The penetration depth in Sr$_2$RuO$_4$: Evidence for orbital dependent superconductivity

Hiroaki Kusunose and Manfred Sigrist
Institut für Theoretische Physik - ETH-Hönggerberg, CH-8093 Zürich, Switzerland

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Abstract. – The apparent $T^2$-temperature dependence of the London penetration depth in Sr$_2$RuO$_4$ is discussed on the basis of a multi-gap model with horizontal line nodes. The influence of the nodes in combination with nonlocal electromagnetic response leads to low-temperature behaviors as $-T^2 \ln T$ in the London limit and as $T^2$ in the Pippard limit. These behaviors appear only at very low temperature. On the other hand, the interplay of the superconductivity of different bands is responsible for the observed $T^2$-like behavior over the wide temperature range. The experimental data can be fitted well with a set of material parameters as was used in the specific heat fitting.

Since the discovery of the superconductivity in the layered perovskite compound Sr$_2$RuO$_4$ [1], much effort has been devoted to the identification of its Cooper pairing symmetry [2]. An early theoretical prediction of the spin-triplet pairing [3] is supported by the NMR Knight shift measurement, which shows no change in the spin susceptibility on passing through the superconducting transition at $T_c (=1.5K)$ [4]. Moreover, $\mu$SR measurements reveal the appearance of an intrinsic magnetism with the onset of superconductivity, which was attributed to pairing with broken time reversal symmetry [5]. These two experiments consistently identify the superconducting phase as the chiral p-wave state $d_1(k) = z \Delta_1(T) (k_x + i k_y)$, an analog to the A-phase of the superfluid $^3$He. Here a representative form of the gap function, $\Delta_{\alpha\beta}(k) = (i\sigma^y \sigma^z)_{\alpha\beta} \cdot d(k)$, is given in the $d$-vector representation [3] [6]. Such a gap function arises in a weak-coupling theory for a two-dimensional Fermi liquid with spin-orbit coupling [3, 7, 8]. This pairing state suggests a nodeless quasiparticle gap, $\Delta_1(T)$.

As sample quality improved, however, power-law temperature dependence was observed in various quantities, such as specific heat [9], NQR relaxation rate [10], thermal conductivity [11, 12] and ultrasonic attenuation [13]. These findings seemed to indicate the unexpected existence of line nodes in the gap and motivated the investigation of models with vertical (pseudo) line nodes [14, 15, 16]. Heat transport measurements, however, gave no evidence of strong anisotropy of the gap in the basal plane, essentially excluding vertical line nodes [11, 12]. If at all, nodes would have to be horizontal, i.e., parallel to the $k_x$-$k_y$-plane [6]. Still such a gap structure alone seemed insufficient to account for all the thermodynamic data.

Motivated by these experimental situations and the fact that Sr$_2$RuO$_4$ has three conduction bands, called $\alpha$, $\beta$ and $\gamma$, Zhitomirsky and Rice (ZR) proposed a multi-gap scenario, in
which the primary nodeless gap, $d_1(k)$, is developed in the single “active” band ($\gamma$-sheet), while the secondary gaps with horizontal line nodes of the form,

$$d_2(k) = z\Delta_2(T)(\hat{k}_x + ik_y)\cos(k_zc/2),$$

(1)

appear in the other (“passive”) bands ($\alpha$, $\beta$-sheets) through an interband Cooper pair scattering [17]. Then this scenario was used to give a very good fit in the specific heat over the whole temperature range. The picture of an orbital dependent superconductivity (ODS) [18] provides a consistent view in many other aspects. Identifying the $\gamma$-band as the dominant band for superconductivity makes sense in view of the fact that ferromagnetic spin fluctuations, beneficial to spin-triplet pairing, are slightly enhanced in this band [19] [20], while the other two bands show very strong spin correlations for the finite incommensurate wave vector $q$. In addition, the chiral $p$-wave state is stabilized by spin-orbit coupling relative to all other triplet states, if the $\gamma$-band is dominant [23].

