New symmetries for Abelian gauge theory
in superfield formulation

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Abstract: We show the existence of some new local, covariant and continuous symmetries for the BRST invariant Lagrangian density of a free two (1 + 1)-dimensional (2D) Abelian $U(1)$ gauge theory in the framework of superfield formalism. The Noether conserved charges corresponding to the above local continuous symmetries find their geometrical origin as the translation generators along the odd (Grassmannian)- and even (bosonic) directions of the four (2 + 2)-dimensional compact supermanifold. Some new discrete symmetries are shown to exist in the superfield formulation. The logical origin for the existence of BRST- and co-BRST symmetries is shown to be encoded in the Hodge decomposed versions (of the 2D fermionic vector fields) that are consistent with the discrete symmetries of the theory.

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1 Introduction

The superfield approach \[1–5\] to Becchi-Rouet-Stora-Tyutin (BRST) formalism is a well-established technique which provides the geometrical origin for the existence of (anti-)BRST charges as the generators of translation along the Grassmannian directions of the compact supermanifold that is parametrized by the spacetime coordinates and two extra anti-commuting (Grassmannian) variables. In fact, in this scheme, the \((p + 1)\)-form super curvature tensor for a \(p\)-form \((p = 1, 2, 3, \ldots)\) gauge theory is restricted to be flat along the Grassmannian directions of the supermanifold. This restriction, popularly known as the horizontality condition \[^1\], leads to the derivation of the (anti-)BRST symmetry transformations for the Lagrangian density of a \(p\)-form gauge theory. In this derivation, the mathematical power of the super exterior derivative \(\tilde{d}\) alone (which is only one of the three de Rham cohomology operators \[^2\] of differential geometry) is exploited when it operates on the super \(p\)-form potential of a \(p\)-form gauge theory to make it a \((p + 1)\)-form curvature tensor through Maurer-Cartan equation. Thus, it is an interesting endeavour to explore the possibility of the existence of some new local symmetries by exploiting the other two super de Rham cohomology operators \((\tilde{\delta}: \text{co-exterior derivative}; \tilde{\Delta}: \text{Laplacian operator})\) of differential geometry and find out their geometrical interpretation in the language of some kind of translation generators on an appropriately chosen compact supermanifold.

The purpose of the present paper is to show the existence of some new local, covariant and continuous symmetries for the free 2D Abelian gauge theory that emerge due to the operation of super co-exterior derivative \(\tilde{\delta}\) \((\tilde{\delta} = -\tilde{\ast} \tilde{d} \tilde{\ast}, \tilde{\delta}^2 = 0, \tilde{\ast} = \text{Hodge duality operation})\) and super Laplacian operator \(\tilde{\Delta}\) \((\tilde{\Delta} = \tilde{d} \tilde{\delta} + \tilde{\delta} \tilde{d})\) on the super one-form connection \(\tilde{A}\) together with the analogue of the horizontality conditions w.r.t. these super de Rham cohomology operators. In fact, we demonstrate that (anti-)co-BRST symmetry- and a bosonic symmetry transformations emerge when we exploit these super cohomological operators on a four \((2 + 2)\)-dimensional compact supermanifold and they turn out to be exactly same as the new local symmetries obtained recently in a set of papers in the Lagrangian formalism alone \[11-15\]. It has been established in these works that the 2D free- as well as interacting (non-)Abelian (one-form) gauge theories provide the field theoretical models for the Hodge theory where all the de Rham cohomology operators find their interpretation as the local Noether charges that generate these new local, continuous and covariant symmetries. Such symmetries and corresponding generators (conserved Noether charges) have also been shown to exist for the four \((3 + 1)\)-dimensional free two-form Abelian gauge theory \[16\]. In these attempts, the local Noether charges have also been shown to refine the BRST coho-

\[^1\] This condition is referred to as the “soul flatness” condition in Ref. \[6\] implying the flatness of the Grassmannian components of the \((p + 1)\)-form super curvature tensor for a \(p\)-form gauge theory.

\[^2\] On an ordinary Minkowskian manifold parametrized by the spacetime coordinate \(x^\mu\), the exterior derivative \(d (d = dx^\mu \partial_\mu, d^2 = 0)\), the co-exterior derivative \(\delta (\delta = \pm \ast \ d \ast, \delta^2 = 0, \ast = \text{Hodge duality operation})\) and the Laplacian operator \(\Delta (\Delta = (d + \delta)^2 = d\delta + \delta d)\) constitute what is popularly known as the set \((d, \delta, \Delta)\) of the de Rham cohomology operators. These geometrical operators obey: \(\delta^2 = 0, d^2 = 0, \{d, \delta\} = \Delta, [\Delta, d] = [\Delta, \delta] = 0\) implying that \(\Delta\) is the Casimir operator for this algebra \[7-10\].
mology [12] and define the analogue of the Hodge decomposition theorem (HDT) in the quantum Hilbert space of states [11-17]. Exploiting these ideas, it has been shown that 2D free (non-)Abelian gauge theories belong to a new class of topological field theories (TFTs) [17]. However, in all the above attempts, the geometrical origin for the existence of these charges has not yet been discussed. In the present work, we show that the (anti-)BRST- and (anti-)co-BRST symmetry generators (conserved and nilpotent Noether charges \((\bar{Q}_b)Q_b\) and \((\bar{Q}_d)Q_d\) respectively) are the translation generators along the Grassmannian (odd) directions of the \((2 + 2)\)-dimensional compact supermanifold and they owe their origin to the super cohomological operators \(\bar{d}\) and \(\bar{\delta}\). A bosonic symmetry, generated by the Casimir operator, turns out to be the translation generator along the bosonic (even) direction of the supermanifold and its origin is encoded in the super operator \(\bar{\Delta}\). This even (bosonic) direction on the supermanifold is equivalent to a couple of intertwined Grassmannian directions. The local conserved charges in the theory provide an analogue of the set \((d, \delta, \Delta)\).

The outline of our present paper is as follows. In Sec. 2, we set up the notations and recapitulate some of the salient features of our earlier works [11-17]. Section 3 is devoted to the derivation of (anti-)BRST symmetry transformations through horizontality condition [3,4]. In Sec. 4, we exploit the super co-exterior derivative and derive the (anti-)co-BRST symmetry transformations exploiting the analogue of the horizontality condition w.r.t. \(\bar{\delta}\). We discuss some interesting discrete symmetries and the Hodge decomposed versions of 2D vectors in Sec. 5. A local bosonic symmetry is obtained in Sec. 6 using the super Laplacian operator \(\Delta\). Finally, we make some concluding remarks in Sec. 7.

