Abstract. Many structural components are subjected to either constant or temporal mechanical loads, such as a suspension bridge bolts and rail tracks. Methods are required to accurately and efficiently measure the stresses experienced by these components to ensure they can continue to operate in an effective and safe manner. Acoustic techniques can be used to monitor the stress in a solid material via the acoustoelastic effect. This is the stress dependence of the acoustic velocity in an elastic media. This work develops a multiphysics computational model to study the acoustoelastic effect in a three point bending system. A simple linear relationship was utilised to represent the stress effect on the acoustic velocity. The simulation results were compared with experimental results and the same general trend was observed. An increase in applied load resulted in a greater difference between the time of flight of two transducers at the top and bottom of a component and perpendicular to the applied load. However, there were quantitative differences between the model and the experiment. The model was used to investigate different ultrasound transducer location and operating frequency, highlighting the benefit of modelling tools for the design of acoustic equipment.

1. Introduction

The acoustoelastic effect is the stress dependence of an acoustic wave’s velocity propagating in an elastic media. By measuring the speed of sound in solid components it is therefore possible to calculate the stresses they are experiencing. This can be used for nondestructive testing on many components such as load bearing bolts on suspension bridges [1], concrete materials [2] steel pipes [3] and rail track [4].

The speed at which a wave propagates within a medium is a function of the density and elasticity of a medium. As stress results in changes in these properties it will have an effect on the propagating wave's speed. The speed of sound in a material can be calculated based on the second and third order elastic constants. When a longitudinal acoustic wave is travelling parallel to the stress in a material:

\[
\rho_0 c_{Lx}^2 = \lambda + 2\mu - \frac{\sigma}{3K_0} \left[2l + \lambda + \frac{\lambda + \mu}{\mu} (4m + 4\lambda + 10\mu) \right]
\]

(1)

Where \(\rho_0\) is the initial density, \(K_0\) is the initial bulk modulus, \(c_{Lx}\) is the longitudinal speed of sound parallel to the applied stress \(\delta\). \(\lambda\) and \(\mu\) are the Lamé or second order elastic constants and \(l\) and \(m\) are Murnaghan’s or third order elastic constants. There is an additional third order elastic constant \(n\) which is required for propagating shear waves.
When the longitudinal wave is travelling perpendicular to the stress this equation becomes:

\[ \rho_0 c_{L_y}^2 = \lambda + 2\mu - \frac{\sigma}{3K_0} \left[ 2l + \lambda + \frac{2\lambda}{\mu} (m + \lambda + 2\mu) \right] \tag{2} \]

Where \( c_{L_y} \) is the longitudinal speed of sound perpendicular to the stress. Often for simplicity a single acoustoelastic constant \( K \) (which is a function of the third order elastic constants) is defined. A different \( K \) must be given for each type of wave and direction of propagation relative to the stress. The resulting modulation on the speed of sound in the material can then be found:

\[ K\sigma = \frac{\Delta C}{C} \tag{3} \]

Where \( \sigma \) is the stress \( C \) is the speed of sound in the unstressed material and \( \Delta C \) is the change in speed of sound. Acoustoelastic constants can be calculated by subjecting materials to different stresses and measuring the speed of sound in those materials. Many components under stress in the real world do not experience either tension or compression in a single direction; therefore methods are required to study them. For example many components experience a three point bending load where there are different regions under either tension or compression.

Computational modelling is a powerful tool when developing ultrasound instruments. They can aid in the design of the system by studying the effect of transducer parameters such as frequency and dimensions and aid with identifying the most suitable location for the transducers. Computational models have been used to study the acoustoelastic effect such as Ferrari who developed a model for wave propagation in metal sheets using EMATs [5] and Elyea et al who studied the effect of bridge bolt dimensions on wave propagation in the presence of tensile stress [6].

In this work we use COMSOL multiphysics to combine structural deformation with acoustic wave propagation to create a 2D model of a component experiencing a three point bending stress. This is representative of a load bearing bolt used in suspensions bridges. Experimental acoustic measurements were made on a bolt and compared to the simulation results. The model was then used to investigate transducer frequency and position as well as the effect that the applied load has on the wave velocity and attenuation.

