Research on two-dimensional hydraulic crack propagation path based on XFEM

Ningbo Li¹,², Yujing Jiang¹, Xingshun Lei¹, Bo-nan Zhang², Ziqi Sun⁴, Xuan Zhang³, Boyu Si³
1. School of Engineering, Nagasaki University, Japan
2. School of Civil Engineering, Shandong University, Jinan, China
3. General Institute of Water Resources and Hydropower Planning and Design Ministry of Water Resources, China
4. Party-masses work Central Route Construction Management Bureau of South to North Water Transfer Project, China

Abstract. The experimental study of hydraulic fracturing, i.e. the experimental study of the interaction between fracturing fluid and rock, was first started by the experiment of plane fracture seepage. In this study, a numerical calculation model is established by applying the net crack water pressure to calculate and analyze the in-plane crack propagation characteristics under in-situ stress conditions. By studying the influence of pre-set crack angle on hydraulic crack propagation behavior, it is found that under the influence of the stress field, stress concentration appears at the crack tip during crack propagation. Under the combined action of confining pressure and water pressure, the fracture surface presents a tension shear state, and its propagation direction is affected by the joint action of tensile stress and shear stress. When there are holes at the bottom of the hydraulic crack, the stress concentration at the tip of the hydraulic crack is greater, and it is easier to crack.

Keywords: hydraulic crack, crack propagation, XFEM, seepage velocity

1. Introduction

Hydraulic fracturing simulation experiment is an important means to study hydraulic fracturing technology, which can accurately simulate the influence of natural defects such as faults and joints in the actual rock mass on the propagation path of hydraulic fractures, and has strong practical engineering significance.

The experimental study of hydraulic fracturing, i.e. the experimental study of the interaction between fracturing fluid and rock, was first started by the experiment of plane fracture seepage. Since then, they have made models of various material types (Louis et al. 1970; Witherspoon et al. 1982; Tian Kaiming 1983; Zhang Youtian et al. 1989; Tao Zhenyu et al. 1993). Among them, the most widely accepted theory to describe fluid flow in fractured rock is called cubic law: \( q = C b^3 \) q is seepage velocity, b is fracture aperture, C is constant coefficient. However, the limitation of cubic law is that the influence of fracture surface roughness on fluid flow is not considered, which leads to the difference between the calculation results and the actual results.

Numerical simulation analysis method is one of the important ways to study hydraulic fracturing. Compared with the indoor physical simulation test, the numerical simulation analysis method can better understand the hydraulic fracturing propagation mechanism. At the same time, due to the long period and high cost of making test block in laboratory physical simulation test, and the test block can not well represent the formation conditions below several thousand meters, many scholars use numerical
simulation analysis method to study hydraulic fracturing. For example, a mathematical model has been established (Zheng Shaohe et al. 1999) to simulate the fracture propagation induced by water drive. It is considered that the blowing ratio and pore displacement rate have an important influence on the fracture propagation, and the fracture propagation can be controlled by them. A pseudo-three-dimensional hydraulic fracturing model is established (Dunat X et al. 1998).

However, for brittle rocks, under the action of high-pressure water, they break down instantaneously. Compared with the single solid stress field, there is a certain deviation in the simulation of the path. Therefore, in this study, a numerical calculation model is established by applying the net crack water pressure to calculate and analyze the in-plane crack propagation characteristics under in-situ stress conditions.

2. Theoretical Method

The traditional finite element description of displacement field is based on element. The displacement between elements can be harmonious or incompatible. However, the displacement field $u^e(x)$ in each element is always expressed by shape function $N^e_k(x)$ and element node displacement $u_k$

$$u^e(x) = \sum_k N^e_k(x)u_k$$

(1)

In this formula, x is the space coordinate, and the subscript $k$ represents the node of the element. In the XFEM method, a set of partition of unity method on the solution domain is constructed by using the shape function of the finite element. The node influence domain composed of adjacent elements is regarded as a set of covers. By defining the displacement mode on each cover, the approximate displacement description of the whole domain can be written as follows.

$$u_{app}(x) = \sum_i N_i(x)\sum_j \beta_{ij} p_j(x)$$

(2)

In this formula, the subscript $i$ represents the node influence domain, $N_i(x)$ is the unit decomposition function defined on the node influence area, which is the combination of shape functions of all elements in the node influence area. $\{p_j(x)\}$ is a set of basis functions on local approximation spaces, which can be continuous or discontinuous. For example, when there is a discontinuous interface passing through the influence domain of a node, the Heaviside function can be selected as one of the terms, so the discontinuous basis vector is used to express the displacement discontinuity caused by the discontinuous interface. When $\{p_j(x)\}$ only contains a constant term, $\beta_{ij}$ is the node displacement, and equation (2) can be reduced to the displacement approximate formula of the traditional finite element method. XFEM, which is based on unit decomposition, improves the ability to describe the complex displacement field, especially increases the flexibility of tracking the evolving discontinuous boundary, and avoids mesh re-division.

