A Hierarchical Game with Strategy Evolution for Mobile Sponsored Content/Service Markets

Wenbo Wang, Zehui Xiong, Dusit Niyato and Ping Wang
School of Computer Engineering, Nanyang Technological University, Singapore 639798

Abstract—The sponsored content/service market is an emerging platform, where the Content/Service Providers (CSPs) pay the Mobile Network Operator (MNO) and subsidize the Mobile Users (MUs) to access their services at a lower price. The sponsoring mechanism leads to a surge in mobile data and service demand, which in return compensates for the sponsoring cost and benefits the CSPs. In this paper, we study the interactions among the three entities in the market, namely, the MUs, the CSPs and the MNO, as a two-level hierarchical game. Our study is featured by the consideration of global network effects owning to consumers’ grouping. We model the service-selection process among the MUs as an evolutionary population sub-game, and the sponsoring-pricing process between the CSPs and the MNO as a non-cooperative sub-game. By investigating the structure of the proposed game, we discover a few important properties regarding the existence of the hierarchical equilibrium, and propose a distributed, projection-based algorithm for iterative equilibrium searching. Simulation results validate the convergence property of the proposed algorithm, and demonstrate how sponsoring helps to improve both the CSPs’ profits and the MUs’ experience.

Index Terms—Sponsored service market, global network effects, evolutionary game, hierarchical game, variational inequalities.

I. INTRODUCTION

The recent few years has witnessed an explosion in the number of mobile applications and daily active Mobile Users (MUs), with which a high data-density market grows at about 38% per year [1]. In 2014, AT&T worked with market portal companies like Aquto and launched a billing platform to allow bill transfer from MUs to their third-party Content/Service Providers (CSPs). Since then, the sponsored content/service market has developed rapidly with more and more companies identifying the business potential. For example, Singtel, in collaboration with Over-The-Top (OTT) communication CSPs (e.g., WeChat), has introduced the sponsored data plans of unlimited OTT service usage at a fixed rate in Singapore. This new market structure encourages the consumers to have a deeper engagement with the CSPs. This is because of not only the lower data/subscription price due to sponsoring, but also the beneficiary global network effect of the services that the CSPs provide. Meanwhile, the CSPs gain more profit from more active use by the MUs, which in return compensates for the sponsorship cost. Further, the Mobile Network Operator (MNO) also obtains more revenue by charging both the CSPs and MUs (Figure 1). Intuitively, such market mechanism promises a win-win situation for all the parties. However, the complexity of analyzing the interactions among the three parties in the market becomes a significant challenge, especially when they seek the optimal market strategies non-cooperatively.

Motivated by this challenge, many studies have attempted to provide insight into the dynamics of the sponsored content and service market [2]–[4]. Therein, the issues of optimal sponsoring strategy searching and its impact on the CSPs and MUs are studied. However, the impact of sponsorship on a large population of users and the influence of global network effect gained from the scale of users generally remain an open issue. In this paper, we study a transparent market with a single MNO, multiple CSPs and a large population of MUs. Our study emphasizes the investigation on the strategy evolution of the user population and the dynamics of CSP sponsoring in the framework of a hierarchical market, with the consideration of both the global network effects and the congestion effect experienced by the users. We propose a two-level hierarchical game-based framework to model the interactions among the different entities in the network. Based on our discoveries on the properties of the Stackelberg Equilibrium (SE) of the two-level game, we cast the SE searching process into a bi-level programming problem, and propose a distributed strategy searching scheme that ensures the convergence to the SE.

II. MARKET MODEL: A STACKELBERG INTERPRETATION

A. System model

We consider a sponsored content/service market (Figure 1) where N MUs are offered with the OTT services by M competing CSPs. The data services connecting the MUs and the CSPs are provided by a monopoly MNO. The MNO determines the unit price of the capacity that each CSP purchases for its operation, and charges each MU for a one-time subscription fee for getting mobile data access to a selected CSP. A CSP chooses independently the level of subsidy for its customers in order to gain more market shares. Each MU makes its decisions independently to select one CSP given the Quality of Experience (QoE) achieved with each CSP and the subscription fee charged by the MNO after discounting the subsidies.