2. Methodology

2.1. Model development

The computational model was developed using COMSOL multiphysics 4.3. The model combined the structural mechanics and acoustics modules. A 2D model was used for this study as the large number of mesh elements required would result in an unacceptable solution time if a 3D model was used.

2.1.1. Geometry. The geometry of the model can be seen in Figure 1. The rectangle has the same length as the bolt used for the experiments and the height is the same as the diameter of the bolt. The two points in the dashed ellipses represent either end of a line where an oscillating pressure is applied. These represent the ultrasound transducers and have a length of 4 mm. This represents the diameter of 4 mm, which was the same dimension as the transducers used for the experiment.
2.1.2. Structural mechanics. The rectangular representation of the bolt was placed on two hemispheres which were fixed in both directions. The bottom surface of the rectangle was allowed to slide along these surfaces as deformation occurred. The load was applied to the top surface of another hemisphere which was in contact with the top surface of the rectangle. The material for all of the components in the model was set as structural steel. For the simulations different forces were applied, ranging between 50 and 300 kN.

2.1.3. Acoustic wave propagation. The acoustic wave was added to the system by applying a pressure wave to the lines which represent a transducer. An analytical function was defined which represented a shaped pulse with known frequency and duration. The amplitude was always set to 1 in the positive direction. For the majority of the simulations a frequency of 1 MHz was used. This is much lower than the 10 MHz transducers used for the experiments. This reduction was required as the frequency controls the element mesh size and simulation time step. A frequency of 10 MHz would have resulted in an unacceptably long time to run the simulations. The acoustoelastic effect was modelled using equation 3 above.

For all simulations the value for the acoustoelastic constant $K$ was selected as 5e-7. Methods do exist for measuring the acoustoelastic constant for specific materials, load and propagation directions [7]. Unfortunately the scope of this work did not allow for the exact measurement to be made at his time. The system was operated in a pulse echo mode so the transmitting transducer was also the receiver. This resulted in an acoustic path length which is twice the length of the bolt.

2.1.4. Finite element mesh. The geometry of the system was discretised using a finite element mesh. A free triangular mesh was used for all models. An important aspect is the size and number of mesh elements utilised as this governs the accuracy, precision and time duration of the simulation. A trade off always exists to this extent. Adopting the method of Mylavarapu and Boddapati [8] the maximum element size used was the ultrasound pulse wavelength divided by ten. This results in ten elements for each cycle of the waveform. The wavelength was calculated from the speed of sound and the transmitted frequency used.

2.1.5. Solver configurations. All simulations had two steps for the solver. The first step was a stationary solver and the second step was a time dependent solver. The stationary step was used to solve the solid mechanics and the results from this (stress) were then used during the second step, which solved the acoustic propagation. The applied load would result in a deformation to the bolt so the resulting deformed mesh was utilised during the second step to account for the changing path
lengths the acoustic waves would now experience. The selection of the simulation time step is important for time dependent studies as it controls the accuracy resolution and simulation duration. Mylavarapu and Boddapati [8] stated that the simulation time step should be less than \((\text{wavelength}/10)/\text{speed of sound}\) to ensure mathematical stability. This approach was used in this work.

2.1.6. Data analysis. The results taken from the simulations were the time of flight and the amplitude of the received acoustic wave. The pressure received at the line representing the transducer was recorded at each time step and exported to MATLAB for post processing. The time of flight was calculated by detecting the time at which the first part of the received pulse over an amplitude value of 0.1 was detected. The amplitude recorded was the maximum amplitude of the received acoustic wave.

2.2. Experimental measurements
The experimental trials were all performed at Strainsonics Ltd, Sheffield, UK. The bolt was placed in a fork and pin connector (Figure 2). This connector was then placed in a load frame and the required load was applied. The bolt has two 10 MHz ultrasound transducers, each with a diameter of 4 mm built into one of the bolt ends. The bolt was orientated so that the transducers were placed at the top and bottom of the bolt both parallel to the applied load. The transducers were attached to a T200 ultrasound pulser receiver device (Tribonics, Sheffield, UK). The time of flight was recorded for the top and bottom transducers, for applied loads between 50 and 300 kN.

