Theory of plastic vortex creep

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We develop a theory for plastic flux creep in a topologically disordered vortex solid phase in type-II superconductors. We propose a detailed description of the plastic vortex creep of the dislocated, amorphous vortex glass in terms of motion of dislocations driven by a transport current $j$. The plastic barriers $U_{pl}(j) \propto j^{-\mu}$ show power-law divergence at small drives with exponents $\mu = 1$ for single dislocation creep and $\mu = 2/5$ for creep of dislocation bundles. The suppression of the creep rate is a hallmark of the transition from the topologically ordered vortex lattice to an amorphous vortex glass, reflecting a jump in $\mu$ from $\mu = 2/11$, characterizing creep in the topologically ordered vortex lattice near the transition, to its plastic values. The lower creep rates explain the observed increase in apparent critical currents in the dislocated vortex glass.

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One of the most fascinating dynamic phenomena of complex systems with internal degrees of freedom is the thermally activated motion of elastic media in a random environment (creep) characterized by a highly nonlinear response to a dc driving force, $F$: $v \propto \exp (-\text{const}/TF^\mu)$, where $v$ is the velocity, $T$ is the temperature, and $\mu$ is the exponent depending on the geometry and the dimensionality of the driven medium. The concept of thermally activated creep, which was originally introduced to describe the driven motion of elastic manifolds through quenched disorder in the context of the dynamics of dislocation and/or domain walls in inhomogeneous environments, was extended later to driven vortex lattices, and has eventually become a paradigm for the thermally activated dynamics of disordered media. The concept of creep proved to be especially successful in describing a wealth of the low temperature transport and relaxation properties of the vortex state of high temperature superconductors. Recent observations on 2D magnets confirmed that domain walls also exhibit creep behavior. Creep dynamics results from the scaling of energy barriers $U(j) \sim j^{-\mu}$ which control thermally activated motion. The derivation of such energy barriers diverging at small drives was based on the elastic behavior of the pinned structures; thus in the common view creep behavior is implicitly attributed to the elastic medium free of topological defects.

The description of thermally activated dynamics of amorphous structures containing a large amount of topological defects remains a major challenge. This is a long outstanding problem in the theory of work hardening and related relaxation processes in dislocated solids. In the context of vortex physics the quest for the description of thermally activated dynamics of a topologically disordered medium was motivated by the observation of the disorder-induced transition between a low-field quasilattice or Bragg glass (BrG), the phase free of topological defects, and a high-field phase characterized by an enhanced apparent critical current. The latter phase was suggested to be a topologically disordered vortex state. It was shown that the low-field BrG phase is indeed stable against dislocation formation. At high magnetic fields the vortex solid undergoes a structural transition which is described as a topological transition between the BrG and a dislocated, amorphous VG (AVG). In a recent series of experiments the phase coexistence characteristic for a first-order transition was established, and creep barriers in the high-field vortex state were shown to diverge faster than creep barriers in the low-field elastic phase.

These recent experimental findings call urgently for a theoretical description of the AVG, and the main theoretical problem is to find an appropriate quantity enabling parameterization of the amorphous phase. A first step towards such a quantitative description was made in Ref. where all phase transitions between vortex lattice phases were described in terms of dislocation-mediated behavior, and a free energy functional $F[n_D]$ for an ensemble of directed dislocations in the presence of thermal fluctuations and quenched disorder was derived ($n_D$ is the areal dislocation density). The BrG-AVG transition was found to be of weak first order in accordance with the experimental results of Ref.: at the BrG-AVG transition dislocations enter with a density $n_D \sim R_a^{-2}$ given by the positional correlation length $R_a$ on which typical vortex displacements are of the order of the lattice spacing $a$.

Upon increasing the magnetic field up to the critical point the dislocation density of the AVG increases to vortex-liquid-like values $n_D \sim a^{-2}$ such that the AVG and vortex liquid phases become thermodynamically indistinguishable at the critical point.

