Program Synthesis Using Example Propagation

Nick Mulleners\textsuperscript{1}\textsuperscript{[0000–0002–7934–6834]}, Johan Jeuring\textsuperscript{1}\textsuperscript{[0000–0001–5645–7681]}, and Bastiaan Heeren\textsuperscript{2}\textsuperscript{[0000–0001–6647–6130]}

\textsuperscript{1} Utrecht University, The Netherlands, \{n.mulleners,j.t.jeuring\}@uu.nl
\textsuperscript{2} Open University of The Netherlands, The Netherlands, bastiaan.heeren@ou.nl

Abstract. We present SCRyBE, an example-based synthesis tool for a statically-typed functional programming language, which combines top-down deductive reasoning in the style of $\lambda^2$ with SMYTH-style live bidirectional evaluation. During synthesis, example constraints are propagated through sketches to prune and guide the search. This enables SCRyBE to make more effective use of functions provided in the context. To evaluate our tool, it is run on the combined, largely disjoint, benchmarks of $\lambda^2$ and MYTH. SCRyBE is able to synthesize most of the combined benchmark tasks.

Keywords: Program Synthesis · Constraint Propagation · Input-Output Examples · Functional Programming

1 Introduction

Type-and-example driven program synthesis is the process of automatically generating a program that adheres to a type and a set of input-output examples. The general idea is that the space of type-correct programs is enumerated, evaluating each program against the input-output examples, until a program is found that does not result in a counterexample. Recent work in this field has aimed to make the enumeration of programs more efficient, using various pruning techniques and other optimizations. Hoogle\textsuperscript{+} \cite{4} and HECTARE \cite{6} explore efficient data structures to represent the search space. Smith and Albarghouthi \cite{12} describe how synthesis procedures can be adapted to only consider programs in normal form. MAGICHASKELL \cite{5} and RESL \cite{11} filter out programs that evaluate to the same result. Instead of only using input-output examples for the verification of generated programs, MYTH \cite{10,9}, SMYTH \cite{7}, and $\lambda^2$ \cite{3} use input-output examples during pruning, by eagerly checking incomplete programs for counterexamples using constraint propagation.

Constraint Propagation Top-down synthesis incrementally builds up a sketch, a program which may contain holes (denoted by $\bullet$). Holes may be annotated with constraints, e.g. type constraints. During synthesis, holes are filled with new sketches (possibly containing more holes) until no holes are left. For example, for type-directed synthesis, let us start from a single hole $\bullet_0$ annotated with a type constraint:

\[ \bullet_0 :: \text{List Nat} \rightarrow \text{List Nat} \]
We may fill $\bullet_0$ using the function $\text{map} :: (a \to b) \to \text{List } a \to \text{List } b$, which applies a function to the elements of a list. This introduces a new hole $\bullet_1$, with a new type constraint:

$$
\bullet_0 :: \text{List } \text{Nat} \to \text{List } \text{Nat} \quad \xrightarrow{\text{map}} \quad \text{map} \ (\bullet_1 :: \text{Nat} \to \text{Nat})
$$

We say that the constraint on $\bullet_0$ is propagated through $\text{map}$ to the hole $\bullet_1$. Note that type information is preserved: the type constraint on $\bullet_0$ is satisfied exactly if the type constraint on $\bullet_1$ is satisfied. We say that the hole filling $\text{map} \ (\bullet_1)$ refines the sketch with regards to its type constraint.

A similar approach is possible for example constraints, which partially specify the behavior of a function using input-output pairs. For example, we may further specify hole $\bullet_0$, to try and synthesize a program that doubles each value in a list:

$$
\bullet_0 \models \{ [0,1,2] \mapsto [0,2,4] \}
$$

Now, when introducing $\text{map}$, we expect its argument $\bullet_1$ to have three example constraints, representing the doubling of a natural number:

$$
\bullet_0 \models \{ [0,1,2] \mapsto [0,2,4] \} \quad \xrightarrow{\text{map}} \quad \text{map} \ (\bullet_1 \models \{ 0 \mapsto 0 \}
\{ 1 \mapsto 2 \}
\{ 2 \mapsto 4 \})
$$

Similar to type constraints, we want example constraints to be correctly propagated through each hole filling, such that example information is preserved. Unlike with type constraints, which are propagated through hole fillings using type checking/inference, it is not obvious how to propagate example constraints through arbitrary functions. Typically, synthesizers define propagation of example constraints for a hand-picked set of functions and language constructs. Feser et al. [3] define example propagation for a set of combinators, including $\text{map}$ and $\text{foldr}$, for their synthesizer $\lambda^2$. Limited to this set of combinators, $\lambda^2$ excels at composition, but lacks in generality. MYTH [10,9], and by extension SMYTH [7], take a more general approach, in exchange for compositionality, defining example propagation for basic language constructs, including constructors and pattern matches.

Presenting SCRYBE In this paper, we explore how the techniques of $\lambda^2$ and SMYTH can be combined to create a general-purpose, compositional example driven synthesizer, which we will call SCRYBE. Figure 1 shows four different interactions with SCRYBE, where the function $\text{dupl}$ is synthesized with different sets of functions. SCRYBE is able to propagate examples through all of the provided functions using live bidirectional evaluation as introduced by Lubin et al. [7] for their synthesizer SMYTH, originally intended to support sketching [13]. By choosing the right set of functions (for example, the set of combinators used in

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3 In this example, as well as in the rest of this paper, we will leave type constraints implicit.
Main Contributions The contributions of this paper are as follows:

- We give an overview of example propagation and how it can be used to perform program synthesis (Section 2).
- We show how live bidirectional evaluation as introduced by Lubin et al. [7] allows arbitrary sets of functions to be used as refinements during program synthesis (Section 3).
- We present SCRYBE, an extension of SMYTH [7] and evaluate it against existing benchmarks from different synthesis domains (Section 4).

