Gravity field and solar component of the precession rate and nutation coefficients of Comet 67P/Churyumov–Gerasimenko

C. Lhotka\textsuperscript{1}\textsuperscript{*}, S. Reimond\textsuperscript{1}, J. Souchay\textsuperscript{2}, O. Baur\textsuperscript{1}

\textsuperscript{1}Space Research Institute, Austrian Academy of Sciences, Schmedlstrasse, 6, 8042 Graz, Austria,
\textsuperscript{2}SYRTE, Observatoire de Paris, PSL Research University, CNRS, Sorbonne Universit\'es,
UPMC Univ. Paris 06, LNE, 61 avenue de l’Observatoire, 75014 Paris, France.

ABSTRACT

The aim of this study is first to determine the gravity field of the comet 67P/Churyumov–Gerasimenko and second to derive the solar component of the precession rate and nutation coefficients of the spin axis of the comet nucleus, i.e. without the direct, usually larger, effect of outgassing. The gravity field, and related moments of inertia, are obtained from two polyhedra, that are provided by the OSIRIS and NAVCAM experiments on Rosetta, and are based on the assumption of uniform density for the comet nucleus. We also calculate the forced precession rate as well as the nutation coefficients on the basis of Kinoshita’s theory of rotation of the rigid Earth and adapted it to be able to indirectly include the effect of outgassing on the rotational parameters. The 2nd degree denormalized Stokes coefficients of comet 67P/C-G turn out to be (bracketed numbers refer to second shape model) $C_{20} \simeq -6.74 \times 10^{-2}$, $C_{22} \simeq 2.60 \times 10^{-2}$ consistent with normalized principal moments of inertia $A/MR^2 \simeq 0.13$, $B/MR^2 \simeq 0.23$, with polar moment $c = C/MR^2 \simeq 0.25$, depending on the choice of the polyhedron model. The obliquity between the rotation axis and the mean orbit normal is $\varepsilon \simeq 52^\circ$, and the precession rate only due to solar torques becomes $\dot{\psi} \in [20,30] \, ''/y$. Oscillations in longitude caused by the gravitational pull of the Sun turn out to be of the order of $\Delta \psi \simeq 1 \, '$, oscillations in obliquity can be estimated to be of the order of $\Delta \varepsilon \simeq 0.5 \, ^\circ$.

Key words: comets: 67P/Churyumov–Gerasimenko – celestial mechanics – gravity field – forced rotational state

1 INTRODUCTION

67P/Churyumov-Gerasimenko (hereafter referred to as 67P in the sequel) is the target of the ESA space mission Rosetta launched on 2 March 2004. This mission provided the opportunity for the first safe touchdown of a lander (Philae) on the surface of a comet nucleus on 12 November 2014.

The Rosetta data allow determining in a precise way physical and dynamical characteristics of the comet. This is the purpose of this paper. First, we derive a gravity field solution starting from a shape model that is based on very precise measurements from the Rosetta mission. Second, we investigate the influence of the gravitational pull of the Sun on the rotation of comet 67P, i.e. we provide the solar component of the precession rate and nutation coefficients of the comet’s spin axis. We remark that the gravitational interaction with the other planets (e.g. Jupiter), and non-gravitational forces generate additional torques that may become orders of magnitudes larger in comparison with solar torques. While we neglect close encounters with the planets (see Souchay et al. 2014, for the effect of close encounters of asteroids with the Earth), outgassing induced effects are important to understand the long-term evolution in time of the rotational parameters. For the general inclusion of non-gravitational torques in rotational cometary dynamics, see e.g. Sidorenko et al. (2008); Neishtadt et al. (2003, 2002); Mysen (2006, 2007). In case of 67P the effect of outgassing has been thoroughly investigated in the pre-era of the Rosetta mission in Gutiérrez et al. (2005). The authors used different shape models and activity patterns to quantify the effect for comet 67P and find typical shifts in the spin period of about $0.1 - 0.8 h$ with typical rates of change of about $0.001 - 0.05 h/d$ (hours per day). Furthermore, in Mottola et al. (2014) the hypothesis has been made that the

* Email: christoph.lhotka@oeaw.ac.at

© 2015 The Authors
rotation rate of 67P may have changed due to cometary activity during its last perihelion passage. Variations to the rotational period due to outgassing are generally accompanied by associated changes in the magnitude and direction of the angular momentum. In Gutiérrez et al. (2005) these changes have been estimated to be of the order of 10°, and at perihelion the angular rate of change corresponding to the motion of the angular momentum vector amounts to about 0.01 – 0.1°/d (degrees per day). The net torque on the rotation strongly depends on the water production rate variation over the surface of 67P that depends itself on the heliocentric distance and insolation conditions of the comet (see, e.g. Keller et al. 2015). In this work the authors describe the necessity of the accurate modeling of the non-gravitational forces on the basis of a sublimation model that determines the gas production rate and temperature, hence the instantaneous force acting on each facet of the real shape model. Therefore, the realistic modeling of the rotation of comets turns out to be a challenging problem of great complexity. The full modeling of the rotation of comet 67P is out of the scope of the present study. However, we aim to demonstrate the importance of the additional torque on cometary rotation due to the gravitational pull of the Sun. In the more active phase of the comet the solar torques are much smaller when compared to outgassing torques, and the effect on the rotation of additional uncertainties associated with the outgassing process is usually much larger than solar induced rotational changes of the comet nucleus. In other words, rotational changes due to solar torques are in the noise when compared with rotational changes and corresponding uncertainties associated with outgassing. However, comets usually spend most of their time in a less active phase of the comet nucleus. Moreover, while solar torques are continuously acting on the rotation, torques due to outgassing are only present for some certain amount of time within one orbital period of the comet. At the present state of knowledge the actual efficiency of these kinds of torques is rather unknown. We therefore claim that the accurate interpretation of the observations of the rotational state of the comet require the perfect knowledge of all torques acting on the rotation over time. Since the Rosetta mission allows the accurate determination of the cometary rotation for the first time it will therefore also allow us to obtain a better insight into these kinds of torques of different origins. Notice that a deterministic rigid-body rotation model for any celestial body (planet, moon, asteroid, or comet) generally serves as a necessary basis for more sophisticated models including non-gravitational torques. It is the purpose of this paper to provide this fundamental rotation model.

