Towards String Predictions

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Abstract

The aim of superstring phenomenology is to develop the tools and methodology needed to confront string theory with experimental data. The first mandatory task is to find string solutions which reproduce the observable data. The subsequent goal is to extract potential signatures beyond the observable data. Recently, by studying exact flat directions of non–Abelian singlet fields, we demonstrated the existence of free fermionic heterotic–string models in which the $SU(3) \times SU(2) \times U(1)_Y$–charged matter spectrum, just below the string scale, consists solely of the MSSM spectrum. In this paper we study the possibility that the exact flat directions leave a $U(1)_{Z'}$ symmetry unbroken at the Planck scale. We demonstrate in a specific example that such unbroken $U(1)_{Z'}$ is in general expected to be not of the GUT type but of intrinsic stringy origin. We study its phenomenological characteristics and the consequences in the case that $U(1)_{Z'}$ remains unbroken down to low energies. We suggest that observation in forthcoming colliders of a $Z'$, with universal couplings for the two light generations but different couplings for the heavy generation may provide evidence for the $Z_2 \times Z_2$ orbifold which underlies the free fermionic models.
1 Introduction

The goal of superstring phenomenology at its present stage is to develop the tools and methodology to connect between string theory and experimental data. It is clear that to understand the mechanism which selects the string vacuum a non-perturbative formulation is needed. However, it is rather likely that detailed confrontation with the experimental data will have to rely on perturbative means. For this purpose the tools to construct realistic string models and the methodology to extract their phenomenological implications must be further developed. The first mandatory task of superstring phenomenology is to produce string solutions which are as realistic as possible with present day technology. The goal in this regard is to construct string models that aim to reproduce the phenomenological data provided by the Standard Model spectrum. Moreover, with the lack of substantial experimental evidence for any extension of the Standard Model, the most desired solution would be one that reproduces solely the Standard Model. Once the first goal is achieved the subsequent goal is to extract possible experimental signatures, beyond the Standard Model, which may provide further evidence for specific string models, in particular, and for string theory, in general. In practice, of course, it makes sense to try to extract the experimental consequences of the theory at every stage of its development. This, for example, was the drive behind much of the superstring inspired activity [1] that followed the seminal Candelas et al. paper [2].

Pursuing the minimalist approach, and taking the Standard Model data as the guide toward understanding the basic building blocks of nature, one is compelled to assess that grand unification structures are relevant in nature. Proton decay constraints then imply the big desert scenario, and gravity becomes important only near the Planck scale. In this eventuality the perturbative heterotic string, which naturally accommodates the grand unification structures with chiral matter, is the relevant framework.

Over the past decade the free fermionic formulation [3] of the heterotic string has been utilized to derive the most realistic string models to date [4, 5, 6]. A large number of three generation models have been constructed, which differ in their detailed phenomenological characteristics. All these models share an underlying $Z_2 \times Z_2$ orbifold structure, which naturally gives rise to three generation models with the standard $SO(10)$ embedding of the Standard Model spectrum [6, 7]. In this respect the phenomenological success of the free fermionic models can be regarded as indicating the relevance of the $Z_2 \times Z_2$ orbifold structure in nature. Furthermore, recently, and for the first time since the advent of superstring phenomenology, it was demonstrated that free fermionic models also produce Minimal Standard Heterotic String Models (MSHSM) [8, 9, 10, 11]. In such models the low energy spectrum, which is charged under the Standard Model gauge group, consists solely of the spectrum of the Minimal Supersymmetric Standard Model. It should be emphasized that it is not suggested that one of the free fermionic models that has been constructed to
date is the correct string vacuum. Indeed, such a claim would at least require a
derivation of the detailed fermion mass spectrum, as well as an understanding of the
dynamics which break supersymmetry. However, a plausible interpretation of the
phenomenological success of the free fermionic models is that the true string vacuum
is in the neighborhood of these models. This interpretation would then single out,
for example, the $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold as the relevant string compactification. The general
lesson in this respect is to extract the string structures which are relevant for the
Standard Model phenomenological data.

Subsequent to achieving the mandatory task of demonstrating the phenomeno-
logical viability of a particular class of heterotic string compactification, trying to
extract possible experimental signatures beyond the Standard Model becomes more
compelling. Various such possible experimental signatures, inspired from string theory,
have been discussed in the past. Among them: additional gauge bosons, exotic
matter, and specific patterns of supersymmetry breaking. Our aim here is to use
the same tools, that have been used to derive realistic string models, to try to ex-
tract the experimental signatures beyond the Standard Model. Such experimental
signatures then have the advantage of being “derived” rather than “inspired” from
string theory. In this paper, we focus on the possibility of additional gauge bosons,
beyond the Standard Model. The possibility of such additional gauge structure has
of course been discussed extensively in the past, mainly in the context of additional
symmetries which arise in $SO(10)$ and $E_6$ grand unification [1, 12]. However, as our
analysis demonstrates the most likely additional gauge bosons to arise from realistic
string models are not of this origin. Therefore, of particular interest in our discussion
will be the additional gauge bosons which are of particular string origin, i.e., those
that do not arise in Grand Unified theories.

2 Free fermionic phenomenology

The analysis of the free fermionic models is conducted in two steps. In the first
step the free fermionic model building rules are used to construct a consistent three
generation string vacuum. Subsequently one extracts the full massless spectrum as
well as the cubic level and higher order non–renormalizable terms in the superpoten-
tial. At this stage the tools used are perturbative heterotic string theory techniques.
The superstring derived three generation models contain numerous massless vector–
like states, some of which carry fractional electric charge. The models typically also
contain a number of additional $U(1)$ symmetries in the observable sector, plus a
hidden gauge group which is a subgroup of the original hidden $E_8$ of the heterotic
string. One additionally finds that many of these three generation models contain an
anomalous $U(1)_A$ symmetry, which generates a Fayet–Iliopoulos term,

