Competition between Hund-Rule Coupling and Kondo Effect

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We investigate a problem about the competition between the hybridization and the Hund-rule coupling by applying the Wilson numerical renormalization-group method to the extended Kondo model where the impurity spin interacts via the Hund-rule coupling, with an extra spin which is isolated from the conduction electrons. It is shown that the Hund-rule coupling is an irrelevant perturbation against the strong coupling fixed point. However, the Hund-rule coupling decreases the characteristic energy $T_K$ drastically to the lower side and the irrelevant operator, which describes the low energy physics, takes a form of ferromagnetic exchange interaction between the extra spin and the Kondo resonance states because of the existence of the Hund-rule coupling.

KEYWORDS: Hund-rule coupling, Kondo effect, numerical renormalization-group, anisotropic hybridization, plural electrons at localized orbitals, transition between high-spin and low-spin state, Uranium based heavy fermion, Ni-doped High-$T_c$ cuprate

§1. Introduction

One of the most important question concerning heavy fermion systems, which exhibit the exotic phenomena such as anisotropic superconductivity and extremely weak antiferromagnetism is to understand how the low energy quasiparticle states are formed, in other words, how the high energy incoherent states are reflected in the low energy physics. Many theoretical attempts have been made to include higher Crystalline Electric Field (CEF) effects both in single impurity and lattice case.

For Ce based compounds this anisotropy is reflected in the formation of highly renormalized quasiparticle band only through the one-body effect: its typical example has been put forward in Ref. 9 to explain anomalous properties observed in CeNiSn. On the other hand, for U based compounds, where $(5f)^2 \text{ or } (5f)^3$ configuration is realized, the anisotropic hybridizations may be reflected on the quasiparticles through a many-body effect, because at least two characteristic energy scales and the Hund-rule coupling are involved in the problem. The “spin of localized electron” tends to be quenched by the Kondo effect, while the Hund-rule coupling stabilizes the high-spin state. Then a competition between the two effects plays a crucial role in determining the low energy physics.

Such a competition is likely to be realized in a variety of physical situations: For example, (i) the magnetic susceptibility in UPd$_2$Al$_3$ can be fitted by a certain CEF scheme under the tetravalent U state $(5f)^2$ configuration, while the Fermi surface measured by dHvA effect is in good agreement with the band structure calculation with the trivalent state $(5f)^3$ configuration, and the photoemission data are reported to be explained by $(5f)^3$ configuration, (ii) in Ni-doped High-$T_c$ cuprates, the spin moment of Ni$^{2+}$ changes from the high-spin to low-spin state with increasing the doping rate of holes, and so on.

In this paper, we discuss how the competition between the effect of anisotropic hybridization and the Hund-rule coupling affects the Kondo screening on the basis of the minimal model including these features. In §2, starting from an extended Anderson model, we derive the usual Kondo exchange model with the extra spin interacting via the Hund-rule coupling. In §3, we discuss the effect of the Hund-rule coupling on the Kondo effect of the model derived in §2 by means of the numerical renormalization-group method. In the final section, we summarize the results and discuss their implications to the realistic problem.

§2. Model

When we discuss the behavior of magnetic impurity in real metals, we have to deal with a degenerate Anderson model including the Hund-rule coupling and CEF as well as the direct Coulomb interaction. In addition to this, we also have to take into account the anisotropy of hybridization specific to each orbitals split by CEF.

In this paper, we make the following simplification to a generalized Anderson model.

- The effect of CEF is implicitly included in such a way that the hybridizations for each orbitals are different according to the guide of the point-group theory.
- The energy levels of those orbitals are assumed to
be the same for simplicity.

- Of these orbitals only the two orbitals are retained.
- The hybridization for one orbital is finite, while for the other one is neglected as the limiting case for different hybridizations.
- Both the Hund-rule coupling \( J_H \) and the direct Coulomb interactions \( U \), which are independent of orbitals, are retained.

