Force and flowfield measurements to understand unsteady aerodynamics of cycloidal rotors in hover at ultra-low Reynolds numbers

Carolyn M Reed, David A Coleman and Moble Benedict

Abstract
This paper provides a fundamental understanding of the unsteady fluid-dynamic phenomena on a cycloidal rotor blade operating at ultra-low Reynolds numbers (Re ~ 18,000) by utilizing a combination of instantaneous blade force and flowfield measurements. The dynamic blade force coefficients were almost double the static ones, indicating the role of dynamic stall. For the dynamic case, the blade lift monotonically increased up to \( \pm 45^\circ \) pitch amplitude; however, for the static case, the flow separated from the leading edge after around 15\(^\circ\) with a large laminar separation bubble. There was significant asymmetry in the lift and drag coefficients between the upper and lower halves of the trajectory due to the flow curvature effects (virtual camber). The particle image velocimetry measured flowfield showed the dynamic stall process during the upper half to be significantly different from the lower half because of the reversal of dynamic virtual camber. Even at such low Reynolds numbers, the pressure forces, as opposed to viscous forces, were found to be dominant on the cyclorotor blade. The power required for rotation (rather than pitching power) dominated the total blade power.

Keywords
Cyclorotor, unsteady aerodynamics, cycloidal rotor, virtual camber, instantaneous forces, instantaneous flowfield

Introduction
The development of efficient, maneuverable, gust tolerant, and sustained hover-capable micro air vehicle (MAV) platforms with expanded flight envelope is the key to the success of many missions in both military and civilian scenarios. During the past decade, there have been many studies on experimental optimization of MAV-scale conventional rotors.\(^1,2\) These studies helped improve the hover figure of merit (FM) of a micro-rotor from an initial value of 0.42 to 0.65. However, this maximum FM is still far below the full-scale helicopter value (~0.8) and is attributed to low Reynolds number (10^4–10^5) aerodynamics, especially the low airfoil lift-to-drag ratios and the complex induced wake distribution below the rotor.\(^2\) Therefore, the vehicles developed using these optimized rotors could only achieve a maximum hover endurance of 15 min which would make them incapable of any realistic missions.\(^1,2\) This clearly indicates the need for a step improvement in hover efficiency, which could only be achieved through a radically different concept to fly at these low Reynolds numbers. A new revolutionary concept of a cyclocopter or a cycloidal rotor-based aircraft (Figure 1) is being investigated.

A cycloidal rotor (or cyclorotor) is a rotating-wing system (Figure 1), where the span of the blades runs parallel to the axis of its rotation. The pitch angle of each blade is varied cyclically by mechanical means such that each blade experiences positive geometric angles of attack at both the top and bottom halves of...
its circular trajectory (Figure 2(a)). The resulting time-varying lift and drag forces produced by each blade are resolved into the vertical and horizontal directions, as shown in Figure 2(a).

With this kind of cyclic blade kinematics, the blades produce a net thrust. Varying the amplitude and phase of the cyclic blade pitch is used to change the magnitude and direction of the net thrust vector produced by the cyclorotor. Pioneering research on the cyclorotor concept for MAV applications has been conducted over the last 10 years.4–16 This body of work represents one of the most comprehensive evaluations ever conducted on cyclorotors at MAV scales and involved systematic performance measurements in both hover4,8–11 and in a wind tunnel,12–14 flowfield studies using particle image velocimetry (PIV),4,8,9 computational fluid dynamic analysis,12,13 and aerelastic modeling.15

These studies established a fundamental understanding of the cyclorotor performance and helped in formulating a set of design principles for an efficient cyclorotor operating at MAV-scale Reynolds numbers (Re < 40,000). This work, along with innovative vehicle design techniques and the development of novel autonomous flight control strategies, has led to the first flying cyclorotor-based aircraft.5 Since then, a wide range of hover-capable cyclorotor aircraft ranging in size from 29 to 800 g have been developed,5–7,16 demonstrating conclusively the feasibility of the cyclorotor concept for MAV applications (Figures 1 and 2(b)). Based on the results from these previous studies,10 it has been shown that this concept has the potential to achieve higher levels of aerodynamic efficiency (almost 50% higher) than a conventional rotor (Figure 3(a)) at MAV-scale Reynolds numbers. The possible reasons for the improved performance could be the uniform spanwise load distribution on the cyclorotor blades and the favorable unsteady aerodynamic mechanisms.

Systematic forward flight studies were also conducted in the wind tunnel, which showed the potential for a cyclorotor to perform efficient high-speed forward flight even beyond an advance ratio of 1.0 by a simple phasing of the cyclic blade-pitch schedule.5,14 As shown in Figure 3(b), the power required decreased by almost 40% from hover to an advance ratio close to unity, which is phenomenal for an MAV-scale rotor. A twin-cyclocopter (Figure 1) has five control degrees of freedom (three RPMs and two thrust directions).5 As a result, the cyclorotor would have greater actuation potential than a typically under-actuated system such as a quad-rotor. In other words, the cyclorotor can potentially command instantaneous accelerations in more directions than a quad-rotor. Another potential advantage of a cyclorotor, especially for indoor reconnaissance missions, is its lower acoustic signature when compared to conventional quad-rotors owing to the lower rotational speed. The only drawback of a cyclorotor is the rotor structural weight, which needs to be significantly reduced in the next generation of cyclorotor designs.

