Estimation of Collision Impact Parameter

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Abstract

We demonstrate that the nuclear collision geometry (i.e. impact parameter) can be determined with 1.5 fm accuracy in an event-by-event analysis by measuring the transverse energy flow in the pseudorapidity region $3 \leq |\eta| \leq 5$ with a minimal dependence on collision dynamics details at the LHC energy scale. Using the HIJING model we have illustrated our calculation by a simulation of events of nucleus-nucleus interactions at the c.m.s energy from 1 up to 5.5 TeV per nucleon and various type of nuclei.
The measurement of the collision impact parameter is a very important practical problem of relativistic heavy ion physics. A part of future experimental programmes on the LHC accelerator will be devoted to relativistic heavy ion collisions\footnote{1}. Significant efforts will be focused on the establishment of the fundamental laws and quite general rules of nucleus-nucleus interactions, the discovery of a new state of QCD matter, i.e. quark-gluon plasma (QGP), and the study of the properties of strongly exited nuclear matter\footnote{2}. It is expected that QGP might be produced in the central nucleus-nucleus collisions at extremely high energy density $\epsilon_0 \sim 1\text{ GeV/fm}^3$ (at the LHC energy scale $\epsilon_0 \sim 1\text{ TeV/fm}^3$).

On the other hand, for studies of diffractive phenomena, properties of a coherent pomeron and collective nuclear effects\footnote{3} it is necessary to select peripheral events with a large collision impact parameter. Assuming that the collision impact parameter is measured the experimentally observed effects can be compared with theoreticall predictions of expected signals of a "new" physics: parton energy losses in nuclear matter, monojet to dijet ratio, quarkonia suppression, correlated jet and $W^{\pm}, Z^0$ production and so on.

In the present paper we demonstrate a method of estimation of a collision impact parameter in the event-by-event analysis.

As the collision impact parameter can not be measured in experiments directly, it would be necessary to find an experimentally measurable $b$-dependent variable. We suggest to use the total transverse energy $E_T$ (total energy $E$) produced in the one event of the nucleus-nucleus collision for the collision impact parameter estimation.

We will show that there is a correlation between the transverse energy and the impact parameter, and will present our numerical calculations performed on the basis of the HIJING model\footnote{4} at the RHIC and LHC energy scales. We argue that the measurement of $E_T$ produced in the forward direction (large (pseudo)rapidity values) allows one to avoid possible uncertainties in the $b$ determination.

The multiplicity and transverse energy production in the nucleus-nucleus collisions in the ultra-relativistic energy domain is considered as a combination of hard processes with $p_T \geq p_0$ and soft particle production. A transverse energy flow produced in hard processes described by perturbative QCD is associated in the main with minijet production, i.e. jets with $p_T \sim 4\text{ GeV}$\footnote{5}.

The average transverse energy carried by (mini)jets in the rapidity intervals $\Delta y$ is related to the collision impact parameter $b$ by the formula:

$$\langle E_T(b, \sqrt{s_{NN}}, p_0, \Delta y) \rangle = T_{AA}(b)\sigma_{jet}(\sqrt{s_{NN}}, p_0)\Delta y \langle E^\Delta y_T \rangle,$$  \hspace{1cm} (1)$$

where $\langle E^\Delta y_T \rangle$ is the average transverse energy per a (mini)jet in $\Delta y$ interval, $\sigma_{jet}$ is the cross-section of minijet production in the parton model at the pp-level. The differential distribution $d\sigma/dE_T$ is:

$$\frac{d\sigma_{jet}}{dE_T}(\sqrt{s_{NN}}, p_0, \Delta y) = \frac{1}{2} K \int_{p_0^2, \Delta y}^{s/4} dp_1^2 dp_2^2 dy_1 dy_2 \sum_{i,j,k,l} x_1 f_i(x_1, p_1^2) x_2 f_j(x_2, p_2^2) \times$$
$$\left[ \frac{d\hat{\sigma}^{ij \rightarrow kl}}{dt}(\hat{s}, \hat{t}, \hat{u}) + \frac{d\hat{\sigma}^{ij \rightarrow kl}}{dt}(\hat{s}, \hat{u}, \hat{t}) \right] \frac{\delta(E_T - p_T)}{1 + \delta_{kl}},$$  \hspace{1cm} (2)$$

