A Role of the Axial Vector Mesons on the Photon Production in Heavy Ion Collisions and Their Relevant Decays

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Abstract

A role of the axial vector mesons, such as $K_1$ and $a_1$, on the emitted photon spectrum in hot hadronic matter is studied through the channels $\pi\rho \rightarrow a_1 \rightarrow \pi\gamma$ and $K\rho \rightarrow K_1 \rightarrow K\gamma$. They are shown to be dominant channels in this spectrum. This study is carried out with an effective chiral lagrangian which includes vector and axial-vector mesons and explains well their relevant decays simultaneously.

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1 Introduction

Recently, the study of dense and hot hadronic matter (HHM) is one of the interesting fields in relativistic heavy ion physics. One of the main goals of this study is to find a trace of phase transition between the hadronic and the quark-gluon plasma (QGP) phases. Photons (or dileptons as massive photons) can be used as a reasonable probe to detect this phenomenon because their mean free paths are much larger than the transverse size of the hot and dense region of the phases. It means that photons escape without rescattering, namely, they retain the physical information in each phase.

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Therefore the observation of the QGP signal through the detection of the photons (or dileptons) could be a plausible choice although there remained many discussions \[1, 2, 3, 4, 5\] to be solved yet, in specific, regarding if the emitted photons are produced from the HHM or QGP phases.

Yet, to our current knowledge, the HHM and QGP produce energetic photons and massive photons nearly equally at fixed temperature, 200 MeV, which is a critical temperature for deconfinement and chiral symmetric phase. Therefore, if any important contributions have been ignored in the emitted photon spectrum, for example, strange particles or heavy mesons, it would be very desirable to investigate those contributions.

Regarding where the photons are emitted from, we consider the photons produced only through the mesons, i.e, baryon free matter\[3\] because baryons are presumed too heavy to be pair produced at the temperature considered in this report. In specific, our interest is aimed to find a role of the axial vector mesons, which usually participated as intermediate states in the emitted photon spectrum by the mesons.

Actually, since the beginning of ’90 \[1, 3, 6\], not only $\rho$ and $\pi$ but also $a_1$ meson’s role is emphasized on the channels via mesons, such as $\rho\pi \rightarrow \pi\gamma$ and $\pi\pi \rightarrow \rho\gamma$. It should be noted that $\rho$ and $a_1$ form a parity doublet, i.e., they are chiral partner just like $\pi$ and $\sigma$. But their masses are not degenerated because of the spontaneous symmetry breaking(SSB) of chiral symmetry ($\chi$SB). In the chiral symmetric phase, therefore, $a_1$ becomes to be as important as $\rho$ meson. For instance, Song \[4\] and Ko \[5\] have shown that $a_1$ meson’s contribution to $\pi\rho \rightarrow \pi\gamma$ and $\pi\pi \rightarrow \rho\gamma$ channels could be dominant within the Massive Yang Mill (MYM) and the gauged linear sigma models, respectively.

But the $K_1$ ($K\rho \rightarrow K_1 \rightarrow K\gamma$) meson which could become to be important as the temperature increases, was not taken into account in their papers. Originally, the contribution of the $K_1$ meson to the radiative decay in a hadron gas was studied by Haglin \[2\], but with a simple phenomenological lagrangian introduced by Xiong et.al \[1\]. Moreover their decay widths are overestimated when they are compared to the experimental data as will be discussed.

In this paper, $K_1$ as well as $a_1$ axial vector mesons’ contributions to the photon spectrum in HHM are calculated and shown to be dominant channels in this spectrum. This study is carried out with an effective chiral lagrangian which includes systematically vector and axial-vector mesons. These fields are introduced by a non-linear realization of chiral symmetry. This scheme reproduces well in a consistent manner axial vector mesons’ decays, such as $a_1 \rightarrow \rho\pi$, $a_1 \rightarrow \pi\gamma$, $K_1 \rightarrow K\rho$ and $K_1 \rightarrow K\gamma$. This successful description was resulted from our systematic extension of the $SU(2)$ group representation to that of $SU(3)$\[6\].

