Effects of Entanglement in Controlled Dephasing

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In controlled dephasing as a result of the interaction of a controlled environment (dephasor) and the system under observation (dephasee) the states of the two subsystems are entangled. Using as an example the “Which Path Detector”, we discuss how the entanglement influences the controlled dephasing. In particular, we calculate the suppression $\nu$ of A-B oscillations as a function of the bias $eV$ applied to the QPC and the coupling $\Gamma$ of the QD to the leads. At low temperatures the entanglement produces a smooth crossover from $\nu \propto (eV/\Gamma)^2$, when $eV \ll \Gamma$ to $\nu \propto eV/\Gamma$, for $eV \gg \Gamma$.

I. INTRODUCTION AND MAIN RESULTS

Recently, the progress in nanofabrication opened technological possibilities to fabricate mesoscopic devices consisting of pairs of capacitively coupled (no particle exchange) mesoscopic systems. The primary motivation was to carry out controlled experimental studies of dephasing in quantum electronic systems. In such experiments one system plays the role of the “dephasor” which causes decoherence in the other system - the “dephasee”. The possibility to experimentally adjust the type of dephasing in quantum electronic systems. In such experiments on mesoscopic systems. The primary motivation (WPD experiment) is the presence of inelastic channels.

In this paper, we will illustrate the previous discussion using the WPD experiment as an example. In the WPD, the flux-sensitive part of the transmission probability through the interferometer is

$$ P_{AB} \propto \text{Re} \left[ t_{ref} t_{QD} e^{2\pi i \Phi/\Phi_0} \right], $$

where $t_{ref}$ is the transmission amplitude through the arm without QD and $t_{QD}$ is the elastic amplitude to cross the arm containing the QD. Referring to the preceding general discussion $t_{QD}$ is proportional to the amplitude $a_{00}$.

It is convenient to characterize the dephasing by means of a positive function $\nu$, defined as

$$ |t_{QD}| = |t_{QD}^0| (1 - \nu (eV, T)) \cdot $$

where $|t_{QD}^0|$ is the transmission amplitude when the interaction with the QPC is switched off, $T$ is the transparency of the QPC, and $eV$ is the bias applied to it.

The bias, transparency, and the temperature dependence of $\nu$ have recently been investigated both experimentally and theoretically. The theoretical results indicated that for temperatures $T < eV$.
Here $\lambda_d$ is the coupling between QD and QPC. It is conventionally expressed as
\[
\lambda_d^2 \simeq \frac{1}{16\pi^2} \left( \frac{\Delta T}{T(1-T)} \right),
\]
where $\Delta T$ is the change in the transparency due to the presence of an electron in the QD. These results were tested experimentally in the following regime

(i) the QD was tuned to the maximum of a Coulomb Blockade (CB) peak and

(ii) the temperature was much higher than the coupling to the leads, $T \gg \Gamma$.

At such temperatures the number of inelastic channels is mainly determined by $T$ and $eV$. In order to investigate the dephasor-dephasee interplay we focus on this work on the opposite low-temperature limit, $T \ll \Gamma, eV$. In Section II we present the details of our calculation and derive a general expression for $\nu$ (Eq. (21)- (23)). Here we summarize our main findings.

According to the standard description of a QPC the bias $eV$ represents the difference in chemical potentials between left and right going scattering states. Therefore, the two energy scales of the problem $\Gamma$ and $eV$ characterize the windows of states in the QD and QPC which can participate in mutual scattering processes leading to dephasing. The idealized “rigid” dephase result is $\Gamma \rightarrow 0$ for which the states in the QPC can only scatter to states with the same energy. The number of open inelastic channels is then proportional to $eV$. Therefore, the calculation of $\nu$ performed in this limit gives the result of Eq. (3). For a finite $\Gamma$ we find that the “rigid” approximation is valid only in the limit of $\Gamma \ll eV$.

As the bias is reduced, the function $\nu$ crossover smoothly to the opposite regime ($eV \ll \Gamma$) where the dephase cannot be approximated as “rigid”. The scattering in QPC can occur between any states in the $eV$ interval and the number of open inelastic channels is proportional to $(eV)^2$. In this case we find accordingly
\[
\nu \simeq 2\lambda_d^2\left(\frac{eV}{\Gamma} \right)^2.
\]

Thus in this regime the dephasor-dephasee interplay significantly modifies the “rigid” dephase result leading in a sense to observable effects of entanglement on dephasing. In practical terms it also means that as a consequence of entanglement the dephasing ability of a dephaser is not universal but depends on the particular properties of the dephasee.

In Section II we study as well the dependence of $\nu$ on the detuning of the QD away from the maximum of a CB peak and show that also in this respect the two regimes discussed above are significantly distinct (cf., Eq (24) and Eq (25)).

