Detecting topological phases via survival probabilities of edge Majorana fermions

Yucheng Wang\textsuperscript{1,2}\textsuperscript{*}

\textsuperscript{1}Beijing National Laboratory for Condensed Matter Physics, Institute of Physics, Chinese Academy of Sciences, Beijing 100190, China
\textsuperscript{2}School of Physical Sciences, University of Chinese Academy of Sciences, Beijing, 100049, China

We investigate the evolutions of edge Majorana fermions (MFs) and unveil that they can be used to characterize different topological phases and study the topological phase transitions. For some limiting cases of the evolution process for the one-dimensional Kitaev model and Su-Schrieffer-Heeger (SSH) model, we give analytical expressions of the survival probabilities of the edge MFs, which indicates that different topological phases correspond to different zero point numbers of the defined survival probabilities at some times. For a general case, we consider a dimerized Kitaev model and the Kitaev chain with disorder chemical potential and numerically calculate the survival probabilities of two edge correlation MFs. Our results show that both of them equal to zero, one of them equals to zero at some times or neither of them equal to zero correspond to the SSH-like topological, topological superconductor and trivial phases respectively.

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I. INTRODUCTION

The topological insulators and topological superconductors have attracted intensive studies in the last decade and a series of outstanding achievements have been achieved \cite{1} which have deepened our understanding of the basic physical properties of solid. The one-dimensional (1D) Kitaev chain \cite{2} is a spinless p-wave superconductor system, which provides a promising candidate to explore Majorana fermions (MFs) \cite{3,4} which fulfill non-Abelian statistics and is a potential application for the topological quantum computing \cite{7}. Su-Schrieffer-Heeger (SSH) model \cite{8} is another famous 1D model including the transitions between topological and trivial phases, which provides a platform to study rich topological phenomena, such as topological soliton excitation and topological edge states \cite{9,11}. The topological property of a system can be characterized by calculating the topological indices in momentum space and the occurrence of topological phase transitions corresponds to the sudden change of topological indices. As the bulk-edge correspondence, one can analyze the edge states in topological insulator or edge MFs in topological superconductors (TSCs) under open boundary conditions (OBC) to study a system’s topological properties.

Dynamics of quantum systems have attracted a great deal of attention \cite{12,13} recently. The survival probability (SP) of an initial state is an important physical quantity in the dynamics research and it characterizes the revival of this state later in time \cite{14–17}. This quantity also plays an important role in studying the extended-localization transition \cite{18,19}. If the initial state is located in a lattice site, the SP of this state approaches to zero in the thermodynamic limit when this system is extended, but tends to a finite value of $O(1)$ for a localized system when the time $t \rightarrow \infty$ \cite{20}. For a topological system, although the bulk states are extended, there exist zero-energy states that are localized at the edges of this system under OBC. If the initial state is taken as the particle located in the edge, one can investigate it’s survival probability after a long time evolution.

It is an interesting problem whether the different topological phases can be distinguished by dynamics, or rather, by the survival probabilities (SPs) of edge states. One natural thought is that setting the edge Majorana zero-energy modes as the initial state and studying it’s SP, which can’t distinguish the SSH-like topological and trivial phases actually, i.e., there exist zero points for the SPs of the initial state in the two phases after a long time evolution. To solve this question, we investigate a dimerized Kitaev model \cite{21,25} which includes the SSH-like topological, TSC and trivial phases. In this work, we introduce the SPs of two edge correlation MFs after a long time evolution, each of them includes two SPs of different topological phases after a long time evolution. To solve this question, we introduce a dimerized Kitaev model \cite{21,25} which includes the SSH-like topological, TSC and trivial phases.

The paper is organized as follows. In section II, we introduce a dimerized Kitaev model, the survival probabilities of four edge MFs and two edge correlation MFs. Fix some parameters, this model can be reduced to the Kitaev model or SSH model and we give analytical expressions of the defined SPs for two limiting cases of the two models. In section III, we numerically study the general evolution cases of the dimerized Kitaev model and the Kitaev model with disorder chemical potential and investigate their topological phase transitions by using the introduced SPs. A brief summary is given in section IV.