It is important to note that the key focus of the previous cyclorotor research was to understand the time-averaged rotor performance (lift, thrust, and power) in both hover and forward flight at Re ~ 40,000,4,8–14 which is right at the median of the estimated operating Reynolds numbers for MAVs, which is Re ~ 10,000–100,000.17–19 As it stands, there is a significant dearth in the understanding of the unsteady blade aerodynamics, which is even more important at the very low MAV Reynolds numbers at which the next generation of pocket-sized flying vehicles for reconnaissance and surveillance will operate. For example, a meso-scale cyclocopter shown in Figure 1 (radius = 1 in., weight = 29 g) is the smallest vehicle of this type built to date and operates at a Reynolds number of around 18,000, which is ultra-low for an MAV-scale Reynolds number. The development of this vehicle is described in Runco et al.3 At these “ultra-low” Reynolds numbers, the steady airfoil performance (lift/drag) would be significantly lower compared to moderately low Reynolds numbers (Re ~40,000).3 Additionally, the flow will be extremely susceptible to separation, and therefore even the smallest perturbation could stall conventional rotor blades. In these types of conditions, we expect the unique unsteady aerodynamics of a cyclorotor to greatly enhance performance, similar to that of a flapping wing. This means, understanding and utilizing the potential of blade unsteady aerodynamics become important at these extremely low Reynolds numbers. Therefore, if the focus of the previous research was on time-averaged performance, the goal of the present study is to understand unsteady blade aerodynamic loads in hover at much lower Reynolds numbers (Re < 20,000). Knowing the forces and flowfield at

---

**Figure 1.** 29-g meso-scale cyclocopter.3
each instant of the blade trajectory can reveal key information about how the blade lift, drag, and pitching moment are affected by blade pitching kinematics and unsteady flow curvature effects, which forms the motivation for the present work.

Obtaining the instantaneous unsteady forces is extremely challenging if the experiments are conducted in air because at high rotational speeds, the aerodynamic forces are corrupted by the large inertial forces. Therefore, the present study conducted at Texas A&M University utilizes a unique experimental setup to measure the instantaneous blade fluid dynamic forces and flowfield (PIV) on a hovering cyclorotor blade in water at matched Reynolds numbers. The goal of the proposed research is to quantify the instantaneous blade forces and flowfield on a cyclorotor blade operating at ultra-low Reynolds numbers ($\text{Re}/C^2 < 18,000$) and high reduced frequencies ($k/C > 0.3$), aerodynamic parameters very similar to that of the meso-scale cyclocopter discussed previously. These experimental results will be used to unravel the key fluid dynamic mechanisms on a cyclorotor blade.

**Experimental methodology**

In order to gain understanding of the unsteady flow phenomena on a cyclorotor blade, the instantaneous blade forces are measured on a single blade in water at matched Reynolds numbers. The reason for conducting these experiments in water is due to the ability to match the Reynolds numbers at significantly lower blade speeds and higher fluid dynamic to inertial force ratio when compared to experiments in air. Additionally, since the purpose of the present work is to quantify the force generation and flowfield characteristics associated with a cyclorotor blade, it is desirable to remove the aerodynamic interaction effects between multiple blades; hence, the reason one blade was utilized. The forces are directly measured at the blade root with a miniature six-component force balance. To obtain just the fluid dynamic forces acting on the cyclorotor blade, the following procedure is followed: first, the total forces are acquired by performing the experiment in water; next, the inertial forces are obtained by repeating the same experiment in air; finally, to calculate the pure fluid dynamic forces, the
inertial forces are subtracted from the total forces. Figure 4(a) shows the three forces in the radial direction as a function of blade azimuthal position. It can be seen that the inertial forces are only a small fraction of the total forces in water.

**Experimental setup**

The experimental setup is shown in Figure 4(b). For this experiment, the forces and moments are directly measured at the blade root using a miniature six-component force balance (ATI Mini 27 Titanium). Instead of using the conventional four-bar based blade pitching mechanism, individual blade control (IBC) is implemented using an analog feedback servo. This allows to electronically couple the blade pitch angle with its azimuthal location (obtained using an encoder) by commanding the servo to provide the required blade pitch kinematics based on the feedback from the blade azimuthal position. The electronic blade pitch control greatly simplifies the blade pitch mechanism. The cyclorotor is 1-bladed with a radius of 3.43 in., has a pitch axis location at the quarter chord, and is rotated by a Maxon EC 22 brushless motor that is equipped with Hall-effect sensors for precise rpm control. The motor is mounted in series with a Maxon Planetary Gearhead with a reduction ratio of 370:1. A 12-channel slip ring is used to transmit the signals from the balance and servo in the rotating frame to the data acquisition equipment in the stationary frame.

The tests are performed in a 3.2 ft long/1.6 ft wide/2.4 ft high rectangular tank, which gives about one rotor diameter or more in every direction azimuthally so as to minimize “ground” effects due to the walls of the tank. The blade is rapid prototyped from ABS plastic and has a 12-in. span and 2-in. chord resulting in an effective aspect ratio of 12 (since there is only one free tip). The blade airfoil is a NACA 0015 and has been sealed with shellac to prevent water absorption due to the nature of the printed material that could corrupt the inertial force measurements.

For conducting detailed flowfield measurements, the same cyclorotor test-rig is implemented with a PIV system shown in Figure 5(a). This system includes an EverGreen dual pulsed laser and power supply, which have been positioned next to the tank. A LaVision Imager sCMOS scientific camera with 5.5 megapixel resolution has been mounted underneath the tank, and pointed normal to the blade tip. The laser has been mounted such that the laser sheet hits midspan on the blade, which allows for visualization of mostly two-dimensional flow (schematic of the PIV setup shown in Figure 5(b)). The images are captured and processed using the LaVision DaVis 8 data acquisition and visualization software.

**Blade force and moment measurements**

Blade forces and moments were measured for both static and dynamic pitch cases. For the static cases, as shown in Figure 6, the blade pitch angle with respect to the tangent of its circular trajectory was held constant. The purpose of these tests was to understand the effect of static angle of attack and steady flow curvature effects such as virtual camber (explained in subsequent sections), due to the curvilinear nature of the flow experienced by the cyclorotor blades. A static pitch angle sweep from $-45^\circ$ to $+45^\circ$ was performed at 40 r/min in water, which corresponds to a Reynolds number of approximately 18,000. The next step was to conduct dynamic blade pitch experiments on the cyclorotor, meaning that the blade was actively pitched using the blade pitch servo to replicate the cyclic blade pitching kinematics shown in Figure 2(a). A pitching amplitude sweep from $\pm5^\circ$ to $\pm45^\circ$ in steps of $5^\circ$ was tested at 40 r/min similar to the static case.
For each test case, data were recorded for 3 min: about 20 s of tare data (rotor not rotating), 2 min of rotor operating at desired RPM, and the rest of the time duration was used for increasing and decreasing the rotor speed. The data were then processed and analyzed using MATLAB. Each time–history curve presented in the paper is an average of 80 waveforms (from 80 consecutive cycles after the rotor has reached the steady state). Each test case was also performed three times, and an average of the three was calculated. This cyclic averaging and test repetition helped minimize any random errors associated with the data.

