Soft-QCD Effects In $B \Rightarrow \gamma\gamma$ Decay: Quark Level Form Factors

A.N. Mitra *
244 Tagore Park, Delhi-110009, India.

5 October 1999

Abstract

Soft-QCD effects of quark-level form factors in $\gamma\gamma$ decay of $B_s$-mesons via $D\bar{D}$ intermediate states, suggested by Ellis et al, are examined in the "Salpeter" model [reinterpreted in terms of the Markov- Yukawa Transversality Principle (MYTP)] when formulated on a covariant light front (null-plane). The gluonic kernel in the infrared regime which generates the constituent mass via standard $DB\chi S$, is calibrated to both $q\bar{q}$ and $qqq$ spectra, meson mass splittings, pion form factor, and other parameters. With this check, an exact evaluation of the $D\bar{D}\gamma$ vertex form factor $F(\alpha)$ (normalized to $F(0) = 1$), gives a multiplicative effect of $F^2(\alpha)$ on the $B_s \Rightarrow \gamma\gamma$ amplitude via $D$-meson triangle loop, where $\alpha = (p^2 + M^2)/M^2$ is the off-shellness parameter of the exchanged $D$-meson of mass $M$ and 4-momentum $p_\mu$, and is a variables of the (internal) loop integration. Unfortunately, the $F^2(\alpha)$ effect decreases the loop amplitude by a factor of 30 w.r.t. the point hadron value, resulting in a reduction of 3 orders of magnitude in long distance (hadronic) contributions to the $B_s \Rightarrow \gamma\gamma$ decay rate, thus greatly impairing the visibility of such modes.
PACS: 11.10.St ; 12.38.Lg ; 13.20.Gd ; 13.40.Hq
Keywords: $B_s$ radiative decay; Soft-QCD; Vertex-Fn; Form-factor; Markov-Yukawa-Transversality Principle; Salpeter eq; Cov. light front

*e.mail: (1) ganmitra@nde.vsnl.net.in ; (2) anmitra@csec.ernet.in
1 Introduction: Hard VS Soft-QCD In B-Decays

Rare B-decays carry signatures of important features of SM (Standard Model), such as FCNC (flavour changing neutral current), CKM (Cabibbo-Kobayashi- Maskawa) in the electro-weak sector; and beyond SM, such as 2-Higgs doublet model, minimum SUSY extensions of SM, etc. Of these decays, particular attention has been paid to $B_s \rightarrow \gamma \gamma$ decay [1-5], which (despite its small branching ratio) has a very clear signal where two monochromatic photons of high energy are produced back-to-back in the rest frame of $B_s$. Its present experimental limit on the branching ratio is $[4] \mathcal{B} < 1.48 \times 10^{-4}$, but its sensitivity to hadronic accelerators (HERA,Tevatron and LHC), as well as to the planned $e^+e^-$ B-factories at KEK and SLAC, is likely to improve this limit significantly in the near future. This necessitates accurate theoretical estimates from different angles. Now within SM, the lowest order short distance contributions (box diagrams, etc) give only $\mathcal{B} \approx 3.8 \times 10^{-7}$ [5], which is not much enhanced by Hard-QCD corrections [2]. This has led to mechanisms beyond SM [3a,3b]. On the other hand, even within SM, other options like a long – distance (hadronic) mechanism has been proposed for this process via $D^* \bar{D}^*$ intermediate states [5], giving big enhancements vis-a-vis short distance mechanisms with and without Hard-QCD corrections [1-2]. Such enhancements call for a closer scrutiny of this mechanism [5], in terms of ”Soft-QCD” corrections (to distinguish it from ”Hard-QCD” corrections [2]), and this is the purpose of this paper.

Now Hard-QCD (leading log) corrections to short-distance effects are compactly achieved by isolating a subsystem/process like $b \rightarrow s \gamma \gamma$ [2]. More elegant methods for Hard-QCD corrections have been developed for heavy meson form factors [6a] under heavy-quark symmetry [6b]. On the other hand, long distance contributions [5], where ”Soft-QCD” effects play the dominant role, are expressed by the $D$-meson loop diagrams, fig.1a, where the $D\bar{D}\gamma$ vertex itself is not a point vertex, but an extended structure consisting of a quark triangle loop with two hadron-quark, plus one quark-photon, vertices as shown in fig.1b. Thus the effect of Soft-QCD corrections boils down to the evaluation of this triangle loop, giving rise to a form factor $F(\alpha)$, where $\alpha$ is the off-shellness parameter $1 + p^2/M^2$ for the exchanged $D$-meson of momentum $p_\mu$ and mass $M$ in fig.1a. Inserting $F(\alpha)$ at either vertex of fig.1a gives a weighting factor of $F^2$ in the $B_s \rightarrow \gamma \gamma$ amplitude via the $D$-meson triangle loop, where $\alpha$ may be conveniently taken as one of the loop variables with its permissible range determined by the kinematics of the decay amplitude. In this respect, the effective $B_s D\bar{D}$ vertex [5] which constitutes the ‘source’ of the $D\bar{D}$ annihilation, is a common background which may be taken over directly from [5]. Further, it should be adequate to consider for simplicity a $D$-meson instead of $D^*$-meson triangle loop for the 2-photon annihilation, since the loop-integral structure is similar for both cases.

