Visual configuration segmentation of quantum states for phase identification in many-body systems

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(Dated: September 15, 2020)

Artificial intelligence provides an unprecedented perspective for studying phases of matter in condensed-matter systems. Image segmentation is a basic technique of computer vision that belongs to a branch of artificial intelligence. In this work, we propose a segmentation scheme named visual configuration segmentation (VCS) to unveil quantum phases and quantum phase transitions in many-body systems. By encoding the information of renormalized quantum states into a color image and segmenting the color image through the VCS, the renormalized quantum states can be visualized, from which quantum phase transitions can be revealed and the corresponding critical points can be identified. Our scheme is benchmarked on several strongly correlated spin systems, which does not depend on the prior knowledge of order parameters of quantum phases. This demonstrates the potential to disclose the underlying structure of quantum phases by the techniques of computer vision.

I. INTRODUCTION

In recent years, artificial intelligence (AI) [1–3] has caused a great impact on physics. One of the main branches of AI is the computer vision [4, 5], where computers or machines can gain high-level understanding from digital images or videos and implement the tasks that the human visual system can do [4, 5]. Until now, vision is still the most important way for human’s brain to obtain and analyze the information [4, 6]. Based on the rapid development of computer vision, various fields including economics, biomecine, face recognition and automatic pilot have been significantly advanced [7–9].

One of the fundamental techniques of computer vision is the image segmentation [10], which can divide the digital image into multiple regions according to some homogeneity criterion. Image segmentation is the first step in many attempts to analyze or interpret an image automatically [11]. The simplest and efficient method of image segmentation is the thresholding method [12]. According the threshold, an image can be transformed to a binary image. Proper binarization of the images is very important for separating the foreground object from the background.

In the same period of AI, the classical simulation of quantum many-body system has also made significant progresses based on the numerical renormalization group methods. From the density matrix renormalization group (DMRG) [13] to tensor networks (TNs) [14–16], one can simulate the ground states of quantum many-body systems approaching to the thermodynamic limit. For a local tensor $T = \{ T_{d=1, \cdots, \sigma}^{m=1, \cdots, \chi_1; n=1, \cdots, \chi_2} \}$ in TNs such as the matrix product states [15], it can be equivalently treated as a color image as the physical index $d$ can be interpreted as the color channel, as shown in Fig. 1, where each channel is related to a gray image with size $\chi_1 \times \chi_2$ [17].

![FIG. 1. (Color online) (a) A 3-index tensor related to matrix product states (MPS), where one unit of MPS is shown. $d$ is physical index with dimension $\sigma$, which represents the local physical space of particles. $m$ and $n$ are the virtual indexes with dimension $\chi_1$ and $\chi_2$, which are related to the entanglement between two subsystems. (b) The RGB color image with random pixels, which consist of three color channels that R, G, B correspond to red, green, blue colors. Each color channel is related to a pixel matrix. So, RGB color image can also be represented by 3-index tensor, where $d_c$ is the color space, $dp_1$ and $dp_2$ are the pixel space.](Image 1)

In this work, we design a visualization scheme of matrix product states (MPS) based image segmentation scheme [18] named visual configuration segmentation (VCS) to reveal the quantum phases and phase transitions. Based on this scheme, renormalized quantum states (the centre tensor of MPS by DMRG) are visualized by binary images. The selection of thresholds is the key for image segmentation [10, 11], which has a direct influence on the final results of image processing such as the edge detection [19]. The traditional way for the binarization of image is first to gray the image and then to choose a threshold [20, 21]. Different from the traditional way for binarization, in our scheme VCS, we first

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choose a threshold in each channel to get binary images, then we calculate the absolute differences of each two binary images to get the finally binary images (see Methods). When the renormalized quantum states are in the same phase, the corresponding binary images look similar. When there is a phase transition, the textures of the corresponding binary images exhibit sharp differences.

Our scheme is benchmarked on one-dimensional (1D) quantum lattice models, where various quantum phases including those within and beyond Landau paradigm. Different from the conventional approaches in many-body physics that often require the information of order parameters to characterize the quantum phases and quantum phase transitions, our scheme shows an intuitive sense of vision of the renormalized quantum states by image segmentation technique, and reveals the quantum phase transitions directly by visual sensing [22]. Our work paves a new way to study quantum many-body systems by revealing the quantum phases and quantum phase transitions through the computer vision techniques.

