Rapidly rotating pulsar radiation in vacuum nonlinear electrodynamics

V.I. Denisov¹, I.P. Denisova², A.B. Pimenov¹, V.A. Soklov⁴,¹

¹Physics Department, Moscow State University, Moscow 119991, Russia
²Moscow Aviation Institute (National Research University) Volokolamskoe Highway 4, Moscow 125993, Russia

Abstract In this paper we investigate vacuum nonlinear electrodynamics corrections on rapidly rotating pulsar radiation and spin-down in the perturbative QED approach (post-Maxwellian approximation). An analytical expression for the pulsar’s radiation intensity has been obtained and analyzed.

1 Introduction

Vacuum nonlinear electrodynamics effects is an object that piques a great interest in contemporary physics [1–4]. First of all it is related to the emerging opportunities of experimental research in terrestrial conditions using extreme laser facilities like Extreme Light Infrastructure (ELI) [5–7], Helmholtz International Beamline for Extreme Fields (HIBEF) [8]. It opens up new possibilities in fundamental physics tests [9–11] with an extremal electromagnetic field intensities and particle accelerations that have never been obtained before.

At the same time investigation of vacuum nonlinear electrodynamics effects in astrophysics gives us an additional opportunity to carry out a versatile research using natural extreme regimes of strong electromagnetic and gravitational fields with intensities unavailable yet in laboratory conditions. Compact astrophysical objects with a strong field, such as pulsars and magnetars are best suited for vacuum nonlinear electrodynamics researches. Nowadays, there are many vacuum nonlinear electrodynamics effects predicted in the pulsar’s neighborhood. For example electron-positron pairs production [12] and photon splitting [13], photon frequency doubling [14], light by light scattering and vacuum birefringence [15], transient radiation ray bending [16, 17] and normal waves delay [18]. Some of predicted effects are indirectly confirmed by astrophysical observations. For instance the evidence of the absence of the high-field (surface fields more then $B_p > 10^{13} G$) radio loud pulsars can be explained by pair-production suppression due to photon splitting [19].

In this paper we calculate vacuum nonlinear electrodynamics corrections to electromagnetic radiation of rapidly rotating pulsar and analyze pulsar spin-down under these corrections.

This paper is organized as follows. In Sec. 2, we present vacuum nonlinear electrodynamics models and discuss their main physical properties and predictions. In Sec. 3 pulsar radiation in post-Maxwellian approximation is calculated. Sec. 4 is devoted to analysis of pulsar spin-down under vacuum nonlinear electrodynamics influence. In the last section we summarize our results.

2 Vacuum nonlinear electrodynamics theoretical models

Modern theoretical models of nonlinear vacuum electrodynamics suppose that electromagnetic field Lagrange function density $L = L(I_{(2)}, I_{(4)})$ depends on both independent invariants $I_{(2)} = F_{ik}F^{ki}$ and $I_{(4)} = F_{ik}F^{kl}F_{lm}F^{mi}$ of the electromagnetic field tensor $F_{ik}$. The specific relationship between Lagrange function and the invariants depends on theoretical model choice. Nowadays the most promising models are Born-Infeld and Heisenberg-Euler electrodynamics.

Born-Infeld electrodynamics is a phenomenological theory originating from the requirement of self-energy finiteness for pointlike electrical charge [20]. In subsequent studies, the attempts of quantization were performed [21, 22] and also it was revealed that Born-Infeld theory describes dynamics of electromagnetic fields on D-branes in string theory [23–25]. As the main features of Born-Infeld electrodynamics one can note the absence of birefringence (however there are modifications of the Born-Infeld theory [26] with the vacuum birefringence predictions) and dichroism.

¹soklov.sev@inbox.ru
for electromagnetic waves propagating in external electromagnetic field [27]. Furthermore, this theory has a distinctive feature – the value of electric field depends on the direction of approach to the point-like charge. This property was noted by the authors of the theory and also eliminated by them in the subsequent model development[28].

