Minimum irrealism as an indicator of the maximum number of interference fringes

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The elements of reality coined by Einstein, Podolsky, and Rosen (EPR) promoted a series of fundamental discussions involving the notion of quantum correlations and physical realism. The superposition principle applied in the double-slit experiment with matter waves highlights the need for a critical review of the adoption of physical realism in the quantum realm. By using a measure of realism, in this work, we use for the first time quantum correlations of noncommuting observables of a single particle to extract information about the behavior of a quantum system. We investigate the role of single particle position-momentum correlations for the degree of irrealism, the interference pattern, wavelike, and particle-like properties in the double-slit setup with matter waves. We find that there is a time of propagation which minimizes position-momentum correlations and this also generates a minimum in the irrealism. Curiously, we show that the maximum number of interference fringes is related with the minimum of the position-momentum correlations and irrealism.

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I. INTRODUCTION

The Loophole-free violation of Bell’s inequalities leaves no doubt that the classical deterministic notion of an objective reality calls for a critical review\textsuperscript{3,7}. One could say that this idea starts with the celebrated work\textsuperscript{1} of EPR, where they introduced a sufficient condition to describe an element of physical reality. With this definition, they argued that quantum mechanics was incomplete, since it would allow simultaneous elements of reality for incompatible observables. For uncorrelated systems, EPR’s criterion makes direct reference to eigenstates of the observables being measured, since it is related to certainly predict the value of some physical property, without disturbing it, then assuming also the condition of causality in space-time\textsuperscript{8}. On the other hand, Bohr’s approach to that was in terms of his complementarity principle\textsuperscript{8}, which says that the elements of reality of incompatible observables cannot be established in the same experiment, but only through mutually excluding experimental arrangements.

The matter waves quantum interference, a notable aspect of nature in which massive particles exhibit spatial delocalization, completely challenging our classical intuition about physical realism, is also a subject of intense research given its importance to the foundations of quantum theory. Today we know that under different circumstances, the same physical system can exhibit either a particle-like or a wave-like behavior, otherwise known as wave-particle duality\textsuperscript{9,11}. Experiments revealing wave-particle duality in the double-slit were performed by Möllenstedt and Jössen for electrons\textsuperscript{12}, by Zeilinger et al. for neutrons\textsuperscript{13}, by Carnal and Mlynek for atoms\textsuperscript{14}, using diffraction gratings by Schöllkopf and Toennies for small molecules\textsuperscript{15}, by Zeilinger et al. for macromolecules\textsuperscript{16}, and electron double-slit diffraction has been experimentally observed in\textsuperscript{17}. Moreover, the Einstein-Bohr debate about the wave-particle duality in the “floating” double-slit gedanken experiment, has recently been explored in\textsuperscript{18}. Using molecules as slits, this provides an experimental proof and theoretical support showing that Doppler marker eliminates the interference pattern, in corroboration with Bohr’s complementary principle\textsuperscript{18}. Interestingly, this consideration goes against the logic initially advocated by EPR and emphasizes the role of correlations generated in the experimental configuration\textsuperscript{3,6,10}.

Conceptually different from quantum correlations between two systems or two different Hilbert Spaces of a single system (for example, spin and position degrees of freedom), position-momentum correlations are quantum correlations that indicate dependence between the position and the momentum of a single particle. In the case of simple Gaussian or minimum-uncertainty wavepacket solution for the Schrödinger equation for a free particle, the position-momentum correlations at $t = 0$ are zero but they appear for later times\textsuperscript{20,21}. On the other hand, more complex states such as squeezed states or linear combination of Gaussian states can exhibit initial correlations, i.e., correlations that do not depend on the time evolution\textsuperscript{22,23}. It was shown that the existence of position-momentum correlations is related with the phases of the wave function Ref.\textsuperscript{20}. The position-momentum correlations can be used to take information about other quantities in physics. It was shown quali-
tative changes in the interference pattern as a function of the increasing in the position-momentum correlations Ref. [26]. The Gouy phase matter waves is directly related to the position-momentum correlations, as studied by the first time in Refs. [27]. It was observed a relation between the position-momentum correlations and the formation of above-threshold ionization (ATI) spectra in the electron-ion scattering in strong laser fields [28]. More recently, it was shown that the maximum of the position-momentum correlations is related with the minimum number of interference fringes in the double-slit experiment [29].