$$\epsilon \equiv \frac{g_s^2 M_P^2}{192\pi^2} \text{Tr} \, Q^{(A)},$$

(2.1)
where $\text{Tr} \, Q^{(A)} \neq 0$, is the trace of the $U(1)_A$ charge over all the massless fields. The Fayet–Iliopoulos term breaks supersymmetry near the Planck scale, and destabilizes the string vacuum. Supersymmetry is restored and the vacuum is stabilized if there exists a direction in the scalar potential $\phi = \sum_i \alpha_i \phi_i$ which is $F$–flat and also $D$–flat with respect to the non–anomalous gauge symmetries and in which $\sum_i Q_i^A |\alpha_i|^2$ and $\epsilon$ are of opposite sign. If such a direction exists it will acquire a vacuum expectation value (VEV) cancelling the anomalous $D$–term, restoring supersymmetry and stabilizing the string vacuum. Since the fields corresponding to such a flat direction typically also carry charges for the non–anomalous $D$–terms, a non–trivial set of constraints,

$$\langle D_A \rangle = \sum_m Q_m^{(A)} |\langle \varphi_m \rangle|^2 + \epsilon = 0,$$

$$\langle D_i \rangle = \sum_m Q_m^{(i)} |\langle \varphi_m \rangle|^2 = 0,$$

on the possible choices of VEVs is imposed. These scalar VEVs will in general break some, or all, of the additional symmetries spontaneously.

Additionally one must insure that the supersymmetric vacuum is also $F$–flat. Each superfield $\Phi_m$ (containing a scalar field $\varphi_m$ and chiral spin–$\frac{1}{2}$ superpartner $\psi_m$) that appears in the superpotential imposes further constraints on the scalar VEVs. $F$–flatness will be broken (thereby destroying spacetime supersymmetry) at the scale of the VEVs unless,

$$\langle F_m \rangle \equiv \langle \frac{\partial W}{\partial \Phi_m} \rangle = 0; \langle W \rangle = 0,$$

where $W$ is the superpotential which contains cubic level and higher order non–renormalizable terms. The higher order terms have the generic form

$$< \Phi_1^f \Phi_2^i \Phi_3^j \cdots \Phi_N^k >.$$

Some of the fields appearing in the non–renormalizable terms will in general acquire a non–vanishing VEV by the anomalous $U(1)$ cancellation mechanism. Thus, in this process some of the non–renormalizable terms induce effective renormalizable operators in the effective low energy field theory wherein either all fields or all fields but one are replaced with VEVs. One must insure that such terms do not violate supersymmetry at an unacceptable level. In practice, however, the studies performed to date have been restricted to the case in which supersymmetry remains unbroken to all orders of non–renormalizable terms.

Thus, the second stage of the string model building analysis is conducted by analyzing the $F$– and $D$–flat directions. An important advance of the last few years has been the development of systematic techniques for the analysis of exact $F$– and $D$–flat directions [13]. Additionally one may impose other phenomenological constraints on the scalar VEVs. For example, in demonstrating the existence of a free fermionic
MSHSM, we required that a set of fields which induces the decoupling of all non-MSSM states, acquire a non-vanishing VEV along the $F$- and $D$-flat directions. The string model building analysis outlined above can be regarded as aiming to achieve the first goal of superstring phenomenology. Namely, to reproduce the data provided by the Standard Model.

3 Additional gauge symmetries in free fermionic models

In this section we discuss the different classes of additional gauge symmetries that are obtained in the free fermionic models prior to the analysis of flat directions. For a given three generation string model, and a specific flat direction, the resulting string vacuum may give rise to additional matter and gauge bosons, which are beyond the Minimal Supersymmetric Standard Model. For example, the hidden sector may give rise to massless matter states, which are not charged with respect to the Standard Model gauge group, and which interact with the Standard Model states only via horizontal $U(1)$ symmetries. Such hidden matter states may have interesting cosmological implications and may serve as dark matter candidates. Similarly, for a given flat direction the string vacuum may contain a combination of the horizontal $U(1)$ symmetries, which remains unbroken. It is this type of unbroken $U(1)$ symmetry that we aim to study in this paper. Thus, for a given flat direction the first task is to extract the combinations of the $U(1)$ symmetries which remain unbroken. Some of the resulting combinations may be entirely hidden. Namely, the Standard Model states will not be charged under them. Such hidden combinations may therefore be less interesting from an experimental point of view. However, there may also exist unbroken combinations of the horizontal $U(1)$ symmetries, under which the Standard Model states are charged. It is precisely this type of unbroken $U(1)$ symmetries that are of enormous experimental and phenomenological interest. Furthermore, in a given string model the charges of the Standard Model states under such an unbroken $U(1)$ symmetry are completely specified. Consequently, the phenomenology of the additional $Z'$ in a given string model is specified, up to some educated assumptions on the scale of $Z'$ breaking and the strength of its coupling. Both of these assumptions will of course eventually be relaxed.

The free fermionic models are constructed by specifying a set of boundary conditions basis vectors and the one-loop GSO projection coefficients [3]. The basis vectors, $b_k$, span a finite additive group $\Xi = \sum_k n_k b_k$ where $n_k = 0, \ldots, N_{z_k} - 1$, with $N_{z_k}$ the smallest positive integer such that $N_{z_k} b_k = 0 \pmod{2}$. The physical massless states in the Hilbert space of a given sector $\alpha \in \Xi$, are obtained by acting on the vacuum with bosonic and fermionic operators and by applying the generalized GSO projections. The $U(1)$ charges, $Q(f)$, with respect to the unbroken Cartan generators of the four dimensional gauge group, which are in one to one correspondence
with the $U(1)$ currents $f^* f$ for each complex fermion $f$, are given by:

$$Q(f) = \frac{1}{2} \alpha(f) + F(f),$$

where $\alpha(f)$ is the boundary condition of the world–sheet fermion $f$ in the sector $\alpha$, and $F_\alpha(f)$ is a fermion number operator counting each mode of $f$ once (and if $f$ is complex, $f^*$ minus once). For periodic fermions, $\alpha(f) = 1$, the vacuum is a spinor in order to represent the Clifford algebra of the corresponding zero modes. For each periodic complex fermion $f$ there are two degenerate vacua $|+\rangle, |-\rangle$, annihilated by the zero modes $f_0$ and $f_0^*$ and with fermion numbers $F(f) = 0, -1$, respectively.

