On the Computation and Approximation of Outage Probability in Satellite Networks with Smart Gateway Diversity

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Abstract—The utilization of extremely high frequency (EHF) bands can achieve very high throughput in satellite networks (SatNets). Nevertheless, the severe rain attenuation at EHF bands imposes strict limitations on the system availability. Smart gateway diversity (SGD) is considered indispensable in order to guarantee the required availability with reasonable cost. In this context, we propose and examine a new SGD architecture, namely, load-sharing SGD (LS-SGD). For this diversity scheme, we define the system outage probability (SOP) using a rigorous probabilistic analysis based on the Poisson binomial distribution (PBD), and taking into consideration the traffic demand as well as the gateway (GW) capacity. Furthermore, we provide several methods for the exact and approximate calculation of SOP. As concerns the exact computation of SOP, a closed-form expression and an efficient algorithm based on a recursive formula are given, both with quadratic worst-case complexity in the number of GWs. Finally, the proposed approximation methods include probability-distribution approximations and Chernoff bounds.

Index Terms—Satellite networks, smart gateway diversity, outage probability, Poisson binomial distribution, recursive formula, probability-distribution approximations, Chernoff bounds.

I. INTRODUCTION

Next-generation broadband SatNets require very high data-rates (up to 1 Tbps) that can be accomplished by utilizing EHF bands (above 30 GHz) in the feeder links. Although the frequency shift from Ka (20/30 GHz) to Q/V (40/50 GHz) or W (75-110 GHz) bands provides more spectrum, the high levels of rain attenuation (tens of dB) cannot be tackled by the standard fade mitigation techniques (FMTs), such as uplink power control (ULPC), adaptive coding and modulation (ACM) and data rate adaptation (DRA). As a result, gateway diversity (GD) is necessary to achieve high system availability, since it is a more effective and powerful FMT (at the expense of installing additional GWs) [1]–[5]. Nevertheless, the conventional GD (where the same signal is transmitted by two or three GWs) is economically prohibitive for reaching the Tbps due to the large number of required GWs [6]. An alternative solution to achieve high availability with reasonable cost is the smart gateway diversity (SGD), where a user beam can be served by different GWs depending on the propagation conditions and the traffic load. In particular, if a GW experiences deep fades then its traffic can be rerouted to other GWs with better propagation conditions.

A. Related Work

In [6], two SGD techniques are examined, namely, the frequency multiplexing diversity and the \( N + P \) diversity. The performance analysis of these schemes is based on a simple probabilistic model, assuming the same outage probability for each GW (although unusual in practice) as well as independent propagation conditions over the GW locations. Moreover, the authors in [7] study the \( N \)-active diversity (with time or frequency multiplexing, taking into account the spatial correlation between the GWs) and the \( N + P \) diversity (where there are \( N \) active plus \( P \) redundant or idle GWs). In the former scheme, all the \( N \) GWs are active and each user beam is served by a group of GWs, whereas in the latter scheme each user beam is served by only one GW and switches to a redundant GW in case of outage.

A novel GW switching scheme for the \( N + P \) scenario is proposed in [8], using a dynamic rain attenuation model and considering two key performance indicators: the average outage probability and the average switching rate. Furthermore, a different SGD scheme, where there is no redundant GWs but each GW should have some spare capacity, is analyzed in [9]. Specifically, in nominal clear-sky conditions all GWs are active and operate using a maximum fraction of their full capacity, while if some GWs experience heavy rain attenuation then their traffic is served by the remaining GWs using their extra capacity. Finally, an extension of the well-known \( N \)-active and \( N + P \) diversity schemes to multiple-input-multiple-output (MIMO) architectures is presented in [10].

B. Contribution

The main contributions of this work, in comparison with existing approaches, are as follows:

- In this paper, we introduce and analyze a new SGD architecture operating in a load-sharing mode, where the GWs do not necessarily have equal outage probabilities.
- Unlike previous research, we present a system-level approach taking into account the traffic demand as well as the GW capacity. In particular, we are interested in the system outage probability (SOP), defined as the probability of not satisfying the overall traffic demand, which is a stricter performance metric than the user outage probability (UOP), i.e., the probability of not satisfying the traffic demand of a specific user.
- Furthermore, we study the performance improvement (in terms of SOP) that can be achieved by increasing the number of GWs in the proposed diversity scheme. For this purpose, we define two comparative metrics, namely, the SOP-improvement factor and the generalized SOP-improvement factor.
- In addition, exact methods for the computation of SOP are given, including a closed-form expression and an
efficient algorithm based on a recursive formula. The worst-case complexity of both methods is quadratic in the number of GWs.

- Finally, we provide some approximation methods for the estimation of SOP. More specifically, the SOP can be approximated by various probability distributions (normal, binomial, Poisson, translated Poisson) as well as Chernoff upper/lower bounds.

C. Paper Organization & Mathematical Notation

The remainder of this article is organized as follows. Firstly, Section II describes and analyzes the new SGD architecture. Moreover, Sections III and IV present exact and approximation methods for calculating the SOP, respectively. In addition, the performance of the proposed diversity scheme as well as the accuracy of the approximation methods are examined in Section V. Finally, concluding remarks are given in Section VI, and some preliminaries on probability distributions are provided in the Appendix.