In this work, we use the facts that the measure introduced by Bilobran and Angelo (BA) \( \Phi \) of the degree of physical (ir)realism of a discrete-spectrum observable for a given quantum system is quantitative, operational, and was further extended for continuous variables in [30], to make formal connections of this quantity, and other measures such as wave-like and particle-like properties in the double-slit experiment with matter waves. Moreover, we verify how the evolution of the position-momentum correlations affects these measures. This paper is structured as follows. In section II, we introduce and briefly discuss the main properties of the measure of the degree of irrealism developed in \( \Phi \). In section III, we model the double-slit experiment with matter waves considering a initially correlated Gaussian wavepacket, which propagates during the time \( t \) from the source to the double-slit and during the time \( \tau \) from the double-slit to the screen. We calculate the wave functions for the passage through each slit using the Green’s function for the free particle to calculate the position-momentum correlations and the irrealism for this system that is a linear combination of the states which passed through each slit. We show that these correlations are minima and the irrealism is also minimum for a given propagation time from the source to the double-slit. In section IV, the irrealism, intensity, visibility, and predictability are analyzed in terms of the maximum and minimum of the position-momentum correlations. In section V we draw our concluding remarks.

II. IRREALISM

In order to discuss the role of position-momentum correlations in the degree of physical realism in the double-slit experiment with matter waves, we review in this section, the measure introduced by Bilobran and Angelo (BA) \( \Phi \), which put forward an operational scheme to assess elements of reality of discrete-spectrum observables in quantum mechanics. The main idea of this measure is constructed under the premise that a measurement establishes the reality of an observable, independently if we have access to the result of this measurement or not. This can be formally stated with the following procedure. They consider a preparation \( \rho \in \mathcal{H}_A \otimes \mathcal{H}_B \) submitted to a protocol of unrevealed measurements (also known as non-selective measurements) of a generic observable \( A = \sum_a a A_a \), with projectors \( A_a = |a\rangle\langle a| \), acting on \( \mathcal{H}_A \). Since the outcome of the measurement is considered not revealed in this protocol, the resulting state is the average over all possible results

\[
\Phi_A(\rho) = \sum_a (A_a \otimes 1_B) \rho (A_a \otimes 1_B) = \sum_a p_a A_a \otimes \rho_B |a\rangle
\]

where \( \rho_B |a\rangle = \langle a|\rho|a\rangle/p_a \) and \( p_a = \text{Tr}[(A_a \otimes 1_B)\rho] \). BA propose to take \( \Phi_A(\rho) \) as a state of reality for \( A \) and \( \rho = \Phi_A(\rho) \) as a formal criterion of reality. Note that this premise of realism also agrees with EPR criterion, since eigenstate preparations are elements of reality for some observable, but this also attempts to generalize EPR in a sense that it also quantifies the degree of realism for mixed states. With that, they compute the degree of irrealism of the observable \( A \) given the preparation \( \rho \) as

\[
\mathcal{I}(\rho) = \mathcal{I}(\Phi_A(\rho)) - S(\rho),
\]

where \( S(\rho) = -\text{Tr}(\rho \log_2 \rho) \) is the von Neumann entropy. Note that, this quantifier is non-negative and vanishes if and only if \( \rho = \Phi_A(\rho) \), thus allowing us to interpret it as an entropic distance between the state \( \rho \) and the state that obeys realism for this observable \( \Phi_A(\rho) \). As discussed in [6, 32], although one could use some other norm, the use of the entropic metric allows one to relate this measure with other quantities of quantum information theory. For example, the above formula can be decomposed as

\[
\mathcal{I}(\rho) = \mathcal{I}(\rho_A) + D_A(\rho),
\]

where \( D_A(\rho) = I_{A:B}(\rho) - I_{A:B}(\Phi_A(\rho)) \) stands for the non-minimized version of the one-way quantum discord (see Refs. [6, 7] for further details). So, the irrealism of \( A \) is the sum of local coherence (that is, the coherence of the reduced state \( \rho_A \)) with quantum correlations associated with measurements of \( A \). Note that, for the single-partite state, \( \rho = \mathcal{H}_A \) or uncorrelated bipartite states, the irrealism measure reduces to the relative entropy of coherence \( \mathcal{I}(\rho_A) \).

