J/ψ and ηc masses in isospin asymmetric hot nuclear matter
– a QCD sum rule approach

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Abstract

We study the in-medium masses of the charmonium states J/ψ and ηc in the nuclear medium using QCD sum rule approach. These mass modifications arise due to modifications of the scalar and the twist-2 gluon condensates in the hot hadronic matter. The scalar gluon condensate, \( \langle \alpha_s \pi G^a_{\mu\nu} G^{a\mu\nu} \rangle \) and the twist-2 tensorial gluon operator, \( \langle i \alpha_s G^a_{\mu\sigma} G^a_{\nu\sigma} \rangle \) in the nuclear medium are calculated from the medium modification of a scalar dilaton field introduced to incorporate trace anomaly of QCD within the chiral SU(3) model used in the present investigation. The effects of isospin asymmetry, density and temperature of the nuclear medium on the in-medium masses of the lowest charmonium states J/ψ and ηc mesons are investigated in the present work. The results of the present investigation are compared with the existing results on the masses of these states. The medium modifications of the masses of these charmonium states (J/ψ and ηc) seem to be appreciable at high densities and should modify the experimental observables arising from the compressed baryonic matter produced in asymmetric heavy ion collision experiments at the future facility of FAIR, GSI.

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I. INTRODUCTION

The study of in-medium hadronic properties is of considerable interest, both experimentally and theoretically in the present day strong interaction physics. The study of the in-medium properties of hadrons has direct relevance in the experiments where hadronic matter is probed at high densities and/or temperatures. The CBM experiment at FAIR, GSI is the one where the dense matter at high densities and moderate temperatures is planned to be produced. The medium modifications of the strange and charm mesons and their effects on the experimental observables are amongst the topics which are intended to be studied extensively in these experiments. Therefore, the topic of study of charm mesons in the medium has gotten considerable interest in the recent past. The medium modifications of the properties of the charm mesons, $D$ and $\bar{D}$ as well as the excited charmonium states can have important consequences on the production of open charm and the suppression of the $J/\psi$ in the heavy-ion collision experiments. The suppression of $J/\psi$ in the heavy-ion collisions may lead to the signature of the quark-gluon-plasma (QGP) \cite{1,2}. Also it is observed that the effect of hadron absorption of $J/\psi$ is not negligible \cite{3,4}. In Ref.\cite{6}, it was reported that the charmonium suppression observed in Pb + Pb collisions of NA50 experiment cannot be simply explained by nucleon absorption, but needs some additional density dependent suppression mechanism. It was suggested in these studies that the co-mover scattering \cite{6-8} can explain the additional suppression of charmonium. An important difference between $J/\psi$ suppression pattern in comovers interaction model and in a deconfining scenario is that, in the former case, the anomalous suppression sets in smoothly from peripheral to central collisions rather than in a sudden way when the deconfining threshold is reached \cite{7}. The $J/\psi$ suppression in nuclear collisions at SPS energies has been studied in covariant transport approach HSD in Ref.\cite{8}. The calculations show that the absorption of $J/\psi$'s by both nucleons and produced mesons can explain reasonably not only the total $J/\psi$ cross-section but also the transverse energy dependence of $J/\psi$ suppression measured in both proton-nucleus and nucleus collisions. In Ref.\cite{9}, the cross section of $J/\psi$ dissociation by gluons is used to calculate the $J/\psi$ suppression in an equilibrating parton gas produced in high energy nuclear collisions. The large average momentum in the hot gluon gas enables gluons to break up the $J/\psi$, while hadron matter at reasonable temperature does not pro-
vide sufficiently hard gluons. The multigluon exchange can lead to an attractive potential between a $c\bar{c}$ meson and a nucleon, such that, for example, the $\eta_c$ could form bound states even with light nuclei \[10, 11\].

The $D$ ($\bar{D}$) mesons are made up of light (u or d) antiquark (quark) and one heavy charm quark (charm antiquark). In the QCD sum rule calculations, the mass modifications of $D(\bar{D})$ mesons in the nuclear medium arise due to interactions of light antiquark (quark) present in the $D(\bar{D})$ mesons with the light quark condensate \[12, 13\]. There is appreciable change in the light quark condensate in the nuclear medium and hence $D(\bar{D})$ meson mass, due to its interaction with the light quark condensate, change appreciably in the hadronic matter. The medium modifications of the $D$ mesons modify the decay widths of the charmonium states, which have been studied in Ref. \[13\]. The charmonium states are made up of a heavy charm quark and a charm antiquark. Within the QCD sum rule calculations, it is suggested that these heavy quarkonium states interact with the nuclear medium through the gluon condensates \[10\] unlike the interaction of the light vector mesons with the nuclear medium which is through the light quark condensates \[14\]. This is because all the heavy quark condensates can be related to the gluon condensates via heavy-quark expansion \[15\].

Also in the nuclear medium there are no valence charm quarks to leading order in density and any interaction with the medium is gluonic. The medium modifications of the gluon condensates are seen to be small and this leads to the mass modifications of $J/\psi$ and $\eta_c$ mesons, which are the lowest charmonium states to be small in the nuclear medium \[10\]. The leading order perturbative calculations \[16\] of the study of the charmonium states also shows that the mass of $J/\psi$ is reduced slightly in the nuclear medium. In Ref. \[17\], the mass modifications of the charmonium states have been studied using QCD second order Stark effect and the linear density approximation for the gluon condensate in the nuclear medium. This shows a small drop for the $J/\psi$ mass at the nuclear matter density, but there is seen to be significant shift in the masses of the excited states of charmonium ($\psi(3686)$ and $\psi(3770)$). Using QCD second order Stark effect, the masses of the charmonium states were also studied Ref. \[18\] in the asymmetric nuclear medium at finite temperatures. These medium modifications were investigated by computing the scalar gluon condensate in the hot nuclear medium from the medium modification of a scalar dilaton field within a chiral
SU(3) model which was introduced to incorporate broken scale invariance of QCD. This investigation showed small drop of the $J/\psi$ mass in the medium, whereas the masses of the excited charmonium states are observed to have appreciable drop at high densities.

In the present investigation, we study the in-medium modifications of the vector meson, $J/\psi$ and the pseudoscalar meson, $\eta_c$, using QCD sum rules [10] and an effective chiral $SU(3)$ model [19]. To apply the QCD sum rules for the study of in-medium modifications of $J/\psi$ and $\eta_c$ mesons, we consider the contributions of the scalar gluon condensates, $\left\langle \frac{\alpha_s}{\pi} G^a_{\mu\nu} G^{a\mu\nu} \right\rangle$ and twist-2 tensorial gluon operator, $\left\langle \frac{\alpha_s}{\pi} G^a_{\mu\sigma} G^{a\nu\sigma} \right\rangle$ up to dimension four [10]. The scalar gluon condensate as well as the twist-2 gluon operator in the nuclear medium are calculated from the medium modification of a scalar dilaton field, $\chi$, introduced within a chiral $SU(3)$ model [19] through a scale symmetry breaking term in the Lagrangian density leading to the QCD trace anomaly. The chiral $SU(3)$ model [19] has been used successfully to study the medium modifications of kaons and antikaons in isospin asymmetric nuclear matter in [20] and in hyperonic matter in [21]. The chiral $SU(3)$ model was generalized to $SU(4)$ to study the mass modifications of $D$-mesons arising from their interactions with the light hadrons in isospin symmetric hot hadronic matter in Ref.[22] and in isospin asymmetric nuclear matter at zero temperature [23] and finite temperatures [18] respectively. The in-medium properties of the vector mesons have also been studied within the model [24, 25]. In the present investigation, we study the in-medium masses of the $J/\psi$ and $\eta_c$ mesons, calculated from the medium modifications of the dilaton field, $\chi$, in the nuclear asymmetric nuclear matter at finite temperatures within the chiral SU(3) model.

The outline of the paper is as follows: In section II, we give a brief introduction of chiral $SU(3)$ model used to study the in-medium masses of charmonium states $J/\psi$ and $\eta_c$, in the present investigation. The medium modifications of these charmonium states, $J/\psi$ and $\eta_c$ mesons arise from the medium modification of the scalar gluon condensate in the nuclear medium, simulated by a scalar dilaton field introduced in the hadronic model to incorporate broken scale invariance of QCD leading to QCD trace anomaly and also due to the medium modification of the expectation value of the twist-2 gluon operator. Section III discusses briefly the QCD sum rule approach used to calculate the masses of the charmonium states $J/\psi$ and $\eta_c$. In section IV, we discuss the results of the present investigation. Section V
summarizes the conclusions of the present work.

