On the Passage from the Quantum theory to the Semi-Classical theory in 2d Dilaton Gravity

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ABSTRACT

In this paper an attempt is made to understand the passage from the exact quantum treatment of the CGHS theory to the semi-classical physics discussed by many authors. We find first that to the order of accuracy to which Hawking effects are calculated in the theory, it is inconsistent to ignore correlations in the dilaton gravity sector. Next the standard Dirac or BRST procedure for implementing the constraints is followed. This leads to a set of physical states, in which however the semi-classical physics of the theory seems to be completely obscured. As an alternative, we construct a coherent state formalism, which is the natural framework for understanding the semi-classical calculations, and argue that it satisfies all necessary requirements of the theory, provided that there exist classical ghost configurations which solve an infinite set of equations. If this is the case it may be interpreted as a spontaneous breakdown of general covariance.

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The simple two dimensional model for understanding questions associated with Hawking radiation [1], proposed by Callan Giddings Harvey and Strominger (CGHS)[2], has recently been the basis of much research activity. In particular it was pointed out by the present author [3], and independently by Bilal and Callan [4], that general covariance implies that the quantum theory has to be a conformal field theory (CFT), and that conformal invariance requires that the classical CGHS action be modified by quantum corrections. Furthermore it was shown in [3, 4] that a class of modifications leads to a solvable Liouville-like CFT, and an example of this class was explicitly worked out. Another example was given in [6]. In both these examples however the transformations to the fields of the Liouville-like theory had a singularity when the number of matter fields $N$ was greater than 24, with the implication that the definition of the exact quantum theory was unclear, since the range of integration (in field space) of the Liouville-like CFT was only half the real line. In a later paper [5] it was pointed out by the author that there is in fact a class of theories in which this problem is absent (even for $N > 24$). Such theories are equivalent to an exact solvable quantum conformal field theory which can be treated by standard techniques.

It should be emphasized at this point, that making some modification of the original CGHS classical action, in order to get a CFT, is not a matter of choice, but a matter of necessity for the consistency of the theory. While there can be more general quantum corrections which lead to CFTs which are not solvable exactly, it is certainly consistent to focus attention on the solvable class. Indeed at the semi-classical level the required modifications all lead to the solvable Liouville-like theory. Indeed one cannot discuss even the semi-classical physics of the CGHS theory in a consistent fashion without including these necessary modifications, since they are of the same order as the anomaly term introduced by CGHS [2] in order to discuss Hawking radiation.${}^\dagger$

Now as pointed out in [9] it does not seem as if the exact quantum theories

${}^\dagger$ This point is discussed in detail in 5.
coming from CGHS are good candidates for discussing Hawking radiation. In
order to model the four dimensional situation, it seems necessary to impose a
boundary in the two dimensional space-time corresponding to the origin of polar
coordinates in $3 + 1$ dimensions. Such a boundary occurs in the models discussed
in [3, 4, 6]. Indeed there is no problem discussing the semi-classical physics of
these theories provided that one imposes a suitable boundary condition as in [6].
However the physics one gets out will depend on the boundary condition one
imposes. More importantly it is not at all clear that this theory defines a consistent
quantum theory, since as we will discuss later in some detail, the argument that
the Liouville like theory is a CFT depends crucially on the assumption that the
range of integration goes over the whole real line.

The main aim of this paper is to understand the passage from the exact quan-
tum treatment to the semi-classical theory. The standard method of treating a
CFT is to impose the physical state conditions. However there is no trace of the
black hole in the space of physical states and it is unclear how to obtain the semi-
classical equations. *In particular the expectation values of field operators in these
states are space time independent.* We suggest therefore that this space should be
enlarged by including coherent states in which the expectation values of the (total)
stress tensor are zero. The independence of the functional integral from the fiducial
metric also requires that products of the stress tensor have vanishing expectation
values as well. At short distances this is guaranteed by the zero central charge
Virasoro algebra, which is a necessary condition, but is by no means sufficient. To
satisfy this condition at finite distances it seems that one has to also impose the
condition that all higher (i.e ¿ 2) conformal spin operators of the theory also have
zero expectation value.