Fig. 1. A schematic example of the sponsored service markets, where all the payments are made to the network operator.
We suppose that an MU perceives the QoE of a service based on two factors: the accessible capacity, i.e., the internal effect, and the social gains that it enjoys owing to the social popularity of the subscribed service, i.e., the network effect. Specifically, a user’s payoff of choosing CSP $j$ can be defined as:

$$
\pi_j^u = u_1(n_j) + u_2(n_j) - (1 - \theta_j)p_u,
$$

where $\theta_j \in [0, 1]$ is the fraction of the subscription fee sponsored by CSP $j$, $p_u$ is the one-time subscription price charged by the MNO, $n_j$ is the total number of MUs subscribing to CSP $j$, $u_1(n_j)$ is a capacity-related QoE function representing the user’s satisfaction level and $u_2(n_j)$ measures the social benefits due to the network effect. We consider that the capacity acquired by CSP $j$ is fairly distributed among its users as $(b_j + b_j)/(o_j + n_j)$, where $o_j \geq 1$ is a constant representing the maintenance overhead, $b_j$ is the basic capacity that CSP $j$ obtains when entering the market and $b_j$ is the extra capacity that SP $j$ purchases from the MNO. We adopt a logarithmic utility function [5] with decreasing marginal return for $u_1(n_j)$:

$$
u_1(n_j) = \left\{
\begin{array}{ll}
\gamma_1 \log \left( \frac{b_j + b_j}{o_j + n_j} \right), & \text{if } n_j > 0, \\
0, & \text{if } n_j = 0,
\end{array}
\right.
$$

where $\gamma_1 > 0$ is the congestion/QoE sensitivity coefficient and $u_1(n_j)$ is piecewise continuous.

Practically, the social benefit (network externalities) of the OTT services, $u_2(n_j)$, is usually affected by the activity of the entire user population. In other words, the increase of users will bring about more value that MUs obtain from a service, and an MU’s local perception of the service’s utility is only driven by the scale of users. Such a phenomenon is known as the global network effect. In comparison to the congestion effect (c.f., (2)), it is usually modeled as a concave function of the user number $n_j$. We adopt the logarithmic model for $u_2(n_j)$:

$$
u_2(n_j) = \gamma_2 \log(1 + n_j),
$$

where $\gamma_2 > 0$ is the network effect sensitivity factor.

On the other hand, a CSP $j$ ($1 \leq j \leq M$) aims to maximize its profit by choosing the sponsorship level $\theta_j \in [0, 1]$ as well as negotiating with the MNO about the demands of additional capacity $b_j$ ($b_j \geq 0$). Note that both the sponsorship level and the capacity provision will influence the number of subscribers, and the revenue of a CSP also possesses the property of decreasing marginal return [6]. Then, we can model CSP $j$’s utility as:

$$
\pi_j^c = \sigma_j \log(1 + n_j) - p_c \theta_j n_j - p_c b_j,
$$

where $\sigma_j$ is the monetary worth of the total active subscriptions for CSP $j$, and $p_c$ is the contract price for capacity $b_j$ charged by the MNO. We consider that each CSP has a budget limit on the total capacity and sponsoring cost as $p_c \theta_j n_j + p_c b_j \leq \tau_j$.

Further, we consider that the MNO applies a uniform price $p_u$ for user subscriptions and an anonymous price $p_c$ for capacity provision to the CSPs. Then, the MNO’s revenue is:

$$
\pi^o = p_c \sum_{j=1}^{M} b_j + p_u \sum_{j=1}^{M} n_j.
$$

### B. Hierarchical Game Formulation

Considering the utilities given in (1)-(5), it is natural to cast the market dynamics described in Section II-A into two stages. In the first stage, the MNO and the CSPs negotiate over the prices $p_c$, $p_u$ and capacity $b_j$, and the CSPs choose the sponsorship level $\theta_j$. In the second stage, each MU determines one CSP to subscribe to, based on the QoE associated with the CSPs. Thereby, we can model the sponsored service market as a two-stage, multi-leader-multi-follower hierarchical game. More specifically, the MNO and $M$ CSPs form a non-cooperative multi-player leader sub-game for price and sponsorship selection, and a large number of MUs form an evolutionary sub-game for CSP subscription. The hierarchical game can be mathematically defined by the following two sub-games:

1) **User-level evolutionary game**: Given a fixed subsidy level $\theta = [\theta_1, \ldots, \theta_M]^T$, capacity vector $b = [b_1, \ldots, b_M]^T$ and price $p_u$, the user-level evolutionary CSP-selection game is defined by a four-tuple: $G_j = (\mathcal{N}^j, M, x, [\pi_j^u(x)]^M)$, where

- $\mathcal{N}^j$ is the set of active MUs with the cardinality $|\mathcal{N}^j| = \mathcal{N}$.
- $M = \{0, 1, \ldots, M\}$ is the set of strategies, where strategy $m = 0$ corresponds to the MUs’ action of subscribing to no CSP and $m \neq 0$ corresponds to the user action of subscribing to CSP $m$. $|\mathcal{M}| = 1 + M$.
- $x = [x_0, x_1, \ldots, x_M]^T$ is the vector of population states, where $x_0$ is the proportion of the user population choosing CSP $j$ ($n_j = x_j N$), and $x$ is defined in the $M$-simplex $\mathcal{X} = \{[x_0, x_1, \ldots, x_M]^T \in \mathbb{R}^{M+1} | \sum_{j=0}^M x_j = 1, x_j \geq 0, \forall j\}$.
- $[\pi_j^u(x)]^M_{j=0}$ is the vector of MUs’ payoffs in population state $x$. $\forall j \neq 0, \pi_j^u(x) = 0$ is given by (1) and $\pi_0^u(x) = 0$.

