Resolving the Hubble tension with Early Dark Energy

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Early dark energy (EDE) offers a solution to the so-called Hubble tension. Recently, it was shown that the constraints on EDE using Markov Chain Monte Carlo are affected by prior volume effects. The goal of this paper is to present constraints on the fraction of EDE, \( f_{\text{EDE}} \), and the Hubble parameter, \( H_0 \), which are not subject to prior volume effects. We conduct a frequentist profile likelihood analysis considering Planck cosmic microwave background, BOSS full-shape galaxy clustering, DES weak lensing, and SH0ES supernova data. Contrary to previous findings, we find that \( H_0 \) for the EDE model is in statistical agreement with the SH0ES direct measurement at \( \leq 1.7 \sigma \) for all data sets. For our baseline data set (Planck + BOSS), we obtain \( f_{\text{EDE}} = 0.087 \pm 0.037 \) and \( H_0 = 70.57 \pm 1.36 \text{ km/s/Mpc} \) at 68\% confidence limit. We conclude that EDE is a viable solution to the Hubble tension.

I. INTRODUCTION

The increasing precision of cosmological measurements revealed a discrepancy known as the Hubble tension (see [1] for a review). The Hubble tension refers to the difference between direct measurements of \( H_0 \) and indirect measurements given a cosmological model. This tension reaches 5\% between the values obtained from the cosmic microwave background (CMB) data from Planck for the A Cold Dark Matter (ΛCDM) model [2], and from the Cepheid-calibrated Type Ia supernovae of the SH0ES project [3].

While systematics are considered as a possible cause for the tension, growing interest has been given to the possibility that this tension points to new physics beyond the ΛCDM model. Among the most well studied proposed solutions to address this tension is the early dark energy (EDE) model [4-6], which introduces a new dark-energy component acting in the early universe.

This model was shown to successfully reduce the tension in \( H_0 \) [7, 8] when analyzed with Planck CMB, Baryon Acoustic Oscillation, Pantheon supernova sample and data from SH0ES [4, 6]. Later it was pointed out in [9, 11] that excluding the SH0ES measurement and including large-scale structure (LSS) probes like galaxy clustering and weak lensing leads to a tight upper limit on the amount of EDE, giving a value of \( H_0 \) compatible with the one from ΛCDM and not being able to solve the Hubble tension. Additionally, it was shown that the so-called \( S_8 \)-tension, a tension in the amplitude of matter clustering, is worsened for the EDE model [9, 11, 12].

However, it was shown in [13], previously hinted in [14, 16] and later confirmed in [17], that the previous analyses of the EDE model using standard Bayesian Markov Chain Monte Carlo (MCMC) methods suffer from marginalization or prior volume effects that can bias the posteriors.

Prior volume effects are common effects in MCMC analyses that appear if the prior is strongly influenced by the prior volume. In the case of the EDE model, the parameter structure of the model leads to large volume differences: When \( f_{\text{EDE}} \) approaches zero, the model reduces to ΛCDM; in this limit, the other parameters of the EDE model are unconstrained, which leads to an enhanced prior volume for ΛCDM and which can drive the posterior towards low fractions of EDE, \( f_{\text{EDE}} \), upon marginalization.

In view of these effects, it was suggested in [13] to use a frequentist profile likelihood. The profile likelihood and the Bayesian MCMC are complementary statistical tools since they address different statistical questions: While MCMC localizes large volumes in parameter space that fit the data well, the profile likelihood is based only on the minimum \( \chi^2 \), i.e. the best fit to the data, regardless of the size of the parameter volume. Therefore, the profile likelihood is reparametrization invariant [18] and, most importantly, is not influenced by prior volume effects.

A profile likelihood of the EDE fraction, \( f_{\text{EDE}} \), resulted in a \( f_{\text{EDE}} = 0.072 \pm 0.036 \) [13] for Planck data [2] and Baryon Oscillation Spectroscopic Survey (BOSS) full-shape likelihood [19, 20], which is considerably higher than the MCMC result for the same data set. A similar analysis with free neutrino mass was performed in [21], with the goal of reducing \( S_8 \), finding a similar constraint (see [22-25] for application to other cases).