3. The influence of stress ratio on the propagation path of the hydraulic crack

3.1. Numerical calculation model

As shown in the figure 1, the side length of the model is 150 mm, $l$ is the length of the preset crack, $\alpha$ is the angle between the preset crack and the minimum horizontal principal stress direction (recorded as the initial angle), and $\beta$ is the angle between the tangent line along the crack propagation direction and the preset crack (denoted as the propagation angle). Therefore, a polar coordinate system with the crack tip as the origin is established, the preset crack tangent line is taken as the polar axis, and the counterclockwise direction is positive. As shown in the figure, $\beta$ at the crack initiation is recorded as $\beta_0$, and the angle between the crack initiation path and the minimum principal stress $\sigma_{II}$ direction is recorded as $\gamma$. 


The characteristic length of the element grid is 1mm and the element type is CPE4. In the first analysis step, the boundary conditions are applied, the horizontal displacement constraint is applied at the midpoint of the upper and lower surfaces, the vertical displacement constraint is applied on the lower surface, the compressive stress $\sigma_H=7.12\text{MPa}$ is applied on the upper and lower surfaces, and the compressive stress $\sigma_h=3.56\text{MPa}$ is applied on the left and right surfaces; in the second analysis step, the hydraulic pressure load of 20MPa is applied to the crack surface for hydraulic fracturing calculation. The elastic modulus of the specimen $E=22\text{GPa}$, Poisson's ratio $\mu=0.2$, and the critical equivalent energy release rate is $G_{IC}=G_{IIc}=6\text{ N/m}$.

### 3.2. Effect of Initial Defects on Hydraulic Crack Propagation Path

Based on the 3.1 calculation model, double cracks are set under the condition of the preset crack inclination angle of 30 ° and the calculation model is shown in Fig. 7. The horizontal internal crack size is 20 mm $\times$ 0.2 mm, and the crack center distance is 15 mm. The net water pressure load of 10MPa is applied to the inclined crack. The boundary conditions and mesh size are the same as those of the model mentioned in 3.1. The crack initiation is calculated.
The maximum principal stress program obtained by calculation is shown in Fig. 8. It can be seen that the preset crack at the lower end of the hydraulic crack has a certain influence on the propagation of the hydraulic crack. The crack initiation angle is about 45 ° and the lower end of the hydraulic crack tends to expand towards the preset crack tip; in addition, the stress concentration at the tip of the preset fracture weakens the stress at the lower end of the hydraulic crack, at this time, the model is no longer symmetrical, and the crack initiation angles of the upper and lower ends of the hydraulic cracks are not completely consistent. At this time, the upper crack initiation angle of the hydraulic crack is about 60 °, and compared with that without crack defects, it is found that the initiation angle of hydraulic cracks with defects is larger, according to the maximum principal stress program, it can be seen that due to the existence of crack defects, hydraulic cracks tend to expand towards (near end of the hydraulic crack) or far away from defect (far end of the hydraulic crack).

3.3. Effect of circular hole defect on the hydraulic crack propagation path

Based on the calculation model in 4.1, the circular hole defect is set under the condition of a preset crack inclination angle of 30 ° and the calculation model is shown in Fig. 4. The distance between the center of the circular hole and the crack center is 15mm, and the radius is 5mm. The net water pressure load of 10MPa is applied to the inclined crack. The boundary conditions and mesh size are the same as those of the model mentioned in 3.1.
The maximum principal stress program obtained by calculation is shown in fig.5. It can be seen that the hole has a certain influence on the energy of the hydraulic crack tip. Firstly, because the hole defect cannot bear the load, the stress concentration at the tip of hydraulic crack is larger and more likely to crack under the same external load condition compared with the models mentioned in 3.1; secondly, the cavity leads to the energy dissipation of the crack tip, so the upper end of the hydraulic crack first expands at the initial moment; in addition, the crack initiation angles of the upper and lower ends of the hydraulic crack are also asymmetric, the upper end tends to expand along the initial crack angle, and the lower end of the hydraulic crack tends to expand toward the top of the hole.

4. Conclusion
The use of sections to divide the text of the paper is optional and left as a decision for the author. Where the author wishes to divide the paper into sections the formatting shown in table 2 should be used.
(1) By studying the influence of pre-set crack angle on hydraulic crack propagation behavior, it is found that under the influence of the stress field, stress concentration appears at the crack tip during crack propagation. Due to the symmetrical distribution of load, the crack propagation path is centrosymmetric with respect to the midpoint of the preset crack.
(2) When there is a horizontal preset crack at the lower end of the hydraulic crack, the hydraulic crack begins to crack from the upper end, and the crack initiation angle at the upper end is about 60° and the lower end of the hydraulic crack tends to expand towards the preset crack tip. Compared with the crack initiation angle without crack defects, the hydraulic crack initiation angle is larger when there are defects.
(3) When there are holes at the bottom of the hydraulic crack, the stress concentration at the tip of the hydraulic crack is greater, and it is easier to crack. At the initial moment, the upper end of the hydraulic crack first expands, and the upper end tends to expand along the angle of the initial crack, and the lower end of the hydraulic crack tends to expand toward the top of the hole.

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Compliance with ethical standards

Conflict of interest: The authors declared that there is no conflict of interest to this work.