2-DOF vortex-induced vibration of two tandem square cylinders with varying natural frequency ratios

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Abstract. In this paper, the two-degree-of-freedom (2-DOF) flow-induced vibrations of two square cylinders in tandem are studied numerically. Both cylinders have a mass ratio of 2 and are placed at a distance of $5D$ in the inline direction. A characteristics-based split (CBS) finite element algorithm is employed to solve the incompressible flow equations. The Reynolds number ($Re$) is kept constant at 100, whereas reduced velocity ($U_r$) ranges from 3-13. Computations are made for three natural frequency ratios i.e. $f_r = f_{max}/f_1 = 1, 1.5$ and 2. Results of the upstream cylinder are similar to that of an isolated square cylinder by other researches; however, the downstream cylinder shows different characteristics due to the effect of wake from the upstream cylinder. All the results for frequency ratios 1 and 1.5 overlapped to each other, indicating that the effect of frequency ratio is inconsiderable, however as $f_r$ increases to 2, the effect is observed in both the cylinders. It is to be noted that the effect is sensitive only in the lock-in region. For all $f_r$, fully developed wake patterns at $U_{cr}$ corresponding to the peak vibration amplitudes are of C(2S) type while that for the other higher $U_{cr}$ are of 2S type. Cylinder motion trajectories are also discussed; except $U_{cr}$ = 7 of $f_r = 2$ at which phase between lift and the lateral displacement is 90°, all the trajectories are appeared to be figure “8” type.

1. Introduction

Increasing practical implication of the square section has led the researchers to study its interaction with fluids. The majority of the earlier articles on tandem square cylinders were concentrated on finding the critical spacing ratio. Based on the experimental results, [1] and [2] stated the critical spacing ratio between two tandem square cylinders as 4. Flow for the spacing ratios lower than the critical one was of reattachment type while for the higher values of spacing ratio, both cylinders shed fully developed vortices in their wakes. The sound generation from square cylinders in tandem arrangement studied by [3] observed the direct relationship of spacing ratio with the intensity of sound generation. In [4], based on the spacing ratio, the author concluded three flow regimes: the single slender-body regime ($L/D<1.5$), the reattachment regime ($L/D<5$) and binary vortex regime ($L/D>5$). Recently, [5] studied the effect of $L/D$ on the transverse vibration response of a square cylinder placed in the wake of a stationary cylinder and the impact of gap flow was found significant.

Flow involving multiple cylinders itself is a complex problem and the flow becomes more complex when one or more cylinders are free to vibrate. In a study by [6], transverse galloping was encountered in the downstream cylinder when it was forced to oscillate transversely. [7] numerically studied the flow in two square cylinders, where, flow situation of the vibration case was altered drastically
compared to that of the stationary cylinders case. [8] and [9] investigated the effect of the Reynolds numbers and reduced velocity respectively in 2-DOF wake induced vibration of the downstream cylinder placed in the wake of a stationary upstream cylinder.

Earlier experimental and numerical studies for tandem square cylinders conducted to date are limited to explain the flow characteristics of one of either both cylinders stationary, both cylinders vibrating in cross-flow direction or only downstream cylinder vibrating case in some extent. Flow around the 2-DOF vortex-induced vibration (VIV) of square cylinders placed in the tandem arrangement is still unknown. The primary focus of the present work is to study the different flow characteristics at a different frequency ratios of two square cylinders vibrating with 2-DOF.

2. Model description and boundary conditions.

The rectangular computational domain and the arrangement of two square cylinders in tandem with necessary boundary conditions are presented in Figure 1. In the model, cylinders are elastically mounted in both inline and transverse directions. The lengths of the computational domains are $100D$ and $70D$ in the streamwise and transverse directions, respectively and the centre of the fluid domain lies in the centre of the downstream cylinder (C2) which is at a distance of $30D$ from the inlet. The upstream cylinder denoted as C1 is placed at $(-5D, 0)$.

The Dirichlet boundary condition of $u_1 = U_\infty = 1$, $u_2 = 0$ is applied at the inlet. Slip condition of $\partial u_1/\partial y = 0.0$, $u_2 = 0.0$ is imposed at sidewalls while no-slip condition of $u_1 = \dot{X}$, $u_2 = \dot{Y}$ is imposed on the cylinder surface. The pressure is set to zero as an outlet boundary condition.

The mass ratio $(M_r)$ of the cylinders is 2 and the cylinders are encouraged to reach their maximum oscillation limit by setting the value of structural damping to zero. The frequency ratio $f_r = f_{nx}/f_{ny}$ is varied from 1 to 2 with an increment of 0.5.

