Azimuthal-angle dependence of charged-pion-interferometry measurements with respect to 2nd- and 3rd-order event planes in Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV

A. Adare,13 S. Afanasiev,30 C. Aidala,43,44 N.N. Ajitanand,62 Y. Akiba,56,57 H. Al-Bataineh,50 J. Alexander,62 K. Aoki,35,56 Y. Aramaki,12 E.T. Atomicsa,36 R. Averbeck,63 T.C. Awes,52 B. Azmoun,7 V. Babintsev,24 M. Bai,8 G. Baksay,20 L. Baksay,20 K.N. Barish,8 B. Bassalleck,49 A.T. Basye,1 S. Bathe,5,8 V. Baulbis,55 C. Baumann,45 A. Bazlevsky,7 S. Belikov,7 R. Belmont,67 R. Bennett,63 A. Berdnikov,59 Y. Berdnikov,59 A.A. Bickley,13 J.S. Bok,71 K. Boyle,63 M.L. Brooks,39 H. Buesching,7 V. Bumazhnov,24 G. Bunce,7,57 S. Butsyk,39 C.M. Camacho,39 S. Campbell,63 C.-H. Chen,63 C.Y. Chi,14 M. Chiu,7 I.J. Choi,71 R.K. Choudhury,4 P. Christiansen,41 T. Chujo,66 P. Chung,62 O. Chvala,8 V. Cianciolo,52 Z. Citron,63 B.A. Cole,14 M. Connors,63 P. Constantin,39 M. Csanád,18 T. Csörgő,70 T. Dahms,63 S. Dairaku,35,56 I. Danchev,67 K. Das,21 A. Datta,43 G. David,7 A. Denisov,24 A. Deshpande,57,63 E.J. Desmond,7 O. Dietzsch,60 A. Dion,63 M. Donadelli,60 O. Drapier,36 A. Drees,63 K.A. Drees,6 J.M. Durham,39,63 A. Durum,24 D. Dutta,4 S. Edwards,21 Y.V. Efremenko,52 E. Ellinghaus,13 T. Engelmore,14 A. Enokizono,38 H. En’yo,56,57 S. Esumi,66 B. Fadem,46 D.E. Fields,49 M. Finger,9 M. Finger, Jr.,17 F. Fleuret,36 S.L. Fokin,34 Z. Fraenkel,69,70 J.E. Frantz,91,63 A. Franz,7 A.D. Frawley,21 K. Fujiwara,56 Y. Fukao,56 T. Fusayasu,48 I. Garishvili,64 A. Glenn,13 H. Gong,63 M. Goniin,36 Y. Goto,56,57 R. Granier de Cassagnac,36 N. Gran,2,14 S.V. Greene,67 M. Grosse Perdekamp,55 T. Gunji,12 H.-Å. Gustafsson,41 J.S. Haggerty,7 K.I. Hahn,19 H. Hamagaki,12 J. Hamble,64 R. Han,54 J. Hanks,14 E.P. Hartouni,38 E. Haslum,41 R. Hayano,12 X. He,22 M. Heffner,38 T.K. Hemmick,63 T. Hester,8 J.C. Hill,28 M. Hohlmann,20 W. Holzmann,14 K. Homma,23 B. Hong,33 T. Horaguchi,23 D. Hornback,64 S. Huang,67 T. Ichihara,56,57 R. Ichiyi,56,57 J. Ide,46 Y. Ikeda,66 K. Imai,29,35,56 M. Inaba,66 D. Isenhower,1 M. Ishihara,56 T. Isobe,12,56 M. Issah,67 A. Isupov,30 D. Ivanischev,55 B.V. Jacak,63 J. Jia,7,62 J. Jin,14 B.M. Johnson,7 K.S. Joo,47 D. Joun,53 D.S. Jumpier,14 F. Kajihara,12 S. Kametani,56 N. Kamiyama,57 J. Kamin,63 J.H. Kang,71 J. Kapustinsky,39 K. Karatsu,35,56 D. Kaval,43,57 M. Kawashima,56,58 A.V. Kazantzès,34 T. Kempel,28 A. Khanzadeev,55 K.M. Kijima,23 B.I. Kim,33 D.H. Kim,47 D.J. Kim,31 E. Kim,61 E.-J. Kim,10 S.H. Kim,71 Y.