Hybrid Beamforming in Frequency Selective Massive MIMO Systems: A Single-Carrier or a Multicarrier Problem?

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Abstract- In this paper, we propose novel hybrid analog and digital beamforming techniques for the uplink of massive multiple-input multiple output (MIMO) systems under frequency-selective and rich scattering propagation conditions. Considering an \(L\)-tap wireless channel, we first derive the closed-forms for single-carrier \(L\)-tap and 1-tap RF beamformers. Then, we demonstrate that the proposed beamformers significantly reduce the root mean square (RMS) delay spread of the effective channel observed at the baseband. We also provide the analytical expressions for the asymptotic sum-rate and illustrate that the performance of the proposed beamformers are comparable to that of digital matched-filtering (MF). Moreover, we evaluate a hybrid beamforming technique using our proposed single-carrier beamforming methods at the RF along with a multicarrier technique using zero-forcing (ZF) at the baseband. Furthermore, we examine the performance of this method for a subconnected architecture, and with the use of digital phase shifters. Our simulation results and analysis indicate that single-carrier RF beamforming with multicarrier baseband techniques is indeed a promising solution for frequency-selective, rich scattering massive MIMO scenarios.

Index Terms

Hybrid beamforming, massive MIMO, frequency-selective channels.

I. INTRODUCTION

The cellular communication systems advancing into the fifth generation (5G) aim to provide very high data rates to support diverse user demands spanning a large range of applications. Considering the bandwidth limitations of cellular communication systems, achieving high spectral efficiency (bits/s/Hz) is of paramount importance in 5G systems. A key methodology to enable spectrally efficient communication in 5G is the massive multiple-input multiple-output (MIMO) technique. The massive MIMO system consists of one or more base stations (BSs) equipped with a large number of antennas serving multiple users. The presence of this large antenna array at the BS not only enables long-range signal transmission through beamforming, but also provides a platform to leverage the
multidimensional signal space for efficient user detection and decoding. With the orthogonal frequency division multiplexing (OFDM) being the primary waveform of the current fourth generation (4G) cellular systems like the long term evolution (LTE), massive MIMO-OFDM scenarios have been extensively discussed in literature \[1\]. It is demonstrated that massive MIMO-OFDM with a dedicated radio frequency (RF) chain per antenna element is capable of providing very high data rates in real frequency selective channels. However, massive MIMO-OFDM is an expensive technology due to the challenges associated with underlying large antenna arrays as well as the OFDM systems \[2\]. For example, OFDM is known to possess a high peak-to-average power ratio (PAPR), which results in decreased energy efficiency of power amplifiers (PAs) used in the system. Moreover, the use of individual RF chains adds to the computational complexity and the price, thereby increasing the overall cost of massive MIMO-OFDM systems.

Alternatively, single-carrier transmission schemes have also been shown to be effective in massive MIMO scenarios. For example, simple time-domain beamforming was shown to be able to achieve a near-optimal performance \[3\] and reduce root mean square (RMS) delay spread of the effective channel \[4\], \[5\]. A smaller RMS delay spread enables the application of a simple time-domain equalizer with smaller number of taps and a shorter cyclic prefix (CP). This reduces the overall system delay, improves the spectral efficiency and also helps in reducing the power consumption \[3\], \[5\], \[6\]. However, using single-carrier transmission over wideband channels would be challenging, often resulting in performance trade-offs. One approach would be to decompose the wideband channel into multiple subbands, such that the channel characteristics do not vary significantly within the individual subbands \[7\] and then apply the narrowband beamforming algorithms. This approach is also compatible with OFDM systems, where the size of the individual subbands can span one or more subcarriers.

As mentioned earlier, massive MIMO systems require a large number of expensive and power hungry RF chains, regardless of whether the targetted application is wideband or narrowband. One of the solutions to alleviate this issue is the use of hybrid analog-and-digital beamformers, where a small number of RF chains are connected to a large number of antennas through a network of low cost phase shifters \[8\]. The switching speed of these phase shifters can dramatically vary from nanoseconds to hundreds of microseconds \[9\], \[10\]. Considering the switching speed aspects, the design complexity and the associated hardware cost, most hybrid beamforming techniques employ a single-tap RF beamformer, which have been demonstrated to improve the performance for narrowband channels. For example, in \[8\], it was shown that narrowband digital and hybrid beamformers result in a similar spectral efficiency in both rich and sparse scattering channels.

The current MIMO-OFDM systems require a fully-digital system, where it is possible to control the phase and amplitude of the signal per subband/subcarrier. Therefore, using a narrowband hybrid beamformer with 1-tap RF beamformer for a MIMO-OFDM reduces the degrees of freedom, since the same phase shift is applied to each subband. This makes it challenging to achieve the desired high data rates or high spectral efficiency. To this end, various wideband hybrid beamforming techniques have been explored in literature \[11\]–\[17\], and reference therein. A hybrid beamforming solution for point-to-point wideband millimeter wave (mmWave) scenario is proposed in \[11\]. The solution relies on RF codebook based analog beamforming and Gram-Schmidt orthogonalization for hybrid precoding. In \[12\], \[18\], a multiuser massive MIMO downlink scenario is evaluated using space-division
multiple-access and orthogonal frequency-division multiple-access (SDMA-OFDMA). A suitable combination of the actual hybrid beamforming design and the associated frequency-domain user scheduling strategy would be necessary for improved performance [13]–[15]. To the best of our knowledge, most works on wideband hybrid beamforming focus on mmWave scenarios [11]–[17]. The algorithms developed in these works exploit the sparsity of the mmWave channels.

