Edge position of object image in projecting noninvariant coherent optical system

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Abstract. Specific features of half-plane image formation in a coherent optical spatially noninvariant system of the 2F-2F telecentric type by the projecting objective with a limited aperture are studied (spatial filter is absent). The dependence of the light intensity on the object position at a point corresponding to the half-plane image edge is found in an analytical form using approximation of the Fresnel integrals by analytical functions. As the half-plane approaches the boundary of the field of system vision determined by the objective aperture diameter, the light intensity is demonstrated to deviate significantly, which may lead to the noticeable measurement errors.

1. Introduction
Telecentric projection systems are widely used in industry for dimensional inspection of various objects in transmitted light. The main advantage of these systems is a weak dependence of the geometric characteristics of object images on their defocusing owing to the telecentric perspective. The general problem of such systems is the influence of vignetting (shading) of light beams because of the limited size of the aperture of the projecting objective (spatial filter is absent). In this case we are dealing with the noninvariant systems [1]. As analytical analysis of the influence of vignetting on the object image properties is rather difficult, it is often assumed that the size of the input field of the system is noticeably smaller than the lens diameter [2]. Up to now, these effects, which result in spatial noninvariance of the system, have been studied only in the optical-geometrical approximation [3]. At large input fields, however, these effects can be fairly well noticeable and should be taken into account under precision measurements of object’s geometrical parameters.

In this work, applied to the dimensional inspection we studied the peculiarities of the image formation of a half-plane (typical fragment of the objects) in telecentric aberration-free coherent optical system. The calculations are based on the use of approximation of the Fresnel integrals of the half-plane by the elementary functions [4].

2. Calculation of the intensity at the edge of the half-plane image
For this procedure, we use the pulse response of the optical 2F-2F imaging system by a projecting objective with the $D_{ob}$ linear aperture (spatial filter is absent). This system is illuminated by plane wave with $E_0$ amplitude and $\lambda$ wavelength (Fig. 1). The point source (the input pulse in plane $P_1$) is
located at \( x \) coordinate. As known [3], output field \( h(x_1, x) \) in plane \( P_3 \) is described by the following equation:

\[
h(x_1, x) = E_0 e^{i(kx_1^2/2d)} e^{ikx^2/2d} h(x_1 - x),
\]

where \( h(x_1 - x) = \sin[\omega_0 (x_1 - x)]/[\pi(x_1 - x)] \) is the pulse response of the invariant coherent optical system, \( \omega_0 = k\theta_0, \quad \theta_0 = D_{ob}/(2F) \) is the objective half-angular aperture \((\theta_0 << 1)\), \( F \) is focal length of the projecting objective \((d = 2F)\).

\[\text{Figure 1. Pulse response of the 2F-2F coherent optical projection system with parallel illumination. The object images are formed by the objective with an aperture } D_{ob}.\]

Noninvariance of this optical system is determined by the first phase term \( e^{ikx^2/2d} \), which is responsible for significant changes in the input working field and frequency characteristics of the diffraction-limited system.

The output amplitude distribution \( g(x_1, x_0) \) in the half-plane image for the case of its arbitrary edge position \( x_0 \) according to Eq. (1) is described by the following integral:

\[
\mathcal{G}(x_1, x_0) = \int_{-\infty}^{+\infty} Y(x - x_0) e^{ikx^2/2d} e^{ikx^2/2d} \sin[\omega_0 (x_1 - x)]/[\pi(x_1 - x)] \, dx,
\]

where \( Y(z) \) is the Heaviside step function, \( \mathcal{G}(x_1, x_0) = g(x_1, x_0)/E_0 \) is the normalized output distribution of the field amplitude in the plane \( P_3 \).

Let us calculate the field at the point \( x_1 = x_0 \) corresponding to the geometrical edge of the object in its image (Fig. 2). Using Eq. (2) after some transformations, we can obtain the following expression for the output field:

\[
\mathcal{G}(x_1, x_0) = \left( j\lambda d \right)^{-1/2} \int_{-\infty}^{+\infty} \tilde{Y}_d(-x_0 - \xi) e^{-jk(x_0 - \xi)^2/2d} \text{rect}(\xi/D_{ob}) \, d\xi = \\
= \left( j\lambda d \right)^{-1/2} \int_{-\infty}^{+\infty} \tilde{Y}_d(t) e^{-jk(t + x_0)^2/2d} \text{rect}(t/D_{ob}) \, dt,
\]

where \( \tilde{Y}_d(x_1) \) is the Fresnel image (Fresnel function) of the stepwise Heaviside function:

\[\tilde{Y}_d(t) = \left( j\lambda d \right)^{-1/2} \int_{-\infty}^{+\infty} Y(t - \xi) e^{ik\xi^2/2d} d\xi.\]

In calculating Eq. (3) we use the approximation of the special Fresnel function using the elementary functions, which were proposed in [4]:
where the parameter \( \hat{p} = (\lambda d)^{1/2} e^{j\pi/4}/\beta \) with \( \beta \to \pi \) at \( t >> (\lambda d)^{1/2} \) and \( \beta \to 2 \) at \( t << (\lambda d)^{1/2} \), and \( \text{sign}(t) = 2Y(t)-1 \) is the sign function.

\[
\tilde{Y}_d(t) = Y(t) - 0.5 \hat{p} \text{ sign}(t) e^{j\beta t^2/2d} / (|\phi| + \hat{p}),
\]

(4)

It can be shown that the output amplitude \( \tilde{G}(x_1 = x_0) \) in the interval \( |x_0| \leq 0.5D_{ob} \) has the form:

\[
\tilde{G}(x_1 = x_0) = 0.5 \left[ 1 - \frac{\sqrt{\lambda d} e^{-j\pi/4}}{\beta} \frac{e^{-j\beta(0.5D_{ob} - x_0)^2/2d}}{0.5D_{ob} - x_0 + \hat{p}} \right] + \frac{1}{2\beta} \ln \left| \frac{0.5D_{ob} + x_0 + \hat{p}}{0.5D_{ob} - x_0 + \hat{p}} \right|,
\]

(5)

where the asterisk * means complex conjugation.

