Iterative model coupled with experimental procedures to determine flow rate in a closed natural circulation circuit

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Abstract. A set of simplified iterative model coupled with experimental procedures has been derived to aid in computing the flow rate of a closed rectangular NC Circuit at low pressure/low power steady state condition. The approach involves partitioning the NC loop into different pressure loses segments depending on the associated factor, thereafter the various pressure drops in each segment is determined with applicable mathematical correlations. The iterative model allows for varying of the outlet temperature, at a predetermined Heating Power (Decay Heat) and inlet temperature such that the flow rate is determined when the pressure drops equals to the driving force ($\Delta P_{\text{drive}} = \sum \Delta P_{\text{loss}}$). The model was applied in computing the flow rate of the NC loop considered based on the assigned parameters with results validated with experimental data showing good agreement. Microsoft excel was employed in the computation procedures to allow for simplicity and easy comprehension.

1. Introduction
The addition of passive safety system features in advanced Nuclear Power Reactors are employed to improve the nuclear reactor safety in the event of loss of primary system pumps so as to prevent accident that may lead to core melt down and subsequent release of radioactivity to the environment. One of such features is the Natural means of Decay Heat Removal without any fluid moving machinery, a phenomenon known as Natural Circulation (NC) flow. A review of modern advances and analysis of the wide applications of natural circulation loops in Nuclear Power has been given by [1].

Due to the absence of any moving part, these systems are relatively maintenance free and cost effective [2] making it one of the good candidates for developing new reactor technologies to improve the nuclear reactor safety in the event of loss of primary system pumps so as to prevent accident that may lead to release of radioactivity to the environment [3]. However, the heat removal capability of these systems strongly depends upon the thermal hydraulic operating conditions such as the flow rate which is largely influence by the geometrical arrangement of the source and the sink along the NC circuit.

The heat applied to the heater section creates the driving force for the flow, the heat supplied will then produce buoyancy and the flow will be created in the hydraulic circuit in such way that the buoyancy is balanced by friction and local pressure loss ($\Delta P_{\text{loss}} = \Delta P_{\text{friction}} + \Delta P_{\text{local}}$) in steady state. If the heating power is increased further, the flow rate will also increase [4]. A study in thermal-
hydraulics loop scaled down and built at IEN, was performed by [5], similar to Pressurized Water Reactor (PWR) passive heat removal system, to acquire natural circulation flow data in validating numerical models. Single-phase natural circulation experiments have also been reported by a large number of researchers [6-10], but due to the complexity in the system codes and various approaches suggested, there was the need to carry out this study to derive a simplified but effective methodology for determining the flow rate in a rectangular closed-circuit Natural Circulation System under single phase condition having its Heat Source and Sink orientation as shown in Figure 1.

2. Methodology
The approaches to achieving the aim of this study was first to study a rectangular natural circulation loop sketched in Figure 1 while taking into consideration the dimensions of the geometry in Table 1, direction of flow, fluid properties and measuring conditions as listed in Table 2. Thereafter correlations leading to the model was derived and the procedure for the flow rate computation presented. Results were further compared with experimental data to validate the derived model.

| Table 1. NC geometry dimensions. | Parameter | Value (m) |
|---------------------------------|-----------|-----------|
| Heated channel Length           | 2.000     |
| Heated Channel Diameter         | 0.008     |
| Riser Length                    | 6.000     |
| Riser/Cooler Diameter           | 0.016     |
| Condenser length                | 2.000     |

| Table 2. Measuring and operating conditions. | Parameter | Value     |
|------------------------------------------------|-----------|-----------|
| System pressure (MPa)                         | 0.3 - 0.8 |
| Heating power (kW)                            | 0 – 10    |
| Inlet Temp. (°C)                              | 20 – 80   |
| Mass flux (kg/m³/s)                           | 400~750   |
| Flow                                           | Natural Circulation |

2.1. Natural circulation loop description
The setup of the experimental NC Loop has been described by [11] having similar geometrical dimensions with the loop under consideration. The Circuit is made up of a primary loop (Thermal-Hydraulic path), Secondary System (cooling section), data acquisition system and auxiliary devices.