![Image](image_url)

**Figure 2.** Fork and Spade connection for experiments (Courtesy of Strainsonics Ltd). The bolt with the built in ultrasound transducers is located in the center of the connection.

3. Results and discussion
The simulation results show that the time of flight is lower for the top transducer indicating a higher speed of sound in this region (Figure 3). This was expected as the top zone was closer to the applied load and a larger stress would be experienced here. This region is experiencing a compressive stress and ultrasound velocity is known to increase with compressive stress [1]. The top section of the bolt is also under compression so the acoustic path length is shortened here, again reducing the time of flight of the ultrasound pulse. The results indicate that an increase in load results in a shorter time of flight for the top transducer and a longer time of flight for the bottom transducer. Again this is an expected result as the increased load will result in an increase in the stress which is modulating the speed of sound in the material. The results for the loads of 100 and 150 kN do not seem to fit with the rest of the results as the time of flight for both transducers are much higher. It is believed that this is a result of how the time of flight is measured. The time of flight is taken as the point at which the received
ultrasound pulse crosses a threshold value of 0.1. In some cases it was noticed that the first cycle of the received pulse was too low to break this threshold so it would be the second cycle from which the time of flight was calculated. This could have been the case for the results for 100 and 150 kN, resulting in this abnormality. The sudden jump in time of flight is of the order of the time period of the wave again supporting this explanation. These errors could be removed by altering the threshold level during the post processing of the results.

**Figure 3.** Simulated ultrasound time of flight for top and bottom horizontal transducers, as function of load.

**Figure 4.** Difference in time of flight for the top and bottom horizontal transducers. Results are presented from the simulation and experiment.

Figure 4 display the difference in acoustic time of flight between the top and bottom transducers in the bolt from the simulation and experiment. Both sets of results show that an increase in load results in an
increase in the difference in time of flight. This result indicates that a simple linear modeling technique is capable of recreating some features of the acoustoelastic effect. The simulation results are higher for all load values than the experimental results. The experimental results display an almost linear relationship between load and delta time of flight. For the simulation results this is almost the case except for the values of 150 and 200 kN. For these values there seem to be no change in delta time of flight. This is most likely a result of how the time of flight is calculated as discussed in the previous paragraph. It was also noticed during the simulations that the propagating waves from the top and bottom transducers would overlap each other and interfere (Figure 5). This would introduce some error into the results. This would not happen for the experiments as the transducer frequency was much higher resulting in much less beam spread. When simulations were performed with higher frequencies this interference was reduced.

![Figure 5. Acoustic pressure field in the system: Left, time = 1.5e-6 (s), right, time = 1.004e-5 s.](image)

The results indicate quantitative differences between the models and the experimental results. It should be noted that the model used to calculate the speed of sound as a function of stress is a simple linear equation. These equations use a single value for the acoustoelastic constant $K$. A simplification to a single value is only valid when the stress is acting in a single direction. For the model we created a three point bending system. This type of structural deformation has regions of tension and compression so a single value for the acoustoelastic constant is not valid and the equation should be further developed to include all three of the third order elastic constants. It should also be remembered that the simulation was a 2D model whereas the experiment was a full 3D system; again this would result in differences between the simulations and experiments.