II. METHODS

For a three-index tensor $T = \{T_{m=1,\ldots,\chi_1; n=1,\ldots,\chi_2}\}$, $T_{m,n}$ is the tensor elements, and the dimensions of the three indexes $m$, $n$, and $d$ are denoted as $\chi_1$, $\chi_2$ and $\sigma$. $T$ can be interpreted as a color image through a map $\mathcal{G}$: $T = \{T_{m,n}\} \xrightarrow{\mathcal{G}} T^\mathcal{G} = \{T^\mathcal{G}_{m,n}\}$, where $T^\mathcal{G}_{m,n}$ represents the pixel value at the position $(m, n)$ in the color channel $d$. Each channel yields a gray image with the size $\chi_1 \times \chi_2$.

Based on the threshold method of image segmentation [10, 11, 18], we design a scheme named VCS to segment image $T^\mathcal{G}$. We take a cat image $T^\mathcal{G}[cat]$ (i.e. the image interpretation of tensor $T[cat]$) shows a shape of cat,) as an instance to illustrate VCS (Fig. 2).

There are two main steps to complete VCS. The first step is the binarization in the color channels, i.e. each gray image $T^\mathcal{G},d[cat]$ is mapped onto a binary image by the map $\mathcal{B}$ as $T^\mathcal{B},d \xrightarrow{\mathcal{B}} T^\mathcal{B},d$ satisfying

$$
T^\mathcal{B},d_{m,n} = \begin{cases} 
1 & T^\mathcal{G},d_{m,n} \geq \alpha^d \\
0 & T^\mathcal{G},d_{m,n} < \alpha^d
\end{cases}
$$

where $\alpha^d$ is the threshold value for $d$-th color channel. We choose the mean value of pixels as threshold value $\alpha^d$, i.e. $\alpha^d = \frac{1}{\chi_1 \times \chi_2} \sum_{m,n} T^\mathcal{G}_{m,n}[cat]$. The binary cat image $T^\mathcal{B},d[cat]$ is shown in Fig. 2 (c), where three binary images corresponding to channels $R$ ($d = 1$), $G$ ($d = 2$), and $B$ ($d = 3$) that contain the original cat.

The second step is to obtain the final binary image $T^{AD,\delta_1,d_2}$ by the map $\mathcal{AD}$ that the absolute difference between any two binary images $T^\mathcal{B},d_1$, $T^\mathcal{B},d_2$: $T^{AD,\delta_1,d_2} \xrightarrow{\mathcal{AD}} T^{AD,\delta_1,d_2}$ :

$$
T^{AD,\delta_1,d_2} = |T^\mathcal{B},d_1 - T^\mathcal{B},d_2|,
$$

with $d_1 \neq d_2$ and $T^{AD,\delta_1,d_2} = T^{AD,\delta_2,d_1}$. The number of the final binary images is $2^{\sigma - 1}$. For the cat image $T^\mathcal{G}[cat]$, three binary images $T^{AD,\delta_1,2}[cat]$, $T^{AD,\delta_1,3}[cat]$ and $T^{AD,\delta_2,3}[cat]$ are shown in Fig. 2 (d).

Both kinds of binary images $T^\mathcal{B}$ and $T^{AD}$ are the segmentation of original colorfull image $T^\mathcal{G}$. However, the concentrated information of the $T^\mathcal{B}$ by map $\mathcal{B}$ and map $\mathcal{AD}$ are different. Taking the cat image $T^\mathcal{G}[cat]$ as example, $T^\mathcal{B}[cat]$ show the whole outlines of the cat in Fig. 2 (c) while $T^{AD}[cat]$ highlights the local key details of cat such as the eyes and ears in Fig. 2 (d). Our scheme manages to segment the details of the cat from the backgrounds.

One shall note that in the existing ways [20], the segmentation of images will be finished after the step of mapping $\mathcal{B}$. However, in our scheme, we make a further binary segmentation by the map $\mathcal{AD}$.