The purpose of doing both static and dynamic pitch experiments was to compare the measured blade forces and flowfield from the two cases and then isolate the unsteady aerodynamic force production mechanisms from the steady ones especially in a curvilinear flow environment, which will be discussed more in the subsequent sections. In the paper, blade radial force, tangential force, and pitching moment are presented. Radial force will be referred to as lift and tangential force as drag. The reason for such a terminology is the fact that the inflow velocity is much smaller than the blade speed due to very low disk loading, and therefore the radial force and lift would be approximately in the same direction.

**PIV flowfield measurements**

As mentioned before, the cyclorotor test-rig has been instrumented with a PIV system, which is used to conduct high resolution flowfield measurements around the blade. The water in the tank is seeded with ~10 μm diameter glass beads. When the laser sheet hits the glass particles, the light is reflected and illuminates the flow. A mirror is also mounted at the back of the tank to reflect the laser light onto the backside of the blade so that the blade shadow is diminished (Figure 5(b)).

PIV measurements were performed at a rotor speed of 40 r/min for static pitch cases of 15°, 30°, and 45° and dynamic pitch cases with amplitudes of ±15°, ±30°, and ±45° to correlate with the force and moment measurement experiments. Phase-locked PIV measurements were conducted around the blade, when the blade reached different azimuthal locations. Azimuthal resolution for the PIV measurements was 10°, which means flowfield measurements around the blade were made at 36 azimuthal locations in one rotor revolution. The size of the interrogation window around the blade was 80 mm × 66 mm. For each azimuthal location (or phase), 70 phase-locked images of the flowfield were taken at a rate of 2/3 Hz (same as the rotational speed); therefore, one image was taken per rotation. These images were processed using the LaVision DaVis 8 data acquisition and visualization software. An average of the 70 images was computed, and a velocity component representing the rotational velocity (ΩR) of 0.365 m/s was subtracted, so that the
resulting flow vectors are in the rotating frame (as seen by the blade). This is how the flowfield is presented in the paper.

**Dynamic virtual camber**

Before the results of the study are presented, a unique phenomenon known as dynamic virtual camber needs to be discussed. The virtual chamber effect will first be explained, followed by what makes it “dynamic” for the cyclorotor blade. This is also addressed in Benedict

The virtual camber effect occurs on airfoils rotating in a circular path about an azimuth, resulting in curvilinear flow over the airfoil. Because the airfoil has a finite chord which is tangent to its trajectory, the velocity at each point along the chord experiences different flow velocity magnitude and direction. This results in a chord-wise variation of the incident velocity angle (or angle of attack) along the airfoil, manifested as an effective camber and incidence. This is known as “virtual camber.” Figure 7(a) shows an example of how curvilinear flow geometry creates a virtual camber effect for a symmetric airfoil at 0° pitch angle. Here, the leading edge of the airfoil experiences a positive angle of attack, while the trailing edge a significant negative angle. This is because, although the angular velocity is the same at all points, the radial distance and direction from the rotation axis to those points on the airfoil is quite different, creating a varying translational velocity along the chord. Hence, a symmetric blade immersed in a curvilinear flow will behave like a cambered blade in a rectilinear flow (Figure 7(a)). Note that this virtual chamber phenomenon is more significant for cyclorotors with a large chord-to-radius ratio (c/R).

The flow over a cyclorotor blade, however, is not purely curvilinear for two reasons. First, when producing thrust, there is an inflow distribution along the chord which effects virtual camber and incidence. Figure 7(b) demonstrates a generalized inflow distribution. Secondly, the blade pitch angle and pitch rate also affect the chord-wise velocity distribution, and therefore virtual camber. Figure 7(c) shows how a nose-up pitch rate causes chord-wise variation of virtual incidence so as to create a positive virtual camber effect. On the other hand, a nose-down pitch causes a negative virtual camber effect. Since these two, thrust and blade pitch kinematics, which affect virtual camber, change with azimuthal locations, the virtual camber changes “dynamically”, hence the term ‘dynamic virtual camber’. This effect is very predominant in cyclorotors because of the time and spatially varying kinematic conditions, making dynamic virtual camber a function of inflow, blade pitch angle, pitch rate, and chord/radius. Note that on the cyclorotor blade the virtual camber would be apparent in the blade forces as variations in the expected aerodynamic forces.

Considering all these effects, we have derived a generalized expression (discussed in detail in Halder et al.) to obtain not only the virtually cambered...
shape of the airfoil but also the resulting additional lift. Figure 8(a) shows the variation of this additional lift coefficient due to virtual camber \( (C_l^0) \) versus azimuthal position for a cyclorotor rotating at 40 r/min \( (Re \sim 18,000) \) and cyclically pitching with a 30° amplitude. Here, the lift coefficient variation due to only the curvilinear flow is constant and negative around the entire azimuth \( (\sim -0.81, \text{ magenta line}) \), while the lift coefficient modification due to pitch angle and rate varies substantially throughout the cycle, creating the dynamically varying lift coefficient, and hence dynamic virtual camber effect. Figure 8(b) shows the final blade pitch angle measured from the shaft encoder after being prescribed a 30° pitching amplitude, and its time derivative is shown in Figure 8(c). The slight variation from sinusoidal motion is a mechanical artifact of the experimental rig, but was determined to be small enough so as to not affect the final measurements.

