We evaluate the cross section of the process $e^+e^- \rightarrow \gamma X(3872)$ in terms of the content of the $D^*\bar{D}^*$ ‘molecular’ component in the wave function of the resonance $X(3872)$. If this component is dominating, the cross section of the reaction $e^+e^- \rightarrow \gamma X(3872)$ can reach up to about $10^{-3}$ of that for $e^+e^- \rightarrow D^*\bar{D}^*$ at energy slightly above the $D^*\bar{D}^*$ threshold, and the considered process can be a realistic source of the $X(3872)$ particles for the studies of this resonance.
The extreme proximity of the mass of the $X(3872)$ resonance\(^1\) to the $D^0\bar{D}^{*0}$ threshold, as well as the co-existence of the decays $X(3872) \rightarrow \pi^+\pi^-J/\psi$ and $X(3872) \rightarrow \pi^+\pi^-\pi^0J/\psi$\(^2\), strongly suggests\(^3\) a significant presence of a ‘molecular’\(^7\) $D^0\bar{D}^{*0} + D^{*0}\bar{D}^0$ component in the wave function of $X(3872)$. Such unusual structure of this hadronic state makes further studies of its properties very promising for understanding the strong dynamics of heavy mesons. At present the $X(3872)$ resonance is observed experimentally only in the decays of $B$ mesons $B \rightarrow X K$\(^8\) and in inclusive production in proton - antiproton collisions at the Tevatron\(^10\)\(^11\). Both these types of processes are quite rare and also present significant challenges for precision measurements of the parameters of the discussed resonance. In particular\(^1\), neither the total width of $X(3872)$ is yet resolved (the current limit is $\Gamma_X < 2.3$ MeV), nor its mass is known with a precision sufficient to determine the binding energy of the molecular component (the current average value $M_X = 3871.2 \pm 0.5$ MeV coincides within the errors with $M(D^0) + M(D^{*0}) = 3871.2 \pm 0.8$ MeV). In this paper we consider the process $e^+e^- \rightarrow \gamma X(3872)$ at the c.m. energy within few MeV of the $D^{*0}\bar{D}^{*0}$ threshold, where the kinematical simplicity of the process would hopefully allow more detailed studies of $X(3872)$. We estimate that the cross section $\sigma[e^+e^- \rightarrow \gamma X(3872)]$ is likely to be at least about $10^{-3}$ of the cross section for the production of $D^{*0}\bar{D}^{*0}$ meson pairs, i.e. in the range of about 1 pb, which makes realistic a study of the discussed here process in a dedicated experiment. Moreover, the energy dependence of the cross section is sensitive to the binding energy of the molecular component. Thus a study of this dependence can provide a better accuracy of determining the mass of $X(3872)$ relative to the $D^0\bar{D}^{*0}$ threshold than a direct mass measurement.

In order to estimate the cross section of the discussed process we calculate the absorptive part of the production amplitude due to the process $e^+e^- \rightarrow D^{*0}\bar{D}^{*0} \rightarrow \gamma X(3872)$ with on-shell $D^*$ mesons, as given by the unitarity relation. We find that this contribution to the amplitude is rapidly changing with the c.m. energy of the process. The contribution of other intermediate states to the amplitude, which potentially could destructively interfere with the calculated amplitude is a slowly varying function of energy, so that such destructive interference cannot occur at all energies in the considered range. Thus the value of the cross section can be estimated as being at least that given by the calculated part of the amplitude.

