A Therometer for the 2D Electron Gas using 1D Thermopower

N. J. Appleyard, J. T. Nicholls, M. Y. Simmons, W. R. Tribe and M. Pepper
Cavendish Laboratory, Madingley Road, Cambridge CB3 0HE, United Kingdom
(March 24, 2022)

We measure the temperature of a 2D electron gas in GaAs from the thermopower of a one-dimensional ballistic constriction, using the Mott relation to confirm the calibration from the electrical conductance. Under hot electron conditions, this technique shows that the power loss by the electrons follows a $T^3$ dependence in the Gruneisen-Bloch regime, as predicted for acoustic phonon emission with a screened piezoelectric interaction. An independent measurement using conventional thermometry based on Shubnikov-de Haas oscillations gives a $T^3$ loss rate; we discuss reasons for this discrepancy.

Accurate electron thermometry is needed in many aspects of low-dimensional semiconductor physics, particularly given the increasing importance of hot electron effects as mesoscopic device dimensions are reduced and electron mobility increases. Surplus heat energy in a two-dimensional electron gas (2DEG) is rapidly shared amongst the carriers through electron-electron interactions, and an effective electron temperature $T_e$ is established which may be considerably higher than the crystal lattice temperature $T_l$, to which both external thermometry and refrigeration are coupled. A measurement of the electron temperature is therefore needed to determine how an electron gas thermalizes with its surroundings. Measurements of the thermoelectric response and thermal conductivity of mesoscopic devices are also interesting in their own right, as they provide fundamental information about electronic properties which is not available from electrical transport measurements alone.

Although many techniques allow the measurement of bulk lattice temperatures, the weak coupling of the electrons to their surroundings has hampered accurate measurement of the electron temperature $T_e$. Previous techniques have employed the visibility of features in the electrical transport, notably Shubnikov-de Haas (SdH) oscillations, but also using the zero field resistance and weak localization corrections. Mesoscopic effects such as Coulomb blockade have also been applied as electron thermometers. In this letter, we introduce a novel technique, where the thermoelectric response (thermopower) of a one-dimensional constriction is used to measure the electron temperature. Self-consistent checks confirm the validity of the technique, which we then employ to deduce the energy relaxation rate of heated electrons in a 2DEG in a GaAs/AlGaAs heterostructure, obtaining good agreement with the theory of acoustic phonon emission in the Gruneisen-Bloch regime.

A schematic of the device is shown in Fig. 1, and is similar to those used in previous thermopower measurements. The area shaded grey shows the etched mesa containing a 2DEG defined at a GaAs/AlGaAs heterojunction, 2770 Å below the surface of a structure grown by molecular beam epitaxy. From measurements of the low-field SdH oscillations and the zero field resistivity, the 2DEG has an electron density $n_e = 2.1 \times 10^{11}$ cm$^{-2}$ and mobility of $\mu = 4.5 \times 10^6$ cm$^2$V$^{-1}$s$^{-1}$ at 1.5 K. Electrons in the heating channel to the left of the device are heated to $T_e$ by an AC electric current $I_H$ at $f_H = 85$ Hz. In a high mobility 2DEG the thermal decay length exceeds 100 μm for $T < 2$ K, and heat is efficiently conducted over the whole heating channel. Hot electrons are prevented by the 1D constrictions A and B from passing into the right-hand side of the 2DEG, which therefore remains at the lattice temperature $T_l$. Each 1D constriction is formed by depleting the 2DEG with a negative voltage $V_g$ applied to a pair of Schottky split-gates (shown in solid black) that are defined on the device surface, with lithographic length $L = 0.4$ μm and gap width $W = 0.8$ μm. On account of its thermoelectric properties, the temperature difference $T_e - T_l$ across a 1D constriction generates a potential difference, which is measured at the harmonic frequency $2f_H$. As the 2DEG is heated directly, without raising the lattice temperature $T_l$, the phonon drag contribution to the thermopower can be ignored and only the contribution of electron diffusion is detected. Figure 2 shows that $\Delta V$ is in fact the voltage measured across the two constrictions; one constriction is kept at fixed width as a reference, whilst the $V_g$ of the other is swept.

