Quantum processing by adiabatic transfer through a manifold of dark states

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We consider a network whose nodes are electromagnetic cavities, each coupled to a single three-level atom. The nodes are connected by optical fibers. Each atom is addressed by a control laser, which along with the cavity field drives atomic transitions. The network can be in the form of chain or two and three dimensional arrays of $N$-cavities connected by $N_B$ fibers. Following the work on two-cavity system by Pellizzari, we find that under certain conditions, the system possesses two kinds of dark states. The first kind are $N$ states corresponding to atomic excitations at each node and these are our logical states for quantum processing. The second kind are $N_B$ degenerate dark states on pairs of sites connected by a fibre. By manipulating intensities and phases of control lasers on the cavities, one can pass adiabatically among these dark states due to their degeneracy. This network operates as a $N$-level quantum system in which one can generate computationally useful states by protocols of external controls. We obtain numerical results for small chains and lattices to demonstrate some quantum operations like the transport of states across the array, generation of W-states and Fourier-like states. We also discuss effects of dissipation and limitations of the model.

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A scalable quantum network is one of the most desired goals for quantum information processing [1–4]. A quantum network consists of nodes which are connected by communication channels. The nodes contain qubits and allow for storage and local processing of quantum information. The communication channels are used to transfer quantum states as well as to create quantum entanglement among nodes. In the past two decades several schemes have been proposed to realize quantum networks, which exploit a variety of physical principles viz optical processes for polarization qubits, ion arrays in laser traps, electromagnetic cavities with trapped atoms, Josephson junction arrays, spin and charge qubits in quantum dots and schemes using NMR [5–7]. The merit of a scheme is guided by several criteria, chief among them being: (i) the ease of physical implementation (ii) robustness against noise and physical defects (iii) scalability to larger networks and (iv) simpler and flexible controls to guide quantum processes for desired ends.

Here we consider a network whose nodes are high-finesse electromagnetic cavities, each coupled to a single atom which has three suitable levels for quantum processing. The cavities are connected by optical fibers. Each atom can be addressed by a control laser. Networks based on cavity QED have been considered by a number of workers in the past, as this system exploits the long term memory and ease of processing of atomic qubits along with efficient communication with photons [8–19, 21, 22]. Experimentally also there has been a remarkable progress which makes these networks amenable to implementation in a variety of ways exploiting besides conventional technologies like optical microcavities, nano-fibre cavities [23–26]. Our proposal is derived from a scheme proposed by Pellizzari [9] for a system of two cavities coupled by a fibre. This scheme achieves quantum state transfer between two three-level atoms by manipulating intensities of the two control lasers. It uses adiabatic passage through a dark state discussed below.

We show that similar dark states also exist in a network of $N$ cavities arranged in a lattice with nearest neighbor cavities connected by $N_B$ fibers. Our considerations apply to many kinds of lattice in one to three dimensions. The full quantum mechanics of the network is very complex, but due to conservation of an excitation number it can be divided into sectors. All our operations and considerations are restricted to a low excitation sector. In this subspace, one finds that there are two kinds of dark states. The first kind are $N$ states that occur on each node involving only the atomic state. We shall use these as the computational states. The other kind are $N_B$ degenerate dark states in which states of atoms in the neighboring cavities and the photonic state in the connecting fibre are involved. By manipulating the intensities and phases of control lasers, one can pass adiabatically through these dark states and create all kinds of superpositions among the first kind of $N$ states. We show that this allows us to execute several computational tasks, though in a way different from the standard application of one-qubit and two-qubit gates.

In Fig. 1, we set up the notation by showing two nodes coupled by a fibre. The atomic levels are denoted by $|a_0\rangle$, $|a_1\rangle$ and $|b\rangle$. One cavity mode is coupled to the transition $|a_1\rangle \leftrightarrow |b\rangle$ and has a frequency $\omega_{a1}$. The external laser treated classically has a frequency $\omega_1$, and couples to the transition $|a_0\rangle \leftrightarrow |b\rangle$ with a Rabi frequency $\Omega$ which is a complex quantity. We first consider the ideal situation in which all dissipative effects including the spontaneous decay of the atomic levels are ignored. The dissipation will be considered later. The Hamiltonian of a single
atom-cavity system is $H = H_0 + H_I$, with
\begin{align*}
H_0 &= \hbar \omega_c \left( C^\dagger C + \frac{1}{2} \right) + \sum_{a=a_0,a_1,b} \hbar \omega_a |a\rangle \langle a|, \\
H_I &= \hbar g \left( |a_1\rangle \langle b | C^\dagger + C |a_1\rangle \right) - \hbar \left[ \Omega e^{-i\omega_L t} |b\rangle \langle a_0| + \Omega^* e^{i\omega_L t} |a_0\rangle \langle b| \right],
\end{align*}
where $C^\dagger$ is the creation operator for the cavity mode. We take the laser to be detuned from the atomic transition $|\Delta| \gg \Omega(t)$, $g$ with $\Delta = \omega - \omega_a + \omega_0$. The detuning satisfies the Raman condition $(\omega_c - \omega) = (\omega_a - \omega_a)$. The first two, to be termed as P-states, are atomic states decoupled by the laser interaction in a trivial way. The second one is regarded as a dark state as it involve no states with cavity photons. This state is used in adiabatic transfer of atomic states \[ \text{from cavity 1 to cavity 2 by varying laser intensities through parameters } s_1 \text{ and } s_2. \]