II. THE MODEL AND CALCULATION

In order to describe the WPD, we can focus on the arm containing the QD. If $\Gamma$ and $eV$ are less than the mean level spacing in the QD we can represent it by a resonant level model
\[
\hat{H}_{QD} = \epsilon_0 \hat{d}_0^\dagger \hat{d}_0 + \sum_{\epsilon_k} \left( \epsilon_k \hat{c}_{\epsilon_k}^\dagger \hat{c}_{\epsilon_k} + (A_{\epsilon_k} \hat{c}_{\epsilon_k}^\dagger \hat{d}_0 + c.c) \right)
\]
where the operators $\hat{c}_{\epsilon_k}^\dagger$ and $\hat{c}_{\epsilon_k}$ refer to states in the left-right leads ($i=L,R$) connected to the QD, and the operators $\hat{d}_0^\dagger$, $\hat{d}_0$ to the resonant level in the QD.

We describe the QPC by means of the standard picture in terms of scattering states $A$. For simplicity we assume that a single transversal channel is open. Therefore, the total Hamiltonian is $\hat{H} = \hat{H}_{QD} + \hat{H}_{QPC} + V$ with
\[
\hat{H}_{QPC} = \sum_{q,j} \epsilon_q \hat{b}_{qj}^\dagger \hat{b}_{qj},
\]
\[
\hat{V} = \hat{d}_0^\dagger \hat{d}_0 \sum_{j,j',q,q'} V_{qq'}(j,j') \hat{b}_{qj}^\dagger \hat{b}_{q'j'},
\]
where the operators $\hat{b}_{qj}^\dagger$ ($\hat{b}_{qj}$) create (annihilate) left-right going scattering states ($j = L,R$). The application of a bias shifts the chemical potentials of the right-left going scattering states such that $\mu_R - \mu_L = eV$. Moreover we take the matrix elements $V_{qq}(j,j)$ to zero if $\epsilon_q < \mu_L$. This corresponds to the inclusion of the equilibrium Hartree shift in the definition of $\epsilon_0$.

We are interested in the modification of $t_{QD}$ due to the presence of the QPC. This transmission amplitude is
\[
t_{QD} = -i\sqrt{4\Gamma_L \Gamma_R} \int d\omega f'(\omega) G_d(\omega),
\]
where $f'$ is the derivative of the Fermi function and $\Gamma_{L,R}$ is the elastic coupling to the left (right) reservoirs ($\Gamma_i = \pi \sum_k |A_{ki}|^2 \delta(\omega - \epsilon_k)$) assumed energy independent.

Information about the QD and its interaction with the QPC is contained in $G_d(\omega)$ which is the Fourier transform of the retarded Green’s function, $G_d(t) = -i \theta(t) \langle \{d_0(t), d_0^\dagger(0)\} \rangle$. In order to calculate it, we use a real time perturbation expansion in the interaction $V$ between the QD and the QPC. The general result is
\[
G_d(\omega) = \frac{1}{(\omega - \epsilon_0) + i\Gamma - \Sigma(\omega)},
\]
where $\Gamma = \Gamma_L + \Gamma_R$ and $\Sigma$ is the proper retarded self-energy.
In the weak coupling limit, we can approximate the self-energy with the diagrams of Fig. 1. We expect this approximation to describe correctly the scattering processes leading to dephasing, provided that at every step of the expansion the broadening of the QD resonant level due to the coupling to the leads is taken into account. Therefore, we calculate the diagrams of Fig. 1 using for the QD the noninteracting Green’s functions

\[
G^<_0(\omega) = i f(\omega) A_0(\omega), \quad G^>_0(\omega) = -i(1 - f(\omega)) A_0(\omega),
\]

where \(A_0(\omega) = 2\Gamma/(\omega - \epsilon_0)^2 + \Gamma^2\) is the noninteracting spectral density. The analogous Green’s functions of the QPC are

\[
g^{<}_{qj}(\omega) = 2\pi i f^{(j)}(\omega) \delta(\omega - \epsilon_q), \quad g^{>}_{qj}(\omega) = -2\pi i \left(1 - f^{(j)}(\omega)\right) \delta(\omega - \epsilon_q),
\]

where the thermal factor \(f^{(j)}(\omega) = f(\omega - \mu_j)\) distinguishes left and right-going scattering states by their chemical potential difference.

The first order term (Fig. 1a) provides a shift of the QD level

\[
\Delta = 2 \sum_{q,j} V_{qj}(j,j') f^{(j)}(\epsilon_q),
\]

the factor of two coming from the summation over spin in the QPC.

