**On the rapid intensification of Hurricane Wilma (2005). Part IV: Inner-core dynamics during the steady radius of maximum wind stage**

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Recent studies show that some hurricanes may undergo rapid intensification (RI) without contracting the radius of maximum wind (RMW). A cloud-resolving mesoscale model prediction of Hurricane Wilma (2005) is used herein to examine what controls the RMW contraction and how a hurricane could undergo RI without contraction. Results show that the processes controlling the RMW contraction are different within and above the planetary boundary layer (PBL). In the PBL, radial inflow contributes to contraction, whereas frictional dissipation acts as an inhibiting factor. Above the PBL, radial outflow and vertical motion are the two main factors governing the RMW contraction, with the former inhibiting it. A budget analysis of absolute angular momentum (AAM) shows that the radial AAM flux convergence is the major process accounting for the spin-up of the maximum rotation in the PBL as the RMW contracts, while the vertical flux divergence of AAM and the friction oppose the spin-up. During the RI stage with no RMW contraction, the local AAM tendencies in the eyewall are however smaller in magnitude and narrower in width than those during the contracting RI stage. In addition, the AAM following the time-dependent RMW decreases with time in the PBL and remains nearly constant aloft during the contracting stage, whereas it increases during the non-contracting stage. These results reveal different constraints for the RMW contraction and RI, and help explain why a hurricane vortex can still intensify after the RMW ceases contraction.

**KEYWORDS**
inner-core dynamics, rapid intensification, steady RMW

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**1 | INTRODUCTION**

Although some promising progress has been made in improving the operational intensity forecasts of tropical cyclones (TCs) in recent years (DeMaria et al., 2014; Tallapragada et al., 2015), our ability to predict the rapid intensification (RI) of TCs, which is defined by Holliday and Thompson (1979) as a deepening rate of 42 hPa (24 h)−1 in the minimum central pressure \(P_{\text{MIN}}\) or by Kaplan and DeMaria (2003) as an increase of greater than 15 m/s (24 h)−1 in the maximum sustained surface wind speed \(V_{\text{MAX}}\), is still limited. The main difficulties in predicting RI are often attributed to deficiencies in our understanding of multiscale processes leading to RI, which range from large-scale environmental influences to inner-core convective structures and ice microphysics (Gray, 1968; Marks and Shay, 1998; Kaplan et al., 2010; Miller et al., 2015; Susca-Lopata et al., 2015).

In a widely accepted convective-ring theory, TC intensification is often accompanied by the inward contraction of the
radius of the maximum wind (RMW) (Schubert and Hack, 1982; Willoughby et al., 1982; Willoughby, 1990b). Zhang et al. (2001), and Montgomery and Smith (2011) showed that the spin-up of swirling winds in the planetary boundary layer (PBL) occurs in the presence of intense radial inflow, because of the inward advection of the absolute angular momentum (AAM) defined by

\[ M = rv_i + \frac{fr^2}{2}, \]

where \( r \) is the radius relative to the TC centre, \( v_i \) is the tangential wind and \( f \) is the Coriolis parameter.

However, Vigh (2010) showed that some major hurricanes, such as Andrew (1992), Lili (2002) and Wilma (2005), ceased eyewall contraction prior to reaching their peak intensities. Later, Kieu (2012) documented the rapid slow-down of eyewall contraction, followed by a nearly steady RMW structure, hereafter referred to as S-RMW, in a series of ensemble simulations of Hurricane Katrina (2005). The rapid slow-down of RMW contraction and the subsequent S-RMW phenomenon are also apparent during the RI of intense TCs, for example, in the simulations of Hurricane Wilma (2005) by Chen et al. (2011, hereafter C11), and Typhoon Megi (2010) by Wang and Wang (2014). More recently, Qin et al. (2016) carried out a statistical analysis of the S-RMW phenomenon associated with rapidly intensifying hurricanes using 25-year Extended Best Track datasets, in order to see to what extent the S-RMW phenomenon, revealed mostly by TC models, occurs in real storms. They found that about 59% of the RI events at 24 h intervals associated with 55 rapidly intensifying hurricanes exhibited the S-RMW phenomenon, and that the S-RMW tends to occur more commonly in more intense hurricanes (i.e. category 2 or higher) and when the RMW contracts to less than 50 km.

Since both the rapid slow-down of the eyewall contraction and the S-RMW phenomenon could not be explained well by the previous theory of TC intensification, a natural question one may ask is: how could TCs undergo intensification without RMW contraction? Kieu (2012) addressed this question by relating the cessation of RMW contraction to the balance between the inward advection of absolute vorticity by radial inflow and the azimuthal frictional dissipation, using the tangential momentum equation. The role of frictional dissipation in preventing the RMW from collapsing is also confirmed by Castaño et al. (2014). In contrast, Wang and Wang (2014) attributed the non-contracting RMW structure during the RI of Typhoon Megi (2010) to the rapid release of latent heat in the outer spiral rain bands interacting with a low-level synoptic depression in which the storm was embedded. Using a kinematic approach, Stern et al. (2015) derived a diagnostic model to investigate the eyewall contraction in idealized simulations, and they attributed the RMW tendency to the radial gradient of the local \( v_i \) tendency and the radial curvature of \( v_i \) at the RMW. However, their kinematic derivation offered little dynamical explanation for why the contraction would continue or stop (Kieu and Zhang, 2017). Apparently, none of the previous studies have offered an explanation for the mechanisms underlying the RI of swirling winds with the constant RMW.

In this study, we attempt to address, among others, the above question by diagnosing a 72-h prediction of Hurricane Wilma (2005) using the Weather Research and Forecasting (WRF) model with the finest resolution of 1 km, which was presented in Part I of this series of articles (C11). (All data used herein are taken from the 1 km resolution WRF model output at 5 min intervals.) The predicted storm, which compares reasonably well against various observations with respect to intensity and track (C11), reaches its peak intensity with a local \( V_{\text{MAX}} \) of 72 m/s and a \( P_{\text{MIN}} \) of 889 hPa at 36 h into the integration, valid at 1200 UTC 19 October 2005 (hereafter dd/hh, 19/12). The simulated storm undergoes a 21 h RI, with a 14 m/s maximum wind increase during the first 5 h and a 13 m/s increase for the remaining 16 h period. C11 showed a slow-down of RMW contraction during Wilma’s RI stage, followed by a period of near steady and even slightly increasing RMW as the storm approaches its maximum intensity (see figure 11 therein). In Part II (Chen and Zhang, 2013, hereafter CZ13), the rapid deepening of \( P_{\text{MIN}} \) is attributed to the generation of intense upper-level warming in the eye from the descending stratospheric air induced by convective bursts in the eyewall. In Part III (Miller et al., 2015), we emphasized the important roles of more numerous convective bursts inside the RMW, co-located with more intense upper-level latent heating associated with depositional growth, in determining the RI of Wilma. In the present study, we wish to examine the generation of the S-RMW phenomenon in relation to the inner-core structures during the RI of Wilma. Specifically, the objectives of this study are to (a) examine the spatio-temporal characteristics of inner-core vortex structures during the S-RMW stage of RI; (b) investigate the mechanisms whereby the RI can continue even with the S-RMW structure; and (c) finally interpret, to the extent possible, why RI could occur in the absence of the RMW contraction.

The next section provides a kinematic description of the inner-core flow structures, especially in the vicinity of the RMW, during Wilma’s RI. Section 3 examines what processes control contraction of the RMW, how the S-RMW could develop and be maintained, and how RI could occur in the absence of RMW contraction. Section 4 provides a more detailed understanding of the inner-core dynamics of RI during the S-RMW stage. A summary and concluding remarks are given in the final section.

