Discussing anthropogenic global warming from an econometric perspective: a change scenario based on the Arima paleoclimate time series model

Gilmar Veriato Fluzer Santos¹†*, Lucas Gamalel Cordeiro¹†, Claudio Antonio Rojo¹†, and Edison Luiz Leismann¹†.

¹Western Paraná State University, Graduate Program in Management/Professional Master's Degree, Cascavel, 85819-110, Brazil.

*Gilmar Santos, mail to: gviriato@hotmail.com

Abstract

Global warming has divided the scientific community worldwide with predominance for anthropogenic alarmism. This article aims to project a climate change scenario using a stochastic model of paleotemperature time series and compare it with the dominant thesis. The ARIMA model – an integrated autoregressive process of moving averages, popularly known as Box-Jenkins - was used for this purpose. The results showed that the estimates of the model parameters were below 1°C for a scenario of 100 years which suggests a period of temperature reduction and a probable cooling, contrary to the prediction of the IPCC and the anthropogenic current of an increase in 1.50° C to 2.0° C by the end of this century. Thus, we hope with this study to contribute to the discussion by adding a statistical element of paleoclimate in counterpoint to the current consensus and to placing the debate in a long-term historical dimension, in line with other research already present in the scientific literature.

Keywords: Global warming. Paleoclimatology. Time series. Arima model. Climate scenarios.

Introduction

The controversies over global warming and its effects on the economy and the environment are the subject of discussion and debate around the world and in some ways determine how governments and companies develop their policies and conduct their business.

The human action according to the followers of anthropogeny and other international bodies such as the IPCC (Intergovernmental Panel on Climate Change - UN), has been responsible for climate change and global warming (greenhouse effect). This is endorsed by most scientific publications by showing that more than 90% of studies on the subject say that the cause of global warming is anthropogenic, established as the "official version" by IPCC advocates (Salzer, Neske & Rojo, 2019; Cook et al. 2013; Bray, 2010; Anderegg et al., 2010; Oreskes, 2004).

Shwed and Bearman (2010) bring an important contribution in the strategy of assessing the state of scientific contestations on certain issues when the scientific community considers a proposition a fact and how the importance of internal dissent in the face of consensus diminishes.

The IPCC Working Group Chair, Jim Skea, states: "Limiting warming to 1.5°C is possible within the laws of chemistry and physics but doing so requires unprecedented changes" (IPCC Special Report, 2019). Supporters of the naturalistic cause, on the other hand, present arguments that
challenge these studies by claiming that anthropogenic global warming is theoretically fragile with calculated misinformation, and its historical sample of only 150 years would be insufficient to establish a consensus often supported by agnotology and metric uncertainties (Molion, 2008; Legates et al. 2015; Legates, Soon & Briggs, 2013; Reinsinger et al. 2010).

What is noticeable is that the more research explores the past, the more the anthropogenic thesis is weakened, as demonstrated by Davis (2017) and Harde (2019) by finding that changes in the atmospheric CO2 concentration did not cause changes in ancient climate temperature and climate change is not related to the carbon cycle, but rather to native impacts. Easterbrook (2016), in his evidence-based book brought data opposing CO2 emissions as the primary source of global warming, the thesis of which has been captured by politics and dubious computer modeling.

Other pro-anthropogenic studies ignore paleoclimatology as a relevant factor in research or have it as a factor of uncertainty, such as that of Haustein et al. (2017), Cook et al. (2013), Mitchell et al. (2017), Medhaug et al. (2017), in addition to those that underpinned the IPCC report (Solomon et al. 2007). However, increasingly scientists are pointing to data which suggests that climate changes are a result of natural cycles, which have been occurring for thousands of years, says Easterbrook (2016).

Thus, it is possible to identify a gap in this debate which is a broader time horizon research and give statistical predictability to climate change. This is the objective of this study, whose essence is to establish a climate prediction scenario based on a series of paleotemperatures of 12 thousand years (Holocene) until nowadays, plus the uncertainties that the data used predict. For this, the integrated autoregressive moving average econometric model (ARIMA - BJ) was adopted as a method whose database was extracted from the article by Kaufman et al. (2020), who applied five statistical methods of thermal reconstruction to ascertain the global average surface temperature (GMST) to the present day, which served as the basis for this research.

