Spontaneous-emission-enabled dynamics at the laser threshold

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Chaos in semiconductor lasers or other optical systems have been intensively studied in past two decades. However, the route from period doubling to chaos is still not sufficiently visible, in particular, in gain-modulated semiconductor lasers. In this article we perform a careful investigation of the route to chaos exhibited by directly modulated semiconductor lasers near the threshold region with various values of modulation frequency and amplitude. Gain nonlinearity is included in the simulation of pulse train generation through gain switching, and a new form of phase space representation is introduced to distinctly display period doubling, tripling, quadrupling and chaos. The irregular behaviour is examined at various modulation frequencies and amplitudes, highlighting the possible route to chaos for very large amplitude modulation in the near-threshold region. The existence of deterministic trajectories below the laser threshold is rendered possible by the presence of the (average component of the) spontaneous emission, a point which has not often been explicitly considered. Furthermore, we report on the existence of a transition between similar attractors characterized by a temporal transient which depends on the amplitude of the modulation driving the pump.

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I. INTRODUCTION

The field of laser physics and chaos theory developed independently until 1975 when Haken discovered a striking analogy between the Lorenz equations that model fluid convection and the Maxwell-Bloch equations modelling light-matter interaction in single mode lasers [11]. Since then, investigations on the Lorenz-Haken chaos in laser systems have attracted a great deal of attention from researchers [2–6], and the analysis of system behaviour, based on chaos theory, has been used to interpret and control many instances of regular and disordered laser output [7]. In addition to their intrinsic physical interests, chaotic oscillations generated by laser systems possess promising applications in secure communications [8], chaotic lidar [9], random number generators [10] and optical time-domain reflectometry [11].

Semiconductor lasers are typical systems far from equilibrium, and abundant nonlinear dynamic behaviour can be observed under the application of a proper external perturbation [12]. From a nonlinear dynamics point of view, semiconductor lasers can be classified as Class-B laser systems [13, 14], where the relatively fast polarization decay is adiabatically eliminated, and the transient behaviour is mainly dominated by the interplay between population inversion and field intensity inside the laser cavity [15]. Therefore, the laser dynamics under single-mode operation can be described by a deterministic differential model accounting for two variables: photon number and population inversion. The introduction of an additional degree of freedom, such as current modulation and/or optical feedback, may lead to instabilities. Although several instabilities have been identified in semiconductor laser systems, some aspects of the period doubling route to chaos deserve a closer look: in particular, the generation of chaotic output in the vicinity of threshold under large signal modulation. This region holds particular interest since, as we will show in the following, deterministic predictions based on the rate equations accounting for the average contribution of the spontaneous emission (through the so-called β-factor), in addition to memory effects, allow for the prolongation of deterministic trajectories in the below-threshold region. This extends the parameter space in which a deterministic behaviour may be observed in a laser, in spite of a partial exploration of the non-lasing region.

The current investigation accounts only for the deterministic trajectories, as a first contribution to dynamical studies at the border between lasing and non-lasing regimes, and establishes the existence of limit cycles also in this border region in parameter space. In addition to its fundamental aspects, the interest for this topic resides in the potential for new applications that may emerge from the generation of different pulsed regimes in macroscopic semiconductor lasers.

Direct modulation of semiconductor lasers has received a great deal of attention both for its potential to generate ultrafast sharp pulses [16, 17] and its rich dynamical scenario. The dynamics generated by current modulation has been principally modeled through using single-mode rate equations [18–20], and the first theoretical work highlighting a period doubling route to chaos
in a semiconductor laser with current modulation was reported in 1985 [21]. A subsequent analysis predicted a similar route to chaos in non self-pulsing single contact laser diodes [2, 6, 22]. The rate equations governing the interrelationship between carrier density and photon density have been successfully applied to semiconductor lasers, for example, in predicting relaxation oscillation under pulsed operation [23]. Earlier work pioneered the use of phase portraits in the electron-density and photon density phase space to gain physical insight into the kinetic mechanism of the lasing process [24]. In this paper, we report on a numerical investigation on the response of a semiconductor laser to the modulation of its injected current near threshold; current bias, modulation depth and frequency are used as parameters to investigate the different dynamical regimes. On the bifurcation diagram, many regions with distinct modulation features are found and analyzed on the basis of a new phase space characterization, which we find particularly helpful in the analysis of the dynamical response at threshold, given its superior performance in identifying the attractors. As will result evident from the ensemble of the results, the average contribution of the spontaneous emission below-threshold, together with the memory effects introduced by the slower relaxation of the population inversion, enables the existence of deterministic attractors even in the non-lasing region.