This paper is organized as follows. In the section 2, our previous lagrangian \[7\] is briefly reviewed. The relevant decays of the axial vector mesons are calculated in section 3 in the framework of this lagrangian and compared to other calculations and experimental data available until now. The photon spectrum
through the channels, $\pi \rho \rightarrow a_1 \rightarrow \pi \gamma$ and $K \rho \rightarrow K_1 \rightarrow K \gamma$ are investigated in section 4. Brief summary is done at the section 5.

2 Lagrangian

Our lagrangian consists of a pseudoscalar meson sector $L(\pi)$, a spin-1 vector and axial vector meson sector $L(V, A)$, and a term of interactions with scalar particles $L_S$, which comes from mass splitting in the SU(3) extension of previous SU(2) lagrangian, i.e.,

$$L = L(\pi) + L(V, A) + L_S.$$  \hfill (1)

The lagrangian for the pseudoscalar meson sector, which is a leading Lagrangian of the chiral perturbation theory (ChPT), is given as

$$L(\pi) = \frac{f_\pi^2}{4} \langle D^\mu U^\dagger D_\mu U \rangle + \frac{f_\pi^2}{4} \langle U^\dagger \chi + \chi U \rangle,$$ \hfill (2)

$$D_\mu U = \partial_\mu U - i(v_\mu + a_\mu)U + iU(v_\mu - a_\mu),$$ \hfill (3)

where bracket denotes a trace in flavor space, $f$ is a pseudoscalar meson decay constant, chiral field is denoted as $U = \exp(i2\pi/f)$ with $\pi = T^a\pi^a$, $T^a = \lambda^a/2 (a = 1, 2, ..., 8)$. External gauge fields are introduced via $v_\mu$ and $a_\mu$. The $\chi$ is defined by $\chi = 2B_0(S + iP)$. Explicit chiral symmetry breaking due to current quark masses can be introduced by treating those masses as if they were uniform external scalar field $S$.

The non-linear realization of chiral symmetry is expressed in terms of $u = \sqrt{U}$ and $h = h(u, g_R, g_L)$ defined from $u \rightarrow g_Ruh^\dagger = hug_L^\dagger$. In this realization, we naturally have the following covariant quantities

$$i\Gamma_\mu = \frac{i}{2}(u^\dagger \partial_\mu u + u\partial_\mu u^\dagger) + \frac{1}{2}u^\dagger(v_\mu + a_\mu)u + \frac{1}{2}u(v_\mu - a_\mu)u^\dagger,$$

$$i\Delta_\mu = \frac{i}{2}(u^\dagger \partial_\mu u - u\partial_\mu u^\dagger) + \frac{1}{2}u^\dagger(v_\mu + a_\mu)u - \frac{1}{2}u(v_\mu - a_\mu)u^\dagger,$$

$$\chi_+ = u^\dagger \chi u^\dagger + u\chi u,$$ \hfill (4)

whose transformations are carried out in terms of $h$, i.e., $\Gamma_\mu \rightarrow h\Gamma_\mu h^\dagger - \partial_\mu h \cdot h^\dagger$, $\Delta_\mu \rightarrow h\Delta_\mu h^\dagger$, and $\chi_+ \rightarrow h\chi_+ h^\dagger$. With these quantities, the Lagrangian in eq.(2) can be rewritten as

$$L(\pi) = f^2\langle i\Delta_\mu i\Delta^\mu \rangle + \frac{f_\pi^2}{4}\langle \chi_+ \rangle.$$ \hfill (5)