1.1 Theoretical Basis for Soft-QCD Corrections

Now, taking account of the state of the Soft-QCD art today, there are not many candidates for a satisfactory calculation of the quantity $F(\alpha)$ in a closed form, with the desired off-shell properties. Among the leading candidates which have received a good deal of attention in the literature are heavy particle symmetry [6], QCD-SR [7] and chiral perturbation theory [8]. However the machinery offered by these do not seem to provide a closed non-perturbative form for the form factor $F(\alpha)$, when looked upon as a triangle loop integral (fig 1b) in terms of hadron-quark vertices. While heavy particle symmetry [6] is
ideally suited to Hard-QCD effects, the actual formulations of the other two [7-8], [QCD-SR with its heavy reliance on the successive ‘twist’ terms in an Wilson expansion; and chiral perturbation theory with its emphasis on expansions of the effective Lagrangians in the momenta], seem to lack the ‘closed form approach’ that is so vital for ensuring a fully analytic form for $F(\alpha)$. [An analytic structure in turn is essential for continuation from one physical region to another].

With the hadron-quark vertex function as the ideal laboratory for testing Soft-QCD ideas, a more promising candidate for a closed form approach is a BSE-SDE framework [9], wherein the starting point is a 4-fermion Lagrangian (current quarks) interacting via an effective gluon propagator in the non-perturbative regime [10]. This framework produces the mass function through the non-trivial solution of the SDE [10] as a concrete realization of dynamical breaking of chiral symmetry ($DB\chi S$), in the sense of a generalized NJL mechanism [11] with space-time extended interaction. The mass function $m(p)$ in turn produces the bulk of the constituent mass in the low momentum limit, via Politzer additivity [12]. We propose to use this framework with the extra ingredient of a 3D support to this 4-fermion interaction, offered by the Markov-Yukawa Transversality Principle (MYTP) [13], whose relative lack of familiarity in the contemporary literature warrants a short background which links it with the Salpeter equation [14].

1.2 MYTP As Covariant Basis For Salpeter Equation

The Salpeter model [14] which is based on the so-called adiabatic (instantaneous) approximation, results from a 3D reduction of the full BSE. Due to its non-covariant nature, its applications (although extensive) had been generally limited to topics like heavy quarkonia, with limited relativistic overtones. However its precise meaning in terms of a physical principle got revealed by the work of the Pervushin Group (Dubna) [15] who showed that the Markov-Yukawa Transversality Principle (MYTP) [13] which decree the pairwise interaction be a function of the relative 4-momentum $\hat{q}$ transverse to the 4-momentum $P$ of the composite, leads exactly to a 3D (Salpeter-like) form. Strictly speaking, such interactions which amount to a 3D support to the BSE kernel, are non-local, but reduce to the Salpeter form [14] in the rest frame of the composite, thus giving this equation [14] a fresh meaning within the MYTP context.

Now MYTP also possesses an interesting dual property, viz., the facility of an exact reconstruction of the 4D BSE through a reversal of steps from the 3D BSE form, as found subsequently by the Delhi Group [16a]. This twin property of MYTP, viz., an ”exact interconnection” between the 3D and 4D forms of the BSE, gives the Salpeter equation a firm theoretical basis, for not only does MYTP reduce to it in the adiabatic limit, but has a wider (Lorentz-covariant) domain of applicability. (Indeed, this twin property of the Salpeter equation [14] had all along been present, but obscured from public view due to its non-covariant form !). Further, the standard SDE-BSE framework [10] characterized by the non-perturbative gluon propagator is easily adapted [16b] to MYTP, so as to produce an MYTP-constrained mass functions $m(\hat{p})$, as a calculational tool for vacuum properties (like condensates) [16b], in a closed form.

The interlinked 3D-4D BSE structure which stems from MYTP, gives rise to a 2-tier mode of applications, the 3D form for the spectra [17], while the 4D form yields the hadron-quark vertex as the basic building block for calculating the 4D quark loop integrals [18], on the lines of the ”dynamical perturbation theory” of Pagels-Stokar [19]
(no criss-cross gluon lines inside the quark loops). These twin applications serve as useful calibrations for the parameters of the model. More importantly, its "exact" structure is ideally suited for imparting a crucial analyticity property to the function $F(\alpha)$, which carries the off-shellness information down to the quark level of compositeness.

