Virtual-image generation in 360-degree viewable image-plane disk-type multiplex holography

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Abstract: By shifting the rotational axis of the recording film and recording the individual image-plane holograms in reversed sequence with the real-image holographic system [Opt. Express 18, 14012 (2010)], the disk-type multiplex hologram can be made to generate virtual image for walk-around viewing if the recording reference source point is maintained on the symmetry axis of hologram disk. Theoretical formulation and numerical simulation show the characteristics of the reconstructed image. Experimental results are also shown for qualitative comparison.

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OCIS codes: (090.4220) Multiplex holography; (090.2870) Holographic display; (090.0090) Holography.

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1. Introduction

Multiplex holography [1], known as a 3D-display method, was originally proposed to overcome the limitation on the subjects which could be used for holographic recording [2]. A series of 2D photographs was taken from a 3D object and then fed into the optical system for synthesizing as a composite hologram. As a result, not only human being and outdoor scene but also computer-generated data can be utilized as objects for recording [3]. Besides the original flat format, multiplex holography was also developed into several formats, including the cylindrical type [4,5], the conical type [6,7], and the disk type [8,9]. Due to the nature of recording, the reconstructed image from the traditional multiplex hologram suffers from the problem, called the picket-fence effect. That is the image is superimposed with a fence structure. In order to alleviate this problem, the image-plane technique was introduced and
applied to all the walk-around viewable multiplex holography [10–12]. The disk-type multiplex hologram, unlike the cylindrical and conical ones, needs not to be bent before image reconstruction and has the potential of mass production utilizing the well developed CD technology. With the recording following the concept of traditional multiplex holography, that is the object beam is perpendicular to the film plane, the resulted image-plane disk-type multiplex hologram can only reconstruct image for one observer only [9]. By tilting the object beam to be off axis and adopting a diverging reference wave, this type of multiplex hologram can be made viewable by the surrounding observers simultaneously [12,13]. In this paper, we show that, by shifting the rotational axis of the recording film while maintaining the position of the reference source point on axis and reversing the holographic recording sequence, this type of hologram can be made to generate virtual image for walk-around viewing. The holographic process is described together with numerical simulation to demonstrate the characteristics of the reconstructed image observed at different locations. Some experimental results are also included for comparison.

2. Holographic recording system and theory of the holographic process

The holographic optical system for recording of the image-plane CD-type multiplex hologram, which is capable of generating virtual image for the surrounding observers simultaneously, is shown in Fig. 1. This holographic recording system differs from that for real-image generation in that the axis of rotation for the recording film is laterally shifted and the top of each 2D image for holographic recording is facing this axis of rotation. However, the location of the diverging reference source point still remains on the symmetric axis of the CD-type hologram.

Light from a He-Ne laser is first split into two beams. The object beam is expanded by the spatial filter SF1 and then is focused by the lens L1 onto the center of the lens L2. This light beam is then imaged by the lens L3 to a distance beyond the recording holographic film. On its way toward the lens L3, the light beam acquires a 2D image from the LCD panel. This image can be one of the images of the original 3D object, placed on a rotational stage, taken by a CCD camera or simply can be generated by computer. In this optical system, the object plane, LCD plane, is directly imaged onto the film plane. The reference beam is diverged from a point, SF2, on the rotational axis of the holographic film to produce interference pattern with the object beam, which is recorded as an individual image-plane hologram. After
one recording, the holographic film is rotated by a small angle and the LCD shows the next 2D image for recording. This process continues until all the 2D images are recorded, which corresponds to both the rotational stage of the 3D object and the recording holographic film are all rotated by a full round.

Figure 2 shows the viewing geometry for virtual-image reconstruction. Either a point white-light source, such as a LED, or a clear light bulb with a line filament could be used for image reconstruction. In contrast to the case of real-image generation in which the image information is generated from the half disk away from the observer, for the present case, the image information is from the other half disk closer to the observer. The 2D image perceived by the right (left) eye of the observer is from the left-hand (right-hand) side of the half disk for the real-image case. However, for the virtual-image case, the right (left) eye sees a 2D image from the right-hand (left-hand) side of the half disk. Hence, the recording of individual holograms for the virtual image case should be in reverse order from that of the real-image case.

