$B_d$ and $B_s$ mixing: mass and width differences and CP violation

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$B_d - \overline{B_d}$ mixing involves three physical parameters: the magnitudes of the off-diagonal elements of the mass and decay matrices and their relative phase. They are related to the mass and width differences between the mass eigenstates and to the CP asymmetry in flavour-specific decays, $a_d$. Introducing a new operator basis I present new, more precise theory predictions for the width differences in the $B_s$ and $B_d$ systems: in the Standard Model one finds $\Delta \Gamma_s = 0.088 \pm 0.017 \text{ ps}^{-1}$ and $\Delta \Gamma_d = (26.7^{+5.8}_{-6.5}) \cdot 10^{-4} \text{ ps}^{-1}$. Updates of the mass differences $\Delta M_d$ and $\Delta M_s$ and of $a_d^0$ and $a_s^0$ are also presented. Then I discuss how various present and future measurements can be combined to constrain new physics. The extraction of a new CP phase $\phi_s^0$ from data on $a_d^0$ also profits from our new operator basis. Confronting our new formulae with DØ data we find that $\sin \phi_s^0$ deviates from zero by 2σ.

1. $B_d - \overline{B_d}$ mixing basics

Loop-induced $b \to s$ transitions are currently receiving much attention from experiment and theory. Present data leave room for new physics with sources of flavour beyond the Cabibbo-Kobayashi-Maskawa (CKM) mechanism [1] of the Standard Model (SM). For example, an extra contribution to $b \to s \gamma$, $q = u, d, s$, decay amplitudes with a new CP phase can alleviate the $\sim 2.6\sigma$ discrepancy between the measured mixing-induced CP asymmetries in these $b \to s$ penguin modes and the Standard Model prediction [2]. If this new contribution involves $b$ and $s$ quarks with the same chirality, no tension with the well-measured $b \to s \gamma$ branching fraction occurs. Supersymmetric grand-unified models can naturally accommodate such new $b \to s$ amplitudes [3]: right-handed quarks reside in the same quintuplets of SU(5) as left-handed neutrinos, so that the large atmospheric neutrino mixing angle could well affect squark-gluino mediated $b \to s$ transitions [4]. Obviously $B_s - \overline{B_s}$ mixing plays an important role in the search for new physics in $b \to s$ FCNC’s. In this talk I present an update of the theory predictions of the physics observables related to $B_s - \overline{B_s}$ mixing. Since the formalism equally applies to $B_d - \overline{B_d}$ mixing, the corresponding quantities for the $B_s$ system are also presented. The presented results are derived with the use of a novel operator basis, which leads to a better precision of the theory predictions. Details can be found in [5].

$B_q - \overline{B_q}$ oscillations are governed by a Schrödinger equation

$\frac{d}{dt} \left( \frac{|B_q(t)|}{|\overline{B_q}(t)|} \right) = \left( M^q - \frac{i}{2} \Gamma^q \right) \left( \frac{|B_q(t)|}{|\overline{B_q}(t)|} \right)$

with the mass matrix $M^q$ and the decay matrix $\Gamma^q$ and $q = d$ or $q = s$. The physical eigenstates $|B_H\rangle$ and $|B_L\rangle$ with the masses $M^q_H$, $M^q_L$ and the decay rates $\Gamma^q_H$, $\Gamma^q_L$ are obtained by diagonalizing $M^q - i\Gamma^q/2$. The $B_q - \overline{B_q}$ oscillations in Eq. 1 involve the three physical quantities $|M^q_{12}|$, $|\Gamma^q_{12}|$ and the CP phase $\phi_q = \arg(-M^q_{12}/\Gamma^q_{12})$ (see e.g. [6]). The mass and width differences between $B_L$ and $B_H$ are related to these quantities as

$\Delta M_q = M^q_H - M^q_L = 2 |M^q_{12}|$, \hspace{1cm} (2)

