MSSM Higgs-boson mass predictions and
two-loop non-supersymmetric counterterms

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Abstract

The evaluation of Yukawa-enhanced two-loop contributions to the
MSSM Higgs-boson mass is considered. We prove the common assumption
that regularization by dimensional reduction preserves supersymmetry at
the required level. Thus generating counterterms by multiplicative renor-
malization is correct. Technically, we identify a suitable Slavnov-Taylor
identity, use a recently developed method to evaluate it at the two-loop
level, and show that it is valid in dimensional reduction.

1 Introduction

The prediction of the mass of the lightest Higgs boson is one of the most striking
features of the Minimal Supersymmetric Standard Model (MSSM). At tree level,
the mass of the lightest MSSM Higgs boson $M_h$ is bound to be smaller than the
mass of the $Z$-boson. At higher orders, $M_h$ is a function of all MSSM param-
eters, but there still is a very restrictive upper bound as long as the masses of
supersymmetric particles are not much higher than 1 TeV.
The LEP exclusion bound of 114.4 GeV [1] for the mass of a standard model (SM)-like Higgs boson allows to derive stringent constraints on the MSSM parameter space. In the future, the direct measurement of the Higgs-boson mass will lead to very small experimental uncertainties of only 200 MeV at the LHC [2] and 50 MeV at the ILC [3]. Comparing the measurement of \( M_h \) to its predicted value amounts to sensitive tests of the MSSM and allows the indirect determination of further MSSM parameters. It is therefore important to know \( M_h \) as a function of the MSSM parameters as precisely as possible.

A lot of effort has been put into the evaluation of \( M_h \) (see Ref. [4] for a review). So far, the one-loop contributions are completely known. The most important two-loop corrections are the ones enhanced by the Yukawa couplings of the top and bottom quark \( \alpha_{t,b} = \frac{y_{t,b}^2}{4\pi} \). The Yukawa-enhanced contributions of \( O(\alpha_t \alpha_s) \) [5–13], \( O(\alpha_t^2) \) [5, 14, 15], \( O(\alpha_b \alpha_s) \) [16, 17], \( O(\alpha_t \alpha_b) \), \( O(\alpha_b^2) \) [18] are known in the on-shell renormalization scheme. All these Yukawa-enhanced contributions can be evaluated in the gauge-less limit, where the electroweak gauge couplings and \( M_W, M_Z \) go to zero with fixed ratio \( M_W/M_Z \) and fixed Higgs vacuum expectation values. Further electroweak two-loop contributions that go beyond the gauge-less limit are known in the \( \overline{DR} \) scheme [19, 20]. The remaining intrinsic error of the Yukawa-enhanced corrections has been estimated to \( \delta M_h^{\text{intr}} = 3 \) GeV [21].

There might be, however, an additional Yukawa-enhanced contribution from non-supersymmetric counterterms at the two-loop level. So far, such terms have been assumed to vanish, but without an explicit investigation. In this paper we close this gap by a detailed evaluation of the counterterm structure at two-loop order.

In all the references listed above, regularization by dimensional reduction (DRED) [22] has been employed and it has been assumed that all symmetries of the MSSM are preserved by this method. According to this assumption, the structure of the counterterms is symmetric and generated by multiplicative renormalization. However, although dimensional reduction has been checked to preserve supersymmetry in many individual cases, there is no general proof.

The traditional checks apply to one-loop self energies [23], one-loop on-shell [24] and off-shell [25, 26] three-point functions and further one-loop relations involving soft and spontaneous symmetry breaking [27]. Further checks concern the renormalization group \( \beta \) functions at the two-loop level [28], corresponding to divergences of two-loop diagrams. Most recently, supersymmetry identities for one-loop four-point functions and two-loop two-point functions (without spontaneous symmetry breaking) have been checked [29]. In all cases, dimensional reduction was in agreement with the required supersymmetry relations. These checks are indeed sufficient to prove that multiplicative renormalization is correct for the one-loop counterterms of the Higgs, quark and squark sectors entering the
two-loop calculation of $M_h$ at the level of one-loop subrenormalization.

These results, however, do not constitute a proof that dimensional reduction preserves supersymmetry at the two-loop level. In particular, they cannot exclude that supersymmetry is broken by a finite amount, which could be relevant for precision calculations of $M_h$.

