Planck Scale Mixing and Neutrino Puzzles

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It is hypothesized that neutrinos are massless and mixed among 3 (ν_µ, ν_e, and ν_τ) flavors with a mixing length of order of \( \alpha^{-1}l_{\text{Planck}}(E_\nu/E_{\text{Planck}})^{-2} \), where \( \alpha \) is a universal coupling constant. It is suggested that this hypothesis mostly reconciles standard solar models with the observations of solar ^8B neutrinos, solar pp neutrinos, and the ratio of atmospheric muon neutrinos to electron neutrinos.

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I. INTRODUCTION

In response to discrepancies that have persistently been reported between predicted and measured neutrino fluxes, some authors have suggested a violation of Lorentz invariance, either via different couplings of neutrinos to gravity [1], or by different indices of refraction of the vacuum to different types of neutrinos [2].

In this Letter we consider another such hypothesis, in which Lorentz invariance is violated by a foam-like vacuum that is granulated near the Planck scale. We consider that neutrinos are mixed over a scale of \( r_d(E_\nu) = \alpha^{-1}l_{\text{Planck}}(E_\nu/E_{\text{Planck}})^{-2} \), where \( E_\nu \) is the neutrino energy in a cosmic rest frame and is conserved by the mixing process, \( l_{\text{Planck}}(E_{\text{Planck}}) \) is the Planck length (energy) and \( \alpha \) is a dimensionless number of order of \( 10^{-2} \). We note that the hypothesis mitigates the discrepancy between SSM predictions and the GALLEX/SAGE, Homestake, and SuperKamiokande (SK) data for solar neutrinos. It also mitigates the reported anomaly in atmospheric neutrinos as well. It is also marginally consistent with limits set by post-beam dump oscillation experiments, which reveal no significant oscillation at neutrino energies of 30 GeV over distances of order of 1/2 km. Physical justification for the hypothesis is not quite known to us at present, and the results are presented here in the spirit of suggestive numerical coincidence: that the energy scale in the expression for \( r_d \) happens to be near the Planck scale. A brief discussion of possible physics that could provide some theoretical justification for this hypothesis is given in section IV.

We will, for convenience, assume that the mixing redistributes neutrinos uniformly and irreversibly among three flavors. Reversible mixing, that is, oscillations, could also be considered at a phenomenological level with an oscillation length that scales as \( E^{-2} \), and would give similar results (except for monochromatic neutrinos), if the coupling among all three flavors were the same. The major feature of the present hypothesis (although we shall refer to it for convenience as an irreversible mixing [IM] hypothesis) is the energy dependence of the mixing length \( r_d \propto E^{-2} \), and not so much the irreversibility.

Consider a case of \( n \) neutrino flavors, all interacting with the foam with the same mean free-path length \( r_d \). In this case the evolution of each flavor number \( y_i \), (Σ\( y_i = N = \text{const} \)), as a function of distance \( r \) from the source is governed by the expression:

\[
y_i = \left( y_i(0) - \frac{N}{n} \right) \exp\left( -\frac{r}{r_d} \right) + \frac{N}{n}, \quad i = 1, n.
\]

Note that, if only a single flavor is emitted (as in the solar neutrino case), then the spectrum of the emitted flavor first softens with time (that is, with distance from source), then returns to its original form at a lower amplitude.

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II. SOLAR NEUTRINOS

The solar neutrino problem may be described as a deficit of the observed neutrino fluxes compared to predicted ones:

a) The SuperKamiokanda experiment\[^3\] presently sets the upper limit to the Boron neutrino flux at $2.44 \times 10^{-6} \text{ cm}^{-2} \text{ s}^{-1}$, more than two times less than the SSM BP98\[^4\] “theoretical” value $5.15 \times 10^{-6} \text{ cm}^{-2} \text{ s}^{-1}$;

b) GALLEX and SAGE experiments\[^5\] measure the reaction rate in $^{71}\text{Ga}$ detector as $72 \pm 5.5 \text{ SNU}$ (figure reported at Neutrino’98) which is 1.8 times less than SSM BP98\[^4\] value 129 SNU, the Gallium rate being caused mainly by low-energy $pp$ neutrinos;

c) The Homestake experiment\[^6\] with $^{37}\text{Cl}$ reports a reaction rate as $2.56 \pm 0.23 \text{ SNU}$ which is three times less than SSM BP98 value 7.56 SNU, here the detected $\nu$’s being mostly energetic Boron neutrinos.

