In this paper, we focus on solving the problems of inertia-free attitude tracking control for spacecraft subject to external disturbance, unknown inertial parameters, and actuator faults. The robust control architecture is designed by using the rotation matrix and neural networks. In the presence of external disturbance and parametric uncertainties, a fault-tolerant control (FTC) scheme synthesized with the minimum-learning-parameter (MLP) algorithm is proposed to improve the reliability of the system when unknown actuator faults occur. These methods are developed based on backstepping to ensure that finite-time convergence is achievable for the entire closed-loop system states with low computational complexity. The validity and advantage of the designed controllers are highlighted by using Lyapunov-based analysis. Finally, the simulation results demonstrate the satisfactory performance of the developed controllers.

1. Introduction

With the rapid development of space engineering, spacecraft attitude control has been studied extensively by researchers for its essential role in various space missions, involving deep space exploration, earth observation, Mars detection, etc. As an important part of spacecraft, control systems determine the success of these missions for the most part. However, designing controllers with satisfactory performance for spacecraft still remains a challenging task, which can be attributed to complex external disturbance, unknown inertial parameters, and actuator faults. Despite these obstacles, numerous methods have been designed to address the attitude tracking control problem, including backstepping control [1–4], sliding mode control [5–7], prescribed performance control [8–10], and event-triggered control [11, 12].

Though fruitful results have been obtained for attitude control of spacecraft during the last decades, it still remains a challenge to design finite-time controllers under the circumstances of the external disturbance and unknown inertial matrix. On the one hand, modern space missions put forward high requirements in fast response and strong robustness for attitude control systems, which could be resorted to finite-time control strategies [13, 14]. On the other hand, inertial parameters are always time-varying with continuing fuel consumption during space missions, which makes it challenging to acquire accurate values of the inertial matrix. On account of the fast response requirement, finite-time control strategies [15–18] and appointed time approaches [19, 20] were adopted in the spacecraft system such that the control objective could be accomplished within a limited time. In [17, 18], terminal sliding mode-based methods were proposed for a single spacecraft and multiple spacecraft, respectively. Different from these two controllers, the prescribed performance control strategy was adopted in [19, 20], where the convergence time of the spacecraft system could be predefined by utilizing a nonlinear function. As
for the unknown system parameters, NN-based control schemes [21, 22] were widely adopted for nonlinear systems, which have also been used to approximate unknown system dynamics in spacecraft systems [23, 24]. As for the stabilization of fractional-order systems with unknown parameters, the T-S fuzzy control theory synthesized with adaptive linear-like control rules shows its superior capability in dealing with uncertainties [25].

Apart from the fast response and unknown system parameters, actuator faults are another important aspect that deserves special attention, especially for the need for high reliability in space activities, such as rendezvous and docking. Complex space environments and disturbances render the spacecraft vulnerable to actuator faults, which will deteriorate the control performance. To address this issue, researchers have introduced FTC schemes for various space applications [26–28]. Niu et al. proposed an observer-based FTC strategy for attitude stabilization of rigid spacecraft when time-varying faults occur [27]. Aiming at attitude synchronization control for multiple spacecraft, Zhou and Xia introduced a fault-tolerant method in the presence of modeling uncertainties [28]. With much more superiority in nonlinear fractional-order systems, Hashtrakhani and Khorosvjerdi proposed a fault-tolerant controller in conjunction with the backstepping terminal sliding mode technique for an uncertain faulty system [29]. Besides, fractional adaptive state feedback control laws based on backstepping were developed in [30] to help promote the stability of the system with actuator failures.

Obviously, controllers in [26–28] have been constructed by using unit quaternion-based attitude descriptions, which have the inherent disadvantage of the unwinding problem. Due to this drawback, researchers focused on the rotation matrix-based attitude description, which can represent attitude motions globally and uniquely [31–33]. By resorting to the rotation matrix, Huang et al. proposed a fault-tolerant attitude tracking controller with finite-time stability [33]. Similarly, several control algorithms have been designed for rigid spacecraft with consideration of the unwinding problem [31, 32]. However, these controllers are not applicable for cases that inertial parameters are unknown. Therefore, further research is required to deal with the actuator fault, unknown system parameters, and unwinding problem.

Inspired by the aforementioned obstacles in controller design for spacecraft, this paper focuses on providing two unwinding-free attitude tracking control architectures for spacecraft when there exist actuator faults and unknown inertial parameters. The main contributions of this paper are given as follows:

(i) A finite-time attitude tracking controller is constructed on the basis of the rotation matrix with unknown inertial parameters, so that finite-time tracking performance could be obtained for spacecraft attitude control. Compared with existing methods [16, 33], inertial parameters could remain unavailable, making the tracking controllers more applicable in aerospace engineering

(ii) Actuator faults and unknown system dynamics could be handled simultaneously with the aid of the FTC strategy proposed in this paper. Based on the MLP algorithm, unknown system dynamics can be approximated with low computational complexity.

