Seismic Reliability of the Shallow Footings against Bearing Failure of the Soil

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Authors’ contributions

This work was carried out in collaboration between two authors. Author SKS designed the study, performed the statistical analysis, wrote the protocol, and wrote the first draft of the manuscript and managed literature searches. Author MMH verified the analyses of the study and literature searches. Both authors read and approved the final manuscript.

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ABSTRACT

The paper presents the reliability of Reinforced Concrete (RC) shallow footings designed following the provision of Bangladesh National Building Code (BNBC) 2006. The principal objective of the study is to evaluate the reliability and corresponding failure probability of shallow footings designed following BNBC 2006. To achieve the objective of the study, three model buildings having different number of stories have been designed following the BNBC 2006. The bearing failure of soil has been used as performance function only. The statistical parameters of the design variables have been selected from available literatures. Monte Carlo Simulation (MCS) method has been used in the study. From the analytical investigation, it is found that the reliability of shallow footings highly depends on the Coefficient of Variation (COV) of bearing capacity of soil. The reliability index varies from 2.29 to 2.46 for COV of soil of 0.40 using a factor of safety of 2.50 under earthquake load. Reliability of footing increases with the decrease of COV of soil. It is also found that the performance of RC shallow footings designed using BNBC 2006 is poor under the earthquake load.

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1. INTRODUCTION

Bangladesh is an earthquake prone country. Earthquakes are always of stochastic nature. Due to existence of active faults in Bangladesh, there is a high probability of occurrence of a large magnitude earthquake in Bangladesh [1]. Therefore it is necessary to predict the probability of failure of structure and their supporting foundation due to future earthquakes. Since, uncertainties are present in different parameters accounting for the analysis and design of any structure; so, it is very difficult to measure absolute safety for any structure using deterministic analysis. Because of the presence of uncertainty in the design parameters, the structural members as well as their foundation are certainly uncertain. Therefore, one of the most important ways to specify a rational criterion for measuring the safety of a structure is its reliability or probability of failure. The reliability of a structure is its ability to fulfill its design purpose for some specified design lifetime [2]. Reliability is often understood to equal the probability that a structure will not fail to perform its intended function. The term failure of structure does not necessarily mean catastrophic failure but is used to indicate that the structure does not perform as desired. However, engineering community, building users and building owners always expect any building or non-building structure and their supporting foundation to be designed with a reasonable margin of safety. In practices, these expectations are considered by following the code requirements. Consequently, many design codes in various parts of the world are now under revision from the allowable or the working stress design format (ASD or WSD) to the Load and Resistance Factor Design format (LRFD) based on reliability. Presently, Norway, Canada, United States of America, United Kingdom follows the reliability based design of structures (or buildings) and other countries are in the process of modifying their standards [3]. So far, the reliability of structure and their foundation designed following BNBC 2006 has not yet been evaluated. So, the principal aim of this study is to evaluate the reliability of shallow footings designed following BNBC 2006.

Probability-based design of structure became practically realizable in the 1970's and its conceptual framework was developed by Ang and Cornell [4], influenced by Freudenthal's pioneering work on structural safety [5]. Applications of reliability concepts in geotechnical engineering have been reported by Ang and Tang [6], Vanmarcke [7,8], Whitman [9], Li and Lumb [10], Oka and Wu [11], Mostyn and Li [12], Tang [13], Christian et al. [14], Chowdhury and Xu [15], Morgenstern [16], Phoon and Kulhawy [17,18], Duncan [19], Phoon et al. [20], Christian [21].

2. STUDY CASE

2.1 Model Buildings

To evaluate the reliability of shallow footing a total three lightly loaded Ready Made Garments (RMG) industries buildings are considered as model buildings. The location of all model buildings is considered at Zone-II of BNBC zoning map in context of Bangladesh. According to BNBC 2006, the building is classified as occupancy G [22]. The structural form of model building is an intermediate moment resisting frame having RC floor panel supported by beam all sides. The beam column grid of model building is presented in Fig. 1 and building geometries are presented in Table 1.

