Impurity Scattering Induced Entanglement of Ballistic Electrons

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We show how entanglement between two conduction electrons is generated in the presence of a localized magnetic impurity embedded in an otherwise ballistic conductor of special geometry. This process is a generalization of beam-splitter mediated entanglement generation schemes with a localized spin placed at the site of the beam splitter. Our entangling scheme is unconditional and robust to randomness of the initial state of the impurity. The entangled state generated manifests itself in noise reduction of spin-dependent currents.

In recent years, entanglement has emerged as a vital resource in quantum information processing [1]. The controlled generation of entanglement between constituents of a condensed matter system would serve as a precursor to large scale quantum computation in that system. In this context, various methods of generating entanglement between spin states of quantum dots [2, 3, 4, 5], electron spins [6], electron numbers [7], nuclear spins [8], persistent current directions [9], cooper-pair numbers [10] and excitons [11], through controlled interactions between relevant quantum systems have recently been proposed. Extraction of entangled electrons from superconductors have been suggested as well [12]. Entanglement generation between continuously interacting spins through the variation of macroscopic parameters such as external fields and temperature have also been studied [13]. Schemes also exist for probing entangled states of electrons in solid state systems, once such a state is available [14, 15].

In this letter, we propose a scheme for entangling the spins of two mobile electrons in a solid state environment by scattering them off a localized magnetic impurity. Being a scattering process, it has the advantage of not requiring the careful switching on and off of interactions as in Refs. [2, 3, 4, 5, 6, 8, 9, 10, 11]. The interaction is automatically active only when the electrons pass the impurity. Moreover, one may actually require mobile entangled electrons for prototype demonstrations of entanglement based quantum communication protocols [1] or for detecting current noise based signatures of entanglement [14]. Most known proposals would have to first generate entangled electrons in an “entangler” such as a coupled dot system [2, 3, 4, 5], a spin resonance transistor [6] or a superconductor [12] and then use additional processes to extract them out into conducting leads as mobile electrons. Our scheme, on the other hand, could in principle, be done entirely inside a single ballistic conductor of special geometry. One suggestion for entangling already mobile electrons does exist [6], but this does not entangle their spin degrees of freedom. Compared to that proposal, ours has the advantage of extremely long spin coherence times of conducting electrons [14, 17], and the potential to interface with spin based solid state quantum computers [2, 3, 4, 5, 6, 8].

A second motivation for our work is the generalization of the existing beam-splitter mediated entanglement generation schemes [18]. In the scheme of Ref. [18], two identical particles from uncorrelated sources can be made entangled by using their indistinguishability and non-absorbing which-way detectors. One merely requires the particles to be incident at a beam splitter simultaneously in oppositely spin polarized (but disentangled) states. Our current proposal can be regarded as a next step generalization of such a scheme with a localized spin placed at the site of the beam splitter. The consequences of this simple generalization, as we will show, are quite profound. Firstly, the entanglement generation becomes deterministic (though the degree depends on the coupling strength of the localized spin with the incoming particles). Secondly, fermionic statistics is used here in a fundamental way to ensure that the incoming particles always exit through separate rails and the which-way detection of the earlier scheme [18] can be dispensed with. In contrast to all other electron entangling methods [2, 3, 4, 5, 6, 8], our scheme, being based on a spin-spin scattering interaction and fermionic statistics, should, with appropriate variations, be also applicable to fermionic atoms and neutrons.

Yet another third motivation for our work is predicting an effect at the interface of two currently fashionable areas of condensed matter physics. One of these areas is the manifestations of fermionic statistics in two electron interference at beam splitter-like mesoscopic struc-
tures [19]. The other is the study of the Kondo effect, a phenomenon arising from the interaction of the conduction electrons in a metal with a localized magnetic impurity, in various mesoscopic systems [24,25,26,27]. Our scheme simply combines fermionic statistics at beam splitter-like structures with Kondo-like scattering of conduction electrons from an impurity (though, operating in a regime different to that of the Kondo effect) to generate spin entangled electrons. Such a mobile entangled state of electrons would then automatically manifest itself through noise reduction of spin-dependent currents in appropriate conductor geometries, as pointed out by Burkard et. al. [4].