2 | SPATIO-TEMPORAL VORTEX STRUCTURES DURING RI

Figure 1a shows the WRF-predicted intensity changes of Wilma represented by the azimuthally averaged maximum tangential and radial winds at the RMW, hereafter denoted
by upper-case letters $V_i$ and $V_r$, respectively, at $z = 20 \text{ m}$ from 18/12 to 19/12. (The $z = 20 \text{ m}$ instead of $z = 10 \text{ m}$ level is used herein because the former occurs between the two lowest model levels at which prognostic variables are defined.) Following a spin-up period, Wilma undergoes an 18 h RI stage from 18/15 to 19/09, with $V_i$ peaking around 19/09 at 70 m/s at $z = 20 \text{ m}$, followed by an eyewall replacement cycle and a subsequent weakening stage, as shown by C11. (Note that the azimuthally averaged maximum $V_i$ is smaller than the point value of $V_{\text{MAX}}$ in C11, and the latter exhibits continued RI until 19/12.) In addition, $V_i$ increases rapidly during the first 5 h following RI onset with an average rate of $4 \text{ m s}^{-1} \text{ h}^{-1}$, and more gradually afterward. $V_i$ encounters a temporary intensification halt coincident with an eyewall merger during 19/03 to 19/05 (see C11). Intense radial inflow, that is, $V_i < 0$, is generated in the PBL where $V_i$ becomes sub-gradient and decreases downward toward the surface layer (Figure 2). The magnitude of $V_i$ at 20 m height trends similarly with $V_r$, showing a sharp increase during the early RI stage and a slow intensification for the rest of RI, ending with a maximum of 24 m/s around the peak intensity time of 19/09 (Figure 1a).

Horizontal distributions of the simulated radial reflectivity and wind fields at 19/06 are given in Figure 1b, showing (a) an axisymmetric inner-core structure with an inner annular eyewall of roughly 15 km width, as represented by the area covered by the radar reflectivity exceeding 40 dBZ roughly inside the radius of 25 km, (b) a remarkably circular wind field with a RMW of less than 15 km, and (c) an outer spiral rain band merging with the eyewall. Considering the dominant symmetric inner-core structures, albeit less so during Wilma’s early RI stage (see figures 7 and 12 in C11), it is appropriate to use azimuthally averaged variables for studying the inner-core process during Wilma’s RI period.

The vertical cross-sectional evolution of the azimuthally averaged $v_t$ and AAM with zoomed-in features in the lowest 3 km layers, given in Figure 2, shows the spin-up of $v_t$ with its peak magnitude increasing from over 30 m/s at 18/12 to above 90 m/s at 19/09, when the $v_t > 50 \text{ m/s}$ contour extends upward to $z = 12 \text{ km}$. In particular, the most intense part of the primary circulation transitions from a broad portion of the lower troposphere to a small core centred at $R = 12 \text{ km}$ and near $z = 0.5 \text{ km}$ (Figure 2c). Note that between 19/00 and 19/09 (and beyond), the RMW below $z = 1 \text{ km}$ remains near $R = 12 \text{ km}$ and does not contract any further. Moreover, the vertical slope of the RMW axis shows an inward tilt at 18/12, and a slightly outward sloping structure at 19/00, which remains so during the rest of RI.

The AAM surfaces closely follow those of $v_t$ inside the RMW below $z = 6 \text{ km}$ (Figure 2). These AAM contours move inward with time, while forming inward-pointing corners known as inward “buckling” near the top of the PBL (Zhang et al., 2001). Note a continued inward displacement of the AAM = $10^6 \text{ m}^2/\text{s}$ contour after the RMW ceases contracting, which will be later examined in detail to gain insight into the RI of the storm. The secondary circulation (“in-up-and-out” flow) becomes more visible during RI than in pre-RI, with a well-defined radial inflow in the lowest layers, an intense updraught core inside the RMW, and a main outflow layer in upper layers. The peak radial inflow in the PBL, located outward from the RMW, increases from less than 10 m/s during the pre-RI stage to over 20 m/s during RI (Figures 2c and 1a), with an increasing inflow depth. At the inner edge of the RMW, where the inflowing air slows sharply and encounters an outflow from the eye region, a convergence zone appears, which is consistent with the fact that the maximum vertical motions stay inside the RMW axis throughout the RI period (Figure 2). The radial wind changes from inflow to outflow along the RMW near the top of the PBL. A low-level outflow jet appears during RI with the peak values of over 2 m/s and 14 m/s at 19/00 and 19/09, respectively. From $z = 2 \text{ km}$ to $z = 10 \text{ km}$ throughout RI, a weak radial inflow exists over a
deep layer, creating a radial wind convergence zone near the RMW axis by meeting with the eyewall radial outflow. This deep inflow accounts for the inner-core intensification of eyewall $v_t$ above the PBL through the inward advection of higher AAM from the outer region (Zhang et al., 2001).

To help further understand the above-mentioned structural changes, Figure 3 shows the temporal evolution of $V_t$ and its associated RMW at $z = 20\text{ m}$, 0.5 km and 5 km corresponding to the surface layer, near the top of the PBL and the mid-troposphere, respectively. We see that Wilma’s intensification rates are broadly similar at the three levels, with all experiencing sharp increases in $V_t$ during the first 5 h of RI, followed by slower intensification and a temporary halt (19/03 to 19/05). The azimuthally averaged $V_t$ decays vertically above $z = 0.5\text{ km}$, with the 0.5–5 km layered difference increasing from 4–7 m/s before 19/00 to about 10 m/s for the remainder of RI. These differences are small relative to the magnitude of $V_t$ at $z = 0.5\text{ km}$, that is, around 10%, which is consistent with the observational and modelling results of Stern and Nolan (2011) showing that the maximum swirling winds decrease slowly with height below $z = 10\text{ km}$.

One can also see clearly the rapid (e.g. at $z = 0.5\text{ km}$ from $R = 36\text{ km}$ at 18/12 to $R = 16\text{ km}$ at 18/18) and slow (e.g. at $z = 0.5\text{ km}$ from $R = 16\text{ km}$ at 18/18 to $R = 12\text{ km}$ at 18/22) contraction of the RMW at the three levels during pre-RI and the early 7 h RI stage, respectively, after which all three of them no longer contract, but remain nearly constant (i.e. the S-RMW stage after 18/22). This result is consistent with the statistical analysis of Qin et al. (2016), who showed that RI processes are not always accompanied by contracting RMW, and that the S-RMW is found more frequently in intense storms and in storms with small RMW.

The above analysis reveals that: (a) the RMW ceases to contract during the later RI stage while the storm continues to

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**FIGURE 2** Radius–height cross-sections of the azimuthally averaged tangential winds ($v_t$, pink-contoured at intervals of 10 m/s), absolute angular momentum (AAM, black-contoured at intervals of $2 \times 10^5 \text{ m}^2/\text{s}$), and radial winds ($v_r$, shaded, m/s), superimposed with in-plane flow vectors (m/s) at (a) 1200 UTC 18; (b) 0000 UTC 19; and (c) 0900 UTC 19 October 2005. Green solid lines denote the vertical profile of the RMW. Note that (i) a larger radial extent is used in (a) than in (b) and (c); and (ii) the lowest 3 km layers, i.e. beneath black dashed lines, have been stretched for the features of interest. A white colour is used to indicate roughly the top of the PBL where transition from inflow to outflow occurs.

**FIGURE 3** Time series of the azimuthally averaged maximum tangential wind ($V_t$, solid, m/s) and the RMW (dashed, km) at $z = 20\text{ m}$, 0.5 km (lines with crosses) and 5 km (lines with dots) during the period of 1200 UTC 18 to 1200 UTC 19 October 2005. Vertical dashed lines indicate RI onset (18/15) and the starting time of the S-RMW stage (18/22), respectively.
intensify; and (b) $V_t$ increases faster during the earlier RI stage with the rapidly contracting RMW than during the later RI stage with an S-RMW. In the next two sections, we diagnose the high-resolution model output of Wilma in order to provide a reasonable understanding of the above characteristics.

