Heterotic Matrix String Theory

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M-theory suggests the large $N$ limit of the matrix description of a collection of $N$ Type IA D-particles should provide a nonperturbative formulation of heterotic string theory. In this paper states in the matrix theory corresponding to fundamental heterotic strings are identified, and their interactions are studied. Comments are made about analogous states in Type IIA string theory, which correspond to bound states of D-particles and D-eightbranes.

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1. Introduction

D-brane techniques [1] provide us with new ways to study the nonperturbative dynamics of string theory. One may use these methods to test the web of strong/weak coupling dualities that have been proposed in recent years [2]. Typically, these dualities have their simplest interpretation when the theories are viewed as compactifications of a hypothetical eleven-dimensional theory known as M-theory [3]. D-brane techniques therefore yield much information about the structure of M-theory. In fact, it has been conjectured the full dynamics of M-theory with the 11th dimension decompactified may be described by the large $N$ matrix quantum mechanics of a system of $N$ D-particles of Type IIA string theory [4]. One can carry the same ideas over to the compactification of M-theory on $S^1/\mathbb{Z}_2$ which has been conjectured to describe the strongly-coupled dynamics of the $E_8 \times E_8$ heterotic string [5], and attempt to describe the dynamics of this system using the large $N$ limit of the matrix quantum mechanics of Type IA D-particles [7–10].

In this paper, we will continue our study of this system [9], elaborating on the spectrum of states and their interactions. Applying a T-duality transformation along the $S^1/\mathbb{Z}_2$ direction, the supersymmetric quantum mechanics is recast into the form of a gauge theory in two dimensions with $(0,8)$ supersymmetry. The coupling constant of this gauge theory $g$ scales with the length, thus in the infrared this gauge theory is expected to flow to some nontrivial conformal field theory, which we identify as a $S_N$ orbifold heterotic sigma-model. The spectrum of the heterotic string is recovered in this formulation, and the interactions of these states are studied. One may also consider the limit in which $g$ is held fixed and one treats the gauge potential as a slowly varying degree of freedom in a Born-Oppenheimer approximation. The states required by equivalence with M-theory on $S^1/\mathbb{Z}_2$ duly appear in this limit. We conclude with some comments about analogous bound states of D-branes in Type II string theory.

2. Matrix Field Theory

Our starting point will be the two-dimensional gauge theory describing the light excitations of $N$ coincident D-strings in Type IB. This system is T-dual to the system of

1 In this paper we will use the notation Type IB to refer to the usual Type I string theory, and Type IA to refer to the theory obtained by T-dualizing Type I on $S^1$ [6], which has previously been referred to as Type I’ string theory.
D-particles in Type IA studied in [7–9]. The relation with $E_8 \times E_8$ heterotic string theory compactified on a circle (the 9 direction) may be seen by the following chain of dualities.

$$
\begin{align*}
\text{IA D–particle} & \overset{T-\text{duality}}{\leftrightarrow} \text{IB D–string winding} \\
E_8 \times E_8 \text{ Heterotic } p_+ & \overset{T-\text{duality}}{\leftrightarrow} \text{SO}(32) \text{ Heterotic winding}
\end{align*}
$$

The two-dimensional gauge theory is obtained from the massless sector of open strings ending on D-strings and D-ninebranes, using conventional D-brane techniques [11]. The interactions of these fields are fixed by the $(0,8)$ supersymmetry. The answer one obtains is the $O(N)$ gauge theory with action

$$
S = \frac{1}{2 \pi} \int \text{Tr} \left( (D_\mu X)^2 - i \theta_+^T D_- \theta_+ + g_s^2 F^2 - i g_s^2 \lambda_- D_+ \lambda_- + 2i \theta_+ \lambda_- \gamma_i X^i - i \chi D_+ \chi + \frac{1}{g_s^2} [X^i, X^j]^2 \right),
$$