| Building ID       | No. of Span in x-direction | No. of span in y-direction | Span length in both direction (m) | Depth of footing (m) | Typical storey height (m) | No. of storey (Nos.) |
|-------------------|---------------------------|----------------------------|----------------------------------|----------------------|--------------------------|----------------------|
| Model building-1  | 3                         | 3                          | 6.0                              | 2.44                 | 3.50                     | 6                    |
| Model building-2  | 3                         | 3                          | 6.0                              | 2.44                 | 3.50                     | 8                    |
| Model building-3  | 3                         | 3                          | 6.0                              | 2.44                 | 3.50                     | 10                   |
2.2 Variability in Loads

2.2.1 Dead load

Dead loads are typically treated as normal random variables. Generally the total dead load remains constant throughout the life of structure [2]. In this study a coefficient of variation COV of 10 percent is assigned to dead load and distribution of dead load is considered as normal distribution [23].

2.2.2 Live load

Live loads are always variable in nature. It is normally idealized as a uniformly distributed load. The statistical parameters of live load depend on the area under consideration. The larger the area which contributes to the live load, the smaller the magnitude of the load intensity [2]. A coefficient of variation COV of 25 percent for the live load in office buildings fit a type I extreme value distribution [23]. For the reliability analysis of shallow footing a wide range of COV for live loads in industrial building is considered in the present study.

2.2.3 Earthquake load

The highly variable earthquake load is considered as random variables in this research. A coefficient of variation COV of 138 percent is assigned to earthquake load and distribution of earthquake is considered as Extreme type I [23].

2.3 Variability of Soil

The COV of mixed soil is 0.41 [24]. The COV of the inherent variability for N value are between 25% and 50% [25] and the probability distribution for N is assumed to be lognormal because: (i) most soil properties can be modeled adequately as lognormal random variables [26] and (ii) negative values of N are inadmissible. However, in this study, the COV of SPT is considered as 40% and the distribution of N value is considered as lognormal.

3. METHODOLOGY

3.1 Reliability Analysis

The objective of the reliability analysis is to determine the probability of failure. The probability of failure $p_f$ is the probability that the realization of the basic variables yield a point in the failure domain, i.e. $p_f=P[G(x)]\leq0$.

Where, $x = \text{vector of basic variable; and } G(x) \text{ limit state function defined such that the region } G(x)\leq0 \text{ corresponds with the failure mode of interest. The corresponding reliability index } \beta \text{ can be calculated from } \beta=\Phi^{-1}(p_f), \text{ where, } \Phi^{-1} \text{ inverse of the standard normal cumulative distribution function. The graphical presentation of reliability index is shown in Fig. 2. Where, Q is the load effects and R is the effect of resistance. The reliability indices, } \beta \text{ for most geotechnical components and systems lie between 1 and 5, corresponding to probabilities of failure ranging from about 0.16 to } 3 \times 10^{-7}, \text{ as shown in Table 2 (US Army Corps of Engineers, 1997).}$

3.2 Random Variables

The nominal mean values are obtained from the deterministic analysis of model buildings. Table 3 presents the list of basic variables that are considered in the study for reliability evaluation of footings.

3.3 Performance Functions

The loads $Q_i$ and resistance $R_i$ are treated as random variables. The limit-state functions $g_i(x)$ for the various failure modes are formulated as $g_i(x) = R_i(x) - Q_i(x)$ where $R_i$ and $Q_i$ denote the modal capacities and demands, respectively. The performance function or limit state of interest for bearing capacity of soil can be defined as $g = (q_u - \gamma z) - \frac{P}{bL}$. Where, $(q_u - \gamma z) = \text{the net ultimate bearing capacity of soil. } \frac{P}{bL} = \text{the upward soil pressure below the base. If } g < 0, \text{ the footing fails and when } g \geq 0 \text{ the footing is safe.}$
**4. RESULTS AND DISCUSSION**

### 4.1 Reliability under Earthquake Loads

The reliability indices of shallow footing against the flexural moment, flexural shear, punching shear, and bearing capacity failure of soil under earthquake loads are presented in Table 4.

From the Table 4, it is seen that the reliability of footings against bearing failure of soil varies from 2.29 to 2.46 under the effect of earthquake load considering FS = 2.50 and COV = 0.40. The reliability of shallow footings decreases with the increase of earthquake loads. In case of all model buildings, the reliability of corner footings under earthquake load is lower than other footings of same building [27].