3. Governing equations

Continuity equation and Navier-stokes (N-S) equations are the governing equations for an incompressible fluid flow problem.

$$\frac{\partial u_i}{\partial t} + (u_j - w_j) \frac{\partial u_i}{\partial x_j} = \frac{1}{Re} \frac{\partial^2 u_i}{\partial x_i \partial x_j} + \frac{\partial p}{\partial x_i} = 0 \tag{1}$$

$$\frac{\partial u_i}{\partial x_i} = 0 \tag{2}$$
Equations (1) and (2) represent the non-dimensional form in Arbitrary Lagrangian-Eulerian (ALE) formulation, where: $X_i$ represent the Cartesian coordinate system ($i = 1, 2$ where 1 and 2 refer to inline($x$) and transverse($y$) direction respectively; $u$ is the velocity component ($u_1, u_2$) corresponding to $x, y$; $W_i$ is the grid velocity component; and $t$ is the time. Reynolds number ($Re$) is defined as $Re = U_\infty D/\nu$, where $D$ is the width of the cylinder, $U_\infty$ is the free stream velocity and $\nu$ represents the fluid kinematic viscosity. In our model, where both the cylinders are vibrating with 2-DOF, equations (3) and (4) describe the motions of the cylinder in the mass-spring system:

$$\frac{\partial^2 X}{\partial t^2} + \frac{4\pi \xi}{U_1} \frac{\partial X}{\partial t} + \left( \frac{2\pi}{U_1} \right)^2 X = \frac{C_D}{2M_1}$$

(3)

$$\frac{\partial^2 Y}{\partial t^2} + \frac{4\pi \xi}{U_1} \frac{\partial Y}{\partial t} + \left( \frac{2\pi}{U_1} \right)^2 Y = \frac{C_L}{2M_1}$$

(4)

$X$ and $Y$ are the non-dimensional displacements of the square cylinder in $x$ and $y$ directions, respectively; $M_i$ is defined as the ratio of cylinder mass to the mass of the fluid displaced; $\xi$ is structural damping coefficient; Reduced velocity ($U_r$) is the inverse of natural frequency ($f_n$) in the transverse direction. The non-dimensional terms, $C_D$ and $C_L$ are the drag and lift coefficients respectively and are defined as $C_D = 2F_D/(\rho U_\infty^2 D)$ and $C_L = 2F_L/(\rho U_\infty^2 D)$, where $F_D$ and $F_L$ are the horizontal and vertical components of the fluid force respectively being applied on the square cylinder.

For a time step ($\Delta t$), by using CBS solution scheme, equations (1) and (2) are solved to obtain the pressure and velocity, this step requires an arbitrary intermediate velocity to be inserted. Equations 3 and 4 are used to calculate the drag and lift coefficients and the associated displacements of the cylinders by using the Newmark- Beta method. The nodal displacement involving Laplace algorithm, update the grid, which substitutes the previous value in the N-S equation unless the stable state is obtained. Discretization steps used for the equations (1) and (2) are the same as those used by [8] whose further details are explained in [10]. It should be noted that the algorithm used in the current study has been used by author for various incompressible fluid flow problems involving circular or square cylinders. The current codes is validated by [8].

4. Results and discussion

4.1. Cylinder amplitude responses

The inline vibration of the cylinder along with the cross-flow vibration brings a drastic changes in the VIV system when $M_i$ is lower than 6 [11]. Figure 3 shows the variation of the maximum inline and transverse vibration amplitudes ($A/D$) of the cylinders with $U_r$ for different $f_r$. For both cylinders, trends of amplitude responses in both inline and transverse directions for $f_r = 1$ and 1.5 are the same, and that becomes relatively higher in the lock-in region for $f_r = 2$. This finding is similar to that observed by [12] for tandem circular cylinders, where the amplitude curves for $f_r$ ranging from 1 to 1.5 collapses together for both cylinders and changes were observed only beyond $f_r = 1.5$. For all the $f_r$, vibration amplitudes are negligible at lower values of $U_r \leq 5$. Vibration amplitudes jump to the higher values due to the onset of lock-in at $U_r = 6$. For the upstream cylinder, amplitude responses at $f_r = 1$ and 1.5 are similar to those observed by [9, 13] for the single square cylinder at the same $Re$. For $f_r = 1$ and 1.5 the transverse vibration amplitudes of the downstream cylinder are two times the magnitudes of the upstream cylinder but at $f_r = 2$, it is 1.43 times. Nevertheless, the inline amplitude responses of both cylinders are similar for all $f_r$. This shows that the wake effect of the upstream cylinder in the inline response of the downstream cylinder is weak at lower $Re = 100$. From the results of the transverse dynamic responses of the downstream cylinder, it can be concluded that the wake effect of the upstream cylinder in the downstream cylinder is dominating over the effect of the frequency ratio.
4.2. Phase difference

Figure 3 shows the phase difference by which the lift force leads the transverse displacement at representative \( f_r \). As shown in Figure 2, the vibration amplitudes of both cylinders at \( U_r = 5 \) are very small for all \( f_r \). The phase differences at \( U_r = 5 \) are not shown in figure because the force and displacement signals are non-sinusoidal restricting the phase calculation. The lift force and the transverse displacement of both cylinders are in phase only up to \( U_r \leq 7 \) while from \( U_r = 7 \) in the upstream cylinder and from \( U_r = 7 \) and 8 in the downstream cylinder involves phase-switching which is also supported by the falling nature of vibration amplitude curve in Figure 2. Interestingly, the downstream cylinder shows an intermediate phase difference of 90° at \( U_r = 8 \) of \( f_r = 2 \). Such intermediate phase difference is related to the higher net work done input in the system [14] and consequently the highest peak amplitude in the current study.