Mathematical notation: $\mathbb{Z}^+ = \{1, 2, 3, \ldots \}, \mathbb{Z}_0^+ = \{0, 1, 2, \ldots \}$, $\mathcal{N} = \{1, 2, \ldots, N\}$ and $\mathcal{N}_0 = \{0, 1, \ldots, N\}$, where $N \in \mathbb{Z}^+$. Moreover, $\mathbb{P}(\cdot)$ and $\mathbb{E}(\cdot)$ denote probability and expectation, respectively. $\lfloor \cdot \rfloor$ and $\lceil \cdot \rceil$ are respectively the floor and ceiling functions. In addition, $|x|$ and $\langle x \rangle = x - |x|$ represent the absolute value and the fractional part of $x \in \mathbb{R}$ respectively ($0 \leq \langle x \rangle < 1$), while $|S|$ stands for the cardinality of a set $S$. $0_N$ and $1_N$ denote the $N$-dimensional all-zeros and all-ones vectors, respectively. Finally, the total variation distance between two (discrete) random variables (RVs) $X$ and $Y$ on $\mathbb{Z}_0^+$ is defined as follows:

$$d_{\text{TV}}(X, Y) = \sup_{A \subseteq \mathbb{Z}_0^+} |\mathbb{P}(X \in A) - \mathbb{P}(Y \in A)| = \frac{1}{2} \sum_{m \in \mathbb{Z}_0^+} |\mathbb{P}(X = m) - \mathbb{P}(Y = m)|$$

Mathematical conventions: $\sum_{i \in \emptyset} a_i = 0$ and $\prod_{i \in \emptyset} a_i = 1$.

II. SMART GATEWAY DIVERSITY ARCHITECTURE

In this section, we describe and analyze a new SGD scheme, namely, load-sharing SGD (LS-SGD), where the unused capacity of available (not in outage) GWs can be exploited to serve the users of the remaining GWs (which are in outage).

A. System Model

Consider a SatNet consisting of a geostationary satellite and a ground network of $N \in \mathbb{Z}^+$ (geographically distributed) GWs, which are denoted by the set $\mathcal{N} = \{1, 2, \ldots, N\}$. All the GWs are connected to a network control center (NCC) through dedicated terrestrial links. The NCC performs, when necessary (in case of deep fading), the traffic switching/rerouting between the GWs. Furthermore, the following analysis focuses on the feeder links (data transmission from the GWs to the satellite), considering ideal (without noise and interference) satellite-user links.

In addition, the distance between any two different GWs is large enough (some hundreds of km), and thus the spatial correlation of the propagation impairments at the GW locations is extremely small [6, 13]. As a result, the rain attenuations/fades experienced by the GWs can be considered (mutually) independent. It is also assumed that there is no ACM, so each feeder link is either available at full capacity or completely unavailable. Therefore, the feeder links can be mathematically modeled as a set $\{X_n\}_{n \in \mathcal{N}}$ of independent, but not necessarily identically distributed, Bernoulli RVs ($X_n \sim \text{Bern}(p_n), \forall n \in \mathcal{N}$), where $p_n \in [0, 1]$ is the outage/exceedance probability of the $n$th link/GW (i.e., the probability that the rain attenuation exceeds a specific threshold); some methods for calculating $p_n$ are discussed in [9]. Moreover, we define the RV $S_N = \sum_{n \in \mathcal{N}} X_n \sim \text{PoisBin}(p_N)$, with $p_N = [p_1, p_2, \ldots, p_N]$, which is the total number of GWs that are in outage in the set $\mathcal{N}$. The expectation, the standard deviation, and the 3rd central moment of $S_N$ are given respectively by:

$$
\mu_N = \mathbb{E}(S_N) = \sum_{n \in \mathcal{N}} p_n \quad (2)
$$

$$
\sigma_N = \sqrt{\mathbb{E}\left((S_N - \mu_N)^2\right)} = \sqrt{\sum_{n \in \mathcal{N}} p_n(1 - p_n)}
$$

$$
\nu_N = \mathbb{E}\left((S_N - \mu_N)^3\right) = \sum_{n \in \mathcal{N}} p_n(1 - p_n)(1 - 2p_n) \quad (4)
$$

Note that $\mu_N \geq \sigma_N^2$, $\mu_N \in [0, N]$, $\sigma^2_N \in [0, N/4]$, and $\nu_N \in [-N/(6\sqrt{3}), N/(6\sqrt{3})]$.

B. System Outage Probability

In the sequel, suppose that the $n$th GW can offer a maximum data-rate (capacity) $R_{\text{tot}}^n > 0$, and the total requested data-rate (traffic demand) is $R_{\text{req}}^t = \sum_{u \in \mathcal{U}} R_{\text{req}}^u > 0$, where $\mathcal{U} = \{1, 2, \ldots, U\}$ is the set of users and $R_{\text{req}}^u \geq 0$ is the requested data-rate of user $u$. Moreover, the operation of NCC ensures the following load-sharing property: all users receive their requested data-rate if and only if (iff) the overall capacity of the available (not in outage) GWs is greater than or equal to the traffic demand. Equivalently, there is at least one user that receives inadequate data-rate iff the overall capacity of the available GWs is less than the traffic demand. In this context, the SOP is defined as follows:

$$
P_{\text{out}}^{\text{sys}} = \sum_{A \subseteq \mathcal{F}} \prod_{i \in A} p_i \prod_{j \in \mathcal{N} \setminus A} (1 - p_j) \quad (5)
$$

where $\mathcal{F} = \left\{ A \subseteq \mathcal{N} : \sum_{j \in \mathcal{N} \setminus A} R_j^{\text{max}} < R_{\text{req}}^t \right\}$. In other words, $\mathcal{F}$ contains all the subsets $A$ of the $N$ GWs such that: if the

1The details on the switching/handover procedure are beyond the scope of this paper; see [6, 8, 11] for more information on this important topic.

2As concerns the downlink of multibeam satellite systems, an energy-efficient power allocation in order to jointly minimize the unmet system capacity and the total radiated power is proposed in [12].
GWs in $A$ are all in outage and the remaining GWs in $\mathcal{N}\setminus A$ are all available (not in outage), then the traffic demand cannot be satisfied by the latter group of GWs. In essence, the SOP expresses the probability of not satisfying the traffic demand of all users (or, equivalently, the probability that there is at least one user that receives inadequate data-rate). Moreover, we can define the system availability (SA) as the probability of the complementary event: $P^{sys}_{\text{out}} = 1 - P^{sys}_{\text{out}}$.