3 | RAPID INTENSIFICATION DURING THE S-RMW STAGE

In this section, we diagnose first the processes governing the changing rates of the RMW, following the framework of Kieu (2012), and then show, using the AAM budget equation, how RI may continue in the absence of RMW contraction.

3.1 | What processes control contraction?

As a first step toward understanding RI in the absence of RMW contraction, it is necessary to see what processes account for the generation of an S-RMW structure. In this regard, Kieu (2012) has derived a formula for the contraction rate of the RMW in the PBL, based on the tangential momentum equation in cylindrical coordinates constrained by the Rankine-vortex structure. For TCs with inherent baroclinic effects associated with the vertical/horizontal diffusions are negligible.

$$\frac{dR}{dt} \approx \frac{1}{\Omega^*} \frac{dV_t}{dt} \bigg|_{\text{RMW}} + \frac{(\Omega^* + f)}{\Omega^*} V_t + W \frac{\partial V_t}{\partial z} - F_h \frac{\Omega^*}{\Omega^*} \tag{2}$$

where $R(t)$ is the time-dependent RMW, $\Omega^*$ is the angular velocity defined at the RMW, $W$ is the vertical motion, and $F_h$ includes the azimuthally averaged frictional and vertical/horizontal diffusion forcings on the acceleration of $V_t$. Note that in Equation (2), the first term on the right-hand side (RHS) denotes the change of $V_t$ with time along the contraction direction that is defined only at $r=R$, not the total derivative along the flow vector field (see appendix for a detailed derivation of Equation (2)). Similarly, all upper-case variables in Equation (2) indicate the values evaluated at $r=R(t)$ for each level.

With the lack of direct term-by-term model output, $F_h$ is often treated as a residual term by computing the difference between the local time tendency of $V_t$ and the sum of the absolute vorticity flux and the vertical advection of $V_t$ in the $V_t$ budget equation (Bui et al., 2009; Abarca and Montgomery, 2014). It is evident from Figure 4d that the residual term $R_{\text{RES}} = F_h/\Omega^*$ is small in layers above the PBL where the frictional effects associated with the vertical/horizontal diffusions are negligible.

Equation (2) states that the RMW tendency ($R_{\text{TEN}}$) on the left-hand side (LHS) depends on four RHS terms (normalized by $\Omega^*$) from left to right: the total change of $V_t$ ($R_{\text{DER}}$), the radial flux of the absolute vorticity ($R_{\text{RAD}}$), the minus vertical advection of $V_t$ ($R_{\text{VER}}$), and the frictional and turbulent dissipation ($R_{\text{FRC}}$), respectively. As Kieu (2012) showed, $R_{\text{RAD}} \approx V_t$ since $\Omega^* \gg f$, which illustrates the direct role that radial flows at the RMW play in determining its contraction. In contrast, $R_{\text{VER}}$ tends to change the position of the RMW by advecting $V_t$ upward once the vortex baroclinicity is taken into account. For $R_{\text{DER}}$, we note that this term does not represent a physical force, but rather, it is consequence of the Rankine-vortex assumption and can be treated as an external term. Practically, $R_{\text{DER}}$ can be determined easily from the time series of $V_t$ as a change of $V_t$ between two time steps, say, at $t=0$ and $t=-6$ h. The magnitude of $R_{\text{DER}}$ turns out to be sufficiently small that it plays a minimal role in the RMW contraction during RI (Figure 4). Clearly, the contraction of the RMW will be halted when all four terms (first three terms above the PBL) on the RHS of Equation (2) are balanced.

To demonstrate the validity of Equation (2) in evaluating the contraction of RMW, Figure 4a,b compare the contraction rates calculated from Equation (2) to the model-diagnosed contraction rates at $z=20$ m and $z=5$ km, respectively, while Figure 4c,d show the contributions of each term on the RHS of Equation (2) to the contraction rates at the two corresponding levels. Despite the presence of oscillating variations during the early RI stage, which could be attributed to asymmetries at $r=R$, the model-diagnosed contraction rate of the RMW in the PBL averages around 3.5 km/h prior to RI onset, after which it decreases and becomes nearly zero after 18/22 (Figure 4a). Apparently, the $R_{\text{TEN}}$ diagnosed by Equation (2) captures the WRF-simulated tendency reasonably well, and it shows that $R_{\text{RAD}}$ and $R_{\text{FRC}}$ represent the two dominant processes controlling $R_{\text{TEN}}$ in the PBL, with the former contributing to RMW contraction and the latter opposing contraction (Figure 4c); this is consistent with the hypothesis of Kieu (2012).

While Equation (2) appears to suggest that the radial inflow directly “adsects” the RMW inward, it should be noted that the RMW is not a material surface that can be advected as a Lagrangian tracer. Instead, the actual physical mechanism underlying the role of the radial inflow in governing the RMW contraction is related to the local increase of inner-core $V_t$. That is, when the inward AAM advection in the inner-core region is much larger than that in the outer region, a faster spin-up of the inner-core $V_t$ would be expected to occur. As a result, a new RMW tends to form inside the previous RMW, giving rise to a picture of the inward RMW advection by the radial inflow as diagnosed by Equation (2). Likewise, the role of friction is not to directly push the RMW outward as Equation (2) may suggest. Instead, frictional dissipation tends to erase any newly-formed RMW inside the previous RMW, thus preventing the RMW from being contracted as discussed in Kieu and Zhang (2017). From this conceptual model, the RMW contraction ceases after the contributions by the radial inflow are balanced by frictional dissipation after 18/22.
FIGURE 4  Time series of (a, b) the diagnosed RMW tendency (km/h) from the WRF model ($R_{\text{TENS}}$, solid) and Equation (2) ($R_{\text{TEND}}$, dashed); and (c, d) contributions of the total derivative of $V_t$ ($R_{\text{DER}}$, double dashed), the radial flux of the absolute vorticity ($R_{\text{RAD}}$, dashed lines with dots), the minus vertical advection of $V_t$ ($R_{\text{VER}}$, dash-dotted), and the frictional dissipation ($R_{\text{FRC}}$, solid line with triangles) on the RHS of Equation (2) divided by the angular velocity to the RMW tendency during the period of 1200 UTC 18 to 1200 UTC 19 October 2005. The left and right columns are evaluated at $z = 20$ m and 5 km, respectively. Grey dashed lines denote the null tendency. Note that (i) $R_{\text{DER}}$ shown in (c) has been multiplied by 10; and (ii) $R_{\text{RES}}$ (long-dashed) in (d) denotes the residual term associated with computational errors.

Note again that the tendency for frictional dissipation to prevent the RMW from contracting is proportional to the squared $V_t$. As such, the role of friction is generally noticeable only after $V_t$ becomes sufficiently large during the RI stage. Regarding the term $R_{\text{DER}}$ evaluated at $r = R$, which is more than one order of magnitude smaller than $R_{\text{RAD}}$ and $R_{\text{FRC}}$ (as indicated by a multiplication factor of 10 in Figure 4c), it generally contributes negatively toward the RMW contraction during RI, and its contribution becomes even smaller after 18/21. For $R_{\text{VER}}$, one may note that it increases after RI onset due to the increasingly more organized secondary circulation (Figure 2), although its contribution toward contraction in the PBL remains smaller than each of the two major terms.

