Turbulent magnetic reconnection in two dimensions

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ABSTRACT

Two-dimensional numerical simulations of the effect of background turbulence on two-dimensional resistive magnetic reconnection are presented. For sufficiently small values of the resistivity (η) and moderate values of the turbulent power (ϵ), the reconnection rate is found to have a much weaker dependence on η than the Sweet–Parker scaling of η^{1/2} and is even consistent with an η independent value. For a given value of η, the dependence of the reconnection rate on the turbulent power exhibits a critical threshold in ϵ above which the reconnection rate is significantly enhanced.

Key words: instabilities – MHD – plasmas – turbulence.

1 INTRODUCTION

Magnetic reconnection is a ubiquitous plasma process whereby oppositely directed magnetic field lines break and rejoin in a different topological configuration – see, e.g. Biskamp (2000) for a review. It is probably the main mechanism behind many spectacular spacecraft and astrophysical phenomena, such as substorms in the Earth’s magnetosphere (Dungey 1961) and solar/stellar (Yokoyama et al. 2001) and accretion disc flares (Goodson, Bohm & Winglee 1999). Reconnection has also been suggested as a possible mechanism behind a number of high-energy astrophysics processes, such as magnetar giant flares (Lyutikov 2003), γ-ray bursts (Giannios & Spruit 2006) and rapid TeV flares in blazars (Giannios, Uzdensky & Begelman 2009). It is believed to lead to heating and non-thermal particle acceleration in astrophysical coronae (Drake et al. 2006) and to play an important role in magnetohydrodynamic (MHD) turbulence and large-scale dynamos (Brandenburg & Subramanian 2005). It is also a key element in sawtooth crashes in tokamaks (Hastie 1998).

Early attempts to understand reconnection in terms of the simplest possible description of the plasma – single-fluid resistive MHD – led to the Sweet–Parker (SP) model (Sweet 1958; Parker 1957) and to the Lundquist number, L/S

where

is the Lundquist number, L is the system size, \( V_A \) is the Alfvén speed and η is the plasma resistivity. In many astrophysical environments, \( S \gg g \) [e.g. \( S \sim 10^{14} \) in solar flares, \( S \sim 10^{18} \) in the interstellar medium (ISM)], resulting in SP reconnection rates which are orders of magnitude slower than that observed in the above mentioned phenomena. The main quest in reconnection research has thus been to identify the key physical mechanisms that are missing from the SP theory and that can explain these fast rates. Following much numerical work, it is now thought that in low-density, collisionless plasmas, non-classical effects [e.g. the Hall effect (Biskamp 2000) or anomalous resistivity (Malyshkin 2005)] can indeed give rise to fast, Petschek-like (Petschek 1964) reconnection. Fast, collisionless reconnection can only take place when the resistive width of the reconnection layer, \( \delta_R \approx L/S^{1/2} \) (Sweet 1958; Parker 1957), is small compared to the relevant kinetic scale [which is frequently the ion collisionless skin-depth (Cassak, Shay & Drake 2005; Yamada et al. 2006; Uzdensky 2007), \( d_i \sim c/\omega_{pi} \) or else \( d_e \sim c/\omega_{pe} \) in the case of pair plasmas]. In many astrophysical situations, for example the solar chromosphere, the ISM, inside stars and accretion discs, and in the high-energy-density environments in central engines of γ-ray bursts (GRB) and core-collapse supernovae (Uzdensky & MacFadyen 2006), the density is so high that the above condition is not satisfied; for example, for the warm ionized ISM, \( \delta_R/d_i \sim 10^3 \) [and even larger for diffuse clouds or molecular clouds (Zweibel 1989)]; in the magnetosphere of the accretion disc in the collapsar scenario of long GRB central engines (Uzdensky & MacFadyen 2006) \( \delta_R/d_e \sim 10^3 \) (here \( d_e \) may be more important than \( d_i \) because, under some circumstances, this is expected to be mostly pair plasma). Therefore, the reconnection layer is collisional and resistive MHD should apply. The prevailing opinion is, however, that in the resistive-MHD case the Petschek mechanism fails (Biskamp 1986; Uzdensky & Kulsrud 2000), and reconnection is negligibly slow, possibly described by the SP theory. Can fast (i.e. η-independent) reconnection happen in these environments?

