Design considerations for a small ducted wind turbine

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Abstract. While ducted wind turbines (DWTs) have not yet enjoyed commercial success, theory has shown their potential and suggests a need to identify and explore key drivers of the design space for DWTs. A governing optimization metric of cost/annual energy production (AEP) was examined and a key cost driver for DWT of turbine thrust identified. A review of generalized actuator disc theory indicated that increasing the axial induction of the duct increases the power/thrust ratio and reduces the cost/AEP. It was shown that the particular selection of the rotor power coefficient, $C_p, \text{Rotor}$ or the total power coefficient $C_p, \text{Total}$, as a design parameter, changes the nature of the AEP optimization, either maximizing AEP with $C_p, \text{Rotor}$ or optimizing $C_p/C_T$ and minimizing turbine size with $C_p, \text{Total}$. Increasing the blade number to raise $C_p$ and lower tip speed ratio optimizes rotor torque as well for generator matching. Iterative interaction between the wind resource, generator specifications, and design elements to minimize cost/AEP is introduced. The relationship between turbine maximum power output and AEP was explored and it was observed that increasing generator size raises AEP/rotor diameter.

Nomenclature

| Symbol | Definition |
|--------|------------|
| AEP    | Annual Energy Production |
| $P$    | Power (W) |
| PDF    | Probability Density Function |
| $C_p, \text{Rotor, Total}$ | Power Coeff. based on rotor area, Duct exit area |
| $C_T, \text{Rotor, Total}$ | Thrust Coeff. based on rotor area, Duct exit area |
| $C_Q$  | Torque coefficient |
| $a$    | Axial induction |
| $A_0$  | Axial induction of duct with zero energy extraction |
| $V_r$  | radial flow field distribution |
| $V_\mu$| mean velocity through rotor |
| $\sigma$ | Rotor solidity |
| $\lambda$ | Tip speed ratio |
| $\alpha$ | Duct airfoil angle of attack |
| $z/C$  | Axial location of rotor plane relative to duct inlet |
| $C$    | Duct airfoil chord length |
| $\Delta r/D$ | Nondimensional Rotor Gap |
| $D$    | Rotor diameter |
| $B$    | Number of blades |
1. Introduction

Ducted wind turbines (DWTs) have historically held a tantalizing promise for increasing the efficiency of horizontal axis wind turbines by capturing a larger stream tube for a given rotor diameter. Despite continued improvements in duct flow modelling [1] and ducted rotor design [2], the expected performance gains have not, to date, resulted in a viable commercial turbine system available on the market. The purpose of this work was to explore generalized actuator disc theory to identify drivers of the DWT design space in order to reduce cost/AEP and improve the commercial potential of DWTs.

To accomplish this, the key parameters in the ducted turbine design space had to be identified and evaluated. A governing principle must first be selected with which to analyze and compare the sensitivity of parameters within the design space. In the context of this work there are two principal design drivers, those relating to performance and those relating to cost. Performance metrics can include nameplate rating, Cost/Watt, Annual Energy Production (AEP), and Levelized Cost of Energy (LCOE). AEP is defined as

\[ \text{AEP} = T \int_0^{U_{\text{max}}} P(U) p(U) dU \]  

where \( T \) is the time in a year, \( P(U) \) is the turbine power curve, and \( p(U) \) is the local wind PDF. LCOE incorporates all capital costs, financing costs and operating expenses, weighed against the revenue generated through energy sales to determine a metric for a wind turbine installation in terms of $/kWh.

The governing metric chosen was unit cost /AEP. Maximizing this value optimizes the commodity produced (energy) at the minimum turbine unit cost. Unit cost was defined as the installed cost of the entire turbine including the tower, foundation and electrical up to the connection to the electrical grid. This metric excludes factors beyond the scope of wind turbine design, such as plant operations, siting, financing, and the balance of system capital costs. Some of these costs, such as turbine control and power management electronics, tower, and foundation costs were included indirectly in considering turbine unit cost by considering the impact of turbine design on the cost of items as available in the marketplace.

