$\rho - \omega$ mixing contribution to the measured $CP$ asymmetry of $B^\pm \to \omega K^\pm$

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Abstract

We study the $CP$ asymmetry of $B^\pm \to \omega K^\pm$ with the inclusion of the $\rho - \omega$ mixing mechanism. It is shown that the $CP$ asymmetry of $B^\pm \to \omega K^\pm$ experimentally measured ($A_{CP}^{\text{exp}}$) and conventionally defined ($A_{CP}^{\text{con}}$) are in fact different, which relation can be illustrated as $A_{CP}^{\text{exp}} = A_{CP}^{\text{con}} + \Delta A_{CP}^{\rho\omega}$, with $\Delta A_{CP}^{\rho\omega}$ the $\rho - \omega$ mixing contribution to $A_{CP}^{\text{exp}}$. $A_{CP}^{\text{exp}}$ is in fact the regional $CP$ asymmetry of $B^\pm \to \pi^+\pi^-\pi^0K^\pm$ when the invariant mass of the three pions lies in the vicinity of the $\omega$ resonance. The numerical value of $\Delta A_{CP}^{\rho\omega}$ is extracted from the experimental data of $B^\pm \to \pi^+\pi^-K^\pm$ and is found to be comparable with $A_{CP}^{\text{exp}}$, hence, nonnegligible. The conventionally defined $CP$ asymmetry, $A_{CP}^{\text{con}}$, is obtained from the values of $A_{CP}^{\text{exp}}$ and $\Delta A_{CP}^{\rho\omega}$, and is compared with the theoretical calculations in the literature.

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It was proposed long time ago that the mixing of the $\rho^0(770)$ and $\omega(782)$ resonances, which is termed as $\rho - \omega$ mixing, can affect $CP$ asymmetries of $B$ meson decays such as $B^\pm \to \pi^\pm \pi^+ \pi^-$ and $B^\pm \to \rho^\pm \pi^+ \pi^-$ [1–3], where $\rho^0 \to \pi^+ \pi^-$ is polluted by $\omega \to \rho^0 \to \pi^+ \pi^-$. However, no definite confirmation of the $\rho - \omega$ mixing effect has ever been provided experimentally [4]. On the contrary, it is usually believed that the $\rho - \omega$ mixing is irrelevant for $B$ meson decays when the $\omega$ meson is involved in the final state, such as $B \to \omega K$ and $B \to \omega \pi$, as the contributions of $B \to \rho^0 K(\pi) \to \omega K(\pi)$ to the amplitudes are negligible. This is true for branching ratios. However, for $CP$ asymmetries, as will be seen, things may change.

In this Letter, we are going to show that the $CP$ asymmetry of $B^\pm \to \omega K^\pm$ measured experimentally is different from that conventionally defined in the literature, where the $\rho - \omega$ mixing effect is absent, which has been calculated theoretically. According to our analysis, this effect turns out to give an important contribution to the measured $CP$ asymmetry of $B^\pm \to \omega K^\pm$.

The difference between the patterns of the $\rho - \omega$ mixing mechanism entering in $B^\pm \to \omega K^\pm$ and that in the previously studied channels such as $B^\pm \to \pi^\pm \pi^+ \pi^-$ and $B^\pm \to \rho^\pm \pi^+ \pi^-$ is apparent. In the latter situation, the weakly produced $\omega$ meson transforms to the $\rho^0$ meson, which finally decays into the $\pi^+ \pi^-$ pair, while for the situation of $B^\pm \to \omega K^\pm$, the process happens inversely, i.e., the weakly produced $\rho^0$ resonance transforms to the $\omega$ meson.

The $CP$ asymmetry of $B^\pm \to \omega K^\pm$ has been measured by Belle and BaBar [5, 6], with the latest world average [7]

$$A_{CP}^{exp} = -0.02 \pm 0.04.$$ (1)

It has also been studied extensively on the theoretical side via QCD factorization [8], perturbative QCD factorization [9], Soft Collinear Effective Theory [10], the flavor diagram approach [11], and the factorization assisted topological amplitude approach [12], with the values $0.221^{+0.137+0.140}_{-0.128-0.130}$, $0.32^{+0.15+0.04}_{-0.17-0.05}$, $0.116^{+0.182+0.011}_{-0.204-0.011}$, $(0.123^{+0.166+0.080}_{-0.173-0.011})$, $0.010 \pm 0.080$, and $0.19 \pm 0.09$, respectively. Although all these results seems consistent with the data thanks to the large uncertainties from both the theory and the experiment, their central values are far away from that of the experimental data and even have different signs.