III. ENTROPY AND CORRELATION FROM BINARY IMAGES

We introduce two concepts that are information entropy and correlation to qualitatively characterize the segmentation. In particular, information entropy introduced by Shannon[23, 24] allows us to measure the amount of the information given by the distribution of binary pixel values. The pixels of binary images have two values 0 and 1, from which the Shannon entropy is calculated by $S = -p(0)\log_2p(0) - p(1)\log_2p(1)$, where $p(0)$ ($p(1)$) is the probability of the pixels being 0 (1).

![FIG. 2. (Color online) Graphically illustrate the image segmentation scheme VCS. (a) The color image $T^\mathcal{G}[cat]$, which is the visualization of 3-index tensor $T[cat]$ by mapping $\mathcal{G}$, where $\mathcal{G} = \mathcal{I}$ is the identical transformation. (b) The gray images $T^{\mathcal{G},d}[cat]$ with $d = 1, 2, 3$ are corresponding to three color channels $R$ (red), $G$ (green), and $B$ (blue). (c) The binary images $T^{\mathcal{B},d}[cat]$ corresponding to three color channels $R$, $G$, and $B$ are obtained by mapping $\mathcal{B}$. (d) The binary images $T^{AD,\delta}[cat]$ are obtained by mapping $\mathcal{AD}$ according to Eq. (2).](image-url)
FIG. 3. (Color online) The color image $\Phi^G$, binary images $\Phi^B$ and binary images $\Phi^{AD,\delta,2}$ of the center tensor $\Phi$ of the ground states of the transverse field Ising model (TFIM) with different transverse fields $h_x = 0.2$, 0.5, and 0.8 are in (a), (b) and (c), respectively. (a) The distinguished details of $\Phi^G$ with different $h_x$ are marked by red circles. (b) The binary images $\Phi^{B,d=1}$ with different $h_x$ have and its textures are always non-regular. (c) The antiferromagnetic (AFM) and paramagnetic (PM) phases are clearly specified by binary images $\Phi^{AD,\delta,2}$. The partial regular texture of $\Phi^{AD,\delta,2}$ when the system at QCP ($h_x = 0.5$) is marked by red triangular. (d) is the histogram that counts the occurring times of the tensor elements of $\Phi$ with $h_x = 0.2$, 0.5, and 0.8. (e) More binary images $\Phi^{AD,\delta,2}$ with different transverse fields that $h_x = 0.1, 0.3, 0.4, 0.6, 0.7$ and 0.9. For DMRG, we take the size of the system $L = 80$ and the dimension cut-off $\chi = 30$. The size of binary images $\Phi^{AD,\delta,2}$ is $\chi \times \chi$.

To reflect the difference of local patterns of the images, we use a finite sized window $W_{a,b}$ to define:

$$T^{W_{x,y}}_{a,b} = \{ T^{AD,1} \}_{x+a-1, y+b-1}^{x, y},$$

where $a$ and $b$ are the length and width of the window respectively, and $T^{AD}_{1,1} = T^{AD}$. We use the window $W_{a,b}$ to scan the binary image $T^{AD}$ by moving one site for each step, so the total number of $T^{W_{x,y}}_{a,b}$ is $(\chi_1 - a + 1)(\chi_2 - b + 1)$. We get the cumulative Shannon entropy $S_{T^{AD}}$ from binary image $T^{AD}$ by:

$$S_{T^{AD}} = \sum_{x=1}^{\chi_1-a+1} \sum_{y=1}^{\chi_2-b+1} S(T^{W_{x,y}}_{a,b}) = \int \chi_1 - a + 1 \right) (\chi_2 - b + 1)$$

where $S(T^{W_{x,y}}_{a,b})$ is the Shannon entropy of $T^{W_{x,y}}_{a,b}$, and $pix$ represents the pixel value. The probability of $pix$ ($pix = 0$ or 1) $P_{W_{x,y}}(pix)$ is $\frac{n_{pix}}{ab}$ with $n_{pix}$ the number of $pix$ in $T^{W_{x,y}}_{a,b}$.