This approach gives a prominent role to the notion of information. Indeed, by employing a model of measurement called monitoring [7, 31], it was deduced a formal connection between information and reality in quantum mechanics, developing a complementarity relation to these concepts [6]. By now, this measure has proven relevant in scenarios involving coherence [14], nonlocality [32–34], weak reality [7], which has also a experimentally verification with photonic weak measurements [35], realism-based entropic uncertainty relations [36], random quantum walk [37], Hardy’s paradox [38], and more recently, from the point of view of a generalized resource theory of information [39]. Nevertheless, all of these works are exclusively applied to discrete spectrum observables. To fill this gap, Freire and Angelo presented a framework in [40], which calculates the degree of realism associated with continuous variables such as position and
momentum by explicitly presenting a formalism through which one can quantify the degree of irrealism associated with a continuous variable for a given quantum state, by showing how to consistently discretize the position and momentum variables in terms of operational resolutions of the measurement apparatus. With that, they implemented an operational notion of projective measurement and a criterion of reality for these quantities.

Interestingly, they introduced a quantifier for the degree of irrealism of a discretized continuous variable which, when applied to pure states, exhibits an uncertainty relation to the conjugated pair position-momentum, as

\[ \mathcal{I}(Q|\rho) + \mathcal{I}(P|\rho) \geq \ln(2\pi e), \]  

meaning that quantum mechanics, equipped with Heisenberg’s uncertainty relation, prevents classical realism for conjugated quantities \[30\]. In what follows we discuss our double-slit setup and discuss how we calculate the degree of irrealism for such system.

### III. Irrealism in the Double-Slit Experiment

In this section we model the double-slit experiment as follow. Before reaching the double-slit setup we consider that a coherent correlated in position and momentum Gaussian wavepacket of initial width \( \sigma_0 \) propagates during a time \( t \) before arriving at a double-slit that divides it into two Gaussian wavepackets. These initial correlations are measured by a parameter \( \gamma \), such that, for \( \gamma \neq 0 \) the state is compressed in position and spread in momentum, but acquire a portion of correlations such that the Robertson-Schrödinger uncertainty relation attains the minimum value \( \hbar^2/4 \). After the double-slit, the two wavepackets propagate during a time \( \tau \) until they reach the detection screen where they are recombined and the interference pattern is observed as a function of the transverse coordinate \( x \). Here, we consider a one dimensional problem such that the \( z \) direction is classical and the quantum effects are observed in the \( x \) direction. As a consequence of the free propagation, which decouples the \( x \), \( y \) and \( z \) dimensions for a given longitudinal location, we can write \( z = v_z t \) for the classical direction. The position and momentum of the particle will be correlated and such correlations will be changed by the evolution and the parameter \( \gamma \). The behavior of these correlations enable us to extract some information about the interference pattern. This model is presented in Fig. 1(a) together with illustrations of the behave that will be find in the results. In Fig. 1(a) the initial wavepacket propagates a time \( \tau_{\text{min}} \) from the source to the double-slit which produces at the detection screen the minimum region of overlap and interference fringes. As we will see later on this propagation time corresponds to the maximum position momentum correlations.

The wavefunction at the time when the wave passes through the upper slit (+) or the lower slit (−) is given by \[20\]

\[ \psi_{\pm}(x, t, \tau) = \int_{-\infty}^{\tau} dx_j \int_{-\infty}^{\infty} dx_i G_2(x, t + \tau; x_j, t) F(x_j \mp d/2) \]

\[ \times G_1(x_j, t; x_i, 0) \psi_0(x_i), \]

where

\[ G_1(x_j, t; x_i, 0) = \sqrt{\frac{m}{2\pi i\hbar t}} \exp \left[ \frac{i m (x_j - x_i)^2}{2\hbar t} \right], \]

\[ G_2(x, t + \tau; x_j, t) = \sqrt{\frac{m}{2\pi i\hbar \tau}} \exp \left[ \frac{i m (x - x_j)^2}{2\hbar \tau} \right], \]

\[ F(x_j \mp d/2) = \frac{1}{\sqrt{\beta \sqrt{\pi}}} \exp \left[ -\frac{i m (x_j \mp d/2)^2}{2\beta^2} \right], \]

