Subdynamics of a many-particle classical system driven from an equilibrium state by an external force

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Abstract
It is shown, that by means of a special projection operator, the Liouville equation for an \( N \)-particle distribution function of classical particles, driven from an equilibrium state by an external field, can be exactly converted into a closed linear homogeneous Generalized Master Equations (GMEs) for an \( s \)-particle \((s < N)\) distribution function. The obtained linear time-convolution and time-convolutionless GMEs define a subdynamics in the \( s \)-particle phase space and contains no inhomogeneous initial correlations terms as compared to the conventional GMEs. No approximation like “molecular chaos” or Bogoliubov’s principle of weakening of initial correlations is needed. The initial correlations are “hidden” in the projection operator and thus they are accounted for in the obtained equations. For the weak interparticle interaction and weak external field, these equations are rewritten in the second order of the perturbation theory. Essentially, that they contain the contribution of initial correlations in the kernel governing the evolution of an \( s \)-particle distribution function. In particular, the evolution equation for a one-particle distribution function is obtained and its connection to the nonlinear Landau and the Fokker-Planck equations is discussed. The obtained results are related to the general issues of statistical physics and to the physical applications (plasma physics).

1 Introduction
One of the basic tasks of the nonequilibrium statistical physics remains the deriving of the appropriate evolution equations for the measurable values (statistical expectations) characterizing a nonequilibrium state of a many-particle system.
It is expected that these equations should generally be evolution equations converting into kinetic (or other irreversible) equations after taking some scaling limits. The principal question is: How to derive such equations rigorously from underlying microscopic reversible classical or quantum dynamical equations? Several approaches are usually used to address this problem commonly starting with the Liouville equation for a distribution function (classical mechanics) or the von-Neumann equation for a statistical operator (quantum mechanics) for the N-particle \((N \gg 1)\) system under consideration. The problem is that it is impossible to resolve these equations due to a huge number of variables on which a distribution function (statistical operator) depends. Fortunately, we do not need such an excessive information, and a distribution function (statistical operator) for a much smaller \(s \ll N\) number of particles is necessary for calculation of the measurable values of interest. But due to the interparticle interaction, the strict derivation of the completely closed equations for these reduced distribution functions (marginals) is still a challenging problem. The approximations allowing to obtain closed evolution equations for marginals, such as the Bogoliubov principle of weakening of initial correlations [1] or RPA, actually imply selecting the specific uncorrelated (factorized) distribution function (statistical operator) as an initial state of the system, and then the problem of the "propagation of chaos" with time should be resolved. Strict justification of the propagation of "molecular chaos" turned out to be not an easy task. It was supposed to be true in the so-called "Boltzmann-Grad limit" [2, 3]. It is also worth noting that the nonlinearity of the kinetic equations, derived from the linear Liouville (or von-Neumann) equation, comes from the nonlinear (factorized) initial state [4], which is not a realistic one [5].

In the approach leading to the so called generalized master equations (GMEs), an \(N\)-particle distribution function \(F_N(x_1, \ldots, x_N; t)\) (we will focus on the classical physics case when \(x_i = (x_i, p_i)\) is the coordinate of the \(i\)-th particle in the phase space) is divided into the essential (relevant) \(\rho_r(x_1, \ldots, x_N; t)\) and inessential (irrelevant) \(\rho_i(x_1, \ldots, x_N; t)\) parts by using some projection operators \(P\) and \(Q = 1 - P\). Applying the projectors \(P\) and \(Q\) to the Liouville equation, one can obtain the time-convolution (non-Markovian) GME (TC-GME) [6], [7], [8] and time-convolutionless (time-local) GME (TCL-GME) [9], [10], which are the exact equations for the relevant part of a distribution function usually selected as a part of a distribution function with no correlations between the selected (by \(P\)) subsystem and remaining part of the system. However, these equations are the inhomogeneous ones, i.e., include a source (irrelevant part) containing all many-particle correlations at the initial instant \(t_0\). In general, these initial correlations pose a problem to deal with, and Bogoliubov’s principle of weakening of initial correlations or simply the factorized initial conditions (RPA) are commonly used to exclude initial many-particle correlations (a source). These approximations result in an approximate closed linear homogeneous GMEs for the relevant part of the distribution function.

The GMEs approach is standard for the important case of a subsystem \(S\) of the total system interacting with the remaining part \(B\) of a full system in an equilibrium state (a thermal bath). In this case, the factorizing initial condition
for the distribution function $F_N(t)$ of the whole $N$-particle system

$$F_N(t_0) = F_S(t_0)F_B^{eq}$$  \hspace{1cm} (1)

is conveniently used ($F_S(t) = \int dx^B F_N(t)$ is a subsystem distribution function defined as the integrated over the thermal bath states the full $N$-particle distribution function, $F_B^{eq}$ is the bath equilibrium distribution function), and the standard projection operator for this case is defined as

$$P(...) = F_B^{eq} \int dx^B (...).$$  \hspace{1cm} (2)

On the other hand, the Prigogine school was developing an approach (see, e.g., [11]) based on the introducing of the projection operator selecting the kinetic part of a distribution function (statistical operator) of a many-particle system. The projector is defined in such a way that the evolution of this kinetic component is described by the closed homogeneous kinetic equation (with no irrelevant initial moment source term) and this process is termed as subdynamics. Such an approach is rather complicated, depends on several unproved conditions, and no specific projection operator of this type was identified. Moreover, the factorized initial state of the whole system was again used in order to obtain the nonlinear kinetic equation [11].