### 2 BRST- and dual BRST symmetries: A brief sketch

Let us start off with the BRST invariant Lagrangian density \(\mathcal{L}_b\) for the free two \((1 + 1)\)-dimensional \(^5\) Abelian gauge theory in the Feynman gauge [6,18-20]

\[
\mathcal{L}_b = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} - \frac{1}{2} (\partial \cdot A)^2 - i \partial_\mu \bar{C} \partial^\mu C \equiv \frac{1}{2} E^2 - \frac{1}{2} (\partial \cdot A)^2 - i \partial_\mu \bar{C} \partial^\mu C, \tag{2.1}
\]

where \(F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu\) is the field strength tensor (curvature two-form) derived from one-form \(A = dx^\mu A_\mu\) (with \(A_\mu = \) vector potential) by the application of the exterior derivative \(d\) (i.e. \(F = dA = \frac{1}{2} dx^\mu \wedge dx^\nu F_{\mu\nu}\)). The gauge-fixing term (zero-form) is derived from one-form \(A\) by the application of the co-exterior derivative \(\delta\) (i.e. \((\partial \cdot A) = \delta A; \delta = -*d*; * = \) Hodge duality operation). Thus, in some sense, \(F = dA\) and \((\partial \cdot A) = \delta A\) are “Hodge dual” to each other. The (anti-)ghost fields \((\bar{C})C\) are anti-commuting \((C^2 = \bar{C}^2 = 0, \bar{C}C + CC = 0)\) in nature. Under the following on-shell \((\Box C = \Box \bar{C} = 0)\) nilpotent \((s_b^2 = 0, \bar{s}_b^2 = 0, s_b \bar{s}_b + \bar{s}_b s_b = 0)\) this \(\mathcal{L}_b\) is a BRST-invariant action.

\(^5\) This theorem states that, on a boundary manifold without a boundary, any arbitrary \(n\)-form \(f_n\), \((n = 0, 1, 2, \ldots)\) can be uniquely written as the sum of a harmonic form \(h_n(\Delta h_n = dh_n = \delta h_n = 0)\), an exact form \(de_{n-1}\) and a co-exact form \(\delta e_{n+1}\) as: \(f_n = h_n + de_{n-1} + \delta e_{n+1}\) [7-10].

\(^4\) We follow here the conventions and notations such that the 2D flat Minkowski metric is: \(\eta_{\mu\nu} = \text{diag} \begin{pmatrix} +1 & -1 \\ -1 & +1 \end{pmatrix}\) and \(\Box = \eta^{\mu\nu} \partial_\mu \partial_\nu = (\partial_0)^2 - (\partial_1)^2, \varepsilon_{\mu\nu} = -\varepsilon^{\mu\nu}, F_{01} = E = -\varepsilon^{\mu\nu} \partial_\mu A_\nu = \partial_0 A_1 - \partial_1 A_0 = F^{10}, \varepsilon_{01} = \varepsilon^{10} = +1\). Here the Greek indices: \(\mu, \nu, \lambda = 0, 1\) correspond to spacetime directions on the 2D manifold.
the key relationships: 

\[ s_b A_\mu = \partial_\mu C, \quad s_b C = 0, \quad s_b \bar{C} = -i (\partial \cdot A), \]
\[ \bar{s}_d A_\mu = -\varepsilon_{\mu\nu} \partial^\nu C, \quad \bar{s}_d C = 0, \quad \bar{s}_d \bar{C} = +i (\partial \cdot A), \]  

(2.2)

the Lagrangian density (2.1) remains invariant. The same Lagrangian density is also invariant under the following on-shell (\( \Box C = \Box C = 0 \)) nilpotent (\( s_d^2 = \bar{s}_d^2 = 0, \ s_d \bar{s}_d + \bar{s}_d s_d = 0 \)) (anti-)dual BRST transformations \( (\bar{s}_d) s_d \) on the basic fields [11,12,17]

\[ s_d A_\mu = -\varepsilon_{\mu\nu} \partial^\nu \bar{C}, \quad s_d \bar{C} = 0, \quad s_d \bar{C} = -i E, \]
\[ \bar{s}_d A_\mu = -\varepsilon_{\mu\nu} \partial^\nu C, \quad \bar{s}_d C = 0, \quad \bar{s}_d \bar{C} = +i E. \]  

(2.3)

We christen the above new continuous, covariant and nilpotent symmetry as the (anti-)dual BRST symmetry because it is the gauge-fixing term \( (\partial \cdot A) = \delta A \) (Hodge dual to the curvature two-form \( F = dA \)) that remains invariant. In contrast, it is the curvature two-form \( F = dA \) that remains invariant under the (anti-)BRST symmetry transformations (2.2). The anti-commutator of the above two symmetries leads to yet another new bosonic type symmetry transformation \( s_w (s_w = \{ s_b, s_d \} = \{ \bar{s}_b, \bar{s}_d \}; s_w^2 \neq 0) \) [12]

\[ s_w A_\mu = \partial_\mu E - \varepsilon_{\mu\nu} \partial^\nu (\partial \cdot A) \equiv -\varepsilon_{\mu\nu} \Box A^\nu, \quad s_w C = 0, \quad s_w \bar{C} = 0, \]  

(2.4)

under which the ghost fields remain invariant. The Noether conserved charges \( (Q_r) \) corresponding to the above continuous symmetries are the generators for the above transformations [11-17]. This statement can be concisely expressed as

\[ s_r \Psi = -i [\Psi, Q_r]_{\pm}, \quad Q_r = Q_b, Q_b, Q_d, \bar{Q}_d, Q_w, Q_g, \]  

(2.5)

where brackets \([ , ]_{\pm}\) stand for the (anti-)commutators for any arbitrary generic field \( \Psi \) being (fermionic) bosonic in nature. Here the ghost charge \( Q_g \) \( (Q_g = -i \int dx [C \bar{C} + \bar{C} C]) \) generates the continuous scale transformations: \( C \rightarrow e^{-\Sigma} C, \bar{C} \rightarrow e^{\Sigma} \bar{C}, A_\mu \rightarrow A_\mu \) (where \( \Sigma \) is a global parameter) for the invariance of the Lagrangian density (2.1).

