Analogue Casimir radiation using an optical parametric oscillator

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Abstract – We establish an explicit analogy between the dynamical Casimir effect and the photon emission of a thin non-linear crystal pumped inside a cavity. This allows us to propose a system based on a type-I optical parametric oscillator (OPO) to simulate a cavity oscillating in vacuum at optical frequencies. The resulting photon flux is expected to be more easily detectable than with a mechanical excitation of the mirrors. We conclude by comparing different theoretical predictions and suggest that our experimental proposal could help discriminate between them.

Any mirror placed in quantum vacuum experiences fluctuations of the vacuum radiation pressure. When moving with non-uniform acceleration, these fluctuations give rise to a dissipative force, opposing itself to the mirror’s motion [1]. As a counterpart, the mirror should emit photons into the free-field vacuum because of energy conservation. Although predicted 30 years ago in the seminal paper by Fulling and Davies, this so-called dynamical Casimir effect has not yet been observed experimentally, mainly because the order of magnitude of the predicted photon flux is very small.

Different attempts have been made in the past to render the dynamical Casimir effect observable, for instance by exploiting the resonant enhancement of radiation inside an oscillating cavity [2–5] or by amplifying the Casimir signal with a sample of superradiant ultracold alkalimetal atoms [6]. Another possibility consists in effectively simulating the displacement of a mirror, for example by rapidly changing or modulating the skin depth of a semiconductor. This has first been discussed in [7] and [8] for linear and non-linear acceleration, respectively and later on it has been implemented in an experimental proposal [9]. Another more recent paper has proposed the generation of Casimir radiation via the coupling of a qubit to a cavity [10]. Noteworthy, related effects based on the same physical phenomenon are the fibre-optical analogue of the event horizon where a light pulse mimics a moving medium [11] or the emission of acoustic Hawking radiation via phonon modulation in Bose-Einstein condensates [12].

In this letter, we propose to generate the analogue of Casimir radiation inside a type-I optical parametric oscillator (OPO). Such a device is commonly used in Quantum Optics [13], but here we intend to drive it in a specific Casimir-like emission regime. We first establish an analogy between photon emission of a coherently pumped non-linear crystal of type $\chi^{(2)}$ inside a cavity and the photon creation via the dynamical Casimir effect. By giving the equivalence relations between the parameters of mechanical motion and the characteristics of the crystal and pump field, we show that the non-linear interaction results in an apparent motion of the mirrors for the field. We give an analytical expression for the photon flux emitted by the model system and discuss orders of magnitude of this Casimir-like radiation, which we find to be easily achievable in a standard experiment.

This is due to the fact that our model system avoids mechanical motion, which is the limiting factor in the oscillating cavity proposals, but uses instead an apparent motion of the mirrors for the field. It rejoins insofar experimental proposals where the mechanical motion is replaced by the optical modulation of the mirrors skin depth [7–9].

Based on the above-mentioned analogy, we then establish a link between the recently introduced concept of time refraction [14,15] and the time varying refractive properties of the pumped crystal, leading to an alternative expression for the photon flux emitted by a cavity oscillating in vacuum. Such as many others, this expression results in an exponential growth of emitted photons at all times. This is not only the case when perfectly
reflecting mirrors are considered [5,16], but also when a damping coefficient $\gamma$ for the energy is introduced [15,17]. We compare these results with our predictions, obtained within the scattering approach where the fields’ transformations are directly evaluated on mirrors with finite reflection coefficients [2,3,18]. The latter procedure leads automatically to a stationary regime with a finite number of photons emitted inside the cavity and constant flux emitted outside. We show that the models predicting an exponential growth are only valid in the short time limit and lead back to our results when accounting for a detailed balance between photons emitted by the mirrors and photons leaving the cavity due to their finite reflectivity.