2 Example Propagation

Example constraints give a specification of a function in terms of input-output pairs. For example, the following constraint represents the function mult that multiplies two numbers.

\[
\begin{align*}
0 & \mapsto 0 \\
1 & \mapsto 1 \\
2 & \mapsto 6
\end{align*}
\]

The constraint consists of three input-output examples. Each arrow (\(\mapsto\)) maps the inputs on its left to the output on its right. A function can be checked against an example constraint by evaluating it on the inputs and matching the results
against the corresponding outputs. During synthesis, we want to check that generated expressions adhere to these constraints. For example, to synthesize \texttt{mult}, we may generate a range of expressions of type \texttt{Nat \rightarrow Nat \rightarrow Nat} and then check each against the example constraint. The expression \(\lambda x \ y . \text{double} \ (\text{plus} \ x \ y)\) will be discarded, as it maps the inputs to 2, 4 and 10, respectively. It would be more efficient, however, to recognize that any expression of the form \(\lambda x \ y . \text{double} \ e\), for some expression \(e\), can be discarded, since there is no natural number whose double is 1.

To discard incorrect expressions as early as possible, we incrementally construct a sketch, where each hole (denoted by \(\bullet\)) is annotated with an example constraint. Each time a hole is filled, the example constraints are propagated to the new holes and checked for contradictions. Let us start from a single hole \(\bullet_0\). We refine the sketch by eta-expansion, binding the inputs to the variables \(x\) and \(y\).

\[
\bullet_0 \models \begin{cases} 
0 & \rightarrow 0 \\
1 & \rightarrow 1 \\
2 & \rightarrow 6 
\end{cases} \quad \xrightarrow{\text{eta-expand}} \quad \lambda x \ y . \ (\bullet_1 \models \begin{cases} 
x & \rightarrow 0 \\
y & \rightarrow 1 \\
1 & \rightarrow 1 \\
1 & \rightarrow 1 \\
2 & \rightarrow 6 
\end{cases})
\]

A new hole \(\bullet_1\) is introduced, annotated with a constraint that captures the values of \(x\) and \(y\). Example propagation through \texttt{double} should be able to recognize that the value 1 is not in the codomain of \texttt{double}, so that the hole filling \(\bullet_1 \rightarrow \text{double} \bullet_2\) can be discarded.

### 2.1 Program Synthesis Using Example Propagation

Program synthesizers based on example propagation iteratively build a program by filling holes. At each iteration, the synthesizer may choose to fill a hole using either a \textit{refinement} or a \textit{guess}. A \textit{refinement} is an expression for which example propagation is defined. For example, eta-expansion is a refinement, as shown in the previous example. To propagate an example constraint through a lambda abstraction, we simply bind the inputs to the newly introduced variables. A \textit{guess} is an expression for which example propagation is \textit{not} defined. The new holes introduced by a guess will not have example constraints. Once you start guessing, you have to keep guessing! Only when all holes introduced by guessing are filled can the expression be checked against the example constraint. In a sense, guessing comes down to brute-force enumerative search.

Refinements are preferred over guesses, since they preserve constraint information, which is needed to prune the search space. It is, however, not feasible to define example propagation for every possible expression. Instead, previous synthesizers define only a hand-picked set of refinements. In the rest of this section, we show how the synthesizers \(\lambda^2\), \textsc{Myth} and \textsc{Smyth} implement and use example propagation.
\[ \begin{align*}
\bullet_0 & = \{ [0, 1, 2] \mapsto [1, 2, 3] \} \\
\bullet_1 & = \{ 0 \mapsto 1, 1 \mapsto 2, 2 \mapsto 3 \}
\end{align*} \]

\[ \begin{align*}
\text{inc} & \quad \{ [0, 0] \mapsto [0] \} \\
\text{compress} & \quad \{ [0, 0, 1] \mapsto [1, 0, 0] \} \\
\text{reverse} & \quad \{ 0 \mapsto 1 \}
\end{align*} \]

Fig. 2. The inputs and outputs for example propagation through the hole filling \( \bullet_0 \mapsto \text{map} \bullet_1 \) for example constraints taken from the functions inc, compress and reverse. The latter two cannot be implemented using map, which is reflected in the contradictory constraints: for compress there is a length mismatch, and for reverse the same input is mapped to different outputs.

2.2 Example Propagation in \( \lambda^2 \)

For the tool \( \lambda^2 \), Feser et al. define deduction rules for a set of combinators, including map, foldr, and filter. In essence, these deduction rules propagate examples through the respective combinators. For example, consider map, which maps a function over a list. Refinement using map replaces a constraint on a list with constraints on its elements, while checking that the input and output lists have equal length, and that no value in the input list is mapped to different values in the output list.