The rest of this paper is organized as follows. In Section 2, the spacecraft model and preliminaries are presented. In Section 3, backstepping controllers are designed by utilizing MLP and FTC. In Section 4, the effectiveness of the proposed algorithms is validated through numerical simulations. Finally, the conclusions of this paper are presented in Section 5.

2. Spacecraft Model and Preliminaries

2.1. Spacecraft Attitude Dynamics. The unit quaternion has been widely used for attitude description in most attitude controllers [9–11]. However, the unit quaternion-based algorithm results in the unwinding problem with high fuel consumption. To address this problem, the rotation matrix is adopted here to describe the attitude dynamics of the spacecraft, which can be defined as follows:

\[
\hat{R} = R\omega^x, \\
J\dot{\omega} = -\omega^x J\omega + \delta u + d,
\]

where \( R \in SO(3) \) denotes the rotation matrix; \( \omega \in R^{3 \times 1} \) is the angular velocity; \( J \in R^{3 \times 3} \) denotes the inertial matrix; \( u \in R^{3 \times 1} \) and \( d \in R^{3 \times 1} \) are the control torque and external disturbance of the spacecraft, respectively; and \( \delta = \text{diag}(\delta_1, \delta_2, \delta_3) \) is the fault coefficient, where \( 0 < \theta < \delta_i \leq 1, i = 1, 2, 3 \), with \( \theta \) being a positive constant. For a vector \( a = [a_1, a_2, a_3]^T \), \( a^x \) is defined as follows:

\[
a^x = \begin{bmatrix} 0 & -a_3 & a_2 \\
         a_3 & 0 & -a_1 \\
        -a_2 & a_1 & 0 \end{bmatrix}.
\]

For the attitude controller design, attitude and angular velocity tracking errors \( \hat{R} \) and \( \hat{\omega} \) are defined in Equations (4) and (5), respectively, where \( R_d \in SO(3) \) and \( \omega_d \in R^{3 \times 1} \) are the commanded rotation matrix and angular velocity, respectively.

\[
\hat{R} = R^T_d R, \\
\hat{\omega} = \omega - \hat{R}^T \omega_d.
\]

Thus, the attitude dynamics can be rewritten as follows:

\[
\dot{\hat{R}} = \hat{R}\omega^x, \\
J\dot{\hat{\omega}} = F + \delta u + d,
\]

where \( F = -(\hat{\omega} + \hat{R}^T\omega_d) J(\hat{\omega} + \hat{R}^T\omega_d) - \hat{J}\hat{R}^T\omega_d + J\hat{\omega}^x\hat{R}^T\omega_d. \)
Angular velocity Assumption 1. The following assumption is necessary for the controller design, the inertial matrix manages the unknown inertial matrix. In order to facilitate learning-parameter algorithm-based NNs are employed to be satisfied if the condition \(|e_R| \leq 1\) is true [31–33]. Consequently, \(|e_R| \leq 1\) must be satisfied during space missions.

Remark 2. The inertial parameters \(J\) are regarded as unknown variables in the controller design. Minimum-learning-parameter algorithm-based NNs are employed to manage the unknown inertial matrix. In order to facilitate the controller design, the inertial matrix \(J\) is assumed to be a positive definite diagonal matrix.

2.2. Assumptions. The following assumption is necessary for the attitude controller design:

**Assumption 1.** The external disturbance \(d\) and commanded angular velocity \(\omega_d\) are bounded and satisfy \(|d| \leq D\) and \(|\omega_d| \leq \bar{\omega}\), where \(D\) and \(\bar{\omega}\) are positive constants.

2.3. Mathematical Notation. In this paper, the notation \(|\cdot|\) denotes the Euclidean norm of a vector or the induced norm of a matrix. For any \(\xi \in \mathbb{R}^n\), \(\text{sign}(\xi)\) is defined as \(\text{sign}(\xi)^\top = [\text{sign}(\xi_1), \text{sign}(\xi_2), \ldots, \text{sign}(\xi_n)]^\top\). For a vector \(\xi = [\xi_1, \xi_2, \ldots, \xi_n]^\top\), \(\text{sign}(\xi)^\top\) is defined as \(\text{sign}(\xi) = [\text{sign}(\xi_1), \text{sign}(\xi_2), \ldots, \text{sign}(\xi_n)]^\top\). Here, \(\chi\) is a constant that satisfies \(0 < \chi < 1\). Furthermore, \(\text{tanh}(\xi) = [\text{tanh}(\xi_1), \text{tanh}(\xi_2), \ldots, \text{tanh}(\xi_n)]^\top\). For a matrix \(A\), the notation \(\lambda_{\text{max}}(A)\) and \(\lambda_{\text{min}}(A)\) represent the maximum eigenvalue and minimum eigenvalue of \(A\), respectively.