On general grounds, however, all scenarios involving line nodes are hampered by the fact that line nodes can appear in a spin triplet pairing state only accidentally, if spin-orbit coupling is present. Blount has shown that, in general, only point nodes appear by symmetry reasons [24]. In order to generate rigorous line nodes the ZR scenario requires exclusively the interlayer pairing for the $\alpha$-$\beta$-bands. Any inplane pairing component would leave point nodes only, in a rigorous sense. Nevertheless, we will assume the presence of horizontal line nodes for the time being, and later discuss consequences of the presence of the inplane pairing components.

In contrast to other thermodynamic quantities, the apparent $T^2$-dependence over wide temperature range below 0.8 K observed in the London penetration depth, $\lambda(T)$, for fields along the $z$-axis, [25] seems incompatible with line nodes, since a simple-minded theory would lead a $T$-linear behavior. Bonalde et al. ascribed $T^2$-behavior in their data to nonlocal effects, as was discussed by Kosztin and Leggett (KL) in the context of the high-temperature superconductors [25] [26]. The KL-theory is based on the fact that for the large limit of the Ginzburg-Landau (GL) parameter $\kappa = \lambda(0)/\xi_0$ (London limit), a small portion of the Fermi surface in the vicinity of the nodes ($\sim \kappa^{-1}$) should be treated in the nonlocal limit. This would alter the power-law from the linear to a $T^2$-behavior. While this discussion is applicable rigorously for very large $\kappa$ and in the very low-temperature regime only, Bonalde et al. argued that the relatively small $\kappa$ ($\approx 2-3$) of Sr$_2$RuO$_4$ could lead to a wider temperature range, even up to $T^* \sim \Delta(0)\kappa^{-1} \sim 0.6T_c$, below which the KL renormalization appears. Although there were several further theoretical explanations for the peculiar power-law behavior [16] [27] [28], a clear understanding has not been achieved yet.

In this Letter, we show that the rigorous asymptotic power-law behaviors in $\lambda(T)$ due to nonlocal effects are restricted to very low temperatures in both the London ($\kappa \gg 1$) and the Pippard ($\kappa \ll 1$) limits. Instead, based on the ODS model the overall $T^2$-dependence and beyond can be fitted well and consistently with the specific heat data. Thereby the non-locality of the electromagnetic response is essential for the fitting of the experimental data and the resultant Ginzburg-Landau (GL) parameter $\kappa = 2.3$ is good agreement with the experimental value.

We restrict our discussion to the clean limit and choose the sample geometry so that the $x$-$z$-plane surface is exposed to magnetic field parallel to the $z$-axis, so that both the vector potential $A$ and the screening currents $j$ are oriented parallel to the $x$-axis. The penetration depth is obtained from the electromagnetic response kernel, $K(q, T) = -(4\pi/c)j(q)/A(q)$, as

$$\lambda(T) = \frac{2}{\pi} \int_0^\infty \frac{dq}{q^2 + K(q, T)},$$

(2)

assuming a specularly reflecting surface.
We only consider intraband pairing so that in the lowest order each band separately contributes to the response kernel,

$$K(q, T) = \frac{1}{\lambda_0} \sum_i \zeta_i \tilde{K}_i(q, T). \tag{3}$$

Here the zero-temperature penetration depth in the London limit has been defined as $\lambda_0 = \sqrt{m e^2/4\pi n e^2}$ with $n/m = \sum_i (n_i/m_i)$, where $n_i$ and $m_i$ are the density and the effective mass of electrons in the band $i$. The ratio, $\zeta_i = (m_i/m_0)/(n_i/m_0)$, gives a measure for the contribution of each band to the current density at $T = 0$. The dimensionless response kernel has the form

$$\tilde{K}_i(q, T) = 2\pi T \sum_{n=-\infty}^{\infty} \sum_{k_z} 2 \int_0^1 d\mu \frac{\sqrt{1 - \mu^2} |d_i|^2}{\sqrt{\omega_n^2 + |d_i|^2 (\omega_n^2 + |d_i|^2 + Q_i^2 \mu^2)}}, \tag{4}$$

