Dirty higher-order Dirac semimetal: Quantum criticality and bulk-boundary correspondence

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We analyze the stability of three-dimensional higher-order topological (HOT) Dirac semimetals (DSMs) and the associated one-dimensional hinge modes in the presence of random pointlike charge impurities. Complementary numerical and renormalization group (RG) analyses suggest that a HOTDSM, while being a stable phase of matter for sufficiently weak disorder, undergoes a continuous quantum phase transition into a metallic phase at finite disorder. However, the corresponding critical exponents (obtained from the scaling of the density of states) are extremely close to the ones found in a dirty, but first-order DSM (supporting two Fermi arcs). This observation suggests an emergent superuniversality in the entire family of dirty DSMs, as also predicted by the RG analysis. As a direct consequence of the bulk-boundary correspondence, the hinge modes gradually fade away with increasing randomness in the system, and completely dissolve in the trivial metallic phase.

\textbf{Introduction.} Traditionally, the bulk-boundary correspondence in a $d$-dimensional topological system refers to the boundary modes residing on a $(d-1)$-dimensional surface\textsuperscript{[11,15]}, also characterized by the codimension $d_\mathrm{c} = d - (d - 1) = 1$. This notion has recently been generalized to encompass boundary modes with $d_\mathrm{c} \in \mathbb{Z}$ (integers)$> 1$, such as zero-dimensional corner ($d_\mathrm{c} = d$) and one-dimensional hinge ($d_\mathrm{c} = d - 1$) modes, which lead to the construction of higher order topological (HOT) phases in insulating (electric and thermal) as well as nodal systems\textsuperscript{[6-27]}. A question regarding the stability of such exotic phase of matter in the presence of interactions and/or disorder arises naturally. Due to a finite gap in the quasiparticle spectra, while the HOT insulators enjoy robustness against sufficiently weak interactions and disorder, their influence on gapless HOT phases demand more careful analyses.

Here we focus on a three-dimensional HOT Dirac semimetal (HOTDSM), supporting one-dimensional hinge modes, and scrutinize its stability when littered with pointlike quenched random charge impurities. Using numerical and field-theoretic renormalization group (RG) analyses, we find that HOTDSM is a stable phase of matter in the presence of sufficiently disorder. But, it undergoes a continuous quantum phase transition (QPT) into a diffusive metallic phase at finite disorder, see Fig.\textsuperscript{[4]} We arrive at these conclusions from the scaling of average density of states (DOS), computed using the kernel polynomial method (KPM)\textsuperscript{[28]}. And as a direct consequence of the bulk-boundary correspondence, the topological hinge modes gradually fade away with increasing randomness and completely melt into a trivial metallic bulk for sufficiently strong disorder, see Fig.\textsuperscript{[2]}

Moreover, numerically extracted values for the dynamical scaling exponent $z$ (DSE) and correlation length exponent $\nu$ (CLE) across a broad range of parameters (that also includes first-order DSMs) appear to be \textit{almost} the same (within the numerical accuracy), see Table\textsuperscript{[1]} and yield satisfactory single-parameter data collapses across the (HOT)DSM-metal continuous QPT, see Fig.\textsuperscript{[3]} This observation strongly promotes the notion of an emergent \textit{superuniversality} near a diffusive quantum critical point (QCP) in the entire family of dirty DSMs, irrespective of their topological order. We also substantiate these findings from a leading-order RG analysis.