For simplicity, we assume that all GWs have the same capacity in the rest of the paper, i.e., $R^\text{max}_{\text{GW}} = R^\text{max}_{\text{GW}} > 0$, $\forall n \in \mathcal{N}$. In this case, $F = \{A \subseteq \mathcal{N} : (N - |A|)R^\text{max}_{\text{GW}} < R^\text{out}_{\text{GW}}\}$. By defining the ratio of the traffic demand to the GW capacity:

$$r = \frac{R^\text{req}}{R^\text{max}_{\text{GW}}} > 0$$

we have that $(N - |A|)R^\text{max}_{\text{GW}} < R^\text{out}_{\text{GW}}$ if and only if $N - |A| < r$. Therefore, $N - |A| < r$ if and only if $|A| > N - [r] + 1$. Furthermore, we can define:

$$L = N - [r] + 1$$

and thus $F = \{A \subseteq \mathcal{N} : |A| \geq L\} = \bigcup_{m=L}^{N} C_m$, where $C_m = \{A \subseteq \mathcal{N} : |A| = m\}$. Consequently, (5) reduces to the following expression:

$$P^{\text{sys}}_{\text{out}} = P^{\text{sys}}_{\text{out}}(L, N) = \sum_{m=L}^{N} \sum_{A \in C_m} \prod_{i \in A} p_i \prod_{j \in \mathcal{N}\setminus A} (1 - p_j)$$

According to Appendix-C, $P^{\text{sys}}_{\text{out}}(L, N) = \sum_{m=L}^{N} \prod_{i \in \mathcal{N}} p_i^{N_i} \prod_{j \in \mathcal{N}\setminus A} (1 - p_j)$ which is obtained when $r = 1$ (since $1 \leq [r] \leq \min(1, N) = 1$ and that $P^{\text{sys}}_{\text{out}}(N, N) > 0$, is defined as follows:

$$I = \frac{P^{\text{sys}}_{\text{out}}(1, 1)}{P^{\text{sys}}_{\text{out}}(N, N)} = \frac{p_1}{\prod_{n=2}^{N} p_n} \geq 1$$

Next, consider a diversity system with $N + K$ GWs ($K \in \mathbb{Z}_+^0$) all of which have the same capacity $R^\text{max}_{\text{GW}} > 0$, and $[r] \in \mathcal{N}$ (since $1 \leq [r] \leq \min(N, N + K) = N$). Furthermore, let $K = \{N + 1, N + 2, \ldots, N + K\}$ be the set of additional GWs, and $p_{\mathcal{N}\cup\mathcal{K}} = p_{\mathcal{N}}$. $p_{\mathcal{K}} = [p_1, p_2, \ldots, p_{N+K}]$ be the vector of GW outage probabilities, where $p_{\mathcal{K}} = [p_{N+1}, p_{N+2}, \ldots, p_{N+K}]$. Suppose also that $\{X_i\}_{i \in \mathcal{N}\cup\mathcal{K}}$ is a set of independent, but not necessarily identically distributed, Bernoulli RVs ($X_i \sim \text{Bern}(p_i)$, $\forall i \in \mathcal{N}\cup\mathcal{K}$). Besides $\mathcal{N}$, we define the RVs $S_{\mathcal{K}} = \sum X_k \sim \text{PoisBin}(p_{\mathcal{K}})$ and $S_{\mathcal{N}\cup\mathcal{K}} = \sum_{i \in \mathcal{N}\cup\mathcal{K}} X_i = S_{\mathcal{N}} + S_{\mathcal{K}} \sim \text{PoisBin}(p_{\mathcal{N}\cup\mathcal{K}})$ denoting the total number of GWs which are in outage in the sets $\mathcal{K}$ and $\mathcal{N}\cup\mathcal{K}$, respectively. For this diversity system $L' = N + K - [r] + 1 = L + K$, $\forall L' \in \{K + 1, K + 2, \ldots, N + K\}$. Finally, $P^{\text{sys}}_{\text{out}} = P^{\text{sys}}_{\text{out}}(L, N) = \mathbb{P}(S_{\mathcal{N}} \leq L)$ and $P^{\text{sys}}_{\text{out}}(L', N) = \mathbb{P}(S_{\mathcal{N}\cup\mathcal{K}} = L')$ stand for the SOP of the $N$-GW and $(N + K)$-GW diversity systems, respectively. Now, by virtue of the law/theorem of total probability, we get:

$$P^{\text{sys}}_{\text{out}}(L, N) = \sum_{j=0}^{K} \mathbb{P}(S_{\mathcal{K}} = j)\mathbb{P}(S_{\mathcal{N}} + S_{\mathcal{K}} \geq L) = \sum_{j=0}^{K} \mathbb{P}(S_{\mathcal{K}} = j)\mathbb{P}(S_{\mathcal{N}} \geq L - K - j) = \sum_{j=0}^{K} \mathbb{P}(S_{\mathcal{K}} = j)\mathbb{P}(S_{\mathcal{N}} \geq L) - \mathbb{P}(S_{\mathcal{N}} \leq L - K - j) \leq \mathbb{P}(S_{\mathcal{N}} \geq L) \leq \mathbb{P}(S_{\mathcal{N}} \geq L) = P^{\text{sys}}_{\text{out}}(N, N) = P^{\text{sys}}_{\text{out}}(L, N)$$

As a result, $P^{\text{sys}}_{\text{out}}(N, N) \leq P^{\text{sys}}_{\text{out}}(N, N)$ In view of this fact, we can generalize the notion of SOP-improvement factor. Specifically, we define the generalized SOP-improvement factor of the $(N + K)$-GW over the $N$-GW diversity system as follows (assuming the same $[r] \in \mathcal{N}$ and that $P^{\text{sys}}_{\text{out}}(N, N) > 0$):

$$I_g = \frac{P^{\text{sys}}_{\text{out}}(N, N)}{P^{\text{sys}}_{\text{out}}(L, N)} = \frac{P^{\text{sys}}_{\text{out}}(L, N)}{P^{\text{sys}}_{\text{out}}(L + K, N + K)} \bigg|_{L = N - [r] + 1} \geq 1$$

Notice that by setting $N = 1$ and $K = N' - 1$ (thus $[r] = 1$ and $L = 1$), we obtain $I_g = \frac{P^{\text{sys}}_{\text{out}}(1, 1)}{P^{\text{sys}}_{\text{out}}(N, N)} = I$. Finally, we would like to emphasize that by increasing the number of GWs the SOP decreases, but higher GW connectivity is required. In other words, there is a trade-off between performance improvement and connectivity complexity.

