The influence of electromagnetic waves and laser fields on conductivity tensor for the case of electron - optical phonon scattering in quantum wells with a parabolic potential

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Abstract. This paper studies the influence of electromagnetic waves and laser fields on conductivity tensor with the case of electron - optical phonon scattering in quantum wells. Because electrons are confined in the quantum well in one direction, the conductivity tensor in the quantum well is different from the conductivity tensor in the bulk semiconductor. The carrier system in a quantum well is located in an electromagnetic wave field and a laser field. Using quantum kinetic equation for electrons in quantum wells in the presence of an electromagnetic wave and a laser field, from the quantum kinetic equation it is possible to calculate the conductivity tensor. The conductivity tensor expression is a function of the electromagnetic wave frequency, laser field frequency and quantum well characteristic parameters. Survey and plot the dependence of conductivity tensor on electromagnetic wave frequency, laser field frequency, laser field amplitude with GaAs / GaAsAl quantum well case.

1. Introduction
The transition from a 3-dimensional structured material system to a low-dimensional structural material system has significantly changed both qualitative and quantitative changes of the material's physical properties such as optical properties, dynamic properties (For example: Electron-phonon scattering, electron-impurity scattering, surface scattering) [1-4]. At the same time, the low-dimensional structure gives rise to many new and advanced features that the 3-dimensional structure does not have. With the scientific significance as well as the applications of the low-dimensional system in life, the low-dimensional system is widely researched today. In volume semiconductors electrons can move throughout the lattice (3-dimensional structures), but in low-dimensional systems the electron's motion will be strictly limited along one or two or three axes [5-8]. The energy spectrum of the carriers is interrupted in these directions. The quantization of the energy spectrum of carriers leads to fundamental changes in material properties such as distribution function, state density, current density, electron-phonon interaction [9-12].

In a quantum well, electrons are confined in one dimension, which leads to changes in the wave function, energy and parameters characteristic of quantum wells [13-18]. This work will calculate the conductivity tensor in a quantum well with the case of electron-optical phonon scattering using quantum
kinetic equations. The conductivity tensor will be a function of the electromagnetic wave frequency, laser field frequency, laser field amplitude and quantum well characteristic parameters. The results will be surveyed and plot the dependence of the conductivity tensor on electromagnetic wave frequency, laser field frequency, and laser field amplitude.

2. Calculating conductivity sensor
Suppose there is a quantum well in which the electron can move freely in the x0y plane and the energy is quantized in the 0z axis. For a quantum well with a parabolic potential, the wave function and the energy of the electron are given by:

\[ \psi(x) = \sum_{n} \psi_n(\vec{R}_i) W(\vec{r} - \vec{R}_i) \]  

\[ \varepsilon_{n,p_\perp} = \hbar \omega_0 \left( n + \frac{1}{2} \right) + \frac{\hbar^2 p^2_\perp}{2m} \]  

Consider the carrier system placed in an electromagnetic wave field with wave vectors:

\[ \vec{E}(t) = E(\vec{c}_{\text{rot}} + \vec{c}_{\text{rot}}^\ast) \]
\[ \vec{H} = [\vec{\sigma}_z, \vec{E}(t)] \]
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and a laser field:

\[ \vec{F}(t) = \vec{F}\sin(\Omega t) \]  

with F is the laser field amplitude \( \omega \) is the electromagnetic wave frequency, \( \Omega \) is the laser field frequency.

The quantum kinetic equation for electron in semiconductor \( (j = 1) [19-20] \):

\[ \frac{\partial f_n(\vec{p}_\perp, t)}{\partial t} + \left( e \vec{E}(t) + \omega_0 \left[ \vec{p}_\perp \cdot \vec{h}(t) \right] \right) \frac{\partial f_n(\vec{p}_\perp, t)}{\partial \vec{p}_\perp} = \]

\[ = 2\pi \sum_{n,n',q} M_{n,n'}(\vec{q}) \sum_{l=\infty}^{\infty} I_{n'}(\vec{q}, \vec{q}) \left[ f_{n'}(\vec{p}_\perp + \vec{q}, t) - f_{n'}(\vec{p}_\perp, t) \right] \delta(\varepsilon_{n',\vec{p}_\perp + \vec{q}} - \varepsilon_{n,\vec{p}_\perp} - \Omega) \]

where

\[ M_{n,n'}(\vec{q}) = \left| C_q \right|^2 I_{n,n'}^2 N_q \]

with the case of electron – optical phonon scattering:

\[ \left| C_q \right|^2 = \frac{2\pi \varepsilon w_q}{\varepsilon_0 V_0 (q^2 + q^2_0)} \left( \frac{1}{\chi_e} - \frac{1}{\chi_0} \right) \; ; \; \varepsilon_0 = \frac{10^{-9}}{36\pi} \]