We also note that the $pp$ neutrino energy spectrum and total flux are claimed\[^4\] to be known much better than those of Boron neutrinos.

A. Chlorine case

Table I presents the results of applying the IM hypothesis to the Solar Chlorine Case. The number of neutrino flavors was taken be equal to 3 with no sterile flavor. Individual yields in the total rate were calculated using files from Web site \[^7\]. Spectral distortion effects are small in the Chlorine case as $pp$ neutrinos give no yield while Boron neutrinos (due to their large energies and strong dependence of IM effect on the neutrino energy) give almost exactly the limiting value $1/n = 1/3$. That is, all Boron electronic neutrinos have been mixed into three active flavors equally.

B. Gallium case

Table II presents the results of applying the IM hypothesis to the Solar Gallium Case. The number of active mixing flavors of neutrinos was taken to be equal to 3 with no sterile flavor. Spectral distortion effects are included in the calculation of rates with the use of files in \[^7\]. Again, as in the Chlorine case, spectral distortion effects are small as $pp$ neutrinos have low energy (and hence large free-path length), while Boron neutrinos (due to their large energies and strong dependence of IM effect on the neutrino energy) give almost exactly the limiting value $1/n = 1/3$.

C. Seasonal variations

IM predicts variation in the solar neutrino fluxes caused by the variation of the Earth-Sun distance. Seasonal variation of the Earth-Sun distance, $\epsilon = \delta r/r = 2e$, ($e = 0.017$ is the mean eccentricity of Earth orbit), leads to relative variation of the total $\nu_{el}$-flux, $\delta F/F = -2\epsilon$ without IM; the IM leads to the following relative variation of the initial flavor $\nu$’s flux:

$$\frac{\delta F}{F} = -\epsilon \left[ 2 + \frac{(n-1)}{n-1+\exp(r/r_d)} \right].$$

(2)

Numerically, taking $n = 3$ and $r = r_d = 1\text{ A.U.}$ for $E_\nu = 1\text{ MeV}$, we get $\delta F/F = 8.2\%$. This value may be approximately applied to $^7\text{Be}$ and $^8\text{pep}$ solar neutrinos with mean energies of about .8 and 1.44 MeV, respectively. For $^8\text{B}$ neutrinos, the mean energy is 6.7 MeV, and, as free-path $r_d$ is proportional to $E_\nu^{-2}$, the variation of the flux is reduced to 6.8\%.

III. ATMOSPHERIC NEUTRINOS

In this field there are two observed "deficits":

a) The flux of upward-going neutrinos (mainly muonic) is about .6 of flux of downward-going neutrinos.

b) The muon-to-electron neutrino relation is observed to be about .6 of the predicted value which is slightly larger than 2.

We discuss both cases.
A. Up-down asymmetry

Up-down asymmetry in muonic and electronic events was observed in a number of experiments and was suggested in [8] as a simple test for discriminating among neutrino oscillation/mixing mechanisms and their modifications.

![Graph of up-down asymmetry for atmospheric muon and electron neutrinos, at the SK site, due to Irreversible Mixing for value of $r_d = 0.1 \, R_{Earth}$ at $E_\nu = 1 \, GeV$.](image1)

In Fig. 1 we present the $E_\nu^{-2}$ mixing-induced up-down asymmetry parameter as a function of $E_\nu$ for both electronic and muonic neutrinos, with $r_d = 0.1 \, R_{Earth}$ at $E_\nu = 1 \, GeV$. We note that the energy dependence of the muon asymmetry and electron asymmetry differs significantly from the cases considered in [8]. We use atmospheric neutrino fluxes from [9] and calculate only fluxes at the SK site but not reaction rates for the SK detector as our aim is to show only main characteristics of up-down asymmetry in the IM model. Note that atmospheric neutrino fluxes are calculated with rather large uncertainties that depend on the solar activity cycle phase, geomagnetic effects etc. These uncertainties may be especially important at low neutrino energies. Inclusion of these and other factors would not drastically change the characteristics of muonic and electronic up-down asymmetries.

