Isospin properties of the $X$ state near the $D\bar{D}^*$ threshold

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Abstract

The $D\bar{D}^*$ scattering amplitude and the production of the final states $\pi^+\pi^-J/\psi$ and $\pi^+\pi^-\pi^0J/\psi$ near the $D^0\bar{D}^{*0}$ threshold are discussed following the recent suggestion that the observed peaks $X(3872)$ and $X(3875)$ in the decays $B \to X K$ are due to a virtual state $X$ in the $D^0\bar{D}^{*0}$ channel. The strong interaction is treated using the small interaction radius approximation. It is shown that the mass difference between the charged and neutral $D^{(*)}$ mesons results in a distinctive behavior of the relevant isotopic amplitudes. In particular, the shape of the peak in the $\pi^+\pi^-J/\psi$ channel should be significantly narrower than in the $\pi^+\pi^-\pi^0J/\psi$ channel, which property can be used for an experimental test of the virtual state hypothesis.
The narrow peak $X(3872)$ currently commands a great interest due to the suggested possibility that it is dominantly a molecular state of charmed mesons ($D^0 \bar{D}^{*0} + D^{*0} \bar{D}^0$), probably of the type discussed in the literature long time ago. Such interpretation, as opposed to considering $X(3872)$ as a regular charmonium resonance, is essentially based on two remarkable observations: the extreme proximity of the mass of $X$ to the $D^0 \bar{D}^{*0}$ threshold and the apparently strong violation of the isotopic symmetry indicated by the co-existence of the decays $X(3872) \rightarrow \pi^+ \pi^- J/\psi$ and $X(3872) \rightarrow \pi^+ \pi^- \pi^0 J/\psi$. The measurement of the position of the $X$ relative to the $D^0 \bar{D}^{*0}$ threshold has been recently improved by the CLEO result for the $D^0$ mass, which places the $D^0 \bar{D}^{*0}$ threshold at $3871.81 \pm 0.36$ MeV, and corresponds to $M(X) - M(D^0 \bar{D}^{*0}) = -0.6 \pm 0.6$ MeV. Due to the exceptional closeness of the peak to the meson-pair threshold one can expect that the mass difference between the pairs of charged and neutral mesons, $\Delta = M(D^+ D^{*-}) - M(D^0 \bar{D}^{*0}) \approx 8.1$ MeV, should naturally give rise to a significant isospin breaking in the properties of the state $X$.

Furthermore, recent experimental study of the $B$ meson decays $B \rightarrow D^0 \bar{D}^0 \pi^0 K$ and $B \rightarrow D^0 \bar{D}^0 \gamma K$ revealed that the invariant mass recoiling against the Kaon displays a significant enhancement with a maximum at approximately 3875 MeV, which is only about 3 MeV above the $D^0 \bar{D}^{*0}$ threshold. The observed events can all be in fact attributed to the process $B \rightarrow (D^0 \bar{D}^{*0} + \bar{D} D^{*0}) K$ since no distinction between the $D^{*0}$ mesons and their decay products was done. Moreover, the yield of the heavy meson pairs within the above-threshold peak is about ten times larger that that of the $\pi^+ \pi^- J/\psi$ and $\pi^+ \pi^- \pi^0 J/\psi$ channels at the peak of $X(3872)$. It has been most recently argued that a very plausible explanation of the observed enhancement of the $D^0 \bar{D}^{*0}$ production combined with the smaller observed $X(3872)$ peak in the $\pi^+ \pi^- J/\psi$ channel is that both these phenomena are due to a virtual state in the $D^0 \bar{D}^{*0}$ channel. In this picture the observed peak in the $\pi^+ \pi^- J/\psi$ and $\pi^+ \pi^- \pi^0 J/\psi$ mass spectra is in fact a cusp with a sharp maximum at the $D^0 \bar{D}^{*0}$ threshold.