Examining Figure 8(a) further shows that when cyclic blade pitching is added to the curvilinear flow, virtual camber lift coefficient decreases (black line), indicating that the effects of curvilinear flow geometry are negated. This effect of blade pitch is more prominent near 90° and 270° azimuth since the pitch angle reaches its peak at those locations (Figure 8(b)). Finally, when blade pitch rate is also considered (blue line), a positive virtual camber lift coefficient is created near 0° azimuth, effectively nullifying the effects of curvilinear geometry; while at 180°, it adds to the negative virtual camber lift coefficient, which, together with curvilinear effects, produces even larger negative lift. In accordance with this final trend, the net lift coefficient is very small at 0° azimuth, while it is significantly negative at 180° azimuth, as will be shown later in the paper. Thus, although pitch angle is near 0° at both 0° and 180° azimuthal locations (Figure 8(b)), pitch rate has the dominant effect on virtual camber since it reaches its maximum at these azimuthal locations (Figure 8(b)).

To better understand the effect of pitch rate on virtual camber lift coefficient, a schematic of the physics is shown in Figure 9(a) at 0° and 180° azimuthal locations. Note the variation in local chordwise velocity of the pitching airfoil, which causes a linear variation in the inflow velocity along the chord. This, together with a constant freestream velocity, \( V_{\infty} \), creates a chordwise variation in angle of attack, and hence lift coefficient. For a positive pitch rate, the result is a positive virtual camber, while the opposite is true for a negative pitch rate. The resulting variation in virtual camber about the rotor azimuth is sketched in Figure 9(b), which shows the actual chord-line of the cyclorotor blade.

---

**Figure 8.** Effect of curvilinear flow, pitch and pitch rate on virtual camber. Adapted from Halder et al.\(^2\) used with permission. (a) Variation of additional lift coefficient due to virtual camber. (b) Prescribed pitch along azimuth. (c) Prescribed pitch rate along azimuth.
along with the virtual chord-line due to virtual camber. As explained, at $0^\circ$ azimuth, virtual camber is at a minimum, producing almost negligible negative lift, while at $180^\circ$ azimuth, there is a large negative virtual camber producing large negative lift. Note that, based on the convention followed in the paper, lift force directed radially inwards is considered negative. The discussion section of the paper will utilize the virtual shape along with the physical airfoil to understand the physics of force production.

**Static pitch experimental results**

Figure 10(a) shows the measured lift coefficient plotted as a function of static pitch angle. A key observation from Figure 10(a) is that the lift curve is highly asymmetric between positive and negative pitch angles even though the airfoil is symmetric. A $0^\circ$ pitch angle produces a non-zero lift force ($C_L = -0.75$) towards the center of the rotor because of the virtual camber effect (Figure 7(a)) explained in the previous section.

The drag coefficient as a function of static pitch angle is shown in Figure 10(b). The asymmetry in drag between the positive and negative pitch angles (negative pitch angle causing more drag that positive ones) is due to the virtual camber effect. The maximum drag produced is larger for negative pitch angles ($C_D = 0.75$) than for positive pitch angles ($C_D = 0.6$). These results clearly show the role of virtual camber effect on the lift and drag production on a static blade experiencing a curvilinear flow. Additionally, these results are also very relevant to a fixed-pitch vertical axis wind turbine blade.

**Dynamic pitching experimental results**

The force and moment results from the dynamic blade pitch experiments are shown in Figure 11(a) to (d). The measured blade pitch angle and lift coefficient as a function of the azimuth for a pitching amplitude sweep of $\pm 5^\circ$ to $\pm 45^\circ$ are shown in Figure 11(a) and (b), respectively. Similar to the static experiments, lift asymmetry can be seen between upper ($0^\circ$–$180^\circ$) and lower ($180^\circ$–$360^\circ$) halves (Figure 11(b)) due to the virtual camber effect. As shown in Figure 9(b), the blade pitch is positive in the upper half, resulting in reverse or negative virtual camber, and hence smaller lift compared to the lower half where the camber is positive (due to negative pitch). The most significant finding from these results is the large values of the

![Figure 9. Pitch rate and virtual chord-line due to dynamic virtual camber. Adapted from Halder et al.22 used with permission.](image-url)

(a) Effect of pitch rate on dynamic virtual camber at two extreme azimuth locations ($0^\circ$ and $180^\circ$). (b) Virtual chord-line due to virtual camber effect along azimuth.

![Figure 10. Lift and drag coefficients for static pitch experiments. (a) Lift coefficient versus fixed pitch angle from experiment. (b) Drag coefficient versus fixed pitch angle from experiment.](image-url)
dynamic lift coefficients shown in Figure 11(b) when compared to the static lift values shown Figure 10(a). The maximum static $C_L$ value for positive pitch is around 0.5 (Figure 10(a)), whereas the maximum dynamic $C_L$ value for positive pitch is around 1.25 ($\pm 45^\circ$ pitching case), which is more than double. The same effect happens for negative pitch where the maximum static $C_L$ is around –1.2 (Figure 10(a)), whereas the maximum dynamic $C_L$ during negative pitch is around –2 ($\pm 45^\circ$ pitching case). It is significant to note that the lift coefficient monotonically increases all the way up to a pitching amplitude of $\pm 45^\circ$, which, as mentioned previously, is not intuitive because one would expect the blade to stall completely at amplitudes much lower than $45^\circ$.

The measured drag coefficient as a function of azimuth for the same pitch amplitudes is plotted in Figure 11(c). Even the drag coefficient is asymmetric between the upper and lower halves and magnitude is significantly higher than the static cases shown in Figure 10(b). To understand the reason for this huge increase in dynamic lift and drag coefficients on a pitching blade as opposed to a static blade in a curvilinear flow, it is important to look at the flowfield around the blade at different azimuthal locations, which is presented in the subsequent sections.

