1 Introduction

Ever since Witten’s paper in Strings’95 [1] there has been a lot of discussion on dualities [2] and hidden dimensions [3]-[8] as properties of an underlying secret theory that includes superstring theory and its super p-brane and D-brane [9] generalizations. The secret theory has been called M-theory [5], F-theory [7], S-theory [10], Y-theory [11] ... where each version emphasizes some aspects of the underlying theory. In this talk I will describe the higher algebraic structures that define S-theory.

S-theory [10] concentrates on certain global aspects of the secret theory based on a superalgebra that is common to all the incarnations of the secret theory. It provides an algebraic approach to study the secret theory through representation theory of the superalgebra. Dualities, hidden dimensions p-branes etc. are some of the properties that are manifest in S-theory.

The superalgebra has 32 fermionic and 528 bosonic generators. The maximum number of supercharges in a physical theory is 32. This constraint comes from 4-dimensions, which admits at the most 8 supercharges, since there can be no supermultiplet of interacting massless particles with spins higher than 2. The collection of all bosonic operators form a $32 \times 32$ symmetric matrix $S$ (528 hermitian generators) given by the anticommutator of the supercharges $\{Q, Q\} \sim S$. The representations of the superalgebra are intimately connected to the properties of $S$. The structure and symmetries of $S$ are related to p-branes, dualities and hidden dimensions. Global properties, certain states and certain non-perturbative properties of the underlying

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theory may be studied by analyzing the representations of this superalgebra. This
line of investigation is called S-theory.

The 528 bosonic generators include momenta $P_\mu$ and central extensions. The
Lorentz scalar central extensions correspond to charges of zero-branes, i.e. particles. The
Lorentz non-scalar central extensions $Z_{\mu_1\cdots \mu_p}^{ab}$ are related to boundaries of
p-branes (for example for an open string, $p=1$, $Z_{\mu_1} = X_{\mu_1}(0) - X_{\mu_1}(\pi)$ is the
difference of end point positions [3]). In the simplest form of S-theory p-branes propagate in flat
space. Then all 528 generators commute with each other and they can be simultaneously
diagonalized. Central extensions are then on an equal footing with momenta, and therefore “momentum space” is enlarged in the secret theory. One may think of
this as a generalization of the concept of flat spacetime. The representations of the
superalgebra are then easy to construct. The space of states include long supermultiplets as well as shorter multiplets of various kinds, corresponding to BPS-like states [10]. Algebraically, the flat version of S-theory corresponds to a contraction of the
superalgebra $\text{OSp}(1/32)$ such that all bosonic generators commute. This is achieved
by rescaling the $\text{OSp}(1/32)$ bosonic generators and then taking the contraction limit.

In this talk I will concentrate mainly on S-theory in flat spacetime. At the end, I
will indicate possible directions that may be pursued to consider S-theory in curved
compactified spaces or curved spacetime, and the relevance of other contractions of
$\text{OSp}(1/32)$, $\text{OSp}(1/64)$, etc...

2 The extended superalgebra

In an arbitrary number of dimensions, we label the 32 supercharges as

$$Q_\alpha^a = \begin{cases} 
  a = 1, 2, \cdots N; & \text{spinor in } c+2 \text{ dims.} \\
  \alpha = \text{spinor in } d \text{ dims.}
\end{cases}$$

(1)

Here $N = 1$, in $d=11$; $N = 2$, in $d=10$; $\cdots$, $N = 8$, in $d=4$, and $c=$number of
compactified string dimensions, such that $d + c = 10$. By interpreting $a$ as the label
of a spinor in $c + 2$ dimensions we begin to see that there are at least 2 hidden
dimensions. $a$ is also a label for an irreducible representation of the group $K$, where
$K$ is the maximal compact subgroup of $U$-duality. For example for $d = 4$, $c = 6$, $N = 8$, the index $a$ is interpreted either as the complex $8$ (or $8^*$) of $K = SU(8) \subset E_{7,7}$

or as the spinors $8_\pm$ of $SO(c+1, 1) = SO(7, 1)$. The $K$ classification gives information
about dualities while the $SO(c+1, 1)$ classification gives information about the hidden
dimensions [3] [10]. The 528 generators as well as all the states of the theory can be
classified in either way. The two classifications are not contained in each other. If
one classifies according to duality one loses track of the hidden dimensions and vice versa. Completeness implies that the collection of representations in one classification
can be expanded into a collection of representations in the other classification. This
observation can be used as a tool for finding some of the states of the secret theory by starting from the well known string states [3][6].