-J. Kim,25 E. Kinney,13 K. Kurilk,13 Å. Kiss,18 E. Kistenev,7 L. Kochenda,55 B. Komkov,55 M. Konno,66 J. Koster,25 D. Kotchetkov,49 A. Kozlov,69 A. Král,15 A. Kravitz,14 G.J. Kunde,39 K. Kurita,56,58 M. Kurosawa,56 Y. Kwon,71 G.S. Kyle,50 R. Lacey,62 Y.S. Lai,14 J.G. Lajoie,28 A. Lebedev,28 D.M. Lee,39 J. Lee,19 K. Lee,61 K.B. Lee,33 K.S. Lee,33 M.J. Leitch,39 M.A.L. Leite,60 E. Leitner,67 B. Lenzi,69 X. Li,11 P. Liebing,57 L.A. Linden Levy,13 T. Liška,15 A. Littvinenko,30 H. Liu,39,50 M.X. Liu,39 B. Love,67 R. Luechenborg,45 D. Lynch,7 C.F. Maguire,67 Y.I. Makdisi,6 A. Malakhov,30 M.D. Malik,49 V.I. Maniko,34 E. Mannel,14 Y. Mao,56 H. Masui,66 F. Matathias,14 M. McCumber,63 P.L. McGaughy,39 N. Meads,63 B. Meredith,25 Y. Miao,56 A.C. Mignerey,42 P. Mikes,92 K. Mikj,56,66 A. Milov,7 M. Mishra,3 J.T. Mitchell,7 A.K. Mohanty,4 Y. Morino,12 A. Morreale,8 D.P. Morrison,47 T.V. Moukhana,34 J. Murata,56 S. Nagamiya,32 J.L. Nagle,43 N. Naglis,69 M.I. Nagy,18 I. Nakagawa,56 Y. Nakamiya,57 T. Nakamura,32 K. Nakano,56,65 J. Newby,38 M. Nguyen,63 T. Niida,66 R. Noutier,7 A.S. Nyain,34 E. O’Brien,7 S.X. Oda,12 C.A. Olgilvie,28 M. Oka,66 K. Okada,57 Y. Onuki,66 A. Oskarsson,41 M. Ouchida,23,56 K. Ozawa,12 R. Pak,7 V. Pantuev,26,63 V. Papavassiliou,50 I.H. Park,19 J. Park,61 S.K. Park,33 W.J. Park,33 S.F. Pate,50 H. Pei,28 J.-C. Peng,25 H. Pereira,16 V. Peresedov,30 D.Yu. Peresounko,34 C. Pinkenburg,7 R.P. Pisani,7 M. Pirois,63 M.L. Purschke,7 A.K. Purwar,39 H. Qu,22 J. Rak,31 A. Rakotozafindrabe,36 I. Ravinovich,69 K.F. Read,52,64 K. Reygers,45 V. Riabov,55 Y. Riabov,55 E. Richardson,42 D. Roach,67 G. Roche,49 S.D. Rohnick,8 M. Rosati,28 C.A. Rosen,13 S.S.E. Rosendahl,41 P. Rosnet,40 P. Rukoyatkin,30 P. Ruzicka,27 B. Sahlmueller,45,63 N. Saito,32 T. Sakaguchi,7 K. Sakashita,56,61 V. Samsonov,55 S. Sano,12,68 T. Sato,66 S. Sawada,32 K. Sedgwick,8 J. Seele,13 R. Seidl,25 A.Yu. Semenov,28 R. Seto,8 D. Sharma,69 I. Shein,24 T.-A. Shibata,56 K. Shigaki,23 M. Shimomura,66 K. Shojo,35,56 P. Shukla,4 A. Sickles,7 C.L. Silva,60 D. Silvermyr,52 C. Silvestre,16 K.S. Sim,33 B.K. Singh,3 C.P. Singh,7 V. Singh,3 M. Slunečka,9 R.A. Soltz,38 W.E. Sondheim,39 S.P. Sorensen,64 I.V. Sourikov,7 N.A. Sparks,7 P.W. Stankus,52 E. Steinfeld,42 S.P. Stoll,7 T. Sugitate,23 A. Sukhanov,7 J. Sziklai,79 E.M. Takagui,90 A. Taketani,56,57 R. Tanabe,66 Y. Tanaka,48 K. Tanida,58,57 M.J. Tannenbaum,7 S. Tarafdar,7 A. Taranenko,62 P. Tarján,17 H. Themann,63 J.T. Thomas,49 T. Todoroki,56 M. Togawa,35,56 A. Toia,63 L. Tomášek,27 H. Torii,23 R.S. Towell,1 I. Tserunnia,69 Y. Tsuchimoto,23 C. Vale,7,28 H. Valle,67 H.W. van Hecke,39 E. Vazquez-Zambrano,14 A. Veicht,25 J. Velkovska,67 R. Vértesi,17,70 A.A. Vinogradov,34 M. Virius,15
V. Vrba,27 E. Vznuzdaev,55 X.R. Wang,50 D. Watanabe,23 K. Watanabe,66 Y. Watanabe,56,57 F. Wei,28 R. Wei,62 J. Wessels,45 S.N. White,7 D. Winter,14 J.P. Wood,1 C.L. Woody,7 R.M. Wright,1 M. Wysocki,13 W. Xie,57 Y.L. Yamaguchi,12 K. Yamaura,23 R. Yang,25 A. Yanovich,24 J. Ying,22 S. Yokkaichi,56,57 Z. You,54 G.R. Young,52 I. Younis,37,49 I.E. Yushmanov,44 W.A. Zajc,14 C. Zhang,52 S. Zhou,11 and L. Zolin30

