Effective mass and decay of $\Theta^+$ in nuclear matter in quark-meson coupling model

C. Y. Ryu$^1$, C. H. Hyun$^{1,2}$, J. Y. Lee$^3$, and S. W. Hong$^1$

$^1$Department of Physics and Institute of Basic Science, Sungkyunkwan University, Suwon 440-746, Korea,
$^2$School of Physics, Seoul National University, Seoul 151-742, Korea,
$^3$Department of Physics, Sejong University, Seoul 143-747, Korea

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Abstract

The in-medium mass of a $\Theta^+$, $m_{\Theta^+}^*$, in cold symmetric nuclear matter is calculated by using the quark-meson coupling model. The $\Theta^+$ is treated as an MIT bag with the quark content $uudd\bar{s}$. Bag parameters for a free $\Theta^+$ are fixed to reproduce the observed mass of the $\Theta^+$. In doing so, we use three different values of the $s$-quark mass since the mass of the $s$-quark is not well known. As usual, the strengths of the $u$ and $d$ quark couplings to $\sigma$- and $\omega$-meson fields are determined to fit the nuclear saturation properties. However, the coupling constant $g^s_{\sigma}$ between the $s$-quark and the $\sigma$-meson cannot be fixed from the saturation properties, and thus we treat $g^s_{\sigma}$ as a free parameter and investigate how $m_{\Theta^+}^*$ depends on $g^s_{\sigma}$. We find that $m_{\Theta^+}^*$ depends significantly on the value of $g^s_{\sigma}$ but not on the mass of the $s$-quark. Chemical potentials of the $\Theta^+$ and the $K+N$ system are calculated to discuss the decay of a $\Theta^+$ in nuclear matter. We calculate the effective mass of a kaon in nuclear matter in two ways; using the optical potential of $K^-$ in matter and using quark model. By comparing the effective masses calculated from these two methods, we find the magnitude of the real part of the optical potential that is consistent with the usual quark model is about 100 MeV.

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I. INTRODUCTION

Since the mass spectra of pentaquark baryons were first studied with the MIT bag model more than two decades ago, they were recently reinvestigated in the framework of chiral soliton model. The experimental mass and decay width of a $\Theta^+$, one of the pentaquark baryons, claimed to be measured by LEPS collaboration seem to agree with the theoretical prediction. The existence of pentaquark baryons is, however, still a matter of controversy. There are experiments done by different groups showing positive results, but some other experiments don’t observe the claimed peak. Though the existence of a $\Theta^+$ remains to be confirmed, in this work we assume it exists with the quark content $uudd\bar{s}$ and take the mass $m_{\Theta^+}$ to be $\approx 1540$ MeV.

To probe the pentaquark baryons in matter, productions of a $\Theta^+$ in relativistic heavy ion collisions are investigated theoretically at the highest energy of RHIC, at the lowest SPS energy, and for the transition phases from quark-gluon plasma to hadronic states. It is believed that the mass of hadrons changes in hot and/or dense systems due to their interaction with the surrounding matter. Results from the measurements of dileptonic decays of $\rho$- and $\omega$- mesons in heavy ion collisions can be interpreted as changes of hadron masses in medium. One can expect that the mass of a $\Theta^+$ in medium, $m_{\Theta^+}^*$, may also be different from its free mass. If such a change in mass occurs indeed, it can influence the production rate of $\Theta^+$ in heavy ion collision experiments.

In this work, as a precursory step to the studies of $m_{\Theta^+}^*$ in hot and/or dense matter, we consider a $\Theta^+$ in cold matter to fix the parameters of the models. Recently, $m_{\Theta^+}^*$ at the nuclear saturation density and zero temperature has been calculated in two independent schemes, whose conclusions are quite different from each other. In Ref. $m_{\Theta^+}^*$ does not nearly change from its free mass, but in Ref. a considerable change in mass is obtained. In this work we treat the dense matter in terms of the quark-meson coupling (QMC) model and calculate the effective mass of a $\Theta^+$ in nuclear matter. In the QMC model, baryons are treated as MIT bags, which interact with each other through the exchange of mesons such as $\sigma$ and $\omega$. Consistently with the QMC model, we also treat the $\Theta^+$ as an MIT bag. Then additional three bag parameters for a $\Theta^+$ need to be fixed; bag radius $R_{\Theta^+}$, bag constant $B_{\Theta^+}$ and a phenomenological constant $Z_{\Theta^+}$. We take $R_{\Theta^+}$ values to be 0.6, 0.8 and 1.0 fm. $B_{\Theta^+}$ and $Z_{\Theta^+}$ are fixed to reproduce the free mass $m_{\Theta^+}$ for given
$R_{\Theta^+}$ values. For the interactions between quarks ($u$, $d$ and $s$) and mesons ($\sigma$ and $\omega$), four quark-meson coupling constants $g_{\sigma}^{u(d)}$, $g_{\sigma}^{s}$, $g_{\omega}^{u(d)}$ and $g_{\omega}^{s}$ are introduced. (We assume $g_{\sigma}^{u} = g_{\sigma}^{d}$ and $g_{\omega}^{u} = g_{\omega}^{d}$.) $g_{\sigma}^{u}$ and $g_{\omega}^{u}$ are fitted as usual to reproduce the binding energy per nucleon $E_b$ ($= 16.0$ MeV) at the nuclear saturation density $\rho_0$ ($= 0.17$ fm$^{-3}$). We also use the so-called modified QMC (MQMC) model [17], in which the bag constant $B$ is assumed to depend on density. These nuclear models will be presented in Section II A.