Lagrangian function in Born-Infeld electrodynamics has the following form:

\[
L = -\frac{1}{4\pi a^2} \left\{ \sqrt{\left[1 - \frac{a^2}{2} I_2(\sigma) - \frac{a^4}{4} (I_4 + \frac{a^6}{8} I_2^2) - 1\right]} \right\},
\] (1)

where \(a\) is a characteristic constant of theory, the inverse value of which has a meaning of maximum electric field for the point-like charge. For this constant only the following estimation is known: \(a^2 < 1.2 \cdot 10^{-32}\) G \(^{-2}\).

The other nonlinear vacuum electrodynamics – Heisenberg-Euler model [15, 29] was derived in quantum field theory and describes one-loop radiative corrections caused by vacuum polarization in strong electromagnetic field. Unlike Born-Infeld electrodynamics, this theoretical model possess vacuum birefringent properties in strong field.

Effective Lagrangian function for Heisenberg-Euler theory has the following form:

\[
L = \frac{I(2)}{16\pi} \cdot \frac{\alpha B^2}{8\pi^2} \int_0^\infty \frac{e^{-\sigma} d\sigma}{\sigma^3} \left[ xy\sigma^2 \text{ctg}(x\sigma)\text{ch}(y\sigma) \right.
\]
\[
+ \frac{\sigma^2}{3} (a^2 - y^2) - 1 \bigg] d\sigma,
\] (2)

where \(B_c = m^2 c^3 / eh = 4.41 \cdot 10^{13}\) G is the value of characteristic field in quantum electrodynamics, \(e\) and \(m\) are the electron charge and mass, \(\alpha = e^2 / hc\) – fine structure constant and for brevity we use the notations

\[
x = -\frac{i}{\sqrt{2} B_c} \left\{ \sqrt{\frac{1}{2} (B^2 - E^2)} + i (B \ E) - \right\}
\]
\[
- \sqrt{\frac{1}{2} (B^2 - E^2)} - i (B \ E) \bigg],
\] (3)

\[
y = \frac{1}{\sqrt{2} B_c} \left\{ \sqrt{\frac{1}{2} (B^2 - E^2)} + i (B \ E) + \right\}
\]
\[
+ \sqrt{\frac{1}{2} (B^2 - E^2)} - i (B \ E) \bigg].
\] (4)

Many attempts to find out the experimental status for each of these theories were taken for a long time, but nowadays it still remains ambiguous. There are experimental evidences in favor of each of them. Heisenberg-Euler electrodynamics predictions were experimentally proved in Delbrück light-by-light scattering [30], nonlinear Compton scattering [31], Schwinger pair production in multiphoton scattering [1]. At the same time the recent astrophysical observations [32, 33] point on the absence of vacuum birefringence effect which favors the Born-Infeld theory prediction. The measurements performed for the speed of light in vacuum show that it doest’n depend on wave polarization with the accuracy \(\delta c / c < 10^{-28}\). So clarification of vacuum nonlinear electrodynamics status requires the expansion of the experimental test list both in terrestrial and astrophysical conditions. The main hopes on this way are assigned to the experiments with ultra-high intensity laser facilities [4] and astrophysical experiments with X-ray polarimetry [34] in pulsars and magnetars neighborhood.

As it follows from Lagrangians (1)-(2) vacuum nonlinear electrodynamics influence becomes valuable only in strong electromagnetic fields comparable to \(E, B \sim 1/\alpha\) for Born-Infeld theory and \(E, B \sim B_c\) for Heisenberg-Euler electrodynamics [35]. In case of relatively weak fields \((E, B < < B_c)\) the exact expressions (1) and (2) can be decomposed and written [36] in the form of unified parametric post-Maxwellian Lagrangian:

\[
L = \frac{1}{3\pi} \left\{ 2I_2(\sigma) + \xi \left[ (\eta_1 - 2\eta_2) I_2^2(\sigma) + 4\eta_2 I_4(\sigma) \right] \right\},
\] (5)

where \(\xi = 1/B_c^2 = 0.5 \cdot 10^{-27}\) G \(^{-2}\), and the post-Maxwellian parameters \(\eta_1\) and \(\eta_2\) depend on choice of theoretical model. In case of Heisenberg-Euler electrodynamics post-Maxwellian parameters \(\eta_1\) and \(\eta_2\) are coupled to fine structure constant \(\alpha [37]\):

\[
\eta_1 = \frac{\alpha}{45\pi} = 5.1 \cdot 10^{-5}, \quad \eta_2 = \frac{7\alpha}{180\pi} = 9.0 \cdot 10^{-5}.
\] (6)

For Born-Infeld electrodynamics these parameters are equal to each other and can be expressed through the field induction \(1/\alpha\) typical of this theory [37]:

\[
\eta_1 = \eta_2 = \frac{a^2 B_c^2}{4} < 4.9 \cdot 10^{-6}.
\] (7)

The electromagnetic field equations for the post-Maxwellian vacuum electrodynamics with the Lagrangian (5) are equivalent [35] to equations of Maxwell electrodynamics of continuous media

\[
\partial_m F_{ik} + \partial_t F_{km} + \partial_k F_{mi} = 0,
\] (8)

\[
\frac{\partial Q^{k}}{\partial x^{l}} = -\frac{4\pi}{c} F^{k l},
\] (9)

with specific nonlinear constitutive relations [18]

\[
Q^{k} = F^{k i} + \xi \left[ (\eta_1 - 2\eta_2) I_2 F^{k i} + 4\eta_2 F_{(3)}^{k i} \right],
\] (10)

where \(F_{(3)}^{k i} = F^{k n} F_{n m} F^{m i}\) is the third power of the electromagnetic field tensor. Tensor \(Q^{k}\) can be separated into two terms \(Q^{k} = F^{k i} + M^{k i}\), one of which \(M^{k i}\) will have a meaning similar to substance polarization tensor in electrodynamics of continuous media.
Also it should be noted that in post-Maxwellian approximation stress-energy tensor $T^{ik}$ and Pointing vector $S$ have a form:

$$T^{ik} = \frac{1}{4\pi} \left\{ (1 + \xi \eta I_{(2)}) F^{ik}_{(2)} - \frac{g^{ik}}{8} [2 f_{(2)} + \xi (\eta_1 + 2 \eta_2) I_{(4)} - 4 \eta_2 \xi I_{(6)}] \right\},$$

where $F^{ik}_{(2)} = g^{im} F_{mn} F^{mk}$ is the second power of the electromagnetic field tensor, $g^{ik}$ is the metric tensor and the Greek index takes a value $\mu = 1, 2, 3$.

As it was shown in [38] that post-Maxwellian approximation turns out to be very convenient for vacuum nonlinear electrodynamics analysis, so we will use this representation (8)-(12) to calculate radiation of the rapidly rotating pulsar.

3 Rapidly rotating pulsar radiation in post-Maxwellian nonlinear electrodynamics

Pulsars are the compact objects best suited for vacuum nonlinear electrodynamics tests in astrophysics. They possess sufficiently strong magnetic field with the strength varying from $B_p \sim 10^8 G$ up to $B_p \sim 10^{14} G$, so as these values are close to $B$, the vacuum nonlinear electrodynamics influence can be manifested. At the same time, the pulsar’s fast rotation may enhance nonlinear influence on its radiation.

Let us consider a pulsar of radius $R$, rotating around an axis passing through its center with the angular velocity $\omega$. We shall suppose that the rotation is fast enough, so the linear velocity for the points on the pulsar’s surface is comparable to speed of light $c \omega R / c \sim 1$. We assume that pulsar’s magnetic dipole moment $m$ is inclined to the rotation axis at the angle $\theta_0$, therefore cartesian coordinates of this vector varies under rotation as $m = \{m_x = m \sin \theta_0 \cos \omega t, m_y = m \sin \theta_0 \sin \omega t, m_z = m \cos \theta_0 \}$.

