Understanding the effect of hammering process on the vibration characteristics of cymbals

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Abstract. Cymbals are thin domed plates used as percussion instruments. When cymbals are struck, they vibrate and radiate sound. Cymbals are made through spin forming, hammering, and lathing. The spin forming creates the basic shape of the cymbal, which determines its basic vibration characteristics. The hammering and lathing produce specific sound adjustments by changing the cymbal’s vibration characteristics. In this study, we study how hammering cymbals affects their vibration characteristics. The hammering produces plastic deformation (small, shallow dents) on the cymbal’s surface, generating residual stresses throughout it. These residual stresses change the vibration characteristics. We perform finite element analysis of a cymbal to obtain its stress distribution and the resulting change in vibration characteristics. To reproduce the stress distribution, we use thermal stress analysis, and then with this stress distribution we perform vibration analysis. These results show that each of the cymbal’s modes has a different sensitivity to the thermal load (i.e., hammering). This difference causes changes in the frequency response and the deflection shape that significantly improves the sound radiation efficiency. In addition, we explain the changes in natural frequencies by the stress and modal strain energy distributions.

1. Introduction
Cymbals are thin domed plates made of bronze, used as percussion instruments. They are played by hitting them with a drumstick or with another cymbal. The sound of a cymbal is complex and inharmonic, containing many frequencies, which is related to the cymbal’s vibration characteristics. The vibration characteristics depend on its shape, size, and material. In addition, manufacturing processes such as hammering and lathing change the vibration characteristics, producing the sound characteristics. To adjust a cymbal’s sound in specific ways, it is important to understand how its vibration characteristics are changed by manufacturing processes.

There has been little research on percussion instruments as compared to wind or string instruments. Rossing et al. [1] reviewed the acoustics of various kinds of percussion instruments. Bretos et al. [2] proposed a process for tuning xylophone and marimba bars by using finite element (FE) analysis and experimental measurements. Wang [3] addressed how integrating FE analysis and experimental modal analysis can be used to design percussion instruments: xylophones, metallophones, and gongs.
Tronchin [4] conducted modal analysis and acoustic radiation measurement of kettledrums, and proposed a new parameter relating the modal pattern to acoustic radiation. Kawai et al. [5] used FE analysis to investigate cymbal’s mode shapes with high and low sound radiation efficiency.

At high vibration amplitudes, cymbals are well known to display nonlinear phenomena and even chaotic behaviour. Chaigne et al. [6] reviewed nonlinear vibration and chaos in cymbals and stated that the nonlinear behaviour depends on its eigenfrequencies. Cymbals vibrate in many different mode shapes and radiate sound with complex overtones. Di Giulio et al. [7] measured the mode shapes of a cymbal by using scanning laser Doppler vibrometer. Perrin et al. [8, 9] identified over a hundred normal modes of cymbals by various experimental methods including electronic speckle pattern interferometry, laser vibrometry, and Chladni sand patterns. They compared the experimental results with predictions from a FE model and showed that the modes couple with others that are close in frequency and that the high density of modes might cause coupling.

As stated in Pinksterboer [10], a cymbal’s sound depends on not only its size and shape but also on the applied manufacturing processes such as hammering and lathing. For example, a cymbal’s sound will be different when it has a regular hammering pattern and when it does not have an obvious regular pattern. However, there is very little available literature on the effect of these manufacturing processes.

In this paper, we study how the hammering process affects the vibration characteristics of cymbals. Hammering produces plastic deformation (small, shallow dents) on the cymbal’s surface and generates a complex distribution of residual stresses within it. To obtain the stress distribution in the cymbal and the resulting change in its vibration characteristics, we performed FE analysis. In our analysis, we use thermal stress analysis to reproduce the stress distribution, and then with this distribution we analyse the vibration characteristics. We compare the frequency responses of cymbals with and without hammering, and we discuss how hammering affects the cymbal’s vibration characteristics. To further demonstrate the effect of hammering, we examine the deflection shapes and their sound radiation efficiencies at the peak of the frequency response.

2. Cymbal

2.1. Anatomy
A cymbal is a thin, round bronze plate with a cross-sectional view as shown in figure 1. The bell is the raised area at the centre of cymbal, the edge is the outer periphery, and the bow is the curved area from the bell to the edge. The size of the bell, the shape of the bow, and the diameter, weight, and material of the whole cymbal affect its sound. Playing the different areas of the cymbal’s surface also varies its sound.

