Fluctuations and phases in baryonic matter

[LB, Kaiser and Weise, Eur. Phys. J. A 57 (2021)]

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Motivation

- QCD vacuum characterized by confinement and spontaneously broken chiral symmetry

- Liquid-gas phase transition to nuclear matter at $\mu = 923\text{MeV}$

- At $\mu \sim 2.6\text{GeV}$ perturbative QCD results imply quark and gluon d.o.f. in color superconducting phase

  [Alford, Rajagopal and Wilczek, Nucl. Phys. B 537 (1999)]

  → **Nature of transition from nuclear matter to color superconductor still unknown**

- Essential for neutron stars with central densities $n \sim 5 – 6\, n_0$

  [Legred et al., Phys. Rev. D 105 (2022)]
Motivation

- For large $\mu$ lattice QCD unavailable because of sign problem

- Chiral effective field theory only valid up to $n \lesssim 2n_0$
  [Holt, Rho and Weise, Phys. Rept. 621 (2016)]

- Chiral model calculations in mean-field approximation find first-order phase transition from spontaneously broken to restored chiral symmetry
  [Rößner et al., Nucl. Phys. A 814 (2008)]

→ Analyse impact of fluctuations beyond mean-field on possible chiral phase transition
  [Drews and Weise, Prog. Part. Nucl. Phys. 93 (2017)]
Chiral nucleon-meson model

- Chiral theory of fermion doublet \( \Psi = (p, n) \) [Floerchinger and Wetterich, Nucl. Phys. A 890–891 (2012)]

- Fermions interacting via chiral boson fields \( \phi = (\sigma, \pi) \), with heavy scalar \( \sigma \) and pion \( \pi \)

\[
\mathcal{L} = \bar{\Psi} \left[ \gamma_{\mu} \partial_{\mu} + g(\sigma + i\gamma_5 \mathbf{T} \cdot \pi) \right] \Psi + \frac{1}{2} \left( \partial_{\mu} \sigma \partial_{\mu} \sigma + \partial_{\mu} \pi \cdot \partial_{\mu} \pi \right) + \mathcal{U}(\sigma, \pi) + \Delta \mathcal{L}
\]

- Boson self-interactions via expansion of chiral invariant, \( \chi \equiv \frac{1}{2} \phi^\dagger \phi = \frac{1}{2} \left( \sigma^2 + \pi^2 \right) \), around vacuum expectation value \( \chi_0 \) plus explicit symmetry breaking term

\[
\mathcal{U}(\sigma, \pi) = \sum_{n=1}^{4} \frac{a_n}{n!} (\chi - \chi_0)^n - m_\pi^2 f_\pi (\sigma - f_\pi)
\]

- Short distance dynamics modeled by massive vector fields \( v_\mu \) and \( w_\mu \)

\[
\Delta \mathcal{L} = -\bar{\Psi}^\dagger \left[ g_v v + g_w \tau_3 w \right] \Psi - \frac{1}{2} m_v^2 \left( v^2 + w^2 \right)
\]
Mean-field (MF) approximation

- Replace chiral fields by their expectation values $\langle \sigma \rangle$ and $\langle \pi \rangle = 0$

- Introduce $T$ and $\mu_{p/n}$ and determine the grand canonical potential

$$\Omega_{MF} = \Omega_F(T, \mu_p, \mu_n; \langle \sigma \rangle, \nu, \omega) + \mathcal{W}(\langle \sigma \rangle, \langle \pi \rangle = 0) - \frac{1}{2} m_\nu^2 (\nu^2 + \omega^2)$$

- Fermionic part with $E = \sqrt{p^2 + M^2(\sigma)}$ and dynamical nucleon mass $M(\sigma) = g \langle \sigma \rangle$

$$\Omega_F = -2 \sum_{i=p,n} \int \frac{d^3p}{(2\pi)^3} \left[ E + \frac{p^2}{3E} \sum_{r=\pm 1} \frac{1}{1 + e^{(E-r\tilde{\mu}_i)/T}} \right]$$

with $\tilde{\mu}_{p/n} = \mu_{p/n} - g_\nu \nu \mp g_\omega \omega$

- Grand canonical potential evaluated at minimum yields thermodynamic observables

$$P = -\Omega_{MF} \quad s = -\frac{\partial \Omega_{MF}}{\partial T} \quad n_i = -\frac{\partial \Omega_{MF}}{\partial \mu_i} \quad \epsilon = -P + \sum_{i=p,n} \mu_i n_i + Ts$$
Extended mean-field (EMF)

- Diverging vacuum term in fermionic contribution

\[ \delta \Omega_{\text{vac}} = -4 \int \frac{d^3p}{(2\pi)^3} E \]

→ Neglected in standard mean-field analyses

- Can be computed via dimensional regularisation [Skokov et al., Phys. Rev. D 82 (2010)]