Recently, it was shown that a modified form of narrowband hybrid beamforming techniques can be applied to frequency-selective scenarios [16], [17]. More precisely, multicarrier techniques are applied at the baseband whereas a modified version of the narrowband RF beamforming methods is applied at the RF domain. In [16], firstly the average of the channels matrices over different subcarriers is calculated, and then the narrowband beamformer of [19] is applied to the average channel. In [17], the RF beamformer is designed according to the steering vectors of the channel when a sparse scattering geometry-based channel model with uniform linear/planar arrays is used. The works in [16], [17] heavily rely on the sparse nature of the mmWave channel, and such methods are not directly applicable to rich scattering channels, e.g., uncorrelated Rayleigh fading. However, the approaches in [16], [17] can significantly simplify the design of hybrid beamformers for frequency selective channels. With this motivation, in this paper we propose a novel hybrid beamformer that is applicable to exponentially decaying independent and identically distributed (i.i.d.) Rayleigh fading channels. In our method, the RF beamformer is treated as a finite-impulse response filter that will employ single-carrier based transmission method. Then, we investigate two scenarios for single-carrier RF beamforming according to the switching time of the phase shifters. First, we consider the case where the RF beamformer can switch the phase of its elements as fast as the delay resolution of the system (i.e., high speed phase shifters). For this scenario, we propose an $L$-tap RF beamformer, where $L$ denotes the number of channel taps. In the second scenario, we consider low-speed phase shifters are only active over the first channel tap and then they are switched off during the rest of the transmission block. This case is referred to as 1-tap beamforming throughout the paper. We provide the closed-form expressions for both the $L$-tap and the 1-tap single-carrier RF beamformers. The analysis of this paper is mostly focused on hybrid beamforming with fully-connected architecture and analog phase shifters as shown in Fig. 1a. However, in order to consider some practical limitations, the impact of using digital phase shifters and subconnected architecture as in Fig. 1b is also studied. More precisely, the main contributions of this paper are summarized as follows:

- We demonstrate that the proposed beamformers can significantly reduce the RMS delay spread of the effective channel\(^{1}\). We present closed-form expressions indicating the reduction in the RMS delay spread and show that the order of reduction is similar to that obtained by using matched-filtering (MF) in a fully-digital system.
- We also provide closed-form expressions for the asymptotic signal-to-interference-plus-noise ratio (SINR) and hence the asymptotic achievable rates when the number of antennas goes to infinity. Using these expressions and Monte Carlo simulations, we illustrate that the same spatial multiplexing is achievable with the proposed methods and the array gain increases with the number of the antennas at relatively low signal-to-noise ratios (SNRs). However, at the high SNR regime, the achievable sum-rate of the proposed single-carrier beamforming

\(^{1}\)Effective channel includes the impact of the propagation channel as well as the time-varying RF beamformer
methods will saturate. As with the previous case, these characteristics are comparable to the performance of MF in a fully-digital system.

- We investigate the sum-capacity of the effective frequency-selective channel. We show that the capacity of the propagation channel with digital beamforming is almost achievable when our single-carrier RF beamforming methods are used in conjecture with multicarrier based beamforming techniques at the baseband. In particular, we apply zero-forcing (ZF) to each subcarrier and demonstrate through simulations that this method eliminates the aforementioned performance saturation at high SNRs that occurs when only the single-carrier RF beamforming is used. Finally, we evaluate the impact of using subconnected structure as well as digital phase shifters on the sum-capacity.

The rest of the paper is organised as follows. In Section II, we provide the system model used in this paper. In Section III, we describe the proposed $L$-tap and 1-tap RF beamformers and provide the closed-form expressions for the reduction in the RMS delay spread and the achievable rates. In Section IV, we present the simulation results validating our propositions and the conclusions are drawn in Section V.

**Notations:** The following notation is used throughout this paper: $\mathbb{R}$ and $\mathbb{C}$ are the field of real and complex numbers. $\mathbf{A}$ and $\mathbf{a}$ represent a matrix and vector. $\mathbf{a}_m$ is the $m$th column of $\mathbf{A}$. $A_{mn}$ and $|A_{mn}|$ denote the $(m, n)$ element of $\mathbf{A}$ and its magnitude. $\mathbf{A}^{-1}$, det$(\mathbf{A})$, $\mathbf{A}^T$ and $\mathbf{A}^H$ denote inverse, determinant, transpose and Hermitian of $\mathbf{A}$, respectively. $\mathcal{CN}(\mathbf{a}, \mathbf{A})$ presents a random vector of complex Gaussian distributed elements with expected value $\mathbf{a}$ and covariance matrix $\mathbf{A}$. Finally, for time-varying matrices $\mathbf{A}(n)$ and $\mathbf{B}(n)$, let $\mathbf{A}(n) * \mathbf{B}(n) = \sum_{l=-\infty}^{\infty} \mathbf{A}(l) \mathbf{B}(n-l)$.