$\Delta \Gamma_q = \Gamma^q_L - \Gamma^q_H = 2 |\Gamma^q_{12}| \cos \phi_q$, \hspace{1cm} (3)

up to numerically irrelevant corrections of order $m^2_b/m^2_W$. $\Delta M_q$ simply equals the frequency of the $B_q - \overline{B_q}$ oscillations. A third quantity providing
independent information on the mixing problem in Eq. (1) is

$$a^q_{\ell s} = \text{Im} \frac{\Gamma^q_{12}}{M^q_{12}} = \frac{|\Gamma^q_{12}|}{|M^q_{12}|} \sin \phi^q.$$  (4)

$a^q_{\ell s}$ is the CP asymmetry in flavour-specific $B_q \rightarrow f$ decays and quantifies CP violation in mixing. The standard way to measure $a^q_{\ell s}$ uses $B_q \rightarrow X_q\ell^{+}\nu_\ell$ decays, so that $a^q_{\ell s}$ is often called the semileptonic CP asymmetry.

## 2. $\Delta M_d$ and $\Delta M_s$

In order to predict $\Delta M_q$ we need to compute $M^q_{12}$. Key quantities entering $M^q_{12}$ are the CKM element $V_{qg}$, the top mass $m_t$, and $f_{B_q}$, $B$, which parameterises the matrix element

$$\langle B_q|Q|\overline{B}_q\rangle = \frac{8}{3} M^q_{B_q} f^q_{B_q} B$$  (5)

of the four-quark operator ($\alpha,\beta = 1, 2, 3$ are colour indices):

$$Q = \overline{\tau}_\alpha \gamma_\mu (1 - \gamma_5) b_\alpha \overline{\tau}_\beta \gamma_\mu (1 - \gamma_5) b_\beta.$$  (6)

While the decay constants $f_{B_d}$ and $f_{B_s}$ differ numerically, no non-perturbative computation of the bag factor $B$ has shown a significant difference for $B_d$ and $B_s$ mesons.

Updating $\Delta M_q$ computed in [7] to $m_t^{\text{pole}} = 171.4 \pm 2.1$ GeV [8], which corresponds to $m_t(m_t) = 163.8 \pm 2.0$ GeV in the $\overline{\text{MS}}$ scheme, gives

$$\Delta M^{\text{SM}}_d = (0.53 \pm 0.02) \text{ ps}^{-1} \left( \frac{|V_{td}|}{0.0082} \right)^2 \cdot \left( \frac{f_{B_d}}{200 \text{ MeV}} \right)^2 \frac{B}{0.85}.$$  (7)

$$\Delta M^{\text{SM}}_s = (19.3 \pm 0.6) \text{ ps}^{-1} \left( \frac{|V_{ts}|}{0.0405} \right)^2 \cdot \left( \frac{f_{B_s}}{240 \text{ MeV}} \right)^2 \frac{B}{0.85}.$$  (7)

$B$ in Eq. (7) and all other bag factors appearing in this talk are evaluated in the $\overline{\text{MS}}$ scheme at the scale $\mu = \overline{m}_b$. The frequently used scheme-invariant bag parameter $B$ is larger than $B$ by a factor of 1.5. Both $\Delta M_d$ and $\Delta M_s$ are well-measured. Here we concentrate on the phenomenological impact of this year’s discovery of $B_s - \overline{B_s}$ mixing and the precise measurement of $\Delta M_s$ [9]:

$$17 \text{ ps}^{-1} \leq \Delta M_s \leq 21 \text{ ps}^{-1} \ @90\% \text{ CL (DO)}$$

$$\Delta M_s = 17.77 \pm 0.10 \pm 0.07 \text{ ps}^{-1} \ (\text{CDF}).$$  (8)

