high strength alloy steel, which has the characteristics of isotropy and uniform deformation. The deformation of fitting contact surface is elastic, so the contact stress of gear and shaft is regarded as plane state [8]. The ratio of addendum circle diameter to mounting hole diameter of gear in transmission is more than or close to 1.2, so the gear can be simplified as a thick-wall cylinder model [9]. Due to the existence of tooth grooves whose volume accounts for 1/2 of the whole tooth area, the deformation and stress distribution of the gear will be affected after interference assembly on the shaft. In order to improve the calculation accuracy, this paper takes the pitch diameter as the outer contour and establishes the theoretical calculation model of gear, as shown in Fig. 2. The radius of the mounting hole in gear is $r_s$, the radius of the pitch circle is $r_a$, the radius of any position between the mounting hole and the pitch circle is $r$, the pressure on the fitting contact surface is $p_1$, and the contact pressure of external part to gear is $p_2$.

![Fig. 2. Theoretical calculation model of gear.](image)

According to the analysis method of thick wall cylinder in elastic-plastic mechanics [8], the stress components and deformation of gear can be calculated by using Lame equation as follows.

$$
\sigma_r = \frac{r_s^2 p_1 r_a^2 p_2}{r_a^2 - r_s^2} \frac{r_s^2 r_a^2 (p_1 - p_2)}{E} \frac{1}{r^2}
$$

(1)

$$
\sigma_\theta = \frac{r_s^2 p_1 r_a^2 p_2}{r_a^2 - r_s^2} \frac{r_s^2 r_a^2 (p_1 - p_2)}{E} \frac{1}{r^2}
$$

(2)

$$
u = \frac{1 + \mu}{E} \frac{r_s^2 (p_1 - p_2)}{r_a^2 - r_s^2} + \frac{1 + \mu}{r} \frac{r_s^2 (p_1 - p_2)}{r_a^2 - r_s^2}
$$

(3)

Where $\sigma_r$ is the radial stress, $\sigma_\theta$ is the circumferential stress, $\nu$ is the radial deformation, $\mu$ is Poisson's ratio of the material, $E$ is Young's modulus of the material.

2.3 Theoretical calculation of interference fit

Assuming that no contact stress is exerted on the gear, $p_2 = 0$. When $r=r_s$, the stress components and deformation at the fitting contact surface of gear can be calculated as follows.

$$
\sigma_r = -p_1
$$

(4)
\[ \sigma_0 = \frac{r_a^2 + r_d^2}{r_a^2 - r_b^2} p_1 \]  \hspace{1cm} (5)

\[ u_1 = \frac{r_a \left( r_a^2 + r_d^2 \right) + \mu}{E} p_1 \]  \hspace{1cm} (6)

When \( r = r_a \), the deformation of pitch circle radius of gear can be calculated as follows.

\[ u_a = \frac{1}{E} \frac{r_a^2}{r_a - r_b} p_1 \]  \hspace{1cm} (7)

The deformation of the shaft outer circle at fitting contact surface can be calculated as follows.

\[ u_2 = \frac{(1+\mu)r_a}{E} p_1 \]  \hspace{1cm} (8)

### 3. Parameter design of interference fit between gear and shaft

#### 3.1 Calculation of the minimum amount of interference required for load transfer

The design principle of interference fit between gear and shaft is not only to make the connection have enough holding force, but also to ensure that the parts will not be damaged under the assembly stress [10]. According to the working conditions and calculation requirements of automobile transmission, the torque transferred by gear is calculated using the maximum limit static load, and the minimum pressure at the fitting contact surface required for load transfer is calculated as follows.

\[ p_{\text{min}} = \frac{\sqrt{E^2 \left( \frac{r_a - r_b}{2L} \right)^2}}{n df} \]  \hspace{1cm} (9)

Where \( p_{\text{min}} \) is the minimum pressure required by the gear to transfer load, \( F \) is the axial force on the gear, \( T \) is the torque transmitted by the gear, \( d \) is the nominal diameter of the fitting surface, \( L \) is the length of the fitting surface, \( f \) is the friction coefficient of the fitting surface. When gear and shaft are assembled by interference fit, the bore of gear and the shaft will have radial elastic deformation, and the sum of the deformation on the diameter is equal to the amount of interference. By bringing the result calculated by Eq. 9 into Eq. 6 and Eq. 8, the minimum amount of interference required for load transfer can be calculated as follows.

\[ \delta_{\text{min}} = 2(u_1 + u_2) = 2\frac{r_a \left( r_a^2 + r_d^2 \right)}{E} \frac{1}{r_a - r_b} + 1 \]  \hspace{1cm} (10)

Where, \( \delta_{\text{min}} \) is the minimum effective amount of interference required for load transfer, \( u_1 \) is the radius deformation of gear bore, \( u_2 \) is the radius deformation of the fitting surface of shaft.