The four dimensional gauge group in the three generation free fermionic models arises as follows. The models can in general be regarded as constructed in two stages. The first stage consists of the NAHE set of boundary conditions basis vectors, which is a set of five boundary condition basis vectors, $\{1, S, b_1, b_2, b_3\}$. The NAHE set is a common set in the three generation models that we discuss here. The gauge group after imposing the GSO projections induced by the NAHE set basis vectors is

$$SO(10) \times SO(6)^3 \times E_8$$

with $N = 1$ supersymmetry. The space–time vector bosons that generate the gauge group arise from the Neveu–Schwarz sector and from the sector $1 + b_1 + b_2 + b_3$. The Neveu–Schwarz sector produces the generators of $SO(10) \times SO(6)^3 \times SO(16)$. The sector $\zeta \equiv 1 + b_1 + b_2 + b_3$ produces the spinorial of $128$ of $SO(16)$ and completes the hidden gauge group to $E_8$. At the level of the NAHE set the sectors $b_1, b_2$ and $b_3$ produce $48$ multiplets, $16$ from each, in the $16$ representation of $SO(10)$.

We remark that in order to understand the origin of the various additional $U(1)$ symmetries that may appear in free fermionic models it is often useful to consider a version of the NAHE set, which is extended by adding the basis vector $X$ with periodic boundary conditions for the right–moving complex fermions $\{\bar{\eta}^{1, \ldots, 5}, \eta^{1, 2, 3}\}$. With this additional boundary basis vector the four dimensional gauge symmetry is extended to

$$E_6 \times U(1)^2 \times SO(4)^3 \times E_8.$$
The second stage of the free fermionic basis construction consists of adding to the NAHE set three (or four) additional boundary condition basis vectors. These additional basis vectors reduce the number of generations to three chiral generation, one from each of the basis vectors \( b_1, b_2 \), and \( b_3 \), and simultaneously break the four dimensional gauge group. The \( SO(10) \) is broken to one of its subgroups \( SU(5) \times U(1) \), \( SO(6) \times SO(4) \) or \( SU(3) \times SU(2) \times U(1)^2 \). Similarly, the hidden \( E_8 \) symmetry is broken to one of its subgroups by the basis vectors which extend the NAHE set. This hidden \( E_8 \) subgroup may, or may not, contain \( U(1) \) factors which are not enhanced to a non–Abelian symmetry. On the other hand, the flavor \( SO(6)^3 \) symmetries in the NAHE–based models are always broken to flavor \( U(1) \) symmetries, as the breaking of these symmetries is correlated with the number of chiral generations. Three such \( U(1)_j \) symmetries are always obtained in the NAHE based free fermionic models, from the subgroup of the observable \( E_8 \), which is orthogonal to \( SO(10) \). These are produced by the world–sheet currents \( \bar{\eta} \eta^* \) \( (j = 1, 2, 3) \), which are part of the Cartan sub–algebra of the observable \( E_8 \). Additional unbroken \( U(1) \) symmetries, denoted typically by \( U(1)_j \) \( (j = 4, 5, \ldots) \), arise by pairing two real fermions from the sets \( \{ \tilde{y}^{3, \ldots, 6} \}, \{ \tilde{y}^{1, 2, \omega^{5, 6}} \} \) and \( \{ \omega^{1, \ldots, 4} \} \). The final observable gauge group depends on the number of such pairings.

Our interest in this paper is in additional gauge bosons that arise from the horizontal flavor symmetries. That is, in additional vector bosons which arise from combinations of world–sheet \( U(1) \) currents of the Cartan subalgebra. The generators of such additional \( U(1) \)'s all arise from the Neveu–Schwarz sector. Before proceeding, however, we briefly discuss possible non–Abelian extensions of the Standard Model in these models, and postpone detailed analysis on these possibilities to future work. It is already clear that from the unbroken subgroup of \( SO(10) \), we can obtain the traditional left–right symmetric extensions of the Standard Model. These originate from the \( SO(6) \times SO(4) \) type models or from left–right symmetric models in which \( SO(10) \) is broken to \( SU(3) \times U(1) \times SU(4) \) at the string level.

Additional sources of possible non–Abelian enhancement may arise from combinations of the boundary conditions basis vectors which extend the NAHE set. In some of the three generation model one finds combinations of the additional basis vectors

\[
Y = n_\alpha \alpha + n_\beta \beta + n_\gamma \gamma, \tag{3.2}
\]

for which \( Y_L \cdot Y_L = 0 \) and \( Y_R \cdot Y_R \leq 8 \). Such a combination may produce additional space–time vectors bosons, depending on the GSO projections. In these cases, some combination of the \( U(1) \) generators of the four dimensional Cartan sub–algebra is enhanced to a non–Abelian gauge symmetry. Often it is found that this is a combination of the flavor symmetries, which is family universal, and combines with the \( U(1)_{B–L} \) generator to produce a baryonic, or leptonic, non–Abelian gauged symmetry \([13]\). Such symmetries may therefore play an important role in insuring proton stability, but their phenomenological viability still needs to be studied. We will not
discuss this type of enhanced symmetries further here and delegate more detailed studies to future work. To summarize, in the spirit of the minimalist approach pursued here, in this paper we are interested in additional gauge bosons that arise from the unbroken Cartan generators of the four dimensional gauge group. In particular, we are interested in possible combinations of the $U(1)$ symmetries, which remain unbroken by a set of flat directions that cancels the anomalous $U(1)$ $D$–term.

4 $Z'$s in free fermionic models

We now turn to discuss the extra $Z'$ symmetries that appear in free fermionic models. The first objective is to study the additional $U(1)$ symmetries which appear prior to the analysis of flat directions. The subsequent objective is to determine which combinations of $U(1)$ symmetries remain unbroken after the analysis of flat directions, and possibly after imposing additional phenomenological constraints that are required by the Standard Model data. Such unbroken $U(1)$ combinations then come closer to being a prediction of the string models. The final goal is of course to extract which possible $U(1)$ combinations remain unbroken after the wealth of Standard Model experimental data is satisfied. Such an extra $U(1)$ combination then is truly a prediction of a specific string vacua. However, short of this ambitious and still unachievable goal, we can already at this stage extract the general characteristics of $U(1)$ combinations that may remain unbroken in detailed $F$– and $D$–flat solutions.

