Rationale for the use of color information on Eddington

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Abstract. For the Eddington mission, the intrinsic stellar variability can be a major source of noise in the detection of extrasolar planets by the transit method. We derive that most detections of terrestrial planets (1–2 \textit{R}_\odot) will occur around G or K stars with 15–16th magnitude. When these stars are 7–12 times more variable than the Sun on a 10 hour timescale, we demonstrate that the detection can be performed with a higher S/N provided composite lightcurves obtained with the combination of two colors are used instead of white ones. The level of 10 hour variability for K stars is quite uncertain. We make two “guess-estimates” of it and find that it could be several times larger than the solar value. If these estimates were relevant, the color information would not provide a significant advantage. Although we do not demonstrate a need for colors, \textit{we point out the risk of an unpleasant surprise regarding the 10 hour stellar variability. Indeed, there is presently no qualified proxy for this variability}. Besides, if Eddington were designed to provide this information at the cost of added complexity but not sensitivity, white photometry by channel summation would still be as efficient. Considering the risk that 10 hour variability is higher than estimated, \textit{the Precaution Imperative points to a study of practical implementations of photometry in different colors before taking irreversible decisions about the Eddington instrument.}

Key words. stars: planetary systems – methods: statistical – techniques: photometric
1. Introduction

The purpose of this research note is to calculate the gain in detectability of extrasolar planets brought by the use of color information instead of the white flux alone, in the case of the Eddington mission (Favata et al. 2000).

The monitoring of the solar variability performed by SOHO/SPM in three bands (Fröhlich et al. 1997) shows that there is a strong correlation between the different colors, the variation in the blue being significantly larger than in the red. On the contrary, a transit is essentially perceived as an achromatic variation of the stellar flux. Thus, a properly weighted linear combination of the colored lightcurves can provide a composite lightcurve almost free of stellar variability. Unfortunately, this has to be traded off against an increase of quantum noise. The following calculation evaluates the variability threshold above which the signal to noise ratio (S/N) is increased by the variability subtraction.

2. White detection S/N

Let us consider the problem from the standpoint of pure detection. Then, the highest S/N is achieved when the lightcurve is averaged over the transit duration, that is to say transits are reduced to one point. The noise figure for Eddington is expected to be dominated by the quantum noise with an additional contribution of the stellar variability noise. If \( N_e \) is the total number of photoelectrons collected during a transit, \( \sigma \), \( N_e \) the standard deviation of the stellar variability, and considering that these two noises as uncorrelated, the white light S/N on a single transit is

\[
(S/N)_w = \left( \frac{R_p}{R_*} \right)^2 \sqrt{\frac{N_e}{1 + \sigma^2 N_e}},
\]

where \( R_p \) and \( R_* \) are the radii of the planet and of its star, respectively.

3. Colored detection S/N

Let us assume now that the stellar flux is observed in two colored channels, denoted B (blue) and R (red), collecting the fractions \( x_B \) and \( x_R \) of the total number of photoelectrons. As the stellar variations are highly correlated at the timescale of a few hours (Fig. 1), one can remove most of the stellar variability noise by combining the relative colored flux variations:

\[
S = \frac{\Delta B}{B} - k \frac{\Delta R}{R},
\]

where \( k \) is a constant adjusted to minimize \( S \) when \( \Delta B \) and \( \Delta R \) are due to the stellar variability alone. Now, we assume that \( S \) is due to the transit of a planet in front of a
Fig. 1. Correlation between the solar relative variabilities in the red (862 nm) and blue (402 nm) narrow filters of SOHO/SPM, at the timescale of a few hours. The coefficient of correlation is 0.913.

star producing a total of \(N_e\) photoelectrons. Denoting by \(N\) the new quantum noise, the S/N in colored light is

\[
(S/N)_c = (k - 1) \left( \frac{R_p}{R_*} \right)^2 \frac{N_e}{x_B^{-1} + k^2 x_R^{-1}}.
\]

Using the SOHO/SPM data and the transmission curve of COROT (Eddington’s precursor, Rouan et al. 2000), one computes \(k = 1.62\), \(x_B = 32\%\) and \(x_R = 42\%\), as being the optimum values for the Sun.

4. Magnitude and spectral type histograms of stars with expected detection

By using a simple transit detection algorithm (Bordé et al. 2001, 2002), we estimate, in the stellar field envisioned for Eddington (\(l_{II} = 70^\circ\), \(b_{II} = 5^\circ\), 7.1 deg\(^2\)), the number of stars with expected planet detections, as a function of their spectral type (Fig. 2) and magnitude (Fig. 3). These computations assume that every star is orbited by a planet with the given radius at 278 K blackbody temperature (1 AU for a G2V star). We conclude that for Earth-size planets most detections occur around K stars with 15–16th magnitude. For larger planets, the detection peak shifts toward G stars still with 15–16th magnitude.

