HYDRODYNAMICAL EVOLUTION OF MERGING CARBON–OXYGEN WHITE DWARFS: THEIR PRE-SUPERNova STRUCTURE AND OBSERVATIONAL COUNTERPARTS

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ABSTRACT

We perform smoothed particle hydrodynamics simulations for merging binary carbon–oxygen (CO) WDs with masses of 1.1 and 1.0 M⊙, until the merger remnant reaches a dynamically steady state. Using these results, we assess whether the binary could induce a thermonuclear explosion, and whether the explosion could be observed as a type Ia supernova (SN Ia). We investigate three explosion mechanisms: a helium-ignition following the dynamical merger (“helium-ignited violent merger model”), a carbon-ignition (“carbon-ignited violent merger model”), and an explosion following the formation of the Chandrasekhar mass WD (“Chandrasekhar mass model”). An explosion of the helium-ignited violent merger model is possible, while we predict that the resulting SN ejecta are highly asymmetric since its companion star is fully intact at the time of the explosion. The carbon-ignited violent merger model can also lead to an explosion. However, the envelope of the exploding WD spreads out to ~0.1 R⊙; it is much larger than that inferred for SN 2011fe (<0.1 R⊙) while much smaller than that for SN 2014J (~1 R⊙). For the particular combination of the WD masses studied in this work, the Chandrasekhar mass model does not successfully lead to an SN Ia explosion. Besides these assessments, we investigate the evolution of unbound materials ejected through the merging process (“merger ejecta”), assuming a case where the SN Ia explosion is not triggered by an carbon-ignition during the merger. The merger ejecta interact with the surrounding interstellar medium and form a shell. The shell has a bolometric luminosity of more than 2 × 10^{35} erg s^{-1}, lasting for ~2 × 10^{4} years. If this is the case, the Milky Way should harbor about 10 such shells at any given time. The detection of the shell(s) can therefore rule out the helium-ignited and carbon-ignited violent merger models as major paths to SN Ia explosions.

Key words: binaries: close – galaxies: evolution – hydrodynamics – supernovae: general – white dwarfs

1. INTRODUCTION

The type Ia supernova (SN Ia) is one of the brightest events in the universe and plays an important role as a cosmological distance indicator. It is widely accepted that an SN Ia is a thermonuclear explosion of a carbon–oxygen (CO) WD, and that the explosion is triggered by an interaction between the CO WD and its companion star (Nomoto et al. 1984; Hillebrandt & Niemeyer 2000). However, it is still controversial whether the companion star is a non-degenerate star (single degenerate scenario, SD; Whelan & Iben 1973; Nomoto 1982), or a degenerate star (double degenerate scenario, DD; Iben & Tutukov 1984; Webink 1984). There are other scenarios, for example, the core degenerate (CD) scenario in which the companion is an asymptotic giant branch core (Kashi & Soker 2011) and the collisional DD scenario in which two CO WDs collide in a dense stellar cluster, or in a multiple stellar system (Aznar-Siguan et al. 2013, 2014).

The SD scenario has been well tested by recent observations (see Maoz et al. 2013, for a review). There are multiple observational indications, some of which are for the SD scenario and others that are against it, for different objects. The observational studies against the SD scenario include the following. Li et al. (2011) have detected no red giant star in the deep pre-explosion images of the site of SN 2011fe. Schaefer & Pagnotta (2012) have reported that no main sequence or red giant stars are observed at the central region of an SN Ia remnant, SNR 0509-67.5. On the other hand, the observations supporting the SD scenario include the following. Dilday et al. (2012) have observed SN Ia PTF 11lx and found evidence of a strong interaction between the SN ejecta and circumstellar matter (CSM). Here, the CSM is thought to originate from a symbiotic nova, which consists of a WD and a red giant star.

As described above, the SD scenario has been directly tested by many studies. On the other hand, most of the “observational” support for the DD scenario indeed come from “non-detection”—it is supported since the SD scenario is ruled out for particular objects. It is therefore necessary to assess the DD scenario directly, based on theoretical predictions of what should be observed if the DD scenario is the case. For this purpose, the following two questions should be answered: (1) whether a CO WD, which accretes materials from a companion CO WD, results in a thermonuclear explosion and (2) whether such an explosion is observed as an SN Ia.

8 However, we should note that the null detection can be explained by spin-up/spin-down models, where the companion star evolved to become a helium WD during the spin-down phase of the CO WD before the delayed carbon ignition occurs in the center (di Stefano et al. 2011; Justham 2011; Hachisu et al. 2012).
9 For CSM in the CD scenario, see Soker et al. (2013).
10 See, however, the earlier footnote for the SD scenario and Soker et al. (2014) for the CD scenario of SN 2011fe.
Many previous studies have focused on the first question, finding many possible paths in which a primary CO WD could explode. These models can be generally divided into the Chandrasekhar mass and sub-Chandrasekhar mass models. In the Chandrasekhar mass model, the CO WD reaches the central density higher than a critical density to ignite explosive carbon burning, whereas it is not the case for the sub-Chandrasekhar mass model.

In the Chandrasekhar mass model, two CO WDs merge, and the merger remnant evolves hydrostatically after the merger toward an explosion. Whether the merger remnant explodes as an SN Ia depends on the structure of the merger remnant. This structure has been intensively investigated by means of two different numerical schemes: smoothed particle hydrodynamics (SPH) simulations (Benz et al. 1990; Guerrero et al. 2004; Yoon et al. 2007; Lorén-Aguilar et al. 2009; Raskin et al. 2012) and mesh-based hydrodynamics simulations (D’Souza et al. 2006; Motl et al. 2007). Among these works, Yoon et al. (2007) have found a path to the SN Ia explosion. Moreover, Zhu et al. (2013) and Dan et al. (2014) have performed large parameter surveys for various binary CO WD parameters, and have searched for the systems that can explode as SNe Ia.

The sub-Chandrasekhar mass model can be subdivided into several categories. First, Pakmor et al. (2010) have suggested a “carbon-ignited violent merger model” (see also Pakmor et al. 2011, 2012a, 2012b). In this model, hotspots appear in the course of the merger of binary CO WDs, and generate carbon detonation, leading to an explosion. Second, Pakmor et al. (2013) have also proposed a “helium-ignited violent merger model.” In this model, a helium layer accreted onto a primary CO WD from a companion WD rises in temperature. It is suggested that the helium detonation occurs at some point in this layer, and then the shock compression triggers the carbon detonation inside the primary CO WD, leading to an explosion. This is an analog to the double detonation model (e.g., Nomoto 1980, 1982; Woosley et al. 1980), except for the nature of the helium donor. Finally, Schwab et al. (2012), Shen et al. (2012), and Ji et al. (2013) have argued that binary CO WDs could explode shortly after the merger due to magnetohydrodynamical effects.

In this paper, we investigate the helium-ignited violent merger, the carbon-ignited violent merger, and Chandrasekhar mass models from two points of view: (1) the first point is whether these models lead to a successful ignition to initiate an explosion. For this purpose, we perform SPH simulations of a merger of binary CO WDs. However, we cannot directly follow the initiation of its explosion; in order to follow the initiation, we need an SPH simulation with impossibly high space resolution, say 1 cm. Instead, we judge success or failure in the explosion from the density and temperature obtained through the SPH simulation. In order to investigate various possibilities in the ignition process, we adopt the following strategy: even when we infer the success of a particular mode of the ignition, we do not stop our simulation, and the SPH simulation results from the subsequent evolution are used to test another mode of the ignition. This is because our inference for a particular ignition mode is not decisive and because we want to test several models with different ignition conditions. As described above, we use temperature to infer the success or failure. However, temperature is vulnerable to random noises in SPH simulations. In order to obtain a robust inference, we carefully treat temperature in the following two ways. First, we adopt two types of temperatures: raw and smoothed temperatures (described in detail later). Second, we do not solve nuclear reactions, which are sensitive to random noises and could increase temperature in an unstable manner.

(2) The second point is whether the expected outcome of the explosion is consistent with observations of SNe Ia. For this purpose, we adopt several observational indications. The first test is the progenitor radii of SN 2011fe and SN 2014J. The former and latter radii are inferred to be less than 0.1R⊙ (Nugent et al. 2011; Bloom et al. 2012; Zheng et al. 2013; Mazzali et al. 2014) and more than 1 R⊙ (Goobar et al. 2015), respectively. As another test, we discuss 56Ni distribution and the ejecta geometry, both of which are thought not to be highly asymmetric for nearby SNe (Maund et al. 2013; Soker et al. 2014) and for supernova remnants (SNRs).

We mainly investigate a merger of a binary consisting of two CO WDs with masses of 1.1 and 1.0 M⊙. For a benchmark, we also follow the evolution of a binary with masses of 0.9 M⊙ CO WD and 0.6 M⊙ CO WD. Other combinations are investigated elsewhere (Sato et al. 2015).

In sum, we find that for the particular binary parameters studied in this paper, the helium- and carbon-ignited violent merger models lead to an explosion, while the Chandrasekhar mass model does not, according to our inference. However, as we mentioned above, our inference is not decisive due to various uncertainties. We therefore suggest a way of evaluating our inference and constraining the fates of WD mergers, based on the insights obtained through the SPH simulations. If the system indeed does not immediately lead to an SN Ia explosion, the binary merger should leave its merger WD remnant and the unbound materials ejected from the system during the dynamical phase of the merger process (hereafter “merger ejecta”). The merger ejecta interact with the interstellar medium (ISM) and form a shell (hereafter “merger shell”), which is analogous to the formation of an SNR. If we find these merger remnants and merger shells, we can dismiss our inference and alternatively we can use these observational counterparts against the violent merger scenarios leading to an SN Ia. We discuss the detectability of such events, in particular, the merger shell. We thereby suggest that it is possible to detect such events.

This discussion has another benefit. The merger shell can be detected not only from pre-explosion images of a site of an SN Ia, but also in the post-explosion observations, if the SN Ia happens in the Chandrasekhar mass model.11 The detection of the merger shell can therefore directly support the Chandrasekhar mass model as a result of the WD merger.

This paper is structured as follows. In Section 2, we describe the methods of our simulations. In Section 3, we show the results from our simulations. In Section 4, we assess whether our binary CO WDs can explode and whether the explosion can be observed as an SN Ia. In Section 5, we discuss the detectability of the merger remnant and merger shell. Finally, we summarize our findings in Section 6.

11 The Chandrasekhar mass model can be successful when binary CO WDs consist of the mass combinations different from the one studied in this paper (see Sato et al. 2015).
2. METHOD

In this section, we describe the methods of our simulations. In Section 2.1, we briefly present schemes in our SPH simulations. In Section 2.2, we define several quantities used throughout the paper. In Section 2.3, we show how to set up an initial condition of binary COWDs. In Section 2.4, we summarize a set of physical and numerical parameters used in this study. In Section 2.5, we introduce our computing environment.

2.1. SPH Simulation

We solve Lagrangian hydrodynamics equations with self gravity by means of SPH simulations. Each SPH particle is evolved by the following equations.

\[ \dot{v}_i = -\frac{\nabla P_i}{\rho_i} + g_i, \]  \hspace{2cm} (1)

\[ \dot{u}_i = -\frac{P_i}{\rho_i^2} (\nabla \cdot v_i), \]  \hspace{2cm} (2)

where \( v_i, u_i, P_i, \) and \( \rho_i \) are the velocity, specific internal energy, pressure, and (mass) density of \( i \)-particle, respectively, and \( g_i \) is gravity exerting on the \( i \)-particle. The over-dots indicate the first time derivative. The nabla symbol means an operator of \( \partial / \partial x, \partial / \partial y, \partial / \partial z \).

We briefly explain our SPH formulations. In our SPH simulations, we solve the “vanilla ice” SPH equations. We adopt a cubic spline kernel for the SPH kernel interpolation. The SPH kernel is modified in the same way as Thomas & Couchman (1992). Similarily to Rosswog et al. (2000), we adopt the treatment of time-dependent artificial viscosity (Morris & Monaghan 1997), combined with a recipe that suppresses the viscosity from shear motion (Balsara 1995).

This is described in detail in the Appendix. We set the length of the SPH kernel of a particle, such that the arithmetic average number of neighbor particles over all the particles, \( \langle n_i \rangle \), is 150. Hereafter, \( \langle \rangle \) indicates an arithmetic average of quantities over all the particles. Neighbor particles of the \( i \)-particle, \( n_i \), are defined as particles whose distances from the \( i \)-particle is less than the kernel length of the \( i \)-particle.