\textbf{Lattice model.} We set out by considering a tight-binding model for a three-dimensional HOTDSM on a cubic lattice $\hat{h}_k^0 = \hat{h}_k^1(t_3_C_3 - m_3 + t_0 \sum_{j=1}^2 (1 - C_j)) \Gamma_3$,

\begin{equation}
\hat{h}_k^1 = t_1 [(C_1 - C_2) \Gamma_4 + S_1 S_2 \Gamma_5],
\end{equation}

$C_3 \equiv \cos(k_3 a)$, $S_j \equiv \sin(k_j a)$, $k_j$ being the compo-
ents of momenta and \( a \) is the lattice spacing that we set to unity. Here, \( i = 1, 2, 3 \) correspond to \( x, y, z \) respectively. The four-component Hermitian \( \Gamma \) matrices, \( \Gamma_1 = \sigma_3 \tau_1 \), \( \Gamma_2 = \sigma_0 \tau_2 \), \( \Gamma_3 = \sigma_0 \tau_3 \), \( \Gamma_4 = \sigma_1 \tau_1 \), \( \Gamma_5 = \sigma_2 \tau_1 \), satisfy the anticommuting Clifford algebra \( \{ \Gamma_i, \Gamma_j \} = 2 \delta_{ij} \), for \( i, j = 1, \ldots, 5 \). Pauli matrices \( \sigma_\mu (\tau_\mu) \) operate on the spin (orbital) degrees of freedom, where \( \mu = 0, \ldots, 3 \). For the rest of this paper we set \( m_z = 0 \) and \( t = t_0 = t_z = 1 \). Then for \( t_1 = 0 \), the above model can be viewed as stacked two-dimensional quantum spin Hall insulators (QSHIs) along the \( k_z \) axis, and each layer accommodates two counter-propagating one-dimensional edge states for opposite spin projections. The resulting three-dimensional system supports two Dirac-points at \( k = (0, 0, \pm \pi) \equiv \pm \mathbf{K} \), and ribbon like edge modes localized on the \( xz \) and \( yz \) planes, yielding two copies of Fermi arcs connecting the Dirac points on the \( k_x k_y \) or \( k_y k_z \) plane. This system describes a first-order DSM, leading to a \( \rho(E) \sim |E|^2 \) scaling of DOS at low energies.

On the other hand, \( \hat{h}_1^k \) acts as a discrete-symmetry breaking, momentum-dependent or Wilson mass in each layer of QSHI. It changes sign under the four-fold rotation and thus acts as a domain wall mass; resulting in four corner-localized zero-energy states (with \( d_c = 2 \)) in each two-dimensional insulating layer, according to a generalized Jackiw-Rebbi index theorem \[29\]. Stacking such layers of two-dimensional HOT insulators in the momentum space along the \( k_z \) axis gives rise to the one-dimensional hinge modes along the \( z \) direction \[26\], see Fig. 2. We then realize a second-order DSM, as \( \hat{h}_1^k \) vanishes at the Dirac nodes (\( \pm \mathbf{K} \)).

As the Wilson mass \( \hat{h}_1^k \) vanishes quadratically with momentum around the Dirac points, it only reduces the DOS without altering its overall \( \rho(E) \sim |E|^2 \) scaling at sufficiently low energies, see Fig. 2 (a). The subdominant influence of the Wilson mass on the DOS suggests that the HOTDSM is stable in the presence of sufficiently weak disorder, and enters into a metallic phase via a QPT at finite disorder, qualitatively similar to the situation in first-order Dirac and Weyl semimetals \[30\]–\[52\], up to exponentially small, but debated rare region effect \[53\]–\[55\]. Next we anchor this anticipation by numerically computing the DOS using the KPM, and investigate the critical properties of such a QPT in HOTDSM. For concreteness, here we focus on pointlike charge impurities, which is the dominant source of elastic scattering in any real material, distributed uniformly and independently within the range \([-W/2, W/2]\) at each site of the cubic lattice. Numerically obtained critical exponents are summarized in Table 1, which we use to perform a finite energy (size) data collapse, shown in Fig. 2 (top) (bottom) \[56\].

