Gluodynamics in external field in dual superconductor approach

M.N. Chernodub

ITEP, B. Cheremushkinskaya 25, Moscow, 117259, Russia and Institute for Theoretical Physics, Kanazawa University, Kanazawa 920-1192, Japan

Abstract

We show that gluodynamics in an external Abelian electromagnetic field should possess a deconfining phase transition at zero temperature. Our analytical estimation of the critical external field is based on the dual superconductor picture which is formulated in the Euclidean space suitable for lattice calculations. A dual superconductor model corresponding to the $SU(2)$ gluodynamics possesses confinement and deconfinement phases below and, respectively, above the critical field. A dual superconductor model for the $SU(3)$ gauge theory predicts a rich phase structure containing confinement, asymmetric confinement and deconfinement phases. The quark bound states in these phases are analyzed. Inside the baryon the strings are $Y$-shaped as predicted by the dual superconductor picture. This shape is geometrically asymmetric in the asymmetric confinement phases. The results of the paper can be used to check the dual superconductor mechanism in gluodynamics.

1. At present there are two popular approaches to the problem of color confinement in gluodynamics. They are based on the Abelian monopole [1] and on the center vortex [2] pictures of the gluodynamics vacuum. In this paper we discuss the Abelian monopole approach which suggests that confining degrees of freedom of the vacuum in an Abelian projection [3] can be described as a dual superconductor. The key element of this picture is the monopole condensate which squeezes a chromoelectric flux to a confining string due to the Meissner effect. The string is an analog of the Abrikosov vortex [4] in an ordinary superconductor while the Abelian monopoles are playing the role of the Cooper pairs. This picture has been confirmed in many numerical simulations on the lattice (for a review see, e.g., Ref. [5]).

Here we investigate the properties of the $SU(2)$ and $SU(3)$ gluodynamics in the external electromagnetic field using the dual superconductor approach. Our study is motivated by the fact that the response of the vacuum of a gauge theory on external fields may provide interesting information about the vacuum structure. The external fields were used to study nonperturbative properties of QCD [6], the baryogenesis in the electroweak theory [7], features of various topological defects in three dimensional models [8], etc.

A common feature of known superconductors is the Meissner effect: the superconductors expel relatively weak external magnetic flux from their interior. Strong enough magnetic field, $H_{\text{ext}} \geq H_{\text{cr}}$, destroys the superconductivity and the superconductor goes in the normal (metal) state. In this paper we estimate analytically the critical electromagnetic fields which break the
dual superconductivity in the $SU(2)$ and $SU(3)$ gluodynamics. Consequently, the confinement is (partially, in the case of $SU(3)$) lost at these critical fields.

We consider the external electromagnetic fields in a particular Abelian gauge, which is used to define the Abelian degrees of freedom. Strictly speaking, these purely Abelian fields are unlikely to be realized in Nature. Nevertheless, the response of the gluodynamics media on these purely Abelian fields can be used for further checks of the dual superconductivity hypothesis in numerical simulations of lattice gluodynamics. Since the dual superconductor picture describes various non-perturbative phenomena such numerical test is physically motivated.

We consider the gluodynamics at zero temperature because in this case the relevant couplings of the corresponding dual superconductor models are already known. Indeed, in the absence of the external field the gluodynamics experiences the deconfining phase transition at sufficiently high temperature. On the other hand, at the critical temperature the dual superconductivity was demonstrated to be destroyed. Thus the couplings of the dual superconductor (at least, the value of the monopole condensate) must depend on the temperature. This dependence is not known at the time being.

Apart from knowledge of the dual couplings, another simplification comes from the Euclidean formulation. The action of the four dimensional dual superconductor is just a trivial dimensional generalization of the (Helmholtz) free energy functional of the ordinary three dimensional superconductor. Thus, from the point of view of static effects — such as a response of the superconductor on a static external field — the Euclidean dual superconductor model of the gluodynamics describes just an infinitely large four dimensional superconducting material. The electric and magnetic components of the electromagnetic field differ only by the orientation of the field strength tensor in the coordinate space with respect to the time axis. However, in the Euclidean formulation at zero temperature no distinguished time–direction exists. In this particular respect there is no difference between external the static electric and the static magnetic fields (this in no more correct in the presence of the external sources such as heavy quarks or monopoles). Therefore in this article we are using the terminology ”electromagnetic (EM) field” for this particular case.

