Relay Control for Full-Duplex Relaying with Wireless Information and Energy Transfer

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Abstract—This study investigates wireless information and energy transfer for dual-hop amplify-and-forward full-duplex relaying systems. By applying time switching as a relay transceiver architecture, the full duplex information relaying can be powered by energy harvested from the source-emitted radio frequency signal. The throughputs are analyzed for three relay control schemes, including the maximum relay, optimal relay, and target relay. Analytical expressions for outage probability and ergodic capacity are also presented for these considered relay control schemes. The time switching factors that maximize the relaying transmission time with collateral energy harvesting benefits are presented for the optimal relay and target relay, which incorporate instantaneous channel information to increase the throughput over the maximum relay. Without requiring channel information for the second-hop, the target relay can ensure competitive performance for the outage probability and ergodic capacity. It is also observed that the throughput decreases dramatically for the target relay when the full-duplex relay moves away from the source toward the destination, while the smallest throughputs are obtained for the maximum relay and optimal relay when the full-duplex relay is placed midway between the source and destination.

Index Terms—Energy harvesting, wireless power transfer, amplify-and-forward relay, full-duplex relay, relay gain control.

I. INTRODUCTION

Energy harvesting (EH) has emerged as a promising approach to prolong the lifetime of energy constrained wireless communications [1]–[7]. Through harvesting energy from natural sources (e.g., solar, wind, thermoelectric effects or other physical phenomena), periodic battery replacement or recharging can be alleviated in conventional EH-aided wireless communications [1]–[4]. However, EH from natural sources is vulnerable to environmental changes so that conventional EH-aided wireless communications are far from convenient, stable, and reliable [4]. With the capability to harvest energy from ambient radio-frequency (RF) signals, simultaneous wireless information and energy transfer (SWIET), also known as simultaneous wireless information and power transfer (SWIPT) provides a more encouraging way than conventional EH-aided wireless communications to function in environments with physical or economic limitations [5]–[11].

The pioneering work on SWIET can be traced back to [5] and [6], where the fundamental tradeoff between capacity and energy was studied for point-to-point communications. Following the assumption that an ideal receiver is capable of observing information and extracting energy from the same received signal, SWIET has been extended to multi-antenna systems [12], [13], multiuser systems [14], and bi-directional communication systems [15]. However, as discussed in [7], a practical circuit for EH from the RF signal can hardly decode the carried information from the same signal. Therefore, two practical receiver architectures, namely, time switching (TS) and power splitting (PS), are proposed in [7]. They are now widely adopted in various wireless systems, such as multiple-input multiple-output (MIMO) systems [16], orthogonal frequency division multiplexing systems [17], and cellular systems [18].

In parallel with the aforementioned studies that mainly deal with single-hop scenarios, employment of cooperative relays to facilitate RF EH and information transfer in wireless cooperative or sensor networks has also drawn significant attention [9]–[11], [18]. Relay-based SWIET not only enables wireless communications over long distances or across barriers, but also keeps the energy-constrained relays active through RF EH. The authors of [11] designed and analyzed the TS and PS relaying protocols for amplify-and-forward (AF) relaying systems, and then extended the results to an adaptive TS relaying protocol [19].

The throughput of the TS and PS relaying protocols for decode-and-forward (DF) relaying was investigated in [10]. Several power allocation schemes for EH relay systems with multiple source-destination pairs were studied in [20]. The outage and diversity of SWIET for cooperative networks with spatially random relays were investigated in [21] and the distributed PS-based SWIET was designed for interference relay systems [22]. More recently, antenna switching and antenna selection for the SWIET relaying systems have been investigated in [23] and [24], respectively. Nevertheless, all these studies are limited to half-duplex relaying (HDR) mode. Since the source-to-relay and relay-to-destination channels are kept orthogonal by either frequency division or time division...
multiplexing, significant loss of spectral efficiency occurs in the HDR mode. As an alternative, full-duplex relaying (FDR) has drawn considerable attention [25]–[30]. Since FDR requires only one channel for the end-to-end transmission, a significant improvement in the spectral efficiency over the HDR can be achieved.

So far, a few studies have been conducted for SWIET in FDR systems. In [27], the throughput of the TS relaying protocol has been analyzed for FDR SWIET systems, in which the EH relay is operated cooperatively. In practice, since the relay node suffers severe self-interference (loop interference) from its own transmit signal, FDR operation is difficult to implement. For example, self-interference of more than 106 dB has to be suppressed by a femto-cell FDR base-station to achieve the link signal-to-noise-ratio (SNR) equal to that of an HDR counterpart [31]. For systems that require higher transmission power, more self-interference suppression is needed [25]. By employing relay EH in the second time phase for the conventional two-phase AF HDR systems, a self-interference immunizing full-duplex relay node was proposed in [32], which can transmit information and extract energy simultaneously via separated transmit and receive antennas. Another challenging problem for SWIET is to determine the EH parameters, i.e., the TS factor and PS factor for the TS and PS relaying protocols, respectively. Determining of the TS factor affects not only the relay-harvested energy but also the effective relaying transmission time in the TS relaying protocol. Compared with the PS relaying protocol, the TS relaying protocol is more practical because of its simplicity. With statistical channel state information (CSI), the numerical optimizations of the TS factor have been presented in [11] and [27]. In delay-limited and delay-tolerant transmissions, the instantaneous CSI can also be applied to optimize the EH parameter, as well as the instantaneous CSI-aided transmission power control in the FDR systems [28], [29].

Motivated by these previous studies, we focus on wireless information and energy transfer for a dual-hop FDR system, in which the TS-based AF relay node is powered via EH from the source-emitted RF signal. Compared with previous research, our study has the following distinct features:

- In [32], the effective information transmission time is the same as that of HDR systems. Therefore, the spectral efficiency improvement is minimal compared with that of the FDR systems. In our study, the spectral efficiency is achieved by the full-duplex information relaying, in which the relay receives and forwards the source information to the destination simultaneously.

- In [27], the authors designed wireless information and energy transfer for FDR systems, in which the relay node adopts maximum relay gain by fully using the given relay transmission power [26], [33]. Then, achieving the maximum throughput becomes a TS optimization problem, which generally does not have a closed-form solution [27]. In our study, the full use of the given relay transmission power is not prerequisite in designing the relay control schemes of the optimal relay and target relay. These relay types will be defined in the following paragraphs. As we will see in a later part of this paper, the TS factors for the relay control schemes of the optimal relay and target relay are obtained in closed-form by harvesting only the necessary energy.

In this paper, three relay control schemes, namely, the maximum relay, optimal relay, and target relay, are investigated for the three transmission schemes of instantaneous transmission, delay-limited transmission, and delay-tolerant transmission. The contributions of this study are summarized as follows:

- The end-to-end signal-to-interference-plus-noise-ratio (e-SINR) is formulated as a non-linear function of the relay gain and TS factor. The optimal relay and target relay are designed to maximize the e-SINR and achieve the target e-SINR, respectively. With the obtained closed-form TS factors for the optimal relay and target relay, instantaneous CSI can be employed to improve the throughput in delay-limited and delay-tolerant transmissions.

- We present analytical expressions for the throughput for the three relay control schemes in different transmissions, such as instantaneous transmission, delay-limited transmission, and delay-tolerant transmission. Specifically, analytical expressions for the outage probability are derived for these three relay control schemes in delay-limited transmission, while analytical expressions for the ergodic capacity are derived for these relay control schemes in delay-tolerant transmission.

- We show the coincidence of the instantaneous throughputs of the maximum relay and optimal relay in the region of high interference-to-noise-ratio (INR). In delay-limited and delay-tolerant transmissions, both the optimal relay and target relay are more competitive than the maximum relay in increasing the throughput in the region of high INR. Both the optimal relay and target relay achieve a better outage performance over the maximum relay in delay-limited transmission. In delay-tolerant transmission, the optimal relay achieves a higher ergodic capacity than the maximum relay in the high SNR region, while the target relay achieves a competitive ergodic capacity without requiring CSI of the second-hop.

- We present numerical results to show that the throughputs of the maximum relay and optimal relay are relatively worse when the full-duplex relay is placed in the middle of the source and destination, while a target relay with a high target e-SINR results in a monotonically decreasing throughput when the full-duplex relay moves toward the destination. In delay-tolerant transmission, it is also shown that for any two relay nodes placed in the symmetric positions of the middle of the source and destination, the relay placed near the source achieves a higher throughput than that of the relay placed near the destination.

The rest of this paper is organized as follows. Section II describes the system model of the considered FDR system.
and formulates the throughput optimization problem. Section III presents the three relay control schemes. The analytical results of the throughput are presented in Section IV. Section V presents numerical results and discusses the system performances of our proposed relay control schemes. Finally, Section VI summarizes the contributions of our study.

II. SYSTEM MODEL

In this paper, we consider a wireless dual-hop FDR system, in which a source node intends to transfer its information to the destination node. Due to large separation or shadowing between the source and destination, a cooperative relay is employed to assist information transmission from the source to the destination. The cooperative relay is assumed to be an energy constrained device such that it has to harvest energy from the source-emitted RF signal to forward the source information to the destination. For simplicity of implementation, the AF relaying scheme and TS transceiver architecture are chosen at the relay node. The channel coefficients from the source to the relay and from the relay to the destination are denoted by $h$ and $g$, respectively. The loopback interference channel at the relay node is denoted by $f$. All the channels are assumed to be frequency non-selective and quasi-static block-fading, following a Rayleigh distribution. The means of the exponential random variables $|f|^2$, $|g|^2$, and $|h|^2$ are denoted by $\lambda_f$, $\lambda_g$, and $\lambda_h$, respectively. Based on the pilot symbols sent from the source over the dual-hop link, the dual-hop CSI can be estimated to facilitate the wireless information transfer [7], [10], [11], [16], [32]. To explore the potential capacity and performance limit of such an AF FDR system, we assume that a centralized control unit with the capability to access global (or partial) CSI computes and updates the relay control parameters.

