Heavy pentaquark states $P_c(4380)$ and $P_c(4450)$ in the $J/\psi$ production induced by pion beams off the nucleon

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Abstract

In this study, we investigate the $J/\psi$ production induced by pion beams off the nucleon, particularly the heavy pentaquarks $P_c(4380)$ and $P_c(4450)$ in intermediate states, based on a hybridized Regge model. The process involving $\rho$ and $\pi$ meson exchange in the $t$ channel is considered as background, and the heavy pentaquark exchange is included in the $s$ channel. The coupling constants such as the $\rho NN$ and $\pi NN$ vertices are taken from the $NN$ potentials, whereas those for the $J/\psi \rho \pi$ and $J/\psi \pi \pi$ vertices are determined by using experimental data based on the branching ratios. In order to estimate the $P_c(4380)$ and $P_c(4450)$ coupling constants, we use the experimental upper limit on the total cross section as a guide for the $\pi N \rightarrow J/\psi N$ reaction. The background total cross section is the order of $10^{-4} - 10^{-3}$ nb. In the vicinity of the heavy pentaquark masses, the total cross section reaches about 1 nb.
I. INTRODUCTION

Finding new exotic hadrons is one of the most important issues for hadron and particle physics. Recently, the LHCb Collaboration announced the observation of two heavy pentaquarks in $\Lambda_b \rightarrow J/\psi K^- p$ decays [1], where the quark content is $uudc\bar{c}$. The significance of these pentaquarks is more than $9\sigma$. The masses and widths were reported as: $M_{P_c} = (4380 \pm 8 \pm 29)$ MeV and $\Gamma_{P_c} = (205 \pm 18 \pm 86)$ MeV for the lower state, whereas $M_{P_c} = (4449.8 \pm 1.7 \pm 2.5)$ MeV and $\Gamma_{P_c} = (39 \pm 5 \pm 19)$ MeV for the higher state. Thus, it is very important to confirm these pentaquark states in other possible reactions. For example, the energy of the pion beam at the Japan Proton Accelerator Research Complex (J-PARC) facility is sufficient to observe them during the $\pi N \rightarrow J/\psi N$ process. The photon beam at the Thomas Jefferson National Accelerator Facility (Jefferson Lab) can also be used to measure the $P_c$ states in $J/\psi$ photoproduction [2–4].

In fact, a recent theoretical study of the $\pi^- p \rightarrow J/\psi n$ reaction with neutral charm pentaquarks $P_c^0$ [5] employed the effective Lagrangian method. Lu et al. [5] assumed that the branching ratios of $P_c \rightarrow J/\psi N$ and $P_c \rightarrow \pi N$ are about 10% and 1%, respectively. The background total cross section is in the order of $10^{-100}$ nb and the total cross section near the pentaquark masses increases to around $1 \mu b$. This indicates that the magnitude of the total cross section for $J/\psi$ production in the vicinity of the pentaquark masses is almost comparable to that of the $\pi N \rightarrow \phi N$ reaction.

In the present study, we consider the contribution of pentaquark resonances to $J/\psi$ production by using previous experimental information on the upper limit of the $\pi N \rightarrow J/\psi N$ reaction [6, 7] and by employing a hybridized Regge model that incorporates the heavy pentaquark states. To estimate the contribution of the heavy pentaquarks to the $\pi N \rightarrow J/\psi N$ reaction, it is crucial to know the background contribution. Recently, the open charm production $\pi^- p \rightarrow D^{*-} \Lambda^+_c$ was analyzed based on a comparison with the associated strangeness production $\pi^- p \rightarrow K^{*0} \Lambda$ [8–10] (see Fig. 1(a)), and thus the parameters for the $\pi^- p \rightarrow D^{*-} \Lambda^+_c$ reaction can be plausibly estimated. A similar approach was also applied to various reactions such as $\bar{p} p \rightarrow \bar{Y}_c Y_c$ and $\bar{p} p \rightarrow \bar{M}_c M_c$, where $Y_c$ and $M_c$ denote $\Lambda^+_c, \Sigma^+_c$ and $D, D^*$ [11].

We adopt the same strategy to study the hidden charm process $\pi^- p \rightarrow J/\psi n$ together with the strangeness process $\pi^- p \rightarrow \phi n$ (see Fig. 1(b)). However, the hidden charm (strangeness) reactions are distinguished from the open charm (strangeness) reactions. In the case of the open charm processes, the exchanged meson in the $t$ channel should be different from that in the open strangeness process, as shown in Fig. 1(a). In addition, $\rho$
meson and pion exchanges play similar roles in both the hidden charm and strangeness reactions because of the Okubo-Zweig-Iizuka (OZI) suppression, which is illustrated in Fig. 1(b). This allows us to obtain the coupling constants more explicitly by using the corresponding experimental data rather than relying on model calculations.

