INTRODUCTION

The polarization data available at present for proton-proton scattering at relatively high energies are compatible with the $s$-channel helicity conservation \cite{1-3} and show that the polarization contribution decreases with increasing energy as it is expected in terms of Regge pole exchanges. The data are insufficiently precise to provide unambiguous conclusion at the higher energy, although a contribution from the helicity-flip amplitude cannot be excluded \cite{2,4-6}. The existing polarized experiments \cite{7-10} at energy lower than LHC allow one to study spin properties of proton-pomeron vertex at intermediate energies while conclusions about spin properties of amplitudes at high $t$ are derived as rule from model extrapolations.

The main conclusion was made that pomeron exchange is expected to produce the observed small spin effects.

Despite a long time interest to spin effects physics in hadron interaction the available data set is not rich in a soft kinematic region. Firstly, the main part of experiments was performed for meson-nucleon and electron-proton interactions. Secondly, these measurements were made at low and intermediate energies. Nevertheless, the available data stimulate permanently the theoretical and phenomenological studies of spin phenomena in the regions of soft and hard kinematic, where different approaches (nonperturbative approaches from early \cite{11} to relatively later and recent \cite{6,12,15}, and perturbative QCD) are exploring \cite{16}.

Unfortunately, the experimental data on spin observable quantities at the highest energies are almost absent, though they may provide additional information about helicity amplitudes properties. As we known only the data at FNAL energies \cite{7-9} and the relatively recent data from RHIC \cite{10} (however, at very small $t$, in Coulomb-nuclear interference region) are available.

However, now thanks to a series of CERN TOTEM proton-proton experiments \cite{17-20} we possess the long-awaited complete set of the precise data for testing various models over a wide energy and momentum transfer range. Proton-proton (antiproton) scattering is the unique possibility to investigate crossing-odd contribution and its spin-flip properties.

The Froissaron and Maximal Odderon model (the FMO model) \cite{21-24} has recently successfully described such an extended set of experimental data. Therefore, the idea to extend the application of this model to consider some of the spin effects, for instance, polarization data of $pp$ scattering at relatively high energy and not too large momentum transfers we found that this model taking into account the spin is available to describe not only the differential, total cross section and $\rho$, but also the existing experimental data on polarization.

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FMO amplitude. In the Section \[\text{III}\] we discuss the original FMO model with spin-flip amplitudes. The results of comparison of both models with the data at $\sqrt{s} > 19$ GeV and at intermediate $t$ is given in the Section \[\text{III}\].

To avoid the inevitable increase in the number of parameters, we use similarly to many authors [1, 2, 12, 14, 15] some simplified assumptions about the spin-non-flip and spin-flip amplitudes and their relationship.

I. DEFINITIONS AND THE FMO APPROACH

A. Helicity amplitudes, observables

Generally the proton-proton and antiproton-proton scattering amplitude reads as

$$A_{pp}(s, t) = A^{(+)}(s, t) \pm A^{(-)}(s, t).$$

(1)

In this model we used the following normalization of the spin-averaged physical amplitudes

$$\sigma_{el}(s) = \frac{1}{\sqrt{s(s - 4m^2)}} \text{Im} A(s, 0),$$

$$\frac{d\sigma_{el}}{dt} = \frac{1}{64\pi ks(s - 4m^2)} |A(s, t)|^2$$

(2)

where $k = 0.3893797$ mb \cdot GeV\(^2\). With this normalization the amplitudes have dimension mb \cdot GeV\(^2\) and all the couplings are given in millibarns.

Taking into account the spin degrees of freedom in nucleon-nucleon elastic scattering still as a not well defined and quite complicated procedure. There are no strict and consequent methods to construct 5 independent helicity amplitudes for elastic nucleon-nucleon interaction:

$$\Phi_1(s, t) = (+ + |T| + +),$$
$$\Phi_2(s, t) = (+ + |T| + -),$$
$$\Phi_3(s, t) = (+ - |T| + +),$$
$$\Phi_4(s, t) = (+ - |T| + -),$$
$$\Phi_5(s, t) = (+ + |T| + +).$$

(3)

Only various phenomenological models for these amplitudes have been constructed and analyzed. Each of these amplitudes has crossing-even and crossing-odd components $\Phi_i(s, t), \ i = 1, 2, ..., 5$

$$\Phi_{pp}(s, t) = \Phi^{(+)}(s, t) \pm \Phi^{(-)}(s, t).$$

(4)

