VARYING ALPHA, MASS GENERATION AND EXTRA DIMENSIONS

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We consider variation of coupling strengths and mass ratios in and beyond the Standard Model, in the light of various mechanisms of mass generation. In four-dimensional unified models, variations in charged particle thresholds, light quark masses and the electron mass can completely alter the (testable) relation between $\Delta \ln \alpha$ and $\Delta \ln \mu$, where $\mu \equiv m_p/m_e$. In extra-dimensional models where a compactification scale below the fundamental scale is varying, definite predictions may result even without unification; we examine some models with Scherk-Schwarz supersymmetry-breaking.

1 Introduction

The recent claim that the fine structure constant $\alpha$ governing QSO absorption spectra in interstellar gas clouds at redshifts $z = 0.5$ to 3 differs from that measured in the laboratory (at 5$\sigma$ level) raises many theoretical and experimental issues. Prominent among these are the possibility that other fundamental parameters of the Standard Model (SM) have also varied, how their variations might be bounded or measured and what theories could be tested thereby. Parameters that are in principle accessible to astronomical or astrophysical observation are, apart from $\alpha$, ratios of particle masses such as $m_q/m_p$ for the light quarks or $m_e/m_p$, and dimensionless ratios such as $m_p^2 G_F \propto (m_p/v_H)^2$, where $v_H$ is the Higgs v.e.v., and $m_p^2 G_N \propto (m_p/M_P)^2$ which measure the strength of weak and gravitational interactions respectively. Any variation in the strong interaction is equivalent, by dimensional transmutation, to a variation in the QCD invariant scale which we denote as $\Lambda_c$ (more properly the ratio of $\Lambda_c$ to some other mass scale), or the proton mass.

The most robust and direct constraints arise from other observations of astronomical spectra at comparable redshifts, which are sensitive to the parameters $m_p/m_e \equiv \mu$ and $g_p$, the gyromagnetic ratio of the proton (in addition to $\alpha$). The second of these observables does not have a well-understood dependence on SM parameters, so we eliminate it in favour of $\alpha$ and $\mu$. Note that the interpretation of such observations depends very little
on the particular form of the cosmological (space-time) evolution of $\alpha$, and is less subject to theoretical uncertainties or degeneracies encountered when studying the dependence of nuclear processes or the CMB on fundamental parameters.

Then the question we will address is the relation between the variation in $\alpha$ and that in $\mu$ in theories that predict the values of, or relations between, the SM parameters. We define the parameter

$$\tilde{R} \equiv \frac{\mu^{-1}\Delta\mu}{\alpha^{-1}\Delta\alpha}$$

which may be compared to observation. Although data constraining $\mu$ are scarce and marginally inconsistent, $\tilde{R}$ should lie in the range $(-10.5, 5.5)$ to stand any chance of agreeing with observation. As we will see, this already rules out many scenarios.

There are two types of theory where predictions of $\tilde{R}$ are possible. First, those with gauge coupling unification, in which the variation of either the unified coupling $g_X$ at the scale $M_X$ (the fundamental scale of the theory, often taken as $M_{\text{GUT}}$ or $M_{\text{Pl}}$), or some ratio of mass scales in the theory induces the observed variation $^7\text{7}^3\text{8}^4\text{10}^4\text{11}^4\text{12}$. Second, those where some or all of the SM fields propagate along (one or more) extra dimensions, where the variation of the radius relative to $M_X^{-1}$ induces variation in $\alpha$. $^13$ In this work we include the full effects of thresholds and light fermion masses, building on the analysis of $^14$. These effects, although formally subleading, can compete with the zeroth-order term, depending on the mechanism of mass generation, and completely alter theoretical expectations. For the case of varying extra-dimensional radius, we generalise formulae obtained $^13$ for extra-dimensional GUT, $^15$ to cases without gauge unification. To obtain a meaningful prediction of $\tilde{R}$ the mechanism of electroweak symmetry-breaking (EWSB) must be accounted for, thus we consider models in which this is closely tied to the presence of extra dimensions. In general, the inclusion of the effects of mass generation, thresholds and fermion masses can bring the value of $\tilde{R}$ closer to the range allowed by observation.

