Scaling of Entanglement-Assisted Communication in Amplified Fiber Links

Sekavčník, Simon and Nötzel Janis

Abstract—Quantum communication technology offers several advanced strategies. However, their practical use is often times still not well understood. In this work we outline the theoretical communication capacity scaling in amplified fiber links. We present a scenario in which the assistance via pre-shared entanglement offers an arbitrary capacity, given enough bandwidth and spatial modes are provided by the fiber. The future capacity advantage over conventional classical techniques as well as non-assisted quantum techniques is potentially infinite. We discuss this theoretical observation in connection to current trends in fiber development.

I. INTRODUCTION

The ever growing complexities of present communication networks are a true testament to human ingenuity. Our reliance on the global network is growing by the day. With the amount of network traffic poised to increase, coupled with high demand for ultra low-latency communication and limit in transmission power allowed by fiber, we are presented with a challenge to reduce the energy consumption while accommodating the required needs. Wast majority of network traffic is serviced by optical fiber links. With current classical technologies the communication capacity is limited by the Shannon capacity. The eventual emergence of Quantum Information Processing (QIP) technologies will - in addition to providing new, never seen before applications and communication protocols - move the theoretical communication capacity bounds.

The fact that capacity bounds are higher for QIP enabled communication channels is well documented [1][2]. Finding use-cases where effects would translate into significant practical communication advantage is a difficult task. Especially difficult is fitting entanglement assisted communication into a real world communication problem. In [3][4] the authors examined the use of QIP for the transceiver design of optical fiber networks and pointed out the possibility to reduce energy consumption of the optical amplifiers. In [5] different capacity limits of amplified optical fiber limits were calculated as well, which accounted for different relations between transmission power and amplifier gains. The work [5] studied the performance under more realistic conditions, taking into account the Kerr-induced phase noise based on the model [6] and a specific design of the transceiver based on the work [7]. Later [8] tried to showcase the advantages offered by entanglement-assisted QIP in high-density, ultra low latency networks. The advantages were clearly laid out, however, it was also pointed out that the parameter region required for meaningful entanglement-assisted QIP advantages are difficult to arrive at and hard to argue for.

In this work we focus on outlining practical communication scenario in which the use of QIP, particularly with entanglement assistance, translates into capacity advantage over purely classical and non-assisted QIP method. In order to showcase the different capacity bounds we consider an optical fiber link with three different transmitter and receiver types.

The differences in the above three capacities stem from the fact that when the product $\tau P/M$ is small enough (for example below one), the QIP enabled system will display an advantage that scales logarithmically with the $\tau P/M$. Additionally the entanglement assisted links will display an advantage over non-assisted QIP links if there is a sufficient amount of noise in the system $\nu \gg 1$, combined with transmitters operating with low power per pulse $P/M \ll 1$.

The questions we are concerned with here are two-fold: First, what the technological meaning of these relations is today, and second, how they might influence future communication system design. Therefore we root our modelling into the current fiber transmission reality by assuming communication at 1550nm. Here, the fiber nonlinearities dictate a power limit of approximately $100mW$ per fiber to current system design, which leads to a value of around $10^{18}$ photons being emitted at the sender per second. We assume this energy is spread over a wide bandwidth $B$ by utilizing several orthogonal channels. The most extreme number $B$ one could think of (based on fibers available today) would be in the order of $B = 10^{13}$, leaving us with $10^5$ photons per slot. It is thus difficult to motivate the operating regime $P/M \ll 1$ based solely on setting $M = B$. However, the recent literature points out the possibility to utilizing multimode fiber to transmit several spatially orthogonal channels over the same fiber, and a number of $N = 10^3$ of such channels has already been demonstrated as technically feasible today [9]. Taking such possibilities into account we may set $M = N - B$ to argue for the very low power per pulse needed to see advantages of entanglement-assisted communication. Since optical amplification is the standard technology enabling high throughput data transmission over more than 100km, and since optical amplification not only regenerates the signal but inevitably also adds noise to it, it is therefore possible to motivate communication in a setting where both $\nu \gg 1$ and $P/M \ll 1$. Our analysis clarifies...
that the condition $\nu \gg 1$ might be relaxed to $\nu > 0$ without changing the general scaling of entanglement-assisted capacity with the value $M$ when amplifiers are used. In our model we assume an amplifier setting which completely regenerates the signal. This is in contrast to the previous work [4] and more in line with the analysis of the quantum limit as in [3], where it has been shown that under the rule of constant amplification (in the sense of being independent of the signal energy), both Shannon- and Holevo capacity approach a limiting value when $M \to \infty$. Our results show that, under the same amplifier rule, the entanglement-assisted capacity can still grow unlimited when $M \to \infty$.