The goal of this paper is to provide robust constraints in the value of \( H_0 \) for the EDE model. We will assess the level of compatibility of the model-dependent \( H_0 \) constraints for the EDE model with the SH0ES direct measurement, revealing whether the EDE model can address the Hubble tension.
II. EARLY DARK ENERGY

The EDE model contains a new component in the energy density of the universe that behaves like dark energy right after matter-radiation equality, but that dilutes away after recombination. The inclusion of this extra energy component decreases the sound horizon at the last scattering surface, which leads to an increase in \( H_0 \).

EDE \cite{26, 28} is the name given to a class of models satisfying the above dynamics (for some examples see \cite{21}). In this work, we use the canonical EDE model \cite{5} which is described by a pseudoscalar field with the potential \( V(\phi) = V_0 \left( 1 - \cos(\phi/f) \right)^n \), where \( V_0 = m^2 f^2 \), \( m \) and \( f \) are the explicit and spontaneous symmetry breaking scales, respectively. Based on previous works \cite{5, 5}, we study here the case of \( n = 3 \), which satisfies the condition that the energy density of EDE dilutes faster than the one for matter.

One can relate the parameters of this model to the phenomenological parameters \( f_{\text{EDE}} \) and \( z_c \), where \( f_{\text{EDE}} \) is the maximum fraction of EDE at the critical redshift \( z_c \). This field has a fixed initial value \( \phi_i \), and becomes dynamical near \( z_c \). These parameters together with the initial dimensionless value of the field \( \theta_i \equiv \phi_i/f \), fully describe the EDE model. This phenomenological description is instrumental in making it clear that a higher \( f_{\text{EDE}} \) indicates a higher \( H_0 \); it was shown that \( f_{\text{EDE}} \sim 0.1 \) is necessary to restore concordance in \( H_0 \) \cite{7, 29}.

III. ANALYSIS METHODS

A. Data and modeling

To model the EDE dynamics, we use the public EDE_CLASS_PT code \cite{30}, an extension of the Einstein-Boltzmann solver CLASS \cite{61, 62}, based on CLASS_EDE \cite{9} and CLASS-PT \cite{31, 32}, a code based on the Effective Field Theory (EFT) of LSS \cite{33, 34} that allows to model the galaxy power spectrum up to mildly nonlinear scales.

We consider the following data sets: Planck 2018 TT, TE, EE, low\( \ell \), lensing \cite{2} (referred to as Planck); the BOSS Data Release 12 \cite{34} full-shape power spectrum with a maximum wavenumber \( k_{\text{max}} = 0.25 \) h/Mpc using a consistent window-function normalization, which we implement along the lines of Beutler and McDonald \cite{37} and which corrects an inconsistency present before (referred to as BOSS); a Gaussian likelihood centered on the clustering amplitude of matter, \( S_8 = \sigma_8 \sqrt{\Omega_m/0.3} = 0.767 \pm 0.017 \), measured by the Dark Energy Survey Year 3 analysis (referred to as DES) \cite{35}; and a Gaussian likelihood centered on \( H_0 = 73.04 \pm 1.04 \) measured by SH0ES \cite{3} (referred to as SH0ES).

We sample the \( \Lambda \)CDM parameters \{\( \omega_b, \omega_{\text{cdm}}, \theta_s, A_s, n_s, \tau_{\text{reio}} \} \), the EDE parameters \{\( f_{\text{EDE}}, \log(z_c), \theta_i \} \), along with the Planck and EFT nuisance parameters. Following the convention of the Planck collaboration \cite{2}, we model the neutrino sector by two massless and one massive neutrino species with \( m_\nu = 0.06 \) eV.

B. Statistical inference: MCMC and profile likelihood

We perform both a Bayesian MCMC and a frequentist profile likelihood analysis using MontePython \cite{39} with the Metropolis-Hastings algorithm \cite{40, 41}. We assume the same priors as \cite{42} on the EFT nuisance parameters, and the same priors as \cite{43} on the EDE parameters. We require the Gelman-Rubin convergence criterion \( R - 1 < 0.05 \).

