Vortex structure and resistive transitions in high-Tc superconductors

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The nature of the resistive transition for a current applied parallel to the magnetic field in high-Tc materials is investigated by numerical simulation on the three dimensional Josephson junction array model. It is shown by using finite size scaling that for samples with disorder the critical temperature $T_p$ for the $c$ axis resistivity corresponds to a percolation phase transition of vortex lines perpendicularly to the applied field. The value of $T_p$ is higher than the critical temperature for $j \perp H$, but decreases with the thickness of the sample and with anisotropy. We predict that critical behavior around $T_p$ should reflect in experimentally accessible quantities, as the $I-V$ curves.

I. INTRODUCTION

Dissipation in high-Tc superconductors caused by motion of vortex lines has become one of the richest fields in the phenomenology of these new materials. From the point of view of applications this dissipation is one of the principal limiting factors when trying to achieve high critical currents. From a purely theoretical point of view, due to the high critical temperatures and to the layered structure, the physics of the vortex structure becomes qualitatively different to that in the low-Tc materials. The magnetic field-temperature ($H-T$) phase diagram is not well described by a mean field approximation. An estimation of the fluctuation-dominated region around the mean field phase transitions in the $H-T$ diagram shows that this is quite accessible experimentally.

One of the new phases of the system that has been observed in the High Tc’s is the so-called liquid phase. This phase is characterized by the fact that the vortex lines are easily movable by applying an external force on them by putting a current $J$, i.e., the resistance of the system, defined as $R = \lim_{t \rightarrow 0} V/I$ is different from zero. This liquid phase is found for high enough temperatures, whereas for $T$ lower than a well defined value $T_i$, the resistance becomes zero. $T_i$ is the superconducting transition temperature in the presence of the magnetic field. The nature of the transition at $T_i$ depends on the disorder present on the system. When the amount of disorder is very small, the low temperature phase is an Abrikosov solid phase and the transition at $T_i$ is a first order melting transition, with the resistivity having a jump at $T_i$. For higher disorder a vortex glass phase has been proposed for $T < T_i$.

In this case the transition at $T_i$ is second order and the resistivity displays critical scaling for $T$ close to $T_i$. In some cases a crossover due to thermal depinning of vortices occurs.

All these features correspond to the case when the magnetic field (which we always supposed applied along the $c$ axis of the material) and the external current are perpendicular to each other. When the current is parallel to the magnetic field a naive argument would say that the dissipation is zero (up to the zero field transition temperature $T_c$) because the Lorentz force on the vortices is zero. However, it has been found experimentally that the transition temperature in this case -which we will call $T_{pr}$- is lower than $T_c$. This behavior is related to the thermal fluctuations of the vortex lines.

The point of wether $T_p$ coincides or is higher than $T_i$ has been highly controversial. It has been found experimentally that for twinned YBaCuO samples $T_p$ turns out to be higher than $T_i$. In other words, the superconducting coherence is lost in two steps when raising the temperature: along the $ab$ plane at $T_i$ and along the $c$ axis at $T_p$. In addition, $T_p$ seems to approach $T_c$ when increasing the thickness of the sample. Theoretically, there have been much discussion about the possibility of the loose of coherence in two successive steps, although the results are not conclusive. Numerical simulations give support to the picture of one or two transitions depending on the model and the parameters used to describe the problem.

It has been realized that a non-zero dissipation for an arbitrary small applied current parallel to the field - i.e., a non-zero resistivity- would require the existence of arbitrary large loops parallel to the $ab$ planes. Arbitrary large isolated loops have infinite energy, and thus the thermal energy which is necessary to create them is infinite. However, the superposition of a large amount of small loops could generate an arbitrarily large one at finite temperatures. The question arises of wether the generation of these large loops is a thermal crossover or a real phase transition occurring at temperature $T_{pr}$.

In this paper we will try to clarify these points, by performing numerical simulations on the three dimensional (3D) Josephson junction array (JJA) model, which has been used previously to study thermodynamical as well as transport properties of high Tc superconductors. As the whole problem is too broad, we will concentrate here on the properties of samples with disorder and not too high anisotropies, leaving the study of the dependence on the amount of disorder for another work. It will be shown than in this case -in coincidence
with experiments- the 3D JJA model for finite values of the thickness has two different transition temperatures $T_i$ and $T_p$ for the onset of the resistivity in the $ab$ plane and along the $c$ axis, respectively. The transition at $T_i$ is -in the case of our model and for the values of parameters used- a thermal depinning of vortex lines, but in a real materials corresponds to a second order phase transition -the vortex glass transition, which has been extensively studied before. The temperature $T_p$ corresponds to the onset of the resistivity along the $c$ axis. We will show \( T_p \) that $T_p$ is the threshold value of a percolation phase transition perpendicularly to the applied field. Moreover, $T_p$ is a well defined thermodynamical temperature for any value of the thickness of the sample, as long as the $ab$ plane can be considered as infinite. The precise value of $T_p$ decreases when increasing the thickness, suggesting that the two transitions collapse onto a single one for a bulk 3D sample.