An important contribution to \(\Sigma\) comes from the second order bubble diagram (Fig. 1b) describing the scattering between electrons in the QD and in the QPC. This contribution can be written as

\[
\Sigma_2(\omega) = i \int \frac{d\omega'}{2\pi} \frac{\Sigma^>_0(\omega') - \Sigma^<_0(\omega')}{\omega - \omega' + i\delta},
\]

where the self energies \(\Sigma^>\) and \(\Sigma^<_0\) can be expressed in terms of the greater and lesser Green’s functions of the QD and of the QPC as

\[
\Sigma^<_0(\omega') = 2 \sum_{i,j,q,q'} |V_{qq'}(j,j')|^2 \int \frac{d\omega_1 d\omega_2}{(2\pi)^2} G^<_0(\omega' - \omega_1) g^>_{qq'}(\omega_2) g^<_{q'j}(\omega_2 - \omega_1),
\]

and a similar equation for \(\Sigma^>\) with interchanged symbols \(<\) and \(>\). Substituting the expressions for the Green’s functions into Eq. (14), after some straightforward algebra, we arrive at the result

\[
\Sigma_2(\omega) = 2 \int \frac{d\omega' d\omega''}{2\pi} \rho_0(\omega'') \left[ \frac{A_0(\omega') f^{(j)}(\omega')}{\omega + \omega'' - \omega' + i\delta} + \frac{A_0(-\omega') f^{(j)}(\omega')}{\omega - \omega'' + \omega' + i\delta} \right],
\]

where the spectral density

\[
\rho_0(\omega) = \sum_{j,j'} \lambda^2 j, j' \int d\omega_1 d\omega_2 \delta(\omega_1 - \omega_2 - \omega) f^{(j)}(\omega_2) \left(1 - f^{(j')}(\omega_1)\right),
\]

describes particle-hole excitations in the QPC. Here we introduced the dimensionless couplings \(\lambda(j,j') = L | V(j,j') | / (2\pi v_F) \ll 1\).

We are interested in \(T \ll \Gamma, eV\) which amounts to taking the limit \(T \to 0\). From Eq. (8),(9) it follows that in order to calculate the transmission amplitude \(t_{QD}\) one needs \(\Sigma(\omega)\) at the Fermi level \(\epsilon_f\) in the the arm containing the QD. In the following we will set \(\epsilon_f = 0\) without loss of generality. The real and imaginary parts of \(\Sigma_2\) at the Fermi level can be written as

\[
\Delta_d = \text{Re}\{\Sigma_2\} = P \int \frac{d\omega' d\omega''}{\pi} \rho_0(\omega'') A^- (\omega') f^{(j)}(\omega'),
\]

\[
\Gamma_d = -\text{Im}\{\Sigma_2\} = \int d\omega' f(\omega') A^+ (\omega') \rho_0(\omega'),
\]

where \(A^\pm (\omega) = A_0(\omega) \pm A_0(-\omega)\).

From Eq. (17) it follows that, because of the symmetry of \(A_0(\omega)\), \(\Delta_d\) goes to zero as \(\epsilon_0\) goes to zero. For \(\epsilon_0 < \Gamma\) we can estimate

\[
\Delta_d \approx -\lambda_c^2 \epsilon_0 (\xi_0 / \Gamma),
\]

where \(\lambda_c = \lambda(j,j)\) and \(\xi_0\) is a cutoff of the order of the Fermi energy in the QPC. Therefore \(\epsilon_0 + \Delta_d \approx (1 - \lambda_c^2 \xi_0 / \Gamma) \epsilon_0\) from which we see that in the weak coupling regime this shift can be neglected.

The imaginary part \(\Gamma_d\) controls the dephasing caused by the interaction with the QPC. At zero temperature and with the QPC in equilibrium the spectral density \(\rho_0\) is finite only for positive frequencies. As expected, this gives vanishing \(\Gamma_d\). If a finite bias is applied to the QPC we have

\[
\rho_0(\omega) = \lambda_d^2 (\omega + eV) \quad \text{for} \quad \omega < 0,
\]

where we used the notation \(\lambda(j,j') = \lambda_d\) for \(j \neq j'\). Substituting Eq. (20) in Eq. (18) and performing the frequency integration we obtain
\[ \Gamma_d = \lambda_d^2 \Gamma \left\{ \ln \left( \frac{(1 + \xi_+^2)^2}{(1 + \xi_+^2)(1 + \xi_-^2)} \right) + 2 \xi_+ [\arctan(\xi_+) - \arctan(\xi_0)] + 2 \xi_- [\arctan(\xi_-) + \arctan(\xi_0)] \right\}, \quad (21) \]

where we introduced the dimensionless variables \( \xi_\pm = (eV \pm \epsilon_0)/\Gamma \) and \( \xi_0 = \epsilon_0/\Gamma \). The dependence of \( \Gamma_d \) on the transparency \( \mathcal{T} \) of the QPC is contained in the coupling constant \( \lambda_d \) (Eq. (3)).

