Optical perturbation of atoms in weak localization

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Abstract. We determine the microscopic transport parameters that are necessary to describe the diffusion process of the atomic gas in optical speckle. We use the self-consistent theory to calculate the self-energy of the atomic gas. We compute the spectral function numerically by an average over disorder realizations in terms of the Greens function. We focus mainly on the behaviour of the energy distribution of the atoms to estimate a correction to the mobility edge. Our results show that the energy distribution of the atoms locates the mobility edge position under the disorder amplitude. This behaviour changes for each disorder parameter. We conclude that the disorder amplitude potential induced modification of the energy distribution of the atoms that plays a major role for the prediction of the mobility edge.

1. Introduction

Disorder has long proven to be a crucial ingredient to understand transport properties for many different kinds of waves: with electromagnetic waves, ultrasonic waves and atomic waves [1-3]. Optical potentials used to produce disorder or confine condensates in particular directions of space offer an unprecedented experimental control. Bose-Einstein condensates thus appear to be ideal systems to study the interplay between disorder and interactions. This is reason why they are extensively used to revisit many problems of condensed-matter physics.

Weak localization leads to an enhanced probability to return back to the origin. It therefore slows down transport, i.e. it reduces the diffusion constant and the conductivity compared to the Boltzmann/Drude incoherent contribution. This effect in condensed matter systems have been widely studied theoretically and experimentally, and signatures have been found in various systems (see the reviews [4-6]). For instance weak localization correction to the conductivity has been studied as a function of temperature (which activates sources of decoherence such as phonons) and magnetic field (which introduces spin-orbit couplings and additional phases to the paths). They have also been studied as a function of disorder parameters, such as impurity concentration for example. The reviews in Ref. [5] present the dependence of the resistance of a thin disordered Mg-film, with magnetic field for several values of the temperature.

In the weak localization regime, the transport is reduced by interference between multiple scattering paths, but particles can still propagate to infinity [6]. However, in our simple model we only considered interference between loop paths that are travelled in one direction or the opposite, but interference effects are to be taken into account for many other pairs of paths, in principle all those which visit the same set of scatterer in particular those which include loops inserted in loops etc... They can lead to the complete suppression of transport, an effect which is called Anderson (or strong) localization. The localized and extended regions of the spectrum are separated by critical energies. The
link between critical energy and the statistical properties of disorder has been studied in experiments with cold atoms in random optical [7- 9]. The wave propagation also is governed by scattering from the random impurities. Three typical energy-dependent length scales can be identified, which characterize three basic effects induced by the disorder. First, single scattering from impurities depletes the $k$-wave states, which can be seen as quasiparticles in the disordered medium, with a finite life-time $\tau_s(k)$. Single scattering hence defines the first length scale, namely the scattering mean-free path, $\ell_s = v \tau_s$, which characterizes the typical length travelled by the wave before it looses the memory of its initial state. Then, multiple scattering defines the second length scale, namely the Boltzmann transport mean-free path, $\ell_B$, which characterizes the typical length travelled by the wave before it looses the memory of its initial direction. In general, several scattering events are necessary to significantly deflect the trajectories so that $\ell_B \geq \ell_s$. The two length scales are found to be equal only in the white-noise limit if it exists, where the wavelength is smaller than the typical size of the impurities [10]. The disorder is then equivalent to a set of randomly distributed Dirac peaks and the scattering is isotropic. In this case the wave looses the memory of its initial state and initial propagation direction at the same time. In the general case, within the distance $\ell_B$, the transport crosses over from ballistic to diffusive. The average squared size of the wave packet increases linearly in time, $\langle r^2 \rangle = 2dD_B t$, with $D_B = v \ell_B / d$ (where $d$ is the space dimension) and $D_B$ is the Boltzmann diffusion constant [11, 12].

