Neutrino masses and proton decay in SO(10)

M. Abud$^{1,2}$, F. Buccella$^2$ and D. Falcone
$^1$Università di Napoli Federico II
$^2$INFN, Sezione di Napoli

Abstract
We consider the constraints on SO(10) unified models coming from the lower limits on proton lifetime and on the scale of B–L symmetry breaking within the framework of the seesaw model for neutrino masses. By upgrading a triangular relationship for the inverse of $\nu_L$ Majorana masses to the experimental situation with non maximal $\theta_{23}$ and non vanishing $\theta_{13}$, we get for the sum of $\nu_L$ masses the upper limit 0.16 eV.

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1 Introduction

It is well known that the minimal $SU(5)$ grand unified model proposed by Georgi and Glashow [1] has met a number of shortcomings, the three running coupling constants do not meet at the same point and, more importantly, the unification scale of the couplings $g_2$ and $g_1$ of the unified electroweak theory is too low to comply with the lower limit on proton lifetime, which scales as to the fourth power of the unification scale. It was soon realized that consistency with experiment was obtainable in the extended $SO(10)$ [2] GUT model, provided the intermediate symmetry breaking pattern comprises a $SU(2)_R$ group, contributing to the weak hypercharge according to $Y = T_{3R} + (B - L)/2$.

In fact, above the intermediate scale the first component belongs to a non-abelian group and the same happens for the second component, if the intermediate symmetry contains the Pati-Salam $SU(4)$ [3], which contains $SU(3)_c \times U(1)_{B-L}$. The change of regime in the RGE provokes the meeting of the gauge constants at a higher scale. This scale is even higher, if $D$ parity, which would imply equal values at the intermediate scale for $g_{2L}$ and $g_{2R}$ is broken at the highest scale [4]. This fact induced a systematic study of the scalar potential with absolute minima in the directions able to provide the desired symmetry breaking, where $B - L$ is broken by the VEV along the $SU(5)$ singlet of the 126 representation, while the breaking of $SO(10)$ at the highest scale is obtained by a VEV of the 54 for the case in which the intermediate symmetry is $SU(4) \times SU(2) \times SU(2) \times D$ [5], and along a particular direction of the 210 in the two-dimensional space of the singlets of $SU(3) \times SU(2) \times SU(2) \times U(1)$ for the three cases where the intermediate symmetry is $SU(4) \times SU(2) \times SU(2)$ [6] or $SU(3) \times SU(2) \times SU(2) \times U(1) \times D$ [7] or just $SU(3) \times SU(2) \times SU(2) \times U(1)$ [8].

In the cases of intermediate symmetry group with a $SU(4)$ factor, a larger (negative) contribution from the gauge bosons imply a more rapid evolution for the coupling constant $g_3$, while in the cases where a discrete $D$ symmetry factor is present, the evolution of $g_{3L}$ is importantly affected by the more copious scalar content, therefore the intermediate symmetry $SU(3) \times SU(2) \times SU(2) \times U(1)$ provides the case with the larger proton lifetime [9]. We assume the Extended Survival Hypothesis (ESH) [10], which limits the contribution of the scalar particles to the RGE only to the multiplets containing the Higgses with the VEV’s responsible for the spontaneous symmetry breaking at a lower scale, namely the electroweak Higgs above $M_Z$ and the multiplet responsible for the spontaneous breaking of the intermediate symmetry between the two highest scales. So we have four different expressions for the two higher scales in terms of $\sin^2 \theta_W$ and $\alpha/\alpha_s$ at the electroweak scale. It has been observed that the scale of $SO(10)$ symmetry breaking, which is related to proton decay, cannot be larger than the expression in terms of the difference between $\sin^2 \theta_W$ and $\alpha/\alpha_s$ that one should get without any change in the evolution. We will give an evaluation of the corrections implied by consider the RGE at the next order.

More recently [11] an extensive analysis has been performed about $SO(10)$ models with two intermediate symmetry groups between $SO(10)$ and the gauge
group of the standard model $SU(3) \times SU(2) \times U(1)$. A general property of these $SO(10)$ models with intermediate scales is that, since the $Q^2$ evolution of $\sin^2 \theta_W$ is more soft just below the scale of $SO(10)$ breaking, there is often a relationship between the scales of $SO(10)$ and the $B - L$ symmetry breaking, higher the first, lower the second. Therefore the models with the highest scale for $M_X$, and therefore longer proton lifetime, is the one with the lowest scale for $B - L$ symmetry breaking, called $M_{B-L}$.