As $T^{AD}$ only includes two pixels that are 0 and 1, we can intuitively view $T^{AD}$ as the classical spin configurations $\tilde{T}^{AD}$ where pixels 0 (1) corresponds to spin up (down). Choosing a finite region of $\tilde{T}^{AD}$ by the window $W_{x,y}$, we get a spin configuration $\tilde{T}^{W_{x,y}}$, and we interpret it as a product state $|\tilde{T}^{W_{x,y}}\rangle$. From this perspective, we define a quantity named virtual correlation $E_{T^{W}}$:

$$E_{T^{W}} = - \frac{\langle \tilde{T}^{W_{x,y}} | \sum_{i<j} \sigma_i^z \sigma_j^z | \tilde{T}^{W_{x,y}} \rangle}{ab \langle \tilde{T}^{W_{x,y}} | \tilde{T}^{W_{x,y}} \rangle},$$

where $\sigma^z$ is Pauli operator, $<i,j>$ means $i$ and $j$ are nearest neighbours.

IV. RESULTS

We firstly examine VCS on the 1D transverse field Ising model (TFIM) [25, 26] with the Hamiltonian $H = \sum_\delta \hat{S}_i^z \hat{S}_{i+1}^z - h_x \sum_i \hat{S}_i^x$, where $\hat{S}_i^z$ and $\hat{S}_i^x$ are the z- and x-component spin operators, respectively, and $h_x$ is the transverse field. With different $h_x$, a Landau-type [27]
quantum phase transition occurs at the quantum critical point (QCP) \( h_x = 0.5 \), which separates the antiferromagnetic (AFM) and paramagnetic (PM) phases. Using DMRG [13, 14] to calculate the ground states for different transverse fields, we thus obtain the colorful image interpretations of the ground states with different transverse fields \( h_x \). However, when the system size \( L = 30 \) and dimension cut-off \( \chi = 30 \), where the QCP is characterized by the visual sense.

This explains why it is hard to classify the quantum phase transition points can be accurately identified as a brick wall in \( \Phi^{AD, \delta_1, 2} \). More interestingly, at the critical transverse field \( h_x = 0.5 \), the corresponding binary image \( \Phi^{AD, \delta_1, 2}[h_x = 0.5] \) shows a mixed pattern of \( \Phi^{AD, \delta_1, 2}[h_x = 0.2] \) and \( \Phi^{AD, \delta_1, 2}[h_x = 0.8] \). As shown in Fig. 3 (c), the upper-left part (marked by a red triangular) of \( \Phi^{AD, \delta_1, 2}[h_x = 0.5] \) is a regular configuration like a brick wall, and the pixels in lower-right part of \( \Phi^{AD, \delta_1, 2}[h_x = 0.5] \) are non-regular distributed like the pixels in \( \Phi^{AD, \delta_1, 2}[h_x = 0.2] \).

From the binary images \( \Phi^{AD, \delta_1, 2}[h_x = 0.2], (0.5, 0.8) \), the quantum phase transition from AFM to PM of TFIM is visualized, and the phase transition can be characterized by the variation of the pattern in \( \Phi^{AD, \delta_1, 2}[h_x = 0.2], (0.5, 0.8) \). More binary images \( \Phi^{AD, \delta_1, 2} \) with different system sizes \( L \), and dimension cut-off \( \chi \) are in the Appendix.

To further demonstrate the validity of characterizing quantum phase transitions by VCS, we turn to the 1D spin-1/2 anisotropic XXZ Heisenberg model (XXZH) \( H = \sum_i (S_i^x S_{i+1}^x + S_i^y S_{i+1}^y) + \Delta \sum_i S_i^z S_{i+1}^z \) with \( \Delta \) the magnetic anisotropy. The system goes through three phases, which are the FM (\( \Delta < 1 \)), XY (\( -1 < \Delta < 1 \)), and AFM (\( \Delta > 1 \)) phases. The canonical central tensor of the ground states is \( \Theta \). According to VCS, the binary images \( \Theta^{AD, \delta_1, 2} \) is obtained by the procedure: \( \Theta \xrightarrow{G, Eq. (8)} \Theta^{G, Eq. (1)} \xrightarrow{B, Eq. (2)} \Theta^{B, Eq. (2)} \xrightarrow{AD, Eq. (2)} \Theta^{AD, \delta_1, 2} \), where the threshold value for the binarization \( B \) is \( \alpha = 0.2, (0.5, 0.8) \).

