Re-analysing the implications of CPT and unitarity for baryogenesis and leptogenesis

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ABSTRACT: In the context of GUT baryogenesis models, it was pointed out by Nanopoulos and Weinberg that CPT conservation and the unitarity of S-matrix ensures that the net CP-violation generated in the decay of a heavy particle by graphs to first order in baryon number (B) violation is zero. We revisit this theorem (which holds for lepton number (L) violation as well) by systematically expanding the S-matrix order by order in B/L-violating couplings and find a re-formulation wherein certain consistent schemes of B/L assignment lead to the presence of both B/L conserving and B/L violating decay modes of a heavy particle. In such schemes, the net CP-violation is shown to be non-zero even with graphs to first order in B/L violation without actually contradicting the theorem. As an application of this result, we construct a model in low-scale leptogenesis.

KEYWORDS: Baryogenesis, Leptogenesis, CP violation
1 Introduction

The asymmetry in the universe between baryonic and anti-baryonic matter is expressed in terms of the ratio,

\[ \eta \equiv \frac{n_B - n_{\bar{B}}}{n_\gamma} \sim 10^{-10}, \]  

(1.1)

where, \( n_B \) and \( n_{\bar{B}} \) represent the baryon and anti-baryon densities respectively, and \( n_\gamma \) represents, similarly, the photon density. The present estimate for this asymmetry has been determined independently from the abundances of light nuclei due to big bang nucleosynthesis and analyses of the Cosmic Microwave Background Radiation (CMBR), confirming that we exist in a universe that is strongly baryon dominated [1, 2]. However, a largely symmetric universe, in terms of matter and anti-matter, is expected from the present understanding of the early universe and our knowledge of extremely tiny amounts of matter-antimatter asymmetry in fundamental particle interactions. While several interesting theories have been proposed to explain the generation of this asymmetry, the actual mechanism by which this occurs in nature remains unknown.

Baryogenesis is one of the mechanisms that attempt to explain the asymmetry, and does so by postulating its dynamic generation in the early universe, during the period between the end of cosmological inflation and reheating, and prior to the onset of nucleosynthesis, via interactions of particles and anti-particles asymmetric in their rates (see refs. [3–5] for detailed reviews). Because it is impossible to generate the observed asymmetry if the standard model of particle interactions is strictly adhered to, baryogenesis typically requires...
contributions from *beyond standard model physics* (BSM) not yet independently confirmed by experiments elsewhere. Often the solution to generating enough asymmetry by baryogenesis lies in looking at decays of super-heavy bosons in models of Grand Unified Theories (GUT). Baryogenesis might also be achieved, without help from GUT, via leptogenesis [6]. This involves the initial generation of an asymmetry in the lepton-antilepton content of the universe and its subsequent conversion to baryon asymmetry by means of sphaleron interactions that violate baryon \((B)\) and lepton \((L)\) numbers simultaneously, while conserving \(B - L\) (see refs. [7, 8] for excellent reviews on the subject). In the present work, we will focus on the constraints that are imposed on models of baryogenesis and leptogenesis by the requirement of some fundamental invariances, namely, CPT and unitarity, in quantum field theories.

The general consequences of CPT-invariance and unitarity of the S-matrix in the context of the generation of baryon asymmetry in GUT models have been explored in the past [9, 10]. In particular, as first pointed out by Nanopoulos and Weinberg [9], while calculating the CP-asymmetry generated in B-violating heavy particle decays, the leading contribution to the asymmetry involves graphs which are to the second-order or higher in B-violating vertices. The contribution of graphs to the first order in \(B/L\) (i.e. \(B\) or \(L\)) violation (and to all orders in \(B/L\) conserving interactions) vanishes as a consequence of CPT invariance and unitarity of the S-matrix in the theory. Henceforth, we shall refer to this result as the Nanopoulos-Weinberg theorem. One crucial input in proving this theorem was that, in the models considered in ref. [9], all decay modes of these heavy bosons were B-violating. Such an assumption is, of course, completely justified when formulating a minimal model satisfying the requirements for GUT-based baryogenesis. However, we note that in the present context of efforts to carry physics beyond the standard model, a wide range of possible models with varying particle content which can provide the seeds for \(B/L\) generation have been studied in the literature. In this wider framework, there may be more than one consistent scheme of \(B/L\) assignment in a given model. It may, thus, be useful to re-examine the physical meaning of the condition specifying the minimum order of \(B/L\) violation required to generate a CP asymmetry. This is because depending on the scheme of \(B/L\) assignment, particles with more than one possible decay modes (leading to final states with different \(B/L\) values) may decay both via channels which could be considered \(B/L\) violating, or via channels which are both \(B/L\) conserving and \(B/L\) violating.

Of course, notwithstanding the scheme of \(B/L\) assignment, physical observables, such as the net \(B\)-asymmetry generated in CP-violating decays, should remain unchanged. It is, however, important to understand that the order in \(B/L\)-violating couplings in which this CP violation occurs may be different in different contexts since this fact is of considerable importance towards finding possible graphs which contribute to the net CP-violation and building appropriate models.

