Optimal Selection Method of Spacecraft Initial Orbits based on Grey Relational Analysis

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Abstract. The spacecraft measure ship need to optimize spacecraft initial orbits in a large number of multi-source orbit data during the spacecraft TT&C mission. To solve this problem, an initial orbits optimization method for spacecraft based on grey relational analysis was proposed, this method is in line with the actual situation of the spacecraft measure ship. By converting the evaluation attributes and combining statistical analysis with past experience, the optimal indicator set was determined. Finally, the evaluation model of the spacecraft’s initial orbit was established. Then using a satellite transfer orbit as an example to prove that the method is reasonable and correct.

1. Introduction

With the rapid development of the aerospace industry, the requirements of the aerospace measurement and control mission for the accuracy of the fixed orbit are getting higher and higher. The research on the method of measuring the initial orbit of the ship has always been an important research direction of marine measurement and control. The measuring vessel has a variety of measuring equipment, which can generate a large amount of multi-source data for orbit determination, and select a set of optimal orbits among a large number of multi-source orbit data. This group of orbits directly determines the orbit determination as the initial orbit of the spacecraft. Precision. At present, the measurement ship only selects the semi-major axis, which is the important orbital component, as the basis for selection. It relies on personnel experience and manual judgment. Usually, the middle value of the long axis of the GPS orbit is selected as the spacecraft's initial orbit. This method discards a large amount of useful data, only considers one orbital component of a single source, and the selection basis is also unreasonable, so that the orbit determination result lacks reliability and scientificity.

All the six orbital components of spacecraft can be used as important reference indexes for orbit determination. Yang Yongan [1] and Wu Jinmei [2] have established optimal initial orbital selection models of spacecraft by using TOPSIS method respectively, and have been verified by examples. But the ideal solution of this method is overly dependent on the "standard orbit" of the pre-budget, the final optimal result is the track with the smallest difference from the "standard orbit ", and this method is computationally intensive. Therefore, this method lacks rigorous logic and scientificity. Yang Yongan [3] applied the grey relational analysis to the spacecraft initial orbit optimization. The principle of this method selection is to select a set of orbital data that is most similar to the theoretical orbit, but does not take into account the actual conditions such as equipment factors, so it also has certain flaws.

In this paper, we use mathematical statistical analysis method, then combined with the experience summarized in the long-term use of equipment to determine the ideal target orbit, we call it the optimal
indicator set, then based on the grey relational analysis method, the spacecraft initial orbit optimization model was established by analyzing the degree of correlation between each data element in the orbit data and the elements in the optimal indicator set. Finally, an example is given to verify the accuracy of the method.

2. Grey Relational Analysis

Grey theory is a kind of system theory initiated by famous scholar Professor Deng Julong. The grey relational analysis method is based on the degree of similarity of the geometrical curves of various factors to judge the degree of correlation or similarity between various factors. By quantitatively analyzing the development trend of dynamic process, this method compares the geometric relationship of time series related statistical data in the system, and finds the gray relational degree between the reference series and each comparison series. The comparison of the comparison series with the reference sequence is closer to the reference sequence and the closer the relationship with the reference sequence.

The specific calculation steps of the grey relational analysis are as follows:

Step 1: Determine the comparison object (evaluation object) and the reference sequence.

There are M evaluation objects, there are N evaluation indicators, the reference number column is \( x_0 = \{ x_{0k} | k = 1, 2, \ldots, n \} \), and the comparison number column is \( x_i = \{ x_{ik} | k = 1, 2, \ldots, n \} \).

Step 2: Determine the weight of each indicator value.

Determine the weight of each indicator object \( w_k = (k = 1, 2, \ldots, n) \) is the weight of the k-th evaluation index.

Step 3: Calculate the grey correlation coefficient.

\[
\xi_i(k) = \frac{\min_{i} \min_{t} |x_{0i}(t) - x_{ti}(t)| + \rho \max_{i} \max_{t} |x_{0i}(t) - x_{ti}(t)|}{|x_{0k}(k) - x_{ik}(k)| + \rho \max_{i} \max_{t} |x_{0i}(t) - x_{ti}(t)|}
\]

Formula (1) is the correlation coefficient of the comparison series \( x_i \) and the reference series \( x_0 \) on the k-th index. \( \rho \) is the resolution coefficient. The smaller \( \rho \), the higher resolution, and when \( \rho \leq 0.5463 \), the resolution is best, usually let \( \rho = 0.5 \).

Step 4: Calculate the weighted relevance.