Proceeding to the calculation of the absorptive part of the amplitude we write the Fock
decomposition of the wave function of \( X(3872) \) in the form
\[
\Psi_X = a_0 \frac{D^0\bar{D}^{*0} + D^{*0}\bar{D}^0}{\sqrt{2}} + \sum_i a_i \psi_i ,
\]
where the molecular component is explicitly separated, and the sum runs over ‘other’ states. The crucial difference between the molecular and ‘other’ components of \( X(3872) \) is that due to a very small binding energy \( w \) the neutral \( D \) and \( D^* \) mesons mostly move at long distances, beyond the range of the strong interaction, while the ‘other’ states are localized at shorter distances typical of the strong interaction. At long distances the coordinate wave function of the meson pair is that of a free \( S \)-wave motion and is proportional to \( \exp(-\kappa r)/r \), where \( \kappa = \sqrt{2m_r w} \approx 44 \text{ MeV} \) determined by the reduced mass \( m_r \approx 966 \text{ MeV} \) in the \( D^0\bar{D}^{*0} \) system and the binding energy \( w \). With the coordinate wave function of this component normalized to one, the coefficient \( a_0 \) in the expansion (1) determines the statistical weight \( |a_0|^2 \) of the molecular state \((D^0\bar{D}^{*0} + D^{*0}\bar{D}^0)/\sqrt{2}\) in the wave function of \( X(3872) \). The notion of the resonance \( X \) being mostly a molecular system corresponds to this statistical weight factor of order one. The numerical value of \( |a_0|^2 \) is presently unknown, in a model calculation\[12\] this weight factor is estimated as 0.7 - 0.8 at \( w = 1 \text{ MeV} \).

In what follows we calculate the contribution of the ‘peripheral’ \( D^0\bar{D}^{*0} + D^{*0}\bar{D}^0 \) component of the \( X(3872) \) resonance to the absorptive part of the amplitude of the process \( e^+e^- \rightarrow D^{*0}\bar{D}^{*0} \rightarrow \gamma X(3872) \), where the latter transition proceeds due to the underlying radiative decay \( D^{*0} \rightarrow D^{0}\gamma \) (\( \bar{D}^{*0} \rightarrow \bar{D}^0\gamma \)), in analogy with the previously discussed\[5, 13\] decays of the \( X \) resonance, \( X \rightarrow D\bar{D}\gamma \). The amplitude of the radiative decay of the vector meson has the form
\[
A(D^{*0} \rightarrow D^{0}\gamma) = \mu \varepsilon_{ijk} k_j a_k ,
\]
with \( \vec{a} \) and \( \vec{\varepsilon} \) being the polarization amplitudes of the vector meson and the photon, and \( \vec{k} \) being the photon momentum.\[1\] The vector meson decay rate \( \Gamma_0 \equiv \Gamma(D^* \rightarrow D\gamma) \) is then related to the transition magnetic parameter \( \mu \) as
\[
\Gamma_0 = \frac{|\mu|^2 \omega_0^3}{3\pi} ,
\]
where \( \omega_0 \approx 137 \text{ MeV} \) is the photon energy in the decay. Numerically the decay rate can be estimated from the data\[11\] as \( \Gamma_0 = 26 \pm 6 \text{ KeV} \). The amplitude of the transition \( D^{*0}\bar{D}^{*0} \rightarrow \)

---

\[1\] The non-relativistic normalization for the heavy meson states is used throughout this paper.
\(\gamma X(3872)\) due to the ‘peripheral’ component of the \(X\) can then be written in terms of the momentum-space wave function \(\phi(\vec{q})\) of this component:

\[
A(D^{*0}\bar{D}^{*0} \to \gamma X) = \frac{\mu a_0}{\sqrt{2}} \varepsilon_{ijk} \varepsilon_i^* \varepsilon_j^* k_j \left[ a_k \left( \vec{b} \cdot \vec{\chi}^* \right) \phi \left( \vec{p} - \frac{\vec{k}}{2} \right) - b_k \left( \vec{a} \cdot \vec{\chi}^* \right) \phi \left( \vec{p} + \frac{\vec{k}}{2} \right) \right],
\]

(4)

where, in addition to the notation conventions in Eq.(2), \(\vec{b}\) stands for the polarization amplitude of the initial \(\bar{D}^*\) meson and \(\vec{\chi}\) is the polarization amplitude of the produced \(X\) resonance. The relative minus sign between the two terms originating from the amplitudes of the processes \(D^{*0} \to D^0\gamma\) and \(\bar{D}^{*0} \to \bar{D}^0\gamma\) is due to the opposite C parity of the \(X\) (\(J^{PC} = 1^{++}\)) and of the photon\(^{[5, 13]}\).