Since the $\omega$ meson is usually reconstructed through the decay channel $\omega \to \pi^+ \pi^- \pi^0$, the $CP$ asymmetry being measured is in fact the regional one of $B^\pm \to \pi^+ \pi^- \pi^0 K^\pm$ when the
invariant mass of the three-pion system lies in the vicinity of the ω resonance, $A_{CP}^{\text{reg}}$,

$$A_{CP}^{\text{exp}} = A_{CP}^{\text{reg}} \left( B^\pm \to \pi^+\pi^-\pi^0 K^\pm \right) |_{s \to m_\omega^2} = \frac{\int (|A|^2 - |A_{CP}|^2) ds_{+0} ds_{-0}}{\int (|A|^2 + |A_{CP}|^2) ds_{+0} ds_{-0}} ds,$$

(2)

where $A$ and $A_{CP}$ are the decay amplitudes of $B^- \to \pi^+\pi^-\pi^0 K^-$ and $B^+ \to \pi^-\pi^+\pi^0 K^+$, respectively, $s_{+0}$, $s_{-0}$, and $s$ are the invariant masses squared of the $\pi^+\pi^0$, $\pi^-\pi^0$, and $3\pi$ systems, respectively, $m_\omega$ is the mass of the $\omega$ meson, the integral with respect to $s$ is performed in the vicinity of $\omega$, which has been taken between $(m_\omega - \Delta_\omega)^2$ and $(m_\omega + \Delta_\omega)^2$, with the cut $\Delta_\omega$ comparable with the decay width of the $\omega$ meson, $\Gamma_\omega$.

For the situations when the invariant mass of the $3\pi$ system lies around the $\omega$ resonance, the decay amplitude of $B^- \to \pi^+\pi^-\pi^0 K^-$ is dominated by the cascade decay $B^- \to \omega(\to \pi^+\pi^-\pi^0) K^-$, and potentially by the decay via the $\rho - \omega$ mixing, $B^- \to \rho^0(\to \omega \to \pi^+\pi^-\pi^0) K^-$. Consequently, the decay amplitude in this region can be expressed as

$$A = \sum_\lambda \left[ A_{B^-\to\omega K^-}^\lambda \right] \delta_{\rho\omega}(s) A_{B^-\to\rho K^-}^\lambda \cdot \frac{\chi_{\omega\to3\pi}^\lambda}{s_\omega},$$

(3)

where $A_{B^-\to\omega K^-}^\lambda$, $A_{B^-\to\rho K^-}^\lambda$, and $A_{\omega\to3\pi}^\lambda$ are the decay amplitudes of $B^- \to \omega K^-$, $B^- \to \rho^0 K^-$, and $\omega \to 3\pi$, respectively, $\lambda$ is the polarization index of the intermediate vector resonances $\omega$ or $\rho^0$, and $\delta_{\rho\omega}(s) \equiv \frac{\Pi_{\rho\omega}(s)}{s_\rho}$, with $\Pi_{\rho\omega}$ the $\rho - \omega$ mixing parameter and $s_\rho = s - m_\rho^2 - i m_\rho \Gamma_\rho$ ($V = \omega, \rho$). The amplitude of CP-conjugate process, $A_{CP}$, is obtained by replacing the CKM matrix elements by their complex conjugates in Eq. (3).