$|V_{cb}| = 0.0415 \pm 0.0010$ determines $|V_{ts}| = 0.0405 \pm 0.0010$, so that CKM uncertainties are not an issue for $\Delta M_s$. However, non-perturbative computations of $f_{B_s}$ and $B$ still cover a wide range. The ballpark of results from lattice QCD [10] and QCD sum rules [11] is represented by $f_{B_s} = 240 \pm 40$ MeV and $B = 0.85 \pm 0.06$, which implies

$$\Delta M^{\text{SM}}_s = (19.30 \pm 0.68) \text{ ps}^{-1}.$$  (9)

## 3. $\Delta \Gamma_d$ and $\Delta \Gamma_s$

In order to predict $\Delta \Gamma_q$ we need to compute $\Gamma^q_{12}$. There are two important differences compared to $M^q_{12}$: first, $\Gamma^q_{12}$ involves an operator product expansion at the scale $\overline{m}_b$, so that one faces two expansion parameters, $\overline{\Lambda}/m_b$ and $\alpha_s(m_b)$, where $\overline{\Lambda} \sim (M_{B_q} - m_b)$ is the relevant hadronic scale of the problem. Second, even in the leading order of $\overline{\Lambda}/m_b$ the prediction of $\Gamma^q_{12}$ involves two operators. In addition to $Q$ in Eq. (6), one encounters

$$Q_S = \overline{\tau}_q (1 + \gamma_5) b_q \overline{\tau}_q (1 + \gamma_5) b_q.$$  (10)

which involves a new bag parameter $B'_q$ in the range $B'_S = 1.34 \pm 0.12$ [10]. $\Gamma^q_{12}$ is known to next-to-leading-order (NLO) in both $\overline{\Lambda}/m_b$ [12] and $\alpha_s(m_b)$ [13–15]. With current values of the input parameters (most relevant are $\overline{m}_b(m_b) = 4.22 \pm 0.08$ GeV, $\overline{m}_c(m_c) = 1.30 \pm 0.05$ GeV and $\alpha_s(M_Z) = 0.1189 \pm 0.0010$), the result of [13] is updated to

$$\Delta \Gamma^{\text{old}}_s = (0.070 \pm 0.042) \text{ ps}^{-1}.$$  (11)

$\Delta \Gamma^{\text{old}}_s$ is pathological in several aspects: both the $\overline{\Lambda}/m_b$ and $\alpha_s$ corrections are large and negative, which limits the accuracy in Eq. (11). Further $\Delta \Gamma^{\text{old}}_s$ is dominated by the matrix element of $Q_S$,
so that hadronic uncertainties do not cancel in the ratio $\Delta \Gamma_s^{\text{had}}/\Delta M_s \propto B_S'/B$.

When computing the leading contribution to $\Gamma_{12}$, one first encounters a third operator,

$$\bar{Q}_S = \bar{f}_a(1 + \gamma_5)b_\alpha \bar{f}_d(1 + \gamma_5)b_\alpha.$$  

(12)

Subsequently $\bar{Q}_S$ is traded for

$$R_0 \equiv Q_S + \alpha_1 \bar{Q}_S + \frac{1}{2} \bar{y}_2 Q,$$  

(13)

which belongs to the sub-leading order in $\bar{X}/m_b$. $\alpha_1$ and $\alpha_2$ are QCD factors [5, 13, 15]. Writing

$$\langle B_s | \bar{Q}_S | B_s \rangle = \frac{1}{3} M_{B_s}^2 f_{B_s} B_S',$$  

(14)

a lattice computation finds $\bar{B}_s' = 1.41 \pm 0.12$ [16].