An important and related question is the emergence of time in quantum gravity.
In principle this question should also have a resolution since we have now an exactly
solvable quantum theory. The physical state condition is the precise statement of
the Wheeler-DeWit equation (in a Fock basis) for the theory. In this case there
is no Schrodinger time evolution. It may also be noted that the existence of a
non-zero boundary Hamiltonian for a classical open system does not make any
difference to the evolution of local operators in the quantum theory. On the other
hand in the coherent state sector one has the time evolution of the expectation
values in accord with the correspondence principle.

Although it was stated already in [3, 4] that the Liouville-like theory is probably
an exact CFT (and indeed in [5] it was observed that the Liouville theory argument
of [8] can be extended to our case), a detailed argument was not given. In the first
part of this paper we will therefore give the justification for this statement. Next
we discuss the physical states and point out that they do not seem to have anything
to do with the semi-classical picture of black hole formation and evaporation. Then
we discuss our alternative and make some concluding remarks.

Let us first prove our assertion that the Liouville-like theory is a CFT. The
action is

\[ S = \frac{1}{4\pi} \int d^2\sigma [\mp \partial_+ X \partial_- X \pm \partial_+ Y \partial_- Y + \sum_i \partial_+ f^i \partial_- f^i + 2\lambda^2 e^{\mp \sqrt{2\pi r|\kappa|}(X \mp Y)}]. \] (1)

The field variables are related to the original variables \( \phi \) (the dilaton) and \( \rho \) (half
the logarithm of the conformal factor) that occur in the CGHS action gauge fixed
to the conformal gauge \( g_{\alpha\beta} = e^{2\rho} \eta_{\alpha\beta} \), by the following relations;

\[ Y = \sqrt{2|\kappa|} [\rho - \kappa^{-1} e^{-2\phi} + \frac{2}{\kappa} \int d\phi e^{-2\phi} \mathcal{H}(\phi)], \] (2)

\[ X = 2 \sqrt{\frac{2}{|\kappa|}} \int d\phi P(\phi), \] (3)

where

\[ P(\phi) = e^{-2\phi} [(1 + \mathcal{H})^2 + \kappa e^{2\phi} (1 + \mathcal{H})]^{\frac{1}{4}}. \] (4)

and \( \kappa = \frac{24-N}{6}, \) \( N \) being the number of matter fields. In (2), (3), the functions
\( h(\phi), \mathcal{H}(\phi) \) parametrize quantum (measure) corrections that may come in when
transforming to the translationally invariant measure (see [3, 4, 5] for details). The statement that the quantum theory has to be independent of the fiducial metric (set equal to $\eta$ in the above) implies that this gauge fixed theory is a CFT. The above solution to this condition was obtained by considering only the leading terms of the beta function equations, but it was conjectured in [3, 4] (on the basis of the resemblance of (1) to the Liouville theory), that it is indeed an exact solution to the conformal invariance conditions. It should be stressed that this latter statement is strictly valid only when $P$ has no zeroes. This implies some restrictions on the possible quantum corrections (as shown in [5] there is a large class which satisfies these conditions).

Putting

$$\zeta^+ \equiv \sqrt{2/|\kappa|} (X + Y) + \ln \sqrt{2/|\kappa|}, \quad \zeta^- \equiv \sqrt{2/|\kappa|} (X - Y),$$

let us define the generating functional for the connected correlation functions of the theory by

$$e^{i|\kappa|W[J]} = \int [d\zeta^+] [d\zeta^-] e^{i|\kappa|S[\zeta^+,\zeta^-]+i|\kappa|} \int d^2\sigma (J_+ \zeta^- + J_- \zeta^+) \, . \tag{5}$$

We have ignored the matter ($f$-fields) and the ghost contributions since they are CFTs by themselves.

