N$^3$LO fits to $xF_3$ data: $\alpha_s$ vs $1/Q^2$ contributions

A. L. Kataev $^a$, G. Parente $^b$ and A. V. Sidorov $^c$

$^a$Institute for Nuclear Research of the Academy of Sciences of Russia, 11312, Moscow, Russia

$^b$Department of Particle Physics, University of Santiago de Compostela, 15706 Santiago de Compostela, Spain

$^c$Bogoliubov Laboratory of Theoretical Physics, Joint Institute for Nuclear Research, 141980 Dubna, Russia

The results of approximate N$^3$LO and detailed NNLO fits to $xF_3$ data of the CCFR’97 collaboration are presented. We demonstrate that $1/Q^2$ non-perturbative corrections to $xF_3$ modeled by three independent procedures are shadowed by perturbative QCD effects, starting at the NNLQO. Special attention is paid to revealing the role of the recently calculated NNLO corrections to the anomalous dimensions and N$^3$LO corrections to the coefficient functions of odd moments of $xF_3$ with $n \leq 13$. The related values of $\alpha_s(M_Z)$ are extracted.

It is known that the leading non-perturbative power suppressed corrections to DIS structure functions (SFs) have the dimension $1/Q^2$. However, it turned out that phenomenological value of non-perturbative effects depend crucially from the order of the corresponding perturbative contributions. It is worth to remind that in 1979, when the data for DIS neutrino-nucleon scattering was not precise enough, the authors of Ref. [1] were unable to separate perturbative $1/\ln(Q^2)$ source of scaling violation from the $1/Q^2$-effects. At present both the precision of $xF_3$ measurements [2] and the information on renormalization-group perturbative QCD evolution of Mellin moments became more precise. The latter ones enter into the Jacobi polynomial formula [3]

$$xF_3(x, Q^2) = w(\alpha, \beta) \sum_{n=0}^{n_{\text{max}}} \Theta_n^{(\alpha, \beta)}(x) \times$$

$$\sum_{j=0}^{n} c_j^{(n)}(\alpha, \beta) M_{j+2}^{\text{TMC}}(Q^2) + \frac{HT}{Q^2}$$

where $w(\alpha, \beta) = x^\alpha(1-x)^\beta$, $c_j^{(n)}(\alpha, \beta)$ contain Euler $\Gamma$ functions from $\alpha$ and $\beta$ and the moments $M_{n}^{\text{TMC}}(Q^2) = M_n(Q^2) + \frac{n(n+1)M_{\text{nucl}}^2}{(n+2)Q^2}$ take into account the leading order target mass corrections. The kinematic contributions of order $1/Q^4$ did not affect the results of our previous less detailed NNLO fits to CCFR’97 data (see Refs.[4, 5]) and the analysis of Ref.[6] described below.

In this talk we will concentrate on the results of the most recent analysis of the CCFR’97 data for $xF_3$ in the NLO, NNLO and approximate N$^3$LO levels of perturbative QCD [6], paying special attention to the possibility of the detection of non-perturbative $1/Q^2$-contributions to $xF_3$. They will be modeled by three independent ways. First, is the infrared-renormalon (IRR) model of Ref. [7]

$$\frac{HT}{Q^2} = w(\alpha, \beta) \sum_{n=0}^{n_{\text{max}}} \Theta_n^{(\alpha, \beta)} \sum_{j=0}^{n} c_j^{(n)}(\alpha, \beta) M_{j+2}^{\text{IRR}}$$

where $M_n^{\text{IRR}} = \tilde{C} M_n(Q^2) A_2^/' Q^2$ and $\tilde{C} = -n + 4 + 2/(n+1) + 4(n+2) + 4S_1(n)$, calculated in Ref. [6] from the single chain of quark loop inser-
tions into the one-gluon contribution to the corresponding Born diagram and $A'_2$ is the arbitrary fitted parameter. Next, following the NNLO Bernstein polynomial fits to CCFR'97 data of Ref. \[11\] (see also Ref. \[8\]) we consider gradient model of twist-4 term, namely

$$M^{HT}_{n,xF_3}(Q^2) = n \frac{B'_n}{Q^2} M_n(Q^2)$$  \tag{4}$$

where $B'_n$ is the free parameter. Another possibility is to choose

$$HT = h(x) \text{ in the model -- independent way}. \tag{5}$$

Here $h(x)$ is defined by free parameters $h_i = h(x_i)$, where $x_i$ are the points in experimental data binning. In our work the following renormalization-group equation for the Mellin moments of $xF_3$ was used:

$$M_n(Q^2) = M_n(Q_0^2) \exp \left[ -\int_{Q_0^2}^{Q^2} \frac{\gamma^{(n)}_{F_3}(t)}{\beta(t)} \, dt \right] \frac{C^{(n)}_{F_3}(A_s(Q_0^2))}{C^{(n)}_{F_3}(A_s(Q^2))} \tag{6}$$

where $A_s = \alpha_s/(4\pi)$ is the $\overline{MS}$-scheme coupling constant and $M_n(Q_0^2)$ is defined in the initial scale as $M_n(Q_0^2) = \int_0^1 x^{-n-2} A(Q_0^2) x^{3n} (1 + \gamma(Q_0^2)) dx$.