As for the massive spin-1 mesons, we include only the mass and kinetic terms

$$L(V, A) = m_V^2 \langle (V_\mu - i\Gamma_\mu/g)^2 \rangle + m_A^2 \langle (A_\mu - i\Delta_\mu/g)^2 \rangle - \frac{1}{2}\langle (gV_{\mu\nu})^2 \rangle - \frac{1}{2}\langle (A_{\mu\nu})^2 \rangle$$ \hfill (6)
with
\[ G_{\mu\nu} = \partial_{\mu}V_{\nu} - \partial_{\nu}V_{\mu} - ig[V_{\mu}, V_{\nu}] - iG[A_{\mu}, A_{\nu}] , \]
\[ A_{\mu\nu} = \partial_{\mu}A_{\nu} - \partial_{\nu}A_{\mu} - ig[V_{\mu}, A_{\nu}] - ig[A_{\mu}, V_{\nu}] , \]  
(7)
where \( V_{\mu} = T^{a}V^{a}_{\mu}(A_{\mu} = T^{a}A^{a}_{\mu}) \) denotes spin-1 vector (axial-vector) meson field and \( g \) denotes a \( V\pi\pi \) coupling constant. The chiral transformation rules of spin-1 fields are expressed in terms of \( h \)
\[ V_{\mu} \rightarrow hV_{\mu}h^{\dagger} - \frac{i}{g} \partial_{\mu}h \cdot h^{\dagger} , \quad A_{\mu} \rightarrow hA_{\mu}h^{\dagger} . \]  
(8)
Note that we have introduced a new form of \( G_{V\mu\nu} \). The chiral symmetry is preserved for any value of \( G \) at the chiral limit in \( G_{V\mu\nu} \), so that the value of \( G \) cannot be determined from the chiral symmetry. If \( G \) is equal to \( g \) as in the HGS approach\[11\], the result may reproduce experimental data by explicitly including other higher order terms.

The introduction of the \( \mathcal{L}_{S} \) term can be found in ref.\[7\]. The resulting Lagrangian is given as
\[ \mathcal{L}_{S} \sim -\frac{1}{2}(s_{\mu}^{a})^{2}(\tilde{M})_{a}^{2} + \frac{1}{2}s_{\mu}M_{a}j^{a} , \]  
(9)
where \( \tilde{M}_{a}^{2} = \frac{1}{6}(2B_{0}\alpha)^{2}\delta_{8a} + M_{a}^{2} \).

For the mixing between axial vector mesons and pion fields, we define \( A'_{\mu} \) as
\[ A^{a}_{\mu} = A'^{a}_{\mu} - \frac{r}{g} \partial_{\mu}p^{a} + \frac{r}{g} f_{abc}B_{c} \]  
\[ = A'^{a}_{\mu} + i \frac{r}{g} \Delta_{\mu}^{a} \]  
(10)
where \( B_{\mu} \) denotes a photon field and \( Q = T^{3} + \frac{Y}{2} \). As for the mixing between vector mesons and pion fields, we also define \( V'_{\mu} \) as
\[ V^{a}_{\mu} = V'^{a}_{\mu} - \frac{G_{f}^{2}}{2g^{2}f_{2}} f_{abc} \partial_{\mu}p^{b} \]  
(11)
Besides the above mixings, we introduced the \( \rho - \omega \) mixing and the mixing between the vector meson and photon field. Since both mixings are explained in detail in ref.\[3\], we skip them here.

This field redefinition of eq.(10), which differs from our previous one, leads to an explicitly manifest gauge invariance of the amplitude relevant to the radiative decay of the axial vector meson. But it does not give any differences in \( V - \pi - \gamma \) reaction in our previous investigations, and describes correctly \( A - V - \pi \) and
$A - \gamma - \pi$ reactions. The lagrangian is, then, simply summerized as