In the present calculation we describe the peripheral component by a wave function of a free-motion with an ultraviolet regularization at large momenta\(^{[14, 13]}\):

\[
\phi(\vec{q}) = \sqrt{8\pi\kappa c} \left( \frac{1}{\vec{q}^2 + \kappa^2} - \frac{1}{\vec{q}^2 + \Lambda^2} \right),
\]

(5)

where the normalization constant \(c\) is given by

\[
c = \sqrt{\Lambda (\Lambda + \kappa) \over \Lambda - \kappa},
\]

(6)

and the regularization parameter \(\Lambda\) is determined by the inverse size of the strong interaction region in the \(X\) resonance. The obvious reason for introducing the cutoff \(\Lambda\) is that the free-motion description of the meson pair inside \(X\) is applicable only at distances beyond the range of the strong interaction and such description generally fails at short distances, where the mesons overlap and the meson pair strongly mixes with the ‘other’ states in the expansion\(^{[11]}\). Thus introducing the parameter \(\Lambda\) is a way of explicitly separating the ‘peripheral’ part from the ‘core’.

The other ingredient in the calculation of the amplitude of the process \(e^+e^- \to D^{*0}\bar{D}^{*0} \to \gamma X(3872)\) is the production amplitude for the \(D^*\bar{D}^*\) pair in \(e^+e^-\) annihilation. At energy \(E = 2M(D^{*0}) + W\) near the threshold this amplitude can be generically written in the form

\[
A(e^+e^- \to D^{*0}\bar{D}^{*0}) = A_0 \left( \vec{j} \cdot \vec{p} \right) \left( \vec{a} \cdot \vec{b} \right)^* + \frac{3}{2} \sqrt{3} A_2 j_i p_k \left[ a_i b_k + a_k b_i - \frac{2}{3} \delta_{ik} (\vec{a} \cdot \vec{b}) \right]^*,
\]

(7)

where \(\vec{j} = (e\gamma\bar{c})\) stands for the current of the incoming electron and positron, \(\vec{p}\) is the momentum of one of the mesons (\(D^{*0}\) for definiteness) in the c.m. frame, and \(A_0\) and \(A_2\)
are the factors corresponding to production of the vector meson pair in the states with respectively the total spin $S = 0$ and $S = 2$. It can be also noted that the amplitude in Eq. (7) describes the production of mesons in the $P$ wave. Another kinematically possible amplitude, the $F$-wave, should be small near the threshold, i.e. at a small $W$. Both $A_0$ and $A_2$ are generally functions of the excitation energy $W$. Furthermore, their dependence on the energy near the threshold is known to be nontrivial due to the $\psi(4040)$ resonance [1], with possible further complications in the immediate vicinity of the threshold [15, 16]. Neither the relative magnitude nor the relative phase of the amplitudes $A_0$ and $A_2$ is presently known, but both of these can be measured from angular correlations [17]. These amplitudes determine the total cross section for production of $D^{*0}\bar{D}^{*0}$ in $e^+e^-$ annihilation:

$$\sigma(e^+e^- \rightarrow D^{*0}\bar{D}^{*0}) = \int |A(e^+e^- \rightarrow D^{*0}\bar{D}^{*0})|^2 \frac{d^3p}{(2\pi)^3} = C \frac{m^3}{2\pi} (|A_0|^2 + |A_2|^2) ,$$

(8)

where $m = M(D^{*0})$, $p = |\vec{p}|$, and $C$ is an overall constant related to the average value of the current $|\vec{j}|^2$. The specific value of the latter constant will not be essential in further calculation, since it cancels in the ratio of the cross sections. It should be pointed out that the nonrelativistic expression for the phase space is used in Eq. (8) corresponding to the nonrelativistic normalization of the states of the heavy mesons.