In quark models, $g_{\sigma}^{s}$ and $g_{\omega}^{s}$ are often set to zero. However, studies of hypernuclei [18, 19] or kaon-nucleon systems [20, 21] seem to indicate that the interaction between the $s$-quark and $u$ ($d$) quarks could be non-negligible. Recently, deeply bound kaonic states are predicted theoretically for light systems [22, 23], and indeed KEK-E471 has revealed similar states identified as deeply bound tribaryon-kaon systems [24, 25]. In some cases [18, 19, 21], an interaction weaker than what is expected by quark models is favored, but in some other cases [20, 22, 23] a strong interaction seems to be needed. Due to this large uncertainty in the interaction of strange sector, we treat $g_{\sigma}^{s}$ as a free parameter and study the dependence of $m_{\Theta^+}^*$ on $g_{\sigma}^{s}$. However, for the interaction between the $s$-quark and the $\omega$-meson, we assume the quark model value $g_{\omega}^{s} = 0$.

Stability of the $\Theta^+$ particle in medium may be discussed by comparing the chemical potential of a $\Theta^+$ with that of a $K_N$ system, which is a possible decay channel of $\Theta^+$. If the chemical potential of a $\Theta^+$ is lower than that of a $K_N$ system, one cannot exclude the possibility of a stable $\Theta^+$ in nuclear matter. In evaluating the chemical potential of $K$, we first need to calculate the effective mass of $K$, $m_K^*$. In subsection III B we show how we calculate $m_K^*$ in two different methods and estimate the magnitude of the optical potential of $K^-$, which is under debate by different authors [18, 19, 20, 21, 22, 23].

In Sect. III, we determine the bag parameters of the nucleon and a $\Theta^+$ from their free masses. The coupling constants between $u$ ($d$) quarks and mesons are adjusted to reproduce the known saturation properties. In Sect. III, we show the results of $m_{\Theta^+}^*$ from QMC and MQMC models. Dependence of the effective mass $m_{\Theta^+}^*$ on the coupling constant $g_{\sigma}^{s}$ between the $s$-quark and $\sigma$ is presented. The in-medium decay of a $\Theta^+$ to kaon-nucleon state is discussed from the consideration of chemical potentials of a $\Theta^+$ and a $K_N$ system. A summary is given in Sect. IV.
II. MODELS

A. QMC and MQMC models for nuclear matter

In the QMC model, the nucleons in nuclear matter are assumed to be described by static MIT bags in which quarks couple to effective meson fields, which are treated as classical in a mean field approximation. The quark field $\psi_q$ inside the bag then satisfies the Dirac equation

$$\left[i\gamma \cdot \partial -(m_q - g^a_q \sigma) - g^a_q \gamma^0 \omega_0 \right] \psi_q = 0,$$

where $m_q$ is the bare mass of the quark and $\sigma$ and $\omega_0$ are the mean fields of $\sigma$- and $\omega$-mesons, respectively. We assume $m_q = 0$ for $q = u$ and $d$ and treat the strange quark mass $m_s$ as a free parameter. The ground state solution to the Dirac equation is given by

$$\psi(r, t) = N_q \exp(-i \epsilon_q t/R) \left( \begin{pmatrix} j_0(x_q r/R) \\ i \beta_q \sigma \cdot \hat{r} j_1(x_q r/R) \end{pmatrix} \right) \frac{x_q}{\sqrt{4\pi}},$$

where $N_q$ is the normalization factor, $R$ is the bag radius and

$$\epsilon_q = \frac{\Omega_q - R m^*_q}{\Omega_q + R m^*_q},$$

$$\beta_q = \sqrt{\frac{\Omega_q - R m^*_q}{\Omega_q + R m^*_q}},$$

$$\Omega_q = \sqrt{x^2_q + (R m^*_q)^2},$$

$$m^*_q = m_q - g^a_q \sigma.$$  

$\chi_q$ is the quark spinor and $x_q$ is determined from the boundary condition on the bag surface

$$j_0(x_q) = \beta_q j_1(x_q).$$

The energy of the nucleon bag with the ground state quarks is given by

$$E_N = \sum_q \frac{\Omega_q}{R} - \frac{Z_N}{R} + \frac{4}{3\pi} R^3 B_N,$$

where $B_N$ and $Z_N$ are the bag constant and a phenomenological constant for the zero-point motion of the nucleon, respectively. The mass of a free nucleon (or the effective mass of a nucleon in matter) is given by