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Missing from the SP picture is that in nature, most, if not all, environments where reconnection takes place are likely to be turbulent (e.g. Retinò 2007). Can background turbulence significantly accelerate reconnection? The aim of this Letter is to address this question.

Turbulent reconnection has been studied previously, both theoretically (Hameiri & Bhattacharjee 1987; Strauss 1986, 1988; Lazarian & Vishniac 1999; Kim & Diamond 2001) and numerically (Matthaeus & Lamkin 1986; Fan, Feng & Xiang 2004; Smith et al. 2004; Watson, Oughton & Craig 2007; Lapenta 2008; Kowal et al. 2009). Given the complexity of the problem, however, a rigorous theory of turbulent reconnection is not yet available and numerical studies have been limited by resolution constraints. Thus, the role of turbulence in reconnection remains controversial. A pioneering numerical study with decaying two-dimensional turbulence by Matthaeus & Lamkin (1986) suggested that a finite level of broad-band MHD fluctuations can enhance reconnection [see also Matthaeus & Montgomery (1981)], but the very limited computing capabilities employed precluded a clear conclusion about the asymptotic regime $\eta \to 0$. Subsequently, in a highly influential paper, Lazarian & Vishniac (hereafter LV) (Lazarian & Vishniac 1999) suggested that turbulence can greatly accelerate reconnection by enabling multiple reconnection sites in the current sheet. The LV picture is an essentially three-dimensional process, as the multiple reconnections mechanism they envision is topologically prohibited in two dimensions. Recent three-dimensional numerical simulations by Kowal et al. (2009) seem to support the LV model; however, due to computational constraints, only moderate scale separations and fairly strong turbulence (but still sub-Alfvénic compared to the reconnecting field) could be probed.

In two dimensions, the LV mechanism should not work and the expectation has been that topological constraints should prevent significant acceleration of reconnection by turbulence. In this Letter, we present evidence contrary to this belief: our numerical results show that background turbulence can have a dramatic effect on two-dimensional magnetic reconnection, yielding reconnection rates that exhibit a much weaker dependence on $S$ than the SP scaling of $S^{-1/2}$ and may even asymptote to an $S$ independent value as $S \to \infty$. These results call for a further theoretical effort to understand magnetic reconnection in the presence of turbulence.

2 NUMERICAL SETUP

Our aim is to investigate how relatively weak MHD turbulence affects reconnection in two dimensions. The incompressible MHD equations, with an external forcing term $F$, are

$$\partial_t u + u \cdot \nabla u = B \cdot \nabla B - \nabla (p + B^2/2) + \nabla^2 u + F,$$

$$\partial_t B + u \cdot \nabla B = B \cdot \nabla u + \eta \nabla^2 B,$$

where $u$ is the velocity, $B$ is the magnetic field and $p$ is the pressure. The pressure gradient is determined by the incompressibility condition $\nabla \cdot u = 0$ (which sets the plasma density $\rho = \text{const}$), so an energy equation is not required. The resistivity of the plasma is denoted by $\eta$ and the viscosity by $\nu$. We have normalized $p$ and $B$ by $\rho$ and $(4\pi \rho)^{1/2}$, respectively. Expressing the two-dimensional $(x, y)$ incompressible velocity field $(\nabla \cdot u = 0)$ in terms of the stream function $\psi$ and the solenoidal magnetic field $(\nabla \cdot B = 0)$ in terms of the flux function $\psi, u = (-\partial \psi, \partial \phi), B = (-\partial \psi, \partial \phi)$, we can rewrite the above equations in terms of $\psi$ and $\phi$ as follows (Strauss 1976):