Our study requires the accurate determination of the principal moments of inertia in a suitable body-fixed reference frame, that can be derived from the knowledge of the low-degree gravity field harmonics of comet 67P. Both the gravity field coefficients and principal moments of inertia of 67P are currently unknown. For this reason we also develop a new gravity field solution on the basis of a recent shape model and the assumption of constant mean bulk density of the comet nucleus.

There is no obvious reason that the density of comet 67P must be uniform. However, little is known about the internal structure and density of cometary nuclei, in general: "Indirect evidences available so far are not compelling and these questions essentially remain a matter of speculation" (see Lamy et al. 2015). The CONSERT radio experiment on the Rosetta spacecraft is due to probe the interior of a comet, i.e. comet 67P, for the first time. No sign of complex interior structure could be revealed so far. On this basis we think, that our uniform density assumption is still the only reasonable one.

We remark that the comparison of our gravity field solution with the gravity field solution obtained from spacecraft orbits of the Rosetta orbiter will indirectly allow to validate the constant density assumption. A similar approach has already been applied in the case of asteroid Eros (see Konopliv et al. 2002), where the comparison of the 'real' gravity field with the gravity field obtained from a constant density assumption shows a nearly homogeneous asteroid despite its irregular shape.

A statistical analysis of the obliquities, precession rates, and nutation coefficients for a set of 100 asteroids has been performed by Lhotka et al. (2013). Moreover the determination of these fundamental rotational parameters for 5 asteroids, that have been targets for past space missions, has been done by Petit et al. (2014). These studies are based on a theory of rigid body dynamics constructed by Kinoshita (1977) and were implemented for an asteroid in the case of Eros in Souchay et al. (2003); Souchay & Bouquillon (2005). The precise modeling and knowledge of the rotational state of celestial bodies allows investigating important physical properties of these objects, in particular, mass & moments of inertia that are related to their composition and internal structure. Moreover, the effect of space-weathering on asteroids and comets cannot be satisfactorily understood without the precise knowledge of the long-term rotational evolution of these celestial bodies.

In the pre-era of the Rosetta mission the shape and rotational state of the comet nucleus have been investigated in detail, e.g. in Lamy et al. (2007); Lowry et al. (2012). A first analysis of the OSIRIS observations can be found in Mottola et al. (2014). First basic characteristics of the rotation of 67P have already been identified by the Rosetta mission, like the spin-direction, and precise rotation period. The comparison of older observations shows that the spin period decreased by about 0.36 hours since (or during) the perihelion passage in 2009 (Sierks et al. 2015). It is important to notice that the spin rate is an important parameter. An increase of the spin rate may induce more cracks in the structure of 67P as they have already been identified in the Anuket region (Rotundi et al. 2015) a first indication that the comet may break up into two pieces in the near future. The Rosetta mission also shows that the nucleus of 67P rotates about the maximum principal axis of inertia, and the longest axis is nearly perpendicular to the axes of the individual lobes of the dual lobe comet. Moreover, the axis with smallest moment of inertia is consistent with being in the equatorial plane (Sierks et al. 2015). The coincidence of the axis of rotation with the axis of maximum moment of inertia suggests that the comet is composed by weakly bonded icy dust aggregates, with porosity being dominant at small scales (Sierks et al. 2015). Most interestingly, the large obliquity of 52° between the rotational axis and the axis normal to the orbital plane...
Table 1. Physical parameters of 67/P Churyumov-Gerasimenko used in our study.

| Physical parameters | Value                                      |
|---------------------|--------------------------------------------|
| rotation period     | 12.4043 ± 0.0007 h                         |
| spin-axis           | α = 69.3 ± 0.1°                            |
|                     | β = 64.1 ± 0.1°                            |
| mass                | 10^{13} kg                                 |
| volume              | 21.4 ± 2 km³                               |
| density             | 470 ± 45 kg/m³                             |

leads to a greater exposure to space-weathering of one of the two hemispheres, implying that the surface structure on the two hemispheres evolved differently over time. The internal structure of 67P is currently unknown although some hypothesis have been made on the basis of the relatively low density in comparison to mass and volume of 67P (e.g., the composition of boulders and rubble pile, Weissman 2015). The rotational state of 67P seems also to play a role on cometary activity since dominant features of the coma’s origin have been found close to the rotational pole exposed by the Sun (Tubiana et al. 2015) while the energy input from the Sun has shown to be smaller in the neck region close to the pole in comparison with the two lobes (Sierks et al. 2015). The further investigation of the complex rotational state of 67P may help to interpret these kinds of observations.

The paper is organized as follows: we derive a new gravity field solution based on the shape model, called 67P/C-G (ESA 2014) in Sec. 2. We calculate mean orbital parameters for 67P based on least square methods in Sec. 3, and investigate the complex rotation of the comet’s nucleus in Sec. 4. Results are summarized in Sec. 5, and a discussion about them can be found at the end of the paper.

2 GRAVITY FIELD, AND MOMENTS OF INERTIA OF 67P

We determine the gravity field up to degree 100, and the moments of inertia along the principal axes of inertia from the polyhedron shape model (referred to as 67P/C-G) (ESA 2014; Jorda et al. 2015). The shape model consists of 62908 facets, and is based on images taken from the OSIRIS and NAVCAM cameras on board of the Rosetta spacecraft. In a first step, we rescaled the shape model such that the volume\(^1\) of the polyhedron is consistent with the estimate \(V = 21.4 \text{ km}^3\). Next, we determined the mass properties and principal axes of the rescaled model (hereafter SHA) using a method derived in Mirtich (1996) under the assumption of a constant mean density \(\rho = 470 \text{ kg/m}^3\). Next, we translated the origin of SHA to coincide with the center of mass, and rotate it such that the z-axis becomes aligned with the polar axis of inertia. The corresponding geometrical transformations can be found in Appendix A. To obtain the expansion of the gravity field in terms of spherical harmonics we implement a method based on Werner & Scheeres (1996); Reimond (2015), and apply it to the case of 67P (see summary in Appendix B). The method requires to choose the radius of a circumscribed sphere around the polyhedron model with the condition that Laplace’s equation is fulfilled in the exterior of this reference surface. We take \(R = 2800\text{m}\) that turns out to be the minimal radius from the center of mass of SHA that covers all facets of the shape model.