in units where $\alpha' = 1$, $g_s$ is the string coupling for $E_8 \times E_8$ heterotic strings, and the $\gamma_i$ are defined as in [12]. Here the eight scalars $X^i$ and their superpartners, the right-moving fermionic fields $\theta_+$ lie in the symmetric rep of $O(N)$. This theory has a $Spin(8)$ R-symmetry group corresponding to the group of rotations in spacetime. The $\theta_+$ fields transform as the $8_c$ while the $X^i$ transform as the $8_v$. The gauge multiplet lies in the adjoint of $O(N)$ and consists of a gauge boson $A_\mu$ and a set of eight left-moving fermionic fields $\lambda_-$ which transform in the $8_c$. The $\chi$ fields are purely left-moving and transform as the fundamental rep of $O(N)$ and in the fundamental of the $SO(32)$ group of spacetime gauge symmetry. The necessity of including the $\chi$ fields may be seen either from the point of view of spacetime anomaly cancellation in Type I string theory [13–14], where a background of 32 ninebranes is required, giving rise to the $\chi$ fields as 1-9 open strings, or from the point of view of anomaly cancellation in the two-dimensional gauge theory [15].

It is natural to conjecture that this theory provides a nonperturbative definition of uncompactified heterotic string theory. However, we know from the work of Polchinski and Witten [16] that Type IB perturbation theory, on which this description is based, will typically break down as the radius of the $S^1$ is varied due to dilaton and graviton tadpoles not cancelling winding number by winding number. The only way to avoid this breakdown of perturbation theory is to choose a Wilson line for the $Spin(32)/\mathbb{Z}_2$ degrees of freedom corresponding to

$$
W = \left( \frac{1}{2} \right)^8, 0^8 \right),
$$

(2.3)
in standard notation. In Type IA language this corresponds to putting eight D-eightbranes on each of the orientifold planes. The $SO(32)$ gauge symmetry is broken to $SO(16) \times SO(16)$. The Wilson line splits the $\chi$ degrees of freedom into two sets of sixteen, which will have boundary conditions differing by a sign as one goes around the $S^1$ direction.

We have expressed the action (2.2) in terms of the string coupling of the heterotic string. This is related to the gauge theory coupling by $g_s = 1/g$ and $g_s$ scales inversely with the worldsheet length scale. The perturbative heterotic string theory should therefore be recovered in the infrared limit. In this limit the gauge theory should flow to some nontrivial superconformal fixed point. We conjecture this superconformal field theory is the heterotic sigma model with $(0,8)$ supersymmetry based on the orbifold theory

$$ (\mathbb{R}^8)^N/(S_N \ltimes (\mathbb{Z}_2)^N) , $$

where the $S_N$ acts by permuting the $\mathbb{R}^8$, their fermionic partners and gauge fermion degrees of freedom. The $\mathbb{Z}_2$’s act by reflecting the different components of the $O(N)$ vectors. In the free-string limit the space of states will be shown to correspond to the Fock space of second-quantized heterotic strings.

3. Spectrum of States

3.1. Heterotic Strings

The techniques of [17–20] carry over straightforwardly to the present case. The Hilbert space of the $S_N$ orbifold is decomposed into twisted sectors for each of the conjugacy classes $[g]$ of $S_N$

$$ \mathcal{H} = \bigoplus_{[g]} \mathcal{H}_{[g]} . $$

These conjugacy classes may be represented as cycle decompositions and then the Hilbert spaces $\mathcal{H}_{[g]}$ may be further decomposed as symmetric products of Hilbert spaces $\mathcal{H}_{(n)}$ where $\mathcal{H}_{(n)}$ is the $\mathbb{Z}_n$ invariant subspace of a single heterotic string on $\mathbb{R}^8 \times S^1$ with winding number $n$ and Wilson line $W$. Modding out by the additional $\mathbb{Z}_2$’s is achieved by extra GSO projections acting on the gauge fermions.