Fig. 4 shows the binary images \( \Theta^{AD, \delta_1, 2} \) in AFM, XY, and FM phases. When the system is in AFM and FM phases, the textures of the \( \Theta^{AD, \delta_1, 2} \) are non-regularly, as shown in Fig. 4 (a) and (c). However, when the system is in XY phase, the textures of the \( \Theta^{AD, \delta_1, 2} \) are regularly distributed like brick walls, as shown in Fig. 4 (b). The phase transition points can be accurately identified as the abrupt change of the image style of \( \Theta^{AD, \delta_1, 2} \), which happens at \( \Delta = 1 \) and \( \Delta = -1 \), respectively. More binary images \( \Theta^{AD, \delta_1, 2} \) with different system sizes \( L \) and dimension cut-off \( \chi \) are in the Appendix.

Determining the topological phases is a challenging task in quantum many-body systems [29–31]. Considering the spin-1 Heisenberg model (spin-1 HM) in a magnetic field with the Hamiltonian \( H = \sum_i \sum_{\gamma=x,y,z} S_i^\gamma S_{i+1}^{\gamma} - h_z \sum_i S_i^z \) we use the VCS to visualize its ground states. For \( h_z < 0.414 \), the system is in a symmetry protect topological phase known as Haldane phase [32–34] with non-trivial boundary excitations and long-range string orders [35, 36]. For \( h_z > 0.414 \), the system enters a topologically trivial magnetic (TTM) phase [31]. The canonical central tensor of the ground states is \( \Omega = \{ \Omega_{d=0, \cdots, d_{max}} \} \), where \( d_{max} = 3 \), and \( \chi = 40 \). We obtain the binary image \( \Omega^{AD, \delta_1, 3} \) by the VCS scheme:
The specific self-similarity of Haldane phase supports the periodicity. The pattern given by $\Omega_{AD}$ is oscillating in magnetic ordered phases, where the distribution of the pixels is non-uniformly. For DMRG, we take the size of the system $L = 80$ and the dimension cut-off $\chi = 30$.

$$\Omega^{G} \rightarrow \Omega^{G'} \xrightarrow{B_{eq}} \Omega^{B_{eq} + A_{D}} \xrightarrow{B_{eq}} \Omega^{A_{D}, \delta_{1}, 3},$$

where $\Omega^{G'} = (1 + \Omega)/2$ and the threshold value for the binarization $B$ is $\alpha^{d=1} = \alpha^{d=2} = 0.5$.

Fig. 5 (a) shows the binary images $\Omega^{A_{D}, \delta_{1}, 3}$ in Haldane phases with $\Delta = 0$ and $\Delta = 0.6$, where a regular pattern like the lattice-window with diamond holes is displayed. Although a few parts exhibits local non-homogeneity, the whole image shows approximate spatial uniformity and periodicity. The pattern given by $\Omega^{A_{D}, \delta_{1}, 3}$ in Haldane phase also exhibit self-similarity. Fig. 5 (b) shows $\Omega^{A_{D}, \delta_{1}, 3}$ at the quantum critical point (QCP) with $\Delta = 0.414$, where the image gives a weak distorted brick wall pattern. The spatial arrangements of the texture become more dense. When the system is in TTM with $\Delta = 0.6$ as shown in Fig. 5 (c), the homogeneous pattern of $\Omega^{A_{D}, \delta_{1}, 3}$ is absent. Phase transition from Haldane to TTM corresponds to the changes of patterns of $\Omega^{A_{D}, \delta_{1}, 2}$ and $\Omega^{A_{D}, \delta_{1}, 3}$, as shown in Fig. A4 in Appendix. The specific self-similarity of Haldane phase supports that topological phases can also be characterized by VCS.

From the perspective of visualization, we note that phase transitions correspond to the drastic change of the binary patterns of $\Phi^{A_{D}}$, $\Theta^{A_{D}}$ and $\Omega^{A_{D}}$. Here, we use two quantities namely the cumulative Shannon entropies and virtual correlations of binary images to quantitatively characterize the phase transitions.