In view of the above considerations, we have re-analysed the Nanopoulos-Weinberg theorem by looking at the S-matrix order-by-order in \(B/L\) violating couplings, and determining the leading order in these couplings at which the net CP-violation generated

\[\text{Here } C, P \text{ and } T \text{ denote the charge-conjugation, parity and time-reversal transformations respectively.}\]
We analyse two possible generic schemes of B/L number assignment leading to either the existence of only B/L violating decay modes of a heavy particle, or, of both B/L conserving and violating ones. Moreover, we go on to demonstrate the physical equivalence of these two pictures at the S-matrix level by comparing the net CP-violating difference in the appropriate partial decay widths occurring at various orders in the two different schemes. A particular upshot of our considerations is that in models where a certain scheme of B/L number assignment leads to the presence of both B/L violating and conserving decay modes of a heavy particle, the net CP-violation calculated with graphs to only first order in B/L violation is non-zero. We should emphasize here that our result is in no way contradictory to the Nanopoulos-Weinberg theorem, but rather a useful re-analysis and extension of the same picture which might be helpful while considering various models of baryogenesis and leptogenesis.

This paper is organised as follows: in section 2, we review the constraints imposed by CPT invariance and unitarity of the S-matrix on the possible generation of CP violation in the decays of heavy particles. In section 3, we find expressions for the B/L asymmetries generated in different schemes of B/L assignment for the decaying particle and demonstrate their equivalence. We also explore the consequences of the re-formulation of the Nanopoulos-Weinberg theorem by constructing an example model of leptogenesis in the same section. The last section contains our conclusions.

2 CP-violation in heavy particle decay

In most models of baryogenesis and leptogenesis, the baryon (B) or lepton (L) asymmetry is generated by the out of equilibrium decays of heavy particles. For example, for GUT baryogenesis models this heavy particle is quite often one of the new gauge bosons with masses at the GUT scale, while in thermal Leptogenesis models it is the heavy Majorana neutrino, which is a singlet under the standard model gauge group. The ingredients necessary for generating a B/L asymmetric universe from an initially symmetric one are given by the Sakharov conditions [11]: i) the presence of B/L violating interactions, ii) C and CP violation in the B-violating interactions, and, iii) the departure from thermal equilibrium for a period of time in the evolution of the universe. All of these conditions have been extensively studied and well established for both baryogenesis and leptogenesis.

The non-conservation of CP in interactions that also violate B is necessary to prevent the corresponding CP conjugate interactions from generating an equivalent amount of anti-baryonic excess to washout the generated baryon asymmetry. This is evident, for instance, in the case where the baryon asymmetry is generated due to the decay of a heavy scalar particle $X$ via the interaction $X \rightarrow qq$ with $\Delta B = B_{qq} - B_X = \alpha$ and the rate $\Gamma(X \rightarrow qq) = r$. If this interaction were CP conserving, the CP conjugate interaction $\bar{X} \rightarrow \bar{q}\bar{q}$ with $\Delta B = -\alpha$ would occur at the same rate, thus generating an excess of $-B$ exactly in keeping with that of $B$ washing any asymmetry out completely. In particular, the amount of CP violation required to explain the observed baryon asymmetry in the universe as having been generated by baryogenesis is more than what the standard model of particles and their interactions can provide, and has lead to the examination of
several models describing physics beyond it. CP violation is, however, constrained by the preservation of CPT symmetry and unitarity in interactions, and we study these constraints in the following paragraphs.

2.1 General implications of CPT invariance and S-matrix unitarity

We first briefly review the general implications of CPT conservation and the unitarity of S-matrix for various interactions [9, 10].

Let us assume that the initial state of a system represented by \( i \) (which represents all the quantum numbers of the system at this state) proceeds via interactions to a final state \( f \). The probability of transition to a state \( f \) from the state \( i \) is given by \( |S_{fi}|^2 \), where

\[
S_{fi} = \langle f | S | i \rangle
\]  

(2.1)

is the so-called S-matrix element. This S-matrix can be decomposed as follows:

\[
S_{fi} = \delta_{fi} + iT_{fi} ,
\]  

(2.2)

where, \( T_{fi} \) represents the \( fi \) element of the T-matrix, which represents the probability amplitude of transition of a system in the initial state \( i \) to a distinct final state \( f \), i.e., without transitioning to itself. The S-matrix must be unitary,

\[
SS^\dagger = 1 = S^\dagger S.
\]  

(2.3)

Written out in terms of the elements after inserting a complete set of states wherever necessary, this gives,

\[
\sum_f |S_{fi}|^2 = 1, \text{ and,}
\]  

(2.4a)

\[
\sum_f |S_{if}|^2 = 1.
\]  

(2.4b)

Equivalently, in terms of the T-matrix this can be expressed as,

\[
\sum_n T_{ni}^* T_{nf} = -i (T_{if} - T_{fi}^*) ,
\]  

(2.5a)

which implies, for \( i = f \),

\[
\sum_n |T_{ni}|^2 = 2 \Im (T_{ii}) ,
\]  

(2.5b)

where, in going from eq. (2.5a) to (2.5b) we have denoted the imaginary part of the complex quantity \( T_{if} \) by \( \Im(T_{if}) \). It is easy to show, along the same lines but starting from eq. (2.4b) instead of (2.4a), that, also

\[
\sum_n |T_{in}|^2 = 2 \Im (T_{ii}) .
\]  

