Analysis of Secondary School Students' Algebraic Thinking and Math-Talk Learning Community to Help Students Learn

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Abstract. This study aims to determine the difficulties of algebraic thinking ability of students in one of secondary school on quadrilateral subject and to describe Math-Talk Learning Community as the alternative way that can be done to overcome the difficulties of the students' algebraic thinking ability. Research conducted by using quantitative approach with descriptive method. The population in this research was all students of that school and twenty three students as the sample that was chosen by purposive sampling technique. Data of algebraic thinking were collected through essay test. The results showed the percentage of achievement of students' algebraic thinking’s indicators on three aspects: a) algebra as generalized arithmetic with the indicators (conceptually based computational strategies and estimation); b) algebra as the language of mathematics (meaning of variables, variable expressions and meaning of solution); c) algebra as a tool for functions and mathematical modelling (representing mathematical ideas using equations, tables, or words and generalizing patterns and rules in real-world contexts) is still low. It is predicted that because the secondary school students was not familiar with the abstract problem and they are still at a semi-concrete stage where the stage of cognitive development is between concrete and abstract. Based on the percentage achievement of each indicators, it can be concluded that the level of achievement of student’s mathematical communication using conventional learning is still low, so students' algebraic thinking ability need to be improved.

1. Introduction

One type of thinking needed in mathematics is algebraic thinking. In the broadest sense, algebraic thinking consist of understandings series which is needed to interpret the world by translating information or events into the language of mathematics in order to explain and predict the phenomena [1]. Theoretically or practically, algebra is found in many areas of life. For example, bankers who study patterns for electronically transferred funds and computer experts use digit patterns and switch to give code for complex statements. The discovery of the usefulness of algebra in various aspects of life and in all areas of mathematics, makes algebraic thinking an important set of thought. In this regard, the National Council of Teachers of Mathematics has set expectations for middle and high school algebra. “In grades 6-8 all students should represent, analyze, and generalize a variety of patterns with tables, graphs, words, and when it possible, they should using symbolic rules”[2].

Algebra is the gateway for high-level mathematics and opportunities [3]. Educators and policy makers also make algebra a focal point, evident from the phrase "Algebra for all" which emphasizes the importance of providing all algebraic access for students [4]. The International Commission on
Mathematical Instruction (ICMI) also shows that algebra is an essential fundamental thinking ability. This was marked by the 12th ICMI research conference focused on algebra with the theme "The Future of the Teaching and Learning of Algebra".

In accordance with the opinions of these experts, it can be stated that algebraic thinking is an essential and fundamental element of mathematical thinking. But apparently, based on the results of previous literature studies, obtained some facts that algebra is one of the material that is difficult to be mastered by students. Secondary school students have difficulties and misconceptions in learning algebraic subjects [5-7]. One fact is that only 22% of California's eighth-graders demonstrate proficiency in an equivalent algebra course [8]. In addition, sixth, seventh and eighth grade students do not have a good understanding of interpretation of equivalence equations and variable concepts [9].

The facts obtained from these countries about the difficulties in algebra, seems doesn’t have much different from the facts found in secondary school students in Indonesia. There is still a problem with the algebraic thinking ability of students who still in secondary school. Yumiati found mistakes made by secondary school students related to the concept of algebra based on a preliminary study conducted in North Jakarta [10]. The result shows that students' algebraic thinking ability is considered low.

This case, of course, indicates that students’ algebraic thinking is still low. Based on these facts, researcher feel that need to make an effort that have aim to determine the extent of algebraic thinking ability of students in secondary school, thus they can find the difficulties faced by students in the problems of algebraic thinking. This effort can help educators to find the ways to solve the problems in students' algebraic thinking ability especially in Indonesia.

2. Experimental Method

Research conducted using quantitative approach with descriptive method. The population in this research was all students of secondary school and 23 students as the sample that was chosen by purposive sampling technique. Data of algebraic thinking were collected through essay test. Furthermore, the data were analyzed with a descriptive way.

3. Result and Discussion

The indicators used in this study and the results achieved by the students on each indicator can be seen on table 1:

| Aspects of Algebraic Thinking | Indicators                                      | Achievement Results of All Students |
|------------------------------|------------------------------------------------|------------------------------------|
| Algebra as generalized       | Conceptually based computational strategies    | 61%                                |
| arithmetic                  | Estimation                                     | 57%                                |
| Algebra as the language of   | Meaning of variables and variable expressions  | 45%                                |
| mathematics                 | Meaning of solution                            | 41%                                |
| Algebra as a tool for        | Representing mathematical ideas using           | 57%                                |
| function and mathematical    | equations, tables or words                     |                                    |
| modeling                    | Seeking activities, generalizing patterns and  | 14%                                |
|                             | rules in real-world contexts                   |                                    |

Table 1 shows that students' algebraic thinking ability is considered low because none of the indicators of algebraic thinking reach 70% or 80% upwards. In fact, some of these indicators are in the range below 50% i.e the meaning of variables and variable expressions indicators, the meaning of solution indicator and seeking activities, generalizing patterns and rules in real-world contexts indicator.
The following will discuss the student difficulties of each indicator based on the answers they provide in the test descriptively. As for question number 1 with basic concept of computational strategies indicator is: “The floor of a room in Mr. Ichsan’s house is rectangular with a length of 6 m and a width of 3 m. The floor will be covered with a tile measuring 30 cm x 30 cm with a price of Rp. 84,000,00 every box. One box contains 25 tiles. What is the cost incurred by Mr. Ichsan to buy the tiles?”