As shown in Figs. 6 (a), (b) and (c), the cumulative Shannon entropies $S_{\Theta^{A_{D}}}$, $S_{\Phi^{A_{D}}}$ and $S_{\Omega^{A_{D}}}$ in different quantum phases display different behaviours. For TFIM, when the system is in the magnetic ordered phases, $S_{\Phi^{A_{D}}}$ shows an oscillatory behaviour, which means the local patterns of $\Phi^{A_{D}}$ are unstable with different transverse field $h_z$. However, when the system is in disordered paramagnetic phase, $S_{\Phi^{A_{D}}}$ is stable and keeps a constant value.

The binary images $\Omega^{A_{D}, \delta_{1}, 3}$ of ground states in the spin-1 Heisenberg antiferromagnetic model with different magnetic field $h_z$.

(a) $\Omega^{A_{D}, \delta_{1}, 3}$ in Haldane phase with $h_z = 0$ and $h_z = 0.1$. The textures are homogeneous and show the regularity of the lattice-window with diamond holes. (b) $\Omega^{A_{D}, \delta_{1}, 3}$ of quantum critical point (QCP) $h_z = 0.414$. The texture is like a weak-distorted brick wall. (c) $\Omega^{A_{D}, \delta_{1}, 3}$ in topological trivial magnetic phase (TTM) with $h_z = 0.6$. The texture are inhomogeneous. For DMRG, we take the size of the system $L = 120$ and the dimension cut-off $\chi = 40$. 

By means of the cumulative Shannon entropies, the
system sizes for the phases of TFIM, XXZHM and spin-1 HM are taken as $L=80$, 80 and 200, respectively. The windows for getting the virtual correlations $E$, due to the large variance of tensor elements. The normalized quantum states and quantum phase transition mapping the ground states by interpreting the canonical central tensor of the ground many-body systems. There are three steps in VCS: first characterize the quantum phases and phase transitions for XXZHM, and spin-1 HM, the textures of $\Phi_{W}$, $\Theta_{W}$ and $\Omega_{W}$ in PM phase, XY phase and TTM phase are more uniform, so the corresponding values of cumulative Shannon entropy are larger.

The virtual correlations $E_{gW}$, $E_{\Theta W}$, and $E_{\Omega W}$ of binary images $\Phi_{AD}$, $\Theta_{AD}$ and $\Omega_{AD}$ are shown in Fig. 6(d), (e) and (f), respectively. Obviously, virtual correlations can also reflect the information of quantum phase transitions.

**Conclusion**—In summary, we have developed an image segmentation scheme named VCS to visualize and characterize the quantum phases and phase transitions for many-body systems. There are three steps in VCS: first interpreting the canonical central tensor of the ground states into a color image by mapping $G$, and second binarizing the color image in each color channel by mapping $B$, then third calculating the absolute differences of the binary images related to color channels by mapping $AD$. The colorful image interpretation of the ground states by mapping $G$ cannot give an effective information of renormalized quantum states and quantum phase transition due to the large variance of tensor elements. The selection of threshold in $B$ is the key in our scheme, and a wrong selection of the threshold value may misinterpret the background pixel and result in a degradation of scheme performance. For the ground states of quantum many-body systems, we chose the threshold value as 0.5, which equals to binarizing the canonical central tensor of the ground states based on signs of tensor elements.

Based on VCS, the renormalized quantum states represented by binary images show different patterns in distinct quantum phases, from which the quantum critical points can be identified. The success of VCS is demonstrated on three 1D quantum many-body spin models that possess various quantum phases, including the conventional phases within Landau paradigm and the topological phases with nonlocal orders. When the TFIM is in PM phase, and the spin-$\frac{1}{2}$ XXZHM is in XY phase, the corresponding binary image patterns show a specific brick wall pattern, while the lattice-window with diamond holes emerges in the texture pattern of the spin-1 Heisenberg model in Haldane phase. These patterns display the spatial homogeneity, self-similarity, periodicity, and directionality, which may be related to the underlying physical properties of the quantum phases. When the systems are at the QCP, the image pattern is a mixture of the patterns of the two adjacent quantum phases.