![Diagram of viewing geometry for virtual image generated from disk-type hologram.](image)

The method of direct object-image relationship, as used frequently in the previous investigation [8–11], is adopted for theoretical formulation. Referring to Fig. 3, consider a generalized object point \( P \) at \((x, y, z)\) in its own coordinate system \( X-Y-Z \). When the object is rotated by an angle \( N \theta_0 \) with respect to the \( Y(Y_o) \)-axis, its position would be at \( x_o = x \cos(N \theta_o) + z \sin(N \theta_o), \quad y_o = y, \) and \( z_o = -x \sin(N \theta_o) + z \cos(N \theta_o) \) in the laboratory coordinate system \( X_o-Y_o-Z_o \), where \( N \) is an integer and \( \theta_o \) is the incremental rotation angle between successive exposures.

![Diagram showing the relationship between the laboratory coordinate system \( X_o-Y_o-Z_o \) and the object coordinate system \( X-Y-Z \).](image)

A CCD camera with its optical axis pointing at the center of the 3D object is inclined at an angle \( \sigma \) with respect to the horizontal plane, as shown in Fig. 4. The position of the object
point under consideration can be expressed in the CCD coordinate system $X_c$-$Y_c$-$Z_c$ through coordinate transformation.

\[
\begin{bmatrix}
x_c \\
y_c \\
z_c
\end{bmatrix} =
\begin{bmatrix}
1 & 0 & 0 \\
0 & \cos \sigma & -\sin \sigma \\
0 & \sin \sigma & \cos \sigma
\end{bmatrix}
\begin{bmatrix}
x_o \\
y_o \\
z_o
\end{bmatrix}.
\] (1)

Simple imaging relationship shows that it would appear at $P_d(x_d = -d_i x_c/(d_o - z_c), y_d = -d_i y_c/(d_o - z_c))$ on the detector plane. Note that, in order to have a clear image, the whole object should lie within the depth of focus of the camera lens.

With a magnification ratio $M$, this object is transmitted to the object plane, LCD, of the recording object system. Its position is at $P_l(x_l = -M x_d, y_l = -M y_d)$. The lens pair $L_2$ and $L_3$ enlarges and directly images this object point onto $P_f(x_f = M x_l, y_f = M y_l)$ on the recording film plane $X_f$-$Y_f$, which is inclined at a preset angle $\theta_1$ with respect to the axis of the optical system. Note that this magnification ratio $M$ is slightly position dependent.

Since the object beam is focused at a distance $d_{fe}$ behind the recording film, the direction cosines of the object ray leaving the object point $P_f$ can be determined by subtracting the coordinates of $P_f$ from that of $P_e$ which is

\[
(\cos \alpha_x, \cos \beta_y, \cos \gamma_z) = \frac{(-x_f, -d_f \sin \theta_1 - y_f, d_f \cos \theta_1)}{\sqrt{x_f^2 + (d_f \sin \theta_1 + y_f)^2 + (d_f \cos \theta_1)^2}}.
\] (2)

The recording reference source point $P_r$ is placed at $(0, A, -D_c)$ in the film coordinate system. Hence, the direction cosines of the reference ray on the object point $P_l$ can similarly be found.
The axis of rotation for the recording film is designated to go through the position \((0, A)\) of the film coordinate system during successive individual hologram recording. Thus, the effective position of the recording reference source point would be on the axis and at a distance \(D_e\) below the hologram disk. The two light rays of Eq. (2) and Eq. (3) produce interference and is recorded on the holographic film. In the reconstruction process, this individual hologram may be rotated to anywhere on the hologram disk. Suppose that it is rotated by an angle \(\theta_v\), hence the object point under consideration would be at the following location \(P_v(x_v, y_v, 0)\) in the observation coordinate system \(X_v-Y_v-Z_v\), where

\[
x_v = \sqrt{x_f^2 + (A - y_f^2)} \sin \left( \theta_v + \tan^{-1} \left( \frac{x_f}{A - y_f} \right) \right),
\]

\[
y_v = \sqrt{x_f^2 + (A - y_f^2)} \cos \left( \theta_v + \tan^{-1} \left( \frac{x_f}{A - y_f} \right) \right).
\]

The retrieved information from the hologram which is of interest to us is the term \(U_i = U_o U_r^* U_c\), where \(U_i\), \(U_o\), and \(U_r\) stand for the image wave, the object wave, and the reference wave, respectively.