We use an equation of state (EOS) as functions of \( \rho_i \) and \( u_i \) in order to get not only \( P_i \), but also raw temperature \( T_i \) and sound speed \( c_{s,i} \). For the EOS, we adopt the Helmholtz EOS (Timmes & Swesty 2000). This EOS includes thermal radiations, an ideal gas of ions, an electron-positron gas with an arbitrary degree of relativity and degeneracy. The EOS requires chemical compositions of fluids. We assume that the chemical composition is uniformly fixed to 50% of carbon, and 50% of oxygen in the number fraction. The chemical composition is fixed during the whole simulation, since we do not consider nuclear reactions.

The gravity is calculated as follows. The gravity \( g_i \) is the sum of the Newtonian gravity on the \( i \)-particle exerted by all of the other particles. We introduce so-called Plummer softening to the Newtonian gravity. Thus, it is expressed as

\[ g_i = \sum_{j\neq i} g_{m,j} \frac{r_{ij}}{\left( r_{ij}^2 + \epsilon^2 \right)^{3/2}}, \]  \hspace{2cm} (3)

where \( m_j \) and \( r_{ij} \) are, respectively, the mass and position vector of \( j \)-particle, \( G \) is the gravitational constant, and \( \epsilon \) is the gravitational softening, fixed to \( \epsilon = 3 \times 10^6 \) cm. In practice, we calculate the gravity with an octree algorithm (e.g., Barnes & Hut 1986). In such algorithms, gravity exerted on the \( i \)-particle by distant particles is approximated as its multipole moment. We consider the multipole moment up to the dipole moment. In order to define whether particles are distant or not, we use Multipole Acceptance Condition (MAC). We choose the same MAC as introduced by Salmon & Warren (1994). The MAC has one accuracy parameter, \( \Delta \), which is the same notation as in Nakasato et al. (2012). The accuracy parameter \( \Delta \) has the dimension of mass divided by square of length. As \( \Delta \) becomes smaller, gravity is calculated with a higher degree of accuracy. At the beginning of our simulation, we determine \( \Delta \) from \( g_i \) at the initial time, such as

\[ \Delta = \frac{\langle |g_i| \rangle}{G}, \]  \hspace{2cm} (4)

where a relative error of gravity on a particle is ensured to be less than \( \eta \). Except for \( t = 0 \), we adopt \( \Delta = 1.7 \times 10^{14} \) g cm\(^{-2}\), which corresponds to \( \eta = 0.01 \). At \( t = 0 \), we adopt \( \Delta \sim 1.0 \times 10^{9} \) g cm\(^{-2}\). The reason why \( \Delta \) at \( t = 0 \) is extremely smaller than \( \Delta \) at \( t > 0 \) is as follows. We do not know the gravity \( g_i \) at \( t = 0 \). Therefore, we do not know what \( \Delta \) corresponds to \( \eta = 0.01 \) at \( t = 0 \). In order to avoid obtaining the gravity with a relative error larger than 0.01, we use sufficiently small \( \Delta \) at \( t = 0 \). As a result of the calculation of the gravity at \( t = 0 \), it becomes clear that \( \Delta \sim 1.0 \times 10^{9} \) g cm\(^{-2}\) corresponds to \( \eta = 6 \times 10^{-8} \). In other words, we calculate the gravity with a relative error much less than 0.01 at \( t = 0 \).

2.2. Definitions

In this section, we define several quantities used throughout this paper.

We define two types of temperatures: raw and smoothed temperatures. The raw temperature of a particle, \( T_{r,i} \), is directly obtained from the Helmholtz EOS as a function of \( \rho_i \) and \( u_i \), as described in Section 2.1. Using \( T_{r,i} \), the smoothed temperature of a particle, \( T_{s,i} \), is calculated as

\[ T_{s,i} = \sum_j T_{r,j} \frac{m_j}{\rho_j} W \left( |r_j - r_i|, h_i \right), \]  \hspace{2cm} (5)

where \( W \) is the SPH kernel. This smoothed temperature is similar to the “SPH-smoothed temperature” in Dan et al. (2014).

We define a timescale of a nuclear reaction, \( t_{nuc} \), as

\[ t_{nuc} = \frac{c_{p,nuc} T}{\epsilon_{nuc}}, \]  \hspace{2cm} (6)

where \( c_p \) is specific heat at constant pressure, \( \epsilon_{nuc} \) is an energy generation rate per unit mass for the nuclear reaction, and \( T \) is either the raw or smoothed temperature. We indicate \( t_{nuc,r} \) and \( t_{nuc,s} \) as the timescale of a nuclear reaction, when we calculate the timescale with the raw and smoothed temperatures, respectively. We replace the subscripts “nuc” in Equation (6) with “3αr” for the triple-alpha reaction, and “cc” for the \(^{12}\text{C} + ^{12}\text{C} \) reaction. We calculate an energy generation rate by the triple-alpha reaction in the same way as Kippenhahn &
Weigert (1990):
\[
\varepsilon_{3a} = q_{he} f_{3a} \rho^2 X^3 T_8^{-3} \exp(-44.027/T_8),
\]
(7)
where \( q_{he} = 5.09 \times 10^{11} \) [erg g\(^{-1}\) s\(^{-1}\)], \( f_{3a} = \exp(2.76 \times 10^{-2} \rho^{1/2} T_8^{3/2}) \) is the weak electron screening factor (Salpeter 1954; Clayton 1968), \( X_4 \) is the mass fraction of helium, and \( T_8 = T/10^8 \), which is the same choice as that of Dan et al. (2014). We also define an energy generation rate by \( ^{12}\text{C} + ^{12}\text{C} \) as
\[
\varepsilon_{cc} = \rho q_e Y_c^2 A_{97} \exp(-Q/T_{9a} + f_{cc}).
\]
(8)
where \( q_e = 4.48 \times 10^{18} \) [erg mol\(^{-1}\)], \( f_{cc} \) is a screening factor (Blinnikov & Khokhlov 1987), \( A_{97} = 8.54 \times 10^{26} T_{9a}^{5/6} T_s^{-3/2} \) [s\(^{-1}\) cm\(^3\)], \( Q = 84.165, \ T_8 = T/(10^8 \text{K}), \) and \( T_{9a} = T_8/(1 + 0.067 T_8) \) (Fowler et al. 1975). We calculate the carbon abundance as \( Y_c = n_C/(n_C + n_H) = 0.033 \) [g g\(^{-1}\)], where \( n_C \) is the number density of carbon and \( n_H \) is the Avogadro constant.

We define the local dynamical timescale as
\[
t_{\text{dyn}} = (24\pi G \rho)^{-1/2},
\]
(9)
which is the same choice as that of Nomoto (1982).

We introduce a shock detector of a particle in order to search for shock-heated regions. The shock detector is defined as
\[
D_{s,i} = -f_i \left[ \frac{h_i}{c_{s,i}} (\nabla \cdot v_i) \right],
\]
(10)
where \( f_i \) is the Balsara switch (see the Appendix). This indicates how strongly a fluid element is compressed. The element is compressed when \( D_{s,i} > 0 \) and is extended when \( D_{s,i} < 0 \). The critical value between shocked and unshocked regions is about unity, but is not severe. Although it is just an indicator, it should be useful to grasp where fluid elements are compressed by shock waves.

We present the definition of merger ejecta as follows. We consider a specific orbital energy of a particle at the time \( t \) as \( b_1 = \phi_1(t) + 0.3|v_1(t)|^2 \), where \( \phi_1(t) \) and \( v_1(t) \) are the specific potential energy and velocity of the particle at the time \( t \). If a particle has \( b_1(t) > 0 \), it is unbound at the time \( t \). A particle that keeps \( b_1(t') > 0 \) during \( t' > t \) is defined as a merger ejecta at the time \( t \). This means that a particle unbound temporarily is not counted as a merger ejecta. We also show the definition of a terminal velocity of a merger ejecta:
\[
v_{\text{ej},i}(t) = (2b_1)^{1/2}.
\]
(11)
We can interpret the terminal velocity as a velocity of a particle at infinity.

**2.3. Initial Condition Setup**

We set up an initial condition in our SPH simulations in three steps. Our setup method is the same as that of Dan et al. (2011), unless otherwise noted. In the first step, we generate two single CO WDs individually. In the second step, we choose particles from each of the single CO WDs and assume that these particles consist of pure helium. In the third step, we combine the two CO WDs in the same frame, and form binary CO WDs. We explain these steps in detail in the following.

We can divide the first step into five substeps. In the first step, we make a one-dimensional density profile of a fully degenerate CO WD with uniform temperature of \( 10^6 \text{K} \). In the second substep, we map particles in such a way that their mass densities and specific internal energies are consistent with the single CO WD. In the third substep, we relax these particles as follows. We evolve these particles for 20 s in the simulation time by means of an SPH simulation which is different from that described in Section 2.1 in two points. One is that a specific internal energy of each particle is fixed. In other words, we do not solve Equation (2). The other is that each particle receives a damping force against its motion. The damping force on the \( i \)-particle is added to the right-hand side of Equation (1), and given by
\[
\left( \dot{v}_i \right)_{\text{damp}} = -\frac{v_i}{\tau_{\text{damp}}},
\]
(12)
where we set \( \tau_{\text{damp}} = C_{\text{damp}} \Delta t \), and \( C_{\text{damp}} = 128 \). We adopt \( C_{\text{damp}} = 128 \) experientially. Nevertheless, the value of \( C_{\text{damp}} \) does not have the impact on the structure of the CO WD. The structure of the CO WD in the case of \( C_{\text{damp}} = 128 \) is almost the same as that in the case of \( C_{\text{damp}} = 64 \). In the fourth substep, we relax these particles again by evolving them during 80 s by means of the SPH simulation described in Section 2.1. The purpose of this substep is to avoid mixing the helium and CO particles. This substep has not been done in Dan et al. (2011). In the fifth substep, we shift the center of mass in the positions of the SPH particles to the origin of coordinates, and the center of mass in the velocities to zero.

In the second step, we choose the outermost particles of each of the single CO WDs, and assume that these particles consist of pure helium. We call these particles “helium particles.” We define a helium fraction, \( f_{\text{hel}} \), as the ratio of helium mass to CO mass in these particles. We note that the EOS of these helium particles is not helium one, but CO one, but it would not introduce a large error for our purposes. We compare the radius of a pure CO WD with that of a CO WD with helium \( (f_{\text{hel}} = 1 \times 10^{-5}, 3 \times 10^{-4}, \) and \( 4 \times 10^{-5} \) \) by performing a one-dimensional hydrostatic calculation, and find that the difference among their radii is at most 1%. Proactively, we show that the helium particles are not mixed CO ones without merging. Figure 1 indicates the distribution of the helium and CO particles. Particularly, as seen in the companion WD which does not accrete materials, the helium particles remain on the surface of the companion WD.

We can divide the third step into two substeps. In the first substep, we put two single CO WDs in the same frame. Hereafter, we call the more massive CO WD “the primary,” and the less massive one “the companion.” We set a separation between the primary and companion, such that the companion’s Roche-lobe radius is \( C_{\text{lobe}} \) times larger than the companion’s radius. We estimate the Roche-lobe radius from an approximate formula of Eggleton (1983). We can express the separation between the primary and companion, \( a_0 \), as
\[
a_0 = C_{\text{lobe}} R_c \left[\frac{0.49 q^{2/3}}{0.6 q^{2/3} + \log(1 + q^{1/3})}\right]^{-1},
\]
(13)
where \( R_c \) is the companion’s radius, and \( q \) is the ratio of the companion’s mass \( (M_c) \) to the primary’s mass \( (M_p) \). We set
lobe in this study. Hereafter, we define the separation between the primary and companion as the distance between their centers of mass. They are on a circular orbit around the origin of coordinates. In the second substep, we relax the configuration of particles composing the binary CO WDs, and simultaneously decay the orbit of the binary. For the relaxation, we again introduce the damping force given by Equation (12).

We decay their orbit at every $\Delta t_{\text{decay}}$ seconds. The extent of the decay is given by

$$\Delta a_{\text{decay}} = \frac{a}{r_{\text{decay}}} \Delta t_{\text{decay}},$$

(14)

where $a$ is the binary separation before the decay. We can write $r_{\text{decay}}$ as

$$r_{\text{decay}} = \frac{1}{C_{\text{decay}}} \frac{1}{\sqrt{G \rho_{c,0}}},$$

(15)

where $\rho_{c,0}$ is the overall mass density of the companion before this relaxation and orbital decay, and written as $\rho_{c,0} = M_c / (4\pi R_c^3 / 3)$. We adopt $\Delta t_{\text{decay}} = 1/64$ s and $C_{\text{decay}} = 0.05$. In this relaxation and orbital decay, we take a co-rotating frame of reference. Therefore, we add centrifugal and Coriolis terms to Equation (1), expressed as

$$\left( \psi \right)_{\text{corotate}} = -\omega \times (\omega \times r) - 2\omega \times v,$$

(16)

where $\omega$ is the angular velocity vector, and $|\omega| = [G(M_p + M_c)/a]^1/2$. We stop this process, $\Delta t_{\text{decay}}$, after their separation is decreased to less than a critical separation $a_{\text{crit}}$. This is different from the setup method of Dan et al. (2011), who stop this process when any particle exceeds the Roche lobe.

