Engineering Floquet Higher Order Topological Insulator by Periodic Driving

Arnob Kumar Ghosh,1, 2, * Ganesh C. Paul,1, 2, † and Arijit Saha1, 2, ‡

1 Institute of Physics, Sachivalaya Marg, Bhubaneswar-751005, India
2 Homi Bhabha National Institute, Training School Complex, Anushakti Nagar, Mumbai 400085, India

We theoretically investigate a periodically driven semimetal in the high frequency regime. The possibility of engineering both Floquet Topological Insulator featuring Floquet edge states and Floquet higher order topological insulating phase, accommodating topological corner modes has been demonstrated starting from the semimetal phase, based on an effective Hamiltonian picture. Topological phase transition takes place in the bulk quasi-energy spectrum with the variation of the drive amplitude where Chern number changes sign from $+1$ to $-1$. This can be attributed to broken time-reversal invariance ($T$) due to circularly polarized light. When the discrete four-fold rotational symmetry ($C_4$) is also broken by adding a Wilson mass term along with broken $T$, higher order topological insulator (HOTI), hosting in-gap modes at all the corners, can be realized. The Floquet quadrupolar moment, calculated with the Floquet states, exhibits quantized value $0.5$ (modulo $1$) identifying the HOTI phase. The appearance of the Floquet corner modes with the external drive is also investigated via the local density of states.

I. INTRODUCTION

Investigation of topological phases of matter1−7 has been at the heart of quantum condensed matter physics for more than a decade from both theoretical and experimental perspective. A wide class of systems harboring topological phases have been discovered, including the Z2 topological insulator (TI)8, 9, Weyl semimetal10−12, Dirac semimetal13 and the topological superconductors14. The fascinating fact about TIs are the emergence of a robust (1D) dimensional boundary states from d dimensional insulating bulk which is the outcome of bulk-boundary correspondance. Very recently, the advent of the higher order topological insulators (HOTI)15−19 has attracted immense interest in the modern condensed matter physics community. In contrast to the prevailing topological insulators (TIs), in an n-th order TI both the d dimensional bulk and $(d-1)$ dimensional boundary remain gapped, whereas, the $(d-n)$ dimensional boundary exhibit gapless modes; where $n$ is the order of the HOTI. In this language, conventional TIs are called first order TI. Thus, in three dimension (3D), one can realize second order TI (SOTI) with gapless states located at the one dimensional (1D) hinges and third order TI with gapless zero dimensional (0D) corner states. Similarly, in two dimension (2D), SOTI exhibits gapless 0D corner modes while the edges remain gapped. These $(d-n)$ dimensional boundary states emerge as quantization of n-th order electric multipole15, 16 moment with evanescing lower modes. Hence, the materials we may have so far identified as trivial due to the absence of $(d-1)$ dimensional boundary states, may turn out to be HOTI20. Following the increasing interest on HOTI, few intriguing models have been proposed so far based on Kagome lattice21, transition metal dichalcogenides (XTe2, X=Mo,W)22, 23, SnTe17 etc. In 2D, SOTIs supporting zero dimensional corner modes have been experimentally realized in acoustic material based on kagome lattice24, photonic crystals25, 26 and electrical circuits27, 28 setup.

Engineering of periodically driven Floquet topological insulators out of trivial system is a field of interest of its own29−31. In this direction, realization of Floquet HOTI has been investigated from different perspectives in recent literature32−34. In recent times, it has been shown that a HOTI phase, hosting 0D corner modes, can be obtained from quantum spin hall insulator (QSHI) via quantum quench35−37. Therefore it is natural to ask whether such phase can also be obtained by periodically driving a semimetallic phase based on a model system. It can also be an interesting fact to investigate whether one can realize first order TI phase as an intermediate phase before turning up to a HOTI. Moreover, the nature and stability of such phases with the enhancement of the driving strength of the external irradiation is worth to explore.

