Inflation and Generalized O’Raifeartaigh SUSY models

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Thermal inflation usually requires an inflationary potential with nonrenormalizable operators (NROs). We demonstrate how O’Raifeartaigh models with or without NROs can provide thermal inflation and a solution to the moduli problem, as well as provide SUSY breaking. We then discuss a scenario where generalized O’Raifeartaigh potentials (with NROs) are included in a SUGRA where the supergravity and O’Raifeartaigh potentials provide negative and a positive contributions to the cosmological constant respectively. Tuning these contributions to nearly cancel can provide the present value of the dark energy.

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There is considerable belief that the fundamental model of particle physics respects local and/or global supersymmetry at high energy. Inflationary cosmology appears to provide further support to this expectation. Due to the ability of supersymmetry to protect against radiative corrections, such models provide powerful means to realize ultra-flat potentials, which are necessary from inflation density perturbation constraints. However, alongside this benefit, cosmological implementations of supergravity and SUSY models generally lead to undesired side effects. In particular for cosmological inflation, whether supercooled or warm, which end at conventional high temperature scales, \( T \gtrsim 10^{10}\text{GeV} \), overabundances of unwanted SUSY particles is a real problem, sometimes termed the moduli problem [2,5].

SUSY must not survive at low energy scales, where physics clearly is not supersymmetric, with current limits set by particle physics experiments indicating SUSY must break above the electroweak scale \( \sim 10^4\text{GeV} \). It is reasonable to expect that symmetry breaking and more specifically SUSY breaking has cosmological implications. For example, one scenario termed thermal inflation [5–9] uses symmetry breaking to overcome the problem of overabundance of unwanted particles created by SUSY at high temperature. A second problem related to SUSY, for which cosmologists are universally and anxiously awaiting an explanation, is the present day cosmological constant \( \rho_\Lambda \). Observation of type IA supernova data have indicated an accelerating universe [10], which could be explained by a cosmological constant of 70% of the critical density, which implies a vacuum energy component \( \rho_\Lambda \sim 10^{-3}\text{eV}^4 \). Recently the first year WMAP data has independently verified the presence of a cosmological constant, finding \( \Omega_\Lambda = 0.73 \pm 0.04 \) [11].

In this Letter, we will demonstrate that generalized O’Raifeartaigh models [12] can realize thermal inflation and solve the present day cosmological constant problem. Recall that spontaneous global SUSY breaking can be accomplished by the O’Raifeartaigh mechanism that requires at least three chiral supermultiplets. The minimal model has a superpotential of the form

\[
W(\phi, \chi, \eta) = a\chi (\phi^2 - M^2) + m\eta \phi. \tag{1}
\]

SUSY is broken since the requirement \( \frac{\partial W}{\partial \phi} = 0 \), with \( \phi_i = \phi, \chi, \eta \), cannot be satisfied for all three fields. In other words the three conditions,

\[
\phi^2 - M^2 = 0, \quad a\chi \phi + m\eta = 0, \tag{2}
\]

cannot be simultaneously satisfied. Our purpose is to demonstrate that within their compact structure, these models contain nontrivial cosmological implications. We will begin with a review of thermal inflation, to understand the relevant scales necessary for such scenarios. Generalizations of the O’Raifeartaigh models are then presented and solutions are derived for thermal inflation and the present day cosmological constant. We then briefly discuss embedding O’Raifeartaigh models in supergravity (SUGRA) and other fundamental theories, as well as particle physics implications of such models.

The thermal inflation scenario is comprised of two phases of inflation. The first phase is the normal one, typically motivated by GUT physics and pictured to end, after reheating, at a high temperature \( T \gtrsim 10^{10}\text{GeV} \). In this phase, the large scale physics is determined, such as density fluctuations. The key new feature that underlies thermal inflation is that it requires the presence of a scalar field \( \phi \), often called the flaton, which has a symmetry breaking potential with the properties that at high temperature, \( T > 10^{10}\text{GeV} \) symmetry is unbroken with \( \phi = 0 \) where the scale of the potential is \( V_0^{1/4} \approx 10^9\text{GeV} \). On the other hand, at \( T = 0 \) symmetry is broken with the minimum now at \( \phi \approx 10^9\text{GeV} \) and with the scalar particles acquiring a mass \( m_\phi \sim 10^{2-3}\text{GeV} \). Given such a potential, a second phase of inflation, termed thermal

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inflation, commences. In this picture, for $T > V_0^{1/4}$ the scalar field finite temperature effective potential locks the flaton field at $\phi = 0$ and the universe is in a hot big bang regime. Once $T < V_0^{1/4}$, the potential energy of this field dominates the energy density of the universe, thereby driving inflation, which to a good approximation is assumed to be an isentropic expansion. Due to the high temperature corrections to the effective potential, in the initial phase of thermal inflation, the scalar field remains locked at its high temperature point, $\phi = 0$. However, since inflationary expansion is rapidly cooling the universe, it implies the effective potential is evolving to its zero temperature form. Eventually, in what is estimated to be $\sim 15$ e-folds, the scalar field VEV no longer is locked at zero, and is able to roll down to its new minimum.