In this paper, we apply the GME approach to considering an evolution in time of a classical many-particle system driven by an external force from an initial equilibrium state of the whole system. We show, that there is a specific projection operator selecting the relevant part of the $N$-particle distribution function which is governed by the exact homogeneous closed linear GMEs. The initial correlations are comprised into the projection operator. These equations are equivalent to the closed evolution equations for the reduced $s$-particle ($s < N$) distribution function. For a small interparticle interaction, the projection operator can be expanded in the interaction series, and thus the corrections, given by initial correlations, can be found. Therefore, we provide the example of the existence of the operator exactly projecting the $N$-particle dynamics on the $s$-particle one and suggest the method of accounting for initial correlation. We apply this approach to the weakly interacting system of classical particles. In particular, the evolution equation for the one-particle distribution function accounting for initial correlations is obtained in the second order approximation in the interparticle interaction and for a weak external field. The connection of this equation to the nonlinear Landau equation and to the Fokker-Planck equation for the weakly interacting gas of classical particles in the space-homogeneous case is discussed. The obtained results seems important for the general problems of the statistical physics as well as for the physical applications, e.g., for the plasma physics.
2 General formalism

Let us consider the Liouville equation for a distribution function \(F_N(t, t_0)\) of \(N\) interacting classical particles

\[
\frac{\partial}{\partial t} F_N(t, t_0) = -L(t) F_N(t, t_0),
\]

\[
\int ... \int dx^N F_N(t, t_0) = 1, \quad dx^N = dx_1...dx_N,
\]

(3)

where \(F_N(t, t_0) = F_N(x_1,...,x_N; t, t_0)\) is a function of \(N\) variables \(x_i = (r_i, p_i)\) (\(i = 1,...,N\)) representing the coordinates and momentum of the particles, \(L(t)\) is the Liouville operator acting on \(F_N(t, t_0)\) as

\[
L(t) F_N(t, t_0) = \{H(t), F_N(t, t_0)\}_P = \sum_{i=1}^{N} \left( \frac{\partial F_N(t, t_0)}{\partial r_i} \frac{\partial H(t)}{\partial p_i} - \frac{\partial F_N(t, t_0)}{\partial p_i} \frac{\partial H(t)}{\partial r_i} \right).
\]

(4)

Here, \(\{H, F_N(t, t_0)\}_P\) is the Poisson bracket and \(H(t)\) is the Hamilton function for the system under consideration generally dependent on time.

The formal solution to Eq. (3) is

\[
F_N(t, t_0) = U(t, t_0) F_N(t_0, t_0),
\]

(5)

where the evolution operator \(U(t, t_0)\) is defined as

\[
U(t, t_0) = \exp[- \int_{t_0}^{t} d\xi L(\xi)],
\]

\[
U(t_0, t_0) = 1.
\]

(6)

It is practically impossible to solve Eq. (3) for a many-particle system. Fortunately, however, in order to calculate the measurable values (statistical expectations) of interest, one usually only needs to know the reduced distribution functions (marginals) \(F_s(t, t_0) = F_s(x_1,...,x_s; t, t_0)\) dependent on much smaller number of variables \(s << N\). In general, the \(s\)-particle (\(s \leq N\)) distribution function is defined as

\[
F_s(t, t_0) = V^s \int ... \int dx^{N-s} F_N(t, t_0), \quad dx^{N-s} = dx_{s+1}...dx_N,
\]

(7)

where \(V\) is the volume of a system. From (3), we have the normalization condition for the reduced distribution functions \(F_s\)

\[
\int ... \int dx^s F_s(t, t_0) = V^s, \quad dx^s = dx_1...dx_s.
\]

(8)

Thus, an average value of a function of the dynamic variables of the group of \(s\) particle is defined by the reduced distribution function \(F_s\) as

\[
< A_s >_{t, t_0} = \int ... \int dx^N A_s F_N(t, t_0) = \int ... \int dx^s A_s \frac{1}{V^s} F_s(t, t_0).
\]

(9)
In order to obtain equations for the reduced distribution function, it is convenient to employ the standard projection operator technique and to break \( F_N(t, t_0) \) into the relevant \( \rho_s^r(t, t_0) \) and irrelevant \( \rho_i(t, t_0) \) parts

\[
F_N(t, t_0) = \rho_s^r(t, t_0) + \rho_i(t, t_0),
\]

\[
\rho_s^r(t, t_0) = P F_N(t, t_0), \quad \rho_i(t, t_0) = Q F_N(t, t_0) = F_N(t, t_0) - \rho_s^r(t, t_0) \tag{10}
\]

with the help of some projection operators \( P \) and \( Q = 1 - P \) \( (P^2 = P, Q^2 = Q, P + Q = 1, P Q = 0) \). We note, that the relevant and irrelevant parts depend on coordinates and momenta of all \( N \) particles in contrast to the reduced distribution functions (like \( F_s(t, t_0) \)). The relevant part \( \rho_s^r(t, t_0) \) is conveniently defined in such a way that it is related to the reduced distribution function of interest \( F_s(t, t_0) \), i.e., the projection operator selects a relevant (“vacuum”) part of \( F_N(t, t_0) \), which contains no correlation between a subsystem (a group of \( s \) particles) and an environment (remaining \( N - s \) particles), and actually describes the evolution of \( F_s(t, t_0) \). Then, the irrelevant part of the distribution function \( \rho_i(t, t_0) \) contains all correlations between a subsystem and an environment.

Applying the projection operators \( P \) and \( Q \) to Eq. (3), it is easy to obtain the equations for the relevant and irrelevant parts of \( F_N(t) \)

\[
\frac{\partial}{\partial t} \rho_s^r(t) = -P L(t)[\rho_s^r(t) + \rho_i(t)],
\]

\[
\frac{\partial}{\partial t} \rho_i(t) = -Q L(t)[\rho_s^r(t) + \rho_i(t)] \tag{11}
\]

(from now on we put \( t_0 = 0 \)). A formal solution to the second Eq. (11) has the form

\[
\rho_i(t) = -\int_0^t U_Q(t, \tau) Q L(\tau) \rho_s^r(\tau) d\tau + U_Q(t, 0) \rho_i(0),
\]

\[
U_Q(t, \tau) = \exp[-\int_\tau^t d\xi Q L(\xi) Q],
\]

\[
\rho_i(0) = F_N(0) - P F_N(0), \quad F_N(0) = F_N(0, 0). \tag{12}
\]

Inserting this solution into the first Eq. (11), we obtain the conventional exact time-convolution generalized master equation (TC-GME) known as the Nakajima-Zwanzig equation for the relevant part of the distribution function

\[
\frac{\partial}{\partial t} \rho_s^r(t) = -P L(t) \rho_s^r(t) + \int_0^t P L(t) U_Q(t, \tau) Q L(\tau) \rho_s^r(\tau) d\tau
\]

\[
- P L(t) U_Q(t, 0) \rho_i(0). \tag{13}
\]