With this result we find the matrix element of $Q_S$ roughly five times smaller than that of $Q_S$, which involves an overall factor of $-5/3$ instead of $1/3$ in Eq. (13). We propose to use Eq. (13) to eliminate $\bar{Q}_S$ from the operator basis, so that the leading term of the operator product expansion involves $Q$ and $\bar{Q}_S$. This change of basis reshuffles the expansions in both $\bar{X}/m_b$ and $\alpha_s(m_b)$. Since terms of order $\alpha_s(m_b) \cdot \bar{X}/m_b$ have not been calculated yet, the central value of our prediction for $\Delta \Gamma_s$ changes within the error bar of the previous predictions in [13, 14]. In our new basis we find that all pathologies vanish: the $\bar{X}/m_b$ and $\alpha_s(m_b)$ corrections have shrunk to their natural sizes and $\Delta \Gamma_s$ is now dominated by the matrix element of $Q$:

$$\Delta \Gamma_s^{\text{SM}} = \left( \frac{f_{B_s}}{240 \text{ MeV}} \right)^2 \left[ (0.105 \pm 0.016) B + (0.024 \pm 0.004) B_S' - 0.027 \right] \text{ps}^{-1}.$$  

(15)

The ratio $\Delta \Gamma_s/\Delta M_s$ is now almost free from hadronic uncertainties. Inserting the numerical ranges for $B$ and $B_S'$ quoted before Eq. (9) and after Eq. (14) yields:

$$\frac{\Delta \Gamma_s^{\text{SM}}}{\Delta M_s^{\text{SM}}} = (49.7 \pm 9.4) \cdot 10^{-4}.$$  

(16)

The progress due to the change of the operator basis is depicted in Fig. [14]. Using the CDF measurement of $\Delta M_s$ in Eq. (5) we predict from Eq. (15):

$$\frac{\Delta \Gamma_s^{\text{SM}}}{\Gamma_s} = 0.088 \pm 0.017 \text{ps}^{-1}$$

$$\frac{\Delta \Gamma_s^{\text{SM}}}{\Gamma_s} = 0.127 \pm 0.024.$$  

(17)

Here $\Gamma_s$ is the average width of the two $B_s$ mass eigenstates and the theoretical relation $\Gamma_s = 1/\tau_{B_d}(1 + O(0.01))$ has been used. Any experimental violation of Eq. (17) will signal new physics in either $\Delta M_s$ or $\Delta \Gamma_s$. The corresponding results for the $B_d$ system are

$$\frac{\Delta \Gamma_d^{\text{SM}}}{\Delta M_d^{\text{SM}}} = \left( 52.6_{-12.8}^{+11.5} \right) \cdot 10^{-4}$$

$$\frac{\Delta \Gamma_d^{\text{SM}}}{\Delta M_d^{\text{SM}}} = \left( 26.7_{-6.5}^{+5.8} \right) \cdot 10^{-4} \text{ps}^{-1}$$

$$\frac{\Delta \Gamma_d^{\text{SM}}}{\Gamma_d} = \left( 40.9_{-9.9}^{+8.9} \right) \cdot 10^{-4},$$  

(18)

where $\Delta M_d^{\text{exp}} = 0.507 \pm 0.004 \text{ps}^{-1}$ and $\tau_{B_d}^{\text{exp}} = 1.530 \pm 0.009$ has been used.

4. $a_{B_s}^q$, $a_{B_s}^e$, $\phi_d$ and $\phi_d$

The CP asymmetries in flavour-specific decays are typically measured using semileptonic decays. No tagging is required, $a_{B_s}^q$ can be found by simply counting positively and negatively charged leptons. It may be worthwhile to study the time evolution of the untagged sample [17],

$$\Gamma[\bar{B}_q \rightarrow X^{-}\ell^+\nu_{\ell}, t] - \Gamma[\bar{B}_q \rightarrow X^{+}\ell^-\bar{\nu}_{\ell}, t]$$

$$\Gamma[\bar{B}_q \rightarrow X^{-}\ell^+\nu_{\ell}, t] + \Gamma[\bar{B}_q \rightarrow X^{+}\ell^-\bar{\nu}_{\ell}, t] = \frac{a_{B_s}^q}{2} \left[ 1 - \cos(\Delta M_q t) \right].$$  

(19)

as it may help to reject fake effects from detection asymmetries and to separate the $B_d$ and $B_s$ samples in hadron collider experiments through $\Delta M_d \neq \Delta M_s$. Further, the time evolution of any (tagged or untagged) decay of $B_q$ mesons contains information on $a_{B_s}^q$ [17], which might be useful to add statistics in the determination of $a_{B_s}^q$ at B factories.

