On Finite Width of Quark Gluon Plasma Bags

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Abstract

Within an exactly solvable model I discuss an influence of the medium dependent finite width of QGP bags on their equation of state. It is shown that inclusion of such a width allows one to naturally resolve two conceptual problems of the QGP statistical description. On the basis of the proposed simple kinetic model for a sequential decay of heavy QGP bags formed in high energy elementary particle collisions it is argued that by measuring the energy dependence of life time of these bags it is possible to distinguish the case of critical point existence from the case of tricritical point.

1. Introduction. – A lot of experimental and theoretical efforts is aimed to determine the equation of state (EoS) of the strongly interacting matter. Despite the great achievements of these efforts [1] even the bulk properties of the quark-gluon plasma (QGP) EoS are not well known. Thus, such important characteristics as the mean volume and life time of QGP bags formed in heavy ion collisions have not caught a necessary attention yet. It is clear, however, that right these quantities may put some new bounds on the spacial and temporal properties of the QGP created in high energy collisions. As shown in [2] it is possible to naturally resolve the HBT puzzles at RHIC energies, if one assumes that the QGP consists of droplets of finite (mean) size. On the other hand the short life time of heavy QGP bags found recently within the Hagedorn-Mott resonance model [3] and within the finite width model (FWM) [4, 5] may not only play an important role in all thermodynamic and hydrodynamic phenomena of the strongly coupled QGP matter, but may also explain the absence of strangelets [6] or, more generally, why the finite QGP bags cannot be observed at energy densities typical for hadronic phase [4] (see below). Therefore, an investigation of the mass and volume distributions along with the life time of the QGP bags and the corresponding consequences for both the experimental observables and theoretical studies is vitally necessary for heavy ion phenomenology. The present paper is devoted to a discussion of these problems in the framework of the FWM.

2. The Finite Width Model. – The FWM employs the most convenient way to study the phase structure of any statistical model by analyzing its isobaric partition [7–10] and to find the rightmost singularities of this partition. Hence, I assume that after the Laplace transform the FWM grand canonical partition $Z(V, T)$ generates the following isobaric partition:

$$
\hat{Z}(\lambda, T) \equiv \int_0^\infty dV \exp(-\lambda V) Z(V, T) = \frac{1}{[\lambda - F(\lambda, T)]},
$$

where the function $F(\lambda, T)$ is a generalized partition

$$
F(\lambda, T) = \int_0^\infty dv \int_0^\infty ds \int_0^\infty dm \rho(m, v, s) \exp(-\lambda v) \phi(T, m)
$$

(2)
of bags of mass \( m \), volume \( v \) and surface \( s \) defined by their mass-volume-surface spectrum \( \rho(m,v,s) \). The partition \( \rho \) is a generalization of the statistical ensembles with fluctuating extensive quantities discussed recently in [11]. Note that one could also introduce in \( \rho \) the perimeter fluctuations, which may play an important role for small hadronic bubbles [12] or for cosmological phase transition [13], but we neglect it because the curvature term has not been seen in such well established models like the Fisher droplet model (FDM) [14, 15], the statistical multifragmentation model (SMM) [16, 17] and many other cluster systems discussed in [8, 9, 10, 15]. A special analysis of the free energy of 2- and 3-dimensional Ising clusters, using the Complement method [18], did not find any traces of the curvature term (see a detailed discussion in [9]). Such a result is directly related to the QGP bags because quantum chromodynamics (QCD) is expected to be in the same universality class [19] as the 3-dimensional Ising model whose clusters were analyzed in [18].

The thermal density of bags of mass \( m \) and a unit degeneracy is given by

\[
\phi(T, m) = \frac{1}{2\pi^2} \int_0^\infty p^2 dp \, \exp \left[ - \frac{(p^2 + m^2)^{1/2}}{T} \right] = \frac{m^2 T}{2\pi^2} \frac{K_2 \left( \frac{m}{T} \right)}{} .
\]

(3)

It is convenient to divide the mass-volume-surface spectrum into the discrete mass-volume spectrum of light hadrons and the continuum contribution of heavy resonances \( \rho(m,v) \)

\[
\rho(m,v,s) = \sum_{j=1}^{N_m} g_j \, \delta(m-m_j)\delta(v-v_j) \, \delta(s) + \Theta(v-V_0) \Theta(m-M_0) \, \delta(s-a_s v^\kappa) \rho(m,v) \rho_1(s),
\]

(4)

The first term on the right hand side (r.h.s.) of (4) represents the contribution of a finite number of low-lying hadron states up to mass \( M_0 \approx 2 \) GeV [4]. This function has no \( \lambda \)-singularities at any temperature \( T \) and can generate only a simple pole of the isobaric partition, whereas the mass-volume spectrum of the bags \( \rho(m,v) \) on the r.h.s of (4) is chosen to generate an essential singularity \( \lambda_Q(T) \equiv \rho_Q(T)/T \) which defines the QGP pressure \( \rho_Q(T) \). For simplicity here I consider the matter with zero baryonic charge.

