The $\eta/s$ of hadrons out of chemical equilibrium

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Abstract. We study how the shear viscosity, $\eta$, entropy density, $s$, and $\eta/s$ depend on growing hadron chemical potentials resulting from the loss of chemical equilibrium during the evolution of a relativistic heavy-ion collision. Our calculations here are for an interacting pion gas in a system of net baryon number zero. Time evolution of the temperature and pion chemical potential are taken from ideal fluid hydrodynamic calculations of RHIC and LHC collisions. We find that the shear viscosity decreases whereas the entropy density increases with increasing pion chemical potential resulting in values of $\eta/s$ that are slightly reduced from the case of chemical potentials being zero when chemical equilibrium prevails. Our results indicate that the inclusion of additional mesons and baryons will likely lead to further reduction in the value of $\eta/s$.

1. Introduction
The objective of this work is to explore the influence of nonzero hadron chemical potentials on the shear viscosity, $\eta$, entropy density $s$, and their ratio $\eta/s$ in the hadronic phase of a relativistic heavy-ion collision. Soon after their formation, hadrons are in a dense medium in which both inelastic and elastic processes serve to keep the chemical potentials of each species zero. As evolution proceeds, the expansion of the system causes the local density to decrease which results in an imbalance of the rates of forward and backward reactions. Consequently, chemical equilibrium will be lost prior to losing thermal equilibrium so that all constituents of the system acquire sizeable chemical potentials as pointed out in Refs. [1, 2]. In an environment where only elastic collisions occur and the expansion of the fluid is adiabatic, the conservation of entropy per particle enables the determination of the chemical potentials of each species (see, e.g., Ref. [2] for a lucid and detailed account). For viscous hydrodynamic expansion in which additional entropy is generated, the ideal fluid approximation serves as a starting guess (for chemical potentials) which we will adopt here.

In our initial study here, we will focus on the situation in which the net baryon number is zero and consider an interacting pion gas in which various resonances are formed (for a list,
see Table I of Ref. [3]). Examples of shear viscosity calculations with finite baryon number (in which conflicting conclusions have been drawn) can be found in Refs. [4, 5, 6]. Our calculations of \( \eta \) here are based on the Chapman-Enskog approach detailed in Refs. [7, 8]. Typos in Ref. [8] are corrected here and more details will be reported elsewhere. Differential cross sections required in the calculation of \( \eta \) are based on the K-matrix cross sections detailed in [3]. Entropy densities are calculated using the virial expansion approach as in Ref. [9]. Time evolution of the pion chemical potential is taken from ideal fluid hydrodynamic calculations for RHIC an LHC.

2. Pion chemical potentials from hydrodynamic simulations

Fig. 1 shows the time evolution of the temperature \( T \) and pion chemical potential \( \mu \) from ideal fluid hydrodynamic simulations for Au+Au collisions of 200 GeV/A at RHIC and Pb+Pb collisions of 2.76 TeV/A at LHC. For the RHIC experiment, the simulation is initialized as in Ref. [10] and for the LHC experiment we use the initialization for a viscous fluid as in Ref. [11]. The calculations are for net baryon number zero and the equation of state is that of the hadron resonance gas below 183 MeV with chemical freeze-out occurring at 150 MeV. When chemical equilibrium prevails, the chemical potentials of all hadrons are, by definition, zero. With the loss of chemical equilibrium, the chemical potentials of stable (with respect to strong interactions) particles with number densities \( n_i \) are chosen such that below the chemical freeze-out temperature, \( n_i/s \), where \( s \) is the total entropy density, is constant [2].

Results for two different centralities, 0-5% (most central collisions) and 40-50% (peripheral collisions), are shown. The hadronic life times of the chosen central and peripheral collisions are slightly different: in the most central collisions, the hadronic phase starts at a somewhat later time compared to that of the peripheral collisions. The times of onset and duration of the hadronic phase are qualitatively similar but quantitatively different at RHIC and LHC. As we can see in the top panels, temperature drops more rapidly as \( \mu \) begins to grow with time compared to the case when \( \mu = 0 \) [12].

A caveat in our use of the above results must be mentioned. In the hadronic phase, the ideal fluid hydrodynamic calculations above employ an equation of state that includes all resonances up to 2 GeV, not just pions. The ensuing results for the time evolution of the temperature and chemical potentials of the various hadronic species are thus more realistic than would be for a single component system of interacting pions that we have chosen for our pedagogical study here. As we aim to follow up this study to include additional mesons and baryons, our results for the pionic component alone should be taken as illustrative but not quantitatively definitive.

