Theory of Dark Matter

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We discuss the hypothesis that the constituents of dark matter in the galactic halo are Primordial Intermediate-Mass Black Holes (PIMBHs). The status of axions and WIMPs is discussed, as are the methods for detecting PIMBHs with emphasis on microlensing. The role of the angular momentum $\mathcal{J}$ of the PIMBHs in their escaping previous detection is considered.

Keywords: dark matter; primordial black holes; wide binaries; CMB distortion; microlensing; Kerr solution

Introduction

Astronomical observations have led to a consensus that the energy make-up of the visible universe is approximately 70% dark energy, 25% dark matter and only 5% normal matter. General discussions of the history and experiments for dark matter are in books authored or edited by Sciama[1], Sanders[2], and Bertone[3]. A recent popular book, The Cosmic Cocktail by Freese[4] is strong on the panoply of unsuccessful WIMP searches. As we shall see, this lack of success may be due to the fact that WIMPs probably do not exist.

The present ignorance of the dark matter sector is put into perspective by looking at the uncertainty in the values of the constituent mass previously considered. The lightest such candidate is the invisible axion with $M = 1\mu eV$. The heaviest such candidate is the intermediate mass black hole (IMBH) with $M = 100,000M_\odot$ which is a staggering seventy-seven orders of magnitude larger. Our aim is to reduce this uncertainty.

An explanation for the neglect of PIMBHs may be that the literature is confusing. At least one study claimed entirely to rule out Eq.(1). We shall attempt to clarify
the situation which actually still permits the whole range in Eq.(1). The present talk is, in part, an attempt to redress the imbalance between the few experimental efforts to search for PIMBHs compared to the extensive WIMP searches.

**Axions**

It is worth reviewing briefly the history of the axion particle now believed, if it exists, to lie in the mass range

\[ 10^{-6} \text{eV} < M < 10^{-3} \text{eV} \]  

The lagrangian originally proposed for Quantum Chromodynamics (QCD) was of the simple form, analogous to Quantum Electrodynamics,

\[ \mathcal{L}_{QCD} = -\frac{1}{4} G_{\mu\nu}^a G_{\mu\nu}^a - \frac{1}{2} \sum_x \bar{q}_{i,a} \gamma^\mu D_{\mu} q_{i,b} \]  

summed over the six quark flavors.

The simplicity of Eq.(3) was only temporary and became more complicated in 1975 by the discovery of instantons which dictated an additional term in the QCD lagrangian must be added

\[ \Delta \mathcal{L}_{QCD} = \frac{\Theta}{64 \pi^2} \tilde{G}_{\mu\nu}^a \tilde{G}_{\mu\nu}^a \]  

where \( \tilde{G}_{\mu\nu} \) is the dual of \( G_{\mu\nu} \).

When the quark masses are complex, an instanton changes not only \( \Theta \) but also the phase of the quark mass matrix \( M_{\text{quark}} \) and the full phase to be considered is

\[ \tilde{\Theta} = \Theta + \arg \det |M_{\text{quark}}| \]  

The additional term, Eq.(4), violates P and CP, and contributes to the neutron electric dipole moment whose upper limit provides a constraint

\[ \tilde{\Theta} < 10^{-9} \]  

which fine-tuning is the strong CP problem. The hypothetical axion particle then arises from an ingenious technique to resolve Eq.(6), although as it turns out it may have been too ingenious.

Over twenty years ago, in 1992, there was pointed out a serious objection to the invisible axion. The point is that the invisible potential is so fine-tuned that adding
gravitational couplings for weak gravitational fields at the dimension-five level requires tuning of a dimensionless coupling \( g \) to be at least as small as \( g < 10^{-40} \), more extreme than the tuning of \( \Theta \) in Eq. (6).

Although a true statement, it is not a way out of this objection to say that we do not know the correct theory of quantum gravity because for weak gravitational fields, as is the case almost everywhere in the visible universe, one can use an effective field theory. To our knowledge, this serious objection to the invisible axion which has been generally ignored since 1992 has not gone away and therefore the invisible axion probably does not exist.

