Structure of baryon states from non-perturbative methods.

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Abstract. In this approach we have adapted non-perturbative many-body methods to describe baryon-like states built from effective quarks, antiquarks degrees of freedom and collective RPA states. As starting point, we have used the QCD Hamiltonian written in the Coulomb gauge, and considered a confining interaction to describe the non-perturbative effects of dynamical gluons. The effective Hamiltonian used is expressed in terms of the effective quark-antiquark, di-quarks and di-antiquark excitations and approximately diagonalized by mapping quark-antiquark pairs and di-quarks (di- antiquarks) onto phonon states. For the structure of the vacuum of the theory, color-scalar- and color-vector-structures are introduced to account for ground state correlations. We present the structure of the baryon-like states, beyond the common three constituent quarks, describe their interactions and present a simple calculation to illustrate the method.

1. Introduction

The low-energy QCD regime has been widely studied during the last four decades. Several aspects have been investigated by the implementation of non-perturbative methods, among them the two more outstanding are the Lattice gauge theory (LGT) and the Dyson-Schwinger Equations (DSEs). The LGT has had several and important advances in the understanding of fundamental phenomena dominated by the low-energy regime of the theory, the two more notorious are: a first principle description of the confinement potential of quarks and gluons [1] and the chiral symmetry breaking. The LGT has provided a satisfactorily description of the low energy meson and baryon spectrum [2–4], but due to the computational limitations, the pion mass used is considerable larger than the experimental one [3] and the accessibility to higher excited states is complicated. The DSEs has had also important and significant advances in the understanding of the low-energy regime of QCD. The description of the dynamical quark and gluon mass generation by the implementation of self-consistent techniques has been outstanding, as well as its contributions to the description of the hadron spectrum [5]. Particularly, the DSEs results for the low energy baryon spectrum is able to describe properly certain aspects of the spectrum, e.g., the Roper resonance description as a radial excitation of the nucleon and its low mass understanding as an effect of a meson cloud that shields the quark core [6]. The models
based on the identification of effective degrees of freedom and their interactions has been also a crucial step in order to describe physical systems like mesons and baryons. In this sense, the many-body methods [7–9] have been a guide during a long time, achieving remarkable success in nuclear and hadron physics. Despite all of these efforts, the low-energy regime of QCD remains unsolved and still more work has to be done in order to get more insights about the physics of different phenomena involved in the proper description of physical states and the interactions between quarks and gluons inside them.

Here, we shall show that the identification of effective degrees of freedom and the use of the Random-Phase-Approximation (RPA) many-body method, implemented in the non-perturbative regime of QCD, can generate a feasible description of the interactions between effective quarks and antiquarks with collective RPA particle-hole (ph), di-quark (di-\(q\)) and di-antiquark (di-\(\bar{q}\)) systems. With all of them a description of the baryon states beyond the common three quarks can be provided. The results look promising and encouraging for further investigation along this line. In this work, the identification of single-particle and hole excitations as well as collective degrees of freedom allows for the construction of effective interactions and discussed their structure in terms of the various couplings allowed by the QCD Hamiltonian in the Coulomb gauge [10]. Several features of the non-perturbative regime of QCD have been explored within the Coulomb gauge framework [11–17]. In [21], the sensitivity of the chiral symmetry breaking has been explored in the excited baryon spectrum, within the Coulomb gauge QCD formalism. Such analysis has been performed in a controlled way in terms of the number of harmonic oscillator (h.o.) shells, similar to that performed in [9,22].

The present description is based on the use of effective quark and antiquark states and quark-antiquark, di-\(q\) and di-\(\bar{q}\) pairs and their interactions. We start from the QCD Hamiltonian written in the Coulomb gauge [12,13] and perform a diagonalization of the kinetic energy term, expressed in the h.o. basis, to get quark and antiquark states. With the effective single-quark and antiquark states, we rewrite the color-charge density interactions and replace the instantaneous color Coulomb interaction by a static potential \(V(r) = (-\alpha/r) + (\beta r)\) [9]. A further diagonalization of the pair-like terms (ph, di-\(q\) and di-\(\bar{q}\)) of the Hamiltonian is performed by applying the RPA method [23], and finally, the remaining terms of the Hamiltonian are treated as a generalized phonon-single-particle- and pair-interactions, keeping in all cases the symmetries of the QCD degrees of freedom at the level of meson and baryon-like states. The main characteristic of the method consist in the use of coloured-pairs configurations of both quarks and antiquarks, in addition to the colorless quark-antiquark pair configurations, to construct baryon-like states by coupling these configurations to quarks or antiquarks. The formalism and general aspects of the model are described in Sections 2, a detailed description of the interactions is presented in Section 4 and the application of the method is illustrated in Section 4.1. The conclusions are drawn in Section 5.