- Extended mean-field (EMF) includes vacuum contribution

\[ \Omega_{\text{EMF}} \equiv \Omega_{\text{MF}} - \frac{(g\sigma)^4}{4\pi^2} \ln \frac{g\sigma}{\Lambda} \]

- Additional fluctuations beyond vacuum contribution (chiral boson and nucleon loops)

→ Include using non-perturbative functional renormalization group (FRG) approach
Functional renormalisation group (FRG)

- Initialize scale-dependent effective action $\Gamma_k[\Phi]$ of chiral-nucleon meson model at $k_{UV} \sim 4\pi f_{\pi}$

- Evolution $k \to 0$ governed by Wetterich’s flow equation
  
  \[ k \frac{\partial \Gamma_k[\Phi]}{\partial k} = \frac{1}{2} \text{Tr} \left[ k \frac{\partial R_k}{\partial k} \cdot \left( \Gamma^{(2)}_k[\Phi] + R_k \right)^{-1} \right] = \frac{1}{2} \]

- $\Gamma_k[\Phi]$ contains all fluctuations with $p^2 \geq k^2$ through regulator $R_k(\rho)$

- Model parameters adjusted to reproduce vacuum properties and nuclear phenomenology
Nuclear thermodynamics

- $\langle \sigma \rangle_{\text{vac}} = f_\pi \approx 93$ MeV
- $M_N = 939$ MeV
- $E/A(n_0) = -16$ MeV, $S(n_0) = 32$ MeV
- Nuclear surface tension $\Sigma = 1.08(6)$ MeV/fm$^2$, Landau mass $M_L^* = 0.7 - 0.8 M_N$ and compression modulus $K = 240(20)$ MeV
- **Nuclear liquid-gas phase transition** with empirical critical parameters [Elliot et al., Phys. Rev. C 87 (2013)]
Phase structure of symmetric nuclear matter

- $\langle \sigma \rangle$ compared to $\langle \sigma \rangle_{\text{vac}} = f_\pi$ serves as order parameter for chiral symmetry, because $M(\sigma) = g \langle \sigma \rangle$

- **Mean-field**: unphysical first-order phase transition to chirally restored phase $\langle \sigma \rangle = 0$ at $n \approx 1.5 n_0$

- **Extended mean-field**: vacuum contribution stabilizes order parameter

- **FRG**: further stabilization through additional fluctuations
Phase structure of pure neutron matter

- Similar results for pure neutron matter
  - First-order phase transition converted to smooth crossover at large densities
- Model adjusted to low-density properties, potential expanded around $\chi_0 = 1/2 \langle \sigma \rangle_{\text{vac}}^2 = 1/2 f_\pi^2$
  - For small $\langle \sigma \rangle/f_\pi$ model no longer applicable
- In FRG $\langle \sigma \rangle/f_\pi$ stays around 40% until $n \sim 6 n_0$ (central densities in heavy neutron stars!)
Further results

- **Similar behaviour in chiral quark-meson models** (with Polyakov loop)
  
  → Chiral restoration seen in mean-field approximation avoided by vacuum fluctuations and in FRG

  [Zacchi and Schaffner-Bielich, Phys. Rev. D 97 (2018)] [Gupta and Tiwari, Phys. Rev. D 85 (2012)]

- **Good agreement with pure neutron matter** $E/A$ from ChEFT calculations (left)

  [Drischler, Hebeler and Schwenk, Phys. Rev. Lett. 122 (2019)]

- **In chiral limit** $m_\pi \to 0$ crossover turns into second-order phase transition (right)
Neutron stars

- Recent multimessenger measurements:
  - General relativistic Shapiro time delays
  - NICER X-ray measurements
  - Gravitational waves from binary neutron star mergers

- Use **Bayesian inference** to constrain speed of sound \(c_s^2 = \frac{\partial P}{\partial \varepsilon}\) inside neutron stars
  [LB, Weise and Kaiser, arXiv:2208.03026 (2022)]

- Strong first-order phase transition inside neutron stars \(M \leq 2M_\odot\) unlikely, crossover still possible

- EoS based solely on nuclear degrees of freedom cannot be ruled out
Summary

- Chiral nucleon-meson model reproduces empirical nuclear properties including liquid-gas phase transition

- **Mean-field approximation**: chiral first-order phase transition at unphysically low densities

- **Extended mean-field**: includes fermionic vacuum contribution
  
  → Chiral symmetry remains spontaneously broken up to higher densities

- **Functional renormalisation group**: additional fluctuations provide an even stronger stabilization against chiral restoration

- Similar results for chiral quark-meson models

  → Fluctuations convert chiral first-order phase transition into smooth crossover at high baryon densities $n \geq 6n_0$

- Based on multimessenger measurements strong first-order phase transition unlikely in neutron stars with $M \leq 2M_\odot$