Disappearance of Transverse Flow in Central Collisions for Heavier Nuclei

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Using the Quantum Molecular Dynamics model, we analyze the disappearance of flow in heavier colliding nuclei. A power law mass dependence (\( \propto \frac{1}{A} \)) is obtained in all cases. Our results are in excellent agreement with experimental data which allows us to predict the balance energy for \(^{238}\text{U} + ^{238}\text{U}\) collision around 37-39 MeV/nucleon.

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Thirty years ago it was predicted by Scheid and Greiner [1] that in heavy ion reactions the nuclei will be compressed and heated and that this yields for non-central reactions to in-plane flow (\( p_{z}^{\text{dir}} \)). More than a decade later, this conjecture was confirmed by the Plastic Ball group [2]. In the following investigations it turned out that this in-plane flow carries information on the nuclear equation of state [3]. With the very recently [11],

\[ \langle p_{z}^{\text{dir}} \rangle \]

is caused by compression will be in the direction of the impact parameter whereas the in-plane flow which creates in-plane flow as well but in opposite direction: Due to the common rotation the nucleons stick together and will be emitted into the direction opposite to the impact parameter whereas the in-plane flow which is caused by compression will be in the direction of the impact parameter.

The maximal density which is reached in a reaction depends on the beam energy as well as on the system size. The lower the beam energy the less is the compression. At very low energies, the repulsive part of the nuclear equation of state, which appears at densities above the normal nuclear matter density, is not tested anymore and the nucleons feel only the attractive mean field. A typical example is the deep inelastic reactions in which the two nuclei rotate around a common center. This rotation creates in-plane flow as well but in opposite direction: Due to the common rotation the nucleons stick together for a while and will be emitted into the direction opposite to the impact parameter whereas the in-plane flow which is caused by compression will be in the direction of the impact parameter.

There is a beam energy at which the in-plane flow disappears when changing from the direction into that opposite to the impact parameter. It has been shown in the simulation of heavy ion reactions that this beam energy called balance energy, \( E_{\text{bal}} \), depends on the nuclear-nucleon (nn) cross-section in the medium as well as on the potential [3, 4]. With the very recently measured \( E_{\text{bal}} \) in \(^{197}\text{Au} + ^{197}\text{Au} \) collisions [5] (earlier only estimated values were available [3]), a renewed interest has emerged in the field [4].

In addition to the Au system, balance energies \( E_{\text{bal}} \) of \(^{12}\text{C} + ^{12}\text{C} \), \(^{20}\text{Ne} + ^{27}\text{Al} \), \(^{36}\text{Ar} + ^{27}\text{Al} \), \(^{40}\text{Ar} + ^{27}\text{Al} \), \(^{40}\text{Ar} + ^{45}\text{Sc} \), \(^{40}\text{Ar} + ^{51}\text{V} \), \(^{64}\text{Zn} + ^{27}\text{Al} \), \(^{40}\text{Ar} + ^{58}\text{Ni} \), \(^{64}\text{Zn} + ^{48}\text{Ti} \), \(^{58}\text{Ni} + ^{58}\text{Ni} \), \(^{64}\text{Zn} + ^{58}\text{Ni} \), \(^{86}\text{Kr} + ^{93}\text{Nb} \), \(^{93}\text{Nb} + ^{93}\text{Nb} \), \(^{129}\text{Xe} + ^{118}\text{Sn} \) and \(^{139}\text{La} + ^{139}\text{La} \) are also available. It is worth mentioning that most of the above studies were for the central collisions only. A few, however, also searched for the impact parameter dependence of the balance energy [6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22].

Apart from the directed in-plane flow, differential as well as elliptic flow has also been predicted very recently [10].

The measurements of the balance energy over wide range of system sizes provide an excellent opportunity to pin down the role of the mass dependence, where only preliminary studies [6, 10] have been performed yet. These preliminary studies suggest a power law dependence \( \propto A^{r} \) of the balance energy on the mass number of the system. Interestingly, most of the theoretical studies are done within the Boltzmann-Uehling-Uhlenbeck (BUU) model [4, 5, 6, 7, 10, 12, 13, 15, 16, 17, 18, 19, 20, 21, 22]. Some attempts, however, also exist within the framework of Quantum Molecular Dynamics (QMD) model [13, 22, 23, 24]. Heavy systems are rather rarely analyzed in these approaches.

