Microwave-induced parametric instability of 2D magnetoplasmons

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Abstract. Recent experiments on microwave-irradiated ultraclean 2DES revealed a colossal narrow photoresistivity peak in the vicinity of the second cyclotron resonance harmonic $2\omega_c$. Here we propose an explanation of this puzzle phenomenon in terms of the parametric instability of 2D magnetoplasmons. We develop the hydrodynamic theory of 2D magnetoplasmons under an inhomogeneous electric field of the MW pumping with frequency $\Omega$. Such a strong inhomogeneous MW field is always induced by lateral metal contacts, which are used to measure photoresistivity. We show that the parametric instability of the 2D magnetoplasmons arises near $\Omega = 2\omega_c$ when the gradient of the electric field of the pumping exceeds a threshold value. The instability leads to heating of 2DES, which in turn leads to the sharp photoresistivity peak.

1. Introduction

Remarkable features of microwave (MW) response of high-mobility two-dimensional electron systems (2DESs) at low temperature in magnetic field have excited much interest the last few years, for review see [1]. The point is that even at low MW power the resistivity of these structures varies dramatically [2, 3]: it oscillates strongly with inverse magnetic field. The period of the oscillations is determined by the relation of MW frequency $\Omega$ to cyclotron frequency $\omega_c$. With the increase of the MW power the oscillation minima become deeper and reach zero values [4, 5].

Recently in ultraclean (mobility $3 \times 10^7$ cm$^2$/Vs) 2DESs a new peculiarity of the resistance was discovered. When the MW frequency is near double cyclotron frequency an extremely high and narrow spike of resistivity was observed [6]-[9]. The origin of the resistance spike is not still clear.

The goal of this article is to propose a possible explanation of the effect in terms of classical approach to the description of 2DES. Our explanation is based on the parametric resonance of 2D magnetoplasmons at the MW irradiation. Under conditions of the parametric resonance amplitudes of plasmon velocity and electron charge density increase exponentially with time, i.e. the plasma instability arises in the system. The instability takes place provided the resonance detuning is sufficiently small and the gradient of the MW field acting on 2D electrons is sufficiently large. The instability leads to heating of 2DES, which in turn leads to the resistance spike. Note that the parametric resonance under different conditions was studied theoretically in [10]-[12].
2. Basic equations
Consider 2DES situated in plane $z = 0$ in constant magnetic field $\vec{B}$ along $OZ$-axis. Electrons are affected by the pumping electric field with the amplitude $E_\Omega$ and frequency $\Omega$. Electric field acting on electrons is always inhomogeneous near metal contacts, so we assume that it depends on the coordinates $E_\Omega = E_\Omega(x)$. We don’t consider a uniform part of the pumping field because it doesn’t effect the parametric resonance, see below. We assume the electric field of the pumping $E_{ext}(x,t) = E_\Omega(x) \cos \Omega t$ is directed along $x$-axis and depends only on coordinate $x$, and the system is uniform along $y$-axis.

Plasmons are treated in the frame of the hydrodynamic approach in terms of the hydrodynamic velocity $\vec{V}(x,t)$, the deviation of 2D charge density $\delta \rho(x,t) = 0$ from the equilibrium $\rho(x,t) = \rho_0$ and the self-consistent potential $\varphi(x,z,t)$.

The hydrodynamic velocity and the self-consistent potential are related by the Euler equation

$$\frac{\partial \vec{V}}{\partial t} + \frac{\vec{V}}{\tau} + (\vec{V}, \nabla)\vec{V} + \frac{e}{m} \nabla \varphi = \frac{e}{m} \vec{E}_{ext}(x,t) + \frac{e}{mc} [\vec{V}, \vec{B}],$$

(1)

where $\tau$ is the phenomenological time that describes the relaxation rate in the system, $e$, $m$ are the electron charge and effective mass, $c$ is the speed of light, $\nabla = (\partial_x, \partial_y, \partial_z)$, $\varphi = \varphi(x,z) = \varphi(x,t)$ is the self-consistent electron potential. Further we assume that the conditions $\Omega \tau \gg 1, \omega_c \tau \gg 1$ take place, i.e. the dissipation is small; $\omega_c = |eB/(mc)|$.

