Study of the Decoupling of Heavy Fermions in a $Z_2$ Scalar-Fermion Model

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Abstract

According to one-loop perturbation theory, fermions whose masses are totally generated from Yukawa couplings do not decouple in the heavy mass limit. We investigate this issue nonperturbatively in a 4-dimensional $Z_2$ scalar-fermion model with staggered fermions. Our data at intermediate and stronger Yukawa couplings on $8^4$ and $12^4$ lattices suggest the nondecoupling of heavy fermions as predicted from one-loop calculation. However, at the strongest Yukawa coupling where a possible multi-critical point may come into play, we cannot be conclusive.

1 Introduction

The decoupling theorem [1] says that when a particle has a mass much higher than the physical scale, it will have very small influence (e.g. radiative corrections) on the “physical world”. However, it can be easily seen at one-loop level that the contribution of the particle to renormalized quantities of other particles will not be suppressed by its huge mass if its mass is generated from the Yukawa coupling, i.e. if its mass is generated through the mechanism of spontaneous symmetry breaking (SSB). This is the so-called phenomenon of nondecoupling.

Nondecoupling of heavy fermions in theories with SSB has been discussed in several papers [2, 3, 4]. Until last year, all arguments were within (one-loop) perturbation theory (except in [4] where large-N expansion was used). Nondecoupling beyond one-loop was still not clear and should be explored.

Recently, we studied this issue in a nonperturbative way in a $U(1)$ scalar-fermion model with explicit mirror-fermions [5]. There, we ignore the gauge field assuming...
that this approximation will not change the picture qualitatively. Hence, the scalar
field is our “physical world”. The relevant quantity to measure to decide decoupling
or not is the ratio of the two renormalized Yukawa couplings: One is the usual
renormalized coupling defined as the renormalized (mirror-)fermion mass divided
by the renormalized vacuum expectation value (VEV); the other is obtained from
the fermion-fermion-scalar 3-point vertex function. In the weak coupling regime,
one-loop calculation shows that the ratio of the two is very close to one. As the bare
Yukawa coupling gets stronger, the renormalized Yukawa coupling defined from the
mass-to-VEV ratio is no longer a good definition for the renormalized coupling.
There, we need to define the renormalized coupling from the appropriate 3-point
vertex function. If the ratio of the two Yukawa couplings (denoted by $R$) still
remains 1.0, we take it as the indication for the nondecoupling of the heavy (mirror-
fermion).

Our main results in [5] are:

(i) Up to the strong Yukawa coupling regime, the ratio $R$ for the mirror-fermion is
always equal to 1.0 within the error, indicating that the heavy mirror-fermion does
have its renormalized Yukawa coupling proportional to the mass and will not decou-
ple from the transverse component of the scalar field. Thus one-loop picture survives
the strong Yukawa coupling limit. The idea of decoupling the mirror partners by
giving them large masses does not work.

(ii) Since the action of the model has a symmetry between fermion and mirror-
fermion, it is obvious that the heavy fermion itself will not decouple either.

(iii) Our data also show that all doublers decouple as expected.

(iv) We think that these conclusions also apply to the $SU(2)$ version of the mirror-
fermion model because its qualitative behaviour appears to be similar to that of the
$U(1)$ model [4].

(v) We conjecture that in other scalar-fermion models, heavy fermions do not de-
couple either.

In order to confirm our conjecture mentioned in (v), we carried out Monte
Carlo simulations on a 4-dimensional $Z_2$ scalar field theory coupled to the stag-
gerred fermion. We now report on our results in this letter.