### III. Exact Methods for Computing SOP

In the sequel, several techniques for the exact computation of SOP are presented. The time complexity of these methods is summarized in Table I.

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4Similar formula is also given in [9] and [14], however, without explicit dependence on the traffic demand and the GW capacity. Herein, this dependence is clearly expressed by [6] and [7]. Note that this SOP definition is a generalization of the classical SOP (i.e., the probability of having all GWs in outage), which is obtained when $[r] = 1 \Rightarrow L = N \Rightarrow P^{\text{sys}}_{\text{out}} = \prod_{n \in \mathcal{N}} p_n$; the classical SOP is used in [15] to select the (globally) minimum number of GWs satisfying SOP-requirements.

5The generalized SOP-improvement factor $I_g$ can be estimated using the approximation methods provided in Section IV.
TABLE I

| Exact Method | Direct Computation | CFE | RF (Algorithm 1) | FFT-based Algorithm [19] |
|--------------|--------------------|-----|-----------------|--------------------------|
| Time Complexity | $O(2^N N)$ | $\Theta(N^2)$ | $\Theta(L(N - L + 1)) = O(N^2)$ | $O(N \log(N)^2)$ |

A. Direct Computation

The direct computation of SOP is based on the analytic formula (9), which requires $\sum_{m=L}^{N} \frac{1}{m!} \left( L + \sum_{n \in \mathbb{N}} \frac{-c^{-n} + 1}{n!} \prod_{n \in \mathbb{N}} (1 + (c^{-n} - 1)r_m) \right)$ (12) where $c = e^{2\pi i/(N+1)}$, with $j = \sqrt{-1}$ being the imaginary unit. It is interesting to note that the CFE comprises a sum of complex numbers, but the overall outcome is a real number in $[0,1]$. The same formula is also derived in [17], using the characteristic function of the PBD as well as the DFT. Furthermore, the computational complexity of (12) is $\Theta(N^2)$.

B. Closed-Form Expression

According to [16], the SOP can be calculated, using polynomial interpolation and discrete Fourier transform (DFT), by the following closed-form expression (CFE):

$$P^{\text{sys}}_{\text{out}}(L, N) = 1 - \frac{1}{\alpha} \left( L + \sum_{n \in \mathbb{N}} \frac{-c^{-n} + 1}{n!} \prod_{n \in \mathbb{N}} (1 + (c^{-n} - 1)r_m) \right)$$ (12)

C. Recursive Formula

In this part, we explore the power and beauty of recursion. More specifically, from the law/theorem of total probability, the SOP $P^{\text{sys}}_{\text{out}}(L, N) = \mathbb{P}(S_N \geq L)$ can be written as follows:

$$P^{\text{sys}}_{\text{out}}(L, N) = \sum_{j=0}^{\infty} \mathbb{P}(X_N = j)\mathbb{P}(S_N \geq L | X_N = j) =$$

$$= \sum_{j=0}^{\infty} \mathbb{P}(X_N = j)\mathbb{P}(S_N \geq L - j) =$$

$$= \mathbb{P}(X_N = 0)\mathbb{P}(S_N \geq L) + \mathbb{P}(X_N = 1)\mathbb{P}(S_{N\setminus\{N\}} \geq L - 1)$$ (13)

where $S_{N\setminus\{N\}} = \sum_{n \in \mathbb{N}\setminus\{N\}} X_n$. Therefore, we have proved the following recursive formula (RF):

$$P^{\text{sys}}_{\text{out}}(L, N) = (1 - px)L^{\text{sys}}_{\text{out}}(L, N - 1) + pxL^{\text{sys}}_{\text{out}}(L - 1, N - 1)$$ (14)

with initial/boundary conditions: a) $P^{\text{sys}}_{\text{out}}(0, N) = 1$ and b) $P^{\text{sys}}_{\text{out}}(N + 1, N) = 0, \forall N \in \mathbb{Z}^+$. It can be verified, using mathematical induction, that (8) is the solution of (14). To the best of our knowledge, this RF is derived for the first time in [18], making use of symmetric switching functions. Our proof, however, is much simpler.

Algorithm 1 presents an efficient method to compute the SOP using the RF, which follows directly from the algorithm given in [18]. The time complexity of Algorithm 1 is $\Theta(L(N - L + 1)) = O(N^2)$, with best-case complexity $\Theta(1)$ for $L = 0$, and worst-case complexity $\Theta(N^2)$ for $L = \lfloor N/2 \rfloor$ and $L = \lceil N/2 \rceil$. Moreover, notice that the complexity is $\Theta(N)$ for $L = 1$ and $L = N$. As a result, Algorithm 1 has lower complexity in some cases than the CFE which requires $\Theta(N^2)$ operations regardless of $L$. Finally, the space complexity of Algorithm 1 is $\Theta(N + L) = \Theta(N)$.

D. FFT-based Algorithm

An even more efficient algorithm for computing the SOP is provided in [19]. In particular, this method recursively applies the fast Fourier transform (FFT) to compute generating-function products, thus achieving an overall complexity of $O(N \log(N)^2)$.