FIG. 1: Sketch of the double-slit experiment. A correlated Gaussian wavepacket of transverse width \( \sigma_0 \) propagates during a time \( t \) before attaining the double-slit and during a time \( \tau \) from the double-slit to the screen. The slit transmission functions are taken to be Gaussian of width \( \beta \) and separated by a distance \( d \). In (a) the initial wavepacket propagates a time \( \tau_{\text{min}} \) from the source to the double-slit which produces at the detection screen the maximum region of overlap and interference fringes. In (b) the initial wavepacket propagates a time \( \tau_{\text{max}} \) from the source to the double-slit which corresponds to the minimum region of overlap and interference fringes.
and
\[
\psi_0(x_i) = \frac{1}{\sigma \sqrt{\pi}} \exp \left[ -\frac{x^2}{2\sigma^2} + i\gamma x_i^2 \right]. \tag{9}
\]

The kernels \(G_1(x_j, t; x_i, 0)\) and \(G_2(x, t+\tau; x_j, t)\) are the free propagators for the particle, the function \(F(x_j + d/2)\) describes the double-slit transmission functions which are taken to be Gaussian of width \(\beta\) separated by a distance \(d\); \(\sigma_0\) is the transverse width of the first slit, where the packet was prepared, \(m\) is the mass of the particle, \(t(\tau)\) is the time of flight from the first slit (double-slit) to the double-slit (screen). The parameter \(\gamma\) ensures that the initial state is correlated in position and momentum. In fact, we obtain for the initial state \(\psi_0(x_i)\) that the position-momentum correlations is \(\sigma_{xp} = \hbar \gamma/2\). For \(\gamma = 0\) we have the simple uncorrelated Gaussian wavepacket and for \(\gamma < 0\) we have a contractive state. In order to obtain analytic expressions for the intensity, visibility, predictability and specially for the position-momentum correlations in the screen of detection we use a Gaussian transmission function instead of a top-hat transmission function because a Gaussian transmission function represents a good approximation to the experimental reality and also because it is mathematically simpler to treat than a top-hat transmission function.

The corresponding wavefunction for the propagation through the upper slit was previously obtained in Ref. [40] and it is given by
\[
\psi_+(x, t, \tau) = \frac{1}{\sqrt{B_\gamma \sqrt{\pi}}} \exp \left[ -\frac{(x - D_\gamma/2)^2}{2B_\gamma^2} \right]
\times \exp \left[ \frac{imx^2}{2\hbar R_\gamma} + i\Delta x + i\theta_\gamma + i\mu_\gamma \right]. \tag{10}
\]
where the wavefunction parameters are displayed in the Appendix 1.

In order to obtain the expressions for the wave function \(\psi_-(x, t, \tau)\) passing through the lower slit, we just have to substitute the parameter \(d\) by \(-d\) in the expressions corresponding to the wave passing through the upper slit. Here, the parameter \(B_\gamma(t, \tau)\) is the beam width for the propagation through one slit, \(R_\gamma(t, \tau)\) is the radius of curvature of the wavefronts for the propagation through one slit, \(b_\gamma(t)\) is the beam width for the free propagation and \(r_\gamma(t)\) is the radius of curvature of the wavefronts for the free propagation. \(D_\gamma(t, \tau)\) is the separation between the wavepackets produced in the double-slit. \(\Delta_\gamma(t, \tau) x\) is a phase which varies linearly with the transverse coordinate. \(\theta_\gamma(t, \tau)\) and \(\mu_\gamma(t, \tau)\) are the time dependent phases and they are relevant only if the slits have different widths. \(\mu_\gamma(t, \tau)\) is the Gouy phase for the propagation through one slit. Different from the results obtained in Ref. [24], all the parameters above are changed by the correlation parameter \(\gamma\). \(\tau_0 = m\sigma_0^2/\hbar\) is one intrinsic time scale which essentially corresponds to the time during which a distance of the order of the wavepacket extension is traversed with a speed corresponding to the dispersion in velocity. It is viewed as a characteristic time for the “aging” of the initial state [26,29] since it is a time from which the evolved state acquires properties completely different from the initial state.