Results generated indicated the fragility of the anthropo-warming thesis, which showed significant divergence from the scenario projected by the IPCC, which in its latest report predicted an increase of more than 1.5°C in the planet's temperature by 2050 (IPCC, 2019).

Therefore, we sought to establish one more variable for the global warming issue, in order to innovate the discussion and enable a technically critical approach, with the intention of comparing it with the consensus that prevails today.

Results

The parameters used to reach at the results were the median and the 5th and 95th percentiles representing the estimate of uncertainties with 90% confidence, as the authors themselves indicate by recommending that

“future users of this reconstruction use the full ensemble when considering the plausible Holocene GMST evolution. By representing the multi-method reconstruction as a single time series, the median of the ensemble may be best along with the 90% range of the ensemble to represent uncertainty.”

(Kaufman et al., 2020, p.04).

For building the results, the data were represented graphically and fed the software IBM - SPSS Statistics, v. 22, for processing the ARIMA methodology - Box Jenkins methodology and the corresponding outputs according to each step of the calculation. Figure 1 shows the evolution of the 12k median of the data set extracted from Kaufmann et al. (2020) on a 100-year scale, with milestone
"0" being the year 2019 (p. 8) calculated from the different reconstruction methods.

Figure 1. Evolution of the Global Median 12k years temperature.
Source: Author elaboration (adapted from Kaufman et al. (2020 p. 06) from CSV file data at https://www.ncdc.noaa.gov/paleo/study/29712).

Figure 2 represents the 5th and 95th percentile range of the set bringing together the various sources of uncertainty, including proxy temperature, chronology, and methodological choices, as per Kaufman et al. (2020 p. 03).

Figure 2. Evolution of the parameters 5th and 95th global percentiles (uncertainties).
Source: Author elaboration (adapted from CSV file data - temp 12K all methods percentiles at https://www.ncdc.noaa.gov/paleo/study/29712).

The average temperature of the 1800-1900 period for each composite was used as the pre-industrial reference period defined by the authors as an anomaly of 0° C and which served as the reference for the IPCC (1850-1900). For this reason, it was removed from each member of the ensemble to avoid issuing individual records and different reconstructions (Kaufman et al, 2020).

Box-Jenkins ARIMA model's objective is to provide a valid basis for forecasting, after
all tests, parameters, and diagnostics have been performed. The forecasts of the two-time series, median and uncertainties, were generated in the IBM - SPSS Statistics software, version 22, in a specific session for ARIMA modeling.

According to the model parameters, predictions for the median were expressed in the form of temperature estimates, for the next 100 years, represented by AR and MA. For statistical reliability purposes, the degree of significance (Marôco, 2018) of the parameters must be measured, being extremely significant in AR and very significant in MA, as described in figure 3.

### Table: Arima model parameters

|                      | Estimate | SE  | t     | Sig.  |
|----------------------|----------|-----|-------|-------|
| MdTempGlob-Model_1   |          |     |       |       |
| MdTempGlob No        |          |     |       |       |
| transformation       |          |     |       |       |
| Constant             | 1.191    | 0.129 | 1.489 | 0.139 |
| AR Lag 1             | 0.932    | 0.032 | 28.799 | 0.000 |
| MA Lag 1             | -0.266   | 0.099 | -2.695 | 0.008 |

**Figure 3:** 100-year scale temperature estimates of AR and MA parameters.
Source: Author elaboration with Software SPSS - Statistics v. 22.
(URL: [https://www.ibm.com/support/pages/spss-statistics-220-available-download](https://www.ibm.com/support/pages/spss-statistics-220-available-download)).

An important condition for model reliability is the residuals of the ACF and PACF correlations, the white noise. For the model to be validated as the most adequate, they should be concentrated around the mean, and the degree of significance is absolute (0 or close), thus represented in figure 4. (Note: Retardo means Lag; “de residuo” means of waste)

**Figure 4:** Residuals of the ACF and PACF correlograms (White noise).
Source: prepared by the author (SPSS - Statistics v. 22).

Thus, once stationarity is achieved (see p. 7-9), we can model it with an autoregressive process (AR), which we will represent by $Y_t$ the Median (Md) at period $t$ (Holocene) as:

$$Y_t - \delta = \varphi Y_{t-1} + u_t$$  \hspace{1cm} (1)

where $\delta$ is the mean of $Y$ and $u_t$ is an uncorrelated random error with zero mean and constant variance $\varphi^2$ (this is white noise), then we will say that $Y_t$ follows a first-order stochastic autoregressive or AR process (1).