There remains the strong CP problem of Eq. (6). One other solution would be a massless up quark but this is disfavored by lattice calculations. For the moment, Eq. (6) must be regarded as fine tuning. We recall that the ratio of any neutrino mass to the top quark mass in the standard model satisfies

\[
\left( \frac{M_\nu}{M_t} \right) < 10^{-12}.
\]

WIMPs

By Weakly Interacting Massive Particle (WIMP) is generally meant an unidentified elementary particle with mass in the range, say, between 10 GeV and 1000 GeV and with scattering cross section with nucleons \( (N) \) satisfying, according to the latest unsuccessful WIMP direct searches,

\[
\sigma_{WIMP-N} < 10^{-44}\text{cm}^2
\]

which is roughly comparable to the characteristic strength of the known weak interaction.

The WIMP particle must be electrically neutral and be stable or have an extremely long lifetime. In model-building, the stability may be achieved by an *ad hoc* discrete symmetry, for example a \( Z_2 \) symmetry under which all the standard model particles are even and others are odd. If the discrete symmetry is unbroken, the lightest odd state must be stable and therefore a candidate for a dark matter. In general, this appears contrived because the discrete symmetry is not otherwise motivated.

By far the most popular WIMP example came from electroweak supersymmetry where a discrete R symmetry has the value \( R=+1 \) for the standard model particles and \( R=-1 \) for all the sparticles. Such an R parity is less *ad hoc* being essential to prevent too-fast proton decay. The lightest R=-1 particle is stable and, if not a
gravitino which has the problem of too-slow decay in the early universe, it was the neutralino, a linear combination of zino, bino and higgsino. The neutralino provided an attractive candidate.

The big problem with the neutralino is that at the LHC where electroweak supersymmetry not many years ago confidently predicted sparticles (gluinos, etc.) at the weak scale $\sim 250 \text{ GeV}$ there is no sign of any additional particle with mass up to at least 2500 GeV and above, so electroweak supersymmetry probably does not exist.

The present run of the LHC is not necessarily doomed if WIMPs and sparticles do not exist. An important question, independent of naturalness but surely related to anomalies, is the understanding of why there are three families of quarks and leptons. For that reason the LHC could discover additional gauge bosons, siblings of the $W^\pm$ and $Z^0$, as occur in e.g. the so-called 331-Model.

**MACHOs**

Massive Compact Halo Objects (MACHOs) are commonly defined by the notion of compact objects used in astrophysics as the end products of stellar evolution when most of the nuclear fuel has been expended. They are usually defined to include white dwarfs, neutron stars, black holes, brown dwarfs and unassociated planets, all equally hard to detect because they do not emit any radiation.

This narrow definition implies, however, that MACHOs are composed of normal matter which is too restrictive in the case of black holes. It is here posited that black holes of mass up to $100,000M_\odot$ (even up to $10^{17}M_\odot$) can be produced.

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*A little history is in order for the identification $DM \equiv PIMBH$ [5].* Our epiphany arrived on July 21, 2015 in the Florentine Duomo during solemn Mass commemorating the quincentennial of Saint Philip Neri. PBHs were first invented in Russia[6] in 1966 then independently in the West[7] in 1974. The idea that PBHs could form the dark matter was first suggested by Chapline[8] in 1975 who, like everybody in the 20th century, assumed the PBHs were much lighter than the Sun. Three decades later, far more massive PBHs, including PIMBHs, were shown to be possible e.g. [9,10] and studies of the entropy of the Universe strongly suggested that far more black holes exist.

One of the most convincing arguments in [5] was the marginalization of the WIMP predicted by the failed theory of weak-scale supersymmetry. We note that [5] appeared on September 30, 2015 four months before the LIGO announcement[11] on February 11, 2016 of the discovery of gravitational waves from a heavy black hole binary, and five months before a copycat paper involving Adam Riess [12] of March 1, 2016 which now has 92 citations while [5] has only 8. This is undoubtedly because Riess has a Nobel prize and chose not to cite [5]. It recalls a situation forty years ago when Sidney Coleman’s copycat paper[13] about vacuum decay in quantum field theory appeared on January 24, 1977 totally identical to, but choosing not to cite, our [14] of September 24, 1976.

By the present time, [13] has 1,684 citations while [14] has only 104 which has been called a *kilocite heist*. The present case is more important because the theory of dark matter invented in [5] may fairly soon be tested by, and possibly shown to agree with, experiment. In that case, the *original* seminal papers for the equation $DM \equiv PIMBH$ are therefore, in chronological order, [8] and [5].
primordially. Nevertheless for the halo the acronym MACHO still nicely applies to dark matter PIMBHs which are massive, compact, and in the halo.

