Manifestations of Space-Time Multidimensionality in Scattering of Scalar Particles

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Abstract

We analyze a possibility of experimental detection of the contribution of the Kaluza-Klein tower of heavy particles to scattering cross-section in a six-dimensional scalar model with two dimensions being compactified to the torus with the radii $R$. It is shown that there is a noticeable effect even for the energies of colliding particles below $R^{-1}$ which may be observed in future collider experiments if $R^{-1}$ is of the order of 1 TeV.

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1 Introduction

Many of theories beyond the standard model exploit two important hypotheses: space-time supersymmetry and multidimensionality (see for example [1]). Unfortunately, the complete realization of these hypotheses in field theoretical or string models meets considerable problems. The most important problem to be solved in SUSY models is to find a mechanism providing SUSY-breaking at the scale of about 1 TeV. Similar problem of finding a mechanism of spontaneous compactification at the energy scale of a few TeV exists in models with extra dimensions. Also, it is well known that the supersymmetry in principle lowers under compactification of a part of space-time dimensions [2]. Thus, it is quite natural to suppose that the SUSY-breaking scale $M_{SUSY} \sim 1 \div 10$ Tev is close to the compactification scale $M_C$. Another argument backing this assumption arises from possible cancellations of non-logarithmic corrections to coupling constant renormalizations, this was discussed in ref. [3]. With such compactification scale one can expect that an evidence of the existence of extra dimensions will be seen in future experiments at supercolliders.

Another serious problem in multidimensional field theoretical calculations is non-renormalizability of the theory. Perhaps the true complete description of fundamental interactions is given by some ultraviolet finite theory. Nowadays there is a common belief that such theory does exist and this is the superstring theory [1]. Unfortunately, calculations in the framework of the superstring theory are very complicated and model dependent. To study low energy effects one has to work within an effective multidimensional non-renormalizable model. In such model physical amplitudes can be calculated by performing renormalizations order by order of perturbation expansion. For this one has to add to the Lagrangian counterterms with higher derivatives, they are supposed to emerge from the complete (superstring) theory as a consequence of the existence of super-heavy particles of Plank mass [4]. Whereas in the framework of a complete finite quantum theory these counterterms of the low energy sector could be calculated explicitly, in the effective non-renormalizable model, we are discussing here, necessary counterterms are added by hand and coupling constants of the new vertices, given by these counterterms, are considered as phenomenological ones. Fortunately, the contributions of these counterterms to the finite part of the amplitudes are of the order $(s/\kappa^2)^n$ [1], where $\sqrt{s}$ is the energy of colliding particles, $\kappa$ can be regarded as the characteristic scale of the complete theory($\kappa \sim M_{Pl}$ in the case of superstrings) and $n \geq 1$. Thus, when $\sqrt{s} \leq M_C \ll \kappa$ these contributions can be neglected.

In the present paper we estimate possible manifestations of effects due to the presence of extra dimensions in a scalar $\Phi^4$-model on six-dimensional space-time $M = M^4 \times T^2$, where $M^4$ is the four-dimensional Minkowski space-time and $T^2$ is the two-dimensional torus. We believe that this simple model captures many features of more realistic multidi-
dimensional models, in particular qualitative behaviour of scattering amplitudes calculated here we expect to be rather generic. We would like to mention also that there are some phenomenologically consistent Kaluza-Klein type models just in six dimensions (see e.g. [1]). Similar calculations were done for a toy scalar field model in (2+2)-dimensional space-time $M = M^2 \times T^2$ in ref. [3].

A few more remarks are in order. The question of physical effects in multidimensional theories but from a different point of view was addressed in [4]. There the authors used the results of the high energy experiments to get the upper bound on the size $R$ of the space of extra dimensions assuming that the first heavy Kaluza-Klein mode had not been yet observed directly. Our approach in the present paper is different in two aspects. First, the authors of [4] considered only one (the first) heavy Kaluza-Klein mode as an exited vector boson. In our calculations here we take into account the whole Kaluza-Klein tower of particles and, hence, all peculiarities of calculations in non-renormalizable theories. Second, we assume that $R^{-1} \sim M_{SUbjective} \sim 1 \div 10 \, TeV$ and look for possible experimental evidence of heavy Kaluza-Klein modes.