The model system that we will consider in analogy with dynamical Casimir radiation is a type-I optical parametric oscillator (OPO) with a $\chi^{(2)}$ non-linear crystal of length $l$ such as schematically shown in fig. 1. As usually this system is conveniently described using Maxwell equations and the coupling of the field with the atoms inside the dielectric crystal. If the pumping field is intense, a non-linear polarization vector $\vec{P}^{(NL)}$ must be added to the usual linear contribution

$$P_i[\vec{r}, \omega] = \varepsilon_0 \chi^{(1)}_{i,j}[\omega] E_j[\vec{r}, \omega] + P_i^{(NL)}[\vec{r}, \omega],$$

$$P_i^{(NL)}[\vec{r}, \omega] = \frac{\varepsilon_0}{2\pi} \int \frac{d\omega'}{2\pi} \chi^{(2)}_{i,j,k}[\omega; \omega', \omega'] \times E_j[\vec{r}, \omega - \omega'] E_k[\vec{r}, \omega'],$$

where we have used the Fourier representation of fields, $i$ and $j$ stand for the projection of vectors on the 3 orthogonal spatial axes $x$, $y$, $z$, and we use the summing convention on repeated indices. $\chi^{(1,2)}$ are, respectively, the linear and non-linear susceptibility tensor. $\chi^{(1)}$ is diagonal if we choose $x$, $y$, $z$ parallel to the proper directions of the crystal, $\chi^{(2)}_{ij}[\omega] = (n_i(\omega)^2 - 1) \delta_{ij}$, $n_i$ being the refraction index of the crystal in direction $i$. For non-centrosymmetrical crystals and reasonable pumping fluxes, the non-linear polarization vector exhibits dominantly a $\chi^{(2)}$ effect. For simplicity, we will consider that the crystal is pumped with a single laser beam of frequency $\Omega$, propagating rightwards along the $x$-axis and linearly polarized along the $p$-axis ($p = y, z$). If we denote $\Phi$ its photon flux and $\theta_p$ its phase at $x = 0$, the pumping field inside the crystal writes $E_p[\vec{r}, t] = E_0 e^{i\theta_p t} e^{-i\Omega t} + c.c.,$ where $E_0 = \sqrt{\frac{h\Omega \Phi}{2\varepsilon_0 c A n_p}}$ is the pump field’s amplitude, and $i$ is the imaginary unit. The constant $A$ represents the transverse extension of the beams.

We consider a type-I interaction inside the crystal. In this case, if we denote “s” the transverse polarization perpendicular to “p”, only the tensorial elements $\chi_{s,s,p}^{(2)} = \chi_{s,p,s}^{(2)}$ of eq. (2) contribute to the non-linear polarization. By assuming that the crystal is non-absorbant and dispersion-free in the spectral range of interest, we can replace the crystal’s linear susceptibility and second-order non-linear susceptibility by their average values $\chi^{(1)}$ and $\chi_{s,s,p}^{(2)}$. In this case, the “s” component of the polarization vector writes $P_s[\vec{r}, t] = \varepsilon_0 \chi^{(1)}(1)(x, t) E_s[\vec{r}, t]$, with $\chi^{(1)}(1)(x, t) = \chi^{(1)} + \kappa \sin[\Omega(t - n_p x/c) - \theta_p]$, $n_p$ being the mean refraction index of the crystal in direction “p” and $\kappa = \sqrt{\frac{\Delta \Omega \Phi}{2\varepsilon_0 c A n_p}} \chi_{s,s,p}^{(2)}$. The crystal thus behaves as a linear dielectric medium in direction “s”, with an effective refraction index $\bar{n}_s(x, t)$ depending on space-time coordinates $x$ and $t$: $\bar{n}_s(x, t)^2 = \chi^{(1)}(1)(x, t) + 1$. The typical order of magnitude of $\kappa$ evaluates to $\kappa \approx 10^{-5}$, if we consider pump beams of power $h\Omega \Phi \approx 1$ W, focalized over an area $A \approx 10^{-10} \text{m}^2$ inside a crystal of mean refraction index $n \approx 1$ and non-linear susceptibility $\chi^{(2)} \approx 10^{-11} \text{m V}^{-1}$. We can thus safely expand the effective refraction index to first order in $\kappa$:

$$\bar{n}_s(x, t) \approx n_s + \frac{\kappa}{2n_s} \sin[\Omega(t - n_p x/c) - \theta_p].$$

If the unperturbed length $l$ of the crystal is small compared to the pump’s wavelength $\Lambda = 2\pi c/\Omega$ (say $l \approx 0.1 \Lambda$), the spatial dependence of $\bar{n}_s(x, t)$ is not significant, and we can define an average time-dependent refraction index $n_s(t) = \bar{n}_s + \frac{\kappa}{2n_s} \sin[\Omega t - \theta_p]$ with

$$\epsilon_{opt} \approx \frac{l}{2n_s} \sqrt{\frac{h\Omega \Phi}{2\varepsilon_0 c A n_p}} \chi_{s,s,p}^{(2)},$$

while $\theta = \theta_p + \Omega n_p (x_0 + l/2)/c - \pi/2$ if the crystal is located between positions $x_0$ and $x_0 + l$. The result of pumping can then be seen as a periodic modulation of the effective length over which the s-polarized fields propagate inside the crystal, that is $l_s(t) \approx n_s(t) \times l = \bar{n}_s l + \epsilon_{opt} \sin[\Omega t - \theta]$. The frequency of this modulation corresponds to the pumping frequency $\Omega$, and its amplitude is given by (4).