Figure 11(d) shows the variation of the pitching moment coefficient as a function of the azimuth. As mentioned previously, as the blade pitches nose-up and -down, the dynamic-stall vortex is swept downstream and the center of pressure shifts along the chord. This causes a large nose-down pitching moment on the blade. From Figure 11(d), it can be seen that the blade experiences moment stall at azimuthal locations of approximately 120$^\circ$ in the upper half and in the lower half at approximately 190$^\circ$. From the flowfield measurements, it can be seen that the local maximum in the upper half is indicative of moment stall due to the formation of a spilled vortex and the local maximum in the lower half occurs during a state of full separation.

To better understand the source of lift and drag forces on a cyclorotor blade operating at ultra-low Reynolds numbers ($Re \sim 18,000$), it is helpful to identify whether the dominating forces are due to pressure or viscous forces. If there are only pressure forces, the resultant force (denoted by $R$ in Figure 12(a)) would be normal to the blade chord. In Figure 12(a), $\beta$ represents the angle between the resultant of the lift and drag forces ($R$) and blade chord. If the forces are indeed dominated by the pressure force, $R$ would be more or less normal to the chord and $\beta$ would be close to 90$^\circ$.

The lift and drag shown in the diagram can be defined as

$$L = R \cos \phi$$

(1)
\[ D = R \sin \phi \]  

(2)

where \( \phi \), the phase angle between the lift vector and the resultant vector is the following

\[ \phi = \tan^{-1}(D/L) \]  

(3)

Finally, the angle \( \beta \) can be computed using equation (4)

\[ \beta = 90 + (\phi - \theta) \]  

(4)

If the forces are dominated mainly by the pressure forces, the difference between \( \phi \) and \( \theta \) will be small, and the angle \( \beta \) will be close to 90°. Deviations from 90° will indicate the presence of viscous forces acting on the blade.

Figure 12(b) shows the variation of \( \beta \) as a function of the azimuth for all of the dynamic pitching amplitudes. As seen from the figure, for all of the amplitudes, the angle \( \beta \) remains close to 90°, which confirms the dominating role of pressure forces as opposed to the shear viscous forces acting parallel to the blade. This is a significant finding considering the ultra-low Reynolds numbers the blade is operating where viscous forces are relatively large.

Static versus dynamic pitching

Since there are significant differences in the lift and drag coefficients between the static and dynamic pitching cases, it is important to compare the flowfield for the two cases at the exact same pitch angles to understand the key reason for the lift enhancement for the dynamic case. Figure 13 shows a comparison of PIV measured velocity vectors and vorticity contours for static and dynamic pitching cases. The static pitch angles include 15°, 30°, and 45°, and the dynamic cases include ±15°, ±30°, and ±45°. For the dynamic pitch case, flowfield at the azimuthal location of 90° is compared because that is where the blade pitch angle reaches +15°, +30°, and +45°, respectively, for the three cases. Therefore, in this comparative study, the flowfields are compared at the same pitch angle, but one subjected to steady flow and the other to unsteady flow conditions.

In Figure 13(a) and (b), the blade is at a low angle of attack of 15°; even then, for the static case, as seen from Figure 13(a), the flow has already separated from the leading edge and re-attaches close to the trailing edge forming a large laminar separation bubble (LSB), which is typical on steady airfoil at very low Reynolds numbers. However, for the dynamic case shown in Figure 13(b), when the blade reaches +15° angle of attack, the flow is still fully attached showing no signs of stall. The static and dynamic cases for the 30° pitch angle are compared in Figure 13(c) and (d), respectively. For the 30° static case (Figure 13(c)), as expected, the flow is fully separated from the leading edge denoting deep stall; however, for the dynamic case (Figure 13(d)), at the same pitch angle of 30°, one can see the initiation of the dynamic stall vortex and the flow is still more or less attached. For the 45° static case shown in Figure 13(e), the flow is fully separated from the leading edge. However, for the dynamic case (Figure 13(f)), the dynamic stall vortex has reached the full strength and is in the process of shedding. Comparing Figure 13(b), (d) and (f) would provide insight into how the pitching amplitude affects the dynamic stall process on a cyclo-rotor blade at ultra-low Reynolds numbers. These PIV results clearly explain the reason for the large lift coefficients measured on a dynamic pitching blade at high amplitudes (Figure 11(b)).

Understanding physics of force production on cyclo-rotor blade

The variation of lift and drag coefficients as a function of azimuth on a blade operating at ±30° pitch
amplitude at 40 r/min is shown in Figure 14(a) and (b). As mentioned before, the lift is the force in the radial direction (positive lift is radially outward) and drag is the force in the tangential direction (positive drag is opposite to the direction of blade motion). Figure 15 shows the measured flowfield around the azimuth (Ψ) at a 10° resolution. On Figure 14(a) and (b), the corresponding PIV figure numbers (Figure 15(i)–(xxxvi)) are provided at a resolution of 10°. Also, to improve clarity in the discussion, each of the flowfield images include the lift vector where the magnitude is proportional to the measured magnitude (Figure 14(a) and (b)) and the direction depends on the sign. Similar to what is shown in Figure 9(b), the computed virtual camber chord-line is superimposed on the physical airfoil in Figure 15 to aid in the explanation of the physics.

