Microscopic calculations of isospin mixing in $N \approx Z$ nuclei and isospin-symmetry-breaking corrections to the superallowed $\beta$-decay

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Abstract

Recently, we applied, for the first time, the angular momentum and isospin-projected nuclear density functional theory to calculate isospin-symmetry breaking (ISB) corrections to superallowed $\beta$-decay. With the calculated set of ISB corrections, we found that $|V_{ud}| = 0.97447(23)$ for the leading element of the Cabibbo–Kobayashi–Maskawa matrix. This is in good agreement with both the recent result of Towner and Hardy (2008 Phys. Rev. C 77 025501) and the central value deduced from the neutron decay. In this paper, we extend our calculations of the ISB corrections covering all superallowed transitions $A, I^z = 0^+, T = 1, T_z \rightarrow A, I^z = 0^+, T = 1$ and $T_z + 1$ where $T_z = −1, 0$, and $A$ ranges from 10 to 74.

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1. Introduction

The isospin symmetry [1, 2] in a nuclear medium is only weakly broken, reflecting the relative weakness of isospin breaking as compared with the isospin-conserving part of the nucleon–nucleon interaction. Hence, the related isotopic spin quantum number, $T$, albeit approximate, remains very useful in labeling nuclear states and in understanding selection rules on different types of nuclear reactions (see [3] and references therein). In particular, for Fermi (vector) and Gamow–Teller (axial) $\beta$-decay, one has $\Delta T = 0$ and $\Delta T = 0, \pm 1$, respectively, with the exception of $I^z = 0^+ \rightarrow I^z = 0^+$ transitions that are strictly forbidden for the Gamow–Teller process [4].

Among pure Fermi transitions, of particular importance are superallowed transitions between the isobaric analogue states $I^z = 0^+, T = 1, T_z \rightarrow I^z = 0^+, T = 1$ and $T_z + 1$ in $N \approx Z$ nuclei. These data are used for performing precision tests of the conserved vector current (CVC) hypothesis, i.e. the nuclear medium independence of the vector current. With the CVC hypothesis being verified, they serve as the most precise source of the $V_{ud}$ element of the Cabibbo–Kobayashi–Maskawa (CKM) matrix which is a key ingredient in investigating its unitarity.

The CVC hypothesis is verified by investigating the nucleus independence of the $\mathcal{F}t$-values:

$$\mathcal{F}t \equiv ft(1 + \delta_R^t)(1 + \delta_{\text{NS}} + \delta_C) = \frac{K}{2G_V^t(1 + \Delta_C^t)} = \text{const},$$

where $K/(hc)^6 = 2\pi^3\hbar \ln 2/(m_e c)^2\delta = 8120.2787(11) \times 10^{-10}$ GeV$^{-4}$ s is a universal constant and $G_V$ stands for the vector coupling constant for the semileptonic weak interaction. The $\mathcal{F}t$-values include empirical reduced lifetimes $ft$ corrected theoretically for radiative processes and isospin-symmetry breaking (ISB). The radiative corrections are routinely divided into the following: a nucleus-independent part $\Delta_C^t = 2.361(38)$% [5], a transition-dependent (Z-dependent) but nuclear-structure-independent part $\delta_R^t$ [5, 6] and a nuclear-structure-dependent part $\delta_{\text{NS}}$ [6, 7]. The ISB correction, $\delta_C$, is defined through the
following nuclear matrix element:

\[ |\langle I = 0, T \approx 1, T_z = \pm 1| \hat{T}_\pm | I = 0, T \approx 1, T_z = 0 \rangle|^2 \approx 2(1 - \delta_C), \quad (2) \]

where \( \hat{T}_\pm \) are the raising and lowering bare isospin operators, respectively.

The application of superallowed \( \beta \)-decay in testing the three-generations-quark standard model of elementary particles requires both high-accuracy empirical \( f_I \) values and a high-quality theory. The aim of this paper is to present new results on the \( \delta_C \) corrections obtained using angular momentum and isospin projected density functional theory (DFT) recently developed by our group [8–14]. The model will be introduced in section 2, the results on \( \delta_C \) will be presented in section 3 and the paper will be briefly summarized in section 4.