This space of states is represented by $n$ copies of the fields $x^I_I(\sigma)$ and $\chi^I_I(\sigma)$ with $I = 1, \ldots, n$ and $\sigma \in [0,2\pi]$, with the cyclic boundary conditions

$$ x^I_I(\sigma + 2\pi) = x^I_{I+1}(\sigma) , $$

$$ \chi^I_I(\sigma + 2\pi) = \epsilon_I O_W \chi^I_I(\sigma) , $$

3
where $\epsilon_I = \pm 1$ corresponds to reflection elements of $O(N)$ and $O_W$ is the action of the Wilson line $W$ on the fermions $\chi$ which is simply multiplication by $-1$ for $r = 1 \cdots 16$ and $+1$ for $r = 17, \cdots, 32$. These fields may be combined into a single set of fields $x^i(\sigma)$ and $\chi^r(\sigma)$ living on $\sigma \in [0, 2\pi n]$ where
\[
\begin{align*}
x(\sigma + 2\pi n) &= x(\sigma) \\
\chi^r(\sigma + 2\pi n) &= \epsilon(O_W)^n \chi^r(\sigma),
\end{align*}
\] (3.3)
and $\epsilon = \pm 1$. We label the sectors with different boundary conditions on the $\chi$'s by $P$ for periodic and $A$ for antiperiodic. Since the first group of 16 $\chi$'s can have different boundary conditions to the second group, we need one label for each group. For $n$ even, we have the $PP$ and $AA$ sectors of the usual heterotic string, while $n$ odd gives the $AP$ and $PA$ sectors. The vacuum energy vanishes in the $AP$ and $PA$ twisted sectors, while in the twisted sector with $PP$ boundary conditions the left-movers make a nonzero contribution equal to $1/n$, and in the $AA$ sector $-1/n$. When we rescale $L_0 + \bar{L}_0$ by a factor of $n$ so it is canonically normalized with respect to $x^i(\sigma)$ the vacuum energy is 1 in the $PP$ sector and $-1$ in the $AA$ sector as expected for a single heterotic string. The usual $E_8 \times E_8$ heterotic string has two GSO projections consisting of keeping states invariant under $(-1)^{F_1}$ and $(-1)^{F_2}$, where $(-1)^{F_1}$ anticommutes with the first group of $\chi^r$ and commutes with the second, and vice-versa for $(-1)^{F_2}$. The $Spin(32)/\mathbb{Z}_2$ heterotic theory on the other hand has just a single GSO projection consisting of $(-1)^{F_1+F_2}$. This projection is to be identified with a $\mathbb{Z}_2$ element of $O(N)$ which acts as $-1$ on vectors $\chi^r$.

Fundamental heterotic strings are obtained in a large $N$ limit, by considering the twisted sector for some cycle of length $n$ with $n/N$ finite. This corresponds to considering a string carrying a finite longitudinal momentum $p_+ = n/N$. Invariance under $\mathbb{Z}_n$ implies that $L_0 - \bar{L}_0$ is a multiple of $n$. The mass of such a state diverges as $N \to \infty$ unless $L_0 - \bar{L}_0$ acting on the state vanishes. In this way, the usual level-matching condition of the heterotic string is recovered.

What happened to the second GSO projection of the $E_8 \times E_8$ heterotic string? The theory we have constructed describes $E_8 \times E_8$ heterotic strings compactified on some large $S^1$ with $n$ quanta of Kaluza-Klein momentum present. This is equivalent via T-duality to an $Spin(32)/\mathbb{Z}_2$ heterotic string with winding number $n$ compactified on a small $S^1$ with the Wilson line $W$ present. The worldsheet fields that emerge from the sigma model are naturally identified with these fields, hence only one GSO projection appears. To take the large $N$ limit it is convenient to T-dualize the sigma model fields, which introduces the second GSO projection, as in the usual $E_8 \times E_8$ theory. We see therefore that the Hilbert space $\mathcal{H}$ corresponds to the second-quantized Fock space of free $E_8 \times E_8$ heterotic strings.
3.2. Type IA BPS Bound States