In this paper such possible virtual state near the $D \bar{D}^*$ threshold is discussed within the approximation of small interaction radius, and a consideration is given to the isospin properties of the scattering and production amplitudes in this energy region. This approach is similar to the previously pursued ‘universal scattering length’ approximation, and differs in including the effect of the nearby threshold for charged mesons $D^+ D^{*-}$. An interesting energy-dependent behavior of the isotopic properties arises from the mere fact of the mass splitting $\Delta$ between the two isospin-related and coupled $D \bar{D}^*$ channels. In particular it will
be argued that the expected pattern of the isospin breaking is consistent with the observed relative yield of $\pi^+\pi^- J/\psi$ and $\pi^+\pi^-\pi^0 J/\psi$ at the peak. Moreover, it will be shown that the production amplitude for the $I = 1$ state $\pi^+\pi^- J/\psi$ in the considered approximation necessarily has a zero between the $D^0\bar{D}^{*0}$ and $D^+D^{*-}$ thresholds, thus reducing the apparent width of the cusp and putting it in line with the experimental limit $[1] \Gamma < 2.3 \text{ MeV}$ on the width of the peak in this particular channel.

It is assumed throughout this paper that the quantum numbers associated with the discussed peaks $X(3872)$ and $X(3875)$ are $J^{PC} = 1^{++}$, corresponding to the $S$ wave motion in the $C$ even state $D^0\bar{D}^{*0} + \bar{D}D^{*0}$ and the coupled channel with charged mesons $D^+D^{*-} + D^-D^{*+}$. These two channels are referred, for brevity, as respectively $n$ and $c$. Considering the nonrelativistic dynamics of the mesons it is convenient to place the origin of the c.m. energy $E$ at the threshold in the $n$ channel: $E = M(D\bar{D}^*) - M(D^0) - M(D^{*0})$. The energy range of interest for the present discussion is from few MeV below the $n$ threshold and up to the $c$ threshold, i.e. up to $E \approx \Delta$. In this range the scale of the c.m. momentum (real and virtual) in either channel is set by $\sqrt{2\mu\Delta} \approx 127 \text{ MeV}$, where $\mu \approx 970 \text{ MeV}$ is the reduced mass for the meson pair. One can apply in this region of soft momenta the standard picture of the strong-interaction scattering (see e.g. in the textbook [19]), where the strong interaction is localized at distances $r < r_0$ such that $r_0\sqrt{2\mu\Delta}$ can be considered as a small parameter. Some well known points of this time-tested approach are repeated here in order to adapt the same treatment to the situation with two closely spaced thresholds in isotopically related channels. Outside the region of the strong interaction the motion the $n$ and $c$ channel is described by the free $S$-wave radial wave function. Considering for definiteness an energy value between the two thresholds, $0 < E < \Delta$, one can write the corresponding wave functions (up to an overall normalization constant) as

$$\chi_n(r) = \sin(k_nr + \delta), \quad \chi_c(r) = \xi \exp(-\kappa_c r),$$

where $k_n = \sqrt{2\mu E}$ and $\kappa_c = \sqrt{2\mu(\Delta - E)}$, $\delta$ is the elastic$^1$ scattering phase in the $n$ channel and the constant $\xi$, generally energy-dependent, describes the relative normalization and phase of the wave function for the two channels.

The wave functions $[11]$ should be matched at $r \approx r_0$ to the solution of the ‘inner’ problem, i.e. that in the region of the strong interaction. In the limit of small $r_0$ all the complexity

$^1$In this consideration the small inelasticity due to the $\pi^+\pi^- J/\psi$ and $\pi^+\pi^-\pi^0 J/\psi$ channels is neglected and will be included later. Also the small width of the $D^*$ mesons is entirely neglected throughout this paper.
of the ‘inner’ problem reduces to only two parameters. Namely, in the region of the strong interaction the \( n \) and \( c \) channels are not independent and get mixed. Due to the isotopic symmetry of the strong interaction the independent are the channels with definite isospin, \( I = 0 \) and \( I = 1 \), corresponding to the functions \( \chi_0 = \chi_n + \chi_c \) and \( \chi_1 = \chi_n - \chi_c \), and the matching parameters are the logarithmic derivatives \(-\kappa_0\) and \(-\kappa_1\) of these functions at \( r = r_0 \). Using the assumption of small \( r_0 \) the matching condition for the functions from Eq. (1) can be shifted to \( r = 0 \), so that one can write the resulting matching equations as

\[
\frac{k_n \cos \delta - \xi \kappa_c}{\sin \delta + \xi} = -\kappa_0 , \quad \frac{k_n \cos \delta + \xi \kappa_c}{\sin \delta - \xi} = -\kappa_1 . \tag{2}
\]

