Primordial Magnetic Fields and the Peccei-Quinn Scale

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A strong primordial magnetic field can induce a relaxation of the present bound on the PQ-constant. We show that, considering the present limits on primordial magnetic fields, a value for the PQ-constant very close to the GUT scale is not excluded. This result naturally opens the possibility for the axion to be defined in the context of the GUT theories.

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After about 30 years, the Peccei-Quinn (PQ) mechanism [1] is still the most appealing solution of the strong-CP problem (for a review see, e.g., Ref. [2]), which consists in explaining the smallness of the CP-violation induced by the QCD Lagrangian. This violation resides in the presence of the so called $\Theta$-term, $\mathcal{L}_\text{CP} = (\alpha_s/8\pi)\Theta GG$, where $\alpha_s$ is the fine structure constant of the strong interactions, while $G$ and $\tilde{G}$ are the gluon field and its dual. Experimental limits on the neutron’s electric dipole moment, lead to the unnatural small upper bound $|\Theta| \lesssim 10^{-10}$.

The fundamental new feature of the PQ-mechanism is the existence of an extra global (axial) symmetry, $U(1)_{\text{PQ}}$, spontaneously broken at some energy scale $f_a$, known as the PQ-scale (or PQ-constant). Therefore, at energies below $f_a$ a new phase degree of freedom $\Theta(x) = a(x)/f_a$ emerges as the Goldstone mode of the $U(1)_{\text{PQ}}$ symmetry. This new field $a$, known as axion, is the most relevant prediction of the PQ-mechanism. Quantum effects (chiral anomaly), which explicitly break the PQ symmetry, generate a non trivial axion-gluon interaction of the form $\mathcal{L}_{ag} = (\alpha_s/8\pi)(a/f_a)GG$, and correspondingly an axion potential $V(a)$. The Vafa-Witten theorem [3] states that this is minimized when the Lagrangian is CP-even, that is when the vacuum expectation value of the axion field cancels the original $\Theta$-term in the QCD Lagrangian, $\langle \Theta \rangle = -\Theta$.

The interaction with gluons induces a mass $m_{\text{QCD}}$ for the axion field, which has the constant value

$$m_{\text{QCD}}(T) = m \simeq 6.2\,eV/(f_a/10^5\text{GeV}) \quad \text{for} \quad T \ll \Lambda, \quad (1)$$

at energies (temperatures) below the QCD scale $\Lambda \sim 200\text{MeV}$, while at higher energies it is suppressed as $[4, 5]

$$m_{\text{QCD}}(T) \simeq 0.1m (\Lambda/T)^{3.7} \quad \text{for} \quad T \gg \Lambda. \quad (2)$$

Besides gluons, axions interact with fermions in a way inversely proportional to $f_a$ and with photons through the electromagnetic anomaly $\mathcal{L}_{\gamma\gamma} = (1/4) g_{\gamma\gamma} aF\tilde{F}$. Here, $F$ is the electromagnetic field, $\tilde{F}$ its dual, and $g_{\gamma\gamma} = \alpha_{\text{em}}\xi/(2\pi f_a)$, with $\alpha_{\text{em}}$ the electromagnetic fine structure constant and $\xi$ an order one, model dependent constant. Therefore, the axion phenomenology is characterized by the PQ-constant, a free parameter of the mechanism. Today, the combined limits from terrestrial experiments, the stellar evolution and the supernova neutrino signal exclude all the values of $f_a$ up to $10^6\text{GeV}$ [6, 7]. On the other hand, cosmological considerations exclude the values for $f_a$ above $10^{12}\text{GeV}$ [8].

