Caching with Unknown Popularity Profiles in Small Cell Networks

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Abstract—A heterogenous network is considered where the base stations (BSs), small base stations (SBSs) and users are distributed according to independent Poisson point processes (PPPs). We let the SBS nodes to posses high storage capacity and are assumed to form a distributed caching network. Popular data files are stored in the local cache of SBS, so that users can download the desired files from one of the SBS in the vicinity subject to availability. The offloading-loss is captured via a cost function that depends on a random caching strategy proposed in this paper. The cost function depends on the popularity profile, which is, in general, unknown. In this work, the popularity profile is estimated at the BS using the available instantaneous demands from the users in a time interval $[0, \tau]$. This is then used to find an estimate of the cost function from which the optimal random caching strategy is devised. The main results of this work are the following: First it is shown that the waiting time $\tau$ to achieve an $\epsilon > 0$ difference between the achieved and optimal costs is finite, provided the user density is greater than a predefined threshold. In this case, $\tau$ is shown to scale as $N^2$, where $N$ is the support of the popularity profile. Secondly, a transfer learning-based approach is proposed to obtain an estimate of the popularity profile used to compute the empirical cost function. A condition is derived under which the proposed transfer learning-based approach performs better than the random caching strategy.

I. INTRODUCTION

The advent of multimedia-capable devices such as smartphones at economical costs has triggered the growth of mobile data traffic at an unprecedented rate. For instance, computationally intensive applications like content-based video analysis alone contributes a large fraction of the total mobile data traffic. This trend is likely to continue at an alarming rate, demanding wireless service providers to reevaluate the design strategies for the next generation wireless infrastructure - providing high data rates for tens of thousands of users, improving the spectral efficiency beyond what is currently available, improving coverage and cell-capacity, and engineering efficient inter-device signalling techniques.

A promising approach to address the wireless infrastructure problem is to deploy small cells which can offload a significant amount of data from the macro BS [2]. A vast majority of research in next generation wireless infrastructure has focussed on the design and implementation of such small cells [3]. However, a potential shortcoming of this infrastructure is that, during peak traffic hours, the backhaul link capacity requirement to support data traffic is enormously high [4]. On the flip side, the cost incurred in designing high capacity backbone network for small cells is quite discouraging especially from an economic standpoint. Therefore, in the grand scheme of things, small cell-based solutions alone will not suffice to efficiently solve the quality of service requirements associated with peak traffic demands; thus, one is motivated to further investigate the advantages offered by heterogeneous networks.

A recent development in this direction is to improve the accessibility of data content to users by storing the most popular data files in the local caches of small cell BSs, with the objective of reducing the peak traffic rates [5] - [7]. This is commonly referred to as “caching” and has attracted significant attention. For instance, in [8], it was shown that, when cached-content demand was uniformly distributed, joint optimization of caching and coded multicast delivery significantly improved the caching gains; this was extended to a more general decentralized setting in [9]. The problem setup considered in [8] was extended to nonuniform distribution on demand in [10]. The tradeoff between the performance gain of coded caching and delivery delay in computationally intensive applications such as video streaming was characterized in [11], [12]; while, in [13], coded caching was achieved for content delivery networks with two layers of caches. A polynomial-time heuristics was proposed in [14] to address the NP-hard optimization problem of maximizing the caching utility of users based on their mobility patterns.

Caching has also made advances in device-to-device (D2D) communications. In [15], the outage-throughput tradeoff was characterized for D2D nodes having access to cached files in a library, but obtained the requested file from another node which had that file in its cache through a D2D link and not via the base station. In [16], the conflict between collaboration-distance and interference was identified among D2D nodes to maximize frequency reuse by exploiting distributed storage of cached content in those nodes. In [17], coded caching was shown to achieve multicast gain in a D2D network, where users had access to linear combinations of packets from cached files. The NP-hard nature of distributed caching was reported in [18], where approximation algorithms were proposed for video content delivery. In [19], the throughput scaling laws of random caching, where users with pre-cached information made arbitrary requests for cached files, was studied.
A. Main contributions of this paper

In the above cited references, the popularity profile of data files was assumed to be known perfectly (cf. [20] - [22], where a learning-based approach was used in which the popularity profile was estimated to devise the caching strategy, without any theoretical analysis of time and sample complexities, to achieve the desired performance). However, in practical applications, this turns out to be a very weak assumption. In this paper, we relax the assumption on the knowledge of the popularity profile to devise a caching strategy.