\textsuperscript{*} wangyc@iphy.ac.cn
II. SURVIVAL PROBABILITY OF MAJORANA FERMIONS AND TOPOLOGICAL PHASE TRANSITIONS

A. Model Hamiltonian

We investigate the dimerized Kitaev superconductor chain with Hamiltonian

\[ H = -J \sum_j [(1 + \lambda)c_{j-1}^\dagger c_j + (1 - \lambda)c_{j+1}^\dagger c_j + H.c.] - \Delta \sum_j [(1 + \lambda)c_{j-1}^\dagger c_j^\dagger + (1 - \lambda)c_{j}^\dagger c_{j+1} + H.c.] + \sum_j (\mu c_{j-1}^\dagger c_{j-1} + \mu c_j^\dagger c_j c_{j+1}), \]

(1)

where \(c_j^\dagger (c_j)\) is the fermionic creation (annihilation) operator, \(J\) denotes the nearest-neighbor hopping strength, \(\mu\) is the on-site chemical potential and \(\Delta\) is the superconducting pairing gap which is taken to be real here. One can easily find that the Hamiltonian becomes the SSH model \([8]\) when \(\Delta = 0\) and \(\mu = 0\) and it reduces to the 1D kitaev model \([2]\) when \(\lambda = 0\). Therefore this model include three phases: SSH-like topological phase, TSC phase and topologically trivial phase.

We set \(|\Delta| \leq 1\) and the size of this system as \(L\). We take \(J = 1\) as the unit of energy and OBC unless otherwise stated.

we introduce the MF operators:

\[ \gamma_{2j-1} = c_j + c_j^\dagger, \gamma_{2j} = \frac{1}{2i}(c_j - c_j^\dagger), \]

(2)

which fulfill

\[ \gamma_j^\dagger \gamma_j = \gamma_j, |\gamma_j| = 2\delta_{jt}. \]

(3)

By using the MF operators, the Hamiltonian \((1)\) can be written as

\[ H = \frac{i}{2} \sum_j [-(J + \Delta)(1 + \lambda)\gamma_{4j-1}\gamma_{4j-2} - (J - \Delta)(1 + \lambda)\gamma_{4j-3}\gamma_{4j} - (J + \Delta)(1 - \lambda)\gamma_{4j+3}\gamma_{4j+1} - (J - \Delta)(1 - \lambda)\gamma_{4j-1}\gamma_{4j+2}], \]

(4)

which is a \(2L \times 2L\) matrix.

B. SP of edge correlation MFs

Given an initial state \(|\Psi(0)\rangle\) at \(t = 0\), the evolution state at time \(t\) can be written as

\[ |\Psi(t)\rangle = e^{-iHt}|\Psi(0)\rangle = \sum_m e^{-iE_m t}|\psi_m\rangle\langle\psi_m|\Psi(0)\rangle, \]

(5)

where \(|\psi_m\rangle\) is the \(m - th\) eigenstate of the Hamiltonian \(H\) and \(E_m\) is the corresponding eigenvalue. The SP of this initial state at the time \(t\) can be defined as

\[ P(t) = |\langle\Psi(0)|\Psi(t)\rangle|^2 = \sum_m e^{-iE_m t}|\langle\psi_m|\Psi(0)\rangle|^2. \]

(6)

FIG. 1. Phase diagram of the 1D Kitaev model (\(\lambda = 0\) for the Hamiltonian \((1)\)). Phase I and phase II are the TSC phase and phase III corresponds to the topologically trivial phase. The coupling cases of MFs in the regions I and II correspond to two limiting cases that \(\Delta = \pm J\) and \(J = \pm J\) with fixed \(\mu = 0\).

which is related to the Loschmidt echo \([14,17,26]\).

If the initial state is set as a MF locating at the first Majorana site of the 2L Majorana chain, i.e., \(|\Psi(0)\rangle = \gamma_{tL}(0)|\Omega\rangle\), where \(|\Omega\rangle\) is the Bogoliubov vacuum \([27]\), its SP can be defined as \(P_{2L}(t) = \langle\gamma_{2L(0)}|\gamma_{2L(t)}\rangle|\Omega\rangle^2\) \([28,30]\). In the case of no causing ambiguity, we omit the \(|\Omega\rangle\). We can further define \(P_{2L}(t) = \langle\gamma_{2L(0)}|\gamma_{2L(t)}\rangle|\Omega\rangle^2\), where the initial state is set as a MF locating at the 2L \(- th\) Majorana site. Fig. 1 show the phase diagram of the 1D Kitaev model, \(P_1(t)\) and \(P_{2L}(t)\) won’t equal to zero at any time \(t\) if the parameters are chosen in the region I \([30]\). In a similar way, the SPs of the second and the \((2L - 1) - th\) Majorana sites can be defined as \(P_{2L-1}(t) = \langle\gamma_{2L-1(0)}|\gamma_{2L-1(t)}\rangle|^2\), which should not equal to zero at any time \(t\) if the parameters are chosen in the region II \([30]\) but will equal to zero at some times \(t\) if we choose the parameters of this system in the regions II or III.