See Figure 2 for examples of example propagation through map, for various example constraints based on common functions on lists. The function inc, which increments each number in a list by one, can be implemented using map. As such, example propagation succeeds, resulting in a constraint which represents incrementing a number by one. The function compress, which removes consecutive duplicates from a list, cannot be implemented using map, since the input and output lists can have different lengths. As such, example propagation fails, as seen in Figure 2. The function reverse, which reverses a list, has input and output lists of the same length. It can, however, not be implemented using map, as map cannot take the positions of elements in a list into account. This is reflected in the example in Figure 2, where the resulting constraint is inconsistent, mapping 0 to two different values.

For each combinator in \( \lambda^2 \), a deduction rule is defined that captures the various properties relevant for example propagation. This allows \( \lambda^2 \) to efficiently synthesize complex functions in terms of these combinators. For example, \( \lambda^2 \) is able to synthesize a function computing the Cartesian product in terms of foldr, believed to be the first functional pearl [2].

\( \lambda^2 \) shows that synthesis using example propagation is feasible, but it is not general purpose. Many synthesis problems require other recursion schemes or are defined over different types. Example propagation can be added for other functions in a similar fashion by adding new deduction rules, but this is very laborious work.
2.3 Example Propagation in MYTH

Osera and Zdancewic [10,9] take a more general approach in their synthesizer MYTH, compared to $\lambda^2$, by focusing on structural recursion, rather than a specific recursion scheme such as foldr. To do so, they describe how example constraints can be propagated through constructors and pattern matches. Note that their language does not contain primitive integers. Rather, literals 0, 1, 2, etc. are syntactic sugar for Peano-style natural numbers.

\[
\text{data} \; \text{Nat} = \text{Zero} \mid \text{Succ Nat}
\]

Constructors To propagate a constraint through a constructor, we have to check that all possible outputs agree with this constructor. For example, the constraint $\{ \text{Zero} \}$ can be refined by the constructor Zero. No constraints need to be propagated, since Zero has no arguments. In the next example, there are multiple possible outputs, depending on the value of the variable $x$.

\[
\bullet_0 \models \begin{cases} \ldots & \text{Succ Zero} \\ \ldots & \text{Succ (Succ Zero)} \end{cases} \quad \xrightarrow{\text{Succ}} \quad \text{Succ} (\bullet_1 \models \begin{cases} \ldots & \text{Succ Zero} \end{cases})
\]

Since every possible output is a successor, the constraint can be propagated through Succ by removing one Succ constructor from each output, i.e. decreasing each output by one. The resulting constraint on $\bullet_1$ cannot be refined by a constructor, since the outputs do not all agree.

Pattern Matching The elimination of constructors (i.e. pattern matching) is a bit more complicated. MYTH describes example propagation through non-nested pattern matches, as long as the scrutinee has no holes. Consider the following example, wherein the sketch double $= \lambda n. \bullet_0$ is refined by propagating the constraint on $\bullet_0$ through a pattern match on the local variable $n$.

\[
\bullet_0 \models \begin{cases} n & 0 \\ 0 & 1 \\ 2 & 2 \\ 4 & 4 \end{cases} \quad \xrightarrow{\text{pattern match}} \quad \text{case} \; n \; \text{of} \\
\text{ZERO} & \rightarrow \bullet_1 \models \{ 0 \} \\
\text{Succ m} & \rightarrow \bullet_2 \models \begin{cases} m & 0 \\ 1 & 2 \end{cases}
\]

Pattern matching on $n$ creates two branches, one for each constructor of Nat, with holes on the right-hand side. The constraint on $\bullet_0$ is propagated to each branch by splitting up the constraint based on the value of $n$. For brevity, we leave $n$ out of the new constraints. The newly introduced variable $m$ is exactly one less than $n$, i.e. one Succ constructor is stripped away.

Structural Recursion With support for structural recursion, MYTH is able to perform general-purpose, propagation-based synthesis. To illustrate this, we
show how MYTH synthesizes the function \texttt{double}, starting from the previous sketches. Hole $\bullet_1$ is easily refined with \texttt{Zero}. Hole $\bullet_2$ can be refined with \texttt{Succ} twice, since every output is at least 2:

\[
\bullet_2 \models \begin{cases} 
  m \\
  0 & 2 \\
  1 & 4 
\end{cases} \quad \text{Succ} \ldots \text{Succ} \quad \text{Succ} \left( \text{Succ} \left( \bullet_3 \models \begin{cases} 
  m \\
  0 & 0 \\
  1 & 2 
\end{cases} \right) \right)
\]

At this point, to tie the knot, MYTH should introduce the recursive call \texttt{double} $m$. Note, however, that \texttt{double} is not yet implemented, so we cannot directly test the correctness of this guess. We can, however, use the original constraint (on $\bullet_0$) as a partial implementation of \texttt{double}. The example constraint on $\bullet_3$ is a subset of this original constraint, with $m$ substituted for $n$. This implies that \texttt{double} $m$ is a valid refinement. This property of example constraints, i.e. that the specification for recursive calls is a subset of the original constraint, is known as \textit{trace completeness} \[9\], and is a prerequisite for synthesizing recursive functions in MYTH.

### 2.4 Example Propagation in SMYTH

In their synthesizer SMYTH, Lubin et al. \[7\] extend MYTH with sketching, i.e. program synthesis starting from a sketch, a program containing holes. A global example constraint is propagated through the sketch using example propagation, after which MYTH-style synthesis takes over using the local example constraints.

