System identification and adaptive control of micro helicopter

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Abstract. This paper copes with the controller design problem of a micro helicopter, which is a multivariable and strongly coupled nonlinear system. The state-space model near the hovering condition of pitch and roll channels is acquired by least-square system identification method, with flight test verification. Based on the identified model, model reference adaptive control is designed as angular rate controller, and PID controller is used to stabilize attitude loop. State-space model helps to decouple the roll and pitch channels, and model reference adaptive control can address external disturbances, model uncertainties and nonlinearities of the dynamics, which can be estimated and further eliminated online. Eventually, both simulations and actual flight tests are conducted to verify the effectiveness of the proposed control scheme.

1. Introduction
Micro helicopter possesses the advantages of small size, light in weight, maneuverability and portability. A micro helicopter can perform various missions which fixed-wing aircraft cannot accomplish because it can take-off and landing vertically, hover and cruise at low-speed. Accordingly, micro helicopter has many potential applications in civil, military and other fields[1]. Tremendous efforts have been made to the development of micro helicopter in recent years[2][3][4][5].

Automatic flight control of a helicopter is one of the important research. However, control system design for micro helicopter is a challenging task because of its nonlinear dynamic behavior, open-loop inherent instability and high degree of inter-axis coupling. Generally speaking, an authentic model of the system is crucial to designing a controller that yields good control performance. There are two ways to obtain a system model: physics-based modelling and system identification. Owing to the unique design of micro helicopter, physics-based modelling is very labor intensive, requiring numerous measurements to calculate unknown parameters of the target dynamic model. Using a system identification method has the advantages of cost-effective and can provide reasonably accurate results. System identification method can obtain the model near the hover flight condition by using small perturbation theory. In this paper, we use system identification method to obtain the dynamic model.

Most unmanned aerial vehicles (UAVs) use traditional control systems such as single-input-single-output PID feedback control system because of its simplicity and ease of implementation. Nevertheless, PID controller is inadequate to deal with multi-input-multi-output(MIMO), nonlinear and time-varying system, and the control accuracy is always unsatisfactory. Micro helicopter is such a MIMO, strong coupling and nonlinear system, owing to its complex mechanism and small size. Therefore, many advanced control methods such as $H\infty$ control[6], fuzzy control[7], neural network[8], robust nonlinear control[9], linear-quadratic regulator (LQR)[10] etc. have been studied and implemented in micro helicopter control design.

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This paper uses model reference adaptive control (MRAC) as angular rate controller to eliminate the estimated unknown disturbance, model inaccuracy and nonlinearity of the system, and decouple the roll and pitch channels. PID controller is also used to stabilize attitude loop.

The paper is organized as follows. Section 2 presents the dynamic model and system identification process of the micro helicopter. Section 3 presents the detail of the control system design, including MRAC and PID controller design, as well as the simulation result of the control system. Section 4 presents the flight testing to validate the control strategy. Section 5 concludes the work.

2. Dynamics modelling and system identification

2.1. Analysis of helicopter dynamics

Compared to conventional fixed-wing aircraft, the establishment of a nonlinear dynamic model of the helicopter needs to consider the complex dynamics of the rotor. Even for linearized models, it is relatively difficult to obtain a helicopter state space equation because the micro helicopter has complex aerodynamics, resulting in strong coupling in the control channel. Similar to a fixed-wing aircraft, the body part of the helicopter is modelled based on a rigid body assumption, and its dynamics are well understood. However, the helicopter's rotor itself acts as a dynamic system coupled with the rigid body dynamics and the surrounding flow field, which introduces complex, unstable aerodynamics, resulting from interaction between rotor and wake flow, as well as rotor and body. Rotor dynamics also show a large degree of coupling between different control channels. Thanks to the lock yaw gyro, there was a yaw rate augmentation control loop, so coupling between the longitude and the lateral channel was mainly considered.

According to research by Mark B. Tischler[11], the lateral and longitudinal flapping dynamics can be represented by the two first-order equations, which are called Tip-Path Plane Model:

\[ \tau_e \dot{b}_x = -\tau_e \omega_x - b_x + A_e a_x + B_e \delta_{lat} + B_{en} \delta_{on} \]
\[ \tau_e \dot{a}_x = -\tau_e \omega_x - a_x + A_e b_x + B_e \delta_{lat} + B_{en} \delta_{on} \]  
(1)

where \( b_x \) and \( a_x \) are the lateral and longitudinal flapping angle; \( \delta_{lat} \) and \( \delta_{on} \) are the cyclic pitch and cyclic roll; \( \tau_e \) was the time constant of the main rotor; \( B_e \) and \( B_{en} \) are the input derivatives; \( A_e \), \( A_s \) are the coupling derivatives of the rotor in pitch and roll.