In the case of supersymmetry-breaking by the method of regularization, extra symmetry-restoring counterterms have to be added as discussed e.g. in Refs. [25–27]. These extra counterterms, which are by themselves not supersymmetric, are required to restore the original symmetry at the quantum level at the considered order. They are finite since all divergences are cancelled by symmetric counterterms derived from multiplicative renormalization. If needed, they would cause a finite shift in the Higgs-boson self-energies and thus would be relevant for precision calculations of $M_h$. It is therefore important to clarify the necessity for such counterterms.

In particular, if the symmetry is broken, the two-loop counterterms for the Higgs-potential have to be modified according to

$$\delta^{(2)} V = \delta^{(2)} V_{\text{sym}} + \delta^{(2)} V_{\text{non-susy}},$$

which contains the usual multiplicative symmetric counterterms $\delta^{(2)} V_{\text{sym}}$ and additional supersymmetry-restoring counterterms $\delta^{(2)} V_{\text{non-susy}}$. The renormalizability of the MSSM [30–33] guarantees that supersymmetry can be restored by a suitable choice of $\delta^{(2)} V_{\text{non-susy}}$. But since the structure of these extra counterterms does not correspond to multiplicative renormalization, they have to be derived from the supersymmetric Slavnov-Taylor identities explicitly evaluated at the two-loop level.

In the present paper we derive the relevant supersymmetric two-loop Slavnov-Taylor identities that are potentially affected by $\delta^{(2)} V_{\text{non-susy}}$ and determine the values of the supersymmetry-restoring counterterms by a direct calculation. Since we are aiming at contributions from the large Yukawa couplings, we restrict ourselves to the gauge-less limit with respect to the electroweak interaction, i.e. we take into account electroweak Yukawa interactions and the strong interaction in terms of supersymmetric QCD.

Our explicit determination makes use of recent developments concerning dimensional reduction [29, 34] (see also Refs. [35, 36] for complementary recent work). In Ref. [29], it has been shown that a slight reformulation of dimensional reduction, which does not affect its application to the calculation of $M_h$, avoids the mathematical inconsistency of Ref. [37]. Based on this reformulation, a dramatic simplification for evaluating supersymmetric Slavnov-Taylor identities has been obtained. The evaluation of such identities constitutes the central part of our calculation of $\delta^{(2)} V_{\text{non-susy}}$. 

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The general strategy of our calculation follows the approach described in Refs. [25–27]. First we discuss general properties of the potential supersymmetry-restoring counterterms that are relevant for the calculation of the Higgs-boson mass. Then we derive suitable Slavnov-Taylor identities that are sensitive to these counterterms. Finally, the Slavnov-Taylor identities are evaluated using the method of Ref. [29].

2 Higgs potential and counterterms

The two-loop evaluation of the Higgs-boson mass spectrum requires three types of counterterms: one-loop counterterms that renormalize divergent one-loop subdiagrams, products of one-loop counterterms that partially renormalize two-loop diagrams, and genuine two-loop counterterms. As mentioned in the introduction, multiplicative renormalization is sufficient for the one-loop counterterms. Here we examine whether this applies also to the genuine two-loop counterterms.

The genuine two-loop counterterms relevant for the evaluation of the Higgs-boson self-energies correspond to the MSSM Higgs potential $V$. At tree level, $V$ is given by

$$V = V_{\text{quadratic}} + V_{\text{quartic}}, \quad (2a)$$
$$V_{\text{quadratic}} = m_1^2 |H_1|^2 + m_2^2 |H_2|^2 + m_3^2 \left( H_1^\dagger H_2^2 - H_2^\dagger H_1^2 + \text{h.c.} \right), \quad (2b)$$
$$V_{\text{quartic}} = \frac{g_1^2 + g_2^2}{8} \left( |H_1|^2 - |H_2|^2 \right)^2 + \frac{g_2^2}{2} \left| H_1^\dagger H_2 \right|^2, \quad (2c)$$

in terms of the Higgs doublets $H_1 = H_1 + (v_1)$, $H_2 = H_2 + (v_2)$, their respective vacuum expectation values $v_{1,2}$, and the $SU(2)$ and $U(1)$ gauge couplings $g_{2,1}$. The mass parameters $m_{1,2}^2$ are combinations of soft supersymmetry-breaking parameters and the $\mu$-parameter, and $m_3^2$ is a soft supersymmetry-breaking parameter.