We consider a heterogeneous network where the users, BS and SBSs are assumed to be distributed according to independent PPPs. Each SBS is assumed to employ a random caching strategy with no caching at the user terminal (see [15]). A protocol model for communications is proposed using which a cost, which captures the backhaul-link overhead that depends on the popularity profile, is derived (Section II). Secondly, assuming a Poisson request model, a centralized cost function to optimize the caching probability. Thus, the profile was estimated to devise the caching strategy, without a learning-based approach was used in which the popularity

\begin{itemize}
  \item [(i)] The waiting time is finite, provided the user density is greater than a threshold.
  \item [(ii)] The waiting time scales as \(N^2\), where \(N\) is the total number of cached data files in the system.
  \item [(iii)] These results are improved by using a transfer learning (TL)-based approach to estimate the popularity profile wherein samples from other domains such as those obtained from a social network are used to improve the estimation accuracy. The minimum number of source domain samples required to achieve a better performance is derived. Employing the TL-based approach, a finite waiting time is achieved for all user density.
  \item [(iv)] In the TL-based approach, the waiting time is a function of the “distance” between the distribution of the files requested and the distribution of the source domain samples (the notion of distance is made precise in the proof of Theorem 3 Section III-B).
\end{itemize}

The following are the main findings of our study:

The rest of the paper is organized as follows. In Section II the system model is first presented followed by a communications protocol for caching. The two methods for estimating the popularity profile and its corresponding waiting time analysis are developed in Section III-A and Section III-B respectively. Concluding remarks are provided in Section IV. The proofs of main theorems of the paper are relegated to appendices.

II. System Model and Communications Protocol

A heterogeneous cellular network is considered where the set of users (denoted \(\Phi_u \subseteq \mathbb{R}^2\)), the set of BSs (denoted \(\Phi_b \subseteq \mathbb{R}^2\)), and the set of SBSs (denoted \(\Phi_s \subseteq \mathbb{R}^2\)) are distributed according to independent PPP with density \(\lambda_u, \lambda_b\) and \(\lambda_s\), respectively [23]. Each user independently requests for a data file of size \(B\) bits in \(\mathcal{F} \triangleq \{f_1, f_2, \ldots, f_N\}\) with popularity \(\mathcal{P} \triangleq \{p_1, \ldots, p_N\}\), \(\sum_{i=1}^N p_i = 1\), and is assumed to be stationery across time. In a typical heterogeneous cellular network, BS fetches the file using its backhaul link to serve the user. During peak data traffic hours, this results in an information-bottleneck both at BS as well as in its backhaul link. To overcome this problem, caching of the most popular files (either at the user nodes or at SBSs) is proposed – the requested file will be served directly by one of the neighboring SBS depending on the availability of the file in its local cache.

The performance of caching depends on the density of SBS nodes, the cache size, the users’ request rate, and the caching strategy. It is assumed that the SBS can cache up to \(M\) files, each of length \(B\) bits. Each SBS \(s \in \Phi_s\) caches its content in an i.i.d. manner by generating \(M\) indices distributed according to \(\Pi \triangleq \{\pi_i : f_i \in \mathcal{F}\}\), \(\sum_{i=1}^N \pi_i = 1\) (see [15]).

The following communication protocol is used in this paper which determines the set of neighbor SBS nodes for any user \(u\). Each SBS \(s \in \Phi_s\) communicates with a user \(u\) at \(x_u \in \Phi_u\) if \(||x_u - x_s|| < \gamma\) (\(\gamma > 0\)); this condition determines the communication radius. Note that, we have ignored the interference constraint. The set of potential neighbors of user \(u\) at \(x_u\) is denoted \(\mathcal{N}_u \triangleq \{y \in \Phi_s : ||y - x_u|| < \gamma\}\).