The helicopter is considered as a rigid body and then Newton's second law can be applied to establish the rigid-body dynamics of the helicopter:

\[ \dot{\omega}_b = \frac{M_b}{T_{aw}} = \frac{(K_e + T_{aw})b + Q_{aw}a}{T_{aw}} \]
\[ \dot{\omega}_r = \frac{M_r}{T_{aw}} = \frac{(K_e + T_{aw})a - Q_{aw}b}{T_{aw}} \]  
(2)

where \( K_e \) denotes the constant stiffness factor of the linear torsional spring which is used to approximate the restraint in the blade attachment to the rotor head; \( h_{aw} \) is the distance between the main rotor head and the helicopter center of gravity; \( T_{aw} \) denotes the aerodynamic torque of the main rotor; \( Q_{aw} \) denotes the reaction torque of the main rotor.

However, flapping angle is unobservable, hence it is not easy to obtain accurate and effective value of it. In this paper, the dynamic of pitch and roll channels model is simplified further, in order to avoid identifying unfaithful flapping angle and simplify the system identification process. And the inaccuracy brought by the model simplification would be considered and compensated by adaptive control. The simplified first-order differential equations of the dynamic model are as follow:

\[ \dot{\omega}_x = A_e \omega_x + B_e \delta_{aw} + B_{en} \delta_{on} \]
\[ \dot{\omega}_y = A_s \omega_y + B_s \delta_{aw} + B_{sn} \delta_{on} \]  
(3)

2.2. System identification

Before controller design, it is important to obtain the parameters in the helicopter dynamic model. In this paper, least-square method is implemented to estimate the parameters in the state space model using real
flight data collected from a TREX-150 micro helicopter. The distinguished feature of least square is easy to achieve and programme and fast convergence. The least square formula is expressed as:

$$\hat{\theta} = (\Phi^T \Phi)^{-1} \Phi^T Y$$

where $\hat{\theta}$ represents the parameters to be estimated, $\Phi$ represents the input and output data series, $Y$ represents the output data series.

In order to improve the fitting result, preconditioning is necessary, which includes eliminating wild value, removing linear trend from data and smoothing data using moving average method. Table 1 represents the system identification result of Least-square method.

|   | $A_1$  | 2.546 | $A_2$  | 2.758 |
|---|---|---|---|---|
| $B_1$  | 8.147 | $B_2$  | 2.836 |
| $B_3$  | -3.133 | $B_4$  | 15.761 |

The accuracy of the identified model is verified by simulation in time domain, using a different set of flight data. Figure 1 and figure 2 represent the tracking results of pitch and roll channel respectively, which shows a good tracking performance. The process of model verification indicates that the system identification result can be used as an approximate model to controller design.

3. Control system design
This paper use MRAC as angular rate controller to eliminate all unknown disturbance, model inaccuracy and nonlinearity of the system, and decouple the roll and pitch channels. PID controller is also used to stabilize attitude loop. The whole helicopter system control block diagram is presented in figure 3.

MRAC system consists of a controller whose parameters (gains) are updated online using an adaptive law. The command also drives the reference model that specifies the desired trajectories for the system to follow. The difference between the reference model output and the system output constitutes the tracking error, which subsequently is sent to the adaptive law for online parameter estimation. Finally, the controller computes its commands based on the reference input, the system output, and the online adjusted parameters from the adaptive law[12]. Figure 4 represents the closed-loop block diagram of MRAC.

Here we consider the helicopter plant as a MIMO nonlinear system:

$$\dot{x} = Ax + BA(u + f(x))$$

The m-dimensional vector function $f(x)$ represents all other unknown possibly nonlinearity in the system dynamics.

$$f(x) = \Phi^T \Phi(x)$$
where $\Theta \in R^{n \times r}$ is a constant matrix of the unknown coefficients and $\Phi(x) = (\varphi_1(x) \ldots \varphi_s(x))^T \in R^n$ is the unknown regressor vector. In this paper, hyperbolic tangent function is chosen to approximate the nonlinearity of the helicopter.