Components of the primary circuit include the heated section, the up riser and downcomer, Pressurizer, Heat sink (heat exchanger), pipes and control valves. Demineralized water is used as the circulating fluid; the fluid leaves the heated tube and flows through the vertical pipe (adiabatic riser). Several thermocouples are attached at the outer surface of the hydraulics circuit to record the changes in temperature. A shell and tube heat exchanger is positioned at a height of 7m between the center of
the heating channel and the heat sink with a counter cooling effect; this helps to lowers the temperature of the circulating fluid in the primary circuit with room temperature water from the controllable secondary cooling tank.

The heat source of circuit signifies the heat dissipated by electrical resistors, through Joule effect, that is transformed in thermal energy. Expansion tank works to ensure the fluid volume expansion due to temperature increase. Several pressure transducers and pressure sensors are to monitor pressure signal of main components. An electromagnetic flow meter installed between the heated channel inlet and pressurizer surge line.

2.2. Model derivation
The closed loop natural circulation follows the conservation of mass and energy. From continuity equation we have.

\[
\dot{m} = \rho_c v_c A = \rho_h v_h A
\]

\[
q = \dot{m} (h_h - h_c)
\]

The Natural Circulation Flow Rate is the flow rate at which entire pressure loses in the loop is equal to the buoyancy or elevation pressure drop which serves as the Driving Force.

\[
\Delta P_{drive} = \Delta P_{loss}
\]

Assumptions:

\[\checkmark\] Heat removed by the cooler equals the heat input from the heater (No heat loss to environment).

\[\checkmark\] Heat Sink (Cooler) is of relatively uniform diameter with rectangular Loop/Pipe.

\[\checkmark\] The model is valid for flow rate under single phase steady state conditions.

2.2.1. Driving force. The driving force for the natural circulation is given by

\[
\Delta P_{drive} = -\dot{\Phi} \rho(z) gdz
\]

Going through the points in the circuit of Figure 1, the summation of the pressure drop will be express as;

\[
-\Delta P_{drive} = \int_0^1 \rho(z) gdz + \int_1^2 \rho_h gdz + \int_2^3 \rho(z) gdz + \int_3^4 \rho_c gdz + \int_4^0 \rho_c gdz
\]

But \( z_0 = z_4 \) and \( z_2 = z_3 \), thus,

\[
-\Delta P_{drive} = \int_0^1 \rho_c + \int_1^2 \rho_h \cdot (z_1 - z_0) + \int_2^3 \rho_c \cdot (z_2 - z_1) + \int_3^4 \rho_h \cdot (z_4 - z_3) + 0 + \rho_c \cdot (z_4 - z_2) + 0
\]

\[
-\Delta P_{drive} = \rho_c g(z_1 - z_0) + \frac{(z_1 + z_0)}{2} (\rho_h - \rho_c) g + \rho_h g(z_2 - z_1) + \rho_c g(z_0 - z_1) + \rho_c g(z_1 - z_2)
\]

\[
\Delta P_{drive} = (\rho_c - \rho_h) g \left[ \frac{z_1 + z_0}{2} + z_2 - z_1 \right]
\]

From Table 1, since \( z_1 = 2, z_0 = 0, z_2 = 8 \), then \( h \) (center of cooler to that of heating channel) = 7, thus,

\[
\Delta P_{drive} = 7(\rho_c - \rho_h) g
\]

\( \rho_h \) and \( \rho_c \) are densities of water from heater to cooler

2.2.2. Total pressure drop of the loop. The total pressure loss (\( \Delta P_{loss} \)) of the loop consists of frictional pressure drop (\( \Delta P_f \)), local pressure drop (\( \Delta P_l \)), and acceleration pressure drop (\( \Delta P_a \)). The \( \Delta P_a \) in a closed loop will be zero. Therefore,

\[
\Delta P_{loss} = \Delta P_f + \Delta P_l
\]

(1) Frictional pressure drop
The frictional pressure drop is given by

\[
\Delta P_f = f \left( \frac{1}{d} \right) \left( \frac{m^2}{2\rho A^2} \right)
\]

Where \( f \) = friction factor and it is based on Reynolds Number (Re) and depends on the relative roughness of the pipe.
To calculate the frictional pressure, drop the loop is divided into Four portions. The first portion is the heater section where internal diameter of heater is 0.008m and length of 2m. Pressure drop is given by

\[
\Delta P_{f,1} = f_1 \left( \frac{2}{0.008} \right) \left( \frac{m^2}{2 \rho \mu 0.00005^2} \right)
\]  