Our new operator basis does not improve the Standard Model prediction of $a_{B_s}^q$ over the one in
[14, 15]. With up-to-date input parameters we find
\[
\alpha_{fs}^{SM,s} = (2.06 \pm 0.57) \times 10^{-5}, \\
\alpha_{fs}^{SM,d} = (-4.8^{+1.0}_{-1.2}) \times 10^{-4}. \tag{20}
\]
\(\alpha_{fs}^{d}\) depends on CKM parameters. The quoted value uses \(\beta = 23^\circ \pm 2^\circ\) for the angle of the unitarity triangle measured in \(B_d \to J/\psi K_S\) and \(R_t = 0.86 \pm 0.11\) for the side of the triangle adjacent to \(\beta\) and opposite to \(\gamma\). The value of \(R_t\) ignores any input from \(\Delta M_d/\Delta M_s\), since our focus is on potential new physics in \(B_s - \bar{B}_s\) mixing. Our predictions in Eq. (20) correspond to the following values of the CP phases appearing in Eq. (3):
\[
\phi_s^{SM} = -0.091^{+0.026}_{-0.038} = -5.2^{+1.5}_{-2.1}^\circ, \\
\phi_s^{SM} = (4.2 \pm 1.4) \times 10^{-3} = 0.24^\circ \pm 0.08^\circ. \tag{21}
\]

5. \(\Delta \Gamma_s, \Delta \Gamma_s/\Delta M_s\) and \(\alpha_{fs}^{s}\) beyond the SM

Looking beyond the Standard Model we now allow for a new CP phase \(\phi_s^\Delta\) which adds to the Standard Model value in Eq. (21). Then \(\alpha_{fs}^{s}\) only only depends on \(\text{Im} \Gamma_{12}^{q}/M_{12}^{q}\), but also on \(\text{Re} \Gamma_{12}^{q}/M_{12}^{q}\), which we can predict in a much better way with our new operator basis. We parameterise the effect of new physics (similarly to [18]) by
\[
M_{12}^{s} \equiv M_{12}^{SM,s} \cdot \Delta_s, \quad \Delta_s \equiv |\Delta_s|e^{i\phi_s^\Delta}. \tag{22}
\]
Then one easily finds
\[
\Delta M_s = |\Delta M_s^{SM}| |\Delta_s|, \\
\Delta \Gamma_s = 2|\Gamma_{12}^{q}| \cos (\phi_s^{SM} + \phi_s^\Delta) \\
\Delta \Gamma_s = |\Gamma_{12}^{q}| \cos (\phi_s^{SM} + \phi_s^\Delta) \\
\Delta \Gamma_s \Delta M_s = |\Gamma_{12}^{q}| |\Delta_s| \cos (\phi_s^{SM} + \phi_s^\Delta) \\
\alpha_{fs}^{s} = |\Gamma_{12}^{q}| |\Delta_s| \sin (\phi_s^{SM} + \phi_s^\Delta). \tag{23}
\]
A further source of information on the phase \(\phi_s^\Delta\) is the angular distribution of the decay \(\bar{B}_s \to J/\psi \phi\), which contains a CP-odd term [19,20]. Recently, the DØ collaboration found \(\phi_s^\Delta - 2\beta_s = -0.79 \pm 0.56 \pm 0.01\) (and mirror solutions in other quadrants of the complex \(\Delta_s\) plane) from this analysis [21]. Here \(\beta_s = 0.020 \pm 0.005 = 1.1^\circ \pm 0.3^\circ\) is the relevant combination of CKM phases. Combining all available experimental information with our new theory predictions (see [5] for details) gives the constraints depicted in Fig. 2. Setting \(\Delta_s \) to its Standard Model value \(|\Delta_s| = 1\) we find with the data from [21] and [22]:
\[
\sin (\phi_s^\Delta - 2\beta_s) = -0.77 \pm 0.04_{(th)} \pm 0.34_{(exp)},
\]
which deviates from the Standard Model value \(\sin (-2\beta_s) = -0.04 \pm 0.01\) by 2 standard deviations.

