Holographic watermarks and steganographic markings for combating the counterfeiting practices of high-value metal products

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Abstract

This paper describes recent advances in laser direct writing of tamper-proof holographic structures on metal surfaces for preventing counterfeiting of high-value metal products, e.g. luxury watches, medical tools and implants, collectible coins, etc. Each of these holographic structures consists of an array of optically-smooth craters arranged in such a way to generate diffractive images comprising, e.g. a company logo and/or a string of alphanumeric characters, providing a unique method for the traceability of genuine products. The craters are less than 10 μm across and less than 500 nm deep. They are generated on metals by UV nanosecond laser pulses (355 nm wavelength and 35 ns pulse duration) that lead to localized melting and evaporation of the material. This paper demonstrates various methods for combining the holographic structures with standard marking patterns, such as QR codes and Data Matrix codes, in order to form aesthetic holographic markings concealing secret messages about the products. By merging a few holographic patterns together it is also possible to generate so-called “holographic watermarks”. Finally, this article describes a few approaches for making the holographic structures particularly difficult to replicate and counterfeit. This includes the generation of multi-level holograms as well as the formation of optically-smooth protrusions in selected places within the holographic structures in order to create hidden identifiers and/or miniature signatures which cannot be detected by the naked eye.

1. Introduction

The report published by the Organization for Economic Co-operation and Development (OCED) and the European Union Intellectual Property Office (EUIPO) in 2016 provides clear evidence that the trade of counterfeit goods is a serious worldwide problem that causes harm to the global economy and society, and significantly affects business of companies and consumers (OECD/EUIPO, 2016). This problem, for instance, has a negative impact on the sale of authentic products by companies that own intellectual property rights (IPR), reduces revenues and profits of firms and manufacturers, and also creates an unfair advantage for those enterprises that exploit the development costs of IPR owners. The production and sale of forged products may also damage a well-established reputation of trademarks and brands if those items are poor quality, defective or harmful. Consumers can also be the victims if they are unaware of buying a fake product, whose quality and real value is often very low. Such a product may be unusable, ineffective or dangerous. Therefore, the full identification and traceability of goods leaving the manufacturing process must be provided in order to ensure the products meet specific requirements and standards of quality, safety and performance.

Markings which are seen daily, directly on the surface of various products, are typically in the form of alphanumeric characters, barcodes, Data Matrix (DM) codes, QR codes as well as images possessing company logos and trademarks. Such “standard” markings can be generated by using different printing methods (e.g. ink-jet printing), mechanical processes (e.g. stamping or indentation), electro-chemical techniques (e.g. etching) or laser-based processes (e.g. engraving by ablation); each of these marking processes is more or less suitable for the material of which the product was made. The description of these marking techniques can be found, for instance, in the papers written by Steen and Mazumder (2010), Dahotre and Hartmink (2008a), Sobotova and Demec (2015) or Han and Gubencu (2008).

Although these “standard” markings provide identification and traceability of products, they can be easily copied onto fake items. Therefore, more sophisticated markings in the form of polymer holographic stickers are also used because they are more robust to local damage and less vulnerable to tampering, as shown by Li (2013). These
markings, however, are manufactured using a replication process that requires complex tooling, and hence making unique holographic stickers in low quantity is not cost-effective. Moreover, some types of sticker can be forged, for instance, using one of the methods described by McGrew (1990). Hence a consumer cannot be sure if an item containing a holographic sticker is authentic or fake. Moreover, polymeric holograms are only attached to a product using adhesive and hence they can come off in particular when they are exposed on high temperatures, excessive wear or exposure to solvents.

Fortunately, ‘hard-to-replicate’ markings exist and they can be generated on metal surfaces, for instance, with the use of lasers. Dusser et al. (2010) reported on the use of a femtosecond laser for the generation of nanostructures containing periodic ripples with controllable orientations (via polarization rotation) that can produce specific color patterns to the naked eye. A few other research groups, including Antończak et al. (2013), Veiko et al. (2014), and Murphy et al. (2015), demonstrated that a low-cost fiber laser can create permanent black and color markings on various grades of stainless steel without ablating the material. Such markings are generated by careful selection of the laser processing parameters, such as pulse energy, pulse repetition rate, laser beam scan speed, hatch distance, laser spot size, or even laser beam scanning strategy, and hence are hard to replicate without detailed process knowledge.

