Experimental and numerical study of heat and hydraulic characteristics of low head natural circulation loops with single-phase flow of working liquid

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Abstract. A 2D numerical simulation of velocity and temperature fields for laminar low flow regimes have been carried out for laboratory experimental natural circulation loop with vertical electrically heated circular tube as flow up section. Calculations have been done for the case of full-length heating of flow up section at constant heat flux density on the heated wall. The variant of loop design with negligibly small hydraulic losses due to local drag reduction and friction on down comer compared with hydraulic losses due to friction in flow up tube has been considered. Laminar flow regime as the regime of most complex friction factor behaviour in buoyancy driven flows was the subject of the analysis. On the basis of calculated velocity and temperature fields in heated zone the longitudinal change of friction factor and heat transfer coefficients have been determined. In general, according to 2D numerical simulation the wall shear stresses are mainly affected by the change of wall velocity gradient due to practically continuous velocity profiles deformation along the whole heated zone. The form of velocity profiles and the extent of their deformation in its turn depend upon the wall heat flux density and the hydraulic diameter. It is shown that in single-phase natural circulation loop where fluid flow is governed exclusively by buoyancy forces wall shear stresses change along the heated zone in a complex way and friction factor for use in 1D calculations can not be described by simple correlations in the form of $\zeta = a/\text{Re}^b$. In all calculated regimes including the lowest considered wall heat flux density the Nusselt numbers exceeds that for stabilized forced flow with constant thermophysical properties. After decreasing with the distance from the inlet to the heated section Nusselt numbers achieve minimum values and then start to increase.

1. Introduction. It is no need to say, that natural circulation loops are heat removal systems which have many industrial applications of different purpose. In recent years natural circulation heat removal mechanism is considered for passive safety systems in new design nuclear power plants. Besides, some advanced nuclear plant designs rely on natural circulation to remove core power under normal operation (startup, normal power operation, and shutdown), and some designs rely on natural circulation to provide cooling of the containment. Many applications including passive safety systems of nuclear power plants operate at low parameters, very often low flow conditions of single phase heat transfer medium are realized. So the availability of the reliable engineering and design calculation
methods of flows driven exclusively by buoyancy forces is an actual problem. In particular it is important for the analysis of start up regimes of passive safety systems of nuclear power plants.

Natural circulation systems may have low driving force and need to be started from the state of rest. Start-up from rest is one of the key issues in assessing the reliability of these systems. There is always a finite time lag before these systems attain their optimum intended performance level. The predicting models applicable for low flow conditions can be quite different from those applicable for high flow conditions. For example, in many cases under low flow conditions the three-dimensional flow effects occur and traditional 1D approach may not predict the behaviour of natural circulation systems with reasonable satisfaction. Also the performance of these systems is strongly dependent on the operating conditions and system geometry [1, 2].

In spite of long year (since early 1950s) experimental, analytical and numerical investigations natural circulation loops remain to be the subject of research till today. The state of the art of correct friction factor calculations can be taken as an example. Figure 1, which is adopted from review [1], presents experimental friction factor $\xi$ versus Reynolds number dependences obtained by different authors in natural single-phase circulation in loops of different configurations.

As it is seen from figure 1 all experimental points are divisibly higher the $\xi$ values of curve 1 that is the $\xi(Re)$ relationship for stabilized laminar isothermal flow. The attempts to generalize the $\xi(Re)$ dependence in the form of $\xi = a/Re^b$ do not result in universal correlation. In fact one have a set of pure empirical functions, which are valid for concrete loop designs within the range of regime parameters limited by the concrete experiments.

![Figure 1. Experimental friction factors at natural circulation of water in round cross-section channels in thermosyphon loops of different configurations (revive [1]); 1) isothermal stabilized laminar flow](image)

Despite the fact that the results presented in figure 1 have been published in [1] more than 35 years ago the state of the problem in general have not been changed till today [2 – 5].

No suitable predicting recommendations for heat transfer and friction for the above regimes have been proposed for engineering practice and correlations for forced flow are commonly used which to large extent overpredict the real flow velocities [1–5]. In particular the predicting friction factor correlations for fully developed isothermal flows is very often used in calculation of loop thermosyphons [2–4].

To our point of view this approach is not fully correct because the form of velocity profiles in real buoyancy-induced circulation flows differs from fully developed isothermal velocity profiles.