2. General aspects and the effective Hamiltonian.

2.1. QCD Hamiltonian in the Coulomb gauge.

As we have mentioned in the Introduction, we start from the QCD Hamiltonian in its canonical Coulomb gauge representation [10],

\[
H^{QCD} = \int \left\{ \frac{1}{2} \left[ J^{-1} \Pi^{tr} \cdot J \Pi^{tr} + B \cdot B \right] - \bar{\psi} \left( -i \gamma \cdot \nabla + m \right) \psi - g \bar{\psi} \gamma \cdot A \psi \right\} dx + \frac{g^2}{2} \int J^{-1} \rho^c(x)\langle c, x | \frac{1}{\nabla \cdot B} (-\nabla^2) \frac{1}{\nabla \cdot B} | c' y \rangle J \rho^c(y) dxdy, \tag{1}
\]

which has been widely studied in the past in its non-perturbative regime [7,9,12–17,21,22,24,25], in order to get insights about the confinement of color-charged particles along with the
constituent quark and gluon masses. Also several bound-states like meson, baryons and others predicted by the theory, e.g. hybrids and glueballs, has been described \[7,8,18–22,26,27\].

The Hamiltonian of Eq. (1) takes into account the interactions between quarks and gluons through the QCD Instantaneous color-Coulomb Interaction (QCD-IcCI) between color-charge-densities of quarks and gluons. At low energy the effects of dynamical gluons in the QCD-IcCI are accounted for by the interaction \( V(|\mathbf{x} - \mathbf{y}|) = -\frac{\alpha}{|\mathbf{x} - \mathbf{y}|} + \beta|\mathbf{x} - \mathbf{y}| \), which is obtained from a self-consistent treatment of the last term of the Hamiltonian \[12,13\].

The quark sector of the Hamiltonian of Eq. (1), is given by

\[
H_{eff}^{QCD} = \int \left\{ \psi^\dagger (\mathbf{x}) (-i \mathbf{\alpha} \cdot \nabla + \beta m) \psi (\mathbf{x}) \right\} d\mathbf{x} - \frac{1}{2} \int \rho_c (\mathbf{x}) V(|\mathbf{x} - \mathbf{y}|) \rho_c^\dagger (\mathbf{y}) d\mathbf{x} d\mathbf{y},
\]

where \( \rho_c^\dagger (\mathbf{x}) = \psi^\dagger (\mathbf{x}) T^c \psi (\mathbf{x}) \) is the quark and antiquark color-charge density. In Eq. (2), the first term is the kinetic energy, while the second term is the QCD-IcCI in its simplified form. The fermion field \( \psi^\dagger (\mathbf{x}) \), whose quantization was explained in \[9\], is expanded in terms of creation and annihilation operators of particles (antiparticles) in a basis of h.o. functions.

The use of the h.o. basis has several advantages, like a considerable simplification in the matrix elements and their calculation, also the use of the h.o. basis represents finite range fields which is something expected for quarks and antiquarks. However, its use requires a pre-diagonalization of the kinetic term, which is performed by means of an unitary transformation between creation (annihilation) operators in the h.o. basis \( q_{\tau(N)}^\dagger \left(q^\tau(N)\right) \) and those belonging to an effective basis \( Q_{M\pi q}^\dagger \left(Q^{M\pi q}\right) \) of quarks, such that

\[
q_{\tau(N)}^\dagger \left(J_q C_q(Y, T_q), M_J q M_{C_q} M_{T_q}\right) = \sum_{\lambda \pi q k} \left(\alpha_{J_q T_q}^{\lambda \pi q}, q\right) \star Q_{\lambda \pi q}^\dagger 
\]

\[\times \left(J_q C_q(Y, T_q), M_J q M_{C_q} M_{T_q}\right) \delta_{\pi q, (1-\tau/2) \pm t}, \]

