Influence of momentum-dependent interactions on balance energy and mass dependence

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Abstract

We aim to study the role of momentum-dependent interactions in transverse flow as well as in its disappearance. For the present study, central collisions involving mass between 24 and 394 are considered. We find that momentum-dependent interactions have different impact in lighter colliding nuclei compared to heavier colliding nuclei. In lighter nuclei, the contribution of mean field towards the flow is smaller compared to heavier nuclei where binary nucleon-nucleon collisions dominate the scene. The inclusion of momentum-dependent interactions also explains the energy of vanishing flow in $^{12}C + ^{12}C$ reaction which was not possible with the static equation of state. An excellent agreement of our theoretical attempt is found for balance energy with experimental data throughout the periodic table.

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1 Introduction

One of the major goals of heavy-ion collisions at intermediate energies is to study the properties of hot and dense nuclear matter formed during a collision. The nuclear equation of state (EOS) has attracted a lot of attention over the past two decades because of its wider usefulness in several branches of physics. However, it is also argued by many authors that the momentum dependent nature of equation of state can also have a significant effect in those situations where nuclear matter is mildly and weakly excited. If matter is highly compressed, the nucleon-nucleon correlations are already broken due to violent and frequent nucleon-nucleon collisions. However, if matter is either weakly or mildly excited, the momentum dependent interactions (MDI) can have sizeable effects. The initial attempts showed drastic effects of momentum dependent interactions on collective flow and particle production [1, 2]. In Ref. [1], pion yield was found to be suppressed by 30% once momentum dependent interactions were included in the evolution of the reaction. Interestingly, the momentum dependent interactions also suppressed the nucleon-nucleon collisions by the same amount. The suppression of nucleon-nucleon collisions due to momentum dependent interactions happens because of the fact that the inclusion of momentum dependent interactions accelerates nucleons in the transverse direction during the initial phase of the reaction leading to the lower density that results into fewer nucleon-nucleon collisions. Similarly, if one goes from soft to hard equation of state, the decrease in pion yield is $\approx 10\%$. Interestingly, a soft equation of state with momentum dependent interactions (dubbed as SMD) yields the same transverse momentum as static hard equation of state [1, 2]. Later on, it was shown in Refs. [3, 4] that for asymmetric reactions (e.g., Ar+Pb), the SMD explains the data better than the hard equation of state. Khoa et al. [5] also showed that the inclusion of momentum dependent interactions leads to the reduced density and temperature whereas transverse momentum observes reverse trend. From the above discussion, it is clear that the momentum dependence of the nuclear mean field plays a major role in determining the nuclear dynamics and is an important feature for the fundamental understanding of nuclear matter properties over a wide range of densities and temperatures. As discussed in detail in Chap. 3, one has tried to understand the influence of momentum dependent interactions on flow as well as on its disappearance using variety of nucleon-nucleon cross sections along with different static equations of state with an aim to pin down the nature
of equation of state as well as strength of nucleon-nucleon cross section. However, if we look carefully at the literature, one has often taken one or two reactions and tried to conclude about the above cited problems. The need of the hour is to look beyond one or two reactions. We shall concentrate here on the role of momentum dependent interactions in collective transverse flow and in its disappearance in heavy-ion collisions throughout the periodic table. Moreover, the conclusions of different authors at the same time are contradictory to each other. For example, Refs. [4, 5] showed that the flow value using SMD is higher compared to the hard equation of state for the reactions of $^{93}$Nb+$^{93}$Nb and $^{40}$Ca+$^{40}$Ca, respectively, at 400 MeV/nucleon, whereas, Ref. [6] demonstrated that the flow value using SMD is smaller compared to hard equation of state for $^{197}$Au+$^{197}$Au at 400 and 800 MeV/nucleon. References [1, 2, 3, 5] showed (for $^{139}$La+$^{139}$La at 400 and 800 MeV/nucleon and $^{93}$Nb+$^{93}$Nb at 400 MeV/nucleon), that flow values are same for the SMD and hard equation of state. On the contrary, Refs. [7, 8] showed that the flow is insensitive to the equation of state irrespective of the momentum dependent interactions. Similarly, Ref. [5] demonstrated (for $^{40}$Ca+$^{40}$Ca at 400 MeV/nucleon) that the soft equation of state gives larger flow compared to hard equation of state whereas Refs. [5, 6, 9] showed (for $^{197}$Au+$^{197}$Au at 200-800 MeV/nucleon, $^{93}$Nb+$^{93}$Nb at 400 MeV/nucleon, and $^{139}$La+$^{139}$La at 800 MeV/nucleon, respectively) that a soft equation of state gives smaller flow. Reference [4] demonstrated better agreement with the data using SMD for Ar+Pb reaction whereas Ref. [6] concluded just the opposite for $^{197}$Au+$^{197}$Au reaction. The balance energy ($E_{\text{bal}}$), (the incident energy at which the transverse flow disappears) using static and momentum dependent interactions, also faces similar contradictions [10, 11, 12]. All the above mentioned examples indicate a need for the systematic study of the influence of momentum dependent interactions on flow as well as on its disappearance throughout the periodic table to pin down the above mentioned questions and universal behaviour of momentum dependent interactions.