The AR process we have just discussed is not just a mechanism that may have generated $Y$. 
In this case, Y may evolve into a first-order moving average process, or an MA (1). If we model Y in this way:

\[ Y_t = \mu + \beta_0 u_t + \beta_1 u_{t-1} \]  \hspace{1cm} (2)

where \( \mu \) is a constant and \( u \), as before, is a white noise stochastic error term. Here Y at period t is equal to a constant plus a moving average of the current and past error terms. More generally, we can represent it like this

\[ Y_t = \mu + \sum_{i=0}^{q} \beta_i u_{t-i} \]  \hspace{1cm} (3)

which is an MA(q) process. In short, a moving average process is just a linear combination of white noise error terms. In this case, most likely Y has characteristics of both AR and MA and is therefore ARMA. Then Yt follows an ARMA (1,1) process, and can be written as

\[ Y_t = \Theta + q_1 Y_{t-1} + \beta_0 u_t + \beta_1 u_{t-1} \]  \hspace{1cm} (4)

because there is an autoregressive term and a moving average term. In the Equation, \( \Theta \) represents a constant term. In general, in an ARMA (p, q) process, there will be p autoregressive terms and q moving average terms.

In the fit chart, shown in figure 5, it is observed that the two lines coincide, almost overlapping, indicating that this is the best of the models tested. The outliers present between 1 and 5 dates were kept in the setup since if we were to remove them, the series would not be robust. This guarantees its impartiality and uncertainty for future events (Stockinger & Dutter, 1987). Note: observado means observed; ajuste means adjust; UCL: the upper control limit; LCL: the lower control limit.

![Graph of the adjusted 12k median series.](image)

Figure 5. Graph of the adjusted 12k median series.

Source: elaborated by the author (SPSS- Statistics).

Regarding the uncertainty results 5th and 95th percentiles, the process follows the same model as the median, whose configuration is described in a supplementary file. The following parameters were generated, according to figure 6:

| ARIMA model parameters | Estimate | SE  | t   | Sig. |
|------------------------|---------|-----|-----|------|
| GLOBALS-Model_1        | GLOBALS| No transformation | Constant | -2,403 | 3,490 | -6,88 | .493 |
| AR                     | Lag 1   | .999 | .003 | 291,464 | .000 |
Figure 6. Parameters of the 5th and 95th percentile temperatures (model uncertainty).
Source: Author elaboration (SPSS - Statistics v. 22).

We then have a set of six different extremely significant temperature results for the estimates of the two models, namely 0.932°C; -0.266°C (fig. 3) and 0.999°C; -0.70°C; 0.996°C; -0.382°C (fig. 6).

To fulfill the objective of this study, it is necessary that a standard measure be calculated and adopted as a reference. The median, extracted from the set of estimates of both models is the most appropriate statistical measure in this case, whose result was 0.333°C (calculated from Microsoft Excel). It is evident, thus, a temperature well below the 1.50% to 2.00% projected by the IPCC by the end of this millennium. The results generated here indicate that as opposed to warming, the scenario being drawn is that the world may go through a period of decreasing temperatures in the next hundred years, which could imply a cooling of global scope.

**Discussion**

Given the results presented, one must ask why there is so much consensus around a scenario that as the evidence shows here, leaves much room for doubt? Another question that arises is why there is so much scientific unanimity around anthropogenic warming (97.2% according to Cook *et al.*, 2013), now called "climate change"?

It is understanding that if we compare recent temperatures to the distribution of global maximum temperatures during the Holocene, there was on average a 1°C increment over the pre-industrial period (1850-1900) and for most members of the ensemble, no 200-year interval during the series exceeded the warmth of the most recent decade (Kaufman *et al.*, p. 5). We see, therefore, that the time horizon of the anthropogenic thesis is recent to the time of man's existence on earth (Holocene) and when compared to the results of this research, lacks substantiation if analyzed in the light of statistical science.