Following the methodology in our previous works \cite{13, 21}, we construct a profile likelihood by fixing the parameter of interest to different values and minimizing \( \chi^2 = -2 \ln \mathcal{L} \) with respect to all other parameters of the model, where \( \mathcal{L} \) denotes the likelihood. The \( \Delta \chi^2 \) as a function of the parameter of interest is the profile likelihood. For the minimization, we adopt a simulated annealing approach based on the method used by Schöneberg et al. \cite{8} (see also \cite{43}). As in our previous work \cite{13}, we construct a confidence interval from the profile likelihood following the prescription by Feldman and Cousins \cite{44}, which extends the procedure by Neyman \cite{45} and is also valid at a physical boundary. We quote confidence intervals obtained from profile likelihoods (MCMC) as bestfit (mean) \( \pm 1 \sigma \).

IV. RESULTS AND DISCUSSION

Fig. 1 and Fig. 2 present the final result of our profile likelihood analysis for \( f_{\text{EDE}} \) and \( H_0 \) for different datasets, with final confidence intervals summarized in Fig. 3 and Table 1.

A. Planck + BOSS full-shape analysis (baseline)

Our baseline data set consists of Planck CMB and BOSS galaxy clustering data (solid teal lines in Figs. 1, 2). The confidence intervals obtained from the profile likelihood are:

\[
f_{\text{EDE}} = 0.087 \pm 0.037, \quad H_0 = 70.57 \pm 1.36 \text{ km/s/Mpc}.
\]

To assess parameter consistency, we report the one-dimensional difference between the bestfits of the two measurements divided by the quadrature sum of the 1 \( \sigma \) errors. We find that \( H_0 \) obtained from the baseline data

\footnote{Using a Gaussian likelihood is an approximation but it was tested in \cite{1} for DES Y1 that the difference to the full likelihood is small for the EDE model.}
The constraints on $f_{\text{EDE}}$ and $H_0$ found here are slightly higher than those from a profile likelihood analysis with the previously widely used BOSS likelihood using an inconsistent normalization ($f_{\text{EDE}} = 0.072 \pm 0.036$ [13]). The consistent window-function normalization leads to higher values of $S_8$. Since $S_8$ is increased in EDE cosmologies compared to ΛCDM, a higher $S_8$ allows for more EDE. This is in agreement with Simon et al. [17], who use MCMC to constrain EDE and find a weaker upper limit on $f_{\text{EDE}}$ with the consistent window-function normalization as compared to the inconsistent normalization.

With the profile likelihood analysis, we also find shifts in other cosmological parameters compared to ΛCDM: the bestfit $n_s$ increases from 0.968 (ΛCDM) to 0.983 (bestfit EDE cosmology, $f_{\text{EDE}} = 0.09$), and $\omega_{\text{cdm}}$ from 0.120 (ΛCDM) to 0.129 ($f_{\text{EDE}} = 0.09$), which can be understood as a compensation of a suppressed early Sachs-Wolfe effect in EDE cosmologies [38]. The most notable change is in $S_8$, which increases from 0.828 (ΛCDM) to 0.840 ($f_{\text{EDE}} = 0.09$), worsening the so-called $S_8$-tension with weak-lensing experiments [49, 50].

B. Baseline + DES

Since EDE cosmologies feature higher $S_8$ [6, 9, 12], including weak lensing measurements into the analysis is