(11)

The second is the frictional pressure drop in the cooler (point 2 to 3) having internal diameter as that of the pipe (0.016) and length 2m which is given by

\[
\Delta P_{f,2} = f_2 \left( \frac{2}{0.016} \right) \left( \frac{m^2}{2 \rho \mu 0.0002^2} \right)
\]  

(12)

In both portions the density of water is assumed to be the average density hence, \( \rho_{avg} = \frac{\rho_c + \rho_h}{2} \)

The third portion is the length of pipe from heater outlet to the inlet of the cooler (point 2 to 3). The length of pipe is 6m with an internal diameter of 0.016m. The frictional pressure drop is given by

\[
\Delta P_{f,3} = f_3 \left( \frac{6}{0.016} \right) \left( \frac{m^2}{2 \rho c 0.0002^2} \right)
\]  

(13)

The fourth portion is from cooler outlet to the inlet of the heater. The length of pipe is 10m (Vertical and horizontal length) with an internal diameter of 0.16m. The frictional pressure drop is given by

\[
\Delta P_{f,4} = f_4 \left( \frac{10}{0.016} \right) \left( \frac{m^2}{2 \rho c 0.0002^2} \right)
\]  

(14)

Thus,

\[
\sum \Delta P_{frictional} = \Delta P_{f,1} + \Delta P_{f,2} + \Delta P_{f,3} + \Delta P_{f,4}
\]  

(15)

(2) Friction factor calculations

The Reynolds number of the flow is given by, \( Re = \frac{\rho v d}{\mu} \)

For a circular section it is also given by, \( Re = \frac{4 m}{\mu D} \)

The friction factor is given in terms of Reynolds number as follows. [12]

\[
f = \begin{cases} 
\frac{64}{Re} & \text{when } Re \leq 2300 \\
0.0025 \left( Re \right)^{\frac{1}{2}} & \text{when } 2300 < Re \leq 4000 \\
0.3164 \left( Re \right)^{\frac{1}{2}} & \text{when } Re > 4000 
\end{cases}
\]

(3) Local pressure drop

The local pressure drop is given by

\[
\Delta P_l = K \left(\frac{m^2}{2 \rho A^2} \right)
\]  

(16)

Where \( K \) = local pressure drop coefficient

The local pressure drops includes the pressure drop due to sudden contraction at the inlet of heating section (point 0), sudden expansion at the outlet of heating section (point 1), four elbows (section a, b, c, d) in the closed as depicted in Figure 1.

\( k_h \) and \( k_c \) are computed using

\[
k_h = \frac{(1-\beta^2)^2}{\beta^2}
\]  

(17)

\[
k_c = \frac{k_h}{\beta^2}
\]  

(18)

Where \( \beta = \frac{d_h}{d_c} \)
The local pressure drop coefficient for sudden contraction at the inlet of heating section is 0.409 at $A_2/A_1 = 0.25$ and that of sudden expansion at the outlet of heating section is 0.5625 at $A_1/A_2 = 0.25$ using eqn (17) and eqn (18) and checked from Figure 3.

$A_1, A_2$ are cross sectional area of the heating section and pipe respectively and calculated by

$$A = \frac{\pi d^2}{4}$$

While local pressure drop coefficient for all the 90° elbows are considered to be flanged and the loss coefficient for a 90° flanged elbow from fluid mechanics [12] is $0.3$.

The local pressure drop is of two parts, with the first part along sudden expansion at the exit of heating section and one elbow at the inlet of cooler where density of the water is same as $\rho_h$. The pressure drop is given by

$$\Delta P_{1, 1} = 0.8625 \left( \frac{m^2}{2\rho_h \cdot 0.0002^2} \right) \quad (19)$$

Second is the local pressure drop in the remaining three elbows and along the sudden contraction at the inlet of heating section. The pressure drop is given by

$$\Delta P_{1, 2} = 1.309 \left( \frac{m^2}{2\rho_h \cdot 0.0002^2} \right) \quad (20)$$

Thus

$$\sum \Delta P_{\text{local}} = \Delta P_{1, 1} + \Delta P_{1, 2} \quad (21)$$

The total pressure loss is the sum of all the frictional pressure drop and local pressure drops and is given by

$$\sum \Delta P_{\text{loss}} = \sum \Delta P_{\text{frictional}} + \sum \Delta P_{\text{local}} \quad (22)$$

The physical properties (such as density, enthalpy, dynamic viscosity, etc.) involved in the calculation are extracted from The International Association for the Properties of Water and Steam (IAPWS) correlations [13] with the aid of the add-Ins feature of the MS Excel.