On the other hand, Kaufman *et al.*, (2020), when relying on the IPCC projections, admit that temperatures for the rest of this century are likely to exceed 1°C if compared to those of the pre-industrial era (1800-1900), which they considered as an anomaly of 0°C. Although the authors claim that the Holocene GMST reconstruction is comparable with the IPCC long-term projections and those seen in the last decade, the results presented here show a different and antagonistic scenario if one considers a hundred-year scale and the historical temporality present in the statistical series.

It is known that one of the villains of anthropogenic genesis, the greenhouse effect, was already unveiled in 1896 by Arrhenius as a natural phenomenon beneficial to the development of biological life on the earth's surface (troposphere) whose subsequent studies were duly confirmed (Miller & Spoolman, 2016). Therefore, reinventing this evidence is something that does not hold up in light of the intuitive and deductive capacity of science, as the proponents of anthropogeny claim in establishing a rationale for global warming.

It does not exclude the impact that human action has brought to recent climate change, which might be important and timely, but seems to be insignificant in the face of the millennial variability.
of the climate, the size and complexity of the universe, and all the natural and astronomical phenomena that interact with the earth in the planetary system.

Lastly, it should be argued that the climate scenario predicted here is not enough to determine which are the true causes of recent climate change, whether natural or anthropogenic, since the two may be complementary, not divergent. For this, new studies on paleoclimate and its variability are needed to corroborate the estimates resulting from this research and to bring more evidence in the search for scientific truth.

It is not credible to submit the world and organizations to the intention of a doubtful thesis with all the consequences that this brings to the strategic planning of political and economic agents. It would condemn humanity to an environmentalist dogmatism and a catastrophism without any support to justify them.

Data and methods

The data for this paper were collected from Kaufmann et al., (2020) unprecedented multi-method reconstruction research of mean land surface temperature (GMST) during the Holocene era (12,000 years) to the present day, "whose database is the most comprehensive global compilation of previously available published Holocene proxy temperature time series" (Kaufman et al., 2020, p. 01).

Extraction of the primary data from this study is available as individual CSV files and merged as a netCDF file at figshare 35 and at NOAA Palaeoclimatology 36 (https://www.ncdc.noaa.gov/paleo/study/29712). A CSV file with the multi-method joint median and 5th and 95th percentiles is also available in both data repositories. All were used as input data to compose the 12k time series of paleotemperatures in the two variables and fed into IBM SPSS-Statistics software (v. 22) for the calculation of parameters and estimates. The data generated for the development of this research are available in supplementary file.

Stochastic Processes and the Stationarity Test

To introduce the development of the forecast, we justify graphically and mathematically the results that the SPSS software generated with their respective outputs, in the two variables of this study, the median and the uncertainty set. To better understand, we will use the graphs in this section and the mathematical formulation of their results as well as the structuring of the uncertainty set (same pattern) in a supplementary file.

First of all, it is necessary to apply two tests to verify the stationarity of the time series: (1) graphical analysis and (2) the correlogram test, since it is a condition for using the ARIMA (BJ) model.

By analyzing Figures 1, 2 and 5, we verify that the series are not stationary, that is, by establishing a mean line for the 12K global temperature median series (Figure 7) we verify that the data do not circulate around it and express a trend. Note: número de sequência means sequence number.
Next, we apply the correlation tests, also called "F" correlation function: ACF (automatic) and ACFP (partial), the next step to make the series stationary, as shown in figures 8 and 10, and their respective reports, represented by figure 9. Note: coeficiente means coefficient. Número de retardo means Lag numbers.

**Figure 8.** Graphical test of autocorrelation (automatic).

### Automatic correlations

| Lag | Autocorrelation | Standard Error | Box-Ljung Statistics |
|-----|-----------------|----------------|----------------------|
| 1   | .956            | .090           | 113,376              |
| 2   | .887            | .089           | 211,775              |
| 3   | .821            | .089           | 296,777              |
| 4   | .759            | .089           | 370,121              |
| 5   | .702            | .088           | 433,357              |
| 16  | .246            | .84            | 723,810              |

| Value | df | Sig. |
|-------|----|------|
| 1     | 1  | .000 |
| 2     | 2  | .000 |
| 3     | 3  | .000 |
| 4     | 4  | .000 |
| 5     | 5  | .000 |
| 16    | 16 | .000 |

a. The underlying process considered is independence (white noise).

b. Based on the asymptotic chi-square approximation.