The velocity field pattern in buoyancy driven flows is in a transforming state in streamwise direction. Besides the flow field is forming under the conditions of considerable variety of thermophysical properties of working liquid. So the velocity field structure is highly complex, especially for laminar flows.

The analysis of measured circulation velocity and longitudinal heated wall temperature distributions in the experimental natural circulation loop in [6, 7] with the use of 1D approach showed that the developing of the appropriate predictive correlations for heat transfer and friction is not a
trivial problem. In general case hydrodynamic pattern of natural convection flows is dependent upon channel cross section configuration, channel hydraulic diameter, channel length to hydraulic diameter ratio and flow orientation relatively to gravity acceleration vector.

In general case for the above conditions friction factor \( \xi \) must be a function of a number of parameters \([8-10]\):

\[
\xi = \xi(z/d_h, Re, Pr, \mu_w/\mu_f, Gr),
\]

where \( d_h \) – hydraulic diameter, \( \mu_w \) and \( \mu_f \) – dynamic viscosity of liquid at wall and bulk liquid temperatures.

Therefore even if the absence of well-grounded recommendation for calculations of pure buoyancy driven flows in tubes makes one to use in 1D calculations correlations for forced flow one must at least take into account such obvious factors as temperature variety of thermo-physical properties of liquid (viscosity first of all) and its influence on friction factor and heat transfer coefficients change under the additional effect of natural convection on forced flow \([8-10]\).

According to definition the friction factor is

\[
\tau_w = \frac{\xi}{8} \rho \Pi^2 = \mu \left( \frac{\partial u}{\partial r} \right)_{r=r_0},
\]

where \( \tau_w \) is wall shear stress, \( \rho \) – liquid density at bulk temperature, \( \bar{u} \) – circulation velocity, \( (\partial \bar{u}/\partial r)_{r=r_0} \) – velocity gradient at the wall.

For the forced flow the effect of physical properties variability on friction factor can be taken into account by applying the correction of the form \([8, 9]\):

\[
\frac{\xi}{\xi_0} = \left( \frac{\mu_w}{\mu_f} \right)^n,
\]

where \( \xi_0 \) is the friction factor at constant properties, \( n > 0 \). In general case \( n = n(z/d_h, Re, \mu_w/\mu_f) \), that is \( n \neq \text{const} \). The most complex \( \xi \) dependence on cross-sectional viscosity varying is at laminar flow regimes.

Unfortunately not many quantitative experimental data on friction-viscosity variability dependence and a few predictive correlations are available for today. All of them are classified among forced flows.

A following correlation for \( n \) in the equation (2) for friction factor for flow up section of the loop can be used in 1D calculations as the first step \([8, 9]\):

\[
n = C \left( Pe \frac{d_h}{l} \right)^m \left( \frac{\mu_w}{\mu_f} \right)^{-0.062},
\]

where \( C = 2.30 \), \( m = -0.3 \) at \( Pe \cdot (d_h/l) \leq 1500 \) and \( C = 0.535 \), \( m = -0.1 \) at \( Pe \cdot (d_h/l) > 1500 \). In equation (3) \( Pe \) is Peclet number, \( l_t \) – heated tube length.

For the same directions of forced and natural convection at \( q_w = \text{const} \) the local Nusselt number at current cross-section can be determined as \([9]\)

\[
\frac{Nu}{Nu_v} = \left( 1 + \frac{Gr_q}{B Re} \right)^{0.25},
\]

where \( Nu_v \) is Nusselt number for viscous flow with constant physical properties;

\[
B = 5.40X^{1.4} + 312X^{0.25} \quad \text{at} \; X \leq 0.07 \quad \text{and} \; B = 240 \quad \text{at} \; X > 0.07; \quad X = \frac{1}{Pe} \frac{z}{d_h} \quad \text{Gr}_q = g\beta d_h^4 q_w/(\nu^2 \lambda) - \text{Grasghof number.}
\]

It is also worth noting that under the influence of thermo-gravitational forces on forced laminar flow the stability loss of such flow and transition to turbulence occurs at lesser Re numbers. For example for circular tube the above transition takes place at \( Re = Re_{cr} < 2300 \).

