New Interpretation of Equivalence Principle in General Relativity from the viewpoint of Micro-Macro duality

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Abstract

We propose a new interpretation of the equivalence principle underlying Einstein’s general relativity: a free-falling frame with gravitational force eliminated locally in a small spacetime region shows the existence of a boundary level, below which gravity is absent and above which gravity emerges as condensation effect of microscopic motions within each such frame and interrelates free-falling frames at different spacetime points. In this picture, gravitational field as a mediator of different free-falling frames shows a remarkable parallelism with an order parameter to specify “degenerate vacua” in different thermodynamic pure phases due to the condensation effects in phase transitions. As the physical basis of general relativity is found in the universality of mass point motions due to the constancy of [inertial m]/[gravitational m], the general relativistic notion of “spacetime” should be meaningful only in the validity regime of this constancy, which is of empirical nature, contrary to the usual consensus. At the end, we comment on the impossibility to observe gravitational waves which would make gravitons and quantum gravity questionable.

Keywords: Equivalence Principle, Gravity, Space-Time Emergence, Micro-Macro Duality
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1 Emergence of spacetime and gravity

The motivating idea of Einstein’s theory of general relativity is strongly of geometric nature and naturally explains his enthusiasm about unsuccessful attempts at “Unified Field Theory” for unifying electromagnetism and gravity. Except for some short period, the same enthusiasm for “Geometrization of Physics” has overwhelmingly dominated in particle physics for more than

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three decades. From this viewpoint, however, the physical origin of gravity and spacetime cannot be unveiled; what about the use of spacetime notion even in the situations with totally indeterminate future??

To overcome such difficulties, we propose a scenario for deriving gravity and spacetime as epigenetic secondary notions emerging from microscopic physics of matter motions. For this purpose, the essence of this report is just to explain the following diagram consisting of the structures relevant to the emergence of special- and general-relativistic spacetimes (see below).

The basic ingredients necessary for this purpose are as follows:

i) Independence = freely falling frames as “sectors” without gravity containing only strong, weak & electromagnetic couplings

ii) Coupling = gravitational force $\Gamma^\lambda_{\mu\nu}$ defined as Levi-Civita connection to connect different free-falling frames as “sectors” at the meta-level, and,

iii) Dependence = the composite system arising from the above physical systems constructed by three kinds of forces (strong, weak & electromagnetic) coupled with each other by the gravitational force.

| Spec = spacetime $\{x^\mu\}$ | Gen. Rel. = freely falling frames at different $\{x^\mu\}$ | General Cov. Functorial Sym $\mathcal{G}$ |
|---------------------------------|----------------------------------------------------------|------------------------------------------|
| $g_{\mu\nu}$                   | $\Gamma^\lambda_{\mu\nu} \sim$                           | $\uparrow$ induced rep $\text{Ind}^\mathcal{H}_H$ |
| $R, R_{\mu\nu}$                | $m_{\text{grav}} = m_{\text{inert}} \uparrow$: Equiv. principle of inside & outside of a sector $x^\mu$ with no gravity | Unbroken Sym $H = \mathcal{P}^+_+$ |
| $\uparrow\downarrow$            |                                                          |                                          |
| Einstein eqn                   | $x$                                                     | local spacetime emergence $1/c$           |
| $R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R$ | $p$ duality                                             |                                          |
| $\parallel$                    | (material path)                                         |                                          |
| $\epsilon_\omega(T_{\mu\nu})$  | $\Wedge$ W-S angle $\theta_W \, \Wedge$               |                                          |
| state $\omega$, $\uparrow\downarrow$ |                                                        |                                          |
| $\uparrow\downarrow$            | Weak Interactions                                       |                                          |
| $T_{\mu\nu}$                   | $\text{Dynamics}$                                       |                                          |
| $\Rightarrow$ Strong Interactions |                                                          |                                          |

By reviewing how general-relativistic spacetime emerges from the physical processes in Micro quantum systems, we clarify here under which condition the notion of “spacetime” can be meaningful from the viewpoint of “Micro-Macro duality”.