Using Eq. (21) together with Eq. (8) and Eq. (1) we finally obtain the transmission amplitude

\[ t_{QD} = \frac{i \sqrt{\Gamma_L \Gamma_R}}{(\epsilon_0 + \Delta) - i(\Gamma + \Gamma_d)}, \quad (22) \]

where \( \Delta = 2\lambda_c eV \) (\( \lambda_c = \lambda(j,j) \)) can be easily obtained from Eq. (23). In this formula both \( \Delta \) and \( \Gamma_d \) renormalize the transmission amplitude. However, the effect of \( \Delta \) is not related to dephasing and in real experiments \( \epsilon_0 \) is typically (e.g., by means of a plunger gate) so that \( \epsilon_0 + \Delta_0 = \epsilon \) is maintained constant.

It is now possible to relate \( \Gamma_d \) to the suppression of Aharonov-Bohm oscillations \( \nu \) defined in Eq. (1). We write \( t_{QD} \) in terms of the renormalized level \( \epsilon \) and expand it up to second order in \( \lambda \). The result is

\[ \nu = \frac{\Gamma^2}{\Gamma^2 + \epsilon^2} \left( \frac{\Gamma_d}{\Gamma} \right). \quad (23) \]

When the amplitude of A-B oscillations is measured, the relevant scale on which \( \epsilon \) is varied near the Fermi level is \( \Gamma \). In this interval one can obtain from Eq. (21) two simple expressions for \( \Gamma_d \) in the opposite limits of large and small bias

\[ \Gamma_d \approx \frac{2\lambda_d^2}{\Gamma^2} \left( \frac{eV^2}{\Gamma} \right) \quad \text{for} \quad eV \ll \Gamma, \quad (24) \]

\[ \Gamma_d \approx 2\pi \lambda_d^2 |eV| \quad \text{for} \quad eV \gg \Gamma. \quad (25) \]

Substituting these expressions in Eq. (23) with \( \epsilon = 0 \) we recover the two results anticipated in Section 1 (Eq. (2) and Eq. (3)). One moreover observes that the QD-QPC entanglement leaves another fingerprint in the dependence on the detuning of the QD resonance \( \epsilon \). Indeed, for \( eV \ll \Gamma \) the function \( \Gamma_d \) is very sensitive to the variation of \( \epsilon \), while in the other case these variations are negligible.

In Eq. (22) the energy scale \( \Gamma_d \) appears as a correction to the width \( \Gamma \). However, we emphasize that \( \Gamma_d \) is the imaginary part of \( \Sigma_2 \) at a specific energy (\( \omega = 0 \)). Examining the energy dependence of \( \text{Im}[\Sigma_2] \) we discover another profound difference between the two regimes. Taking for simplicity \( \epsilon_0 = 0 \) we obtain \( |\text{Im}[\Sigma_2(\omega)]| \approx \Gamma_d + 2\lambda_d^2 \omega^2/\Gamma \) for \( |\omega| \lesssim \Gamma \). In this interval the relative variation of \( \text{Im}[\Sigma_2] \), defined as \( \delta \equiv |\text{Im}[\Sigma_2(\Gamma)] + \Gamma_d | / \Gamma_d \), is given by

\[ \delta \approx \left( \frac{\Gamma}{eV} \right)^2 \gg 1 \quad \text{for} \quad eV \ll \Gamma \quad (26) \]

\[ \delta \approx \frac{\Gamma}{|eV|} \ll 1 \quad \text{for} \quad eV \gg \Gamma \quad (27) \]

It therefore follows that only in the large bias case ("rigid" dephasee limit) \( \text{Im}[\Sigma_2] \) is well approximated by a constant \( \Gamma_d \). Only then it is possible to interpret \( \Gamma_d \) as a dephasing rate and the dephasing-induced time dependence of the QD Green's function is exponential \((G_d(t) \propto e^{-|-(\Gamma + \Gamma_d) t|})\). In the opposite regime the strong variations of \( \text{Im}[\Sigma_2] \) with \( \omega \) imply that the characteristic of the dephasing by dephasing rate \( \Gamma_d \) is impossible, i.e., the dephasing is not described by a simple exponential decay of \( G_d(t) \).

We would like to express special thanks to Y. Levinson for many useful and stimulating discussions. We are also thankful to Y. Imry, M. Heiblum, B. Rosenow, D. Sprinzak, I. Ussishkin, M. Rokni, and M. Schechter for valuable discussions. We acknowledge the support of the DIP grant no.DIP-C7.1.

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