Four design parameter types were defined for classification as indicated in Table 1: performance drivers, cost drivers, optimization drivers, and tuning parameters. Tuning parameters are those which do not drive the design, but which can be optimized once a design is established to either maximize AEP or minimize cost. Pure performance or cost drivers are driven to their maximum/minimum values within the design space. Parameters which affect both performance and cost must be optimized, and it is this optimization process which defines the resulting design space.

Table 1. DWT design parameters.

| Parameter Type       | Effect on Design Space          | Example               |
|----------------------|---------------------------------|-----------------------|
| Performance Driver   | Affects turbine AEP             | Duct Angle (\( \alpha \)) |
| Cost Driver          | Affects turbine Cost            | Thrust coefficient (\( C_T \)) |
| Optimization Driver  | Nonlinear effect on AEP and cost| Rotor Placement (\( z/C \)) |
| Tuning Parameter     | Optimized to maximize AEP      | Rotor Gap (\( \Delta r/D \)) |

Retail turbine system costs for several small wind turbines (SWTs) were compiled from manufacturers MSRP and retailers’ advertised prices, along with the advertised AEP of each as shown in Figure 1. When power curves were available AEP was calculated with Equation (1) using a 5 m/s Rayleigh wind PDF. Otherwise the AEP is as stated by the manufacturer for either a 5 or 6 m/s Rayleigh wind resource. The trend lines shown have a cost/AEP of $2.31/kWh at 5 m/s and $1.89/kWh at 6 m/s. Clarkson University has developed an operational DWT with a cost/AEP of $1.46/kWh at 5 m/s, exceeding the
performance of the open rotor turbines (ORTs) shown at 6 m/s average wind conditions. A live view of the turbine can be found here: https://resources.clarkson.edu/cameras/wind_turbine.php. Various commercial DWT manufacturers were investigated, including Ogin, WindTamer, and Halo Energy, but cost data was unavailable, or unreliable, and thus they are not included in Figure 1.

![Figure 1](image1.png)

**Figure 1.** Comparison of SWT Cost/AEP.

The turbine costs, as percentage of total cost, are shown in Figure 2. These data have been compiled from confidential industry data by NREL for utility scale wind turbines [3] and by DWEA for Small Wind Turbines (SWT) [4]. Comprehensive cost data for DWTs does not exist, so cost data for a single turbine design, developed at Clarkson University, is presented here as a preliminary reference only.

![Figure 2](image2.png)

**Figure 2.** Wind turbine costs as percentage of total cost [3] [4].
Figure 2 identifies the top three respective cost sets as Nacelle, Rotor, and Electrical for a utility scale turbine, Electrical, Tower, and Nacelle for a SWT, and Electrical, Tower, and rotor for a DWT. The duct of a DWT was considered part of the rotor, as it is part of the aerodynamics package which creates torque for the generator. The nacelle consists of the generator, generator support systems (gearbox, cooling etc..) the fairing, and the structure required to support these systems and transfer the loads to the tower top. For DWTs, the duct support structure was considered in the nacelle cost. Scale effects of differing turbine sizes were not considered in this comparison. The trends of Figure 2 show that electrical and tower are key cost drivers in all classes, and that only for DWTs are rotor costs preeminent over nacelle costs. This suggests that it is the cost of providing the DWT’s aerodynamic advantage that is of primary concern, rather than the structural cost that the DWT adds to the nacelle. Structural costs (primarily tower/foundation) are driven by two primary factors: turbine weight and rotor thrust. Of these two, thrust is of primary concern, as it is the turbine thrust which generates the primary bending moment on the tower and torque at the foundation and is multiplied by tower height.