IV. APPROXIMATION METHODS FOR ESTIMATING SOP

Afterwards, we introduce some useful methods to approximate the SOP, exploiting the fact that $P^{\text{sys}}_{\text{out}}(L, N) = \mathbb{P}(S_N \geq L) = 1 - \mathbb{P}(S_N \leq L - 1), \forall L \in \mathbb{N}_{0}$. These techniques are divided into two categories: probability-distribution approximations (PDA) as well as Chernoff upper/lower bounds (CUB/CLB). For convenience, a summary of approximation methods is given in Table II

A. Probability-Distribution Approximations

1) Normal Approximation (NA): According to [20], the central limit theorem (CLT) for the PBD states that:

$$\lim_{N \to \infty} \Delta_N = 0 \quad \text{(asymptotic normality of } (S_N - \mu_N)\sigma_N^{-1})$$

if and only if (iff) $\lim_{N \to \infty} \sigma_N^2 = \infty$, where $\Delta_N = \sup_{s \in \mathbb{R}} |\mathbb{P}(S_N \leq s) - \Phi \left( (s - \mu_N)\sigma_N^{-1} \right)|$ and $\Phi(\cdot)$ is the CDF of the standard normal distribution (see Appendix-F). Therefore, by applying a continuity correction [18], the SOP can be approximated by:

$$P^{\text{sys}}_{\text{out}}(L, N) \approx 1 - \Phi(\zeta) = Q(\zeta)$$ (15)

where $\zeta = (L - \mu_N - 0.5)\sigma_N^{-1}$ and $Q(\cdot)$ is defined in Appendix-F.

4In probability theory, a continuity correction is an adjustment that is made when a discrete (probability) distribution is approximated by a continuous distribution. In particular, suppose that the continuous RV $Y$ approximates the discrete RV $X$. Then, $\mathbb{P}(X \leq m) = \mathbb{P}(Y \leq m + 0.5) \approx F(Y \leq m + 0.5), \forall m \in \mathbb{Z}$.
2) Refined Normal Approximation (RNA): Consider the following function:
\[
G(x) = \Phi(x) + \nu_N(6\sigma_N^2)^{-1}(1-x^2)\varphi(x)
\]
where \(\Phi(\cdot)\) and \(\varphi(\cdot)\) are respectively the CDF and PDF of the standard normal distribution (see Appendix-F). The added term improves the accuracy of the NA even for relatively small \(N\). In particular, according to [20]–[22], there exists a (universal) constant \(C < \infty\) such that:
\[
\Delta_N = \sup_{x \in \mathbb{R}}|\mathbb{P}(S_N \leq x) - G((s - \mu_N, \sigma_N^{-1})|) - C\sigma_N^{-2} = O(\sigma_N^{-2}).
\]
Observe that \(\lim_{N \to \infty} \Delta_N = 0\), when \(\lim_{N \to \infty} \sigma_N^{-2} = \infty\). As a result, by applying the continuity correction once more, we obtain the following approximation:
\[
\hat{P}_{\text{out}}^{\text{sys}}(L, N) \approx \min \left( \max \left( \hat{P}_{\text{out}}^{\text{sys}}(L, N), 0 \right), 1 \right)
\]
where \(\hat{P}_{\text{out}}^{\text{sys}}(L, N) = 1 - G(\zeta)\) and \(\zeta = (L - \mu_N - 0.5)\sigma_N^{-1}\).

Note that we make use of the above min-max formula in order to ensure that \(\hat{P}_{\text{out}}^{\text{sys}}(L, N) \in [0, 1]\), because \(\hat{P}_{\text{out}}^{\text{sys}}(L, N)\) may be outside the interval \([0, 1]\) in some cases.

3) Binomial Approximation (BA): The PBD can be approximated by the binomial distribution in the following sense [23], defining \(\bar{p} = \frac{1}{N} \sum_{n \in N} p_n = \frac{\mu_N}{N}, \ bar{q} = 1 - \bar{p}\), and assuming \(p_n \in (0, 1)\): a) \(d_{\text{TV}}(S_N, Y) \leq (N/(N + 1))(1 - \bar{p}^{-N+1} - \bar{q}^{-N+1})\delta_N\), where \(Y \sim \text{Bin}(N, \bar{p})\) and \(\delta_N = 1 - (N\bar{p}\bar{q})^{-1}\sigma_N^{-2}\), and b) \(d_{\text{TV}}(S_N, Y) \to 0 \iff \delta_N \to 0\) (or, equivalently, \((N\bar{p}\bar{q})^{-1} \to 0\)). It is interesting to note that when \(p_n = \bar{p}\), \(\forall n \in N\), it holds that: \(\sigma_N^2 = N\bar{p}\bar{q} \Rightarrow \delta_N = 0 \Rightarrow d_{\text{TV}}(S_N, Y) = 0 \Rightarrow S_N \sim \text{Bin}(N, \bar{p})\), which is in agreement with Appendix-B. Consequently, the BA is given by:
\[
\hat{P}_{\text{out}}^{\text{sys}}(L, N) \approx 1 - \mathbb{P}(Y \leq L - 1) = 1 - \sum_{m=0}^{L-1} \binom{N}{m} \bar{p}^m(1-\bar{p})^{N-m}
\]
2) Translated Poisson Approximation (TPA): As stated in [24], the translated Poisson distribution can approximate the PBD as follows:
\[
d_{\text{TV}}(S_N, W) \leq \left( \sum_{n \in N} p_n(1-p_n) + 2 \right)\sigma_N^{-2}, \text{where } W \sim \text{TrPois}(\mu_N, \sigma_N^2).
\]

i.e., according to Appendix-E, \(W = [\mu_N - \sigma_N^2] + H\) with \(H \sim \text{Pois}(\vartheta)\) and \(\vartheta = \sigma_N^2 + (\mu_N - \sigma_N^2)/N\). Due to the fact that \(\sum_{n \in N} p_n(1-p_n) \leq p_{\text{max}}^{\text{sys}} \sum_{n \in N} p_n(1-p_n) = p_{\text{max}}^{\text{sys}} \sigma_N^{-2} \leq \sigma_N^{-2}\), where \(p_{\text{max}} = \max_{n \in N} \{p_n\}\), we obtain \(d_{\text{TV}}(S_N, W) \leq (p_{\text{max}}^{\text{sys}} + 2)\sigma_N^{-2} \leq (\sigma_N^2 + 2)\sigma_N^{-2} = O(\sigma_N^{-1})\). Furthermore, observe that if \(\sigma_N \to \infty\), then \(d_{\text{TV}}(S_N, W) \to 0\). Therefore, the SOP is approximately equal to:
\[
\hat{P}_{\text{out}}^{\text{sys}}(L, N) \approx 1 - \mathbb{P}(W \leq L - 1) = 1 - e^{-\vartheta} \sum_{m=0}^{\infty} \bar{p}^m(m!)^{-1}
\]
where \(Z \sim \text{Pois}(\mu_N)\). As reported in [25], Le Cam’s theorem/inequality admits various proofs using different techniques. Hence, we have that:
\[
\hat{P}_{\text{out}}^{\text{sys}}(L, N) \approx 1 - \mathbb{P}(Z \leq L - 1) = 1 - e^{-\vartheta} \sum_{m=0}^{L-1} \bar{p}^m(m!)^{-1}
\]