For the spin-averaged initial protons (antiprotons) the following equations for the total and differential cross sections and polarization are used

$$\sigma_{tot} = \frac{1}{\sqrt{s(s - 4m^2)}} \text{Im} (\Phi_1 + \Phi_3),$$

$$\frac{d\sigma_{el}}{dt} = \frac{1}{16\pi ks(s - 4m^2)} \times (|\Phi_1|^2 + |\Phi_2|^2 + |\Phi_3|^2 + |\Phi_4|^2 + 4|\Phi_5|^2),$$

(5)

$$P(t) = -2 \frac{\text{Im}[\Phi_1 + \Phi_2 + \Phi_3 - \Phi_4] \Phi_5^*}{|\Phi_1|^2 + |\Phi_2|^2 + |\Phi_3|^2 + |\Phi_4|^2 + 4|\Phi_5|^2}. $$

(7)

Therefore in what follows we consider the case with $\Phi_2 = \Phi_3 = \Phi_4 = 0$, assuming in accordance with a common opinion that other spin amplitudes are small at high energies [1, 2, 4, 6, 12, 14, 15].

B. FMO approach

Let’s comment shortly the Foissaron and Maximal Odderon approach to elastic proton-proton (antiproton) elastic scattering. Two realizations of it are explored in the paper.

Historically it was formulated in [21] and applied [22, 24] in description of high energy data on $pp$ and $\bar{p}p$ scattering including the newest TOTEM data at 13 TeV.

The maximal growth of the even-under-crossing amplitude $A^{(+)}$ allowed by unitarity:

$$A^{(+)}(s, t = 0) \propto is \ln^2 \bar{s}, \quad \bar{s} = -is/s_0, \quad s_0 = 1 \text{ GeV}^2$$

leads via the optical theorem to the difference of the antiparticle-particle and particle-particle total cross sections

$$\Delta \sigma \propto \ln(s/s_0)$$

(11)

which grows, in absolute value, with energy. However, the sign of $\Delta \sigma$ is not fixed by general principles.

Such a behaviour [1] is often referred to as “Foissaron”, while the behaviour [10] is termed as “Maximal Odderon”. At $t = 0$, they corresponds in the $j$-plane to a triple pole located at $j = 1$. It was shown in [22] that Froissaron and Maximal Odderon model describes perfectly all the forward scattering TOTEM data [18, 20], including the surprisingly small value of the ratio $\rho$ at $\sqrt{s} = 13$ TeV. Moreover, fit to the data at $t = 0$ of the model with a more general form of FMO [23] shows that one returns to the solution with Froissaron and Maximal Odderon. Extension of the FMO model for $t \neq 0$ was suggested in [24] and led to quite good fit in the region $\sqrt{s} \geq 5$ GeV at $t = 0$ and $19 \text{ GeV} \sqrt{s} \leq 13$ TeV at $0 < |t| \leq 5 \text{ GeV}^2$. Thus, it would be interesting to consider at least some of the spin effects within this approach.
II. THE MODELS FOR SPIN-NON-FLIP AND SPIN-FLIP AMPLITUDES

As noted above, our aim is to study the spin-flip effects, assuming that they are determined mainly by the \( \Phi_5 \) term. Therefore, we apply here the assumption here the assumption \( \Phi_2 = \Phi_3 = \Phi_4 = 0 \) to the models considered. If \( \Phi_3 \neq 0 \) but has the same functional form as \( \Phi_1 \), then just redefinition of the couplings in \( \Phi_1 \) can be applied.

A. Simplified FMO model (Model A)

Only the main terms of Froissar and Maximal Odderon amplitudes in FMO model [24] are taken into account in the model A. At \( t = 0 \) they correspond to triple poles in the \( s \)-plane. This truncated crossing-even and crossing-odd terms of FMO model are noted as \( P_i^T(s,t) \) and \( O_i^T(s,t) \), respectively.

Thus, in the model A the crossing-even and crossing-odd parts of the both spin amplitudes at high energy are chosen as follows:

\[
\begin{align*}
\Phi_1^{(e)}(s,t) &= P_1(s,t) + R_1^z(s,t) \pm [O_1(s,t) + R_1^z(s,t)], \\
P_1(s,t) &= P_1^T(s,t) + P_1^D(s,t) + P_1^S(s,t), \\
O_1(s,t) &= O_1^T(s,t) + O_1^D(s,t) + O_1^S(s,t),
\end{align*}
\]

where \( i = 1, 5 \). The full explicit expressions for the model A are given in Appendix A.

