Low-energy Dipole Excitations in Nuclei at the $N = 50, 82$ and $Z = 50$ Shell Closures as Signatures for a Neutron Skin.

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Abstract. Low-energy dipole excitations have been investigated theoretically in $N=50$ $^{88}$Sr and $^{90}$Zr, several $N=82$ isotones and the $Z = 50$ Sn isotopes. For this purpose a method incorporating both HFB and multi-phonon QPM theory is applied. A concentration of one-phonon dipole strength located below the neutron emission threshold has been calculated in these nuclei. The analysis of the corresponding neutron and proton dipole transition densities allows to assign a genuine pattern to the low-energy excitations and making them distinct from the conventional GDR modes. Analyzing also the QRPA wave functions of the states we can identify these excitations as Pygmy Dipole Resonance (PDR) modes, recently studied also in Sn and $N=82$ nuclei. The results for $N = 50$ are exploratory for an experimental project designed for the bremsstrahlung facility at the ELBE accelerator.

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1. Introduction

The progress in nuclear structure physics is closely connected with the extended experimental possibilities of the new facilities [1]. In particular, the experiments with rare isotope beams give us the opportunity to investigate nuclei far from stability. One of the most interesting results was the discovery of a new dipole mode at low-excitation energy in nuclei with high isospin asymmetry [1, 2, 3]. Typically, one observes in nuclei with a neutron excess, $N > Z$, a concentration of electric dipole strength of predominantly electric character at or close to the particle emission threshold. Since this bunching of $1^-$ states resembles spectral structures, otherwise known to indicate resonance phenomena, these states have been named Pygmy Dipole Resonance (PDR). However, only a tiny fraction, less than 1% of the total Thomas-Reiche-Kuhn energy weighted dipole sum rule strength is found in the PDR region. These studies have various astrophysical applications, e.g., explosive nucleosynthesis and the neutron stars.

Here, we present our investigations on the dipole excitations in many nuclei from N=50, 82 and Z=50 regions. For this purpose a method based on Hartree-Fock-Bogoljubov (HFB) description of the ground state is applied [6]. The excited states are calculated with the Quasiparticle-Phonon Model (QPM) [4].

2. The Model

The model Hamiltonian [4]:

$$ H = H_{MF} + H_{M}^{ph} + H_{SM}^{ph} + H_{M}^{pp} $$

is built from the HFB term $H_{MF} = H_{sp} + H_{pair}$ containing two parts: $H_{sp}$ describes the motion of protons and neutrons in a static, spherically-symmetric mean-field, taken as a Wood-Saxon (WS) potential. The parameters of the WS potential are derived from fully microscopic HFB calculations of the ground state [5, 6, 9], separately for every nucleus under consideration, which is different from the standard QPM scheme given in [4]; $H_{pair}$ accounts for the monopole pairing between isospin identical particles with coupling constants extracted from the data [7]. The last three terms present the residual interaction $H_{res} = H_{M}^{ph} + H_{SM}^{ph} + H_{M}^{pp}$ and refer to the multipole-multipole $H_{M}^{ph}$ and spin-multipole interactions $H_{SM}^{ph}$ of isoscalar and isovector type in the particle-hole and multipole pairing $H_{M}^{pp}$ in the particle-particle channels.

In the QPM the residual interaction is taken in a separable form

$$ R_{\lambda}(r_1, r_2) = \kappa_{\lambda} R_{\lambda}(r_1) R_{\lambda}(r_2), $$

where $R_{\lambda}(r)$ is a radial form factor, which is usually chosen as $r^\alpha$; $\kappa_{\lambda} = (\kappa_{\lambda}^0, \kappa_{\lambda}^1)$ are empirical isoscalar and isovector coupling constants, which are obtained by a fitting procedure [8].

