The M-sigma Relation of Super Massive Black Holes from the Scalar Field Dark Matter

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We explain the M-sigma relation between the mass of super massive black holes in galaxies and the velocity dispersions of their bulges in the scalar field or the Bose-Einstein condensate dark matter model. The gravity of the central black holes changes boundary conditions of the scalar field at the galactic centers. Owing to the wave nature of the dark matter this significantly changes the galactic halo profiles even though the black holes are much lighter than the bulges. As a result the heavier the black holes are, the more compact the bulges are, and hence the larger the velocity dispersions are. This tendency is verified by a numerical study. The M-sigma relation is well reproduced with the dark matter particle mass \( m \approx 5 \times 10^{-22} \text{eV} \).
I. INTRODUCTION

The M-sigma relation is a tight correlation between the mass $M_{bh}$ of a central super massive black hole (SMBH) of a galaxy and the stellar velocity dispersion $\sigma$ in its bulge [1, 2], that is, $M_{bh} \propto \sigma^\beta$ with $\beta \approx 4 - 5$. The relation remains a mystery, because SMBHs are usually quite small and light compared to the host bulges, and the relation is tighter than the relation between $\sigma$ and the mass or the luminosity of the bulge. Furthermore, it is hard to understand how this relation survives galaxy mergers, even if the relation was established in the early universe. A possible solution to this problem could be a feedback mechanism acting during the galaxy evolution. For example, Silk and Rees [3] suggested that the SMBHs drive a wind against the accretion flow which disturbs the bulge growth, while King [4] proposed a momentum-driven stellar wind to explain the normalization coefficient of the relation.

However, galaxies are basically dark matter (DM) dominated objects and visible matter resides in a gravitational potential well generated by the DM distribution. Considering the complicated galaxy evolution history and the varieties of galaxy types, it is plausible to think that the M-sigma relation is from some dynamical equilibrium conditions of the DM distribution not from the properties of visible matter.

In this paper, we propose a new physical mechanism behind the M-sigma relation based on the Bose-Einstein condensate (BEC) or the scalar field dark matter (SFDM) model which is proposed and studied by many authors [5–25]. In this model the galactic halo DM is in a BEC state of ultra-light scalar particles with mass $m \sim 10^{-22} eV$. (For a review see [25–30]). This DM has a wave-like nature [31] and its large Compton wavelength suppresses small scale structure formation [32]. Beyond the galactic scale the SFDM behaves like cold dark matter (CDM), and hence solves the problems of the CDM such as the missing satellites problem and the cusp problem [33–36]. It has been also shown that the BEC/SFDM can explain the rotation curves [12, 37–39], the large scale structures of the universe [40], and the spiral arms [31].

Sin [11] suggested that the DM in galactic halos can be described by a nonlinear Schr"{o}dinger equation, and that the uncertainty principle prevents halos from collapses. Lee and Koh [8] proposed that the DM halos are giant boson stars [41] made of a complex scalar field $\tilde{\psi}$ with a typical action

$$S = \int d^4x \sqrt{-g} \frac{c^4}{16\pi G} R - \frac{g^{\mu\nu}}{2} \bar{\tilde{\psi}} \gamma_\mu \tilde{\psi}_\nu - U(\tilde{\psi}),$$

(1)

where $c$ is the light velocity, and $U(\tilde{\psi}) = \frac{\hbar^2}{2m} |\tilde{\psi}|^2$ is a field potential. (In this paper, we only consider a free scalar field.) From the action one can obtain the well-known Einstein-Klein-Gordon (EKG) equation

$$\Box \tilde{\psi}^* = \frac{dU}{d\tilde{\psi}},$$

(2)

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu},$$

which can describe the galactic halos made of BEC/SFDM.

The aim of this paper is to find the relation between $\sigma$ and $M_{bh}$ in the context of the BEC/SFDM. In Sec. 2 we study the approximate profile of the BEC/SFDM halos with central BHs. In Sec. 3 we present results of our numerical study. The section 4 contains discussion.