(PHENIX Collaboration)

1 Ablene Christian University, Abilene, Texas 79699, USA
2 Department of Physics, Augustana College, Sioux Falls, South Dakota 57197, USA
3 Department of Physics, Banaras Hindu University, Varanasi 221005, India
4 Bhabha Atomic Research Centre, Bombay 400 085, India
5 Baruch College, City University of New York, New York, New York, 10010 USA
6 Collider-Accelerator Department, Brookhaven National Laboratory, Upton, New York 11973-5000, USA
7 Physics Department, Brookhaven National Laboratory, Upton, New York 11973-5000, USA
8 University of California - Riverside, Riverside, California 92521, USA
9 Charles University, Ovocný trh 5, Praha 1, 116 36, Prague, Czech Republic
10 Chonbuk National University, Jeonju, 561-756, Korea

11 Science and Technology on Nuclear Data Laboratory, China Institute of Atomic Energy, Beijing 102413, P. R. China
12 Center for Nuclear Study, Graduate School of Science, University of Tokyo, 7-3-1 Hongo, Bunkyo, Tokyo 113-0033, Japan
13 University of Colorado, Boulder, Colorado 80309, USA
14 Columbia University, New York, New York 10027 and Nevis Laboratories, Irvington, New York 10533, USA
15 Czech Technical University, Zikova 4, 166 36 Prague 6, Czech Republic
16 Dapnia, CEA Saclay, F-91191, Gif-sur-Yvette, France
17 Debrecen University, H-4010 Debrecen, Egyetem tér 1, Hungary
18 ELTE, Eötvös Loránd University, H - 1117 Budapest, Pázmány P. s. 1/A, Hungary
19 Ewha Womans University, Seoul 120-750, Korea
20 Florida Institute of Technology, Melbourne, Florida 32901, USA
21 Florida State University, Tallahassee, Florida 32306, USA
22 Georgia State University, Atlanta, Georgia 30303, USA
23 Hiroshima University, Kagamiyama, Higashi-Hiroshima 739-8526, Japan
24 IHEP Protvino, State Research Center of Russian Federation, Institute for High Energy Physics, Protvino, 142281, Russia
25 University of Illinois at Urbana-Champaign, Urbana, Illinois 61801, USA
26 Institute for Nuclear Research of the Russian Academy of Sciences, prospekt 60-letiya Oktyabrya 7a, Moscow 117312, Russia
27 Institute of Physics, Academy of Sciences of the Czech Republic, Na Slovance 2, 182 21 Prague 8, Czech Republic
28 Iowa State University, Ames, Iowa 50011, USA
29 Advanced Science Research Center, Japan Atomic Energy Agency, 2-4 Shirakata Shirane, Tokai-mura, Naka-gun, Ibaraki 319-1195, Japan
30 Joint Institute for Nuclear Research, 141980 Dubna, Moscow Region, Russia
31 Helsinki Institute of Physics and University of Jyväskylä, P.O.Box 35, FI-40014 Jyväskylä, Finland
32 KEK, High Energy Accelerator Research Organization, Tsukuba, Ibaraki 305-0801, Japan
33 Korea University, Seoul, 136-701, Korea
34 Russian Research Center “Kurchatov Institute”, Moscow, 123098 Russia
35 Kyoto University, Kyoto 606-8502, Japan
36 Laboratoire Leprince-Ringuet, Ecole Polytechnique, CNRS-IN2P3, Route de Saclay, F-91128, Palaiseau, France
37 Physics Department, Lahore University of Management Sciences, Lahore, Pakistan
38 Lawrence Livermore National Laboratory, Livermore, California 94550, USA
39 Los Alamos National Laboratory, Los Alamos, New Mexico 87545, USA
40 LPC, Université Blaise Pascal, CNRS-IN2P3, Clermont-Fd, 63177 Aubiere Cedex, France
41 Department of Physics, Lund University, Box 118, SE-221 00 Lund, Sweden
42 University of Maryland, College Park, Maryland 20742, USA
43 Department of Physics, University of Massachusetts, Amherst, Massachusetts 01003-9337, USA
44 Department of Physics, University of Michigan, Ann Arbor, Michigan 48109-1040, USA
45 Institut für Kernphysik, University of Muenster, D-48149 Muenster, Germany
46 Muhlenberg College, Allentown, Pennsylvania 18104-5586, USA
47 Myongji University, Yongin, Kyonggido 449-728, Korea
48 Nagasaki Institute of Applied Science, Nagasaki-shi, Nagasaki 851-0193, Japan
49 University of New Mexico, Albuquerque, New Mexico 87131, USA
50 New Mexico State University, Las Cruces, New Mexico 88003, USA
51 Department of Physics and Astronomy, Ohio University, Athens, Ohio 45701, USA
52 Oak Ridge National Laboratory, Oak Ridge, Tennessee 37831, USA
53 IPN-Orsay, Universite Paris Sud, CNRS-IN2P3, BP1, F-91406, Orsay, France
54 Peking University, Beijing 100871, P. R. China
55 PNPI, Petersburg Nuclear Physics Institute, Gatchina, Leningrad region, 188300, Russia
56 RIKEN Nishina Center for Accelerator-Based Science, Wako, Saitama 351-0198, Japan
with the picture that the final distribution still retains the initial distributions. Previous results are consistent with the planar geometry and may even reverse the major and minor axes of eccentricity of the spatial distribution in the transverse direction. This phenomenon, elliptic flow, reduces the shape to a stronger expansion of the source within the in-plane plane, and this leads to that along the major axis (out of plane), and this leads to that along the major axis (out of plane). These initial spatial fluctuations may be preserved until freeze-out and that the 3rd-order oscillations are largely dominated by the dynamical effects from triangular flow.

