Isospin effect on baryon and charge fluctuations from the pNJL model

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We have studied the possible isospin corrections on the skewness and kurtosis of net-baryon and net-charge fluctuations in the isospin asymmetric matter formed in Au+Au collisions at RHIC-BES energies, based on a 3-flavor polyakov-looped Nambu-Jona-Lasinio model. With typical scalar-isovector and vector-isovector couplings leading to the splitting of $u$ and $d$ quark chiral phase transition boundaries and critical points, we have observed dramatic isospin effects on the susceptibilities, especially those of net-charge fluctuations. Reliable experimental measurements at even lower collision energies are encouraged to confirm the observed isospin effects.

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Studying the hadron-quark phase transition and exploring the phase structure of quantum chromodynamics (QCD) matter are the fundamental goals of relativistic heavy-ion collision experiments. Lattice QCD simulations predict that the transition between the hadronic phase and the partonic phase is a smooth crossover at nearly zero baryon chemical potential ($\mu_B \sim 0$) \cite{1}. At larger $\mu_B$, the transition can be a first-order one based on investigations from theoretical models, see, e.g., studies from the Nambu-Jona-Lasinio (NJL) model and its extensions \cite{1–7}. The knowledge of the QCD critical point (CP) in-between the smooth crossover and the first-order phase transition boundaries is important in mapping out the whole QCD phase diagram \cite{5}. In order to find the signature of the QCD CP at finite baryon chemical potentials, the Beam Energy Scan (BES) program at the Relativistic Heavy-Ion Collider (RHIC) \cite{9} has been carrying out “low-energy” relativistic heavy-ion collisions and obtained many interesting results. However, heavy-ion collisions with neutron-rich beams produce isospin asymmetric quark matter consisting of different net numbers of $u$ and $d$ quarks, and the isospin degree of freedom is expected to be increasingly important at lower collision energies with larger net baryon densities, related to the QCD phase structure at finite isospin chemical potentials $\mu_I$ \cite{10}. Although lattice QCD can explore the phase transition at finite $\mu_I$ \cite{11–13}, it suffers from the fermion sign problem at finite $\mu_B$ \cite{14–16}. Based on the NJL model studies, the isovector couplings may lead to the isospin splittings of chiral phase transition boundaries and critical points for $u$ and $d$ quarks in isospin asymmetric quark matter \cite{17–20}. This may influence the search of QCD CP signals in heavy-ion collisions at RHIC-BES energies.

It was proposed \cite{21} that the non-Gaussian fluctuations of observables in relativistic heavy-ion collisions can be a signal of the phase transition critical point, and the moment of these fluctuations can be used to measure the magnitude of the correlation length. Typically, the susceptibilities of conserved quantities carry information of the QCD phase boundary as well as the position of the critical point \cite{22}. The above findings have stimulated measurements of the kurtosis and skewness of net-baryon, net-charge, and net-strangeness fluctuations from $\sqrt{s_{NN}} = 7.7$ to 200 GeV at RHIC \cite{23–27}, as well as lattice QCD calculations \cite{28,29}. For recent reviews on this topic, we refer the reader to Refs. \cite{30,31}. However, to the best of our knowledge, the isospin effect on these fluctuations was assumed to be small and has never been addressed. Actually, based on studies from the statistis model \cite{32–34}, the isospin chemical potential can be as larger as 10 MeV at the chemical freeze-out stage of heavy-ion collisions at RHIC-BES energies. Taking this isospin effect into consideration, it is expected that the isospin splitting of the chiral phase transition boundaries and the critical points may affect the moments of fluctuations, especially for the net-charge fluctuations. In the present study, we investigate the isospin effects based on the 3-flavor polyakov-looped NJL (pNJL) model with scalar-isovector and vector-isovector couplings incorporated in Ref. \cite{20}.