In order to show these properties of the transition at $T_p$, we perform finite size scaling on the three 3D JJA model. We find that the system exhibits critical behavior around $T_p$, which reflects, in particular, in a critical scaling of the $I - V$ curves. The values of the critical exponents are different to those find for the vortex glass transition at $T_i$ and agree with those found for the $c$ axis $V - I$ curves of real materials.

The remainder of the paper is organized as follows. In the next section we describe the 3D JJA model that we use for a detailed investigation of the finite size scaling of the transition at $T_p$. In section III a simplified model for the percolation transition is introduced. This model allows to derive quantitative expressions for the finite size scaling that can be checked against the numerical simulations, and we do that in section IV, showing that the numerical data scale as expected for a percolation transition. In section V we focus on the consequences of a percolation transition on the experiments, and indicate that these consequences are indeed observed. Finally, in section VI we summarize and conclude.

II. DESCRIPTION OF THE MODEL

For the numerical simulations, we have worked on the isotropic three dimensional-resistively shunted-Josephson junction array model on a cubic lattice. This model has been extensively used to derive sensible results for thermodynamical as well as transport properties of the vortex structure in high-Tc superconductors. When looking at equilibrium properties it reduces to the 3D uniformly frustrated XY model.

The 3D-JJA is conformed by a set of Josephson junctions connecting nearest neighbor nodes of -in our case- a cubic lattice. Each junction consists of an ideal Josephson junction of critical current $I_c$, shunted by a normal resistance $R$, and a Johnson noise generator, which accounts for the temperature. Thus, the current $i$ through each junction is given by:

$$ i = I_c \sin \Delta \phi - \frac{1}{R} \frac{\partial \Delta \phi}{\partial t} + \eta(t) $$

(1)

where $\Delta \phi$ is the difference between the superconducting phases on the two nodes connected by the link. A vortex in a given plaquette of the model is characterized by a value of $2\pi$ when following the variation of the superconducting phase around this plaquette. An external current applied between two opposite sides of the sample produces a Lorentz force on the vortices of the system, and may give rise to dissipation, which is characterized by a finite value of the potential difference $\Delta V$ between the two contacts. This potential difference is calculated in terms of the phases of the system by using the Josephson equation

$$ \Delta V \sim \left\langle \frac{\partial (\varphi_1 - \varphi_2)}{\partial t} \right\rangle $$

(2)

$\varphi_1$ and $\varphi_2$ being the phases of the contacts, and $\langle \ldots \rangle$ indicating a time average.

The system of equations (1) are complemented by the condition of conservation of currents on each node. The equations are numerically integrated in time and different magnitudes are calculated after thermalization at each temperature.

The boundary conditions (BC) are taken open in the $ab$ plane. However, if open BC in the $c$ axis are used, there will be a finite force on an isolated vortex at finite temperature if the top and bottom ends of the vortex are not aligned. The dissipation -that is non-zero even in the linear regime- caused by this net force turns out to be independent of the thickness of the sample, and in this sense, it is only a surface effect. In order to eliminate this spurious surface effect it is crucial to use BC for the $c$ direction that assure that each vortex line leaving the sample at a given point of the bottom plane re-enters at the same point of the top plane. Strict periodic BC on the phases $\varphi$ have this property, however we would obtain that the voltage difference between top and bottom planes is identically zero.