2.4. Related Lemmas. For the satisfactory performance in approximating functions, RBF NNs have been widely used in controller design for nonlinear systems. Therefore, RBF NNs are employed herein to address the unknown system dynamics caused by the unavailable inertial matrix. The following lemmas are introduced to facilitate the design process.

**Lemma 3** [20]. For a random continuous function \(f(x)\), an ideal weight vector \(W\) exists that satisfies the following function:

\[
\psi(R) = 2 - \sqrt{1 + \text{tr}(R)}.
\]

Here, \(\psi(R)\) denotes the inverse operation of \(\times\). Combining Equations (1)–(9), error dynamics can be reconstructed as follows:

\[
\dot{e}_R = E\hat{\omega},
\]

\[
J\hat{\omega} = F + \delta u + d,
\]

\[
E = \frac{1}{2\sqrt{1 + \text{tr}(R)}} \left(\text{tr}(R)I - R^\top + 2e_R e_R^\top\right),
\]

\[
F = -\left(\hat{\omega} + R^\top\omega_d\right)^\top J\left(\hat{\omega} + R^\top\omega_d\right) - J R^\top \omega_d + J\omega^s R^\top \omega_d.
\]

**Remark 1.** To ensure Equation (10) is valid, \(\text{tr}(R) \neq -1\) must be satisfied. As stated by Huang et al., \(\text{tr}(R) \neq -1\) can be derived if the condition \(|e_R| \leq 1\) is true [31–33]. Consequently, \(|e_R| \leq 1\) must be satisfied during space missions.

**Remark 2.** The inertial parameters \(J\) are regarded as unknown variables in the controller design. Minimum-learning-parameter algorithm-based NNs are employed to manage the unknown inertial matrix. In order to facilitate the controller design, the inertial matrix \(J\) is assumed to be a positive definite diagonal matrix.

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**Lemma 3** [20]. For a random continuous function \(f(x)\), an ideal weight vector \(W\) exists that satisfies the following function:

\[
f(x) = W^\top h(x) + \varepsilon,
\]

where \(W = [w_1, w_2, \ldots, w_p]^\top\) and \(x = [x_1, x_2, \ldots, x_m]\) are Ge-Lee vectors defining weight and input vectors, respectively; \(p\) and \(m\) are the node and input number, respectively; \(\varepsilon\) is the approximation error; and \(h(x) = [h_1(x), \ldots, h_p(x)]^\top\) is the Gaussian basis function vector with the following form:

\[
h_i(x) = \exp \left(-\frac{||x - c_i||^2}{2\sigma^2}\right), \quad i = 1, \ldots, p,
\]

where \(c_i \in \mathbb{R}^m\) and \(b_i \in \mathbb{R}\) are the center vector and Gaussian basis function vector of \(h_i(x)\), respectively.

**Lemma 4** [33]. If a vector \(x \in \mathbb{R}^n\) satisfies \(R^\top R = \exp(x^\top)\), the inequality is always tenable. The term \(E\) defined in Equation (12) satisfies \(\|E\| = 1/2\). Additionally, if \(\|x\| \neq n\), \(E\) is always invertible.

**Lemma 5** [33]. Considering positive constants \(\rho\) and \(\alpha_i\), \(i = 1, 2, \ldots, n\), if the inequality \(1 < \rho < 2\) holds, the following equation is obtained:

\[
(a_1^2 + \cdots + a_n^2)^\rho \leq (a_1^\rho + \cdots + a_n^\rho)^2.
\]

**Lemma 6** [33]. For the system Equations (19) and (20), if a Lyapunov function \(V_1\) exists such that it satisfies the following equation:

\[
\dot{V}_1 \leq -a V_1^a,
\]

where \(a\) and \(\alpha\) are positive constants satisfying \(0 < \alpha < 1\), then the system converges to the origin within a finite time.

**Lemma 7** [34, 35]. Given positive scalers \(x \in \mathbb{R}\) and \(\sigma > 0\), the following key inequality can be derived:

\[
0 \leq |x| - \frac{|x|^2}{\sqrt{x^2 + \sigma^2}} \leq |x| - \frac{|x|^2}{|x| + \sigma} < \sigma.
\]

3. Design of the Controller

In this section, a fault-tolerant attitude tracking controller is presented to ensure the stability of the closed-loop system with finite-time convergence. For better applicability, MLP and adaptive laws are proposed to estimate unknown system information. Besides, a backstepping-based FTC scheme is proposed with an auxiliary system, thus improving system reliability when an unknown actuator fault occurs.
To prove that $x_2 = E\hat{\omega} - \hat{\omega}_{p,1}$, the derivative of $V_1$ can be derived as follows:

$$
\dot{V}_1 = x_1^T x_1 = x_1^T E\hat{\omega} + x_1^T (x_2 + \hat{\omega}_{p,1})
$$

$$
\leq -k_1 \frac{x_1}{\sqrt{\|x_1\|^2 + \sigma^2}} - k_2 x_1 + x_1^T x_2.
$$

Using Lemma 7, the following equation is derived:

$$
\dot{V}_1 = -k_1 \frac{x_1}{\sqrt{\|x_1\|^2 + \sigma^2}} - k_2 x_1 + x_1^T x_2
$$

$$
\leq -k_1 \|x_1\| - \left( k_2 - \frac{1}{2} \right) \|x_1\|^2 + \frac{\|x_2\|^2}{2} + k_1 \sigma.
$$