The thermodynamic potential of the 3-flavor pNJL model with scalar-isovector and vector-isovector couplings at temperature $T$ can be expressed as \cite{20}

\begin{equation}
\Omega_{\text{pNJL}} = \mathcal{U}(\Phi, \tilde{\Phi}, T) - 2N_c \sum_{i=u,d,s} \int_{0}^{\Lambda} \frac{d^{3}p}{(2\pi)^{3}} p_{i} E_{i}\nonumber
\end{equation}

\begin{equation}
- 2T \sum_{i=u,d,s} \int \frac{d^{3}p}{(2\pi)^{3}} [\ln(1 + e^{-3\beta (E_{i} - \tilde{\mu}_{i})}) + 3\Phi e^{-\beta (E_{i} - \tilde{\mu}_{i})} + 3\tilde{\Phi} e^{-2\beta (E_{i} - \tilde{\mu}_{i})} + \ln(1 + e^{-3\beta (E_{i} + \tilde{\mu}_{i})}) + 3\Phi e^{-\beta (E_{i} + \tilde{\mu}_{i})} + 3\tilde{\Phi} e^{-2\beta (E_{i} + \tilde{\mu}_{i})})] + G_{S}(\sigma_{u}^{2} + \sigma_{d}^{2} + \sigma_{s}^{2}) + 4K_{s} \sigma_{u} \sigma_{d} + G_{V}(\rho_{u}^{2} + \rho_{d}^{2} + \rho_{s}^{2}) + G_{IS}(\sigma_{u} - \sigma_{d})^{2} + G_{IV}(\rho_{u} - \rho_{d})^{2}. \tag{1}
\end{equation}

In the above, the temperature-dependent effective potential $\mathcal{U}(\Phi, \tilde{\Phi}, T)$ as a function of the polyakov loop $\Phi$ and
\( \Phi \) is expressed as \[ U(\Phi, \bar{\Phi}, T) = -b \cdot T \{ 54e^{-a/T\Phi} + \ln[1 - 6\Phi] \\ - 3(\Phi \bar{\Phi})^2 + 4(\Phi^3 + \bar{\Phi}^3) \}, \] (2)

with \( a = 664 \text{ MeV} \) and \( b = 0.015 \Lambda^3 \) \[ 5 \], where \( \Lambda = 750 \text{ MeV} \) is the cutoff value in the momentum integral of the second term in Eq. (1). The factor \( 2N_c \) with \( N_c = 3 \) represents the spin and color degeneracy, and \( \beta = 1/T \) represents the temperature. \( G_S \) and \( G_V \) are respectively the scalar and vector coupling constants, \( K \) is the coupling constant of the six-point Kobayashi-Maskawa-t Hooft (KMT) interaction, and \( G_{IS} \) is the reduced strength of the scalar-isovector and vector-isovector couplings that break the SU(3) symmetry while keeping the isoscalar symmetry. For the ease of discussions, we define \( R_{IS} = G_{IS}/G_S \) and \( R_{IV} = G_{IV}/G_S \) as the reduced strength of the scalar-isovector and vector-isovector coupling. The energy \( E \) of quarks with flavor \( i \) is expressed as \( E_i(p) = \sqrt{p^2 + M_i^2} \), where \( M_i \) is the constituent quark mass. In the mean-field approximation, quarks can be considered as quasiparticles with constituent masses \( M_i \) determined by the gap equation

\[ M_i = m_i - 2G_S\sigma_i + 2K_3\sigma_k - 2G_{IS}\tau_3(\sigma_u - \sigma_d) \] (3)

where \( m_i \) is the current quark mass, \( \sigma_i \) stands for quark condensate, \( (i, j, k) \) is any permutation of \( (u, d, s) \), and \( \tau_3 \) is the isospin quantum number of quark flavor \( i \), i.e., \( \tau_{3u} = 1, \tau_{3d} = -1, \) and \( \tau_{3s} = 0 \). As shown in Eq. (3), \( \sigma_d \) and \( \sigma_s \) contribute to the constituent quark mass of \( u \) quarks as a result of the six-point interaction and the scalar-isovector coupling. Similarly, the effective chemical potential expressed as

\[ \tilde{\mu}_i = \mu_i + 2G_V\rho_i + 2G_{IV}\tau_3(\rho_u - \rho_d) \] (4)

