Neutrons from Multiplicity-Selected
Au-Au Collisions at 150, 250, 400, and 650 AMeV

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We measured neutron triple-differential cross sections from multiplicity-selected Au-Au collisions at 150, 250, 400, and 650 AMeV. The reaction plane for each collision was estimated from the summed transverse velocity vector of the charged fragments emitted in the collision. We examined the azimuthal distribution of the triple-differential cross sections as a function of the polar angle and the neutron rapidity. We extracted the average in-plane transverse momentum $\langle P_x \rangle$ and the normalized observable $\langle P_x / P_\perp \rangle$, where $P_\perp$ is the neutron transverse momentum, as a function of the neutron center-of-mass rapidity, and we examined the dependence of these observables on beam energy. These collective flow observables for neutrons, which are consistent with those of protons plus bound nucleons from the Plastic Ball Group, agree with the Boltzmann–Uehling–Uhlenbeck (BUU) calculations with a momentum–dependent interaction. Also, we calculated the polar-angle-integrated maximum azimuthal anisotropy ratio $R$ from the value of $\langle P_x / P_\perp \rangle$.

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I. INTRODUCTION

Predictions [1–4] of collective flow effects in relativistic heavy-ion collisions have been confirmed by experiments [5–17]. Components of collective flow, such as the side-splash of participants, the bounce-off of spectators, and the out-of-plane squeeze-out [15,17] are extracted by means of various analysis techniques. Different methods are used to investigate the onset of collective flow [18–22]. Several observables have been extracted to describe collective flow, such as the flow angle [5], the average in–plane transverse momentum [7–9,13,14], the flow [8], and the scale invariant flow [24]; however, the azimuthal correlations of collective flow have not been studied fully. A series of measurements were made recently with the FOPI spectrometer at GSI and the time projection chamber at the Lawrence Berkeley Laboratory with a view toward examining the detailed azimuthal correlations and the triple-differential cross sections of charged particles in the full phase space [17,23].

In this paper, we report measurements of the triple-differential cross sections and collective flow observables of neutrons from semi-central Au-Au collisions at beam energies of 150, 250, 400, and 650 AMeV.

II. APPARATUS

This experiment (E848H) was carried out in 1988 with the Bevalac accelerator at the Lawrence Berkeley Laboratory. Neutron spectra were measured by the time-of-flight technique with (NE–102 plastic scintillator) neutron detectors at 18 polar angles from 3° to 90°. All neutron detectors were 1–m long and 10–cm thick. The widths of the neutron detectors ranged from 2.5 to 50.8 cm. The momentum and the rapidity of the neutrons were calculated from the measured flight time over paths that ranged from 5.9 to 8.4 m. The flight path, the polar angle, and the width of each neutron detector are listed in Table 1. The properties of the neutron detectors were studied thoroughly [28–30]; here only the resolutions and detection efficiencies of a typical 25–cm wide detector are listed. Its time resolution was 390 ± 70
ps, and the corresponding energy resolutions were about 4 and 38 MeV for neutrons of 150 and 650 MeV, respectively, with a flight path of 8.4 m. The threshold cut on the pulse height of the neutron detectors was selected to be 16 MeVee (equivalent electron energy).

The neutron detection efficiencies of the detector at this threshold, as calculated with the Monte Carlo code of Cecil et al. [28], varied from 7.4% at 150 AMeV to 6.3% at 650 AMeV. The efficiencies at this analysis threshold were insensitive to the relatively low hardware threshold of 4 MeVee. Particles incident on each of the 18 neutron detectors were vetoed with a thin anticoincidence plastic scintillator in front of the neutron detector. Simultaneous incidence of a charged particle with a neutron would veto that neutron. The false veto rates were estimated from auxiliary measurements of double-hit rates of charged particles. At the beam energy of 650 AMeV, the rates were about 15% for detectors close to the beam line and a few percent for those at wide angles. The measurement of the triple-differential cross sections was corrected for the false veto rates. The neutron double-hit rates for the 18 detectors were negligible at all energies because of low neutron detection efficiencies. Because one of the neutron detectors (at the polar angle of 7°) malfunctioned, the data from this detector were discarded.