2) **Provider-level non-cooperative game**: Given the user subscriptions, the MNO and the CSPs form a non-cooperative game as defined by a three-tuple $G_k = (\mathcal{K}, A, \pi)$, where

- $\mathcal{K} = \{0, 1, \ldots, M\}$ is the set of players, where $k = 0$ represents the MNO and $k \neq 0$ represents CSP $k$.
- $A = \times_{k=0}^{M} \mathcal{A}_k$ is the Cartesian product of the action spaces for player $k \in \mathcal{K}$, $\mathcal{A}_0 = \{(p_u, p_c) | p_u > 0, p_c > 0\}$ and $\mathcal{A}_k = \{[\theta_k, b_k] | \theta_k \in [0, 1], p_c \theta_k x_k N + p_c b_k - \tau_k \leq 0\}$. We denote the vector of joint player actions as $a = [a_k]_{k=0}^{M} \in A$.
- $\pi = [\pi^u(a), \pi^c_1(a), \ldots, \pi^c_M(a)]^T$ is the vector of payoffs. The MNO’s payoff $\pi_0 = \pi^u(a)$ is given by (5), and the CSP $k$’s payoff ($k \neq 0$), $\pi_k = \pi^c_k(a)$, is given by (4).

We consider that an MU is able to perceive the other MUs’ payoff in the market through side information, and the CSPs and the MNO interact as rational, autonomous players to maximize their own utilities. Then, the SE is composed of the sub-game Nash Equilibria (NE) $x^*$ and $a^*$, where

- $x^*$ is an NE to $a^*$ in the user-level sub-game. Namely, $\forall x_j^* > 0, j \in \mathcal{M}$,

$$
\pi_j^u(x^*, a^*) \geq \pi_j^u(x, a^*), \forall i \neq j.
$$

- $a^*$ is the NE of the provider-level sub-game as well as the best response to $x^*$. $a^*$ is given by

$$
\begin{align}
\pi_j^c(a^*_j, x^*, a^*_{\neq j}) \geq & \pi_j^c(a_j, x^*, a^*_{\neq j}), \forall a_j \in \mathcal{A}_j \\
\pi_j^u(a^*_j, a^*_{\neq j}, x^*) \geq & \pi_j^u(a_j, a^*_{\neq j}, x^*), \forall a_j \in \mathcal{A}_j,
\end{align}
$$

where $\pi_j^c(a_j, x^*, a^*_{\neq j}) = \sigma_j \log(1 + x_j) - p_c \theta_j x_j - p_c b_j$ and $\pi_j^u(a_j, x^*, a^*_{\neq j}) = \log(1 + x_j)$.
where \(1 \leq j \leq M\), and \(a_{-j}\) represents the joint adversary actions of player \(i\), \(\forall i \in K\). Here, we abuse slightly the notations of \(\pi_j^u, \pi_j^h\) and \(\pi_j^\pi\) in (1), (4) and (5), respectively.

### III. Game Analysis and Stackelberg Equilibrium

#### A. Evolutionary Stable Strategies in the User-Level Game

With (1) and (6), we obtain the average payoff of all the MUs as \(\pi^u_j(x, a) = \sum_{j=0}^M x_j \pi_j^u(x, a)\). Then, by the pairwise proportional imitation protocol [7], the replicator dynamics yields the following Ordinary Differential Equations (ODEs) \(\forall j \in M\), which indicates that the subscription growth rate of a CSP is in proportion to the subscription’s excess payoff:

\[
\frac{dx_j}{dt} = v_j(x, a) = x_j(\pi_j^u(x, a) - \pi^h_j(x, a)),
\]

where we omit \(t\) in \(x_j(t)\) for simplicity. It is well-known by the folk theorem in the evolutionary game theory that with the replicator dynamics, state \(x^*\) is the NE of the user sub-game \(G_f\) if and only if \(x^*\) is a stable rest point of the ODEs [7]. Therefore, we are interested in identifying the stability property of \(G_f\). From (1) we note that when \(a\) is fixed, \(\pi_j^u(x, a)\) is only determined by the population state \(x_j\). Then, we can verify that \(G_f\) is an evolutionary population potential game [7].