In Figure 2, black dots show the potential energies of particles in the frame corotating with the binary. This figure corresponds to Figure 3 of Dan et al. (2011). The blue curves $(\Phi_{\text{app}})$ indicate the potential energies of the field, where the primary and companion are approximated to be the point mass at their center of mass. The potential energies are expressed as

$$\Phi_{\text{app}} = - \frac{GM_p}{|r - r_p|} - \frac{GM_c}{|r - r_c|} - \frac{1}{2} (\omega \times r)^2,$$

(17)

where $r_p$ and $r_c$ are the position of the center of mass of the primary and companion, respectively. The red curves $(\Phi_{\text{num}})$ indicate the potential energies of the field, where the potential energies among particles are calculated numerically. The potential energies are expressed as

$$\Phi_{\text{app}} = \Phi(r) - \frac{1}{2} (\omega \times r)^2,$$

(18)

where $\Phi(r)$ is the potential energies among particles. The binary is tidally locked.

2.4. Simulation Run

We simulate mergers of two types of binary CO WDs: pairs of $1.1M_\odot$ and $1.0M_\odot$ CO WDs, and $0.9M_\odot$ and $0.6M_\odot$ CO WDs. The latter is used for a benchmark, since such a pair of CO WDs has been widely investigated in various studies, such as Yoon et al. (2007).

We perform the simulations for binary CO WDs with masses of $1.1M_\odot$ and $1.0M_\odot$ as follows. We set a separation between the primary and companion, such that $a_{\text{crit}} = 1.5 \times 10^9$ cm. We choose the helium fraction as $f_{\text{He}} = 4 \times 10^{-5}$, $3 \times 10^{-4}$, and

Figure 1. Distribution of helium and carbon–oxygen particles at $t = 0$, 10, 30, and 50 s in model 5.5M. The red and blue points indicate the helium particles of the primary and companion WDs, respectively. The yellow and light blue points show the carbon–oxygen particles of the primary and companion WDs, respectively.

Figure 2. Potential energies of particles as a function of their $x$-coordinate. The definition of $\Phi_{\text{app}}$ and $\Phi_{\text{num}}$ are in the main text.
1 × 10⁻³ for both the primary and companion. We adopt various mass resolutions, where the numbers of SPH particles used to resolve 0.1M☉ are 64k, 128k, 256k, and 512k (where
1 k = 2¹⁰ = 1024). The total numbers of particles in each model are therefore about 1.4, 2.8, 5.5, and 11 × 10⁶. We name these models “model xM” (x = 1.4, 2.8, 5.5, and 11) after the total number of particles used in each run. We follow the evolutions of binary CO WDs for 500 s of the simulation time, except for model 11M. In all of the models, they merge after the binary components orbit around each other several times. In all models except for model 11M, the simulation is followed until the merger remnants reach a dynamically steady state. We stop the simulation of 11M just after their merger, since the simulation time is quite long.

For binary CO WDs with masses of 0.9 M☉ and 0.6 M☉, we separate the primary and companion by a = 2.45 × 10⁶ cm. The number of SPH particles to resolve 0.1 M☉ is 64k; the total number of particles is about 980 × 10³. We evolve it for 1000 s. At that time, it reaches a dynamically steady state.

2.5. Computing Environment

We use a code called OTOO (Nakasato et al. 2012), which stands for “OctTree On Opencl.” The OTOO code (hereafter, OTOO) supports a variety of astronomical particle simulations, such as N-body and SPH simulations. OTOO utilizes an octree algorithm (e.g., Barnes & Hut 1986) for fast calculations of particle–particle interactions. It is optimized to multi- and many-core architectures on shared-memory environments. It is relatively machine-independent, since it is implemented with OpenCL.

We perform each SPH simulation on a single node of a supercomputer HA-PACS at the Center for Computational Sciences, University of Tsukuba. A single node of HA-PACS consists of two CPUs plus four GPUs. The CPUs are Intel Sandy Bridge-EP-8, and the GPUs are NVIDIA Tesla M2090. In this configuration, OTOO spends 1.7, 3.3, 6.5, and 13 s for every timestep for models 1.4M, 2.8M, 5.5M, and 11M, respectively. The total wall-clock times for these models are about 160, 360, 670, and 1200 hr.

3. SIMULATION RESULTS

In this section, we show results of our simulations. In Section 3.1, we overview the time evolution of the binary CO WDs. In Section 3.2, we compare our results with previous studies.

3.1. Overview

In Figure 3, we present the time evolution of the binary CO WDs in models 1.4M, 2.8M, 5.5M, and 11M. In the top panels, we show the separations of the binaries. In the second top panels, we present mass distribution of the binaries. For this purpose, we use “x percent Lagrangian radii” defined as follows: each is a radius of a sphere that encloses x percent of the total mass of the binary CO WDs, and whose center is the center of mass of the primary. In the second top panel, we draw 10%, 50%, 60%, 65%, 70%, 75%, 80%, 90%, 99%, 99.9%, 99.99%, and 99.999% Lagrangian radii from bottom to top. In the second bottom and bottom panels, we depict the maximum T₉,i and T₉,i, respectively, in a range of mass density of 10⁻¹₂ < ρ₉/(g cm⁻³) < 10⁻¹⁺⁰.5. These maximum T₉,i and T₉,i are, respectively, indicated as T₉,max,i and T₉,max,i. We sample all the above quantities at every 1 s.

We can see in the top panels that the separation is steeply decreased at t ∼ 270, 180, 120, and 100 s for models 1.4M, 2.8M, 5.5M, and 11M, respectively. At these times, the binaries merge. We call these times “merger times.” The binaries rotate around each other at least five times before the dynamical merger, since their periods are about 22 s. It is difficult to avoid such different merger times for different resolutions, since the mass transfer from the companion to the primary is quite unstable and chaotic. The separation oscillates with the periods of the binaries from the initial time to the merger time. This is due to non-zero eccentricities of the orbits of these binaries.

In our simulation, the merger time is relatively smaller than previous studies, such as those of Dan et al. (2011). This may be because we stop the relaxation process of the binary when the binary separation becomes smaller than a critical separation. We may make the binary separation too small. In Section 4, we discuss this effect on whether the thermonuclear explosion becomes successful or not.

We follow the time evolution of the inner mass distribution (<90% Lagrangian radii) of model 5.5M as an example. This is instructive, since the mass distribution in other models is similar to that in model 5.5M, except for their merger times. In model 5.5M, the 10% and 50% Lagrangian radii are not changed throughout the simulation. These radii represent the primary’s material. This means that the merger has little effect on the mass distribution of the primary. On the other hand, the 60%–90% Lagrangian radii are drastically decreased around the merger time (t ∼ 120 s). These radii contain the companion’s mass. This means that most of the companion’s mass is accreted to the primary at once at the merger time. During about 50 s after the merger time, the mass distribution of the accreted material is still changing. During this time, the 60%, 65%, and 70% Lagrangian radii decrease gradually, whereas the 80% and 90% Lagrangian radii expand rapidly. After this time, the materials below the 70% Lagrangian radii keep constant in their radii, while the materials farther out than this in the mass coordinate expand slowly. We conclude that the mass distribution, in particular, the interior of the “merger remnant,” is dynamically steady at the end of our simulation (t = 500 s).

Below, the evolution of the outer mass distribution, above than 90% Lagrangian radii, is explained for model 5.5M. The 99.99% and 99.999% Lagrangian radii rapidly increase well in advance of the merger time. Similarly, the 99% and 99.9% Lagrangian radii rapidly increase around the merger time. The evolution of these Lagrangian radii means that a substantial amount of materials is ejected before/at the merging process, which surround the binary system. This matter can/should affect observations of outcome of the merger, e.g., an explosion, as discussed in detail in Section 4.

The highest raw temperature is achieved for 20 s following the merger time in each model. Hereafter, this peak of temperature is called the “first peak.” The temperature at the first peak is 2.4 × 10⁷ K, 3.6 × 10⁷ K, 3.8 × 10⁹ K, and 3.8 × 10⁹ K for models 1.4M, 2.8M, 5.5M, and 11M, respectively. This temperature seems to converge to 3.8 × 10⁹ K toward the higher resolution. The smoothed temperature also reaches a high value around the first peak. In fact, the smoothed temperature at the first peak is the highest
among the values obtained for the whole evolution of the system, in each of the models 2.8M, 5.5M, and 11M. It is $1.4 \times 10^9$ K, $1.6 \times 10^9$ K, and $2.1 \times 10^9$ K for models 2.8M, 5.5M, and 11M, respectively. This temperature, however, does not converge even with the highest resolution among our runs. Moreover, this smoothed temperature is much lower than the raw temperature at the first peak. If the mass resolution becomes extremely high, the smoothed temperature at the first peak may converge and may become consistent with the raw temperature at the first peak. We do not discuss the convergence of the smoothed temperature anymore in this paper, but we keep in mind that we may underestimate temperature when we adopt the smoothed temperature.

Both the raw and smoothed temperatures in each model have another peak about 20 s after the first peak. Hereafter, this peak is called the “second peak.” This peak is achieved for materials in the range of density $10^7 - 10^{7.5}$ g cm$^{-3}$ in all of the models. At the second peak, model 1.4M achieves the highest smoothed temperature. The raw and smoothed temperature at the second peak converge to $2.2 \times 10^9$ K and $1.5 \times 10^9$ K, respectively.

Table 1 summarizes the properties of the first and second peaks. The first peaks of the raw and smoothed temperatures

![Figure 3](image-url)
are not coincident. Since the time lag between the first peaks is at most 5 s, being sufficiently smaller than the orbital time (22 s), these peaks appear at the same merging phases. This is true for the second peaks of the raw and smoothed temperatures, except for model 11M. However, in model 11M, the smoothed temperature at the time of the second peak of the raw temperature gets as high as \(1.4 \times 10^9 \, \text{K}\), comparable to the smoothed temperature at its second peak. Accidentally, the former smoothed temperature is slightly smaller than the latter smoothed temperature.

At \(t = 500 \, \text{s}\), the raw and smoothed temperatures converge to \(9 \times 10^8 \, \text{K}\) and \(8 \times 10^8 \, \text{K}\), respectively, for materials whose density is in the range of \(10^3-10^7 \, \text{g cm}^{-3}\). As for the range of density \(10^7-10^{2.5} \, \text{g cm}^{-3}\), the smoothed temperature converges to \(6 \times 10^8 \, \text{K}\), while the raw temperature does not. This is because the raw temperature in model 5.5M jumps up at \(t = 350 \, \text{s}\). This jump-up is due to the effect of an artificial viscosity adopted in the SPH simulations. Even a slight amount of the artificial viscosity can highly increase the temperature of a particle in a high density region, since temperature is sensitive to an internal energy especially in the high density region. This effect is not seen in models 1.4M and 2.8M, and therefore the raw temperature in these models, rather than in model 5.5M, should be correct at \(t = 500 \, \text{s}\). Therefore, the raw temperature is \(8 \times 10^8 \, \text{K}\) at the density of \(10^5-10^7 \, \text{g cm}^{-3}\) at \(t = 500 \, \text{s}\). When we compare temperatures of materials at the density \(<10^7 \, \text{g cm}^{-3}\) and \(>10^7 \, \text{g cm}^{-3}\), the former temperature is higher than the latter temperature. This is because a lower density region is shock-heated more strongly during the merger event.

Below, we discuss the properties of merger ejecta in model 5.5M. Figure 4 shows the time evolution of the mass and kinetic energy of the merger ejecta. The mass is small at the merger time (\(t = 120 \, \text{s}\)), being only \(1.3 \times 10^{-5} \, M_\odot\). The ejecta mass then increases rapidly during \(t = 140-150 \, \text{s}\). Finally, the ejecta mass reaches \(3.9 \times 10^{-3} \, M_\odot\) at \(t = 500 \, \text{s}\). At \(t = 500 \, \text{s}\), the ejecta stops growing in mass. The ejecta mass reaches a constant value at about \(t = 500 \, \text{s}\), while the kinetic energy does so at an earlier time, about \(t = 200 \, \text{s}\). This is because the merger ejecta with higher velocities are formed and ejected at an earlier time. Eventually, their total kinetic energy at infinity reaches \(\sim 3.2 \times 10^{47} \, \text{erg}\).