Now we are in position to discuss how we evaluate the irrealism for the wavefunction at the detection screen. First we note that, for single partite pure states, it follows that \(S(\rho) = 0\), and the von Neumann entropy of this state when applied the unread measurement map reduces to \(S(\Phi_{Q(p)}(\rho)) = H_{Q(p)}\), where \(H_{Q(p)}\) is the Shannon entropy associated with probability distributions for the variables \(q(p)\) (see [31] for more details). To construct the Shannon distribution of the continuous variable \(x(p)\), we follow the work [11] to write the discretized probability distribution of a position measurement \(p_m\) in terms of experimental resolutions for positions measurement of \(\delta q\) as
\[
p_m = \int_{m\delta q}^{(m+1)\delta q} dx \rho(x, t, \tau, \gamma), \tag{11}
\]
where \(\rho(x, t, \tau, \gamma) = \psi(x, t, \tau, \gamma) \times \psi^*(x, t, \tau, \gamma)\) is the probability density in position and
\[
\psi(x, t, \tau, \gamma) = \frac{\psi_+(x, t, \tau) + \psi_-(x, t, \tau)}{\sqrt{2 + 2 \exp \left[ -\frac{B_\gamma^2}{2\hbar^2} - \Delta_\gamma^2 B_\gamma^2 \right]}}. \tag{12}
\]
the normalized wavefunction in the screen of detection.

Note that we made a discretization of the probability distribution in terms of the experimental resolution \(\delta q\) which is assumed to be a constant. With that, we can calculate the Shannon entropy as
\[
H_Q(t, \tau, \gamma) = - \sum_{m=-\infty}^{\infty} p_m \ln p_m. \tag{13}
\]
Now it is straightforward to calculate the corresponding degree of irrealism for the position of the state [12], explicitly by
\[
\mathcal{I}(Q|\rho) = H_Q(t, \tau, \gamma) = - \sum_{m=-\infty}^{\infty} p_m \ln p_m. \tag{14}
\]

For the momentum in terms of the wavevector \(k\), we follow the same strategy to write
\[
\mathcal{I}(P|\rho) = H_P(t, \tau, \gamma) = - \sum_{n=-\infty}^{\infty} p_n \ln p_n, \tag{15}
\]
with
\[
p_n = \int_{n\delta k}^{(n+1)\delta k} dk \tilde{\rho}(k, t, \tau, \gamma), \tag{16}
\]
where \(\tilde{\rho}(k, t, \tau, \gamma)\) is obtained with the corresponding Fourier transform of \(\psi(x, t, \tau, \gamma)\), and \(p_n\) is the discretized
probability for momentum measurements with resolution \(\delta k\).

Now, we intend to explore how the time evolution of the position-momentum correlation for an initial contractive state affects the irrealism for the discretized position and momentum of the particle in terms of the respectively experimental resolutions.

### A. Minimum position-momentum correlations coincides with the minimum irrealism

In this section we calculate the position-momentum correlations and the irrealism in the screen of detection. We observe that the position-momentum correlations have a minimum and a maximum as a function of the propagation time \(\tau\) for a negative value of the correlation parameter \(\gamma\) (contractive state). For positive or null values of \(\gamma\) such correlations have only a maximum they do not have a minimum. The relationship between the minimum number of interference fringes as well as the wave and particle properties with the maximum of the position-momentum correlations was obtained in Ref. \[29\].

For the normalized wavefunction Eq. (12) we calculate the position-momentum correlations and obtain

\[
\sigma_{xp}(t,\tau,\gamma) = \frac{mB^2}{2R_\gamma} + \frac{(mD^2/R_\gamma)}{4 + 4\exp[-\frac{D^2}{4B^2} - \Delta^2B^2_\gamma]} - \frac{\hbar\Delta_g D_\gamma}{2} - \frac{(m\Delta^2 B^2_\gamma/R_\gamma)}{1 + \exp[\frac{D^2}{4B^2} + \Delta^2B^2_\gamma]} \tag{17}
\]