II. THE HADRONIC CHIRAL $SU(3) \times SU(3)$ MODEL

We use an effective chiral $SU(3)$ model for the present investigation \[19\]. The model is based on the nonlinear realization of chiral symmetry \[26-28\] and broken scale invariance \[19, 24, 25\]. This model has been used successfully to describe nuclear matter, finite nuclei, hypernuclei and neutron stars. The effective hadronic chiral Lagrangian density contains the following terms

$$\mathcal{L} = \mathcal{L}_{\text{kin}} + \sum_{W=X,Y,V,A,u} \mathcal{L}_{BW} + \mathcal{L}_{\text{vec}} + \mathcal{L}_0 + \mathcal{L}_{SB}$$

In Eq. (1), $\mathcal{L}_{\text{kin}}$ is kinetic energy term, $\mathcal{L}_{BW}$ is the baryon-meson interaction term in which the baryon-spin-0 meson interaction term generates the vacuum baryon masses. $\mathcal{L}_{\text{vec}}$ describes the dynamical mass generation of the vector mesons via couplings to the scalar mesons and contain additionally quartic self-interactions of the vector fields. $\mathcal{L}_0$ contains the meson-meson interaction terms inducing the spontaneous breaking of chiral symmetry as well as a scale invariance breaking logarithmic potential. $\mathcal{L}_{SB}$ describes the explicit chiral symmetry breaking.

To study the hadron properties at finite temperature and densities in the present investigation, we use the mean field approximation, where all the meson fields are treated as classical fields. In this approximation, only the scalar and the vector fields contribute to the baryon-meson interaction, $\mathcal{L}_{BW}$ since for all the other mesons, the expectation values are zero. The interactions of the scalar mesons and vector mesons with the baryons are given as

$$\mathcal{L}_{B_{\text{scal}}} + \mathcal{L}_{B_{\text{vec}}} = -\sum_i \bar{\psi}_i \left[m_i^* + g_{\omega i} \gamma_0 \omega + g_{\rho i} \gamma_0 \rho + g_{\phi i} \gamma_0 \phi \right] \psi_i.$$  

(2)

The interaction of the vector mesons, of the scalar fields and the interaction corresponding to the explicitly symmetry breaking in the mean field approximation are given as

$$\mathcal{L}_{\text{vec}} = \frac{1}{2} \left(m_{\omega}^2 \omega^2 + m_{\rho}^2 \rho^2 + m_{\phi}^2 \phi^2 \right) \frac{\chi^2}{\lambda_0^2} + g_4 (\omega^4 + 6 \omega^2 \rho^2 + \rho^4 + 2 \phi^4),$$

(3)

$$\mathcal{L}_0 = -\frac{1}{2} k_0 \chi^2 \left(\sigma^2 + \zeta^2 + \delta^2 \right) + k_1 \left(\sigma^2 + \zeta^2 + \delta^2 \right)^2.$$
\[ + k_2 \left( \frac{\sigma^4}{2} + \frac{\delta^4}{2} + 3\sigma^2\delta^2 + \zeta^4 \right) + k_3 \chi \left( \sigma^2 - \delta^2 \right) \zeta \]
\[- k_4 \chi^4 - \frac{1}{4} \chi^4 \ln \frac{\chi^4}{\chi_0^4} + \frac{d}{3} \chi^4 \ln \left( \frac{\left( \frac{\sigma^2 - \delta^2}{\sigma_0^2\zeta_0} \right)^2 \frac{\chi}{\chi_0}}{\chi_0^3} \right), \tag{4}\]

and
\[ \mathcal{L}_{SB} = -\left( \frac{\chi}{\chi_0} \right)^2 \left[ m_{\pi}^2 f_{\pi}^2 \sigma + (\sqrt{2} m_{k}^2 f_{k} - \frac{1}{\sqrt{2}} m_{\pi}^2 f_{\pi}) \zeta \right]. \tag{5}\]

The effective mass of the baryon of species \( i \) is given as
\[ m_i^* = -(g_{\sigma i} \sigma + g_{\zeta i} \zeta + g_{\delta i} \delta) \tag{6}\]

The baryon-scalar meson interactions, as can be seen from equation (6), generate the baryon masses through the coupling of baryons to the non-strange \( \sigma \), the strange \( \zeta \) scalar mesons and also to scalar-isovector meson \( \delta \). In analogy to the baryon-scalar meson couplings, there exist two independent baryon-vector meson interaction terms corresponding to the F-type (antisymmetric) and D-type (symmetric) couplings. Here antisymmetric coupling is used because the universality principle \[29\] and vector meson dominance model suggest small symmetric couplings. Additionally, we choose the parameters \[19, 20\] so as to decouple the strange vector field \( \varphi_\mu \sim \bar{s}\gamma_\mu s \) from the nucleon, corresponding to an ideal mixing between \( \omega \) and \( \phi \) mesons. A small deviation of the mixing angle from ideal mixing \[30–32\] has not been taken into account in the present investigation.

The concept of broken scale invariance leading to the trace anomaly in (massless) QCD, \( \theta^\mu_\mu = \frac{3G_{\mu \nu} G^{\mu \nu}}{2g} \), where \( G_{\mu \nu} \) is the gluon field strength tensor of QCD, is simulated in the effective Lagrangian at tree level \[33\] through the introduction of the scale breaking terms
\[ \mathcal{L}_{\text{scalebreaking}} = -\frac{1}{4} \chi^4 \ln \left( \frac{\chi^4}{\chi_0^4} \right) + \frac{d}{3} \chi^4 \ln \left( \frac{I_3}{\det(\langle X \rangle_0)} \left( \frac{\chi}{\chi_0} \right)^3 \right), \tag{7}\]

where \( I_3 = \det\langle X \rangle_0 \), with \( X \) as the multiplet for the scalar mesons. These scale breaking terms, in the mean field approximation, are given by the last two terms of the Lagrangian density, \( \mathcal{L}_0 \) given by equation (4) \[34\]. Within the chiral SU(3) model used in the present investigation, the scalar gluon condensate \( \langle \frac{1}{2}G_{\mu \nu} G^{\mu \nu} \rangle \), as well as the twist-2 gluon operator, \( \langle \frac{1}{2}G_{\mu \nu} G_{\sigma \tau}^{\mu \nu} \rangle \), are simulated by the scalar dilaton field, \( \chi \). These are obtained from the energy momentum tensor
\[ T_{\mu \nu} = \left( \partial_\mu \chi \right) \left( \frac{\partial \mathcal{L}_\chi}{\partial (\partial_\nu \chi)} \right) - g_{\mu \nu} \mathcal{L}_\chi, \tag{8}\]
derived from the Lagrangian density for the dilaton field, given as

\[
\mathcal{L}_\chi = \frac{1}{2} (\partial_\mu \chi) (\partial^\mu \chi) - k_4 \chi^4 - \frac{1}{4} \chi^4 \ln \left( \frac{\chi^4}{\chi_0^4} \right) - \frac{d}{3} \chi^4 \ln \left( \left( \frac{\sigma^2 - \delta^2}{\sigma_0^2 \zeta_0} \right) \left( \frac{\chi}{\chi_0} \right)^3 \right),
\]

(9)

In massless QCD, the energy momentum tensor can be written as \[3,5,36\]

\[
T_{\mu\nu} = -ST(G^a_{\mu\sigma} G^a_{\nu}^\sigma) + \frac{g_{\mu\nu} \beta_{\text{QCD}}}{2g} G^a_{\sigma\kappa} G^{a\sigma\kappa}
\]

(10)

where the first term is the symmetric traceless part and second term is the trace part of the energy momentum tensor. Writing

\[
\langle \alpha G^a_{\mu\sigma} G^a_{\nu}^\sigma \rangle = \left( u_\mu u_\nu - \frac{g_{\mu\nu}}{4} \right) G_2,
\]

(11)

where \( u_\mu \) is the 4-velocity of the nuclear medium, taken as \( u_\mu = (1,0,0,0) \), we obtain the energy momentum tensor in QCD as

\[
T_{\mu\nu} = -\left( \frac{\pi}{\alpha_s} \right) \left( u_\mu u_\nu - \frac{g_{\mu\nu}}{4} \right) G_2 + \frac{g_{\mu\nu} \beta_{\text{QCD}}}{2g} G^a_{\sigma\kappa} G^{a\sigma\kappa}
\]

(12)

Equating the energy-momentum tensors given by equations (8) and (12) and multiplying by \((u_\mu u_\nu - \frac{g_{\mu\nu}}{4})\), we obtain the expression for \( G_2 \) as

\[
G_2 = -\frac{\alpha_s}{\pi} \left( \partial_\alpha \chi \right) \left( \frac{\partial \mathcal{L}_\chi}{\partial (\partial_\alpha \chi)} \right) + \frac{4}{3} \left( \partial_i \chi \right) \left( \partial_i \chi \right).
\]

(13)

We might note here that by multiplying the energy momentum tensor of QCD given by equation (12) by \((u_\mu u_\nu - \frac{g_{\mu\nu}}{4})\), we project out the traceless part given by the first term of the energy momentum tensor, described by the function, \( G_2 \). This is because \( g_{\mu\nu}(u_\mu u_\nu - \frac{g_{\mu\nu}}{4}) = 0 \), and hence, there is no contribution from the trace part of the energy momentum tensor in QCD, when we multiply the same by \((u_\mu u_\nu - \frac{g_{\mu\nu}}{4})\). Similarly, by multiplying the energy momentum tensor given by equation (12) by \( g_{\mu\nu} \), the first part gives zero and only the second term contributes to the trace of the energy momentum tensor. The effect of the logarithmic terms in the chiral SU(3) model, given by equation (9), is to break the scale invariance. Multiplying equation (8) by \( g_{\mu\nu} \), we obtain the trace of the energy momentum tensor within the chiral SU(3) model as

\[
T_{\mu}^\mu = \left( \partial_\mu \chi \right) \left( \frac{\partial \mathcal{L}_\chi}{\partial (\partial_\mu \chi)} \right) - 4\mathcal{L}_\chi.
\]