When the right-movers of the heterotic strings are in their ground states, the states found above will be BPS saturated and using the 9-11 flip duality (2.1) may be reinterpreted as BPS bound states of Type IA D-particles (and tensor products of such states). The states we will be interested in here break half of the spacetime supersymmetry and correspond to bound states at threshold of Type IA D-particles. The infrared limit of the gauge theory amounts to considering the Born-Oppenheimer approximation in the supersymmetric quantum mechanics of D-particles on the branch of the moduli space when they are stuck to one of the orientifold planes. The above results for the BPS heterotic string states found above carry over immediately to this case. The BPS bound states of \( N \) D-particles are to be identified with the massless states in the \( \mathbb{Z}_N \) twisted sector. For \( N \) even one finds the adjoint of \( SO(16) \times SO(16) \) in the \( 8_v + 8_s \), and a gauge singlet in the \( 8_v \times (8_v + 8_s) \) coming from the AA sector, while for \( N \) odd the \( (1, 128) + (128, 1) \) in the \( 8_v + 8_s \), is found in the \( AP \) and \( PA \) sectors.

One may also consider the branch when the D-particles move off of the orientifold plane. On this branch of the moduli space the scalar \( A \) coming from the gauge field up in two dimensions is treated as a slow variable in the Born-Oppenheimer approximation. For the moment, let us focus on the case of two D-particles considered in [9]. There are two distinct limits to be considered. When \( l_{11} \ll A \ll l_s \), where \( l_s \) is the string length, the relevant quantum mechanics is obtained by dimensionally reducing the \( N = 2 \) system in two dimensions. Different boundary conditions for the \( \chi \) fields in two dimensions give rise to different sectors of the SUSY quantum mechanics with different Hamiltonians. In the sector where all the \( \chi \) fields satisfy antiperiodic boundary conditions the states found [9] included the adjoint of \( SO(16) \times SO(16) \) in the

\[
8_v + 8_s \tag{3.4}
\]

and a gauge singlet in the

\[
8_v \times (8_v + 8_s) . \tag{3.5}
\]

The quantum numbers of these states match the massless states coming from the AA sector of the CFT discussed above. This is a necessary condition for the exact wavefunctions to smoothly match as one moves from one branch of the moduli space to the other. The one-loop corrections to the effective kinetic term for \( A \) [7] are of the form \( gL_A \dot{A}^2/A^3 \), indicating that corrections to this approximation set in at the eleven-dimensional Planck scale \( l_{11} \). For \( l_{11} \ll A \ll l_s \) the wavefunction of these states should vary as \( c_1 A + c_2 \) where \( c_1 \) and \( c_2 \) are constants, which can in principle be determined by matching onto the behavior as \( A \rightarrow l_{11} \).
and as $A \to l_s$. We know these states should spread onto the branch of moduli space at $A = 0$, so we expect $c_2$ to be nonzero. In the limit $A \gg l_s$, the dominant interaction is closed string exchange, and we expect the Hamiltonian to flow to that describing a Type IIA D-particle, interacting with its mirror image [7]. The $\chi$ modes will become massive, and completely decouple from this Hamiltonian. The zero energy states will correspond to a singlet of the gauge group in the $(8_v + 8_s) \times (8_v + 8_c)$ of $Spin(8)$ which are the quantum numbers of the ground state of a single Type IIA D-particle. This state does not match onto the charged states found for $A \ll l_s$, so the wavefunction of (3.4) should vanish when $A \gg l_s$. In this way, we see the states (3.4) are localized near the orientifold plane.

In the sector when all the $\chi$ fields satisfy periodic boundary conditions one finds all states become massive, and this sector decouples from the low-energy description of the D-particles. When half of the $\chi$’s satisfy periodic boundary conditions, and half are antiperiodic, a similar calculation to the one in [7] shows the normal-ordering terms appearing in the term in the Hamiltonian linear in $A$ vanish. In particular, the $\chi$ fields become massive and may be integrated out leaving states that are singlets under $SO(16) \times SO(16)$. The states found in the Born-Oppenheimer approximation arise from quantizing the fermion zero modes arising from the trace part of $\theta^+$ and the $\lambda_-$, the left-moving superpartners of $A$. This gives rise to the $(8_v + 8_s) \times (8_v + 8_c)$ of $Spin(8)$ which are the quantum numbers of the ground state of a single Type IIA D-particle - as expected from duality with M-theory. Note in this case the Hamiltonian describing these degrees of freedom is the same for all $A \gg l_{11}$. These states are not localized near the orientifold planes.