Scaling. To formulate the scaling theory for DOS, we concentrate around the dirty QCP at \( W = W_c \), and parametrize the distance from it by \( \delta = (W - W_c)/W_c \). The number of states \( N(E, L) \) in a \( d \)-dimensional system of linear size \( L \) below some energy \( E \) is in general a function of two dimensionless parameters, \( L/\xi \) and \( E/E_0 \). Here \( \xi \) is the correlation length, which diverges at the QCP as \( \xi \sim \delta^{-\nu} \), and \( E_0 \sim \xi^{-\nu} \) is the corresponding energy scale. Since the number of states is proportional to \( L^d \), the functional form of \( N(E, L) \) ought to be \[33\]–\[42\]

\[
N(E, L) = (L/\xi)^d \ F(E \xi^z, L/\xi),
\]

(2)

where \( F(x, y) \) is an unknown, but universal function of its arguments. The DOS is then given by

\[
\rho(E) = \frac{1}{L^d} \frac{dN(E, L)}{dE} = \delta^{(d-z)\nu} \ G(|E| \delta^{-z\nu}, L^{1/\nu} \delta).
\]

(3)

To investigate the scaling behavior of the universal function \( G \), we consider the scaling of \( \rho(E) \) at low energies.
inside the HOTDSM and metallic phases, as well as inside the critical regime around the QCP at $W = W_c$. For now we assume $L$ to be sufficiently large, such that the $L$-dependence of $G$ can be neglected.

For linearly dispersing HOT Dirac fermions the DOS at zero energy is finite, leading to
\[
\rho(E) \sim \delta (d - z)^{\nu} (|E| \delta^{1 - \nu z})^0 = \delta (d - z)^{\nu},
\]
for $d > 0$ or $W > W_c$. Lastly, at the critical point ($d = 0$) $\xi$ diverges, and therefore the $\xi$ independence of $G$ implies
\[
\rho(E) \sim \delta (d - z)^{\nu} (|E| \delta^{1 - \nu z})^0 = |E|^{d - z}/z.
\]

The fact that the DOS at zero energy vanishes for $W \leq W_c$ and becomes finite in the metallic phase, allows one to treat $\rho(0)$ as an order-parameter in dirty HOTDSM.

Numerically we reconstruct the average DOS by using the KPM in a cubic system of linear dimension $L = 200$ for various choices of $t$ and $t_1$. First, we identify the critical disorder strength $W_c$, where $\rho(0)$ deviates from zero. Subsequently, we compute (a) the DSE $z$ from the scaling of $\rho(E)$ around $W = W_c$ [see Eq. (6)], and (b) the order-parameter exponent $\beta \equiv (d - z)^{\nu}$ from the scaling of $\rho(0)$ inside the metallic phase [see Eq. (5)]. Finally, from the known values of $z$ and $\beta$, we compute the CLE $\nu$. The results are summarized in Table I and the details of the numerical analysis are shown in the Supplemental Materials [54]. Across a wide range of hopping parameters ($t$ and $t_1$) we find that $z \approx 1.5$ and $\nu \approx 1.0$, which are fairly close (within the numerical accuracy) to the ones found for first-order Dirac and Weyl semimetals.

With the numerically extracted values of the critical exponents, we obtain convincing data collapses by comparing $\rho(E) |E|^{\nu(d - z)}$ vs $E^{\nu(d - z)}$, see Fig. 3 (top row). All data points collapse onto three curves, corresponding to the DSM (lower ones), a metal (upper left ones) and the quantum critical regime (upper right ones). Finally, from the same set of exponents we obtain excellent finite size data collapses by comparing $\rho(0)L^{1/\nu}\delta$ inside the metallic phase for $100 \leq L \leq 200$, as shown in Fig. 3 (bottom row). All together our extensive numerical analyses strongly suggest an emergent superuniversality across the DSM-metal QPT irrespective of their topological order (first or second), which now we substantiate from a leading-order RG analysis.