The framework of the TS relaying protocol is illustrated in Fig. 1(a), in which each time block $T$ is divided into two phases. Denoting the TS factor by $\alpha$ ($0 < \alpha < 1$), we use the first phase assigned with a duration of $\alpha T$ for energy transfer from the source to the relay. The second phase assigned with the remaining duration of $(1-\alpha)T$ is used for full-duplex information relaying via the dual-hop channel. The relay-received RF signal in the two time phases are sent to the EH receiver and full-duplex transceiver, respectively, as illustrated in Fig. 1(b). Since the relay node does not transmit during the first time phase, loop interference is not introduced during the EH period. The harvested energy at the relay is given by

$$E_h = \frac{\eta P_s |h|^2}{d_1^m} \alpha T,$$

where $P_s$ is the source transmission power, $d_1$ is the distance between the source and relay, $m$ is the path loss exponent, and $\eta$ is the energy conversion efficiency that depends on the EH circuitry and rectification process. In the FDR mode, the relay concurrently receives the signal $y_r(t)$ and transmits the signal $x_r(t)$ on the same frequency. As depicted in Fig. 2(b), the full-duplex transceiver down-converts the received RF signal to the baseband, processes the baseband signal, and up-converts the processed baseband signal. In Fig. 2(b), $\tilde{n}_a^{[r]}(t)$ and $\tilde{n}_c^{[r]}(t)$ are the narrow-band Gaussian noises introduced by the receive and transmit antennas, respectively. In addition, $\tilde{n}_c^{[r]}(t)$ and $\tilde{n}_c^{[r]}(t)$ are the baseband additive white Gaussian noise (AWGNs) caused by down-conversion and up-conversion, respectively [7]. For simplicity, the equivalent baseband noise composing both $\tilde{n}_a^{[r]}(t)$ and $\tilde{n}_a^{[r]}(t)$ is modeled by the zero mean AWGN $n_a^{[r]}(t)$ with the variance $\sigma_a^2$, and the equivalent baseband noise composing both $\tilde{n}_c^{[r]}(t)$ and $\tilde{n}_c^{[r]}(t)$ is modeled by the zero mean AWGN $n_c^{[r]}(t)$ with the variance $\sigma_c^2$. Therefore, the overall AWGN at the relay node can be modeled as the zero mean AWGN $\eta_r(k) \triangleq n_a^{[r]}(k) + n_c^{[r]}(k)$ with the variance $\sigma_r^2 \triangleq \sigma_a^2 + \sigma_c^2$. At the relay node, the sampled baseband signal is given by

$$y_r(k) = \sqrt{\frac{P_s}{d_1^m}} h s(k) + f x_r(k) + n_r(k),$$

where $k$ denotes the symbol index, $s(k)$ is the sampled and normalized information signal from the source, $x_r(k)$ is the sampled signal of $x_r(t)$, and the second term on the right side of (2) is the loop interference. In the present study, we do not investigate any self-interference cancellation schemes such that $f$ may represent the loopback channel of the relay with or without self-interference cancellation.

Using the harvested energy, the relay amplifies the received signal by a relay gain $\beta$. Then, the transmitted signal at the relay can be expressed as

$$x_r(k) = \sqrt{\beta y_r(k - \tau)},$$
where $\tau \geq 1$ is the processing delay at the relay. By recursively substituting (4) and (3), we have the following expression for the transmitted signal at the relay:

$$x_r(k) = \sqrt{\beta} \sum_{j=1}^{\infty} (f \sqrt{\beta})^{-1} \times \left( \sqrt{\frac{P}{d_2}} h s(k-j\tau) + n_r(k-j\tau) \right).$$

(4)

The sampled received signal at the destination is given by

$$y_d(k) = \frac{1}{\sqrt{d_2}} g x_r(k) + n_0^{[d]}(k) + n_e^{[d]}(k),$$

(5)

where $d_2$ is the distance between the relay and destination and $n_0^{[d]}(k)$ and $n_e^{[d]}(k)$ are the antenna and down-conversion AWGNs, respectively. The variances of $n_0^{[d]}(k)$ and $n_e^{[d]}(k)$ are $\sigma_d^2$ and $\sigma_e^2$, respectively. Substituting (4) into (5), we have

$$y_d(k) = \sqrt{\frac{P}{d_2}} g h \sum_{j=1}^{\infty} (f \sqrt{\beta})^{-1} s(k-j\tau)$$

$$+ \sqrt{\frac{1}{d_2}} g \sum_{j=1}^{\infty} (f \sqrt{\beta})^{-1} n_r(k-j\tau) + n_d(k),$$

(6)

where $n_d(k) = n_0^{[d]}(k) + n_e^{[d]}(k)$ is the zero mean AWGN at the destination with the variance $\sigma_d^2 \triangleq \sigma_d^2 + \sigma_e^2$.

In the following, we derive the end-to-end signal power under the condition of employing cooperative non-oscillating relays. By assuming that all the signal and noise samples are mutually independent, we calculate the relay transmission power from (4) as

$$\mathbb{E}\{|x_r(k)|^2\} = \beta \sum_{j=1}^{\infty} (|f|^2 \beta)^{-1} \left( (d_2^m)^{-1} P_s |h|^2 + \sigma_d^2 \right)$$

$$= \beta \frac{(d_2^m)^{-1} P_s |h|^2 + \sigma_d^2}{1 - |f|^2 \beta}.$$  

(7)

To prevent oscillation and guarantee finite relay transmission power, the relay gain is limited by

$$\beta < \frac{1}{|f|^2}.$$  

(8)

Given the relay-harvested energy, the maximum relay transmission power is expressed as

$$P_r = \frac{E_h}{(1-\alpha) T} = \frac{\mu P_s |h|^2}{d_2^m},$$  

(9)

where $\mu \triangleq \frac{\alpha}{\alpha + \beta}$. The actual relay transmission power should be less than or equal to the maximum relay transmission power, i.e.,

$$\mathbb{E}\{|x_r(k)|^2\} \leq P_r.$$  

(10)

When (7) and (9) are substituted into (10), the relay gain under the maximum relay transmission power is limited by

$$\beta \leq \frac{\mu}{1 + \gamma_{SR}^{-1} \alpha + \mu |f|^2},$$  

(11)

where the channel SNR of the first-hop link is defined as $\gamma_{SR} \triangleq \frac{P_s |h|^2}{d_2^m \sigma_d^2}$. At symbol index $k$, the destination node can employ any standard detection procedure to decode the desired signal $s(k-\tau)$, and the rest of the received signal components act as interference and noise. Again based on the assumption that signal and noise are independent of each other, the received signal power at the destination is calculated from (5) as

$$\mathbb{E}\{|y_d(k)|^2\} = \left( (d_2^m)^{-1} P_s |h|^2 \beta \right)^2$$

$$+ \left( (d_2^m)^{-1} P_s |h|^2 + \sigma_d^2 \right) \left( (d_2^m)^{-1} P_s |h|^2 + \sigma_d^2 \right)$$

$$+ \left( (d_2^m)^{-1} |g|^2 \sigma_d^2 + \sigma_e^2 \right).$$  

(12)

Based on (12), the e-SINR at the destination is given by

$$\gamma = \frac{\gamma_{SR} \gamma_{RD}}{\gamma_{SR} + \gamma_{RD} + (\gamma_{SR} + 1) \gamma_{RD}},$$  

(13)

where the channel SNR of the second-hop link is defined as $\gamma_{RD} \triangleq \frac{P_s |h|^2}{d_2^m \sigma_d^2}$. The channel SNR of the second-hop link is defined as $\gamma_{RD} \triangleq \frac{P_s |h|^2}{d_2^m \sigma_d^2}$.

The throughputs of the three considered transmission schemes: instantaneous transmission, delay-limited transmission, and delay-tolerant transmission are, respectively, given by

$$R_I(\alpha, \beta) = (1-\alpha) \log_2 (1+\gamma_{RD})$$

$$R_{DL}(\alpha, \beta) = (1-\alpha) \log_2 (1-P_{out}) R,$$  

(14a)

$$R_{DT}(\alpha, \beta) = (1-\alpha) C_E,$$  

(14b)

where $P_{out} = \Pr(\gamma < \gamma_{th})$ is the outage probability, $R = \log_2 (1+\gamma_{th})$ is the fixed transmit rate, $C_E = \mathbb{E} \{\log_2 (1+\gamma_{th})\}$ is the ergodic capacity, and $\gamma_{th}$ is an e-SINR threshold for correct data detection at the destination.

The design goal of the relay control scheme is to maximize the throughput by optimizing the control parameters $\{\alpha, \beta\}$. The optimal control parameters $\{\alpha^*, \beta^*\}$ can be obtained by solving the following optimization problem:

$$\{\alpha^*, \beta^*\} = \underset{\alpha, \beta}{\arg \max} R_M(\alpha, \beta)$$  

(15)

subject to $0 < \alpha < 1$ and $0 < \beta \leq \frac{\mu}{1 + \gamma_{SR}^{-1} \alpha + \mu |f|^2}$, where $R_M(\alpha, \beta)$ represents $R_I(\alpha, \beta)$, $R_{DL}(\alpha, \beta)$, and $R_{DT}(\alpha, \beta)$ for instantaneous transmission, delay-limited transmission, and delay-tolerant transmission, respectively. For a given expression of $R_M(\alpha, \beta)$, the optimal $\{\alpha^*, \beta^*\}$ can be obtained by exhaustively searching for all the possible combinations of $\{\alpha, \beta\}$.

