The Starobinsky model within the $f(R, T)$ formalism as a cosmological model

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Abstract In this paper we derive a cosmological model from the $f(R, T)$ theory of gravity, for which $R$ is the Ricci scalar and $T$ is the trace of the energy-momentum tensor. We consider $f(R, T) = f(R) + f(T)$, with $f(R)$ being the Starobinsky model $R + \alpha R^2$ and $f(T) = \gamma T$, with $\alpha$ and $\gamma$ constants. We find that from such a functional form, it is possible to describe the cosmological scenario of a radiation-dominated universe, which has shown to be a non-trivial feature within the $f(R, T)$ formalism.

Keywords $f(R, T)$ gravity · Starobinsky model · radiation era · cosmological models

1 Introduction

The $f(R)$ theories of gravity [1,2,8] are an optimistic alternative to the shortcomings General Relativity (GR) faces as the underlying gravitational theory for a cosmological model [4,5,6]. They can account for the cosmic acceleration [7,8], providing a great match between theory and cosmological observations [9,10,11], and also for inflation [12,13,14,15,16,17,18] and dark matter issues [19,20,21].

One of the crucial troubles surrounding GR is that apparently it cannot be quantized, although attempts to do so have been proposed, as String Theory [22,23,24] (check also [25,26] for reviews on the topic), and can, in future, provide us a robust and trustworthy model of gravity - quantum mechanics unification.

Meanwhile it is worthwhile to attempt to consider the presence of quantum effects in gravitational theories. Those effects can rise from the consideration of terms proportional to the trace of the energy-momentum tensor $T$ in the gravitational part of the $f(R)$ action, yielding the $f(R, T)$ gravity theories [27]. Those theories were also motivated by the fact that although $f(R)$ gravity is well behaved in cosmological scales, the Solar System regime seems to rule out most of the $f(R)$ models proposed so far [28,29,30,31].

Despite its recent elaboration, $f(R, T)$ gravity has already been applied to a number of areas, such as Cosmology [32,33,34,35,36,37,38,39,40,41,42,43,44,45,46] and Astrophysics [47,48,49,50,51,52].

By deeply investigating the outcomes and features of an $f(R, T)$ or $f(R)$ model, one realizes the strong relation they have with the functional form of the chosen functions and free parameter values. In fact, a reliable method to constraint those “free” parameters to values that yield realistic models can bee seen in [53] and [54] for $f(R, T)$ and $f(R)$ models, respectively.

In $f(R)$ gravity a reliable and reputed functional form was proposed by A.A. Starobinsky as [55]

$$f(R) = R + \alpha R^2,$$

which is known as Starobinsky Model (SM), with $\alpha$ a constant. It predicts quadratic corrections of the Ricci scalar to be inserted in the gravitational part of the Einstein-Hilbert action.

An analysis of matter density perturbations in SM was presented in [56]. Black hole studies were made for $R^2$ gravity in [57]. The consideration of wormholes in such theories can be appreciated in [58,59].

Our proposal in this paper is to construct a cosmological scenario from an $f(R, T)$ functional form whose $R$–dependence is the same as in the SM, i.e., with a quadratic extra contribution of $R$, as in Eq. [1]. The
describe a radiation-dominated universe.

Despite the high number of considerations of the SM in $f(R)$ cosmology (check also [60,61,62]), it has not been considered for the $R$–dependence in $f(R,T)$ models for cosmological purposes so far, only in the study of astrophysical compact objects [50,51,52]. We believe this is due to the expected high nonlinearity of the resulting differential equation for the scale factor. Anyhow, the consideration of quantum corrections together with quadratic geometrical terms can imply interesting outcomes in a cosmological perspective as it did in the astrophysical level (check [50,51,52]). Therefore we present here a reliable and well referenced method to obtain solutions for such a cosmological scenario.

