Kalb–Ramond dipole solution in
low-energy bosonic string theory

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Abstract
We construct a new solution subspace for the bosonic string theory toroidally compactified to 3 dimensions. This subspace corresponds to the complex harmonic scalar field coupled to the effective 3–dimensional gravity. We calculate a class of the asymptotically flat and free of the Dirac string peculiarity solutions which describes a Kalb–Ramond dipole source with the generally nontrivial dilaton charge.
1 Introduction

The main part of the valid string theory results was obtained using a perturbative approach \[1\]. Moreover, the most nonperturbative information about string theory was derived from the perturbative one by the help of the conjectured nonperturbative string dualities \[2\]. In the framework of the perturbative approach one reduces the quantum string dynamics at low energies to the classical field theory of the string low mass excitation modes \[3\]. A solution spectrum of these effective string theories provide the main tool for the following quantum string theory investigation \[4\].

The field theories of the string excitation modes are essentially nonlinear. To construct wide classes of exact solutions of these theories one must use both the consistent ansatz search and the symmetry technique application \[5\]. The best strategy for generation of the new classical solutions consists of the straightforward construction of the simple solution of the interested type with its following generalization by the help of the symmetry transformations. Finally, one obtains a symmetry invariant solution class of the same type if the symmetries used for the generalization form a subgroup preserving the type under consideration (see \[6\] for the asymptotically flat fields in string theories).

In this paper we deal with the effective field theory describing the low–energy dynamics of the bosonic string theory massless excitation modes. These modes include the scalar dilaton field and the tensor Kalb–Ramond and graviton (metric) ones living in the multidimensional space–time. The equations of the theory have a highly nonlinear form and correspond to the multidimensional General Relativity \[7\] with the nontrivially coupled dilaton and Kalb–Ramond matter fields. In this paper, using the ansatze approach, we construct a class of the asymptotically flat solutions which possesses the invariance property under the action of the symmetry subgroup preserving the ansatze taken.

The paper is organized as follows: in section 2 we define a consistent truncation of the bosonic string theory and reformulate the resulting system in terms of the matrix–valued symmetric space model \[8\]. In section 3 we construct an ansatze which corresponds to the complex harmonic scalar field coupled to the effective gravity in 3 dimensions. In section 4 we construct a special axisymmetric solution of the ansatze equations which is defined by the harmonic function of the Coulomb form with the complex “charge” and the imaginary “location” on the symmetry axis. The corresponding solution of the bosonic string theory describes a Kalb–Ramond dipole with the dilaton charge. In the general case it possesses a Dirac string peculiarity in the general case; we establish all the special situations where this peculiarity vanishes. In Conclusion we discuss the possible symmetry generalization of the constructed solution in the framework of the bosonic and heterotic string theories and also consider the general perspectives of the approach applied in this paper.
2 A truncation

Let $X^M (M = 1, \ldots, D)$ be the coordinates of the space–time with the signature $(- + \cdots +)$. Let $\Phi, B_{MN} = -B_{NM}$ and $G_{MN} = G_{NM}$ denote the dilaton, Kalb–Ramond and metric fields correspondingly. Then the action of the classical field theory which describes the low–energy dynamics of these massless modes of the bosonic string theory reads [3]:

$$S_D = \int d^D X \sqrt{-\det G_{MN}} e^{-\Phi} \left( R_D + \Phi_M \Phi^M - \frac{1}{12} H_{MNB} H^{MNB} \right),$$

where $H_{MNB} = \partial_M B_{NB} + \partial_N B_{MB} + \partial_B B_{MN}$. In this paper we deal with the special truncation of the theory (1) which can be performed using two steps. Let us put $D = d + 3$ and denote $y^m = X^m (m = 1, \ldots, d)$ and $x^\mu = X^{d+\mu} (\mu = 1, 2, 3)$. We impose the following consistent restrictions at the first step:

$$B_{mn} = G_{m,d+\mu} = 0.$$  

(2)