2 Action and Renormalized Quantities

The lattice action of the 4-dimensional $Z_2$ scalar-fermion model with staggered
fermions is

$$S = -2\kappa \sum_x \sum_{\mu=1}^{4} \phi_x \phi_{x+\mu} + \lambda \sum_x (\phi_x^2 - 1)^2 + \sum_x \phi_x^2$$

$$+ \frac{1}{2} \sum_x \sum_{\mu=1}^{4} \tilde{\chi}_x \eta_{x,\mu} (\chi_{x+\mu} - \chi_{x\mu}) + S_{\text{int}}$$

where $\phi_x$ is the scalar field with only one real component, $\chi_x$ and $\tilde{\chi}_x$ are the one-
component staggered fermion fields, $\eta_{x,\mu}$ being the phase factor associated with
staggered fermions, $\eta_{x,1} = 1$ and $\eta_{x,\mu} = (-1)^{x_1 + \cdots + x_{\mu-1}}$ for $2 \leq \mu \leq 4$, $S_{\text{int}}$ represents the term for the Yukawa interaction. In this report, we study the model with the so-called “non-overlapping” Yukawa coupling introduced in [7]. Hence,

$$S_{\text{int}} = G \sum_x \bar{\chi}_x \Phi_x \chi_x , \quad \Phi_x \equiv \frac{1}{16} \sum_{x' \in \text{hypercube}(x)} \phi_{x'}$$

where the set of sites $x'$ is obtained by

$$x'_\mu = 2 \left[ \frac{x_\mu}{2} \right] + \delta_\mu , \quad \delta_\mu = 0, 1 ,$$

and $[.]$ denotes the largest integer part of the argument. The boundary condition is chosen such that it is periodic for the scalar field and the spatial directions of the fermion fields, and antiperiodic along the temporal (4th) direction of the fermion fields. Since each species of staggered fermions will generate four Dirac fermions in the continuum with opposite chiralities, the physical spectrum will be vector-like.

The above action has the following discrete symmetry:

$$\phi_x \to -\phi_x , \quad \chi_x \to i(-1)^{\sum_\mu x_\mu} \chi_x , \quad \bar{\chi}_x \to i(-1)^{\sum_\mu x_\mu} \bar{\chi}_x . \quad (2)$$

For the study of the decoupling of heavy fermions, the relevant phase (called the FM phase from now on) is the one where $Z_2$ symmetry in eq.(2) is spontaneously broken. Notice that we do not have the bare fermion mass term in the action. This means that the renormalized fermion mass will be totally generated by the Yukawa coupling in the FM phase. Definitions of various renormalized quantities there are as follows.

The renormalized scalar mass $m_R$ and the scalar wavefunction renormalization constant $Z_\phi$ are defined as

$$\frac{Z_\phi}{\hat{p}_4^2 + m_R^2} \equiv \lim_{p_4 \to p_{4\text{min}}} \langle \phi(p_4) \phi^*(p_4) \rangle_c$$

where subscript $c$ means the connected part of the propagator,

$$\tilde{\phi}(p_4) = \frac{1}{\sqrt{\Omega}} \sum_x e^{-ip_4 x_4} \phi_x , \quad \hat{p}_4^2 = 4 \sin^2 \left( \frac{p_4}{2} \right) ,$$

$\Omega$ is the total number of lattice sites, $x$ means $(\vec{x}, x_4)$, and $\tilde{\phi}^*(p_4)$ is the complex conjugation of $\tilde{\phi}(p_4)$. In our simulation on a finite $L^4$ lattice, we set $p_{4\text{min}} = 2\pi/L$. The renormalized VEV is defined as

$$v_R \equiv \left< |\phi| \right> , \quad |\phi| \equiv \frac{1}{\Omega} \sum_x \phi_x . \quad (4)$$

Similarly, the renormalized fermion mass $\mu_R$ and the fermion wavefunction renormalization constant $Z_F$ are defined as

$$\frac{Z_F}{\mu_R + i \sin(p_4)} \equiv \sum_y e^{-ip_4 y_4} \langle \chi_y \bar{\chi}_0 \rangle \equiv Z_F(\tilde{\Gamma}_R(p_4))^{-1}$$