2 High-scale unification and mass generation

The essential feature of GUT-like theories is that gauge couplings are determined by a single degree of freedom $g_X$ at the fundamental scale $M_X$ and the renormalization group (RG) evolution between $M_X$ and observable energies. We fix notation by quoting (the solution of) the one-loop RG equation for gauge couplings, including the threshold contribution of a charged field with
decoupling mass $m^{th}(m^{th})$:

$$\alpha^{-1}_i(\mu^-) = \alpha^{-1}_i(\mu^+) - \frac{b_i^-}{2\pi} \ln \left( \frac{\mu^-}{\mu^+} \right) - \frac{b_i^{th}}{2\pi} \ln \left( \frac{m^{th}}{\mu^+} \right)$$

(2)

where $b_i$ is negative for asymptotically free groups and $b_i^{th} \equiv b_i^+ - b_i^-$, the beta-function coefficient being $b_i^+$ above the threshold and $b_i^-$ below, with tree-level matching at $m^{th}$.

For $\alpha$, we have in general

$$\frac{\Delta \alpha}{\alpha} = \sum_{i=1,2} \frac{\alpha}{\alpha_i(\mu^+)} \frac{\Delta \alpha_i(\mu^+)}{\alpha_i(\mu^+)} + \alpha \sum_{th} Q_{th}^2 \frac{f^{th}}{2\pi} \Delta \ln \frac{m^{th}}{\mu^+}$$

(3)

where $\alpha_1 = g^2/4\pi$, the second sum is over all charged fields, $f^{th}$ is 2/3 per chiral (or Majorana) fermion, 1/3 per complex scalar and 11/3 per vector boson. For the light quarks, the QCD scale $\Lambda_c$ provides a dynamical cutoff. The first (direct) term gives $\Delta \alpha/\alpha|_{direct} = (8\alpha/3\alpha_X) \Delta \ln \alpha_X \simeq 0.47 \Delta \ln \alpha_X$ due to the hypercharge normalization in GUTs, where we take the SUSY-GUT value $\alpha_X \simeq 1/24$.a The remaining threshold contribution is

$$\frac{\Delta \alpha}{\alpha}|_{th} = \frac{\alpha_1}{2\pi} \left[ 8 \beta_A \Delta \ln \alpha_X + \frac{4}{3} \left( \frac{2}{3} \Delta \ln m_s m_t M_\mu^2 + \frac{1}{3} \Delta \ln \frac{m^q}{M_X} + 2 \Delta \ln \frac{m^l}{M_X} \right) \right]$$

$$+ \frac{21}{3} \Delta \ln \frac{M_W}{M_X} + 8 \Delta \ln \frac{\tilde{m}}{M_X} + \frac{1}{3} \Delta \ln \frac{m_H}{M_X} \right]$$

(4)

$$= \frac{\alpha}{2\pi} \left[ \frac{8}{3} \beta_A + \beta_v + \frac{25}{3} \beta_S \right] \Delta \ln \alpha_X$$

(5)

where $m_l$ is an averaged lepton mass and $\tilde{m}$ and $m_H$ stand for geometric averages over superpartner and heavy Higgs masses respectively, where such fields exist. The parameters $\beta_v$ and $\beta_S$ describe the dependence of the Higgs v.e.v. and superpartner masses on the unified coupling as

$$\Delta \ln \frac{\mu^H}{M_X} = \beta_A \Delta \ln \alpha_X, \ \Delta \ln \frac{\tilde{m}}{M_X} = \beta_S \Delta \ln \alpha_X$$

while $\beta_A$ is defined analogously for the ratio $\Lambda_c/M_X$ (to be derived shortly). We neglect variations in fermion Yukawa couplings.

For the QCD invariant scale $\Lambda_c \equiv M e^{-2\pi/9\alpha_3(M)}$ where $m_s < M < m_c$ we have, including superpartners (squarks of geometric average mass $m_\tilde{q}$ and

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a Since single-step non-supersymmetric unification is ruled out, in what follows we will take the ‘non-supersymmetric case’ to denote the results when superpartner thresholds are ignored.
gluinos of mass $m_\tilde{g}$, for $\mu^+ > m_t, m_\tilde{q}, m_\tilde{g}$,

$$\frac{\Lambda_c}{\mu^+} = e^{-2\pi/9\alpha_3(\mu^+)} \left( \frac{m_c m_b m_t}{\mu^{+3}} \right)^{2/27} \left( \frac{m_\tilde{g} m_\tilde{q}}{\mu^{+2}} \right)^{2/9} \tag{6}$$

where for non-supersymmetric theories the last term is to be set to unity. Thus we find

$$\Delta \ln \frac{\Lambda_c}{M_X} \equiv \beta_\lambda \frac{\Delta \alpha_3(M_X)}{\alpha_3(M_X)} = \left( \frac{2\pi}{9\alpha_3(M_X)} + \frac{2}{9} \beta_v + \frac{4}{9} \beta_S \right) \frac{\Delta \alpha_3(M_X)}{\alpha_3(M_X)} \tag{7}$$

where we set $\beta_S = 0$ for a theory without superpartners. The threshold terms are of higher order in $\alpha$ or $\alpha_3$, thus formally they ought to be grouped with the power-law correction to $\Lambda_c$ from two-loop running, which we have neglected. However, when the Higgs v.e.v. is varying rapidly compared to $\alpha$, as is generic in theories with high fundamental scale\footnote{The threshold terms can become dominant, in contrast to the two-loop term, which is model-independent.}