It is worth mentioning that the mass dependence studies have been performed in a variety of problems in heavy-ion physics. For example, the study of mass dependence in the evolution of density and temperature reveals that maximum density scales with the size of the system [5, 9, 13] whereas maximum temperature is insensitive towards the mass of the system. Similarly, multifragmentation, particle production, and collective flow (the most sensitive observable) also depend strongly on the mass of the system [13].
It has been discussed by several authors that the above mentioned observables are mass dependent and the agreement/disagreement with experimental observations depends upon the size of the reacting partner. In Chap. 3, we presented a complete theoretical analysis of balance energy using quantum molecular dynamics model where model was confronted against balance energy observed in $^{20}\text{Ne}+^{27}\text{Al}$ [14], $^{36}\text{Ar}+^{27}\text{Al}$ [15, 16], $^{40}\text{Ar}+^{27}\text{Al}$ [10], $^{40}\text{Ar}+^{45}\text{Sc}$ [14, 17, 18], $^{40}\text{Ar}+^{51}\text{V}$ [11, 19], $^{64}\text{Zn}+^{27}\text{Al}$ [12], $^{40}\text{Ar}+^{58}\text{Ni}$ [20], $^{64}\text{Zn}+^{48}\text{Ti}$ [16], $^{58}\text{Ni}+^{58}\text{Ni}$ [18, 20, 21, 22], $^{58}\text{Fe}+^{58}\text{Fe}$ [21, 22], $^{64}\text{Zn}+^{58}\text{Ni}$ [16], $^{86}\text{Kr}+^{93}\text{Nb}$ [14, 18], $^{93}\text{Nb}+^{93}\text{Nb}$ [23], $^{129}\text{Xe}+^{118}\text{Sn}$ [20], $^{139}\text{La}+^{139}\text{La}$ [23], and $^{197}\text{Au}+^{197}\text{Au}$ systems [18, 24, 25]. Apart from the above mentioned and extensively analyzed reactions, $^{12}\text{C}+^{12}\text{C}$, though, has also been subjected to balance energy, interestingly, is the content out of the discussion [14]. Therefore, one also needs to understand the dynamics involved in the balance energy of $^{12}\text{C}+^{12}\text{C}$ system. As mentioned above, some attempts are made in literature where equation of state with momentum dependent interactions is employed in heavy-ion collisions to study the balance energy [26, 27]. However, very few attempts yet exist where mass dependence of the balance energy is reported in the literature using momentum dependent interactions [26]. The mass dependence using momentum dependent interactions may have interesting physics since, the surface contribution in lighter nuclei is much larger compared to the heavier nuclei. For example, the ratio of the surface to radius is 0.12 in $^{12}\text{C}+^{12}\text{C}$ system whereas it is 0.022 in $^{197}\text{Au}+^{197}\text{Au}$ system (the surface is defined as the radial distance marked with density between 90% and 10% of its central value whereas radius is the distance where density falls to 50% of the central density). In other words, the surface effects and surface to volume ratio will be much stronger in light nuclei compared to heavy ones.