3. Shear viscosity with finite chemical potentials

We employ the Chapman-Enskog approach, generalized to include relativistic kinematics and quantum statistics, for the calculation of shear viscosity. An interacting pion gas will be chosen to explore the effects of nonzero chemical potentials. We follow closely the formalism of Refs. [7, 8] in which the shear viscosity to first order is expressed as

\[
\eta = \frac{T}{10} \frac{\gamma_0^2}{\gamma_0}. \tag{1}
\]

Above,

\[
\gamma_0 = -10 \frac{S_3^{(-2)}}{S_2^{(-1)}}, \quad \text{with} \quad S_n^{(\alpha)} = \sum_{k=1}^{\infty} \exp \left[ \frac{k m \mu}{T} \right] k^{\alpha} K_n(kz), \tag{2}
\]

where \( m \) is the mass of the particle, \( \mu \) is the reduced (with the mass \( m \)) chemical potential, \( T \) is the temperature, \( z = m/T \) is the relativity parameter, and \( K_\nu(z) \) is the modified Bessel
Figure 1: Temperature $T$ and pion chemical potential $\mu$ from ideal fluid hydrodynamical simulations of Ref. [13]. The left panels are for Au+Au collisions of 200 GeV/A at the RHIC and the right panels are for Pb+Pb collisions of 2.76 TeV at the LHC. In both cases, results are shown for 40-50% and 0-5% centralities.

function of order $\nu$. Apart from the numerical factor of $-10$, $\gamma_0$ represents the reduced enthalpy per particle of an ideal Boltzmann gas. The denominator in Eq. (1) is composed of terms involving the differential cross section between particles in the heat bath weighted by thermal occupation factors:

$$c_{00} = C_{00} + 2z^{-1}B_{11} + \frac{8}{3}z^{-2}A_{22}.$$  

The terms $C_{00}$, $B_{11}$, and $A_{22}$ are given by

$$C_{00} = \frac{16z^4 e^{(2m\mu/T)}}{[S_2^{-1}]^2} \int_0^\infty d\chi \int_0^\infty d\psi \int_0^\pi d\theta \int_0^\pi d\Theta \int_0^{2\pi} d\phi \sigma(\psi, \Theta) e^{-2z \cosh \psi \cosh \chi} \times \sinh^2 \chi \cosh^3 \psi \sinh^7 \psi \sin \theta \left(1 - \cos^2 \Theta\right) \sin \Theta \left(\frac{1}{B}\right),$$

$$B_{11} = \frac{8z^5 e^{(2m\mu/T)}}{[S_2^{-1}]^2} \int_0^\infty d\chi \int_0^\infty d\psi \int_0^\pi d\theta \int_0^\pi d\Theta \int_0^{2\pi} d\phi \sigma(\psi, \Theta) e^{-2z \cosh \psi \cosh \chi}$$
\[ A_{22} = \frac{2 z^6 e^{(2 m \mu / T)}}{S^2} \int_0^\infty d\chi \int_0^\infty d\psi \int_0^\pi d\theta \int_0^{2\pi} d\phi \sigma(\psi, \Theta) e^{-2 z \cosh \psi \cosh \chi} \]
\[ \times \sinh^4 \chi \cosh^3 \psi \sinh^7 \psi \sin \theta \left( \cos^2 \theta - 2 \cos \theta \cos \theta' \cos \Theta + \cos^2 \theta' \right) \sin \Theta \left( \frac{1}{B} \right), \]
\[ \times \sinh^6 \chi \cosh^3 \psi \sinh^7 \psi \sin \theta \left( \cos^2 \theta - \cos^2 \theta' \right) \sin \Theta \left( \frac{1}{B} \right), \] (4)

with
\[ B = 1 + \exp \left[ \frac{2 m \mu}{T} - 2 z \cosh \psi \cosh \chi \right] \]
\[ - \exp \left[ \frac{m \mu}{T} - z \left( \cosh \psi \cosh \chi - \sinh \psi \sinh \chi \cos \theta \right) \right] \]
\[ - \exp \left[ \frac{m \mu}{T} - z \left( \cosh \psi \cosh \chi + \sinh \psi \sinh \chi \cos \theta \right) \right] \] (5)

and
\[ \cos \theta' \equiv \cos \theta \cos \Theta - \sin \theta \sin \Theta \cos \phi. \] (6)

Hyperbolic functions of the quantity \( \psi \) characterize the relative momentum and invariant center-of-mass energy of the two scattering particles:
\[ \sinh \psi = \frac{g}{m}, \quad \cosh \psi = \frac{P}{m}, \] (7)

where \( g = \sqrt{(p_1 - p_2)^2 / 2} \), \( P = \sqrt{(p_1 + p_2)^2} \), and \( p_1 \) and \( p_2 \) are the initial four momenta of the colliding hadrons. Hyperbolic functions of the quantity \( \chi \) are components of the fluid four-velocity \( U^\alpha = (\cosh \chi, -\hat{e} \sinh \chi) \). The symbol \( \sigma(\psi, \Theta) \) denotes the center-of-mass differential cross section. Expressions for higher orders can be found in Refs. [7, 8], but are not required for the application on hand. The result for a Boltzmann gas is obtained by retaining only the \( k = 1 \) term in Eq. (2) and appropriate reductions of Eqs. (4) and (5) by taking the Boltzmann limit of the Bose distribution function in the kernels. As noted in Ref. [14], \( \eta \) is independent of \( \mu \) for a Boltzmann gas. Explicit \( \mu \) dependence of \( \eta \) arises only when Bose or Fermi statistics is relevant.