At the N$^3$LO the expression for $C^{(n)}_{F_3}(A_s)$ can be presented in the following form

$$C^{(n)}_{F_3} = 1 + C^{(1)}(n) A_s + C^{(2)}(n) A_s^2 + C^{(3)}(n) A_s^3$$  \tag{7}$$

where the NNLO correction $C^{(2)}(n)$ can be obtained for any $n$ from the results of Ref. \[10\], which were checked with the help of other methods in Ref. \[11\]. The N$^3$LO contributions to Eq. \(6\) were analytically calculated in Ref. \[12\] for odd $n \leq 13$. The N$^3$LO expansion of the anomalous dimension term has the following form

$$\exp \left[ \int A_s(Q^2) \frac{\gamma^{(n)}_{F_3}(t)}{\beta(t)} \, dt \right] =$$

$$= (A_s(Q^2))^{\gamma^{(n)}_{F_3}/\beta_0} \left[ 1 + p(n) A_s(Q^2) + q(n) A_s(Q^2)^2 + r(n) A_s(Q^2)^3 \right]$$  \tag{8}$$

where $p(n)$, $q(n)$ and $r(n)$ are defined through coefficients of QCD $\beta$-function and anomalous dimension $\gamma^{(n)}_{F_3}$ (see Ref. \[13\]). Note, that on the contrary to the QCD $\beta$-function, analytically calculated in Ref. \[13\] at the N$^3$LO level, the expression for $\gamma^{(n)}_{F_3}$ is known up to NNLO order. Moreover, its NNLO corrections were calculated in case of odd $n \leq 13$ only. In order to fix the numerical values of the NNLO corrections to $\gamma^{(n)}_{F_3}$ (and thus the term $q(n)$) for even $n$ inside the interval $3 \leq n \leq 13$ we used the smooth interpolation procedure, proposed in Ref. \[15\], and supplemented it by fine-tuning of definite NNLO coefficients of $\gamma^{(n)}_{F_3}$. The application of this procedure for estimating NNLO coefficients of $\gamma^{(n)}_{F_3}$ with even $n$ and $N_f = 4$ result in the numbers, which differ from the corresponding NNLO terms of non-singlet contributions to $\gamma^{(n)}_{F_3}$ \[15\] in the 4th significant digit. The NNLO correction to $\gamma^{(2)}_{F_3}$ was estimated by us using extrapolation technique, which has definite theoretical uncertainties. As to the applicability of the smooth interpolation procedure, we checked that it is reproducing the known even contributions to $C^{(2)}(n)$ with satisfactory precision \[13\]. That is why we consider the results of its application, including the estimates of even terms of $C^{(3)}(n)$, as really reliable.

Fixing by this way the N$^3$LO coefficients $C^{(3)}(n)$ and estimating N$^3$LO correction $r(n)$ to Eq. \(8\) by means of $[1/1]$ Padé approximation technique, previously used in perturbative QCD e.g. in Ref. \[16\], we can use Eq. \(1\) for performing approximate N$^3$LO fits to $xF_3$ data. At the next page the results, obtained in Ref. \[8\] in the case of combining Eq. \(1\) with the IRR model of Eq. \(3\), are presented for $N_{max} = 6$, first studied in Refs. \[12\] \& \[1\], and $N_{max} = 9$ (see Table 1).

Looking at Table 1 we arrive at the following conclusions:

1) The NLO fits seem to support the IRR model of Ref. \[8\] by the foundation of the negative values of $A_2$.

These values are in agreement with the results of the previous fits of Refs. \[16\] \& \[1\] and with the ones, obtained in Ref. \[17\] using the NLO DGLAP analysis of the same set of CCFR'97 data and the parton distributions set (PDFs) of Ref. \[18\]. The similar value $A_2 = -0.104 \pm 0.005$ GeV$^2$ was also found in the NLO fits to the combined
The cases of different $Q_0^2$ and $N_{\text{max}}$ are considered.