Thus, we are left with an extended Anderson model as,

\[
H = \sum_{k\sigma} \epsilon_k a_{k\sigma}^\dagger a_{k\sigma} + H_{\text{mix}} + H_f, \tag{2.1}
\]

\[
H_{\text{mix}} = \sum_{k\sigma} \left( V_k a_{k\sigma}^\dagger f_{i\sigma} + \text{h.c.} \right), \tag{2.2}
\]

\[
H_f = E_f \sum_{m\sigma} f_{m\sigma}^\dagger f_{m\sigma} + \frac{U}{2} \sum_{m\sigma \sigma'} f_{m\sigma}^\dagger f_{m\sigma}^\dagger f_{m'\sigma'} f_{m'\sigma'} + \frac{J_J}{2} \sum_{m\sigma \sigma'} \sum_{m'\sigma'} f_{m\sigma}^\dagger f_{m'\sigma'}^\dagger f_{m'\sigma'} f_{m'\sigma}, \tag{2.3}
\]

where \( f_{m\sigma}^\dagger \) denotes the creation operator for the localized electron with spin \( \sigma \) \((\uparrow, \downarrow)\) on the orbital \( m \) \((1, 2)\) deformed by the CEF, and \( a_{k\sigma}^\dagger \) for conduction electron with the wave number \( k \) and the spin \( \sigma \) on the band hybridizing only with the localized orbital \((m = 1)\) via the hybridization \( V_k \).

We can rewrite the local part \( H_f \), (2.3), of the Hamiltonian as

\[
H_f = E_f n_f + \frac{U}{2} (n_f^2 - n_f) - J_H (S_f^2 + \frac{1}{4} n_f^2 - n_f). \tag{2.4}
\]

in terms of the number and the total spin of localized electrons defined by

\[
n_f = \sum_m n_{mf} = \sum_m \sum_{\sigma} f_{m\sigma}^\dagger f_{m\sigma}, \tag{2.5}
\]

\[
S_f = \sum_m S_{mf} = \frac{1}{2} \sum_{\sigma \sigma'} f_{m\sigma}^\dagger \sigma \cdot \sigma' f_{m\sigma'}, \tag{2.6}
\]

where \( \sigma \) is the vector of the Pauli matrices.

In order to simplify this model furthermore, we restrict our discussions within the case of strong Coulomb interaction. Then we can treat the hybridization term \( H_{\text{mix}} \) in eq. (2.1) within the second order perturbation by restricting the Hilbert space in such a way that the number of localized electrons on ground state is given by \( \langle n_f \rangle = 2 \). Such a restriction is valid when \( (E_f + U)(E_f + 2U) < 0 \) and \( E_f, U \gg J_H \). Thus we can get the effective Hamiltonian (as shown in Appendix):

\[
H = \sum_{k\sigma} \epsilon_k a_{k\sigma}^\dagger a_{k\sigma} - J_H (S_f^2 + \frac{1}{4} n_f^2 - n_f) + J_J \sum_{f,i} |f\rangle \langle i| \langle f | S_{1f} | i \rangle + \sum_{k\sigma \sigma' k' \sigma'} a_{k\sigma}^\dagger \sigma \cdot \sigma' a_{k' \sigma'} \tag{2.7}
\]

where \(|i\rangle\) and \(|f\rangle\) are initial and final states of the localized electrons and the exchange coupling \( J \) is given by

\[
J = \left[ \frac{1}{E_f + 2U} - \frac{1}{E_f + U} \right] |V_{Kf}|^2 > 0. \tag{2.8}
\]

Now we consider the two limiting cases, (i) \( J_H \ll J \) and (ii) \( J_H \gg J \), in which the second term in (2.7), the Hund-rule coupling, takes a simple form.