$$m^*_N = \sqrt{E_N^2 - \sum_q \left( \frac{x_q}{R} \right)^2}.$$
We consider three values of $R_0$ (= 0.6, 0.8 and 1.0 fm) as the bag radius of a free nucleon. For each $R_0$ value, $B_N$ and $Z_N$ can be determined from the minimum condition

$$\frac{\partial m^*_N}{\partial R} = 0$$  \hspace{1cm} (10)

by taking $m^*_N$ as $m_N = 939$ MeV. The values of $B_N^{1/4}$ and $Z_N$ are determined to be (188.1 MeV, 2.03), (157.5 MeV, 1.628) and (136.3 MeV, 1.153) for $R_0=0.6$, 0.8, and 1.0 fm, respectively. When Eq. (10) is applied for a nucleon in matter, we obtain the effective bag radius $R$ as well as the effective mass of nucleons $m^*_N$.

The self-consistency condition (SCC) for $\sigma$-meson fields is obtained through the thermodynamic condition

$$\frac{\partial \varepsilon}{\partial \sigma} = 0$$  \hspace{1cm} (11)

with the energy density of the matter

$$\varepsilon = \frac{1}{2} m^2_{\sigma} \sigma^2 + \frac{1}{2} m^2_{\omega} \omega^2_0 + 4 \int_0^{k_F} \frac{d^3 k}{(2\pi)^3} \sqrt{k^2 + m^*_N},$$  \hspace{1cm} (12)

where $k_F$ is the Fermi momentum of the nucleons at a given density. Then the effective mass $m^*_N$ can be calculated self-consistently by solving the boundary condition of Eq. (7), the minimum condition of Eq. (10), and the SCC of Eq. (11). The quark-meson coupling constants $g^u_\sigma$ and $g^u_\omega$ are fixed to reproduce the saturation properties; $\rho_0$ and $E_b$. The coupling constants thus fixed, the resulting $m^*_N$, and the compression modulus $K$ from the QMC model are listed in Table I.

The MQMC model for nuclear matter takes into account the density dependence of $B_N$, which may be expressed as

$$B_N(\sigma) = B_N \left(1 - g^N_\sigma \frac{4 \sigma}{\delta m_N}\right)^\delta.$$  \hspace{1cm} (13)
We take the limit $\delta \to \infty$, in which case
\[ B_N(\sigma) = B_N \exp(-4g_\sigma^N \sigma/m_N). \] (14)
In this model we have an extra parameter $g_\sigma^N$ to be determined. We use the following two constraints at the saturation to fix $g_\sigma^N$.

\[ m_N^* = (0.7 - 0.8)m_N, \quad K = (200 - 300) \text{MeV}. \]

For all $R_0$ values, we have used a fixed value of $g_\sigma^u = 1$ in this work.

The self-consistency condition for the MQMC model is also obtained from Eq. (11). It has additional terms due to the density dependent bag constant. The coupling constants $g_\sigma^N$ and $g_\sigma^u$ and the resulting $m_N^*$ and $K$ in the MQMC model are also listed in Table II.

**B. QMC and MQMC models for $\Theta^+$ in medium**

We may regard a $\Theta^+$ in medium also as an MIT bag, and write its effective mass as
\[ m_{\Theta^+}^* = \sqrt{E_{\Theta^+}^2 - \sum_{i=q,d} \left( \frac{x_i^2}{R} \right)^2}, \] (15)
where the bag energy of a $\Theta^+$ is given by
\[ E_{\Theta^+} = \sum_{i=q,d} \frac{\Omega_i}{R} - \frac{Z_{\Theta^+}}{R} + \frac{4}{3} \pi R^3 B_{\Theta^+} \] (16)
\[ \Omega_i = \sqrt{x_i^2 + (Rm_i^*)^2}, \] (17)
\[ m_i^* = m_i - g_\sigma^i \sigma, \] (18)
in which $i$ includes $\bar{s}$ as well as $u$ and $d$ quarks. For fixed $R_0$ values, $B_{\Theta^+}$ and $Z_{\Theta^+}$ can be determined from the observed value of the free $\Theta^+$ mass ($m_{\Theta^+} = 1540 \text{MeV}$) and a minimum condition similar to Eq. (10) but for $\Theta^+$. Since the value of $m_s$ is not well known, we extract $B_{\Theta^+}$ and $Z_{\Theta^+}$ for a few different values of $m_s$ and $R_0$. The results are shown in Table III. After fixing the bag parameters for $\Theta^+$ as above, we calculate its effective mass $m_{\Theta^+}^*$ in nuclear matter by using Eq. (15). $m_{\Theta^+}^*$ deviates from $m_{\Theta^+} = 1540 \text{MeV}$ since $\sigma$ in Eq. (18) is non-zero.