Here let us stress that the $f(R,T)$ formalism exhibits a sort of shortcoming for a specific era of the Universe evolution. One could ask what are the predictions of $f(R,T)$ gravity in the regime $T = 0$. It is natural to think that for different functional forms formulated to the $f(R,T)$ function, the regime $T = 0$ makes $f(R,T)$ gravity to recover $f(R)$ theories. The regime $T = 0$ is achieved for $p = \rho/3$, with $p$ and $\rho$ being the pressure and density of the Universe, respectively, which is the equation of state (EoS) of radiation. Therefore, from a cosmological perspective it becomes intuitive to think that $f(R,T)$ gravity itself is not able to describe the era in which the Universe was dominated by radiation.\footnote{In [33] it was deeply discussed that this non-contribution regime of $f(R,T)$ gravity can also be expected in vacuum; for instance, in the study of gravitational waves propagation.} It would only recover the $f(R)$ outcomes.

The $T = 0$ issue surrounding the $f(R,T)$ formalism was already investigated in [33,36,37,63,64]. In [33], in order to be able to describe the radiation era of the Universe, a scalar field was invoked in $f(R,T)$ gravity, namely the $f(R,T^2)$ gravity. In [36] such a description became possible only in a five-dimensional space-time, while in [37] the speed of light was considered a variable and an alternative scenario to inflation was obtained. Here, instead, one of our goals is to check if restrictively the choice of the SM for the $R$ dependence in the $f(R,T)$ function is able to make $f(R,T)$ formalism to describe a radiation-dominated universe.

The SM in $f(R)$ formalism is known to successfully describe the accelerated periods of the Universe evolution, namely the inflationary and dark energy eras [53,54,55,65,66,67,68]. Would it also be a powerful tool to help $f(R,T)$ gravity to be able to describe the radiation era of the Universe? Let us address this question in the next sections.

2 An overview of the $f(R,T)$ formalism

Originally proposed as a generalization of the $f(R)$ theories, the $f(R,T)$ gravity considers the gravitational part of the model action to be dependent not only on a general function of the Ricci scalar $R$, but also on a general function of the trace of the energy-momentum tensor $T$, as

\[ S_{\text{grav}} = \frac{1}{16\pi} \int d^4x \sqrt{-g} f(R,T), \] (3)

with $g$ being the determinant of the metric and $f(R,T)$ the function of $R$ and $T$. Moreover, throughout this article we will consider natural units.

By varying action (3) with respect to the metric $g_{\mu \nu}$, one obtains the following field equations:

\[ f_R(R,T)R_{\mu \nu} - \frac{1}{2} f(R,T)g_{\mu \nu} + (g_{\mu \nu} \Box - \nabla_\mu \nabla_\nu), \] (4)

\[ f_R(R,T) = 8\pi T_{\mu \nu} - f_T(R,T)T_{\mu \nu} - f_T(R,T)\Theta_{\mu \nu}. \] (5)

In [3], $R_{\mu \nu}$ is the Ricci tensor, $f_R(R,T) = \partial f(R,T)/\partial R$, $f_T(R,T) = \partial f(R,T)/\partial T$, $\Box$ is the D’Alambert operator, $\nabla_\mu$ is the covariant derivative and $\Theta_{\mu \nu} = -2T_{\mu \nu} - pg_{\mu \nu}$, with the energy-momentum tensor $T_{\mu \nu}$ being considered the one of a perfect fluid.

Moreover, the covariant divergence of the energy-momentum tensor in $f(R,T)$ gravity reads [69,70]

\[ \nabla_\nu T_{\mu \nu} = \frac{f_T(R,T)}{8\pi - f_T(R,T)} [(T_{\mu \nu} + \Theta_{\mu \nu})\nabla_\nu \ln f_T(R,T) + \nabla_\nu \Theta_{\mu \nu} - (1/2)g_{\mu \nu} \nabla_\nu T]. \] (6)

3 The $f(R,T) = R + \alpha R^2 + \gamma T$ model

3.1 Field equations

By substituting Eq. (2) in Eq. (4) yields the following field equations:

\[ (2\alpha R + 1)G_{\mu \nu} - \alpha R^2 g_{\mu \nu} = (8\pi + \gamma)T_{\mu \nu} + \frac{1}{2} (\rho - p)g_{\mu \nu}. \] (7)

In Eq. (7), $G_{\mu \nu}$ is the usual Einstein tensor and we have already taken the trace of the energy-momentum tensor of a perfect fluid to be $\rho - 3p$. The elegant form in which Eq. (7) is presented makes straightforward to recover GR when $\alpha, \gamma \to 0$. 
3.2 Friedmann-like equations