(i) \( J_H \ll J \)

We can treat the Hund-rule coupling as a perturbation on the Hamiltonian eq. (2.7) with \( J_H = 0 \). We can show the following identities valid in the restricted Hilbert space such that \( \langle n_{1f} \rangle = \langle n_{2f} \rangle = 1 \),

\[
\sum_{i,f} |f\rangle \langle i| \langle f | S_{1f} | i \rangle = S \tag{2.9}
\]

\[
-J_H \left( S_f^2 + \frac{1}{4} n_f^2 - n_f \right) = -2J_H S \cdot S' - \frac{J_H}{2} \tag{2.10}
\]

where \( S \) and \( S' \) are the spin matrices of \( S = S' = 1/2 \), respectively. Thus, we get the \( S = 1/2 \) s-d exchange Hamiltonian interacting with a spin of \( S' = 1/2 \) via the Hund-rule coupling:

\[
H = \sum_{k\sigma} \epsilon_k a_{k\sigma}^\dagger a_{k\sigma} + J_S \sum_{k\sigma k' \sigma'} a_{k\sigma}^\dagger \sigma \cdot \sigma' a_{k' \sigma'} - 2J_H S \cdot S'. \tag{2.11}
\]

(ii) \( J_H \gg J \)

In this case, the spin triplet state is realized as a local ground state, and then the matrix element for the localized electron, interacting with conduction electrons, can be related to the spin of \( S = 1 \) as

\[
\sum_{i,f} |f\rangle \langle i| \langle f | S_{1f} | i \rangle = \frac{1}{2} S. \tag{2.12}
\]

Thus we can get the \( S = 1 \) s-d exchange Hamiltonian as a model in this limit:

\[
H = \sum_{k\sigma} \epsilon_k a_{k\sigma}^\dagger a_{k\sigma} + \frac{J_S}{2} S \sum_{k\sigma k' \sigma'} a_{k\sigma}^\dagger \sigma \cdot \sigma' a_{k' \sigma'}. \tag{2.13}
\]

It is noted that the exchange coupling constant is devied by a factor 2 because only the half of spin \( S \), i.e., \( S_{1f} \), interacts with the conduction electrons. Therefore the characteristic energy \( T_K/D \sim e^{-1/\rho J} \) for this model is far less than \( T_K/D \sim e^{-1/\rho J} \) for a usual \( S = 1 \) exchange model unless \( T_K^0 \) is comparable to \( D \). Here, \( D \) is half of the bandwidth and \( \rho \) is the density of states of conduction electrons at the Fermi level.

\section{Effect of Hund-rule coupling}

We investigate the truncated model (2.1) in order to study the effect of the Hund-rule coupling on the Kondo effect. We put no restriction on the couplings \( J \) and \( J_H \), despite the model is derived under the condition \( J_H \ll J \). Practically, we calculate the RG flow of excitation energies and the temperature dependence of the magnetic susceptibility for the impurity spin by using the Wilson numerical renormalization-group (NRG) method \cite{Wilson1975}.

In order to use NRG method, we translate the kinetic energy part in the Hamiltonian (2.1) into a tridiagonal form as usual:

\[
H_N = \Lambda^{(N-1)/2} \sum_{\sigma \sigma'} \sum_{n=0}^{N-1} \Lambda^{-n/2} \left( f_{n\sigma}^\dagger f_{n+1\sigma} + \text{h.c.} \right) + J_S \sum_{\sigma \sigma'} f_{0\sigma}^\dagger \sigma \cdot \sigma' f_{0\sigma'} - J_H S \cdot S'. \tag{3.1}
\]
electron of the Wannier representation with radial extent $k_F^{-1} \Lambda^{n/2}$, and the energy scales are expanded by a factor $\Lambda^{(N-1)/2}$ and remeasured in a unit of $(1 + \Lambda^{-1})D/2$. We use $\Lambda = 2$ throughout this paper.

The flow of excitation energies is shown in Fig. 1 for $J = 0.4$ and $J_H = 0.04$. The labels attached at the right side of each lines denote the states with the number of particle $Q$ and the total spin $S$ as shown in the figure. The excitation energies are measured from that of the state, $e_0$ for even iterations and $\alpha_1$ for odd iterations, as it will turn out to be convenient later.

It is remarked that the flow lines gradually converge to those at the fixed point. We can reproduce the level structure of the excitation energies at the fixed point by combining with one-particle excitations which are determined by the exchange Hamiltonian (3.1) with $J = \infty$ and $J_H = 0$; in other words, the couplings $J$ and $J_H$ approach $J^* = \infty$ and $J_H^* = 0$, respectively, as the renormalization step proceeds. In order to see this, we plot the flow lines denoted by dots for $J = 20.0$ and $J_H = 0$.