If we consider the case when the bag constant $B_{\Theta^+}$ may depend on the matter density, we refer to it as the MQMC model for a $\Theta^+$. Using the form of Eq. (14), we express $B_{\Theta^+}$ as
\[ B_{\Theta^+}(\sigma) = B_{\Theta^+} \exp(-4g_\sigma^{R_B} \sum_{q=u,d} n_q \sigma/m_{\Theta^+}) \] (19)
TABLE II: \( B_{\Theta^+} \) and \( Z_{\Theta^+} \) values for a \( \Theta^+ \) when \( R_0 = 0.6, 0.8 \) and 1.0 fm and \( m_s = 50, 150 \) and 300 MeV. \( B_{\Theta^+}^{1/4} \) is in MeV. In all the cases, \( m_u = m_d = 0 \).

| \( R_0 (\text{fm}) \) | \( m_s = 50 \) MeV | \( m_s = 150 \) MeV | \( m_s = 300 \) MeV |
|----------------------|-------------------|-------------------|-------------------|
|                      | \( B_{\Theta^+}^{1/4} \) | \( Z_{\Theta^+} \) | \( B_{\Theta^+}^{1/4} \) | \( Z_{\Theta^+} \) |
| 0.6                  | 219.6             | 4.557             | 217.9             | 4.653             |
| 0.8                  | 182.3             | 3.805             | 180.2             | 3.941             |
| 1.0                  | 156.8             | 2.943             | 155.4             | 3.122             |

TABLE III: \( m_{\Theta^+}^*/m_{\Theta^+} \) at the saturation density for three different bare masses of the strange quark. The table shows \( m_{\Theta^+}^*/m_{\Theta^+} \) is insensitive to the choice of \( m_s \) and \( R_0 \). \( g_s^* = 0 \) is used for the calculations shown in this table. \( m_s \) is in MeV.

| \( R_0 (\text{fm}) \) | \( m_s = 50 \) | \( m_s = 150 \) | \( m_s = 300 \) | \( m_s = 50 \) | \( m_s = 150 \) | \( m_s = 300 \) |
|----------------------|----------------|----------------|----------------|----------------|----------------|----------------|
|                      | \( m_{\Theta^+}^*/m_{\Theta^+} \) in QMC | \( m_{\Theta^+}^*/m_{\Theta^+} \) in MQMC |
| 0.6                  | 0.90            | 0.90            | 0.90            | 0.82            | 0.83            | 0.84            |
| 0.8                  | 0.91            | 0.91            | 0.91            | 0.82            | 0.83            | 0.84            |
| 1.0                  | 0.92            | 0.92            | 0.92            | 0.82            | 0.83            | 0.84            |

where \( \sum n_q = 4 \) for a \( \Theta^+ \) bag and the extra parameter \( g_s^B \) can be related to \( g_s^N \) of Eq. (14) by \( g_s^B = g_s^N / 3 \) with \( g_s^N \) given in Table I.

As noted in the Introduction, the nuclear matter properties at the saturation can fix the parameters related to \( u- \) and \( d \)-quarks, but not \( s \)-quarks. We thus treat \( g_s^s \) as a free parameter in the following and show our results for three choices of \( g_s^s \) values.

III. RESULTS

A. Effective mass of \( \Theta^+ \)

Table III shows \( m_{\Theta^+}^* \) at the saturation density divided by its free mass \( m_{\Theta^+} \) for different bag radii \( R_0 \) and \( s \)-quark masses \( m_s \). \( m_{\Theta^+}^*/m_{\Theta^+} \) values are rather stable against the variation of \( R_0 \) and \( m_s \). In the present models, a change in mass is caused by the non-zero value of
FIG. 1: Ratios of the effective radius $R$ of a $\Theta^+$ bag to $R_0$ for $m_s = 150$ MeV. $R/R_0$ from QMC(MQMC) calculations are plotted by thick(thin) curves. $g_\sigma^s = 0$ is used for calculations in this figure.

the $\sigma$-field. Table II shows that $m_N^*/m_N$ at the saturation density turns out to be more or less independent of $R_0$, which reflects the fact that the value of $\sigma$-field is nearly independent of $R_0$ in both QMC and MQMC models. The effective bag radius $R$ in matter deviates from $R_0$ and $x_q$ also changes from that of a free bag due to non-zero value of the $\sigma$-field. However, as Fig. I shows, the ratio $R/R_0$ depends little on the choice of $R_0$. We also find that the change of $x_q$ value in medium is only a few % from its value for a free bag. Since these three density-dependent quantities, $\sigma$-field, $R/R_0$, and $x_q$ are rather independent of $R_0$ and $m_s$, $m_{\sigma+}^*$ remains more or less the same regardless of the values of $R_0$ and $m_s$ for both QMC and MQMC models.

Fig. I also shows that the effective radii of $\Theta^+$ from MQMC (QMC) models increase (decrease) with density of matter. The increase of bag radii for MQMC models is of course due to the change of the bag constant in medium. The radius increases roughly by 15% at the saturation density, which may be compared with the results from previous studies for nucleons [28, 29, 30].