For the interpretation of our results it is important to note: the key assumption to the analysis is that future fiber networks will need to obey a per-fiber power limit $P$, while at the same time being able to transport an increasing number of orthogonal channels, so that the assumption $M \to \infty$ is justified.

II. SYSTEM MODEL

Following the exposition of [10] we use a modelling approach here which starts with a narrowband, linearly polarized optical signal coming in the form of uniformly spaced pulses in temporal slots of duration $B^{-1}$. Due to the modulation being independently applied to these slots, the Fourier transform shows that such a system occupies a spectral bandwidth proportional to the slot rate $B$. Such a system can quantum-mechanically be modelled by assigning a quantum state $|\alpha\rangle \in F$ to every slot; $F$ being the Fock space which has a countably-infinite orthonormal basis $\{|n\rangle\}_{n=0}^{\infty}$ called the number-state basis. A system in state $|n\rangle$ consists of exactly $n$ photons, and these number states can be used to express a system in a coherent state $|\alpha\rangle$ as

$$|\alpha\rangle = \exp^{-|\alpha|^2/2} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle. \quad (1)$$

The expected energy of a coherent state $|\alpha\rangle$ is then equal to $|\alpha|^2$. If these pulses are detected and then converted to electronic values in a slot-by-slot fashion, and if thermal noise with an average number of $\nu$ photons per slot is present at the receiver, one recovers the well-known Shannon formula for the capacity of the channel:

$$C_S(M, P, \tau, \nu) = M \cdot \log \left(1 + \frac{\tau \cdot P}{M(1 + \nu)}\right) \quad (2)$$

Motivated by the work [11][12] of Holevo, researchers have also studied the capacity of such an optical data transmission system when arbitrary (quantum) measurements are allowed at the receiver. When it comes to concrete realization of this concept, one speaks of the so-called Joint Detection Receiver (JDR), where a first proposal was made in [7] and a second one in [13]. Both proposals use an interferometric setup to interfere a series of consecutive pulses (slots) with each other prior to detection. This setup is ideal to harness the advantage of JDRs, which is in the region where the received signal energy per slot is very weak. Due to the relation

$$P = \mathbb{E}(|\alpha_j|^2) B h f_c \quad (3)$$

where $P$ is the average optical power carried by the signal, $h = 6.626 \cdot 10^{-34} J \cdot s$, $f_c$ is the carrier frequency and the complex numbers $\alpha_j$ specify the symbols that are generated at the sender, it is possible to reach the limit of small received energy per slot by keeping $P$ constant while increasing $B$. The ultimate capacity for a communication system employing a JDR is given by the Holevo capacity:

$$C_F(M, P, \tau, \nu) = M \cdot \left(g(\tau \cdot P \cdot M^{-1} + \nu) - g(\nu)\right) \quad (4)$$

where $g(x) = (x + 1) \log(x + 1) - x \log x$. Note that we assumed a noise level $\nu$ which is independent of the power $P/M$ of the individual pulses both in Eq. (2) and in Eq. (4).

This assumption is actually the method by which a separation between the data transmission concepts outlined so far on the one side and entanglement-assisted data transmission on the other can be achieved when $M \to \infty$.

Finally, let us introduce the third major data transmission concept that emerged from the work [14]. It is the concept of entanglement-assisted communication. Here, it is assumed that at the point in time when classical information is encoded into a quantum state at the sender side, this quantum state is not a simple coherent state $|\alpha\rangle$ but rather an arbitrary quantum state shared between the sender and receiver. This implies the act of sharing said entangled state is to be taken into account in the system design. The work [15] described such data transmission system as one where idle periods are used to execute the task of sharing entangled states, which is then consumed when data needs to be transmitted by an application. The states that can be utilized for this task can for example be so-called squeezed states

$$|s_\tau\rangle = \frac{1}{\cosh \tau} \sum_{n=0}^{\infty} \tanh(r)^n |n\rangle_I |n\rangle_D, \quad (5)$$

where the index $I$ stands for the idler mode which is sent prior to data transmission, and the index $D$ for the data mode which is used to encode the data when it arrives at the sender and which is then transmitted to the receiver. The capacity of the above described system when the idler modes are shared and it is used in entanglement-assisted mode is given by

$$C_E(M, P, \tau, \nu) = M \sum_{x=0}^{1} g\left(\frac{P}{M} + x\nu\right) - g(d_\tau(\tau, \frac{P}{M}, \nu)). \quad (6)$$