2. The model

The degree of isospin mixing in atomic nuclei (isospin impurity) is predominantly a result of the subtle interplay between short-range isospin-symmetry-conserving strong interaction and the long-range ISB Coulomb force. The Coulomb force polarizes wave functions of all participating protons and, in turn, neutrons, thereby creating a number of conceptual and technical difficulties within the nuclear shell model (SM) that have rather profound consequences for calculations of \( \delta_C \) corrections [6, 15–18]. The SM can be used to compute only a part of the correction related to configuration mixing \( \delta_C^1 \). The second part, \( \delta_C^2 \), related to the radial mismatch of the single-particle wave functions, must be calculated independently using a mean field. Both corrections are treated as additive: \( \delta_C = \delta_C^1 + \delta_C^2 \). The consequences of this rather artificial division include, for example, the necessity of using effective isospin operators which violate the SU(2) commutation rules. This problem was recently noticed and discussed extensively by Miller and Schwenk [19] who, however, did not provide any quantitative estimate of the impact of these deficiencies on the \( \delta_C \) results of Towner and Hardy (TH) [6] whose calculations have set the standard in this field.

In contrast, Hartree–Fock (HF) and DFT are free from these specific problems. Here all nucleons participate on an equal footing and a balance between long- and short-range effects is treated in a self-consistent manner. It is well known, however, that these approaches break spontaneously fundamental nuclear symmetries including rotational and isospin symmetry. In fact, the isospin symmetry is violated both explicitly, by virtue of charge-dependent interactions, and spontaneously, which leads to unphysical quenching of true isospin mixing.

In order to avoid spontaneous mixing and to compute matrix element (2) in a fully quantum mechanical way by using bare isospin operators, we have recently developed the angular-momentum and isospin projected scheme on top of the Skyrme-DFT approach. The approach is based on re-diagonalization of the entire Hamiltonian including the Coulomb interaction in good angular momentum and good isospin basis

\[ |\psi; IMK; TT_T; C_{\hat{T}_T}, \hat{P}_M^{\hat{T}_T}| \approx \hat{P}_M^{\hat{T}_T} \hat{P}_M^{\hat{T}_T} |\psi\rangle, \quad (3) \]

projected of the self-consistent Slater determinant \(|\psi\rangle\). Here \( \hat{P}_M^{\hat{T}_T} \) and \( \hat{P}_M^{\hat{T}_T} \) are the isospin and angular momentum projection operators (for further details see [20]).

For the Fermi matrix element (2) the normalized bra and ket states are calculated in the following way. The \(|I = 0, T \approx 1, T_z = \pm 1\rangle\) state in the even–even nucleus is projected of the self-consistent Slater determinant, \(|\psi\rangle\), representing the ground state in this nucleus:

\[ |\langle I = 0, T \approx 1, T_z = \pm 1| = \sum_{T \geq 1} c_T^{(\psi)} \hat{P}_M^{\hat{T}_T} \hat{P}_I^{I=0} |\psi\rangle. \quad (4) \]

The state \(|\psi\rangle\) is unambiguously defined. The \(|I = 0, T \approx 1, T_z = 0\rangle\), on the other hand, is projected of the self-consistent Slater determinant, \(|\psi\rangle\), representing the so-called anti-aligned configuration \(|\psi\rangle \approx |T \otimes \pi \rangle \) (or \(|\nu \otimes \pi \rangle \), selected by placing the odd neutron and odd proton in the lowest available time-reversed (or signature-reversed) single-particle orbits:

\[ |\langle I = 0, T \approx 1, T_z = 0| = \sum_{T \geq 0} c_T^{(\psi)} \hat{P}_M^{\hat{T}_T} \hat{P}_I^{I=0} |\psi\rangle. \quad (5) \]

The selected single-particle configuration \(|\nu \otimes \pi \rangle \) manifestly breaks the isospin symmetry (see figure 1 in [11]). This is essentially the only way of reaching the \(|T \approx 1, I = 0\rangle\) states in odd–odd \( N \neq Z \) nuclei. Indeed, only the \( T = 0 \) states in \( N \approx Z \) nuclei can be represented by a single Slater determinant. The final expression for the nuclear matrix element (2) will be given in our forthcoming paper.