Because the MFs always appear in pairs, we consider the SPs of two edge correlation MFs that \(\Psi^1(0) = \gamma_{tL}^\dagger(0)|\Omega\rangle\) and \(\Psi^2(0) = \gamma_{2L-tL}^\dagger(0)|\Omega\rangle\) \([23]\) after a long time evolution \([28]\) that

\[ G^1(t) = \langle\gamma_{2L(0)}|\gamma_{tL}(0)\gamma_{tL}(t)|\gamma_{2L(t)}\rangle = \langle\gamma_{tL}(0)\gamma_{tL}(t)\rangle|\gamma_{2L(0)}\gamma_{2L(t)}\rangle, \]

(7)

as our above definition, \(G^1(t) = \sqrt{P_1(t)P_{2L}(t)}\) and \(G^2(t) = \sqrt{P_{2L-1}(t)P_{2L-2}(t)}\). Both \(G^1\) and \(G^2\) aren’t equal to zero at any time \(t\) means that there exists a Dirac fermion which robustly locate at the each end of this chain. If one of them is equal to zero at some times \(t\) but the other one never equals to zero at any time, there exists a MF that robustly localize at the each end of this chain and this system is TSC. If both of them simultaneously equal to zero at some times \(t\), this system is topologically trivial. In order to obtain an intuitive understanding, we first consider two limiting cases of the Kitaev
FIG. 2. (Color online) The SP of the MFs $P_1$ and $P_{2L}$ for the 1D Kitaev model as a function of time $t$ for (a) $\Delta = -1$ and $\mu = 0$, (b) $\Delta = J = 0$ and $\mu = 0.05$. $P_1$, $P_2$, $P_{2L-1}$ and $P_{2L}$ for the SSH model as a function of time $t$ for (c) $\lambda = -1$ and $J = 0.025$, (d) $\lambda = 1$ and $J = 0.025$. The size of them are $L = 600$.

C. One dimensional Kitaev model ($\lambda = 0$)

When $\lambda = 0$, the Hamiltonian (1) reduces to the 1D Kitaev model [2]:

$$H = \sum_{j=1}^{L-1} (-J c_j^\dagger c_{j+1} - \Delta c_j^\dagger c_j^\dagger + H.c.)$$

$$+ \sum_{j=1}^{L} \mu (c_j^\dagger c_j - \frac{1}{2}).$$

(8)

The phase diagram of this system is shown in Fig. 1, which is trivial when $|\frac{\mu}{J}| > 1$ and is TSC when $|\frac{\mu}{J}| < 1$.

By using the MF operators in Eq. (2), the Hamiltonian (8) can be written as

$$H = \frac{i}{2} \sum_{j} [(-J - \Delta) \gamma_{2j-1} \gamma_{2j+2} + (J - \Delta) \gamma_{2j} \gamma_{2j+1}]$$

$$- \frac{i}{2} \sum_{j} \mu \gamma_{2j-1} \gamma_{2j}.$$ 

(9)

In the first case, we fix $\Delta = -J$ and $\mu = 0$, then the Hamiltonian (9) becomes $H = i J \gamma_{2j} \gamma_{2j+1}$. From Fig. 1, this system is at TSC phase. We can calculate the evolution of $\gamma_1$ and $\gamma_{2L}$,

$$\frac{d\gamma_1}{dt} = \frac{1}{i}[\gamma_1, H] = 0,$$

$$\frac{d\gamma_2}{dt} = \frac{1}{i}[\gamma_2, H] = \mu \gamma_1.$$ 

(10)

and $\frac{d\gamma_{2L}}{dt} = 0$. If there exists one MF located at the first or $2L - th$ Majorana site of this Majorana chain at $t = 0$, the corresponding SP of this MF $P_i$ ($P_{2L}$) will be 1 all the time, as showed in Fig. 2(a), where we show the $P_1$ and $P_{2L}$ as a function of the time $t$ for this system with $\Delta = -J$ and $\mu = 0$. One can see that $P_1(t) = P_{2L}(t)$ at any time, so we have $G^1(t) = P_1(t) = P_{2L}(t)$ as the above definition and $G^1(t)$ is always 1 for this case. If we take $\Delta = J$ and $\mu = 0$, one can easily verify that $G^1(t)$ is always 1 at any time.