Yet another simplification is due to the fact that the vacuum of $SU(2)$ and $SU(3)$ Yang–Mills theories in the Abelian projection is known to be close to the border between type–I and type–II dual superconductors. At the borderline — called also ”the Bogomol’ny limit” — analytical results for the string tension are available. Below we consider the dual models for both theories in the Bogomol’ny limit. We assume that the external field does not change the couplings of the dual superconductor model.

2. Let us first consider the $SU(2)$ gauge theory in the $4D$ Euclidean space. The infrared properties of the vacuum of this model can be described by the Abelian Higgs (or, Ginzburg–Landau) Lagrangian:

$$\mathcal{L}_{GL}[B, \Phi] = \frac{1}{4} F_{\mu \nu}^2 + \frac{1}{2} |D_B(\Phi)|^2 + \lambda \left( |\Phi|^2 - \eta^2 \right)^2, \quad (1)$$

Note, however, that a general non–Abelian field may have a non–zero projection on the Abelian subspace discussed in the paper.

2This assumption works well in the macroscopic (Ginzburg–Landau) description of the ordinary superconductivity.
where \( F_{\mu\nu} = \partial_{\mu}B_{\nu} - \partial_{\nu}B_{\mu} \) is the field strength for the dual gauge field \( B_{\mu} \), \( \Phi \) is the monopole field with the magnetic charge \( g_M \) and \( D_{\mu} = \partial_{\mu} + ig_M B_{\mu} \) is the covariant derivative. The gauge field \( B_{\mu} \) is dual to the third component of the gluon field in an Abelian gauge. The model possesses the dual \( U(1) \) gauge symmetry, \( B_{\mu} \rightarrow B_{\mu} - \partial_{\mu} \alpha, \Phi \rightarrow e^{ig_M \alpha} \Phi \). The form of the potential implies the existence of the monopole condensate, \( |\langle \Phi \rangle| = \eta \) with \( \eta^2 > 0 \), and, consequently, non-zero masses of the dual gauge, \( m_B = g\eta \), and monopole, \( m_\Phi = 2\sqrt{2}\lambda\eta \), fields.

The Bogomol'ny limit corresponds to a region of the coupling space where the masses of the monopole and gauge fields are the same. In our notations (1) the Bogomol’ny limit is defined by the condition

\[
g_M^2/\lambda = 8. \tag{2}
\]

The properties of the Ginzburg–Landau model (1) are very well known. Below we briefly derive the value of the critical field in the \( U(1) \) model (1) and later we apply this method to a more complicated \([U(1)]^2\) case corresponding to the \( SU(3) \) gluodynamics.

Consider the four-dimensional sample of the (dual) superconductor occupying half-space, \( x_2 \geq 0 \). Let us apply the constant external EM field \( F_{\mu\nu}^{\text{ext}} = \varepsilon_{\mu\nu\alpha\beta}H_{\alpha\beta}^{\text{ext}} \) to the boundary of the superconductor. A weak external field penetrates inside the sample up to the distance \( \sim m_B^{-1} \). The screening of the field (the Meissner effect) is realized due to the induced superconducting current,

\[
J_{\mu} = \Im m(\Phi^{\ast} D_{\mu}(B)\Phi) \equiv |\Phi|^2 \cdot v_{\mu}, \quad v_{\mu} = \partial_{\mu} \varphi + g_M B_{\mu}, \tag{3}
\]

where we have set \( \Phi = |\Phi|e^{i\varphi} \). The current is parallel to the boundary of the superconductor. The monopole kinetic term in Eq. (1) can be written as \( |D_{\mu}\Phi|^2 = (\partial_{\mu}|\Phi|)^2 + |\Phi|^2 v_{\mu}^2 \). Clearly, a non-zero current provides an additional positively-defined term in the Lagrangian \( \propto |\Phi|^2 \). As a result, the external field lowers the value of the monopole condensate. We are looking for the critical value of the EM field which destroys the monopole condensate and, consequently, confinement.

