Study of Asymmetric Nuclear Matter Using Skyrme Potential

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Abstract. The binding energy, symmetry energy, pressure, velocity of sound and the incompressibility are calculated for asymmetric nuclear matter using Skyrme interaction. The behavior of these physical quantities is studied for different values of the asymmetry parameter $\alpha$, the density $\rho$ and the temperature $T$.

Key word: Nuclear matter, Neutron matter, binding energy, Skyrme interaction

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I. Introduction

Static properties of nuclear matter, e.g., binding energy, symmetry energy, incompressibility, etc; can be determined by the equation of state (EOS). In the last 10 years, the study of EOS of nuclear matter has a great interest in nuclear physics and astrophysics [1-3]. The equation of state of nuclear matter NM is closely related to the study of nuclear fission, heavy ion reactions and hot neutron stars in astrophysics. It is also interesting to study the thermal properties of NM, e.g. free energy, pressure, entropy, effective mass and chemical potential and all possible phases in which the matter may exist. Besides using a realistic NN interaction (see e.g. Friedman and Pandharipande FP [4]), many calculations were carried out by the use of effective NN interactions. Most of the calculations of hot NM considered the symmetric case [5, 6], or the asymmetric one [7, 8]. The polarized NM has also been studied by Mansour et al. [9, 10] at zero and finite temperatures. Skyrme effective interaction potential has been widely used to investigate the properties of nuclear matter. This is because it produces simple analytical expressions to deal with. The parameters of the Skyrme force have been calculated by many authors [11, 12]. Nuclear matter and neutron matter properties were used to put constraints on the parameters which are not well determined from the nuclear data.

In the present work a Skyrme interaction [13] has been chosen to calculate the energy, symmetry energy, pressure, velocity of sound and incompressibility. The calculations for the physical quantities are presented as a function of the density $\rho$, the temperature $T$ and the asymmetry parameter $\alpha$. The main aim of the present work is to study the behavior of nuclear matter properties with extreme neutron excess and a more realistic neutron matter with a proton mixture. The reason for this is that in the last stage of type II supernova [14, 15], because of electron capture processes, a highly asymmetric nuclear matter is formed and the star reaches the proton - to neutron ratio of $Z/N = \frac{1}{2}$. This value stays almost constant during the collapse time until the core bounces and the shock wave is formed.
Figure 1. The energy per nucleon (a), the pressure (b), the velocity of sound (c) and the incompressibility (d) at zero temperature are shown as a function of the density for different values of $\alpha \tau$.

Figure 2. The energy per nucleon (a), the pressure (b), the velocity of sound (c) and the incompressibility (d) are shown as a function of $\alpha \tau$ at temperature $T = 0$ for different values of the density.

Figure 3. The free energy (a), the pressure (b), the velocity of sound (c) and the incompressibility (d) at $\rho_o$ are shown as a function of $\alpha \tau$ for different values of the temperature.

Figure 4. The symmetry free energy as a function of the density at temperatures $T = 0, 5$ and $10$ MeV.
Results and discussion

Nuclear matter is composed of \( N \) neutrons and \( Z \) protons with \( N \neq Z \) for asymmetric nuclear matter. All the nucleons are contained in a periodicity box of volume \( \Omega \). The composition of the system is characterized by

\[
A = N + Z,
\]  
(1)

The neutron excess parameter is defined by

\[
\alpha_{\tau} = \frac{(N - Z)}{A}.
\]  
(2)

In our calculation we consider the Skyrme interaction which is given by [16].

\[
V_{\text{Skyrme}} = t_\omega (1 + x_\omega P_\omega) \delta + \frac{1}{2} t_\omega (1 + x_\omega P_\omega) (\vec{k}^2 \delta + \vec{\delta k}^2)
\]

\[
+ t_\omega (1 + x_\omega P_\omega) \vec{k}^\tau \cdot \delta \vec{k} + \frac{1}{6} t_\omega (1 + x_\omega P_\omega) \rho^\sigma (R) \delta,
\]  
(3)

Where \( \delta = \delta(r_i - r_j) \), \( \vec{k} = \frac{1}{2i} (\nabla_i - \nabla_j) \), is the relative momentum operator acting on the wave function to the right and \( \vec{k}^\tau \) is the adjoint of \( \vec{k} \). \( P_\omega \) is the spin-exchange operator and \( R = (r_i + r_j) / 2 \).

For the detailed theory and method of calculation we refer the reader to our previous works [9, 10].

In the present work the properties of the nuclear matter as a function of the density \( \rho \), the neutron excess parameter \( \alpha_{\tau} \), and the temperature \( T \) are calculated. The range of \( \alpha_{\tau} \) varies between 0 and 1 i.e. from symmetric nuclear matter to a highly asymmetric case which has a neutron excess, then to the other extreme case of pure neutron matter. Figure (1) gives the values of the energy per nucleon \( E/A \), pressure \( P \), velocity of sound \( V/C \) and the incompressibility \( \kappa \) at zero temperature for different values of \( \rho \) and \( \alpha_{\tau} \). The energy shows that it has a pronounced minimum at \( \rho_o = 0.158 \text{ fm}^{-3} \) for \( \alpha_{\tau} = 0 \) with similar trends for larger values of \( \alpha_{\tau} \) except for \( \alpha_{\tau} = 1 \) it shows no bound states and the energy in this case lies above the symmetric nuclear matter case. For the other three physical quantities \( P, V/C \) and \( \kappa \), they are presented in figure (1) b, c and d. They increase with increasing \( \rho \) as well as with increasing \( \alpha_{\tau} \). Figure (2) gives the behavior of \( E/A, P, V/C \) and \( \kappa \) as a function of \( \alpha_{\tau} \) for different values of the density at \( T = 0 \). They slightly increase with the values of \( \alpha_{\tau} \), and at higher densities the physical quantities are larger than those for small \( \rho \). Figure (3) gives the change of the four physical quantities as a function of \( \alpha_{\tau} \) and \( T \) at \( \rho_o \). The free energy and incompressibility increase with increasing \( \alpha_{\tau} \), but they decrease by increasing the temperature. For the pressure and velocity of sound it is clear that they increase with increasing \( \alpha_{\tau} \), whereas they increase by increasing the temperature. The symmetry energy which is related to the difference between the energy at \( \alpha_{\tau} = 0 \) and \( \alpha_{\tau} \) by

\[
E(\rho, \alpha_{\tau}) = E(\rho, 0) + E_{\text{sym}}(\rho) \alpha_{\tau}^2
\]

is shown in figure (4). Figure (4) gives the free symmetry energy as a function of the density for different temperatures. It is clear that the symmetry energy increases as the density increases as well as when temperature is increased. The results obtained in this work and the behavior of the physical quantities agrees with previous theatrical estimates [4] using realistic potentials.

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