Unlike the axion and WIMP elementary particles which would have a definite mass, the black holes will have a range of masses. The lightest PBH which has survived for the age of the universe has a lower mass limit

$$M_{PBH} > 10^{-18} M_\odot \sim 10^{36} TeV$$

already thirty-six orders of magnitude heavier than the heaviest would-be WIMP. This lower limit comes from the lifetime formula derivable from Hawking radiation

$$\tau_{BH}(M_{BH}) \sim \frac{G^2 M_{BH}^3}{\hbar c^4} \sim 10^{64} \left( \frac{M_{BH}}{M_\odot} \right)^3 \text{years}$$

Because of observational constraints the dark matter constituents must generally be another twenty orders of magnitude more massive than the lower limit in Eq. (9).

We assert that most dark matter black holes are in the mass range between ten and one hundred thousand times the solar mass. The name primordial intermediate mass black holes (PIMBHs) is appropriate because they lie in mass above stellar-mass black holes and below the supermassive black holes which reside in galactic cores.

Let us discuss three methods (there may be more) which could be used to search for dark matter PIMBHs. While so doing we shall clarify what limits, if any, can be deduced from present observational knowledge.

Before proceeding, it is appropriate first to mention the important Xu-Ostriker upper bound of about a million solar masses from galactic disk stability for any MACHO residing inside the galaxy.

**Wide binaries**

There exist in the Milky Way pairs of stars which are gravitationally bound binaries with a separation more than 0.1pc. These wide binaries retain their original orbital parameters unless compelled to change them by gravitational influences, for example, due to nearby IMBHs.

Because of their very low binding energy, wide binaries are particularly sensitive to gravitational perturbations and can be used to place an upper limit on, or to detect, IMBHs. The history of employing this ingenious technique is regretfully checkered. In 2004 a fatally strong constraint was claimed by an Ohio State University group
in a paper [15] entitled "End of the MACHO Era" so that, for researchers who have time to read only titles and abstracts, stellar and higher mass constituents of dark matter appeared to be totally excluded.

Five years later in 2009, however, another group [16] this time from Cambridge University reanalyzed the available data on wide binaries and reached a quite different conclusion. They questioned whether any rigorous constraint on MACHOs could yet be claimed, especially as one of the important binaries in the earlier sample had been misidentified.

Because of this checkered history, it seems wisest to proceed with caution but to recognize that wide binaries represent a potentially useful source both of constraints on, and the possible discovery of, dark matter IMBHs.

**Distortion of the CMB**

This approach hinges on the phenomenon of accretion of gas onto the PIMBHs. The X-rays emitted by such accretion of gas are downgraded in frequency by cosmic expansion and by Thomson scattering becoming microwaves which distort the CMB, both with regard to its spectrum and to its anisotropy.

One impressive calculation [17] by Ricotti, Ostriker and Mack (ROM) in 2008 of this effect employs a specific model for the accretion, the Bondi-Hoyle model, and carries through the computation all the way up to a point of comparison with data from FIRAS on CMB spectral distortions, where FIRAS was a sensitive device attached to the COBE satellite.

Unfortunately the Bondi-Hoyle model was invented for a static object and assumes spherically symmetric purely s-wave accretion. Studies of the SMBH in the giant galaxy M87 have shown since 2014 that the higher angular momenta strongly dominate, not surprising as the SMBH possesses a gigantic spin angular momentum in natural units.

The results from M87 suggest the upper limits on MACHOs imposed by ROM were too severe by some 4 or 5 orders of magnitude and that up to 100% of the dark matter is permitted by arguments about CMB distortion to be in the form of PIMBHs.

**Microlensing**

Microlensing is the most direct experimental method and has the big advantage that it has successfully found examples of MACHOs. The MACHO Collaboration
used a method which had been proposed by Paczynski where the amplification of a distant source by an intermediate gravitational lens is observed. The MACHO Collaboration discovered several striking microlensing events whose light curves are exhibited in its 2000 paper. The method certainly worked well for $M < 25M_\odot$ and so should work equally well for $M > 25M_\odot$ provided one can devise a suitable algorithm and computer program to scan enough sources.