In the next section we describe our model and renormalization condition. In sect.3 the results of 1-loop calculations for amplitudes and scattering cross section are presented. Sect.4 is devoted to the analysis of these results. Sect.5 contains some conclusions.

2 The model and renormalization

We consider here a model of one real scalar field $\Phi(x, y)$ on the six-dimensional space-time $M^4 \times T^2$ with the radii of the torus being equal to $R = M_C^{-1}$. The action is given by

$$S = \int d^4x d^2y \left\{ \frac{1}{2} g^{MN} \partial_M \Phi(x, y) \partial_N \Phi(x, y) - \frac{m^2}{2} \Phi^2(x, y) - \frac{g_6}{4!} \Phi^4(x, y) - \frac{h_6}{4!} \Phi^2(x, y) \Box \Phi^2(x, y) + \ldots \right\}, \quad (1)$$

where $M, N = 0, ..., 5$; the metric $g_{MN} = \text{diag}(-1, 1, ..., 1)$, $g_6, h_6$ are six-dimensional bare coupling constants, dots in the integrand denote possible higher derivative terms. The last term is necessary for the renormalization of the 4-point Green function at one loop order. The field $\Phi(x, y)$ is periodic in coordinates $y^1, y^2$:

$$\Phi(x, y + 2\pi R) = \Phi(x, y); \quad R = M_C^{-1},$$

and can be expanded in Fourier series:

$$\Phi(x, y) = \frac{1}{(2\pi R)^2} \sum_{n_1, n_2 = -\infty}^\infty \varphi_n(x) \exp\left\{i\bar{y}n/R \right\},$$
\[ \vec{n} = (n_1, n_2), \quad \vec{y} = (y^1, y^2), \]
\[ \varphi_{-\vec{n}} = \varphi_{\vec{n}}^*. \]

Substituting the Fourier expansion into the action (1) we get

\[
S = \int d^4x \left[ \frac{1}{2} \sum_{\vec{n}} \varphi_{-\vec{n}}(\partial^2 - M_n^2)\varphi_{\vec{n}} + \frac{g_B}{4!} \sum_{\vec{n}, \vec{k}, \vec{p}} \varphi_{\vec{n}} \varphi_{\vec{k}} \varphi_{\vec{p}} \varphi_{-(\vec{n}+\vec{k}+\vec{p})}^* + \right.
\]
\[ + \frac{h_B}{4!} \sum_{\vec{n}, \vec{k}, \vec{p}} \varphi_{\vec{n}} \varphi_{\vec{k}} \Box (4) \varphi_{\vec{p}} \varphi_{-(\vec{n}+\vec{k}+\vec{p})}^* - \frac{h_B}{4!} \sum_{\vec{n}, \vec{k}, \vec{p}} \frac{(\vec{n}, \vec{k})}{R^2} \varphi_{\vec{n}} \varphi_{\vec{k}} \varphi_{\vec{p}} \varphi_{-(\vec{n}+\vec{k}+\vec{p})}^* + \ldots \right],
\]

where \((\vec{n}, \vec{k}) = n_1 k_1 + n_2 k_2\) is the scalar product,

\[
M_n^2 = m^2 + \frac{n^2}{R^2} = m^2 + M_C^2 n^2, \quad n^2 = n_1^2 + n_2^2
\]

and

\[
g_B = \frac{g_6}{(2\pi R)^2}, \quad h_6 = \frac{h_6}{(2\pi R)^2}
\]

are four-dimensional bare coupling constants.

The field \(\varphi_0 \equiv \varphi_0(x)\) with the light mass \(m \ll M_C\) in our simple scalar model would be an analog of the usual vector fields and fermion fields in the Standard Model, whereas the Kaluza-Klein tower of fields \(\{\varphi_{\vec{n}} \neq 0\}\) with heavy masses \(M_n\) would be an analog of a set of fields of a multidimensional extension of the Standard Model. The action (3) describes the four-dimensional theory with infinite number of massive fields, and it is important to notice that, as a consequence of the six-dimensional nature of the model, interactions between modes are determined by a few coupling constants only.