(5)
where the spatial 3-momentum is set to zero, $p_4$ is chosen to be $\pi/L$ for the fermion fields, and $\tilde{\Gamma}_R(p_4)$ is the renormalized fermion 2-point vertex function in momentum space. The renormalized Yukawa coupling $G_R$ is defined as

$$G_R = \frac{\mu_R}{v_R}. \quad (6)$$

The relevant quantity to tell decoupling from nondecoupling of heavy fermions in this $Z_2$ scalar-fermion model of staggered fermions with nonoverlapping Yukawa coupling is still the ratio of the 3-point renormalized Yukawa coupling and the one defined in eq.(3). The 3-point renormalized Yukawa coupling coupled to the scalar field is defined as

$$G^{(3)}_R \delta_{k,-p+q} = -\frac{\hat{k}_4^2 + m_R^2}{Z_F \sqrt{Z_\phi}} \tilde{\Gamma}_R(p_4) \tilde{\Gamma}_R(q_4), \quad (7)$$

where $k_4, p_4, q_4$ are the 4th components of the momenta of scalar field, fermion and anti-fermion, respectively, and $\hat{k}_4^2$ is $4 \sin^2(k_4/2)$. We have set the spatial components of all momenta to zero. The appearance of the Kronecker-delta above is due to energy-momentum conservation. $G^{(c)}$ is the connected part of the $\phi$-\(\chi\)-\(\bar{\chi}\) 3-point Green’s function and is

$$G^{(c)} = \frac{1}{L^4} \sum_{x,y,z} e^{-ik_4 x_4} e^{-ip_4 y_4} e^{iq_4 z_4} \langle \phi_x \chi_y \bar{\chi}_z \rangle_c. \quad (8)$$

In our simulations on $L^4$ lattices we choose

$$k_4 = \frac{2\pi}{L}, \quad p_4 = -\frac{\pi}{L}, \quad q_4 = \frac{\pi}{L}.$$\\

The ratio $R$ is defined to be $R = G^{(3)}_R/G_R$.

Notice that SSB only occurs in the zero-momentum mode, the above connected 3-point Green’s function $G^{(c)}$ is equal to the disconnected one, because it is defined at $k = (0, k_4)$ with $k_4 \neq 0$. In the $Z_2$ scalar-fermion models, there are no massless Goldstone bosons in the FM phase since no continuous symmetry is spontaneously broken. We simply treat the scalar field itself as the “physical world” and assume that the issue of the decoupling of heavy fermions does not depend on whether the broken symmetry is discrete or continuous. Notice that reflection positivity for scalar-fermion models with staggered fermions cannot be proven. We assume that no ghost particles are present in the spectrum.

### 3 Phase Structure and Numerical Simulation

The phase structure of the model defined in eq.(4) at $\lambda = 0.01$ has been explored and was presented in figure 2 of [1]. At $G = 0$, the model reduces to a one-component pure scalar $\lambda \phi^4$ theory which has FM, symmetric (PM) phases and an anti-ferromagnetic (AFM) phase. At $\lambda = 0.01$, the transition point between FM and PM phases occurs around $\kappa = 0.120$ and is a Gaussian fixed point. Due to the symmetry: $\phi_x \rightarrow (-1)^x \phi_x, \kappa \rightarrow -1\kappa$, the transition between PM and AFM
phases occurs around $\kappa = -0.120$. As $G$ is gradually turned on, the FM-PM phase transition occurs at smaller and smaller (and eventually negative) $\kappa$ values because dynamical fermions favour the FM phase. This feature has been observed in all scalar-fermion models studied so far [4, 5]. As the value of $G$ is further increased, some new phase, like the ferrimagnetic (FI) phase, shows up and the phase structure will be (slightly) dependent on the model [5]. For the $Z_2$ scalar-fermion model investigated in this letter, the FI phase and a new PM phase were found at strong and infinite $G$ [7]. Around $G = 1.7$, there is a point (called point B in [7], and will be called as such from now on) around which four phases (FM, PM, AFM and FI) may coexist. We have explored the phase structure around that point B on $4^3 \cdot 8$ lattice, and find that point B is located around $G = 1.7$ and $\kappa = -0.125$, consistent with previous estimate [7, 9].

The physically interesting phase transition is the one between the FM and PM (called S1 in [7]) phases (curve AB in figure 2 of [7]). It was found to be consistent with a second-order phase transition on which the cutoff can be removed. We therefore carry out Monte Carlo simulations at $\lambda = 0.01$ in the FM phase, but in the vicinity of this FM-PM phase transition line.