Table 2 shows the tidal mass, and the masses and kinetic energies of the total ejecta in models 1.4M, 2.8M, and 5.5M. We cannot investigate these properties of the total ejecta in model 11M, since we do not follow the evolution of this model until a dynamical steady state is reached. The properties of the ejecta are independent of the mass resolution.

The merger ejecta can be divided into two groups, depending on their formation mechanism. The merger ejecta in the first group can be seen in Figure 5. They are generated from tidal tails of the primary and companion, and so they are called “tidal ejecta.” They become unbound, since they receive orbital angular momenta from the rotating bar potential formed by the binary system. They are formed only before \(t = 130 \, \text{s}\). Their total mass is \(8.6 \times 10^{-5} \, M_\odot\). The tidal ejecta do not become unbound due to a shock wave, since they are far away from shocked regions. The shocked region is identical to that creating hotspots at Pakmor’s time (see Section 4.2). They are not strong shocks, since their shock detectors are a bit smaller than unity.

In Figure 6, we show the moment when a part of the merger ejecta in the second group is formed. They are generated in a

### Table 1

| Model | Peak | Type | \(T\) (10^9 K) | \(\rho\) (10^6 g cm^{-3}) | Time (s) |
|-------|------|------|----------------|----------------|---------|
| 1.4M  | 1st  | \(T_{\text{ini}}\) | 2.5            | 1.7            | 284     |
|       | 1st  | \(T_{\text{ini}}\) | 1.3            | 6.4            | 279     |
|       | 2nd  | \(T_{\text{ini}}\) | 1.9            | 13             | 344     |
|       | 2nd  | \(T_{\text{ini}}\) | 1.4            | 6.4            | 346     |
| 2.8M  | 1st  | \(T_{\text{ini}}\) | 3.4            | 2.9            | 194     |
|       | 1st  | \(T_{\text{ini}}\) | 1.5            | 3.6            | 195     |
|       | 2nd  | \(T_{\text{ini}}\) | 2.1            | 13             | 230     |
|       | 2nd  | \(T_{\text{ini}}\) | 1.3            | 5.0            | 226     |
| 5.5M  | 1st  | \(T_{\text{ini}}\) | 3.8            | 2.9            | 134     |
|       | 1st  | \(T_{\text{ini}}\) | 1.6            | 1.9            | 133     |
|       | 2nd  | \(T_{\text{ini}}\) | 2.1            | 19             | 171     |
|       | 2nd  | \(T_{\text{ini}}\) | 1.5            | 9.2            | 173     |
| 11M   | 1st  | \(T_{\text{ini}}\) | 3.8            | 3.8            | 110     |
|       | 1st  | \(T_{\text{ini}}\) | 2.1            | 3.7            | 111     |
|       | 2nd  | \(T_{\text{ini}}\) | 2.2            | 13             | 138     |
|       | 2nd  | \(T_{\text{ini}}\) | 1.4            | 10             | 153     |

### Figure 4

Time evolution of mass (top) and kinetic energy (bottom) of merger ejecta in model 5.5M.

### Table 2

| Model | Tidal Ejecta Mass | Total Mass \((M_\odot)\) | Total Kinetic Energy (erg) |
|-------|-------------------|--------------------------|----------------------------|
| 1.4M  | \(2.1 \times 10^{-5}\) | \(5.0 \times 10^{-3}\) | \(3.0 \times 10^{47}\) |
| 2.8M  | \(2.1 \times 10^{-5}\) | \(3.9 \times 10^{-3}\) | \(3.1 \times 10^{47}\) |
| 5.5M  | \(1.3 \times 10^{-5}\) | \(3.9 \times 10^{-3}\) | \(3.2 \times 10^{47}\) |
| 11M   | \(\ldots\)        | \(\ldots\)               | \(\ldots\)                |
shocked region (see the top panels). Hereafter, they are called “shocked ejecta.” The shocked region arises from a collision between the main body of the system and a tidal tail. The merger ejecta are prevented from traveling toward the directions of the orbital plane, which can be seen in the bottom right panel. This is because the tidal tail acts as an obstacle. The shocked ejecta are formed several times through the above mechanism after \( t = 130 \) s. They dominate the total mass of the merger ejecta.

The terminal velocities of the tidal ejecta are typically \( 3-4 \times 10^8 \text{cm s}^{-1} \), similar to the relative velocity between the primary and companion just before their merger. Those of the shocked ejecta range from \( 10^7 \) to \( 10^9 \) cm s\(^{-1} \). These velocities are the largest just after the merger and decrease gradually.

### 3.2. Comparison with Previous Studies

In this section, we compare our results with those obtained by previous studies. First, we use the results of a binary with masses of 1.1 \( M_\odot \) and 1.0 \( M_\odot \), and next use those of a binary with masses of 0.9 \( M_\odot \) and 0.6 \( M_\odot \).

For a binary with masses of 1.1 \( M_\odot \) and 1.0 \( M_\odot \), we focus on the raw temperature and the mass of the merger ejecta at its final state. We also check the smoothed temperature, if it is available for comparison in any of the previous studies. Not all of the previous studies treat binary CO WDs exactly with masses of 1.1 and 1.0 \( M_\odot \), and for such cases we adopt, for comparison to work out, previous studies with mass combinations similar to the one in our simulations. In our results, the maximum raw and smoothed temperatures are, respectively, \( 9 \times 10^8 \text{K} \) and \( 8 \times 10^8 \text{K} \) (see Section 3.1). The corresponding temperature in previous studies is as follows. In Dan et al. (2014), the maximum raw and smoothed temperatures are, respectively, \( 11 \times 10^8 \text{K} \) and \( 8.5 \times 10^8 \text{K} \) in the case of binary CO WDs with masses of 1.05 and 1 \( M_\odot \) (see their Table A1). In Zhu et al. (2013), their maximum raw temperature is \( 9.4 \times 10^8 \text{K} \) in binary CO WDs with masses of 1.0 and 1.0 \( M_\odot \) (see the column of \( T_{\text{max}}^\circ \) in their Table 2). In Raskin et al. (2012), the maximum raw temperature is about \( 12.5 \times 10^8 \text{K} \) for a binary with masses of 1.06 and 0.96 \( M_\odot \) (see their Figure 9). Our results are in good agreement with theirs.

The total mass of the merger ejecta in our simulation is \( 3.9 \times 10^{-3} M_\odot \). In Dan et al. (2014), it is \( 7.64 \times 10^{-4} M_\odot \) for a binary with masses of 1.0 \( M_\odot \) and 1.0 \( M_\odot \), 8.50 \( \times 10^{-4} M_\odot \) for 1.05\( M_\odot \) and 1.0 \( M_\odot \), and 1.453 \( \times 10^{-3} M_\odot \) for 1.05 \( M_\odot \) and 1.05 \( M_\odot \). Our mass is consistent with those in Dan et al. (2014) to the first order.

In our results, the total mass of the tidal ejecta is \( 8.6 \times 10^{-5} M_\odot \). In Raskin & Kasen (2013), the mass of the tidal ejecta is \( 4.7 \times 10^{-3} M_\odot \) for a binary with masses of 1.06 \( M_\odot \) and 1.06 \( M_\odot \), and \( 3.3 \times 10^{-3} M_\odot \) for 1.20\( M_\odot \) and 1.06 \( M_\odot \) (see their Table 1). According to the fitting formula of Dan et al. (2014; see their Equation (A11)), the mass of the tidal ejecta is \( \sim 1 \times 10^{-3} M_\odot \) in the case of a binary with masses of 1.1 \( M_\odot \) and 1.0 \( M_\odot \). Our mass is smaller than those in these previous studies by an order of magnitude. The small merger time of model 5.5M should not affect the small mass of the tidal ejecta. In model 1.4M, the CO WDs orbit around each other more than 10 times, but the mass of the tidal ejecta is less than \( 2.1 \times 10^{-5} M_\odot \), which is almost the same as in model 5.5M (see Table 2). Since the tidal ejecta are a minor component, their formation would be sensitive to the detail of the setup of simulations. We do not discuss this discrepancy anymore, but we should keep in mind that we may underestimate the total mass of the tidal ejecta.

For a binary with masses of 0.9 \( M_\odot \) and 0.6 \( M_\odot \), we show the time evolution of the raw temperature in Figure 7. The maximum raw temperature after the merger event (\( t = 1000 \) s) is \( 6 \times 10^8 \text{K} \) or less depending on the position of the materials within the final merger remnant. This is consistent with those found in previous studies (Yoon et al. 2007; Zhu et al. 2013; Dan et al. 2014). In Nomoto et al. (2013), the corresponding temperature was reported to be \( 8.5 \times 10^8 \text{K} \). This is a bit higher than our results although Nomoto et al. (2013) adopted the same simulation code as ours. We find that this difference comes from the difference in the recipes making the initial conditions. We relax a configuration of the binary CO WDs, while Nomoto et al. (2013) did not. This is consistent with the argument by Dan et al. (2011), in which the maximum raw temperature at the final state becomes higher without the relaxation.

For this set of the binary parameters (0.9 \( M_\odot \) and 0.6 \( M_\odot \)), the total mass in the merger ejecta is about \( 8.7 \times 10^{-4} M_\odot \), which consists mainly of tidal ejecta. The ejecta mass of a binary with masses of 0.9 \( M_\odot \) and 0.6 \( M_\odot \) is much smaller than that of a binary with masses of 1.1 \( M_\odot \) and 1.0 \( M_\odot \), since the former merger is less violent than the latter. The corresponding masses are \( 2.0 \times 10^{-3} M_\odot \) in binary CO WDs with masses of 0.96 and 0.64 \( M_\odot \) in Raskin & Kasen (2013), and \( 1.0 \times 10^{-3} M_\odot \) for a combination of 0.9 and 0.65 \( M_\odot \) in Dan et al. (2014) from their Table A1. Our result is almost consistent with the previous studies.

In summary, our raw temperature in the final state is consistent with those of the previous studies. The total mass of the merger ejecta is also in good agreement with the previous studies. However, the mass of the tidal ejecta is different from those found in previous studies by an order of magnitude. Since
the tidal ejecta are a minor component, their formation would be sensitive to the detail of the setup of simulations.

4. ASSESSMENT OF EXPLOSION MODELS

In this section, we assess explosion models based on the results of our SPH simulations. These models are the helium-ignited violent merger, the carbon-ignited violent merger, and the Chandrasekhar mass (see Section 1). We also discuss other branches of models that are not categorized in any of these models. We investigate these explosion models in chronological order: the helium-ignited violent merger model (Section 4.1), the carbon-ignited violent merger model (Section 4.2), the other models (Section 4.3), and the Chandrasekhar mass model (Section 4.4). In Sections 4.1, 4.2, and 4.4, we investigate whether binary CO WDs can lead to an explosion, and whether the explosion can be observed as an SN Ia. In Section 4.3, we only discuss what an explosion should look like, assuming the explosion is triggered by these models.

4.1. Helium-ignited Violent Merger Model

In this model, the helium detonation at the surface of the primary induces compression of the core materials of the primary. This triggers the carbon detonation in the core, subsequently leading to an explosion. In order to assess the possibility of such a mode in the explosion, we provide two check points; whether the helium detonation is initiated, and whether it triggers the carbon detonation.

We define conditions of initiation of the helium detonation. The helium detonation is a supersonic flame powered by helium burning, i.e., the triple-alpha reaction. In order for the triple-alpha reaction to power the flame, the triple-alpha reaction needs to proceed faster than cooling due to an adiabatic expansion. In other words, the timescale of the triple-alpha reaction should be shorter than the local dynamical timescale of the fluid under consideration. Therefore, we adopt $t_{\alpha < t_{\alpha}}$ as a criterion for the initiation condition of the helium detonation. We should keep in mind that this is only a necessary condition. For a necessary and sufficient condition, a condition that the flame propagates supersonically must be satisfied. However, we do not consider such a condition in this paper.
Figure 8 shows the time evolution of the total mass of helium particles with $t_{\text{He, t}} < t_{\text{dyn}}$ (top) and $t_{\text{He, s}} < t_{\text{dyn}}$ (bottom).

Figure 8 shows the time evolution of the total mass of helium particles with $t_{\text{He, t}} < t_{\text{dyn}}$ in model 5.5M. The total mass becomes non-zero before the merger time ($t = 120$ s), regardless of a choice of $t_{\text{He, t}}$ or $t_{\text{He, s}}$, and regardless of the value of $f_{\text{He}}$. Therefore, the system generally satisfies the necessary condition to initiate the helium detonation before the merger time.