**II. SYSTEM MODEL**

We consider the uplink of a single-cell massive MIMO scenario where $K$ single-antenna user-equipment at time index $n$ transmit the signal vector $\mathbf{x}(n) \in \mathbb{C}^{K \times 1}$ to the base station with $M$ antennas. The elements of $\mathbf{x}(n)$ are

![Block diagram of antenna selection techniques.](image)
i.i.d. with $E[x(n)x^H(n)] = P_l I_K$ where $P_l$ is the transmit power of each user. The $L$-tap frequency-selective channel matrix $H \in \mathbb{C}^{M \times K}$ is modeled as a finite impulse response (FIR) filter

$$H(n) = \sum_{l=0}^{L-1} H_l \delta(n-l) = \sum_{l=0}^{L-1} H_{wl} D_l^{1/2} \delta(n-l),$$

(1)

where $\delta(n)$ and $H_l \in \mathbb{C}^{M \times K}$ denote discrete-time unit impulse function and the channel matrix of the $l$-th tap, respectively. The channel matrix $H_l$ consists of slow fading and fast fading parameters, denoted by $D_l \in \mathbb{R}^{K \times K}$ and $H_{wl} \in \mathbb{C}^{M \times K}$, respectively. The fast fading channel matrix, $H_{wl}$, is modeled with i.i.d. $CN(0,1)$ distributed elements, as in [3]. The elements of the diagonal matrix $D_l$ are represented as $d_{lk}$. The relationship between $x(n)$ and the received signal vector $y(n) \in \mathbb{C}^{M \times 1}$ is

$$y(n) = \sum_{l=0}^{L-1} H(l)x(n-l) + z(n),$$

(2)

where $z \in \mathbb{C}^{M \times 1}$ denotes the i.i.d. zero-mean additive Gaussian noise vector with variance $E[z(n)z^H(n)] = \sigma_z^2 I_M$. The receiver will employ the combiner matrix $W(n) = \sum_{l=0}^{L-1} W_l \delta(n-l)$ where $W_l \in \mathbb{C}^{K \times M}$. Then, the transmitted vector as seen by the detector, $\hat{x} \in \mathbb{C}^{K \times 1}$ is given by

$$\hat{x}(n) = W(n) * H(n) * x(n) + W(n) * z(n)$$

$$= H_c(n) * x(n) + z_c(n),$$

(3)

where we refer to $H_c \in \mathbb{C}^{K \times K}$ and $z_c(n) \in \mathbb{C}^{K \times 1}$ as the effective channel matrix and the effective noise vector, respectively. Moreover, the variance of the elements of $z_c$ is denoted by $\sigma_{z_c}^2$.

A. Receiver combiner

In the combiner design and performance evaluation, it is assumed that the base station has perfect knowledge of the channel state information (CSI), while the users do not have CSI and cannot collaborate. The base station is equipped with a hybrid beamformer where the large number of the antennas, $M$, are connected to a small number of the RF chains through a network of phase shifters as shown in Fig. 4. It is assumed that the number of the RF chains is equal to the number of users, i.e., $K$. This assumption is suitable for the scenarios where achieving high spatial multiplexing is desired [3], [8], [15]–[17].

For the hybrid beamformer, the combiner matrix is modeled as $W(n) = W_{BB}(n) * W_{RF}(n)$ where $W_{BB} \in \mathbb{C}^{K \times K}$ and $W_{RF} \in \mathbb{C}^{K \times M}$ are the baseband combiner and RF beamformer matrices, respectively. While the baseband combiner can be designed to apply either single-carrier or multicarrier signal processing techniques, the design of the RF beamformer is a more challenging task. To the best of our knowledge, all state-of-the-art hybrid beamformers consider a scenario where the RF beamformer provides the same phase shift for all the subbands. This is equivalent to expressing $W_{RF}(n)$ as a 1-tap FIR filter where the effective channel at the baseband becomes a simple matrix multiplication as

$$H_c(n) = W_{RF}H(n).$$

(4)
The $l$-th tap of the fully-connected RF beamforming matrix $\mathbf{W}_{RF,l}$ is

$$\mathbf{W}_{RF,l} = \begin{pmatrix}
e^{j\theta_{l11}} & e^{j\theta_{l12}} & \ldots & e^{j\theta_{l1M}} \\
e^{j\theta_{l21}} & e^{j\theta_{l22}} & \ldots & e^{j\theta_{l2M}} \\
\vdots & \vdots & \ddots & \vdots \\
e^{j\theta_{lK1}} & e^{j\theta_{lK2}} & \ldots & e^{j\theta_{lKM}}
\end{pmatrix},$$

(5)

where $\forall \theta_{lkm} \in \Theta, \forall l \in \{1, ..., L\}$,

$$\forall k \in \{1, ..., K\}, \forall m \in \{1, ..., M\},$$

where $\Theta = [0, 2\pi)$ for analog phase shifters and $\Theta = \{0, 2\pi/2^B, \ldots, (2^B - 1)2\pi/2^B\}$ for digital phase shifters with $B$ bits of resolution. The RF beamformer for the subconnected structure $\mathbf{W}_{sub,l}$ is a block diagonal matrix and its elements are expressed as

$$\mathbf{W}_{\text{sub},lk} = \begin{cases} 
e^{j\angle \theta_{lkm}} & \text{if } m \in \mathcal{I}_k, \\
0 & \text{if } m \notin \mathcal{I}_k, 
\end{cases}$$

(6)

where $\theta_{lkm} \in \Theta$ and $\mathcal{I}_k = \{\frac{M}{K}(k - 1) + 1, \ldots, \frac{M}{K}k\}$.

### B. Performance metrics

In this paper, we use two the following two metrics for evaluating the performance of our hybrid beamforming technique - a) the RMS delay spread and b) the achievable sum-rate of the effective channel. The study of RMS delay spread of the effective channel enables us to evaluate the possibility of using single-carrier based transmission schemes.

1) **RMS delay spread**: Let $P(n)$ denote the channel power delay profile (PDP) of a single-input single-output channel. Then, the RMS delay spread $\tau$ is expressed as

$$\tau = \sqrt{\frac{\sum_{n=-\infty}^{\infty} P(n)n^2}{\sum_{n=-\infty}^{\infty} P(n)} - \left(\frac{\sum_{n=-\infty}^{\infty} P(n)n}{\sum_{n=-\infty}^{\infty} P(n)}\right)^2}. $$

(7)

2) **Achievable sum-rate**: When the interference is treated as additive white Gaussian noise, the sum-rate of the system is defined as

$$R = \sum_{k=1}^{K} \log_2 \left(1 + \gamma_k\right),$$

(8)

where $\gamma_k$ denotes the SINR for the symbol of user $k$ after combining. It is to be noted that the interference term in our analysis consists of inter-symbol interference (ISI) plus the multiuser interference (MUI).