The remaining nontrivial field components consist of $G_{mn}, B_{m,d+\mu}$ (we combine them into the $d \times d$ and $d \times 1$ matrices $G$ and $b_\mu$ correspondingly), and also the fields $\Phi, G_{d+\mu,d+\nu}$ and $B_{d+\mu,d+\nu}$. At the second step we suppose that these fields are independent on the coordinates $y^m$, i.e. we perform the toroidal compactification of the first $d$ dimensions [9], [10]. The resulting dynamical system admits a consistent restriction

$$B_{d+\mu,d+\nu} = 0,$$  

(3)

and can be naturally expressed in terms of the fields $G, \phi, v$ and $h_{\mu\nu}$, where

$$\phi = \Phi - \ln \sqrt{-\det G},$$  

(4)

$$h_{\mu\nu} = e^{-2\phi} G_{d+\mu,d+\nu},$$  

(5)

and $v$ is defined on shell by the differential relation

$$\nabla v = e^{-2\phi} G^{-1} \nabla \times \vec{b}.$$  

(6)

In Eq. (6) $(\vec{b})_\mu = b_\mu$; also in this equation and below all the differential operations are related to the coordinates $x^\mu$ and the 3-dimensional indeces are lowered and raised using the metric $h_{\mu\nu}$ and its inverse one $h^{\mu\nu}$. Finally the motion equations for the truncated theory of the bosonic string take the following form:

$$\nabla J = 0,$$  

(7)
\[ R_{3\mu\nu} = \frac{1}{4} \text{Tr} (J_\mu J_\nu), \quad (8) \]

where the Ricci tensor \( R_{3\mu\nu} \) is constructed using the 3–metric \( h_{\mu\nu} \), \( J = \nabla G G^{-1} \) and

\[
G = \begin{pmatrix} -e^{-2\phi} + v^T G v & v^T G \\
G v & G \end{pmatrix}, \quad G^{-1} = \begin{pmatrix} -e^{2\phi} & e^{2\phi} v^T \\
e^{2\phi} v & G^{-1} - e^{2\phi} v v^T \end{pmatrix}. \quad (9)
\]

Using a solution of the system (7)–(8) and performing Eq. (9) one can easily calculate the multidimensional dilaton field \( \Phi \) and the metric \( ds_{d+3}^2 \); the result reads:

\[
e^{\Phi} = \sqrt{\text{det} G} e^{2\phi}, \quad ds_{d+3}^2 = dy^T G dy + e^{2\phi} ds_3^2, \quad (10)
\]

where \( y \) is the \( d \times 1 \) column with the components \( y^m \) and

\[ ds_3^2 = h_{\mu\nu} dx^\mu dx^\nu. \quad (11) \]

From Eq. (10) it follows that

\[ G = G_{22}, \quad e^{2\phi} = -\left( G^{-1} \right)_{11}, \quad (12) \]

where the indeces enumerate the corresponding matrix blocks. To calculate the Kalb–Ramond field component \( B_{m,d+\mu} \) it is convenient to introduce the vector matrix potential \( \vec{\Omega} \) accordingly the relation

\[ \nabla \vec{\Omega} = J. \quad (13) \]

This potential exists on shell of Eq. (7); using its definition and Eq. (1) it is easy to prove that

\[ B_{m,d+\mu} = -\left( \vec{\Omega}_{21} \right)_{m\mu}, \quad (14) \]

Thus, the strategy for construction of a concrete solution for the theory (1) truncated accordingly the presented procedure consists of the calculation of the quantities \( G, G^{-1}, \text{det} G \) and \( \vec{\Omega} \) and of the following application of Eqs. (10), (12) and (13). Before this application one can generalize the obtained solution using the transformation

\[ G \to C^T G C, \quad \vec{\Omega} \to C^T \vec{\Omega} C^{-1}, \quad (15) \]

which gives a symmetry of the theory, see Eqs. (10), (12) and (13).
3 A subspace

The matrix \( G \) introduced in the previous section must be a symmetric nondegenerated matrix of the signature \((- - + \cdots +)\). The simplest solution of the motion equations (7)–(8) possessing these properties is the matrix \( G_0 = \text{diag}(-1, -1, 1, \ldots, 1) \); the corresponding 3–metric \( h_{\mu\nu} \) will describe the flat Euclidean 3–space. Let us now consider an ansatze with