Recent results of the 3rd-order dependence indicate that the initial eccentricity is reduced during the medium evolution, and that the 3rd-order oscillations are largely dominated by the dynamical effects from triangular flow.

Charged-pion-interferometry measurements were made with respect to the 2nd- and 3rd-order event planes for Au+Au collisions at \( \sqrt{s_{NN}} = 200 \text{ GeV} \). A strong azimuthal-angle dependence of the extracted Gaussian-source radii was observed with respect to both the 2nd- and 3rd-order event planes. The results for the 2nd-order dependence indicate that the initial eccentricity is reduced within the medium evolution, but not reversed in the final state, which is consistent with previous results. In contrast, the results for the 3rd-order dependence indicate that the initial triangular shape is significantly reduced and potentially reversed by the end of the medium evolution, and that the 3rd-order oscillations are largely dominated by the dynamical effects from triangular flow.

Quantum-statistical interferometry of two identical particles, also known as Hanbury Brown and Twiss (HBT) interferometry [7, 8], provides information on the space-time extent of the particle-emitting source. In heavy-ion collisions, hadron interferometry is sensitive to the space-time extent of the hadronic system at the time of the last scattering, referred to as kinetic freeze-out. In noncentral collisions of like nuclei, the initial density distribution is predominantly elliptical in shape, with additional fluctuations [9]. There is a larger pressure gradient along the minor axis (in plane) of the ellipse, compared to that along the major axis (out of plane), and this leads to a stronger expansion of the source within the in-plane direction. This phenomenon, elliptic flow, reduces the eccentricity of the spatial distribution in the transverse plane, and may even reverse the major and minor axes of the initial distributions. Previous results are consistent with the picture that the final distribution still retains the initial elliptical orientation, although with a smaller eccentricity upon freeze-out [10].

The full set of anisotropic moments of the flow is characterized by the Fourier coefficients of the azimuthal distribution of emitted particles: \( dN/d\phi \propto 1 + 2 \sum v_n \cos[n(\phi - \Psi_n)] \), where \( \phi \) is the azimuthal angle of the particle, \( v_n \) is the strength of \( n \)th-order flow harmonic, and \( \Psi_n \) is the \( n \)th-order event plane, where \( \Psi_2 \) and \( \Psi_3 \) are independent [11]. Elliptic flow is defined by the 2nd-order coefficient \( (n = 2) \), but triangular \( (n = 3) \), quadrangular \( (n = 4) \), and higher-order moments are also present and have been measured in both the spatial and momentum distributions in heavy ion collisions [11, 13]. While the higher-order even moments are needed to accurately describe the original elliptic shape, the odd moments arise solely through fluctuations in the initial spatial distribution. Depending on strength of the fluctuations, flow profile, expansion time, and shear viscosity, these initial spatial fluctuations may be preserved until freeze-out [14, 15].

In relativistic heavy ion collisions, HBT interferometry with respect to different order event planes uniquely probes the magnitude of the initial-state fluctuations and the subsequent space-time evolution, thereby providing important constraints on the dynamics of the QGP. Here, we present results of azimuthal HBT measurements of charged pions with respect to 2nd-order event plane, as well as the first results with respect to the 3rd-order event plane in Au+Au collisions at \( \sqrt{s_{NN}} = 200 \text{ GeV} \) at cen-
tral rapidity. The centrality and transverse momentum dependence are also presented. This analysis is based on data collected in 2007 with the PHENIX detector [16]. Collision centrality was determined using the measured charge distribution in the beam-beam counters (3.0 < |η| < 3.9) [17]. The event planes, $\Psi_n$, were determined using the reaction plane detector (RXNP) covering forward and backward angles 1.0 < |η| < 2.8 [18]. The event plane resolution $\text{Res}(\Psi_n)$ was estimated by the two-subevent method [19] using the $\Psi_n$ correlation between the RXNP at forward and backward angles, where $\text{Res}(\Psi_n)$ is defined as $\langle \cos [n(\Psi_n - \Psi_n, \text{real})] \rangle$. Track and momentum reconstruction of charged particles was performed by combining hits from the drift chamber and pad chambers in the central spectrometers (|η| < 0.35), where the momentum resolution is $\delta p/p \approx 1.3\% \pm 1.2\% \times p$ [20]. Charged pions were identified by combining time-of-flight from the electromagnetic calorimeters [21] covering azimuthal angle $\Delta \phi = \pi/2$, with reconstructed momentum and trajectory in the magnetic field. Particles within two standard deviations of the peak of charged pions in mass-squared distributions were identified as pions up to a momentum of $\sim 1$ GeV/c.