The tendency that the exchange coupling is renormalized to the strong coupling one and the Hund-rule coupling is weaken can be seen also by the perturbational renormalization-group argument. Indeed, the lowest non-vanishing renormalization of $J$ and $J_H$ are given by the scaling equations

\begin{equation}
\frac{dJ}{d \ln (D/D_0)} = -J^2, \tag{3.2}
\end{equation}

\begin{equation}
\frac{dJ_H}{d \ln (D/D_0)} = J^2 J_H, \tag{3.3}
\end{equation}

which arises from the processes shown in Fig. 2. Solutions of eqs. (3.2) and (3.3) are given by

\begin{equation}
J = \frac{J_0}{1 + J_0 \ln (D/D_0)}, \tag{3.4}
\end{equation}

\begin{equation}
J_H = J_0^0 e^{-(J-J_0)}. \tag{3.5}
\end{equation}

As a result, the exchange coupling $J$ approaches the strong coupling and the Hund-rule coupling $J_H$ is renormalized rapidly downward simultaneously.

The way to approach the fixed point is quite gradual as compared with the way in the case of $S = 1/2$, because the irrelevant operator consists dominantly of the ferromagnetic exchange interaction due to the presence of the Hund-rule coupling. Actually, we can describe the excitations near the fixed point by the effective Hamiltonian with the ferromagnetic exchange interaction,

\begin{equation}
H_N = \Lambda^{(N-1)/2} \left[ \sum_{l=1}^{N-1} \sum_{\sigma} \Lambda^{-n/2} (f_{n+1}^\dagger f_n^\sigma + \text{h.c.)} - J_{\text{eff}}(N) S' \cdot \sum_{\sigma \sigma'} f_{\sigma \sigma'}^\dagger \sigma_{\sigma'}^\alpha f_{\sigma'}^\alpha \right], \tag{3.6}
\end{equation}

where $f_{\sigma \sigma'}^\dagger$ represents the quasiparticle associated with Kondo resonance state, reflecting the singlet formation between the spin $S$ and the conduction electrons. It is noted that the spin $S'$, which is left after the spin $S$ is compensated by the conduction electrons, couples with the conduction electrons at the "site" $n = 1$ because the conduction electrons at $n = 0$ are wiped out due to the singlet formation with $S$. We can estimate the $N$-dependence of the ferromagnetic exchange coupling by using the first order perturbation with respect to $J_{\text{eff}}(N)$. $f_{\sigma \sigma'^\dagger}$ is expressed in terms of the eigenstates $g_l$ and $h_l$ of the $N$-site free chain Hamiltonian, the first term in eq. (3.6), as

\begin{equation}
J_{\text{eff}}(N) = \frac{\alpha J_0}{1 + J_0 \ln (\Lambda (N-N_0)/2)}, \tag{3.10}
\end{equation}

where we put $T/D = \Lambda^{-(N-1)/2}$ and fitting parameters are put $\alpha = 1.306$, $J_0 = 0.3027$ and $N_0 = 20$, respectively. We also confirm that the higher excitations can be fitted by means of the effective exchange $J_{\text{eff}}(N)$.

This result implies that $J$ and $J_H$ are renormalized as $J \to \infty$ and $J_H \to 0$ even before $N \sim 25$ iteration and the effective ferromagnetic exchange interaction $J_{\text{eff}}(N)$ between the conduction electrons, and after $N \sim 25$ iteration the spin $S'$ rules the excitations as the irrelevant operator around the fixed point $J = \infty$, $J_H = 0$, and $J_{\text{eff}}(N) = 0$. This is understood as follows. When $J(T) \gg J_H(T)$, "local" spin $S'$ forms singlet with "0th" conduction electron at "0th" site and there are two degeneracies with respect to the degrees of freedom of the extra spin $S'$. If the virtual hopping process between the "1st" and "0th" site is taken into account, the renormalized Hund-rule coupling $J_H(T)$ lifts the degeneracy leaving the state, where the spin $S'$ and the spin of conduction electron at the "1st" site are parallel, be lower energy than that of anti-parallel. This mechanism is similar to that discussed in Ref. 20 for the origin of anomaly of multichannel Kondo effect.