As a process imitating the present possibilities of collider experiments it is reasonable to consider \((2 \text{ light particles}) \to (2 \text{ light particles})\) scattering assuming that \(m \ll M_C\) and the scattering energy satisfies \(E^2 \leq 4M_1^2\). Since \(h_B\) is expected to be proportional to \(\kappa^{-2} \sim M_1^2\) (see discussion in the Introduction) it is natural to assume that \(h_B M_C^2 \sim g_B^2\). So, for the process under consideration the interaction term with the coupling constant \(h_B\) will be taken into account in the next-to-leading order only. Then the scattering amplitude of the light \(\varphi_0\)-particles in the leading approximation is defined by the usual tree contribution of the vertex \(\sim \varphi_0^4\). One can see that heavy modes do not contribute at this level. The reason is that there are no vertices with only one heavy field in the sum in eq. (2) as a consequence of rotational symmetry on the torus \(T^2\). This property is rather generic and holds for other types of scalar interactions and all compact spaces of extra dimensions. Hence, the lowest order where we can hope to find some new effects due to the Kaluza-Klein tower of modes is the one loop order.
To perform 1-loop calculations for scattering of light particles besides the vertex

\[ \frac{g_B}{4!} \phi_0^4 \]  \hspace{1cm} (4)

with the light fields only the following relevant vertices must be taken into account

\[ \frac{g_B}{2} \phi_0^2 \sum_{n} \phi_{\vec{n}} \phi_{-\vec{n}} + \frac{h_B}{4!} \phi_0^2 \Box(4) \phi_0^2. \]  \hspace{1cm} (5)

We will show that due to these vertices the massive Kaluza-Klein fields give considerable contribution to the cross section of the scattering process under investigation.

Regularized two light particle scattering amplitude to the one-loop order in our model has the following form (see, for example, [7]):

\[ A_{\text{reg}}(p_{12}^2, p_{13}^2, p_{14}^2) = g_B \left( 1 - \frac{g_B}{32 \pi^2} \sum_{\vec{n}, \vec{n}' < K^2} \left[ I_n^{\text{reg}}(p_{12}^2) + I_n^{\text{reg}}(p_{13}^2) + I_n^{\text{reg}}(p_{14}^2) \right] \right) \]

\[ + \frac{h_B}{12} \frac{p_{12}^2 + p_{13}^2 + p_{14}^2}{p_{12}^2}, \]  \hspace{1cm} (6)

where \( p_{ij}^2 = (p_i + p_j)^2 \), \( p_i, i = 1, 2, 3, 4 \) are the external momenta of the 1-loop diagram satisfying the conservation law \( p_1 + p_2 + p_3 + p_4 = 0 \) and \( I_n^{\text{reg}}(p_{12}^2) \), \( I_n^{\text{reg}}(p_{13}^2) \), \( I_n^{\text{reg}}(p_{14}^2) \) are regularized 1-loop contributions of the \( \vec{n} \)th mode given by the usual four-dimensional momentum integral

\[ I_n^{\text{reg}}(p^2) = \frac{i}{\pi^2} \int_{\Lambda^2} d^4k \frac{1}{[(p - k)^2 - M_n^2 + i\varepsilon][(k^2 - M_n^2 + i\varepsilon)]} \]  \hspace{1cm} (7)

The last integral is regularized by momentum cut-off, other appropriate regularizations (e.g. the dimensional regularization) will also work here. Thus, we have two regulators in the model: the momentum cut-off \( \Lambda \) regularizing the four-dimensional integrals and the cut-off \( K \) regularizing the sum over \( \vec{n} \).