So, the PQ-scale is not related to any of the relevant scales in high energy physics, being well above the electroweak scale, $T_{\text{ew}} \simeq 250\text{GeV}$, but also largely below the scale of the Grand Unification Theory (GUT), $T_{\text{GUT}} \sim 10^{15-16}\text{GeV}$. It is therefore not very plausible that the PQ-mechanism could be related to the physics at these scales, and this is a rather unattractive feature of this elegant mechanism. What is the origin and meaning of this new scale? Is it possible to relax the bounds on the PQ-constant to a more meaningful scale? We refer to these questions as the PQ-scale problem. A discussion of such a problem might seem premature when there is no experimental evidence that the PQ-mechanism is effectively realized in nature. However, the relevance of the axion in cosmology is universally accepted. Excluding supersymmetric particles, axions are certainly the best candidate for the dark matter component of the universe. It is, therefore, a rather unpleasant result that, for axions to represent the dark matter, it is necessary to fix the PQ-constant to the nowadays meaningless scale $f_a \simeq 10^{12}\text{GeV}$. Understanding the origin of this new scale, or trying to relax this toward an already known scale, is certainly one of the most relevant problem of axion physics. This explains the number of papers that have been addressed to it (see, e.g., Ref. [9, 10, 11]).

The aim of this paper is to investigate the possibility of relaxing the upper bound on the axion-constant through the interaction of the axion with an intense, primordial magnetic field [12].

Before proceeding, we shall analyze the present limits on the intensity of a cosmological magnetic field at the scales relevant to our problem. Though the origin of the observed large scale magnetic fields is still unclear, we cannot exclude the existence of primordial magnetic fields in the early universe [13, 14], as long as their presence does not invalidate the predictions of the standard cosmological model. In particular, they must satisfy
energy density, in the radiation era, is less than the energy
constraint corresponds to having a magnetic field whose en-
where $R \propto g_s^3 T^{-1}$ is the expansion factor, and $g_s$ counts the total number of effectively massless degrees of freedom referring to the entropy density of the universe. It is convenient to parameterize the magnetic field as
$$B = b(T) T^2. \quad (3)$$

In the following we shall assume that a uniform magnetic field is present during the evolution of the axion between the electroweak scale and a few times the QCD-scale, say $T \simeq 1 \text{ GeV}$. Since a magnetic field correlated on the Hubble scale at $T = 1 \text{ GeV}$ can be considered as uniform for $1 \text{ GeV} \lesssim T \lesssim T_{ew}$, we will analyze, to be conservative, the limit on magnetic fields correlated on the Hubble scale at that time, which corresponds to a comoving scale $\xi_B \simeq 6 \times 10^{-2} \text{ pc}$. Unfortunately, no limits coming from the BBN and CMB on these scales exist. The limit coming from BBN refers to uniform magnetic fields at that time or, equivalently, correlated on the Hubble comoving scale $L \simeq 1 \text{ kpc}$. The upper bound is given in Ref. [13]:
$$B(T_{BBN}, L \simeq 1 \text{ kpc}) \lesssim 1 \times 10^{11} \text{ G},$$
where $T_{BBN} = 10^9 K \simeq 0.1 \text{ MeV}$. The strongest limit on small-scale magnetic fields from CMB are given in Ref. [10]. There, it is deduced the limit $B(T_0, L = 400 \text{ pc}) \lesssim 3 \times 10^{-8} \text{ G}$, $T_0$ being the actual temperature, on a magnetic field correlated on a comoving scale $L = 400 \text{ pc}$. In order to convert the above limits in a constraint on magnetic fields correlated on smaller scales (in particular on the scale $\xi_B$) we must perform a suitable average over the magnetic domains. Following the standard procedure (see, e.g. Ref. [14] and references therein) we can write $B(T, L) \equiv B(T, \xi_B)/N^p$, where $N = L/\xi_B$, with $L$ the comoving scales on which we want average and $\xi_B$ the comoving correlation length. Here, $p = 1/2, 1, 3/2$, depending on the statistical properties of the tangled magnetic field [14].