We use boundary conditions in the $c$ direction that reduce the surfaces effects drastically -although does not eliminate them completely. We proceed as follows: we numerically integrate the equations of motion for an open system one time step, and find the value of the phases in the bottom and top plane $\varphi_1^B$ and $\varphi_1^T$. Then the phases in this two planes are modified to $\tilde{\varphi}_1^B = (\varphi_1^B + \varphi_1^T) / 2 + \bar{\varphi} / 2$ and $\tilde{\varphi}_1^T = (\varphi_1^B + \varphi_1^T) / 2 - \bar{\varphi} / 2$, with $\bar{\varphi} = \varphi_1^B - \varphi_1^T$ being the mean difference between the phases of top and bottom planes. At this stage the equations are integrated another time step. This process guarantees that the vortex configuration is periodic along the $c$ axis, allowing at the same time to calculate the voltage difference when we apply a current. The relation between $c$ axis resistivity and percolation described in Section IV would not be observable if open boundary conditions were used.
Flux conservation implies that every flux line going into a unit cell of our lattice also goes out of the cell. When two vortices go into the same elemental cell we cannot tell which one of the two outgoing vortices correspond to each one of the ongoing vortices. We interpret this situation as the meeting of two vortex lines. In a real material this corresponds to two vortex lines being at a distance lower than the core size of the vortex. At high enough temperatures the vortex structure is heavily interconnected, and we can follow a vortex line starting from one side of the sample and arriving to the opposite side. In this case we will say that the vortex structure has percolated perpendicularly to the applied field. Due to the finite size of the systems used, and to the dynamical evolution, percolation is not expected to occur at every time, but only at a given fraction of the total time, which depends on temperature. We evaluate the probability $P$ that there exists a vortex line crossing the system from one side to the opposite as a function of the temperature. This probability $P$ will be one of the most important variables in our analysis.

The ideal Abrikosov (triangular) vortex lattice is frustrated on the subyacent square lattice. The thermodynamical properties of the uniformly frustrated XY model on a square lattice have strong and non-monotonic variations as a function of the field, due to commensurability effects. This is a spurious effect for us, because we are interested in the simulation of systems as close to the real samples as possible. In this work we concentrate on the case of disordered samples, in which an Abrikosov -or any ordered- lattice does not exist at all, even at low temperatures. In real samples this may be due to the existence of twinned boundaries or point defects, and the physics of the low temperature phase may be that of a vortex glass or simply a frozen (disordered) set of flux lines.

In the numerical calculations we simulate the disorder by considering the 3D JJA system with a random (non-correlated) distribution of critical currents of the junctions. The energy of a vortex in a single loop with one Josephson junction of critical current $I_c$ is proportional to $I_c$, so when taking a random distribution of critical currents the links with lowest $I_c$ act as pinning centers of vortices. If the disorder -i.e, the dispersion of $I_c$- is high enough, then the configuration of the ground state will be disordered, and not related to the commensurability of the vortex lattice parameter and the subyacent square lattice. In Fig. 1 we show a top view of the vortex lattice at zero temperature, as obtained when slowly cooling down the system in the presence of an external field of value 0.2 in units of flux quanta per plaquette. Two different results after the annealing process for the same distribution of disorder are shown. They correspond to a system of $16 \times 16 \times 8$ sites, with a squared random distribution of critical currents ranged between 0.2 and 1.8, which is the same distribution that we use throughout the paper. As can be seen, the vortex structure is not the corresponding to an ordered system, and more-over the vortex lines are not necessarily straight. Using this disordered JJA system we have checked we obtain a quite smooth behavior of the quantities we calculate (for example the resistivities) as a function of the field. It must be keep in mind that the situation for samples without defects, especially in the case of a triangular lattice of Josephson junctions, is different to the one described here, and will be addressed in another work.

### III. PERCOLATION TRANSITION AND FINITE SIZE SCALING.

The transition at $T_i$ of our model corresponds to a depinning of vortex lines from their equilibrium positions due to thermal activation. This reflects in the form of the resistivity vs temperature curves (current along the $ab$ plane) which show a typical thermally activated behavior. In addition, the process is well described by a single vortex model, indicating that collective effects are of minor importance.

On the other hand, the transition at $T_p$ is a collective effect. As we said before, for $T > T_p$ there exist some paths crossing the sample perpendicularly to the applied field, that are free to move when a current parallel to the field is applied. We will show now that these paths appear at $T_p$ due to a percolation phase transition of vortex lines.