![Figure 1. Cross-sectional view of a cymbal.](image)

2.2. Manufacturing process
Every cymbal company produces cymbals with a unique process. This section briefly explains the manufacturing process used by Koide Cymbals in Japan.

First, an ingot of bronze is formed, cut, and rolled into a blank metal disc by Osaka Alloying Works in Japan. After the disc is heated, the bell is pressed in, and then the cymbal is quenched. The cymbal is lathed, decreasing its weight, and then the bow is shaped using a spin-forming machine, producing a cymbal with a basic shape. The cymbal is then hammered, as shown in figure 2. The hammering compresses the metal in local areas, producing small, shallow dents on the surface, as shown in figure 3. Finally, sound grooves are cut using a lathe.
3. Analysis method

3.1. Equation of motion

The equation of motion for a cymbal with hysteretic (or material) damping can be written as

$$[M]\{\ddot{u}\} + ([K] + j[D])\{u\} = \{f\}$$

(1)

where $[M]$, $[K]$, and $[D]$ are the mass, stiffness, and damping matrices. $\{u\}$ and $\{f\}$ are the displacement and external force vectors. For a hammered cymbal, its mass distribution may not change, but its stiffness changes considerably because hammering can produce a complex distribution of residual stresses throughout the cymbal. The equation of motion for the hammered cymbal becomes

$$[M]\{\ddot{\tilde{u}}\} + ([K] + [K^\sigma] + j[D])\{\tilde{u}\} = \{f\}$$

(2)

where $[K^\sigma]$ is called an initial stress stiffness matrix, which represents the stiffening or weakening of the cymbal due to in-plane stress. $\{\tilde{u}\}$ is the displacement vector after hammering. The element initial stress stiffness matrix $[K^\sigma]^e$ for the shell element with in-plane stress is given by [11]

$$[K^\sigma]^e = \int_{V} [B_d]^T \begin{bmatrix} \sigma_x & \sigma_y & Sym. \\ \tau_{xy} & \sigma_x & \sigma_y \\ 0 & 0 & \tau_{xy} \end{bmatrix} [B_d] dV$$

(3)

where $\sigma_x$ and $\sigma_y$ are the normal stresses in the $x$ and $y$ directions of the element coordinate system, and $\tau_{xy}$ is the shear stress. $[B_d]$ is a matrix relating the node displacements and the derivatives of displacements. $V$ is the volume of the element.

3.2. Stress analysis

Obtaining $[K^\sigma]$ requires the stress distribution of the hammered cymbal. To reduce the computational effort, we apply the technique used in Kuratani and Yano [12]. This technique calculates the stress distribution by using thermal stress analysis instead of plastic deformation stress analysis. To obtain the temperature distribution for a thermal load, the following Gaussian distribution function is used:

$$T(r) = T_{max} \exp\left(-\frac{(r-R_c)^2}{2s^2}\right)$$

(4)
where $T(r)$ is the temperature at distance $r$ from the centre $R_c$ of each thermal load, $T_{\text{max}}$ is the temperature at the centre, and $s$ is the standard deviation of the temperature distribution. $R_c$ corresponds to the centre of each dent, and $T_{\text{max}}$ and $s$ must be determined from the diameter and depth of the shallow dent produced by hammering. This technique was validated by Kimura and Ando [13] using the example of a disc.

3.3. Vibration Analysis

In this paper, we consider the steady state of a cymbal excited by a harmonic force to investigate its vibration characteristics in the frequency domain.

The natural frequencies and mode shapes of a hammered cymbal with a stress distribution are obtained by solving the following eigenvalue problem:

$$( [K] - \omega^2 [M] ) \{ \hat{\phi} \} = \{ 0 \}$$

where $[\hat{K}] = [K] + [K^\alpha]$, and $\omega$ and $\{ \hat{\phi} \}$ are the natural angular frequency and the mode shape of the hammered cymbal. Because a cymbal is made of bronze and its material damping is small, we assume that the damping matrix is directly proportional to the stiffness matrix:

$$[D] = \eta [\hat{K}]$$

where $\eta$ is the loss factor of the material. The equation of motion for the hammered cymbal with harmonic excitation becomes

$$[M] \ddot{\{ \hat{u} \}} + ((1 + j \eta) [\hat{K}]) \{ \hat{u} \} = \{ F \} e^{j \omega t}$$