where $p_0$ is the pQCD cut-off parameter, $x_1$ and $x_2$ are the fractional momenta of the initial par-
tons \( i \) and \( j \), and \( y_1 \) and \( y_2 \) are the rapidities of outgoing partons, \( d\hat{\sigma}^{ij\rightarrow kl}/d\hat{t} \) is the parton-parton scattering cross section. The summation runs over all parton species and the factor \( K \approx 2 \) is used to correct the lowest order pQCD rates for the effects of next-to-leading order terms. The value of \( \sigma_{jet}(\sqrt{s_{NN}}, p_0)\Delta y \langle E_T^{\Delta y} \rangle \) then is:

\[
\sigma_{jet}(\sqrt{s_{NN}}, p_0)\Delta y \langle E_T^{\Delta y} \rangle = \frac{1}{2} K \int_{p_0^2}^{s/4} dp_T^2 dy_1 dy_2 \sum_{i,j,k,l} x_1 f_i(x_1, p_T^2) x_2 f_j(x_2, p_T^2) \times 
\frac{d\hat{\sigma}^{ij\rightarrow kl}}{d\hat{t}}(\hat{s}, \hat{t}, \hat{u}) + \frac{d\hat{\sigma}^{ij\rightarrow kl}}{d\hat{t}}(\hat{s}, \hat{u}, \hat{t}) \frac{p_t}{1 + \delta_{kl}}.
\]

Experimentally known effects of modification of quark and gluon structure functions by nuclear medium called parton shadowing \[6\] has not been included here. To take into account the shadowing effect we should modify the formula (3) by multiplying parton distributions by a corresponding correction term:

\[
f_i(x_1, p_T^2) f_j(x_2, p_T^2) \rightarrow R^{A}_{i,j}(x, p_T^2) f_i(x_1, p_T^2) R^{A}_{j,k}(x, p_T^2) f_j(x_2, p_T^2),
\]

here the ratio \( R^{A}_{i,j} \equiv f_{i,j/A}(x, p_T^2)/f_{i,j/N}(x, p_T^2) \), where \( f_{i,j/N}(x, p_T^2) \) is the parton structure function for a free nucleon and \( f_{i,j/A}(x, p_T^2) \) is the corresponding parton distribution in a proton inside the nucleus.

The nuclear density overlap function of two colliding nuclei at a given impact parameter \( T_{AA}(b) \) calculated in the assumption of the Wood-Saxon nuclear density distribution \( \rho_A(r) \):

\[
T_{AA}(|\vec{b}|) = \int d^2\vec{r} T_A(\vec{r}) T_A(\vec{b} - \vec{r}),
\]

where \( \vec{r} \) is a 2-dimensional vector defining the interaction point. The nuclear thickness function is

\[
T_A(|\vec{r}|) = \int dz \rho_A(\sqrt{|\vec{r}|^2 + z^2}).
\]

The expression (1) relating \( E_T \) and \( b \) includes a term arisen from hard processes with \( p_t \geq 2 \text{ GeV} \) only. For a more correct estimation of the collision impact parameter we should take into account in addition the part of a total transverse energy flow produced in soft interactions.

\[
\langle E_T \rangle^{total} = \langle E_T \rangle^{jet} + \langle E_T \rangle^{soft},
\]

\[
\langle E_T \rangle^{soft} = T_{AA}(b) \sigma_{soft\text{incl}} \langle E_T^{\Delta y} \rangle^{soft},
\]

where \( \langle E_T^{\Delta y} \rangle^{soft} \) is the averaged transverse energy per particle produced by soft interactions. But soft processes can not be calculated by pQCD applications and we should to use some phenomenological models \[7, 8\] for the estimation of a soft part of the total energy flow.

We think that a permissible pseudorapidity region for an the measuring energy is limited by the following reason. Main signatures of a possible QGP production are expected in the central
(pseudo)rapidity region. One of the discussed features of such a state of nuclear matter is energy losses of scattered partons in final state interactions with a dense nuclear matter called jet quenching [9]. Among other effects originated by jet quenching one may expect a significant modification in the distributions of the transverse energy flow and charged multiplicity $dE_T/d\eta$, $dE_T^*/d\eta$, and $dn_{ch}/d\eta$ [10].