We start by introducing a simple model which will allow us to check the proposed percolation phase transition by using standard finite size scaling. Let us consider the bond percolation problem on a cubic structure of size $L_{ab} \times L_{ab} \times L_c$, but with one important modification: the vertical bonds are supposed to be present with probability one, whereas the horizontal bonds are present with probability $p$. This means that in all cases (for all values of $p$) a rigid squared lattice of lines piercing the sample along the $c$ axis (representing the vortex lines at $T = 0$) is present. We will consider the value of $p$ as a function of temperature (see below), and look for the probability $P$ that there exists a path connecting two opposite sides of the sample perpendicularly to the rigid lines. This model has a percolation phase transition that can be easily characterized in terms of the percolation transition of the corresponding two dimensional bond percolation problem on the square lattice. For example, if we look at the percolation probability $P_{2D}$ on a two–dimensional sample of size $L_{ab} \times L_{ab}$, it satisfies (in the limit of large $L_{ab}$) a scaling relation of the form:

$$P_{2D}(p_{2D}) = f \left( (p_{2D} - p_c) L_{ab}^{1/\nu} \right)$$

where $f$ is a universal function. The values of $p_c$ and $\nu$ are known to be $p_c = 0.599$ and $\nu = 1.33$. The exponent $\nu$ is the one controlling the divergency of a correlation length $\xi$, i.e., $\xi \sim (p_{2D} - p_c)^{-\nu}$. This correlation length measures the typical size of a cluster in the system.
The value of $P$ for our model can be read out from the solution for the percolation probability of the two dimensional case, simply noting that two neighbor rigid lines are connected if there is at least one horizontal segment between them. This corresponds to replace $p_{2D}$ in Eq. (3) by $1 - (1 - p)^{L_c}$. So, we obtain

$$P(p) = f\left(\tilde{x} L_{ab}^{1/\nu}\right)$$

with $\tilde{x} = 1 - p_c - (1 - p)^{L_c}$. Similar scaling expressions for other quantities are readily derived.

We will tentatively apply these scaling expressions to the vortex structure. Before showing the results, it is important to point out the differences between the two situations: the percolation of a real vortex structure is a complicated interacting percolation problem, which proceeds via thermal fluctuations of vortex lines and thermally generated loops of different sizes and energy scales. Moreover, only paths which conserve the direction of the magnetic flux has to be considered. In the simplify model, instead, we use a single probability $p$, which is related to the temperature by a Boltzmann factor, i.e., $p \sim \exp(-\Delta/kT)/(1 + \exp(-\Delta/kT))$, with $\Delta$ being an energy scale that is roughly given by the energy of a vortex loop connecting two nearest vortices in the real system. In terms of the temperature, the scaling variable $\tilde{x}$ reads

$$\tilde{x} = 1 - p_c - (1 + \exp(-\Delta/kB T))^{-L_c}.$$

IV. NUMERICAL FINITE SIZE SCALING

We first show in Fig. 2 the results for the percolation probability $P$ obtained for the JJ model as a function of temperature -in units of the mean Josephson energy of each junction- for an external magnetic field $H = 0.2$ flux quanta per plaquette, which is the value that we used in all simulations. The results for different values of $L_{ab}$ and $L_c$, as well as the best fitting to equations (4) and (5) are shown. The free parameters are $p_c$, $\Delta$, and $\nu$. As can be seen in Fig. 2(c), the scaling is quite good in spite of the approximations involved, giving confidence on the correctness of our ideas. The values obtained for the parameters are $\nu = 1.1 \pm 0.1$, $p_c = 0.5 \pm 0.1$ and $\Delta = 3.60 \pm 0.05$. Although these parameters were considered as free variables, the relation of $p_c$ and $\nu$ with the values corresponding to the 2D model ($p_c = 0.599$, $\nu = 1.33$) is noteworthy.

The size effects due to the finite values of $L_{ab}$ and $L_c$ have very different effects that can be read out from the plots in Fig. 2(a) and (b): when $L_{ab}$ increases for a given value of $L_c$, the percolation transition as a function of temperature becomes steeper, indicating that we have a well defined percolation transition in the limit $L_{ab} \rightarrow \infty$. This is the usual finite size effect of a two-dimensional second order phase transition. Note that this occurs for a fixed (finite) value of the thicknesses $L_c$, and that the transition temperature $T_p$ -defined as the temperature at which there is a jump in the percolation probability for $L_{ab} \rightarrow \infty$- is well defined. The threshold value $T_p$ depends on the thickness $L_c$, as can be seen in Fig. 2(b) $T_p$ decreases as $L_c$ increases. For $L_c \gg 1$ the form of this dependence can be inferred by putting $\tilde{x} = 0$ in Eq. (6):

$$k_B T_p \sim \Delta / \ln(L_c).$$

This indicates a weak decrease of the transition temperature for thicker samples. This is easy to understand: the thicker the sample, the probability of having a vortex loop connecting two nearest vortices increases due to the fact that there are more places for creating the loop, the mathematical expression for this fact being contained in equation (6).