The first type of $Z'$ symmetry that has been considered in the context of free fermionic models [16] has been the $U(1)$ combination

$$Q_{Z'} = \frac{B - L}{2} - \frac{2}{3} T_{3R}, \quad (4.1)$$

which is embedded in $SO(10)$ and is orthogonal to the Standard Model weak–hypercharge. The phenomenology of this class of extra $U(1)$'s, as well as its family–universal extensions in the context of $E_6$ string inspired phenomenology, have been discussed extensively in the past [1,2]. As we discussed above, in the free fermionic models the additional $U(1)$ symmetry (aside from (4.1) which is embedded in $E_6$ is given by the family universal combination of the horizontal symmetries, given by

$$U(1)_{E_6} = U(1)_1 + U(1)_2 + U(1)_3. \quad (4.2)$$

However, there are several reasons to argue that these particular $U(1)$ combinations, (4.1) and (4.2), in the free fermionic models cannot remain unbroken to low energies. In the first place, one often finds (although not always [17]) that the family universal $U(1)$ which is embedded in $E_6$ is anomalous and is therefore broken by the flat direction VEVs. Second, the scale of the breaking of the $U(1)$ symmetry which is embedded in $SO(10)$ is associated with the see–saw scale, which is needed to suppress the left–handed neutrino masses. Thus, the requirement of sufficiently small neutrino
masses implies that this particular $U(1)$ symmetry cannot remain unbroken to low energies.

The natural question is then which additional $U(1)$ symmetries, beyond the weak–hypercharge of the Standard Model, can remain unbroken to low energies. As discussed in the introduction the string models under considerations often contain an anomalous $U(1)$ symmetry. In those cases most, or all, of the horizontal $U(1)$ symmetries in the observable sector are broken by the choices of flat directions. Additionally, one has to impose plausible phenomenological constraints, like the decoupling of exotic fractionally charged states and quasi–realistic fermion mass spectrum, which may further result in the breaking of the observable horizontal symmetries. The choice of flat directions may leave unbroken $U(1)$ symmetries in the hidden sector, but those are of less interest from an experimental and phenomenological perspective. We further remark that in left–right symmetric models, with $SU(3) \times U(1) \times SU(2)_L \times SU(2)_R$ as the unbroken subgroup of $SO(10)$ at the string scale, one finds models in which all the horizontal $U(1)$ symmetries are anomaly free \cite{17}. That is, in these models there is no anomalous $U(1)$ symmetry. Consequently these semi–realistic string vacua are supersymmetric and anomaly free without the need for scalar VEVs which break some of the horizontal $U(1)$ symmetries. However, the phenomenology of this class of string models has not been studied extensively and one may expect that imposing plausible phenomenological constraints will necessitate some Planck scale VEVs. Therefore, in this paper we focus on string models that do contain an anomalous $U(1)$ symmetry.

5 $Z'$ in the FNY model

As our concrete illustrative example of a $Z'$ appearing in a string model we consider the string derived model of ref. \cite{5}. We will refer to this model as the FNY model. The $F$– and $D$–flat directions in this model were studied in detail in refs. \cite{8, 9, 10, 11}. There it was shown that there exist for this model flat directions which result in the decoupling of all the massless exotic fractionally charged states by the scalar VEVs. This is achieved due to the fact that in this model there exist cubic level superpotential terms, in which the exotic fractionally charged states are coupled to a set of $SO(10)$ singlets. Thus, assigning non–vanishing VEVs to this set of $SO(10)$ singlets results in all of the fractionally charged exotic states receiving mass of the order of the Fayet–Iliopoulos term. It was further shown that for these flat directions all the additional states beyond the spectrum of the Minimal Supersymmetric Standard Model receive masses from up to quintic order terms in the superpotential. Therefore, in this model all the states that are beyond the MSSM and which are charged with respect to the Standard Model gauge group decouple from the massless spectrum at or slightly below the string scale. This string model therefore provides the first known example of a Minimal Standard Heterotic–String Model (MSHSM). It should be emphasized that this does not indicate that the FNY string model is the correct string vacuum,
nor is it our intention to claim that the FNY model passes all of the phenomenological constraints imposed by the Standard Model data. However, what we think is a reasonable lesson to extract is that the success of producing a MSSM, as well as the other unique phenomenological characteristics of the free fermionic models, like the standard \( SO(10) \) embedding of the weak–hypercharge, may be taken as suggesting that the correct string vacuum may indeed exist in the vicinity of the free fermionic point in the string moduli space. The details of the FNY string model, its massless spectrum, and superpotential terms up to sixth order are given in ref. [5, 8, 9]. Here, for completeness, we only discuss the features of the model which are relevant for our discussion, and give in Table 1 the relevant states and charges in the effective low energy field theory.

Prior to the analysis of flat directions the observable gauge group of the FNY model is: \( SU(3)_C \times SU(2)_L \times U(1)_C \times U(1)_L \times U(1)_B \) and the hidden gauge group is: \( SO(4) \times SU(2) \times U(1) \). The Standard Model weak–hypercharge is given by \( U(1)_Y = \frac{1}{3} U(1)_C + \frac{1}{2} U(1)_L \). The sectors \( b_1, b_2 \) and \( b_3 \) produce the three light generations. Electroweak Higgs doublets \( \{ h_1, h_2, h_3, \bar{h}_1, \bar{h}_2, \bar{h}_3 \} \) arise from the Neveu–Schwarz sector, and \( H_{34}, H_{41} \) from the sectors \( b_3 + \alpha \pm \beta \) and \( b_1 + b_2 + b_4 + \alpha \pm \beta \).