Our aim, in the present chapter, therefore, is at least twofold.

1. To study the $^{12}\text{C}+^{12}\text{C}$ system for the collective transverse flow and its disappearance.

2. To understand the role of momentum dependent interactions in the collective transverse flow as well as in its disappearance throughout the periodic table in central collisions and to analyze whether mass dependence can be presented in terms of some scaling relation or not.
2 The model

We simulate the nucleons within the framework of quantum molecular dynamics (QMD) model. In the QMD model,[37,?] each nucleon propagates under the influence of mutual interactions. The propagation is governed by the classical equations of motion:

\[ \dot{r}_i = \frac{\partial H}{\partial p_i}; \quad \dot{p}_i = -\frac{\partial H}{\partial r_i}, \]  

where \( H \) stands for the Hamiltonian which is given by:

\[ H = \sum_{i} A \frac{p_i^2}{2m_i} + \sum_{i} (V_i^{\text{Skyrme}} + V_i^{\text{Yuk}} + V_i^{\text{Coul}} + V_i^{\text{MDI}}). \]  

The \( V_i^{\text{Skyrme}} \), \( V_i^{\text{Yuk}} \), \( V_i^{\text{Coul}} \), and \( V_i^{\text{MDI}} \) in Eq. (2) are, respectively, the Skyrme, Yukawa, Coulomb and momentum-dependent potentials.

The momentum dependent interactions are obtained by parameterizing the term taken from the measured energy dependence of the nucleon-nucleus optical potential. It can be parameterized as

\[ V_{ij}^{\text{MDI}} = t_4 \ln^2[t_5 (p_i - p_j)^2 + 1] \delta(r_i - r_j). \]  

Here \( t_4 = 1.57 \) MeV and \( t_5 = 5 \times 10^{-4} \) MeV\(^{-2}\). The final form of the momentum dependent potential reads as

\[ U^{\text{MDI}} = \delta \ln^2[\epsilon(\rho/\rho_0)^{2/3} + 1]\rho/\rho_0. \]  

A parameterized form of the local plus momentum dependent potential is given by

\[ U = \alpha \left( \frac{\rho}{\rho_0} \right) + \beta \left( \frac{\rho}{\rho_0} \right)^\gamma + \delta \ln^2[\epsilon(\rho/\rho_0)^{2/3} + 1]\rho/\rho_0. \]  

It can be used to compute the corresponding density dependence of the compressional energy per nucleon which is shown in Fig. ?? for the soft (dubbed as Soft) and hard (dubbed as Hard) local Skyrme potentials and for the interactions with a momentum dependent term which is denoted by SMD and HMD.
3 Results and Discussion

We simulated 1000-3000 events for each of $^{12}$C+$^{12}$C ($b/b_{\text{max}} = 0.4$), $^{20}$Ne+$^{27}$Al ($b/b_{\text{max}} = 0.4$), $^{36}$Ar+$^{27}$Al ($b = 2$ fm), $^{40}$Ar+$^{27}$Al ($b = 1.6$ fm), $^{40}$Ar+$^{45}$Sc ($b/b_{\text{max}} = 0.4$), $^{40}$Ar+$^{51}$V ($b/b_{\text{max}} = 0.3$), $^{40}$Ar+$^{58}$Ni ($b = 0$-3 fm), $^{64}$Zn+$^{48}$Ti ($b = 2$ fm), $^{58}$Ni+$^{58}$Ni ($b/b_{\text{max}} = 0.28$), $^{64}$Zn+$^{58}$Ni ($b = 2$ fm), $^{80}$Kr+$^{93}$Nb ($b/b_{\text{max}} = 0.4$), $^{93}$Nb+$^{93}$Nb ($b/b_{\text{max}} = 0.3$), $^{129}$Xe+$^{nat}$Sn ($b = 0$-3 fm), $^{139}$La+$^{139}$La ($b/b_{\text{max}} = 0.3$), and $^{197}$Au+$^{197}$Au ($b = 2.5$ fm) using HMD and hard equations of state at various incident energies between 30 MeV/nucleon and 800 MeV/nucleon in small steps. The constant cross sections of 40, 50, and 55 mb strength are used at each such incident energy. The impact parameters are taken from the experimental extractions [10, 11, 12, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25]. The directed transverse momentum is calculated using

$$\langle p_{x}^{\text{dir}} \rangle = \frac{1}{A} \sum_{i} \text{sgn}\{Y(i)\}p_{x}(i),$$

where $Y(i)$ and $p_{x}(i)$ are, respectively, the rapidity distribution and transverse momentum of $i$th particle.