where $\omega$ is the angular frequency of excitation. Using the modal superposition method, the displacements $\{ \hat{u} \}$ can be expressed as a linear combination of mode shapes $\{ \hat{\phi} \}$ as

$$\{ \hat{u} \} = \sum_{r=1}^{N} \{ \hat{\phi}_r \} q_r$$

where $q_r$ is a modal coordinate. By using equation (8), equation (7) can be transformed into a set of uncoupled equations. For the $r$th mode, the equation of motion for a single degree of freedom is

$$- \omega^2 \hat{m}_r \ddot{q}_r + \hat{k}_r (1 + j \eta) q_r = \hat{\phi}_r^T \{ F \} e^{j \omega t}$$

where $\hat{m}_r$ and $\hat{k}_r$ are the modal mass and modal stiffness. Solving equation (9) for $q_r$ gives

$$q_r = \frac{\hat{\phi}_r^T \{ F \}}{- \omega^2 \hat{m}_r + \hat{k}_r (1 + j \eta)} e^{j \omega t}$$

When the mode shapes are mass normalized, $q_r$ is useful for identifying the dominant modes contributing to the frequency response. Substituting equation (10) into equation (8) and differentiating with respect to $t$, we obtain the velocity response $\{ v \}$ of the cymbal as

$$\{ v \} = j \omega \sum_{r=1}^{N} \frac{\hat{\phi}_r^T \{ F \} \{ \hat{\phi}_r \}}{- \omega^2 \hat{m}_r + \hat{k}_r (1 + j \eta)} e^{j \omega t}$$
3.4. Sound radiation efficiency

The sound radiation efficiency of a cymbal depends on its deflection shape, which is expressed as a linear combination of the mode shapes. In this section, we derive the sound radiation efficiency for a cymbal to examine how its deflection shape influences its radiation efficiency. We consider the steady state of a cymbal excited by harmonic force in the same way as in section 3.3.

A cymbal is assumed to be baffled to eliminate the acoustic interaction between surface radiation from the top and bottom. The cymbal is divided into small elements, and each element can be regarded as a point source of sound. The radiated sound pressure caused by the vibrating cymbal at any point of observation can be described by Rayleigh’s integral as

\[
p(x, y, z) = \frac{j \rho_0 \omega}{2\pi} \int_S v(\vec{x}, \vec{y}, \vec{z}) \frac{e^{-jkS}}{r_j} dS
\]

where \( p(x, y, z) \) is the sound pressure at the observation location \((x, y, z)\), \( v(\vec{x}, \vec{y}, \vec{z}) \) is the velocity normal to the surface at the location \((\vec{x}, \vec{y}, \vec{z})\) on the cymbal, and \( r_j \) is the distance from \((\vec{x}, \vec{y}, \vec{z})\) to \((x, y, z)\). \( \omega \) is the angular frequency of excitation, \( \rho_0 \) is the density of air, \( S \) is the surface area of the cymbal, and \( k \) is the acoustic wavenumber. The radiated sound power is calculated as the integral of sound intensity over the cymbal’s surface as

\[
W = \frac{1}{2} \text{Re} \left( \int_S p(\vec{x}, \vec{y}, \vec{z}) v^*(\vec{x}, \vec{y}, \vec{z}) dS \right)
\]

where * denotes the complex conjugate. Substituting equation (12) into equation (13) and rewriting it in discrete form, the sound power can be computed by summing over all elements as

\[
W = \frac{\omega \rho_0}{4\pi} \sum_{i=1}^{N_e} \sum_{j=1}^{N_i} v_i S_j \frac{\sin kr_j}{r_j} v^*_i S_i
\]

where \( N_e \) is the number of elements in the FE model of the cymbal and \( S_i \) is the area of \( i \)th element. \( v_i \) is the velocity normal to the surface at the centre of \( i \)th element, where \( v_i \) is calculated from the nodal velocities obtained by equation (11). The sound radiation efficiency is defined as the ratio of the radiated sound power to the reference sound power. Its general expression is given by

\[
\sigma = \frac{W}{\rho_0 c S \langle v^2 \rangle}
\]

where \( \langle v^2 \rangle \) is the spatially averaged mean square velocity normal to the cymbal’s surface, and \( c \) is the speed of sound.