\( \chi_e \) and \( \chi_0 \) are high-frequency dielectric constant and the static dielectric constant;

\( I_{n,n'} = \sum_{j=0}^{J} \exp(iq_j z) \psi_n(z - j\delta) \psi_n(z - j\delta) \) dz is form factor;

\( N_q = \frac{k_B T}{\hbar \omega_q} \) is the time-independent component of distribution function of optical phonon.

\( f_{n'}(\vec{p}_\perp, t) \) is distribution function of electrons;

\( \vec{h}(t) = \frac{\vec{H}(t)}{H} \) is the unit vector in the magnetic field direction; \( \vec{q} \) is momentum of phonon;

\( \vec{p} = \vec{p}_\perp + \vec{p}_z \) is momentum of electron;

\[ \varepsilon_{n,\vec{p}_\perp} = \frac{(\vec{p}_\perp)^2}{2m} + \hbar \omega_0 (n + \frac{1}{2}) ; \varepsilon_{n,\vec{p}_\perp + \vec{q}} = \frac{(\vec{p}_\perp + \vec{q})^2}{2m} + \hbar \omega_0 (n + \frac{1}{2}) \]
\( J_1^2(\tilde{a}, \tilde{q}) \) is the Bessel function of real argument; \( \tilde{a} = \frac{e \tilde{F}}{m \Omega}; \; \omega_n = \frac{e \tilde{H}}{mc} \)

For simplicity, we limit the problem to the case of \( l = 0, \pm 1 \); \( J_0 \approx 1 \); \( J_1^2 = \frac{(\tilde{a}, \tilde{q})^2}{4} \);

\[
f_n (\tilde{p}_\perp, t) = f_0 (\epsilon_{n,p}) + f_1 (\tilde{p}_\perp, t); \quad f_0 (\epsilon_{n,p}) = \theta (\epsilon_{p} - \epsilon_{n,p}) ; \quad f_1 (\tilde{p}_\perp, t) = -\tilde{p}_\perp \tilde{\chi} (t) f_0 (\epsilon_{n,p}) ;
\]

\[
f_0 (\epsilon_{n,p}) = - \frac{\partial f_0 (\epsilon_{n,p})}{\partial \tilde{p}_\perp}; \quad f_1 (\tilde{p}_\perp, t) = f_1 (\tilde{p}_\perp) \text{e}^{-i \omega t} + f_1^* (\tilde{p}_\perp) \text{e}^{i \omega t} ; \quad f_1 (\tilde{p}_\perp) = -\tilde{p}_\perp \tilde{\chi} f_0 (\epsilon_{n,p})
\]

\[
\tilde{\chi} (t) = \tilde{\chi} \text{e}^{-i \omega t} + \tilde{\chi}^* \text{e}^{i \omega t} ; \quad \tilde{\chi} = \frac{e \tilde{E}}{m} \frac{\tau (\epsilon)}{1 - i \omega \tau (\epsilon)} ;
\]

\[
\frac{\partial f_0 (\tilde{p}_\perp, t)}{\partial t} = \frac{\partial}{\partial t} \left[ f_0 (\epsilon_{n,p}) + f_1 (\tilde{p}_\perp) \text{e}^{-i \omega t} + f_1^* (\tilde{p}_\perp) \text{e}^{i \omega t} \right] = - i \omega f_1 (\tilde{p}_\perp) \text{e}^{-i \omega t} + i \omega f_1^* (\tilde{p}_\perp) \text{e}^{i \omega t}
\]