**Definition 1.** Consider a generalized single-population game \(G = (\mathcal{N}, \mathcal{M}, \mathcal{X}, F(x) = [F_j(x)]_{j \in \mathcal{M}})\) with payoff function \(F(x) : \mathbb{R}^+ \to \mathbb{R}\). \(G\) is a full potential game if there exists a continuously differentiable function \(f(x) : \mathbb{R}^+ \to \mathbb{R}\) satisfying

\[
\frac{\partial f(x)}{\partial x_j} = F_j(x), \forall j \in \mathcal{M}.
\]

**Lemma 1.** Given any fixed action \(a\) by the MNO and the CPs, the user-level sub-game \(G_f\) is a population potential game.

**Proof.** From our observation in (1) that \(\pi_j^u(x, a)\) is only determined by the population state \(x_j\), we can easily derive

\[
\frac{\partial \pi_j^u(x, a)}{\partial x_i} = \frac{\partial \pi_j^u(x, a)}{\partial x_j} = 0, \forall j \neq i,
\]

which satisfies the property of full externality symmetry by [7]. Then, according to (20) in [7], \(G_f\) is a potential game. □

**Lemma 2.** Under the replicator dynamics given in (8), the population state \(x(t)\) always falls into the \(M\)-simplex \(X^*\) from any \(x(0) \in X\) with a sum of changing rate given by

\[
\sum_{j=0}^M \frac{dx_j}{dt} = \sum_{j=0}^M x_j(t)(\pi_j^u(x(t), a) - \pi^h_j(x(t), a)) = 0.
\]

**Proof.** See [8] for details. □

It is worth noting that a rest point of the replicator dynamics given by (8) may not necessarily be the Evolutionary Stationary State (ESS) (c.f., Section VII.3 in [7]). Based on Lemmas 1 and 2, we obtain Theorem 1 regarding the game \(G_f\)’s equilibria:

**Theorem 1.** Every NE of \(G_f\) is evolutionarily stable in the interior of \(X' = X \cap \{x_0 = 0\}\).

**Proof.** By Definition 1 and Lemma 1, we can find a potential function \(f(x)\) such that \(\frac{\partial f(x)}{\partial x_j} = \pi_j^u(x), \forall j \in M\). Note that \(\pi_j^u\) only depends on the local state \(x_j\). By Proposition 3.1 in [9], for a potential game, the set of NE points \(x^*\) coincides with the solution set of the following problem:

\[
x^* = \arg \max_{x \in X'} \left( f(x) = \sum_{j \in M} \int_0^{x_j} \pi_j^u(z, a)dz \right).
\]

For each local maximum solution \(x^*\) of (11), \(\exists \epsilon > 0\) such that \(f(x^*) \geq f(x)\) for any \(x \in C = B(x^*) \cap X'\), where \(B(x^*)\) is the \(\epsilon\)-ball centered at \(x^*\). Thereby, we are able to design a Lyapunov function \(L(x(t)) = f(x^*) - f(x(t))\) such that \(L(x(t)) \geq 0, \forall x(t) \in C\). Following the Lyapunov Theorem, we have

\[
\frac{dL(x(t))}{dt} = -\frac{\partial f(x(t))}{\partial x_j} \frac{dx_j}{dt} = \frac{dL(x(t))}{dt} = \frac{dL(x(t))}{dt} = \frac{dx_j}{dt} = 0,
\]

where \(\frac{\partial f(x(t))}{\partial x_j} = [\pi_1^u(x(t), a), \ldots, \pi_M^u(x(t), a)]\) and \(\frac{dx_j}{dt} = [a(x(t), a), \ldots, \mu_j(x(t), a)]\). From (8) and \(\mu_j(x(t), a)\) for conciseness, then, we have:

\[
\frac{dL(x(t))}{dt} = -\sum_{j \in M} \pi_j^u \frac{dx_j}{dt} = -\sum_{j \in M} x_j \pi_j^u \pi_j^u - \pi^u \pi^u \sum_{j \in M} x_j \pi_j^u
\]

Thus, we are able to design a Lyapunov function \(L(x(t)) = f(x^*) - f(x(t))\) such that \(L(x(t)) \geq 0, \forall x(t) \in X'\). Following the Lyapunov Theorem, we have

\[
\frac{dL(x(t))}{dt} = -\sum_{j \in M} \pi_j^u \frac{dx_j}{dt} = -\sum_{j \in M} x_j \pi_j^u \pi_j^u - \pi^u \pi^u \sum_{j \in M} x_j \pi_j^u
\]

\[
= -\sum_{j \in M} x_j \pi_j^u \pi_j^u - \pi^u \pi^u \sum_{j \in M} x_j \pi_j^u
\]