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set of equations, where \( \lambda_r \) is the wavelength in the reconstruction process and \( \lambda_c \) is the wavelength for the hologram-forming process:

\[
\cos \alpha_i = \cos \alpha + \frac{\lambda_r}{\lambda_c} \cos \alpha - \frac{\lambda_r}{\lambda_c} \cos \alpha_r,
\]

\[
\cos \beta_i = \cos \beta + \frac{\lambda_r}{\lambda_c} \cos \beta - \frac{\lambda_r}{\lambda_c} \cos \beta_r.
\]

According to our design, the retrieved information from the hologram would produce an observation ring with radius \( d_{fe} \cos \theta_1 + A \) and is at a distance \( d_{fe} \sin \theta_1 \) above the hologram disk for the wavelength of the light during hologram recording. For light of longer (shorter) wavelength, the observation ring appears to be lower (higher) due to the dispersion effect (diffraction) of the holographic grating. When the observer’s eyes are placed in the vicinity of the observation zone, the diffracted light ray for the object point under consideration may reach one of his eyes through one individual hologram. The line joining the center of that eye pupil and the object point on the hologram plane determines the line of sight (Fig. 2) for that eye. Similarly, the other eye of the observer can see the same object point through another individual hologram. The intersection point of these two lines of sight then determines the final image position. All the image points of the original 3D object can be considered in exactly the same way using the same equations described above to obtain the final 3D image distribution in the observation coordinate system.

4. Numerical simulation and experimental result

A large set of holographic parameters can be used for numerical simulation to bring out the characteristics of the reconstructed image. Since this type of hologram has the potential of mass production utilizing the well developed CD technology, we report here a case with its radius \( A = 5 \)cm (defined as the distance from the center of hologram disk to the origin of any individual hologram), which is comparable to that of a real CD. A cube of length 2.5cm on each side is taken as the original object for numerical simulation with the following holographic parameters: \( \theta_1 = 45^\circ, \theta_0 = 0.36^\circ, d_0 = 67 \)cm, \( M_{cl} = 9.8 \), \( f_2 = 20 \)cm, \( f_3 = 30 \)cm, \( P_2 = 11 \)cm, \( d_{23} = 40 \)cm, \( q_3 = 56 \)cm, \( d_{fe} = 60 \)cm, \( M = 1.9 \), \( \theta_1 = 45^\circ, \theta_2 = 27.7^\circ, D_c = 19.4 \)cm, \( (A^2 + D_c^2)^{1/2} = 20 \)cm. For an observer with his eyes, each of a diameter 3mm, placed on the designated viewing ring for the wavelength of the light during hologram recording, the numerically simulated reconstructed image on the hologram plane is shown in Fig. 7(b). ABCD is the top surface of the cube image while EFGH is bottom surface. AD (FG) is the edge of the cube image closest to (farthest from) the observer since the image is perceived at an angle \( \theta_1 = 45^\circ \). The whole image is generated from a single individual hologram numbered \(-12\) as designed. The observed average wavelength for each image point is close to that for hologram formation which is 632.8nm, and the wavelength bandwidth is on the order of 4 to 6 nm. This is due to the fact that an individual hologram focuses the incident light of one particular wavelength into a point for observation and this focal point sweeps across the eye vertically owing to the dispersion effect of the hologram if a white-light point source is utilized for image reconstruction. When the observer moves his eyes down to 42.5 degree with respect to the horizontal plane, the observed image becomes that in Fig. 7(a). The mean wavelength for each object point is increased considerably, however, the wavelength bandwidth stays approximately the same. Different object points start to be observed through different individual holograms. For instance, object point E is observed through individual hologram numbered \(-10\). As the observation angle, defined as the angle of the plane of lines of sight and the horizontal plane, is increased the observed mean wavelengths of all the object points are decreased due to dispersion effect of the hologram, as shown from Fig. 7(c) to Fig. 7(f). The more deviation of the observation angle from the designated observation angle, the greater the wavelength variation from the top to the bottom of the image. For example, at observation angle equal to 55 degrees, the wavelength variation is more than 100nm.
Meanwhile, the variation of the observed individual hologram for various object points becomes more significant. As seen from the same figure, Fig. 7(f), the object point E is now seen through individual hologram numbered -22.

