Natural inflation and flavor mixing from Peccei–Quinn symmetry breaking

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We propose a left–right symmetric model to simultaneously give natural inflation and flavor mixing from a Peccei–Quinn symmetry breaking at the Planck scale. Our model can be embedded into SO(10) grand unification theories.

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1. Introduction

The Peccei–Quinn [1] (PQ) symmetry predicts the existence of an axion [1,2] which would solve the strong CP problem. In addition the axion could be dark matter and the PQ symmetry could have various other interesting implications for particle physics and cosmology [3]. For example [4], in an SU(5) grand unification theory (GUT), the PQ symmetry breaking at the Planck scale can result in a pseudo Nambu–Goldstone boson (pNGB) with a Coleman–Weinberg potential [5] to drive inflation [6]. One can also relate the PQ symmetry breaking to the neutrino mass-generation [7]. In particular, the PQ symmetry can be embedded into the SU(3) × SU(2) × SU(2) × U(1)B−L left–right symmetric theories [8] if the left–right symmetry is charge-conjugation [9]. From the PQ and left–right symmetry breaking, we [9] can elegantly realize the universal [10] seesaw [11] or the double [12] and linear [13] seesaw.

In this paper we propose a novel left–right symmetric scenario where the PQ symmetry is realized such that it naturally leads simultaneously to natural inflation [14] and flavor mixing simultaneously. The specific model which we will discuss can be embedded into SO(10) GUTs, where the particle content emerges from a bigger picture. However, we will not discuss this embedding further and we will focus on the main mechanism within the left–right framework. At the left–right level our model contains one Higgs bi-doublet for each family, two Higgs doublets, two leptoquark doublets and six complex singlets in the scalar sector while three neutral singlets and three generations of lepton and quark doublets in the fermion sector. The six scalar singlets are responsible for a U(1)B global symmetry breaking at the Planck scale. Because of the Yukawa interactions between the scalar and fermion singlets, the U(1)B symmetry is explicitly broken down to a U(1)3 symmetry [15]. Three Nambu–Goldstone bosons (NGBs) will obtain heavy masses through the Coleman–Weinberg potential [5] while the other three will pick up tiny masses through the color anomaly [16]. The heavy and light pNGBs can act as the inflaton and the axion, respectively. This inflationary scenario can also avoid the cosmological domain wall problem [17]. In the absence of any off-diagonal Yukawa couplings involving the lepton and quark doublets, we can make use of the mixed fermion singlets to induce the lepton mixing by tree-level seesaw [18] and the quark mixing by one-loop diagrams.

2. The model

The scalar sector includes one Higgs bi-doublet for each family,

\[ \phi_{i(1,2,2^*,0)} = \begin{bmatrix} \phi_{i1}^0 \\ \phi_{i1}^- \\ \phi_{i2}^+ \\ \phi_{i2}^{-} \end{bmatrix}, \]

(1)

two Higgs doublets,

\[ \chi_{L(1,2,1,-1)} = \begin{bmatrix} \chi_{L}^0 \\ \chi_{L}^{-} \end{bmatrix}, \]

\[ \chi_{R(1,2,1,-1)} = \begin{bmatrix} \chi_{R}^0 \\ \chi_{R}^{-} \end{bmatrix}, \]

(2)

two leptoquark doublets,

\[ \eta_{L(3,2,1,1/3)} = \begin{bmatrix} \eta_{L}^{2/3} \\ \eta_{L}^{-1/3} \end{bmatrix}. \]
\[ \eta_R \left( 3, 1, 2, \frac{1}{3} \right) = \left[ \eta_R \right]^{2/3} \]

and six complex singlets,
\[ \sigma_{ij}^{(1, 1, 1, 0)} = \sigma_{ji}^{(1, 1, 1, 0)} \]

In the fermion sector, there are three neutral fermion singlets,
\[ S_{R_i}^{(1, 1, 1, 0)} \]

and three generations of quark and lepton doublets,
\[ q_{L_i} \left( 3, 2, 1, \frac{1}{3} \right) = \left[ \frac{u_{L_i}}{d_{L_i}} \right] \]
\[ q_{R_i} \left( 3, 2, 1, \frac{1}{3} \right) = \left[ \frac{u_{R_i}}{d_{R_i}} \right] \]
\[ l_{L_i} \left( 1, 2, 1, -1 \right) = \left[ \frac{v_{L_i}}{e_{L_i}} \right] \]
\[ l_{R_i} \left( 1, 2, 1, -1 \right) = \left[ \frac{v_{R_i}}{e_{R_i}} \right] \]