At 0° azimuth (Figure 15(i)), even though the airfoil is symmetric and pitch is zero, the small negative virtual camber shown in the figure creates a small negative lift. This can also be seen in Figure 14(a). The direction of lift is also very evident from the flowfield around the airfoil. As seen from Figure 14(b), the drag is very small and positive. In Figure 15(ii), since the pitch is increasing in the positive direction, the negative lift is decreasing. It is interesting because the lift is downwards or negative due to the virtual camber as shown in the figure. In other words, here the virtual camber is in the reverse direction of the conventional camber and hence it will be called ‘negative camber’. Typically, because of the positive pitch angle, the lift should be upwards or positive. However, in Figure 15(ii), the negative camber effect is more dominating than the effect of positive pitch angle and hence results in a negative lift. Again, as before, the direction of lift is evident from the flowfield as well. The drag increases due to the increase in pitch angle (Figure 14(b)).
At the 20°/C14 azimuth location (Figure 15(iii)), the pitch angle has increased further, and the lift is close to zero because the negative lift from negative virtual camber is nullified by positive pitch. This could be thought of as the zero-lift angle of attack for the virtually cambered airfoil. It is important to note that, as shown in Figures 14(a) and 9(b), the camber of the virtual airfoil changes from one azimuthal location to another. At 30°/C14 azimuth (Figure 15(iv)), the blade pitch is further increased and now positive lift is generated, which means the pitch is high enough to dominate negative virtual camber. At this point, there is significant negative virtual camber and incidence, as well as a strong dynamic stall vortex forming on the leading edge. The flow remains more or less attached on the top even at such high pitch angles. The lift drops slightly, possibly due to the negative camber and incidence that is more effective than the additional vortex lift.

Starting at 60°/C14 azimuth, the lift starts dropping even though the pitch angle is increasing. This may be because of the large increase in negative virtual camber and negative virtual incidence, which is reducing the effective angle of attack and the lift produced. However, more prominent than the drop in lift is the sudden local drop in drag. The reason for this can be traced back to the flowfield shown in Figure 15(vii), which shows the initiation of a vortex at the leading edge causing leading edge suction, which could reduce the net profile drag. Lift further decreases for the 70° azimuth (Figure 15(viii)) even though the pitch is increasing. Again, this occurs because of the large negative virtual camber and incidence from the reduced pitch rate. Drag drops further because of the increased leading edge suction as seen from the vorticity contours in the flowfield. From the 70° to 80° azimuth (Figure 15(ix)), the lift stays almost constant (Figure 14(a)). However, from the flowfield, a strong dynamic stall vortex can be seen. At this point, it is unclear why the vortex lift does not significantly increase the blade lift (Figure 14(a)). However, there is a slight increase in drag (Figure 14(b)). At the 90° azimuthal location (Figure 15(xi)), the blade reaches its maximum pitch angle. At this point, the lift and drag changes from positive to negative (Figures 14(a) and (b)). It is also significant to note the large negative virtual camber and incidence for these cases. From the 110° to 120° azimuth, the direction of lift and drag changes from positive to negative (Figures 14(a) and (b) and 15(xii) and (xiii)). This is because the negative virtual camber and incidence have increased to such an extent that it dominates even large positive pitch angles. This clearly shows the strong effect of dynamic virtual camber on the force production of a cyclorotor blade. Interestingly, the drag (tangential force) becomes negative, indicating power extraction from the 115° to 170° azimuthal locations.

From 150° (Figure 15(xvi)) to 180° (Figure 15(xix)), there is large negative lift, which continues to increase due to the strong negative virtual camber. As seen from Figures 7(a), 9(b) and 15(xix), the maximum virtual camber occurs at the 180° azimuth, causing a huge negative lift force, even with a symmetric airfoil and physical pitch angle of zero. Again, the direction of lift is evident from the flowfield. The drag also increases as shown in Figure 14(b). At the 190° azimuth, as seen from Figure 15(xx), the camber is consistent with the blade pitch angle and is called ‘positive camber’. In the entire lower half (180° to 360°), the camber will not oppose the pitch angle, which is the case in the upper half (0° to 180°). In the entire lower half, the lift will be negative because it will be acting radially inwards.
towards the center of the rotor. From the 190° to 220° azimuthal locations (Figures 14(a), 15(xx) to (xxiii)), the magnitude of negative lift keeps increasing due to the positive camber. Also, from the flowfield, a strong trailing edge separation can be seen, which increases with increasing pitch angle.

From the 220° to 270° azimuth, the magnitude of negative lift starts decreasing, but the drag coefficient

**Figure 15.** PIV measured flow velocity vectors and vorticity contours at different azimuthal locations for ±30° dynamic pitching.
keeps increasing until it reaches a maximum value of 0.6 at the 270° azimuth. At this position, the blade attains the maximum pitch angle. The 230° azimuth (Figure 15(xxiv)) shows a weak dynamic stall vortex beginning to appear on the leading edge. It is weaker in nature mostly because virtual camber is positive and hence decreases flow separation (unlike the upper half where the camber was negative or reverse camber).
For azimuths of 240°–260° (Figure 15(xxv) to (xxvii)), the dynamic stall vortex builds up strength. There is mild shedding at approximately 270° (Figure 15(xxviii)). The vortex is fully separated at the 290° azimuth (Figure 15(XXX)). During this time, the magnitude of effective angle of attack is very high causing large lift and drag. It is also important to note that even though the pitching is symmetric, the dynamic stall process in the upper and lower half of the circular trajectory is completely different because of the complete reversal of dynamic virtual camber from the upper to lower half. From the 300° to 320° azimuth (Figure 15(XXXI) to (XXXIII)), the flow again starts re-attaching as pitch decreases. The magnitude of lift and drag decreases as seen from Figure 14(a) and (b). However, from the 330° to 350° azimuth (Figure 15(XXXIV) to (XXXVI)), the shedding of a secondary leading edge vortex can be seen. The reason for this is not completely understood at this point. Finally, at 360° (Figure 15(i)), the flow reattaches again.

A key insight gained from these measurements is the interplay of blade pitch angle and dynamic virtual camber in the blade force production. In the entire upper half, the pitch angle and virtual camber oppose each other, unlike the lower half where they act in the same direction. This leads to completely different dynamic stall process in the upper and lower halves. This can also explain why a huge improvement in cyclorotor performance is obtained using asymmetric blade pitching, where the pitch angle in the upper half increased and decreased in the lower half.