The effect of the second phase of inflation is to lower the temperature of the universe from $T \sim 10^9$GeV to $T \sim 10^4$GeV. This alone does not solve any overabundance problems since the abundance ratios $n/s$ for all species remain constant. However, subsequent to thermal inflation the scalar field oscillates, thereby producing scalar particles of mass $m_\phi \sim 10^2 - 3$GeV and lighter. These particles eventually decay, producing a huge increase in entropy, thereby adequately diluting the abundances of unwanted relics. Finally, in order not to affect the success of hot big bang nucleosynthesis, the temperature after decay of scalar particles is constrained to be above $\sim 10$MeV. Note, the desired features of thermal inflation could also occur for a continuous phase transition and a nonisentropic, warm-inflationary type expansion, which dampens the flaton’s motion during its evolution to its new minimum [4,13].

The details of the thermal inflation scenario outlined above can be found in [5–9]. The key point demonstrated in these papers is that all the desired features of this scenario follow, provided a potential with the properties described above is present. Considerable work on thermal inflation studies the consequences of such potentials, but many fewer works attempt to find explicit models of such potentials. Thermal inflation is typically carried out with potentials containing higher (>4) dimension operators that are suppressed by powers of the Planck mass. In most studies of thermal inflation [5–9], SUSY breaking is handled separately, for example, through nonperturbative means, such as the Affleck-Dine mechanism. Here we observe that a generalization of the O’Raifeartaigh potential, with one term replaced by a higher dimension operator can provide SUSY breaking, thermal inflation, and potentially, the presentday cosmological constant. Aside from the compactness of this solution, another advantage is that SUSY breaking terms are calculable at the tree level in the renormalizable O’Raifeartaigh model, so one has more control in model building. For the generalized O’Raifeartaigh model, loop level calculations would diverge. However, the basic motivation of the higher dimension operators is string theory which would serve to cut off all divergences and still leaves the model with some degree of control.

To treat the cosmological moduli and cosmological constant problems, consider the generalization of the O’Raifeartaigh model superpotential,

$$W(\phi, \chi, \eta) = a\chi [\phi^2 - M^2] + \lambda \eta \frac{\phi^{n+1}}{m_{pl}}.$$  \hfill (3)

The $\frac{\partial W}{\partial \phi_1} = 0$ conditions now become

$$\frac{\partial W}{\partial \chi} = a[\phi^2 - M^2] = 0, \quad \frac{\partial W}{\partial \eta} = \lambda \frac{\phi^{n+1}}{m_{pl}} = 0,$$

and since these cannot be simultaneously satisfied, SUSY is broken. To carry out the calculation of thermal inflation and the cosmological constant, we need the Higgs potential $V = \left( \frac{\partial W}{\partial \chi} \right)^+ \left( \frac{\partial W}{\partial \eta} \right)$, which is

$$V = a^2 |\phi|^2 - M^2|\phi|^2 + \lambda^2 \frac{m_{pl}^4 \phi^{2(n+1)}}{m_{pl}^{2(n+1)}} + 2a\phi \chi + (n + 1)\lambda \frac{\phi^n}{m_{pl}^{n-1}} |\phi|^2. \hfill (5)$$

The first objective is to show at zero temperature this potential has the correct features and scales for thermal inflation and the presentday cosmological constant. For this the minimum of $V$ is required. There is a single family parameter of minima with $\langle \phi \rangle \neq 0$, given by setting the third term in the potential to zero. This gives the condition $\langle \eta \rangle = x(\chi)$, with $x = -2am_{pl}^{-n-1}/(n + 1)\lambda(\phi)^{n-1}$. A number of possibilities exist for this direction. First if the flat direction is uncorrected by the full theory, then there will be a massless boson $b = \chi + x\eta$. If this boson couples sufficiently weakly to standard model fields, it does not upset the cosmology. In particular, although it will not thermalize, it still redshifts away. For a more strongly coupled $b$, particle physics familiar limits apply [14]. If the full theory corrects the O’Raifeartaigh potential, the mass generated for $b$ will allow it to contribute to the dark matter density or $b$ could generate an additional moduli problem at lower scale. A further implication is that corrections from outside the O’Raifeartaigh potential could allow the overconstrained set of conditions on the VEVs to be relaxed in a way that spoils the O’Raifeartaigh mechanism and could restore SUSY. We assume this does not happen or if it did we would have to modify the O’Raifeartaigh potential to again make the system overconstrained and thus break SUSY.
This model also will have a goldstone fermion (goldstino) once global $N = 1$ SUSY is broken. Nevertheless, should such particles be produced, they will redshift away like radiation. However, the other fermions generally will have mass and this leads to an interesting possibility. These fermionic components could be identified with right-handed neutrinos, for example if the $U(1)$ symmetry of the generalized O’Raifeartaigh model was identified with $B - L$. In this case a leptonic asymmetry can be produced, which can lead to baryogenesis based on the scenario of [15].