As the vacuum nonlinear electrodynamics influence in post-Maxwellian approximation has the character of a small correction to Maxwell theory one can represent the total electromagnetic field tensor $F^{ik}$ in form

$$F^{ik} = F^{ik}_{(0)} + f^{ik},$$

where $F^{ik}_{(0)}$ is the electromagnetic field tensor of the rotating magnetic dipole $m$ in Maxwell electrodynamics and $f^{ik}$ is the vacuum nonlinear correction. Substituting (13) in to (10) and retaining only the terms linear in small value $f^{ik}$ it can be found that:

$$Q^{ik} \simeq f^{ik} + F^{ik}_{(0)} + M^{ik}_{(0)},$$

where $M^{ik}_{(0)} = M^{ik}(F^{ij}_{(0)})$ - polarization tensor calculated in approximation of the Maxwell electrodynamics field $F^{ij}_{(0)}$.

Electromagnetic field equations (8)-(9) with the account of (13)-(14) then will take a form:

$$\partial_n F^{ik}_{(0)} + \partial_k F^{ni}_{(0)} + \partial_m f_{ik} + \partial_l f_{km} + \partial_l f_{lm} = 0,$$

$$\frac{\partial f^{ik}}{\partial x^l} + \frac{\partial F^{ik}_{(0)}}{\partial x^l} = - \frac{4\pi}{c} j^k.$$  

The solution of these equation may be obtained by successive approximation method. In initial approximation we assume that $F^{ik}_{(0)}$ is the solution of Maxwell electrodynamics equations

$$\partial_m f_{ik} + \partial_k f_{im} + \partial_l f_{lm} = 0,$$

$$\frac{\partial f^{ki}}{\partial x^l} + \frac{\partial M^{ik}_{(0)}}{\partial x^l} = 0.$$  

To satisfy homogeneous equation (17) electromagnetic potential $A^k$ should be introduced $f_{ik} = \partial_k A_i - \partial_i A_k$. Using this potential the inhomogeneous equation (18) under the Lorentz gauge will take a form

$$\partial_n f^{ik} A_k = - \frac{\partial M^{ik}_{(0)}}{\partial x^l}.$$  

It is more convenient to rewrite the last equation in terms of the antisymmetric Hertz tensor $\Pi^{ik}$ defined as:

$$A^k = - \frac{\partial \Pi^{ki}}{\partial x^l}.$$  

In this case equation (19) will take a simple form

$$- \partial_n \partial^a \Pi^{ki} = \Box \Pi^{ki} = M^{ki}_{(0)},$$  

where $\Box = - \partial_n \partial^a$ is D’Alembert operator. Six independent equations in (21) may be expressed in vector form by introducing Hertz electric $\Pi$ and magnetic $\mathbf{Z}$ potentials [39]:

$$\Pi^{a} = \Pi^{a0}, \quad \mathbf{Z}^{a} = \frac{1}{2} \varepsilon^{a\mu\nu} \Pi_{\mu\nu},$$  

where $\varepsilon^{a\mu\nu}$ - Levi-Civita symbol and all of the indexes take values $\alpha, \mu, \nu = 1, 2, 3$. In terms of these potentials equations (21) can be rewritten:

$$\Box \Pi = \Pi^{a0}, \quad \Box \mathbf{Z} = \mathbf{M}_a,$$  

where the source vectors $\Pi^{a0}$ and $\mathbf{M}_a$ are expressed from polarization tensor $M^{ik}_{(0)}$ by equalities:

$$\Pi^{a0} = M^{a0}_{(0)}, \quad \mathbf{M}_a = \frac{1}{2} \varepsilon^{a\mu\nu} M^{\mu\nu}_{(0)}.$$  