Next, we discuss the temperature dependence of the susceptibility for the impurity spin which is defined by

\begin{equation}
T_{\chi_{\text{imp}}} = \lim_{N \to \infty} \left[ \frac{\text{Tr} S_{\sigma N}^2 e^{-\beta_H H_N}}{\text{Tr} e^{-\beta_H H_N}} - \frac{\text{Tr} S_{\sigma N}^2 e^{-\beta_H H_N^0}}{\text{Tr} e^{-\beta_H H_N^0}} \right], \tag{3.11}
\end{equation}
where \( S_{z,N} \) is the \( z \)-component of the total spin

\[
S_{z,N} = S_z + S'_z + \sum_{n=0}^{N} \sum_{\sigma} \frac{1}{2} f_{n\sigma}^\dagger \sigma_{\sigma} f_{n\sigma},
\]

and the second term of (3.11) represents the contribution of non-interacting system, and \( \beta_N \) is defined as

\[
\beta_N = \Lambda^{-(N-1)/2}/T.
\]

By setting \( T = T_N \equiv \Lambda^{-(N-1)/2} \), we can determine the susceptibility with a good accuracy because the excited states with the energy \( \beta_N \sim 1 \), which contribute dominantly to the thermodynamic quantities are obtained with a good accuracy in NRG calculation.

The temperature dependence of the susceptibility \( \chi_{\text{imp}} \) is shown in Fig. 4. One can see that the Hund-rule coupling makes the characteristic energy, \( T_K \), lower and the \( T \)-dependence of \( T \chi_{\text{imp}} \) in the limit of \( J_H \to \infty \) is equivalent to that of \( S = 1 \) exchange model, eq. (2.13), with \( J/2 = 0.2 \), half of \( J = 0.4 \) in the model (3.1). It is important to note that the characteristic energy \( T_K \) decreases remarkably by three orders of magnitude even if \( J_H \sim J \). \( T \chi_{\text{imp}} \) approaches toward \( 1/4 \), the value for the case of free spin \( S = 1/2 \), as temperature decreases well below \( T_K \). This is consistent with the fact that the fixed point is given by \( J^* = \infty \) and \( J_H^* = 0 \). It is remarked that the fixed point remains the same as the case in the absence of the Hund-rule coupling while the Hund-rule coupling changes \( T_K \) drastically to the lower value. The way to approach the fixed point becomes gradual with the increase of \( J_H \) as shown in Fig. 4. In the case \( J_H = 0 \), \( T \chi_{\text{imp}} \) obviously approaches the fixed point in the same way as the conventional antiferromagnetic Kondo effect, apart from the contribution of extra spin \( S' \), i.e., \( 1/4 \). In the case \( J_H \gg J \), on the other hand, the temperature dependence of \( T \chi_{\text{imp}} \) becomes coincident with that of \( S = 1 \) Kondo effect.

We can show that the way to approach the fixed point of \( S = 1 \) Kondo effect is equivalent to that of \( S = 1/2 \) ferromagnetic Kondo effect as shown in Fig. 5, in which the temperature dependence of \( T \chi_{\text{imp}} \) for \( J = 0.4 \) and \( J_H = 0.04 \) in eq. (3.1) is compared with the magnetic \( S = 1/2 \) Kondo effect with \( J = -7.4 \). This result shows that the ferromagnetic exchange interaction behaves as an irrelevant operator in consistent with the previous discussions about the excitation level scheme in the presence of the Hund-rule coupling \( J_H \).