When CSI is not available at the transmit antennas, the capacity of frequency-selective channels is expressed as

$$C = \frac{1}{N_e} \sum_{n_s=1}^{N_s} \log_2 \det \left(1 + \rho \mathbf{H}(n_s)\mathbf{H}^\dagger(n_s)\right),$$

(9)

where $N_s$ denotes the number of the subcarriers and $\mathbf{H}(n_s) = \sum_{l=1}^{L} \mathbf{H}_l \exp(-j\frac{2\pi n_s}{N_e})$, $n_s = 1, \ldots, N_s$. When the impact of the receiver combiner is included, the capacity of the effective channel matrix $\mathbf{H}_e = \mathbf{W}(n) \ast \mathbf{H}(n)$ is derived by replacing $\mathbf{H}$ with $\mathbf{H}_e$ in (9).
In this paper, we will investigate two scenarios based on the modelling strategy used for the RF beamformer - a) the conventional 1-tap FIR filter and b) time-varying \( L \)-tap FIR filter. For the latter, it could be easily verified that the frequency response of the FIR filter is not flat any-more. This corresponds to the scenario where the phase shifters are fast enough to switch their phases according to the delay resolution of the channel\(^2\). In our approach, the RF beamformer will always use single-carrier based signal processing techniques whereas the baseband combiner applies OFDM. In the following, we will investigate the impact of 1-tap and \( L \)-tap RF beamformers on the RMS delay spread, array gain and spectral efficiency of the effective channel that is observed at the baseband.

III. TIME-VARYING RF BEAMFORMING

The proposed hybrid beamformer consists of two stages. In the first stage, a single-carrier type transmission is performed at the RF beamformer to maximize the SNR of each user. In the second stage, a multicarrier based method will be applied at the baseband to remove the residual interference if needed. In this section, we will focus on the performance of the RF beamformer.

When the constant modulus constraint of the phase shifters is relaxed, MF equalization with \( W_{\text{MF}}(n) = H^H(-n) \) can be used to maximize SNR [3]. MF achieves a near optimal spectral efficiency in the low SNR regime. At the high SNR regime, however, the achievable rates will saturate and does not increase with SNR [3]. It is also known that the RMS delay spread of the effective channel will reduce with \( 1/\sqrt{M} \) [5].

When the constant modulus constraint of the \( L \)-tap RF beamformer is included, the channel matrix for the effective channel \( H_e \in \mathbb{C}^{K \times K} \) observed at the baseband is

\[
H_e(n) = \sum_{l=0}^{L-1} W_{n-l} H_l. \tag{10}
\]

Then, the desired signal level from user \( k \) is related to

\[
P_{kk}(n) = |h_{e,kk}(n)|^2 = \left| \sum_{m=1}^{M} \sum_{l=0}^{L-1} \bar{w}_{n-l}^{(m)} h_{lmk} \right|^2 \tag{11}
\]

\[
= \left| \sum_{l=0}^{L-1} \sum_{m=1}^{M} h_{lmk} e^{-j\theta_{(n-l)km}} \right|^2 \leq \left| \sum_{l=0}^{L-1} \sum_{m=1}^{M} |h_{lmk}| \right|^2,
\]

where the equality holds if \( \theta_{lmk} = -\angle h_{lmk} \). Hence, the optimal \( L \)-tap RF beamformer that maximizes the received SNR is

\[
W_{\text{RF}}(n) = \exp(-j\angle H^H(-n)). \tag{12}
\]

Assuming that the first channel tap has the highest gain, the corresponding 1-tap RF beamformer that maximizes the received SNR is obtained by setting

\[
W_{\text{RF}}(n) = \exp(-j\angle H^H_{0}(0)) \delta(n). \tag{13}
\]

In the following, we evaluate the performance of the RF beamformers in (12) and (13).

\(^2\)The delay resolution of the channel depends on the sampling time, \( T_s \). We adopt the standard methodology of setting \( T_s = 1/B \), where \( B \) is the system bandwidth.
**Proposition 1:** When the proposed RF beamformers in (12) and (13) are used, the RMS delay spread of \( P_{kk}(n) \) reduces with \( 1/\sqrt{M} \) when \( M \to \infty \).

**Proof:** We only present the proof for the \( L \)-tap beamformer as the same steps could be repeated for the 1-tap scenario.

Let \( f_{lmkn} = h_{lmk} e^{-j \angle h(n+l)m} \), then

\[
P_{kk}(n) = \begin{cases} \left| \sum_{m=1}^{M} \sum_{l=0}^{L-1+n} f_{lmkn} \right|^2, & -L + 1 \leq n < 0, \\ \left| \sum_{l=0}^{L} \sum_{m=1}^{M} |h_{lmk}| \right|^2, & n = 0, \\ \left| \sum_{m=1}^{M} \sum_{l=n}^{L-1} f_{lmkn} \right|^2, & 0 < n \leq L - 1. \end{cases}
\]

The RMS delay spread \( \tau_e \) of \( P_{kk}(n) \) is

\[
\tau_e = \sqrt{\frac{\sum_{n=-\infty}^{\infty} P_{kk}(n)n^2}{\sum_{n=-\infty}^{\infty} P_{kk}(n)}} - \left( \frac{\sum_{n=-\infty}^{\infty} P_{kk}(n)}{\sum_{n=-\infty}^{\infty} P_{kk}(n)} \right)^2.
\]