Our scheme gives a perceptual analysis of the renormalized quantum states in many-body systems based on the image segmentation. Other techniques in computer vision such as edge detection [37], digital image processing [38] may also be utilized in this scheme to obtain a
better performance of the image interpretation of quantum states. This would motivate people to seek for techniques in computer vision to unveil more novel properties of quantum states of many-body systems.

ACKNOWLEDGMENTS

This work is supported in part by the NSFC (Grant No. 11834014), the National Key R&D Program of China (Grant No. 2018FYA0305804), the Strategic Priority Research Program of the Chinese Academy of Sciences (Grant No. XDB28000000), and Beijing Municipal Science and Technology Commission (Grant No. Z118100004218001). SJR is supported by Beijing Natural Science Foundation (No. 1192005 and No. Z180013), and the Academy for Multidisciplinary Studies, Capital Normal University.

Appendix A: Binary images representation of quantum states with different system sizes

By our designed image segmentation scheme VCS, the quantum states of many-body systems can be represented by binary images. Below, we show the textures of binary image representation of quantum states with different system size \( L \) and dimension cut-off \( \chi \) to illustrate that the special binary textures of quantum states are not from the finite-size effects.

As shown in Fig. A1, the ground states of the transverse field Ising model (TFIM) are represented by binary images \( \Phi^{AD,\chi_2} \), where the ground states are calculated by density matrix renormalization group (DMRG) method with system size \( L = 80, 90, 100 \) and dimension cut-off \( \chi = 20, 40, 60 \). The sizes of binary images are \( \chi \times \chi \). When the dimension cut-off \( \chi \) is fixed at 40, the textures of \( \Phi^{AD,\chi_2} \) with different system sizes \( L = 80, 90, 100 \) keep a consistency, which indicate the specific binary textures of \( \Phi^{AD,\chi_2} \) are not from the finite-size effect. Whenever the system is in antiferromagnetic (AFM) phase, quantum paramagnetic (PM) phase or at the quantum critical point (QCP), the textures of \( \Phi^{AD,\chi_2} \) show specific self-similarity with increasing \( \chi \) from 20, 40 to 60 when system size \( L \) is fixed at 80. Self-similarity is the fundamental character of fractal geometry [?]. As the binary images \( \Phi^{AD,\chi_2} \) are directly related to the sign of the local tensor elements in ground states, the self-similarity may be related to some latent physics of the quantum mechanic wavefunctions.

In Fig. A2, we show the binary images \( \Theta^{AD,\chi_2} \), which is the interpretation of the ground states of the spin-1 XXZ anisotropy Heisenberg chain model. We fix the system size at \( L = 80 \), and show the various of the textures of \( \Theta^{AD,\chi_2} \) with different dimension cut-off \( \chi = 20, 40, \) and 60. When the system is in some quantum phases, the textures of \( \Theta^{AD,\chi_2} \) show a self-similarity with the increase of image size \( \chi \times \chi \). By taking the standard XY phase (\( \Delta = 0 \)) as an example, we may see that the brick-wall pattern of the textures does not change with the increase of \( \chi \).

Appendix B: Binary images of other local tensors in matrix product states

The ground states \( \left| \Psi \rightangle \) of the TFIM in the form of MPS is in the main text (Eq. (7)), where \( \left| \Psi \rightangle = \sum_{\{s\}} A^{[1]} \cdots A^{[s_{i-1}]} B^{[s_i]} B^{[s_{i+1}]} \cdots B^{[s_L]} \) and \( \left| \Psi \rightangle \) is in the mixed canonical form. \( \Phi \) is the canonical central tensor and \( \{A^i\} \ (\{B^j\}) \) are the left- (right-) normalized. In the main text, we treat \( \Phi \) as the feature of the ground states and use its binary images \( \Phi^{AD,\chi_2} \) to characterize quantum phase transitions based on scheme VCS.

Here, we treat other local tensors \( \{A^i\} \ (\{B^j\}) \) as the feature of \( \left| \Psi \rightangle \), and use VCS to obtain its binary images as shown in Fig. A3. Although each local tensor is different, but its binary images \( \{A^i\}^{AD,\chi_2} \) along the MPS chain show the same textures, which may be related to a kind of symmetry of the ground states.