Next we consider $\Delta = J = 0$, then this Hamiltonian becomes $H = -\frac{\mu}{2} \sum_{j} \gamma_{2j-1} \gamma_{2j}$. We have

$$\frac{d\gamma_1}{dt} = \frac{1}{i}[\gamma_1, H] = -\mu \gamma_2,$$

$$\frac{d\gamma_2}{dt} = \frac{1}{i}[\gamma_2, H] = \mu \gamma_1.$$ 

(11a)

Therefore $\frac{d\gamma_1}{dt} = -\mu^2 \gamma_1$, then it can be easily obtained that $\gamma_1(t) = a \cos(\mu t) + b \sin(\mu t)$, where $a$ and $b$ are the undetermined operators. In a similar way, we obtain $\gamma_2(t) = c \cos(\mu t) + d \sin(\mu t)$, where $c$ and $d$ are the undetermined operators. If the initial state is set as $|\Psi(0)\rangle = \gamma_1(0)|\Omega\rangle$, we have $a = \gamma_1(0)$ and $c = 0$. From Eq. (11a), we can obtain $d = \gamma_1(0)$ and $b = 0$. Therefore, the SP of this MF $P_1$ will become $\cos^2(\mu t)$, which oscillates as the time period $T = \frac{\pi}{\mu}$ and it becomes zero at the time $t = n\frac{\pi}{\mu}$, where $n = 1, 2, 3, \cdots$. Similarly, we can obtain $P_{2L} = \cos^2(\mu t)$ when setting the MFs located at the $2L - th$ Majorana sites of the Majorana chain as the initial state. Fig. 2(b) displays $P_1$ and $P_{2L}$ versus $t$ for this model with $\Delta = J = 0$ and $\mu = 0.05$. It can be seen that $P_1$ and $P_{2L}$ oscillate in synchrony with the time period $T = \frac{\pi}{\mu} \approx 62.8$ and $P_1$, $P_{2L}$ and $G^1$ become zero at the same time $t = n\frac{\pi}{\mu}$, which is consistent with our results. If the MFs located at the second and $(2L - 1) - th$ Majorana sites of the Majorana chain is set as the initial state, one can easily verify that there exist same oscillation for $P_2$, $P_{2L-1}$, $G^2$ and they become zero at the same time $t = n\frac{\pi}{2\mu}$, which means that the message of the initial edge MFs completely disappear at these times and this system is topologically trivial.

D. SSH model ($\Delta = 0$ and $\mu = 0$)

When $\Delta = 0$ and $\mu = 0$, the Hamiltonian (11) becomes the SSH model [8], which is written as:

$$H = -J \sum_{j=1}^{L/2} [(1 + \lambda) c_{2j-1}^\dagger c_{2j} + H.c.]$$

$$- J \sum_{j=1}^{L/2-1} [(1 - \lambda) c_{2j+1}^\dagger c_{2j+2} + H.c.]$$

(12)

This system is topological when $\lambda < 0$ and it is trivial when $\lambda > 0$.

After introducing the Majorana operators as showed in
We then consider two limiting cases. In the first case, we fix \( \lambda = -1 \), then the Hamiltonian (12) becomes
\[
H = -iJ(1 + \lambda) \sum_{j=1}^{L/2} (\gamma_{4j-3}\gamma_{4j} + \gamma_{4j-1}\gamma_{4j-2})
\]
\[
+ -iJ(1 - \lambda) \sum_{j=1}^{L/2-1} (\gamma_{4j-1}\gamma_{4j+2} + \gamma_{4j+1}\gamma_{4j}).
\]  
(13)

The second limiting case is \( \lambda = 1 \), then the Hamiltonian (12) becomes
\[
H = -iJ \sum_{j=1}^{L/2} (\gamma_{4j-3}\gamma_{4j} + \gamma_{4j-1}\gamma_{4j-2}).
\]  
It can be easily proved that \( \gamma_n(t) = \gamma_n(0)\cos(2Jt) \). The corresponding SP \( P_m \) will become \( \cos^2(2Jt) \), which oscillates as the time period \( T = \frac{\gamma}{2J} \) and it equals to zero at the time \( t = n\frac{\gamma}{2J} \), where \( n \) is a positive integer. Fig. 2(d) shows that \( P_m \) oscillate as the time period \( T \) and they are equal to zero at the time \( t = n\frac{\gamma}{2J} \). As the above definition, \( G_1 \) and \( G_2 \) simultaneously equal to zero at the time \( t = n\frac{\gamma}{2J} \), which means that the initial edge MFs will completely disappear at these times and this system is topologically trivial.

