Forced oscillations in relativistic accretion disks and QPOs

J. Pétri

Max-Planck-Institut für Kernphysik, Saupfercheckweg 1, 69117 Heidelberg - Germany

Abstract. In this work we explore the idea that the high frequency QPOs observed in LMXBs may be explained as a resonant coupling between the neutron star spin and epicyclic modes of accretion disk oscillations. We propose a new model for these QPOs based on forced oscillations induced in the accretion disk due to a stellar asymmetric rotating gravitational or magnetic field. It is shown that particles evolving in a rotating non-axisymmetric field are subject to three kinds of resonances: a corotation resonance, a Lindblad resonance due to a driving force, and a parametric resonance due to the time varying epicyclic frequencies. These results are extends by means of 2D numerical simulations of a simplified version of the accretion disk. The simulations are performed for the Newtonian gravitational potential, as well as for a pseudo-general relativistic potential, which enables us to explore the behavior of the resonances around both rotating neutron stars and black holes. Density perturbations are only significant in the region located close to the inner edge of the disk near the ISCO where the gravitational or magnetic perturbation is maximal. It is argued that the nearly periodic motion induced in the disk will produce high quality factor QPOs.

1. Introduction

Quasi-periodic oscillations (QPOs) have been observed in accretion disks around neutron stars, black holes, and white dwarf binaries with frequencies ranging from a few 0.1 Hz up to 1300 Hz. Recent observations have shown a strong correlation between the low and high frequency QPOs (Mauche 2002, Psaltis et al. 1999). This relation holds over more than 6 orders of magnitude in frequency and strongly supports the idea that the QPO phenomenon is a universal physical process independent of the nature of the compact object.

To date, quasi-periodic oscillations (QPOs) have been observed in about twenty Low Mass X-ray Binaries (LMXBs) containing an accreting neutron star. Among these systems, the high-frequency QPOs (kHz-QPOs) which mainly show up in pairs, denoted by frequencies $\nu_1$ and $\nu_2 > \nu_1$, possess strong similarities in their frequencies, ranging from 300 Hz to about 1300 Hz, as well as in their shapes (van der Klis 2000).

Several models have been proposed to explain the kHz-QPOs in LMXBs. A beat-frequency model was introduced to explain the commensurability between the twin kHz-QPOs frequency difference and the neutron star rotation. This interaction between the orbital motion and the star rotation happens at some preferred radius. Alpar & Shaham (1985) and Shaham (1987) proposed the magnetospheric radius to be the preferred radius. The sonic-point beat-frequency model was suggested by Miller et al. (1998). In this model, the preferred radius is the point where the radial inflow becomes supersonic.

The relativistic precession model introduced by Stella & Vietri (1998, 1999) makes use of the motion of a single particle in the Kerr-spacetime. In this model, the kHz-QPOs frequency difference is related to the relativistic periastron precession of weakly elliptic orbits while the low-frequencies QPOs are interpreted as a consequence of the Lense-Thirring precession. Abramowicz & Kluzniak (2001) introduced a resonance between orbital and epicyclic motion that can account for the 3:2 ratio around Kerr black holes leading to an estimate of their mass and spin. The 3:2 ratio of black hole QPOs frequencies is well established (McClintock & Remillard, 2003). In other models, the QPOs are identified with gravity or pressure oscillation modes in the accretion disk (Titarchuk et al. 1998, Wagoner et al. 2001). Rezzolla et al. (2003) suggested that the high frequency QPOs in black hole binaries are related to p-mode oscillations in a non Keplerian torus.

We propose a new explanation of this phenomenon based on a resonance in the star-disk system arising from the response of the accretion disk to either a non-
axisymmetric rotating gravitational field, for a hydrodynamical disk, or to a non-axisymmetric rotating magnetic field, for an MHD disk.

2. Hydrodynamical disk

We first consider a hydrodynamical disk evolving in an asymmetric gravitation field imposed by the rotating accreting compact object. A simplified linear analysis reveals the main characteristic of the perturbed motion in the disk. This is then confirmed by full non-linear numerical simulations.

2.1. Linear analysis

By perturbing the set of hydrodynamical equations governing the evolution in the disk around the equatorial plane, and introducing the Lagrangian displacement $\xi$, the weak oscillations in the radial and vertical directions can be cast into a partial differential equation for the radial and vertical displacement respectively. Neglecting the sound propagation term (not important because not leading to resonance conditions), the Laplacian displacement $\xi_{r/z}$ satisfies:

\[
\frac{D^2\xi_r}{Dt^2} + \frac{\kappa_r^2}{\rho} \xi_r + \frac{1}{\rho} \frac{\partial}{\partial r} (r \rho \xi_r) \delta g_r = \delta g_r \tag{1}
\]

\[
\frac{D^2\xi_z}{Dt^2} + \frac{\kappa_z^2}{\rho} \xi_z + \frac{1}{\rho} \frac{\partial}{\partial z} (\rho \xi_z) \delta g_z = \delta g_z \tag{2}
\]

We have introduced the radial/vertical epicyclic frequency $\kappa_{r/z}$ and the perturbation in radial/vertical gravitational field by $\delta g_{r/z}$. The convective derivative is denoted by $D/Dt = \partial_t + \Omega \partial_{\phi}$ where $\Omega$ is the unperturbed local orbital frequency in the disk. The 3 terms in Eq. ([1] and [2]) are: a harmonic oscillator at frequency $\kappa_{r/z}$, second term on the left hand side, with a periodically perturbed eigenfrequency, third term on the LHS, and a periodic driven source, on the right hand side.