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2 We cite $\chi^2$ and bestfit parameters for the EDE cosmology with

| Data set | $\chi^2$(ΛCDM) | $\chi^2$(EDE) | $\Delta \chi^2$ | $\Delta AIC$ | $f_{\text{EDE}}$ | $H_0$ (consistency w. SH0ES) |
|----------|----------------|--------------|-----------------|--------------|----------------|-------------------------------|
| Planck   | 2774.24        | 2770.72      | -3.52           | +2.48        | 0.072 ± 0.039 | 69.97 ± 1.52 (1.7σ)          |
| Planck+BOSS (base) | 3045.65        | 3039.98      | -5.67           | +0.33        | 0.087 ± 0.037 | 70.57 ± 1.36 (1.4σ)          |
| Baseline + DES | 3052.06        | 3049.13      | -2.93           | +3.07        | 0.061±0.035  | 70.28 ± 1.33 (1.6σ)          |
| Baseline + SH0ES  | 3068.44        | 3042.08      | -26.36          | -20.36       | 0.127±0.023   | 72.12 ± 0.82 (0.69σ)         |

The $\chi^2$ values of the ΛCDM and bestfit EDE models, the difference $\Delta \chi^2 = \chi^2(\text{EDE}) - \chi^2(\text{ΛCDM})$, the Akaike information criterion (AIC), the constraints on $f_{\text{EDE}}$ and $H_0$, and the compatibility with the SH0ES measurement in units of $\sigma$ for the different data sets considered in this work.
an important test for EDE. In this section, we include a Gaussian likelihood from DES$^3$ with $S_8 = 0.776 \pm 0.017$ along with the baseline data set (blue dashed lines in Figs. 1, 2). The profile likelihood analysis yields:

$$f_{EDE} = 0.061^{+0.035}_{-0.034}, \quad H_0 = 70.28 \pm 1.33 \text{ km/s/Mpc}. \quad (2)$$

As expected, we find smaller $f_{EDE}$ and $H_0$ than those from the baseline data set, but $H_0$ is still consistent with SH0ES at $1.6\sigma$. The improvement of the fit compared to ΛCDM, $\Delta \chi^2 = -2.93$, is smaller than for the baseline result. The worsening can be attributed mainly to the contribution from the $S_8$ likelihood. The bestfit $S_8$ for ΛCDM, $S_8 = 0.812$, and the bestfit EDE model $f_{EDE} = 0.06$, $S_8 = 0.817$, are comparable but both are higher than the DES measurement, $S_8 = 0.776$. The AIC shows a mild preference for ΛCDM over EDE, $\Delta \text{AIC} = +3.07$.

The trend of a decreasing $f_{EDE}$ and $H_0$ when including an $S_8$ likelihood is similar as in previous MCMC analyses$[6,9,12]$ but the effect in the profile likelihood is less pronounced since it is not overlaid by prior volume effects. While the MCMC results suggest that EDE is not able to solve the $H_0$ tension, the profile-likelihood result for $H_0$ from the baseline + DES data set is in statistical agreement with the SH0ES measurement.

### C. Baseline + SH0ES

Given that the value of $H_0$ for the EDE baseline data set is consistent with the SH0ES measurement at $1.4\sigma$, it is sensible to combine both data sets. A profile-likelihood analysis of the baseline data set with a Gaussian likelihood centered on the measurement by the SH0ES experiment, $H_0 = 73.04 \pm 1.04$ (yellow dashed lines in Figs. 1, 2), yields:

$$f_{EDE} = 0.127 \pm 0.023, \quad H_0 = 72.12 \pm 0.82 \text{ km/s/Mpc}. \quad (3)$$

This constraint of $H_0$ is consistent with SH0ES at $0.69\sigma$. We find an improvement of fit of the EDE model compared to ΛCDM by $\Delta \chi^2 = -26.36$, where the main contribution to the $\Delta \chi^2$ comes from the SH0ES-$H_0$ likelihood, $\Delta \chi^2_{\text{SH0ES}} = -18.47$. The AIC shows a strong preference for the EDE model over ΛCDM, $\Delta \text{AIC} = -20.36$. The profile likelihood constraints are consistent with previous MCMC constraints including SH0ES data$[6,9,11,15]$, at $< 1\sigma$.