The self-consistent electron potential $\varphi(x,z,t)$ and fluctuations of the 2D charge density $\rho(x,t)$ in quasi-static approximation are related by the Poisson equation

$$\Delta_{3D} \varphi(x,z,t) = -\frac{4\pi}{\varkappa} \rho(x,t) \delta(z),$$

(2)

where $\delta(z)$ is the Dirac delta function, $\varkappa$ is a background static dielectric constant, $\Delta_{3D} = (\partial_x^2 + \partial_y^2 + \partial_z^2)$.

Finally, $\vec{V} = \vec{V}(x,t)$ and $\rho = \rho(x,t)$ are related by the continuity equation

$$\partial_t \rho + \text{div}(\varepsilon \rho_0 + \rho) \vec{V} = 0.$$  

(3)

Thus, we obtain system of Eqs. (1)-(3) for $\varphi$, $\rho$, and $\vec{V}$.

3. Scheme of solution
Suppose that solutions of Eqs. (1)-(3) can be found in the form

$$\varphi(x,t) = \varphi_0(x,t) + \delta \varphi(x,t),$$

$$\rho(x,t) = \rho_0(x,t) + \delta \rho(x,t),$$

$$\vec{V}(x,t) = \vec{V}_0(x,t) + \delta \vec{V}(x,t),$$

(4)

where $\varphi_0(x,t), \rho_0(x,t)$, and $\vec{V}_0(x,t)$ are the solutions of Eqs. (1)-(3) linearized with respect to $\varphi$, $\rho$, and $\vec{V}$.

In high-mobility 2D systems with $\omega_c \tau \gg 1, \omega \tau \gg 1$, the Fourier components of $\vec{V}_0(x,t)$, $\rho_0(x,t)$, $\varphi_0(x,t)$ have the form

$$V_{0x}(q,\omega) = \frac{e}{m} E_{ext}(q,\omega) \frac{-i\omega}{-\omega^2 + \omega_{mp}^2(q)^2}, \quad V_{0y}(q,\omega) = \frac{\omega_c}{i\omega} V_{0x}(q,\omega),$$

$$\rho_0(q,\omega) = \frac{e \rho_0 q}{i\omega} V_{0x}(q,\omega), \quad \varphi_0(q,\omega) = \frac{2\pi}{\varkappa |q|} \rho_0(q,\omega).$$

(5)
Here $\omega_{mp}^2(q) = \omega_s^2 + \omega_p^2(q)$ is the square magnetoplasma frequency for a given $q$. One can write down Eqs. (5) for $\vec{E}_{ext}(x,t) = \vec{E}_\Omega(x) \cos \Omega t$ in the space-time representation as

$$V_{0x}(x,t) = V_{sx}(x) \sin(\Omega t), \quad V_{0y}(x,t) = V_{cy}(x) \cos(\Omega t), \quad \rho_0(x,t) = \rho_c(x) \cos(\Omega t). \quad (6)$$

After substitution of Eqs. (4) into Eqs. (1)-(3), linearization with respect to fluctuations $\delta \varphi(x,t)$, $\delta \vec{V}(x,t)$, $\delta \rho(x,t)$ and exclusion of $\delta \varphi(x,t)$, $\delta V_g(x,t)$, one can derive equations for $\delta \rho(x,t)$ and $\delta V_x(x,t)$. We don’t give an explicit form of these cumbersome equations. The Eqs. contain coefficients periodic with frequency $\Omega$, therefore, their solutions obey the Floquet-Bloch theorem. According to [13] the solution near the fundamental parametric resonance (when $\Omega$ is near $2\omega_c$) can be found in the two-wave approximation in the form

$$\delta V_x = \left( A(x) \cos \left( \frac{\Omega t}{2} \right) + B(x) \sin \left( \frac{\Omega t}{2} \right) \right) e^{\lambda t}, \quad (7)$$

$$\delta \rho = \left( A_\rho(x) \cos \left( \frac{\Omega t}{2} \right) + B_\rho(x) \sin \left( \frac{\Omega t}{2} \right) \right) e^{\lambda t}, \quad (8)$$

where $s$ is the amplification coefficient. We assume that $s$ is a real constant. Condition $s > 1/\tau$ must be fulfilled to achieve unstable solutions. Equation $s = 1/\tau$ takes place at the threshold.