Step 2. With Remark 2 and the result in Equation (24), consider the Lyapunov function as follows:

$$
V_2 = V_1 + \frac{1}{2} x_2^T J x_2.
$$

Then, the derivative of $V_2$ can be calculated as follows:

$$
\dot{V}_2 = \dot{V}_1 + x_2^T J x_2 = -k_1 \|x_1\| - \left( k_2 - \frac{1}{2} \right) \|x_1\|^2 + \frac{\|x_2\|^2}{2}
$$

$$
+ k_1 \sigma + x_2^T \dot{E}\hat{\omega} + x_2^T \left[ \left( \hat{\omega} + \hat{R}^T \omega_d \right) \times \left( \hat{\omega} + \hat{R}^T \omega_d \right) \right]
$$

$$
- J \hat{R}^T \omega_d + f \hat{\omega} \times \hat{R}^T \omega_d + \dot{d} + u - E^T f \hat{\omega}_{p,1}.
$$

The notation is selected as follows:

$$
H = J\hat{E}\hat{\omega} - f \hat{\omega}_{p,1} + E \left[ \left( \hat{\omega} + \hat{R}^T \omega_d \right) \times \left( \hat{\omega} + \hat{R}^T \omega_d \right) \right]
$$

$$
- J\hat{R}^T \omega_d + f \hat{\omega} \times \hat{R}^T \omega_d.
$$

From Assumption 1 and Lemma 4, the term $x_2^T Ed$ is known to satisfy the following condition:

$$
\|x_2^T Ed\| \leq \frac{D}{2} \|x_2\|.
$$

Thus, Equation (27) is rewritten as follows:

$$
\dot{V}_2 \leq -\sqrt{2k_1} V_1^{1/2} - (2k_2 - 1) V_1 + \frac{\|x_2\|^2}{2} + k_1 \sigma + x_2^T (W^T h + \varepsilon) + \frac{D}{2} \|x_2\| + x_2^T Eu.
$$

Observing Lemma 3, $H$ can be expressed as follows:

$$
H = W^T h + \varepsilon,
$$

where $W = [W_1, W_2, W_3]^T$ is a Ge-Lee matrix defining the weight matrix, $h = [h_1, h_2, h_3]^T$ is a Ge-Lee matrix defining the radial basis function matrix, and $\varepsilon$ is the approximation error satisfying $\|\varepsilon\| \leq \varepsilon_d$ with $\varepsilon_d > 0$. $W_i, i = 1, 2, 3$, and $h_i$ are three-dimensional Ge-Lee vectors. Combining Equations (29)–(31) results in

$$
\dot{V}_2 \leq -\sqrt{2k_1} V_1^{1/2} - (2k_2 - 1) V_1 + \frac{\|x_2\|^2}{2} + k_1 \sigma + \frac{D}{2} \|x_2\| + x_2^T Eu.
$$

Here, the term $x_2^T W^T h$ can be defined as follows:

$$
\|x_2^T W^T h\| = \sum_{i=1}^{3} x_{2i} W_{i}^* T h_i.
$$

Equation (31) is further equivalent to the following:

$$
\dot{V}_2 \leq -\sqrt{2k_1} V_1^{1/2} - (2k_2 - 1) V_1 + \frac{\|x_2\|^2}{2} + k_1 \sigma
$$

$$
+ \frac{3}{2} \|x_{2i}\|^2 \|h_i\|^2 + \frac{3\eta}{2} + \|x_2\| \left( \frac{D}{2} + \varepsilon_d \right) + x_2^T Eu,
$$

where $\beta = \max_{1 \leq i \leq 3} \{ ||W_i^*||^2 \}$ and $\eta$ is a positive constant. Thus, $\beta$ is easier to estimate than $W^*$. To simplify the design, we introduce a constant $\tilde{D}$ satisfying $\tilde{D} = (D/2) + \varepsilon_d$.

Step 3. In the healthy condition of the actuator, $\delta_i = 1$ is ensured. However, in practical applications, external
Remark 8. Inertial parameters are totally unknown to designers in this paper. To deal with the unmodeled dynamics, the MLP method is adopted to ensure finite-time convergence for the spacecraft system. In Equation (35), the term $k_3x_2 + k_4\text{sig}(x_2)^3$ is introduced for finite-time convergence while $(\beta x_2 \circ (h \circ h))/2\eta$ and $\hat{D}\tan (x_2/\mu_2)$ are designed to estimate unknown dynamics and the upper bound of external disturbances.