This equation is quite general and valid for any initial distribution function \( F_N(0) \). Serving as a basis for many applications, Eq. (13), nevertheless, contains
the undesirable and in general non-negligible inhomogeneous term (the last term in the right hand side of (13), which depends via \( \rho_i(0) \) on the same large number of variables as the distribution function at the initial instant \( F_N(0) \) and includes all initial correlations. Therefore, Eq. (13) does not provide for a complete reduced description of a multiparticle system in terms of the relevant (reduced) distribution function. Applying Bogoliubov’s principle of weakening of initial correlations (allowing to eliminate the influence of \( \rho_i(0) \) on the large enough time scale \( t \gg t_{cor} \)) or using a factorized initial condition (see (1)), when \( \rho_i(0) = QF_N(0) = 0 \) (i.e., e.g., \( F_N(0) = \rho_s^i(0) \)), one can achieve the above-mentioned goal and obtain the homogeneous GME for \( \rho_s^i(t) \), i.e. Eq. (13) with no initial condition term. However, obtained in such a way homogeneous GME is either approximate and valid only on a large enough time scale (when all initial correlations vanish) or applicable only for a rather artificial (actually unreal, as pointed in [5]) initial conditions (no correlations at an initial instant of time). In addition, Eq. (13) poses the problem to deal with due to its time-nonlocality. However, it is possible to obtain the time-local equation for the relevant part of the subsystem distribution function [12, 9, 10] (see also below).

Then, the interesting question arises: Is it possible to obtain exact and completely closed (homogeneous) GME, i.e. the equation with no inhomogeneous initial correlations term? It means that the initial correlations would be contained in the kernel governing the evolution of the relevant part of the distribution function. As we already mentioned in the Introduction, there are the ways to include initial correlations into consideration on an equal footing with collisions (see [13, 14, 15, 16]. The initial equilibrium state for the whole system provides the new opportunities for obtaining the homogeneous GMEs. It was demonstrated in the recent works [17, 18], where the quantum case of the subsystem (like an electron) interacting with a heat bath (like the equilibrium boson field) was considered and the projection operator \( P \) is of a standard form (the quantum version of (2)) was employed.

3 Completely closed (homogeneous) equations for a reduced distribution function

One can consider the following problem: Is it possible to introduce such a projection operator which allows for converting the Liouville equation (3) into a completely closed (homogeneous) GME? If so, then this projection operator should comprise in some way the initial correlations. This idea is reminiscent of an approach, which had been developing by Prigogine with coworkers (see, e.g. [11]). They assert that it is possible to formally introduce some projection operator \( \Pi \) allowing for the exact dividing of the \( N \)-particle distribution function into the kinetic \( f_k(t) \) and nonkinetic parts (both include "vacuum" and correlated terms), and the "vacuum" part of the kinetic part \( Vf_k(t) \) (the projection operator \( V \) selects the "vacuum" part of \( f_k(t) \) with no correlations)
satisfies the completely closed (homogeneous) kinetic (irreversible) equation like Eq. (13) but with no source term (the third term on the r.h.s. of (13)) and the infinite \( \infty \) upper limit of the integration over \( \tau \). Such an evolution of \( V f_k(t) \) was termed subdynamics. However, this approach is very formal, includes some not generally proven conditions, and contains no explicit form of the projection operators \( \Pi \) and \( V \) for an arbitrary \( N \)-particle system.

Now we will show for the \( N \)-particle classical system driven from an initial equilibrium state \( \rho_{eq} \) by an external force that there is an explicit projection operator enabling one to obtain homogeneous closed time-convolution and time-convolutionless (time local) generalized master equations for a relevant part of \( N \)-particle distribution function.

Thus, we now suppose that up to the moment of time \( t = 0 \) the system is in an equilibrium state with the Gibbs distribution function

\[
F_N(t \leq 0) = \rho_{eq} = Z^{-1} \exp(-\beta H), \beta = 1/k_B T, Z = \int ... \int dx^N \exp(-\beta H), \quad (14)
\]

but just after \( t = 0 \) (at \( t > 0 \)) an external force is applied to a system. For what follows, it is convenient to present the system’s Hamilton function as

\[
H(t) = H_s + H_\Sigma + \bar{H}_{s\Sigma} + H^F(t),
\]

\[
H^F(t) = 0, \quad t \leq 0, \quad (15)
\]

Here, we selected the group of \( s \) particles (described by \( H_s \)), which interacts (through \( \bar{H}_{s\Sigma} \)) with an environment \( \Sigma \) of other \( N - s \) particles (described by \( H_\Sigma \)), and \( H^F \) defines the influence of an external field.

Then, we the Liouville equation (3) can be written as

\[
\frac{\partial}{\partial t} F_N(t) = -L(t) F_N(t), \quad t > 0,
\]

\[
\frac{\partial}{\partial t} F_N(t) = 0, \quad t \leq 0, \quad (16)
\]

and the Liouville operator \( L(t) = L_s + L_\Sigma + \bar{L}_{s\Sigma} + L^F(t) \). The formal solution to Eq. (15) is

\[
F_N(t) = U(t) \rho_{eq}, \quad t \geq 0,
\]

\[
F_N(0) = \rho_{eq}, \quad (17)
\]

where the evolution operator \( U(t) \) is defined by \( \rho \) (at \( t_0 = 0 \)).

Conventionally, a natural choice for a projection operator in Eq. (13) is the operator of the type given by (2). For a system of classical particles under consideration, such projection operator can be defined as

\[
P(\ldots) = \rho_\Sigma \int ... \int dx^{N-s}(\ldots), Q(\ldots) = 1 - P(\ldots),
\]

\[
\rho_\Sigma = \frac{1}{Z_\Sigma} \exp(-\beta H_\Sigma), Z_\Sigma = \int ... \int dx^{N-s} \exp(-\beta H_\Sigma), dx^{N-s} = dx_{s+1}...dx_N, \quad (18)
\]
and the relevant and irrelevant parts of $F_N(t)$ are

$$\rho^r_s(t) = \rho_s \frac{1}{V_s} F_S(t),$$
$$\rho^i(t) = F_N(t) - \rho_s \frac{1}{V_s} F_S(t).$$

(19)

We see from Eqs. (13), (14), that for such a choice of the projector, $\rho_i(0) = \rho_{eq} - \rho_s \frac{1}{V_s} F_S(0) \neq 0$. It is also worth noting, that formal introducing of the distribution function $\rho_S$ in (18) does not generally mean that the environment of $N-s$ particles is in the equilibrium state.