Here, the functions $d_\tau$ are defined via

$$d_\tau(\tau, n, \nu) = (d(\tau, n, \nu) - 1 - (-1)^d((\tau - 1)n + \nu))/2 \quad (7)$$

$$d(\tau, n, \nu) = \sqrt{((1 + \tau)n + \nu + 1)^2 - 4\tau n(n + 1)}. \quad (8)$$

We choose to divide a link into $K$ segments of equal length $L$, as presented in Figure [1]. At the end of each segment the
There are two different possible implementations which we study here. In both implementations, there is a receiver at the end of the last segment. The implementations differ in the sense that one uses a pre-amplifier directly in front of the receiver and the other does not. The two corresponding capacities for that one uses a pre-amplifier directly in front of the receiver and the other does not. The two corresponding capacities for the three receiver types ($X \in \{S, J, D\}$) can then be computed as

\[
C_{X,\text{np}}(M, P, \tau) = C_X(M, P, \tau, 1 - \tau^{L-1})
\]

when no pre-amplifier is used in front of the receiver and as

\[
C_{X,p}(M, P, \tau) = C_X(M, P, 1, \tau^{-1} - \tau^{L-1})
\]

if pre-amplification is used. These formulas arise from the calculation of overall loss for the setting without pre-amplification as

\[
\tau_L = \tau \prod_{i=1}^{K-1} G \tau = \tau
\]

where $G = 1/\tau$ was used, and as $\tau_L = 1$ if pre-amplification is used. Moreover, as the noise photons injected by the amplification process are subject to loss as well, the overall noise (for $K \geq 2$) is calculated using the geometric series as

\[
\nu_K = \sum_{i=1}^{K-1} \tau^{K-i}(G - 1) = (1 - \tau)\frac{1 - \tau^{K-1}}{1 - \tau} = 1 - \tau^{K-1}.
\]

When pre-amplification is used, the last amplifier injects an additional number of $\tau^{-1} - 1$ noise photons. In this case, the noise level satisfies $\nu \gg 1$. As one can see we have, in both cases, an amount of noise which is strictly larger than 0. We discuss our approach in Subsection III-A.

### III. RESULTS AND DISCUSSION

**Theorem 1:** The scaling of the entanglement-assisted capacity for $M \to \infty$ is given by the formula

\[
C_E(M, \tau, P, \nu) = \mathcal{O}(T \cdot P \cdot \log M)
\]

where $T = \frac{1+\nu-\tau}{1+\nu-\tau^2}$.

The logarithmic scaling of the capacity with the signal energy has already been observed early on in [16]. The added value of this work is thus in the concrete phrasing in terms of conventionally used system parameters allowing for comparison with the other two technologies, and in the interpretation of formula [13] in Subsection III-A.

Using formula (13) we may take any constant $c > 0$ modelling - for example the capacity limit of a given amplified link, specified by values $P$ and $\nu$ - and ask what values of $M$ are needed such that, for example, $C_E(M, \tau, P, \nu) \geq 10 \cdot c$. The approximate answer, which is increasingly exact as $P/M \to 0$, is then given by

\[
\log M = 10 \cdot c \cdot \frac{1 + \nu}{P(1 + \nu - \tau)}.
\]

To show the applicability of this formula, we compute its solution for two particular cases of a 160km link and a 10.000km link, both with $P = 10^{10}$.

![Preamplified final receiver](image)

Fig. 2: Scaling of the capacity in the setting when pre-amplification is used at the receiver. The bandwidth $B$ is set to $10^{13}$ and only the number $N$ of spatial modes is varied. Starting from a value of $N = 10^5$, the system operates under the condition $PM^{-1} < 1$.

