ADAMMC: A Model Checker for Petri Nets with Transits against Flow-LTL (Full Version)∗†

Bernd Finkbeiner¹, Manuel Gieseking², Jesko Hecking-Harbusch¹, and Ernst-Rüdiger Olderog²

¹ Saarland University, Saarbrücken, Germany {finkbeiner,hecking-harbusch}@react.uni-saarland.de
² University of Oldenburg, Oldenburg, Germany {gieseking,olderog}@informatik.uni-oldenburg.de

Abstract. The correctness of networks is often described in terms of the individual data flow of components instead of their global behavior. In software-defined networks, it is far more convenient to specify the correct behavior of packets than the global behavior of the entire network. Petri nets with transits extend Petri nets and Flow-LTL extends LTL such that the data flows of tokens can be tracked. We present the tool ADAMMC as the first model checker for Petri nets with transits against Flow-LTL. We describe how ADAMMC can automatically encode concurrent updates of software-defined networks as Petri nets with transits and how common network specifications can be expressed in Flow-LTL. Underlying ADAMMC is a reduction to a circuit model checking problem. We introduce a new reduction method that results in tremendous performance improvements compared to a previous prototype. Thereby, ADAMMC can handle software-defined networks with up to 82 switches.

1 Introduction

In networks, it is difficult to specify correctness in terms of the global behavior of the entire system. Instead, the individual flow of components is far more convenient to specify correct behavior. For example, loop and drop freedom can be easily specified for the flow of each packet. Petri nets and LTL lack this local view. Petri nets with transits and Flow-LTL have been introduced to overcome this restriction [10]. A transit relation is introduced to follow the flow induced by tokens. Flow-LTL is a temporal logic to specify both the local flow of data and the global behavior of markings. The global behavior as in Petri nets and LTL is still important for maximality and fairness assumptions. In this paper,

†This is the full version of [13].
∗This work was supported by the German Research Foundation (DFG) Grant Petri Games (392735815) and the Collaborative Research Center Foundations of Perspicuous Software Systems (TRR 248, 389792660), and by the European Research Council (ERC) Grant OSARES (683300).
we present the tool \textsc{AdamMC} as the first model checker for Petri nets with transits against Flow-LTL and its application to software-defined networking.

In Fig. 1 we present an example of a Petri net with transits that models the security check at an airport where passengers are checked by a security guard. The number of passengers entering the airport is unknown in advance. Rather than introducing the complexity of an infinite number of tokens, we use a fixed number of tokens to model possibly infinitely many flow chains. This is done by the transit relation which is depicted with colored arrows.

The left-hand side of Fig. 1 models passengers who want to reach the terminal. There are three tokens in the places \textit{airport}, \textit{queue}, and \textit{terminal}. Thus, transitions \textit{start} and \textit{en} are always enabled. Each firing of \textit{start} creates a new flow chain as depicted by the green arrow. This models a new person arriving at the airport. Meanwhile, the double-headed blue arrow maintains all flow chains that are still in place \textit{airport}. Passengers have to enter the \textit{queue} and wait until the security \textit{check} is performed. Therefore, transition \textit{en} continues every flow chain in \textit{airport} to \textit{queue}. Checking the passengers is carried out by transition \textit{check} which becomes enabled if the security guard \textit{work}s. Thus, passengers residing in \textit{queue} have to wait until the guard \textit{checks} them. Afterwards, they reach the \textit{terminal}. The security guard is modeled on the right-hand side of Fig. 1. By firing \textit{comeToWork} and thus moving the token in place \textit{home}, her flow chain starts and she can repeatedly either \textit{idle} or \textit{work}, \textit{check} passengers, and \textit{return}. Her transit relation is depicted in orange and models exactly one flow chain.

In Fig. 1 we define the checkpoints \textit{cp}$_1$ and \textit{cp}$_2$ and the \textit{booth} as a security zone and require that passengers never enter the security zone and eventually reach the \textit{terminal}. The flow formula $\varphi = A(airport \rightarrow (\square (\neg (cp_1 \lor cp_2 \lor booth) \land \Diamond \text{terminal})))$ specifies this. \textsc{AdamMC} verifies the example from Fig. 1 against the formula $\square \Diamond \text{check} \rightarrow \varphi$ specifying that if passengers are checked regularly then they cannot access the security zone and eventually reach the terminal.

In this paper, we present \textsc{AdamMC} as a full-fledged tool. First, \textsc{AdamMC} can handle Petri nets with transits and Flow-LTL formulas in general. Second, \textsc{AdamMC} has an input interface for a concurrent update and a software-defined network and encodes both of them as a Petri nets with transits. Common as-

\footnote{\textsc{AdamMC} is available online at \url{https://uol.de/en/csd/adammc}.}
sumptions on fairness and requirements for network correctness are also provided as Flow-LTL formulas. This allows users of the tool to model check the correctness of concurrent updates and to prevent packet loss, routing loops, and network congestion. Third, ADAMMC provides algorithms to check safe Petri nets against LTL with both places and transitions as atomic propositions which makes it especially easy to specify fairness and maximality assumptions.

The tool reduces the model checking problem for safe Petri nets with transits against Flow-LTL to the model checking problem for safe Petri nets against LTL. We develop the new parallel approach to check global and local behavior in parallel instead of sequentially. This approach yields a tremendous speed-up for a few local requirements and realistic fairness assumptions in comparison to the sequential approach of a previous prototype [10]. In general, the parallel approach has worst-case complexity inferior to the sequential approach even though the complexities of both approaches are the same when using only one flow formula.

As last step, ADAMMC reduces the model checking problem of safe Petri nets against LTL to a circuit model checking problem. This is solved by ABC [2,4] with effective verification techniques like IC3 and bounded model checking. ADAMMC verifies concurrent updates of software-defined networks with up to 38 switches (31 more than the prototype) and falsifies concurrent updates of software-defined networks with up to 82 switches (44 more than the prototype).

The paper is structured as follows: In Sec. 2, we recall Petri nets with transits and Flow-LTL. In Sec. 3, we outline the three application areas of ADAMMC: checking safe Petri nets with transits against Flow-LTL, checking concurrent updates of software-defined networks against common assumptions and specifications, and checking safe Petri nets against LTL. In Sec. 4, we algorithmically encode concurrent updates of software-defined networks in Petri nets with transits. In Sec. 5, we introduce the parallel approach for the underlying circuit model checking problem. In Sec. 6, we present our experimental evaluation.

2 Petri Nets With Transits and Flow-LTL

A safe Petri net with transits $\mathcal{N} = (\mathcal{P}, \mathcal{T}, \mathcal{F}, \text{In}, \mathcal{Y})$ [10] contains the set of places $\mathcal{P}$, the set of transitions $\mathcal{T}$, the flow relation $\mathcal{F} \subseteq (\mathcal{P} \times \mathcal{T}) \cup (\mathcal{T} \times \mathcal{P})$, and the initial marking $\text{In} \subseteq \mathcal{P}$ as in safe Petri nets [27]. In a safe Petri net, reachable markings contain at most one token per place. The transit relation $\mathcal{Y}$ is for every transition $t \in \mathcal{T}$ of type $\mathcal{Y}(t) \subseteq (\text{pre}^{-1}(t) \cup \{\epsilon\}) \times \text{post}^{+}(t)$. With $p \mathcal{Y}(t) q$, we define that firing transition $t$ transits the flow in place $p$ to place $q$. The symbol $\triangleright$ denotes a start and $\triangleright \mathcal{Y}(t) q$ defines that firing transition $t$ starts a new flow for the token in place $q$. Note that the transit relation can split, merge, and end flows. A sequence of flows leads to a flow chain which is a sequence of the current place and the fired outgoing transition. Thus, Petri nets with transits can describe both the global progress of tokens and the local flow of data.

Flow-LTL [10] extends Linear-time Temporal Logic (LTL) and uses places and transitions as atomic propositions. It introduces $\&$ as a new operator which uses LTL to specify the flow of data for all flow chains. For Fig. 1, the formula
4 B. Finkbeiner et al.

Fig. 2: Overview of the workflow of AdamMC: The application areas of the tool are given by three different input domains: software-defined network / Flow-LTL (Input I), Petri nets with transits / Flow-LTL (Input II), and Petri nets / LTL (Input III). AdamMC performs all unlabeled steps. MCHyper creates the final circuit which ABC checks to answer the initial model checking problem.

\( A(booth \rightarrow \Diamond \text{check}) \) specifies that the guard performs at least one check. We call formulas starting with \( A \) flow formulas. Formulas around flow formulas specify the global progress of tokens in the form of markings and fired transitions to formalize maximality and fairness assumptions. These formulas are called run formulas. Often, Flow-LTL formulas have the form run formula \( \rightarrow \) flow formula.