By assuming the seesaw model \[12\], which comes out naturally in the framework of $SO(10)$ unification and accounts for the order of magnitude of left-handed neutrino masses, one has the relation

$$\det m_L \det M_R = \det(m_D)^2$$

(1)

where $m_L$ and $M_R$ are the Majorana mass matrices for left and right handed neutrinos respectively, and $m_D$ is the Dirac neutrino mass matrix. The relation (1), which is crucial for the lower bound that we will get for the scale of $B - L$ symmetry breaking, $M_{B-L}$, holds if one neglects type II seesaw. Therefore our result do not apply in presence of non-negligible $SU(2)_L$ triplet VEV's. We do not know $m_D$, but it is reasonably to assume within the $SO(10)$ model that

$$\det m_D \det m_d = \det m_l \det m_u,$$

(2)

at least approximately. This implies

$$\det m_D = 2 \cdot 10^7 \text{ MeV}^3$$

(3)

so the seesaw model gives

$$\det m_L \det M_R = \det(m_D)^2 = 4 \cdot 10^{14} \text{ MeV}^6$$

(4)

with the upper limit from cosmology \[13\] $\det m_L < 8 \cdot 10^{-3} \text{ eV}^3$, and a corresponding lower limit

$$\det M_R > \frac{4 \cdot 10^{14} \text{ MeV}^6}{8 \cdot 10^{-3} \text{ eV}^3} = 0.5 \cdot 10^{26} \text{ GeV}^3.$$ 

(5)

We expect $\det M_R$ to be less than the cube of the scale of $B - L$ symmetry breaking and so for this scale we find the lower limit $3.68 \times 10^8 \text{ GeV}$.

In the four models we are considering, one predicts the values of the two higher scales of spontaneous symmetry breaking, the highest one being related to proton decay. Then, the lower limit on proton lifetime implies a corresponding lower limit for that scale, while the second scale would imply a lower limit on the product of left-handed neutrino masses. From the general properties of the $b$’s for the RGE one sees that the trend is such that to a higher scale for the lepto-quarks mediating proton decay corresponds a lower scale for the spontaneous symmetry breaking of $B - L$ and consequently a higher lower limit for the product of left-handed neutrino masses. In conclusion proton decay and cosmological neutrino masses provide two conflicting limits on the $SO(10)$ models described here.
2 Symmetry breaking scales

The values of $\sin^2 \theta_W$ and $\alpha_s$ are $0.23116 \pm 0.00013$ and $0.1184 \pm 0.0007$, respectively [14]. In the four models with intermediate symmetry containing $SU(2)_R$ we have at first order:

- Model with Higgses in the 54 and intermediate symmetry $SU(4) \times SU(2) \times SU(2) \times D$

\[
3/8 - \sin^2 \theta_W = \alpha/2\pi [109/24 \ln(M_{B-L}/M_Z) - 49/24 \ln(M_X/M_{B-L})]
\]

- Model with Higgses in the 210 and intermediate symmetry $SU(4) \times SU(2) \times SU(2) \times U(1)_{B-L} \times D$

\[
3/8 - \sin^2 \theta_W = \alpha/2\pi [109/24 \ln(M_{B-L}/M_Z) + 11/8 \ln(M_X/M_{B-L})]
\]

- Model with Higgses in the 210 and intermediate symmetry $SU(3) \times SU(2) \times SU(2) \times SU(2) \times U(1)_{B-L} \times D$

\[
3/8 - \sin^2 \theta_W = \alpha/2\pi [109/24 \ln(M_{B-L}/M_Z) + 19/8 \ln(M_X/M_{B-L})]
\]

- Model with Higgses in the 210 and intermediate symmetry $SU(3) \times SU(2) \times SU(2) \times U(1)_{B-L} \times D$

\[
3/8 - \sin^2 \theta_W = \alpha/2\pi [109/24 \ln(M_{B-L}/M_Z) + 29/12 \ln(M_X/M_{B-L})]
\]

By these equations we can obtain the intermediate scale (also the scale of the mass of RH neutrino) and the unification scale, which is the scale of leptoquark allowing proton decay. Of course, we should consider the two-loop equations which were reported for the first time by Jones [15].