In quark models, the $s$-quark – meson coupling constants are usually set to zero, which leads to the relation between the coupling constants, $g_{MY} = \frac{2}{3} g_{MN}$ where $g_{MY}$ is the meson-
FIG. 2: $m_{s+}^{*}/m_{s+}$ is plotted against density for $g_\sigma^s = -2, 0, 2$. Thick (thin) curves denote $m_{s+}^{*}/m_{s+}$ calculated from QMC (MQMC) for both $\Theta^+$ and nuclear matter.

The hyperon coupling constant and $g_{MN}$ the meson-nucleon constant. However, as mentioned in the Introduction, there are studies [18, 19, 20, 21] indicating that $g_{MY}$ may have a value different from $\frac{2}{3} g_{MN}$, which means $g_\sigma^s$ or $g_\omega^s$ can be non-zero. In view of this uncertainty in the strange sector, we choose $g_\sigma^s$ as $-2, 0$ and $2$ and calculate $m_{s+}^{*}$ for these choices. Some consequences of the non-zero $g_\sigma^s$ value will be discussed in the next subsection in connection with the optical potential of a kaon and its effective mass in nuclear medium.

Fig. 2 shows $m_{s+}^{*}/m_{s+}$ calculated for $m_s = 150$ MeV and $R_0 = 1$ fm at nuclear densities up to $2.5 \rho_0$. (As shown in Table III, other choices of $m_s$ or $R_0$ values give us similar results of $m_{s+}^{*}$.) The solid, dashed and dotted lines correspond to the results with $g_\sigma^s = -2, 0$ and $2$, respectively. The upper three thick lines represent the results from QMC, and the lower three thin lines are those from MQMC. The biggest change in the mass from the QMC model is about 12 % in the region of densities we consider. $m_{s+}^{*}$ from the QMC model does not change much with density and $g_\sigma^s$. This weak dependence on the density and $g_\sigma^s$ is due to small magnitudes of the $\sigma$-field. However, this small value of $\sigma$-fields cannot account for the spin-orbit phenomenology in nuclei. The MQMC model [17] increases the value of $\sigma$-fields as much large as in quantum hadrodynamics [31]. Due to the large $\sigma$-fields, $m_{s+}^{*}$ from the MQMC model drops rapidly with density, and the dependence of $m_{s+}^{*}$ on the $g_\sigma^s$
value is pronounced. Another reason for the mass reduction in MQMC is the following. As the $\sigma$-field increases with density, the bag constant in Eq. [19] decreases. Since the bag constant term contributes positively to the effective mass, a smaller bag constant will result in a smaller effective mass.

### B. Kaonic decay of $\Theta^+$ in medium

Let us consider the in-medium decay of a $\Theta^+$ into $KN$ channels ($K^+n$ and $K^0p$). Since $K^+ (u\bar{s})$ and its isospin partner $K^0 (d\bar{s})$ interact identically with symmetric nuclear matter, we do not distinguish the two decay channels. If a $\Theta^+$ at rest in free space decays into $KN$, the momenta of $K$ and $N$ are $|\vec{p}_K| = |\vec{p}_N| \approx 270$ MeV/c. If the masses and energies of $K, N$ and $\Theta^+$ do not change in medium from those in free space, a $\Theta^+$ in matter at the saturation density may decay into $KN$ because the momentum of an outgoing $N$ is above the Fermi momentum. However, since the properties of $K, N$ and $\Theta^+$ may change, the chemical potentials of these particles in medium need to be considered. If the chemical potential of a $\Theta^+$ is larger than the sum of those of $K$ and $N$, a $\Theta^+$ in medium can decay into $KN$ channels. In this subsection, we use only the MQMC model in treating both nuclear matter and $\Theta^+$.

In calculating the chemical potential and the effective mass of a kaon, we can treat a kaon not only as a point particle but also as a meson bag. By doing that, we may associate the optical potential of a kaon with $g_{\sigma K}$, the coupling constant between a kaon and a $\sigma$-meson, as we will explain shortly. Among the mean field Lagrangians [32, 33] suggested for the in-medium kaons, we employ the Lagrangian given by Ref. [33]

$$\mathcal{L}_K = D^*_\mu K^* D^\mu K - m^*_K K^* K,$$

where $K$ denotes the isospin doublet kaon field. The covariant derivative in the symmetric matter $D_\mu = \partial_\mu + ig_{\omega K} \omega_\mu$ couples the kaon field to the $\omega$-meson fields, and $g_{\omega K}$ is assumed to follow the quark counting rules, i.e., $g_{\omega K} = g_{\omega}^u$. The kaon effective mass $m^*_K$ includes the interaction of $K$ with the $\sigma$-meson and is calculated in the following two methods.