Our setup consists of a ballistic conductor with four rails 1, 2, 3 and 4 meeting at a common junction as shown in Fig. 1. In order to generate entanglement, two conduction electrons are injected in the rails 1 and 2. Both the electrons must have the same spin orientation, which can be achieved by the use of a spin filter as part of the injection mechanism. The spot where the rails 1 and 2 meet has a localized impurity atom as shown in Fig. 1. In the regime where the impurity atom has a net magnetic moment, the conduction electrons will be magnetically scattered by the impurity. As a result, the two output electrons always exit the scattering region through separate rails 3 and 4 and are always entangled. The degree of entanglement depends on the strength of the scattering interaction. Moreover, if the spin of the magnetic impurity can be measured, then, conditional on a spin-flip interaction. Moreover, if the spin of the magnetic impurity is favored. In this case the Anderson impurity may be described by the Kondo Hamiltonian [24]. For certain values of the parameters, the formation of a localized magnetic moment at the site of the impurity is favored. In this case the Anderson Hamiltonian can be, by means of the Schrieffer-Wolff transformation [24], be cast in the form

\[ H_{s-d} = \sum_{\mathbf{k},\sigma} \varepsilon(\mathbf{k}) a_{\mathbf{k}\sigma}^\dagger a_{\mathbf{k}\sigma} + J \sum_{\mathbf{k},\mathbf{k}'} \tilde{S}_{\mathbf{k}\mathbf{k}'} c_{\mathbf{k}\mathbf{k}'}^\dagger c_{\mathbf{k}'\mathbf{k}'}, \]

where \( \varepsilon(\mathbf{k}) \) is the dispersion relation for the conduction electron gas, \( a_{\mathbf{k}\sigma}^\dagger \) creates a conduction electron with spin \( \sigma \) and wave vector \( \mathbf{k} \), \( \tilde{S} \) is the spin operator of the impurity, \( \mathbf{\sigma} \) is the vector formed by the Pauli matrices, \( a_{\mathbf{k}\mathbf{k}'}^\dagger = (a_{\mathbf{k}\mathbf{k}^1}^\dagger, a_{\mathbf{k}\mathbf{k}^2}^\dagger) \), and \( J \) is the exchange coupling per atom between the localized spin and the conduction electrons’ spin [24].

Defining the spin raising and lowering operators in the usual way: \( \sigma^+ = a_{\mathbf{k}\mathbf{k}^1}^\dagger \), \( \sigma^- = a_{\mathbf{k}\mathbf{k}^1}^\dagger, S^+ = \mathbf{S}^+ + i\mathbf{S}^y, S^- = \mathbf{S}^x - i\mathbf{S}^y \), one can write the interaction term \( \mathcal{V} \) as a sum of a “longitudinal” and a “transverse” part, \( \mathcal{V} = \mathcal{V}_l + \mathcal{V}_t \). The longitudinal part \( \mathcal{V}_l = S^+ \mathcal{V}^- + S^- \mathcal{V}^+ \) is responsible for spin-flip processes, in which the spin of a conduction electron and of the localized state are both flipped.

We are interested in what happens to a conduction electron propagating through the system depicted in Fig. 1. We will assume that the rails are of reduced transverse dimensions, such that there are only very few conducting states near the Fermi level. This allows us to simplify the analysis of the conduction process by assuming that only one channel is important in their description. If we disregard, for the moment, the presence of the magnetic impurity, the geometry of the rails is that of a “beam splitter.” Following Loudon [26,27], the propagation of the conduction electrons through this kind of system can be described by a scattering matrix \( s \), which relates the “incoming” states (electrons propagating on rails 1 and 2) to the “outgoing” states (electrons propagating on rails 3 and 4), \( a_l = \sum_{m=3,4} \rho_{lm} a_m \), where \( a_l^\dagger \) creates an electron with spin \( \sigma \) in the propagating channel of rail \( l \). In principle, the only requirement on \( s \) is unitarity, and the explicit form of its elements is determined by the transmission properties of the system.

The role of the magnetic impurity will be described by the \( s-d \) Hamiltonian [14] taking into account the specific characteristics of the proposed geometry. In the end we will rewrite the final state in terms of the operators \( a_{l\sigma, \mathbf{k}}^\dagger \), which create conduction electrons with spin \( \sigma \) propagating in rail \( l \) with Fermi wave vector \( \mathbf{k}_F \). We shall assume that all the other electronic states in the rails are occupied, and play no role in the propagation process. Thus, the scattering of the conduction electrons by the impurity may be described by the T-matrix associated with \( V \) in the first Born approximation [28],

\[ T^{(1)} = V = J \sum_{l=1,2, \mathbf{k}, \mathbf{k}'} \left\{ S^+ a_{l \mathbf{k}^1}^\dagger a_{l \mathbf{k}^1}^\dagger + S^- a_{l \mathbf{k}^2}^\dagger a_{l \mathbf{k}^2}^\dagger + \right. \]

\[ \left. S^z [a_{l \mathbf{k}^1}^\dagger a_{l \mathbf{k}^1}^\dagger - a_{l \mathbf{k}^1}^\dagger a_{l \mathbf{k}^1}^\dagger] \right\}. \]