#### 3.2 Calculation of the maximum amount of interference allowed for the fitting surface without plastic deformation

The interference fit of gear and shaft must ensure that the fitting contact surface only has elastic deformation. Therefore, the contact stress at the fitting surface should be less than the allowable stress of the material. According to the calculation of plane stress of elastic material [8], the two-dimensional stress state of the fitting surface can be calculated as follows.

\[
\begin{align*}
\sigma_1 &= \frac{\sigma_0 + \sigma_3}{2} + \sqrt{\left(\frac{\sigma_0 - \sigma_3}{2}\right)^2 + \sigma_{\text{eff}}^2} \\
\sigma_3 &= \frac{\sigma_0 + \sigma_1}{2} - \sqrt{\left(\frac{\sigma_0 - \sigma_1}{2}\right)^2 + \sigma_{\text{eff}}^2}
\end{align*}
\]  \hspace{1cm} (11)

Where \( \sigma_1 \) and \( \sigma_3 \) are the maximum principal stress and minimum principal stress of the material under complex stress state, \( \sigma_{\text{eff}} = fP_1 \), \( f \) is the friction coefficient of the fitting surface. According to the third strength theory, the strength criterion for calibrating the elastic failure of gear and shaft is as follows.

\[ \sigma_{3r} = \sigma_1 \cdot \sigma_3 \leq \frac{[\sigma]}{n} \]  \hspace{1cm} (12)
Where $[\sigma]$ is the allowable stress of the material, $n$ is the safety factor. By bringing the stress components $\sigma_r$ and $\sigma_\theta$ calculated by Eq. 4 and Eq. 5 into Eq. 11 and Eq. 12, the maximum contact pressure allowed for the fitting surface of gear without plastic deformation can be calculated as follows.

$$P_{gmax} = \frac{[\sigma]}{2n} \frac{r_a^2}{\sqrt{(r_a^2-r_s^2)}} f^2$$ (13)

The maximum contact pressure allowed for the fitting surfaces of shaft without plastic deformation can be calculated as follows [8].

$$P_{smax} = \frac{[\sigma]}{2n}$$ (14)

Taking the smaller of the results of Eq. 13 and Eq. 14, the maximum contact pressure allowed for the fitting surface of gear and shaft without plastic deformation can be calculated as follows.

$$P_{emax} = \min\{P_{gmax}, P_{smax}\}$$ (15)

Using equations Eq. 6, Eq. 8 and Eq. 15, the maximum amount of interference allowed for gear and shaft without plastic deformation can be calculated as follows.

$$\delta_{emax} = \frac{2\pi}{E} \left(\frac{r_a^2}{r_a^2+r_s^2} + 1\right) P_{emax}$$ (16)

3.3 Calculation of the maximum amount of interference allowed for controlling tooth deformation

The interference assembly of gear and shaft will lead to the change of gear profile and gear-pair backlash, which will have a great impact on the vibration, noise and working life of the automobile transmission [11]. This paper take the involute gear commonly used in transmission as the research object. The calculation model of single tooth is established.

Fig. 3. Diagram of involute profile deformation of single tooth.

In Fig. 3, $r_b$ is the radius of base circle, and $r_a$ is the radius of pitch circle. The deformation of normal tooth pitch on the pitch circle can be calculated as follows according to Fig. 3.

$$\Delta f_p = u_a \sin \alpha$$ (17)

Where $\alpha$ is the pressure angle on gear pitch circle. Assuming that the maximum deformation allowed for normal tooth pitch on the pitch circle is $\Delta f_{pmax}$, the maximum contact pressure for controlling tooth deformation can be calculated using Eq. 7 and Eq. 17 as follows.

$$P_{pmax} = \frac{\pi(r_a^2-r_s^2)}{r_a^2+r_s^2 \sin \alpha} \Delta f_{pmax}$$ (18)

Using Eq. 6, Eq. 8 and Eq. 18, the maximum amount of interference allowed for controlling tooth deformation can be calculated as follows.
\[ \delta_{p_{\text{max}}} = \frac{2r_a}{E} \left( \frac{r_a^2 + r_s^2}{r_a^2 + r_s^2} + 1 \right) p_{p_{\text{max}}} \]  \hspace{1cm} (19)

### 3.4 Determination of parameters for interference fit

In order to ensure that the gear will not slip on the shaft, the amount of interference should be \( \delta_{\text{min}} \) at least. At the same time, for ensuring that the fitting surface has no plastic deformation and the deformation of tooth profile is controlled within the allowable range, the maximum amount of interference should not exceed \( \delta_{\text{max}} \). The value of \( \delta_{\text{max}} \) can be calculated as follows.

\[ \delta_{\text{max}} = \min\{\delta_{\text{emax}}, \delta_{p_{\text{max}}}\} \]  \hspace{1cm} (20)

In order to determine the amount of interference and the tolerance zone of hole and shaft, we can first calculate the basic amount of interference between gear and shaft, and then select them from the relevant tolerance fit standard. The basic amount of interference can be calculated as follows.

\[ \delta_b = \frac{\delta_{\text{min}} + \delta_{\text{max}}}{2} \]  \hspace{1cm} (21)