6. Conclusions

We have improved the theoretical prediction of \(\Re \Gamma_{12}^{q}/M_{12}^{q}\) (with \(q = d\) or \(q = s\)) by introducing a new operator basis. This quantity enters the predictions of the width difference \(\Delta \Gamma_q\) in the \(B_q - \bar{B}_q\) system and the prediction of the CP asymmetry in flavour-specific decays, \(\alpha_{fs}^{s}\), in scenarios of physics beyond the Standard Model. Applying our formulae to DØ data we find that the \(B_s - \bar{B}_s\) mixing phase deviates from the Standard Model value by \(2\sigma\). While this is not statistically significant, it shows that current experiments are reaching the sensitivity to probe any new physics entering the \(B_s - \bar{B}_s\) mixing phase.

REFERENCES
1. N. Cabibbo, Phys. Rev. Lett. 10, 531 (1963); M. Kobayashi and T. Maskawa, Prog. Theor. Phys. 49, 652 (1973).
2. The experimental situation is summarised in: M. Hazumi, plenary talk at 33rd International Conference On High Energy Physics (ICHEP 06), 26 Jul – 2 Aug 2006, Moscow, Russia.
3. D. Chang, A. Masiero and H. Murayama, Phys. Rev. D 67, 075013 (2003) [arXiv:hep-ph/0205111].
4. Effects of the model in [3] on $B_s - \bar{B}_s$ mixing have been studied in: S. Jäger and U. Nierste, Eur. Phys. J. C 33, S256 (2004) [arXiv:hep-ph/0312145]; S. Jäger and U. Nierste, in Proceedings of the 12th International Conference On Supersymmetry And Unification Of Fundamental Interactions (SUSY 04), 17-23 Jun 2004, Tsukuba, Japan, p. 675-678, Ed. K. Hagiwara, J. Kan- zaki, N. Okada [hep-ph/0410360]; S. Jäger, hep-ph/0505243, to appear in the Proceedings of the XLth Rencontres de Moriond, Electroweak Interactions and Unified Theories, 5-12 Mar 2005, La Thuile, Aosta Valley, Italy, Ed. J. Tran Thanh Van.

5. A. Lenz and U. Nierste, arXiv:hep-ph/0612167.

6. K. Anikeev et al., *B physics at the Tevatron: Run II and beyond*, hep-ph/021071, Chapters 1.3 and 8.3.

7. A.J. Buras, M. Jamin and P.H. Weisz, Nucl. Phys. B347, 491 (1990).

8. E. Brubaker et al. [Tevatron Electroweak Working Group], arXiv:hep-ex/0608032.

9. Talks by D. Glenzinski (plenary), T. Moulik and S. Giagu at 33rd International Conference On High Energy Physics (ICHEP 06), 26 Jul – 2 Aug 2006, Moscow, Russia. Talks by P. Tamburello [DØ] and A. Belloni [CDF] at Beauty 2006, 25-29 Sep 2006, Oxford, England, Nucl. Phys. Proc. Suppl. 170 (2007) 123 and ibid. 129.

10. S. Hashimoto and T. Onogi, arXiv:hep-ph/0407221; S. Hashimoto, Int. J. Mod. Phys. A 20 (2005) 5133 [arXiv:hep-ph/0411126]; M. Okamoto, PoS LAT2005 (2006) 013 [arXiv:hep-lat/0510113]; A. Ali Khan et al. [CP-PACS Collaboration], Phys. Rev. D 64 (2001) 054504 [arXiv:hep-lat/0103020]; A. Ali Khan et al. [CP-PACS Collaboration], Phys. Rev. D 64 (2001) 034505 [arXiv:hep-lat/0010009]; C. Bernard et al. [MILC Collaboration], Phys. Rev. D 66 (2002) 094501 [arXiv:hep-lat/0206016]; S. Collins, C. T. H. Davies, U. M. Heller, A. Ali Khan, J. Shigemitsu, J. H. Sloan and C. Morningstar, Phys. Rev. D 60 (1999) 074504 [arXiv:hep-lat/9901001].