Let us discuss now the renormalization of our amplitude. The sum of the 1-loop integrals in eq. (6) corresponds to a 1-loop integral of the type (7) in the original six-dimensional theory (1) which is quadratically divergent. Thus, if the calculations were carried out directly in six dimensions one would have to make two subtractions to obtain finite result. Since compactification does not influence the ultraviolet properties of the theory also two subtractions are needed in general to make the amplitude (6) in four-dimensions to be finite. The additional divergence, which is due to the six-dimensional nature of the complete model, reveals itself in the divergence of the sum in (6) after the logarithmical divergence of each term \( I_n^{\text{reg}} \) is removed. It is for this reason that
the regulator $K$ have been introduced. Similar to usual quadratic divergencies in four dimensional models this additional divergence gives rise to quadratic corrections to the renormalization of the coupling constant thus reducing considerably the range of validity of the perturbation theory and, because of this, is quite undesirable. Fortunately, in the case under consideration the quadratic divergencies vanish in the sum of three channels in the amplitude on the mass shell if the renormalization is carried out according to standard condition
\[ A_{\text{reg}}(\mu_s^2, \mu_t^2, \mu_u^2) = g \] at the physical subtraction point satisfying
\[ \mu_s^2 + \mu_t^2 + \mu_u^2 = 4m^2. \]

Here $g$ is the physical (renormalized) coupling constant.

Indeed, expressing $g_B$ in terms of the physical coupling constant the amplitude can be written as
\[ A_{\text{reg}}(p_{12}^2, p_{13}^2, p_{14}^2) = g \left[ 1 - \frac{g}{32\pi^2} \sum_{\vec{n}^2 < K^2} (I_{\text{reg}}^g(p_{12}^2) - I_{\text{reg}}^g(\mu_s^2)) + I_{\text{reg}}^g(p_{13}^2) - I_{\text{reg}}^g(\mu_t^2) 
+ I_{\text{reg}}^g(p_{14}^2) - I_{\text{reg}}^g(\mu_u^2) \right] + h_B \frac{p_{12}^2 + p_{13}^2 + p_{14}^2 - \mu_s^2 - \mu_t^2 - \mu_u^2}{12}. \] Of course, to renormalize the coupling constant $h_B$ additional condition must be imposed. However, this is not necessary for our purpose. The differences of the type $I_{\text{reg}}^g(p_{12}^2) - I_{\text{reg}}^g(\mu_s^2)$ are finite and do not depend on the regulator $\Lambda$. We will represent them as
\[ I_{\text{reg}}^g(p_s^2) - I_{\text{reg}}^g(\mu_s^2) = J \left( \frac{p_s^2}{4M_n^2} \right) - J \left( \frac{\mu_s^2}{4M_n^2} \right), \]
where $J(z)$ is the finite part of the 1-loop integral
\[ J(z) = 2 + \int_0^1 dx \ln (1 - 4zx(1-x)), \]
which is equal to
\[ J(z) = -i\pi \sqrt{\frac{z-1}{z}} + 2 \sqrt{\frac{z-1}{z}} \ln(\sqrt{z} + \sqrt{z-1}) \quad \text{if } z > 1, \]
\[ J(z) = 2 \sqrt{\frac{1-z}{z}} \arctan \sqrt{\frac{z}{1-z}} \quad \text{if } 0 < z \leq 1, \]
\[ J(z) = 2 \sqrt{\frac{z-1}{z}} \ln(\sqrt{1-z} + \sqrt{-z}) \quad \text{if } z \leq 0. \]
It is easy to see that for small \( z \) the expressions (13) and (14) have the following Taylor expansion

\[
J(z) = 2 \left( 1 - \frac{z}{3} - \frac{2z^2}{15} \right) + \mathcal{O}(z^3).
\] (15)

Let us show now that the scattering amplitude (10) is actually finite and does not depend on the cut-off \( K \) on the mass shell. Taking formulas (11) and (15) into account one gets

\[
A_{\text{reg}}(p^2_{12}, p^2_{13}, p^2_{14}) = g \left\{ 1 - \frac{g}{32\pi^2} \sum_{\vec{n}} \int_{\vec{n}^2 < K^2} \left[ -\frac{2}{3} p^2_{12} + p^2_{13} + p^2_{14} - \mu^2 - \mu^2 - \mu^2 \right] \right\}
\] (16)

\[
- \frac{4}{15} \frac{p^4_{12} + p^4_{13} + p^4_{14} - \mu^4 - \mu^4 - \mu^4}{(4M_n^2)^2} + \mathcal{O}\left( \frac{1}{M_n^6} \right)
\]

\[
+ h_B \frac{p^2_{12} + p^2_{13} + p^2_{14} - \mu^2 - \mu^2 - \mu^2}{12}
\]