This Letter is organized as follows. In Section II, we explain the general formalism of a hybridized Regge model and we show how to determine the relevant parameters for the $\pi$ and $\rho$ Reggeons and the $P_c$ pentaquark states. In Section III, we first examine the assumption made in previous studies regarding the branching ratios of $P_c\to J/\psi N$ and $P_c\to \pi N$, where we compute the total cross section for the $\pi N\to J/\psi N$. We also present numerical results for the total cross-section and differential cross sections. In Section IV, we give our conclusions and a summary of the present study.

II. GENERAL FORMALISM

In this study, we employ a hybridized Regge model for the charm production $\pi^- p\to J/\psi n$ in order to consider the contribution of the charm pentaquark states $P_c^0$ together with the strangeness production $\pi^- p\to \phi n$. Figure 2(a) shows a generic t-channel tree diagram for $\pi^- p\to V(\phi, J/\psi)n$, and Fig. 2(b) depicts $P_c^0$ exchange in the s channel only for the $\pi^- p\to J/\psi n$ process. The initial momenta of the pion and the proton are denoted by $k_1$ and $p_1$, respectively, and the final momenta of the vector meson and the neutron are denoted by $k_2$ and $p_2$, respectively.

The effective Lagrangians for the exchanges of the $\rho$ and $\pi$ mesons are expressed as

$$\mathcal{L}_{V\rho\pi} = \frac{g_{V\rho\pi}}{M_V} \varepsilon^{\mu\nu\alpha\beta} \partial_\mu V_\nu \partial_\alpha \rho_\beta \cdot \pi,$$

$$\mathcal{L}_{V\pi\pi} = -ig_{V\pi\pi}(\pi^- \partial_\mu \pi^+ - \partial_\mu \pi^- \pi^+)V^\mu,$$

where $V = (\phi, J/\psi)$ and $\pi$, $\rho$, $\phi$, and $J/\psi$ denote the fields corresponding to the $\pi(140, 0^-)$, $\rho(770, 1^-)$, $\phi(1020, 1^-)$, and $J/\psi(3097, 1^-)$ mesons, respectively. The coupling constants for $\rho$ exchange are determined by using the decay width of the corresponding vector mesons [12]

$$\Gamma(V\to \rho\pi) = \frac{1}{4\pi} \frac{1}{M_V^2} |\vec{k}|^3 |g_{V\rho\pi}|^2,$$
where $|\vec{q}| = \sqrt{M_V^2 - (M_\rho + M_\pi)^2}/(2M_V)$. Similarly, the coupling constants $g_{V\pi\pi}$ are derived from experimental data on the decay width \[12\]

$$
\Gamma(V \to \pi^+\pi^-) = \frac{1}{6\pi} \frac{1}{M_V^2} |q|^2 g_{V\pi\pi}^2, \tag{4}
$$

where $|q| = \sqrt{M_V^2 - 4M_\pi^2}/2$. All of the relevant numerical values for the couplings of $\phi$ and $J/\psi$ mesons \[12\] are summarized in Table I. In fact, the Particle Data Group provides the branching ratio $\Gamma(\phi \to (\rho \pi + \pi^+\pi^-))/\Gamma(\phi) = 15.32\% \[12\]$. However, we ignore this part of the $3\pi$ decay in the data, where we assume that it is smaller than the $\rho\pi$ channel. The $\rho$ meson has a rather large width, but we find that its effect is very small, as stated previously \[13\]. Hence, we neglect the effect of the $\rho$ meson decay width when calculating the coupling constant $g_{V\rho\pi}$.

| $V$ | $V_\gamma$[MeV] | $V_\gamma\to\pi\pi/V_\gamma[\%]$ | $V_\gamma\to\rho\pi/\text{keV}$ | $g_{V\rho\pi}$ | $V_\gamma\to\pi^+\pi^-/V_\gamma[\%]$ | $V_\gamma\to\pi^+\pi^-/\text{eV}$ | $g_{V\pi\pi}$ |
|-----|----------------|-------------------------------|-----------------------------|------------|-------------------------------|-----------------------------|------------|
| $\phi$ | 4.266 | 15.32 ± 0.32 | 654 | 1.25 | $(7.4 \pm 1.3) \cdot 10^{-3}$ | 316 | 7.24 \cdot 10^{-3} |
| $J/\psi$ | 0.0929 | 1.69 ± 0.15 | 1.57 | 7.90 \cdot 10^{-3} | $(1.47 \pm 0.14) \cdot 10^{-2}$ | 13.7 | 8.20 \cdot 10^{-4} |

**TABLE I.** Coupling constants $g_{V\rho\pi}$ and $g_{V\pi\pi}$, based on the branching ratios of $V(\phi, J/\psi)$ relative to $\rho\pi$ and to $\pi^+\pi^-$ \[12\].

