A reduced dimension multiple signal classification–based direct location algorithm with dense arrays

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Abstract
Aiming at the issue of parameter matching in conventional two-step location, a reduced dimension multiple signal classification direct position determination algorithm based on multi-array is proposed. Based on the idea of dimension reduction, the algorithm avoids multi-dimensional search in spatial domain and attenuation coefficient domain and reduces the search complexity. Simulation results show that the performance of the algorithm is better than the traditional angle of arrival two-step localization algorithm and subspace data fusion direct localization algorithm.

Keywords
Direct position determination, multi-array, passive localization

Introduction
Nowadays, with the rapid development of intelligent nodes and Internet-of-Things (IOT) technology, the electromagnetic space is becoming more and more complex, and the signal source location technology is becoming more and more important.¹ At present, it has become an indispensable part in many fields, such as intelligent driving, radio supervision, and so on.²,³

With the increasingly complex radio environment, the existing single-emitter location methods cannot meet the needs of location. Therefore, a new location technology for multi-source location is urgently needed.⁴,⁵ At present, most multi-source location methods still belong to two-step location methods.⁶ First, intermediate parameters related to the source position are extracted from the received signals,⁷,⁸ including angle of arrival (AOA), time of arrival (TOA), time difference of arrival (TDOA), and so on. Then, a position equation is established according to the source position and the observation base station position, and the equation is optimized to obtain the target position. It should be noted that due to the existence of intermediate parameters, one-to-one correspondence of parameters is also required for multiple sources.⁹ In some cases, when the parameter matching process cannot be used effectively, such as the source is too far from the observation station, the clustering algorithm¹⁰ begins to be used to eliminate false location points in multi-source location. However, due to the use of two-step positioning system, it is necessary to estimate the parameters first, and then locate the emitter. There will be information loss in each step, and the final positioning accuracy will be affected by the accumulated loss of

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information. Although the positioning accuracy can be improved to a certain extent by improving the method of parameter estimation. For example, Wen et al. studied the multi-parameter estimation of coherent targets in bistatic electromagnetic vector sensor (EMVS) multi-input multi-output (MIMO) radar. In Zheng et al.’s study, the altitude measurement of meter wave polarization MIMO radar is mainly studied. In Shi et al.'s study, for the problem of parameter identifiability, the element spacing is further introduced to establish a complete parameter identifiability scheme. In addition, the conditions of bistatic MIMO radar are derived from a two-dimensional point of view. In Xu et al.’s study, a new and improved signal subspace model is introduced, and the signal subspace is refined using the model, so as to improve the performance of direction of arrival (DOA) estimation. Jirhandeh et al. proposed an effective DOA estimation method for broadband sources based on the concept of super-resolution. However, these improvements cannot avoid the accumulation of two-step positioning error. It is necessary to study a new positioning system to improve the positioning accuracy.

Aiming at the deficiency of two-step positioning, a direct positioning technology is developed. Direct positioning technology directly processes the original data and estimates the target position. The positioning accuracy is better than two-step positioning. In addition, the direct location algorithm does not need intermediate parameter estimation, so the issue of parameter matching in multi-source location scenario is avoided. It has become a research hotspot in wireless location. Although this technology also has its shortcomings, that is, it directly processes the original data and has high information processing capacity for the system, with the development of the information processing system, the equipment has developed rapidly, which can meet the requirements of this technology.

In Amar and Weiss’s study, direct position determination (DPD) is extended to multi-source scenes using the idea of decoherent subspace decomposition when constructing the cost function. In Wang et al.’s study, a direct location method based on cross-correlation matrix (CCM) under multi-antenna array (DPS) is proposed. In Yin et al.’s study, a direct location algorithm is proposed, which uses the angle and Doppler shift information to construct the cost function. Subspace data fusion (SDF) method is a more computationally efficient method for multi-transmitter scenarios, in which all data are collected by mobile antenna arrays, and the cost function is an improvement of the traditional multi-signal classification spectrum. In Ma et al., multiple moving arrays are used to intercept static emitter signals, ignoring TDOA, and a moving multi-station direct positioning method based only on AOA and frequency difference of arrival (FDOA) is proposed. In Wu et al.’s study, a direct positioning method is proposed, which fully combines the noise subspace and signal subspace; therefore, the positioning accuracy of the algorithm is better than that of minimum variance distortion-less response (MVDR) or multiple signal classification (MUSIC) spectrum. In Li et al.’s study, the SDF direct positioning method is improved, and two weighted DPD methods are proposed to improve the positioning accuracy.