First, we may treat a kaon as a meson bag. Then the effective mass of a kaon can be determined from the following equations:

$$m^*_K = \sqrt{E^2 - \sum_{i=q\bar{q}} \left( \frac{x_i}{R} \right)^2},$$
FIG. 3: The effective masses of a kaon as a point particle and a meson bag are plotted by the solid and the dashed curves, respectively. The dashed and dotted curves denotes $m_{K}^{*}$ when we choose $U_{K}(\rho_{0})$ as $-100$ and $-150$ MeV, respectively.

\[
E_K = \sum_{i=q,\bar{q}} \frac{\Omega_i}{R} - \frac{Z_K}{R} + \frac{4}{3} \pi R^3 B_K. \tag{22}
\]

Here we assume $g_{\sigma}^{s} = 0$ and choose the bag radius of a kaon as $R_{K0} = 0.6$ fm and the $s$-quark mass as $m_s = 150$ MeV. (The results for other $g_{\sigma}^{s}$ values may be inferred from Fig. 2.) By requiring the mass of a free $K^{+}$ to be $m_K = 494$ MeV, we get $B_{K}^{1/4} = 141.65$ MeV and $Z_K = 1.086$. For a kaon in matter, the bag constant for the kaon may be assumed to be

\[
B_K(\sigma) = B_K \exp \left( -\frac{4}{3} g_{\sigma}^{N} \frac{\sigma}{m_K} \right), \tag{23}
\]

where the factor 3 in denominator is from the quark counting rule. Using the same MQMC model parameters for nuclear matter as given in Table I we obtain $m_{K}^{*}$ plotted by the solid curve in Fig. 3.

Let us now treat a kaon as a point particle. Then the effective mass is given as

\[
m_{K}^{*} = m_K - g_{\sigma K} \sigma. \tag{24}\]

To fix $g_{\sigma K}$, the optical potential for an antikaon ($K^{-}$) in medium is employed. Note that $g_{\sigma K}$ for $K^{+}$ is the same as that for $K^{-}$ as can be seen from $\mathcal{L}_K$ in Eq. (20). The real part
of the optical potential for antikaons at the saturation density $U_{\bar{K}}(\rho_0)$ is given by

$$U_{\bar{K}}(\rho_0) = -g_{\sigma\bar{K}} \sigma(\rho_0) - g_{\omega\bar{K}} \omega_0(\rho_0)$$  \hspace{1cm} (25)$$

in our model. With $g_{\omega\bar{K}} = g_{\omega}^u$ (See Table I.), we may fix $g_{\sigma\bar{K}}$ so that $U_{\bar{K}}(\rho_0)$ calculated by Eq. (25) becomes a certain value that we choose. As mentioned in the Introduction, the value of the real part of $K^-$ optical potential in matter is a matter of debate in relation to the existence of a deeply bound kaonic nuclear system. We choose $U_{\bar{K}}(\rho_0) = -100$ MeV because this value makes $m_{\bar{K}}^*$ from Eq. (24) agree with $m_{\bar{K}}^*$ of Eq. (21), which treats a kaon as a meson bag. $U_{\bar{K}}(\rho_0) = -100$ MeV gives us $g_{\sigma\bar{K}} = 1.997$ from Eq. (25).

In Fig. 3, $m_{\bar{K}}^*$ obtained for a point kaon with $g_{\sigma\bar{K}} = 1.997$ is shown by the dashed curve. The dashed curve almost overlaps with the solid curve, which represents $m_{\bar{K}}^*$ for a kaon bag with $g_{\sigma}^s = 0$. Roughly speaking, the optical potential value $U_{\bar{K}}(\rho_0) = -100$ MeV corresponds to the $g_{\sigma}^s$ and $g_{\omega}^u$ values given by the quark model. A larger magnitude of $U_{\bar{K}}(\rho_0)$ value corresponds to a larger $g_{\sigma}^s$ value, and vice versa. Fig. 3 also shows by the dotted curve $m_{\bar{K}}^*$ of a point particle kaon when we choose $U_{\bar{K}} = -150$ MeV. $m_{\bar{K}}^*$ for $U_{\bar{K}} = -150$ MeV at the saturation is smaller than $m_{\bar{K}}^*$ for $U_{\bar{K}}(\rho_0) = -100$ MeV by about 10 %, and the difference gets larger as the density increases. This shows $m_{\bar{K}}^*$ is sensitive to the value of the optical potential at the saturation. We find that the $g_{\sigma}^s$ value that gives us $m_{\bar{K}}^*$ nearly overlapping with the dotted curve is $g_{\sigma}^s = 2.6$. $m_{\bar{K}}^*$ plotted by the dotted curve is expected to get extremely small at higher densities, and may cause kaon condensation in a neutron star. Such phenomena will be discussed elsewhere.