II. RATE EQUATIONS

The single mode rate equations for the carrier density \( N \) and photon density \( S \) of a semiconductor laser can be described by [26]:

\[
\begin{align*}
\frac{dN}{dt} &= \frac{I_{\text{tot}}}{qV} - \frac{N}{\tau_n} - G(N - N_{\text{tran}}) \frac{S}{1 + \varepsilon S} , \quad (1) \\
\frac{dS}{dt} &= \Gamma G(N - N_{\text{tran}}) \frac{S}{1 + \varepsilon S} - \frac{S}{\tau_s} + \Gamma \beta BN^2 \quad (2)
\end{align*}
\]

where \( N \) represents the carrier density and \( S \) the photon density. On the right hand side of eq. (1), the first term represents the modulated injection current, \( I_{\text{tot}} = I_{\text{dc}} + I_{\text{m}} \sin(2\pi f_m t) \), where \( I_{\text{dc}} \) and \( I_{\text{m}} \) are the bias current and modulation amplitude, respectively, and \( f_m \) is the modulation frequency; \( q \) is the electric charge, and \( V \) is the active volume. The second term is the carrier spontaneous decay with a lifetime \( \tau_n \). The last term describes the stimulated emission of radiation where \( G \) indicates the gain coefficient, and \( N_{\text{tran}} \) is the carrier density at transparency. In order to fit more realistic laser characteristics, the nonlinear gain compression (factor \( \varepsilon \)) is introduced for better agreement with experimental results [18]. In eq. (1), the parameter of \( \Gamma \) represents optical confinement factor, \( \tau_s \) is the photon lifetime, and \( \beta \) denotes the spontaneous emission coupling emission factor. It is this last term which introduces the average contribution of the spontaneous emission to the photon number, thus enabling the establishment of deterministic trajectories across the lasing threshold. In principle, the carrier lifetime is a function of \( N \) according to [27] for which we use the following functional form:

\[
\frac{1}{\tau_n} = A + BN + CN^2
\]

where \( A \) is the recombination coefficient, \( B \) the bimolecular recombination coefficient, \( C \) the Auger recombination coefficient and \( D \) the carrier leakage coefficient. Detailed parameter values are summarized in Table 1. It is worth remembering that laser modulation through the pump requires much larger amplitudes than modulation through other parameters, such as cavity losses [25].

III. RESULTS AND DISCUSSIONS

A. Free running laser characterization

We first investigate the emission properties of semiconductor laser under free running operation. Fig. 1 shows the light-in light-out (L-L) characteristics in double-logarithmic representation, the “S-curve”, obtained by taking averaged photon numbers from the temporal data sequence. It exhibits a narrow transition between spontaneous and stimulated emission, indicating the typical threshold feature of low-\( \beta \) laser. Fig. 1b displays a typical radio frequency (RF) spectrum of the free running laser obtained at 1.14\( I_{\text{th}} \), which indicates a broad peak and a relaxation frequency around 2.00 GHz.

| Parameter                        | Symbol | Value           | Unit |
|----------------------------------|--------|-----------------|------|
| Active volume                    | \( V \) | \( 6.75 \times 10^{-11} \) | cm\(^2\) |
| Gain coefficient                 | \( G \) | \( 1.15 \times 10^{-4} \) | cm\(^3\) s\(^{-1}\) |
| Transparent carrier density      | \( N_0 \) | \( 1.1 \times 10^{18} \) | cm\(^{-3}\) |
| Gain compression factor \( \varepsilon \) | | \( 1 \times 10^{-17} \) | cm\(^3\) |
| Optical coefficient factor \( \Gamma \) | | \( 0.35 \) | - |
| Photon lifetime \( \tau_p \)     | | \( 1.75 \times 10^{-12} \) | s |
| Spontaneous emission factor \( \beta \) | | \( 1 \times 10^{-7} \) | - |
| Recombination coefficient \( A \) | | \( 1 \times 10^{8} \) | s\(^{-1}\) |
| Bimolecular recombination coefficient \( B \) | | \( 1.25 \times 10^{-10} \) | cm\(^3\) s\(^{-1}\) |
| Auger recombination coefficient | \( C \) | \( 3.5 \times 10^{-20} \) | cm\(^3\) s\(^{-1}\) |