4) Poisson Approximation (PA): In 1960, Le Cam [24] established a remarkable inequality: \(d_{\text{TV}}(S_N, Z) \leq \sum_{n \in N} p_n\), which holds \(\forall L \in \{\lfloor \mu_N \rfloor + 1, \lfloor \mu_N \rfloor + 2, \ldots, N\}\), since \(\delta > 0 \iff L > \mu_N \iff L > \lfloor \mu_N \rfloor \iff L \geq \lfloor \mu_N \rfloor + 1\).

\[
\hat{P}_{\text{out}}^{\text{sys}}(L, N) \leq (\mu_N/L) \cdot e^{-\mu_N}
\]
2) Chernoff Lower Bound (CLB): As reported in [27], we know that: \( \mathbb{P}(S_N \leq (1 - \delta)\mu_N) \leq \left( e^{-\delta/(1 - \delta)^{1 - \delta}} \right)^{\mu_N} \), \( \forall \delta \in (0, 1) \). Now, by setting \((1 - \delta)\mu_N = L - 1\) and assuming \(\mu_N > 0\), we have the following CLB:

\[
P_{\text{sys}}^\text{out}(L, N) \geq 1 - (\mu_N/(L - 1))^{L - 1} e^{L - \mu_N - 1}
\]

(22)

which is true \( \forall L \in \{2, 3, \ldots, \lceil \mu_N \rceil \} \), since \( \delta > 0 \Leftrightarrow L - 1 < \mu_N \Leftrightarrow L - 1 < \lceil \mu_N \rceil \Leftrightarrow L - 1 \leq \lceil \mu_N \rceil - 1 \Leftrightarrow L \leq \lceil \mu_N \rceil \), and \( \delta < 1 \Leftrightarrow L > 1 \Leftrightarrow L \geq 2 \).

V. NUMERICAL RESULTS AND DISCUSSION

In this section, all results present statistical averages derived from 10^3 independent system configurations, where the GW outage probabilities \{\( p_i \)\}_{i \in N} are uniformly distributed in (0, 1). In order to evaluate the accuracy of a PDA method and the tightness/sharpness of Chernoff bounds, we define the maximum absolute error (maxAE), the root-mean-square error (RMSE), and the mean absolute error (MAE) as follows:

\[
\epsilon_{\text{max}}(N) = \max_{L \in S} \left| P_{\text{sys}}^\text{out}(L, N) - \tilde{P}_{\text{sys}}^\text{out}(L, N) \right|
\]

(23)

\[
\epsilon_{\text{rms}}(N) = \sqrt{\frac{1}{|S|} \sum_{L \in S} \left( P_{\text{sys}}^\text{out}(L, N) - \tilde{P}_{\text{sys}}^\text{out}(L, N) \right)^2}
\]

(24)

\[
\epsilon_{\text{mean}}(N) = \frac{1}{|S|} \sum_{L \in S} \left| P_{\text{sys}}^\text{out}(L, N) - \tilde{P}_{\text{sys}}^\text{out}(L, N) \right|
\]

(25)

where \( \tilde{P}_{\text{sys}}^\text{out}(L, N) \) is the approximate SOP. Moreover, for PDA methods \( S = N_0 \) (with \(|S| = N + 1\)), for CUB \( S = \{\lceil \mu_N \rceil + 1, \lceil \mu_N \rceil + 2, \ldots, N \} \) (with \(|S| = N - \lceil \mu_N \rceil \geq 1\)), and for CLB \( S = \{2, 3, \ldots, \lceil \mu_N \rceil \} \) (with \(|S| = \lceil \mu_N \rceil - 1 \geq 1\)). In general, it holds that \( \epsilon_{\text{max}}(N) \geq \epsilon_{\text{rms}}(N) \geq \epsilon_{\text{mean}}(N) \).

Firstly, we study the SOP as a function of the number of GWs, \( N \), and the ratio of the traffic demand to the GW capacity, \( r \). As shown in Fig. 1, the SOP increases with \( \lceil r \rceil \) for all values of \( N \), which is in accordance with the last sentence of Section II-B. Moreover, for any fixed \( \lceil r \rceil \), we can observe that the SOP decreases with the increase of \( N \). Nevertheless, as mentioned at the end of Section II-C, this SOP improvement is achieved in exchange for higher connectivity complexity.

Secondly, we examine the performance enhancement achieved by a \((10 + K)\)-GW compared to a 10-GW diversity system by means of the generalized SOP-improvement factor (where \( K \in \{1, 2, 3, 4, 5\} \) is the number of additional GWs). Specifically, as illustrated in Fig. 2, \( I_g \) decreases rapidly with the increase of \( \lceil r \rceil \) for every value of \( K \). Furthermore, for a given \( \lceil r \rceil \), larger number of additional GWs results in higher performance improvement.

Last but not least, Fig. 3 presents the accuracy of approximation methods versus the number of GWs. It can be observed that maxAE, RMSE, and MAE decrease as \( N \) increases for all approximation techniques (except for the maxAE of CUB/CLB which slightly increases with \( N \)). In addition, the approximation methods in descending-performance (or, equivalently, ascending-error) order are as follows: \{RNA, NA, TPA, BA, PA, CLB, CUB\}. This order is probably explained by the fact that these methods take advantage of different information about the PBD. In particular, RNA exploits the quantities \( \{\mu_N, \sigma_N ; \mu_N\} \), NA and TPA take into consideration \( \{\mu_N, \sigma_N\} \), while BA, PA and CLB/CUB use only \( \{\mu_N\} \).