where $\omega_n = (2n + 1)\pi T$ is the fermionic Matsubara frequency. Here, we have assumed cylindrical Fermi surfaces for simplicity and used $(\partial |d_i|^2/\partial k) \cdot \mathbf{q} = 0$ for the present geometry. The nonlocality appears via $Q_i = q v_F/2 = (\pi \tilde{q}/2 \kappa_{0l}) \Delta_i(0)$ where the dimensionless variables $\tilde{q} = q_0/\lambda_0$ and $\kappa_{0l} = \lambda_0/\xi_{0l}$ with the coherence length for each bands, $\xi_{0l} = v_F/\pi \Delta_i(0)$, were introduced. For nodeless gaps or gaps with horizontal line-nodes, the angular integral in eq. (4) yields

$$\tilde{K}_i(q, T) = \frac{2\pi T}{Q_i^2} \sum_{n=-\infty}^{\infty} \sum_{k_z} |d_i|^2 g \left( \frac{\sqrt{\omega_n^2 + |d_i|^2}}{Q_i} \right), \tag{5}$$

where $g(z) = (\sqrt{1 + z^{-2}} - 1)/z$. At zero temperature, replacing $2\pi T \sum_n$ by $\int_{-\infty}^{\infty} d\omega$, we obtain

$$\tilde{K}_i(q, 0) = \sum_{k_z} g_0 \left( \frac{Q_i}{|d_i|} \right), \tag{6}$$

where $g_0(z) = (2/z) \tan^{-1}(z) - \ln(1 + z^2)/z^2$.

We consider now the contributions from the bands with horizontal line nodes, i.e., the passive bands in the ZR scenario ($\lambda = 2$), since their quasiparticles would dominate the low-temperature behavior, while those in the totally gapped dominant band are inactive. Analogous to the KL theory, we evaluate the average over $k_z$ for an approximate gap form in the vicinity of the nodes. It leads to

$$\delta \tilde{K}_i(q, T) = \tilde{K}_i(q, T) - \tilde{K}_i(q, 0) = \delta \tilde{K}_i(0, T) F(\tilde{q}/t), \tag{7}$$

where $\delta \tilde{K}_i(0, T) = -2 \ln 2 (T/\Delta_2(0))$ is the usual $T$-linear contribution to the kernel and $t = T/T_L$ with $T_L = Q_2/\tilde{q} = (\pi/2 \kappa_{02}) \Delta_2(0)$. For horizontal line nodes the universal function $F(z)$ has a form different from the KL-expression:

$$F(z) = 1 - \frac{1}{\ln 2} \int_0^z dx f(x) \left( 1 - \frac{x}{z} \right)^2, \tag{8}$$

and $f(x) = 1/(e^x + 1)$. Note that $F(z)$ has the asymptotic form $F(z) \approx \pi^2/6 \ln 2$ for $z \gg 1$ and $1 - z/6 \ln 2$ for $z \ll 1$. Since $\tilde{K}_2(q, 0) = 1 - \pi \tilde{q}/6 \xi_{02} \approx 1$ in the London limit ($\kappa_{02} \gg 1$) (eq. (10)), the deviation of $\lambda(T)$ from its zero-temperature value, $\lambda(0) (= \lambda_0)$, is given by

$$\frac{\Delta \lambda(T)}{\lambda_0} = \frac{\lambda(T) - \lambda(0)}{\lambda_0} = -\frac{2}{\pi} \int_0^{\infty} d\tilde{q} \frac{\delta \tilde{K}_2(q, T)}{(\tilde{q}^2 + 1)^2}. \tag{9}$$
An appropriate estimate of eq. 9) gives the following asymptotic expression for \( T < 0.05T_L \),

\[
\frac{\Delta \lambda(T)}{\lambda_0} = \frac{\pi^2}{3 \kappa_{02}} \left( \frac{T}{T_L} \right)^2 \left( 1.0 + \ln \frac{T}{T_L} \right).
\] (10)