An array of 184 scintillation detectors, 4.5–m high and 5–m wide, for detection of charged particles was located at a mean distance of 4.4 m from the target. The size of the detectors varied from 15 cm by 15 cm to 50 cm by 60 cm. The purpose of this array was to estimate the azimuthal angle $\phi_R$ of the reaction plane for each collision, and to record the multiplicity of charged particles, which is an indication of the centrality of the collision. Based on a simulation with FREESCO [26], the double-hit rate of charged particles was estimated to be about 20% in an individual detector for a beam energy of 650 AMeV and smaller at lower beam energies. Only 10% of the charged particles were undetected with this double-hit rate. Neglect of this small percentage of charged particles did not affect the measurement of the reaction plane significantly. For the bombarding energies of 150, 250, 400, and 650 AMeV, the target thicknesses were 0.59, 1.1, 1.7, and 1.7 g/cm$^2$, respectively, with (hwhm) energy spreads of 17, 14, 11, and 6%. 
III. DETERMINATION OF REACTION PLANE

We used the transverse-velocity method reported by Fai et al. [27] to determine the reaction plane. This technique is an adaptation of the transverse-momentum method of Danielewicz and Odyniec [7]. The detailed description of the determination is presented elsewhere [16, 21]. For Au-Au collisions reported in this paper, we determined \( \phi_R \) with charged fragments above a normalized rapidity \( \alpha \equiv Y/Y_{P} \) \( cm = 0.3 \), where \( \alpha \) is defined as the charged particle rapidity divided by the projectile rapidity \( Y_{P} \) in the center-of-mass system; for bombarding energies of 150, 250, 400, and 650 AMeV, we observed dispersions \( \Delta \phi_{R} \) of 39.4\(^\circ\), 31.3\(^\circ\), 29.5\(^\circ\), and 25.4\(^\circ\), respectively, with uncertainties of about 5%.