Substituting Eqs. (7), (8) to the system of equations mentioned above, then equating coefficients at $\sin(\Omega t/2)$ and $\cos(\Omega t/2)$, and neglecting other harmonics, equations for $A(x), B(x), A_\rho(x), B_\rho(x)$ can be derived. Then we exclude $A_\rho(x), B_\rho(x)$ neglecting the terms containing the second or the higher powers of $E_\Omega, s$ and $\tau^{-1}$. The resulting system is as follows

$$\begin{pmatrix}
\hat{\omega}_p^2/\Omega - \Delta \omega + \hat{U}(x) & s \\
\hat{\omega}_p^2/\Omega + \Delta \omega + \hat{U}(x) & -s
\end{pmatrix}
\begin{pmatrix}
A(x) \\
B(x)
\end{pmatrix}
= \begin{pmatrix}
0 \\
0
\end{pmatrix}, \quad (9)$$

where $\Delta \omega = \Omega/2 - \omega_c$ is the resonance detuning. The linear in the external field ”perturbation” operator $\hat{U}$ is determined as follows:

$$\hat{U}(x) = \Delta(x) + \frac{\hat{\omega}_p^2}{2en_0\Omega}\rho_c(x) - [\hat{\omega}_p^2, V_{sx}(x)] \frac{\partial_x}{\Omega^2}, \quad (10)$$

where $\Delta(x) = \partial_x(V_{sx}(x) + V_{cy}(x)2\omega_c/\Omega)/4$, $\hat{\omega}_p^2 = -(e^2n_0/m) \partial_x^2 \hat{G}$ is the operator in the coordinate representation. The operator $\hat{G}$ acts as follows $F_0[\hat{G}f(x)] = G(q)f(q)$, where $F_0[g(x)] = g(q)$ is the Fourier transform of function $g(x)$, $G(q) = 2\pi/(\pi|q|)$; $[\hat{\omega}_p^2, V_{sx}(x)] = \hat{\omega}_p^2 V_{sx}(x) - V_{sx}(x)\hat{\omega}_p^2$ is the commutator of operator $\hat{\omega}_p^2$ and function $V_{sx}(x)$. Operator of derivative and $\hat{\omega}_p^2$ act on all the functions behind them.

It should be noted that if $\vec{E}_{ext}(x,t)$ is uniform then $\hat{U} = 0$, and the instability doesn’t appear. The system loses stability as electric field acting on electrons becomes more inhomogeneous.

Let us estimate the form of the solutions of Eqs. (9) in simple models.

4. Solution of Eqs. (9) for models

Solution without the self-consistent field. Consider the problem without taking into account the self-consistent field. This is the case when the electron-electron interaction in 2DES is suppressed, for example, via the background dielectric constant $\varepsilon$. When $\varepsilon$ is sufficiently high one can neglect $\hat{G}$ and $\hat{\omega}_p^2$ in (9). Then $\hat{U}(x) = \Delta(x)$ and the system of eqs. (9) becomes as follows

$$\begin{pmatrix}
-\Delta \omega + \Delta(x) & s \\
\Delta \omega + \Delta(x) & -s
\end{pmatrix}
\begin{pmatrix}
A(x) \\
B(x)
\end{pmatrix}
= \begin{pmatrix}
0 \\
0
\end{pmatrix}. \quad (11)$$
If there is at least one point in which condition \( \Delta^2(x) = s^2 + (\Delta \omega)^2 \) is satisfied then the system of local equations (11) has nontrivial solutions. After replacement of \( s \) by its minimum value \( \tau^{-1} \) we obtain the conditions for the local derivative of the electric field:

\[
\left| \frac{e E'_{\Omega}(x)}{2m\Omega} \right| = \sqrt{(1/\tau)^2 + (\Delta \omega)^2}.
\] (12)

The solutions have local behavior:

\[
\begin{pmatrix} A(x) \\ B(x) \end{pmatrix} = \sum_n C_n \left( \frac{1}{\Delta \omega - \Delta(x_n)} \right) \delta(x - x_n),
\] (13)

where \( x_n \) are the positions in which condition (12) is satisfied, \( C_n \) are an arbitrary constants.