In the following, we plot the curves for the position-momentum correlations and irrealism as a function of the time \(t/\tau_0\) for neutrons. The reason to consider neutrons relies in their experimental reality, which is most close to our model for interference with completely coherent matter waves, although we still have loss of coherence as discussed in Ref. \[42\]. We adopt the following parameters: mass \(m = 1.67 \times 10^{-27}\) kg, initial width of the packet \(\sigma_0 = 7.8\) \(\mu\)m (which corresponds to the effective width of \(2\sqrt{2}\sigma_0 \approx 22\) \(\mu\)m), slit width \(\beta = 7.8\) \(\mu\)m, separation between the slits \(d = 125\) \(\mu\)m and de Broglie wavelength \(\lambda = 2\) nm. These same parameters were used previously in double-slit experiments with neutrons by A. Zeilinger et al. \[13\]. In Fig. 2 we show the plot of the position momentum correlations as a function of \(t/\tau_0\) for \(\tau = 18\tau_0\) and for a initial contractive state \(\gamma = -1.0\). As we can observe the correlations have a point of minimum and a point of maximum which we calculate and obtain, respectively, \(t_{min} = 0.49\tau_0\) and \(t_{max} = 1.36\tau_0\). A result very interesting is that despite the difference for the times of maximum and minimum correlations is smaller than a unity in terms of \(\tau_0\), these values of time produce interference fringes completely distinct as we will see later on. In Fig. 3 we show the plot of the irrealism in position as a function of \(t/\tau_0\) for two values of experimental resolution \(\delta q\), \(\tau = 18\tau_0\) and \(\gamma = -1.0\). We consider the experimental resolution of \(\delta q = 2.5\) \(\mu\)m for the left curve and \(\delta q = 2.5\) nm for the right one. The irrealism is minimum at the same time for which the correlations are minimum, i.e., \(t_{min} = 0.49\tau_0\). We also obtain that the time for the minimum irrealism does not change when we increase the experimental resolution. On the other hand, when the experimental resolution is increased the minimum irrealism also increases. In Fig. 4 we show the plot of the irrealism in momentum as a function of \(t/\tau_0\) for two values of experimental resolution \(\delta k\), \(\tau = 18\tau_0\) and \(\gamma = -1.0\). We consider \(\delta k = 140\) \(m^{-1}\) for the left curve and \(\delta k = 0.14\) \(m^{-1}\) for the right one. The irrealism in momentum has a minimum which also coincides with the minimum of the correlations. We can observe a similar behave as the irrealism in position, i.e., the time for the minimum irrealism in momentum does not change with the experimental resolution and the minimum irrealism increases when the experimental resolution is increased. Now, it is interesting to discuss what means the minimum correlations in terms of the region of overlap between
the two wavepackets generated in the double-slit. The correlations at the time of minimum is governed by the first term of equation (17), i.e., \( \sigma_{xp}(t_{\text{min}}) \approx mB_\gamma^2/2R_\gamma \). Therefore, we have \( B_\gamma^2(t_{\text{min}}) \gg D_\gamma^2(t_{\text{min}}) \). Since \( B_\gamma(t, \tau) \) is the width of the wavepacket and \( D_\gamma(t, \tau) \) the separation between the wavepackets at the screen, we have a bigger region of overlap between the two packets for the minimum correlations which also means the minimum irrealism. Therefore, when the superposition created in the double-slit is localized in the detection screen to have a large region of overlap which means that one cannot distinguish the packets in the superposition the irrealism tends to be minimum. These results are also reflected in the interference pattern as we can observe in the next section.

IV. MINIMUM IRREALISM AND THE MAXIMUM NUMBER OF INTERFERENCE FRINGES

Here, we calculate the relative intensity, visibility and predictability to analyze the interference pattern as well as the wave and particle properties from the knowledge of the minimum of the position-momentum correlations and irrealism. Such analysis shows the role of the single particle position momentum correlations in understanding the quantum behavior of a particle in the double-slit experiment. The minimum correlations which means minimum irrealism is characterized by a maximum number of interference fringes. We also study the interference pattern in the time of maximum of the correlations.

The intensity on the screen is given by

\[
I(x, t, \tau) = F(x, t, \tau) + F(x, t, \tau) \frac{\cos(2\Delta, x)}{\cosh(D_\gamma x)} ,
\]

where

\[
F(x, t, \tau) = I_0 \exp \left[ -\frac{x^2 + (D_\gamma x)^2}{B_\gamma^2} \right] \cosh \left( \frac{D_\gamma x}{B_\gamma} \right) .
\]

The first term in equation (18) is the single slits envelope and the second term is the interference (29).

In Fig. 4 we show half of the symmetrical plot for the relative intensity as functions of \( x \) for an initial contractive state. In Fig. 5, we consider the time for which the correlations are minima \( t_{\text{min}} = 0.49\tau_0 \) and in Fig. 5, we consider the time for which they are maxima \( t_{\text{max}} = 1.36\tau_0 \). We fixed the propagation time from the double-slit to the screen in \( \tau = 18\tau_0 \). We observe a large number of interference fringes associated with the minimum correlations and minimum irrealism and a small number of interference fringes associated with the maximum correlations. The maximum of the correlations is closed to the maximum irrealism but they do not coincide.