3 Application Areas

AdamMC consists of modules for three application areas: checking safe Petri nets with transits against Flow-LTL, checking concurrent updates of software-defined networks against common assumptions and specifications, and checking safe Petri nets against LTL. The general architecture and workflow of the model checking procedure is given in Fig. 2. AdamMC is based on the tool Adam [14].

Petri Nets with Transits Petri nets with transits follow the progress of tokens and the flow of data. Flow-LTL allows to specify requirements on both. For Petri nets with transits and Flow-LTL (Input II), AdamMC extends a parser for Petri nets provided by APT [30], provides a parser for Flow-LTL, and implements two reduction methods to create a safe Petri net and an LTL formula. The sequential approach is outlined in [10] and the parallel approach in Sec. 5.

Software-Defined Networks Concurrent updates of software-defined networks are the second application area of AdamMC. The tool automatically encodes an initially configured network topology and a concurrent update as a Petri net with transits. The concurrent update renews the forwarding table. We provide parsers for the network topology, the initial configuration, the concurrent update, and Flow-LTL (Input I). In Sec. 4, we present the creation of a Petri net with transits from the input and Flow-LTL formulas for common network properties like connectivity, loop freedom, drop freedom, and packet coherence.

Petri Nets AdamMC supports the model checking of safe Petri nets against LTL with both places and transitions as atomic propositions. It provides dedicated algorithms to check interleaving-maximal runs of the system. A run is
interleaving-maximal if a transition is fired whenever a transition is enabled. Furthermore, ADAMMC allows a concurrent view on runs and can check concurrency-maximal runs which demand that each subprocess of the system has to progress maximally rather than only the entire system. State-of-the-art tools like LoLA [32] and ITS-Tools [29] are restricted to interleaving-maximal runs and places as atomic propositions. For Petri net model checking (Input III), we allow Petri nets in APT and PNML format as input and provide a parser for LTL formulas.

The construction of the circuit in Aiger format [3] is defined in [11]. MCHyper [15] is used to create a circuit from a given circuit and an LTL formula. This circuit is given to ABC [2,4] which provides a toolbox of modern hardware verification algorithms like IC3 and bounded model checking to decide the initial model checking question. As output for all three modules, ADAMMC transforms a possible counterexample (CEX) from ABC into a counterexample to the Petri net (with transits) and visualizes the net with Graphviz and the dot language [9]. When no counterexample exists, ADAMMC verified the input successfully.

4 Verifying Updates of Software Defined Networks

We show how ADAMMC can check concurrent updates of realistic examples from software-defined networking (SDN) against typical specifications [19]. SDN [25,6] separates the data plane for forwarding packets and the control plane for the routing configuration. A central controller initiates updates which can cause problems like routing loops or packet loss. ADAMMC provides an input interface to automatically encode software-defined networks and concurrent updates of their configuration as Petri nets with transits. The tool checks requirements like loop and drop freedom to find erroneous updates before they are deployed.

4.1 Network Topology, Configurations, and Updates

A network topology \( T = (Sw, Con) \) is an undirected graph with switches as vertices and connections between switches as edges. Packets enter the network at ingress switches and they leave at egress switches. Forwarding rules are of the form \( x.fwd(y) \) with \( x, y \in Sw \). A concurrent update has the following syntax:

- switch update ::= \( \text{upd}(x.fwd(y/z)) \mid \text{upd}(x.fwd(y/-)) \mid \text{upd}(x.fwd(-/z)) \)
- sequential update ::= (update \( >> \) update \( >> \) ... \( >> \) update)
- parallel update ::= (update \( || \) update \( || \) ... \( || \) update)
- update ::= switch update \( || \) sequential update \( || \) parallel update

where a switch update can renew the forwarding rule of switch \( x \) from switch \( z \) to switch \( y \), introduce a new forwarding rule from switch \( x \) to switch \( y \), or remove an existing forwarding rule from switch \( x \) to switch \( z \).

4.2 Data Plane and Control Plane as Petri Net with Transits

For a network topology \( T = (Sw, Con) \), a set of ingress switches, a set of egress switches, an initial forwarding table, and a concurrent update, we show how data
and control plane are encoded as Petri net with transits. Switches are modeled by tokens remaining in corresponding places $s$ whereas the flow of packets is modeled by the transit relation $T$. Specific transitions $i_s$ model ingress switches where new data flows begin. Tokens in places of the form $x.fwd(y)$ configure the forwarding. Data flows are extended by firing transitions $(x,y)$ corresponding to configured forwarding without moving any tokens. Thus, we model any order of newly generated packets and their forwarding. Assuming that each existing direction of a connection between two switches is explicitly given in $Con$, we obtain Algorithm 1 which calls Algorithm 2 to obtain the control plane.

**Algorithm 1: Data plane**

**Input:** $T = (Sw, Con)$, ingress, forwarding, update

**Output:** Petri net with transits $N = (P, T, F, I, T)$ for update of topology $T$ with ingress and forwarding

1. Create empty $N = (P, T, F, I, T)$
2. For switch update $u \in SwU$
   - Add place $s$ to $P$
   - If $s \in ingress$
     1. Add transition $i_s$ to $T$
     2. Add $s$ to $pre(i_s)$, $post(i_s)$
     3. Add creating data flow $\uparrow T(i_s) \cdot s$ to $T$
     4. Add maintaining data flow $\downarrow T(i_s) \cdot s$ to $T$
3. For connection $(x, y) \in Con$
   - Add place $x.fwd(y)$ to $P$
   - If $x.fwd(y) \in forwarding$
     1. Add place $x.fwd(y)$ to $I$
     2. Add transition $(x, y)$ to $T$
     3. Add $x, y, x.fwd(y)$ to $pre((x, y))$
     4. Add $x.fwd(y)$ to $pre((x, y))$
     5. Add connecting data flow $x \uparrow T((x, y)) \cdot y$ to $T$
     6. Add maintaining data flow $y \downarrow T((x, y)) \cdot y$ to $T$
4. For parallel update $p \in PaU$
   - Add places $p^f, p^l$ to $P$
   - Add transitions $p^f, p^l$ to $T$
   - Add $p^f$ to $pre(p^f)$, $p^l$ to $post(p^l)$
5. For sub-update $u_i$ of $p$
   - Add $u_i^f$ to $post(p^f)$, $u_i^l$ to $pre(p^l)$

**Algorithm 2: Control plane**

**Input:** $T = (Sw, Con)$, update, $N$

**Output:** $N = (P, T, F, I, T)$

1. For switch update $u \in SwU$
   - If $u \neq t$
     1. Add $x.fwd(z)$ to $pre(u)$
   - If $y \neq t$
     1. Add $x.fwd(y)$ to $post(u)$
2. For sequential update $s \in SeU$
   - If $s = |s_1, \ldots, s_{|s|}|
     1. Add places $s^s, s^f$ to $P$
     2. For $i \in \{0, \ldots, |s|\}$
       1. Add transition $s^s$ to $T$
       2. If $i == 0$
          1. Add $s^s$ to $pre(s^s)$
        3. Else
          1. Add $s^f$ to $pre(s^f)$
          2. If $i = |s|$
             1. Add $s^f$ to $post(s^f)$
          3. Else
             1. Add $s_i$ to $post(s^f)$
3. For parallel update $p \in PaU$
   - Add places $p^f, p^l$ to $P$
   - Add transitions $p^f, p^l$ to $T$
   - Add $p^f$ to $pre(p^f)$, $p^l$ to $post(p^l)$
   - For sub-update $u_i$ of $p$
     1. Add $u_i^f$ to $post(p^f)$, $u_i^l$ to $pre(p^l)$

For the $update$, let $SwU$ be the set of switch updates in it, $SeU$ the set of sequential updates in it, and $PaU$ the set of parallel updates in it. Depending on $update$'s type, it is also added to the respective set. The subnet for the $update$ has an empty transit relation but moves tokens from and to places of the form $x.fwd(y)$. Tokens in these places correspond to the forwarding table. The order of the switch updates is defined by the nesting of sequential and parallel updates.
Fig. 3: Overview of the sequential approach: Each firing of a transition of the original net is split into first firing a transition in the subnet for the run formula and subsequently firing a transition in each subnet tracking a flow formula. The constructed LTL formula skips the additional steps with until operators.

The update is realized by a specific token moving through unique places of the form $u^s, u^f, s^s, s^f, p^s, p^f$ for start and finish of each switch update $u \in SwU$, each sequential update $s \in SeU$, and each parallel update $p \in PaU$. A parallel update temporarily increases the number of tokens and reduces it upon completion to one. Algorithm 2 defines the update behavior between start and finish places and connects finish and start places depending on the subexpression structure.

