Configuration entropy of the Cosmic Web: Can voids mimic the dark energy?

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ABSTRACT

We propose an alternative physical mechanism to explain the observed accelerated expansion of the Universe based on the configuration entropy of the cosmic web and its evolution. We show that the sheets, filaments and clusters in the cosmic web act as sinks whereas the voids act as the sources of information. The differential entropy of the cosmic velocity field increases with time and also acts as a source of entropy. The growth of non-linear structures and the emergence of the cosmic web may lead to a situation where the overall dissipation rate of information at the sinks are about to dominate the generation rate of information from the sources. Consequently, the Universe either requires a dispersal of the overdense non-linear structures or an accelerated expansion of the underdense voids to prevent a violation of the second law of thermodynamics. We argue that this accelerated expansion of the voids inside the cosmic web may mimic the behaviour of dark energy.

Key words: methods: analytical - cosmology: theory - large scale structure of the Universe.

1 INTRODUCTION

The current accelerated expansion of the Universe remains one of the major unsolved puzzle in cosmology. Observations imply that we live in an expanding Universe (Hubble 1929) which is currently expanding at an accelerating rate (Riess et al. 1998; Perlmutter et al. 1999). The accelerated expansion is unforeseen in an Universe in the presence of matter and gravity. The observed acceleration can be explained by the general theory of relativity only if a hypothetical entity called dark energy with a negative pressure co-exists along with the matter and radiation in the Universe.

Einstein introduced the cosmological constant \( \Lambda \) in his field equations to counterbalance the effect of gravity and achieve a stationary Universe. But such a time-independent cosmological constant can be also regarded as a candidate for dark energy. The cosmological constant is very often identified as the energy of the vacuum which remains constant despite the expansion of the Universe and eventually becomes the most dominant component leading to the observed acceleration. Unfortunately, the theoretical value of vacuum energy predicted by the quantum field theories exceeds the observed value of the cosmological constant by a factor of \( 10^{60} \) to \( 10^{120} \). The huge discrepancy between the predicted theoretical value and the observational value indicates that the nature and physical origin of the dark energy still remain largely elusive.

Many alternative models of dark energy have been proposed by either modifying the matter side (e.g. quintessence (Ratra & Peebles 1988; Caldwell et al. 1998) and k-essence (Armendariz-Picon et al. 2001)) or the geometric side (e.g. f(R) gravity (Buchdahl 1970) and scalar tensor theories (Brans & Dicke 1961)) of Einstein’s field equations. Some other alternatives have been also introduced based on some interesting physically motivated ideas such as the backreaction mechanism (Buchert 2000), effect of a large local void (Tomita 2001; Hunt & Sarkar 2010), entropic force (Easson et al. 2011), extra-dimension (Milton 2003), entropy maximization (Radicella & Pavón 2012; Pavón & Radicella 2013), information storage in the spacetime (Padmanabhan 2017; Padmanabhan & Padmanabhan 2017) and configuration entropy of the Universe (Pandey 2017). Copeland et al. (2006) and Amendola & Tsujikawa (2010) provide a detailed review on some of these models and the possible ways to confront them with observations.

The \( \Lambda \)CDM model with a time-independent cosmological constant still stands out as the best bet among all the possible scenarios proposed in the literature. The cold dark matter (CDM) model was put forth by Peebles (1982) in the early 1980s. The CDM model very soon became the central ingredient in understanding the cosmic structure formation (Bond et al. 1982; Blumenthal et al. 1984; Davis et al. 1985). Subsequent observations of the cosmic microwave background radiation (CMBR) suggested a critical density universe with a flat FRW geometry
(Komatsu et al. 2011; Planck Collaboration et al. 2016). On the other hand the total mass density of the Universe was constrained to $\Omega_m = 0.3$ by many other observations (Carlberg et al. 1996; Mohr et al. 1999; Benjamin et al. 2007; Fu et al. 2008; Eisenstein et al. 2005; Hawkins et al. 2003; Tegmark et al. 2004; Reid et al. 2010; Percival et al. 2010). The requirement of a flat critical density universe with $\Omega_m = 0.3$ imply that the density parameter associated with the cosmological constant $\Lambda$ must have to be $\Omega_\Lambda = 0.7$. The existence of such a dark energy component is also supported by independent observations (Riess et al. 1998; Perlmutter et al. 1997; Carlberg et al. 1989; Eisenstein et al. 2005; Wang 2006).

