Spacetime Instability and the Problems with Low Energy Quantum Gravity

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ABSTRACT: In this paper we discuss spacetime instability problems in effective field theories of the quantum gravity (QG). The effective action of the gravity requires higher-derivative curvature terms $R^2, R_{\mu\nu}R^{\mu\nu}, R_{\mu\nu\kappa\lambda}R^{\mu\nu\kappa\lambda} \ldots$ and they are leading quantum gravitational corrections. Although these higher-curvature terms are indispensable for the construction of the semiclassical or quantum gravity, they lead to several pathologies. We clearly show that even if they are Planck-suppressed operators they lead to serious consequences of the spacetime stability and, de Sitter or radiation dominated Universe are highly unstable for the Hubble perturbation. Furthermore, these curvature terms also violate the null energy condition (NEC) which is required for the self-consistent theories or system. The standard effective field theory of the gravity might fail to describe the observed Universe.
1 Introduction

Constructing quantum field theory (QFT) of the gravity has many serious problems. Famously, the standard general relativity is not renormalizable which is a reasonable requirement on a fundamental theory. Adding higher-order curvature terms \( R^2, R_{\mu\nu}R^{\mu\nu}, R_{\mu\nu\kappa\lambda}R^{\mu\nu\kappa\lambda} \) \([1]\) to the Einstein-Hilbert action makes them renormalizable or even superrenormalizable \([2, 3]\) and express the leading quantum corrections of the gravity. However, they lead to unphysical massive ghosts and spacetime instability. The spin-2 massive ghost brings a notorious unitary problem about the gravitational \( S \)-matrix \([4]\) although further higher-curvature corrections might save such a unitary problem \([5–9]\). Furthermore, these higher-order curvature corrections destabilize the classical spacetime on the tiny perturbations \([10–14]\) and provide unstable de Sitter solutions \([15, 16]\). Although they are actually indispensable for the renormalization and the self-consistent framework of semiclassical gravity or quantum gravity theories, these higher-curvature terms lead to the undesired pathology and there has been considerable debate about these facts in the framework of QG.

On the other hand, if one considers effective field theory (EFT) approaches of QG or string theories \([5–8]\), these higher-order curvatures appear as the leading quantum corrections in Einstein-Hilbert action and the effective action of the gravity can be
given as follows \[17\],

$$\Gamma_{\text{eff}}[g_{\mu\nu}] = -\frac{1}{16\pi G_N} \int d^4 x \sqrt{-g} (R + 2\Lambda + c_1 R^2$$

$$+ c_2 R_{\mu\nu}R^{\mu\nu} + c_3 R_{\mu\nu\lambda\rho}R^{\mu\nu\lambda\rho} + c_4 \Box R + \cdots), \quad (1.1)$$

where the parameters of the effective action like cosmological constant $\Lambda$ and Newton’s constant $G_N$ can be determined by experiments and observations. The higher-derivative curvature terms $R^2, R_{\mu\nu}R^{\mu\nu}, R_{\mu\nu\lambda\rho}R^{\mu\nu\lambda\rho}$ are the leading quantum corrections and express gravitational vacuum polarization or quantum particle creation. For the effective field approaches, these coefficients can be expected to be $c_{1,2\ldots} \sim \frac{N}{M_P}$ ($N$ is a particle number for theory) and are strongly suppressed by the Planck mass or reduced Planck mass: $M_P^2 = 1/8\pi G_N$. Therefore, one might consider that the low-energy physics and the Planck-scale physics are safely sequestered. For instance, the Newtonian potential for the gravitational interactions of two heavy objects can be described as follows \[18\],

$$V(r) = -\frac{G_N m_1 m_2}{r} \left[1 - \frac{G_N (m_1 + m_2)}{r} - \frac{135}{30\pi^2} \frac{G_N}{r^2} + \cdots \right]. \quad (1.2)$$

The first correction of order $G_N (m_1 + m_2)/r$ comes from ordinary general relativity, whereas the second correction express true quantum corrections which are derived by the higher-derivative curvatures $R^2, R_{\mu\nu}R^{\mu\nu}, R_{\mu\nu\lambda\rho}R^{\mu\nu\lambda\rho}$. Now we found that the quantum corrections of the gravity are safely negligible for the Newtonian potential and the effects of QG appear only at the very short distance observations. The above argument matches the heart of effective field theory which is a standard paradigm of particle physics \[19\], and therefore, one expect that these higher-derivative quantum corrections are irrelevant in the low-energy physics and our universe would not receive such effects of QG except very early stage like the singularity \[20\–22\] because there is a huge difference between the cosmological scale and the Planck scale.

However, there are several reasons that this argument is not necessarily correct and the universe strongly receives the influence of QG. For instance, these higher-curvature corrections modify the Einstein equations as the higher-derivative equations and, consequently the gravitational system has the well-known Ostrogradski instability \[23\]. Even if these higher-derivative corrections are suppressed by the Planck mass, the spacetime dynamics would drastically changes compared with the ordinary general relativity \[24\]. Although these issues about higher derivative quantum gravity or so-called $f(R)$-gravity theories have been investigated thoroughly (see, e.g. Refs. [25–29] for review), the spacetime instability induced by the Planck-suppressed operators has not been clarified and well understood. However, this would provide a serious UV/IR mixing puzzle for quantum gravity theories.

The present paper addresses this problem involving the effective field theory of QG and discuss the spacetime instability induced by the higher-derivative quantum
corrections. We consider the effective action of the gravity and derive modified Einstein equations. Solving the Einstein equations, we explore the instability of the Friedmann-Lemaître-Robertson-Walker (FLRW) spacetime in various initial conditions. We found that the de Sitter spacetime is unstable against small perturbations of the Hubble parameter \(^1\) and the de Sitter expansion rolls down to the Planckian stage or terminates even in one normalization time \(\tau = H_0 \cdot t\) where \(H_0\) is the initial value of the Hubble parameter. The instabilities are consistent with the early results of Refs. [13, 14]. The radiation dominated FLRW spacetimes are also unstable and drastically change in one normalization time \(\tau\). Although it is inconsistent with the paradigm of the effective field theory we confirm that the instability induced by the Planck-suppressed quantum corrections is serious and the cosmological influence is inevitable. Furthermore, the effective field theory of the gravity violates the null energy condition (NEC) and this approach might not be appropriate to describe the spacetime of the Universe.