Micro-Macro Duality & Quadrality Scheme

1) Micro-Macro duality [I] as a mathematical version of “quantum-classical correspondence” between microscopic sectors defined by quasi-equivalence (= unitary equiv. up to multiplicity) classes of factor states of observable algebra & macroscopic inter-sectorial level described by ge-
ometrical structures on the central spectrum $Spec(3)$:

| Visible | Macro | independent objects | \( \cdots \) | \( \gamma \) | \( \gamma_2 \) | \( \gamma_1 \) | Intersectorial | \( Spec(3) \) |
|---------|-------|---------------------|-------------|---------|---------|---------|-------------|----------------|
| \( \cdots \) | \( \gamma_N \) | sectors | \( \cdots \) | \( \gamma \) | \( \gamma_2 \) | \( \gamma_1 \) | \( \uparrow \) | \( \downarrow \) |
| \( \cdots \) | \( \pi_{\gamma_N} \) | \( \pi_{\gamma} \) | \( \pi_{\gamma_2} \) | \( \pi_{\gamma_1} \) | \( \pi \) | \( \pi \) | \( \pi \) | \( \pi \) |
| \( \cdots \) | \( \vdots \) | \( \vdots \) | \( \vdots \) | \( \vdots \) | \( \vdots \) | \( \vdots \) | \( \vdots \) | \( \vdots \) |

According to Fourier & Galois dualities, these Micro & Macro are in duality, meaning that the data given at Macro level is derived from the analysis on Micro and vice versa.

2) Quadrality Scheme: As a combination of two kinds of Micro-Macro dualities in “horizontal” and “vertical” directions, a general methodological framework can be formulated for theoretical description of physical phenomena which I call quadrality scheme:

| Micro: visible levels | \( Spec = \) classifying space |
|----------------------|--------------------------------|
| \( \text{classification} \) \( \text{/emergence} \) | \( \uparrow \) dual \( \downarrow \) |
| \( States \& Rep. ’s \) | dual \( \leftrightarrow \) Fourier-Galois dualities |
| \( \uparrow \) dual \( \uparrow \downarrow \) | \( \leftrightarrow \) Algebra of observables |
| \( \text{Dynamics} \) | object system : Micro |

In a sense, the essence of this scheme overlaps with the basic structure controlling the four interactions as follows:

| Gravity | \( \text{Electromagnetism} \) | Thermality \( k_B \) |
|---------|-------------------------------|-----------------|
| \( \leftrightarrow \) “Quantum” \( \text{ℏ} \) | Weak forces |
| \( \text{Strong force} \) | | |

Namely, the meaning of “unification of four forces” need not be restricted to a simple-minded version like the currently prevailing one with their convergence into a single entity, but, the mutual relations among them may well alternatively be understood in such a form of their integrated organization that they occupy mutually different places in nature and in theoretical frameworks, playing different roles inherent in each, through which the unified totality of nature and its theoretical explanations can be attained. The standard picture of “unification” pursued in the context of “geometrization of physics” seems to lack systematically considerations into this kind of as-
3) Fourier-Galois Duality: Bi-directionality between Induction & Deduction

Essence of duality in Fourier transform \((\mathcal{F}f)(\gamma) = \int \gamma(g)f(g)dg\) \((f \in L^1(G))\) is formulated by Fourier-Pontryagin duality \(G \leftrightarrow \hat{G}\) between a locally compact abelian group \(G\) and its dual group \(\hat{G}\) consisting of all the characters \(\chi : G \rightarrow \mathbb{T}\). Via extension to compact cases due to Tannaka & Krein, the most general form can now be found in Tatsuuma-Enock-Schwartz theorem of the duality between a locally compact non-abelian group \(G\) and its representation category \(\text{Rep}(G)\) consisting of “all” the representations. The corresponding version is formulated by Takesaki (and/or Takai) for dynamical system with a (non-commutative) algebra \(\mathcal{F}\) and with an action \(\tau : \mathcal{F} \times G\) of \(G\) on \(\mathcal{F}\) in such a form (in C*- or W*-versions, respectively) as