We disregard the quantum fluctuations of the fields in the model (1) treating this system classically. To derive the critical EM field it is convenient to rewrite the action of the model (1) as a two-dimensional integral. The first direction of the two-dimensional plane is obviously the depth of the dual superconductor, \( x_2 \), while the second is given by the direction of the current (2). We choose \( J_{\mu} \propto \delta_{\mu,1} \) and rewrite the model (1) in the (1,2) plane. The first term of eq. (1) is \( F_{\mu\nu}^2/4 = H^2/2 \) while the second term can be rewritten with the help of relations:

\[
|D_{\mu}\Phi|^2 = \sum_{\alpha=1,2} |D_{\alpha}\Phi|^2 = \left| \left( D_1 \pm iD_2 \right) \Phi \right|^2 \mp 2\varepsilon_{\alpha\beta}\partial_{\alpha}J_{\beta} \pm i\Phi^{\ast}[D_1, D_2]\Phi, \tag{4}
\]

The last term in this equation can also be represented as \( i\Phi^{\ast}[D_1, D_2]\Phi = -g_M H |\Phi|^2 \). We get the following action in the (1,2) plane:

\[
S_{GL} = L_3 L_4 \int d^2x \left[ \frac{1}{2} H^2 + \frac{1}{2} \left| \left( D_1 \pm iD_2 \right) \Phi \right|^2 - \frac{g_M}{2} H |\Phi|^2 + \lambda \left( |\Phi|^2 - \eta^2 \right)^2 \right] + S_J, \tag{5}
\]
where $L_i$ is the (infinite large) length of the dual superconductor in $i$th direction and $S_J$ is the action of the surface current:

$$S_J = L_3 L_4 \int d^2 x \varepsilon_{\alpha\beta} \partial_{\alpha} J_\beta = L_1 L_3 L_4 \cdot J_1(x_2 = 0).$$  

(6)

Using condition (2) we rewrite Eq. (5) further:

$$S = \frac{1}{2} L_3 L_4 \int d^2 x \left[ \left| (D_1 \pm i D_2) \Phi \right|^2 + \left[ H \mp \frac{g_M}{2} \left( |\Phi|^2 - \eta^2 \right) \right] \right] \mp S_{flux} + S_J. $$  

(7)

The sign in this equation is dictated by the "flux" action, $S_{flux} = g_M L_3 L_4 \eta^2 \int d^2 x H/2$, which must be positive. We choose $H > 0$ and get two Bogomol’ny equations which minimize the action (7):

$$(D_1 - D_2) \Phi = 0, \quad H + \frac{g_M}{2} \left( |\Phi|^2 - \eta^2 \right) = 0.$$  

The second equation gives the value of the monopole condensate at the boundary

$$|\Phi(x_2 = 0)|^2 = \eta^2 - 2H^{ext}/g_M.$$  

(8)

The condensate disappears at the critical value of the EM field, $H^{ext} = H_{cr} = g_M \eta^2/2$. At this value the superconducting current at the boundary vanishes, $J_1 = 0$, giving $S_J = 0$. The total action is given by the flux contribution, $S = S_{flux} = V ol \cdot g_M^2 \eta^4/4$ which is the sum of the free energy of the normal state, $S_n \equiv S_{GL}[B = 0, \Phi = 0]$ and the free energy of the EM field, $S_H$: $S_n = S_H = V ol \cdot g_M^2 \eta^4/8$.

In the Bogomol’ny limit the tension of the string spanned on trajectories of the fundamental charges can be evaluated exactly\footnote{\cite{11, 12}, $\sigma = \pi \eta^2$.} $\sigma = \pi \eta^2$. Using the Dirac relation between magnetic ($g_M$) and electric ($g$) charges, $g_M g = 2\pi$, we get the exact value of the critical EM field in terms of the string tension:

$$gH_{cr}/\sigma = 1, \quad \text{for SU}(2).$$  

(9)

This equation can be used in numerical simulations to check independently the closeness of the gluodynamics vacuum to the boundary between type–I and type–II superconductors.

