Low-Energy Probes of a Warped Extra Dimension

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Abstract

We investigate a natural realization of a light Abelian hidden sector in an extended Randall-Sundrum (RS) model. In addition to the usual RS bulk we consider a second warped space containing a bulk $U(1)_x$ gauge theory with a characteristic IR scale of order a GeV. This Abelian hidden sector can couple to the standard model via gauge kinetic mixing on a common UV brane. We show that if such a coupling induces significant mixing between the lightest $U(1)_x$ gauge mode and the standard model photon and Z, it can also induce significant mixing with the heavier $U(1)_x$ Kaluza-Klein (KK) modes. As a result it might be possible to probe several KK modes in upcoming fixed-target experiments and meson factories, thereby offering a new way to investigate the structure of an extra spacetime dimension.
1 Introduction

New physics beyond the standard model (SM) should either be kinematically inaccessible or couple very weakly to the SM in order to have evaded experimental efforts to date. The first possibility, of heavy new physics, has received the most attention and has pushed us to build ever larger colliders in an attempt to directly probe new energy frontiers [1, 2, 3]. It has also motivated the construction of precise lower-energy probes that search for the indirect effects of heavy new physics [4, 5, 6]. The second possibility, of relatively light new physics which couples very weakly to the SM, has not been explored to the same extent. Such new physics can also be sought in high-energy particle colliders, but in many cases a more promising route is to use lower-energy colliders with an enormous luminosity.

Hidden sectors containing new particles coupled very weakly to the SM comprise an interesting class of new physics scenarios that emerge naturally in a number of extensions of the SM. For example, theories of supersymmetry breaking typically contain a hidden sector where the breaking actually occurs. This hidden sector only couples to the SM via a set of heavy mediator particles [7] and in some cases can give rise to very light states [8]. Several recently proposed theories of dark matter, motivated by astrophysical measurements such as PAMELA [9] and Fermi [10], also contain light hidden sectors [11]. New gauge groups under which the SM fields are singlets also arise frequently in string-theoretic constructions [12, 13].

If the characteristic mass scale of a hidden sector is at or below a few GeV, it can potentially be discovered in current and upcoming fixed-target [14, 15, 16, 17, 18] and \(e^+e^-\) colliders [19, 20, 21, 22, 23, 24]. It is therefore timely and interesting to study hidden sector models where the GeV scale emerges in a (technically) natural way. Previous works have shown that the GeV scale can arise naturally in supersymmetric models provided supersymmetry breaking in the hidden sector is suppressed relative to the SM [25]. Similarly, models with composite hidden states can naturally realize a light hidden sector via dimensional transmutation [26], and have been studied in the context of hidden valleys [27, 28] and unparticles [29, 30].

In this work we realize a light Abelian hidden sector in an extended Randall-Sundrum (RS) model [31]. RS models provide a geometric means by which to naturally generate mass hierarchies and can readily realize sub-Planckian scales as simple redshifted incarnates of Planck scale input parameters. The specific model we consider consists of the standard bulk RS scenario together with a second hidden bulk space. The SM propagates exclusively in one of the bulks with the Higgs localized on the IR brane to realize the weak/Planck hierarchy. The second hidden bulk shares the same UV brane but has an independent IR scale. We take the hidden IR scale to be near a GeV and, as a minimal scenario, consider a \(U(1)_x\) gauge theory propagating in the hidden bulk. An illustration of the setup appears in Figure 1.

At energies near a GeV the minimal spectrum consists of the SM along with towers of hidden GeV-spaced gauge and gravity KK modes. The SM couples to the hidden KK vectors primarily via a localized kinetic mixing operator connecting \(U(1)_x\) and \(U(1)_Y\) on the shared UV brane. This kinetic mixing induces a suppressed coupling between the hidden vectors and SM matter which, for the lightest hidden KK modes, is proportional to SM electric...
charge \[32\]. Relative to the lightest \(U(1)_x\) mode, the higher modes have a coupling that is suppressed, but only by a moderate logarithmic factor. This can potentially allow the creation of several hidden gauge KK modes at high luminosity low-energy particle colliders. Kinetic mixing also allows these modes to decay back to the SM and we show that such channels can dominate, at least for the first few KK states. Fixed-target experiments and meson factories therefore have the potential to explore the geometry of a warped extra dimension beyond what might be possible at the LHC.

Although there is nothing fundamental about our choice of a GeV for the hidden IR scale, we feel there is a strong phenomenological motivation to study models of this type. RS models can be thought of as effective theory frameworks that capture some of the gross features of the warped throats found in string theory. The simplifications of the RS model include neglecting transverse throat structure\(^1\) and shrinking the rest of the compactification manifold to a single point (represented by the UV brane). String compactifications can also contain multiple warped throats with distinct characteristic IR scales \[12, 13, 34\]. This situation can be modeled with multiple RS bulks sharing a common UV brane \[35\]. The essential point is that if the SM is localized in one throat and has similar couplings to other throats, the throats with the lowest IR scales are typically the easiest to detect.

RS models are also thought to be dual to certain classes of 4D conformal field theories (CFTs) with conformal symmetry breaking at energies corresponding to the UV and IR scales \[36\]. In the multi-throat picture the hidden throats are dual to additional hidden 4D CFTs that share a common UV breaking of conformal invariance. These conformal breaking interactions can induce couplings between the different CFTs which are modeled by UV localized operators in the 5D picture. Each of the hidden CFTs has an, in principle, independent IR scale at which the conformality of the given CFT is broken. These IR scales set the characteristic scale for low-energy composites of the CFTs. We shall use the language of the 5D multi-throat picture in the present work but one should keep in mind that our investigations also encompass potentially observable effects from a broad class of hidden CFTs that couple to the SM. As with multiple warped throats, in a theory with multiple hidden CFTs we typically expect those with light characteristic mass scales to be the most

\(^1\)Phenomenological extensions that model transverse structure are possible; see e.g. \[33\].
amenable to discovery.

Multiple warped bulk spaces have been considered previously in a number of works. Our effective theory description follows that of Refs. \[35, 37\]. Various aspects of multi-throat physics from a string-motivated perspective appear in Refs. \[34\], and phenomenological applications can be found in Refs. \[38, 39, 40\]. Kinetic mixing in warped spaces was investigated in Ref. \[41\] as well as in certain string compactifications in Refs. \[42, 43\]. Our work also has overlap with several studies of hidden valleys \[27, 28\], composite dark sectors \[26\], and unparticles \[28, 29, 30\]. However, relative to these earlier studies, our specific construction and the application to low-energy probes is new to the best of our knowledge.

The outline of this paper is as follows. In Section 2 we describe the gravitational background of our model. Next, we derive the spectrum of physical vectors and their couplings to SM matter in Section 3. In Section 4 we consider the constraints on this scenario and discuss the prospects for new signals in upcoming fixed-target and meson factory experiments. Finally, Section 5 is reserved for our conclusions. Some technical details including the KK graviton spectrum and details of the gauge boson mass diagonalization and couplings appear in the Appendix.

2 Gravitational Background and Modes

The extended RS model that we consider consists of two independent warped bulk spaces, or \textit{throats}, sharing a common UV brane. We use the coordinates \(z_i \in [k^{-1}, R_i], i = 1, 2\), for the extra dimension of the \(i\)-th throat and take the metric in the \(i\)-th throat to be

\[
d s_i^2 = \frac{1}{(k z_i)^2} \eta_{\mu \nu} d x^\mu d x^\nu - d z_i^2 = G_i^{MN} d x^M d x^N, \tag{1}
\]

where \(k\) is a throat-independent curvature\(^2\). The common UV brane, with characteristic mass scale \(\sim k\), is located at \(z_1 = z_2 = k^{-1}\) and IR branes with characteristic scales \(R_i^{-1}\) reside at \(z_i = R_i\). The IR scales are suppressed relative to the curvature, as is readily seen in the non-conformal coordinates\(^3\) in terms of which one has \(R_i^{-1} = e^{-k \pi r_i} k \ll k\). When sourced by a bulk cosmological constant and appropriate brane tensions the metric of Eq. (1) is a solution to the 5D Einstein equations \[35\].