In the following, all three relay control schemes are presented. Then, their throughputs are analyzed for the considered three transmissions.
III. RELAY CONTROL SCHEME

In this section, we investigate how to compute the relay gain and TS factor for the three relay control schemes, namely, maximum relay, optimal relay, and target relay. In designing the relay control scheme, we assume that exact knowledge of CSI is available at the control unit.

A. Maximum Relay

A simple and popular relay control scheme involves setting the relay gain at the maximum relay transmission power [22], [27], [33]. In contrast to these works, our study considers the maximum relay targeting to maximize the throughput in the region of high INRs. For a given $\alpha$ in the range $(0, 1)$, the relay-harvested energy and maximum relay transmission power are determined, so that the relay gain is given by the following according to (11):

$$\beta_{\text{max}} = \frac{\mu \gamma_{\text{SR}}}{1 + \gamma_{\text{SR}} + \mu \gamma_{\text{RD}}/f^2},$$

which guarantees that (3) holds. Substituting (16) into (13), the e-SINR achieved by the maximum relay is given by

$$\gamma_{\text{max}} = \frac{\mu \gamma_{\text{SR}} \gamma_{\text{RD}}}{\gamma_{\text{SR}} + (\mu \gamma_{\text{SR}}/f^2 + 1)(\mu \gamma_{\text{RD}} + 1)}.$$  

Since the relay can apply the maximum relay transmission power without exact knowledge of CSI, the relay gain of the maximum relay is easy to realize. Also, since both the maximum relay transmission power and effective relaying transmission time are determined by the TS factor, searching the optimal TS factor becomes critical for the throughput maximization problem in (15). The optimized TS factor that maximizes the throughput is discussed in the next section along with the throughput analysis.

B. Optimal Relay

When the TS factor is not optimized, the maximum relay is not optimal in maximizing the e-SINR, which in turn may have negative effects on the throughput. Although the throughput is affected by both the relaying transmission time and e-SINR, the relay gain of the optimal relay is first determined to maximize the e-SINR. Then, the relaying transmission time is determined with the TS factor that harvests only the necessary energy.

According to (12), given the received signal power at the destination node as a function of the relay gain, the desired signal power is linear, but the loop interference power is nonlinear. Consequently, increasing the relay gain can increase the loop interference power faster than the desired signal power and lead to a reduced e-SINR. We can show that (15) has a single maximum point for $\beta \in (0, 1/\sqrt{f^2})$. Thus, by setting the derivative of (15) equal to zero, the optimal relay that maximizes the e-SINR is obtained as

$$\beta_{\text{opt}} = \frac{\gamma_{\text{SR}}}{\gamma_{\text{SR}}/f^2 + \sqrt{\gamma_{\text{SR}}(\gamma_{\text{SR}} + 1)\gamma_{\text{RD}}f^2}.$$  

which also satisfies the non-oscillatory condition in (8). Substituting (18) into (13), we can express the e-SINR as

$$\gamma_{\text{opt}} = \frac{\gamma_{\text{SR}} \gamma_{\text{RD}}}{\gamma_{\text{SR}}/f^2 + \gamma_{\text{RD}} + 2\sqrt{\gamma_{\text{SR}}(\gamma_{\text{SR}} + 1)\gamma_{\text{RD}}f^2}}.$$  

Obviously, fully utilizing the relay-harvested energy is not prerequisite in designing the relay gain $\beta_{\text{opt}}$. For example, the redundant energy can be harvested by the relay in addition to the necessary energy harvested to support the relay gain $\beta_{\text{opt}}$. To extend the relaying transmission time as long as possible, the TS factor is designed as small as possible such that the relay-harvested energy is just enough to implement the relay gain $\beta_{\text{opt}}$. Therefore, by solving $\beta_{\text{opt}} = \beta_{\text{max}}$ for any $\alpha$ $(0 < \alpha < 1)$, we can provide the TS factor for the optimal relay as

$$\alpha_{\text{opt}} = \frac{\sqrt{\gamma_{\text{SR}} + 1}}{\gamma_{\text{SR}} + 1 + \eta \sqrt{\gamma_{\text{SR}} \gamma_{\text{RD}}f^2}}.$$  

Since no redundant energy has been harvested by employing $\alpha_{\text{opt}}$, the relaying transmission time $(1 - \alpha_{\text{opt}})T$ is longer than those of other $\alpha$ satisfying $\alpha > \alpha_{\text{opt}}$. The TS factor $\alpha_{\text{opt}}$ can be computed at the destination, or at the relay locally when the relay can access the global CSI.

C. Target Relay

When the optimal relay is employed, exact knowledge of the channel SNR $\gamma_{\text{RD}}$ has to be exploited, which can be estimated only at the destination. A feedback channel is then required if $\alpha_{\text{opt}}$ is computed locally at the relay. Therefore, a simplified relay control scheme that aims to achieve a target e-SINR $\hat{\gamma}$ is proposed. To avoid using knowledge of $\gamma_{\text{RD}}$, the target e-SINR should satisfy $\hat{\gamma} < \gamma_{\text{SR}}$. Denoting the e-SINR achieved by the target relay as $\gamma_{\text{tar}} (\gamma_{\text{tar}} = \hat{\gamma})$, the target relay is designed to make $\gamma_{\text{tar}}$ optimal, i.e., $\gamma_{\text{tar}} = \gamma_{\text{opt}}$. Denoting the TS factor for the target relay by $\alpha_{\text{tar}}$ and substituting $\mu_{\text{tar}} = \frac{\alpha_{\text{tar}}}{\gamma_{\text{SR}}}$ into (16) and (17), we can write the relay gain and e-SINR as

$$\beta_{\text{tar}} = \frac{\mu \gamma_{\text{SR}} \gamma_{\text{RD}}}{1 + \gamma_{\text{SR}} + \mu \gamma_{\text{SR}}/f^2}.$$  

and

$$\gamma_{\text{tar}} = \frac{\mu \gamma_{\text{SR}} \gamma_{\text{RD}}}{\gamma_{\text{SR}} + (\mu_{\text{tar}} \gamma_{\text{SR}}/f^2 + 1)(\mu_{\text{tar}} \gamma_{\text{RD}} + 1)},$$  

respectively. For a given $\hat{\gamma}$, by eliminating $\gamma_{\text{RD}}$ from the equation pair $\{\gamma_{\text{tar}} = \hat{\gamma}, \gamma_{\text{opt}} = \hat{\gamma}\}$, the TS factor is given by

$$\alpha_{\text{tar}} = \frac{\gamma_{\text{SR}}(\gamma_{\text{SR}} - \gamma + \eta \sqrt{\gamma_{\text{SR}} f^2}) + \gamma_{\text{SR}} f^2 \sqrt{\gamma(\gamma + 1) \gamma_{\text{SR}} (\gamma_{\text{SR}} + 1)}}{\gamma_{\text{SR}}(\gamma_{\text{SR}} + 1) (\gamma_{\text{SR}} - \gamma + \eta \sqrt{\gamma_{\text{SR}} f^2}) + \gamma_{\text{SR}} f^2 \sqrt{\gamma(\gamma + 1) \gamma_{\text{SR}} (\gamma_{\text{SR}} + 1)}}.$$  

Since $\alpha_{\text{tar}}$ only harvests the necessary energy to support the relay gain $\beta_{\text{tar}}$, the relaying transmission time $(1 - \alpha_{\text{tar}})T$ is longer than those of other $\alpha$ satisfying $\alpha > \alpha_{\text{tar}}$. Also, since $\hat{\gamma} < \gamma_{\text{SR}}$, (23) guarantees that $0 < \alpha_{\text{tar}} < 1$. When $\hat{\gamma} \geq \gamma_{\text{SR}}$, we have $\alpha_{\text{tar}} \leq 0$. In this case, no time is assigned for EH such that information relaying fails due to the lack of power. Alternatively, we can reset the TS factor by $\alpha_{\text{tar}} = 1$ such that only EH is implemented for the entire time block.
IV. THROUGHPUT ANALYSIS

In this section, the THROUGHPUTS of the three considered relay control schemes are investigated for instantaneous transmission, delay-limited transmission, and delay-tolerant transmission, respectively.

A. Instantaneous Transmission

When the optimal relay and target relay are employed, the instantaneous throughput of the FDR system can be computed by using $R_t = (1-\alpha) \log_2(1+\gamma)$, where $\alpha$ is given by (20) and (23), respectively, and the e-SINR is given by (19) and (22), respectively. For the optimal relay, $\alpha_{\text{opt}}$ can be computed at the destination or at the relay with the global CSI assumption. For the target relay, given that no exact knowledge of $\gamma_{\text{RD}}$ is required, the TS factor can be calculated at the relay to reduce the feedback from the destination. Moreover, when $\hat{\gamma} \geq \gamma_{\text{SR}}$, the instantaneous throughput achieved by the target relay is zero because of the failure of the effective relaying.

When the maximum relay is applied, the optimized TS factor can be obtained by solving the following optimization problem:

$$\alpha^* = \arg \max_{\alpha} R_t(\alpha)$$

subject to $0 < \alpha < 1$. \hspace{1cm} (24)

In (24), since $R_t(\alpha) = (1-\alpha) \log_2(1+\gamma_{\max})$ is concave with respect to $\alpha$, the optimized $\alpha^*$ can be obtained by solving the equation $\frac{d R_t(\alpha)}{d \alpha} = 0$. However, given the complicated expression for $\frac{d R_t(\alpha)}{d \alpha} = 0$, a closed-form solution is difficult to obtain. Thus, we obtain the optimized $\alpha^*$ by using the built-in function “NSolve” of Mathematica as in [27].