It should be keep in mind that the dependence of the transition temperature on thickness is not the usual dependence of a pseudo-critical temperature on the system size, because here we do have a sharp percolation transition at any $L_c$ (as long as $L_{ab} \rightarrow \infty$), and in fact the transition is of a two dimensional character.

From the experimental point of view, it would be interesting to have a relation between the percolation transition and some directly accessible quantity. This quantity turns out to be the $c$ axis resistivity of the sample. The relation between percolation and $c$ axis resistivity is more clearly seen in the model in which the cosine interactions of the Josephson junctions are replaced by Villain interactions. In this case, the model can be exactly mapped onto a problem in which -after integrating out a gaussian part, related to spin waves- the only degrees of freedom that remain correspond to the positions of the vortex lines. Moreover, the voltage between two points $A$ and $B$ of the sample is due to the vortex movement and can be calculated in the following way: each time a vortex line crosses a path joining $A$ and $B$, the phase difference $\varphi_{AB}$ between points $A$ and $B$ changes in $\pm 2\pi$, the sign depending on the sign of $\vec{v} \times \vec{H}$, where $\vec{H}$ represents the direction of the magnetic field within the vortex, and $\vec{v}$ is its velocity. The mean value between $A$ and $B$ is given by $\varphi_{AB}/t$, where $t$ is the time of measurement. (Nota: The argument is asintotically correct only when $t \rightarrow \infty$, in other case the result may depend on the chosen path joining $A$ and $B$).

The relation between percolation and $c$ axis dissipation can now be understood in the following way: if there are mobile vortex lines crossing the sample perpendicularly to the $c$ axis, any current along this axis dissipates, i.e., percolation of mobile vortex lines is a sufficient condition for dissipation. If there are no stationary vortex lines percolating perpendicularly to the $c$ axis, still vortex loops can be thermally generated in the sample. These loops can cut and reconnect and also blow out and leave the sample. The movement of a vortex loop will generate a net voltage only in the case in which the loop is blown
out of the sample, in any other case only fluctuations in the voltage are possible. If a vortex loop grow to infinity, it implies that it generated during some time interval a percolation path across the sample, so percolation is a necessary condition for dissipation. Note that the argument is not true in the case of using open boundary condition along the $c$ direction, because in this case a single vortex can give rise to dissipation performing only slight displacements from its equilibrium position.

The relation between percolation and $c$ axis resistivity was shown before in Ref. [17]. It was argued that the behavior of $\rho_c$ vs $T$ near the threshold is similar to that of $S/(L^{2}_{ab}L^{2}_{c})$, where $S$ is the volume of the percolation cluster (The percolation cluster $S$ is defined as the number of elemental segments of vortex lines which are linked to (at least) one percolation path). The argument that led to this conclusion was a counting of the vortex lines that are involved in the dissipation process, as well as the assumption of the validity of a viscous motion description of the vortex lines. Here we put it in the language of scaling near the transition temperature. In the same way as we derived expression (4), we can write a scaled relation for the $c$ axis resistivity which reads

$$L^{\gamma_1}_{ab}L^{\gamma_2}_{c}\rho_c(T) = g\left(\hat{x}_{ab} L^{1/\nu_{c}}\right)$$

with $g$ a new universal function, and $\gamma_1$ and $\gamma_2$ being new critical exponents (the value of $\nu$ in this equation should be the same as in Eq. (4), if there is only one divergent length scale $\xi \sim (T - T_p)^{-\nu}$ close to $T_p$). The $c$ axis resistivity for different values of $L_{ab}$ as well as the scaling according to Eq. (4) are shown in Fig. 2a for a system with $L_z = 8$ (the current used to calculate the resistivity is always 1/100 of the critical current at zero temperature in the corresponding direction). The values for the exponents are $\nu_2 = 0.40 \pm 0.05$ and $\nu_2 = 1.6 \pm 0.2$ (the value of $\nu_2$ is the one used to do the fitting of Fig. 2a). The critical behavior of the resistivity in the limit $L_{ab} \to \infty$ is given by

$$\rho \sim (T - T_p)^{\eta},$$

where $\eta$ is the exponent that determines the asymptotic behavior of the function $g$ for large values of its argument, i.e., $\lim_{x \to \infty} g(x) \sim x^{\eta}$. In the limit of very large $L_{ab}$, the dependence on this length should cancel out in Eq. (4), and we get $\eta = \nu \tau_1$. With the previously found value of $\nu$ and the value of $\tau_1$ from Fig. 2(b) we find $\eta = 0.45 \pm 0.1$. The asymptotic behavior of $\rho(T)$ is shown in Fig. 2(a) as a dotted line.