\[
G = G_0 + \sum_k \lambda_k C_k C_k^T, \tag{16}
\]

where \( \lambda_k = \lambda_k(x^\mu) \) are the (nonmatrix) functions, \( C_k \) are the constant columns restricted by the relations

\[
C_k^T G_0 C_k = \sigma_k \delta_{kl}, \tag{17}
\]

where \( \sigma_k = 0, \pm 1 \), and \( k = 1, \ldots, N \). This ansatze includes the simplest solution mentioned above for the choice \( \lambda_k = 0 \) and provides the necessary matrix \( G \) properties at least for the small \( \lambda_k \) values. Eq. (17) means that the columns \( C_k \) are normalized and mutually orthogonal in respect to the “metric” \( G_0 \). The normalization conditions do not restrict the ansatze generality because the nonzero norms \( C_k^T G_0 C_k \) can be absorbed by the functions \( \lambda_k \), as it is seen from Eq. (16). More precisely, the column \( C_k \) restricted by Eq. (17) is normalized for \( \sigma_k = \pm 1 \) and selforthogonal for \( \sigma_k = 0 \).

Using the relations (17) one can calculate the matrices \( G^{-1} \) and \( J \). The result reads:

\[
G^{-1} = G_0 + \sum_k \mu_k C_k C_k^T G_0, \tag{18}
\]

where

\[
\mu_k = -\frac{\lambda_k}{1 + \sigma_k \lambda_k}, \tag{19}
\]

and

\[
J = \sum_k \frac{\nabla \lambda_k}{1 + \sigma_k \lambda_k} C_k C_k^T G_0. \tag{20}
\]

Let us now introduce the functions \( \xi_k \) as

\[
\xi_k = \int_0^{\lambda_k} \frac{d\lambda}{1 + \sigma_k \lambda}. \tag{21}
\]
Then for the functions $\lambda_k$ and $\mu_k$ one obtains the following symmetric expressions:

$$
\lambda_k = \frac{e^{\sigma_k \xi_k} - 1}{\sigma_k}, \quad \mu_k = \frac{e^{-\sigma_k \xi_k} - 1}{\sigma_k},
$$

where the case of $\sigma_k = 0$ is understood in the limit l’Hopitalle sense (i.e., for example, $\lambda_k = \xi_k$ if $\sigma_k = 0$).

It is easy to see that Eq. (7) is satisfied if

$$
\nabla^2 \xi_k = 0,
$$

i.e. if the functions $\xi_k$ are harmonic. Using the relations (17) and Eqs. (20), (21) one can prove that the Einstein equation (8) reduces to the following one

$$
R_{\mu\nu} = \frac{1}{4} \sum_k \sigma_k^2 \xi_k,\mu \xi_k,\nu.
$$

Eqs. (23) and (24) complete our ansatze definition; finally it is defined by the relations (16), (17), (22)–(24). In this section we will not give a concretezation of the columns $C_k$, functions $\xi_k$ and metric $h_{\mu\nu}$ and will only mean that the relations mentioned above are satisfied.

Now let us define the set of vector functions $\vec{\nu}_k$ on shell of Eq. (23) as

$$
\nabla \times \vec{\nu}_k = \nabla \xi_k.
$$

Than for the matrix $\tilde{\Omega}$ one obtains the following expression:

$$
\tilde{\Omega} = \sum_k \vec{\nu}_k C_k C_k^T \mathcal{G}_0,
$$

see Eqs. (13), (20), (24) and (25). To calculate $\det \mathcal{G}$ it is convenient to rewrite the matrix $\mathcal{G}$ in the form of