The nature of the phase transition at the critical EM field can be understood as follows. The EM field, applied to the boundary of the media with the $SU(2)$ gauge fields, lowers the value of the monopole condensate according to Eq. (8). For the external fields $H^{ext} < H_{cr}$ the monopole condensate is non-zero, $|\Phi(x_2 = 0)| > 0$. This implies, that the dual photon mass at the boundary is non-zero as well, $m_B(x_2 = 0) = g_M |\Phi(x_2 = 0)|$. Due to the Meissner effect the external field diminishes as we go deeper into the media. At the distances of the order of the correlation length, $\sim m_\Phi^{-1}$, the monopole condensate restores at bulk value, $\Phi \sim \eta$. Consequently, at these distances the dual photon mass is restored as well, $m_B \sim g_M \eta$ and the external EM field diminishes exponentially, $H(x_2) \sim e^{-m_B x_2}$ for $x_2 \gtrsim m_\Phi^{-1}$. Thus, no EM field is present in the bulk of the system.\footnote{We stress that this result is obtained at zero temperature in the absence of the external fields.}
Now suppose that the external field reaches its critical value, \( H^{\text{ext}} = H_{\text{cr}} \). Then both the monopole condensate and the dual photon mass are zero at the boundary. The last fact implies that the field penetrates inside the media for an infinitesimally small distance \( \delta x_2 \) without any suppression because the Meissner effect is absent. At the distance \( \delta x_2 \) the situation repeats again: the condensate and the photon mass are zero and the field penetrates deeper into the media for another infinitesimal step, \( \text{etc.} \) Thus, the external field "eats" the condensate step by step and finally the condensate disappears in the whole space. Due to this mechanism the EM field originated at the boundary can destroy the monopole condensate (and, consequently, the confinement) in the bulk.

3. Now let us consider the Lagrangian of \([U(1)]^2\) Higgs model corresponding to \(SU(3)\) gluodynamics \([13]\):

\[
\mathcal{L} = \frac{1}{4} F_{\mu \nu}^{a} F^{a, \mu \nu} + \sum_{i=1}^{3} \left[ \frac{1}{2} |D_{\mu}^{(i)} \Phi_i|^2 + \lambda \left( |\Phi_i|^2 - \eta^2 \right)^2 \right],
\]

(10)

where \( F_{\mu \nu}^{a} = \partial_{\mu} B_{\nu}^{a} - \partial_{\nu} B_{\mu}^{a} \) is the field strength for the gauge fields \( B_{\mu}^{a}, a = 3, 8 \), \( D_{\mu}^{(i)} = \partial_{\mu} + ig_{M} \varepsilon_{a}^{i} B_{\mu}^{a} \) is the covariant derivative acting on the monopole fields \( \Phi_i, i = 1, 2, 3 \). The \( \varepsilon \)'s are the root vectors of the group \( SU(3) \): \( \bar{\varepsilon}_1 = (1, 0), \bar{\varepsilon}_2 = (-1/2, -\sqrt{3}/2), \bar{\varepsilon}_3 = (-1/2, \sqrt{3}/2) \). No summation over the Latin index \( i \) is implied.

The gauge fields \( B_{3,8}^{a} \) are dual to the diagonal components \( a = 3, 8 \) of the gluon field \( A_{\mu}^{a} \). Lagrangian \([14]\) respects the dual \([U(1)]^2\) gauge invariance: \( B_{\mu}^{a} \rightarrow B_{\mu}^{a} + \partial_{\mu} \alpha^{a}, \theta_i \rightarrow \theta_i + g_{M}(\varepsilon_{3}^{i} \alpha^{3} + \varepsilon_{8}^{i} \alpha^{8}), a = 3, 8, i = 1, 2, 3 \), where \( \alpha^{3} \) and \( \alpha^{8} \) are the parameters of the gauge transformation. The phases of the monopole fields satisfy the relation

\[
\sum_{i=1}^{3} \arg \Phi_i = 0,
\]

(11)

which plays an important role in the formation of the quark bound states within the dual superconductor formalism \([14, 15]\).