4. Hammering effect

To evaluate and understand how hammering a cymbal affects its vibration characteristics, we perform stress analysis and vibration analysis with a stress distribution.

In this paper, we consider a 16-inch cymbal. Its outer diameter is 404 mm, its inner diameter is 12 mm, and its bell is 120 mm in diameter and 37 mm in height. Figure 4 shows the FE model of the cymbal, built using ANSYS FE software. The element type used is a four-node shell element (Shell 181). The cymbal is meshed with a maximum element size of 2 mm in the radial direction. For the circumferential direction, the bell is divided into 200 elements, and the bow is divided into 600 elements. The maximum element size in the circumferential direction is approximately 2 mm at the outer edge. The nodes on the interface between the bell and bow are connected using multi-point constraints because they have different mesh sizes. All of the elements have the same thickness, 1.5 mm. The material properties are as follows: Young’s modulus \( E=85.1 \) GPa, density \( \rho=8700 \) kg/m\(^3\),
Poisson’s ratio $\nu=0.36$, and coefficient of thermal expansion $\alpha=1.76\times10^{-5}/^\circ\text{C}$. The free boundary condition is assumed because the cymbal is supported by a washer covered with felt at its centre and the cymbal can move and rotate freely.

After modal analysis, the natural frequencies and mode shapes obtained from the FE model are transferred to a MATLAB program to calculate the frequency responses and sound radiation efficiencies.

![Finite element model of the cymbal.](image)

**Figure 4.** Finite element model of the cymbal.

4.1. Natural frequency and mode shape

Figure 5 shows the natural frequencies obtained from the FE model without thermal load, and figure 6 illustrates some examples of mode shapes. The modes are designated by $(m, n)$, where $m$ is the number of nodal diameters and $n$ is the number of nodal circles. If the cymbal has axisymmetric geometry and uniform material properties, then modes containing at least one nodal diameter $(m>0)$ have repeated natural frequencies. The model in figure 4 has 244 modes up to 5000 Hz, consisting of six rigid body modes, eight modes with zero nodal diameter, and 115 doublet modes that have repeated natural frequencies. Because the model has two doublet modes for which the bell vibrates locally, the eight modes with zero nodal diameter and the 113 doublet modes are shown in figure 5. Figure 5 indicates that the mode count increases with increasing natural frequency. In figure 6, the mode shapes at lower frequencies are simple, while those at frequencies above 3000 Hz are complicated.

![Natural frequencies for the model without thermal load.](image)

**Figure 5.** Natural frequencies for the model without thermal load.
4.2. Stress analysis

First, we consider the case of hammering in grids of 12 in the radial direction and 50 in the circumferential direction. Figure 7 shows the thermal loads corresponding to this hammering, where the parameters of the temperature distribution in equation (4) are $T_{\text{max}}=500 \, ^\circ\text{C}$ and $s=2.5 \, \text{mm}$. These values were determined to roughly reproduce the stresses produced by the hammering. Figure 8 shows the stress distribution within this cymbal obtained from the thermal stress analysis. Figure 8(a) shows the principal tensile stress, where red and yellow indicate areas of high tensile stress and blue indicates areas of no tensile stress. Figure 8(b) shows the principal compressive stress, where blue and yellow indicate areas of high compressive stress and red indicates areas of no compressive stress.

Comparing figures 7 and 8, we find that the compressive stresses exist at the locations of thermal loads, while the tensile stresses exist around the thermal loads. In figure 8(a), the tensile stresses are widely distributed from the bell to the edge, and large tensile stresses appear at a slight distance from the bell; in figure 8(b), large compressive stresses appear near the bell.
4.3. Frequency response

To evaluate how hammering affected the cymbal’s vibration characteristics, we compare its frequency responses before and after hammering, that is, with and without thermal load. When the cymbal was excited at 20 mm from the outer edge with a vertical force of 1 N, we calculated the vertical velocity at the same point by equation (11). The excitation frequency was varied from 1 to 5000 Hz in 1 Hz steps. The loss factor was set to 0.001 for all frequencies based on the dynamic properties of copper.