Let us now generalize these results to arbitrary numbers of D-particles. For $N$ even, we can choose $A$ to break $O(N) \to U(1) \times SU(N/2)$. The $U(1)$ component of $A$ may be treated in a similar way to the $N = 2$ case already discussed. In the sector where the $\chi$’s satisfy $AP$ or $PA$ boundary conditions, the other light degrees of freedom correspond to the $SU(N/2)$ quantum mechanics of Type IIA D-particles with fixed center of mass. The Hamiltonian for these slow modes computed in the Born-Oppenheimer approximation matches that of $N/2$ Type IIA D-particles. This is to be expected from the M-theory point of view – far from the ends of interval, the states should match that of uncompactified M-theory. In section 5 we will comment on finding BPS bound states for this system. In the sector where the $\chi$’s obey $AA$ boundary conditions the $U(1)$ component of $A$ behaves as for the $N = 2$ case, and again the other degrees of freedom again take the form of the $SU(N/2)$ quantum mechanics of $N/2$ Type IIA D-particles. Assuming this $SU(N/2)$ quantum mechanics yields one $L^2$ normalizable bound state, one finds BPS bound states which match onto the charged states appearing on the other branch of the moduli space (i.e. the adjoint and gauge singlet states) and will be peaked near the orientifold plane.
by the argument given above. Finally when $\chi$’s obey $PP$ boundary conditions all states become massive and hence non-BPS.

For $N = 1$ the D-particle is always stuck to the orientifold, and the BPS states are just the massless states of the CFT found in the previous section. Only $AP$ and $PA$ boundary conditions are possible for this case and as mentioned before, one finds the $(1, 128) + (128, 1)$ of $SO(16) \times SO(16)$. For general $N > 1$ odd, $A$ can be chosen to break $O(N) \to \mathbb{Z}_2 \times U(1) \times SU((N - 1)/2)$. This branch of the moduli space describes one D-particle stuck on one of the orientifold planes, and $(N - 1)/2$ off of the orientifold. The slow modes consist of the $\chi$ modes arising from open strings running between D-eightbranes and the D-particle on the orientifold plane, and modes identical to $(N - 1)/2$ Type IIA D-particles. The $\chi$ zero modes imply such states will always carry the spinor charges of $SO(16) \times SO(16)$. The quantum numbers of these match the $N$ odd states on the other branch discussed above. The $U(1)$ component of the gauge multiplet will behave in the same way as the $N = 2$ case above, and the wavefunctions of these states will be localized near the orientifold planes.

4. Heterotic String Interactions

The two-dimensional gauge theory (2.2) in the infrared limit gives rise to the Fock space of free heterotic strings. Interaction terms should correspond to irrelevant perturbations of the orbifold sigma model. Dijkgraaf, Verlinde, Verlinde [18] considered such terms in the Type II case, and much of their argumentation applies here. The idea is that the interactions correspond to transpositions of the $x_I$ eigenvalues when they coincide. In the heterotic case one must also match the gauge fermion degrees of freedom, and transpose these at the same time. This will give rise to splitting and joining interactions of the long strings discussed above.

For the right-moving sector, the construction of the interaction vertex precisely mimics that of [18]. We consider two eigenvalues $x_1$ and $x_2$ which are interchanged under a $\mathbb{Z}_2$ twist. In terms of the linear combinations $x_\pm = x_1 \pm x_2$, the $\mathbb{Z}_2$ flips the sign of $x_-$ (and likewise for the right-moving fermionic modes $\theta$). This $\mathbb{Z}_2$ orbifold is well-known, and the twist operators are defined by the operator product relations

$$
\partial x^i_-(z) \cdot \sigma(0) \sim z^{-\frac{1}{2}} \tau^i(0) \\
\theta^a_-(z) \cdot \Sigma^i(0) \sim z^{-\frac{1}{2}} \gamma^i a \Sigma^a \\
\theta^a_-(z) \cdot \Sigma^a(0) \sim z^{-\frac{1}{2}} \gamma^a i \Sigma^i .
$$

(4.1)
One can then show that the operator
\[ V_R = \tau^i \Sigma^i , \] (4.2)
is the unique, least irrelevant perturbation that preserves \( Spin(8) \) spacetime rotations and spacetime supersymmetry. This operator has conformal weight \((0, \frac{3}{2})\).