Motivated by the above mentioned questions, in this work, we consider a two dimensional semimetal in presence of external irradiation. We show that the semimetal becomes Floquet topological insulator (FTI), characterized by quantized non-zero Chern number, under the influence of an external irradiation for e.g. circularly polarized light which breaks time-reversal symmetry $\mathcal{T}$. We derive the quasi-static Floquet Hamiltonian employing Brillouin-Wigner (BW) perturbation theory44, 45. The sharp transition of the Chern number46 from $+1$ to $-1$ takes place with concomitant band gap closing in the bulk of the quasi energy spectrum. In non-irradiated case, breaking the crystalline $C_4$ symmetry by adding a mass perturbation, we show that the system becomes a static SOTI having in-gap corner modes while the bulk and the edges remain gapped. This SOTI phase is identified by vanishing dipole moment and half integer quadrupole moment $Q_{xy}^{(0)} = 0.5$ (modulo $1$). As a result of periodic drive on the SOTI phase, one can realize a Floquet second order topological insulator (FSOTI) phase where the Floquet corner modes are found to be pristine at all four corners of our system. Numerically computed Floquet quadrupole moment $Q_{xy}$ also turns out to be half integer value (modulo $1$) for the driven case.

The remainder of the paper is organized as follows.
In Sec. II, we describe the model Hamiltonian for our setup and present a brief outline of the derivation of the effective Hamiltonian in the high frequency regime. The results are presented in Sec. III where we discuss the static semimetal Hamiltonian, the first order FTI and then both static SOTI and FSOTI phases. Finally, we summarize our results and conclude in Sec. IV.

II. MODEL AND METHOD

A. Static Hamiltonian

We begin with a 2D square lattice which describes a semimetal, for which the Hamiltonian reads

$$H_{SM} = \sum_{j,k,l} \left[ c_{j,k}^\dagger T_x c_{j+1,k} + c_{j,k}^\dagger T_y c_{j,k+1} \right] ,$$

(1)

with

$$T_x = \frac{ilT_1}{2} \Gamma_1 + \frac{t_2}{2} \Gamma_3 ,$$

$$T_y = \frac{ilT_1}{2} \Gamma_2 + \frac{t_2}{2} \Gamma_3 .$$

(2)

Here, $l = \pm 1$ represents nearest neighbours and $t_1, t_2$ are the amplitudes of nearest-neighbour hopping, $c_{j,k}$ is a four component spinor $\{A_1, B_1, A_2, B_2\}^T$ where $A, B$ are the orbitals and $i, j$ run along $x$ and $y$-directions respectively. The mutually anti-commuting hermitian $\Gamma$ matrices are: $\Gamma_1 = \sigma_1 \tau_1, \Gamma_2 = \sigma_0 \tau_2, \Gamma_3 = \sigma_0 \tau_3$. Two sets of Pauli matrices $\tau$ and $\sigma$ respectively indicate the orbital and spin degrees of freedom of the system.

B. Effective Hamiltonian: Brillouin-Wigner perturbation expansion

The schematic of our geometry in presence of external irradiation is demonstrated in Fig. 1. We use Brillouin-Wigner (BW) perturbation theory\textsuperscript{44} to obtain the effective Hamiltonian for periodically driven system in the high-frequency limit i.e., frequency is large compared to the bandwidth ($\omega \gg t$). We consider circularly polarized light of the form $\mathbf{A}(t) = \mathbf{A}(\cos(\omega t), \sin(\omega t))$ and the beam spot is much larger than the system in order to get rid of any spatial dependency. The purpose of choosing circularly polarised light instead of linearly polarised light, is to break the time reversal symmetry and enabling us to explore non-trivial topological phases\textsuperscript{45,47,48}. We begin with a time periodic Hamiltonian $H(t + T) = H(t)$ given by its Fourier components as

$$\mathcal{H}_n = \int_0^T dt H(t) e^{in\omega t} ,$$

(3)

where $T = 2\pi/\omega$ is the period of drive. Following Mikami \textit{et al.}\textsuperscript{44}, the effective Hamiltonian can be obtained in powers of $1/\omega$ using the BW perturbation theory. Although we consider here, terms only upto the order of $1/\omega$, for simplicity, it is important to note that the essential physics can be extracted curtailing the Hamiltonian upto $1/\omega$ in the high frequency limit. The effective Hamiltonian can be written as

$$\mathcal{H}_{BW} = \sum_{r=0}^\infty \mathcal{H}_n^{(r)} ,$$

(4)

with

$$\mathcal{H}_n^{(0)} = \mathcal{H}_0 ,$$

$$\mathcal{H}_n^{(1)} = \sum_{n \neq 0} \frac{\mathcal{H}_{-n} \mathcal{H}_n}{n\omega} ,$$

$$\mathcal{H}_n^{(2)} = O \left( \frac{1}{\omega^2} \right) .$$

(5)

The zeroth order Hamiltonian $\mathcal{H}_0$ contains terms as the unperturbed one, but with modulated hoppings whereas, the term with $O(1/\omega)$ manifests new kind of hopping elements originating due to the effect of irradiation in the high frequency limit\textsuperscript{44,45,47}.

