Effect of motion of the scatterers on localization: quasi localization and quasi mobility edge

E. Kogan

Jack and Pearl Resnick Institute, Physics Department, Bar Ilan University, Ramat Gan 52900, Israel
(Dated: February 8, 2020)

We study kinetics of electrons, scattered by heavy particles undergoing slow diffusive motion. In a three-dimensional space we claim the existence of the crossover region (on the energy axis), which separates the states with fast diffusion and the states with slow diffusion; the latter is determined by the dephasing time. In a two-dimensional space the diffusion coefficient for any value of energy is determined by the dephasing time.

PACS numbers:

Consider electrons in an inhomogeneous media. When discussing the kinetics of the electrons, the quantity, we usually start from in the theoretical description, is the transport relaxation time \( \tau \), calculated in Born approximation. Using Boltzmann equation we can obtain the relation between this relaxation time and the diffusion coefficient

\[
D_0 = \frac{1}{3} v^2 \tau, \quad (1)
\]

where \( v \) is the electron velocity. However, the Boltzmann equation is valid provided \( E \tau \gg 1 \). If we take into account that \( 1/\tau(E) \) typically decreases slower than the first power of energy when the latter goes to zero, we see that however weak is the scattering, the condition of the applicability of Boltzmann equation is broken near the bottom. It is well known since the seminal works of N. F. Mott \([1]\), about the existence of the mobility edge \( E_m \), that is the energy which separates the states with finite diffusion coefficient and states with the diffusion coefficient being exactly equal to zero. All this is true provided the disorder is static. Natural question arises: what happens with this picture when the scatterers slowly move.

To answer this question we need some theory of localization. As such we’ll use the self-consistent localization theory by Vollhard and Wölfle \([2]\). Of crucial importance in the above mentioned theory are maximally crossed diagrams (the sum of all such diagrams is called Cooperon) for the two-particle Green function. The calculations of these diagrams for the case of moving scatterers were done in the paper by Golubentsev \([3]\). So in the first part of the present paper we reproduce the results by Golubentsev (plus some additional interpretation). In the second part we use the results for the Cooperon as an input for the self-consistent localization theory, which we modify to take into account the slow motion of scatterers. In the third part we discuss the results obtained.

The electrons are scattered by the potential

\[
V(r, t) = V \sum_a \delta (r - r_a(t)). \quad (2)
\]

Define the correlator

\[
K(r - r', t - t') = \langle V(r, t)V(r', t') \rangle. \quad (3)
\]

In the leading approximation in the scatterers density we have for the Fourier component of the correlator

\[
K(q, t) = V^2 \left( \int \exp \{i q(r - r') \} \times drdr' \sum_a \delta (r - r_a(t)) \sum_{a'} \delta (r' - r_a'(0)) \right)
= V^2 \sum_a \langle \exp \{i q(r_a(t) - r_a(0)) \} \rangle = n V^2 f(q, t), \quad (4)
\]

where \( n \) is the scatterers density. We consider the case when the scatterers undergo slow diffusive motion. In the ballistic case

\[
f(q, t) = \exp \left( - \frac{q^2}{6} < v^2 > t^2 \right), \quad |t| \ll \tau_{imp}, \quad (5)
\]

In the diffusive case

\[
f(q, t) = \exp \left( - \frac{q^2}{2} D_{imp}|t| \right), \quad |t| \gg \tau_{imp}, \quad (6)
\]

where \( D_{imp} = < v^2 > \tau_{imp}/3 \), and \( \tau_{imp} \) is the scatters free path time. For the Cooperon we get \([3]\)

\[
C_{E}(q) = \int_{0}^{\infty} \exp \left\{ - D(E) q^2 t - \frac{1}{\tau} \int_{0}^{t} (1 - f_{t'}) dt' \right\} dt. \quad (7)
\]

where \( E \) is the energy of each of the two electron lines in Cooperon diagram, and \( q \) is the sum their momenta (see Fig. 1). Also

\[
\frac{1}{\tau} = n V^2 \frac{k^2}{\pi v}, \quad (8)
\]

and

\[
f_t = \int \frac{ds'}{4\pi} f(k_0(s - s'), t)
= \left\{ \begin{array}{ll}
y \left( \frac{t}{\tau_{imp}} \right) & |t| \ll \tau_{imp} \\
y \left( \frac{|t| \tau_{imp}}{\tau} \right) & |t| \gg \tau_{imp}
\end{array} \right. \quad (9)
\]
where

\[ y(x) = \frac{1 - e^{-x}}{x}, \quad \tau_\lambda = \left(\frac{2}{3} k^2 < v_{\text{imp}}^2 > \right)^{-1/2}. \]  

(10)

Eq. (10) can be easily understood if we compare diagrams for the Diffuson (the sum of all ladder diagrams) and the Cooperon on Fig. 1. The Diffuson does not have any mass because of Ward identity. In the case of the Cooperon, the Ward identity is broken, and the difference \([1 - f(t)]\) shows how strongly. The interaction line which dresses single particle propagator is given by static correlator, and interaction line which connects two different propagators in a ladder is given by dynamic correlator. The time-reversal invariance in the system we are considering is broken due to dephasing; the diffusion pole of the particle-particle propagator disappears, although particle-hole propagator still has a diffusion pole, which is guaranteed by particle number conservation.