5. Variability threshold

The variability threshold \(\sigma_{\nu \nu}^{\text{min}}\) above which one benefits from the colored detection is given by the condition

\[
(S/N)_c \geq (S/N)_w
\]

\[
\Rightarrow \sigma_{\nu \nu}^2 \geq \frac{1}{N_e} \left\{ \frac{x_B^{-1} + k^2 x_R^{-1}}{(k - 1)^2} - 1 \right\}
\]

i.e. \(\sigma_{\nu \nu}^{\text{min}} \approx \frac{4.8}{\sqrt{N_e}}\).
Fig. 2. Histograms of expected detections vs. the parent star spectral type for $R_p = 1.0$, 1.25, 1.5 and 2.0 $R_{\odot}$. Every star is assumed to be orbited by a planet of the given size at 278 K blackbody temperature (1 AU for a G2V star).

Fig. 3. Same as Fig. 2 vs. the visual magnitude of the parent star.

Thus, the colored detection becomes more efficient than the white one when the stellar variability noise overcomes the quantum noise by a factor of $\approx 5$.

Now, let us adopt the Astrium concept for Eddington: 4 Schmidt telescopes with $D = 0.6$ m pupils pointing at the same direction. We assume that all telescopes are equipped with dichroic plates allowing the simultaneous measure of blue and red signals.
Instead of dichroic plates, one may use a dispersive device, for instance derived from that by Chao & Chi [1998]. For a G2 star and a planet at 1 AU, Table I lists the values obtained for $\sigma^\text{min}_\star$ as a function of the star visual magnitude, $m_V$. These values are converted into solar variability levels using high-pass filtered SOHO/SPM data. At the time scale of a transit, 11.2 h for an Earth-like planet, $\sigma_\odot \approx 6.3 \times 10^{-5}$.

| $m_V$ | $N_e$ | $\sigma^\text{min}_\star$ | $\sigma^\text{min}_\star/\sigma_\odot$ |
|-------|-------|-----------------|-----------------|
| 10    | $1.0 \times 10^{10}$ | $4.7 \times 10^{-5}$ | 0.8 |
| 11    | $4.1 \times 10^{9}$ | $7.5 \times 10^{-5}$ | 1.2 |
| 12    | $1.6 \times 10^{9}$ | $1.2 \times 10^{-4}$ | 1.9 |
| 13    | $6.5 \times 10^{8}$ | $1.9 \times 10^{-4}$ | 3.0 |
| 14    | $2.6 \times 10^{8}$ | $3.0 \times 10^{-4}$ | 4.7 |
| 15    | $1.0 \times 10^{8}$ | $4.7 \times 10^{-4}$ | 7.5 |
| 16    | $4.1 \times 10^{7}$ | $7.5 \times 10^{-4}$ | 11.9 |

Table 1. Stellar variability thresholds for a planet orbiting a G2 star at 1 AU.

6. Estimate of stellar variability on a 10 hour timescale

To address the question of stellar variability on a 10 hour timescale, the only star for which we have data with sufficient precision is the Sun, as observed by SOHO. Work by the St Andrews group and co-workers (see for instance http://capella.st-and.ac.uk/~acc4/corot_no_bid.pdf) as well as Carpano, Aigrain & Favata (2002) indicates that:

(i) the amplitude of variability on timescales less than one day does not vary strongly during the solar cycle;

(ii) the color signature of the variability on these short timescales (from high-pass filtered VIRGO/SPM data) is indistinguishable from that of either spots or faculae.

This leads to the conclusion that whatever phenomenon produces the 10 hour timescale variability, it does not scale with the overall coverage of active regions (solar spots) and therefore variability on a 10 day timescale.

SOHO/MDI continuum images taken around solar minimum have been examined, searching for low-contrast features that might be responsible for this variability. The only features that showed up were the bright network elements dotted around the edges of supergranules (see http://capella.st-and.ac.uk/~jrb3/sunmovie3.gif). They are limb-brightened in the same way as faculae, showing up strongly only when they are near the limb. If they are the source of the hour timescale variability, this explains the color signature very nicely. Since the network elements appear to be a “magnetic carpet”
phenomenon, this would also explain why the amount of variability on these timescales
does not vary much during the solar cycle, a distinct physical process.