These equations determine both the scattering phase \( \delta \) and the constant \( \xi \) as

\[
\cot \delta = -\frac{\kappa_{\text{eff}}}{k_n} \tag{3}
\]

with

\[
\kappa_{\text{eff}} = \frac{2\kappa_0 \kappa_1 - \kappa_c \kappa_1 - \kappa_c \kappa_0}{\kappa_0 + \kappa_1 - 2\kappa_c} , \tag{4}
\]

and

\[
\xi = \frac{\kappa_0 - \kappa_1}{2\kappa_c - \kappa_1 - \kappa_0} \sin \delta . \tag{5}
\]

The nonrelativistic scattering amplitude in the \( n \) channel is therefore given by [19]

\[
f = -\frac{1}{\kappa_{\text{eff}} + i k_n} , \tag{6}
\]

and the scattering length \( a \) is thus found from the \( E = 0 \) limit of this expression as

\[
a = \frac{1}{\kappa_{\text{eff}} \big|_{E=0}} = \frac{\kappa_0 + \kappa_1 - 2\sqrt{2\mu \Delta}}{2\kappa_0 \kappa_1 - (\kappa_0 + \kappa_1)\sqrt{2\mu \Delta}} . \tag{7}
\]

The whole approach considered here is applicable if the scattering length is large in the scale of strong interaction. A large positive value of \( a \) implies an existence of a shallow bound state, while a large negative \( a \) corresponds to the situation with a virtual state [19]. According to the estimates of Ref. [15] the required by the data scattering length in the problem considered is \(-3 - 4 \) fm, corresponding to a negative and quite small indeed parameter \( \kappa_{\text{eff}} (E = 0) \approx (50 - 60) \) MeV.

The physical picture, consistent with a small \( \kappa_{\text{eff}} \), and which could be argued on general grounds [8] is that an attraction in the \( I = 0 \) channel is strong enough to provide a small value of \( \kappa_0 \), while the interaction in the \( I = 1 \) channel is either a weak attraction or, more
likely, a repulsion. In both cases the absolute value of \( \kappa_1 \) is large, i.e. of a normal strong interaction scale, with the sign being respectively negative or positive. Another, purely phenomenological, argument in favor of large \( |\kappa_1| \) is that no peculiar near-threshold behavior is observed in the production of the \( I = 1 \) charged states, e.g. \( D^0D^*^- \). At large \( |\kappa_1| \) the expression (4) simplifies and takes the approximate form

\[
\kappa_{\text{eff}} \approx 2\kappa_0 - \kappa_c. \tag{8}
\]

Using this approximation, one can readily see that in order for \( \kappa_{\text{eff}}(E = 0) \) to be negative and small the parameter \( \kappa_0 \) has to be positive and quite small:

\[
\kappa_0 < \sqrt{\mu\Delta/2} \approx 63 \text{ MeV}. \tag{9}
\]

It is interesting to note that in the suggested picture the interaction in the \( I = 0 \) state is strong enough by itself to produce a shallow bound state in the limit of exact isospin symmetry, i.e. at \( \Delta \to 0 \). In reality the isospin breaking by the mass difference between the charged and neutral charmed mesons turns out to be sufficiently significant to deform the bound state into a virtual one, i.e. to shift the pole of the scattering amplitude from the first sheet to the second sheet of the Riemann surface for the amplitude as a complex function of the energy \( E \).

One can also notice that within the approximation in Eq.(8) the scattering amplitude (6) can be written in the form

\[
f = \frac{1}{2\kappa_0 - \kappa_c + i\kappa_n}, \tag{10}
\]

which corresponds to equal coupling of the virtual state to the \( n \) and \( c \) channels. Such behavior, assumed in Ref.[15] on the grounds of isotopic symmetry, turns out to be applicable, as long as \( |\kappa_1| \) is large, even if the isospin breaking by the mass difference is essential.