### 4.2 Effect of COV of Soil on the Reliability of Footings

Effect of COV of soil on the bearing capacity reliability of shallow footings considering the effect of earthquake loads of model building-1, model building-2 and model building-3 are presented in Table 5, Table 6 and Table 7, respectively.

From Fig. 3, Fig. 4 and Fig. 5 it is seen that the reliability of footing increases with the decrease of COV of soil. When the COV of soil ≤ 30%, the reliability of shallow footing is *above average* under the earthquake loads using a FS=2.50. On the other hand, it is also observed that if the COV of soil ≥ 40%, the reliability of shallow footing is *unsatisfactory to poor* using a FS = 2.50 [27].

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**Table 2. The range of geotechnical reliability index (US Army Corps of Engineers 1997)**

| Reliability Index | Probability of failure $p_f = \Phi(-\beta)$ | Expected performance level |
|-------------------|---------------------------------------------|----------------------------|
| 1.0               | 0.16                                        | Hazardous                  |
| 1.5               | 0.07                                        | Unsatisfactory             |
| 2.0               | 0.023                                       | Poor                       |
| 2.5               | 0.006                                       | Below average              |
| 3.0               | 0.0001                                      | Above average              |
| 4.0               | 0.00003                                     | Good                       |
| 5.0               | 0.0000003                                   | High                       |

**Table 3. List of random variables**

| $X_i$ | Description | Distribution          | Mean | COV   | References               |
|-------|-------------|-----------------------|------|-------|--------------------------|
| DL    | Dead Load   | Normal                | Nominal | 0.1  | Ellingwood, et al. 1980 [23] |
| LL    | Live Load   | Extreme type I        | Nominal | 0.25 | Ellingwood, et al. 1980 [23] |
| EQ    | Earthquake Load | Extreme type I      | Nominal | 1.38 | Ellingwood, et al. 1980 [23] |
| $\gamma$ | Unit weight of soil | Normal           | Nominal | 0.10 | Lee et al. 1983        |
| $q_u$ | Bearing capacity of soil | Lognormal | 1.0 | 0.25-0.50 | Phoon and Kulhawy 1999a [17] |
Table 4. Reliability indices of footings under seismic loads

| Model building ID | Footing ID | Gravity loads DL (kN) | Gravity loads LL (kN) | EQ load (kN) | Width of footing B (m) | Factor of safety (FS) | COV of soil (%) | Bearing capacity reliability index (β) |
|-------------------|-----------|-----------------------|-----------------------|--------------|-----------------------|---------------------|----------------|--------------------------------------|
| 1                 | F1        | 810                   | 354                   | 68           | 2.02                  | 2.50                | 0.40           | 2.34                                 |
|                   | F2        | 1261                  | 635                   | 84           | 2.58                  | 2.50                | 0.40           | 2.38                                 |
|                   | F3        | 1505                  | 1150                  | 1            | 3.08                  | 2.50                | 0.40           | 2.44                                 |
| 2                 | F1        | 1146                  | 506                   | 94           | 2.41                  | 2.50                | 0.40           | 2.32                                 |
|                   | F2        | 1725                  | 990                   | 127          | 3.09                  | 2.50                | 0.40           | 2.36                                 |
|                   | F3        | 2005                  | 1820                  | 05           | 3.68                  | 2.50                | 0.40           | 2.43                                 |
| 3                 | F1        | 1491                  | 657                   | 137          | 2.74                  | 2.50                | 0.40           | 2.29                                 |
|                   | F2        | 2192                  | 1271                  | 180          | 3.48                  | 2.50                | 0.40           | 2.35                                 |
|                   | F3        | 2537                  | 2271                  | 10           | 4.10                  | 2.50                | 0.40           | 2.46                                 |

Table 5. Effect of COV of soil on the reliability of footings considering earthquake load for model building-1

| Footing ID | Loads DL (kN) | EQ load (kN) | Width of footing B (m) | Factor of safety (FS) | COV of soil (%) | Bearing capacity reliability index (β) |
|------------|---------------|--------------|------------------------|-----------------------|----------------|--------------------------------------|
| F1         | 810           | 354          | 68                     | 2.02                  | 2.50           | 0.40                                 |
| F2         | 1261          | 635          | 84                     | 2.58                  | 2.50           | 0.40                                 |
| F3         | 1505          | 1150         | 01                     | 3.08                  | 2.50           | 0.40                                 |