The experimentally measured correlation function is defined as $A(q)/B(q)$, where $A(q)$ is the relative-momentum distribution of all combinations of identified pion pairs in the same event, and $B(q)$ is the event-mixed background distribution of pairs formed from pions from different events, but with similar event centralities, vertex positions, and 2$^{\text{nd}}$ (3$^{\text{rd}}$-order) event planes. To remove ghost tracks and detector inefficiencies, pairs with either $\Delta z < 5$ cm and $\Delta \phi < 0.07$ or $\Delta z < 70$ cm and $\Delta \phi < 0.02$ at the drift chamber were removed from the analysis, as were tracks separated by less than 17 cm at the front face of the electromagnetic calorimeters. The correlation functions were also binned according to the centrality of the event and the momentum of the pion pair. Positive and negative pion pairs were combined to cancel charge-dependent acceptance effect [22].

A three-dimensional analysis was performed with the Bertsch-Pratt parameterization assuming a Gaussian source [23, 24]:

$$G = \exp(-R_k^2 q_k^2 - R_\rho^2 q_\rho^2 - R_l^2 q_l^2 - 2R_{2s} q_s q_o). \quad (1)$$

In this framework, the relative momentum $q$ is decomposed into $q_k$, $q_\rho$, and $q_s$, where $q_k$ denotes the beam direction, $q_\rho$ is perpendicular to $q_k$ and parallel to the mean transverse momentum of the pair $\vec{k}_T = (\vec{p}_{1T} + \vec{p}_{2T})/2$, and $q_s$ is perpendicular to both $q_k$ and $q_\rho$. The $R_k$ ($\mu = s, o, l$) Gaussian parameters provide information on the size of the emission region in each direction, but $R_\rho$ and (to a lesser extent) $R_l$ include contributions from the emission duration and all are influenced by position-momentum correlations. The $R_{2s}$ is a cross term that arises from asymmetries in the emission region [25]. The analysis was performed in the longitudinally co-moving system, where $p_{1z} = -p_{2z}$. The measured correlation functions were fit by:

$$C_2 = N[(\lambda(1+G))F_e + (1-\lambda)], \quad (2)$$

where $N$ is a normalization factor and $F_e$ is the Coulomb correction factor evaluated using a Coulomb wave function. Equation (2) is based on the core-halo model [26, 27], which divides the source into two regions: a central core that contributes to the quantum interference, and a long-range component that includes the decay of long-lived particles having a negligible Coulomb interaction and a quantum statistical interference that occurs in a relative momentum range that is too small to be resolved experimentally. The fraction of pairs in the core is given by $\lambda$.

Finite event-plane resolution reduces the oscillation amplitude of HBT radii relative to the event plane. In this analysis, a model-independent correction suggested in [28] was applied to $A(q)$ and $B(q)$. The correction factor is 54% (32%) for the 2$^{\text{nd}}$-order (3$^{\text{rd}}$-order) event planes in 0%–10% centrality. As a crosscheck, the oscillation amplitude was also corrected by dividing by $\text{Res}(\Psi_n)$ [29]. Both methods applied to the 2$^{\text{nd}}$- and 3$^{\text{rd}}$-order event-plane dependence are consistent within systematic uncertainties. The effect of momentum resolution was studied using GEANT simulations following previous analyses [22, 30] and its impact is negligible on the extracted radii (<1%).

Systematic uncertainties were estimated by the variation of single track cuts, pair selection cuts, and input source size for the Coulomb wave function. Also incorporated were the variations when using alternate event plane definitions from the forward, backward, and combined RXNPs. Total systematic uncertainties for $R_{2s}$ and $R_{2o}$ are not more than 5% (12%) and 7% (17%) for the 2$^{\text{nd}}$-order (3$^{\text{rd}}$-order) event plane, respectively.