B. Modulation frequency dependence

We first analyze the laser response to the modulation frequency for fixed amplitude and bias current. Fig. 2

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2. Equations have been numbered consecutively in parentheses to facilitate reference. Table 1 has been formatted in a tabular format to clearly show the parameters, symbols, values, and units for each. Figures and equations have been included to illustrate the concepts discussed.
doubling distinctly appears for 1.65 GHz phase portraits for laser modulation at and 4.22 GHz). Finally, the spectrum indicates that the laser large modulation amplitude ($I_m$) when $f$ shows the RF spectral map at $I_{dc} = 1.14I_{th}$. 

shows the RF spectral map at $I_{dc} = 1.14I_{th}$ as a function of modulation frequency for a modulation amplitude $I_m = 1.10I_{th}$. The higher-order harmonics, generated when $f_m < 1.65\text{GHz}$, are clearly visible. The period doubling distinctly appears for $1.65\text{GHz} < f_m < 2.30\text{GHz}$, followed by a narrow period quadrupling and period eight regions when the modulation frequency in the range of $2.30\text{GHz} < f_m < 2.45\text{GHz}$ and $2.45\text{GHz} < f_m < 2.50\text{GHz}$, respectively. In the range $2.55\text{GHz} < f_m < 2.90\text{GHz}$, the whole RF spectrum broadens, suggesting chaotic oscillation. The broad spectral region is followed by a period 3 and then 6 response, succeeded by more transitions to oscillations with wide spectral features ($3.85\text{GHz} < f_m < 3.91\text{GHz}$ and $4.14\text{GHz} < f_m < 4.22\text{GHz}$). Finally, the spectrum indicates that the laser follows the sinusoidal modulation when $f_m > 5.00\text{GHz}$.

Fig. 3 shows the temporal dynamics, RF spectra and phase portraits for laser modulation at $I_{dc} = 1.14I_{th}$ and $f_m = 1.5, 2.0, 2.4$ and $2.8\text{GHz}$, respectively. The large modulation amplitude ($I_m = 1.10I_{th}$) enables gain-switching operation: the laser is periodically driven far below threshold, with the generation of regular, sharp pulses (Fig. 3h1) recognizable by the higher order harmonics of the modulation frequency (Fig. 3h2). The RF frequency spectrum (Fig. 3b) shows a peculiar feature of this choice of modulation whose dynamical response presents an approximate symmetry relative to the diagonal (laser output oscillation vs. current modulation). Its origin rests on the large amplitude modulation around threshold which carries with it the generation of sharp pulses (Fig. 3), thus an approximate repetition of the dynamical spectral features at higher harmonics. Physically, the large amplitude driving extends the attractor well below threshold, where memory is maintained through the tiny contribution of the average spontaneous emission ($\Gamma\beta BN^2$ term in eq. (2)). In spite of the smallness of this term, due to the value of $\beta$ in a macroscopic laser, it enables the system to maintain a trajectory even in the region where the photon number is negligibly small and is contributed by the average spontaneous emission. This is the reason why a phase space portrait based on a logarithmic representation (cf. later) is most suited and capable of identifying the trajectories, as opposed to the traditional linear representation in photon number. It also represents a peculiarity of the laser system perturbed by large amplitude modulation around threshold, a point which has, so far, not been broadly studied.

At the resonance frequency ($f_m \approx 2.0\text{GHz}$, close to the laser relaxation oscillations [28]), the pulse amplitude is enhanced (Fig. 3b1), in a similar way to what observed in a mesoscale laser [29]. Here, we remark on the appearance of a spectral subharmonic component (at $1.0\text{GHz}$, Fig. 3b2), consistent with previous observations [21]. Period-doubling is confirmed by the phase portrait (Fig. 3b3–logarithmic vertical scale), where a double-loop is observed. The pulse quality degrades when increasing the modulation frequency and at $f_m = 2.04\text{GHz}$ a period quadrupling bifurcation occurs (Fig. 3b2). Due to the fact that most of the dynamics takes place at very low photon numbers, the phase portrait presents a quadruple-loop structure (Fig. 3b3). Finally, when the modulation frequency reaches to 2.8 GHz, the temporal pulses appear to become random and their amplitude irregular (Fig. 3b1). The spectrum broadens (Fig. 3b2) and the phase space is compatible with a temporal behaviour of a chaotic nature. These simulations show that a period-doubling structure appears when the driving frequency is close to the above-threshold relaxation oscillation even when large amplitude driving brings the laser far below threshold, in transient.