(2.6)
Further, conservation of CPT ensures that the probability of transition of an initial state $i$ to a final state $f$ is equivalent to that of the transition of the corresponding CP conjugate states $\bar{f}$ to $\bar{i}$

$$T_{fi} = T_{\bar{i}\bar{f}}.$$  

(2.7)

The consequence of unitarity as expressed in Eq. (2.5) and (2.6), along with CPT invariance ensures that

$$\sum_f |T_{fi}|^2 = \sum_f |T_{fi}|^2.  \tag{2.8}$$

Therefore, the probability of a system in a state $i$ transitioning to all possible final states $f$ is identical to the probability of the system in the CP conjugate state $\bar{i}$ transitioning to all possible final states $\bar{f}$. This is an important consequence of CPT conservation and unitarity and it tells us, among other things, that the total decay width of a particle and its CP conjugate (anti-particle) are necessarily identical.

**CP violating amplitudes and unitarity.** We now pose the question: what constraint does unitarity impose on CP-violating amplitudes? If the interaction that generates the transition amplitude $T_{fi}$ is CP non-conserving, then the difference between the probabilities of the CP conjugate processes $i \rightarrow f$ and $\bar{i} \rightarrow \bar{f}$, or equivalently between $i \rightarrow f$ and $f \rightarrow i$ is finite and non-zero. Indeed, using eq. (2.5b) in the form

$$T_{if} = \sum_n T_{in} T_{fn}^* + T_{fi}^*,$$  

(2.9)

it is straightforward to obtain an expression for the difference in the probabilities for the CP conjugate interactions

$$|T_{\bar{f}\bar{i}}|^2 - |T_{fi}|^2 = \left| T_{fi} \right|^2 - \left| T_{fi}^* \right|^2$$

$$= -2\Im\sum_n \left( T_{in} T_{fn}^* \right) T_{fi} + \left| \sum_n T_{in} T_{fn}^* \right|^2 \tag{2.10}$$

This equation implies that CP-violating differences are generated by the interference of tree and loop graphs, where the intermediate states in the loop are on-shell [10].

**The Nanopoulos-Weinberg theorem** In the context of GUT baryogenesis models, in Ref. [9], general formulae were given for the mean baryon excess $\Delta B$ produced in the decays of super-heavy $X$ bosons and their anti-particles. This calculation was simplified considerably by a general theorem proved by the authors, which states that graphs to first order in B-violating interactions but to arbitrary order in baryon-conserving interactions make no contribution to $\Delta B$. In particular, it was shown that when decay amplitudes are calculated using graphs to first-order in B-violating interactions, CPT invariance requires that the decay rate for a particle $X$ into all final states with a given baryon number $B$ equals the rate for the corresponding decay of the anti-particle $\bar{X}$ into all states with baryon number $-B$. Therefore, this theorem indicates that one must consider graphs to at least second order in B-violating interactions. For details of the proof given by Nanopoulos and Weinberg please see Ref. [9]. In this paper, the authors considered models where the
super-heavy boson giving non-zero contribution to the net baryon asymmetry had only B-violating decay modes. This assumption was incorporated in the proof of this theorem by demanding that in the absence of B-violating interactions, the wave-function of $X$, $\psi_X$ is a one-particle state. In section 3, we shall implement this assumption at the S-matrix level by demanding that in the case where the heavy boson $X$ decays only via B-violating interactions, the S-matrix elements $(S_0)_{fX} = \delta_{fX}$, where $S_0$ denotes the part of the S-matrix which contains only B-conserving interactions. Expanding the S-matrix order-by-order in B-violating couplings $\alpha_B$, we then show that the net CP-violation generated is zero to $O(\alpha_B^2)$, in accordance with the theorem.

3 Systematic expansion of the S-matrix in B/L-violating couplings

In general, it can be shown that heavy bosons from GUT models, be it a vector or scalar particle, couple to the standard fermions (both quarks and leptons) either via channels with a unique value of B/L, or to two or more channels with different B and L. In the simplest example of the first case the initial particle $X$ can only decay via a single channel $X \rightarrow \alpha\beta$. Thus, it can always be assigned a baryon number and a lepton number so as to preserve both B/L symmetry in the interaction. It, therefore, contributes nothing to the asymmetry and thus, particles which can only decay via a single channel are not interesting from the point of view of baryogenesis or leptogenesis. Where the initial particle $X$ can decay through channels leading to final states with differing B/L, it is impossible to assign B/L to $X$ in a way that conserves B/L in both the possible channels. Therefore, in such cases, irrespective of the B/L assignment of $X$, there will always be a mode of decay that would violate the B/L symmetry and could lead to successful baryogenesis or leptogenesis. Consider, e.g., the scenario where the initial scalar boson $X$ can decay to both a lepton and a quark, and two quarks:

$$X \rightarrow lq'$$  \hspace{1cm} (3.1a)

$$X \rightarrow \bar{q}q',$$  \hspace{1cm} (3.1b)

where, $l$ denotes a lepton and $q$ and $q'$ denote quarks. In the first case (3.1a), the baryon asymmetry generated is $\Delta B_1 = B(q') - B(X)$, while in the second (3.1b) it is $\Delta B_2 = -2B(q) - B(X)$. It is clear that irrespective of the value of $B(X)$, at least one of these interactions will be B-violating. It is only in such cases that $X$ is said to have B-violating modes of decay, and it is by interactions such as these that baryogenesis can proceed.