$$
\mathcal{G} = \hat{S} \mathcal{G}_0,
$$

where the “evolutionary operator” $\hat{S}$ can be represented as

$$
\hat{S} = \exp \left( \sum_k \xi_k C_k C_k^T \mathcal{G}_0 \right).
$$

Finally, after the additional application of Eq. (17), one obtains that

$$
\det \mathcal{G} = \exp \left( \sum_k \sigma_k \xi_k \right).
$$
Eqs. (16), (18), (22), (26) and (29) give the complete information necessary for calculation of the nonzero components of the bosonic string theory truncated in section 2. To write down these components in the explicit form, let us parametrize the columns \( C_k \) as

\[
C_k = \begin{pmatrix} p_k \\ q_k \end{pmatrix},
\]

where \( p_k \) are the numbers, whereas \( q_k \) are the \( d \times 1 \) columns. Then, using some algebra and applying Eqs. (10), (12) and (14), one finally obtains the following result:

\[
\begin{align*}
\mathcal{G}_0 + \sum_k q_k q_k^T \frac{e^\sigma_k \xi_k - 1}{\sigma_k} dy + \left( 1 - \sum_k p_k^2 e^{-\sigma_k \xi_k} - \frac{1}{\sigma_k} \right) ds^2_3, \\
e^\Phi = \exp \left( \sum_k \sigma_k \xi_k / 2 \right) + \sum_k p_k^2 \sinh \sigma_k \xi_k / 2, \\
B_{m,d+\mu} = \sum_k p_k q_k m \nu_{k\mu}.
\end{align*}
\]

(31)

Here the constants \( p_k \) and \( q_k \) must satisfy the relations

\[
- p_k p_l + q_k^T G_0 q_l = \sigma_k \delta_{kl},
\]

(32)

where \( G_0 \) is the “22” block component of the matrix \( G_0 \), i.e., \( G_0 = \text{diag}(-1, 1, \ldots, 1) \).

Now let us put \( N = 2 \) and restrict the following consideration by three special cases with \( \sigma_1 = \sigma_2 = \sigma \) and \( \sigma = 0, \pm 1 \). Let us combine the functional pair \((\xi_1, \xi_2)\) into the single complex potential

\[
\xi = \xi_1 + i\xi_2.
\]

(33)

Then the motion equations (23) and (24) take the following form:

\[
\nabla^2 \xi = 0,
\]

(34)

\[
R_{3 \mu\nu} = \frac{\sigma^2}{4} \left( \xi_\mu \xi_\nu + \xi_\nu \xi_\mu \right).
\]

(35)

Eqs. (34) and (35) describe the harmonic complex scalar field coupled to the 3–dimensional gravity. From Eq. (35) one concludes that \(|\sigma|\) plays the role of the effective coupling constant. In the case of \( \sigma = 0 \) the coupling vanishes and one obtains the flat 3–space (the extremal
case, see also [11]), whereas for \( \sigma_k = \pm 1 \) this 3-space is curved (the nonextremal case). In the next section we construct a concrete special solution for the system (34)–(35) and calculate the corresponding complex vector field

\[
\vec{\nu} = \vec{\nu}_1 + i\vec{\nu}_2,
\]

which satisfies the relation

\[
\nabla \times \vec{\nu} = \nabla \xi,
\]

as it follows from Eqs. (25) and (33). We also give the explicit expressions for the columns \( C_1 \) and \( C_2 \); this last step defines the multidimensional bosonic string theory fields completely, see Eqs. (31). Thus, in the rest part of this paper we consider the 2–functional subspace of the previously truncated bosonic string theory written in the compact complex form of Eqs. (34), (35) and (37).