The spectrum of the gravity field in terms of normalized coefficients (see, e.g. Toge & Müller 2012) is shown in Figure 1. We find good convergence of the series. The low degree, denormalized, coefficients, up to harmonic degree 3, are summarized in Table 2. Due to our choice of a proper coordinate system, we find the coefficients \(C_{10}, C_{11}, S_{11}, C_{21}, S_{21}\) and \(S_{22}\) to be of the orders of \(10^{-13} \ldots 10^{-15}\), and the principal coefficients \(C_{20} = -6.74 \times 10^{-2}\), \(C_{22} = 2.60 \times 10^{-2}\). The sole numerical error in the calculation of the coefficients can be estimated to be of the same order as \(C_{11}\) (that would be zero without numerical errors due to the proper choice of a suitable body fixed reference system). Since the shape model 67P/C-G comes without any error bars we are unable to determine physical error bars on the gravity coefficients which could be large because of large errors on the shape. To estimate the influence of positional errors in the shape model on the gravity coefficients given in Table 2 we performed the following test: we first constructed a simplified shape model on the basis of 67P/C-G consisting of 1000 facets only. The simplification has been done by making use of a quadric edge collapse decimation algorithm\(^2\) with preservation of boundary, surface normal and topology of the original mesh. The parameters have been chosen to allow for offsets of the positions of the vertices within ±5% of the positions of the vertices of the original shape model. These offsets would therefore correspond to virtual positioning errors of the vertices of the order of ±140μm. Next, we repeated the procedure to determine the gravity field coefficients on the basis of this simplified shape model and found agreement of the lower degree Stokes coefficients within ±1% of the values published in Table 2. We notice that knowing the resolution of the camera and the pointing error, coming from Rosetta

---

\(^1\) Fundamental physical parameters used in this study are summarized in Table 1

\(^2\) MeshLab (http://meshlab.sourceforge.net/)
orbit miss-modeling, would allow to estimate the real possibly systematic error. With our numerical experiment we are able to show that if the real error of the positioning of the vertices of the shape model is less than 140 m the shape model induced error on our results is less than 1%.

Recently, a new shape model together with different values for volume \( V = 18.7 \times 10^6 \text{km}^3 \), and density \( \rho = 535 \pm 35 \text{kg/m}^3 \) have been published (see Preusker et al. 2015). We therefore repeat our study on the basis of the new shape model, as described above, and find \( C_{20} \approx -7.93 \times 10^{-2} \), and \( C_{22} = 2.71 \times 10^{-2} \). The difference (96% agreement in \( C_{22} \), but only 85% agreement in \( C_{20} \)) is consistent with the different topography of the new shape model: the mass loss due to the thinned out part of the new shape model close to the neck region has a much bigger influence on the mass distribution along the \( z \)-axis (and therefore also on \( C_{20} \)) while it has a smaller influence on the mass distribution along the \( x \)-axis (and therefore on \( C_{22} \)). In the following we present our results within the range of possible values for the principal gravity harmonics derived on the basis of both shape models. All values based on the more recent shape model are given in square brackets.

Using the relations \( J_2 MR^2 = C - (A + B)/2 \), \( C_{22} MR^2 = (B - A)/4 \), \( C = c MR^2 \) (with \( J_2 = -C_{20} \)) we find the principal moments of inertia \( A, B, C \) as functions of the normalized polar moment of inertia \( c \). We determine the quantity \( c \approx 0.25 \) from the requirement that the principal moments of inertia \( A < B < C \) (on the diagonal) of the inertia matrix, that can independently be calculated from Mirtich (1996), are consistent with the non-vanishing 2nd degree gravity harmonics that we obtain with the method developed in Reimond (2015) (see also Appendix A). Using this value, the other normalized moments of inertia turn out to be \( A/MR^2 \approx 0.13 \) [0.11] and \( B/MR^2 \approx 0.23 \) [0.22]. We remark, that the parametric study in \( c \) is important to allow to adapt our results easily once the interior structure of 67P is known more accurately as it is usually done in planetary studies, e.g. the case of Mercury (see e.g. Noyelles & Lhotka 2013).

### 3 MEAN ORBITAL ELEMENTS OF 67P

Orbital data for 67/P Churyumov-Gerasimenko is obtained from ephemeris service (NASA 2014) based on DE431, and

Table 2. Denormalized, low order spherical harmonics up to degree 3 of shape model SHA. For comparison we provide values obtained from a different shape model (Preusker et al. 2015), with \( V = 18.7 \pm 1.2 \text{km}^3 \) and \( \rho = 535 \pm 35 \text{kg/m}^3 \) in square brackets.

| Gravity field | \( l \) | \( m \) | \( C_{lm} \) | \( S_{lm} \) |
|---------------|------|-----|----------|----------|
| 0 0 | 0.0 | 0.0 | 0.0 | 0.0 |
| 1 0 | \( \approx 10^{-13} \) | \( \approx 10^{-14} \) | 0.0 | 0.0 |
| 1 1 | \( \approx 10^{-13} \) | \( \approx 10^{-14} \) | 0.0 | 0.0 |
| 2 0 | \( \approx 0.67 \times 10^{-2} \) | \( \approx 10^{-3} \) | \( \approx 10^{-4} \) | \( \approx 10^{-5} \) |
| 2 1 | \( \approx 10^{-3} \) | \( \approx 10^{-4} \) | \( \approx 10^{-5} \) | \( \approx 10^{-6} \) |
| 3 0 | \( \approx 2.60 \times 10^{-2} \) | \( \approx 10^{-3} \) | \( \approx 10^{-4} \) | \( \approx 10^{-5} \) |
| 3 2 | \( \approx 10^{-3} \) | \( \approx 10^{-4} \) | \( \approx 10^{-5} \) | \( \approx 10^{-6} \) |
| 3 3 | \( \approx 10^{-4} \) | \( \approx 10^{-5} \) | \( \approx 10^{-6} \) | \( \approx 10^{-7} \) |