The amplitude ratio was calculated by the following equation:

$$AR = \frac{A_{\text{top}} - A_{\text{bottom}}}{A_{\text{top}}}$$

Where $A_{\text{top}}$ is the maximum amplitude of the received pulse from the top transducer. $A_{\text{bottom}}$ is the maximum amplitude of the received pulse from the bottom transducer. This equation results in a ratio of the received amplitudes relative to the top transducers amplitude. It was decided to make the ratio relative to the top transducers received amplitude as this was always found to be higher in the simulation results. The results in Figure 6 indicate that an increase in applied load results in an increase in the difference in the received pulse amplitude. This relationship appears to be almost linear with the only outlying result the one for a load of 100 kN. This could be due to the interfering waves. The amplitude of the received pulses was not recorded from the experiments so a direct comparison
with them cannot be made. These simulation results indicate that in addition to measuring the time of
flight and calculating the speed of sound in a material experiencing a stress, amplitude measurements
may also be a viable method for measuring stress provided that appropriate models are developed.
Using different measurements would increase the versatility of an ultrasound inspection system.

Figure 6. Simulated amplitude ratio of received acoustic waves from the top and bottom horizontal
transducers, as a function of load. The acoustic waves had a frequency of 1 MHZ.

Figure 7. The effect of acoustic wave frequency on the change in time of flight for the top and bottom
horizontal transducer. Simulation results at a load of 150 kN.

The frequency of the transmitted acoustic wave was varied for some simulations (Figure 7). The
frequency was varied from 0.25 to 1.5 MHz. No frequency was simulated above this value due to the
required increase in number of mesh elements and the reduction in simulation time step resulting in
too long a simulation duration with the current computational capabilities. The Simulation results
show that the higher the acoustic wave frequency the greater the difference in time of flight. This indicates that higher frequencies are more suitable for detecting stress and for identifying smaller changes in stress. High frequency waves will experience much less beam spread reducing the chance of interference and sampling a smaller area of stress. It should always be remembered that attenuation does increase with frequency and a trade-off always exists with this respect. The result for the effect of frequency on the amplitude ratio again showed that higher frequencies are more preferable as the difference was greater for the higher frequencies (Figure 8). This relationship almost appears to be linear.

![Figure 8](image.png)

**Figure 8.** Simulated amplitude ratio of received acoustic waves from the top and bottom horizontal transducers, for different acoustic wave frequencies. The applied load was 150 kN.

One benefit of using computational modelling when developing ultrasound instruments is to investigate the location of the transducers. This is done easily in a model where it only requires the location of the oscillating boundary to be moved. Transducer locations can be tested automatically with parameterised solvers to find the precise optimum location. In addition to the two horizontal transducers at the top and bottom of the bolt another transducer location was defined along the bottom edge with a wave travelling parallel to the applied load (Figure 1). It should be noted that for the model where the wave is propagating in the same direction as the applied load for convenience the same value for the acoustoelastic constant was used. This would not be the case in real life.

The results for the time of flight indicate that a larger applied load result in a longer time of flight (Figure 9). This result is a good linear fit. This is unlike the previous results where measurements were made based on two propagating wave. This potentially indicates the observed abnormalities were a result of the two waves interfering. The wave travelling in the vertical direction experiences regions of both compression and tension so it is interesting to observe the overall effect on the time of flight.
The amplitude difference for the single vertical transducer was calculated by deducting the maximum positive amplitude of the received pulse from 1, which was the transmitted pulse positive amplitude. The results for the amplitude difference as a function of load indicates that with an increase in load the difference in amplitude between the transmitted and received pulse reduces (Figure 10). This relationship is not linear and shows very little change between 50 and 150 kN before it reduces with increased load. Throughout the range of loads applied the change is only small (0.05) suggesting that when the ultrasound propagation in parallel to the applied load, the change is not great.

4. Conclusion
This work has developed a multiphysics model combining structural analysis with acoustic wave propagation to study the acoustoelastic effect in a three point bending system. Although a greatly simplified model was used with a linear relationship between applied stresses and speed of sound the key features of the acoustoelastic effect were simulated and qualitatively the simulation results shared
the key features of experiment. This model showed that increasing the load applied to the system would result in a greater difference in time of flight and received acoustic wave amplitude for two transducers in different location but both perpendicular to the applied load. The effect of transmitted wave frequency and transducer orientation was also studied. The results indicate modeling techniques such as this can be powerful design tools. Further work would require more detailed modelling of the acoustoelastic effect at higher frequencies and in three dimensions.

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