**A. Discussion**

There are four major themes which impact the technological plausibility of our analysis. First, this is the amplification model. Second, this is the assumed growth of the value $B$. Third, this is the assumed power limit $P$ imposed on the entire optical link. Fourth, the growth of $N$ is highly relevant. We will discuss these aspects separately.
While entanglement-assisted communication yields unbounded (logarithmic) growth.

2) Growth of $M$ through $B$: In the work [17], a growth of $B$ over time has been observed which seems to be well approximated by the formula $B = 10^{13}(1 + j - 2000)$, where $j$ is the year and the current experiments operating around a value of $200 \cdot 10^9$ pulses per second are taken as the value of today. The growth of $B$ is hindered by many practical challenges, among which is the conversion of light between different spectral domains. Fortunately the recent literature [18] demonstrates that new ways of accessing more optical spectrum in a cost-effective way have been found. Taking only the $C,L$ and $O$ band in the existing SMF-28 fiber therefore already justifies that a value of $B = 10^{13}$ might become accessible. In addition, the development of the hollow-core fiber [19] brings about new fiber types with large transmission windows which are essentially homogeneous in terms of the physical properties affecting communications, thereby simplifying system design. However, even the very high value of $B = 10^{13}$ is not sufficient to motivate the condition $PM^{-1} \ll 1$. Therefore, in this work we also consider a growth of $M$ induced by other factors, such as spatial multiplexing.

**B. Power Limit $P$**

Optical fiber systems are subject to a power limit induced by the nonlinear response of the fiber to the energy which is emitted into the fiber per second [20]. In practice, this results in systems utilizing around $P = 100mW$ spread over large numbers of individual channels (the ITU lists 144 channels for communication in the C-band). While the Shannon formula, Eq. [2], predicts a logarithmic capacity growth with the transmission power, the nonlinear response of the fiber leads to a decrease of the system capacity when the emitted power rises above a threshold.

This fact leads to the prediction of a so-called capacity crunch where fiber networks are not able to support the (exponentially) growing demand for data transmission anymore. As a solution to this problem, one can simply put more fiber, use more clever ways of handling the data to prevent exponential growth in the demand or change the fiber design. Among the activities in the latter direction is the above-mentioned development of hollow-core fibers [19]. Another direction of research pursues multi-mode fiber in general, where one of the research directions pursues approaches to increase the core diameter of the fiber so that it supports multiple spatial modes. The different approaches to multi-mode fiber design are well described in [9].

Due to the novelty of the newly proposed fiber designs, analytical treatments from the perspective of (quantum) communication systems design are scarce. The analytical prediction that entanglement-assisted communication may at some point outperform conventional and even joint detection receiver technologies however rests on the presence of a power limit. While the question of an existence of a power limit for fiber transmission is not on the table, the question of how
that power limit depends on the number $N$ of spatial modes is largely open [9]. Although it might decrease the value of this work, we would like to note the following: some results [21] seem to indicate that nonlinearities (which are the major reason for the presence of a global power limit such as [15] in optical transmission systems) might couple the power limit to the number $N$ in a way where an increase from $N$ to $xN$ would not inevitably necessitate a reduction of power per channel by a factor of $x$ anymore.

1) Growth of $M$ through $N$: As pointed out in [9], the number of spatial modes $N$ in a fiber of sufficient core diameter can already today easily reach values of $N = 1000$. This new degree of freedom may separate future network design from the current one, where standard single-mode fiber is the technology standard for long-haul transmission systems.

IV. DERIVATION OF APPROXIMATING FORMULA FOR $C_E$

Our goal here is to derive an approximation to $C_E$ which allows us to study, with high accuracy, its asymptotic scale when the number $M$ of orthogonal channels, which are subject to a global power constraint

$$\sum_{m=1}^{M} P_m \leq P,$$  

increases. The scaling of the overall system capacity with $M$ is, in this case, described by the quantities $C_{X,x}(M,P,\tau)$. For $X = C$ and $X = J$ the limit of these quantities has been stated in [3], where the formula $\lim_{M \to \infty} C_{C,x}(M,P,\tau) = P \cdot \tau \cdot \log(1+\nu)\nu^{-1}$ can be found with $\tau$ being the transmittivity of the last segment (which is equal to 1 if pre-amplification is used) and $\nu$ being equal to $1 - \tau K^{-1}$ for $x = n$ and $\nu_K = \tau^{-1} - \tau K^{-1}$ for $x = p$. This value sets the true benchmark for entanglement-assisted communication to provide an advantage in terms of data transmission capacity. The quality is thus for which values of $(M,P,\tau)$ the entanglement-assisted capacity can outperform the non-assisted ones by a given factor.