Lasers also enable the generation of miniature markings that can be hardly seen on a product by the naked eye. This was demonstrated by Wlodarczyk et al. (2014) who used a picosecond laser and a spatial light modulator (SLM) for the generation of 20 × 20 pixel DM codes whose overall dimensions did not exceed 320 μm by 320 μm. More recently, Wlodarczyk et al. (2015) also presented a different laser-based approach for the generation of markings on metals. These markings, in the form of phase holographic structures, were produced with the use of UV nanosecond laser pulses that enabled the creation of optically-smooth craters of a specific depth and diameter on a flat surface of stainless steel. By producing such craters in specific locations on the metal surface, holographic structures can be generated. These structures are hence tamper-proof and robust to damage unlike polymer holographic stickers. The laser process was further developed and improved in last two years. It was demonstrated that the holographic structures containing up to 300 × 300 pixels (overall size: 2.4 mm by 2.4 mm) can be generated in less than 1 min on the surface of various metals, such as stainless steel, nickel, and some selected Inconel alloys (Wlodarczyk et al., 2017), but also on glass substrates, such as fused silica and Borofloat®33, by using a CO2 laser (Wlodarczyk et al., 2016).

The localized laser-induced melting and laser-induced vaporization process used for direct write of optically-smooth phase holographic structures in metals requires a combination of a suitable laser wavelength (λ = 355 nm), which provides a relatively high linear absorption, and pulse length (τ = 35 ns) that is sufficiently long to produce a melt pool that can flow under surface tension forces producing either smooth craters or protrusions, depending on the temperature coefficient of the surface tension (dY/dT) (Wlodarczyk et al., 2017). Shorter (e.g. picosecond) laser pulses lead to the formation of rough surfaces (Leitz et al., 2011), whereas laser pulses of longer wavelengths (e.g. 1064 nm) do not provide sufficient control over the material vaporization due to an increase of the reflectivity (Dahotre and Harimkar, 2008b).

In this paper, we exploit the holographic structures for the formation of aesthetic holographic markings in the form of watermarks, DM codes and QR codes, and describe various approaches for making these markings extremely difficult to replicate by a counterfeiter. Although aesthetic holographic watermarks have already been presented by Martinez et al. (2013, 2012), our work is different because here we use a different type of computer-generated holograms (CGHs), i.e. non-detour phase holograms instead of detour phase (Lohmann-type) holograms, a different approach for arranging holograms into watermarks, as well as a different technique for the generation of these structures on a workpiece. The difference between the non-detour phase holograms and the detour phase holograms has been well explained by Tricoles (1987).

2. Equipment and methodology

2.1. Laser processing workstation

All holographic structures demonstrated in this work were generated by using a custom-made laser marking system that was constructed around a 10 W Q-switched diode-pumped UV laser (JDSU Q-series). This laser delivered horizontally polarized 35 ns long pulses (FWHM) of wavelength 355 nm to the workpiece via a 2-axis galvo-scanning system (Raylase SuperScan-III-10). The galvo scanner was coupled with a 108 mm focal length F-theta lens which focused the laser beam to an 11 ± 2.2 μm diameter spot (measured at 1/e² of its maximum intensity). Software control of the laser and galvo scanner enables delivery of laser pulses on demand to specific locations on the workpiece to generate hologram designs (pixel-by-pixel) onto the workpiece surface with a marking speed of up to 1250 pixels/s and positioning accuracy of less than 1.5 μm.

