Next-to-Leading Order Evolution
of Polarized Fragmentation Functions

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Abstract
We present a brief description of the determination of the two-loop spin-dependent time-like splitting functions relevant for the NLO evolution of polarized fragmentation functions. Our calculation based on the analytic continuation of the corresponding space-like results obtained within the light-cone gauge method proposed by Curci, Furmanski, and Petronzio. As an application we present an analysis of polarized Λ production in $e^+e^-$ and $ep$ collisions.

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NLO Evolution of Polarized Fragmentation Functions

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Abstract. We present a brief description of the determination of the two-loop spin-dependent time-like splitting functions relevant for the NLO evolution of polarized fragmentation functions. Our calculation based on the analytic continuation of the corresponding space-like results obtained within the light-cone gauge method proposed by Curci, Furmanski, and Petronzio. As an application we present an analysis of polarized Λ production in $e^+ e^-$ and $ep$ collisions.

GENERAL FRAMEWORK

In analogy to the familiar space-like (S) structure function $g_1^{(S)}(x, Q^2)$ appearing in longitudinally polarized DIS one can define a similar time-like (T) structure function $g_1^{(T)}(z, Q^2)$, where $q^2 \equiv Q^2 > 0$, which describes the helicity transfer in single-particle inclusive $e^+ e^-$ annihilation processes (SIA), e.g.,

$$
\frac{d\sigma_{e^+ e^- \rightarrow H X}}{dy dz} = \frac{d\sigma_{e^+ e^- \rightarrow H^+} - d\sigma_{e^+ e^- \rightarrow H^-}}{dy dz} = \frac{6\pi\alpha^2}{Q^2} (2y - 1) g_1^{(T)}(z, Q^2).
$$

where $z = 2p_H \cdot q/Q^2$, $y = p_H \cdot p_{e^-}/p_H \cdot q$ and $(\pm)$ denotes the helicities of the $e^-$ and $H$. In terms of the spin-dependent fragmentation functions $\Delta D^H_f(z, Q^2) \equiv D^{H(+)\rightarrow H(+)f(+)}_f(z, Q^2) - D^{H(+)\rightarrow H(+)f(-)}_f(z, Q^2)$, where $D^{H(+)\rightarrow H(+)f(+)}_f(z, Q^2)$ ($D^{H(+)\rightarrow H(+)f(-)}_f(z, Q^2)$) is the probability for finding a hadron $H$ with positive (negative) helicity in a parton $f$ with positive helicity at a mass scale $Q$, carrying a fraction $z$ of the parent parton’s momentum, we can write $g_1^{(T)}$ to NLO as [1] (the symbol $\otimes$ denotes the usual convolution)

$$
g_1^{(T)}(z, Q^2) = \sum_q e_q^2 \left\{ [\Delta D^H_q + \Delta \Sigma_q] \otimes \Delta C^{(T)}_q + 2 \Delta D^H_g \otimes \Delta C^{(T)}_g \right\}(z, Q^2).
$$

The $Q^2$-evolution of the $\Delta D^H_f$, predicted by QCD, is again similar to the space-like case. The singlet evolution equation, e.g., schematically reads

$$
\frac{d}{d\ln Q^2} \left( \frac{\Delta D^H_q}{\Delta D^H_q} \right)(z, Q^2) = \left[ \Delta \hat{P}^{(T)} \otimes \left( \frac{\Delta D^H_q}{\Delta D^H_q} \right) \right](z, Q^2),
$$

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with $\Delta D_{\Sigma}^H \equiv \sum_q (\Delta D_q^H + \Delta D_{\bar{q}}^H)$. It should be recalled that the off-diagonal entries in the singlet evolution matrices $\Delta \hat{P}^{(S,T)}$ interchange their role when going from the space- to the time-like case, see, e.g., [2,1].

