System analysis of resource interaction processes in the biotechnological system

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Abstract. The paper provides mathematical modeling of processes occurring in the biotechnological system, using the example of the interaction of functional microorganisms with contaminating microflora, which is currently a promising scientific direction. The analysis showed that there are no existing studies on this issue, as well as systematic and integrated approaches to solving this problem that allow describing the processes under consideration. Therefore, it is necessary to analyze and build a system model that reflects the full range of interactions between useful and extraneous populations of microorganisms in the production of bakery products. System analysis and mathematical modeling of microbiological processes is a complex task. It is necessary to take into account a lot of factors, such as the simultaneous flow of several non-stationary processes; a lot of state parameters, including the analysis of relationships between them; changes in real-time technological parameters, in particular the main resource, taking into account various properties of microbial populations that affect the behavior of microbiological systems (mortality, birth rate, the existence of extraneous microflora, etc.), and other factors that directly affect the quality of the finished product. These factors characterize the complexity of forming a mathematical model that allows you to take into account all the main features of the production of bakery products. The existing mathematical apparatus and the system analysis of this kind of production carried out before considered the behavior of the population of beneficial microorganisms, but did not contain an analysis of the effect of contaminating microflora on it. The problem can be reduced to a system of differential equations describing the microbiological process as a whole. To describe the General model needed to describe an unknown function, including constraints on the parameters. In addition, you must set the initial and boundary conditions and determine the coefficients of the model. The paper offers a system model for analyzing the effect of plant additives on biotechnological properties and test quality indicators. The presented system model allows you to continue the study of the issues raised in the work, by further studying its components.
1. Introduction
System analysis and mathematical modeling of microbiological processes is a complex task. It is necessary to take into account a lot of factors, such as the simultaneous occurrence of several non-stationary processes; a variety of state parameters, including the analysis of relationships between them; changes in real-time technological parameters, in particular the main resource, taking into account various properties of microbial populations that affect the behavior of microbiological systems (mortality, birth rate, the existence of extraneous microflora, etc.), and other factors that directly affect the quality of the finished product. These factors characterize the complexity of formalizing a mathematical model that allows you to take into account all the main features of the production of bakery products. The lifting power of yeast is an indicator of its ability to ferment carbohydrates contained in the main raw materials: fructose, sucrose and glucose. It is defined as the time it takes for the dough ball to float, that is, the less time it takes for the ball to float, the more active the gas formation process, and therefore the fermentation process is more intense. In modern baking, there are various ways to intensify fermentation processes, one of them is the introduction of nutritional supplements of plant origin. This work is devoted to the formalization of a mathematical model of the dependence of the lifting force of baking yeast on the nature and amount of non-traditional additives of plant origin, powdered semi-finished products: mountain ash (PSFC), walnut partitions (PSFWP), sea buckthorn leaves (PSFLSB).

2. The theoretical part
The processes occurring in the production of bakery products can be formalized as a system of two differential equations. We describe the vital functions of microorganisms using two initial conditions that set their initial number. The following model parameters are considered: \( \mu \) - specific growth rate of microorganisms, \( C \) - their mortality rate, \( \alpha \) - coefficient describing the interaction of microorganisms with each other. For \( K \), we take the biomass of the environment, consisting of useful and contaminating biomass. We believe that \( K \) is described by the economic coefficient \( a \), which takes into account the share of the consumed resource in the production of new biomass.

The vector of performance criteria: \( Q = Q(q_1, \ldots, q_s) \), \( s \in S \), will allow us to analyze the quality of the finished product. As an indicator of effectiveness would be to apply the principle of Pareto optimality.

In connection with the designations we have introduced, we can present a system model describing the microbiological processes in the production of bakery products [1].