In extreme cases, from Eq. (7) we obtain

\[ C_E(q) = \int_0^\infty \exp \left[ -D(E)q^2 t - t^3 / \tau_\lambda^3 (E) \right] dt, \]  

(11)

\[ C_E(q) = \int_0^\infty \exp \left[ -D(E)q^2 t - t^2 / \tau_\varphi^2 (E) \right] dt, \]  

(12)

where in the ballistic case

\[ \tau_\varphi = (3 \tau_\lambda^2)^{1/3} \]  

(13)

and in the diffusive case

\[ \tau_\varphi = (2 \tau_\lambda^2 \tau_{\text{imp}}^{-1})^{1/2} \]  

(14)

Thus we obtain the crucial parameter - the dephasing time \(\tau_\varphi\).

The results for the dephasing time (up to a numerical factors of order of one) can be understood using simple qualitative arguments. Consider ballistic regime. If a single collision leads to the electron energy change \(\delta E\), the dephasing time could be obtained using relation (4)

\[ \tau_\varphi \delta E \sqrt{\frac{\tau_\varphi}{\tau}} \sim 2\pi, \]  

(15)

where \(\tau_\varphi / \tau\) is just the number of scattering acts during the time \(\tau_\varphi\). So in this case

\[ \frac{1}{\tau_\varphi^3} \sim \frac{(\delta E)^2}{\tau}. \]  

(16)

If we notice that \(1 / \tau_\lambda\) is the averaged electron energy change in a single scattering act \(\delta E\), we immediately regain Eq. (15).

Inserting Eq. (11) into the self-consistent equation, for the diffusion coefficient \(D\) we obtain equation

\[ \frac{D_0(E)}{D(E)} = 1 + \frac{1}{4 \pi^2 m k} \sum_q C_E(q), \]  

(17)

where \(D_0\) is the diffusion coefficient calculated in Born approximation (Eq. (11)) and the momentum cut-off \(|q| < 1 / \ell\) is implied, where \(l = k \tau / m\) is the mean free path. Thus we obtain

\[ \frac{D_0}{D} = 1 + \frac{1}{\pi m k} \int_0^\infty dt \int_0^{1/\tau} dq q^2 \exp \left[ -Dq^2 t - g(t) \right], \]  

(18)

where

\[ g(t) = \frac{1}{\tau} \int_0^t (1 - f_r) dt'. \]  

(19)

(In the particular case of ballistic regime \(g(t) = t^3 / \tau_3^3\), and in the diffusive regime \(g(t) = t^2 / \tau_\varphi^2\).) The Eq. (18) can be presented as

\[ \frac{D_0}{D} = 1 + X_{\text{IR}} \frac{D_0}{D} \]  

\[ \int_0^\infty dx \int_0^1 dy y^2 \exp \left[ -xy^2 - g \left( \frac{x^2}{D} \right) \right], \]  

(20)

where

\[ X_{\text{IR}} = \frac{3}{4 \pi \tau^2 \tau_\varphi} \]  

(21)

is the Iofe-Regel parameter. Further on we’ll consider Eq. (20) for

\[ \tau_\varphi \gg \tau. \]  

(22)

To solve the equation we take into account that the function is equal to zero for \(t = 0\) and assume that it becomes of the order of \(1 / \tau_\varphi\), where we introduced \(D_\varphi = \ell^2 / \tau_\varphi\). Notice, that the condition (22) can be presented as \(D_\varphi \ll D_0\).

Let us analyze the behavior of the r.h.s. of Eq. (20) as a function of \(D\). For \(D \gg D_\varphi\) the second term in the exponent in Eq. (20) (which represents dephasing) becomes irrelevant, and we get

\[ \text{r.h.s.}(20) \rightarrow 1 + X_{\text{IR}} \frac{D_0}{D}. \]  