\[ \mathcal{L} = \frac{1}{2} m_{V_a}^2 V_a V^\mu + \frac{1}{2} m_{A_a}^2 A_\mu A^\mu \\
+ \frac{m_{V_a}^2}{2gf_f^2} (1 - \frac{G \rho^2}{g}) f_{abc} V_\mu^a \pi^b \partial^\mu \pi^c \\
+ e Q f_{abc} B_\mu^a \pi^b \partial^\mu \pi^c \\
- \frac{1}{4} (\partial_\mu V_\nu - \partial_\nu V_\mu)^2 - \frac{1}{4} (e g)^2 Q^2 (\partial_\mu B_\nu - \partial_\nu B_\mu)^2 \\
- \frac{e Q}{2g} (\partial_\mu V_\nu - \partial_0 V_\mu)(\partial^\mu B_\nu - \partial^\nu B^\mu) \\
+ \frac{1}{2} G \rho f_{abc} (\partial_\mu V_\nu^a - \partial_\nu V_\mu^a)(A^{\mu b} \partial^\nu \pi^c + \partial^\mu \pi^b A^{\nu c}) \\
+ \frac{1}{2} G \rho f_{abc} (\partial_\mu B_\nu^a - \partial_\nu B_\mu^a)(A^{\mu b} \partial^\nu \pi^c + \partial^\mu \pi^b A^{\nu c}) \\
- \frac{1}{4} (\partial_\mu A_\nu - \partial_\nu A_\mu)^2 \\
+ \frac{1}{2} r f_{abc} (\partial_\mu A_\nu^a - \partial_\nu A_\mu^a)(V^{\mu b} \partial^\nu \pi^c + \partial^\mu \pi^b V^{\nu c}) \\
- \frac{1}{2} r e f_{abc} (\partial_\mu A_\nu^a - \partial_\nu A_\mu^a) \partial^\mu B^{\nu c} - \partial^\nu B^{\mu c}) \\
- \frac{1}{2} m_{\pi a}^2 \pi^a \pi^a + \frac{1}{2} \partial^\mu \pi^a \partial^\mu \pi^a, \quad (12) \]

where $m_{V_a}^2 = g^2 (f_f^2 + s_m s_a M_a)$, $m_{A_a}^2 = g^2 (f_f^2 + s_m s_a M_a)$ and $V_\mu$ and $A_\mu$ stand for redefined fields $V_\mu'$ and $A_\mu'$. The relation of our chiral effective lagrangian to the other lagrangians was discussed at the ref.\[7, 8\]. The 9 th and 12 th terms, and the 8 th and 11 th terms in the above lagrangian, which were omitted in our previous lagrangian, corresponds to the $A - \gamma - \pi$ and $A - V - \pi$ interactions, respectively. Detailed discussions concerning the lagranigans and applications to decay modes will be done at the next section. In order to determine pseudoscalar meson mass and decay constants, we exploit the following covariant quantities

\[ m_{\pi a}^2 = (M_a + \frac{(s_m)^2 M_a^2}{f}) Z_{\pi a}^2, \quad f_a = Z_{\pi a} f \\
with \quad Z_{\pi a}^2 = (1 + s_m s_a M_a f/a)^2. \quad (13) \]

Mass splitting between non-strange particles and strange particles is generated from the interaction of the meson fields with scalar particle fields.
3 Axial vector meson decay

The Lagrangian for $A - V - \pi$ reaction is given as

$$\mathcal{L}_{AV\pi} = \frac{1}{2} Gf f_{abc}(\partial_\mu V^a_\mu - \partial_\nu V^a_\nu)(A^{\mu b} \partial^\nu \pi^c + \partial^\mu \pi^b A^{\nu c})$$

$$+ \frac{1}{2} f f_{abc}(\partial_\mu A^a_\mu - \partial_\nu A^a_\nu)(V^{\mu b} \partial^\nu \pi^c + \partial^\mu \pi^b V^{\nu c}).$$  \hspace{1cm} (14)

The 1st term corresponds to the lagrangian used by Xiong[1] and Haglin[2]. The 2nd term comes from the $\partial_\mu \pi$ in the mixing of eq.(10), which avoids a direct coupling of the pion to the axial vector mesons. From the above Lagrangian, the partial decay width of $a_1$ meson to $\rho \pi$ is calculated as follows

$$\Gamma_{a_1^\pm \rightarrow \rho \pi} = \Gamma_{a_1^\pm \rightarrow \rho \pi^0} + \Gamma_{a_1^\pm \rightarrow \rho \pi^\pm}$$

$$\Gamma_{a_1^\pm \rightarrow \rho \pi^0(a_1^\pm \rightarrow \rho \pi^\pm)} = \frac{1}{24\pi} \frac{\bar{q}^2}{m_a^2} |\mathcal{M}_{a_1^\pm \rightarrow \rho \pi^0(a_1^\pm \rightarrow \rho \pi^\pm)}|^2,$$  \hspace{1cm} (15)