(14)
Using the Euler-Lagrange’s equation for the $\chi$ field, the trace of the energy momentum tensor in the chiral SU(3) model can be expressed as \[ \chi \frac{\partial L}{\partial \chi} - 4L = -(1 - d)\chi^4. \] (15)

Multiplying equation (12) by $g^{\mu\nu}$, we obtain the trace of the energy momentum tensor in QCD as

$$T_\mu^\mu = \langle \beta_{QCD}^{2} g_{a\sigma\kappa} G_{a\sigma\kappa} \rangle$$ (16)

Using the Euler-Lagrange’s equation for $\chi$ and dropping a total divergence term in equation (13), the expression for $G_2$ can be written as

$$G_2 = \frac{\alpha_s}{\pi} \left[ \chi \frac{\partial L}{\partial \chi} - \frac{4}{3} (\partial_i \chi)(\partial_i \chi) \right] = \frac{\alpha_s}{\pi} \left[ - (1 - d + 4k_4)\chi^4 - \chi^4 \ln \left( \frac{\chi^4}{\chi_0^4} \right) + \frac{4}{3} d\chi^4 \ln \left( \left( \frac{\sigma^2 - \delta^2}{\sigma_0^2 - \delta_0^2} \right) \left( \frac{\chi}{\chi_0} \right)^3 \right) - \frac{4}{3} (\partial_i \chi)(\partial_i \chi) \right],$$ (17)

The twist-2 gluon operator has only contribution in the nuclear medium and is zero in vacuum \[10\]. Hence $(G_2)_{\text{vac}} = 0$, which implies that

$$- (1 - d + 4k_4)\chi_0^4 - \frac{4}{3} \langle (\partial_i \chi)(\partial_i \chi) \rangle_{\text{vac}} = 0$$ (18)

Assuming the glueball field, $\chi$ to be non-relativistic, and hence assuming that $\langle (\partial_i \chi)(\partial_i \chi) \rangle_{\text{medium}} \simeq \langle (\partial_i \chi)(\partial_i \chi) \rangle_{\text{vac}}$ and using equation (18), the expression for $G_2$ is obtained from the equation (17) as

$$G_2 = \frac{\alpha_s}{\pi} \left[ - (1 - d + 4k_4)\chi^4 - \chi^4 \ln \left( \frac{\chi^4}{\chi_0^4} \right) + \frac{4}{3} d\chi^4 \ln \left( \left( \frac{\sigma^2 - \delta^2}{\sigma_0^2 - \delta_0^2} \right) \left( \frac{\chi}{\chi_0} \right)^3 \right) \right].$$ (19)

The scalar gluon condensate and the twist-2 gluon operator, described in terms of the function, $G_2$, given by equations (15) and (19) are thus related to the $\chi$ field, which is solved from the coupled equations of motion of the scalar fields within the chiral SU(3) model. These medium dependent gluon condensates are then related.
The coupled equations of motion for the non-strange scalar field $\sigma$, the strange scalar field $\zeta$, the scalar-isovector field $\delta$ and the dilaton field $\chi$, are derived from the Lagrangian density, and are given as

\begin{align*}
  k_0 \chi^2 \sigma - 4k_1 \left( \sigma^2 + \zeta^2 + \delta^2 \right) \sigma - 2k_2 \left( \sigma^3 + 3\sigma\delta^2 \right) - 2k_3 \chi \sigma \zeta \\
  - \frac{d}{3} \chi^4 \left( \frac{2\sigma}{\sigma^2 - \delta^2} \right) + \left( \frac{\chi}{\chi_0} \right)^2 m_{\pi}^2 f_{\pi} - \sum g_{\sigma_i} \rho_i^s = 0 \quad (20)
\end{align*}

\begin{align*}
  k_0 \chi^2 \zeta - 4k_1 \left( \sigma^2 + \zeta^2 + \delta^2 \right) \zeta - 4k_2 \zeta^3 - k_3 \chi \left( \sigma^2 - \delta^2 \right) \\
  - \frac{d}{3} \chi^4 + \left( \frac{\chi}{\chi_0} \right)^2 \left[ \sqrt{2} m_k^2 f_k - \frac{1}{\sqrt{2}} m_{\pi}^2 f_{\pi} \right] - \sum g_{\zeta_i} \rho_i^s = 0 \quad (21)
\end{align*}

\begin{align*}
  k_0 \chi^2 \delta - 4k_1 \left( \sigma^2 + \zeta^2 + \delta^2 \right) \delta - 2k_2 \left( \delta^3 + 3\sigma^2 \delta \right) + k_3 \chi \delta \zeta \\
  + \frac{2}{3} \frac{d}{\chi^4} \left( \frac{\delta}{\sigma^2 - \delta^2} \right) - \sum g_{\delta_i} \rho_i^s = 0 \quad (22)
\end{align*}

\begin{align*}
  k_0 \chi \left( \sigma^2 + \zeta^2 + \delta^2 \right) - k_3 \left( \sigma^2 - \delta^2 \right) \zeta + \chi^3 \left[ 1 + \ln \left( \frac{\chi^4}{\chi_0^4} \right) \right] + (4k_4 - d) \chi^3 \\
  - \frac{4}{3} d \chi^3 \ln \left( \left( \frac{\sigma^2 - \delta^2}{\sigma_0^2 \zeta_0} \right) \left( \frac{\chi}{\chi_0} \right)^3 \right) + \frac{2\chi}{\chi_0} \left[ m_{\pi}^2 f_{\pi} \sigma + \left( \sqrt{2} m_k^2 f_k - \frac{1}{\sqrt{2}} m_{\pi}^2 f_{\pi} \right) \zeta \right] = 0 \quad (23)
\end{align*}

In the above, $\rho_i^s$ are the scalar densities for the baryons, given as

\begin{equation}
  \rho_i^s = \gamma_i \int \frac{d^3k}{(2\pi)^3} \frac{m_i^*}{E_i^*(k)} \left( \frac{1}{e^{(E_i^*(k) - \mu_i^s)/T} + 1} + \frac{1}{e^{(E_i^*(k) + \mu_i^s)/T} + 1} \right) \quad (24)
\end{equation}

where, $E_i^*(k) = (k^2 + m_i^{*2})^{1/2}$, and, $\mu_i^s = \mu_i - g_{\omega_i \omega} - g_{\rho_i \rho} - g_{\phi_i \phi}$, are the single particle energy and the effective chemical potential for the baryon of species $i$, and, $\gamma_i = 2$ is the spin degeneracy factor [20].

The above coupled equations of motion are solved to obtain the density and temperature dependent values of the scalar fields ($\sigma$, $\zeta$ and $\delta$) and the dilaton field, $\chi$, in the isospin asymmetric hot nuclear medium. As has been already mentioned, the value of the $\chi$ is related to the scalar gluon condensate as well as the twist-2 gluon operator in the hot hadronic medium, and is used to compute the in-medium masses of charmonium states, in the present investigation. The isospin asymmetry in the medium is introduced through the scalar-isovector field $\delta$ and therefore the dilaton field obtained after solving the above equations
is also dependent on the isospin asymmetry parameter, \( \eta \) defined as
\[
\eta = \frac{\rho_n - \rho_p}{2\rho_B},
\]
where \( \rho_n \) and \( \rho_p \) are the number densities of the neutron and the proton and \( \rho_B \) is the baryon density. In the present investigation, we study the effect of isospin asymmetry of the medium on the masses of the charmonium states \( J/\psi \) and \( \eta_c \).

The comparison of the trace of the energy momentum tensor arising from the trace anomaly of QCD with that of the present chiral model given by equations (15) and (16), gives the relation of the dilaton field to the scalar gluon condensate. We have, in the limit of massless quarks [37],
\[
T^\mu_\mu = \left( \frac{\beta_{QCD}}{2g} G^a_{\mu\nu} G^{\mu\nu a} \right) \equiv -(1 - d) \chi^4 \quad (25)
\]
In the case of finite quark masses, equation (25) gets modified to
\[
T^\mu_\mu = \sum_i m_i \bar{q}_i q_i + \left( \frac{\beta_{QCD}}{2g} G^a_{\mu\nu} G^{\mu\nu a} \right) \equiv -(1 - d) \chi^4,
\]
where the first term of the energy-momentum tensor, within the chiral SU(3) model is the negative of the explicit chiral symmetry breaking term, \( \mathcal{L}_{SB} \) given by equation (5).