(23)
Substituting this result into Eq. (20) we obtain the solution

$$D = D_0(1 - X_{IR}). \quad (24)$$

This solution is valid, provided that $X_{IR} < 1$, that is $E > E_c$, where the mobility edge $E_c$ is obtained from the equation

$$E_c \tau (E_c) = \sqrt{3/4\pi}, \quad (25)$$

which is just the Iofe-Regel criterium For $X_{IR} > 1$, thinking in terms of graphical method of solution, we see that the curve, representing the asymptotic formula (23), is above the curve, representing the l.h.s. of Eq. (20), and they never cross. However, Eq. (23) is no longer valid for $D \leq D_c$. In particular, when $D \ll D_c$, it is the first term in the exponent which is irrelevant, and we get

$$\text{r.h.s.} \sim \frac{D_0}{D_c} \quad (26)$$

This equation guarantees the existence of solution for $X_{IR} > 1$. Because the only scale parameter in the r.h.s. is the quantity $D/D_c$, the deviations from the asymptotic formula appear only for $D \leq D_c$, and the solution for $X_{IR} > 1$ is

$$D = \frac{D_c}{\alpha}, \quad (27)$$

where $\alpha$ is the solution of equation

$$1 = X_{IR} \int_0^\infty dx \int_0^1 dy y^2 \exp\left[-\frac{xy^2}{\alpha} - g(\alpha x \tau_c)\right]. \quad (28)$$

In particular, deep in the ”dielectric region” ($E \tau \ll 1$), the solution of the self-consistent equations is

$$D = \frac{4\pi}{\int_0^\infty \exp[-g(z \tau_c)] dz} E^2 \tau^2 D_c. \quad (29)$$

Eqs. (24) and (27) cover the whole range of change of the Iofe-Regel parameter save the narrow cross-over region near $X_{IR} \approx 1$.

The influence of static disorder in the spaces with dimensionality 2 and 3 is drastically different from that in the space with the dimensionality 3, considered above. In one- and two-dimensional spaces all the states are localized citeeiffetov (provided the disorder is static). Again we ask ourselves, what happens with this picture when the scatterers undergo slow diffusive motion. According to the self-consistent localization theory, for the space of dimensionality $d$ the Eq. (24) becomes

$$\int_0^\infty dx \int_0^1 dy y^{d-1} \exp\left[-\frac{xy^2}{D} - g\left(\frac{x^2}{D}\right)\right], \quad (30)$$

and the Eq. (1) is $D_0 = \frac{1}{4} \nu^2 \tau$. Analyzing behavior of the r.h.s., for $D \gg D_c$ we obtain

$$\text{r.h.s.}(30) = 1 + X_{IR} \frac{D_0}{D} \ln\left(\frac{D}{D_c}\right). \quad (31)$$

Again thinking in terms of graphical method of solution, we see that the curve, representing the asymptotic formula (31), is for any $X_{IR}$ above the curve, representing the l.h.s. of Eq. (30), unless $D \sim D_c$. Thus the solution for any value of the Iofe-Regel parameter is

$$D = \beta D_c, \quad (32)$$

where $\beta$ is the solution of the equation

$$1 = X_{IR} \int_0^\infty dx \int_0^1 dy y \exp\left[-\frac{xy^2}{2} - g(\beta x \tau_c)\right]. \quad (33)$$

**CONCLUSIONS**

We considered the influence of slow diffusive motion of scatterers on the localization of electrons. In this case, like in the case of purely elastic scattering, the diffusion coefficient drastically differs for the energies below and above the mobility edge, the latter being found from the Iofe-Regel criterium. Above the mobility edge we have fast diffusion, and the defasing is irrelevant. Below the mobility edge the diffusion coefficient is inversly proportional to the diffusion time.

Now we would like to mention some possible generalization and applications of these ideas. First, they are completely applicable for the case of ballistic motion of scatterers, say to gas, consisting of heavy classical particles and electrons. The results are identical to those obtained above in the ballistic regime. Second, in the previous publication [6], we studied the influence of dephasing on the Anderson localization of the electrons in magnetic semiconductors, driven by spin fluctuations of magnetic ions. There the role of heavy particles was played by magnons; complete spin polarization of conduction electrons prevented magnon emission or absorption processes, and only the processes of electron-magnon scattering being allowed. Finally, the results obtained can be applied for studying kinetics of classical waves, especially light. This last application is particularly appealing, taking into account that the results of Golubentsev [9] were obtained for light waves.

* Electronic address: kogan@mail.biu.ac.il

[1] N. F. Mott, Adv. Phys. 16, 49 (1967).
[2] P. Wölfle and D. Vollhardt, in Anderson Localization, edited by Y. Nagaoka and H. Fukuyama (Springer, Berlin, 1982).
[3] A. A. Golubentsev, Zh. Eksp. Teor. Fiz. 86, 47 (1984) [Sov. Phys. JETP 59, 26 (1984)].
[4] B. L. Altshuller, A. G. Aronov, and D. E. Khmelnitskii, J. Phys. C 15, 7367 (1982).
[5] K. Efetov, Supersymmetry in Disorder and Chaos, (Cambridge University Press, Cambridge, 199 ).
[6] E. M. Kogan, M. Auslender, and M. Kaveh, Eur. Phys. J. 9, 373 (1999).