Let us now compare the chemical potential of a $\Theta^+$, $\mu_{\Theta^+}$, with $\mu_K + \mu_N$ of the $KN$ system. The equation of motion for the kaon can be written as $[D_\mu D^\mu + m_{\bar{K}}^*]^2 K = 0$. Using a plane wave for the kaon field, one obtains the following dispersion relation with the upper (lower) sign for kaons (antikaons) in uniform and symmetric matter

$$\epsilon_{K,\bar{K}} = \sqrt{k_K^2 + m_{\bar{K}}^2} \pm g_{\omega K} \omega_0,$$  \hspace{1cm} (26)$$

where $k_K$ is the momentum of the kaon. The distortion of the kaon field in matter will smear $\epsilon_K$ around this value. We can then calculate the chemical potential of kaons as $\mu_K = \epsilon_K$. The chemical potential of a nucleon and a $\Theta^+$ can be written as

$$\mu_N = g_{\omega N} \omega_0 + \sqrt{k_N^2 + m_N^*},$$

$$\mu_{\Theta^+} = g_{\omega \Theta^+} \omega_0 + \sqrt{k_{\Theta^+}^2 + m_{\Theta^+}^*},$$
where $g_{\omega N} = 3g_{u\omega}$ and $g_{\omega\Theta^+} = 4g_{u\omega}$ are used following the quark counting rule. Here we assume $k_{\Theta^+} = 0$ and consider only a stationary $\Theta^+$ particle for simplicity.

FIG. 4 shows that $\mu_{\Theta^+}$ (the solid curve) decreases with density at low densities due to the decrease of $m_{\Theta^+}^*$ as shown in Fig. 2. At low densities both $\sigma$- and $\omega$-meson fields are approximately proportional to the density, and competition between attraction and repulsion produces the nuclear saturation. At high enough densities, $\sigma$-meson fields are roughly proportional to $\rho^{2/3}$, while $\omega$-meson fields are still proportional to $\rho$. Thus as the density increases, repulsion becomes stronger than attraction. As a result, $\mu_{\Theta^+}$ eventually increases at high densities as shown in Fig. 4. For the chemical potential of $KN$ system, $\mu_N + \mu_K$, we show two cases; one for $k_K = 0$ and the other for $k_K = k_F$ where $k_F$ is the Fermi momentum of nucleons. $k_K = 0$ gives us the minimum value of $\mu_N + \mu_K$ plotted by the dotted curve. $\mu_N + \mu_K$ with $k_K = k_F$ is plotted by the dashed curve. In the decay of a single $\Theta^+$ to $KN$, a kaon should have momentum at least equal to $k_F$ of nucleons due to Pauli blocking and momentum conservation. However, if we consider the decay of a $\Theta^+$ through $N\Theta^+ \to KN$ or $N\Theta^+ \to NNK$, a kaon can have zero momentum without violating the momentum
Let us denote the density at the intersection between the solid curve and the dashed (dotted) curve by $\rho_1$ ($\rho_2$): $\rho_1 \approx 0.75 \rho_0$ and $\rho_2 \approx 1.95 \rho_0$. At low densities where $\rho < \rho_1$, $\mu_{\Theta^+}$ is greater than $\mu_K + \mu_N$ with $k_K = k_F$, and thus both $\Theta^+ \to KN$ and $N\Theta^+ \to KNN$ channels are opened. At $\rho_1 < \rho < \rho_2$, $\Theta^+ \to KN$ channel is forbidden due to Pauli blocking, but $N\Theta^+ \to KNN$ channel is open. Thus, if a $\Theta^+$ is created inside a heavy nucleus with normal nuclear density, a $\Theta^+$ can only decay through $N\Theta^+ \to KNN$. But if a $\Theta^+$ is created at the surface of a nucleus where $\rho < \rho_1$, $\Theta^+ \to KN$ may occur. At higher densities where $\rho > \rho_2$, a $\Theta^+$ may be found to be stable because both $KN$ and $KNN$ decay channels are forbidden. However, if a $\Theta^+$ has non-zero momentum, $\mu_{\Theta^+}$ increases in the form of $\sqrt{k_{\Theta^+}^2 + m_{\Theta^+}^*}$ and the $\mu_{\Theta^+}$ curve will move upward. In that case, both $\rho_1$ and $\rho_2$ will increase, and there will be significant changes in the decay scheme. Stability of a $\Theta^+$ depends not only on the various interactions but also on the kinematic conditions.

IV. SUMMARY

In this work we have considered the in-medium mass of the pentaquark baryon $\Theta^+$ and its decay in matter. We have employed the quark-meson coupling model and the modified quark-meson coupling model for the description of nuclear matter. As usual, the bag model parameters $B_N$, $Z_N$, $B_{\Theta^+}$, $Z_{\Theta^+}$, $B_K$ and $Z_K$ are fixed to reproduce the masses of $N$, $\Theta^+$ and $K$, respectively, and other QMC model parameters ($g_{\sigma}^n$, $g_{\omega}^n$ and $g_{\sigma}^N$) are determined from the nuclear saturation properties. Coupling constants related to the $\omega$-meson $g_{\omega N}$, $g_{\omega K}$, and $g_{\omega \Theta^+}$ are fixed by the quark counting rules. We treat the bag radius ($R_0$), the valence quark mass of the $s$-quark ($m_s$), and the $\sigma$-meson – $s$-quark coupling constant ($g_{\sigma}^s$) as free parameters. The effective mass of a $\Theta^+$ remains more or less constant against the variation of $R_0$ and $m_s$, but it changes considerably with $g_{\sigma}^s$. The mass shift is about 10% or more at the saturation density, and the amount of mass reduction varies widely depending on the models.