![FIG. 1. (Color online) Schematic of our square lattice geometry is shown in presence of external irradiation. Here, the magenta (light grey) solid circles represent a square lattice and the blue (dark grey) helix denotes the external circularly polarized light modeled as a vector potential $\mathbf{A}(t)$.](image)

The effect of periodic drive is taken into account by considering a Peierls phase in the hopping elements as

$$T_x \rightarrow T_x e^{-ilA \cos(\omega t)} ,$$

$$T_y \rightarrow T_y e^{-ilA \sin(\omega t)} .$$

(6)

Using BW expansion, we compute the effective Floquet Hamiltonian for the semimetal in presence of the drive which is given by Eq.(4), where

\[ \mathcal{H}_{BW} = \sum_{r=0}^\infty \mathcal{H}_n^{(r)} , \]

\[ \mathcal{H}_n^{(0)} = \mathcal{H}_0 , \]

\[ \mathcal{H}_n^{(1)} = \sum_{n \neq 0} \frac{\mathcal{H}_{-n} \mathcal{H}_n}{n\omega} , \]

\[ \mathcal{H}_n^{(2)} = O \left( \frac{1}{\omega^2} \right) . \]
Here, we will present our results involving static semimetal phase, we discuss the FTI and then analyse both static and Floquet SOTI. We have considered both \(t_1, t_2 = 1\) for all our numerical results and all the parameters having the dimension of energy are scaled with respect to the hopping strength. For all the results shown below, we have chosen \(\omega = 10\).

\[
\mathcal{H}_{\text{BW}}^{(0)} = \sum_{j,k} \left[ c_{j,k}^\dagger T_1 c_{j+1,k} + c_{j,k}^\dagger T_2 c_{j,k+1} + \text{h.c.} \right],
\]

\[
\mathcal{H}_{\text{BW}}^{(1)} = \sum_{j,k} c_{j,k}^\dagger M c_{j,k} + \sum_{j,k} \left[ c_{j,k}^\dagger T_3 c_{j+1,k+1} + c_{j,k}^\dagger T_4 c_{j+2,k} + c_{j,k}^\dagger T_5 c_{j,k+2} + c_{j,k}^\dagger T_6 c_{j-1,k+1} + \text{h.c.} \right],
\]

with

\[
M = J_2 (t_1^2 + t_2^2) \sigma_0 \tau_0,
\]

\[
T_1 = \frac{J_0(A)}{2} (i t_1 \sigma_3 \tau_1 + t_2 \sigma_0 \tau_3),
\]

\[
T_2 = \frac{J_0(A)}{2} (i t_1 \sigma_0 \tau_2 + t_2 \sigma_0 \tau_3),
\]

\[
T_3 = J_{c1} t_2^2 \sigma_0 \tau_0 + J_{s1} (t_1^2 \sigma_3 \tau_1 + i t_1 t_2 \sigma_3 \tau_2 + i t_1 t_2 \sigma_0 \tau_1),
\]

\[
T_4 = J_1 (t_2^2 - t_1^2) \sigma_0 \tau_0,
\]

\[
T_5 = J_1 (t_2^2 - t_1^2) \sigma_0 \tau_0,
\]

\[
T_6 = J_{c2} t_2^2 \sigma_0 \tau_0 - J_{s2} (t_1^2 \sigma_3 \tau_3 + i t_1 t_2 \sigma_3 \tau_2 - i t_1 t_2 \sigma_0 \tau_1),
\]

where,

\[
J_1 = \sum_{n \neq 0} \frac{(-1)^n J_n^2(A)}{4n\omega}, \quad J_{c1} = \sum_{n \neq 0} \frac{(-1)^n J_n^2(A) \cos \left( \frac{n\pi}{2} \right)}{2n\omega}, \quad J_{s1} = \sum_{n \neq 0} \frac{(-1)^n J_n^2(A) \sin \left( \frac{n\pi}{2} \right)}{2n\omega},
\]

\[
J_2 = \sum_{n \neq 0} \frac{J_n^2(A)}{n\omega}, \quad J_{c2} = \sum_{n \neq 0} \frac{J_n^2(A) \cos \left( \frac{n\pi}{2} \right)}{2n\omega}, \quad J_{s2} = \sum_{n \neq 0} \frac{J_n^2(A) \sin \left( \frac{n\pi}{2} \right)}{2n\omega}.
\]

Here, \(J_n\) is the Bessel function of first kind and \(A\) is the amplitude of drive.