We define the initiation time of the helium detonation as the time after which the total mass of the helium particle with $t_{\text{He, t}} < t_{\text{dyn}}$ raises more than $10^{-9} M_\odot$ and keeps to be so, in order to exclude numerical noises. The initiation time in all of the cases is summarized in Table 3. The initiation time is earlier when we adopt $t_{\text{He, t}}$ and a larger value of $f_{\text{He}}$. We can also see in Figure 9 that a few helium particles have $t_{\text{He, t}} < t_{\text{dyn}}$ at the initiation times of the helium detonation in all of the cases.

We ignore the contribution of the alpha process, such as $^{12}\text{C} \rightarrow ^{16}\text{O}$, to the initiation of the helium detonation. This is because the triple-alpha reaction is much faster than the alpha process when temperature is lower than $10^{9}$ K (e.g., Shen & Moore 2014). The alpha process is important for the propagation of the helium detonation, but unimportant for the initiation of the helium detonation.

The small merger time in our simulation may artificially increase the temperature of the helium particles, since the merging process may start suddenly. We thus note that the helium detonation might easily be initiated artificially. However, we do not consider this effect in the following discussion.

Next, we consider whether the helium detonation can trigger the carbon detonation in the core of the primary. The helium detonation is thought to trigger the carbon detonation in either of two ways. In one way, the helium detonation hits the core of the primary, and directly ignites the carbon detonation there. This is called a “direct drive” mechanism. In the other way, the shock wave created by the helium detonation propagates into the core with little or no carbon burning. If the helium detonation region encloses the core the shock wave then could converge somewhere in the core, and could become sufficiently strong to initiate the carbon detonation. This is called a “converging shock” mechanism. For the direct drive mechanism, a substantial amount of helium is required (e.g., Moll & Woosley 2013). It is unlikely that this mechanism triggers the carbon detonation in our models, since $f_{\text{He}}$ is small. We therefore focus on the converging shock mechanism, to assess if our system could lead to an explosion by this mode, i.e., the helium-ignited violent merger.

For the converging shock mechanism to successfully operate, a helium layer needs to enclose the core of the primary. We investigate the distribution of helium at the initiation time of the helium detonation, as shown in Figure 10. In the cases of $f_{\text{He}} = 1 \times 10^{-3}$ and $3 \times 10^{-4}$, the helium particles enclose the core of the primary, regardless of a choice of the raw or smoothed temperature to define the initiation time.

![Figure 8](image-url)

**Figure 8.** Time evolution of the total mass of helium particles with $t_{\text{He, t}} < t_{\text{dyn}}$ (top) and $t_{\text{He, s}} < t_{\text{dyn}}$ (bottom).

![Figure 9](image-url)

**Figure 9.** Mass density and temperature of helium particles at the time when the helium detonation is initiated. In the left and right panels, vertical axes indicate the raw and smoothed temperatures, respectively. From top to bottom, $f_{\text{He}} = 1 \times 10^{-3}$, $3 \times 10^{-4}$, and $4 \times 10^{-5}$. A curve in each panel shows mass density and temperature at which the timescale of the triple-alpha reaction is equal to the local dynamical timescale.

### Table 3

| Temperature | $f_{\text{He}}$ | Time (s) | $^{56}\text{Ni}$ ($M_\odot$) |
|-------------|-----------------|---------|-------------------------------|
| Raw         | $1 \times 10^{-3}$ | 29 | $1.1 \times 10^{-6}$ |
| Raw         | $3 \times 10^{-4}$ | 48 | $1.1 \times 10^{-6}$ |
| Raw         | $4 \times 10^{-5}$ | 76 | $3.8 \times 10^{-7}$ |
| Smoothed    | $1 \times 10^{-3}$ | 64 | $1.1 \times 10^{-5}$ |
| Smoothed    | $3 \times 10^{-4}$ | 73 | $3.4 \times 10^{-6}$ |
| Smoothed    | $4 \times 10^{-5}$ | 87 | $3.8 \times 10^{-7}$ |
of the helium detonation. However, in the cases of $f_{\text{He}} = 4 \times 10^{-3}$, the helium particles are sparse on the orbital plane, again regardless of a choice of the raw or smoothed temperature.

A reason for this sparseness can be explained as follows. The mass accretion from the companion to the primary proceeds in the following way; the helium particles first fall, and subsequently the CO particles do so. The CO particles have already started falling onto the surface of the primary at the initiation time of the helium detonation. These CO particles hitting the surface of the primary on the orbital plane push the helium particles away. When $f_{\text{He}}$ is smaller, the helium particles are distributed more sparsely on the orbital plane at the initiation time of the helium detonation for two reasons: first, because of the smaller $f_{\text{He}}$, all of the helium particles are more easily pushed away from the orbital plane. Second, as $f_{\text{He}}$ becomes smaller, the initiation time becomes closer to the merger time (see Table 3). As the initiation time is closer to the merger time, the CO particles are accreted by the primary more violently (see Figure 3), pushing the helium particles away more easily.

From the above, we conclude that only in the case of $f_{\text{He}} \gtrsim 3 \times 10^{-4}$, the helium detonation can potentially succeed in triggering the core carbon detonation, and can lead to an explosion. In Pakmor et al. (2013), the helium detonation encloses the core of their primary in $f_{\text{He}} = 0.01$. This result is consistent with our results.

As described above, our binary can potentially explode through the helium-ignited violent merger model, if $f_{\text{He}} \gtrsim 3 \times 10^{-4}$. Hereafter we consider the expected observational outcome, focusing on brightness just after the explosion, hereafter called the “early brightness.” In this phase, the optical photons are basically powered by the thermal energy content produced by the shock heating following the SN explosion, in the envelope of the exploding progenitor. Energy reservoir is sensitive to the structure of the binary, especially to the size of the envelope of the exploding progenitor. Figure 11 shows the matter distribution of our binary system at the initiation time of the helium detonation. It has a tidal-tail structure at $t = 29$ and 48 s, and a disk structure at $t = 64$ and 73 s. These materials spread out to $\sim 0.1 R_\odot$ away from the center of the primary.

The early brightness is also affected by a radius of the companion, through the interaction between the SN ejecta and the companion. The radius of the companion at the initiation time of the helium detonation is similar to that at the initial time. Then, the companion radius is about $5 \times 10^{9}$ cm ($7 \times 10^{-3} R_\odot$).

Moreover, the early brightness can be partly powered by the radioactive decay of $^{56}\text{Ni}$ synthesized by the helium detonation. We estimate an amount of $^{56}\text{Ni}$ as follows. The helium detonation synthesizes $^{56}\text{Ni}$ when the density is higher than a critical density, $\rho_c > 10^6$ g cm$^{-3}$, after the helium detonation passes. Taking into account the shock compression, we assume that the helium particles with $\rho_c > 10^6$ g cm$^{-3}$ at the initiation time of the helium detonation are converted to $^{56}\text{Ni}$. We show the amount of $^{56}\text{Ni}$ as estimated in this way, in all the cases in the fourth column of Table 3.

We qualitatively compare the expected early brightness resulting from the helium-ignited violent merger model of our binary with the early brightness of SN 2011fe and SN 2014J, in terms of radii of the primary and companion at the time of the helium ignition, and $^{56}\text{Ni}$ mass after the helium detonation. The expected light curve will be much fainter than the light curve of SN 2014J. Following the analysis of its early brightness (Goobar et al. 2015), SN 2014J is suggested to have either a large primary radius ($\sim 1 R_\odot$), a larger companion radius ($\sim 4 R_\odot$), or a large amount of $^{56}\text{Ni}$ mass near the surface ($10^{-3} M_\odot$). All of these are larger than those found in the results of our simulation. Another test is provided by SN 2011fe. The expected early brightness may likely be brighter than the light curve of SN 2011fe; the progenitor of SN 2011fe is suggested to have a small radius, <0.1 $R_\odot$ (Nugent et al. 2011; Bloom et al. 2012; Zheng et al. 2013; Mazzali et al. 2014). However, we should keep in mind that our “envelope” has tidal-tail and disk-like structures, which has not been taken into account in models to connect the size of the progenitor and the early brightness. This structure should affect the expected early brightness; the light curve may be fainter than we expect above because of the small opening angle of the envelope from the primary, and may be consistent with that of SN 2011fe. In order to quantitatively compare the early brightness resulting from our binary with those of SNe 2014J and 2011fe, we have to follow the explosion of our binary by means of numerical simulations. This is our future work.

Another test is provided by properties of SNRs. Papish et al. (2015) have investigated expected properties of SN ejecta from
large fraction of SN Ia remnants, which tend to be spherically symmetric.

This section is summarized as follows: when \( f_{\text{He}} \gtrsim 3 \times 10^{-4} \), our binary can explode through the helium-ignited violent merger model. However, the expected explosion has different features from SN Ia in several regards. The explosion has an early brightness that is much brighter than that of SN 2011fe, and much fainter than that of SN 2014J. Note that the light curve may be fainter than we expect, and may be consistent with that of SN 2011fe. In addition, the explosion will lead to an SNR with a non-spherically symmetric shape, which is inconsistent with shapes of a large fraction of SN Ia remnants. We note that it is unclear whether the other mass combinations can explode through this mode, and whether the explosions can be observed as an SN Ia.

### 4.2. Carbon-ignited Violent Merger Model

If the system survives with no or insufficient energy injection from the helium detonation, there is a chance that the carbon detonation initiated at a hotspot leads to an explosion, i.e., the carbon-ignited violent merger model. A successful explosion in this mode depends on whether the hotspot appears, in which case the nuclear reaction proceeds rapidly to lead to the carbon detonation. A necessary condition for this mode to lead to a successful explosion is the following: for the carbon detonation to take place, the \( ^{12}\text{C} + ^{12}\text{C} \) reaction should proceed rapidly to lead to thermonuclear runaway. Therefore, a heating rate by the \( ^{12}\text{C} + ^{12}\text{C} \) reaction should exceed a cooling rate due to an adiabatic expansion. In other words, the timescale of the \( ^{12}\text{C} + ^{12}\text{C} \) reaction should be shorter than the local dynamical timescale, i.e., \( t_{\text{cc}} < t_{\text{dyn}} \). We call materials that satisfy this condition the hotspots.

We search for hotspots in our simulation at the time when any particle has the density of \( >2 \times 10^6 \) g cm\(^{-3}\) and the raw temperature of \( >2.5 \times 10^9 \) K, and at the time of the first peak (see Section 3.1). The former time is the same as the time of the creation of the hotspots defined by Pakmor et al. (2012a), which is based on results of Seitenzahl et al. (2009). We call this time “Pakmor’s time.” In Figure 12, we show the densities and temperatures of particles at Pakmor’s time (left) and the time of the first peak (right) in model 11M. Curves in the top and bottom panels indicate contours of \( t_{\text{cc}}/t_{\text{dyn}} \). Note that the criterion of Pakmor et al. (2012a) are stronger than even \( t_{\text{cc,r}} < 0.1t_{\text{dyn}} \), since it considers the decay of the carbon detonation. Although we use \( t_{\text{cc,r}} < t_{\text{dyn}} \) for the criterion of the hotspots below, we also consider whether the hotspots appear if we choose the constraints of Seitenzahl et al. (2009).

At Pakmor’s time, there are particles satisfying \( t_{\text{cc,r}} < t_{\text{dyn}} \) (the top left panel), but none with \( t_{\text{cc,s}} > t_{\text{dyn}} \) (the bottom left panel), where \( t_{\text{cc,r}} \) and \( t_{\text{cc,s}} \) are evaluated using the raw and smoothed temperatures, respectively. On the other hand, at the time of the first peak, \( t_{\text{cc,r}} < t_{\text{dyn}} \) (the top right panel) and \( t_{\text{cc,s}} < t_{\text{dyn}} \) (the bottom right panel). Therefore, the creation of the hotspots at Pakmor’s time depends on the numerical treatment of the temperature, while at the time of the first peak the hotspots are created robustly irrespective of the treatment of temperature in an SPH simulation. We note that, if we use the criterion of Pakmor et al. (2012a) for the creation of the hotspots, the hotspots are not created even at the first peak from the point of view of the smoothed temperature.

**Figure 11.** Material distribution of a binary with masses of \( 1.1 M_\odot \) and \( 1.0 M_\odot \) in model 5.5M. We draw only particles separated from the center of the primary by \( 2 \times 10^5 \) cm. The time is indicated at the top right in each panel. The time is the initiation time of the helium detonation in the cases of \( f_{\text{He}} = 1 \times 10^{-3} \) and the raw temperature (two top-left panels), \( f_{\text{He}} = 1 \times 10^{-3} \) and the smoothed temperature (two top-right panels), \( f_{\text{He}} = 3 \times 10^{-4} \) and the raw temperature (two bottom-left panels), and \( f_{\text{He}} = 3 \times 10^{-4} \) and the smoothed temperature (two bottom-right panels). Dashed curves indicate \( 0.1 R_\odot \) from the center of the primary.
In our simulation, the merger time is relatively shorter than in previous studies. However, the time to the merger does not affect the peak temperature throughout the merging process (Pakmor et al. 2012b). Therefore, the above discussion on whether the hotspots are created is robust.