The effective mass of a kaon as a meson bag is compared with that as a point particle kaon. They are in good agreement with each other if the real part of the optical potential of antikaon, $U_{\bar{K}}(\rho_0)$, is chosen as $-100$ MeV and if usual quark model coupling constants ($g_{\sigma}^s = 0$) are used. In this case, we find $g_{\sigma K} = 1.997$. The chemical potentials of the
$KN$-system and $\Theta^+$ are calculated to discuss the stability of a $\Theta^+$ in medium. Our results indicate that a stable state of a $\Theta^+$ in nuclear medium is not excluded, but the stability is sensitive to the coupling constants ($g_{\sigma}^s$, $g_{\sigma K}$, $g_{\omega K}$, $g_{\omega \Theta^+}$) and the momenta of particles in the initial and final states. In this work, we have considered only $KN$ channels as decay modes, but other possible modes have to be included for a better understanding.

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[1] D. Strottman, Phys. Rev. D 20 (1979) 748.
[2] D. Diakonov, V. Petrov and M. V. Polyakov, Z. Phys. A 359 (1997) 305.
[3] T. Nakano et al., Phys. Rev. Lett. 91 (2003) 012002.
[4] V. V. Barmin et al., Phys. Atom. Nucl. 66 (2003) 1715.
[5] S. Stepanyan et al., Phys. Rev. Lett. 91 (2003) 252001.
[6] J. Barth et al., Phys. Lett. B572 (2003) 127.
[7] K. Hicks, hep-ex/0412048.
[8] J. Randrup, Phys. Rev. C 68 (2003) 031903.
[9] J. Letessier, G. Torrieri, S. Steinke and J. Rafelski, Phys. Rev. C 68 (2003) 061901(R).
[10] L. W. Chen, V. Greco, C. M. Ko, S. H. Lee and W. Liu, Phys. Lett. B601 (2004) 34.
[11] G. Agakichiev et al., Phys. Lett. B422 (1998) 405.
[12] K. Ozawa et al., Phys. Rev. Lett. 86 (2001) 5019.
[13] D. Trnka et al., nucl-ex/0504010.
[14] H.-Ch. Kim, C.-H. Lee and H.-J. Lee, J. Korean Phys. Soc. 46 (2005) 393.
[15] H. Shen and H. Toki, nucl-th/0410072.
[16] P. A. M. Guichon, Phys. Lett. B200 (1988) 235.
[17] X. Jin and B. K. Jennings, Phys. Lett. B374 (1996) 13; Phys. Rev. C 54 (1996) 1427.
[18] H. Shen and H. Toki, Nucl. Phys. A707 (2002) 469.
[19] Y. Fujiwara, C. Nakamoto and Y. Suzuki, Prog. Theor. Phys. 94 (1995) 65; Phys. Rev. Lett. 76 (1996) 2242.
[20] E. Friedman, A. Gal and C. J. Batty, Nucl. Phys. A579 (1994) 578.
[21] A. Ramos and E. Oset, Nucl. Phys. A671, (2000) 481.
[22] Y. Akaishi and T. Yamazaki, Phy. Rev. C 65 (2002) 044005.
[23] A. Dote, H. Horiuchi, Y. Akaishi and T. Yamazaki, Phys. Rev. C 70 (2004) 044313.
[24] T. Suzuki et al., Phys. Lett. B597 (2004) 263.
[25] T. Suzuki et al., nucl-ex/0501013
[26] S. Fleck, W. Bentz, K. Shimizu and K. Yazaki, Nucl. Phys. A510 (1990) 731.
[27] S. Pal, M. Hanauske, I. Zakout, H. Stöcker and W. Greiner, Phys. Rev. C 60 (1999) 015802.
[28] P. B. Siegel, W. B. Kaufmann and W. R. Gibbs, Phys. Rev. C 31 (1985) 2184.
[29] G. E. Brown, C. B. Dover, P. B. Siegel and W. Weise, Phys. Rev. Lett. 60 (1988) 2723.
[30] Y. Mardor et al., Phys. Rev. Lett. 65 (1990) 2110.
[31] J. Boguta and A. R. Bodmer, Nucl. Phys. A292 (1977) 413.
[32] J. Schaffner, I. N. Mishustin and J. Bondorf, Nucl. Phys. A625 (1997) 325.
[33] N. K. Glendenning and J. Schaffner, Phys. Rev. C 60 (1999) 025803.