B. Delay-limited Transmission

In delay-limited transmission, the source transmits at a fixed rate $R$ to satisfy some outage criteria. The average throughput is determined by $R_{DL}(\alpha) = (1-\alpha)(1-P_{out})R$.

**Proposition 1:** The outage probability achieved by the maximum relay is given by

$$P_{out} = 1 - \frac{1}{\lambda_f \lambda_h} \int_{w=0}^{\mu v h} \int_{u=0}^{\nu v h} e^{-\left(\frac{w^2 + c w^2 v u}{(c v^2 + 2 c w) u^2}\right)} e^{-\frac{w^2}{\nu v h}} dw du$$

$$\approx 1 - \frac{1}{\lambda_f \lambda_h} \int_{0}^{\frac{1}{\lambda_f}} p K_1(p) e^{-\frac{p}{\lambda_f}} dp, \hspace{1cm} \text{(AHS)} \hspace{1cm} (25b)$$

where $\alpha \triangleq P_d \sigma_d \mu \gamma_{\text{th}}(1+\mu w)$, $b \triangleq \int_{0}^{\nu v h} \int_{u=0}^{\mu v h} e^{-\left(\frac{w^2 + c w^2 v u}{(c v^2 + 2 c w) u^2}\right)} e^{-\frac{w^2}{\nu v h}} dw du$, $c \triangleq P_d \sigma_d \mu \gamma_{\text{th}}$, $\beta \triangleq \sqrt{\frac{4 h^2}{c v^2}}$, $K_1(\cdot)$ is the first-order modified Bessel function of the second kind [34] Eq. (8.432)], and AHS represents an approximation in the region of high SINR values.

**Proof:** A proof is provided in Appendix A.

**Proposition 2:** The outage probability achieved by the optimal relay is given by

$$P_{out} = 1 - \frac{1}{\lambda_f \lambda_h} \int_{0}^{\frac{1}{\lambda_f}} \left(\frac{a w + b \sqrt{w} + c}{(P_d \sigma_d)^2 \lambda_f \lambda_h} + \frac{1}{\lambda_f} \right) e^{-\frac{w}{\nu v h}} dw$$

$$\approx 1 - e^{-\frac{w}{\nu v h}} (\rho - \frac{w}{\nu v h})^\gamma, \hspace{1cm} \text{(AHS)}$$

where $\alpha \triangleq P_d \sigma_d \mu \gamma_{\text{th}}(1+\mu w)$, $b \triangleq \int_{0}^{\nu v h} \int_{u=0}^{\mu v h} e^{-\left(\frac{w^2 + c w^2 v u}{(c v^2 + 2 c w) u^2}\right)} e^{-\frac{w^2}{\nu v h}} dw du$, $c \triangleq 2 \int_{0}^{\nu v h} \int_{u=0}^{\mu v h} e^{-\left(\frac{w^2 + c w^2 v u}{(c v^2 + 2 c w) u^2}\right)} e^{-\frac{w^2}{\nu v h}} dw du$, $\beta \triangleq \lambda_f (\alpha + c v^2) - 2 \lambda_f P_d \sigma_d \mu \gamma_{\text{th}}$, and $E_i(\cdot)$ is the exponential integral function [34] Eq. (8.214)].

**Proof:** A proof is provided in Appendix B.

**Proposition 3:** The outage probability achieved by the target relay is given by

$$P_{out} = 1 - \frac{1}{\lambda_f \lambda_h} \int_{w=0}^{\infty} \int_{u=0}^{\infty} e^{-\left(\frac{w^2}{\nu v h} + \frac{u^2}{\nu v h}\right)} e^{-\frac{w}{\nu v h}} dw du$$

$$\times e^{-\frac{u}{\nu v h}} \left(1 + \omega e^\omega (\text{Chi}(\omega) - \text{Shi}(\omega))\right), \hspace{1cm} \text{(AHS)} \hspace{1cm} (27b)$$

where $a \triangleq \int_{0}^{\nu v h} \int_{u=0}^{\mu v h} e^{-\left(\frac{w^2 + c w^2 v u}{(c v^2 + 2 c w) u^2}\right)} e^{-\frac{w^2}{\nu v h}} dw du$, $b \triangleq \int_{0}^{\nu v h} \int_{u=0}^{\mu v h} e^{-\left(\frac{w^2 + c w^2 v u}{(c v^2 + 2 c w) u^2}\right)} e^{-\frac{w^2}{\nu v h}} dw du$, $c \triangleq \gamma_{\text{th}} \sqrt{P_d \gamma_{\text{th}}(1+\gamma_{\text{th}})}$, $d \triangleq P_d \gamma_{\text{th}}$, $\gamma \triangleq \sqrt{P_d \gamma_{\text{th}} + \int_{0}^{\nu v h} \int_{u=0}^{\mu v h} e^{-\left(\frac{w^2 + c w^2 v u}{(c v^2 + 2 c w) u^2}\right)} e^{-\frac{w^2}{\nu v h}} dw du}$, and $\text{Shi}(\cdot)$ and $\text{Chi}(\cdot)$ are the hyperbolic sine integral function and hyperbolic cosine integral function, respectively [34] Eq. (8.221)].

**Proof:** A proof of this proposition is similar to the proof of Proposition 2.

When the maximum relay is employed, the optimized TS factor can be obtained by solving

$$\alpha^* = \arg \max_{\alpha} (1-\alpha)(1-P_{out})R$$

subject to $0 < \alpha < 1$. \hspace{1cm} (28)

Given the complicated expression for the derivative of $(1-\alpha)(1-P_{out})R$, the optimized $\alpha^*$ is also obtained numerically. Since $P_{out}$ in Proposition 1 contains only the statistical CSI, the instantaneous CSI has not been utilized to optimize the TS factor for the maximum relay.

Although the TS factor does not appear directly in the expressions for $P_{out}$ in Proposition 2 and Proposition 3, the expressions in (20) and (23) for the TS factors have been employed in the derivation of the outage probabilities of the optimal relay and target relay. Moreover, $\alpha_{\text{opt}}$ and $\alpha_{\text{tar}}$ are computed with the instantaneous CSI. Thus, the average throughput of the FDR system employing the optimal relay and target relay can be evaluated by

$$R_{DL}(\hat{\alpha}) = (1-\hat{\alpha})(1-P_{out})R, \hspace{1cm} (29)$$

where $\hat{\alpha} = \text{E}(\alpha_{\text{opt}})$ (or $\hat{\alpha} = \text{E}(\alpha_{\text{tar}})$) when the optimal relay (or the target relay) is employed and the expectation value is obtained by the simulations. Moreover, when $\hat{\gamma} \geq \gamma_{\text{SR}}$, information relaying fails for the target relay and the corresponding
contribution to the average throughput should be set as zero. Since (29) involves both \( \hat{\alpha} \) and the analytical expression for \( P_{\text{out}} \), it is a quasi-analytic method.

C. Delay-tolerant Transmission

In delay-tolerant transmission, the codeword length is very large compared with the channel block time, so that the codeword could experience all possible realizations of the channel. Thus, the ergodic capacity becomes a measure in determining the throughput and the source can transmit at any rate upper-bounded by the ergodic capacity.

**Proposition 4:** The ergodic capacity achieved by the maximum relay is given by

\[
C_E = \frac{1}{\ln 2} \int_0^1 \int what is the Meijer G-function [34], Eq. (9.301).

**Proof:** A proof is provided in Appendix C.

The expression in Proposition 4 involves a triple integral. To facilitate further processing, the following upper bound on the ergodic capacity is presented.

**Corollary 1:** The ergodic capacity achieved by the maximum relay is upper-bounded by

\[
C_E^{\text{up}} = \frac{1}{\ln 2} \int_0^1 \int \int \cdots
\]

where \( \xi(v, w, z) = \log_2 \left( 1 + \frac{P_z + \int \frac{d\sigma}{\sigma} \int \frac{d\gamma}{\gamma} \xi(v, w, z)dzdv, (30) \right) \)

and \( G_{p,q}^{m,n}(x) \) is the Meijer G-function [36].

**Proof:** A proof is similar to the proof of Corollary 1.

For the target relay, the effective EH and relaying transmission fail when \( \hat{\gamma} \geq \gamma_{SR} \). In this case, the e-SINR \( \gamma_{tar} \) does not exist because of an impractical \( \mu_{tar} \). Although the derivative of \( P_{out} \) in Proposition 3 can still be obtained by the mathematical manipulation, it cannot be used to represent the PDF of \( \gamma_{tar} \) because of the discontinuity of \( \gamma_{tar} \). Therefore, finding the PDF of \( \gamma_{tar} \) to evaluate \( \mathbb{E}(\log_2(1 + \gamma_{tar})) \) is difficult. As an alternative, the ergodic capacity achieved by the target relay can be expressed by

\[
C_E = \mathbb{E}(\log_2(1 + \gamma_{tar})) - \mathbb{E}(\log_2(1 + \gamma_{tar,2})),
\]

where \( \gamma_{tar,1} = \frac{P_x}{d_t \mu_{SR}^2 \sigma_\gamma^2} + \frac{P_x}{d_t \mu_{SR}^2 \sigma_\gamma^2} + \frac{P_x}{d_t \mu_{SR}^2 \sigma_\gamma^2} + \frac{P_x}{d_t \mu_{SR}^2 \sigma_\gamma^2} \), and \( \gamma_{tar,2} \) is the gamma distribution [36].