Hereafter we adopt model 5.5M to investigate the formation mechanism of these hotspots. In model 5.5M, Pakmor’s time is \( t = 118 \) s as shown in Figure 13. Black dots indicate the hotspots. The hotspots are formed as follows. Just before \( t = 118 \) s, the companion is tidally disrupted. Subsequently, a large amount of the disrupted debris is rapidly accreted onto the primary. This can be seen in the top panel. The center of the primary is located on the red region, and the debris extends in the direction of the top left from the primary in this figure. The accretion of the debris forms a shocked region at the surface of the primary. The shocked region can be seen from \((x/10^9 \text{ cm}, y/10^9 \text{ cm}) = (-0.5, -0.5)\) to \((0, 0.5)\) in the bottom panel. In this region, the materials are compressed, and the resulting high raw temperatures lead to the creation of the hotspots.

This behavior is qualitatively in agreement with the results by Pakmor et al. (2012a) where a system of binary CO WDs with masses of 1.1 and 0.9 \( M_\odot\), similar to our case, is considered. As seen in their Figure 1, their hotspots are generated at the primary’s surface at the time just after the companion is tidally disrupted.

In order to investigate the effects of the alpha process, we draw the positions of the helium particles when \( f_{\text{He}} = 4 \times 10^{-5} \). The reason why we choose \( f_{\text{He}} = 4 \times 10^{-5} \) is that the binary explodes in the case of \( f_{\text{He}} \geq 3 \times 10^{-4} \) before this time through the helium-ignited violent merger mode (see Section 4.1). As seen in the top panel of Figure 13, the helium particles are far away from the hot particles indicated by the black dots. The alpha process does not affect the carbon-ignited violent merger mode at this time.

Next, we investigate the creation of the hotspots at the time of the first peak. Actually, at the time of the first peak in model 5.5M, even the particles with the highest temperature do not satisfy the necessary condition to form the hotspots if we adopt the smoothed temperature. Despite the absence of hotspots in model 5.5M (for the smoothed temperature), we adopt this model for further investigation for the following reasons: we unfortunately do not follow the evolution of model 11M until its merger remnant reaches a dynamically steady state, since the simulation of model 11M is highly time-consuming. As a result, we cannot assess the Chandrasekhar model and investigate merger ejecta, using model 11M. In order to assess all of the explosion models with the same simulation model, we assess the carbon-ignited violent merger model, using model 5.5M. Indeed, the particles in model 5.5M obtain high smoothed temperatures through the same mechanism as the hotspots found in model 11M. Regarding these particles with the high smoothed temperature as the hotspots (which should satisfy the condition for the hotspots in the corresponding higher-resolution simulation), we investigate the formation mechanism of these particles.

Figure 14 shows the states of particles around at the time of the first peak. The black dots indicate particles with the smoothed temperature exceeding \( 1.5 \times 10^9 \) K at \( t = 133 \) s. The number of these particles is 4. We regard these particles at
Figure 14. Mass densities (top) and shock detectors (bottom) of particles at \( t = 128 \), 130, and 133 s from left to right in model 5.5M. The particles are separated from the orbital plane by \( <0.1 \times 10^3 \) cm. They are colored in the same way as Figure 13. Black dots indicate particles with smoothed temperatures more than \( 1.5 \times 10^9 \) K at \( t = 133 \) s. The numbers of these particles are 4. In each panel, black dots look like only one dot. This is because these particles are located on almost the same positions. In the top right panel, black crosses indicate the helium particles when \( \rho_{\text{He}} = 4 \times 10^{-5} \).

\( t = 133 \) s as the hotspots. Note that these particles are different from those regarded as the hotspots at Pakmor’s time.

Similarly to Pakmor’s time (see Figure 13), we draw the helium particles (\( \rho_{\text{He}} = 4 \times 10^{-5} \)) on the top right panel of Figure 14. The reason why we choose \( \rho_{\text{He}} = 4 \times 10^{-5} \) is the same as the case of Pakmor’s time. Similarly to Pakmor’s time, the helium particles are far away from the hot particles indicated by the black dots. Therefore, the alpha process does not affect the carbon-ignited violent merger mode at this time.

We follow trajectories of these particles. At \( t = 128 \) s, they are caught between the primary and a tidal tail. A collision between the primary and the tidal tail forms a shocked region. When the particles pass across the shocked region, their kinetic energies are converted to internal energies. After \( t = 128 \) s, these particles orbit around the primary. At \( t = 133 \) s, these particles irrupt into a clump created by the debris of the tidally disrupted companion. The clump has a high density, \( 10^{6.5} < \rho / (\text{g cm}^{-3}) < 10^{6.875} \), despite the fact that it is separated from the center of the primary by \( \sim 0.5 \times 10^9 \) cm. Its surroundings have a lower density, \( 10^6 < \rho / (\text{g cm}^{-3}) < 10^{6.5} \). When the particles irrupt into the clump, they are compressed (nearly adiabatically), and achieve the highest smoothed temperatures.

Figure 15 shows the creation and evolution of the clump quantitatively. The horizontal axes in all but the top right panel indicate \( \phi \), which is an angle between a line segment connecting the coordinate origin and a given point, and one connecting the coordinate origin and the initial position of the center of mass of the primary. At \( t = 0 \) s, the densities are almost independent of \( \phi \) at a density exceeding \( 10^6 \) g cm\(^{-3} \). However, it is not the case at \( t = 130, 150, \) and \( 170 \) s, during several tens of seconds after the merger time. The clump is present at \( \phi = 5 \) (\( t = 130 \) s), 4 (\( t = 150 \) s), and 0 radian (\( t = 170 \) s). At \( t = 250 \) s, the density becomes independent of \( \phi \) again, announcing that the clump has disappeared.

Kashyap et al. (2015) have found a hotspot formed through a spiral mode instability in the accretion disk consisting of the debris of the companion. Such a hotspot and spiral possibly appear in our simulation. We can see a spiral in the bottom panels of Figure 14. Also, the particle with the highest temperature at the second peak in our simulation is similar to the hotspot in Kashyap et al. (2015); our particle has a density of \( 2 \times 10^7 \) g cm\(^{-3} \) and temperature of \( 2.1 \times 10^9 \) K in 5.5M (see Table 1), while the hotspot has a density of \( 10^7 \) g cm\(^{-3} \) and temperature of \( 3 \times 10^9 \) K. Our particle has a slightly lower temperature than the hotspot, since nuclear reactions are not solved in our simulation. We do not discuss the hotspot (or the particle with the highest temperature at the second peak) anymore.

Since we consider only the necessary condition to lead to the initiation of the carbon detonation and do not deal with subsequent evolution following the detonation, it is not clear whether this binary system explodes in the end or not. Also, it is uncertain whether the explosion is initiated at Pakmor’s time or at the time of the first peak—both are possible but our understanding is currently limited by numerical difficulties. Nevertheless, we investigate what the explosion should look like as we did for the helium-ignited violent merger model, especially considering two situations where the explosion...
occurs either at Pakmor’s time or at the time of the first peak. We again focus on the expected early brightness and 56Ni distribution within the hypothesized SN ejecta.

Figure 15 shows the material distribution of our binary at Pakmor’s time and at the time of the first peak. The materials spread out beyond 0.1 $R_\odot$, and reach up to $\sim 0.3 R_\odot$. The system has a disk structure, similar to the material just before an explosion in the helium-ignited violent merger model (see Section 4.1). However, the disk in this case is more massive and thicker than in the case of the helium-ignited violent merger model.

Following the same argument as presented in Section 4.1 but applied to the expected pre-SN structure for the carbon-ignited violent merger model, we expect that this mode results in the early brightness much brighter than that of SN 2011fe, and much fainter than that of SN 2014J. This is because the envelope has a radius of $\sim 0.3 R_\odot$, which is much larger than that inferred for SN 2011fe ($< 0.1 R_\odot$), but much smaller than that for SN 2014J ($> 1 R_\odot$). The situation is similar to that for the helium-ignited violent merger model, and again our comparison suffers from the limitation of the spherically symmetric structure assumed in the estimates of the progenitor radii for these SNe. The envelope has disk-like structure (see Figure 16), and this should be taken into account for detailed comparison. Qualitatively, the small opening angle of the envelope from the primary should make the early brightness fainter than we expected above, and the light curve may be consistent with that of SN 2011fe. This should be quantitatively investigated by means of numerical simulations, similarly to the early brightness resulting from the helium-ignited violent merger model (see Section 4.1).

Next we discuss 56Ni distribution synthesized at the hypothesized explosion. If the explosion is initiated at Pakmor’s time, its distribution is similar to that of Pakmor et al. (2012a). Briefly speaking, 56Ni is expected to be absent in the central region of the SN ejecta for the following reason: since the companion is burned later than the primary, the ashes of the companion are expected to be located at the central region in the SN ejecta. At the same time, the low density there results in a small amount of 56Ni. On the other hand, if the explosion is initiated at the time of the first peak, 56Ni is present at the central region of the explosion for the following reason. Since the companion has been largely disrupted already, the system is more spherically symmetric in this case than at Pakmor’s time. The hotspots are created at the surface of the primary, and the explosion is initiated at the off-center region. This configuration is similar to that in the gravitationally confined detonation model (Jordan et al. 2008; Meakin et al. 2009) or the off-center delayed-detonation model (Kasen et al. 2009; Maeda et al. 2010; Seitenzahl et al. 2013), in which a large amount of 56Ni is synthesized near the center of the SN ejecta. The difference of 56Ni distribution comes from whether the companion is disrupted or not at the time of the explosion.

From an observational viewpoint, a model where the explosion is initiated at the time of the first peak is more favorable, since SN Ia contains 56Ni (or other Fe-peak elements) near the center of the explosion. However, even in this case, after the explosion the 56Ni distribution is expected to evolve to an hourglass-like shape, since 56Ni is prevented from moving toward the direction of the orbital plane as blocked by a debris of the companion (Raskin et al. 2014). This is not

**Figure 15.** Mass density on a circle on the orbital plane. The circle is centered on the center of the primary. As seen in the top right panel, a position on the circle is depicted as a radius $R$ and angle $\phi$, where the origin is the center of the primary. In the other panels, mass densities are shown on circles with $R = 0.010, 0.018, 0.032, 0.056, 0.10, 0.18, 0.32, 0.56$ in units of $10^9$ cm from top to bottom at $t = 0, 130, 150, 170,$ and 250 s.

**Figure 16.** Material distribution of a binary with 1.1$M_\odot$ and 1.0 $M_\odot$ in model 5.5M. We draw only particles separated from the center of the primary by $2 \times 10^9$ cm. The time is indicated at the bottom left in each panel. The time is the initiation time of the carbon detonation in the cases of the raw temperature (two left panels) and the smoothed temperature (two right panels). Dashed curves indicate $0.1 R_\odot$ from the center of the primary.
consistent with the $^{56}$Ni distribution generally inferred for SN Ia which is a spherically symmetric shape (Maund et al. 2013; Soker et al. 2014).

This section is summarized as follows: in the carbon-ignited violent merger model, we confirmed that the hotspots appear. Therefore, an explosion can potentially occur through this mode. However, it is expected that the resulting early brightness is much brighter than SN 2011fe and much fainter than SN 2014J. Note that the small opening angle of the envelope from the primary should make the light curve fainter than we expect, and that the light curve may be consistent with that of SN 2011fe. Moreover, it is expected that the $^{56}$Ni distribution does not have a spherically symmetric shape, also being inconsistent with the $^{56}$Ni distribution generally inferred for SN Ia. The expected observational outcome will apply to any explosions through this mode, since the explosions can potentially occur only from the WD mass combinations similar to the one studied in this paper (Sato et al. 2015).

4.3. Other Models

An explosion in the carbon-ignited violent merger model happens around the merger time, while an explosion in the Chandrasekhar mass model happens ~$10^4$ years after the merger time (e.g., Yoon et al. 2007). Between these two epochs, an explosion through other models possibly happens, given that the system does not experience the explosion through the helium- and carbon-ignited violent merger models. For example, Schwab et al. (2012) and Ji et al. (2013) have suggested an explosion triggered by magnetohydrodynamical effects in this phase. In this section, we do not assess whether the explosion happens in such a model as it is beyond what we can discuss based on our pure hydrodynamic simulation, but discuss what the explosion should look like, assuming that the explosion does happen.