Let us now introduce the following projection operators $P_s$ and $Q_s$

$$P_s(...) = \rho_s^s \int \ldots \int d^{N-s}(...) Q_s(...) = 1 - P_s(...),$$
$$\rho_s^s = \frac{1}{Z_s} \exp[-\beta(H_S + \tilde{H}_s)],$$
$$Z_s = \int \ldots \int d^{N-s} \exp[-\beta(H_S + \tilde{H}_s)].$$

(20)

It is not difficult to see that $P_s^2 = P_s$, $Q_s^2 = Q_s$, $P_s Q_s = 0$. Then, we can divide $F_N(t)$ into the relevant $f^r_s(t)$ and irrelevant $f^i(t)$ components as (compare with (19))

$$F_N(t) = f^r_s(t) + f^i(t)$$
$$f^r_s(t) = P_s F_N(t) = \rho_s^s \frac{1}{V_s} F_S(t),$$
$$f^i(t) = Q_s F_N(t) = F_N(t) - \rho_s^s \frac{1}{V_s} F_S(t)$$

(21)

Note, that in the case of the projector (20), we use the notations $f^r_s(t)$ and $f^i(t)$ for the relevant and irrelevant components, correspondingly, while for the conventional projector $P$ (18) we leave the notations $\rho^r_s(t)$, $\rho^i(t)$ and Eq. (13).

It is not difficult to see that the dynamics of the average value (9) is completely defined by the relevant part $f^r_s(t)$ of $F_N(t)$, i.e.,

$$< A_S > = \int \ldots \int d^s A_s \frac{1}{V_s} F_S(t) = \int \ldots \int d^N A_s f^r_s(t).$$

(22)

The projection operator $P_s$ (20) has an interesting property, namely,

$$P_s \rho_{eq} = \rho_{eq}, Q_s F_N(0) = 0.$$

(23)

### 3.1 Time-convolution homogeneous GME

Thus, by applying the introduced projection operators $P_s$ and $Q_s$ to the Liouville
equation (16), we arrive at the following homogeneous time-convolution GME (compare with (13))

\[
\frac{\partial}{\partial t} f^{s}_{r}(t) = -P^{s}_{L}(t)f^{s}_{r}(t) + \int_{0}^{t} P^{s}_{L}(t)U_{Q^{s}_{L}}(t, \tau)Q^{s}_{L}(\tau)f^{s}_{r}(\tau)d\tau,
\]

\[U_{Q^{s}_{L}}(t, \tau) = \exp\left[-\int_{\tau}^{t} d\xi Q^{s}_{L}(\xi)\right].\] (24)

Equation (24) shows, that in the considered case, the dynamics of the \(N\)-particle distribution function can be exactly projected on the dynamics within its relevant part subspace. It follows, that the evolution of the selected complex of \(s\) particles can be described by the linear equation in the subspace of the corresponding coordinates \(x_{i} = (r_{i}, p_{i})\) \((i = 1, ..., s)\) if we rewrite Eq. (24) as the equation for an \(s\)-particle distribution function

\[
\frac{\partial}{\partial t} F^{s}_{S}(t) = -\left[ \int ... \int dx^{N-s} L(t) \rho^{s}_{S} \right] F^{s}_{S}(t) + \left[ \int ... \int dx^{N-s} L(t) \int_{0}^{t} d\tau U_{Q_{s}}(t, \tau)Q_{s}L(\tau)\rho^{s}_{S} \right] F^{s}_{S}(\tau).
\] (25)

### 3.2 Time-convolutionless homogeneous GME

Generally, the evolution equation (24) poses some problem to deal with due to its time-nonlocality. It is possible, however, to obtain the exact homogeneous time-local equation for the relevant part of the distribution function. The idea is to take advantage of the evolution of the distribution function, defined by (17), which leads to the relation

\[
F^{N}_{N}(\tau) = U^{-1}(t, \tau)F^{N}_{N}(t),
\]

\[U^{-1}(t, \tau) = \exp\left[\int_{\tau}^{t} d\xi L(\xi)\right].\] (26)

Using (26) and the conventional projection operator (18), the well known time-convolutionless equation for a relevant distribution function \(\rho^{s}_{s}(t)\), which contains the undesirable inhomogeneous term \(\rho_{i}(0)\) comprising the initial correlations, can be obtained. (see [12, 9, 10]). We will show now, that the use of the projector (20) instead of (18) leads to the completely closed homogeneous time-convolutionless GME for the relevant part of the distribution function. We will briefly conduct the derivation which is a rather standard one. First, we apply the projector (20) to (26) and obtain

\[
f^{s}_{r}(\tau) = P^{s}_{r}U^{-1}(t, \tau)[f^{s}_{r}(t) + f_{i}(t)].\] (27)
We also have the equation for the irrelevant part \( f_i(t) \) (see (12) and (23))

\[
f_i(t) = - \int_0^t U_Q_s(t, \tau) Q_s L(\tau) f_r(\tau) d\tau,
\]

(28)

where \( U_Q_s(t, \tau) \) is given by (24). From two equations (27) and (28) one finds that

\[
f_i(t) = [1 - \alpha(t)]^{-1} \alpha(t) f_r(t),
\]

\[
\alpha(t) = - \int_0^t U_Q_s(t, \tau) Q_s L(\tau) P_s U^{-1}(t, \tau) d\tau.
\]

(29)

Substituting \( f_i(t) \) (29) into the projected by \( P_s \) equation (16)

\[
\frac{\partial}{\partial t} f_s(t) = - P_s L(t) [f_r(t) + f_i(t)],
\]

(30)

we finally obtain

\[
\frac{\partial}{\partial t} f_r(t) = - P_s L(t) [1 - \alpha(t)]^{-1} f_r(t).
\]

(31)

If it is possible to expand the operator \([1 - \alpha(t)]^{-1}\) into the series in \( \alpha(t) \), then the first two terms of this expansion results in the following time-local equation (compare with (24))

\[
\frac{\partial}{\partial t} f_r(t) = - P_s L(t) f_r(t) + P_s L(t) \int_0^t d\tau U_Q_s(t, \tau) Q_s L(\tau) P_s U^{-1}(t, \tau) f_r(t).
\]

(32)

Equations (24) and (31) present the main result of this section. They show that the projector (20) allows for selecting the relevant part \( f_r(t) \) of the multi-particle distribution function \( F_N(t) \) which satisfies the completely closed linear time-convolution and time-convolutionless equations when a system is driven from an initial equilibrium state \( (14) \) by an external force. They, in fact, describe the evolution of the \( s \)-particles marginals on the arbitrary timescale. Thus, one remains in the scope of the linear evolution given by the Liouville equation (3) but should pay for this simplification by accounting for initial correlations, which are conveniently ignored. It is also worth noting that the developed formalism only works in the framework of classical physics (when the terms of the Hamilton function (15) commutes with each other). For quantum physics a different approach is needed (see [17, 18]).
4 Simplified homogeneous GMEs