II. SCALAR FIELD DARK MATTER WITH BLACK HOLES

In the weak field limit the EKG with a spherical symmetric metric $ds^2 = -(1 + \frac{2\bar{V}}{c^2}) (cdt)^2 + (1 - \frac{2\bar{V}}{c^2}) d\bar{r}^2$ is reduced to the following Schrödinger-Poisson equations (SPE),

$$i\hbar \partial_t \tilde{\psi} = \frac{\hbar^2}{2m} \nabla^2 \tilde{\psi} + m\bar{V} \tilde{\psi},$$

(3)

$$\nabla^2 \bar{V} = \frac{4\pi G}{c^2} T_{00} = \frac{4\pi G}{c^2} (\rho_d + \rho_{vis}).$$

In this paper the gravitational potential $\bar{V} = \bar{V}_d - \bar{r}_s/\bar{r}$ is the sum of the DM contribution $\bar{V}_d$ and the black hole (BH) contribution $-\bar{r}_s/\bar{r}$. The half of the Schwarzschild radius is $\bar{r}_s = GM_{bh}/c^2 \approx 4.78 \times 10^{-14} \frac{M_{bh}}{pc}, \rho_d = m|\tilde{\psi}|^2$ is the DM density, and $\rho_{vis}$ is the visible matter density. The SPE can be also derived from the mean field approximation of the BEC Hamiltonian.

We consider a ground state (a boson star) of the SFDM with a central BH as a model of a galaxy with mass $M$. This ground state model is adequate for small or medium size galaxies. The high precision numerical study [42]
indicates that large galaxies in the SFDM model consist of soliton-like cores and outer tails similar to CDM profiles, thus the inner parts of the large galaxies resemble the ground state. For the spherical symmetric case we can ignore the outer part to calculate $\sigma$. For simplicity we also ignore $\rho_{vis}$ in the SPE from now on.

For a numerical study it is useful to introduce dimensionless variables using the relations $\bar{r} = hr/mc, \bar{t} = ht/mc^2$, and

$$\bar{\psi} = \frac{c^2}{\hbar} \sqrt{\frac{m}{4\pi G}} e^{-i\bar{E}\bar{t}/\hbar} \psi,$$

where $\psi$ is a real field. Then, a dimensionless form of the SPE is

$$\begin{cases}
\nabla^2_\bar{r} \bar{\psi} = 2(V_d - \frac{\psi_0^2}{3\bar{r}}) \bar{\psi} \\
\nabla^2_\bar{r}(V_d - \frac{\psi_0^2}{3\bar{r}}) = \psi_0^2
\end{cases}$$

where $\nabla^2_\bar{r} = \partial^2_\bar{r} + 2\partial_\bar{r}/\bar{r}$. In this unit physical quantities can be recovered by the relations $\bar{E} = Emc^2, M_{bh} = hcr_s/mG$ and so on.

If there is no central BH (i.e., $r_s = 0$), a natural boundary condition is $d\psi/dr|_{r=0} = 0$, which gives the core-like density profile observed in dwarf galaxies [14, 43]. On the other hand, if there is a central BH, the gravity of the BH changes the boundary condition of the field at the center and gives a cuspy central density. From the first equation of Eq. (5) one can see that as $r \to 0$, only $1/r$ dependent terms dominate and they give a different boundary condition [44]

$$\partial_r \psi|_{r_0} = -r_s \psi,$$

which means the bigger the black hole is, the steeper the central field profile slope is. (Here $r_0$ is a central point for the boundary condition where we can use the Newtonian approximation while the gravity of the BH still influences the DM scalar field.) Though $r_s$ is usually very small, this small change of the boundary condition at the center could result in a huge change in the overall DM density profile due to the wave nature of the BEC/SFDM. This phenomenon might be the key to understand the M-sigma relation.

We arbitrary choose $r_0 = 100r_s$ in this paper, and we focus on the region where $r \geq r_0$ so that we do not need to care about any relativistic effect or details of BH physics. This means that the effect of the BH on the relation is simply to give the boundary condition for the SFDM, which also implies that any super massive compact object at the galactic center has a similar relation.

Since there is no known analytic solution of the SPE in Eq. (5), we study approximate solutions and numerical solutions to see the effect of the black holes. The approximate solutions of the SPE we find are

$$V_d(r) \simeq V_0 + \psi_0^2 r^2 \frac{r_s^2}{6},$$

$$\psi(r) \simeq \psi_0 \left(1 - r_s r + \frac{3[V_0 - E]r^2}{3r_s^2}\right),$$

which can be checked by inserting them into the SPE. For example, $\nabla^2_\bar{r}(V_d - \frac{\psi_0^2}{3\bar{r}}) \simeq \psi_0^2$ [44].