\[
\mathcal{F}^G \rtimes \hat{G} = \mathcal{F} : \text{Recovery of } \mathcal{F} \text{ from } G\text{-invariants } \mathcal{F}^G;
\]

\[
\mathcal{F} \rtimes \tau G = \mathcal{F}^G \otimes K(L^2(G)) \text{ or } \mathcal{F}^G \otimes B(L^2(G)).
\]

In all cases, the Kac-Takesaki operators play crucial roles, in term of which a method for constructing composite system can be developed systematically via coupling between object system and reference system. Unfortunately, we should omit them here for lack of space.

4) Symmetry Breaking, condensed states & induced representations \(\rightarrow\) condensation and degenerate vacua

Breakdown of a symmetry \(G\) of a dynamical system \(\mathcal{F} \times G\) in a state \(\omega \in E_{\mathcal{F}}\) is characterized \([2]\) by non-invariance of the “central extension” of \(\omega\) on the centre \(Z_{\pi}(\mathcal{F}) := \pi_{\omega}(\mathcal{F})' \cap \pi_{\omega}(\mathcal{F})'\) under the corresponding \(G\)-action on \(Z_{\pi}(\mathcal{F})\). In this case, Galois closedness of \(\mathcal{F}^G\) is broken, which is recovered by dynamical system \(\mathcal{F} \times H\) described by a compact Lie subgroup \(H\) of \(G\) corresponding to unbroken symmetry: \(\mathcal{F} = \mathcal{F}^H \times \hat{H}\). Then, the sector structure is determined by factor spectrum \(\hat{H} = \text{Spec}(3(\mathcal{F}^H)) = \hat{H}\): group dual consisting of irreducible unitary rep.’s of \(H\).

5) Emergence of Macro by condensation effects of Micro \(\rightarrow\) physical application of forcing method (which implies Born rule \([3]\))

With \(\widetilde{\mathcal{F}} := \mathcal{F}^H \times \hat{G} = \mathcal{F} \times (H\backslash G)\) called an augmented algebra \([2]\), we have a split bundle exact sequence \(\mathcal{F}^H \overset{\widetilde{m}}{\twoheadrightarrow} \widetilde{\mathcal{F}} \overset{\iota}{\hookrightarrow} \mathcal{F}^H \simeq \hat{G}\).

In this situation, minimality of \(G\) and \(\widetilde{\mathcal{F}}\) is guaranteed by \(G\)-central ergodicity, i.e., \(G\)-ergodicity of centre \(Z_{\pi}(\mathcal{F})\) in the rep. \(\pi\) given by GNS rep. of \(\omega_0 \circ \widetilde{m}\) induced from vac. state \(\omega_0\) of \(\mathcal{F}^H\) \([2]\), and we have the following commutativity diagrams:
The above diagram for algebra extension is dual to the following one for sectors:

| \[ \tilde{\mathcal{F}}^G = \tilde{\mathcal{F}}^H \simeq \tilde{H} \] | unbroken sectors |
|---|---|
| \[ \tilde{\mathcal{F}} \] | \[ \tilde{\mathcal{F}}^H \simeq G \times \hat{H} \] |
| \[ \downarrow \text{onto} \] | \[ \downarrow \text{onto} \] |
| \[ \mathcal{F} \] | \[ \mathcal{F}^H \] |
| \[ \downarrow \text{onto} \] | \[ \downarrow \text{onto} \] |
| \[ \mathcal{H} \] | \[ \mathcal{G} \] |
| \[ \leftarrow \] | \[ \leftarrow \] |
| \[ \mathcal{F}/\mathcal{H} \] | \[ \mathcal{G}/\mathcal{H} \] |

where \( \tilde{\mathcal{F}} = \text{Spec}(\mathfrak{F}(\mathcal{F})) \) denotes the factor spectrum of \( \mathcal{F} \), etc.