The parameter \( d \) in equation (26) originates from the second logarithmic term of equation (7). To get an insight into the value of the parameter \( d \), we recall that the QCD \( \beta \) function at one loop level, for \( N_c \) colors and \( N_f \) flavors is given by
\[
\beta_{QCD} (g) = -\frac{11N_c g^3}{48\pi^2} \left( 1 - \frac{2N_f}{11N_c} \right) + O(g^5) \quad (27)
\]
In the above equation, the first term in the parentheses arises from the (antiscreening) self-interaction of the gluons and the second term, proportional to \( N_f \), arises from the (screening) contribution of quark pairs. For massless quarks, the equations (25) and (27) suggest the value of \( d \) to be 6/33 for three flavors and three colors, and for the case of three colors and two flavors, the value of \( d \) turns out to be 4/33, to be consistent with the one loop estimate of QCD \( \beta \) function. These values give the order of magnitude about which the parameter \( d \) can be taken [34], since one cannot rely on the one-loop estimate for \( \beta_{QCD}(g) \).

In the present investigation of the in-medium properties of the charmonium states due to the medium modification of the dilaton field within chiral \( SU(3) \) model, we use the value of \( d=0.064 \) [23]. This parameter, along with the other parameters corresponding to the scalar Lagrangian density, \( \mathcal{L}_0 \) given by (4), are fitted so as to ensure extrema in the vacuum for
the $\sigma$, $\zeta$ and $\chi$ field equations, to reproduce the vacuum masses of the $\eta$ and $\eta'$ mesons, the mass of the $\sigma$ meson around 500 MeV, and, pressure, $p(\rho_0)=0$, with $\rho_0$ as the nuclear matter saturation density \[19, 23\].

The trace of the energy-momentum tensor in QCD, using the one loop beta function given by equation (27), for $N_c=3$ and $N_f=3$, and accounting for the finite quark masses \[37\] is given as,

$$ T_\mu^\mu = -\frac{9}{8} \frac{\alpha_s}{\pi} G^{a}_{\mu\nu}G^{a\mu\nu} + \left( \frac{\chi}{\chi_0} \right)^2 \left( m^2_{\pi} f_{\pi} \sigma + (\sqrt{2} m^2_k f_k - \frac{1}{\sqrt{2}} m^2_{\pi} f_{\pi}) \zeta \right). $$

(28)

Using equations (25) and (28), we can write

$$ \langle \frac{\alpha_s}{\pi} G^{a}_{\mu\nu}G^{a\mu\nu} \rangle = \frac{8}{9} \left[ (1 - d) c^4 + \left( \frac{\chi}{\chi_0} \right)^2 \left( m^2_{\pi} f_{\pi} \sigma + (\sqrt{2} m^2_k f_k - \frac{1}{\sqrt{2}} m^2_{\pi} f_{\pi}) \zeta \right) \right]. $$

(29)

We thus see from the equation (29) that the scalar gluon condensate $\langle \frac{\alpha_s}{\pi} G^{a}_{\mu\nu}G^{a\mu\nu} \rangle$ is related to the dilaton field $\chi$. For massless quarks, since the second term in (29) arising from explicit symmetry breaking is absent, the scalar gluon condensate becomes proportional to the fourth power of the dilaton field, $\chi$, in the chiral SU(3) model. As mentioned earlier, the in-medium masses of charmonium states are modified due to the scalar gluon condensate and the twist-2 gluon operators, which are calculated from the modification of the $\chi$ field.

### III. QCD SUM RULE APPROACH AND IN-MEDIUM MASSES OF $J/\psi$ AND $\eta_c$

In the present section, we shall use the medium modifications of the gluon condensate, calculated from the dilaton field in the chiral effective model, to compute the masses of the charmonium states $J/\psi$ and $\eta_c$ in isospin asymmetric hot nuclear matter. Using QCD sum rules \[10\] the in-medium masses of the lowest charmonium states can be written as

$$ m^2 \simeq \frac{M_{n-1}^J(\xi)}{M_n^J(\xi)} - 4m_c^2 \xi $$

(30)

where $M_n^J$ is the $n$th moment of the meson and $\xi$ is the normalization scale. Using operator product expansion, the moment $M_n^J$ can be written as \[10\]

$$ M_n^J(\xi) = A_n^J(\xi) \left[ 1 + a_n^J(\xi) \alpha_s + b_n^J(\xi) \phi_b + c_n^J(\xi) \phi_c \right], $$

(31)
where \( A_n^J(\xi) \), \( a_n^J(\xi) \), \( b_n^J(\xi) \) and \( c_n^J(\xi) \) are the Wilson coefficients. The common factor \( A_n^J \) results from the bare loop diagram. The coefficients \( a_n^J \) take into account perturbative radiative corrections, while the coefficients \( b_n^J \) are associated with the scalar gluon condensate term

\[
\phi_b = \frac{4\pi^2}{9} \frac{\langle \frac{2\pi}{\xi} G_{\mu\nu}^{a} G^{a\mu\nu} \rangle}{(4m_c^2)^2}
\]  

(32)

As already mentioned, the contribution of the scalar gluon condensate is taken through the dilaton field within the chiral \( SU(3) \) model used in the present investigation. Using equation (29), the above equation can be rewritten in terms of the dilaton field \( \chi \), as

\[
\phi_b = \frac{32\pi^2}{81(4m_c^2)^2} \left[ (1 - d) \chi^4 + \left( \frac{\chi}{\chi_0} \right)^2 \left( m_{\pi}^2 f_{\pi} \sigma + (\sqrt{2}m_k f_k - \frac{1}{\sqrt{2}}m_{\pi} f_{\pi}) \zeta \right) \right].
\]  

(33)

The coefficients \( A_n^J, a_n^J, \) and \( b_n^J \) are listed in Ref.\[38\]. The coefficients \( c_n^J \) are associated with the value of \( \phi_c \), which gives the contribution from twist-2 gluon operator and is given as

\[
\phi_c = \frac{4\pi^2}{3(4m_c^2)^2} G_2,
\]  

(34)

where \( G_2 \) is given by equation (19). We shall calculate the in-medium masses of the charmonium states \( J/\psi \) and \( \eta_c \) in the hot asymmetric nuclear matter and shall compare the results with the contribution from the twist-2 gluon operator as calculated in the linear density approximation. In the low density approximation, the term \( \phi_c \) is given as [10]

\[
\phi_c = -\frac{2\pi^2}{3} \frac{\langle \frac{2\pi}{\xi} A_G \rangle}{(4m_c^2)^2} m_N \rho_B.
\]  

(35)

In the above equation, \( A_G \) represents twice the momentum fraction carried by gluons in the nucleon and is set equal to 0.9 [10]. \( m_N \) and \( \rho_B \) are the nucleon mass and baryon density respectively. The Wilson coefficients, \( c_n^J \) in the vector channel (for \( J/\psi \)) and the pseudoscalar channel (for \( \eta_c \)) can be found in Ref. [38]. The parameters \( m_c \) and \( \alpha_s \) are the running charm quark mass and running coupling constant and are \( \xi \) dependent [38]. These are given by

\[
\frac{m_c(\xi)}{m_c} = 1 - \frac{\alpha_s}{\pi} \left\{ \frac{2 + \xi}{1 + \xi} \ln(2 + \xi) - 2\ln 2 \right\}
\]  

(36)

where, \( m_c \equiv m_c(p^2 = -m_c^2) = 1.26 \text{ GeV} \) [39], and

\[
\alpha_s \left( Q_0^2 + 4m_c^2 \right) = \alpha_s \left( 4m_c^2 \right) / \left( 1 + \frac{25}{12\pi} \alpha_s \left( 4m_c^2 \right) \ln \frac{Q_0^2 + 4m_c^2}{4m_c^2} \right)
\]  

(37)
with, $\alpha_s(4m_c^2) \simeq 0.3$ and $Q_0^2 = 4m_c^2\xi$.

In the next section, we present and discuss the results of our present work of the investigation of the in-medium masses of $J/\Psi$ and $\eta_c$ in isospin asymmetric hot nuclear matter.

IV. RESULTS AND DISCUSSIONS

In this section, we first investigate the effects of density, isospin-asymmetry and temperature of the nuclear medium on the dilaton field $\chi$ in the chiral SU(3) model, from which we obtain the expectation value of the scalar gluon condensate in the medium. Using the QCD sum rule approach, the in-medium masses of charmonium states $J/\psi$ and $\eta_c$ are calculated from the medium dependence of the gluon condensates. The medium dependent dilaton field, $\chi$ is obtained by solving the equations of motion of the scalar fields, $\sigma$, $\zeta$, $\delta$ and $\chi$ given by equations (20) to (23). The values of the parameters used in the present investigation, are: $k_0 = 2.54, k_1 = 1.35, k_2 = -4.78, k_3 = -2.77, k_4 = -0.22$ and $d = 0.064$, which are the parameters occurring in the scalar meson interactions defined in equation (4).

The vacuum values of the scalar isoscalar fields, $\sigma$ and $\zeta$ and the dilaton field $\chi$ are $-93.3$ MeV, $-106.6$ MeV and $409.77$ MeV respectively. The values, $g_{\sigma N} = 10.6$ and $g_{\zeta N} = -0.47$ are determined by fitting to the vacuum baryon masses. The other parameters fitted to the asymmetric nuclear matter saturation properties in the mean-field approximation are: $g_{\omega N} = 13.3, g_{\rho p} = 5.5, g_4 = 79.7, g_{\delta p} = 2.5, m_\zeta = 1024.5$ MeV, $m_\sigma = 466.5$ MeV and $m_\delta = 899.5$ MeV. The nuclear matter saturation density used in the present investigation is 0.15 fm$^{-3}$.