4.3 Assumptions and Requirements

We use the run formula $\Diamond \Box \text{pre}(t) \rightarrow \Box \Diamond t$ to assume weak fairness for every transition $t$ in our encoding $\mathcal{N}$. Transitions, which are always enabled after some point, are ensured to fire infinitely often. Thus, packets are eventually forwarded and the routing table is eventually updated. We use flow formulas to test specific requirements for all packets. Connectivity ($\mathbb{A}(\Diamond \bigvee_{s \in \text{egress}} s)$) ensures that all packets reach an egress switch. Packet coherence ($\mathbb{A}(\Box (\bigvee_{s \in \text{initial}} s) \lor \Box (\bigvee_{s \in \text{final}} s))$) tests that packets are either routed according to the initial or final configuration. Drop freedom ($\mathbb{A}(\Box (\bigwedge_{e \in \text{egress}} \neg e \rightarrow \bigvee_{f \in \text{Con}} f)$) forbids dropped packets whereas loop freedom ($\mathbb{A}(\Box (\bigwedge_{s \in Sw \setminus \text{egress}} s \rightarrow (s \cup \Box \neg s)))$) forbids routing loops. We combine run and flow formula into fairness $\rightarrow$ requirement.

5 Algorithms and Optimizations

Central to model checking a Petri net with transits $\mathcal{N}$ against a Flow-LTL formula $\varphi$ is the reduction to a safe Petri net $\mathcal{N}^>$ and an LTL formula $\varphi^>$. The infinite state space of the Petri net with transits due to possibly infinitely many flow chains is reduced to a finite state model. The key idea is to guess and track a violating flow chain for each flow subformula $\mathbb{A} \psi_i$, for $i \in \{1, \ldots, n\}$, and to only once check the equivalent future of flow chains merging into a common place.

AdamMC provides two approaches for this reduction: Fig. 3 and Fig. 4 give an overview of the sequential approach and the parallel approach, respectively. Both algorithms create one subnet $\mathcal{N}^>_i$ for each flow subformula $\mathbb{A} \psi_i$ to track the corresponding flow chain and have one subnet $\mathcal{N}^>_O$ to check the run part of the formula. The places of $\mathcal{N}^>_O$ are copies of the places in $\mathcal{N}$ such that the current state of the system can be memorized. The subnets $\mathcal{N}^>_i$ also consist of the original places of $\mathcal{N}$ but only use one token (initially residing on an additional
place) to track the current state of the considered flow chain. The approaches differ in how these nets are connected to obtain $\mathcal{N}^\triangleright$.

**Sequential Approach** The places in each subnet $\mathcal{N}^\triangleright_i$ are connected with one transition for each transit ($\mathcal{T}_{\mathcal{F}_i} = \bigcup_{t \in \mathcal{T}} \mathcal{Y}(t)$). An additional token iterates sequentially through the subnets to activate or deactivate the subnet. This allows each subnet to track a flow chain corresponding to firing a transition in $\mathcal{N}^\triangleright_i$. The formula $\varphi^\triangleright$ takes care of these additional steps by means of the until operator: In the run part of the formula, all steps corresponding to moves in a subnet $\mathcal{N}^\triangleright_i$ are skipped and, for each subformula $\mathcal{A}_i \psi_i$, all steps are skipped until the next transition of the corresponding subnet is fired which transits the tracked flow chain. This technique results in a polynomial increase of the size of the Petri net and the formula: $\mathcal{N}^\triangleright$ has $O(|\mathcal{N}| \cdot n + |\mathcal{N}|)$ places and $O(|\mathcal{N}|^3 \cdot n + |\mathcal{N}|)$ transitions and the size of $\varphi^\triangleright$ is in $O(|\mathcal{N}|^3 \cdot n \cdot |\varphi| + |\varphi|)$. We refer to [11] for formal details.

**Parallel Approach** The $n$ subnets are connected such that the current chain of each subnet is tracked simultaneously while firing an original transition $t \in \mathcal{T}$. Thus, there are $((|\mathcal{Y}(t)| + 1)^n$ transitions. Each of these transitions stands for exactly one combination of which subnet is tracking which (or no) transit. Hence, firing one transition of the original net is directly tracked in one step for all subnets. This significantly reduces the complexity of the run part of the constructed formula, because no until operator is needed to skip sequential steps. A disjunction over all transitions corresponding to an original transition suffices to ensure correctness of the construction. Transitions and next operators in the flow parts of the formula still have to be replaced by means of the until operator to ensure that the next step of the tracked flow chain is checked at the corresponding step of the global timeline of $\varphi^\triangleright$. In general, the parallel approach results in an exponential blow-up of the net and the formula: $\mathcal{N}^\triangleright$ has $O(|\mathcal{N}| \cdot n + |\mathcal{N}|)$ places and $O(|\mathcal{N}|^3 \cdot n + |\mathcal{N}|)$ transitions and the size of $\varphi^\triangleright$ is in $O(|\mathcal{N}|^3 \cdot n \cdot |\varphi| + |\varphi|)$. For the practical examples, however, the parallel approach allows for model checking Flow-LTL with few flow subformulas with a tremendous speed-up in comparison to the sequential approach. We refer to App. A for formal details.

**Optimizations** Various optimizations parameters can be applied to the model checking routine described in Sec. 3 to tweak the performance. Table 1 gives an overview of the major parameters. We found that the versions of the sequential and the parallel approach with inhibitor arcs to track flow chains are generally...
Table 1: Overview of optimization parameters of AdamMC: The three reduction steps depicted in the first column can each be executed by different algorithms. The first step allows to combine the optimizations of the first and second row.

| Step | Petri Net with Transits $\rightarrow$ Petri Net | sequential | parallel |
|------|-----------------------------------------------|------------|----------|
|      | 1) Petri Net with Transits $\rightarrow$ Petri Net | inhib. | act. token |
|      | 2) Petri Net $\rightarrow$ Circuit | inhibitor | act. token |
|      | 3) Circuit $\rightarrow$ Circuit | explicit | logarithmic |

faster than the versions without. Furthermore, the reduction step from a Petri net into a circuit with logarithmically encoded transitions had oftentimes better performance than the same step with explicitly encoded transitions. However, several possibilities to reduce the number of gates of the created circuit worsened the performance of some benchmark families and improved the performance of others. Consequently, all parameters are selectable by the user and a script is provided to compare different settings. An overview of the selectable optimization parameters can be found in the documentation of AdamMC [12]. Our main improvement claims can be retraced by the case study in Sec. 6.

6 Evaluation

We conduct a case study based on SDN with a corresponding artifact [16]. The performance improvements of AdamMC compared to the prototype [10] are summarized in Table 2. For realistic software-defined networks [19], one ingress and one egress switch are chosen at random. Two forwarding tables between the two switches and an update from the first to the second configuration are chosen at random. AdamMC verifies that the update maintained connectivity between ingress and egress switch. The results are depicted in rows starting with T. For rows starting with F, we required connectivity of a random switch which is not in the forwarding tables. AdamMC falsified this requirement for the update.

The prototype implementation based on an explicit encoding can verify updates of networks with 7 switches and falsify updates of networks with 38 switches. We optimize the explicit encoding to a logarithmic encoding and the number of switches for which updates can be verified increases to 17. More significantly, the parallel approach in combination with the logarithmic encoding leads to tremendous performance gains. The performance gains of an approach with inferior worst-case complexity are mainly due to the smaller complexity of the LTL formula created by the reduction. The encoding of SDN requires fairness assumptions for each transition. These assumptions (encoded in the run part of the formula) experience a blow-up with until operators by the sequential approach but only need a disjunction in the parallel approach. Hence, the size of networks for which AdamMC can verify updates increases to 38 switches and the size for which it can falsify updates increases to 82 switches. For rather small networks, the tool needs only a few seconds to verify and falsify updates which makes it a great option for operators when updating networks.
Table 2: We compare the explicit and logarithmic encoding of the sequential approach with the parallel approach. The results are the average over five runs from an Intel i7-2700K CPU with 3.50 GHz, 32 GB RAM, and a timeout (TO) of 30 minutes. The runtimes are given in seconds.