**RG analysis.** To perform the RG analysis in a dirty HOTDSM, we consider the following low-energy model
\[
\hat{h}(p) = \sum_{j=1}^{2} \Gamma_j p_j + v_3 p_3 + b \sum_{j=4}^{5} d_j(p) \Gamma_j,
\]
obtained by expanding $\hat{h}_k$ around one of the Dirac points at $-K$ with $p = -K + k$. Here $\Gamma_j = ta$, $v_3 = t_2 a$, $b = t_1 a^2/2$, $d_4(p) = p_2^2 - p_1^2$, $d_5(p) = 2p_1p_2$. Even though in the bare theory $v_3 = v_\perp \equiv v$ (for $t = t_2$), in general their RG flows are different when $|b| > 0$, as it breaks the rotational symmetry between $p_\perp = (p_1, p_2)$ and $p_3$. To incorporate the effects of disorder we consider the following imaginary time ($\tau$) Euclidean action in $d$ dimensions 46 [48]
\[
S = \int d\tau d^d x \Psi^\dagger \left[ \partial_\tau + \hat{h}(p) \Psi \right] - i \nabla \Phi - \Phi \right] \Psi + \frac{1}{2\Delta} \int d^3 x \Phi |\nabla|^{
u} \Phi.
\]
Here $\Phi$ is the disorder field that minimally couples to the four-component fermionic fields ($\Psi$) like a gauge field.
The two-point correlator for the disorder fields in the real and momentum space are respectively
\[
\langle \Phi(x)\Phi(y) \rangle = \frac{\Delta}{|x-y|^{d-m}}, \quad \langle \Phi(q)\Phi(0) \rangle = \frac{\Delta}{|q|^m}.
\]
As \(m \to 0\), we recover Gaussian white noise distribution.

The scaling dimensions of momentum and frequency are \(|q| = 1\) and \(|\omega| = \nu\), respectively. The scale invariance of the action \(S\) implies \(|\Psi| = d/2\), \(|\nu| = z - 1\), \(|b| = z - 2\), and \(|\Phi| = z + \eta_F\), where \(\eta_F\) is the anomalous dimension of the disorder field, yielding \(|\Delta| = 2(z + \eta_F) - (d - m)\).

At the clean HOTDSM fixed point \(z = 1\) due to linearly dispersing excitations at sufficiently low energies (the regime of ultimate interest), and \(\eta_F = 0\) (due to the gauge invariance of \(S\)). Therefore \(|\Delta| = m - 1\) in \(d = 3\), showing that (a) for Gaussian white noise distribution (\(m = 0\)) disorder is an irrelevant perturbation at the HOTDSM fixed point (since \(|\Delta| = -1\)), and (b) strong disorder phenomena (such as a QPT to a metal) can be addressed by performing a controlled RG analysis in terms of a small parameter \(c = 1 - m\), about \(c = 0\) for which disorder is marginal, following the general spirit of \(c\) expansion, even though ultimately we set \(c = 1\).

To derive the RG flow equations, we integrate out a thin momentum shell \(|\Delta c^\epsilon, \Delta|\], where \(\epsilon(>0)\) is the logarithm of the RG scale and \(\Delta\) is the ultraviolet cutoff. The relevant Feynman diagrams are shown in Fig. 4(c) and (d)]. After accounting for the quantum corrections up to the leading-order, the RG flow equations read [56]

\[
\frac{dv_3}{d\ell} = g v_3 \left[ 2 f_1(x) - f_3(x) \right], \quad \frac{d\nu}{d\ell} = -v_3, \quad \frac{dx}{d\ell} = -x,
\]

where \(g = \Delta c^\epsilon/(2\pi^2v_3^2)\) is the dimensionless disorder coupling, \(x = b\Delta/\nu\) is also dimensionless, and

\[
f_1(x) = \frac{1}{2} \int_0^{\pi} d\theta \frac{S_\theta}{1 + x^2S_\theta^2}, \quad f_3(x) = \int_0^{\pi} d\theta \frac{S_\theta^2}{(1 + x^2S_\theta^2)^2},
\]

\[
f_\perp(x) = \frac{1}{2} \int_0^{\pi} d\theta \frac{S_\theta^3}{(1 + x^2S_\theta^2)^2}.
\]