The explicit components of these vectors may be easily obtained in Minkowski space-time with the using of (10) and (24):

\[
P_0 = 2\xi \left\{ \eta_1 (E_0^2 - B_0^2)E_0 + 2\eta_2 (B_0 E_0)B_0 \right\},
\]

\[
M_0 = 2\xi \left\{ \eta_1 (E_0^2 - B_0^2)E_0 - 2\eta_2 (B_0 E_0)E_0 \right\},
\]

where \(E_0\) and \(B_0\) are the electromagnetic field components of the rotating magnetic dipole in Maxwell electrodynamics, the expressions for which are well described in literature [40] and the field vectors themselves have the form:

\[
B_0(r,t) = \frac{3}{r^5} \left\{ \frac{\hat{m}(\tau)}{r} r - \frac{r^2 \hat{m}(\tau)}{c^2} \right\} + \left[ \frac{\hat{m}(\tau)}{c^2} \right] \sum_{n=1}^{\infty} \frac{1}{\omega_n c^2} \int_{u_0}^{\infty} du u^{n-3/2} = \frac{3}{r^5} \left\{ \frac{\hat{m}(\tau)}{r} r - \frac{r^2 \hat{m}(\tau)}{c^2} \right\},
\]

\[
E_0(r,t) = \frac{r \cdot \hat{m}(\tau)}{c^2 r^2} + \left[ \frac{\hat{m}(\tau)}{c^2} \right] \sum_{n=1}^{\infty} \frac{1}{\omega_n c^2} \int_{u_0}^{\infty} du u^{n-3/2} = \frac{r \cdot \hat{m}(\tau)}{c^2 r^2} + \left[ \frac{\hat{m}(\tau)}{c^2} \right] \sum_{n=1}^{\infty} \frac{1}{\omega_n c^2} \int_{u_0}^{\infty} du u^{n-3/2}.
\]

where \(\tau = t - r/c\) is the retarded time and the dot corresponds to the derivative of the magnetic dipole moment \(\hat{m}(\tau)\) with the respect to the retarded time \(\tau\). Therefore, the right hand side of the equations (23) can be obtained by using of (25)–(28). The equations (23) themselves are the unhomogeneous hyperbolic equations the exact solution methods of which are well developed and described in literature [41–43]. Since we are interested only with the radiative solutions for the pulsar’s field, when solving equations (23) one should retain only the terms decreasing not faster than \(\sim 1/r\) with the distance to the pulsar. At the same time there is no restrictions on the rotational velocity so \(\omega R_p/c \sim 1\).

Due to excessive unwieldiness here we will not represent the whole solutions for the Hertz potentials \(\Pi\) and \(Z\), but we will use the results for them to find the components of the electromagnetic field tensor \(\epsilon_{ik}\) and radiation properties such as Pointing vector \(\mathbf{S}\) and toal intensity \(I\). The Pointing vector components represented by (12) in post-Maxwellian electrodynamics, can be simplified by the radiative asymptotic condition \(S^0 \sim 1/r^2\) which actually means that for radiation description we can use the Maxwellian expression for this vector:

\[
S^0 = c T_0^0 \sim \frac{c}{4\pi} T_0^0 (2).
\]

Finally, the total intensity can be obtained by integrating of Pointing vector by the surface with the normal \(\mathbf{n}\) directed to the observer located at the large distance \(r >> R_p\) from the pulsar:

\[
I = \int (\mathbf{S} \cdot \mathbf{n}) r^2 d\Omega,
\]

where \(d\Omega\) is the solid angle.