The longevity of a given lensing event is proportional to the square root of the lensing mass and numerically is given by ($\hat{t}$ is longevity)

$$\hat{t} \simeq 0.2 \text{yr} \left( \frac{M_{\text{lens}}}{M_\odot} \right)^{1/2}$$  \hspace{1cm} (11)

where a transit velocity 200 $\text{km/s}$ is assumed for the lensing object.

The MACHO Collaboration [18] investigated lensing events with longevities ranging between about two hours and one year. From Eq. (11) this corresponds to MACHO masses between approximately $10^{-6}M_\odot$ and $25M_\odot$.

The total number and masses of objects discovered by the MACHO Collaboration could not account for all the dark matter known to exist in the Milky Way. At most 10% could be explained. To our knowledge, the experiment ran out of money and was essentially abandoned in about the year 2000. But perhaps the MACHO Collaboration and its funding agency were too easily discouraged.

What is being suggested is that the other 90% of the dark matter in the Milky Way is in the form of MACHOs which are more massive than those detected by the MACHO Collaboration, and which almost certainly could be detected by a straightforward extension of their techniques. In particular, the expected microlensing events have a duration ranging up to two centuries.

Microlensing experiments involve systematic scans of millions of distant star sources because it requires accurate alignment of the star and the intermediate lensing MACHO. Because the experiments are already highly computer intensive, it makes us more optimistic that the higher longevity events can be successfully analyzed. Study of an event lasting two centuries should not necessitate that long an amount of observation time. It does require suitably ingenious computer programming to track light curves and distinguish them from other variable sources. This experiment is undoubtedly extremely challenging, but there seems no obvious reason it is impracticable.

\[\text{\textsuperscript{b}We have read that such gravitational lensing was later found to have been calculated in unpublished 1912 notes by Einstein who did not publish perhaps because at that time he considered its experimental measurement impracticable.}\]
Intermediate discussion

Axions probably do not exist for theoretical reasons discovered in 1992. Electroweak supersymmetry probably does not exist for the experimental reason of its non-discovery in Run 1 of the LHC. The idea that dark matter experiences weak interactions (WIMPs) came historically from the appearance of an appealing DM constituent, the neutralino, in the theory of electroweak supersymmetry for which there is no experimental evidence.

The only interaction which we know for certain to be experienced by dark matter is gravity and the simplest assumption is that gravity is the only force coupled to dark matter. Why should the dark matter experience the weak interaction when it does not experience the strong and electromagnetic interactions?

All terrestrial experiments searching for dark matter by either direct detection or production may be doomed to failure.

We began with four candidates for dark matter constituent: (1) axions; (2) WIMPs; (3) baryonic MACHOs; (4) PIMBHs. We eliminated the first two by hopefully persuasive arguments, made within the context of an overview of particle phenomenology including a combination of old and new results. We eliminated the third by the upper limit on baryons imposed by robust Big Bang Nucleosynthesis (BBN) calculations.

We assert that PIMBHs can constitute almost all dark matter while maintaining consistency with the BBN calculations. This is an important point because distinguished astronomers have written an opposite assertion e.g. Begelman and Rees state that black holes cannot form more than 20% of dark matter because the remainder is non-baryonic.

These authors are making an implicit assumption which does not apply to the PIMBHs which we assert comprise almost all dark matter. That assumption is that black holes can be formed only as the result of the gravitational collapse of baryonic stars. We are claiming, on the contrary, that dark matter black holes can be, and the majority must be, formed primordially in the early universe as calculated and demonstrated in FKTY(2010) and independently by CKSY(2010).

Our proposal is that the Milky Way contains between ten million and ten billion massive black holes each with between a hundred and a hundred thousand times the solar mass. Assuming the halo is a sphere of radius a hundred thousand light years the typical separation is between one hundred and one thousand light years which is also the most probable distance of the nearest PIMBH to the Earth. At first sight, it may be surprising that such a huge number of PIMBHs
– the plums in a “PIMBH plum pudding” – (c.f. Thomson 1904) could remain undetected.

[111 years after Thomson; 31 powers of ten bigger; not replaceable by a nuclear halo.]

However, the mean separation of the plums is at least a hundred light years and the plum size is smaller than the Sun.