Our present aim is therefore to study the mass dependence of the balance energy in heavy colliding nuclei and to predict for the first time the disappearance of the collective in-plane flow in central \(^{238}\text{U} + ^{238}\text{U}\) collision. We shall show that the mass dependence of \( E_{\text{bal}} \) for heavier nuclei scales approximately more as \( \frac{1}{A} \) rather than as \( A^{-\frac{1}{3}} \) as has been suggested for light and medium colliding nuclei [10]. The present study is made within the framework of QMD model which is described in detail in refs. [22, 23, 24, 25].

In the QMD model, each nucleon propagates under the influence of mutual interactions. The propagation is governed by the classical equations of motion:

\[
\dot{\mathbf{r}}_i = \frac{\partial H}{\partial \mathbf{p}_i}; \quad \dot{\mathbf{p}}_i = -\frac{\partial H}{\partial \mathbf{r}_i},
\]

(1)

where \( H \) stands for the Hamiltonian which is given by:

\[
H = \sum_i \frac{\mathbf{p}_i^2}{2m_i} + \sum_i (V_{i}^{\text{Skyrme}} + V_{i}^{\text{Yuk}} + V_{i}^{\text{Coul}}). \quad (2)
\]

Here \( V_{i}^{\text{Skyrme}} \), \( V_{i}^{\text{Yuk}} \) and \( V_{i}^{\text{Coul}} \) are, respectively, the Skyrme, Yukawa and Coulomb potentials. Momentum dependent interactions are not important at these low beam energies as Zhou et al. [21] have shown. It is worth mentioning that both the soft and hard equations of state have been employed in the literature. Following [4, 5, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22].
we shall also use a hard equation of state. For the nn cross-section we chose an isotropic and energy independent cross-section of 40 mb for the present analysis. This seems to us a reasonable choice in view of the fact that cross-sections based on G-Matrix calculations differ widely and because at this energy most of the collisions are Pauli blocked. Hence different cross-sections do not produce a large effect if they are not too small.

Since we plan to study the heavier colliding nuclei, different nn cross-sections should not have much effect. Further, it has been shown in ref. [18] that the nucleons in the present energy domain (≤ 80 MeV/nucleon) collide with average σ=55 mb. The isotropy of the cross-section also does not affect the reaction dynamics. Similar assumptions were also made in refs. [13, 14, 15, 16, 20, 21, 22, 23, 24, 27]. One should, however, keep in mind that different nn cross-sections may affect the dynamics in lighter systems.

Using the above description, we simulated the central collisions with system mass A (= A_T + A_P; A_T being the target mass, and A_P being the projectile mass) ≥ 175. In particular, we simulated 93Kr + 93Nb (b = 4.07 fm) [10], 93Nb + 93Nb (b = 3.104 fm) [10], 129Xe + 118Sn (b = 0-3 fm) [10, 139La + 139La (b = 3.549 fm) [10], 197Au + 197Au (b = 2.5 fm) [10] and 238U + 238U (b = 0-3 fm) at incident energies between 30 MeV/nucleon and 80 MeV/nucleon at a step of 10 MeV. A straight line interpolation between steps was used to calculate the energy of vanishing flow E_bal. The reaction was followed till transverse flow saturates which is close to 300 fm/c for heavier colliding nuclei whereas it is ≤ 200 fm/c for lighter colliding nuclei.

In fig. 1, we display the average directed transverse momentum \( \langle p_x^{\text{dir}} \rangle \) defined as:

\[
\langle p_x^{\text{dir}} \rangle = \frac{1}{A} \sum_{i=1}^{A} \text{sign}(Y(i))p_x(i),
\]

where \( Y(i) \) and \( p_x(i) \) are, respectively, the rapidity and the transverse momentum of the \( i \)th particle for the reactions 93Nb + 93Nb, 139La + 139La, 197Au + 197Au and 238U + 238U. We see that the collective in-plane flow for 93Nb + 93Nb changes sign between 55-60 MeV/nucleon whereas it is already positive around 40 MeV/nucleon for 238U + 238U system. Further, the saturation time increases with the size of the colliding nuclei. The early onset of the flow in heavier colliding nuclei is due to the Coulomb forces that are much stronger in heavier systems compared to lighter nuclei. In addition, a large collision rate in heavier systems also contributes towards the early onset of the flow.