Two major drawbacks of the model in its present formulation are (i) the lack of pairing correlations and (ii) the use of an old-fashioned and low-quality SV parametrization of the Skyrme force. The latter deficiency pertains to the angular momentum projection, which is known to be ill-defined for density-dependent modern Skyrme and Gogny energy density functionals (EDF) [21–25]. At present, the SV interaction augmented by a tensor term is essentially the only available Skyrme interaction originating from the true Hamiltonian which can be safely used in connection with the angular momentum projection without any further regularization [25]. It is worth mentioning here that the isospin impurities calculated using modern Skyrme forces in the isospin-projected variant of our model [10, 12] are consistent with the recent data extracted from the giant dipole resonance decay studies in \(^{80}\text{Zr}\) [26] and the isospin-forbidden E1 decay in \(^{64}\text{Ge}\) [27]. This indicates that our model is, in principle, capable of quantitatively capturing the amount of isospin mixing that is important in the context of making reliable calculations of \( \delta_C \) in spite of the fact that \( \delta_C \) is mostly sensitive to the difference between the isospin impurities of parent and daughter nuclei [28, 29].

3. The results

The isospin-breaking corrections \( \delta_C \) were computed by different groups and with diverse nuclear models [6, 29–32].
Our calculations were published recently in [14] and were compared to the TH ones [6]. A comparison of δCs shows that although individual values differ, both sets of calculations follow a similar trend with increasing A. The differences between individual values are stronger in light nuclei and can be traced back to the poor spectroscopic quality of the SV parametrization. It is interesting to note that our results are considerably larger as compared with the results of [32], which are based on the relativistic Hartree (RH) + random phase approximation (RPA) formalism.

The average value of the nucleus-independent reduced lifetime (1) calculated for 12 out of 13 high-precision superallowed β-decays (excluding the 38K → 38Ar transition) equals 0.3704(9) s, which is consistent with the CVC hypothesis. This value was obtained using the Gaussian-distribution-weighted formula to conform with the standards of the Particle Data Group (PDG) and using our adopted δC listed in table 1 of [14]. In the calculations, we used the radiative corrections and f(t)-values taken from [6] and [18], respectively. The calculated 0.07447(23), which is in good agreement with the TH result 0.07448(26) and a central value deduced from the neutron decay 0.0746(19). Combining our Vval with the results of 0.2252(9) and 0.0038(44) adopted from the most recent PDG compilation [33] leads to |Vval|² + |Vud|² + |Vub|² = 1.000 31(61). This result implies that the CKM unitarity inferred from its first row is fulfilled with 0.1% precision.

The confidence level (CL) of our calculations can be assessed by performing the test proposed recently in [18]. The test indicates that the CL of our model is lower than the CL of other models analyzed in [18]. It should be stressed, however, that the low CL of our model primarily results from a very small δC for a single transition in A = 62. This seems to be a part of the deeper problem faced not only by our model but also, in fact, all models involving a self-consistent mean field. Indeed, these models have problems in reproducing quantitatively the rapid increase in δC expected to occur already at A ~ 62; see [18]. In our model, the increase in δC takes place at A = 70, similar to the RH + RPA calculations of [32]. To a lesser extent this problem is also seen in the SM + HF calculations of [17, 34].

The low confidence level of our results reflects the poor spectroscopic quality (ordering and energy of the single-particle levels) of the SV EDF. Global characteristics of the functional are also unsatisfactory. In particular, it reproduces the nuclear binding energies of even–even superallowed β-decays (excluding the 38K → 38Ar transition) equals 0.3704(9) s, which is consistent with the CVC hypothesis. This value was obtained using the Gaussian-distribution-weighted formula to conform with the standards of the Particle Data Group (PDG) and using our adopted δC listed in table 1 of [14]. In the calculations, we used the radiative corrections and f(t)-values taken from [6] and [18], respectively. The calculated 0.07447(23), which is in good agreement with the TH result 0.07448(26) and a central value deduced from the neutron decay 0.0746(19). Combining our Vval with the results of 0.2252(9) and 0.0038(44) adopted from the most recent PDG compilation [33] leads to |Vval|² + |Vud|² + |Vub|² = 1.000 31(61). This result implies that the CKM unitarity inferred from its first row is fulfilled with 0.1% precision.