The effective quark \( b^\dagger \) and antiquark \( d \) operators are obtained via

\[
Q_{-\frac{1}{2}k\pi q}^\dagger \left(J_q C_q(Y, T_q), M_J q M_{C_q} M_{T_q}\right) = d_{k\pi q} \left(J_q C_q(Y, T_q), M_J q M_{C_q} M_{T_q}\right), \]

and

\[
Q_{\frac{1}{2}k\pi q}^\dagger \left(J_q C_q(Y, T_q), M_J q M_{C_q} M_{T_q}\right) = b_{k\pi q} \left(J_q C_q(Y, T_q), M_J q M_{C_q} M_{T_q}\right). \]

The sub-index \( \pi q = \pm \) indicates the parity of the effective quark or antiquark and \( k = 1, 2, \ldots \) runs over all orbital states. The index \( J_q = \frac{1}{2}, \frac{3}{2}, \ldots \) indicates the total \( (J_q = I \pm \frac{1}{2}) \) single-particle spin. The flavor hypercharge and isospin quantum numbers for quarks are given by \( (Y_q, T_q) = \left(\frac{1}{2}, \frac{1}{2}\right), \left(-\frac{1}{2}, 0\right), \left(-\frac{1}{2}, \frac{1}{2}\right)\). The quarks and antiquarks belong to a triplet \( C_q = (10) \) and anti-triplet \( \bar{C_q} = \bar{C_q} = (01) \) color irreducible representations (irreps), respectively, which are conjugate representations. From now on, we will use the short-hand notation \( \gamma_q = \{J_q, C_q, (Y_q, T_q)\} \) for the quarks and \( \gamma_q = \{J_q, \bar{C_q}, (\bar{Y_q}, \bar{T_q})\} \) for antiquarks irreps with \( \bar{Y_q} = -Y_q \), while \( \mu_q = \{M_J q, M_{C_q}, M_{T_q}\} \) and \( \bar{\mu_q} = \{M_J q, \bar{M_{C_q}}, \bar{M_{T_q}}\} \) will be used for the magnetic numbers with \( M_{J_q} = -M_{J_q} \) and \( M_{T_q} = -M_{T_q} \).

The kinetic \( (K) \) term rewritten in terms of effective quarks and antiquarks operators has the following structure \[9\]

\[
K = \sum_{k\pi q \gamma q} \sum_{\mu_q} \left( b_{k\pi q \gamma_q \mu_q} \bar{b}_{k\pi q \gamma_q \bar{\mu_q}} - d_{k\pi q \gamma_q \mu_q} \bar{d}_{k\pi q \gamma_q \bar{\mu_q}} \right),
\]
where the rules to raise and lower indices are taken from [28].

The QCD-icCI term, in its simplified form \( H_{\text{Coul}} \), rewritten in terms of effective quark and antiquark operators is given by [9]

\[
H_{\text{Coul}} = -\frac{1}{2} \sum L \sum q_i \sum \left[ \left( \mathcal{F}_{\lambda_1 q_1, \lambda_2 q_2; \gamma_f} \mathcal{G}_{\lambda_3 q_3, \lambda_4 q_4; \gamma_f} \right)^\dagger_{\mu_f} + \left( \mathcal{F}_{\lambda_1 q_1, \lambda_2 q_2; \gamma_f} \mathcal{G}_{\lambda_3 q_3, \lambda_4 q_4; \gamma_f} \right)_{\mu_f} \right]_{\gamma_f}
\]

\[
+ \left[ \mathcal{G}_{\lambda_1 q_1, \lambda_2 q_2; \gamma_f} \mathcal{G}_{\lambda_3 q_3, \lambda_4 q_4; \gamma_f} \right]_{\mu_f} + \left[ \mathcal{G}_{\lambda_1 q_1, \lambda_2 q_2; \gamma_f} \mathcal{G}_{\lambda_3 q_3, \lambda_4 q_4; \gamma_f} \right]_{\mu_f},
\]