In terms of the $\zeta$ fields the action plus source terms take the form,

$$\frac{1}{4\pi} \int d^2\sigma [\partial_+ \zeta_+ \partial_- \zeta_- + 2\lambda^2 e^{\zeta^+} + 4\pi (J_+ \zeta_- + J_- \zeta^+)]. \tag{6}$$

The equations of motion are

$$-\partial_+ \partial_- \zeta_- + 2\lambda^2 e^{\zeta^+} + 4\pi J_- = 0 \tag{7}$$

$$-\partial_+ \partial_- \zeta_+ + 4\pi J_+ = 0.$$

The effective action is defined by the following equations.
\[ \Gamma[\zeta_{\pm c}] = W[J] - \int d^2 \sigma (J_+ \zeta_{-c} + J_- \zeta_{+c}), \quad (8) \]

\[ \zeta_{\pm c} = \frac{\delta W}{\delta J_{\mp}} = <\zeta_{\pm}>_{J} \equiv \frac{\int [d\zeta_{\pm}] \zeta_{\pm} e^{iS+iJ \zeta}}{\int [d\zeta_{\pm}] e^{iS+iJ \zeta}} \quad (9) \]

It follows that

\[ \frac{\delta \Gamma}{\delta \zeta_{\pm c}} = -J_\mp \quad (10) \]

Since the measures \([d\zeta_{\pm}]\) are translationally invariant we may use the equations of motion inside the functional integral to get

\[ \partial_+ \partial_- \frac{\delta W}{\delta J_-} = \partial_+ \partial_- \zeta_{+c} = 4\pi J_+, \quad (11) \]

\[ \partial_+ \partial_- \frac{\delta W}{\delta J_+} = \partial_+ \partial_- \zeta_{-c} = 2\lambda^2 <\zeta^+> + 4\pi J_- \quad (12) \]

From (11) we have

\[ W[J] = \int d^2 \sigma d^2 \sigma' J_-(\sigma) \Delta_F(\sigma - \sigma') J_+(\sigma) + \int d^2 \sigma \zeta_0^0 J_- + F[J_+], \quad (13) \]

where the last term is a functional (to be determined below) of \(J_+\) only, and \(\zeta_0^0\) is a c-number solution of the homogeneous equation. \(\Delta_F\) is the Feynman-Green function defined as the solution of \(\partial_+ \partial_- \Delta_F(\sigma - \sigma') = 4\pi \delta^2(\sigma - \sigma')\) with the Feynman boundary conditions which come from the fact that the path integral actually must contain the initial and final (\(t \to \pm \infty\)) wave functionals. From equation (13) for \(W\) we have, for the connected correlation functions,

\[ <\zeta_+(\sigma)\zeta_+(\sigma')>_{\text{conn,J}} = (i|\kappa|)^{-1} \frac{\delta^2 W[J]}{\delta J_-(\sigma) \delta J_-(\sigma')} = 0 \quad (14) \]

and
< ζ(σ)ζ(σ') >_{\text{conn}, J} = (i|\kappa|)^{-1} \frac{\delta^2 W[J]}{\delta J_-(\sigma) \delta J_+(\sigma')} = (i|\kappa|)^{-1} \Delta_F(\sigma - \sigma'). \quad (15)

From (14) we see that < ζ_1(σ_1) \ldots ζ_n(σ_n) >= ζ_{c+}(σ_1) \ldots ζ_{c+}(σ_n) and hence we may rewrite (12) as

\partial_+ \partial_- ζ_{c-} = 2 \lambda^2 e^{ζ_{c+}} + 4\pi J_. \quad (16)

From (11), (16), and (10), we find the effective action,

\Gamma(ζ_{c+}, ζ_{c-}) = \frac{1}{4\pi} \int d^2\sigma [\partial_+ ζ_{c+} \partial_- ζ_{c-} + 2 \lambda^2 e^{ζ_{c+}}]. \quad (17)