Fig. 1: (Colour on-line) Type-I optical parametric oscillator with one thin crystal slab stuck on the left mirror of the cavity, and equivalent oscillating cavity.
Suppose now that we use a p-polarized laser beam of frequency $\Omega$ to pump the system sketched in fig. 1. Then the temporal change in the refractive index results in a modulation of the cavity length for s-polarized fields which is equivalent to an apparent motion of the mirrors: 

$$L(t) = L_0 + \delta L(t),$$

with $L_0 = L_{cav} + l(\pi_s - 1)$ and $\delta L(t) = l \times \delta n_s(t)$. This modulation writes

$$L(t) = L_0 + e_{opt} \sin[\Omega t - \theta],$$

meaning that optical parametric oscillators (OPO) can indeed reproduce changes in boundary conditions equivalent to those generating the dynamical Casimir effect. Accordingly we expect the OPO model system to amplify the parametric fluorescence of the crystal in a Casimir-like oscillation regime entailing the creation of pairs of s-polarized correlated photons from vacuum, with frequencies $\omega$ and $\omega'$ satisfying energy conservation: $h(\omega + \omega') = h\Omega$. The resulting signals are resonantly enhanced when $\omega$ and $\omega'$ correspond to cavity modes.

As we will discuss more precisely in a forthcoming paper, a single field mode of frequency $\omega$, propagating right- or leftward ($A_{in}$ and $A_{in}$) undergoes an analogous transformation when interacting either with the composed “mirror-crystal” system, pumped at frequency $\Omega$, or with a mirror mechanically oscillating at frequency $\Omega$ around a mean position $x_0$:

$$A_{out}(\omega) = \sqrt{1 - r} A_{in}(\omega) + \sqrt{r} e^{-i\omega/\Omega} A_{in}(\omega) + \sqrt{r} e^{-i\omega/\Omega} \left[ e^{i\theta} \left( 1 - \frac{\omega}{\Omega} \right) \beta_{opt/mech} A_{in}(\omega - \Omega) + e^{-i\theta} \left( 1 + \frac{\omega}{\Omega} \right) \beta_{opt/mech} A_{in}(\omega + \Omega) \right].$$

$r = e^{-2p}$ is the mirror’s reflection coefficient. For simplicity, it is chosen to be frequency independent and equal for both mirrors of the cavity. The only difference between mechanical and non-linear optical excitation lies in the expression of the parameter $\beta_{opt/mech}$: for the mechanical excitation it corresponds to the mirror’s maximum velocity $v_{max}$ with respect to the speed of light $c$ while for the non-linear optical interaction it is a function of the crystal’s non-linearity and of the pump field’s intensity and frequency

$$\beta_{mech} = \frac{v_{max}}{c} = \frac{\epsilon_{mech}}{c},$$

$$\beta_{opt} = \frac{\Omega}{2c\pi_s} \sqrt{\frac{h\Omega \Phi}{2e}} \frac{\chi^{(2)}_{s,s,p}}{\chi^{(2)}_{s,p,p}} \epsilon_{opt} \Omega.$$  

As the field transformation in both cases are strictly analogous, all results for physical quantities such as emitted photon flux or photon statistics can be transposed from one system to the other with the appropriate choice for $\beta$. In both systems $\beta$ measures the efficiency of the coupling and gives a measure of the number of emitted photons produced via these processes. The mechanical oscillation is limited to frequencies in the GHz regime with maximum amplitudes in the range of less than a nanometer. At best $\beta_{mech}$ thus evaluates to $\beta_{mech} \leq 10^{-3}$.