"Sector Bundle' associated with Broken Symmetry

The physical essence of extension \( \mathcal{F}^G \implies \mathcal{F}^H \) from the \( \mathcal{G} \)-fixed point subalgebra \( \mathcal{F}^G \) to \( \mathcal{H} \)-fixed one \( \mathcal{F}^H \) can now be interpreted as "extension of coefficient algebra \( \mathcal{F}^G \)" by (the dual of) \( \mathcal{G}/\mathcal{H} \) to parametrize degenerate vacua: \( \mathcal{F}^H = \tilde{\mathcal{F}}^G = \left[ (\mathcal{F} \rtimes (\mathcal{H}/\mathcal{G}))^G = \mathcal{F}^G \rtimes (\mathcal{H}/\mathcal{G}) \right] \).

In this extension, a part \( \mathcal{G}/\mathcal{H} \) of originally invisible \( \mathcal{G} \) has become visible through the emergence of degenerate vacua parametrized by \( \mathcal{G}/\mathcal{H} \) due to condensation of order parameter \( \in \mathcal{G}/\mathcal{H} \) associated with symmetry breaking of \( \mathcal{G} \) to \( \mathcal{H} \).

As a result, observables \( A \in \mathcal{A} \) acquire \( \mathcal{G}/\mathcal{H} \)-dependence: \( \tilde{A} = (\mathcal{G}/\mathcal{H} \ni \dot{g} \mapsto \tilde{A}(\dot{g}) \in \mathcal{A} \rtimes (\mathcal{H}/\mathcal{G})) \), which should just be interpreted as an example of logical extension \([4]\) transforming a "constant object" \( \tilde{A} \in \mathcal{A} \rtimes (\mathcal{H}/\mathcal{G}) \) into a "variable object" \( \tilde{A} \in \mathcal{A} \rtimes (\mathcal{H}/\mathcal{G}) \) having functional dependence on the universal classifying space \( \mathcal{G}/\mathcal{H} \) for multi-valued semantics, as is familiar in non-standard and Boolean-valued analysis.

6) Emergence of Spacetime as Symmetry Breaking

By replacing \( \mathcal{G}/\mathcal{H} \) with spacetime, the above situation can be regarded as a prototype for the origin of functional dependence of physical quantities on spacetime coordinates, due to the physical emergence of spacetime from microscopic physical world.

Along this line, we prescribe the similar logical extension procedure on
the observable algebra $\mathcal{F}^H$ adding $G/H$-dependence: $\mathcal{F}^H \times (H\backslash G) = (\mathcal{F} \times (H\backslash G))^H = \tilde{\mathcal{F}}^H$. The whole sector structure of $\tilde{\mathcal{F}}^H = (\mathcal{F}^H \times (H\backslash G)^H)$ can be identified with its factor spectrum $\tilde{\mathcal{F}}^H = G \times \hat{H}$; this constitutes a sector bundle, $\hat{H} \hookrightarrow \tilde{\mathcal{F}}^H = G \times \hat{H} \to G/H$, consisting of the classifying space $G/H$ of degenerate vacua, each fibre over which describes the sector structure $\hat{H}$ of unbroken remaining symmetry $H$ (or, more precisely, the conjugated group $gHg^{-1}$ for the vacuum parametrized by $\dot{g} = gHg \in G/H$).

