Accuracy standards should be imposed on model waveform and detector calibration accuracies:
- to prevent a significant rate of missed detections,
- to prevent accuracy losses in measurements,
- to avoid unnecessary costs of achieving excess accuracy.

This talk will describe possible abuses of the standards, and ways to avoid them.
Waveform and Calibration Accuracy Standards:

- Combined waveform and calibration accuracy standards:

\[
\sqrt{\langle \delta h_m | \delta h_m \rangle} + \sqrt{\langle \delta h_R | \delta h_R \rangle} < \left\{ \frac{1}{\rho \sqrt{2\epsilon_{\text{max}}}} \right\}
\]

- \( \delta h_m = h_m - h_e \)  
  Model waveform error.
- \( \delta h_R \)  
  Errors from calibration inaccuracies.

- Standards are written in terms of the noise-weighted inner product:

\[
\langle h_e | h_m \rangle = 2 \int_0^{\infty} \frac{h_e^*(f)h_m(f) + h_e(f)h_m^*(f)}{S_n(f)} df,
\]

where \( S_n(f) \) is the power spectral density of the detector noise.

- The maximum allowed errors are determined by \( \rho \), the signal to noise ratio, and \( \epsilon_{\text{max}} \) which determines the missed detection loss rate (typically set to \( \epsilon_{\text{max}} = 0.005 \)).
Waveform accuracy standards can be re-written as:

\[ \frac{\sqrt{\langle \delta h_m | \delta h_m \rangle}}{\rho} = \sqrt{\delta \chi_m^2 + \delta \Phi_m^2} < \begin{cases} \frac{1}{(2\rho_{\text{max}})} & \text{measurement,} \\ \sqrt{2\epsilon_{\text{max}}} & \text{detection.} \end{cases} \]

- Amplitude \( \delta \chi_m \) and phase \( \delta \Phi_m \) errors are defined as

\[ \delta h_m = h_e e^{\delta \chi_m + i\delta \Phi_m} - h_e \approx h_e (\delta \chi_m + i\delta \Phi_m). \]

- Signal-weighted average errors are defined as

\[ \overline{\delta \chi_m^2} = \int_0^\infty \delta \chi_m^2 \frac{4|h_e|^2}{\rho^2 S_n} df, \quad \text{and} \quad \overline{\delta \Phi_m^2} = \int_0^\infty \delta \Phi_m^2 \frac{4|h_e|^2}{\rho^2 S_n} df. \]
More Intuitive Waveform Accuracy Standards

- Waveform accuracy standards can be re-written as:

\[
\sqrt{\frac{\langle \delta h_m | \delta h_m \rangle}{\rho}} = \sqrt{\delta \chi_m^2 + \delta \Phi_m^2} < \begin{cases} 
\frac{1}{(2 \rho_{\text{max}})} & \text{measurement,} \\
\frac{1}{\sqrt{2 \epsilon_{\text{max}}}} & \text{detection.}
\end{cases}
\]

- Amplitude \( \delta \chi_m \) and phase \( \delta \Phi_m \) errors are defined as

\[
\delta h_m = h_e e^{\delta \chi_m + i \delta \Phi_m} - h_e \approx h_e (\delta \chi_m + i \delta \Phi_m).
\]

- Signal-weighted average errors are defined as

\[
\overline{\delta \chi_m^2} = \int_0^\infty \delta \chi_m^2 \frac{4 |h_e|^2}{\rho^2 S_n} \, df, \quad \text{and} \quad \overline{\delta \Phi_m^2} = \int_0^\infty \delta \Phi_m^2 \frac{4 |h_e|^2}{\rho^2 S_n} \, df.
\]

- How do you relate \( \delta \chi_m(f) \) and \( \delta \Phi_m(f) \) to the time-domain waveform errors that arise in waveform modeling?

- How do you estimate these errors reliably?
Maximum Error Fallacy

- Some NR groups have estimated the maximum time-domain waveform errors $\max|\delta \chi_t|$ and $\max|\delta \Phi_t|$, and compared them with the standards for $|\delta \chi_m|$ and $|\delta \Phi_m|$.
- Is this good enough?
Maximum Error Fallacy

- Some NR groups have estimated the maximum time-domain waveform errors $\max|\delta \chi_t|$ and $\max|\delta \Phi_t|$, and compared them with the standards for $|\delta \chi_m|$ and $|\delta \Phi_m|$.