The adjusted length, at which the viscous-gravitational flow loses stability, that is the length of stability loss \( \chi \) can be estimated by the empirical correlation \([9]\)
\[ X_* = 12.9 (Gr/R_e)^{0.8}. \]

The transition to turbulent flow takes place at somewhat greater \( X \) value [9]:

\[ X = X_{cr} \approx 1.3 X_* . \]

To verify the validity of correlations (2) – (4) for the conditions of buoyancy driven flows the additional information is necessary. This information can be obtained from physical experiments for different natural circulation loops as well as from the results of numerical simulation of the velocity and temperature fields in heated zone.

In present work the 2D numerical simulation of velocity and temperature fields in circular tube with reference to laboratory natural circulation loop [6, 7] has been carried out. The above loop was designed for studying natural circulation of boiling liquid at low pressures. At low pressures boiling the single-phase convection region can be relatively very long so low flow single-phase experiments and their analysis were the part of the investigation. The present article presents the numerical results for laminar flow.

2. Numerical model and mathematical formulation of the problem. The laboratory experimental natural circulation loop of rectangular configuration described in details in [6, 7] has been considered as the object of numerical modeling. The flow up (heated) and flow down legs of the loop were joined to the cooling section at the top of the loop. One of the structural features of the experimental loop was large aspect ratio between the down leg cross sectional area and that of the heated leg. This detail of construction made it possible to substantially reduce pressure losses in flow down line (down comer). So the flow characteristics can be considered as somewhat upper limiting case for buoyancy driven flow for given heat flux for specific cross sectional geometry and hydraulic diameter of the heated leg.

A diagram of the corresponding numerical model of the loop is shown in figure 2. The simulated loop was a closed by liquid ensemble from annular and circular tube. Circular (heated) tube was placed coaxially inside the annular tube. Annular tube was the down comer of the simulated loop. The heated tube height was specified equal to the loop test section height, and cross-sectional area of annular gap was equal to cross sectional area of the experimental loop down leg. Thus the simulated circulation conditions were made more realistic in terms of the circulation in experimental loop. So the axially symmetric 2D problem has been formulated.

For steady state axially symmetrical laminar flow of incompressible liquid with the variable thermophysical properties the conservation equations of mass, momentum and energy in the absence of inner heat sources and energy dissipation being written in cylindrical coordinate system has the form (\( U_z \) and \( U_r \) – projections of the velocity vector on coordinate axes, \( z \)-axe is directed vertically, \( r \)-axe is directed along the tube radius):
The temperature of liquid at the inlet to the heated zone was set in calculations equal to 23°C. The velocity profiles deformation is accompanied by the increase of velocity gradient in near wall zone, the extent of velocity profile deformation is noticeable along the heated flow region and the increase of the wall shear stress as a consequence. Moreover as it follows from figure 3,c where the longitudinal change of absolute value of wall velocity gradient is presented the velocity gradient increase takes place along the full length of heated tube. That is the evidence of the absence of velocity profile stabilization along the heated zone.

Here \( \rho, h, T, p \) are density, enthalpy, temperature and pressure, \( F_{Bz} \) is the projection of buoyancy force vector \( \mathbf{F}_B \) on z-axis.

The expression for buoyancy force vector, which appears in the system of conservation equations and which is the driving force of motion can be written in general case as 
\[
\mathbf{F}_B(r,z) = \rho g(r,z) - \rho \mathbf{F}_B(r,z) \]
where \( g \) is vector of gravity acceleration and \( \rho_{1} \) is local reference density. For inner gravity flows (in closed space bounded by solid walls) the reference density is chosen according to the type of inner flow under consideration [11]. In our calculations \( \rho_{1}(z) \) was considered as cross-section averaged density at current coordinate \( z \) in down leg of the loop.

Boundary conditions were formulated according to the experimental ones in [6, 7]. They are the constant heat flux density on the wall of the heated section and adiabatic conditions on the other surfaces except the cooler zone. The loop cooler was simulated as volumetric heat sink of the cylindrical shape which height was equal to the upper gap in figure 2. The total capacity of the cooler was equal to the input heat power.

The system of conservation equations (7) – (10) was solved numerically with the use of control volume method within the ANES CFD-code [12].

3. 2D simulation results.

3.1. Velocity profiles and friction factors. The 2D numerical simulation have been carried out for distilled water circulation at heat flux densities which correspond to laminar flow regimes in the experimental loop [6, 7]. The flow was considered to be laminar along the entire length of the heated zone when adjusted coordinate \( X \) at the outlet cross section was less than \( X_{eq} \), the latter was estimated according to equations (5) – (6). The calculation results presented in this paper have been obtained for flow up tube diameters: 5.4, 7, 8, 9.1 and 15 mm. The height of heated section varied from 1.3 to 2 m. The temperature of liquid at the inlet to the heated zone was set in calculations equal to 23°C.