Let us specify the Hamilton function \( H(t) \) for the case of the identical particles with the two-body interparticle interaction \( V_{ij} \) as

\[
H(t) = H_S + H_\Sigma + \tilde{H}_\Sigma + H^F(t),
\]

\[
H_s = \sum_{i=1}^s \frac{p_i^2}{2m} + \sum_{1 \leq i < j \leq s} V_{ij}(|r_i - r_j|) + \langle H_\Sigma \rangle, \qquad H_\Sigma = \sum_{i=s+1}^N \frac{p_i^2}{2m} + \sum_{s+1 \leq i < j \leq N} V_{ij}(|r_i - r_j|),
\]

\[
\tilde{H}_\Sigma = H_\Sigma - \langle H_\Sigma \rangle, H_\Sigma = \sum_{i=1}^s \sum_{j=s+1}^N V_{ij}(|r_i - r_j|),
\]

\[
H^F(t) = \sum_{i=1}^N V_i(r_i, t), t > t_0, H^F(t) = 0, t \leq t_0. \tag{33}
\]

Here, for simplicity, we take an external force \( H^F(t) \) dependent only on \( r_i \), and, for convenience, introduce the energy of the mean field \( \langle H_\Sigma \rangle \) acting on the \( s \)-complex by the "equilibrium" environment

\[
\langle H_\Sigma \rangle = \int \ldots \int dx N^{-s} \rho_\Sigma H_s, \tag{34}
\]

where \( \rho_\Sigma \) is given by (18). Note, that \( \langle H_\Sigma \rangle \) depends only on the coordinates of \( s \) selected particles \( r_i \) \( (i = 1, \ldots, s) \). For a space-homogeneous case, this mean field does not depend on \( r_i \) \( (i = 1, \ldots, s) \).

The corresponding to (33) Liouville operator \( L(t) \) is

\[
L(t) = L_s + L_\Sigma + \tilde{L}_\Sigma + L^F(t),
\]

\[
L_s = \sum_{i=1}^s \left[ \mathbf{v}_i \nabla_i - (\nabla_i H_\Sigma) \right] \frac{\partial}{\partial p_i} - \sum_{1 \leq i < j \leq s} (\nabla_i V_{ij}) \cdot \left( \frac{\partial}{\partial p_i} - \frac{\partial}{\partial p_j} \right),
\]

\[
L_\Sigma = \sum_{i=s+1}^N \mathbf{v}_i \nabla_i - \sum_{s+1 \leq i < j \leq N} (\nabla_i V_{ij}) \cdot \left( \frac{\partial}{\partial p_i} - \frac{\partial}{\partial p_j} \right),
\]

\[
\tilde{L}_\Sigma = -\sum_{i=1}^s \sum_{j=s+1}^N (\nabla_i V_{ij}) \cdot \left( \frac{\partial}{\partial p_i} - \frac{\partial}{\partial p_j} \right) + \sum_{i=1}^s (\nabla_i < H_\Sigma \rangle) \frac{\partial}{\partial p_i},
\]

\[
L^F(t) = \sum_{i=1}^N L^F_i(t), L^F_i(t) = -[\nabla_i V_i(r_i, t)] \frac{\partial}{\partial p_i},
\]

\[
\mathbf{v}_i = p_i/m, \nabla_i = \frac{\partial}{\partial r_i}, V_{ij} = V_{ij}(|r_i - r_j|). \tag{35}
\]
Equation (24) and (31) can be rewritten and simplified if we take into account the following operator properties:

\[ P_s L^F(t) = P_s L^F(t) P_s = \sum_{i=1}^{s} L_i^F(t) P_s, \]

\[ P_s L^F(t) Q_s = 0, \quad Q_s L^F(t) P_s = \sum_{i=s+1}^{N} L_i^F(t) P_s, \]

\[ P_s L_s = P_s L_s P_s = \rho_s L_s \frac{\partial}{\partial \rho_s} \sum_{i=1}^{s} \langle F_i \rangle_s, \]

\[ Q_s L_s P_s = L_s P_s - \rho_s L_s \frac{\partial}{\partial \rho_s} \sum_{i=s+1}^{N} \langle F_i \rangle_s, \]

\[ P_s L_{s\Sigma} = 0, \quad Q_s L_{s\Sigma} P_s = 0, \quad Q_s L_{s\Sigma} = L_{s\Sigma} P_s, \]

\[ P_s \tilde{L}_{s\Sigma} P_s = \sum_{i=1}^{s} \langle F_i \rangle_{s\Sigma} - \langle F_i \rangle_{\Sigma} \frac{\partial}{\partial \rho_i} \rho_s, \]

where

\[ F_i = - \sum_{j=s+1}^{N} (\nabla_i V_{ij}), \quad \langle F_i \rangle_{s\Sigma} = \int \ldots \int dx^{N-s} (\rho_s \rho_{s\Sigma}), \]

i.e., \( F_i \) is the force acting on the \( i \)-particle (\( i = 1, \ldots, s \)) from the "environment" of \( N-s \) particles. Here and further on, we use, as usual, that all functions \( \Phi(x_1, \ldots, x_N; t) \), defined on the phase space, and their derivatives vanish at the boundaries of the configurational space and at \( p_i = \pm \infty \).

It is evident, that initial correlations are "hidden" in the projection operators \( P_s, \quad Q_s = 1 - P_s \). In the case of a small inter-particle interaction \( V_{ij} \), we can expand \( P_s \) into series in the corresponding small parameter. Thus, if we suppose that

\[ \langle p_i^2/2m \rangle \sim k_B T \gg V_{ij}, \]

i.e., the interparticle interaction is small as compared to the average particle kinetic energy. In particular, we assume, that \( \beta \tilde{H}_{s\Sigma} \) is proportional to a small parameter. Then, approximately, in the linear approximation in \( \beta \tilde{H}_{s\Sigma} \),

\[ P_s = P_s^1 = \frac{e^{-\beta \tilde{H}_{s\Sigma}}(1 - \beta \tilde{H}_{s\Sigma})}{\int \ldots \int dx^{N-s} e^{-\beta \tilde{H}_{s\Sigma}}(1 - \beta \tilde{H}_{s\Sigma}) \ldots \int dx^{N-s}} \]

\[ = (1 - \beta \tilde{H}_{s\Sigma}) \rho_i \int \ldots \int dx^{N-s} \]

\[ = P - \beta \tilde{H}_{s\Sigma} P, \]

\[ Q_s = Q_s^1 = Q + \beta \tilde{H}_{s\Sigma} P, \]

where \( P \) is the conventional projection operator. It is not difficult to verify that the operator \( P_s^1 \) is the projector also. The relevant function \( (21) \) is now given by
rewritten as the deviation of the projector on the r.h.s. of (43) represents the correction due to initial correlations (i.e., to terms which result from the term $-\sum_i$). We can see that the first two terms on the r.h.s. of (43) are the conventional one for (42), (43) and for the collision (the second on the r.h.s.) term in (24), where $P_1$.