In figure 1, we show the variation of dilaton field $\chi$, with temperature, for both zero and finite baryon densities, and for selected values of the isospin asymmetry parameter, $\eta = 0, 0.1, 0.3$ and 0.5. At zero baryon density, it is observed that the value of the dilaton field remains almost a constant up to a temperature of about 130 MeV above which it is seen to drop with increase in temperature. However, the drop in the dilaton field is seen to be very small up to a temperature of around 175 MeV above which the drop is seen to be larger. The value of the dilaton field is seen to change from 409.8 MeV at $T=0$ to about 409.7 MeV, 409.3 MeV and 405.76 MeV at $T=150$ MeV, 175 MeV and 200 MeV respectively. The thermal distribution functions have an effect of increasing the scalar densities at zero baryon density, i.e., for $\mu_i^+=0$, as can be seen from the expression of the scalar densities, given by
FIG. 1: (Color online) The dilaton field \( \chi \) plotted as a function of the temperature, at given baryon densities, for different values of the isospin asymmetry parameter, \( \eta \).
This effect seems to be negligible up to a temperature of about 130 MeV. This leads to a decrease in the magnitudes of scalar fields, $\sigma$ and $\zeta$. This behaviour of the scalar fields is reflected in the value of $\chi$, which is solved from the coupled equations of motion of the scalar fields, given by equations (20), (21), (22) and (23), as a drop as we increase the temperature above a temperature of about 130 MeV. The scalar densities attaining nonzero values at high temperatures, even at zero baryon density, indicates the presence of baryon-antibaryon pairs in the thermal bath and has already been observed in the literature [25, 40]. This leads to the baryon masses to be different from their vacuum masses above this temperature, arising from modifications of the scalar fields $\sigma$ and $\zeta$.

For finite density situations, the behaviour of the $\chi$ field with temperature is seen to be very different from the zero density case, as can be seen from the subplots (b), (c) and (d) of figure I where the $\chi$ field is plotted as a function of the temperature for densities $\rho_0$, $2\rho_0$ and $4\rho_0$ respectively. At finite densities, one observes first a rise and then a decrease of the dilaton field with temperature. This is related to the fact that at finite densities, the magnitude of the $\sigma$ field (as well as of the $\zeta$ field) first show an increase and then a drop with further increase of the temperature [18] which is reflected in the behaviour of $\chi$ field, since it is solved from the coupled equations of the scalar fields. The reason for the different behaviour of the scalar fields ($\sigma$ and $\zeta$) at zero and finite densities can be understood in the following manner [25]. As has already been mentioned, the thermal distribution functions in (24) have an effect of increasing the scalar densities at zero baryon density, i.e., for $\mu_i^* = 0$. However, at finite densities, i.e., for nonzero values of the effective chemical potential, $\mu_i^*$, for increasing temperature, there are contributions also from higher momenta, thereby, increasing the denominator of the integrand on the right hand side of the equation (24). This leads to a decrease in the scalar density. The competing effects of the thermal distribution functions and the contributions of the higher momenta states give rise to the observed effect of the scalar density and hence of the $\sigma$ and $\zeta$ fields with temperature at finite baryon densities [25]. This kind of behaviour of the scalar $\sigma$ field on temperature at finite densities has also been observed in the Walecka model by Li and Ko [41], which was reflected as an increase in the mass of the nucleon with temperature at finite densities in the mean field calculations. The effects of the behaviour of the scalar fields on the value...
of the $\chi$ field, obtained from solving the coupled equations (20) to (23) for the scalar fields, are shown in figure 1.

In figure 1, it is observed that for a given value of isospin asymmetry parameter $\eta$, the dilaton field $\chi$ decreases with increase in the density of the nuclear medium. The drop in the value of $\chi$ with density is seen to be much larger as compared to its modification with temperature at a given density. For isospin symmetric nuclear medium ($\eta = 0$) at temperature $T = 0$, the reduction in the dilaton field $\chi$ from its vacuum value ($\chi_0 = 409.8$ MeV), is seen to be about 3 MeV at $\rho_B = \rho_0$ and about 13 MeV, for $\rho_B = 4\rho_0$. As we move from isospin symmetric medium, with $\eta = 0$, to isospin asymmetric medium, at temperature $T = 0$, and, for a given value of density, there is seen to be an increase in the value of the dilaton field $\chi$. However, the effect of isospin asymmetry of the medium on the value of the dilaton field is observed to be negligible upto about a density of nuclear matter saturation density, and is appreciable only at higher values of densities as can be seen in figure 1. At nuclear matter saturation density, $\rho_0$, the value of dilaton field $\chi$ changes from 406.4 MeV in symmetric nuclear medium ($\eta = 0$) to 406.5 MeV in the isospin asymmetric nuclear medium ($\eta = 0.5$). At a density of about $4\rho_0$, the values of the dilaton field are modified to 396.7 MeV and 398 MeV at $\eta = 0$ and 0.5, respectively. Thus the increase in the dilaton field $\chi$ with isospin asymmetry of the medium is seen to be more at zero temperature as we move to higher densities.

At a finite density, $\rho_B$, and for given isospin asymmetry parameter $\eta$, the dilaton field $\chi$ is seen to first increase with temperature and above a particular value of the temperature, it is seen to decrease with further increase in temperature. At the nuclear matter saturation density $\rho_B = \rho_0$ and in isospin symmetric nuclear medium ($\eta = 0$) the value of the dilaton field $\chi$ increases upto a temperature of about $T = 145$ MeV, above which there is a drop in the dilaton field. For $\rho_B = \rho_0$ in the asymmetric nuclear matter with $\eta = 0.5$, there is seen to be a rise in the value of $\chi$ upto a temperature of about 120 MeV, above which it starts decreasing. As has already been mentioned, at zero temperature and for a given value of density, the dilaton field $\chi$ is found to increase with increase in the isospin asymmetry of the nuclear medium. But from figure 1, it is observed that at high temperatures and for a given density, the value of the dilaton field $\chi$ becomes higher in symmetric nuclear medium as
TABLE I: The mass shifts of $J/\psi$ and $\eta_c$ are shown at densities of $\rho^0$, $2\rho^0$ and $4\rho^0$ at values of the isospin asymmetric parameter, $\eta=0$ and 0.5 for $\xi=0.874$. This value of $\xi$ reproduces the vacuum mass of $J/\psi$ as 3097 MeV.

Compared to isospin asymmetric nuclear medium e.g. at nuclear saturation density $\rho_B=\rho^0$ and temperature $T=150$ MeV the values of dilaton field $\chi$ are 407.3 MeV and 407 MeV at $\eta=0$ and 0.5 respectively. At density $\rho_B=4\rho^0$, $T=150$ MeV the values of dilaton field $\chi$ are seen to be 399.1 MeV and 398.7 MeV for $\eta=0$ and 0.5 respectively. This observed behaviour of the $\chi$ is related to the fact that at finite densities and for isospin asymmetric matter, there are contributions from the scalar isovector $\delta$ field, whose magnitude is seen to decrease for higher temperatures for given densities, whereas $\delta$ field has zero contribution for isospin symmetric matter.

In figure 2 we show the variation of the trace and non-trace parts of the energy momentum tensor given by equation (12), with temperature, for different values of the baryon density and isospin asymmetry parameter, $\eta$. The trace part, $G^0 = \langle \frac{1}{2} g_{\mu\nu} G^\mu_\nu \rangle$, is given by equation (29) and $G_2$, which is related to the nontrace part of the energy momentum tensor is given by the equation (19), both obtained from the SU(3) model used in the present investigation. The value of the trace part, $G^0$ is plotted as a function with temperature, for a densities, $\rho_B=0$, $\rho^0$ and $4\rho^0$ in the subplots (a), (c) and (e) in figure 2 plotted in figure (2). For zero density, there is seen to be an increase of $G^0$ with temperature up to a temperature of about 175 MeV, and then a drop with further increase in the temperature. The values of $G^0$ are obtained as $1.9361 \times 10^{-2}$ GeV$^4$, $1.9362 \times 10^{-2}$ GeV$^4$, $1.9381 \times 10^{-2}$ GeV$^4$ and $1.88 \times 10^{-2}$ GeV$^4$ at values of the temperature, $T=0$, 100, 150 and 200 MeV respectively. We might note here that the calculations of the scalar gluon condensate, $G_0$, given by equation (29), have been performed by accounting for the finite quark masses in the
trace anomaly. In the absence of finite quark masses, the scalar gluon condensate becomes proportional to the fourth power of the dilaton field, as can be seen from equation (29). The dilaton field is seen to decrease with temperature at zero baryon density, as can be seen from figure [1]. The values of $G_0$, for the limit of zero quark masses, also decrease accordingly with temperature for $\rho_B=0$, with the values of $G_0$ given as $2.3455 \times 10^{-2}$ GeV$^4$, $2.34547 \times 10^{-2}$ GeV$^4$, $2.3437 \times 10^{-2}$ GeV$^4$ and $2.323 \times 10^{-2}$ GeV$^4$ at values of temperature, $T$ as 0, 100, 150 and 200 MeV respectively. A similar behaviour of $G_0$ with temperature at zero baryon density has also been observed earlier in Ref. [36]. In the present investigation, the finite quark mass term leads to a decrease in the value of $G_0$, as can be seen from equation (29).