where $\bar{q}$ is an incoming pion momentum in the $a_1$ rest frame given as $\bar{q}^2 = \frac{(m_a^2 + m_\rho^2 - m_\pi^2)^2}{4m_a^2} - m_\pi^2$. Invariant amplitude $\mathcal{M}[a_1(p, \epsilon_A) \rightarrow \rho(k, \epsilon_\rho)\pi(q)]$ and $|\mathcal{M}|^2$ are given as

$$\mathcal{M}_{a_1 \rightarrow \rho \pi} = f_1 \epsilon_\rho \epsilon_\pi (\langle q \cdot k \rangle g^\nu_\mu - k_\mu q^\nu)^c A + f_2 \epsilon_\rho \epsilon_\pi ((\langle p \cdot q \rangle g^\nu_\mu - p_\mu q^\nu)\epsilon_A,$$  \hspace{1cm} (16)

$$|\mathcal{M}_{a_1 \rightarrow \rho \pi^\pm}|^2 = |\mathcal{M}_{a_1 \rightarrow \rho \pi^\pm}|^2 = 4[f_1(2(k \cdot q)^2 + m_\rho^2(m_\rho^2 - \bar{q}^2))]$$

$$+ f_2(2(p \cdot q)^2 + (q \cdot k)^2) + 6f_1f_2(p \cdot q)(k \cdot q).$$  \hspace{1cm} (17)

Coupling constants $f_1, f_2$ are defined as $f_1 = \frac{1}{2Gf}$, $f_2 = \frac{1}{2Gf}$. $p$, $q$ and $k$ stand for the momenta of the axial-vector meson, the vector meson, and the pion, respectively. The partial decay width of $K_1$ meson to $\rho K$ is also calculated in the same manner,

$$\Gamma_{K_1^\pm \rightarrow \rho K} = \Gamma_{K_1^\pm \rightarrow \rho K^0} + \Gamma_{K_1^\pm \rightarrow \rho K^\pm}$$

$$\Gamma_{K_1^\pm \rightarrow \rho K} = \frac{1}{24\pi} \frac{\bar{q}^2}{m_{K_1}^2} |\mathcal{M}_{K_1^\pm \rightarrow \rho K}|^2,$$  \hspace{1cm} (18)

where invariant amplitude $\mathcal{M}[K_1(p, \epsilon_A) \rightarrow \rho(k, \epsilon_\rho)K(q)]$ and $|\mathcal{M}|^2$ are given as

$$\mathcal{M}_{K_1 \rightarrow \rho K} = f_{K_1} \epsilon_\rho \epsilon_\pi (\langle q \cdot k \rangle g^\nu_\mu - k_\mu q^\nu)^c A + f_{K_2} \epsilon_\rho \epsilon_\pi ((\langle p \cdot q \rangle g^\nu_\mu - p_\mu q^\nu)\epsilon_A,$$  \hspace{1cm} (19)
\begin{align*}
|\mathcal{M}_{K_1^+ \rightarrow \rho K^0}|^2 &= \left| f_{K_1}(2(k \cdot q)^2 + m_{K_1}^2(q^2 - q^{'2})) 
+ f_{K_2}(2(p \cdot q)^2 + (q \cdot k)^2) + 6f_{K_1}f_{K_2}(p \cdot q)(k \cdot q) \right|
\\)
|\mathcal{M}_{K_1^+ \rightarrow \rho^+ K^0}|^2 &= \left| 2f_{K_1}(2(k \cdot q)^2 + m_{K_1}^2(q^2 - q^{'2})) 
+ f_{K_2}(2(p \cdot q)^2 + (q \cdot k)^2) + 6f_{K_1}f_{K_2}(p \cdot q)(k \cdot q) \right|
\end{align*}

where \( f_{K_1}, f_{K_2} \) are defined as \( f_{K_1} = \frac{1}{2} G_\rho, f_{K_2} = \frac{1}{2} f_\pi \), with kaon decay constant \( f_K \). In this case, incoming momentum \( q^2 \) is \( q^2 = \frac{(m_K^2 + m_{\pi}^2 - m_{\rho}^2)^2}{4m_K^2} - m_K^2 \), and \( p, q, k \) are \( K_1 \) meson, \( \rho \) meson, kaon momentum, respectively.