For $P_1$, we are going to simplify Eq. (24) using the relations (41) and restricting ourselves up to the second order approximation in the interparticle interaction. The left-hand-side of Eq. (24) for projection operator $P_1$ turns to

$$\frac{\partial}{\partial t} f^s_1(t) = (1 - \beta \bar{H}_{s\Sigma}) \frac{1}{\rho_0} F_s(t)$$

where $\rho_0(t)$ is defined by (19). In this approximation, Eqs. (39) look like

$$P_s^1 L^F(t) = P_s^1 L^F(t) P_s^1 = (1 - \beta \bar{H}_{s\Sigma}) \sum_{i=1}^{s} L^F_i(t) P_s$$

$$P_s^1 L^F(t) Q_s^1 = 0, Q_s^1 L^F(t) P_s^1 = (1 - \beta \bar{H}_{s\Sigma}) \sum_{i=s+1}^{N} L^F_i(t) P_s$$

$$P_s^1 L_s = P_s^1 L_s P_s^1 = (1 - \beta \bar{H}_{s\Sigma}) L_s P_s, P_s^1 L_s Q_s^1 = 0, Q_s^1 L_s P_s^1 = \beta [\bar{H}_{s\Sigma}, L_s] P_s$$

$$P_s^1 L_{s\Sigma} = 0, P_s^1 L_{s\Sigma} Q_s^1 = 0, Q_s^1 L_{s\Sigma} P_s^1 = L_{s\Sigma} P_s^1 = -\beta L_{s\Sigma} \bar{H}_{s\Sigma} P_s$$

$$P_s^1 \bar{L}_{s\Sigma} P_s^1 = -(1 - \beta \bar{H}_{s\Sigma}) \sum_{i=1}^{s} <F_i \beta \bar{H}_{s\Sigma} > \Sigma \frac{\partial}{\partial P_i} P_s$$

Now, we are going to simplify Eq. (24) using the relations (41) and restricting ourselves up to the second order approximation in the interparticle interaction. The left-hand-side of Eq. (24) for projection operator $P_s^1$ turns to

$$\frac{\partial}{\partial t} f^s_1(t) = (1 - \beta \bar{H}_{s\Sigma}) \frac{\partial}{\partial t} \rho^s_1(t).$$

For $P_s = P_s^1$, the first term on the right-hand-side of (24) is as following

$$-P_s^1 L(t) f^s_1(t) = -(1 - \beta \bar{H}_{s\Sigma}) \sum_{i=1}^{s} L^F_i(t)$$

$$+ L_s - \sum_{i=1}^{s} <F_i \beta \bar{H}_{s\Sigma} > \Sigma \frac{\partial}{\partial P_i} \rho^s_1(t).$$

Taking into account that the factor $(1 - \beta \bar{H}_{s\Sigma})$ can be cancelled as the mutual one for (42), (13) and for the collision (the second on the r.h.s.) term in (24), we can see that the first two terms on the r.h.s. of (13) are the conventional terms which result from the term $-PL(t) \rho^s_1(t)$ of Eq. (13), and the third term on the r.h.s. of (13) represents the correction due to initial correlations (i.e., to the deviation of the projector $P_s^1$ from the conventional one $P$).

Employing Eqs. (11), the second on the r.h.s. of (24) collision term can be rewritten as

$$C(t) = \int_0^t P_s L(t) U_{Q_s}(t, \tau) Q_s L(\tau) f^s_1(\tau) d\tau = \int_0^t P_s^1 \bar{L}_{s\Sigma} U_{Q_s^1}(t, \tau) Q_s^1 [\bar{L}_{s\Sigma} + L^F(\tau)$$

$$+ L_s + L_{s\Sigma}] P_s^1 f^s_1(\tau) d\tau.$$
As we will see later, the third and fourth terms in the r.h.s. of (44) appear due to initial correlations, while the first and the second ones are the same as for Eq. (13) with $\rho(0) = 0$.

For simplicity, from now on, we will consider the case of a weak external field (linear response regime), i.e., only the terms of the order $(V^{ij})^m V_i$ ($m = 0, 1$) will be accounted for. Then, in the second order approximation for the collision integral in the weak interaction and for a weak external field, $U_{Q=1}(t, \tau)$ could be taken in the zero order approximation in $V^{ij}$

$$U_{Q=1}^0(t, \tau) = \exp[-(L^0_s + L^0_\Sigma)(t - \tau)],$$

$$L^0_s = \sum_{i=1}^s v_i \nabla_i, L^0_\Sigma = \sum_{i=s+1}^N v_i \nabla_i$$

(45)

taking into account that the other factors under the integral in (44) are already of the second order in the interaction. Then, for any function of the particles coordinates $\Phi(r_1, ..., r_N)$

$$\exp[-(L^0_s + L^0_\Sigma)\tau] \Phi(r_1, ..., r_N) = \Phi(r_1 - v_1 \tau, ..., r_N - v_N \tau).$$

(46)

To remain within the adopted accuracy, the distribution $\rho_\Sigma$ in (48) and (44) should be taken in the zero approximation in the interaction

$$\rho^0_\Sigma = \exp(-\beta \sum_{i=s+1}^N \frac{p_i^2}{2m}) \prod_{i=s+1}^N \exp(-\beta \frac{p_i^2}{2m}) = \frac{\prod_{i=s+1}^N \exp(-\beta \frac{p_i^2}{2m})}{\int \prod_{i=s+1}^N dp_i |f_s r(t)|d\tau}.$$  

(47)