Figure 1 shows the answers of most students. They know the basic initial concept, but they made a mistake when looking for incurred cost. Students have no problem with calculation (arithmetic), but they have problem to determine how many boxes of tiles are needed. Only 6 students (26%) of 23 students answered the question correctly, while others showed the correct process but did not reach the right result.

Figure 1. Student answer for number 1 with conceptually based computational strategies indicator

Then the question number 2 with the estimation indicator is: “On the tiled floor, there is a wake as shown below. Estimated area of the wake area is”

Figure 2. Question number 2

The answer in Figure 3 shows the students' difficulty in estimating half-full and more than half-full squares. Then there are also some students who count the less full square, whereas the less full square is not counted as one full square. Of the 23 students, only 5 students (22%) answered correctly while the rest only showed the correct process.
Figure 3. Student answer for number 2 with the estimation indicator

Next the question number 3 with meaning of variable and variable expressions is: “The height of a PQRS parallelogram is 1/3 times the base length. It is known that the width of the parallelogram is 75 cm\(^2\). Determine the length of the base and height of the parallelogram.” Based on the student’s answer in Figure 4, we can see the student's difficulties on the meaning of variable indicator. There are students who know the expression of the variable but can not complete the calculation until the end and most students do not know the meaning of the variable in question. Only 4 students (17%) of 23 students answered correctly.

![Figure 4. Student answer for number 3 with meaning of variables and variable expressions](image)

The question number 4 with meaning of solution indicator is: “Andi has 2 cartons in the shape of a diamond and a kite. The diamond-shaped carton is inside the kite as in the picture. Determine the shaded area.”

![Figure 5. Picture number 4 with meaning of solution indicator](image)

Figure 6 shows that students know the process that must be passed to get results. They have no difficulty in determining the extent of the kite and the rhomb, but they have difficulty when determine the value of the “y” variable. Only 3 students (13%) of 23 students answered the question correctly.

![Figure 6. Student answer for number 3 with meaning of solution indicator](image)
Next question number 5a and 5b with represent mathematical ideas using equations, tables or words and seeking activities, generalizing patterns and rules in real-world contexts, are: “Gardens are framed single row of tiles as illustrated here. a) How many border tiles are required for a garden of length 18? b) How many border tiles are required for a garden of length “n”?

Figure 7. Picture of question number 5a and 5b

Figure 8 shows students’ answers in representing mathematical ideas using tables, but students have difficulty to understanding the problem. Students errors have been seen at the time of the park with a length of 1 meter, the tiles around the park are 2, whereas it is clear in the case that the garden with a length of 1 meter is surrounded by 8 tiles. A total of 12 students (52%) of the 23 students answered the question correctly. Most of them respond with words and pictures.

Figure 8. Student answer for number 5a

Figure 9 shows students’ difficulties in generalizing patterns. It can be seen from the answer that students only identify the right side and the left side. While the top tile and bottom tiles are not identified because they do not understand the meaning of "taman dengan panjang sisi n meters". From the total number 23 students, only 2 students (9%) answered the question correctly while most of them left it empty.

Figure 9. Student answer for number 5b

Based on the results of the analysis, many students have difficulty in indicating the meaning of variables and variable expressions indicators, the meaning of solution and generalize the pattern. While on the problem aspect: algebra as arithmetic generalization, there are not many students have difficulty. For representing mathematical ideas using equations, tables or words indicator, there are not many students have difficulty also. Most of them represent ideas with images. Researcher believe the images provide a link between numeric and algebra for students. This is reasonable considering that secondary school students are still at a semi-concrete stage where the stage of cognitive development is between concrete and abstract (according to Piaget).

One effort that can be done by educators in improving students’ algebraic thinking ability is to use appropriate learning model. Learning model that is supposed to improve students’ algebraic thinking ability is Math-Talk Learning Community, "Math-Talk Learning Community as a community in which one assists mathematical discourse" [11]. The statement provides the definition that math talk learning as a learning process framework that provides an opportunity for every individual both students and
teachers to interact or help other students in learning mathematics with a meaningful mathematical discourse community so that the learning process (mathematical discourse) will be achieved as the goal of learning.

The four main components of Math-Talk Learning are the process of learning development from time to time: questioning, explanation of mathematical thinking, source mathematical ideas, and responsibility for learning. Teacher builds the Math-Talk inquiry environment and encourages constructive discussion of problem solving methods through well-structured classroom activities based on the four Math-Talk components. Bednarz, Kieran, and Lee in Windsor argue that a classroom environment that values and promotes collaborative learning situations, student discourse, and provides students with opportunities to communicate mathematical ideas and conjectures can facilitate algebraic thinking ability better [12]. One of the learning that creates and builds mathematical discourse in the classroom is Math-Talk Learning Community.

4. Conclusion
Based on the achievement of each indicator, it can be concluded that students' algebraic thinking ability is still low with the average of each indicator is 46%, so the students' algebraic thinking ability need to be improved. The efforts that can always be done is to develop the competence of teachers in educating, in addition the effort to choose the appropriate model or learning strategy do not be stopped also. And one of the learning models that can be used by teachers and allegedly can improve students' algebraic thinking ability is Math-Talk Learning Community. Furthermore, this research can be developed next by using the learning model with the aim to improve students' algebra thinking ability.

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