The Determination of Damping Ratio Based on Cross-correlation Function Method

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Abstract: For a long time, the regular, even sinusoidal dynamic load is used to study the development of pore pressure and strain in soil dynamic tests. But in fact, the dynamic loads in the study of soil dynamic characteristics are mostly irregular and even random. At present, the equivalent linear model method is used to calculate the dynamic characteristics of soil under random loads. However, this method does not take into account the plastic deformation of soil which is contrary to the actual deformation of soil. In this paper, the stochastic process theory is introduced into the measurement and analysis of soil dynamic characteristics. The dynamic stress time history, dynamic strain time history and dynamic pore pressure time history are all regarded as random processes. So a cross-correlation function method based on stochastic theory is proposed, and the feasibility of determining the damping ratio of soil by this method is verified by means of experiments and numerical analysis. The results show that the cross-correlation function method can not only solve the damping ratio of soil under periodic load, but also determine the damping ratio of soil under random load.

1. Introduction
Soil dynamic damping ratio is an important parameter of the dynamic property analysis of soil [1]. Hardin B. O et al [2-14] who studied the damping ratio of soil by using the equivalent linear model. They analyzed the dynamic damping ratio change pattern with dynamic strain, cycle times, void ratio and the other factors of the different types of soil. WANG Y L [15] used an approximate-calculation method which utilized the hysteresis curve to approximate polygon to calculate the damping ratio of soil. LIU B J [16] also suggested not to consider the influence of factors such as the soil property, the types of consolidation, the types of dynamic loads, frequency and so on in calculating the damping ratio of soil with cross-correlation function method.

At present, the equivalent linear model is used to calculate the damping ratio of the soil in most studies. However, there are few studies on calculating the damping ratio of soil by using cross-correlation function method and the damping ratio of soil under random load. Therefore, it is necessary to carry out research in this area. In this paper, the similarities and differences between the equivalent linear model method and the cross-correlation function method are compared and the feasibility of using the cross-correlation function method to calculate the damping ratio of soil under random load is verified. It provides a reference for determining the damping ratio of soil under different types of dynamic loads.
2. Test Content

2.1  specimen opreparation
The sample in experiment is the saturated remodeling sand. The basic physical property parameters of the soil are shown in table 1. The GDS hollow cylindrical torsional shear test system is used in the test. The sample size is 50 mm in diameter and 100 mm in height.

| dry density $\rho_d$ (g.cm$^{-3}$) | Maximum void ratio $e_{\max}$ | Minimum void ratio $e_{\min}$ | Relative Density $D_r$ (%) | The quality percentage in different size of the soil sample $p_i$ (%) |
|-----------------------------------|-------------------------------|-------------------------------|-----------------------------|-----------------------------|
| 1.64                              | 0.75                          | 0.47                          | 45                          | <0.25                       |

2.2 Test process
The dynamic triaxial test module of GDS hollow cylinder torsional shear test system of Changan University was used. Periodic load
Dynamic triaxial test of sand under cyclic load: Consolidation phase, $K = \sigma_i / \sigma_3 = 1, 1.5, 2$, consolidation confining pressures are 100kPa, 200kPa, 300kPa. Dynamic load stage: dynamic load phase which frequency is 1 Hz and the vibration waveform is sine wave. The dynamic load in the experiment using stress controlled set the samples in dynamic vibration under 10 times per level and then imposed the dynamic step by step.

Random load
Seismic wave with VELACS model 1 at dynamic loading stage has a duration of 10 s. The other test sets are the same as the periodic load’s.

3. Results and Analysis
The specific calculation methods are as follows:
1) equivalent linear model method
In the dynamic triaxial test, each vibration period of the corresponding axial dynamic stress $\sigma_d$ and strain $\varepsilon_d$ are depicted in the $\sigma_d - \varepsilon_d$ coordinate, as shown in Fig.1.

![Fig.1. the hysteresis curve](image)

the dynamic damping ratio of soil calculation is as follows:

$$\lambda = \frac{1}{4\pi} \frac{A_0}{A_1} = \frac{1}{\pi} \frac{A_0}{A_r}$$

In formula 1, $A_0$ is the area of the hysteresis curve, $A_1$ is the area of the triangle $OAB$, $A_r$ is the area of the triangle $ACD$. Calculated by the method of dynamic damping ratio is designated as $\lambda_i$.

