Thermal dependence of the zero-bias conductance through a nanostructure

A. C. Seridonio\textsuperscript{1,2}, M. Yoshida\textsuperscript{3} and L. N. Oliveira\textsuperscript{1(a)}

\textsuperscript{1} Departamento de Física e Informática, Instituto de Física de São Carlos, Universidade de São Paulo
369, São Carlos, SP, Brazil

\textsuperscript{2} ICCMP - International Center for Condensed Matter Physics, Universidade de Brasília - 04513, Brasília, DF, Brazil

\textsuperscript{3} Departamento de Física, Instituto de Geociências e Ciências Exatas, Universidade Estadual Paulista
13500, Rio Claro, SP, Brazil

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Abstract - We show that the conductance of a quantum wire side-coupled to a quantum dot, with a gate potential favoring the formation of a dot magnetic moment, is a universal function of the temperature. Universality prevails even if the currents through the dot and the wire interfere. We apply this result to the experimental data of Sato \textit{et al.} (\textit{Phys. Rev. Lett.}, 95 (2005) 066801).

Nanodevices owe much of their development to the theory of many-body phenomena. Consider, e.g., the single electron transistor (SET), a quantum dot bridging two otherwise independent two-dimensional electron gases \cite{1,2}. The competition between the Coulomb blockade, which bars transport through the dot, and the Kondo screening of the dot magnetic moment by the electron gases \cite{3}, which favors low-temperature conduction, was discussed on blackboards \cite{4} a decade before it surfaced in the laboratory \cite{1}. By the time the first device was developed, quantitatively accurate theoretical results were available. Chiefly important was the universal conductance curve \(G_{\text{SET}}(T)\) for the symmetric Anderson model \cite{5,6}, which was shown to match the temperature dependence of the zero-bias conductances in SETs and analogous devices.

More recently, experiment has leaped ahead of theory. The development of complex structures, such as the side-coupled device \cite{7–12}, has motivated only qualitative predictions. As fig. 1 shows, the current in the side-coupled device is carried by electrons that can either traverse the quantum wire or hop to a quantum dot to skip the central section \cite{12}. A Fano parameter \(q\) \cite{13}, defined below, measures the amplitude for the latter process relative to that for the former. The limit \(q \to \infty\) emulates a SET. For smaller \(q\)'s, the wire bypasses the Coulomb blockade and allows high-temperature conduction. Below the Kondo temperature \(T_K\), the screening of the magnetic moment enhances the electronic flux through the dot and allows interference with the flow along the central portion of the wire \cite{14,15}.

Fig. 1: Side-coupled device. The gate potential \(V_g\) controls the energy \(\varepsilon_d\) of the quantum-dot level \(c_d\). The open circle depicts the Wannier orbital \(f_0\).

Attention to the diversity of experimental findings, we have applied numerical renormalization-group (NRG) tools to an Anderson Hamiltonian modeling the side-coupled device. The resulting essentially exact numerical data for the temperature-dependent conductance \(G_q(T)\) will be detailed elsewhere \cite{16}. Here, we focus the relation between \(G_q(T)\) and the universal curve \(G_{\text{SET}}^q(T)\). Our central result covers the Kondo domain, the set of dot energies and dot-wire couplings that favor the formation of a Kondo screening cloud.

Since the thermodynamical properties of the Kondo crossover are universal functions of the temperature

\(\text{[a]}\)E-mail: luizno@usp.br
scaled by $T_K$ [2,17–25], a mathematical relation between $G_q(T/T_K)$ and the universal function $G^S_{SET}(T/T_K)$ is hardly surprising. Nonetheless, the diversity manifest in conductances that, in contrast with $G^S_{SET}(T)$, rise with temperature and in conductance profiles (fixed-$T$ conductance vs. gate-voltage plots) that display antiresonances [12] rules out a proportionality between $G_q(T)$ and $G^S_{SET}(T)$. Instead, we will show that a linear mapping binds the two functions:

$$G_q\left(\frac{T}{T_K}\right) = \frac{G_2}{2} = \left(G^S_{SET}\left(\frac{T}{T_K}\right) - \frac{G_2}{2}\right) \cos 2\delta,$$  

