Fibre optic strain measurements for bond modelling of prestressed near-surface-mounted iron-based shape memory alloy bars

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HIGHLIGHTS
- Fibre optic measurements were performed on near-surface-mounted memory-steel bars.
- Decrease in bar stiffness led to a changed behaviour in active bond length.
- An active bond length after prestressing of 200 mm was determined.
- Post-processing of distributed strain delivered distributed stress and slip curves.
- Novel fiber optic strain measurements delivered bond-slip law parameters.

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ABSTRACT
Near-surface-mounted (NSM) iron-based shape memory alloy (memory-steel) bars represent a novel type of reinforcement for prestressed strengthening of concrete structures. To examine the bond behaviour of these embedded reinforcements in detail, fibre optic (FO) methods can be used. In this study, distributed FO strain measurements were performed in small-scale bond experiments and large-scale structural experiments using prestressed and non-prestressed NSM memory-steel bars. The experiments demonstrated the FO measurement technique’s feasibility on memory-steel bars, provided insights into the complex strain profiles before and after prestressing, and revealed the active bond lengths. When the memory-steel bar was subjected to an external load, a reduction in the bar’s stiffness resulted in an increase in active bond length. The results furthermore highlighted the importance of rigid bar fixations to avoid prestress loss. The obtained strain distributions were also processed to calculate the distributed slip and axial bar stress, and to investigate the parameters of three different bond shear stress–slip laws (BSSLs). A linear increasing BSSL with a nonlinear bar stiffness led to a realistic calculation of the axial stress in the bar, while resulting in acceptable slip and strain. The implementation of a constant Young’s modulus consistently overestimated the axial bar stress.

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1. Introduction

The use of iron-based shape memory alloy (Fe-SMA) reinforcements in structural engineering has been the subject of several studies in the last decade. In particular, their application for strengthening deteriorated concrete structures has become a focus of several research groups. It has been demonstrated that the shape memory effect (SME) of Fe-SMAs can be used effectively for prestressed retrofitting of beams, slabs, columns, and other structural elements. The SME of this type of alloy is based on a reversible phase transformation between an austenitic and a martensitic lattice structure induced by mechanical deformation (prestraining) followed by heating and cooling (activation) [1]. Prestress can be obtained by fixing the SMA element to a building component after it has been prestrained at the factory followed by activation [2]. Furthermore, fixation in combination with the activated SME results in mechanical stress in the SMA element, which is transferred into the building component [3] and termed as recovery stress [4].
SME can also be found in other SMAs, such as nickel-titanium [5,6] alloys. The lower material cost of Fe-SMAs makes them more appealing for larger elements, such as reinforcement bars or reinforcement strips. A type of Fe-SMA was developed at the Swiss Federal Laboratories for Materials Science and Technology (Empa) [7,8] to specifically exhibit transformation temperatures that did not damage adjacent concrete components and resulted in high recovery stress. This Fe-SMA was later named memory-steel and is commercially available [9]. Subsequent studies focused on further increasing the recovery stress heat treatment [10].

The effectiveness of memory-steel reinforcements for strengthening concrete structures has been demonstrated in several studies. Memory-steel strips were used to strengthen concrete structures [11] for flexural [3,12,13], and shear loads [14]. These reinforcements were also used to enhance the performance of steel elements [15] and structures [16,17].

Owing to the novelty of ribbed memory-steel reinforcement bars, studies on this type of reinforcement are limited. The material properties of this material investigated in [18] demonstrated high ductility of up to 35% and a high tensile strength of 880 MPa. The bond behaviour of memory-steel bars embedded in concrete was similar to conventional steel bars if installed with a large cover depth [19]. Other studies investigated the application of memory-steel bars in flexural [20] and shear strengthening [21].

One strengthening technique that uses this type of reinforcement is represented by the near-surface-mounted (NSM) method [22,23]. In this method, grooves are cut into the concrete structure followed by the installation of bars using cementitious mortar under a small cover depth. Small to medium-scale bond investigations conducted in [24,25] studied the effects of ductility, heating, prestress, groove geometry, and material properties. The studies confirmed that the bars exhibited excellent bond performance without causing detrimental effects on the bond due to the heating and prestressing process.

In a subsequent study [26], ribbed memory-steel bars were used to strengthen concrete slabs, which led to a significant increase in cracking, yielding, and ultimate load while maintaining a ductile load–deflection behaviour. The prestressing process, including resistive heating of the memory-steel bars, did not damage the cementitious end-anchorage. This proven effectiveness of memory-steel reinforcements has also led to numerous site applications in the construction industry [27,28].

For analytical modelling of the bond behaviour, the differential equation of bond behaviour is generally used with an implemented bond shear stress–slip law [29]. Conventionally, simplified laws, such as constant, linear, bilinear, or nonlinear laws, are used [30]. The authors of [31] presented a novel approach, where the parameters of bond shear stress–slip laws were obtained via a fitting process. The specific solution of the differential equation for slip was fitted to digital image correlation (DIC) slip measurements on externally bonded carbon fibre reinforced polymer strips. This process can be applied only when the reinforcement is applied externally and not embedded in the parent structure.

In previous studies, simplified bilinear bond shear stress–slip laws were implemented to solve the differential equation of bond for pull-out specimens with NSM memory-steel bars on concrete blocks [24,25], and exhibited good agreement with the experimental results.

The analytical modelling procedures are generally based on the results of conventional measurement techniques using load cells for force measurement, linear variable differential transformers for displacement measurement, or DIC systems for measuring surface deformations, strains, and displacements. These measurement techniques suffer from the disadvantage of only being able to measure the specimen externally, i.e., internal behaviour, such as strains in the embedded bar section, depends on calculations.

Conventionally, internal strain measurements were conducted by installing strain gauges on the embedded bar section [32]. In particular, when conducting studies on specimens with short bond lengths, the limited number of discrete measurement points, presence of cables, and required manipulation of the specimen may influence the results and lead to the limited applicability of this measurement technique.

In contrast, a novel measurement technique termed distributed fibre optic (FO) strain measurement enables internal strain measurement on embedded elements with negligible disturbance to bond behaviour. In this measurement technique, glass fibres are installed on the specimens, followed by sending laser light into the fiber reflected and processed in the measurement unit (interferometer). As an output, a distributed strain profile along the fibre length is generated, enabling a high strain
measurement resolution of up to ~1500 strain measurement points per meter. In addition to the high resolution, the used glass fibres are chemically inert and not affected by magnetic fields, electricity, or moisture.

The fibres are generally installed using the same adhesives as those for installing strain gauges, i.e., epoxy adhesives, methylmetacrylates, or cyanoacrylates. Although different types of fibres can be found in the market, glass fibres with polyamide and nylon coatings are used predominantly. Polyamide coatings may enable accurate strain measurements, eliminate slip between the fibre core and coating, and small fibre geometry. Nylon coatings may cause slippage between the glass fibre core and fibre jacket, which can be beneficial in the presence of cracks in the investigated specimen due to a reduction in peak strain [33,34].