Now we go on to consider lattices of coupled cavities as described above. Though these considerations apply to chains and other lattices in two and three dimensions, here we shall present explicit results for linear chain and square lattice. So we write below the effective Hamiltonian generalized from the two-cavity case, for a square lattice.

\begin{equation}
H = \sum_{i=1}^N \left[ s_i(t) |a_1, 1\rangle \langle a_0, 0| C_i + w(C_i + C_i + C_{i+\delta_x}) X_i^1 + w(C_i + C_i + C_{i+\delta_y}) Y_i^1 \right] + H.c.,
\end{equation}
where $i + \delta_x$ and $i + \delta_y$ denote the right and upper neighbours of the site $i$ respectively. $X_i^1(Y_i^1)$ create photons in fibers connecting sites $i$ and $i + \delta_x(i + \delta_y)$. The basis set for the wave function is written in the following notation, $\prod_{i=1}^N |\mu_i, n_i\rangle \prod_{b=1}^{|M_b|} |m_b\rangle$, where $\mu_i$ takes values 0 and 1, $n_i$ denotes the photon number in $i^{th}$ cavity and $m_b$ denotes the photon number in the $b^{th}$ fibre. The quantity $M = \sum_i |1 - \mu_i \rangle + n_i + \sum_b m_b$ is conserved by the Hamiltonian. We shall work in the $M = 1$ sector, which has either zero photons, one atom in $a_0$ state and the rest in $a_1$ state or with one photon and all atoms in $a_1$ state. Thus only the following basis functions occur, which we denote by a condensed notation. $|p_i\rangle = |a_1, 0\rangle_{1|a_0, 0\rangle_2} |a_0, 0\rangle_{3|a_1, 0\rangle_4 ... |a_0, 0\rangle_{N}|a_0, 0\rangle_{N+1}|0\rangle_b ... |0\rangle_B$; $|q_i\rangle = |a_1, 0\rangle_{1|a_0, 0\rangle_2} |a_0, 0\rangle_{3|a_1, 0\rangle_4 ... |a_0, 0\rangle_{N}|a_0, 0\rangle_{N+1}|0\rangle_b ... |0\rangle_B$; $|f_{i+\delta_x}\rangle = |a_1, 0\rangle_{1|a_0, 0\rangle_2} |a_0, 0\rangle_{3|a_1, 0\rangle_4 ... |a_0, 0\rangle_{N}|a_0, 0\rangle_{N+1}|1\rangle_{i+\delta_x} ... |0\rangle_B$, with $\alpha = x, y$. The general wave function in this sector can be written as
\begin{equation}
|\Psi(t)\rangle = \sum_{i} \left[ (A_i(t)|p_i\rangle + B_i(t)|q_i\rangle + F_{i+\delta_x}(t)|f_{i+\delta_x}\rangle \right] + F_{i+\delta_y}(t)|f_{i+\delta_y}\rangle \right]
\end{equation}
The equation of motion are
\begin{align*}
\frac{i}{\hbar} \frac{\partial}{\partial t} A_i(t) &= s_i^* (t) B_i(t) \\
\frac{i}{\hbar} \frac{\partial}{\partial t} B_i(t) &= s_i (t) A_i(t) + w \left[ F_{i-\delta_x} + F_{i+\delta_y} + F_{i-\delta_y} \right] \\
\frac{i}{\hbar} \frac{\partial}{\partial t} F_{i+\delta_x}(t) &= w \left[ B_i(t) + B_{i+\delta_x}(t) \right] \\
\frac{i}{\hbar} \frac{\partial}{\partial t} F_{i+\delta_y}(t) &= w \left[ B_i(t) + B_{i+\delta_y}(t) \right].
\end{align*}
We again look for ‘Dark states’ corresponding to zero energy. It is easy to check that the state,
\begin{equation}
D_{i+\delta_x} = \left[ \frac{1}{s_i} |p_i\rangle - \frac{1}{w} |f_{i+\delta_x}\rangle + \frac{1}{s_{i+\delta_x}} |p_{i+\delta_x}\rangle \right].
\end{equation}
is an eigenstate of zero energy. Number of these states is equal to $B$. We have here a manifold of degenerate ‘Dark states’ termed as D-states. When the cavity-fibre coupling is large, $w >> \{s_j\}$, the dark state is largely a superposition of two atomic excitations. One would like to operate the network in this regime.