The present paper is organized as follows. In Section 2, we review the renormalization in semiclassical or QG and explain why the higher-derivative curvature terms are indispensable for the gravitational effective action. In Section 3, we numerically investigate the spacetime instability induced by the higher-derivative gravitational corrections. We clearly show that the de Sitter spacetime is generally unstable against the small Hubble perturbations and radiation-dominated Universe inhibits the same behavior. In Section 4, we clearly show that the effective field theory of the gravity breaks the NEC and discuss the pathological problem. Finally, in Section 5 we draw the conclusion of our work.

2 Renormalization and effective action for gravity

The general relativity is the most standard theory to describes the universe or the gravity sysytem and it is based on the Einstein-Hilbert action. However, the renormalization requires more complicated action including the higher-derivative terms for the metric. The simplest renormalizable gravitational action can be given by [1],

\[
S[g_{\mu\nu}] \equiv -\frac{1}{16\pi G_N} \int d^4x \sqrt{-g} \left( R + 2\Lambda \right) + S_{\text{HG}}[g_{\mu\nu}] + S_{\text{matter}},
\]

where \(S_{\text{HG}}\) is the higher-derivative gravitational action,

\[
S_{\text{HG}}[g_{\mu\nu}] = \int d^4x \sqrt{-g} \left( a_1 R^2 + a_2 R_{\mu\nu} R^{\mu\nu} + a_3 R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} + a_4 \Box R + \cdots \right),
\]

with the higher-derivative curvature couplings \(a_{1,2,3,\ldots}\) and \(S_{\text{matter}}\) is the matter action. The gravitational quantum corrections express gravitational vacuum polarization or

\(^1\) The similar instability of de Sitter spacetime from higher-derivative quantum gravity has been discussed in Ref. [25].
quantum particle creation and they are usually written as the divergent quantum corrections. These higher-derivative terms and couplings \(a_1, a_2, a_3, \ldots\) are indispensable for the renormalization to eliminate the one-loop divergences.

For instance, one-loop divergent corrections from the scalar field are calculated by using Schwinger-DeWitt method and dimensional regularization as follows \([30]\):

\[
\Gamma^{(1\text{-loop})}_{\text{eff}} = -\frac{1}{2(4\pi)^2} \int d^4 x \sqrt{-g} \left\{ \ln \left( \frac{m^4}{\mu^2} \right) - \frac{1}{\epsilon} - \log 4\pi + \gamma + \cdots \right\} \times \left[ \frac{1}{2} m^4 + m^2 \left( \xi - \frac{1}{6} \right) R \\
- \frac{1}{6} \left( \xi - \frac{1}{6} \right) \Box R + \frac{1}{2} \left( \xi - \frac{1}{6} \right)^2 R^2 + \frac{1}{180} \left( R_{\mu\nu\lambda\rho} R^{\mu\nu\lambda\rho} - R_{\mu\nu} R^{\mu\nu} - \Box R \right) \right\}, \tag{2.3}
\]

where \(\mu\) is the subtraction scale, \(\epsilon\) is the regularization parameter and \(\gamma\) is the Euler’s constant, and \(m\) and \(\xi\) is the mass and non-minimal coupling of the scalar field. These divergences can be absorbed by the (bare) coupling constants of the gravitational action, Eq. (2.1) and Eq. (2.2), and therefore, one can get the renormalized finite constants. For instance, proceeding to the renormalization, we obtain the renormalized cosmological constant as follows,

\[
\frac{\Lambda_{\text{ren}}}{8\pi G_N^{\text{ren}}} = \frac{\Lambda(\mu)}{8\pi G_N(\mu)} + \frac{m^4}{64\pi^2} \left[ \ln \left( \frac{m^4}{\mu^2} \right) + \text{finite constant} \right], \tag{2.4}
\]

which express physical cosmological constant and \(\mu\) express the renormalization scale.

Recalling that the renormalized cosmological constant \(\Lambda_{\text{ren}}\) does not depend on the scale \(\mu\), we can get the renormalization group equations for the cosmological constant,

\[
\mu \frac{d}{d\mu} \left( \frac{\Lambda(\mu)}{8\pi G_N(\mu)} \right) = \beta_{\Lambda} = \frac{m^4}{2(4\pi)^2}, \tag{2.5}
\]

where \(\beta_{\Lambda}\) is one-loop \(\beta\)-function for the cosmological constant. Similarly, we can obtain the renormalization group equations for other gravitational coupling constants. If we consider \(N_s\) real scalars with \(m_s\), \(N_f\) Dirac spinors with \(m_f\) and \(N_b\) massless vector bosons gravitational one-loop \(\beta\)-functions are given as follows \([31]\),

\[
\mu \frac{d}{d\mu} \left( \frac{\Lambda(\mu)}{8\pi G_N(\mu)} \right) = \beta_{\Lambda} = \frac{N_s m_s^4}{2(4\pi)^2} - \frac{N_f m_f^4}{(4\pi)^2} \\
\mu \frac{d}{d\mu} \left( -\frac{1}{16\pi G_N(\mu)} \right) = \beta_{G_N} = \frac{N_s m_s^2}{(4\pi)^2} \left( \xi - \frac{1}{6} \right) + \frac{N_f m_f^2}{3(4\pi)^2} \\
\mu \frac{d a_1(\mu)}{d\mu} = \beta_1 = \frac{N_s}{2(4\pi)^2} \left( \xi - \frac{1}{6} \right)^2 - \frac{5N_f + 50N_b}{360(4\pi)^2}, \quad \mu \frac{d a_2(\mu)}{d\mu} = \beta_2 = -\frac{N_s + 4N_f + 88N_b}{180(4\pi)^2} \\
\mu \frac{d a_3(\mu)}{d\mu} = \beta_3 = \frac{2N_s + 7N_f - 26N_b}{360(4\pi)^2}, \quad \mu \frac{d a_4(\mu)}{d\mu} = \beta_4 = \frac{N_s + 6N_f - 18N_b}{180(4\pi)^2} \tag{2.6}
\]