If one parameterizes the decay amplitudes of $B^- \to \omega K^-$ and $B^- \to \rho^0 K^-$ as

$$A_{B^-\to\omega K^-}^\lambda = \mathcal{F}_{B^-\to\omega K^-} \epsilon^\star(q, \lambda) \cdot p_B$$

and $A_{B^-\to\rho K^-}^\lambda = \mathcal{F}_{B^-\to\rho K^-} \epsilon^\star(q, \lambda) \cdot p_B$, respectively, with $p_B$ the momentum of the $B^-$ meson, $\epsilon$ the polarization vector of the $\omega$ or $\rho^0$ resonances, $\mathcal{F}_{B^-\to\omega K^-}$ and $\mathcal{F}_{B^-\to\rho K^-}$ the rest parts of these two amplitudes, respectively, then the decay amplitude in Eq. (3) can be cast into

$$A = \left[ \mathcal{F}_{B^-\to\omega K^-} + \delta_{\rho\omega}(s) \mathcal{F}_{B^-\to\rho K^-} \right] \cdot \frac{\chi_{\omega\to3\pi}}{s_\omega},$$

(4)

where $\chi_{\omega\to3\pi} \equiv \sum_\lambda A_{\omega\to3\pi}^\lambda \epsilon^\star(q, \lambda) \cdot p_B$. Once Eq. (4) is substituted into Eq. (2), the experimentally measured CP asymmetry can be expressed as

$$A_{CP}^{\text{exp}} \approx \frac{\left( \mathcal{F}_{B^-\to\omega K^-} + \delta_{\rho\omega} \mathcal{F}_{B^-\to\rho K^-} \right)^2 - \left( \mathcal{F}_{B^+\to\omega K^+} + \delta_{\rho\omega} \mathcal{F}_{B^+\to\rho K^+} \right)^2}{\left( \mathcal{F}_{B^-\to\omega K^-} + \delta_{\rho\omega} \mathcal{F}_{B^-\to\rho K^-} \right)^2 + \left( \mathcal{F}_{B^+\to\omega K^+} + \delta_{\rho\omega} \mathcal{F}_{B^+\to\rho K^+} \right)^2},$$

(5)
where $\hat{\delta}_{\rho\omega} \equiv \delta_{\rho\omega}(m_\omega^2)$. In deriving Eq. (5), we have neglected all the smooth dependence on $s$ by replacing it simply by $m_\omega^2$, as the integral with respect to $s$ is performed in a narrow interval around $m_\omega^2$. On the contrary, the $s$-dependence of $1/s_\omega$ survived because that this dependence is sharp. However, the corresponding integrals $\int 1/|s_\omega|^2 ds$ in the numerator and the denominator have been cancelled out.

The real and imaginary parts of the $\rho - \omega$ mixing parameter $\Pi_{\rho\omega}$ when $s = m_\rho^2$ is fitted to be [13]

\[ \Re \left( \Pi_{\rho\omega}(m_\rho^2) \right) = -4620 \pm 220_{\text{model}} \pm 170_{\text{data}} \text{ MeV}^2, \] \[ \Im \left( \Pi_{\rho\omega}(m_\rho^2) \right) = -6100 \pm 1800_{\text{model}} \pm 1110_{\text{data}} \text{ MeV}^2. \] (6) (7)

Neglecting the $s$-dependence of $\Pi_{\rho\omega}$, one can easily see that $\hat{\delta}_{\rho\omega}$ is numerically very small,

\[ \hat{\delta}_{\rho\omega} \approx (0.049 \pm 0.018) + (0.045 \pm 0.003)i, \] (8)

which is the main reason why the $\rho - \omega$ mixing is negligible for branching ratios of $B$ meson decays with $\omega$ involved in the final states. However, its contribution to the measured $CP$ asymmetry, $A_{CP}^{\text{exp}}$, is not negligible in spite of the smallness of $\hat{\delta}_{\rho\omega}$. To see this, let us make a Taylor expansion of $A_{CP}^{\text{exp}}$ up to $O(\hat{\delta}_{\rho\omega})$, which reads

\[ A_{CP}^{\text{exp}} = A_{CP}^{\text{con}} + \Delta A_{CP}^{\rho\omega}, \] (9)

where

\[ A_{CP}^{\text{con}} = \frac{|F_{B^-\to\omega K^-}|^2 - |F_{B^+\to\omega K^+}|^2}{|F_{B^-\to\omega K^-}|^2 + |F_{B^+\to\omega K^+}|^2}, \] (10)

is the conventionally defined $CP$ asymmetry of $B^\pm \to \omega K^\pm$ in the literature, and

\[ \Delta A_{CP}^{\rho\omega} = (1 - A_{CP}^{\text{con}}^2) \times \Re \left[ \left( \frac{F_{B^-\to\rho K^-}}{F_{B^-\to\omega K^-}} - \frac{F_{B^+\to\rho K^+}}{F_{B^+\to\omega K^+}} \right) \hat{\delta}_{\rho\omega} \right], \] (11)

measures the contribution of the $\rho - \omega$ mixing effect to $A_{CP}^{\text{exp}}$.