has also the contribution from quarks of other isospin states through the vector-isovector coupling. The net quark number density of flavor \( i \) can be calculated from

\[ \rho_i = 2N_c \int (f_i - \bar{f}_i) \frac{d^3p}{(2\pi)^3} \] (5)

where

\[ f_i = \frac{1 + 2\Phi\xi_i + \Phi\xi_i^2}{1 + 3\Phi\xi_i + 3\Phi\xi_i^2 + \xi_i^3} \] (6)

and

\[ \bar{f}_i = \frac{1 + 2\Phi\xi_i + \Phi\xi_i^2}{1 + 3\Phi\xi_i + 3\Phi\xi_i^2 + \xi_i^3} \] (7)

are the effective phase-space distribution functions for quarks and antiquarks in the pNJL model with \( \xi_i = e^{(E_i - \tilde{\mu}_i)/T} \) and \( \xi_i' = e^{(E_i + \tilde{\mu}_i)/T} \).

In the present study, we adopt the values of parameters \[ 7, 35 \] as \( m_u = m_d = 3.6 \text{ MeV} \), \( m_s = 87 \text{ MeV} \), \( G_S\Lambda^2 = 3.6 \), and \( K\Lambda^3 = 8.9 \). Since the purpose is not to study the effect of \( G_V \) on the structure of phase diagram \[ 4, 7 \], it is set to 0 in the present study. The following equations

\[ \frac{\partial\Omega_{pNJL}}{\partial\sigma_u} = \frac{\partial\Omega_{pNJL}}{\partial\sigma_d} = \frac{\partial\Omega_{pNJL}}{\partial\sigma_s} = \frac{\partial\Omega_{pNJL}}{\partial\Phi} = \frac{\partial\Omega_{pNJL}}{\partial\bar{\Phi}} = 0 \] (8)

are used to obtain the values of \( \sigma_u, \sigma_d, \sigma_s, \Phi, \) and \( \bar{\Phi} \) at the minimum thermodynamic potential in the pNJL model.

In the following, we consider fluctuation moments of conserved quantities, such as net-baryon and net-charge fluctuations, from the above 3-flavor pNJL model. The n-order susceptibility representing the cumulant of a given conserved quantity in the grand ensemble can be expressed as the derivative of the thermodynamic potential as

\[ \chi^{(n)}_{X} = \frac{\partial^n (-\Omega/T)}{\partial(\mu_X/T)^n}, \] (9)

where \( \mu_X \) represents the baryon (\( \mu_B \)) or the charge (\( \mu_Q \)) chemical potential. Numerically, the isospin chemical potential \( \mu_I \) and the charge chemical potential \( \mu_Q \) are equal to each other \[ 28, 39 \]. In the present study, we use the empirical relation between the isospin chemical potential and the baryon chemical potential, i.e., \( \mu_I = -0.293 - 0.0264\mu_B \) (MeV), determined from the statistical model fits of the experimental data from Au+Au collisions from center-of-mass energy \( \sqrt{s_{NN}} = 200 \text{ GeV} \) \[ 32, 34 \]. For the strangeness chemical potential, we take the empirical relation as \( \mu_S = 1.032 + 0.232\mu_B \) (MeV) \[ 32, 34 \]. Note that the chemical potentials of \( u, d, \) and \( s \) quarks in the 3-flavor pNJL model can be expressed in terms of \( \mu_B, \mu_I, \) and \( \mu_S \). The higher-order susceptibilities are related to the skewness \( S \) and kurtosis \( \kappa \) measured experimentally in relativistic heavy-ion collisions through the relations

\[ S\sigma = \frac{\chi^{(3)}}{\chi^{(2)}}, \quad \kappa\sigma^2 = \frac{\chi^{(4)}}{\chi^{(2)}} \] (10)

where \( \sigma \) is the variance of the conserved quantity. The subscript of the net baryon (B) or the net charge (Q) is omitted in the above equation.