Still another property of MYTP which stems from the characteristic $D \times \phi$ structure of the 4D vertex function [16a], ($D$ and $\phi$ being the 3D denominator fn and 3D wave fn respectively, which together satisfy the 3D BSE $D\phi = fK\phi$), concerns 'quark confinement'. Thus in a 4D loop integral, such as fig.(1b), the $D$-fn causes a cancellation [16a] of the Landau-Cutkowsky singularities arising from integration over the loop variable, and thus prevents an otherwise free propagation of quarks inside the loops, a standard disease that usually plagues a quark loop calculation in terms of point vertex structures.

### 1.3 MYTP w.r.t. The Covariant Null Plane

MYTP still suffers from a serious problem, viz., a Lorentz mismatch of the participating vertex functions in loop integrals with 3 or more quark lines, (e.g., fig.(1b)), leading to ill-defined integrals for the pion form factor and related applications [20], due to the appearance of time-like momenta in the 3D (gaussian) wave fns $\phi$. This shows up through a "complexity" in such amplitudes [20], where there is no physical reason for such behaviour. (On the other hand, 2-quark loops such as off-shell $\rho-\omega$ mixing [18a] and strong SU(2)-breaking for hadron mass splittings [18b] just escape this pathology). This problem has recently found a simple solution through an obvious extension [21] of MYTP, so that the "transversality" w.r.t. the composite 4-momentum $P$ is defined covariantly on a null-plane whose orientation is specified by 2 dual 4-vectors $n_\mu, \tilde{n}_\mu$, with $n^2 = \tilde{n}^2 = 0$; and $n.\tilde{n} = 1$. The necessary details, making use of standard null-plane technology [22] adapted to the covariant light-front, are given in [21]; it is shown that the reduced 3D BSE has a formally identical structure to that of the standard MYTP [13-16], so that the spectral predictions [17] remain unchanged. With this check as a "control" at the 3D level of spectroscopy, the application to 4D triangle loop integrals with the new vertex structures (albeit light-front orientation dependent), shows that they no longer suffer from the Lorentz mismatch disease, so that after the 'pole' integrations over the time-like momenta, the loop integrals reduce to well-defined 3D forms, where the 3rd component depends covariantly on the null-plane orientation. In ref.[21], this new MYTP method is compared with covariant light-front (Null-plane) methods of Kadychevsky-Karmanov, and the Wilson Group’s [23a-c], (collectively reviewed in [24]). And as a basic application to the pion e.m. form factor, it is shown how a simple device of "Lorentz-completion" yields a Lorentz-invariant structure with the correct asymptotic $1/k^2$ behaviour, when considered in a BSE-SDE framework [10] with a non-perturbative gluon propagator [16b]. The new MYTP method is ideally suited to the triangle loop integral for fig.(1b) which corresponds to the e.m. form factor of a $D$-meson, except that its domain of off-shellness differs from that of the pion form factor [21].

In Sec.2, we collect some essential elements of the BSE model with 3D kernel support [15-16] in accordance with MYTP [13], as extended to the Covariant null-plane [21], on the lines indicated above. We write down the full structure of the $B_s \to \gamma\gamma$ amplitude $G_{\mu\nu}$ [5] in terms of the form factors $F(\alpha)$ of the two composite $D\bar{D}\gamma$ vertices involved in the $D$-meson triangle loop [5]. In Sect 3, we sketch the derivation of $F(\alpha)$ in an exact analytic fashion, and use this result in Sect 4 to evaluate $G_{\mu\nu}$ analytically over the
all kinematically allowed range of $\alpha$, resulting in a 30 times reduced amplitude over
the point hadron value [5], hence about a 1000-fold reduction in the corresponding decay rate. Sect 5 summarises our findings, and notes the significance of this result vis-a-vis the observability of such radiative decay modes [5].

2 MYTP-Based BSE Model: 3D-4D Interlinkage

In this Section we collect some essential ingredients of the MYTP [13]- governed Bethe-Salpeter framework in preparation for the calculation of the e.m. form factor of a hadron with unequal quark masses in the notation and phase convention of ref.[16], but adapted to the covariant null-plane (CNPA for short) [21]. To emphasize the close similarity of the old Covariant Instantaneity Ansatz (CIA) [16a], and the new CNPA [21] forms, especially the exact 3D-4D interconnection among the respective wave functions, we start with the principal ingredient, namely the hadron- quark vertex function which has the structure [16, 21]:

$$\gamma_D \Gamma(\hat{q}) = N_n(P)D_n(\hat{q})\phi(\hat{q})\gamma_D/(2\pi)^{5/2}$$

(2.1)

Here $\hat{q}_\mu$ can be given a common meaning to cover both the CIA [16] and CNPA [21] situations: In CIA [16], it is the relative momentum transverse to the hadronic 4-momentum $P_\mu$, i.e., $\hat{q} = q - q.P/P^2$; while in CNPA [21] it is transverse to the null-plane $n_\mu$, i.e., $\hat{q} = q - q.n + n.qP/2n.P$ [21], where $n_\mu$ and $\tilde{n}_\mu$ are the null-plane 4-direction and its dual respectively, normalized to $n^2 = \tilde{n}^2 = 0$ and $n.\tilde{n} = 1$ [21]. In the standard ± notation [22], $\hat{q}$ translates exactly to to a 3-vector, viz., $q_\perp, q_3$ where $q_3 = Mq_+/P_+$, with $P^2 = -M^2$ on the hadron mass shell. For off-shell hadron propagation (as is pertinent for the problem on hand) on the other hand, $M^2$ should be replaced by $M^2(1 - \alpha)$, where $\alpha = (P^2 + M^2)/M^2$ measures off-shellness. (The general $n_\mu$ dependence [21] keeps track of formal covariance, but for calculational purposes it is simpler (and faster) to use the old-fashioned notation [22]).