| order/$N_{\text{max}}$ | $Q_0^2 = $ | 5 GeV$^2$ | 20 GeV$^2$ | 100 GeV$^2$ |
|-------------------------|-------------|------------|------------|------------|
| NLO/6                   | $\Lambda^{(4)}_{\text{MS}}$ | 370$\pm$38 | 369$\pm$41 | 367$\pm$38 |
|                         | $\chi^2/\text{nep}$ | 80.2/86   | 80.4/86   | 79.9/86   |
|                         | $A_2^\prime$     | $-0.121\pm0.052$ | $-0.121\pm0.053$ | $-0.120\pm0.052$ |
| NNLO/6                  | $\Lambda^{(4)}_{\text{MS}}$ | 379$\pm$41 | 376$\pm$39 | 374$\pm$42 |
|                         | $\chi^2/\text{nep}$ | 78.6/86   | 79.5/86   | 79.0/86   |
|                         | $A_2^\prime$     | $-0.125\pm0.053$ | $-0.125\pm0.053$ | $-0.124\pm0.053$ |
| NNLO/9                  | $\Lambda^{(4)}_{\text{MS}}$ | 331$\pm$33 | 332$\pm$35 | 331$\pm$35 |
|                         | $\chi^2/\text{nep}$ | 73.1/86   | 75.7/86   | 76.9/86   |
|                         | $A_2^\prime$     | $-0.013\pm0.051$ | $-0.015\pm0.051$ | $-0.016\pm0.051$ |
| N$^3$LO/6               | $\Lambda^{(4)}_{\text{MS}}$ | 305$\pm$29 | 327$\pm$34 | 326$\pm$34 |
|                         | $\chi^2/\text{nep}$ | 76.0/86   | 76.2/86   | 78.5/86   |
|                         | $A_2^\prime$     | $0.036\pm0.051$ | $0.033\pm0.052$ | $0.029\pm0.052$ |
| N$^3$LO/9               | $\Lambda^{(4)}_{\text{MS}}$ | 333$\pm$34 | 328$\pm$33 | 328$\pm$38 |
|                         | $\chi^2/\text{nep}$ | 73.8/86   | 75.9/86   | 76.4/86   |
|                         | $A_2^\prime$     | $0.038\pm0.052$ | $0.035\pm0.052$ | $0.034\pm0.052$ |

For $F_2$ data [19] using MRS(R2) PDFs [20].

2) At the NNLO the values of $A_2$ are comparable with zero within statistical error bars. The similar small value, namely $A_2 = -0.0065 \pm 0.0059$, was obtained from the NNLO fits to $F_2$ data in Ref. [19].

3) The inclusion of the $N^3$LO corrections make $A_2$ positive. However, it has the statistical uncertainties twice as large as the central value.

Thus we conclude that starting from the NNLO the IRR-model corrections to $xF_3$ can not be extracted from CCFR’97 data with reasonable precision and are shadowed by perturbative QCD effects.

4) The values of $\chi^2$ decrease from NLO up to NNLO and at the $N^3$LO it almost coincide with the ones obtained at the NNLO. Moreover, $\chi^2$ decreases with the increase of $N_{\text{max}}$. This is the welcome feature of including in the fits more detailed information on the perturbative theory contributions both to the coefficient functions and the anomalous dimensions of $xF_3$ moments.

5) For $N_{\text{max}} = 9$ $\Lambda^{(4)}_{\text{MS}}$ and $A_2$ are rather stable to variation of $Q_0^2$ not only at the NLO but at the NNLO and $N^3$LO as well. This property gives favor of our new results from Ref. [21] in comparison with the ones obtained in Ref. [5] for $N_{\text{max}} = 6$ and $Q_0^2 = 20$ GeV$^2$ using more approximate model for $\gamma_F^{(2)}(n)$ and Padé approximation for $C_F^{(3)}(n)$.

To transform $\Lambda^{(4)}_{\text{MS}}$ into the values of $\alpha_s(M_Z)$ we first used the $\overline{\text{MS}}$-scheme matching condition of Ref. [22] with the matching point chosen as $m_b^2 \leq M_b^2 \leq 36m_b^2$ following the proposal of Ref. [22]. This gives us the possibility to estimate threshold uncertainties in $\alpha_s(M_Z)$. Varying the factorization and renormalization scales $\mu_R^2 = \mu_F^{2,\text{AC}} = \mu^2$ in the interval $\mu_F^{2,\text{MS}}/4 \leq \mu_F^{2,\text{MS}} \leq 4\mu_F^{2,\text{MS}}$ we estimated the scale-dependent uncertainties. As the result we obtained the following values of $\alpha_s(M_Z)$ extracted from the fits to $xF_3$...
CCFR’97 data with $1/Q^2$ non-perturbative corrections modeled using the IRR approach [6]:

$$NLO \quad \alpha_s(M_Z) = 0.120 \pm 0.002 \ (stat)$$
$$\pm 0.005 \ (syst) \pm 0.002 \ (thresh) \pm 0.010 \ (scale)$$

$$NNLO \quad \alpha_s(M_Z) = 0.119 \pm 0.002 \ (stat)$$
$$\pm 0.005 \ (syst) \pm 0.002 \ (thresh) \pm 0.004 \ (scale)$$

$$N^3LO \quad \alpha_s(M_Z) = 0.119 \pm 0.002 \ (stat)$$
$$\pm 0.005 \ (syst) \pm 0.002 \ (thresh) \pm 0.002 \ (scale)$$