Now we wish to discuss some of the discrete symmetries present in the theory. It is interesting to note that \( s_b \leftrightarrow \bar{s}_b \) under the discrete symmetry transformations : \( C \leftrightarrow \bar{C}, \ (\partial \cdot A) \leftrightarrow -(\partial \cdot A) \). On the other hand, \( s_d \leftrightarrow \bar{s}_d \) when we take: \( C \leftrightarrow \bar{C}, \ E \leftrightarrow -E \). Under yet another discrete symmetry transformations [17,15]

\[ C \rightarrow \pm i \bar{C}, \bar{C} \rightarrow \pm i C, \ E \rightarrow \pm i (\partial \cdot A), \ (\partial \cdot A) \rightarrow \pm i E, \ A_\mu \rightarrow A_\mu, \partial_\mu \rightarrow \pm i \varepsilon_{\mu\nu} \partial^\nu, \]  

(2.6)

the Lagrangian density (2.1) remains form-invariant and the symmetry transformations (2.2) and (2.3) are related to one-another. Furthermore, this discrete symmetry turns out to be the analogue of the Hodge * duality operations of the differential geometry as one of the key relationships: \( s_d \Psi = \pm * s_b * \Psi, (\bar{s}_d \Psi = \pm * \bar{s}_b * \Psi) \) exists for any arbitrary

\[ \text{Here the notations, followed in Ref. [20], are adopted. In fact, in its totality, a BRST transformation} \delta_B \text{ is the product of an anti-commuting spacetime independent parameter} \eta \text{ and} \ s_b \text{ (i.e.} \delta_B = \eta \ s_b). \]
generic field $\Psi$ of the theory $[15,17]$. The (±) sign in this relationship is dictated by the existence of the corresponding sign in the operation: $\ast (\ast \Psi) = \pm \Psi$ where $\ast$ is nothing but the discrete transformations (2.6). The Lagrangian density (2.1) and the corresponding symmetric energy-momentum tensor $T_{\mu\nu}$ can be expressed, modulo some total derivatives, as $[11,12,17]$

$$L_b = \{Q_d, S_1\} + \{Q_b, S_2\} \equiv s_d (i S_1) + s_b (i S_2),$$

$$T_{\mu\nu} = \{Q_d, V_{\mu\nu}^{(1)}\} + \{Q_b, V_{\mu\nu}^{(2)}\} \equiv s_d (i V_{\mu\nu}^{(1)}) + s_b (i V_{\mu\nu}^{(2)}),$$

(2.7)

where $S_1 = \frac{1}{2} EC$, $S_2 = -\frac{1}{2} (\partial \cdot A) C$ and the local field dependent expressions for $V'$s are

$$V_{\mu\nu}^{(1)} = \frac{1}{4} \left[ (\partial_\mu C) \varepsilon_{\nu\lambda} A^\lambda + (\partial_\nu C) \varepsilon_{\mu\lambda} A^\lambda - \eta_{\mu\nu} EC \right],$$

$$V_{\mu\nu}^{(2)} = \frac{1}{2} \left[ (\partial_\mu C) A_\nu + (\partial_\nu C) A_\mu + \eta_{\mu\nu} (\partial \cdot A) \bar{C} \right].$$

(2.8)

The expressions in (2.7) establish the topological nature of 2D free Abelian gauge theory as topological invariants and their recursion relations have been obtained in Ref. [17]. The algebra amongst the conserved charges of the theory are reminiscent of the algebra obeyed by the de Rham cohomology operators of differential geometry. Thus, the present theory is a field theoretic model for the Hodge theory and it represents a new class of topological field theory which captures some of the key features of Witten- and Schwarz type TFTs.

### 3 Super exterior derivative and (anti-)BRST symmetry transformations

We begin with the definition of a super exterior derivative ($\tilde{d}$) and a super one-form connection ($\tilde{A}$) on a $(2 + 2)$-dimensional compact supermanifold as $[21]$

$$\tilde{d} = dZ^M \partial_M = dx^\mu \partial_\mu + d\theta \partial_\theta + d\bar{\theta} \partial_{\bar{\theta}},$$

$$\tilde{A} = dZ^M A_M = dx^\mu B_\mu (x, \theta, \bar{\theta}) + d\theta \bar{\Phi} (x, \theta, \bar{\theta}) + d\bar{\theta} \Phi (x, \theta, \bar{\theta}),$$

(3.1)

where supermanifold is parametrized by the superspace coordinates $Z^M = (x^\mu, \theta, \bar{\theta})$ with two c-number (commuting) spacetime co-ordinates $x^\mu$ (with $\mu = 0, 1$) and two Grassmann (anti-commuting) variables $\theta$ and $\bar{\theta}$ (with $\theta^2 = \bar{\theta}^2 = 0, \theta \bar{\theta} + \bar{\theta} \theta = 0$) and partial derivatives, with respect to these superspace coordinates, are

$$\partial_M = \frac{\partial}{\partial Z^M}, \quad \partial_\mu = \frac{\partial}{\partial x^\mu}, \quad \partial_\theta = \frac{\partial}{\partial \theta}, \quad \partial_{\bar{\theta}} = \frac{\partial}{\partial \bar{\theta}}.$$  

(3.2)

The bosonic (commuting) superfield $B_\mu (x, \theta, \bar{\theta})$ and the fermionic (anti-commuting) superfields: $\Phi (x, \theta, \bar{\theta})$, $\bar{\Phi} (x, \theta, \bar{\theta})$, constitute the component multiplet of a supervector superfield $V_\lambda$, defined on the four-dimensional compact supermanifold, as $[3,4]$

$$V_\lambda = (B_\mu (x, \theta, \bar{\theta}), \phi (x, \theta, \bar{\theta}), \bar{\phi} (x, \theta, \bar{\theta})).$$

(3.3)

The above superfields can be expanded in terms of the superspace coordinates $(x^\mu, \theta, \bar{\theta})$, the field variables of the Lagrangian density (2.1) and some extra (secondary) fields, as

$$B_\mu (x, \theta, \bar{\theta}) = A_\mu (x) + \theta R_\mu (x) + \bar{\theta} R_\mu (x) + i \theta \bar{\theta} S_\mu (x),$$

$$\Phi (x, \theta, \bar{\theta}) = C (x) + i \theta (\partial \cdot A) (x) - i \bar{\theta} E (x) + i \theta \bar{\theta} s (x),$$

$$\bar{\Phi} (x, \theta, \bar{\theta}) = \bar{C} (x) + i \theta E (x) - i \bar{\theta} (\partial \cdot A) (x) + i \theta \bar{\theta} \bar{s} (x).$$

(3.4)
Here the signs in the expansion are chosen for the later convenience. It is straightforward to see that the local fields \( R_\mu(x), \tilde{R}_\mu(x), C(x), \tilde{C}(x), s(x), \tilde{s}(x) \) are fermionic (anti-commuting) in nature and the bosonic (commuting) local fields are: \( A_\mu(x), S_\mu(x), \pm E(x), \pm (\partial \cdot A)(x) \) in the above expansion so that bosonic- and fermionic degrees of freedom can match. It is interesting to note that the above expansion is such that: \( \Phi(x, \theta, \tilde{\theta}) \) and \( \tilde{\Phi}(x, \theta, \tilde{\theta}) \) are chosen such that the condition: 
\[
\Phi(x, \theta, \tilde{\theta}) + \tilde{\Phi}(x, \theta, \tilde{\theta}) = 0 \quad \text{and} \quad [B_\mu(x, \theta, \tilde{\theta}), B_\nu(x, \theta, \tilde{\theta})] = 0.
\]

As a consequence, it is straightforward to verify that \( \tilde{A} \wedge \tilde{A} = \frac{1}{2}[\tilde{A}, \tilde{A}] = 0 \).