Velocity and temperature fields in heated zone at different wall heat flux densities have been calculated as the result. Based on the calculated velocity profiles according to formulas (1) wall shear stresses and friction factors have been determined. Examples of calculated velocity profiles at different distances from the inlet to the heated zone and longitudinal wall shear stresses change are shown in figure 3.

Velocity profiles calculated for water circulation near the inlet to the heated zone (\( z/d = 3.85 \)) and almost at outlet from it (\( z/d = 139.0 \)) for the tube of \( d = 9.1 \) mm and two wall heat flux densities \( q_w \) = 7.0 and 20.0 kW/m² are shown in figures 3,a and 3,b.

As it is seen from figures 3,a–b the form of velocity profiles noticeably changes along the heated zone, the extent of velocity profile deformation increasing with the wall heat flux density increase.

The velocity profiles deformation is accompanied by the increase of velocity gradient in near wall flow region and the increase of the wall shear stress as a consequence.

More over as it follows from figure 3,c where the longitudinal change of absolute value of wall velocity gradient is presented the velocity gradient increase takes place along the full length of heated tube. That is the evidence of the absence of velocity profile stabilization along the heated zone.
a) – b) – velocity profiles at two cross sections of heated tube with 9.1 mm inner diameter, calculated for two wall heat flux densities at water circulation; c) – longitudinal change of absolute value of wall velocity gradient corresponding to these regimes; d) – change of liquid viscosity in near wall, region due to wall temperature increase; a): $q_w = 7.0 \cdot 10^3 \text{ kW/m}^2$, 1 – $z/d = 3.85$, 2 – $z/d = 139.0$; b): $q_w = 20.0 \cdot 10^3 \text{ kW/m}^2$, 1 – $z/d = 3.85$, 2 – $z/d = 139.0$; c), d): 1 – $q_w = 7.0 \cdot 10^3 \text{ kW/m}^2$, 2 – $q_w = 20.0 \cdot 10^3 \text{ kW/m}^2$

**Figure 3.** Calculated longitudinal change of velocity profiles, liquid viscosity and wall velocity gradient.

The wall velocity gradient increase is balanced out by the liquid viscosity decrease with temperature in near wall flow region (see figure 3,d). So the outcome longitudinal wall shear stress behaviour is a function of more strong effect at current flow cross-section. Moreover the result effect depends not only on wall heat flux density but also on channel cross-section form and hydraulic diameter.

Calculated friction factors as a function of adjusted length for two tubes which differs by inner diameters are shown in figure 4,a. The Reynolds number, which was used for determining the adjusted length was calculated with the account of viscosity change with temperature along the heated section. Because of the strong temperature dependence the viscosity of liquid changes considerably along the heated zone (figure 3,d). This naturally leads to strong change along the flow of such dimensionless parameters as Gr and Re numbers. As a whole according to calculations the Re number inlet and outlet values under heating conditions can differ from each other from 20 to 40 % or more depending on heat flux density and hydraulic diameter, the less the diameter the higher the Re number change.
The heating conditions of regimes presented in figure 4 are characterized by close values of the Reynolds number. In the first case (curve 1 in figure 4,a) the Re number calculated according to physical properties at local bulk liquid temperature for $q_w = 15 \text{ kW/m}^2$ and $d = 5.4 \text{ mm}$ changed from 540 at the inlet to the heated section to 1070 at the outlet from it. In the second case (curve 3 in figure 4,a) the corresponding Re number varied for $q_w = 10 \text{ kW/m}^2$ and $d = 9.1 \text{ mm}$ from 720 at the inlet to 1040 at the outlet from the heated zone.

One can clearly see from figure 4,a the qualitatively different $\xi(\chi)$ behaviour for two diameters. The $\xi$ values calculated according to equation (2) are also shown if figure 4,a (lines 2 and 4). As one can see, formula (2) noticeably underpredicts longitudinal $\xi$ distribution. In both cases simulated $\xi$ values always remain higher than that calculated according to equation (2) if Re number is determined at local bulk liquid temperature. The discrepancy increases downstream and for higher hydraulic diameter one can observe different qualitative behaviour.