The key point of this paper is that by adiabatic passage through the D-states one can generate any linear combination of P-states and pass from one combination to another. The D-states are not orthogonal, so it is difficult to treat the dynamics analytically for large $N$. However their nature helps us devise protocols for control lasers which take us from an initial state to a target state. Here all operations are controlled externally, unlike most other schemes where a particular unitary transformation depends on fixing a precise time for the Hamiltonian evolution.

Since this sector has a single atomic excitation ($a_1$ to $a_0$), it does not provide us with single node or two-node qubits as in conventional networks [1]. On the other hand it offers us a highly manipulable $N$-state quantum system, in which several computationally useful states can be generated by simple interpretation. We can associate sites of the lattice with numbers $x$ from 1 to $N$. For parallel processing, the state used in algorithms like those of Shor [29] or Grover [30] is: $|\psi\rangle = \frac{1}{\sqrt{N}} \sum_{x=1}^{N} |x\rangle$. For this state and its Fourier transform $\hat{Q} = \frac{1}{\sqrt{N}} \sum_{x=1}^{N} |x\rangle \exp(i\hat{Q}_{\theta}(x))$, where $\hat{R}(x)$ denotes the site site assigned $x$, protocols can be devised which seem easy to implement, as shown below.

We now present some typical results of numerical simulations of Eqs.(7). We use parameters that are commonly achieved in recent experimental setups [11, 23, 25, 28].

The time variation of Rabi frequencies is of the form $\tan(r(t + t_0))$. In Fig. 2 we show a successive transfer of atomic excitation $a_0$ on the nodes of a chain of six sites. The protocol for the variation of Rabi frequencies...
are shown in Fig. 2(a). The probabilities |A_i|^2 for states |p_i⟩ are shown in Fig. 2(b). Fig. 2(c) takes an initial state (|p1⟩ + |p2⟩)/√2 to states of the form (|p1⟩ + |p2⟩)/√2 as shown by magnitudes of amplitudes in Fig. 2(d). In Fig. 3, we show the protocol (Fig. 3(a)) and results for the generation of states |ψ⟩ and |Q⟩ for a chain of eight nodes. From the initial state |p1⟩ our protocol generates a state of equal amplitude on sites 2 to 8 as seen in Fig. 3(b) and equispaced phases θ_j as seen in Fig. 3(c). This requires taking Rabi frequencies in the form |Ω_i⟩e^{iθ_0}. If we suppress the phases, one obtains the |ψ⟩ state. In Fig. 4, results on a lattice of 9 sites is exhibited. The protocol shown in Fig. 4(a) takes an initial state |p1⟩ to a superposed state in which the magnitudes of A_1 and A_3 have one value and the remaining amplitudes are equal in magnitude with another value as seen in Fig. 4(b). The phases of these amplitudes are equispaced as seen in Fig. 4(c). The protocol of Fig. 4(d) achieves a transfer of a superposed state (|p1⟩ + |p2⟩)/√2 to (|p3⟩ + |p4⟩)/√2 as seen by the magnitudes of the amplitudes in Fig. 4(e). This demonstrates the control and the flexibility of this scheme.

The effect of dissipation is a key feature for the viability of quantum processing. The main sources of dissipation are: (i) Spontaneous decay of the level |b⟩. (ii) Loss of photons from cavities mode. (iii) Decay of the photons in fibers. (iv) Motion of atoms. The effects of (i) and (ii) are minimized in this scheme as the dark states used involve only states with zero number of photons in the cavities and no population in upper levels |b⟩. However, this is contingent on doing operations adiabatically and in manner that the system does not transfer out of M = 1 subspace. The decay of photons in fibers should be minimized by having high quality short fibers and by increasing the cavity-fibre coupling, as that decreases the weightage of states with photons in fibers.

FIG. 5: (color online) Effects of dissipation are shown through fidelity of transfer across a chain of four sites for two cavity-fibre couplings (a) w = 5 g and (b) w = 10 g and a range of cavity decay rates κ. Black solid line corresponds to κ_f = 0, red dashed line is represents to κ_f = 0.01 g and cyan dotted line is represents to κ_f = 0.1 g and γ = 0.1 g. [1] M. A. Nielsen and I. L. Chuang, Quantum Computation and Quantum Information (Cambridge University Press, Cambridge, UK, 2000).
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