The effective Lagrangians for the $\rho NN$ and $\pi NN$ vertices are as follows

$$
\mathcal{L}_{\rho NN} = -g_{\rho NN} \left[ \tilde{N} \gamma_\mu \tau N - \frac{\kappa_{\rho NN}}{2M_N} \tilde{N} \sigma_{\mu\nu} \tau N \partial^\nu \right] \cdot \rho^\mu, \tag{5}
$$

$$
\mathcal{L}_{\pi NN} = -ig_{\pi NN} \tilde{N} \gamma_5 \tau \cdot \pi N,
$$

where $N$ denotes the nucleon and the coupling constants are taken from the $NN$ potentials, i.e., $g_{\rho NN} = 3.36$, $\kappa_{\rho NN} = 6.1$ and $g_{\pi NN} = 13.3$ (e.g., see \[14\]).

The invariant amplitudes are derived in the form of

$$
\mathcal{M} = \bar{u}_n \mathcal{M}^\mu \epsilon^*_\mu u_p, \tag{6}
$$

where

$$
\mathcal{M}_\rho^\mu = I_\rho \frac{g_{V\rho\pi} g_{\rho NN}}{t - M_\rho^2} \epsilon^{\mu\nu\alpha\beta} \gamma_\nu - \frac{ig_{\rho NN}}{2M_N} \sigma_{\nu\lambda} (k_2 - k_1)^\lambda \left[ k_{2\alpha} k_{1\beta} \right],
$$

$$
\mathcal{M}_\pi^\mu = -2I_\pi \frac{ig_{V\pi\pi} g_{\pi NN}}{t - M_\pi^2} \gamma_5 k_1^\mu. \tag{7}
$$

$u_p$ and $u_n$ denote the Dirac spinors of the incoming and outgoing nucleons, respectively, and $\epsilon^*_\mu$ is the polarization vector of the final vector meson. The isospin factors are given by $I_\rho = I_\pi = \sqrt{2}$.

We now replace the Feynman propagators in Eq. (7) with the Regge propagators $R(t)$ as \[15\]

$$
\frac{1}{t - M_\rho^2} \to R_\rho(t) = \left( \frac{s}{s_\rho} \right)^{\alpha_\rho(t)-1} \frac{g_{\rho NN}}{t - M_\rho^2} \left( \frac{1}{\sin[\pi\alpha_\rho(t)]} \frac{\pi\alpha_\rho'}{2\Gamma[\alpha_\rho(t)]} \right),
$$

$$
\frac{1}{t - M_\pi^2} \to R_\pi(t) = \left( \frac{s}{s_\pi} \right)^{\alpha_\pi(t)} \frac{g_{\pi NN}}{t - M_\pi^2} \left( \frac{1}{\sin[\pi\alpha_\pi(t)]} \frac{\pi\alpha_\pi'}{2\Gamma[\alpha_\pi(t)]} \right). \tag{8}
$$
where the Regge trajectories are given by $\alpha_\rho(t) = 0.55 + 0.8t$ and $\alpha_\pi(t) = 0.7(t - M_\pi^2)$. The energy scale parameters are selected as $s_\rho = s_\pi = 1\text{GeV}^2$ for simplicity. For the signature factor, we select a constant phase in both the Regge propagators because the $\rho$ and $\pi$ mesons are degenerate in pion photoproduction within a Regge model. Though a phase factor, $\exp(-i\pi\alpha_\rho(t))$, can also be included, we find that the results change only slightly.

The Regge amplitude $T_R$ can be expressed in terms of the individual invariant amplitudes combined with the Regge propagators

$$T_R = \mathcal{M}_\rho \cdot (t - M_\rho^2) \cdot R_\rho(t) \cdot C_\rho(t) + \mathcal{M}_\pi \cdot (t - M_\pi^2) \cdot R_\pi(t) \cdot C_\pi(t),$$

where $C_\rho(t)$ and $C_\pi(t)$ are called scale factors, which are employed to fit the experimental data based on the $\pi^- p \to \phi n$ reaction at high energies.

