New feature in the differential cross sections at 13 TeV be measured at the LHC

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Analysis of dσ/dt of the TOTEM Collaboration data, carried out without model assumptions, showed the existence of a new effect in the behavior of the hadron scattering amplitude at a small momentum transfer at a high confidence level. The quantitatively description of the data in the framework of the HEGS model support such phenomenon which can be connect with quark potentials at large distances.

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Research into the structure of the elastic hadron scattering amplitude at superhigh energies and small momentum transfer - t can give a connection between experimental knowledge and the basic asymptotic theorems, which are based on first principles[1]. It gives information about hadron interaction at large distances where the perturbative QCD does not work, and a new theory, as, for example, instantons or string theories, must be developed. There were many works in which the consequences of breaking the Pomeranchuk theorem were investigated[2]. It was shown[3] that if the Pomeranchuk theorem was broken and the scattering amplitude grew to a maximal possible extent, but not break the Froissart boundary, many zeros in the scattering amplitude should be available in the nearest domain of t → 0 in the limit s → ∞. Hence, with increasing energy of colliding beams, some new effects[4] in differential cross sections can be discovered at small t[5]. Such phenomena were explored in some works[6,7]. They had represent an explanation of some separate (individual) effects in the definite domain of momentum transfer and with a period proportional to \( \Delta t \). In the previous paper[8], it was shown “AKM oscillations” could exist in high-precision experimental data of the UA4/4 Collaboration. Now we examined this effect assuming the existence of the potential of hadron-hadron interactions at large distance and made a new quantitative treatment of experimental data at a high confidence level.

The standard fast falling potentials of the Gaussian type lead to the exponential dependence of the scattering amplitude in the range of small momentum transfer. During a long time there have been different attempts to find unusual behavior of the amplitude of the elastic hadron scattering. In[9], it was shown that peripheral contributions of the inelastic diffraction processes lead to the appearance in the elastic cross sections of large and small periodical structures over t. Note also, that in[10], was shown that the potential of the rigid string leads to oscillations of the pion elastic form factors at large distances (q = 10 - 20 fm\(^{-1}\)). In the case of cutting the potential, for example by the \( \theta \)-function, we shall get a strongly oscillating expression as t → 0 with the period, which is approximately proportional t, and with the amplitude, which is suppressed as t grows. As, example, note the potentials with peculiarities on the determined distance, considered in[11], for example

\[
gV(r) = g[1 - \exp(\mu(r - r_0))]^{-1},
\]

lead to similar behavior of the additional term. In contrast with the potential of the multi-gluon exchange there is no necessity to enter the additional cut of the potential. But on the other hand, generally speaking, we have to get eikonalization of the Born terms. At first, we need to calculate the eikonal and then the amplitude in the t-representation together with the leading eikonal. Numerical integration allows one to calculate an additional term in both the direct and eikonal approach. The results of calculations are shown in Fig.1.

The usual method of minimization \( \chi^2 \) in this situation often works poorly. On the one hand, we should define a certain model for part of the scattering amplitude having zeros in the domain of small t. However, this model may slightly differ from a real physical picture. On the other hand, the effect is rather small and gives an insignificant change in the sum of \( \chi^2 \). Therefore, in this work let us apply another method in first, namely, the method of comparison of two statistically independent choices, for example[12]. If we have two statistically independent choices \( x_n \) and \( x'_n \) of values of the quantity X distributed around a definite value of A with the standard error equal to \( 1 \), we can try to find the difference between \( x_n \) and \( x'_n \). For that we can compare the arithmetic mean of these choices

\[
\Delta X = (x_1 + x'_1 + ... x_{n1})/n1 - (x_1 + x'_2 + ... x_{n2})/n2 = x_{n1} - x_{n2}.
\]

The standard deviation for that case will be

\[
\delta_\Delta = [1/n1 + 1/n2]^{1/2}.
\]

And if \( \Delta X/\delta_\Delta \) is larger than 1, we can say that the difference between these two choices has the 99% probability.

The deviations \( \Delta R_t \) of experimental data from these theoretical cross sections we will be measured in units of experimental error for an appropriate point

\[
\Delta R_t = [(d\sigma/dt)_{exp} - (d\sigma/dt)_{th}] / \delta_{i}^{exp},
\]

where \( \delta_{i}^{exp} \) is an experimental error. To take this effect into account, we break the whole studied interval of momentum transfer into k equal pieces \( k\delta t = (t_{max} - t_{min}) / k \), where sum \( \Delta R_t \) separately over even and odd pieces. Thus, we receive two sums \( S^{up} \) and \( S^{dn} \) for \( n_1 \) even and \( n_2 \) odd interval. At this \( n_1 + n_2 = k \) and \( |n_1 - n_2| = 0 \).
or 1

\[ S^{up} = \sum_{j=1}^{n_1} \left( \sum_{i=1}^{N} \Delta R_i \right) |\delta q(2j-1) - \delta q(2j)|, \]

\[ S^{dn} = \sum_{j=1}^{n_2} \left( \sum_{i=1}^{N} \Delta R_i \right) |\delta q(2j) - \delta q(2j+1)|. \]  