The nuclear excited states are constructed of Quasiparticle-Random-Phase-Approximation (QRPA) phonons, defined as a linear combination of two-quasiparticle
creation and annihilation operators as follows:

\[ Q_{\lambda \mu}^+ = \frac{1}{2} \sum_{\tau} \sum_{jj'} (\psi_{jj'}^{\lambda i} A_{\lambda \mu}^+(jj') - \varphi_{jj'}^{\lambda i} A_{\lambda \mu}(jj')) , \]  

where \( A_{\lambda \mu}^+ \) and \( A_{\lambda \mu} \) are time-forward and time-backward operators, coupling proton and neutron two-quasiparticle creation or annihilation operators to a total angular momentum \( \lambda \) with projection \( \mu \) by means of the Clebsch-Gordan coefficients \( C_{jmjm'm'}^{\lambda \mu} = \langle jm|\lambda\mu\rangle \). Correspondingly,

\[ A_{\lambda \mu}^+(j_1 j_2 q) = \left[ \alpha_{j_1 j_2 q}^+ \alpha_{j_1 j_2 q}^+ \right]_{\lambda \mu} = \sum_{m_1 m_2} C_{j_1 m_1 j_2 m_2}^{\lambda \mu} \alpha_{j_1}^+ \alpha_{j_2}^+ \]  

The QRPA phonon operators obey the equation of motion

\[ \left[ H, Q_{\alpha}^+ \right] = E_{\alpha} Q_{\alpha}^- , \]  

which solves the eigenvalue problem, giving the excitation energies \( E_{\alpha} \) and the wave functions of the excited states, defined by the time-forward and time-backward amplitudes \( \psi_{jj'}^{\lambda i} \) and \( \varphi_{jj'}^{\lambda i} \), respectively.

The spacial structure of a nuclear excitation becomes accessible by analyzing the one-body transition densities \( \delta \rho(\vec{r}) \), which are the non-diagonal elements of the nuclear one-body density matrix. In the QRPA theory the one-phonon transition density is given by the coherent sum over two-quasiparticle transition densities entering in the wave function of a phonon by the relation:

\[ \rho_{\lambda i}(r) = \sum_{j_1 \geq j_2} \rho_{j_1 j_2}^{(\lambda)}(r) g_{j_1 j_2}^{\lambda i} , \]

where the radial parts are given by the radial single particle wave functions and reduced matrix elements

\[ \rho_{j_1 j_2 q}^{\lambda}(r) = R_{j_1 q}(r) R_{j_2 q}(r) \frac{1}{\lambda} \langle j_1 | i^{\lambda} Y_{\lambda} | j_2 \rangle , \]

with \( \lambda = \sqrt{2\lambda + 1} \). The BCS quasiparticle properties and QRPA state amplitudes are contained in

\[ g_{j_1 j_2}^{\lambda i} = \frac{\psi_{j_1 j_2}^{\lambda i} + \varphi_{j_1 j_2}^{\lambda i}}{1 + \delta_{j_1 j_2}} (u_{j_1} v_{j_2} + u_{j_2} v_{j_1}) . \]

3. Application to PDR Excitations

The calculated neutron and proton ground state densities are presented in Fig.1 for \( N=50 \) and \( Z=50 \) nuclei. Of special importance for our investigations are the surface regions, where the formation of a skin takes place. For the \( N=50 \) nuclei, the neutron skin decreases from \( ^{88}\text{Sr} \) to \( ^{90}\text{Zr} \), when the number of the protons increases. In the \( Z=50 \) Sn isotopes we find, that for \( A \geq 106 \) the neutron distributions begin to extend beyond the proton density and the effect continues to increase with the neutron excess, up to \( ^{132}\text{Sn} \).