The DWT potential cost drivers identified were the generator and inverter, the aero components (duct/rotor), and the tower & foundation, which taken together account for 85% of the turbine cost. DWT and ORT generator and electrical costs are the same for a given power output, so there is no added cost in this area for a DWT. The combined tower and foundation costs are 38% of total turbine cost, suggesting that turbine thrust is the largest cost driver in the DWT design. A potential cost driver unique to the DWT is the complexity of installation, given the requirement to erect a turbine with a duct installed.

Factors beyond the turbine design were identified that influence the cost drivers and cause discontinuities in the cost curve, or inflection points. One such inflection point occurs when a turbine can no longer be self-erecting and requires a crane for installation. Another is the rotor size at which cambered and twisted flat plate airfoils, utilized in the current Clarkson design, lose their effectiveness relative to full thickness airfoils. The rotor size break point in certification regulations between small and full-size wind turbines is another inflection point, requiring more thorough and costly design and certification requirements for turbines with a rotor swept area >200 m². In addition, the maximum size at which a DWT does not require a yaw drive, and can instead rely on its inherent (if properly designed) passive yaw stability, is also an inflection point. A fourth example is the array of foundation types, including ballasted, guyed, helical pile, and poured monopole tower and foundation types, and the design limitations for each case. Inflection points are not parameters themselves, however they may cause discontinuities in design parameters. These inflection points break the design space into domains with separate optimization results, some of which may not be competitive with ORT designs.

Given the governing metric of cost/AEP, it was found that a DWT with a 5 m/s Rayleigh wind PDF can outperform ORT trends at 5 m/s and even at 6 m/s Rayleigh PDF wind resources. In comparing classes of wind turbine, it was found that the cost of the duct tends to drive overall turbine cost, rather than the cost of structure to support the duct. It was also noted that thrust drives tower and foundation cost, has greater impact in DWT, and as percentage of total cost, greater weight than duct cost. Differences in inflection points between ORTs and DWTs contribute to their relative cost effectiveness. These factors influence the cost competitiveness of a commercial DWT and show that the key parameters minimize thrust/AEP, duct cost relative to performance, and optimize the location of inflection points on the cost curve.
2. Limit state analysis

To identify these aforementioned key parameters, and even others, it is important to first examine the physics of the underlying flow field itself. The well-known work by Betz identifies the performance limits of an ideal open rotor turbine based on an inviscid flow actuator disc model. In 2008 Jamieson expanded on this work and the work of van Bussel [5] to develop a generalized actuator disc model, shown in Figure 3, that applies equally well to open rotor (a) and ducted rotor (b) flow [6]. In this analysis Jamieson found several results which help to give direction to the DWT design space.

![Figure 3. Actuator disc flow models a) open rotor b) ducted rotor [6].](image)

The axial induction in open rotor flow is defined as the reduction in the far upstream flow velocity due to energy extraction by the actuator disk. For a DWT, a second induction term, $a_0$, is added to describe the increase in velocity at the rotor plane generated by the duct. The term $a_0$ is negative to match the sign convention of $a$, for which a velocity decrease is positive. The results of Jamieson’s analysis show that while open rotor $C_p,_{\text{max}}$ is $16/27$, for augmented flow the maximum energy extraction efficiency is

$$C_p,_{\text{max}} = \frac{16}{27} (1 - a_0) \tag{2}$$

This shows that the limit of energy extraction is governed by the maximum flow augmentation achievable by the augmentation device. Jamieson also noted that rotor thrust coefficient is unaffected by flow augmentation and remains

$$C_{T, \text{Rotor@}_C_p,_{\text{max}}} = 8/9 \tag{3}$$
whether ducted or open rotor. However, this generalized actuator disc model does not account for flows external to the stream tube, and results from Phillips [8] indicate a real rotor may be optimally loaded above 8/9, although the additional disc loading does not appear to be significant. The implications of Jamieson’s theory are that based on equal rotor area the addition of a properly designed duct has a positive influence on the power coefficient without a significant increase of thrust coefficient. Jamieson performed the same analysis with equal source areas (stream tube inlet), instead of equal rotor areas. In this case, the downstream area, velocity, and energy extracted are the same, but the rotor diameter will be smaller. His results are given in Table 2, below.