On the mass shell \( p^2_{12} = s, p^2_{13} = t, p^2_{14} = u \), where \( s, t \) and \( u \) are the Mandelstam variables satisfying the well known kinematical relation

\[
s + t + u = 4m^2.
\]

Together with eq. (9) this amounts to vanishing of the first term in the square brackets and the term proportional to \( h_B \). The rest of the one-loop contribution in the expression (16) is equal to the finite sum so that the cut-off \( K \) can be sent to infinity. We also see that the amplitude renormalized by the condition (8) is independent of the coupling constant \( h_B \) on the mass shell. Finally we have

\[
A_{\text{mass shell}}(s,t) = g \left\{ 1 - \frac{g}{32\pi^2} \sum_{\vec{n}} \left[ J \left( \frac{s}{4M_n^2} \right) - J \left( \frac{\mu^2}{4M_n^2} \right) + J \left( \frac{t}{4M_n^2} \right) - J \left( \frac{\mu^2}{4M_n^2} \right) \right] \right\}
\] (17)

\[
+ J \left( \frac{u}{4M_n^2} \right) - J \left( \frac{\mu^2}{4M_n^2} \right) \}
\]

where \( u = 4m^2 - s - t \) and the subtraction points are related by eq. (9).

3 Heavy modes contribution to the total cross section

In this section we will study the contribution of heavy Kaluza-Klein particles to the total cross section of the scattering of two light particles (zero modes). The most interesting
range of energies is above the threshold of the light particle and below the threshold of the first heavy mode: \( 4m^2 \leq s \leq 4(m^2 + M_C^2) \). Indeed, due to decoupling of heavy modes at low energies (see [14]) the contribution of the Kaluza-Klein tower below \( s = 4m^2 \) is likely to be negligible. On the other hand, if \( M_C \sim M_{SUSY} \) it is very unlikely that the energies of colliders will exceed the threshold of the first heavy particle in the near future. For the purpose of our analysis we found convenient to introduce the following quantity

\[
\Delta^{(N,0)}(s) = 16\pi^2 \frac{\sigma^{(N)}(s) - \sigma^{(0)}(s)}{g \sigma^{(0)}(s)} .
\] (18)

Here \( \sigma^{(N)}(s) \) is the total cross section of the \((2\text{ light particles}) \rightarrow (2\text{ light particles})\) scattering calculated using the amplitude of the type (17) but with summation over \( \vec{n} \) satisfying \( 0 \leq |\vec{n}| \leq N \) only. Physically this means that only a finite number of heavy particles of the Kaluza-Klein tower with \( |\vec{n}| \leq N \) contribute to the 1-loop correction to the total cross section \( \sigma^{(N)}(s) \). The cross section calculated in the complete six-dimensional theory is equal to \( \sigma^{(\infty)}(s) \). Notice that to compute the leading order of \( \Delta^{(N,0)}(s) \) in \( g \) only tree approximation for \( \sigma^{(0)}(s) \) is used in the denominator of (18).

We calculate the total cross sections using formulas (13), (14) and (17) and get the following expression for \( \Delta^{(N,0)} \)

\[
\Delta^{(N,0)}(s) = - \sum_{n>0, \vec{n}^2 \leq N} \left[ J(z_n) - 2 + 2J(y_n) + \frac{J^2(y_n)}{2(1 - y_n)} - J(\tilde{z}_n) - 2J(\tilde{y}_n) \right] ,
\] (19)

where

\[
\begin{align*}
  z_n &= \frac{s}{4m^2} \xi_n ; \quad y_n = \xi_n - z_n < 0 ; \\
  \tilde{z}_n &= \frac{\mu_n^2}{4m^2} \xi_n ; \quad \tilde{y}_n = (\xi_n - \tilde{z}_n)/2 ; \\
  \xi_n &= \left( 1 + \frac{M_C^2}{m^2 \vec{n}^2} \right)^{-1} ,
\end{align*}
\]

and we put \( \mu_t^2 = \mu_u^2 \). For numerical evaluations we take

\[
\frac{m^2}{M_C^2} = 10^{-4} , \quad \frac{\mu_u^2}{m^2} = 10^{-2} .
\]