Aiming at the advantages of high resolution of music algorithm, some articles have improved it. In Zheng et al.’s study, the problem of split beam in SVS-MUSIC is found, and the solution is given. The proposed algorithm is also extended to long dipole and large-loop MIMO radar. In Zheng and Song’s study, a generalized MUSIC algorithm is proposed. Zuo et al. proposed a new model-based two-dimensional MUSIC damage identification algorithm for plate structures. In Bao et al.’s study, a new synthetic aperture MUSIC algorithm is proposed for damage diagnosis of array error compensation. In Xu et al.’s study, a focused MUSIC algorithm is proposed for baseline free Lamb wave damage location on isotropic materials. In Zhong et al.’s study, a new near-field two-dimensional music method based on piezoelectric sensor array is proposed.

In this article, a direct location algorithm based on reduced dimension MUSIC (RD-MUSIC) is proposed based on the multi-signal source localization problem in multi-array. First, based on the multi-array positioning scene, the received signal model under multi-array is constructed. Then, the signals received by multiple arrays are fused, and the signal covariance matrix is obtained. Then, the signal subspace and noise subspace are obtained according to the covariance matrix. Finally, the objective function is used to search the grid in the spatial region, which avoids the multi-dimensional search in the spatial domain and attenuation coefficient domain and reduces the complexity of the search. In summary, the main contributions of this article are as follows:

1. In this article, a reduced dimension MUSIC-based direct localization algorithm with dense arrays is proposed, which avoids multi-dimensional search, reduces the search complexity of cost function, and realizes the direct location of signal sources under multiple arrays.

2. The proposed algorithm is simulated and compared with the traditional AOA two-step location method and SDF-DOA algorithm, the results show that the performance of the RD-MUSIC direct location algorithm is improved to a certain extent.
Signal model

Considering the system model shown in Figure 1, assuming that there are $K$ signal sources in the space, and the number of emitters $K$ has been estimated by the source number estimation algorithm, the position of the emission source is $p_k = [x_k, y_k]^T$, $L$ precisely known observation points are located at $u_l = [x_{l,1}, y_{l,1}]^T$. Each observation point is equipped with a uniform linear array of $M$ array elements, and the array element spacing is $d$.

It is assumed that the emitter signals are far-field narrow-band signals, and the signal of the $k$th emitter is $s_{l,k}(t)$, then it can be obtained that the received signal of the $l$th observation array at the time $t$ is as shown in equation (1)

$$x_l(t) = \sum_{k=1}^{K} a_{l,k} \alpha_{l,k} s_{l,k}(t) + n_l(t)$$

where $\alpha_{l,k}$ represents the fading coefficient, $n_l(t)$ is independent zero mean additive Gaussian white noise, the variance is $\sigma^2_n I_M$, and the noise and signal are not related to each other. $a_{l,k} = [e^{j\phi_{l,k}}, \ldots, e^{j\phi_{l,k}|M]}^T$ is the array guidance vector, where $d_n$ is the position vector of the $n$th array element of the array relative to the reference array element, and $k_{l,k}$ is the wave number vector at the observation position $u_l$, expressed as

$$k_{l,k} = \frac{2\pi}{\lambda} \left( p_k - u_l \right)$$

Therefore, the received signal of $J$ snapshots is

$$X_l = A_l(p)\Phi S_l + N_l$$

where $A_l(p) = [a_{l,1}, \ldots, a_{l,M}]$ is the array manifold matrix and $p = [p_1^T, \ldots, p_M^T]^T$ is the position vector composed of the position coordinates of $K$ signal sources. $S_l = [s_{l,1}, \ldots, s_{l,K}]^T$, where $s_{l,k} = [s_{l,k}(1), s_{l,k}(2), \ldots, s_{l,k}(J)]^T$, $N_l = [n_l(1), n_l(2), \ldots, n_l(J)]$. Matrix $\Phi_l$ is a diagonal matrix composed of $K$ power fading and propagation delay coefficients, expressed as

$$\Phi_l = diag(\alpha_{l,1}, \alpha_{l,2}, \ldots, \alpha_{l,K})$$

After the array collects the received signals of $J$ snapshots, the output expression of the array can be obtained as

$$X_l = [X_l(1), \ldots, X_l(J)] = A_l(p)S_l + N_l$$

where $s_{l,k} = [s_{l,k}(1), s_{l,k}(2), \ldots, s_{l,k}(J)]^T$ and $N_l = [n_l(1), n_l(2), \ldots, n_l(J)]$.