As one can see from (6), it is helpful to understand the individual scaling of the involved four terms. In this analysis, it is helpful to study the terms $(g(\tau \cdot P \cdot M^{-1}) - g(d_+(\tau, P \cdot M^{-1}, \nu))) M$ and $(g(\tau \cdot P \cdot M^{-1} + \nu) - g(d_-(\tau, P \cdot M^{-1}, \nu))) M$ separately.

Our approximation rests on the following observations: For every $\tau \in [0,1]$ and $\nu \geq 0$ it holds $d_-(\tau,0,\nu) = \nu + 1$, which then implies

$$d_-(\tau,0,\nu) = 0.$$  

Obviously it holds $P \cdot M^{-1} \to 0$ as $M \to \infty$. Our second observation is that the derivative of $d_-(1,n,\nu)$ at $n = 0$ satisfies

$$\partial_n d_-(\tau,n,\nu)|_{n=0} = (1 + \nu - \tau) \cdot (1 + \nu)^{-1}. $$

Since $\lim_{n \to 0} g(\nu + \epsilon) = g(\nu)$, $\lim_{M \to \infty} d_+(\tau, P \cdot M^{-1}, \nu) = \nu$ and $\partial_n d_+(\tau, P \cdot M^{-1}, \nu)|_{n=0} = \nu/(1 + \nu)$ we conclude by L’Hospital’s rule that the term $(g(\tau \cdot P \cdot M^{-1} + \nu) - g(d_+(\tau, P \cdot M^{-1}, \nu))) M$ asymptotically approaches a constant given by $(1 - \tau\nu/(1 + \nu)\log(1 + \nu)/\nu$. Since we are aiming to study the asymptotic growth of $C_E$ we neglect this constant. Thus, the asymptotic growth of $C_E$ must be well approximated by

$$C_E(M,\tau,P,\nu) \approx M \left( g(\tau PM^{-1}) - g(d_-(\tau, PM^{-1}, \nu)) \right).$$  

For small values of $\epsilon > 0$ it holds $g(\epsilon) \approx (1 + \epsilon)\epsilon - \epsilon \log(\epsilon)$. Therefore, setting $T := (1 + \nu - \tau)/(1 + \nu)$ and abbreviating $PM^{-1}$ as $\epsilon$ we arrive at

$$C_E(M,\tau,P,\nu) \approx P \left( (g(\epsilon) - g(T\epsilon)) / \epsilon \right)$$

$$\approx \frac{\tau \nu}{\epsilon} ((1 + \epsilon)\epsilon - (1 + T\epsilon)\epsilon \log \epsilon + T\epsilon \log T\epsilon)$$

$$= P \left(-\epsilon \log \epsilon + \epsilon \log T\epsilon + (1-T)\epsilon \log T\epsilon \right) / \epsilon$$

$$= P \left((1-T)\epsilon \log T\epsilon \right)$$

$$\approx P(1-T)\epsilon \log T\epsilon.$$  

Here we subsequently neglected all terms converging to constants.

This formula applies to both the pre-amplified and the not pre-amplified case, where the difference between the two cases is handled by the term $1 - T$. It can be set in comparison to existing ones, and it shows that entanglement-assisted communication can enable infinite growth even for amplified optical networks. However, when numbers for existing networks are inserted, this formula also shows that the parameter regions where entanglement-assisted communication might be helpful have not yet been reached.

\[ \text{Fig. 4: Quality of approximation formula. As the power per pulse per mode } P/(BN) \text{ approaches 0, the quality of approximation improves, with the approximation function } C_{EA}^\text{NA} \text{ converging the formula } C_{EA}^\text{NA} \text{ which we are trying to approximate.} \]

V. CONCLUSIONS

We produced a formula to calculate the threshold after which a desired factor (for example a factor of 10) of an improvement
in capacity can be expected from the use of entanglement-assisted links, where the formula involves total transmission power \( P \) over all channels, number of spatial channels \( M \) and the bandwidth \( B \) consumed by the system.

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