The values of $\beta_v$ and $\beta_S$ are determined by the mechanisms of mass generation and SUSY-breaking. Technicolour and radiative EWSB with hidden-sector SUSY-breaking have in common the generation of an exponentially small scale by strong running in an asymptotically-free gauge group:

$$\frac{v_H}{M_X} = k_\alpha^n \alpha X^{-2\pi m/b_h}\alpha X$$

where $k$ is a numerical constant and $n$ parameterises a possible power-law dependence. Here we take the most natural case when the coupling of the the “hidden” gauge group responsible for generating the electroweak scale $\alpha_H$ is unified with the SM gauge couplings at the scale $M_X$. Our estimates are valid more generally if the variation in $\alpha^{-1}_H(M_X)$ equals that in $\alpha^{-1}_X$. In the case of theories without fundamental Higgs, this equation simply parameterises whatever condensate breaks SU(2)$\times$U(1). Thus,

$$\frac{M_X}{v_H} \Delta \frac{v_H}{M_X} \simeq \left( n + \frac{2\pi m}{b_h}\alpha X \right) \frac{\Delta \alpha_X}{\alpha_X}. \tag{9}$$

Assuming that $k$ and $n$ are order 1, we neglect (the logarithm of) their contribution to Eq. (9) and find

$$\frac{2\pi m}{b_h}\alpha X \simeq \ln[M_X/v_H \simeq (2 \times 10^{16})/(2 \times 10^2)] \simeq 32$$

where we identified $M_X$ with the SUSY-GUT value $2.4 \times 10^{16}$ GeV. Thus the RHS of Eq. (9) is approximately $(n + 32)\Delta \ln \alpha_X$, consistent with neglecting $n$ of order 1. If $M_X$ is the heterotic string scale $M_X \sim 4 \times 10^{17}$ GeV the
prefactor is \((n + 35)\), thus the uncertainty introduced by \(n\) is small and we estimate in general \(\beta_v = (34 \pm 2)\). Similarly, SUSY-breaking masses vary as

\[
\frac{\tilde{m}}{M_X} = k' \alpha_X^n e^{-2\pi m/b_0 \alpha_X}
\]

where \(m_q, m_s\) etc. vary as \(\tilde{m}\), thus their variation can also be estimated as \(\beta_S = 34 \pm 2\) given that superpartner masses are around the electroweak scale.

Radiative EWSB may, however, depend sensitively on a combination of the top Yukawa coupling and ratios of SUSY-breaking soft mass terms \([17, 17]\), which are exceedingly model-dependent. To cover potential uncertainties we allow the Higgs v. e. v. to have a different fractional variation compared to the superpartner masses, thus we allow \(\beta_s \neq \beta_S\) in general. Since there is also the possibility that the variation of the “hidden” gauge kinetic function \(\alpha_H(M_X)^{-1}\) may differ from that of \(\alpha_X^{-1}\), we keep \(\beta_s\) and \(\beta_v\) as free parameters in the calculation and take the values \(\beta_v = \beta_S = 34 \pm 2\) as illustrative only. For these values, using the dependence of \(\Lambda_c\) from Eq. (7) in Eq. (4) we find

\[
\Delta \ln \alpha|_{\text{th}} \simeq (0.11 \pm 0.01) \Delta \ln \alpha_X\ [\text{non-SUSY}], \ (0.49 \pm 0.03) \Delta \ln \alpha_X\ [\text{SUSY}]
\]

which is a non-negligible correction to the direct contribution \(0.47 \Delta \ln \alpha_X\).