3. Estimation of the intensity level in the half-plane image at its edge position

From the measurement point of view it is very important to know the behavior of the edge position of half-plane image \( \tilde{G}(x_1 = x_0) \) under the object’s displacements within the field \( |x_0| < 0.5D_{ob} \). Let us first consider the situation where the half-plane edge coincides with the optical axis, i.e., at \( x_0 = 0 \).

Taking into account that \( D_{ob} >> |p| = (\lambda d)^{1/2} / \beta \), and \( \beta \to \pi \), one can obtain from Eq. (5) the following expression for the normalized output intensity:

\[
\bar{I}(0) = 0.25 - \cos(kD_{ob}^2 / 8d + \pi/4)(\pi N)^{-1} + (\pi^2 N^2)^{-1},
\]

(6)

where \( N = D_{ob}(\lambda d)^{1/2} \) is a fundamental parameters characterizing the Fresnel diffraction on the objective aperture (in our case \( N >> 1 \)). Neglecting the last term and taking into account the oscillatory character of the function \( \cos[kD_{ob}^2(8d)^{-1} + 0.25\pi] \), we find the maximum and minimum for \( \bar{I}(0) \):

\[
\bar{I}(0) \approx 0.25.
\]
\[
\tilde{I}(0) = 0.25 \pm (\pi N)^{-1} = I_{thr} \pm \delta_0,
\]
where \(\delta_0 = (\pi N)^{-1}\) is a constant addition to the standard threshold \(I_{thr} = 0.25\) (Fig. 3). Equation (7) describes the deviation of the light intensity from the standard one at the point \(x_1 = 0\) owing to the influence of the objective aperture.

Let us estimate the value of the addition \(\delta_0\). For instance, for \(N = 220\) \((D_{ob} = 50\) mm, \(d = 100\) mm, and \(\lambda = 0.5\) \(\mu\)m), \(\delta_0/I_{thr}\) is equal to \(\pm 0.6\)%\. If the half-plane displacement parameter is assumed to be \(x_0 \gg (\lambda d)^{1/2}\), it can be demonstrated that the expression for the normalized intensity takes the form

\[
\tilde{I}(x_0) = 0.25 + \tilde{\delta}_x(x_0),
\]
where \(\tilde{\delta}_x(x_0) = \delta_0 + \tilde{\delta}(x_0)\) with \(\tilde{\delta}(x_0) = x_0^2 (\pi^2)^{-1}\) and \(x_0 = x_0 (0.5 D_{ob})^{-1}\) is the relative displacement of the half-boundary edge. For \(N^{-1} \ll R \ll 1\), the variable addition to the threshold \(\delta(x_0)\) has a quadratic dependence. If the edge displacement \(x_0\) is chosen to be \(0.25 D_{ob}\), the value of \(\tilde{\delta}_x(x_0)\) increases up to 50%.

Obviously, the use of the standard threshold \(I_{thr} = 0.25\) for precision determination of the object edge position by the threshold processing can lead to significant measurement errors \(\varepsilon(x_0)\) (see Fig. 3). The error level is determined both by the value of the addition \(\tilde{\delta}_x(x_0)\), and by the slope angle \(\tan \alpha\) of the function \(\tilde{I}(x_1)\) at the point \(x_1 = x_{th}\), i.e., by its derivative \(\tilde{I}'(x_0)\). As its estimate, we choose the value of the derivative for the spatially invariant system with the aperture \(2 \theta_0\), namely, \(\tan \alpha = \omega_0 \pi^{-1} = 20 \lambda^{-1}\) \[2\]\. As a result, the error \(\varepsilon(x_0)\) is described by the following expression:

\[
\varepsilon(x_0) = \tilde{\delta}_x(x_0) / \tan \alpha = \tilde{\delta}_x(x_0) \lambda / (2 \theta_0)
\]

If, for instance, we choose the half-plane displacement \(x_0 = 0.4 D_{ob}\), \(2 \theta_0 = 0.02\) and \(\lambda = 0.5\) \(\mu\)m then the error according to Eq. (9) is more than 3 \(\mu\)m. This error is systematic. Taking into account this error one can increase the accuracy of measurement of the object edge position being shifted with respect to the center of the optical system axis of the 2F-2F system. It should be noted that an alternative way to increase the accuracy of dimensional measurements is to introduce a correcting addition \(\delta_\Sigma(x_0)\) according to Eq. (8) to the standard threshold.

4. Conclusions
The present paper describes the results of studying the specific features of half-plane image formation in a standard 2F-2F telecentric coherent optical system with allowance for the aperture effect (without spatial filter). As this system is spatially noninvariant one, there is shifting of the object edge in determining its position by means of processing the half-plane image by the threshold method.

The light intensity at the point corresponding to the half-plane edge in its image depending on the edge position was determined analytically for the first time on the basis of approximating of the Fresnel functions by elementary functions. As the half-plane approaches the boundary of the field of system vision (determined by the objective aperture diameter), the light intensity appreciably deviates, which may lead to noticeable measurement errors (some microns) under dimensional accurate inspection by the projecting method in transmitted light.

5. References
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