where we have compacted the single particle orbital number, parity and irreps, into the shorthand notation \( q_i = k_i \pi_{\eta_i \gamma_{\eta_i}} \), and use for the (flavorless) quantum numbers of the interaction the label \( \gamma_f = \{L, (11), (0,0)\} \) and for their magnetic projections \( \mu_f = \{M_L, M_C, 0\} \), respectively. The conjugate representations satisfy \( \bar{\gamma}_f = \gamma_f \) and \( \bar{\mu}_f = \{-M_L, M_C, 0\} \). Then for the total couplings (upper index) and magnetic numbers (lower index) of the interaction, we have used \( \gamma_0 = \{0, (00), (0,0)\} \) and \( \mu_0 = \{0, 0, 0\} \) respectively. The operators \( \mathcal{F} \) and \( \mathcal{G} \) are written

\[
\mathcal{F}_{\lambda_1 q_1, \lambda_2 q_2; \gamma_f} = \frac{1}{\sqrt{2}} \left\{ \delta_{\lambda_1, -1} \delta_{\lambda_2, 1} \left[ b^\dagger_{q_1} \otimes b_{q_2} \right]_{\gamma_f} - \delta_{\lambda_1, -1} \delta_{\lambda_2, -1} \left[ d^\dagger_{q_1} \otimes d_{q_2} \right]_{\gamma_f} \right\}
\]

\[
\mathcal{G}_{\lambda_1 q_1, \lambda_2 q_2; \gamma_f} = \frac{1}{\sqrt{2}} \left\{ \delta_{\lambda_1, -1} \delta_{\lambda_2, 1} \left[ d^\dagger_{q_1} \otimes d_{q_2} \right]_{\gamma_f} - \delta_{\lambda_1, -1} \delta_{\lambda_2, -1} \left[ d^\dagger_{q_1} \otimes d_{q_2} \right]_{\gamma_f} \right\}.
\]

The \( \mathcal{F}^2 \) and \( \mathcal{G}^2 \) terms describe the interactions with two creation and two annihilation operators \( H_{22} \), as well as the creation \( H_{40} \) and four annihilation operators \( H_{04} \), which are the structures relevant for the RPA bosonization. The RPA method leaves out the Hamiltonian terms \( H_{31} \) and \( H_{13} \), i.e. the \( \mathcal{FG} \) and \( \mathcal{GF} \) terms, corresponding to three creation and one annihilation operators and their Hermitian conjugate terms. These terms are taken into account in the mapped Hamiltonian of Section 4.

The Hamiltonian expressed in the effective quark and antiquark operators, Eqs. (6) and (7), describes the interactions of these effective particles in bound states like mesons and baryons. In [9] the low energy meson states with \( J^P = 0^+, 1^- \) were represented by particle(quarks)-hole(antiquarks) (or ph) phonons by means of the Tamm-Dancoff-Approximation (TDA) and the RPA methods. The ph structure of the interactions in the Hamiltonian are shown in Figure 1a). The TDA and RPA methods provided some insights about the low-energy meson spectrum and the influence of ph-correlations in the vacuum. The ph-correlations considered in the vacuum

![Figure 1](image_url)

**Figure 1.** \( H_{22} \), \( H_{40} \) and \( H_{04} \) interaction terms for a) particle-hole and b) di-quark and di-antiquarks subspaces, respectively.
were colorless in order to describe meson-states as one-phonon excitations. In the present work, we go further and consider color-correlations in a more general description of the vacuum. These color correlations, viewed as excitations of the vacuum, can not describe physical states but they may be coupled to color-vector configurations to describe bound states e.g., baryon states, which are constructed such that the overall color is zero.

3. Basic operators and representations.

In the following, for the vacuum, meson-like (or \( ph \)-pair), di-\( q \) and di-\( \bar{q} \) states we use a similar short-hand notation \( \gamma_n = \{ J_n, C_n, (Y_n, T_n) \} \). The sub-index \( n = 0, q, p, a, r \) reads for the vacuum, quarks, mesons, di-\( q \) and di-\( \bar{q} \) states, respectively. In this work we only consider quarks and antiquarks with total spin \( J_q = \frac{1}{2} \). Higher single particle spin states are left for a further publication. For the baryon state representations and magnetic components we will use \( \gamma_B = \{ J_B, C_B, (Y_B, T_B) \} \) and \( \mu_B = \{ M_{J_B}, M_{C_B}, M_{T_B} \} \), respectively.