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where these higher-derivative couplings $a_{1,2,3,4}$ cannot be fixed to be zero due the renormalization group (RG) running. Note that a large number of particle species $\mathcal{N}$ brings the fine-tuning problems to these gravitational couplings, and therefore, they are expected to be

$$\frac{\Lambda}{8\pi G_N} \sim \mathcal{N} \Lambda_{UV}^4, \quad \frac{1}{16\pi G_N} \sim \mathcal{N} \Lambda_{UV}^2, \quad a_{1,2,3,4} \sim \mathcal{N},$$

(2.7)

where $\Lambda_{UV}$ is the cut-off scale. Clearly, the cosmological constant $\Lambda$ and the Newton’s constant $G_N$ must permit a hard fine-tuning against the quantum corrections.

On the other hand, these higher-gravitational terms are interpreted as the gravitational vacuum polarization or the quantum particle production from the gravity. Indeed, the particle creation ratio $p_{\text{creation}}$ for the scalar field in the FLRW spacetime can be expressed by these higher-derivative terms [32],

$$p_{\text{creation}} \simeq 2 \cdot \text{Im} \Gamma_{\text{eff}}^{(1-\text{loop})} \simeq \frac{1}{16\pi^2} \int d^4x \sqrt{-g} \left( \frac{1}{180} R_{\mu\nu\kappa\lambda} R^{\mu\nu\kappa\lambda} - \frac{1}{180} R_{\mu\nu} R^{\mu\nu} \right) + \frac{1}{2} \left( \frac{1}{6} - \xi \right)^2 R^2 + \mathcal{O}(R^3),$$

(2.8)

which is also consistent with the mode-mixing Bogolyubov technique. Clearly, these terms can not be regarded as the low-energy decoupling effects of the cosmological constant $\Lambda$ and the Newton’s constant $G_N$ unlike the QED case (see e.g. the detailed discussion in [30]). The higher-derivative curvature terms $R^2, R_{\mu\nu} R^{\mu\nu}, R_{\mu\nu\kappa\lambda} R^{\mu\nu\kappa\lambda}$ or couplings are indispensable for the gravitational effective action and usually appears for semiclassical, quantum gravity and string theories.

### 3 Quantum gravitational spacetime instability

In this section, we investigate the spacetime instability using the gravitational effective action of Eq. (2.1) and Eq. (2.2). The higher-derivative correction terms modify the Einstein’s equations and destabilize the classical solutions of the spacetime even for the small Hubble perturbations [10–14]. Although the higher-derivative curvatures are expressed as the Planck-suppressed operators, they non-trivially affect the spacetime through differential equations of the system. Here, we investigate the instability for the gravitational system with various conditions and seek the stability condition for the FLRW spacetime.

The effective action of Eq. (2.1) and Eq. (2.2) which includes gravitational vacuum polarization and quantum particle creation effects derives the following modified Einstein’s equations [33],

$$\frac{1}{8\pi G_N} \left( R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} + \Lambda g_{\mu\nu} \right) + a_1 H_{\mu\nu}^{(1)} + a_2 H_{\mu\nu}^{(2)} + a_3 H_{\mu\nu} = \langle T_{\mu\nu} \rangle,$$

(3.1)
where \( \langle T_{\mu\nu} \rangle \) is the vacuum expectation value of the energy momentum tensor and,

\[
H_{\mu\nu}^{(1)} \equiv \frac{1}{\sqrt{-g}} \frac{\delta}{\delta g_{\mu\nu}} \int d^4x \sqrt{-g} R^2 = 2 \nabla_\nu \nabla_\mu R - 2 g_{\mu\nu} \Box R - \frac{1}{2} g_{\mu\nu} R^2 + 2 \Box R_{\mu\nu},
\]

\[
H_{\mu\nu}^{(2)} \equiv \frac{1}{\sqrt{-g}} \frac{\delta}{\delta g_{\mu\nu}} \int d^4x \sqrt{-g} R_{\mu\rho\nu} R^{\mu\rho\nu} = 2 \nabla_\alpha \nabla_\beta R_{\mu\alpha\beta} - \Box R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} \Box R - \frac{1}{2} g_{\mu\nu} R_{\alpha\beta} R^{\alpha\beta} + 2 R_{\mu} R_{\nu},
\]

\[
H_{\mu\nu} \equiv \frac{1}{\sqrt{-g}} \frac{\delta}{\delta g_{\mu\nu}} \int d^4x \sqrt{-g} R_{\mu\nu\kappa\lambda} R^{\mu\nu\kappa\lambda} = - H_{\mu\nu}^{(1)} + 4 H_{\mu\nu}^{(2)}.
\]

Since left-hand side of Eq. (3.1) is covariantly conserved, the quantum energy momentum tensor must satisfy the covariant conservation:

\[
\nabla_\mu \langle T_{\mu\nu} \rangle = 0.
\]