Power calculations

Also of interest is the power consumption of a dynamic pitching cyclorotor blade. Figure 16(a) shows the measured instantaneous blade power breakdown for the ±45° dynamic pitching case as a function of azimuth. Included in the plot are the power required for the blade to rotate, the power required for the blade to pitch, and the total power. These are calculated using the following equations

\[ Rotation\ power (P_{\text{ROT}}) = D \Omega R \]  

\[ Pitching\ power (P_{\text{PITCH}}) = M_z \dot{\theta} \]  

\[ Total\ power = P_{\text{ROT}} + P_{\text{PITCH}} \]  

where \( D \) is the measured tangential force, which is referred to as the drag force in this paper, \( \Omega R \) is the rotational velocity, \( M_z \) is the measured pitching moment, and \( \dot{\theta} \) is the measured pitch rate.

It is clear from Figure 16(a) that most of the power is required for the blade rotation and not pitching. The present study conclusively proves that it only takes up a very small fraction of the total aerodynamic power to dynamically pitch the blades at least at such low Reynolds numbers. Figure 16(b) shows the instantaneous total blade power versus the azimuth for all of the dynamic pitching cases. It can be seen that, as expected, the power continues to increase as the dynamic pitching amplitude increases. In fact for a small part of the azimuthal cycle, the blade power is negative, which indicates the blade is extracting power.

The cycle-averaged power versus the pitching amplitude for all the dynamic pitching cases is plotted in Figure 17(a). As seen from the figure, the power increases quadratically with the pitching amplitude. For the pitching amplitudes of 5° and 10°, the power is slightly negative denoting power extraction or negative induced power in a cycle-averaged sense. Similarly, for 0°, the average power is close to zero may be because the component of lift vector in the forward direction (causing negative induced power) cancels out the profile power. From 20° to 45°, the power increases rapidly until it reaches a maximum of about 0.24 W.
In Figure 17(b), the cycle-averaged thrust is plotted against the dynamic pitching amplitude. The cycle-averaged thrust \( T \) is computed as follows

\[
T = \sqrt{F_Z^2 + F_Y^2}
\]  (8)

where \( F_Z \) and \( F_Y \) are vertical and horizontal components (in the fixed frame, refer Figure 2) of measured radial and tangential forces. As seen from the figure, there is a non-linear relationship between the average thrust produced by the rotor and the dynamic pitching amplitude.

Figure 18(a) has the cycle-averaged thrust to power ratio as a function of pitching amplitudes of 25° to 45°. It can be seen that the power loading decreases with respect to the pitching amplitude. The unusually high power loading values can be attributed to extremely low disk loading at which the present rotor is operating. For the 45° pitching amplitude, based on the rectangular projected area of the rotor, the disk loading is around 16 N/m² (0.33 lb/ft²), which is extremely low. The decrease in power loading with pitch amplitude can be attributed to the fact that the disk loading is increasing and the \( FM \) is decreasing with pitch amplitude. The \( FM \) is defined as follows

\[
FM = \frac{P_{\text{IDEAL}}}{P_{\text{MEASURED}}}
\]  (9)

\( P_{\text{IDEAL}} \), the ideal power, is given as

\[
P_{\text{IDEAL}} = T v_i
\]  (10)

where \( v_i \), the induced velocity calculated as follows

\[
v_i = \sqrt{\frac{T}{2\rho A}}
\]  (11)

where \( \rho \) is the density of water and \( A \) is the rectangular projected area of the rotor \((A = b \times d)\), where \( b \) is the blade span and \( d \) is the cyclorotor diameter.

The \( FM \) is plotted a function of pitch amplitude in Figure 18(b). It can be seen that \( FM \) drops with pitch amplitude; however, for a pitch amplitude of ±25°, the \( FM \) is around 0.65, which is very high considering the ultra-low Reynolds numbers (Re ~ 18,000) at which the blade is operating. However, as seen from Figure 18(b), for a pitch amplitude of ±45°, the \( FM \) is
around 0.34. This decrease in $FM$ is mainly due to the higher drag coefficient (Figure 11c) which the blade experiences at larger blade pitching angles.

**Summary and conclusions**

The present study provides an in-depth understanding of the unsteady aerodynamic mechanisms on a cyclo-rotor blade operating at ultra-low Reynolds numbers ($Re \sim 18,000$). This is accomplished by utilizing a combination of force and flowfield measurements. This is the first time the instantaneous blade fluid dynamic forces on a cyclorotor blade were measured, which, along with PIV-based high resolution flowfield measurements around the blade at different azimuthal locations, revealed the key fluid dynamic mechanisms acting on the blade. Studies were performed with both static and dynamic blade pitching.

Direct comparison of the static and dynamic pitch experimental results helped isolate the unsteady aerodynamic phenomena from the steady effects.

Specific conclusions from this study are as follows:

1. Large dynamic virtual camber induced by the inherent flow curvature and blade pitch rate caused asymmetry in lift and drag coefficients between positive and negative pitch for both the static and dynamic pitching cases.
2. The unsteady blade force coefficients were almost double the static ones clearly, indicating the role of unsteady aerodynamic mechanisms on the force production on cyclorotor blades. This explains the ability of a cyclorotor to produce large thrust at relatively lower rotational speeds, which was a key inference from the previous performance studies.
3. For the dynamic case, the blade lift coefficient monotonically increased even up to ±45° pitch amplitude due to dynamic stall phenomenon, which kept the flow attached until higher pitch angles. On the other hand, for the static case, the flow separated from the leading edge after around 15° with a large laminar separation bubble and eventually completely separating at higher pitch angles.
4. Dynamic stall processes in the upper and lower halves of the circular blade trajectory were completely different because of the reversal of the virtual blade camber from upper to lower half.
5. The measured resultant forces were mostly normal to the chord for the dynamic pitch cases indicating that the pressure force, as opposed to viscous force, is dominant on a cyclorotor blade (even at these ultra-low Reynolds numbers).
6. The power required for blade rotation (rather than pitching power) is the significant component of the total power required for dynamic pitching cyclorotor blade.
7. The cyclorotor $FM$ drops at higher pitch amplitudes due to higher dynamic drag coefficients.

**Acknowledgements**

The authors would also like to thank undergraduate student Farid Saemi for his contributions to the experimental setup.