2) The cross-correlation function method
Using cross-correlation function method to solve the damping ratio of soil have the following steps:
① Sorting out the dynamic stress process curve $x(t)$ and the dynamic strain process curve $y(t)$ through the test data. ② Calculating the cross-correlation function of $x(t)$ and $y(t)$, among them $R_{xy}(\tau) = \lim_{\tau \to \infty} \int x(t)y(t+\tau)dt$. ③ Finding out the time $\tau$ when $R_{xy}(\tau)$ get the maximum value. ④ Using $\varphi = 2\pi \tau$ to calculate the phase angle value $\varphi$ which the dynamic strain lags the dynamic stress. ⑤ Plug the phase angle value $\varphi$ into $\lambda = \tan \varphi / 2$ to calculate the damping ratio $\lambda$ of soil, calculated by this method of dynamic damping ratio is designated as $\lambda_1$.

3.1 Calculation of damping ratio under cyclic load
The dynamic stress-strain relationship of hysteresis loop is shown in Fig.2. In the process of calculating the damping ratio of soil by using the method of equivalent linear model. The relation curve between damping ratio and dynamic strain is obtained by using formula 1 and shown in Fig.3. The relationship between damping ratio and dynamic strain is obtained by using cross-correlation function method which is shown in Fig.4.

Through the analysis of the above curves, the following conclusions can be drawn:

(1) It can be seen from Fig.3 and Fig.4 that the damping ratio of sand increases with the increase of dynamic strain. The damping ratio of soil calculated by cross-correlation function method is always larger than that by using equivalent linear model method. The reason is that the plastic deformation of soil is not taken into account in the calculation of damping ratio of soil by using equivalent linear model method.

(2) It can be seen from Fig.4 that the damping ratio of soil calculated by cross-correlation function method increases with the increase of dynamic strain and tends to steady gradually.

Fig.2. dynamic stress-dynamic strain hysteresis curve

Fig.3. Curves of damping ratio and dynamic stress
3.2 The analysis of the dynamic triaxial test under the random load

The dynamic stress curve and the dynamic strain curve can be obtained by inputting the seismic wave of VELACS model 1 by computer. The dynamic stress curves and the dynamic strain curve are shown in Fig. 5 and Fig. 6.

It can be seen from Fig. 5 that in the local enlarge figure that under the random load the measured values can basically reach the set value. But the whole measured curve is a little hysteretic to the set curve. This is the instrument itself a delay between the sending the command and the executing the command. The correlation coefficient between the measured curve and the pre-set curve is 0.9684 which shows the correlation of measured value and pre-set value is very well.

From Fig. 6 you can see that under the random load the sample’s axial displacement change trend is consistent with the dynamic stress move trend. It is the starting point of the stress cycle which is shown in Fig. 5 and Fig. 6 as the arrow. Then we choose a stress circle and draw the first cycle of dynamic stress and dynamic strain relationship curve. According to the above method we draw the second and the third dynamic stress and dynamic strain relationship curve, as shown in Fig. 7. It can be seen from Fig. 7 that under the random load the dynamic stress-strain curve of the soil is not oval or elliptical in a cycle, but some irregular graphics. So we can't use the equivalent linear model which see the soil as a viscoelastic material to solve the dynamic damping ratio of soil under this kind of random load. However we can use cross-correlation function method to calculate the dynamic damping ratio.
The relationship between the damping ratio and the dynamic strain of the soil is shown in Fig. 8 that the damping ratio is calculated by using the cross-correlation function method. The standard deviation of the dynamic strain history curve, which can reflect the fluctuation degree of the actual dynamic strain, is used as the dynamic strain in the statistical sense.

4. Conclusions
The dynamic triaxial tests of remolded saturated sand under cyclic load and random load were studied. The damping ratio of soil was calculated by using the equivalent linear model method and the cross-correlation function method. The conclusions are as follows.

(1) The damping ratio of soil under cyclic load and random load is calculated by using the equivalent linear model method and the cross-correlation function method respectively. But plastic deformation of the soil is neglected in the calculation of the damping ratio of the soil by the equivalent linear model method. The dynamic stress-strain hysteretic curve needs to be fitted and analyzed by MATLAB, so the calculation is complex and has errors. However, cross-correlation function method only needs to calculate the lag time between dynamic strain and dynamic stress by MATLAB, which is much simpler, faster and more accurate.

(2) The experimental results show that no matter the dynamic load is a cyclic load or a random load, the damping ratio of the soil increases with the increase of the amplitude of the dynamic stress, and increases with the increase of the dynamic strain and eventually tends to a stable value. With the increase of consolidation confining pressure, it decreases and eventually tends to a stable value, and decreases with the increase of consolidation stress ratio. Under the same cyclic load, the damping ratio of soil calculated by cross-correlation function method is larger than that calculated by equivalent linear model method.

(3) The analysis shows that the cross-correlation function method can not only calculate the damping ratio of soil under periodic load, but also can calculate the damping ratio of soil under random load.

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