Now, how can these preliminary conclusions be extrapolated to other stars? One would
predict that if the hour timescale variability is a granulation/network phenomenon, its
amplitude would not change much from year to year, even for more active stars.

The estimate of the variability amplitude is much more uncertain, especially for latter
type stars where the contrast of the filament network to quiet photosphere can be higher.
In fact, there is no qualified proxy for the 10 hour stellar activity. For any star, the
luminosity 10 hour variability depends upon the total number of filaments, \( n \), and their
individual brightness, \( \delta \), as a function \( f(n, \delta) \). For instance, if most of the variation comes
from Poissonian fluctuations of \( n \), \( f(n, \delta) = \sqrt{n \delta} \).

Let us consider two estimates:

1. One can assume that the number and brightness of network elements is independent
of spectral type, i.e. \( n = n_\odot \), \( \delta = \delta_\odot \) and \( f = f_\odot \). Then the standard deviation of the
relative change of the stellar luminosity is
\[
\sigma_{\ast,1} = \frac{f(n_\odot, \delta_\odot)}{L_\ast} = \frac{\sigma_\odot}{L_\odot} \frac{L_\odot}{L_\ast}.
\]
(7)
The corresponding values for different spectral types are given in Table 2 second
column;

2. Alternatively, one can assume that a proxy indicator for the total luminosity of the
network elements is the lowest Ca II HK fluxes seen in main-sequence stars as a func-
tion of their spectral type. Noyes et al. (1984) show in their Fig. 2 that \( \log R'(HK) \),
i.e. the chromospheric Ca II HK emission flux as a fraction of the bolometric lumin-
osity, has a lower envelope ranging from \(-5.2\) for solar spectral type to \(-4.9\) for M0V
spectral type.

If all network elements have comparable brightness, the basal \( R'(HK) \) implies that
there must be twice as many of them per unit bolometric luminosity on an M0 star as
on the Sun. But the M0 star has \( \approx 1/16 \) the solar bolometric luminosity, so the total
number of elements on the M0 star would be 8 times fewer than on the Sun. The
Poissonian fluctuations in their contribution to the stellar flux would be \( \sqrt{5} \) times
less than on the Sun, but the stellar luminosity on which they are superimposed is 16
times less. Overall, the fluctuations \( \Delta f/f \) on the M0 star could be \( 16 / \sqrt{16/2} \approx 6 \)
times greater than \( \Delta f/f \) for the Sun. The estimates for other spectral types proceed
in a similar way and are reported in Table 2 third column.

### 7. Conclusion and discussion

For the Sun, the 10 hour variability and 10 day one are decoupled indicating different
physical origins. For other stars, the same decoupling is expected. The fact that a signifi-
Table 2. Stellar 10 hour variability as estimated by method (1) and (2).

| Sp  | $\sigma_{1,*}/\sigma_\odot$ | $\sigma_{2,*}/\sigma_\odot$ |
|-----|-----------------------------|-----------------------------|
| G2  | 1.0                         | 1.0                         |
| G5  | 1.3                         | 1.2                         |
| K0  | 2.5                         | 2.0                         |
| K5  | 6.3                         | 3.5                         |
| M0  | 16                          | 6.0                         |

cant fraction of them are much more active than the Sun on long periods does not imply a similar variability on 10 hour timescale. Estimating the latter is presently quite uncertain, as well from the theoretical point of view (very little is known about its physical origin) as from the observational one (amplitudes are too small to be observed from the ground). One can only conjecture about it. We have considered two such “guess-estimates”. For spectral type and magnitude around which most telluric planet detections are expected, G and K stars with $m_V = 15 − 16$, the 10 hour variability would be 2–6 times that of the Sun, whereas the threshold where the photometry in different colors is more efficient is 7–12 times it. If these estimates are relevant, the colored photometry would not provide a significant advantage.

On the other hand, if the colored information is obtained thanks to an instrument that does not lose many photons, as a dichroic and two CCD sets or a dispersive system (e.g. derived from the proposal by Chao & Chi (1998)) and a single CCD, photometry in white is still possible and as efficient as in the absence of color information. Then, the Precaution Imperative stresses the advantage of having the capability of photometry in different colors in case our present estimates of the 10 hour variability are incorrect, and stars are significantly more variable than expected. The main possible disadvantage of the corresponding instrument would be cost and complexity, at a level that has to be estimated.

In conclusion, it is our opinion that it is worth studying practical implementations of photometry in different colors for the Eddington mission before taking irreversible decisions.

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