Thus far the presence of any inelastic channels was neglected in the discussion of the \( D\bar{D}^* \) scattering amplitude. Such channels certainly exist and include the observed ones \( \pi^+\pi^-J/\psi (\rho J/\psi), \pi^+\pi^-\pi^0J/\psi (\omega J/\psi), \gamma J/\psi \) and probably other, which are yet to be found in experiment. The inelasticity however appears to be reasonably small, as one can infer from the observed [13, 14] dominance of the \( D^0\bar{D}^{*0} \) production in the threshold region, and can be parametrized by a small imaginary shift \( i\gamma \) of the denominator of the scattering amplitude in Eq.(6) or Eq.(10):

\[
f = -\frac{1}{\kappa_{\text{eff}} + i\kappa_n + i\gamma} \approx -\frac{1}{2\kappa_0 - \kappa_c + i\kappa_n + i\gamma}. \tag{11}
\]
The latter expression is similar to the one used in Ref.\[15\] and differs in that a term in the denominator, quadratic in \( k_n \), being neglected, as appropriate in the considered here small interaction radius approximation. The previous analysis \[15\], based on the Breit-Wigner type description, includes such quadratic term and concludes that its contribution is very small in the discussed region of parameters. Furthermore, the relatively small value of the inelasticity is estimated there in terms of the scattering length as \( \text{Im} \ a / \text{Re} \ a \sim 0.1 \) or less.

The width parameter \( \gamma \) in Eq.\( (11) \) is the total sum over the inelastic channels, and in what follows the contributions of the \( \omega J/\psi \) and \( \rho J/\psi \) channels, \( \gamma_\omega \) and \( \gamma_\rho \), will be addressed in some detail. It is further assumed here \[20 \ 15\] that the ‘seed’ decay \( B \rightarrow XK \) is a short-distance process, so that the yield in each final channel coupled to \( X \) is proportional to that channel’s contribution to the unitary cut of the amplitude \( f \). This implies in particular that

\[
B[B \rightarrow (D^0 \bar{D}^{*0} + \bar{D} D^{*0}) \ K] : B(B \rightarrow \omega J/\psi \ K) : B(B \rightarrow \rho J/\psi \ K) = k_n |f|^2 : \gamma_\omega |f|^2 : \gamma_\rho |f|^2 ,
\]

where the specific expression for \( |f|^2 \) depends on the value of the energy \( E \) relative to the \( n \) and \( c \) thresholds, as given by Eq.\( (11) \) and its analytical continuation across the thresholds. Besides the energy dependence of the overall factor \( |f|^2 \), the heavy meson channel contains the phase space factor \( k_n \), while for the \( \omega J/\psi \) and \( \rho J/\psi \) yields an additional dependence on the energy arises from the factors \( \gamma_\omega \) and \( \gamma_\rho \).

A certain variation of the width parameter \( \gamma_\omega \) for the \( \pi^+ \pi^- \pi^0 J/\psi \) channel in the discussed range of energy is of a well known kinematical origin. Indeed, the central value of the mass of the \( \omega \) resonance puts the threshold for the channel \( \omega J/\psi \) at 3878.5 MeV, which corresponds to \( E \approx 6.7 \) MeV in our conventions, i.e. squarely between the \( n \) and \( c \) thresholds. Any production of the \( \pi^+ \pi^- \pi^0 J/\psi \) states at smaller invariant mass is a sub-threshold process, possible due to the width \( \Gamma_\omega \) of the \( \omega \) resonance. In other words, the energy dependence of the width factor \( \gamma_\omega \) can be estimated as \[20 \ 15\]

\[
\gamma_\omega = |A_\omega|^2 q_{\text{eff}}^{(\omega)} , \tag{13}
\]

where \( A_\omega \) is the amplitude factor for the coupling to the \( \omega J/\psi \) channel and \( q_{\text{eff}}^{(\omega)} \) is the effective momentum of \( \omega \) at the invariant mass \( M \) calculated as

\[
q_{\text{eff}}^{(\omega)} (M) = \frac{m_\omega \Gamma_\omega}{(m^2 - m_\omega^2)^2 + m_\omega^2 \Gamma_\omega^2} \frac{dm^2}{\pi} \tag{14}
\]

with the c.m. momentum \( |\vec{q}(m)| \) found in the standard way:

\[
|\vec{q}(m)| = \sqrt{\frac{(M - m_{J/\psi})^2 - m^2}{2M} \left[ (M + m_{J/\psi})^2 - m^2 \right]} \tag{15}
\]
The lower limit \( m_0 \) in the integral in Eq.(14) can be chosen anywhere sufficiently below \( m_\omega - \Gamma_\omega \), since the Breit-Wigner curve in the integrand rapidly falls off away from the resonance.