$$\partial_t \psi + \nabla^2 \psi + [\phi, \nabla^2 \phi] = \{ \psi, \nabla^2 \psi \} + \nu \nabla^2 \psi + f,$$

$$\partial_t \phi + [\psi, \phi] = \eta \nabla^2 (\psi - \psi_{eq}),$$

where the Poisson brackets are denoted by $\{ \phi, \psi \} = \partial_t \phi \partial_t \psi - \partial_t \partial_t \phi \psi$. These are the equations that we solve with our numerical code. The dimensions of length are set by the box size $(L_x, L_y) = (2\pi, 1.5\pi)$ and the dimensions of magnetic field are set by the equilibrium configuration: we use a tearing-mode unstable equilibrium, $\psi_{eq} = \psi_0 / \cosh^2(x)$, where $\psi_0 = 3 \sqrt{3}/4$, so that the maximum value of the equilibrium magnetic field, $B_0 = (d\psi_{eq}/dx)_{\text{max}} = 1$. The background density is normalized to 1 and the dimensions of velocity are given by setting the Alfvén speed $V_A = B_0 q$. Thus, time is measured in Alfvénic units, $q_L/(2\pi B_0)$. Diffusion of the equilibrium magnetic field is prevented by the addition of an external electric field to the right-hand side of equation (4) – the term $-\eta \nabla^2 \psi_{eq}$. The externally imposed (turbulent) forcing $f = \hat{z} \cdot (\nabla \times F)$ (random, white noise in time) is characterized by two parameters: $\epsilon = (u \cdot F)$, the power input per unit area and $k_f$, the forcing wave number. Note that the forcing is applied only to the momentum equation, and does not break the frozen-flux constraint. The equations are solved in a doubly periodic box using a pseudo-spectral, symplectic algorithm (Loureiro & Hammett 2008) (which can be used in either semi-implicit or explicit timestepping mode: the former is employed in the laminar simulations; the latter in the turbulent ones). Our ultimate goal is to characterize the basic behaviour of the effective reconnection rate $E_{\text{eff}}$ (defined below) in the four-dimensional parameter space: $E_{\text{eff}}(\eta, \nu, \epsilon, k_f)$. However, in the present study, we focus only on two parameters, $\eta = \nu$ (i.e. the magnetic Prandtl number $Pm = \nu/\eta = 1$) and $\epsilon$. Start our simulations without turbulence following a standard tearing mode evolution. After the linear (Furth, Killeen & Rosenbluth 1963), Rutherford (1973) and X-point collapse stages (Loureiro et al. 2005), a thin SP current layer forms between two large magnetic islands. For our parameters, this happens when the value of the reconnected flux at the X-point is $\psi_x \approx 0.4$, whereupon the length of the current sheet is $L \approx 1$, so we define $S = LV_A^2/\eta = 1/\eta$. Since our intention is to focus solely on the effect of turbulence on resistive reconnection, we use the laminar SP configuration with $\psi_x \approx 0.4$ as a starting point for our turbulent runs and switch on the driving term $f$ in equation (3) at this stage. Furthermore, when $\psi_x \approx 0.9$, the SP stage gives way to saturation (Loureiro et al. 2005). Thus, we restrict our analysis of the numerical data to the time interval, where $0.4 \lesssim \psi_x \lesssim 0.9$.

Our runs are characterized by the following parameters. We scan the range from $\epsilon = 3 \times 10^{-3}$ to 0.1, and from $\eta = 10^{-3}$ to $6.5 \times 10^{-5}$. The forcing wave number is $k_f = 10$, giving turbulent motions with characteristic scales a few times smaller than the current sheet length ($l_f \approx 0.3$). For $\eta = 6.5 \times 10^{-5}$, the width of the laminar current sheet is $\delta_{CS} \approx 0.014$, so we have a reasonably good scale separation in these simulations. Resolutions up to 8192$^2$ were used. Such resolutions would not have been possible in three dimensions. Convergence tests were performed on selected runs to ensure that increasing the resolution did not change the results.