Multiply both sides Eq. (1) by \(- \frac{e}{m} \tilde{p} \delta (\epsilon - \epsilon_{n,p})\) and then sum over \( \tilde{p} \) to get expression:

\[
- \frac{e}{m} \sum_{n,p} \left[ \frac{\partial f_n (\tilde{p}_\perp, t)}{\partial t} + \left( e \tilde{E}(t) + \omega_n [\tilde{p}_\perp \tilde{h}(t)], \frac{\partial f_n (\tilde{p}_\perp, t)}{\partial \tilde{p}_\perp} \right) \right] \delta (\epsilon - \epsilon_{n,p}) = 0
\]

Calculate the left side of equation (12):

The first term

\[
- \frac{e}{m} \sum_{n,p} \tilde{p}_\perp \frac{\partial f_n (\tilde{p}_\perp, t)}{\partial t} \delta (\epsilon - \epsilon_{n,p}) = - i \omega \tilde{R} (\epsilon) \text{e}^{-i \omega t} + i \omega \tilde{R}^* (\epsilon) \text{e}^{i \omega t}
\]

with

\[
\tilde{R} (\epsilon) = - \frac{e}{m} \sum_{n,p} \tilde{p}_\perp f_1 (\tilde{p}_\perp) \delta (\epsilon - \epsilon_{n,p})
\]

\[
\tilde{R}^* (\epsilon) = - \frac{e}{m} \sum_{n,p} \tilde{p}_\perp f_1^* (\tilde{p}_\perp) \delta (\epsilon - \epsilon_{n,p})
\]

The second term:

\[
- \frac{e}{m} \sum_{n,p} \tilde{p}_\perp \left( e \tilde{E}(t), \frac{\partial f_n (\tilde{p}_\perp, t)}{\partial \tilde{p}_\perp} \right) \delta (\epsilon - \epsilon_{n,p}) = - Q (\epsilon) \text{e}^{-i \omega t} + e^{i \omega t}
\]

with

\[
Q = \frac{e^2}{m} \sum_{n,p} \left( E, \frac{\partial f_n (\tilde{p}_\perp, t)}{\partial \tilde{p}_\perp} \right) \tilde{p}_\perp \delta (\epsilon - \epsilon_{n,p})
\]

The third term:

\[
- \frac{e}{m} \sum_{n,p} \omega_n [\tilde{p}_\perp \tilde{h}(t)], \frac{\partial f_n (\tilde{p}_\perp, t)}{\partial \tilde{p}_\perp} \delta (\epsilon - \epsilon_{n,p}) =
\]

\[
= - \omega_n \left[ \tilde{R} (\epsilon), \tilde{h} \right] - \omega_n \left[ \tilde{R}^* (\epsilon), \tilde{h} \right] e^{2i \omega t} - \omega_n \left[ \tilde{R} (\epsilon), \tilde{h} \right] e^{2i \omega t}
\]

Calculate the right side of equation (12):

Consider the case that \( l = 0 \):