Appendix C: Color images and binary images of the spin-1 chain

In the main text, we use the binary image \( \Omega^{AD,\chi_3} \) to show different quantum phases of the spin-1 antiferromagnetic chain model. The dimension of physical bond \( \Omega \) is 3, for each \( \Omega \) under the fixed magnetic field \( h_z \), we can get three binary images such as \( \Omega^{AD,\chi_2} \), \( \Omega^{AD,\chi_3} \), and \( \Omega^{AD,\chi_3} \). Here, we show the color images \( \Omega^G \) and binary images \( \Omega^{AD,\chi_2} \) (\( \Omega^{AD,\chi_3} \)) with different \( h_z \) in Fig. A4.

The visual sense of \( \Omega^G \) is the same as the color image \( \Phi^G \) of the TFIM in Fig. 3 of main text, where most of the pixels are gray except a few distinguishable pixels in parts marked by red circles, from which the quantum phase transition from Haldane phase to topological trivial magnetic (TTM) phase can hardly be identified. The texture of binary images \( \Omega^{AD,\chi_2} \) (\( \Omega^{AD,\chi_3} \)) in Haldane phase with \( h_z = 0.1 \) shows the pattern of many diamond-shaped clusters. As the textures of \( \Omega^{AD,\chi_3} \) in Haldane phase are the lattice-window with diamond holes, \( \Omega^{AD,\chi_3} \) and \( \Omega^{AD,\chi_2} \) (\( \Omega^{AD,\chi_3} \)) display the complementary symmetry. When the system is at the QCP \( (h_z = 0.414) \), such specific textures are broken, and when the system is in the TTM phase, the textures are non-regular.

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FIG. A2. (Color online) Binary images $\Theta^{A_{D},A_{1}}$ with fixed system size $L = 80$ and different dimension cut-off $\chi = 20$, 40, and 60 are given in (a), (b), and (c), respectively. In each case of $L$ and $\chi$, $\Theta^{A_{D},A_{1}}$ with XXZ anisotropy $\Delta = -1.4, -1, 0, 1, 1.4$ are presented, which correspond to FM phase, QCP (FM to XY), XY phase, QCP (XY to AFM), and AFM phase, respectively. The size of the binary images $\Theta^{A_{D},A_{1}}$ is $\chi \times \chi$.

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FIG. A3. (Color online) (a) The ground states $\psi$ of the transverse field Ising model (TFIM) are represented in form of matrix product states. $\psi$ is in the mixed canonical form, where $A^i$ and $B^j$ are left- and right- normalized. (b) By the scheme of VCS, $\{A^i\}$ and $\{B^j\}$ are visualized by binary images with $i = 10, 20, 30$ and $j = 50, 60, 70$, where $i$ ($j$) is the position of local tensor on MPS. Three cases for the TFIM in AFM phase ($h_x = 0.2$), PM phase ($h_x = 0.8$) and at the QCP ($h_x = 0.5$) are presented. For DMRG, we take the size of system $L = 80$ and dimension cut-off $\chi = 30$. The size of binary images of $\{A^i\}$ and $\{B^j\}$ is $\chi \times \chi$.

FIG. A4. (Color online) The ground states of the spin-1 antiferromagnetic Heisenberg model in Haldane phase ($h_z = 0.1$), topological trivial magnetic (TTM) phase ($h_z = 0.6$), and at the quantum critical point (QCP) ($h_z = 0.414$) are visualized by color image $\Omega^G$ and binary image $\Omega^{AD,\delta_{1,2}}$ ($\Omega^{AD,\delta_{2,3}}$) based on the VCS. The distinguishable details of $\Omega^G$ with different $h_z$ are marked by red circles. The system size $L = 120$, and dimension cut-off in DMRG $\chi = 40$. The sizes of color images $\Omega^G$ and binary images $\Omega^{AD,\delta_{1,2}}$ ($\Omega^{AD,\delta_{2,3}}$) are $3 \times \chi \times \chi$ and $\chi \times \chi$. 