A careful analysis of this equation shows the emergence of three kinds of resonance corresponding to:

- a corotation resonance;
- an inner and outer Lindblad resonance;
- a parametric resonance when $m (\Omega_* - \Omega) = 2 \kappa_{r/z}/n$, where $\Omega_*$ is the stellar spin, $\kappa_{r/z}$ the radial and vertical epicyclic frequencies, $n$ a natural integer and $m$ the azimuthal mode of the gravitational perturbation.

For a Newtonian disk, we have $\Omega = \kappa_r = \kappa_z$, and the parametric resonance condition simplifies into:

\[
\frac{\Omega}{\Omega_*} = \frac{m}{m \pm 2/n} \tag{3}
\]

As a consequence, the resonances are all located in the frequency range $\Omega \in [\Omega_*/3, 3 \Omega_*]$. For the general-relativistic disk, it splits into the two following cases:

\[
\Omega(r, a) \pm \frac{2 \kappa_{r/z}(r, a)}{m n} = \Omega_* \tag{4}
\]

For a typical neutron star, we choose:

- mass $M_* = 1.4 M_\odot$;
- angular velocity $\nu_0 = \Omega_*/2\pi = 300 - 600$ Hz;
- moment of inertia $I_* = 10^{38}$ kg m$^2$;
- angular momentum $a_* = c I_*/G M_*^2$.

The angular momentum is then given by $a_* = 5.79 \times 10^{-5}$ $\Omega_*$. For the selected spin rate of the star we find $a_* = 0.109 - 0.218$ and so the vertical epicyclic frequency is close to the orbital one $\kappa_z \approx \Omega$. Thus for the vertical resonance, we are still close to the Newtonian case given by Eq. ([3]).

The results for an accretion disk evolving in a Newtonian and a Kerr spacetime are shown in Table 1 and 2 (Pétri 2006).

### Table 1. Orbital frequencies at the parametric vertical resonance for the first three order n in the case of a Newtonian gravitational potential. The value on the left of the symbol / corresponds to the absolute value sign taken to be - and on the right to be +.

| Mode | $\nu_0 = 600$ Hz | $\nu_0 = 300$ Hz |
|------|-----------------|-----------------|
| $m$  | $n = 1$         | $n = 2$         |
| $n$  | $n = 1$         | $n = 2$         |
| 1    | -600 / 200      | -300 / 100      |
| 2    | -100 / 200      | 600 / 200       |
| 3    | 900 / 450       | 225             |

### Table 2. Same as Tab. 1 but for the general-relativistic disk.

| Mode | $\nu_0 = 600$ Hz | $\nu_0 = 300$ Hz |
|------|-----------------|-----------------|
| $m$  | $n = 1$         | $n = 2$         |
| $n$  | $n = 1$         | $n = 2$         |
| 1    | -200 / 100      | -100 / 50       |
| 2    | 100 / 50        | 200             |
| 3    | 450             | 225             |

2.2. Two-dimensional simulations

From the analytical analysis of the linear response of a thin accretion disk in the 2D limit, we know that waves are launched at the aforementioned resonance loci. They propagate in some permitted regions inside the disk, according to the dispersion relation obtained by a WKB analysis. We confirm and extend these results by performing non-linear hydrodynamical numerical simulations using a pseudospectral code solving Euler’s equations in a 2D cylindrical coordinate frame. Simulations were performed for the...
Fig. 1. Final snapshot of the density perturbation $\delta\rho/\rho_0$ in the accretion disk evolving in a quadrupolar perturbed Newtonian potential. Time is normalized to the spin period. The $m = 2$ structure emerges in relation with the $m = 2$ quadrupolar potential perturbation.

Fig. 2. Final snapshot of the density perturbation in the accretion disk evolving in a perturbed Newtonian potential having many azimuthal modes $m$.

3. MHD disk

We extend the previous idea to a magnetised accretion disk evolving in the rotating asymmetric magnetic field field of the accreting compact object (linear analysis and numerical simulations) (Pétri 2005a). This case is well suited for binaries containing a neutron star or even for white dwarfs in cataclysmic variables for which the QPO-phenomenology seems to be identical (Warner et al. 2003).