Solutions of equations (23) with the right hand side (25), (26) lead to the following expression for the pulsar radiation intensity:

\[
I = \frac{2\omega^4 B_p^2 B_c^6}{3c^3} \sin^2 \theta_0 \left\{ 1 + \frac{2}{35Y} \frac{B_p^2}{B_c^2} \left[ (24Y)^9 \left[ \frac{1}{15} \eta_1 - \eta_2 \right] \right. \right.
\]

\[
\left. \times C_i(2Y) \sin^2 \theta_0 + 4Y \left[ \eta_2 - \frac{311}{45} \eta_1 \right] C_i(2Y) \right. + Y^3 \left[ \eta_1 - \frac{15}{5} \eta_2 \right] (2Y^4 - 3Y^2) - 18\eta_1 - 10\eta_2 \cos(2Y) \sin^2 \theta_0
\]

\[
+ \frac{Y}{6} \left[ \frac{45\eta_2 - 311\eta_1}{15} (2Y^6 - 3Y^4) + (172\eta_1 - 60\eta_2)Y^2 \right]
\]

\[
- 336\eta_1 \cos(2Y) + Y^2 \left[ \frac{30\eta_2 - 2\eta_1}{5} (2Y^6 - 3Y^4) \right]
\]

\[
- \frac{1}{5} \left[ \frac{311\eta_1 - 45\eta_2}{15} (2Y^8 - 3Y^4) \right]
\]

\[
+ \left( \frac{15\eta_2 - 141\eta_1}{} \right) Y^2 + 84\eta_1 \sin(2Y) \right\},
\]

where the following notations are used for brevity: \(k = \omega/c\) and \(Y = kr\), also \(B_p\) is the magnetic field. It is obvious that obtained intensity can be represented in form which distinguishes Maxwell radiation intensity and vacuum nonlinear electrodynamics correction. In this representation it is convenient to introduce the "correction function" \(\Phi(\theta_0, Y)\) which is a multiplier before the scaling factor \(B_p^2/B_c^2\) determining how strong the vacuum nonlinear electrodynamics influence on the pulsar radiation is:

\[
I = \frac{2\omega^4 B_p^2 B_c^6}{3c^3} \sin^2 \theta_0 \left\{ 1 + \frac{B_p^2}{B_c^2} \Phi(\theta_0, Y) \right\}.
\]

For the most known rapidly rotating pulsars [44] with \(Y \sim 1\) the factor \(B_p^2/B_c^2 \ll 1\) is small which matches the requirements of post-Maxwellian approximation. At the same time this means that vacuum nonlinear electrodynamics corrections will be sufficiently suppressed in comparison with Maxwell electrodynamics radiation. However, this assessment may be waived for special sources of so called Fast Radio Bursts (FRB’s), six cases of which have recently been discovered [46]. One of the hypotheses explaining the nature of FRBs assumes that their source is a rapidly rotating neutron star with the strong surface magnetic field \(B_p > B_c\) called blitar [45]. In this case vacuum nonlinear electrodynamics corrections to pulsar radiation became significant but at the same time this makes strict solution (31) unapplicable because it was obtained in low-field limit. So our further evaluations will be applied to the case of the typical rapidly rotating pulsar, for instance PSR B1937+21 with \(B_p \sim 4.2 \cdot 10^8 G \ll B_c\), and maybe for blitar but with the restriction \(B_p < B_c\). The main purpose of our analysis will be in identification of new qualitative features of the pulsar radiation and comparing vacuum nonlinear corrections to the electromagnetic radiation with the other weak energy loss mechanisms.
Let’s investigate the properties of the correction function $\Phi(\theta_0, Y)$. First of all, it should be noted that there is no radiation when the pulsar dipole moment is coaxial with the rotation axis i.e. when $\theta_0$ is zero. The correction function depends both on the angle $\theta_0$ and the angular velocity through $Y = \omega R/c$, so the $\Phi(\theta_0, Y)$ may be represented as a surface defined in the region where it’s coordinates take values $0 \leq Y < 1$ and $0 \leq \theta_0 \leq \pi/2$. Some isolines – the relations $\Phi(Y)$ at which this surface takes a constant value $\Phi(\theta_0, Y) = \text{const}$ are represented at the Fig.1, the numerical values for which were obtained with the $\eta_1$ and $\eta_2$ from the Heisenberg-Euler theory.