By defining the quantity

\[ \Phi = \Phi(t) \equiv \left( \frac{\ddot{a}}{a} \right)^2 + \frac{\dot{a}}{a}, \]  

(8)

with \( a = a(t) \) being the scale factor and dots representing time derivatives, the non-null components of Eq. (7) for a flat Friedmann-Robertson-Walker metric are:

\[ \Phi - \frac{\ddot{a}}{a} - 12\alpha \Phi \left( \Phi - \frac{\ddot{a}}{a} + \Phi^2 \right) = \frac{1}{6} \left[ (16\pi + 3\gamma) \rho + \gamma p \right], \]

(9)

\[ \Phi + \frac{\ddot{a}}{a} - 12\alpha \Phi \left( \Phi + \frac{\ddot{a}}{a} - 3\Phi^2 \right) = -\frac{1}{2} \left[ (16\pi - 3\gamma) p + \gamma \rho \right]. \]

(10)

It is worthwhile reinforcing that, as required, the limits \( \alpha, \gamma \to 0 \) in Eqs. (9)-(10) retrieve GR predictions.

Moreover, in this context, Eq. (6) is written as

\[ \dot{\rho} + 3 \frac{\dot{a}}{a} (\rho + p) = \gamma (\dot{\rho} - \dot{p}), \]

(11)

where \( \gamma \equiv \gamma / [2(1 - 2\gamma)] \) with \( \gamma \equiv \gamma / (\gamma - 8\pi) \). It is worth mentioning that by making \( \gamma \to 0 \), GR is once again recovered.

4 Analytical solutions for the scale factor and their cosmological consequences

As we can see in the previous section, the equations (9)-(10) are nonlinear second-order differential equations. It is worth pointing out that nowadays the nonlinearity is found in many areas of Physics, including Condensed Matter \[72,73,74\], Field Theory \[68,76,77,78\] and also Cosmology \[79,80,81\]. In a cosmological context, the nonlinear effects can play an important role to understand the dynamics of the Universe. For instance, in a recent work it has been shown that in a cosmological scenario with Lorentz symmetry breaking, the so-called oscillons \[82\] in the early Universe have passed through a phase transition that changed their internal structure \[83\].

Unfortunately, as a consequence of the nonlinearity, in general we lose the capability of getting the complete solutions. However, in this section we will show that

\[ 12\alpha \Phi^2 (\Phi - 1) + \Phi = \frac{8\pi}{3} \rho + \left( \frac{5\gamma}{3} - 8\pi \right) p. \]

(12)

It is important to remark that there is no restriction in adding these equations and that such a mathematical approach was shown to be very useful \[84\]. Also, in \[77\], it was used in order to find a class of traveling solitons in Lorentz and CPT breaking systems.

Now we will focus on getting analytical solutions for Eq. (12). Looking at it, it is natural to think that the functions \( \rho \) and \( p \) can be represented by polynomial functions of third degree in \( \Phi \). In fact, such a representation is constantly used in studies concerning oscillon theories \[85,86,87,88,89,90\]. In those cases, this mathematical procedure allows to obtain the fundamental characteristics of the oscillons, such as their field configuration, lifetime, amplitude and rate of decaying. By using this approach we will have a specific class of solutions, but with the great advantage of its analytical form.

Therefore, with the above motivation, we assume that \( \rho \) and \( p \) are related by a general polytropic equation of state \[91\]:

\[ p(t) = K [\rho(t)]^m. \]

(13)

In \[13\], \( K \) and \( \gamma_0 \) are constants.

By substituting the above form of \( p \) in Eq. (11), we obtain the following constraints

\[ \rho(t) = A_0 (a(t))^{-3/\Gamma_0}, \]

(14)

where \( A_0 \) is an arbitrary constant of integration. Moreover, we are using the following definition

\[ \Gamma_0 \equiv \frac{(1 - \bar{\gamma}) [1 - \bar{\gamma}K / (1 - \bar{\gamma})]}{1 + K}. \]

(15)

It is important to remark that, in order to avoid singularities, we must impose that \( \Gamma_0 < 0 \).