Figure 9 compares the velocity frequency responses of cymbals with no thermal load and thermal loads of $T_{\text{max}} = 100 \, ^\circ\text{C}$ and $500 \, ^\circ\text{C}$. Changing the thermal load shifted the peak frequencies and significantly changed the corresponding peak values. For example, the peak values near 1700 Hz, 2400 Hz, and 3200 Hz for the thermal load $T_{\text{max}} = 100 \, ^\circ\text{C}$ are larger than those for no thermal load. Thus, thermal loading (i.e., hammering) affects the cymbal’s frequency response.

Figure 10 shows the expanded frequency responses in the range of 3100–3400 Hz with a frequency resolution of 0.1 Hz. Figure 10(a) is for no thermal load, figure 10(b) is for the thermal load $T_{\text{max}} = 100 \, ^\circ\text{C}$, and figure 10(c) is for $T_{\text{max}} = 500 \, ^\circ\text{C}$. The frequency responses for no thermal load and $T_{\text{max}} = 100 \, ^\circ\text{C}$ are similar, except for the peak value near 3200 Hz, while the result for $T_{\text{max}} = 500 \, ^\circ\text{C}$ is very different from the other two.

![Figure 9. Frequency responses for various thermal loads.](image)

![Figure 10. Frequency responses.](image)
Next, we clarify the reason for the difference in the frequency response, especially the highest peak near 3200 Hz in figure 10. When the mode count is low at low frequencies, the response at each resonance can be dominated by one mode. The deflection shape at each resonance frequency is approximately equal to the mode shape. In contrast, when the mode count is high at higher frequencies, the response at each peak depends on two or more modes because these modes have almost the same natural frequency. The deflection shape at each peak does not coincide with any of the mode shapes; rather, it is a combination of their mode shapes (i.e., mode mixing).

As shown in figure 5, the mode count becomes high (five modes or more) at frequencies above 1500 Hz. Table 1 shows the dominant modes at the highest peak near 3200 Hz, determined from equation (10). (17, 1), (2, 6), and (8, 4) correspond to the mode shapes shown in figure 6. For no thermal load and \( T_{\text{max}} = 100 ^\circ \text{C} \), the (17, 1) and (2, 6) modes are dominant, while for \( T_{\text{max}} = 500 ^\circ \text{C} \), the (17, 1) and (8, 4) modes are dominant. Thus, the dominant modes contributing significantly to the frequency response change when the thermal load is high. This behaviour occurs because the natural frequencies are changed by the thermal load. As \( T_{\text{max}} \) increases, the natural frequency of the (17, 1) mode increases while the natural frequencies of the (2, 6) and (8, 4) modes decrease, especially the frequency of the (8, 4) mode drastically decreases. This indicates that each mode has a different sensitivity to the thermal load; that is, some natural frequencies increase and some natural frequencies decrease.

Regarding the difference in peak value near 3200 Hz between no thermal load and \( T_{\text{max}} = 100 ^\circ \text{C} \), the natural frequencies for the (17, 1) and (2, 6) modes are slightly different at no thermal load, while they are close at \( T_{\text{max}} = 100 ^\circ \text{C} \). This behaviour causes the difference in peak value: as two natural frequencies come closer, the peak value of the frequency response increases.

### Table 1. Dominant modes at the highest peak near 3200 Hz.

| Thermal load       | No thermal load | \( T_{\text{max}} = 100 ^\circ \text{C} \) | \( T_{\text{max}} = 500 ^\circ \text{C} \) |
|--------------------|----------------|---------------------------------|---------------------------------|
| Dominant mode      | (17, 1), (2, 6) | (17, 1), (2, 6)                 | (17, 1), (8, 4)                |
| Natural Frequency  | (17, 1)         | 3214 Hz                         | 3216 Hz                        | 3224 Hz                        |
|                    | (2, 6)          | 3217 Hz                         | 3217 Hz                        | 3209 Hz                        |
|                    | (8, 4)          | 3261 Hz                         | 3253 Hz                        | 3221 Hz                        |
| Radiation efficiency | 1.77%          | 1.87%                           | 0.79%                           |

### 4.4. Sound radiation efficiency

Now we examine how mode mixing affects the radiation efficiency. Figure 11 shows the deflection shape at the highest peak near 3200 Hz. The deflection shapes for no thermal load and \( T_{\text{max}} = 100 ^\circ \text{C} \) seem to be combination of the shapes of the (17, 1) and (2, 6) modes, which are slightly different. The deflection shape for \( T_{\text{max}} = 500 ^\circ \text{C} \) is a combination of the (17, 1) and (8, 4) mode shapes. Table 1 shows the sound radiation efficiency for each of the deflection shapes in figure 11. The efficiency for \( T_{\text{max}} = 100 ^\circ \text{C} \) is slightly higher than that for no thermal load, and the lowest is for \( T_{\text{max}} = 500 ^\circ \text{C} \).