Minor Planet’s Data Center (MPC 2014). In order to implement the rotation theory in the subsequent section we make use of mean orbital elements instead of osculating ones. The mean orbital elements are summarized in Table 3. The osculating ones are obtained in the following way: we first transform the time series of heliocentric ecliptic osculating elements from NASA (2014) to osculating Keplerian elements using the mass parameter of the Sun \( \mu = 2.959 \times 10^{-5} \text{au}^3 \text{d}^{-2} \). Next, for each orbital element we fit models of the form:

\[
X + Y(t - t_0) + Z(t - t_0)^2,
\]

through each time series, centered around \( t_0=2013-01-01 \) within the time window \( \pm 5 \text{y} \). Then, from \( X \) we immediately obtain the values of mean semi-major axis \( a \), eccentricity \( e \), orbital inclination (wrt. to the ecliptic) \( i \), argument of perihelion \( \omega \), and longitude of the ascending node \( \Omega \). We validate \( X \) from the linear rates of change \( Y \) that turn out to be of the order of \( 10^{-6} \) for \( a \), \( e \), \( i \), and the order of \( 10^{-7} \) for \( \omega \) and \( \Omega \). The mean motion \( n \) can be directly obtained from constant \( Y \) in the fit for mean anomaly \( M \), whereas we use \( Z \) to validate \( Y \) that turns out to be of the order of \( 10^{-8} \).

### 4 ROTATIONAL PARAMETERS OF COMET 67P

In this section we investigate the effect of solar torques on the rotation of the nucleus of comet 67P by taking into account the possible variations of the spin period and direction of angular momentum due to non-gravitational effects (Gutiérrez et al. 2005; Keller et al. 2015). Precisely, we calculate the secular component characterizing the precession, and its short-period oscillations characterizing the nutation of the spin axis.

We assume here in first approximation that 67P can be assimilated to a rigid body of ellipsoidal shape by means of moments of inertia \( A \leq B < C \), with respect to the semi-major axes of the ellipsoid \( \alpha' > \beta' > \gamma' \). Moreover, as demonstrated by Souchay & Bouquillon (2005) in the case of the asteroid 433 Eros, we consider that the effect of the triaxial shape of 67P on the rotation is small enough to be neglected for the purpose of determination of precession rates and nutation coefficients. The influence of higher degree harmonics (e.g. \( C_{30} \) in Table 2), and non-rigid effects on the rotation are subject of a follow-on study.

We also assume that the rotation axis is very close to the figure axis such that 67P is considered to be in a short axis mode. This has been confirmed by the Rosetta mission (Sierks et al. 2015). Indeed this mode characterizes the very large majority of small bodies of the solar system such as asteroids and comets and represents the natural state resulting from dissipative processes occurring at relatively short time scales in their history (Burns & Safronov 1973). In this
context we only take into account in our computations the gravitational effect of the Sun on the rotation of the comet.

In the following, after determination of the obliquity with respect to the mean orbit of 67P, we express and calculate the precession rate as well as the nutations in longitude and in obliquity of the comet in the range of parameters that include possible offsets due to outgassing effects.

4.1 Obliquity

The obliquity \( \varepsilon \) can be obtained from the simple equation:

\[
\cos \varepsilon = \vec{\delta} \cdot \vec{f}.
\]

Here, the rectangular coordinates of the spin-pole \( \vec{f} \) are

\[
\vec{f} = (\cos \beta \cos \lambda, \cos \beta \sin \lambda, \sin \beta),
\]

where \( \lambda \) and \( \beta \) stand respectively for the ecliptic longitude and latitude of \( \vec{f} \). The rectangular coordinates of the orbit-pole \( \vec{\delta} \) are themselves directly determined from the inclination \( i \) and the longitude of the node \( \Omega \) of the orbit:

\[
\vec{\delta} = \left( \sin i \sin \Omega, -\cos \Omega \sin i, \cos i \right).
\]

As the spin-pole is given by its equatorial coordinates \((\alpha, \delta)\) whereas we are doing our calculations in an ecliptic frame, we make use of the transformation from equatorial to ecliptic coordinates

\[
\begin{pmatrix}
q_1 \\
q_2 \\
q_3
\end{pmatrix}
= R_1(-\varepsilon_E)
\begin{pmatrix}
\cos \alpha \cos \delta \\
\sin \alpha \cos \delta \\
\sin \delta
\end{pmatrix},
\]

with rotation matrix \( R_1 \) around the \( x \)-axis. Here \( \varepsilon_E \) represents the nominal value of the Earth obliquity. Thus we calculate \((\lambda, \beta)\) from

\[
\lambda = \tan^{-1}\left(\frac{q_2}{q_1}\right), \quad \beta = \sin^{-1} q_3.
\]

Using \( \varepsilon_E = 23.26^{\circ}/21.448^{\circ} \) (J2000) and \((\alpha, \delta) \simeq (69.3^{\circ}, 64.1^{\circ})\) we find \((\lambda, \beta) \simeq (78^{\circ}, 42^{\circ})\), and therefore we find the obliquity of 67P to be \( \varepsilon \simeq 52^{\circ}/0.626 \), in agreement with the published value of 52\(^{\circ}/0\) in ESA (2015); Sierks et al. (2015). In the following we allow for an offset \( \delta \varepsilon \) of 10\(^{\circ}\) in \( \varepsilon \) to include possible variations of the spin-axis due to changes in the direction of the angular momentum vector during the rendezvous of the Rosetta mission (Gutiérrez et al. 2005).