This effective action has already been derived by Russo, Susskind, and Thorlacius [6], by formally doing the ζ_- integral in (5) which gives a delta functional that allows one to do the ζ_+ integral. We have however obtained this by a long winded route because it is important for our purposes to demonstrate exactly and rigorously what the correlation functions are. From the effective action we obtain, by doing the inverse Legendre transform, the generating functional for connected correlation functions as (13), with

\[ F[J] = \frac{\lambda^2}{2\pi} \int d^2\sigma e^{[\int d^2\sigma' \Delta_F(\sigma - \sigma') J_+(\sigma') + ζ^0_{c+}].} \]

Using this we have the final correlation function

< ζ(σ)ζ(σ') >_{\text{conn}, J} = \frac{\lambda^2}{2\pi |\kappa|} \int d^2\sigma'' \Delta(\sigma - \sigma'') e^{<ζ_{c+}(σ'')>_J} \Delta(σ'' - \sigma').

It should be stressed however that the above arguments depend crucially on using the equations of motion inside the functional integral. i.e. we have used the
formula
\[
\int \left[ d\zeta \frac{\delta}{\delta \zeta(\sigma)} \right] e^{iS + iJ \zeta} = 0,
\]
which will not be valid unless the functional integration ranges over the whole real line. This in turn means that the argument is valid only for quantum theories in which \( P(\phi) \) has no zeroes so that the integration range for \( \zeta \) is not cut off. Now we are not claiming that the theories in which there is a so-called quantum singularity are necessarily inconsistent, but only that it is very difficult to establish that they are a correct representation of the quantum theory, in so far as it is not at all clear that they are CFTs. The present author’s hunch is that this singularity is spurious. That this may be the case is also acknowledged in [6].

It follows also that there is an operator formulation of the theory with operator equations of motion

\[
\partial_+ \partial_- \hat{\zeta}_+ = 0, \quad \partial_+ \partial_- \hat{\zeta}_- = 2\lambda^2 e^{\hat{\zeta}_+}
\]

and equal time commutation relations,

\[
[\hat{\zeta}_+(\sigma), \hat{\zeta}_-(\sigma')]\delta(t - t') = \frac{8\pi i}{|\kappa|} \delta^2(\sigma - \sigma')
\]

\[
[\hat{\zeta}_-(\sigma), \hat{\zeta}_+(\sigma')]\delta(t - t') = \frac{8\pi i}{|\kappa|} \delta^2(\sigma - \sigma')
\]

with all other commutators vanishing. One could have arrived at this directly by means of a canonical quantization of the action (1) with the conjugate momenta \( \Pi_\pm = \frac{\partial}{\partial \sigma} \hat{\zeta}_\pm \). However the path integral formulation highlights the fact that these operator equations are valid only for those theories for which the field has a range which extends over the whole real line. From now on we will deal with the operator formulation and will drop the hats on operators.
The stress tensors may be expressed in terms of the canonical variables using the equations of motion to rewrite the second order time derivative terms, and we get

\[ T_{\pm\pm} = \frac{|\kappa|}{4} \left( \partial_{\pm} \zeta_{\pm} \partial_{\pm} \zeta_{\pm} \right) + \partial_{\pm}^2 (\zeta_{\pm} - \zeta_{\pm}) \]

\[ \Rightarrow T_{\pm\pm}^0 - \frac{|\kappa|}{2} \lambda^2 e^{\zeta_{\pm}}, \]

where \( T^0 \) is the stress tensor of the free \((\lambda^2 = 0)\) theory expressed in terms of the canonical variables. Then it easily follows from the fact that the exponential term is a dimension \((1, 1)\) operator with respect to \( T^0 \) [3, 4], which has no correlations with itself (14), that the full stress tensor generates a Virasoro algebra with central charge \( c_{\zeta} = 2 + 6\kappa \). In addition it follows from the equations of motion that

\[ T_{+-} = 0, \quad \partial_\mp T_{\pm\pm} = 0, \]

as operator statements. Thus we have the complete operator formulation of a CFT. The matter and ghost sectors (in conformal gauge) are free CFTs with central charges \( c_f = N, \ c_{gh} = -26 \), so that the quantum CGHS theory is a CFT with zero central charge for \( \kappa = \frac{24 - N}{6} \).