In several studies [35–37], different fibres were applied on concrete surfaces in combination with FO sensors embedded in the concrete. The studies demonstrated successful strain measurement on the concrete surface and crack detection. In one study [38], optical fibres were installed on reinforcement bars along the longitudinal ridge in the rib geometry using cyanoacrylate to successfully investigate the shrinkage strains after casting the concrete or tension stiffening behaviour during loading based on crack measurements. In another study [33], the same technique was utilised to investigate the effects of corrosion on the strain profile of internal steel reinforcement. The authors of [39] installed optical sensors based on the fiber Bragg grating technology inside prestressing strands to monitor the post-tensioning process and long-term loss of prestress in a concrete bridge, thereby enabling comparison with current design guidelines. The same method was also used in another study [40] to investigate the transfer length related to the prestressing process with steel strands. In [34], distributed FO sensing with higher measurement resolution was used on embedded reinforcement bars to investigate the bond behaviour under an external load successfully.

In the current investigation, distributed FO measurements were conducted during medium-scale pull-out and large-scale RC slab experiments using ribbed memory-steel bars with a cementitious end-anchorage. The aim was to obtain insights into the form of distributed strain in memory-steel bars in an NSM configuration during external loading or activation of the SME (prestressing). Until now, this was not possible using well-established measurement techniques, including strain gauges and DIC. The derived active bond length, distributed axial stress, slip, and bond shear stress curves along with the obtained bond shear stress–slip laws and subsequent analytical modelling of the bond behaviour of ribbed memory-steel bars formed the focus of the current investigation. In preceding studies [24,25], analytical modelling of NSM memory-steel bars’ bond behavior could only be based on estimated strain curves and strain measurements outside the bond length as the actual distributed strain could not be measured using conventional techniques. Owing to the fact that this study represents the first application of distributed FO measurements on NSM memory-steel bars with and without activation, an important aspect of this study further involved investigating the feasibility and validation of strain measurements using conventional measurement techniques. Additionally, the study of different bond shear stress–slip laws and connected parametric studies formed a vital part of this investigation during post-processing of the obtained experimental data. This investigation marks an important contribution to bond research on internal reinforcement because the observation of distributed strain and subsequent derivation of bond shear stress–slip laws directly from the measurements was impossible. This study also aims to indicate the possibilities and limitations of the distributed FO measurement technique for internal reinforcement.

2. Experiments

The aim of these experiments was to investigate the distributed strain profile of NSM memory-steel bars representing end-anchorages of a strengthening system for concrete structures. The experiments were divided into two studies. The first study focussed on an anchorage-scale investigation, where memory-steel bars were installed on concrete blocks to study the effects of external load, heating, prestress, and anchorage rigidity on the strain profile of the embedded memory-steel bars. These experiments were part of an investigation, as presented in [25]. The second study focussed on the system-scale behaviour of NSM memory-steel bars. The experiments were part of an investigation presented in [26].

2.1. Anchorage-scale experiments

The specimens consisted of NSM memory-steel bars, as described in Section 2.3, installed on concrete blocks with dimensions of 950 × 740 × 300 mm on one bar end and fixed with mechanical fixations on the other bar end. An illustration of the experimental setup is shown in Fig. 1. One bar was subjected to an external load via a hydraulic load jack without activating the SME (Specimen 1-nonact).

Two other bars were activated via resistive heating to approximately 190 °C without external loading. The activated bars were fixed on the unbonded end using two different fixations. One consisted of a wedge-barrel system termed as compliant fixation (Specimen 2-act-comp), whereas the other consisted of a rigid threaded fixation (Specimen 3-act-rig).

The two different fixations were designed to investigate the effect of compliant and rigid fixations on the magnitude of the recovery stress. The experimental matrix is presented in Table 1. Grooves having a width of 34 mm and depth of 27 mm were cut into the top surfaces of the concrete blocks using a diamond saw. The concrete block specimens offered a mean compressive strength (f_{cm}) of 45.8 MPa on the day of the experiment with a w/c ratio of 0.5, a maximum aggregate size of 16 mm, and cement type CEM II/A-LL 42.5 N Vlg.

The bars were bonded using a fast-curing cementitious mortar (SikaGrout 314 N) with a bond length of 800 mm. The mortar had a compressive strength of 71.5 MPa in Specimen 1-nonact and 80.6 MPa in Specimens 2-act-comp and 3-act-rig. The maximum aggregate size measured 4 mm and the water/powder ratio of the mix was 0.124 – 0.132 l according to the data sheet [41]. After placement a thin mortar layer in the groove to fill the voids, the bars were placed and the groove was filled, resulting in a mortar cover of 15 mm. The mortar was cured at room temperature for 3 d before the experiments. The detailed information regarding the specimens and manufacturing process is provided in [25].

2.2. System-scale experiment

A system-scale experiment was conducted to investigate the behaviour of cementitious end-anchorages of an NSM prestressed memory-steel bar used for strengthening an RC slab. The slab was installed as a cantilever in a two-support setup, as depicted in Fig. 2.

Five grooves were cut into the top surface followed by bonding one memory-steel bar in each groove at the bar-end at a length of 500 mm, as shown in Fig. 3 (a). Three FO sensors were installed on three memory-steel bars, respectively. The memory-steel bars used in the current investigation are described in Section 2.3, and the groove geometry and mortar were the same to that used in the anchorage-scale experiments presented in Section 2.1.
To activate SME and attain prestress, heating clamps were attached to the memory-steel bars in sequence. As observed in [25], activation of an NSM memory-steel bar, including the free and bonded length, can result in a longitudinal splitting crack in the mortar cover of the bonded length. However, it was observed that this crack did not have a significant effect on the load capacity of the bonded joint. Although the crack width of this longitudinal splitting crack was negligible, it was decided not to activate the bar section inside the cementitious anchorage in subsequent studies, as described in Section 3.2. A heating device, provided by re-fer AG, was used for the resistive heating process using a current of 400 A (3.85 A/mm²) and energy of 3000 kWs.