For flat FLRW universe, the geometrical tensors \( H_{\mu\nu}^{(1)} \) and \( H_{\mu\nu}^{(2)} \) are related with \( H_{\mu\nu}^{(1)} = 3 H_{\mu\nu}^{(2)} \). Thus, we can obtain the following relation,

\[
a_1 H_{\mu\nu}^{(1)} + a_2 H_{\mu\nu}^{(2)} + a_3 H_{\mu\nu} = \left( a_1 + \frac{1}{3} a_2 - \frac{7}{3} a_3 \right) H_{\mu\nu}^{(1)} = \alpha_1 H_{\mu\nu}^{(1)}, \tag{3.2}
\]

in which we introduce \( \alpha_1 = a_1 + \frac{1}{3} a_2 - \frac{7}{3} a_3 \). However, the quantum energy momentum tensor \( \langle T_{\mu\nu} \rangle \) requires more additional geometric tensors (for the detailed discussions see Ref [33]). For instance, the renormalized vacuum energy momentum tensor for a massless conformal coupled field is given as follows,

\[
\langle T_{\mu\nu} \rangle_{\text{conformal}} = \frac{1}{2880 \pi^2} \left( -\frac{1}{6} H_{\mu\nu}^{(1)} + H_{\mu\nu}^{(3)} \right) \tag{3.3}
\]

where

\[
H_{\mu\nu}^{(3)} \equiv \frac{1}{12} R_{\mu\nu} R - R^\rho_\sigma R_{\rho\mu\nu\sigma} = R_{\rho\mu} R_{\rho\nu} - \frac{2}{3} \Box R_{\mu\nu} - \frac{1}{2} R_{\rho\sigma} R^\rho_\sigma g_{\mu\nu} + \frac{1}{4} R^2 g_{\mu\nu},
\]

Furthermore, we must introduce an additional geometric tensor \( H_{\mu\nu}^{(4)} \) which depends on the quantum state (see Ref [33] for the details). From here we drop the brackets of \( \langle T_{\mu\nu} \rangle \) and simply neglect \( H_{\mu\nu}^{(4)} \). Hence, we obtain the following modified Einstein’s equations taking account of quantum gravitational effects,

\[
\frac{1}{8 \pi G_N} \left( R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} \right) + \alpha_1 H_{\mu\nu}^{(1)} + \alpha_3 H_{\mu\nu}^{(3)} = T_{\mu\nu}, \tag{3.4}
\]

Thus, we can get a differential equation for the flat FLRW spacetime,

\[
\frac{\dot{a}^2}{a^2} = \frac{\Lambda}{3} - 8 \pi G_N \frac{18 \alpha_1}{3} \left( \frac{2}{a^2} - \frac{\dot{a}^2}{a^2} + \frac{2}{a^2} \ddot{a}^2 - 3 \frac{\dot{a}^4}{a^4} \right)
+ 8 \pi G_N \alpha_3 \left( \frac{\dot{a}^4}{a^4} \right) + \frac{8 \pi G_N}{3} \rho_{\text{matter}}, \tag{3.5}
\]

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Then, we rewrite Eq. (3.5) with respect to the Hubble parameter \(^2\),

\[
H^2 = \frac{\Lambda}{3} - 48\pi G_N \alpha_1 \left(6H^2 \dot{H} + 2H \ddot{H} - \dot{H}^2\right) \\
+ 8\pi G_N \alpha_3 H^4 + \frac{8\pi G_N}{3} \rho_{\text{matter}},
\]

where the energy density of matter satisfy the covariant conservation law,

\[
\dot{\rho}_{\text{matter}} = -3H (\rho_{\text{matter}} + P_{\text{matter}}) = -3H (1 + \omega) \rho_{\text{matter}},
\]

in which \(w = P/\rho\) is an equation-of-state parameter. For the non-relativistic, relativistic matter or vacuum state we get \(w = 0, 1/3, -1\) respectively. The standard Einstein’s equations have no additional terms and the de Sitter spacetime is defined to be the vacuum spacetime: \(H^2 = \Lambda/3\)

### 3.1 De Sitter spacetime solutions from quantum corrections

The modified Einstein’s equations are formally written as the higher-derivative equations, and therefore, they do not necessarily follow the standard description of the general relativity. Actually, the vacuum state \(w = -1\) of the effective Einstein’s equations leads two classical and quantum de Sitter spacetime solution [15]. To discard time-derivative terms of Eq. (3.6), we get stationary solutions for the Hubble parameter as follows:

\[
H^2 = \left(\frac{1}{16\pi G_N \alpha_3}\right) \pm \frac{1}{\alpha_3} \sqrt{\left(\frac{1}{16\pi G_N}\right)^2 + \frac{\Lambda \alpha_3}{24\pi G_N}}.
\]

For \(\alpha_3 > 0\) and relatively small cosmological constant \(M_P^2 \gg \frac{4\alpha_3 \Lambda}{3}\), we can get two de Sitter spacetime solutions [15, 34–38],

\[
H_C \simeq \sqrt{\frac{\Lambda}{3}}, \quad H_Q \simeq \sqrt{\frac{1}{8\pi G_N \alpha_3}},
\]

where \(H_C\) turns out to be classical de Sitter solution which is the same as the general relativity and \(H_Q\) is quantum driven de Sitter solution. On the other hand, the gravity theory has no quantum de Sitter solutions for \(\alpha_3 < 0\). These two de Sitter solutions are generally unstable for the small perturbation [39] and any other spacetime derived from Eq. (3.4) would have the instability. For instance, the spacetime evolution in the radiation-dominated Universe for the \(R^2\)-gravity theory is considered in Ref. [40]. Hereafter, we consider the stability of the FLRW spacetime using Eq. (3.6) and Eq. (3.7) numerically.