The super curvature tensor (two-form \( \tilde{F} \)) for the gauge theory can be constructed by exploiting (3.1) (i.e. \( \tilde{F} = \tilde{d} \tilde{A} + \tilde{A} \wedge \tilde{A} \)). For the \( U(1) \) gauge theory

\[
\tilde{F} = \tilde{d} \tilde{A} = (dx^\mu \wedge dx^\nu)(\tilde{\partial}_\mu B_\nu) - (d\theta \wedge d\tilde{\theta})(\tilde{\partial}_\theta \tilde{\Phi}) + (dx^\mu \wedge d\theta)(\tilde{\partial}_\mu \tilde{\Phi} - \tilde{\partial}_\theta B_\mu)
\]

where use has been made of the fact that the nilpotency of the super exterior derivative \( \tilde{d}^2 = 0 \) implies the following relations for the wedge products on the supermanifold \((dx^\mu \wedge dx^\nu) = -(dx^\nu \wedge dx^\mu), (dx^\mu \wedge d\theta) = -(d\theta \wedge dx^\mu), (d\theta \wedge d\tilde{\theta}) = +(d\theta \wedge d\tilde{\theta}) \) etc. Now the soul-flatness (or horizontality) condition imposes the following restriction

\[
\tilde{F} = \tilde{d} \tilde{A} = \frac{1}{2} (dZ^M \wedge dZ^N) \tilde{F}_{MN} \equiv F = dA = \frac{1}{2} (dx^\mu \wedge dx^\nu) F_{\mu\nu}.
\]

In the language of the component superfields of (3.3), the above condition implies

\[
\partial_\theta \tilde{\Phi} = 0, \quad \partial_{\bar{\theta}} \Phi = 0, \quad \partial_\theta \Phi + \partial_{\bar{\theta}} \bar{\Phi} = 0, \quad \partial_\mu \tilde{\Phi} = \partial_\theta B_\mu, \quad \partial_\mu \Phi = \partial_{\bar{\theta}} B_\mu,
\]

and the following conditions on the local component fields of the superfield \( B_\mu(x, \theta, \bar{\theta}) \)

\[
\partial_\mu \tilde{R}_\nu - \partial_\nu \tilde{R}_\mu = 0, \quad \partial_\mu R_\nu - \partial_\nu R_\mu = 0, \quad \partial_\mu S_\nu - \partial_\nu S_\mu = 0.
\]

The conditions (3.7) lead to the following solutions

\[
\begin{align*}
R_\mu(x) &= \partial_\mu C(x), & \tilde{R}_\mu(x) &= \partial_\mu \tilde{C}(x), & S(x) = 0, \\
S_\mu(x) &= -\partial_\mu (\partial \cdot A)(x), & \tilde{s}(x) = 0, & E(x) = 0.
\end{align*}
\]

It will be noticed that the signs in the expansion (3.4) are chosen such that the condition: \( \partial_\theta \Phi + \partial_{\bar{\theta}} \bar{\Phi} = 0 \) is satisfied trivially. Furthermore, the solutions in (3.9) automatically satisfy the conditions in (3.8). Now the expansion in (3.4) can be expressed as

\[
\begin{align*}
B_\mu(x, \theta, \bar{\theta}) &= A_\mu(x) + \theta (\bar{s}_b A_\mu(x)) + \bar{\theta} (s_b A_\mu(x)) + \theta \tilde{\theta} (s_b \bar{s}_b A_\mu(x)), \\
\Phi(x, \theta, \bar{\theta}) &= C(x) + \theta (\bar{s}_b C(x)), \\
\tilde{\Phi}(x, \theta, \bar{\theta}) &= \tilde{C}(x) + \tilde{\theta} (s_b \tilde{C}(x)).
\end{align*}
\]

We conclude that the horizontality condition on the super two-form curvature tensor for the \( U(1) \) Abelian gauge theory leads to the derivation of BRST- and anti-BRST symmetries for the Lagrangian density (2.1). The corresponding conserved and nilpotent charges find their geometrical origin as the translation generators along the Grassmannian directions.
of the supermanifold. In other words, it is the power of \( \tilde{d} \) that provides the geometrical interpretation for \( Q_b \) and \( \bar{Q}_b \) as translation generators (cf. (2.5)). Thus, the mapping is: \( \tilde{d} \Leftrightarrow (Q_b, \bar{Q}_b) \) but the ordinary exterior derivative \( d \) is identified with \( Q_b \) alone because the latter increases the ghost number of a state by one [11,12,17] as \( d \) increases the degree of a form by one on which it operates [7-10].

4 Super co-exterior derivative and (anti-)co-BRST symmetry transformations

We operate the super co-exterior derivative \( \tilde{\delta} = -\ast \tilde{d} \ast \) on the super one-form connection \( \tilde{A} \) of (3.1), with the Hodge duality operation \( \ast \) defined on the differentials and their wedge products (for the case of \((2 + 2)\)-dimensional compact supermanifold), as

\[
\ast (dx^\mu) = \varepsilon^{\mu\nu}(dx^\nu), \quad \ast (d\theta) = (d\theta), \quad \ast (d\bar{\theta}) = (d\theta),
\]

\[
\ast (dx^\mu \wedge dx^\nu) = \varepsilon^{\mu\nu}, \quad \ast (dx^\mu \wedge d\theta) = \varepsilon^{\mu\theta}, \quad \ast (dx^\mu \wedge d\bar{\theta}) = \varepsilon^{\mu\bar{\theta}}, \quad \ast (d\theta \wedge d\bar{\theta}) = s^{\theta\bar{\theta}}, \quad \ast (d\theta \wedge d\bar{\theta}) = s^{\theta\bar{\theta}},
\]

where \( \varepsilon^{\mu\theta} = -\varepsilon^{\theta\mu}, \varepsilon^{\mu\bar{\theta}} = -\varepsilon^{\bar{\theta}\mu} \) and \( s^{\theta\bar{\theta}} = s^{\bar{\theta}\theta} \) etc. It is obvious that the operation \( \tilde{\delta}\tilde{A} \) would result in a superscalar (zero-form) superfield (as \( \tilde{\delta} \) reduces the degree of a super form by one on which it operates). The explicit expression for this superfield is

\[
\tilde{\delta}\tilde{A} = (\partial \cdot B) + s^{\theta\bar{\theta}}(\partial_\theta \Phi) + s^{\bar{\theta}\theta}(\partial_{\bar{\theta}} \bar{\Phi}) + s^{\theta\bar{\theta}}(\partial_\theta \bar{\Phi} + \partial_{\bar{\theta}} \Phi) - \varepsilon^{\mu\theta}(\partial_\mu \Phi + \varepsilon_{\mu\nu}\partial_\nu B^\nu) - \varepsilon^{\mu\bar{\theta}}(\partial_\mu \bar{\Phi} + \varepsilon_{\mu\nu}\partial_\nu B^\nu).
\]