The obtained isolines differ from each other by the absolute value of the correction function but all of them have pronounced extremum at some point which lies on the red line. This means that for each fixed angle $\theta_0$ between the pulsar dipole moment and rotation axis there is an angular velocity at which vacuum nonlinear electrodynamics corrections become the most pronounced. Increasing $Y$ at the constant $\theta_0$ up to the value marked by the red line increases correction of vacuum nonlinear electrodynamics. The subsequent $Y$ and angular velocity increase becomes ineffective because the vacuum corrections in this case will be reduced. It should be noted, that increasing $Y \rightarrow 1$ also will enhance total pulsar luminosity which is $I \approx \omega^4 \sin^4 \theta_0$ but at the same time, as it was mentioned, this will decrease vacuum nonlinear electrodynamics correction on the Maxwell radiation background. For instance, if $\theta_0 \sim \pi/2$ the correction will most significantly stand out for the pulsars with $Y \sim 0.5$. So the correction function $\Phi(\theta_0, Y)$ plays a role of a contrast. And the red line in Fig.1 marks the relation between $\theta_0$ and $Y$ for the best contrast.

Another distinctive feature of the pulsar radiation is manifested in sophisticated, non-polynomial dependence between the radiation intensity (31) and the angular velocity, which greatly complicates the analysis. Performing power-law approximation of (31) will allow us to describe vacuum nonlinear electrodynamics influence on the pulsar spin-down, in traditional terms of braking-indexes and torque-functions [47]. It also provides a possibility for comparison of the pulsar spin-down caused by different non-electromagnetic dissipative factors and the power-law relation between the radiation intensity and angular velocity, for instance with the quadrupole gravitational radiation. Let us investigate the features of the pulsar spin-down as a result of the radiation, with the amendments of vacuum nonlinear electrodynamics.

### 4 Pulsar spin-down

The observed spin-down rate [47] can be expressed by derivative of the angular velocity as

$$
\dot{\omega} = -\frac{I}{J R^2} = -\frac{2B_2^2R_4^4}{3J} \sin^2 \theta_0 \left\{ Y^3 + \frac{B_4^2}{R_4^2} Y^3 \Phi \right\},
$$

(33)

where $J$ is the pulsar’s inertia momentum and the dot means the time derivative. For description in terms of torque functions, the right hand side of equation (33) should be represented in polynomial form of angular velocity

$$
Y^3 \Phi(\theta_0, Y) = \sum_{n} \alpha_n(\theta_0) Y^n = \sum_{n} \alpha_n(\theta_0) \left( \frac{\omega R}{c} \right)^n,
$$

(34)

where $\alpha_n$ are decomposition coefficients and the number of the terms $N$ should be selected sufficient to ensure the required accuracy of the decomposition. We will take the number of terms in the expansion (34) equal to $N = 8$. This choice ensures the accuracy of power-law approximation for the pulsars with the $Y > 0.6$ better then 0.1%. It should be noted, that the series does not converge at $Y \sim 1$ but its replacement by the partial sum with the specially selected number of terms allows to accomplish the polynomial approximation which provides a good match with the exact expression near $Y \sim 1$, but leads to significant errors when $Y < 1$. In this case the expansion coefficients (with the $\eta_1$ and $\eta_2$ from the Heisenberg-Euler theory) for the terms providing the largest contribution are represented in the Table 1. The coefficients not listed in the table are small and can be neglected in further consideration. For quantitative analysis, we will take the inclination angle is equal to $\theta_0 = \pi/2$.