Now consider the left-moving sector, and bosonize the gauge fermion modes \( \chi \), to yield sixteen bosonic coordinates \( x^M \), with \( M = 1, \cdots, 16 \) living on the Cartan torus of the gauge group. We define the \( \mathbb{Z}_2 \) eigenvectors \( x^M_{\pm} = x^M_1 \pm x^M_2 \). The twist operator for the left-moving spacetime coordinates \( x_-(\bar{z}) \) is constructed as above. One can also define the twist operators for the \( x^M_{-} \) in a similar way, but now one must remember the \( x^M \) are compact bosonic coordinates. The action \( x^M_{-} \rightarrow -x^M_{-} \) has two fixed points, so we have two twist operators \( \bar{\sigma}_{\pm} \) for each \( x^M \). We now wish to construct the least irrelevant perturbation to the CFT preserving \( Spin(8) \) rotations, spacetime supersymmetry, and gauge invariance. We also demand that the perturbation be invariant under \( z \rightarrow e^{i \theta} z \), which means the conformal weight of the left-moving piece should be \((\frac{3}{2}, 0)\). The combination of left-moving twist operators which satisfies these conditions is
\[ V_L = \sum \bar{\sigma}(0) \cdot \prod_{M=1}^{16} \bar{\sigma}_s^M , \] (4.3)
where \( s_M = \pm \) and the sum is over all permutations of the \( s_M \).

The final interaction term that appears in the sigma model is then a sum over pairs of eigenvalues that may be interchanged
\[ S_{int} = \lambda \sum_{I<J} \int d^2 \bar{z} (V_L \otimes V_R)_{IJ} , \] (4.4)
where \( I, J = 1, \cdots, N \). The coupling \( \lambda \) will scale inversely with the worldsheet length scale, therefore \( \lambda \) will scale linearly with \( g_s \). The interaction corrections contained in the supersymmetric gauge theory \((2.2)\) therefore reproduce heterotic string interactions to first order in \( g_s \).

5. Comments on Type II Bound States

5.1. Bound states of \( N \) D-particles

We saw above that the quantum mechanics describing Type IA D-particles far from the orientifold plane reduced to that of a collection of Type IIA D-particles. The quantum
mechanics of Type IIA D-particles has previously been studied in [21–25]. For M(atrix) theory to be correct for the case of eleven uncompactified dimensions, these Type IIA D-particles must form bound states at threshold. The prediction is that one normalizable bound state at threshold of $N$ D-particles appears for every $N$, and this bound state lies in an ultra-short multiplet of the ten-dimensional Type IIA supergravity.

If such a normalizable bound state appears in uncompactified Type IIA, then it should show up as a normalizable bound state when we further compactify on a torus. We can then recast the problem into a statement about the ground states of $d = 2$ $SU(N)$ Yang-Mills theory with $(8,8)$ supersymmetry (here we have factored out the $U(1)$ describing the center of mass degrees of freedom). That is, there should be one $L^2$ normalizable zero-energy ground state which appears regardless of the boundary conditions [26].

We can make a heuristic argument for the existence of such a state as follows. Since the state is normalizable (in fact it should fall off as $1/r^7$ up in ten dimensions), it should remain a bound state if we add a perturbation that only effects the large distance behavior of the fields. In fact, if we choose the perturbation judiciously, the bound state at threshold should become a bound state with mass gap.