Now if \(m_\mu\) is well-approximated by a constant times \(\Lambda_c\), we find

\[
\frac{\Delta \mu}{\mu} = (\beta_\Lambda - \beta_v) \frac{\Delta \alpha_X}{\alpha_X} = (-10 \pm \text{few}) \frac{\Delta \alpha_X}{\alpha_X}\ [\text{non-SUSY}], \ (5 \pm \text{few}) \frac{\Delta \alpha_X}{\alpha_X}\ [\text{SUSY}].
\]

If the electron Yukawa coupling also varies, due to some dynamics of flavour structure, these values may be changed to, for example, \(-13\) (non-SUSY) or \(+2\) (SUSY) if we have \(y_e \propto \alpha_X^3\). These values differ considerably from the cases where Higgs v. e. v. and superpartner masses are fixed relative to \(M_X\), giving \(\Delta \ln \mu \simeq 17 \Delta \ln \alpha_X\), or where \(v_H\) varies as in \([17]\) but the resulting effects on and of QCD thresholds are neglected, giving \(\Delta \ln \mu/\mu \simeq -17 \Delta \ln \alpha_X\). The values of \(\bar{R}\) following from Eq. (11) are \(-17 \pm 3\) (non-SUSY) and \(4 \pm 3\) (SUSY).

One may also include the effect of varying \(m_u, m_d\) and \(m_s\) relative to \(\Lambda_c\) on the proton mass: we find \([10]\)

\[
\Delta \ln \frac{m_p}{M} \approx 0.78 \Delta \ln \frac{\Lambda_c}{M} + (0.12 \pm 0.1) \Delta \ln \frac{m_s}{M} + 0.048 \sum_{u,d} \Delta \ln \frac{m_q}{M}
\]

where the strange term introduces the largest uncertainty, the other terms having errors of order 10%. Thus our final estimate is

\[
\bar{R} \approx \frac{0.54 \alpha_X^{-1} + (-0.61 \pm 0.12) \beta_v + 0.35 \beta_S}{0.022 \alpha_X^{-2} + 0.0018 \beta_v + 0.011 \beta_S}
\]
Without variation in the electroweak and superpartner scales, setting \( \beta_v = \beta_S = 0 \) we find the rather model-independent result \( \bar{R} = 25 \pm (\text{few}) \). This differs from the expectation \( \bar{R} \approx 36 \) obtained by neglecting the effects of light quark masses on \( \Lambda_c \) and the effect of varying \( \Lambda_c/M_X \) on \( \alpha \). If we set \( \beta_v \approx 34 \) and \( \beta_S = 0 \), we obtain \( \bar{R} = -13 \pm 7 \). Finally for \( \beta_S = \beta_v = 34 \pm 2 \) we find \( \bar{R} = 4 \pm 5 \).

3 Varying extra-dimensional radii

In a model which is described by \((4+\delta)\)-dimensional field theory over some range of energies, there is a hierarchy between the (inverse) compactification radius or radii \( R_i^{-1} \) and the ultraviolet cutoff of the higher-dimensional theory \( \Lambda_D \), where \( D = 4+\delta \). This makes it a well-defined problem to compute the effect of varying radius on the 4D low-energy theory, since one can take physics above the scale \( R_i^{-1} \) to be unchanged by such variation (apart from the change in masses of Kaluza-Klein modes), consistent with decoupling. The varying dimensionless quantity is the radius normalized to the cutoff of the extra-dimensional field theory, \( R \Lambda_D \). By dimensional reduction at the energy scale \( M_\delta \equiv R^{-1} \) we find the simple formulae

\[
\Delta \ln \alpha_i(M_\delta) = \delta_i \Delta \ln \frac{M_\delta}{\Lambda_D}, \quad \Delta \ln y(M_\delta) = \delta_i \left( \frac{p_y}{2} + c \right) \Delta \ln \frac{M_\delta}{\Lambda_D},
\]

which are good approximations in the perturbative regime where \( y \) is a Yukawa coupling for an operator with \( p_y = 0, 1, 2, 3 \) fields propagating in extra dimension and \( (3-p_y) \) fields localized “on the brane” and \( \delta_i \) is the number of extra dimensions around which the gauge or matter fields propagate. For simplicity we will take one varying dimension. The integer \( c \) takes the value 0 for operators localized in the extra dimension and 1 for a bulk coupling when \( p_y = 3 \). For a supersymmetric Higgs mass term \( \mu S H_1 H_2 \) localized on the brane with Higgs in the bulk, \( \mu_S \) is directly proportional to \( M_\delta \), \( \mu_S/M_\delta = \text{constant} \); for a bulk \( \mu \)-term or for localized Higgses \( \mu_S \) arises from a mass term in the \( D = 5 \) theory and varies as \( \mu_S/M_\delta \propto \left( M_\delta/\Lambda_D \right)^{-1} \).