A rough but insightful order estimation gives $\Delta A_{CP}^{\rho\omega} \sim \sqrt{B_{B^\pm\to\rho K^\pm}/B_{B^+\to\omega K^+}} \hat{\delta}_{\rho\omega}$, with $B_{B^\pm\to\rho K^\pm}$ and $B_{B^+\to\omega K^+}$ the $CP$-averaged branching ratios of $B^\pm \to \rho K^\pm$ and $B^\pm \to \omega K^\pm$, respectively, from which one can see that $\Delta A_{CP}^{\rho\omega}$ and $\hat{\delta}_{\rho\omega}$ are of the same order, provided that no strong cancellation happens between the two amplitude ratios in Eq. (11). In fact, a strong cancellation between these two terms is very unlikely because the observed large $CP$ asymmetry in $B^\pm \to \rho^0 K^\pm$ [14] already indicates a considerable difference between
\[ \mathcal{F}_{B^\pm \to \rho K^\mp} \text{ and } \mathcal{F}_{B^\pm \to \rho K^\pm}. \] Since \( \hat{\delta}_{\rho\omega} \) is numerically the same order as the central value of the latest experimentally measured \( A_{CP}^{\text{exp}} \), it follows that the contribution of \( \Delta A_{CP}^{\rho\omega} \) to \( A_{CP}^{\text{exp}} \) is not negligible.

Eq. (9) represents the difference between the experimentally measured \( CP \) asymmetry \( A_{CP}^{\text{exp}} \) and the conventionally defined one \( A_{CP}^{\text{con}} \), which is the basic viewpoint of this Letter. To reconcile this contradiction, one can extract the conventionally defined \( CP \) asymmetry of \( B^\pm \to \omega K^\pm \) by a combination of the measurements of \( A_{CP}^{\text{exp}} \) and \( \Delta A_{CP}^{\rho\omega} \), where the latter can be extracted from an amplitude analysis of the experimental data of \( B^\pm \to \pi^+\pi^-K^\pm \). In practice, \( \Delta A_{CP}^{\rho\omega} \) can be approximated as \( \Delta A_{CP}^{\rho\omega} \approx \Re \left[ \left( \frac{\mathcal{F}_{B^\pm \to \rho K^\pm}}{\mathcal{F}_{B^\pm \to \omega K^\pm}} - \frac{\mathcal{F}_{B^\pm \to \omega K^\pm}}{\mathcal{F}_{B^\pm \to \omega K^\pm}} \right) \hat{\delta}_{\rho\omega} \right] \), where, according to the fitted amplitudes in Ref. [14], \( \mathcal{F}_{B^\pm \to \omega K^\pm} = \mathcal{F}_{B^\pm \to \omega K^\pm} \propto g_{\omega\pi\pi}^\text{eff}(x\omega + iy\omega) \), \( \mathcal{F}_{B^\pm \to \rho K^\pm} \propto g_{\rho\pi\pi}^\text{eff}(x\rho + iy\rho) \), and \( g_{\omega\pi\pi}^\text{eff} \) and \( g_{\rho\pi\pi}^\text{eff} \) are the coupling constants of \( \omega \to \pi^+\pi^- \) [15] and \( \rho^0 \to \pi^+\pi^- \), respectively, \( x\omega = -0.058 \pm 0.067 \pm 0.018 \pm 0.053 \), \( y\omega = 0.100 \pm 0.051 \pm 0.010 \pm 0.033 \), \( \Delta x\rho = -0.160 \pm 0.049 \pm 0.024 \pm 0.094 \), and \( \Delta y\rho = 0.169 \pm 0.096 \pm 0.057 \pm 0.133 \). Then, the \( \rho - \omega \) mixing effect contributing to \( A_{CP}^{\text{exp}} \) is extracted to be