Take for example the constraint \{ $[0, 1, 2] \mapsto [0, 2, 4]$ \}, which represents doubling each number in a list. The programmer may provide the sketch \texttt{map} $\bullet$ as a starting point for the synthesis procedure. In order to perform MYTH-style synthesis, the constraint has to be propagated through \texttt{map}, but unlike $\lambda^2$, SMYTH does not provide a handcrafted rule for \texttt{map}. Instead, SMYTH determines how examples are propagated through functions based on their implementation.

The crucial idea is that the sketch is first evaluated, essentially inlining all function calls\footnote{Note that function calls within the branches of a stuck pattern match are not inlined.} until only simple language constructs remain, each of which supports example propagation. Omar et al. \[8\] describe how to evaluate an expression containing holes using live evaluation. The sketch is applied to the provided input, after which \texttt{map} is inlined and evaluated as far as possible:

\[
\text{map} \bullet [0, 1, 2] \sim [\bullet 0, \bullet 1, \bullet 2]
\]

At this point, the constraint can be propagated through the resulting expression. Lubin et al. \[7\] extend MYTH-style example propagation to work for the primitives returned by live evaluation. The constraint is propagated through the result of live evaluation.

\[
[\bullet 0, \bullet 1, \bullet 2] = \{ [0, 2, 4] \} \quad \longrightarrow^* \quad \begin{cases} 
  \bullet \models \{ 0 \mapsto 0 \} & 0 \\
  \bullet \models \{ 1 \mapsto 2 \} & 1 \\
  \bullet \models \{ 2 \mapsto 4 \} & 2 
\end{cases}
\]
The constraints propagated to the different occurrences of \( \bullet \) in the evaluated expression can then be collected and combined to compute a constraint for \( \bullet \) in the input sketch.

\[
\text{map}(\bullet) = \begin{cases} 
0 \mapsto 0 \\
1 \mapsto 2 \\
2 \mapsto 4 
\end{cases}
\]

This kind of example propagation based on evaluation is called live bidirectional evaluation. For a full description, see Lubin et al. [7]. SMYTH uses live bidirectional evaluation to extend MYTH with sketching. Note, however, that SMYTH does not use live bidirectional evaluation to introduce refinements during synthesis.

### 3 Program Synthesis Using Example Propagation

We define our synthesis problem as finding an expression of type \( \tau \) in the environment \( \Gamma \) that adheres to example constraint \( \varphi \). Inspired by Smith and Al-barghouthi [12], we give a high-level overview of the synthesis procedure as a set of guarded rules that can be applied non-deterministically, shown in Figure 3.

We keep track of a set of candidate expressions \( \mathcal{E} \), which is initialized by the rule \texttt{init} and then expanded by the rule \texttt{expand} until the rule \texttt{final} applies, returning a solution.

The rule \texttt{init} initializes \( \mathcal{E} \) with a single hole \( \bullet_0 \), constrained by the synthesis parameters. Each invocation of the rule \texttt{expand} non-deterministically picks an expression \( e \) from \( \mathcal{E} \) and a hole \( \bullet_i \) in \( e \) to fill, by generating a hole filling \( hf \) using the context and type of \( \bullet_i \). If the resulting expression \( e' \) does not conflict with \( \varphi \), it is considered a valid candidate and added to \( \mathcal{E} \). As an invariant, \( \mathcal{E} \) only contains expressions that do not conflict with \( \varphi \). As such, a solution to the synthesis problem is simply any expression that has no holes.

To implement a synthesizer according to these rules, we have to make the non-deterministic choices explicit: we have to decide in which order expressions are expanded \( (e \in \mathcal{E}) \); which holes are selected for expansion \( (\bullet \in \text{holes}(e)) \); and how hole fillings are generated based on the hole’s type and environment \( (\Gamma \vdash hf : \tau) \). Additionally, we describe how expressions containing holes are checked against the constraint \( \varphi \).

#### 3.1 Expression Order

To decide in which order candidate expressions are selected for expansion, we define an order on expressions by assigning a weight to each expression. We keep track of all expressions in a priority queue and expand expressions in increasing order of their weight. The weight of an expression is computed as follows: we assign a weight of 1 to each application, as well as to each pattern match and each call to a recursion scheme. Additionally, the weight of scrutinees is doubled, to disincentivize pattern matching on large scrutinees.
\[\Gamma \vdash b_0 : \tau \vdash \varphi\]
\[E \leftarrow \{b_0\}\]  
\[\text{INIT}\]

\[e \in E \quad \bullet_i \in \text{holes}(e) \quad \Gamma_i \vdash hf : \tau_i\]
\[e' = [\bullet_i \mapsto hf]e \quad e' \vdash \varphi\]
\[E \leftarrow E \cup \{e'\}\]
\[\text{EXPAND}\]

\[e \in E \quad \text{holes}(e) = \emptyset\]
\[\text{FINAL}\]

Fig. 3. Program synthesis using example propagation as a set of guarded rules that can be applied non-deterministically.

3.2 Hole Order

Choosing in which order holes are filled during synthesis is a bit more involved. Consider, for example, the hole fillings in Figure 4, synthesizing the expression \(\lambda x. \text{map} (\lambda x. \text{Succ} x) x\) starting from \(\lambda x. b_0\). There are three different synthesis paths that lead to this result, depending on which holes are filled first. More specifically, \(b_2\) can be filled independently of \(b_1\) and \(b_3\), so it could be filled before, between, or after them. To avoid generating the same expression three times, we should fix the order in which holes are filled, so that there is a unique path to every possible expression.