When \( M \to \infty \), the law of large numbers implies that

\[
P_{kk}(0) = \left| \sum_{l=0}^{L} \sum_{m=1}^{M} |h_{lmk}| \right|^2 = M^2 \left| \sum_{l=0}^{L-1} \mathbb{E}_M [ |h_{lmk}| ] \right|^2
\]

\[
= M^2 \left( \sum_{l=0}^{L-1} E_M [ d_{kk}^{1/2} h_{w,lmk} ] \right)^2
\]

\[
= \frac{\pi M^2}{4} \left( \sum_{l=0}^{L-1} d_{kk}^{1/2} \right)^2,
\]

\[
\text{where } E_M[|h_{lmk}|] \text{ denotes the expected value of } |h_{lmk}| \text{ with respect to } M, \text{ and } E_M[|h_{w,lmk}|] = \sqrt{\pi/2} \text{ [8].}
\]

Since \( h_{lmk} \) is a zero-mean i.i.d random variable, its phase-shifted version, \( f_{lmkn} \), is also uncorrelated zero-mean i.i.d random variable. For \(-L + 1 \leq n < 0 \) and \( M \to \infty \), the law of large numbers leads to

\[
P_{kk}(n) = M \left[ \frac{\sum_{l=0}^{L-1+n} \sum_{m=1}^{M} f_{lmkn}}{\sqrt{M}} \right]^2
\]

\[
= M \sum_{l=0}^{L-1+n} E_M [ |f_{lmkn}|^2 ] = M \sum_{l=0}^{L-1+n} \text{Var}_M (f_{lmkn})
\]

\[
= M \sum_{l=0}^{L-1+n} d_{lk},
\]

where \( \text{Var}_M[a] \) denotes the variance of \( a \) with respect to \( M \). Similarly, \( \forall 0 < n \leq L - 1 \) it could be shown that

\[
P_{kk}(n) = M \sum_{l=0}^{L-1} d_{lk},
\]

which indicates that \( P_{kk}(n \neq 0) \) grows with \( M \). Since the ratio of the of the numerator to the denominator of (15) is \( 1/M \), the RMS delay spread \( \tau_e \) reduces with \( 1/\sqrt{M} \). \( \square \)

In the following propositions, we consider a scenario where interference is considered as additive noise and investigate the SINR behaviour of the proposed methods.
Proposition 2: When the $L$-tap RF beamformer in [12] is used, for $M \to \infty$, the achievable sum-rate $R_{\text{sum}}^{L}$ is given by

$$R_{\text{sum}}^{L} = \sum_{k=1}^{K} \log_2 (1 + \gamma_k^L),$$

(19)

where the SINR $\gamma_k^L$ of the $k$-th user is

$$\gamma_k^L = \frac{1}{L + P_i(KL - 1)} \times \frac{\pi P_i M}{4} \left( \sum_{l=0}^{L-1} d_{lk} \right)^2.$$

(20)

Proof: From (16), the desired signal level from user $k$ is

$$S_k = P_i P_{kk}(0) = P_i M^2 \pi \frac{2}{4} \left( \sum_{l=0}^{L-1} d_{lk} \right)^2.$$

(21)

The interference consists of intersymbol $I_{\text{ISI}} = P_i \sum_{n \neq m} P_{kk}(n)$ and interuser $I_{\text{MUI}} = P_i \sum_{k \neq k'} \sum_{n} P_{kk'}(n), \forall k, k' \in \{1, ..., K\}$ and $k \neq k'$ terms. As a result of (17) and (18), the ISI term is expressed as

$$I_{\text{ISI}} = MP_i \left( \sum_{n = -L}^{L-1} \sum_{l = 0}^{L-1} d_{lk} + \sum_{n=1}^{L-1} \sum_{l=0}^{L-1} d_{lk} \right)$$

(22)

$$= MP_i \left( \sum_{n=1}^{L-1} \sum_{l=0}^{L-1} d_{lk} + \sum_{n=1}^{L-1} \sum_{l=0}^{L-1} d_{lk} \right)$$

$$= MP_i \left( \sum_{n=1}^{L-1} \sum_{l=0}^{L-1} d_{lk} \right) = (L-1)MP_i.$$

For $I_{\text{MUI}}$, using the law of large numbers, we calculate the asymptotic value of $P_{kk'}(n)$ as

$$P_{kk'}(n) = \left| \sum_{m=0}^{M} h_{lmk} e^{-j \theta_{(n+i)mk}} \right|^2 = ME_M \left[ |f'_{lmkn}|^2 \right]$$

(23)

$$= M \sum_{l=0}^{L-1} \text{Var}_M \left( f'_{lmkn} \right) = M \sum_{l=0}^{L-1} d_{lk},$$

where $f'_{lmkn} = e^{-j \theta_{(n+i)mk}} h_{lmk}$ is an i.i.d. zero-mean random variable with respect to $m$, $\forall n \in \{-L+1, ..., -1\}$.