The discussed here absorptive part of the amplitude of the process $e^+e^- \rightarrow \gamma X(3872)$ due to the $D^{*0}\bar{D}^{*0}$ intermediate state is found from the unitarity relation and the amplitudes (4) and (7) in the standard way:

$$A_{Abs}(e^+e^- \rightarrow \gamma X) = \frac{1}{2} \sum_{pol} A(e^+e^- \rightarrow D^{*0}\bar{D}^{*0}) A(D^{*0}\bar{D}^{*0} \rightarrow \gamma X) 2\pi \delta \left(W - \frac{p^2}{m}\right) \frac{d^3p}{(2\pi)^3} = \mu a_0 \frac{pm}{2\omega^2} \sqrt{\frac{\kappa}{\pi}} F \epsilon_{ijk} \epsilon_i^* \epsilon_j^* \left[ \chi_k^* (\vec{j} \cdot \vec{k}) \left(A_0 - \frac{A_2}{\sqrt{5}}\right) + j_k (\vec{k} \cdot \vec{x}) \frac{3}{2} \frac{3}{\sqrt{5}} A_2 \right].$$

(9)

In the latter expression $\omega = |\vec{k}|$ is the energy of the photon, the sum goes over the polarizations of the vector mesons in the intermediate state, and $F$ stands for the dimensionless form factor:

$$F = \frac{1}{\sqrt{8\pi\kappa}} \int_{-1}^{1} (\vec{p} \cdot \vec{k}) \phi \left(\vec{p} - \frac{\vec{k}}{2}\right) d\cos \theta ,$$

(10)

where $\theta$ is the angle between the vectors $\vec{p}$ and $\vec{k}$. Using the expression (9) for the ‘peripheral’
wave function, one readily finds the form factor as

\[ F = \frac{c}{p\omega} \left( \frac{p^2 + \omega^2/4 + \kappa^2}{\sqrt{2}} \right) \ln \left( \frac{(p + \omega/2)^2 + \kappa^2}{(p - \omega/2)^2 + \kappa^2} \right) - \left( \frac{p^2 + \omega^2/4 + \Lambda^2}{\sqrt{2}} \right) \ln \left( \frac{(p + \omega/2)^2 + \Lambda^2}{(p - \omega/2)^2 + \Lambda^2} \right) \]

(11)

with the normalization coefficient \( c \) given by Eq.(6).

The absorptive part of the amplitude in Eq.(9) corresponds to the cross section

\[ \sigma_{\text{Abs}}(e^+e^- \to D^{*0}\bar{D}^{*0} \to \gamma X) = \int |A_{\text{Abs}}(e^+e^- \to \gamma X)|^2 \frac{2\pi}{2} \delta(\omega - |\vec{k}|) \frac{d^3k}{(2\pi)^3 2\omega} = C |\mu|^2 |a_0|^2 p^2 \omega \kappa F^2 \frac{12\pi^2}{|A_0 - A_2/\sqrt{3}|^2 + 9/20 |A_2|^2} \]

where the overall constant \( C \) in the latter expression is the same as in Eq.(8). Thus using also Eq.(3) one finds the formula for the ratio of the cross sections:

\[ \frac{\sigma_{\text{Abs}}(e^+e^- \to D^{*0}\bar{D}^{*0} \to \gamma X)}{\sigma(e^+e^- \to D^{*0}D^{*0})} = |a_0|^2 \frac{\Gamma_0 \frac{m\omega\kappa}{2\omega_0^3}}{p} F^2 \frac{12\pi^2}{|A_0 - A_2/\sqrt{3}|^2 + 9/20 |A_2|^2} \]

(12)

The so-defined cross section \( \sigma_{\text{Abs}} \) is (most likely) not the actual value of the cross section, since the amplitude of the process \( e^+e^- \to \gamma X \) can receive contribution from other mechanisms. Nevertheless, it is instructive to examine the numerical value and the behavior with energy of this quantity as given by Eq.(13). The dependence on the c.m. energy of the factor \((\kappa/p) F^2\) is shown in Fig.1 for two representative values of the ‘molecular’ binding energy in \(X(3872), w = 1\,\text{MeV} \quad (\kappa \approx 44\,\text{MeV})\) and \(w = 0.3\,\text{MeV} \quad (\kappa \approx 24\,\text{MeV})\). This factor peaks at the energy where \(p \approx \omega/2 \approx 70\,\text{MeV}\). The appearance of this peak is easily understood qualitatively: at \(\vec{p} \approx \vec{k}/2\) the \(D^0\) meson emerging from the emission of the photon in \(D^{*0} \to D^0\gamma\) moves slowly relative to the \(\bar{D}^{*0}\) and forms a loosely bound state.\(^2\) The width of the peak is clearly determined by the parameter \(\kappa\).