We note that all the unstable rest points in (8) violating the NE occur on the boundary of \(X\) [9]. By the Karush-Kuhn-Tucker (KKT) Theorem, for a stable point \(x^*\) in the interior of \(X\), we have the following KKT system \(\forall j \in M\):

\[
\left\{ \begin{array}{l}
\pi_j^u(x_j^*, a) = \mu^u_j - \lambda_j^u,

\lambda_j^u x_j^* = 0, \lambda_j^u \geq 0,
\end{array} \right.
\]

where \(\mu_j^u\) is the Lagrange multiplier for the active constraint \(\sum_{j \in M} x_j = 1\) and \(\lambda_j^u\) is the multiplier for the constraint \(x_j \geq 0\). Since \(\forall j \neq 0, x_j \neq 0\) in the interior of \(X'\), we have \(\lambda_j^u = 0\) in (13). Therefore, the solution of (13) is equivalent to the solution of \(E_{\pi_j^u} = x_j x_j^* \pi_j^u - \pi^u \pi^u \sum_{j \in M} x_j \pi_j^u \pi_j^u = 0\). By applying Proposition 3.1 of [9] again, we know that the equality in (12) holds only when \(x = x^*\), so \(x^*\) is either locally uniformly asymptotically stable (if \(x^*\) is an isolated solution) or Lyapunov stable in a locally asymptotically attractor area (if \(x^*\) is in a connected set of solutions). From (11), (12) and (13), we note that \(x^*\) is in a subset of the solutions to the following system \(\forall i, j \in M\):

\[
\left\{ \begin{array}{l}
\pi_j^u(x_j^*, a) = \mu^u_j - \lambda_j^u,

\lambda_j^u x_j^* = 0, \lambda_j^u \geq 0,
\end{array} \right.
\]

which can be converted into a system of polynomial equations. By Bézout’s Theorem, the solutions of (14) are of finite number and isolated. Therefore, the proof of Theorem 1 is completed. □

Further, since \(\forall j \in M\), \(\pi_j^u(x, a)\) is upper bounded, by the Weierstrass Theorem, there exists a global maximum of \(f(x)\) on \(X'\). Suppose that this global maximum is achieved at \(x^*\). Following the same technique of proving Theorem 1, we can find another Lyapunov function \(L(x(t)) = f(x^*) - f(x(t))\) such that \(L(x(t)) \geq 0, \forall x(t) \in X'\). Moreover, \(L(x(t)) = 0\) only holds...
at the NE. By employing (12) again, we know that $\hat{L}(x(t))$ is strictly decreasing at the non-equilibrium points (excluding the boundary). Then, we know that there is no unstable equilibrium that attracts an evolutionary trajectory on the interior of $X^1$.

**Corollary 1.** On the interior of $X^1$, the replicator dynamics in (8) for $G_f$ always converges to an ESS.

Furthermore, the condition for game $G_f$’s NE to be unique can be obtained through the analysis of the maximization problem in (11) based on the concavity of the potential function:

**Corollary 2.** For the replicator dynamics in (8), a unique interior ESS exists if the following holds:

$$
\gamma_1(1+Nx_j) > \gamma_2(o_j+Nx_j), \quad \forall x_j \in X, \forall j \in M \setminus \{0\}.
$$

**Proof.** The condition of a unique NE in the sub-game $G_f$ is equivalent to the condition of a unique solution to the problem given in (11). Therefore, we only need to check the condition for $f(x)$ to be strictly concave. Since $\pi_j(x,a)$ is determined only by the local state $x_j$, and $\nabla^2 f(x,a) = \text{Diag}(\frac{\partial \pi_j(x,a)}{\partial x_1}, \ldots, \frac{\partial \pi_M(x,a)}{\partial x_M})$, it suffices to show that the following diagonal Hessian matrix of $f(x)$,

$$\frac{\partial^2 f(x,a)}{\partial x^2} = \text{Diag}(\frac{\partial \pi_j(x,a)}{\partial x_1}, \ldots, \frac{\partial \pi_M(x,a)}{\partial x_M}),
$$