The Bogomol'ny limit is defined by condition \([16, 17]\)

\[
g_{M}^2 / \lambda = 16/3,
\]

(12)

and the equations of motion in this limit are given by

\[
\left( D_{1}^{(i)} \pm i D_{2}^{(i)} \right) \chi_i = 0, \quad H^{(i)} = \frac{3g_{M}}{4} \left( |\chi_i|^2 - \eta^2 \right) = 0; \quad i = 1, 2, 3,
\]

(13)

where

\[
H^{(i)} = \sum_{a=3,8} \varepsilon_{a}^{i} H^{a},
\]

(14)

are the EM fields projected on the (3, 8)-charges of the monopole fields. One can imagine these three fields as the dual red, dual blue and dual green EM fields.
The second equation in (13) gives the same critical value for all components of the $H^{(i)}$ fields:

$$H_{cr}^{(i)} \equiv \tilde{H}_{cr} = \frac{3g_M \eta^2}{4}.$$  \hspace{1cm} (15)

The critical values are equivalent due to the Weyl symmetry of the dual model \[17, 18\] which states that the $[U(1)]^2$ Lagrangian (10) is invariant under the transformations of the dual gauge fields $B^{3,8}$ corresponding to the mutual permutations of the $H^{(i)}$ fields (14).

The string tension spanned between the fundamental charges (quarks) in the Bogomol’ny limit (12) of the $SU(3)$ gluodynamics is \[16, 17\] $\sigma = 2\pi \eta^2$. Using the Dirac quantization condition we get the critical field $\tilde{H}_{cr}$ in units of the string tension:

$$g\tilde{H}_{cr}/\sigma = \frac{3}{4}, \quad \text{for SU}(3).$$  \hspace{1cm} (16)

When the strength of the EM component $H^{(i)}$ reaches the $\tilde{H}_{cr}$ value then the expectation value of the corresponding component of the monopole field, $\Phi_i$, gets vanished. Note, however, that the fields $H^{(i)}$ play an auxiliary role because they are not independent according to Eq. (14). Expressing the auxiliary fields $H^{(i)}$ in terms of the components the EM field, $H^{3,8}$, and using Eqs.(15,16) we get the phase diagram depicted in Figure 1.

The phase diagram contains confinement (C), deconfinement (D) and the asymmetric confinement phases (A). The position of the phase transition depends not only on the absolute value of the EM field but also on the ("color") orientation of this field in the Cartan subgroup. At low values of the field the model is always confining regardless of the color orientation. However, as the absolute value of the field is increased, the model enters – depending on color orientation – one of six ($A_{12}$, $A_{13}$, $A_{23}$, $A_1$, $A_2$ or $A_3$) asymmetric confinement phases. In the $A_{ij}$ phase the $i$th and $j$th components of the monopole field are condensed while the expectation value of the third component is zero. In the phase $A_i$ the $i$th component is condensed while the others two components are not. With the further increase of the field the model either enters the deconfinement phase, $D$, or stays in one of the three asymmetric confinement phases, $A_1$, $A_2$ or $A_3$.

4. The quark bound states in the Abelian projection approach are classified with the help of the states of the strings spanned between the constituent quarks. The string configuration in the baryon was extensively studied both in the Abelian projection approach \[14, 15\] as well as in a gauge independent formalism \[19\]. The dual superconductor model predicts \[15\] the existence of the $Y$-shaped string configuration in agreement with most of Ref. \[19\]. Here we discuss the quark bound states in the presence of the external Abelian field.