Namely, sector bundle, $\hat{H} \hookrightarrow \tilde{\mathcal{F}}^H = G \times \hat{H} \to G/H$, can be seen as the connection= splitting of bundle exact sequence dual to $\tilde{\mathcal{F}}^H = \hat{H} \hookrightarrow \tilde{\mathcal{F}}^H = G \times \hat{H} \hookrightarrow G/H$ of observable triples, $\mathcal{F}^H \hookrightarrow \tilde{\mathcal{F}}^H = \mathcal{F}^H \times (H\backslash G) \to (H\backslash G)!$

Now we apply the above scheme to the situation with group $G$ containing both external (= spacetime) and internal symmetries. For simplicity, the latter component described by a subgroup $H$ of $G$ is assumed to be unbroken, and hence, the broken symmetry described by $G/H$ represents the spacetime structure. It would be convenient to take $H$ as a normal subgroup of $G$, while not essential. To be precise, $G/H$ may contain such non-commutative components as spatial rotations (and Lorentz boosts) acting on spacetime, but, we simply neglect this aspect to identify $G/H$ as spacetime itself (from which the corresponding transformation group can easily be recovered).

Then, by identifying $G/H$ with a spacetime domain $\mathcal{R}$, we find an impressive parallelism between the commutative diagram in the previous section and the diagram in Doplicher-Roberts reconstruction [5] of local field net $\mathcal{R} \to \mathcal{F}(\mathcal{R})$ from local observable net $\mathcal{R} \to \mathcal{A}(\mathcal{R})$ (without the two bottom lines) as follows:

\[
\begin{array}{c}
\hat{H} \hookrightarrow \tilde{\mathcal{F}}^H = \mathcal{F}^G \times_{G/H} \tilde{\mathcal{F}}^H \quad \mathcal{G} \hookrightarrow \mathcal{O}_d = \mathcal{O}_d^G \times_{\mathcal{R}} \mathcal{A}(\mathcal{R}) \\
\downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \\
\hat{G} \leftarrow \tilde{\mathcal{G}} \leftarrow \tilde{\mathcal{G}} \leftarrow \mathcal{G} \times \mathcal{R} \leftarrow \mathcal{R}
\end{array}
\]

where $\mathcal{O}_d$ is a Cuntz algebra of $d$-isometries.

Thus we have arrived at the stage just before gravity to be switched on, to enter General Relativity via Equivalence Principle. This can naturally be formulated and understood by the above scheme in combination with induced representation. So, we should recall here the diagram at the beginning.
2 Physical meaning of Equivalence Principle in General Relativity in the emergence process

We consider processes of spacetime emergence taking place in parallel under the influence of strong and electro-weak interactions other than gravity, each of which results in a “fiber” (= sector= pure phase) parametrized by spacetime coordinates \( x^\mu \). The word “fiber” here means a flat tangent space as a fiber \((T_x(M))\) of a tangent bundle \((T(M))\) on each point \( x^\mu \) of the base space as a “spacetime manifold” (which would be called \( M \) but which cannot be recognized yet as such); as its physics is controlled by the three interactions other than the gravity, this fiber describes a free-falling frame without any gravitational force (the last of which has not emerged yet). In connection with our discussion up to this point, the word “sector” should be more appropriate than “fiber”, we accept the latter use in order to avoid the misunderstanding of what we are concerned with here. To be precise, what we know up to now is only the simultaneous processes of spacetime emergence at many “fiber” points \( x^\mu \) each of which consists of the physical world of Poincaré covariant quantum fields governed by the strong and electro-weak interactions in the Minkowski spacetime but we do not know anything about the mutual relations among different fibers. By picking up just one specific “fiber”, we focus on the local physics described by the Poincaré covariant QFT developed inside of the “gravitation-less free falling system”, which is nothing but the physical contents of “tangential world” equipped with local Lorentz structure, on (or “in”? ) a point \( (x^\mu) \) in the emergent “base space”.