(1)

where $\delta$ is the ground-state phase shift of the wire, electrons, and $G_2 \equiv 2e^2/h$, the quantum conductance. While the mapping (1) is universal, the phase shift and Kondo temperature are model-parameter dependent. At fixed temperature, the Fano parameter $q$ controls the functional dependence of the conductance on the gate voltage $V_g$. As $q$ grows, valleys in the conductance profiles evolve into plateaus, a result in qualitative agreement with measurements. Most importantly, eq. (1) affords quantitative comparison with experiment; as an illustration, we will present curves that reproduce the temperature-dependent conductances reported by Sato et al. [8]; show that the dot moment was fully screened; and extract $T_K$ and $\delta$ from the data. Our results justify mathematically the authors’ phenomenological treatment of their results.

**Overview.** – Preliminary to the formalism, we present an overview of conduction in the side-coupled device. We consider weak coupling to the wire, so that the dot occupation $n_d$ is nearly conserved, and illustrate the discussion with NRG plots of the temperature-dependent conductance.

The device has three characteristic energy scales, set by i) the coupling to the wire, which broadens the dot levels; ii) the electrostatic barrier $\Delta N$ between adjacent dot occupations $n_d = N - 1$ and $n_d = N$; and iii) the Kondo temperature $T_K$, below which the wire screens the dot moment. The first two scales catch the eye in conductance profiles. The third one defines the thermal regimes $T \gg T_K$ and $T \ll T_K$ displayed schematically in the top and bottom panels of fig. 2a, respectively.

**Left:** $q \gg 1$. If $T \gg T_K$ (top), the Coulomb blockade impedes conduction; to defeat it, the gate voltage must be raised, so that the $n_d = N$ and the $n_d = N + 1$ ground states are nearly degenerate. The conductance profile is hence a sequence of narrow resonances. Upon cooling (bottom), little changes if $V_g$ makes $n_d$ even. For odd $n_d$, however, the Kondo hybridization between the wire and the dot states allows conduction. The conductance $G(V_g)$ alternates between insulating valleys ($G = 0$ for even $n_d$) and Kondo plateaus ($G = G_2$ for odd $n_d$) [26].

**Right:** $q \approx 0$. The pattern is reversed. If $T \gg T_K$ (top), except at the resonant voltages, the flux through the wire is ballistic. On resonance, the strong coupling to the dot blocks conduction, and $G(V_g)$ dips to zero [27]. For $T \ll T_K$ (bottom) and odd $n_d$, the screening cloud blocks the wire. The conductance thus alternates between ballistic plateaus and Kondo valleys [14].

**Center:** $q = 1$. There are now two conduction paths. If $T \approx T_K$ (top), Fano antiresonances near the resonant voltages signal interference between the two currents. Off resonance, the electrons flow only through the wire, and the conductance remains close to $G_2/2$. For $T \ll T_K$ (bottom) and odd $n_d$, again the Kondo hybridization to the wire allows conduction through the dot. The Kondo cloud nonetheless blocks conduction through the wire, so that the off-resonance conductance is again close to $G_2/2$.

For each voltage identified by two crosses, squares, or circles in fig. 2a, fig. 2b displays our NRG results for the conductance as a function of $T$ normalized by $T_K$ —the temperature at which the conductance is half the quantum conductance, $G_q(T = T_K) \equiv G_2/2$. The agreement with the solid lines representing eq. (1) is very good: even at the limits of the Kondo regime (crosses and circles, $T = 50T_K$) the absolute deviations are smaller than 0.01 $e^2/h$.