The SM (hidden sector) resides in the throat \(i = 1 (i = 2)\) which we refer to as the visible (hidden) throat. The standard bulk RS picture is employed in the visible throat with SM gauge and matter fields propagating in the bulk and the SM Higgs localized at \(z_1 = R_1\). Consistency with precision electroweak bounds requires \(R_1^{-1} \gtrsim \text{TeV}\) as per usual in RS models \[44, 45\]. In principle the hidden throat could have a complicated gauge structure and could contain matter charged under the hidden gauge symmetries. However, in order

\(^2\)The curvature may differ in the two throats but will be approximately equal provided the throat localized energy densities are dominated by a common bulk cosmological constant. Allowing non-hierarchical differences will not significantly modify our results.

\(^3\)These are defined by \(y_i = k^{-1} \log(k z_i)\), \(y_i \in [0, \pi r_i]\) with \(r_i = (\pi k)^{-1} \log(k R_i)\).
to determine the likely low-energy probes of a light hidden sector it suffices to consider a minimal gauge sector. We therefore assume that the hidden throat contains an Abelian $U(1)_x$ gauge group that is broken, either spontaneously by an explicit Higgs at $z_2 = R_2$ or through Higgsless IR boundary conditions [16]. The mass scale on the hidden IR brane is taken to be $R_2^{-1} \lesssim \text{GeV}$ and the setup is illustrated in Figure 1.

Interactions between the SM and the hidden sector will be mediated by gravity and by local operators on the UV brane. Among the set of local operators, we focus primarily on a UV-localized gauge kinetic mixing interaction

\begin{equation}
S_{\text{UV}} \supset - \frac{\epsilon_s}{2M_*} \int_{\text{UV}} d^4x \sqrt{-\text{g}} g^{\mu\alpha} g^{\nu\beta} B_{\mu\nu} X_{\alpha\beta},
\end{equation}

where $B_{\mu\nu}$ is the 5D hypercharge gauge field strength, $X_{\mu\nu}$ is the 5D $U(1)_x$ field strength and $M_*$ is the bulk gravity scale. This interaction produces the only significant renormalizable coupling between the SM and hidden sector fields in the effective 4D theory and is responsible for the low-energy effects of interest in this work.

The graviton KK spectrum contains a zero mode that extends into both throats and reproduces Newtonian gravity in the effective 4D theory. Its coupling is universal and dictated by the effective 4D Planck mass,

\begin{equation}
M_{Pl}^2 = \sum_i M_*^3 \left[ 1 - \frac{1}{(kR_i)^2} \right] \approx \frac{M_*^3}{k}.
\end{equation}

For $k \sim M_*$ one has $M_{Pl} \sim M_*$ and we also take $k/M_* \lesssim 0.1$ to ensure the gravitational description can be trusted. In addition to the zero mode there are massive KK gravitons which split up naturally into a pair of KK towers; see Appendix A for details. The first (second) tower has spacings on the order of $R_2^{-1} \sim \text{GeV} \ (R_1^{-1} \sim \text{TeV})$ and is strongly localized towards the IR of the hidden (visible) throat. One can show that the GeV-spaced KK modes couple extremely weakly to matter on the UV brane and to the SM in the visible throat. These light KK modes are therefore experimentally viable, which should come as no surprise to those familiar with the viability of gravity in RS2 [47]; the KK graviton spectrum here is essentially the RS2 spectrum with the sub-TeV modes removed on one side and the sub-GeV modes removed on the other. The GeV-scale gravitons do however couple significantly to fields in the hidden bulk with important effects on the phenomenology of the hidden vectors.

Metric fluctuations in the throats also give rise to a pair of physical scalar modes (radions), that only acquire mass once a stabilization mechanism is specified. We will not discuss radius stabilization in this work, but expect that it is straightforward to extend the Goldberger-Wise mechanism [48] to stabilize both throats. The ratio $R_2/R_1 \sim 10^3$ translates into the rather mild hierarchy of $r_1/r_2 \approx 0.8$ in the non-conformal coordinates, which is easily achieved with small differences in the input parameters. Beyond the tuning required to fix the 4D cosmological constant, the setup is therefore devoid of fine tunings.
3 Gauge Bosons and Kinetic Mixing

In the setup described above the SM is sequestered from the hidden sector, up to gravity and local operators on the common UV brane. For the most part, the detailed bulk realization of the SM is not important for the low-energy effects we are interested in. It will likely require a custodially-extended electroweak gauge sector [44] and may include additional flavor symmetries [49]. The one assumption we make is that the hyper charge gauge factor extends to the UV brane, and that it is the only Abelian factor to do so. Custodially-extended RS models typically invoke Dirichlet boundary conditions on the UV brane, leaving $U(1)_Y$ as the only Abelian factor to extend to the UV [44]. Our assumption is therefore consistent with standard bulk RS constructs.

In what follows we derive the spectrum of hidden KK vectors and determine their couplings to the SM. As we consider the low-energy physics our results would be qualitatively the same if the SM was simply placed on the UV brane with an alternative mechanism like supersymmetry stabilizing the weak/Planck hierarchy [50].

3.1 Gauge Boson KK Modes

On the hidden side we consider two cases, differentiated by the breaking of the $U(1)_x$ gauge symmetry. In the first case $U(1)_x$ is broken by the vacuum expectation value (VEV) of a hidden Higgs $H_x$ confined to the hidden IR brane with $\langle H_x \rangle \ll R^{−1}_2$. The second case employs Higgsless breaking of $U(1)_x$ via a Dirichlet boundary condition on the hidden IR brane [46]. Increasing $\langle H_x \rangle$ towards and above $R^{−1}_2$ smoothly interpolates between these two cases [46]. Neumann boundary conditions are employed for both the SM and the hidden gauge fields on the UV brane, and we treat the kinetic mixing operator as a small perturbation.

The full bulk action for the Abelian gauge factors in unitary gauge with $B_5 = X_5 = 0$ is

$$S \supset -\frac{1}{4} \int d^4x \, dz_1 \, \sqrt{G_1} \, G_1^{MA} G_1^{NB} B_{MN} B_{AB}$$

$$-\frac{1}{4} \int d^4x \, dz_2 \, \sqrt{G_2} \, G_2^{MA} G_2^{NB} X_{MN} X_{AB}$$

$$-\frac{\epsilon_*}{2M_*} \int_{UV} d^4x \, g^{\mu\nu} g^{\alpha\beta} B_{\mu\nu} X_{\alpha\beta}.$$  

We assume perturbative values of the mixing parameter $\epsilon_* \ll 1$, which could be generated by integrating out a set of UV localized Dirac fermions charged under both hypercharge and $U(1)_x$. Regardless of its origin this term is consistent with the gauge symmetries of the theory and we therefore include it in our effective theory description.