### Table 2. Wind Turbine optimum performance parameters [6].

| Parameter                     | ORT Flow | Augmented Flow          |
|-------------------------------|----------|-------------------------|
| Maximum $C_p$                 | 16/27    | 16/27(1-$a_0$)          |
| Axial induction @ $C_p_{\text{max}}$ | 1/3      | $(1 + 2a_0) / 3$        |
| Thrust coefficient @ $C_p_{\text{max}}$ | 8/9      | 8/9                     |
| $\Delta P$ across rotor       | 4/9$pV_0^2$ | 4/9$pV_0^2$           |

By comparing Jamieson’s power coefficients at equal power output and flow conditions, the area ratio of the two turbine rotors is noted to be:

$$A_{\text{ducted}} = A_{\text{open}}/(1-a_0).$$

(4)

And thus the associated thrust becomes:

$$T_{\text{ducted}} = T_{\text{open}}/(1-a_0).$$

(5)

Therefore, within the generalized actuator disc model there are two complementary conclusions. First, given equal power output, the rotor area and thrust of a DWT will be smaller. Second, given equal rotor area, the maximum power of a DWT will be larger, and the thrust loads equal. In either case, the power to thrust ratio will be higher. Since the AEP is a function of the power curve and a primary cost driver is the thrust, a higher power to thrust ratio suggests a reduction in cost/AEP.

These observations were then tested by examining recent work by Sadeghi et al [7] on the optimization of Clarkson’s 2.5 m rotor diameter, 3.3 m duct diameter DWT. That work investigated a number of parameters for turbine optimization, including $C_{T, \text{Rotor}}$, $\alpha_{\text{duct}}$, and axial placement of the rotor within the duct for an optimal duct design. The data in Sadeghi’s work was used to examine the effect of duct chord length to rotor diameter (essentially a fineness ratio for DWTs) on $C_p$ and $C_t$. A linear trend was identified when $C_{T, \text{Total}}$ was plotted against $C_p$ for several optimization cases, shown in Figure 4. However, a parabolic trend was found for the same data when $C_{T, \text{Total}}$ was plotted against $C_p$. The data callouts in Figures 4 and 5 indicate the c/D ratio associated with each optimized case. The trendlines indicate that when optimizing on $C_{p, \text{Rotor}}$, $C_{T, \text{Total}}$ increased with rotor diameter. However, if the optimization is based on $C_{p, \text{Total}}$, an optimum $C_{p, \text{Total}}$ of ~0.64 was identified at an associated $C_{T, \text{Total}}$ of ~0.975.
Figure 4. $C_{P, \text{Rotor}}$ vs $C_{T, \text{Total}}$ trend analysis.

Figure 5. $C_{P, \text{Total}}$ vs $C_{T, \text{Total}}$ trend analysis.

The cause of the differing trends was also found in Sadeghi’s work, in which he stated “The rotor power coefficient was [...] insensitive to the rotor’s axial location for positions ranging from upstream of the throat to nearly half the distance down the duct.” The effect of optimizing for $C_{P, \text{Rotor}}$ as in Figure 4 is to place the rotor where the duct is most effective and the rotor is smallest because $C_{P, \text{Rotor}} \propto P/A_{\text{Rotor}}$. $C_{P, \text{Rotor}}$ is a maximum near the inlet of the duct and maximizes turbine size for a given rotor diameter. Optimizing for $C_{P, \text{Total}}$ as in Figure 5 reduces the total area because $C_{P, \text{Total}} \propto P/A_{\text{Total}}$. This places the rotor deeper into the expansion of the duct at $\sim0.75$ chord as shown in Figure 8, which reduces the duct exit area relative to the rotor area. The implications of these results are that maximizing $C_{P, \text{Rotor}}$ increases the AEP in our governing metric, while minimizing turbine size and optimizing thrust results in a lower AEP and lower cost. Therefore, optimizing cost/AEP requires the comparison of these alternative design spaces.
3. Selected elements of the design space