We begin our discussion with the higher-order fluctuations of net baryon and net charge from the 3-flavor pNJL model in the \( \mu_B - T \) plane with various isovector coupling constants, as shown in Fig. 1. As shown in Fig. 3 of Ref. \[ 20 \], \( R_{IS} = 0.14 \) and \( R_{IV} = 0.5 \) lead to the splittings of \( u \) and \( d \) quark chiral phase transition boundaries as well as their critical points, which are plotted in all panels of Fig. 1 for references. We note that the empirical relation \( \mu_I = -0.293 - 0.0264\mu_B \) (MeV) is only well valid near the phase boundary, and this is exactly the region where we are interested. As seen in Fig. 1, chiral phase transition boundaries separate the red and blue areas for skewness results, representing respectively
FIG. 1: (color online) Skewness and kurtosis of net-baryon (B) (left) and net-charge (Q) (right) fluctuations in the $\mu_B - T$ plane with finite scalar-isovector (upper panels) and vector-isovector (lower panels) coupling constants and the empirical relation $\mu_I = -0.293 - 0.0264 \mu_B$ (MeV). The chiral phase transition boundaries and the corresponding critical points (CP) of $u$ (black) and $d$ (white) quarks are plotted in all panels for reference.

In relativistic heavy-ion collision experiments, the net-baryon and net-charge fluctuations are measured at chemical freeze-out. It is well-known that the phase transition temperature from lattice QCD calculations at zero baryon density is the same as that extracted from the statistical model at top RHIC energy or LHC energy, leading to the conclusion that the chemical freeze-out happens right after hadron-quark phase transition in extreme relativistic heavy-ion collisions. In collisions at RHIC-BES energies, when the chemical freeze-out happens is not known. There are several empirical criteria for chemical freeze-out in relativistic heavy-ion collisions, such as fixed energy per particle, baryon-antibaryon density, normalized entropy density, as well as percolation model and so on (see Ref. [37] and references therein). In order to compare qualitatively the higher-order susceptibilities from the pNJL model with experimental results, we obtain the hypothetical chemical freeze-out lines by rescaling $\mu_B$ of

the positive and negative values of $S\sigma$. For kurtosis results, however, the chiral phase transition boundaries go through the blue areas, and the critical points stand at the ends of the blue areas. It is also interesting to see that the skewness of net-charge fluctuations gives the different orders of the red and blue areas compared to that of net-baryon fluctuations. Using the empirical relation between $\mu_I$ and $\mu_B$, the splitting of the chiral phase transition boundaries from $R_{IV} = 0.5$ is much weaker compared to that observed in Ref. [21].

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the averaged chiral phase transition boundaries of $u$ and $d$ quarks with factors of 0.98, 0.95, and 0.90, corresponding respectively to the dash-dotted, dashed, and dotted curves in Fig. 2. We note that a similar assumption of the chemical freeze-out lines was made in Ref. [38], and the present hypothetical chemical freeze-out lines are always below the chiral phase transition boundaries of both $u$ and $d$ quarks. For the net-baryon susceptibility shown in the upper panels of Fig. 2, it is seen that $S\sigma(B)$ has one positive peak while $\kappa\sigma^2(B)$ has a positive and a negative peak along the chemical freeze-out lines, and the peaks are sharper if the hypothetical chemical freeze-out lines are closer to the chiral phase transition boundary. The critical point of the $d$ quark chiral phase transition is always at the low-temperature or low-energy side of the positive peaks for both $S\sigma(B)$ and $\kappa\sigma^2(B)$, and the distance between the positive peaks and the critical point is related to that between the chemical freeze-out line and the chiral phase transition boundary. For the net-charge susceptibility shown in the lower panels of Fig. 2, we have also observed sharper peaks if the hypothetical chemical freeze-out line is very close to the chiral phase transition boundary. Again, the
FIG. 2: (color online) Density plot of $S\sigma$ (left) and $\kappa\sigma^2$ (right) in the $\mu_B - T$ plane as well as those along the different hypothetical chemical freeze-out lines with the scalar-isovector coupling constant $R_{IS} = 0.14$ and the empirical relation $\mu_I = -0.293 - 0.0264\mu_B$ (MeV). The solid line represents the averaged chiral phase transition boundary for $u$ and $d$ quarks, while the other lines are hypothetical chemical freeze-out curves by rescaling $\mu_B$ of the solid line with factors of 0.98, 0.95, and 0.90. The upper panels are the net-baryon susceptibilities while the lower panels are the net-charge susceptibilities.

critical point of the $d$ quark chiral phase transition is at the low-temperature or low-energy side of the negative peak for $S\sigma(Q)$ or the positive peak for $\kappa\sigma^2(Q)$.