To get into account of the two loop contributions, we correct the lowest order coefficients by just multiplying them by the ratios of the second to the first order values [16], where the values of the $b_i$'s have only the slight difference of considering two Higgs doublets at the electroweak scale, while here to avoid flavour changing neutral currents we have only one. Also the values of $\sin^2 \theta_W$ and $\alpha_s$ are slightly changed, with a relevant impact on the first order values for the scales and a less important consequence on the ratios between the second order and first order values for the scales. Also the corresponding error on the scales is negligible with respect to the uncertainties on the masses of the scalars, as it is shown in [16]. So we get for the scales for the four model considered the following values:
to be compared with the lower limit $M_X = 10^{15.431} \text{GeV}$ coming from the corresponding lower limit $8 \times 10^{33} \text{ years}$ on the rate $\tau(p \to e^+ + \pi^0)$ [17] and the formula

$$\tau(p \to e^+ + \pi^0) = 8 \times 10^{33} [M_X / 10^{15.431} \text{GeV}]^4 \text{ ys.} \quad (14)$$

The first model gives a lifetime more than an order of magnitude shorter than the lower limit, which also keeping into account the uncertainties on our evaluation of the scales strongly disfavours it. The third one gives a lifetime about a factor two larger than the present lower limit, while the second and the fourth imply lifetimes not accessible for the next decades. It is important to stress that due to the fast dependance on $M_X$, which has an exponential dependence on the values of $\sin^2 \theta_W$ and $\alpha_s$ at the scale $M_Z$ with a relatively large uncertainty on the value of $\alpha_s$, the conclusions may be changed with larger values for these constants, which would imply longer lifetimes, while smaller values would have the opposite effect of shorter lifetimes. Also the uncertainties associated to the masses of the scalars neglected for the evolution in the ESH limit the sharpness of our conclusions.

For the four models discussed here we plot in Fig.’s 1-4 the values of the constants $\sin^2 \theta_W$ and $\alpha_s$ with their uncertainties and the allowed zones for them consistent with the lower limits for the scale $M_X$ and $M_R$ coming from the lower limit on proton lifetime and the upper limit on the sum of the left-handed neutrino masses, which within the see-saw model (see eq(2)) and assuming eq(3) determine the lower limit for $M_R$, respectively. The fourth model, the one with the highest $M_X$, implies for neutrino masses values near to the present upper limits, while for the third one both proton decay and neutrino masses are not so far from the present limits. Finally one may observe that the present values of the electro-weak couplings are just in the region, where for the model with $SU(4) \times SU(2)_L \times SU(2)_R$ intermediate symmetry experimental signatures are not expected in the near future.

As long as for the large class of models considered by [11] beyond the obvious limitation to the models with $n_U \geq 15.431$ the restriction $n_1 \geq 8.57$ is affecting also the model with the largest $n_U$, also excluding a large part of the values of $n_1$ for the two last models defined in Table IV of [11]. If $B-L$ is spontaneously broken by a vev of the 16, one should have for the mass of the Majorana mass of the right-handed states the expression [18]

$$M_{\nu_R} = \left( \frac{\alpha}{\pi} \right)^2 Y_{10} M_{B-L}^2 M_{X}$$

\(^{1}\text{For graphic reasons the central bar corresponds to three standard deviations for } \sin^2 \theta_W \text{ and only one for } \alpha_s .\)
Y_{10} being the Yukawa coupling to the 10. The above equation implies that right-handed neutrino masses are several orders of magnitude smaller than the scale of spontaneous breaking of $B - L$ and, within our hypotheses, a higher lower limit for that scale. In conclusion one may say that the models with the largest $M_X$ and therefore longest proton lifetime are the ones, where one expects the largest signals for $m_{ee}$ (the parameter appearing in neutrinoless double beta decay) and $m_{\nu_e}$ (the kinematical neutrino mass related to the final part of the electron energy spectrum in tritium decay). Our limits on $M_{B-L}$ depend of course on our assumption for $\det m_D$, but on the other side we expect that $\det M_R$ is smaller than $M_{B-L}^3$, which would make the lower limit on $M_{B-L}$ more restrictive.