The confining properties of both the confinement and the deconfinement phases are standard. In the confinement phase all three monopole fields, $\Phi_i$, $i = 1, 2, 3$, are condensed and the $[U(1)]^2$ model possesses three types of vortex solutions \[15, 18, 20\] which confine all quarks into bound states. Each vortex solution is characterized by the winding number $n_i$, $\Phi_i \propto e^{i n_i \phi_i}$, where $\phi_i$ is the azimuth angle. The winding numbers of the strings are subjected to the constraint $n_1 + n_2 + n_3 = 0$ coming from Eq. (11). In the deconfinement phase all monopole fields are not condensed and the bound stated are not formed.\footnote{In this paper we disregard weakly bounded states which might appear due to the exchange of the perturbative gluons between quarks. We also disregard the role of the Jacobian \[21\] arising in the string model.}
Let us discuss the bound states in the asymmetric confinement phases. Consider, for example, $A_{12}$ phase. In this case the model possesses only two condensates, $\langle \Phi_{1,2} \rangle \neq 0$. Consequently, on the classical level only two types of the chromoelectric strings can be formed with $n_{1,2} \neq 0$. Despite the third field has zero expectation value, $\langle \Phi_i \rangle = 0$, its phase may fluctuate and the (classically tensionless) $n_3 \neq 0$ string-like configurations may appear. This implies that in addition to the purely classical $\vec{n} = (1, -1, 0)$ string configuration there exist also $(1, 0, -1)$ and $(0, 1, -1)$ configurations. These additional configurations are composed from the classical strings with either $n_1 \neq 0$ or $n_2 \neq 0$ and a string-like (tensionless) quantum excitation with $n_3 \neq 0$. The stability of the quantum components of such configurations is guaranteed by Eq. (11).

The $\vec{n} = (1, -1, 0)$ string configuration must have bigger string tension than $(-1, 0, 1)$ and $(0, 1, -1)$ configurations. Thus in the $A_{12}$ phase we expect the existence of the relatively heavy meson composed from red quarks and two lighter mesons made of blue and green quarks.\footnote{Here we adopt the classification of Ref. \cite{15} assuming that the quarks in a meson state are connected with each other by a pair of the strings with winding numbers $\vec{n} = (1, -1, 0)$, $(-1, 0, 1)$ and $(0, 1, -1)$ for $RR$, $BB$ and $GG$ states respectively.}

The observed color asymmetry in the meson states is caused by the breaking of the Weyl
symmetry [17, 18] of the Lagrangian (10) by the external field.

The baryon state also exists in the $A_{12}$ phase but it should be lighter than the baryon in the confinement phase. The quarks in the baryon are connected to each other by all three types of the string configurations. It is known [15] that in the absence of the external field these strings form a symmetric $Y$–shaped profile. In the $A_{12}$ phase the tension of the $(1, -1, 0)$ string configuration is heavier than that of $(-1, 0, 1)$ and $(0, 1, -1)$ configurations. Therefore in this phase the strings in the baryon state must form an asymmetric $Y$–profile, see Figure 2.

![Color asymmetric phase $A_{12}$](image)

Confinement

![Color asymmetric phase $A_{12}$](image)

Figure 2: The meson and baryon bound states in confinement, deconfinement and $A_{12}, A_{1}$ asymmetric confinement phases. The solid lines correspond to the strings existing on the classical level while the dashed lines represent the quantum string excitations.

The properties of the $A_i$ phases are different from those of the $A_{ij}$ phases. In particular, in the $A_1$ phase the string with the winding number $n_1 \neq 0$ has a non–zero string tension while the strings with $n_{2,3} \neq 0$ are classically tensionless. Thus in this phase only $(1, -1, 0)$ and $(1, 0, -1)$ string configurations have a non–zero string tension. This implies the existence of the mesons composed of red and blue quarks while the meson made of green quarks is absent. The existing mesons must be lighter than those in the confinement phase. All possible meson and baryon states in confinement, deconfinement and asymmetric confinement phases are depicted in Figure 4.

5. Summarizing, we have explored the phase structure of the $SU(2)$ and $SU(3)$ gauge theories in the external electromagnetic fields at zero temperature. Both theories are considered in the dual superconductor formalism formulated in the Euclidean space (suitable for lattice calculations) and in the Bogomol’ny limit (as confirmed by various lattice calculations).