The appearance of the explosion will be affected by the nature of merger ejecta. Figure 17 shows the spatial and velocity distributions of the merger ejecta at 500 s. The merger ejecta spread almost isotropically, except that they are relatively deficient on the orbital plane (see also Figure 6). Nevertheless, the covering factor of the merger ejecta around the merger remnant is almost at unity.

Raskin & Kasen (2013) have also studied effects of merger ejecta on observations of a putative SN taking place in this phase/ mode (see also Levanon et al. 2015). However, they have focused only on the tidal ejecta for the merger of binary CO WDs with masses of 0.96$M_\odot$ and 0.64$M_\odot$. On the other hand, our binary model consists of 1.1$M_\odot$ and 1.0 $M_\odot$ CO WDs. Since our binary has more massive components and a mass ratio closer to unity, our binary merges more violently than theirs (Marsh et al. 2004). Consequently, our merger ejecta are dominated by the shocked ejecta (see Section 3.1). Therefore, the expected effects of the merger ejecta on the observational features are partly different from those described by Raskin & Kasen (2013).

Raskin & Kasen (2013) have shown that NaID absorption features are potentially observed with a probability of 10%–50% if the explosion occurs $10^3$–$10^5$ years after the merger time. The probability corresponds to the covering factor of their merger ejecta. However, in the case of our binary, the NaID absorption features are potentially observed in almost all of the cases. This result is complementary to the discussion in Sections 3.4 and 3.6 of Raskin & Kasen (2013).

Finally, we point out that, if an explosion does happen at $t = 500$ s, the explosion might look like SN 2014J in the early brightness. At $t = 500$ s, the envelope of our binary model spreads out beyond 1.0$R_\odot$, as seen in Figure 18. The horizontal axis indicates the spherical radius.

4.4. Chandrasekhar Mass Model

In this section, we explore the possibility of an explosion in the Chandrasekhar mass model, which might take place if the system does not undergo an explosion through the mechanisms investigated by the previous sections. After the remnant reaches a dynamically steady state, it gradually loses the thermal energy by neutrino cooling, and increases its central density and temperature. If its central density and temperature exceed critical values as set by the balance between the nuclear reaction timescale and dynamical or convection timescale, the remnant is likely to explode as an SN Ia (“the Chandrasekhar mass model”) assuming the central region consists of carbon. On the other hand, it likely collapses to a neutron star as triggered by the electron capture, if the remnant has become an oxygen–neon–magnesium WD before reaching this phase.

The presence of carbon in the remnant depends on whether carbon is quiescently burned. All of the carbon in the remnant is burned if the $^{12}$C + $^{12}$C reactions proceed faster than the neutrino cooling at the time when the remnant reaches a dynamically steady state (Saio & Nomoto 1985, 1998, 2004).
Figure 18. Distribution of the mass density of particles in model 5.5M at \(t = 500\) s. They are mapped into a one-dimensional profile; the horizontal axis indicates the spherical radius. The vertical dotted lines indicate a distance of \(0.1R_\odot\) and \(1.0R_\odot\) from the center of the merger remnant.

Figure 19 shows mass density and temperature of particles at \(t = 500\) s in model 5.5M. At that time, the merger remnant reaches a dynamically steady state. In many particles, the timescale of the \(^{12}\text{C} + ^{12}\text{C}\) reaction is shorter than the timescale of the neutrino cooling, where we calculate the neutrino cooling rate using a public code available at F. X. Timmes website\(^{13}\) which is based on Itoh et al. (1996). However, the timescale of the \(^{12}\text{C} + ^{12}\text{C}\) reaction is longer than the dynamical timescale. Therefore, the \(^{12}\text{C} + ^{12}\text{C}\) reaction does not trigger the carbon detonation, but will convert the merger remnant to an oxygen–neon–magnesium WD on a thermal timescale. We conclude that, after the remnant evolves on a thermal timescale, the remnant would not explode in the Chandrasekhar mass model, rather it collapses to a neutron star.

We should keep in mind that this result may be affected by the short merger time in our simulation. According to Dan et al. (2011), the maximum temperature in the merger remnant becomes high when the binary suddenly merges. Therefore, if the time to the merger is longer, the maximum temperature in the merger remnant may be lower, and the \(^{12}\text{C} + ^{12}\text{C}\) reaction rate is lower than the neutrino cooling rate. Then, the remnant could explode in the Chandrasekhar mass model.

5. DETECTABILITY OF THE MERGER REMNANT AND THE MERGER SHELL

If binary CO WDs (or its merger remnant) fail to explode in all but the Chandrasekhar mass model, the merger ejecta have at least \(10^7\) years of time during which the ejecta expand into the ISM. An expanding shell is formed as the merger ejecta sweep up its surrounding ISM, analogous to an SNR. Also, the merger remnant still exists during this phase, analogous to a neutron star in a core-collapse SNR. For this situation, we estimate a detectability of the merger remnant and merger shell.

In particular, we focus on the merger shell, estimating its luminosity.

The evolution of a merger shell is divided into three phases. The first phase is a “free expansion phase,” where the amount of ISM swept up by the merger ejecta is negligible. Once the shell sweeps up ISM mass comparable to the mass of the merger ejecta, the shell is substantially decelerated entering into the “Sedov phase.” Materials are thermalized behind the shock wave. Because of the high temperature, the shell loses only a negligible fraction of energy through radiation approximately conserving the total energy content. Once the shell is cooled down, a significant amount of energy is lost by radiation, where the momentum is approximately conserved. This phase is called the “snowplow phase.”

The luminosity of the shell is notated by \(L_{\text{shell}}\), and given by

\[
L_{\text{shell}} = \Lambda V_{\text{shell}},
\]

where \(\Lambda\) is a cooling function, and \(V_{\text{shell}}\) is the volume of the shell. The volume \(V_{\text{shell}}\) can be expressed as

\[
V_{\text{shell}} = 4\pi R_s^2 \Delta_{\text{shell}},
\]

where \(R_s\) is the radius of the shock wave in front of the shell, and \(\Delta_{\text{shell}}\) is the thickness of the shell. We give the cooling function \(\Lambda\) as follows:

\[
\Lambda = \beta T_{\text{shell}}^{0.7} n_{e,\text{shell}} n_{H,\text{shell}},
\]

where \(n_{e,\text{shell}}\) and \(n_{H,\text{shell}}\) are the number densities of electrons and hydrogen atoms in the shell, \(T_{\text{shell}}\) is the temperature in the shell, and \(\beta = 1.7 \times 10^{-18} \text{erg cm}^{-3} \text{s}^{-1} \text{K}^{0.7}\) (Draine 2011). Note that this cooling function takes into account metal emission lines, and is applicable in the range of \(10^5 < T_{\text{shell}} / K < 10^{7.3}\). Since hydrogen atoms are perfectly ionized after they pass through the shock wave, the number densities \(n_{e,\text{shell}}\) and \(n_{H,\text{shell}}\) can be expressed as \(n_{H,\text{ism}}(\gamma + 1)/(\gamma - 1)\).

We estimate the shell luminosity at the free expansion. At the free expansion phase, the radius of the shock wave \(R_s \propto t\), and the thickness \(\Delta_{\text{shell}}\) is constant. Then, \(V_{\text{shell}} \propto t^3\), and the temperature in the shell, \(T_{\text{shell}}\), is constant. Eventually, \(L_{\text{shell}} \propto t^3\).

We focus on the Sedov phase, since the luminosity \(L_{\text{shell}}\) reaches to the peak around the end of the Sedov phase (see below). Although, at the free expansion phase, we show only the proportional relation between \(L_{\text{shell}}\) and \(t\), we give the equation expressing the relationship between \(L_{\text{shell}}\) and \(t\) at the Sedov phase. Assuming a strong shock wave, the radius \((R_s)\) and the thickness \((\Delta_{\text{shell}})\) can be written as

\[
R_s = \left[ \frac{75 \gamma (\gamma - 1)(\gamma + 1)^2 E_{\text{keq}}}{16\pi (3\gamma - 1) m_{\text{H,ism}}} \right]^{0.2} m^{0.4}
\]

\[
\Delta_{\text{shell}} = \frac{(\gamma - 1)}{3(\gamma + 1)} R_s,
\]

\(^{13}\) http://cococubed.asu.edu/code_pages/nulloss.shtml
where \( E_{k, ej} \) is the total kinetic energy of the merger ejecta, \( n_{\text{ism}} \) is the number density of the ISM in front of the shock wave, \( m_H \) is the mass of a hydrogen atom, and \( \gamma \) is the adiabatic index (Cavaliere & Messina 1976). Note that we assume that the ISM consists only of a hydrogen atom here to provide a first order estimate.

The shell temperature \( T_{\text{shell}} \) can be obtained as follows. From Rankine–Hugoniot conditions, the shell pressure \( p_{\text{shell}} \) is given by

\[
p_{\text{shell}} = m_H n_{\text{ism}} \dot{R}_s^2 / (\gamma + 1),
\]

where \( \dot{R}_s \) is the speed of the shock wave. On the other hand, the speed of the shock wave is set to the average velocity of the ISM gains from the shock wave. Since \( \dot{R}_s \) is the speed of the shock wave, \( p_{\text{shell}} \) can be written from an EOS of an ideal gas as

\[
p_{\text{shell}} = n_{H, \text{shell}} k_B T_{\text{shell}},
\]

where \( k_B \) is the Boltzmann constant. Using Equations (23) and (24), we obtain the shell temperature:

\[
T_{\text{shell}} = \frac{2(\gamma - 1)}{(\gamma + 1)^2} m_H k_B^{-1} \dot{R}_s^2.
\]

(25)

At the Sedov phase, the speed of the shock wave \( \dot{R}_s \) can be written as

\[
\dot{R}_s = \frac{3}{4\pi} \frac{(\gamma - 1)(\gamma + 1)^2}{(3\gamma - 1)} \frac{E_{k, ej}}{m_H n_{\text{ism}}} \frac{R_{\text{initial}}^{0.5}}{R_s^{1.5}} \left[ \text{cm} \text{s}^{-1} \right],
\]

\[
Cavaliere & Messina 1976.
\]

Then, we can rewrite Equations (19) and (21) as

\[
L_{\text{shell}} = 2.6 \times 10^{36} \left( \frac{n_{\text{ism}}}{1 \text{ cm}^{-3}} \right)^{1.68} \left( \frac{E_{k, ej}}{3.2 \times 10^{47} \text{ erg}} \right)^{0.32}
\]

\[
\times \left( \frac{t}{10^4 \text{ years}} \right)^{2.04} \left[ \text{erg s}^{-1} \right],
\]

(27)

\[
R_s = 2.6 \left( \frac{n_{\text{ism}}}{1 \text{ cm}^{-3}} \right)^{-0.2}
\]

\[
\times \left( \frac{E_{k, ej}}{3.2 \times 10^{47} \text{ erg}} \right)^{0.2} \left( \frac{t}{10^4 \text{ years}} \right)^{0.4} \left[ \text{pc} \right].
\]

(28)

where we set \( \gamma = 5/3 \). As seen in the power of \( t \) in Equation (27), the shell luminosity keeps increasing as time goes by. We roughly estimate the time when the Sedov phase is terminated, \( t_{\text{cool}} \), using the shell luminosity \( L_{\text{shell}} \) and the kinetic energy of the merger ejecta \( E_{k, ej} \) as

\[
E_{k, ej} = \int_0^{t_{\text{cool}}} L_{\text{shell}} dt.
\]

(29)

Solving Equation (29), we obtain \( t_{\text{cool}} \) as follows.

\[
t_{\text{cool}} = 1.1 \times 10^4 \left( \frac{n_{\text{ism}}}{1 \text{ cm}^{-3}} \right)^{0.553} \left( \frac{E_{k, ej}}{3.2 \times 10^{47} \text{ erg}} \right)^{0.224} \left[ \text{year} \right].
\]

(30)

The cooling function \( \Lambda \) is appropriate only when \( 10^5 < T_{\text{shell}} / K < 10^7.3 \). From Equation (25), we obtain the following expression.

\[
T_{\text{shell}} = 2.3 \times 10^5 \left( \frac{n_{\text{ism}}}{1 \text{ cm}^{-3}} \right)^{-0.4}
\]

\[
\times \left( \frac{E_{k, ej}}{3.2 \times 10^{47} \text{ erg}} \right)^{0.4} \left( \frac{t}{10^4 \text{ years}} \right)^{-1.2} \left[ \text{K} \right].
\]

(31)

From Equation (31), the cooling function can be applied from a few \( 10^2 \) years to a few \( 10^4 \) years.