4. Results
The scaled entropy density, \( s/T^3 \), of the pion gas is shown in Fig. 2 as a function of temperature \( T \) for the Boltzmann and Bose cases with pion chemical potentials \( \mu = 0, 25, \) and 50 MeV, respectively. As expected, nonzero chemical potentials yield larger values of \( s/T^3 \) in both cases, that for the Bose gas being slightly larger than the result for the Boltzmann gas at all \( \mu/T \).

In Fig. 3, the shear viscosity \( \eta \) is shown as a function of temperature for \( \mu = 0 \) and 50 MeV, respectively. With increasing \( T \), \( \eta \) for the Bose case falls below that of the Boltzmann result. For the Bose gas, the magnitude of \( \eta \) for \( \mu = 50 \) MeV is slightly smaller than that for \( \mu = 0 \), whereas the Boltzmann result does not depend on \( \mu \) in first order [7, 14].

In Fig. 4, the ratio of \( \eta/s \) vs. \( T \) is shown for \( \mu = 0 \) and 50 MeV, respectively. For both values of \( \mu \), results for the Bose gas are smaller than those for the Boltzmann gas and the differences grow with increasing \( T \). Despite the fact that \( \eta \) is independent of \( \mu \) for the Boltzmann case, \( \eta/s \) decreases with \( \mu \) because of the increase in \( s \) with increasing \( \mu \). For the Bose gas, a nonzero \( \mu \) increases \( s \) and decreases \( \eta \) relative to their values at \( \mu = 0 \). In conjunction, both of these
effects render $\frac{\eta}{s} (\mu \neq 0) > \frac{\eta}{s} (\mu = 0)$ for all $T$. We observe, however, that for the range of temperatures of interest to hadron hydrodynamics, the decrease in $\frac{\eta}{s}$ due to finite $\mu$'s is not significantly large.

In Fig. 5, the entropy density, $s$, shear viscosity, $\eta$, and $\frac{\eta}{s}$ are shown as functions of time for RHIC and LHC experiments at centralities of 0-5% (central collisions) and 40-50% (peripheral collisions). Results here are for temperature and chemical potential profiles of the hadronic phase from hydrodynamic simulations shown in Fig. 1. Each panel of Fig. 5 contains comparisons between results when Boltzmann and Bose distribution functions are used for the calculations of $\eta$ and $s$. For Au+Au collisions of 200 GeV/A at RHIC, $\frac{\eta}{s}$ for RHIC lies in the range 0.36-0.5 for both centralities. In the case of Pb+Pb collisions of 7 TeV/A at LHC, $\frac{\eta}{s}$ is in the range 0.36-0.6 (not too different from RHIC). All four panels in Fig. 5 exhibit qualitatively similar features. The nearly linear rise in $\frac{\eta}{s}$ during the time that chemical equilibrium prevails ($T < 150$ MeV) stems from the time evolutions of $\eta$ and $s$ which in turn depend on the time.
Figure 4: Ratio of shear viscosity to entropy density, $\eta/s$, of an interacting pion gas. Solid (dash-dotted) curves are for a Bose (Boltzmann) gas with pion chemical potentials $\mu = 0$ and 50 MeV, respectively.

Figure 5: Entropy density, $s$, shear viscosity, $\eta$ and $\eta/s$ as functions of time for RHIC (left panels) and LHC (right panels) experiments for centralities of 0-5% (central collisions) and 40-50% (peripheral collisions). Solid (dash-dotted) curves are for Bose (Boltzmann) gases.
evolution of $T$. With the loss of chemical equilibrium, the time evolutions of $\eta$ and $s$ depart from their earlier behaviors as an additional time dependence enters through the time evolution of $\mu$. Noteworthy is the feature that $\eta/s$ exhibits a minimum before rising monotonically as time progresses. The minimum is a consequence of the different rates at which $\eta$ and $s$ fall with time, that of $\eta$ being a combination of the rate at which $T$ falls with $t$ and $\mu$ grows with $t$ (see Fig. 1).

5. Summary and conclusion
We have explored the influence of finite chemical potentials arising from the expansion and hence the dilution of the hadronic system in relativistic heavy-ion collisions of relevance to experiments at RHIC and LHC. Our study was limited to the case of an interacting pion gas in which chemical equilibrium is lost prior to thermal freeze-out. In contrast to the case when Boltzmann statistics is used and $\eta$ is independent of $\mu$, use of Bose statistics causes $\eta$ to decrease with increasing $\mu$. Additionally, Bose statistics leads to larger values of $s$ than Boltzmann statistics. Consequently, $\eta/s$ decreases as the pion chemical potential $\mu$ increases toward freeze-out. The effects of nonzero pion chemical potentials on time evolving $\eta/s$ are not very large. However, our results indicate that the inclusion of additional mesons will likely lead to a further reduction in $\eta/s$. Larger reductions may be expected for nonzero net baryon and strangeness numbers. Calculations in this regard are in progress to complement those in the literature [4, 5, 6].

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