The performance of the caching scheme depends on the metric used to optimize the strategy. We focus on the performance of a typical user located at the origin. Since the main goal is to reduce the information-bottleneck at BS, the objective is to minimize the time overhead due to the unavailability of the requested file. The following “offloading loss” metric is used to optimize the caching strategy proposed in this paper:

\[
\mathcal{T}(\Pi, \mathcal{P}) \triangleq \frac{B}{R_0} \mathbb{E}[1\{f_i \notin \mathcal{N}_u\}],
\]

which captures the unavailability of the requested file from a typical user at the origin. \(R_0\) is the rate supported by BS to the user at the origin, and \(\frac{B}{R_0}\) is the time overhead incurred in transmitting the file from BS to the user. The expectation is with respect to \(\Phi_u, \Phi_s\) and \(\mathcal{P}\). The indicator function \(1\{A\}\) is equal to one if event \(A\) occurs, and zero otherwise. Better performance can be achieved by minimizing \(\mathcal{T}(\Pi, \mathcal{P})\), i.e.,

\[
\min_{\Pi \geq 0} \mathcal{T}(\Pi, \mathcal{P})
\]

subject to

\[
\sum_{i=1}^N \pi_i = 1, \quad \pi_i \geq 0,
\]

for \(i = 1, \ldots, N\). To solve the optimization setup (2), we need an expression for \(\mathcal{T}(\Pi, \mathcal{P})\), which is derived in the following.
Theorem 1: For the caching strategy proposed in this paper, the average offloading loss is given by

\[
\mathcal{T}(\Pi, \mathcal{P}) = \frac{B}{R_0} \left[ \sum_{i=1}^{N} \exp\left(-\lambda_u \pi \gamma^2 (1 - \pi_i)^{M_i}\right) \right].
\]  

(3)

Proof: See Appendix A.

In general, the optimization problem in (2) is non-convex. More importantly, the popularity profile \( \mathcal{P} \) is unknown, and is estimated from the available data. Denoting the estimated popularity profile by \( \hat{\mathcal{P}} \triangleq \{\hat{p}_1, \ldots, \hat{p}_N\} \) and the corresponding offloading loss by \( \mathcal{T}(\Pi, \hat{\mathcal{P}}) \), the optimization problem in (4) can be reformulated as follows:

\[
\min_{\Pi \geq 0} \mathcal{T}(\Pi, \hat{\mathcal{P}})
\]

subject to \( \sum_{i=1}^{N} \pi_i = 1 \).

(4)

The solution to the setup (4) differs from that of the original problem in (2). Assuming both (2) and (4) can be solved efficiently, the following situation is analyzed. Let \( \Pi^* \) and \( \Pi^\ast \) denote the optimal solutions to the problems in (2) and (4), respectively. The throughput achieved using \( \Pi^* \) is denoted \( \mathcal{T}^* \triangleq \mathcal{T}(\Pi^*, \mathcal{P}) \). We analyze the offloading loss difference, i.e., \( \mathcal{T}^\ast - \mathcal{T}^* \), where \( \mathcal{T}^\ast \triangleq \mathcal{T}(\Pi^\ast, \mathcal{P}) \).

III. LEARNING COMPLEXITY

In this section, we study two quantities of interest, namely, the time complexity involved in obtaining the samples and estimating the empirical risk to obtain a performance within an error \( \epsilon > 0 \) of the optimal solution. The efficiency of the estimate \( \hat{\mathcal{P}} \) of the popularity profile depends on the number of available samples, which in turn depends on the number of requests made by the users. We first define the user-request model.

Definition 1: (Request Model) Each user requests for a file \( f \in \mathcal{F} \) at a random time \( t \in [0, \infty) \) following an independent Poisson arrival process with density \( \lambda_r > 0 \).

For notational convenience, the density \( \lambda_r > 0 \) is assumed across all the users. A centralized scheme is used where BS collects the requests from all the users in its coverage area in the time interval \([0, \tau]\), to estimate the popularity profile of the requested files. Let the number of users in the coverage area of BS \( b \) be \( \Phi_b \), which is distributed according to a PPP with density \( \lambda_u \). Let the number of requests made by the user \( u \in \Phi_b \) be \( k_u \), which is a two-dimensional ball of radius \( R \) centered at \( 0 \). We assume that requests across the users are known at BS. The requests from the user \( u \) are denoted \( X_u \triangleq \{X^{(1)}_u, \ldots, X^{(k_u)}_u\} \), where \( X^{(l)}_u \in \{1, \ldots, N\} \) denotes the indices of the files in \( \mathcal{F} \), \( l = 0, \ldots, k_u \). After receiving \( X_u \), \( u \in \Phi_b \), in the time interval \([0, \tau]\), BS computes an estimate of the popularity profile as follows:

\[
\hat{p}_i = \frac{1}{\sum_{u \in \Phi_b} k_u} \sum_{l=0}^{k_u} \sum_{u \in \Phi_b} \mathbb{1}\{X^{(l)}_u = i\},
\]