At the snowplow phase, the ISM that passes through the shock wave emits energy almost instantly. The shell luminosity is written as \( L_{\text{shell}} \propto \dot{m}_s e_s \), where \( \dot{m}_s \) is the rate of the ISM mass passing the shock wave, and \( e_s \) is the specific energy that the ISM gains from the shock wave. Since \( R_s \propto t^{2/7} \) and \( \dot{R}_s \propto t^{-5/7} \), \( \dot{m}_s \propto t^{-1/7} \). From Rankine–Hugoniot conditions, \( e_s \propto \dot{R}_s^2 \propto t^{-10/7} \). Then, \( L_{\text{shell}} \propto t^{-11/7} \) at the snowplow phase.

In Figure 20, we illustrate the time evolution of the shell luminosity for a set of typical parameters corresponding to our system; \( E_{k, ej} = 3.2 \times 10^{47} \text{ erg} \), assuming \( n_{\text{ism}} = 1 \text{ cm}^{-3} \). We apply Equation (27) for the Sedov phase. We define the time when the Sedov phase begins as the time when the shell has swept up the ISM mass comparable to the ejecta mass. The speed of the shock wave is set to the average velocity of the merger ejecta, \( \sim 3 \times 10^8 \text{ cm} \text{ s}^{-1} \) at the free expansion phase. For the time when the Sedov phase ends, we adopt \( t_{\text{cool}} \) in Equation (30).

As seen in Figure 20, the shell luminosity reaches to the peak \( L_{\text{shell, peak}} \) at the time when the Sedov phase ends, \( t_{\text{cool}} \). Substituting \( t_{\text{cool}} \) in Equation (30) into \( t \) in Equation (27), we obtain the dependence of the peak luminosity of the shell on the kinetic energy of the merger ejecta as

\[
L_{\text{shell, peak}} = 3.1 \times 10^{36} \left( \frac{n_{\text{ism}}}{1 \text{ cm}^{-3}} \right)^{1.81}
\]

\[
\times \left( \frac{E_{k, ej}}{3.2 \times 10^{47} \text{ erg}} \right)^{0.776} \left[ \text{erg s}^{-1} \right].
\]

(32)

We define a lifetime of the shell, \( T_{\text{life}} \), during which the shell has more than half of the peak luminosity. Then, the lifetime is
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given by

\[ T_{\text{life}} = 4r_{\text{cool}}. \]  

(33)

The above estimate on a peak luminosity and lifetime of a merger shell is based on the results of a merger of WDs with masses of 1.1\(M_\odot\) and 1.0\(M_\odot\). However, such a massive binary system is rare. To connect the predictions to observations, we therefore scale the above result to a more common situation of a binary whose total mass just exceeds the Chandrasekhar mass. Specifically, we consider a merger of WDs with masses of 0.9\(M_\odot\) and 0.6\(M_\odot\). This system is representative of the one in which the merging process proceeds in the least violent manner among the systems potentially leading later to an explosion through the Chandrasekhar mass model in the ignition mode. According to our simulation for merging binary CO WDs with masses of 0.9 and 0.6\(M_\odot\) (see Section 3.2), its merger ejecta have the total kinetic energy of \(L_{\text{shell,peak}} = 2 \times 10^{35}\) erg s\(^{-1}\), and the lifetime of \(T_{\text{life}} = 2 \times 10^4\) years. We regard these values as the lower limits of the peak luminosity and lifetime of merger shells, which would be created by any combination of binary WD masses if the total mass exceeds the Chandrasekhar mass (hereafter “super-Chandrasekhar binaries”).

The shell could be more luminous than estimated above. This is because the shell could be illuminated by the merger remnant. Since the merger remnant has a high temperature for a while after the merger, it would also have a substantial luminosity. This is analogous to a planetary nebula. However, we do not consider this effect, but simply note that our estimate on the merger shell detectability should be regarded as a lower limit.

The merger shells will emit ultraviolet and soft X-ray photons. We estimate the number of the merger shells in the Milky Way, \(N_{\text{shell}}\), assuming that super-Chandrasekhar binaries explode only in the Chandrasekhar mass model, or fail to explode in all of the explosion models. Then, \(N_{\text{shell}}\) is given by

\[
N_{\text{shell}} \sim 10 \left( \frac{\Gamma_{\text{merge}}}{10^{-14}\ \text{years}^{-1} M_\odot^{-1}} \right) \times \left( \frac{M_{\text{MW}}}{6 \times 10^{10} M_\odot} \right) \left( \frac{T_{\text{life}}}{2 \times 10^4\ \text{years}} \right). \]  

(34)

where \(\Gamma_{\text{merge}}\) is a merger rate of super-Chandrasekhar binaries per unit mass in the Milky Way, \(M_{\text{MW}}\) is the mass of the Milky Way. We adopt \(\Gamma_{\text{merge}}\) in Badenes & Maoz (2012), and \(M_{\text{MW}}\) in McMillan (2011) and Licquia & Newman (2014).

We note difficulties in distinguishing merger shells from SNRs and nova shells. Since their explosion energies are different by several orders of magnitudes, their sizes and luminosities can be distinguished, if the number density of the ISM is known. However, the number density is usually unknown. One way to overcome this difficulty to identify the merger shells is to use the information about the central compact object.

We search for any hint of the merger shell in the literature dealing with pre-explosion images of SNe Ia in order to check whether these SNe Ia involve merger shells. Nielsen et al. (2012; see also Liu et al. 2012; Nielsen et al. 2014) have constrained the upper limit of bolometric luminosities of nearby SNe Ia with pre-explosion images. The most stringent limit is on SN 2011fe, \(\sim 10^{36}\) erg s\(^{-1}\). This is still larger than the estimated luminosities of merger shells. Unfortunately, these images are not deep enough to constraint the presence of the merger shells.

As seen in Equation (28), a merger shell has a parsec-scale size. When an SN occurs within the merger shell, the merger shell is not disturbed by the SN ejecta during the first 10\(^2\) years after the SN explosion. Therefore, existence of the merger shell could be also tested by observations of an SN Ia after the explosion. We postpone such a study for future investigation.

6. SUMMARY

We have performed SPH simulations for merging binary CO WDs with masses of 1.1 and 1.0\(M_\odot\), until the merger remnant reaches a dynamically steady state. Using these results, we assess whether the binary could induce a thermonuclear explosion, and whether the explosion could be observed as an SN Ia. We investigate three explosion mechanisms: a helium-ignition following the dynamical merger (“helium-ignited violent merger model”), a carbon-ignition (“carbon-ignited violent merger model”), and an explosion following the formation of the Chandrasekhar mass WD (“Chandrasekhar mass model”). In addition to the evaluation if the resulting system satisfies requirements set in each mode. We have discussed whether the resulting explosions, through different ignition modes, would look like SNe Ia.

Our results are summarized as follows.

1. In the helium-ignited violent merger model, our binary can explode if the mass fraction of helium exceeds a critical value, i.e., \(f_{\text{He}} \gtrsim 3 \times 10^{-4}\). However, the expected early brightness is likely different from those of SN 2011fe and SN 2014J, since materials of our binary spread out to \(\sim 0.1 R_\odot\), which does not fit to what were inferred for these SNe. Moreover, the explosion likely results in an SNR with an extremely asymmetric symmetric shape, which is unusual for SN Ia.

2. In the carbon-ignited violent merger model, our binary can explode. However, the explosion likely results in the early brightness dissimilar to those of SN 2011fe and SN 2014J for the same reason as for the helium-ignited violent merger model. Moreover, we predict that the explosion will synthesize \(^{56}\text{Ni}\) whose distribution is extremely aspherical. This is inconsistent with the \(^{56}\text{Ni}\) distribution generally inferred for SN Ia. Note that the \(^{56}\text{Ni}\) distribution will depend on our choice of the raw and smoothed temperatures for the \(^{12}\text{C} + ^{12}\text{C}\) reaction. In the case of the raw temperature, \(^{56}\text{Ni}\) is absent in the center of the explosion. On the other hand, in the case of the smoothed temperature, \(^{56}\text{Ni}\) distribution is similar to a hourglass.

3. If our binary explodes a few hundred seconds after its merger by some mechanism (i.e., the “other model”), the explosion may have the early brightness consistent with that of SN 2014J. At that time, materials of our binary spread out beyond 1\(R_\odot\).

4. For the particular set of binary parameters examined in this paper, the binary would not lead to an SN Ia explosion through the Chandrasekhar mass model. Rather, the merger remnant should be converted to an oxygen–neon–magnesium WD, and then will experience an accretion-induced collapse to become a neutron star.
Binary CO WDs generate the merger ejecta before and after its merger time. The merger ejecta will interact with its surrounding ISM, and form a merger shell. We estimate a bolometric luminosity of the merger shell; the luminosity is more than $2 \times 10^{35}$ erg s$^{-1}$ at its peak, if the total mass of the binary CO WDs exceeds the Chandrasekhar mass. Suppose that all of the super-Chandrasekhar binaries explode in the Chandrasekhar mass model or fail to explode at all, the number of the merger shells in the Milky Way is estimated to be $\sim 10$. The detection of such merger shells can rule out the helium-ignited and carbon-ignited violent merger models. If an explosion is initiated in the Chandrasekhar mass model, a merger shell can be detected not only from pre-explosion images of a site of an SN Ia, but also in the post-explosion observations. Unfortunately, we have not found merger shells from pre-explosion images of previous SNe Ia, since the lower limit to these observations is at best $\sim 10^{36}$ erg s$^{-1}$. In the future, the merger shells would be found from post-explosion images of sites of nearby SNe Ia.

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**APPENDIX**

**ARTIFICIAL VISCOSITY**

We describe our chosen artificial viscosity, and its numerical parameters. In this section, variables are defined again with the same notations as those in the main text. The viscosity exerted on the i-particle by the j-particle is indicated as $\Pi_{ij}$. As described in the main text, we adopt a shear-free viscosity term (Balsara 1995) combined with time dependent viscosity parameters (Morris & Monaghan 1997). Then, the term is given by

$$\Pi_{ij} = \tilde{\Pi}_{ij} \tilde{\Pi}_{ij,max}, \quad (35)$$

where $\tilde{\Pi}_{ij}$ is the so-called Balsara switch and $\tilde{\Pi}_{ij,max}$ is a sort of bulk and von-Neumann–Richtmyer viscosities. Hereafter, we define a combination of the overline and subscript “ij” as

$$\tilde{X}_{ij} = \frac{1}{2}(X_{i} + X_{j}). \quad (36)$$

The viscosity term $\tilde{\Pi}_{ij,max}$ is expressed as

$$\tilde{\Pi}_{ij,max} = \max \left( \tilde{\Pi}_{ij,i}, \tilde{\Pi}_{ij,j} \right),$$

$$\tilde{\Pi}_{ij,k} = \begin{cases} -\alpha_{k} \tilde{X}_{ij} \mu_{ij} + \beta_{k} \mu_{ij}^{2} & (r_{ij} \cdot v_{ij} < 0), \\ 0 & (r_{ij} \cdot v_{ij} \geq 0) \end{cases} \quad (38)$$

where $\rho_{i}$ and $c_{i,j}$ are the mass density and sound speed of the i-particle, respectively. The vectors $r_{i}$ and $v_{i}$ are, respectively, the position and velocity of i-particle, and $r_{ij} = r_{j} - r_{i}$ and $v_{ij} = v_{j} - v_{i}$. The variable $\mu_{ij}$ is given by

$$\mu_{ij} = \left| r_{ij} \right| + \delta_{ij} h_{ij}, \quad (39)$$

where $h_{ij}$ is the kernel length of the i-particle, and $\delta_{ij} = 0.01$ is chosen. The viscosity parameter $\alpha_{i}$ is time-dependent. It is evolved as

$$\alpha_{i} = -\frac{\alpha_{\min}}{h_{i}/(\xi c_{i,j})} + \max \left[ -(\nabla \cdot v_{i}) (\alpha_{\max} - \alpha_{i}), 0 \right], \quad (40)$$

where $\alpha_{\max} = 1.5$, $\alpha_{\min} = 0.05$, and $\xi = 0.25$. Another viscosity parameter $\beta_{i}$ is proportional to $\alpha_{i}$, such that $\beta_{i} = 2\alpha_{i}$.

The Balsara switch can be written by $f_{i}$ and $f_{j}$ with Equation (36), and $f_{i}$ is given by

$$f_{i} = \left[ |\nabla \cdot v_{i}| + |\nabla \times v_{i}| + \delta_{2} c_{i,j}/h_{i} \right], \quad (41)$$

where we adopt $\delta_{2} = 0.0001$.

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