Matter-induced hadronic processes 1, 2

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Abstract. Two examples of “exotic” phenomena which become possible and important in the presence of nuclear matter are discussed: $\omega \to \pi\pi$ decay, and $\rho - \omega$ mixing. Significance of these processes for the low-mass dilepton production in relativistic heavy-ion collisions is indicated.

INTRODUCTION

An interesting factor brought in by the presence of the medium is that processes which are forbidden in the vacuum by symmetry principles now become possible. The constraints of Lorentz-invariance, $G$-parity, or isospin invariance, are no longer effective. Below we discuss two examples of such processes: $\omega \to \pi\pi$ decay, and $\rho - \omega$ mixing. Strictly speaking, $\rho$ and $\omega$ mix in the vacuum but this is a negligible effect caused by the small explicit breaking of the isospin symmetry. Similarly, the partial width for the decay $\omega \to \pi\pi$ is only $\sim 0.2\text{MeV}$ in the vacuum, which is again a tiny isospin-violation effect.

In this paper we summarize the results of Refs. [1–3] which extend the work presented in Refs. [4] and [5]. We emphasize that the matter-induced width for the $\omega \to \pi\pi$ decay is large: for $\omega$ moving with respect to the medium with a momentum above $\sim 200\text{MeV}$ the corresponding width, at the nuclear saturation density, is of the order of 100MeV. We also show that even a moderate excess of neutrons over protons in nuclear matter, such as in $^{208}\text{Pb}$, can lead to large $\rho - \omega$ mixing.

The in-medium broadening of the $\omega$ meson, as well as the shifts of the positions of the resonances (due to their mixing) are examples of the so-called in-medium modifications of hadron properties, which are predicted in a variety of theoretical calculations [6–19]. The recent interest in studying such modifications (for a review see [20]) has been triggered by the experimental observation of the enhanced

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production of low-mass dileptons in relativistic heavy-ion collisions [21,22]. The data are most easily described by the assumption that either the masses of vector mesons decrease in medium or their widths become larger.

The non-vanishing amplitude for the decay \( \omega \rightarrow \pi\pi \) indicates that the processes of pion annihilation into dilepton pairs in the \( \omega \) channel are also possible, as first pointed in Ref. [4]. However, due to the smallness of the \( \omega\gamma \) coupling they cannot compete with the annihilation occurring in the \( \rho \) channel [2,4]. Nevertheless, the large width of the \( \omega \) mesons should cause a depletion in their population. In our opinion such an effect should be included in simulations of heavy-ion collisions. In fact, the results of some recent transport calculations [28,29] show an excess in the dilepton yield at \( q^2 = m_{\omega}^2 \), attributed to the direct \( \omega \rightarrow e^+e^- \) decay. With an increased hadronic width of the \( \omega \) a better agreement with the data may be achieved.

The effects of the isospin asymmetry in nuclear matter for the \( \rho - \omega \) mixing were studied in Ref. [5] in the framework of the Walecka model. The results of Ref. [5] indicate that at asymmetries such as in \(^{208}\)Pb and at nuclear saturation density, the \( \rho \) and \( \omega \) mix with an angle of about \( \sim 2\% \). In our approach, performed on a broader footing, we show that the matter-induced \( \rho - \omega \) mixing can be in fact much larger. We expect that it may show up, among other medium-induced effects, in future high-accuracy relativistic heavy-ion collisions.

\( \omega \rightarrow \pi\pi \) DECAY IN NUCLEAR MATTER

Our calculation of the \( \omega \rightarrow \pi\pi \) width is done in the framework of an effective hadronic theory. Mesons (\( \omega, \sigma, \pi \)) interact with nucleons and \( \Delta(1232) \). We work to the leading order in nuclear density, hence only the diagrams shown in Fig. 1 are taken into account. The “bubble” diagram (a) was studied by Wolf, Friman, and Soyeur [4], who pointed out the significance of the \( \omega - \sigma \) mixing for the in-medium \( \omega \rightarrow \pi\pi \) decay. The “triangle” diagrams (equally important in any formal scheme) were taken into consideration in Ref. [1]. The complete set of diagrams (a-d) was included in Ref. [2].