Spin-flip amplitudes are written in the model A in the following form:

\[
\begin{align*}
P_5^{(T,D,S,f)}(s,t) &= \frac{\sqrt{-t}}{2m} \lambda_+(t) P_{1}^{(T,D,S,f)}(s,t), \\
O_5^{(T,D,S,\omega)}(s,t) &= \frac{\sqrt{-t}}{2m} \lambda_-(t) O_{1}^{(T,D,S,\omega)}(s,t), \\
P_1^f &= R_1^f, \quad O_1^\omega = R_1^\omega.
\end{align*}
\]

For \( \lambda_{\pm}(t) \) we try three variants in the fit to experimental data:

V.1 \( \lambda_{\pm}(t) = 1 \);
V.2 \( \lambda_{\pm}(t) = 1 + p_{\pm} t \);
V.3 \( \lambda_{\pm}(t) = 1 + p_{1,\pm} t + p_{2,\pm} t^2 \).

Matching the model with experimental data are given in Sect. 11.

B. Full FMO model (Model B)

For spin-non-flip amplitudes we use the FMO model at \( t \neq 0 \) [24]. The only difference with [24] is a notation for constants and functions.

\[
\begin{align*}
\Phi_1^+(z_i, t) &= P_{1}^{(F)}(z_i, t) + P_{1}^{(1)}(z_i, t) + P_{1}^{(2)}(z_i, t) \\
&+ P_{1}^{(h)}(z_i, t) + R_1^+(z_i, t), \\
\Phi_1^-(z_i, t) &= O_{1}^{(M)}(z_i, t) + O_{1}^{(1)}(z_i, t) + O_{1}^{(2)}(z_i, t) \\
&+ O_{1}^{(h)}(z_i, t) + R_1^-(z_i, t),
\end{align*}
\]

where \( z_i = -1 + 2s/(4m^2 - t) \approx 2s/(4m^2 - t) \) and \( i = 1, 5 \).

In Eq. (15) \( P_{1}^{(F)}, O_{1}^{(M)} \) are the Froissaron and Maximal Odderon contributions, \( P_{1}^{(1)}, O_{1}^{(1)} \) are the standard (single \( j \)-pole) pomeran and odderon contributions and \( R_1^+, R_1^- \) are effective \( f \) and \( \omega \) single \( j \)-pole contributions, where \( j \) is an angular momentum of these reggeons. \( P_{1}^{(2)} \) and \( O_{1}^{(2)} \) are double \( PP, OO \) and \( PO \) cuts, respectively. We take into account the ”hard” pomeran and odderon \( P_{1}^{(h)}, O_{1}^{(h)} \) as well.

We consider the model at \( t \neq 0 \) and at energy \( \sqrt{s} > 19 \text{ GeV} \), so we neglect the rescatterings of secondary reggeons \( R_1^+, R_1^- \) with \( P \) and \( O \). In the considered kinematical region they are small. Besides, because \( f \) and \( \omega \) are effective, they can take into account small effects from the cuts via their parameters.

The explicit expressions for the spin-non-flip amplitudes in the full FMO model are given in Appendix B.

We used the following forms of spin-flip amplitudes for the fit:

\[
\begin{align*}
\Phi_5^+(z_i, t) &= \frac{\sqrt{-t}}{2m} \lambda_+(t) \\
&\times \left\{ P_{5}^{(F)}(z_i, t) + P_{5}^{(e,f)}(z_i, t) + R_5^+(z_i, t) \right\}, \\
\Phi_5^-(z_i, t) &= \frac{\sqrt{-t}}{2m} \lambda_-(t) \\
&\times \left\{ O_{5}^{(M)}(z_i, t) + O_{5}^{(e,f)}(z_i, t) + R_5^-(z_i, t) \right\}.
\end{align*}
\]

For the sake of reduced number of free parameters we consider the sum of all standard pomeran (odderon) terms as one effective pomeran (odderon):

\[
\begin{align*}
P_{1}^{(e,f)}(z_i, t) &= P_{1}^{(1)}(z_i, t) + P_{1}^{(2)}(z_i, t) + P_{1}^{(h)}(z_i, t), \\
O_{1}^{(e,f)}(z_i, t) &= O_{1}^{(1)}(z_i, t) + O_{1}^{(2)}(z_i, t) + O_{1}^{(h)}(z_i, t).
\end{align*}
\]