| T / F | Network     | #Sw | expl. enc. Alg. Time | log. enc. Alg. Time | parallel appr. Alg. Time |
|-------|-------------|-----|----------------------|---------------------|------------------------|
| T     | Arpanet196912 | 4   | IC3 12.08 ✔          | IC3 9.89 ✔          | IC3 2.18 ✔             |
| T     | Napnet      | 6   | IC3 146.49 ✔         | IC3 96.06 ✔         | IC3 4.75 ✔             |
|       |             |     | ...                  | ...                 | ...                    |
| T     | Heanet      | 7   | IC3 806.81 ✔         | IC3 84.62 ✔         | IC3 30.30 ✔            |
| T     | HiberniaIreland | 7 | - TO ?             | - TO ?             | IC3 26.58 ✔            |
| T     | Arpanet19706 | 9   | - TO ?             | IC3 362.21 ✔        | IC3 11.33 ✔            |
| T     | Nordu2005   | 9   | - TO ?             | - TO ?             | IC3 12.67 ✔            |
|       |             |     | ...                  | ...                 | ...                    |
| T     | Fatman      | 17  | - TO ?             | IC3 1543.34 ✔       | IC3 162.17 ✔           |
| T     | Myren       | 37  | - TO ?             | IC3 1309.23 ✔       | IC3 1261.32 ✔          |
| T     | KentmanJan2011 | 38 | - TO ?             | IC3 1469.34 ✔       | IC3 1742.17 ✔          |
| F     | Arpanet196912 | 4  | BMC3 2.18 x         | BMC3 1.85 ✔         | BMC3 1.97 ✔            |
| F     | Napnet      | 6   | BMC2 4.17 x         | BMC2 5.22 x         | BMC3 1.48 x            |
|       |             |     | ...                  | ...                 | ...                    |
| F     | Fatman      | 17  | BMC3 168.78 x        | BMC3 169.82 x       | BMC3 6.72 x            |
| F     | Belnet2009  | 21  | BMC2 1146.26 x       | BMC2 611.81 x       | BMC3 24.26 x           |
|       |             |     | ...                  | ...                 | ...                    |
| F     | KentmanJan2011 | 38 | BMC3 167.92 x        | BMC3 86.44 x        | BMC2 9.35 x            |
|       |             |     | ...                  | ...                 | ...                    |
| F     | Latnet      | 69  | - TO ?             | - TO ?             | BMC2 209.20 x          |
| F     | Ulaknet     | 82  | - TO ?             | - TO ?             | BMC2 1043.74 x         |

Sum of runtimes (in hours): 82.99 79.15 30.31
Nb of TOs (of 230 exper.): 146   138  6

7 Related Work

We refer to [21] for an introduction to SDN. Solutions for correctness of updates of software-defined networks include consistent updates [28,17], dynamic scheduling [17], and incremental updates [18]. Both explicit and SMT-based model checking [5,22,23,31,1,26] is used to verify software-defined networks. Closest to our approach are models of networks as Kripke structures to use model checking for synthesis of correct network updates [8,24]. The model checking subroutine of the synthesizer assumes that each packet sees at most one updated switch. Our model checking routine does not make such an assumption.

There is a significant number of model checking tools (e.g., [32,35]) for Petri nets and an annual model checking contest [20]. ADAMMC is restricted to safe Petri nets whereas other tools can handle bounded and colored Petri nets. At the same time, only ADAMMC accepts LTL formulas with places and transitions as atomic propositions. This is essential to express fairness in our SDN encoding.
8 Conclusion

We presented the tool ADAMC with its three application domains: checking safe Petri nets with transits against Flow-LTL, checking concurrent updates of software-defined networks against common assumptions and specifications, and checking safe Petri nets against LTL. New algorithms allow ADAMC to model check software-defined networks of realistic size: it can verify updates of networks with up to 38 switches and can falsify updates of networks with up to 82 switches.

References

1. Ball, T., Bjørner, N., Gember, A., Itzhaky, S., Karbyshev, A., Sagiv, M., Schapira, M., Valadarsky, A.: Vericon: towards verifying controller programs in software-defined networks. In: Proceedings of PLDI. pp. 282–293 (2014), http://doi.acm.org/10.1145/2594291.2594317
2. Berkeley Logic Synthesis and Verification Group: ABC: A system for sequential synthesis and verification, http://www.eecs.berkeley.edu/~alanmi/abc/, version 1.01 81030
3. Biere, A., Heljanko, K., Wieringa, S.: AIGER 1.9 and beyond. Tech. rep., Johannes Kepler University, Linz (2011)
4. Brayton, R.K., Mishchenko, A.: ABC: an academic industrial-strength verification tool. In: Proceedings of CAV. pp. 24–40 (2010), https://doi.org/10.1007/978-3-642-14295-6_5
5. Canini, M., Venzano, D., Peresini, P., Kostic, D., Rexford, J.: A NICE way to test openflow applications. In: Proceedings of NSDI. pp. 127–140 (2012), https://www.usenix.org/conference/nsdi12/technical-sessions/presentation/canini
6. Casado, M., Foster, N., Guha, A.: Abstractions for software-defined networks. Commun. ACM 57(10), 86–95 (2014), http://doi.acm.org/10.1145/2661061.2661063
7. Cerný, P., Foster, N., Jagani, N., McClurg, J.: Optimal consistent network updates in polynomial time. In: Proceedings of DISC. pp. 114–128 (2016), https://doi.org/10.1007/978-3-662-53426-7_9
8. El-Hassany, A., Tsankov, P., Vanbever, L., Vechev, M.T.: Network-wide configuration synthesis. In: Proceedings of CAV. pp. 261–281 (2017), https://doi.org/10.1007/978-3-319-63390-9_14
9. Ellison, J., Gansner, E.R., Koutsofios, E., North, S.C., Woodhull, G.: Graphviz and dynagraph - static and dynamic graph drawing tools. In: Graph Drawing Software, pp. 127–148. Springer (2004). https://doi.org/10.1007/978-3-642-18638-7_6
10. Finkbeiner, B., Giese, M., Hecking-Harbusch, J., Olderog, E.: Model checking data flows in concurrent network updates. In: Proceedings of ATVA. pp. 515–533 (2019), https://doi.org/10.1007/978-3-030-31784-3_30
11. Finkbeiner, B., Giese, M., Hecking-Harbusch, J., Olderog, E.: Model checking data flows in concurrent network updates (full version). Tech. rep. (2019), http://arxiv.org/abs/1907.11061
12. Finkbeiner, B., Giese, M., Hecking-Harbusch, J., Olderog, E.: ADAMC – A Model Checker for Petri Nets with Transits against Flow-LTL. University of Oldenburg and Saarland University, https://uol.de/en/csd/adammc (2020)
13. Finkbeiner, B., Giese, M., Hecking-Harbusch, J., Olderog, E.: ADAMC: A model checker for petri nets with transits against Flow-LTL. In: Proceedings of CAV (2020)
14. Finkbeiner, B., Gieseking, M., Olderog, E.: Adam: Causality-based synthesis of distributed systems. In: Proceedings of CAV. pp. 433–439 (2015), https://doi.org/10.1007/978-3-319-21690-4_25

15. Finkbeiner, B., Rabe, M.N., Sánchez, C.: Algorithms for model checking HyperLTL and HyperCTL*. In: Proceedings of CAV. pp. 30–48 (2015), https://doi.org/10.1007/978-3-319-21690-4_3

16. Gieseking, M., Hecking-Harbusch, J.: AdamMC: A Model Checker for Petri Nets with Transits against Flow-LTL (Artifact) (2020), https://doi.org/10.6084/m9.figshare.11676171

17. Jin, X., Liu, H.H., Gandhi, R., Kundula, S., Mahajan, R., Zhang, M., Rexford, J., Wattenhofer, R.: Dynamic scheduling of network updates. In: Proceedings of SIGCOMM. pp. 539–550 (2014), https://doi.org/10.1145/2619239.2626307

18. Katta, N.P., Rexford, J., Walker, D.: Incremental consistent updates. In: Proceedings of HotSDN. pp. 49–54 (2013), https://doi.org/10.1145/2491185.2491191

19. Knight, S., Nguyen, H.X., Falkner, N., Bowden, R.A., Roughan, M.: The internet topology zoo. IEEE Journal on Selected Areas in Communications 29(9), 1765–1775 (2011), https://doi.org/10.1109/JSAC.2011.1111002

20. Kordon, F., Garavel, H., Hillah, L.M., Hulin-Hubard, F., Amparore, E., Beccuti, M., Berthomieu, B., Ciardo, G., Dal Zilio, S., Liebke, T., Li, S., Meijer, J., Miner, A., Srba, J., Thierry-Mieg, Y., van de Pol, J., van Dirk, T., Wolf, K.: Complete Results for the 2019 Edition of the Model Checking Contest. http://mcc.lip6.fr/2019/results.php (April 2019)

21. Kreutz, D., Ramos, F.M.V., Veríssimo, P.J.E., Rothenberg, C.E., Azodolmolky, S., Uhlig, S.: Software-defined networking: A comprehensive survey. Proceedings of the IEEE 103(1), 14–76 (2015), https://doi.org/10.1109/JPROC.2014.2371999

22. Mai, H., Khurshid, A., Agarwal, R., Caesar, M., Godfrey, B., King, S.T.: Debugging the data plane with anteater. In: Proceedings of SIGCOMM. pp. 290–301 (2011), https://doi.org/10.1145/2018436.2018470

23. Majumdar, R., Tetali, S.D., Wang, Z.: Kuai: A model checker for software-defined networks. In: Proceedings of FMCAD. pp. 163–170 (2014), https://doi.org/10.1109/FMCAD.2014.6987609

24. McClurg, J., Hojjat, H., Cerný, P.: Synchronization synthesis for network programs. In: Proceedings of CAV. pp. 301–321 (2017), https://doi.org/10.1007/978-3-319-63990-9_16