The solid line in Fig. 1 denotes the in-medium nucleon propagator [23]

\[
G(k) = (k + M) \left[ \frac{1}{k^2 - M^2 + i\varepsilon} + \frac{i\pi}{E_k} \delta(k_0 - E_k)\theta(k_F - |k|) \right],
\]

(1)

where \( k \) is the nucleon four-momentum, \( M \) denotes the nucleon mass, \( E_k = \sqrt{M^2 + k^2} \), and \( k_F \) is the Fermi momentum. Diagram (a) involves the intermediate \( \sigma \)-meson propagator, which we take in the form

\[
G_\sigma(k) = \frac{1}{k^2 - m_\sigma^2 + i m_\sigma \Gamma_\sigma - \frac{1}{4} \Gamma_\sigma^2}.
\]

(2)

Here the mass and the width of the \( \sigma \) meson are chosen in such a way that they reproduce effectively the experimental \( \pi\pi \) scattering length at \( q^2 = m_\omega^2 = (780\text{MeV})^2 \),
which is the relevant kinematic point for the process at hand. From this fit we find $m_\sigma = 789\text{MeV}$ and $\Gamma_\sigma = 237\text{MeV}$. Note that $m_\omega$ and $m_\sigma$ are very close to each other, which enhances the amplitude obtained from diagram (a) \cite{4}.

The double line in diagrams (c-d) denotes the $\Delta$ propagator

$$G_{\Delta\alpha\beta}^{\alpha\beta}(k) = \frac{k + M_\Delta}{k^2 - M_\Delta^2 + i M_\Delta \Gamma_\Delta - \frac{1}{4} \Gamma_\Delta^2} \left[ -g^{\alpha\beta} + \frac{1}{3} \gamma^\alpha \gamma^\beta + \frac{2}{3} M_\Delta \gamma^\alpha k^\beta - \frac{\Gamma_\Delta}{3 M_\Delta} \gamma^\alpha \gamma^\beta \right].$$

(3)

This formula corresponds to the usual Rarita-Schwinger definition \cite{24,25} with the denominator modified in order to account for the finite width of the $\Delta$ resonance, $\Gamma_\Delta = 120\text{MeV}$.

We assume that the $\omega NN$ and $\omega \Delta \Delta$ vertices have the form which follows from the minimum-substitution prescription and vector-meson dominance applied to the nucleon and the Rarita-Schwinger \cite{24} Lagrangians:

$$V_{\omega NN}^\mu = g_\omega \gamma^\mu,$$

$$V_{\omega \Delta \Delta}^{\mu\alpha\beta} = g_\omega \left[ -\gamma^\mu g^{\alpha\beta} + g^{\alpha\mu} \gamma^\beta + g^{\beta\mu} \gamma^\alpha + \gamma^\alpha \gamma^\mu \gamma^\beta \right].$$

(4)

(5)

The results presented below do not qualitatively depend on the form of the coupling, as long as it remains strong. The coupling constant $g_\omega$ can be estimated from

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig1.png}
\caption{Diagrams contributing to the $\omega \to \pi \pi$ amplitude in nuclear medium. The incoming $\omega$ has momentum $q$ and polarization $\epsilon$. The outgoing pions have momenta $p$ and $q - p$. Diagrams (b-d) have corresponding crossed diagrams, not displayed.}
\end{figure}
the vector dominance model. We use $g_\omega = 9$. For the $\pi NN$ vertex we use the pseudoscalar coupling, with the coupling constant $g_{\pi NN} = 12.7$. The same value is used for $g_{\sigma NN}$. The $\sigma \pi \pi$ coupling constant is taken to be equal to $g_{\sigma \pi \pi} = 12.8 m_\pi$, where $m_\pi = 139.6 \text{MeV}$ is the physical pion mass (this value follows from the fit done to $\pi \pi$ scattering phase shifts done in Ref. [4]). The $\pi N\Delta$ vertex has the form $V_{\pi N\Delta}^\mu = (f_{\pi N\Delta}/m_\pi)p^\mu T$, where $p^\mu$ is the pion momentum, $T$ is the $\frac{1}{2} \rightarrow \frac{3}{2}$ isospin transition matrix, and the coupling constant $f_{\pi N\Delta} = 2.1$ [26].

The amplitude, evaluated according to the diagrams depicted in Fig. 1 (a-d) can be uniquely decomposed in the following Lorentz-invariant way:

$$\mathcal{M} = \epsilon^\mu (A p_\mu + B u_\mu + C q_\mu),$$

where $p$ is the four-momentum of one of the pions, $q$ is the four-momentum of the $\omega$ meson, $u$ is the four-velocity of nuclear matter, and $\epsilon$ specifies the polarization of $\omega$. Our calculation is performed in the rest frame of nuclear matter, where $u = (1, 0, 0, 0)$. In this reference frame the amplitude $\mathcal{M}$ vanishes for vanishing 3-momentum $q$, as requested by rotational invariance. Hence, the process $\omega \rightarrow \pi \pi$ occurs only when the $\omega$ moves with respect to the medium.