In fig. 2, we display the energy of vanishing flow \( E_{\text{bal}} \) as a function of the combined mass of the system. As stated earlier, \( E_{\text{bal}} \) is extracted using a straight line interpolation between the calculated values of the in-plane flow. Here open squares represent our calculations whereas solid stars are the experimental findings. The dotted and dash-double-dotted lines are, respectively, the power law fits \( (\propto A^p) \) to the theoretical values including 238U + 238U in one case and excluding 238U + 238U in the other case. The fit to the experimental points is represented by a solid line. All fits are obtained with \( \chi^2 \) minimization. The values of \( E_{\text{bal}} \) (obtained with a stiff equation of state and 40 mb cross-section) are very close to the experimentally measured \( E_{\text{bal}} \). The fit to the experimental data yields \( \tau = -0.5245 \pm 0.06261 \), whereas that to theory yields \( \tau = -0.5326 \pm 0.16373 \). Once 238U + 238U is included, the \( \tau \) decreases to \( -0.5087 \pm 0.10883 \). In other words, we observe a dependence in the \( E_{\text{bal}} \). Similar dependence can also be obtained with a least square fit. Based on the above findings, we predict \( E_{\text{bal}} \) for the central 238U + 238U reaction around 37-39 MeV/nucleon (according to power law fits, it is around 37-38 MeV/nucleon whereas QMD simulation predicts around 39 MeV/nucleon). It is worth mentioning that most of the earlier mass dependence calculations [5, 11, 21] could not reproduce the experimentally extracted slopes [5, 10]. Zhou et al. [21] could reproduce the slope, however their analysis was only done for lighter nuclei ≤ 200. Our calculations can reproduce the experimentally extracted slope very closely, therefore, we can predict the energy of vanishing flow in the 238U + 238U system. The large deviation of our \( \tau \) value from the standard value \( (\approx -\frac{1}{4}) \) reflects the increasing importance of the Coulomb repulsion with the size of the system, as noted in ref. [5]. There the \( \tau \) value is close to \( -\frac{1}{4} \) for masses ≤ 200 [10, 21], whereas it increases to \( \approx -0.45 \) when heavier systems like 139La + 139La and 197Au + 197Au are included. In the present analysis, we took only heavier nuclei (A ≥ 175). Therefore the slope is steeper than the above cited values. If one also takes the lighter nuclei into consideration, our value also decreases to \( -0.4 \). In other words, for lighter and medium nuclei, the balance energy \( E_{\text{bal}} \) emerges due to the interplay between the mean field and nucleon scattering. However, for heavier colliding nuclei, Coulomb interaction is as well an important factor. It is of interest to see the contributions of the mean field (that includes the Coulomb interaction) and nn collisions towards the transverse flow at the balance energy \( E_{\text{bal}} \). Following ref. [22], we decomposed both these contributions in the simulations itself. At each time step during the reaction, the momentum transferred due to the two-body collisions and mutual mean field potential is calculated separately using equation (3). The separation was done at each simulated incident energy and a straight line interpolation was used. The decomposition is plotted in fig. 3 as a function of the total mass of the system. We find that the contribution of mean field towards transverse momentum is negative whereas it turns repulsive for the collision part. Both these contributions again obey a power law behavior with \( \tau = -0.70931 \pm 0.24234 \).

Summarizing, we present the disappearance of flow in heavier colliding nuclei with a prediction of balance energy for 238U + 238U around 37-39 MeV/nucleon.
Our calculations (with a stiff equation of state and \( \sigma = 40 \text{ mb} \)) are in a very close agreement with experimentally extracted values (\( \tau_{th} = -0.53261 \pm 0.16373; \tau_{expt} = -0.52451 \pm 0.06261 \)). Both these findings suggest a power law mass dependence \( \propto \frac{1}{\sqrt{A}} \). The contribution of the mean field towards flow at \( E_{\text{bal}} \) is negative whereas it is positive for the collision part. Both contributions can be parameterized in terms of a power law.