At finite densities, the dilaton field $\chi$ is seen to increase up to a temperature above which it starts decreasing, as can be seen from figure [1]. Accounting for the finite quark masses, we get a positive contribution to $G_0$ from the temperature effects from the second term in equation (29), leading to an increase in the scalar condensate up to a temperature above which there is seen to be a decrease with further rise in temperature. For baryon densities of $\rho_B = \rho_0$ and $4\rho_0$, the values up to which $G_0$ increases with temperature are about 145 MeV and 175 MeV respectively. In isospin symmetric nuclear matter, for $\rho_B = \rho_0$, the values of $G_0$ are observed to be $1.90646 \times 10^{-2}$ GeV$^4$, $1.91755 \times 10^{-2}$ GeV$^4$, $1.92 \times 10^{-2}$ GeV$^4$ and $1.8554 \times 10^{-2}$ GeV$^4$ for temperatures of 0, 100, 150 and 200 MeV respectively. For the same values of the temperature, in the absence of finite quark masses, the values of $G_0$ are observed to be $2.269 \times 10^{-2}$ GeV$^4$, $2.2857 \times 10^{-2}$ GeV$^4$, $2.29 \times 10^{-2}$ GeV$^4$ and $2.2 \times 10^{-2}$ GeV$^4$ for $\rho_B = \rho_0$ and $\eta=0$. In isospin symmetric nuclear matter for $\rho_B = 4\rho_0$, the values of $G_0$ are given as $1.7367 \times 10^{-2}$ GeV$^4$ ($2.06 \times 10^{-2}$ GeV$^4$), $1.7656 \times 10^{-2}$ GeV$^4$ ($2.094 \times 10^{-2}$ GeV$^4$), $1.78 \times 10^{-2}$ GeV$^4$ ($2.112 \times 10^{-2}$ GeV$^4$) and $1.7626 \times 10^{-2}$ GeV$^4$ ($2.09 \times 10^{-2}$ GeV$^4$) for values of temperature, $T = 0$, 100, 150 and 200 MeV respectively, for the cases of the finite (zero) quark masses in the trace anomaly.

The non-trace part of the energy momentum tensor, $G_2$, is plotted as a function of temperature in subplots (b), (d) and (f) of figure [2] for densities, $\rho_B=0$, $\rho_0$ and $4\rho_0$. It may be noted that value of $G_2$ is zero in vacuum and this has a nonzero contribution only for finite density and/or temperature. The magnitude of the quantity, $G_2$ is observed to increase with increase in the temperature of the nuclear medium for zero density, with the values of $G_2$ at
\( \rho_B = 0 \), given as \(-7.106 \times 10^{-12} \text{ GeV}^4, -2.386 \times 10^{-7} \text{ GeV}^4 \) and \(-1.106 \times 10^{-5} \text{ GeV}^4 \) and \(-3.528 \times 10^{-5} \text{ GeV}^4 \) at values of temperature, \( T \) as 50, 100, 150 and 200 MeV respectively. The observed behaviour of the magnitude of \( G_2 \) increasing as a function of temperature at zero baryon density has also been observed in Ref. [36]. At nuclear saturation density, \( \rho_B = \rho_0 \) there is seen to be a decrease in the magnitude of \( G_2 \) with temperature and then an increase with further rise in temperature. In isospin symmetric medium, for \( \rho_B = \rho_0 \), the values of \( G_2 \) are given as \(-1.181 \times 10^{-4} \text{ GeV}^4, -1.130 \times 10^{-4} \text{ GeV}^4, -1.069 \times 10^{-4} \text{ GeV}^4, -1.034 \times 10^{-4} \text{ GeV}^4 \) and \(-1.4527 \times 10^{-4} \text{ GeV}^4 \) at values of temperature, \( T = 0, 50, 100, 150 \) and 200 MeV respectively. For density \( 4\rho_0 \) and \( \eta=0 \), the values of \( G_2 \) are given as \(-1.63 \times 10^{-4} \text{ GeV}^4, -1.626 \times 10^{-4} \text{ GeV}^4, -1.613 \times 10^{-4} \text{ GeV}^4, -1.5992 \times 10^{-4} \text{ GeV}^4 \) and \(-1.6156 \times 10^{-4} \text{ GeV}^4 \) for \( T = 0, 50, 100, 150 \) and 200 MeV respectively. In isospin asymmetric medium, \( \eta = 0.5 \), at \( \rho_B = 4\rho_0 \), the values of \( G_2 \) are \(-1.602 \times 10^{-4} \text{ GeV}^4, -1.598 \times 10^{-4} \text{ GeV}^4, -1.591 \times 10^{-4} \text{ GeV}^4, -1.5991 \times 10^{-4} \text{ GeV}^4 \) and \(-1.6265 \times 10^{-4} \text{ GeV}^4 \) at temperature, \( T = 0, 50, 100, 150 \) and 200 MeV respectively. In the present investigation, the effects of isospin asymmetry and temperature of the nuclear medium on the values of \( G_0 \) and \( G_2 \) are observed to be small and the effect of density seems to be the dominant effect. This is related to the fact that the dilaton field and the scalar fields, \( \sigma \), \( \zeta \) and \( \delta \) in the hot isospin asymmetric nuclear medium are strongly dependent on the density of the medium and the effects of temperature and isospin asymmetry on these scalar fields are much smaller as compared to the density effects.

After obtaining the medium modification of the scalar gluon condensate from the value of the dilaton field using equation (29), and of the twist-2 gluon operator, by using equations (11) and (19), we next determine the in-medium mass shift of \( J/\psi \) and \( \eta_c \) mesons using QCD sum rule approach. We use the moments in the range \( 5 \leq n \leq 12 \) and fix the value of parameter \( \xi = 0.874 \), so that we can reproduce the vacuum value of mass of \( J/\psi \), \( m_{J/\psi} = 3097 \text{ MeV} \). For this value of \( \xi \), the parameters \( \alpha_s \) and the running quark mass \( m_c \) turn out to be \( 0.2667 \) and \( 1.232 \times 10^3 \text{ MeV} \) respectively. We consider the contributions from scalar gluon condensate \( \left\langle \frac{\alpha_s}{\pi} G_{\sigma \kappa} G^\sigma G^\kappa \right\rangle \) and the twist-2 tensorial gluon operator \( \left\langle \frac{\alpha_s}{\pi} G_{\mu \sigma} G^\mu G^\sigma \right\rangle \) through the dilaton field \( \chi \) within the chiral \( SU(3) \) model used in the present investigation. We obtain the value of \( \phi_b \) which is related to the scalar gluon condensate by equation (32), from the
medium dependent $\chi$ field using the equation $^{[33]}$. The value for $\phi_c$ arising from the twist-2 tensorial gluon operator is calculated within the chiral SU(3) model by using equation $^{[34]}$. We also compare our results with the twist-2 gluon operator as calculated from the formula obtained in the low density approximation as given by equation $^{[35]}$. The value of $\phi_c$ at nuclear saturation density, $\rho_0 = 0.15 \text{ fm}^{-3}$ is calculated to be $-4.2158 \times 10^{-5}$ within the SU(3) chiral model used in the present investigation, which may be compared to the value of $-1.4685 \times 10^{-5}$ in the linear density approximation $^{[10]}$. In table I we summarize the results for the mass shifts of $J/\psi$ and $\eta_c$, as obtained in the present investigation, at zero temperature, for values of the baryon densities as $\rho_0$, $2\rho_0$ and $4\rho_0$, and for isospin asymmetry parameter as $\eta = 0$ and 0.5, for the value of $\xi=0.874$. As has already been mentioned this value of $\xi$ is fixed so as to obtain the observed vacuum mass of $J/\psi$ as 3097 MeV.