### Table 1
Experimental matrix of the anchorage-scale experiments.

| Specimen   | d_b [mm] | l_b [mm] | Activation | f_{ccm} [MPa] | f_{mcm} [MPa] | Fixation |
|------------|---------|---------|------------|--------------|--------------|----------|
| 1-nonact   | 11.5    | 800     | No         | 45.8         | 71.5         | Wedge    |
| 2-act-comp | 11.5    | 800     | Yes        | 45.8         | 80.6         | Wedge    |
| 3-act-rig  | 11.5    | 800     | Yes        | 45.8         | 80.6         | Threaded |

- d_b – Bar diameter
- l_b – Bond length
- f_{ccm} – Mean concrete compressive strength
- f_{mcm} – Mean mortar compressive strength

![Fig. 1. Experimental setup of anchorage-scale experiments.](image)

![Fig. 2. Experimental setup of system-scale experiment (units in mm).](image)

![Fig. 3. (a) Cross-section of strengthened RC slab from [26]; (b) End-anchored memory-steel bars with DIC speckle pattern.](image)
The bars were heated to a maximum temperature of approximately 200 °C. The completed activation process (heating and subsequent cooling to room temperature) resulted in tensile stress in the memory-steel bars, which effectively subjected the bar ends in the cementitious anchorages to an axial tensile stress similar to the load case of Specimen 1-nonact described in Section 2.1. The end anchorages are depicted in Fig. 3(b). In the system-scale experiment, a 5 m long concrete slab having a cross-section of 1.00 × 0.23 m was used.

The slab included 5 × Ø12 mm steel bars for internal tensile and compressive reinforcement, resulting in a tensile reinforcement ratio of 0.0025. The compressive strength of concrete on the day of the experiment was 70.2 MPa with a maximum aggregate size of 16 mm and a w/c ratio of 0.43.

In a separate investigation, measurements were performed with additional specimens and multiple FO sensors during quasi-static loading with and without activating the memory-steel bars.

2.3. Memory-steel bars

Memory-steel bars with a nominal bar diameter of 12 mm (actual diameter ~ 11.5 mm) and rib geometry as per British Standard 6744:2001 [42] were obtained from re-fer AG. The alloy comprised of Fe-17Mn-5Si-10Cr-4Ni-1 (V,C) (mass-%) had a tensile strength of approximately 780 MPa and an ultimate strain of ~ 23%. The bars were prestrained at the factory to an extent of 4–4.5% to enable activation of the SME during the experiments. The stress–strain behaviour of the prestrained memory-steel bars is depicted in Fig. 7(b). Detailed information regarding the used memory-steel bars is provided in [18,43].

2.4. Distributed FO strain sensing

2.4.1. Measurement system and optical fibres

For this investigation, the commercially available measurement system ODiSI 6000 series (LUNA Innovations Incorporated) was used (Fig. 4(a)). The measurement system offers a maximum measurement frequency of 250 Hz, an instrument accuracy of ± 1 με, system accuracy of ± 30 με, and maximum resolution (distance between the strain measurement points) of 0.65 mm [44]. Optical glass fibres having a maximum length of 50 m can be used as sensors. The measurement frequency, resolution, and fibre length are interdependent, i.e., an increase in one of these parameters requires the other parameters to decrease.

For the current setup, a resolution of 0.65 mm could be realised owing to a maximum fibre length of 10 m (system-scale) and 2 m (anchorage-scale), and measurement frequency below 5 Hz.

Bare fibres with polyamide coating were selected because slippage may have occurred between the fibre core and fibre jacket if nylon coated fibres were used. The fibres, depicted in Fig. 4(b), had a diameter of approximately 0.16 mm. To use them as FO sensors, the fibres were equipped with a connector and a termination at the fibre end. The ultimate strain of the fibres was approximately 3% according to the manufacturer. The LUNA software limits strain measurements to 1.2%.

2.4.2. Installation process of fibres

The FO sensors were installed on the bars along the longitudinal ridge of the rib geometry, as illustrated in Fig. 5(a). The memory-steel bars were prestrained prior to the installation of the FO sensors. To prepare the bars for installation of the FO sensors, the surface was sanded manually to remove coarse imperfections due to production, and followed by roughening of the smooth surface with fine sand paper. The bare fibres were then temporarily fixed to the bar using tape with subsequent application of adhesive. The epoxy adhesive Duralco 4525 N (Cotronics) was utilised [45] owing to its high temperature resistance and excellent bonding properties to polyamide coatings and metal surfaces. The adhesive consisted of a resin and a hardener with Al2O3 filler, which offered a tensile shear strength of 69 MPa.

According to the manufacturer, the adhesive offered a maximum operating temperature of 260 °C and a degradation temperature of 310 °C. Detailed information regarding the glass transition temperature was not available. However, preliminary investigations by the authors showed that the adhesive remained solid while heating up to temperatures of 215 °C after curing.

The curing process of the adhesive was executed according to the data sheet at room temperature for 16 h. The accelerated curing option of 120 °C for a duration of 5 min was not feasible because it would have activated the SME of the prestrained memory-steel bars. The installation was executed according to the recommendations of the manufacturer.

3. Results and discussion

3.1. Anchorage-scale experiment

3.1.1. Strain measurement without activation

Fig. 6 shows strain development in the memory-steel bar of Specimen 1-nonact for increasing axial stress due to external load-
The black dashed lines indicate the start (loaded end) and end (free end) of the bond length. Three parts can be distinguished in the strain profiles up to a stress of 414 MPa. The first part towards the load represents a section where the fibre was not bonded to the memory-steel bar; therefore, a constant strain value was evident. The second section shows a plateau strain, which represents the strain in the free memory-steel bar where it was not bonded to mortar. The third and main part represents the bonded length of the bar between the loaded end and free end, exhibiting a non-linear strain profile.

Fig. 6 shows multiple selected strain profiles at different increasing axial stress levels induced by the hydraulic load jack. The figure indicates an increase in axial strain with increasing axial stress up to a software limitation of approximately 1.2%. Therefore, the peak strains at axial stresses higher than ~577 MPa could not be measured. The position of the peak strain is expected to have progressed towards the load free end based on bond theory [25].

Fig. 6 also depicts an increase in the active bond length as the axial stress is increased. The active bond length is defined as the length between the start of the bond length at the loaded end.
and the point at which the strain approaches zero. While examining the strain profile at the maximum stress of ~712 MPa, it was evident that parts of the strain signal were missing beyond a strain of ~0.6%. Also in other curves, parts of the bond length did not deliver strain measurements. This may be attributed to the fact that when strain levels exceed 1.2% in a section of the fibre, the light signal may be disturbed, leading to problems in the measurement system.

Valuable information, such as the active bond length or shape of the strain curves, could be extracted and processed further despite incomplete strain profiles, as described in Section 4. Fig. 6 also shows the last strain profile before bond failure when the stress decreased to ~674 MPa. The final failure occurred when the strain at the end of the bond length at the load free end started to increase.