1. A mathematical model:
\[
q = q(q_1, \ldots, q_s) n_i m_j R_p \in \Phi \rightarrow \text{opt},
q(n_i)n_i \in \Phi \rightarrow \text{max},
q(m_i)K \in \Phi \rightarrow \text{min},
q(K)K \in \Phi \rightarrow \text{max}
\] (1)

2. value search area:
\[
N = n(n_1, \ldots, n_i, \ldots, n_I), i = 1, I,
M = m(m_1, \ldots, m_j, \ldots, m_J), j = 1, J,
D = D(d_1, d_2, \ldots, d_P) = 1, P,
d_p = d(d_{op}, x, y, x, t), p = 1, P, t \in T,
d_p \vee t = 0 = d_{op}(x, y, x, t), p \in P,
Q_p(d_p, t) = \{Q_p(d_p, t), D_p > D_{op} 0, D_p \leq D_{op},
\]
\[ D_{op} = D_o \left(d_p, x, y, z, t\right), p \in P, t \in T, \]
\[ \frac{dp}{dt} = D_p - \alpha_i \mu_i n_i - \alpha_j \mu_j n_j, \]
\[ \frac{dn_i}{dt} = \mu_i n_i - \mu_i \frac{n_i m_j}{m_j} - c_i n_i, \]
\[ \frac{dm_j}{dt} = \mu_j m_j - \mu_j \frac{m_j n_i}{m_j} - c_j n_j, \]
\[ \frac{1}{K} \frac{dK}{dt} = \mu K - \frac{\mu K^2}{K_m} - cK, \]
\[ K = n_i + m_j, i \in I, j \in J, t \in T, \]
\[ m_j \forall_{t=0} = m_{oi}(x, y, x), \]
\[ n_i \forall_{n=0} = n_{oi}(x, y, x), \]
\[ K \forall_{t=t} = K_m, \]
\[ m_j \forall_{t=t} = m_{jm}, \]
\[ n_i \forall_{n=t} = n_{3i}, \]
\[ n = \{n_i, D_p > D_{op} 0, D_p \leq D_{op}, m_j = \{m_j, D_p > D_{op} 0, D_p \leq D_{op}, \}
\[ \alpha_i \geq 0, \alpha_j \geq 0, \mu_i \geq 0, \mu_j \geq 0, c_i \geq 0, c_j \geq 0 \] (2)

where: N is a set of functional microorganisms, M is a set of contaminating microorganisms. D is a resource function defined in the vector form \( D = D(D_1, ..., dp) \), \( p = 1...P \), where \( d = d(X, Y, Z, t) \) is a vector of resource parameters, \( X, Y, Z \) are spatial coordinates, \( t \) is time \( T \) is a time interval, \( n_{im}, m_{jm} \) are the maximum values of population power, \( K_m \) is the maximum value of biomass growth power.

3. Practical research
As an object of research, pressed baking yeast (GOST 54731-2011), powdered semi-finished products: from Rowan berries (GOST R 56637-2015), sea buckthorn leaves (GOST R 51074-2003), walnut partitions (GOST 32874-2014) were used. The lifting force of yeast was determined by the accelerated method of "ball ascent" (GOST 171-40). For this purpose, the dough was prepared with the introduction of powdered semi-finished products instead of part of wheat flour and a control dough according to the method presented in the regulatory documentation.

The results of measuring the lifting force of baking yeast with the introduction of various amounts of PSF, PSF PSP, PSFLSB are shown in figure 1.

In this paper, the regression model was built using the “data Analysis ” add-in in the MS Excel environment and is a development of previous studies [1-5].

Two variants of the model were considered for the indicator of the lifting force of baking yeast: the

First variant in the form of \( Y_i = \beta_0 + \beta_1 x_1 + \epsilon_i \) (3)

The second option is in the form \( Y_i = \beta_0 + \beta_1 x_1 + \beta_2 \left(x_1^2\right) + \epsilon_i \) (4)

The impact of unrecorded factors and random measurement errors is determined using residual variance. The sample estimate is (standard error):

\[ S_e^2 = \frac{\sum_{i=1}^{n} \epsilon_i^2}{n-(p+1)} \] (5)
The value of the R-square coefficient shows how much percent variation of the dependent variable is described by this model. The significance of the multiple regression equation was evaluated using the F-criterion. The equation is considered significant if

\[ F = \frac{Q_R(n-p-1)}{Q_e} > F_{table} \]  \hspace{1cm} (6)

where

\[ Q_R = \sum_{i=1}^{n} y_i - (\bar{y})^2 \] - sum of squared deviations due to regression \hspace{1cm} (7)

\[ Q_e = \sum_{i=1}^{n} e_i^2 \] - the residual sum of squares that characterizes the influence of unaccounted factors (8).