In Fig. 1, we display $\langle p_{x}^{\text{dir}} \rangle$ as a function of incident energy ranging between 30 MeV/nucleon and 800 MeV/nucleon, for the reactions of $^{12}$C+$^{12}$C, $^{40}$Ar+$^{45}$Sc, $^{93}$Nb+$^{93}$Nb, and $^{197}$Au+$^{197}$Au. The open (solid) diamonds denote the $\langle p_{x}^{\text{dir}} \rangle$ values for HMD (Hard) equation of state. A constant cross section of 55 mb has been used in this figure. The lines are to guide the eye. In all the cases, transverse momentum is negative at lower incident energies which turns positive at relatively higher incident energies. The value of the abscissa at zero value of $\langle p_{x}^{\text{dir}} \rangle$ corresponds to the energy of vanishing flow (EVF) or, alternatively, the balance energy ($E_{\text{bal}}$). The following important results emerge from the graph. (a) The transverse momentum increases monotonically with increase in the incident energy. The increase in transverse flow $\langle p_{x}^{\text{dir}} \rangle$ is sharp at smaller incident energies (up to 200 MeV/nucleon) compared to higher incident energies where it starts saturating. Its slope decreases at higher incident energies that finally saturates depending upon the mass of the colliding nuclei.

(2) At higher energies (eg., above 400 MeV/nucleon), the repulsion due to momentum dependent interactions is stronger during the early phase of reaction and transverse momentum increases sharply. However, the overall effect...
depends on the mass of colliding nuclei. The difference of $\langle p_x^{\text{dir}} \rangle$ between HMD and hard equations of state decreases as one goes from lighter to heavier systems. For example, the difference $(\langle p_x^{\text{dir}} \rangle_{\text{HMD}}-\langle p_x^{\text{dir}} \rangle_{\text{Hard}})$ at 400 MeV/nucleon is approximately, 11, 7.3, 2, and -3.2 MeV/c, respectively, for the reactions of $^{12}\text{C}+^{12}\text{C}$, $^{40}\text{Ar}+^{45}\text{Sc}$, $^{93}\text{Nb}+^{93}\text{Nb}$, and $^{197}\text{Au}+^{197}\text{Au}$. If one calculates in terms of normalized percentage [i.e., $(\langle p_x^{\text{dir}} \rangle_{\text{HMD}}-\langle p_x^{\text{dir}} \rangle_{\text{Hard}})/\langle p_x^{\text{dir}} \rangle_{\text{Hard}} \times 100$], these numbers are modified to 56.35, 15.44, 3.04, and -4.31%, respectively, for the reactions of $^{12}\text{C}+^{12}\text{C}$, $^{40}\text{Ar}+^{45}\text{Sc}$, $^{93}\text{Nb}+^{93}\text{Nb}$, and $^{197}\text{Au}+^{197}\text{Au}$. Note that the lighter colliding nuclei now show a huge variation compared to heavy ones where effects are insignificant. (3) If one looks carefully at the reac-
Figure 2: The decomposition of ⟨p_{x}^{dir}⟩ into mean field (left triangles) and collision part (right triangles) as a function of incident energy. Again, solid (open) triangles represent hard (HMD) equation of state.