\(^2\) The derivative of Eq. (3.6) yields the following differential equation,

\[
\dot{H} = 16\pi G_N \alpha_3 H^2 \dot{H} - 48\pi G_N \alpha_1 \\
\times \left(6\dot{H}^2 + 3H \ddot{H} + \dddot{H}\right) - 4\pi G_N (1 + \omega) \rho_{\text{matter}},
\]

which includes the covariant conservation law of Eq. (3.7) and takes account of the matter effects.
3.2 Numerical estimation for spacetime instability

First, let us start the de Sitter spacetime under the small Hubble perturbation and consider the effects of the gravity only. We rewrite Eq. (3.6) and Eq. (3.7) in terms of dimensionless quantities and obtain the following differential equation,

$$h^2 = -xh^4 - y \left( 6h^2 h' + 2hh'' - h'^2 \right) + z,$$
$$z' = -3h (1 + \omega) z. \quad (3.10)$$

where we introduce

$$\tau = H_0 t, \quad h = H/H_0, \quad x = -8\pi G N \alpha_3 H_0^2, \quad y = 48\pi G N \alpha_1 H_0^2,$$
$$z = \Lambda/3H_0^2 + 8\pi G N \rho_{\text{matter}}/3H_0^2$$

and $H_0$ is the initial Hubble parameter at some time $t_0$. For instance we can expect the following cosmological relations,

$$H_0 \sim 10^{14} \text{GeV}, \quad M_P \sim 10^{18} \text{GeV}, \quad \alpha_{1,3} \sim 10^{-2} \implies x,y \sim 10^{-10}$$
$$H_0 \sim 10^{-42} \text{GeV}, \quad M_P \sim 10^{18} \text{GeV}, \quad \alpha_{1,3} \sim 10^{-2} \implies x,y \sim 10^{-122} \quad (3.11)$$

where the former corresponds to the typical Hubble parameter during inflation from the current bound [41] and the latter is consistent with the current Hubble parameter dominated by the dark energy. The dynamics of the dimensionless Hubble parameter $h$ with the vacuum state $w = -1$ and $\rho_{\text{matter}} = 0$ is determined by the following equation,

$$h^2 = -xh^4 - y \left( 6h^2 h' + 2hh'' - h'^2 \right) + z. \quad (3.12)$$

where prime express the derivative with respect to dimensionless time $\tau$. The natural de Sitter initial conditions are given by

$$\tau_0 = 1, \quad h_0 = 1, \quad h'_0 = 0, \quad z_0 = 1. \quad (3.13)$$

We investigate the system of equations starting at $\tau_0 = 1$ with various conditions and perturbations. We find out that the numerical solutions of the system of equations show the stability or instability which can be roughly understood by some analytical estimates. In Fig.1(a) and 1(b), we present the numerical results for the dimensionless Hubble parameter $h$ determined from Eq. (3.12) with the following initial conditions,

Fig.1(a): $h_0 = 1 + 0.9, \quad h'_0 = 0, \quad x = 10^{-1,-3,-5}, \quad y = 10^{-1,-3,-5},$
Fig.1(b): $h_0 = 1 + 0.1, \quad h'_0 = 0, \quad x = 10^{-1,-3,-5}, \quad y = 10^{-1,-3,-5}, \quad (3.14)$

and we compare them with the de Sitter solution $h(\tau) = 1$ from the general relativity. Fig.1(a) and 1(b) show that the de Sitter spacetime is stable for the perturbation of the Hubble expansion and the variation converges for a few normalization times $\tau$. It is found that the Hubble oscillation becomes faster for the small values of $x,y$ and the spacetime dynamics for $x,y > 0$ or $x < 0, y > 0$ shows the same results. In
Figure 1. For $y > 0$, numerical solution of Eq. (3.12) with the de Sitter initial conditions and the higher-derivative couplings of Eq. (3.14). These figures show that the dynamics of the dimensionless Hubble parameter $h(\tau)$ in a few normalization time $\tau$. The dashed line shows the usual de Sitter solution $h(\tau) = 1$ from the general relativity.

Fig.2(a), 2(b), 3(a) and 3(b), we show the numerical results for the dynamics of the dimensionless Hubble parameter $h$ for $y < 0$ and take the following conditions,

Fig.2(a): $h_0 = 1$, $h'_0 = 0$, $x = 10^{-1.0, -1.3, -1.5, -1.7, -1.9, -2.1}$, $y = -10^{-1.0, -1.3, -1.5, -1.7, -1.9, -2.1}$,

Fig.2(b): $h_0 = 1$, $h'_0 = 0$, $x = 10^{-9.8, -10.0, -10.2}$, $y = -10^{-9.8, -10.0, -10.2}$,

Fig.3(a): $h_0 = 1 - 10^{-1.3}$, $h'_0 = 0$, $x = 10^{-1.0, -1.1, -1.2, -1.3, -1.4, -1.5}$, $y = -10^{-1.0, -1.1, -1.2, -1.3, -1.4, -1.5}$,

Fig.3(b): $h_0 = 1 - 10^{-4.3}$, $h'_0 = 0$, $x = 10^{-10.0, -10.3, -10.6, -10.9}$, $y = -10^{-10.0, -10.3, -10.6, -10.9}$.

We found that the de Sitter spacetime is destabilized by the Planck-suppressed quantum corrections in one normalization time $\tau$ even if we set the tiny values of $x, y$. Rather, the smallness of $x, y$ amplifies the spacetime instability and this case is inconsistent with the usual general relativity. In other words, the de Sitter spacetime is highly unstable for $|y| \ll 1$, whereas the instability can be alleviated for $|y| \approx 1$. However, it means a serious UV/IR mixing problem. Therefore, the higher-derivative curvature corrections can not be ignored when they are Planck-suppressed operators and the standard inflation can not be realized unless the higher-derivative curvature couplings for the gravitational action are rather large.

Next, we investigate the system at the radiation-dominated Universe with the relativistic state $w = 1/3$. The natural radiation initial conditions are given by,

$$\tau_0 = 1/2, \quad h_0 = 1, \quad h'_0 = -2, \quad z_0 = 1.$$
(a) $y = -10^{-1.0}, -1.3, -1.5, -1.7, -1.9, -2.1$

Figure 2. For $y < 0$, numerical solution of Eq. (3.12) with the de Sitter initial conditions and the higher-derivative couplings of Eq. (3.15). These figures show the instability for the dimensionless Hubble parameter $h(\tau)$ in a few normalization time $\tau$. It is found that the small values of $x, y$ amplify the spacetime instability.

