Sacrificing Accuracy for Reduced Computation: Cascaded Inference Based on Softmax Confidence

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Abstract

We study the tradeoff between computational effort and accuracy in a cascade of deep neural networks. During inference, early termination in the cascade is controlled by confidence levels derived directly from the softmax outputs of intermediate classifiers. The advantage of early termination is that classification is performed using less computation, thus adjusting the computational effort to the complexity of the input. Moreover, dynamic modification of confidence thresholds allows one to trade accuracy for computational effort without requiring retraining. Basing of early termination on softmax classifier outputs is justified by experimentation that demonstrates an almost linear relation between confidence levels in intermediate classifiers and accuracy. Our experimentation with architectures based on ResNet obtained the following results. (i) A speedup of 1.5 that sacrifices 1.4% accuracy with respect to the CIFAR-10 test set. (ii) A speedup of 1.19 that sacrifices 0.7% accuracy with respect to the CIFAR-100 test set. (iii) A speedup of 2.16 that sacrifices 1.4% accuracy with respect to the SVHN test set.

1 Introduction

The quest for high accuracy in Deep Neural Networks (DNNs) has lead to the design of large network consisting of hundreds of layers with millions of trainable weights. Computation in such DNNs requires billions of multiply-accumulate operations (MACs) for a single image classification [SCYE17]. The natural question that arises is whether this amount of computation is indeed required [PSR16].

In this paper, we focus on the computational effort spent on inference in DNNs. (For simplicity, we measure the computational effort in the number of multiply-accumulate operations (MACs)). Many conjecture that the computational effort required for classifying images is not constant and depends on the image [BBPP15 Gra16 FCZ+17 PSR16 TMK16]. We claim that the required computational effort for classification is an intrinsic yet hidden property of the images. Namely, some images are much easier to classify than others, but the required computational effort needed for classification is hard to predict before classification is completed.

The desire to spend the “right” computational effort in classification leads to the first goal in this work.

Goal 1.1. Provide an architecture in which the computational effort is proportional to the complexity of the input.

We also consider a setting in which the system’s power consumption or throughput is not fixed. Examples of such settings are: (1) As the battery drains in a mobile device, one would like to enter a “power saving mode” in which less power is spent per classification. (2) If the input rate increases in a real-time system (e.g., due to a burst of inputs), then one must spend less time per input [CZC+10].
(3) Timely processing in a data center during spikes in query arrival rates may require reducing the computational effort per query [Bod10].

Dynamic changes in the computational effort or the throughput leads to the second goal in this work.

**Goal 1.2.** Introduce the ability to dynamically control the computational effort while sacrificing accuracy as little as possible. Such changes in the computational effort should not involve retraining of the DNN.

### 1.1 Techniques

We propose an architecture that is based on a cascade of DNNs [BWDS17] depicted in Figure 1. The cascade comprises multiple DNNs (e.g., three DNNs), called *component DNNs*. Classification takes place by invoking the component DNNs in increasing complexity order, and stopping the computation as soon as the confidence level reached a desired level.

Our component DNNs are not disjoint, namely, levels of previous components are part of the processing of the consecutive component. The advantage of this approach is that the next component reuses the computational outcome of the previous component and further refines it.

The decision of choosing the final DNN component invoked in the computation is based on the softmax output of the invoked DNNs. We define a simple confidence threshold, based on the softmax output, that allows for trading off (a little) accuracy for (a substantial) computational effort. Basing the stopping condition on the softmax output has two advantages over previous methods [PSR16, BWDS17]: (1) Simplicity, as we do not require an additional training step for configuring the control that selects the output. (2) Improved tradeoffs of computational effort vs. accuracy.

### 1.2 Environment assumptions

We focus on the task of classification of images. Our setting is applicable to other data as we do not have any limitations on the number of classes or the distribution of the input data.

The cascading of DNNs allows for the usage of various DNN layers, including convolutional layers, fully connected layers, alongside with batch-normalization and pooling layers. We do not limit the non-linear functions employed by the neurons, and only considered classification networks terminating with a softmax function.