At $z = 5$ km, by comparison, most terms in Equation (2) are one order of magnitude smaller, with larger variations in $R_{\text{VER}}$ and $R_{\text{RAD}}$ (cf. Figure 4d,c). In particular, the processes dominating $R_{\text{TEN}}$ at $z = 5$ km differ from those at $z = 20$ m due to the absence of frictional effects. That is, $R_{\text{RAD}}$ reverses its role from promoting to inhibiting the RMW contraction as the RMW-evaluated radial flow changes from inflow in the PBL to outflow above the PBL due to the outward tilting of the eyewall. In the absence of frictional effects, the radial and vertical terms are opposite in sign and similar in magnitude (Figure 4d). Thus, $R_{\text{VER}}$ contributes to RMW contraction above the PBL by transporting higher angular momentum upward to compensate for negative $R_{\text{RAD}}$ contributions. Note that both $R_{\text{RAD}}$ and $R_{\text{VER}}$ become substantially smaller from 18/18 to 19/03 due to the merging of an outer rain band into the eyewall (see C11 for more details). Regarding the term $R_{\text{DER}}$, its time series is similar to that at $z = 20$ m, with the term keeping a small positive sign until 18/22, and then staying near zero thereafter. Even though the diagnosed $R_{\text{TEN}}$ from Equation (2) underestimates 0.5–1 km/h of the model-diagnosed contraction trends prior to 18/22, it captures the model-diagnosed S-RMW for the later RI stage, and exhibits disagreement only for the eyewall merger period around 19/05.
### 3.2 How could RI continue in the absence of contraction?

Next, let us attempt to understand how swirling winds can intensify without the contracting RMW. Our analyses in the preceding subsection show that the RMW contraction ceases when the advective tendency on $V_t$ at $r=R$ is cancelled by frictional dissipation, thus preventing the formation of a new RMW inside the previous RMW. However, this balance for the RMW contraction is not the same as the balance for the tangential wind $V_t$ because the governing equations for the two quantities are different. As such, any explanation of the continued strengthening of $V_t$ at the location of an S-RMW has to be based on the inward AAM advection. To analyse the AAM budget at the location of the S-RMW, we rewrite Equation (1) as the following equation:

$$\frac{\partial v_t}{\partial t} = \frac{1}{r} \frac{\partial M}{\partial r}$$  \hspace{1cm} (3)

where $M$ is the AAM. It is apparent that $v_t$ can be intensified as the RMW becomes smaller, assuming the same positive local AAM tendency $\partial M/\partial t$. Following Zhang et al. (2001), the axisymmetric local AAM tendency ($M_T$) in cylindrical $(r, z)$ coordinates can be obtained from the AAM budget equation,

$$\frac{\partial M}{\partial t} = -v_t \frac{\partial M}{\partial r} - w \frac{\partial M}{\partial z} + r F_\lambda$$  \hspace{1cm} (4)

Invoking the anelastic approximation, Equation (4) can be rewritten as follows,

$$\frac{\partial M}{\partial t} = \frac{1}{r} \frac{\partial (r v_T M)}{\partial r} - \frac{\partial (\rho_0 w M)}{\partial z} + r F_\lambda$$  \hspace{1cm} (5)

where $\rho_0 (z)$ is the air density as a function of $z$ only. Contributions to $M_T$ appear on the RHS of Equation 5, from the left to right, as the azimuthally averaged horizontal flux divergence (FDMH) and vertical flux divergence (FDMV) of AAM, and residual term ($M_{\text{RES}}$), respectively. Note that like $R_{\text{RES}}$, $M_{\text{RES}}$ includes not only the PBL friction and numerical diffusion, but also computational errors, such as those arising from the interpolation from model to cylindrical coordinates. All budget terms in Equation (5) are calculated from the 1 km resolution model domain at 5 min intervals, using centred differencing for the local tendency, and they are then smoothed using a 30 min running mean.

Now focusing on the processes that account for the positive local AAM tendency, Figures 5 and 6 show the radius–height plots of $M_T$, FDMH, FDMV and $M_{\text{RES}}$ in the inner-core region that represent the contracting (18/18) and the S-RMW (19/00) stages, respectively. Note first that $M_{\text{RES}}$, representing the residues of $M_T$ minus the sum of FDMH and FDMV, is at least one order of magnitude smaller than the latter two terms above the PBL (cf. Figures 5a–c and 6a–c). However, organized positive $M_{\text{RES}}$ is similar in magnitude to $M_T$ in the upper outflow layer, where the vertical mixing of AAM could be large due to the presence of strong vertical shear, while $M_{\text{RES}}$ in the layers below the outflow layer but above the PBL is on average about 20% of $M_T$. These findings are consistent with those shown in the AAM study of Zhang et al. (2001), in which each budget term is directly output during the model integrations. Since the local tendency term is always small, we may consider that computational errors in our budget analysis are negligible and that $F_\lambda$ is the primary contributor to $M_{\text{RES}}$ in the PBL. The capacity of the PBL to act as an AAM sink increases rapidly inward toward the RMW due to the increased frictional dissipation associated with the squared $v_t$, while a relatively large source of AAM is located in its buckled layer coinciding with a low-level outflow jet (cf. Figures 5a, 6a and 2).

The horizontal and vertical AAM flux divergence terms (Figures 5b,c and 6b,c) are the primary contributors to Wilma’s intensification above the PBL. Zhang et al. (2001) showed negative horizontal advection of AAM across the RMW above $z = 1$ km (see their figure 2b), which results from up-gradient AAM transport by super-gradient outflows in the eyewall. However, after converting the AAM budget equation to flux form, $M_T$ depends on the total flux divergence of AAM, which is similar in magnitude but opposite in sign between $\text{FDMH}$ and $\text{FDMV}$, and the AAM budget terms take on different patterns from those shown in Zhang et al. (2001). Thus, $\text{FDMH}$ above the PBL is positive outward from and negative inward from the slantwise updraught core (Figures 5b and 6b), whereas the opposite is true for $\text{FDMV}$ (Figures 5c and 6c). Note that the patterns of $\text{FDMH}$ are similar to those of the horizontal velocity divergence except for opposite signs (cf. figure 2g in Liu et al., 1999, and Figures 5b and 6b herein) due to the inclusion of a minus sign in $\text{FDMV}$. The vertical divergence pattern could be understood as resulting from latent heat release in the sloping eyewall, causing horizontal velocity convergence below and divergence above the updraught core. In the PBL, positive $\text{FDMH}$ increases inward rapidly towards the RMW due to the increasing radial inflow and a sharpening radial AAM gradient (Figure 2), and its amplitude is much greater than $\text{FDMV}$ although both terms are still opposite in sign. Furthermore, the maximum $\text{FDMH}$ in the PBL is found just inside the RMW during both the contracting and S-RMW stages of RI, where the radial inflow is mostly convergent, namely, it decreases in magnitude inward from the RMW axis (Figure 2).

Because of the pronounced compensation effects between $\text{FDMH}$ and $\text{FDMV}$, the net $M_T$ is a small difference between the two (three) large terms above (in) the PBL. Figures 5d and 6d show well-organized positive $M_T$ in the eyewall, with the ridge axis closely following the updraught core vertically. This result is consistent with the intensifying $V_t$ as shown in Figure 3, and confirms that a key to TC intensification is the continuous inward advection and/or convergence of high AAM that must overcompensate for AAM losses associated with the frictional dissipation in the PBL (Eliassen, 1951; Shapiro and Willoughby, 1982; Zhang et al., 2001; Montgomery and Smith, 2011). Above the PBL, the positive $\text{FDMH}$ contributes to the positive $M_T$ along the RMW.
FIGURE 5  Radius–height cross-sections of the azimuthally averaged AAM budget equation terms, showing (a) residual ($M_{\text{RES}}$, contoured, $10^5$ m$^2$ s$^{-1}$ h$^{-1}$), (b) horizontal AAM flux divergence ($FDM_H$, contoured, $10^5$ m$^2$ s$^{-1}$ h$^{-1}$), (c) vertical AAM flux divergence ($FDM_V$, contoured, $10^5$ m$^2$ s$^{-1}$ h$^{-1}$), and (d) local AAM tendency ($M_T$, shaded, $10^5$ m$^2$ s$^{-1}$ h$^{-1}$) superimposed with in-plane flow vectors (grey, m/s) at 1800 UTC 18 October 2005. Both (b) and (c) are contoured at $\pm 5.0$, $\pm 10$, $\pm 20$, $\pm 30$, $\pm 50$ and $\pm 100 \times 10^5$ m$^2$ s$^{-1}$ h$^{-1}$, while (a) is contoured at $\pm 0.2$, $\pm 0.6$, $\pm 1.0$, $\pm 1.4$, $\pm 5.0$, $\pm 10$ and $\pm 20 \times 10^5$ m$^2$ s$^{-1}$ h$^{-1}$.