In the absence of well-grounded recommendations a formula for friction factor for stabilized flows at constant properties are often used in engineering hydraulic calculations of natural circulation loops. For laminar flows in circular tubes it is

$$\xi = \frac{64}{Re}. \quad (11)$$

In all calculated cases simulated $\xi$ values were always higher than that calculated according to formula (11).

It follows from the comparison of curves 1 – 4 in figure 4 that under the conditions of exclusively buoyancy induced circulation wall friction changes along the heated zone in a complex way and friction factor distribution can not be described by a simple relationships of the form $\xi = a/Re^b$. In most cases the effect of velocity profiles deformation is predominant as related to the effect of viscosity change. The shape of velocity profiles and the extent of their deformation in its turn depend upon hydraulic diameter.

The shape of velocity profiles for two tube diameters (5.4 mm and 9.1 mm) is shown in figure 4,b as an example. Though qualitatively the shape of both profiles is M-like it is clearly seen from figure 4,b that the deformation of the velocity profiles becomes more intensive with inner tube diameter increase.

More detailed idea of influence on friction factor geometric characteristics and regime parameters of the loop gives figure 5. Figure 5 represents plots of $\xi$ change along the heated section for different tubes and wall heat flux densities. The influence of hydraulic diameter on friction factor is presented in figure 5,a. Plots, shown in figure 5,a have been calculated at $q_w = \text{ idem } = 2 \text{ kW/m}^2$. 

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a) – calculated friction factors as a function of adjusted length; b) – calculated velocity profiles at the outlet from the heated zone ($z=1.28 \text{ m}$); 1) $q_w = 15 \text{ kW/m}^2$, $d = 5.4 \text{ mm}$, numerical simulation, 2) eq. (2) for the regime 1 conditions; 3) $q_w = 10 \text{ kW/m}^2$, $d = 9.1 \text{ mm}$, numerical simulation; 4) eq. (2) for the regime 3 conditions; b): 5) $q_w = 10 \text{ kW/m}^2$, $d = 9.1 \text{ mm}$; 6) $q_w = 15 \text{ kW/m}^2$, $d = 5.4 \text{ mm}$.

Figure 4. Calculated friction factors and velocity profiles for different inner diameters of heated tubes.
for tubes with different hydraulic diameters within the range of (5.4 – 15) mm. As \(d_h\) increases \(\xi\) decreases. One can observe nonmonotonous change of friction factors along the flow. This nonmonotony becomes more explicit with the hydraulic diameter increase. The nonmonotony of longitudinal \(\xi\) change is clearly seen in figure 5,b where the plots of \(\xi - z/d\) dependences are presented in larger scale for two tubes with the same inner diameter \(d_h = 9.1\) mm, but with different heated lengths \(L_h\) (1.3 and 2 m).

Change of heated tube length (in given case change of heated section height) means in practice change of overall dimensions of the loop and change possible driving head. As heated section height increases (line 2 in figure 5,b) at \(q_w = \text{idem}\) and \(d_h = \text{idem}\) the total input heat power increases. Flow rate and circulation velocity increase as a consequence. This in its turn leads to \(\xi\) decrease.

Friction factor change along the heated zone for different wall heat flux densities at \(d_h = \text{idem}\) and \(L_h = \text{idem}\) is shown in figure 5,c. As wall heat flux density increases the circulation velocity increases and friction factor decreases.

**3.2. Heat transfer coefficients.** In this subsection heat transfer coefficients and corresponding to them Nusselt numbers change along the heated zone under the conditions of continuously transforming velocity field will be considered.
Calculated heat transfer coefficients for different tube diameters at relatively low wall heat flux density \( q_w = 2 \text{ kW/m}^2 \) is shown in figure 6.a. As distance from the inlet to the heated section increases heat transfer coefficients firstly decrease in monotone way up to some value of longitudinal coordinate \( z \) and then with further \( z \) increase heat transfer coefficients practically does not change. As it is seen from figure 6.a at \( q_w = \text{idem} \) heat transfer coefficients decrease with tube inner diameter increase. For the same inner diameter of the heated tube the higher the \( q_w \) the higher heat transfer coefficients.

![Diagram](image1.png)

1 – \( d = 15 \text{ mm} \); 2 – \( d = 9.1 \text{ mm} \); 3 – \( d = 5.4 \text{ mm} \); \( \text{Nu}^* = 4.36 \) – stabilized Nusselt number for forced flow with constant physical properties at \( q_w = \text{const} \).