Because our techniques rely heavily on evaluation, we let evaluation guide the hole order. After filling hole \(b_0\), we live evaluate.

\[
\text{map} (\lambda x. b_1) b_2 \sim \text{case } b_2 \text{ of } \ldots
\]

At this point, evaluation cannot continue, because we do not know which pattern \(b_2\) will be matched on. We say that \(b_2\) blocks the evaluation. By filling \(b_2\), the pattern match may resolve and generate new example constraints for \(b_1\). Conversely, filling \(b_1\) does not introduce any new constraints. Hence, we always fill blocking holes first. Blocking holes are easily computed by live evaluating the expression against the example constraints.

3.3 Generating Hole Fillings

Hole fillings depend on the local context and the type of a hole and may consist of constructors, pattern matches, variables and function calls. To avoid synthesizing multiple equivalent expressions, we will only generate expressions in \(\beta\)-normal, \(\eta\)-long form. An expression is in \(\beta\)-normal, \(\eta\)-long form exactly if no \(\eta\)-expansions or \(\beta\)-reductions are possible. During synthesis, we guarantee \(\beta\)-normal, \(\eta\)-long form by greedily \(\eta\)-expanding newly introduced holes and always fully applying functions, variables and constructors. Consider, for example, the function \text{map}. 

Fig. 4. A set of hole fillings synthesizing an expression that increments each value in a list, starting from the sketch \(\lambda x. b_0\).
To use \texttt{map} as a refinement, it is applied to two holes, the first of which is \(\eta\)-expanded:

\[
\text{map}\ (\lambda x. \bullet_0) \bullet_1
\]

Pattern matches can be handled in the same way by interpreting them as eliminator functions, which are equivalent in expressiveness.

Furthermore, we add some syntactic restrictions to the generated expressions: we only allow the recursive argument of recursion schemes such as \texttt{foldr} to be variables. This is similar to the restriction on structural recursion in \textsc{Myth} [10] and \textsc{Smyth} [4]. Additionally, we disallow expressions that are not in normal form, somewhat similar to equivalence reduction as described by Smith and Albarghouthi [12]. Currently, our tool provides a handcrafted set of expressions that are not in normal form, which are prohibited during synthesis. Ideally, these sets of disallowed expressions would be taken from an existing data set (such as \textsc{HLint}[^5]), or approximated using evaluation-based techniques such as \textsc{QuickSpec} [1].

### 3.4 Pruning Expressions

For an expression \(e \in \mathcal{E}\) and a hole \(\bullet_i \in \text{holes}(e)\), we generate a set of possible hole fillings based on the hole context \(\Gamma_i\) and the hole type \(\tau_i\). For each of these hole fillings, we try to apply the \texttt{expand} rule. To do so, we must check that the resulting expression \(e'\) does not conflict with the example constraint \(\varphi\). We use \textsc{Smyth}-style example propagation to compute hole constraints for \(e'\). If example propagation fails, we do not add \(e'\) to \(\mathcal{E}\), essentially pruning the search space.

**Diverging Constraints** Unfortunately, example propagation is not feasible for all possible expressions. Consider, for instance, the function \texttt{sum}. If we try to propagate a constraint through \texttt{sum} \(\bullet\), we first use live evaluation, resulting in the following partially evaluated result, with \(\bullet\) in a scrutinized position:

\[
\text{case } \bullet\text{ of } \\
[ ] \rightarrow 0 \\
x : xs \rightarrow \text{\texttt{plus}}\ x\ (\text{\texttt{sum}}\ xs)
\]

Unlike \textsc{Myth}, \textsc{Smyth} allows examples to be propagated through pattern matches whose scrutinee may contain holes, considering each branch separately under the assumption that the scrutinee evaluates to the corresponding pattern. This introduces disjunctions in the example constraint. Propagating \(\{ \text{\texttt{Zero}} \}\) through the previous expression results in a constraint that cannot be finitely captured in our constraint language:

\[
(\bullet = \{ [ ] \}) \lor (\bullet = \{ [\text{\texttt{Zero}}] \}) \lor (\bullet = \{ [\text{\texttt{Zero}}, \text{\texttt{Zero}}] \}) \lor \ldots
\]

Without extending the constraint language it is impossible to compute such a constraint. Instead, we try to recognize that example propagation diverges, by

[^5]: \url{https://github.com/ndmitchell/hlint}
setting a maximum to the amount of recursive calls allowed during example propagation. If the maximum recursion depth is reached, we cancel example propagation.