As a manifestation of the so-called Gribov-Lipatov relation [3] the space-and time-like splitting functions are equal in LO. Furthermore they are related by analytic continuation of the space-like splitting functions (Drell-Levy-Yan relation (ACR) [4]) which can be schematically expressed as ($z < 1$)

$$\Delta P_{ij}^{(T)}(z) = z \mathcal{AC} \left[ \Delta P_{ji}^{(S)}(x = \frac{1}{z}) \right], \quad (4)$$

where the operation $\mathcal{AC}$ analytically continues any function to $x \to 1/z > 1$ and correctly adjusts the color factor and the sign depending on the splitting function under consideration [1], e.g.,

$$\Delta P_{qq}^{(T)}(z) = -z \Delta P_{qq}^{(S)}(\frac{1}{z}) , \quad \Delta P_{gq}^{(T)}(z) = \frac{C_F}{2T_f} z \Delta P_{gq}^{(S)}(\frac{1}{z}) , \ldots \quad (5)$$

These LO relations are based on symmetries of tree diagrams under crossing, and should be therefore no longer valid when going to NLO. Fortunately [5], the breakdown of the ACR arising beyond LO is essentially due to kinematics and can be rather straightforwardly detected within the light-cone gauge method used in [2,5–7] to calculate the space-like splitting functions.

**UNDERSTANDING OF THE ACR BREAKING**

The general strategy of the light-cone gauge method is based on a rearrangement of the perturbative expansion of a partonic cross section into a finite hard part and a process-independent part $\Gamma_{ij}$ which contains all (and only) mass singularities [8,5]. Using dimensional regularization the MS evolution kernels appear order by order as the residues of the single $1/\epsilon$ poles in $\Gamma_{ij}$ [5]. The difference between the space- and the time-like $\Gamma_{ij}$-kernels essentially amounts to relative extra phase space factors $(z^{-2\epsilon})$ in the time-like case[1]. All this gives on aggregate for $z < 1$ for the AC of the spin-dependent space-like kernels [1]

$$\Delta \Gamma_{qq}^{(T)}(z, \frac{1}{\epsilon}) = -z^{1-2\epsilon} \Delta \Gamma_{qq}^{(S)}(\frac{1}{z}, \frac{1}{\epsilon}) , \quad \Delta \Gamma_{gq}^{(T)}(z, \frac{1}{\epsilon}) = \frac{C_F}{2T_f} z^{1-2\epsilon} \Delta \Gamma_{gq}^{(S)}(\frac{1}{z}, \frac{1}{\epsilon}) , \ldots \quad (6)$$

Obviously, higher $(1/\epsilon^2)$ pole terms in $\Delta \Gamma_{ij}^{(S)}$ will generate additional contributions to the relevant single pole part of $\Delta \Gamma_{ji}^{(T)}$ if combined with the factors $z^{-2\epsilon}$ in (6). To extract the contributions that break the ACR one can easily go through the NLO calculation of the $\Delta \Gamma_{ij}^{(S)}$ [7] graph by graph picking.

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1) In the unpolarized case a further difference arises [1] through the necessary adjustment of the different spin-average factors for gluons and quarks in $d = 4 - 2\epsilon$ dimensions.
up all $1/\epsilon^2$ pole terms. Apart from the slightly subtle case of the so-called ‘ladder-subtraction’ topology \[7\] this is a rather straightforward task \[1\]. To obtain the desired final result for $\Delta P_{ij}^{(T)}$ \[1\] one has to combine the results of the AC of the NLO splitting functions $\Delta F_{ij}^{(S)}$ \[9,7\], by using the operation $\mathcal{AC}[\ldots]$ defined above, with the contributions that break the ACR. Finally, the missing endpoint contributions, i.e., the terms $\sim \delta(1-z)$, are exactly the same as in the space-like situation \[5,2\]. In the same way the hard subprocess cross sections $\Delta C_i^{(T)}$ in (2) can be obtained via analytic continuation (see ref. \[1\] for details). The final results for the $\Delta P_{ij}^{(T)}$ and $\Delta C_i^{(T)}$ are too long to be listed here but can be found in ref. \[1\]2.