25. McKeown, N., Anderson, T., Balakrishnan, H., Parulkar, G.M., Peterson, L.L., Rexford, J., Shenker, S., Turner, J.S.: Openflow: enabling innovation in campus networks. Computer Communication Review 38(2), 69–74 (2008), http://doi.acm.org/10.1145/1355734.1355746

26. Padon, O., Immerman, N., Karbyshev, A., Lahav, O., Sagiv, M., Shoham, S.: Decentralizing SDN policies. In: Proceedings of POPL. pp. 663–676 (2015), https://doi.org/10.1145/2676726.2676990

27. Reisig, W.: Petri Nets: An Introduction. Springer (1985), https://doi.org/10.1007/978-3-642-69968-9

28. Reitblatt, M., Foster, N., Rexford, J., Schlesinger, C., Walker, D.: Abstractions for network update. In: Proceedings of SIGCOMM. pp. 323–334 (2012), http://doi.acm.org/10.1145/2342356.2342427

29. Thierry-Mieg, Y.: Symbolic model-checking using ITS-tools. In: Proceedings of TACAS. pp. 231–237 (2015), https://doi.org/10.1007/978-3-662-46681-0_20

30. University of Oldenburg: APT – Analyse von Petri-Netzen und Transitionssystemen. https://github.com/CvO-Theory/apt (2012)
Appendix

A Technical Details

In this part of the appendix, details of the parallel approach, i.e., the construction of the Petri net $N^>$ and the construction of the LTL formula $\varphi^>$, are given.

A.1 Construction of the Net Transformation (Parallel Approach)

Let $\mathbb{ID}$ be a set of unique identifiers and $\nu_N: \mathcal{P} \cup \mathcal{T} \to \mathbb{ID}$ an injective naming function which uniquely identifies every place and transition of a given Petri net $N$ (or of a Petri net with transits). We omit the subscript if the net is clear from the context. To keep the presentation clear, we often directly use identifier for a node $n \in \mathcal{P} \cup \mathcal{T}$ with $\nu(n) = \text{identifier}$.

The construction of a Petri net with transits to a standard P/T Petri net is given by the following definition.

**Definition 1 (Petri Net with Transits to a P/T Petri Net).** For a Petri net with transits $N = (\mathcal{P}, \mathcal{T}, F, \text{In}, \Upsilon)$ and a Flow-LTL formula $\varphi$ with $n$ subformulas, a Petri net $N^> = (\mathcal{P}^>, \mathcal{T}^>, F^>, F_I^>, \text{In}^>)$ with inhibitor arcs (denoted by $F_I^>$) and a labeling function $\lambda: \mathcal{T}^> \to \mathcal{T}$ are defined as follows:

1. The places of the original net $N$ are copied $n + 1$ times:
   $$\mathcal{P}^> = \mathcal{P} \cup \bigcup_{\{1, \ldots, n\}} ([\iota]\_i \cup \{[p]\_i \mid p \in \mathcal{P}\})$$

2. For every transition $t \in \mathcal{T}$ and every combination of which subnet is tracking which transit (or no transit with marker $\circ$), there is one transition in $N^>$. Each transition is connected to the original part of the net according to $t$. For the subnet part, if it either a) moves the token from the initial place according to the transit, b) moves the token from the corresponding ingoing place of the transition according to the transit, or, in the case that the subnet is not involved in any of the transits, c) is connected by inhibitor arcs to all ingoing places of the transition $t$.

   $$\forall t \in \mathcal{T}: \forall c = ((x_1, p_1), \ldots, (x_n, p_n)) \in (\Upsilon(t) \cup \{\circ\})^n: \exists t^> \in \mathcal{T}^>: (p, t) \in \mathcal{F} \land \nu(t^>) = \nu(t) \land \lambda(t^>) = \nu(t) \land$$

   $$\forall i \in \{1, \ldots, n\}: x_i = \triangleright \Rightarrow ([x_i]\_i \_t^>, [p_i]\_i) \in \mathcal{F}^> \land$$

   $$x_i, p_i \in \mathcal{P} \Rightarrow ([x_i]\_i \_t^>, [p_i]\_i) \in \mathcal{F}^> \land$$

   $$(x_i, p_i) = \circ \Rightarrow (p, t) \in \mathcal{F} : ([p_i]\_i \_t^>) \in \mathcal{F}_I^>$$
14 B. Finkbeiner et al.

Fig. 5: A possible sequence of the global timeline $\tau$ and the timelines of the possible infinite number of flow chains $\beta_i$. A filled time step for a timeline of a flow chain indicates that the fired transition has a transit which extends this flow chain.

(I) The initial marking is given by $I^{\tau} = I^n \cup \{[\iota]_i \mid i \in \{1, \ldots, n\}\}$. The sets $F^\tau$, $F^\tau_A$, and $F^\tau_I$ are defined as the smallest sets fulfilling condition (t). The identifiers with the square brackets and those with a combination $\iota$ in their index are fresh identifiers.

The results regarding the size of the constructed net directly follow from the definition and that there are $|P| \cdot |T| \cdot |P| + |T| \cdot |P|$ transits in the worst-case.

Lemma 1 (Size of the Constructed Net). The constructed Petri net $N^\tau$ has $\mathcal{O}(|N| \cdot n + |N|)$ places and $\mathcal{O}(|N|^3 n + |N|)$ transitions.

A.2 Construction of the Formula Transformation (Parallel Approach)

We create an LTL formula $\varphi^\tau$ to the Petri net $N^\tau$ (created by Def. 1 of a Petri net with transits $N = (P, T, F, I^n, \tau)$) from a Flow-LTL formula $\varphi$ with $n \in \mathbb{N}$ flow subformulas $\varphi_{F_i} = \text{a} \psi_i$.

The intricate part of the construction is to deal with the different timelines. On the one hand, there is the global timeline of the Petri net $N$. This timeline can be used to check the run part of the formula. On the other hand, there are the different timelines of the possible infinite flow chains. For the flow chains, the global steps not concerning the chain have to be adequately skipped with until operators. Figure 5 gives an overview of a possible sequence of different timelines.

We define the set of transitions tracking a chain of a specific subnet $i \in \{1, \ldots, n\}$ by $T_i^\tau = \{t \in T^\tau \mid \exists p \in P : (\{p\}_i, t) \in F^\tau \lor (t, \{p\}_i) \in F^\tau \}$ and the set of all other transitions by $O_i = T^\tau \setminus T_i^\tau$. For a transition $t \in T$, the set $M_i(t) = T_i^\tau \cap \{t^\tau \in T^\tau \mid \lambda(t^\tau) = t\}$ collects all corresponding transitions tracking a chain of the subnet.

First, the places of the flow subformulas have to be substituted by the corresponding places tracking the chain, i.e., all occurrences of a place $p \in P$ in a
flow subformula \( \varphi_F \) are simultaneously replaced by \([p]_i\). Second, the transitions of the flow subformulas have to be substituted such that all steps of the global timeline which do not involve the tracked flow chain are skipped until a transition involving the flow chain is fired, i.e., all occurrences of a transition \( t \in \mathcal{T} \) in a flow subformula \( \varphi_F \) are simultaneously substituted by \((\bigvee_{t_o \in O_t} t_o) \cup (\bigvee_{t_m \in M(t)} t_m)\).

Similarly, the next operator of the flow subformulas have to be substituted such that the steps of the global timeline are skipped until a step involving the tracking subnet is taken. Here two cases have to be considered: either the chain ends, i.e., no transition of the subnet is ever fired again, then the formula has to directly hold in the stuttering part, or there is a transition of the subnet, then the formula has to hold in the direct successor state. This means all occurrences of a subformula \( \bigcirc \phi \) in a flow subformula \( \varphi_F \) are replaced from the innermost to the outermost occurrence by \(((\bigvee_{t \in O_t} t) \cup ((\bigvee_{t \in \mathcal{T}> t} \bigwedge \bigcirc \phi)) \lor (\Box (\neg(\bigvee_{t \in \mathcal{T}> t})) \land \phi)\).

For the run part of the formula, we can directly use the global timeline, i.e., the next operator needs no substitution. Further, the places are already correctly named. Only the transitions \( t \in \mathcal{T} \) in the run part of \( \varphi \) have to be substituted simultaneously by \( \bigvee_{t \in \mathcal{T}> t} t' \) to allow for all transitions corresponding to \( t \).

Finally, the flow subformulas are simultaneously substituted by \(([i]_i \bigvee (\neg[i]_i) \land \psi_t')\) (where \( \psi_t' \) is the result of the above mentioned substitutions within a flow subformula) such that all steps of the global timeline are skipped until a flow chain is created and tracked. Table 3 gives an overview of these substitutions.