Decomposing the five-dimensional $U(1)_x$ gauge field into KK modes according to

$$X_\mu(x, z) = \sum_n f^{(n)}(z) \tilde{X}_\mu(x),$$

we get

\footnote{In the CFT picture this corresponds to the SM being fundamental states, external to a hidden CFT.}

\footnote{We drop the subscript in this section so $z = z_2$.}
the bulk wave functions satisfy the equation of motion
\[
\left[z^2 \partial_z^2 - z \partial_z + z^2 m_n^2\right] f^{(n)}(z) = 0,
\]
and the following orthogonality conditions
\[
\int \frac{dz}{(kz)} f^{(n)}(z) f^{(m)}(z) = \delta_{nm}.
\]
In the weakly Higgsed case there is a zero mode with a constant profile \( f^{(0)} = \sqrt{k/\log(kR_2)} \).

Above this mode, for both the Higgses and Higgsless cases, we have the wavefunctions
\[
f^{(n)}(z) = \frac{(kz)}{N_n} [J_1(m_nz) + \beta_n Y_1(m_nz)],
\]
where \( J_1 \) and \( Y_1 \) are Bessel functions, and the Neumann boundary condition at \( z = k^{-1} \) gives
\[
\beta_n = -\frac{J_0(m_n/k)}{Y_0(m_n/k)} \approx \frac{\pi}{2} \frac{1}{\log(2k/m_n) - \gamma},
\]
where \( \gamma \approx 0.5778 \) is the Euler-Mascheroni constant. The eigenvalues \( m_n \) are fixed by applying the IR brane boundary conditions, and give
\[
\beta_n = -\frac{J_0(m_nR_2)}{Y_0(m_nR_2)} \quad \text{(Neumann at } z = R_2),
\]
or
\[
\beta_n = -\frac{J_1(m_nR_2)}{Y_1(m_nR_2)} \quad \text{(Dirichlet at } z = R_2).
\]

For \( n \) greater than a few the KK masses \( m_n \) and normalization factors \( N_n \) are well approximated by
\[
m_n \simeq \frac{\pi}{R_2} (n + 1/4),
\]
and
\[
N_n^{-1} \simeq \frac{1}{(n + 1/4)^{1/2}} \frac{m_n}{\sqrt{k}}
\]
where the minus (plus) sign corresponds to the Higgsed (Higgsless) case. The mass of the lowest mode in the Higgsless case, which we label as “0”, is suppressed relative to the hidden IR scale,
\[
m_0 \simeq \frac{1}{R_2} \sqrt{\frac{2}{\log(2kR_2) - \gamma}},
\]
while its wavefunction is given by Eq. (10) with \( N_0^{-1} \simeq \sqrt{2/kR_2^2} \).
Putting these results together, and treating the $\gamma$ and $Z$ components of the SM hypercharge gauge boson as simple zero modes with profiles $\sqrt{k/\log(kR_1)}$, the effective kinetic mixing operator is

$$\mathcal{L}_{\text{eff}} \supset -\frac{1}{2} \sum_n \epsilon_n X_n^{\mu\nu}(c_W \tilde{F}_{\mu\nu} - s_W \tilde{Z}_{\mu\nu}),$$

(17)

where $s_W$ and $c_W$ refer to the weak mixing angle, and the kinetic mixing parameters $\epsilon_n$ are given by

$$\epsilon_n = \epsilon_* \frac{k}{M_*} \frac{1}{\sqrt{\log(kR_1)}} \frac{f(n)(z = k^{-1})}{\sqrt{k}}.$$

(18)

For the zero modes, the expressions derived above in the weakly Higgsed scenario give

$$\epsilon_0 \simeq \epsilon_* \frac{k}{M_*} \frac{1}{\sqrt{\log(kR_1)}} \frac{1}{\log(kR_2)},$$

(Higgsed)

(19)

while for the Higgsless case one has

$$\epsilon_0 \simeq \epsilon_* \frac{k}{M_*} \frac{1}{\sqrt{\log(kR_1)}} \frac{1}{[\log(m_0/2k) + \gamma]},$$

(Higgsless).

(20)

For the higher modes we find

$$\epsilon_n \simeq \epsilon_* \frac{k}{M_*} \frac{1}{\sqrt{\log(kR_1)}} \frac{1}{[\log(m_n/2k) + \gamma]}(n \mp 1/4)^{-1/2}, \quad n \geq 1,$$

(21)

where the minus (plus) sign again applies to the Higgsed (Higgsless) case. These expressions show that the higher KK modes have a suppressed kinetic mixing with the SM fields relative to the lowest mode, but that this suppression is only logarithmic.

### 3.2 Neutral Gauge Boson Mass and Kinetic Mixing

In the preceding we have motivated a tower of neutral vectors that kinetically mix with the SM in an extended RS framework. Having determined the mass spectrum and kinetic mixing parameters of the hidden vectors in the effective 4D theory we now present the transformations that diagonalize both the kinetic and mass mixing. These operations induce couplings between SM matter and the hidden vectors and also modify the couplings of the SM $Z$ boson, thereby providing experimental means by which to explore the present scenario. We note that despite having motivated the tower of spin-one modes via a warped extra dimension, the methodology we develop in this section is more general and can be employed in any scenario containing a tower of vectors that kinetically mix with SM hypercharge. We therefore keep the discussion somewhat general. For the time being we shall treat all KK modes as narrow resonances and will return to this point in Section 4.
We consider a tower of vectors $\tilde{X}_n$ with mass $m_n$ that kinetically mix with SM hypercharge. The tower is labeled by the integer $n \in [0, n_\Lambda]$ and the heaviest mode has mass $m_{n_\Lambda} \sim \Lambda$ where $\Lambda \gg m_Z$ is a high-energy cutoff. In the present context we are primarily interested in the low-energy physics and do not concern ourselves with the KK modes of the SM photon and $Z$. We therefore truncate the hidden KK tower at the usual RS KK scale, $\Lambda \sim \text{few TeV}$. For convenience we define the integer $n_z$ such that

$$m_n < m_Z \quad \text{for} \quad n \leq n_z, \quad \text{(22)}$$

$$m_n > m_Z \quad \text{for} \quad n_z < n \leq n_\Lambda, \quad \text{(23)}$$

where $m_Z$ is the SM value of the $Z$ mass. The mixed kinetic Lagrangian is

$$- 4 \mathcal{L}_{\text{Kin}} = \tilde{F}_{\mu\nu} \tilde{F}^{\mu\nu} + \tilde{Z}_{\mu\nu} \tilde{Z}^{\mu\nu} + \sum_{n=0}^{n_\Lambda} \tilde{X}_n^{\mu\nu} \tilde{X}_n^{\mu\nu} + \sum_{n=0}^{n_\Lambda} 2\epsilon_n (c_W \tilde{F}_{\mu\nu} - s_W \tilde{Z}_{\mu\nu}) \tilde{X}_n^{\mu\nu}, \quad \text{(24)}$$

where we label the mixed fields with a tilde and $(c_W, s_W) = (\cos \theta_W, \sin \theta_W)$ refer to the weak mixing angle.

In a general theory containing two vectors $V_1, V_2$ with masses $m_1 < m_2$ and kinetic mixing parameter $\epsilon \ll 1$ the kinetic Lagrangian may be diagonalized to $O(\epsilon^3)$ with the following field redefinitions:

$$V_1 \rightarrow V_1 - \epsilon V_2,$$

$$V_2 \rightarrow (1 + \epsilon^2/2)V_2. \quad \text{(25)}$$

This shift is asymmetric between $V_1$ and $V_2$ and the diagonalization can similarly be achieved by instead shifting the heavier field $V_2$. However, by shifting the lighter field (and simply rescaling the heavier one) the mass mixing induced by the shift is proportional to the lighter mass scale $m_1$. Consequently the mass mixing angle is suppressed relative to the kinetic mixing parameter, $\sim \epsilon m_1^2/m_2^2$, and in this basis mass mixing effects are subdominant.