### PIMBHs and PSMBHs

Focusing on the Milky Way halo where we can most easily detect the PBHs, we already know from earlier searches, especially the MACHO Collaboration that masses \( M \leq 20 M_\odot \) can make up no more that 10% of the halo dark matter. At the high mass end, we know from Xu-Ostriker that MACHOs with \( M \geq 10^5 M_\odot \) endanger disk stability. For the Milky Way halo one is led to consider intermediate mass PIMBHs in the mass range

\[
20 M_\odot \leq M_{\text{PIMBH}} \leq 10^5 M_\odot \tag{12}
\]

for the DM constituents. This leads to a plum pudding model for the Milky Way halo, named after Thomson’s atomic model, where for the DM halo the plums are PIMBHs with masses satisfying Eq. (12) and the pudding is rarefied gas, dust and a few luminous stars.

The formation of PBHs with masses as large as Eq. (12) and much larger is known to be mathematically possible during the radiation era. An existence theorem is provided by hybrid inflationary models. One specific prediction of hybrid inflation is a sharply-peaked PBH mass function. If we need a specific PIMBH mass, we shall use a calligraphic \( \mathcal{P}_{\text{IMBH}} \) defined by \( M_{\mathcal{P}_{\text{IMBH}}} \equiv 100 M_\odot \) exactly. This is merely an example and extension to the whole range of Eq. (12) can also be discussed.

The cosmic time \( t_{\text{PBH}} \) at which a PBH is formed has been estimated to be

\[
t_{\text{PBH}} \simeq \left( \frac{M_{\text{PBH}}}{10^5 M_\odot} \right) \text{ seconds} \tag{13}
\]

so that the PIMBHs in Eq. (12) are formed in the time window \( 0.0002 s \leq t_{\text{PIMBH}} \leq 1.0 s \) with the special case \( t_{\mathcal{P}_{\text{IMBH}}} \simeq 0.001 s \). In terms of red shift \( (Z) \), this corresponds to

\[
5 \times 10^{11} \geq Z_{\text{PIMBH}} \geq 5 \times 10^9 \tag{14}
\]

with the special case \( Z_{\mathcal{P}_{\text{IMBH}}} \simeq 2 \times 10^{11} \).
The formation of BHs which are not primordial, which we shall denote without an initial $P$ or $\mathcal{P}$, necessarily occurs after star formation which conservatively occurs certainly only for very different redshifts satisfying

$$Z_{BH} \leq 100$$

(15)

The sharp difference in the red-shifts of Eq. (14) and Eq. (15) will become important when we discuss the reasons for previous non-detection, the angular momentum of PIMBHs and BHs, and the central issue of possible CMB distortion by X-rays.

As already mentioned, by using the mathematical models, it is possible to form PBHs not only in the PIMBH mass range of Eq. (12) but also Primordial Super Massive Black Hole (PSMBHs) in the mass range

$$10^5 M_\odot \leq M_{PSMBH} \leq 10^{17} M_\odot$$

(16)

where the upper limit derives from the formation time $t_{PSMBH}$ given by Eq. (13) staying within the radiation-dominated era. We shall discuss the higher mass range Eq (16) later in the paper.

Finally for this Introduction, we recall that in a microlensing experiment, e.g. using the LMC or SMC for convenient sources, microlensing by halo PIMBHs, and assuming a typical transit velocity $200 km.s^{-1}$, the time duration of the microlensing light curve can be estimated to be approximately

$$\tau \simeq \left( \frac{M_{PIMBH}}{25 M_\odot} \right)^2 \text{ years}$$

(17)

which we note is close to one year and two years, respectively, for lens masses $25 M_\odot$ and $100 M_\odot$. For reference, the highest duration such light curve detected by the MACHO Collaboration which published in the year 2000 corresponded to $M_{PIMBH} \simeq 20 M_\odot$.

Nevertheless, if longer duration microlensing light curves can be detected of two years or more, the only known explanation will be the existence of Kerr black holes in the halo with many solar masses.