Figure 1 shows $R_{2s}$, $R_{2o}$, $R_{2l}$, and $R_{2s}$ for pions as functions of azimuthal angle $\phi$ with respect to $\Psi_2$ and $\Psi_3$ for two centrality bins, where $\langle k_T \rangle \approx 0.53$ GeV/c. The filled symbols show the extracted HBT radii and the open symbols are reflected by symmetry around $\phi - \Psi_n = 0$. For the 0%–10% bin, $R_{2s}$ shows a very weak oscillation relative to both $\Psi_2$ and $\Psi_3$, while $R_{2o}$ clearly exhibits a stronger oscillation. For the 20%–30% bin, $R_{2s}$ and $R_{2o}$ for $\Psi_2$ show opposite-sign oscillations, as expected for an elliptical source viewed from in-plane and out-of-plane axes [10]. For $\Psi_3$, $R_{2l}$ shows a weaker angular dependence of the same sign as $R_{2o}$.

The oscillation amplitudes were extracted by fitting the angular dependence of $R_{2s}$ to the functional form:

$$R_{2s} = R_{2s, 0} + 2 \sum_{m=n, 2m} R_{2s, n} \cos[n(\phi - \Psi_m)] \quad (\mu = s, o, l),$$

$$R_{2o} = 2 \sum_{m=n, 2m} R_{2o, n} \sin[n(\phi - \Psi_m)] \quad (\mu = os), \quad (3)$$

The oscillation amplitudes were extracted by fitting the angular dependence of $R_{2s}$ to the functional form:
FIG. 1: (Color online) The azimuthal dependence of $R_2$, $R_2^o$, $R_2^l$, and $R_{2 os}$ for charged pions in $0.2 < k_T < 2.0 \text{ GeV}/c$ with respect to 2nd (a-d) and 3rd-order (e-h) event plane in Au+Au collisions at $\sqrt{s_{NN}} = 200 \text{ GeV}$. The $R_{2 os}$ is plotted relative to dashed lines representing $R_{2 os} = 0$. The filled symbols show the extracted HBT radii and the open symbols are reflected by symmetry around $\phi - \Psi_n = 0$. Bands of two thin lines show the systematic uncertainties and dashed lines show the fit lines by Eq. (3).

FIG. 2: (Color online) The solid points are the oscillation amplitudes relative to the average of HBT radii as a function of initial spatial anisotropy ($\varepsilon_n$), which are calculated using the Glauber model. Boxes show the systematic uncertainties. Open star symbols are the $\varepsilon_{\text{final}}$ from STAR [10]. Dashed lines indicate the line of $\varepsilon_n = |2R_{\mu,n}^2/R_{\nu,0}^2|$.

Figure 2 shows the amplitudes relative to the average of $R_s$, $R_o$, and $R_{os}$, $2R_{\mu,n}^2/R_{\nu,0}^2$, as functions of initial eccentricity ($\varepsilon_2$) and triangularity ($\varepsilon_3$). Each $\varepsilon_n$ is calculated by Monte-Carlo Glauber simulation as given in Ref. [15, 33] and decreases with increasing centrality, however the centrality dependence of $\varepsilon_3$ is weaker than that of $\varepsilon_2$.

The $2R_{\mu,n}^2/R_{\nu,0}^2$ (Fig. 2(a)) is sensitive to the final source eccentricity ($\varepsilon_{\text{final}}$) at freeze-out [28], and approaches the whole source eccentricity in the limit of $k_T = 0$. Our results for the $\Psi_2$ dependence are consistent with the STAR experiment [10]. We note that the $\varepsilon_{\text{final}}$ defined from $R_s$ has a systematic uncertainty of 30% due to the assumption of space-momentum correlation in the Blast-Wave model [28]. The positive value of $\varepsilon_{\text{final}}$ indicates that the source shape still retains the initial shape extended out-of-plane, though reduced in magnitude. Other combinations of $|2R_{\mu,n}^2/R_{\nu,0}^2|$ also have similar $\varepsilon_n$ dependence, but are larger than $2R_{s,3}^2/R_{s,0}^2$.

They include contributions from the emission duration and will have different sensitivity to the dynamics [34]. The $2R_{s,3}^2/R_{s,0}^2$ are less than or equal to zero, which seems to be an opposite trend to other combinations,
as noted already in Fig. 1. For all amplitudes, the values for 3rd-order are small compared to those for 2nd-order.

It is well known that the HBT radii are influenced by the presence of dynamical correlations between momentum and spatial distributions at the time of freeze-out [33, 39], as evident in the transverse pair momentum $k_T$ dependence of the radii. Figure 3 shows these results for the 3rd-order oscillation amplitudes. The $R^{2}_{o,3}/R^{2}_{o,0}$ decreases with $k_T$, whereas $R^{2}_{s,3}/R^{2}_{s,0}$ does not show a significant dependence.

Although the reduced 3rd-order anisotropy in Fig. 3 may indicate small triangular deformation at freeze-out, its interpretation is complicated by the influence of dynamical correlations from the triangular flow [32]. To illustrate the different contributions of these effects we show separately the $k_T$ dependence for a source with radial symmetry and triangular flow ($\epsilon_3=0$, $\overline{\epsilon}_3=0.25$) and a source with triangular deformation and radial flow ($\epsilon_3=0.25$, $\overline{\epsilon}_3=0$) [37]. The model curves are taken from [32], but the radii are scaled by 0.3 to fit within the range of the data. The $R^{2}_{o,3}$ favors the deformed flow scenario, while the $R^{2}_{s,3}$ matches the deformed flow only at lower $k_T$.