B. $\nu_\mu/\nu_{el}$ relation

SK experiment[10] shows a deficit of muonic events compared to predicted values which was claimed as evidence of neutrino oscillations. IM with values of free-path length about $r_d = 0.1 \, R_{Earth}$ or less at $E_\nu = 1 GeV$, as it is shown in Fig. 2, may reduce initial muonic-to-electronic-neutrino ratio down to observed values.

![Graph of muon-to-electron neutrino flux ratio for atmospheric neutrinos due to IM, at SK site, for two values of $r_d$ at $E_\nu = 1 \, GeV$, .1 (+) and .02 (*).](image2)

In Fig. 2 we present the muon-to-electron neutrino flux ratio for atmospheric neutrinos due to IM, at SK site, for two values of $r_d$ at $E_\nu = 1 \, GeV$, .1 (+) and .02 (*). Ordinates are ratio-of-ratios, that is, $\nu_\mu/\nu_{el}$ ratio with account of IM divided by $\nu_\mu/\nu_{el}$ ratio without IM. Also shown are SK[10] data on muonic-to-electronic events ratio.

IV. IS THERE ANY PHYSICAL BASIS?

As Lorentz invariance seems to be a symmetry in our low energy world, it is reasonable to suppose that if it is broken, such breaking takes place at high energy, e.g. by a foam-like condensate of quantum black holes, or a spaghetti-like condensate of flux tubes at the GUT scale. Low energy neutrino scattering off small scale inhomogeneities so formed would most likely have an scattering cross section $\propto E^2$, as for any low energy neutrino scattering[11]. Also, it could
be argued that because Lorentz invariant equations work as well as they do in describing the observed world, with the modest price of containing infinities, then any violation of Lorentz invariance probably enters as an anomaly via the choice of regularization procedure. The difficulty is that a foam-like condensate that breaks Lorentz invariance could also break translational invariance, and this would imply a scattering that is apparently ruled out by observations of the neutrinos from Supernova 1987A. Also, at least for charged particles, it is ruled out by the survival times of particle beams in storage rings. We have no solution to this difficulty at present; any such solution, it appears to us, would probably require a geometric interpretation of the different neutrino flavors as well as a model for why momentum and angular momentum are exactly conserved quantities whereas neutrino flavor is not. Alternatively, the foam may be entirely virtual, that is, unable to increase the entropy of real particles, in which case the mixing would be reversible. But then it is harder to justify the $E_{\nu}^{-2}$ dependence of mixing length.

V. DISCUSSION AND CONCLUSION

Neutrino mixing considered in this Letter has only one free parameter (mean free-path length $r_d$) given the number of flavors. In spite of this, $E_{\nu}^2$ mixing leads to a marginally acceptable account of various deficits observed for the solar and atmospheric neutrinos. Taking $r_d(E)\propto E^{-2}$ and $r_d = \eta \times 2$ A.U. at energy $E = 1$ MeV characteristic of the solar neutrino problem, then at energy $E = 1$ GeV, characteristic of the atmospheric neutrino problem, we have $r_d = (\eta/20) R_{\text{Earth}}$. At $E = 30$ GeV, characteristic of the post-beam neutrino experiments, we have $r_d = (\eta/3) km$, so the Fermilab mixing experiments[12] probably imply $\eta$ larger about 2. The calculations presented in this Letter show that for $\eta$ about 2, the predicted count rate for the Homestake detector is reduced from 7.5 to 3.5 SNU, the predicted rate for GALLEX is reduced from 129 to 113 SNU, and the atmospheric $\nu_{\mu}/\nu_{\tau}$ ratio is reduced by a factor of .6 at energies above .6 GeV. Although the reductions of the predicted count rate for the radiochemical detectors, Homestake, GALLEX and SAGE are not quite enough to completely reconcile the reported observations with the standard solar model, the overall trend in these reductions for both atmospheric and solar neutrino experiments is perhaps suggestive enough to make the hypothesis worthy of consideration at present.