**Kerr metric and period $\tau$**

The PIMBHs are described by a Kerr metric which has the form in Boyer-Lindquist $(t, r, \theta, \phi)$ coordinates, after defining $\alpha = \frac{J}{M}$, $\rho^2 = r^2 + \alpha^2 \cos^2 \theta$ and $\Delta = r^2 - 2Mr + \alpha^2$. 

\[ ds^2 = -\left(1 - \frac{2Mr}{\rho^2}\right)dt^2 - \left(\frac{4Mar\sin^2 \theta}{\rho^2}\right)d\phi dt + \left(\frac{\rho^2}{\Delta}\right)dr^2 + \rho^2 d\theta^2 + \left(r^2 + \alpha^2 + \frac{2Mr^2\sin^2 \theta}{\rho^2}\right)\sin^2 \theta d\phi^2 \]  

(18)

In Eq. (18), there are two free parameters, \( M \) and \( J \). Analytic calculations building on Eq. (18) can be difficult, usually leading to numerical techniques. In this talk, we shall need only order-of-magnitude estimates for the rotational period \( \tau \) and, in the next Section, for the angular momentum \( J \). These will suffice to make our point about concomitant X-ray emission. The solution is axially symmetric and the radius at the pole \( \theta = \frac{\pi}{2} \) is the same as the Schwarzschild radius \( R = 2M \). For other values of \( \theta \) the black hole radius is smaller than the static one and the rest of the static would-be sphere is filled out by an ergosphere whose equatorial radius is also \( R = 2M \).

For the primordial black holes of interest, there is no reason to expect that the radiation will collapse in a spherically symmetric fashion to a static Schwarzschild black hole when the PBH formation necessarily occurs in an environment of extreme fluctuations and inhomogeneities. The black holes must be described by the Kerr metric in Eq. (18) with \( \alpha \) having a value anything up to the maximal Kerr solution which corresponds to an equatorial speed \( V \) equal to the speed of light. The range of \( V \) is thus \( 0 \leq V \leq c \).

We do not know observationally any black hole which is primordial with certainty although many of the observed black holes, including those in the binary coalescences observed by LIGO, could be primordial. For illustration of black hole observations, let us consider the well-studied binary GRS1915+105 of a star and a black hole.

The black hole mass in GRS1915+105 has been established as \( M \simeq 13M_\odot \) and hence its Schwarzschild radius is \( r_s \simeq 39 \text{ km} \). Its rotation occurs 1,150 times per second which translates to an equatorial speed \( V \simeq 0.94c \), remarkably close to maximal. We mention this example to show that such high \( V \) Kerr black holes are known to exist and although we cannot derive the value of \( V \) arising from PBH formation it is to be expected that all values \( V \) up to the maximum can occur. For our present qualitative purposes, to be conservative, we employ \( V = 0.1c \).

To proceed with our estimate we shall therefore take the equatorial velocity of the ergosphere to have magnitude \( V = 0.1c \) and use Newtonian mechanics to estimate the rotation period \( \tau \) as simply
\[ \tau = \left( \frac{2\pi R}{V} \right) \]  \hspace{1cm} (19)

For the Sun, we have \(2M_\odot \simeq 3 \text{ km} \) so that for a black hole of mass \( M = \eta M_\odot \) and therefore radius \( R \simeq 3\eta \text{ km} \) Eq. (19) is, for \( V = 0.1c = 3 \times 10^4 \text{ km.s}^{-1} \),

\[ \tau = \left( 2 \times 10^{-4}\pi \eta \right) \text{ seconds} \]  \hspace{1cm} (20)

Some values of \( \tau \), estimated by this method, are shown in the third column of the Table and angular momentum \( J \) (discussed later) is in the last column.

| Astrophysical object | Mass solar masses | Period \( \tau \) seconds | Angular Momentum \( J \) kg.km\(^2\).s\(^{-1}\) |
|----------------------|-------------------|---------------------------|-----------------------------------------------|
| Earth               | \( M_\oplus = 6 \times 10^{24} \text{ kg} \) | 24 hours | \( 1.1 \times 10^{27} \) |
| Sun                 | \( M_\odot = 2 \times 10^{30} \text{ kg} \) | 25 days | \( 1.1 \times 10^{46} \) |
| PIMBH               | \( 20M_\odot \) | 0.013s | \( 3.0 \times 10^{17} \) |
| PIMBH (100M_\odot)  | 0.063s | \( 7.2 \times 10^{38} \) |
| PIMBH               | \( 10^4M_\odot \) | 0.63s | \( 7.2 \times 10^{40} \) |
| PIMBH               | \( 10^6M_\odot \) | 6.3s | \( 7.2 \times 10^{42} \) |
| PIMBH               | \( 10^9M_\odot \) | 63s | \( 7.2 \times 10^{44} \) |
| PSMBH (M87)         | \( 6 \times 10^9M_\odot \) | 3.8 \times 10^6s | \( 2.6 \times 10^{54} \) |