- Is this good enough?

- Consider a model waveform: $h_m(t)$ with errors of the form:

$$h_m(t) = A_e(t) \left[ 1 + \max|\delta \chi_t| g\chi(t) \right] \cos \left[ \Phi_e(t) + \max|\delta \Phi_t| g\Phi(t) \right],$$

with $g\chi = g\Phi = \cos[\lambda \Phi_e(t)]$. 

Bad News! Limiting $\max|\delta \chi_t|$ and $\max|\delta \Phi_t|$ is not sufficient.
Maximum Error Fallacy

Some NR groups have estimated the maximum time-domain waveform errors \( \max|\delta \chi_t| \) and \( \max|\delta \Phi_t| \), and compared them with the standards for \( |\delta \chi_m| \) and \( |\delta \Phi_m| \).

Is this good enough?

Consider a model waveform: \( h_m(t) \) with errors of the form:

\[
h_m(t) = A_e(t) \left[ 1 + \max|\delta \chi_t| g_\chi(t) \right] \cos \left[ \Phi_e(t) + \max|\delta \Phi_t| g_\Phi(t) \right],
\]

with \( g_\chi = g_\Phi = \cos[\lambda \Phi_e(t)] \).

Compute ratio of frequency- to time-domain error measures,

\[
R = \sqrt{\frac{\delta \chi_m^2 + \delta \Phi_m^2}{\max(|\delta \chi_t|^2 + |\delta \Phi_t|^2)}}
\]

using the PN+Caltech/Cornell waveform for \( A_e \) and \( \Phi_e \).
Some NR groups have estimated the maximum time-domain waveform errors $\max|\delta \chi_t|$ and $\max|\delta \Phi_t|$, and compared them with the standards for $|\overline{\delta \chi}_m|$ and $|\overline{\delta \Phi}_m|$. Is this good enough?

Consider a model waveform: $h_m(t)$ with errors of the form:

$$h_m(t) = A_e(t) \left[ 1 + \max|\delta \chi_t| g_\chi(t) \right] \cos \left[ \Phi_e(t) + \max|\delta \Phi_t| g_\Phi(t) \right],$$

with $g_\chi = g_\Phi = \cos[\lambda \Phi_e(t)]$.

Compute ratio of frequency- to time-domain error measures,

$$R = \sqrt{\frac{\delta \chi_m^2 + \delta \Phi_m^2}{\max(|\delta \chi_t|^2 + |\delta \Phi_t|^2)}}$$

using the PN+Caltech/Cornell waveform for $A_e$ and $\Phi_e$.

Bad News! Limiting $\max|\delta \chi_t|$ and $\max|\delta \Phi_t|$ is not sufficient.
Error Envelope Fallacy

- Additional knowledge of the full waveform errors, \( \max|\delta \chi_t| g_\chi(t) \) and \( \max|\delta \Phi_t| g_\Phi(t) \), is needed. Unfortunately the exact time dependencies, \( g_\chi(t) \) and \( g_\Phi(t) \), will never be known.
- Is a partial knowledge of \( g_\chi(t) \) and \( g_\Phi(t) \) sufficient?
Error Envelope Fallacy

- Additional knowledge of the full waveform errors, \( \max |\delta \chi_t| g_\chi(t) \) and \( \max |\delta \Phi_t| g_\Phi(t) \), is needed. Unfortunately the exact time dependencies, \( g_\chi(t) \) and \( g_\Phi(t) \), will never be known.

- Is a partial knowledge of \( g_\chi(t) \) and \( g_\Phi(t) \) sufficient?