The reason why the observed mean wavelength and the individual hologram number of each object point is varied in the aforementioned way may be understood through a close look at the observation rings belonging to various wavelengths. Since the diffracted wave from an each object point is varied in the aforementioned way may be understood through a close look at the observation rings belonging to various wavelengths. Since the diffracted wave from an...
individual hologram becomes more astigmatic as the wavelength of the light deviates more from that in hologram formation, we separate into orthogonal directions, i.e., horizontal and vertical directions, to describe where the light of different wavelength is focused. Figure 8(a) shows the location and the spot size of the focal point for each wavelength in the horizontal direction while Fig. 8(b) shows those in the vertical direction. The observation ring in space can be conceived by rotating these figures about the $Z_v$-axis, i.e., $X_v = 0$ for left figure and $Y_v = 0$ for right figure. One can see that, as the wavelength goes up to 700nm, the astigmatic effect of the focal point is rather gentle since the wavelength deviation is only about 70nm. However, as the wavelength goes down to 500nm, the horizontal focus is approximately at $(y_v, z_v) = (-188, 232)$ while the vertical focus is at the other side of the hologram at $(y_v, z_v) = (188, -232)$. In this case, the horizontal focus is a real image and the vertical focus is a virtual image. When the wavelength goes to the shorter end of the visible spectrum, 400nm, both of the focal points become virtual images, one is at $(y_v, z_v) = (62, -105)$ and the other is at $(y_v, z_v) = (26, -49)$. Consider first in the vertical direction, if the focal point of the observation point is further away, Fig. 7(c) to Fig. 7(f), the upper part of the image points need more dispersion effect in order to reach the eye pupil. Hence, the observed wavelength for each image point is increased. On the contrary, for the lower part, less dispersion effect is needed and the observed wavelength is decreased. For the case when the vertical focus is nearer to the hologram than the observer, the situation is reversed. The lower-part (upper-part) image points require more (less) dispersion effect from the hologram in order for the light to reach the observer’s eye. Hence, the observed wavelength is increased (decreased), as shown in Fig. 7(a).

When the observer moves closer to the hologram, he can see an image of smaller region limited by his eye pupil and the viewing distance. The more violation of the viewing distance from the designated one, the less horizontal extent of the image can be observed. However, due the dispersion effect of white light source from holographic diffraction, he can always see the whole image in the vertical direction. Hence, the observed image from an individual hologram becomes a vertical strip. Referring to Fig. 9(a), the number of the individual hologram through which the center of the image is observed is changed to approximately 17 due to oblique observation of the eye. The right side of the image must be seen through individual hologram from even more left-hand side. Hence, for each image point, the number of individual hologram becomes more negative. On the contrary, each image point on the left hand side of the cube image requires the individual hologram at more right-hand side to be observed. Hence, the number of individual hologram becomes more positive for each of them. From Fig. 9(c) to Fig. 9(e), the situation is reversed since the designated viewing focus is in
front of the observer. The image points at right-hand side need to be observed through individual hologram at more right-hand side than that for the central image point. Hence, their individual hologram numbers becomes more positive. The reverse situation goes for image points on the left-hand side of the cube image.

![Hologram Plane](image)

Table 9. Observed wavelength bandwidth and number of individual hologram for the corners of the cube image at different distances along the designated viewing direction: (a) 40 cm (b) 60 cm (c) 90 cm (d) 130 cm (e) 180 cm.

Since at different locations other than the designated one, all the object points are observed through a set of different individual holograms, these result in that the observed 3D

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Fig. 9. Observed wavelength bandwidth and number of individual hologram for the corners of the cube image at different distances along the designated viewing direction: (a) 40 cm (b) 60 cm (c) 90 cm (d) 130 cm (e) 180 cm.
images are distorted by different amounts. Figure 10(a) shows the width-to-height ratio of the observed image as a function of the observation distances in the designated viewing direction. The holographic process without distortion compensation produces 3D image of width-to-height ratio equal to about 1.13 at the designated viewing location (60cm). When the observer moves closer to the hologram, the width-to-height ratio of the observed image becomes smaller, i.e. the observed image becomes skinnier. On the contrary, when the observer goes away from the hologram, the observed image becomes fatter with larger width-to-height ratio. The hologram, when viewed at different angles, generates images of different colors much like the situation of viewing a rainbow hologram. Figure 10(b) shows the width-to-height ratio of the observed image as a function of the viewing angle at the designated viewing distance. The image of longer wavelength (less observation angle) appears to be fatter with larger width-to-height ratio. On the contrary, the image of less observed wavelength appears to be skinner when observer at steeper viewing angle. It is interesting to note that, although without image pre-distortion compensation, the observed image can be of correct perspective at observation angle roughly equal to 47.5 degree.