Table 3: An overview of the necessary substitutions to create \( \varphi^> \) from \( \varphi \). The next operator is substituted from the innermost to the outermost occurrence, the other subformulas are substituted simultaneously.

| Run part of \( \varphi \) | Flow subformula \( \bigvee \psi_t \) part of \( \varphi \) |
|--------------------------|---------------------------------|
| \( p \in \mathcal{P} \) | \([p]_i\) |
| \( t \in \mathcal{T} \) | \( (\bigvee_{t_o \in O_t} t_o) \cup (\bigvee_{t_m \in M(t)} t_m)\) |
| \( \bigcirc \phi \) | \( (\bigvee_{t \in O_t} t) \cup ((\bigvee_{t \in \mathcal{T}> t} \bigwedge \bigcirc \phi)) \lor (\Box (\neg(\bigvee_{t \in \mathcal{T}> t})) \land \phi)\) |
| \( \bigwedge \psi_t \) | \( [i]_i \bigvee (\neg[i]_i) \land \psi_t'\) |

The size of the constructed formula directly results from the blow-up of the number of transition during the creation of \( \mathcal{N}^> \) and the substitutions introducing the disjunctions over these transition in the creation of \( \varphi^> \).

**Lemma 2 (Size of the Constructed Formula).** The size of the constructed LTL formula \( \varphi^> \) is in \( \mathcal{O}(|\mathcal{N}|^3n \cdot |\varphi| + |\varphi|)\).

Note that there is only a significant blow-up in the formula when transitions are used as atomic propositions in either the flow or the run part of the formula or when the next operator is used in the flow part of the formula. Moreover, even the usage of transitions as atomic propositions in the run part of the formula only results in a blow-up by all combinations of transits of this transition regarding the subnets. In practical applications, this makes a huge difference compared to the sequential approach, because model checking is exponential in the size of the
formula and many examples need fairness assumptions, i.e., transitions in the run part of the formula, and have only few local requirements.

The proof of the correctness of the transformations for the parallel approach is very similar to the one of the sequential approach presented in [11]. We again can mutually transform the counterexample to show the contraposition $N \nvdash \varphi$ iff $N^\triangleright \nvdash \text{LTL } \varphi^\triangleright$. Here we do not have to pump up the firing sequence serving as counterexample for $N \models \varphi$, but have to replace each transition by a transition which adequately extends all flow chains of the counterexample. For the other direction, we can replace the transitions of the counterexample by the labels of the transitions and, analog to the sequential approach, iteratively concatenate the transitions and places of the subnets to gain the flow chains serving as counterexamples for the subformula part. The complicated parts of the structural induction, i.e., adequately skipping the global time steps for the flow subformulas, can be done analogously because the formulas of the parallel approach and the sequential approach are similar in this case and fit to the different structure of the net.

B Complete Results

Table 4: We compare the explicit and the logarithmic encoding of the sequential approach with the parallel approach. The results are the average over 5 runs from an Intel i7-2700K CPU with 3.50 GHz, 32 GB RAM, and a timeout of 30 minutes. We report the runtimes of IC3 to verify (T) updates of software-defined networks and the runtimes of both BMC2 and BMC3 to falsify (F) updates of software-defined networks all with respect to connectivity between randomly chosen ingress and egress switches and forwarding tables.