We have found that the phase diagram of the dual superconductor model corresponding to the $SU(2)$ gluodynamics contains confinement and deconfinement phases which are located below and, respectively, above the critical field. The critical electromagnetic field is analytically
estimated in terms of the string tension in Eq. (9).

The phase diagram of the \([U(1)]^2\) dual superconductor corresponding to the \(SU(3)\) gluodynamics contains 8 phases (confinement, deconfinement and 6 asymmetric confinement phases). The reach phase structure – shown in Figure 1 – comes from the dependence of the monopole condensate on the color orientation of the external field. This finding is supported by the observation of Ref. [22] that the properties of the Abelian monopoles depend on the color orientation of the monopole. Three of six asymmetric confinement phases \((A_{12}, A_{13} \text{ and } A_{23})\) contain one baryon and three meson states. Two of the meson states are lighter then the third one. The strings in the baryon form the asymmetric \(Y\)–shaped profile. These phases can be still regarded as confinement phases since the quarks of all three colors are confined.

The other three asymmetric confinement phases \((A_1, A_2 \text{ and } A_3)\) may contain only two light meson states while the baryon state is absent at all. The quarks carrying a particular (phase–dependent) color are not confined in these phases.

Note, that our results were obtained for the Abelian external fields which are applied in a fixed Abelian projection\(^6\). In particular, this means that the recent results of Refs. [24] for the phase diagram in the external Abelian fields can not be compared with our predictions because these results were obtained without the gauge fixing.

It would be interesting to check the predictions of this paper by numerical simulations performed in the Maximal Abelian projection of the \(SU(2)\) and \(SU(3)\) gluodynamics. As it is shown above the dual superconductor hypothesis predicts a particular phase diagram in the external electromagnetic fields. The numerical investigation of this diagram can be used for further checks of the dual superconductivity of the vacuum. Moreover, the value of the critical fields corresponding to the phase transitions (if exist) could be used to determine the closeness of parameters of the dual superconductor to the type–I/II boundary.

Acknowledgments

The author is grateful to V.G. Bornyakov and Y. Koma for interesting discussions. The work is supported by the JSPS Fellowship P01023.

References

[1] Y. Nambu, Phys. Rev. D10 (1974) 4262; G. ’t Hooft, in High Energy Physics, ed. A. Zichichi, EPS International Conference, Palermo (1975); S. Mandelstam, Phys. Rept. 23, 245 (1976).

[2] L. Del Debbio, M. Faber, J. Greensite and S. Olejnik, Phys. Rev. D 55 (1997) 2298.

[3] G. ’t Hooft, Nucl. Phys. B 190, 455 (1981).

\(^6\)Here we do not discuss a controversy topic of the (in)dependence of the dual superconductor picture on chosen Abelian projection [23]. We assume that this picture is realized in the Maximal Abelian projection, see Ref. [3] for a review.
[4] A. A. Abrikosov, Sov. Phys. JETP, 32 (1957) 1442; H. B. Nielsen and P. Olesen, Nucl. Phys. B 61 (1973) 45.

[5] M. N. Chernodub and M. I. Polikarpov, in "Confinement, Duality and Non-perturbative Aspects of QCD", p.387, Plenum Press, 1998, [hep-th/9710203]; R. W. Haymaker, Phys. Rept. 315, 153 (1999).

[6] D. Kabat, K. Y. Lee and E. Weinberg, Phys. Rev. D 66, 014004 (2002); N. O. Agasian, Phys. Lett. B488 (2000) 39; I. A. Shushpanov and A. V. Smilga, Phys. Lett. B402 (1997) 351; A. I. Vainshtein et al., Sov. J. Nucl. Phys. 39 (1984) 77.

[7] K. Kajantie et al., Nucl. Phys. B544 (1999) 357; D. Comelli, D. Grasso, M. Pietroni and A. Riotto, Phys. Lett. B458 (1999) 304; V. Skalozub and M. Bordag, Int. J. Mod. Phys. A15 (2000) 349.

[8] P. H. Damgaard and U. M. Heller, Phys. Rev. Lett. 60 (1988) 1246; K. Kajantie, M. Laine, T. Neuhaus, A. Rajantie and K. Rummukainen, Nucl. Phys. B 559 (1999) 395; M. N. Chernodub, E. M. Ilgenfritz and A. Schiller, Phys. Rev. D 64 (2001) 114502.