A clean prediction, however, is that longer terrestrial lengths for post beam dump neutrino experiments must reveal mixing. A null result over the CERN-MACRO distance or the Fermilab-Wisconsin distance would unequivocally rule out the hypothesis.

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[1] A. Halprin and C. N. Leung, Phys. Rev. Lett. 67 (1991) 1833.
[2] S. Coleman and S. L. Glashow, Phys. Lett. B 405 (1997) 249.
[3] Y. Fukuda et al., Phys. Rev. Lett. 77 (1996) 1683.
[4] J. N. Bahcall, astro-ph/9808162; J. N. Bahcall, S. Basu and M. H. Pinsonneault, Phys. Lett. B 433 (1998) 1.
We refer to the SSM BP98 as a representative of a number of more or less "concordant" SSM’s (see references in BP98). However we should mention that there is also the "discordant" SSM DS98: A. Dar, G. Shaviv, astro-ph/9808008; we do not attempt any judgment of their relative merits.
[5] GALLEX Collaboration, P. Anselman et al., Phys. Lett. B 342 (1995) 400; SAGE Collaboration, J. N. Abdurashitov et al., Phys. Rev. Lett. 77 (1996) 4708.
[6] B. T. Cleveland et al., Astrophys. J. 496 (1998) 505.
[7] J. N. Bahcall, www.sns.ias.edu/jnb.
[8] J. W. Flanagan, J. G. Learned and S. Paksava, Phys. Rev. D 57 (1998) R2649.
[9] M. Honda et al., Phys. Rev. D 52 (1995) 4985.
[10] Y. Fukuda et al., Phys. Rev. Lett. 81 (1998) 1562; Y. Fukuda et al., Phys. Lett. B 433 (1998) 9.
[11] D. Eichler, astro-ph/9811023.
[12] N. Ushida et al., Phys. Rev. Lett. 57 (1986) 2897.
TABLE I. Irreversible Mixing, with SSM BP98 predictions for Chlorine reaction rates, at n=3 flavors for some values of the free-path length \( r_d \) at \( E_\nu = 1 \text{ MeV} \). Case \( r_d = \infty \) corresponds to SSM BP98 predictions.

| Source | Rates (SNU) | \( r_d \) (A.U.) |
|--------|-------------|-----------------|
|        | \( \infty \) | 5   | 4   | 3   |
| 1      | \( pp \)    | 0   | 0   | 0   |
| 2      | \( pep \)   | .22 | .17 | .16 | .15 |
| 3      | \( ^7Be \)  | 1.15| 1.05| 1.05| 1.00|
| 4      | \( ^8B \)   | 5.72| 1.91| 1.91| 1.91|
| 5      | \( ^{13}N \)| .10 | .09 | .09 | .08 |
| 6      | \( ^{15}O \)| .36 | .32 | .31 | .29 |
| Total  |             | 7.56| 3.54| 3.50| 3.43|

TABLE II. Irreversible Mixing, with SSM BP98 predictions, for Gallium reaction rates at n=3 flavors for some values of the free-path length \( r_d \) at \( E_\nu = 1 \text{ MeV} \). Case \( r_d = \infty \) corresponds to SSM BP98 predictions.

| Source | Rates (SNU) | \( r_d \) (A.U.) |
|--------|-------------|-----------------|
|        | \( \infty \) | 4   | 3   | 2   |
| 1      | \( pp \)    | 69.6| 67.6| 68.0| 67.1|
| 2      | \( pep \)   | 2.8 | 2.1 | 1.9 | 1.6 |
| 3      | \( ^7Be \)  | 34.4| 30.9| 29.9| 27.9|
| 4      | \( ^8B \)   | 12.4| 4.1 | 4.1 | 4.1 |
| 5      | \( ^{13}N \)| 3.7 | 3.4 | 3.3 | 3.1 |
| 6      | \( ^{15}O \)| 6.0 | 5.2 | 4.9 | 4.5 |
| Total  |             | 129.0| 113.3| 112.1| 108.3|