The constraints of $H_0$ and $f_{EDE}$ within the EDE model for the baseline + SH0ES data set are consistent with

\[ \text{FIG. 3: Constraints of } H_0 \text{ within the EDE model for different data sets. The top four errorbars show constraints from the profile likelihood, whereas the bottom errorbar shows the constraint from MCMC. For comparison, the red shaded area corresponds to the 1 } \sigma \text{ and 2 } \sigma \text{ constraint from Planck, the grey shaded area to the 1 } \sigma \text{ and 2 } \sigma \text{ constraint from SH0ES.} \]

### D. Planck-only constraint and comparison to ACT

Lastly, we probe the constraining power of the Planck CMB data alone. We find

$$f_{EDE} = 0.072 \pm 0.039, \quad H_0 = 69.97 \pm 1.52 \text{ km/s/Mpc}. \quad (4)$$

The $H_0$ constraint is consistent with SH0ES at $1.7\sigma$. We find an improvement of fit of $\Delta \chi^2 = -3.52$. This improvement is dominated by the Planck high-$\ell$ likelihood with $\Delta \chi^2_{\text{high-}\ell} = -2.90$. The AIC shows a mild preference of ΛCDM over EDE, $\Delta \text{AIC} = +2.48$.

The relatively high $f_{EDE}$ preferred by Planck in the profile likelihood analysis is interesting in light of the preference for $f_{EDE}$ in an MCMC analysis of Atacama Cosmology Telescope (ACT) CMB data$[52]$. The profile likelihood constraints of $f_{EDE}$ from Planck are consistent at $< 1\sigma$ with MCMC constraints from ACT ($f_{EDE} = 0.091^{+0.020}_{-0.030}$ for the baseline data set in [54], see also [55, 56]). The difference between the results from Planck and ACT from MCMC analyses is likely due to prior volume effects in the MCMC analysis for Planck. The strong preference for the EDE model over ΛCDM that was found for ACT$[54]$ seems to indicate that the constraints from this data set are less affected by prior volume effects.

### V. Conclusion

In this paper, we obtained constraints on the value of $H_0$ for the EDE model, which are not subject to prior volume effects, using a frequentist profile likelihood and assessed the viability of EDE as a solution to the Hubble tension.

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$^3$ We did not include likelihoods for HSC$[51]$ and KiDS$[52]$ simultaneously since there is non-negligible cross-correlation between the data sets. Using a combined weak-lensing likelihood would be an important further check.

$^4$ With the exception of the result from D'Amico et al.$[11]$ for Planck+BAO+SNeIa(Pantheon)+BOSS full-shape power spectrum+SH0ES, which is consistent with our result at $\sim 2\sigma$. 

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It was previously concluded from MCMC analyses that EDE is not able to resolve the $H_0$ tension and simultaneously fit different cosmological data. We find a similar result from the MCMC analysis of our baseline data set (bottom errorbar in Fig. [3]). As was previously shown in [13], MCMC analyses of the EDE model are affected by marginalization or prior volume effects. Therefore, we used the profile likelihood to obtain confidence intervals for $H_0$ (Fig. [3]) and to assess consistency with other measurements and the resolution of the tension.

We assessed whether the data prefers EDE over $\Lambda$CDM using the AIC, which takes into account that the EDE model has three additional parameters compared to $\Lambda$CDM. The AIC shows a mild preference for $\Lambda$CDM for the baseline data set, the baseline + DES and the Planck-only data sets. Only when adding SH0ES, there is a clear preference for the EDE model over $\Lambda$CDM. Therefore, EDE presents a good fit to CMB and LSS but is a clear preference for the EDE model over $\Lambda$CDM. The AIC shows a mild preference for $\Lambda$CDM for baseline data alone is compatible with SH0ES, and interestingly also in agreement with previous works performing an MCMC analysis with ACT data. Considering the relative $\chi^2$ contributions for all likelihoods considered in this work, we find that (apart from SH0ES), the Planck high-$\ell$ likelihood dominates the improvement of fit compared to all other data sets.

For all data combinations, the $H_0$ value obtained with the profile likelihood analysis is consistent with the measurement from SH0ES at $\leq 1.7\sigma$. Therefore, the values of $H_0$ for the EDE model are in agreement with SH0ES. We conclude that the EDE model provides a resolution of the Hubble tension.

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