IV. ESTIMATION OF BACKGROUNDS

The target-unrelated background from collisions of the Au projectiles with air and other materials were estimated by measurements without a target in place. The target–induced background resulting from neutrons scattered from the floor, ceiling, and air were measured with shadow shields of about 1–m long between the target and the neutron detectors. The function of the shadow shields was to attenuate neutrons that were emitted from the target and traveled directly to the neutron detectors. The numbers of neutrons derived by subtracting the yields measured with shadow shields from the yields without shadow shields were the basic data for extraction of the triple–differential cross sections. The shadow shields blocked neutrons emitted from the target, but they were also sources that generated secondary neutrons which might hit the neutron detectors, causing an overestimation of the background. As an illustration of this overestimation, we display in Fig. 1 the azimuthal distribution of neutrons plus background (crosses), background (squares), and neutrons (circles) with rapidities \( \alpha \) between \(-0.2 \) to \(+0.2 \) at a polar angle of 18\(^\circ\) for a beam energy of 650 AMeV. We see from Fig. 1(a) that the anisotropy in the azimuthal distribution of the background is stronger than that of neutrons plus background and that the inferred az-
imuthal distribution of neutrons shows negative yields (without background corrections) at azimuthal angles ($\phi - \phi_R$) in the vicinity of 0°. The stronger anisotropy of the background and the negative yields indicate an overestimation of the background at azimuthal angles ($\phi - \phi_R$) in the vicinity of 0°. We emphasize that this overestimation of the background is significant in only a limited kinematic region (See below). The shadow shields located at $\phi = 0°$ and 180° blocked the direct path of neutrons from the target to the detectors, but at the same time, provided extra material at $\phi = 0°$ and 180° to interact with fragments emitted from the collisions. These secondary interactions resulted in the production of excess neutrons. Collisions where the difference between the azimuthal angles of the reaction plane and the detector $\phi_R - \phi_d$ was near 0° or 180° would produce more excess neutrons than those with this difference away from 0° and 180°. The excess neutrons produced in the collisions with the difference $\phi_R - \phi_d$ close to 180° would not affect the background measurements significantly because these neutrons were produced far away from the neutron detectors being studied; however, because excess neutrons produced in the collisions with the difference $\phi_R - \phi_d$ close to 0° could hit the neutron detectors being studied, these excess neutrons could distort the azimuthal anisotropy of the background. We made a correction for this distortion by utilizing the observed azimuthal distributions of both neutrons plus background and the background. First, we used a function $\sigma_{nb}^{180}[1 + A_{nb}(\phi - \phi_R)]$ to describe the observed azimuthal distribution of neutrons plus background, where $\sigma_{nb}^{180}$ is the cross section of neutrons plus background at $\phi - \phi_R = \pm 180°$ and $A_{nb}(\phi - \phi_R)$ is an anisotropic function which is equal to zero at $\phi - \phi_R = \pm 180°$. Then, we assumed that the correct azimuthal distribution of the background had a form similar to that of neutrons plus background, i.e., $\sigma_{b}^{180}[1 + A_{b}(\phi - \phi_R)]$, where $\sigma_{b}$ is the cross section of the background at $\phi - \phi_R = \pm 180°$, and $A_{b}$ is an anisotropic function to be determined. The correct background rate at $\phi - \phi_R = \pm 180°$ should be close to the observed rate, as per the reasoning given above. Finally, we assumed that the relation between the two anisotropic functions is $A_{b}(\phi - \phi_R) = K A_{nb}(\phi - \phi_R)$, where $0 \leq K \leq 1$ is a constant; the cases $K = 0$ and 1 correspond a flat background and a background with the strongest plausible correlation between
the signal and the background, respectively. The constant $K$ was chosen to be $0.5 \pm 0.5$ when applying the correction. This correction and the uncertainty in the correction are not significant at low beam energies, at wide polar angles, or at high rapidities, where $\sigma_b$ is much smaller than $\sigma_{nb}$, because this correction and its uncertainty are proportional to $\sigma_b$; however, if $\sigma_b$ is close to $\sigma_{nb}$, the correction and its uncertainty become significant as in the case shown in Fig. 1(a). The same three distributions of Fig. 1 (a) are replotted in Fig. 1 (b) with this correction for overestimating the background; we notice from Fig. 1(b) that the corrected neutron spectrum no longer has negative yields.

V. TRIPLE-DIFFERENTIAL CROSS SECTIONS

Figure 2 shows the charged multiplicities with (solid line) and without (dashed line) a Au target in place for collisions at a beam energy of 650 AMeV. From Fig. 2, we see that collisions with a target are contaminated by collisions without a target only at low charged multiplicities. We selected collisions with a charged multiplicity $M \geq M_o = 30, 32, 34, 36$ for beam energies of 150, 250, 400, and 650 AMeV, respectively, to reduce the background from collisions of Au projectiles with air and other materials to less than 5% of the signal. These selections correspond, in a simple geometrical picture, to maximum impact parameters of about 0.55, 0.62, 0.63, and 0.68 times the diameter of the Au nucleus, with an uncertainty of about 10%. We realize that the above values of the impact parameter do not agree with the percentages of the total geometric cross section that we estimated earlier [21]. The earlier percentages were underestimated because in that estimate we subtracted more background than we should have. To make a reliable estimation, we needed a proper data sample with triggers not involving any neutrons. Unfortunately, we did not take such a data sample in 1988; therefore, an overestimation was hard to avoid. We collected data necessary for the estimation of the impact parameters later in an extension of E848H in 1991. The impact parameters listed above were estimated with that data sample.