By fusing the received signals of all observation positions, the total received signal vector of each position can be obtained as

$$X = B(p)S + N$$

where matrix $B(p)$ is a joint matrix vertically synthesized by matrix $B_i(p)$, $i = 1, \ldots, M_L$. Similarly, matrix $S$ is also a joint signal matrix composed of received signals of $L$ arrays.

Proposed DPD algorithm

The covariance matrix of the signal obtained from equation (5) is

$$R_{xx} = E[XX^H] = BR_p B^H + \sigma^2_n I_{(M \times L)}$$

where $(\cdot)^H$ represents the conjugate transpose operation of the matrix, $R$ is a signal correlation matrix, $\sigma_n^2$ is noise power, and $I_{(M \times L)}$ is the identity matrix.

Due to the limited sampling length of the actual received signal, in practice, the sampling covariance matrix composed of the sampling data of $J$ snapshots is used to replace the real covariance matrix, and the sampling covariance matrix of the received signal is expressed as

$$\hat{R}_{xx} = \frac{1}{J} \sum_{j=1}^{J} X(t)X(t)^H$$

where $\hat{R}_{xx}$ is a symmetric matrix. By eigenvalue decomposition of $\hat{R}_{xx}$, we can get:

$$\hat{R}_{xx} = U\Sigma U^H = \sum_{i=1}^{ML} \lambda_i e_i e_i^H$$

$$U = [e_1, \ldots, e_{ML}]$$

$$\Sigma = diag\{\lambda_1, \lambda_2, \ldots, \lambda_{ML}\}$$

where $U$ is the matrix composed of eigenvectors, $\Sigma$ is the diagonal matrix composed of eigenvalues $\lambda_i$, and $\lambda_i$ is the $i$th eigenvalue of the correlation matrix $R_{xx}$. Note
that \( \lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_K \geq \lambda_K + 1 = \lambda_K + 2 = \ldots = \lambda_{ML} = \sigma^2 \), and \( e_i \) is the eigenvector corresponding to \( \lambda_i \). Let the joint signal subspace be as follows

\[
U_S = [e_1, \ldots, e_K]
\]

(10)

Let the joint noise subspace be as follows

\[
U_N = [e_K + 1, \ldots, e_{ML}]
\]

(11)

According to equation (8)

\[
\hat{R}_eU_N = BR_bB^HU_N + \sigma^2eU_N - \sigma^2U_N
\]

(12)

When the matrix \( \hat{R}_e \) is a full-rank matrix, we can obtain

\[
B^HU_N = 0
\]

(13)

It shows that each column vector in the matrix \( B \) is orthogonal to the noise subspace, so it can be obtained

\[
U^H_b(p_k) = 0, k = 1, 2, \ldots, K
\]

(14)

where \( b(p_k) \) is the \( k \)th column of the matrix \( B \), then for the source position \( p_k \), the vector \( b(p_k) \) is orthogonal to the noise subspace \( U_N \) in equation (14), so the cost function can be established

\[
\min_p b(p)^H U_N U^H b(p)
\]

(15)

It should be noted that in the cost function of the above objectives, there is also the attenuation coefficient of the position, and the search complexity is very high. Expand \( b(p) \) to obtain

\[
b(p) = \begin{bmatrix} a_1(p) & a_2(p) & \ldots & a_L(p) \end{bmatrix}^T
\]

(16)

\[
= \begin{bmatrix} a_1(p) \\ a_2(p) \\ \vdots \\ a_L(p) \end{bmatrix}
\]

where \( \alpha = [\alpha_1, \alpha_2, \ldots, \alpha_L]^T \) is the attenuation coefficient vector.

We construct the following cost function

\[
L(p, \alpha) = \alpha^H Q(p) \alpha - \lambda (e^H \alpha - 1)
\]

(19)

where \( \lambda \) is a constant, according to equation (18), it can be obtained

\[
\frac{\partial}{\partial (\alpha)} L(p, \alpha) = 2Q(p) \alpha + \lambda e_1 = 0
\]

(20)

According to equation (19), \( \alpha = (Q(p))^{-1} e_1 / e^H_1(Q(p))^{-1} e_1 \). Finally, the grid search of the target space area is carried out, and the position estimation of the target is obtained by obtaining the maximum value of the \((1,1)\)th element of \( Q^{-1}(p) \).