We assume a discrete left–right symmetry which is connected to charge-conjugation and the fields thus will transform as
\[ \phi_i \leftrightarrow \phi_i^T, \quad \chi_L \leftrightarrow \chi_R^* \]
\[ \eta_L \leftrightarrow \eta_R^* \quad \sigma_{ij} \leftrightarrow \sigma_{ji} \]
\[ q_{L_i} \leftrightarrow q_{R_i}^\dagger, \quad l_{L_i} \leftrightarrow l_{R_i}^\dagger, \quad S_{R_i} \leftrightarrow S_{L_i} \]

In the presence of the above left–right symmetry, we can impose a family symmetry \( U(1)_F \), under which the left- and right-handed fermion doublets carry an equal but opposite charge for each family, i.e.
\[ (-\delta_{11}, -\delta_{12}, -\delta_{13}) \quad \text{ for } l_{L_i} \leftrightarrow l_{R_i}^\dagger \] and \( q_{L_i} \leftrightarrow q_{R_i}^\dagger \).
We also assign the \( U(1)_F \) charges for other fields,
\[ (-\delta_{11}, -\delta_{12}, -\delta_{13}) \quad \text{ for } S_{R_i} \]
\[ (\delta_{11} + \delta_{12}, \delta_{12} + \delta_{12}, \delta_{13} + \delta_{13}) \quad \text{ for } \sigma_{ij} \]
\[ (-2\delta_{11}, -2\delta_{12}, -2\delta_{13}) \quad \text{ for } \phi_i \]
\[ (0, 0, 0) \quad \text{ for } \chi_{L,R} \quad \eta_{L,R} \]

We further introduce a global symmetry \( U(1)_C \), under which the fields carry the following quantum numbers,
\[ 2 \quad \text{ for } \sigma_{ij} \]
\[ 1 \quad \text{ for } \chi_{L,R} \quad \eta_{L,R} \quad \sigma_{ij} \quad S_{R_i}^c \]
\[ 0 \quad \text{ for } l_{L_i} \leftrightarrow l_{R_i}^\dagger \quad q_{L_i} \leftrightarrow q_{R_i}^\dagger \]

Next we specify the allowed Yukawa interactions,
\[ L_Y = -y_{\phi, q_L} \bar{q}_L \phi q_R - y_{\phi, l_L} \bar{l}_L \phi q_R - f_1 \left( \bar{l}_L \chi_L S_{R_i} + \bar{\chi}_R^* S_{R_i} \right) - f_2 \left( \bar{q}_L \eta_L S_{R_i} + \bar{\eta}_R^* S_{R_i} \right) - \frac{1}{2} g_{ij} \sigma_{ij} S_{R_i}^c S_{R_i} + \text{H.c.} \]

The full scalar potential should be
\[ V = \mu_{\phi_i}^2 |\sigma_{ij}|^2 + \mu_{\chi}^2 (|\chi_L|^2 + |\chi_R|^2) + \mu_{\eta}^2 (|\eta_L|^2 + |\eta_R|^2)
+ \mu_{\phi_i}^2 (|\phi_i|^2 + |\phi_j|^2)
+ \lambda_{\phi_i} |\sigma_{ij}|^2 |\sigma_{ij}|^2
+ \lambda_{\chi} (|\chi_L|^4 + |\chi_R|^4)
+ \lambda_{\eta} (|\eta_L|^4 + |\eta_R|^4)
+ \lambda_{\phi} (|\phi_i|^2 |\eta_L|^2 + |\phi_j|^2 |\eta_R|^2)
+ \lambda_{\chi} (|\chi_L|^2 |\chi_R|^2 + |\chi_R|^2 |\chi_L|^2)\]

Clearly, the lepton and quark doublets, the Higgs bi-doublets, the fermion singlets and the scalar singlets can, respectively, belong to the \( 16_{R_i} \), \( 16_{L_i} \), \( 10_{R_i} \), \( 1_{F_i} \) and \( 1_{H_i} \) representation in SO(10) GUTs.