In this Letter, building on the aforementioned ideas, we propose a quantitative description of plastic creep in terms of the dislocation degrees of freedom. We establish and find a critical plastic current $j_{pl}$ below which dislocations are collectively pinned and plastic creep occurs via the activated motion of collectively pinned dislocation lines. The critical plastic current is lower than the critical current for vortex depinning $j_{pl} < j_c$, hence plastic motion of depinned dislocations sets in before viscous flow of the entire vortex lattice can occur. We calculate the associated plastic plastic creep energy barriers $U_{pl}(j) \sim j^{-\mu_{pl}}$.
dissipation infinitely at \( j \to 0 \). We treat dislocations in the pinned vortex lattice as directed elastic strings subject to a pinning force which we derive from the Peach-Köhler force exerted on vortices by the pinning centers. We show that an external current sent through a dislocated vortex lattice generates a Peach-Köhler force with a driving component for dislocation glide. Knowledge of the pinning and driving forces on the elastic dislocation string enables us finally to study the glassy dynamics of the dislocation, in particular the depinning threshold for dislocation glide and the energy barriers for plastic creep below the depinning threshold.

The energy of a single straight dislocation of length \( L \) and with Burger’s vector \( \mathbf{b} \) in the FLL consists of the core energy and of the logarithmically diverging contribution from the long-range elastic strains [19]:

\[
E_0 = LE_D(c_D + \ln(L_1/a)),
\]

where \( L_1 \) is the lateral system size, \( E_D = K\beta^2/4\pi \), \( K = \sqrt{\epsilon_{44}e_{66}} \) is the isotropized elastic constant in the rescaled coordinate \( z = \frac{2}{\pi} \sqrt{\epsilon_{44}/e_{66}} \), and \( \epsilon_D \approx 1 \) is found numerically (\( \epsilon_{44} \) and \( e_{66} \) are the tilt and shear moduli of the vortex lattice, respectively). Bending of the dislocation line costs an elastic energy associated with its stiffness \( \epsilon_D \). Hence, the single directed dislocation line – parameterized by its displacement field \( \mathbf{u}_D(z) \) – is described by the Hamiltonian

\[
H_D[\mathbf{u}_D] = E_0 + \int dz \frac{1}{2} \epsilon_D(\partial_z \mathbf{u}_D)^2
\]

where the stiffness \( \epsilon_D \approx E_D \ln(1/k_2a) \) has a logarithmic dispersion due to the long-range strain field.

Let us now consider the driving force on an edge dislocation with \( b \parallel x \) if a transport current \( j \parallel y \) is sent through the sample. Due to Maxwell’s equation \( \nabla \times \mathbf{B} = \frac{4\pi}{c} \mathbf{j} \) the driving current creates a magnetization gradient which, in turn, gives rise to shear strains in the vortex lattice:

\[
\partial_z u_y = \partial_x a = a \frac{\pi}{2k} \mathbf{j}.
\]

Resulting shear stresses give rise to a glide-component of the driving Peach-Köhler force [19]

\[
F_x^{\text{drive}} = \sigma_{yx}b = bK a \frac{2\pi}{c} \frac{j}{B}
\]

per length (compression stress can only give rise to dislocation climb which can be neglected as slow process requiring diffusion of interstitials [20]).

The displacements induced by the magnetization gradient can only be accommodated by the creation of a stationary superstructure of regularly spaced dislocation bands with Burger’s vectors with a \( y \)-component [21] in which our single “test”-dislocation is moving. The superstructure is similar to the grain boundaries formed in an bent atomic crystal [19]. Because such dislocation bands are essentially free of shear stresses [19] our test-dislocation experiences the purely elastic driving force [2]

everywhere in between the bands such that the presence of the superstructure does not affect the glide motion.