Angular momentum \( J \)

Let us define the dimensionless angular momentum \( J \equiv J/kg.km^2.sec^{-1} \). We are interested in order of magnitude estimates of \( J \) for the PIMBHs and PSMBHs. The value of \( J \) for astrophysical objects is necessarily a large number so to set the scene we shall estimate \( J \) for the Earth \( J_\oplus \) and for the Sun \( J_\odot \).

The parameters for the Earth are radius \( R_\oplus \simeq 6300 \text{ km} \), period \( \tau_\oplus \simeq 86400 \text{ s} \), mass \( M_\oplus \simeq 6 \times 10^{24} \text{ kg} \), hence angular velocity \( \omega_\oplus = 2\pi/\tau_\oplus \) and moment of inertia \( I_\oplus = \frac{2}{5}M_\oplus R_\oplus^2 \) so an estimate is \( J_\oplus \sim I_\oplus \omega_\oplus \simeq 1.1 \times 10^{27} \). For the Sun the
similar calculation using \( R_\odot \simeq 700,000 \text{km}, \tau_\odot \simeq 25 \text{days}, M_\odot \simeq 2 \times 10^{30} \text{kg} \) gives \( J_\odot \simeq 1.1 \times 10^{36} \).

For the black holes, the value of \( J \) is proportional to \( \eta^2 \) where \( \eta = (M/M_\odot) \). A similar estimate to that for the Earth and Sun gives \( J \simeq 7.2 \times 10^{34} \eta^2 \), which provides the remaining entries in the Table.

CMB distortion revisited

Because of rotational invariance, angular momentum is conserved. The \( J \) of a compact astrophysical object will not change dramatically unless there is an extremely unlikely event like a major collision. For example, the Earth and the Sun in the first two rows of our Table were formed 4.6 billion years ago. Their respective angular momenta \( J_\oplus \) and \( J_\odot \) have remained essentially constant all of that time. According to Eq. (13), the PIMBHs listed in the next five rows of our Table were all formed at time \( t \leq 1 \text{s} \) and their angular momenta have therefore remained roughly constant for the last 13.8 billion years since then.

In detecting the dark matter, let us focus on the special case \( \text{PIMBH} \) with \( M = 100M_\odot \). The \( \text{PIMBH} \) was formed, according to Eq. (13), at time \( t = 10^{-3} \text{s} \) and rotates with period \( t \simeq 63 \text{ms} \), thus rotating \( \sim 16 \) times per second and with an absolute angular momentum \( \sim 6 \times 10^{11} \) times that of the Earth and \( \sim 600 \) times that of the Sun. There is no known reason that \( J_{\text{PIMBH}} \) would change significantly after its formation.

These remarks about angular momentum are salient to resolving the contradiction between the PIMBH dark matter proposal and the limits on halo MACHOs derived earlier by Ricotti, Ostriker and Mack (ROM) on the basis of X-ray emission and CMB distortion.

The PIMBH proposal was made that the Milky Way dark halo is a plum pudding with, as “plums”, PIMBHs in the mass range of Eq. (12) making up 100% of the dark matter. On the other hand, in Figure 9 of ROM, there is displayed an upper limit of less than 0.01% of the dark matter for this mass range of MACHOs. Thus, it would seem that at least one must be incorrect? The conclusion of the present talk is that ROM is correct for stellar-collapse black holes but is not applicable to a model which employs primordial black holes.

This ROM upper limit arises from the lack of any observed departure of the CMB spectrum from the predicted black-body curve or of any CMB anisotropy. ROM calculated the accretion of matter on to the MACHOs, the emission of X-rays by the accreted matter and then the downgrading of these X-rays to microwaves by cosmic expansion and more importantly by Compton scattering from electrons.
A crucial assumption made by ROM is that the accretion on to the MACHO can be modeled as if the MACHO has zero angular momentum $J = 0$. The justification for this assumption is based on earlier work by Loeb [19] who studied the collapse of gas clouds at redshifts $200 \leq Z \leq 1400$. Such collapse can form compact objects, eventually black holes, but during the collapse angular momentum is damped out from the electrons by Compton scattering with the CMB.