}\]
\[
\frac{-e}{m} \sum_{n,a,q} 2\pi \sum_{\ell_i} M_{n,a}(q) \delta_{\ell_i}(a, q) [f_1(\hat{p}_\perp + q, t) - f_n(\hat{p}_\perp, t)] \delta(\varepsilon_{n,\ell_i} - \varepsilon_{n,p}) \delta(\varepsilon - \varepsilon_{n,\ell_i}) = 
\]
\[
= -\frac{1}{\tau(\varepsilon)}(\bar{R}(\varepsilon)e^{-i\omega t} + \tilde{R}^*(\varepsilon)e^{i\omega t})
\]
Consider the case that \(l = \pm 1\):
\[
\frac{-e}{m} \sum_{n,a,q} M_{n,a}(q) \sum_{\ell_i} \left[ 2\pi \sum_{l=1}^{\infty} \delta_{l}(a, q) [f_1(\hat{p}_\perp + q, t) - f_n(\hat{p}_\perp, t)] \delta(\varepsilon_{n,\ell_i} - \varepsilon_{n,p}) \right] 
\]
\[
= \bar{S}(\varepsilon)e^{-i\omega t} + \tilde{S}^*(\varepsilon)e^{i\omega t}
\]
with
\[
\bar{S}(\varepsilon) = -\frac{e}{m} 2\pi \sum_{n,a,q} M_{n,a}(q) \frac{\langle \hat{a}, q \rangle^2}{4} \sum_{\ell_i} f_1(\hat{p}_\perp) \times
\]
\[
\times \left[ \delta(\varepsilon_{n,\ell_i} - \varepsilon_{n,p}) + \delta(\varepsilon_{n,\ell_i} - \varepsilon_{n,p} + \Omega) \right] 
\]
\[
\times \left[ (\hat{p}_\perp + q) \delta(\varepsilon - \varepsilon_{n,\ell_i} - \varepsilon_{n,p}) - (\hat{p}_\perp) \delta(\varepsilon - \varepsilon_{n,p}) \right]
\]
\[
= -2\pi n_n \delta(\varepsilon - \varepsilon_{n,p}) \left\{ \lambda \delta(\varepsilon - \Omega) - A \delta(\varepsilon - \varepsilon_P) \right\}
\]
where
\[
A = \frac{e^2 F^2}{2m^2} M_{n,a}(\sqrt{2m}\Omega) \sqrt{2m} \left[ e_P - \omega_0 \left( n + \frac{1}{2} \right) \right]
\]
\[
\lambda = \frac{e^2 F^2}{2m^2} M_{n,a}(\sqrt{2m}\Omega) \left[ 2m \left[ e_P - \omega_0 \left( n + \frac{1}{2} \right) \right] \left[ \sqrt{2m} \left( \Omega - \omega_0 \left( n + \frac{1}{2} \right) \right) - 1 \right] \right]
\]
\[
M_{n,a}(\sqrt{2\pi}\Omega) = C_q r^2 \int_{n,a} N_q = \frac{e^2}{e_v V_0 \Omega} \left( 1 - \frac{1}{\chi_0} \right) \frac{k_B T}{h} t^2
\]
Put the expressions (13), (16), (18), (19), (20) on (12), then unify the terms of both sides to get the system of equations:
\[
\left[ i\omega + \frac{1}{\tau(\varepsilon)} \right] \bar{R}^*(\varepsilon) = \bar{Q} + \bar{S}(\varepsilon)
\]
from equations (19) obtained:
\[
\bar{R}(\varepsilon) = \frac{\tau(\varepsilon)}{1 - i\omega\tau(\varepsilon)} (\bar{Q} + \bar{S}(\varepsilon))
\]
n_0 is particle density; m is the effective mass of electron; \(e = 1.6 \times 10^{-19} \text{C}\); \(\tau(\varepsilon)\) is the momentum relaxation time in absence of laser radiation.
At time \(t = 0\), the density of current [7]:
\[
\bar{j}(t = 0) = \int (\bar{R}(\varepsilon) + \bar{R}^*(\varepsilon)) d\varepsilon = \bar{I}_0 + \bar{I}_0^*
\]
\[
= \frac{4e^2 n_0 \tau(e_P)}{m} \left[ e_P - \omega_0 \left( n + \frac{1}{2} \right) \right] + \frac{\tau(\Omega)}{1 + \omega^2 \tau^2(\varepsilon_P)} \left[ 1 - \omega^2 \tau(\Omega) \tau(e_P) \right] + \frac{A}{1 + \omega^2 \tau^2(\varepsilon_P)} \bar{E}
\]
\[ j(t) = \sigma E(t) = 2E \]

so

\[ \sigma_{ik} = \frac{2e^2n_0}{m} \frac{\tau(\varepsilon_p)}{1 + \omega^2 \tau^2(\varepsilon_p)} \left[ \varepsilon_p - \omega_0 \left( n + \frac{1}{2} \right) \right] \delta_{ij} + \frac{\tau(\Omega) \left[ 1 - \omega^2 \tau(\Omega) \tau(\varepsilon_p) \right]}{1 + \omega^2 \tau^2(\Omega)} A + \]

\[ - \frac{\tau(\varepsilon_p) \left[ 1 - \omega^2 \tau^2(\varepsilon_p) \right]}{1 + \omega^2 \tau^2(\varepsilon_p)} A \] (28)

This is the expression for calculating the conductivity tensor in a quantum well with a parabolic potential hole for electron scattering - optical phonons. The expression shows the dependence of the conductivity tensor on the electromagnetic wave frequency, laser field frequency, laser field amplitude, with \( \delta_{ik} \) is the Kronecker symbol, \( h\omega_0 = 36.25 \text{MeV} \).