In some models with $B-L$ number conservation, $B-L$ quantum numbers can be uniquely assigned (upto a common multiplicative constant for all the particles), but not B or L individually. Since to start with, we need B/L violation in a sector of our theory, there is no unique way to fix these global quantum numbers. It could yet happen that, at the energies we are considering, the B/L assignments of the particles in the effective theory might be pre-determined by certain symmetries in the high scale theory, notwithstanding that these symmetries might be absent from the effective low-energy theory itself. Thus, in the end we might end up having either only B-violating modes for $X$, or both B-violating
and conserving modes. In what follows, by explicitly considering these two scenarios we
demonstrate that, though stemming from different possible B-number assignments, they
are equivalent from the point of view of physical observables; in particular, we will establish
this for the difference between the partial decay widths of the CP conjugate processes in
the two scenarios.

We first split the S-matrix into two parts,

\[ S = S_0 + i\tilde{T}, \]

(3.2)

where \( S_0 \) includes the identity element of the total S-matrix and also processes represented
by graphs with only B-conserving interactions. \( \tilde{T} \) contains processes described by graphs
with B-violating interactions to first order or higher and B-conserving interactions to all
orders. Using this expansion in eq. (2.3) we arrive at the following relation between
\( S_0 \) and \( \tilde{T} \)

\[ \tilde{T} = S_0\tilde{T}^\dagger S_0 + iS_0\tilde{T}^\dagger\tilde{T} = S_0\tilde{T}^\dagger S \]

(3.3)

In terms of the elements of the S and T matrices, we therefore see that

\[ \tilde{T}_{Xf} = \sum_{i,j} (S_0)_{Xi} (\tilde{T}^\dagger)_{ij} S_{jf} \]

(3.4)

From eq. (3.4) we get

\[ |\tilde{T}_{Xf}|^2 = \sum_{i,j,k,m} (S_0)_{Xi} \tilde{T}^*_{ji} S_{jf} (S_0)_{Xk} \tilde{T}_{mk} S^*_{mf} \]

(3.5)

Denoting all B-violating coupling constants by \( \alpha_B \), we expand the quantity \( \tilde{T} \) in a
perturbation series in this coupling constant

\[ \tilde{T} = \alpha_B \tilde{T}_1 + \alpha_B^2 \tilde{T}_2 + \cdots, \]

(3.6)

where the quantities \( \tilde{T}_1 \) and \( \tilde{T}_2 \) themselves do not contain any factors of the B-violating
coupling constant \( \alpha_B \). Thus,

\[ S = S_0 + i\left( \alpha_B \tilde{T}_1 + \alpha_B^2 \tilde{T}_2 \right) + O(\alpha_B^3) \]

(3.7a)

\[ \Rightarrow S_{f\bar{X}} = S_{Xf} = (S_0)_{Xf} + i\left( \alpha_B \tilde{T}_1 + \alpha_B^2 \tilde{T}_2 \right)_{Xf} + O(\alpha_B^3), \]

(3.7b)

where in eq. (3.7b) we have used CPT conservation, as usual, to rewrite \( S_{f\bar{X}} \) as \( S_{Xf} \).

3.1 Case 1: Where the initial particle decays only by B-violating interactions.
If the initial particle \( X \) and its CP conjugate particle \( \bar{X} \) decay only via B-violating inter-
actions, i.e.,

\[ (S_0)_{f\bar{X}} = (S_0)_{Xf} = \delta_{Xf}, \]

(3.8)
we get, using eq. (3.8) in (3.5),
\[\sum_{f \in B} |\tilde{T}_{fX}|^2 = \sum_{f \in B} \sum_{j,m} \tilde{T}_{fX}^* S_{ijf} \tilde{T}_{mX} S_{mf}^* \] (3.9a)
\[= \sum_{f \in B} \left( \tilde{T}_{fX}^* \tilde{T}_{fX} \right) - i \sum_{m} \tilde{T}_{fX}^* \tilde{T}_{mX} \tilde{T}_{mf}^* + i \sum_{m} \tilde{T}_{mX} \tilde{T}_{mf} \tilde{T}_{fX} \]
\[+ \sum_{j,m} \tilde{T}_{fX}^* \tilde{T}_{jX} \tilde{T}_{mX} \tilde{T}_{mf}^* \] (3.9b)

where \(\sum_{f \in B}\) represents the sum over all final states \(f\) with a given baryon number \(B\). In going from eq. (3.9a) to (3.9b) we have expanded \(S\) in accordance with eq. (3.2) and summed over the \(\delta_{\alpha\beta}\) as appropriate. We can carry over the first sum in the R.H.S. of eq. (3.9b) to the other side of the equality, and use CPT as required, to obtain the important difference in the partial decay widths of the CP conjugate processes violating baryon numbers by \(\Delta B = B - B(\chi)\) and \(\Delta \bar{B} = -B - B(\bar{\chi})\) units respectively.

\[\sum_{\bar{f} \in -B} |\tilde{T}_{\bar{f}X}|^2 - \sum_{f \in B} |\tilde{T}_{fX}|^2 = \sum_{f \in B} \left( -i \sum_{m} \tilde{T}_{fX}^* \tilde{T}_{mX} \tilde{T}_{mf}^* + i \sum_{m} \tilde{T}_{mX} \tilde{T}_{mf} \tilde{T}_{fX} \right) \]
\[+ \sum_{j,m} \tilde{T}_{fX}^* \tilde{T}_{jX} \tilde{T}_{mX} \tilde{T}_{mf}^* \] (3.10)

We now expand \(\tilde{T}\) order-by-order in \(\alpha_B\) according to eq. (3.6) and evaluate this difference. The results of the calculation to \(O(\alpha_B^2)\) and \(O(\alpha_B^3)\) are enumerated below.