Then

\[
\begin{align*}
P_{5}^{(F)}(z_i, t) &= h_5 e^{\beta_5(F)\tau_p} P_{1}^{(F)}(z_i, t), \\
\tau_p &= 2m - \sqrt{4m^2 - t}, \\
P_{5}^{(e,f)}(z_i, t) &= g_5^{(F)} e^{\beta_5^{(F)}\tau_p} P_{1}^{(e,f)}(z_i, t), \\
R_5^+(z_i, t) &= g_5^{+} e^{\beta_5^{+}\tau_p} R_1^+(z_i, t),
\end{align*}
\]
TABLE I. Data description in the Simplified FMO model (S-FMO) and original FMO model (FMO) with the three choices of the functions \(\lambda_\pm(t)\) in the spin-flip amplitudes. \(N\) is the total number of experimental points in the fit. Comments on the number of free parameters in fits are given in Appendix C.

| Process | Observable | \(N\) | S-FMO model, \(\chi^2/N_0\) | FMO model, \(\chi^2/N_0\) |
|---------|------------|------|-----------------------------|-----------------------------|
| \(pp \rightarrow pp\) | \(\sigma_{\text{tot}}\) | 110  | 0.899 0.864 0.865 | 0.880 0.890 0.894 |
| \(p\bar{p} \rightarrow p\bar{p}\) | \(\sigma_{\text{tot}}\) | 58   | 2.193 1.271 1.130 | 0.896 0.900 0.868 |
| \(pp \rightarrow pp\) | \(\rho\) | 67   | 1.788 1.586 1.587 | 1.563 1.562 1.564 |
| \(p\bar{p} \rightarrow p\bar{p}\) | \(\rho\) | 12   | 1.267 0.658 0.595 | 0.389 0.379 0.394 |
| \(pp \rightarrow pp\) | \(d\sigma/dt\) | 1701 | 1.728 1.630 1.587 | 1.522 1.513 1.514 |
| \(\bar{p}\bar{p} \rightarrow \bar{p}\bar{p}\) | \(d\sigma/dt\) | 389  | 1.359 0.943 0.919 | 1.106 1.063 1.094 |
| \(pp \rightarrow pp\) | \(P(t)\) | 49   | 0.920 1.742 1.483 | 0.908 0.882 1.105 |
| \(p\bar{p} \rightarrow p\bar{p}\) | \(P(t)\) | 35   | 1.648 1.493 1.449 | 1.410 1.405 1.405 |

\(n_{\text{par}}\), number of free parameters
\(\chi^2/\text{NDF} = \chi^2/(\sum N - n_{\text{par}})\)

III. RESULTS

Free parameters of the models were determined from the fit to the data on \(\sigma_{\text{tot}}(s)\), \(\rho(s,0)\), \(d\sigma(s,t)/dt\) and \(P(t)\) in the region

For the functions \(\lambda_\pm(t)\) we have explored the same variants as used in the model A (Sect. II A).

\[
\begin{align*}
O_5^{(M)}(z_t,t) &= \alpha_5 e^{g_5(M)\tau_0} O_1^{(M)}(z_t,t), \\
\tau_0 &= 3m_\pi - \sqrt{9m_\pi^2 - \sqrt{s}}, \\
O_5^{(eff)}(z_t,t) &= g_5^{(O)} e^{g_5(O)\tau_0} O_1^{(eff)}(z_t,t), \\
R_5^{-}(z_t,t) &= g_5 e^{g_5\tau_0} R_1^{-}(z_t,t).
\end{align*}
\]
The data on $\sigma_{\text{tot}}$ and $\rho$ are taken from the Particle Data Group site [26]. The recent TOTEM data [17–20] also were added. We did not include to the fit two ATLAS points on $\sigma_{\text{tot}}$ at 7 and 8 TeV (see discussion in [23]). The data on differential cross sections were collected from the big number of the papers from FNAL, ISR, CERN and other experimental collaborations. They can be found at the Repository for publication-related High-Energy Physics data [27]. Data on polarization included in the data set for fit are taken from Refs. [8, 9].