**Figure 6.** Change of the heat transfer coefficients (a) and \( \text{Nu}/\text{Nu}^* \) ratio (b) along the heated zone at \( q_w = 2 \text{ kW/m}^2 \) for different inner tube diameters

The calculated \( \text{Nu}/\text{Nu}^* \) ratio as a function of longitudinal coordinate \( z \) are presented in figure 6.b. Here \( \text{Nu} \) is local Nusselt number at buoyancy driven flow in natural circulation loop and \( \text{Nu}^* = 4.36 \) is stabilized \( \text{Nu} \) value at forced flow of liquid in circular pipe at constant thermophysical properties at wall boundary condition \( q_w = \text{const} \). In all calculated regimes including the lowest considered wall heat flux density the Nusselt numbers exceed that for stabilized forced flow with constant thermophysical properties. (figure 6,b)

The calculated \( \text{Nu}/\text{Nu}^* \) ratios as a function of dimensionless longitudinal coordinate \( z/d \) for the conditions of relatively high wall heat flux densities (\( q_w = 15 \) and \( 10 \text{ kW/m}^2 \)) and different flow up (heated) sections including sufficiently long ones (with dimensionless lengths \( z/d \) up to 400) are presented in figure 7. As it is seen from figure 7 at relatively high \( q_w \) the plot \( \text{Nu}/\text{Nu}^* \) as a function of \( (z/d) \) passes through the minimum that is in general case at some distance from the inlet to the heated section the heat transfer coefficients start to increase. According to calculation results this effect manifests itself appreciably at relatively high wall heat flux densities \( q_w \) for long enough heated sections or less inner diameters.

At low \( q_w \) the domain of minimum on the \( \text{Nu} \) versus \( z \) curve is highly wide and in the case of short tubes it can exist the interval, where practically \( \alpha \) does not change along the tube. For short tubes this apparent "stabilization" of heat transfer coefficients corresponded to the adjusted coordinates

\[
X = \frac{1}{\text{Pe}} \frac{z}{d_h} \quad \text{half as much as that for forced flow with constant physical properties.}
\]

As a whole the obtained results are quite expectable. Heat transfer intensity in discussed above flows is governed by mechanism of velocity profiles transforming and the conditions of liquid passing over the wall surface. According to calculation results velocity profiles change continuously along the full length of the heated zone and at some distance from the inlet liquid in near wall region starts to accelerate (the velocity profiles take the M-like form – figures. 3 – 4).
Near wall liquid velocity increase must lead to heat transfer enhancement and the increase of heat transfer coefficients. More over in the case of very long tubes laminar flow regime loses stability when adjusted coordinate $X$ achieves the $X^*$ value (see eq. (6)) and transition to turbulent flow regime takes place. It also enhances heat transfer. Analysis of natural circulation in the loop at turbulent flow is the subject of subsequent stage of the investigation.

**Acknowledgements.** This work was supported by Russian Fund of Basic Research, grant No 19-08-01044

**Nomenclature**

- $d, d_h$ – inner tube diameter, hydraulic diameter, m; for circular tube $d = d_h$
- $g$ – gravity acceleration, m/s$^2$
- $l$ – length of the heated zone, m
- $Nu$ – Nusselt number, $Nu = \alpha d_h / \lambda$,
  dimensionless
- $Pr$ – Prantl number, dimensionless
- $Pe$ – Peclet number, $Pe = Re Pr$, dimensionless
- $q$ – heat flux density, W/m$^2$
- $r$ – radial coordinate, m
- $r_0$ – inner tube radius, m
- $Re$ – Reynolds number, dimensionless
- $t$ – temperature, °C, K
- $\bar{t}_l$ – bulk liquid temperature, °C, K

**Greek symbols**

- $\alpha$ – heat transfer coefficient, W/(m$^2$·K)
- $\beta$ – thermal expansion coefficient, 1/K
- $\lambda$ – heat conduction, W/(m·K)
- $\nu$ – cinematic viscosity, m$^2$/s
- $\xi$ – friction factor, dimensionless

$$ X = \frac{1}{Pe} \frac{z}{d_h} \text{ or } X = \frac{1}{Re} \frac{z}{d_h} \text{ – adjusted} $$

**Subscripts**

- $w$ – wall
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