The sound radiation efficiencies of the (17, 1), (2, 6), and (8, 4) modes are \( 1 \times 10^{-4} \text{%} \), 1.93%, and 0.13%; the efficiency for the (17, 1) mode is quite low. As shown in figure 9(a), the frequency response for the (17, 1) mode (3214 Hz) is much larger than those for the other two modes (3217 Hz and 3261 Hz). This difference indicates that the (17, 1) mode has a large response but low efficiency. In contrast, the (2, 6) and (8, 4) modes have small responses but high efficiencies. Thus, combining these modes improves the radiation efficiency and the response amplitude. Table 1 shows that combining the (17, 1) and (8, 4) modes improves the radiation efficiency from \( 1 \times 10^{-4} \text{%} \) and 0.13% to
0.79% ($T_{\text{max}}=500$ °C) and combining the (17, 1) and (2, 6) modes changes the efficiency from $1 \times 10^{-4}$% and 1.93% to 1.87% ($T_{\text{max}}=100$ °C). In addition, figure 9 shows that the value of the highest peak near 3200 Hz is much larger for $T_{\text{max}}=100$ °C than for no thermal load and that there is little difference in the responses between $T_{\text{max}}=500$ °C and no thermal load. This improvement in response and efficiency comes from the hammering.

(a) No thermal load           (b) Thermal load $T_{\text{max}}=100$ °C    (c) Thermal load $T_{\text{max}}=500$ °C

Figure 11. Deflection shapes at the highest peak near 3200 Hz.

4.5. Modal strain energy and stress distribution

The natural frequencies of the cymbal change because of the stresses produced by hammering, and these changes depend on modal strain energy. The modal strain energy is calculated as the product of the elemental stiffness matrix and the second power of the mode shape component.

Figure 12 shows the modal strain energy distribution for the (17, 1), (2, 6), and (8, 4) modes, where red and yellow indicate areas of high strain energy. For the (17, 1) mode, the strain energy is distributed around the outer edge, while for the (2, 6) and (8, 4) modes, the energies are distributed from the bell to the edge. In more detail: for the (2, 6) mode, the higher strain energy is widely distributed near the outer edge, while for the (8, 4) mode, the highest energy exists near the bell. As mentioned in section 4.2, the higher compressive stresses exist near the bell, while the higher tensile stresses exist at a slight distance from the bell, and the tensile stresses are widely distributed from the bell to the edge. This distribution implies that the natural frequency of the (17, 1) mode increases owing to the high strain energy and the tensile stresses distributed over a large area around the outer edge. In contrast, the natural frequency of the (8, 4) mode drastically decreases, owing to the high strain energy and the high compressive stress near the bell. The natural frequency of the (2, 6) mode probably changed little because the strain energy, tensile stress, and compressive stress are widely distributed throughout the cymbal. Thus, the changes in the natural frequencies are explained by the stress and modal strain energy distributions.

(17, 1) mode                             (2, 6) mode                              (8, 4) mode

Figure 12. Modal strain energy distribution.
5. Conclusions
A cymbal’s sound depends on not only its size and shape but also the applied manufacturing process such as hammering and lathing. In this paper, we studied how hammering cymbals affects their vibration characteristics. Hammering produces plastic deformation (small, shallow dents) on the cymbal’s surface and generates a complex distribution of residual stresses within it. To obtain the cymbal’s stress distribution and the resulting change in its vibration characteristics, we performed FE analysis. In this analysis, we used thermal stress analysis to reproduce the stress distribution, and then with this stress distribution we performed vibration analysis. We compared the frequency responses of cymbals with and without thermal load (i.e., hammering), and we discussed how the hammering affected the cymbal’s vibration characteristics. To further demonstrate the effect of hammering, we examined the deflection shapes and their sound radiation efficiencies at the peak of the frequency response. These results show that each mode of the cymbal has a different sensitivity to the thermal load. This difference changes the frequency response and the deflection shape that significantly improves the sound radiation efficiency. In addition, we explained the changes in natural frequencies by the stress and modal strain energy distribution.

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