3.1. Vacuum state.

In [9] the correlated vacuum included colorless \( ph \) correlations, constructed explicitly by using the RPA formalism, thus the RPA solutions aimed to describe meson-like states. In general, the ground state can include also colored correlations by means of (11)-\( ph \)-pairs and (20), (01)-di-\( q \) and (02), (10)-di-\( \bar{q} \) operators. Up to second order in the quark-antiquarks operators, we can relate the new ground state to the vacuum state \( |0 \rangle \) via

\[
|RPA\rangle = |\tilde{0} \rangle + \sum_{p\mu} \sum_{q,a} Z^{p\mu\gamma}_{\phi q,a} \times \left[ b_{q,1}^{\dagger} \otimes d_{q,2}^{\dagger} \right]_{\mu_1}^{\gamma_2} \left[ b_{a,1}^{\dagger} \otimes d_{a,2}^{\dagger} \right]_{\mu_2}^{\gamma_1} |\tilde{0} \rangle.
\]

The vacuum coefficients \( Z^{p\mu\gamma}_{\phi q,a} \) expression have been shown and explained in [29].

The terms \( b_{q,1}^{\dagger} \otimes d_{q,2}^{\dagger} \) and \( d_{a,1}^{\dagger} \otimes d_{a,2}^{\dagger} \), represent the \( ph \)-pair operators (or quark-antiquark operators), the di-\( q \) and di-\( \bar{q} \) operators, respectively. Their explicit expressions have been shown in [29]. These operators are the building blocks of the RPA operators, as we will show in Section 3.2, and will be crucial for the mapping of the effective Hamiltonian, Eqs. (6)-(7), to a Hamiltonian that describes the propagation of the effective quarks, antiquarks, the RPA-pair and their interactions.

3.2. \( ph \)-, di-\( q \)- and di-\( \bar{q} \) phonon operators.

The one-phonon creation-operators of the \( ph \)-, di-\( q \)- and di-\( \bar{q} \)-type are defined as follows

\[
\Gamma_{p\mu\gamma}^{\dagger} = \sum_{q,a} \left\{ X_{q,a;2\mu\gamma} b_{q,2}^{\dagger} \otimes d_{a,2}^{\dagger} \right\}_{\mu_2}^{\gamma_2} - Y_{q,a;2\mu\gamma} (-1)^{\delta_{q\mu}^{\gamma_2}} \left[ d_{q,2}^{\dagger} \otimes b_{a,2}^{\dagger} \right]_{\mu_2}^{\gamma_2}
\]

(10)

\[
\Gamma_{a\mu\gamma}^{\dagger} = \sum_{q,a} \left\{ X_{q,a;2\mu\gamma} b_{a,2}^{\dagger} \otimes b_{q,2}^{\dagger} \right\}_{\mu_2}^{\gamma_2} - Y_{q,a;2\mu\gamma} (-1)^{\delta_{\gamma_2}^{\mu_2}} \left[ d_{q,2}^{\dagger} \otimes d_{a,2}^{\dagger} \right]_{\mu_2}^{\gamma_2}
\]

(11)

and

\[
\Gamma_{r\mu\gamma}^{\dagger} = \sum_{q,a} \left\{ X_{q,a;2\mu\gamma} d_{q,2}^{\dagger} \otimes d_{a,2}^{\dagger} \right\}_{\mu_2}^{\gamma_2} - Y_{q,a;2\mu\gamma} (-1)^{\delta_{\gamma_2}^{\mu_2}} \left[ b_{q,2}^{\dagger} \otimes b_{a,2}^{\dagger} \right]_{\mu_2}^{\gamma_2}
\]

(12)
respectively, where the amplitudes $X$ and $Y$ are the forward- and backward-going amplitudes, in each case. The one-phonon annihilation operators $\Gamma^{n\pi n, \gamma n, \mu n}$ satisfy $\Gamma^{n\pi n, \gamma n, \mu n} \langle RPA \rangle = 0$, for $n = p, a, r$. The phase $(-1)^{\phi_{\gamma n, \mu n}} = (-1)^{J_n - M_n} (-1)^{\chi_{C_n} - T_n - M_T}$ in each phonon operator $n = p, a, r$ accounts for the scalar couplings in the vacuum, Eq. (9).