Finally, it is interesting to note that RNA and NA significantly outperform the other methods (the achieved errors are of the order of \( 10^{-3} \) or \( 10^{-4} \)), while CUB and CLB exhibit by far the lowest accuracy.

VI. CONCLUSION

In this paper, we have studied a new SGD scheme in SatNets, which operates in a load-sharing mode. Furthermore, a number of useful mathematical tools have been presented in order to compute and approximate the SOP. Finally, based on the numerical results, we conclude that the SOP can be well approximated by NA and RNA, since these PDA methods achieve remarkable accuracy. Such approximations may be useful for simplifying and solving hard optimization problems with SOP-constraints in SGD-based SatNets.
Appendix
Preliminaries on Probability Distributions

In this part, we summarize some probability distributions of random variables (RVs), which are used throughout the paper.

A. Bernoulli Distribution

A binary (0/1) RV follows a Bernoulli distribution with parameter \( p \in [0, 1] \), \( X \sim \text{Bern}(p) \), if and only if (iff) its probability mass function (PMF) is given by:

\[
P(X = 1) = 1 - P(X = 0) = p \tag{26}
\]

B. Binomial Distribution

A discrete (integer-valued) RV \( X \sim \text{Bin}(N, p) \), where \( N \in \mathbb{Z}^+ \) and \( p \in [0, 1] \), iff its PMF is:

\[
P(X = m) = \binom{N}{m} p^m (1 - p)^{N - m}, \ \forall m \in \mathbb{N}_0 \tag{27}
\]

The binomial distribution is a generalization of the Bernoulli distribution, because \( \text{Bin}(1, p) \equiv \text{Bern}(p) \). Furthermore, if \( \{X_n\}_{n \in \mathcal{N}} \) is a set of independent and identically distributed (i.i.d.) Bernoulli RVs \( (X_n \sim \text{Bern}(p), \ \forall n \in \mathcal{N}) \), then \( S = \sum_{n \in \mathcal{N}} X_n \sim \text{Bin}(N, p) \).

C. Poisson Binomial Distribution

A discrete RV \( X \sim \text{PoisBin}(p) \), where \( p = [p_1, p_2, \ldots, p_N] \in [0, 1]^N \) with \( N \in \mathbb{Z}^+ \), iff its PMF is:

\[
P(X = m) = \sum_{\mathcal{A} \subseteq \mathcal{N}: |\mathcal{A}| = m} \prod_{i \in \mathcal{A}} p_i \prod_{j \in \mathcal{N} \setminus \mathcal{A}} (1 - p_j), \ \forall m \in \mathbb{N}_0 \tag{28}
\]

where \( \mathcal{C}_m = \{ \mathcal{A} \subseteq \mathcal{N}: |\mathcal{A}| = m \} \) (i.e., the set of all subsets of \( \mathcal{N} \) having \( m \) elements) with \( |\mathcal{C}_m| = \binom{N}{m} = \frac{N!}{m!(N - m)!} \).

The binomial distribution is a special case of the PBD, since \( \text{PoisBin}(p1_{\mathcal{N}}) \equiv \text{Bin}(N, p) \). Moreover, if \( \{X_n\}_{n \in \mathcal{N}} \) is a set of independent, but not necessarily identically distributed, Bernoulli RVs \( (X_n \sim \text{Bern}(p_n), \ \forall n \in \mathcal{N}) \), then \( S = \sum_{n \in \mathcal{N}} X_n \sim \text{PoisBin}(p) \).

D. Poisson Distribution

A discrete RV \( X \sim \text{Pois}(\lambda) \), where \( \lambda \geq 0 \), iff its PMF is expressed by:

\[
P(X = m) = e^{-\lambda} \lambda^m (m!)^{-1}, \ \forall m \in \mathbb{Z}_0^+ \tag{29}
\]

E. Translated Poisson Distribution

According to \[26\], a discrete RV \( X \sim \text{TrPois}(\mu, \sigma^2) \), where \( \mu \in \mathbb{R} \) and \( \sigma^2 \geq 0 \), iff \( X \) can be written as:

\[
X = [\mu - \sigma^2] + Y \tag{30}
\]

where \( Y \sim \text{Pois}(\sigma^2 + (\mu - \sigma^2)^2) \). Observe that \( \text{TrPois}(\sigma^2, \sigma^2) \equiv \text{Pois}(\sigma^2) \). If \( \mu \geq \sigma^2 \), then \( X \) takes values on \( \{\eta, \eta + 1, \eta + 2, \ldots\} \subseteq \mathbb{Z}_0^+ \), where \( \eta = [\mu - \sigma^2] \geq 0 \).
F. Normal Distribution

A continuous (real-valued) RV $X \sim \text{Norm}(\mu, \sigma^2)$, where $\mu \in \mathbb{R}$ and $\sigma^2 > 0$, if its probability density function (PDF) is expressed by:

$$f(x) = \left(\frac{\sigma}{\sqrt{2\pi}}\right)^{-1} e^{-0.5(x-\mu)^2/\sigma^2}, \forall x \in \mathbb{R} \quad (31)$$

In the special case where $\mu = 0$ and $\sigma = 1$, we obtain the standard normal distribution, $\text{Norm}(0, 1)$, with PDF:

$$\varphi(x) = \left(\frac{\sqrt{2\pi}}{\sigma}\right)^{-1} e^{-0.5x^2}, \forall x \in \mathbb{R} \quad (32)$$

cumulative distribution function (CDF):

$$\Phi(x) = \left(\frac{\sqrt{2\pi}}{\sigma}\right)^{-1} \int_{-\infty}^{x} e^{-0.5u^2} du \quad (33)$$

and complementary CDF (CCDF) given by the $Q$-function:

$$Q(x) = 1 - \Phi(x) = \left(\frac{\sqrt{2\pi}}{\sigma}\right)^{-1} \int_{x}^{+\infty} e^{-0.5u^2} du \quad (34)$$

In addition, if $X \sim \text{Norm}(\mu, \sigma^2)$, then $Z = (X - \mu)\sigma^{-1} \sim \text{Norm}(0, 1)$.