(b) $y = -10^{-9.8}, -10.0, -10.2$

Figure 3. Numerical solution of Eq. (3.12) with the de Sitter conditions and the higher-derivative couplings of Eq. (3.15). These figures show the instability for the dimensionless Hubble parameter $h(\tau)$ in a few normalization time $\tau$. It is found that for $h_0 < 1$ the de Sitter expansion terminates.

(a) $y = -10^{-1.0}, -1.1, -1.2, -1.3, -1.4, -1.5$

(b) $y = -10^{-10.0}, -10.3, -10.6, -10.9$
Figure 4. We compare the numerical solution of Eq. (3.12) with the conditions of Eq. (3.17) and the standard solution $h(\tau) = 1/2 \cdot \tau$ form the general relativity. Fig.4(a) show that de Sitter spacetime is stable under the Hubble perturbations. Fig.4(b) show the instability for radiation-dominated Universe and the solutions do not follow the general relativity.

Figure 5. Numerical solution of Eq. (3.12) with the conditions of Eq. (3.17) where we set $x = 0$. These figures show the instability for the dimensionless Hubble parameter $h(\tau)$ in a few normalization time $\tau$. For $y > 0$, the dynamics of the normalized Hubble parameter $h(\tau)$ is different from Fig.4(b).
where we rewrite \( z = 8\pi G_N \rho_{\text{matter}}/3H_0^2 \) and take \( \Lambda = 0 \). In Fig.4(a) and 4(b) we investigate the system of equations starting at \( \tau_0 = 1/2 \) by using Eq. (3.10) with the relativistic conditions and the higher-derivative parameters as follows,

Fig.4(a): \( h_0 = 1 + 0.5, \ h'_0 = -2, \ x = 10^{-1, -3, -5}, \ y = 10^{-1, -3, -5}, \)
Fig.4(b): \( h_0 = 1, \ h'_0 = -2, \ x = 10^{-1, 0, -1.3, -1.6, -1.9, -2.2}, \ y = -10^{-1.0, -1.3, -1.6, -1.9, -2.2}, \)
Fig.5(a): \( h_0 = 1 + 0.5, \ h'_0 = -2, \ x = 0, \ y = 10^{-1, -3, -5}, \)
Fig.5(b): \( h_0 = 1, \ h'_0 = -2, \ x = 0, \ y = 10^{-1.0, -2.0, -3.0}, \)

and compare them with the radiation-dominated solution \( h(\tau) = 1/2 \cdot \tau \). Fig.4(a) show that the de Sitter spacetime is stable for \( y > 0 \) under the Hubble perturbations and the variations converge to the solutions of the general relativity. In this case we found that the Hubble oscillations are faster for the small values of \( x, y \). On the other hand, Fig.4(b) show that the higher-derivative curvature corrections lead to the instability and for \( y < 0 \) the solutions do not follow the usual general relativity. As we saw in the case of the de Sitter spacetime the smallness of \( x, y \) amplify the spacetime instability [13, 14]. Therefore, the Planck-suppressed curvature corrections strongly affect the spacetime dynamics.

For \( x = 0 \), we demonstrate the dynamics of the normalized Hubble parameter \( h(\tau) \) in Fig.5(a) and Fig.5(b). Clearly, these figures shows the instability of radiation-dominated Universe. For \( y > 0 \), the higher-derivative curvature corrections might not destabilize the classical spacetime since the solutions approach to the usual general relativity. Thus, the stability condition for the effective field theory of QG could be written as follows,

\[
y(\mu) > 0 \implies 8\pi G_N(\mu) \left( \frac{18a_1(\mu)}{3} \right) > 0 \implies a_1(\mu) + \frac{1}{3}a_2(\mu) - \frac{7}{3}a_3(\mu) > 0.
\]

(3.18)

where \( \mu \) is the renormalization scale. However, it is not easy to satisfy that condition because the higher-derivative curvature couplings changes for the cosmological scale. The hard fine-tuning is required for the couplings from the past to the future, and moreover, the above condition is only effective for the one-loop perturbative corrections. Any higher-loop corrections require such conditions and that is not desired. The problems of the spacetime instability have been widely discussed and there are several proposals in the literature [42–46]. For instance, the higher-derivative curvature terms \( R^2, R_{\mu\nu}R^{\mu\nu}, R_{\mu\nu\rho\lambda}R^{\mu\nu\rho\lambda} \) are one-loop perturbative corrections for the gravity and non-perturbative effects might ensure the stability [44]. For the effective field theories, these curvature terms appear only as the approximation terms from the unknown quantum gravity theories and the UV completion of QG might not have such higher-derivative curvature terms. It would be one of the most decent solutions.
for the spacetime stability although we have no clear answer. After all, the standard effective field approaches of QG might have failed to describe the spacetime dynamics of the Universe. In the next section we will see that the effective field theory of the gravity causes the violation of the null energy condition.

4 Null energy condition and quantum gravity

In previous section, we have shown that the effective field theory of QG or higher derivative quantum gravity inhabit the spacetime instability and they sometimes conflict with the usual general relativity. The higher-derivative gravitational corrections destabilize the classical spacetime solutions, and also violate the null energy condition (NEC). Roughly speaking, the instability corresponds to the NEC violation. The NEC is the weakest but most standard energy conditions to restricts the pathological spacetimes for the general relativity and states that

\[ T_{\mu\nu}k^\mu k^\nu \geq 0, \]  

for any null (light-like) vector \( k^\mu \). For a perfect fluid with the positive energy, the NEC yields the relation: \( P + \rho \geq 0 \). The condition preclude undesired consequences such as wormhole, spacetime instabilities, superluminal propagation and unitary violations [47–51] for general relativity and it is consistent with gravitational thermodynamics [52–55]. It is widely believed that any physical systems or theory should respect the condition and the violation leads to the pathology.