Prior to rotating the anomaly into a single \( U(1)_A \), six of the FNY model’s twelve \( U(1) \) symmetries are anomalous: \( \text{Tr} U_1 = -24, \text{Tr} U_2 = -30, \text{Tr} U_3 = 18, \text{Tr} U_5 = 6, \text{Tr} U_6 = 6 \) and \( \text{Tr} U_8 = 12 \). Thus, the total anomaly can be rotated into a single \( U(1)_A \) defined by

\[
U_A \equiv -4 U_1 - 5 U_2 + 3 U_3 + U_5 + U_6 + 2 U_8. \tag{5.1}
\]

The five orthogonal linear combinations,

\[
\begin{align*}
U'_1 &= 2 U_1 - U_2 + U_3 ; & U'_2 &= -U_1 + 5 U_2 + 7 U_3 ; \\
U'_3 &= U_5 - U_6 ; & U'_4 &= U_5 + U_6 - U_8 ; \\
U'_5 &= 12 U_1 + 15 U_2 - 9 U_3 + 25 U_5 + 25 U_6 + 50 U_8 ,
\end{align*}
\tag{5.2}
\]

are all traceless.

A particular flat solution in the FNY model is given by the set of fields

\[
\{ \Phi_{12}, \Phi_{23}, \Phi_{56}, \Phi_4, \Phi'_4, \Phi'_3, H_{31}, H_{38}, H_{23}, V_{40}, H_{28}, V_{37} \}. \tag{5.3}
\]

As discussed in ref. [10, 11] with this set of VEVs all of the exotic states beyond the MSSM receive heavy mass from cubic or quintic order terms in the superpotential.

Detailed investigation of the fermion mass texture which is generated by the \( F^- \)– and \( D^- \)-flat solutions has been performed in ref. [10]. The analysis was performed for flat directions which utilize only non–Abelian singlet VEVs. The solution in Eq. (5.3) also contains non–Abelian fields, and was shown to be flat to all orders in ref. [10]. Quick examination of the non–renormalizable terms suggests that the fermion mass textures generated by this flat direction are similar to those found in ref. [10]. We

\[U(1)_C = \frac{4}{7} U(1)_{B-L}; U(1)_L = 2 U(1)_{3R}.\]
then have that the light Higgs representations consist of $h_1$ and a combination of $h_1$ and $h_3$. One then finds that the leading mass terms are $Q_1 u_1^c h_1$ and $Q_3 d_3^c h_3$. These mass textures are therefore not phenomenologically viable as the left-handed component of the top and bottom quarks live in different multiplets. A plausible solution is to find a flat direction for which $h_3$ is not part of the surviving light Higgs combination. In which case a mass term for the bottom quark can appear, for example, from the quartic term $Q_1 d_3^c H_{41} H_{21}^*$. For the purpose of our discussion here we make the assumption that the sector $b_1$ produces the heavy generation states and $b_{2,3}$ produce the two light generations. A more detailed study of the phenomenology of the non–Abelian flat directions will be reported in ref. [18].

We next turn to discuss whether any combination, and which, of the $U(1)$ symmetries of the FNY model remains unbroken by the choice of VEVs in Eq. (5.3). Subsequently, we will examine the phenomenological characteristics of the unbroken combinations. The first observation is that the family universal $U(1)_{Z'}$, which is embedded in $SO(10)$ is broken by this choice of VEVs. Similarly, the family universal $U(1)$ combination which is embedded in $E_6$ is broken at the string scale. As we discussed above, our general expectation is that in fact these particular $U(1)$ symmetries cannot remain unbroken to low energies. The set of VEVs in Eq. (5.3) leaves two $U(1)$ combination unbroken at the string scale. The first is given by the combination

$$U(1) = 3U(1)_7 + U_h,$$

while the second unbroken combination is given by

$$U(1)_{Z'} = 5U(1)'_3 + U(1)_7 - 3U_h.$$

The $U(1)$ generators appearing in the first combination are from the Cartan sub-algebra of the hidden $E_8$. Therefore, the three Standard Model generations from the sectors $b_1, b_2$ and $b_3$ are not charged with respect to this $U(1)$ combination and it is consequently not of interest from the point of view of low energy experiments. In the second unbroken combination $U(1)'_3$ appears and consequently the Standard Model states are charged under this unbroken $U(1)_{Z'}$ symmetry.

6 Phenomenological characteristics

As we illustrated in the previous section, the $F$– and $D$–flat solution Eq. (5.3) leaves the $U(1)_{Z'}$ combination, Eq. (5.5), unbroken at the string scale. Several issues are important to consider in regard to the possible low energy phenomenological implications. Furthermore, many of the issues which are crucial for fully extracting the phenomenological consequences, like the fermion identification, are still not under complete control. Consequently, prior to embarking on a detailed phenomenological analysis, we have to try to isolate those characteristics which are independent of the details about which we are ignorant at this stage. If such an extraction is possible then
the discussion becomes more substantial. This is the price we have to pay for trying to extract phenomenological consequences from a theory whose natural scale is vastly separated from the experiments’ natural scale. Similarly, to this level we have found a $U(1)$ combination which remains unbroken at the string scale. It is quite plausible that supersymmetry breaking requires the existence of an intermediate energy scale. Of course, one can devise various scenarios, like radiative breaking, by which the $U(1)_{Z'}$ will be broken just at the right scale, namely near the electroweak scale. The $U(1)_{Z'}$ breaking can be generated due to the VEV of one of the remaining light Standard Model singlets, which are charged under $U(1)_{Z'}$, for example $\Phi'_{56}$ and $\bar{\Phi}'_{56}$. But at this stage we regard the possibility that the $U(1)_{Z'}$ remains unbroken down to low energies as an assumption and extract the phenomenological implications from there. We see from Table 1 that the charges of the three generations and the Higgs multiplets under the $U(1)_{Z'}$ are completely specified. Then, up to the caveat stated above, the phenomenological implications are completely fixed.

Several observations are interesting to note. First from Eq. (5.3) we see that indeed the unbroken $U(1)_{Z'}$ is not of $E_6$ or $SO(10)$ origin. Moreover, the unbroken $U(1)$ combination does not arise from the $U(1)$ generators of the observable $E_8$, but rather from $U(1)$ symmetries which arise from the compactified Narain lattice. Thus, the unbroken $U(1)$ symmetries that we may expect to arise from string vacua are not of the GUT type. Furthermore, as the fermion charges are related to the particular type of compactification, $U(1)_{Z'}$ experimental data may contain information on the underlying compactified manifold.