From Eq. (8) one can find that $\psi = \psi_0/2$ at

$$r_h = \frac{\sqrt{r_s^2 + 2w} - r_s}{2w},$$

where $w = (E - V_0)/3 > 0$. If the virial theorem holds $w = |V_0|/6$, and $r_h \simeq \sqrt{3/|V_0|} - 3r_s/|V_0|$. $r_h$ is roughly the size of the bulge, which is a decreasing function of $r_s$. For a fixed total mass $M \propto \int d^3r |\psi|^2$, $\psi_0$ should increase to compensate the reduction of the halo size. This means that the halo with a big BH has a narrow wave function and hence a compact DM density profile and a compact bulge. (See Fig. 1) Interestingly, this behavior is also consistent with observations of galaxies. Graham et al. [15] have found that the bulge light concentration $C_{r_e}$ positively correlates with $\sigma$ and $M_{bh}$, which means that a galaxy with a big BH has a compact bulge. The correlation is as strong as the M-sigma relation, which implies they have a common physical origin. This fact supports our hypothesis.

Let us roughly derive the M-sigma relation using the approximate solutions. The stellar rotation velocity is proportional to the square root of the potential depth. From Eq. (8) we can define a size of the halo $r_f \simeq 1/\sqrt{w} = \sqrt{6/|V_0|}$ using the condition $\psi(r_f) = 0$. From the condition $V_d(r_f) = 0 = V_0 + \psi_0^2/|V_0|$ one can see $\psi_0 = |V_0|$. In Eq. (23) of Ref. [44] it was shown that $\psi_0 \propto r_s^{1/2}$. As a result $|V_0| \propto r_s^{1/2}$, and hence $\sigma^2 \propto |V_0|^2 \propto r_s$, which means $\beta = 4$. However, this is a rough estimation based on the approximation.
mass and the size of galaxies. Let us use this parameterization example, Gebhardt et al. [47] suggested relation. The numerical results support the semi-analytical arguments above. We first need to fix \( m \). The heavier BH is, the more compact the bulge is, and more compact bulges means a larger \( r_{bh} \) on the scalar field. The heavier BH is, the steeper the central slope of the DM density is. Fig. 2 clearly reveals central field value increases as \( r_{bh} \) shows the rescaled DM density profile as a function of 10\(^{-22}\) eV, respectively. For this \( m \) the unit length \(( r = 1)\) is 0.0127 pc. The galaxy with a heavier BH has a more cuspy halo.

\[
\psi_0 = (V_d(r_h) - V_d(r_0))^{1/2} c. 
\]

(10)

In this equation we have assumed that the rotation velocity of visible matter is determined by the depth of the potential well generated only by the DM. This approximation can be justified because the sphere of influence of the BH is usually small compared to the bulges and almost all visible matter in the bulges remains far from the BH and hardly falls into the BH potential trap. (However, this approximation fails for \( M_{bh} \approx M_h \).

Figs. 1-2 show the result of our numerical study with the parameters \( V_d(r_0) \approx -4.3 \times 10^{-7} \) and \( E = -4.7 \times 10^{-8} \). The total mass \( M \) within \( r = 4000 \) including the BH is \( O(10^8 M_\odot) \) for all 3 cases with \( m = 5 \times 10^{-22} \) eV. Fig. 1 shows the rescaled DM density profile as a function of \( r \) for several different \( M_{bh} \). The graphs show the effect of the BH on the scalar field. The heavier BH is, the steeper the central slope of the DM density is. Fig. 2 clearly reveals this tendency. The effective radius \( r_h \) of the bulges is inversely correlated with \( r_s \) as predicted in Eq. (9), and the central field value increases as \( r_s \). We assumed \( \psi_0 \approx \psi(r_0) \) for the interpretation of the numerical work. Therefore, the heavier BH is, the more compact the bulge is, and more compact bulges means a larger \( \sigma \), and hence the M-sigma relation. The numerical results support the semi-analytical arguments above.