Since \( \mu_u^2 \ll m^2 \ll M_C^2 \) the function (18) practically depends on the variable

\[
\frac{s}{4M_C^2} \simeq z_1 = \frac{s}{4M_t^2}
\]

only. The plot of \( \Delta^{(\infty)} \) as a function of \( z \) is presented in Fig. 1.
The behaviour of this function shows that the contribution of the Kaluza-Klein tower of particles increases rapidly when the energy of colliding particles grows. We see that accumulation of these contributions is considerable and gives a quite noticeable effect even for energies much below the threshold of the first heavy mode. Thus, \( \Delta^{(\infty,0)} \approx 0.76 \) for \( s = 0.5(4M_C^2) \) and \( \Delta^{(\infty,0)} \approx 0.17 \) for \( s = 0.25(4M_C^2) \). Even for \( s = 0.1(4M_C^2) \) the effect is not that small: \( \Delta^{(\infty,0)} \approx 0.03 \).

It is clear, of course, that due to the convergence of the sum in (19) the heavier the particle of the Kaluza-Klein tower the less it contributes to this sum. Hence the curve \( \Delta^{(\infty,0)} \) actually represents the contributions of a few first modes. (To draw the plot in Fig. 1 we approximated it by \( \Delta^{(20,0)} \). We found that this approximation is sufficiently good, for example, \(|\Delta^{(21,0)}(s) - \Delta^{(20,0)}(s)| < 10^{-9} \) for all \( s \) in the range under investigation.) It is useful to evaluate relative contributions of a few first heavy modes. In a more realistic model comparison of these kind of estimates with experimental data would allow us to conclude how many heavy particles one actually "detects" in the experiment (see discussion in the next Section). To analyse this we calculated the function \( \Delta^{(1,0)} \), representing the contribution of the first heavy mode, for the same range of energies (Fig. 1). From the curves in Fig. 1 one can clearly distinguish the "infinite" number of modes and just one first mode for the energies below its threshold. Thus, the difference between them is about 0.23 for \( s = 0.5(4M_C^2) \) and about 0.05 for \( s = 0.3(4M_C^2) \).

To have more illustrative characteristics let us introduce the quantities

\[
\epsilon_N(z) \equiv \frac{\Delta^{(N,0)}(4zM^2)}{\Delta^{(\infty,0)}(4zM^2)},
\]

which show the relative contribution of the first \( N \) heavy Kaluza-Klein modes, and the quantities

\[
\delta \epsilon_N(z) \equiv \epsilon_N(z) - \epsilon_{N-1}(z),
\]

which show the relative contribution of the \( N \)th mode. The plots for some \( \epsilon_N(s) \) and \( \delta \epsilon_N(s) \) for the same range of energies as before are presented in Fig.2 and Fig.3. Combining the results presented in Fig. 1 - Fig. 3 we conclude that with accuracy about 5 \( \div \) 10% the function \( \Delta^{(\infty,0)}(s) \) in the energy region \( s \sim 0.5M^2 \div 0.9M^2 \) shows the presence of at least 3 \( \div \) 4 first heavy modes in the theory.

### 4 Discussion and conclusions

We have shown that effective field theories, obtained from models in \( 4 + d \) dimensions, provide self-consistent way to calculate possible effects related to their multidimensional nature.
Though our model is not physical we believe that the results (for example, rapid growth of \( \Delta^{(\infty,0)}(s) \) above the threshold of the light particle) capture some general features in the more realistic theories. In the latter case analogous results could be \textit{in principle} used for comparison with experiments as follows. We assume that the low-energy sector of the theory, which is the sector consisting the field \( \varphi_0 \) only, is already well determined. Thus, the value of the coupling constant \( g \) is known (notice, that due to the multidimensional nature of the complete theory the coupling constant of the self-interaction of \( \varphi_0 \) and coupling constants of the interactions of the zero mode with non-zero ones are the same, see \((3), (4), (5)\)), and the total cross section \( \sigma^{(0)}(s) \) of the (2 light particles) \( \rightarrow \) (2 light particles) scattering within the low energy sector can be calculated with sufficient accuracy. Experimentally we measure the total cross section \( \sigma^{\exp}(s) \) and compute the quantity

\[
\Delta^{\exp}(s) = -16\pi^2 \frac{\sigma^{\exp}(s) - \sigma^{(0)}(s)}{g\sigma^{(0)}(s)}.
\]