Fig. 7 depicts the development of the active bond length \( l_{act} \) at increasing axial stress. The active bond length was defined as the length between the start of the bond length on the load side and the coordinate at which the strain in the bar decreases to approximately zero [31]. Two linear regression fits, which were separated at an axial stress of 647 MPa, were performed in the commercially available software Matlab to demonstrate their behaviour clearly. The equations for each fit were as follows:

\[
\sigma_{\text{bar,1}} = 1.498 \ l_{act} \quad \text{for} \quad l_{act} \leq 430 \ mm
\]

\[
\sigma_{\text{bar,2}} = 0.1798 \ l_{act} + 566.3 \quad \text{for} \quad l_{act} > 430 \ mm
\]

Between approximately 600 MPa and 700 MPa, the slope of the linear correlation decreased significantly from 1.50 MPa/mm to 0.18 MPa/mm. When the trend was compared to the stress–strain behaviour of the prestrained memory-steel bars, depicted in Fig. 7 (b), a connection between the yielding point and decrease in slope can be made. The term yielding is not completely correct in the case of prestrained memory-steel bars because the stress level at change in stiffness is influenced by the maximum stress and strain level at prestraining. However, yielding is still used herein for simplicity. The influence of yielding on bond deterioration and associated decrease in bond stress was also reported in [46] and described in current design standards [47]. Therefore, it is expected that the decrease in the slope of the relationship between axial stress and active bond length is a consequence of yielding of the prestrained bars. The level of axial stress at change of slope in Fig. 7(a) and (b) does not comply entirely.

However, it is assumed that this delay results from the fact that the highest axial stress and associated reduction in cross-section is related to transferring the respective tensile stress, as shown in Fig. 7, and Fig. 7. Based on the data, it was determined that the respective active bond length was approximately 210 mm. Therefore, the present anchorage length of 800 mm would exhibit a safety factor of 3.8 if it was used in a prestressing application.

Fig. 7 also implies that a bond length larger than ~400 mm would not result in a significant gain in pull-out capacity. A similar statement was found in [25], where an anchorage with a bond length of 400 mm for the same materials resulted in a pull-out load similar to that measured for a bond length of 800 mm.

In Fig. 8, the FO measurements and DIC measurements on the free length of the bar before the starting point of the bond length are compared. The two measurements were compared by means of the measured strain versus axial stress obtained from the axial force, which was measured using a load cell during loading. The strain from FO measurement was obtained by collecting the strain values located at the longitudinal coordinate at the starting point of the bond length on the load side. The strains representing the DIC technique were obtained by measuring the average strain on the unbonded bar surface using a 3D DIC system. The DIC and FO measurements showed good agreement. Owing to the lower measurement frequency of the DIC system compared to the FO measurement, the inconsistencies in the FO measurement curve at 225 MPa and 381 MPa created by the actuation of the manual hydraulic pump are not visible in the measurement curve of the DIC system. Detailed information about the respective DIC measurements can be found in [25].

### 3.1.2. Strain measurements after activation

Fig. 9 shows the strain profiles of Specimens 2-act-comp and 3-act-rig after activation via resistive heating and cooling back to room temperature. The required strain profile is displayed along the fibre length. The vertical dashed lines represent the start and end of the embedded bar length. The section of interest contained (from the fixed end towards the free end from right to left in Fig. 9): a part of the fibre that was not bonded to the bar, a part where the fibre was installed on the unbonded bar section, a part where the fibre was installed on the bonded bar section, a part...
where the fibre was installed on the unbonded bar exiting the embedded length (free end), and a short part of the fibre not bonded to the bar. The comparison between the strain profiles of Specimens 2-act-comp and 3-act-rig indicates the effect of a compliant and rigid mechanical fixation during activation of the shape-memory effect. In the unbonded bar section near the fixed end, negative strain was observed. This negative strain, which is commonly known as recovery strain, was caused by a contraction in the bar.

The compliant fixation of Specimen 2-act-comp resulted in a significantly higher absolute value of recovery strain in the unbonded bar section. The magnitude of the recovery strain measuring −0.208% was lower in the case of Specimen 2-act-comp because the compliant fixation allowed shortening of the bar.

A negative recovery strain of −0.096% was observed in the case of Specimen 3-act-rig despite the fact that the rigid fixation did not allow shortening of the bar during activation. As stated in [25], it was assumed that this recovery strain originated from residual displacements and rigidity in the overall test setup. In the same study, negative recovery strain values were measured using a DIC system on the surface of the memory-steel bar, resulting in a strain of −0.19% in the case of Specimen 2-act-comp and −0.06% in the case of Specimen 3-act-rig. The values measured using the FO system and DIC system were in good agreement considering that the values of the DIC system were obtained by evaluating the reduction in the distance between two points on the bar surface, thereby effectively averaging the strain within this measurement length.

In both specimens, the strain increased from negative values in the unbonded length to a positive tensile strain peak at the beginning of the cementitious anchorage. However, the magnitude of these strain peaks varied significantly owing to the compliant and rigid behaviour of the fixations. The positive strain peaks indicated the tensile stress in the memory-steel bar due to the SME. As described in a previous investigation [25], the compliant fixation of Specimen 2-act-comp resulted in a reduced prestress of 193 MPa, resulting in lower tensile loading on the bonded bar compared to the rigid fixation of Specimen 3-act-rig, where a prestress of 261 MPa was observed. These axial stresses were obtained via force measurements using a load cell adjacent to the mechanical fixations.

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The positive strain peak measured using the FO system in the current investigation was 0.264%. Based on this strain measurement and examined stress–strain behaviour of prestrained memory-steel bars (Fig. 7(b)), the axial stress can also be estimated to be approximately 278 MPa. The deviation of 17 MPa for the load-cell measurement corresponds to an axial force of 1.8 kN, which is equal to 6% of the measured prestress, and can be accepted considering that this value is similar to the accuracy of the load cell of 1 kN. Overall, it can be stated that the FO strain measurements were successfully validated using two independent measurement systems.

The strain profile of Specimen 2-act-comp showed a lower peak strain originating from tensile stress and a discontinuity within the first 25 mm of the bond length. The discontinuity was expected to be caused by local debonding of the fibre from the bar while delivering strain signals. Therefore, a reduced peak strain was measured, which was not used to evaluate the tensile load in the bar. In the main part of the bond length, both specimens exhibited strains fluctuating around zero. This was expected because of the rigid bond with the adjacent mortar and concrete. Towards the free end of the memory-steel bar, both specimens exhibited a steep drop in strain, representing contraction in memory-steel due to the recovery strain.

3.2. System-scale experiment

Fig. 10 depicts the strain profile in the centre memory-steel bar after activation. It should be noted that the bar was only bonded at the ends via 500 mm long cementitious anchorages, whereas the bar section between the anchorages was not bonded. The measured distributed strain profile exhibited two pronounced peaks at the position where the bar entered the cementitious anchorages. These peaks result from an axial tensile stress generated by activating the SME and obtained recovery stress (prestress).

Between the anchorages, fluctuations in the strain signal were evident. These fluctuations are expected to be caused by a combination of two reasons. The fibre was installed only on one side of the bar facing the groove in addition to the bar showing initial curvatures and eccentricities after installation.