In the presence of a random pinning potential \( V_{\text{pin}}(r) \) in the vortex array also the dislocation experiences Peach-Köhler-type pinning forces. To calculate these we have to determine the random stress exerted by the pinning potential in a frozen-in elastic displacement configuration \( u_{\text{el}}(R, z) \) of the vortex lattice: \( V_{\text{pin}}(R + u_{\text{el}}, z) = \sigma_{ij}^{\text{pin}}(R + u_{\text{el}}, z)\nabla_i u_{\text{el}, j} \). The distribution of the pinning stresses is given by the quenched distribution of the elastic displacements \( u_{\text{el}} \). This distribution is determined by the different spatial scaling regimes of the dislocation-free collectively pinned vortex array: (i) Small distances where vortex displacements \( u \) are smaller than the coherence length \( \xi \) and perturbation theory applies [22].

(ii) Intermediate scales where \( \xi < u < \xi \) and disorder potentials seen by different FLs are effectively uncorrelated. This regime is captured in so-called random manifold (RM) models [10], leading to a roughness \( \tilde{G}(r) \Rightarrow (u_{\text{el}}(r) - u_{\text{el}}, 0)^2) \approx a^2(r/R_\text{a})^{2\zeta_{\text{RM}}} \) where \( \zeta_{\text{RM}} \approx 1/5 \) for the \( d = 3 \) dimensional RM with two displacement components. The crossover scale to the asymptotic behaviour is the positional correlation length \( R_a \) where the average displacement is of the order of the FL spacing: \( u \approx a \). (iii) The asymptotic Bragg glass regime where the \( a \)-periodicity of the FL array becomes important for the coupling to the disorder and the array is effectively subject to a periodic pinning potential with period \( a \) [10]. Here the logarithmic roughness \( \tilde{G}(r) \approx (a/\pi)^2 \ln (er/R_a) \), i.e., \( \zeta_{\text{BrG}} = O(\log) \) [10] takes over.

For the physics of dislocations on scales \( \gg a \) only the RM and BrG regimes are relevant. We obtain approximately Gaussian distributed quenched stresses with \( \sigma_{ij}^{\text{pin}} = 0 \) and \( \sigma_{ij}^{\text{pin}}(k)\sigma_{ij}^{\text{pin}}(k') = \Sigma^{\text{pin}}(k)(2\pi)^3\delta(k + k') \) with \( \Sigma^{\text{pin}}(k) = K^2k^2G(k) \), i.e.,

\[
\Sigma^{\text{pin}}(k) = K^22\pi a^2k^{-1}
\]

\[
\begin{cases}
\text{BrG:} & 1 \\
\text{RM:} & B_{\text{RM}}(kR_a)^{-2\zeta_{\text{RM}}}
\end{cases}
\]

determined by the elastic correlations \( G(k) \) with a numerical constant \( B_{\text{RM}} \). The RM-result holds for \( kR_a > 1 \) the BrG-result for \( kR_a < 1 \).

To derive the correct Peach-Köhler pinning force on the dislocation it is crucial not only to consider the “di-
rect” quenched pinning stresses $\sigma_i(r)$ but also the elastic stresses $\sigma_i^0$ themselves which are responding to the same pinning potential and hence tend to relax (longitudinal) components of the stress. A lengthy calculation shows that if both components are properly added the pinning Peacock-Köhler force on a dislocation element $dR$ is rotation-free ($\nabla \times F_{\text{pin}}^\text{rot} = 0$) such that its potential $\mathcal{H}_{\text{pin}}[\mathbf{u}_D]$ can be defined:

$$\mathcal{H}_{\text{pin}}[\mathbf{u}_D] = \int dz \frac{d\mathbf{u}_D}{dz} \delta_{kk} \epsilon_{\mathbf{k}} \delta_{\mathbf{r}}$$

The free energy $F_{\text{pin}}[\mathbf{u}_D] = \mathcal{H}_{\text{pin}}[\mathbf{u}_D] - \int dz F_{\text{drive}} \cdot \mathbf{u}_D$ can be calculated from Ref. 8.