### 4. Design example of interference fit between gear and shaft

#### 4.1 Theoretical calculation of interference fit

In a design example of transmission, the gear module is \( m = 1.44 \text{mm} \), the number of teeth is \( z = 22 \), the pressure angle is \( \alpha = 18.5^\circ \), the diameter of the fitting surface is \( d = 21 \text{ mm} \), the length of the fitting surface is \( L = 33 \text{ mm} \), the static maximum limit torque is \( T = 265 \text{ N.m} \). The material of gear and shaft is alloy steel, whose Poisson's ratio is \( \mu = 0.3 \), Young's modulus is \( E = 2.1 \times 10^5 \text{ Mpa} \), yield limit is \( \sigma_s = 850 \text{ MPa} \). In theoretical calculation, the allowable stress of material is \( [\sigma] = 850 \text{ MPa} \), the safety factor is \( n = 1.2 \), the friction factor is \( f = 0.2 \), and the allowable maximum deformation of normal tooth pitch on the pitch circle is \( \Delta f_{p_{\text{max}}} = 0.0035 \text{ mm} \). The results of theoretical calculation are shown in Table 1, which shows that \( \delta_{p_{\text{max}}} \) is less than \( \delta_{\text{emax}} \). If the amount of interference between gear and shaft is designed as \( \delta_{\text{emax}} \), although the fitting surface has no plastic deformation, but the deformation of gear tooth profile is too large, which will reduce the accuracy of gear and affect the working performance of the transmission.

| Projects of calculation                                      | Results            |
|--------------------------------------------------------------|--------------------|
| Minimum contact pressure required for load transfer \( p_{\text{min}} \) | 55.9 Mpa           |
| Minimum amount of interference required for load transfer \( \delta_{\text{min}} \) | 0.02mm             |
| Maximum contact pressure allowed without plastic deformation \( p_{\text{emax}} \) | 215.2 Mpa          |
| Maximum amount of interference allowed without plastic deformation \( \delta_{\text{emax}} \) | 0.077mm            |
| Maximum contact pressure for controlling tooth deformation \( p_{p_{\text{max}}} \) | 186.6 Mpa          |
| Maximum amount of interference for controlling tooth deformation \( \delta_{p_{\text{max}}} \) | 0.067mm            |
| Basic amount of interference between gear and shaft \( \delta_b \) | 0.044mm            |
4.2 Finite element simulation analysis

The finite element analysis model of gear and shaft is constructed according to the parameters of the design example in the previous section. Then the calculation results $\delta_{\text{min}}$, $\delta_{\text{emax}}$ and $\delta_{\text{pmax}}$ in Table 1 are taken as the amount of interference between gear and shaft respectively for finite element analysis. The contact pressure distributions of gear are shown in Fig. 4, Fig. 5 and Fig. 6 respectively.

According to the contact mechanics, there is edge effect in the interference joint with unequal length of shaft and hub, which inevitably leads to stress concentration at the contact boundary [12]. Therefore, the contact pressure and stress value should be taken from the middle area of the gear fitting surface for comparative analysis. The contact pressure in the middle area is 47.4 MPa in Fig. 4, which is close to the result $p_{\text{min}} = 55.99$ MPa in Table 1. But in Fig. 5 and Fig. 6, the contact pressure in the middle area of fitting surface is 188.4 MPa and 163.6 Mpa, which is slightly different from $p_{\text{emax}} = 215.2$ Mpa and $p_{\text{pmax}} = 186.6$ Mpa respectively in Table 1. The main reason is there are tooth grooves accounting for nearly 1/2 of the tooth area, which will affect the stress distribution of the gear after interference assembly. In this paper, when the gear is transformed into theoretical calculation model of thick wall cylinder, the gear pitch circle is taken as the outer circle of the model. Although the error is larger compared with the normal thick wall cylinder model, the calculation accuracy is higher than that of taking the gear addendum circle as the outer circle of the theoretical model. Therefore, how to reasonably determine the outer circle diameter of the theoretical calculation model remains to be further studied.

Fig. 4. The contact pressure distribution when $\delta_{\text{min}}=0.02$mm.

Fig. 5. The contact pressure distribution when $\delta_{\text{min}}=0.077$mm.
Fig. 6. The contact pressure distribution when $\delta_{\text{min}}=0.067\text{mm}$.

5. Conclusions
According to the elastic-plastic mechanics, the analytical equations calculated for contact stress and radial displacement of the fitting contact surface of the gear and shaft are established. In order to calculate the amount of interference and design the parameters of interference fit, three limiting conditions such as load transfer capacity of gear, no plastic deformation of fitting surface and control of gear tooth deformation are brought into the calculation. Then the theoretical calculation and finite element simulation analysis are carried out through the design example. The results of calculation and analysis show that the interference fit of gear and shaft will cause a significant deformation of tooth profile and affect the gear transmission accuracy. It is necessary to design appropriate amount of interference and strictly control the deformation of tooth profile when designing the interference fit. At the same time appropriate measures should be taken in the production process to reduce the adverse effects of interference fit between gear and shaft on the transmission performance.

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