\[
\Delta A_{CP}^{\rho\omega} \approx -2\Re \left[ \frac{\Delta x\rho + i\Delta y\rho}{x\omega + iy\omega} \cdot \frac{\hat{\Pi}_{\rho\omega}(m_\omega^2)\hat{\Pi}_\omega^*(m_\rho^2)}{(m_\omega^2 - m_\rho^2 + im_\rho\Gamma_\rho)^2} \right] \\
\approx 0.016^{+0.017}_{-0.036}(\text{BaBar}_{B^\pm \to \pi^+\pi^-K^\pm}) \pm 0.007(\Pi_{\rho\omega}), \tag{12}
\]

where \( \hat{\Pi}_{\rho\omega}(s) = s\rho g_{\omega\pi\pi}^\text{eff}/g_{\rho\pi\pi}^\text{eff} \) with \( s \) taken to be \( m_\omega^2 \) is the effective \( \rho - \omega \) mixing parameter with the inclusion of the direct decay \( \omega \to \pi^+\pi^- \) and has been taken the same value as \( \Pi_{\rho\omega} \) in our numerical calculation, the first uncertainty is estimated based on the extracted amplitudes by BaBar, and the second one comes from \( \Pi_{\rho\omega} \). From a comparison of Eqs. (1) and (12), it is concluded that the \( \rho - \omega \) mixing effect is indeed not negligible.

Combining Eq. (12) with the experimental data for \( A_{CP}^{\text{exp}} \), the conventionally defined \( CP \) asymmetry of \( B^\pm \to \omega K^\pm \) is obtained accordingly,

\[
A_{CP}^{\text{con}} = -0.036 \pm 0.040(\text{PDG}_{A_{CP}^{\text{exp}}})^{+0.036}_{-0.017}(\text{BaBar}_{B^\pm \to \pi^+\pi^-K^\pm}) \pm 0.007(\Pi_{\rho\omega}). \tag{13}
\]

This number should be compared with the theoretical predictions of the \( CP \) asymmetry of \( B^\pm \to \omega K^\pm \) via different approaches, which are roughly consistent with each other, although there are tensions between the central values. Note however, one should not take the number in Eq. (13) too seriously, as the extracted amplitudes of BaBar in Ref. [14] are not that reliable for our purpose for at least two reasons. First of all, the \( CP \) asymmetry of \( B^\pm \to \omega K^\pm \) is set to be zero by hand in Ref. [14]. Secondly, the extracted amplitudes of
$B^\pm \to \omega K^\pm$ suffer from large uncertainties, as their fractions in that of $B^\pm \to \pi^+\pi^-K^\pm$ are quite small. Future extractions of the amplitudes with smaller uncertainties and more in line with our purpose are needed both for $A_{CP}^{\exp}$ and $\Delta A_{CP}^{\rho\omega}$, so as to give a more accurate result for the genuine $CP$ asymmetry of $B^\pm \to \omega K^\pm$, $A_{CP}^{\con}$. Of course, a theoretical calculation of $A_{CP}^{\exp}$ with the explicit inclusion of the $\rho - \omega$ mixing effect is also desirable.

Before concluding, one subtle point should be discussed. For obtaining $A_{CP}^{\con}$ alone, the definition in Eq. (10) tells us that the extraction of $F_{B^\pm \to \omega K^\pm}$ directly from the amplitude analyses of channels such as $B^\pm \to \pi^+\pi^-K^\pm$ are sufficient, needless to work out from $A_{CP}^{\exp}$ and $\Delta A_{CP}^{\rho\omega}$ instead. However, for the explicit demonstration of the difference between $A_{CP}^{\exp}$ and $A_{CP}^{\con}$, the simultaneous extraction of all the four amplitudes in Eq. (11) is essential, where $B^\pm \to \pi^+\pi^-K^\pm$ is currently the only process in the literature which fits this purpose [16].