With this in mind the strategy for decoupling the kinetic mixing in Eq. (24) is to always shift the lightest fields and thereby minimize the effects of mass mixing. One first performs a shift of the photon field to decouple the kinetic mixing between $\tilde{A}$ and all the $\tilde{X}_n$. Then one decouples the mixing between $\tilde{Z}$ and $\tilde{X}_n$ by shifting $\tilde{X}_n$ for $n \leq n_z$ and shifting $\tilde{Z}$ for $n > n_z$. The result of this multi-step shift can be combined into the following:

$$\tilde{Z}^\mu \rightarrow \left(1 + \frac{s_W^2}{2} \sum_{n=0}^{n_z} \epsilon_n^2\right) \tilde{Z}^\mu + s_W \sum_{n=n_z+1}^{n_\Lambda} \epsilon_n \tilde{X}_n^{\mu},$$

$$\tilde{X}_n^{\mu} \rightarrow \begin{cases} (1 + c_W^2/2) \tilde{X}_n^{\mu} + \epsilon_n s_W \tilde{Z}^\mu & n \leq n_z, \\ (1 + \epsilon_n^2/2) \tilde{X}_n^{\mu} & n > n_z, \end{cases} \quad \text{(26)}$$

$$\tilde{A}^\mu \rightarrow A^\mu - \sum_{n=0}^{n_z} \epsilon_n c_W \tilde{X}_n^{\mu} - c_W s_W \left(\sum_{n=0}^{n_z} \epsilon_n^2\right) \tilde{Z}^\mu.$$

These shifts diagonalize the kinetic terms for $\tilde{Z}$ and $A$ up to $O(\epsilon^3)$. Subleading corrections of the form $\epsilon_n \epsilon_m \tilde{X}_n \tilde{X}_m$ remain but can be consistently neglected for $|\epsilon_n| \ll 1$. 9
Performing the field redefinitions of Eq. (26) in the vector-mass Lagrangian induces mass mixing between $\tilde{Z}$ and $\tilde{X}_n$, while $A$ is the physical massless photon. The full $\tilde{Z}$-$\tilde{X}_n$ mass matrix is given in Appendix B and can be diagonalized by a series of orthogonal transformations. To leading non-trivial order in $\epsilon$ (meaning $O(\epsilon)$ in off-diagonal terms and $O(\epsilon^2)$ on the diagonal), a single independent orthogonal rotation is needed for each KK level. The corresponding mixing angle at level $n$ is

$$
\eta_n = \epsilon_n s_W \times \frac{m_<^2}{m_Z^2 - m_n^2},
$$

where

$$
m_< = \begin{cases} 
m_n & \text{for } n \leq n_z \\
m_Z & \text{for } n > n_z
\end{cases}.
$$

To $O(\epsilon^2)$ the mixed fields $\tilde{Z}$, $\tilde{X}_n$ are related to the mass eigenstates $Z$, $X_n$ as

$$
\tilde{Z}^\mu \simeq \left(1 - \sum_{n=0}^{nA} \eta_n^2 \right) Z^\mu - \sum_{n=0}^{nA} \eta_n X_n^\mu,
$$

$$
\tilde{X}_n^\mu \simeq \left(1 - \frac{\eta_n^2}{2} \right) X_n^\mu + \eta_n Z^\mu.
$$

We give the corresponding mass eigenvalues in Appendix B. With the above one readily obtains the coupling of the physical vectors $X_n$ to SM matter and the induced modification of the SM $Z$ coupling. We present these couplings in Appendix C.

We note the following features of the above. For $n \ll n_z$ one has $m_n^2 \ll m_Z^2$ and the mixing angle $\eta_n$ is mass-suppressed relative to the kinetic mixing parameter, $|\eta_n| \ll |\epsilon_n|$. Similarly $\eta_n$ is mass-suppressed for $n \gg n_z$. Thus mass mixing effects are subdominant to the pure $\epsilon_n^2$ corrections from direct kinetic mixing unless $m_n \sim m_Z$. Modes with $\Delta m = |m_n - m_Z| \ll m_Z$ ($n \sim n_z$) can have a “resonant enhancement” relative to the kinetic mixing $\epsilon_n$:

$$
|\eta_n| \simeq \frac{s_W \epsilon_n}{2} \left(\frac{m_Z}{\Delta m}\right) \simeq \epsilon_n \left(\frac{22 \text{ GeV}}{\Delta m}\right),
$$

and for these modes the corrections from mass mixing effects can dominate.

A similar discussion follows for the couplings of the KK modes $X_n$ to SM matter, which are given in Appendix C. For modes with $m_n^2 \ll m_Z^2$ the mass suppression of $\eta_n$ ensures that the dominant coupling of $X_n$ to the SM is via the electromagnetic current $J_{em}^\mu$. Thus the coupling relevant for low-energy probes of our scenario is $-c_W \epsilon_n Q_{em} \epsilon$. Similarly for $n \gg n_z$ the dominant coupling to the SM is via the hypercharge current $J_Y^\mu$. For $n$ in the immediate vicinity of $n_z$ the dominant coupling of $X_n$ is via the SM $Z$ current $J_Z^\mu$, while for $|n - n_z|$ of order a few the coupling is via a linear combination of $J_Z^\mu$ and either $J_{em}^\mu$ or $J_Y^\mu$, depending on the sign of $(n - n_z)$. 

10
4 Signals and Signatures

A warped hidden sector mixing kinetically with the SM can give rise to new signals at the luminosity frontier achieved by fixed-target experiments and meson factories. Since the couplings of hidden KK vectors to the SM are similar, it may be possible to produce multiple KK resonances in relatively low-energy collisions. Thus arises the interesting possibility that existing and forthcoming low-energy experiments may directly probe an extra spacetime dimension. In this section we estimate the current constraints and the discovery prospects for this class of models. Our analysis here is preliminary and a more detailed investigation will be presented in an upcoming work [51].

4.1 Hidden KK Mode Decays

The constraints and signals of a warped $U(1)_x$ gauge sector depend sensitively on how the gauge KK modes decay. Therefore we must determine the most likely decay channels for the KK excitations. In addition to kinetic mixing with the SM on the UV brane, the vector KK modes also couple to the KK gravitons localized in the hidden bulk space. The vector excitations can also couple to an explicit Higgs field on the GeV brane, or through higher-dimensional operators.

Kinetic mixing with hypercharge on the UV brane allows the gauge KK modes to decay to the SM. Using the results of Ref. [20] the corresponding decay width of the $n$-th mode to a pair of SM leptons $\ell \bar{\ell}$ is

$$\Gamma(X_n \to \ell \bar{\ell}) = \frac{\epsilon_n^2 m_n}{12\pi} \left(1 + \frac{2m_\ell^2}{m_n^2}\right) \left(1 - \frac{m_\ell^2}{m_n^2}\right)^{1/2}.$$  \hspace{1cm} (32)

The decay width to hadrons is similar [20]. Since $\epsilon_n^2 \sim 1/n$ and $m_n \sim n$ the total decay width to the SM is roughly independent of mode number, up to a growth in the number of kinematically accessible SM final states for heavier modes.