When both charge and heat are exclusively carried by electrons, it has been shown both for diffusive and ballistic transport that the thermopower $S$ is related to the energy derivative of the conductance $G$,

$$ S = \frac{\Delta V}{T_e - T_l} \bigg|_{t=0} = -\frac{\pi^2 k_B^2}{3e} (T_e + T_l) \frac{\partial (\ln G)}{\partial \mu}, \quad (1) $$

where $\mu$ is the chemical potential of the contacts relative to the 1D subbands. It is assumed that features in the thermopower are not subject to thermal broadening, and that the electrons are non-interacting. The electrical transport properties of ballistic 1D constrictions are well understood. The conductance is quantized at $G = i(2e^2/h)$, when there are $i$ transmitted 1D subbands.
As the gate voltage \( V_g \) on the split-gate is made more negative, the constriction is narrowed and the number of 1D subbands is reduced, and the conductance drops in steps of \( 2e^2/h \). The potential in the constriction can be well described by a saddle point and tunneling through the device has a characteristic broadening energy \( \hbar \omega_x \). As the conductance \( G \) rises from \( i(2e^2/h) \) to \((i+1)(2e^2/h)\) a peak is observed in the thermopower voltage \( \Delta V \).

The height of the \( i^{th} \) peak in \( \Delta V \) is expected to be

\[
\Delta V_{pk}^i = - \frac{C(T_e^2 - T_l^2)}{(i + \frac{1}{2})},
\]

where the constant \( C \) depends on the intrinsic peak broadening. In the saddle-point model, \( C = \sqrt{2\pi^2 k_B^2/2 e \hbar \omega_x} \) for AC measurements in linear response.

A simultaneous measurement is made of the four-terminal resistance \( R \) of the constriction, using a small current of \( I_R = 2 \) nA at a different frequency \( f_R = 18 \) Hz. Figure 3 shows the conductance, \( G(V_g) = 1/R \), of constriction A, as its width is swept whilst constriction B is held fixed as a reference. The steps in conductance between the quantized values lead to peaks in the measured thermopower, and we also plot the thermopower behavior predicted from the Mott relation (1), \( d(\ln G)/dV_g \).

The thermal voltage is in good agreement with this prediction, except in the region close to pinch-off \((i = 0)\) which will be discussed elsewhere.

Figure 4 shows thermopower data for constriction A at different lattice temperatures \( T_l \) and heating currents \( I_H \). The traces \( \Delta V(V_g) \) collapse onto a single curve when normalized by the \( i = 1 \) peak height \( \Delta V_{pk}^1 \); a similar data collapse was seen for constriction B. The dotted line is representative of measurements taken at high lattice temperatures and currents, where thermal broadening causes the breakdown of the Mott relation and the use of the constriction as a thermometer. This broadening occurs at \( T_c \approx 3 \) K for constriction A, and \( 1.7 \) K for constriction B. The inset to the figure confirms the prediction for the height of the \( i^{th} \) peak in the thermopower voltage, \( -\Delta V_{pk}^i \propto 1/(i + \frac{1}{2}) \), for many 1D subbands.

From Eq. 3 we use the \( i = 1 \) thermopower peak height to determine the temperature \( T_e \) of the hot electrons in the heating channel. The fitting parameter \( C \) yields \( \hbar \omega_x = (1.6 \pm 0.2) \) meV and \( (3.0 \pm 0.3) \) meV for constructions A and B, respectively. Calibrating the gate voltage in terms of energy using the method of Patel et al. (1,2,3) and fitting the shape of the conductance characteristic between the \( i = 1 \) and \( i = 2 \) conductance plateaux to the saddle-point model gives the independent values \( \hbar \omega_x = (2.2 \pm 1.0) \) meV and \( (2.0 \pm 0.6) \) meV. This provides independent confirmation that the thermopower follows the Mott prediction, and that our calibration of the electron temperature scale is accurate.

Figure 5 shows \( -\Delta V_{pk}^1 \) as construction B is swept at several lattice temperatures \( T_l \), as a function of the power dissipated per electron \( P = I_H^2 R_H/n_e A \). The resistance \( R_H = 61.5 \) \( \Omega \) is the measured along the heating channel, which has an area \( A = (800 \times 100)(\mu m)^2 \). Electrons heated to a temperature \( T_e \) exchange energy with the lattice at \( T_l \), and the net power transfer is expected to follow

\[
P = \dot{Q}(T_e) - \dot{Q}(T_l).
\]