### 3. Numerical results and discussion

The conductivity tensor expression in the quantum well shows its dependence on electromagnetic waves and laser fields, which will be investigated, plotted and discussed in this section. The parameters used in the calculations are as follows [8-9]: \( m = 0.067m_0 \) (\( m_0 \) is the mass of free electron); \( \varepsilon_F = 50 \text{meV} \); \( \tau(\varepsilon_F) \sim 10^{-11} \text{s}^{-1} \); \( n_0 = 10^{23} \text{m}^{-3} \); \( E = 106 \text{V/m} \); \( \chi_e = 10.8 \) and \( \chi_0 = 13.1 \) [19-23].

![Figure 1](image1.png)  \( \text{Figure 1. The dependence of } \sigma \text{ on the frequency } \omega \text{ with different values of } \Omega \text{ when } \omega \text{ at low frequencies.} \)

![Figure 2](image2.png)  \( \text{Figure 2. The dependence of } \sigma \text{ on the frequency } \omega \text{ with different values of } \Omega \text{ when } \omega \text{ at frequencies from } 10^4-10^5. \)

Figure 1 shows the dependence of the conductivity tensor on the electromagnetic wave frequency when \( \omega < 10^4 \text{s}^{-1} \). With this frequency range the figure shows that the conductivity tensor does not depend on the electromagnetic wave frequency, the value of the tensor is constant when changing the electromagnetic wave frequency and with three different values for the laser field frequency, but the plot is nearly identical.
Figure 3. The dependence of $\sigma$ on the frequency $\omega$ with different values of $\Omega$ when $\omega$ at frequencies from $10^{12} - 5.10^{13}$.

Figure 4. The dependence of $\sigma$ on the frequency $\omega$ with different values of $\Omega$ when $\omega$ at low frequencies.

Figure 5. The dependence of $\sigma$ on the frequency $\Omega$ with different values of $\omega$ at high frequencies.

Figure 6. The dependence of $\sigma$ on the amplitude $F$ of the laser radiation.

Figure 2 shows the dependence of the conductivity tensor on the electromagnetic wave frequency in the range $10^6 s^{-1}$ to $10^9 s^{-1}$, when the electromagnetic wave is increased, the conductivity tensor will decrease, however this decrease is slow. Unlike figure 2, in figure 3, conductivity tensor decreases very quickly.
with increasing electromagnetic wave frequency. This shows that for each different interval of the electromagnetic wave frequency, the variation of the conductivity tensor is different.

Figure 4 and 5 show the dependence of the conductivity tensor on the laser field frequency, the dependence of the conductivity tensor on the laser field frequency is almost negligible, at the frequency corresponding to the laser wave the conductivity tensor almost unchanged as the frequency increases.

The dependence of the conductivity tensor on the laser field amplitude is shown in graph 6. The graph shows that the conductivity tensor depends a lot on the laser field amplitude, which is shown that as the amplitude increases then conductivity tensor also increases rapidly.

4. Conclusions

Conductivity tensor is a familiar concept in the study of material properties. Conductivity tensor calculation has been carried out in many studies. From the conductivity tensor it is possible to compute a variety of dynamic effects in materials. Because the electromagnetic wave carries both energy and momentum, as it propagates in the material it will interact with the carriers, thus affecting the carrier's properties as well as giving rise to many effects.

The paper studies the effects of electromagnetic waves and laser fields on conductive tensors in quantum wells with the case of electron scattering - optical phonons. The conductivity tensor expression shows its dependence on electromagnetic wave and laser fields. A graph of the dependence of the conductivity tensor on electromagnetic wave frequency, laser field frequency and laser field amplitude was plotted. This tensor is less dependent on the laser field frequency but very much dependent on its amplitude. For electromagnetic waves, there is a range of frequencies that causes tensors to increase, but also ranges of frequencies that cause tensors to decrease. The change in conductivity tensor with electromagnetic wave frequency is more evident at high frequencies (about $10^{12}$s$^{-1}$).

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