The symmetric counterterms used in the literature are generated by the multiplicative renormalization transformation

$$m_i^2 \rightarrow m_i^2 + \delta^{(1)} m_i^2 + \delta^{(2)} m_i^2, \quad (3a)$$
$$g_i \rightarrow g_i + \delta^{(1)} g_i + \delta^{(2)} g_i, \quad (3b)$$
$$v_i \rightarrow v_i + \delta^{(1)} v_i + \delta^{(2)} v_i, \quad (3c)$$
$$H_i \rightarrow (1 + \delta^{(1)} Z_{H_i} + \delta^{(2)} Z_{H_i})^{1/2} H_i, \quad (3d)$$

where the notation $\delta^{(l)}$ refers to a renormalization constant of $l$-loop order. Thus, the genuine two-loop symmetric counterterms have the structure

$$\delta^{(2)} V_{\text{sym}} = \left[ \delta^{(2)} m_i^2 \partial_{m_i^2} + \delta^{(2)} g_i \partial_{g_i} + \frac{1}{2} H_i \delta^{(2)} Z_{H_i} \partial_{H_i} + \delta^{(2)} v_i \partial_{v_i} \right] V, \quad (4)$$
which is sufficient only if DRED preserves supersymmetry at the two-loop level in the gauge-less limit.

If this is not the case, $\delta^{(2)}V$ has to be augmented by supersymmetry-restoring counterterms, as indicated in (1). A priori, $\delta^{(2)}V_{\text{non-susy}}$ contains quadratic and quartic terms,

$$\delta^{(2)}V_{\text{non-susy}} = \delta^{(2)}V_{\text{quadratic non-susy}} + \delta^{(2)}V_{\text{quartic non-susy}}. \tag{5}$$

Two simple properties of $\delta^{(2)}V_{\text{non-susy}}$ can immediately be derived. First, $\delta^{(2)}V_{\text{non-susy}}$ respects global $SU(2) \times U(1)$ gauge invariance. The reason is that DRED$^1$ preserves global gauge invariance as can easily be obtained e.g. from the gauge invariance of the regularized Lagrangian using the quantum action principle [29]. As a consequence, $\delta^{(2)}V_{\text{non-susy}}$ depends on the Higgs doublets $H_{1,2}$ and $v_{1,2}$ only via the combinations $\mathcal{H}_{1,2}$, similar to $V$ itself. Second, the quadratic terms in $\delta^{(2)}V_{\text{non-susy}}$ are not necessary since the quadratic terms in the Higgs potential (2b) are not restricted by supersymmetry in the first place, and no supersymmetry-breaking could modify their structure. Therefore, we have

$$\delta^{(2)}V_{\text{quadratic non-susy}} = 0. \tag{6}$$

### 3 Relevant Slavnov-Taylor identities

Any non-zero counterterm quartic in $\mathcal{H}_1, \mathcal{H}_2$ modifies both the Higgs-boson four-point and two-point vertex functions and can thus contribute to the Higgs-boson mass prediction. For example, the term $|\mathcal{H}_1|^4 = |H_1|^4 + 2v_1^2 |H_1|^2 + 4v_1^4 (\text{Re}H_1)^2 + \ldots$ would contribute to the four-point function $\Gamma_{H_1 H_1 H_1 H_1}$ and to the two-point functions involving $H_1$. On the other hand, owing to (6), vanishing non-supersymmetric quartic counterterms would imply also the absence of non-supersymmetric counterterms for the self-energies. It is therefore sufficient to evaluate the quartic non-supersymmetric counterterms $\delta^{(2)}V_{\text{quartic non-susy}}$ by considering their contribution to four-point functions.

The starting point for the evaluation is the basic requirement that after renormalization, i.e. in particular after adding $\delta^{(2)}V_{\text{quartic non-susy}}$, the supersymmetric Slavnov-Taylor identity of the MSSM [33] holds,

$$S(\Gamma) = 0. \tag{7}$$

Here $\Gamma$ denotes the renormalized effective action, the generating functional of the renormalized one-particle irreducible vertex functions.