The self-consistent theory, first formulated by Vollhardt and Wolfle in 1981 for 2D electron conductivity [13, 14], was the first work that explicitly calculated quantum corrections to the classical “Drude”-conductance. Beyond this theory, the diffusion of the BEC in a random potential has been studied [15–18] and it was pointed out that only some fraction of the condensate diffuses away, while the rest stays localized near its initial location (Anderson transition for BEC) [17, 18]. The macroscopic transport properties of a coherent wave in a disordered medium, namely diffusion and weak/strong localization, closely depend on the microscopic and statistical properties of the disorder itself. In this paper we study quantum transport of matterwaves in disordered potentials with isotropic correlations in 3d. The isotropy provides a great simplification, and other methods, such as the self-consistent Born approximation, can be used to calculate the self-energy. Many experiments reported the observation of an Anderson transition in momentum space using cold-atom kick-rotor setups [19, 20], study of classical diffusion in two-dimensional (2D) speckle potentials [21] which is revelant for current experiments on ultra-cold atoms in speckle disorder. The calculation of any measurable quantity is specific to the particular realization of the disorder. Therefore, meaningful quantities correspond to statistical averages over realizations of disordered potentials. When averaging over disorder realizations, some quantities can be written in terms of the average Green function $\langle G(E) \rangle$ it is the case the spectral function $A(E, k)$ defined by [22,23]. It contains all the information about the spectrum of the disordered medium. For a particle in disorder-free space, it is given by $A(E, k = 2\pi (\delta(E - \epsilon(k))) \text{with} \, \epsilon(k) = \frac{\hbar k^2}{2m}$. The average density of states (per unit volume) reads: $N(E) = \int \frac{dk}{2\pi d} \frac{A(E, k)}{2\pi}$. This paper uses the first Born Approximation to study the behavior of the energy distribution of the atoms in an optical speckle for two values of disorder amplitude. We expect finding the true mobility edge and possibly leading to the Anderson localization phenomenon.

2. Model
We Consider a weakly interacting Bose-Einstein condensate of $N \gg 1$ atoms of mass $m$ expanding in a three-dimensional random potential $V(r)$. The origin of time is chosen within the third stage of the toy model described in Ref [24] which ensures that interactions are neglect. We calculate the density profile of the condensate inserting the expression of the random potential see in Ref [25]. The potential disorder obtained inside an integrating sphere lit with a laser beam, the real-space correlation
function of which reads $C(r) = V_R^2 \frac{(\sin r)^2}{r^2}$ with $r$ normalized by $\sigma$. $\sigma$ is the correlation length the correlation [24].

The spectral function permits to relate the energy distribution $\phi_E$ and the momentum distribution $\phi_k$ of the stationary particles in the disorder: $\phi_E = \int \frac{dk}{2\pi a} A(E, k) \phi_k$ which is normalized by $\int dE \frac{A(E, k)}{2\pi} = 1$. The calculation of the energy distribution therefore requires the knowledge of the full spectral function, i.e. of the real and imaginary parts of the self-energy see in this equation:

$$A(E, k) = \frac{(- 1 \text{Im} \Sigma(\epsilon, k))}{(E - \epsilon(k) - \text{Re} \Sigma(\epsilon, k))^2 + (\text{Im} \Sigma(\epsilon, k))^2}$$

With $\text{Re} \Sigma(\epsilon, k)$ and $\text{Im} \Sigma(\epsilon, k)$ are the real and imaginary parts of the self-energy $\Sigma$ respectively. The quantity $\text{Re} \Sigma(\epsilon, k)$ thus describes the shift in energy of the free-particle modes when they are dressed by the disorder. The quantity $\text{Im} \Sigma(\epsilon, k)$ is the energy width of the spectral function, which defines the scattering mean free time $\tau_s = -\frac{\hbar}{2\text{Im} \Sigma(\epsilon, k)}$. Therefore the spectral function contains all the information about the relative weight, the energy, and the lifetime of the quasi-particles, i.e. the particles dressed by the disordered medium, which on average define an effective medium.

The microscopic parameters in these systems are precisely known and in many cases, which paves the way to direct comparison between experiments and theory [25]. This is a great advantage of ultracold atoms, compared to traditional condensed matter systems. Major advances reported so far are the observation of one-dimensional (1D) Anderson localization of matter waves [26, 27], and studies of the effects of weak [28-31] and strong [32] interactions in disordered gases. All these studies have much benefited from the close interplay between theory and experiments.

The aim of this work is to discuss the transport and localization properties of matter waves in 3D isotropic optical speckle potentials. We will focus on the mobility edge $\epsilon_m$, which is the solution of $E - \epsilon(k) - \text{Re} \Sigma(\epsilon, k) - \text{Im} \Sigma(\epsilon, k) = 0$. Here, we incorporate $\text{Re} \Sigma(\epsilon, k)$ and $\text{Im} \Sigma(\epsilon, k)$ into the spectral function in first approximation.

3. Results and discussion

In this section, we discuss the numerical results on the distribution of energy expressed from the self-energy by using Born approximation. The angular integral of the self-energy $\Sigma(\epsilon, k)$ was calculated analytically for first Born approximation [32] and we also calculate it numerically.

Figure 1 shows the imaginary part of $\Sigma(\epsilon, k)$ for three values of the energy $\epsilon$. The curve shows that the behavior of $\text{Im} \Sigma(\epsilon, k)$ is different for three values of $\epsilon$ and it is clearly negative. This shifts the band edge of the energy spectrum to $\Delta \epsilon = 0.2$ (normalized by $\epsilon_m$).