### 2 Related work

The two principle techniques that we employ are *cascaded classification* and *confidence estimation*. Cascaded classification is suggested in the seminal work of [VJ01]. As opposed to voting or stacking ensembles in which a classification is derived from the outputs of multiple experts (e.g., majority), the decision in a cascaded architecture is based on the last expert. Uncertainty measures of classifiers are discussed in [CSTV95, SSV00]. These works address the issue of the degree of confidence that a classifier has about its output. We elaborate on recent usage of these techniques hereinafter.

#### 2.1 Multi-stage classification

A cascaded neural network architecture for computer vision is presented in [HCL+17]. In their work, as the of the input increases, the evaluation is performed with increased resolution and increasing the number of component DNNs in the cascade. The work of [WYDG17] presented the SkipNet approach, where each input can take a path composed of a subset of layers of the original architecture. Skipping of layers requires training of switches that decide whether skipping of layers takes place. The work by [LDBC+17] presented the idea of early stopping in a setting in which the cascaded DNNs are distributed among multiple devices.

Reinforcement learning is employed in [OLO17] in a cascade of meta-layers to train controllers that select computational modules per meta-layer.
2.2 Confidence estimation

Confidence of an assembly of algorithms is investigated by Fagin et al. [LBG+16] in a general setup. Fagin et al. define instance optimality and suggest to terminate the execution according to a criterion based on a threshold.

Rejection refers to the event that a classifier is not confident about its outcome, and hence, the output is rendered unreliable. In [GEY17] a selective classification technique in which a classifier and a rejection-function are trained together. The goal is obtain coverage (i.e., at least one classifier does not reject) while controlling the risk via rejection functions. They proposed a softmax-response mechanism for deriving the rejection function and discussed how the true-risk of a classifier (i.e., the average loss of all the non-rejected samples) can be traded-off with its coverage (i.e., the mass of the non-rejected region in the input space). Our work adopts the usage of the softmax response as a confidence rate function, however it differs in a way we apply the confidence threshold. Namely, we propose a cascade of classifiers that terminates as soon as a desired confidence threshold is reached.

2.3 Combined approach: multi-stage, confidence based inference

The work by [CZC+10] presented an additive ensemble machine learning approach with early exits in a context of web document ranking. In the additive approach, the sum of the outputs of a prefix of the classifiers provides the current output confidence.

The work by [TMK16] presented the BranchyNet approach, in which a network architecture has multiple branches, each branch consists of a few convolutional layers terminated by a classifier and a softmax function. Confidence of an output vector \( y \) in BranchyNet is derived from the entropy function \( \text{entropy}(y) = -\sum_c y_c \log y_c \). Our approach attempts to reduce the amount of computation that takes place outside the "main path" so that computations that take place in rejected branches are a negligible fraction of the total of the computation. In addition, we derive the confidence by taking the maximum over the softmax. Finally, in [TMK16], automatic setting of threshold levels is not developed.

Cascaded classification with dedicated linear confidence estimations (rather than softmax) appears in the Conditional Deep Learning (CDL) of [PSR16]. Cascaded classification with confidence estimation appears also in the SACT mechanism of [FCZ+17], an extension of the prior work by [Gra16] that deal with recurrent neural networks. Confidence estimation is based on summation of the halting scores. Computation is terminated as soon the cumulative halting score reaches a threshold. An interesting aspect of the SACT architecture, is the feature of spatial adaptivity. Namely, different computational efforts are spent on different regions of the input image.

The recent work by [BWDS17] proposes an adaptive-early-exit cascaded classification architecture. The computation may terminate after each convolutional layer. For every convolutional layer \( k \), a special decision function \( \gamma_k \) is trained to whether an exit should be chosen. We conjecture that it is hard to decide whether an exit should be chosen based on a convolutional layer without using a classifier. Indeed, in [BWDS17], a speedup of roughly 10% is achieved for every doubling of the accuracy loss.

3 Cascaded Inference (CI)

Table 1 depicts the parameters and notations used in this paper.