For $q \gg 1$ (left, $\delta \approx 0$), the device mimics a SET, the mapping (1) reduces to an identity, and the conductance decays from ballistic to zero along the universal curve $G^S_{SET}(T)$. The opposite extreme, $q = 0$ (right, $\delta \approx \pi/2$),

![Fig. 2](67006-p2)
reverses the pattern: ballistic conductance at $T \gg T_K$, and
perfect insulation at $T = 0$. As intuition would dictate, and as
general [8] and mathematical [28] arguments suggest, the
temperature dependence complements the universal function:
$G_q(T) = G_2 - G_{SET}^q(T)$. For $q \sim 1$ (center, $\delta \approx \pi/4$),
three voltages $V_q$ are highlighted. In each case,
the conductance interpolates monotonically the high-
to the low-temperature limits in fig. 2a. For $2\varepsilon_d + U = 0$,
in particular, fig. 2a shows that $G_q(T \gg T_K) = G_q(T = 0) = G_2/2$; the conductance must therefore be constant,
$G_q(T) = G_2/2$, and this is what eq. (1) predicts for $\delta = \pi/4$.

**Model.** – Hamiltonians describing side-coupled
nanostuctures, in the geometry of fig. 1 [12,14,27–29], or other
arrangements [30–32] have appeared in print. While our
analysis could start from any of the former, to keep the
presentation self-contained, we define the alternative
three-component Hamiltonian $H = H_d + H_w + H_\text{w}$. Here, $H_d \equiv H_{\text{q}} n_q + H_{\text{n}} n_q$ models the quantum-dot,
where $n_d \equiv c_d^\dagger c_d$ is the dot-level occupation; $\varepsilon_d$ is the dot energy, controlled by
the potential $V_d$; and $U$ is the Coulomb repulsion.
The second term, $H_w \equiv \sum_k \varepsilon_k c_k^\dagger c_k + (K/N) \sum_k \varepsilon_k c_k^\dagger c_k^\dagger$, models the wire, of length $L$. The energies $\varepsilon_k$, measured from the Fermi level, form a symmetric, structureless, half-
filled conduction band of width $2D$ comprising $N$ levels
separated by the splitting $\Delta \equiv 2D/N$ (see footnote 1). The scattering
Kondo model describes (the fixed point) potential applied
to the wire [12].

The last component, $H_w \equiv \sum_k (V_k/\sqrt{N}) c_k^\dagger c_d + H_c$, couples the wire to the dot. The relatively small thermal energies $k_B T \ll D$ authorize retention of the two leading terms in the expansion of the coupling $V_k$ in powers of $\varepsilon_k$ (see footnote 2): $V_k \approx V_k + \varepsilon_k (\Delta V/\delta \varepsilon_k) k$. We rewrite this approximation as $V_k = V + \pi p q V \varepsilon_k$, where $p \approx 1/2D$, to define the coupling $V$ and the Fano
parameter $q$ [13]. Following NRG tradition, we introduce the shorthand $f_0 \equiv \sum_k \varepsilon_k/\sqrt{N}$. For future reference, we note that, in this notation, the (particle-hole) symmetric ($q = K = \varepsilon_d + U = 0$) Anderson Hamiltonian reads

$$H_0^S = \sum_k \varepsilon_k c_k^\dagger c_k + V(f_0 c_d + H_c) - \frac{U}{2} (n_d^t - n_d)^2. \quad (2)$$

The conductance is more easily computed on a basis
constituted of $f_0$ and $N - 1$ other conduction
operators $a_p \equiv \sum_k a_{pk} c_k$, where $p = \pm 2\pi n_p/\sqrt{N}$ [$n_p = -N/2 + 1, \ldots, N/2 - 1$], such that $f_0 f_0^\dagger + a_p a_p^\dagger = \delta_{pp}$ (see footnote 3). With the shorthand
$f_1 = \sum_p a_p/\sqrt{N} - 1$, the model Hamiltonian becomes

$$H_A = \sum_p \varepsilon_p a_p^\dagger a_p + K_L M f_0 f_0^\dagger, \quad (3)$$

where $N_q \equiv \sqrt{1 + (\pi p V q)^2}$, $N_q d_q \equiv f_0 + \pi p q V c_d$; $t_0 \equiv D N_q/\sqrt{3}$; and the energies $\varepsilon_k$ are the conduction energies $\varepsilon_k$ phase shifted by $\pi/2$, i.e., $\varepsilon_k \equiv \varepsilon_k - \Delta/2$.