The analogue of the horizontality condition with the super co-exterior derivative \( \tilde{\delta} \) is to equate Eqn. (4.2) to the gauge-fixing term \( \delta A = (\partial \cdot A) \) (i.e., \( \tilde{\delta}\tilde{A} = \delta A \)). This restriction leads to the following conditions on the superfields

\[
\partial_\theta \bar{\Phi} + \partial_{\bar{\theta}} \Phi = 0, \quad \partial_\theta \Phi = 0, \quad \partial_{\bar{\theta}} \bar{\Phi} = 0,
\]

\[
\partial_\mu \Phi + \varepsilon_{\mu\nu}\partial_\nu B^\nu = 0, \quad \partial_\mu \bar{\Phi} + \varepsilon_{\mu\nu}\partial_\nu B^\nu = 0,
\]

and an additional restriction on the local field components of the expansion (3.4) for the bosonic superfield \( B_\mu(x, \theta, \bar{\theta}) \). The latter conditions are

\[
\partial \cdot \bar{R} = 0, \quad \partial \cdot R = 0, \quad \partial \cdot S = 0.
\]

The solutions for the restriction (4.3) are listed below

\[
R_\mu (x) = -\varepsilon_{\mu\nu} \partial^\nu \bar{C}(x), \quad \bar{R}_\mu (x) = -\varepsilon_{\mu\nu} \partial^\nu C(x), \quad \bar{s} (x) = 0,
\]

\[
S_\mu (x) = +\varepsilon_{\mu\nu} \partial^\nu E(x), \quad s (x) = 0, \quad (\partial \cdot A) (x) = 0,
\]

which automatically satisfy the restrictions (4.4). It will be noticed that the choice of the signs in the expansion (3.4) are such that the restriction \( \partial_\theta \bar{\Phi} + \partial_{\bar{\theta}} \Phi = 0 \) is satisfied trivially.

In terms of solutions (4.5), the expansion (3.4) can be re-expressed as

\[
B_\mu (x, \theta, \bar{\theta}) = A_\mu (x) - \theta \varepsilon_{\mu\nu} \partial^\nu C(x) - \bar{\theta} \varepsilon_{\mu\nu} \partial^\nu \bar{C}(x) + i \theta \bar{\theta} \varepsilon_{\mu\nu} \partial^\nu E(x),
\]

\[
\Phi (x, \theta, \bar{\theta}) = C(x) - i\bar{\theta} E(x),
\]

\[
\bar{\Phi} (x, \theta, \bar{\theta}) = \bar{C}(x) + i \theta E(x).
\]

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It is worth pointing out that the above expansion can be directly obtained from the definition of $\ast$ operation in Section 2 (cf. Eqn. (2.6)). Now exploiting dual- and anti-dual BRST symmetries (discussed in Sec. 2), we can rewrite Eqn. (4.6) as

$$
B_\mu (x, \theta, \bar{\theta}) = A_\mu (x) + \theta (s_d A_\mu (x)) + \bar{\theta} (s_d A_\mu (x)) + \theta (s_d s_d A_\mu (x)), \\
\Phi (x, \theta, \bar{\theta}) = C(x) + \theta (s_d C(x)), \\
\bar{\Phi} (x, \theta, \bar{\theta}) = \bar{C}(x) + \bar{\theta} (s_d \bar{C}(x)),
$$

(4.7)

which is the analogue of Eqn. (3.10) of the previous section. We summarize this section with the following comments: (i) (anti-) co-BRST symmetry transformations are generated along the $\theta$- and $\bar{\theta}$ directions of the supermanifold. (ii) The translation generators along the Grassmannian directions of the supermanifold are the conserved and nilpotent (anti-)co-BRST charges. (iii) For the odd (fermionic) superfields, the translations are either along $\theta$ or $\bar{\theta}$ directions (unlike the bosonic superfield where translations are along both $\theta$ as well as $\bar{\theta}$ directions). (iv) Comparison between (3.10) and (4.7) shows that the (anti-)BRST transformations are along $(\theta)\bar{\theta}$ directions for the odd fields $(C)\bar{C}$. On the contrary, the (anti-) co-BRST transformations are the other way around. (v) A single restriction $\delta A = \delta A$ produces co-BRST- and anti-co-BRST symmetry transformations for the Lagrangian density (2.1). Thus, the mapping is: $\delta \leftrightarrow (Q_d, \bar{Q}_d)$ but the ordinary co-exterior derivative $\delta$ is identified with $Q_d$ alone because it decreases the ghost number of a state by one [11,12,17] as $\delta$ reduces the degree of a given form by one on which it operates [7-10].

5 Discrete symmetries

We have discussed a few discrete symmetries at the fag end of Sec. 2. Now we exploit these discrete symmetries vis-a-vis our superfield expansion (3.4). We emphasize the fact that, for the BRST- and dual BRST symmetries, we have shown that: $s(x) = 0, \bar{s}(x) = 0$ in the expansion (3.4). Thus, we shall now be concentrating on (3.4) only for this case. First of all, it is straightforward to verify that under the following discrete transformations

$$
C \rightarrow \pm i \bar{C}, \quad \bar{C} \rightarrow \pm i C, \quad E \rightarrow \pm i (\partial \cdot A), \quad (\partial \cdot A) \rightarrow \pm i E, \quad \partial_\mu \rightarrow \pm i \varepsilon_{\mu\nu} \partial^\nu, \quad s_\mu \rightarrow \pm i \varepsilon_{\mu\nu} \partial^\nu, \quad A_\mu \rightarrow A_\mu,
$$