The knowledge of both “particle” and “wave” in an interferometric experiment is given by the Greenberger and Yasin expression \( P(\theta)^2 + V(\theta)^2 \leq 1 \), where \( P \) stands for particle property and \( V \) for wave property. The parameter \( (\theta) \) is used to vary from full particle to full wave knowledge preserving the general case in which one can have considerable knowledge of both. The equality is ensured for pure quantum mechanical states and the inequality for mixed states (43). We calculate the predictability and visibility for our experimental setup and obtain

\[
P(x) = \frac{\left| \psi_1 \right|^2 - \left| \psi_2 \right|^2}{\left| \psi_1 \right|^2 + \left| \psi_2 \right|^2} = \tanh \left( \frac{D_\gamma x}{B_\gamma^2} \right) ,
\]

and

\[
V(x) = \frac{I_{\text{max}} - I_{\text{min}}}{I_{\text{max}} + I_{\text{min}}} = \frac{1}{\cosh \left( \frac{D_\gamma x}{B_\gamma^2} \right)} ,
\]

where \( I_{\text{max}} \) is the intensity for \( \cos(2\Delta, x) = 1 \) and \( I_{\text{min}} \) is the intensity for \( \cos(2\Delta, x) = -1 \) (29). Similar results were obtained previously in Ref. (44).

In Fig. 6, we show half of the symmetrical plot of the visibility (solid line) and predictability (dotted line)
as functions of $x$ for the time for which the correlations and irrealism are minimum and in Fig. 6 we show half of the symmetrical plot of the visibility (solid line) and predictability (dotted line) as functions of $x$ for the time for which the correlations are maximum. As before, we fixed the propagation time from the double-slit to the screen in $\tau = 187\gamma$.

These results show clearly the relationship between the minimum and maximum position-momentum correlations and irrealism with the number of interference fringes and the wave and particle properties in the double-slit experiment. The maximum number of interference fringes occurs when the correlations in the screen of detection are minima and the minimum number when such correlations are maxima. The wave property is predominant in a larger region of the axis $x$ when the correlations are minima and the particle property is dominant when the correlations are maxima as we can observe by comparing the curves of visibility and predictability for the minima and maxima correlations. Therefore, the knowledge of the correlations tells us if the particle sent by the source will behave more as wave-like or particle-like on the screen in a given value of $x$, excluding only the values near $x = 0$.

We can also understand why the visibility (predictability) dominate when the correlations are minima (maxima) by observing that there is a value

$$x_0 = \ln(1 + \sqrt{2})B_\gamma^2/D_{\gamma}$$

for which the wave and particle properties are equal, i.e., $P_\gamma^2(x_0) = V_\gamma^2(x_0) = 0.5$. For the time of minimum correlations (irrealism) $t_{\text{min}}$ we have a large value for $x_0$ since for this time the region of overlap is large and is dominated by the width of the wavepacket followed by a small separation of the wavepackets, i.e., $B_\gamma(t_{\text{min}}) \gg D_\gamma(t_{\text{min}})$. This large region of overlap is associated with localization of the superposition state which is difficult to distinguish each superposition state. In this sense localization is associated with minimum irrealism. On the other hand, for the time of maximum correlations $t_{\text{max}}$ we have a small value of $x_0$ since for this time the region of overlap is small and is dominated by the separation between the wavepackets, where $D_\gamma(t_{\text{max}}) \gg B_\gamma(t_{\text{max}})$. Here, we can say that as more the superposition state is delocalized, more irrealism we have for that system.

\section{Conclusions}

The purely epistemic uncertainty present in classical statistical physics does not reflect that we can not assume realism to the physical properties that describe these systems. Besides our statistical ignorance, these properties are already predetermined before any measurement. On the other hand, quantum superposition, which lies at the heart of quantum theory, and gives rise to many of its contra-intuitive interpretations, since not only microscopic but even molecules exhibits this condition, points out that reality seems to be in suspension in such cases, that is, physical properties do not have well-defined values that supports physical realism. The ontological uncertainty of quantum mechanics implies that the classical notion of realism is forbidden in general for two non-commuting observables such as position and momentum.