For future reference we also note the following. The solutions to the equations of motion are

\[ \zeta_{\pm}(\sigma, \tau) = g_{\pm}(\sigma^+) + g_-(\sigma^-) \]

\[ \zeta_{\mp}(\sigma, \tau) = u_{\pm}(\sigma^+) + u_-(\sigma^-) + 2\lambda^2 \chi_{\pm}(\sigma^+) \chi_{\pm}(\sigma^-) \]  

where, \( \chi_{\pm}(\sigma^\pm) = \int^{\sigma^\pm} g^{\zeta}, \) and \( \sigma^\pm = \tau \pm \sigma. \)

Also from the equal-time commutators and these solutions one obtains the following light-cone commutator,
\[ [\partial_{\pm} \zeta(\sigma^\pm), \chi_{\pm}(\sigma'^\pm)] = \frac{8\pi i}{|\kappa|} \theta(\sigma^\pm - \sigma'^\pm) \partial_{\pm} \chi_{\pm}(\sigma) \] (21)

Now the general covariance of the theory is reflected in the conformal gauge by the constraints, which classically amount to the statement that the total stress tensor must vanish. In the quantum theory general covariance has the implication that the gauge fixed path integral be independent of the fiducial metric \( \hat{g} \) [3,5].\(^*\)

In the semi-classical analyses of the theory [2, 3, 4, 5, 6, 7, etc.] this was only implemented in the form

\[ <\Psi | T_{\pm \pm} | \Psi > = 0 \] (22)

where,

\[ T = T^{dg} + T^f + T^{gh} \]

is the total stress tensor of dilaton gravity \( dg \), matter \( f \), and diffeomorphism ghosts \( gh \). Once the fiducial metric is fixed this constraint gives a relation between (the expectation values of) matter field and the dilaton gravity fields.\(^\dagger\) One then discusses Hawking radiation by transforming to coordinates appropriate to an asymptotic observer. As pointed out in [5, 9] this procedure already has some ambiguities stemming from the fact that it is only the total stress tensor which transforms as a tensor, and that it is not possible to give a coordinate invariant justification for the usual argument. The situation becomes even more confusing if one follows the standard prescription for an exact quantum treatment. The reason is that the although equation (22) is a necessary condition it is by no means a

\(^*\) The physical metric, which is integrated over, can always be put in the form \( g = e^{2\rho} \hat{g} \) in two dimensions.

\(^\dagger\) We may set the expectation value of the ghost stress tensor to zero in a particular conformal frame, for example the Kruskal one in which \( \rho = \phi \), see [5].
sufficient expression of the constraints of coming from general covariance. In fact the complete statement of the constraints in the quantum theory (i.e. the independence of the quantum theory from the fiducial metric) is that the physical states of the theory must be such that the expectation value of an arbitrary product of stress tensors must vanish.

\[ < \Psi | \prod_{r=1}^{n} T_{\pm\pm}(\sigma_r) | \Psi >_{\text{conn}} = 0 \]  

(23)

Since we have a Virasoro Algebra with zero central charge the leading singularities of (23) are guaranteed to vanish by (22). But in general the physical states of the quantum theory belong to a smaller subspace than those allowed by (22). In fact it is possible to argue that it is inconsistent to ignore the \( n = 2 \) constraint in (23) when discussing Hawking radiation, which is an \( O(\hbar) \) effect in (22). To see this let us note that we may introduce the analog of the Einstein tensor \( G_E \) in 4d by writing \( T^{dg} = N G_E \), so that (22) takes the form \( N < G^{dg} > = - < T^{f} > \).\(^\dagger\) Hawking radiation is an \( O(N\hbar) \) contribution to the right hand side which must be balanced by an \( O(\hbar) \) contribution to \( G_E \). Now the \( n = 2 \) equation in (23) may be written \( N^2 < G_E G_E > = < T^{f} T^{f} > \). The correlations of the \( N \) matter fields give \( O(N\hbar) \) contribution on the right hand side which can only be balanced by the \( O(N^{-1}\hbar) \) correlations (see (15)) of the dilaton gravity fields on the left hand side. In other words the constraints inform us that to the same order to which we are calculating the back reaction to Hawking radiation in (22) we must also take into account the \( n = 2 \) equation in (23).