is negative definite. Then, a simple derivation leads to (15). □

**B. Stackelberg Equilibria in the Hierarchical Game**

In the leader sub-game, each player aims to maximize its individual profit through non-cooperative price and sponsorship level selection, given the joint MUs’ population state $x$. The derivation of the SE requires that the follower sub-game in the form of potential function optimization (see (11)) to be solved as a parametric optimization problem. Namely, the ESS of the follower sub-game is drawn from a best reply-based point-to-point mapping from the leaders’ strategies. Therefore, the SE can be expressed in the form of a sequence of bi-level optimization problems of the CSPs and the MNO, with the KKT system in (12) being the lower-level constraint for each of them. From (4), the best response for CSP $j$ can be expressed as:

$$
(\theta^*_j, b^*_j) = \arg \max_{(\theta_j, b_j)} \sigma_j \log(1 + x_j N) - p_u \theta_j x_j N - p_v b_j,
$$

s.t. $p_u \theta_j x_j N + p_v b_j - \tau_j \leq 0,
\pi_j^u(x_j, \theta_j, b_j) = \mu^u - \lambda_j^u,
\lambda_j^u x_j = 0, \quad \lambda_j^u \geq 0,
\sum_{j \in M} x_j = 1.
$$

Similarly, from (5) we have for the MNO,

$$
(p^*_u, p^*_v) = \arg \max_{(p_u, p_v)} p_u \sum_{j=1}^{M} b_j + p_v N \sum_{j=1}^{M} x_j,
$$

s.t. $\forall j \in M : \pi_j^u(x_j, \theta_j, b_j) = \mu^u - \lambda_j^u,
\forall j \in M : \lambda_j^u x_j = 0, \lambda_j^u \geq 0,
\sum_{j \in M} x_j = 1.
$$

The optimization problems jointly defined by (17) and (18) help to convert the original hierarchical game into a single-level game with equilibrium constraints. We note that the objective functions in (17) and (18) are linear with respect to local actions $a_j = (\theta_j, b_j)$ ($1 \leq j \leq M$) and $a_0 = (p_u, p_v)$, respectively. Then, we can interpret such an auxiliary game as a Generalized Nash Equilibrium Problem (GNEP) [10] with the shared constraint being dependent of not only the adversaries of player $j \in K$, but also the equilibrium population state of the MUs in the original game. In Theorem 2, we provide a sufficient condition for the existence of the SE:

**Theorem 2.** Suppose that the potential function $f(x,a)$ defined in (11) is strictly concave with respect to $x$, namely, $\gamma_1(1+Nx_j) > \gamma_2(o_j+Nx_j), \forall j \in M \setminus \{0\}$. Then, at least one SE exists in the hierarchical game.

**Sketch of Proof.** The proof of Theorem 2 can be achieved by proving the existence of the solution to the equivalent Generalized Quasi-Variational inequalities (GQVI) problem of the GNEP problem [10] defined by (17) and (18). By Proposition 12.7 in [10], the set of solution to the equivalent GQVI problem is non-empty as long as the gradient-based mapping $F(x,a) = \left[-\nabla a_i \pi^u(x,a), (-\nabla a_j \pi^c(x,a))_{1 \leq j \leq M}\right]^T$ is continuous with respect to the local actions of each player, and the local action space is compact and convex for every $x$. The strict concavity condition given in Theorem 2 ensures the existence condition for the solution of the GQVI problem. See [8] for details. □

**C. Equilibrium Searching in the Stackelberg Game**

According to our analysis of the follower sub-game, as long as the average payoff information is available, the population evolution starting from the interior of $X$ will always reach an ESS. Therefore, we can assume the existence of a Central Information Provider (CIP), and describe the replicator dynamics for CSP selection by Algorithm 1. When the prices asked by the MNO and the MUs’ population states are fixed, we note from the constraints in (17) that the strategy space of the CSPs has a Cartesian product structure. Then, by reformulating the constrained leader sub-game into a GNEP problem (c.f., Theorem 2), we introduce the distributed projection-based method (c.f., [11]) for the leaders’ strategy searching in Algorithm 2.
Algorithm 2 Distributed provider strategy updating

Require: Randomly initialize the user population states as $x(0) \in \mathcal{X}$ and the leader strategies as $\alpha(0)$.

1: while the provider strategies $\alpha(t)$ do not converge do
2: Update the population state $x(t+1)$ by Algorithm 1.
3: Update the CSPs and the MNO’s strategies as follows:

$$
\begin{align*}
\{ a_j(t + \frac{1}{2}) & = a_j(t) + \alpha \nabla_\alpha \pi_\alpha(x(t + 1), \alpha(t)), j = 0, \\
\{ a_j(t + \frac{1}{2}) & = a_j(t) + \alpha \nabla_\alpha \pi_j^c(x(t + 1), \alpha(t)), j \neq 0, \\
\end{align*}
$$

\text{(19)}

where $\alpha > 0$ is the updating step size and $j \in \mathcal{K}$.