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$^1$The version of DRED used in the literature is the one defined in Ref. [29] with anticommuting $\gamma_5$. 
The following derivative of the Slavnov-Taylor identity directly constrains the desired Higgs-boson four-point functions:

\[ 0 = \frac{\delta^5 S(\Gamma)}{\delta \phi_a \delta \phi_b \delta \phi_c \delta H^i_{kL} \delta \bar{\epsilon}_L}, \]  

where \( \phi_i \) denote any components of the MSSM Higgs bosons \( H^i_j \), \( H^i_j \), and \( \tilde{H}^i_k \) is the Higgsino partner of \( H^i_k \). \( \epsilon \) denotes the supersymmetry ghost. The index \( L \) denotes left-handed spinors,

\[ \frac{\delta}{\delta \bar{\epsilon}_L} = P_L \frac{\delta}{\delta \epsilon}, \quad \frac{\delta}{\delta H^i_{kL}} = \frac{\delta}{\delta \tilde{H}^i_k} P_L \]  

with \( P_L = \frac{1}{2} (1 - \gamma_5) \). In the gauge-less limit, identity (8) is equivalent to the following relation between renormalized vertex functions,

\[ 0 = \sum_{\phi_i} \left[ \Gamma_{\tilde{H}^i_{kL}} Y_{\phi_i \epsilon_L} \Gamma_{\phi_a \phi_b \phi_c \phi_i} + \Gamma_{\phi_a \phi_b \phi_c \tilde{H}^i_{kL}} Y_{\phi_i \epsilon_L} \Gamma_{\phi_i} \right. \]

\[ + \left( \Gamma_{\phi_a \tilde{H}^i_{kL} Y_{\phi_i \epsilon_L} \Gamma_{\phi_b \phi_c \phi_i} + \Gamma_{\phi_a \phi_b \phi_c \tilde{H}^i_{kL} Y_{\phi_i \epsilon_L} \Gamma_{\phi_i}} + \text{perm} \right) \]

\[ - \sum_{i,j} \left[ Y^i_{\epsilon_L} \Gamma_{\phi_a \phi_b \phi_c \tilde{H}^i_{kL} \tilde{H}^j_i} + \Gamma_{\phi_a \phi_b \phi_c \tilde{H}^i_{kL} \tilde{H}^j_i} \Gamma_{\phi_i \epsilon_L} \Gamma_{\phi_i} \right. \]

\[ + \left( \Gamma_{\phi_a Y^i_{\epsilon_L} \Gamma_{\phi_b \phi_c \tilde{H}^i_{kL} \tilde{H}^j_i} + \Gamma_{\phi_a \phi_b \phi_c Y^i_{\epsilon_L} \Gamma_{\phi_c \tilde{H}^i_{kL} \tilde{H}^j_i} + \text{perm} \right) \]

\[ + \left( \Gamma_{\phi_a \phi_b Y^i_{\epsilon_L} \Gamma_{\phi_c \tilde{H}^i_{kL} \tilde{H}^j_i} + \Gamma_{\phi_a \phi_b \phi_c Y^i_{\epsilon_L} \Gamma_{\phi_c \tilde{H}^i_{kL} \tilde{H}^j_i} + \text{perm} \right) \]

\[ + \left( \Gamma_{\phi_a \phi_b \phi_c \phi_i Y^i_{\epsilon_L} \Gamma_{\tilde{H}^i_{kL} \tilde{H}^j_i} + \Gamma_{\phi_a \phi_b \phi_c \phi_i Y^i_{\epsilon_L} \Gamma_{\tilde{H}^i_{kL} \tilde{H}^j_i} + \text{perm} \right) \]

\[ + \sqrt{2} P_L f_0 \Gamma_{\phi_a \phi_b \phi_c \tilde{H}^i_{kL} \chi_L}. \]  