Now, we pose a question: what does it mean to impose the physical requirement of “equivalence principle” between gravitational and inertial masses, \( m_{\text{grav}} = m_{\text{inert}} \) on the situation after the “individual” processes of free-falling systems arising from the emergence of special-relativistic local spacetime? While the notion of “inertial mass” already exists in the “standard” physics formulated within the free-falling frames without gravity, it does not apply to the case of “gravitational mass” before our starting to discuss the situations governed by the gravitational interaction. It can be meaningful only in the context where such an attribute is assigned to a(n asymptotically) free mass point on the mass-shell, as generating the gravitational force or field as the forth one other than strong and electro-weak forces, through Einstein’s gravitational equation:

\[
R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \kappa T_{\mu\nu}.
\]

When we find the first (or, the 0-th approximated) roles of gravity in regulating the mutual relations among different fibers= sectors as free-falling frames, the proper range of action of the gravitational mass \( m_{\text{grav}} \) is at the
level of “inter-fiber= inter-sectorial relations”, but, in contrast, that of the inertial one $m_{\text{inert}}$ is in the physics within each “fiber” (or sector). Therefore, the equivalence principle qualitatively controls in a bi-directional way the mutual duality relation between the inside and the outside of “fibers” (or, sectors) we suppose that the inter-fiber relation of free-falling frames on the “neighbouring” points $x^\mu$ and $x^\mu + \delta x^\mu$ is controlled by the connection coefficients $\Gamma^\lambda_{\mu\nu}$, as is indicated in the diagram at the beginning, which results in a force proportional to the gravitational mass $m_{\text{grav}}$ acting on the inertial mass $m_{\text{inert}}$. Then, the Newtonian equation of motion of the mass point $m_{\text{inert}}$ with the velocity vector $v^\lambda := \frac{dx^\lambda}{d\tau}$ can be written as,

$$m_{\text{inert}} dv^\lambda = -v^\mu (m_{\text{grav}} \Gamma^\lambda_{\mu\nu} dx^\nu) = m_{\text{grav}} v^\mu \nabla_\mu dx^\lambda. \tag{1}$$

By the requirement of equivalence principle $m_{\text{grav}} = m_{\text{inert}}$, this reduces to the geodesic equation, $\frac{dv^\lambda}{d\tau} + \Gamma^\lambda_{\mu\nu} v^\mu v^\nu = 0$, whose purely geometric form and independence of the specific mass values ensure the universality of the mass-point motions. Namely, through the validity of equivalence principle $m_{\text{grav}} = m_{\text{inert}}$, the spacetime notion $x^\mu$ acquires its own abstract universal meaning, independently of its physical origin in the mutual relations among different “fibers” of local physics consisting of three interactions, to such an extreme extent that space and “time” exist in themselves, extending from the past, the present and even the future! Eventually, the physical motions of mass points are now absorbed into a (small) part of spacetime geometry in the form of geodesic motions, without exhibiting their individuality. Owing to this mechanism, we can easily forget about the physical origin of spacetime, which can, however, exhibit its existence in the situation where the validity of equivalence principle is threatened. It is also interesting to note that the above equation of motion can be rewritten in terms of momentum $p^\lambda = mv^\lambda$ into

$$dp^\lambda = p^\mu \nabla_\mu dx^\lambda,$$

which explains that mass-point motions as geodesic motion can be absorbed into the covariance (of physical motions) under the (covariantized) general coordinate transformations. If the above discussion is compared with the standard mathematical treatment of bundle structures in differential geometry, we understand that ours go from physics in the (standard) fiber to the mathematical structure of the bundle and base spaces in the opposite direction to the latter and that the mathematical essence of the equivalence

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1 From the viewpoint of emergence as a process of phase separation, the roles played by the free-falling frame in each “fiber” and by “base space” can be compared with $H$ and $G/H$ whose duality relation can be seen in the form of “Helgason duality”. In this sense, the gravitational equivalence principle is analogous to “Helgason duality”.

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principle lies in the $G$-structure of the tangent and frame bundles of the spacetime $M$ with $G$ being identified with the Lorentz group [6].