The CMBR observations suggest that the Universe was highly smooth and regular at earlier times. The level of anisotropy observed in the CMBR is $\sim 10^{-5}$. On the other hand, the present day mass distribution in the Universe is highly irregular and clumpy due to the structure formation. Pandey (2017) proposed an alternative scenario where the dissipation of the configuration entropy in the Universe due to structure formation can lead to an accelerated expansion of the Universe. If the other entropy generation mechanisms are not as efficient as the dissipation then the Universe requires a mechanism to cease the dissipation by suppressing the growth of structures. Interestingly, introducing a cosmological constant $\Lambda$ in the setting shut off the growth of structures on large scales and terminate the dissipation of the configuration entropy. However, this is not true for the non-linear structures for which the collapse has already started. The anisotropic gravitational collapse (Zeldovich 1970) leads to the formation of a complex network known as the “Cosmic Web” (Bond et al. 1996). The earlier analysis by Pandey (2017) was limited to the linear regime. In the present work, we extend the analysis beyond the linear regime using the Zeldovich approximation (Zeldovich 1970) and propose a possible mechanism for the cosmic acceleration driven by the information flow inside the cosmic web.

2 CONFIGURATION ENTROPY OF THE COSMIC WEB AND ITS EVOLUTION

We consider a large volume $V$ of the Universe and treat its matter content as a fluid. The configuration entropy of the fluid in that volume is then defined (Pandey 2017) as,

$$S_c(t) = -\int \rho(\vec{x},t) \log \rho(\vec{x},t) dV$$

where $dV$ is a small element of volume and $\rho(\vec{x},t)$ is the density inside the volume element centered at $\vec{x}$. The definition is motivated by the idea of the information entropy originally proposed by Shannon (1948).

Pandey (2017) show that the configuration entropy in a volume $V$ of the fluid will evolve as,

$$\frac{dS_c}{dt} = \int \rho(\vec{x},t) \nabla \cdot \vec{v}(\vec{x},t) dV$$

where $\vec{v}(\vec{x},t)$ is the peculiar velocity of the fluid contained inside the volume element $dV$ at time $t$.

Zeldovich approximation (Zeldovich 1970), a first order Lagrangian perturbation theory provides an elegant description of the formation of the cosmic web. It provides a scheme for mapping the Lagrangian positions of the particles to their Eulerian co-ordinates. If $\vec{x}(t)$ is the Eulerian co-ordinate of a particle at time $t$ then it is related to its initial Lagrangian co-ordinate $\vec{q}$ at $t \to 0$ as,

$$\vec{x}(t) = a(t)[\vec{q} + D_s(t)\hat{S}(\vec{q})]$$

where $a(t)$ is the scale factor, $D_s(t)$ is the growing mode of density perturbations and $\hat{S}(\vec{q})$ is a time-independent vector field. It assumes that the particles continue to move along the initial directions.

Considering the conservation of mass, one can obtain the Eulerian density $\rho(\vec{x},t)$ as,

$$\rho(\vec{x},t) = \frac{\hat{\rho}}{|\partial x_j / \partial q_j|}$$

where $\hat{\rho}$ is the mean density at time $t$ and $\partial x_j / \partial q_j$ is the Jacobian of the transformation between $\vec{x}(t)$ and $\vec{q}$. This is often known as the deformation tensor which accounts for the gravitational evolution of the fluid. The vector field $\hat{S}(\vec{q})$ is irrotational since the density perturbations originate from the growing mode. This allows one to write it in terms of a potential. Consequently, the deformation tensor becomes a real symmetric matrix which is diagonalizable in some co-ordinate system. The Eulerian density can be written as,