(cf. \((18)\)). If above the threshold of the light particle \( \Delta^{\exp}(s) = 0 \), there are no any evidence of heavy Kaluza-Klein modes at given energies. If \( \Delta^{\exp}(s) \neq 0 \), there is an evidence of the existence of heavier particles. Next step would be to see which curve \( \Delta^{(N,0)}(s) \) fits the experimental data best. If it is the curve with \( N = \infty \) or sufficiently large \( N \), then this might be considered as an indirect evidence of the multidimensional nature of the interactions, at least in the framework of the given model and given type of compactification. Our calculations suggest that the effect is rather noticeable even for energies below the threshold of the first heavy particle (see Fig. 1). Thus, \( \Delta^{(\infty,0)} \approx 0.03 \) for \( s = 0.1(4M_C^2) \) that, with our supposition \( M_C \sim M_{SUSY} \sim (1 \div 10)TeV \) corresponds to the energy of colliding particles \( \sqrt{s} \sim 0.63(1 \div 10)TeV \). Our numerical estimations show also that a few (\( \sim 3 \div 4 \)) first Kaluza-Klein modes can be ”seen” in the range \( 0.5 < s/(4M_C^2) < 1 \) with \( 5 \div 10\% \) accuracy.

We would like to mention that we also computed the differential cross section of the (2 light particles) \( \rightarrow \) (2 light particles) scattering. However it does not add essentially new information about the relative contributions of the heavy Kaluza-Klein particles.

An interesting question is whether different types of compactification can be distinguished at these energies. The main difference comes from the structure of the spectrum of the states in the theory, namely from the multiplicities of states with the same mass and the rate of the growth of the mass with \( n \) (see \((3)\)). This issue is under investigation now. Also it would be interesting to understand whether one can distinguish between Kaluza-Klein type models and certain models with infinite tower of composite particles.

Two more remarks are in order. Here we considered effects when the energies of the scattering particles are less then the threshold of the first Kaluza-Klein particle. In this case momentum behaviour of the effective (running) coupling constant is mainly defined by the contribution of the lightest mode only \[.8, .9, .10\]. Note that renormalization
group equation for the coupling constant $g$, which is physically important at (relatively low) energies $s \sim M_c^2$, is independent of the coupling constant $h$ of the vertices with derivatives (the latter is important for renormalization only) [11].

Another remark is that there are two cases when heavy modes may give more considerable contributions. The first case includes multidimensional models with non-scalar particles. In such models for certain spaces of extra dimensions some of the heavy Kaluza-Klein modes may give non-zero contributions in the tree approximation of perturbation theory. The second possibility is to consider specific processes for which the tree approximation is absent and the leading contribution is given by the 1-loop diagrams even in the zero mode sector of the theory. Such processes are obviously more sensitive to the heavy Kaluza-Klein modes. These possibilities are also under investigation.

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Figure captions

Fig. 1 Plots of $\Delta^{(\infty,0)}$ and $\Delta^{(1,0)}$ as functions of $z \equiv s/(4M_1^2)$.

Fig. 2 Plots of functions $\epsilon_N(z) = \frac{\Delta^{(N,0)}(4zM_1^2)}{\Delta^{(\infty,0)}(4zM_1^2)}$ for $N = 1, 2, 3, 4, 5$; $\epsilon_\infty \equiv 1$.

Fig. 3 Plots of functions $\delta\epsilon_N(z) = \epsilon_N(z) - \epsilon_{N-1}(z)$ for $N = 2, 3, 4, 5$; $\delta\epsilon_\infty \equiv 0$. 
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