Table 1 shows the values of the standard error and the R-square for the two variants of models considered in the work on the indicator of the lifting force of baking yeast.

The values of the standard error coefficients and the R-square of the two model variants for the yeast lift index with the addition of powdery semi-finished chokeberry, powdery semi-finished product walnut partitions, and powdery semi-finished leaves of sea-buckthorn are shown in table 1.

As can be seen from table 1, the best according to these characteristics is the second version of the model in terms of the lifting force of baking yeast when applying various powdered semi-finished products.
Table 1.

| Value of coefficient | Variant of model |
|----------------------|------------------|
|                      | 1                | 2                  |
| with the addition of powdery semi-finished chokeberry |               |                   |
| Standard error       | 0.753563         | 0.783764           |
| $R^2$                | 0.209938         | 0.359006           |
| with the addition of powdery semi-finished product walnut partitions |               |                   |
| Standard error       | 1.013011         | 1.005047           |
| $R^2$                | 0.296327         | 0.48051            |
| with the addition of powdery semi-finished leaves of sea-buckthorn |               |                   |
| Standard error       | 1.045398         | 0.414039           |
| $R^2$                | 0.000816         | 0.882449           |

For the second option (with the introduction of powdery semi-finished chokeberry), the following dependency was obtained: $Y_i = 6.36 + 0.35x_1 - 0.11x_1^2$.

For the model in question $Q_R = 1.032143$, $Q_e = 1.842857$, and the value $F = 0.840116$.

For the second option (with the introduction of powdery semi-finished product walnut partitions), the following dependency was obtained: $Y_i = 6.55 + 0.53x_1 - 0.17x_1^2$.

For the model in question $Q_R = 2.802976$, $Q_e = 3.030357$, and the value $F = 1387448$.

For the second option (with the introduction of powdery semi-finished leaves of sea-buckthorn), the following dependency was obtained: $Y_i = 5.86 - 1.62x_1 + 0.32x_1^2$.

For the model in question, $Q_R = 3.860714$, $Q_e = 0.514286$, and the value $F = 11.26042$. The table value $F_{table} = 9.55$.

Since the regression equations are significant in all variants, this may be due to the fact that a small number of observations were made. In our case, the value is approximately equal to and is the probability of obtaining these results for a random sample from a population in which Y does not depend on independent variables.

4. Conclusions

From the obtained studies, it can be concluded that the introduction of vegetable powdered semi-finished products can correct the lifting force of pressed baking yeast during the technological process. In the case of using powdered semi-finished products of various nature, it is necessary to strictly monitor the ratio of wheat flour and vegetable additives, since with the introduction of powdered semi-finished products of chokeberry in the dosage of 1, 2 and 3%, the lifting force increased by 17, 17 and 8%, respectively, which will negatively affect the quality of finished products, and the dosage of 4% decreased by 17%, which is likely to have a beneficial effect on the course of the technological process, and consequently the quality of finished products. With the introduction of powdered semi-finished product for the walnut partitions in a dosage of 1, 2 and 3% of the lifting force increased by 33, 8% and 17% respectively, at a dosage of 4% and 5% decreased by 17% and 8% respectively; with the introduction of prefabricated powdered leaves of Seabuckthorn in the dose of 1, 2, 3, 4 and 5% of the lifting force is decreased by 25, 42, 33, 17 and 8%, respectively. From the above, it is clearly evident that the use of powdered semi-finished sea buckthorn leaves is optimal for adjusting the lifting force of yeast, which affects product safety [6, 7].

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