Assume that the amplitude of the electric field in the sample area \( x > 0 \) is described as a power law \( E_{\Omega}(x) = E_c l/(x + l) \) or as an exponential law \( E_{\Omega}(x) = E_c \exp(-x/l) \), where from the contact situated at \( x = 0 \) and \( l \) is the length characterising the decrease of the electric field. Using condition (12) the expression for the threshold electric field can be presented in the form

\[
\left| \frac{e E'_{\Omega}}{2m\Omega} \right| = \frac{1}{\tau} \left( \frac{1}{\tau} \right)^2 + (\Delta \omega)^2.
\] (14)

Obviously the threshold field increases with the increase of the resonance detuning \( \Delta \omega \) and with the decrease of the relaxation time \( \tau \).

**Smooth pumping field.** We exclude component \( B(x) \) from Eqs. (9) and obtain the equation for \( A(x) \):

\[
\begin{pmatrix} \omega_p^2 - \omega \\\n \omega_p^2 - \omega \end{pmatrix}^2 + s^2 + \frac{1}{\Omega} \left[ \omega_p^2, \hat{U}(x) \right] - \hat{U}^2(x) \right) A(x) = 0.
\] (15)

Assume that \( V_{\omega x}(x) = \alpha x \), i.e. that the screened electric field \( E_{\Omega}^{\text{scr}} \), which is proportional to velocity \( V_{\omega x}(x) \), is a linear function of \( x \). Then \( [\omega_p^2, \hat{U}(x)] = 0 \) and Eq. (15) after the Fourier transformation is reduced to the algebraic one

\[
\left( \frac{\omega_p^2(q)}{\Omega^2} - \frac{\alpha^2}{4\Omega^2} \right) - \frac{2\omega_p^2(q)}{\Omega} \left( \Delta \omega - \Delta \frac{\alpha}{2\Omega} \right) + (\Delta \omega)^2 + s^2 - \Delta^2 \right) A(q) = 0.
\] (16)

Leaving only the largest terms in the coefficients of Eq.(16), we obtain the condition for existence of unstable solutions in this approximation:

\[
\left| \frac{e (E_{\Omega}^{\text{scr}})'}{2m\Omega} \right| \geq \sqrt{(1/\tau)^2 + (\Delta \omega)^2}, \quad \Delta \omega \leq 0,
\] (17)

\[
\left| \frac{e (E_{\Omega}^{\text{scr}})'}{2m\Omega} \right| \geq \frac{1}{\tau}, \quad \Delta \omega > 0,
\]

where \( (E_{\Omega}^{\text{scr}})' \) is the spatial derivative of the screened pumping field. Functions \( A(x), B(x) \) oscillate in space. Notice that the condition (17) for \( \Delta \omega \leq 0 \) is in agreement with the condition (12).
5. Conclusion
We developed the hydrodynamic theory of 2D magnetoplasmons under an inhomogeneous electric MW field oscillating with frequency $\Omega$. In fact, the strong heterogeneity of the MW field is always induced by the lateral metal contacts which are used to measure photoresistivity. We use the linear approximation on the plasmon variables such as the hydrodynamic velocity and the 2D charge density and the self-consistent potential, but take into account nonlinear terms in the MW pumping. It is shown that magnetoplasmon variables obey equations containing inhomogeneous parametric terms with frequency $\Omega$, therefore the Floquet-Bloch theorem is valid for them. If $\Omega$ is near $2\omega_c$, then the instability of magnetoplasmons was found to occur, i.e. their amplitudes start to grow exponentially with time with increasing microwave pumping. The parametric instability of the 2D magnetoplasmons arises when the gradient of pumping electrical field exceeds a threshold value. Conditions for the instability are analysed in simple approximations, and the expressions (12), (17) for the threshold are derived. The threshold field decreases with the increase of 2DES mobility and with the decrease of the resonance detuning $\Delta\omega = \Omega/2 - \omega_c$. The instability leads to heating of 2DES, which in turn leads to an increase of the sample resistance. Thus the sharp photoresistivity peak occurs when $\Omega$ is near $2\omega_c$.

Let us estimate the threshold field for the following parameters: $\mu = 10^7 cm^2/V s$, $\Omega = 2\pi 10^{11} rad/s$, $m = 0.066m_0$, $l = 0.1 \mu m$, $\Delta\omega = 0$, here $m$ is the electron effective mass of GaAs, $\mu = e\tau/m$. From the expression (14) we get $E_{th}^\Omega \approx 0.03 V/cm$ and this is less than the amplitude of electric fields of MW radiation in the experiments [6]-[9].

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