Now, we consider the resonance contribution from the pentaquark states $P_c^0(4380)$ and $P_c^0(4450)$. The exact quantum numbers of these two pentaquark states are not known. However, a previous study suggested that the spins and parities for $P_c(4380)$ and $P_c(4450)$ should be $(3/2^-, 5/2^+)$, respectively, for the best fit in partial-wave analysis. In addition, the combinations of $(3/2^+, 5/2^-)$ and $(5/2^+, 3/2^-)$ provide acceptable solutions for the pentaquark states. We also consider the case of $(5/2^-, 3/2^+)$ in our calculation.

The effective Lagrangians for the $P_c N\pi$ vertex shown in Fig. 2(b) are given as

$$\mathcal{L}_{P_c N\pi}^{3/2^\pm} = \frac{g_{P_c N\pi}}{M_\pi^2} \tilde{N} \Gamma^{(\pm)} \tau \cdot \partial_\mu \pi P_{c\mu} + \text{H.c.},$$

$$\mathcal{L}_{P_c N\pi}^{5/2^\pm} = i \frac{g_{P_c N\pi}}{M_\pi^2} \tilde{N} \Gamma^{(\pm)} \tau \cdot \partial_\mu \partial_\nu \pi P_{c\mu\nu} + \text{H.c.},$$

where we ignore the off-shell part of the Rarita-Schwinger fields because the resonances are almost on mass shell. The following notations are used

$$\Gamma^{(\pm)} = \begin{pmatrix} \gamma_5 & \\ -1 & \end{pmatrix}, \quad \Gamma^{(\pm)} = \begin{pmatrix} \gamma_\mu & \\ \gamma_\mu & \end{pmatrix}. \quad (11)$$

After we obtain the branching ratio of $P_c \to N\pi$, we can easily determine the coupling constants $g_{P_c N\pi}$ from the decay widths

$$\Gamma(P_c^{3/2^\pm} \to N\pi) = \frac{g_{P_c N\pi}^2}{4\pi} \frac{p_N^3}{M_\pi^2 M_{P_c}} (E_N \pm M_N),$$

$$\Gamma(P_c^{5/2^\pm} \to N\pi) = \frac{2 g_{P_c N\pi}^2}{5} \frac{p_N^5}{M_\pi^2 M_{P_c}^2} (E_N \mp M_N), \quad (12)$$

where $E_N = (M_{P_c}^2 + M_N^2 - M_\pi^2)/(2M_{P_c})$ and $p_N = \sqrt{E_N^2 - M_N^2}$. Unfortunately, the branching ratios are not known at present, so we have to rely on previous experimental information on the upper limit on the total cross section for $\pi N \to J/\psi N$.

The effective Lagrangians for the $P_c N J/\psi$ vertex can be expressed as

$$\mathcal{L}_{P_c N J/\psi}^{3/2^\pm} = i \bar{P}_{c\mu} \left[ \frac{g_1}{2M_N} \Gamma^{(\pm)} N \mp \frac{i g_2}{(2M_N)^2} \Gamma^{(\pm)} \partial_\nu N \pm \frac{i g_3}{(2M_N)^3} \Gamma^{(\pm)} \partial_\nu N \right] \psi^{\mu\nu} + \text{H.c.},$$

$$\mathcal{L}_{P_c N J/\psi}^{5/2^\pm} = \bar{P}_{c\mu\nu} \left[ \frac{g_1}{(2M_N)^2} \Gamma^{(\pm)} N \mp \frac{i g_2}{(2M_N)^3} \Gamma^{(\pm)} \partial_\nu N \pm \frac{i g_3}{(2M_N)^3} \Gamma^{(\pm)} \partial_\nu N \right] \partial^{\mu} \psi^{\mu\nu} + \text{H.c.}. \quad (13)$$
The heavy pentaquark states play a dominant role near the threshold region, so we only consider the first term in Eq. (13). Thus, the decay widths for the heavy pentaquarks are written as

$$
\Gamma(P_c^{3/2^+} \to NJ/\psi) = \frac{g_{P_c N J/\psi}^2}{12\pi} \frac{p_N}{M_{P_c}} (E_N \mp M_N) \\
\times [2E_N(E_N \pm M_N) + (M_{P_c} \pm M_N)^2 + 2M_{J/\psi}^2], \\
\Gamma(P_c^{5/2^+} \to NJ/\psi) = \frac{g_{P_c N J/\psi}^2}{60\pi} \frac{p_N^2}{M_{P_c}} (E_N \pm M_N) \\
\times [4E_N(E_N \mp M_N) + (M_{P_c} \pm M_N)^2 + 4M_{J/\psi}^2],
$$

which we can use to determine the coupling constants for the $P_c$ states. The kinematic variables $E_N$ and $p_N$ in Eq. (14) are defined as

$$
E_N = (M_{P_c}^2 + M_N^2 - M_{J/\psi}^2)/(2M_{P_c}) \quad \text{and} \quad p_N = \sqrt{E_N^2 - M_N^2}.
$$