It could be easily verified that $P_{kk'}(0) = M$ due to the normalization $\sum_{l=0}^{L-1} d_{lk} = 1$, and $P_{kk'}(n > 0) = \sum_{l=0}^{L-1} d_{lk}$. Similar to (22), the multiuser interference term $I_{\text{MUI}}$ is related to

$$\sum_{n=-L}^{L+1} P_{kk'}(n) = P_{kk'}(0) + \sum_{n=-L}^{L+1} P_{kk'}(n) = \sum_{n=1}^{L-1} \sum_{l=0}^{L-1} d_{lk}.$$

(24)

$$= M + M \left( \sum_{l=0}^{L-1} d_{lk} \right) = ML.$$

Due to the symmetry of the problem, the total interference from $K-1$ users on user $k$’s signal is

$$I_{\text{MUI}} = (K-1)LM P_i.$$

(25)

Since noise is a zero-mean i.i.d. random variable, the power $|z_m(0)|^2$ and variance of the effective noise at the baseband are equal. At $n = 0$, $|z_m(0)|^2$ is

$$\left| \sum_{l=0}^{L-1} \sum_{m=1}^{M} w_{lmn} z_{lm} \right|^2 = M \sum_{l=0}^{L-1} E_M \left[ |z_m|^2 \right] = LM.$$

(26)
Finally, it could be easily verified that the SINR, given by $\gamma_L^{\text{tap}} = \frac{S_k}{|z_e(0)|^2 + I_{\text{ISI}} + I_{\text{MUI}}}$, grows with $M$ and Proposition 2 is proved.  

**Proposition 3:** When the 1-tap RF beamformer in (13) is used, and for $M \to \infty$, the achievable sum-rate $R_{\text{sum}}^{1\text{-tap}}$ is

$$R_{\text{sum}}^{1\text{-tap}} = \sum_{k=1}^{K} \log_2(1 + \gamma_k^{1\text{-tap}}),$$

where the SINR $\gamma_k^{1\text{-tap}}$ of the $k$-th user is

$$\gamma_k^{1\text{-tap}} = \frac{d_{0k}}{1 + P_t \sum_{n=1}^{L-1} d_{nk} + P_t(K - 1) \times \pi P_t M \frac{d_0}{4}}.$$

**Proof:** Equations (4), (14) and (21) indicate that the power the desired signal for user $k$ is

$$S_k = P_t P_{kk}(0) = \left| \sum_{m=1}^{M} |h_{0mk}| \right|^2 = \frac{P_t M^2 d_0}{4},$$

In addition, when $M \to \infty$ the inter-symbol interference is

$$I_{\text{ISI}} = P_t \sum_{n \neq 0} P_{kk}(n) = P_t M \sum_{l=1}^{L-1} d_{nk},$$

and the interference form user $k'$ is

$$I_{\text{MUI},k'} = P_t \sum_{n=0}^{L-1} P_{kk'}(n) = \left| \sum_{m=0}^{M} \sum_{l=0}^{L-1} h_{lmk} e^{-j \angle h_{0mk}} \right|^2$$

$$= MP_t \sum_{l=0}^{L-1+n} \text{Var}_M(h_{lmk'}) = MP_t.$$

Finally, the effective noise power at $n = 0$ is $|z_e(n)|^2 = |\sum_{m=1}^{M} z_{lm}|^2 = M$, and the Proposition is proved.

Propositions 1-3 indicate that the behaviour of the proposed RF beamformers in terms of the RMS delay spread and the SINR growth is similar to that digital MF [3], [5].

Hitherto, we considered a fully-connected beamforming architecture and analog phase shifters. In general, most of the low-cost commercial phase shifters with low insertion losses have discrete resolution and analog phase shifting is not possible. Next, we discuss the performance of the proposed RF beamformer in a fully-connected architecture when digital phase shifters are used. Then, we provide the closed-form expressions of our RF beamformer for the subconnected architecture.

**A. Digital phase shifters**

When the impact of digital phase shifters is taken into account, the design of hybrid beamformers turns into a computationally expensive combinatorial problem with a huge search space. For an $L = 4$ tap RF beamformer with $M = 100$ antennas, $K = 10$ RF chains and $B = 3$ bits of resolution per phase shifters, there are $2^{LMKB} = 2^{12000}$ possible phases. One approach is to use a predefined set of phases, known as RF codebooks, as in [22] and references therein. However, the disadvantages of codebook based approaches are that - 1) they are just applicable to fixed type channel conditions and specific criteria, e.g. mmWave channels and 2) the codebook size grows very large with number of the antennas, which can make it difficult to use them in real-time applications. Hence, it is desired
to find the RF beamforming weights of digital phase shifters such that it requires low computational complexity and without relying on RF codebooks. For narrowband systems, Lemma 4 in [8] expresses that hybrid beamformers with low-resolution digital phase shifters can almost achieve the performance of analog phase shifting by simply rounding the phase of the optimal RF beamformer with analog phase shifters to the closest available phase that the digital phase shifter can provide. Following a similar line of thinking, we employ this approach to the time-varying $L$-tap RF beamformer. Then, the weight of the $(k, m)$-th digital phase shifter at $l$-th tap is

$$\theta_{lkm}^d = \arg \min_{\theta_{lkm}} |\angle H(-l)km - \theta_{lkm}|,$$

s.t. $\theta_{lkm} \in \{0, ..., (2^B - 1)2\pi/2^B\}$,

where $l = 1, ..., L - 1$. As it will be shown in the simulation results section, 3 bits of resolution will be able to provide a similar performance as analog phase shifting.