In figure 3 we show the variation of masses of $J/\psi$ and $\eta_c$ mesons with $n$, for fixed value of baryon density, $\rho_B = \rho_0$, and for different values of isospin asymmetry parameter, $\eta$. We show the results for values of temperature, $T = 0, 100$ and 150 MeV. In symmetric nuclear matter, at nuclear matter saturation density, $\rho_B = \rho_0$, and at temperature $T = 0$, we obtain the mass shifts for $J/\psi$ and $\eta_c$ mesons to be equal to $-4.48$ MeV and $-5.21$ MeV respectively, as can be seen from table IV. These values of mass shifts for for $J/\psi$ and $\eta_c$ mesons may be compared with the mass shifts of $-7$ MeV and $-5$ MeV respectively obtained in the linear density approximation in Ref. $^{[10]}$. In the present investigation, we calculate the values of $\phi_b$ and $\phi_c$ from the medium modification of the dilaton field, $\chi$, within the chiral $SU(3)$ model, by using equations $^{[33]}$ and $^{[34]}$. In isospin symmetric nuclear medium, at baryon densities, $\rho_B = 0$ and $\rho_0$, the values of the dilaton field, $\chi$ are 409.76 and 406.38 MeV respectively and hence using the equation $^{[29]}$, the values of the scalar gluon condensate $\langle \frac{\alpha_s}{\pi} G_{\mu\nu} G^{\mu\nu} \rangle$ turn out to be $(373 \text{ MeV})^4$ and $(371.6 \text{ MeV})^4$ for densities $\rho_B = 0$ and $\rho_0$ respectively. We might note here that when we neglect the quark masses in the trace anomaly, the values of the scalar gluon condensate at these densities are modified to $(391 \text{ MeV})^4$ and $(388 \text{ MeV})^4$ respectively. We thus observe an increase of the values of the scalar gluon condensate by about 20% when we do not account for the finite masses of the quarks. The values of $\phi_b$, accounting for the finite quark masses, turn out to be $2.3 \times 10^{-3}$ and $2.27 \times 10^{-3}$ in the vacuum and at nuclear matter saturation density, $\rho_0$, respectively. These may be compared
FIG. 2: (Color online) The functions $G_0$ and $G_2$ describing the trace and non-trace parts of the energy momentum tensor are plotted as functions of the density at different temperatures and for different values of the isospin asymmetry parameter, $\eta$. 
FIG. 3: (Color online) The in-medium masses of the $J/\psi$ and $\eta_c$ mesons plotted as functions of $n$, for nuclear matter saturation density, $\rho_0$ at different temperatures and for different values of the isospin asymmetry parameter, $\eta$. The value of parameter $\xi$ is taken as 0.874 which reproduces the vacuum mass of $J/\psi$ as 3097 MeV.
FIG. 4: (Color online) The in-medium masses of the $J/\psi$ and $\eta_c$ mesons plotted as functions of $n$, for baryon density, $\rho_B = 2\rho_0$ at different temperatures and for different values of the isospin asymmetry parameter, $\eta$. The value of parameter $\xi$ is taken as 0.874 which reproduces the vacuum mass of $J/\psi$ as 3097 MeV.
with the values of $\phi_b$ to be equal to $1.7 \times 10^{-3}$ and $1.6 \times 10^{-3}$ respectively for $\rho_B = 0$ and for $\rho_B = \rho_0$, obtained from the values of scalar gluon condensate of $(350 \text{ MeV})^4$ and $(344.81 \text{ MeV})^4$ respectively in vacuum and at nuclear saturation density, $\rho_0$ in Ref. [10] in the linear density approximation. We might note here that the value of nuclear matter saturation density used in the present calculations is $0.15 \text{ fm}^{-3}$ and in Ref. [10], it was taken to be $0.17 \text{ fm}^{-3}$. When the quark masses are neglected, and $\phi_c$ as calculated in the chiral SU(3) model used in the present investigation, the values of the mass shift for $J/\psi$ and $\eta_c$ turn out to be $-8.01 \text{ MeV}$ and $-5.13 \text{ MeV}$ respectively. When we calculate $\phi_b$ within the chiral SU(3) model, but calculate the contribution of the twist-2 operator through $\phi_c$ calculated in the linear density approximation given by equation (35) [10], we obtain the mass shifts for $J/\psi$ and $\eta_c$ at $\rho_B = \rho_0$ for symmetric nuclear matter at zero temperature to be given as $-2.88 \text{ MeV}$ and $-2.02 \text{ MeV}$ respectively. The value of the mass shift of $J/\psi$ of about $-4.48 \text{ MeV}$ at the nuclear matter density $\rho_0$ in symmetric nuclear matter at zero temperature obtained in the present investigation may be compared to the values of the mass shift of $-8 \text{ MeV}$ obtained using QCD second order Stark effect using the value of the scalar gluon condensate obtained using a linear density approximation [17] as well as a value of $-8.6 \text{ MeV}$, when the scalar gluon condensate was obtained from the expectation value of the scalar dilaton field in a chiral SU(3) model [18]. We observe in figure 3 that the isospin dependence of the mass shifts of $J/\psi$ and $\eta_c$ are very small. This is due to the fact that the dependence of $\chi$ on the isospin asymmetry is very small, as can be seen from figure 1.

Figures 4 and 5 show the mass shifts of $J/\psi$ and $\eta_c$ for baryon densities $\rho_B = 2\rho_0$ and $4\rho_0$ respectively, at different temperatures and different values of the isospin asymmetry parameter, $\eta$. In isospin symmetric nuclear medium, at density $\rho_B = 2\rho_0$, temperature $T = 0$, the mass shifts for $J/\psi$ and $\eta_c$ mesons are observed to be $-10 \text{ MeV}$ and $-9.14 \text{ MeV}$ respectively. The effects of isospin asymmetry of the medium on the mass shift of the $J/\psi$ and $\eta_c$ mesons are seen to be almost negligible, as can be seen from table I. This is due to the very small changes in the dilaton field with the isospin asymmetry of the medium, as can be seen from figure 1. In isospin asymmetric nuclear medium ($\eta = 0.5$), at nuclear saturation density $\rho_0$, the mass shifts in $J/\psi$ and $\eta_c$ mesons at zero temperature are observed to be $-4.34 \text{ MeV}$ and $-5.06 \text{ MeV}$ from their vacuum values, which may be compared with the
values of $-4.48$ MeV and $-5.21$ MeV respectively for the isospin symmetric nuclear matter. At values of the baryon density $\rho_B$ as $2\rho_0$ and $4\rho_0$, as can be seen from table II, the isospin dependence of the mass shifts for $J/\psi$ and $\eta_c$ mesons are seen to be negligibly small.

The effects of temperature on the dilaton field $\chi$ is very small and this is reflected in the small change in the mass shifts of $J/\psi$ and $\eta_c$ mesons with temperature $T$. In isospin symmetric nuclear medium ($\eta = 0$), at nuclear saturation density $\rho_B = \rho_0$, the mass shifts for $J/\psi$ mesons, from their vacuum values, are observed to be $-4.01$, $-3.5$ and $-3.23$ MeV at temperatures $T = 50, 100$ and $150$ MeV respectively. At baryon density, $\rho_B = 4\rho_0$, the values of mass shift for $J/\psi$ meson changes to $-16.13$, $-14.82$ and $-13.76$ MeV at temperatures $T = 50, 100$ and $150$ MeV respectively. The value of the mass shift obtained at finite value of temperature is observed to be smaller as compared to the zero temperature case. This is because, at finite value of baryon density $\rho_B$, the dilaton field $\chi$ increases with increase in the temperature of the nuclear medium, but the increase is very small. In isospin asymmetric nuclear medium, $\eta = 0.5$, at density $\rho_B = 4\rho_0$, the mass shifts for $J/\psi$ mesons, from their vacuum values are $-14.82$, $-14.16$ and $-14.36$ MeV at temperatures $T = 50, 100$ and $150$ MeV respectively.

For the pseudoscalar meson $\eta_c$, the mass shifts at nuclear saturation density $\rho_0$, in nuclear medium with $\eta=0$ (0.5), are $-4.81(-4.73)$, $-4.352(-4.345)$ and $-4.1(-4.54)$ MeV at temperatures $T = 50, 100$ and $150$ MeV respectively. At density $\rho_B = 4\rho_0$, with $\eta=0$ (0.5), these values are modified to $-12.77(-11.98)$, $-12.02(-11.6)$ and $-11.41(-11.73)$ MeV respectively for $T=50, 100$ and $150$ MeV. It may be noted that at high values of temperatures e.g. at $T = 150$ MeV, the mass shift is more in the isospin asymmetric nuclear medium ($\eta = 0.5$) as compared to isospin symmetric nuclear medium ($\eta = 0$). This is opposite to the zero temperature case. The reason is that at high temperatures the dilaton field $\chi$ has larger drop in the isospin asymmetric nuclear medium ($\eta = 0.5$) as compared to the isospin symmetric nuclear medium ($\eta = 0$), as can be seen in figure II.

As mentioned earlier, for the above calculations we had fixed the value of parameter $\xi$ so as to reproduce the vacuum value of $J/\psi$ mass. However, with this value of $\xi$, the vacuum value of $\eta_c$ meson comes out to be 2955.6 MeV. We can reproduce the vacuum value of pseudoscalar meson $\eta_c = 2980.5$ MeV, if we fix the value of $\xi = 0.8995$. For this value
FIG. 5: (Color online) The in-medium masses of the $J/\psi$ and $\eta_c$ mesons plotted as functions of $n$, for baryon density, $\rho_B = 4\rho_0$ at different temperatures and for different values of the isospin asymmetry parameter, $\eta$. The value of parameter $\xi$ is taken as $0.874$ which reproduces the vacuum mass of $J/\psi$ as $3097$ MeV.
of $\xi$, the parameters $\alpha_s$ and the running charm quark mass $m_c$ turn out to be 0.266 and $1.2313 \times 10^3$ MeV respectively. For these values of parameters, the mass shifts for $J/\psi$ and $\eta_c$ mesons, in nuclear medium at zero temperature, at densities $\rho_0, 2\rho_0$ and $4\rho_0$ for $\eta=0$ (0.5) are summarized in table II.