During activation, the curvatures of the bar were reduced by SME, thereby subjecting some sections to bending strains. The negative strain may be attributed to the recovery strain generated by small slippage in the anchorages, elastic deformations of the strengthened structure, and geometric eccentricities in the bar. The elastic deformation in the short sections between the heating clamps and cementitious anchorage also resulted in overall contraction of the bar. Another cause for the fluctuations in the unbonded length was that the fibre was partly in contact with the rough groove surface, which might have caused local strain peaks. As this investigation focused on the strain in the bonded bar sections, the strain profiles in the unbonded length were not studied further.

Fig. 11 shows the strains in the memory-steel bar inside both cementitious anchorages with maxima of 0.34% (right anchorage) and 0.26% (left anchorage). Fig. 11 also indicates an active bond length of approximately 200 mm for both anchorages. It is evident that the strain in both anchorages attained different maxima; however, several aspects have to be considered.

Firstly, small amounts of mortar flowed outside the scaffolding while casting the anchorages, resulting in the bar being partly bonded in the first ~30 mm of the free length. This required the heating clamps to be placed at a distance from the anchorages. Therefore, the sections in between were neither fully activated (only by conduction of thermal energy) nor fully bonded when subjected to tensile stress from the fully activated free length. Moreover, the length of these sections varied at the left and right anchorages, resulting in unequal boundary conditions for the bars at the left and right anchorages.

When the strain profiles in Fig. 11 were studied, it was evident that the slope of the strain curves tended to decrease towards the maximum. This behaviour was not observed when pull-out Specimen 1-nonact was subjected to an external load (Fig. 6). This behaviour was expected to be caused by minor damages in the cementitious bond in the first ~15 mm, thereby not providing full restraint to the bar.

The measurements obtained using two other FO sensors on two other bars did not deliver useable strain signals or stopped at an early stage. This is attributed to local failure of the fibres or interface failure between the resin and bar. Therefore, the strain profile of the centre bar was investigated further and presented in the following sections.

4. Post-processing and analytical modelling

The measured strain distributions can be used to determine the distributed axial stress, slip, and bond shear stress curves. This was not possible via conventional measurement techniques. Furthermore, the analytical procedures allow the implementation of different bond shear stress–slip laws and direct comparison of the obtained strain and slip.

4.1. Axial stress in anchorage due to prestress

Based on the experimentally determined stress–strain behaviour of the prestrained memory-steel bars (Fig. 7(b)), the measured strain in the memory-steel bars (Fig. 11) can be utilised to obtain the stress inside the anchorages via linear interpolation. The resulting stress profiles are shown in Fig. 12. The stresses in the anchorage indicate the prestress generated by activating the memory-steel bars in the unbonded length. In the left anchorage, a stress of 300 MPa was determined, whereas in the right anchorage, a stress of 352 MPa was calculated. The possible reasons for this inconsistency have been discussed previously.

The mean value of both measurements resulted in a prestress of 326 MPa. In a previous study on the same specimen [26], prestress was calculated based on separate measurements. The calculations resulted in an estimated prestress of 314 MPa, which is similar to what was found in the current study based on FO measurements.

4.2. Calculation of slip from distributed strain measurement

The measured strains shown in Fig. 11 can be utilised to calculate the relative displacement between the memory-steel bar and surrounding mortar, commonly known as slip. Excessive slip in the anchorages would enable free conraction of the bar and result in a loss of prestress via recovery strain. The slip of the memory-steel bar in the anchorages can be calculated by obtaining the cumulative sum of the strains. For this calculation, the command cumsum [48] was used in the commercially available software Matlab. The results are illustrated in Fig. 13.

In the presence of large concrete deformations, the implementation of concrete strain for slip calculation can be significant. However, this requires different forms of the differential equation of bond behaviour [49]. As large deformations were not present in the current study (see [26]), strains in the adjacent cementitious materials were not considered.

Owing to the higher strain in the right anchorage, the maximum slip in the right anchorage (0.138 mm) was greater than that in the left anchorage (0.122 mm) with a difference of 0.016 mm.
As the full load capacity of the anchorage was not attained, the slip decreased to zero before the end of the anchorage, resulting in an active bond length of approximately 200 mm.

In addition to the FO system, the slip of the memory-steel bar in the left anchorage was also measured using a DIC system [26]. The maximum slip in the left anchorage according to DIC measurement fluctuated between 0.124 and 0.130 mm, whereas the maximum slip in the left anchorage calculated based on FO measurement was 0.122 mm, i.e., a deviation of 0.005 mm from the mean value of the DIC measurement (0.127 mm). Thus, there was good agreement between the independent measurement techniques. To estimate the effect of slip on the obtained prestress, the sum of both slips was calculated (0.26 mm) and divided by the free bar length (3800 mm) to obtain the contraction in the bar (~0.007%) due to activation.
Based on the literature [4], the recovery stress loss due to this contraction delivered a value of 4.2 MPa. This value can be better interpreted if it is related to a full prestress of approximately 320 MPa, which delivers a ratio of 1.3%. Therefore, the anchorages represent compliant fixations, as discussed in Section 3.1.2.

4.3. Determination of bond shear stress–slip law

4.3.1. Process

As a basis for the analytical bond models, a form of the differential equation of bond behaviour presented in Equation (3) was utilised, with $\tau_b$ representing the slip, $x$ the longitudinal coordinate, $d_b$ the bar diameter, $E_b$ the Young’s modulus of the bar, and $s_b$ the bond shear stress.

$$\frac{d^2 s_b}{d x^2} - \frac{4}{d_b E_b} \tau_b(x) = 0 \tag{3}$$

This form of the differential equation does not consider deformations in the adjacent cementitious material for a simpler solving process. As explained in Section 4.2, this approach was found plausible due to negligible strains in the surrounding substrate.

To obtain specific solutions of the differential equation of bond behaviour, a bond shear stress–slip law is required [24,25]. In this section, a novel approach is presented, where the parameters of different bond shear stress–slip laws are found by fitting the specific solution of the differential equation for axial strain to the measured strain curves (Fig. 12). The parameters were then used as input for specific solutions of slip and bond shear stress.

$$s_b'' - \frac{4}{d_b E_b} \tau_b(s_b) = 0 \tag{4}$$

To solve this equation, one of the bond shear stress–slip laws displayed in Fig. 14 can be used. Owing to the dependency of the secant modulus on the first derivative of slip, the specific solutions for this equation can only be obtained numerically. However, for the curve-fitting process, the secant modulus was defined as a vector with discrete values for each strain value. The values were obtained by calculating the differential quotient of the stress–strain curve shown in Fig. 7(b). The strain distributions in each anchorage (Fig. 12), enabled assigning a secant modulus to each coordinate point of the bond length. This resulted in Equation (5), as illustrated in Fig. 15.

$$s_b'' - \frac{4}{d_b E_b(x)} \tau_b(s_b) = 0 \tag{5}$$

Fig. 15 shows the distribution of the secant modulus, which was modified using a smoothing algorithm to eliminate fluctuations induced by fluctuations in the measured strain profile.