Taking the minimum of the Higgs potential gives
\[
dV{d}\phi = 0 = -4a^2M^2\phi + 4a^2\phi^3 + 2(n + 1)\lambda^2m^4 \phi^{2n+1} m^2_{pl} 2^{2n+2}.
\]

Defining $a \equiv M/m_{pl}$, we are interested in the regime $a, \lambda \ll 1$, for which the solution to Eq. (6) is
\[
\langle \phi_{n_{min}}^2 \rangle \approx M^2[1 - \frac{(n + 1)}{2}2^{2n-4}].
\]

At this minima
\[
V_{min} = V(\langle \phi_{n_{min}}^2 \rangle) \approx \lambda^2 a^{2(n+1)}m^4_{pl},
\]
\[
m^2_{\phi} \equiv V''(\phi_{n_{min}}) \approx 8a^2M^2,
\]
and
\[
V_0 \equiv V(\phi = 0) = a^2M^4.
\]

Choosing the scale $M \sim 10^{10-11}$GeV leads to
\[
V_0^{1/4} \approx 10^{5-8}$GeV
\]
\[
m_{\phi} \approx 10^{2-4}$GeV,
\]
which are the desired properties for the thermal inflation zero temperature potential. Moreover, $\lambda$ remains a free parameter along with a choice for the index $n$ of the higher dimensional operator. This implies the value of $V_{min}$ remains at our discretion, and it can be chosen to give the desired scale of the presentday cosmological constant. In particular, for $V_{min} = \rho_\Lambda \approx 10^{-10}eV^4$ it implies the condition $\lambda \approx 10^{-53+8n}$. So, for example, for $n = 2$ it requires $\lambda \approx 10^{-37}$ whereas for $n = 6$, $\lambda \approx 10^{-5}$. It is interesting that both the moduli and cosmological constant problems can be solved by this model, but parametrically neither of these two examples are particularly desirable. $\lambda$ must be highly fine tuned in the $n = 2$ case, and although $\lambda$ is a typical coupling for the lepton sector of the standard model when $n = 6$, the relevant term in the O’Raifeartaigh model Higgs potential is of order $\phi^{14}$. Another undesirable feature of the model in its present form is, since it only respects global SUSY, after symmetry breaking for $\phi$, since $V_{min} \approx 0$, SUSY remains only very weakly broken [16] and so uninteresting for particle physics. Later we will propose a scenario where incorporating this model within a local supersymmetric theory can overcome all these problems, yet preserve those features attractive for solving cosmological problems.

At low-temperature the O’Raifeartaigh potential has the shape and scales necessary for thermal inflation and at $T = 0$ its minima can be chosen to give the scale of the presentday cosmological constant. For thermal inflation, it still must be confirmed that at high temperature, $T > a^{1/2}M$, thermal corrections to the effective potential stabilize $\phi$ at zero. Lowest order finite temperature corrections to SUSY models shift the mass as shown in [17], and this argument can be modified to the generalized O’Raifeartaigh models. Since $\lambda$ is tiny, the dominant high temperature corrections will come in Eq. (5) from the terms $a^2\phi^4$ and $4a^2\phi^2\chi^2$, which lead to high-T terms $\sim a^2T^2\phi^2$ and $\sim a^2T^2\chi^2$. Thus, for $T > a^{1/2}M$ the minimum of the effective potential will be as desired at $\phi > T = \eta > T = \eta > T = 0$.

It is interesting to note that independent of the thermal inflation problem, for an appropriate choice of scales, the potential Eq. (5) can be implemented just to address the cosmological constant problem. In particular, the minimum scale necessary to obtain adequate vacuum energy is $M \sim \Lambda_{QCD}$. An interesting case is when $M \sim 10^6$GeV, the electroweak scale, where for the simplest nonnormalizable potential $n = 2$,
\[
V(\phi_{min} = M) = \lambda^210^{16}eV^4,
\]
which is at the scale of $\rho_\Lambda$ for $\lambda \sim 10^{-13}$. Moreover independent of $\lambda$, at the minimum $m_{\phi} \equiv \sqrt{V''(\phi_{min})} \approx 10^{-3-5}$eV. This is an interesting scale as it is in the neighborhood of the neutrino mass mixing parameters.