The above analysis can be potentially generalized to other $B$ meson decays with the involvement of the $\omega$ meson as a final state particle. For example, the similar pattern shows up in the decay $B^\pm \to \omega \pi^\pm$, where the $CP$ asymmetry is also comparable with $\hat{\delta}_{\rho\omega}$ [4, 5]. In general, for the decay process $B \to \omega X$ ($X$ represents one or more particles) and its $CP$ conjugate $\overline{B} \to \omega \overline{X}$, the difference between the regional $CP$ asymmetry of $B \to \pi^+\pi^-\pi^0X$ with the invariant mass of the three pions lying around the $\omega$ resonance and that of $B \to \omega X$ will be

$$\Delta A_{CP}^{\rho\omega}(B \to \omega X) = (1 - A_{CP,B \to \omega X}^{\con}) \Re \left( \left( \frac{A_{\rho}}{A_{\omega}} - \frac{\overline{A}_{\rho}}{\overline{A}_{\omega}} \right) \hat{\delta}_{\rho\omega} \right), \quad (14)$$

where $A_V$ and $\overline{A}_V$ are the decay amplitudes of $B \to VX$ and $\overline{B} \to V\overline{X}$, respectively, $A_{CP,B \to \omega X}^{\con}$ is the conventionally defined $CP$ asymmetry of $B \to \omega X$ without the contribution of $\rho - \omega$ mixing. Strictly speaking, careful analysis of the $\rho - \omega$ mixing contributions should be performed for $CP$ asymmetries of most (if not all) $B$ meson decays with $\omega$ appears in the final states, since $\hat{\delta}_{\rho\omega}$ may give considerable contributions to the $CP$ asymmetries.

To sum up, it is concluded that the $CP$ asymmetry of $B^- \to \omega K^-$ being measured experimentally $A_{CP}^{\exp}$ is in fact the regional one of $B^\pm \to \pi^+\pi^-\pi^0K^\pm$ when the invariant mass of the $3\pi$ system lies around the $\omega$ resonance, which is different from the conventionally defined one, $A_{CP}^{\con}$. This conclusion is expressed as $A_{CP}^{\exp} = A_{CP}^{\con} + \Delta A_{CP}^{\rho\omega}$, where the $\rho - \omega$ mixing contribution, $\Delta A_{CP}^{\rho\omega}$, is not negligible. This is supported by the amplitude analysis of the channel $B^\pm \to \pi^+\pi^-K^\pm$, from which $A_{CP}^{\con}$ is also extracted and compared with the
theoretical calculations.

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[15] The coupling $g_{\omega\pi\pi}^{\text{eff}}$ includes both the direct decay $\omega \to \pi^+\pi^-$ and the decay via $\rho - \omega$ mixing, $\omega \to \rho \to \pi^+\pi^-$. 
[16] Note that $B^\pm \to \pi^+\pi^-\pi^0K^\pm$ is not a suitable channel for the amplitude analysis either for $A_{CP}^{\text{con}}$
or \( \Delta A_{CP}^{\omega} \). The key reason is the presence of the factor \( 1/s_\omega \) in Eq. (3). The narrowness of \( 1/s_\omega \) indicates that the expression of Eq. (3) is only valid for \( s \sim m_\omega^2 \), around which the amplitude analysis is to be performed. Moreover, since \( 1/s_\omega \) appears as a common factor in both the two amplitude ratios in Eq. (3), this makes it almost impossible to distinguish the contributions of these two terms experimentally. In the situation of \( B \to \pi^+ \pi^- K \), on the other hand, the decay amplitude is approximated as

\[
A(B \to \pi^+ \pi^- K)|_{s \sim m_\omega^2} \propto \left( F_{B \to \rho K} + F_{B \to \omega K} \tilde{\Pi}_{\rho\omega}/s_\omega \right)/s_\rho
\]

when the invariant mass squared of the two pions \( s \) lies around \( m_\omega^2 \). In this region, one has \( \tilde{\Pi}_{\rho\omega}/s_\omega \sim \mathcal{O}(1) \), indicating that the two terms in the parenthesis of the above equation are comparable. The presence of the extra factor \( 1/s_\omega \) in the second term leads to an easy-to-observe \( s \)-dependent interference effect which allows for the extraction of the two terms in the above equation from the experimental data.