From Eq.(1), it is evident that only nearest neighbour hoppings are present in the static Hamiltonian describing a semimetal. Here, \(T_1\) and \(T_2\) are the renormalized amplitudes of such hoppings. On the other hand, \(T_3, T_4, T_5\) and \(T_6\) are the newly generated next nearest neighbour hoppings in different directions. We have shown all these hoppings schematically in Fig. 2.

It is worth to mention here that irradiation effect in the high frequency limit can also be taken into account by using other perturbation expansion methods i.e., Floquet Magnus\textsuperscript{49,50}, van Veick perturbation theory\textsuperscript{51}. Floquet Magnus expansion harbors unwanted driving phase dependence as a result of which the effective Hamiltonian also contains the driving phase. On the other hand, it is much more complicated to write higher order terms in van Veick perturbative expansion series.

\[\text{III. RESULTS}\]

In this section, we present our results involving static phases as well as Floquet phases. Starting from the static semimetal phase, we discuss the FTI and then analyse both static and Floquet SOTI. We have considered both \(t_1, t_2 = 1\) for all our numerical results and all the parameters having the dimension of energy are scaled with respect to the hopping strength. For all the results shown below, we have chosen \(\omega = 10\).

\[\text{FIG. 2. (Color online) Directions of newly generated effective hoppings are depicted by different arrows in the square lattice system. Here, \(T_1\) and \(T_2\) represent the renormalized nearest-neighbour hopping elements, whereas \(T_3, T_4, T_5\) and \(T_6\) denote the drive induced next nearest neighbour hoppings in different directions within the square lattice.}\]

\[\text{A. Trivial static semimetal without drive}\]

The bulk band structure of the static Hamiltonian given by Eq.(1) is shown in Fig. 3(a) and the corresponding total density of states (DOS) is presented in Fig. 3(b). As we see from Fig. 3(a), the conduction and
valence band meet at four points in the first Brillouin zone: \( \Gamma = (0,0), X = (0, \pi), Y = (\pi,0) \) and \( S = (\pi, \pi) \), describing a semimetallic behavior. The low energy spectrum at those four points exhibits a massless Dirac like dispersion. Therefore, this phase corresponds to a trivial semimetal with Chern number 0. The DOS plot resembles similar to that of graphene\(^{52}\) as it is vanishingly small near the zero of the Fermi energy (Dirac point in case of graphene) and scales linearly around it.

![Image](image_url)

**FIG. 3.** (Color online) (a) Bulk band spectrum for the static semimetal phase is shown. (b) Total DOS of the semimetal phase is demonstrated.

### B. First order topological insulator with drive

We now demonstrate the outcome of the external irradiation on the semimetal Hamiltonian (Eq.(1)). As the circularly polarized light breaks time-reversal symmetry \( T \) in the semimetal phase, it becomes a FTI with chiral edge states in presence of the external irradiation. In the high frequency regime, the effective Floquet Hamiltonian in presence of the external drive is given by Eq.(7).

#### 1. Band topology

In presence of the external drive, the band topology of the system exhibits very intriguing behavior. To understand this, we explore the bulk quasi-energy spectrum along the high symmetry line: \( \Gamma - X - S - Y - \Gamma \) for three different values of driving amplitude \( A \). They are illustrated in Fig. 4(a),(b),(c). Finite band gap is present in bulk quasi-energy spectrum for \( A = 3.0 \) as shown in Fig. 4(a), while the band gap closes at \( A = 3.35 \) (see Fig. 4(b)). The band gap again starts to reopen which is depicted in Fig. 4(c) for \( A = 3.8 \). The value of \( A \) at which the bulk band gap closes, Chern number (\( C_n \)) changes sign (see text for discussion). The band inversion process, with the rise of the strength of the external drive, can be understood as a Floquet topological phase transition in the bulk quasi-energy spectrum. Similar band inversion again takes place near \( A = 7.3 \) (not shown here) where \( C_n \) changes sign (see Fig 4(d)).