In the simulations, besides $\lambda = 0.01$, we set $G$ to be 0.3, 0.6, 1.0, 1.2 and 1.8. The lattices we use are $8^4$ and $12^4$. We then tune the scalar mass parameter $\kappa$ to have $m_R$ around 1.0 on $8^4$ lattice to reduce finite size effects and at the same time maintain a reasonable cutoff.

In this $Z_2$ scalar-fermion model, no continuous symmetry is spontaneously broken in the FM phase, so there is no massless particle in the spectrum. Finite size effects should be small except on small lattices where vacuum tunnelings happen. It is well known that on a finite lattice, the ground state is nondegenerate. This means that there is no SSB on a finite lattice. The unique ground has a vanishing VEV and is a symmetric linear combination of two states, which are peaked at the positive and negative minima respectively. It is these two states that will converge to the two degenerate ground states in the thermodynamic limit [10]. Once tunnelings happen on a finite lattice, we will be in the nondegenerate ground state of the system. Finite size effects will be dominated by an instanton-like equation [10] and may not be small on small lattices. If the system is not too close to criticality such that it is trapped in one of the minima, then finite volume effects will be dominated by perturbative effects rather than tunneling. In the presence of dynamical fermions, the precision of our data will not be good enough. To analyze finite size effects dominated by tunnelings is therefore too demanding for the moment. So, when we study the properties of the FM phase of the model on a finite lattice and eventually extrapolate to get the infinite volume limit, we actually do not wish to see tunneling events.

Without loss of generality in our simulations, we always set up the initial condition such that the system starts in the positive minimum. We define the time-slice average of the scalar field as

$$\phi_s(t) \equiv \frac{1}{L^3} \sum_{\bar{x}} \phi_{\bar{x},t}.$$  

By observing values of those $\phi_s(t)$, we will know whether tunnelings happen or not.
The Monte Carlo simulations are performed by the unbiased Hybrid Monte Carlo method [11]. Therefore, the fermions have to be doubled by taking the adjoint of the fermion matrix for the second species. (The fermionic part in eq.(1) is given for a single species of staggered fermions.) We will have eight degenerate Dirac fermions in the continuum limit. The number of leapfrog steps per molecular dynamics trajectory was chosen randomly between 3 and 10. The step size was tuned to maintain an acceptance rate around 75%. The necessary inversions of the fermion matrix were done by the conjugate gradient iteration, until the residuum was smaller than some small value times the length square of the input vector. We find that this value has to be $10^{-12}$ on the $8^4$ and $12^4$ lattices. We use Creutz observable

$$\exp(-\delta H)$$



to decide whether the system has equilibrated or not [12] where $\delta H$ is the difference between the new and old Hamiltonians in the Hybrid Monte Carlo update. In equilibrium, we should have

$$\langle e^{-\delta H} \rangle = 1.$$  

### 4 Conclusion and Discussion

Our numerical data are presented in table 1. Our data on the renormalized fermion mass and bare VEV at $G = 1.2$ and 1.8 agree with those published in [7]. We also find that the renormalized fermion mass has the mean field behaviour $\mu_R = G \langle |\phi| \rangle$. The renormalized fermion mass presented here can be as high as 700 to 800 GeV at strong Yukawa couplings. (The physical scale is set by $v_R = 246$ GeV.) At all points where we did simulations, no vacuum tunneling events were observed. We think this is partly due to the fact that dynamical fermions tend to increase the height of the barrier between the two minima. It is also because Hybrid Monte Carlo is a local updating algorithm. (In the presence of dynamical fermions, we cannot use cluster algorithms to perform global updates.) Thus it is not easy for the system to tunnel even on an $8^3$ spatial lattice.

According to table 1, data on $R$ from $G = 0.1$ to 1.2 are all consistent with 1.0 within errors, indicating a universal behaviour of the nondecoupling of heavy fermions as expected.