\( \dagger \) The time reversed operator is defined as \( \tilde{A}_{\lambda \mu} = (-)^{\lambda-\mu} A_{\lambda - \mu} \).
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Thus, these nuclei have a neutron skin. The situation reverses in $^{100-102}$Sn, where a tiny proton skin appears at the nuclear surface. We find, that the properties of the ground states reflect directly off the low-energy dipole excitations. From QRPA calculations in $N=50$, $N=82$ [10] and $^{112-132}$Sn [6, 9] nuclei a sequence of low-lying one-phonon dipole states of almost pure neutron structure, located below the particle threshold is obtained. The analysis of the dipole transition densities at $E^* \leq 8$ MeV in $Z=50$ (Fig.4 left), $N=82$ (Fig.3 left) and at $E^* \leq 9$ MeV in $N=50$ (Fig.2 left) reveal in-phase oscillation of protons and neutrons in the nuclear interior, while at the surface only neutrons contribute. These states we have identified with a neutron PDR. The states in the region $E^* = 8-8.5$ MeV in $Z=50$ and $N=82$ and $E^* = 9-9.5$ MeV in $N=50$ nuclei carry a different signature, being compatible with the low-energy part of the GDR. At $E^* = 9-20$ MeV a strong, isovector oscillation, corresponding to the excitation of the GDR is obtained. An interesting observation is the most exotic $^{100}$Sn nucleus, where at $E^* = 8.3$ MeV a state with a proton structure is found. The analysis on dipole transition densities for different excitation energy regions in $^{100}$Sn is presented in Fig.5, illustrating the proton surface oscillations at $E^* \leq 8.3$ MeV. This mode could indicate a proton PDR. The dependence of the calculated total PDR strength on the mass number in $N=50, 82$ and $^{100-132}$Sn is compared to the relative difference between the neutron and proton rms radii

$$\delta r = \sqrt{\langle r_n^2 \rangle} - \sqrt{\langle r_p^2 \rangle}$$

in the right hand side part of Fig.2, Fig.3 and Fig.4, respectively. In the case of $N=50$ and $N=82$ isotones we keep the neutron number fixed and change the proton number only. This affects the thickness of the neutron skin (see Fig.1 left) as well.

Figure 1. Ground state densities of $N=50$ and $Z=50$ isotopes used in the QPM calculations.
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and respectively the total PDR strength (Fig. 2 right) and (Fig. 3 right) decreases with increasing proton number. The results obtained for $^{100-132}$Sn nuclei, where the neutron number increases from $N=50$ to $N=82$ are in agreement with these obtained for $N=50,82$ isotones considered above. Accordingly, the total PDR strength increases (Fig. 4 right), when $\delta r$ increases and correspondingly the neutron or proton skin thicknesses increase (Fig. 4 right).

4. Conclusions

In the isotones with $N=50$, $N=82$ and the $Z=50$ isotopes low-energy dipole states, identified with PDR were obtained. A close connection between the total PDR strengths and the neutron skin thickness defined by the relative difference of neutron and proton rms radii was found. These observations agree very well with our previous results for the $Z = 50$ isotopes and the $N = 82$ isotones. In the most exotic nuclei $^{100-104}$Sn lowest dipole states of almost pure proton structure are identified. They are related to oscillations of weakly bound protons, indicating a proton-driven PDR. The interesting point is, that these states are predicted to exist in heavy nuclei with $N$ slightly larger or equal to $Z$. We suggest, that the effect is due to Coulomb repulsion, that pushes the weakly bound protons orbitals into the nuclear surface. The results for Sn isotopes and $N=82$ nuclei are in a good agreement with available data [10, 11, 12].

At present, extended investigations on the fragmentation pattern of the low-
Figure 3. One-phonon dipole transition densities in N=82 nuclei (left). The total PDR strength is compared to the nuclear skin thickness $\delta r$, eq.9, as a function of the mass number in N=82 nuclei (right).

Figure 4. Dipole one-phonon transition densities in Z=50 nuclei (left). The total PDR strength is compared to the nuclear skin thickness $\delta r$, eq.9, as a function of the mass number in Z=50 nuclei (right).
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Figure 5. QPM results for the one-phonon dipole transition densities in $^{100}$Sn.

energy dipole excitations are in progress. The QPM calculations will be performed in considerably larger configuration spaces and using microscopically derived interactions, thus enabling a detailed description of data on the dipole response of stable and exotic nuclei to be expected for the near future from ELBE and the experiments planned at GSI and for FAIR.

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