The continuous part of the spectrum \( \rho(m,v,s) \) introduced in [4] is parameterized as

\[
\rho(m,v) = \frac{N_f}{\Gamma(v)} \frac{m^{\alpha+2}}{m_0^{\alpha+2}} \exp \left[ \frac{m}{T_H} - \frac{(m-Bv)^2}{2\Gamma(v)} \right], \quad \text{and} \quad \rho_1 \left( \frac{v^\kappa}{v^\kappa} \right) = \frac{f(T)}{v^\kappa} \exp \left[ -\frac{\sigma(T)}{T} \right].
\]

(5)

As it is seen from (5) the mass spectrum \( \rho(m,v) \) has a Hagedorn like parameterization and the Gaussian attenuation around the bag mass \( Bv \) ( \( B \) is the mass density of a bag of a vanishing width) with the volume dependent Gaussian width \( \Gamma(v) \) or width hereafter. I will distinguish it from the true width defined as \( \Gamma_R = \alpha \Gamma(v) \) (\( \alpha \equiv 2\sqrt{2\ln 2} \)). It is necessary to stress that the Breit-Wigner attenuation of a resonance mass cannot be used in the spectrum (5) because in case of finite width it would lead to a divergency of the mass integral in (2) above \( T_H \) [4, 5].

The normalization factor obeys the condition \( N_f^{-1} = \int_{M_0}^\infty \frac{dm}{\Gamma(v)} \exp \left[ -\frac{(m-Bv)^2}{2\Gamma(v)} \right] \). The constants \( a > 0 \) and \( b > 0 \) define the Fisher exponent \( \tau \equiv a + b \) [4, 5] (also see later).

3. Important Features of the FWM Spectrum. – The spectrum in (5) contains the surface free energy \( (\kappa = 2/3) \) with the \( T \)-dependent surface tension which is parameterized as \( \sigma(T) = \sigma_0 \cdot \frac{T_0-T}{T_0^2} \cdot 2^{l+1} \) \( (l = 0, 1, 2, \ldots) \) [9, 20], where \( \sigma_0 > 0 \) can be a smooth function of temperature. For \( T \) not above the tricritical temperature \( T_c \) such a parameterization is justified by the usual cluster models like the FDM [14, 15] and SMM [16, 17], whereas the
general case for any $T$ can be derived from the surface partitions of the Hills and Dales model [20]. Note that the Hills and Dales model [20] explicitly accounts for all possible surface deformations which correspond to the same cluster volume and, therefore, it is another example of the statistical partition with fluctuating extensive quantity, which in this case is cluster surface.

In Ref. [9] it was rigorously proven that at low baryonic densities the first order deconfinement phase transition degenerates into a cross-over just because of negative surface tension coefficient for $T > T_c$. The other consequences of the present surface tension parameterization and the discussion of the absence of the curvature free energy in (5) can be found in Refs. [9, 18, 21].

The power $\kappa < 1$ which describes the bag’s effective surface is a constant that, in principle, can differ from the typical FDM and SMM value $\kappa = \frac{2}{3}$ for which the coefficient $a_s$ is $a_s \equiv (36\pi)^{\frac{1}{3}}$. This is so because near the deconfinement phase transition region the QGP has low density and, hence, like in the low density nuclear matter [22], the non-spherical bags (spaghetti-like or lasagna-like [22]) can be favorable (see also [12, 13] for the bubbles of complicated shapes). A similar idea of “polymerization” of gluonic quasiparticles was introduced recently [23].

Note that in contrast to the continuous part of the spectrum (4) its discrete part does not contain the surface free energy because according to the present days status of the statistical model of hadron gas this is not necessary [24].

The spectrum (5) has a simple form, but is rather general since both the width $\Gamma(v)$ and the bag’s mass density $B$ can be medium dependent. In [4] it is shown that the FWM has no contradiction, if $\Gamma(v) \equiv \Gamma_1 = \gamma \sqrt{m}$ only ($\gamma = \text{const}$ of $v$).