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[1] W. Scheid, R. Ligensa, and W. Greiner, Phys. Rev. Lett. 21, 1479 (1968).
[2] H.A. Gustafsson et al., Phys. Rev. Lett. 52, 1590 (1984).
[3] H. Stöcker and W. Greiner, Phys. Rep. 137, 277 (1986).
[4] D. Krofcheck et al., Phys. Rev. C 46, 1416 (1992).
[5] J.J. Molitoris and H. Stöcker, Phys. Lett. B 162, 47 (1985); G.F. Bertsch, W.G. Lynch, and M.B. Tsang, Phys. Lett. B 189, 384 (1987).
[6] C.A. Ogilvie et al., Phys. Rev. C 42, R10 (1990).
[7] D.J. Magestro, W. Bauer, O. Bjarki, J.D. Crispin, M.L. Miller, M.B. Tonjes, A.M. Vander Molen, G.D. Westfall, R. Pak, and E. Norbeck, Phys. Rev. C 61, 021602(R) (2000); D.J. Magestro, W. Bauer, and G.D. Westfall, ibid. 62, 041603(R) (2000); G.D. Westfall, Nucl. Phys. A681, 343c (2001).
[8] W.M. Zhang et al., Phys. Rev. C 42, R491 (1990); M.D. Partlan et al., Phys. Rev. Lett. 75, 2100 (1995); P. Crochet et al., Nucl. Phys. A624, 755 (1997).
[9] D. Cusso et al., Phys. Rev. C 65, 044604 (2002).
[10] G.D. Westfall et al., Phys. Rev. Lett. 71, 1986 (1993).
[11] A. Buta et al., Nucl. Phys. A584, 397 (1995).
[12] J.P. Sullivan et al., Phys. Lett. B 249, 8 (1990).
[13] R. Pak et al., Phys. Rev. C 54, 2457 (1996); R. Pak et al., ibid. 53, R1469 (1996).
[14] D. Krofcheck et al., Phys. Rev. C 43, 350 (1991).
[15] Z.Y. He et al., Nucl. Phys. A598, 248 (1996).
[16] Y.M. Zheng, C.M. Ko, B.A. Li, and B. Zhang, Phys. Rev. Lett. 83, 2534 (1999); B.A. Li and A.T. Sustich, ibid. 82, 5004 (1999).
[17] R. Pak et al., Phys. Rev. Lett. 78, 1022 (1997).
[18] B.A. Li, Phys. Rev. C 48, 2415 (1993).
[19] V. de la Mota, F. Sebille, M. Farine, B. Remaud, and P. Schuck, Phys. Rev. C 46, 677 (1992).
[20] H.M. Xu, Phys. Rev. Lett. 67, 2769 (1991); H.M. Xu, Phys. Rev. C 46, R389 (1992).
[21] H. Zhou, Z. Li, and Y. Zhuo, Phys. Rev. C 50, R2664 (1994).
[22] E. Lehmann, A. Fuessler, J. Zipprich, R.K. Puri, and S.W. Huang, Z. Phys. A 355, 55 (1996).
[23] S. Soff, S.A. Bass, C. Hartnack, H. Stöcker, and W. Greiner, Phys. Rev. C 51, 3320 (1995).
[24] S. Kumar, M.K. Sharma, R.K. Puri, K.P. Singh, and I.M. Govil, Phys. Rev. C 58, 3494 (1998).
[25] J. Aichelin, Phys. Rep. 202, 233 (1991).
[26] C. Hartnack, R.K. Puri, J. Aichelin, J. Konopka, S.A. Bass, H. Stöcker, and W. Greiner, Eur. Phys. J. A 1, 151 (1998).
[27] J.J. Molitoris and H. Stöcker, Phys. Rev. C 32, 346 (1985).
[28] S. Kumar, R.K. Puri, and J. Aichelin, Phys. Rev. C 58, 1618 (1998).
[29] H.W. Barz, J.P. Bondorf, D. Idier, and I.N. Mishustin, Phys. Lett. B 382, 343 (1996).
Figure Captions

FIG. 1. The time evolution of $\langle p^\text{dir}_x \rangle$ for four different reactions: (a) $^{93}\text{Nb} + ^{93}\text{Nb}$, (b) $^{139}\text{La} + ^{139}\text{La}$, (c) $^{197}\text{Au} + ^{197}\text{Au}$ and (d) $^{238}\text{U} + ^{238}\text{U}$. Here a stiff equation of state along with constant nn cross-section of 40 mb strength is used.

FIG. 2. The $E_{\text{bal}}$ as a function of the total mass of the system. Solid stars are the experimental data whereas open squares are the present theoretical results. The solid line is a $\chi^2$ minimization fit of power law ($\propto A^\tau$) for experimental data whereas dash-double-dotted line is a fit for the corresponding theoretical result. The theoretical fit that includes $^{238}\text{U} + ^{238}\text{U}$ reaction is represented by dotted line.

FIG. 3. The decomposition of $\langle p^\text{dir}_x \rangle$ at $E_{\text{bal}}$ into mutual mean field part and collision part as a function of the system size. The lines are the $\chi^2$ fit of power law $\propto c.A^\tau$. 
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\[ \langle p_x^{\text{dir}} \rangle \ (\text{MeV/c}) \]

(a) collision part

(b) mean field part

\[ \tau = -0.70931 \pm 0.24234 \]

\[ \propto A^\tau \]