3 RESULTS

A typical snapshot of these simulations is shown in Fig. 1. The large-scale reconnecting configuration is manifest; the turbulent motions, which are also clearly visible, are not sufficiently strong.
to destroy it,¹ so the reconnection problem is still well posed. In
the presence of turbulence, the current sheet is no longer straight
as in the laminar case, but now wiggles about its original (i.e.
laminar) location (x = 0): see Fig. 2. We find that secondary islands
(plasmoids) are constantly present, being continuously generated
and expelled from the sheet (whereas the laminar simulations at
most display only one plasmoid). The size of these plasmoids increases
with ϵ, as does the turbulent distortion of the sheet.

We would like to measure the reconnection rate, $E_{\text{eff}}$, in the pre-
sence of turbulence. In the laminar case, $E_{\text{eff}} = d\psi_x/dt$. However,
in turbulent simulations, the wandering of the current sheet makes
the reconnection rate difficult to define. To quantify the effective
reconnection rate, smoothed over the rapid turbulent fluctuations,
we employ the following procedure. Consider the representative
timetraces of $\psi(x = 0, y = 0)$ of Fig. 3. We see that they can be
described by mostly upward fluctuations on top of a monotonically
increasing baseline.² We have checked that the minima of these
curves correspond to maxima of the current. The reason is that,
because of the turbulence, the actual very thin current sheet wanders
around the original (laminar) X-point location – see Fig. 2. As it
deviates from this location, the value of $\psi_x$ increases, since the lam-
inar $\psi$ profile has a minimum at the X-point. When the current sheet
crosses the location of the laminar X-point, the measured current is
maximum, and $\psi$ is again minimum. This suggests that the slope
of a linear fit to the local minima of these timetraces is an accurate
diagnostic of the reconnection rate and that is what we use. Error
bars are estimated by reducing the interval of values of $\psi$ where the
minima are selected by 0.05 at both ends. Although the turbulence
is switched on at $\psi_x \approx 0.4$, we only apply this diagnostic for $\psi \gtrsim
0.5$ so that the turbulence can reach at a saturated stage before we
assess its effect on the reconnection rate. We emphasize that this is

¹ There are regions with qualitatively different turbulence. This is because,
whereas our turbulent forcing is statistically homogeneous, the background
magnetic and velocity fields associated with the large-scale reconnecting
configuration are not.

² Occasional downward fluctuations are due to plasmoids crossing the ($x =
0, y = 0$) location.
on $S$ that is significantly shallower than the SP dependence of $S^{-1/2}$ and may even consistent with a $S$ independent value as $S \to \infty$. Also remarkable is that, common to all finite-$k_f$ sequences, there is a clear transition that does not seem to depend on $\epsilon$, always taking place around $S \approx 2 \times 10^3$. Specifically, at values of $S$ below this, the reconnection rate in the turbulent runs is enhanced over the SP value, but only modestly, and still scales as $\sim S^{-1/2}$. Above this critical value turbulence has a much stronger effect on reconnection.

Shown in Fig. 5 is the dependence of the reconnection rate enhancement factor $E_{eff}/E_{SP}$ on the injected power $\epsilon$. There is a distinct transition at $\epsilon \approx 0.003$. For smaller $\epsilon$, the dependence of the reconnection rate on $\epsilon$ is very weak, roughly consistent $E_{eff}/E_{SP} \sim 1$; for $\epsilon \gtrsim 0.003$, the enhancement factor increases with $\epsilon$ in a way which is consistent both with a power law $E_{eff}/E_{SP} \sim \epsilon^{\alpha}$, where $\alpha \approx 0.15 - 0.25$, and with a logarithmic dependence. Note that the power-law exponent is significantly shallower than the LV prediction of $\epsilon^{1/2}$ for the three-dimensional case.