Remark 9. When $0 < \delta_1 < 1$ holds, the spacecraft encounters the partial actuator effectiveness loss fault, which will cause considerable performance degradation. To this end, the fault-tolerant controller is proposed as Equation (34). Here, $u_i$ implies the norm part of this controller dedicating to forcing spacecraft towards the desired attitude. $u_i$ defines the compensation part for the actuator fault. With this control strategy, finite-time stability is achievable for tracking errors even when actuator faults occur.

Theorem 10. For the attitude error dynamics expressed in Equations (10), (11), (19), and (20) with Assumption 1, then, the virtual angular velocity $\hat{\omega}_{ps}$ is derived as Equation (21). If the virtual control law and actual control law are designed as in Equations (21) and (34)–(39), the following conclusions can be obtained:

(i) The region of attraction is represented as follows:

\[
x^T_1(0)x_1(0) + x^T_2(0)x_2(0) + \frac{1}{c_1}\hat{\beta}^2(0) + \frac{\theta}{c_2}\hat{\gamma}^2(0) + \frac{1}{c_3}\hat{D}^2(0) < 1.
\]

(ii) $x_2$ will be stabilized in the region $\{ \sqrt{2/(\lambda_{\text{min}}(J))}(\Delta_J \rho_2), \sqrt{2/(\lambda_{\text{min}}(J))}(\Delta_J \rho_2)^{(k+1)/2}) \}$ within a finite time. Here, $\rho_2 = \min \{ \sqrt{2k_1,k_2/(\lambda_{\text{min}}(J))}^{(1+k)/2}, (c_1c_2(2k_2 + 1)/k_1)\}^{1/2}, (c_1c_2(2k_2 - 1)/k_1)\}^{1/2}, ((\dot{c}_6c_4(2k_7 - 1))/k_7)^{1/2}$ and $\Delta_i = (c_6k_5/2)^{2} + (c_6k_7/2)^{2} + (3\delta_5\mu_1 + \mu_2\gamma_0) + k_1\sigma + (3\eta_i/2)$, $k_i, i = 5, 6, 7$, are positive constants satisfying $k_i > 0.5$

Proof. The Lyapunov function can be established as follows:

\[
V_3 = V_2^* + \frac{1}{2c_1}\hat{\beta}^2 + \frac{\theta}{c_2}\hat{\gamma}^2 + \frac{1}{2c_3}\hat{D}^2.
\]

Considering Equations (33) and (34), the derivation can be calculated as follows:

\[
\dot{V}_3 \leq -(2k_2 - 1)V_1 + \frac{\|x_2\|^2}{2} + k_1\sigma + \frac{3\beta x_2^2||h_1^2||^2}{2\eta} + \frac{3\eta}{2} + x_2^T(I - \delta)u_1 + x_2^T u_1 + x_2^T\delta u_2 + ||x_2||\left(\frac{D}{2} + \varepsilon\right)
\]

\[
- \frac{1}{c_1}\hat{\beta}\hat{\beta} - \frac{\theta}{c_2}\hat{\gamma}\hat{\gamma} - \frac{1}{c_3}\hat{D}\hat{D}.
\]

Then, learning from Equations (35) and (36), one has

\[
\dot{V}_3 \leq -(2k_2 - 1)V_1 + \frac{\|x_2\|^2}{2} + k_1\sigma + \sum_{i=1}^{3}\frac{\beta x_2^2||h_1^2||^2}{2\eta} + \frac{3\eta}{2}
\]

\[
+ x_2^T(I - \delta)u_1 - k_3||x_2||^{1+k} - x_2^T\frac{\beta x_2 \circ (h \circ h)}{2\eta} - \hat{D}x_2^T\tan (x_2/\mu_2) - \gamma\hat{\theta}x_2^T\mu_1||u_1|| + \frac{x_2^T||u_1||}{\mu_2}
\]

\[
+ \hat{D}||x_2|| - \frac{1}{c_1}\hat{\beta}\hat{\beta} - \frac{\theta}{c_2}\hat{\gamma}\hat{\gamma} - \frac{1}{c_3}\hat{D}\hat{D}
\]

\[
- (k_3 - \frac{1}{2})||x_2||^2 + k_1\sigma + \sum_{i=1}^{3}\frac{\beta x_2^2||h_1^2||^2}{2\eta} + \frac{3\eta}{2}
\]

\[
+ (1 - \theta)||x_2||||u_1|| - \hat{D}||x_2|| + 3\mu_2\hat{\gamma}\hat{\theta} + \hat{D}||x_2|| + \frac{1}{c_1}\hat{\beta}\hat{\beta} - \frac{\theta}{c_2}\hat{\gamma}\hat{\gamma} - \frac{1}{c_3}\hat{D}\hat{D}
\]