**Proposition 6:** The PDF of \( \gamma_{tar} \) in [27] is provided by.

**Proof:** A proof is similar to the proof of Corollary 1.

**Proposition 5:** The ergodic capacity achieved by the optimal relay is given by

\[
C_E = - \frac{1}{\lambda_f \lambda_h} \int_0^1 \int \frac{d\gamma}{\lambda_f \lambda_h} \int \frac{d\sigma}{\sigma} \left( \frac{e^{\sigma \gamma} + e^{\sigma \gamma}}{\gamma_{a \gamma, \gamma}^{-1} + \gamma_{a \gamma, \gamma}^{-1}} \right) \times e^{\gamma \gamma} \log_2(1 + \gamma) dz d\gamma
\]

**Proof:** A proof is similar to the proof of Corollary 1.
By employing the bounds presented in Proposition 6, the ergodic capacity achieved by the target relay can be approximated by

$$C_E \approx \log_2 \left( 1 + \frac{P_r}{d_1^2 \sigma_d^2 + \sigma_d^2} e^u + \frac{P_r^2 (1+\gamma - \sqrt{\gamma (1+\gamma)})}{d_1^2 d_2 \gamma \sigma_d^2} e^v \right).$$

(37)

Although the above expression is rough-bounded, we will see that its changing trend is coincident with the simulation results.

When the maximum relay is employed, the optimized $\alpha^*$ can be obtained by solving the following problem:

$$\alpha^* = \arg \max (1 - \alpha) C_E$$

subject to $0 < \alpha < 1$. 

(38)

Also, only the statistical CSI has been utilized in optimizing the TS factor for the maximum relay. When the optimal relay and target relay are employed, the quasi-analytic method is applied to evaluate the throughput by

$$R_{DT}(\tilde{\alpha}) = (1 - \tilde{\alpha}) C_E.$$

(39)

V. NUMERICAL RESULTS

This section presents some numerical results to validate the analytical expressions developed in the previous section and discuss the throughputs for the considered relay control schemes. Unless otherwise stated, the source transmission rate is set to be $R = 3$ bps/Hz and the e-SINR threshold causing outage is given by $\gamma_{th} = 2^R - 1 = 7$. The path loss exponent is set to be $m = 3$, whereas the distance $d_1$ and $d_2$ are normalized to unit value. The energy harvesting efficiency is set to be $\eta = 0.4$. The means of the dual-hop channel gains are set as $\lambda_h = \lambda_g = 1$. The source transmit SNR is defined as $\text{SNR} \triangleq P_s/\sigma_d^2$. For the loop interference channel, the instantaneous channel INR and average channel INR are defined as $\gamma_{LI} \triangleq |h|^2/\sigma_d^2$ and $\bar{\gamma}_{LI} \triangleq \lambda_h/\sigma_d^2$, respectively. For simplicity, similar noise variances are assumed at the relay and destination, i.e., $\sigma_d^2 = \sigma_d^2 = \sigma^2$.

Fig. 2 shows the impact of the TS factor on the instantaneous throughput. In Fig.2(a), we focus on a single frame with the following channel setting: $|h|^2 = 1.898$, $|g|^2 = 0.986$, and $|f|^2 = 1.3368$. When SNR = 15 dB, 25 dB, and 35 dB, the target e-SINRs for the target relay are set by $\hat{\gamma} = 8$ dB, 12 dB, and 14 dB, respectively. As observed, the TS factor decreases for all the considered relay control schemes when SNR increases. Although the optimal relay does not achieve the maximum throughput, its throughput is very close to the maximum throughput of the maximum relay. The target relay also achieves a competitive throughput. In addition, $\alpha_{opt}$ moves toward to the optimized $\alpha^*$ when SNR increases. When SNR decreases, both $\alpha_{opt}$ and the optimized $\alpha^*$ increase, in which $\alpha_{opt}$ increases more quickly than the optimized $\alpha^*$. In such a case, the optimal relay results in a shorter relaying transmission time than that of the maximum relay employing the optimized $\alpha^*$. In Fig.2(b), we focus on a single frame with the following setting: $|h|^2 = 1.898$, $|g|^2 = 0.986$, $|f|^2 = \gamma_{LI}\sigma^2$, and SNR = 30 dB. The numerical results in Fig.2(b) also verify that a competitive throughput can be achieved by the optimal relay. Moreover, $\alpha_{opt}$ decreases when

![Fig. 2. Instantaneous throughput versus $\gamma_{LI}$.](image-url)
γ_{LI} increases, which implies that a relatively long relaying transmission time is achieved for the optimal relay in the region of high γ_{LI}. In the rest of the simulations, all the throughputs for the maximum relay are obtained with the numerically optimized α∗.

Fig. 3 illustrates the impacts of SNR and γ_{LI} on the instantaneous throughput. In Fig. 3(a), we set SNR = 35 dB and ˆγ = 22. As a result, the optimal relay achieves the same throughput as that of the maximum relay in the region of high γ_{LI}. However, in the region of low γ_{LI}, the throughput of the optimal relay is worse than that of the maximum relay. This result is coincident with the fact that (1 − α_{opt})T becomes small when γ_{LI} decreases, as previously shown in Fig. 2(b). Fig. 3(a) also shows that the throughput of the target relay can not only catch up but also surpass that of the optimal relay. Thus, the target relay is more suitable than the optimal relay in scenarios for which the channel knowledge γ_{RD} is unavailable. The curves of instantaneous throughput versus SNR are plotted in Fig. 3(b), where we fixed γ_{LI} = 35 dB. In the entire SNR region, the optimal relay achieves almost the same throughput as that of the maximum relay. Given that the optimized α∗ is numerically obtained while α_{opt} has a closed-form expression, the optimal relay is more preferable than the maximum relay in the region of high γ_{LI} for all SNRs.

Fig. 4 displays the impact of ˆγ on the instantaneous throughput for the target relay. The two-hop channel gains are set by |h|2 = 1.898 and |g|2 = 0.986, whereas the loop interference channel is set by |f|2 = γ_{LI}σ2_{g}. The maximum throughput of the target relay is always achieved with the target e-SINRs satisfying ˆγ < γ_{SR}. For example, when SNR = 35 dB and γ_{LI} = 25 dB, the channel SNR of the first-hop link is γ_{SR} = 37.78 dB, whereas the target e-SINR that achieves the maximum throughput is ˆγ = 23 dB, which satisfies ˆγ < γ_{SR}. However, an approximately 2 bps/Hz throughput decrease happens when ˆγ moves by approximately 7.5 dB away from ˆγ = 23 dB. Thus, the target relay is unsuitable for scenarios in which the priority is throughput maximization.

Fig. 5 examines the analytical results for the outage probability versus SNR. In the simulation, 20,000 random channel realizations are applied. Both the analytical and the analytical approximation (defined in figure as “Anal. Approx.”) expressions for the outage probability are evaluated. As shown in Fig. 5, the analytical and simulation results match in the entire SNR region with the fixed ˆγ_{LI}. The results verify the analytical
expressions for $P_{\text{out}}$ presented in Propositions 1, 2, and 3. Fig. 5 also shows that the optimal relay always has an equal or lower outage probability than the maximum relay, whereas the target relay achieves a better outage probability performance than the maximum relay in the region of high SNR. The reason for this observation is that a higher SNR results in a higher $\gamma_{SR}$, which in turn leads to a larger upper bound of $\hat{\gamma} (\hat{\gamma} < \gamma_{SR})$.

Fig. 6 shows the analytical results for the outage probability versus $\gamma_{LI}$. For the target relay, the target e-SINR is set as $\hat{\gamma} = 8$ dB. In the region of high $\gamma_{LI}$, the analytical and simulation results for all the relay control schemes match well. However, in the region of low $\gamma_{LI}$, the deviation between the analytical and simulation results happens for both the optimal relay and target relay. In practice, FDR systems in most cases encounter a high level of loop interference, which validates the effectiveness in applying the derived analytical expressions.

Fig. 7 examines the analytical results for the ergodic capacity versus SNR. For the target relay, the target e-SINR is set as $\hat{\gamma} = 15$ dB. For $C_E$ of the maximum relay and optimal relay, the analytical expressions match well with the simulation results. Fig. 7 also verifies the effectiveness of the upper bound on $C_E$ for the maximum relay. For $C_E$ of the target relay, although a deviation is observed between the bounded approximation of (37) and the simulation results, their changing trends are coincident. Moreover, the ergodic capacity achieved by the optimal relay is higher than that of the maximum relay, whereas the ergodic capacity achieved by the target relay is higher than that of the maximum relay in the SNR region corresponding to the target ergodic capacity. For example, the target ergodic capacity is $E\{\log_2(1+\hat{\gamma})\} \approx 5.03$ bps/Hz when $\hat{\gamma} = 15$ dB (linear $\hat{\gamma} \approx 31.62$). When the ergodic capacity is 5.03 bps/Hz and $\gamma_{LI} = 15$ dB, the target relay achieves a gain of approximately 6 dB SNR relative to the maximum relay.

Fig. 8 examines the ergodic capacity versus $\gamma_{LI}$. For the target relay, the target e-SINR is set as $\hat{\gamma} = 8$ dB. The analytical expressions match well with the simulation results. The achieved ergodic capacities for the considered relay control schemes decrease when $\gamma_{LI}$ increases. Although the upper bound on $C_E$ for the maximum relay is higher than $C_E$ for the optimal relay in the region of high $\gamma_{LI}$, $C_E$ for the maximum relay is always lower than that of the optimal relay, since the e-SINR for the maximum relay is always smaller than that of the optimal relay. Similar to Fig. 7, Fig. 8 also shows that
the bounded approximation of $C_E$ for the target relay can not match well with the simulation result, whereas their changing trends are coincident.