In the two cases referring to BBN and CMB, $N \simeq 2 \times 10^4$ and $N \simeq 6 \times 10^3$, respectively. Therefore, the BBN and CMB limits translate to $B_{\text{max}}(T_{BBN}, \xi_B) \simeq 1 \times 10^{13} \text{ G}$, and $B_{\text{max}}(T_0, \xi_B) \simeq 2 \times 10^{-6} \text{ G}$, respectively, where we have considered the most conservative case $p = 1/2$. Evolving adiabatically the above maximum values of the magnetic field back in time, we get that the limit coming from the CMB analysis is more stringent with respect to the one from BBN of about two order of magnitude, and gives $b_{\text{max}}(T) \simeq 1.2 g_{s}^{2/3}(T)$. In particular, the maximum allowed value at the electroweak scale is $B_{\text{max}}(T_{ew}, \xi_B) \simeq 2 \times 10^{22} \text{ G}$. Essentially, the above constraint corresponds to having a magnetic field whose energy density, in the radiation era, is less than the energy density of the universe, $\rho = (\pi^2/30) g_*(T) T^4$, where $g_*$ counts the total numbers of effectively massless degrees of freedom referring to the energy density of the universe. In fact, imposing that $\rho_B = B^2/2 \lesssim \rho$, we get $b(T) \lesssim 0.8 g_{s}^{1/2}(T)$. (Note that in the range of interest, $1 \text{ GeV} \lesssim T \lesssim T_{ew}$, the quantities $g_*$ and $g_{s}^{1/2}$ are equal [3].)

Coming back to the axion’s cosmology, there are two phenomenological aspects that should be considered, i) an external magnetic field allows the axion to mix with one photon. However, this does not lead to a relevant change of the axion’s cosmology [17, 18, 19] (see also note 2) and will not be considered in this paper; ii) in an external magnetic field the axion has a contribution $m_B$ to its mass [20, 21], so that $m^2 = m_{\text{QCD}}^2 + m_{B}^2$. As we will show, this has at least two possible phenomenological consequences. First, the magnetic field can induce a breaking of the PQ-symmetry independent of the QCD dynamics, and this might spoil the PQ-mechanism. This in principle sets an upper limit on the intensity of the primordial magnetic field. However, as we shall see, this is not competitive with the bounds from BBN and CMB. Second, the magnetic induced mass can force the cosmological axion production mechanism to start earlier, therefore changing the expected axion relic abundance. We will show that, if the primordial magnetic field is sufficiently intense, this is indeed the case. A consequence is a relaxation of the cosmological bound on the PQ-constant. Moreover, the present limits on the intensity of the magnetic field cannot exclude a value of the PQ-constant very close to the GUT scale.

In order to show what is stated above, let us consider the cosmological evolution of the axion field $\Theta$. Today $\Theta$ is settled in the CP-conserving minimum $\Theta_{\text{today}} = \overline{\Theta}$. However, just after the PQ symmetry breaking, at temperatures of order of the PQ-scale, the axion potential is flat and the value of the phase $\Theta$ is chosen stochastically. We shall indicate this as $\Theta_i$. The misalignment between the two angles $\Theta_i$ and $\overline{\Theta}$ is at the origin of an efficient mechanism for the generation of axions, known as the axion misalignment production [3]. The evolution of $\Theta$ is described by the equation of motion
$$\ddot{\Theta} + 3H \dot{\Theta} + m_{\text{QCD}}^2 (\Theta - \overline{\Theta}) = 0, \quad (4)$$
where $H \simeq 1.66 g_{s}^{1/2} T^2 / m_{\text{Pl}}$ is the Hubble parameter with $m_{\text{Pl}}$ the Planck mass.