In addition to SM decays, the heavier KK vectors can decay to lighter vector and graviton modes. For $n > (m + a)$ the decay $X_n \to X_m + h^{(a)}$ is kinematically permitted\(\text{\footnote{For the Higgsed case this assumes } v_x \ll R_2^{-1} \text{ so that } X_0 \text{ is light.}}\) for $n \geq 2$ and has width

$$\Gamma(X_n \to X_m + h^{(a)}) \sim \frac{1}{8\pi} \left(\frac{k}{M_*}\right)^2 B_{a,mn} m_n,$$  \hspace{1cm} (33)

where $B_{a,mn} \lesssim 1$ is a dimensionless coefficient that depends on wavefunction overlaps and is suppressed if the integers $a, m, n$ are vastly different. (See Appendix $\text{\ref{app:graviton-vector}}$ for the graviton-vector couplings). Comparing the coupling of Eq. (33) to Eq. (21), we see that gauge KK modes with $n \geq 2$ typically decay to a lower gauge KK mode and a graviton mode rather than going directly to SM final states.
If an explicit Higgs field with $U(1)_x$ charge $x_H = +1$ resides on the GeV brane and develops a vacuum expectation value, $H_x \rightarrow (v_x + \phi_x)/\sqrt{2}$ (with canonical normalization), the vector zero mode will acquire a mass $m_0 = g_x v_x$, and the hidden Higgs boson $\phi_x$ will couple to the vector KK modes as

$$- \mathcal{L}_{\text{eff}} \supset \lambda_{mn} \phi_x X_m X_n,$$

with

$$\lambda_{mn} = \frac{g_x^2 v_x}{k} \log(k R_2) \left. f^{(m)} f^{(n)} \right|_{z=R_2}.$$

When kinematically allowed this coupling will induce $X_n \rightarrow \phi_x X_m$, and will also allow the Higgs itself to decay to a pair of gauge bosons. If the Higgs is lighter than the lightest gauge mode $X_0$, it will decay to the SM either through a loop or via off-shell gauge bosons as in the 4D case discussed in Ref. [20]. While we do not consider radion excitations here, such a mode would participate in KK mode decays in much the same way as a hidden IR brane Higgs.

Higher-dimensional operators can also contribute to the decays of vector KK modes. However the leading operator of this type has the form $\int d^5x (X_{\mu\nu})^4/M_5^5$ and is typically less important than decay channels involving gravitons.

Putting these pieces together we see that the lightest gauge KK modes $X_0$ and $X_1$ may potentially decay primarily to pairs of SM fermions. For $n \geq 2$ the decays to a lighter gauge mode and a graviton, such as $X_2 \rightarrow X_0 h^{(1)}$, will typically dominate over the direct SM modes. The lightest KK graviton will also decay mainly through the gauge-graviton coupling, either $h^{(1)} \rightarrow X_0 X_0$ or $h^{(1)} \rightarrow X_1 X_1$ (with $X_1$ off shell or in a loop), and will eventually produce multiple SM fermions. A hidden Higgs may also decay to the SM through gauge KK modes.

The dominance of decays to lower KK modes rather than to the SM coincides with the discussion of unparticles and hidden valleys in Refs. [28, 52]. Going to higher modes, the decay width increases due the larger mass as well the larger number of final states. This width scales as

$$\Gamma_n \sim \frac{1}{8\pi} g_s(X_n) \left( \frac{k}{M_{\text{Pl}}} \right)^2 m_n,$$

where $g_s(X_n)$ is the number of significant decay channels. The results of Refs. [28, 52] suggest that $g_s(X_n) \sim n$, and we will assume this to be the case here. For $k/M_{\text{Pl}} \ll 1$ the decay width is less than the mode separation for $n < (M_{\text{Pl}}/k)/g_s(X_n)$, while for higher modes the KK resonances begin to overlap. Thus high-energy probes initiated at the UV brane (such as SM initial states) will excite a continuum of overlapping bulk modes, much like in RS2 or unparticle scenarios [28, 30]. These higher modes will cascade down to the the lightest hidden KK modes, which will then decay back to the SM producing a high multiplicity of soft final states [52]. This behavior is simply that of the hidden valley paradigm [28].

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Note that the form of the brane-to-bulk propagator suppresses couplings to modes with $m_n \ll q$, where $q$ is the momentum transferred in the process [36].
4.2 Constraints on a Hidden KK Tower

Our scenario extends the vector spectrum of Abelian hidden sector models considered previously with an entire KK tower of new kinetically-mixed states. We present here a preliminary estimate of the current experimental bounds on such a tower, deferring a detailed analysis to a future work [51]. For the time being we assume there is no explicit Higgs-like state with mass below the lightest gauge mode for which the bounds can be even stronger [16, 18]. We find that the lightest gauge modes are constrained primarily by low-energy probes. Bounds on heavier KK modes are less strict except for those with masses near the $Z$, which as discussed above can have a resonant enhancement in their mixing. We find that low-energy probes provide the strongest constraints, and the allowed parameter space in minimal models is very similar to that of a single Abelian hidden vector.

Low-energy probes are typically the most sensitive to the lightest gauge mode for two reasons. First, Eq. (21) shows that the relative coupling of the higher modes to the SM is somewhat smaller, $\epsilon_n \sim \epsilon_0 / \sqrt{n}$. Second, the higher modes are heavier and can receive an additional kinematic suppression. For example, the constraint from the anomalous magnetic moment of the muon $(g-2)_\mu$ scales as $\epsilon^2_n / m^2_n$ [19], and similarly for bounds from atomic parity violation [53]. Bounds from rare meson decays are also weakened by a reduction in the final state phase space [19, 22]. For these reasons, the parameter space in the $m_0 - \epsilon_0$ plane for the full KK tower that is consistent with low-energy tests is nearly identical to that of a single hidden Abelian gauge boson with $\epsilon = \epsilon_0$ and $m_V = m_0$ [15, 17, 19]. With $0.3 \text{ GeV} \lesssim m_0 \lesssim 10 \text{ GeV}$, this corresponds to $\epsilon_0 \lesssim 3 \times 10^{-3}$ [15].

High-energy probes can also be sensitive to a light warped hidden sector. Note that since the SM couples to the hidden sector only via the UV brane, we can still sensibly compute processes with energies well above the hidden IR cutoff provided they are initiated by SM states [36]. Among existing higher-energy probes, precision electroweak measurements near the $Z$ pole put the strongest constraints on gauge kinetic mixing [54]. As shown in Section 3, mixing between the SM $Z$ and the hidden sector is resonantly enhanced for modes with $m_n \sim m_Z$. The treatment of that section is appropriate for narrow $X_n$ and $Z$ states. When the mass difference $|m_n - m_Z|$ of an unstable gauge boson is less than the larger of the widths $\Gamma_Z, \Gamma_n$, it is more convenient to compute the effect of the hidden tower on $e^+e^-$ collisions by treating the kinetic mixing operator as an interaction.

Near the $Z$-pole the leading effect comes from two kinetic mixing insertions on a $Z$ propagator. This is reminiscent of an oblique correction, but we will see that it depends in an essential way on the finite widths of the KK resonances. Ignoring initial- and final-state fermion masses, the $Z$ propagator in the scattering amplitude is modified to

$$
\frac{1}{(s - m^2_Z) + im_Z \Gamma_Z} \left[ 1 + \sum_n \frac{s^2_W \epsilon^2_n s^2}{(s - m^2_Z + im_Z \Gamma_Z)(s - m^2_n + im_n \Gamma_n)} + \ldots \right],
$$

where $\Gamma_n$ is the decay width of $X_n$. This correction is enhanced relative to $\epsilon^2_n$ for modes with $m_n \sim m_Z$ when $s \sim m^2_Z$, coinciding with the resonant enhancement of the mixing angle

Note that the $\epsilon$ defined in Ref. [15] corresponds to $\epsilon_n c_W$ here.
found when diagonalizing the full gauge boson mass matrix. However, we see in Eq. (37) that the mixing is regulated by the finite decay widths. This correction only modifies the $Z$ propagator and cancels out of asymmetries. (Subleading mixing with the photon will modify the asymmetries.) It can, however, modify the shape and peak location of the $Z$ resonance, which were measured carefully by the LEP collaborations [6]. These effects will be studied in more detail in Ref. [51], but a preliminary analysis, using Eq. (36) to estimate the widths $\Gamma_n$, finds that the fractional shift in the lineshape over the entire $Z$ resonance is always well below $10^{-4}$ for $\epsilon_0 < 3 \times 10^{-3}$ and KK mode spacings less than a few GeV. Given the precision of the combined electroweak measurements, this is safely small [6].