4 A solution

Let \( x^\mu = (\rho, z, \varphi) \) be the Weil canonical coordinates. We plan to construct an axisymmetric solution of the system (34)–(35). In the axisymmetric case \( \partial \varphi \sim 0 \) on all the functions and the 3–dimensional metric can be taken in the Lewis–Papapetrou form [12]

\[
ds_3^2 = e^{\gamma} (d\rho^2 + dz^2) + \rho^2 d\varphi^2.
\]

For this metric Eq. (34) transforms into the one

\[
(\rho \xi,_{\rho})_{,\rho} + (\rho \xi,_{z})_{,z} = 0,
\]

whereas Eq. (33) converts into the system

\[
\begin{align*}
\gamma_{,\rho} &= \frac{\rho}{4}(\xi,_{\rho} \bar{\xi},_{z} + \xi,_{z} \bar{\xi},_{\rho}), \\
\gamma_{,z} &= \frac{\rho}{4}(\xi,_{\rho} \bar{\xi},_{\varphi} - \xi,_{\varphi} \bar{\xi},_{\rho}).
\end{align*}
\]

From Eq. (32) it follows that the 3–space becomes flat in the case of \( \gamma = 0 \). Then, Eq. (40) is \( \gamma \)-independent, so one can take an arbitrary axisymmetric solution of the “usual” flat Laplace equation as a solution of Eq. (39). Then, the system (40) is consistent for
the arbitrary solution of Eq. (39), so one can calculate the function $\gamma$ for the arbitrary axisymmetric harmonic $\xi$. We take

$$
\xi = \frac{e}{[\rho^2 + (z - ia)^2]^{1/2}},
$$

(41)

where

$$
e = e_1 + ie_2
$$

(42)

and the parameters $e_1, e_2$ and $a$ are the arbitrary real constants. The harmonic function (42) formally corresponds to the complex Coulomb charge $e$ “located” on the symmetry axis at the imaginary “position” $z = ia$. Solving Eq. (40), one obtains that

$$
\gamma = \frac{|\sigma e|^2}{16a^2} \left\{ 1 - \frac{\rho^2 + z^2 + a^2}{[(\rho^2 + z^2 + a^2)^2 - 4a^2\rho^2]^{1/2}} \right\},
$$

(43)

where the integration constant had been chosen to obtain $\gamma = 0$ at the spatial infinity.

To rewrite the solution (41), (43) in the root–free form, let us introduce the prolonged spheroidal coordinates $(r, \theta, \phi) = x^\mu$ accordingly the relations

$$
\rho = \sqrt{r^2 + a^2 \sin \theta},
$$

$$
z = r \cos \theta.
$$

(44)

Then the function $\xi$ and the metric $ds^2_3$ take the form of

$$
\xi = \frac{e}{r - ia \cos \theta},
$$

(45)

$$
\begin{align*}
ds^2_3 &= \exp \left( -\frac{|\sigma e|^2}{8} \frac{\sin^2 \theta}{r^2 + a^2 \cos^2 \theta} \right) \left[ \frac{r^2 + a^2 \cos^2 \theta}{r^2 + a^2} dr^2 + (r^2 + a^2 \cos^2 \theta) d\theta^2 + (r^2 + a^2) \sin^2 \theta d\phi^2 \right] + \\
&+ (r^2 + a^2) \sin^2 \theta d\phi^2.
\end{align*}
$$

(46)

The last step is to calculate the vector field $\vec{\nu}$ using Eqs. (37), (43) and (44). The result for the nonzero components reads:

$$
\nu_3 = \nu_\phi = e \frac{r \cos \theta - ia}{r - ia \cos \theta}.
$$

(47)
Using Eqs. (45) and (47) it is easy to obtain the components $\xi_k$ and $\nu_k$, the result is:

$$\xi_1 = \frac{e_1 r - e_2 a \cos \theta}{r^2 + a^2 \cos^2 \theta}, \quad \xi_2 = \frac{e_1 a \cos \theta + e_2 r}{r^2 + a^2 \cos^2 \theta},$$

$$\nu_{\varphi 1} = e_1 \cos \theta + a \sin^2 \theta \frac{e_1 a \cos \theta + e_2 r}{r^2 + a^2 \cos^2 \theta},$$

$$\nu_{\varphi 2} = e_2 \cos \theta - a \sin^2 \theta \frac{e_1 r - e_2 a \cos \theta}{r^2 + a^2 \cos^2 \theta}. \quad (48)$$