The threshold cut on the neutron kinetic energy was generally 60 MeV except for neutrons
in the seven detectors with polar angles less than 21° for the bombarding energy of 650 AMeV; at this highest beam energy, cuts on the neutron kinetic energy from 200 to 380 MeV were needed to eliminate background contamination.

Shown in Figs. 3 to 6 are the triple-differential cross sections of neutrons at target-like ($-1.0 \leq \alpha < -0.2$), middle ($-0.2 \leq \alpha < +0.2$), forward ($+0.2 \leq \alpha < +0.7$), and projectile-like ($+0.7 \leq \alpha \leq +1.2$) rapidities, respectively. In each of the four figures, 36 spectra are presented in an array of four rows (for the four bombarding energies) and nine columns (for nine out of the 17 polar angles, selected because the neutrons detected at these angles dominate the observed collective flow effects at the corresponding rapidity). The closed circles and the open triangles in these figures show the results with and without correction for the overestimation of the target–induced background, respectively. The overestimation in the target-like and projectile-like rapidities is negligible as seen in Fig. 6; however, the overestimation is serious in middle and forward rapidities, especially at high beam energies and at small polar angles, as seen in Figs. 3 to 5. For the beam energy of 650 AMeV, the spectra at 30° in Figs. 3 to 5 are void because of electronic faults during data acquisition, and the three plots in Fig. 4 at 9°, 12°, and 15° are empty because of the high cuts on the energy described earlier. The remaining 12 void spectra at the two lowest beam energies in Figs. 3 and 4 are a consequence of a cut of 60 MeV on neutron energy. For each of the rapidity bins in Figs. 3 to 5, we notice that the anisotropy in the azimuthal distribution at a given polar angle does not reveal much sensitivity to the bombarding energy. This insensitivity and its significance will be discussed in the next section. Also, the curves shown in Fig. 2 to 6 are the theoretical predictions from the BUU approach [25]. We present BUU predictions for neutrons at projectile-like rapidities for the four beam energies of 150, 250, 400, and 650 AMeV; for neutrons at other rapidities, we present predictions only for the beam energy of 400 AMeV to illustrate how well data and theory agree with each other at rapidities away from projectile-like rapidities.
VI. FLOW OBSERVABLES

We calculated the average in–plane transverse momentum \( \langle P_x \rangle \) of neutrons with normalized rapidities \( \alpha \) between \(-1.0\) and \(1.2\) based on the triple–differential cross sections measured with much finer rapidity bins than that shown in Figs. 3 to 6. First, we averaged the measured in–plane transverse momentum over the whole space and over the rapidities:

\[
\langle P_x \rangle_m = \frac{\int P(\theta, \alpha) \sin \theta \cos \phi \frac{d\sigma}{d\theta d\phi d\alpha} d\cos \theta d\phi d\alpha}{\int \frac{d\sigma}{d\theta d\phi d\alpha} d\cos \theta d\phi d\alpha} \\
\approx \frac{\sum P(\theta = \theta_d, \alpha) \sin \theta_d \cos \phi \left( \frac{d\sigma}{d\theta d\phi d\alpha} \right)_{\theta = \theta_d} \Delta \cos \theta \Delta \phi \Delta \alpha}{\sum \left( \frac{d\sigma}{d\theta d\phi d\alpha} \right)_{\theta = \theta_d} \Delta \cos \theta \Delta \phi \Delta \alpha},
\]

where \( \theta, \phi, \) and \( \alpha \) are the polar angle, the azimuthal angle, and the normalized rapidity of the neutrons, respectively; the neutron momentum \( P \) is a function of \( \theta \) and \( \alpha \); and \( \theta_d \) is the polar angle of the neutron detector. Then, we derived the in–plane average transverse momentum \( \langle P_x \rangle \) by correcting the measured \( \langle P_x \rangle_m \) for the dispersion of \( \Delta \phi_R \) described in Sec. III,

\[
\langle P_x \rangle = \frac{\langle P_x \rangle_m}{\cos(\Delta \phi_R)}.
\]