\[
= -(2k_2 - 1)V_1 - (k_3 - \frac{1}{2})||x_2||^2 + \sum_{i=1}^{3}\frac{\beta x_2^2||h_1^2||^2}{2\eta}
\]

\[
+ \theta\hat{\gamma}||x_2||||u_1|| + \hat{D}||x_2|| - \frac{1}{c_1}\hat{\beta}\hat{\beta} - \frac{\theta}{c_2}\hat{\gamma}\hat{\gamma} - \frac{1}{c_3}\hat{D}\hat{D}
\]

\[
+ 3k_2\hat{D}u_1 + \mu_2\hat{\gamma}\hat{\theta} + k_1\sigma + 3\eta/2.
\]
Combining adaptive laws in Equations (37)–(39), one can obtain the following result:

\[
V_3 \leq -(2k_2 - 1) V_1 - \left( k_3 - \frac{1}{2} \right) \|x_2\|^2 + \sum_{i=1}^{3} \beta \|h_i\|^2 + \frac{3}{2\eta} x_2^T \dot{h}_i - c_4 \beta \\
+ \theta \|x_2\| \|u_1\| + D \|x_2\| - \beta \left( \sum_{i=1}^{3} \frac{x_2^T h_i}{2\eta} - c_4 \beta \right) \\
- \theta \left( \|u_1\| \|x_2\| \tanh \left( \frac{\|u_1\| \|x_2\|}{\mu_2} \right) - c_5 \beta \right) \\
- \tilde{D} \left( \|x_2\| \tan \left( \frac{\|x_2\|}{\mu_1} \right) - c_6 \tilde{D} \right) + 3\kappa(\dot{D} u_1 + \mu_2 \dot{\gamma} \theta) \\
+ k_1 \sigma + \frac{3}{2} \mu_3 \leq -(2k_2 - 1) V_1 - \left( k_3 - \frac{1}{2} \right) \|x_2\|^2 + c_4 \beta \beta \\
+ 3\kappa \mu_3 \dot{\gamma} \theta + c_7 \dot{\gamma} \beta \gamma + 3\mu_4 \dot{D} \beta + c_8 \dot{D} \beta + 3\kappa(\dot{D} u_1 + \mu_2 \dot{\gamma} \gamma) \\
+ k_1 \sigma + \frac{3}{2} \mu_3 \leq -(2k_2 - 1) V_1 - \left( k_3 - \frac{1}{2} \right) \|x_2\|^2 - c_4 \beta \beta \\
- \frac{c_4 \theta}{2} \gamma^2 - c_6 \beta \gamma + c_4 \beta \beta + c_6 \beta \beta + c_8 \beta \beta \\
+ 3\kappa(\dot{D} u_1 + \mu_2 \dot{\gamma} \gamma) + k_1 \sigma + \frac{3}{2} \mu_3 \leq -\rho_3 V_3 + \Delta_3, \\
\]

(44)

where \( \rho_3 = \min \{2k_2 - 1, (2k_2 - 1)/(\lambda_{\max}(J)) \}, c_1, c_2, c_3, c_4, c_5, c_6 \) and 
\( \Delta_3 = (c_4/2) \beta \beta + (c_4/2) \beta \gamma + (c_4/2) \beta \gamma + k_1 \sigma + (3\mu_3/2) \kappa(\dot{D} u_1 + \mu_2 \dot{\gamma} \gamma) \).

Consequently, \( V_3 \) is asymptotically stable and converges to a region containing the origin. To proceed, the derivation of \( V_3 \) in Equation (42) can be rewritten as follows:

\[
V_4 \leq \sqrt{2k_1} V_1^{1/2} + \frac{\|x_2\|^2}{2} + k_1 \sigma + \frac{3}{2} \beta \|h_i\|^2 + \frac{3}{2} \mu_3 \\
+ x_2^T (I - \delta) u_1 + x_2^T u_1 + x_2^T \delta u_2 + \|x_2\| \left( \frac{D}{2} + \epsilon \right) \\
- \frac{1}{c_1} \beta \tilde{\gamma} - \frac{1}{c_6} \beta \tilde{D} \tilde{D} - \frac{\mu_2}{c_2} \gamma \tilde{\gamma}. \\
\]

(45)

Combining the control laws Equations (34)–(39), the following expression is obtained:

\[
V_4 \leq \sqrt{2k_1} V_1^{1/2} - k_4 \|x_2\|^{1/4} + c_4 \beta \beta + c_4 \theta \beta \gamma + c_4 \dot{D} \beta \\
+ 3\kappa(\dot{D} u_1 + \mu_2 \dot{\gamma} \gamma) + k_1 \sigma + \frac{3}{2} \mu_3 = \sqrt{2k_1} \left( \frac{1}{2} x_2^T x_2 \right)^{1/2} \\
- k_4 \left( \frac{2}{\lambda_{\min}(J)} \right)^{1/2} \left( \frac{1}{2} x_2^T J x_2 \right)^{1/2} + c_4 \beta \beta + c_4 \theta \gamma \\
+ c_6 \dot{D} \beta + 3\kappa(\dot{D} u_1 + \mu_2 \dot{\gamma} \gamma) + k_1 \sigma + \frac{3}{2} \mu_3. \\
\]