4.3.3. Constant bond shear stress–slip law

With a constant bond shear stress–slip law that requires $\tau_b(x) = \tau_b = \text{const.}$, and boundary conditions

$$s_b(l_r) = 0 \tag{6}$$

$$s_b'(l_r) = 0 \tag{7}$$

the specific solutions for slip

$$s_b(x) = \frac{2 \tau_b (l_r^2 - 2 l_r x + x^2)}{d_b E_b} \tag{8}$$

and strain

A constant, a linear increasing and a bilinear bond shear stress–slip law were used to enable closed-form specific solutions. The implemented laws are illustrated in Fig. 14.

In current design guidelines, such as [50], nonlinear bond shear stress–slip laws are proposed. Solving the differential equation using the nonlinear law requires a numerical solving process. Additionally, the use of a nonlinear bond shear stress–slip law was not expected to deliver significant advantage owing to the low load level and predominantly elastic behaviour. Therefore, the following section focuses on the implementation of constant, linear, and bilinear bond shear stress–slip laws.

4.3.2. Constant and variable stiffness

As investigated previously [25], the use of a constant Young’s modulus or variable secant modulus impacts the obtained curves of strain, slip, and bond shear stress. If a variable secant modulus is used in Equation (3), it can be expressed as a function of the first derivative of slip, which is equal to the strain:

$$s_b'' - \frac{4}{d_b E_b(s_b)} \tau_b(s_b) = 0$$

Fig. 13. Slip in anchorages of prestressed memory-steel bar calculated via FO measurement.

Fig. 14. Different bond shear stress–slip laws: (a) Constant; (b) Linear increasing; (c) Bilinear.
can be obtained, where \( l_i \) is equal to the coordinate \( x \) along the bond length, and strain and slip is reduced to zero. Furthermore, this coordinate is equal to the end of the active bond length. The parameters \( s_b \) and \( l_i \) were obtained using the nonlinear regression algorithm nlinfit in the commercially available software Matlab to fit Equation (9) to the measured strains (Fig. 11). Fig. 16 shows the results of the fitting process, and the results of the left anchorage are displayed as a representative example. The measured slip curves were obtained via numerical integration of the measured strains, as explained in Section 4.2.

The fitting was performed using a constant Young’s modulus and a variable secant modulus (Fig. 15). Fig. 16 shows that the constant Young’s modulus resulted in a linear strain curve, as expected from constant bond shear stresses, whereas the nonlinear variable second modulus resulted in a nonlinear strain curve. When the fitted curves were compared to the measurements, it was evident that the use of a constant Young’s modulus delivered relatively good results although it underestimated the active bond length. In contrast, the use of a variable secant modulus overestimated the maximum strain and slip although it showed better agreement in terms of active bond length. However, both procedures tend to underestimate the active bond length. It is also evident that using a constant Young’s modulus resulted in approximately twice the constant bond shear stress compared to a variable secant modulus. This fact originates from the decreasing values of \( E_b \) at increasing strain values.

To evaluate the fitted constant bond shear stress, a plausibility check was performed to calculate the average bond shear stress based on axial stresses found in Section 4.1 and an estimated active bond length of 200 mm. An axial stress of 300 MPa delivers an average bond shear stress of 4.3 MPa (left anchorage), whereas an axial stress of 352 MPa delivers an average bond shear stress of 5.1 MPa (right anchorage). By comparing these values to the fitted bond shear stress in Fig. 16, the results obtained using a constant \( E_b \) (\( s_b = 11.8 \) MPa) appeared unrealistically high, whereas the results obtained with a variable \( E_b \) (\( s_b = 3.9 \) MPa) delivered more plausible values. Furthermore, the high bond shear stress resulting from a constant \( E_b \) would require a short active bond length of ~72 mm in the plausibility calculation.

4.3.4. Linear increasing bond shear stress–slip law

For a linear increasing bond shear stress–slip law, the bond shear stress is defined as

\[
s_b(x) = s_b(x) = \frac{4}{d_b E_b} \left( \frac{\tau_b(x) - \tau_b}{\tau_b} \right)
\]

where \( a \) denotes the slope of the increasing relationship. Implementation in Equation (3) and introduction of the term

![Graphs showing fitting results, constant bond shear stress–slip law, left anchorage.](image-url)
provides the modified differential equation
\[ \frac{d^2 s_b}{dx^2} - s_b \alpha^2 = 0 \]  

By using the boundary conditions
\[ s_b(x) = \frac{e_{\text{measured}} \cosh(x\omega)}{\sinh(\omega l)} \]  
\[ s_b'(x) = e_{\text{measured}} \frac{\sinh(x\omega)}{\sinh(\omega l)} \]  
\[ \tau_b(x) = \frac{d_b E_b}{4} s_b(x) = \frac{e_{\text{measured}} \cosh(x\omega)}{\sinh(\omega l)} \]

The parameter \( \alpha \) in Equation (16) was then fitted to match the measured strain curves. The results of the fitting process for the left anchorage are depicted in Fig. 17. It is evident that the implementation of a linear increasing bond shear stress–slip law results in a larger active bond length compared to that obtained if a constant bond shear stress–slip law was used. However, the equations tend to overestimate the active bond length. As the maximum measured strain was used as a boundary condition, the curves fit perfectly at the coordinate \( x = 500 \text{ mm} \). The assumption of a variable secant modulus results in a lower maximum slip and bond shear stress. Moreover, the derived bond shear stress–slip law shows a lower slope for a variable secant modulus. For the right anchorage, the equations seem to deliver more accurate fitting results and more realistic active bond lengths compared to that of the left anchorage.

4.3.5. Bilinear bond shear stress–slip law

The solution of the differential equation based on the bilinear bond shear stress–slip law has been widely discussed in previous studies [24,25]. For this type of law, the differential equation must be solved twice. The specific solutions are connected at a specific coordinate \( x_0 \), which separates the elastic zone from the bond damage zone. Due to the extent of the equations involved, the authors refer to a previous study [24] for detailed derivation and specific solutions. Once the specific solutions for strain, slip, and bond shear stress were obtained, the input parameters \( x_0, s_{b,el}, \) and \( s_{b,\text{max}} \) were iterated for the obtained strain curve to match the measured strains. The iteration of \( s_{b,\text{max}} \), the parameter representing the maximum slip at failure of the bonded anchorage, was not feasible because the anchorage remained predominantly elastic. Therefore, \( s_{b,\text{max}} \) was defined as 2.5 mm based on a previous study [24].