In the Pippard limit (\( \kappa_{02} \ll 1 \)) we use \( g(z) = z^{-2} \) in eq. 9. Then, the Matsubara sum

\[
\tilde{K}_2(q, T) = \frac{\pi}{Q_2} \sum_{k_z} |d_2| \tanh \left( \frac{|d_2|}{2T} \right).
\] (11)

From eq. 2 we obtain

\[
\frac{\lambda(T)}{\lambda(0)} = \left[ \frac{\pi}{2} \sum_{k_z} \frac{|d_2|}{\Delta_2(0)} \tanh \left( \frac{|d_2|}{2T} \right) \right]^{-1/3},
\] (12)

where \( \lambda(0)/\lambda_0 = (2\pi/3\sqrt{3})(\pi/4\kappa_{02})^{1/3} \). At low temperature the main contribution in eq. 11 comes from \( k_z \sim \pm \pi/c \). Evaluating the \( k_z \)-average in eq. 11 or using the large-\( q \) limit of eqs. 10 and 11, the low-temperature expression is given by

\[
\frac{\Delta \lambda(T)}{\lambda(0)} = \frac{1}{18} \left( \frac{T}{T_P} \right)^2 + \frac{83}{3240} \left( \frac{T}{T_P} \right)^4,
\] (13)

where \( T_P = \Delta_2(0)/\pi \). The deviation from the \( T^2 \)-dependence becomes visible for \( T \) larger than \( 0.3T_P \). Note that the prefactor of the \( T^4 \) term in both limits is determined by the inverse of the characteristic temperatures, i.e. the smaller \( \Delta_2(0) \) gives the faster increase of \( \lambda(T) \) close to \( T = 0 \).

In conclusion, the power-law behaviors due to nonlocal effect are restricted to temperature below \( T^* \). For any \( \kappa \), \( T^* \) has a value between both limits, i.e., \( 0.05T_L < T^* < 0.3T_P \), which is much lower than \( \Delta_2(0) \). Since \( T^* < 0.17T_L \) even with the maximal gap, \( \Delta_2(0) = \Delta_{BCS} \) (= 1.76\( T_L \)), it already excludes a reasonable fitting of the \( T^2 \)-behavior with a single-gap model including horizontal line nodes for \( \text{Sr}_2\text{RuO}_4 \).

We now turn to the multi-gap scenario for \( \text{Sr}_2\text{RuO}_4 \), in which the temperature dependence of the gap magnitudes are given by an effective two-band model: \( \Delta_{\alpha} = \Delta_{\beta} = \Delta_2(T) \) and \( \Delta_{\gamma} = \Delta_1(T) \). In the gap equations, the pairing interactions are defined as

\[
V_{11} = -g_1k \cdot k', \quad V_{22} = -2g_2(k \cdot k') \cos(k_c c/2) \cos(k'_c c/2) \text{ for intraband coupling and}
\]

\[
V_{12} = -\sqrt{2}g_1(k \cdot k') \cos(k'_c c/2) \text{ for the interband Cooper pair scattering.}
\]

We estimate various other material parameters from the de Haas-van Alphen measurement [8]: the Fermi velocities, \( v_{F,\alpha} \sim v_{F,\beta} = 0.68v_{F,\gamma} \), the (Fermi surface averaged) band masses over their carrier densities, \( \xi_{\alpha} : \xi_{\beta} : \xi_{\gamma} = 0.20 : 0.44 : 0.36 \) and the densities of states at the Fermi level, \( N_{01} : N_{02} = 0.57 : 0.43 \). We fix the parameters as \( g_1 = 0.4 \) (in units of the inverse total density of states), \( g_2/g_1 = 0.85 \) and \( \kappa_{02}/\kappa_{01} = (\Delta_2(0)/\Delta_1(0))/0.68 \). This leaves two free parameters for fitting: \( g_3/g_1 \) and \( \kappa_{01} \).