4.3. Mean force coefficients

In most cases, imposed forces when fluid passes through a bluff body cause the body to oscillate periodically. Figure 4 shows the mean force coefficients (drag and lift) versus \( U_r \). The disturbance is created in the downstream cylinder is due to the changes in the flow patterns from upstream cylinder with the change in \( U_r \). This causes the higher fluctuation in the mean drag of the downstream cylinder as shown in the Figure (b). Comparatively, no significant effect (zero) of \( f_r \) is noticed in the mean lift as these zero mean lifts are related to symmetrical vortex shedding in the wake of the cylinders.
4.4. Flow configurations

The dependence of vortex pattern on the $U_r$ at $f_r = 1$ and 2 is shown in Figure 5. The 2S ($S =$ single vortex) mode is the dominating vortex shedding pattern for all the $f_r$; however, its intensity and configuration are found to be different due to change in the amplitude of the cylinders motion with the $U_r$. At small $U_r$ (i.e., $U_r = 3$), classical Karman vortexes are seen at both $f_r$ indicating a small oscillation of the cylinders. Interestingly when $U_r$ increases to 5, shear layers from the upstream cylinder enlarge in such a way that they enclose the gap between the upstream and downstream cylinders as shown for $U_r = 5$ in Figure 5, which leads the flow to reach the reattachment state and ultimately brings the downstream cylinder to steady state. When $U_r$ further increases to 6, the flow synchronizes with the cylinders motion. This increases the vibration amplitude of the cylinders and leads to shed C(2S) modes of vortices as shown in Figure 5. Compared to $f_r = 1$, the coalescence of the vortices in the wake at $f_r = 2$ occurs at a farther distance signifying the strong vortices corresponding to the higher oscillation amplitude. The spacing between two parallel rows of vortices become much wider at $U_r = 7$ of $f_r = 1$ at which the downstream cylinder achieves its peak oscillation amplitude. On the other hand, for $U_r$ corresponding the peak value of vibration amplitude at $f_r = 2$ (i.e., $U_r = 7$), vortex shedding pattern seems unsymmetrical about the horizontal axis which is possibly due to the intermediate value of phase difference observed between the lift and the transverse displacements.
4.5. Orbital trajectories

Figures 6 and 7 present the trajectories of the cylinders at selected $U_r$ of $f_r = 1$ and 2, respectively. In general, the periodic forces excite the body in such a way that its oscillation frequency in the inline direction is twice of that in the transverse direction, resulting to the formation of figure “8” trajectories. Except for $U_r = 7$ of $f_r = 2$, all the trajectories have figure “8” shape. From figures, it can be seen that the centres of all trajectories in the transverse direction lie at the zero vertical axis indicating zero mean lift while in the inline direction, those centres lie at different non-zero positions indicating non-zero mean drag as discussed in the previous section. It should also be noted that the stream wise portion of the trajectories is much smaller than that of the transverse portion, which supports the fact that the inline oscillation is much smaller when compared to the transverse oscillation. Quantitatively, no transparent differences are noticed between the trajectories of two $f_r$, however orientation of the tip of the trajectories at $f_r = 1$ and $f_r = 2$ are different for most of the $U_r$, which is related with the phase between the inline and transverse vibrations and hence the energy transformation in the system [15].
5. Conclusion

Effects of the inline to transverse frequency ratio ranging from 1 to 2 are studied numerically on 2-DOF vortex-induced vibrations of two square cylinders arranged in tandem. The Reynolds number is kept constant at 100 and the $U_r$ ranged between 3 and 13. Flow governing equations are solved by using a characteristics-based-split (CBS) finite element algorithm. Based on the results, key findings are as follow:

The phase difference between the lift force and transverse displacement is around zero in the lower $U_r$ for all cases. At $U_r = 7$ of $f_r = 2$, the phase has an intermediate value of $90^\circ$ while all other higher reduced velocities have a phase difference of $180^\circ$.

The dynamic responses of both cylinders in the pre-lock-in and after lock-in regions are unaffected due to the change in frequency ratio, but relatively higher dynamic responses are observed in the lock-in region when $f_r$ increases to 2.

Regarding the flow patterns, static, 2S and C(2S) types of the vortex shedding patterns are observed. The vortex shedding corresponding to the maximum vibration amplitude of $f_r = 1$ and 1.5 have two parallel rows of vortices, while, in the case of $f_r = 2$, the vorticity shows an unsymmetrical nature.

Except for $U_r = 7$ of $f_r = 2$, all the motion trajectories are observed to be figure “8” shape. However, their orientations for most of the $U_r$ are not similar at $f_r = 1$ and $f_r = 2$, which is associated with the phase difference between the inline and transverse displacements.

Similar trends of flow characteristics for different frequency ratios in a wide range of reduced velocity show minor role of frequency ratio on flow characteristics, however, the wake from the upstream cylinder has significant effects on the flow characteristics of the downstream cylinder.

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