The standard way of ensuring that (23) is satisfied is to implement the physical state condition i.e.,

\(^\dagger\) We ignore the ghosts at this point, and note that \( N \) plays the role of the inverse Newton constant here.
where \( T^+ \) is the positive frequency part of the stress tensor and \( Q_B \) is the BRST charge. Using standard string theory methods it is possible to obtain the solution to these conditions. One first constructs an operator (essentially the so-called DDF operator of string theory [10]) by dressing the matter field with the dilaton gravity field \( \chi \), which it should be noted (see (20) and (14)) has no correlations with itself.

\[
A^i_\pm(\omega) = \frac{1}{2\pi} \int d\sigma^\pm (\lambda \chi^\pm)^i \omega \partial^\pm f^i
\]  

(25)

It is easily shown (using (21)) that these operators commute with the total stress tensor. Furthermore they satisfy the same commutation relations as the mode operators of the \( f \) field. Thus the action of products of these operators (with negative frequencies) on the Fock vacuum \(|0>\) (\( T^+ |0> = 0 \)) give solutions to the physical state conditions. i.e.

\[
|\Psi> = \prod A |0> .
\]  

(26)

It is probably the case that the standard argument that these states are complete, in the space of physical states up to spurious (i.e. BRST trivial) states, is also valid here modulo technicalities involving the infinite length of the space, but we will not discuss this further, since it is not germane to the main point of this paper.

The question we wish to address is the interpretation of the semi-classical equations that have been discussed in the literature. As long as one did not have an

\( \text{§} \) For the case \( N = 24 \) this result has already been obtained in [11]

\( \text{¶} \) A detailed argument to this effect has been given for the case \( N = 24 \), with reflecting boundary conditions and an infra-red cut off, in [11].
exact solution or even a formulation of quantum gravity, one is free to speculate as to what it might be. However the model that we have is a well-defined and solvable theory of quantum gravity, albeit in two dimensional space time. Nevertheless as we will see, if we impose the standard rules for this theory (i.e. (24)), it is very difficult to extract the semi-classical equations which have been used to discuss issues like Hawking radiation.

We begin with an elementary observation. The physical state condition (24) implies in particular that

\[ H|\Psi> = 0, \quad P|\Psi> = 0, \] (27)

where the Hamiltonian and momentum operators are given by

\[ H = \int d\sigma^+ T_{++} + \int d\sigma^- T_{--}, \quad P = \int d\sigma^+ T_{++} + \int d\sigma^- T_{--}. \]

Now the semi-classical theory deals with space time dependent classical fields (for both dilaton gravity and matter), which are to be interpreted as the expectation values of field operators in some quantum state. Clearly such a state cannot be one satisfying (24) since from (27) \(<\Psi|\hat{\phi}|\Psi>\) is independent of space-time. From our explicit construction it is also clear that there is no limit in which we can recover anything like a semi-classical state. This is in contrast to the usual argument that there must be some limit, \(G_N << 1\) in four dimensions, \(N \rightarrow \infty\) in the CGHS case, in which quantum gravity should yield a semi-classical formulation. Instead what we have established is that the principal difference between the semi-classical theory and the usual formulation of the quantum theory lies in the way the constraints are imposed. In the former case one imposes the constraint as an expectation value (22) in the latter as a condition on the states (24).

Is there an alternative to imposing the physical state conditions (24)? Since the physical requirement is actually (23), and since (24), though a sufficient condition,
has not been shown to be necessary, one might ask whether it is too strong a condition. In other words the question is whether there are states which satisfy (23) exactly without satisfying (24). This is tantamount to asking whether general covariance can be broken spontaneously.\* We believe that the following is a possible solution to this problem.