The formula for calculating the gray weighted correlation degree is:

\[
r_i = \sum_{k=1}^{n} w_i \xi_i(k)
\]

In Formula (2): The gray weighted relevance of the i-th evaluation object to the ideal object.

Step 5: Sort the relevance.

According to the size of the gray weighted association degree, each evaluation object will be sorted, and then the correlation order of the evaluation object will be established. The greater the degree of association, the better the evaluation results.

3. Application and Implementation of Grey Relational Analysis Method in the Initial Orbit Optimization

There are 6 indicators of spacecraft orbital, and we cannot give scientifically accurate weight ratios for each indicator. Based on this situation, the six orbital elements of the spacecraft can be converted into the position and velocity of the spacecraft orbit. In the following, we take position and velocity as the evaluation index of spacecraft initial orbit optimization, and discuss the realization and application of spacecraft initial orbit optimization based on grey relational analysis method on the spacecraft measure ship.
3.1. Data standardization
First, convert the orbit elements to the position and velocity in the J2000.0 inertial coordinate system as (X, Y, Z, Vx, Vy, Vz), the attribute vectors formed by the orbit data generated by different devices are recorded as (Xi1, Xi2, Xi3, Xi4, Xi5, Xi6). Each data element in the different sets of attribute vectors is corresponding. We can group the attribute vectors of different tracks into a normalized matrix, denoted as X, as in formula (3).

\[
X = \begin{pmatrix}
X_{11} & X_{12} & \cdots & X_{16} \\
X_{21} & X_{22} & \cdots & X_{26} \\
\vdots & \vdots & \ddots & \vdots \\
X_{n1} & X_{n2} & \cdots & X_{n6}
\end{pmatrix}
\]

3.2. Determining the optimal indicator set
The optimal indicator set is a data set that is referenced by other data sets, before using the grey relational analysis method, we must first determine an ideal set of optimal indicators. The basic principle of our orbit determination is to find the orbit that is most similar to the determined optimal indicator set. Therefore, it is necessary to ensure the accuracy of the optimal indicator set to ensure the accuracy and reliability of the orbit determination results.

Multi-source data comes from a variety of devices, so the accuracy of data from different sources is also different. In the measurement ship, GPS data is generally considered to be more reliable and accurate, telemetry data and tracking data are slightly less accurate.

We have counted 20 orbital results in recent years. Then compare the orbital results of different data sources with the orbital results of the exact calculations after the task, we use the most representative semi-major axis as the contrast element, and the difference between the orbital results calculated accurately after the task yields the results in Table 1 (the calculation process is omitted here).

| data source | semi-major axis (a) maximum error / Unit (km) |
|-------------|---------------------------------------------|
| GPS data    | 7.79                                        |
| telemetry data | 65.21                                      |
| tracking data  | 70.93                                       |

Based on the maximum error data in Table 1 and the actual orbit determination, we determined the weight ratio of GPS, telemetry and external measurement orbit data in determining the optimal indicator set to 64:8:7. After normalization, it is 0.81:0.10:0.09. If there are n sets of external test data participating in determining the optimal index set, the weight of each set of external test data is 0.09/n. We weighted the orbital results of each data source in the task to obtain the optimal index set, denoted as A.

3.3. Calculate relevance
After determining the optimal set of indicators, the correlation coefficient is obtained using the gray relation analysis formula with the matrix determined in formula (3), as in formula (4).
Then, according to formula (5), the correlation degree of each group of orbits is obtained.

\[ r_i = \sum_{k=1}^{n} w_i \xi_i(k) \]

Because the elements in the orbit data are position and velocity, the values of the elements are the same, so let A, then calculate the gray correlation of each set of orbital data. Then sort according to the gray correlation degree r. The greater the correlation degree r, the more similar the representative is to the optimal indicator set, that is, its evaluation result is better.

4. Example calculation and analysis

In order to verify the accuracy of the grey relational analysis method in the initial orbital optimization of spacecraft, we take the example of a satellite transfer orbit in reference [2] as an example. The initial orbit elements for measuring different data sources in the measure ship is shown in Table 2. Convert the initial orbit elements in Table 2 to the position and velocity in the J2000.0 inertial coordinate system, as shown in Table 3. The seventh set of data in Table 3 is the optimal set of indicators calculated by the weighting method. Taking the optimal indicator data set calculated by the weighted average orbit as a reference, the gray correlation value of each indicator object in Table 4 is calculated. Taking the accurate calculation orbit after the task as a reference, the gray relation value of each indicator object in Table 5 is calculated. Then, according to formula (4) and formula (5), the gray relation degree with reference to the optimal index data set and the accurate calculation orbit after the task is obtained, as shown in Table 6 and Table 7. (Note: replace the first significant digit with * in the table)