Table 2. MIMO MRAC design equations.

| Equation                                    |
|---------------------------------------------|
| Open-loop plant                             |
| $\dot{x} = Ax + BA\left(u + \Theta^T\Phi(x)\right)$ |
| Reference model                             |
| $\dot{x}_r = A_{rm}x_r + B_{rm}r$           |
| Matching conditions                         |
| $A + BAK^T = A_{mr}$, $BAK^T = B_{mr}$      |
| Tracking error                              |
| $e = x - x_r$                               |
| Control input                               |
| $u = \hat{K}_e x + \hat{K}_r r - \hat{\Theta}^T\Phi(x)$ |
| Algebraic Lyapunov equation                 |
| $PA + \dot{A} = -Q$                         |
| $\dot{K}_r = -I_x e^TPB$                   |
| $\dot{\Theta} = \Gamma_{\theta}(x)e^TPB$   |

Considering formulas in table 2, $K_r$ and $K_e$ represent the ideal feedback and feedforward gains, respectively. The uncertainty in Matrix $A$ is introduced to model control failures or modelling errors, in the sense that there may exist uncertain control gains or the designer may have incorrectly estimated the system control effectiveness. $\Gamma_{\theta}$, $\Gamma$, and $\Gamma_r$ are the rates of adaptation.

The stability of the MRAC laws has been proved using Lyapunov method\cite{12}. The closed-loop error dynamics are uniformly stable according to Lyapunov design approach, which shows the time derivative of Lyapunov function is non-positive. It is proved that the tracking error and the parameter estimation error are uniformly bounded. The fact that the system tracking error $e$ asymptotically tends to zero does not automatically imply that $\hat{\Theta}$ converges to its ideal unknown parameter. What is certain is that the estimated parameters will remain uniformly bounded during tracking.

Nevertheless, there are cases when parameter convergence will take place alongside the desired tracking. A sufficient condition for parameter convergence is given by the persistency of excitation, which imposes certain restrictions on the commanded signal $r$.

As in any other control design method, MRAC has its own parameters to be tuned. They are the rates of adaptations. As seen from MRAC laws, the larger the rates, the faster the adaptive laws will evolve. One may conjecture that large rates would result in better and faster closed-loop tracking performance. This is partially true. Indeed, large rates of adaptation will yield fast tracking. However, this will also lead to undesirable oscillations during transient times, when the system regulated output is trying to get closer to its command. The trade-off between fast tracking and smooth transients presents a design challenge.

According to MRAC design equations, the simulation framework is established in Simulink, which are represented in figure 5. The plant subsystem in the framework is the state space model obtained from system identification above, and the command comes from real flight data, which increase the credibility of the simulation. The selection of reference model plays an important role in the controller design process because it is the key to system performance. It should be selected according to the bandwidth of actual flight platform.

The simulation result in figure 6 and figure 7 indicate that the errors between reference model and plant converge to zero gradually, which means that the plant can perform according to our expectation, owing to the MRAC laws.

4. Flight experiment

This section describes how the experiment was implemented and the process of flight data collection. During the entire experiment, TREX-150 was used as the micro helicopter platform (figure 8). The length of the
fuselage is 23 centimeters, and the diameter of main rotor is 27 centimeters. The weight of the whole helicopter is only 81g.

![Figure 5. MRAC simulation framework.](image)

![Figure 6. Roll channel simulation result.](image)

![Figure 7. Pitch channel simulation result.](image)

![Figure 8. TREX-150 micro helicopter platform.](image)

The platform, which is equipped with a 3-axis gyro, a 3-axis acceleration sensor, a compass, can save the flight data of angular rate, Euler angle and control command of each channel into SD card through STM32 processor. The sampling frequency of the flight data is 100Hz. The yaw channel is stabilized by gyro and performed well, thus it is not under consideration.

The angular rate tracking performance is shown in figure 9 and figure 10, while figure 11 and figure 12 represent the Euler angle tracking performance. It is proved that this control strategy works well.

![Figure 9. Roll rate tracking performance.](image)

![Figure 10. Pitch rate tracking performance.](image)

![Figure 11. Roll tracking performance.](image)

![Figure 12. Pitch tracking performance.](image)
5. Conclusion
Micro helicopter is a multivariable and strongly coupled nonlinear system, which needs more advanced control methods in stand of traditional control method. MRAC is a model-based control method, which is capable of eliminating unknown disturbance.

In this paper, the dynamic model of helicopter are analysed and simplified, and least-square method is used to identify the system parameters of the state space model of roll and pitch channels. Based on the identified model, MRAC and PID controller are used to stabilize angular loop and angle loop. Both simulation and flight experiment show a good performance.

This paper provides a compact method to design a micro helicopter controller based on simplified first-order model, and use adaptive control to eliminate unknown factors of the system, which has been proved feasible. Moreover, this method is easy to conduct because of the simplification during the design process.

In the future, more complex helicopter dynamic model will be consider during the controller design process. In addition, the efficiency of eliminating disturbance will be evaluated.

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