Note that the $q_-$ component does not appear in $\hat{q}$. $\gamma_D$ is a Dirac matrix which equals $\gamma_5$ for pseudoscalar, $i\gamma_\mu$ for vector, etc hadrons [22c]; $D_n$ and $\phi$ are the 3D denominator and 3D wave function respectively [16]; $N_n(P)$ is the BS normalizer. The explicit structures [16,21,22c] of the various symbols, generalized to off-shell hadron propagation, in the covariant NPA [21] are:

$$q = \tilde{m}_2p_1 - \tilde{m}_1p_2; \quad 2\tilde{m}_{1,2} = 1 \pm (m_1^2 - m_2^2)/M^2;$$

(2.2)

$$D_n(\hat{q}) = 2P.n[q^2_\perp + M^2(1 - \alpha)q^2_\perp/P^2_+ - \lambda(M^2(1 - \alpha), m_1^2, m_2^2)/M^2(1 - \alpha)]$$

(2.3)

where $\lambda$ is the standard triangle function of its arguments. The corresponding denominator function $D_n(\hat{q}')$ is the same as (2.3) above, except for the replacements $P, q \rightarrow P', q'$ and $\alpha = 0$. $\phi = \exp(-\hat{q}^2/2\beta^2)$ is the 3D wave function whose inverse range parameter $\beta$ is a dynamical function [21] of the basic constants of the (input) BS kernel [17]; and similarly for $\phi'$.

We shall make use of this framework to calculate the quark level form factor $F(\alpha)$ at each of the two $D\bar{D}\gamma$ vertices of the hadron triangle (fig. 1a) for 2-photon decay of $B_s$, a la fig.1b. Here $\alpha$ is the off-shellness parameter for the exchanged $D_s$-meson in fig.1a which corresponds to fig.1 of ref.[5]. Inserting these two form factors in the corresponding amplitude $G_{\mu\nu}$ of ref.[5] for $B_s \rightarrow \gamma\gamma$, the latter in our momentum notation (fig.1a) and
euclidean convention, reads [5]

\[ G_{\mu\nu} = e^2 \mathcal{G} f \int \frac{d^4 p}{(2\pi)^4} \left[ (K^2 - P_1^2)f_+ + P_2^2 f_- \right] R^2(\alpha) \]

\[ \frac{(4\mu\nu - \delta_{\mu\nu})(p^2 + M^2)}{(p^2 + M^2)(p + k_1)^2 + M^2)((p - k_2)^2 + M^2)} + [1 \Rightarrow 2] \]

where \( K = P_1 + P_2 = k_1 + k_2 \) is the total 4-momentum, and the weak interaction parameters \( \mathcal{G}, f, f_\pm \) are as defined in ref.[5]; \( p \) is the 4-momentum of the exchanged \( D_s \)-meson of mass \( M \), so that \( p^2 + M^2 = M^2\alpha \). The effect of the contact \( \gamma \gamma \) interaction in the hadron loop, viz., \( G^{(3)}_{\mu\nu} \) of ref.[5], may be recognized in this 'master expression', via the term proportional to \( \delta_{\mu\nu} \) in the numerator of the integrand on the right, which has been simplified by dropping some terms that vanish on contracting with the photon polarizations: \( e_1 k_1 = e_2 k_2 = 0 \).

Our next task is to write down the expression for \( F(\alpha) \), when the exchanged hadron \( p_\mu \) (fig.1a) is off-shell. To that end we temporarily relabel \( p_\mu \) as \( P_\mu \), with \( P^2 = -M^2(1 - \alpha) \), and \( P_1 = P' \) with \( P'^2 = -M^2 \), so as to conform to fig.1b, and the following expression for \( F(\alpha) \) emerges in a fairly standard fashion [21, 22c]

\[ 2e \bar{P}_\mu iF(\alpha) = (2\pi)^{-4}eN_n(P)N_n(P') \int d^4p_2D_nD'_n\delta^4\phi\phi'4T_\mu/[\Delta_1\Delta'_1\Delta_2] + [1' \Rightarrow 2'] \]

\[ 4T_\mu = \Delta_1\Delta'_1\Delta_2 \times Tr[\gamma_5 S_F(p_1)i\gamma_\mu S_F(p'_1)\gamma_5 S_F(-p_2)] \]