**To \(O(\alpha_B^2)\):** It is easy to see that each of the three sums in the R.H.S. of eq. (3.10) gives a contribution that is at least to \(O(\alpha_B^3)\). Hence, the \(O(\alpha_B^2)\) contribution to the L.H.S. is zero. Since the tree graph must contain one \(B\)-violating vertex, an \(O(\alpha_B^3)\) contribution to the difference in \(|T_{fX}|^2\) can only come from the interference of such a tree graph with a loop graph also containing, at most, one \(B\)-violating vertex. Thus this result is consistent with the results of the Nanopoulos-Weinberg theorem, and shows that graphs to the first order in \(\alpha_B\) do not contribute to the CP-violating difference.

**To \(O(\alpha_B^3)\) and higher:** The \(O(\alpha_B^3)\) contribution to the CP violating difference comes from the first two sums in the R.H.S., and is given by
\[\sum_{f \in -B} |\tilde{T}_{\bar{f}X}|^2 - \sum_{f \in B} |\tilde{T}_{fX}|^2 = \alpha_B^3 \sum_{f \in B} \sum_{m} \left( \tilde{T}_{fX}^* (\tilde{T}_{\bar{f}})_{mX} (\tilde{T}_{\bar{f}})_{mf} \right) \]
\[+ O(\alpha_B^4) \] (3.11)

\(^2\)The third sum contributes only to \(O(\alpha_B^4)\) and higher.
The leading contribution in $\alpha_B$ to the CP violating difference is, therefore, to the third order and, as is evident from eq. (3.11), comes due to the interference of a tree level graph with its only vertex being B-violating and a loop graph with two B-violating vertices.

### 3.2 Case 2: Where the initial particle can decay both through B-conserving and B-violating interactions.

We now study, in a similar context, the case where the initial particle $X$ may decay via B-conserving as well as B-violating channels to the final states. This translates, in terms of S-matrix elements, to the condition

$$
(S_0)_{fX} = (S_0)_{Xf} = \delta_{Xf} + i(T_0)_{Xf},
$$

(3.12)

with $(T_0)_{Xf} \neq 0$, and likewise for $(S_0)_{fX}$. We carry out a similar calculation as in Sec.3.1, using eq. (3.12) in (3.5). We expand $\sum_{f \in -B} |T_{fX}|^2 - \sum_{f \in B} |\tilde{T}_{fX}|^2$ order by order in $\alpha_B$ and read out the $O(\alpha_B^2)$ terms in the expansion. Thus, to $O(\alpha_B^2)$, we find that

$$
\sum_{f \in -B} |\tilde{T}_{fX}|^2 - \sum_{f \in B} |T_{fX}|^2 = -i\alpha_B^2 \sum_{f \in B} (\tilde{T}_{1})_{fX} \sum_{m} \left( (T_0)_{Xm}(\tilde{T}_{1})^*_m + (\tilde{T}_{1})^*_{mX}(T_0)_{mf} \right) + i\alpha_B^2 \sum_{f \in B} (\tilde{T}_{1})^*_{fX} \sum_{m} \left( (T_0)_{mX}(\tilde{T}_{1})^*_m + (\tilde{T}_{1})^*_f m \right).
$$

(3.13)

We would now like to compare the CP-violating difference in case 2 given by eq. (3.13) with that found for case 1 given by eq. (3.11). Since, to start with, we have assumed that $(T_0)_{Xm} \neq 0$, and since $T_0$ contains only B-conserving interactions, we have $B(X) = B(m)$. But, as $B$ has to be finally violated in the decay of $X$, $B(X) \neq B(f)$. Therefore, we arrive at the conclusion that $B(m) \neq B(f)$ which implies that $(T_0)_{mf} = 0 = (T_0)_{mX}^*$. Using this result, we find that

$$
\sum_{f \in -B} |\tilde{T}_{fX}|^2 - \sum_{f \in B} |T_{fX}|^2 = \alpha_B^2 \sum_{f \in B} \sum_{m} 23 \left( (\tilde{T}_{1})^*_{fX} (T_0)_{mX}(\tilde{T}_{1})^*_m \right) + O(\alpha_B^3),
$$