Fig. 10. (a) Width-to-height ratio of the reconstructed image as a function of the viewing distance along the designated viewing direction. (b) Width-to-height ratio of the reconstructed image as a function of the observation angle at the designated viewing distance.

How good are these 3D images in focus? Two lines of sight of the observer always intersect at the axis of the hologram disk if each line of sight goes through the origin of an individual hologram. Hence, the center of each 3D image is quite in registry. But, how is the shortest distance between two lines of sight varied for any generalized image point? Figure 11 shows the shortest distance $D_{hl}$ for the upper-corner image point B, which is closer to the center of the hologram disk, as a function of the size of the cube image. For the image size we considered above, $D_{hl}$ is 0.425mm, which is approximately 3.5 times of the resolution element at the designated viewing distance. This image spans roughly from 3cm for the inner radius to 7cm for the outer radius of our CD-type multiplex hologram.
In the experiment, a 25 mW He-Ne laser (wavelength 632.8nm) with linear polarization perpendicular to the optical table is utilized as the light source for holographic recording. The LCD panel in the object beam is a product from Seiko-Epson Company (L3P10X-45G00) which has a dimension about 2cmx1.5cm with pixel size 19.5μmx19.5μm and total pixels 1028x772. This LCD module is connected to the desk-top computer for input of the 2D images. Instead of using a real object for holographic recording, we numerically mimic the viewing of a cube object, of dimension 2.5cm on each side, at a distance 60cm away. All the 2D images for holographic recording are obtained through viewing the 3D object with successive rotation angle of 0.36°, so totally it needs one thousand 2D images in order to produce a CD-type hologram. These images are stored in the desk-top computer and are displayed sequentially on the LCD through the “ACD See” software at successive time interval of 5 seconds. In order to obtain high contrast interference fringes for holographic recording, the path length of the object beam should be made equal to that of the reference beam, the intensity of image at recording plane should be equal to that of the reference wave, and the polarization direction should be in parallel for two interfering beams. Since the polarization state for the object wave is affected by the LCD panel, we put a linear polarizer with transmission axis perpendicular the optical table right behind the LCD to ensure the polarization direction of the transmitted light to be parallel to that of the reference beam. The contrast of the 2D image on LCD depends on the polarization state of the incident light, hence, a half-wave plate is placed in front of the pinhole assembly SF1 to rotate the polarization direction. In our experiment, a rotation of the polarization direction by about 40° for the incident light on LCD yields best observable image contrast with respect to the background. Multiple spectra of the 2D image, due to the grid structure of the LCD, are displayed at the plane of the lens L2. Both the horizontal and the vertical periods are about 6.7mm. A card board with a hole of linear dimension of 6mm is placed in this plane to act as a low-pass filter to get rid of the grid structure of the 2D image. This filtered 2D information travels downstream with enlargement factor M = 1.9 and is directly imaged onto the recording holographic film. In this experiment, we use PFG03c film produced by the Slavich company which has resolving power more than 5000lines/mm. The total intensity of the combined object and reference waves is about 19μW/cm². An exposure time of 0.6seconds in each ACD imaging frame gives the exposure energy of about 11μJ/cm² for each individual image-plane hologram. Since the image width on the recording film is approximately 2.5 to 3cm and the lateral shifting of successive individual image-plane hologram is about 0.3mm, the number of exposures on every location on the holographic film is about 80 to 100. This gives the total exposure energy for the holographic film to be about 1mJ/cm². After multiple image-plane holographic recording, the light-exposed area becomes a donut-shaped region. Then, the light-exposed holographic film is treated with the standard chemical procedure [14] given by the Slavich company. The procedure is following steps: 6min. in Hardener, 2min. in water, 2min. in JD-2 solution, 2min. in water, 5min. in PBU-Amidol solution, 2min. in water,
2min. in Stop Bath, 2min. in water. Then the processed film is waited to dry. The finished disk-type multiplex hologram is then illuminated by a white-light LED (Fig. 2) of luminous area of approximately 0.8mmx0.8mm, which is taken from a hand light (Fire Monster CREE.F27).