4. Conclusions and Discussions

We have investigated the effect of the Hund-rule coupling by using the extended Kondo model in which the impurity spin interacts with the conduction electrons and the other spin, which itself is decoupled from conduction electrons, via the Hund-rule coupling as well. We have derived this model from the generalized Anderson model which has two orbitals, where the one orbital has more localized character than the other and its hybridization with the conduction electrons is neglected. It means that we restrict our concern within the temperature range above the lower characteristic energy corresponding to the smaller hybridization. We have concluded that Kondo effect is always dominant even if the exchange coupling is much smaller than the Hund-rule coupling. However, we have shown two prominent effects of the Hund-rule coupling beyond the conventional Kondo model.

First, the characteristic energy is drastically decreased from the value of the usual \( S = 1/2 \) Kondo model to that of the \( S = 1 \) Kondo model with half of the exchange coupling, as the Hund-rule coupling increases. This gives us a hint on general question how the lower characteristic energy scales are realized in uranium based heavy fermions with the hybridization larger than that of the Ce based compounds. Namely, if there exists a localized orbital, which less hybridizes with ligand conduction electrons due to the symmetry reason, it can work to reduce the characteristic energy scale of the localized orbital, which well hybridizes with conduction electrons, through the Hund-rule coupling.

Second, the Hund-rule coupling turns into the ferromagnetic exchange interaction between the extra spin \( S' \) and conduction electrons at “1st” site. In other words, the quasiparticle associated with the Kondo resonance state, consisting of the spin \( S \) and the conduction electrons, interact with \( S' \) in the ferromagnetic exchange form. This irrelevant operator does not change the fixed point but affects the way to approach the fixed point. Namely, it takes the form of the ferromagnetic Kondo effect as the Hund-rule coupling increases. However, such ferromagnetic Kondo effect may not occur in real systems because of at least the following two reasons: (i) if we take into account the direct interaction between the conduction electrons and the localized spin \( S' \), no matter how small it is, the antiferromagnetic Kondo effect leads to form the another Kondo resonance state against the Hund-rule coupling. (ii) if we consider more complicated CEF model, the internal degrees of freedom at the impurity site cannot be described as the simple spin, e.g. in the case of CEF singlet for \( f^2 \) configuration, the irrelevant operator may have the tensor form in general.

The results suggest that the Kondo resonance states are individually formed in each channels and the interactions between them are irrelevant. This is contrary to the picture that the high-spin state due to the Hund-rule coupling is compensated by the conduction electrons in multi channels. Thus, the consideration of a more realistic picture for the CEF structure and the anisotropic hybridization will give us a more solid picture for the quasiparticles.

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Appendix

Here we derive the exchange part of the effective Hamiltonian, (2.7), from the extended Anderson model, (2.1), by the second order perturbation with respect to $H_{\text{mix}}$ in eq. (2.1).

The eigenstates of the local Hamiltonian $H_f$, (2.4), with $f^n$ configuration and total $f$ spin, $S_f$, for the have the eigen energy,

$$I(n, S_f) = E_f n + \frac{U}{2} (n^2 - n) - J_H \left[ S_f (S_f + 1) + \frac{1}{4} n^2 - n \right].$$

(A.1)

When we consider $n = 2$ ground state, which is valid when $(E_f + U)(E_f + 2U) < 0$ and $E_f U >> J_H$, the Schrieffer-Wolff transformation gives us the following effective exchange interaction between the conduction electrons and the ground states $|i\rangle, |f\rangle$ of localized electrons,

$$H_{\text{ex}} = \sum_{i,f,kk\sigma\sigma'} \sum_r \left[ \frac{(i|f\rangle \langle r|f\rangle |i\rangle)}{\epsilon - \{ I(3,1/2) - I(2, S_f) - \epsilon_k \}} a_{k\sigma}^\dagger a_{k'\sigma'}. 
+ \frac{(i|f\rangle \langle i|f\rangle |i\rangle)}{\epsilon - \{ I(1,1/2) - I(2, S_f) + \epsilon_k \}} a_{k\sigma} a_{k'\sigma}. \right] |i\rangle \langle f| V_k V_{f}^*. \right. \left. \right) \right]$$

(A.2)