(3.14)

i.e., a non-zero contribution to $O(\alpha_B^2)$, unlike in eq. (3.11) where we had only obtained non-zero contributions to $O(\alpha_B^3)$ and higher. Here we have used the approximate equality of $(\tilde{T}_{1})^*_{ij}$ and $(\tilde{T}_{1})_{ji}$, since their difference is higher order in $\alpha_B$ and similarly for $(T_0)_{ij}$ and $(T_0)_{ji}$. The results in eq. (3.11) and (3.14) are actually equivalent, considering that the only change between the two scenarios is that a baryon number violating vertex in case 1 has been replaced by a baryon number conserving vertex in case 2, thereby inducing a corresponding change in the transition amplitudes $\alpha_B \tilde{T}_1 \rightarrow T_0$ in the respective expressions. This establishes that irrespective of the scheme of baryon number assignment, the CP violation from processes taking the initial particle $X$ to all possible final states with a given baryon number $B$ and their conjugate remains unchanged.
3.3 An example in Leptogenesis

The result derived in the above paragraph has important consequences for the purpose of constructing models of baryogenesis and leptogenesis. It allows the presence of B/L-conserving decay channels to contribute to the generation of B/L asymmetry by ways that might have earlier been ignored in view of the strong requirements of the Nanopoulos-Weinberg theorem. While a proper and thorough examination of models in light of this equivalence is beyond the scope of the present work, we demonstrate its utility in the case of one particular model for leptogenesis at the electroweak symmetry breaking (EWSB) scale. The advantage of models at such low energies is their obvious testability at present or upcoming experiments.

We re-analyse, in particular, the model for leptogenesis explored in [12], where leptonic asymmetry was generated at the EW energies by the decay of a Majorana neutrino $N_1$ into a left-handed charged lepton and a charged Higgs field, $N_1 \rightarrow \phi^- \ell^+$. Being its own anti-particle, it also decays to the CP conjugate final states $N_1 \rightarrow \phi^+ \ell^-$. It is clear that irrespective of the lepton number assigned to $N_1$, at least one of the two modes would be lepton number violating. We assign a lepton number of 0 to the particle $N_1$ for all our following analysis.\(^3\) The model discussed in [12] requires, in addition to the heavy RH neutrinos, the presence of three scalar $SU(2)_L \times U(1)_Y$ doublets. However, in this model, both the tree level graph and the possible one-loop graphs for the process $N_1 \rightarrow \phi^- \ell^+$ contain just one L-violating vertex each. In keeping with the Nanopoulos-Weinberg theorem, therefore, the contribution of the interference of these diagrams can only generate an $O(\alpha^2_B)$ term, which, as established in section 3.1 will lead to a vanishing CP-asymmetry. Therefore, the stringent requirements of the Nanopoulos-Weinberg theorem, prevent the generation of lepton asymmetries from the interference of one-loop graphs with tree graphs, and it becomes necessary to use two-loop graphs and a particular hierarchy of masses of the scalar doublets and the heavy neutrinos to generate the required leptonic asymmetry in this model [12].

Based on the results proved in sec. 3, we propose a model for leptogenesis along the lines of the electroweak scale model considered in ref. [12], where one can now generate the required CP-asymmetry at the one-loop level itself. In addition to the particle content of the standard model (SM) including the SM Higgs doublet $\phi$, we require two heavy right-handed neutrinos with Majorana masses. We denote them by $N_1$ and $N_2$, and assume their masses to be hierarchical with $M_{N_1} > M_{N_2}$. The SM Higgs doublet is given by

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix},$$

where the neutral member of the Higgs doublet develops a vacuum expectation value at the electroweak scale, $\langle \phi^0 \rangle \equiv v$ with $v \sim 246$ GeV. We denote a generic SM left-handed

\(^3\)As noted before, there can be other possible consistent assignments of lepton number for $N_i$, and they would lead to exactly same results for the relevant physical observables.
lepton doublet of a given flavour by

\[ L = \begin{pmatrix} l^- \\ \nu \end{pmatrix} \]

The relevant terms in the interaction part of the Lagrangian, which are of interest in our following discussion are given by,

\[ \mathcal{L} \supset g_1 N_1 N_2 \phi^0 + g_2 N_1 l^+ \phi^- + g_3 N_2 l^+ \phi^- + g_4 \phi^0 \phi^+ \phi^- + h.c. \] (3.15)

In order to obtain the first interaction term of the form \( g_1 N_1 N_2 \phi^0 \), we can first couple the SM singlet RH neutrinos \( N_1 \) and \( N_2 \) to another SM singlet real scalar field \( S \) via a term of the form \( \mu N_1 N_2 S \), where \( \mu \) is a dimensionless parameter. This field \( S \) can couple to the doublet Higgs via terms of the form \( (\alpha S + \beta S^2)\phi^\dagger \phi \), leading to mixing between \( S \) and the neutral component of \( \phi \) once electroweak symmetry is broken. This will then give rise to the desired \( g_1 N_1 N_2 \phi^0 \) coupling, with the coupling strength proportional to the degree of singlet-doublet mixing. The second and third terms in the Lagrangian stem from the usual Yukawa couplings of the left-handed charged leptons, the charged Higgs and the RH neutrinos. Finally, the last term in the Lagrangian arises from the quartic Higgs self-coupling \( (\phi^\dagger \phi)^2 \), after one of the neutral Higgs fields gets a vev. Thus the coupling constant in this term is proportional to \( v \) and therefore can be considerably large. In the above Lagrangian, the coupling constants \( g_1 \) and \( g_4 \) are constrained to be real, while the other two, viz., \( g_2 \) and \( g_3 \) are complex in general. The interactions in this Lagrangian allow for one lepton number violating decay channel for the heavier Majorana \( N_1 \):

\[ N_1 \rightarrow l^+ + \phi^- \]

Figure 1. Decay of the heavier Majorana neutrino \( N_1 \) to an \( l^+ \) and a \( \phi^- \) via a tree level interaction and a first order loop correction involving the lighter Majorana \( N_2 \).