C. Modulation amplitude dependence

We now examine the laser response to variations in modulation amplitude. Anticipating on the content of this subsection, we establish two interesting findings. On the one hand, as observed in the modulation frequency sweep, a cascade of bifurcations is observed as a function of modulation amplitude. The phenomenon is not entirely trivial (as for the frequency modulation), since the laser is modulated across threshold with a very large modulation amplitude: the system is therefore driven very far below threshold for approximately half the cycle and only the (tiny) contribution of the average spontaneous emission enables the survival of trajectories
FIG. 3: Temporal dynamics, RF spectra and phase space trajectories as function of modulation frequency for a semiconductor laser which is modulated at $I_{dc} = 1.14 I_{th}$, and the modulation amplitude $I_m = 1.10 I_{th}$: a1-a3, 1.5 GHz; b1-b3, 2.0 GHz; c1-c3, 2.4 GHz; d1-d3, 2.8 GHz.

(at extremely low photon numbers). On the other hand, we find a very unusual transition between attractors, which we characterize in some detail, at the beginning of the bifurcation cascade. No final conclusions are drawn on this observation and further work is needed to better understand the attractor features in this part of the phase space.

Fig. 4 shows the results (vertical axis in log scale) obtained by sampling the peak values of temporal pulses for the laser modulated at $f_m = 2.8$ GHz and $I_m = 1.14 I_{th}$. The diagram shows a complex structure as a function of modulation amplitude. Some of the finer features of doubling become apparent using the logarithmic representation used here. For $I_m \lesssim 0.37 I_{th}$, the laser follows the external modulation, to then present a period doubling bifurcation which evolves in the shaded area into a complex structure, which we examine in detail in the following (cf. inset). A sustained limit cycle of period $\frac{2}{f_m}$ and amplitude nearly constant persists until $I_m \approx 0.58 I_{th}$, where a further bifurcation appears, followed by a cascade similar to the one observed as a function of modulation frequency (Fig. 2 and 3).

The shaded area highlights the parameter region which presents unusual topological features. Starting from the period-doubled solution (double branch present for $I_m > 0.37 I_{th}$) a complex evolution of the amplitude takes place, illustrated in the inset of Fig. 4. A high-resolution scan shows the succession of peak heights which develops a steady growth until the high branch (main figure) is attained. Further details can be gathered from the phase space picture and temporal evolution of the oscillation (Fig. 5).

As clearly illustrated by the temporal evolution of the signal at one of the amplitude levels ($I_m = 0.4334 I_{th}$, Fig. 5), the Poincaré sections shown in the inset of Fig. 4 display the evolution of the transient which leads from a first, smaller attractor (inner one in Fig. 5) towards a larger attractor (outer trajectory in the phase space representation, Fig. 5). In the example shown, the transient takes more than 500 oscillation cycles to reach the larger trajectory, which has evolved from the smaller one. The transition between the two takes place without hysteresis, thus suggesting the disappearance
FIG. 5: Phase space trajectories (a), temporal dynamics within 300 ns (b) and 20 ns (c) duration for laser is modulated by $0.4334I_{th}$; (d) duration time of transient pulses as function of modulation amplitude.

of the first in favor of the second attractor. Fig. 5 shows a detail of the transition between the two types of oscillation: the smaller one with a broader pedestal and the larger one consisting of sharper and larger peaks. Both oscillations take place at $\frac{f_m}{2}$, as shown by the RF spectra of Fig. 6. Specifically, Fig. 6a and 6c show the spectra for the two different attractors. The one with smaller amplitude (Fig. 6a) presents alternating heights for the frequency components, with a slight preference for the bifurcated ones (odd orders) and a clear reduction in the amplitudes of the harmonics (even the minima drop). The larger amplitude attractor (Fig. 6d) instead is characterized by a remarkable closeness between the $f_m$ and $\frac{f_m}{2}$ components and a much less pronounced amplitude decrease for the higher harmonics. The transient, presented in panels (b) and (c) for two slightly different values of $I_{th}$, possesses more complex spectral features which reflect the transition between the two kinds of oscillation (cf. also inset of Fig. 4 for comparison). The last interesting feature of this unusual attractor transition emerges from the dependence of duration of the transient $\tau$ as a function of $I_{th}$ (Fig. 5d). A sharp increase in the transient duration occurs as the modulation amplitude approaches the value for which the smaller attractor is stable, even though no scaling law appears to be identifiable.