Indeed, an indication is found for ultrarelativistic energy domain concerning the appearance of a wide bump in the interval $-2 \leq \eta \leq 2$ over a pseudorapidity plateau of such distributions due to jet quenching. Fig.1 demonstrates the evolution of the effect with collision impact parameter variation. It is interesting to note that a secondary interaction effect such as a jet quenching modifies only the central rapidity part, while both pseudorapidity regions ($3 \leq |\eta| \leq 5$) remain practically unchanged. Therefore the large $\eta$ region can be used for the collision impact parameter estimation with minimal dependence on possible signals of new physics in the central (pseudo)rapidity region.

The total cross-section of AA collisions has been calculated in the framework of a HIJING hybrid model of nucleus-nucleus interactions [4], where the cross-section of hard processes has been defined by the formula (3). The contribution of the soft part of a produced particle spectrum, i.e. small $p_T$-processes has been simulated by using the FRITIOF and DPM models [7, 8]. In these models hadrons are considered as relativistic strings exited at hadron interactions. Calculations shows that for central PbPb collisions at the LHC energy the inclusive cross-section of soft interactions at the pp-level is equal to 57.0 mb while the inclusive cross-section of hard processes is equal to 54.3 mb. The parton shadowing effect has been taken into account.

However, it is obvious that the average transverse energy of partons produced in hard processes $\langle E_T \rangle_{jet}^{3 \div 5 \text{ GeV}}$ ($3 \div 5 \text{ GeV}$ for $|\eta| \leq 0.5$) is larger than the average transverse energy of soft partons $\langle E_T \rangle_{soft} \sim 0.4 \text{ GeV}$. This fact reduces more strongly the relative contribution of soft processes in the total transverse energy production. In the LHC energy domain already 80 % of the total transverse energy is calculated by perturbative QCD application. This allows one to reduce ambiguities of $E_T$ calculations induced by the use of phenomenology models to take into account small $p_T$-processes.

In the framework of the HIJING model we have simulated 10,000 events of PbPb, NbNb, CaCa interactions at 5.5 TeV per nucleon. The dependence of the total transverse energy produced in the pseudorapidity interval $3 \leq |\eta| \leq 5$ on the collision impact parameter is presented in fig.2. The variable $E_T$ is connected with $b$. Some fluctuations in nucleus-nucleus collision dynamics limit the impact parameter estimation to a 1.5 fm precision.

The correlation of the same type is presented in fig.3 with variation of a collision energy. It is shown that the increase of the collision energy leads to an improvement of the accuracy the impact parameter definition.

The correlation curve for the total energy flow is of the same shape. The bulk of the energy produced in the forward direction run up to $10 \div 100 \text{ TeV}$. This allows one to measure the total energy with high accuracy and to reduce experimental errors for the $b$ estimation.

We should remark that uncertainties are associated with the use of various parton shadowing models and the sets of structure functions lead to an ambiguity in $E_T - b$ correlation. In more detail the influence of these aspects on the total (transverse) energy flow is discussed in ref [11].
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Figure 1: Differential distribution of the total transverse energy $dE_T/d\eta$ (GeV) over pseudorapidity $\eta$ for 10000 minimum bias PbPb collisions at $\sqrt{s_{NN}} = 5.5$ TeV/nucleon with various impact parameter. Normalized per number of events.
Figure 2: Correlation between the transverse energy flow per collision $E_T$ (GeV) in the pseudorapidity direction ($3 \leq |\eta| \leq 5$) and collision impact parameter $b$ (fm). From top to bottom: PbPb, NbNb, CaCa collisions at $\sqrt{s_{NN}}=5.5$ TeV/nucleon.

Figure 3: Correlation between the transverse energy flow per collision $E_T$ (GeV) in the pseudorapidity direction ($3 \leq |\eta| \leq 5$) and collision impact parameter $b$ (fm) for PbPb collision at $\sqrt{s_{NN}}=5.5$, 3, 1, 0.5 TeV/nucleon.