(5)
i = 1, \ldots, N. Given the number \( n_R \) of users in the coverage area of BS, the term \( \sum_{u \in \Phi_b} k_u \) is a PPP with density \( n_R \lambda_u \). Also, \( E \{\bar{p}_i|\{\Phi_u \bigcap \mathcal{B}(0, R)\} = n_R\} = p_i \), from which we conclude that \( \hat{p}_i \) is an unbiased estimator. The estimated popularity profile \( \hat{p}_i \) is shared with every SBS in the coverage area of BS, and then used in (4) to find the optimal caching probability. The derived estimator can be improved by using samples from other related domains, for example, a social network. The term “target domain” is used when samples are obtained only from users in the coverage of BS. Next, we derive the minimum waiting time \( \tau \) required to achieve the desired accuracy of \( \epsilon \).

A. Target domain-only learning

In this subsection, we derive a lower bound on the waiting time \( \tau \) corresponding to the estimator in (5).

Theorem 2: For any \( \epsilon > 0 \), with a probability of at least \( 1 - \delta \), a throughput of \( \mathcal{T}^\ast \leq \mathcal{T}^* + \epsilon \) can be achieved using the estimate in (5) provided

\[
\tau \geq \left\{ \frac{1}{\lambda_r \pi} \log \left( \frac{1}{\epsilon} \right) \right\} + \frac{1}{\lambda_u \pi} \log \left( \frac{1}{\epsilon} \right),
\]

(6)

if \( \lambda_u > \mathcal{L} \), otherwise,

where \( \{x\}^+ \triangleq \max\{x, 0\} \), \( g^* \triangleq (1 - \exp(-2\pi^2)) \), \( \mathcal{L} \triangleq \frac{\pi R}{\log(\frac{2K}{\delta})} \) and

\[
\varepsilon \triangleq \frac{R_0\epsilon}{2B\sup_{\Pi} \sum_{i=1}^{N} g(\pi_i)}.
\]

(7)

Proof: See Appendix B.

To achieve a finite waiting time that results in an accuracy of \( \epsilon > 0 \), the user density \( \lambda_u \) has to be greater than a threshold. Further insights into (6) are obtained by making the following approximation: \( 1 - x \leq e^{-x} \) for all \( x \geq 0 \). This is combined with \( \sup_{\Pi} \mathcal{L} \geq 0 \) to yield the following upper bound on \( \tau \):

\[
\tau \geq \frac{2B^2}{\pi R^2 \lambda_u \lambda_r R_0^2 \epsilon^2} N^2 \log \left( \frac{2N}{\delta} \right).
\]

(8)

The following observations are in order:

(1) The waiting time \( \tau \) to achieve an \( \epsilon \)-offloading loss difference scales as \( N^2 \);

(2) \( \tau \) is inversely proportional to \( (\lambda_u, \lambda_r) \);

(3) as the radius of coverage increases, the delay decreases as \( 1/R^2 \); and

(4) as the file size \( B \) increases, the waiting time scales as \( B^2 \).

The above result is a lower bound on the waiting time per request per user, since the offloading loss is derived for a given request per user. There is an average \( \lambda_r \) requests per unit time per user. Thus, to obtain the waiting time per user, the offloading loss has to be multiplied by \( \lambda_r \). This amounts to replacing \( \epsilon \) by \( \epsilon/\lambda_r \). Therefore, (8) becomes

\[
\tau \geq \frac{2B^2 \lambda_r}{\pi R^2 \lambda_u R_0^2 \epsilon^2} N^2 \log \left( \frac{2N}{\delta} \right).
\]

(9)
It can be observed that the waiting time scales linearly with \( \lambda \). Although the waiting time per user per request tends to zero as \( \lambda \to \infty \), the waiting time per user tends to \( \infty \). This is because the number of requests per unit time approaches \( \infty \), and thus, a small fraction of errors results in an infinite difference in offloading loss leading to an infinite waiting time. In the following subsection, a TL-based approach is devised to improve the waiting time.