![FIG. 4: Scaling of DOS with (a) the \(C_4\) symmetry breaking Wilson mass \((t_1)\), yielding a HOTDSM, for fixed \(t\) or Fermi velocity and (b) \(t\) for fixed \(t_1\) in the clean system of \(L = 200\) for \(W = 0.05\). With increasing \(t_1\) or \(t\) the DOS decreases, without altering the \(\rho(E) \sim |E|^2\) scaling, leading to the enhancement of critical disorder strength (\(W_c\)) for the onset of metallicity, see Fig. 1 and Table I. Feynman diagrams contributing to the leading-order RG analysis are shown in (c) and (d). Here solid (dashed) lines represent fermion (disorder) fields.](image)
with $S_0 \equiv \sin \theta$, $C_\theta \equiv \cos \theta$. Since we are interested in the leading-order RG analysis, (1) the quantum corrections are computed by setting $v_\perp = v_3 = v$, and (2) only the engineering dimension of $x$ has been taken into account. The flow equation of $v_3$ is fixed by its scaling dimension, yielding dynamic scaling exponent $z = 1 + g [2 f_1(x) - f_3(x)]$. The flow equation for $g$ then supports two fixed points: (1) an infrared stable one at $g = 0$, describing a clean HOTDSM, and (2) an infrared unstable QCP at $g = g_{\ast} \equiv \epsilon/2[2 f_1(x) - f_3(x)]$. The latter one controls the QPT into a metallic phase at finite disorder, where $z = 1 + \epsilon/2 = 3/2$ for Gaussian white noise distribution ($\epsilon = 1$ for any $x$). The CLE, defined as $\nu^{-1} = d(dg/d\ell)/dg|_{g=g_{\ast}} = \epsilon$ at the dirty QCP. Note that $(g_{\ast}, x)$ determines the phase boundary between the DSMs and a metal [the blue line in Fig. 1(b)], which is symmetric about $x = 0$, as all the functions in Eq. (11) are symmetric under $x \rightarrow -x$. In principle, one can also obtain DSE $z$ from the flow equation of $v_\perp$, which however does not alter any outcome qualitatively.

Finally we note that the four-fold symmetry breaking Wilson mass $(x)$ is an irrelevant parameter. Thus in the deep infrared regime ($\ell \rightarrow \infty$) $x \rightarrow 0$, where $f_\perp(0) = f_3(0) = 2/3$, and the velocity anisotropy becomes marginal. Consequently, the ratio $v_\perp/v_3$ ultimately flows to its bare value, set by the hopping parameters $t$ and $t_\perp$. Therefore, the Wilson mass $(x)$ only changes the location of the dirty QCP $(g_{\ast})$ without altering the universality class of the semimetal-metal QPT (determined by $\nu$ and $z$; see Table I), giving rise to a superuniversality in the entire family of dirty DSMs, that includes its first- and second-order cousins.

**Discussion.** Here we investigate the stability of a HOTDSM in the presence of quenched charge impurities, and identify a semimetal-metal QPT at finite disorder. While the topological hinge modes gradually melt across this transition (similar to the Fermi arcs in conventional dirty Weyl semimetals [57]), see Fig. 2, we come to the conclusion that the topological order (first or second) does not affect its universality class, see Table I. Since real materials are inherently dirty, the stability of HOTDSM is critical for its experimental realization and observation of the hinge modes (via scanning tunneling microscopy, for example) in sufficiently clean systems. We also note that HOTDSM in general is more stable than its first-order counterpart (due to the suppression of DOS by the Wilson mass), see Fig. 1. In the future, it will be worthwhile to investigate the nature of the critical wavefunctions in dirty HOTDSM, and search for measurable signatures of the discrete symmetry breaking.