If $K \to \infty$, the scattering potential decouples $f_0$ from the other states, freezes its occupation at $f_0 f_0^\dagger = 0$, and
forces the current through the dot. The condition $2\varepsilon_d + U = 0$ reduces $H_A$ to the symmetric-SET Hamiltonian

$$H_{SET}^S = \sum_p \varepsilon_p a_p^\dagger a_p + V(f_0 c_d + H_c) - \frac{U}{2} (n_d^t - n_d)^2, \quad (4)$$

where $V_q \equiv \pi q V/\sqrt{3}$. Given the analogous definitions $f_0 \sim \sum k \varepsilon_k$ and $f_1 \sim \sum p a_p$, we see that only the phase shifted energies $\varepsilon_k = \varepsilon_k - \Delta/2$ distinguish $H_{SET}^S$ from $H_0^S$.

The first term within parentheses on the right-hand side of eq. (3), which couples the conduction states $f_1 \sim \sum p a_p$ to $d_q \sim f_0 + \pi p q V c_d$, is the mathematical expression of the two conduction paths in fig. 1, one of which runs through the central wire-orbital $f_0$, and the other, with relative amplitude $\pi p q V$, through the dot orbital $c_d$. To flow from the left to the right side of the wire, the current must traverse $d_q$. Accordingly, the Linear-
Response Theory [33] shows that $\rho_q$, the spectral density
for the operator $d_q$, controls the zero-bias conductance:

$$G_q(T) = G_2 N_q^2 \int_{-D}^{D} \frac{\rho_q(\epsilon, T)}{\rho} \left[ \frac{-\partial f(\epsilon)}{\partial \epsilon} \right] d\epsilon. \quad (5)$$

Here $f(\epsilon)$ is the Fermi function, and

$$\rho_q(\epsilon, T) = \frac{1}{2} \sum_{mn} e^{-\beta E_m} | \langle m | d_q^\dagger | n \rangle |^2 \delta(\epsilon_{m,n} - \epsilon), \quad (6)$$

where $Z$ is the partition function, and $\epsilon_{m,n} \equiv E_m - E_n$.

**Analysis.** – We are chiefly interested in the Kondo
regime, the set of temperatures and model parameters favoring unitary dot occupation, i.e., such that the energies $|\varepsilon_d| (|\varepsilon_d + U|)$ to remove the dot electron (add a second electron) dwarf the energy $k_B T$ and dot-level width $\Gamma = \pi p V^2$. The Kondo temperature $T_K$ then sets the energy scale.

At high temperatures, $T \gg T_K$, the Hamiltonian (3) lies close to an unstable local-moment fixed point (LM), which comprises a noninteracting spin-1/2 variable (the dot spin) decoupled from a conduction Hamiltonian [34]

$$H_{LM} = \sum_k \varepsilon_k c_k^\dagger c_k + K_L M f_0 f_0^\dagger, \quad (7)$$

where the effective scattering potential $K_L M$ depends on the model parameters. For $H_A = H_0^S$ ($H_{SET}^S$, in particular, $K_L M = 0$ ($K_L M \to \infty$).