which simplifies to

\[ T_\mu = \bar{P}_\mu[\Delta_2 + \Delta_1/2 + \Delta'_1/2 - x_2k^2/2 - (1 - \hat{m}_2)(\delta m^2 + P^2/2 - M^2/2)] \]

where \( \Delta_i = p_i^2 + m_i^2, i = 1, 2 \), are the inverse propagators, and the substitution \( \hat{m}_2 \approx p_2.bar P/P^2 \), as the fraction of \( p_2 \) in the direction of \( \bar{P} \), has been made in the last term of \( T_\mu \), eq.(2.7). The BS normalizers \( N_n(P, P') \) which correspond to both hadrons on-shell \( (\alpha = 0) \) and photon 4-momentum \( k_\mu = 0 \), which are expressible, by Lorentz covariance, in

\[ Figure 1: \]

\[ \gamma(k_1) \]

\[ \gamma(k_2) \]

\[ p = P_1 - k_1 \]

\[ D(P_1) \]

\[ D(P_2) \]

\[ B_s(K) \]

(a) \( B_s \Rightarrow \gamma\gamma \)

\[ p' \]

\[ p_1 \]

\[ p_1 = P' \]

\[ p_2 \]

\[ -p_2 \]

(b) Triangle loop for vertex ‘O’ in (a)
terms of the invariant hadron normalizers $N_H$ as $N_n(P, P') = N_H(M/P_+, M/P'_+)$, may be inferred from (2.5-7) [21], and the condition $F(0) \equiv 1$ (see eq.(3.7) below, for an explicit formula), so that the value of $F(\alpha)$ away from $F(0)$ is a direct measure of the form factor effect on the point $D\bar{D}\gamma$ vertex [5]. The kinematical range of $\alpha$ which can be related to the cosine of the scattering angle in the subprocess $D\bar{D} \rightarrow \gamma\gamma$, fig.1a, works out as

$$\alpha_{\text{min,max}} = \frac{M_s^2 \pm M_s \sqrt{M_s^2 - 4M^2}}{2M^2}$$  \hspace{1cm} (2.8)$$

where the energetics are controlled by the mass $M_s$ of the ‘source’ hadron $B_s$. Substituting for the respective masses [25], the limits work out as $1.20 < \alpha < 6.26$.

We now turn to the evaluation of $F(\alpha)$, eq.(2.5), in Sect.3, and use this quantity for evaluating $G_{\mu\nu}$, eq.(2.4), in Sect.4 next.

### 3 Exact Evaluation of $F(\alpha)$

The evaluation of $F(\alpha)$, eq.(2.5), follows closely a recent calculation of the pion form factor [21], except that one of the hadrons ($P_\mu$) is now off shell, and the photon is on shell, while in the pion form factor case [21], it was the other way around. To recount the main steps, note first that since $q_-$ is absent from $\phi(\bar{q})$, all the ‘pole’ singularities in the $q_-\text{-plane}$ are contained only in the 3 quark propagators in eq.(2.5). The detailed techniques of pole residues may be found in [22c], but we summarize the formulae [21]:

$$f \left\{ \frac{dp_{2-}}{2 \Delta_2} \left[ \frac{1}{\Delta_1}; \frac{1}{\Delta_1}; \frac{1}{\Delta_1}\Delta_1 \right] \right\} = \frac{2i\pi}{2} \left[ \frac{1}{D_+}; \frac{1}{D'_+}; \frac{2p_{2+}}{D_+D'_+} \right]$$  \hspace{1cm} (3.1)$$

in the standard $\pm$-component notation [21,22] for the $(D_n, D'_n)$-functions associated with the hadronic 4-momenta $P_\mu$ (off-shell) and $P'_\mu$ (on-shell) respectively. Next, we collect some definitions and the results of some simplification after integration over $dp_{2-}$

$$q_+ + q'_+ = 2\bar{q}_+; \hspace{0.5cm} z_2 \equiv \bar{q}_+ / \bar{P}_+; \hspace{0.5cm} p_{2+} = \hat{m}_2 P_+ - \bar{q}_+;$$  \hspace{1cm} (3.2)$$

$$\phi\phi' = \exp[-q_+^2 / \beta^2 - W^2 z_2^2 / \beta^2]; \hspace{0.5cm} W^2 \equiv M^2(1-\alpha/2)$$  \hspace{1cm} (3.3)$$