Results of the fits in all considered variants of the models A and B are shown in Table I. Because of the relatively small differences in $\chi^2$ we have plotted the Figures 1 - 5 with theoretical curves for both the models in the simplest variants V1 with $\lambda = 1$. The values and errors of free parameters in the chosen variant V1 are given in Appendix C in Tables II and Table III. One can see from the Figures a quite good agreement with the data on cross sections and polarization in the kinematic region (20).

We would like to note here a little bit overestimated total cross sections at the highest energies and as consequence lower absolute values of the ratio $\rho$ in the Simplified FMO model. Additional terms, the “hard” pomeron and odderon (exactly as it is written in the original FMO model) can fix the problem (then $\chi^2$ and curves for $\sigma_{\text{tot}}$, $\rho$ in both the models almost coincide). We do not discuss here this solution.

Fig. 2 demonstrates that the ratios of spin-flip amplitudes to spin-non-flip ones are more important at small and middle $|t|$, decreasing with $|t|$. The S-FMO model predicts more strong spin effects, in particular, in the region around dip in the differential cross sections and at higher $|t|$. One can see in Fig. 7 the influence of odderon in the spin-non-flip and spin-flip amplitudes in the considered models. Contribution of the Maximal Odderon in non-flip amplitudes at not highest energies is stronger.

![Fig. 2. Differential $pp \rightarrow pp$ cross sections at the FNAL and ISR energies](image)

![Fig. 3. Differential $pp \rightarrow pp$ cross sections at the LHC energies](image)

![Fig. 4. Differential $\bar{p}p \rightarrow \bar{p}p$ cross sections at the FNAL, ISR, SPS and Tevatron energies](image)
in the original FMO model (red lines) and decreases with $|t|$ slower than in the S-FMO model (blue lines). In the $^9$TeV region situation is opposite. The Maxi-
mal Odderon to Froissaron terms ratio in the spin-flip components (dashed lines) ($|\Phi^-|/|\Phi^+_5| \approx 10\%$ at the $|t| \gtrsim 1 - 2 \text{ GeV}^2$) in these models is higher than it was

The last comments in fact give the answers to the ques-
tions addressed in Introduction about the role and mag-
nitude of spin effects in the models within the FMO ap-
proach.

We confirm conclusions of Refs. [1, 2, 4, 6, 14] obtained
in more traditional pomeron and odderon models that
the crossing-even and crossing-odd spin-flip amplitudes
are important for agreement with experimental data. It
would be not worse to note that polarization in FMO
model goes to 0 with rising $|t|$, while in S-FMO it does
not vanish at least at a few GeV$^2$.

Thus, concluding the brief discussion of the results we
would like to emphasize that considered FMO models
taking into account the spin-flip effects are in agreement
with available experimental data. They predict more im-
portant role of the spin-flip amplitudes at higher ener-
gies and momenta transferred than it is predicted by the
traditional Regge pole models. Unfortunately, however,
there are no data at high energy to confirm or deny these
predictions.

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Appendix A: Simplified FMO model

The simple and dipole Pomerons (Odderon) for both components have a conventional form:

\[ P_1^{(s)}(s,t) = -g_1^{(S)} s_{\alpha_p}(t) e^{\beta_1^{(S)} \tau_p}, \]
\[ P_1^{(D)}(s,t) = -g_1^{(D)} s_{\alpha_p}(t) e^{\beta_1^{(D)} \tau_p}, \]  
\[ \tau_p = 2m_p^2 - \sqrt{4m_p^2 - t}, \]
\[ \alpha_p(t) = 1 + \alpha_p' t, \]
\[ \alpha_o(t) = 1 + \alpha_o' t. \]  

(A1)

Contributions of the secondary reggeons, \( f \) and \( \omega \) have a standard form

\[ R_1^{(f)}(s,t) = -g_1^{(f)} s_{\alpha_f}(t) e^{\beta_1^{(f)} \tau_f}, \]
\[ R_5^{(f)}(s,t) = -g_5^{(f)} \sqrt{-t} \lambda_p(t) s_{\alpha_f}(t) e^{\beta_5^{(f)} \tau_f}, \]  
\[ \alpha_f(t) = \alpha_f(0) + \alpha_f' t, \]
\[ \alpha_o(t) = \alpha_o(0) + \alpha_o' t. \]  

(A8)

Appendix B: Original FMO model

We write here the explicit spin-non-flip terms of the FMO model at \( t \leq 0 \) just for reader’s convenience. The only difference of [24] is a notation for constants and functions. Spin-flip terms are presented and discussed in Section IIIB.