For high temperatures, the mass term in Eq. (4) is negligibly small compared to the friction (Hubble) term, so the axion remains frozen to its initial value $\Theta_i$. However, as the axion mass becomes dominant over the friction

1 This last aspect was considered in Ref. [22], however without accounting for the temperature effects.
term, $\Theta$ begins to oscillate with the frequency $m_{\text{QCD}}$ and will approach the CP-conserving limit $\Theta_{\text{today}} \sim \Theta$. During this period of coherent oscillations, if axions are not interacting, their number, in a comoving volume, remains constant, so the axion relic abundance today can be easily calculated as

$$\Omega_a \simeq 1.6 \, \Theta_i^2 \, g_{i1}^{-1/2} f_{12} \, (\text{GeV}/T_1),$$

where $g_{i1} = g_i(T_1)$ and $f_{12} = f_{a}/(10^{12}\text{GeV})$. Here, the temperature $T_1$ is such that $m_{\text{QCD}}(T_1) = 3H(T_1)$ and represents, approximately, the time when the oscillations start. $^2$ If the only contribution to the axion mass were given by the QCD effects $^2$, then $T_1 \simeq 0.9 \Lambda_{200} \, f_{12}^{-0.175} \, \text{GeV}$, where $\Lambda_{200} = \Lambda/(200\text{MeV})$. As a consequence, Eq. $^4$ reduces to $\Omega_a \simeq 0.2 \Lambda_{200} \, f_{12}^{-0.175}$. Assuming $\Theta_i \simeq 1$, $^3$ we get $\Omega_a \simeq 0.3$ (i.e. the expected dark matter abundance) for $f_{12} \simeq 1$. Much larger values of $f_{12}$ would cause too much axion production and are therefore excluded. This observation leads to the upper limit on the PQ-constant discussed in the literature, $f_a < 10^{12}\text{GeV}$ $^5$ $^8$.

However, if a strong external magnetic field is present, the axion mass has a magnetic contribution, which in the range of interest for the problem at hand, $1\text{GeV} \lesssim T \lesssim T_{\text{ew}} \simeq 250\text{GeV}$, is $^2$

$$m_B \simeq g_{a1}B \simeq 7.5 \times 10^{-3} \xi b \, \Lambda_{200}^2 \, m \, (T/\Lambda)^2,$$

where in the last equality we used Eq. $^6$. In order to compare the electromagnetic and QCD axion masses, it is useful to introduce the temperature $T_*$ such that $m_B(T_*) = m_{\text{QCD}}(T_*)$. It results $T_* \simeq 1.6 \xi^{-0.18} b^{-0.18} \Lambda_{200}^{-0.35} \, \Lambda$. From the above equation we see that for strong magnetic fields, say $b \sim 1$, the temperature at which the QCD and electromagnetic axion masses are equal is about $T_* \simeq f_{\text{ew}} \times \Lambda$ (see Fig. 1). This means that a strong enough magnetic field induces a contribution to the axion mass which would dominate the standard QCD one sufficiently above the QCD phase transition.

Now, since both $m_B$ and $H$ scale as $T^2$, we can distinguish two cases. If $m_B < 3H$, that is if $b < b_{\text{th}} \simeq 3.5 \times 10^{-4}\xi^{-1} g_{i1}^{1/2} f_{12}$, the (Hubble) friction is always greater then the electromagnetic mass. Therefore, the axion coherent oscillations would start only when the QCD mass equals the friction term. In other words, $b_{\text{th}}$ indicates a threshold value for the parameter $b$, and therefore for the magnetic field, such that if $b < b_{\text{th}}$ the standard analysis of the axion misalignment production applies. On the other hand, for $b > b_{\text{th}}$, the electromagnetic mass term is always greater than the friction term and, consequently, the axion coherent oscillations start at the time when the magnetic field is generated. In the following, to avoid inessential complications, we shall assume that a magnetic field is generated above or during the electroweak phase transition. In this case, if $b > b_{\text{th}}$, the axion coherent oscillations would start at $T_{\text{ew}}$ (above that the magnetic-induced axion mass vanishes $^2$), and the present axion relic abundance would be $\Omega_a \simeq 0.6 \times 10^{-3} \Theta_i^2 f_{12}$. From the above equation we can deduce the maximum value of the Peccei-Quinn constant corresponding to the maximum value allowed for the axion relic abundance, $\Omega_a \simeq 0.3$, which implies that the dark matter component of the universe is composed by cold axions. Taking $\Theta_i \simeq \xi \simeq 1$, we see that this value for the axion abundance is compatible with a PQ-constant equal to $f_a \simeq 0.5 \times 10^{15}\text{GeV}$, a scale very close to the GUT scale. This value of $f_a$ corresponds to $b \gtrsim 1.7$, that is $B \gtrsim 1.5 \times 10^{24}\text{G}$ at the electroweak phase transition. As discussed above, this is compatible with all constraints coming from cosmological and astrophysical analysis and observations.