As is seen from the plots of Fig.1 the numerical value of the factor \((\kappa/p) F^2\) near its peak is of order one. Another factor in Eq.(13), \(\Gamma_0 m\omega/(2\omega_0^3) \approx \Gamma_0 m/(2\omega_0^3) \approx 1.5 \times 10^{-3}\), sets the overall scale of the discussed cross section. The factor in Eq.(13) depending on the presently unknown ratio of the (generally complex) amplitudes \(A_0/A_2\), takes values between 0.34 (at \(A_0/A_2 \approx 0.68\)) and 1.31 (at \(A_0/A_2 \approx 1.47\)), and can thus be considered as being of order one. Finally, the statistical weight factor \(|a_0|^2\), as discussed, is likely to be large fraction of one. Summarizing these numerical estimates, the value of the ratio in Eq.(13) at the peak can be estimated as being of order \(10^{-3}\), although the uncertainty is presently large.

\(^2\)The same situation arises at \(\vec{p} \approx \vec{k}/2\) for the \(D^0\) meson emerging from \(\bar{D}^{*0} \to \bar{D}^0\gamma\).
Figure 1: The factor $\kappa F^2/p$ vs. the excitation energy $W$ above the $D^{*0}\bar{D}^{*0}$ threshold at representative values of the binding energy $w$ in $X(3872)$ and the ultraviolet cutoff parameter $\Lambda$: $w = 1$ MeV, $\Lambda = 200$ MeV (solid), $w = 1$ MeV, $\Lambda = 300$ MeV (dashed), $w = 0.3$ MeV, $\Lambda = 200$ MeV (dashdot), and $w = 0.3$ MeV, $\Lambda = 300$ MeV (dotted).

In absolute terms, the measured\cite{15} cross section $\sigma(e^+e^- \rightarrow D^{*0}\bar{D}^{*0})$ at $E = 4015$ MeV, i.e. at the energy above the $D^{*0}\bar{D}^{*0}$ threshold $W \approx 1.6$ MeV is about $0.15$ nb. This cross section grows from the threshold as $p^3$. With this factor taken into account the peak of the quantity $\sigma_{\text{Abs}}(e^+e^- \rightarrow \gamma X)$ shifts to a slightly higher value of $p$, $p \approx 100$ MeV, corresponding to $W \approx 5$ MeV, where according to Eq.\cite{12} and the presented estimates, it should be numerically of the order of $1$ pb.

The considered mechanism of the process $e^+e^- \rightarrow \gamma X$ describes a ‘soft’ production of its peripheral $D^0\bar{D}^{*0} + D^{*0}\bar{D}^0$ component in radiative transitions from slow $D^{*0}\bar{D}^{*0}$ pairs. Generally, one can also expect a presence of states with charged mesons, $D^+D^{*-} + D^-D^{*+}$, within the $X$ resonance. Therefore a contribution of the mechanism $e^+e^- \rightarrow D^{*+}D^{*-} \rightarrow \gamma X$ to the considered here process merits discussion. However, this contribution in fact should be very small for at least three reasons\cite{13}: The mass of the pair of charged mesons is by $\delta \approx 8$ MeV heavier than that of the neutral ones. For this reason the charged mesons in this component are separated by shorter distances: the corresponding parameter $\kappa$ is
approximately $\sqrt{2m_r\delta} \approx 125$ MeV, and as a result the statistical weight of such state should be significantly smaller than for the pair of neutral mesons. Furthermore, the C-conjugate processes $D^{*+} \rightarrow D^+\gamma$ and $D^{*-} \rightarrow D^-\gamma$ destructively interfere in $D^{*+}D^{*-} \rightarrow \gamma X$ (cf. the minus sign between the two terms in Eq.\[4\]). The (negative) interference is enhanced for the more closely separated charged mesons in $X(3872)$. Finally, the transition magnetic coupling $\mu$ is noticeably weaker for the charged mesons than for the neutral ones: $\Gamma(D^{*+} \rightarrow D^+\gamma) = 1.5 \pm 0.5$ KeV.