Resonantly enhanced kinetic mixing can also induce non-standard $Z$ decays. More precisely, the relevant process is $e^+e^- \rightarrow \text{(hidden)}$, with the hidden sector states decaying in a cascade down to the lightest hidden KK modes, which subsequently decay to the SM. This produces a spherical distribution of multiple soft SM final states [52]. Indeed, this picture is precisely that of a hidden valley discussed in Refs. [27, 28]. The dominant contribution comes from an $s$-channel $Z$ mixing kinetically into an $X_n$ state with $m_n \sim m_Z$, which then decays to lighter graviton and gauge KK modes. The corresponding “branching fraction”, defined as a ratio of rates on the $Z$-pole, is estimated to go like

$$BR(Z \rightarrow \text{hidden}) \sim \sum_n \frac{s_{1W}^2 \epsilon_n^2 m_Z^4}{(m_Z^2 - m_n^2)^2 + m_n^2 \Gamma_n^2} \left( \frac{\Gamma_n}{\Gamma_Z} \right).$$

(38)

For $\epsilon_0 < 3 \times 10^{-3}$ we find that this fraction is always less than about $3 \times 10^{-6}$. This is at the edge of sensitivity of the LEP experiments to exotic $Z$ decays, and the typical final state consisting of a high multiplicity of soft leptons and pions was not searched for directly [6, 55].

4.3 New Low-Energy Signals

We have argued that a closely-spaced tower of hidden KK modes can be consistent with current bounds for $\epsilon_0 \lesssim 3 \times 10^{-3}$ and a mode spacing on the order of or below a GeV. For couplings not too much smaller than this, it may be possible to discover the lightest hidden KK modes directly in proposed fixed-target and meson factory searches. Most interestingly, several KK modes could potentially be found this way.

The lowest hidden gauge mode will typically be the easiest to find in a fixed-target experiment. Production of this and higher modes will proceed as discussed in Refs. [21]. However, relative to the lowest mode, the production of higher modes will be suppressed by factors of $\epsilon_n^2/\epsilon_0^2 \sim 1/36 n$. Despite this suppression, the potential reach of proposed future fixed-target experiments can exceed $\epsilon \sim 10^{-6}$ [15], which may be enough to discover the lowest and the first excited hidden KK modes provided they both decay primarily to the SM. Note that it will be possible to reconstruct both resonances provided their mass spacing is not too small.

Higher resonances that decay primarily to lighter hidden states will be more challenging to identify in fixed-target experiments. On top of a reduced production rate, the methods proposed in Ref. [15] rely on using specific geometries tailored to the distribution of signal
events to reduce backgrounds. In a multi-step decay process it will be more difficult to collect all the decay products without increasing the detector acceptance. On the other hand, the multiple sets of tracks produced in the decay of a higher KK mode could be used to reduce backgrounds.

Lower-energy $e^+e^-$ colliders such as BaBar, Belle, and DAΦNE may also permit the discovery of one or more light hidden vectors. Searches for a hidden vector based on continuum $\gamma X$ production at these colliders currently bound $\epsilon_0 \lesssim 3 \times 10^{-3}$ for $0.3 \lesssim m_0 \lesssim 10$ GeV [15]. This limit is adapted from a BaBar search for $\Upsilon(3s) \rightarrow \gamma a^0$, where $a^0$ is a light pseudoscalar decaying to $\mu^+\mu^-$ [56]. By expanding the search to include $\Upsilon(4s)$ data from Belle as well as multi-lepton final states it will be possible to investigate a significantly larger range of hidden vector models [20, 21, 22], including multiple hidden vector KK modes. Further improvements can also be expected from other low-energy searches such as the KLOE experiment at DAΦNE [15, 22, 23].

In addition to continuum $\gamma X$ production at $e^+e^-$ colliders, higher hidden KK vectors can also be produced resonantly in the $s$-channel for KK masses near the center-of-mass energies of the $B$-factories ($\sqrt{s} \sim 10.5$ GeV) and other meson factories such as DAΦNE ($\sqrt{s} \sim 1.0$ GeV) [21]. These rates can potentially be significant when the resonant KK vectors are able decay efficiently into lighter hidden-sector modes. Compared to minimal Abelian hidden scenarios where the hidden vector is only able to decay to the SM, resonant production in this case is proportional to $\epsilon^2$ rather than $\epsilon^4$, and the heavier KK modes can be relatively broad making them more likely to overlap with the center-of-mass energy. The typical signature of a heavier vector KK mode will be multiple charged tracks with a relatively spherical distribution, and can be similar to non-Abelian hidden sectors [21]. An analysis of the signals from resonant hidden KK mode production will be be presented in Ref. [51].

4.4 Dark Matter and Cosmology

Light hidden sectors have received attention recently in the context of models of dark matter (DM) motivated by new results from PAMELA, Fermi, and DAMA. All three of these experiments observe signals that can potentially be due to DM. However, the standard picture of a weakly-interacting massive particle (WIMP) undergoing thermal freeze-out does not appear to explain the data [57]. Instead a DM particle with mass above a few hundred GeV coupling to a light hidden force with an enhanced annihilation (or decay) rate in the local region can account for the signals [11].

This scenario has been realized in the context of supersymmetric hidden sectors [25], and with a few modifications can also apply in the present scenario. A simple option is for the DM to consist of a Dirac fermion confined to the UV brane with a mass near the electroweak scale and charged only under $U(1)_x$. This state will annihilate primarily to pairs of $U(1)_x$ zero-mode gauge bosons with a significant rate that depends on the fermion mass and charge, and possibly enhanced at late times through the Sommerfeld mechanism. These gauge bosons will in turn decay to SM states. This DM candidate will also acquire a direct
coupling to the SM $Z$ through gauge kinetic mixing, although with a strongly suppressed coupling. Let us also point out that to be consistent with bounds from DM direct detection searches, the gauge kinetic mixing must be somewhat small, $\epsilon_0 \lesssim 10^{-6}(m_0/\text{GeV})^2$ [58].

The primary modification required for this DM picture is in the mechanism to generate the DM density in the early universe. At temperatures well above the hidden IR scale the five-dimensional RS geometry is replaced by AdS-Schwarzschild [59]. The transition from this thermal state to the usual truncated AdS RS spacetime occurs at temperatures near the IR brane scale. Depending on the mechanism of radius stabilization this transition can be too slow to ever complete in the calculable regime of $k/M_{\text{Pl}} \ll 1$ [60]. This possibility will be avoided in the present scenario if primordial inflation never reheated above the hidden IR brane scale, so that the AdS-Schwarzschild geometry was never realized after inflation. This is consistent with primordial nucleosynthesis provided the reheating temperature was greater than about 5 MeV [61]. With even a very low reheating temperature the DM density could arise non-thermally from inflaton (or related) decays [62]. At higher reheating temperatures approaching a GeV, the DM could have partially rethermalized enough to generate a significant relic abundance [63].