The non-Markovian Eq. (24) can be turned to a time-local form if we take $f_\tau^s(\tau)$ in (44) in the zero order approximation

$$f_\tau^s(\tau) = e^{L^0_\tau(t-\tau)} f_\tau^s(t)$$

(48)
in order to remain in the adopted second order in interaction approximation. Now, after changing the variable $t - \tau \rightarrow \tau$ under the integral, the approximate collision term is

$$C(t) = \int_0^t P_s^1 \bar{L}_\Sigma e^{-(L^0_s + L^0_\Sigma)\tau} Q^1_s \bar{L}_\Sigma + L^F(t - \tau) + L_s + L_\Sigma] P_s^1 e^{L_\tau^0 f_\tau^s(t)}d\tau.$$  

(49)

In the same way we can simplify Eq. (32) in the case of a small interparticle interaction (see (38)) and a small external field by using the approximate value $P_s^1$ (49) for the projection operator $P_s$ and restricting ourselves to the second order in the interaction and to the terms of the order $(V^{ij})^m V_i$ ($m = 0, 1$). Using
we see that the l.h.s. and the first term on the r.h.s. of (32) are given by (42) and (43). From relations (41), (45) and (47), in the adopted approximation, the collision term (the second on the r.h.s. of (32)) reads as

\[ C(t) = \int_0^t d\tau P_s^1 \tilde{L}_s \Sigma e^{-(L_s^0 + L_s^0)\tau} Q_s^1 [\tilde{L}_s \Sigma + L^F(t - \tau) + L_s + L^\Sigma_s] P_s^1 e^{(L_s^0 + L_s^0)\tau} f_s^* (t). \]

(50)

where \( f_s^* (t) \) should be approximated as \( f_s^* (t) = \rho_s^0 \frac{1}{V} F_s (t) \). Now one can see that \( e^{L_s^0 \tau} f_s^* (t) = f_s^* (t) \), and (50) coincides with (49).

5 Equation for a one-particle distribution function

A one-particle distribution function \( F_1 (x_1, t) \) is often mainly of interest. This case is less involved, although the developed formalism can also be specified for two-, three-, and more-particle distribution functions. Let us write down the terms of Eq. (32), given by (42), (43) and (49) for \( s = 1 \). Omitting the common factor \( (1 - \beta \tilde{H}_1 \Sigma) \rho_1 \Sigma / V \) and using Eqs. (33) - (35), (39), and (45) - (47), we obtain

\[ L_1^F (t) \rho_1^1 (x_1, t) = -[\nabla_1 V_1 (r_1, t)] \frac{\partial}{\partial \rho_1} F_1 (x_1, t), \]

\[ L_1 \rho_1^1 (x_1, t) = [v_1 \nabla_1 - (\nabla_1 < H_1 \Sigma > \Sigma) \frac{\partial}{\partial \rho_1}] F_1 (x_1, t), \]

\[ < F_1 \beta \tilde{H}_1 \Sigma > \Sigma \frac{\partial}{\partial \rho_1} \rho_1^1 (x_1, t) = -\beta n \int dr_2 (\nabla_1 V_1) V_{12} \frac{\partial}{\partial \rho_1} F_1 (x_1, t), \]

(51)

where \( n = N / V \) (here, as usual, the limiting procedure, \( N \rightarrow \infty, V \rightarrow \infty \) with \( n \) remaining fixed is assumed). Here we also used that in the first approximation in \( V_{ij} \)

\[ < H_1 \Sigma > \Sigma = \prod_{i=2}^N \int dr_i \int d\rho_i^0 \sum_{j=2}^N V_{ij} \]

\[ = n \int dr_2 V_{12}. \]

(52)

The first term in the collision integral (50) for the case under consideration
(s = 1) is as following

\[
C_L(x_1, t) = \int_0^t d\tau P_0 \tilde{L}_s \Sigma U_{Q_1}(\tau) Q_1 \tilde{L}_s \Sigma P_0 e^{L_2^s \tau} f_r^1(t)
\]

\[
= n \int_0^t d\tau \int d\mathbf{p}_2 \partial_1 G_L(x_1, \mathbf{g}_{12}; \tau)(\partial_{12} + \frac{\tau}{m} \nabla_1)\rho_{\Sigma}^{(2)} F_1(x_1, t),
\]

(53)

where

\[
G_L(x_1, \mathbf{g}_{12}; \tau) = \int d\mathbf{r}_2 F_{12}(0) F_{12}(\tau),
\]

\[
F_{12}(\tau) = -\nabla_1 V(\mathbf{r}_1 - \mathbf{r}_2 - \mathbf{g}_{12}\tau), \mathbf{g}_{12} = \mathbf{v}_1 - \mathbf{v}_2,
\]

\[
\rho_{\Sigma}^{(2)} = \frac{\exp(-\frac{\beta \mathbf{p}_2^2}{2m})}{\int d\mathbf{p}_2 \exp(-\frac{\beta \mathbf{p}_2^2}{2m})}, \partial_1 = \frac{\partial}{\partial \mathbf{p}_1}, \partial_{12} = \frac{\partial}{\partial \mathbf{p}_1} - \frac{\partial}{\partial \mathbf{p}_2}.
\]

(54)

The second term in (50), related to the external field which acts on the particles of a reservoir and due to the interparticle interaction influences the targeted particle, looks as

\[
C_F(x_1, t) = n \int_0^t d\tau \int d\mathbf{p}_2 G_F(x_1, \mathbf{v}_2; \tau) \partial_2 \rho_{\Sigma}^{(2)} \partial_1 F_1(x_1, t),
\]

\[
G_F(x_1, \mathbf{v}_2; \tau) = \int d\mathbf{r}_2 F_{12}(0) F_{2}^F(\tau),
\]

\[
F_{2}^F(\tau) = -\nabla_2 V_2(\mathbf{r}_2 - \mathbf{v}_2\tau, t - \tau), \partial_2 = \frac{\partial}{\partial \mathbf{p}_2}.
\]

(55)

It is not difficult to see, that in the case of a homogeneous external force (e.g., \(V_i(\mathbf{r}_i, t) = -e \mathbf{E}(t) \mathbf{r}_i\)), \(F_{2}^F(\tau)\) does not depend on \(\mathbf{v}_2\) and, therefore, \(C_F(x_1, t) = 0\) (see definition for \(\rho_{\Sigma}^{(2)}\)).