### B. Transfer learning to improve the waiting time

In general, the minimum waiting time required to achieve an accuracy of \( \epsilon > 0 \) can be very large. A recent approach to overcome this drawback is to utilize the knowledge obtained from users’ interactions with a social community (termed “source domain”). Specifically, by cleverly combining samples from the source domain and users’ request pattern (i.e., the target domain), one can potentially reduce the waiting time. However, the accuracy is indicative of the dependency between the source and target domains. Thus, TL-based approach and its impact on the time complexity to achieve a given performance accuracy is of paramount importance. TL-based approach was also employed in [24] to negotiate over-fitting problems in estimating the content popularity profile matrix. However, in this paper, we are interested in finding the minimum waiting time to achieve a desired performance accuracy. Furthermore, the model we consider is quite different from that considered in [24].

TL-based approach considered here comprises of two sources from which the samples are obtained, namely, source domain and target domain. An estimate of the popularity profile is obtained as follows. First, using target domain samples, the following parameter is estimated at BS

\[
\hat{S}^{(\text{tar})}_i \triangleq \frac{1}{k_u} \sum_{l=0}^{k_u} \sum_{i \in B(0,R) \cap \Phi_u} 1\{X_u^{(l)} = i\},
\]

\( i = 1, \ldots, N \), similar to [5]. We denote the source domain samples \( \mathcal{X}^s \triangleq \{X_1^s, \ldots, X_N^s\} \), which is drawn i.i.d from a distribution \( \mathcal{Q} \). Here, \( X^s_i = i \) \( (i = 1, \ldots, N) \) denotes that the user corresponding to the \( i \)’th sample has requested the file \( f_i \). Using this, BS computes

\[
\hat{S}^s_i \triangleq \sum_{k=1}^{m} 1\{X^s_i = i\}, \quad i = 1, 2, \ldots, N.
\]

Now, BS uses (10) and (11) to compute an estimate of \( \hat{p}^{(tl)}_i \) (the superscript \( (tl) \) indicates transfer learning) given by

\[
\hat{p}^{(tl)}_i \triangleq \frac{\hat{S}^{(\text{tar})}_i + \hat{S}^s_i}{\sum_{u \in B(0,R) \cap \Phi_u} k_u + m}.
\]

Using this estimate, we provide a bound on the time complexity in the following theorem.

**Theorem 3:** For any accuracy \( \epsilon > 0 \), with a probability of at least \( 1 - \delta \), a throughput of \( \bar{T}^* \leq T^* + \epsilon \) can be achieved using the estimate in (12) provided

\[
\tau \geq \left\{ \frac{1}{\lambda_u (1 - e^{-2p q \epsilon})} \log \left( \frac{1}{\epsilon} \right) + \infty \right\}
\]

\[
\text{if } \lambda_u > \rho, \quad (14)
\]

\[
\text{otherwise},
\]

where \( \rho \triangleq \frac{1}{\pi R^2} \left( \log \frac{2N}{\delta} - 2\epsilon^2 \right), \epsilon \triangleq \bar{\epsilon} - \|P - Q\|_H, \Lambda \triangleq \frac{1}{\lambda_u \pi R^2} \left( \log \frac{2N}{\delta} - 2\epsilon^2 m \right), \quad (15)
\]

\[
\bar{\epsilon} \triangleq \frac{2B \sup_{\Pi \in \mathcal{H}} \left( \sum_{g=1}^{G} \sigma_g \right)}{\pi R^2} \text{ and } g(\pi_i) \triangleq \exp \left( -\lambda_u \pi \gamma^2 (1 - \pi_i) \right) M \}.
\]

**Proof:** See Appendix C.

From Theorem 3 we see that under suitable conditions the TL-based approach performs better than the source domain sample-based agnostic approach. The following inferences are drawn:

1. The minimum user density to achieve a finite delay is reduced by a positive offset of \( 2\epsilon^2 \) in large enough sample-based agnostic approach.

2. The finite delay achieved is smaller compared to source domain sample-based agnostic approach for large enough source samples, and the distributions are “close.” This is made more precise in the following corollary.