Although \( P_1(t) \) (\( P_2(t) \)) equal to \( P_{2L}(t) \) (\( P_{2L-1}(t) \)) at any time for the above cases, there also exist some cases that \( P_1(t) \) (\( P_2(t) \)) may not equal to \( P_{2L}(t) \) (\( P_{2L-1}(t) \)), e.g., for the case that the system size \( L \) is odd for the SSH model. Therefore, we need consider the four SPs \( P_1, P_2, P_{2L-1} \) and \( P_{2L} \) or just consider the \( G_1 \) and \( G_2 \) to investigate the topological properties of a system, which give the same results. For simplicity, we will just consider \( G_1 \) and \( G_2 \).

III. NUMERICAL STUDY FOR GENERAL CASES

In this section, we investigate the general cases that the dimerized Kitaev model and the Kitaev model with disorder chemical potential. Although no analytical solution can be found for these cases, we can still explore whether the presence or absence of the zeros of the SPs \( G_1 \) and \( G_2 \) at some times can still serve as a characteristic signature of topological nontrivial or trivial phases by numerically analyzing the evolution of edge MFs.

A. Topological phase transitions of the dimerized Kitaev model

When the chemical potential is fixed as \( \mu = 0 \), the phase diagram of this system is presented in Fig. 3(a), which can be obtained by calculating the topological numbers under periodic boundary conditions \( \{ \} \) or two edge correlation functions of MFs under OBC \( \{ \} \). The SPs of the edge correlation MFs \( G_1 \) and \( G_2 \) versus time \( t \) for the topologically trivial phase, TSC and SSH-like topological phase are showed in Fig. 3(b), Fig. 3(c) and Fig. 3(d) respectively. From these figures, we see that if the system is at the topologically trivial phase, both of the SPs can reach nearby zero after a finite time evolution, which means that the messages of the initial edge Majorana states completely disappear. If the system is at the TSC phase, one of the SPs can reach nearby zero but the other one approaches a nonzero constant after a finite time evolution. If the system is at the SSH-like topological phase, both of the SPs never approach zero after a long time evolution.

From Fig. 3 we see that the change of \( G_1 \) and \( G_2 \) versus the time are oscillations during a short time and then almost become constants after a long time evolution, which is simi-
ilar to the dynamical evolution in the Anderson localized system with the initial state located one lattice site [18,19]. One can easily find that $G^1$ and $G^2$ hardly change over time when $t > 100$ from this figure. In Fig. 4 we present the minimum values of $G^1$ and $G^2$ for $t \in [100, 300]$ as a function of $\lambda$ with fixed $\Delta = 0.5, L = 600$. Actually, the minimum values approximately equal to the average values of $G^1$ and $G^2$ for $t \in [100, 300]$, since they hardly oscillate in that time. We see that both $G^1_{\text{min}}$ and $G^2_{\text{min}}$ don’t equal to zero when $\lambda < -0.5$, this system is at the SSH-like topological phase, $G^2_{\text{min}} = 0$ and $G^1_{\text{min}} \neq 0$ when $-0.5 < \lambda < 0.5$, this system is at TSC phase and both $G^1_{\text{min}}$ and $G^2_{\text{min}}$ equal to zero when $\lambda > 0.5$, this system is at topologically trivial phase.

B. Kitaev model with disorder chemical potential

We add the uniformly distributed random potential $\mu_j \in [\frac{-\mu}{2}, \frac{\mu}{2}]$ to investigate the effect of disorder on this model. Fig. 5(a) and (b) show that $G^1$ and $G^2$ as a function of time $t$ for this system with $w = 0.1$ at the SSH-like topological phase and TSC phase respectively. We see that the SSH-like topological phase is sensitive but the TSC phase is robust against the disorder, which is consistent with the results obtained from the energy spectrum [21,34]. When added a small chemical potential, the zero-energy states of the SSH-like topological phase split, while the zero energy of the TSC phase is robust, because this disorder chemical potential breaks the sublattice symmetry but doesn’t break the particle-hole symmetry of the superconductivity [21]. Even the disorder strength $w$ is added to 3, as showed in Fig. 5(c), $G^1$ never approaches zero after a long time evolution, which means that the MFs are located at the two edges of this system and it is still TSC.