#### 2. Chern number and edge modes

To identify the Floquet topological phase transition, we calculate the Chern number \( C_n \) of the bulk band following Ref. [46]. The Chern number of the upper band is shown as a function of driving amplitude \( A \) in Fig. 4(d). As the periodic drive is turned on \((A \neq 0)\), due to broken \( T \), we see a sudden jump of \( C_n \) from 0 to 1 where the system becomes a FTI with counter propagating dispersive edge modes. We present the band structure in a \( y \)-directed slab geometry in Fig. 4(c) where the edge modes are clearly visible for \( A = 1.0 \). Similar edge modes have been found in case of graphene which becomes a FTI in presence of external periodic drive\(^{47,48}\).

The first jump of \( C_n \) from +1 to −1 takes place for \( A = 3.35 \) where the band inversion occurs. Similar Floquet topological phase transition also occurs for higher values of \( A \) \((A = 7.3, \text{ and can be seen from Fig. } \text{4(d)}, \text{ which again brings the concomitant band inversion. Note that, the value of } C_n \text{ is ill defined near the points } A = 2.4 \text{ and } 5.5 \text{ where the Hamiltonian itself is vanishingly small due to the vanishing of Bessel function } J_0 \text{. Those points are marked by red dots in Fig. 4(d).}

#### C. Higher order topological phase with drive

We now move to the main part of the paper where we describe the appearance of the HOTI phase. We add a Wilson mass term of the form\(^{43,53}\) \( H_B = \Delta [\cos k_x - \cos k_y] \sigma_1 \tau_1 \) with the initial Hamiltonian \( H_{SM} \) given by Eq.(1). \( H_B \) breaks both \( C_4 \) rotational symmetry and \( T \) but preserves the combined symmetry operation \( C_4 T \). We note that \( \{H_{SM}, H_B\} = 0 \). \( H_B \) changes sign across the corners under \( C_4 \) rotation and thus the presence of four corner modes are ensured by a generalized Jackiw-Rebbi index theorem\(^{54}\). A static second order topological insulator accommodating in-gap corner modes at all the four corners can thus be realized in the square lattice system.

In presence of the irradiation, the effective Floquet Hamiltonian is given by Eq.(4) and Eq.(7) including the new terms arising due to the \( C_4 \) symmetry breaking mass perturbation. Different hopping amplitudes in Eq.(8) are now modified by the mass parameter \( \Delta \) and given by,
FIG. 4. (Color online) Panel (a),(b),(c): Bulk band structure is shown along the Γ − X − S − Y − Γ line, indicating the presence of a bulk band gap, gap closing and reopening for \( A = 3, 3.35, 3.8 \) respectively. (d): Chern number is shown as a function of driving amplitude \( A \). Unphysical points where the Hamiltonian vanishes are indicated by red dots. (e): Band structure for a \( y \)-directed slab with counter propagating dispersive Floquet edge modes is presented for \( A = 1.0 \) and \( N_x = 100 \). Floquet edge modes around \( k_y = 0 \) are marked by the red circle.

where different parameters have their usual meaning as defined earlier.

1. Quadrupole moment

To establish the HOTI phase in our model we first investigate the topological invariant. From the existing literature\(^{15,16}\), it is established that SOTIs are distinguished by vanishing dipole moment but exhibiting quantized quadrupole moment \( Q_{xy} = 0.5 \) \((\text{modulo } 1)\)^\(^{43,53,55,56}\). Here we present the brief outline how to calculate quadrupolar moment numerically for HOTI phase. The macroscopic quadrupole moment for a crystal obeying periodic boundary condition is defined as following\(^{53,56}\):

\[
Q_{xy}^{(0)} = \frac{1}{2\pi} \text{Im} \ln \langle \Psi_0 | e^{2\pi i \sum \hat{q}_{xy}(\mathbf{r})} | \Psi_0 \rangle \quad \text{.} \tag{10}
\]

