1. Introduction

In this paper we propose a new variation in the class of laser-generated fast electron focussing and guiding schemes based on exploiting resistivity gradients produced by using different \( Z \) materials. The fast electron beams produced by laser irradiation at relativistic intensities (\( >10^{18} \text{ W cm}^{-2} \text{ \mu m}^2 \)) are essential to a wide range of phenomena associated with ultra-intense laser-solid interactions including ion acceleration, x-ray generation and target heating. Fast electron beams are of particular importance in the fast ignition (FI) variant of inertial confinement fusion \([1, 2]\). There is a general need to develop means to guide and perhaps focus these fast electron beams, as well as guiding and focusing being critical to some proposed applications (e.g. FI \([3–5]\)). Numerical \([6, 7]\) and experimental \([8–10]\) studies of fast electron generation generally indicate that the divergence of these fast electron beams can be substantial and well above the tolerances for certain applications.

Since the idea that resistivity gradients might be exploited for confinement and guiding of fast electron beams \([11–13]\) along a single embedded wire, it was recognized that certain applications would not allow a guide element to be included throughout the target. This is particularly the case in cone-guided fast ignition (see \([14]\) and references therein). This encouraged, amongst other avenues of research \([3, 15–18]\), the development of alternative schemes for exploiting resistivity gradients in which the fast electrons would be focussed or collimated before propagating in homogeneous material. In this paper we show that a low-angle conical element may also have high efficacy in producing a collimated flow. Although the conical element does not have the geometric focusing properties of the ellipsoidal configuration, the conical element will tend to reduce the angular spread of the fast electrons through reducing their propagation angle on each successive bounce.

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(Some figures may appear in colour only in the online journal)
for microfabricators to fashion such targets in comparison to those with ellipsoidal guide elements. For clarity, we should emphasize that, like the elliptical configuration, the conical structure we propose here is inverse tapering (i.e. expanding) as one moves away from the laser source. This is quite different from the ‘Tongari’ cone-tip proposed by Johzaki and co-workers [22], which tapers (narrows) as one moves away from the fast electron source. The schematic shown in figure 1 illustrates the conical guide that we we propose. The experimental study of guiding could be considerably facilitated by a configuration that is much cheaper to produce than the ellipsoidal guide. We discuss the theoretical basis for why a conical guide element can be highly effective and we present 3D hybrid simulations which demonstrate its capabilities in a sub-kJ configuration.

2. Theory

The guiding of fast electrons in targets with resistivity gradients is based on the resistive generation of magnetic fields. The electrons are then collimated by this predominantly azimuthal magnetic field which grows due to the combination of gradients in the fast electron current density and resistivity gradients inside the target. One can assume both charge and current neutrality, i.e. \( j_i + j_b = 0 \), where \( j_i \) is the fast electron current and \( j_b \) is the background electron current. Calculating the background current through Ohm’s law, \( \eta j_b = E \), where \( \eta \) is the conductivity, Faraday’s law can be written as [23]

\[
\frac{1}{c} \frac{dB}{dt} = \eta \nabla \times j_e + (\nabla \eta) \times j_e
\]

The first term on the right-hand side generates a magnetic field that directs electrons towards regions of higher current density and thus acts to collimate the fast electron beam. The second term, which forms the basis of the elliptical mirror concept, generates a magnetic field at resistivity gradients which acts to keep the fast electrons within regions of higher resistivity. The resistivity gradient is created by a transition between two materials with different \( Z \). The high energy electrons are generated within the high-\( Z \) material and the magnetic field at the material interface will deflect the electrons and keep them inside the high-\( Z \) domain.

When the magnetic flux density and field width reaches sufficiently high values, the fast electrons can be thought of as specularly reflecting off these ‘magnetic walls’. Since the magnetic fields grow very strongly at the imposed resistivity gradients, the geometrical arrangement of the ‘magnetic walls’ is established by how one engineers the material interfaces. Consider a conical magnetic wall. Let the magnetic wall have an angle with respect to the target axis of \( \alpha \). Now consider a fast electron incident on this magnetic wall. Prior to striking the wall, the fast electron is travelling at an angle of \( \theta \) to target axis. The angle that it travels on after specularly reflecting can be obtained from the vectorial law of specular reflection,

\[
v = u - 2(u, \hat{n})\hat{n},
\]  

where \( v \) is the propagation vector after reflection, \( u \) is the propagation vector prior to reflection and \( \hat{n} \) is the unit vector normal to the reflecting surface. From this we find that,

\[
\tan \theta' = \frac{\tan 2\alpha - \tan \theta}{1 + \tan 2\alpha \tan \theta}.
\]

Using the relation for the tangent of a sum, equation (2) can be identified as \( \tan \theta' = \tan (2\alpha - \theta) \) and from this we have,

\[
\theta' = 2\alpha - \theta.
\]

This, of course, might be deduced somewhat faster if just treated as a standard geometry problem. From this we immediately observe that there is a particular angle, \( \theta = 2\alpha \), at which the reflected electron travels parallel to the target axis and that the absolute angle that the electron propagates at with respect to the target axis is lowered by \( 2\alpha \) if \( \theta > 2\alpha \). This is shown in figure 2.