The key steps of the proposed algorithm are as follows:

1. The observation array receives the signal at position \( p_i = [x_i, y_i]^T \) and samples it to obtain the received signal \( x(t), t = 1, \ldots, J \) with the number of snapshots \( J \);
2. Fuse the received signals of multiple observation arrays to obtain the total received signal vector of \( L \) positions shown in equation (5);
3. According to equation (8), the sampling covariance matrix \( \hat{R}_e \) can be obtained. After decomposing the sampling covariance matrix according to equations (10) and (11), the first \( K \) large eigenvalues \( \lambda_i, i = 1, \ldots, K \) and \( ML - K \) small eigenvalues \( \lambda_i, i = K + 1, \ldots, ML \) of \( \hat{R}_e \) are obtained;
4. According to equation (15), the cost function is constructed and simplified to obtain equation (18). Based on the objective function, the grid search is carried out on the spatial region to obtain the maximum value of the \((1,1)\)th element of \( Q^{-1}(p) \) and estimate the position estimation of the target.

### Complexity analysis

This section studies the complexity comparison between the proposed reduced dimension music direct location algorithm and the traditional direct location algorithm (SDM-MUSIC), only considering the comparison of multiple times in the algorithm. The algorithm complexity analysis in this section is not only related to the number of signals \( K \), arrays \( L \), and array elements \( M \), but also related to the number of spectral function search points along the \( x \) and \( y \) directions, which are recorded as \( F_x \) and \( F_y \) respectively, the number of search points in the attenuation coefficient domain is recorded as \( G \). The complexity of the proposed algorithm is mainly related to the covariance calculation of the received signal, eigenvalue decomposition, and the
as can be seen from Table 2, the running time of the proposed algorithm is the shortest, followed by the SDF direct positioning algorithm, and the running time of the non-dimensionality reduction algorithm is the longest, which shows that the time complexity of the dimensionality reduction algorithm is effectively reduced.

Figure 2 shows the complexity comparison of the two algorithms under specific parameters. The abscissa is the number of points searched along the x direction, from 100 to 500, and other parameters are set as follows: the number of arrays $L = 4$, the number of signals $K = 3$, the number of array elements $M = 7$, and the number of snapshots $J = 100$.

**Simulation results**

In this article, Monte Carlo simulation experiments will be used to evaluate the signal estimation performance of the algorithm, and root mean square error (RMSE) will be used to measure the positioning performance of the algorithm, which is defined as follows

$$RMSE = \frac{1}{K} \sqrt{\frac{1}{Q} \sum_{i=1}^{Q} \sum_{k=1}^{K} ||\hat{p}_{k,i} - p_{k,i}||^2}$$

(21)

where $Q$ is the number of Monte Carlo simulation experiments, and $\hat{p}_{k,i}$ is the position estimation result of the $i$th signal source in the Monte Carlo simulation experiment. In this article, the simulation number $Q$ is set to 300.

**Simulation 1**

Figures 3 and 4, respectively, show the positioning peak diagram and corresponding contour diagram of the proposed RD-MUSIC algorithm under the condition of signal-to-noise ratio (SNR) = 10 dB. Besides, the number of signal sources is $K = 2$, their real positions are (100, 600)$m$, (700, 200)$m$, (600, 400)$m$, (400, 900)$m$, and (300, 500)$m$, respectively, the number of observation arrays is $L = 4$, and the positions of each array are (0, 0)$m$, (1000, 0)$m$, (0, 1000)$m$, and (1000, 1000)$m$, respectively. Each observation point is configured with a uniform linear array with array elements $M = 7$ and
the number of snapshots $J = 500$. Two obvious spectral peaks can be seen from the simulation results. From the contour map, it can be seen that the position of the spectral peak corresponds to the real position of the target to be estimated, which shows that the proposed algorithm can realize the simultaneous location of multiple sources and has good location performance.