Fig. 7 displays the typical temporal dynamics, RF spectra and phase space trajectories for modulation amplitudes $I_{th} = 0.35I_{th}$, $0.50I_{th}$, $0.60I_{th}$ and $0.677I_{th}$, respectively. In specific, for $I_{th} = 0.35I_{th}$ regular, small amplitude pulses appear (Fig. 7a1) at the modulation frequency producing the limit cycle of Fig. 7a3. The response is, however, nonlinear as highlighted by the RF spectrum (Fig. 7a2). Then when $I_{th} = 0.50I_{th}$, a significantly enhanced pulsing dynamics (Fig. 7b1) with period doubling (Fig. 7b2) emerges. In phase space, we find a closed loop containing a second loop (Fig. 7b3). At $I_{th} = 0.60I_{th}$ the laser is driven further below threshold, generating stronger and sharper pulses (Fig. 7c1) and presenting an additional subharmonic component ($T/4$, Fig. 7c2) and a double-looped phase space portrait (Fig. 7c3). Further increase in modulation amplitude ($I_{th} = 0.677I_{th}$) extends the emission modifications to the peak heights (Fig. 7d1) with the appearance of an additional bifurcation (Fig. 7d2) and four similar structures in the phase portrait (Fig. 7d3).

We now examine the details of a region in which the transition towards a regime compatible with chaotic emission takes place (cf. Fig. 4). For $I_{th} = 0.695I_{th}$ the pulse amplitude sequence starts to become more complex, with a partial decrease of the overall amplitude (Fig. 8a1), maintaining a high subharmonic and harmonic content (Fig. 8a2). The phase space trajectory separates into trajectories which substantially run along one another (Fig. 8a3). A slight increase in modulation amplitude ($I_{th} = 0.712I_{th}$) enhances the low amplitude components (Fig. 8b1) with no substantial modifications of the RF spectrum (Fig. 8b2) or of the phase space portrait, which becomes more broadened (Fig. 8b3). When $I_{th}$ reaches to $0.75I_{th}$, small peaks become more obvious (Fig. 8c1), but subharmonic content starts disappear, indicating by RF spectrum (Fig. 8c2). The phase space plotting indicates more trajectories with similar structure are presented (Fig. 8c3). At $I_{th} = 1.10I_{th}$ higher amplitude, random-looking pulses occur (Fig. 8d1) accompanied by a broad RF spectrum (Fig. 8d2) and a dense phase space filling (Fig. 8d3), compatible with a broadband chaotic emission.
FIG. 7: Temporal dynamics, RF spectra and phase space trajectories as function of modulation amplitude (from \( I_m = 0.35I_{th} \) to \( 0.677I_{th} \)) for a semiconductor laser at \( I_{dc} = 1.14I_{th} \) under 2.8 GHz modulation frequency: a1-a3, \( 0.35I_{th} \); b1-b3, \( 0.50I_{th} \); c1-c3, \( 0.60I_{th} \); d1-d3, \( 0.677I_{th} \).

FIG. 8: Temporal dynamics, RF spectra and phase portraits as function of modulation amplitude (from \( I_m = 0.695I_{th} \) to \( 1.10I_{th} \)) for a semiconductor laser at \( I_{dc} = 1.14I_{th} \) under 2.8 GHz modulation frequency: a1-a3, \( 0.695I_{th} \); b1-b3, \( 0.712I_{th} \); c1-c3, \( 0.75I_{th} \); d1-d2, \( 1.10I_{th} \).

IV. CONCLUSIONS

In conclusion, we presented the existence of a bifurcation structure which appears for a semiconductor laser for across-threshold, large amplitude pump modulation. The features are similar to those of what
emerges for above-threshold simulations, but for the use of a logarithmic representation which allows for a clear trajectory identification at very small photon numbers. In a traditional linear scale phase portrait the below-threshold features would merge together blurring the picture and preventing the identification of distinct attractors. The existence of such trajectories when the laser is driven far below threshold comes from the inclusion of the average contribution of the spontaneous emission which, in concomitant with the memory effects imprinted into the carrier density, permits the prolongation of average, deterministic trajectories even when the laser is (practically) not emitting. In reality, a small quantity of photons is always present in the cavity, as reflected by the current results.

We have also identified the existence of a somewhat unusual transition between attractors of different amplitude but otherwise similar properties, characterized by different phase portraits and RF spectra. The transition takes place as a continuous, bistability-free, temporal transient whose duration is controlled by the amplitude of the sinusoidal component modulating the pump.

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