However, at $G = 1.8$, $\kappa = -0.04$, data on $R$ on $8^4$ lattice (i.e. point g in table 1) is clearly smaller than 1.0 by several standard deviations, while data on $12^4$ lattice are too noisy to tell. Therefore, we cannot be certain whether heavy fermions are still coupled to the scalar field at $G = 1.8$. At the moment, we would like to say that there could be a possibility that heavy fermions do decouple as the decoupling theorem says, although their masses are totally generated from Yukawa coupling. (In the action of our model, we do not have a bare fermion mass term.) This possibility exists because points f, F, g, G in table 1 are lying slightly to the right of point B and may very well be affected by it. Since point B might be a multi-critical point and may have, in principle, renormalization properties different from those dictated by the Gaussian fixed point at $G = 0$, it should not be too surprising that heavy fermions decouple in the vicinity of point B. Besides, if we stay at $G = 1.8$ and keep reducing the value of $\kappa$, we will be approaching the phase transition line between the FM and FI phases. This FM-FI transition line is the place where bare VEV is still nonzero. Thus, it should not be a physically relevant phase transition line.
At this point, we would like to conclude that at least along the phase transition line between the FM and PM phases from $G = 0.1$ to $G = 1.2$ where the system is still governed by the infrared stable Gaussian fixed point at $G = 0$, heavy fermions whose masses are totally generated by the Yukawa coupling do not decouple. One-loop picture is qualitatively correct throughout this region. Although data presented in this letter are at $\lambda = 0.01$, we believe that the above conclusion holds at all values of $\lambda$ between zero and infinity. As we go to even stronger Yukawa coupling where a possible multi-critical point may come into play, the decoupling of heavy fermions remains a possibility. However, our present data cannot give a conclusive signal for decoupling. Whether heavy fermions really decouple or not there depends on the properties of that possible multi-critical point. Due to limited computer resources, this issue will be left for the future study.

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Table 1: Our Monte Carlo data are presented here. Points with lower-case letters are obtained on $8^4$ lattice while points E and F are on $12^4$ lattice. Each point has about 30000 molecular dynamics trajectories for equilibration, and around 100000 to 300000 for measurements. But point G has only 30000 trajectories for measurements.

|   | $G$ | $\kappa$ | $v_R$  | $m_R$  | $\mu_R$ | $G_R^2$ | $G_R^{(3)}$ | $R$   |
|---|-----|---------|--------|--------|---------|---------|------------|-----|
| a | 0.1 | 0.129   | 0.600(3) | 0.578(8) | 0.115(1) | 0.191(2) | 0.205(43) | 1.07(9) |
| b | 0.3 | 0.128   | 0.942(3) | 0.928(11) | 0.525(2) | 0.557(2) | 0.546(60) | 0.98(7) |
| c | 0.6 | 0.110   | 0.654(2) | 0.90(1) | 0.709(4) | 1.083(4) | 1.10(10) | 1.02(8) |
| d | 1.0 | 0.080   | 0.496(2) | 1.27(3) | 0.876(2) | 1.766(6) | 1.88(17) | 1.06(9) |
| e | 1.2 | 0.060   | 0.437(2) | 1.45(3) | 0.907(2) | 2.08(2) | 2.07(21) | 0.99(9) |
| E | 1.2 | 0.060   | 0.430(2) | 1.34(14) | 0.906(3) | 2.11(2) | 2.17(66) | 1.03(20) |
| f | 1.8 | 0.00    | 0.362(2) | 2.29(6) | 1.142(2) | 3.15(4) | 2.97(44) | 0.94(5) |
| F | 1.8 | 0.00    | 0.368(4) | 2.21(54) | 1.140(2) | 3.10(8) | 2.2(1.2) | 0.71(40) |
| g | 1.8 | -0.04   | 0.296(1) | 1.37(3) | 0.766(2) | 2.59(2) | 2.15(24) | 0.83(5) |
| G | 1.8 | -0.04   | 0.275(7) | 1.47(36) | 0.768(7) | 2.79(10) | 2.9(1.4) | 1.03(46) |