(5.1)

the superfields in (3.4) undergo the following change

$$
\Phi(x, \theta, \bar{\theta}) \rightarrow \pm i \bar{\Phi}(x, \theta, \bar{\theta}), \quad \bar{\Phi}(x, \theta, \bar{\theta}) \rightarrow \pm i \Phi(x, \theta, \bar{\theta}), \quad B_\mu (x, \theta, \bar{\theta}) \rightarrow B_\mu (x, \theta, \bar{\theta}).
$$

(5.2)

Furthermore, it can be trivially checked that the above transformations still satisfy: $\Phi^2 = 0, \bar{\Phi}^2 = 0, \Phi \bar{\Phi} + \bar{\Phi} \Phi = 0$ and $[B_\mu, B_\nu] = 0$. Yet another interesting point is to see that in the limit: $\theta \rightarrow 0, \bar{\theta} \rightarrow 0$, the above transformations reduce to: $C \rightarrow \pm i \bar{C}, \bar{C} \rightarrow \pm i C, A_\mu \rightarrow A_\mu$. Thus, transformations (5.2) are the generalization of the discrete symmetry (2.6). A close look at the expressions for $R_\mu, \bar{R}_\mu, S_\mu$ in equations (3.9) and (4.5) allows us to write down the Hodge decomposed versions for these 2D fermionic ($R_\mu, \bar{R}_\mu$)- and bosonic ($S_\mu$) vectors.
expression for discrete transformations (2.6). However, it is interesting to note that the r.h.s. of the
expression for the existence of (anti-)BRST- and (anti-)co-BRST symmetries for the Lagrangian density (2.1). In the language of the BRST cohomology and HDT, this is the
origin for the existence of the (anti-)BRST- and (anti-)co-BRST transformations. The generalized version of these symmetries, vis-a-vis our superfield expansion (3.4), is:
\begin{align}
C & \leftrightarrow \bar{C}, \quad (\partial \cdot A) \leftrightarrow -(\partial \cdot A), \quad E \leftrightarrow -E,
\theta & \leftrightarrow \bar{\theta}, \quad R_{\mu} \leftrightarrow -\bar{R}_{\mu}, \quad \bar{R}_{\mu} \leftrightarrow -R_{\mu}, \quad S_{\mu} \leftrightarrow S_{\mu},
\end{align}
under which the superfields transform as
\begin{align}
\Phi & \leftrightarrow \bar{\Phi}, \quad (\partial \cdot B) \leftrightarrow -(\partial \cdot B), \quad -\varepsilon^{\mu\nu}\partial_{\mu}B_{\nu} \leftrightarrow \varepsilon^{\mu\nu}\partial_{\mu}B_{\nu}.
\end{align}
It will be noticed that in the limit $\theta \to 0, \bar{\theta} \to 0$, we get back our original discrete symmetries: $C \leftrightarrow \bar{C}, (\partial \cdot A) \leftrightarrow -(\partial \cdot A), E \leftrightarrow -E$. It is interesting to point out that the solutions (5.3) are no longer the appropriate solutions for the present case. In fact, taking the help of (3.9) and (4.5), now the solutions for the 2D fermionic vectors are
\begin{align}
R_{\mu} = \partial_{\mu}C - \varepsilon^{\mu\nu}\partial^{\nu}C, \quad \bar{R}_{\mu} = -\partial_{\mu}C + \varepsilon^{\mu\nu}\partial^{\nu}C,
\end{align}
which are nothing but the orthogonal Hodge decomposed version of the corresponding solution in (5.3). Now, for the present case where $(\partial \cdot A) \leftrightarrow -(\partial \cdot A), E \leftrightarrow -E, \varepsilon_{\mu\nu}$, it is clear that any arbitrary linear combination: $S_{\mu} = P \partial_{\mu}(\partial \cdot A) + Q \varepsilon_{\mu\nu}\partial^{\nu}E$ (where $P$ and $Q$ are some c-number constants) would lead to $S_{\mu} = 0$ for the requirement $S_{\mu} \to S_{\mu}$ (cf. (5.4)) to be satisfied. The origin for the existence of the (anti-)BRST- and (anti-)co-BRST symmetries in the theory is encoded in the orthogonal relations (5.3) and (5.6) for the Hodge decomposed versions of $R_{\mu}$ and $R_{\mu}$. In fact, these relations show that $\partial_{\mu}C(\partial_{\mu}C)$ and $\varepsilon_{\mu\nu}\partial^{\nu}C(\varepsilon_{\mu\nu}\partial^{\nu}C)$ are the separate and independent symmetry transformations for the Lagrangian density (2.1). In the language of the BRST cohomology and HDT, this is the logical explanation for the existence of (anti-)BRST- and (anti-)co-BRST symmetries for the Lagrangian density (2.1) of a free 2D Abelian gauge theory.

6 Super Laplacian operator and bosonic symmetry

For the sake of brevity, we shall consider the expansion (3.4) for the case $s(x) = \bar{s}(x) = 0$. The analogue of the horizontality condition w.r.t. super Laplacian operator $\Delta$ is
\begin{align}
\Delta \bar{A} = \Delta A, \quad \Delta = \delta \bar{\delta} + \bar{\delta} \delta, \quad \Delta = \delta \bar{\delta} + \bar{\delta} \delta.
\end{align}
It is obvious that $\Delta A = dx^\mu \left[ \partial_\mu (\partial \cdot A) - \varepsilon_{\mu
u} \partial^\nu E \right] = dx^\mu \Box A_\mu$. Now we can check that the l.h.s. of (6.1) (with $\delta = -\bar{\delta} \dagger \bar{\delta}$) can be rewritten as

\[
\begin{align*}
\bar{d} (\bar{\delta} \bar{A}) &= dx^\rho \partial_\rho (\bar{d} \bar{A}) + d\theta \partial_\theta (\bar{d} \bar{A}) + d\bar{\theta} \partial_{\bar{\theta}} (\bar{d} \bar{A}), \\
\bar{\delta} (\bar{d} \bar{A}) &= dx^\rho \varepsilon_{\rho\lambda} \partial^\lambda \left[ \bar{\delta}(\bar{d} \bar{A}) \right] - d\theta \partial_\theta [ \bar{\delta}(\bar{d} \bar{A}) ] - d\bar{\theta} \partial_{\bar{\theta}} [ \bar{\delta}(\bar{d} \bar{A}) ],
\end{align*}
\] (6.2)

where the explicit expression for the term in the square bracket is

\[
\bar{\delta}(\bar{d} \bar{A}) = \varepsilon^{\mu\nu} \partial_\rho B_\nu + \varepsilon^{\mu\nu} (\partial_\rho \bar{\Phi} - \partial_\rho B_\mu) + \varepsilon^{\mu\nu} (\partial_\rho \Phi - \partial_\rho B_\mu) - s^{\rho\theta} \partial_{\rho\theta} \Phi.
\] (6.3)