If we now take the Higgs v. e. v. and masses of SM fermions (and superpartners, if any) to vary proportional to \( M_\delta \), which is expected to be a reasonable approximation for Scherk-Schwarz symmetry-breaking, we find

\[
\Delta \ln \alpha = \left( \delta_Y \cos^2 \theta_W + \delta_z \sin^2 \theta_W + \frac{\alpha(M_Z)}{2\pi} (\delta_Y b_Y + \delta_z b_2) \ln \frac{M_Z}{M_\delta} \right) \Delta \ln \frac{M_\delta}{\Lambda_D},
\]

\[
\Delta \ln \mu = \frac{2\pi \delta_3}{9} \left( \frac{\alpha^{-1}(M_Z)}{2\pi} + \frac{b_3}{2\pi} \ln \frac{M_Z}{M_\delta} \right) \Delta \ln \frac{M_\delta}{\Lambda_D}
\]

\( (14) \)
thus without superpartners we have

\[ \bar{R} \approx \delta_3 \left( 5.9 - \frac{7}{9} \ln \frac{M_Z}{M_\delta} \right) \left[ \delta_Y \cos^2 \theta_W + \delta_2 \sin^2 \theta_W + \frac{1}{804} \left( \frac{41}{6} \delta_Y - \frac{19}{6} \delta_2 \right) \ln \frac{M_Z}{M_\delta} \right]^{-1} \]

and with superpartners (in the case \( M_\delta \gg \tilde{m} \))

\[ \bar{R} \approx \delta_3 \left( 5.9 - \frac{3}{9} \ln \frac{M_Z}{M_\delta} \right) \left[ \delta_Y \cos^2 \theta_W + \delta_2 \sin^2 \theta_W + \frac{1}{804} \left( 11 \delta_Y + \delta_2 \right) \ln \frac{M_Z}{M_\delta} \right]^{-1}. \]

Enforcing \( \delta_Y = \delta_2 = \delta_3 \) as required by an extra-dimensional GUT we obtain

\[ \bar{R} \approx \frac{5.9 - \frac{7}{9} \ln(M_Z/M_\delta)}{1 + \frac{1}{804} \cdot \frac{14}{3} \ln(M_Z/M_\delta)} \text{ [non-SUSY]}, \quad \frac{5.9 - \frac{3}{9} \ln(M_Z/M_\delta)}{1 + \frac{1}{804} \cdot 12 \ln(M_Z/M_\delta)} \text{ [SUSY].} \]

If we use the more detailed treatment of the proton mass Eq. (12) then the value of \( \bar{R} \) is simply reduced by a factor 0.78. Given Eq. (17), in the small radius limit \( M_\delta \to 2.4 \times 10^{16} \text{GeV} \) we recover the SUSY-GUT expectation \( \bar{R} \approx 33 \). In the opposite limit where \( M_\delta \) approaches \( M_Z \) we would obtain \( \bar{R} \gg 6 \). This suppression of \( \bar{R} \) is consistent with [13].

In more general “brane world” models, one has freedom to choose the integers \( \delta_i \) and \( p_y \). Clearly if \( \delta_3 = 0 \) we have \( \Delta \ln \mu = 0 \), up to effects (so far neglected) of thresholds below \( M_\delta \) and variation in \( m_e/M_\delta \). The strong force does not “feel” the variation because it is not propagating round the varying dimension. If one wishes to fit the value \( \bar{R} = 0 \) this choice of \( \delta_3 \) is obvious, if somewhat arbitrary in the absence of a concrete model.

Above, we neglected any variation in the electron mass \( m_e/M_\delta \). But for small values of \( \bar{R} \) even a mild power-law dependence of \( m_e/M_\delta \) may be significant. We considered some concrete models of EWSB in extra dimensions [14, 20], the model of [14] does not lead to any definite prediction due to the arbitrary choice of the value of \( \mu_S \) and the sensitivity of the variation of \( v_H/M_\delta \) to this parameter. The model of [20] predicts the dependence \( m_e/M_\delta \propto \text{constant} \times (M_\delta/\Lambda_D) \), which simply subtracts 1 from the denominator of Eq. (17). This feeble dependence justifies neglecting the variation of other fermion masses. Taking into account the compactification scale required by the model we find \( \bar{R} = 6.3 \pm 1 \).

4 Conclusions

If the finding of a nonzero variation in \( \alpha \) persists, improved constraints on \( \mu \) may be a powerful tool to discriminate between models of physics beyond the
SM. We showed that it is important to include effects of varying mass ratios which were previously neglected in models with high-scale unification. We also showed that it is possible to obtain predictions without unification if the observed variation is due to a varying extra dimension.

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