The preceding section illustrates the manner in which the geometry of the DWT design drives the design’s performance through $C_p$ and cost through $C_T$. The next step is to outline the design process steps, identify parameters relevant to this process, and divide them into the parameter types of Table 1. An example parameter of each type is identified as part of the duct design and discussion of these follows. It was found that the design of the duct defines the actuator disc properties, which define elements of the rotor design space. One factor within the rotor design space, the blade number, was then analyzed to determine the parameters that make up that factor within the rotor design space and how these parameters interact with the turbine design in an iterative manner.

3.1 Duct Design Versus Rotor Design

Sadeghi [7] outlined the key parameters for duct optimization, shown in Figure 6 below. These include chord to rotor diameter ratio ($c/D$), duct angle ($\alpha$), rotor gap to diameter ratio ($\Delta r/D$), and the rotor’s axial placement within the duct ($z/c$). The parameter $\alpha$ is a design driver, as it is one of the parameters that define the diameter of the duct and strongly affect the turbine performance as shown in Figure 7. At some point, increasing $\alpha$ causes the duct to “stall” and performance drops rapidly.

The parameter $z/c$ is an optimization driver as it drives duct size as well as contributing to the optimization of thrust. $\Delta r/D$ is a tuning parameter as it will affect the performance optimization but will not affect either duct or rotor design in a significant way. Figure 8 illustrates that the parameter $z/c$ can be a design or optimization driver depending on the type of analysis performed. If $C_{p, \text{Total}}$ is the basis of analysis, it drives duct size as well as contributing to the optimization of thrust as previously shown. However, if $C_{p, \text{Rotor}}$ is selected as a basis, $z/c$ has a weak influence (<5%) on the power coefficient up to ~60% of chord length, and then has an increasingly detrimental effect on performance.

![Figure 6. Duct design parameters [7].](image-url)
Several elements of the rotor design space are fixed by the duct design. These include the rotor thrust coefficient, the radial distribution of the flow velocity through the rotor, and the velocity’s mean value. Additional elements of the rotor design to be optimized are the blade number, solidity, tip speed ratio, and rotor RPM. These parameters, along with those of the blade airfoil(s), define the blade geometry and the resulting torque coefficient which interacts strongly with the generator design.

3.2 Blade Number Effect Examples
The effect of blade number (B) on power and more specifically torque of a DWT was studied by Wang and Chen [9]. Two fixed blade designs were analyzed, where for each design increasing the blade count increased solidity. In Figures 9 and 10 their results show that increasing blade number increases the torque coefficient $C_Q$ and increases $C_P$ to a point, while reducing $\lambda$ at the optimal point. The optimal points of these figures were plotted against each other in Figure 11 to show $C_P$ vs $C_Q$ at the optimal $\lambda$ (shown in brackets) for each blade number. The trendline suggests an optimal point between 4 and 6 blades exists (likely either 4 or 5 blades) with a corresponding $\lambda$ between 4.7 and 6.
The foregoing analysis does not address the effect of solidity on power and torque. Visser and Duquette [10] reported that for optimized blades on an ORT, $C_p$ increased with blade number, the optimum solidity increased with blade count, and that increasing the solidity lowered $\lambda$ at $C_{p, \text{max}}$, as shown in Figure 12 below. It can also be seen from both studies that $C_{Q, \text{max}}$, which equals $C_p/\lambda$, likely has an optimum value within the design space based not only on maximizing $C_p$, but also optimizing $C_Q$ based on generator specifications. The result is an iterative process to optimize AEP with the generator and rotor design elements as inputs and solidity and tip speed ratio as outputs. The rotor design elements $C_p$, $C_Q$, $\sigma$, and $\lambda$ interact with the specification of generator RPM and torque curve, and therefore the AEP based on the local wind PDF. The outputs include $C_p$, $C_T$, $C_Q$, and $\lambda$. These outputs in turn define the AEP and affect the generator specifications ($C_Q$ and $\lambda$), and tower design ($C_T$). An iterative process is thus required to converge the optimization problem, with AEP as the objective function to maximize.
Figure 12. Maximum $C_p$ vs $\lambda$ for 3, 6, and 12 blades (optimum BEM analysis) [10].
4. Generator Performance and Sizing Example