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| A | Tz = 0 | Tz = 1 | Tz = 0 | Tz = 1 | Tz = 0 | Tz = 1 | Tz = 0 | Tz = 1 | Tz = 0 | Tz = 1 |
|---|---|---|---|---|---|---|---|---|---|---|
| 10 | 0.559 | 0.497 | 0.610 | 0.767 |
| 14 | 0.290 | 0.189 | 0.386 | 0.486 |
| 18 | 2.031 | 1.819 | 0.602 | 0.460 |
| 22 | 0.243 | 0.255 | 0.805 | 0.622 |
| 26 | 0.399 | 0.308 | 5.828 | 4.235 |
| 30 | 1.260 | 0.974 | 62 | 1.739 | 0.854 |
| 34 | 0.865 | 0.679 | 66 | 1.200 | 0.850 |
| 38 | 8.315 | 9.826 | 70 | 1.527 | 1.516 |

Figure 1. Differences between theoretical and experimental Q-values for the superallowed β-decays $I^\pi = 0^−, T = 1, T_0 \rightarrow I^\pi = 0^+, T = 1$ and $T = 0$ versus the atomic number $A$. Black and white symbols represent transitions in mirror-symmetric pairs corresponding to $T_z = -1$ and $T_z = 0$, respectively.

The table shows that these difficulties are seen in both the odd–odd and pathologically large mixing in $A = 2$. In particular, it is unclear how to cure this anomaly or how to estimate its influence on isospin mixing.

Table 1. Calculated values of $\delta_C$ for the superallowed $\beta$-decays $I^\pi = 0^−, T = 1, T_0 \rightarrow I^\pi = 0^+, T = 1$ and $T = 1$ with the atomic number $A$. Black and white symbols represent transitions in mirror-symmetric pairs corresponding to $T_z = -1$ and $T_z = 0$, respectively.

Table 1. Calculated values of $\delta_C$ for the superallowed $\beta$-decays $I^\pi = 0^−, T = 1, T_0 \rightarrow I^\pi = 0^+, T = 1$ and $T = 1$ with the atomic number $A$. Black and white symbols represent transitions in mirror-symmetric pairs corresponding to $T_z = -1$ and $T_z = 0$, respectively.
in mirror-symmetric transitions \( T_z = -1 \rightarrow T_z = 0 \) and \( T_z = 0 \rightarrow T_z = 1 \) follow a similar trend with increasing \( A \) but the individual values are different. In some cases the differences are sizeable. It is interesting to observe that the largest differences between \( \delta_C \) in mirror-symmetric pairs occur in \( A = 62 \) and \( A = 66 \), i.e. exactly in the region where the transition from small to large values of \( \delta_C \) is predicted to occur in the SM + Woods–Saxon potential calculations \([6, 18]\). This result indicates that configuration mixing around \( A = 62 \) is very fragile in the self-consistent calculations. The effect may be sensitive to various characteristics of the underlying EDF and/or to missing correlations, and requires further study.

4. Summary and conclusions

We have extended the isospin and angular momentum projected DFT calculations of the isospin-breaking corrections \( \delta_C \) to all theoretically possible superallowed \( \beta \)-decays in nuclei ranging from \( A = 10 \) to \( A = 74 \) in order to assess the stability of our predictions. The calculations reveal that values of \( \delta_C \) in mirror-symmetric decays \( T_z = -1 \rightarrow T_z = 0 \) and \( T_z = 0 \rightarrow T_z = 1 \) follow a similar trend versus the atomic number \( A \). The individual corrections in mirror-symmetric decays are shown to be different, in some cases sizeably, reflecting differences in configuration mixing in \( T_z = \pm 1 \) nuclei. The largest differences were found for the cases \( A = 62 \) and \( 66 \), i.e. in the region where the SM plus Woods–Saxon potential calculations \([6, 18]\) predict a rapid transition from small to large values of \( \delta_C \). This transition is delayed in the self-consistent calculations.

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