To complete the definition of the bosonic string theory fields, one must give an explicit expression for the columns $C_1$ and $C_2$. These columns must yield the quadratic algebraical restrictions (17). Solving them, one obtains that

$$C_1 = \begin{pmatrix} \sqrt{s_1^2 - \sigma \cos(\beta + \frac{\alpha}{2})} \\ \sqrt{s_1^2 - \sigma \sin(\beta + \frac{\alpha}{2})} \\ s_1 n_1 \end{pmatrix}, \quad C_2 = \begin{pmatrix} \sqrt{s_2^2 - \sigma \cos(\beta - \frac{\alpha}{2})} \\ \sqrt{s_2^2 - \sigma \sin(\beta - \frac{\alpha}{2})} \\ s_2 n_2 \end{pmatrix}, \quad (49)$$

where

$$n_1^T n_1 = n_2^T n_2 = 1 \quad (50)$$

and

$$\cos \alpha = \frac{s_1 s_2}{\sqrt{(s_1^2 - \sigma)(s_2^2 - \sigma)}} n_1^T n_2, \quad (51)$$

whereas the angle parameter $\beta$ is arbitrary.

Eqs. (31), (46) and (48)-(51) completely define the constructed solution. Let us now discuss some its properties. First of all, from Eqs. (31) and (48) it follows that the multi-dimensional metric is asymptotically flat, i.e., it describes the Minkowskian space–time at $r \to \infty$. Then, the dilaton field vanishes at the spatial infinity, so there is a sense to define and compute its charge. We define the dilaton charge by the relation

$$\Phi \sim \frac{D}{r}, \quad r \to \infty; \quad (52)$$

then from Eqs. (31) and (48) it follows that

$$D = e_1 (p_1^2 + \frac{\sigma}{2}) + e_2 (p_2^2 + \frac{\sigma}{2}). \quad (53)$$

10
Taking into account Eq. (49), one finally concludes that

\[ D = e_1 [(s_1^2 - \sigma) \cos^2(\beta + \frac{\alpha}{2}) + \frac{\sigma}{2}] + e_2 [(s_2^2 - \sigma) \cos^2(\beta - \frac{\alpha}{2}) + \frac{\sigma}{2}] . \]  

(54)

Then, from Eqs. (31) and (48) it follows that the constructed solution possesses the Dirac string peculiarity for the Kalb–Ramond field components \( B_{m,d+3} \). The corresponding term does not vanish at \( r \to \infty \) and contains the NUT–like (13) \( r \)–independent component

\[ B_{m,d+3} \sim (e_1 p_1 q_{1m} + e_2 p_2 q_{2m}) \cos \theta . \]

(55)

To remove this peculiarity and to obtain the completely asymptotically flat solution one must restrict the solution parameters by the relation

\[ e_1 p_1 q_1 + e_2 p_2 q_2 = 0 . \]

(56)

After some algebra one concludes that Eq. (56) can not be satisfied in the case of \( \sigma = 1 \) if the Kalb–Ramond field remains nontrivial. For the case of \( \sigma = 0 \) the solution of Eq. (56) reads:

\[ s_1^{-1} C_1 = s_2^{-1} C_2 = \left( \begin{array}{c} \cos \beta \\ \sin \beta \\ n \end{array} \right) , \]

(57)

where \( n^T n = 1 \). In this case

\[ e_1 = \frac{s_2}{s_1} e_0 , \quad e_2 = -\frac{s_1}{s_2} e_0 , \]

(58)

where the parameter \( e_0 \) is arbitrary; from Eq. (53) it follows that \( D = 0 \) here. For the case of \( \sigma = -1 \) one obtains three distinct special situations:

a) the “symmetric” charge configuration with \( e_1 = e_2 = e_0 \), \( D = 0 \) and

\[ C_1 = \left( \begin{array}{c} \cos \beta \\ \sqrt{1 + s^2 \sin \beta} \\ sn \sin \beta \end{array} \right) , \quad C_2 = \left( \begin{array}{c} -\sin \beta \\ \sqrt{1 + s^2 \cos \beta} \\ sn \cos \beta \end{array} \right) ; \]