To analyze the effect of scaling the size of the turbine’s generator on a turbine’s output power, the experimental wind tunnel data from Clarkson’s 2.5 m DWT [11] was extrapolated to generate a series of power curves up to 3500 W, as shown in Figure 13. This effectively increases the “nameplate rating” of the turbine by increasing the generator size. As can be seen, increasing the nameplate rating by 67% from 1.2 kW to 2.0 kW increases AEP by only 20% (from 3931 kWh to 4785 kWh), assuming a 5 m/s Rayleigh PDF, also shown in Figure 13. Increasing the generator again by a further 75% from 2.0 kW to 3.5 kW increases the AEP by only a further 10%. Hence, while increasing the generator size can increase the advertised power output of a wind turbine, it will not significantly improve the AEP of the turbine if it is poorly matched to the available wind resource.

![Figure 13. DWT generator sizing effects (2.5 m rotor.) [11]](image-url)

Specifying a turbine’s performance in terms of AEP instead of power incorporates the available wind resource and gives a more realistic description of the turbine’s comparative performance, particularly for DWTs. When the extrapolated AEP was plotted against rotor diameter for a variety of generator sizes, interesting trends emerged as shown in Figure 14. DWTs outperform ORTs significantly for the same rotor size, even when comparing the AEP of an ORT having a diameter equal to duct diameter of the DWT in Figure 14 (3.3 m for 2.5 m rotor, 3.7 m for 3 m rotor etc.) AEP/Droter, the slope of the trend line, is also improved for DWTs. Lastly, increasing generator size for each rotor shows a trend of diminishing returns, while the generator capacity lines show that increasing the generator capacity for DWT increases the slope of the trend line.
5. Key Conclusions

In analyzing DWT costs, it was found that turbine thrust is the key cost driver in DWT, affecting the tower, foundation, and nacelle costs and demonstrating that if thrust is lower in DWTs than ORTs of comparable AEP, DWTs will outperform ORTs. In investigating the limit state of wind turbines, it was shown that given an equal power output the rotor area and rotor thrust of a DWT will be smaller, resulting in a higher power/thrust ratio, and by extension, lower cost/AEP. It was found by comparing $C_P, \text{Rotor}$ to $C_P, \text{Total}$ that maximizing $C_P, \text{Rotor}$ will maximize AEP, and alternately maximizing $C_P, \text{Total}$ will both reduce overall turbine size and optimize the corresponding thrust. It was found that it is the duct design that sets the $C_P$ of a DWT, and the optimization of the rotor design matches the rotor to the duct performance and generator sizing. From the rotor design observations, $C_P$ increased with blade number, optimum solidity increased with blade count, increasing the solidity lowered $\lambda$ at $C_P, \text{max}$, and $C_Q, \text{max}$ likely has an optimum value based maximizing $C_P$ while optimizing $C_Q$ for generator matching. The final conclusion was that the nameplate rating of a turbine has a weak correlation with a wind turbine’s purpose of generating energy, while the AEP has a strong correlation. Increasing generator size to increase nameplate rating has diminishing returns, but does increase the AEP/D$_{\text{rotor}}$ which encourages the selection of larger diameter turbines with increased generator capacity.

Figure 14. Comparison of annual energy output trends.
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