The systematic errors are fixed from separate consideration of these experimental uncertainties of the CCFR’97 collaboration. Notice, that the inclusion of higher-order perturbative QCD corrections into Eq. (6) minimize essentially the scale-dependence uncertainties. They are in agreement with the similar estimates, obtained in Ref. [23] in the process of the fits to the definite model of $xF_3$ data. Our NNLO value of Eq. (10), obtained in Ref. [6], within existing error-bars is in agreement with the results of NNLO Bernstein polynomial fits to the CCFR’97 data, namely

$$\alpha_s(M_Z) = 0.1153 \pm 0.0041 \ (exp) \pm 0.0061 \ (theor)$$

and

$$\alpha_s(M_Z) = 0.1196^{+0.0027}_{-0.0031}$$

In Table 2 we present the outcomes of our NLO and NNLO fits to the subset of CCFR’97 data, analyzed in Ref. [8]. The twist-4 term was fixed with the help of the gradient model of Eq. (4).

Table 2
The results of the fits to the subset of CCFR’97 data with HT defined by the gradient model with the coefficient $B'_2 \ [\text{GeV}^2]$.

| order | $\Lambda^{(Q)^2}_{\text{MS}} \ [\text{MeV}]$ | $B'_2(\text{HT})$ | $\chi^2/\text{nep}$ |
|-------|---------------------|-----------------|-----------------|
| NLO   | 371$\pm$72          | -0.135$\pm$0.113 | 75$\pm$74       |
| NNLO  | 316$\pm$51          | -0.031$\pm$0.088 | 64$\pm$74       |

At the NLO the value of $B'_2$ is in agreement with the value of the IRR model parameter $A'_2$. At the NNLO $B'_2$ is comparable with zero. Thus we confirm the existence of the effect of the shadowing of the dynamical $1/Q^2$-corrections to $xF_3$ SF at the NNLO of perturbative QCD. This effect was first observed using the IRR model in Ref. [3].

If we use model-independent parameterization of the twist-4 contribution (see Eq. (5)), this effect is becoming even more vivid. The results of extraction of $h(x)$ in different orders of perturbative QCD and for different $N_{max}$ are presented at Fig.1, taken from Ref. [8].

![Figure 1](image-url)

The $x$-shape of $h(x)$ extracted from the fits of CCFR’97 for $Q^2_0 = 20 \text{ GeV}^2$.

Several comments are in order.

1) The $x$-shape of $h(x)$, obtained at the LO and NLO, is in satisfactory agreement with the prediction of the IRR model of Ref. [7].

2) In all orders of perturbation theory the results are rather stable to the variation of $N_{max}$.

3) The $x$-shape of $h(x)$, obtained during the NNLO and approximate $N^3LO$ fits, demonstrate oscillation-type behavior with large error-bars. Thus we conclude, that starting from the NNLO the $x$-shape of $h(x)$ is strongly correlated with higher-order perturbative QCD corrections to Eq. (6).

However, it is possible that more detailed understanding of the NNLO behavior of $h(x)/Q^2$ contribution to $xF_3$ will be obtained after NNLO analysis with taking into account systematic uncertainties of the data [24].
To conclude, we demonstrated that the inclusion into the fits to CCFR’97 $xF_3$ data of the NNLO perturbative QCD effects is leading to effective shadowing of the dynamical $1/Q^2$ corrections. However, it might be possible, that they will be detected in future even at the NNLO. This might happen in case more precise experimental data for the SFs of $\nu N$ DIS will be obtained, say at the future neutrino factories (for detailed discussions see Ref. [25]).

Note also, that while considering massive-dependent perturbative series for the Adler function of $e^+e^-$-scattering [26] and re-extracting the gluon condensate value from the charmonium sum rules [27] with the calculated in Ref. [28] three-loop massive corrections to their spectral function the effects of influence of higher-order perturbative QCD effects to the value of the gluon condensate, introduced in Ref. [29], were observed. Thus, the problems of correlations between perturbative and non-perturbative terms, discussed in our talk, seem to be typical to other cases also, when it is necessary to analyse the expansions in both $\alpha_s$ and inverse powers of $Q$. The similar point of view was expressed at this Symposium in the talk of Ref. [30].

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