Since example propagation through \( \text{sum} \) always diverges, we could decide to disallow it as a hole filling. This is, however, too restrictive, as example propagation becomes feasible again when the length of the argument to \( \text{sum} \) is no longer unrestricted. Take, for example, the following constraint, representing counting the number of \text{True}s in a list, and a possible series of hole fillings:

\[
\begin{array}{c|c}
{\bullet}_0 \mapsto & \{ \text{False} \} \\
{\bullet}_1 \mapsto & \{ \text{False, True} \} \\
{\bullet}_2 \mapsto & \{ \text{True, True} \}
\end{array}
\]

Trying to propagate through \( {\bullet}_0 \mapsto \text{sum} {\bullet}_1 \) diverges, since \( {\bullet}_1 \) could be a list of any length. At this point, we could decide to disregard this hole filling, but this would incorrectly prune away a valid solution. Instead, we allow synthesis to continue guessing hole fillings, until we get back on the right track: after guessing \( {\bullet}_1 \mapsto \text{map} (\lambda x. {\bullet}_2) {\bullet}_3 \) and \( {\bullet}_3 \mapsto {\bullet}_8 \), the length of the argument to \( \text{sum} \) becomes restricted and example propagation no longer diverges:

\[
\text{sum} (\text{map} (\lambda x. {\bullet}_2) {\bullet}_3) \equiv \sum_{n} \{ \text{True}, \text{True}, \ldots \}
\]

At this point, synthesis easily finishes by pattern matching on \( x \). Note that, unlike \( \lambda^2 \), MYTH and SMYTH, SCRyBE is able to interleave refinements and guesses.

**Exponential Constraints** Even if example propagation does not diverge, it still might take too long to compute or generate a disproportionally large constraint, slowing down the synthesis procedure. Lubin et al. [7] compute the falsifiability of an example constraint by first transforming it to disjunctive normal form (DNF), which may lead to exponential growth of the constraint size. For example, consider the function \text{or}, defined as follows:

\[
\text{or} = \lambda a \ b. \ \text{case} \ a \ \text{of} \\
\text{False} & \mapsto b \\
\text{True} & \mapsto \text{True}
\]

Propagating the example constraint \( \{ \text{True} \} \) through the expression \( \text{or} {\bullet}_0 {\bullet}_1 \) puts the hole \( {\bullet}_0 \) in a scrutinized position, resulting in the following constraint:

\[
({\bullet}_0 \vdash \{ \text{False} \}) \land {\bullet}_1 \vdash \{ \text{True} \}) \lor {\bullet}_0 \vdash \{ \text{True} \}
\]

This constraint has size three (the number of hole occurrences). We can extend this example by mapping it over a list of length \( n \) as follows:

\[
\text{map} (\lambda x. \text{or} {\bullet}_0 {\bullet}_1) \{0, 1, 2, \ldots\} \vdash \{ \text{True, True, True,} \ldots \}
\]
Propagation generates a conjunction of $n$ constraints that are all exactly the same apart from their local context, which differs in the value of $x$. This constraint, unsurprisingly, has size $3n$. Computing the disjunctive normal form of this constraint, however, results in a constraint of size of $2^n \times \frac{1}{4}n$, which is exponential.

In some cases, generating such a large constraint may cause example propagation to reach the maximum recursion depth. In other cases, example propagation succeeds, but returns such a large constraint that subsequent refinements will take too long to compute. In both cases, we treat it the same as diverging example propagation.

4 Evaluation

To evaluate SCRYBE, we combine the benchmarks of Myth \cite{Myth} and $\lambda^2$ \cite{lambda2}. This evaluation is not intended to compare our technique directly with previous techniques in terms of efficiency, but rather to show the wide range of synthesis problems that SCRYBE can handle. Additionally, we get some insight in the effectiveness of example propagation as a pruning technique.

For ease of readability, the benchmark suite is split up into a set of functions operating on lists (Table 1, Appendix A) and a set of functions operating on binary trees (Table 2, Appendix A). We have excluded functions operating on just booleans or natural numbers, as these are all trivial and synthesize in a few milliseconds. For consistency, and to avoid naming conflicts, the names of some of the benchmarks are changed to reflect the corresponding functions in the Haskell prelude. To avoid confusion, each benchmark function comes with a short description.

Each row describes a single synthesis problem in terms of a function that needs to be synthesized. The first two columns give the name and a short description of this function. The third and fourth columns show, in milliseconds, the average time our synthesizer takes to correctly synthesize the function with example propagation (EP) and without example propagation (NoEP), respectively. Some functions may fail to synthesize ($\bot$) within 5 seconds and some cannot straightforwardly be represented in our language (-). The last three columns show, for Myth, Smyth, and $\lambda^2$, respectively, whether the function synthesizes (✓), fails to synthesize (✗), or is not included in their benchmark (-).

The benchmarks list_head, list_tail, list_init and list_last are all partial functions (marked †). We do not support partial functions, and therefore these functions are replaced by their total equivalents, by wrapping their return type in Maybe. For example, list_last is defined as follows, where the outlined hole filling is the result returned by SCRYBE (input-output constraints are omitted
for brevity):

```
{-# USE foldr #-}
list_last :: List a → Maybe a
list_last xs = ⬧
  ⬧ → foldr (λx r. case r of Nothing → Just x) Nothing xs
```

The benchmarks list_drop, list_index, list_take and tree_level (marked ‡) all recurse over two datatypes at the same time. As such, they cannot be implemented using foldr as it is used in Section 3.3. Instead, we provide a specialized version of foldr that takes an extra argument:

```
{-# USE foldr :: (a → (c → b) → (c → b)) → (c → b) → List a → c → b #-}
list_take :: Nat → List a → List a
list_take n xs = ⬧
  ⬧ → foldr (λx m. case m of Zero → [] Succ o → (x : r o)) (λ_. []) xs n
```

A few functions (marked *) could not straightforwardly be translated to our approach:

- Function list_delete_mins requires a total function in scope that returns the minimum number in a list. This is not possible for natural numbers, as there is no obvious number to return for empty lists.
- Function list_swap uses nested pattern matching on the input list, which is not possible to mimic using a fold.
- Function list_reverse combines a set of benchmarks from MYTH that synthesize reverse using different techniques, which are not easily translated to our language.