**Proposition 1:** For any \( \epsilon > 0 \) and \( \delta \in [0,1] \), the TL-based approach performs better than the source sample-based agnostic approach provided the number of source samples satisfies

\[
m \geq \frac{1}{2\epsilon^2} \left[ \log \left( \frac{2N}{\delta} \right) - F \right]^+, \quad (16)
\]

and the distributions satisfy the following condition

\[
\|P - Q\|_H < \frac{\epsilon R_0}{2B \lambda_u \pi \gamma^2 N}, \quad (17)
\]

where

\[
F \triangleq \lambda_u \pi R^2 \left( 1 - \exp \left\{ \frac{1 - e^{-2\epsilon^2 \rho}}{1 - e^{-2\epsilon^2 \rho}} \right\} (1 - \mathcal{L}) \right) \quad (18)
\]

and \( \mathcal{L} \triangleq \frac{1}{\lambda_u \pi R^2} \log \left( \frac{2N}{2\epsilon^2} \right) \).

### IV. Concluding Remarks

This paper considered the problem of caching in a distributed heterogenous cellular network, when the popularity profiles of cached data files were unknown. A heterogenous network was considered where BS, SBSs and users were distributed according to independent PPPs. SBSs stored high data-rate content, which could be downloaded directly by one of the users from a SBS in the vicinity. A metric was proposed that captured the offloading-loss, which was used to optimize a random caching strategy. This metric was shown to be a function of the popularity profile (assumed unknown).

The popularity profile was estimated at BS using the available instantaneous demands from users in a time interval \([0, \tau]\).
We showed that a waiting time $\tau$ to achieve an $\epsilon > 0$ difference between the achieved cost and the optimal cost was finite, provided the user density was greater than a threshold. In this case, $\tau$ was shown to scale as $N^2$, where $N$ was the support of the popularity profile. A TL-based approach was proposed to estimate the popularity profile, which was then used to compute the empirical cost. A condition was derived under which the TL-based approach performed better than the random caching strategy.

Although TL-based approach performs better, the improvement is not without a price. The error that is achieved in (14) depends on $\|P - Q\|_H$, suggesting that lower the distance between the two distributions better the TL scheme performs. If $\|P - Q\|_H = 0$, the TL-based approach performs significantly better than the other proposed scheme. However, from Proposition 1, the benefits of using target domain samples can only be realized with the knowledge of the distance $\|P - Q\|_H$. In practice, an accurate estimate of this distance can never be obtained. Thus, it is important to find an efficient estimate of $\|P - Q\|_H$, and is relegated to the future work.

Other potential avenues for future research for the problem setup considered in this paper include:

1. The popularity profile was assumed to be constant across users and stationary in time, which, in practice, may not be justifiable. Therefore, analysis of the performance of the caching strategy by relaxing these assumptions merits investigation.

2. Characterizing the performance of TL-based caching when the popularity profile is estimated using linear combinations of source domain and target domain samples is another interesting topic for research.

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APPENDIX A

PROOF OF THEOREM 1

Consider the first term in (11):

$$E[1\{f_i \notin N_u\}] = \underbrace{E_{n_u} \Pr \{f_i \notin N_{u_i} | N_u = n_u\}}_{(a)} = \underbrace{E_{n_u} \Pr \{f_i \notin a, \text{ for some } a \in N_{u_i}\}}_{(b)} = \underbrace{E_{n_u} (1 - \pi_i)^{n_u M}}_{(c)} = \underbrace{E_{n_u} \sum_{j=0}^{\infty} (1 - \pi_i)^j M e^{-(\lambda \pi_i \gamma^2)} \frac{(\lambda \pi_i \gamma^2)^j}{j!}}_{(d)} = \sum_{i=1}^{\infty} \exp \{-U\} \pi_i,$$

where $U \triangleq \lambda \pi \gamma^2 \left[1 - (1 - \pi) M\right]$. In the above exposition, (a) follows from the fact that the random caching proposed in this paper is independent across users, (b) follows since the cache size is fixed to be $M$, and (c) follows due to the fact that $n_u$ is a PPP with mean $\lambda \pi \gamma^2$ since $n_u$ is the number of SBs in a circular area of radius $\gamma$. ■
Appendix B
Proof of Theorem 2

First, for any $\epsilon > 0$, the following inequality is proved (25):

$$
\Pr\{ \hat{T}^* \leq T^* + \epsilon \} \leq \Pr \left\{ \frac{2}{\lambda} \sup_{1 \leq i \leq 1^{\frac{1}{2}} \Pi: \Pi = 1} |\Delta T| > \epsilon \right\},
$$