$$\rho(\vec{x},t) = \frac{\hat{\rho}}{(1 - D_s(t)\lambda_1(q))(1 - D_s(t)\lambda_2(q))(1 - D_s(t)\lambda_3(q))}$$

where $\lambda_i = \partial x_j / \partial q_j$ are the eigenvalues of the deformation tensor. $\lambda_1(q)$, $\lambda_2(q)$ and $\lambda_3(q)$ are the three eigenvalues which give contraction or expansion along the three eigenvectors. If the eigenvalues are ordered as $\lambda_1(q) > \lambda_2(q) > \lambda_3(q)$ then the gravitational collapse would first occur along the shortest axis corresponding to the largest eigenvalue. The first singularity occurs at $q$ when $D_s(t) = 1$. The contraction along the shortest principal axes leads to a sheet-like structure which are believed to be the first non-linear structure formed by gravitational clustering. Doroshkevich (1970) show that simultaneous collapse along multiple axes is unlikely to occur. The subsequent collapse along the medium and the longest principal axes would produce a filament and a cluster respectively.

The Zeldovich approximation predicts the emergence of the observed cosmic web (Doroshkevich et al. 1980; Pauls & Melott 1995; Sarkar & Pandey 2018) and provides a fairly good match to the structures predicted by the N-body simulations (Buchert 1989; Coles et al. 1993; Yoshisato et al. 2006; Tassev & Zaldarriaga 2012). Some reviews of the Zeldovich approximation can be found in Shandarin & Zeldovich (1989), Sahni & Coles (1995), Hidding et al. (2014) and White (2014).

The sheets, filaments and clusters in the cosmic web would emerge at a given location depending on the signs of the eigenvalues $\lambda_1(q)$, $\lambda_2(q)$ and $\lambda_3(q)$. When one of the eigenvalues is positive and the other two are negative then the anisotropic gravitational collapse would produce a sheetlike structure. Similarly two positive and one negative eigenvalues would produce a filament and all three positive eigenvalues would eventually produce a cluster. In all of the above three cases the density $\rho(\vec{x},t)$ at a location $\vec{x}$ increases with time eventually reaching singularity when collapse occurs along one or multiple eigenvectors. On the other hand, if all three eigenvalues of the deformation tensor are negative then the density at a location $\vec{x}$ would continuously decrease producing a large underdensity or void.

The divergence of the peculiar velocity field $\nabla \cdot \vec{v}$ would be negative at the locations where sheets, filaments or clusters are formed. This is simply due to the inflow of mass towards these overdensities from their surroundings. The nature of flow would be different around these structures but the sign of $\nabla \cdot \vec{v}$ would be always negative for these structural elements of the cosmic web. On the other hand, the mass outflow from the underdensities or the voids towards the neighboring overdensities at their periphery would always give rise to a positive $\nabla \cdot \vec{v}$. Combining these information with Equation 2
indicates that the configuration entropy $S_c(t)$ would always dissipate from the underdense regions whereas it would increase inside the underdense regions. The decrease and increase in the configuration entropy in different parts of the cosmic web could play a crucial role in governing the dynamics of the cosmic web.

One should also take into account the change in the information entropy of the cosmic velocity field besides the change in the configuration entropy of the mass distribution inside the cosmic web. It is difficult to analytically predict the detailed flow patterns around the sheets, filaments and clusters in the cosmic web. It would require N-body simulations to fully track the details of the cosmic velocity field. Even with the N-body simulations, the collisionless dark matter makes the task complicated due to the multi-valued nature of the velocity field after shell crossing and discontinuities at caustics (Hahn et al. 2015).