This T-matrix may now be used to calculate the final scattering state. In momentum space [28], \( \mathbf{k}_F^\dagger = \sum_{l=1,2, \mathbf{k}} S_{l \mathbf{k} \mathbf{k}'} \mathbf{k} \), and \( \mathbf{k} \) is the initial state \( \mathbf{k}_F = a_{l \mathbf{k}^1} a_{l \mathbf{k}^2}^\dagger |0\rangle |\downarrow\rangle - 2\sqrt{2}iJ |\mathbf{\psi}^\dagger\rangle |\uparrow\rangle \). A straightforward calculation shows that, if one takes as the initial state \( \mathbf{k}_F = a_{l \mathbf{k}^1} a_{l \mathbf{k}^2}^\dagger |0\rangle |\downarrow\rangle \), with \( \downarrow = d_{l}^\dagger |0\rangle \), the (unnormalized) final state,

\[ |\mathbf{k}_F^\dagger\rangle = (1 + iJ) |\uparrow\rangle \otimes |\downarrow\rangle - 2\sqrt{2}iJ |\mathbf{\psi}^\dagger\rangle |\uparrow\rangle \]

where \( \alpha_{l\sigma} = a_{l \mathbf{k} \mathbf{k}^1}, J = \pi J \rho(\epsilon_F), |\uparrow\rangle = \alpha_{l\uparrow}^\dagger a_{l\uparrow}^\dagger |0\rangle, |\mathbf{\psi}^\dagger\rangle = \frac{1}{\sqrt{2}}(\alpha_{l\uparrow}^\dagger a_{l\downarrow}^\dagger + \alpha_{l\downarrow}^\dagger a_{l\uparrow}^\dagger) |0\rangle \) and \( \rho(\epsilon_F) \) is the density of states for the conduction electrons at the Fermi level in
the rails. It should be noted that we are assuming that the temperature is larger than Kondo temperature, in which case we do not need to worry about the breakdown of the perturbation expansion of the $T$ matrix.

If one considers higher order terms in the $T$ matrix expansion the result is not qualitatively different. It can be easily shown that the contributions from the higher orders factor out, and the final state still has a component corresponding to the initial state plus a maximally entangled component with a weight of the order of $J \rho(\epsilon F)$.

The value of $J \rho(\epsilon F)$ can be estimated as follows [23]: bearing in mind that all the energies involved are of the order of the width of the conduction band $D$, one readily verifies that $J \sim D$. $\rho(\epsilon F)$ can be roughly estimated for a normal metal by considering a constant density of states extending over the band width $D$, which gives $\rho(\epsilon F) \sim \frac{1}{D}$. Thus, $J = \pi J \rho(\epsilon F) \sim 1$, and the weight of the entangled part of the final state is of the same order as that of the non-entangled component.

The method of obtaining the highest amount of entanglement would now be to measure the spin of the impurity atom after the electrons have scattered. If the spin of the impurity is measured and found flipped, the electrons in the rails 3 and 4 will be projected onto the maximally entangled state $|\psi^+\rangle$. The probability for this to happen is $8J^2/(1 + 9J^2)$, which is finite for any non-zero $J$. However, as far as the generation of entanglement is concerned, our scheme is unconditional. Even if the impurity spin was not measured at all, the electrons are projected onto the mixed state

$$\Lambda = \frac{1 + J^2}{1 + 9J^2} |\uparrow\uparrow\rangle\langle\uparrow\uparrow| + \frac{8J^2}{1 + 9J^2} |\psi^+\rangle\langle\psi^+|,$$  

(4)

which is entangled irrespective of the value of $J$. The entanglement of the above state for a range of feasible (of the order of unity) values of $J$, can be calculated from a formula by Wootters [23] and is shown in Fig. 2. The plot clearly shows that entanglement is already above 0.8 for $J \sim 3$. Apart from being unconditional, our scheme also shows robustness to uncertainty in the initial spin direction of the impurity. In fact, if the impurity spin is initially in the completely random state $|\uparrow\rangle|\uparrow\rangle|\downarrow\rangle|\downarrow\rangle\rangle$, the final state $\Lambda' = (1 + 5J^2)/(1 + 9J^2) |\uparrow\downarrow\rangle\langle\uparrow\downarrow| + 4J^2/(1 + 9J^2) |\psi^+\rangle\langle\psi^+|$, of the electrons is still entangled. This is also plotted in Fig. 2 as a dashed line.