**Figure 9.** LJung Box statistical report (Ho and H₁ hypotheses).
Figure 10. Graphical test of partial autocorrelation - PCA. Graphical and correlation analysis indicates that we have to normalize the series making it stationary. The process occurs with the choice of the first lag (lag), which exceeded the confidence interval in both tests and whose degree of graphical significance is higher, i.e., has the highest correlation and the lowest value according to the Ljung-Box statistic. The lag that meets these criteria, therefore, is number 1, highlighted in fig. 9.

From these results, we can graphically represent (figure 11) the stationarity adjusted, as a function of the first differentiation (lag 1):

Figure 11. Adjusted stationarity as a function of lag 1. So, we can then replicate this modeling for the probabilistic analysis of the uncertainties, represented by the 5th and 95th percentiles, at a 90% confidence level, since it assumed the same stationarity criteria and tests (graph and correlogram) of the median. The graphical representation of the uncertainty set is described in the supplementary file.

Applying the Box- Jenkins model

Box-Jenkins’s method aims (figure 12) is to estimate a statistical model and interpret it according to the sample data. If this estimated model is used for forecasting, we should assume that its characteristics are constant over the period and particularly in future periods. A simple reason for requiring the stationary data is that any model that is inferred based on that data can be interpreted as stationary or stable and therefore provides a valid basis for prediction (Pokorny, 1987, Gujarati and Porter, 2011).
1. Identification of the model
   (Choosing tentative p, d, q)

2. Parameter estimation of
   the chosen model

3. Diagnostic checking:
   Are the estimated residuals white noise?
   Yes (Go to Step 4)      No (Return to Step 1)

4. Forecasting

**Figure 12.** The Box–Jenkins methodology. About step 4, Forecasting: One of the reasons for the popularity of the ARIMA modeling is its success in forecasting. In many cases, the forecasts obtained by this method are more reliable than those obtained from the traditional econometric modeling, particularly for short-term forecasts. Of course, each case must be checked (Gujarati and Porter, 2011, p. 778).

We concluded that the MedTempGlobal (as described in the data/figures) time series model was not stationary and we had to normalize it, making it stationary with constant mean and variance and its covariance invariant over time. Therefore, it is an integrated time series, i.e., it combines the two autoregressive processes (AR and MA) in the same set.

An important point to note is that when using the Box-Jenkins methodology, we must have both a stationary time series and a time series that is stationary after one or more differentiations (Gujarati and Porter, 2011).

Then, we can state that if a time series is integrated of order 1, therefore, it is $I(1)$, after differentiating it becomes $I(0)$, that is, stationary. In general, if a time series is $I(d)$, after differentiating it $d$ times, we get an $I(0)$ series.

If one has to differentiate a time series $d$ times to make it stationary and apply the ARMA ($p$, $q$) model to it, one will say that the original time series is $ARIMA (p, d, q)$, that is, it is a moving average **integrated autoregressive time series**, where $p$ denotes the numbers of the autoregressive terms, $d$ the number of times the series must be differentiated before it becomes stationary, and $q$ the number of moving average terms.

We, therefore, have in this time series an ARIMA (1,0,1) model, as it was differentiated once ($d = 1$) before becoming stationary (of first difference), and can be modeled as an ARMA ($1,1$) process, as it has an AR term and an MA post stationarity.

Finally, it is important to emphasize that to optimize the results, it was necessary to run in the software SPSS - Statistics all the possible combinations of the ARIMA model ($p,d,q$) in the two parameters, to arrive at the statistically optimal model after the decomposition of the data and meeting the criteria of analysis and execution.

**References**
1. Salzer, E. Neske, D. A. L. & Rojo, C. A. Global warming: bias analysis in divergent strategic scenarios. Journal Multi-Science Research (Msr), Vitoria, v. 2, n. 2, p. 144-158. Semestral. Disponível em: https://msrreview.multivix.edu.br/index.php/msr/article/view/43 (2019).

2. IBM – SPSS Statistics v. 22. https://www.ibm.com/support/pages/spss-statistics-220-available-download (2020).

3. Cook J. et al 2013 Environ. Res. Lett. 8 024024. https://iopscience.iop.org/article/10.1088/1748-9326/8/2/024024 (2013).