Here, \( \hat{q}_{xy}(\mathbf{r}) = \frac{\nabla^2}{\nabla^2} \hat{n}(\mathbf{r}) \) is the microscopic quadrupole moment at a lattice site \( \mathbf{r} \) with respect to \( x = y = 0 \), while \( L \) is the lattice size and \( | \Psi_0 \rangle \) is a many-body ground state which can be defined using the occupied Bloch states through the Slater determinant\(^{57}\).
To evaluate the quadrupole moment using Eq. (10), we construct a \( N \times N_o \) dimensional matrix \( \mathcal{U} \) by columnwise marshaling of \( N_o \) eigenvectors according to their energy. Here \( N \) is the total number of states while, \( N_o \) is the number of occupied states. Afterwards, we formulate another matrix operator \( \mathcal{W} \):

\[
\mathcal{W}_{\alpha,i,j} = \exp \left[ \frac{2\pi}{L^2} f(x_{\alpha,i}, y_{\alpha,i}) \right] \mathcal{U}_{\alpha,i,j} , \quad (11)
\]

Here, \( \alpha \) includes both the orbital and spin index. We consider \( f(x_{\alpha,i}, y_{\alpha,i}) = x_{\alpha,i} y_{\alpha,i} \). Therefore, \( Q_{xy}^{(0)} \), defined in Eq. (10), can be recasted into the following form using the above mentioned matrices as

\[
Q_{xy}^{(0)} = -\frac{1}{2\pi} \text{Im} \left[ \text{Tr} \ln (\mathcal{U}^\dagger \mathcal{W}) \right] . \quad (12)
\]

We obtain the value of quadrupole moment only up to modulo 1 and now on we omit the term “modulo 1” but whenever we present the value of the quadrupole moment, we intend it as modulo 1. We compute \( Q_{xy}^{(0)} \) when \( C_4 \) is broken along with \( \mathcal{T} \) where the concomitant topological static corner modes arise in the system. We find that \( Q_{xy}^{(0)} \) is always quantized with the value 0.5 (within numerical accuracy) for our system being in second order topological insulating phase.

With the inclusion of the external periodic drive, we now compute the Floquet quadrupole moment which is given by\(^{43,53,55,56} \)

\[
Q_{xy} = Q_{xy}^{(0)} - Q_{xy}^{(1)} \quad \text{where} \quad Q_{xy}^{(1)} = \sum_{\alpha} \frac{1}{2} f(x_{\alpha,i}, y_{\alpha,i}) \text{ is the value of } Q_{xy} \text{ in the atomic limit.}
\]

To numerically evaluate the quadrupole moment \( Q_{xy} \) from the Floquet Hamiltonian given by Eq. (7) along with the new hopping parameters (see Eq. (9)), we use the same prescription mentioned above. We find that \( Q_{xy} \) is always quantized having a value 0.5 for any nonzero value of \( A \) except when \( J_0(A) \) become zero which in turn makes the Hamiltonian vanishingly small and thus making numerical evaluation of \( Q_{xy} \) ill defined.

Note that as the drive is turned on, an onsite potential term is generated which is given by \( M \) in the effective Floquet Hamiltonian in Eq. (7). This term indicates the shifting of the band spectra in the quasi-energy space. The effect of the drive generated onsite potential can always be nullified by implementing an extra suitable gate voltage connected to the 2D system. We have neglected this term in our calculation for simplicity and for better understanding of the engineering of the Floquet HOTI phase. The latter has also been reported very recently via the application of quench.\(^{43} \)

2. Floquet corner modes

The static SOTI phase hosts in gap corner modes.\(^{15,16} \) Similarly, the FSOTI is characterized by the appearance of Floquet corner modes.\(^{16} \) The signature of these zero energy corner modes appear in the local density of states (LDOS). The LDOS as a function of energy \( E \) at \( i-th \) site of a lattice is defined as

\[
\rho_i(E) = \sum_\lambda |\langle i | n_\lambda \rangle|^2 \delta(E - \lambda) , \quad (13)
\]

where, \( |i \rangle \) and \( |n_\lambda \rangle \) are the eigenstates of the Hamiltonian. In order to probe the zero energy corner modes we have calculated LDOS at \( E = 0 \). The Floquet corner modes that arise in our square lattice system with periodic drive (\( A \neq 0 \)) and \( C_4 \) symmetry breaking term, are shown in Fig. 5(b), where the LDOS is depicted along two spatial directions (\( L_x, L_y \)) of the sample. One can observe that the corner modes are almost (due to finite
system size) localized at the four corners of the system. In Fig. 5(a), we have shown the eigenvalue spectrum of the Floquet Hamiltonian. The in-gap corner modes (at zero energy) are shown by the red dot while the blue line indicates the continuum bulk. Four corner modes, all of which are at zero energy, are depicted in the inset of Fig. 5(a). We emphasize the fact that the 0D Floquet corner modes are robust against the high frequency drive and are pinned at zero energy.