We observed that $\text{Im} \Sigma(\epsilon, k)$ is nonzero only for $0 \leq \epsilon \leq 2.78$, at $\epsilon = 0.5$ comparing with the other two values. This behavior assumes that the number of atoms expanding in the random disorder increases and it can be seen that the first Born approximation significantly overestimates the amount of scattering in this case. This behavior is qualitatively similar to that obtained in Ref [33] but with $\epsilon = 1$ with being another consequence of long-range correlations.
Figure 1. Imaginary part of the self-energy (in units of $\varepsilon_\xi$) of an atom in a speckle potential with $\varepsilon=0.5, 1, and 1.5$, as a function of $k$ in first Born approximation.

To find the spectral $A(\varepsilon,k)$ we go beyond the for first Born approximation [20, 26] at energy $\varepsilon$. At larger energies $\varepsilon > 3$, the spectral function behaves as a free space expression $A(\varepsilon,k) \propto \delta(\varepsilon - \varepsilon_k)$ strongly peaked near $k = \sqrt{\varepsilon}$.

For a short-range correlations, the self-energy is independent of $k$ and the spectral function has a large energy tail ($A(\varepsilon,k) \propto 1/\varepsilon^{3/2}$) and atoms with high energies exhibit diffusion behavior ($\mu > \varepsilon$) [21]. Thus, a potential with correlation reduces the large energy tail of $A(\varepsilon,k)$ and seems to help in localizing more atoms.

We go beyond the for first Born approximation to solve numerically the spectral function $A(\varepsilon,k)$ for $V_R = 1$ and $V_R = 2$. The curve observed in figure 2 shows that the edges of the spectrum are different, one with positive energy, the atom expands from zero to 2. Other one with positive energy exhibits a peak with small energy width. This peak related to the real part of the self-energy assume us that $\int A(\varepsilon,k) \, d\varepsilon = 1$ with a constant amplitude of disorder.

Figure 2. The distribution of energy expressed from the self energy by using Born approximation at $\varepsilon = 1$, for $V_R = 1$ and $V_R = 2$.

We solve by iterations the distribution of energy expressed from spectral function. The energy spectrum becomes continuous and larger. It has a large weight and extends up to $\varepsilon = 2$, many atoms achieve energies $\varepsilon$. These results are studied with predictions based in Ref [21] whose found 45%
atoms localized for $V_R = 1$ at small chemical potential. In Figure 3, we observed the distribution of energy $N(E)$ which is the probability to find an atom with energy $\varepsilon$ but on a logarithmic scale. In the presence of the random potential, one find at $\varepsilon = 0$ the probability remains finite and nonzero, and they obtain 35% atoms delocalized [24].

Our numerical results show that the first Born approximation locates the band edge at larger energies and extends up $\varepsilon = 2$. The amplitude disorder takes an important place on energy spectrum when we include the first order corrections in Born approximation. We expect that the localization occurs near the mobility edge which is located at $\varepsilon = -0.34$.

We predict the possible presence of a mobility edge in this region. We observe that the disorder amplitude provided by the optical disordered potential causes a shift of the band edge towards the negative energy.

This work may expect to find the true mobility edge position in a particular way to measure the exact value of $\varepsilon_c$, and to determine critical exponents for localization.

4. Conclusion

In this work, we discuss the optical perturbation of atoms in weak localization for the first order Born approximation. We calculate imaginary part of the self-energy for different value of the disorder amplitude to study the behavior of self-energy on the distribution of energy. The self-energy $\Sigma(\varepsilon, k)$ is a complex function, we found that its imaginary part shifts the band edge of the energy spectrum. This part of the self-energy has an important role on the energies distribution. In the first order Born approximation, the energy spectrum of the spectral function has extend from $\varepsilon = 0$, to $\varepsilon = 2$, it can be seen that the first order Born approximation significantly overestimates the amount of scattering in this case. We predict a possible presence of the mobility edge in this region. We observe that the disorder amplitude provided by the optical disordered potential changes the form of the curve and shifts the band edge to a negative energy. Also, we examine the behavior of the spectral function for two values of constant disorder amplitude to locate the mobility edge position, we find out some remarkable differences between them. We assumes that the number of atoms expanding in the random disorder increases in this case the energy spectrum becomes continuous with a maximum equal to 0.64 for $V_R = 2$. The behavior of the density of states is different compared to the density of states of free particles. By this way we have plotted the probability of distribution of energy at logarithmic scale to locate the mobility edge. We find that the disorder amplitude provided by the optical speckle is responsible for the position of the mobility edge.

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