The extended superalgebra in \(d\)-dimensions has the form

\[
\{ Q^{a}, Q^{b} \} = ( S )^{ab}_{\alpha \beta} + \sum_{p=0,1,2,\cdots} \gamma^{\mu_{1}\cdots \mu_{p}} \delta_{\alpha \beta} \gamma_{\mu_{1}\cdots \mu_{p}} Z^{ab}_{\mu_{1}\cdots \mu_{p}},
\]

where \( P_{\mu} \) is the momentum operator and \( Z^{ab}_{\mu_{1}\cdots \mu_{p}} \) are the central extensions. It can be shown that (for \( d \leq 9 \)) this can be derived by compactifying either a type-A superalgebra in 12D or a type-B superalgebra in 10D \( \otimes \) 3D, both embedded in the spinor space of 13D [10]. Hence S-theory is related to both M-theory (A-type,11D) and F-theory (B-type, 12D \( \rightarrow \) 10D). The spinor space of 13D with (11,2) signature has 64 spinors. There are two distinct projections that have 32 real spinors. The type-A projection distinguishes the 13th dimension, leaving an SO(10,2) covariance. The type-B projection distinguishes 10D from 3D, leaving an SO(9,1) \( \otimes \) SO(2,1) covariance. Under compactification, the type-A(B) reduces to the type-A(B) superalgebra of type-IIA(B) string. The geometrical origin of the SL(2,R) duality symmetry of type-IIB string is the extra 3D. The T-duality symmetry between types A,B involves the mixing of the 13th dimension with the others. So there are traces of up to 13 hidden dimensions. Note that it is the spinor space of 13D that is relevant so far. As emphasized in the introduction, the meaning of “dimensions” and of “spacetime” get generalized in unusual ways (not just 13D, but 528 bosonic generators), and as noted below, there is only one time coordinate corresponding to the trace of \( S \) [10].

In 12D the extended superalgebra takes the form [3]

\[
\{ Q^{a}, Q^{b} \} = \gamma_{\alpha \beta}^{M_{1}M_{2}} Z^{M_{1}M_{2}} + \gamma_{\alpha \beta}^{M_{1}\cdots M_{6}} Z_{M_{1}\cdots M_{6}}^{+} \] (3)

where \( M = 0', \mu, \) and \( \mu = 0,1,2,\cdots 10, \) with two time-like dimensions denoted by \( M = 0', 0. \) The six-index tensor is self dual in 12D. By compactifying the 0' coordinate the 11D extended superalgebra [3] emerges

\[
Z^{M_{1}M_{2}} \rightarrow P_{\mu} \oplus Z_{\mu_{1}\mu_{2}} \quad 66 = 11 + 55
\]

\[
Z_{M_{1}\cdots M_{6}}^{+} \rightarrow X_{\mu_{1}\cdots \mu_{5}} \quad 462 = 462 \] (4)

Note that there is no translation operator \( P_{M} \) in the (10,2) version. Therefore the extension of the theory from (10,1) to (10,2) is not the naive extension that would have implied two time coordinates, since the corresponding canonical conjugate momenta are not present. There is only one time translation operator, hence there is only one time coordinate that can be recognized only in the 11D (or lower D) notation.

\[a\] One may be tempted to distinguish the 0'-direction instead of the 13th and get a projection with (11,1) signature, but this projection gives a complex 32-component spinor, which is really equivalent to 64 supercharges. This would contradict the basic theorem in 4D that prevents more than 32 supercharges.
Another remark is that the 12D Lorentz generator $L_{MN}$ does not appear on the right hand side; $Z_{M_1M_2}$ or $Z^{ab}_{\mu_1\mu_2}$ are not related to $L_{MN}$. The fermionic or bosonic generators of the superalgebra do not commute with $L_{MN}$ since they are classified as spinors or tensors under Lorentz transformations. Hence, including $L_{MN}$ would enlarge the algebra further in a trivial form.

The type-B superalgebra in 10D $\otimes$ 3D can be written as

$$\{Q^\alpha_{\bar{a}}, Q^\beta_{\bar{b}}\} = (S_B)^{\bar{a}\bar{b}}_{\alpha\beta} \tilde{g}_{\bar{a}\bar{b}} (c_{\bar{\tau}_{i}})^{\alpha\beta} P_{i}^{\mu} + \tilde{\gamma}_{\bar{a}\bar{b}}^{\bar{\mu}_{1}\bar{\mu}_{2}\bar{\mu}_{3}} c^{\bar{a}\bar{b}} Y_{\bar{\mu}_{1}\bar{\mu}_{2}\bar{\mu}_{3}} + \tilde{\gamma}_{\bar{a}\bar{b}}^{\bar{\mu}_{1}\bar{\mu}_{2}\bar{\mu}_{3}\bar{\mu}_{4}\bar{\mu}_{5}} (c_{\bar{\tau}_{i}})^{\bar{a}\bar{b}} X_{i}^{\mu_{1}\cdots\mu_{5}}. \quad (5)$$