3.1 Cascaded architecture

A cascade of DNNs is a chain of convolutional layers with a branching between layers to a classifiers (see Figure 1). Early termination in cascaded DNN components means that intermediate feature maps are evaluated by classifiers. These classifiers attempt to classify the feature map, and output a confidence measurement of their classification. If the confidence level is above a threshold, then execution terminates, and the classification of the intermediate feature map is output. See Figure 1 for an example of a cascaded architecture based on three convolutional layers. In our experimentation, we employ ResNet-modules [HZRS15a] as component DNNs in our cascade.
Table 1: Notations and definitions used in this paper

| Notation | Domain | Semantics |
|----------|--------|-----------|
| \( n_e \) | \( \mathbb{N} \) | Number of training epochs |
| \( n_m \) | \( \mathbb{N} \) | Number of component DNNs in the cascade |
| \( n_c \) | \( \mathbb{N} \) | Number of classes in the classification task |
| \( n \) | \( \mathbb{N} \) | Number of ResNet-blocks in a ResNet-module |
| \( T \) | | Labeled training set, containing pairs of inputs and corresponding labels |
| \( M \) | | Set of component DNNs that form a cascade (\(|M| = n_m\)) |
| \( M_m \) | | The \( m \)th component in the cascade, \( m \in \{0, \ldots, n_m - 1\} \) |
| \( \theta_{\text{conv}} \) | | Weights and biases of the convolutional layers in component \( M_m \) |
| \( \Theta_{\text{conv}} \) | | Weights and biases of the convolutional layers in the cascade |
| \( \Theta_{\text{fc}} \) | | Weights and biases of the fully connected layers of the cascade |
| \( \theta_{\text{fc}} \) | | Weights and biases of the fully connected layers of component \( M_m \) |
| \( \text{out}_m(x) \) | \( \{0, \ldots, n_c - 1\} \) | Class predicted by component \( M_m \) for input \( x \) |
| \( \delta_m(x) \) | \([0, 1]\) | Confidence output by component \( M_m \) for input \( x \) |
| \( \hat{\delta}_m \) | \([0, 1]\) | Confidence threshold of component \( m \) |

Each component in a cascaded architecture consists of convolutional layers followed by a branching that leads to (1) a classifier, and (2) the next component.

![Image](Figure_1.png)

Figure 1: An example of a cascaded architecture of three component DNNs with early termination. A cascade of convolutional layers \((\text{CONV}_0, \ldots, \text{CONV}_2)\) ends with a fully connected layer \(\text{FC}_2\). Early termination is enabled by introducing fully connected networks \(\text{FC}_i\) after convolutional layers. The output of each fully connected layer consists of a classification \(\text{out}_i\) and a confidence measurement \(\delta_i\).

### 3.2 Early termination based on confidence threshold

The usage of the threshold for determining early termination in the cascade is listed as Algorithm 1.

The algorithm applies the component DNNs one by one, and stops as soon as the confidence measure reaches the confidence threshold of this component. This approach differs from previous cascaded architectures in which a combination (e.g., sum) of the confidence measures of the components is used to control the execution \([\text{FCZ}+17, \text{CZC}+10]\).

**Algorithm 1**  
\(\text{CL}(M, \hat{\delta}, x)\)- An algorithm for sequential execution of DNN components in a cascaded architecture. Early termination takes place as soon as the confidence level reaches the confidence threshold. The parameters are: the cascade architecture \(M\), the confidence threshold \(\hat{\delta} = (\hat{\delta}_0, \ldots, \hat{\delta}_{n_m - 1})\), and the input \(x\).

1. For \( m = 0 \) to \( n_m - 1 \) do
   
   (a) \((\text{out}_m(x), \delta_m(x)) \leftarrow M_m(x)\)

   (b) If \(\delta_m(x) \geq \hat{\delta}_m\) then return \(\text{out}_m(x)\)

2. Return \(\text{out}_{n_m - 1}(x)\)
3.3 Softmax confidence

Every classifier consists of one or more fully connected layers followed by a softmax function. Let \( z_m \in \mathbb{R}^r \) denote the input to the softmax function in the \( m \)'th classifier of the cascade. Let \( s_m \in [0,1]^n \) denote the softmax value in the \( m \)'th classifier. The softmax value is defined as follows:

**Definition 3.1 (softmax).** \( s_m[i] = \frac{e^{z_m[i]}}{\sum_{c=0}^{n_c} e^{z_m[c]}} \)

The output \( out_m \in \{0, ..., n_c - 1\} \) and the confidence measure \( \delta_m \in [0,1] \) with respect to this output are defined as follows:

**Definition 3.2.** \( out_m \triangleq \arg \max_c \{s_m[c] \mid 0 \leq c \leq n_c - 1\} \)

**Definition 3.3.** \( \delta_m \triangleq \max_c \{s_m[c] \mid 0 \leq c \leq n_c - 1\} \)

4 Training procedure

In this section we present the training procedure of the DNN components and classifiers.