In order to explore the effective QCD Hamiltonian by means of the RPA method within colorless, like in [9], and colored subspaces like those spanned by the di-$q$ and di-$\bar{q}$ phonon operators, it is important to distinguish the interactions, see Figure 1, that will contribute to the $ph$ subspace and those to the di-$q$ (di-$\bar{q}$) subspaces of RPA equation,

$$
\langle RPA \mid \hat{T}^{n'\pi n, \gamma n, \mu n}, \left[ \hat{H}^{QCD}_{eff}, \Gamma^{n\pi n, \gamma n, \mu n} \right] \mid RPA \rangle = \omega_{n\pi n, \gamma n} \delta_{n,n'},
$$

(13)

with $\Gamma^{n\pi n, \gamma n, \mu n}$ the collective RPA operators and $\omega_{n\pi n, \gamma n}$ the eigenvalues for the corresponding $ph$- di-$q$- di-$\bar{q}$-subspace. The latter implies four color subspaces of diagonalization i.e., $C_p = (00), (11)$ and $C_a = (20), (01)$, since the corresponding diagonalizations in the subspaces $C_r = (02) and C_r = (10)$ are symmetric with respect to $C_a = (20)$ and $C_r = (01)$ respectively.

### 3.3. Baryon-like operators and representations.

We now introduce the baryon representations obtained from the effective quarks and the phonon operators described above.

The simplest baryon-like operators in this scheme are given by

$$
\hat{B}^{b_1}_{B_1} = \left[ b^{\dagger}_{q_1} \otimes b^{\dagger}_{q_2} \otimes b^{\dagger}_{q_3} \right]_{\mu B_1},
$$

(14)

where $\pi_{B_1} = \prod_i \pi_i$, and $\gamma_{B_1} = \{ J_{B_1}, (Y_{B_1}, T_{B_1}), C_{B_1} \}$. These baryon-like operators do not account for any ground state correlations.

The first baryon-like operators that account for ground state correlations are

$$
\hat{B}^{b_1}_{B_2} = \left[ \hat{B}^{b_1}_{B_1} \otimes \Gamma^{\dagger_{\mu B_1}}_{\mu B_2} \right]_{\mu B_2}.
$$

(15)

Clearly the number of baryon-like representations for the $\hat{B}^{b_1}_{B_2}$ operator increases considerable respect to the $\hat{B}^{b_1}_{B_1}$ operator, due to all the possibilities to couple to baryon quantum numbers. In Section 4.1, we present the results for the lowest baryon-like states and discuss the more important features.

The other baryon-like operator that accounts for ground state correlations is

$$
\hat{B}^{b_1}_{B_3} = \left[ \left[ b^{\dagger}_{q_1} \otimes b^{\dagger}_{q_2} \otimes d^{\dagger}_{q_3} \right]_{\mu B_3} \right]_{\mu B_3},
$$

(16)

where this operator can be seen as a quark in the presence of a simple scalar $ph$-pair and a di-$q$ phonon. Therefore, in order to describe a positive parity baryon-like state, the quark denoted by $q_1$ has the same parity as the di-$q$ phonon operator $\pi_{q_1} = \pi_0$.

With the description of the baryon-like operators in terms of the effective degrees, we can analyze the effective Hamiltonian, searching for those interactions that will be relevant for the description of a more general baryon-like state.
4. The mapped Hamiltonian.