(20)

where $\Delta T \triangleq \mathcal{T}(\Pi, \hat{P}) - \mathcal{T}(\Pi, P)$. We have,

$$
\hat{T}^* - T^* = \hat{T}^* - \inf_{i} \mathcal{T}(\Pi, P) \leq \hat{T} - T + \sup_{i} \left| \mathcal{T}(\Pi, \hat{P}) - \mathcal{T}(\Pi, P) \right| \leq 2 \sup_{i} \left| \mathcal{T}(\Pi, \hat{P}) - \mathcal{T}(\Pi, P) \right|,
$$

(21)

where $\hat{T} \triangleq \mathcal{T}(\Pi, \hat{P})$, thus proving (20). Now, consider the right hand side of (20) after substituting for $\mathcal{T}(\Pi, \hat{P})$ and $\mathcal{T}(\Pi, P)$ from (3) to get

$$
\Pr \left\{ \sup_{i} \left| \sum_{i=1}^{N} g(\pi_i) (\hat{p}_i - p_i) \right| > \epsilon \right\},
$$

which can be upper bounded as follows:

$$
\Pr \left\{ \sup_{i} \left| \sum_{i=1}^{N} g(\pi_i) \hat{\delta}_p \right| > \epsilon \right\} \leq \Pr \left\{ \max_{i=1,2,\ldots,N} \hat{\delta}_p > \epsilon \right\} \leq \sum_{i=1}^{N} \Pr \{ \hat{\delta}_p > \epsilon \} \leq 2 N \mathbb{E} \left\{ \exp \left\{ -2 \epsilon^2 n_p \right\} \right\}
$$

(22)

where $\hat{\delta}_p \triangleq |\hat{p}_i - p_i|$, $\epsilon \triangleq \frac{\epsilon \lambda}{2B}$, $\hat{\epsilon} \triangleq \sup_{i} \left( \sum_{i=1}^{N} g(\pi_i) \right)$, and $g(\pi_i) \triangleq \exp \left\{ -\lambda_{\epsilon} \pi \gamma^2 (1 - \pi_i) \right\}$. The last inequality above follows by applying Hoeffdings inequality (25), since the estimator $\hat{P}$ is unbiased and $\pi_i, \pi \in [0,1]$. The expectation in (22) is with respect to $n_p$. Conditioned on the number of users $n_R$ in the coverage area of BS, $n_R$ is a Poisson distributed random variable with density $\lambda n_R \tau$. We let $\tilde{g} \triangleq (2 \epsilon^2 k + \lambda n_R \tau)$, and $g^* \triangleq (1 - \exp \left\{ -2 \epsilon^2 \right\})$. Using this, we get

$$
2 N \mathbb{E} \sum_{n=0}^{\infty} \exp \left\{ -\tilde{g} \right\} \left( \frac{\lambda n_R \tau^2}{n!} \right)^n = 2 N \mathbb{E}_{n_R} \exp \left\{ -\lambda n_R \tau g^* \right\}.
$$

(23)

This can be further simplified as

$$
2 N \sum_{k=0}^{\infty} \exp \left\{ -\lambda \tau k \gamma \right\} \exp \left\{ -\lambda \pi n_R^2 \right\} \left( \frac{\lambda n_R \tau^2}{k!} \right)^k = 2 N \exp \left\{ -\lambda \pi n_R^2 (1 - \exp \left\{ -\lambda \tau \gamma^* \right\}) \right\}.
$$

(24)

Now, it is easy to see that $\Pr \left\{ \sup_{i} \left| \Delta T \right| > \frac{\epsilon}{2} \right\} \leq \delta$ if the expression in (24) is upper bounded by $\delta$, which implies that

$$
\tau \geq \frac{1}{\lambda \pi \gamma^*} \log \left( \frac{1}{1 - \frac{2 \epsilon^2}{\lambda \pi n_R^2 \log \frac{2N}{\delta}}} \right)
$$

(25)

provided $\lambda > \frac{1}{2 \epsilon^2} \log \frac{2N}{\delta}$, otherwise $\tau = \infty$. This completes the proof of the theorem.