After thermal inflation, once the flaton $\phi$ is near its minimum, it will oscillate and thereby enter a reheating phase similar to that after supercooled inflation. The particle production history that develops has the same range of possibilities and outcomes as studied in other thermal inflation works [5–9]. For example, if $\phi$ is a gauge singlet as in Eq. (3), then reheating will create $\phi$-bosons. Also, $\phi$ can couple to charged scalars which can mediate decay into gauge particles. On the other hand, it is possible to easily generalize our O’Raifeartaigh models by letting $\phi$ be in the adjoint representation of some gauge group $G$, while keeping $\chi$ and $\eta$ as singlets of $G$. Thus, making the replacement $\phi^p \rightarrow \text{Tr}(A^p)$, a nonvanishing VEV for $A$ can break $G$ to a set of degenerate minima, although gravity will lift the degeneracy (see below). For example for $G = SU(N)$, $(A)$ can be diagonalized by a $SU(N)$ transformation so $G$ may break to subgroups of the form
\[
H = \prod_i SU(N_i) \times U^p(1),
\]
where $\sum_i(N_i - 1) + p = N - 1$, i.e., $H$ has the same
rank as $G$. This form of O’Raifeartaigh models has more possibilities of dissipating the vacuum energy.

The O’Raifeartaigh type models we have been discussing up to now have global SUSY. In this case, the symmetry breaking considered above does not lead to SUSY breaking at scales of interest to particle physics, since the vacuum energy at the minimum is essentially zero. The full theory is expected to start off locally supersymmetric, thus be a supergravity theory. It is well known that global SUSY models can have many degenerate minima as long as SUSY is unbroken. SUGRA lifts this degeneracy [18] and only one minimum can have zero energy, with the others having negative energy. These results discussed in [18] were explicitly stated to exclude models of the O’Raifeartaigh [12] and Fayet-Iliopoulos [19] type. If O’Raifeartaigh potentials are included, they break SUSY and make positive contributions to vacuum energy. It is thus quite possible that one of the negative vacuum energy SUGRA minima receives an additional positive vacuum energy contribution from the O’Raifeartaigh sector. Thus, while both the (+) and (−) contributions are large, the residual vacuum energy can be small and positive. This could be the true vacuum energy of the universe, and so explain the observed cosmological constant. For example consider the parameters necessary for near balancing vacuum contributions at a scale relevant to particle physics SUSY symmetry breaking, $\sim 10^{-10}$GeV. For the scale considered in Eq. (11), $M \sim 10^{-10}$-11GeV, for $n = 2$ to obtain $V^{1/4} \lesssim 10^9$GeV, it requires $\lambda \lesssim 10^{-5}$, which is a realistic value. It remains a model building challenge to realize this effect through a natural mechanism.

It appears the inclusion of O’Raifeartaigh superpotentials in the full SUGRA has interest for both particle physics and cosmology. While breaking SUSY adequately to generate potentially interesting phenomenological particle spectra, the O’Raifeartaigh potential can also shift the vacuum to a small positive value, generating the cosmological constant and from our above treatment, the same model can solve the moduli problem by permitting realization of thermal inflation. This scenario is promising and it seems worth further developing toward a realistic model. An initial step is to understand the origin of such models from fundamental theories. It is known that various compactifications of string theory have a number of light scalar singlets in their spectrum. For instance many models obtained from type IIB strings via orbifolding $\mathrm{AdS}_5 \times S^5$ lead to such scalars. The form of the superpotential is certainly model dependent. For example, it will depend on the initial string theory, or more generally the initial region of parameter space in M-theory, and details of the compactification. However, the occurrence of scalars are generic, so O’Raifeartaigh potentials can naturally arise in SUGRA and thus lead to our cosmology.

To summarize, in this Letter we have shown that generalized O’Raifeartaigh models can have powerful implications both for cosmology and uniting cosmology with particle physics. Within the compact structure of these models, we have shown that they can solve the moduli problem and potentially lead to a solution of the cosmological constant problem. In an attempt to unify the symmetry breaking necessary to solve these cosmological problems with that necessary to break SUSY in particle physics models, a new interpretation of the presentday accelerating universe emerges providing dark matter and a “balanced” residual vacuum energy. This is an intriguing coincidence of solutions, given that O’Raifeartaigh type models may arise generically from fundamental theories.

While we believe our scenario is provocative, more work needs to be done if it is to be developed into a completely satisfying model. Some way to avoid fine tuning of positive and negative contributions to the vacuum energy, or at least the renormalization of the fine tuning parameters (This is already provided for in the superpotential above the SUSY scale.) would be an important step. Another avenue to follow would be to develop a similar scenario for Fayet-Iliopoulos [19] D-term SUSY breaking.

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