The third term in (50), related to \(L_s\) \((s = 1)\), looks like

\[
C_1(x_1, t) = n \beta \int d\mathbf{p}_2 \partial_1 G_1(x_1, \mathbf{g}_{12}; t) \rho_{\Sigma}^{(2)} F_1(x_1, t),
\]

\[
G_1(x_1, \mathbf{g}_{12}; t) = - \int_0^t d\tau \int d\mathbf{r}_2 F_{12}(0)[\nabla_1 V(\mathbf{r}_1 - \mathbf{r}_2 - \mathbf{g}_{12}\tau)].
\]

(56)

In the space-homogeneous case, when \(F_1(x_1, t) = \varphi(\mathbf{p}_1, t)\), \(G_1(x_1, \mathbf{g}_{12}; t) = \int_0^t d\tau G_L(x_1, \mathbf{g}_{12}; \tau)\) (the second term of the commutator in (47) \(V(\mathbf{r}_1 - \mathbf{r}_2 - \mathbf{g}_{12}\tau)\nabla_1 \varphi(\mathbf{p}_1, t) = 0\).
And the last term in the collision integral (50), stipulated by $L_{\Sigma}$, at $s = 1$ is

$$C_2(x_1, t) = -n\beta \int_0^t d\tau \int dp_2 \partial_1 v_2 G_L(x_1, g_{12}; \tau) \rho_{\Sigma}^{(2)} F_1(x_1, t),$$ \hspace{1cm} (57)$$

where we used that $\nabla_2 V_{12} = -\nabla_1 V_{12}$.

Collecting all above obtained results, we finally obtain from (50) the following equation for a one-particle distribution function in the second order approximation in the inter-particle interaction

$$\frac{\partial}{\partial t} F_1(x_1, t) = \left[ \nabla_1 U_1(r_1, t) + n(\nabla_1 \int dr_2 V_{12}) - \beta n \int dr_2 (\nabla_1 V_{12}) V_{12} \right] \frac{\partial}{\partial p_1} F_1(x_1, t)
$$

$$- v_1 \nabla_1 F_1(x_1, t) \right] \right]$$

$$\quad - v_1 \nabla_1 F_1(x_1, t) + [C_L(x_1, t) + C_F(x_1, t) + C_1(x_1, t) + C_2(x_1, t)]$$

$$\quad - v_1 \nabla_1 F_1(x_1, t) \right] \right]$$

Equation (58) is the main result of this section. We stress again that it is a linear equation strictly obtained in the second order approximation in $V_{ij}$ and is valid for all timescales. The collision integral $C_L(x_1, t)$ coincides with the corresponding collision integral in the nonlinear equation for inhomogeneous system of classical particles (see [11]) if in the latter, the distribution function for the particle, with which the targeted particle collides, is replaced by the equilibrium distribution function for this particle $\rho_{\Sigma}^{(2)}$. However, for such a coincidence, the integral over $d\tau$ should be extended to infinity. It can be done, if the interaction is rather a short-range one and if for a timescale

$$t > t_{\text{cor}} \sim r_{\text{cor}} / v$$

the force acting on the particle vanishes ($F_{12}(t) = 0$), where $r_{\text{cor}}$ is a radius of the inter-particle interaction $V_{ij}$ and $v$ is the average particle velocity. The third term in the first line of (58) (proportional to $\beta$), as well as the collision integrals $C_1(x_1, t)$ and $C_2(x_1, t)$, appear due to the initial correlation contribution.

Equation (58) acquires more simple form in the space-homogeneous case when an external force is a coordinate-independent one and $F_1(x_1, t) = \varphi(p_1, t)$. In this case Eq. (58) reads as follows

$$\frac{\partial}{\partial t} \varphi(p_1, t) = \left[ \nabla_1 U_1(r_1, t) \frac{\partial}{\partial p_1} + n \int dp_2 \partial_1 g_{12} G_L(g_{12}, t) \rho_{\Sigma}^{(2)} \right] \varphi(p_1, t)
$$

$$+ n\beta \int dp_2 \partial_1 g_{12} G_L(g_{12}, t) \rho_{\Sigma}^{(2)} \varphi(p_1, t),
$$

$$\overline{G}_L(g_{12}, t) = \int_0^t d\tau \int dr |\nabla_1 V(r)| |\nabla_1 V(r - g_{12} \tau)|.$$ \hspace{1cm} (60)$$

Note, that Eqs. (58) and (60) are the reversible in time ones. They become irreversible in the scale limit $t \to \infty$ (when the latter exists), e.g., in the case of the rapid vanishing of the interparticle interaction.
It is interesting to compare Eq. (60) with the Landau irreversible in time nonlinear equation for a homogeneous gas of classical particles, which can be derived from the the Boltzmann equation for the weak inter-particle interaction (see, e.g. [11]). The collision integral in the linear Eq. (60) will coincide with the Landau collision integral only if we disregard the contribution of initial correlations (the third term on the r.h.s. of Eq. (60)) and in the Landau equation replace the distribution function for the second targeted particle \( \varphi(\mathbf{p}_2, t) \) by the equilibrium distribution function \( \rho^{(2)}_\Sigma \). However, this coincidence will be valid only on the timescale (59), when the integral in \( G_L(\mathbf{g}_{12}, t) \) can be extended to infinity. At the same time, the linear collision integral of Eq. (60) at \( t \to \infty \) can be rewritten in the form coincided with that of the Fokker-Planck equation (see [11]). We would like to stress again, that linear Eqs. (58) and (60) are obtained strictly in the second order of the perturbation theory from the homogeneous exact GMEs (24) and (31), and that they are valid on the arbitrary timescale.

6 Conclusion

For a system of \( N \) \( (N >> 1) \) classical particles driven from the initial equilibrium state by an external field, the exact new time-convolution and time-convolutionless homogeneous (completely closed) linear GMEs for an \( s \)-particle \( (s < N) \) distribution functions valid on the arbitrary timescale have been obtained. It has become possible due to the special projection operator (20) which accounts for initial correlations in the initial equilibrium distribution function of the whole system. No assumption like ”molecular chaos” (factorized initial state) or Bogoliubov’s principle of weakening of initial correlations, which results in converting the linear Liouville equation into the nonlinear evolution equation, has been used. By expanding the kernels of the obtained equations in the perturbation series in the interparticle interaction, the corresponding equation in the second order approximation for a weak interaction and small external field is obtained. This equation is specified for a one-particle distribution function in the space-inhomogeneous (Eq. (58)) and space-homogeneous (Eq. (60)) cases. Both these equations contain contribution of initial correlations which have not been conveniently disregarded. The collision integral in (60) is the linear Landau (Fokker-Planck) collision integral on the large timescale.

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