Froissaron and Maximal Odderon are written in the following form

\[ \frac{1}{iz} P_1^{(F)}(z_e, t) = h_{1,1} \xi^2 e^{2J_1(t + \tau \xi)} e^{\beta_1^{(R)} \tau_p} \]
\[ + h_{1,3} \xi \sin(r + \tau \xi) e^{\beta_1^{(G)} \tau_p}, \]  
\[ \frac{1}{iz} O_5^{(M)}(z_e, t) = o_{1,1} \xi^2 e^{2J_1(t + \tau \xi)} e^{\beta_1^{(O)} \tau_o} \]
\[ + o_{1,3} \xi \sin(r + \tau \xi) e^{\beta_1^{(O)} \tau_o}, \]  
\[ \tau = \sqrt{-t}, \quad t_o = 1 \text{ GeV}^2, \quad r = r - \delta_r, \quad \delta_r \geq 0, \]  
\[ \text{was found in Ref. [24] that the best data description is achieved for minimal allowed} \]  
\[ \delta_r \geq 0. \]  
\[ \text{The standard Regge pole contributions have the form} \]
\[ R_1^{(K)}(z_e, t) = -\frac{1}{iz} 2m² g_1^{(K)} (-iz_e) \alpha_K(t) \]
\[ \times \left[ d_k e^{\beta_1^{(K)} t} + (1 - d_k) e^{\beta_1^{(K)} t} \right], \]  
\[ \alpha_K(t) = \alpha_K(0) + \alpha_K' t, \]
\[ K = P, O, f, \omega, \quad k = p, o, \quad d_k = 1. \]  

They take into account a possibility of a non pure exponential behaviour of the vertex functions for the standard pomeron and odderon [23].

The factor \( 2m² \) is inserted in amplitudes \( R_1^{(K)}(z_e, t) \) in order to have the normalization for amplitudes and dimension of coupling constants (in millibarns) coinciding
with those in \[22\]. The same is made for all other amplitudes.

We have added in the FMO the double pomeron and odderon cuts, \(PP, OO, PO\) in their usual standard form without any new parameters as well. Namely,

\[
P_1^{(PP)}(z_t, t) = P_1^{(PO)}(z_t, t) + P_1^{(OO)}(z_t, t),
\]

\[
O_1^{(PP)}(z_t, t) = O_1^{(PO)}(z_t, t),
\]

\[
P_1^{(PP)}(z_t, t) = -i \frac{2m^2(z_t g_1^{(P)})^2}{16\pi s \sqrt{1 - 4m^2/s}} \left\{ \frac{d_{p}^{2}}{2B_{1}^{p}} \exp(tB_{1}^{p}/2) + 2d_{p}(1 - d_{p}) \exp(tB_{1}^{p}/B_{1}^{p} + B_{2}^{p}) + (1 - d_{p})^2 \exp(tB_{2}^{p}/2) \right\},
\]

\[
P_1^{(OO)}(z_t, t) = -i \frac{2m^2(z_t g_1^{(O)})^2}{16\pi s \sqrt{1 - 4m^2/s}} \left\{ \frac{d_{o}^{2}}{2B_{1}^{o}} \exp(tB_{1}^{o}/2) + 2d_{o}(1 - d_{o}) \exp(tB_{1}^{o}/B_{1}^{o} + B_{2}^{o}) + (1 - d_{o})^2 \exp(tB_{2}^{o}/2) \right\},
\]

\[
P_1^{(PO)}(z_t, t) = \frac{2m^2(z_t g_1^{(P)} g_1^{(O)})}{16\pi s \sqrt{1 - 4m^2/s}} \left\{ \frac{d_{p}d_{o}}{2B_{1}^{p}B_{1}^{o}} \exp(tB_{1}^{p}B_{1}^{o}/2) \right\} \times \left\{ \frac{d_{p}(1 - d_{p})}{2B_{1}^{p}B_{2}^{o}} \exp(tB_{1}^{p}B_{2}^{o}/B_{1}^{p} + B_{2}^{o}) \right\}
\]

\[
(1 - d_{p})d_{o} \exp(tB_{2}^{p}B_{1}^{o}/B_{1}^{p} + B_{2}^{o}) + (1 - d_{p})(1 - d_{o}) \exp(tB_{2}^{p}B_{2}^{o}/2B_{1}^{o} + B_{2}^{o}) \right\},
\]

where \(B_{k}^{p,o} = b_{k}^{p,o} + \alpha_{r}^{f} \ln(-iz_t), \ k = 1, 2, \ b_{1,k}^{p,o}\) are the constants from single pomeron and odderon contributions.