Note that shadings in (a–c) indicate larger magnitudes (of over $20 \times 10^5$ m$^2$ s$^{-1}$ h$^{-1}$). Green solid lines in (a–d) indicate the RMW. The positive $FDM_H$ and $FDM_V$ indicate flux convergence, due to the inclusion of a negative sign on the RHS of Equation (5).

Given similarities between Figures 5 and 6, one may wonder to what extent the AAM budgets differ between the contracting and S-RMW RI stages. The most obvious difference is that both $FDM_H$ and $FDM_V$ during the S-RMW stage are much greater in amplitude than those during the contracting RMW stage (cf. Figures 5b,c and 6b,c), which is consistent with the intensities and the gradient of $v_t$, $v_r$ and updraughts at the corresponding stages. A second difference is that the region of positive $M_T$ in the eyewall during the S-RMW stage is smaller in magnitude and narrower in width than that during the contracting RMW stage. The difference in width could be attributed to the fact that $M_T$ calculated in centred time differences (of a 10 min period) includes the leading and trailing portions of AAM changes where the RMW contracts, whereas these changes occur in nearly the same volume following the storm movement during the S-RMW stage.

To see a more complete picture of the AAM budgets, Figures 7 and 8 show time–radius plots of the four budget terms at $z = 20$ m and 5 km, respectively, within a 30 km radial range. We see that both $FDM_H$ and $FDM_V$ increase with time in their absolute magnitudes during RI, reaching their peaks at 19/09. Again, $M_{\text{RES}}$ in Figure 7a exhibits the pronounced sink of AAM that is centred slightly inside the RMW, whereas $M_{\text{RES}}$ in Figure 8a shows its negligible contributions to $M_T$ in the mid-troposphere, as compared to $FDM_H$ and $FDM_V$, except for the short period of 19/00–19/03. Indeed, we see from Figure 7b (Figure 8b) the large inward flux convergence of AAM across the RMW by convergent $v_r$; the opposite is true for $FDM_V$ (Figures 7c and 8c). Likewise, one notices the divergence of AAM inside the RMW due to the divergence of $v_t$. 
As in Figures 5d and 6d, $M_T$ remains largely positive over a wide radial range during the contracting stage, but subsequently becomes small and positive at $r = R$ until Wilma reaches its azimuthal peak intensity near 19/09 (Figures 7d and 8d), as mentioned before. This magnitude difference in $M_T$ is consistent with Equation (3), which requires a larger (smaller) positive local AAM tendency for large (small) radii during the contracting (S-RMW) stage to maintain $V_t$ intensification. Again, the positive $M_T$ results from the excess of $F_{DMH}$ with respect to $F_{DMV}$ and $M_{RES}$ (Figures 7a–c and 8a–c), which is a small differenced field among the three large terms. Of relevance to this study is that the RMW at $z = 20\,\text{m}$ displaces from higher to lower AAM surfaces during the contracting stage, but then displaces toward higher AAM surfaces during the S-RMW RI stage.

In contrast, at $z = 5\,\text{km}$, the AAM at $r = R$ keeps steady during 18/16–18/22, but then increases as the air is being replaced by air parcels with higher AAM. (A sharp increase in AAM around 19/05 is associated with the eyewall merger mentioned earlier.) This indicates that during the contracting stage, although $M_T$ shows large positive values near the RMW, the AAM following the time-dependent RMW decreases at $z = 20\,\text{m}$ but remains nearly constant at $z = 5\,\text{km}$. By comparison, during much of the S-RMW stage, the RMW for both levels is characterized by a slightly positive $M_T$ and increasing AAM. Therefore, the RI of $V_t$ is closely related to the RMW contraction during the early RI stage, whereas it requires the continuing flux convergence of AAM during the S-RMW stage. Figure 9 summarizes these characteristics at $r = R$, and its implication for RI will be further discussed in the next section.

It should be mentioned that we have examined asymmetric contributions to changes in $M_T$ by separating each flow variable into an azimuthal mean and a perturbation, and found much smaller asymmetric contributions than symmetric ones, except during the first 12 h spin-up period (not shown). This result does not contradict the major findings of CZ13, who showed the importance of asymmetrically distributed convective bursts in the RI of Wilma. Apparently, the asymmetric distribution of convective bursts (or diabatic heating) in the eyewall does not imply the same degree of asymmetry in the flow field because of rapid axisymmetrization under the influence of extreme intense rotation (e.g. see figure 5 in Zhang et al., 2002).
FIGURE 7  Time–radius cross-sections of the azimuthally averaged (a) residual term ($M_{\text{RES}}$, shaded, $10^5$ m$^2$ s$^{-1}$ h$^{-1}$) superimposed with the azimuthal mean tangential wind (contoured at intervals of 10 m/s); (b) horizontal flux divergence of the AAM ($FDM_H$, shaded, $10^5$ m$^2$ s$^{-1}$ h$^{-1}$) superimposed with the azimuthal mean radial wind (contoured at intervals of 10 m/s), (c) vertical flux divergence of the AAM ($FDM_V$, shaded, $10^5$ m$^2$ s$^{-1}$ h$^{-1}$) superimposed with the azimuthal mean vertical motion (contoured at intervals of 4 cm/s), and (d) local AAM tendency ($M_T$, shaded, $10^5$ m$^2$ s$^{-1}$ h$^{-1}$) superimposed with the azimuthal mean AAM (contoured at intervals of $2 \times 10^5$ m$^2$/s), at $z = 20$ m during the period of 18/12 to 19/12. Black solid lines in (a–d) indicate the RMW. Note the different definitions of colour bars used among the four panels.

4 | INNER-CORE DYNAMICS DURING THE S-RMW STAGE

Our understanding of RI and S-RMW will not be complete without applying a closed set of primitive equations consisting of the horizontal momentum and continuity equations, given the vertical motion field (Kieu and Zhang, 2009; 2010). In particular, in the preceding section, we have seen the importance of the radial AAM flux convergence and the radial vorticity flux in determining RI and RMW contraction, respectively, which involves both the magnitude and radial gradients of the radial inflow. Thus, it is desirable to see...
how the use of the axisymmetric radial momentum equation in cylindrical coordinates, as given below, could improve our understanding of the dynamics of RI during the S-RMW stage,

$$\frac{d\psi}{dr} = \frac{v^2}{r} + f v_r - \frac{1}{\rho} \frac{\partial p}{\partial r} + F_r \quad (6)$$

where $F_r$ denotes the radial frictional effects. Equation (6) states that the acceleration of radial flows is determined by the sum of the centrifugal and Coriolis forces (CCF), the radial pressure gradient force (PGF) and friction, on the RHS. In general, the Coriolis force in the inner-core region is small (Zhang et al., 2001) and can be neglected under a scale analysis.