In the vicinity of balance energy, one sees that the momentum dependent interactions suppress the transverse momentum and hence enhance the balance energy in agreement with the results of [30, 27]. The enhancement in the balance energy due to momentum dependent interactions over static one is, respectively, 13.29, 10.11, and 2.6 MeV/nucleon for the reactions of 40Ar+45Sc, 93Nb+93Nb, and 197Au+197Au. On the contrary, momentum dependent interactions reduce the energy of vanishing flow in 12C+12C by 9.9 MeV/nucleon. All the above mentioned findings can be understood by decomposing the total transverse momentum into contributions due to mean field and two-body nucleon-nucleon collisions (as shown in Fig. 2). This decomposition has been explained in details in Ref. [31]. The left triangles represent mean field contribution whereas collision contribu-
Figure 3: Total \( \langle p_x^{\text{dir}} \rangle \) (diamonds) and its decomposition into mean field (left triangles) and collision part (right triangles) as a function of the mass of the system at the balance energy of hard equation of state. Solid (open) symbols represent hard (HMD) equation of state. One notices that the flow due to binary nucleon-nucleon collisions increases almost linearly with increase in the incident energy in all the above listed reactions. The mean field flow, however, increases sharply up to a couple of hundred MeV/nucleon then saturates. This trend is slower in lighter colliding nuclei compared to heavy ones. In other words, the continuous increase in the total transverse momentum in high energy tail results due to the frequent nucleon-nucleon binary collisions alone. Under this saturation, the overall trend of the role of momentum dependent interactions in transverse flow is decided by the impact of the mean field.
Figure 4: (a) The total number of allowed nucleon-nucleon collisions as a function of total mass of the system at the balance energy of hard equation of state. Solid (open) pentagons represent hard (HMD) equation of state. The lines are a power law fit of form $cA^\tau$. Solid (dotted) line represents power law fit for hard (HMD) equation of state. (b) The percentage difference $\Delta N_{\text{coll}}$ as a function of mass of the system.

The striking result is for the case of $^{12}\text{C}+^{12}\text{C}$ reaction, where the contribution of the mean field towards collective transverse flow using momentum dependent interactions dominates the decrease in the collective flow due to nucleon-nucleon collisions using momentum dependent interactions resulting in the net increased transverse flow due to HMD compared to the static hard equation of state. Whereas, for medium mass nuclei (e.g., $^{93}\text{Nb}+^{93}\text{Nb}$), the increase in the flow due to mean field is almost balanced by the decrease in flow due to nucleon-nucleon binary collisions, therefore neutralizing the net effect. For the heavier systems, a very little swing can be noticed. In other
Figure 5: The rate of allowed nucleon-nucleon collisions $dN_{\text{coll}}/dt$ versus reaction time for the reactions of (a) $^{12}$C+$^{12}$C, (b) $^{40}$Ar+$^{45}$Sc, (c) $^{93}$Nb+$^{93}$Nb, and (d) $^{197}$Au+$^{197}$Au. Solid (dotted) lines represent hard (HMD) equation of state.