On the other hand, the lagrangian for \( A - \gamma - \pi \) reaction is obtained from eq. (12) as follows

\[
\mathcal{L}_{A,\gamma,\pi} = \frac{G_F}{2} \left( \frac{\alpha}{\sqrt{\pi}} \right) Q f_{abc}(\partial_\mu B_\nu^a - \partial_\nu B_\mu^a)(A^{\mu b} F^{\nu c} + \partial_\nu F^{\mu b} A^{\nu c})
- \frac{1}{2} \left( \frac{\alpha}{\sqrt{\pi}} \right) Q f_{abc}(\partial_\mu A_\nu^a - \partial_\nu A_\mu^a)\pi^b(\partial_\nu F^{\mu b} - \partial_\mu F^{\nu b}).
\] (21)

The 1st term in lagrangian of eq.(21) resembles the lagrangian of Xiong[1], but in our lagrangian, the 2nd term appears, which originated from the 3rd term in the mixing of eq.(10). This mixing term allows a direct coupling of the photon to the pion and causes small deviation from the conventional vector meson dominance (VMD) model (see the Figure 2 and 3 in ref.[7]). Moreover it makes the relevant amplitude gauge invariant as shown below.

Partial decay width of \( a_1 \) meson to \( \gamma \pi \) and \( K_1 \) meson to \( \gamma K \) are expressed as

\[
\Gamma_{a_1^+ \rightarrow \gamma \pi^\pm} = \frac{1}{24\pi m_{a_1}^2} |\mathcal{M}_{a_1^+ \rightarrow \gamma \pi^\pm}|^2
\]
\[
\Gamma_{K_1^+ \rightarrow \gamma K^\pm} = \frac{1}{24\pi m_{K_1}^2} |\mathcal{M}_{K_1^+ \rightarrow \gamma K^\pm}|^2,
\]

(22)

where invariant amplitude \( \mathcal{M}[a_1(p, \epsilon_A) \rightarrow \gamma(k, \epsilon_\gamma)\pi(q)] \) and \( |\mathcal{M}|^2 \) are given as

\[
\mathcal{M}_{a_1 \rightarrow \gamma \pi} = h_1 \epsilon_\gamma \left( (q \cdot k)g_\mu^\nu - k_\mu q_\nu \right) \epsilon_A^\nu - h_2 \epsilon_\gamma \left( (p \cdot k)g_\mu^\nu - k_\mu p_\nu \right) \epsilon_A^\nu,
\]

(23)

\[
|\mathcal{M}_{a_1^+ \rightarrow \gamma \pi^\pm(K_1^+ \rightarrow \gamma K^\pm)}|^2 = 4[2h_1(k \cdot q)^2 + 2h_2(k \cdot p)^2 - 4h_1 h_2 (k \cdot q)(k \cdot p)],
\]

(24)

where \( q^2 = \frac{(m_{a_1}^2 - m_{\rho^0}^2)^2}{2m_{a_1}^2} \) and \( h_1 = \frac{\epsilon}{g} f_1, h_2 = \frac{\epsilon}{g} f_2 \). \( p, q \) and \( k \) are axial-vector meson, vector meson, photon momentum, respectively. It is easy to check that the above amplitude is fully gauge invariant.

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By exploiting the above equations, we determine values of $G$, $r$ and $g$ by fitting our numerical predictions of eqs. (15), (18) and (22) to the experimental data. The results and experimental data are tabulated in table 1 with the comparison to other cases. The masses of axial vector meson used here are $a_1(1260) = 1210\text{MeV}$, $K_3(1270) = 1280\text{MeV}$, respectively.

As shown in table 1, the theoretical value of $\Gamma_{a_1 \rightarrow \pi \gamma}$ in Haglin’s paper[2] is overestimated about 2 times. Consequently, $\Gamma_{K_1 \rightarrow K \gamma}$ in our prediction is much smaller than that of Haglin. It leads to a smaller contribution of $K_1$ to the photon spectrum compared to that of $a_1$ as will be shown.