2.2. Design and generation of phase holograms on metals

Fig. 1 shows the general procedure used for the design and generation of non-detour phase holograms on the surface of a workpiece. The target image that is aimed to be generated by a hologram was designed by using a simple image drawing tool (MS Paint). This image was saved as a monochrome (black and white) bitmap and then was imported to custom software written in Matlab. The software enabled the design of holograms by employing an Iterative Fourier Transform Algorithm (IFTA) whose principles have been described in many

![Fig. 1. General procedure used for the design and generation of holograms on a metal surface.](image-url)
publications, e.g. (Wyrowski and Bryngdahl, 1988). As shown in Fig. 1, the target image may contain a string of alphanumeric characters, but in general it may contain any monochrome graphical image. At this point it must be highlighted that the number of pixels within the target image design strictly determines the physical dimension of a single period of the hologram that will be generated on a workpiece. Our software was written in such a way that the hologram design containing a single period had always the same number of pixels as the target image design. To increase the physical aperture of a holographic structure, our practice was to copy the hologram design several times and tile the copies in an ‘m × n’ arrangement.

The holographic patterns were in the form of non-detour phase computer-generated holograms (CGHs). The software enables the design of both two-level (binary) and multi-level phase CGHs. The binary CGH designs contain an array of black and white pixels, as can be seen in Fig. 1, in which the white pixels indicate areas responsible for introducing a phase shift of coherent light when reflected from the holographic structure. A characteristic feature of binary holograms is that they generate a diffractive image containing a target image as well as a secondary image (so-called “twin image”) rotated by 180° with respect to the target image, as can be seen in Fig. 2(a). The multi-level CGH designs, in turn, include additional grey pixels; each shade of grey indicates an area responsible for introducing a different phase shift (in the range from 0 to 2π). As shown in Fig. 2(b), multi-level CGHs are capable of generating diffractive images very similar to the target image, i.e. without the twin image.

The hologram designs were mapped onto the workpiece surface by using the laser system described in Section 2.1. The mapping procedure relied on the generation of optically-smooth craters and/or bulges of specific levels in locations that corresponded to the position of white (and grey) pixels in the CGH designs. Optical smoothness of the deformations provided an appropriate phase shift and minimal light scatter. The depth (or height) of the laser-induced deformations used for the construction of the holograms rarely exceeded 500 nm, and was controlled with ± 25 nm accuracy by using single laser pulses of predefined values of energy. The typical diameter of these deformations was less than 11 μm, i.e. less than the diameter of the focused laser beam. In general, the diameter of the deformations along with the number of pixels in the CGH design defined the physical dimensions of the laser-generated hologram.

2.3. Projection of diffractive images generated by holograms

The diffractive image produced by a laser-generated hologram can be revealed by illuminating the hologram using a coherent beam (e.g. laser pointer), as shown in Fig. 3, and projecting the diffractive image onto a screen located in the far field, i.e. at the distance \( L > w^2/\lambda \), where \( w \) and \( \lambda \) are the radius and the wavelength of a laser beam used for revealing the diffractive image. To generate a readable diffractive image, the laser spot must illuminate at least one entire period of the holographic structure.

For the experiments described in this paper, a He-Ne laser

\[ \lambda = 632.8 \text{ nm} \]

producing a beam of diameter 1.8 mm (measured at 1/e² of its maximum intensity) was used to reveal the holographic images. For this laser, readable diffractive images appeared at a distance of approximately 1 m from the hologram surface. To capture diffractive images generated by the holograms we used an SLR camera (Canon EOS 400D).

The quality (sharpness and intensity) of the holographic image depends on the initial surface quality of the metal on which the hologram is generated. If the initial surface of the metal is rough, curved or irregular, the image is highly distorted. Therefore, it is necessary to have a polished surface before writing holograms. Fortunately, the level of polish required is only moderate and hence easy to achieve.

2.4. Design procedure of holographic watermarks

Fig. 4 illustrates a concept that we have developed for the generation of “holographic watermarks” on the surface of metals. Here we exploit a characteristic feature of CGH designs, i.e. their noticeable orientation, which was found to depend on the orientation and position of the text within the target image. The holographic watermark shown in Fig. 4 was generated by arranging two CGH designs of two different orientations in such a way to form a pattern containing the word “HI”. Here, it should be noted that the CGH designs (CGH #1 and CGH #2) comprise four identical patterns tiled in a 2 × 2 arrangement, and hence they contain four times as many pixels as the target image designs (100 × 100 pixels rather than 50 × 50 pixels).
2.5. Design procedure of steganographic patterns

Steganography refers to a practice of concealing a secret message with an ordinary, easily-readable message. Fig. 5 shows a concept for designing a “steganographic” pattern with our directly-written CGHs. In this method, CGH designs are used for the creation of patterns of conventional markings, such as QR codes and Data Matrices (DMs). In the example of Fig. 5, single elements of the Data Matrix (i.e. black squares) were replaced by the CGH designs. The holograms in this arrangement acted as hidden security markings, whilst the DM pattern provided an ordinary, easily-readable marking.