Diagonalization of the quadratic form (7) yields $N$
eigenoperators $g_k$ and eigenvalues $\varepsilon_k = \varepsilon_k - \delta_{LM} \Delta/\pi$:}

$$H_{LM} = \sum_k \varepsilon_k g_k^\dagger g_k. \quad (8)$$

Near the Fermi level, the phase shifts $\delta_{LM}$ are uniform [34].
Table 1: Properties of the symmetric Hamiltonians.

| $H_A$ | $\delta_{LM}$ | $g_k$ | $\phi_0$ | $D_{\phi_1}$ | $\rho_{\phi_0}$ | $\rho_{\phi_1}$ |
|-------|----------------|------|----------|--------------|----------------|----------------|
| $H_0^S$ | 0 | $c_k$ | $f_0$ | $D_{f_1}$ | $\rho^S_0$ | $\rho^S_{\phi_0}$ |
| $H_{SET}^S$ | $\pi/2$ | $a_k$ | $f_1$ | $\sqrt{N} \sum_p \epsilon_p a_p$ | $\rho^S_{\phi_1}$ | $\rho^S_{\phi_0}$ |

Table 2: Conductance at the two fixed points.

| Fixed Point | Phase | $G_q$ | $G^S_0$ | $G^S_{SET}$ |
|-------------|-------|-------|---------|-------------|
| LM          | $\delta + \pi/2$ | $G_2 \sin^2 \delta$ | $G_2$ | 0 |
| FL          | $\delta$ | $G_2 \cos^2 \delta$ | $G_2$ | 0 |

In analogy with the definitions $f_0 \sim \sum_k c_k$ and $f_1 \sim \sum_k c_k \epsilon_c k$, we define the mutually orthogonal combinations of eigenoperators $\phi_0 = \sum_k g_k / \sqrt{N}$ and $\phi_1 = \sqrt{3N} \sum_k (\epsilon_c / D) g_k$. For any pair of constants $\alpha_0$ and $\alpha_1$,

$$\alpha_0 f_0 + \alpha_1 f_1 = \beta_0 \phi_0 + \beta_1 \phi_1,$$

where $\beta_0$ and $\beta_1$ are linear combinations of $\alpha_0$ and $\alpha_1$, with coefficients fixed by $\delta_{LM}$.

Table 1 collects results for the symmetric (eq. (2)) and the symmetric-SET (eq. (4)) Hamiltonians. While columns 3–5 follow from the definitions of $g_k$, $\phi_0$ and $\phi_1$, columns 5 and 6 require explanations. For $H_A = H_0^S \lbrace i.e., q = K = 2 \epsilon_d + U = 0 \rbrace$, the operator $d_q$ reduces to $f_0 = \phi_0$. Its spectral density $\rho_0 = 2 \omega_T$ is therefore equal to $\rho_{\phi_0}$.

For $H_A = H_{SET}^S$, $d_q \rightarrow -\pi \rho q V c_d / N_q$. To relate its spectral density to $\rho_{\phi_1}$, choose two eigenstates $|m\rangle$, $|n\rangle$ of $H_{SET}^S$ with $E_m, E_n \approx \hbar \omega_T \ll D$. In the identity

$$|m| |n\rangle \sum_p \frac{a_p H_{SET}^S |n\rangle}{\sqrt{N}} = \frac{D(m | \phi_1 | n)}{\sqrt{3}} + V_q |m | c_d | n\rangle,$$

the left-hand side is then much smaller than each matrix element on the right. This shows that $|m | \phi_1 | n\rangle \approx -\pi \rho q V |m | c_d | n\rangle$, a result equivalent to $|m | \phi_1 | n\rangle = -N_q |m | d_q | n\rangle$, and hence to the last column in table 1.

As the system is cooled, the wire electrons screen the dot moment, and $H_A$ crosses over from the LM to a stable frozen-level fixed point (FL). The latter is but the conduction band resulting from letting $K_{LM} \rightarrow 1/(\pi^2 \rho^2 K_{LM})$ in eq. (7). Diagonalization of the FL Hamiltonian yields $N$ eigenvalues $\epsilon_k = \epsilon_k - \Delta \omega / \pi$, where, in conformity with Friedel’s sum rule [35], the phase shift $\delta = \delta_{LM} - \pi/2$.