11. M. Jamin and B. O. Lange, Phys. Rev. D 65 (2002) 056005 [arXiv:hep-ph/0108133]; J. G. Korner, A. I. Onishchenko, A. A. Petrov and A. A. Pivovarov, Phys. Rev. Lett. 91 (2003) 192002 [arXiv:hep-ph/0306032].

12. M. Beneke, G. Buchalla and I. Dunietz, Phys. Rev. D54, 4419 (1996).

13. M. Beneke, G. Buchalla, C. Greub, A. Lenz and U. Nierste, Phys. Lett. B 459 (1999) 631 [arXiv:hep-ph/9808385].

14. M. Ciuchini, E. Franco, V. Lubicz, F. Mescia and C. Tarantino, JHEP 0308 (2003) 031 [arXiv:hep-ph/0308029].

15. M. Beneke, G. Buchalla, A. Lenz and U. Nierste, Phys. Lett. B 576 (2003) 173 [arXiv:hep-ph/0307344].

16. D. Becirevic, V. Gimenez, G. Martinelli, M. Papinutto and J. Reyes, JHEP 0204 (2002) 025 [arXiv:hep-lat/0110091].

17. U. Nierste, [hep-ph/0406300], in: *Proceedings of the XXXIXth Rencontres de Moriond, Electroweak Interactions and Unified Theories*, 21-28 Mar 2004, La Thuile, Aosta Valley, Italy, Ed. J. Tran Thanh Van.

18. Y. Grossman, Y. Nir and M. P. Worah, Phys. Lett. B 407 (1997) 307 [arXiv:hep-ph/9704287].

19. A. S. Dighe, I. Dunietz, H. J. Lipkin and
20. I. Dunietz, R. Fleischer and U. Nierste, Phys. Rev. D 63 (2001) 114015 [arXiv:hep-ph/0012219].

21. DØ collaboration, conference note 5144, http://www-do.fnal.gov/.

22. DØ collaboration, conference note 5143, http://www-do.fnal.gov/ V. Abazov [DØ Collaboration], arXiv:hep-ex/0609014.
Figure 1. Uncertainty budget for $\Delta \Gamma_s/\Delta M_s$. The largest uncertainties stem from the renormalisation scale $\mu_1$ of the $\Delta B = 1$ operators and the bag parameter $B_{R_2} = 1.0 \pm 0.5$ of one of the $1/m_b$-suppressed operators. $B_{R_0} = 1.0 \pm 0.5$ is the bag parameter of $R_0$ in Eq. (13) and $z = m_c^2/m_b^2$. The transparent segment of the right pie chart shows the improvement with respect to the old result on the left.

Figure 2. Current experimental bounds in the complex $\Delta_s$-plane. The bound from $\Delta M_s$ is the red (dark-grey) annulus around the origin. The bound from $|\Delta \Gamma_s|/\Delta M_s$ corresponds to the yellow (light-grey) region and the bound from $\alpha_s^2$ is given by the light-blue (grey) region. The angle $\phi_s^\Delta$ can be extracted from $|\Delta \Gamma_s|$ (solid lines) with a four-fold ambiguity — each of the four regions is bounded by a solid ray and the x-axis — or from the angular analysis in $B_s \to J/\Psi \phi$ (dashed line). (No mirror solutions from discrete ambiguities are shown for the latter.) The current experimental situation shows a $2\sigma$ deviation from the Standard Model case $\Delta_s = 1$. 