**Declaration of conflicting interests**

The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

**Funding**

The author(s) disclosed receipt of the following financial support for the research, authorship, and/or publication of this article: This research was supported by the U.S. Army’s Micro Autonomous Systems and Technology-Collaborative Technology Alliance (MAST-CTA) with Chris Kroninger and Brett Piekarski (Vehicle Technology Directorate, Army Research Laboratory) as Technical Monitors for grant W911NF-08-2-0004.

**ORCID iD**

Carolyn M Reed http://orcid.org/0000-0003-3863-5346

**References**

1. Pines and Bohorquez. Challenges facing future micro-air vehicle development. *J Aircraft* 2006; 43: 290–305.
2. Hein B and Chopra I. Hover performance of a micro air vehicle: rotors at low Reynolds number. *J Am Helicopter Soc* 2007; 52: 254–262.
3. Runco C, Coleman D and Benedict M. Development of the world’s smallest cyclocopter. In: *Proceedings of the 72nd annual AHS international forum and technology display meeting*, West Palm Beach, FL, 16–19 May 2016.
4. Benedict M, Ramasamy M, Chopra I, et al. Performance of a cycloidal rotor concept for micro air vehicle applications. *J Am Helicopter Society* 2010; 55: 022002-1–022002-14.
5. Benedict M, Gupta R and Chopra I. Design, development and flight testing of a twin-rotor cyclocopter micro air vehicle. *J Am Helicopter Soc* 2013; 58: 1–10.
6. Elena S, Harishkeshavan V, Benedict M, et al. Development of control strategies and flight testing of a twin-cyclocopter in forward flight. In: *Proceedings of the 70th annual national forum of the American Helicopter Society*, Montreal, Quebec, Canada, 20–22 May.
7. Benedict M, Mullins J, Harishkeshavan V, et al. Development of a quad cycloidal-rotor unmanned aerial vehicle. *J Am Helicopter Soc* 2016; 61: 1–12.
8. Benedict M. Fundamental understanding of the cycloidal-rotor concept for micro air vehicle applications. PhD
9. Benedict M, Ramasamy M and Chopra I. Improving the aerodynamic performance of micro-air-vehicle-scale cycloidal rotor: an experimental approach. *J Aircraft* 2010; 47: 1117–1125.

10. Benedict M, Jarugumilli T and Chopra I. Experimental optimization of MAV-scale cycloidal rotor performance. *J Am Helicopter Society* 2011; 56: 022005-1–022005-11.

11. Benedict M, Jarugumilli T and Chopra I. Effect of rotor geometry and blade kinematics on cycloidal rotor hover performance. *J Aircraft* 2013; 50: 1340–1352.

12. Benedict M, Jarugumilli T, Lakshminarayan VK, et al. Effect of flow curvature on the forward flight performance of a MAV-scale cycloidal rotor. *AIAA J* 2014; 52: 1159–1169.

13. Lind AH, Jarugumilli T, Benedict M, et al. Flowfield studies on a micro-air-vehicle-scale cycloidal rotor in forward flight. *Exp Fluids* 2014; 55: 1–17.

14. Jarugumilli T, Benedict M and Chopra I. Wind tunnel studies on a micro air vehicle-scale cycloidal rotor. *J Am Helicopter Soc* 2014; 59: 1–10.

15. Benedict M, Mattaboni M, Chopra I, et al. Aerodynamic Analysis of a micro-air-vehicle-scale cycloidal rotor in hover. *AIAA J* 2011; 49: 2430–2443.

16. Zachary HA, Benedict M, Hrishikeshavan V, et al. Design, development, and flight test of a small-scale cyclogyro UAV utilizing a novel cam-based passive blade pitching mechanism. *Int J Micro Air Veh* 2013; 5: 145–162.

17. McMichael JM and Francis MS. Micro air vehicles – toward a new dimension in flight, www.darpa.mil/tto/MAV/mavAuvisi.html (1997, accessed 1 September 2004).

18. Shyy W, Lian Y, Tang J, et al. Aerodynamics of low Reynolds number flyers. Cambridge: Cambridge University Press, 2008.

19. Mueller TJ. An overview of micro air vehicle aerodynamics. Fixed and flapping wing aerodynamics for micro air vehicle applications. In: TJ Mueller (ed.) *Progress in aeronautics and astronautics*. Vol.195. Reston, VA: AIAA, 2001, pp.483–500.

20. Leishman JG. *Principles of helicopter aerodynamics*. Cambridge: Cambridge University Press, 2006, pp.258–259.

21. Halder A, Walther C and Benedict M. Unsteady hydrodynamic modeling of cycloidal propeller. In: *Proceedings of the fifth international symposium on marine propulsors*, Helsinki, Finland, 12–15 June 2017.

22. Halder A, Walther C and Benedict M. Unsteady hydrodynamic modeling of a cycloidal propeller. *Ocean Eng* 2018; 154: 94–105.

### Appendix

#### Notation

- $A$ rectangular projected rotor area
- $b$ blade span
- $c$ blade chord length
- $C_L$ coefficient of lift
- $C_{LO}$ additional lift coefficient due to virtual camber
- $C_D$ coefficient of drag
- $C_{MZ}$ pitching moment coefficient
- $d$ cyclorotor diameter
- $D$ drag force
- $F_y$ horizontal force component
- $F_z$ vertical force component
- $L$ lift force
- $M_Z$ pitching moment
- $P_{IDEAL}$ ideal power
- $P_{PITCH}$ pitching power
- $P_{ROT}$ rotational power
- $R$ rotor radius
- $R$ resultant force
- $RPM$ revolutions per minute
- $Re$ Reynolds number
- $T$ thrust
- $v_i$ induced velocity
- $\beta$ angle between chord and resultant force
- $\theta$ blade pitch angle
- $\dot{\theta}$ blade pitch rate
- $\rho$ density
- $\Psi$ azimuthal position of blade
- $\Omega$ rotational speed of rotor