3. Pseudo Nambu–Goldstone bosons

Each scalar singlet \( \sigma_{ij} \) has an independent phase transformation to perform a \( U(1)^6 \) symmetry. However, the six scalar singlets have the Yukawa interactions with the three fermion singlets [the g-terms in Eq. (11)] so that the \( U(1)^6 \) symmetry should be explicitly broken down to a \( U(1)^2 \) symmetry. After the six scalar singlets develop their vacuum expectation values (VEVs), there will be six NGBs, i.e.
\[ \sigma_{ij} = \frac{1}{\sqrt{2}} (f_{ij} + \epsilon_{ij} \phi_i) e^{i \phi_i / \phi_i} \]

The fermion singlets then obtain their Majorana masses
\[ M_{ij} = \frac{1}{\sqrt{2}} g_{ij} f_{ij} e^{i \phi_i / \phi_i} = M_{ij} e^{i \phi_i / \phi_i} \]

which will result in a Coleman–Weinberg potential \([5,15]\],
\[ V = \frac{1}{32\pi^2} \text{Tr} \left( \left( \hat{M}_{ij} \hat{M}_{ij}^\dagger \right) \ln \left( \frac{\lambda^2}{\hat{M}_{ij}^2} \right) \right) \]

with \( \lambda \) being the ultraviolet cutoff.

Only three NGBs can exist in the Coleman–Weinberg potential while the other three can be absorbed by the three fermion singlets. For example, we can take
\[ S_{R_i} e^{i \phi_i / \phi_i} \Rightarrow S_{R_i} \]
and then \( \hat{M}_{ij} = M_{ij} e^{i \phi_i / \phi_i} \)

with \( \phi_i / \phi_i = \phi_i / \phi_i - \frac{\phi_i}{\phi_i} \)

Clearly, we have \( \phi_i = 0 \) for \( i = j \). By taking a reasonable simplification on the logarithm
\[ \ln \left( \frac{\lambda^2}{\hat{M}_{ij}^2} \right) \approx \text{constant} = O(1) \]

the Coleman–Weinberg potential can be expanded by
\[ V = \frac{1}{32\pi^2} \left( \left( M_{11}^2 + M_{12}^2 + M_{13}^2 \right)^2 + \left( M_{21}^2 + M_{22}^2 \right)^2 + \left( M_{31}^2 + M_{32}^2 + M_{33}^2 \right)^2 + 2M_{11}^2 M_{12}^2 \right) \]
\[ + M_{13}^2 M_{23}^2 + M_{23}^2 M_{32}^2 + 2M_{11}^2 M_{13}^2 + M_{12}^2 M_{23}^2 \]
\[ + M_{13}^2 M_{23}^2 + 2M_{12}^2 M_{13}^2 + M_{23}^2 M_{32}^2 \]
A combination of $\varphi_{12}$, $\varphi_{13}$ and $\varphi_{23}$ can have a potential of the form as below,

$$V = \mu^4 \left( 1 \pm \cos \frac{\varphi}{F} \right) \text{ with } \mu = O(M_{ij}), \quad f = O(f_{ij}). \tag{19}$$

The pNGB $\varphi$ can realize the natural inflation for $\mu = O(10^{15} \text{ GeV})$ and $f = O(M_{R})$ [14]. Note that the Majorana masses $M_{ij}$ should be determined by the Yukawa couplings $g_{ij} = O(10^{-5})$ for the given symmetry breaking scales $f_{ij} = O(M_{R})$. Actually, the three pNGBs accommodate a multi-field inflation scenario where a non-Gaussianity in the statistics of primordial perturbations is potential to discovery [19]. We will study the details elsewhere.

On the other hand, the Yukawa interactions (11) mean that the three NGBs absorbed in the three fermion singlets can have derivative couplings with the quarks,

$$\mathcal{L} = -\frac{1}{2} \mathcal{M}_{ii} \bar{q}_{L_{i}} \gamma^{\mu} q_{L_{i}} \tilde{\varphi}_{ii} \gamma_{\mu} \varphi_{ii} + \text{H.c.}$$

Therefore, the NGBs $\varphi_{ii}$ can obtain their tiny masses through the color anomaly. Clearly, the pNGBs $\varphi_{ii}$ play the role of the invisible axion [20,21] while the family symmetry $U(1)_{F}$ is identified with the PQ symmetry. Benefited from the inflation, our model can escape from the cosmological domain wall problem [17]. Furthermore, for the PQ symmetry breaking at the Planck scale, we can choose the initial value of the misalignment angle by the anthropic argument [22,23] to give a desired dark matter relic density [4,3,24].