$$E_{\text{pin}}^2(L, u_D) \approx \frac{\rho^2}{a^2} R^2 \ln \frac{L}{R}$$

whereas the corresponding elastic bending energy of the dislocation is $E_{\text{el}}(L, u_D) \approx \frac{\rho^2}{a^2} R^2 \ln \frac{L}{R}$. Optimization gives a dislocation roughness

$$u_D(L) \approx L \left\{ \begin{array}{ll} \text{RM}: & \frac{L}{R} = \frac{1}{2} \zeta_{\text{RM}} \left( 3 - 2 \zeta_{\text{RM}} \right) \mathcal{O}(\log(L/R)) \\ \text{BrG}: & \frac{L}{R} = \frac{1}{4} \zeta_{\text{RM}} \end{array} \right.$$  

i.e., exponents $\zeta_D \approx \frac{1}{2}$ for RM scaling ($L < R$) and $\zeta_D \approx 1 - \log^2/3$ for BrG scaling. An instability with respect to dislocation proliferation is signaled by anomalous energy gains if $\zeta_D > 1$, i.e., in the RM-regime. In the BrG-regime the energy balance is more subtle and to conclude $\zeta_D < 1$ or stability one can convert the result into an approximate renormalization (RG) scheme: the energy gain due to roughening is $\Delta E \sim E_0 L \ln^{-1/3}(L/a) \sim E_0 L \zeta_D L^{-1/3}$, where the logarithmic correction is identical to the dimensionless line tension $\epsilon_D = \epsilon_D/L$ on the scale $L$. Interpreting $\Delta E/E_0 L$ as disorder correction to the line tension $\epsilon_D(L)$ on the scale $L$ and summing these corrections successively on each scale, together with the bare tension $\epsilon_D^0(L) = \ln (L/a)$, one obtains an integral RG equation

$$\epsilon_D(L) = \int_0^{\ln L} dt (1 \pm \epsilon_D(t)^{-1/3})$$

equivalent to the result of Ref. 12. Integration shows that corrections to $\epsilon_D^0(L)$ are irrelevant and hence the stability of the BrG regime with respect to dislocation formation is the analogue of the Larkin pinning length $L_c \approx \xi \delta^{-1/3}$ of the single vortex where $\delta$ is the dimensionless pinning strength of Ref. 3. To depin the dislocation the driving force has to exceed the pinning force $F_{\text{pin}}(L) \approx \frac{\epsilon_D}{\delta} L \left( \frac{a}{R} \right)^{2 \zeta_{\text{RM}}/3}$ on a segment of length $L_{\text{pl}}$. This determines a critical plastic current $j_{\text{pl}}$

$$j_{\text{pl}} \approx \frac{e}{8 \pi^2} \left( \frac{a}{R} \right)^{2 \zeta_{\text{RM}}/3} \ln \left( \frac{2 \pi \hbar^2}{B} \right)^{-7/4} \delta^{1/8}$$

where $j_0 \approx \frac{e a}{\hbar} \delta_{\text{drive}} \zeta_{\text{pin}}$ is the depinning current. Comparing this result to the depinning current $j_c \approx j_0 \delta_{\text{drive}}^{1/3}$ for a single vortex, one finds $j_{\text{pl}} \ll j_c$ for typical dislocation strengths $\delta \approx 10^{-3}$. This shows that plastic motion of dislocations will set in even if the vortex lines are still pinned. Thus plastic motion is the dominating transport mechanism in the dislocated AVG phase.

However, the existence of a current $j_{\text{pl}} > 0$ also demonstrates that dislocations can be pinned at low currents and plastic motion for $j < j_{\text{pl}}$ will only occur through activation over diverging plastic energy barriers $U_{\text{pl}}(j) \sim j^{-\mu_{\text{pl}}}$ giving rise to plastic creep. The typical segment size $L(j)$ for activated motion at $j < j_{\text{pl}}$ is determined by balancing the energy gain by the driving force $F_{\text{drive}} L \epsilon_D$ against the pinning energy $E_{\text{pin}}(L, u_D(L)) \approx E_0 L^{2 \zeta_{\text{RM}} - 1}$ of the rough dislocation line. This yields

$$U_{\text{pl}}(j) \approx E_0 a \left( \frac{a}{R} \right)^{2 \zeta_{\text{RM}}/3} \left( \frac{a}{j} \right)^{(2 \zeta_{\text{RM}} - 1)/(2 - \zeta_{\text{pl}})}$$