(46)

For the term \( c_4 \beta \beta \), the following condition always exists:

\[
c_4 \beta \beta = c_4 \left( \beta \beta + \kappa \beta \beta \right) \leq c_4 \left( \beta \beta + \frac{1}{2k_5} \beta \beta \right) \\
\leq -\frac{c_4 (2k_5 - 1)}{2k_5} \beta \beta + \frac{c_4 k_5 \beta \beta}{2}, \\
\]

(47)

where \( k_5 \) is a positive constant. If it satisfies \( (c_4 (2k_5 - 1))/2k_5 \beta \beta \geq 1 \), the following equation is obtained:

\[
\left( \frac{c_4 (2k_5 - 1)}{2k_5} \beta \beta \right)^{1/2} + c_4 \beta \beta \leq \frac{c_4 (2k_5 - 1)}{2k_5} \beta \beta + c_4 \beta \beta \leq \frac{c_4 k_5 \beta \beta}{2}, \\
\]

(48)
If \((c_4(2k_5 - 1))/2k_5 \beta^2 < 1\) exists, the following equation is obtained:

\[
\left( \frac{c_4(2k_5 - 1)}{2k_5} \beta \right)^{1/2} \left| \frac{(c_4(2k_5 - 1))/2k_5 \beta^2 < 1}{(c_4(2k_5 - 1))/2k_5 \beta^2 \geq 1} \right.
\]

Then, it follows

\[
\left( \frac{c_4(2k_5 - 1)}{2k_5} \beta \right)^{1/2} + c_4 \beta \hat{\beta} \leq \frac{c_4 k_5}{2} \beta^2.
\]

The same results for \(c_4 \theta \hat{\gamma} \hat{\gamma}\) and \(c_4 \tilde{D} \tilde{D}\) are obtained. Consequently, Equation (46) can be rewritten as follows:

\[
V_4 \leq -\sqrt{2k_1} \left( \frac{1}{2} x_1^T J x_1 \right)^{1/2} - k_4 \left( \frac{2}{\lambda_{\text{min}}(f)} \right)^{(1/2)} \left( \frac{1}{2} x_1^T f x_1 \right)^{(1/2)}
- \left( \frac{c_4(c_1(2k_5 - 1))}{k_5} \right)^{1/2} \left( \frac{1}{2} \beta^2 \right)^{1/2} - \left( \frac{c_4(c_1(2k_5 - 1))}{\theta k_5} \right)^{1/2} \left( \frac{\theta}{2c_2} \right)^{1/2}
- \left( \frac{c_4(c_1(2k_5 - 1))}{k_7} \right)^{1/2} \left( \frac{1}{2} \hat{D}^2 \hat{D} \right)^{1/2} + \left( \frac{c_4(c_1(2k_5 - 1))}{2k_5} \beta \right)^{1/2}
+ \left( \frac{c_4(2k_5 - 1)}{2k_5} \hat{\gamma} \right)^{1/2} + \left( \frac{c_4(2k_5 - 1)}{2k_5} \hat{\gamma} \right)^{1/2} + c_4 \beta \hat{\beta}
+ c_4 \theta \hat{\gamma} \hat{\gamma} + c_4 \tilde{D} \tilde{D} + 3k(D \mu + \mu_2 \theta) + k_1 \sigma + \frac{3\eta}{2}
\leq -\rho_4 \min \left\{ V_4^{1/2}, V_4^{(1/2)} \right\} + \left( \frac{c_4(2k_5 - 1)}{2k_5} \beta \right)^{1/2}
+ \left( \frac{c_4(2k_5 - 1)}{2k_5} \hat{D} \hat{D} \right)^{1/2} + c_4 \beta \hat{\beta}
+ c_4 \theta \hat{\gamma} \hat{\gamma} + c_4 \tilde{D} \tilde{D} + 3k(D \mu + \mu_2 \theta) + k_1 \sigma + \frac{3\eta}{2}
\]
In terms of Equation (50), we have

\[
\dot{V}_4 \leq -\rho_4 \min \left\{ V_{1/4}^2, V_{1/4}^{(1+4)/2} \right\} + \frac{c_{1} k_{2}}{2} \beta^2 + \frac{c_{2} k_{2} \gamma^2}{2} + \frac{3 \kappa \left( \mu_2 \gamma^2 + \mu_2 \delta^2 \right)}{2} + \frac{3 \eta}{2} V_x^2
\]

\[
= -\rho_4 \min \left\{ V_{1/4}^2, V_{1/4}^{(1+4)/2} \right\} + \Delta_t.
\]