Other intermediate states with charmed meson pairs, i.e. $D\bar{D}$ and $D\bar{D}^*$ ($\bar{D}D^*$), can potentially contribute to the discussed process $e^+e^- \rightarrow \gamma X$. Indeed, the suitable final state arises in the chain $e^+e^- \rightarrow D\bar{D} \rightarrow \gamma (D\bar{D}^* + D^*\bar{D})$ through the radiative transition $D \rightarrow \gamma D^*$ ($D \rightarrow \gamma D^*$) as well as in the chain $e^+e^- \rightarrow D\bar{D}^* + D\bar{D}^* \rightarrow \gamma (D\bar{D}^* + D^*\bar{D})$ through an elastic emission of a photon by $D^*$ ($\bar{D}^*$). However, one can readily see that in either of these processes the charmed meson emerging after the emission of the photon is very far off the mass shell in the scale of $\kappa$. Thus neither of these processes can proceed due to the long-distance peripheral component of the $X(3872)$ resonance, but rather is determined by the short-distance dynamics of the ‘core’ of $X$. For this reason these contributions, as well as other possible mechanisms related to the ‘core’ dynamics, should be smooth functions of the c.m. energy on the scale of few MeV around the $D^{*0}\bar{D}^{*0}$ threshold, where the amplitude given by Eq.\[9\] experiences a significant variation. Therefore even under the most conservative (and quite unlikely) assumption that these mechanisms cancel the contribution of the latter amplitude near its maximum, such cancellation cannot take place at all energies in the considered range. Thus the cross section of the process $e^+e^- \rightarrow \gamma X$ at an energy within few MeV of the $D^{*0}\bar{D}^{*0}$ threshold has to be at least as large as the above estimates for $\sigma_{Abs}$ near its maximum i.e. of the order of 1 pb. The latter is a conservative estimate, since we cannot exclude that the contribution of those ‘other’ mechanisms exceeds the calculated amplitude and that the actual cross section is larger than $\sigma_{Abs}$.

The work of MBV is supported, in part, by the DOE grant DE-FG02-94ER40823.

References

[1] W. M. Yao et al. [Particle Data Group], J. Phys. G 33, 1 (2006).

[2] K. Abe et.al., [Belle Coll], Report BELLE-CONF-0540, May 2005, [hep-ex/0505037].
[3] F.E. Close and P.R. Page, Phys.Lett.B 578, 119 (2004).

[4] S. Pakvasa and M. Suzuki, Phys.Lett.B. 579, 67 (2004).

[5] M.B. Voloshin, Phys.Lett.B. 579, 316 (2004).

[6] N.A. Törnqvist, Phys.Lett.B 590, 209 (2004); [hep-ph/0402237] and [hep-ph/0308277].

[7] M.B. Voloshin and L.B. Okun, JETP Lett. 23, 333 (1976).

[8] S. K. Choi et al. [Belle Collaboration], Phys. Rev. Lett. 91, 262001 (2003) [arXiv:hep-ex/0309032].

[9] B. Aubert et al. [BABAR Collaboration], Phys. Rev. D 73, 011101 (2006) [arXiv:hep-ex/0507090].

[10] D. Acosta et al. [CDF II Collaboration], Phys. Rev. Lett. 93, 072001 (2004) [arXiv:hep-ex/0312021].

[11] V. M. Abazov et al. [D0 Collaboration], Phys. Rev. Lett. 93, 162002 (2004) [arXiv:hep-ex/0405004].

[12] E. S. Swanson, Phys. Lett. B 588, 189 (2004) [arXiv:hep-ph/0311229].

[13] M. B. Voloshin, Int. J. Mod. Phys. A 21, 1239 (2006) [arXiv:hep-ph/0509192].

[14] M. Suzuki, Phys. Rev. D 72, 114013 (2005) [arXiv:hep-ph/0508258].

[15] R. Poling, eConf C060409, 005 (2006) [arXiv:hep-ex/0606016].

[16] S. Dubynskiy and M. B. Voloshin, [arXiv:hep-ph/0608179]

[17] M. B. Voloshin, eConf C060409, 014 (2006) [arXiv:hep-ph/0605063].