We associate a lepton number of 0 to both the Majorana neutrinos \( N_1 \) and \( N_2 \); it is clear, with such assignments, that the only lepton number violating vertex in the loop graph in figure 1 is that with the coupling constant \( g_3 \), and thus this loop graph represents an amplitude that is to the first order in lepton number violation. It is now straightforward to calculate the contribution to the interaction width due to the interference of the loop and the tree graphs (which we denote by \( \Gamma \))

\[ \Gamma (N_1 \rightarrow l^+ \phi^-) = g_1 g_2 g_3 g_4 I + g_2^* g_1 g_3 g_4 I^*, \] (3.16)
and, therefore,
\[ \Gamma (N_1 \to l^+ \phi^-) - \Gamma (N_1 \to l^- \phi^+) = -4 \Im(I) \Im(g_2 g_1 g_3^* g_4), \] (3.17)

where, \( I \) represents, as in appendix A the phase space factor obtained by carrying out the integration over the free loop momentum in the loop graph of figure 1. The factor \( I \) can, in general, have a non-zero imaginary part if the intermediate states in the loop graph for \( N_1 \) decay are on-shell (this can happen only if \( M_{N_1} > M_{N_2} + M_{\phi} \)). Eq. (3.17) shows that the contribution from the decay of \( N_1 \) to leptogenesis, which is proportional to the CP violation in the B/L violating decay, \( \Gamma (N_1 \to l^+ \phi^-) - \Gamma (N_1 \to l^- \phi^+) \), is non-zero even though we have included terms only to the first order in lepton number violation in the loop amplitude. This becomes possible due to the presence of the lepton number conserving interaction \( N_1 \to N_2 \phi^0 \) in the model and demonstrates an application of the re-analysis of the Nanopoulos-Weinberg theorem in the presence of B/L conserving decays of the initial particle as discussed in section 3.2. Further, it turns out that the CP violation from such decays, of \( N_1 \to \phi^+ \tau^- \), for example, is proportional to the scalar quartic couplings and independent of the Yukawa coupling constants which are rather constrained [12].

\[ \epsilon_{CP}(N_1 \to \tau \phi) \sim |g_4| \left( \frac{m_\tau}{M_1} \right)^2 \frac{1}{1 + \eta^2} \ln \left( \frac{1 - \eta + \eta^2}{\eta} \right), \] (3.18)

where we have assumed that the scalar particles are approximately of the same mass \( m_H \), and, \( \eta = m_H^2/\Lambda^2 \). Since the asymmetry is proportional to the magnitude of the unconstrained quartic scalar coupling \( g_4 \), the required amount of asymmetry may be generated by requiring this to be large, without any dependence on the other coupling constants.

Our model does face the generic problem of competing with the onset of exponential damping of the sphaleron interactions after EWSB, that convert the generated lepton number asymmetry to baryon asymmetry. While a detailed analysis of the model and its issues is beyond the scope of the present work, we note here that this problem can be overcome by postulating the generation of large amounts of CP violation which is possible in this case without depending on unusually large values of the Yukawa couplings. This is because the CP violation is proportional to the (yet) unconstrained quartic couplings instead.

4 Remarks and Conclusion

We have done a thorough analysis of the Nanopoulos-Weinberg theorem and demonstrated its applicability in various scenarios of B/L assignment to the decaying particle. Having expanded the interaction amplitude in a perturbation series in the B/L-violating coupling \( \alpha_B \), we have shown the non-trivial implication of the theorem in the case where B/L assignments are such that the initial particle may decay by B/L-conserving interactions in addition to B/L-violating interactions. In particular, it turns out that in such cases, the asymmetry generated due to B/L-violating decays may be augmented by B/L-conserving interactions in the loop graphs, in a way that appears contrary to the consequences of the Nanopoulos-Weinberg theorem. This re-interpretation of the theorem has significant
implications for models of baryogenesis and leptogenesis — by opening up channels of CP violation generation that might have been earlier ignored with the intention of subscribing to the theorem’s stringent requirements. In addition to setting up new models for B/L-genesis employing B/L conserving channels as we have shown, it might be an interesting exercise to re-analyse some currently proposed models of Baryogenesis and Leptogenesis in the light of this interpretation. As an example of this approach, we have considered a recently proposed model of leptogenesis which generates a CP asymmetry only at the two-loop level. By studying a simple variation of this model, which allows B/L conserving decays, we have shown that it is possible to generate sufficient CP asymmetry at the one loop level.

Acknowledgments

The authors would like to thank Ashoke Sen for his invaluable suggestions clarifying several key points in the present work. The authors are also grateful to Boris Kayser for discussions leading to the present work. SM would like to thank Kaoru Hagiwara for useful discussions and the KEK theory group for warm hospitality in a period during which part of this work was carried out. RG and AB thank M. K. Parida for discussions relating to the work. RG also thanks the CERN Theory Division and the University of Wisconsin – Madison phenomenology group for hospitality while the work was in progress.