4.2 Determination of the precession rate &
nutation coefficients

The general theoretical framework to model the rotation of a given celestial body as the comet 67P has been constructed by Kinoshita (1977). Starting from this framework Petit et al. (2014) have proposed formulae, valid up to order 4 in \( e \), for the precession rate \( \psi \), the nutation of the longitude of the node \( \Delta \psi \), and the oscillations of the obliquity \( \Delta \varepsilon \). These formulae were successfully applied on celestial bodies with well defined physical constraints, as (1) Ceres, (4) Vesta, (433) Eros, (2867) Steins and (25143) Itokawa. Since the eccentricity, \( e = 0.64 \), of 67P is much larger than for these kinds of objects we need to develop these formulae to much higher order in \( e \) to be able to apply them for the present case of 67P as well. At this point, we provide the general formula for the precession rate up to order 16 in \( e \), while we only summarize the formulae for \( \Delta \psi \), \( \Delta \varepsilon \) up to 4th order in \( e \) (but still use 16th order formulae in our calculations). The precession rate up to order 16, according to Petit et al. (2014), is given by:

\[
\psi = \left[ 1 + \frac{3}{2} e^2 + \frac{15}{8} e^4 + \frac{35}{16} e^6 + \frac{121}{8} e^8 + \frac{603}{256} e^{10} + \frac{3003}{1024} e^{12} + \frac{6435}{2048} e^{14} + \frac{109395}{32768} e^{16} \right] \times \frac{K}{2} \cos \varepsilon.
\]

where the constant \( K \) together with its possible ranges during one orbital period of the comet is given by

\[
K = \frac{3n^2}{\omega_s^2} \times H_d \simeq \frac{8.9}{10.5} \frac{9/\gamma}{c}.
\]

Here, \( n = 0.1529^{\circ}/d \) is the mean motion of the comet, and \( \omega_s \in (644.569, 757.609)^{\circ}/d \) is the spin frequency consistent with the rotation period \( T = 12.4043 \pm 1h \) that includes possible variations due to outgassing of the order of \( \pm 1h \). The expression also contains the dynamical ellipticity \( H_d \) that is related to the moments of inertia (and therefore 2nd degree gravitational harmonics) by:

\[
H_d = \frac{2C - A - B}{2C} \simeq \frac{0.0674}{0.0793}.
\]

Since the polar moment of inertia may be sensitive to the structure models of the interior of the comet - that we do not take into account at the present time - we provide a parameter study of \( \psi \) for different values of polar moments of inertia \( c \) in Figure 2. As we can see the current precession rate may vary between 12\(^{\circ}/y \) to 30\(^{\circ}/y \) within the interval 0.2 \( \leq c \leq 0.5 \). For \( c = 0.25 \) the actual value of \( \psi \) turns out to lie in the interval [20, 30]\(^{\circ}/y \). This precession rate due to the sole gravitational forcing of the Sun is comparable to the corresponding precession rate of the Earth (1/3 of the total lunisolar part) that is to say 10\(^{\circ}/y \). The nutation in longitude \( \Delta \psi \) and in obliquity \( \Delta \varepsilon \) can be expressed starting from the mean anomaly \( M \) and its sole harmonics as follows:

\[
\Delta \psi = \left[ \left( C' \left( \frac{\epsilon^3}{12} + 27e^3 \right) + 3e \right) \frac{\sin(M)}{n} + \right. \\
\left. \left( C' \frac{41e^4}{48} + 5e^2 \right) \frac{\sin(2M)}{2n} + \right. \\
\left. \left( C' \frac{123e^5}{16} + 7e^3 \right) \frac{\sin(3M)}{3n} + \right. \\
\left. \left( C' \frac{115e^6}{6} + 17e^4 \right) \frac{\sin(4M)}{4n} \right]
\]

Figure 2. Mean constant precession rate of 67P for \( \varepsilon = 52 \pm 10^\circ \) and \( \omega_s = 12.4043 \pm 1h \) for different polar moments of inertia \( c \).
and
\[
\Delta \varepsilon = \left[ \frac{\varepsilon^3}{12} - \frac{\varepsilon}{2} \right] \frac{S^2 \sin (M)}{n} - \left[ \frac{\varepsilon^3}{24} - \frac{\varepsilon}{2} \right] \frac{C^2 \cos (M)}{n} + 
\left( \frac{43e^4}{48} - \frac{5e^2}{2} + 1 \right) \frac{S^3 \sin (2M)}{2n} - 
\left( \frac{37e^4}{48} - \frac{5e^2}{2} + 1 \right) \frac{C^3 \cos (2M)}{2n} + 
\left( \frac{7e}{2} - \frac{123e^3}{16} \right) \frac{S^2 \sin (3M) - C^2 \cos (3M)}{3n} + 
\left( \frac{17e^2}{2} - \frac{115e^4}{6} \right) \frac{S^2 \sin (4M) - C^2 \cos (4M)}{4n} + 
\frac{845e^3}{48} \frac{S^2 \sin (5M) - C^2 \cos (5M)}{5n} + 
\frac{533e^4}{16} \frac{S^2 \sin (6M) - C^2 \cos (6M)}{6n} \right] \times K \sin \varepsilon \frac{2}{2}.
\]

In these formulæ the coefficients $C'$, $S'$ are obtained from $C' = \cos (2\omega - 2\Lambda)$ and $S' = \sin (2\omega - 2\Lambda)$. Here $\Lambda$ stands for the angle along the orbital plane between the equinox $\overrightarrow{\Gamma}$ and the ascending node $\overrightarrow{N} = (\cos \Omega, \sin \Omega, 0)$ of the orbital plane with respect to the inertial plane of ecliptic J2000.0, where the unit vector $\overrightarrow{\Gamma}$ is given by the following vectorial product:
\[
\overrightarrow{\Gamma} = \frac{1}{\sin \varepsilon} \overrightarrow{f} \times \overrightarrow{a}.
\]

Notice, that in (4) and (5) the orbital longitude $\lambda$ of the perturbing body, i.e. the Sun, is counted from the “equinox” $\overrightarrow{\Gamma}$ of the comet which is the ascending node of the relative orbit of the Sun (as determined from the comet) with respect to the comet’s equatorial plane. Therefore: $\lambda = 180^\circ + \omega + \nu - \Lambda$, where $\nu$ is the true anomaly. The application of the expressions above leads to: $\Lambda = -57^\circ$ and $C' = -0.76114$ and $S' = 0.6485$. We provide the nutation series, up to order 16 in eccentricity $e$, for the parameters of Table 1 and $c = 0.25$ in Table 4. We clearly observe that $M$ is the fundamental period of the nutation motion.