Appendix C
Proof of Theorem 3

As in the proof of Theorem 2 it is easy to see that

$$
\Pr \{ \hat{T}^* \geq T^* + \epsilon \} \leq \Pr \left\{ \sup_{1 \leq i \leq N} \left| \hat{p}_i^{(t)} - p_i \right| > \epsilon \right\},
$$

(26)

where $\epsilon \triangleq \frac{\epsilon \lambda}{2B \sup_{i} \left( \sum_{i=1}^{N} g(\pi_i) \right)}$, and $g(\pi_i) \triangleq \exp \left\{ -\lambda_{\epsilon} \pi \gamma^2 (1 - \pi_i) \right\}$. Denote by $n_p$ the total number of requests in the coverage area of BS. As mentioned previously, conditioned on the number of users $n_R$ in the coverage area of BS, $n_R$ is a Poisson distributed random variable with density $\lambda n_R \tau$. Further, $E \left\{ p_i^{(t)} \right\} | n_p = \frac{n_p}{n_p + \tau R} \tau + \frac{m}{n_p + m} \tau R$. Using this, we can write

$$
\Pr \left\{ \sup_{1 \leq i \leq N} \left| \hat{p}_i^{(t)} - E \hat{p}_i^{(t)} + E \hat{p}_i^{(t)} - p_i \right| > \epsilon \right\}
$$

$$
\leq \Pr \left\{ \sup_{1 \leq i \leq N} \left| \hat{p}_i^{(t)} - E \hat{p}_i^{(t)} \right| + E \hat{p}_i^{(t)} - p_i \right\} > \epsilon \right\}
$$

(27)

$$
\leq \Pr \left\{ \sup_{1 \leq i \leq N} \left| \hat{p}_i^{(t)} - E \hat{p}_i^{(t)} \right| > \epsilon - \frac{m}{n_p + m} - \left| P - Q \right| \right\}
$$

$$
\leq \mathbb{E}_{n_p} \Pr \left\{ \sup_{1 \leq i \leq N} \left| \hat{p}_i^{(t)} - E \hat{p}_i^{(t)} \right| > \epsilon - \left| P - Q \right| \right\} | n_p \right\}
$$

where $\epsilon > \left| P - Q \right| \epsilon$, where $\left| P - Q \right| \epsilon \triangleq \sup_{i \in [1, N]} |q_i - p_i|$. Now, using Hoeffdings inequality, the following holds:

$$
2 N \mathbb{E}_{n_p} \exp \left\{ -2 \epsilon^2 n_p \right\}
$$

$$
= 2 N \mathbb{E}_{n_p} \exp \left\{ -2 \epsilon^2 n_p + \lambda n_R \pi \right\} \sum_{k=0}^{\infty} \frac{\epsilon^k}{k!},
$$

$$
= 2 N \exp \left\{ -2 \epsilon^2 \pi^2 m \right\} \mathbb{E}_{n_p} \exp \left\{ -\tilde{g}_{pR} \right\}
$$

$$
= 2 N \exp \left\{ -2 \epsilon^2 \pi^2 m \right\} \exp \left\{ -\lambda \pi n_R^2 \tau \right\} \times \sum_{l=0}^{\infty} \exp \left\{ -\lambda \pi n_R^2 \tau \left( 1 - \exp \left\{ -2 \epsilon^2 \pi^2 \right\} \right) \right\}
$$

(28)

where $a \triangleq \lambda n_R \pi \tau \exp \left\{ -2 \epsilon^2 \pi^2 \right\}$, $\epsilon_p \triangleq \epsilon - \left| P - Q \right|$ and $\tilde{g}_{pR} \triangleq \lambda \pi n_R^2 \tau \left( 1 - \exp \left\{ -2 \epsilon^2 \pi^2 \right\} \right)$. Therefore, we have

$$
2 N \mathbb{E}_{n_p} \exp \left\{ -2 \epsilon^2 \pi^2 (n_p + m) \right\}
$$

$$
= 2 \exp \left\{ -2 \epsilon^2 \pi^2 m \right\} \exp \left\{ -\lambda \pi n_R^2 \tau \right\},
$$

provided

$$
\lambda > \frac{1}{\pi^2} \left( \log \frac{2N}{\delta} - 2 \epsilon^2 \pi^2 m \right),
$$

(29)

otherwise $\tau = \infty$. This completes the proof of Theorem 3.