- Probably the most we will ever know will be local-in-time error envelope-functions \( G_\chi(t) \) and \( G_\Phi(t) \), that satisfy

  \[
  |g_\chi(t)| \leq G_\chi(t) \leq 1, \quad \text{and} \quad |g_\Phi(t)| \leq G_\Phi(t) \leq 1.
  \]

- Do time-domain bounds imply frequency-domain bounds, i.e., does \( |g(t)| \leq G(t) \) imply \( |g(f)| \leq G(f) \)?
Error Envelope Fallacy

- Additional knowledge of the full waveform errors, \( \max|\delta \chi_t| g_\chi(t) \) and \( \max|\delta \Phi_t| g_\Phi(t) \), is needed. Unfortunately the exact time dependencies, \( g_\chi(t) \) and \( g_\Phi(t) \), will never be known.

- Is a partial knowledge of \( g_\chi(t) \) and \( g_\Phi(t) \) sufficient?

- Probably the most we will ever know will be local-in-time error envelope-functions \( G_\chi(t) \) and \( G_\Phi(t) \), that satisfy

  \[
  |g_\chi(t)| \leq G_\chi(t) \leq 1, \quad \text{and} \quad |g_\Phi(t)| \leq G_\Phi(t) \leq 1.
  \]

- Do time-domain bounds imply frequency-domain bounds, i.e., does \( |g(t)| \leq G(t) \) imply \( |g(f)| \leq G(f) \)?

- No!

- It is not possible to verify the accuracy of a waveform using a time-domain error-envelope function.
Time Domain Accuracy Standards

- An alternate form of the accuracy standards can be written in terms of the time domain $L^2$ norm $\| \delta h_m(t) \|^2 = \int_{-\infty}^{\infty} |\delta h_m|^2 dt$.

- This alternate standard has the form:

$$\frac{\| \delta h(f) \|}{\| h_m(f) \|} = \frac{\| \delta h(t) \|}{\| h_m(t) \|} < \frac{C}{2\rho},$$

where $C$, is a scale invariant ratio of two signal-to-noise measures

$$C^2 = \frac{\rho^2}{2\| h_m(f) \|^2 / \min S_n(f)} \leq 1.$$

- The error envelope functions, $\max |\delta \chi_t| G_\chi(t)$ and $\max |\delta \Phi_t| G_\Phi(t)$, provide strict upper limits for these error measures.
Combined accuracy standards now exist for waveform accuracy and calibration. The model waveform standards can be written as:

\[
\sqrt{\delta \chi_m^2 + \delta \Phi_m^2} < \begin{cases} 
\frac{1}{2\rho_{\text{max}}} & \text{measurement,} \\
\sqrt{2\epsilon_{\text{max}}} & \text{detection.}
\end{cases}
\]

The basic standards are difficult (impossible?) to enforce directly, so easier to enforce time-domain conditions have been derived:

\[
\frac{\|\delta h_m(t)\|}{\|h_m(t)\|} \leq \sqrt{\int_{-\infty}^{\infty} A_m^2 \left( \max |\delta \chi_t|^2 G_{\chi}^2 + \max |\delta \Phi_t|^2 G_{\Phi}^2 \right) dt} \lesssim \left\{ \frac{C}{2\rho_{\text{max}}} \right\} \left\{ \frac{C}{\sqrt{2\epsilon_{\text{max}}}} \right\}
\]
Combined accuracy standards now exist for waveform accuracy and calibration. The model waveform standards can be written as:

$$\sqrt{\delta \chi_m^2 + \delta \Phi_m^2} < \left\{ \begin{array}{l} \frac{1}{(2\rho_{\text{max}})} \text{ measurement,} \\ \frac{1}{\sqrt{2\epsilon_{\text{max}}}} \text{ detection.} \end{array} \right.$$

The basic standards are difficult (impossible?) to enforce directly, so easier to enforce time-domain conditions have been derived:

$$\frac{\|\delta h_m(t)\|}{\|h_m(t)\|} \leq \sqrt{\int_{-\infty}^{\infty} A_m^2 \left( \max |\delta \chi_t|^2 G_{\chi}^2 + \max |\delta \Phi_t|^2 G_{\Phi}^2 \right) dt} \leq \left\{ \begin{array}{l} C/(2\rho_{\text{max}}) \\ C\sqrt{2\epsilon_{\text{max}}} \end{array} \right.$$

- How well do the calibration and search template accuracies currently being used by LIGO satisfy these requirements?
- How well do the waveforms produced by various NR groups satisfy these requirements?