Numerically, the effective momentum \( q_{\text{eff}}^{(\omega)} \) can be estimated as varying from approximately 20 MeV to 50 MeV between the \( n \) and \( c \) thresholds, i.e. when \( E \) changes from \( E = 0 \) to \( E = \Delta \). As will be argued here, the amplitude \( A_\omega \) should vary only slowly in the considered energy range, so that the shape of the threshold cusp in the \( \omega J/\psi \) channel is determined by the behavior of the scattering amplitude factor \(|f|^2\) and by the estimated kinematical effect.

In the \( \rho J/\psi \) channel the expected energy behavior of the yield is quite different. If one writes the corresponding width factor \( \gamma_\rho \) similarly to Eq.(13) as

\[
\gamma_\rho = |A_\rho|^2 q_{\text{eff}}^{(\rho)},
\]

the effective momentum \( q_{\text{eff}}^{(\rho)} \) can be estimated as varying only slightly due to the large width of the \( \rho \) resonance: \( q_{\text{eff}}^{(\rho)} \approx (125 \div 135) \) MeV as the energy changes between \( E = 0 \) and \( E = \Delta \). On the contrary, as will be argued, the amplitude \( A_\rho \) should experience a significant variation in this energy range and in fact cross zero between the \( n \) and \( c \) thresholds.

In order to argue the claimed properties of the amplitudes \( A_\omega \) and \( A_\rho \) it can be first noticed that the inelastic processes \( D\bar{D}^* \rightarrow \omega J/\psi \) and \( D\bar{D}^* \rightarrow \rho J/\psi \) involve a rearrangement of the heavy and light quarks and therefore cannot be due to peripheral interactions at long distances, but are determined by the dynamics at a typical range of the strong interaction. In other words the amplitudes of these processes are sensitive to the behavior of the heavy meson pair wave function at short strong-interaction distances. Thus for the \( I = 0 \) state \( \omega J/\psi \) the amplitude is related to the short-distance part of the function \( \chi_0 \), while the amplitude for the \( I = 1 \) channel \( \rho J/\psi \) is determined by the function \( \chi_1 \). Assuming also as previously, that the \( B \) decays are also determined by short distances, one can write for the decay amplitudes the expressions

\[
A(B \rightarrow \omega J/\psi K) = \int F_\omega \chi_0(r) \, dr \quad \text{and} \quad A(B \rightarrow \rho J/\psi K) = \int F_\rho \chi_1(r) \, dr,
\]

where \( F_\omega \) and \( F_\rho \) are the weight functions in the respective channels, and their support, as argued, is limited to typical strong-interaction distances. In the limit of vanishing interaction range these functions should each be replaced by a \( \delta \)-function: \( F_{\omega,\rho} \rightarrow \Phi_{\omega,\rho} \delta(r) \), so

\[2\]Certainly, at present any details of these functions are not known. It can only be mentioned that it is likely that \( F_\rho \approx F_\omega \) due to the \( \rho - \omega \) universality.
that, using the expressions in Eq. (11) and the results in the equations (3), (4) and (5) for the solution to the matching conditions, one finds

$$A(B \to \omega J/\psi K) = 2 \frac{\kappa_1 - \kappa_c}{\kappa_0 + \kappa_1 - 2 \kappa_c} \Phi_\omega \sin \delta \approx 2 \Phi_\omega \sin \delta$$

$$A(B \to \rho J/\psi K) = 2 \frac{\kappa_0 - \kappa_c}{\kappa_0 + \kappa_1 - 2 \kappa_c} \Phi_\rho \sin \delta \approx 2 \frac{\kappa_0 - \kappa_c}{\kappa_1} \Phi_\rho \sin \delta,$$