We also compute the sole nutational part as a function of time resulting in a bi-dimensional motion ($\Delta \psi \cdot \sin \varepsilon$, $\Delta \varepsilon$) projected to the equatorial plane as shown in Figure 3: the spin axis describes a complex multi-periodic closed loop whose amplitude varies within 0.5$^\circ$ to 1$^\circ$ within one orbital period 0 $\leq M \leq 360^\circ$. We also provide, for reference, a solution with $c = 0.2$ and $c = 0.3$ and see that the amplitudes in ($\Delta \psi \cdot \sin \varepsilon$, $\Delta \varepsilon$) decrease for larger values of the moment of inertia $c$. Typical amplitudes of nutation in $\Delta \psi \cdot \sin \varepsilon$ roughly range within ($-50^\circ$, 65 $^\circ$), for $c = 0.2$, from $-35^\circ \leq \Delta \psi \cdot \sin \varepsilon \leq 45^\circ$, for $c = 0.3$, and within ($-40^\circ$, 50 $^\circ$) for $c = 0.25$. The corresponding amplitudes in $\Delta \varepsilon$ range from about $-35^\circ \leq \Delta \varepsilon \leq 30^\circ$, for $c = 0.2$, from $-25^\circ \leq \Delta \varepsilon \leq 20^\circ$, for $c = 0.3$, and from $-30^\circ \leq \Delta \varepsilon \leq 25^\circ$ for $c = 0.25$.
If we repeat our study for various values of $c$ in the interval (0.2, 0.5) we obtain the results summarized in Figure 4, where we show maximum and minimum values for nutation coefficients defined as follows: we calculate the time series of the nutation, for different $c$, over one full revolution period of 67P. The time series therefore is the superposition of various trigonometric terms with different periods. Since the different harmonics may sum up or cancel out each other - depending on the actual value of the mean anomaly $M$ - we calculate the furthest points along the $\Delta \psi \cdot \sin \varepsilon$ and $\Delta \varepsilon$ - directions, and denote by $mx(\Delta \psi \cdot \sin \varepsilon)$, $mx(\Delta \varepsilon)$ the maximal values into the positive, and by $mn(\Delta \psi \cdot \sin \varepsilon)$, $mn(\Delta \varepsilon)$ the maximal values into the negative directions. We notice in particular that amplitudes in oscillations in longitude are typically significantly larger than oscillations in obliquity, and that nutation coefficients decrease with increasing polar moment of inertia $c$.

**4.3 Influence of outgassing-induced effects**

In this section we investigate the influence of outgassing-induced effects on the time series of the nutation parameters of comet 67P using the proposed values for the changes in spin period and direction of angular momentum after Gutiérrez et al. (2005); Keller et al. (2015). We follow our approach to obtain the results for the precessional motion and allow offsets $\delta \varepsilon$ of $\pm 10^\circ$ from the nominal value of the obliquity $\varepsilon$ as well as offsets $\delta \omega_s$ of $\pm 1h$ in the rotation period of the comet. As a consequence the constants $K$, $C'$, and $S'$ in (4), (5) will change in well determined intervals too. To account for the full ranges of the intervals we therefore look, for each value of $M$, for the minimum and maximum values of $\Delta \psi$ and $\Delta \varepsilon$, respectively. The results for $c = 0.25$ are shown in Figure 5. We observe that the nutation amplitude in $\Delta \psi$ is about twice as big as the amplitude in $\Delta \varepsilon$. We also clearly see that the maximal nutation amplitudes are found close to $M = 0$ that corresponds to the time of next perihelion passage (on August 13, 2015 MPC 2014), while the amplitudes decrease by orders of magnitude close to aphelion ($M = 180^\circ$). The variations in obliquity and rotation periods are more present in longitudinal directions.

**5 CONCLUSIONS**

In this paper we have determined for the first time the coefficients of the gravity field of the comet 67P by using a shape model based on very precise data measurements from the Rosetta mission. Then we have investigated the motion of its spin-axis due the gravitational forcing of the Sun. We have found that this motion is rather complex due to both the irregular shape and the high value of the orbital eccentricity of the comet. With a value of $c = 0.25$ we have calculated a precession rate $\dot{\psi} \in [20, 30] \degree/\text{y}$, comparable to the solar part of the precession of the Earth (roughly $15^\circ$/y). Moreover we have found a maximum amplitude in nutation in longitude of about $\Delta \psi \approx 1^\circ$, and a maximum nutation amplitude in nutation in latitude of about $\Delta \varepsilon \approx 0.5^\circ$. We notice that these nutation amplitudes are much larger than the corresponding ones for the Earth (respectively roughly $10^\circ$ to $20^\circ$). As can be seen in (4)–(5) the reason lies partly in the large eccentricity, but also in the large value of the dynamical ellipticity $H_q$ of the comet and consequently of the scaling factor $K$.

In Sierks et al. (2015) the nucleus structure and activity of comet 67P have been investigated based on data of the OSIRIS scientific imaging system on board the Rosetta space-craft. The authors found no obvious evidence for complex rotation of the comet nucleus and were able to constrain any motion of the spin-axis to $< 0.3^\circ$ over 55 days. In this work we predict a complex motion of the spin axis of 67P over the comet’s orbital period of 6.4 years, that is in agreement with the bounds given by Sierks et al. (2015), namely in terms of the precession rate and nutation coefficients of the comet’s spin axis based on a rigid body approximation. This preliminary work looks necessary to any further work dealing with the short or long term evolution of the rotational state of comet 67P, in particular concerning the variations of its spin axis in space.

We also performed a parametric study in normalized polar moment of inertia linked to rotational parameters. The relatively small value of the moment of inertia factor $c$ is consistent with the thinned out part of 67P along the spin-axis direction. However, the small value of $c$ may also indicate a possible differentiated interior structure, and our parametric study should allow to validate different density profiles. For this purpose, the extended Rosetta mission period will be crucial to improve the chances to detect the precession rate and nutation coefficients of comet 67P.

Our study is focussed on the influence of solar torques on the rotational parameters of the comet. We investigated the interplay between these torques and the outgassing-induced effects on the basis of recent Rosetta findings (Keller et al. 2015). With this we are able to provide a better insight into the sensitivity of the solar component to the specific rotational state of the comet. Our study may therefore serve as a good starting point for better models of cometary rotational dynamics.

**Acknowledgments**

We thank an anonymous reviewer for valuable sugges-
tions, and F. Preusker from DLR for providing us the most accurate shape model for comet 67P (Preusker et al. 2015).

