Measurement of vortex spectrum in a purely degenerate vortices array

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Abstract. The article presents a new method for determining the basic characteristics (square of amplitude and initial phase) of purely degenerate arrays of optical vortices using intensity moments. Under purely degenerate arrays, a complex beam is assumed, in which the magnitude of the square of the amplitude of the vortices, opposite in sign of the charge, is equal, and the phases take only two values 0 and π. When the frequency parameter changes in hundredths of decimal places, the internal structure of the complex beam can be significantly changed, which is well reflected by the results obtained.

1. Introduction

The possibility of separating the combined paraxial beams, including optical vortices of various orders, is currently one of the pressing issues in the field of singular optics. In connection with the opened prospects of their use in the lines of compacted transmission of optical information [1-3], in the nodes of quantum computers [4,5], in optical cryptography [6], etc.

The key element in these systems are the devices for the formation of combined beams and their division into elementary paraxial beams that carry higher order optical vortices. This problem, first raised by V. A. Soifer and M. A. Golub in the mid 80s of the 20th century (see, for example, [7] and the list of references), was continued in the landmark articles by E. Abramochkin and V. Volostnikov [8,9], who set the task of restoring images through the holographic formation of an optical vortex skeleton in combined singular beams, as well as in the works of V. Kotlyar, S. Khonina and co-authors [10–13] who developed a complex of high-quality diffractive optical elements (DOEs) and holographic filters for the generation of combined singular beams and their decomposition into The spatial spectrum of standard vortex beams of various orders, just as a prism spreads white light.

At the same time, the wide technical application of combined beams carrying optical vortices should be based on reliable methods for measuring and analyzing the structure of partial beams.

2. Beam model and measurement method

Thus, the authors of [14] proposed a new method for measuring the square of amplitudes of partial beams in an array of vortices. This method is based on recording the intensity moments of higher orders of the light beam as without destroying its internal structure. It is important to note that the general expression for the intensity moments allows us to simplify the analysis of the propagation of paraxial beams in an optical system (see, for example, [15] and references). For example, moments J_{0,1} and J_{1,0}
characterizes the "center of gravity" of the beam in the cross section, the moments \( J_{2,0} \) and \( J_{2,0} \) define

the radii of the beam waist along the axis \( x \) and \( y \), the combination of moments \( J_{3,0} \) and \( J_{0,3} \) sets

the beam twist along the axes \( x \) and \( y \). By composing a system of linear equations based on a

combination of intensity moments, we can measure the main characteristics of an array of combined

singular beams, where intensity moments \( J_{p,q} \) are on the left side of the equations, which are measured

in the experiment, and the desired quantities \( |C_n|^2 \) on the right side. But, this approach has a limitation

due to the fact that the moments of intensity are degenerate with respect to the sign of the topological

charge of optical vortices. For that reason this method cannot be used for degenerate arrays of vortices.

But there is an exception, which we called a purely degenerate state, when \( |C_n|^2 = |C_{-n}|^2 \) and the initial

phase takes two values \( \beta_m = 0, \pi \). In this case, this method is works.

The Laguerre-Gauss laser beam model \( (LG_{m,0}^n) \) in the waist plane was chosen as the basis of the

combined beam with an array of optical vortices. The complex amplitude of this beam is described by the expression

\[
\Psi(r,\varphi, z = 0) = \sum_{m=-N}^{N} C_m LG_{m,0}^n = \sum_{m=-N}^{N} \frac{C_m r^{|m|} e^{i(m\varphi+\beta_m)}}{N_m} G(r),
\]

where \( G = \exp(-r^2) \) – the Gauss function, \( \varphi \) – the azimuth angle, \( r = \sqrt{x^2 + y^2} \) – normalized radial coordinate, \( N_m = \sqrt{2^{-m-1} m! \pi} \) are the normalization factor, \( C_m \) and \( \beta_m \) – amplitudes and initial phases

of the partial beams.

3. Experiment and discussion of results

Experimental measurements of the square of the amplitudes in purely degenerate arrays of vortices were carried out on an experimental setup, which was considered detail in [14].

The magnitudes of the amplitudes were given by the expression \( C_m = (-1)^m \sin^n (am) \), where \( a \) is the frequency parameter, \( n = 1,2,3,... \). The sign \( - \) indicates a phase change on \( \pi \). The results of the theoretical calculation and experiment of the array with \( N = 21 \) are presented in Figures 1-3: Figures 1-3 illustrate the theoretical intensity distribution \( \Im(x, y) \) when the frequency parameter \( a \) changes in the hundredth decimal place and \( n = 7 \). As can be seen from Fig. 1-3, a small change in the frequency parameter \( a \) significantly changes the intensity distribution, which in turn makes is displayed to the spectrum shift of the square amplitudes of this array. Figures 4-6 illustrate intensity \( \Im(x, y) \) reconstructed after the SLM modulator, using a holographic amplitude grating, which carries the basic parameters of the array. Figures 7-9 set the experimental and theoretical vortex spectrum of the partial beams; negative values of the squares of the amplitude modulus correspond to a phase jump of \( \pi \). Note that even very small changes in the parameter are recorded experimentally in the spectrum of the vortices.
To estimate the measurement error, we used the correlation function between the initial intensity $\mathcal{I}_m(x,y)$ distribution and the experimental values $\mathcal{I}_e(x,y)$ with the vortex spectrum obtained in the experiment, according to the relation
\[ \eta = \frac{\iint S_n(x, y) S_{\exp}(x, y) dS}{\int_{00}^{\infty} j_{00}^{\exp}}. \]  

The results of the degree of correlation are located in the interval \( \eta = 0.83 \pm 0.91 \), which indicates good agreement between theory and experiment.

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