4. Bray, D. The scientific consensus of climate change revisited. Ciência e política ambiental , 13 (5), 340-350, (2010). https://doi.org/10.1038/nature22315

5. Anderegg W. R. L., Prall J.W., Harold J., Schneider S. H. Expert Credibility in Climate Change. Proceedings of the National Academy of Sciences Jul 2010, 107 (27) 12107-12109; DOI: 10.1073 / pnas.1003187107 (2010).

6. Oreskes, N. Science. Vol. 306, Issue 5702, p.1686. DOI: 10.1126/science.1103618 (2004).

7. Shwed, U., & Bearman, P. The Temporal Structure of Scientific Consensus Formation. American Sociological Review, 75(6), 817-840. doi:10.2307/25782168 (2010).

8. IPCC - Intergovernmental Panel on Climate Change. United Nations. N.Y. https://www.ipcc.ch/sr15/ (2019).

9. Molion, L. C. B. Aquecimento global: Uma visão crítica. Revista brasileira de climatologia, 3. https://revistas.ufpr.br/revistaabclima/article/view/25404
DOI: http://dx.doi.org/10.5380/abclima.v3i0.25404 (2008).

10. Legates, D. R., Soon, W, &. Briggs, W. M. & C. Monckton of Brenchley. Consenso Climático e 'Desinformação': Uma Tréplica à Agnotologia, Consenso Científico e o Ensino e Aprendizagem das Mudanças Climáticas . Sci & Educ 24, 299-318. https://doi-org.ez89.periodicos.capes.gov.br/10.1007/s11191-013-9647-9 (2015).

11. Legates, D. R., Soon, W. & Briggs, WM Learning and Teaching Climate Science: The Perils of Consensus Knowledge Using Agnotology. Sci & Educ 22, 2007–2017 (2013). https://doi-org.ez89.periodicos.capes.gov.br/10.1007/s11191-013-9588-3 (2013).

12. Reisinger, A. , Meinhausen, M., Manning, M. & Bodeker, G. Uncertainties of global warming metrics: CO2 and CH4, Geophys. Res. Lett. , 37 , L14707, doi: 10.1029 / 2010GL043803 (2010).

13. Davis, W.J. The Relationship between Atmospheric Carbon Dioxide Concentration and Global Temperature for the Last 425 Million Years. Climate 2017, 5, 76. https://doi.org/10.3390/cli5040076 (2017).

14. Harde, H. What Humans Contribute to Atmospheric CO2: Comparison of Carbon Cycle Models with Observations. Earth Sciences. Vol. 8, No. 3, 2019, pp. 139-158. doi: 10.11648/j.earth.20190803.13 (2019).

15. Easterbrook, D. (Ed.). (2016). Evidence-based climate science: data opposing CO2 emissions as the primary source of global warming. Elsevier (2016).

16. Haustein, K., Allen, MR, Forster, PM et al. A real-time Global Warming Index. Sci Rep 7, 15417. https://doi-org.ez89.periodicos.capes.gov.br/10.1038/s41598-017-14828-5 (2017).

17. Mitchell, D., James, R., Forster, P. et al. Realizing the impacts of a 1.5 °C warmer world. Nature Clim Change 6, 735–737 (2016). https://doi.org/10.1038/nclimate3055 (2016).

18. Medhaug, I., Stolpe, M., Fischer, E. et al. Reconciling controversies about the 'global warming hiatus'. Nature 545, 41–47 (2017).

19. Solomon, S. et al. Climate Change 2007: The Physical Science Basis. Working Group I Contribution to the Fourth Assessment Report of the IPCC (Cambridge University Press, 2007).

20. Kaufman, D., McKay, N., Routson, C. et al. Holocene global mean surface temperature, a multi-method reconstruction approach. Sci Data 7, 201 (2020). https://doi.org/10.1038/s41597-020-0530-7 (2020).