Indeed, there has been continuously debated for the validity [56–58] and there are several violating examples of the QFT such as the squeezed states [59], Casimir effect [60], Hawking evaporation [64] and conformal anomaly [57, 65, 66]. It is known that the inflation and dark energy are generally difficult if higher dimensional theories like Kaluza-Klein theory or string theory satisfy the NEC [67]. However, self-consistent theories for QG are expected to satisfy the condition and it is problematic to permit the pathology at the UV completion. Now, there are unclear points whether the effective field theories of QG or string theory can break it if the UV competition satisfy the condition. That would be only possible if the higher-derivative terms for the low-energy effective action are apparent and there are no such terms at the UV completion. Therefore, the NEC violation phenomena could not occur if the UV completion of QG satisfy the condition. In this section, we clearly show that the effective field theory of the gravity violates the NEC and discuss whether these approaches are reliable to describe the Universe.

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3 The Casimir energy might be a clear instances violating the NEC from quantum effects. The Casimir effect is usually interpreted as the negative electromagnetic zero-point energy [61] between two parallel conducting plates. However, there exists no consensus about this interpretation since standard perturbative QED [62] and van der Waals interactions [63] can describe this phenomenon without invoking the zero-point energy.
For the cosmological framework of the flat FLRW universe, the Friedmann equations yields a simple equation,

$$\dot{H} = -4\pi G_N (P + \rho). \quad (4.2)$$

where the Hubble parameter decreases with time or stays constant if the null energy condition $P + \rho \geq 0$ is satisfied. Thus, the flat FLRW spacetime always decelerates and finally terminates the expansion. The de Sitter spacetime is always stable and the cosmological constant $\Lambda$ satisfy the relation $P + \rho = 0$. The time-evolution of the Universe can be classified as,

$$H \longrightarrow \begin{cases} 
0 & (P + \rho > 0) \\
\text{const} & (P + \rho = 0) \\
\infty & (P + \rho < 0)
\end{cases} \quad (4.3)$$

For the slow-roll inflation driven by a inflaton field $\phi$, we have $\dot{H} = -4\pi G_N \dot{\phi}^2 < 0$ which is consistent with one’ intuition. The ghost field has negative kinetic terms which provides serious problems in the QFT and leads to the relation $\dot{H} = 4\pi G_N \dot{\phi}^2 > 0$. Thus, we can expect that the Hubble expansion ratio always decelerates or stays for the ordinary cosmological theories.

Let us consider the semiclassical gravity which includes the backreaction effects of the quantum fluctuations or quantum particle creations onto the spacetime. The de Sitter spacetime can be interoprated as one observer is surrounded by thermal radiation at the Hawking temperature $T_H = H/2\pi$ [68] from the horizon. The energy density or pressure including the thermal de Sitter radiation can be written as

$$\rho_{\text{dS}} = \rho_{\Lambda} + \frac{H^4}{480\pi^2}, \quad P_{\text{dS}} = P_{\Lambda} + \frac{1}{3} \frac{H^4}{480\pi^2}. \quad (4.4)$$

The backreaction of the thermal de Sitter radiation satisfy the NEC: $P_{\text{dS}} + \rho_{\text{dS}} \geq 0$ and terminates the expansion as follows [69]

$$\dot{H} = -\frac{G_N H^4}{720\pi^2} < 0 \implies H = \frac{H_0}{\left(\frac{G_N H_0^4}{240\pi^2} + 1\right)^{1/3}}. \quad (4.5)$$

where it is not surprise that the thermal backreaction of Eq. (4.4) satisfy the NEC since we regards the quantum corrections as the classic matters. However, the above thermal interpretation of the de Sitter particle creations is not exact and it is necessary for a detail consideration based on the QFT approach. In order to take account of the gravitational vacuum polarization and quantum particle creation we usually consider the vacuum expectation values of the energy momentum tensor $\langle T_{\mu\nu}\rangle$. For massless minimally coupled scalar field, the renormalized energy momentum tensor
is computed approximately for the Bunch-Davies vacuum as follows [70]:

$$
\langle T_{\mu\nu} \rangle = \frac{1}{2880\pi^2} \left( -\frac{1}{6} H_{\mu\nu}^{(1)} + H_{\mu\nu}^{(3)} \right) - \frac{H_{\mu\nu}^{(1)}}{1152\pi^2} \log \left( \frac{R}{\mu^2} \right)
$$

(4.6)

$$
+ \frac{1}{13824\pi^2} \left[ -32\nabla_{\nu} \nabla_{\mu} R + 56\Box R_{\mu\nu} - 8RR_{\mu\nu} + 11R^2 g_{\mu\nu} \right]
$$

For simplicity, let us consider massless conformal coupled fields and the renormalized energy momentum tensor is computed analytically as follows:

$$
\langle T_{\mu\nu} \rangle_{\text{conformal}} = \frac{1}{2880\pi^2} \left( -\frac{1}{6} H_{\mu\nu}^{(1)} + H_{\mu\nu}^{(3)} \right)
$$

(4.7)

which correspond to the conformal anomaly [71–74], and the corresponding energy density or pressure are given by

$$
\rho_{\text{conformal}} + p_{\text{conformal}} = \frac{H^2 \dot{H}}{720\pi^2} + \frac{6\dot{H}^2 + 3H\ddot{H} + \dddot{H}}{1440\pi^2} \geq 0,
$$