For large bag volumes ($v \gg M_0/B > 0$) the normalization factor $N_\Gamma$ can be found to be $N_\Gamma \approx 1/\sqrt{2\pi}$. Similarly, one can show that for heavy free bags ($m \gg BV_0, V_0 \approx 1\, \text{fm}^3$ [4], ignoring the hard core repulsion and thermostat)

$$
\rho(m) \equiv \int_{V_0}^{\infty} dv \rho(m, v) \approx \frac{\rho_\text{QGP}}{B} m^{\frac{1}{2}} \exp \left( \frac{m}{t_H} \right).
$$

It originates in the fact that for heavy bags the Gaussian in (5) acts like a Dirac $\delta$-function for $\Gamma_1$. Thus, the Hagedorn form of (6) has a clear physical meaning and, hence, it gives an additional argument in favor of the FWM.

Similarly to (6), it is possible to estimate the width of heavy free bags averaged over bag volumes and get $\Gamma(v) \approx \Gamma_1(m/B) = \gamma \sqrt{m/B}$. Thus, the mass spectrum of heavy free QGP bags must be the Hagedorn-like one with the property that heavy resonances should have the large mean width because of which they would be hard to be observed.

The FWM allows one to express the pressure of large bags in terms of their most probable mass and width. Comparing the high and low temperature FWM pressures [4, 5] with the lattice QCD data [25, 26, 27], it was possible to estimate the minimal resonance width at zero temperature $\Gamma_R(V_0, T = 0) \approx 600$ MeV and the width at the Hagedorn temperature $\Gamma_R(V_0, T = T_H) = \sqrt{12} \Gamma_R(V_0, T = 0) \approx 2000$ MeV. It was also found that these values of the width are almost independent of the number of the lattice QCD elementary degrees of freedom [5]. Clearly, so large widths can naturally explain the huge deficit of the heavy hadronic resonances in the Particle Data Group compared to the exponential mass spectrum used to describe the QGP EoS. Applying the same line of arguments to the strangelets, I conclude that, if their mean volume is a few cubic fermis or larger, they should survive
for a very short time. Such a conclusion is similar to the results of Ref. [6] predicting an instability of the strangelets.

Also it is remarkable that at the temperatures below the half of the Hagedorn one the QGP bag pressure of the FWM acquires the linear $T$ dependence, i.e. $p(T < 0.5T_H) = -T \frac{E^2}{2\pi^2}$, which is clearly seen in the recent lattice QCD data [27] in the range $T \in [202.5; 419.09]$ MeV [5].

As shown in [5] the relation between the resonance width and the mean mass of the FWM bags at high temperatures obeys the upper bound for the Regge trajectory asymptotic behavior found in [28], whereas a similar relation at low temperatures exactly corresponds to lower bound for the Regge trajectory asymptotic form [28].

4. The Life Time of a Protofireball. – The found FWM width values allow one to get the rough estimates of the life time of the protofireballs suggested in [29] to explain the hadron multiplicities measured in the elementary particle collisions at high energies. The microcanoncal analysis of the thermostatic properties of heavy resonances with exponential mass spectrum [30, 21] teaches us two principal facts: first, even a single heavy resonance with the exponential mass spectrum imparts the Hagedorn temperature to any other hadron being in a thermal contact with it, and, second, the splitting of a single heavy resonance into several heavy pieces with mass above $M_0$ practically does not alter the latter conclusion. These two facts allow us to greatly simplify a treatment of the sequential decay of the heavy QGP bags formed in the elementary particle collisions at high energies. Indeed, the first fact allows one to consider the decay products with the mean kinetic energy which corresponds to the Hagedorn temperature (i.e. $3T_H$ for pions and $3/2T_H$ for heavier particles), and the second fact enables us to study the decay of several QGP bags independently of each other. Moreover, these both facts combined with the low particle densities formed in the elementary particle collisions allow one to neglect the treatment of daughter hadrons which may absorb on the decaying bags.

To simplify the problem I study the two particle decays only and consider the evolution of the heaviest QGP bag. The assumption of two particle decay is not too restrictive because it is possible to effectively account for three, four and more particle decays by representing them as the two particle sequential decays with shorter life-time. Therefore, in the rest frame of decaying bag of mass $M$ the mean change of mass of the heaviest of two daughter particles of energies $E_1$ and $E_2$ is $\Delta M \approx M - \max(E_1, E_2)_M = \min(E_1, E_2)_M$, where the bar means the averaging over all possible combinations which obey the energy conservation. Then the mass evolution equation for the heaviest QGP bag can be cast as

$$\frac{\Delta M}{\Delta t} \approx -q \Gamma_M \frac{\min(E_1, E_2)_M}{M},$$

where $\Delta t$ is the time change, $\Gamma_M = \Gamma_R(V_0, T = 0) \sqrt{\frac{M}{M_0}} \equiv \Gamma_0 \sqrt{\frac{M}{M_0}} \approx 600 \cdot \sqrt{\frac{M}{M_0}}$ MeV is an average resonance width in the vacuum. Here the constant $q = \frac{\ln N}{\ln 2}$ accounts for the mean number of daughter particles $N > 2$ in a decay.