4. Lepton masses and mixing

The charged leptons have a diagonal $3 \times 3$ mass matrix, i.e.

$$\mathcal{L} = -\tilde{m}_{e} \bar{e}_{L} e_{R} + \text{H.c.} \quad \text{with } (\tilde{m}_{e})_{i} = y_{l_{i}} \langle \phi_{12} \rangle. \tag{21}$$

The mass terms involving the neutral leptons are given by

$$\mathcal{L} = -\tilde{m}_{\nu} \bar{\nu}_{L} \nu_{R} - f(\chi_{R}^{0}) \bar{\nu}_{L} S_{R} - f(\chi_{R}^{0}) \bar{\nu}_{R} S_{L}$$

$$- \frac{1}{2} \mathcal{M}^{\nu} \bar{\nu}_{R} \nu_{L} + \text{H.c.} \quad \text{with } (\tilde{m}_{\nu})_{i} = y_{\nu_{l_{i}}} \langle \phi_{12} \rangle. \tag{22}$$

For $f(\chi_{R}^{0})$ and/or $\tilde{M}$ much bigger than $\tilde{m}_{\nu}$ and $f(\chi_{R}^{0})$, we can make use of the seesaw formula [11] to derive the neutrino masses, i.e.
Here we have defined
\[ \delta^2_{2/3} = \frac{1}{\sqrt{2}} c_f u_i (\phi^0_i) + a (X^0_i) (X^0_i), \]
\[ \delta^2_{1/3} = \frac{1}{\sqrt{2}} c_f u_i (\phi^0_i). \]

We further rotate the fermion singlets to diagonalize their Majorana mass matrix,
\[ U^* M U^T = \text{diag} (M_{S1}, M_{S2}, M_{S3}) \]

We then give the loop-induced quark masses as below,
\[
\bar{m}_{ij}^{1} = \frac{\sin 2\theta_{1/3}}{32\pi^2} h_i h_j U^T_{uk} U^T_{kj} M_{uk} \left( \frac{M^2_{1/3}}{M^2_{1/3} - M^2_{3k} \ln \frac{M^2_{1/3}}{M^2_{3k}}} \right) \sim \frac{\sin^2 2\theta_{1/3}}{16\pi^2} \frac{h_i h_j M^*_{1/3}}{M^3_{1/3}} \]
\[
\bar{m}_{ij}^{2} = \frac{\sin 2\theta_{2/3}}{32\pi^2} h_i h_j U^T_{uk} U^T_{kj} M_{uk} \left( \frac{M^2_{2/3}}{M^2_{2/3} - M^2_{3k} \ln \frac{M^2_{2/3}}{M^2_{3k}}} \right) \sim \frac{\sin^2 2\theta_{2/3}}{16\pi^2} \frac{h_i h_j M^*_{2/3}}{M^3_{2/3}} \]

It is easy to check that the tree-level and loop-order quark mass matrices can induce the desired quark masses and mixing for a proper parameter choice.

6. Summary

We discussed a left-right symmetric model with PQ symmetry with the aim to combine in a natural way inflation and flavor mixing. A key ingredient is the Yukawa interactions between the six scalar singlets and the three fermion singlets which explicitly break a $U(1)^6$ symmetry to a $U(1)^3$ symmetry. Among the six NGBs, three can obtain heavy masses through the Coleman-Weinberg potential to drive the natural inflation while the other three can pick up tiny masses through the color anomaly to solve the strong CP problem and to explain dark matter. Although the PQ symmetry forbids any off-diagonal Yukawa couplings involving lepton and quark doublets, we can mediate the mixing of the fermion singlets to the neutrino sector by tree-level seesaw and to the quark sector by one-loop diagrams. Our model can be embedded into SO(10) GUTs, but we did not discuss the details which do not change the proposed mechanism.

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