In \[24\] it was noted that for a better description of the data it is advisable to add to the amplitudes the contributions that mimic some properties of “hard” pomeron \(\langle P^{h}\rangle\) and odderon \(\langle O^{h}\rangle\). We take them in the simplest form

\[
P_1^{(h)}(t) = i2m^2z_t \frac{g_{h,p}}{(1 - t/t_{h,p})^4},
\]

\[
O_1^{(h)}(t) = 2m^2z_t \frac{g_{h,o}}{(1 - t/t_{h,o})^4}.
\]

### Appendix C: Parameters in the models

The number of free parameters is varied in various fits, because we put a reasonable (in our opinion) limits for slopes in the exponential vertexes \(0 \leq b, \beta \leq 20\) GeV\(^{-1}\) and for those of trajectories \(0.8 \leq \alpha_{r}^{f} \leq 1.1\) GeV\(^{-2}\). If during fitting a parameter goes to the limit value, then it is fixed at the corresponding limit.

| Name | Dimension | Value | Error |
|------|------------|-------|-------|
| \(g_1^{(P)}\) | mb | 0.29343E+01 | 0.22854E-03 |
| \(\rho_1^{(P)}\) | GeV\(^{-1}\) | 0.51823E+01 | 0.25651E-02 |
| \(r_p\) | GeV\(^{-1}\) | 0.29661E+00 | 0.57101E-04 |
| \(\alpha_{r}^{f}\) | GeV\(^{-2}\) | 0.16361E+00 | 0.39431E-03 |
| \(g_1^{(O)}\) | mb | -0.76106E+00 | 0.11876E-02 |
| \(\rho_1^{(O)}\) | GeV\(^{-1}\) | 0.19434E+01 | 0.32444E-02 |
| \(g_1^{(D)}\) | mb | 0.27402E+02 | 0.40164E-01 |
| \(\rho_1^{(D)}\) | GeV\(^{-1}\) | 0.41323E+01 | 0.38042E-02 |
| \(r_o\) | GeV\(^{-2}\) | 0.12120E+01 | 0.45999E-03 |
| \(\alpha_{r}^{o}\) | mb | 0.20536E+00 | 0.25831E-02 |
| \(\alpha_{r}^{f}\) | GeV\(^{-1}\) | 0.65245E+01 | 0.35911E-01 |
| \(\alpha_{r}^{o}\) | mb | 0.13783E+01 | 0.14165E-01 |
| \(\alpha_{r}^{f}\) | GeV\(^{-1}\) | 0.34116E+01 | 0.12406E-01 |
| \(\alpha_{r}^{o}\) | GeV\(^{-2}\) | 0.76576E+00 | 0.15048E-02 |
| \(\alpha_{r}^{f}\) | mb | 0.35167E+02 | 0.24488E+00 |
| \(\alpha_{r}^{o}\) | GeV\(^{-1}\) | 0.00000E+00 | fixed at limit |
| \(\alpha_{r}^{f}\) | mb | 0.61636E+00 | 0.58521E-02 |
| \(\alpha_{r}^{o}\) | GeV\(^{-2}\) | 0.80000E+00 | fixed at limit |
| \(\alpha_{r}^{f}\) | mb | 0.24366E+02 | 0.35434E+00 |
| \(\rho_1^{(P)}\) | GeV\(^{-1}\) | 0.20000E+02 | fixed at limit |
| \(\rho_1^{(O)}\) | mb | 0.98237E+00 | 0.29269E-02 |
| \(\rho_1^{(D)}\) | GeV\(^{-1}\) | 0.97449E+01 | 0.11045E-01 |
| \(\rho_1^{(P)}\) | mb | -0.23787E+02 | 0.71732E-01 |
| \(\rho_1^{(O)}\) | GeV\(^{-1}\) | 0.12183E+02 | 0.11698E-01 |
| \(\rho_1^{(D)}\) | mb | 0.29036E+03 | 0.70315E+00 |
| \(\rho_1^{(P)}\) | GeV\(^{-1}\) | 0.97486E+01 | 0.80005E-02 |
| \(\rho_1^{(O)}\) | mb | 0.57721E-03 | 0.57266E-03 |
| \(\rho_1^{(D)}\) | GeV\(^{-1}\) | 0.26306E+01 | 0.96362E+00 |
| \(\rho_1^{(P)}\) | mb | 0.77319E+01 | 0.13988E+01 |
| \(\rho_1^{(O)}\) | GeV\(^{-1}\) | 0.14185E+02 | 0.63904E-01 |
| \(\rho_1^{(D)}\) | mb | -0.69382E+02 | 0.14333E+01 |
| \(\rho_1^{(P)}\) | GeV\(^{-1}\) | 0.12588E+02 | 0.79085E-01 |
| \(\rho_1^{(O)}\) | mb | -0.58675E+03 | 0.12276E+02 |