The close relation between $c$ axis resistivity and percolation can be also seen in the following way. By combining Eqs. (4) and (6) we obtain

$$L^{\gamma_1}_{ab}L^{\gamma_2}_{c}\rho_c(T) = g\left(f^{-1}(P)\right)$$

In fact, for all the sizes considered, this scaling relation is satisfied, as shown in Fig. 4. This figure also shows clearly that the $c$ axis resistivity is different from zero only if the percolation probability is different from zero.

The previous scaling theory can be successfully generalized to account for the possibility of (weak) anisotropy in the system. The only change is that now the typical energy of an excitation $\Delta$ should be replaced by a value which depends on anisotropy. We introduce anisotropy in our JJA system by diminishing the $c$ axis mean critical current of the junctions and at the same time increasing the $c$ axis elemental resistance by the same factor $a$. In a mean field approach, the dependence of $\Delta$ on the parameter $a$ is of the form $\Delta = \Delta_0/\sqrt{a}$, with $\Delta_0$ the energy parameter for the isotropic system. Eq. (6) becomes

$$k_B T_p \sim \Delta_0/\sqrt{a} \ln(L_c),$$

making clear that anisotropy gives a much stronger decrease of $T_p$ than that obtained when varying $L_c$.

The validity of Eq. (10) is limited at very large $L_c$ or very high anisotropies, where $T_p$ may become close to the transition temperature $T_i$ along the $ab$ direction (see below). For the range of sizes studied, the $ab$ plane resistivity when a current of value 0.01 is applied is shown in Fig. 5. In all cases $T_i \approx 0.6$, which is always lower than the corresponding value of $T_p$, i.e., we are in the region where the previous scaling is expected to be valid.

V. PHYSICAL CONSEQUENCES OF THE PERCOLATION TRANSITION

The fact that the system has only one relevant length scale $\xi$ that diverges at $T_p$, has some consequences that can be checked experimentally, and in fact, be the key point in order to test the adequacy of the theory to the experiments.

From the experimental point of view, the transition at $T_i$ is a vortex glass to liquid transition or a depinning of individual vortex lines depending on the strength of the disorder. In our simulations $T_i$ always corresponds to a thermal depinning. The value of $T_i$ is rather thickness independent for the sizes of the sample studied (see Fig. 6, and also Ref. 13, Fig. 3(b)).

The dependence of the percolation temperature $T_p$ and thus, of the threshold value for the $c$ axis resistivity as a function of the thickness of the sample and the anisotropy is given in Eq. (10). As we said above, all the results presented here correspond to the case where the percolation temperature $T_p$ is greater than $T_i$, and in fact this is a condition for the previous scaling expressions to be valid. This is because, in order to have dissipation along the $c$ direction we must have a percolation path across the sample, but it is also necessary that this path is free to move. As the path consists of segments that go through the horizontal planes, this implies that the temperature should be greater than the value $T_i$ for the thermal depinning of vortices in the planes. This indicates that percolation is necessary although not
sufficient to have dissipation in the $c$ direction. Our previous scaling expressions and the relation between $\rho_c$ and the percolation probability are valid only if the condition $T_p > T_i$ is satisfied.

From the point of view of the numerical simulations, it is very difficult to make $T_p$ close to $T_i$ by increasing the thickness, due to its logarithmic dependence on $L_c$, but it is possible by changing the anisotropy $a$. However, when changing the anisotropy, a new feature appears: if there is no correlated disorder in the planes, the vortex configuration in the limit $a \to 0$ will be dominated by the independent pinning on each plane, with the Josephson vortices between planes -which in this limit have an energy $E \to 0$- forming an entangled set of vortex lines. If, on the other hand, the disorder is zero, when $T < T_0$ the vortex lines are still straight lines. These different possibilities reflect on the relative values of $T_p$ and $T_i$.

In Fig. 1 we plot the values of $T_i$ and $T_p$, as a function of the anisotropy for a $24 \times 24 \times 16$ sample. The value of $T_p$ is given from the onset in the $c$ axis resistivity and from the threshold of the percolation transition. The first point to be noted is the finite value to which $T_i$ tends for very high anisotropies. This value is the depinning temperature of a single plane. The two ways of determining $T_p$ give similar results as long as $T_p > T_i$, and this happens if the anisotropy is lower than a critical value $a_{cr}$. For $a > a_{cr}$ the threshold for the percolation transition moves to very low temperatures, whereas the onset of the resistivity behaves smoothly. The form of the curve $T_p$ vs $a$ for $a < a_{cr}$ follows the form $a^{-1/2}$ in agreement with Eq. 1.