It would also be of interest to compare the above situation of gravity with that of electromagnetism: in this case, once the (field strength of) electromagnetic field $F_{\mu\nu}$ as a universal quantity is generated, $F_{\mu\nu} \leftarrow J_{\mu}$, via the Maxwell equation, $\partial^\nu F_{\mu\nu} = eJ_{\mu}$, from the electric current $J_{\mu}$ arising from the microscopic matter motions, the resulting $F_{\mu\nu}$ starts to control all the matter motions with a 4-velocity $v^\mu$ by the Lorentz force $eF_{\mu\nu}v^\nu$ acting on them, through which the coupled system of electromagnetic field and matter motions is equationally closed. In the direction from matter motions to universal quantities, $R_{\mu\nu} \& g_{\mu\nu} \leftarrow T_{\mu\nu}$ [: to Macro, or meta-level from Micro], the case of gravity shares the common features with the above electromagnetism, through a well-known form of the Einstein equation,

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \kappa T_{\mu\nu},$$

where geometric quantities, $R_{\mu\nu} \& g_{\mu\nu}$, related with gravity is generated from the energy-momentum tensor $T_{\mu\nu}$ of matter motions. In the opposite direction of the gravitational field $R_{\mu\nu} \& g_{\mu\nu}$ to exert its counter-action on matter motions as its sources, however, our considerations above exhibit certain complicated aspects involved, in such forms as the essential roles played by the formation of spacetime points $x^\mu$ and the action of $\Gamma^\lambda_{\mu\nu}$, whose essence cannot be exhausted in the direction from Macro to Micro levels. In other words, what determines how the generated gravity counter-acts on matter motions is not only the spacetime point $x^\mu$ emerging as the indices of a family of free-falling systems, but also the equivalence principle, $m_{\text{grav}} = m_{\text{inert}}$, to equate the gravitational mass $m_{\text{grav}}$ appearing as the source of gravitation field in the Einstein equation and the inertial mass $m_{\text{inert}}$ characterizing the Newtonian-mechanical mass point summarizing the (micro-)physical contents in the free-falling system indexed by $x^\mu$. By this equivalence principle, the latter physical meaning related with Newtonian mechanics within each fiber (: Micro) is absorbed into the purely geometric context of geodesics on Macro level, which finally settles the physical and geometric meanings of general-relativistic “spacetime” and “gravity”.

The final pictures attained in both cases, however, are quite similar, in such forms as

$$dp_\mu = p^\nu \frac{e}{m}(F_{\nu\mu}d\tau); \quad dp^\lambda = p^\mu (\nabla_\mu dx^\lambda) \quad (\text{if } m_{\text{grav}} = m_{\text{inert}}),$$

in the cases of electromagnetic Lorentz force and gravitational force, respectively. Their common features can be seen in the parallelism with the left action of a Lie group $G$ on the homogeneous space $G/H$ by the left $G$-shift: $G/H \ni sH \mapsto g(sH) = gsH \in G/H$, which can physically be interpreted as the action of a broken symmetry $G$ with its subgroup $H$ remaining unbroken taken care of by the Goldstone modes $\sim G/H$ acting on the space $G/H$ of degenerate vacua arising from the condensation effect due to the
symmetry breaking. Namely, both the electromagnetic force $F_{\mu\nu}$ and the gravitational force $\Gamma^\lambda_{\mu\nu}$ behave as Goldstone-like modes acting transitively on the emergent classical Macro objects arising from the condensation effects due to some symmetry breakings. While the question may be subtle as to what kind of symmetry is broken in electromagnetism, something related to local gauge invariance is broken, triggering the emergence of the Minkowski spacetime as the condensation, whose Goldstone mode is given by $F_{\mu\nu}$ (or, $\varepsilon F_{\mu\nu}/m$). In the case of gravity, what is broken is the invariance under the general coordinate transformations, the emerging condensation is the spacetime coordinates $x^\mu$ to parametrize the free-falling frames, with Goldstone mode being the Levi-Civita connection $\Gamma^\lambda_{\mu\nu}$ (or, $\nabla_\mu dx^\lambda = -\Gamma^\lambda_{\mu\nu} dx^\nu$).