Fig. 3. Effect of COV of soil on the reliability of footings of model building-1 under earthquake loads
Table 6. Effect of COV of soil on the reliability of footings considering earthquake load for model building-2

| Footing ID | Loads  | Width of footing, B (m) | Factor of safety (FS) | COV of soil (%) | Bearing capacity reliability (β) |
|------------|--------|-------------------------|-----------------------|----------------|----------------------------------|
|            | DL (kN) | LL (kN) | EQ (kN) |            |                    |                                 |
| F1         | 1146    | 506     | 94      | 2.41       | 2.5               | 2.05                            |
|            | 30      | 3.08    |          |            |                    |                                 |
|            | 35      | 2.70    |          |            |                    |                                 |
|            | 40      | 2.34    |          |            |                    |                                 |
|            | 45      | 2.05    |          |            |                    |                                 |
|            | 50      | 1.91    |          |            |                    |                                 |
| F2         | 1725    | 990     | 127     | 3.09       | 2.5               | 2.67                            |
|            | 30      | 3.05    |          |            |                    |                                 |
|            | 35      | 2.67    |          |            |                    |                                 |
|            | 40      | 2.36    |          |            |                    |                                 |
|            | 45      | 2.10    |          |            |                    |                                 |
|            | 50      | 1.90    |          |            |                    |                                 |
| F3         | 2005    | 1820    | 05      | 3.68       | 2.5               | 2.45                            |
|            | 30      | 3.16    |          |            |                    |                                 |
|            | 35      | 2.76    |          |            |                    |                                 |
|            | 40      | 2.45    |          |            |                    |                                 |
|            | 45      | 2.20    |          |            |                    |                                 |
|            | 50      | 1.96    |          |            |                    |                                 |

Fig. 4. Effect of COV of soil on the reliability of footings of model building-2 under earthquake loads

COV of EQ load = 138%  
FS = 2.50  
Mean soil capacity = 292 kPa

Fig. 5. Effect of COV of soil on the reliability of footings of model building-3 under earthquake loads

COV of EQ load = 138%  
FS = 2.50  
Mean soil capacity = 292 kPa
Table 7. Effect of COV of soil on the reliability of footings considering earthquake load for model building-3

| Footing ID | Loads (kN) | Width of footing, B (m) | Factor of safety (FS) | COV of soil (%) | Bearing capacity reliability (β) |
|------------|------------|-------------------------|----------------------|-----------------|----------------------------------|
| F1         | DL 1491    | LL 657                  | 137                  | 2.74            | 30                               |
|            |            |                         |                      |                 | 35                               |
|            |            |                         |                      |                 | 40                               |
|            |            |                         |                      |                 | 45                               |
|            |            |                         |                      |                 | 50                               |
| F2         | DL 2192    | LL 1271                 | 180                  | 3.48            | 30                               |
|            |            |                         |                      |                 | 35                               |
|            |            |                         |                      |                 | 40                               |
|            |            |                         |                      |                 | 45                               |
|            |            |                         |                      |                 | 50                               |
| F3         | DL 2537    | LL 2271                 | 10                   | 4.10            | 30                               |
|            |            |                         |                      |                 | 35                               |
|            |            |                         |                      |                 | 40                               |
|            |            |                         |                      |                 | 45                               |
|            |            |                         |                      |                 | 50                               |

5. CONCLUSION

The reliability index against bearing failure of soil varies from 2.29 to 2.46 under the effect of earthquake load when FS=2.50 and COV of soil is 0.40. However the reliability of footing increases with the decrease of COV of soil. When the COV of soil ≤ 30%, the reliability of shallow footing is above average under the earthquake loads using a FS = 2.50. On the other hand, if the COV of bearing capacity of soil ≥ 40%, the reliability of shallow footing under earthquake is unsatisfactory to poor using a FS = 2.50. The reliability of corner footings under earthquake load is lower than other footings of same building. In the case of interior footings where earthquake load is very lower or negligible, the reliability of footings against bearing failure of soil is approximately similar in both cases of gravity loads and for earthquake load.

DISCLAIMER

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