And the trace factor $Tr_+ \equiv T_+/P_\mu$ in Eq.(2.6) is

$$Tr_+ = 2\bar{P}_+ \left[ q_+^2 + W^2(z_2^2 - 1/4) + (m_1^2 + m_2^2/2 - (\Delta m^2)^2 W^2 / 4M^4(1-\alpha)) \right] + 2p_{2+}\bar{m}_1[W^2 - \delta m^2]$$  \hspace{1cm} (3.4)$$

where $\Delta m^2 = m_1^2 - m_2^2$. Collecting all the pieces of $F(\alpha)$ from eqs.(3.1-4) and putting them in (2.5) yields a simple quadrature:

$$F(\alpha) = 2N_H^2 \frac{M^2}{P_+P'_+} \int d^2q_+ d\phi' \int d^2z_2 Tr_+ \phi'$$  \hspace{1cm} (3.5)$$

the result of whose integration gives the explicit formula

$$F(\alpha) = \frac{4N_H^2 M^2}{(2\pi)^3W} (\pi \beta^2)^{3/2} \left[ \frac{3\beta^2}{2} + \frac{(m_1^2 + m_2^2)}{2} - W^2/4 + M^2\sigma^2\theta + 2\hat{m}_1\hat{m}_2(W^2 - \delta m^2) \right]$$  \hspace{1cm} (3.6)$$
where the simplification $P_+ = P_+^t = \bar{P}_+$ has been employed in view of the on-shellness of the emitted photon ($k^2 = 0$). $\sigma$ is defined as $\Delta m^2/2M^2$, and $\theta = (1 - \alpha/2)/(\alpha - 1)$. From this the normalizer $N_H$ is inferred in the limit $\alpha = 0$ as

$$N_H^{-2} = \frac{4M}{(2\pi)^3}(\pi \beta^2)^{3/2}[3\beta^2/2 + (m_1^2 + m_2^2)/2 - M^2/4 - M^2\sigma^2 + 2\hat{m}_1\hat{m}_2(M^2 - \delta m^2)] \quad (3.7)$$

which is symmetrical in $(m_1, m_2)$ and ensures $F(0) \equiv 1$.

Since form factors like $F(\alpha)$ are a typical feature of hadron compositeness, additional singularities should be expected in the integrand for $G_{\mu\nu}$, vis-a-vis a point-hadron description [5] where its only singularity is the pole $\alpha = 0$, corresponding to the on-shell value of the hadron propagator. Indeed (3.6) shows a branch point at $\alpha = 2$, but its effect on $G_{\mu\nu}$ is harmless; see Sect.4 below.

4 Exact Evaluation Of $G_{\mu\nu}$

Our next task is to evaluate the decay amplitude (2.4) by inserting in it the value (3.6-7) for $F(\alpha)$, with the kinematical range for $\alpha$ as $1.2 < \alpha < 6.26$. The ratio of this amplitude to one with a point $DD\gamma$ vertex ($F(\alpha) = 1$) will measure the effect of hadronic compositeness on the ‘long-distance’ (soft-QCD) contribution to this decay process [5]. The strategy is thus to evaluate (2.4) with and without the $F^2(\alpha)$ factor under the same approximation as in ref.[5], viz., replacing the two propagators $[P_{1,2}^2 + M^2]^{-1}$, on the rhs of (2.4) by their on-shell values $i\pi\delta(P_{1,2}^2 + M^2)$. Next, explicit gauge invariance is achieved by ensuring the proportionality of $G_{\mu\nu}$ to $Q^2\delta_{\mu\nu} - 2Q_{\mu}Q_{\nu}$, where $2Q = k_1 - k_2$, since it vanishes on contraction separately with $k_{1\mu}$, or with $k_{2\nu}$, and using the results $k_1Q = 2Q^2$, etc. To extract this combination from the integrand at the earliest, the $d^4p$ integration in (2.4) may be arranged in accordance with the resolution of the 4-vector $p_{\mu}$ in 4 mutually perpendicular directions: $p_{\mu} = p_{\perp\mu} + p.QQ_{\mu}/Q^2 + p.KK_{\mu}/K^2$, where $K = k_1 + k_2$, $Q = (k_1 - k_2)/2$, and $Q.K = 0$.

Now the 4D measure may be expressed as

$$d^4p = d^2p_{\perp} d(p.Q)d(p.K)/[M_s\sqrt{Q^2}] \quad (4.1)$$

and in association with the absorptive parts (delta-fns) of the two $D$-propagators, the 4D measure gives the net result

$$d^4p\pi^2\delta(p^2 + 2p.Q + M^2)\delta(p.K)/2 = d^2p_{\perp} \pi^2/[4M_s\sqrt{Q^2}]; \quad p.K = 0; \quad 2p.Q = -M^2\alpha \quad (4.2)$$

Thus effectively $p_{\mu}$ and $Q_{\mu}$ are 3-vectors, and the off-shell parameter $\alpha$ corresponds to the angle between them. And the 2D measure $d^2p_{\perp}$, on integration w.r.t. the azimuthal angle (not involved in the denominators), becomes $\pi d(p_{\perp}^2)$, where $p_{\perp}^2$ is entirely expressible in terms of $\alpha$:

$$p_{\perp}^2 = M^2(\alpha - 1) - M^4\alpha^2/4Q^2; \quad d^2p_{\perp} = M^2(1 - M^2\alpha/2Q^2) d\alpha \quad (4.3)$$

which reduces the 4D integration to a simple quadrature in $\alpha$ only.