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TABLE II – Continued from previous page

| Name      | Dimension | Value         | Error          |
|-----------|-----------|---------------|----------------|
| $\beta^{(1)}_2$ | GeV$^{-1}$ | 0.92251E+01  | 0.12837E+00    |
| $\beta_1^{(0)}$ | mb       | 0.29637E+02  | 0.17742E+01    |
| $\beta_2^{(0)}$ | GeV$^{-1}$ | 0.00000E+00  | fixed at limit |

TABLE III: Parameters of the FMO model (B)

| Name     | Dimension | Value         | Error          |
|----------|-----------|---------------|----------------|
| $h_{1,1}$ | mb        | 0.14330E+00  | 0.11569E-03    |
| $h_{1,2}$ | mb        | 0.29852E+01  | 0.54259E-02    |
| $h_{1,3}$ | mb        | 0.09096E+01  | 0.10925E+00    |
| $r_+=mb$  | 0.26288E+00 | 0.54777E-04 |
| $\beta^{(1)}_1$ | GeV$^{-1}$ | 0.23291E+00 | 0.13574E-02    |
| $\beta^{(2)}_1$ | GeV$^{-1}$ | 0.39343E+01 | 0.44429E-02    |
| $\beta^{(3)}_1$ | GeV$^{-1}$ | 0.15649E+02 | 0.67054E+00    |
| $\alpha_1$ | mb        | -0.42452E+01 | 0.16579E-03    |
| $\omega_1$ | mb        | 0.94686E+00  | 0.54389E-02    |
| $\omega_3$ | mb        | -0.14200E+02 | 0.10239E+00    |
| $\beta^{(1)}_2$ | GeV$^{-1}$ | 0.15717E+01 | 0.34223E-02    |
| $\beta^{(2)}_2$ | GeV$^{-1}$ | 0.56167E+01 | 0.14887E+00    |
| $\beta^{(3)}_2$ | GeV$^{-1}$ | 0.31265E+01 | 0.93369E-02    |
| $\alpha'_p$ | GeV$^{-2}$ | 0.17802E+00 | 0.15904E+03    |
| $g^{(1)}_p$ | mb        | 0.71055E+02  | 0.56702E-01    |
| $d_p$ | mb        | 0.73915E+00  | 0.74388E-03    |
| $b^{(1)}_1$ | GeV$^{-2}$ | 0.52838E+01 | 0.49458E-02    |
| $b^{(2)}_1$ | GeV$^{-2}$ | 0.21387E+01 | 0.29922E-02    |
| $\alpha'_c$ | GeV$^{-2}$ | 0.41252E+02 | 0.15522E-03    |
| $g^{(0)}_c$ | mb        | 0.27241E+02  | 0.36231E-01    |

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TABLE III – Continued from previous page

| Name     | Dimension | Value         | Error          |
|----------|-----------|---------------|----------------|
| $\omega_5$ | mb        | 0.38312E+01  | 0.15409E+00    |
| $\beta^{(0)}_5$ | GeV$^{-1}$ | 0.66434E+01 | 0.13773E+00    |
| $g^{(1)}_5$ | mb        | 0.28732E+01  | 0.34589E+00    |
| $\omega_5$ | mb        | 0.13172E+01  | 0.69818E+00    |
| $\omega_5$ | mb        | 0.18954E+02  | 0.16931E+01    |
| $\omega_5$ | mb        | 0.20000E+02  | fixed at limit |
| $\omega_5$ | mb        | 0.29711E+01  | 0.17843E+00    |
| $\omega_5$ | mb        | 0.00000E+00  | fixed at limit |

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