Fig. 9 illustrates the throughput of different transmissions. In the evaluation of Fig. 9, we set $\bar{\gamma}_{\text{LI}} = 45$ dB and $\bar{\gamma} = 22$ dB. Both the maximum relay and optimal relay achieve the largest instantaneous throughput. Owing to an available closed-form TS factor, the optimal relay appears to be more preferable than the maximum relay in instantaneous transmission. The optimal relay in delay-tolerant transmission also achieves the largest throughput, whereas the maximum relay and target relay do not. Moreover, Fig. 9 shows that the throughput for the target relay is larger than that for the maximum relay in certain SNR regions. As expected, the throughput in delay-limited transmission is upper-bounded by the constant transmission rate $R = 3$ bps/Hz. We also observe that in delay-limited transmission, the throughput for the optimal relay is larger than that for the maximum relay. The curves of throughput versus $\bar{\gamma}_{\text{LI}}$ for the considered three relay control schemes in delay-limited and delay-tolerant transmissions are plotted in Fig. 10, in which we set SNR = 35 dB and $\bar{\gamma} = 15$ dB. The throughput for the optimal relay is larger than that for the maximum relay in the region of high $\bar{\gamma}_{\text{LI}}$. However, in the region of low $\bar{\gamma}_{\text{LI}}$, the throughput for the optimal relay is lower than that for the maximum relay. Although the optimal relay achieves a better outage probability and ergodic capacity, the throughput is also affected by the relaying transmission time $(1 - \alpha)T$. Recalling from Fig. 2(b) that $\alpha_{\text{opt}}$ is larger than $\alpha^*$, it can be concluded that the relaying transmission time for the optimal relay is smaller than that for the maximum relay. As a result, the optimal relay achieves a lower throughput in the region of low $\bar{\gamma}_{\text{LI}}$. Fig. 10 also shows that the largest throughput achieved by the target relay is competitive to that of the maximum relay. Given that a small $\alpha_{\text{tar}}$ can be obtained with higher SNRs and higher $\bar{\gamma}_{\text{LI}}$ (recall the results in Fig. 2), an interested target throughput could be obtained by simply tuning the target e-SINR $\bar{\gamma}$.

Fig. 11 illustrates the throughput versus the relay location. In the evaluation, we set SNR = 50 dB, $\bar{\gamma}_{\text{LI}} = 35$ dB, $d_1 + d_2 = 2$, and $d_1$ varies from 0.1 to 0.9. As shown in Fig. 11(a), in delay-tolerant transmission, the throughput for the maximum relay and optimal relay decrease initially and then increase with the increase of $d_1$. For the maximum relay and optimal relay, the lowest throughput is obtained when the relay is placed in the middle of the source and destination because the maximum e-SINR can be achieved only when the channel SNRs of the first and second hops are balanced, i.e., $d_1$ and $d_2$ are equal. When the maximum relay and optimal relay are employed, for any two relay nodes whose positions are symmetric about the middle of the source and destination, the throughput of the relay placed near the source is higher than that of the relay placed near the destination because a longer distance between the source and relay entails a less relay-harvested energy due to the path loss effect. When the target e-SINR is quite smaller than $\gamma_{\text{SR}}$, the throughput achieved by the target relay is a constant irrespective of $d_1$. The reason for this scenario is that $\gamma_{\text{SR}}$ is high enough such that $\gamma_{\text{tar}} = \bar{\gamma}$ can be exactly achieved with sufficient EH irrespective of $d_1$. However, at a higher $\gamma$, a long distance $d_1$ results in insufficient EH, and thus the throughput decreases. For a higher $\bar{\gamma}$ closer to $\gamma_{\text{SR}}$, the throughput for the target relay decreases dramatically. The results in Fig. 11(b) also verify that the throughput for the target relay decreases dramatically for a higher $\bar{\gamma}$ when the relay moves from the source toward the destination. For the maximum relay and optimal relay in delay-limited transmission, relative worse throughputs are also obtained by the relay placed in the middle of the source and relay. In instantaneous transmission, similar phenomena happen for all of the considered relay control schemes (results are not provided).

VI. Conclusion

This study has investigated wireless information and energy transfer for AF FDR systems, in which the unavoidable high loop interference degrades the system performance. Three relay control schemes, namely, maximum relay, optimal relay, and target relay have been investigated in different transmissions. Analytical expressions for the outage probability and ergodic capacity have been derived for the considered relay control schemes in delay-limited and delay-tolerant transmissions. For the optimal relay and target relay, the TS factor, the optimal relay appears to be more preferable than the maximum relay in instantaneous transmission. The optimal relay in delay-tolerant transmission also achieves the largest throughput, whereas the maximum relay and target relay do not. Moreover, Fig. 9 shows that the throughput for the target relay is larger than that for the maximum relay in certain SNR regions. As expected, the throughput in delay-limited transmission is upper-bounded by the constant transmission rate $R = 3$ bps/Hz. We also observe that in delay-limited transmission, the throughput for the optimal relay is larger than that for the maximum relay.

The curves of throughput versus $\bar{\gamma}_{\text{LI}}$ for the considered three relay control schemes in delay-limited and delay-tolerant transmissions are plotted in Fig. 10, in which we set SNR = 35 dB and $\bar{\gamma} = 15$ dB. The throughput for the optimal relay is larger than that for the maximum relay in the region of high $\bar{\gamma}_{\text{LI}}$. However, in the region of low $\bar{\gamma}_{\text{LI}}$, the throughput for the optimal relay is lower than that for the maximum relay. Although the optimal relay achieves a better outage probability and ergodic capacity, the throughput is also affected by the relaying transmission time $(1 - \alpha)T$. Recalling from Fig. 2(b) that $\alpha_{\text{opt}}$ is larger than $\alpha^*$, it can be concluded that the relaying transmission time for the optimal relay is smaller than that for the maximum relay. As a result, the optimal relay achieves a lower throughput in the region of low $\bar{\gamma}_{\text{LI}}$. Fig. 10 also shows that the largest throughput achieved by the target relay is competitive to that of the maximum relay. Given that a small $\alpha_{\text{tar}}$ can be obtained with higher SNRs and higher $\bar{\gamma}_{\text{LI}}$ (recall the results in Fig. 2), an interested target throughput could be obtained by simply tuning the target e-SINR $\bar{\gamma}$.

Fig. 11 illustrates the throughput versus the relay location. In the evaluation, we set SNR = 50 dB, $\bar{\gamma}_{\text{LI}} = 35$ dB, $d_1 + d_2 = 2$, and $d_1$ varies from 0.1 to 0.9. As shown in Fig. 11(a), in delay-tolerant transmission, the throughput for the maximum relay and optimal relay decrease initially and then increase with the increase of $d_1$. For the maximum relay and optimal relay, the lowest throughput is obtained when the relay is placed in the middle of the source and destination because the maximum e-SINR can be achieved only when the channel SNRs of the first and second hops are balanced, i.e., $d_1$ and $d_2$ are equal. When the maximum relay and optimal relay are employed, for any two relay nodes whose positions are symmetric about the middle of the source and destination, the throughput of the relay placed near the source is higher than that of the relay placed near the destination because a longer distance between the source and relay entails a less relay-harvested energy due to the path loss effect. When the target e-SINR is quite smaller than $\gamma_{\text{SR}}$, the throughput achieved by the target relay is a constant irrespective of $d_1$. The reason for this scenario is that $\gamma_{\text{SR}}$ is high enough such that $\gamma_{\text{tar}} = \bar{\gamma}$ can be exactly achieved with sufficient EH irrespective of $d_1$. However, at a higher $\gamma$, a long distance $d_1$ results in insufficient EH, and thus the throughput decreases. For a higher $\bar{\gamma}$ closer to $\gamma_{\text{SR}}$, the throughput for the target relay decreases dramatically. The results in Fig. 11(b) also verify that the throughput for the target relay decreases dramatically for a higher $\bar{\gamma}$ when the relay moves from the source toward the destination. For the maximum relay and optimal relay in delay-limited transmission, relative worse throughputs are also obtained by the relay placed in the middle of the source and relay. In instantaneous transmission, similar phenomena happen for all of the considered relay control schemes (results are not provided).