REFERENCES

Burns J. A., Satorov V. S., 1973, MNRAS, 165, 403
ESA 2014, http://blogs.esa.int/rosetta/2014/10/03/measuring-comet-67pcg-
www
ESA 2015, http://www.esa.int/spaceinimages/Images/2015/01/Comet_vital_statistics. www
Gutiérrez P. J., Jordà L., Samarasinha N. H., Lamy P., 2005, Planet. Space Sci., 53, 1135
Jordà L., Gaskell R., Hviid S., Capanna C., Preusker F., Scholten F., Gutierrez P., in prep. 2015, SHAPE MODELS OF 67P/CHURYUMOV-GERASIMENKO. RO-C-OSINAC/O/OSWAC-5-67P-SHAPE-V1.0. NASA Planetary Data System and ESA Planetary Science Archive
Keller H. U., Mottola S., Skorov Y., Jordà L., 2015, A&A, 579, L5
Kinoshita H., 1977, Celest. Mech., 15, 277
Konopliv A. S., Miller J. K., Owen W. M., Yeomans D. K., Giorgini J. D., Gaskell R., Hviid S., Capanna C., Preusker et al. in prep. 2015, SHAPE
Miret J., 2007, Planet. Space Sci., 53, 1135
Mysen E., 2006, Planet. Space Sci., 53, 1135
Mysen E., 2007, Space Sci. Rev., 128, 23
Lamy P., Herique A., Toth I., 2015, Space Science Reviews, pp 1–15
Lhoˆtka C., Souchay J., Shahsavari A., 2013, A&A, 556, A8
Lowry S., Duddy S. R., Rozitis B., Green S. F., Fitzsimmons A., Snodgrass C., Hsieh H., Hainaut O., 2012, A&A, 548, A12
MPC 2014, http://www.minorplanetcenter.net/ . www
Mirtich B., 1996, J. Graph. Tools, 1, 31
Mottola S., Lowry S., Snodgrass C., 46 coauthors 2014, A&A, 569, L2 (5p.)
Mysen E., 2006, MNRAS, 372, 1345
Mysen E., 2007, MNRAS, 381, 301
NASA 2014, http://ssd.jpl.nasa.gov/horizons.cgi. www
Neishtadt A. I., Scheeres D. J., Sidorenko V. V., Vasiliev A. A., 2002, Icarus, 157, 205
Neishtadt A. I., Scheeres D. J., Sidorenko V. V., Stooke P. J., Vasiliev A. A., 2003, Celestial Mechanics and Dynamical Astronomy, 86, 249
Noyelles B., Lhoˆtka C., 2013, Adv. Space Research, 52, 2085
Petit A., Souchay J., Lhoˆtka C., 2014, A&A, 565, A79
Preusker F., et al., 2015, A&A
Reimond S., 2015, Representation of the gravity field of irregularly shaped bodies. TU-Graz, Austria
Rotundi A., et al., 2015, Science, 347
Sidorenko V., Scheeres D., Byram S., 2008, Celestial Mechanics and Dynamical Astronomy, 102, 133
Siersk H., et al., 2015, Science, 347
Souchay J., Bouquillon S., 2005, A&A, 433, 375
Souchay J., Kinoshita H., Nakai H., Roux S., 2003, Icarus, 166, 285
Souchay J., Souami D., Lhoˆtka C., Puente V., Folgueira M., 2014, A&A, 563, A24
Stumpf K., Meffroy J., 1973, Himmelsmechanik. Bd.1: Das Zweikörperproblem und die Methoden der Bahnbestimmung der Planeten und Kometen. VEB Deutscher Verlag der Wissenschaften
Torge W., Müller J., 2012, Geodesy. De Gruyter
Tubiana C., et al., 2015, A&A, 573, A62
Weissman P. R., 2015, in American Astronomical Society Meeting Abstracts. p. #134.01
Werner R., Scheeres D., 1996, Cel. Mech. Dyn. Astr., 65, 313

APPENDIX A: GEOMETRIC TRANSFORMATIONS

We implement MatLab and Mathematica programs to rescale the original shape model 67P/C-G such that the mass, volume, and density are consistent with Table 1, and translate its center of mass to the origin. The inertia tensor I of the physical shape model is calculated on the basis of Mirtich (1996) to find the principal axes. Using the eigen-system of I we diagonalize the inertia matrix. The center of mass of the rescaled shape model before translation turns out to be:

$$\begin{pmatrix} X_0, Y_0, Z_0 \end{pmatrix} = (2.7525 \times 10^2 m, 1.0197 \times 10^2 m, 0.1093 \times 10^2 m).$$

The rotation matrix composed by the eigenvectors becomes:

$$PA = \begin{pmatrix} -0.9974958 & 0.00498726 & 0.07054984 \\ -0.02099773 & -0.9734178 & -0.2280722 \\ 0.06753702 & -0.2289824 & 0.9710849 \end{pmatrix}.$$ 

The rescaled, translated and rotated shape model SHA is the basis for the calculation of the gravitational field provided in Table 2.

Remark. The center of mass and inertia matrix can also be derived from the spherical harmonic coefficients of degree 1 and 2. Let A, B, C, D, E, F be the diagonal & off-diagonal matrix elements of the mass-inertia tensor (see in more detail Torge & Müller 2012):

$$A = \int V (Y^2 + Z^2) dm \quad B = \int V (X^2 + Z^2) dm,$$

$$C = \int V (X^2 + Y^2) dm \quad D = \int V YZ dm,$$

$$E = \int V XZ dm \quad F = \int V XY dm.$$ 

Using the standard definitions $C_{lm}$, $S_{lm}$ with $l \leq m \leq l$, and of the center of mass and inertia matrix can be put into

$$\begin{pmatrix} X_0, Y_0, Z_0 \end{pmatrix} = R(C_{11}, S_{11}, C_{10}) ,$$

and

$$I = M R^2 \cdot \begin{pmatrix} e_1, e_2, e_3 \end{pmatrix}$$

with

$$e_1 = \begin{pmatrix} C_{11} S_{11} - 2S_{22} \\ C_{10} C_{11} - C_{21} \end{pmatrix},$$

$$e_2 = \begin{pmatrix} C_{11} S_{11} - 2S_{22} \\ -C_{10} C_{11} + C_{21} + 2C_{22} \end{pmatrix},$$

$$e_3 = \begin{pmatrix} C_{10} C_{11} - C_{21} \\ C_{10} S_{11} - S_{21} \\ -C_{11}^2 + C_{22} \end{pmatrix},$$

where we make use of the parametrization $C = cM R^2$. Inserting the values of Table 2 into (A1), and equating with I obtained directly with the method proposed in Mirtich (1996) allows to obtain $c \approx 0.25.$
APPENDIX B: DETERMINATION OF THE GRAVITY FIELD