We further consider the topological phase transition of the Kitaev model induced by disorder chemical potential, i.e., we set $\lambda = 0$ for the Hamiltonian (8), which have been widely studied [35–40] and we now investigate it from dynamics. From Fig. 5(c), $G^1$ and $G^2$ don’t approach a constant after a long time evolution, so we take the minimum values of $G^1$ and $G^2$ during a long time. If both of them are nearby zero, this system is topologically trivial. If one of them is nearby zero but the other one doesn’t approach zero for any time $t$, this system is TSC. To decrease the oscillation, we take some samples to obtain the averaged minimum values of $G^1$ and $G^2$. Fig. 5 show the averaged values of the minimum $G^1$ and $G^2$ as a function of the disorder strength $w$ for the system with $\Delta = 0.5$. We see that $\langle G^1_{\text{min}} \rangle \neq 0$ and $\langle G^2_{\text{min}} \rangle = 0$ when the disorder strength is weak, which means that the system is at TSC. Both $\langle G^1_{\text{min}} \rangle$ and $\langle G^2_{\text{min}} \rangle$ approximately equal to zero when $w$ is plenty big enough, which means that this system enters into the trivial phase. The transition point from TSC to the topologically trivial phase $w_c$ should satisfy that $\langle G^1_{\text{min}} \rangle > \langle G^2_{\text{min}} \rangle$ when $w < w_c$ and $\langle G^1_{\text{min}} \rangle \leq \langle G^2_{\text{min}} \rangle$ when $w_c + \delta w$, where $\delta w$ is a infinitely small quantity. The transition point is about $w_c = 7 - 7.4$, which is consistent with previous results [40].

![FIG. 4](image-url) (Color online) The minimum values of the SPs of the edge correlation MFs $G^1$ and $G^2$ for $t \in [100, 300]$ as a function of $\lambda$ with fixed $\Delta = 0.5, L = 600$.

![FIG. 5](image-url) (Color online) The SPs of the correlative Majorana fermions $G^1$ and $G^2$ as a function of the time $t$ with fixed $\Delta = 0.5, L = 600$ and (a) $\lambda = -0.7$, $w = 0.1$, (b) $\lambda = 0$, $w = 0.1$, (c) $\lambda = 0$, $w = 3$. 
the topological phase transitions of a dimerized Kitaev model. When $\lambda = 0$, this model reduces to the Kitaev model and we discuss two limiting cases $\Delta = -J$, $\mu = 0$ and $\Delta = J = 0$. We obtain the analytical expressions of the SPs of edge MFs, which suggest that if there exist a series of zero points at some times $t$, the system is trivial, otherwise it is TSC. When $\Delta = 0$ and $\mu = 0$, this model reduces to the SSH model and we discuss two limiting cases $\lambda = -1$ and $\lambda = 1$. Our results show that all of them equal to zero at some time points $t$ for the trivial phase and all of them aren’t equal to zero at any time corresponding to the SSH-like topological phase. We further numerically investigate the SPs of edge correlation MFs for a general dimerized Kitaev model and the Kitaev chain with disorder chemical potential. Our results show that if both of them aren’t equal to zero at any time, the system is a SSH-like topological phase, if one of them is equal to zero at some times but the other one isn’t, the system is at TSC phase and if both of them equal to zero at some times, this system is topologically trivial.

IV. SUMMARY

In summary, we have introduced the SPs of edge MFs and edge correlation MFs after a long time evolution to describe the topological phase transitions of a dimerized Kitaev model.

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FIG. 6. (Color online) The averaged minimum values of $G^1$ and $G^2$ as a function of the disorder strength $w$ with fixed $\lambda = 0$, $\Delta = 0.5$, sample size=1000 and $L = 400$. The inset shows the enlarged region from $w = 5$ to $w = 7.5$. 

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\[ |\Psi(0)|^2 |\Psi(t)|^2 \] is actually the ground state of this system and \( E_0 \) is the corresponding eigenvalue.

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