In the case of some difference of experimental data from the theoretical behavior expected by us or incorrectly determined parameters, these two sums will deviate from zero; however, their sizes should coincide within experimental errors. However, this will be so in the case if experimental data have no any periodic structure or a sharp effect coincides with one interval. We assume that such a periodic structure is available and its period coincides with the chosen interval 2σ. In this case, the sum \( S^{up} \) will contain, say, all positive half-cycles; and the sum \( S^{dn} \), all negative half-cycles. The difference between \( S^{up} \) and \( S^{dn} \) will show the magnitude of an additional effect summed over the whole researched domain.

Our method does not require exact representation of the oscillatory part of the scattering amplitude, and now let us apply it to new LHC data of the TOTEM Collaboration at 13 TeV [13, 14]. It gives us two sets of the data: one \( 0.000879 < |t| < 0.201041 \) GeV\(^2\) includes the Coulomb-hadron interference range and the other \( 0.0384 < |t| < 3.822 \) GeV\(^2\) for the large \( t \). Both sets have the overlapping region. We find that three first points of the second set hardly differ from the data of the first set and we removed them. For the first analysis we do not include the region of the diffraction minimum as we try, using this new method, to examine some additional oscillation behaviour with minimum model assumption, but the region of the dip requires some model for the description of the elastic scattering in a wide region of \( t \).

Here and below we use only statistical errors, and systematic errors are taken into account as an additional normalization coefficient for both the sets. For the basic (non-oscillating) amplitude we use the standard exponential form with three slopes multiplied by \( t, t^{2}, t^{3} \). In Fig.2, such sums in eq.(4) are represented for the period that is proportional to \( \sqrt{-t} \). Then we move the segments by one-half the segment and calculate these sums again. The results are shown in Fig.2 by the long-dashed and dotted lines. The large difference between the first and second cases clearly shows the existence of some oscillation contributions in the scattering amplitude especially at large \( t \). Now let us calculate such sums with the period that is proportional \( t \). The results are shown in Fig.3 a,b. To evaluate the size of a possible effect, one should examine the difference of the arithmetic mean values \( \Delta S \) and the corresponding dispersion - \( \delta S \) [12]

\[ \Delta S = S^{up} - S^{dn}; \quad \delta S = (1/[1/n_1 + 1/n_2]^{1/2})/N. \]  

Let us calculate the sum of \( \Delta R_i \) and its arithmetic mean

\[ S_t = \sum_{i=1}^{325} \Delta R_i = 285; \quad \overline{S_t} = S_t/325 = 0.877 \pm 0.028. \]

Obviously, it is show the existence of the oscillatory contributions on high confidence level.

Now let us try to find the form of such additional oscillation contribution to the basic elastic scattering amplitude. As a basis, take our high energy generalized structure (HEGS) model [13, 18] which quantitatively describes, with only a few parameters, the differential cross section of \( pp \) and \( p\bar{p} \) from \( \sqrt{s} = 9 \) GeV up to 13 TeV, includes the Coulomb-hadron interference region and the high-\( |t| \) region up to \( |t| = 15 \) GeV\(^2\) and quantitatively well describes the energy dependence of the form of the diffraction minimum [17]. However, to avoid possible problems connected with the low-energy region, we consider here only the asymptotic variant of the model [18]. The total elastic amplitude in general receives five helicity contributions, but at high energy it is enough to write it as \( F(s, t) = F^{h}(s, t) + F^{m}(s, t)e^{\nu(s, t)} \), where \( F^{h}(s, t) \) comes from the strong interactions, \( F^{m}(s, t) \) from the

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig1.png}
\caption{The Born term of the amplitude, appropriate to expression (2), calculated at different \( r_0 \) (long-dashed line with \( r_0 = r_0^0 \) and hard line for \( r_0 = 5r_0^0 \) with \( \mu = 1 \) GeV; dashed line - for \( r_0 = 5r_0^0 \) with \( \mu = 0.1 \) GeV.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig2.png}
\caption{a) Left] Sums \( S^{up} \) and \( S^{dn} \) calculated with additional normalization \( n_i = 1.1 \) for \( q_0 = 0.039 \) GeV and \( \delta q = 0.000831 \) GeV (full and dashed lines); and for \( q_0 - \delta q/2 \) (long-dashed and dots lines).}
\end{figure}
FIG. 3: \(S^{up}\) and \(S^{dn}\) calculated with additional normalization \(n_i = 1.1\) for \(t_0 = 9 \times 10^{-3} \text{ GeV}^2\) and \(\delta t = 0.0498 \text{ GeV}^2\) (full and dashed lines); b) [bottom] the same for \(q_0 - \delta q/2\) (long-dashed and dots lines).