On the other hand, by applying Eqs. (13) and (14) into (12), we find the equation

\[ \dot{\Phi}^3 + \dot{\Phi}^2 - (1/12\alpha) \Phi = 0, \]

(16)

where we are using the indentation

\[ K \equiv \frac{8\pi}{24\pi - 5\gamma}. \]

(17)
Now, in order to solve Eq. (16) and consequently find a class of analytical solutions for the scale factor, we impose that $a = 1/3$. Thus, we can see from Eq. (16) that there are two different roots for $\Phi$, which are given by

$$\Phi_1 = 0, \quad \Phi_2 = \frac{1}{2}. \quad (18)$$

Thus, after some mathematical manipulations, we can obtain the following analytical solutions for the scale factor

$$a_1(t) = \sqrt{A_1 t + B_1}, \quad \quad (20)$$

$$a_2(t) = A_2 e^{-t/2} \sqrt{B_2 e^{2t} + C}, \quad \quad (21)$$

where $A_i$, $B_i$ and $C$ are arbitrary constants of integration, with $i = 1, 2$.

To interpret these solutions, we will construct the referred Hubble and deceleration parameters. The Hubble parameter, expressed by $H = \dot{a}/a$, shows us the expansion rate of the Universe in time, whereas the deceleration parameter, expressed by $q = -\ddot{a}/a^2$, is such that negative values stand for an accelerated expansion while positive values, for a decelerated expansion.

Let us start by analysing solution (20). Such a scale factor evolves in time according to Fig. 1 below.

The referred Hubble parameter reads

$$H_1 = \frac{A_1}{2( A_1 t + B_1 )}, \quad \quad (22)$$

which is depicted in Fig. 2.

Moreover, independently of the values of the constants $A_1$ and $B_1$, Eq. (20) yields $q_1 = 1$.

The behaviour of the cosmological parameters $a$, $H$ and $q$ obtained above are in agreement with a universe dominated by radiation. In order to verify this, let us recall that the standard Friedmann equations are obtained in the present model by making $\alpha, \gamma = 0$ in (9) and read

$$3 \left( \frac{\dot{a}}{a} \right)^2 = 8\pi \rho, \quad \quad (23)$$

$$2\frac{\ddot{a}}{a} + \left( \frac{\dot{a}}{a} \right)^2 = -8\pi p. \quad \quad (24)$$

In order to make standard Friedmann equations above to describe a radiation-dominated universe, one usually assumes $p = \rho/3$ as the EoS of the Universe in (24). Such an assumption yields the solution $a(t) \sim t^2$, exactly as in Eq. (20), obtained from the $f(R,T)$ formalism.

Furthermore, Fig. 2 shows that $a \neq 0$ as $t \to 0$. In fact, a null value for $a$ would indicate the origin of the Universe. However, since we are treating the radiation dominated universe, $t = 0$ does not describe the Big-Bang. Rather, it describes the time in which radiation starts dominating the Universe dynamics. In this way, the fact that $a \neq 0$ for low values of time is in agreement with a radiation dominated universe. We can also see that $a$ increases with time, corroborating an expanding universe.

The Hubble parameter behaviour of Figure 2 also strengthens our argument. Firstly, we can see that it decreases with time, as it should happen in an expanding universe. Secondly, since $H \sim t_H^{-1}$, with $t_H$ being the Hubble time, at the end of the stage in which the Universe dynamics was dominated by radiation, $H$ must
be $\neq 0$. High values of time in Fig. 3 (and also in Fig. 1) indicate the end of the radiation era rather than the present or future epochs of the Universe, in which $H$ asymptotically tends to 0. Such an asymptotically behaviour for $H$ can be seen, for instance, in [33,36], for which high values of time stand for present and future epochs of the Universe evolution.

Moreover, the value which we obtained for the deceleration parameter, i.e., $q = 1$, also is in accordance with a radiation dominated universe. The fact that it is positive means that during this stage, the Universe expansion was decelerating (in fact, the expansion started to accelerate some few billion years ago [71]). Also, from the time proportionality obtained for $a$ from the standard Friedmann equations above, i.e., $a \sim t^2$, the deceleration parameter definition $-\dot{a}a/\dot{t}^2$ yields exactly 1, i.e., our model has the same features of a standard cosmology radiation-dominated universe.