(4.8)

which breaks the NEC and lead to the expansion $\dot{H} > 0$ [39]. Although the semiclassical gravity does not quantize the metric it takes into account the backreaction of the quantum matter fields properly. However, the semiclassical gravity suffers from the spacetime instability and has the NEC violation pathology [24]. Similarly, it found that the effective field theory of the gravity with Eq. (2.1) and Eq. (2.2) also violate the NEC and the relations can be given by

$$
\dot{H} = 8\pi G_N 2\alpha_3 H^2 \dot{H} - 8\pi G_N 6\alpha_1 \left( 6\dot{H}^2 + 3H\dot{H} + \dddot{H} \right)
$$

$$
- 4\pi G_N \left( P_{\text{matter}} + \rho_{\text{matter}} \right) \geq 0,
$$

(4.9)

where we can regard the higher-derivative gravitational corrections as the quantum matter. It is clear that the effective field theory violate the NEC from the higher-derivative gravitational corrections and the Hubble expansion ratio can increase with various conditions

\[4\]

We can see the similar consequences by using de Sitter entropy. The thermal character of the event horizon in de Sitter spacetime is summarized by the de Sitter entropy [68] and the time-evolution is written as follows,

$$
dS_{\text{dS}} \frac{dt}{dt} = -\frac{2\pi H^{-3} \dot{H}}{G_N} \iff dS_{\text{dS}} \frac{dN_{\text{tot}}}{dN_{\text{tot}}} = -\frac{2\pi H^{-4} \dot{H}}{G_N}
$$

To incorporate the de Sitter thermal radiation $P_{\text{dS}} + \rho_{\text{dS}} \geq 0$ from the horizon the de Sitter entropy always increases,

$$
dS_{\text{dS}} \frac{dt}{dt} = \frac{H}{360\pi} > 0 \iff dS_{\text{dS}} \frac{dN_{\text{tot}}}{dN_{\text{tot}}} = \frac{1}{360\pi} > 0
$$

On the other hand, for the effective field theory of the gravity, the de Sitter entropy decreases with various conditions as follows,

$$
dS_{\text{dS}} \frac{dt}{dt} = -32\pi^2 \alpha_3 H^{-1} \dot{H} + 96\pi^2 \alpha_1 \left( 6H^{-3} \dot{H}^2 + 3H^{-2} \dddot{H} + H^{-3} \dddot{H} \right) \geq 0
$$

(4.10)

which is inconsistent with gravitational thermodynamics and corresponds to the NEC violation [52].
If the UV competition completely satisfy the NEC or do not have the higher-derivative terms, the NEC violation or stability of the gravitational effective action might have no serious problem since the violation is seeming. However, how can we say that quantum gravity theories have such satisfactory characteristics. Although it would be one of the most desirable solutions, there are no clear proofs and the effective action of the gravity of Eq. (1.1) always includes the higher-curvature terms which are general and invariant under general coordinate transformations. Therefore, the standard effective field theories of the gravity might fail to describe the dynamics of the Universe although they allow the anomaly induced inflation [15] or the avoidance of the singularity problem [20]. Even if we permit the NEC violation from the higher-derivative curvatures they lead to the spacetime instabilities, which are generally inconsistent with the results of the general relativity as seen in Section 3, and the theories must satisfy the stable conditions for any-loop orders and disallow the Hubble perturbations from the past to the future.

5 Conclusion

In this paper we have discussed the spacetime instability problems in the effective field theories of QG. The effective action of the gravity requires higher-derivative curvature terms \( R^2, R_{\mu\nu}R^{\mu\nu}, R_{\mu\nu\kappa\lambda}R^{\mu\nu\kappa\lambda} \ldots \) and they are leading quantum corrections. Although these higher-derivative curvatures are indispensable for the renormalization and the construction of the semiclassical or quantum gravity theories, they lead to many pathologies like the spacetime instability or the NEC violation.

We have investigated the spacetime instability from the higher-derivative curvature corrections thoroughly compared with the previous works [10–14]. We found out that even if they are expressed as the Planck-suppressed operators they lead to the instability for the de Sitter spacetime or radiation dominated Universe. Rather, the gravitational couplings must be large \( a_{1,2,3} \gg 1 \) and \( |y| \approx 1 \) for the instability. Furthermore, We found out that for \( y > 0 \) the higher-curvature solutions can approach the solutions of the general relativity and the one-loop stable condition is given by Eq. (3.18). The instabilities are generally inconsistent with the results of the general relativity, and the standard effective field theory of the quantum gravity might fail to describe the observed Universe.

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A Geometrical tensors for FLRW spacetime

Here, we provide geometrical tensors for FLRW spacetime. In this paper we take the FLRW line element as follows,

\[ ds^2 = dt^2 - a(t)^2 \sum_{i,j=1}^{3} h_{ij} dx^i dx^j, \]  
(A.1)

in which \( a = a(t) \) express the scale factor with the cosmic time \( t \) and,

\[ \sum_{i,j=1}^{3} h_{ij} dx^i dx^j = \frac{1}{1 - Kr^2} dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2), \]  
(A.2)

where \( K \) is the spatial curvature parameter. For simplicity, we consider spatially flat spacetime \( K = 0 \). The conformal time parameter \( \eta \) is given by,

\[ d\eta = \frac{dt}{a(t)} \]  
(A.3)

whose line element is given by

\[ ds^2 = a^2(\eta) \left( d\eta^2 - \sum_{i,j=1}^{3} h_{ij} dx^i dx^j \right), \]  
(A.4)

We introduce \( C(\eta) = a^2(\eta) \) and \( D(\eta) = C(\eta)' / C(\eta) \) in which the prime \( ' \) express the derivative of \( \eta \). The Ricci tensor, Ricci scalar and other geometrical tensors are given by [33],

\[ R_{00} = \frac{3}{2} D', \quad R_{11} = -\frac{1}{2} (D' + D^2), \quad R = \frac{3}{C} \left( D' + \frac{1}{2} D^2 \right), \]  
(A.5)

\[ H_{00}^{(1)} = \frac{9}{C} \left( -D'' D + \frac{1}{2} D^2 + \frac{3}{8} D^4 \right), \]  
(A.6)

\[ H_{00}^{(3)} = \frac{3}{C} \left( \frac{1}{16} D^4 \right). \]  
(A.7)

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