As a representative example, the results obtained for the left anchorage are shown in Fig. 18. The use of a constant Young’s modulus and variable secant modulus both provided good results. Equations with a constant Young’s modulus seem to provide better results in terms of strain and slip, which might have originated from inaccuracies of the implemented \( E_b(x) \). However, when the delivered bond shear stress curves were compared, the constant Young’s modulus seemed to deliver unrealistically high peak bond shear stresses. It should be noted that the obtained bond shear stress–slip laws only reflect the behaviour for the current load level corresponding to prestress. If laws were to be expressed for the maximum load level at failure, the procedure would deliver different results.

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**Fig. 17.** Fitting results, linear increasing bond shear stress–slip law, left anchorage.
4.3.6. Summary and discussion

A summary of the results presented in Section 4.3 is provided in Table 2. When the goodness of the fits were evaluated using the mean squared error (MSE), Table 2 suggests that the best accordance was obtained for the bilinear bond shear stress–slip law with a constant $E_b$ with an MSE of $1.7262 \times 10^{-3}$ for the left anchorage. The comparison also showed that the largest MSE of $1.5689 \times 10^{-3}$ was obtained using a constant bond shear stress–slip law with a constant $E_b$ for the right anchorage.

Generally, the data suggest that a variable $E_b$ does not deliver substantially better fitting results than a constant $E_b$. However, this fact seems plausible considering the low load level and low nonlinearity of the memory-steel material.

$E_b$ showed significant influence when the fitted bond shear stress parameters $s_b$ and $s_{b\max}$ were studied, as evident from the constant and bilinear bond shear stress–slip laws. However, the use of a constant $E_b$ consistently results in significantly higher values therein.

By comparing the mean values of the MSEs for the left ($5.7757 \times 10^{-3}$) and right anchorages ($5.6987 \times 10^{-3}$), it was observed that the fitting process resulted in similar goodness of fits.

Another possibility to evaluate the obtained bond shear stress–slip laws is the axial load, which can be calculated by integrating the bond shear stress $s_b(x)$ along the bond length $l_b$ and multiplying with the bar diameter $d_b$ and $\pi$. To obtain the axial stress $\sigma_{\text{bar}}$, the calculated force was divided by the cross-sectional area of the bar to obtain

$$\sigma_{\text{bar}} = \frac{4}{d_b} \int_0^{l_b} s_b(x) dx$$

Fig. 18. Fitting results, bilinear bond shear stress–slip law, left anchorage.

Table 2

| S | Anch | $E_b$ [GPa] | $s_b$ [MPa] | $l_b$ [mm] | $l$ [TPa] | $x_0$ [mm] | $s_{b\text{init}}$ [mm] | $s_{b\max}$ [MPa] | $s_{b\max}$ [mm] | MSE [-] | $\sigma_{\text{bar,init}}$ [MPa] | $\sigma_{\text{bar,final}}$ [MPa] |
|---|------|-------------|-------------|------------|-----------|------------|----------------|----------------|----------------|--------|----------------|----------------|
| Constant | L | 162.7 | 11.8195 | 408.0392 | 6.6172 | 10^{-3} | 382 |
| Constant | L | Variable | 3.9473 | 367.0368 | 6.6172 | 10^{-3} | 382 |
| Linear | L | 162.7 | 217.8434 | 4.7820 | 10^{-3} | 184 |
| Linear | L | Variable | 191.6055 | 3.41 | 10^{-3} | 184 |
| Bilinear | L | 162.7 | 466.7970 | 0.0496 | 17.3065 | 2.5 | 423 |
| Bilinear | L | Variable | 457.1725 | 0.0457 | 6.7342 | 2.5 | 423 |
| Constant | R | 162.7 | 18.1741 | 426.7829 | 1.5689 | 10^{-3} | 233 |
| Constant | R | Variable | 4.6424 | 381.4433 | 1.5689 | 10^{-3} | 233 |
| Linear | R | 162.7 | 386.0740 | 2.8120 | 10^{-3} | 410 |
| Linear | R | Variable | 310.6804 | 7.6914 | 10^{-3} | 410 |
| Bilinear | R | 162.7 | 488.7054 | 0.0813 | 36.2962 | 2.5 | 301 |
| Bilinear | R | Variable | 445.6762 | 0.0263 | 7.1079 | 2.5 | 301 |
| Calculated according to Section 4.1 | | | | | | | |
| Calculated according to [26] | | | | | | | |

1 Calculated according to Equation (18).
By applying Equation (18) to the bond shear stress curves presented in Fig. 16, Fig. 17, and Fig. 18, the values presented in Table 2 were obtained. It is evident that using a constant Young’s modulus results in a higher axial stress than that obtained by using a variable secant modulus owing to overestimation of bar stiffness. For a constant Young’s modulus, the use of a bilinear bond shear stress–slip law results in the highest axial stress of 560 MPa. For the variable secant modulus, the highest values were obtained by implementing a linear bond shear stress–slip law. A comparison between the current calculated values and axial stresses calculated from the FO strain measurements, as presented in Section 4.1, shows that the use of a constant Young’s modulus tends to overestimate the axial stress in the bar significantly, with the constant bond shear stress–slip law delivering the closest proximity. In contrast, the variable secant modulus delivers higher or lower values depending on the implemented law. The values calculated in Section 4.1 and [26] should be considered as estimations for the prestress because the actual prestress could not be measured directly. However, prestresses between 270 MPa and 350 MPa can be considered as realistic estimations. Furthermore, the comparison shows that the use of a linear increasing bond shear stress–slip law with variable secant modulus delivered a realistic axial stress of 341 MPa.

It should be noted that the obtained bond shear stress–slip laws only represent the current geometries, materials, and load level. Therefore, they do not represent the failure load level and other configurations. Furthermore, owing to the multi-parametric fitting procedures, the actual fitted parameter values may not be realistic.
in terms of axial stress although they can provide acceptable results in terms of strain and slip, as shown in Sections 4.3.3 to 4.3.5. The goodness of the fit using a variable $E_b$ also depends on the definition of $E_b$. As it was obtained based on fitting to the derivative of the stress–strain behaviour, inaccuracies may have occurred, thereby resulting in different strain, slip, and bond shear stress curves.

4.4. Parametric studies

To better understand the effects of the parameters of the bond shear stress–slip laws, parametric studies were conducted by varying the implemented values by focusing on the maximum bond shear stress $\tau_{b,\text{max}}$ and the modulus $E_b$. As a representative bond shear stress–slip law, the bilinear law was selected.