The uncertainty of less than 5\% in \( \Delta \phi_R \) is small compared to the other systematic uncertainties in \( \langle P_x \rangle \). Shown in Fig. 7 for the three beam energies of 150, 250, and 400 AMeV is the average in–plane transverse momentum of neutrons versus the normalized neutron rapidity \( \alpha \). The \( \langle P_x \rangle \) for the beam energy of 650 AMeV is not presented because the relatively high and differing cuts on the energy for neutrons in the seven detectors with small polar angles would distort the spectrum of the average \( \langle P_x \rangle \). The uncertainties in the triple–differential cross sections and in the dispersion were propagated to the uncertainty in \( \langle P_x \rangle \) shown in Fig. 7. The approximation in Eq. (1) introduced a large uncertainty in the calculation because of the large \( \Delta \theta \) of about several degrees; this uncertainty was folded into the systematic uncertainties in \( \langle P_x \rangle \). The shift of the \( \langle P_x \rangle \) curve toward low rapidities, as shown in Fig. 7, was attributed to the large systematic uncertainty in \( \langle P_x \rangle \) from the large \( \Delta \theta \). The average \( \langle P_x \rangle \) decreases in the vicinity of the projectile-like rapidities. This drop is
explained by the fact that both evaporation [31] and bounce-off neutrons are included in the
calculation of the average; both types of neutrons are emitted at projectile-like rapidities
and usually have low transverse momentum. The slope of the average in–plane transverse
momentum at negative rapidities is steeper than that at positive rapidities because the (60
MeV) cut on the neutron energy rejects neutrons with low transverse momenta at negative
rapidities. The data below rapidities $\alpha$ of 0.0, −0.1, and −0.2 are affected by the cut on the
neutron energy for beam energies of 150, 250, and 400 AMeV, respectively. We extracted
the slope at mid-rapidity (up to $\alpha = 0.5$) with a linear fit to $\langle P_x \rangle$ in the region unaffected by
the cut on the neutron energy; the uncertainty in the fit is included in the error bar. Doss
et al. [8] defined this slope as the flow $F$. Plotted in Fig. 8 with open squares are the flow
$F$ of neutrons for the three bombarding energies; also displayed with circles are the results
for the same three energies for protons plus bound nucleons from the Plastic Ball Group [8].
The relatively large uncertainties in the measured neutron flow $F$ reflect the effect on
the determination of the flow $F$ by the shift of the $\langle P_x \rangle$ curve seen in Fig. 7. The results of
Plastic Ball Group are obtained without any multiplicity cuts. Our multiplicity cuts selected
the majority of the events (See Fig. 2). Neglect of low multiplicity events would reduce the
flow $F$ by a few MeV/c, which is small compared to the systematic uncertainties in the flow $F$. We did not correct the flow $F$ for this small reduction; instead, we included this reduction
amount in the systematic uncertainties in the flow $F$. From Fig. 8, we see that the extracted
neutron flow $F$ is consistent within uncertainties with that of protons plus bound nucleons.

Also, we calculated the average of $P_x/P_\perp$, where $P_\perp$ is the transverse momentum of
neutrons. The average $\langle P_x/P_\perp \rangle$ is plotted as a function of the neutron rapidity in Fig. 9.
The Plastic Ball Group measured the average $\langle P_x/P_\perp \rangle$ for protons plus bound nucleons at
the beam energy of 200 AMeV [9]. By comparing Fig. 9 in this paper and Fig. 2 in Ref. 9,
we see that the interpolated $\langle P_x/P_\perp \rangle$ of neutrons corresponding to a beam energy of 200
AMeV is almost equal to that of protons plus bound nucleons observed in the Plastic Ball
at positive rapidities. At projectile-like rapidities, the neutron value is a little lower than
the value for protons plus bound nucleons because of the inclusion of evaporation of free
neutrons in our analysis.