Examining then the $U(1)_{Z'}$ charges in Table 1 we see that mass mixing of the $Z'$-gauge boson with the Standard Model $Z$ would not arise if the light electroweak doublets are composed only of doublets from the Neveu–Schwarz sector. This in fact would be the general case if the unbroken $U(1)$ is solely a combination of the Cartan generators arising from the Narain lattice, and possibly hidden sector generators. That is, if it does not contain $U(1)$ currents from the observable $E_8$. In this case, as is seen from Table 1, all Neveu–Schwarz electroweak doublets are neutral with respect to $U(1)_{Z'}$. $Z - Z'$ mass mixing could arise if the light electroweak Higgs doublets contain a state which arises from the twisted sectors. In the case of the FNY model those are $H_{34}$ and $H_{41}$ in Table 1. In general, the states of this type, arising from twisted sectors, are charged with respect to the $U(1)$ currents which arise from the Narain lattice. However, here it is found that also $H_{34}$ and $H_{41}$ are neutral under the particular unbroken $U(1)$ combination given in Eq. (5.3). Therefore, here all the electroweak Higgs doublets are neutral under $U(1)_{Z'}$ and $Z - Z'$ mass mixing cannot arise.

Possible kinetic mixing, arising from one–loop oblique corrections to the gauge boson propagator, is also highly suppressed. This follows from our assumption that the sector $b_1$ produces the heavy generation. The heavy generation states are therefore neutral under $U(1)_{Z'}$, and do not contribute to the one–loop corrections. For the two light generations, arising from the sectors $b_2$ and $b_3$, we see from Table 1
that the charges of the states are equal in magnitude and opposite in sign, and would therefore cancel. For Standard Model fermions the kinetic mixing can therefore only be of the order of \( \ln(m^2/m_Z^2) \sim \ln(m^2/m_{Z'}^2) \), which is highly suppressed even for \( M_{Z'} \sim 500 \) GeV. Gauginos, Higgsinos and the light Higgs cannot contribute to kinetic mixing because they are all neutral under this particular \( U(1)_{Z'} \).

We now turn to the supersymmetric scalar sector. Since under our assumption the sector \( b_1 \) produces the heavy generation, which is neutral under \( U(1)_{Z'} \), only the two light generation can contribute. However, up to light fermion mass corrections, and assuming universality, the two light scalar generations are degenerate in mass. Nonuniversality, could arise due to the \( U(1)_{Z'} \) \( D \)-term contribution. However, \( D_{Z'} \) vanishes if the VEVs of the two fields which break \( U(1)_{Z'} \), say \( \Phi^{' 56} \) and \( \bar{\Phi}^{56} \), are equal in magnitude. Therefore, under this assumption, the scalar contribution to the scalar masses is also negligible, and kinetic mixing is highly suppressed for this particular \( Z' \) combination.

We now give a rough estimate of the phenomenological constraints on \( M_{Z'} \). For this purpose we have to normalize \( U(1)_{Z'} \) so that it has the correct normalization to produce the correct conformal dimension, \( \tilde{h} = 1 \), for the massless states. From Eq. (5.3) we deduce that the normalization factor is \( N = 1/\sqrt{78} \). Estimating the beta function coefficients from the charges given in Table 1, we obtain \( b_{Z'} \approx 2.4 \), where we have taken the spectrum to consist of three MSSM generations, excluding the three right–handed neutrinos. Taking \( \alpha^{-1}_{\text{GUT}} \approx 25 \), and extrapolating from \( M_{\text{GUT}} \approx 10^{17} \) GeV to \( M_Z \), we obtain \( \alpha_{Z'}^2(M_Z) \approx 40 \). As seen from Table 1 the charges of the two light generations, while equal in magnitude are opposite in charge, and consequently not universal. Therefore, the strongest constraint is from Flavor Changing Neutral Currents, which arises here from fermion mixing. To estimate this constraint we use

\[
\Gamma(K_L^0 \to \mu^+\mu^-) \approx 10^{-8} \Gamma(K^+ \to \mu^+\nu_{\mu}).
\]  

Estimating the tree diagrams we obtain

\[
Q_{Z'}^2\alpha_{Z'}^2 \cos^2\theta_C \sin^2\theta_C/M_{Z'}^2 = 10^{-8} \alpha_2^2 \sin^2\theta_C/(4M_W^4).
\]  

With the appropriately normalized charges for \( Q_{Z'} \), we obtain \( M_{Z'} \approx 25M_W \approx 2 \) TeV. We remark that the additional suppression of the \( Z' \) interaction is obtained because of the \( U(1)_{Z'} \) normalization factor that we calculated above. This reflects the fact that the \( U(1)_{Z'} \) combination contains Cartan generators of the hidden \( E_6 \) under which the Standard Model states are not charged. The consequence is that there is roughly an order of magnitude suppression of the \( Q_{Z'} \) charges of the Standard Model states.

A more stringent constraint arises by considering the mixing in the \( K_0 - \bar{K}_0 \) system parametrized by the mass difference \( \Delta M_K = 3.5 \times 10^{-12} \) MeV [19]. Treating the \( Z' \) as a contact interaction we have that \( \Delta M_K \sim G_2 M_K f_K \), where \( f_K \approx 1.2m_\pi \) is the kaon decay constant, \( M_K \approx 0.5 \) GeV is the kaon mass, and \( G_2 = (Q_{Z'}^2/M_{Z'}^2)4\pi\alpha_{Z'}(\cos\theta_C\sin\theta_C)^2 \) is the \( Z' \) contact interaction term. We then
find that $G_2 \leq 10^{-7} G_F$, where $G_F$ is the Fermi constant. From this we obtain $M_{Z'} > 30$ TeV. Considering the corresponding mass difference in the $B$–meson system, $\Delta M_B \approx 3.12 \cdot 10^{-4}$eV [14], imposes only $m_{Z'} \geq 500$GeV, and is therefore less restrictive, where we have used the Standard Model value for $V_{td}$. We do not estimate here constraints arising from FCNC in the lepton sector as the leptonic mixing parameters are not known.