If one is willing to give up on explaining the leptonic signals of PAMELA or Fermi with DM, it is also possible to have a light DM state within the hidden sector that is produced thermally [64, 65]. Such candidates have been considered as possible explanations for DAMA [66] and the INTEGRAL 511 keV line [67]. A simple option is a Dirac fermion on the hidden IR brane, but other possibilities exist. For example, instead of a minimal hidden gauge group of $U(1)$ one could consider a larger hidden sector; one possibility would be a warped implementation of the exact-parity or mirror matter models [68]. If the mirror sector resides in the hidden bulk then the DM may be stabilized by an approximate hidden baryon number symmetry and additional couplings between the two sectors could result [69].

5 Conclusions

In this work we have investigated a light Abelian hidden sector in an extended RS model. The hidden $U(1)_x$ symmetry propagates in a separate warped bulk and couples to the SM only via localized operators on a common UV brane. With a hidden IR scale of order a GeV the low-energy spectrum consists of the SM plus a tower of GeV spaced hidden KK vectors. The latter acquire a coupling to the SM via UV-localized mixing with SM hypercharge. Relative to the vector zero mode, the couplings of the heavier KK modes are only suppressed by a moderate logarithmic factor suggesting that they too can give rise to observable signals.

Although we defer a detailed analysis of the bounds on such a scenario to a future work [51], a preliminary analysis suggests that such a spectrum is consistent with existing constraints. We find that the lowest KK modes can potentially decay primarily to the SM. Higher modes will decay in a hidden cascade down to the lowest modes, which then decay back to the SM. In principle the lightest modes can be reconstructed directly as resonances, while the heavier modes can be reconstructed indirectly from their multi-body final states. This offers the interesting possibility that lower-energy experiments operating at
the luminosity frontier may observe light hidden KK vectors and thereby probe the structure of an extra spacetime dimension.

Note added: While this manuscript was in preparation Ref. [70] appeared, in which the authors obtain a GeV-scale hidden Abelian vector in the RS framework by an alternative means. The phenomenology of their model differs from ours as the KK modes of the hidden vector appear at the TeV scale in their case. See also Ref. [71].

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A Kaluza-Klein Gravitons

In RS models the mass of the lightest KK graviton modes is set by the IR scale. In multiple-throat constructions one expects that the KK graviton spectrum will contain modes with masses set by the local IR scales. In this appendix we derive the graviton KK spectrum for a pair of warped throats sharing a common UV brane as discussed in the text. We confirm the general expectation and show that this setup contains two towers of graviton modes whose masses and splittings are set by the IR scales $1/R_1 \sim \text{TeV}$ and $1/R_2 \sim \text{GeV}$. We show further that the GeV-scale modes are strongly localized in the hidden GeV throat, and the TeV-scale modes are strongly localized in the TeV throat. Several aspects of this discussion were presented previously in Ref. [35].

The 5D graviton arises from the metric perturbation,
\[ G^{(i)}_{\mu\nu} \rightarrow (kz_i)^{-2} \left[ \eta_{\mu\nu} + \frac{2}{M^3_\ast} h_{\mu\nu}(x, z_i) \right], \quad i = 1, 2, \] (39)
with $\partial^\mu h_{\mu\nu} = 0 = h_{\mu\nu}$. Performing a KK expansion in each of the throats,
\[ h_{\mu\nu}(x, z_i) = \sum_n h^{(n)}_{\mu\nu}(x) f^{(n)}_{h_i}(z_i), \] (40)
the Einstein equations require the profile in the $i$th throat to satisfy the equation of motion
\[ \left[ z_i^2 \partial_i^2 - 3z_i \partial_i + n_i^2 z_i^2 \right] f^{(n)}_{h_i}(z_i) = 0, \] (41)
and the following orthogonality condition,
\[ \sum_i \int \frac{dz_i}{(kz_i)^2} f^{(n)}_{h_i}(z_i) f^{(m)}_{h_i}(z_i) = \delta^{nm}. \] (42)
The solutions to (41) are of the usual RS form [72],

\[ f_{h,i}^{(n)}(z_i) = C^{(n)} \frac{(kz_i)^2}{N_i^{(n)}} \left\{ J_2(m_nz_i) + \beta_i^{(n)}Y_2(m_nz_i) \right\}, \quad (43) \]

where \( C^{(n)} \) is a throat-independent normalization constant determined by (42), \( \beta_i^{(n)} \) is a constant and we have factored out a throat-dependent constant \( N_i^{(n)} \). Imposing the usual Neumann BC at the IR brane of the \( i \)th throat gives

\[ \text{IR brane : } \beta_i^{(n)} = - \frac{J_1(m_nR_i)}{Y_1(m_nR_i)} \quad (44) \]

and the demand that the metric be continuous at the common UV brane,

\[ f_{h,1}^{(n)}(k^{-1}) = f_{h,2}^{(n)}(k^{-1}), \quad (45) \]

determines the constants \( N_i^{(n)} \) as

\[ N_i^{(n)} = J_2(m_n/k) + \beta_i^{(n)}Y_2(m_n/k). \quad (46) \]

One must also impose the generalized Israel junction condition at the UV brane [35]:

\[ \sum_i \left. \partial_i f_{h,i}^{(n)}(z_i) \right|_{UV} = 0, \quad (47) \]

which gives

\[ \sum_i \frac{1}{N_i^{(n)}} \left\{ J_1(m_n/k) + \beta_i^{(n)}Y_1(m_n/k) \right\} = 0, \quad (48) \]

the solutions to which determine the KK masses \( m_n \). The spectrum contains a massless mode with the throat-independent wave function

\[ f_{h,i}^{(0)}(z_i) = \sqrt{\frac{2k}{\sum_j(1 - (kR_i)^{-2})}} \simeq \sqrt{k}, \quad (49) \]

which corresponds to the usual 4D graviton. For the lighter KK modes one can simplify Eq. (48) by expanding in \( m_n/k \ll 1 \). This gives

\[ \beta_1^{(n)} \beta_2^{(n)} = \frac{\pi m_n^2}{8k^2} (\beta_1^{(n)} + \beta_2^{(n)}), \quad (50) \]

which, for the low-lying KK masses, is well approximated by

\[ \Pi_i J_1(m_nR_i) \simeq 0. \quad (51) \]

The usual (approximate) expression for graviton KK masses in RS models is \( J_1(m_nR_1) \simeq 0 \) [72] and these same \( \sim \) TeV KK modes are contained in the spectrum (51). In addition to
these RS-like modes the spectrum contains modes with $\sim$ GeV masses set by $J_1(m_{h,n} R_2) \simeq 0$. Thus, the spectrum splits naturally into a pair of KK towers with $\Delta m \sim 1/R_1 \sim$ TeV in one tower and $\Delta m \sim 1/R_2 \sim$ GeV in the other, with the lightest masses being $m_n R_{1,2} \simeq 3.83, 7.02, \ldots$. Note that for generic $R_1, R_2 \ll k$, it is highly unlikely for both $\beta_1^{(n)}$ and $\beta_2^{(n)}$ to be simultaneously on the order of $(m_n/k)^2 \ll 1$, which would induce mixing between the towers.

The modes in the GeV-spaced (TeV-spaced) tower are strongly localized in the hidden (visible) throat. As a result, the only graviton modes with a significant coupling to the SM are those lying in the visible throat. This is easy to see for GeV-spaced modes considerably lighter than a TeV. The wavefunctions of these modes are approximately flat in the visible throat and peak towards the IR brane in the GeV throat. Computing the normalization, the KK graviton wavefunction in the TeV throat is approximately $f_1(z_1) \simeq \sqrt{k/kR_2}$. This amplitude is parametrically smaller than even the zero mode graviton, which has $f_1(z_1) \simeq \sqrt{k}$. The couplings of GeV-mass gravitons to the SM can therefore be safely neglected. This conclusion also applies to much higher modes in the GeV tower provided $|\beta_1^{(n)}|$ is parametrically larger than $|\beta_2^{(n)}|$, which is certainly the case for $m_n \lesssim$ TeV. A similar argument applies to the TeV-spaced modes, which typically only have very weak couplings to the hidden sector in the GeV throat.