### 4 Photon Production Rate

In this section, we show a brief formalism regarding the emitted photon spectrum in a hadronic gas. In principle, including the axial vector mesons as intermediate states can add other channel processes in photon production. However, we consider only the s-channel contribution as in figure 1, because the contributions of other channels turned out to be small compared to that of s-channel in the hadronic gas of $T = 100 \sim 200\text{MeV}$[1, 2].

For a reaction of the mesons, $1 \rightarrow 2 \rightarrow 3 \rightarrow \gamma$ the photon production rate with temperature $T$[12] is given by

$$
E \frac{dR}{d^3p} = \frac{N}{16(2\pi)^7} E \int_0^\infty ds \int_{t_{min}}^{t_{max}} dt |M|^2 \int dE_1 \\
\times \int dE_2 f(E_1) f(E_2) [1 + f(E_3)], \quad (25)
$$

where

$$
\begin{align*}
a &= -(s + t - m_2^2 - m_3^2) , \\
b &= E_1(s + t - m_2^2 - m_3^2)(m_2^2 - t) + E_\gamma [(s + t - m_2^2 - m_3^2)(s - m_1^2 - m_2^2) \\
&\quad - 2m_1^2(m_2^2 - t)] , \\
c &= -E_1^2(m_2^2 - t) - 2E_1E_\gamma[2m_2^2(s + t - m_2^2 - m_3^2) - (m_2^2 - t)(s - m_1^2 - m_2^2)] \\
&\quad - E_\gamma^2[(s - m_1^2 - m_2^2)^2 - 4m_1^2m_2^2] - (s + t - m_2^2 - m_3^2)(m_2^2 - t) \\
&\quad \times (s - m_1^2 - m_2^2) + m_2^2(s + t - m_2^3 - m_3^2)^2 + m_1^2(m_2^3 - t)^2 , \\
E_{1_{min}} &= \frac{(s + t - m_2^2 - m_3^2)^2}{4E_\gamma} + \frac{E_\gamma m_3^2}{s + t - m_2^2 - m_3^2} , \\
E_{2_{min}} &= \frac{E_\gamma m_2^2}{m_2^2 - t} + \frac{m_2^2 - t}{4E_\gamma} , \\
E_{2_{max}} &= -\frac{b}{a} + \frac{b^2 - ac}{a} ,
\end{align*}
$$

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where $\mathcal{N}$ is the overall degeneracy of the particles 1 and 2, $\mathcal{M}$ is the invariant amplitude of the reactions considered in this paper (summed over final states and averaged over initial states), and $s, t, u$ are the usual Mandelstam variables. In the above equation, the indices 1, 2, 3 and $\gamma$ mean for the incident pion, incident $\rho$ meson, outgoing pion and outgoing photon, respectively. They are allowed to vary in a whole range to take off-shell property of the relevant particles into account. $f(E) = \frac{1}{(e^{E/T} - 1)}$ is the Bose-Einstein distribution function.

The invariant amplitude $M$ for the diagram in figure 1 is calculated as

$$M = 2\epsilon V \alpha \left[ f_1((q \cdot k)g_\mu^\alpha - k_\mu q^\alpha) + f_2((p \cdot q)g_\mu^\alpha - p_\mu q^\alpha)\right]D^{\mu\beta} \times 2\epsilon \gamma \left[ h_1((i \cdot j)g_\beta^\gamma - j_\beta i^\gamma) - h_2((p \cdot j)g_\beta^\gamma - j_\beta p^\gamma)\right],$$  \hspace{1cm} (26)

where $k$ and $q$ are momenta of the incoming $\rho$ and $\pi$ mesons, $i, j$ are outgoing photon and $\pi$ meson momentum and $p$ is the axial vector meson’s momentum, respectively. $D^{\mu\beta}$ is the propagator for the axial vector meson

$$D^{\mu\beta} = (g^{\mu\beta} - p^\mu p^\beta) \frac{1}{p^2 - m_{a_1(K_1)}^2 - \text{Im} a_{a_1(K_1)} \Gamma_{a_1(K_1)}}.$$  \hspace{1cm} (27)