The natural quantum states for representing classical configurations are coherent states. In order to understand the essence of the problem it is helpful to consider first the example of the harmonic oscillator. Introducing creation and annihilation operators \( a, a^\dagger \), satisfying the commutation relation \([a, a^\dagger] = 1\), a (normalized) coherent state of the oscillator is given by \(|z\rangle = e^{-\frac{z^2}{2}} e^{za^\dagger} |0\rangle\). This has the property that \( a|z\rangle = z|z\rangle \), so that in particular the position operator has the expectation value \( \Re z \) and \( <z| : h(a^\dagger, a) : |z\rangle = h(\bar{z}, z) \). The analog of the physical state constraint (24) would be a requirement that only (say) the unit energy eigenstate \( |1\rangle \) of the Hamiltonian \( a^\dagger a \) is “physical”. This state is of course an infinite superposition of the coherent states,

\[
|1\rangle = \int dz d\bar{z} |z\rangle <z| 1\rangle .
\] (28)

Now semi-classical physics in our case picks up (the analog of) one state from this infinite superposition so although the “physical state” itself will not represent space time dependent configurations each member of the superposition separately may. Indeed since \(|<z'|z\rangle| = e^{-\frac{(|z|^2+|z'|^2)}{2}} e^{\Re(\bar{z'}z)} \to 0\) for large \(|z|\) one may argue that large field configurations will exhibit classical behaviour and one will not be able to detect interference with other members of the superposition (28). It should be noted that according to the “physical state” condition the system must be in \(|1\rangle\) which is not a complete set of states for the whole Hilbert space. In particular a coherent state cannot be expressed as a superposition of physical states.

\* The Hilbert space of the theory has negative norm states and therefore the usual injunction against spontaneous symmetry breaking in 2d will not apply for the same reason that there is no Goldstone theorem in a gauge theory. Of course in 2d there are no gravitons either!

\*\* The zero energy state is trivially equal to the \( z = 0 \) coherent state so a non-zero energy state gives a better analogy.
Let us now get back to dilaton gravity. The constraint equations of the semi-classical theory are easily obtained as exact statements by taking $|\Psi >$ in to be a coherent state in (22). In the following $u$ and $g$ are classical $c$-number fields such that the classical solutions to the equations of motion $\zeta_{cl} = < \zeta >$ are given in terms of them by (20). Working in the Kruskal coordinate system where $\zeta_{cl} = g = 0$, we take

$$|\Psi > = NV_f V_{\zeta} V_{gh} |0 >$$

(29)

where $N$ is a normalization factor and,

$$V_f = e^{\frac{1}{4\pi\alpha}} \int d\sigma f^{(-)} \partial f^{(+)}$$

$$V_{\zeta} = e^{\frac{1}{4\pi\alpha}} \int d\sigma \zeta^{(-)} \partial u^{(+)}$$

with a similar expression for the ghost sector involving classical ghost configurations (Grassmann valued functions) $b_{cl}, c_{cl}$.** These states do not satisfy the physical state condition (24). But, as we discussed in our harmonic oscillator example any physical state can be expressed as a superposition of these coherent states. For large field configurations one may argue again that interference with other members of the superposition will not be detectable. The semi-classical analysis however also requires that (22) be imposed. If one is starting with a physical state however, this equation (actually all of the equations in (23)) are satisfied automatically. There is no reason why any given member of the superposition should be made to satisfy (22) separately. Indeed the physical state condition would be satisfied through cancelations between different members of the superposition of coherent states.