For low currents we measure \( \Delta V_{pk}^1 \propto P \), demonstrating that the thermopower is in the linear response regime, \((T_e - T_l) < T_l \). At high currents, the lattice temperature becomes irrelevant to the net loss rate, and the data converges to \( \Delta V_{pk}^1 \propto P^{0.4 \pm 0.02} \), suggesting that \( \dot{Q} \propto T_e^{0.5 \pm 0.3} \). A least-squares fit based on this behavior is shown as a dashed line in Figure 5. Using the value of \( C \) determined earlier, the thermopower peak heights collapse onto a single line of the form given in Eq. 3 using \( \dot{Q}(T) = (61 \pm 10)T^5 + (9 \pm 3)T^2 \) eVs\(^{-1} \) (where \( T \) is in units of K); this form is independent of which of the two constructions is used as the thermometer. The \( T^2 \) term has been included to represent heat leakage through the Ohmic contacts of the device, with an effective thermal conductance \( \kappa \approx 1.0 \) nWK\(^{-2} \cdot T \), equivalent to a conductance \( G \approx (20 \Omega)^{-1} \). This term dominates at low temperatures, where cooling of the electrons can be achieved only by thermal conduction through the contact wires. The magnitude of the thermal conduction term is of crucial importance in device design for low temperature measurements, where it has proved difficult in the past to effectively cool a 2DEG below 50 mK.

The \( T^5 \) dependence of \( \dot{Q} \) is expected from the stimulated emission of acoustic phonons by hot 2D electrons. Price showed that at low temperatures, in the Gruneisen-Bloch (GB) regime, phonon emission produces small-angle scattering of the electrons and the wavenumber-dependence of the interaction becomes important. Coupling through a screened deformation potential (DP) yields \( \dot{Q} \propto T^7 \), whereas a screened piezoelectric (PZ) coupling gives \( \dot{Q} \propto T^9 \), and so PZ coupling should dominate at the lowest temperatures in a polar material such as GaAs. Phonon dispersion anisotropy reduces the prediction by a factor 0.77 to \( \dot{Q} \approx 270 T^5 n_e^{-3/2} \) (where \( T \) is in units of K and \( n_e \) in \( 10^{11} \text{cm}^{-2} \)). Our measurements using the 1D thermopower for thermometry give \( \dot{Q} \approx 61 T^5 \), in excellent agreement with the prediction \( \dot{Q} \approx 88 T^5 \). Our measurements therefore confirm both the theories of 1D thermopower and of 2D energy loss by phonon emission.

The crossover from PZ to DP-dominated coupling is predicted to occur at \( T_e \approx 2 \) K in GaAs/AlGaAs heterostructures, and so it is at first surprising that there is no deviation from \( \dot{Q} \propto T^5 \) behavior at high temperatures. However, interpolation between the equipartition and GB regimes showed that the onset of DP-coupled scattering merges with the demise of PZ coupling, extending the \( T^5 \) behavior to higher temperatures, and hence no \( T^7 \) behavior is expected. We are unable to
extend our measurements of this sample to higher temperatures, to test the prediction that $Q \propto T$ in the equipartition regime, $T > 6$ K, as thermal broadening of the thermopower peaks invalidates Eq. 2 and our method of thermometry.

There have been few measurements of the energy relaxation from GaAs 2DEGs below 10 K in zero magnetic field. Using the 2DEG resistance for thermometry, Wennberg et al. reported $Q \propto T^3$ behavior, but with a measured prefactor two orders of magnitude larger than expected. In contrast, a photoconductivity measurement by Verevkin et al. measured prefactor two orders of magnitude larger than expected. In contrast, a photoconductivity measurement by Verevkin et al. gave agreement with theory at the onset of the GB regime. Recently, Mittal et al. have measured both the $B_\perp = 0$ resistance and the weak localization correction, and reported agreement with Price’s model around $T_e = 0.2$ K, though the data presented are insufficient to establish the full temperature dependence.

In a number of experiments, the electron temperature $T_e$ has been deduced from the damping of SdH oscillations when the electrons are heated by varying $T_1$ or a heating current $I_H$. These have shown that the power loss rate $Q$ varies as $T^3$ at low temperatures, and a low-field measurement of our sample by this technique also shows $Q = I^2_H R(B_\perp)/N_e \propto T^3$, as shown in Fig. 2. Whilst some authors have attributed this to the crossover between the GB and equipartition regimes, our data show that the two thermometry techniques yield different temperatures under the same heating conditions. We propose, therefore, that the energy relaxation mechanism at finite field differs from that at $B_\perp = 0$, as the electron wavefunctions are those appropriate to Landau levels, and the momentum transfer in the plane is restricted to $\sim h/l_e$, where $l_e$ is the cyclotron length. Scattering is then restricted to phonons in a narrow cone perpendicular to the 2DEG, resulting in a weaker temperature dependence of the energy relaxation rate. We note that Chow et al. have observed $Q(T, B_\perp) \approx 800 T^{-2} h^{-3/2}$ using the SdH technique for $T_e < 700$ mK at $B_\perp \approx 0.14$ T, where the thermal cut-off of available phonon energies occurs before the momentum cut-off comes into effect. There is some evidence of a crossover at $T_e \sim 0.5$ K both in their data and our own, and this is an area which merits further investigation.