From Loeb’s discussion, the resultant black holes will have $J = 0$ and this appears to underly why ROM used the Bondi-Hoyle model which presumes spherical symmetry for accretion. This is justified for stellar-collapse black holes by the arguments of Loeb and therefore the upper bounds derived by ROM are applicable.

There is evidence that the Bondi-Hoyle model of accretion is not, by contrast, applicable to spinning PSMBHs, in particular the one at the centre of the large galaxy M87. In recent analyses Bondi-Hoyle was used to calculate the number of X-rays expected from the accreted material near M87. In the case of M87 the X-rays are experimentally measured. The conclusion is striking: that the measured X-rays are less by several orders of magnitude than predicted by Bondi-Hoyle theory.

This supports the idea that the SMBHs such as that in M87 are primordial, so we list PSMBH(M87) in the final row of our Table. The ROM constraints apply to black holes which originate from gravity collapse of baryonic stars. Collecting this fact, together with the ROM limit of $\leq 10^{-4}$ of the dark matter for MACHOs, implies that 99.99% of the dark matter black holes are primordial, formed during the radiation era.

**Final discussion**

The dark matter and its explanation is a pressing problem which impacts on both high-energy physics and on cosmology. It is indisputable that over 80% of the Milky Way’s mass lies in a dark approximately spherical halo surrounding the luminous more planar spiral. The results in the present Letter strongly support the model involving billions of PIMBHs.

The plum pudding model for the dark halo proposed in arose from a confluence of theoretical threads including study of the entropy of the universe and the knowledge of how to form PBHs with many solar masses as in Eqs. [12] and [16]. Nevertheless it was the weakening of the argument for WIMPs which was most decisive.

The strongest objection to the PIMBHs has been based on the X-rays and the CMB distortion as calculated by ROM. In the present talk we have attempted to lay this criticism to rest by noting that ROM assumed $J = 0$ and that the putative PIMBHs have not only many times the Solar mass but also many times the Solar angular
momentum. This appears to us to render the ROM constraints inapplicable to the PIMBHs. On the other hand, they do apply to stellar-collapse black holes which implies that almost none (≤ 0.01%) of the dark matter black holes are of that type. To decide whether dark matter really is PIMBHs will require their detection by a dedicated microlensing experiment.

Examples of PSMBHs may already have been observed in galactic cores and quasars. Other PSMBHs can play the role of dark matter in clusters and may well be detectable by other future lensing experiments. There is also the upper mass range contained in Eq.(16). Although masses of PSMBHs up to a few times $10^{10}M_\odot$ may have already been observed in quasars, there are what could be called Primordial Ultra Massive Black Holes (PUMBHs) with masses between $10^{11}$ and $10^{17}$ solar masses which might exist within the visible universe.

PUMBHs remain speculative but what can in the near future be examined experimentally is the existence of PIMBHs in the halo. A positive result would solve the 83-year-old problem of the dark matter and explain ∼ 26.7% of the total stress-energy tensor of the visible universe. It would presumably put a stop to searches for WIMPs because the scientific community would accept that WIMPs, like low-energy supersymmetry, do not exist. Searches for axions would perhaps continue but purely within the particle physics domain with no notion that axions, if they exist, can form more than a very tiny fraction of dark matter.

The identification of the dark matter constituents as PIMBHs can revolutionize astronomy and cosmology. To give just one example, the formation of stars which takes place at redshifts $Z \leq 8$ becomes as if only a minor “afterthought” with regard to all the earlier large scale structure formation which would take place in a Universe containing only dark matter in the form of PIMBHs. In this sense, the result of this experiment can diminish the cosmological significance of normal matter.

Of the detection methods discussed, extended microlensing observations seem the most promising and an experiment to detect higher longevity microlensing events is being actively pursued. The wide-field telescope must be in the Southern Hemisphere to use the Magellanic Clouds (LMC and SMC) for sources.

The most appropriate active telescope has been identified as the Blanco 4m at Cerro Tololo in Chile. This telescope was named after the late Victor Blanco the Puerto Rican astronomer who was the CTIO Director. A bigger telescope which can confirm the high-duration light curves is the Large Synoptic Survey Telescope (LSST) under construction, also in Chile, expected to take first light in 2022.
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