The mass distribution of the heaviest bag can be constructed from the auxiliary function

$$\Omega_M(E_1) \equiv N_\Omega \rho(E_1) \int_{E_{min}}^M dE_2 \rho(E_2) \delta \left(1 - \frac{E_1 + E_2}{M}\right),$$

where the density of states of hadrons of energy $E$ is denoted as $\rho(E)$, the Dirac delta function accounts for the energy conservation, and the normalization factor $N_\Omega$ is given by
Here the first (second) term corresponds to the fact that the fragment of energy \( E_1 \) (\( E_2 \)) is the heaviest one, and \( E_{\min} \approx 3T_H \) is the minimal energy of the lightest fragment. Using (9), one finds an averaged minimal mass of the daughter fragment as

\[
\text{min}(E_1,E_2)_M \approx \int_{M/2}^{M-E_{\min}} dE_1 \rho(E_1) [M - E_1] \rho(M - E_1) \left[ \int_{M/2}^{M-E_{\min}} dE_1 \rho(E_1) \rho(M - E_1) \right]^{-1}.
\]

In principle the density of states should be defined from the convolution of the spectrum (3) and Boltzmann density (3) with

\[
\rho \approx \frac{C}{m} \left[ \frac{M}{m} \right]^{\tau+3/2} \exp\left( \frac{m}{T_H} \right).
\]

The energy conservation \( E = m + \frac{3}{2}T_H + \bar{\sigma}_0 m^\kappa \) relates the mean energy of daughter hadron and its mass. Neglecting the surface energy, I get the resulting energy spectrum of the daughter hadron as

\[
\rho(E) \approx C \left[ \frac{M}{E_F} \right]^{\tau} \left[ e^{E/E_F} \rho_{\text{H}} \right]^{\frac{3}{2}}. \tag{10}
\]

As one can see, the leading exponentials cancelled each other and, thus,

\[
\text{min}(E_1,E_2)_M \approx E_{\min} \left[ \frac{M}{E_{\min}} \right]^{2-\tau} \ln \left( \frac{M}{E_{\min}} \right)^{2\tau-3} \text{ for } \tau \geq 1.
\]

Hence the life time of the protofireball of mass \( M_F \) created in the high energy collision of elementary particles is

\[
t_{\tau=\frac{3}{2}} (M_F) \approx \frac{G_0^{-1}}{q (\tau - \frac{3}{2})} \sqrt{E_{\min} \left[ \frac{E_F}{M_f} \right]^{3/2-\tau} \left[ \frac{M_f}{M_0} \right]^{3/2-\tau}}, \quad \text{and } t_{\tau=\frac{3}{2}} (M_F) \approx \frac{G_0^{-1}}{q} \ln \left[ \frac{E_{\min}}{E_0} \right]. \tag{11}
\]

Since for \( \frac{3}{2} < \tau \leq 2 \) at nonzero baryonic chemical potentials the deconfinement transition is of the first order and for \( \frac{3}{2} < \tau \leq 2 \) it degenerates to the second order and in either case there exists the tricritical point [8], whereas there are some arguments that for \( \tau > 2 \) there should exist the critical point with the first order deconfinement [8]. Therefore, I conclude that the measurements of the energy dependence of the life time (hadronization time) of the protofireballs created in the elementary particle collisions may allow us to distinguish the critical point existence from the tricritical one.

5. **Conclusions.** – Here I discuss some new ideas on the basic properties of the QGP EoS. The main attention is paid to the role of the medium dependent width of heavy QGP bags. Their large width explains a huge deficit of experimental mass spectrum of heavy hadronic resonances and enlightens some important thermodynamic aspects of the color confinement in finite systems. The proposed simple kinetic model for a sequential decay of heavy QGP bags formed in the elementary particle collisions at high energies allows one to distinguish the case of critical point existence from the tricritical one by measuring the energy dependence of the life time of these bags. Of course, the freeze-out process [32] may some what change these conclusions, but the simple kinetic arguments for the nucleus-nucleus collisions [33, 34] teach us that such changes are just a few per cent only and, hence, they cannot affect the result obtained.

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