| T / F | Network         | #Sw | expl. enc. Alg. | Time $\models$ | log. enc. Alg. | Time $\models$ | parallel appr. Alg. | Time $\models$ |
|-------|-----------------|-----|-----------------|---------------|----------------|---------------|---------------------|---------------|
| T     | Arpanet196912   | 4   | IC3            | 12.0760 ✓     | IC3            | 9.8872 ✓      | IC3                 | 2.1760 ✓      |
| T     | Napnet          | 6   | IC3            | 146.4920 ✓    | IC3            | 96.0640 ✓     | IC3                 | 4.7448 ✓      |
| T     | Epoch           | 6   | IC3            | 240.5720 ✓    | IC3            | 214.6960 ✓    | IC3                 | 6.7800 ✓      |
| T     | Telecomserbia   | 6   | IC3            | 1182.4320 ✓   | IC3            | 912.7560 ✓    | IC3                 | 12.1232 ✓     |
| T     | Layer42         | 6   | IC3            | 133.1992 ✓    | IC3            | 131.6824 ✓    | IC3                 | 6.2624 ✓      |
| T     | Dataxchange     | 6   | -              | TO ?          | IC3            | 380.1976 ✓    | IC3                 | 19.9968 ✓     |
| T     | Sanren          | 7   | IC3            | 304.6368 ✓    | IC3            | 437.0128 ✓    | IC3                 | 16.6776 ✓     |
| T     | Getnet          | 7   | IC3            | 940.4160 ✓    | IC3            | 103.0960 ✓    | IC3                 | 11.0480 ✓     |
| T     | Netrail         | 7   | IC3            | 171.5952 ✓    | IC3            | 531.5576 ✓    | IC3                 | 31.9800 ✓     |
| T     | Heanet          | 7   | IC3            | 806.8144 ✓    | IC3            | 84.6160 ✓     | IC3                 | 30.3008 ✓     |
| T     | HiberniaIreland | 7   | -              | TO ?          | -              | TO ?          | IC3                 | 26.5824 ✓     |
| T     | Arpanet19706    | 9   | -              | TO ?          | IC3            | 362.2056 ✓    | IC3                 | 11.3304 ✓     |
| T     | Nordu2005       | 9   | -              | TO ?          | -              | TO ?          | IC3                 | 12.6688 ✓     |
| T     | Nsfcnet         | 10  | -              | TO ?          | -              | TO ?          | IC3                 | 5.5448 ✓      |
| T | Sprint | 11 | - | TO ? | - | TO ? | IC3 | 745.7408 ✓ |
| T | TLex  | 12 | - | TO ? | - | TO ? | IC3 | 17.1296 ✓ |
| T | Compuserve | 13 | - | TO ? | - | TO ? | IC3 | 107.9464 ✓ |
| T | Enet  | 13 | - | TO ? | - | TO ? | IC3 | 40.3456 ✓ |
| T | HiberniaCanada | 13 | - | TO ? | - | TO ? | IC3 | 107.6000 ✓ |
| T | Naviga | 13 | - | TO ? | - | TO ? | IC3 | 360.5248 ✓ |
| T | Nsnet | 13 | - | TO ? | - | TO ? | IC3 | 181.3240 ✓ |
| T | Uninet | 13 | - | TO ? | - | TO ? | IC3 | 1336.1420 ✓ |
| T | Eunetworks | 14 | - | TO ? | - | TO ? | IC3 | 80.3952 ✓ |
| T | Ilan | 14 | - | TO ? | - | TO ? | IC3 | 137.8408 ✓ |
| T | Claranet | 15 | - | TO ? | - | TO ? | IC3 | 128.0024 ✓ |
| T | HiberniaUk | 15 | - | TO ? | - | TO ? | IC3 | 184.5888 ✓ |
| T | Spiralight | 15 | - | TO ? | - | TO ? | IC3 | 153.2312 ✓ |
| T | Garr199901 | 16 | - | TO ? | - | TO ? | IC3 | 164.5248 ✓ |
| T | KentmanJul2005 | 16 | - | TO ? | - | TO ? | IC3 | 163.2448 ✓ |
| T | Marwan | 16 | - | TO ? | - | TO ? | IC3 | 136.5992 ✓ |
| T | Peer1 | 16 | - | TO ? | - | TO ? | IC3 | 357.0224 ✓ |
| T | Rhnet | 16 | - | TO ? | - | TO ? | IC3 | 62.6520 ✓ |
| T | Fatman | 17 | - | TO ? | - | TO ? | IC3 | 1543.3360 ✓ |
| T | Nextgen | 17 | - | TO ? | - | TO ? | IC3 | 403.3296 ✓ |
| T | Nordu2010 | 18 | - | TO ? | - | TO ? | IC3 | 50.1136 ✓ |
| T | Pacificwave | 18 | - | TO ? | - | TO ? | IC3 | 932.5960 ✓ |
| T | Ans | 18 | - | TO ? | - | TO ? | IC3 | 1511.3020 ✓ |
| T | Arpanet19719 | 18 | - | TO ? | - | TO ? | IC3 | 840.6400 ✓ |
| T | BsonetEurope | 18 | - | TO ? | - | TO ? | IC3 | 496.2936 ✓ |
| T | HiberniaNireland | 18 | - | TO ? | - | TO ? | IC3 | 229.0768 ✓ |
| T | Noel | 19 | - | TO ? | - | TO ? | IC3 | 402.8256 ✓ |
| T | Restena | 19 | - | TO ? | - | TO ? | IC3 | 698.4024 ✓ |
| T | Savvis | 19 | - | TO ? | - | TO ? | IC3 | 1382.3480 ✓ |
| T | Twaren | 20 | - | TO ? | - | TO ? | IC3 | 1167.9080 ✓ |
| T | Janetlense | 20 | - | TO ? | - | TO ? | IC3 | 730.6448 ✓ |
| T | BtAsiaPac | 20 | - | TO ? | - | TO ? | IC3 | 1311.1100 ✓ |
| T | Oxford | 20 | - | TO ? | - | TO ? | IC3 | 678.3344 ✓ |
| T | Harnet | 21 | - | TO ? | - | TO ? | IC3 | 347.0536 ✓ |
| T | Belnet2009 | 21 | - | TO ? | - | TO ? | IC3 | 1604.5860 ✓ |
| T | GtsRomania | 21 | - | TO ? | - | TO ? | IC3 | 236.7912 ✓ |
| T | Packetexchange | 21 | - | TO ? | - | TO ? | IC3 | 688.5176 ✓ |
| T | Garr200404 | 22 | - | TO ? | - | TO ? | IC3 | 184.8440 ✓ |
| T | Belnet2010 | 22 | - | TO ? | - | TO ? | IC3 | 346.6064 ✓ |
| T | Garr200109 | 22 | - | TO ? | - | TO ? | IC3 | 1499.7620 ✓ |
| T | KentmanApr2007 | 22 | - | TO ? | - | TO ? | IC3 | 429.7848 ✓ |
| T         | Istar 23 | - TO ? | - TO ? | IC3 169.0848 ✓ | T         | Garr199905 23 | - TO ? | - TO ? | IC3 440.1864 ✓ | T         | Garr199904 23 | - TO ? | - TO ? | IC3 590.0240 ✓ | T         | Cesnet2001 23 | - TO ? | - TO ? | IC3 308.9808 ✓ | T         | Fccn 23 | - TO ? | - TO ? | IC3 752.9816 ✓ | T         | Uran 24 | - TO ? | - TO ? | IC3 82.6056 ✓ | T         | Garr200112 24 | - TO ? | - TO ? | IC3 1731.2440 ✓ |
| T         | Psinet 24 | - TO ? | - TO ? | IC3 226.0680 ✓ | T         | Arpanet199723 25 | - TO ? | - TO ? | IC3 218.3872 ✓ | T         | Vinaren 25 | - TO ? | - TO ? | IC3 1084.5340 ✓ | T         | KentmanFeb2008 26 | - TO ? | - TO ? | IC3 421.7424 ✓ | T         | Garr200212 27 | - TO ? | - TO ? | IC3 1205.4020 ✓ | T         | Bbnplanet 27 | - TO ? | - TO ? | IC3 896.9776 ✓ | T         | Darkstrand 27 | - TO ? | - TO ? | IC3 1466.4960 ✓ | T         | KentmanAug2005 28 | - TO ? | - TO ? | IC3 278.9248 ✓ | T         | Myren 37 | - TO ? | - TO ? | IC3 1309.2280 ✓ | T         | KentmanJan2011 38 | - TO ? | - TO ? | IC3 1261.3220 ✓ |
| F         | Arpanet196912 4 | BMC2 2.3528 × BMC2 2.0952 × BMC2 1.1992 × | F         | Arpanet196912 4 | BMC3 2.1768 × BMC3 1.8528 × BMC3 1.1968 × | F         | Napnet 6 | BMC2 4.1688 × BMC2 5.2240 × BMC2 1.6408 × | F         | Napnet 6 | BMC3 5.7072 × BMC3 5.4368 × BMC3 1.4808 × | F         | Epoch 6 | BMC2 20.7584 × BMC2 14.3200 × BMC2 2.6328 × | F         | Epoch 6 | BMC3 15.4112 × BMC3 13.5632 × BMC3 2.3912 × | F         | Telecomserbia 6 | BMC2 45.1120 × BMC2 39.7160 × BMC2 13.2600 × | F         | Telecomserbia 6 | BMC3 37.5104 × BMC3 41.4688 × BMC3 12.8704 × | F         | Layer42 6 | BMC2 9.4880 × BMC2 11.8768 × BMC2 2.0744 × | F         | Layer42 6 | BMC3 11.4400 × BMC3 6.7544 × BMC3 2.5560 × | F         | Sanren 7 | BMC2 64.8976 × BMC2 134.8184 × BMC2 8.2312 × | F         | Sanren 7 | BMC3 173.9256 × BMC3 81.2960 × BMC3 3.8832 × | F         | Getnet 7 | BMC2 7.3792 × BMC2 9.2480 × BMC2 1.6872 × | F         | Getnet 7 | BMC3 7.5144 × BMC3 7.2872 × BMC3 1.5248 × | F         | Netrail 7 | BMC2 80.8968 × BMC2 50.0872 × BMC2 5.6976 × | F         | Netrail 7 | BMC3 63.8416 × BMC3 72.6552 × BMC3 3.5632 × | F         | Heanet 7 | BMC2 57.2528 × BMC2 66.6016 × BMC2 5.6632 × | F         | Heanet 7 | BMC3 54.5128 × BMC3 27.9272 × BMC3 5.5848 × | F         | Arpanet19706 9 | BMC2 52.5888 × BMC2 44.3200 × BMC2 7.7576 × | F         | Arpanet19706 9 | BMC3 58.1392 × BMC3 33.6264 × BMC3 4.5640 × | F         | Nordu2005 9 | BMC2 35.9496 × BMC2 33.2320 × BMC2 6.0872 × | F         | Nordu2005 9 | BMC3 38.6664 × BMC3 25.6184 × BMC3 3.2904 × | F         | Nsfcnet 10 | BMC2 14.3520 × BMC2 13.2688 × BMC2 2.1520 × | F         | Nsfcnet 10 | BMC3 13.5312 × BMC3 4.8568 × BMC3 2.0184 × |
| F     | Sprint 11 | - | TO ? | - | TO ? | BMC2 567.1048 | X |
|-------|-----------|---|-------|---|-------|---------------|---|
| F     | Sprint 11 | - | TO ? | - | TO ? | BMC3 582.5752 | X |
| F     | TLex 12   | BMC2 92.9472 | X | BMC2 81.4936 | X | BMC2 4.9240 | X |
| F     | TLex 12   | BMC3 89.6536 | X | BMC3 49.1896 | X | BMC3 7.0464 | X |
| F     | Compuserve 13 | - | TO ? | - | TO ? | BMC2 544.4312 | X |
| F     | Compuserve 13 | - | TO ? | - | TO ? | BMC3 460.1632 | X |
| F     | Eenet 13  | BMC2 249.1056 | X | BMC2 238.0224 | X | BMC2 21.8432 | X |