The two-dimensional super Yang-Mills theory can be thought of as the dimensional reduction of a $d = 4$ $\mathcal{N} = 4$ theory. The vector multiplet in four dimensions can be decomposed into a single $\mathcal{N} = 1$ vector multiplet and three $\mathcal{N} = 1$ chiral multiplets which we denote by $X$, $Y$ and $Z$, all in the adjoint rep of the gauge group. Let us assume the perturbation of the superpotential parametrized by $m$

$$W = \text{Tr} \left( X[Y, Z] - \frac{1}{2} m(X^2 + Y^2 + Z^2) \right), \quad (5.1)$$

behaves in the way described. We may now use a variant of the arguments of [27]. Since the ground state we are interested in now has a mass gap, we can actually treat $m$ as being large, and use semiclassical methods.

The semiclassical vacua correspond to critical points of the superpotential (5.1) which are solutions of

$$[X, Y] = mZ$$
$$[Y, Z] = mX$$
$$[Z, X] = mY. \quad (5.2)$$

These equations are just the commutation relations of $SU(2)$. There three are classes of solutions to these equations: (i) trivial solution for which $X = Y = Z = 0$, (ii) non-trivial reducible embedding of $SU(2)$ into $SU(N)$, (iii) irreducible embedding of $SU(2)$ into $SU(N)$. For cases (i) and (ii) part of the gauge symmetry is unbroken and one is
left with a two-dimensional gauge theory with (2, 2) supersymmetry. These vacua will not have mass gaps due to massless fermions. Case (iii) however leads to a solution which completely breaks the gauge symmetry. In this case all fields become massive and one is left with a single bosonic ground state. Including the fermion zero modes coming from the center of mass degrees of freedom, one obtains a single bound state of $N$ D-particles with the expected spacetime quantum numbers.

5.2. Bound states of $N$ D-particles and $M$ D-eightbranes

Studies of solitonic solutions of Type IIA supergravity do not show any sign of supersymmetric bound states of D-particles and D-eightbranes [28]. We can now understand this from the point of view of the two-dimensional supersymmetric gauge theory describing this system. As in the Type IA case, we have a theory with (0, 8) supersymmetry, this time with $U(N)$ gauge multiplet $(A, \lambda_-)$. The matter content consists of multiplets $(X^i, \theta_+)$ in the adjoint of $U(N)$, and left-moving fermions in the $(N, \bar{M}) + (\bar{N}, M)$. Consider the conformal field theory this will flow to in the infrared. One will be left with the fields living on the Cartan torus of $U(N)$ corresponding to the diagonal components of $X^i$, $\theta_+$ and $\lambda_-$ together with the other left-moving fermions. For all $N$ and $M$ this conformal field theory will not have a massless ground state, hence no BPS bound state is possible. Massless ground states are possible if one considers for example two sets of separated D-eightbranes. The condition then is that there is a choice of boundary conditions on the left-moving fundamental fermions which gives zero contribution to the vacuum energy.

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Note added

While this manuscript was being completed the preprint [29] appeared in which closely related results are reported. I understand related ideas have also been considered by R. Dijkgraaf and S.J. Rey and by P. Horava.
References

[1] J. Polchinski, “TASI Lectures on D-Branes,” hep-th/9611050; J. Polchinski, S. Chaudhuri and C. Johnson, “Notes on D-Branes,” hep-th/9602052.

[2] E. Witten, “String Theory Dynamics in Various Dimensions,” Nucl. Phys. B443 (1995) 85, hep-th/9503124; C. Hull and P. Townsend “Unity of Superstring Dualities,” Nucl. Phys. B438 (1995) 109, hep-th/9410167.

[3] J. Schwarz, “The Power of M-Theory,” Phys. Lett. B367 (1996) 97, hep-th/9510086.

[4] T. Banks, W. Fischler, S. Shenker and L. Susskind, “M-Theory as a Matrix Model: A Conjecture,” hep-th/9610043.

[5] P. Horava and E. Witten, “Heterotic and Type I String Dynamics from Eleven Dimensions,” Nucl. Phys. B460 (1996) 506, hep-th/9510209.