In conclusion, we present measurements of the thermopower of a 1D constriction, including the linear regime, that validate the single-particle model both by comparison to simultaneous resistance measurements through the Mott relation, and using independently obtained values for 1D subband broadening. We have demonstrated a new method of electron thermometry, with potential for application to the study of non-equilibrium effects in low-dimensional semiconductor structures. This also paves the way for accurate measurements of the thermal conductance of a 1D system and a test of theoretical predictions that the Wiedemann-Franz law will be violated by interacting electrons. The measured energy relaxation rate from the 2DEG shows unequivocally that power losses in the absence of a magnetic field scale as $T^3$, in good agreement with the theory of coupling to acoustic phonons, whereas the theory is no longer valid in the presence of a small magnetic field.

We thank the Engineering and Physical Sciences Research Council (UK) for supporting this work, and JTN acknowledges an Advanced EPSRC Fellowship. We thank Dr A.S. Dzurak for useful discussions.

1. B. K. Ridley, Rep. Prog. Phys. 54, 169 (1991).
2. Y. Ma et al., Phys. Rev. B 43, 9033 (1991).
3. S. J. Manion et al., Phys. Rev. B 35, 9203 (1987).
4. A. M. Kreschuk et al., Solid State Commun. 65, 1181 (1988).
5. D. R. Leadley, R. J. Nicholas, J. J. Harris, and C. T. Foxon, Semicond. Sci. Technol. 4, 879 (1989).
6. N. Balkan, H. Çelik, A. J. Vickers, and M. Cankurtaran, Phys. Rev. B 52, 17210 (1995).
7. A. K. M. Wennberg et al., Phys. Rev. B 34, 4409 (1986).
8. A. Mittal et al., Surf. Sci. 361, 537 (1996).
9. M. Pekola, K. P. Hirvi, J. P. Kauppinnen, and M. A. Papaefremov, Phys. Rev. Lett. 73, 2903 (1994).
10. L. W. Molenkamp et al., Phys. Rev. Lett. 65, 1052 (1990).
11. A. S. Dzurak et al., J. Phys.: Cond. Matt. 5, 8055 (1993).
12. K. Ja. Thomas et al., Appl. Phys. Lett. 67, 109 (1995).
13. This signal is sensitive only to thermal effects, which vary as $T^3_H$.
14. D. G. Cantrell and P. N. Butcher, J. Phys. C 19, L429 (1986).
15. N. F. Mott and H. Jones, The Theory of the Properties of Metals and Alloys, 1st ed. (Clarendon, Oxford, 1936).
16. U. Sivan and Y. Imry, Phys. Rev. B 33, 551 (1986).
17. C. W. J. Beenakker and H. van Houten, in Solid State Physics, edited by H. Ehrenreich and D. Turnbull (Academic Press, New York, 1991).
18. H. A. Fertig and B. I. Halperin, Phys. Rev. B 36, 7969 (1987).
19. L. Martín-Moreno, J. T. Nicholls, N. K. Patel, and M. Pepper, J. Phys.: Cond. Matt. 4, 1323 (1992).
20. C. R. Proetto, Phys. Rev. B 44, 9096 (1991).
21. N. K. Patel et al., Phys. Rev. B 44, 13549 (1991).
22. P. J. Price, J. Appl. Phys. 53, 6863 (1982), we correct an error of 2 in the calculation.
23. C. Jasiukiewicz and V. Karpuš, Semicond. Sci. Technol. 11, 1777 (1996).
24. A. A. Verevkin et al., Phys. Rev. B 53, 7592 (1996).
25. D. J. McKitterick, A. Shik, A. J. Kent, and M. Henini, Phys. Rev. B 49, 2585 (1994).
26. E. Chow, H. P. Wei, S. M. Girvin, and M. Shayegan, Phys. Rev. Lett. 77, 1143 (1996).
27. C. L. Kane and M. P. A. Fisher, Phys. Rev. Lett. 76, 3192 (1996).
28. R. Fazio, F. W. J. Hekking, and D. E. Khmelnitskii, Phys. Rev. Lett. 80, 5611 (1998).
FIG. 1. Schematic of the device and measurement circuit. The etched mesa, shown in grey, consists of a heating channel and two voltage probes, where the two 1D constrictions are defined. The four-terminal resistance $R$ is measured simultaneously with the thermopower $S$, but at a different frequency. Magnified view: the two pairs of split-gates defining the constrictions A and B are shown in solid black.