To quantify the nature of the force balance between the PGF and the CCF, Figure 10 plots the time series of inward PGF and outward CCF in the same manner as that shown in
FIGURE 9 Time series of the predicted AAM ($10^5 \text{m}^2/\text{s}$) along the time-dependent RMW at $z = 20 \text{ m}$ (dashed) and $z = 5 \text{ km}$ (solid), taken from Figures 7d and 8d, respectively, during the period of 18/12 to 19/12. Vertical dashed (dash-dotted) lines indicate AAM = $7 (9.5) \times 10^5 \text{m}^2/\text{s}$

FIGURE 10 Time series of the azimuthally averaged sum of centrifugal and Coriolis forces (CCF, grey solid, $\text{m s}^{-1} \text{h}^{-1}$) and radial pressure gradient force (PGF, black solid, $\text{m s}^{-1} \text{h}^{-1}$) both calculated at the time-dependent RMW (dashed, km), and of the azimuthally averaged radius of the maximum CCF (RCCF, double dashed, km) and the maximum PGF (RPGF, dash-dotted, km) from 1200 UTC 18 to 1200 UTC 19 October 2005, at (a) $z = 20 \text{ m}$; and (b) $z = 5 \text{ km}$. Vertical dashed lines indicate RI onset (18/15) and the starting time of the S-RMW stage (18/22), respectively.
between the PGF and \( R \) for a given AAM value can be obtained as

\[
PGF|_R = \frac{M^2}{R^3} - \frac{1}{4} r^2 R
\]  

Equation (7) states that: (a) at a small RMW, slight contraction will cause a large PGF increase due to the cubic power of the RMW; and (b) when the RMW ceases to contract, the PGF at \( r = R \) can only keep intensifying when higher AAM could be continuously advected towards the time-dependent RMW. Note that during the RMW contracting stage, both the AAM advection and RMW contraction contribute to increases in the PGF. They are different from those occurring during the S-RMW stage, as observed in the AAM budget.

To help illustrate the above points, Figure 11 plots the modelled PGF (solid dots with given times) taken from Figure 10b, the diagnostic PGF (crosses) from Equation (7) using the predicted AAM at \( r = R \), and the idealized PGF (curve in green) from Equation (7) for a given value of AAM (i.e. \( 9.5 \times 10^5 \text{ m}^2/\text{s} \)) as a function of the RMW at \( z = 5 \text{ km} \). As can be seen, all three types of PGFs evolve similarly until 18/21 when the RMW contracts 15 km, implying that the gradient wind balance is maintained during the contracting stage, as expected. (The RMW = 14 km during 19/02–19/05 is not considered in the definition of the S-RMW stage herein because it is affected by the eyewall merger process mentioned earlier.) In fact, Figure 9 shows that the AAM at \( z = 5 \text{ km} \) evaluated at the modelled time-dependent RMW remains nearly constant (\( \sim 9.5 \times 10^5 \text{ m}^2/\text{s} \)) from 18/16 to 18/21. However, the idealized curve indicates that any further increase in PGF (and \( V_t \)) must be accompanied by RMW contraction to less than \( R = 15 \text{ km} \), whereas the modelled PGF continues to intensify during the contracting stage; the RMW even expands during 19/06–19/08 (cf. Figures 3, 10b and 11). This apparent “contradiction” results from the fact that Equation (7) is essentially valid for a quasi-steady state, inviscid, two-dimensional TC vortex, whereas the model-simulated increasing PGF above the PBL occurs in the presence of the AAM convergence near to \( r = R \), as described in the preceding section. Clearly, the hurricane vortex after 18/22 must have become so stiff, or inertially stable, such that it cannot contract any further, even as the inward-directed PGF and \( V_t \) keep intensifying. In the absence of RMW contraction, Equations (3) and (7) state that the intensification of \( V_t \) and PGF can only occur if the local AAM becomes larger due to AAM flux convergence at \( r = R \). This is indeed the case, as Figure 9 shows that the AAM at \( r = R \) increases from \( 9.5 \times 10^5 \text{ m}^2/\text{s} \) at 18/21 to \( 10.0 \times 10^5 \text{ m}^2/\text{s} \) at 18/22, and then to \( 13.2 \times 10^5 \text{ m}^2/\text{s} \) at 19/07 with an average rate of \( 0.37 \times 10^5 \text{ m}^2 \text{ s}^{-1} \text{ h}^{-1} \) as shown in Figure 6d. This increase of local AAM results from the surplus of the radial flux convergence over the vertical flux divergence of AAM (Figures 8b,c).

Similar scenarios occur at \( z = 20 \text{ m} \), except that AAM is not conserved due to the PBL friction (Figure 9), which plays an important role in influencing the RMW contraction and AAM transport. Thus, the AAM at \( r = R \) decreases from 18/16 to 18/21 because the RMW contracts more rapidly than the “contraction” of the AAM surfaces caused by frictional dissipation. Subsequently, higher AAM is transported to the RMW until 19/06, which coincides with increasing radial inflow from about 20 to 30 m/s (cf. Figures 9 and 7b). A balance among FDM_{H}, FDM_{V} and \( M_{RES} \) appears to be reached when the RMW remains at a steady state.

5 | SUMMARY AND CONCLUDING REMARKS

In this study, the inner-core dynamics of a rapidly intensifying hurricane during the contracting and steady RMW stages, as well as the processes leading to the generation of the S-RMW structures during RI, are examined by applying a set of dynamical budget equations to the WRF-predicted Hurricane Wilma (2005) with the finest grid spacing of 1 km at 5 min output intervals. The predicted hurricane vortex exhibits clearly the rapid contraction of the RMW prior to RI onset, followed by the slow-down and the cessation of the contraction coinciding with the rapid and slower intensification of \( V_t \) during the early and later RI stages, respectively.

By extending Kieu’s (2012) RMW tendency equation to the three-dimensional real-data vortex, it is shown that the contracting rates of the RMW capture well the model-diagnosed initial rapid and later slower contraction, and the final S-RMW stage. Results show that: (a) the contraction rates during the early RI stage are determined mostly by the inward advection of AAM that offsets the frictional dissipation in the PBL, and the updraughts that overcompensate radial (super-gradient) outflows above the PBL; and (b) the above
opposing processes are in near balance during the S-RMW stage. The AAM budgets are calculated to examine the RI during both the contracting and S-RMW stages. In general, the radial flux convergence of AAM, resulting from both the PBL friction and latent heating in the eywall (Zhang and Kieu, 2006), is the major process accounting for the spin-up of $V_r$, whereas the vertical flux divergence of AAM and frictional dissipation oppose the spin-up in the PBL. Spatially, the radial flux convergence increases rapidly inward towards the RMW as a result of increasing radial inflow and a sharpening radial AAM gradient. This flux convergence overcompensates the vertical flux divergence and the PBL frictional sinks, leading to pronounced positive local AAM tendency during the contracting stage. Although the magnitudes of the above processes in the inner-core region during the S-RMW stage are much greater than those during the contracting stage, the net local $M_T$ is smaller in amplitude and narrower in width. This is attributable to little change in the eyewall width and local AAM values compared to the contracting stage. However, the AAM following the time-dependent RMW decreases slightly with time in the PBL due to the fact that the inward transport of AAM could not offset the frictional dissipation that is proportional to the squared $V_r$. In contrast, the AAM remains nearly constant above the PBL during the contracting stage due to the presence of little friction. By comparison, the local AAM in and above the PBL continues to increase during the S-RMW phase as a result of the greater radial flux convergence of AAM when the radial inflow (outflow) is in balance with frictional dissipation (weighted updraughts) in (above) the PBL.

An analysis of the force balance in the radial momentum equation shows that the intensification of the PGF and CCF trend similarly to that of $V_r$ in and above the PBL, namely, they increase slowly prior to RI onset, rapidly during the early RI stage, and moderately after reaching the S-RMW stage. In general, for air parcels forming the RMW, the PGF offsets the CCF and frictional dissipation in the PBL in order to maintain the frictional radial inflow, whereas the PGF and CCF above the PBL are in gradient wind balance. It is found that the radii of the maximum PGF and CCF are located inside the RMW during the pre-RI and early RI periods, and they become aligned with the S-RMW during the later RI stage.