| NO. | Orbital data source | Semi-long axis (a) | Eccentricity (e) | Orbital inclination (i) | Ascending node (Ω) | Eccentric Anomaly (ω) | Perigee (M) |
|-----|---------------------|-------------------|-----------------|------------------------|-------------------|-----------------------|-------------|
| 1   | cutoff point       | *4 416.083 0      | .730 371 00    | *0.500 000             | *50.335 0         | *78.598 0             | *.180 0     |
| 2   | Platform second node | *4 423.187 0     | .730 448 00    | *0.500 000             | *50.334 0         | *78.600 0             | *.179 0     |
| 3   | Inertia second node | *4 514.686 0     | .731 457 00    | *0.479 000             | *50.958 0         | *77.992 0             | *.181 0     |
| 4   | GPS                | *4 518.967 0     | *731 452 00    | *0.539 000             | *50.222 0         | *78.624 0             | *.181 0     |
| 5   | Pulse radar        | *4 513.452 1     | .731 388 00    | *0.544 000             | *50.223 0         | *78.651 0             | *.178 0     |
| 6   | USB                | *4 504.557 0     | *731 270 00    | *0.520 000             | *50.210 0         | *78.730 0             | *.169 0     |
| 7   | Accurate post-mission calculation | *4 517.830 9 | *731 438 89    | *0.538 998             | *50.221 3         | *78.622 5             | *.181 0     |
### Table 3. Position and velocity after conversion

| NO. | Orbital data source | X       | Y       | Z       | Vx      | Vy      | Vz      |
|-----|---------------------|---------|---------|---------|---------|---------|---------|
| 1   | cutoff point        | -*617   | 184.839 | 593.201 | 259.326 | 627.82  | 500.76  |
|     |                     | -70     | 201.32  | 31.32   | 31.32   | 31.32   | 31.32   |
| 2   | Platform second node| -*617   | 195.837 | 223.704 | 165.519 | 533.92  | 910.32  |
|     |                     | -70     | 201.32  | 31.32   | 31.32   | 31.32   | 31.32   |
| 3   | Inertia second node | -*618   | 570.34  | 580.168 | 887.343 | 630.09  | 628.09  |
|     |                     | -30.85  | 201.32  | 31.32   | 31.32   | 31.32   | 31.32   |
| 4   | GPS                 | -*618   | 868.310 | 729.071 | 364.066 | 809.05  | 217.05  |
|     |                     | 533     | 201.32  | 31.32   | 31.32   | 31.32   | 31.32   |
| 5   | Pulse radar         | -*618   | 680.817 | 150.583 | 246.163 | 750.06  | 267.06  |
|     |                     | 353     | 201.32  | 31.32   | 31.32   | 31.32   | 31.32   |
| 6   | USB                 | -*618   | 499.104 | 543.095 | 235.690 | 915.06  | 703.06  |
|     |                     | 353     | 201.32  | 31.32   | 31.32   | 31.32   | 31.32   |
| 7   | Optimal indicator set| -*618 | 721.461 | 233.328 | 81.983 | 418.983 | 418.983 |
|     |                     | 343     | 201.32  | 31.32   | 31.32   | 31.32   | 31.32   |
| 8   | Accurate post-mission calculation | -*618 | 897.354 | 390.881 | 269.953 | 764.06  | 691.06  |
|     |                     | 353     | 201.32  | 31.32   | 31.32   | 31.32   | 31.32   |

### Table 4. Gray relation table for each data object - (reference series is the optimal indicator set)

| NO. | Orbital data source | X       | Y       | Z       | Vx      | Vy      | Vz      |
|-----|---------------------|---------|---------|---------|---------|---------|---------|
| 1   | cutoff point        | 0.891   | 0.958   | 0.767   | 0.998   | 0.998   | 0.995   |
|     |                     | 398     | 351     | 355     | 419     | 419     | 351     |
| 2   | Platform second node| 0.891   | 0.927   | 0.762   | 0.998   | 0.998   | 0.999   |
|     |                     | 922     | 972     | 208     | 080     | 080     | 073     |
| 3   | Inertia second node | 0.988   | 0.435   | 0.333   | 0.994   | 0.994   | 0.996   |
|     |                     | 164     | 367     | 677     | 822     | 822     | 556     |
| 4   | GPS                 | 0.988   | 0.961   | 0.907   | 0.999   | 0.999   | 0.999   |
|     |                     | 496     | 165     | 339     | 941     | 941     | 767     |
| 5   | Pulse radar         | 0.986   | 0.920   | 0.915   | 0.999   | 0.999   | 0.999   |
|     |                     | 802     | 985     | 521     | 985     | 985     | 302     |
| 6   | USB                 | 0.982   | 0.773   | 0.987   | 0.998   | 0.998   | 0.998   |
|     |                     | 673     | 447     | 284     | 754     | 754     | 866     |