3 Absence of gravitational sink and of gravitational waves

At the end, we add brief comments on a new observation about the absence of gravitational waves, which can be seen as follows:

1) In the general theory of relativity and in all modern physics, the notion of spacetime point $x^\mu$ occupies the most fundamental position to parametrize all the events taking place in macroscopic nature which can be compared with the “elements” in set theory, upon which (almost) all the structures are built. Combining this aspect with the processes of emergence from Micro to Macro, spacetime points $x^\mu$ can be interpreted as “initial objects” in category theory characterized by the uniqueness of arrow emanating from it, with all other arrows convergent to it: in this context, there is such a sharp asymmetry between the two sides of a spacetime point $x^\mu$, that there are many arrows to $x^\mu$ involving three interactions other than gravity but that arrows involving gravity are only those unique ones emanating from each $x^\mu$.

2) The above asymmetry implies the absence of gravitational sinks to which many arrows involving gravity would converge. Without this asymmetry, the qualification of $x^\mu$ as set-theoretical elements could not be consistent with the regularity axiom in set theory denying the substructure of elements. This also implies the universal alternative choice of gravity being either attractive or repulsive, which explains (up to sign!) why the gravity is universally attractive.

3) The above conclusion does not exclude the possibility to interpret black holes as gravitational sinks because they are singularity points. In the usual non-singular physical regions, however, physical detection of gravitational waves is made impossible by the absence of gravitational sinks.

\textsuperscript{2}The importance of the notions of initial and terminal objects has been emphasized by Dr. H. Saigo in combination with that of the category of sets, to whom I am very grateful for useful discussions.
which concludes the absence of gravitational waves at the experimentally observable levels.

4) Then, the notion of “gravitons” and “quantization of gravity” are become physically meaningless, in view of the essential roles played by such duality relations as between wave and particle natures in the context of quantum theory.

As for the detailed accounts of the above remarks, please see my joint paper [7] with Dr. H. Saigo, who has reformulated my idea of the absence of gravitational sinks into that of gravitational “black bodies” to absorb gravitational field, according to which it becomes evident that “gravitational wave” cannot be used in the double slit experiment (because of the absence of “slits” for it), and hence, that it cannot exhibit the interference effects. This implies the absence of wave characters in the gravitational field.

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References

[1] Ojima, I., Micro-macro duality in quantum physics, 143-161, Proc. Intern. Conf. “Stochastic Analysis: Classical and Quantum –Perspectives of White Noise Theory” ed. by T. Hida, World Scientific (2005), arXiv:math-ph/0502038.

[2] Ojima, I., A unified scheme for generalized sectors based on selection criteria –Order parameters of symmetries and of thermality and physical meanings of adjunctions–, Open Systems and Information Dynamics, 10 (2003), 235-279 (math-ph/0303009).

[3] Ojima, I., Space(-Time) Emergence as Symmetry Breaking Effect, Invited talk at International Conference in QIBC (= Quantum Bio-Informatics Center, Tokyo University of Sciences) 2010; Dilation and Emergence in Physical Sciences, Invited talk at International Conference, “Advances in Quantum Theory” at Linnaeus University, June 2010.

[4] Ojima, I. and Ozawa, M., Unitary representations of the hyperfinite Heisenberg group and the logical extension methods in physics, Open Systems and Information Dynamics 2, 107-128 (1993).

[5] Doplicher, S. and Roberts, J.E., Why there is a field algebra with a compact gauge group describing the superselection structure in particle physics, Comm. Math. Phys. 131 (1990), 51-107; Endomorphism of C*-algebras, cross products and duality for compact groups, Ann. Math. 130
(1989), 75-119; A new duality theory for compact groups, Inventiones Math. 98 (1989), 157-218.

[6] Dieudonne, J., *Treatise on Analysis*, Vol.4, Ch.20, Academic Press, 1974.

[7] Ojima, I. and Saigo, H., in preparation.