In summary, the role of the momentum dependent interactions in mean field contribution, depends on the mass of the system. In lighter systems, the role of momentum dependent interactions is larger which decreases for the heavier colliding nuclei. Let us now examine the mass dependence of momentum dependent interactions in balance energy → A point in the energy scale corresponding to vanishing flow. A careful look at Fig. 2 shows that the balance energy is, respectively, 142.88 (133), 70.11 (83.4), 47.18 (57.29), and 37.44 (40.1) MeV/nucleon for the reactions of $^{12}$C+$^{12}$C, $^{40}$Ar+$^{45}$Sc, $^{93}$Nb+$^{93}$Nb, and $^{197}$Au+$^{197}$Au using hard (HMD) equation of state. All the reactions, except $^{12}$C+$^{12}$C, show a uniform trend, i.e., the inclusion of momentum dependent interactions reduces the transverse flow at low incident energies,
therefore pushing the balance energy towards higher end. However, $^{12}\text{C}+^{12}\text{C}$ shows just the opposite. To understand this, let us divide the total transverse momentum at the energy of vanishing flow into contributions resulting from the mean field as well as collision parts. In Fig. 3, we plot the decomposition for all the reactions reported in the introduction where balance energy has been measured and reported experimentally. As discussed above, the difference between the transverse flow contributions due to mean field of HMD and hard equations of state is maximal for lighter colliding nuclei which gets suppressed for the heavier colliding nuclei. It was argued by Zhou et al. \cite{26} that for the lighter systems like $^{12}\text{C}+^{12}\text{C}$, a larger value of the balance energy results in stronger momentum dependent repulsion that enhances the transverse momentum and hence suppresses the balance energy. Our findings are also pointing towards the same effects. However,
one should also keep in mind that this trend is not universal in the mass range. Looking at Fig. 1, one notices that the collisions of heavier colliding nuclei at the balance energy of $^{12}\text{C}+^{12}\text{C}$ reaction do not yield any significant difference or trend shown by the $^{12}\text{C}+^{12}\text{C}$ reaction. Let us check whether the inclusion of momentum dependent interactions reduces the frequency of nucleon-nucleon binary collisions or not. We display in Fig. 4(a), the total number of nucleon-nucleon collisions observed in the simulations using HMD and hard equations of state as a function of the mass of the system at their corresponding theoretical balance energy for a hard equation of state. The aim to simulate the reactions of HMD also at the balance energy of static
hard equation of state is to eliminate any variation that might result due to different balance energy in HMD compared to hard equation of state. The solid line corresponds to hard equation of state whereas dotted line is showing the outcome for the HMD equation of state. As is evident, the inclusion of momentum dependent interactions suppresses the binary collisions by as much as 35-45% throughout the mass range which is in close agreement with Ref. [1]. Further, these can be parameterized by a power law of the form \( cA^\tau \) with \( \tau = 0.92 \pm 0.014 \) and \( \tau = 0.91 \pm 0.021 \), respectively, for the HMD and hard equations of state. The similar values of power law parameter \( \tau \) indicate towards universal suppression of binary collisions due to inclusion of momentum dependent interaction. To look more carefully, we plot in Fig. 4(b), the percentage difference of binary nucleon-nucleon collisions defined as

\[ \Delta N_{\text{coll}} = \left| \frac{N_{\text{coll}}^{\text{HMD}} - N_{\text{coll}}^{\text{Hard}}}{N_{\text{coll}}^{\text{Hard}}} \right| \times 100. \]