and we obtain the scaling law $\mu_{\text{pl}} = (2 \zeta_{\text{RM}} - 1)/(2 - \zeta_{\text{pl}})$ as for single vortex creep \(\text{(Ref. 12)}\) relating the plastic creep exponent $\mu_{\text{pl}}$ to the dislocation roughness. We find $\mu_{\text{pl}} = 17/12$ in the RG regime ($L(j) < R$) and $\mu_{\text{pl}} = 1$ in the BrG regime ($L(j) > R$). Both exponents are considerably larger than their counterpart $\mu = 2/11$ for elastic single vortex creep showing that plastic creep rates are much smaller than elastic creep rates.

So far we focused on a single dislocation. Now we turn to interacting dislocations. On large scales exceeding the dislocation spacing $R_D$, which varies from $R_D \approx R_a$ at the AVG-BrG transition to $R_D \approx a$ at the critical
point interactions become essential and plastic creep is governed by motion of dislocation bundles in a glide plane (xz-plane). Deformations \( u_D(x,z) \) of such a 2D bundle can be described by an elastic Hamiltonian with tilt modulus \( K_z \simeq E_D/R_D \) and the compression modulus \( K_x \simeq R_D^2/R_D f(R_D) \) which can be calculated from the dislocation free energy \( f(R_D) \), see Ref. [14] (in the absence of disorder one finds \( K_z \simeq E_D/R_D \)). Including the pinning energies we obtain the Hamiltonian

\[
\mathcal{H}[u_D(x,z)] = \int dx dz \left\{ \frac{1}{2} \left[ K_x (\partial_x u_D)^2 + K_z (\partial_z u_D)^2 \right] + \sum_i \int dz \mathcal{H}_{D,i}^\text{pin} \right\}
\]

(12)

The bundle description contains dislocations of opposite signs with the same density to avoid the accumulation of stress. Hence the sum over the dislocation index \( i \) in [12] goes over alternating Burger’s vectors \( \mathbf{b}_i \parallel \{x\} \). On scales \( L_x \gg R_D \) dislocations couple effectively as dipoles to disorder and we obtain for the bundle disorder energy fluctuations \( E_{D,i}^\text{pin}(L_x, u_D) \simeq E_{\text{pin}}(L_z, u_D) \) with \( E_{\text{pin}}(L_z, u_D) \) from (11). This has to be balanced against the elastic energy \( E_{\text{el}}(L_x, L_z, u_D) \simeq \sqrt{K_z K_x} u_D^2 \) with \( L_z \sim \sqrt{K_z/K_x} L_x \) resulting in a roughness

\[
u_D(L_z) \sim L_z^{1/3} R_D^{2/3} \left\{ \begin{array}{l}
\text{RM:} \quad \left( \frac{L^1/3 R^{2/3}}{R_a} \right)^{2/3} \\
\text{BrG:} \quad 1
\end{array} \right. \]

(13)

i.e., a reduced dislocation bundle roughness \( \zeta_D \approx \frac{5}{6} \) for RM scaling \( (L < R_a) \) and \( \zeta_D \approx \frac{3}{2} \) for BrG scaling. It is easy to establish the corresponding plastic creep exponents for bundle creep along the same lines as for the single dislocation, in particular one finds the same scaling relation \( \mu_{pl} = (2 \zeta_D + d - 2)/(2 - \zeta_D) \) as for d-dimensional vortex bundles [3]. This gives \( \mu_{pl} = \frac{1}{2} \) in the RM-regime and \( \mu_{pl} = \frac{3}{2} \) in the BrG-regime. The crossover from single dislocation to bundle scaling happens for currents \( j < j^b \) where \( L_j^b \simeq \sqrt{K_z/K_x} R_D \). For \( R_D \simeq a \) one finds \( j^b \approx j_{pl} \), which means that only plastic bundle creep can be measured above the critical point defined by \( R_D \simeq a \). Though we do think that our results apply rather to the YBCO-compound than the strongly layered BSCCO-compound, in Ref. [8] a creep exponent strikingly close to \( \mu_{pl} = \frac{1}{2} \) has been measured within the AVG phase.