4.4.1. Bond shear stress

The variation of $\tau_{b,\text{max}}$ in the bilinear bond shear stress–slip law is illustrated in Fig. 19. To investigate the effects of the parametric
study, Equation (3) was solved numerically using the bilinear law and the following boundary conditions:

\[ s_b'(0) = 0 \]  

and  

\[ s_b'(l_b) = 0.005 \]  

By implementing fixed strain values as boundary conditions, i.e., the same approach as in Section 4.3.5, where the calculations were based on the measured strain at the starting point of the bond length on the load side \((x = l_b)\). Only the strain at the end of the bond length \((x = 0)\) was defined as zero. In this parametric study, \(\tau_{b,max}\) was varied from 7 MPa to 25 MPa with \(s_{b,el}\) and \(s_{b,max}\) being fixed to 0.23 mm and 2.5 mm, respectively. The values for \(s_{b,el}\) and \(s_{b,max}\) were found in related investigations [24,25], and therefore used in the current study.

The results are shown in Fig. 20. It is evident that by varying \(\tau_{b,max}\) while keeping other parameters constant, including the Young’s modulus, significant changes occurred in the obtained curves. It was observed that the point of inflection, representing the separation between the elastic and damage zones, shifted towards \(x = 500\) when the maximum bond shear stress was increased. This shift of the respective coordinate, also known as \(x_0\), was enabled by the numerical solving process.

As \(E_b\) remained constant for all values of \(\tau_{b,max}\), the area under the bond shear stress curve, and therefore the axial stress and pull-out load, must remain constant. This was verified by integrating the bond shear stress curves according to Section 4.3.6, which yielded the same force for every \(\tau_{b,max}\) value.

Therefore, the area under the bond shear stress–slip laws also remained constant with varying \(\tau_{b,max}\) values.

When examining the obtained slip and bond shear stress curves in Fig. 20, an indirect proportional relationship between \(\tau_{b,max}\) and maximum slip was evident. Therefore, the largest \(\tau_{b,max}\) value resulted in the lowest maximum slip. It was also observed that lower values of \(\tau_{b,max}\) resulted in a substantial increase in end slip (slip at \(x = 0\)). This fact is significant because an increase in end slip often indicates bond failure. This study also showed that the active bond length decreased with increasing \(\tau_{b,max}\) values.

### 4.4.2. Constant and variable Eb

In the second parametric study, the effect of \(E_b\) on the strain, slip, bond shear stress curves, and bond shear stress–slip law was studied. Therefore, \(E_b\) is defined as follows:

\[ E_b\left(s_b(x)ight) = E_0 - k s_b(x) \]  

where \(E_0\) was set to be 160 GPa and \(k\) represented the decrease in stiffness, as depicted in Fig. 21. The parameter \(k\) was varied from zero (constant \(E_0\)) to 16,000 GPa. The bilinear bond shear stress–slip law was defined using the parameters \(\tau_{b,max} = 13\) MPa, \(s_{b,el} = 0.23\) mm, and \(s_{b,max} = 2.5\) mm according to [24].

Furthermore, the boundary conditions were modified for clearer results:

\[ s_b'(0) = 0 \]  

and  

\[ s_b'(l_b) = 0.007 \]  

The results are shown in Fig. 22. In contrast to maintaining the same \(E_b\) value, a varying \(E_b\) value does not require an equal integral of the bond shear stress curves, which is emphasised by the figure. This relationship becomes more obvious considering that the strain at \(x = l_b\) is fixed to a certain value. The equation with a lower \(E_b\) value satisfies this boundary condition at a lower load level compared to an equation with a higher \(E_b\) value.

Thus, the axial stress according to Equation (18) is not constant for different \(E_b\) values, thereby representing the relationship that a less stiff material requires lower axial stress to reach the same strain as a more stiff material. The calculated axial stress with varying \(k\) values is listed in Table 3. As the maxima of the bond shear stress curves are fixed at 13 MPa, the area under the curve increases when the peak value shifts towards \(x = 0\).

The decrease in axial stress also leads to a decrease in the area under the bond shear stress slip curves (note that colour-coded curves for bond–slip in Fig. 22 end at different slips). The figure indicates that a higher stiffness will result in higher active bond length. Furthermore, a higher stiffness results in an increased slip if the strain at \(x = l_b\) is fixed as a boundary condition. This behaviour seems plausible because the lowest \(k\) value also results in the lowest axial force. If the strain at \(l_b\) was not fixed, the opposite behaviour would be true, as demonstrated in [25]. Thus, the obtained curves and relationship to the parameters highly depend on the chosen boundary conditions.

### 5. Conclusions

The following conclusions were made based on the investigation:

- The distributed FO measurement technique using glass fibres as sensors enabled valuable insights into the bond behaviour of embedded memory-steel reinforcement. These insights could not have been obtained with conventional measurement techniques such as digital image correlation and strain gauge measurements.
- When subjecting a non-activated NSM memory-steel bar to an external load, it was observed that the linear correlation between axial stress and active bond length showed a significant decrease in slope at \(\tau = 650\) MPa. The results suggested that this phenomenon was caused by a change in the stiffness of the memory-steel bar.
- Prestressing the memory-steel bars by activating the shape-memory effect resulted in an active bond length of \(~ 200\) mm, as indicated by FO measurements. A prestress loss due to slip in the cementitious anchorages of the bar was estimated to be 1.3% of the full prestress, thereby exhibiting satisfactory performance.
- It was observed that a compliant fixation resulted in a decrease in prestress of 26%, compared to a rigid fixation. Installation of the fibres on the memory-steel bar surface without a protective groove delivered satisfactory measurements. The placement of fibres inside a groove in the reinforcement is expected to provide more robust specimens.
- The FO strain measurements were validated via load cell and DIC measurements.

### Table 3

Calculated axial stress in the bar with fixed bond shear stress–slip parameters depending on the stiffness parameter \(k\).

| \(k\) [GPa] | \(\sigma_{b,max}\) [MPa] |
|---|---|
| 0 | 1137 |
| 4000 | 1039 |
| 8000 | 942 |
| 12,000 | 844 |
| 16,000 | 746 |
The FO strain measurements enabled the calculation of distributed axial stress and slip of the bars inside the anchorage areas. The calculated values were in good agreement with the independent measurements, as reported by the authors.

Comparisons of the obtained MSE suggest that the best fitting results between measured and simulated strain curves were obtained using a bilinear bond shear stress–slip law with a constant Young's modulus. The use of a constant Young's modulus consistently delivered significantly higher bond shear stress parameters compared to a variable secant modulus. A linear increasing bond shear stress–slip law with variable secant modulus delivered the most realistic axial stress of 341 MPa when compared to previous studies. However, the constant or bilinear laws delivered unrealistic axial stresses.

The implementation of a constant Young's modulus consistently delivered higher axial stresses in the memory-steel bar compared to a variable secant modulus. The relationship between the constant Young's modulus or variable secant modulus and the obtained strain, slip, and bond shear stress curves highly depends on the boundary conditions used to solve the differential equation. A constant modulus may result in a higher slip than that obtained using a variable modulus if the strain on the load side is fixed by the boundary condition. If the strain is not fixed, the variable modulus may result in a higher strain.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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