To disentangle the relative contributions of spatial and flow anisotropy to the azimuthal dependence of HBT radii, we have performed a Monte-Carlo simulation introducing the spatial anisotropy and collective flow with anisotropic modulation at freeze-out. The assumptions of this model are similar to those adopted in the Blast-Wave (BW) model [28, 38], generalized for 3rd-order modulation, and do not include effects such as viscosity and source opacity. The particle distributions in the transverse plane were parameterized with a Woods-Saxon function, $\Omega(r) = 1/(1 + \exp[(r-R)/a])$. To control the final source triangularity, we introduced a parameter $\epsilon_3$ into the radius parameter $R$ in $\Omega(r)$ as follows:

\[
R = R_0 \left(1 - 2\epsilon_3 \cos[3(\phi - \Phi)]\right),
\]

\[
\beta_T = \beta_0 \left(1 + 2\beta_3 \cos[3(\phi - \Phi)]\right),
\]

where $\phi$ is the azimuthal angle of particle positions, $\Phi$ is reference angle of the spatial anisotropy and triangular flow, and $R_0$ is average radius. To take the collective flow into account, generated particles were boosted in the transverse radial direction with a velocity $\beta_T$ in addition to their thermal velocities. We used a similar definition to the BW model [28, 38] as the flow rapidity $\rho(r) = (r/R) \tanh^{-1}(\beta_T)$. In Eq. (5), $\beta_0$ represents the average of radial flow and $\beta_3$ is used to control the flow anisotropy. We assume that the particles are emitted with a Gaussian time distribution with $\Delta T$ standard deviation, which affects $R_o$, but not $R_s$. The effect of HBT interference was calculated by $\cos(3\mathbf{x} \cdot \mathbf{q})$, where $\Delta \mathbf{x}$ and $\mathbf{q}$ are 4-vectors for relative distance and relative momentum of the pair. All other parameters except $\epsilon_3$ and $\beta_3$ were tuned to reproduce the strength of radial flow

measured by $m_T$ spectra [39] and the averages of HBT radii shown in Fig. 1. For this analysis $\Delta T$ was set to 3.5 fm/c (2.7 fm/c) for 0%–10% (20%–30%) to achieve better agreement with the average of $R^2_s$. A simulation result with $\epsilon_3=0$ and $\beta_3=0.12$ is shown in Fig. 3, displaying qualitatively consistent with the [32].

![FIG. 4: (Color online) $\chi^2$ contours representing the difference between data and simulation in 2$R^2_{o,2}/R^2_{o,0}$ ($\mu=s, o$), as functions of $\epsilon_3$ and $\beta_3$. Shaded areas represent $\chi^2$ less than unity and constrained by the experimental uncertainty.](image)

To understand how the data may constrain these values, we have performed a least-square fit for $\epsilon_3$ and $\beta_3$. Figure 4 shows the contour plots of $\chi^2$ defined by

\[
\chi^2 = \frac{1}{\sigma^2} \left(\frac{1}{N} \sum_{i=1}^{N} \left(\frac{2R^2_{o,2}/R^2_{o,0}^{\text{sim}} - [2R^2_{o,2}/R^2_{o,0}^{\text{exp}}]}{E}\right)^2\right)
\]

where $\sigma$ is the experimental uncertainty. The value of $\epsilon_3$ is well constrained by the measured value of $R^2_o$, and indicates that the final triangularity is very close to zero. The inclusion of $R^2_o$ favors a positive value for $\beta_3$ for 0%–10%, but does not add much information to 20%–30%, where a slightly negative value of $\epsilon_3$ is favored by $R^2_s$. We note that the discrepancy at high $k_T$ remains, but the data integrated over $k_T$ is primarily influenced by lower $k_T$ pairs. Detailed comparison with a realistic hydrodynamic model (e.g., [32, 30]) will be a key to fully understand the results.

In summary, we have presented results on the azimuthal dependence of charged-pion HBT radii with respect to 2rd- and 3rd-order event planes in Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV. The results for the 2rd-order event plane dependence indicate that in noncentral collisions the source starts with an initial elliptical distribution and ends with an elliptical distribution at freeze-out, but with a diluted eccentricity due to the medium expansion. For the 3rd-order event plane results, the observed $R^2_o$ oscillation may come from flow anisotropy, but the small $R^2_s$ oscillation with the same sign as $R^2_o$ in noncentral collisions may imply that the source expansion with triangular flow inverts the initial triangular shape. A Monte-Carlo simulation for an expanding triangular transverse distribution produces results consistent with this interpretation. Comparisons with an event-by-event hydrodynamic model will be needed to reveal the relation of spatial and hydrodynamical flow anisotropy at
freeze-out, as well as to provide further constraints on the hydrodynamic evolution in relativistic heavy ion collisions.