Although this identity is quite involved, it is easy to see that it relates the desired quantity, the \( \phi^4 \)-interaction, which appears as the first term in the first line, to many other vertex functions. In the first sum, \( \phi_i \) runs over all Higgs boson components \( H^i_j \), \( H^i_j \), and in the second sum \( i, j \) take the values 1, 2. The abbreviation “perm” denotes terms corresponding to all possible permutations of \( \phi_a, \phi_b, \phi_c \). The conventions for the Slavnov-Taylor identity of the MSSM have been adapted to four-spinors as e.g. in Refs. [26, 27]. The symbols \( Y_{\phi_i} \) and \( \tilde{Y}^i \) denote the sources of the BRS variations of \( \phi_i \) and \( \tilde{H}^i_i \), respectively; \( \chi \) and \( f_0 \) are the spinorial and constant components of the spurion superfield introduced to describe soft supersymmetry breaking [32].
At the two-loop level, each term of the form \( \Gamma_A \Gamma_B \) can be decomposed as follows,

\[
\Gamma_A^{(0)} \left( \Gamma_B^{(2), \text{DRED}} + \Gamma_B^{(2), \text{ct}} \right) + \Gamma_A^{(1)} \Gamma_B^{(1)} + \left( \Gamma_A^{(2), \text{DRED}} + \Gamma_A^{(2), \text{ct}} \right) \Gamma_B^{(0)},
\]

where \( \Gamma^{(l)} \) denotes the \( l \)-loop order contribution. Thereby, the two-loop contribution has been split into the regularized vertex functions including one-loop but excluding two-loop counterterms (denoted by the upper index “DRED”), and the corresponding genuine two-loop counterterms (index “ct”).

Likewise, identity (8) or (10), evaluated at 2-loop order, can be rewritten in the form

\[
0 = \Delta^{(2), \text{ct}} + \Delta^{(2), \text{DRED}},
\]

where \( \Delta^{(2), \text{ct}} \) is the contribution from genuine two-loop counterterms and \( \Delta^{(2), \text{DRED}} \) is the remaining contribution, involving the regularized vertex functions and one-loop counterterms. In the gauge-less limit, the first contribution in (12) is given by

\[
\Delta^{(2), \text{ct}} = \sum_{\phi_i} \left[ \Gamma^{(0)}_{\phi_i} \bar{H}_{kL} Y_{\phi_i} \epsilon_L \Gamma^{(2), \text{ct}}_{\phi_a \phi_b \phi_c \phi_i} + \Gamma^{(2), \text{ct}}_{\phi_a \phi_b \phi_c \phi_i} \Gamma^{(0)}_{\phi_i} \right] + \sum_{i,j} \left[ \Gamma^{(2), \text{ct}}_{\phi_a \phi_b \phi_i} \Gamma^{(0)}_{\phi_c \bar{H}_{kL} \epsilon_L} + \Gamma^{(2), \text{ct}}_{\phi_a \phi_b \phi_i} \Gamma^{(0)}_{\phi_c \bar{H}_{kL} \epsilon_L} \right] + \text{perm}.
\]

All terms in (11) that do not re-appear in (13) vanish since they involve Green functions of power-counting dimension \( \geq 5 \), for which there are neither classical nor counterterm contributions. Moreover, most of the terms in (13) vanish due to global gauge invariance or as a result of the gauge-less limit. The only term that can be non-zero is the first term for \( \phi_i = \phi_d \equiv H_{kL} \). The result for \( \Gamma^{(0)}_{\phi_i} \bar{H}_{kL} Y_{\phi_d} \epsilon_L \) is well known (see e.g. [26, 27]), and we obtain

\[
\Delta^{(2), \text{ct}} = \sqrt{2} P_L \Gamma^{(2), \text{ct}}_{\phi_a \phi_b \phi_c \phi_d}.
\]

The second contribution in (12) has the same form as the right-hand side of (11), but the products \( \Gamma_A \Gamma_B \) have to be evaluated at the two-loop level without the genuine two-loop counterterms. \( \Delta^{(2), \text{DRED}} \) can be written in the following simple form,

\[
\Delta^{(2), \text{DRED}} = \left[ \frac{\delta^5 S(\Gamma^{(2), \text{DRED}})}{\delta \phi_a \delta \phi_b \delta \epsilon_c \delta \phi_d \delta \epsilon_L} \right]_{2\text{-loop}}.
\]
where \( \Gamma^{(2),DRED} \) denotes the regularized vertex functional including one-loop but excluding 2-loop counterterms.