(59)

b) the “nonsymmetric” charge configuration with \( e_1 = e_0 \), \( e_2 = 0 \), \( D = -e_0 / 2 \) and

\[ C_1 = \cosh \beta_1 \left( \begin{array}{c} 0 \\ 1 \\ n \tanh \beta_1 \end{array} \right) , \quad C_2 = \cosh \beta_2 \left( \begin{array}{c} \frac{\sqrt{1 + \sinh^2 \beta_1 + \sinh^2 \beta_2}}{\cosh \beta_1 \cosh \beta_2} \\ \tanh \beta_1 \tanh \beta_2 \\ n \tanh \beta_2 \end{array} \right) ; \]

(60)
c) and the another "nonsymmetric" charge configuration which can be obtained from the previous one using the replacement 1 ↔ 2.

Finally, Eqs. (46), (48) together with Eqs. (49)–(51) (in the case of the possible Dirac string existence) and Eqs. (57)–(60) (in the Dirac peculiarity free case) give the total concretization of the solution expressed by Eq. (31). We leave the study of its regularity as well as the horizon analysis to the forthcoming articles.

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5 Conclusion

In this paper we have constructed a special class of asymptotically flat solutions of the consistently truncated effective field theory of the bosonic string. This class belongs to the two–functional solution subspace which admits a compact complex form. Our concrete solution is related to the special complex Coulomb–like axisymmetric solution of the 3–dimensional Laplace equation. Our physical interpretation of the obtained solution is based on its asymptotical behaviour near to the spatial infinity. It is shown that using an appropriate choice of the solution parameters one can remove a Dirac string and obtain a solution describing the point–like source with the nontrivial Kalb–Ramond dipole moment and the dilaton charge vanishing in the extremal case and possessing both zero and nonzero values for the nonextremal configurations. The constructed solution class can be transformed into the corresponding one for the nonstatic Kaluza–Klein theory coupled to the dilaton field using the duality relations given in [11]. Moreover, in the case of $d = 2$ it is possible to rewrite our solutions as the solutions of the complete 5–dimensional bosonic string theory compactified on a 3–torus. In the framework of the $(d + 3)$–dimensional effective string theories one can apply the charging symmetry subgroup of U–dualities to generate both the bosonic and heterotic string theory degrees of freedom “missed” in our present “truncated” consideration.

Let us now discuss the performed analysis and its possible generalizations. In fact, the crucial steps had been done in section 3, where we have introduced the matrices

$$\Pi_{kk} = C_k C^\dagger_k.$$  \hfill (61)

These matrices form a set of the mutually orthogonal projectional operators; this statement
is supported by the list of their products. In the considered case of $k = 1, 2$ this list reads:

$$
\Pi_{11}^2 = \sigma_1 \Pi_{11}, \quad \Pi_{22}^2 = \sigma_2 \Pi_{22}, \quad \Pi_{11} \Pi_{22} = 0.
$$

In this paper we have taken the equal signs $\sigma_1 = \sigma_2 = \sigma$, and have studied the dependence of the Dirac string removing in our concrete solution on this common sign. In the context of the scheme presented it will be interesting to develop the general formalism based on the use of $\mathcal{N}$ arbitrary columns $C_k$ and to relate the solution subspace with the corresponding projection operators $\Pi_{kl} = C_k C_l^T$. In this general situation one works again with the closed table of the mutual operator products, and obtains a real possibility to calculate the nontrivial asymptotically flat solutions of the type more general than presented in this paper. It will be interesting to establish the general role of this projectional formalism in the framework of the theories possessing the $\sigma$–model representation.

As the nearest interesting perspective it will be important to develop the projectional formalism in the situation, where the general toroidally compactified heterotic string theory will be projected to the stationary and axisymmetric Einstein–Maxwell theory. This problem seems solvable in the above discussed generalized projectional approach and, in the case of its realization, opens new wide possibilities in extension of the all known solutions of the Einstein–Maxwell theory, including supersymmetric ones [14], to the (super)string theory field.

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