21. Maroco, J. Análise estatística com o SPSS Statistics. 7ª. ed. 54-60 (Pero Pinheiro, 2018).
22. Stockinger N., Dutter R. Robust time series analysis: a survey Kybernetika, Vol. 23 (1987), No. Suppl, (1), 3-88 Persistent URL: http://dml.cz/dmlcz/124955.
23. Miller, G. T., & Spoolman, S. E. Environmental Science. 22-24 (Cengage Learning, 2016).
24. Gujarati, D. N. & Porter, D. C. Basic Econometrics, 5th edition. 767-778. AMGH Editora, (2011).
25. Box, G. P. & Jenkins, G. M. Time series analysis: forecasting and control. Ed. rev. holden day, sào francisco: holden, (1978). Document shared on www.docsity.com.
26. Box, G. E. P., Jenkins, G. M. & Reinsel, G. C. (2008) Time Series Analysis: Forecasting and Control. 4ª Edição, Wiley, Oxford. http://dx.doi.org/10.1002/9781118619193.
27. Pokorny, M. An introduction to econometrics. (ed. Blackwell, B.) p. 343 (Basil Blackwell, 1987).
28. Routson, CC, McKay, NP, Kaufman, D. S. et al. Mid-latitude net precipitation decreased with Arctic warming during the Holocene. Nature 568, 83–87. https://doi.org/10.1038/s41586-019-1060-3 (2019).
29. PAGES 2k Consortium. Consistent multi-decadal variability in global temperature reconstructions and simulations over the Common Era. Nat. Geosci. 12, 643–649 (2019).
30. Marcott, S. A., Shakun, J. D., Clark, P. U. & Mix, A. C. A reconstruction of regional and global temperature for the past 11,300 years. Science 339, 1198 (2013).

Supplementary Materials

Mathematical rationale and statistical guidelines used in the methodology (Ref. fig. 7 to10, pg. 8 - 9).

When translating this analysis into a mathematical expression, the FAC correlation denoted by k as a function of k is defined as:

\[ \rho_k = \frac{\overline{Y}_k}{\overline{Y}_0} = \frac{\text{covariance with lag } k}{\text{Variance}} \]  
(1)

where the value of k is the chosen delay (1), and the system considers covariance with lag k and variance already calculated according to the data. In practice, as we have a stochastic uniequational series, we can compute the function of sample correlation, \( \hat{\rho}_k \). For such, we first need to calculate the covariance of the sample with lag \( k \), \( \hat{\gamma}_k \), and the variance of the sample, \( \hat{\gamma}_0 \) defined as:

\[ \hat{\gamma}_k = \frac{\sum(Y_i - \overline{Y})(Y_{i+k} - \overline{Y})}{n} \\
\hat{\gamma}_0 = \frac{\sum(Y_i - \overline{Y})^2}{n} \]  
(2)

where \( n \) is the sample size and \( \overline{Y} \) is the sample mean. Thus, the function of sample correlation, with lag \( k \) is:

\[ \hat{\rho}_k = \frac{\hat{\gamma}_k}{\hat{\gamma}_0} \]  
(3)

which is simply the ratio of sample covariance (with \( k \)lag) and sample variance. The \( \hat{\rho}_k \) k-versus-k graph is known as a sample correlogram.

Modeling the time series according to the methods (AR) of moving averages (MA) and ARIMA (p. 07 to 10).

Once the parking period is conquered, we can model it with an autorregressive process (AR), which we will represent by \( Y_t \), the Median (Md) in the t period (Holocene) as:

\[ (Y_t - \delta) = \alpha_1(Y_{t-1} - \delta) + \eta_t \]  
(4)
in which $\delta$ is the mean of $Y$ and $u_t$ is an error that's not correlated with $m$ is day zero and constant variance $\sigma^2$ (this is a white noise), so we will say that $Y_t$ follows a first-order stochastic autoregressive process or AR (1). Here the value of $Y$ in period $t$ depends on its value in the previous period and on a random term; $y$ values are expressed as deviations based on an average value. In other words, this model states that the predicted value of $Y$ in period $t$ is simply some proportion ($=q$) plus a random shock or disturbance in the $t$-period; again, the $Y$ values are expressed around their mean values.

The AR process we just discussed is not just a mechanism that may have generated $Y$. In this case, $Y$ can evolve into a first-order moving average process, or an MA (1). If we model $Y$ this way:

$$Y_t = \mu + \beta_0 u_t + \beta_1 u_{t-1}$$

(5)

in which $\mu$ is a constant and $u$, as before, is a stochastic error term of white noise. Here $Y$ in period $t$ is equal to a constant plus a moving average of the current and past terms of error. In a more general way, we can represent

$$Y_t = \mu + \beta_0 u_t + \beta_1 u_{t-1} + \beta_2 u_{t-2} + \cdots + \beta_q u_{t-q}$$

(6)

which is an MA process($q$). In short, a moving average process is just a linear combination of white noise error terms.