4: Denote $y(t + \frac{1}{2}) = (x(t+1), \alpha(t + \frac{1}{2}))$. Project the averaged updating result of the strategies onto the feasible set $\mathcal{C}(x(t+1))$:

$$
\begin{align*}
y(t+1) &= \Pi_\mathcal{C}(x(t+1)) \left[ y(t) + \xi (y(t + \frac{1}{2}) - y(t)) \right], \quad \text{(20)}
\end{align*}
$$

where $\Pi_\mathcal{C}()$ is the projection operator onto the convex set $\mathcal{C}$, and $0 < \xi < 1$ is an averaging factor.

5: end while

The convergence condition for the strategy updating process in Algorithm 2 is provided in Theorem 3.

Theorem 3. Assume that the condition given in (15) holds. Algorithm 2 converges to an SE from any feasible initial strategies, if the step size $\alpha$ in (19) is small enough.

Sketch of Proof. By Corollary 2, Algorithm 1 ensures the convergence to the unique ESS $x^*$ when the condition in (15) is satisfied. Let $y = (x, \alpha)$ denote the joint strategy, and

$$
H(y) = \left[-\nabla_\alpha \pi_\alpha(y), (-\nabla_j \pi_j^c(y))_{1 \leq j \leq M}, (-\nabla_j \pi_j^c(y))_{j \in \mathcal{M}_j}\right]^{\top}
$$

denote the joint gradient-based mapping. We can show that the equivalent GQVI solution for (17) and (18) is:

$$
(y - y^*)^\top H(y^*) \geq 0, \forall y = (x, \alpha), x \in \mathcal{X}, \alpha \in \mathcal{C}. \quad \text{(21)}
$$

The proof of Theorem 3 relies on the co-coercivity property of the gradient-based mapping $H(y)$ (c.f. 2.9.25 in [11]) as well as the uniqueness condition of the follower sub-game ESS in (15). Since the projection onto a convex set is non-expansive, we can employ the co-coercivity property of $H(y)$ to prove that without strategy averaging in (20), Algorithm 2 forms a non-expansive mapping of the joint strategies sequence $\{y(t)\}$ when the learning step $\alpha$ is sufficiently small. Then, the perturbation with averaging to a non-expansive mapping guarantees the convergence to the solution of the GQVI problem in (21) [10]. The details of the proof are provided in [8].

IV. SIMULATION RESULTS

For the purpose of visualization, we use the case of 3 CSPs to demonstrate the convergence property of the proposed strategy searching scheme in Algorithm 2 (see Figure 2). We choose the simulation parameters as $N = 8000$, $\tilde{b}_1 = 1.4\text{MHz}$, $\tilde{b}_2 = 2\text{MHz}$, $\tilde{b}_3 = 1.5\text{MHz}$, $a_1 = 2$, $a_2 = 5$, $a_3 = 4$, $r_j = 10^4$, $\gamma_1 = 1.7$ and $\gamma_2 = 1.1$. We note that the set of parameters satisfy the condition in (15) and create a market of heterogeneous CSPs with differentiable and imperfectly substitutable services. Figures 2(c) and 2(d) provide important insight into the projection-based searching scheme in Algorithm 2. Theoretically, the joint strategy-projection mechanism in (20) prevents the gradient-based mechanism in (19) to linearly increase the capacity price asked by the MNO and decrease the sponsorship level offered by the CSPs. The strategy evolution in Figures 2(c) and 2(d) matches well with our theoretical findings. Moreover, Figures 2(c) indicates that the CSP with a lower basic capacity $\tilde{b}_j$ tends to take more advantage of the global network effect to improve its profit. In order to compensate for the disadvantage in QoE due to a smaller $\tilde{b}_j$ and attract users, it will attempt to offer a higher level of sponsorship. Figure 2(b) indicates that although the CSPs providing a lower basic capacity (i.e., CSP 1 and CSP 3) are not able to increase their user utility to the same level as CSP 2 does due to the sponsoring cost, they do succeed in attracting more users by increasing their sponsorship levels. In comparison, thanks to a higher internal effect, SP 2 mainly relies on its better QoE among the CSPs to attract subscribers and does not need to offer a high level of subsidy.