B. Subconnected structure

In this section, we present the closed-form expression of the performance when subconnected architecture is used. The asymptotically optimal narrowband hybrid beamforming with subconnected phase shifter network and the closed-form expression of its performance was presented in [23]. It was shown that the beamforming weights can be directly derived from the fully-connected scenario. Following a similar approach, the $L$-tap RF beamforming matrix can be expressed as

$$W_{\text{sub}, lkm} = \begin{cases} 
eq & \text{if } m \in \mathcal{I}_k, \\ 0 & \text{if } m \notin \mathcal{I}_k. 
\end{cases}
$$

When $M \to \infty$ and the proposed hybrid beamformer in (33) is used, then a) the RMS delay spread of $P_{kk}(n)$ reduces with $1/\sqrt{M}$ and b) the closed-form of the achievable sum-rate for the $L$-tap and 1-tap RF beamformer is achieved by replacing $\gamma^L_{\text{sub}, k}$ in (8) with $\gamma^L_{\text{sub}, k}$ or $\gamma^1_{\text{sub}, k}$, given by the following equations:

$$\gamma^L_{\text{sub}, k} = \frac{1}{L + P_t(KL - 1)} \times \frac{\pi P_t M}{4K} \left| \sum_{l=0}^{L-1} \frac{d_{lkk}}{2} \right|^2,$$

where $\gamma^L_{\text{sub}, k}$ represents the SINR for the $k$-th user after the $L$-tap beamformer, and

$$\gamma^1_{\text{sub}, k} = \frac{d_{0kk}}{1 + P_t \sum_{n=1}^{L} d_{nk} + P_t(K - 1)} \times \frac{\pi P_t M}{4K},$$

is the SINR of the $k$-th user after 1-tap RF beamformer.

Remark: The proof is straightforward following the same steps as in the proofs of Proposition 1-3.

It should be noted that the difference between the performance of the subconnected and fully-connected techniques is attributed to a loss of $M/K$ in the antenna array gain.

C. Baseband combiner matrix

Using single-carrier based transmission techniques at the RF beamformer could firstly reduce the RMS delay spread and secondly increase the SNR of the signal for the intended user. In order to mitigate the residual inter-user
interference, we apply multicarrier beamforming techniques at the baseband. In particular, we apply ZF to each subcarrier in this paper.

In terms of complexity, the key tasks in the baseband of a massive MIMO system, such as, fast Fourier transform (FFT) and channel inversion per subcarrier, scale with the number of RF chains. Typically, in a fully digital system, the number of RF chains is equal to the number of antennas, \( M \). However, with our proposed hybrid beamforming techniques, the number of RF chains is equal to the number of users, \( K \leq M \). The use of a smaller number of RF chains is possible because the RF beamformer will operate on the initial signal stream over \( M \) antennas, adjust the phases and then generate only \( K \) signal streams for the baseband. Therefore, the original \( N_s \)-point FFT operations for \( M \) RF chains, would now be required only over \( K \) RF chains. Similarly, the pseudo-inverse of a rectangular \( M \times K \) channel matrix per subcarrier would be replaced by that of \( K \times K \) matrix inversion, thereby reducing the baseband complexity.

In the next section, we present the simulation results and analysis of the proposed beamforming techniques.

IV. SIMULATION RESULTS

In this section, we evaluate the performance of the proposed \( L \)-tap and 1-tap beamformers with analog and digital phase shifters. The accuracy of the closed-form expressions in Propositions 1-3 with respect to the Monte Carlo simulations will also be examined. Then, the impact of the baseband signal processing will be investigated.

We performed the simulations over 1000 channel realizations. Unless specifically mentioned, we set \( M = 100 \) and \( N_s = 128 \). The remaining parameters were set according to [3], i.e., \( K = 10, L = 4 \) and the slow fading parameter of the channel model

\[
d_{lk} = \exp(-\psi_k l) / \sum_{l=0}^{3} \exp(-\psi_k l),
\]

where \( \psi_k = (k-1)/5, \forall k \in \{1, \ldots, K\} \).

Fig. 2 presents the RMS delay spread of the effective channel averaged over the users. Proposition 1 stated that the reduction rate of the RMS delay spread of the effective channel is related to \( 1/\sqrt{M} \). In Fig. 2, we use \( 1/\sqrt{M} \) and \( 7/\sqrt{M} \) curves as lower and upper bounds to validate Proposition 1 for different cases (\( L \)-tap/1-tap, fully-connected/subconnected)\(^3\). As expected, the subconnected case has a higher RMS delay spread than the fully-connected case, owing to the reduction in the array gain. Also, for both fully-connected and subconnected structures, the \( L \)-tap RF beamformer has a lower RMS delay spread than its 1-tap counterpart, since it accounts for the channel imperfections in an improved manner. Moreover, in Fig. 2, we observe that the performance gap between the \( L \)-tap and the 1-tap beamformers decreases with increasing \( M \). This indicates that the 1-tap RF beamformer enabling simplified single-carrier transmission can be effective in massive MIMO scenarios.

Fig. 3 shows the cumulative distribution function (CDF) of the RMS delay observed at the baseband. It is seen that the CDFs of \( L \)-tap and 1-tap beamformers are comparable to that of the digital MF. Also, the \( L \)-tap RF beamformer has a lower spread than the 1-tap case. However, the 1-tap beamformer provides significant gains in terms of reducing the RMS delay spread (as indicated by the average value in Fig. 2 and the CDF in Fig. 3) when

\(^3\)It should be noted that 1 and 7 are arbitrary numbers that are used to show the general \( 1/\sqrt{M} \) behaviour.
Fig. 2. Expected value of RMS delay spreads for SISO scenario, the effective channel with digital MF, $L$-tap and 1-tap RF beamformers with fully-connected and subconnected structures.

Fig. 3. CDF of RMS delay spreads of SISO scenario, the effective channel with digital matched-filtering, $L$ and 1 tap RF beamformers with fully-connected structure.

compared to the single-input single-output (SISO) scenario, thereby supporting its employment for massive MIMO hybrid beamforming.

Fig. 4 presents the simulation and analysis results for the achievable sum-rate performance of digital MF, 1-tap and $L$-tap single-carrier RF beamformers with fully-connected and subconnected phase shifter networks. It is observed that the performance of all methods get saturated at around $P_t/\sigma_z^2 = 0$ dB. Moreover, there is a good match between simulations and the closed-form expressions in Propositions 2-3.