We use the method of least squares adjustment to determine in Table 2 the gravitational field coefficients $C_{lm}$, $S_{lm}$ in the spherical harmonics expansion of the gravitational potential (Torge & Müller 2012). Let $I$ be the vector of evaluations of the potential, $A$ be the design matrix, and $x$ be the parameter vector of gravity harmonics:

$$I = [U_1 U_2 \ldots U_K]^T, \quad A = [A_C \, A_S], \quad x = [xc \, xs]^T.$$

The gravitational potential values $U_k$ of the shape model SHA, with $k = 1, \ldots, K$, are computed for evenly distributed points on the surface of the reference sphere using the algorithm presented in Werner & Schaefer (1996). In order to guarantee for good coverage and highly overdetermined equation systems, a Reuter grid on the reference sphere of radius $R = 2800m$ with 200 meridional points is used, which yields in total $K = 50831$ values in $I$. The design matrix establishes the relation between the evaluations of the potential and the unknown coefficients of the potential. It is given by the partial derivatives of the spherical harmonics expansion with respect to the coefficients $C_{lm}$ ($A_C$) and $S_{lm}$ ($A_S$):

$$A_C = \begin{bmatrix}
\partial U_1/\partial C_{0,0} & \partial U_1/\partial C_{1,0} & \ldots & \partial U_1/\partial C_{N,N} \\
\partial U_2/\partial C_{0,0} & \partial U_2/\partial C_{1,0} & \ldots & \partial U_2/\partial C_{N,N} \\
\vdots & \vdots & \ddots & \vdots \\
\partial U_K/\partial C_{0,0} & \partial U_K/\partial C_{1,0} & \ldots & \partial U_K/\partial C_{N,N}
\end{bmatrix},$$

$$A_S = \begin{bmatrix}
\partial U_1/\partial S_{1,1} & \partial U_1/\partial S_{2,1} & \ldots & \partial U_1/\partial S_{N,N} \\
\partial U_2/\partial S_{1,1} & \partial U_2/\partial S_{2,1} & \ldots & \partial U_2/\partial S_{N,N} \\
\vdots & \vdots & \ddots & \vdots \\
\partial U_K/\partial S_{1,1} & \partial U_K/\partial S_{2,1} & \ldots & \partial U_K/\partial S_{N,N}
\end{bmatrix}.$$

The coefficients in $x$ are of course ordered accordingly inside the parameter vector:

$$xc = [C_{0,0} \ C_{10} \ C_{11} \ \ldots \ C_{NN}], \quad xs = [S_{11} \ S_{21} \ S_{22} \ \ldots \ S_{NN}].$$

We determine the spherical harmonics up to degree $N = 100$ and provide them in Figure 1. Additional information on the determination of the gravity field can be found in Reimond (2015).

APPENDIX C: SERIES IN NUTATION COEFFICIENTS

The presence of a large eccentricity of 67P requires to develop (3)–(5) up to high orders in $\epsilon$: we start from (see, e.g. Petit et al. 2014):

$$\Delta \psi = \frac{K}{2} \cos (I) \times \int \left[ \left( \frac{a}{r} \right)^3 \sin (2\lambda - 2h) \right]_{per} \, dt,$$

$$\Delta \varepsilon = -\frac{K}{2} \sin (I) \times \int \left[ \left( \frac{a}{r} \right)^3 \sin (2\lambda - 2h) \right]_{per} \, dt \quad (C1).$$

Here, $I = -\varepsilon$ is the obliquity angle, $h = -\psi$ is the precession angle, and $r$, $\lambda$ are the distance between the Sun and 67P, and the orbital longitude of the Sun, respectively. Let the angle $\lambda = 180^\circ + \omega + \nu - \Lambda$, $C^\prime = \cos (2\omega - 2\Lambda)$ and $S^\prime = \sin (2\omega - 2\Lambda)$. Since $\omega$, $\Lambda$ are slowly varying angles with time (in comparison to true anomaly $\nu$) we assume that $C^\prime$, $S^\prime$ are constant from now on. Using the identities

$$\cos (2\omega + 2\nu - 2\Lambda - 2h) = C^\prime \cos (2\nu) - S^\prime \sin (2\nu),$$

$$\sin (2\omega + 2\nu - 2\Lambda - 2h) = S^\prime \cos (2\nu) + C^\prime \sin (2\nu),$$

we are able to express the integrands in (C1), by means of trigonometric terms in $2\nu$ instead of $2\lambda - 2h$. Making use of basic trigonometric identities, and standard series expansions of $\cos \nu$, $\sin \nu$, and $(a/r)^3$ (see, e.g. Stumpff & Meffroy 1973) the integrands can also be expressed in terms of mean anomaly $M = nt$. The integration with respect to time $t$ provides $\Delta \psi, \Delta \varepsilon$ in terms of

$$\Delta \psi = \sum_{k}^{N} \left[ c_k (S^\prime, n, e) \cos (kM) + s_k (C^\prime, n, e) \sin (kM) \right],$$

$$\Delta \varepsilon = \sum_{k}^{N} \left[ c_k^\prime (C^\prime, n, e) \cos (kM) + s_k^\prime (S^\prime, n, e) \sin (kM) \right].$$

We provide the numerical values of the coefficients $c_k$, $s_k$, and $c_k^\prime$, $s_k^\prime$ of the nutation series for 67P, with $c = 0.25$, in Table 4. The secular part in the integral of the first equation of (C1) (not depending on mean anomaly $M$) can directly be identified with $\psi$ - that gives (3).

This paper has been typeset from a TeX/\LaTeX\ file prepared by the author.