As stated above, we see that the average suppression is of the order of 40% throughout the periodic table masses. Since above discussion was for the final stage collisions, it will be of further interest to see how collisions are affected during the course of the reaction. To see how momentum dependent interactions can affect the frequency of the binary collisions, we show in Fig. 5 the rate of change of allowed collisions \( dN/dt \) as a function of time for \(^{12}\text{C}+^{12}\text{C}, ^{40}\text{Ar}+^{45}\text{Sc}, ^{93}\text{Nb}+^{93}\text{Nb}, \) and \(^{197}\text{Au}+^{197}\text{Au}\. This rate is after eliminating the Pauli blocked collisions. Interestingly, during early phase of the reaction, the inclusion of momentum dependent interactions has drastic effect on the collision rate compared to hard equation of state in lighter colliding nuclei. This trend is reversed in the case of heavier nuclei where very little effect can be seen. This also points towards the stronger effect of momentum dependent interactions in lighter nuclei compared to heavy colliding nuclei. To understand the suppression of the flow using momentum dependent interactions, we display in Fig. 6 the time evolution of the rescaled density, for the reactions of \(^{12}\text{C}+^{12}\text{C}, ^{40}\text{Ar}+^{45}\text{Sc}, ^{93}\text{Nb}+^{93}\text{Nb}, \) and \(^{197}\text{Au}+^{197}\text{Au}\. One notices that the momentum dependent interactions suppress the high dense phase of the reaction even at low incident energies like the balance energy. The reduction in the maximal value of the average density indicates a reduction in the number of nucleon-nucleon collisions. Note that the reduction in the maximal value of the average density is nearly the same in all the reactions irrespective of quite different energy of vanishing flow indicating
nearly the same reduction in number of binary nucleon-nucleon collisions and hence collision transverse flow for all the reacting systems. One also notices a faster decomposition of the compressed system using momentum dependent interactions as has been reported in Ref. [9]. In our previous works [31], we had reported the mass dependence of the disappearance of transverse flow for the whole range ranging from \(^{20}\text{Ne}+^{27}\text{Al}\) to heavier masses \(^{238}\text{U}+^{238}\text{U}\). Our comparison with experimental data suggested a preference for the hard equation of state. Also nucleon-nucleon cross section of 35-40 mb explained the data throughout the mass range quite nicely. The experimental balance energy yielded \(\tau_{\text{expt}} = -0.42 \pm 0.05\) whereas our results predicted \(\tau_{35} = -0.43 \pm 0.09\) and \(\tau_{40} = -0.42 \pm 0.08\) for the cross sections of 35 and 40 mb, respectively. We here extend the above study to also include the \(^{12}\text{C}+^{12}\text{C}\) reaction, therefore, increasing the mass range from 24 to 394. In addition, we shall also study the role of momentum dependent interactions. In Fig. 7 we display the balance energy as a function of the total mass of the system from \(^{12}\text{C}+^{12}\text{C}\) to \(^{197}\text{Au}+^{197}\text{Au}\). We display the results for a hard equation of state (open square) and HMD (inverted triangles) with \(\sigma = 40\) mb. An enhanced nucleon-nucleon cross section with \(\sigma = 50\) mb is also used in the case of momentum dependent interactions (open hexagon). Experimental data are displayed by stars. The lines are the power law fits of the form \(\propto A^\tau\). The dashed, dotted, dash-double-dotted, and solid lines represent, respectively, the power law fit for Hard\(^{40}\), HMD\(^{40}\), HMD\(^{50}\), and experimental data. The value of \(\tau\) for experimental data is \(\tau_{\text{expt}} = -0.37 \pm 0.031\) whereas \(\tau_{\text{Hard}}^{40} = -0.7 \pm 0.06\), \(\tau_{\text{HMD}}^{40} = -0.38 \pm 0.029\), and \(\tau_{\text{HMD}}^{50} = -0.4 \pm 0.003\). Once momentum dependent interactions are included, value of the \(\tau_{\text{HMD}}^{40}\) comes very close to the \(\tau_{\text{expt}}\). As noted in upper part of the figure, inclusion of momentum dependent interactions improves the agreement for \(^{12}\text{C}+^{12}\text{C}\) system. One still notices that in most of the medium and heavy masses, it rather overestimates the balance energy. To improve the agreement further, we also simulated the reactions at \(\sigma = 50\) mb using momentum dependent interactions. The comparison is displayed in the lower part of the figure. We see excellent agreement of HMD\(^{50}\) with experimental balance energy throughout the periodic table with mass between 24 and 394. Obviously, the effect is larger for lighter nuclei where incident energy is higher compared to heavy nuclei having lower incident energies. The failure of hard equation of state is due to the fact that it fails badly to explain the data for \(^{12}\text{C}+^{12}\text{C}\) reaction. It seems that the static equation of state is not able to generate enough repulsion in terms of transverse momentum in lighter colliding nuclei. Though
as reported in Ref. [31], other balance energies can be nicely explained. One possibility is to also use enhanced cross section for the static hard equation of state. This, however, worsens the balance energy agreement for medium and heavy nuclei.

4 Summary

We have studied the role of momentum dependent interactions in transverse flow as well as in its disappearance for central collisions over a wide range of masses between 24 and 394. We find that even for the central collisions at low incident energies, the role of momentum dependent interactions is significant and is not uniform over the mass range. The impact of momentum dependent interactions is different in lighter colliding nuclei compared to heavier colliding nuclei. In lighter nuclei, the contribution of mean field towards transverse flow is much smaller compared to heavier nuclei where binary nucleon-nucleon collisions dominate the scene. We also find that the inclusion of momentum dependent interactions explains the energy of vanishing flow in $^{12}$C+$^{12}$C which, otherwise, was not possible with static hard equation of state. In this work, we have taken a constant NN cross-section and stiff equation of state along with its momentum dependence. The energy and isospin dependence of the cross-section may also affect the results. It has been shown in [32] that these effects are small in the Fermi energy domain. Further, the addition of asymmetric potential may alter the equation of state and balance energy for heavy nuclei, though, it should have no role for lighter and medium mass colliding nuclei. It should be further noted that the hard equation of state does not agree with the experimental data of fragmentation at low energies and particle production at relativistic energies.

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