REFERENCES

[1] A. D. Panagopoulos et al., “Satellite communications at Ku, Ka, and V bands: Propagation impairments and mitigation techniques,” IEEE Commun. Surv. & Tutor., vol. 6, no. 3, pp. 2-14, Third Quarter 2004.

[2] A. D. Panagopoulos et al., “Long-term rain attenuation probability and site diversity gain prediction formulas,” IEEE Trans. Antennas Propag., vol. 53, no. 7, pp. 2307-2313, July 2005.

[3] C. I. Kourogiorgas et al., “On the earth-space site diversity modeling: A novel physical-mathematical outage prediction model,” IEEE Trans. Antennas Propag., vol. 60, no. 9, pp. 4391-4397, Sept. 2012.

[4] G. A. Karagiannidis et al., “Multidimensional rain attenuation stochastic dynamic modeling: Application to earth-space diversity systems,” IEEE Trans. Antennas Propag., vol. 60, no. 11, pp. 5400-5411, Nov. 2012.

[5] A. D. Panagopoulos, “Propagation phenomena and modeling for fixed satellite systems: Evaluation of fade mitigation techniques,” in Radio wave propagation and channel modeling for earth-space systems, Taylor & Francis Group, CRC Press, 2016.

[6] N. Jeannin et al., “Smart gateways for terabit’s satellite,” Int. J. Satell. Commun. Netw., vol. 32, no. 2, pp. 93-106, March 2014.

[7] A. Kyrigiazos, B. G. Evans and P. Thompson, “On the gateway diversity for high throughput broadband satellite systems,” IEEE Trans. Wireless Commun., vol. 13, no. 10, pp. 5411-5426, Oct. 2014.

[8] A. Gharanjik et al., “Multiple gateway transmit diversity in Q/V band feeder links,” IEEE Trans. Commun., vol. 63, no. 3, pp. 916-926, March 2015.

[9] T. Rossi, M. De Sanctis and F. Maggio, “Evaluation of outage probability for satellite systems exploiting smart gateway configurations,” IEEE Commun. Lett., vol. 21, no. 7, pp. 1541-1544, July 2017.

[10] T. Delamotte and A. Knopp, “Smart diversity through MIMO satellite Q/V-band feeder links,” IEEE Trans. Aerosp. Electron. Syst., vol. 56, no. 1, pp. 285-300, Feb. 2020.

[11] M. Muhammad, G. Giambene and T. de Cola, “QoS support in SGD-based high throughput satellite networks,” IEEE Trans. Wireless Commun., vol. 15, no. 12, pp. 8477-8491, Dec. 2016.

[12] C. N. Efrem and A. D. Panagopoulos, “Dynamic energy-efficient power allocation in multibeam satellite systems,” IEEE Wireless Commun. Lett., vol. 9, no. 2, pp. 228-231, Feb. 2020.

[13] A. Z. Papafragkakis et al., “ALPHASAT site diversity experiments in Greece and the UK at Ka band: Comparison of 2-years’ results,” ITU Journal: ICT Discoveries, vol. 2, no. 1, Nov. 2019.

[14] T. Rossi et al., “Smart gateway diversity optimization for EHF satellite networks,” IEEE Trans. Aerosp. Electron. Syst., vol. 56, no. 1, pp. 130-141, Feb. 2020.

[15] C. N. Efrem and A. D. Panagopoulos, “Globally optimal selection of ground stations in satellite systems with site diversity,” IEEE Wireless Commun. Lett., to be published.

[16] M. Fernandez and S. Williams, “Closed-form expression for the Poisson-binomial probability density function,” IEEE Trans. Aerosp. Electron. Syst., vol. 46, no. 2, pp. 803-817, April 2010.

[17] Y. Hong, “On computing the distribution function for the Poisson binomial distribution,” Computational Statistics & Data Analysis, vol. 59, pp. 41-51, March 2013.

[18] A. M. Rushdi, “Utilization of symmetric switching functions in the computation of k-out-of-n system reliability,” Microelectron. Reliab., vol. 26, no. 5, pp. 973-987, 1986.

[19] L. A. Belfiore, “An $O(n\log_2 n)^2$ algorithm for computing the reliability of k-out-of-n:G and k-to-1-out-of-n:G systems,” IEEE Trans. Reliab., vol. 44, no. 1, pp. 132-136, March 1995.

[20] P. Deheuvels, M. L. Puri, and S. S. Ralescu, “Asymptotic expansions for sums of nonidentically distributed Bernoulli random variables,” Journal of Multivariate Analysis, vol. 28, no. 2, pp. 282-303, 1989.

[21] V. G. Mikhailov, “On a refinement of the central limit theorem for sums of independent random indicators,” Theory Probab. Appl., vol. 38, no. 3, pp. 479-489, 1994.

[22] A. Yu. Volkova, “A refinement of the central limit theorem for sums of independent random indicators,” Theory Probab. Appl., vol. 40, no. 4, pp. 791-794, 1996.

[23] W. Ehm, “Binomial approximation to the Poisson binomial distribution,” Statist. & Probab. Lett., vol. 11, no. 1, pp. 7-16, Jan. 1991.

[24] L. Le Cam, “An approximation theorem for the Poisson binomial distribution,” Pacific J. Math., vol. 10, no. 4, pp. 1181-1197, 1960.

[25] J. M. Steele, “Le Cam’s inequality and Poisson approximations,” The American Mathematical Monthly, vol. 101, no. 1, pp. 48-54, 1994.

[26] A. R"{o}llin, “Translated Poisson approximation using exchangeable pair couplings,” Ann. Appl. Probab., vol. 17, no. 5/6, pp. 1596-1614, 2007.

[27] T. Hagerup, and C. R"{u}b, “A guided tour of Chernoff bounds,” Information Processing Letters, vol. 33, no. 6, pp. 305-308, Feb. 1990.