Equation (6.1) can be expressed in a more transparent way as follows

\[
dx^\rho \left[ \partial_\rho (\bar{d} \bar{A}) + \varepsilon_{\rho\lambda} \partial^\lambda \left\{ \bar{\delta}(\bar{d} \bar{A}) \right\} \right] = dx^\rho \Box A_\rho,
\] (6.4)

\[
d\theta \left[ \partial_\theta (\bar{d} \bar{A}) - \partial_\theta \left\{ \bar{\delta}(\bar{d} \bar{A}) \right\} \right] = 0,
\] (6.5)

\[
d\bar{\theta} \left[ \partial_{\bar{\theta}} (\bar{d} \bar{A}) - \partial_{\bar{\theta}} \left\{ \bar{\delta}(\bar{d} \bar{A}) \right\} \right] = 0.
\] (6.6)

The last requirement in the above equation leads to the following restrictions

\[
\partial \cdot S = 0, \quad \varepsilon^{\mu\nu} \partial_\mu S_\nu = 0, \quad \partial \cdot R = \varepsilon^{\mu\nu} \partial_\mu \bar{R}_\nu, \quad S_\mu = -\partial_\mu (\partial \cdot A), \quad S_\mu = \varepsilon_{\mu\nu} \partial^\nu E.
\] (6.7)

It is clear that $R_\mu = \varepsilon_{\mu\nu} \bar{R}_\nu$ and the two expressions for $S_\mu$ lead to

\[
S_\mu = -\frac{1}{2} \left[ \partial_\mu (\partial \cdot A) - \varepsilon_{\mu\nu} \partial^\nu E \right], \quad \partial_\mu (\partial \cdot A) + \varepsilon_{\mu\nu} \partial^\nu E = 0,
\] (6.8)

where the r.h.s. of $S_\mu$ is nothing but the equation of motion for the 2D free photon. The latter equation is not invariant under the “duality” transformations (2.6) and $R_\mu = \varepsilon_{\mu\nu} \bar{R}_\nu$ is satisfied for the solutions (5.3) as well as (5.6). The condition (6.5) leads to

\[
\partial \cdot S = 0, \quad \varepsilon^{\mu\nu} \partial_\mu S_\nu = 0, \quad \partial \cdot \bar{R} = \varepsilon^{\mu\nu} \partial_\mu R_\nu, \quad S_\mu = -\partial_\mu (\partial \cdot A), \quad S_\mu = \varepsilon_{\mu\nu} \partial^\nu E.
\] (6.9)

It is evident that now $\bar{R}_\mu = \varepsilon_{\mu\nu} \bar{R}_\nu$ and the two expressions for $S_\mu$ lead to the same conclusions as in (6.8). In fact, equation (6.8) implies that all the conditions on $S_\mu$ (i.e. $\partial \cdot S = 0, \quad \varepsilon^{\mu\nu} \partial_\mu S_\nu = 0, \quad \Box S_\mu = 0$) are satisfied because $\Box (\partial \cdot A) = 0$ and $\Box E = 0$. The consistency with the equation of motion, however, implies that $S_\mu = 0$ on the on-shell. Furthermore, the requirement of duality invariance of the latter equation in (6.8) forces us to choose: $\partial_\mu (\partial \cdot A) = 0, \quad \varepsilon_{\mu\nu} \partial^\nu E = 0$. As an operator equation, the more stringent restrictions: $\partial_\mu (\partial \cdot A) = 0$ and $E = 0$ are expected because if we choose the harmonic states to be the physical state of the theory then $Q_b|\text{Phys} >= 0$ (with $Q_b = \int dx [\partial_\mu (\partial \cdot A) C - (\partial \cdot A) \hat{C}]$ and $Q_d|\text{Phys} >= 0$ (with $Q_d = \int dx [E \hat{C} - \hat{E} C]$) imply that $(\partial \cdot A)|\text{Phys} >= 0$ and $E|\text{Phys} >= 0$ [11,12,17]. Now, Eqn.(6.4) yields the relations: $dx^\rho \Box B_\rho = dx^\rho \Box A_\rho \leftrightarrow \Box R_\rho = \Box \bar{R}_\rho = \Box S_\rho = 0$. Setting the coefficients of $(dx^\rho s^{\rho\theta}),(dx^\rho s^{\rho\bar{\theta}})$ equal to zero leads to: $\partial_\rho (\partial \cdot A) - \varepsilon_{\rho\lambda} \partial^\lambda E = 0$ which, once again, establishes the fact that $S_\mu = 0$ in (6.8). The operator equations: $(\partial \cdot A) = 0, \quad E = 0$ also imply
the same. Note that the coefficient of \((dx^\rho s^{\theta_2})\) leads to no new restrictions as choice of signs in the expansion (3.4) satisfies it trivially. Lastly, setting the coefficients of \((dx^\rho \varepsilon^{\mu \theta})\) and \((dx^\rho \varepsilon^{\mu \theta})\) equal to zero leads to
\[
\begin{align*}
\partial_\mu (\varepsilon_{\rho \lambda} \partial^\lambda C - \partial_\rho \tilde{C}) &= \varepsilon_{\rho \lambda} \partial^\lambda R_\mu + \partial_\rho (\varepsilon_{\mu \nu} R^\nu), \\
\partial_\mu (\varepsilon_{\rho \lambda} \partial^\lambda C - \partial_\rho C) &= \varepsilon_{\rho \lambda} \partial^\lambda R_\mu + \partial_\rho (\varepsilon_{\mu \nu} \tilde{R}^\nu), \\
\partial_\mu [\varepsilon_{\rho \lambda} \partial^\lambda (\partial \cdot A) - \partial_\rho E] &= -[\varepsilon_{\rho \lambda} \partial^\lambda S_\mu + \partial_\rho (\varepsilon_{\mu \nu} S^\nu)].
\end{align*}
\] (6.10)