(18)

where the latter expressions for each amplitude are written in the limit of large $|\kappa_1|$. One can readily see that in this limit the amplitude for production of the isoscalar state $\omega J/\psi$ is finite, while that for the isovector state $\rho J/\psi$ is suppressed by $1/\kappa_1$. Furthermore, it should be noted that, generally, in the limit of small interaction range it would be incorrect to retain such suppressed term without taking also into account effects of finite range. In order to fix this deficiency in the second line in Eq. (18) one should go beyond the $r = 0$ point approximation for the second integral in Eq. (17) and use the first two terms of the Taylor expansion of the function $\chi_1(r)$ at $r = 0$ rather than only the first term:

$$\chi_1(r) = \sin \delta - \xi + (k n \cos \delta + \xi \kappa_c) r + O(r^2) = 2 \frac{\kappa_0 - \kappa_c}{\kappa_0 + \kappa_1 - 2 \kappa_c} (1 - \kappa_1 r) \sin \delta + O(r^2).$$

(19)

Then the improved estimate of the second integral in Eq. (17) can be written in terms of the effective radius $R$ of the weight function $F_\rho$,

$$R = \frac{\int F_\rho r \, dr}{\int F_\rho \, dr}$$

(20)

as

$$A(B \to \rho J/\psi K) = 2 \frac{\kappa_0 - \kappa_c}{\kappa_0 + \kappa_1 - 2 \kappa_c} (1 - \kappa_1 R) \Phi_\rho \sin \delta \approx 2 \left( \frac{\kappa_0 - \kappa_c}{\kappa_1} \right) \Phi_\rho \sin \delta,$$

(21)

where, as discussed, the parameter $\kappa_1 R$ should be considered as being of order one.

Clearly, the factor $\sin \delta$ in each amplitude is proportional to the scattering amplitude factor $f$, which is accounted for separately in the yield of each of the discussed channels (Eq. (12)). Therefore this factor should be omitted in the amplitudes $A_\omega$ and $A_\rho$, and one can conclude that the amplitude $A_\omega$ is a smooth slowly varying function of the energy inasmuch as $\kappa_1$ is large. Furthermore, the ratio of the $\rho J/\psi$ and $\omega J/\psi$ production amplitudes is estimated as

$$\frac{A_\rho}{A_\omega} = \frac{\kappa_0 - \kappa_c}{\kappa_1 - \kappa_c} (1 - \kappa_1 R) \frac{\Phi_\rho}{\Phi_\omega},$$

(22)
Since the situation where the $X$ peak is a virtual state corresponds to a small positive $\kappa_0$ satisfying the condition (19), the amplitude $A_\rho$ described by Eq.(22) should necessarily change sign between the $n$ threshold, where $\kappa_c = \sqrt{\frac{1}{2}\mu^2}$, and the $c$ threshold, where $\kappa_c = 0$. It can be noticed that the existence of the zero of the amplitude $A_\rho$ results from both first terms of the expansion (19) for the function $\chi_1(r)$ being proportional to $\kappa_0 - \kappa_c$.

![Figure 1](image.png)

**Figure 1**: The expected shape (in arbitrary units) of the virtual state peak in the yield of $\pi^+\pi^-J/\psi$ (solid) and $\pi^+\pi^-\pi^0J/\psi$ (dashed) channels.

The expected difference in the shape of the cusp in the $\rho J/\psi$ and $\omega J/\psi$ channels is illustrated in Fig.1. In these plots the parameters of the virtual state correspond to the scattering length $a = -(4 + 0.5 i)\ \text{fm}$, which is close to the possible fit values of the scattering length found in Ref.[15]. In the limit of large $\kappa_1$ this value of $a$ translates into $\kappa_0 \approx 38\ \text{MeV}$ and $\gamma \approx 6\ \text{MeV}$. One can see from Fig.1 that due to the discussed zero of the amplitude the peak in the $\pi^+\pi^-J/\psi$ is expected to be quite narrow in agreement with the experimental limit on the width of $X(3872)$. The plots in Fig.1 are normalized to the same total yield in each channel over the shown energy range in order to approximate the experimentally observed relative yield. Such normalization corresponds to setting

$$\left| \frac{\kappa_1}{1 - \kappa_1 R \frac{\Phi_\omega}{\Phi_\rho}} \right| \approx 175\ \text{MeV},$$
which value does not appear to be abnormal, even though at present we have no means of independently estimating this quantity.

Clearly, the suggested significant difference of the shape of the virtual state peak in the two discussed channels provides with a way of discriminating between the options of $X$ being a virtual state and a bound state.

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