Finally, we have the following expressions for the $s$ channel

$$
M_{P_c(3/2^+)}^s = i g_{P_c N J/\psi} g_{P_c N \pi} \frac{1}{2M_N} \frac{M_{P_c}}{s - M_{P_c}^2} \gamma_5 \gamma_\mu (k_2^\alpha g^{\mu \nu} - k_2^\nu g^{\mu \alpha}) \Delta_{\alpha \beta}(P_c, k_1 + p_1) k_1^\beta,
$$

$$
M_{P_c(3/2^-)}^s = i g_{P_c N J/\psi} g_{P_c N \pi} \frac{1}{2M_N} \frac{M_{P_c}}{s - M_{P_c}^2} \gamma_\mu (k_2^\alpha g^{\mu \nu} - k_2^\nu g^{\mu \alpha}) \Delta_{\alpha \beta}(P_c, k_1 + p_1) k_1^\beta \gamma_5,
$$

$$
M_{P_c(5/2^-)}^s = -i g_{P_c N J/\psi} g_{P_c N \pi} \frac{1}{(2M_N)^2} \frac{M_{P_c}^2}{s - M_{P_c}^2} \gamma_\mu k_2^{\alpha_2} (k_2^\alpha g^{\mu \nu} - k_2^\nu g^{\alpha_1 \mu}) \\
\times \Delta_{\alpha_1 \alpha_2; \beta_1 \beta_2}(P_c, k_1 + p_1) k_1^{\beta_1} k_1^{\beta_2} \gamma_5,
$$

$$
M_{P_c(5/2^-)}^s = -i g_{P_c N J/\psi} g_{P_c N \pi} \frac{1}{(2M_N)^2} \frac{M_{P_c}^2}{s - M_{P_c}^2} \gamma_\mu k_2^{\alpha_2} (k_2^\alpha g^{\mu \nu} - k_2^\nu g^{\alpha_1 \mu}) \\
\times \Delta_{\alpha_1 \alpha_2; \beta_1 \beta_2}(P_c, k_1 + p_1) k_1^{\beta_1} k_1^{\beta_2},
$$

where the different spins and parities of the $P_c$ states are assumed. The isospin factors are given by $I_{P_c} = \sqrt{2}$. Given the decay widths of the $P_c$ states, the propagators of the pentaquark states should be modified to $M_{P_c} \to (M_{P_c} - i\Gamma_{P_c}/2)$. Previous studies [17, 18] provided the explicit expressions for $\Delta_{\alpha \beta}$ and $\Delta_{\alpha_1 \alpha_2; \beta_1 \beta_2}$ in Eq. (15). The relevant hadrons are spatially extended, so we consider the phenomenological form factors in the $s$ channel

$$
F_{P_c}(s) = \left( \frac{\Lambda^4}{\Lambda^4 + (s - M_{P_c}^2)^2} \right)^2,
$$

where the cutoff masses are selected as $\Lambda = 1.0$ GeV. The cutoff masses actually play no crucial roles in this calculation because the pentaquark states lie almost near the threshold region.

### III. RESULTS

First, we study the background contribution ($\rho$ and $\pi$ Reggeon exchanges) to the total cross sections for both the $\pi^-p \to J/\psi n$ and $\pi N \to J/\psi N$ reactions. Figure 3 shows the effects of $\rho$ and $\pi$ Reggeon exchanges on both the reactions. Note that the total cross...
sections are drawn as a function of \( s/s_{th} \) so we can easily compare the total cross section of the \( \pi^{-}p \to \phi n \) reaction with that of \( J/\psi \) production. \( s_{th} \) denotes the threshold value of \( s \), i.e., \( s_{th} = (M_{\phi} + M_{n})^2 = 3.84 \text{ GeV}^2 \) and \( s_{th}^\pi = (M_{J/\psi} + M_{n})^2 = 16.3 \text{ GeV}^2 \) for the \( \pi^{-}p \to \phi n \) and \( \pi^{-}p \to J/\psi n \) reactions, respectively. The \( \rho \) Reggeon exchange dominates the \( \pi \) Reggeon exchange, which is expected due to the relatively smaller values of \( g_{V\pi\pi} \) compared with those of \( g_{V\rho\pi} \), where \( V \) generically represents \( \phi \) and \( J/\psi \) mesons (see Table I). The Regge approach is known to describe the experimental data well at higher energies, so we fit the scale factor \( C_{\rho(\pi)}(t) \) defined in Eq. (9) such that it explains the total cross section for \( \pi^{-}p \to \phi n \) \cite{19} in the higher energy region. We use the form of \( C_{\rho(\pi)}(t) = 0.5/(1 - t/\Lambda^2)^2 \) with the cutoff mass \( \Lambda = 1 \text{ GeV} \) fixed in Eq. (16) to avoid additional ambiguity. We use the same form of the scale factor to obtain the total cross section for the \( \pi^{-}p \to J/\psi n \) reaction. The results of the background contribution lie below the experimental upper limit \cite{6, 7} on the total cross section of the \( \pi^{-}p \to J/\psi n \) reaction.