We consider a peculiar velocity field $\mathbf{v}(\mathbf{x})$ which is smoothed over a radius $R$ using a spherical top hat or a Gaussian window function of size $R$. The smoothed peculiar velocity field is given by,

$$\mathbf{v}_R(\mathbf{x}) = \int \mathbf{v}(\mathbf{x}') W_{R}(\mathbf{x} - \mathbf{x}') d^3 x'$$

(6)

where $W_R(\mathbf{x} - \mathbf{x}')$ is the window function used for smoothing. The radius $R$ is chosen such that $R << L$, where $L = \left(\frac{3V}{4}\pi\right)^{1/3}$ and $V$ is the volume considered.

The line of sight component of this smoothed peculiar velocity field is $v_R(\mathbf{x}) = \mathbf{v}_R(\mathbf{x}) \cdot \mathbf{\hat{n}}$. The dispersion of $v_R(\mathbf{x})$ can be written as (Seto & Yokoyama 1998),

$$X(L, R) = \frac{1}{V} \int v_R^2(\mathbf{x}) d^3 x.$$  

(7)

The normalized dispersion of the line of sight component of the peculiar velocity field is given by,

$$\sigma_R^2 = \frac{(X^2(L, R)) - (X(L, R))^2}{(X(L, R))^2}.$$  

(8)

The cosmic velocity field can be described by a Gaussian distribution in the mildly non-linear regime (Nusser & Dekel 1993; Ciecielg et al. 2003). The differential entropy of a Gaussian distribution is $\ln(\sigma R \sqrt{2\pi e})$ where $\sigma$ is the standard deviation of the Gaussian. In the present context, if the distribution of the line of sight component of the peculiar velocity can be described by a Gaussian then the entropy associated with it would be $\ln(\sigma_R R \sqrt{2\pi e})$, where $\sigma_R$ is given by Equation 8. The value of $\sigma_R$ increases with the growth of non-linear structures resulting in an increase in the entropy associated with the velocity field.

### 3 DISCUSSION AND CONJECTURE

Let us imagine a sufficiently large volume $V$ over which the Universe can be treated as homogeneous and isotropic. If we consider the Universe to be divided into many such volumes then there will be no net mass exchange across these volumes. Initially the mass distribution would be highly uniform in each of these volumes leading to a high configuration entropy for the mass distribution. But over time, the tiny density fluctuations present in them would be amplified by gravitational instability leading to the formation of the cosmic web. The segregation of mass into different morphological components such as sheets, filaments and clusters would produce a highly non-uniform distribution inside each of these volumes. The mass which was earlier distributed uniformly across the entire volume $V$ now only occupies a small fraction of it and is distributed in a complex filamentary cosmic web. This leads to an overall decrease in the configuration entropy of the mass distribution inside the volume $V$. The Equation 5 tells us that the density would increase at the regions where sheets, filaments and clusters are formed. The $\nabla \cdot \mathbf{\hat{v}}$ would be negative at these regions due to the inflow of mass towards them. Combining these information in Equation 2, we find that the structural elements like sheets, filaments and clusters would act as a sink of configuration entropy. The configuration entropy dissipates through these structural elements of the cosmic web. On the other hand, although the density decreases in the underdense regions but the $\nabla \cdot \mathbf{\hat{v}}$ remains a positive quantity due to the mass outflow from these regions. The Equation 2 under such a situation would always lead to an increase in the configuration entropy. So the underdense regions or the voids can be regarded as the source which generates configuration entropy.

Many earlier studies (Kauffmann & Fairall 1991; El-Ad et al. 1996; Hoyle & Vogeley 2002; Plionis & Basilakos 2002; Platen et al. 2007) indicate that voids in the galaxy distribution occupy ~ 95% of the total volume. Colberg et al. (2005) find that the void volume fraction in a set of GIF2 simulations of the CDM model increases from 2.7% at redshift 3 to 61.2% at redshift 0. A recent analysis of the Millennium simulations (Springel et al. 2005; Boylan-Kolchin et al. 2009) by Cautun et al. (2014) using the NEXUS algorithm finds that at present the voids occupy the largest volume fraction (~ 77%) in the Universe but contain only ~ 15% of the total mass content. Their findings show that the filaments host the greatest share of mass (~ 50%) occupying only ~ 6% of the volume in the Universe. The sheets occupy ~ 18% volume and ~ 24% of the mass content. The nodes or the clusters occupy the least amount of volume (~ 0.1%) while hosting ~ 11% of the mass content in the Universe. These statistics imply that the overdensities like sheets, filaments and the clusters together host the majority of the mass content (~ 85%) of the Universe while occupying only ~ 33% of the volume of the Universe. The $\rho(\mathbf{x}, t) dV$ term in the Equation 2 represents the mass content inside each of the volume element $dV$. The Equation 2 suggests that a negative divergence of the peculiar velocity field in the overdense structures like sheets, filaments and clusters would result in a large decrease in the configuration entropy of the mass distribution.