Since the azimuthal angle is involved only in the factor $4p_{\mu}p_{\nu}$ in (2.4), its integration gives

$$<4p_{\mu}p_{\nu}> = 2\Theta_{\mu\nu}p_{\perp}^2 + 4x^2Q_{\mu}Q_{\nu}; \quad x = p.Q/Q^2; \quad \Theta_{\mu\nu} = \delta_{\mu\nu} - 2Q_{\mu}Q_{\nu}/Q^2 \quad (4.4)$$
To ensure explicit gauge invariance, we employ a pedagogical method which checks exactly
with the final result of ref.[5] in the point hadron limit. This consists in ‘regularizing’
a certain (apparently non-gauge invariant) portion from \[ \delta_{\mu\nu}(p^2 + M^2) - 4p_{\mu}p_{\nu} \],
following the classical Bethe-Schweber treatment of vacuum polarization in QED [26]. Of
this the piece \( \Theta_{\mu\nu} \) is gauge invariant by itself, but the ‘unwanted’ piece \[ \delta_{\mu\nu}(p^2 + M^2) - 2x^2Q^2 \] must be isolated and ‘regularized’. The net result which gives \( G_{\mu\nu} \) as a product \( \Theta_{\mu\nu} \times A \), is a simple quadrature in \( \alpha \):

\[
A = C \int d\alpha \frac{F^2(\alpha)}{\alpha} (1 - \lambda\alpha/2)(\lambda\alpha^2/2 + 1 - \alpha); \quad \lambda = M^2/Q^2
\]  

(4.5)

where the constant \( C \) represents the effect of the electroweak factors as employed in ref.[5]:

\[
C = M^2 G f c^2 \frac{[(M^2 - M^2)f_+ - M^2f_-]}{16\pi}
\]

and \( F(\alpha) \) is given by (3.6-7). And except for a slightly different integration strategy to
accommodate the form factor \( F(\alpha) \), this formula checks with ref.[5] in the point hadron
limit. The constant \( C \) may now be dropped as we are interested only in checking the
relative effect of \( F(\alpha) \) on the point-hadron result of ref.[5].

The integration w.r.t. \( \alpha \) in \( 1.20 < \alpha < 6.26 \) is elementary even after the inclusion of
\( F(\alpha) \). In the point hadron limit [5], its value is

\[
A_0 = J_0(6.26) - J_0(1.20) = -0.4824; \quad J_0(x) = \ln x - (1 + \lambda/2)x + \lambda x^2/2 - \lambda^2x^3/12
\]  

(4.6)

In the composite hadron case, eq.(3.6) suggests a branch point singularity at \( \alpha = 2 \) which
is the limit up to which the (gaussian) integral for \( F(\alpha) \) is defined. This gives a simple pole structure for \( F^2(\alpha) \) at \( \alpha = 2 \); beyond this point \( F^2(\alpha) \) may be defined by analytic
continuation. [The singularity at \( \alpha = 1 \) is outside its range of integration]. The integral
(as a Principal value) after all substitutions is

\[
A = \int_{1.20}^{6.26} \frac{dx}{x(1 - x/2)} (1 - x + x^2\lambda/2)[a - (1 - x/2)(b + c/(1 - x))]^2 = -0.01464
\]  

(4.7)

where \( \alpha \) has been renamed as \( x \) and

\[
aN = 3/2\beta^2 + (m_1^2 + m_2^2)/2 - 2\tilde{m}_1\tilde{m}_2\delta m^2
\]

\[
bN = M^2/4 - 2\tilde{m}_1\tilde{m}_2M^2; \quad cN = M^2\sigma^2
\]

\[
N = 3/2\beta^2 + m_2^2 - (M^2 - m_1^2 + m_2^2)^2/4M^2 + 2\tilde{m}_1\tilde{m}_2(M^2 - \delta m^2)
\]

(4.8)

The numerical values of these quantities were obtained by substitution from the spectro-
scopic values of their basic ingredients [17,21]:

\[
a = +0.9446; \quad b = -0.2370; \quad c = +0.2882
\]  

(4.9)

leading to the final value \((-0.01464)\) for \( A \), eq.(4.7). As a result of this exercise, the ratio
of the amplitudes for the composite (4.7) to the point hadron limit (4.6) is \( 0.03035 \), giving
a reduction in the decay rate by the factor \( \rho \approx 900 \), i.e., by 3 orders of magnitude.
5 Resume And Conclusion