| F     | Eenet 13  | BMC3 271.4032 | X | BMC3 218.4312 | X | BMC3 53.5256 | X |
| F     | HiberniaCanada 13 | - | TO ? | - | TO ? | BMC2 440.9632 | X |
| F     | HiberniaCanada 13 | - | TO ? | - | TO ? | BMC3 319.2552 | X |
| F     | Navigata 13 | - | TO ? | - | TO ? | BMC2 159.0920 | X |
| F     | Navigata 13 | - | TO ? | - | TO ? | BMC3 177.6008 | X |
| F     | Eunetworks 14 | BMC2 1039.4640 | X | BMC2 1104.7060 | X | BMC2 38.2784 | X |
| F     | Eunetworks 14 | BMC3 1417.8800 | X | BMC3 1056.0080 | X | BMC3 35.9568 | X |
| F     | Claranet 15 | BMC2 189.4912 | X | BMC2 165.1504 | X | BMC2 10.2744 | X |
| F     | Claranet 15 | BMC3 160.3808 | X | BMC3 150.6000 | X | BMC3 7.9720 | X |
| F     | Spirailight 15 | - | TO ? | - | TO ? | BMC2 1249.4900 | X |
| F     | Spirailight 15 | - | TO ? | - | TO ? | BMC3 1734.8840 | X |
| F     | Garr199901 16 | BMC2 625.8648 | X | BMC2 432.1856 | X | BMC2 31.3630 | X |
| F     | Garr199901 16 | BMC3 743.3096 | X | BMC3 370.4792 | X | BMC3 61.6488 | X |
| F     | KentmanJul2005 16 | BMC2 1391.2280 | X | BMC2 1243.6260 | X | BMC2 175.4600 | X |
| F     | KentmanJul2005 16 | BMC3 1405.8180 | X | BMC3 1030.0400 | X | BMC3 171.0152 | X |
| F     | Marwan 16 | - | TO ? | - | TO ? | BMC2 696.8736 | X |
| F     | Marwan 16 | - | TO ? | - | TO ? | BMC3 799.7816 | X |
| F     | Peer1 16 | - | TO ? | - | TO ? | - | TO ? | X |
| F     | Peer1 16 | - | TO ? | - | TO ? | BMC3 1551.8120 | X |
| F     | Rhnet 16 | - | TO ? | - | TO ? | BMC2 105.4688 | X |
| F     | Rhnet 16 | - | TO ? | - | TO ? | BMC3 49.9360 | X |
| F     | Fatman 17 | BMC2 193.7456 | X | BMC2 200.3976 | X | BMC2 18.1704 | X |
| F     | Fatman 17 | BMC3 168.7768 | X | BMC3 169.8224 | X | BMC3 6.7232 | X |
| F     | Goodnet 17 | - | TO ? | - | TO ? | BMC2 410.3936 | X |
| F     | Goodnet 17 | - | TO ? | - | TO ? | BMC3 378.1480 | X |
| F     | Nextgen 17 | - | TO ? | - | TO ? | - | TO ? | X |
| F     | Nextgen 17 | - | TO ? | - | TO ? | BMC3 5014.4240 | X |
| F     | Nordu2010 18 | BMC2 183.4608 | X | BMC2 140.1384 | X | BMC2 13.6824 | X |
| F     | Nordu2010 18 | BMC3 116.1192 | X | BMC3 80.7856 | X | BMC3 6.9120 | X |
| F     | Pacificwave 18 | BMC2 1761.4720 | X | BMC2 1035.8420 | X | BMC2 95.0104 | X |
| F     | Pacificwave 18 | BMC3 1166.0460 | X | BMC3 1545.4200 | X | BMC3 64.5768 | X |
| Prefix | Number | BMC2 | BMC3 |
|--------|--------|------|------|
| BsonetEurope | 18 | - | - |
| BsonetEurope | 18 | - | - |
| Highwinds | 18 | - | - |
| Highwinds | 18 | - | - |
| Noel | 19 | - | - |
| Noel | 19 | - | - |
| Restena | 19 | - | - |
| Restena | 19 | - | - |
| Twaren | 20 | BMC2 560.8456 | BMC2 429.7516 |
| Twaren | 20 | BMC2 650.5672 | BMC3 349.0816 |
| Marnet | 20 | BMC2 858.9728 | BMC2 557.1536 |
| Marnet | 20 | BMC3 846.0584 | BMC3 475.7168 |
| Janetlense | 20 | BMC2 735.1432 | BMC2 721.8584 |
| Janetlense | 20 | BMC3 492.5848 | BMC3 616.7248 |
| BtAsiaPac | 20 | - | - |
| BtAsiaPac | 20 | - | - |
| Oxford | 20 | - | - |
| Oxford | 20 | - | - |
| Harnet | 21 | BMC2 961.5872 | BMC2 873.2056 |
| Harnet | 21 | BMC3 1410.2540 | BMC3 735.7256 |
| Belnet2009 | 21 | BMC2 1146.2600 | BMC2 611.8096 |
| Belnet2009 | 21 | - | - |
| Garr200404 | 22 | BMC2 61.5632 | BMC2 70.9440 |
| Garr200404 | 22 | BMC3 45.9016 | BMC3 49.3768 |
| Bandcon | 22 | - | - |
| Bandcon | 22 | - | - |
| KentmanApr2007 | 22 | - | - |
| KentmanApr2007 | 22 | - | - |
| Istar | 23 | BMC2 574.5056 | BMC2 302.8448 |
| Istar | 23 | BMC3 221.1632 | BMC3 236.4752 |
| Garr199905 | 23 | BMC2 188.2176 | BMC2 155.1984 |
| Garr199905 | 23 | BMC3 272.3944 | BMC3 78.0928 |
| Garr199904 | 23 | BMC2 478.9360 | BMC2 342.0088 |
| Garr199904 | 23 | BMC3 304.7032 | BMC3 385.6456 |
| Aconet | 23 | - | - |
| Aconet | 23 | - | - |
| Belnet2003 | 23 | - | - |
| Belnet2003 | 23 | - | - |
| Belnet2005 | 23 | - | - |
| Belnet2005 | 23 | - | - |
| Name            | Year | Total Cost | Subsequent | Final Cost | Subsequent |
|-----------------|------|------------|------------|------------|------------|
| Cesnet2001      | 23   | -          | TO ?       | BMC2 1439.4880 | BMC2 277.3880 |
| Cesnet2001      | 23   | -          | TO ?       | BMC3 1508.9240 | BMC3 169.1864 |
| Fccn            | 23   | -          | TO ?       | -          | TO ?       | BMC2 772.9776 |
| Fccn            | 23   | -          | TO ?       | -          | TO ?       | BMC3 785.8392 |
| Uran            | 24   | BMC2 87.9600 | BMC2 126.0488 | BMC2 4.0680 |
| Uran            | 24   | BMC3 71.7760 | BMC3 72.4816 | BMC3 3.8056 |
| BtEurope        | 24   | -          | TO ?       | -          | TO ?       | BMC2 5035.2740 |
| BtEurope        | 24   | -          | TO ?       | -          | TO ?       | BMC3 538.3072 |
| Garr200112      | 24   | -          | TO ?       | -          | TO ?       | BMC2 294.9824 |
| Garr200112      | 24   | -          | TO ?       | -          | TO ?       | BMC3 1774.0800 |
| Arpanet19723    | 25   | -          | TO ?       | -          | TO ?       | BMC2 1079.5160 |
| Arpanet19723    | 25   | -          | TO ?       | -          | TO ?       | BMC3 142.0280 |
| Vinaren         | 25   | BMC2 1537.2920 | BMC2 799.1280 | BMC2 138.3432 |
| Vinaren         | 25   | BMC3 1093.3200 | BMC3 1171.3140 | BMC3 93.6536 |
| KentmanFeb2008  | 26   | BMC2 152.6880 | BMC2 230.8040 | BMC2 7.1976 |
| KentmanFeb2008  | 26   | BMC3 83.3048 | BMC3 89.7208 | BMC3 4.4288 |
| Garr200212      | 27   | BMC2 416.9272 | BMC2 405.2304 | BMC2 29.7792 |
| Garr200212      | 27   | BMC3 418.8768 | BMC3 301.6184 | BMC3 14.4280 |
| Gambia          | 28   | -          | TO ?       | -          | TO ?       | BMC2 916.3864 |
| Gambia          | 28   | -          | TO ?       | -          | TO ?       | BMC3 586.2224 |
| KentmanAug2005  | 28   | -          | TO ?       | -          | TO ?       | BMC2 799.1960 |
| KentmanAug2005  | 28   | -          | TO ?       | -          | TO ?       | BMC3 935.1016 |
| Ernet           | 30   | -          | TO ?       | -          | TO ?       | BMC2 462.7160 |
| Ernet           | 30   | -          | TO ?       | -          | TO ?       | BMC3 361.0344 |
| WideJpn         | 30   | -          | TO ?       | -          | TO ?       | BMC2 336.8296 |
| WideJpn         | 30   | -          | TO ?       | -          | TO ?       | BMC3 75.9296 |
| Inet            | 31   | BMC2 1380.3760 | BMC2 1217.9420 | BMC2 76.8728 |
| Inet            | 31   | BMC3 1468.8020 | BMC3 701.2528 | BMC3 97.9888 |
| CrlNetworkServices | 33   | -          | TO ?       | -          | TO ?       | BMC2 1534.4540 |
| CrlNetworkServices | 33   | -          | TO ?       | -          | TO ?       | BMC3 591.3928 |
| GtsSlovakia     | 35   | -          | TO ?       | -          | TO ?       | BMC2 938.1456 |
| GtsSlovakia     | 35   | -          | TO ?       | -          | TO ?       | BMC3 1014.7040 |
| Bren            | 37   | -          | TO ?       | -          | TO ?       | BMC2 346.8384 |
| Bren            | 37   | -          | TO ?       | -          | TO ?       | BMC3 799.3200 |
| Myren           | 37   | BMC2 183.0128 | BMC2 154.1552 | BMC2 10.1816 |
| Myren           | 37   | BMC3 142.0280 | BMC3 75.9296 | BMC3 12.9468 |
| KentmanJan2011  | 38   | BMC2 237.3152 | BMC2 192.5768 | BMC2 9.3536 |
| KentmanJan2011  | 38   | BMC3 167.9208 | BMC3 86.4384 | BMC3 10.5464 |
| Cesnet200511    | 39   | -          | TO ?       | -          | TO ?       | BMC2 496.0712 |
| Cesnet200511    | 39   | -          | TO ?       | -          | TO ?       | BMC3 249.1456 |
| Country          | Code | BMC2 | BMC3  |
|------------------|------|------|-------|
| F Litnet         | 43   | TO   | 234.4000 |
| F Litnet         | 43   | TO   | 223.7856 |
| F Bellsouth      | 51   | TO   | 173.6720 |
| F Bellsouth      | 51   | TO   | 184.7136 |
| F BtLatinAmerica | 51   | TO   |       |
| F BtLatinAmerica | 51   | TO   |       |
| F Garr201103     | 58   | TO   | 1346.7440 |
| F Garr201103     | 58   | TO   | 89.1304 |
| F Forthnet       | 62   | TO   | 1040.0800 |
| F Forthnet       | 62   | TO   |       |
| F Latnet         | 69   | TO   | 209.1984 |
| F Latnet         | 69   | TO   | 235.7168 |
| F Ulaknet        | 82   | TO   | 1043.7440 |
| F Ulaknet        | 82   | TO   |       |