We also show the results for the mass modifications of $J/\psi$ and $\eta_c$ mesons, if we consider the value of parameters, $\xi = 1.10$, leading to the values of $\alpha_s$ and $m_c$ as 0.21 and $1.24 \times 10^3$ MeV respectively. Figures 6, 7 and 8 show the temperature and isospin asymmetry dependence of the mass modifications of $J/\psi$ mesons and $\eta_c$ mesons, for baryon densities of $\rho_0, 2\rho_0$ and $4\rho_0$ respectively, with parameter $\xi = 1$. We observe that with $\xi=1$, the vacuum values of the masses of $J/\psi$ and $\eta_c$ mesons are given as 3196.56 and 3066.57 MeV respectively. With $\xi=1$, the results for the mass shifts for $J/\psi$ and $\eta_c$ at different densities with $\eta=0$ and 0.5, and zero temperature, obtained in the present investigation are summarized in table III. The values of the mass shifts for $J/\psi$ meson in isospin symmetric medium, with $\xi = 1$, are shown at densities of $\rho_0, 2\rho_0$ and $4\rho_0$ at values of the isospin asymmetric parameter, $\eta=0$ and 0.5 for $\xi=0.8995$. This value of $\xi$ reproduces the vacuum mass of $\eta_c$ as 2980.5 MeV.

| $\rho_B$ | $\eta = 0$ | $\eta = 0.5$ | $\eta = 0$ | $\eta = 0.5$ |
|---------|-------------|-------------|-------------|-------------|
| $\rho_0$ | -4.27 | -4.14 | -5.69 | -5.54 |
| $2\rho_0$ | -9.55 | -8.87 | -9.39 | -8.92 |
| $4\rho_0$ | -16.02 | -14.51 | -13.12 | -12.25 |

TABLE II: The mass shifts of $J/\psi$ and $\eta_c$ are shown at densities of $\rho_0, 2\rho_0$ and $4\rho_0$ at values of the isospin asymmetric parameter, $\eta=0$ and 0.5 for $\xi=0.8995$. This value of $\xi$ reproduces the vacuum mass of $\eta_c$ as 2980.5 MeV.

| $\rho_B$ | $\eta = 0$ | $\eta = 0.5$ | $\eta = 0$ | $\eta = 0.5$ |
|---------|-------------|-------------|-------------|-------------|
| $\rho_0$ | -4.43 | -4.28 | -3.8 | -3.66 |
| $2\rho_0$ | -10.43 | -9.66 | -7.67 | -7.18 |
| $4\rho_0$ | -17.93 | -16.19 | -11.85 | -10.87 |

TABLE III: The mass shifts of $J/\psi$ and $\eta_c$ are shown at densities of $\rho_0, 2\rho_0$ and $4\rho_0$ at values of the isospin asymmetric parameter, $\eta=0$ and 0.5 for $\xi=1$. The values of the mass shifts for $J/\psi$ meson in isospin symmetric medium, with $\xi = 1$, are shown at densities of $\rho_0, 2\rho_0$ and $4\rho_0$ at values of the isospin asymmetric parameter, $\eta=0$ and 0.5 for $\xi=0.8995$. This value of $\xi$ reproduces the vacuum mass of $\eta_c$ as 2980.5 MeV.
at nuclear saturation density $\rho_B = \rho_0$ are observed to be $-3.92$, $-3.38$ and $-3.1$ MeV for $T = 50, 100$ and 150 MeV respectively. At baryon density $\rho_B = 4\rho_0$, these values of the mass shift change to $-17.23$, $-15.77$ and $-14.59$ MeV at temperature $T = 50, 100$ and 150 MeV respectively. For pseudoscalar meson $\eta_c$, the mass shifts at $\rho_B = \rho_0$ are obtained to be $-3.42$, $-3.06$ and $-2.91$ MeV for $T = 50, 100$ and 150 MeV respectively, whereas at $\rho_B = 4\rho_0$ these values of mass shift are seen to be modified to $-11.47$, $-10.68$ and $-10.04$ MeV respectively.

In Ref. [15] the operator product expansion was carried out up to dimension six and mass shift for $J/\psi$ was found to be $-4$ MeV at nuclear saturation density $\rho_0$ and temperature $T = 0$. The effect of temperature on the $J/\psi$ in deconfinement phase was studied in Ref. [42, 43]. In these investigations, it was reported that $J/\psi$ mass is essentially constant in a wide range of temperatures and above a particular value of the temperature, $T$, there is a sharp change in the mass of $J/\psi$ in the deconfined phase. In Ref. [44] the mass shift for $J/\psi$ was reported to be about 200 MeV at $T = 1.05 T_c$. The pseudoscalar charmonium spectral function for different temperatures was studied using a screened potential in [45]. The effect of rising temperature was observed to melt the higher excited states by $1.1 T_C$ and to shift the continuum threshold to lower energies. In these studies, it was observed that the charmonium $\eta_c$ survives even in the deconfined phase. In Ref. [43, 46], the effect of temperature on $\eta_c$ in the deconfinement phase was studied. In Ref. [46], it was reported that the $J/\psi$ and $\eta_c$ survive as distinct resonances in the plasma even up to $T \simeq 1.6 T_c$ and that they eventually dissociate between $1.6 T_c$ and $1.9 T_c$. This suggests that the deconfined plasma is non-perturbative enough to hold heavy-quark bound states. In the present work, we have studied the effects of temperature on the mass modifications of $J/\psi$ and $\eta_c$ mesons in the confined phase due to modifications of the scalar gluon condensate and twist-2 tensorial gluon operator, simulated by a medium dependent scalar dilaton field in chiral SU(3) model and the temperature effects are found to be very small as compared to the density effects.

V. SUMMARY

In summary, in the present investigation, we have studied the mass modifications of the charmonium states, $J/\psi$ and $\eta_c$ in the nuclear medium using QCD sum rule approach and
FIG. 6: (Color online) The in-medium masses of the $J/\psi$ and $\eta_c$ mesons plotted as functions of $n$, for nuclear matter saturation density, $\rho_0$ at different temperatures and for different values of the isospin asymmetry parameter, $\eta$, with $\xi=1$. 

\[ \text{meV} \]

\[ \text{MeV} \]

\[ n \]

\[ T = 0 \]

\[ T = 100 \text{ MeV} \]

\[ T = 150 \text{ MeV} \]
FIG. 7: (Color online) The in-medium masses of the $J/\psi$ and $\eta_c$ mesons plotted as functions of $n$, for baryon density of $2\rho_0$, at different temperatures and for different values of the isospin asymmetry parameter, $\eta$, with $\xi=1$. 
FIG. 8: (Color online) The in-medium masses of the $J/\psi$ and $\eta_c$ mesons plotted as functions of $n$, for baryon density of $4\rho_0$ at different temperatures and for different values of the isospin asymmetry parameter, $\eta$, with $\xi=1$. 
using modification of a dilaton field (which simulates the gluon condensates) within a chiral $SU(3)$ model. The in-medium modifications of the $J/\psi$ and $\eta_c$ are studied as arising due to changes in the scalar and twist-2 gluon condensates in the nuclear medium, obtained from the medium modification of the $\chi$ field. The value of the dilaton field in the hot nuclear matter is obtained by solving the coupled equations (20) to (23), which are the equations of motion of $\sigma$, $\zeta$, $\delta$ and $\chi$ fields. The dilaton field, $\chi$, thus depends on the scalar isovector field, $\delta$, which is related to the isospin asymmetry of the nuclear medium. The isospin asymmetry dependence of the $\chi$, in turn, leads to the isospin asymmetry dependence of the charmonium states, $J/\psi$ and $\eta_c$. The modification of the $\chi$ field is observed to be small with the isospin asymmetry of the medium, as can be seen from figure 1. This is related to the fact that the magnitude of the obtained value of $\delta$ after solving the coupled equations for the scalar fields turns out to be much smaller (about few percent) as compared to the magnitudes of $\sigma$ and $\zeta$ and hence the isospin asymmetry (through $\delta$) only gives rise to a very small modification of the dilaton field $\chi$ [18]. It is observed that the temperature effect on the $\chi$ field is also very small and the modification of the dilaton field with density is seen to be the dominant medium effect in the present investigation. The negligible dependence of the dilaton field on isospin asymmetry as well as on temperature is reflected in the small isospin/temperature dependence of the masses of the $J/\psi$ and $\eta_c$ states in the nuclear medium. Experimentally, measurements of dileptons (diphotons) in heavy-ion collisions may provide a clue to the properties of vector (pseudo-scalar) mesons in hot/dense matter [46]. The present study of the in-medium properties of $J/\psi$ and $\eta_c$ mesons will be helpful for the experiments in the future facility of the FAIR, GSI, where the compressed baryonic matter at high densities and moderate temperature will be produced.

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