The situation for $a > a_{cr}$ is not so clear. The fact that the vortex loops percolates at very low temperatures indicates that vortex lines are entangled even for $T \to 0$, due to the disorder within the planes. The dissipation along the $c$ direction for $T < T_i$ may be due to the movement of vortex lines lying entirely between two neighbor planes. In our simulations disorder is a fundamental ingredient in order to have $T_p < T_i$. In ordered samples $T_i$ and $T_p$ collapse onto a single value for high anisotropies, because the vortices remain straight for $T \to 0$. In this case the threshold of the percolation transition and the onset of the $c$ axis resistivity coincide for all anisotropies.

Experiments performed in twinned YBaCuO samples show a decreasing of $T_p$ with thickness. This is qualitatively similar to the predictions of Eq. 1. For the more anisotropic BiSrCaCuO samples, $T_p$ is equal to $T_i$, at least when looking at them by resistivity measurements, whereas measurability measurements suggest that $T_p < T_i$, but the value of $\rho_c$ for $T_p < T < T_i$ is too small to be detected. This may be similar to the behavior in Fig. 1 for $a > a_{cr}$, although more precise experimental as well as numerical work is needed in order to make the point clearer.

In what follows we discuss some experiments that can be used to discriminate between a crossover or a real phase transition occurring at $T_p$. A percolation transition at $T_p$ should reflect in the fact that the $I-V$ curves scale onto two universal curves for $T$ higher or lower than $T_p$. This kind of scaling has been a convincing proof of the vortex glass phase transition at $T_i$.

In our picture of a percolation transition we expect -according to scaling arguments near a second order phase transition- that the $I-V$ curves for different temperatures plotted as $V/I (T-T_p)^\nu$ vs $I/(T-T_p)^\nu$ show a universal behavior -i.e., independent of the particular value of $T_-$, depending only on whether $T$ is greater or lower than $T_p$. The exponents $\eta$ and $\nu$ can be extracted from the results obtained in the previous sections. The value at which $V/I (T-T_p)^\nu$ tends for $I \to 0$ when $T > T_p$ should be independent of temperature, so $V/I \sim (T-T_p)^\nu$ for low currents. Thus we see that $\eta$ is the critical exponent for the resistivity as defined in Eq. 3. We now show that $\nu$ is the same exponent defined in section III. The value of $I/(T-T_p)^\nu$ for which the dissipation starts to be appreciable when $T < T_p$ should again be temperature independent. This value corresponds to the minimum current $I_{cr}$ that is necessary to blow out the largest vortex loops perpendicular to the applied current that are present in thermal equilibrium. If these largest loops have linear dimensions $\sim \xi$, and thus an area $\sim \xi^2$, the energy $E_1$ of this loops in the presence of the current $I$ is roughly given by $E_1 = a\xi - bI\xi^2$, with $a$ and $b$ numerical constants, and it has a maximum of $E_1^M = a^2/4bI$ for $\xi = a/2bI$. The energy $E_0$ when there is no current is simply $E_0 = a\xi$. The loop will blow out when $E_0 > E_1^M$, and we get $I_{cr}\xi = \text{cte}$. As the typical length $\xi$ scales as $(T-T_p)^{-\nu}$ we conclude that $I/(T-T_p)^{-\nu}$ is the appropriate scaling combination, thus justifying the use of $\nu$ for the exponent. It should be noticed that the same conclusion can be obtained using dimensional analysis, just keeping in mind that in our case the relevant dimensionality of our diverging clusters is two, instead of three.

If the proposed mechanism for the transition at $T_p$ in the 3D JJA model is in fact the one occurring in experiments, the values obtained for the exponents $\nu$ and $\eta$ in both cases should be similar. Preliminary results in YBaCuO samples indicate that this scaling is rather good with values for the exponents that agree with ours. Note that the values $\nu \simeq 1.1$ and $\eta \simeq 0.45$ that we found for the exponents are certainly different than those obtained for the vortex-glass transition along the $ab$ plane ($\nu \sim 1.7$ and $\eta \sim 7$).