Now, using Eq. (21), we find the following results for the cosmological parameters

$$H_2 = \frac{A_2 e^{-\frac{2}{3}t} (B_2 e^{2t} + C)}{2 \sqrt{C - B_2 e^{2t} t}}, \tag{25}$$

$$q_2 = \frac{C(6B_2 e^{2t} - 1) + B_2^2 e^{4t}}{(C + B_2 e^{2t})^2}. \tag{26}$$

The evolution of these quantities in time can be appreciated in Figs. 3 and 4 below.

![Fig. 3](image_url) Time evolution of the Hubble parameter from Equation (24). The (blue) dotted line stands for $A_2 = 2$ and $B_2 = 1.5$, the (green) dot-dashed stands for $A_2 = 3$ and $B_2 = 1.9$ and (red) solid lines stand for $A_2 = B_2 = 1$. In all curves, $C = -0.9$.

![Fig. 4](image_url) Time evolution of the deceleration parameter from Equation (25). The (blue) dotted line stands for $A_2 = 2$ and $B_2 = 1.5$, the (green) dot-dashed stands for $A_2 = 3$ and $B_2 = 1.9$ and (red) solid lines stand for $A_2 = B_2 = 1$. In all curves, $C = -0.9$.

5 Discussion

It is known that for a small but non-negligible period of time the dynamics of the early universe was dominated by radiation. During this epoch, the density and temperature of photons were high enough to prevent atoms, (and consequently) stars and galaxies to form.

In such a stage, the EoS of the Universe is written as $p = \rho/3$. For a perfect fluid, such an EoS yields a null trace of the energy-momentum tensor and therefore one expects, in this regime, $f(R, T)$ gravity to simply retrieve $f(R)$ gravity. Indeed, no contributions from the former are expected since the dependence on $T$ disappears.

Such an $f(R, T)$ formalism shortcoming has generated some important discussions. In [33], in order to surpass such an unpleasant feature, the authors have formulated a cosmological scenario for the $f(R, T^o)$ gravity, with $\phi$ being a scalar field. They have showed that even in the regime $T = 0$, the field equations of the model present extra contributions, when compared to those from $f(R)$ gravity, coming from the trace of the energy-momentum tensor of the scalar field. Such a formalism originated the possibility of studying grav-
itational waves in \( f(R, T) \) gravity [49] (recall that the \( T = 0 \) regime is also obtained in vacuum).

Here, instead, we have proposed a quadratic correction for the \( R \)-dependence of the \( f(R, T) \) function. Motivated by the application of the SM in \( f(R) \) cosmology [55][60][61][62] and \( f(R, T) \) astrophysics [50][51][52], we intended here to check if from the \( f(R, T) = R + \alpha R^2 + \gamma T \) theory, one could derive a healthy cosmological scenario.

In constructing our model, we have obtained a highly nonlinear set of differential equations for the scale factor \( a \), from which important and informative cosmological parameters are obtained.

Remarkably, for small values of time, the values of our scale factor solution presented in Fig[1] are not close to 0. The restriction of this model to the radiation era of the Universe can be checked also in Fig[2] in which we can see that for high values of time (end of radiation era) the Hubble parameter does not tends asymptotically to 0, which is expected in a recent universe (check, for instance, [53]).

We have presented from solution [20] a formalism which makes \( f(R, T) \) gravity able to generate a cosmological scenario in which radiation dominates the dynamics of the Universe. The relevance of such a construction lies on the fact that one does not expect \( f(R, T) \) gravity to be capable of describing such a stage of the Universe without simply recovering \( f(R) \) gravity. Here, instead, we have shown that besides predicting a variety of well behaved cosmological and astrophysical scenarios in \( f(R) \) gravity, the SM within the \( f(R, T) \) gravity solves the \( T = 0 \) issue of \( f(R, T) \) theories.

On the other hand, solution [21] is related to a more complete cosmological scenario. It predicts, from the analysis of the referred deceleration parameter, the radiation, matter and dark energy-dominated eras, as well as the transition among these stages, in a continuous form, which is certainly a milestone in theoretical cosmology.

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