where $\bar{\alpha}, \bar{\beta} = 1, 2, \cdots, 16$ and $\bar{a}, \bar{b} = 1, 2$ while $\bar{\mu} = 0, 1, \cdots, 9$ and $i = 0', 1', 2'$. The $X_{i}^{\mu_{1}\cdots\mu_{5}}$ are self dual in 10D. Here $\tilde{\gamma}_{\bar{a}\bar{b}}^{\bar{\mu}}$ are 16$\times$16 10D gamma matrices and the $c_{\bar{\tau}_{i}}^{ab}$ are 2$\times$2 gamma matrices in some hidden 3D Minkowski space, with $c^{ab} = i\sigma^{ab}_{2} = \varepsilon^{ab}$. This algebra is covariant under SO(1,9) $\otimes$ SO(1, 2). Using the SO(11,2) gamma matrices in [10] one can see that it can be embedded in 13 dimensions.

All bosonic operators commute with each other. In this sense they all behave like the momentum operators, and hence can be simultaneously diagonalized. The type A,B superalgebras are two versions of S-theory that are embedded in contractions of OSp(1/32) with different choices of bases that have different isometries.

### 3 Representations, BPS states, predictions

A basic hypothesis is that the superalgebra that defines S-theory is valid as a dynamical structure of the secret theory (including broken symmetries). Then certain global properties of the secret theory can be described non-perturbatively as properties of the superalgebra itself. For example the supermultiplet structure of the spectrum of the theory, relations among correlation functions or scattering amplitudes, coupling constants, etc. can be computed directly as the properties of the representations of the superalgebra. In spirit there is a similarity to the theory of current algebras used in the 60’s to explore the properties of strong and weak interactions, without knowing the details of the underlying theory now known as the Standard Model.

The physical states of the secret theory (i.e. after gauge freedom and auxiliary fields have been eliminated) must form multiplets of the superalgebra consistent with its isometries in various dimensions. For the flat version of S-theory one can work in a basis in which all 528 bosonic generators are diagonal (in the more familiar states, such as string states, most of these eigenvalues vanish, leaving only spacetime momentum, Kaluza-Klein compactified momentum and winding numbers, as the non-zero eigenvalues that specify the “base”). Simultaneously, one can specify a representation of the isometries which may be reducible (in string theory this arises by applying oscillators on the base; then at a fixed level there is a collection of representations of the rotation group and/or other compactification isometries). Then one can start
with a reference state of the form

\[ |528 \text{ eigenvalues}; \text{ representation of isometries at some level } l > \]  \quad (6)

and apply all possible polynomials of the fermionic generators in order to obtain a supermultiplet. This construction is analogous to the construction of representations of standard supersymmetry, and hence the familiar supersymmetry multiplets get generalized by replacing the momentum by the 528 bosonic generators. In the generic case one obtains the long supermultiplet of dimension \( 2^{32/2} = 2^{15}_{\text{bosons}} + 2^{15}_{\text{fermions}} \). However, under special conditions on the eigenvalues there are shorter supermultiplets. The condition for shorter multiplets is \( \det S = 0 \). If the multiplicity of the zero eigenvalue of \( S \) is \( 2n \), then \( 2n \) supercharges vanish on the reference state, and the remaining supercharges act non-trivially, producing a supermultiplet of dimension \( 2^{(32-2n)/2} \). These are the BPS-type states. In the context of S-theory there is a much richer spectrum of BPS-type states than the ones discussed previously in M- or F-theories because of the inclusion of Lorentz non-singlet central extensions with non-zero eigenvalues \([10]\).

The supermultiplet should include string states of a given level for consistency with string theory, but in addition, the collection of representations should be consistent with being expressible as a set of representations of \( K \) (duality) or as a set of representations of \( SO(c+1,1) \) (hidden dimensions). This triple constraint is extremely strong and it typically demands the inclusion of additional states, beyond the string states of a given level, in order to have a complete supermultiplet. The additional states are predictions about the secret theory. In some examples \([8]\) they have been interpreted as D-branes \([12]\), but it is not clear that the full interpretation has been given so far.

As an outcome of representation theory one can give some new results that have not been obtained in other approaches.

- It is well known that by keeping only the Lorentz singlet charges one describes black holes as BPS states. The degeneracy of these states coincides with the entropy of the black holes. This entropy is is a function of only the Lorentz singlet central extensions \([13]\) and can be expressed in terms of natural invariants of U-duality, such as \( E_{7,7}(E_{6,6}) \) in \( d = 4 \) \((5)\). What was not noticed before is that the entropy is also invariant under transformations involving hidden dimensions. This is seen by reclassifying \([3]\) the central extensions as representations of \( SO(c+1,1) \) that includes two hidden dimensions. Then the blackhole entropy can be rewritten explicitly as an invariant of \( SO(7,1) \) in \( d = 4 \) and \( SO(6,1) \) in \( d = 5 \), etc., and therefore provides evidence for 2 hidden dimensions, one spacelike and the other timelike \([15]\). The \( SO(c+1) \) subgroup is also a subgroup of \( K \subset U \), so that it is already included in U-duality. This part which indicates the presence of the 11th dimension is already expected on the basis of M-theory. However, the additional hidden timelike 12th dimension
implied in $SO(c + 1, 1)$ is not included in U-duality or in M-theory, it is a new feature discovered in the context of S-theory.