It is therefore expected that a $Z'$ with non–universal charges for the two light generations is constrained to be above the reach of the LHC. Nevertheless, as we discuss below, a $Z'$ with universal couplings for the first two light generations and with different couplings to the heavy generation may also arise from the free fermionic models and may in fact be a signature of the $Z_2 \times Z_2$ orbifold which underlies the free fermionic models. We note that if a $Z'$ gauge boson of the type that we discussed above is in the region accessible to future hadron colliders, it will yield spectacular signatures. Namely, in the case of the particular $U(1)^{Z'}$ combination that we examined here, it will result in enhancement in the production of the two light generations, whereas a parallel enhancement in the production of the heavy generation will not be observed. Similarly, for this particular $U(1)^{Z'}$ combination, production of Higgs doublets and gauginos in the $Z'$ channel will not be observed. While it is not our aim to argue that the particular $U(1)^{Z'}$ combination examined here is necessarily phenomenologically viable, what we see is that in a given string model, and for a given flat direction, the phenomenological consequences and possible production and decay channels are completely specified and yield distinctive signatures.

7 Discussion

We emphasize that it is not our intent to argue here that the particular $U(1)^{Z'}$ combination that we examined is necessarily “the” phenomenologically viable combination that may be seen in future collider experiments. What we have shown is that in a specific string model the $U(1)^{Z'}$ combinations which remains unbroken for specific flat directions are given. Consequently, the charges of the Standard Model fermions are specified and, in the case that the $U(1)^{Z'}$ symmetry remains unbroken down to low energies, the phenomenological implications are determined. The $U(1)^{Z'}$ that we examined here provides an illustrative example. However, we believe that more general lessons can be extracted. The first is that we anticipate that the $U(1)$ combinations which remain unbroken after analysis of the flat directions are not of the type which appear in $SO(10)$ or $E_6$ grand unifying theories. Therefore, it is anticipated that if a $U(1)$ combination remains unbroken down to low energies, it contains $U(1)$ factors which are external to the GUT gauge group.

The second important lesson arises by examining the various $U(1)$ charges given in Table 1. We see that a common feature is precisely the flavor non–universality of the different $U(1)$ combinations. Thus, we see, for example that for $U'_1$, $U'_4$, and $U_4$ the charges of the two light generations are universal and differ from the charges...
of the heavy generation. Flat directions which preserve one of these $U(1)$’s as a component of an unbroken $U(1)$ symmetry, may therefore yield a $Z'$ gauge boson which is less severely restricted by FCNC constraints. Nevertheless, the distinctive collider signatures of a $Z'$ arising from any of those $U(1)$ symmetries will be a non-universality in the production of the different generations. Thus, for example, $U'_4$ would predict enhancement in the production of the two light families, without a corresponding enhancement in production of the heavy family, whereas $U_4$ would predict exactly the opposite.

A $Z'$ with this characteristic may in fact be a consequence of the $Z_2 \times Z_2$ orbifold with standard embedding, which underlies the free fermionic formulation for the following reason. Take, for example, the case in which the anomalous $U(1)$ is a combination which is embedded in $E_6$ and is given by $U_A = U_1 + U_2 + U_3$ in the notation of Section 3. The two anomaly free orthogonal combinations can be taken as $U'_1 = U_1 - U_2$ and $U'_2 = U_1 + U_2 - 2U_3$. The states of each generation from each sector $b_j$ have charge $+1/2$ under $U(1)_j$ and are neutral with respect to the other two. Consequently, $U'_1$ produces charges which are equal in magnitude and opposite in sign for two generation, whereas one generation is neutral under it. This yields the same type of $Z'$ that we examined here and is strongly constrained by FCNC. On the other hand $U'_2$ is universal with respect to two families and produces different charges for the third family. This situation may, in fact, be a unique consequence of the $Z_2 \times Z_2$ orbifold twisting, due to its cyclic permutation symmetry. From Table 1 we see that, in fact, this type of charge assignment is also frequently preserved in the three generation models. What we argue is that if a $Z'$ with universal couplings for the two light generations and different couplings for the heavy generation is observed in future experiments, it may be a key piece of evidence for the $Z_2 \times Z_2$ orbifold compactification. In the case of a $Z'$ with charges equal in magnitude but opposite in sign for the first two generations, we may expect it to be outside the reach of forthcoming hadron colliders. However, if it is not too far above their reach, we may expect novel FCNC phenomena, and potentially new sources for CP violation. We note that an additional $Z'$ of the type that we discussed here has also been advocated as playing a role in suppressing proton decay in supersymmetric extensions of the Standard Model [20]. We also remark that very recently it has been suggested that there exists evidence for a $Z'$ with these characteristics in electroweak precision data [21]. All in all, nature may eventually prove to be kind for her patient and obedient servants.

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## A Quantum Number of FNY Massless Fields