In contrast, the amplitude of the apparent cavity length modulation depends on the frequency and OPOs are commonly used in the optical frequency range. Equation (4) shows that the order of magnitude of $\epsilon_{opt}$ will be given by $\epsilon_{opt} \approx k \Omega \sim 10^{-5}$. Therefore the parameter $\beta_{opt}$ for the non-linear optical process can be by many orders of magnitude larger than for the dynamical Casimir effect based on mechanical motion, e.g. $\beta_{opt} \approx 10^3$ for a pump frequency of $\Omega/2\pi \sim 3 \times 10^{14}$ Hz.

Let us now consider a high finesse cavity ($F \approx \pi/2\rho \gg 1$), with one mirror oscillating at frequency $\Omega$. If the cavity length is $L_0$, a resonant enhancement of the Casimir radiation will occur when $\Omega = 2mc^2/L_0$, i.e. when the oscillation frequency is chosen to be a multiple integer of the fundamental cavity mode. The number of photons emitted inside and outside the cavity via the dynamical Casimir effect after a time $t \gg 1/\Omega$ has been evaluated within the scattering approach in the past [2,3] and we will just recall the result here:

$$\langle N(t) \rangle \approx \frac{2m}{3} \beta_{opt/mech}^2 \frac{F}{\pi},$$

$$\langle N_{out}(t) \rangle \approx \frac{2}{3} \beta_{opt/mech} \frac{F}{\pi} \times \frac{\Omega t}{2\pi}.$$  

These expressions have been obtained far below the oscillation threshold which is given by $\beta_{opt/mech} = \pi/2F$ and by assuming a perfect tuning between $\Omega$ and the cavity modes. The effect of detuning has been studied in detail in [19].

As we have analogous field transformations for the model OPO system and the cavity with a single oscillating mirror, we can apply the above equations to both systems. With the above discussion on the value of the parameter $\beta$, it is straightforward to discuss the different orders of magnitude for the dynamical Casimir radiation, on the one hand, and the Casimir-like photon emission, on the other hand. For a cavity of finesse $F = 10^4$ the mechanical motion gives rise to a photon flux $\langle N_{out}(t) \rangle / t$ of about $10^{-6}$ photons/s. For the optical analogue we obtain $\beta_{opt} \approx 10^{-6}$, if we consider a laser beam of power $h\Omega \Phi = 1\text{W}$, at resonance inside the cavity, pumping a crystal of length $l = 0.1\mu$m over an area $A = 10^{-10}\text{m}^2$ at a frequency $\Omega/2\pi \approx 3 \times 10^{14}\text{Hz}$. We then expect an extracavity radiation of $\langle N_{out}(t) \rangle \approx 10^5$ photons per second excited from vacuum, which largely exceeds predictions for Casimir radiation in systems based on mechanical motion. This is simply due to the fact that the model OPO system avoids direct mechanical motion but uses an apparent modulation of the cavity length instead. Even though the OPO’s oscillation regime reproducing the dynamical Casimir effect is not standard, we expect the Casimir-like photons to be emitted by the usual parametric conversion processes taking place inside the crystal. The analogue
Casimir radiation is thus not an additional but the only signal emitted by the OPO in this regime. Let us also underline that our proposal is different from usual OPO experiments because the non-linear crystal is required to be thinner than the pumps’ wavelength, in order to have a suitable Casimir-like oscillation regime. This should relax the dispersion and phase-matching constraints, and permit the simultaneous oscillation of several parametric resonances.

We finally want to discuss the difference between our predictions, which imply the existence of a stationary regime for the emitted photon flux, in comparison with other approaches which predict exponential growth [5,14–17]. Obviously, it is of crucial importance for any experimental observation of the Casimir radiation to know whether the extracavity photon flux increases linearly or exponentially in time. To point out the discrepancy and explain its origin we apply in the reminder of the paper our set-up to the theoretical framework developed recently in [5,14,15] for cavities with time-dependent refractive media. There the resulting number of Casimir-like photons inside a perfectly reflecting cavity takes the form

\[ \langle N(t) \rangle = \sum_k \sinh^2[r_k(t)], \]  

where \( r_k(t) \) represents the squeezing factor for photons emitted in a given cavity mode \( \omega_k(t) = \frac{k}{2t} \). Let us consider the same sinusoidal length change \( L(t) \) as before with \( \epsilon \ll L_0 \), and use the mean cavity modes \( \omega_{k,0} = \frac{k}{2t} \). For a modulation frequency \( \Omega = 2m\pi c/L_0 \), pairs of Casimir photons are mainly emitted into the degenerate cavity mode \( \omega_{m,0} = \Omega/2 = m\pi c/L_0 \), with a squeezing function \( r_m(t) \approx \nu_0 t \approx \frac{2m}{\pi c} \) [15]. The parameter \( \nu_0 \) measures as \( \beta \) the efficiency of photon creation and corresponds to \( \beta = \nu_0 \tau \), where \( \tau = L_0/c \) is the time of flight for photons over the mean cavity length.