As shown in Fig. 3, we find that the magnitude of the background contribution to the total cross section for \( \pi^{-}p \to J/\psi n \) is about \( 10^6 \) times smaller than that for \( \pi^{-}p \to \phi n \). This is due mostly to the greatly suppressed value of \( g_{\phi\rho\pi}/M_{\phi} \) in the case of dominant \( \rho \)-meson Reggeon exchange: \( (g_{\phi\rho\pi}/M_{\phi})^2 \simeq 2.5 \times 10^5 \times (g_{\phi\rho\pi}/M_{J/\psi})^2 \). It is interesting to compare the current results with those of previous theoretical studies. For example, Kodaira and Sasaki \cite{20} estimated the total cross section for the \( \pi^{-}p \to J/\psi n \) reaction by using generalized Veneziano models many years ago. The results were obtained as \( \sigma(\pi^{-}p \to J/\psi n) = (1.1, 0.44) \text{ pb} \) at \( p_{lab} = (50, 100) \text{ GeV/c} \), respectively, and we obtained \( (0.17, 0.071) \text{ pb} \) at the corresponding momenta. Thus, the orders of magnitude appear to be similar to each other. However, Wu and Lee \cite{21} predicted about 1.5 nb near the threshold region (\( W \sim 4.2 \text{ GeV} \)) within a coupled-channel model. Lu et al. \cite{5} computed the total cross section for the \( \pi^{-}p \to J/\psi n \) reaction by considering the heavy pentaquark states,
where they determined the background contribution to the total cross section in the vicinity of the $P_c$ resonances as in the order of $10 - 100$ nb, which is about $10^4 - 10^5$ times larger than those obtained in the present study.

In the case of the $\pi N \rightarrow \phi N$ reaction, the contributions of the nucleon resonances have roles at low energies, as studied by Xie et al. [22], who focused on the role of $N^*(1535)$. Similarly, the heavy pentaquark resonances will make a specific contribution to the $\pi^- p \rightarrow J/\psi n$ reaction near the threshold. When considering the heavy pentaquark resonances, we encounter a problem with determining the coupling constants due to the unknown branching ratios of the $P_c$ states relative to any other channels. Wu et al. [23] predicted new resonances with the $c\bar{c}$ component based on a unitarized coupled-channel formalism before observations of the $P_c$ states were reported. The branching ratios of the predicted states for the $J/\psi N$ and $\pi N$ decay channels were proposed as about 40 % and 6 %, respectively. Assuming that they are the heavy pentaquark states announced recently, we find that the magnitude of the total cross section reaches about $10^4$ nb, as depicted in Fig. 4, which is even closer to the magnitude of the total cross section for the $\pi^- p \rightarrow \phi n$ reaction. Moreover, it exceeds the experimental upper limit on the $\pi^- p \rightarrow J/\psi n$ reaction [6, 7] by approximately $10^4$ times. Note that we have set the quantum numbers of the $P_c$ states as $J^P = (3/2^-, 5/2^+)$ for $(P_c^0(4380), P_c^0(4450))$, respectively. The notations for the experimental data are the same as those given in Fig. 3.