The numerical solutions of the dimensionless SPE are independent of \( m \). To compare the results with observations we first need to fix \( m \). The particle mass \( m \approx 10^{-22} \) eV is required to solve the cusp problem and to suppress the small-scale power [14], and in Ref. [40] it was shown that the SFDM with \( m \approx 5 \times 10^{-22} \) eV can explain the minimum mass and the size of galaxies. Let us use this \( m \) for the fitting below.

Fig. 3 shows our \( M_{bh} - \sigma \) relation obtained from the numerical calculation. We check the relation with the parameterization

\[
\log_{10} M_{bh} = \alpha + \beta \log_{10} \sigma, 
\]

(11)

where \( M_{bh} \) is in \( M_\odot \) unit and \( \sigma \) is in \( \text{km} \, \text{s}^{-1} \). There is still a controversy about the value of the exponent \( \beta \). For example, Gebhardt et al. [47] suggested \( \beta = 3.75 \pm 0.3 \) while Ferrarese and Merritt [2] had reported \((\alpha, \beta) = (-2.9 \pm 1.3, 4.8 \pm 0.54)\). For \( m = 5 \times 10^{-22} \) eV and \( \beta = 4.8 \), we obtain the best fitting value \( \alpha = -2.3 \), which is
comparable to the observed value. On the other hand, our best fitting value for free \((\alpha, \beta)\) is \((\alpha, \beta) = (-3.4, 5.3)\), which is also quite similar to the recently inferred value \((\alpha, \beta) = (-3.4 \pm 0.86, 5.12 \pm 0.36)\) [48]. This is a remarkable consistency despite a few free parameters and the simple arguments we have used so far.

Since \(\alpha\) is very sensitive to \(m\) while \(\beta\) is not, we can constrain \(m\) from the fitting. For \(m = 10^{-22} eV\) our fitting gives \(\alpha = -2.7\) while for \(m = 10^{-21} eV\) \(\alpha = -3.7\). Therefore, it is interesting that the DM particle mass \(m \simeq 5 \times 10^{-22} eV\) required to reproduce the correct \(\alpha\) is just that for explaining the minimum size and the mass of galaxies [46, 49]. The BEC/SFDM can explain not only the exponent but also the normalization of the M-sigma relation.

IV. DISCUSSION

The fitting in the previous section was done only for the \(\sigma\) range where the simple power-law holds. For \(M_{bh} \ll M\) the effect of the BH on \(M_{bh}\) is negligible and we expect a large scatter of the M-sigma relation as often found in the M-sigma relation graphs [48]. On the other hand, for \(M_{bh} \sim M\) the gravitational force exerted by the BH is much larger than that by the DM and we expect \(\sigma\) to follow not the M-sigma relation but a position dependent Kepler-law like velocity dispersion \(\sigma \simeq \sqrt{GM_{bh}/r}\). Interestingly, the recent observation of the over-massive black hole in NGC 1277 [50] showed this behavior of \(\sigma\). In this galaxy the mass of the central black hole \((M_{bh} = 1.7 \times 10^9 M_\odot)\) is 59\% of its bulge mass, which is quite different from the value \(M_{bh} \simeq 2 \times 10^9 M_\odot\) estimated from the M-sigma relation. In our model the simple power-law M-sigma relation is no longer valid for a relatively large or small \(M_{bh}\) compared to \(M\) as expected theoretically. This is different from the prediction of the feedback mechanisms proposed so far. The numerical result in Fig. 3 also shows this tendency. To study the effect of very heavy SMBHs we need a SFDM halo profile for a big galaxy, which is unclear yet.

In conclusion we have shown that BEC/SFDM model with \(m = 5 \times 10^{-22} eV\) can explain the M-sigma relation between the SMBH mass and the velocity dispersions in galaxies as well as other cosmological constraints. This resolution does not need any feedback mechanism, and the M-sigma relation is robust against mergers, and hence universal. Since the M-sigma relation is established by the DM wave dynamics, the tight relation is automatically and rapidly readjusted after the mergers. This model also seems to explain the relation between the bulge size and the BH mass. Therefore, the BEC/SFDM model seems to give a new hint to the BH-galaxy coevolution. More detailed analytical and numerical studies including visible matter are desirable from this perspective.

ACKNOWLEDGMENTS

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