Figure 4 shows that the temperature dependence of the gaps \( \Delta_{i}(T) \) for different interband couplings \( g_3/g_1 \). In the absence of interband coupling (\( g_3 = 0 \)) a second phase transition occurs for \( \Delta_{\gamma} \) at temperature \( T/T_c \sim 0.2 \) yielding a gap of order \( \Delta_2(0)/\Delta_{BCS} \sim 0.2 \). The stronger the interband coupling the less visible the feature of this transition becomes. For strong interband coupling all gaps are tied together and act like a superconductor with a single
Fig. 1 – The temperature dependence of the gap functions scaled by the BCS gap at $T = 0$. The interband couplings are varied, while the other parameters are fixed as $g_1 = 0.4$ and $g_2/g_1 = 0.85$.

order parameter. The ZR fitting of the specific heat suggests that we are in an intermediate regime of interband coupling, $g_3/g_1 = 0.07$.

In Fig. 2 we compare the calculated penetration depth for various values of $g_3/g_1$ and $\kappa_{01} = 2.0$ with the experimental data by Bonalde et al. [25] with $\xi_{01} = 700$ Å [29]. For the weak interband coupling, the penetration depth increases rapidly at low temperature below $T/T_c \sim 0.2$, which reflects both the smallness of the passive-band gap, $\Delta_2(0)$, and the rapid decrease of $\Delta_2(T)$ at $T/T_c \sim 0.2$. For the strong interband coupling, the $T^2$-increase due to the passive bands is easily overshadowed by the active-band contribution, showing a monotonous

Fig. 2 – The temperature dependence of the penetration depth for the different $g_3/g_1$. The other parameters are fixed as $g_1 = 0.4$, $g_2/g_1 = 0.85$ and $\kappa_{01} = 2.0$. A moderate interband coupling fits well the observed temperature dependence.
increase with concave shape. Intermediate interband couplings give a temperature dependence with a weak convexo-concave shape by subtly combining the contributions from both bands. The experimental data indeed exhibit this type of behavior. The moderate interband coupling, \( g_3/g_1 = 0.07 \), which is exactly the same parameter as ZR to fit the specific heat measurement, also reproduces the observed temperature dependence over the wide temperature range.

Next we fix the interband coupling constant to the value optimized by ZR \( g_3/g_1 = 0.07 \) and use \( \kappa_{01} \) as the only free fitting parameter. Figure 3 shows curves for various values of \( \kappa_{01} \), where the resultant GL parameter for the active band, \( \kappa_1 = \lambda(0)/\xi_{01} \), is shown in the parenthesis. The thin solid line is for the nodeless gap in passive bands (see text in detail).

![Graph](image)

Fig. 3 — The \( \kappa_{01} \) dependence for \( g_1 = 0.4 \), \( g_2/g_1 = 0.85 \) and \( g_3/g_1 = 0.07 \). The resultant Ginzburg-Landau parameters, \( \kappa_1 = \lambda(0)/\xi_{01} \) are shown in the parenthesis. The thin solid line is for the nodeless gap in passive bands (see text in detail).

While the difference is small in \( \lambda(T) \), the removal of the nodes would be more easily visible in the specific heat data.
We have shown that the nonlocal response for the horizontal line node yields $-T^2 \ln T$ behavior in the London limit, while $T^2$ power law in the Pippard limit yet at quite low temperature. On the other hand, the ODS model in combination with nonlocal response naturally explains the apparent $T^2$-dependence in a quantitatively consistent way with the specific heat fitting. Concerning the question of nodes, it seems that the London penetration depth data are less affected by nodes than the specific heat data. Eventually, a more anisotropic but nodeless gap shape would likely be sufficient for good fits in any case. In view of the complicated quasiparticle spectrum in the multi-gap compound Sr$_2$RuO$_4$, it is difficult from present experimental data to come to conclusive answer concerning the detailed gap shape.

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