IV. SUMMARY AND CONCLUSIONS

To summarize, in this article, we have explored the possibility of obtaining both conventional topological insulator (first order) and higher order topological insulator (second order) from two dimensional trivial semimetal via Floquet engineering. We have first formulated the Floquet Hamiltonian using Brillouin-Wigner perturbation theory expansion in the high frequency limit (Floquet Hamiltonian using Brillouin-Wigner perturbation theory expansion). We have then formulated the (second order) from two dimensional trivial semimetal insulator (first order) and higher order topological insulator and are pinned at zero energy.

In Fig. 5(a), we have shown the eigenvalue spectrum of the Floquet Hamiltonian. The in-gap corner modes (at zero energy) are shown by the red dot while the blue line indicates being broken due to circular nature of the polarized light and the system becomes a FTI accommodating two counter-propagating edge modes. This topological phase transition is identified by the abrupt change of the Chern number of the quasi-energy bands from 0 to 1. With the enhancement of the driving strength, we observe another topological phase transition near $A = 3.35$ which occurs with concomitant band gap closing in the bulk. This Floquet topological phase transition from semimetal to FTI is similar to the well known Floquet topological insulator formation by driving pristine Graphene.

As the crystalline $C_4$ symmetry is broken in the square lattice system by adding a Wilson mass term, we realize a static second order topological insulating phase hosting corner modes at the four corners. These in-gap corner modes are demonstrated by both eigenvalue spectrum as well as LDOS. The static HOTI phase is identified by quantized quadrupolar moment $Q_{xy}^{(0)}$ which always exhibits the value 0.5. We numerically compute $Q_{xy}^{(0)}$ which comes out to be 0.5 within numerical accuracy for our model as soon as the Wilson mass term is added to the Hamiltonian i.e., $C_4$ is broken alongwith $T$. When the external drive is turned on, the system becomes Floquet HOTI accommodating Floquet corner modes. We evaluate the Floquet quadrupole moment $Q_{xy}$ which still comes out to be the same quantized value 0.5 for any strength of the driving amplitude $A$ and consequently Floquet corner modes appear localized at the four corners of the system. We illustrate these corner modes in the LDOS behavior.

The Floquet edge and corner modes that appear in respective first order TI and SOTI phase of our model system is still quasi-static in nature as we have formulated our problem based on the effective Hamiltonian picture in the high frequency regime. In this limit, only virtual photon transition have been taken into account. Technically, the full Floquet Hamiltonian in the extended Sambe space is projected back to the zero photon subspace using a high-frequency expansion based on the BW perturbation theory. The investigation of dynamical nature of these corner modes in the intermediate frequency regime ($\omega \sim t_1, t_2$) considering real photon transition within the Floquet sub-bands, still remains a open question and will be presented elsewhere for our model.

Throughout our calculations, we have treated our square lattice model to be disorder free and at zero temperature. Although, this might not be the case in practical situation. Nevertheless, corner modes should be persistent in presence of weak disorder and at finite temperature with the disorder and temperature scale being smaller than the bulk band gap of the system. However, the effect of strong disorder with its strength being comparable to the bandwidth and in presence of external irradiation, can be very interesting and is beyond the scope of the present work.

As far as practical realization of our model is concerned, 2D SOTI has recently been realized in kagome lattice using acoustic measurements, in photonic crystals setup with near field scanning measurement technique, and in electrical circuits setup using spectroscopic measurements. Our 2D model, therefore, may also be possible to engineer in such systems and may be a platform to understand and discover the Floquet higher order topological phases and Floquet corner modes using local measurements (e.g. STM) in presence of high frequency periodic drive with circular polarization.

ACKNOWLEDGMENTS

We acknowledge Adhip Agarwala and Tanay Nag for helpful discussions. We acknowledge SAMKHYA: High Performance Computing Facility provided by Institute of Physics, Bhubaneswar, for our numerical computation.

Author Contributions

AKG and GCP have contributed equally for this work.