Also, we examined another observable associated with the azimuthal distribution about a reaction plane for the emitted neutrons, the polar-angle-integrated maximum azimuthal anisotropy R. The polar-angle-integrated maximum azimuthal anisotropy R was defined and extracted by Madey et al. [32] for semi-central Nb-Nb and Au-Au collisions at 400 AMeV. The relation between the average $\langle P_x/P_\perp \rangle$ and the polar-angle-integrated maximum azimuthal anisotropy R can be expressed as

$$R = \frac{(1 + 2|\langle P_x/P_\perp \rangle|)}{(1 - 2|\langle P_x/P_\perp \rangle|)}.$$  

We calculated the polar-angle-integrated maximum azimuthal anisotropy R from Eq. (3); the results are shown in Fig. 10 for the beam energies of 150, 250, and 400 AMeV.

The difference in the three spectra of the average $\langle P_x/P_\perp \rangle$ and the polar-angle-integrated maximum azimuthal anisotropy R for the three energies is much smaller than that in the spectra of the average in–plane transverse momentum $\langle P_x \rangle$. In the previous section, we noticed the insensitivity of the azimuthal anisotropy to the beam energy; this fact is revealed clearly by the spectra of the average $\langle P_x/P_\perp \rangle$ in Fig. 9 and the polar-angle-integrated maximum azimuthal anisotropy R in Fig. 10. Previously we observed a weak dependence of R on the mass from Au–Au and Nb–Nb collisions [33]. Normalized observables, such as the average $\langle P_x/P_\perp \rangle$ reported here, and scaled observables, such as the scale invariant flow [24], have a weaker dependence on the beam energy and mass than observables without scaling and normalization. This weaker dependence of scaled observables agrees with the observations by Lambrecht et al. [14] for neutrons from Au–Au collisions.

**VII. THEORETICAL INTERPRETATION**

For theoretical interpretation, we rely here on the BUU approach [25] with a momentum–dependent nuclear mean field, $U(\rho, \vec{p})$, as parameterized in Ref. [35]. The momentum-dependent interaction is essential not only from a theoretical standpoint [36], but also has
important observable implications \cite{37,38}. The BUU calculations simulated the experiment by using the maximum impact parameters that were estimated from the multiplicity selection criteria described in Sec. V, and imposing a neutron threshold energy of 60 MeV also as described in Sec. V. Also, we set the incompressibility modulus K to be 215 MeV, and we subtracted contributions to the cross sections from composite fragments by rejecting neutrons when the distance between the neutron and another nucleon from the same BUU ensemble \cite{25} is less than a critical distance \cite{16,39}. This critical distance was determined to be 3.3, 3.0, 3.2, and 2.7 fm for the energies of 150, 250, 400, and 650 AMeV, respectively, by adjusting the BUU predictions to fit the double–differential cross sections $d\sigma/d\Omega$ for free neutrons. The double-differential cross sections for the beam energies of 150, 250, 400, and 650 AMeV are shown in Fig. 11. In Fig. 11, symbols represent the data, dashed lines represent the BUU predictions with all neutrons, and solid lines represent neutrons that are not in clusters, as defined by the critical distance. In general, the results obtained using coalescence prescriptions in conjunction with one–body models like the BUU will depend on the time during the reaction when the coalescence model is applied. A cluster contains nucleons correlated both in coordinate space and momentum space. It is inappropriate to apply such pictures at times that are too early during the reaction, as two–body collisions are still too numerous and will disrupt the cluster as well as form new ones. Here the coalescence picture was applied at a time corresponding to the one when the momentum distributions have just about relaxed to their asymptotic values; in this case, we can use a single parameter in coordinate space because nucleons with widely differing momenta will have separated anyway; in other words, at that time, nucleons close together in coordinate space are in fact together in momentum space. We verified this picture quantitatively, by spanning the momenta of the nucleons temporarily assigned to a cluster and by rejecting those for which it was kinematically impossible to belong to a common Fermi sphere. This last criterion brings modifications of the straight coordinate space picture at the level of 0.1%. We recognize that the above simple criterion for clustering is approximate; nevertheless, it does provide a basis for adequate phenomenology. We observe a weak energy dependence in the
coalescence parameter probably because we treat only the coordinate-space part. Improvements are being made to the above approach. It is not clear that other transport theories such as QMD provide a sound theoretical foundation for the generation of composites. The calculated triple–differential cross sections are shown in Fig. 6 for neutrons in the vicinity of the projectile-like rapidities for beam energies of 150, 250, 400, and 650 AMeV. From Fig. 6, we see that the BUU calculations agree generally with the data at 400 and 650 AMeV except for the most forward polar angles \((i.e., 3^\circ, 5^\circ, \text{and } 9^\circ \text{ for } 400 \text{ AMeV and } 3^\circ \text{ and } 5^\circ \text{ for } 650 \text{ AMeV})\); however, at the two lower beam energies of 150 and 250 AMeV, the BUU calculations overestimate the triple–differential cross sections at polar angles above 15° and underestimate below 15°. The comparison at the most forward angles suffers from the fact that the experimental data at angles below 15° include evaporation neutrons. The failure to obtain agreement at the two lower beam energies for polar angles above 15° may indicate that the prescription for subtracting composites in the BUU calculation needs to be placed on a more solid basis.