A Examples in Baryogenesis to demonstrate the Nanopoulos-Weinberg theorem

Typically, the contribution to baryon asymmetry generated by the particle $X$ with baryon number $B(X) = B_X$ and total decay width $\Gamma_X$, due to its transition to final states $f$ with $B(f) = B_f \neq B_X$, is given by

$$\epsilon_X = \sum_f (B_f - B_X) \frac{\Gamma(X \to f) - \Gamma(\bar{X} \to \bar{f})}{\Gamma_X} \propto \sum_f (B_f - B_X) \sum_m \Im(T_{fX}^* T_{mX} T_{mf}).$$  \hspace{1cm} (A.1)

We consider two examples to illustrate the implications of the Nanopoulos-Weinberg theorem. First, we consider a model in which a heavy scalar boson $X$ with baryon number $B_X = 0$ can decay via a B-violating interaction to a pair of fermions $f_1$ and $f_2$, while another scalar heavy boson $Y$ can decay only via separate B-conserving interactions to both the fermions. The Lagrangian for the model is given below:

$$\mathcal{L}_a = g_1 X f_1^\dagger f_1 + g_2 Y f_1^\dagger f_1 + g_3 Y f_2^\dagger f_2 + h.c.$$  \hspace{1cm} (A.2)

The possible tree and one-loop diagrams for the decay process $X \to \bar{f}_1 f_2$ are shown in figure 2. Both the tree and one-loop graph have one B-violating vertex each (vertex with coupling constant $g_1$ in both graphs). One can easily calculate the asymmetry generated,
Figure 2. Tree and one-loop graphs for the decay $X \rightarrow \bar{f}_1 f_2$ due to the Lagrangian $\mathcal{L}_a$.

$\epsilon_X$, in the decay of $X$ due to the interference of the two graphs and find that

$$\Gamma(X \rightarrow \bar{f}_1 f_2) = |g_1|^2 g_3 (I_{XY} + I_{XY}^*),$$

(A.3a)

and,

$$\Gamma(\bar{X} \rightarrow f_1 \bar{f}_2) = |g_1|^2 g_3 (I_{XY} + I_{XY}^*),$$

(A.3b)

which means,

$$\epsilon_X \propto \Gamma(X \rightarrow \bar{f}_1 f_2) - \Gamma(\bar{X} \rightarrow f_1 \bar{f}_2) = 0.$$  

(A.3c)

Here, we have represented only the contribution to the decay width arising due to the interference between a one-loop graph and a tree graph by $\Gamma$. The kinematic factor arising out of the integral over the loop-momentum is denoted by $I_{XY}$, which can be complex if the fermions in the loop are kinematically allowed to go on-shell. As a result of eq. (A.3c), the asymmetry generated due to $X$ decays in this model, which is proportional to the CP violation, also becomes zero. This is, clearly, what we expect from the Nanopoulos-Weinberg theorem, as the only contributions to the B-violating decay $X \rightarrow \bar{f}_1 f_2$ come from processes represented by graphs to the first order in B-violation.

We next consider a model in which both the super-heavy bosons $X$ and $Y$ can decay only via B-violating interactions to fermion pairs. The interaction Lagrangian for this model is given by:

$$\mathcal{L}_b = g_1 X f_2^\dagger f_1 + g_2 X f_1^\dagger f_3 + g_3 Y f_1^\dagger f_3 + g_4 Y f_1^\dagger f_4 + h.c.,$$

(A.4)

where each fermion $f_i$ has a different and unique B-number $B_i$. The baryon asymmetry generated out of the decays of the super-heavy scalars $X$ and $Y$ in this model has been extensively studied in the literature (see e.g., [13]). The graphs at the tree and one-loop levels that contribute to the decay $X \rightarrow \bar{f}_1 f_2$ are shown in figure 3; the loop graph in this case has three B-violating vertices. It is easy to see that the asymmetry generated in this case is non-zero:

$$\epsilon_X^{12} = \frac{4 (B_2 - B_1) \Im (I_{XY}) \Im (g_1^* g_2 g_3^* g_4)}{\Gamma_X},$$

(A.5)

where, as usual, $I_{XY}$ denotes a factor arising out of integration over the loop momentum. One can similarly see that the asymmetry generated due to the decay $X \rightarrow \bar{f}_3 f_4$ is given by

$$\epsilon_X^{34} = \frac{4 (B_4 - B_3) \Im (I_{XY}) \Im (g_1 g_2^* g_3 g_4^*)}{\Gamma_X}.$$  

(A.6)
Figure 3. Tree and one-loop graphs for the decay $X \to f_1 f_2$ due to the Lagrangian $\mathcal{L}_b$.

The total asymmetry due to all possible B-violating decays of $X$ is, thus,

$$\epsilon_X = \epsilon_X^{34} + \epsilon_X^{12} = \frac{4\left((B_4 - B_3) - (B_2 - B_1)\right)\Im(I_{XY})\Im(g_1^* g_2 g_3^* g_4^*)}{\Gamma_X} \neq 0.$$  \hspace{1cm} (A.7)

This is also what is expected from the Nanopoulos-Weinberg theorem, since the one-loop contribution to the B-violating decays in this case are of the third order in B-violation.

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