Simulation 2

Figures 5 and 6 show the comparison of the positioning performance of the proposed algorithm with that of other algorithms and the SNR varies from $-10$ dB to $10$ dB. The algorithms involved in the figure are the SDF-DPD algorithm and the two-step positioning algorithm based on AOA. Because the traditional two-step localization algorithm requires additional information matching in multi-source localization, we assume that the multi-source information has been matched well in this article. The simulation parameters in Figure 5 are set as follows: the number of signal sources is $K = 3$, and real position is $(400, 200)m$, $(200, 700)m$, and $(600, 400)m$. The number of observation arrays $L = 4$ and the positions of each array are $(0, 0)m$, $(1000, 0)m$, $(0, 1000)m$, and $(1000, 1000)m$. A uniform linear array with array element number $M = 7$ is configured at each observation point, and the number of snapshots is $J = 500$. The simulation parameters in Figure 5 are set as follows: the number of signal sources is $K = 4$, and their real positions are $(400, 200)m$, $(200, 700)m$, $(600, 400)m$, and $(300, 400)m$. The parameter setting of the array antenna is the same as Figure 5. According to Xiong et al., Cramer-Rao Bound (CRB) comparison results are added in Figures 5 and 6.

From the simulation results, it can be seen that the positioning performance of these algorithms continues to improve with the increase of SNR. The algorithm proposed in this article takes into account the influence of channel attenuation in the objective function. Therefore, in the multi-source location scenario, the location accuracy of the proposed algorithm is better than the traditional AOA two-step location algorithm and clustering direct location algorithm (SDF). Simulation results show the effectiveness of the proposed algorithm.

Conclusion

In this article, the simultaneous location of multiple unknown signal sources in multi-array is discussed, and
a reduced dimension music direct location method is proposed. The proposed method takes advantage of the inherent advantages of direct location, including avoiding additional data association in multi-source scenes. Because the direct positioning method directly processes the data domain and the performance of the traditional two-step method is limited by the intermediate parameter estimation error, the proposed method obtains better positioning accuracy than the traditional AOA two-step method. At the same time, compared with the original DPD method, the proposed location method has higher location estimation accuracy for multi-source target locations.

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**References**

1. Gupta AK and Johari R. IOT based electrical device surveillance and control system. In: Proceedings of the 2019 4th international conference on internet of things: smart innovation and usages (IoT-SIU), Ghaziabad, India, 18–19 April 2019. New York: IEEE.

2. Benarous L, Bitam S and Mellouk A. CSLPPS: concerted silence-based location privacy preserving scheme for internet of vehicles. *IEEE Trans Veh Technol* 2021; 70: 7153–7160.

3. Lu L and Wu HC. Novel robust direction-of-arrival-based source localization algorithm for wideband signals. *IEEE Trans Veh Commun* 2012; 11(11): 3850–3859.

4. Ma S, Liu Q and Sheu PCY. Foglight: visible lightweight indoor localization system for low-power IoT devices. *IEEE Intern Things J* 2018; 5(1): 175–185.

5. Zhu Q, Li H, Fu Y, et al. A novel 3D non-stationary wireless MIMO channel simulator and hardware emulator. *IEEE Trans Commun* 2018; 66(9): 3865–3878.

6. Wang G and Ho KC. Convex relaxation methods for unified near-field and far-field TDOA-based localization. *IEEE Trans Wirel Commun* 2019; 18(4): 2346–2360.

7. Wu X, Zhu WP, Yan J, et al. A spatial filtering based gridless DOA estimation method for coherent sources. *IEEE Access* 2018; 6: 56402–56410.

8. Shi F. Two dimensional direction-of-arrival estimation using compressive measurements. *IEEE Access* 2019; 7: 20863–20868.

9. Li Y, Li S, Song Q, et al. Fast and robust data association using posterior based approximate joint compatibility test. *IEEE Trans Ind Inform* 2014; 10(1): 331–339.

10. Liao Q, Sun D and Andreasson H. FuzzyPSReg: strategies of fuzzy cluster-based point set registration. *IEEE Trans Robot*. Epub ahead of print 22 November 2021. DOI: 10.1109/TRO.2021.3123898.

11. Wen F, Shi J and Zhang Z. Generalized spatial smoothing in bistatic EMVS-MIMO radar. *Signal Process* 2022; 193: 108406.

12. Zheng G, Song Y and Chen C. Height measurement with meter wave polarimetric MIMO radar: signal model and MUSIC-like algorithm. *Signal Process* 2022; 190: 108344.