In a previous paper [19] from the following assumptions:

1) See-saw model for neutrino masses;
2) A value for the highest eigenvalue of the neutrino Dirac mass matrix $m_D$ of the order of the top-quark mass and a form for the matrix $V^L$, which appears in the biunitary transformation which diagonalizes $m$, similar to $V_{CKM}$;
3) The upper limit for the mass of the heaviest right-handed neutrino given by the scale of $B - L$ symmetry breaking in nonsupersymmetric $SO(10)$ model, which, as it is shown in this paper, is around $10^{11}$ GeV,

we derived the sum rule for the inverse of the Majorana masses of the left-handed neutrinos:

$$\sum_i \frac{|U_{\tau i}|^2}{m_i} = 0 \quad (16)$$

and we considered $\theta_{23}$ maximal and a vanishing $\theta_{13}$. A finite value of $\theta_{13}$ and eq(16) have been shown to play an important role [20] for the realization of the leptogenesis scenario for baryogenesis [21].

The Daya Bay [22] and Reno [23] results confirmed what was deduced by a global analysis of existing data [24]. With the parameters in Table 1 of [25], namely $\Delta m^2_2 = 7.54 \cdot 10^{-5}$ eV$^2$, $\Delta m^2_3 = 2.43 \cdot 10^{-3}$ eV$^2$, and $\sin^2 \theta_{12} = 0.307$, $\sin^2 \theta_{13} = 0.0241$, and $\sin^2 \theta_{23} = 0.386$, $\delta = 1.08\pi$ one has:

$$|U_{\tau 1}|^2 = 0.196, \quad |U_{\tau 2}|^2 = 0.204, \quad |U_{\tau 3}|^2 = 0.599, \quad (17)$$

with important consequences. It implies the normal hierarchy for the masses of $\nu_L$’s and for $|m_1|$ the range: $6.3 \cdot 10^{-3} \leq |m_1| \leq 4.4 \cdot 10^{-2}$ eV.

One gets an upper limit of $1.28 \cdot 10^{-4}$ eV$^3$ for $|\det m_L|$ and of 0.16 eV for the sum of the masses of left-handed neutrinos, even smaller than the most severe bound, 0.2 eV in [13]. From eq(4) the smaller upper limit implies the higher lower limit for $M_{B-L} = 1.46 \cdot 10^9$ GeV.

In Figures 5, 6, 7 we plot for all the models here considered, excepted the first one (where the the two lines are external to the diagram) the constraints following by this stronger limit together with the one corresponding to $M_X$ to a higher lower limit on proton lifetime, $1.2 \cdot 10^{34}$ years. With these constraints, the third model would be almost excluded by mentioned limit on proton decay. The fourth model would be almost excluded, on the other side, by the limit on $M_{B-L}$. 

5
By the way the second model, which is the only one fully consistent with these more severe constraints, corresponds to a very stable minimum of the Higgs potential [6]. For this model, with the scale of $B - L$ symmetry breaking around $10^{11}$ GeV, possible $\Delta(B - L) = -2$ decays as the ones related to the $d = 7$ effective operators described in [20], may be the signal of baryon non-conservation.

In Figures 8, 9 and 10 we plot $|m_i|$ and their sum, $|m_{\nu_e}|$ and $|m_{ee}|$, and $\det(m_L)$ in the allowed range for $|m_1|$ given by eq (18).

In conclusion, we obtain the following bounds (all values in meV)

$$63 \leq \Sigma m_i \leq 155,$$

$$11 \leq m_{\nu_e} \leq 45,$$

$$8.6 \leq m_{ee} \leq 44.7.$$

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The allowed zones in the \((\alpha_s, \sin^2 \theta_w)\)-plane for models I-IV are the ones in the region between the two lines. The blue line is due to the limit on \(M_X\), while the red line to the limit on \(M_R\).
Same as for figures 2-4 for the more restrictive lower limits for $M_X$ and $M_{B-L}$. 
Fig. 8
Values of $m_1$ (black), $m_2$ (green), $m_3$ (blue) and $\Sigma m_i$ (red) vs. $|m_1|$ in meV.

Fig. 9
Values of $m_{\nu_e}$ (blue) and $|m_{ee}|$ (red) vs. $|m_1|$ in meV.

Fig. 10
Value of $\det(m_L)$ ($10^{-4}$ eV$^3$) in the allowed region for $|m_1|$ (meV).