We calculated the \(\langle P_x \rangle\), the flow \(F\), and the \(\langle P_x/P_\perp \rangle\) of neutrons with the BUU theory, and the results are presented along with the data in Figs. 7, 8, and 9, respectively. From Fig. 7, we see that the BUU calculations agree with the results of \(\langle P_x \rangle\) at middle rapidities for the two lower energies and that the data would agree with the BUU calculations at 400 AMeV if the spectrum of the data were shifted slightly to the center. The BUU calculations (solid squares) of the flow \(F\) agree within uncertainties with the data (open squares) in Fig. 8; and in Fig. 9, the BUU calculations agree with the data for the average \(\langle P_x/P_\perp \rangle\). The sensitivity of the flow to the parameters of the equation-of-state has been explored in detail by Zhang et al. [40].

VIII. SUMMARY AND CONCLUSIONS

We measured neutron triple-differential cross sections from multiplicity-selected Au-Au collisions at 150, 250, 400, and 650 AMeV, and extracted the neutron collective flow observ-
ables of the average $\langle P_x \rangle$, the flow $F$, and the average $\langle P_x/P_\perp \rangle$. The BUU calculations of the triple–differential cross sections for free neutrons agree generally with the data at 400 and 650 AMeV except for the most forward angles where the data include evaporation neutrons. At the two lower beam energies, the BUU calculations overestimate the cross sections at polar angles above $15^\circ$. This discrepancy indicates that the prescription for calculating free neutrons is in fact approximate and may need improvement for the two lower energies. It is in fact well known that the composite “contamination” in the BUU grows as the bombarding energy is lowered. The measured neutron flow observables agree with the calculations from the BUU theory. Also, collective flow results for neutrons are consistent with those for protons plus bound nucleons from the Plastic Ball Group.

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FIGURES

FIG. 1. The azimuthal distribution of neutrons plus background, background, and neutrons emitted with midrapidities at a polar angle of $18^\circ$ from Au–Au collisions at 650 AMeV (a) with a correction and (b) without a correction for the overestimation of the background.

FIG. 2. Charged particles multiplicities with (solid line) and without (dashed line) a target in place for 400 AMeV Au beam.

FIG. 3. Triple-differential cross sections of neutrons with the target-like rapidities $(-1.0 \leq \alpha < -0.2)$ emitted at nine selected polar angles from Au–Au collisions at 150, 250, 400 and 650 AMeV. Closed circles and open triangles represent the measurements with and without a correction for the overestimation of background, respectively, and lines represent the calculations from the BUU theory for the beam energy of 400 AMeV with an incompressibility modulus $K = 215$ MeV.