In a first stage the RPA method leaves out the Hamiltonian terms $H_{31}$ and $H_{13}$, i.e. second and third terms in Eq. (7), corresponding to three creation and one annihilation operators and their hermitian conjugate terms. These terms are taken into account by the transformed Hamiltonian of Eq. (17). Once the RPA method is implemented and their solutions known, the effective QCD Hamiltonian can be mapped onto a Hamiltonian that describes the propagation of the original fermionic particles (quarks and antiquarks) as well as the boson-like particles ($ph$, $di-q$ and $di-q$ phonons) and the interaction between them. The procedure to get the vertex functions ($\Lambda_{n\pi\gamma,qq}$) is rather straightforward: one needs to return to the effective Hamiltonian, Eqs. (6)-(8), and replace the simple $ph$-pair, $di-q$ and $di-q$ operators by linear combinations of phonon operators. In Figure 2, we illustrate the mapping of two $H_{31}$ terms by replacing a $ph$-pair and a $di-q$ by a linear combination of phonon operators. The latter is done by inverting Eqs. (10), (11) and (12), where the pseudo-norm $X^2 - Y^2 = 1$ is satisfied by the forward and backward amplitudes of each phonon operator.

The general form of the mapped effective Hamiltonian is given by

$$
H_{QCD}^{eff, map} = \sum_{q\mu q} \varepsilon_q \left( a^\dagger_{q\mu q} a_{q\mu q} + d^\dagger_{q\mu q} d_{q\mu q} \right) + \sum \sum \omega_{n,\pi,\gamma,\mu} \Gamma_{n,\pi,\gamma,\mu}^\dagger \frac{(-1)^{\gamma - \mu}}{\sqrt{\text{dim}(\gamma)}} \left( \left[ b^\dagger_{q\mu} \otimes b_{q\mu} \right]_{\gamma}^\dagger + \left[ d^\dagger_{q\mu} \otimes d_{q\mu} \right]_{\gamma}^\dagger \right) + h.c. 
$$

$$
+ \sum \sum \Lambda_{n\pi\gamma,qq}^{(1)} \sum_{\mu} \frac{(-1)^{\gamma - \mu}}{\sqrt{\text{dim}(\gamma)}} \left( \Gamma_{n,\pi,\gamma,\mu}^\dagger \left[ b^\dagger_{q\mu} \otimes b_{q\mu} \right]_{\gamma}^\dagger + \left[ d^\dagger_{q\mu} \otimes d_{q\mu} \right]_{\gamma}^\dagger \right) + h.c. 
$$

$$
+ \sum \sum \Lambda_{n\pi\gamma,qq}^{(2)} \sum_{\mu} \frac{(-1)^{\gamma - \mu}}{\sqrt{\text{dim}(\gamma)}} \left( \Gamma_{n,\pi,\gamma,\mu}^\dagger \left[ d^\dagger_{q\mu} \otimes b_{q\mu} \right]_{\gamma}^\dagger + h.c. \right) 
$$

$$
+ \sum \sum \Lambda_{n\pi\gamma,qq}^{(3)} \sum_{\mu} \frac{(-1)^{\gamma - \mu}}{\sqrt{\text{dim}(\gamma)}} \left( \left[ b^\dagger_{q\mu} \otimes d_{q\mu} \right]_{\gamma}^\dagger \right) + h.c. \right). 
$$

Figure 2. Schematic representation for the mapping of the $H_{31}$ terms onto their bosonized representation. a) the creation of a $ph$-pair is expressed in terms of $ph$-RPA phonons, b) the creation of a $di-q$ is replaced by its phonon counterpart.

The schematic representation of the matrix elements of the mapped Hamiltonian in the baryon-like states basis spanned by the operators given in Eqs. (14)-(16),

$$
|B\rangle = b_1|B_1\rangle + b_2|B_2\rangle + b_3|B_3\rangle ,
$$

are shown in Figure 3.
4.1. Simple calculation and results.

In order to get some insights about the baryon spectrum by using the mapped Hamiltonian and the effective degrees of freedom described above, we focused on the description of the low energy baryons, i.e. p, n and Δ states and their quantum numbers $T_{\pi B} = \frac{1}{2}$, $\frac{3}{2}$ and $C_B = (00)$. For such purpose, we have considered the radial and angular excitations up to $N = 1$ and $l = 1$ and restricted to single quark states to total spin $J_q = \frac{1}{2}$.