Consider a cascaded architecture \( n_m \) components. We denote this cascade by \( M = (M_0, \ldots, M_{n_m-1}) \), where \( M_m \) denotes the \( m \)'th component in the cascade. Let \( \theta_{conv_m}, \theta_{fc_m}, \text{resp.} \) denote the weights and biases of the convolutional (fully connected, resp.) layers in component \( M_m \). Let \( \Theta_{conv} = \{\theta_{conv_0}, \ldots, \theta_{conv_{n_m-1}}\} \) and \( \Theta_{fc} = \{\theta_{fc_0}, \ldots, \theta_{fc_{n_m-1}}\} \) denote the weights and biases of the convolutional layers and fully connected layers, respectively.

Let \( L_M(out_m, T) \) denote a loss function of the cascade \( M \) with respect to the output of the \( m \)'th component, averaged over the labeled dataset \( T \). In order to train the cascade \( M \), we propose a backtrack-training algorithm \( \text{BT}(M, T, n_c) \). We emphasize that the training procedure first optimizes all the convolutional weights together with the weights of the last fully connected layers. Only then, do we optimize the weights of the fully connected levels of the remaining components, one by one. Our approach differs from previous training procedures [MK16, WYG17] in which the loss functions associated with all the classifiers were jointly optimized.

**Algorithm 2 BT(\( M, T, n_c \)) - An algorithm for performing a backtrack training of the cascade \( M = (M_0, \ldots, M_{n_m-1}) \), each component DNN trains for \( n_c \) epochs over the training set \( T \). The algorithm outputs the trained weights of the cascade \( M \)**

1. Optimize \( \Theta_{conv} \cup \theta_{fc_{n_m-1}} \) with \( L_M(out_{m-1}, T) \) for 1.25\( n_c \) epochs
2. For \( m = 0 \) to \( n_m - 2 \)
   3.1 Optimize \( \theta_{fc_m} \) with \( L_M(out_m, T) \) for \( n_c \) epochs
3. Return \( \Theta_{conv} \cup \Theta_{fc} \)

In our experiments, we noticed that the “long path” of the cascade required a larger number of training epochs. This why the number of training epochs in Line[1] of BT(\( M, T, n_c \)) is 1.25\( n_c \).

5 Setting of confidence threshold

In this section we present an automatic methodology for setting the confidence threshold \( \hat{\delta}_m \) per component \( M_m \). Early termination is chosen if the confidence level reaches the threshold. The important feature of the automatic setting of the confidence thresholds is that one can change them on the fly during the inference stage.

Let \( T_m(\delta) \) denote the subset of inputs for which the confidence measure of the \( m \)th component is at least \( \delta \).

\[
T_m(\delta) \triangleq \{(x,y) \mid \delta_m(x) \geq \delta\}.
\]
We performed the training with respect to algorithm 2. A simple data augmentation was employed. We transformed the regular dataset...& architecture used for CIFAR-10 and CIFAR-100 are identical except for the last FC layer in each classifier, containing 10 and 100 outputs respectively, which caused a relatively negligible addition in parameter count. The weights were initialized at random from \( N(0, \sqrt{2/k}) \) where \( k \) is the number of inputs to a neuron, as proposed by \[HZRS15b]. All the models used cross-entropy loss regularized by an L2 loss with a coefficient 1e-4. Following the practice of \[IS15\], no dropout was applied. The CIFAR and the SVHN models were trained with Stochastic Gradient Descent (SGD) for 160 and 50 epochs per classifier, respectively. Learning rate was scheduled as described in \[HZRS15a].
6.2 Confidence threshold effect

We trained the CI-ResNet(18) model and evaluated its performance using various \( \epsilon \) values. The tradeoff between the test-accuracy and the number of MACs required for a single inference is shown in Figure 3. The MAC counts were obtained analytically by summing up the linear operations in the convolutional layers and the fully connected layers, excluding activations and batch normalization. Quantitative results appear in Table 2. Similar gains are reported using SkipNet with ResNet110 [WYDG17]. Note however that our approach does not incur extra computation for gating.