It should be noted that Fig. 5 presents the achievable sum-rates when there is only single-carrier RF beamforming. In order to investigate the impact of baseband processing, the capacities of the different RF beamforming techniques (indicated in Eqn. (9)) for frequency-selective effective channels at the baseband are presented in Fig. 5a and Fig. 5b.
Fig. 4. Achievable rates for the single-carrier transmission versus $P/\sigma^2_z$, lines correspond to simulations whereas the markers +, *, ⋄ and ◦ present the closed-forms in Propositions 2-3, respectively.

for $K = 10$ and $K = 20$, respectively. Firstly, it is observed that full spatial multiplexing is achieved by all of techniques and structures. Secondly, Fig. 5a indicates that a hybrid single-carrier RF and multicarrier baseband transmission can result in a promising performance.

For a fully-digital system, it is observed that the MF almost achieves the channel capacity, i.e., $C(\mathbf{H}, \mathbf{z})$ and $C(\mathbf{H}^{\text{MF}}_c, \mathbf{z}_{c,\text{MF}})$ are almost equal. For the proposed beamformers, we see that $C(\mathbf{H}^{L\text{-tap}}_c, \mathbf{z}^{L\text{-tap}}_c)$ and $C(\mathbf{H}^{1\text{-tap}}_c, \mathbf{z}^{1\text{-tap}}_c)$ for the fully-connected, and $C(\mathbf{H}^{L\text{-tap}}_{c,\text{sub}}, \mathbf{z}^{L\text{-tap}}_{c,\text{sub}})$ and $C(\mathbf{H}^{1\text{-tap}}_{c,\text{sub}}, \mathbf{z}^{1\text{-tap}}_{c,\text{sub}})$ for the subconnected structure, have similar performances. Moreover, the performance gap with respect to MF is not too large for the fully-connected case.

Furthermore, an interesting characteristic emerges on comparing Fig. 5a where $K = 10$ and Fig. 5b where $K = 20$. We see that the 1-tap beamformer has a slightly better performance than the $L$-tap RF beamformer when the number of users increases. This can be attributed to the channel model parameter for slow fading, $d_{lk}$, adopted in this work (indicated by Eqn. 36). The method used for obtaining $d_{lk}$ (similar to that used in [3]) provides a channel PDP such that first channel tap has increased power as the number of users increases, i.e., a large number of user channels tend to become single-tap channels with increasing $K$. Therefore, using an $L$-tap beamformer results in accumulating more noise, leading to a degradation in performance when compared to the 1-tap beamformer.

Considering the results in Fig. 5a and Fig. 5b for rich scattering channels, in addition to the results in [16], [17] for sparse scattering channels, we infer that 1-tap RF equalization, which is also known as narrowband RF beamforming, can provide a promising solution for massive MIMO hybrid beamforming.

Fig. 6 shows the impact of applying digital ZF in the baseband. For a fully-digital system, it is observed that applying ZF per subcarrier almost achieves the capacity of the channel. However, there is a small loss in spectral efficiency when single-carrier MF is combined with multicarrier based ZF. The figure also shows that performing a second stage ZF can significantly improve the achievable sum-rates, unlike the case where only the single-carrier RF transmission was used (which resulted in the saturation in achievable sum-rate as indicated by Fig. 4). Moreover, Fig. 6 indicates that the combination of proposed RF beamforming methods with the multicarrier based ZF result
Fig. 5. Capacity of frequency-selective $H$, $H_{r, MF}$, $H_{r, L-tap}$, $H_{e,1-tap}$, $H_{e,sub,L-tap}$ and $H_{e,sub,1-tap}$ channels according to (9) with $N_s = 128$ and $P_t/\sigma^2$ indicating SNR per subcarrier.

Finally, we evaluate the effect of using digital phase shifters along with our proposed RF beamforming techniques in Fig. 7. For this evaluation, we use multicarrier ZF at the baseband. It is observed that the performance of digital phase shifters with three bits of resolution is comparable to that of analog phase shifting, supporting the practical use of our proposed beamformers.

V. CONCLUSION

In this paper, we investigated the hybrid beamforming problem for rich scattering frequency-selective channels in massive MIMO systems. It was shown that for fully-connected and subconnected structures, with both analog and digital phase shifters, the proposed methods can provide a promising performance despite their simplicity and low computational complexity. The advantages of this approach are multifold - a) the reduction in RMS
delay spread obtained using our solutions results in reduced computational complexity and shorter CP length in the massive MIMO-OFDM system, when compared to its conventional fully-digital counterpart, b) in terms of spectral efficiency, similar achievable sum-rates are observed when compared to a fully-digital system with ZF per subcarrier, c) the computational complexity and power consumption of our hybrid approach is much less compared to digital systems as the phase shifters network reduce the dimensions of the propagation channel while the total sum-capacity is not significantly affected and d) with 1-tap RF beamforming demonstrating similar performance as \( L \)-tap beamforming, it allows the use of phase shifters with low switching-speed.

Considering our results for rich scattering channels, and state-of-the-art methods for mmWave systems, we conclude that a hybrid beamforming technique using the proposed single-carrier RF beamforming methods along with a multicarrier baseband technique, is a promising solution for massive MIMO scenarios. Moreover, the problem can also be treated as a hybrid narrowband and wideband beamforming at the RF and baseband, respectively, when
1-tap RF beamforming is applied. Finally, further research is required to optimize and investigate the performance of the proposed methods in other scenarios, for instance, using different channel models.

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