We may follow another suggestion proposed by Wang et al. [2], where $B(P_c^+ \rightarrow J/\Psi p) = 5$ % was assumed for the study of $J/\psi$ photoproduction. As a result, Wang et al. [2] were able to describe the old experimental data even when the heavy pentaquark states were considered. Following the same line of reasoning, we consider the experimental upper limit for the $\pi N \rightarrow J/\psi N$ reaction, which is given as around 1 nb [6]. This implies that the branching ratio of $P_c \rightarrow \pi N$ should be very small: $B(P_c^0 \rightarrow \pi N) \sim 10^{-5}$. Thus, we can

![Diagram](image-url)
obtain the corresponding coupling constants using Eqs. (12) and (14). The results are listed in Table II.

| Coupling | State       | 3/2^+ | 3/2^- | 5/2^+ | 5/2^- |
|----------|-------------|-------|-------|-------|-------|
| g_{PcN\pi} | P_c(4380)  | 2.74 \times 10^{-4} | 4.23 \times 10^{-4} | 4.47 \times 10^{-5} | 2.89 \times 10^{-5} |
|          | P_c(4450)  | 1.17 \times 10^{-4} | 1.79 \times 10^{-4} | 1.86 \times 10^{-5} | 1.21 \times 10^{-5} |
| g_{PcNJ/\psi} | P_c(4380)  | 0.771 | 0.345 | 1.53  | 3.61  |
|          | P_c(4450)  | 0.291 | 0.141 | 0.568 | 1.24  |

TABLE II. Coupling constants for the heavy pentaquark state s with each J^P assignment. The branching ratios for the pentaquarks are assumed to be B(P_c^0 \rightarrow \pi N) = 10^{-5} and B(P_c^0 \rightarrow J/\Psi N) = 0.05.

Figure 5 shows our results for the total cross section as a function of the center of mass (CM) energy W, where different combinations of the spin and parity are considered for the heavy pentaquark states. As illustrated in Fig. 5, the results are not sensitive to the selection of the spin and parity for the heavy pentaquark states. The peaks of the P_c states reach the experimental upper limit, i.e., about 1 nb. P_c(4380) has a broad width (\Gamma_{P_c(4380)} \approx 210 \text{ MeV}), so its peak overlaps with that of P_c(4450), where the width is \Gamma_{P_c(4450)} \approx 40 \text{ MeV}. 

![Figure 5](image_url)
Thus, it may be very difficult to directly distinguish $P_c(4380)$ from $P_c(4450)$ based on the $\pi^-p \to J/\psi n$ reaction.

We note that a larger value of the branching ratio could be considered for the $P_c \to J/\Psi$ decay, where we suppress that of $P_c^0 \to \pi N$ to examine the dependence of the total cross section on them. For example, we can choose $\mathcal{B}(P_c^0 \to J/\Psi N) = 0.5$, which is 10 times larger than the value used in the present study. By contrast, we may take $\mathcal{B}(P_c^0 \to \pi N) = 10^{-6}$, which is 10 times smaller than the value in the present study. However, we obtain almost the same numerical results.

In addition to the hidden-charm production process investigated in this study, it is also very interesting to investigate the production of the heavy pentaquarks in the open-charm channel. Previously, we performed studies based on the $\pi^-p \to D^{*+}\Lambda_c^+$ reactions without considering the pentaquark states. We compare the corresponding results for the total cross sections with those obtained in the present study in Fig. 6. As expected, due to OZI suppression, the total cross section for $J/\Psi n$ production is approximately $10^2 - 10^4$ smaller than those for $D^{*+}\Lambda_c^+$ and $D^{-}\Lambda_c^+$ production, excluding the resonance region. This indicates that if the pentaquark states are strongly coupled to the $D^{*+}\Lambda_c^+$ and/or $D^{-}\Lambda_c^+$ channels, it might be easier to find evidence for the existence of the heavy pentaquarks in open-charm processes.

![Figure 6](image)

**FIG. 6.** (color online) Total cross section for $\pi^-p \to J/\psi n$. The notation is the same as that employed in Fig. 5. The spin and parity are given as $J^P = (3/2^- , 5/2^+)$ for $(P_c^0(4380), P_c^0(4450))$, respectively. The experimental data on the upper limit come from Jenkins et al. [6] (blue circle) and Chiang et al. [7] (blue square). The black triangle represents the upper limit on $D^{-}\Lambda_c^+$ production [25]. The dotted curve depicts the results of the total cross section for $\pi^-p \to D^{-}\Lambda_c^+$, and the dot-dashed curve illustrates that for $\pi^-p \to D^{*+}\Lambda_c^+$. The contribution of the heavy pentaquark is not included in the results for these two reactions.