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Table 5. Gray relation table for each data object - (reference series is Accurate post-mission calculation)

| NO. | Orbital data source | X     | Y     | Z     | Vx    | Vy    | Vz    |
|-----|---------------------|-------|-------|-------|-------|-------|-------|
| 1   | cutoff point        | 0.885| 0.856| 0.724| 0.999| 0.999| 0.999|
|     |                     | 0.095| 0.933 | 2.238 |       | 267   | 411   | 527   |
| 2   | Platform second node| 0.885| 0.878| 0.721| 0.999| 0.999| 0.999|
|     |                     | 0.749 | 0.808 | 0.414 |       | 504   | 904   | 008   | 277   |
| 3   | Inertia second node | 0.975| 0.434| 0.333| 0.999| 0.999| 0.999|
|     |                     | 0.809 | 2.010 | 3.333 |       | 707   | 97    | 402   | 926   |
| 4   | GPS                 | 0.997| 0.975| 0.992| 0.999| 0.999| 0.999|
|     |                     | 0.803 | 0.003 | 0.916 |       | 966   | 957   | 957   | 000   |
| 5   | Pulse radar         | 0.983| 0.982| 0.998| 0.999| 0.999| 0.999|
|     |                     | 0.850 | 0.109 | 0.999 |       | 931   | 841   | 941   | 000   |
| 6   | USB                 | 0.970| 0.822| 0.927| 0.999| 0.999| 0.999|
|     |                     | 0.694 | 2.442 | 753   |       | 528   | 577   | 745   | 362   |

Table 6. Grey relation degree of data sources - (reference series is the optimal indicator set)

| NO. | 1                | 2                | 3                | 4                | 5                | 6                |
|-----|------------------|------------------|------------------|------------------|------------------|------------------|
|     | cutoff point     | Platform second node | Inertia second node | GPS              | Pulse radar      | USB              |
| grey relational degree | 0.926 699 881 | 0.930 182 034 762 | 0.972 950 947 261 | 0.972 159 459 610 | 0.957 261 909    |

Table 7. Grey relation degree of data sources - (reference series is Accurate post-mission calculation)

| NO. | 1                | 2                | 3                | 4                | 5                | 6                |
|-----|------------------|------------------|------------------|------------------|------------------|------------------|
|     | cutoff point     | Platform second node | Inertia second node | GPS              | Pulse radar      | USB              |
| grey relational degree | 0.910 542 956 | 0.913 946 700 762 | 0.972 950 947 261 | 0.972 159 459 610 | 0.957 261 909    |

According to Table 6, we can get the relation order of the gray relation degree, that is, 4>5>6>2>1>3. The fourth group (GPS) has the highest degree of similarity with the optimal indicator set, and it can also be considered that the GPS initial orbit result is optimal. This result is consistent with the results we manually selected in the task, so we can think that the calculation results of the gray relation analysis method are reasonable. In order to further verify the rationality of the selection of the optimal indicator set, we use the accurate post-mission calculation orbit result as the reference series to calculate the data shown in Table 7. The correlation order of the gray correlation degree is 4>5>6>2>1>3, this is consistent with the results obtained by using the optimal indicator set as a reference series. It can be shown that the set of indicators obtained by the weighted average method is also reasonable. In summary, on the spacecraft measure ship, the spacecraft initial orbit optimization model established by the grey relational analysis method is reasonable and correct.

5. Conclusion

This paper converts the six orbital elements of the spacecraft into the position and velocity of the orbit. Then combined with the measurement accuracy of the equipment, summed up the past task experience, determine the weight ratio of each data source in the calculation of the most deterministic concentration, and reasonably determine the optimal indicator set. Using the gray correlation analysis method, the converted satellite orbit is evaluated and analyzed. The preliminary evaluation model of the spacecraft
initial orbit based on the grey relational analysis method is established, which solves the practical problem of the measurement ship in the initial orbital optimization of the spacecraft. Finally, the example verification shows that the verification results and the actual situation of the task show that the method is reasonable in the primary selection of spacecraft orbit, and the optimal results are ideal. It has practical value in the practical application of spacecraft initial orbit.

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