Self-regressive process of moving averages (ARMA)

It is most likely that $Y$ has both AR and MA characteristics and is therefore ARMA. So, $Y_t$ follows an ARMA process (1, 1), and can be written as

$$Y_t = \theta + \alpha_1 Y_{t-1} + \beta_0 u_t + \beta_1 u_{t-1}$$

(7)

because there is an autoregressive term and a moving average term. In the Equation, $\theta$ represents a constant term. In general, in an ARMA process ($p$, $q$), there will be autoregressive terms $p$ and moving average terms $q$.

Graphical configuration of the uncertainty set (p. 5 to 10, v. Data and methods).

![Graphical configuration of the uncertainty set](image)

**Figure 1.** Graphic description of the 5th and 95th percentiles decomposed (uncertainties)

Next, we represent the graphs of the correlations of both models, the ACF and ACFp, and their subsequent differentiation for parking.
Figure 2. 5th percentile Autocorrelation Graphic Test (Automatic ACF)  
Source: Prepared by the authors (SPSS - Statistics v. 22)

Figure 3. Partial autocorrelation graphic test 5th percentile - ACFP.  
Source: prepared by the authors (SPSS- Statistics v. 22).
**Figure 4.** 95th percentile autocorrelation graph test (automatic ACF)
Source: Prepared by the authors (SPSS - Statistics v. 22)

**Figure 5.** 95th percentile partial autocorrelation graph test - ACFP.
Source: prepared by the authors (SPSS- Statistics v. 22).
Figure 6. Residues of the ACF and PACF correlograms – 5th and 95th percentiles (White noise).

Figure 7. Adjustment chart of the 5th and 95th percentiles parameters. Source: SPSS output - Statistics powered by the authors.
Data Records

The data that led to this research were reused by Kaufman et al., 2020, as already referenced in the text. After treatment, the data fed the IBM-SPSS Statistics v. 22, at https://www.ibm.com/docs/en/spss-statistics/SaaS?topic=reference-arima, for the development of this research and the generation of results. They can be found in the figshare repository: https://figshare.com/articles/dataset/ArimaMedTempGlobal_spv/14429006;https://figshare.com/articles/dataset/Spreadsheet_for_entering_and_processing_paleoclimate_data_and_graphs_with_the_results_of_the_model_/14429273;https://figshare.com/articles/dataset/Mathematical_and_operational_foundations_of_the_model_mediana_and_uncertainties_/14442701.

Technical Validation

All validations aiming to verify the technical quality and accuracy of the results were done in the ARIMA platform of SPSS-Statistics, and are described in the body of the text and in the data repository. For space reasons, only the data from the model that satisfied the research methodology was sent, according to the foundations found in the specific literature.

Acknowledgements

The scientific foundation on which the results of this paper are based, without ideological or political bias, and the timing of the research data are important to consider. No less important is the legacy left by the authors cited and researched in this work, besides Kaufman et al. (2020): Routson et al. (2019), Cook et al. (2013), PAGES 2k Consortium, Marcott et al. (2013), Harde (2019), Box & Jenkins (1978), Gujarati & Porter (2011). Special gratitude and acknowledgement to professors Claudio Antonio Rojo and Edison Luiz Leismann, who gave all the trust and support necessary to carry out this research.

Author contributions statement

G.V.F.S directed the project, wrote the manuscript and developed the methodology, with input from all authors. E.L.L provided version 22 of the IBM Statistics software to conduct the research and guide the methodology. L.G.R participated in the elaboration of the theoretical framework and in the structuring of the texts. C.A.R oriented the line of research and the projection of scenarios. All authors have reviewed the manuscript.

Competing interests and funding sources

The author declare no competing interests and have not received any specific grants from funding agencies in the public, commercial, or non-profit sectors to do this research.

Additional information

Correspondence and requests for materials should be addressed to G.V. F. S.

Author’s information

G.V.F.S: https://orcid.org/0000-0001-6216-2126 mail to: gviriato@hotmail.com
L.G.C: https://orcid.org/0000-0002-9043-7024
C. A. R: https://orcid.org/0000-0003-4484-9033
E. L. L: https://orcid.org/0000-0002-4112-8241