We can take the sum over the intermediate states $|r\rangle$ as forming a complete set. Then, with the use of the anti-commutation relations for $f_{m\sigma}$ and $a_{k\sigma}$, and the identity, $\delta_{\sigma_1,\sigma_2}\delta_{\sigma_2,\sigma_3} = (\sigma_1,\sigma_2,\sigma_3 + \delta_{\sigma_1,\sigma_2}\delta_{\sigma_2,\sigma_3})/2$, (A.2) can be reduced to the following form:

$$H_{\text{ex}} = J \sum_i |i\rangle \langle i| S_{1f} |i\rangle \sum_{k\sigma} a_{k\sigma}^\dagger \sigma_{\sigma\sigma'} a_{k'\sigma'}^\dagger + V \sum_i |i\rangle \langle i| \sum_{kk'\sigma} a_{k\sigma}^\dagger a_{k'\sigma} + \Sigma_f,$$

(A.3)

where we have taken $\epsilon = \epsilon_k = -\epsilon_{k'} = \epsilon_{k'}$, and the exchange interaction $J$, the strength of potential scattering $V$ and the self-energy $\Sigma_f$ of localized electrons are given by

$$J = \left[ \frac{1}{E_f + 2U} - \frac{1}{E_f + U} \right] |V_{k\sigma}|^2$$

(A.4)

$$V = \left[ \frac{1}{E_f + 2U} + \frac{1}{E_f + U} \right] |V_{k\sigma}|^2,$$

(A.5)

$$\Sigma_f = \sum_k \left| \frac{|V_{k\sigma}|^2}{E_f + 2U} \right| \sum_i |i\rangle n_{i1} |i\rangle,$$

(A.6)

respectively. Here we have neglected $J_H$ compared to $U$. Ignoring the potential scattering and the self-energy, we obtain the exchange Hamiltonian eq. (2.7).

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Figure Captions

Fig. 1 The flow of excitation energies for $J = 0.4$ and $J_H = 0.04$. The labels at the right side of each lines denote the states with the number of particle $Q$ and the total spin $S$.

Fig. 2 The Feynman diagrams giving the lowest non-vanishing renormalization of the Hund-rule coupling $J_H$ and the exchange coupling $J$. The solid line denotes the conduction electron, the broken line the pseudo fermion representing the spin $S$, and the dotted line the pseudo fermion of $S'$. 

Fig. 3 N-dependence of effective ferromagnetic interaction between Fermi liquid at strong fixed point and the extra spin $S'$. The dots are determined from the first excitation energies in Fig. 1. The solid line represents the scaling form (3.10).

Fig. 4 Temperature dependence of the susceptibility for the impurity spin described by (3.1) for various amount of $J_H$ with $J = 0.4$. The solid line represents $T \chi_{\text{imp}}$ for $S = 1$ Kondo model, (2.13), with $J/2 = 0.2$.

Fig. 5 Comparison between $T \chi_{\text{imp}}$ of $S = 1$ antiferromagnetic Kondo model and those of $S = 1/2$ ferromagnetic Kondo model.
Fig. 1

\[ J = 0.4, J_H = 0.04 \]
\[ J = 20.0, J_H = 0 \]

Excited energy

| \( Q \) | 0 | ±1 | ±1 | 0 | ±1 | 0 | ∓1 |
|-------|---|----|----|---|----|---|-----|
| \( S \) | 1/2 | 1 | 0 | 3/2 | 1/2 | 1/2 | 1/2 |

Fig. 2

\[ J \]
\[ J \]
\[ J_H \]
\[ \Lambda = 2, J = 0.4, J_H = 0.04 \]

Fig. 3

\[ \frac{\alpha J_0}{1 + J_0 \ln(N - N_0)/2} \]
\[ \alpha = 1.306 \]
\[ J_0 = 0.3027 \]
\[ N_0 = 20 \]

Fig. 4

\[ \Lambda = 2.0, J = 0.4 \]
\[ S = 1 \text{ Kondo model} \]
\[ (J/2 = 0.2) \]
Fig. 5

\[
\chi_{\text{imp}}(T) = 2.0
\]

- \( J = 0.4, J_H = 0.04 \)
- \( J = -7.4 \) (ferromagnetic Kodno model)