Before concluding, it is worth observing that the PQ-mechanism is not spoiled by the presence of a magnetic field within the limits allowed by cosmology. This is not obvious since both the magnetic field and QCD effects break the PQ-symmetry, with the former not necessarily toward the CP-conserving minimum. If $B$ is large enough, a temperature $T_* \ll T_{\text{ew}}$, the QCD contribution to the axion potential $\mathcal{L}_{\text{CP}} + \mathcal{L}_{\text{aq}}$ would be negligible with respect to the magnetic one. The Vafa-Witten theorem $^2$, applied to the

\[\begin{align*}
\text{FIG. 1: Solid line refers to the QCD axion mass, Eq. (2), while dotted, dashed, and long-dashed lines refer to the electromagnetic contribution to the axion mass, Eq. (6), for } b = 10, \quad b = 1, \quad b = 0.1, \text{ respectively. Here, we have taken } \xi = \Lambda_{200} = 1.
\end{align*}\]
Lagrangian with the only electromagnetic contribution, \( \mathcal{L}_{\text{em}} \), leads to the inequality \( V[0] \leq V[a] \) for the axion potential energy \( V[a] \). Therefore, the axion dynamically evolves toward that state \( a = 0 \) which minimizes its potential. However, at temperatures sufficiently below \( T_* \), the role of gluons becomes prominent with respect to the electromagnetic contribution and therefore the term \( \mathcal{L}_{\text{em}} \) can be neglected with respect to \( \mathcal{L}_{\text{QCD}} + \mathcal{L}_a \). In this case, \( V[a] \) satisfies \( V[\mathcal{T}] \leq V[a] \) and so the axion field evolves toward the CP-even minimum of the Lagrangian, \( a = -f_a \mathcal{T} \). The PQ-mechanism is therefore effective for the solution of the strong CP-problem, unless the magnetic-induced axion mass dominates the QCD mass until very recent times (\( T_* \sim T_{\text{today}} \)). This in principle sets a bound on the possible intensity of a primordial magnetic field. However, it is clear from the discussion above that, if the magnetic field satisfies the bounds from BBN and CMB, the magnetic mass \( m_B \) is always negligible with respect to \( m_{\text{QCD}} \) under the QCD-phase transition. Therefore, we can safely conclude that no magnetic field allowed by the standard cosmology can spoil the PQ-mechanism.

In conclusion, our analysis shows that a sufficiently intense cosmological magnetic field could considerably modify the expected axion relic abundance. For example, assuming the common value \( f_a \sim 10^{12}\text{GeV} \), we find a threshold value for the magnetic field at, say, the electroweak scale, \( B_{\text{ew}}(T_{\text{ew}}) \sim 3 \times 10^{21}\text{G} \). A magnetic field more intense than that would cause a reduction of the expected axion density. This result is relevant for axion physics and cosmology in general, since axions are widely believed to be a relevant fraction of the dark matter in the universe. In addition, we have shown that a sufficiently intense primordial magnetic field (\( B \gtrsim 10^{24}\text{G} \) at the electroweak scale) would allow the PQ-constant to be \( f_a \sim 10^{15}\text{GeV} \), a scale that could easily be related to the GUT scale. This would solve the widely discussed problem of the meaning of the PQ-scale, requiring no physics beyond the standard model. This result also shows that the present experiments for the axion search at scales \( f_a \lesssim 10^{12}\text{GeV} \) would not be able to exclude the existence of the axion field, unless it would also be possible to prove the non-existence of such an intense primordial magnetic field.

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