This suggests that to effectively exploit the conical mirror we should therefore use a relatively low \( \alpha \) and a fairly long cone. In this case a majority of the injected fast electrons will lie in the \( \theta > 2\alpha \) region and will therefore there will be an overall improvement in the beam divergence from a single
bounce. In addition to this, if the cone is sufficiently long, the electron may experience further bounces and, while $\theta > 2\alpha$ it will be shallower by $2\alpha$ each time.

In summary, provided one accepts the validity of the assumption of specular reflectivity, one is lead to the conclusion that each ‘bounce’ from the ‘walls’ of the guide element must reduce the angle of the fast electron with respect to the target axis. Overall this must lead to ‘guiding’ in the sense of divergence angle with respect to a particular direction. In the following section we proceed to demonstrate this effect in numerical simulations.

3. Simulations

In the previous section it was deduced that a conical guide element should provide guiding of a fast electron beam through divergence reduction on each ‘bounce’ off the magnetic fields that develop at the guide element interfaces. In this section we proceed to demonstrate this effect. The scenario we have chosen to do this in is the case where a picosecond PW laser irradiates an initially cold solid density target. This target is in contact with a hot, solid-density hydrogenic target. We then examine the propagation of fast electrons through this hot, low-Z plasma where self-generation of magnetic fields should be weak. Any guiding that occurs should be dominated by the effect of the conical guide element.

3.1. Set-up

Simulations were performed using the 3D particle hybrid code ZEPHYROS. The ‘standard’ run used was set up as follows: a $200 \times 200 \times 200$ grid was used with a $1 \mu m$ cell size in the $x$-direction and a $0.5 \mu m$ cell size in the $y$- and $z$-directions. The target consisted of a CH$_2$ substrate of $57 \mu m$ thickness, within which a carbon conical guide element was embedded. The guide element had the form of a truncated cone and was colinear with the $x$-axis and centred on $y = z = 50 \mu m$. The initial radius of the truncated cone was always $5 \mu m$ and the angle of the cone wall with respect to the target axis was varied. For clarity, the cone radius expands on moving along the axis, i.e. it inverse tapers. The remainder of the domain (i.e. $x > 57 \mu m$) consists of hydrogen at $1$ g cc$^{-1}$. The background temperature is initially set to $1$ eV in the foil and $500$ eV in the dense hydrogen region. The background resistivity was described by the model which closely follows Lee and More, but with the minimum electron mean free path taken to be $5 r_e$, where $r_e$ is the interatomic spacing. The temporal profile of the injected fast electron beam is a top-hat function of $t_e = 2$ ps duration and the transverse profile was also a top-hat of $5 \mu m$ radius. The injected fast electron beam models irradiation at an intensity of $I_L = 1 \times 10^{20}$ W cm$^{-2}$, with the assumption of $30\%$ conversion efficiency. The fast electron angular distribution was chosen to be $\cos^2 \theta$ distribution. The fast electron temperature used was set to, which was chosen to model irradiation at $\lambda_L = 0.5 \mu m$, with $T_f = 0.6 T_L$ (following Sherlock’s numerical simulations [24]), where $T_p$ is the ponderomotive force obtained from the Ponderomotive Scaling proposed by Wilks,

$$T_p = 0.511 \left[ \sqrt{1 + \frac{I_p \lambda_e^2}{1.38 \times 10^{18} \text{Wcm}^{-2}}} - 1 \right] \text{MeV.} \tag{4}$$

A total of six simulations were carried out and they are labelled A–F. Runs A–D use the conical guides with different cone angles. Run E is a comparator simulation in which we replace the conical guide with a straight wire, with a constant radius of $5 \mu m$. Run F is also a comparator simulation in which there is no carbon guide element at all, only the plain CH substrate. We have tabulated the details of each simulation in table 1. Note that cone angle is always taken to mean the angle between the cone wall and the $x$-axis, i.e. it is a ‘half’ angle and not a ‘full’ cone angle.

3.2. Results

The results of all six simulations are summarized in figure 3 below. In figure 3 we show the angular distributions of the fast electrons, $dN/d\Omega$, at $3$ ps all on a single plot.

The case of no guide element (run F) is shown as a dashed black line and one can immediately compare this to the results of runs A and B where we use $5^\circ$ and $10^\circ$ conical guides respectively. The results of run F are very close to the distribution of the injected beam, so self-generated magnetic fields have done little to reduce the angular spread of the beam. It is therefore clear that the effect of the conical guide has been to reduce the angular spread of the beam substantially and the comparison to run F makes this clear. One might also question how this compares to a straight wire. According to the discussion of section 2, the straight wire should not produce any reduction in the angular spread of the fast electron beam. That statement assumes that the magnetic fields behave as perfect specular reflectors. In reality, they may not behave quite so close to this ideal and some mitigation of the angular spread might be produced. Indeed, figure 3 shows that, compared to the un-guided case (run F) there is some clear reduction in the angular spread. However the reduction in angular spread in run E (straight wire) is not as much as the reduction that is seen in runs A and B ($5^\circ$ and $10^\circ$ conical guides).