The underdense regions or the voids occupy most of the volumes but contain little amount of mass. The density inside the voids decreases as matters stream out of them and accumulate around their periphery. The density within the voids gradually increases outward from their centre. Consequently, the matter in the void centre moves outward faster than matter near their periphery. The faster evacuation from the central regions of the voids leads to a uniform low density region at their interior (Goldberg & Vogeley 2004) which gradually evolve towards $\delta = -1$. The divergence of the peculiar velocity field $\nabla \cdot \mathbf{\hat{v}}$ always remains positive inside the underdense regions or the voids. Initially the configuration entropy would increase faster inside the voids and then slow down when the matter evacuation from the central region would turn it into a uniform low density region.

The configuration entropy rate $\frac{dS_c}{dt}$ is negative at the overdense regions like sheets, filaments and clusters whereas it remains positive at the underdense regions or the voids. The decrease and the increase of the configuration entropy from the different parts of the cosmic web continues with time. At any given instant of time, the increase in the configuration entropy at the voids plus the increase in the differential entropy of the velocity field should be larger than the overall decrease of the configuration entropy at the sheets, filaments and clusters. This is particularly true in the absence of any

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other major source of entropy. We expect the Universe as a whole to behave like a thermodynamical system and the total entropy of the Universe must always increase with time.

When structure formation in the Universe enters the non-linear regime, the growth of the non-linear structures and the emergence of the cosmic web would accelerate the dissipation of the configuration entropy. The Universe soon reaches a stage when the dissipation rate of the configuration entropy from the overdense regions is about to overcome its growth rate from the underdense regions and the cosmic velocity field. The situation can be reversed only if the non-linear structures are dissolved or dispersed uniformly or the underdense regions or the voids are uniformly stretched at an accelerated rate. The non-linear structures can not be dispersed due to the presence of their gravity and hence can not be a viable option. However due to their repulsive and outward peculiar gravitational acceleration, the voids naturally expand faster than the Hubble flow. It may be noted that the voids are very low density regions where the divergence of the peculiar velocity field \( \nabla \cdot \vec{v} \propto -\frac{\partial \delta}{\partial t} \) would be a small positive quantity resulting in a slower increase in the configuration entropy despite the huge volume occupied by them. Interestingly, if the voids undergo an accelerated expansion it would lead to a larger \( \nabla \cdot \vec{v} \) inside them due to a larger magnitude of \( \frac{\partial \delta}{\partial t} \). It should be noted that this does not happen due to the normal evacuation of matter from the voids due to the gravitational field of the non-linear structures at their periphery. The increase in the divergence of the peculiar velocity field at the voids purely arises due to a response of the Universe to the dissipation of the configuration entropy at the overdense non-linear structures. Incidentally, the accelerated expansion of the Universe started in the near past when the non-linear structure formation leads to the emergence of the cosmic web. The accelerated expansion of the underdense regions or the voids suppress any further growth of structures on the linear scales. So the freeze out of the growth of structures on linear scales in the near past may be a consequence of this accelerated expansion of the voids.

In the present work, we propose a physical mechanism which may lead to the observed accelerated expansion of the Universe. Admittedly, this does not rule out the possibility of the existence of the dark energy but provides an alternative which may explain the accelerated expansion of the Universe without requiring any such fiducial component.

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