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* Deceased

† PHENIX Co-Spokesperson: morrison@bnl.gov

‡ PHENIX Co-Spokesperson: janie.nagle@colorado.edu

[1] K. Adcox et al. (PHENIX Collaboration), Nucl. Phys. A 757, 184 (2005).
[2] J. Adams et al. (STAR Collaboration), Nucl. Phys. A 757, 102 (2005).
[3] B. B. Back et al. (PHOBOS Collaboration), Nucl. Phys. A 757, 28 (2005).
[4] I. Arsene et al. (BRAHMS Collaboration), Nucl. Phys. A 757, 1 (2005).
[5] S. Pratt, Phys. Rev. Lett. 102, 232301 (2009).
[6] R. A. Soltz, I. Garishvili, M. Cheng, B. Abelev, A. Glenn, J. Newby, L. A. L. Levy, and S. Pratt, Phys. Rev. C 87, 044901 (2013).
[7] R. H. Brown and R. Q. Twiss, Nature 178, 1046 (1956).
[8] G. Goldhaber, S. Goldhaber, W. Lee, and A. Pais, Phys. Rev. 120, 300 (1960).
[9] U. Heinz and R.Snellings, Ann. Rev. Nucl. Part. Sci. 63, 123 (2013).
[10] J. Adams et al. (STAR Collaboration), Phys. Rev. Lett. 93, 012301 (2004).
[11] A. Adare et al. (PHENIX Collaboration), Phys. Rev. Lett. 107, 252301 (2011).
[12] K. Aamodt et al. (ALICE Collaboration), Phys. Rev. Lett. 107, 032301 (2011).
[13] G. Aad et al. (ATLAS Collaboration), Phys. Rev. C 86, 014907 (2012).
[14] S. A. Voloshin, J. Phys. G 38, 124007 (2011).
[15] B. Alver and G. Roland, Phys. Rev. C 81, 054905 (2010).
[16] K. Adcox et al. (PHENIX Collaboration), Nucl. Instrum. Meth. A 499, 469 (2003).
[17] M. Allen et al. (PHENIX Collaboration), Nucl. Instrum. Meth. A 499, 549 (2003).
[18] E. Richardson et al. (PHENIX Collaboration), Nucl. Instrum. Methods A 636, 99 (2011).
[19] A. M. Poskanzer and S. A. Voloshin, Phys. Rev. C 58, 1671 (1998).
[20] A. Adare et al. (PHENIX Collaboration), Phys. Rev. C 85, 064914 (2012).
[21] L. Apecetche et al. (PHENIX Collaboration), Nucl. Instrum. Meth. A 499, 521 (2003).
[22] J. Adams et al. (STAR Collaboration), Phys. Rev. C 71, 044906 (2005).
[23] S. Pratt, Phys. Rev. D 33, 72 (1986).
[24] G. Bertsch, M. Gong, and M. Tohyama, Phys. Rev. C 37, 1896 (1988).
[25] M. Lisa, S. Pratt, R. Soltz, and U. Wiedemann, Ann. Rev. Nucl. Part. Sci. 55, 357 (2005).
[26] M. G. Bowler, Phys. Lett. B 270, 69 (1991).
[27] Y. M. Sinyukov, R. Lednickya, S. V. Akkelin, J. Plutta, and B. Erazmus, Phys. Lett. B 432, 248 (1998).
[28] F. Retière and M. A. Lisa, Phys. Rev. C 70, 044907 (2004).
[29] D. Adamová et al., Phys. Lett. B 432, 248 (1998).
[30] C. Alt et al. (NA49 Collaboration), Phys. Rev. C 77, 064908 (2008).
[31] The $R_{n,n}$ as a fitting parameter could be negative, which means a different phase of the cosine function.
[32] C. J. Plumberg, C. Shen, and U. Heinz, Phys. Rev. C 88, 044914 (2013).
[33] M. L. Miller, K. Reygers, S. J. Sanders, and P. Steinberg, Ann. Rev. Nucl. Part. Sci. 57, 205 (2007).
[34] U. Heinz and P. F. Kolb, Phys. Lett. B 542, 216 (2002).
[35] Y. Hama and S. S. Padula, Phys. Rev. C 37, 3237 (1998).
[36] S. V. Akkelin and Y. M. Sinyukov, Phys. Lett. B 356, 525 (1995).
[37] $\tilde{c}_3$ parameterizes the triangular spatial source deformation and $\tilde{v}_3$ represents the strength of a triangular flow in the model of [32].
[38] S. S. Adler et al. (PHENIX Collaboration), Phys. Rev. C 72, 014903 (2005).
[39] S. S. Adler et al. (PHENIX Collaboration), Phys. Rev. C 69, 034909 (2004).
[40] P. Bozek, arXiv:1401.4894.