With the above finding, a dynamical (idealized) relationship between the RMW and PGF at the RMW above the PBL is derived by substituting AAM into the gradient wind balance equation. Results show that during the contracting stage, the idealized PGF tracks closely with the modelled PGF at the RMW, regardless of whether the model-diagnosed AAM at the RMW, or an appropriately chosen constant AAM, is used for this relationship. However, after the RMW ceases contracting, the modelled PGF can continue to intensify, whereas holding a constant AAM would require continued RMW contraction in order for the PGF to keep intensifying. It follows that the intensification of the PGF and $V_r$ at the RMW in the mid-troposphere can be largely maintained within the framework of AAM conservation and gradient wind balance during the contracting stage. In contrast, the intensification can be maintained during the S-RMW stage only if the radial flux convergence of AAM could overcompensate for the vertical flux divergence of AAM at the RMW. The above results are remarkable because they reveal that (a) during the contracting stage, RI can still happen with a constant AAM at the RMW, and (b) AAM at the RMW must increase with time through the replacement of air parcels with the higher AAM originating from the “buckling” layer near the top of the PBL (or larger radius) in order for the PGF (and $V_r$) at the RMW to keep increasing during the S-RMW stage.

It should be mentioned that the processes of increasing AAM at the time-dependent RMW during the S-RMW stage described herein are limited to the present case of highly symmetric flow fields in the eyewall. In many other intense TCs in which the eyewalls are highly asymmetric during their RI stages, asymmetric dynamics may play a more important role than symmetric dynamics in determining the contraction of RMW. In this regard, more case-studies are needed in order to fully understand the different inner-core dynamics during the S-RMW stage.

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REFERENCES

Abarca, S.F. and Montgomery, M.T. (2014) Departures from axisymmetric balance dynamics during secondary eyewall formation. *Journal of the Atmospheric Sciences*, 71, 3723–3738.

Bui, H.H., Smith, R.K., Montgomery, M.T. and Peng, J. (2009) Balanced and unbalanced aspects of tropical cyclone intensification. *Quarterly Journal of the Royal Meteorological Society*, 135, 1715–1731.

Castro, D., Navarro, M.C. and Herrero, H. (2014) Thermocconvective vortices in a cylindrical annulus with varying inner radius. *Chaos*, 24, 043116.

Chen, H. and Zhang, D.-L. (2013) On the rapid intensification of Hurricane Wilma (2005). Part II: Convective bursts and the upper-level warm core. *Journal of the Atmospheric Sciences*, 70, 146–162.

Chen, H., Zhang, D.-L., Carton, J. and Atlas, R. (2011) On the rapid intensification of Hurricane Wilma (2005). Part I: Model prediction and structural changes. *Weather and Forecasting*, 26, 885–901.

DeMaria, M., Sampson, C.R., Knaff, J.A. and Miusgravre, D. (2014) Is tropical cyclone intensity guidance improving? *Bulletin of the American Meteorological Society*, 95, 387–398.
Eliassen, A. (1951) Slow thermally or frictionally controlled meridional circulation in circular vortex. *Astrophysica Norvegica*, 5, 19–60.

Emanuel, K.A. (1986) An air–sea interaction theory for tropical cyclones. Part I: Steady-state maintenance. *Journal of the Atmospheric Sciences*, 43, 585–604.

Emanuel, K.A. (1995) Sensitivity of tropical cyclones to surface exchange coefficients and a revised steady-state model incorporating eye dynamics. *Journal of the Atmospheric Sciences*, 52, 3969–3976.

Gray, M.W. (1968) Global view of the origin of tropical disturbances and storms. *Monthly Weather Review*, 96, 669–700.

Hacc, J.J. and Schubert, W.H. (1986) Nonlinear response of atmospheric vortices to heating by organized cumulus convection. *Journal of the Atmospheric Sciences*, 43, 1559–1573.

Holland, G.J., Belanger, J.I. and Fritz, A. (2010) A revised model for radial profiles of hurricane winds. *Monthly Weather Review*, 138, 4393–4401.

Holliday, C.R. and Thompson, A.H. (1979) Climatological characteristics of rapidly intensifying typhoons. *Monthly Weather Review*, 107, 1022–1034.

Kaplan, J. and DeMaria, M. (2003) Large-scale characteristics of rapidly intensifying tropical cyclones in the North Atlantic basin. *Weather and Forecasting*, 18, 1093–1108.

Kaplan, J., DeMaria, M. and Knaff, A. (2010) A revised tropical cyclone rapid intensification index for the Atlantic and eastern North Pacific basins. *Weather and Forecasting*, 25, 220–241.

Kieu, C.Q. (2012) An investigation into the contraction of the hurricane radius of maximum wind. *Meteorology and Atmospheric Physics*, 115, 47–56.

Kieu, C.Q. and Zhang, D.-L. (2009) An analytical model for the rapid intensification of tropical cyclones. *Quarterly Journal of the Royal Meteorological Society*, 135, 1336–1349.

Kieu, C.Q. and Zhang, D.-L. (2010) On the consistency between dynamical and thermodynamic equations with prescribed vertical motion in an analytical tropical cyclone model. *Quarterly Journal of the Royal Meteorological Society*, 136, 1927–1930.

Kieu, C.Q. and Zhang, D.-L. (2017) Comments on “Revisiting the relationship between eyewall contraction and intensification”. *Journal of the Atmospheric Sciences*, 74, 4265–4274.

Liu, Y., Zhang, D.-L. and Yau, M.K. (1999) A multiscale numerical study of Hurricane Andrew (1992). Part II: Kinematics and inner-core structures. *Monthly Weather Review*, 127, 2597–2616.

Mallen, K., Montgomery, M.T. and Wang, B. (2005) Reexamining the near-core radial structure of the tropical cyclone primary circulation: implications for vortex resiliency. *Journal of the Atmospheric Sciences*, 62, 408–425.

Marks, F.D. and Shay, L.K. (1998) Landfalling tropical cyclones: forecast problems and associated research opportunities. *Bulletin of the American Meteorological Society*, 79, 305–323.

Miller, W., Chen, H. and Zhang, D.-L. (2015) On the rapid intensification of Hurricane Wilma (2005). Part III: Effects of latent heat of fusion. *Journal of the Atmospheric Sciences*, 72, 3829–3849.

Montgomery, M.T. and Smith, R. (2011) Paradigms for tropical-cyclone intensification. *Quarterly Journal of the Royal Meteorological Society*, 137, 1–31.

Petterssen, S. (1956) *Motion and Motion Systems*, Vol. I, *Weather Analysis and Forecasting*. New York, NY: McGraw-Hill.

Qin, N., Zhang, D.-L. and Li, Y. (2016) A statistical analysis of steady eyewall sizes associated with rapidly intensifying hurricanes. *Weather and Forecasting*, 31, 737–742.

Rotunno, R. and Emanuel, K.A. (1987) An air–sea interaction theory for tropical cyclones. Part II: Evolutionary study using a nonhydrostatic axisymmetric numerical model. *Journal of the Atmospheric Sciences*, 44, 542–561.

Schubert, W.V. and Hack, J.J. (1982) Inertial stability and tropical cyclone development. *Journal of the Atmospheric Sciences*, 39, 1687–1697.

Shapiro, L.J. and Willoughby, H.E. (1982) The response of the balanced hurricanes to local sources of heat and momentum. *Journal of the Atmospheric Sciences*, 39, 378–394.

Stern, D.P. and Nolan, D.S. (2011) On the vertical decay of the maximum tangential winds in tropical cyclones. *Journal of the Atmospheric Sciences*, 68, 2073–2094.