APPENDIX A: A PROOF OF PROPOSITION 1

Substituting (17) into $P_{\text{out}} = \Pr(\gamma_{\text{max}} < \gamma_{\text{th}})$, the outage probability is given by

$$P_{\text{out}} = \Pr\left(\frac{\gamma_{\text{SR}} + (\mu_{\gamma_{\text{RD}}} f + 1)(\mu_{\gamma_{\text{RD}}} + 1)}{\gamma_{\text{SR}} + (\mu_{\gamma_{\text{RD}}} f + 1)(\mu_{\gamma_{\text{RD}}} + 1)} < \gamma_{\text{th}}\right) = \Pr\left(|g|^2 < \frac{P_s d_1^m d_2^m \sigma_2^2 \gamma_{\text{tar}}}{P_s d_1^m d_2^m \sigma_2^2 \gamma_{\text{SR}} (1 + |f|^2) |h|^2 + d_1^m d_2^m \sigma_2^2 \gamma_{\text{SR}} |h|^2} \right) = \Pr\left(|g|^2 < \frac{\bar{a} |h|^2 + b}{c |h|^2 + d} \right), \tag{A.1}\right.$$
always greater than a negative number, $P_{\text{out}}$ can be simplified as

$$P_{\text{out}} = \begin{cases} \Pr \left( |g|^2 < \frac{\bar{a}|h|^2+b}{\bar{c} \sqrt{1-d|h|^2}} \right), & |g|^2 < \frac{1}{\mu_{\gamma_h}} \text{ and } |h|^2 > \frac{\bar{a}}{\bar{c}} \\ 1, & |g|^2 < \frac{1}{\mu_{\gamma_h}} \text{ and } |h|^2 < \frac{\bar{a}}{\bar{c}} \\ 1, & |g|^2 > \frac{1}{\mu_{\gamma_h}} \text{ and } |h|^2 > 0 \end{cases} \quad (A.2)$$

Denote the probability density functions (PDFs) of the exponential random variable $|h|^2$ and $|f|^2$ by $f_{|h|^2}(z) \triangleq \frac{1}{\lambda_h} e^{-\frac{z}{\lambda_h}}$ and $f_{|f|^2}(z) \triangleq \frac{1}{\lambda_f} e^{-\frac{z}{\lambda_f}}$, respectively, where $\lambda_h$ and $\lambda_f$ are the means of $|h|^2$ and $|f|^2$, respectively. Denote the cumulative distribution function (CDF) of the exponential random variable $|g|^2$ by $F_{|g|^2}(z) \triangleq \Pr(|g|^2 < z) = 1 - e^{-\frac{z}{\lambda_g}}$, where $\lambda_g$ is the mean of $|g|^2$. Following (A.2), $P_{\text{out}}$ is given by

$$P_{\text{out}} = \int_{w=\frac{\bar{c}}{\bar{c} \lambda_h}}^{\infty} \int_{z=0}^{\frac{\lambda_f}{\lambda_h}} f_{|f|^2}(w) f_{|h|^2}(z) dzdw$$

$$+ \int_{w=\bar{a} \lambda_f}^{\infty} \int_{z=0}^{\frac{\lambda_f}{\lambda_h}} f_{|f|^2}(w) f_{|h|^2}(z) dzdw$$

$$+ \int_{w=0}^{\infty} \int_{z=\frac{\lambda_f}{\lambda_h}}^{\lambda_h} f_{|h|^2}(z) f_{|f|^2}(w) dzdw$$

$$= e^{-\frac{1}{\lambda_f \lambda_h}} + \frac{1}{\lambda_f \lambda_h} \int_{w=0}^{\frac{\lambda_f}{\lambda_h}} \left( \int_{z=0}^{\frac{\lambda_f}{\lambda_h}} f_{|f|^2}(w) dz \right)$$

$$+ \int_{z=\frac{\lambda_f}{\lambda_h}}^{\lambda_h} \left( 1 - e^{-\frac{\bar{a} \lambda_f + \bar{c} \lambda_h}{\bar{c} \lambda_h^2} \lambda_g} \right) dz$$

$$= 1 - \frac{1}{\lambda_f \lambda_h} \int_{w=0}^{\infty} e^{-\left( \frac{\lambda_f}{\lambda_h} + \frac{\lambda_f}{\lambda_h} \lambda_g \right) w} e^{-\bar{c} \sqrt{1-d|h|^2} \lambda_g} dw, \quad (A.3)$$

where $\bar{a} \triangleq P_s d_1^{m_1} d_2^{m_2} \sigma_d^{2 \gamma_{th}} (1+\mu_w)$ and $\bar{b} \triangleq P_s^{2} \mu (1-\gamma_{th}w)$. The analytical expression for $P_{\text{out}}$ in (A.3) cannot be further simplified. However, at the high SINRs, $\frac{\bar{a} \lambda_f + \bar{c} \lambda_h}{\bar{c} \lambda_h^2} \lambda_g$ has an approximation as $\frac{\bar{a} \lambda_f}{\bar{c} \lambda_h^2}$, so that $P_{\text{out}}$ can be approximated as

$$P_{\text{out}} \approx 1 - \frac{1}{\lambda_f \lambda_h} \int_{w=0}^{\infty} e^{-\left( \frac{\bar{a} \lambda_f}{\bar{c} \lambda_h^2} + \frac{\lambda_f}{\lambda_h} \lambda_g \right) w} e^{-\bar{c} \sqrt{1-d|h|^2} \lambda_g} dw$$

$$z^{\bar{a} \lambda_f - d} = 1 - \frac{1}{\lambda_f \lambda_h} \int_{w=0}^{\frac{\lambda_f}{\lambda_h}} \int_{z=0}^{\lambda_h} e^{-\left( \frac{\lambda_f}{\lambda_h} + \frac{\lambda_f}{\lambda_h} \lambda_g \right) w} e^{-\bar{c} \sqrt{1-d|h|^2} \lambda_g} dw$$

$$= 1 - \frac{1}{\lambda_f \lambda_h} \int_{w=0}^{\frac{\lambda_f}{\lambda_h}} \rho K_1(\rho) e^{-\frac{\lambda_f}{\lambda_h} \bar{c} \sqrt{1-d|h|^2} \lambda_g} dw, \quad (A.4)$$

where $\rho \triangleq \sqrt{\frac{4\bar{a}}{\bar{c} \lambda_h^2 \lambda_g}}$ and $K_1(\cdot)$ is the first-order modified Bessel function of the second kind [34 Eq. (8.432)]. The last equality in (A.4) is obtained by applying $\int_0^\infty e^{-\frac{\lambda_f}{\lambda_h} \bar{c} \sqrt{1-d|h|^2} \lambda_g} dw = \sqrt{\frac{2}{\lambda_f \lambda_h}} K_1(\sqrt{\frac{\lambda_f}{\lambda_h} \bar{c} \sqrt{1-d|h|^2} \lambda_g})$.

**APPENDIX B: A PROOF OF PROPOSITION 2**

Substituting (19) into $P_{\text{out}} = \Pr(\gamma_{\text{opt}} < \gamma_{\text{th}})$, the outage probability is given by

$$P_{\text{out}} = \Pr \left( \frac{\gamma_{\text{SR}} \gamma_{\text{RD}}}{\gamma_{\text{SR}}(\bar{b} \gamma_{\text{RD}} + \gamma_{\text{RD}} \gamma_{\text{SR}} + 1) \gamma_{\text{RD}}} < \gamma_{\text{th}} \right)$$

$$= \Pr \left( |g|^2 < \frac{\bar{a} \lambda_f + \bar{c} \lambda_h \sqrt{1-d|h|^2} \lambda_g}{\lambda_f \lambda_h} \right)$$

$$\frac{1}{\lambda_f \lambda_h} \int_{w=0}^{\frac{\lambda_f}{\lambda_h}} e^{-\left( \frac{\bar{a} \lambda_f}{\bar{c} \lambda_h^2} + \frac{\lambda_f}{\lambda_h} \lambda_g \right) w} e^{-\bar{c} \sqrt{1-d|h|^2} \lambda_g} dw$$

$$= 1 - \frac{1}{\lambda_f \lambda_h} \int_{w=0}^{\lambda_h} e^{-\left( \frac{\lambda_f}{\lambda_h} \lambda_g + \frac{\lambda_f}{\lambda_h} \lambda_g \right) w} e^{-\bar{c} \sqrt{1-d|h|^2} \lambda_g} dw, \quad (B.1)$$

where $\bar{a} \triangleq P_s d_1^{m_1} d_2^{m_2} \sigma_d^{2 \gamma_{th}} (1+\mu_w)$ and $\bar{b} \triangleq \frac{\bar{a} \lambda_f}{\bar{c} \lambda_h^2} \lambda_g$. The expression in (B.3) can be approximated by $b \approx 0$, $\bar{c} \sqrt{|h|^2} (P_s |h|^2 + d)$, and $(P_s |h|^2 - \gamma_{th}d)^2 \approx P_s^2 |h|^2 - \gamma_{th}d^2$, respectively, where $u \triangleq 2P_s d_1^{m_1} \sigma_d^{2 \gamma_{th}}$. Thus, $P_{\text{out}}$ in (B.1) can be approximated by

$$P_{\text{out}} \approx \begin{cases} \Pr \left( |g|^2 < \frac{\bar{a} \lambda_f + \bar{c} \lambda_h \sqrt{1-d|h|^2} \lambda_g}{\lambda_f \lambda_h} \right), & |h|^2 > \frac{2\bar{a} \lambda_f + \bar{c} \lambda_h \sqrt{1-d|h|^2} \lambda_g}{\lambda_f \lambda_h} \\ 1, & |h|^2 < \frac{2\bar{a} \lambda_f + \bar{c} \lambda_h \sqrt{1-d|h|^2} \lambda_g}{\lambda_f \lambda_h} \end{cases} \quad (B.3)$$
Following (B.3), and after some mathematical manipulations, $P_{\text{out}}$ can be written as

$$P_{\text{out}} \approx \int_{0}^{\infty} \int_{0}^{\infty} f_{f|f}(w) f_{h|w}(z) \, dz \, dw$$

$$+ \int_{0}^{\infty} \int_{0}^{\infty} f_{f|f}(w) f_{h|w}(z) \Pr \left( |g|^2 < \frac{\alpha+\sqrt{\gamma}}{P_h^2 |h|^2 - \alpha} \right) \, dz \, dw$$

$$= \frac{1}{\lambda_f \lambda_h} \int_{0}^{\infty} \int_{0}^{\infty} \left( \frac{\alpha+c \sqrt{\gamma}}{(P_h^2 + u) \lambda_h} + \frac{1}{\lambda_f} \right) e^{-\frac{x}{\lambda_h}} \, dx$$

$$= 1 - e^{-\frac{x}{\lambda_h}} - \left( \rho + \frac{\sigma\sqrt{\gamma}}{\lambda_f \lambda_h^2} \right) e^{\rho} E_i(-\rho - \frac{\sigma\sqrt{\gamma}}{\lambda_h}) . \tag{B.4}$$

where $\rho = \lambda_f (a+c \sqrt{\gamma}) - \lambda_h u$, $v = \frac{2 \sigma^2}{P_h^2 \lambda_h^2}$, and $E_i(\cdot)$ is the exponential integral function \cite{1} Eq. (8.214).