3.1. Linear analysis

Here again, applying the linear perturbation theory to the set of MHD equations governing the motion in the disk and focusing only on resonance terms in the radial and vertical directions, the Laplacian displacement $\xi_{r/z}$ satisfies:

$$\frac{D^2 \xi_z}{Dt^2} + \left[ \kappa_z^2 - \frac{\partial}{\partial z} \left( \frac{\delta B_r^\ast}{\mu_0 \rho} \frac{\partial B_r}{\partial z} \right) \right] \xi_z = -\frac{\partial}{\partial z} \left( \frac{B_r \delta B_r^\ast}{\mu_0 \rho} \right)$$

$$\frac{D^2 \xi_r}{Dt^2} + \left[ \kappa_r^2 - \frac{\partial}{\partial r} \left( \frac{\delta B_z^\ast}{\mu_0 \rho} \frac{\partial B_z}{\partial r} \right) \right] \xi_r = -\frac{\partial}{\partial r} \left( \frac{B_z \delta B_z^\ast}{\mu_0 \rho} \right)$$

Now, the perturbation in magnetic field $\delta B_{r/z}$ replaces the perturbation in gravitational field. We recognise again a harmonic oscillator with periodically varying eigenfrequency superposed on a driven source term. Therefore, despite the presence of a magnetic field, the behavior of the accretion disk, at least in the linear stage of its evolution, agrees with the study already made for a hydrodynamical disk. The orbital frequencies for resonance are therefore also given by tables 1 and 2 for a Newtonian and a relativistic disk respectively.

3.2. Two-dimensional simulations

We again performed 2D numerical simulations by solving the magnetohydrodynamical equations for the accretion disk. This is done by extending the previous pseudo-spectral method to a simplified version of the 2D MHD accretion disk.

Before the time $t = 0$, the disk stays in its axisymmetric equilibrium state and possesses only azimuthal motion. At $t = 0$, we switch on the perturbation by adding an asymmetric rotating component to the magnetic field. We then let the system evolve during more than one thousand orbital revolutions of the inner edge of the disk.

We ran a simulation in which the rotation of the star is taken into account. This shifts the location of the ISCO
closer to the surface of the neutron star as compared to the non-rotating case.

We chose a star with an angular momentum of $a_\ast = 0.5$. Therefore, the disk inner boundary corresponding to the marginally stable circular orbit approaches the horizon. An example of the density perturbation in the pseudo-Kerr metric is shown in Fig. 3 for the $m = 2$ mode. The inner Lindblad radius is clearly identified while the outer Lindblad radius is outside the simulation box.

4. Slow vs fast rotator

Focusing on accreting neutron stars in LMXBs, observations reveal that they can be divided into two categories (van der Klis [2004]) : the slow rotators possessing a rotation rate $v_\ast \approx 300$ Hz, and for which the frequency difference between the two peaks is around $\Delta \nu \approx v_\ast$ and $v_2 \approx 3 v_1 / 2$ and the fast rotators having $v_\ast \approx 600$ Hz, for which this difference is around $\Delta \nu \approx v_\ast / 2$.

The model presented in this work can account for this segregation if the innermost stable circular orbit (ISCO) is taken into account. Indeed, for a typical neutron star, the orbital frequency at the ISCO is $v_{isco} = 1571$ Hz which is therefore the upper limit for any QPO frequency. Discarding the resonance frequencies in the relativistic disk which are higher than $v_{isco}$, we conclude from Table 2 that:

- for slow rotators, the two highest frequencies are less than $v_{isco}$ and given by $v_1 = 599$ Hz and $v_2 = 898$ Hz, therefore $\Delta \nu = 299$ Hz which is very close to $v_\ast$. Moreover $v_2 \approx 3 v_1 / 2$ in accordance with observations;
- for fast rotators, the highest frequency is not observed because the resonance is located inside the radius of the ISCO ($v_{isco} < 1790$ Hz). Therefore the two highest observable frequencies are $v_1 = 899$ Hz and $v_2 = 1198$ Hz, having a difference $\Delta \nu = 299$ Hz which is close to $v_\ast / 2$.

As already claimed in the previous section, these conclusions apply to hydrodynamical (Pétrí [2005a]) as well as to magnetised (Pétrí [2005c]) accretion disks. Observations therefore strongly support our resonance model.

5. Conclusion

The consequences of a weakly rotating asymmetric stellar gravitational or magnetic field on the evolution of a thin accretion disk are as follows. Corotation, driven and parametric resonances are excited at some preferred radii. The kHz-QPOs are interpreted as the orbital frequency of the disk at locations where the vertical response to the resonances is maximal. The 3:2 ratio is predicted for the strongest modes and a clear distinction exists between slow and fast rotators, a direct consequence of the presence of an ISCO. Nevertheless general relativistic effects are not required to excite these resonances. They behave identically in the Newtonian as well as in the Kerr field. Therefore the QPO phenomenology is explained by the same picture, irrespective of the nature of the compact object (black hole, neutron star or white dwarf). Indeed the presence or the absence of a solid surface, a magnetic field or an event horizon plays no relevant role in the production of the X-ray variability.

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