3. Results

3.1. Holograms

Fig. 6(a) shows a two-level phase holographic structure that was generated on the surface of stainless steel (grade ST304LD) according to the procedure described in Section 2.2. This structure contains optically-smooth craters of a specific depth, as shown in Fig. 6(b). The diffractive image generated by this hologram contains the target image and the rotated “twin” image of the same intensity, as can be seen in Fig. 6(c). The relatively strong undiffracted 0th order beam in the diffractive image is due to the rounded bowl shape of the hologram pixels, as already explained in (Wlodarczyk et al., 2015).

The multi-level phase holograms, i.e. structures containing optically-smooth craters of at least two different depths, are able to generate diffractive images with a suppressed ‘twin’ image, as demonstrated in Fig. 6(d). Since such craters introduce an asymmetry in the structure, the intensity symmetry between the target image and the ‘twin’ image is broken. By measuring the optical power in different areas of the diffractive image, it was found that the optical power gathered within the target image can be up to 3 times as high as the optical power within the “twin” image. This optical ratio depends on
the depth of the craters within the holographic structure.

3.2. Holographic watermarks

Fig. 7 shows a holographic watermark (7.2 mm × 7.2 mm) that was generated on the surface of stainless steel in accordance with the design procedure described in Section 2.4. The time required to generate this holographic watermark was approximately 6 min. The structure contains 900 × 900 pixels. It was constructed from two different binary CGH designs that were arranged in such a way to form the word “HI”. This word can be seen by the naked eye or a camera (see Fig. 7) when the hologram is tilted at an angle. This structure generates two distinct holographic images, dependent on which part of the watermark is illuminated by the laser. In this case, one image contains the initials “EPS” and is generated by illuminating only part of either “H” or “I” letters in the word “HI”, whilst the other image containing the initials “HWU” is produced by illuminating only the watermark background. If the viewing laser beam illuminates part of a letter and the background, the holographic image comprises both inscriptions, as can be seen in Fig. 7.

3.3. Steganographic markings

Fig. 8 shows three different variations of the steganographic markings that were generated on the surface of a stainless steel mirror. The marking shown in Fig. 8(a) was produced following the design procedure described in Section 2.5. The marking shown in Fig. 8(b) is an inverted version of the marking from Fig. 8(a); it was generated by writing CGHs in the areas of the DM code background. The last marking shown in Fig. 8(c) was produced by embedding CGHs into the DM code pattern, while the DM code background was filled in an array of evenly spaced optically-smooth craters. In this way, a steganographic marking with “watermark” appearance was generated.

The reason for generating three different variations of steganographic markings was to investigate whether all of these can be directly read by an ordinary smartphone (tested using Sony Xperia Z3) with installed NeoReader® software used for decoding DMs and QR codes. Single elements of the DM patterns (i.e. squares) had a dimension of 0.5 mm × 0.5 mm and contained at least one whole period of the CGH. The holograms were designed in a way to generate a diffractive image with the inscription “HWU”. The DM patterns were large enough (approximately 9 mm × 9 mm) to be read by the smartphone camera. We observed good quality diffractive images from all three steganographic markings, as can be seen in Fig. 8, when an area containing the holograms was illuminated by the He-Ne laser beam. However, only the first two markings shown in Fig. 8(a) and (b) were successfully decoded by NeoReader® software. In both cases, the smartphone decoded the DM in a few seconds and generated a link to the official website of Heriot-Watt University (http://www.hw.ac.uk). The direct readout of the third DM code was not possible because the contrast between DM squares was not sufficiently high. However, after taking a photo of the DM code and enhancing its contrast, it was possible to
successfully decode the pattern using NeoReader® software.