In Fig. 2 we present our numerical results at the nuclear saturation density, $\rho_B = \rho_0 = 0.17 \text{fm}^{-3}$. We show the width of longitudinally polarized $\omega$ mesons, $\Gamma^L$, as a function of $|q|$. In our calculation, we reduce the value of the in-medium nucleon mass to 70% of its vacuum value, $M^* = 0.7 M$, which is a typical number at the nuclear saturation density. We also reduce by the same factor the mass of the $\Delta$, i.e. $M^*_\Delta = 0.7 M_\Delta$, since it is expected to behave similarly to the nucleon.

![FIGURE 2](image-url)  
**FIGURE 2.** The in-medium width of the $\omega$ meson plotted as a function of its 3-momentum $|q|$. The labels (a), (a-b) and (a-d) refer to Fig.1. They indicate the diagrams included in the calculation.
The labels indicate which diagrams of Fig. 1 have been included. The complete result corresponds to the case (a-d). The case (a) reproduces the result of Ref. [4]. The width of the transverse modes (∼ 1 MeV) is negligible and we do not show it in the plot.

In Fig. 1 the mass of ω is kept at the vacuum value. For $m^*_ω = 0.70 m_ω$ our results decrease by a factor of 2.5. Still, the widths remain substantial and the effects discussed above are important.

**ρ − ω MIXING IN ASYMMETRIC NUCLEAR MATTER**

For vanishing 3-momentum $q$, the vector-meson correlator in the coupled $ρ^0$ and $ω$ channels has the following structure

$$Π^αβ(ν, q = 0) = \begin{pmatrix} Π_ρ^α(ν) & Π_ρ^β(ν) \\ Π_ω^α(ν) & Π_ω^β(ν) \end{pmatrix}, \quad (7)$$

where $ν$ is the energy variable. For the diagonal parts of (7) we choose a simple form which can mimic the results of various calculations of in-medium vector mesons:

$$Π_ν^α(ν) = Z_ν^{-1} ((ν - iΓ_ν^*/2)^2 - m_ν^{*2}) T^{αβ}, \quad ν = ρ, ω. \quad (8)$$

The asterisk denotes here the in-medium values of the resonance position, $m_ν$, width, $Γ_ν$, and the wave-function renormalization, $Z_ν$. In the case $q = 0$ the tensor $T^{αβ}$ has a simple form, $T^{αβ} = \text{diag}(0, 1, 1, 1)$ [9,23]. Our parameterization incorporates basic features of mesons propagating in nuclear medium, such as the shift of the resonance position, broadening, and wave-function renormalization.

Applying the same formalism [23] as in the previous Section, we find that the off-diagonal matrix element in Eq. (7), describing the mixing of $ρ^0$ and $ω$ (at $q = 0$) is given by expression

$$Π_ρ^α(ν) = -i \int \frac{d^4k}{(2π)^4} \left\{ \text{Tr}[V_ρ^α(ν)G_ρ^D(k^0 + ν, k)V_ω^β(-ν)G_ω^F(k)] - \text{Tr}[V_ρ^α(ν)G_ρ^n(k^0 + ν, k)V_ω^β(-ν)G_ω^n(k)] \right\} + (F \leftrightarrow D) \equiv T^{αβ}Π_ρ(ν), \quad (9)$$

where $G_ρ^n(k)$ and $G_ω^n(k)$ denote the density part and the free part of the Dirac propagator for the proton and neutron (compare our notation in Eq. (1)). The quantity

$$V_{ν,α} = g_ν \left( γ_α - \frac{κ_ν}{2M} σ_{αβ}σ^{βγ} \right) \quad (10)$$

is the vector-mesons nucleon vertex which includes the tensor coupling $κ$. Following Ref. [12] we use two parameter sets:
I: $g_p = 2.63, \kappa_\rho = 6.0, g_\omega = 10.1, \kappa_\omega = 0.12,$
II: $g_p = 2.72, \kappa_\rho = 3.7, g_\omega = 10.1, \kappa_\omega = 0.12.$

This parameterization follows from the vector meson dominance model [30]. The basic difference between the two sets is the value of $\kappa_\rho$ [31].