Hence, identity (8) leads to

\[
P_L \Gamma^{(2),ct}_{\phi_a \phi_b \phi_c \phi_d} = -\frac{1}{\sqrt{2}} \left[ \frac{\delta^5 S(\Gamma^{(2),DRED})}{\delta \phi_a \delta \phi_b \delta \phi_c \delta \phi_d \delta \bar{\epsilon}_L} \right]_{2\text{-loop}},
\]

at the two-loop level in the gauge-less limit. This is an equation that directly determines the counterterm for any \( \phi^4 \)-interaction. Since the multiplicative counterterm to \( \Gamma^{ct}_{\phi_a \phi_b \phi_c \phi_d} \) vanishes in the gauge-less limit, we can rewrite this equation in the form

\[
\frac{\delta^4}{\delta \phi_a \delta \phi_b \delta \phi_c \delta \phi_d} \int d^D x P_L \delta^{(2)}_{\text{quartic non-susy}} = \frac{1}{\sqrt{2}} \left[ \frac{\delta^5 S(\Gamma^{(2),DRED})}{\delta \phi_a \delta \phi_b \delta \phi_c \delta \phi_d \delta \bar{\epsilon}_L} \right]_{2\text{-loop}}.
\]

Obviously, by choosing all possible combinations of \( \phi_i \), this equation, together with its complex conjugate, completely determines the non-supersymmetric counterterms at the desired order.

### 4 Evaluation of the Slavnov-Taylor identities

The remaining task is to evaluate the right-hand side of (17). Traditionally, we would have to evaluate all Green functions appearing in (10), in particular, also 5-point functions at the two-loop level. We can simplify the evaluation by using the results of Ref. [29], according to which the right-hand side of (17) is equal to

\[
\Delta^{(2),DRED} = -iP_L \left[ ([S(\Gamma^{(0)})] \cdot \Gamma^{DRED})_{\phi_a \phi_b \phi_c \bar{R}_k \bar{\epsilon}} \right]_{2\text{-loop}} P_L,
\]

involving an insertion of the operator \([S(\Gamma^{(0)})]\) at the two-loop level in the gauge-less limit.

In Ref. [29], the operator \([S(\Gamma^{(0)})]\) has been given for a general supersymmetric gauge theory without spontaneous breaking of gauge invariance and soft supersymmetry breaking. For the purpose of the present paper we have rederived this operator for the case of the MSSM, taking into account both symmetry breakings in the way defined in Ref. [33]. It turns out that those parts that can play a role in (18) are not modified compared to the case without symmetry breaking. All relevant terms in \([S(\Gamma^{(0)})]\) are four-fermion operators involving \( \bar{\epsilon} \) and three other fermions (here either gluinos, Higgsinos, or quarks).

Fig. 1 shows the Feynman diagrams contributing in the gauge-less limit to (18), i.e. to the vertex function with external \( \phi_{a,b,c}, H_k, \bar{\epsilon} \) and the insertion of the composite operator \([S(\Gamma^{(0)})]\). The insertion of \([S(\Gamma^{(0)})]\) is marked by a cross, and
the three Higgs fields have to be attached in all possible ways. In all diagrams, the basic fermion-loop topology is “Topology (c)” from Ref. [29]; i.e. the open fermion line involves $\bar{\epsilon}$ and quarks or Higgsinos, but no gauginos.

It is useful to denote the chain of $\gamma$-matrices corresponding to the open fermion line as $A$, and the $\gamma$-chain corresponding to the closed fermion loop as $B$. Then the Feynman rules for the diagrams involve terms like the following ones,

$$P_L \gamma^\mu B \gamma_\mu P_L A P_L, \quad P_L A P_L Tr(P_R B), \ldots$$

We do not need the detailed Feynman rules but only two properties that can be easily read off from the rules given in Ref. [29] combined with the explicit factors $P_L$ appearing in (18):

1. a diagram vanishes if the number of $\gamma^\mu$-matrices in $AB$ is odd.

2. a diagram vanishes if the number of $\gamma^\mu$-matrices in $B$ is smaller than four.

After carrying out the two loop integrals in diagrams of the form Fig. 1a,b, the only covariants that can appear in the $\gamma$-chain $B$ are $p_{a,b,c}$, where $p_{a,b,c}$ are the incoming momenta of the Higgs fields $\phi_{a,b,c}$. Hence the $\gamma$-chain $B$ can be simplified to terms involving at most three factors of $p_i$ and no other $\gamma^\mu$-matrices. As a result of property 2 above, these diagrams all vanish.