The coupling between a pair of hidden KK vectors and a hidden KK graviton is given by

$$\mathcal{L}_{\text{eff}} \supset \frac{k}{M_{\text{Pl}}} \sum_{a,m,n} \eta^{\rho\sigma} \eta^{\alpha\beta} \eta^{a} \eta^{(a)}_{\rho\sigma} \eta^{(a)}_{\alpha\beta} \left[ f_{h,1}, f_{h,2} f_m f_n \right] \left( \zeta_{a,m,n} \eta^{\mu\nu} X^m_{\mu\nu} X^n_{\alpha\beta} - \xi_{a,m,n} X^m_{\mu\nu} X^n_{\alpha\beta} \right),$$

with

$$\zeta_{a,m,n} = \frac{1}{k^{3/2}} \int \frac{dz_2}{(z_2^2)} f_{h,1} f_{h,2} f_m f_n,$$  \hspace{1cm} (53)

$$\xi_{a,m,n} = \frac{1}{k^{3/2}} \int \frac{dz_2}{(z_2^2)} f_{h,1} f_{h,2} \partial_z f_m \partial_z f_n,$$  \hspace{1cm} (54)

where $f_m, f_n$ are the KK vector profiles.

## B Vector Mass Mixing

The mass Lagrangian,

$$\mathcal{L}_{\text{mass}} = \frac{1}{2} m_Z^2 Z_\mu Z^\mu + \frac{1}{2} \sum_{n=0}^{n_A} m_n^2 \tilde{X}_n \tilde{X}_n,$$

where $m_Z$ is the SM value of the $Z$ mass, can be written in terms of the shifted fields as

$$\mathcal{L}_{\text{mass}} = \frac{1}{2} \gamma^\mu M^2 \gamma_\mu,$$  \hspace{1cm} (55)
where the basis vector and mass matrix are, respectively,
\[
\mathcal{V}^\mu = (\tilde{Z}^\mu, \tilde{X}_0^\mu, \tilde{X}_1^\mu, \ldots, \tilde{X}_{n_\Lambda}^\mu),
\]
\[
\mathcal{M}^2 = \begin{pmatrix} M_{ZZ} & M_{mix} \\ M_{mix}^T & M_{KK} \end{pmatrix}.
\]

Here we have defined
\[
M_{ZZ} = m_Z^2 \left( 1 + s_W^2 \sum_{n=0}^{n_z} \epsilon_n^2 \left[ 1 + \frac{m_n^2}{m_Z^2} \right] \right),
\]
and
\[
M_{KK} = \text{diag}(\bar{m}_0^2, \bar{m}_1^2, \ldots, \bar{m}_{n_\Lambda}^2),
\]
with
\[
\bar{m}_n^2 \equiv \begin{cases} (1 + \epsilon_n^2)m_n^2 & \text{for } n \leq n_z \\ (1 + \epsilon_n^2)m_n^2 & \text{for } n > n_z. \end{cases}
\]

The \(n\)th element of the mixing vector \(M_{mix}\) is given by
\[
(M_{Mix})_n = s_W \times \begin{cases} \epsilon_n m_n^2 & \text{for } 0 \leq n \leq n_z \\ \epsilon_n m_n^2 & \text{for } n_z < n \leq n_\Lambda. \end{cases}
\]

Diagonalization proceeds as given in the text with the mass of the physical \(Z\) boson being
\[
M_Z^2 \simeq m_Z^2 \left( 1 + s_W^2 \sum_{n=0}^{n_z} \epsilon_n^2 \left[ 1 + \frac{m_n^2}{m_Z^2} \right] - \sum_{n=0}^{n_\Lambda} \eta_n^2 \left[ \frac{1}{2} - \frac{m_n^2}{m_Z^2} \right] \right) + \sum_{n=0}^{n_\Lambda} \frac{2s_W^2 \epsilon_n^2 m_n^2}{m_Z^2 - m_n^2},
\]
where \(m_\omega\) is defined in (28). The tower of physical hidden vectors \(X_n\) have mass
\[
M_n^2 \simeq \begin{cases} m_n^2 \left( 1 + \epsilon_n^2 \epsilon_n^2 - \eta_n^2 - \frac{2s_W^2 \epsilon_n^2 m_n^2}{m_Z^2 - m_n^2} \right) + \eta_n^2 m_Z^2 & \text{for } n \leq n_z, \\ m_n^2 \left( 1 + \epsilon_n^2 - \eta_n^2 \left[ 1 - \frac{m_n^2}{m_Z^2} \right] + \frac{m_Z^2}{m_n^2} \times \frac{2s_W^2 \epsilon_n^2 m_n^2}{m_n^2 - m_Z^2} \right) & \text{for } n > n_z. \end{cases}
\]

With the above one readily obtains the coupling of \(X_n\) to SM matter and the induced modification of the SM \(Z\) coupling.

### C Gauge Boson Couplings

In the field basis where gauge kinetic mixing appears explicitly, the couplings of the neutral gauge bosons to matter are given by
\[
- \mathcal{L} \ni J^\mu_{em} A^\mu + J^\mu_Z Z^\mu + \sum_n J^\mu_n X_{n,\mu}
\]

Shifting the fields as in Eq. (26) and rotating them to the mass eigenbasis found above, we obtain
\[ - \mathcal{L} \supset A_\mu J^\mu_{em} \]
\[ + Z_\mu \left[ \left( 1 + \frac{1}{2} \sum_{n=0}^{n_z} s_W^2 \epsilon_n^2 + \sum_{n=n_z+1}^{n_A} s_W \epsilon_n \eta_n - \frac{1}{2} \sum_{n=0}^{n_A} \eta_n^2 \right) J^\mu_Z - \left( \sum_{n=0}^{n_z} c_W s_W^2 \epsilon_n^2 + \sum_{n=0}^{n_A} c_W \epsilon_n \eta_n \right) J^\mu_{em} \right. \]
\[ + \sum_{n=0}^{n_A} \left( s_W \epsilon_n + \eta_n \right) J_n + \sum_{n=n_z+1}^{n_A} \eta_n J_n \]
\[ + \sum_{n=0}^{n_z} X_{n\mu} \left[ \left( 1 + \frac{1}{2} \epsilon_n^2 - s_W \epsilon_n \eta_n - \frac{1}{2} \eta_n^2 \right) J^\mu_n - c_W \epsilon_n J^\mu_{em} - \eta_n J^\mu_Z \right] \]
\[ + \sum_{n=n_z+1}^{n_A} X_{n\mu} \left[ \left( 1 + \frac{1}{2} \epsilon_n^2 - \frac{1}{2} \eta_n^2 \right) J^\mu_n - c_W \epsilon_n J^\mu_{em} + \left( s_W \epsilon_n - \eta_n \right) J^\mu_Z \right]. \]

Note that for high modes with \( n \gg n_z \) (and \( m_n \gg m_Z \)), \( \eta_n \) becomes very small so that \( X_n \) couples primarily to the hypercharge current, \( J_Y = (c_W J_{em} - s_W J_Z) \). Similarly, for \( m_n \ll m_Z \) the corresponding state couples primarily to the electromagnetic current.

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