In order to account for cavity losses, it has been repeatedly proposed [6,15] to introduce a linear damping rate \( \gamma \). If we assume that cavity losses are mainly due to photons escaping the cavity because of partially transmitting mirrors, then the linear damping rate should be linked to the cavity finesse as \( \gamma \tau = \frac{1}{\pi} \). In the limiting case of a high finesse cavity the number of emitted photons (10) should be affected by a decay factor \( \exp(-\gamma t) \) [15], leading to an exponential growth in the photon emission inside and outside the cavity in the long time limit

\[ \langle N_m(t) \rangle = \sinh^2(\nu_0 t)e^{-\gamma t} \approx \frac{1}{4}(2^{2
u_0}-\gamma t)^2, \]

\[ \langle N_{m+1}(t) \rangle = \sinh^2(\nu_0 t)(1-e^{-\gamma t}) \approx \frac{1}{4}e^{2\nu_0 t}. \]  

Clearly, equations (8) and (12) give quantitatively different predictions for the photon emission rate via the dynamical Casimir effect. This can however be easily understood by considering the number of intracavity photons far below threshold, i.e. for \( 2\nu_0 \ll \gamma \). In the short time limit \( \nu_0 t \ll 1 \), we can write \( \langle N_m(t) \rangle \approx (\nu_0 t)^2 e^{-\gamma t} \).

This function reaches a maximum value at \( t = 2/\gamma \), and decreases afterwards such as shown in fig. 2. The reason for this is that when \( \gamma t \rightarrow 2 \), or equivalently when the number of round trips \( t/2\tau \) performed by photons inside the cavity approaches \( F/\pi \), the losses balance the amplification effect. The oscillating cavity should thus reach a stationary state with a constant number \( \langle N_m \rangle \) of intracavity Casimir photons given by its maximum value

\[ \langle N_m \rangle \approx \frac{1}{e^{2\nu_0^2}} \left( \frac{2\nu_0^2}{\gamma} \right)^2 = \frac{1}{e^{2\nu_0^2}}(\beta_{opt} F/\pi)^2. \]  

The discrepancy between this result and the one given by eq. (8) now mainly consists in a factor \( m \). An explanation could be that the calculations performed in [15] only account for the contribution of degenerate pairs of photons emitted in the same mode \( \omega_{m,0} = m\pi c/L_0 \). In contrast, according to our previous works [2,3], all pairs of photons \( (\omega_{k,0}, \omega_{k',0}) \) satisfying \( \omega_{k,0} + \omega_{k',0} = \Omega \) (i.e. \( k + k' = 2m \)) should contribute to the emitted spectrum. When accounting for all parametric resonances the “\( m \)” factor should be recovered.

In conclusion we argue that the exponential growth of photon flux is valid only in the short time limit while our method remains valid in the long time limit, as it includes right from the beginning a non-unitary reflection coefficient \( r \) for the mirrors. Accordingly one observes a stationary regime with a constant number of Casimir photons inside the cavity. The difference between this method and the introduction of an energy loss \( \gamma \) is that only by using reflection coefficients the phase relations for the fields are explicitly taken into account and automatically respected.

While the dynamical Casimir effect is still unobserved today, an intermediate step in order to achieve this task would be to use the above OPO model system which obeys rigorously the same field transformations as a cavity with a single oscillating mirror. This could give considerable additional insight into the dynamical Casimir effect especially as far as experimental questions are concerned. It already permits to test a number of important predictions for the dynamical Casimir effect in easily achievable conditions. In particular it would allow to test whether the
The present prediction for the emitted photon flux is correct, i.e. that the system evolves into a stationary state with a constant number of Casimir photons inside the cavity and an extracavity radiation growing linearly in time or if these quantities grow exponentially. The clarification of this point would be of greatest importance for any experimental set-up aiming at observing the dynamical Casimir radiation due to direct mechanical motion.

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