** The superscript $(-)$ is an instruction to take the negative frequency part of the corresponding field.
To make sense of the semi-classical equations then one has to try to satisfy (23) on coherent states. For \( n = 1 \) i.e. (22), one has the equations used in the semi-classical analyses, [2, to 7]

\[
\partial^2_{\pm} u + \partial_{\pm} f_{\text{cl}} \partial_{\pm} f_{\text{cl}} + t_{\pm \pm} = 0
\]

where \( t \) is the classical ghost stress tensor. Now while this is an exact equation in the quantum theory (because of our use of coherent states) the statement of general covariance in the quantum theory is by no means exhausted by this. One has to also satisfy the infinite set of equations (23). In the semi-classical discussion only the first of these was satisfied. When \( n > 1 \) in (23) the leading singularities in the correlation functions are also guaranteed to vanish by the Virasoro algebra with zero central charge and (22). However the subleading terms (including the non-singular ones) of the operator product expansion for products of stress tensors gives an infinite number of constraints of the form \(< W_n >= 0\), where \( W_n \) are the generators of the \( W_\infty \) algebra of the system. It should be noted that since the central charge of the Virasoro algebra is zero so is the central charge of the \( W \) algebra so that there is no c-number term coming from multiple \((n > 2)\) operator products [12]. These additional constraints imply that the classical ghost configurations in our coherent state satisfies an infinite set of conditions. It is not clear that a solution exists, but on the other hand there is no obvious reason why this is ruled out either. If such a solution exists then we would have achieved a spontaneous breakdown of general covariance.

The CGHS theories in which the field space is unrestricted, are exact solvable quantum CFTs. Therefore although they are not good models for understanding Hawking radiation in 3 + 1 dimensions, in so far as they are precisely defined theories of two dimensional quantum gravity, they may be used to elucidate the conceptual problems associated with quantization of geometry. In fact what we have established is that in our simple theory, the so-called semi-classical equations
have nothing to do with taking a large \( N \) limit.** In the standard treatment of the quantum theory using Dirac quantization or BRST techniques the semi-classical picture of black hole formation and decay seems to be completely hidden from view. Presumably this is because one is considering quantum states which are generally covariant and are a superposition of all metrics.***** In this picture then one needs some mechanism for collapsing the geometrical part of the wave function to one metric configuration.

An alternative to the standard picture has been suggested here by weakening the constraint on the states (which is a sufficient but not a necessary condition for the general covariance of expectation values). This leads to a formalism using coherent states which gives the usual semi-classical equations, and indeed in our simple theory the latter would be exact. However it is not clear that the formalism is completely consistent in that we are unable to prove that the infinite set of equations for the ghost configurations can be satisfied.

The original motivation for studying the CGHS theory was to provide a simple toy model in which the question of information loss and the breakdown of unitary evolution could be decided one way or the other. All the arguments on either side of this issue even in this simplified context have been semi-classical with the differences between the two (or three) camps depending on the assumptions made about the quantum gravity regime. Indeed in a recent paper Susskind, Thorlacius, and Uglum [7] have argued that the separation of scales that is usually used to justify semi-classical physics is not valid in the analysis of Hawking radiation. Furthermore they propose a principle of complementarity between the physics obtained by an asymptotic observer and that of one falling into the black hole.

** Of course if the theory with a boundary [6] is a consistent quantum theory, i.e. is a CFT, then it is likely to have very complicated higher order effects which would be suppressed at \( N \to \infty \). Nonetheless insofar as it must be a CFT, the basic problem of how to impose the constraints, will be exactly as in the simple theory discussed in this paper.

***** In this sense the state is similar to one in the soliton sector in a quantum treatment of field theories with soliton solutions, where the underlying translational symmetry of the theory is restored by integrating over collective coordinates. [13]
What we have proposed here is that one should try to test such hypothesis within the well defined context of quantum dilaton gravity. Indeed given that there is no experimental test which can decide these issues it is imperative that the logical basis of the arguments used are as firm as possible. In particular since underlying reality is quantum mechanical, and geometry itself must be quantum mechanical, the logical problem becomes one of deriving the observed classical (or semi-classical) world from the quantum mechanical one. As we have seen, it is very difficult to do this even in our simple theory. This is probably an indication that classical (or semi-classical) intuition may break down in unexpected ways.

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