### 4.1 Results

Scrybe is able to synthesize most of the combined benchmarks of MYTH and λ², with a median runtime of 15.95 milliseconds. Furthermore, synthesis with example propagation is on average 5.22 times as fast as without example propagation, disregarding the benchmarks where synthesis without example propagation failed. λ² noticed a similar improvement (6 times as fast) for example propagation based on automated deduction, which indicates that example propagation using live-bidirectional evaluation is similar in strength, while being more general.

Some functions benefit especially from example propagation, in particular problems that are composed of multiple synthesis problems. Take, for example, tree_snoc, which effectively synthesizes mapTree and snoc from foldTree and foldr
respectively. Without example propagation, it is not tractable to automatically decompose this synthesis problem into these two parts.

\[
\{-\# \text{ USE foldTree, foldr \#-}\} \\
tree\_\text{snoc} :: a \rightarrow \text{Tree} (\text{List} a) \rightarrow \text{Tree} (\text{List} a) \\
tree\_\text{snoc} x t = \bullet \\
  \bullet \rightarrow \text{foldTree} (\lambda l \_xs r. \text{Node} l (\text{foldr} (\lambda x q. x : q) \_x [x] \_xs) r) \text{LEAF} t
\]

On the other hand, for some functions, such as tree\_search, synthesis is noticeably faster without example propagation, showing that the overhead of example propagation sometimes outweighs the benefits. This indicates that it might be helpful to use some heuristics to decide when example propagation is beneficial. A few functions that fail to synthesize, such as list\_compress, do synthesize when a simple sketch is provided:

\[
\{-\# \text{ USE foldr, (≡) \#-}\} \\
\text{compress} :: \text{List Nat} \rightarrow \text{List Nat} \\
\text{compress} \_xs = \text{foldr} (\lambda x r. 0 : 0 \rightarrow \_x \rightarrow x : \text{case} r \text{ of} \_x \rightarrow r \_y : \_y \rightarrow \text{if} x \equiv y \text{ then} \_y \text{ else} r)
\]

Since our evaluation was not aimed at sketching, we still consider list\_compress to fail (\bot).

5 Conclusion

We presented an approach to program synthesis using example propagation that specializes in compositionality, by allowing arbitrary functions to be used as refinement steps. One of the key ideas is holding on to constraint information as long as possible, rather than resorting to brute-force, enumerative search. Our experiments show that we are able to synthesize a wide range of synthesis problems from different synthesis domains.

There are many avenues for future research. One direction we wish to explore is to replace the currently ad hoc constraint solver with a more general purpose SMT solver. Our hope is that this paves the way for the addition of primitive data types such as integers and floating point numbers.

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## Appendix

| Function        | Description                                   | EP (ms) | NoEP (ms) | Myth Smyth | $\lambda^2$ |
|-----------------|-----------------------------------------------|---------|-----------|------------|-------------|
| list_add        | Increment each value in a list by $n$         | 4.96    | 11.60     | -          | - ✓         |
| list_append     | Append two lists                              | 4.90    | 18.30     | ✓ ✓ ✓      | ✓ ✓ ✓       |
| list_cartesian  | The cartesian product                         | 449.00  | - -       | ✓ X -      | ✓ ✓ ✓       |
| list_compress   | Remove consecutive duplicates from a list     | -       | - -       | ✓ -        | - - ✓       |
| list_flatten    | Flatten a list of lists                       | 3.53    | 3.00      | ✓ ✓ ✓      | ✓ ✓ ✓       |
| list_copy_first | Replace each element in a list with the first  | 40.10   | 82.90     | - - ✓      | - ✓ ✓       |
| list_copy_last  | Replace each element in a list with the last  | 38.20   | 106.00    | - - ✓      | - ✓ ✓       |
| list_delete_max | Remove the largest numbers from a list        | 38.60   | 127.00    | - - ✓      | - ✓ ✓       |
| list_delete_mins*| Remove the smallest numbers from a list of lists | - | - -   | ✓ -        | ✓ ✓ ✓       |
| list_drop†      | All but the first $n$ elements of a list       | 192.00  | 473.00    | ✓ ✓ -      | ✓ ✓ ✓       |
| list_even_parity| Whether a list has an odd number of Trues     | 15.30   | 99.90     | ✓ X -      | ✓ ✓ ✓       |
| list_index‡     | Index a list starting at 0                     | 0.65    | 0.70      | ✓ ✓ -      | ✓ ✓ ✓       |
| list_inc        | Increment each value in a list by one         | 3.30    | 221.00    | ✓ ✓ -      | ✓ ✓ ✓       |
| list_inc§       | Increment each value in a list of lists by one | 9.16    | 23.40     | - - ✓      | - ✓ ✓       |
| list_index‡     | Index a list starting at 0                     | 51.80   | 167.00    | ✓ ✓ -      | ✓ ✓ ✓       |
| list_init†      | All but the last element of a list             | 869.00  | - -       | - - ✓      | - ✓ ✓       |
| list_last†      | The last element of a list                    | 167.00  | 123.00    | ✓ ✓ ✓      | ✓ ✓ ✓       |
| list_length     | The number of elements in a list              | 0.62    | 2.86      | ✓ ✓ ✓      | ✓ ✓ ✓       |
| list_map        | Map a function over a list                    | 1.38    | 2.03      | ✓ ✓ -      | ✓ ✓ ✓       |
| list_maximum    | The largest number in a list                  | 26.20   | 303.00    | - - ✓      | - ✓ ✓       |
| list_member     | Whether a number occurs in a list             | 873.00  | 4090.00   | - - ✓      | - ✓ ✓       |
| list_nub        | Remove duplicates from a list                 | -       | - -       | ✓ ✓ ✓      | ✓ ✓ ✓       |
| list_swap*      | Swap the elements in a list pairwise          | -       | - -       | ✓ ✓ ✓      | ✓ ✓ ✓       |
| list_reverse*   | Reverse a list                                | 1.67    | 2.57      | ✓ ✓ ✓      | ✓ ✓ ✓       |
| list_shiftl     | Shift all elements in a list to the left       | 69.00   | 366.00    | - - ✓      | - ✓ ✓       |
| list_shiftr     | Shift all elements in a list to the right      | 89.20   | 708.00    | - - ✓      | - ✓ ✓       |
| list_snoc       | Add an element to the end of a list           | 69.00   | 366.00    | ✓ ✓ ✓      | ✓ ✓ ✓       |
| list_set_insert | Insert an element in a set                    | -       | - -       | ✓ ✓ ✓      | ✓ ✓ ✓       |
| list_dupli      | Duplicate each element in a list              | 2.44    | 3.33      | ✓ ✓ ✓      | ✓ ✓ ✓       |
| list_sum        | The sum of all numbers in a list              | 3.59    | 19.20     | ✓ ✓ ✓      | ✓ ✓ ✓       |
| list_sums       | The sum of each nested list in a list of lists | 607.00  | - -       | - - ✓      | - ✓ ✓       |
| list_tail†      | All but the first element of a list           | 0.85    | 1.33      | ✓ ✓ ✓      | ✓ ✓ ✓       |
| list_take‡      | The first $n$ elements of a list              | 182.00  | 3690.00   | ✓ ✓ -      | ✓ ✓ ✓       |
| list_to_set     | Sort a list, removing duplicates              | 458.00  | - -       | ✓ ✓ ✓      | ✓ ✓ ✓       |