(52)

From Equation (52), it can be concluded that the result \( \dot{V}_4 \leq 0 \) always holds. Consequently, \( x_1, x_2, \tilde{\beta}, \tilde{\gamma}, \) and \( \Delta_t \) are all bounded. To keep \( \|x_1\|^2 < 1 \), one can obtain

\[
\dot{V}_4 \leq V_4(0) \leq \frac{1}{2} x_1^T(0)x_1(0) + \frac{1}{2} x_2^T(0)x_2(0) + \frac{1}{2} \beta^2(0)
\]

\[
+ \frac{1}{2 c_1} D^2(0) < \frac{1}{2}.
\]

(53)

Therefore, (i) has been proved.

Meanwhile, according to Lemma 6, \( x_2 \) will be stabilized to the region max \( \{ \sqrt{2/(\lambda_{\min}(J))} (\Delta_t/\rho_4), \sqrt{2/(\lambda_{\min}(J))} (\Delta_t/\rho_4)^{1/(1+i)} \} \) within a finite time. Thus, Theorem 10 is proven.

4. Simulation Results

This section is dedicated to authenticating the stability and validity of the controllers presented in the theoretical analysis. The main parameters, inertial matrix, disturbance torque, initial state, desired rotation matrix, and angular velocity are selected as follows: \( J = \begin{bmatrix} 22.7 & 0 & 0 \\ 0 & 23.3 & 0 \\ 0 & 0 & 24.5 \end{bmatrix} \) kg \( \cdot \) m\(^2\), \( \omega(0) = [0.1, 0.1, 0.1]^T \) rad/s, \( R(0) = \begin{bmatrix} 0.4977 & -0.8674 & -0.0007 \\ 0 & -0.0008 & 1.0000 \\ -0.8674 & -0.4977 & -0.0004 \end{bmatrix} \), \( s \), \( R_d(0) = I_{3 \times 3}, \) \( \omega_d = 0.1 \times [\sin(t/40), -\cos(t/50), -\sin(t/60)]^T \) rad/s, and \( d = 0.02 \times [\sin(0.1t), \cos(0.2t), \sin(0.2t)]^T \), respectively.

To show the adverse effect of the actuator faults, the control performance of the controller in [22] is shown under two scenarios \( \delta = \begin{cases} \text{diag} (0.3, 0.3, 0.3), & \text{if } t > 80 \text{s} \\ \text{diag} (1, 1, 1), & \text{else} \end{cases} \)

\( \delta = \begin{cases} \text{diag} (0.2, 0.2, 0.2), & \text{if } t > 80 \text{s} \\ \text{diag} (1, 1, 1), & \text{else} \end{cases} \), where \( d = 0.05 \times [\sin(0.1t), \cos(0.2t), \sin(0.2t)]^T \) N \( \cdot \) m. Attitude tracking errors are presented in Figure 1, which implies that system performance degradation will be caused as \( \delta \) increases. Thus, it concludes that fault-tolerant control strategies are necessary for spacecraft during space missions in case of actuator faults.

Then, the actuator faults are set as follows: \( \delta = \begin{cases} \text{diag} (0.5, 0.4, 0.3), & \text{if } t > 100 \text{s} \\ \text{diag} (1, 1, 1), & \text{else} \end{cases} \). The design parameters of the fault-tolerant controller are determined as follows: \( k_1 = 0.05, \) \( k_2 = 0.5, \) \( k_3 = 2, \) \( k_4 = 1, \) \( c_1 = 0.001, \) \( c_2 = 0.001, \) \( c_3 = 0.001, \) \( c_4 = 2, \) \( c_5 = 2, \) \( c_6 = 2, \) \( \mu_1 = 1, \) \( \mu_2 = 1, \) and \( \lambda = 0.9. \) The following parameters are selected for the RBF NN: \( b_1 = 2, \) \( p = 20, \) and the input vectors of the RBF NN satisfy \( \omega_1 \in [-0.5, 0.5] \) rad/s, \( \omega_1 \in [-0.5, 0.5] \) rad/s, \( \omega_1 \in [-0.3, 0.3], \) \( x_2 \in [-0.5, 0.5] \) rad/s, and \( x_2 \in [-0.5, 0.5] \) rad/s. Simulation results for this method are presented in Figures 2–8. From the simulation results, we can conclude that the control objective can be achieved within 60 s in the case of actuator faults. Additionally, the chattering problem does not appear in the control torque. Thus, the points mentioned in Theorem 10 are illustrated by the simulation results.
5. Conclusions

In this paper, the unwinding-free problem is solved for spacecraft attitude tracking control missions. The proposed control schemes not only exhibit excellent robustness against actuator faults and disturbance but also address the unknown inertial parameters. Additionally, the application of the MLP method results in a controller with less computational complexity. The simulation results prove the effectiveness of the proposed methods.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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