The processes we are going to study are $\pi \rho \rightarrow a_1 \rightarrow \pi \gamma$ and $K \rho \rightarrow K_1 \rightarrow K \gamma$ in figure 1. Using the approximation by W. Greiner [10], we numerically compute a three dimensional integral in eq.(25) and get the photon spectra at various temperatures. Results are shown in figure 2. $a_1$ and $K_1$ resonance’s contributions are presented as the solid and dot-dashed lines, respectively. The spectra are calculated at three different temperatures, $T = 200, 150, 100$ MeV from the uppermost, respectively. To investigate a dependence of the spectrum on the given lagrangian, previous results [1, 2], which are reproduced by switching off the 2nd terms in eqs.(14) and (21), are presented by the dotted lines. Likewise to the discussion in reference [3], there does not appear such a discernable difference due to the different lagrangian. Namely, in the case of $a_1$, our results show predictions more or less similar to those of Haglin. But maximum values in the spectrum are increased within 10% at each temperature. It is also noticeable that the peak positions of $E_\gamma$ in the spectrum are shifted a little bit backward compared to the previous predictions.

The $K_1$ contribution, as expected, turned out to be smaller in high $E_\gamma$ region at least one order than that of $a_1$. But at low $E_\gamma$ region, it shows a comparable behavior to that of $a_1$. Therefore, in the low $E_\gamma$ region below 0.5 GeV, both $a_1$ and $K_1$ axial vector mesons show competitive roles in this spectrum. But the $K_1$ meson’s role in the high $E_\gamma$ region is decreased up to $3 \sim 4$ GeV region.

5 Conclusion

Based on our previous $SU_L(3) \otimes SU_R(3)$ chiral lagrangian [3], $K_1$ and $a_1$ meson’s contributions to $K \rho \rightarrow K_1 \rightarrow K \gamma$ and $\pi \rho \rightarrow a_1 \rightarrow \pi \gamma$ channels are investigated for the emitted photon spectrum in a hot hadronic matter. Before the
calculation, the relevant decay widths are quite well reproduced within experimental errors. In specific, the radiative and hadronic decays of the axial vector meson $a_1$ and $K_1$ are shown to be consistently explained in our lagrangian. Our emitted photon spectrum shows that $K_1$ and $a_1$ mesons could be dominant channels in low $E_\gamma$ region below 0.5 GeV, while the role of $K_1$ meson is decreased in high $E_\gamma$ region.

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|                  | Xiong | Haglin | C. Song | P. Ko | Ours | Experiment         |
|------------------|-------|--------|---------|-------|------|-------------------|
| \( m_{a_1} (1260) \) | 1230  | 1230   | 1260    | 1210  | 1230 \( \pm 40 \) MeV |
| \( m_{K_1} (1270) \) | 1273  | .      | .       | 1280  | 1273 \( \pm 7 \) MeV |
| \( \Gamma_{a_1 \to \rho \pi} \) | 400   | 400    | 328     | 488   | 200\~600MeV       |
| \( \Gamma_{a_1 \to \pi \gamma} \) | 1.4   | .      | 0.67    | 0.688 | 0.64\~0.28MeV     |
| \( \Gamma_{K_1 \to \rho K} \) | 37.8  | .      | .       | 47    | 57\~5 MeV         |
| \( \Gamma_{K_1 \to K \gamma} \) | 1.5   | .      | .       | 0.350 |                   |

Table 1: Masses and decay widths of the relevant axial vector mesons.

\[ \begin{aligned} 
\pi & \quad (K) \\
\rho & \quad (K) \\
\end{aligned} \]

Figure 1: Feynman diagram of \( \pi \rho \to \pi \gamma (K \rho \to K \gamma) \) through \( a_1(K_1) \) resonance
Figure 2: Photon production rate at $T=100 \sim 200$ MeV. From the uppermost $T = 200, 150, 100$, respectively.