Table 1. Benchmark for functions acting on lists. Each row describes a single benchmark task and the time it takes for each function to synthesize with example propagation (EP) and without (NoEP) respectively. Some tasks cannot be synthesized within 5 seconds (⊥) and others are omitted, since they cannot straightforwardly be translated to our language (-).
| Function         | Description                                                                 | EP (ms) | NoEP (ms) | Myth Smyth | $\lambda^2$ |
|------------------|------------------------------------------------------------------------------|---------|-----------|------------|-------------|
| tree_cons        | Add an element to the front of each node in a tree of lists                 | 6.92    | ⊥         | -          | ✓           |
| tree_flatten     | Flatten a tree of lists into a list                                          | 20.80   | 25.20     | -          | ✓           |
| tree_height      | The height of a tree                                                         | 7.71    | 27.00     | -          | ✓           |
| tree_inc         | Increment each element in a tree by one                                     | 6.14    | ⊥         | -          | ✓           |
| tree_inorder     | Inorder traversal of a tree                                                 | 11.50   | 9.17      | ✓          | ✓           |
| tree_insert      | Insert an element in a binary tree                                          | ⊥       | ⊥         | ✓          | ✗           |
| tree_leaves      | The number of leaves in a tree                                               | 24.50   | 40.00     | ✓          | ✓          |
| tree_maximum     | The largest number in a tree                                                | 31.90   | 157.00    | -          | ✓           |
| tree_map         | Map a function over a tree                                                  | 2.61    | 6.84      | ✓          | ✓          |
| tree_member      | Whether a number occurs in a tree                                           | 597.00  | ⊥         | -          | ✓           |
| tree_level$^\dagger$ | The number of nodes at depth $n$                                      | ⊥       | ⊥         | ✓          | ✗          |
| tree_postorder   | Postorder traversal of a tree                                               | 19.60   | 24.40     | ✓          | ✗          |
| tree_preorder    | Preorder traversal of a tree                                                | 7.49    | 15.40     | ✓          | ✓          |
| tree_search      | Whether a number occurs in a tree of lists                                 | 964.00  | 307.00    | -          | ✓          |
| tree_select      | All nodes in a tree that satisfy $p$                                         | 773.00  | 3170.00   | -          | ✓           |
| tree_size        | The number of nodes in a tree                                               | 16.60   | 39.50     | ✓          | ✓          |
| tree_snoc        | Add an element to the end of each node in a tree of lists                   | 81.60   | ⊥         | -          | ✓           |
| tree_sum         | The sum of all nodes in a tree                                              | 16.70   | 104.00    | -          | ✓           |
| tree_sum_lists   | The sum of each list in a tree of lists                                      | 7.13    | ⊥         | -          | ✓           |
| tree_sum_trees   | The sum of each tree in a list of trees                                      | 28.40   | ⊥         | -          | ✓           |

**Table 2.** Benchmark for functions acting on binary trees. Each row describes a single benchmark task and the time it takes for each function to synthesize with example propagation (EP) and without (NoEP) respectively. Some tasks cannot be synthesized within 5 seconds (⊥) and others are omitted, since they cannot straightforwardly be translated to our language (-).