[9] M. N. Chernodub, M. I. Polikarpov and A. I. Veselov, JETP Lett. 63 (1996) 411; Phys. Lett. B399 (1997) 267; A. Di Giacomo and G. Paffuti Phys. Rev. D 56 (1997) 6816; N. Nakamura, V. Bornyakov, S. Ejiri, S. Kitahara, Y. Matsubara and T. Suzuki, Nucl. Phys. Proc. Suppl. 53 (1997) 512.

[10] M. Baker, J.S. Ball and F. Zachariasen, Phys. Rev. D31 (1985) 2575; ibid. 41 (1990) 2612; S. Maedan, Y. Matsubara and T. Suzuki, Prog. Theor. Phys. 84 (1990) 130; V. Singh, D. A. Browne and R. W. Haymaker, Phys. Lett. B 306, 115 (1993); Y. Matsubara, S. Ejiri and T. Suzuki, Nucl. Phys. Proc. Suppl. 34 (1994) 176; G. S. Bali, C. Schlichter and K. Schilling, Prog. Theor. Phys. Suppl. 131 (1998) 645.

[11] E. B. Bogomolny, Sov. J. Nucl. Phys. 24 (1976) 449 [Yad. Fiz. 24 (1976) 861].

[12] H. J. de Vega and F. A. Schaposnik, Phys. Rev. D 14 (1976) 1100.

[13] S. Maedan and T. Suzuki, Prog. Theor. Phys. 81 (1989) 229.

[14] S. Kamizawa, Y. Matsubara, H. Shiba and T. Suzuki, Nucl. Phys. B 389 (1993) 563; Y. Koma, M. Koma, D. Ebert and H. Toki, Effective string action for the $U(1) \times U(1)$ dual Ginzburg-Landau theory beyond the London limit, [hep-th/0206074].

[15] M. N. Chernodub and D. A. Komarov, JETP Lett. 68, 117 (1998).

[16] M. N. Chernodub, Phys. Lett. B 474 (2000) 73.

[17] Y. Koma and H. Toki, Phys. Rev. D 62 (2000) 054027;

[18] Y. Koma, E. M. Ilgenfritz, T. Suzuki and H. Toki, Phys. Rev. D 64 (2001) 014015.
[19] N. Brambilla, G. M. Prosperi, A. Vairo, Phys. Lett. B 362 (1995) 113; D. S. Kuzmenko, Y. A. Simonov, Phys. Atom. Nucl. 64 (2001) 107; Triangular and Y-shaped hadrons in QCD, hep-ph/0202277; Y. A. Simonov, Nonperturbative quark dynamics in a baryon, hep-ph/0205334; T.T. Takahashi, H. Matsufuru, Y. Nemoto and H. Suganuma, Phys. Rev. Lett. 86 (2001) 18; C. Alexandrou, P. de Forcrand and A. Tsapalis, Probing hadron wave functions in lattice QCD, hep-lat/0206026.

[20] D. Antonov and D. Ebert, Phys.Lett. B444 (1998) 208.

[21] E. T. Akhmedov, M. N. Chernodub, M. I. Polikarpov and M. A. Zubkov, Phys. Rev. D 53 (1996) 2087.

[22] P. Cea and L. Cosmai, Phys. Rev. D 62 (2000) 094510.

[23] M. N. Chernodub, M. I. Polikarpov and A. I. Veselov, Phys. Lett. B 342 (1995) 303; S. Fujimoto, S. Kato, T. Suzuki and T. Tsunemi, Prog. Theor. Phys. Suppl. 138 (2000) 36; J. M. Carmona, M. D’Elia, A. Di Giacomo, B. Lucini and G. Paffuti, Phys. Rev. D 64 (2001) 114507.

[24] P. Cea and L. Cosmai, Abelian chromomagnetic fields and confinement”, hep-lat/0204023; Phys. Rev. D 60 (1999) 094506