| state | $U_E$ | $(C, L)_Y$ | $U_A$ | $U_C$ | $U_L$ | $U'_1$ | $U'_2$ | $U'_3$ | $U'_4$ | $U'_5$ | $U'_4$ | $(3, 2, 2')_H$ | $U_7$ | $U_H$ | $U_9$ |
|-------|-------|------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-----------|-------|-------|-------|
| $Q_1$ | $\frac{2-1}{3}$ | $(3, 2)_{\frac{1}{6}}$ | 8 & 2 & 0 & -4 & 2 & 0 & 0 & -24 & 2 | $(1, 1)$ | 0 & 0 & 0 |
| $Q_2$ | $\frac{2-1}{3}$ | $(3, 2)_{\frac{1}{3}}$ | 12 & 2 & 0 & 2 & -10 & 2 & 2 & 20 & 0 | $(1, 1)$ | 0 & 0 & 0 |
| $Q_3$ | $\frac{2-1}{3}$ | $(3, 2)_{\frac{1}{2}}$ | 8 & 2 & 0 & 2 & 14 & -2 & 2 & 32 & 0 | $(1, 1)$ | 0 & 0 & 0 |
| $d_1^c$ | $\frac{1-2}{3}$ | $(3, 1)_{\frac{1}{3}}$ | 8 & -2 & 4 & -4 & 2 & 0 & 0 & -24 & -2 | $(1, 1)$ | 0 & 0 & 0 |
| $d_2^c$ | $\frac{1-2}{3}$ | $(3, 1)_{\frac{1}{3}}$ | 8 & -2 & 4 & 2 & -10 & -2 & -2 & -80 & 0 | $(1, 1)$ | 0 & 0 & 0 |
| $d_3^c$ | $\frac{1-2}{3}$ | $(3, 1)_{\frac{1}{3}}$ | 4 & -2 & 4 & 2 & 14 & 2 & -2 & -68 & 0 | $(1, 1)$ | 0 & 0 & 0 |
| $u_1^c$ | $\frac{-1-2}{3}$ | $(3, 1)_{-\frac{1}{3}}$ | 8 & -2 & -4 & -4 & 2 & 0 & 0 & -24 & -2 | $(1, 1)$ | 0 & 0 & 0 |
| $u_2^c$ | $\frac{-1-2}{3}$ | $(3, 1)_{-\frac{1}{3}}$ | 12 & -2 & -4 & 2 & -10 & 2 & 2 & 20 & 0 | $(1, 1)$ | 0 & 0 & 0 |
| $u_3^c$ | $\frac{-1-2}{3}$ | $(3, 1)_{-\frac{1}{3}}$ | 8 & -2 & -4 & 2 & 14 & -2 & 2 & 32 & 0 | $(1, 1)$ | 0 & 0 & 0 |
| $e_1^c$ | 1 | $(1, 1)_{\frac{1}{1}}$ | 8 & 6 & 4 & -4 & 2 & 0 & 0 & -24 & -2 | $(1, 1)$ | 0 & 0 & 0 |
| $e_2^c$ | 1 | $(1, 1)_{\frac{1}{1}}$ | 12 & 6 & 4 & 2 & -10 & 2 & 2 & 20 & 0 | $(1, 1)$ | 0 & 0 & 0 |
| $e_3^c$ | 1 | $(1, 1)_{\frac{1}{1}}$ | 8 & 6 & 4 & 2 & 14 & -2 & 2 & 32 & 0 | $(1, 1)$ | 0 & 0 & 0 |
| $N_{c1}^c$ | 0 | $(1, 1)_{\frac{0}{0}}$ | 8 & 6 & -4 & -4 & 2 & 0 & 0 & -24 & -2 | $(1, 1)$ | 0 & 0 & 0 |
| $N_{c2}^c$ | 0 | $(1, 1)_{\frac{1}{0}}$ | 8 & 6 & -4 & 2 & -10 & -2 & -2 & -80 & 0 | $(1, 1)$ | 0 & 0 & 0 |
| $N_{c3}^c$ | 0 | $(1, 1)_{0}$ | 8 & 6 & -4 & 2 & -10 & -2 & -2 & -80 & 0 | $(1, 1)$ | 0 & 0 & 0 |
| $L_1$ | 0 & $(1, 2)_{\frac{1}{2}}$ | 8 & -6 & 0 & -4 & 2 & 0 & 0 & -24 & 2 | $(1, 1)$ | 0 & 0 & 0 |
| $L_2$ | 0 & $(1, 2)_{\frac{1}{2}}$ | 8 & -6 & 0 & 2 & -10 & -2 & -2 & -80 & 0 | $(1, 1)$ | 0 & 0 & 0 |
| $L_3$ | 0 & $(1, 2)_{\frac{1}{2}}$ | 4 & -6 & 0 & 2 & 14 & 2 & -2 & -68 & 0 | $(1, 1)$ | 0 & 0 & 0 |
| $h_1$ | 0 & $(1, 2)_{\frac{1}{2}}$ | 16 & 0 & -4 & -8 & 4 & 0 & 0 & -48 & 0 | $(1, 1)$ | 0 & 0 & 0 |
| $h_2$ | 0 & $(1, 2)_{\frac{1}{2}}$ | -20 & 0 & -4 & -4 & 20 & 0 & 0 & 60 & 0 | $(1, 1)$ | 0 & 0 & 0 |
| $h_3$ | 0 & $(1, 2)_{\frac{1}{2}}$ | -12 & 0 & -4 & -4 & 28 & 0 & 0 & 36 & 0 | $(1, 1)$ | 0 & 0 & 0 |
| $h_1$ | 1 & $(1, 2)_{\frac{1}{2}}$ | -16 & 0 & 4 & 8 & -4 & 0 & 0 & 48 & 0 | $(1, 1)$ | 0 & 0 & 0 |
| $h_2$ | 1 & $(1, 2)_{\frac{1}{2}}$ | 20 & 0 & 4 & 4 & -20 & 0 & 0 & -60 & 0 | $(1, 1)$ | 0 & 0 & 0 |
| $h_3$ | 1 & $(1, 2)_{\frac{1}{2}}$ | 12 & 0 & 4 & 4 & 28 & 0 & 0 & -36 & 0 | $(1, 1)$ | 0 & 0 & 0 |
| $H_{34}$ | 1 | $(1, 2)_{\frac{1}{2}}$ | 8 & 3 & 2 & -2 & 11 & 2 & -4 & 32 & 0 | $(1, 1)$ | 1 & 0 & 0 |
| $H_{41}$ | 1 | $(1, 2)_{\frac{1}{2}}$ | 0 & -3 & -2 & 2 & -13 & -2 & -4 & 56 & 0 | $(1, 1)$ | 1 & -3 & 0 |

Table 1: Gauge Charges of FNY three generation and Higgs sectors. The names of the states appear in the first column, with the states’ various charges appearing in the other columns. The entries under $(C, L)_Y$ denote Standard Model charges, while the entries under $(3, 2, 2')$ denote hidden sector $SU(3)_H \times SU(2)_H \times SU(2)'_H$ charges. All $U(1)$ charges are multiplied by a factor of 4 relative to the definition in Eqs. (5.1) and (5.2).
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