In order to compare the higher-order susceptibilities with and without the isospin effect, we display in Fig. 3 the skewness and kurtosis of net-baryon and net-charge fluctuations with four different scenarios of isospin chemical potentials and isovector couplings: $\mu_I = 0, R_{IS} = 0, R_{IV} = 0, \mu_I = -0.293 - 0.0264\mu_B$ (MeV), $R_{IS} = 0, R_{IV} = 0, \mu_I = -0.293 - 0.0264\mu_B$ (MeV), $R_{IS} = 0.14, R_{IV} = 0, \mu_I = -0.293 - 0.0264\mu_B$ (MeV), $R_{IS} = 0, R_{IV} = 0.5$. In order to illustrate the largest possible isospin effect, the susceptibilities are calculated along the closest hypothetical chemical freeze-out line to the phase boundary, i.e., using the rescaling factor of 0.98. It is seen that even without isovector couplings, the skewness and kurtosis can be slightly different for $\mu_I = 0$ and $\mu_I \neq 0$, especially for net-charge susceptibilities. With finite isovector coupling constants, the isospin effect is largely enhanced. The peaks of the net-baryon susceptibilities move to the low-temperature or low-energy side, especially for $R_{IS} = 0.14$. The general shape of the net-baryon susceptibility is qualitatively consistent with the experimental data [24, 39]. The net-baryon susceptibility is not a unique probe of the isospin effect, since it is largely affected by other effects as well, such as the vector coupling. On the other hand, the isospin couplings affect dramatically the net-charge susceptibilities. It is seen that the isovector couplings lead to a negative peak at lower temperatures/energies for $S\sigma(Q)$. For $\kappa\sigma^2(Q)$, the isovector couplings lead to two peaks and $R_{IS} = 0.14$ is the only scenario that leads to negative values. The experimental results from STAR and PHENIX Collaborations for net-charge susceptibilities are not consistent with each other yet [25, 26]. So far the experimental results for $S\sigma(Q)$ seem to be positive above $\sqrt{s_{NN}} = 7.7$ GeV from both STAR and PHENIX measurements, and it is of great interest to distinguish different scenarios if reliable measurements are done at even lower collision energies. For the $\kappa\sigma^2(Q)$ results, the STAR results lead to negative values at lower $\sqrt{s_{NN}}$ [25] while the PHENIX results remain positive at all energies [26]. It is again of great interest to check experimentally whether another peak appears at even lower collision energies, as seen from the $\kappa\sigma^2(Q)$ results with $R_{IS} = 0.14$.

To summarize, based on the 3-flavor polyakov-looped Nambu-Jona-Lasinio model with the scalar-isovector and vector-isovector couplings, we have studied the higher-order susceptibilities of net-baryon and net-charge fluctuations in isospin asymmetric matter formed in Au+Au collisions at RHIC-BES energies. Although the general features remain the same, the isovector couplings move
the peaks of the net-baryon susceptibilities to the low-temperature or low-energy side. On the other hand, the shape of the net-charge susceptibilities is largely changed with the isovector couplings, i.e., an additional negative peak appear in the skewness results and two positive peaks peaks appear in the kurtosis results along the hypothetical chemical freeze-out line, if it is very close to the phase boundary. It is of great interest to confirm our findings by measuring the net-charge susceptibility at even lower collision energies, such as that in the future FAIR-CBM program. Such analysis may also be helpful in extracting the strength of the isovector couplings or even the information of the isospin dependence of the QCD phase diagram.

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