electromagnetic interactions and \(\varphi(s, t)\) is the interference phase factor between the electromagnetic and strong interactions \[19\]. The Born term of the elastic hadron amplitude at large energy can be written as a sum of two pomeron and odderon contributions,

\[
F_P(s, t) = s^{\alpha_0}(C_0 F_1^2(t) + C_0' A^2(t)) \ , \ (6)
\]
\[
F_0(s, t) = i s^{\alpha_0 + \frac{1}{4}}(C_0 + C_0' t/(1 - r_0^2 t)) A^2(t) . \ (7)
\]

All terms are supposed to have the same intercept \(\alpha_0 = 1 + \epsilon_0 = 1.11\), and the pomeron slope is fixed at \(\alpha' = 0.24 \text{ GeV}^{-2}\). The model takes into account the two hadron form factors \(F_1(t)\) and \(A(t)\), which correspond to the charge and matter distributions \[20\]. Both form factors are calculated as the first and second moments of the same Generalized Parton Distributions (GPDs). As a probe for the oscillatory function take

\[
f_{osc}(t) = h_{osc}(i + p_{osc}) J_1(\tau) / \tau ; \ \tau = \pi (\phi_0 - t)/t_0 , \ (8)
\]

here \(J_1(\tau)\) is the Bessel function of the first order. This form has only a few additional fitting parameters and allows one to represent a wide range of possible oscillation functions. After the fitting procedure, we obtain \(\chi^2/n.d.f. = 1.24\) (remember that we used only statistical errors). One should note that the last points of the second set above \(-t = 2.8 \text{ GeV}^2\) show an essentially different slope, and we removed them. The total number of experimental points of both sets equals 415. If we remove the oscillatory function, then \(\chi^2/n.d.f. = 2.7\), so an increase is more than two times. If we make a new fit without \(f_{osc}\), then \(\chi^2/n.d.f. = 2.5\) decreases but remains large. Our model calculations are represented in Fig.4(a,b,c). It can be seen that the model gives a beautiful description of the differential cross section in both the Coulomb-hadron interference region and the region of the diffraction dip.

To see the oscillations in the differential cross sections,
FIG. 5: $R\Delta th$ of eq.(9a) (the hard line) and $R\Delta exp$ eq.(9b) (the tiny line) [top - region of small $t$; bottom - large $t$] at $\sqrt{s} = 13$ TeV.

let us determine two values - one is pure theoretical and the other with experimental data

\begin{align*}
R\Delta th &= \frac{d\sigma/dt_{th0+osc} - d\sigma/dt_{th0}}{d\sigma/dt_{th0}}, \\
R\Delta Exp &= \frac{d\sigma/dt_{Exp} - d\sigma/dt_{th0}}{d\sigma/dt_{th0}}. 
\end{align*} \tag{9}

The corresponding values calculated from the fit of two sets of the TOTEM data at 13 TeV are presented in Fig.5. At small $t$ there is a large noise; however, the oscillation contributions can be seen. This corresponds to the small size of the $SuSd$ values in Fig.3; however, at large $-t > 0.1$ GeV$^2$ we can see that $R\Delta th$ is similar to the value $R\Delta exp$. The oscillation contribution is small; however, the noise of the background decreases at this $t$ and does not dump the oscillation part.

We show that HEGS model describe on quantitatively level the new experimental data at 13 TeV with taking into account only statistical errors. However, only adding the small oscillatory term allow obtain $\chi^2_{d.n.f.} = 1.25$. The phenomenon of oscillations of the elastic scattering amplitude will give us the important information about the behavior of the hadron interaction potential at large distances. We have shown the existence of such oscillations at the statistical level by three methods: a) the method of statistically independent selection; b) the comparison of the $\chi^2$ without oscillation ($\sum \chi^2 = 1140$) and with oscillation ($\sum \chi^2 = 515$); c) the comparison of $R\Delta th$ and $R\Delta exp$, Fig.5). All three methods show the presence of the oscillatory behavior. Very likely that such effects exist also in experimental data at essentially smaller energies \[23\] but, maybe, they have a more complicated form (with two different periods, for example). The phenomena of oscillations are also related to the asymptotic properties of the scattering amplitude. They can impact the determination of the sizes of the total cross sections, the ratio of the elastic to the total cross sections and the size of the $\rho(s, t)$ - the ratio of the real to imaginary part of the elastic scattering amplitude.

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