Stern, D.P., Vigh, J.L., Nolan, D.S. and Zhang, F. (2015) Revisiting the relationship between eyewall contraction and intensification. *Journal of the Atmospheric Sciences*, 72, 1283–1306.

Susca-Lopata, G., Zawislak, J., Zipser, E.J. and Rogers, R.F. (2015) The role of observed environmental conditions and precipitation evolution in the rapid intensification of Hurricane Earl (2010). *Monthly Weather Review*, 143, 2207–2223.

Tallapragada, V., Kieu, C., Trahan, S., Zhang, Z., Liu, Q., Wang, W., Tong, M., Zhang, B. and Strahl, B. (2015) Forecasting tropical cyclones in the western North Pacific basin using the NCEP operational HWRF model: model upgrades and evaluation of real-time performance in 2013. *Weather and Forecasting*, 30, 1355–1373.

Vigh, J.L. (2010) *Formation of the hurricane eye*. PhD Dissertation, Colorado State University, pp 378, 473, 475 and 498. Available at: http://www.ral.ucar.edu/staff/jvigh/documents/vigh2010_dissertation_corrected_color_hyperlinks.pdf (accessed October 4, 2018).

Wang, Y. and Holland, G.J. (1996) The beta drift of baroclinic vortices. Part I: Adiabatic vortices. *Journal of the Atmospheric Sciences*, 53, 411–427.

Wang, H. and Wang, Y. (2014) A numerical study of Typhoon Megi (2010). Part I: Rapid intensification. *Monthly Weather Review*, 142, 29–48.

Willoughby, H.E. (1990a) Gradient balance in tropical cyclones. *Journal of the Atmospheric Sciences*, 47, 265–274.

Willoughby, H.E. (1990b) Temporal changes of the primary circulation in tropical cyclones. *Journal of the Atmospheric Sciences*, 47, 242–264.

Willoughby, H.E., Clos, J.A. and Shoreibah, M.G. (1982) Concentric eye walls, secondary wind maxima, and the evolution of the hurricane vortex. *Journal of the Atmospheric Sciences*, 39, 395–411.

Zhang, D.-L. and Kieu, C.Q. (2006) Potential vorticity diagnosis of a simulated hurricane. Part II: Quasi-balanced contributions to forced secondary circulations. *Journal of the Atmospheric Sciences*, 63, 2989–2914.

Zhang, D.-L., Liu, Y. and Yau, M.K. (2001) A multiscale numerical study of Hurricane Andrew (1992). Part IV: Unbalanced flows. *Monthly Weather Review*, 129, 92–107.

Zhang, D.-L., Liu, Y. and Yau, M.K. (2002) A multiscale numerical study of Hurricane Andrew (1992). Part V: Inner-core thermodynamics. *Monthly Weather Review*, 130, 2745–2763.

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APPENDIX 1. THE RMW CONTRACTION RATE FOR A HURRICANE-LIKE VORTEX

To derive an RMW contraction equation for a hurricane-like vortex, we extend the derivations of Kieu (2012) by introducing a baroclinic structure to the modified Rankine vortex. This implies that unlike a barotropic vortex, the RMW contraction rate is no longer constant with height. Consider a baroclinic hurricane-like vortex and assume that its azimuthally averaged tangential wind \( v_r \) in the inner-core region in the cylindrical \((r, z)\) coordinates may be expressed by

\[
\begin{align*}
\Omega(z, t) r & \quad \forall r < R \\
\frac{\partial v_r(r, z, t)}{\partial r} & = 0 \quad \text{at } r = R
\end{align*}
\]

where \( R \) is the RMW, and \( \Omega(z, t) \) is the time-dependent angular velocity that could vary with height \( z \). It should be noted that the linear profile inside the inner core region does not conflict with the second condition \( \partial v_r / \partial r = 0 \) at the RMW in Equation (A1), because they are valid in different domains (see figure 3 in Kieu and Zhang, 2017 for a function that can ensure both conditions in Equation (A1)). The linear relationship in the inner-core region at various heights and periods...
as indicated by Equation (A1) has been widely accepted in various observational and modelling studies (e.g. Rotunno and Emanuel, 1987; Willoughby, 1990b; Wang and Holland, 1996; Mallen et al., 2005; Holland et al., 2010). Our next step is to see how the modified Rankine profile given by Equation (A1) would evolve with time.

To this end, we recall the axisymmetric tangential momentum equation in the cylindrical coordinates as follows:

$$\frac{\partial v_t}{\partial t} = -v_r \frac{\partial v_t}{\partial r} - \frac{v_r v_t}{r} - w \frac{\partial v_t}{\partial z} - f v_t + F_\lambda \quad (A2)$$

where \( v_r \) and \( w \) are the radial and vertical winds, respectively, \( f \) is the Coriolis parameter, and \( F_\lambda \) is the friction/diffusions. Substituting Equation (A1) into Equation (A2), and noting that \( \frac{\partial v_t}{\partial t} = \frac{r}{r} \frac{\partial \Omega}{\partial t} \) since the partial derivative with respect to \( t \) will not act on the coordinate variables \( (r, z) \), we obtain

$$\frac{\partial \Omega}{\partial t} r = -v_r \frac{\partial v_t}{\partial r} - w \frac{\partial v_t}{\partial z} - (\Omega + f) v_t + F_\lambda \quad \forall r < R \quad (A3)$$

Since Equation A3 is valid for \( \forall r < R \), taking a limit of Equation (A3) \( r \to R \) gives

$$\frac{\partial \Omega^*}{\partial t} R = -(\Omega^* + f) V_t - W \frac{\partial V_t}{\partial z} + F_\lambda, \quad r = R \quad (A4)$$

where \( V_t(R, z, t) = v_t(r, z, t)_{r=R} \), and \( \Omega^*(z, t) \) is an angular parameter defined exactly at \( r=R \) such that \( V_t(R, z, t) = V_t(r, z, t)_{r=R} = \Omega^*(z, t)R \). The difference between \( \Omega^*(z, t) \) and \( \Omega(z, t) \) in Equation (A1) is small, as clarified in appendix 1 of Kieu and Zhang (2017). The capital letters used herein represent the variables evaluated at the RMW.

To estimate the RMW contraction rate, we trace the evolution of the RMW as a horizontal curve in the \((r, z)\) plane such that the curve can be parametrized as \( R(t) \) at each level \( z \). Using the definition of directional derivative along this curve \( R(t) \) as presented in Kieu and Zhang (2017), the so-called derivative along the moving frame of reference in Petterssen (1956), we have:

$$\frac{d\Omega^*(z, t)R(t)}{dt} \bigg|_{\text{RMW}} = \frac{d\Omega^*(z, t)R(t)}{dt} - R(t) + \Omega^*(z, t) \frac{dR(t)}{dt} \quad (A5)$$

Note in the above directional derivative along the contracting direction that there is no derivative in the \( z \) direction, because the contraction rate at each level is assumed to be along the radial direction only. Namely, the RMW is not a material surface following a flow trajectory, but simply the change of \( V_t \) with time along the contraction direction. Practically, the total change \( \frac{d\Omega^*}{dt} \bigg|_{\text{RMW}} \) on the LHS of Equation (A5) is simply a difference of the maximum wind speed between two different times at each level. That is, one can just take the difference of the maximum 10 m wind at two adjacent times (see figure 1 in Kieu and Zhang (2017) and related discussion therein for detailed discussion of this term). Plugging Equation (A5) into Equation (A4) leads to

$$\frac{dR}{dt} = \frac{\frac{dV_t}{dt} + (\Omega^* + f) V_t + W \frac{\partial V_t}{\partial z} - F_\lambda}{\Omega^*}, \quad at \ r = R, \quad (A6)$$

which is Equation (2) in the main text.