The last equation is satisfied due to \((\partial \cdot A) = 0, E = 0, \partial \cdot S = 0, \varepsilon^{\mu \nu} \partial_\mu S_\nu = 0\). It is clear that for \(\partial \cdot A = 0, E = 0\), we obtain \(S_\mu = 0\) in (6.8). However, there is another choice \(S_\mu = -\varepsilon_{\mu \nu} \Box A^\nu\) that remains invariant under both the discrete symmetries (5.1) and (5.4) but vanishes on the on-shell \((\Box A_\mu = 0)\). The other two coupled equations for the fermionic vectors (with \(R_\mu = \varepsilon_{\mu \nu} \tilde{R}^\nu, \tilde{R}_\mu = \varepsilon_{\mu \nu} R^\nu\)) are satisfied for the choice of Hodge decomposed versions (5.6) with the restrictions \(R_\mu = \tilde{R}_\mu = 0\). More precisely, these equations lead to: \(R_\mu = \partial_\mu C - \varepsilon_{\mu \nu} \partial^\nu \tilde{C} = 0\) and \(\partial_\mu R_\nu + \varepsilon_{\mu \lambda} \partial^\lambda \tilde{R}_\nu = 0\). Thus, ultimately, we have obtained: \(R_\mu = 0, \tilde{R}_\mu = 0, S_\mu = -\varepsilon_{\mu \nu} \Box A^\nu, \partial \cdot A = 0, E = 0\). With these values together with \(s(x) = \bar{s}(x) = 0\) and the observation that \((s_w A_\mu = -\varepsilon_{\mu \nu} \Box A^\nu)\), we have expansion (3.4) as
\[
B_\mu (x, \theta, \bar{\theta}) = A_\mu (x) + i \theta \bar{\theta}(s_w A_\mu (x)), \quad \Phi (x, \theta, \bar{\theta}) = C(x), \quad \bar{\Phi} (x, \theta, \bar{\theta}) = C(x), \quad (6.11)
\]
which shows that there are no transformations for the (anti-)ghost fields but the gauge field \(A_\mu\) alone transforms to its own equation of motion (cf. Sec. 2) along the \((\theta \bar{\theta})\) direction.

7 Conclusions

We have demonstrated the existence of some new local symmetries by exploiting the mathematical power of the super de Rham cohomology operators of differential geometry defined on a \((2 + 2)\)-dimensional compact supermanifold. As conserved and nilpotent (anti-)BRST charges \((\tilde{Q}_b)Q_b\) are connected with the super exterior derivative \(\tilde{d}\) [3,4], in a similar fashion (anti-)co-BRST charges \((\tilde{Q}_d)Q_d\) are connected with the super co-exterior derivative \(\tilde{\delta}\). These nilpotent charges turn out to be the translation generators along the Grassmannian directions of the supermanifold. A bosonic charge \(Q_w\) is shown to be related with the super Laplacian operator \(\tilde{\Delta}\). This charge turns out to be the translation generator along the bosonic direction (which is equivalent to a couple of intertwined Grassmannian directions) of the supermanifold. The mapping between super operators \((\tilde{d}, \tilde{\delta}, \tilde{\Delta})\) and the local conserved charges is: \(\tilde{d} \Leftrightarrow (Q_b, \tilde{Q}_b), \tilde{\delta} \Leftrightarrow (Q_d, \tilde{Q}_d), \tilde{\Delta} \Leftrightarrow Q_w\). The analogy between the ghost number of a state in the quantum Hilbert space and the degree of a differential form allows one to relate the ordinary de Rham cohomology operators \((d, \delta, \Delta)\) with the conserved charges as: \(d \Leftrightarrow (Q_b, \tilde{Q}_b), \delta \Leftrightarrow (Q_d, \tilde{Q}_d), \Delta \Leftrightarrow Q_w = \{Q_b, Q_d\} = \{\tilde{Q}_b, \tilde{Q}_d\}\). In the setting of the superfield formulation, the above mappings find their geometrical interpretation. The interplay between the discrete- and continuous symmetries of the theory allows one to write down the Hodge decomposed versions for the 2D fermionic vectors which provide an unambiguous explanation for the existence of (anti-)BRST- and (anti-)co-BRST symmetries in the theory. It would be nice to extend these ideas to the interacting case [14,15].
References

[1] J. Thierry-Mieg, J. Math. Phys. 21 (1980) 2834; Nuovo Cim. 56 A (1980) 396.
[2] M. Quiros, F. J. De Urries, J. Hoyos, M. L. Mazou and E. Rodrigues, J. Math. Phys. 22 (1981) 767.
[3] L. Bonora and M. Tonin, Phys. Lett. B 98 (1981) 48.
L. Bonora, P. Pasti and M. Tonin, Nuovo Cim. 63 A (1981) 353.
[4] L. Baulieu and J. Thierry-Mieg, Nucl. Phys. B 197 (1982) 477; ibid. B 228 (1982) 259;
L. Alvarez-Gaumé and L. Baulieu, ibid. B 212 (1983) 255.
[5] D. S. Huang and C.-Y. Lee, J. Math. Phys. 38 (1997) 30.
[6] N. Nakanishi and I. Ojima, Covariant Operator Formalism of Gauge Theories and Quantum Gravity (World Scientific, Singapore, 1990).
[7] T. Eguchi, P. B. Gilkey and A. J. Hanson, Phys. Rep. 66 (1980) 213.
[8] S. Mukhi and N. Mukunda, Introduction to Topology, Differential Geometry and Group Theory for Physicists (Wiley Eastern Ltd., New Delhi, 1990).
[9] J. W. van Holten, Phys. Rev. Lett. 64 (1990) 2863; Nucl. Phys. B 339 (1990) 258
[10] K. Nishijima, Prog. Theo. Phys. 80 (1988) 897, 905; H. Aratyn, J. Math. Phys. 31 (1990) 1240; G. Fülöp and R. Marnelius, Nucl. Phys. B 456 (1995) 442.
[11] R. P. Malik, J. Phys. A: Math. Gen. 33 (2000) 2437 [hep-th/9902146].
[12] R. P. Malik, Int. J. Mod. Phys. A 15 (2000) 1685 [hep-th/9808040].
[13] R. P Malik, Mod. Phys. Lett. A 14 (1999) 1937 [hep-th/9903121].
[14] R. P. Malik, Mod. Phys. Lett. A 15 (2000) 2079 [hep-th/0003128].
[15] R. P. Malik, Mod. Phys. Lett. A 16 (2001) 477 [hep-th/9711050].
[16] E. Harikumar, R. P. Malik and M. Sivakumar, J. Phys. A: Math. Gen. 33 (2000) 7149 [hep-th/0004145].
[17] R. P. Malik, J. Phys. A: Math. Gen. 34 (2001) 4167 [hep-th/0012085].
[18] K. Nishijima, in: Progress in Quantum Field Theory, ed. H. Ezawa and S. Kamefuchi (North-Holland, Amsterdam, 1986).
[19] M. Heannahx and C. Teitelboim, Quantization of Gauge Systems (Princeton University Press, New Jersey, Princeton, 1992).
[20] S. Weinberg, The Quantum Theory of Fields: Modern Applications, Vol. 2 (Cambridge University Press, Cambridge, 1996).
[21] J. Wess and J. Bagger, Supersymmetry and Supergravity, (2nd Edition) (Princeton University Press, New Jersey, Princeton, 1991).