2.3. Experimental procedure for validation

To validate the derived model, experiment was conducted using a NC loop described by [11] and summarized in section 2.1 with operating conditions stated in table 2. Figure 2 shows the pictorial view of the Experimental facility.

The operation was initiated by forced circulation with a circulation pump that provides the initial force for the natural circulation low. When temperature difference of cold and hot section have been built, the circulation pump was then disconnected and flow was redirected via a by-pass line, the primary loop’s flow rate was then supported by natural circulation’s driving head. The water temperature entering the heated section ($T_{\text{inter}}$) are regulated by changing the flow rate of cooling water of the secondary loop till it reaches the predetermined value of interest. Flow phenomena were then monitored via a graphical interface on computer screen for data acquisition and recording. Result is presented and validated with the derived model in section 3.2.

![Figure 3. Coefficient of Contraction and Expansion Losses [12].](image-url)
3. Results and discussion

3.1. Iterative model
The derived Model was used to compute the flow rates iteratively at the stated conditions in Table 2. The iterative process will continue and flow rate is recorded at the points where the driving force equal to the total pressure drop in the loop. The flow rates recorded at 2kW power with inlet temperatures, $T_{in}$ of heater (varied from 20-60°C) is plotted against the inlet temperatures, $T_{inlet}$ in Figure 4. The heated power was then increased to 4kW and the determined flow rate is also depicted, the system pressure is kept at 5 ATM (absolute pressure, 0.5MPa) in all cases. From both results, the flow rate for each operating power steadily increases over time as the inlet temperature changes; this will subsequently cause variation in the outlet temperature thus creating a temperature difference along the hydraulic circuit.

![Figure 4](image1)

**Figure 4.** Plot of flow rate against $T_{inlet}$ at different heating power.

The average flow rate at 2kW Heating Power with the same $T_{inlet}$ values as stated above was computed to be 0.018498884 $\approx$ **0.0185 kg/s**. As the heating power was raised to 4kW while maintaining the same range of $T_{inlet}$, the average flow rate was 0.024701111 $\approx$ **0.0247 kg/s**, showing a noticeable higher flow rate.

3.2. Model validation
The phenomenon presented by the model was compared with experimental result as depicted in Figure 5. The trend shows that flow rates determined at different sub-cooled temperatures using the derived model are in agreement as they follow the same trend with experimental results conducted at low pressure/low power under steady state single phase flow.

![Figure 5](image2)

**Figure 5.** Validation of the derived model with experimental result.

The experimental and the theoretical models follow similar trend with an onset variation (starting at 20°C) of the flow rate with positive gradient. At temperature around 50°C (Almost the middle of temperature range considered (20°C-80°C)), there is an intersection of the variation with the experimental variations drifting below the theoretical (iterative model) after the point of intersection.
With the line of best fit, one can infer that the negative drift (after the intersection point) balances with the positive drift (before the intersection point), thus the experiment variation gives a curve in close agreement with the iterative model. In summary, the temperature differences, $\Delta T$ caused by increase in heating power or $T_{\text{inter}}$ will lead to continues density drop and in turn an increasing flow rate will be achieved.

4. Conclusions
An iterative model has been derived to determine the flow rate of a closed rectangular Natural Circulation Loop with orientation similar to the Passive Heat Removal System of a Nuclear Reactor. The model involves segmenting the flow path based on the flow characteristics along the loop. The iterative model allows for varying of the Heating Power and Temperature depending on the operational conditions of interest such that the flow rate is determined when the pressure drops equals to the frictional Pressure drop. The model was applied to computing the flow rate of the reviewed NC hydraulic circuit at the stated predetermined flow conditions with results in good agreement with the experimental data. This presents a positive contribution of NC phenomena and acquisition of information useful for future scaling purposes.

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