The fixed-point physical properties are independent of temperature and energy. In particular, the LM and FL conductances are trigonometric functions of the phase shift. The expressions, which result from an extension of Langreth’s argument [35], are recorded in table 2.

The effective antiferromagnetic interaction $H_J = JS \sigma_{\mu \nu} \phi_0 \phi_0 \phi_0$, [18,34,36], between the dot spin $S$ and the spin of the localized orbital $\phi_0$, drives the Hamiltonian from the LM to the FL. The NRG trajectory is universal: scaled by $\hbar k_B T_K$, the eigenvalues of $H_A$ are universal, and so are the corresponding eigenstates on the basis of the $\{g_k\}$ [17,34]4. The spectral densities $\rho_0$ and $\rho_1$, in table 1 are therefore universal functions of the ratios $\hbar k_B T_K$ and $\epsilon / \hbar k_B T_K$. To highlight these findings, we define the scaled energy $\hbar \omega \equiv \epsilon / \hbar k_B T_K$ and temperature $T \equiv \hbar k_B T_K$.

Next, we turn to the asymmetric Kondo-domain Hamiltonians. To relate the conductance $G_q$ to the universal function $G^S_{SET}$, we add to $H_A$ an infinitesimal harmonic perturbation, frequency $\omega$, coupling the spectrum of $H_A$ to an auxiliary orbital $\tilde{\sigma}$ at the Fermi level:

$$H_\eta = \eta \tilde{\sigma} d_q e^{-i\omega t} + H.c.$$

The golden rule shows that the spectral density $\rho_\eta$ is the response function for the thermally averaged transition rate $j_{\eta \rightarrow \Lambda}$ induced by $H_\eta$:

$$\langle j_{\eta \rightarrow \Lambda} (\omega) \rangle_T = (\pi \eta^2 / \hbar) \rho_\eta (\hbar \omega, T).$$

Consider, then, the perturbative effects of $H_\eta$ upon the Kondo crossover [18]. Close to a Fermi-liquid fixed point $H^*$, that is, for $H = H^* + \delta H$, one can always construct an effective Hamiltonian $H_{eff}$ that reproduces the spectrum of $H$ to linear order in $\delta H$. Here, to follow a pedestrian route, we subject the sum $H_A + H_\eta$ to the Schrieffer-Wolff transformation: $H_A^\eta + H_{eff}^\eta \equiv e^{\delta} (H_A + H_\eta) e^{-\delta}$ [36]. Besides substituting the spin-spin interaction $H_J$ for the dot-wire coupling $H_{dot}$, this casts eq. (11) in the form

$$H_{eff}^\eta = (\eta / N_q) \tilde{d}^\dagger (\alpha \tilde{\sigma} \tilde{c}_d + \alpha_0 f_0 + \alpha_1 f_1) e^{-i\omega t} + H.c.$$

where $\alpha_d, \alpha_0$, and $\alpha_1$ depend on $\omega$, $\epsilon_d$, $U$, and $K$.

The unperturbed effective Hamiltonian $H_A^\eta$ commutes with $n_d$. Of its eigenstates, only those with $n_d = 1$ are energetically accessible at the LM. It is safe to disregard the perturbation proportional to $\alpha_d$ in eq. (13), which couples them to the subspaces $n_d = 0$ and $n_d = 2$.

The other two perturbations conserve $n_d$ and are important even at very low energies. It is convenient to project them upon $\phi_0$ and $\phi_1$, because the spectral densities for $f_0$ and $f_1$ are phase-shift dependent. Substitution of eq. (9) for $\alpha_0 f_0 + \alpha_1 f_1$ brings eq. (13) to the form

$$H_{eff}^\eta = (\eta / N_q) \tilde{d}^\dagger (\beta_0 \phi_0 + \beta_1 \phi_1) + H.c.$$

In the (LM to FL) crossover, just as the Kondo Hamiltonian $H_A^\eta = e^{\delta} H_A e^{-\delta}$ is equivalent to $H_A$ and the diagonalization of $H_A^\eta$ yields the physical properties for $H_A$, the effective perturbation $H_{eff}^\eta$ is equivalent to $H_\eta$ and