In Figure 3, we provide further analysis about the impact of the sponsorship levels on the MUs’ and the CSPs’ payoffs. Again, for the purpose of visualization, we consider a market of two CSPs and 50000 MUs, and fix the MNO prices as $p_u = 0.3$ and $p_c = 0$. We set $\gamma_1 = 0.1$ and $\gamma_2 = 0.05$ and assume that the two CSPs provide identical services by fixing the capacity parameters as $b_j = 0\text{Hz}$ and $\tilde{b}_j = 1\text{MHz}$. From Figures 3(a) and 3(b), we can observe that when fixing the sponsorship level of one CSP, the probability of MUs subscribing to the other CSP will increase as the other CSP increases its sponsorship level. Consequently, from Figures 3(c) and 3(d) we note that when fixing one CSP’s sponsorship level, increasing the other CSP’s sponsorship level will lead to a higher equilibrium payoff of the MUs, regardless of the CSP that they choose. From the perspective of a CSP, when fixing its own sponsorship level, its payoff will decrease when the other CSP provides a higher sponsorship level (Figures 3(e) and 3(f)). In the case that the subscription price asked by the MNO is relatively high and the budget limit of CSP is sufficiently large, such a competition between the CSPs will result in a situation of “arm-racing” (namely, prisoner dilemma), and both the CSPs will end up in offering full subsidies to the MUs (Figure 3(g)). However, an extremely high $p_u$ will drain the profit of the CSPs due to the soaring cost of providing subsidies for all the users. In this case, both CSPs will stop offering any subsidies to the MUs.

V. CONCLUSION

In this paper, we have studied a sponsored content/service market with multiple providers in mobile networks. The market is featured by the global network effect in the perceived utility of a large population of users. We have proposed a hierarchical game-based framework to model the interactions among the mobile network operator, the service providers and the mobile users. In the proposed hierarchical game, we have modeled the user-level sub-game as an evolutionary game and the provider-level sub-game as a non-cooperative game. We have discovered a few important properties regarding the Stackelberg equilib-
rrium of the game. We have designed a distributed, iterative scheme for equilibrium searching. Both the theoretical analysis and numerical simulation results have shown the effectiveness of the proposed algorithm. The simulation results have provided important insight into the formation of the market equilibrium as well as the impact of sponsorship levels on the payoffs of both the mobile users and the service providers.

VI. ACKNOWLEDGEMENT

This work was supported in part by Singapore MOE Tier 1 (RG33/16 and RG18/13) and MOE Tier 2 (MOE2014-T2-2-015 ARC4/15 and MOE2013-T2-2-070 ARC16/14).

REFERENCES

[1] Cisco, “Cisco visual networking index: Global mobile data traffic forecast update, 2015-2020,” Tech. Rep., Feb. 2016.
[2] M. Andrews, “Implementing sponsored content in wireless data networks,” in 51st Annual Allerton Conference on Communication, Control, and Computing, Monticello, IL, Oct. 2013, pp. 1208–1212.
[3] L. Zhang, W. Wu, and D. Wang, “Tds: Time-dependent sponsored data plan for wireless data traffic market,” in IEEE International Conference on Computer Communications, San Francisco, CA, Apr. 2016.
[4] C. Joe-Wong, S. Ha, and M. Chiang, “Sponsoring mobile data: An economic analysis of the impact on users and content providers,” in IEEE International Conference on Computer Communications, Kowloon, Apr. 2015, pp. 1499–1507.
[5] Q. T. Nguyen-Vuong, Y. Ghamri-Doudane, and N. Agoulmine, “On utility models for access network selection in wireless heterogeneous networks,” in IEEE Network Operations and Management Symposium, Salvador, Bahia, Apr. 2008, pp. 144–151.
[6] Y. Chen, B. Li, and Q. Zhang, “Incentivizing crowdsourcing systems with network effects,” in IEEE International Conference on Computer Communications, San Francisco, CA, Apr. 2016.
[7] W. H. Sandholm, “Evolutionary game theory,” in Encyclopedia of Complexity and Systems Science, R. A. Meyers, Ed. New York, NY: Springer New York, 2009, pp. 3176–3205.
[8] W. Wang, X. Zehui, N. Dusit, P. Wang, and Z. Han, “A hierarchical game with strategy evolution for mobile sponsored content and service markets,” Submitted to IEEE Transactions on Communications, 2017.
[9] W. H. Sandholm, “Potential games with continuous player sets,” Journal of Economic Theory, vol. 97, no. 1, pp. 81 – 108, Mar. 2001.
[10] F. Facchinei and J.-S. Pang, “Nash equilibria: the variational approach,” in Convex optimization in signal processing and communications, D. P. Palomar and Y. C. Eldar, Eds. Cambridge University Press, 2010, ch. 12, p. 443.
[11] F. Facchinei and J.-S. Pang, Eds., Finite-Dimensional Variational Inequalities and Complementarity Problems. New York, NY: Springer New York, 2003.