In this work, we have taken a step further on this issue by exploring the connection between irrealism and position-momentum correlations in the intensity, visibility, and predictability of the wavepackets interference. We saw that, although apparently contradictory, we have a maximum interference accompanied by a minimum of irrealism. This is so because we have a bigger region of overlap between the two packets for the minimum correlations, which indicates that at the same time when the wavepackets are closer, and more likely to be real, this helps to create interference, in reference to typical wave behavior. However, it is important to note that Realism is never fully defined because the measure does not go to zero at any time, indicating the quantum nature of this mechanism of interference and the lack of reality of that systems, in contrast to interference experiments in classical optics which exhibits interference but has its physical properties well-defined before any measurements. It is also important to note that increasing the experimental resolution, increases the corresponding value of irrealism, but the behavior of the temporal evolution stills the same, the propagation time which minimizes the correlations and the irrealism in position or in momentum is experimental resolution independent. Also, we saw that the behavior of the irrealism for position and momentum variables is similar, this is due to the fact that, besides being non-commutative, both properties are able to encode the path taken by the system in this experimental setup.

Finally, the results developed here is important to propose a successful interference experiment, indicating how to define values for parameters that can produce a maximum number of fringes with better visibility. This is
also important from the perspective of the foundations of quantum theory, indicating that there is a minimum value for the irrealism of the studied system which produces the maximum interference in the context of matter waves, which allows us to reinterpret the double-slit experiment by employing a notion of a state with fundamental physical indefiniteness, instead of thinking of a particle traveling as a definite wave. We hope that our results encourage experimentalists to implement a direct measurement of irrealism to matter waves.

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VI. APPENDIX 1: WAVEFUNCTION PARAMETERS

Here we displayed the wavefunction parameters of Eq. (10) corresponding to the free propagation of a correlated Gaussian wavepacket [10].

\[
R_\gamma(t, \tau) = \tau \frac{\left(\frac{1}{\beta^2} + \frac{1}{\tau^2}\right)}{\sqrt{\frac{1}{\beta^2} + \frac{1}{\tau^2} + \frac{\sigma^2}{\sigma^2(1+\gamma^2+2\sigma^2\beta)}}},
\]

\[
C = \left[\frac{\tau_0^2 + t\tau_0^2}{\tau} + \tau_0^2 \gamma^2 + \frac{\tau_0^2 \gamma^2}{\tau} + \frac{t^2 \tau_0^2 \gamma^2}{\tau} + \frac{2\tau_0^2 \sigma^2}{\beta^2}\right],
\]

\[
B_\gamma^2(t, \tau) = \left(\frac{1}{\beta} + \frac{1}{\tau}\right)^2 + \frac{\sigma^2}{\sigma^2} \left(\frac{1}{\beta} + \frac{1}{\tau}\right)^2,
\]

\[
\Delta_\gamma(t, \tau) = \frac{\tau \sigma_0^2 d}{2\tau_0 \beta^2 B^2},
\]

\[
D_\gamma(t, \tau) = d \frac{1 + \frac{\tau}{\gamma}}{1 + \frac{\tau^2}{\gamma^2}},
\]

\[
\theta_\gamma(t, \tau) = \frac{md^2 \left(\frac{1}{\beta} + \frac{1}{\tau}\right)}{8\hbar\beta^4 \left(\frac{1}{\beta^2} + \frac{1}{\tau^2}\right)^2 + \frac{m^2}{\beta^2} \left(\frac{1}{\beta} + \frac{1}{\tau}\right)^2},
\]

\[
\mu_\gamma(t, \tau) = -\frac{1}{2} \arctan \left[\frac{t + \tau \left(1 + \frac{\sigma^2}{\beta^2}\right) + \frac{t\gamma}{\tau\sigma^2}}{\tau_0 \left(1 - \frac{t\gamma}{\tau_0\sigma^2}\right) + \gamma (t + \tau)}\right],
\]

\[
b_\gamma(t) = \frac{\sigma_0}{\tau_0} \left[\frac{t^2 + \tau_0^2 + 2t\tau_0 \gamma + t^2 \gamma^2}{\gamma}\right]^{\frac{1}{2}},
\]

\[
r_\gamma(t) = \frac{(t^2 + \tau_0^2 + 2t\tau_0 \gamma + t^2 \gamma^2)}{[t(1 + \gamma^2) + \gamma \tau_0]},
\]

and

\[
\tau_0 = \frac{m\sigma_0^2}{\hbar}.
\]

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