Assuming that ρ-meson exchange is dominant, we find that the other isospin channels for the $\pi N \to J/\psi N$ reactions are related to each other by the isospin factors given as follows

$$\sigma(\pi^-p \to J/\psi n) = \sigma(\pi^+n \to J/\psi p) = 2\sigma(\pi^0n \to J/\psi n) = 2\sigma(\pi^0p \to J/\psi p). \quad (17)$$
However, considering that $J/\psi$ cannot decay into two neutral pions and charged mesons are not allowed to be exchanged, then the mechanism of the $\pi^0 N \to J/\psi N$ reactions should differ from that of the $\pi^\pm N \to J/\psi N$ processes. Note that the $\pi^0$ beam is not suitable for use in the experimental production of any hadrons because of its neutral nature and short lifetime. However, the $\pi^+ n \to J/\psi p$ reaction provides an opportunity to study the existence of the charged heavy pentaquark $P_c^+$, and the present study considers the neutral $P_c^0$.

In Fig. 7 we show the results of differential cross sections for the $\pi^- p \to J/\psi n$ reaction at four different CM energies $W$ near the threshold. As expected from the results obtained for the total cross section, the contribution of the $P_c$ states has a dominant influence on the differential cross section in the vicinity of the energies corresponding to the $P_c$ resonances, so the differential cross section is almost independent of the scattering angle at $W = 4.38$ GeV. As $W$ increases, the magnitude of the differential cross section drops drastically. At $W = 4.75$ GeV, which is above the energies corresponding to the $P_c$ resonances, the differential cross section exhibits a forward peak, where the dependence on $\theta$ is rather weak as $\theta$ increases.

![Graphs showing differential cross sections](image)

**Fig. 7.** Differential cross sections for $\pi^- p \to J/\psi n$ at four different CM energies. The spin-parity is selected as $J^P = (3/2^-, 5/2^+)$ for $(P_c^0(4380), P_c^0(4450))$.

**IV. CONCLUSION AND SUMMARY**

In the present study, we aimed to investigate the production mechanism for the $\pi^- p \to J/\psi n$ reaction based on the hybridized Regge model by including the contributions of the
two heavy pentaquark $P_c(4380)$ and $P_c(4450)$ states. First, we considered the effects of $\rho$ and $\pi$ Reggeon exchanges, which yielded the background contribution to the total cross section for the $\pi^-p \rightarrow J/\psi n$ reaction. The scale factor for the $\pi^-p \rightarrow \phi n$ was determined by fitting it to experimental data on the total cross section. In order to avoid ambiguity, we employed the same form for the scale factor to describe the $\pi^-p \rightarrow J/\psi n$ reaction by using the same numerical values for the parameters used. Thus, we found that the background contribution to the total cross section for the $\pi^-p \rightarrow J/\psi n$ process is approximately $10^6$ times smaller than that for the $\pi^-p \rightarrow \phi n$ reaction. This is mainly explained by the highly suppressed value of the coupling constant $g_{J/\psi \rho \pi}$ compared with that of $g_{\phi \rho \pi}$ in the case of dominant $\rho$-meson Reggeon exchange.

No information is available regarding the branching ratios of the decays for heavy pentaquark resonances, so we carefully studied the assumptions and suggestions proposed in previous theoretical studies. First, we examined the branching ratios proposed by Wu et al. [23], where we assumed that the predicted states in Wu et al. [23] correspond to the heavy pentaquark states observed by the LHCb Collaboration. Wu et al. [23] proposed branching ratios for the pentaquark states of the $J/\psi N$ and $\pi N$ decay channels of about 40 % and 6 %, respectively. Considering these values for the branching ratios, the results for the total cross section for $\pi^-p \rightarrow J/\psi n$ near the resonance region are $10^4$ nb, which is approximately of the same order as that for $\pi^-p \rightarrow \phi n$. However, if we use the branching ratio proposed by Wang et al. [2], the results obtained for the total cross section decrease dramatically, where the maximum values are in the order of 1 nb. This is consistent with the experimental upper limit [6, 7] on the total cross section for the $\pi^-p \rightarrow J/\psi n$ reaction. The results are not sensitive to the selection of different spins and parities for the heavy pentaquark states.

In the present study, we examined the two neutral pentaquark states $P_c^0$, but the other isosymmetric reaction $\pi^+n \rightarrow J/\psi p$ is as appropriate for the study of charged pentaquark states $P_c^+$ as for the intermediate states. The total cross section of the $\pi^+n \rightarrow J/\psi p$ process is simply the same as that of $\pi^-p \rightarrow J/\psi n$. However, for $J/\psi$ photoproduction, we could have a large isospin asymmetry because of the different photocouplings between $P_c^+$ and its neutral partner. We also obtained the results for differential cross sections at four different energies in the CM frame in the vicinity of the energies corresponding to the heavy pentaquark resonance states.

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