So from figure 3, we can clearly conclude that there is a strong reduction of angular spread in the low angle conical guide simulations (A and B) by comparison to the un-guided case (run F). It is also clear that these low angle conical guides are far more effective than the straight wire (which we expect to be ineffective in the ideal limit). What then about the higher
angle conical guides of 15° and 20° (runs C and D)? Figure 3 shows that runs C and D have produced some mitigation of the angular spread relative to the un-guided case (run F), but substantially poorer mitigation compared to the case of a straight wire (run E). The performance of runs C and D is poor in comparison to runs A and B. In order to see this more quantitatively we have tabulated the mean angle for each distribution in Table 2.

The results shown in Table 2 re-iterate the previous discussion: runs A and B perform much better than the comparator simulations, yielding substantial improvement in the angular spread. The straight wire exhibits some improvement in the angular spread. Runs C and D show a slight improvement in the angular spread, but are poor in comparison to runs A, B and E.

We can now proceed to interpret some of these results. Firstly we note that, in general, we see the same pattern in magnetic field generation in these simulations as we observed in previous work. At early times the $\mathbf{V} \times \mathbf{j} \eta$ term dominates the growth of magnetic fields and at later times the $\mathbf{V} \times \mathbf{j} \eta \times \mathbf{j} \mathbf{f}$ terms becomes dominant and reinforces the growth of confining magnetic fields that developed at material interfaces [11]. This is particularly the case in runs A, B and E.

In runs C and D there is another process at work. This is filamentation inside the cone, i.e. not directly involving the material interfaces. Clearly filamentation disrupts the working of the conical guide and consequently destroys any reduction in the angular spread that might have been achieved. This filamentation does not occur in runs A and B. To show this, a fast electron density plot from run D is shown in Figure 4 and one from run A is shown in Figure 5. The background Z plot is shown along in both cases for reference.

Figure 4 clearly shows that strong and highly disruptive filamentation has developed in run D. This is cause of the poor performance of the conical guide in this case. In contrast, no such filamentation has developed in the case of run A. Run C exhibits similar behaviour to run D and run B exhibits similar behaviour to run A. It is therefore clear that runs A and B, being free from filamentation, have such good performance in terms of reducing angular spread, whereas runs C and D do not. The extent to which filamentation in larger...
angle conical guides might be avoided requires further investigation, for now we can only note that it occurs and that it has severely degraded the performance of runs C and D in this simulation set.

Finally we might also ask why the straight wire target (run E) is unexpectedly good at reducing the angular spread of the fast electrons (although not as good as the low angle conical guides in runs A and B). As previously mentioned, from the point of view of the ideal specular reflector, the straight wire should not reduce the angular spread of the fast electron beam. In order to understand this result, we looked at how the angular distributions from runs A, E and F evolved in time. Up to about 1.6 ps the angular distribution from E moved only slightly away from that in F, whereas that in run A exhibited quite a marked reduction in angular spread. Noticeable reduction in the angular spread in E only occurred after this time up to 3 ps. Although it is relatively straightforward to determine when this occurs, the cause is harder to diagnose. Some magnetic field does develop both inside the wire and at the tip of the wire in the hydrogenic region. These fields are relatively weak (20–100 T) but it may well be the case that they are sufficient to reduce the angular spread of the fast electrons in this time period.

4. Conclusions

In this paper we have proposed that guiding laser-generated fast electron beams into a homogeneous region of low resistivity can be done with a conical guide elements and not just an ellipsoidal guide element. For the sake of clarity we reiterate the point that these conical guide elements are expanding radially as one moves away from the fast electron source, which means they are fundamentally different from the ‘Tongari’ cone tip targets studied by Johzaki and co-workers [22]. The guiding is not superior to the ellipsoidal case (there is no convergence for one thing), but conical guide elements are easier and cheaper to fabricate, so these can facilitate the experimental studying of fast electron guiding based on resistivity gradients. The conical element works on the basis of specular reflection, i.e. fast electrons striking the oblique surface will have their angle of propagation with respect to the target axis reduced by twice the angle of the cone wall with respect to the target axis.

We have carried out a number of 3D hybrid simulations employing the conical guiding element where the fast electron beam propagates from a cold foil into a region of hot, dense hydrogen (homogeneous, low resistivity). On comparison to the case without a guide element, it is clear that the low angle (5–10°) conical guides substantially reduce the angular spread. In terms of the mean angle of the distribution this is a reduction of up to 25° in the divergence half-angle. We also compared these low-angle conical guides to a straight wire target and showed that they produced substantially more reduction in angular spread than the straight wire target. One problem that interferes with the operation of the conical guide is the development of filamentation inside the conical guide element. This does not occur in the 5–10° guides, but does for larger angles. The filamentation has such a severe effect that there is very little improvement in the angular spread of the fast electrons in the case of the larger angle conical guides. Nonetheless the low-angle conical guides have been shown to be highly effective by this set of simulations and currently appear to provide the optimum divergence reduction for a non-curved guide.

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