4Projected on the $\{g_k\}$ basis, $H_A^\eta / \hbar k_B T_K$ has universal eigenvalues and eigenvectors. Thermally averaged expectation values involving only the operators $\phi_0 \sim \sum_k g_k$ and $\phi_1 \sim \sum_k \epsilon_c k g_k$ are therefore universal functions.
application of the golden rule to $H^\text{eff}_q$ yields the transition rate induced by $H_q$. Out of the terms then resulting from eq. (14), only those proportional to $|\langle m|\phi_q^0|m\rangle|^2$ and $|\langle m|\phi_q^1|m\rangle|^2$ contribute to $G_q$ (see footnote 5). The cross-terms disregarded, the last two columns in table 1 lead to

$$\langle j \tau \rightarrow A \rangle_T = \frac{\pi\eta^2}{h} \left( \frac{\beta_2^2}{N_q^0} \rho_0^S + \beta_2^2 \rho_{SET}^S \right).$$

Comparison with eq. (12) then shows that

$$\rho_q(\mathcal{E}, T) = (\beta_2^2/N_q^0) \rho_0^S(\mathcal{E}, T) + \beta_2^2 \rho_{SET}^S(\mathcal{E}, T),$$

and eq. (5) yields an expression for the conductance:

$$G_q(T) = \beta_2^2 G_0^S(T) + N_q^0 \beta_2^2 \cos^2 \delta.$$  

Substitution of the expressions in the LM line of table 2 for $G_q$, $G_0^S$, and $G_{SET}^S$ shows that $\beta_2^2 = \sin^2 \delta$, while the expressions in the FL line show that $N_q^0 \beta_2^2 = \cos^2 \delta$. With this, eq. (17) becomes

$$G_q(T) = G_0^S(T) \sin^2 \delta + G_{SET}^S(T) \cos^2 \delta,$$

and to complete the derivation of eq. (1), we only have to recall that $G_0^S = G_2 - G_{SET}^S$.

**Discussion.** – The phase shift $\delta$ and Kondo temperature $T_K$ in eq. (1) depend on the model parameters. Almost invariably, they have to be computed numerically, because the perturbative expressions for $\delta$ and $T_K$ are accurate only in corners of the parametrical space [3, 17, 20, 34, 37]. Exceptions are the particle-hole symmetric Hamiltonians $H_0^H$ (eq. (2)) and $H_0^S$ (eq. (4)), for which $\delta = \pi/2$ and $\delta = 0$, so that eq. (1) reduces to $G_q(T) = G_2 - G_{SET}^S(T) = G_0^S(T)$ and $G_q(T) = G_{SET}^S(T)$, depicted in the right and left panels of fig. 2b, respectively.

In other regions of the Kondo domain, $\delta$ lies between $-\pi/2$ and $\pi/2$. In the Kondo crossover, the conductance on the left-hand side of eq. (1) changes by less than $G_2$, from $G_0 \sin^2 \delta$ to $G_2 \cos^2 \delta$. The rise or decay of $G_q(T)$ is centered at $G_2/2$ and proportional to $G_{SET}^S - G_2/2$.

The central panel of fig 2b shows examples. The solid lines are eq. (1) with $\delta$ (table 3) extracted from the FL

5The contribution to $\rho_q$ of the cross-terms, $\rho_{01}(\epsilon) = \sum_m e^{-\beta E_m} f(\epsilon_m) \langle m|\phi_q^0|\epsilon\rangle \langle \epsilon|\phi_q^1|m\rangle \delta(\epsilon - \epsilon_m) + c.c.$, is universal (see footnote 4). We can therefore choose $H_0^A = H_0^H$ to compute it. Given that $H_0^H$ is invariant under particle-hole transformations, while $\rho_{01}(\epsilon) \rightarrow -\rho_{01}(-\epsilon)$, we see that $\rho_{01}(\epsilon)$ is an odd function of the energy; substituted for $\rho_q(\epsilon)$ in eq. (5), it makes no contribution to $G_q$.  