13. Shi J, Yang Z and Liu Y. On parameter identifiability of diversity-smoothing-based MIMO radar. *IEEE Trans Aerosp Electron Syst*. Epub ahead of print 15 November 2021. DOI: 10.1109/TAES.2021.3126370.

14. Xu K, Xing M, Zhang R, et al. High-accuracy DOA estimation algorithm at low SNR through exploiting a supervised index. *IEEE Trans Aerosp Electron Syst*. Epub ahead of print 25 January 2022. DOI: 10.1109/TAES.2022.3144121.

15. Jirhandeh MJ, Hezaveh H and Kahaei MH. Super-resolution DOA estimation for wideband signals using non-uniform linear arrays with no focusing matrix. *IEEE Wirel Commun Lett* 2022; 11: 641–644.

16. Tirer T and Weiss AJ. High resolution direct position determination of radio frequency sources. *IEEE Signal Process Lett* 2016; 23(2): 192–196.

17. Bosse J, Ferreol A and Larzabal P. Performance analysis of passive localization strategies: direct one step approach versus 2 steps approach. In: *Proceedings of the 2011 IEEE statistical signal processing workshop (SSP)*, Nice, 28–30 June 2011. New York: IEEE.

18. Londhe A, Bhalaroa V, Ghodey S, et al. Data division and replication approach for improving security and availability of cloud storage. In: *Proceedings of the 2018 4th international conference on computing communication control and automation (ICCUBEA)*, Pune, India, 16–18 August 2018, pp.1–4. New York: IEEE.

19. Amar A and Weiss AJ. Direct position determination (DPD) of multiple known and unknown radio-frequency signals. In: *Proceedings of the 2004 12th European signal processing conference*, Vienna, 6–10 September 2004. New York: IEEE.

20. Wang G, Gao C, Razul SG, et al. A new direct position determination algorithm using multiple arrays. In: *Proceedings of the 2018 IEEE 23rd international conference on digital signal processing (DSP)*, Shanghai, China, 19–21 November 2018. New York: IEEE.

21. Yin J, Wang D, Wu Y, et al. Direct localization of multiple stationary narrowband sources based on angle and Doppler. *IEEE Commun Lett* 2017; 21(12): 2630–2633.
22. Demissie B, Oispuu M and Ruthotto E. Localization of multiple sources with a moving array using subspace data fusion. In: Proceedings of the 2008 11th international conference on information fusion, Cologne, 30 June–3 July 2008. New York: IEEE.

23. Ma F, Guo F and Yang L. Direct position determination of moving sources based on delay and Doppler. IEEE Sens J 2020; 20(14): 7859–7869.

24. Wu GZ, Zhang M and Guo FC. High-resolution direct position determination based on eigenspace using a single moving ULA. Signal Imag Vid Process 2019; 13(5): 887–894.

25. Li J, He Y, Zhang X, et al. Simultaneous localization of multiple unknown emitters based on UAV monitoring big data. IEEE Trans Ind Inform 2021; 17: 6303–6313.

26. Zheng G and Song Y. Signal model and method for joint angle and range estimation of low-elevation target in meter-wave FDA-MIMO radar. IEEE Commun Lett 2022; 26: 449–453.

27. Zuo H, Yang Z, Xu C, et al. Damage identification for plate-like structures using ultrasonic guided wave based on improved MUSIC method. Compos Struct 2018; 203: 164–171.

28. Bao Q, Yuan S and Guo F. A new synthesis aperture-MUSIC algorithm for damage diagnosis on complex aircraft structures. Mech Syst Signal Process 2020; 136: 106491.

29. Xu C, Wang J, Yin S, et al. A focusing MUSIC algorithm for baseline-free Lamb wave damage localization. Mech Syst Signal Process 2022; 164: 108242.

30. Zhong Y, Yuan S and Qiu L. Multiple damage detection on aircraft composite structures using near-field MUSIC algorithm. Sens Actuat A Phys 2014; 214: 234–244.

31. Zhang X, Xu L, Xu L, et al. Direction of departure (DOD) and direction of arrival (DOA) estimation in MIMO radar with reduced-dimension MUSIC. IEEE Commun Lett 2010; 14(12): 1161–1163.

32. Xiong Y, Wu N, Shen Y, et al. Cooperative network synchronization: asymptotic analysis. IEEE Trans Signal Process 2018; 66(3): 757–772.