- It is well known that the special form of $S_{\alpha\beta} = \gamma_{\alpha\beta}^\mu p_\mu$, with the 11D momentum $p^2 = 0$, corresponds to a reference state (6) that gives the 11D supergravity states $2^7_{\text{bosons}} + 2^7_{\text{fermions}}$. The fields corresponding to these states depend on 11 momenta or coordinates (some of which may be compactified). Their interactions is described by 11D supergravity, which follows by requiring local supersymmetry. According to S-theory there are generalized supergravity theories that follow from other special forms of $S$. One such example is 

$$S_{\alpha\beta} = \gamma_{\alpha\beta}^{MN} (p_M p'_N - p_N p'_M), \quad p^2 = 0, \quad p \cdot p' = 0,$$

(7)

where $p, p'$ are 12D “momenta”. This special form of $S$ is related to the 11D one by an $SO(10, 2)$ boost that brings $p$ or $p'$ to a standard form. The supermultiplet based on this reference state is also the 11D supergravity multiplet, but this construction shows that this multiplet is also a basis for 12D supertransformations involving the new $SO(10, 2)$ covariant superalgebra. Hence it implies that the 11D supergravity fields can be extended to 12D by allowing them to depend on a pair of 12D “momenta” or coordinates, with the proper constraints. All this implies that, among other things, there should exist a 12D extension of supergravity of the type described here (possibly including auxiliary non-propagating fields), as a prediction of S-theory. It is therefore advisable to revive old attempts that almost succeeded to construct an $SO(10, 2)$ supergravity theory. Apparently there are renewed efforts in this direction.

- The examples above are special solutions of the condition $\det S = 0$ for the existence of BPS-like states. Other new solutions are provided in [10]. The previous 12D example, as well as the other new solutions, include central extensions that are Lorentz non-singlets. This implies that the theory that can support such a superalgebra must include p-branes. The classification of all solutions of $\det S = 0$ is tantamount to a classification of all the BPS-like states of the secret theory.

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8 After the talks that relate to the present paper and [10], the algebra (7) has been recognized to be at the basis of new supersymmetric models as reported in [16] [17]. In their case one of the vectors $p, p'$ is taken as constant, thus breaking the 12D covariance. More generally, I emphasize that the superalgebra (7) permits these vectors to be dynamical operators. From this point of view the models in [16] [17] are constructed for a fixed eigenvalue. Therefore I expect that there are generalizations of the models in [16] [17] that are fully 12D covariant. Generalized models may be constructed by using a pair of “momenta” or coordinates.
4 Curved spacetime

The examples above illustrate the methods of S-theory in flat spacetime. In more complicated versions of the secret theory p-branes propagate in curved compactified spaces (and/or non-compactified curved spacetime). Then from the point of view of S-theory some or all of the 528 generators do not commute with each other or with the fermionic generators (for example momenta do not commute in curved spacetime). The structure of the commutation rules is related to the geometry of the background. The representation theory is technically harder and more interesting, but the underlying physical concepts are similar to the flat case. Some non-Abelian versions of S-theory would be related to more intricate contractions of OSp(1/32). This can be achieved by different rescalings of various sets of bosonic and fermionic generators, and then taking various contraction limits. Details will be reported elsewhere [15].

The superalgebra may be further generalized by admitting fermionic central extensions [19]. In this case one may consider the contractions of superalgebras larger than OSp(1/32). A natural extension involves the 64 dimensional spinor space of 13D used in S-theory [10] (thus unifying types A,B superalgebras via a duality transformation involving the 13th dimension). One may then consider superalgebras with 64 fermionic generators, such as OSp(1/64) or SU(1/32) contracted to the desired form, in order to describe a further generalized version of S-theory in curved space, now involving fermionic central extensions.

Recall that in the flat spacetime the Lorentz generators do not appear on the right hand side of the anticommutator of the supercharges (i.e. they are outside of the OSp(1/32) generators). When larger (contracted) superalgebras are used for S-theory the Lorentz generators (or other isometries of the curved spacetime) may be included as part of the enlarged algebra.

It is evident that there is much to be studied algebraically in S-theory (and in its representations via field theoretic or string-like toy models) in order to learn the global properties of the secret theory. In this talk I have described just the beginning.

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