FIG. 2. Experimental traces of the conductance $G$ (derived from $R$) and the thermopower voltage $-\Delta V$ from constriction A, using a heating current $I_H = 1.5 \, \mu A$ at a lattice temperature $T_l = 305 \, \text{mK}$, so that $T_e \approx 600 \, \text{mK}$. The dashed line shows the predicted thermopower signal $-\Delta V(V_g) \sim d(\ln G)/dV_g$ from the Mott relation (Eq. 1).

FIG. 3. Collapse of nineteen traces of the thermopower for constriction A, for lattice temperatures $0.3 \, \text{K} < T_l < 1.35 \, \text{K}$, and heating currents $0.7 \, \mu A < I_H < 20 \, \mu A$. Each trace has been divided by the $i = 1$ peak height $-\Delta V_{pk}$, which varies from $0.1 - 4.7 \, \mu V$. The dotted trace, for which $T_l = 1.39 \, \text{K}$ and $I_H = 20 \, \mu A$, shows the effect of thermal broadening; such data have been excluded from subsequent analysis. Inset: The thermopower peak heights $-\Delta V_{pk}$ follow the predictions of Eq. 2, which is shown as a solid line.

FIG. 4. The $i = 1$ thermopower peak height of constriction B as a function of dissipated power $P$, measured at different lattice temperatures $T_l$. Linear response is achieved even at the lowest temperatures. The solid lines show the fit $\dot{Q} = 61 \, T^{5} + 9 \, T^{2} \, \text{eV} \, \text{s}^{-1}$ (deduced from Fig. 5).

FIG. 5. Universal behaviour of $\dot{Q}$ from thermopower data for the two 1D constrictions. The rms electron temperature is calculated from the $i = 1$ thermopower peak height $\Delta V_{pk}$. The solid curve shows $\dot{Q}(T_e)$ and the points $\dot{Q}(T_l) + P$ are calculated from the heating current $I_H$. The dashed line is the theoretical prediction $\dot{Q} = 88 \, T^{5}$, with no fitting parameters. Electron temperatures for the lower data set were obtained from the amplitude of SdH oscillations for $T_e > T_l = 305 \, \text{mK}$, and were compared to the dependence of the low-current amplitude on $T_l$. The dot-dash line is the fit $\dot{Q} = 14 \, T^{3}$. 
Appleyard et al.  Fig. 1

- $100 \text{k}\Omega$
- $f_H = 85 \text{ Hz}$
- $0.03-5 \text{ V}_{\text{rms}}$

- $50 \text{ M}\Omega$
- $f_R = 18 \text{ Hz}$
- $1.0 \text{ V}_{\text{rms}}$
Appleyard et al  Figure 2

![Graph showing thermal voltage and conductance changes with gate voltage.](image)
Appleyard et al. Figure 3

Thermal Voltage $\Delta V$ (rescaled) vs. Gate Voltage $V_g(V)$

Inset: $-\Delta V_{pk}^i$ (µV) vs. Subband Index $i$

Graph showing the relationship between thermal voltage and gate voltage, with an inset graph showing the relationship between $-\Delta V_{pk}^i$ and subband index $i$. The data points are plotted as circles, and the trend is indicated by a line. The x-axis ranges from -2.9 to -2.4 V, and the y-axis ranges from 0 to 1.5 (rescaled). The inset graph shows a logarithmic scale, with values ranging from 0.01 to 10 for the y-axis and 5 to 15 for the x-axis.
\[ \dot{Q}(T) = 61 T^5 + 9 T^2 \text{ eV/s} \]

from \( B=0 \) thermopower

- ■ ■ ■ A : \( S_{pk}= 7.58 \text{ T } \mu \text{V/K} \)
- □ □ □ B : \( S_{pk}= 4.12 \text{ T } \mu \text{V/K} \)

\[ \dot{Q}(T) = 14 T^3 \text{ eV/s} \]

from SdH oscillations

- + + + \( B_{\perp} = 0.20 \text{ T} \)
- × × × \( B_{\perp} = 0.28 \text{ T} \)