Table 2: Cascaded inference with early termination - Accuracies of a cascade of \( i \) components is listed in columns \( M_{0,...,i-1} \). Accuracies and speedups using early termination based on confidence thresholds \( \delta_m(\epsilon) \) are depicted for five values of \( \epsilon \). Speedup is relative to the computational effort of \( M_{0,1,2} \) component.

| Dataset   | \( M_0 \) | \( M_{0,1} \) | \( M_{0,1,2} \) | \( \epsilon = 0\% \) | \( \epsilon = 1\% \) | \( \epsilon = 2\% \) | \( \epsilon = 4\% \) | \( \epsilon = 8\% \) |
|-----------|-----------|-----------|-----------|----------------|----------------|----------------|----------------|----------------|
| CIFAR-10  | 77.5\%    | 81.4\%    | 93.1\%    | 93.1\%         | 92.7\%         | 91.9\%         | 91.1\%         | 86.4\%         |
| CIFAR-100 | 48.1\%    | 50.0\%    | 70.5\%    | 70.5\%         | 70.65\%        | 70.5\%         | 70.3\%         | 69.8\%         |
| SVHN      | 89.8\%    | 85.2\%    | 97.0\%    | 97.0\%         | 95.6\%         | 94.0\%         | 91.3\%         | 89.8\%         |

6.3 Softmax as a confidence measure

For the CI-ResNet(18) architecture, we measured the accuracy \( \alpha_m(\delta) \) (see definition in Section 5) of each classifier independently. This time, the \( \alpha_m(\delta) \) was determined with respect to the test-set rather than to the training set. The plots in Figure 4 show how the choice of the threshold provides a control over the test accuracy. In addition, we examined the frequency of the different \( \delta \) values, which is shown as a bar-plot in Figure 4. The distribution of the first two components of the cascade is relatively uniform. The distribution of the confidences of the last classifier has no importance since
We showed that using a softmax output as a confidence measure in a cascade of DNNs can provide a \( \alpha \in \{20\%, \ldots, 1\%, 0\%\} \) in our inference approach the confidence threshold of the last classifier is set to \( \hat{\delta}_{\alpha_{m-1}} = 0 \). Note that the range of \( \alpha_m(\delta) \) starts with the accuracy of \( M_m \) and ends with the accuracy that corresponds to the highest confidence measure. The almost linear behavior of \( \alpha_m(\delta) \) as a function of \( \delta \) justifies basing the confidence threshold on the softmax output.

![Figure 3: Cascaded inference with early termination](image)

*Figure 3: Cascaded inference with early termination* test accuracy vs. average number of MAC operations per inference. The curves correspond to confidence threshold vectors chosen w.r.t. \( \epsilon \in \{20\%, \ldots, 1\%, 0\%\} \).

![Figure 4: Softmax as a confidence measure](image)

*Figure 4: Softmax as a confidence measure.* The line plots show the accuracy \( \alpha_m(\delta) \) of each classifier in the cascade independently. The bar plot presents the frequency of the different confidence levels sampled over the test set. All plots were obtained by separately testing the three component DNNs of CI-ResNet(18) architecture w.r.t. the test sets of the CIFAR and the SVHN datasets.

### 7 Discussion and future work

As a further research, cascading can be applied to RNNs [BBPP15] or alternatively, the impact of depth of feedforward DNNs on the confidence estimation can be investigated. A gap between the allowed accuracy degradation (\( \epsilon \)) and the actual test accuracy degradation was especially evident in the CIFAR-100 dataset. We believe that if one determines the thresholds with respect to a validation set, rather than the training set, this gap will be reduced.

From a digital hardware point of view, we see two interesting directions to investigate in the context of cascaded inference. First, the impact on cache memory performance can be observed as a function of the confidence threshold adjustment. Second, an innovative hardware architecture can be proposed to support cascaded inference and to provide high throughput via allocation of resources to each component DNN and taking advantage of the locality at each component.

### 8 Conclusions

We showed that using a softmax output as a confidence measure in a cascade of DNNs can provide a \( \times 1.2 \) to \( \times 2 \) speedup at a cost of 1\% of classification accuracy loss. This approach is both simple and requires no retraining when the confidence thresholds must be adjusted after the network was trained.
In addition, fascinating properties of the softmax function as a confidence measure were revealed. We showed that if trained properly, a cascade of DNN components can reliably indicate its confidence level directly through the softmax output.

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