We have tried to estimate the effect of quark compositeness on the long distance contributions to $B_s \rightarrow \gamma\gamma$ [5], within a BSE framework under the Markov-Yukawa Transversality Principle (MYTP) on the BSE kernel for $q\bar{q}$ interaction, which gives an exact interconnection between the 3D and 4D forms of the BS amplitude. A major advantage of this formalism (which is fully calibrated to the spectra [17] and other observables [18,21]), vis-a-vis several other candidates [6-8] in the field, is that it provides an exact analytic structure for the $D\bar{D}\gamma$ form factor in terms of the off-shell energy variable $\alpha$ of the exchanged hadron. Further, when folded into the $D$-triangle loop for the $B_s \rightarrow \gamma\gamma$ amplitude, it gives an analytic structure of the integral in this variable, and thus offers a basic confidence in the reliability of the quark compositeness effect due to its prior calibration to spectroscopy. The result for the decay rate is a reduction by three orders of magnitude over the point hadron value [5]. And although the calculation was made for the $D\bar{D}$ annihilation mode for simplicity, the mechanism is general enough to apply to the more pertinent $D^*\bar{D}^*$ mode as well, as it seems to be the more promising candidate for the measurability of such long distance modes of radiative $B_s$ decays in hadronic B-factories (HERA-B, CDF, D0, LHC-B), as and when available [5].

I acknowledge useful comments from Prof S.R.Choudhury on this paper.

REFERENCES

[1] G.-L.Lin, J.Liu and Y.P.Yao, Mod.Phys.Lett.A6, 1333 (1991); H.Simma and D.Wyler, Nucl.Phys.B344, 283 (1990); S.Herrlich and J.Kalinowski, Nucl.Phys.B381, 502 (1992).

[2] T.M.Aliev and G.Turan, Phys.Rev.D48, 1176 (1993); C.-H.V.Chang et al, Phys.Lett.B415, 395 (1997);

[3] (a) T.M.Aliev and E.O.Iltan, Phys.Rev.D58, 095014-(1-11) (1998); (b) G.G.Devidze and G.R.Jibuti LANL hep-ph/9810343

[4] M.Acciarri et al.(L3 Collab), Phys.Lett.B363, 127 (1995).

[5] D.Choudhury and John Ellis Phys.Lett.B433, 102 (1998)

[6] (a) A.F.Falk et al, Harvard HUTP-90/A011 2/90; (b) N.Isgur and M.B.Wise, Phys.Lett.B232, 113 (1989); (c) H.Georgi, Harvard HUTP-90/A007.

[7] E.g., B.L.Ioffe and A.V.Smigla, Nucl.Phys.B232, 109 (1984)

[8] I.Gasser and H.Leutwyler, Ann.Phys.(N.Y.)158, 142 (1984).

[9] Review: C.D.Roberts and A.G.Williams, Prog.Part.Nucl.Phys.33, 477 (1994)

[10] S.L.Adler and A.C.Davies, Nucl.Phys.B244, 469 (1984).

[11] Y.Nambu and G.Jona-Lasino, Phys.Rev.122, 123 (1961).

[12] H.D.Politzer, Nucl.Phys.B117, 397 (1976).
[13] M.A. Markov, Sov. J. Phys. 3, 452 (1940); H. Yukawa, Phys. Rev. 77, 219 (1950).

[14] E.E. Salpeter, Phys. Rev. 87, 328 (1952).

[15] Yu.L. Kalinowski et al, Phys. Lett. B 231, 288 (1989).

[16] (a) A.N. Mitra and S. Bhatnagar, Int. J. Mod. Phys. A 7, 121 (1992). (b) A.N. Mitra and B.M. Sodermark, Int. J. Mod. Phys. A 9, 915 (1994).

[17] (a) K.K. Gupta et al, Phys. Rev. D 42, 1604 (1990); (b) Anju Sharma et al, ibid D 50, 454 (1994).

[18] (a) A.N. Mitra and K.-C. Yang, Phys. Rev. C 51, 3404 (1995); (b) A.N. Mitra, Intl J Mod Phys. A 11, 5245 (1996).

[19] H. Pagels and S. Stokar, Phys. Rev. D 20, 2947 (1979).

[20] Pion ff: S.R. Choudhury et al, Delhi Univ. Preprint (1991), unpublished; \(\rho\pi\pi\): I. Santhanam, et al, Intl. J. Mod. Phys. E 2, 219 (1993).

[21] A.N. Mitra, LANL Preprint hep-ph/9812404; Phys. Lett. B (1999)-in press.

[22] (a) J. Kogut and L. Susskind, Phys. Rep. 8C, 75 (1973); (b) H. Leutwyler and J. Stern, Ann. Phys. (N.Y) 112, 94 (1978); (c) S. Chakrabarty et al, Prog. Part. Nucl. Phys. 22, 43-180 (1989).

[23] (a) V. Kadychevsky, Nucl. Phys. B 6, 125 (1968); (b) V. Karmanov, Nucl. Phys. B 166, 378 (1980); (c) R. Perry, A. Harindranath, K. Wilson, Phys. Rev. Lett. 65, 2959 (1990).

[24] J. Carbonell et al, Phys. Rep. 400, 215 (1998).

[25] Particle Data Group, Phys. Rev. D 54, July 1 - Part I (1996).

[26] S. Schweber, H. A. Bethe, F. de Hoffmann, Mesons and Fields I, Row, Peterson and Co Inc Evanston Il 1955.