[6] P. Horava, “Strings on Worldsheet Orbifolds,” Nucl. Phys. B327 (1989) 461; J. Dai, R.G. Leigh and J. Polchinski, “New Connections Between String Theories,” Mod. Phys. Lett. A4 (1989) 2073; P. Horava, “Background Duality of Open-String Models,” Phys. Lett. B231 (1989) 251.

[7] U. Danielsson and G. Ferretti, “The Heterotic Life of the D-particle,” hep-th/9610082.

[8] S. Kachru and E. Silverstein, “On Gauge Bosons in the Matrix Model Approach to M Theory,” hep-th/9612162.

[9] D.A. Lowe, “Bound States of Type I′ D-particles and Enhanced Gauge Symmetry,” hep-th/9702006.

[10] N. Kim and S.J. Rey, “M(atrix) Theory on an Orbifold and Twisted Membrane,” hep-th/9701139.

[11] M.R. Douglas, “Branes within Branes,” hep-th/9512077.

[12] M. Green, J. Schwarz and E. Witten, “Superstring Theory,” Cambridge University Press, 1987.

[13] M. Green and J. Schwarz, “The hexagon gauge anomaly in Type I superstring theory,” Nucl. Phys. B255 (1985) 93.

[14] Y. Arakane, H. Itoyama, H. Kunitomo and A. Tokura, “Infinity Cancellation, Type I′ Compactification and String S-Matrix Functional,” hep-th/9609151.

[15] T. Banks, S. Seiberg and E. Silverstein, “Zero and One-dimensional Probes with N=8 Supersymmetry,” hep-th/9703052.

[16] J. Polchinski and E. Witten, “Evidence for Heterotic-Type I String Duality,” Nucl. Phys. B460 (1996) 525, hep-th/9510169.

[17] R. Dijkgraaf, G. Moore, E. Verlinde and H. Verlinde, “Elliptic Genera of Symmetric Products and Second Quantized Strings,” hep-th/9608096.

[18] R. Dijkgraaf, E. Verlinde and H. Verlinde, “Matrix String Theory,” hep-th/9703030.

[19] T. Banks and N. Seiberg, “Strings from Matrices,” hep-th/9702187.

[20] L. Motl, “Proposals on Nonperturbative Superstring Interactions,” hep-th/9701023.
[21] U. Danielsson, G. Ferretti and B. Sundborg, “D-particle Dynamics and Bound States,” *Int. Jour. Mod. Phys.* **A11** (1996) 5463, hep-th/9603081.

[22] D. Kabat and P. Pouliot, “A Comment on Zerobrane Quantum Mechanics,” *Phys. Rev. Lett.* **77** (1996) 1004, hep-th/9603127.

[23] M.R. Douglas, D. Kabat, P. Pouliot and S.H. Shenker, “D-branes and Short Distances in String Theory,” *Nucl. Phys.* **B485** (1997) 85, hep-th/9608024.

[24] M. Claudson and M.B. Halpern, “Supersymmetric Ground State Wave Functions,” *Nucl. Phys.* **B250** (1985) 689; M. Baake, P. Reinicke and V. Rittenberg, “Fierz Identities for Real Clifford Algebras and the Number of Supercharges,” *J. Math. Phys.* **26** (1985) 1070; R. Flume, “On Quantum Mechanics with Extended Supersymmetry and Nonabelian Gauge Constraints,” *Ann. Phys.* **164** (1985) 189.

[25] B. de Wit, J. Hoppe, H. Nicolai, “On the Quantum Mechanics of Supermembranes,” *Nucl. Phys.* **B305** (1988) 545.

[26] A. Sen, “A Note on Marginally Stable Bound States in Type II String Theory,” *Phys. Rev.* **D54** (1996) 2964; hep-th/9510229.

[27] E. Witten, “Bound States of Strings and p-Branes,” *Nucl. Phys.* **B460** (1996) 335, hep-th/9510135.

[28] G. Papadopoulos and P.K. Townsend, “Kaluza-Klein on the Brane,” *Phys. Lett.* **B393** (1997) 59, hep-th/9609097.

[29] T. Banks and L. Motl, “Heterotic Strings from Matrices,” hep-th/9703218.