Similarly, it can be easily seen that diagrams of the form Fig. 1c: vanish if at most two of the three Higgs fields are attached to the fermion loop. The only remaining case is the one shown in Fig. 2 where all three Higgs fields are attached to the fermion loop. This case is now discussed in more detail. Due to the structure $\not{q}_i + m_i$ of the numerators of the fermion propagators, the $\gamma$-chain $B$
in the diagram of Fig. 2 is a sum of terms involving between zero and five factors of $q_i$, while the terms of the $\gamma$-chain $A$ involve either zero or one such factor.

After carrying out the fermion-loop integral, $B$ can contain up to four covariants $q_i \in \{p_{a,b,c}, k\}$, where $k$ is the second loop momentum. Hence in those terms of $B$ that contain five factors of $q_i$, actually two of the $q_i$ must be equal, and the product of these five $q_i$ can be reduced to products of only three different $q_i$. Hence such terms of $B$ contribute zero to the diagrams.

Those terms of $B$ that contain four factors of $q_i$ can contribute only together with terms of $A$ containing no such factor, due to property 1 above. In such a situation, the $k$-dependence of the second loop integral has the following form,

$$
\int \frac{d^Dk}{(2\pi)^D} \frac{k f(k^2, kp_a, kp_b, kp_c)}{[(k + p_a + p_b + p_c)^2 - m_q^2][k^2 - m_q^2]},
$$

(20)

where the scalar function $f$ results from the fermion-loop integral. As a result of the loop integral (20), the factor $k$ within $B$ is replaced by a linear combination of $p_{a,b,c}$. In this way, each term within $B$ can be simplified to a term involving at most two $q_i$-factors.

Taken together, there are no terms in the $\gamma$-chain $B$ that can possibly give a contribution of diagrams of the form Fig. 1c. Therefore, all diagrams in Fig. 1 vanish, and thus we obtain

$$
\Delta^{(2), DRED} = 0,
$$

(21)

and, with the help of (15) and (17), we find the relation

$$
\delta^{(2)} V_{\text{quartic-non-susy}} = 0,
$$

(22)

which excludes the presence of non-symmetric counterterms.
5 Conclusions

We have considered the Yukawa-enhanced two-loop contributions to Higgs-boson masses in the MSSM, i.e. contributions of $O(\alpha_t \alpha_s)$, $O(\alpha_t^2 \alpha_s)$, $O(\alpha_t \alpha_b)$. In the literature these contributions have been evaluated under the assumption that DRED is a supersymmetric regularization up to the required order. In the present article we have verified this assumption.

The question of necessity for supersymmetry-restoring counterterms was studied utilizing the algebraic method of supersymmetric Slavnov-Taylor identities. A Slavnov-Taylor identity has been found that unambiguously determines the potentially necessary non-symmetric counterterms. It involves up to two-loop 5-point functions, but it could be explicitly evaluated using the method of Ref. [29]. The identity turned out to be valid at the regularized level, which means that supersymmetry is preserved by DRED, multiplicative renormalization is sufficient, and therefore no extra supersymmetry-restoring counterterms are necessary.

The restriction to the Yukawa-enhanced contributions and the gauge-less limit is of crucial importance. If the gauge-less limit is relaxed, more terms of the operator $[S(T^{(0)})]$ can contribute and can lead to more complicated Feynman diagrams. For example, there are diagrams with a topology like in Fig. 1a, but with a vector boson instead of the virtual squark (and e.g. the $\tilde{H} - q - \bar{q}$-vertex replaced by a $\tilde{H} - \tilde{H} - V$-vertex). The $\gamma$-strings $A$ and $B$ in such diagrams have one $\gamma$-matrix more, and the arguments of section 4 cannot be applied.

It is, therefore, not straightforward to extrapolate the results of the present paper to the full electroweak two-loop contributions and/or to the three-loop level. The future experimental accuracy of Higgs-boson mass measurements, however, requires to bring the full two-loop and leading three-loop contributions under control. A dedicated study of the supersymmetry-properties of DRED for these more complicated cases will be an indispensable step in this program.

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