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Table 3: Parameters for the five runs in fig. 2b. $U = 0.20D$

| $-\varepsilon_A/D$ | $V/D$ | $-K/D$ | $q$ | $\delta/\pi$ | $k_BT_K/D$ |
|---------------------|--------|---------|-----|-------------|------------|
| 0.10                | 3.6×10^{-4} | 100.0  | 100 | 0.00        | 1.3×10^{-5} |
| 0.17                | 0.021  | 0.315   | 1   | 0.22        | 1.1×10^{-5} |
| 0.10                | 0.021  | 0.315   | 1   | 0.25        | 1.1×10^{-9} |
| 0.022               | 0.021  | 0.315   | 1   | 0.28        | 1.1×10^{-5} |
| 0.10                | 0.056  | 0.000   | 0   | 0.5         | 1.4×10^{-5} |

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Fig. 3: (Colour on-line) Comparison with experiment [8]. The crosses and squares are the conductances $G$ at the indicated gate potentials $V_g$; the temperatures measured below 100 mK carry large uncertainties [8]. The dash-dotted and solid lines depict eq. (19). The inset shows that, for each $V_g$, the appropriate $T_K$ straightens the plot of $G$ vs. $G$, and yields eq. (19).

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The central panel of fig 2b shows examples. The solid lines are eq. (1) with $\delta$ (table 3) extracted from the FL

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Comparison with experiment. – To allow for the inevitable background currents, we exploit the linearity in eq. (1). Given a set of $N$ experimental pairs $\{G_i, T_i\}$, from a trial Kondo temperature $T_K'$ we generate the dimensionless temperatures $T_i = T_i/T_K'$ ($i = 1, \ldots, N$). Next, we invert the function $G_i(T)$ to determine the universal conductance $G_{s_i}$ for each $T_i$. If the plot of $G_i$ vs. $G_{s_i}$ is straight, $T_K'$ is the Kondo temperature. If it is not, we iterate. Visual inspection (numerical evaluation of the curvature) determines $T_K'$ within 10% error (to an accuracy limited only by the experimental dispersion).

As an example, the inset of fig. 3 treats the conductances in fig. 3b of ref. [8], measured with the gate potentials $V_g = -795$ mV (squares) and $-775$ mV (crosses), and yields the
Kondo temperatures $T_K = 850 \text{ mK}$ and $1230 \text{ mK}$, as well as the linear relation

$$G(T) = 1.9e^2/h - 0.5G_{\text{SET}}^S(T).$$ (19)

For $T \gg T_K$, $G_{\text{SET}}^S(T) \rightarrow 0$, so that $G \rightarrow 1.9e^2/h$, close to the measured off-resonance conductance $(1.8e^2/h)$ [8].

The agreement with the solid and the dash-dotted lines representing eq. (19) in fig. 3 shows that the data are in the Kondo regime and the contact resistance is negligible. There is, however, a background current: at $T = 1$, eq. (19) yields $G(T = 1) = 1.4e^2/h$; the excess $G_0 = 0.4e^2/h$ over $G(T) = e^2/h$, predicted by eq. (1) is the background conductance [8]. If we define $\tilde{G} \equiv G - G_0$, then eq. (19) takes the universal form $\tilde{G} - e^2/h = -0.5(G_{\text{SET}}^S - e^2/h)$, from which we find the shift: $\cos 2\delta = -0.5(\delta \approx \pi/3)$.

In conclusion, we have shown that, in the Kondo regime, measured from $G_2/2$ and scaled by $\cos 2\delta$, the conductance of side-coupled devices is a universal function of $T/T_K$. Our application to experimental data identified unequivocally the measured thermal dependences with Kondo screening; detected a background current; and determined the ground-state phase shift.

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