In addition to the experimental investigation of the scaling of the $I-V$ curves, it would be interesting to study the same scaling numerically in the 3D JJA model. However, this is a difficult task because of the size effects due to the finite value of $L_{ab}$. We have done only two partial checks of the scaling: we found that the current $I_{cr}$ for which the dissipation starts to be appreciable scales as $I_{cr} \sim (T_p-T)^\nu$, with the value of $\nu$ in agreement with the value obtained independently before.
VI. SUMMARY AND CONCLUSIONS

In this paper we presented a description of the $c$ axis resistive transition in the mixed state of high-Tc superconductors for the case of samples where (due to disorder and for no so high values of anisotropy) the superconducting coherence is lost along the $c$ axis at a temperature $T_p$ higher than the corresponding to the $ab$ plane. We showed that the idea of a percolation phase transition perpendicularly to the applied field as the mechanism driving the $c$ axis to a linearly dissipative (resistive) state is supported by finite size scaling in the 3D JJA model as well as transport measurements in YBaCuO samples.

The transition is a well defined thermodynamic transition as long as $L_{ab} \to \infty$, irrespective of the value of the thickness $L_c$. However, the threshold value $T_p$ decreases logarithmically with $L_c$. The analysis of the $\rho_{ab}$ vs $\rho_c$ curves give support to the idea of a real phase transition at $T_p$, against the possibility of a crossover due to finite thickness with only one single transition at $T_i$. Also, preliminary results for the critical scaling of the $I-V$ curves are consistent with a phase transition at $T_p$, the critical exponents being close to those found in the numerical simulations, and certainly different than those found for the vortex glass transition.

As pointed out before, the case corresponding to clean samples (that can be numerically simulated using triangular lattices) gives results qualitatively different to the ones described here. In particular, the interplay between the transitions for the $c$ axis and for the $ab$ plane gives rise, in some cases, to a single first order transition. If disorder is then put into the system, the transition remains first order up to a critical value of the disorder. If the disorder is increased further then two separate transitions (with $T_p > T_i$ if the anisotropy is weak) are recovered.  

The case of very anisotropic samples, where possibly $T_p < T_i$, still needs further experimental as well as theoretical clarification. Resistivity measurements in BiSrCaCuO samples have not been able to detect a region where $\rho_c \neq 0$, and at the same time $\rho_{ab} = 0$. However this could be due to the very high aspect ratio of the samples, which prevents an accurate determination of $\rho_c$. On the other hand, our simulations in the 3D JJA model suggest that the case $T_p < T_i$ is obtained only in the case where the vortex lattice -due to the effects of disorder- has percolated even for $T \to 0$. For samples without disorder simulations suggest $T_p = T_i$ for very anisotropic samples.

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FIG. 1. Top view of the configuration of vortices in a lattice of 18×18×8 for H = 0.2 and a random distribution of disorder in the critical currents of the junctions (0.2 < L_c < 1.8). Two different states after an annealing process are shown.

FIG. 2. Percolation probability for different values of L_{ab} (a) and L_c (b) as a function of temperature (in units of the mean Josephson energy of the junctions). Panel (c): the curves from (a) and (b) scaled according to the given formula (see text for explanation) with parameters \nu = 1.1, \rho_c = 0.5 and \Delta = 3.6.

FIG. 3. (a) Scaled c axis resistivity as a function of temperature for different values of L_{ab} and (b) the same curves plotted against the scaled variable \tilde{x}. The values of the exponents \tau_1 and \tau_2 are \tau_1 = 0.4 and \tau_2 = 1.6. In (a) the limit for large L_{ab} -namely \rho_c = (T - T_p)^{\nu\tau_1}- is also shown.

FIG. 4. Scaled c axis resistivity vs percolation probability for all sizes quoted in Fig. 2(a) and (b).

FIG. 5. ab plane resistivity for all system sizes considered (as quoted in Fig. 2(a) and (b)). All curves have the critical temperature around T_c \approx 0.6.

FIG. 6. Critical temperatures for the c axis (T_p) and for the ab plane (T_c) as a function of anisotropy for a 24×24×16 sample. For T_p, circles correspond to the onset of the resistivity, whereas stars indicate the onset of the percolation. For a ≥ 10 the onset of percolation moves towards T = 0.
\[ \tilde{x} = L_{ab}^{1/\nu} \left( 1 - p_c - \left( 1 + \exp \left( -\frac{\Delta}{T} \right) \right)^{-L_c} \right) \]
(Fig. 3)

\( \rho_c \) (arb. units)

\( L_c : 8 \)

\( L_{ab} : 8 \)

\( 16 \)

\( 24 \)

\( 32 \)

\( \tau_1 L_c \)

\( \tau_2 L_c \)

\( \sim \)

\( T \)

\( T_p \)

(a)

(b)
(Fig. 4)
(Fig. 6)