Explicit evaluation gives
\[
\Pi_{\rho\omega}(\nu) = \frac{2}{3} g_\rho g_\omega \int \frac{d^3 k}{(2\pi)^3 E_k^*} \frac{\theta(k_n - |k|) - \theta(k_p - |k|)}{\nu^2 - 4(E_k^*)^2} \times 
\left[ 8(E_k^*)^2 + 4M^* + 3(\kappa_\rho + \kappa_\omega) \frac{M^*}{M} \nu^2 + \kappa_\rho \kappa_\omega \frac{(E_k^*)^2 + 2M^*}{M^2} \nu^2 \right],
\]
\[ (11) \]
where $k_p$ and $k_n$ are the proton and neutron Fermi momenta and $M^*$ is the nucleon mass in medium. In symmetric matter, where $k_p = k_n = k_F$, the proton and neutron contribution to Eqs. (9) and (11) cancel, and $\Pi_{\rho\omega}(\nu)$ vanishes. In asymmetric matter $k_n > k_p$, and we get a net contribution to $\Pi_{\rho\omega}(\nu)$. We note that the proton and neutron densities are equal to $\rho_{p,n} = k_{p,n}/(3\pi^2)$, and the baryon density $\rho_B$ and the isospin asymmetry $x$ are equal to $\rho_B = \rho_p + \rho_n$ and $x = (\rho_n - \rho_p)/\rho_B$. At low $x$ it can be easily shown that $\Pi_{\rho\omega}(\nu)$ is linear in $x$. It remains linear for asymmetries accessible in heavy-ion collisions. If in addition we expand Eq. (11) at small $\rho_B$, we notice that $\Pi_{\rho\omega}(\nu) \sim x \rho_B = \rho_n - \rho_p$, in agreement with the low-density theorem for the scattering amplitude.

Finding the eigenvalues of the matrix (7) is equivalent to solving the following equation
\[
\text{Det} \left( \begin{array}{c}
(\nu - i\Gamma_\rho/2)^2 - m_\rho^2 \\
\sqrt{Z_\rho^* Z_\omega^*} \Pi_{\rho\omega}(\nu)
\end{array} \right) = 0.
\]
\[ (12) \]
Equation (12) yields eigenvalues $\nu_1$ and $\nu_2$, and the corresponding eigenstates $|1\rangle$ and $|2\rangle$. Our convention is that in the absence of mixing, i.e. for $x = 0$, we have $|1\rangle = |\rho\rangle$ and $|2\rangle = |\omega\rangle$. A commonly used measure of mixing of states is the mixing angle. Since the problem (12) is not hermitian, the eigenstates $|1\rangle$ and $|2\rangle$ are not orthogonal and we cannot define a single mixing angle. We find it useful to introduce two mixing angles, $\theta_1$ and $\theta_2$, through the relations
\[
|1\rangle = \cos \theta_1 |\rho\rangle + \sin \theta_1 |\omega\rangle, \quad |2\rangle = -\sin \theta_2 |\rho\rangle + \cos \theta_2 |\omega\rangle.
\]
\[ (13) \]
Since the matrix in (12) is complex, the mixing angles are also complex.

Our results are shown in Table I, which contains 8 representative cases for $\rho_B = 2\rho_0$. We assume that at this density $M^*/M = 0.5$. The table should be read from top to bottom. The first row labels the case. Five input rows contain $m_\rho^*, m_\omega^*, \Gamma_\omega^*, \Gamma_\rho^*$ and $\sqrt{Z_\rho^* Z_\omega^*}$ for symmetric matter of density $\rho_B$.

To summarize, we observe that the mixing effects are sizable for all sensible cases, with mixing angles of the order of $10^\circ$, or larger. As a consequence, the
resonance positions and widths of the vector mesons are shifted significantly. Our analysis shows also (see [3] for more details) that the mixing effect will continue to be important at moderate temperatures. Therefore we expect that our results of large $\rho$-$\omega$ mixing may show up, among other possible medium-induced effects, in future high-accuracy relativistic heavy-ion experiments. In particular, the results to be obtained with the HADES spectrometer at the SIS accelerator at GSI, whose anticipated mass resolution in the discussed region will reach 1% [32], should be influenced by the phenomenon of $\rho$-$\omega$ mixing.

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