The rather simple and transparent structure of the ACR breaking part is a hint that there could be a more straightforward way of linking the time- and the analytically continued space-like NLO quantities. The key observation for such considerations is to notice that although the usual 4-dim. LO splitting functions obey the ACR rule, the rule must break down for their $d = (4-2\epsilon)$-dimensional counterparts as an immediate consequence of eq.(6) \[1\]. It turns out \[1\] that the breakdown of the ACR beyond LO in the MS scheme is entirely driven by this breaking such that it can be accounted for by a simple factorization scheme transformation \[1\].

Finally, our results for $\Delta P_{ij}^{(T)}$ (as well as the corresponding unpol. results given in \[5,2\]) fulfill the so-called SUSY relation which links all singlet splitting functions in a remarkably simple way in the limit $C_F = N_C = 2T_f \equiv N$ (cf. \[9,7\]) if they are properly transformed to a regularization method which respects SUSY such as dimensional reduction \[10\]. The validity of the SUSY relation \[1\] serves as an important check for the correctness of our results.

**APPLICATION: POLARIZED $\Lambda$ PRODUCTION**

The most likely candidate for a measurement of polarized fragmentation functions is the $\Lambda$ baryon due to its self-analyzing decay $\Lambda \to p\pi^-$. In \[11\] a strategy was proposed for extracting the $\Delta D^A_\Lambda$ in SIA $e^+e^- \to \Lambda X$. At high enough energies, no beam polarization is needed since the parity-violating coupling $q\bar{q}Z$ automatically generates a net polarization of the quarks. In Fig. 1a we compare the first results for such a measurement of the cross section asymmetry $A_{SA}^{\Lambda}$ on the $Z$-pole by ALEPH and DELPHI \[12\] with two conceivable NLO models for the $\Delta D^A_\Lambda$ (the corresponding LO results are very similar) \[13\]. Scenario 1, where $\Delta D^A_{s}(z,\mu^2) = z^a D^A_{s}(z,\mu^2)$, $\Delta D^A_{t \neq s}(z,\mu^2) = 0$, is based on the naive quark model whereas for scenario 2 we also allow for a non-vanishing $\Delta D^A_{u,d}$ input as discussed in \[11\]. In both cases the inputs for the evolution are specified at some low starting scale $\mu^2 \lesssim \mathcal{O}(0.3\text{ GeV}^2)$ (for more details see ref. \[13\] where also a model for the unpolarized $D^A_f$ can be found).

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2) We also have recalculated the unpolarized NLO time-like Altarelli-Parisi kernels where we fully agree with the results given in \[5,2\].
Apart from SIA the possibility of extracting the $\Delta D_f^\Lambda$ in semi-inclusive DIS (SIDIS), i.e., $ep \rightarrow e\Lambda X$, has also been discussed recently [14]. Here, either a longitudinally polarized lepton beam or a polarized nucleon target would be required. Such kind of measurement appears possible e.g. for the HERMES experiment [15]. In Fig. 1b we present LO and NLO predictions for the cross section asymmetry $A_{SIDIS}^\Lambda$ in $\vec{e}p \rightarrow \vec{\Lambda}X$ as a function of $x$ at $Q^2 = 3$ GeV$^2$ and where we have integrated over $0.3 \leq z \leq 0.9$ [13]. Such a measurement by HERMES would allow, in principle, to discriminate between the two proposed scenarios for the polarized $\Lambda$ fragmentation functions $\Delta D_f^\Lambda$.

![Figure 1](image_url)

**FIGURE 1.** a) Comparison of recent data for $A_{SIA}^\Lambda$ [13] with the results obtained for two conceivable scenarios for the $\Delta D_f^\Lambda$; b) LO and NLO predictions for $A_{SIDIS}^\Lambda$ in $\vec{e}p \rightarrow \vec{\Lambda}X$.

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