An interesting limit of the lattice model in Eq. (1) is when $t = 0$. The system then describes a double first-order DSM, supporting four Fermi arcs in the $k_x,k_z$ or $k,y,k_z$ plane. However, due to the $|E|$-linear DOS this system is unstable in the presence of infinitesimal disorder. A finite $t$ converts the double first-order DSM into a second-order DSM, where DOS vanishes as $\rho \sim |E|^2$, and stabilizes the system against sufficiently weak disorder.

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[1] M. Z. Hasan and C. L. Kane, Rev. Mod. Phys. 82, 3045 (2010).
[2] X.-L. Qi and S.-C. Zhang, Rev. Mod. Phys. 83, 1057 (2011).
[3] B. A. Bernevig and T. L. Hughes, Topological Insulators and Topological Superconductors (Princeton University Press, Princeton, New Jersey, 2013).
[4] C.-K. Chiu, J. C. Y. Teo, A. P. Schnyder, and S. Ryu, Rev. Mod. Phys. 88, 035005 (2016).
[5] N. P. Armitage, E. J. Mele, and A. Vishwanath, Rev. Mod. Phys. 90, 15001 (2018).
[6] W. A. Benalcazar, B. A. Bernevig, and T. L. Hughes, Science 357, 61 (2017).
[7] F. Schindler, Z. Wang, M. G. Vergniory, A. M. Cook, A. Murani, S. Sengupta, A. Y. Kasumov, R. Debloch, S. Jeon, I. Drozdov, H. Bouchiat, S. Guron, A. Yazdani, B. A. Bernevig, and T. Neupert, Nat. Phys. 14, 918 (2018).
[8] Z. Song, Z. Fang, and C. Fang, Phys. Rev. Lett. 119, 246402 (2017).
[9] W. A. Benalcazar, B. A. Bernevig, and T. L. Hughes, Phys. Rev. B 96, 245115 (2017).
[10] J. Langbehn, Y. Peng, L. Trifunovic, F. von Oppen, and P. W. Brouwer, Phys. Rev. Lett. 119, 246401 (2017).
[11] S. Franca, J. van den Brink, and I. C. Fulga, Phys. Rev. B 98, 201144(R) (2018).
[12] F. Schindler, A. M. Cook, M. G. Vergniory, Z. Wang, S. P. Parkin, B. A. Bernevig, and T. Neupert, Sci. Adv. 4, eaat0346 (2018).
[13] Z. Wang, B. J. Wieder, J. Li, B. Yan, and B. A. Bernevig, Phys. Rev. Lett. 123, 186401 (2019).
[14] C.-H. Hsu, P. Stano, J. Klímovája, and D. Loss, Phys. Rev. Lett. 121, 196801 (2018).
[15] Y. Wang, M. Lin, and T. L. Hughes, Phys. Rev. B 98, 165144 (2018).
[16] M. Lin and T. L. Hughes, Phys. Rev. B 98, 241103(R) (2018).
[17] D. Çăăgărău, V. Jurićić, and B. Roy, Phys. Rev. B 99, 041301(R) (2019).
[18] S. A. A. Ghorashi, X. Hu, T. L. Hughes, and E. Rossi, Phys. Rev. B 100, 020509 (2019).
[19] Y. Volpe, D. Loss, and J. Klímovája, Phys. Rev. Lett. 122, 126402 (2019).
[20] T. Nag, V. Jurićić, and B. Roy, Phys. Rev. Research 1, 032045 (2019).
[21] K. Plekhanov, M. Thakurathi, D. Loss, and J. Klímovája, Phys. Rev. Research 1, 032013 (2019).
[22] Z. Yan, Phys. Rev. Lett. 123, 177001 (2019).
[23] B. Roy, Phys. Rev. Research 1, 032048 (2019).