It is clear that the steganographic markings must contain a large number of the CGH designs in order to create a DM code. This means that such markings can generate a holographic image even if some parts of the DM code are damaged, e.g. due to mechanical abrasion. The redundant parts of the DM code can also be replaced by some other patterns.

Fig. 9 shows a QR code that was generated using a very similar design procedure applied to produce the holographic DM code shown in Fig. 8(a). This time, however, the code contains some extra elements that can act as additional security features for the verification of the marking originality. One of the extras is a logo that was generated in the center of the QR code. This logo is visible to the naked eye and can act as a simple identifier. The second addition is less visible and not so obvious to a counterfeiter because the QR code contains miniature signatures (the inscriptions “HWU” with a dimension of 120 μm × 55 μm) that were embedded into some of the CGH designs. These signatures cannot be seen by the naked eye, but can be detected when the entire structure is carefully inspected under an optical microscope.

As shown in Fig. 9, the QR code with the embedded additional security features generated a diffractive image with the inscription “HWU” as well as it was readable by NeoReader® software.
4. Discussion

Phase holographic structures can also be generated by creating optically-smooth bulges or protrusions (instead of craters) provided that their height introduces similar phase shift as the crater of a specific depth. Our research has shown that the shape of the surface deformations generated on metals by the UV nanosecond laser pulses depends on the peak temperature and the temperature gradients generated by the laser beam as well as the physical and chemical properties of the material (Weston et al., 2012; Wlodarczyk et al., 2017, 2015). On some metals, such as nickel or Inconel®625 alloy, optically-smooth protrusions can be generated by using single laser pulses of certain energy, while on stainless steel they can be generated by processing the metal under CO$_2$ shielding gas.

Fig. 10 shows an example of deformations created on the surface of nickel. These deformations were generated by using single laser pulses of two different energies. In both cases, the metal was processed in air, i.e. without using any shielding gas. Since the height of the protrusion is equivalent to the depth of the crater, these two deformations can be used alternately for the generation of binary holograms on nickel by utilizing the same CGH design. Our simulations have shown that for a laser beam with wavelength ($\lambda$), a reflective binary hologram containing only protrusions of height ($\lambda/4$) would generate a diffractive image with very similar diffraction efficiency as a reflective binary hologram containing only craters of depth ($\lambda/4$).

The ability to generate on demand two different kinds of deformation with very similar phase shifts provides the possibility of producing an enormous number of unique holograms that can be applied on individual items. By generating holographic structures consisting of a combination of craters and protrusions, whose location may be known only by the manufacturer, it is possible to generate markings with a hidden level of encryption. By recording in a database the pattern of craters and protrusions within a given CGH design, this pattern can act as an additional identifier to verify the authenticity of a product. Although in theory a counterfeiter could take actions to inspect the entire hologram, e.g. by using a 3D surface profilometer, in order to find the location of craters and protrusions (bulges), the level of effort, time and costs involved in the replication of such holograms on forged products would be substantial.

5. Conclusions

In this paper, we demonstrate the incorporation of additional security features into indelible, directly-written holographic structures generated using UV nanosecond laser pulses. These features enhance the applicability of the holograms as anti-counterfeiting markings for metal products. The holograms can be merged together to form aesthetic watermarks or they can be embedded into patterns of standard
markings (such as Data Matrix codes or QR codes) to act as hidden security markings. Furthermore, it is possible to create additional hidden features into the holograms, including protrusions in place of some craters or miniature signatures/logos. Potential industrial applications include jewellery, luxury watches, automotive and aerospace parts, medical tools and implants, or even collectible coins. Currently, the laser process enables the generation of approximately a thousand hologram pixels per second. This means that 2.4 mm × 2.4 mm holograms (with a pixel diameter of 8 μm) can be generated in less than a minute. Such a high processing speed is already very attractive; however, there is potential to further increase this, e.g. by generating laser pulses of constant energy with a higher repetition rate (> 1.25 kHz, in our case this would require the use of a different laser) or using multiple parallel laser beams for direct writing several periods of a holographic structure simultaneously.

Acknowledgements

This work was supported by the Engineering and Physical Sciences Research Council, UK [Grant number EP/K030884/1].

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