In the high-field regime above the critical point there is no melting phase transition but a crossover in the resistive behavior due to thermal depinning of dislocations will still be observable. The single pinned dislocation line thermally depins if \( u_D^2(R_D) \simeq a^2 \), i.e., when the thermal fluctuations of a segment of the size of the plastic pinning length increase beyond a lattice spacing which leads to considerable smoothing of the pinning potential. This happens at a plastic depinning temperature \( T_{pl} \simeq E_D a^2/R_a^{2/3} \). Using results of Ref. [14], we find \( T_{pl} \simeq T_m \simeq 1.6 E_D a \) at the critical point. This suggests that at the critical point the first-order melting line terminates and transforms into a thermal depinning line \( T_{pl}(B) \sim B^{-3/8} \) describing the dynamic crossover to reduced depinning currents and thermal activation over all plastic barriers \( U_{pl}(j) < T \).

In conclusion, we have developed a theory of plastic creep in terms of the actual dislocation dynamics in the pinned vortex lattice. We find infinite barriers for plastic vortex transport in agreement with experimentally observed low creep rates or apparently high critical currents. The obtained results are relevant for and are easily extended to other systems where glassy dynamics is controlled by topological defects, for example, charge density waves in disordered crystals and/or work-hardened solids.

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[1] L. B. Ioffe and V. M. Vinokur, J. Phys. C 20, 6149 (1987)
[2] M.V. Feigelman et al., Phys. Rev. Lett. 63, 2303 (1989).
[3] M.P.A. Fisher, Phys. Rev. Lett. 62, 1415 (1989).
[4] T. Nattermann, Phys. Rev. Lett. 64, 2454 (1990).
[5] G. Blatter, M.V. Feigelman, V.B. Geshkenbein, A.I. Larkin, and V.M. Vinokur, Rev. Mod. Phys. 66, 1125 (1994).
[6] S. Lemerle et al., Phys. Rev. Lett. 80, 849 (1998).
[7] V.N. Kopylov et al., Physica C 162-164, 1143 (1989), ibid. 170, 291 (1990); M. Daeumling et al., Nature (London) 346, 332 (1990); N. Chikumoto et al., Physica C 185-189, 2201 (1991), Phys. Rev. Lett. 69, 1260 (1992); B. Khaykovich et al., Phys. Rev. Lett. 76, 2555 (1996).
[8] M. Konczykowski et al., preprint cond-mat/9912228.
[9] C.J. van der Beek et al., Phys. Rev. Lett. 84, 4196 (2000).
[10] T. Giamarchi and P. Le Doussal, Phys. Rev. B 52, 1242 (1995).
[11] J. Kierfeld, T. Nattermann, and T. Hwa, Phys. Rev. B 55, 626 (1997).
[12] D.S. Fisher, Phys. Rev. Lett. 78, 1964 (1997).
[13] D. Ertaş and D.R. Nelson, Physica C 272, 79 (1996); T. Giamarchi and P. Le Doussal, Phys. Rev. B 55 6577 (1997); A.E. Koshelev and V. Vinokur, ibid. 57, 8026 (1998).
[14] V. Vinokur et al., Physica C 295, 209 (1998).
[15] J. Kierfeld, Physica C 300, 171 (1998).
[16] J. Kierfeld and V. Vinokur, Phys. Rev. B 61, R14928 (2000).
[17] D. Lopez et al., Phys. Rev. Lett. 80, 1070 (1998); C. Marcenat et al., unpublished.
[18] D.R. Nelson, Phys. Rev. Lett. 60, 1973 (1988).
[19] J.P. Hirth and J. Lothe, Theory of Dislocations (McGraw-Hill, New York, 1968).
[20] M.C. Marchetti and D.R. Nelson , Phys. Rev. B 41, 1910 (1990).
[21] D.W. Braun et al., Phys. Rev. Lett. 76, 831 (1996).
[22] A.I. Larkin, Sov. Phys. JETP 31, 784 (1970).