[24] B. J. Wieder, Z. Wang, J. Cano, X. Dai, L. M. Schoop, B. Bradlyn, and B. A. Bernevig, Nat. Commun. 11, 627 (2020).
[25] H. Hu, B. Huang, E. Zhao, and W. V. Liu, Phys. Rev.
[26] A. Szabó, R. Moessner, and B. Roy, arXiv:1907.12568
[27] R.-X. Zhang, Y.-T. Hsu, and S. Das Sarma, arXiv:1909.07980
[28] A. Weiße, G. Wellein, A. Alverman, and H. Feshke, Rev. Mod. Phys. 78, 275 (2006).
[29] R. Jackiw and C. Rebbi, Phys. Rev. D 13, 3398 (1976).
[30] E. Fradkin, Phys. Rev. B 33, 3263 (1986).
[31] P. Goswami and S. Chakravarty, Phys. Rev. Lett. 107, 196803 (2011).
[32] Y. Ominato and M. Koshino, Phys. Rev. B 89, 054202 (2014).
[33] K. Kobayashi, T. Ohtsuki, K.-I. Imura, and I. F. Herbut, Phys. Rev. Lett. 112, 016402 (2014).
[34] B. Roy and S. Das Sarma, Phys. Rev. B 90, 241112(R) (2014); ibid. 93, 119911(E) (2016).
[35] B. Sbierski, G. Pohl, E. J. Bergholtz, and P. W. Brouwer, Phys. Rev. Lett. 113, 026602 (2014).
[36] S. V. Syzranov, L. Radzihovsky, and V. Gurarie, Phys. Rev. Lett. 114, 166601 (2015).
[37] B. Sbierski, E. J. Bergholtz, and P. W. Brouwer, Phys. Rev. B 92, 115145 (2015).
[38] J. H. Pixley, P. Goswami, and S. Das Sarma, Phys. Rev. Lett. 115, 076601 (2015).
[39] S. V. Syzranov, P. M. Ostrovsky, V. Gurarie, and L. Radzihovsky, Phys. Rev. B 93, 155113 (2016).
[40] J. H. Pixley, P. Goswami, and S. Das Sarma, Phys. Rev. B 93, 085103 (2016).
[41] S. Liu, T. Ohtsuki, and R. Shindou, Phys. Rev. Lett. 116, 066401 (2016).
[42] S. Bera, J. D. Sau, and B. Roy, Phys. Rev. B 93, 201302(R) (2016).
[43] B. Roy, V. Juričić, and S. Das Sarma, Sci. Rep. 6, 32446 (2016).
[44] T. Louvet, D. Carpentier, and A. A. Fedorenko, Phys. Rev. B 94, 220201(R) (2016).
[45] P. Goswami and S. Chakravarty, Phys. Rev. B 95, 075131 (2017).
[46] T. Louvet, D. Carpentier, and A. A. Fedorenko, Phys. Rev. B 95, 014204 (2017).
[47] T. S. Sikkenk and L. Fritz, Phys. Rev. B 96, 155121 (2017).
[48] B. Roy, R.-J. Slager, V. Juričić, Phys. Rev. X 8, 031076 (2018).
[49] I. Balog, D. Carpentier, and A. A. Fedorenko, Phys. Rev. Lett. 121, 166402 (2018).
[50] J. Klier, I. V. Gornyi, and A. D. Mirlin, Phys. Rev. B 100, 125160 (2019).
[51] E. Brillaux, D. Carpentier, and A. A. Fedorenko, Phys. Rev. B 100, 134204 (2019).
[52] T. Hirosawa, H. Maebashi, M. Ogata, arXiv:1912.11996
[53] R. Nandkishore, D. A. Huse, and S. L. Sondhi, Phys. Rev. B 89, 245110(2014).
[54] J. H. Pixley, D. A. Huse, and S. Das Sarma, Phys. Rev. X 6, 021042 (2016).
[55] M. Buchhold, S. Diehl, and A. Altland, Phys. Rev. Lett. 121, 215301 (2018).
[56] See Supplementary Materials at XXX-XXXX for the details of the RG analysis and scaling analysis of DOS.
[57] R.-J. Slager, V. Juričić, and B. Roy, Phys.Rev. B 96, 201401 (2017).