In the case of the $B_2$ operator, Eq. (15), the baryon-like irreps related to the nucleon and Δ states, i.e., $J_{B_2} = \frac{1}{2}^+$ and $\frac{3}{2}^+$, $(Y_B T_B) = \left(\frac{1}{2}, \frac{1}{2}\right)$ and $(\frac{1}{2}, \frac{3}{2})$ and $C_{B_2} = (00)$, the positive parity presents a particular scenario. Since the lowest ph-RPA solution corresponds to that given by the pion-like one, the $B_2$ baryon-like operator in Eq. (15), must have negative parity. Thus the negative parity coming from the $B_2$ baryon-like operator is due to a radial excitation of one the effective quarks.

For the $B_3$ baryon-like operator, Eq. (16), the lowest di-q phonon solutions corresponds to $T_a (J_a^q) = 0 (0^+)$ [29], which is the lowest RPA solutions within the SU(3) color subspace.

Figure 3. Schematic representation of the matrix elements of the mapped Hamiltonian Eq. (17) in the baryon-like basis, Eqs. (14)-(16).

Figure 4. Comparison between calculated energies $E_i$ of baryon-like states (solid lines) with experimental values (dashed-lines) taken from [30].
(01), needed in order to satisfy the colorless condition of a physical state. Therefore, the $B_3$ baryon-like operators contribute only to the nucleon description but not to the $\Delta$-baryon.

For the spectrum shown in Figure 4, we have used effective vertex-coefficients, $\Lambda_{n\pi\gamma}^{(i)}$ → $\Lambda^{(i)}$, and work in the minimal space of quark(antiquark) configurations. The latter translated in the use of three interaction parameters $\Lambda^{(1)}$, $\Lambda^{(2)}$ and $\Lambda^{(3)}$. The first two were used to calculate the nucleon spectrum up to 2.5 GeV and the third used to calculate the $\Delta$ spectrum up to 2 GeV. Considering the simplicity of the calculation, the spectrum resulting encouraged, since several features described by the experimental data could be observed in our results. One of them is the number of states obtained at about 2.5GeV, i.e., six nucleon-like solutions and three $\Delta$-like solutions. The energy range for these solutions were also satisfactory. However, the energy gaps between the experimental states were not reproduced exactly, but from our experience with the implementation of the many-body methods, we expect that a larger quark(antiquark) configurational space generates more and more collective RPA-solution, due to the larger number of such pairs in the vacuum of the model, and with it lower energy phonon states. The latter should provide more notorious energy gaps, e.g. between the N(1710) and N(1880).

Within this calculation, the Roper resonance (N(1440)) was not well reproduced. However, that was something expectable, since it corresponds to a radial excitation of the nucleon and its low energy is associated with a meson cloud that shields the quark core [6]. In our calculation, we did not considered radial and angular excitations higher than $N \leq 1$ and $l \leq 1$, with the restriction of total single quark spin $J_q = \frac{3}{2}$. The first radial excitation corresponding to a positive parity involves $N = 2$ which of course should improve the description of the Roper resonance and well as the implementation of a larger configurational space, that will generate a more collective $pb$-phonon state. The $pb$-phonon in Eq. (15) plays the role of a meson-like contribution to the baryon-like description.

5. Conclusions.
We have presented a description of baryon-like states by identifying effective degrees of freedom within an effective QCD Hamiltonian. The main conjecture of the model is that colored pairs may contribute to the structure of the vacuum, describing part of the ground state correlations. We started from the QCD Hamiltonian within the Coulomb gauge and restricted to the pure quark sector with a confining interaction. We identified effective quarks and antiquarks which led to a coupling scheme of effective degrees of freedom. We applied the RPA method to partially diagonalized the effective Hamiltonian within collective pairs subspaces. We went further and showed how to incorporate those interactions that were left out in the RPA method, i.e. the $H_{31}$ and $H_{13}$ terms, and from them constructed a mapped Hamiltonian that described the interaction between effective quark(antiquarks) and the RPA-phonon states. Finally, we constructed the baryon-like basis, beyond the common three quarks description, and diagonalize the mapped Hamiltonian in a reduced configurational space. The results are quite encouraging, since they show that the use of non-perturbative many-body techniques are powerful enough to simplify the task of treating many quark and antiquark configurations within the real QCD, as needed for the microscopic interpretation of baryon states at low energy.

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