**APPENDIX C: A PROOF OF PROPOSITION 4**

The ergodic capacity $C_E = \mathbb{E}\{\log_2(1 + \gamma_{\text{max}})\}$ can be expressed as

$$C_E = \mathbb{E}\{\log_2(1 + \mu \gamma_{\text{RD}})\} + \mathbb{E}\{\log_2(1 + \gamma_{\text{SR}} + \mu \gamma_{\text{SR}}|f|^2)\}$$

$$- \mathbb{E}\{\log_2(1 + \gamma_{\text{SR}} + \mu \gamma_{\text{RD}} + \mu \gamma_{\text{SR}}|f|^2 + \mu^2 \gamma_{\text{SR}} \gamma_{\text{RD}}|f|^2)\} . \tag{C.1}$$

Define $x \triangleq |h|^2 |g|^2$ and $y \triangleq |h|^2 (1 + \mu |f|^2)$, where the PDFs of $x$ and $y$ are given by $f(x) = \frac{2}{\lambda_h \sqrt{\pi}} K_0 \left( 2 \sqrt{\lambda_h x} \right)$ and $f(y) = \frac{2}{\lambda_h \lambda_f \sqrt{\pi}} K_0 \left( 2 \sqrt{\lambda_h \lambda_f y} \right)$, respectively. Then, the first term can be evaluated as

$$\mathbb{E}\{\log_2(1 + \mu \gamma_{\text{RD}})\} = \frac{2}{\lambda_h \lambda_f \ln 2} \int_{0}^{\infty} \log \left( 1 + \frac{\mu \rho x}{d_1^2 \sigma_2^2} \right) K_0 \left( 2 \sqrt{\lambda_h x} \right) \, dx$$

$$= \frac{2}{\lambda_h \lambda_f \ln 2} \int_{0}^{\infty} G_{2.5}^{1.2} \left( \frac{\mu \rho x}{d_1^2 \sigma_2^2}, 1.1 \right) K_0 \left( 2 \sqrt{\lambda_h x} \right) \, dx$$

$$= \frac{1}{\ln 2} G_{4.2}^{1.4} \left( \frac{\mu \rho x}{d_1^2 \sigma_2^2}, 0.01, 1.1 \right). \tag{C.2}$$

In (C.2), we have used the relationship \cite{13} Eq. (8.4.6.5)] in the step (a) and the integral identity \cite{14} Eq. (7.821.3) in the step (b), respectively. Now, the second term can be shown similarly as

$$\mathbb{E}\{\log_2(1 + \gamma_{\text{SR}} + \mu \gamma_{\text{SR}}|f|^2)\} = \frac{1}{\ln 2} G_{4.2}^{1.4} \left( \frac{\mu \rho \lambda_h (1 + \mu \lambda_f)}{d_1^2 \sigma_2^2}, 0.01, 1.1 \right). \tag{C.3}$$

The third term can be evaluated as

$$\mathbb{E}\{\log_2(1 + \gamma_{\text{SR}} + \mu \gamma_{\text{RD}} + \mu \gamma_{\text{SR}}|f|^2 + \mu^2 \gamma_{\text{SR}} \gamma_{\text{RD}}|f|^2)\}$$

$$= \frac{1}{\lambda_h \lambda_f \lambda_f} \int_{0}^{\infty} \int_{0}^{\infty} e^{-\frac{x}{\lambda_h}} - \frac{\rho}{\lambda_h} - \frac{\sigma \sqrt{\gamma}}{\lambda_h} \xi(v, w, z) \, dz \, dw \, dv . \tag{C.4}$$

where $\xi(v, w, z) \triangleq \log_2 \left( 1 + \frac{P_s z}{d_1^2 \sigma_2^2} + \frac{\mu P_s z w}{d_1^2 \sigma_2^2} + \frac{\mu P_s z v}{d_1^2 \sigma_2^2} + \frac{\mu^2 P_s z w v}{d_1^2 \sigma_2^2} \right)$.

Then, the desired result follows immediately.

**APPENDIX D: A PROOF OF PROPOSITION 5**

According to (B.2), the CDF of $\gamma_{\text{opt}}$, $F_{\gamma_{\text{opt}}}(\gamma)$, is given by

$$F_{\gamma_{\text{opt}}}(\gamma) = 1 - \frac{1}{\lambda_f \lambda_h \sqrt{\gamma}} \int_{0}^{\infty} \left( \frac{z + b + c \sqrt{\gamma}}{(P_z - \gamma d^2 \lambda_h^2)^{\lambda_h}} + \frac{1}{\lambda_f} \right) e^{-\frac{\gamma}{\lambda_h}} \, dz . \tag{D.1}$$

where $a \triangleq P_s d_1^2 \sigma_2^2 \gamma^2 (1 + 2 \gamma)$, $b \triangleq P_s d_1^2 \sigma_2^2 \gamma^2 2$, and $c \triangleq 2 d_1^2 \sigma_2^2 \gamma^2 \sqrt{P_f \gamma} (1 + \gamma)$, and $d \triangleq \sqrt{\sigma^2 \gamma}$. Then, the PDF of $\gamma_{\text{opt}}$ can be computed by

$$f_{\gamma_{\text{opt}}}(\gamma) = \frac{\partial F_{\gamma_{\text{opt}}}(\gamma)}{\partial \gamma}$$

$$= \frac{1}{\lambda_f \lambda_h \sqrt{\gamma}} \int_{0}^{\infty} \frac{\partial}{\partial \gamma} \left( \frac{z + b + c \sqrt{\gamma}}{(P_z - \gamma d^2 \lambda_h^2)^{\lambda_h}} + \frac{1}{\lambda_f} \right) e^{-\frac{\gamma}{\lambda_h}} \, dz . \tag{D.2}$$

Using (D.2), the ergodic capacity $C_E = \mathbb{E}\{\log_2(1 + \gamma)\}$ can be expressed as

$$C_E = \int_{0}^{\infty} f_{\gamma_{\text{opt}}}(\gamma) \log_2(1 + \gamma) \, d\gamma$$

$$= \frac{1}{\lambda_f \lambda_h \sqrt{\gamma}} \int_{0}^{\infty} \int_{0}^{\infty} \frac{\partial}{\partial \gamma} \left( \frac{z + b + c \sqrt{\gamma}}{(P_z - \gamma d^2 \lambda_h^2)^{\lambda_h}} + \frac{1}{\lambda_f} \right) e^{-\frac{\gamma}{\lambda_h}} \, dz \, d\gamma . \tag{D.3}$$

The analytical result presented in (D.3) has a very large expression and its close-form can hardly be obtained. However, an approximation in the high SNIR region can be applied as in Appendix B. Following (B.3) and (B.4), the approximation of (D.1) can be expressed as

$$F_{\gamma_{\text{opt}}}(\gamma) \approx 1 - e^{-\frac{\gamma}{\lambda_h}} - \rho^\rho (\rho + \frac{\rho \sqrt{\gamma}}{\lambda_h}) E_i(-\rho - \frac{\rho \sqrt{\gamma}}{\lambda_h}) , \tag{D.4}$$

where $\rho = \frac{2 \sigma^2 \gamma}{P_h^2 \lambda_h}$ and $\rho = \lambda_f (a+c \sqrt{\gamma}) - \lambda_h u$. By evaluating the derivative of $F_{\gamma_{\text{opt}}}(\gamma)$ with respect to $\gamma$, the approximate PDF of $\gamma_{\text{opt}}$ can be written as

$$f_{\gamma_{\text{opt}}}(\gamma) = \frac{\rho}{\lambda_h} e^{-\rho \gamma} - \rho^\rho e^{-\rho \gamma}$$

$$- \left( \frac{\rho}{\lambda_h} + \frac{\rho \sqrt{\gamma}}{\lambda_h} \right) e^{-\rho \gamma} + \rho^\rho \gamma E_i(-\rho - \frac{\rho \sqrt{\gamma}}{\lambda_h}) , \tag{D.5}$$

where $\rho \triangleq \frac{1}{(1 + \gamma + \sqrt{(1 + \gamma)(\lambda_h \rho + \gamma))}}} \frac{1}{(1 + \gamma + \sqrt{(1 + \gamma)(\lambda_h \rho + \gamma))}}$ and $\rho \triangleq \frac{1}{(1 + \gamma + \sqrt{(1 + \gamma)(\lambda_h \rho + \gamma))}} \frac{1}{(1 + \gamma + \sqrt{(1 + \gamma)(\lambda_h \rho + \gamma))}}$. Then, the approximate ergodic capacity achieved by the optimal relay can be expressed as

$$C_E \approx \int_{0}^{\infty} \left( \frac{\rho}{\gamma \lambda_h} e^{-\rho \gamma} - \rho^\rho e^{-\rho \gamma} - \rho^\rho \gamma E_i(-\rho - \frac{\rho \sqrt{\gamma}}{\lambda_h}) \right) \log_2(1 + \gamma) \, d\gamma . \tag{D.6}$$
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