FIG. 4. Triple-differential cross sections of neutrons with the middle rapidities $(-0.2 \leq \alpha < +0.2)$ emitted at nine selected polar angles from Au–Au collisions at 150, 250, 400, and 650 AMeV. Closed circles and open triangles represent the measurements with and without a correction for the overestimation of background, respectively, and lines represent the calculations from the BUU theory for the beam energy of 400 AMeV with an incompressibility modulus $K = 215$ MeV.

FIG. 5. Triple-differential cross sections of neutrons with the forward rapidities $(+0.2 \leq \alpha < +0.7)$ emitted at nine selected polar angles from Au–Au collisions at 150, 250, 400, and 650 AMeV. Closed circles and open triangles represent the measurements with and without a correction for the overestimation of background, respectively, and lines represent the calculations from the BUU theory for the beam energy of 400 AMeV with an incompressibility modulus $K = 215$ MeV.
FIG. 6. Triple-differential cross sections of neutrons the projectile-like rapidities \((+0.7 \leq \alpha \leq +1.2)\) at nine selected polar angles from Au–Au collisions at 150, 250, 400, and 650 AMeV. Closed circles and open triangles represent the measurements with and without a correction for the overestimation of background, respectively, and lines represent the calculations from the BUU theory with an incompressibility modulus \(K = 215\) MeV.

FIG. 7. Average in–plane transverse momentum of neutrons as a function of the rapidity for 150, 250, and 400 AMeV Au-Au collisions. Symbols represent the data and lines represent the calculations from the BUU theory with an incompressibility modulus \(K = 215\) MeV

FIG. 8. Flow of neutrons (open squares) and protons plus bound nucleons (circles) from Au–Au collisions as a function of the beam energy. The solid squares represent the calculations from the BUU theory.

FIG. 9. Normalized average \(\langle P_x/P_\perp \rangle\) of neutrons as a function of the rapidity for 150, 250, and 400 AMeV Au-Au collisions. Symbols represent the data and lines represent the calculations from the BUU theory.

FIG. 10. Polar-angle-integrated maximum azimuthal anisotropy \(R\) of neutrons as a function of the rapidity for 150, 250, and 400 AMeV Au-Au collisions.

FIG. 11. Double-differential cross sections for neutrons emitted from Au–Au collisions at 150, 250, 400, and 650AMeV. Symbols represent the data and lines represent the calculations from the BUU theory: dashed lines represent all neutrons and solid lines represent neutrons that are not in clusters.
TABLE I. Flight path, polar angle, and width of each neutron detector

| Detector number | Polar angle (deg) | Flight path (m) | Width (cm) |
|-----------------|------------------|-----------------|------------|
| 1               | 3                | 8.32            | 2.5        |
| 2               | 5                | 8.31            | 2.5        |
| 3               | 7                | 8.38            | 12.5       |
| 4               | 9                | 8.38            | 12.5       |
| 5               | 12               | 8.36            | 25.4       |
| 6               | 15               | 8.36            | 25.4       |
| 7               | 18               | 8.37            | 25.4       |
| 8               | 21               | 8.36            | 25.4       |
| 9               | 24               | 8.38            | 25.4       |
| 10              | 27               | 8.38            | 25.4       |
| 11              | 30               | 8.35            | 25.4       |
| 12              | 36               | 8.34            | 50.8       |
| 13              | 45               | 8.33            | 50.8       |
| 14              | 54               | 7.92            | 50.8       |
| 15              | 63               | 7.42            | 50.8       |
| 16              | 72               | 6.91            | 50.8       |
| 17              | 81               | 6.41            | 50.8       |
| 18              | 90               | 5.91            | 50.8       |
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