Figure 12 shows two images observed at the designated viewing distance (60cm) and viewing direction (45°) but with different azimuthal angles. These images are taken by a digital camera (Canon EOS 600D) which mimics the viewing of the observer by setting the aperture diameter to be about 3mm. The images observed along the designated viewing direction but at different viewing distances are shown in Fig. 13. These images reveal that, at smaller (larger) observation distance, the image appears to be skinnier (fatter) as expected from Fig. 10(a). Also, all these images appear to be quite monochromatic which is consistent with the theoretical result shown in Fig. 9.

Fig. 12. Typical images observed by both eyes of the observer at the designated viewing distance but with different azimuthal angles.

Fig. 13. Images observed at different viewing distances along the designated viewing angle of 45°.

Images observed at the designated viewing distance (60cm) but at different viewing angles with respect to the horizontal plane are shown in Fig. 14. As expected from Fig. 10(b), at larger (smaller) angle, image appears to be skinnier (fatter). Note that the image observed at 47.5° has the width-to-height ratio equal to one and hence appears to be more like a perfect cube. Comparing with the spectral characteristic of the theoretical result shown in Fig. 7, these three images show the consistency with that of Fig. 7(c), Fig. 7(d), and Fig. 7(f).

Fig. 14. Images observed at the designated viewing distance (60cm) but at different viewing angles with respect to the horizontal plane.
4. Conclusion

The CD-type image-plane multiplex hologram can produce clear image quite away from the hologram plane with illumination of white light from convenient light source. Utilizing the well developed CD technology, it has the potential to be mass produced. Further, with the advent of the holographic material of high sensitivity, it may finally be evolved into one form of the holographic printer.

Previously, we proposed this type of hologram to generate 3D image for walk-around viewing (Ref.12). In this paper, with some modification of the previous holographic optical system, the resulted image-plane disk-type multiplex hologram can be made to display virtual image for walk-around viewing as well. The axis of rotation for the recording film should be laterally shifted while the reference source point is maintained on this axis. The top of all the 2D images for holographic recording should be near the inner radius of the hologram disk. Moreover, the holographic recording for all the individual image-plane holograms should be in reverse order from that for real-image generation. Theoretical formulation following the previous method of “direct object-image relationship” is described. A generalized object point on the original 3D object is taken by a CCD camera and is imaged by optical system onto the recording film. The position on the recording film and the direction cosines of this image point can be found through geometrical ray tracing. In image reconstruction, the image ray of some spectral bandwidth may reach one eye of the observer. Similarly, it can reach another eye of the observer through another individual hologram. The intersection point determines the final image location perceived by the observer. All the image point can be considered in the same process to obtain the final reconstructed 3D image.

Numerical simulation for a disk-type hologram with its radius comparable to the usual CD is presented. The finite size of radius limits the image size to be around 2.5cm. The images observed at the designated viewing distance but at various angles as well as at the designated viewing direction but at different distances are presented. At the designated viewing location, the observed wavelength bandwidth for each corner of the cube image is approximately 4 to 6nm. When the observation position deviates from the designated one, the observed spectrum and the number of individual hologram belonging to different corners of the cube image are varied to some different degree. Close investigation of the observation rings belonging to different wavelengths show that they are astigmatically focused. Beyond particular wavelength, the observation ring becomes virtual image (i.e., not focused as a real-image ring). With this characteristic, the variation of the observed wavelength bandwidth and individual hologram of different corners of the cube image is explained. The width-to-height ratio of the reconstructed image is increased as the observation distance is increased. At the designated viewing distance, the width-to-height ratio of the reconstructed image is decreased as the observation angle is increased. Some experimental results are provided which show consistency with those from the numerical simulation.

It is generally believed that such a vast number of multiple exposures for this type of hologram would reduce the diffraction efficiency drastically (inversely proportional to the number of multiple exposures). However, experimentally, quite bright reconstructed image can still be observed under room-light illumination. One way to increase the diffraction efficiency of the hologram is to copy the hologram as another hologram. Due to the elimination of the bias-build problem and hence the full utilization of the dynamic range of the recording film, the diffraction efficiency of this CD-type image-plane multiplex hologram can be significantly increased.

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