### 4 Discussion and Conclusion

In summary, we have performed a detailed numerical study of the effect of background turbulence on two-dimensional incompressible resistive magnetic reconnection. Our key finding is that, at sufficiently large values of the Lundquist number and moderate values of the injected (turbulent) power, the scaling of the reconnection rate with $S$ is much weaker than the SP scaling of $S^{-1/2}$ and is, in fact, consistent with asymptotically fast reconnection (independent of $S$; see Fig. 4). Other important conclusions are: (i) the transition from slow to fast reconnection happens at a fixed value of $\epsilon$ of the Lundquist number, $S \approx 2 \times 10^3$ and (ii) the enhancement of the reconnection rate over its laminar (SP) value exhibits a clear threshold in the injected turbulent power, $\epsilon \approx 0.003$, below which turbulence has little influence on the reconnection rate and above which significant enhancement of the reconnection by turbulence exists – see Fig. 5. In our simulations, we find that the rms turbulent velocity $u_{rms}$ (averaged over the simulation box) is closely described by $u_{rms} \approx 2.44\epsilon/k_{f}^{3/2}$. Thus, the Alfvén Mach number associated with the transition to fast reconnection is $M_A \approx u_{rms}/V_A \approx 0.16$ (note that the dependence of our results on $k_f$ has not yet been tested; this can influence the numerical values of both the $S$ and the $\epsilon$ transitions).

Our results imply that a novel mechanism of turbulent enhancement of reconnection exists which is operative already in two dimensions. While our results support the basic claim by LV of turbulent enhancement of reconnection, we emphasize that, because they are two dimensional, they cannot be explained by any present theory, including theirs. One speculative possibility is that the enhancement of the reconnection rate is related to the formation of multiple plasmoids (Louereiro, Schekochihin & Cowley 2007; Loureiro et al. 2009; Samtaney et al. 2009). This hypothesis is strengthened by the fact that the transition from slow to fast reconnection (see Fig. 4) is independent of $\epsilon$ and occurs at a value of $S$ consistent with the critical Lundquist number for plasmoid generation (Loureiro et al. 2005). As $S$ increases, turbulence might facilitate frequent plasmoid formation by the local, transient, enhancement of the tearing mode instability parameter $\Delta^*$ (Furth et al. 1963) (i.e., the local enhancement of the aspect ratio of the current sheet), but whether plasmoids can indeed provide the enhancement of the reconnection rate that we observe is not yet known.

In this study, the effect of viscosity has not been addressed. Our modelling choice has been to keep the magnetic Prandtl number fixed and equal to 1. This is not characteristic of most astrophysical systems, where either $Pm \gg 1$ (e.g. warm interstellar and intracluster medium, as well as some accretion discs) or $Pm \ll 1$ (e.g. stars, planets and liquid-metal laboratory dynamos) (Brandenburg & Subramanian 2005). We have adopted $Pm \approx 1$ to avoid additional complications and to present a clear demonstration of principle – that turbulence can accelerate reconnection in two dimensions. Changing $Pm$ from 1 will result in two effects: (i) different viscous cut-off scale; however, since our results appear to depend only on the outer scale features of the turbulence (i.e. $\epsilon$, and possibly also $k_f$), we do not expect this effect to be significant; (ii) different laminar solution, with a broader current sheet and slower outflows (Park, Monticello & White 1984). This might affect the instability of the current sheet to plasmoids and could potentially impact on the reconnection rate in the presence of turbulence. Investigations of how Prandtl number (and forcing scale) might affect our results are left for future work.

A final caveat is that, in this work, we have neglected two-fluid effects, whose importance in reconnection is now widely recognized.
There are many astrophysical environments where these effects will doubtlessly play an important role (e.g. solar flares); a future publication will assess their importance in turbulent reconnection.

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