For detection of the above entangled mixed state $\Lambda$ (or $\Lambda'$), one can use a simple modification of the current noise based method suggested by Burkard et. al. [24]. As in Ref. [24], the electrons in rails 3 and 4 should be brought together to interfere at a beam splitter. The currents in the outputs of the beam splitter (say, rails 5 and 6) will be completely noiseless for the state $\Lambda$ we have produced in the rails 3 and 4. This fact, in itself, is not sufficient to guarantee that the state $\Lambda$ is entangled. One then needs to measure the spin correlation $\langle S_z(5)S_z(6)\rangle$ of the electrons coming through the rails 5 and 6. No disentangled mixed state (a state of the form $\sum_i p_i |\psi_i\rangle \otimes |\phi_i\rangle \langle\phi_i|$ in the rails 3 and 4 can produce zero current noise in the outputs 5 and 6, unless $|\psi_i\rangle = |\phi_i\rangle$ for all $i$. This means that noiseless current in rails 5 and 6 is consistent with a disentangled state in rails 3 and 4 only when $\langle S_z(5)S_z(6)\rangle = 1$. However, for our state $\Lambda$, we will measure $\langle S_z(5)S_z(6)\rangle$ to be $1 - 7J^2/(1 + 9J^2)$, which guarantees its entanglement for any nonzero $J$.

We now discuss the issue of feasibility. We can choose the rails for the electrons to be carbon nanotubes, which are ballistic [30, 31] and spin coherent [17] conductors. Cross junctions of carbon nanotubes, as required by us, have been fabricated [32, 33]. In the context of Kondo effect experiments in nanotubes, recently cobalt clusters have been embedded in nanotubes as magnetic impurities [23]. If one can similarly place a single cobalt atom, it would serve the purpose of the localized spin in our experiment. In general, the effect of impurities in carbon nanotubes are averaged over the circumference of the entire tube [24] and sudden narrowing of the tube at the site of the localized spin could increase the scattering strength (our $J$). Alternatively, a junction between a nanotube and a very small conductor of a real metal can be made (such as the electrode-nanotube junctions in Ref. [6]), and the impurity can be placed in this small length of metal. Another option comes from the recent implementation of Kondo effect in quantum dot carbon nanotubes [21]. Conducting nanotubes can be connected to a quantum dot nanotube, whose spin effectively serves as our magnetic impurity. Quite apart from carbon nanotubes, one can have an all semiconductor implementation of our proposal by modifying the setup of Ref. [1] by placing a quantum dot with spin with the site of the beam-splitter. Indeed, such semiconductor quantum dots have served as localized magnetic spins in recent Kondo experiments [2]. Moreover, an all metal implementation is also possible if small enough gold wires can be fabricated so that electron transport in them is ballistic. This will then have to be combined with the deposition of a single
cobalt atom on a gold substrate (as in Ref. [22]) to obtain our setup. Quite outside the realm of electrons, ballistic waveguides and beam splitter like microstructures have been recently fabricated for tests of atomic statistics [23]. If one can localize an atomic spin at the beam splitter, then fermionic atoms could also be used to carry out our experiment.

In this letter, we have presented a scheme for entangling the spins of two conducting electrons which uses the combined effects of magnetic scattering and fermionic statistics. It has advantages of obtaining the entangled electrons mobile in separate wires, not requiring control over spin-spin interactions, not requiring non-absorbing measurements of electron paths, and is applicable to all fermions. The entangling is successful irrespective of the final state of the impurity and is robust to uncertainty about the initial state of the impurity. Further work can focus on the possibility of using electron scattering from successive magnetic impurities to implement a two qubit logic gate between the impurity spins. Prototype implementations of quantum communications through magnetic scattering and entangled electrons could also be studied. In particular, some quantum information processing protocols using statistics in a fundamental way have been proposed recently [35]. Extensions of these schemes with an additional magnetic scattering should be interesting.

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