| $\theta_0$ | $\alpha_1 \cdot 10^4$ | $\alpha_2 \cdot 10^4$ | $\alpha_3 \cdot 10^4$ | $\alpha_4 \cdot 10^4$ | $\alpha_5 \cdot 10^4$ |
|-----------|---------------------|---------------------|---------------------|---------------------|---------------------|
| $\pi/2$   | 1.7                | -4.0               | 2.4                | -0.3               | -0.2               |
| $\pi/3$   | 1.5                | -3.8               | 2.7                | -0.5               | -0.3               |
| $\pi/6$   | 1.1                | -3.6               | 3.3                | -0.9               | -0.5               |
of nonlinear electrodynamics spin-down with the other nonlinear electrodynamics dissipative factors, it gives the upper limit of nonlinear electrodynamics influence.

After the expansion, the right hand side of the spin-down equation (33) will take the form:

$$\omega = K_M + \sum K_n \omega_n^n,$$

where $K_M$ corresponds to the torque function of the dipole magnetic radiation in Maxwell electrodynamics [48, 49]:

$$K_M = -\frac{2B_p^2R_c^6}{3Je^3}\sin^2 \theta_0,$$

(36)

and $K_n$ are the torques originating form nonlinear vacuum electrodynamics:

$$K_n = \alpha_n(\theta_0)K_M \left( \frac{B_p}{B_c} \right)^2 \left( \frac{R_c}{c} \right)^n \omega^{-3}.$$  

(37)

Let us compare pulsar spin-down caused by nonlinear vacuum electrodynamics and dissipation caused by gravitational waves radiation. Among several possible ways of gravitational radiation by an isolated pulsar we will choose two most relevant scenario – quadrupole mass radiation [50] and the radiation caused by Rossby waves [51], called r-modes.

Quadrupole gravitational radiation can be originated by the strain caused by the pulsar rotation, which is especially likely for rapidly rotating pulsars. The spin-down under this kind of radiation can be represented as:

$$\dot{\omega} = K_\nu \omega^5 = -\frac{32GJe^2}{5c^3} \omega^5,$$

(38)

where $G$ is a gravitational constant and $\epsilon$ is the pulsar ellipticity, which is in accordance with modern representations $\epsilon < 10^{-4}$ [47].

Another reason for gravitational waves emission by isolated pulsar are the oscillations modes induced by the pulsar rotation. Gravitational radiation is caused by instability of such oscillations. As it was shown by Owen et al. [52] for young rapidly rotation pulsars spin-down caused by r-modes can be expressed in form:

$$\dot{\omega} = K_\nu \omega^7 = -\frac{217\pi F^2GM^2R^6\beta^2}{3^55^2Jc^4} \omega^7,$$

(39)

where $M$ and $R$ are the pulsar mass and it’s radius, the r-mode oscillations saturation amplitude $10^{-7} \leq \beta_{sat} \leq 10^{-5}$ was defined by [53], and dimensionless constant $F$ as it has been shown in [54] is to be strictly bounded within $1/(20\pi) \leq F \leq 3/(28\pi)$.

So the torque function for the quadrupole gravitational radiation $K_\nu$ can be compared with nonlinear electrodynamics torque $K_5$ and the r-modes radiation torque $K_R$ can be compared with the torque $K_7$. For this comparison we suppose the pulsar with the typical radius $R_c = 30 \text{ km}$, mass $M = 2M_\odot$ and inertia moment $J = 10^{45} \text{ g} \cdot \text{cm}^2$. Also we assume, that dipole moment inclination is $\theta_0 = \pi/2$ and post-Maxwellian parameters correspond to Heisenberg-Euler theory (choice of Born-Infeld parameters in first estimation gives the similar order).

For the pulsar with the surface magnetic field $B_p \sim 10^{13}G$, which ellipticity reaches the maximum value $\epsilon \sim 10^{-4}$, r-mode saturation amplitude $\beta_{sat} \sim 10^{-6}$, and $F = 1/(20\pi)$ the following estimation takes place $K_5/K_\nu \sim 1.3 \cdot 10^{-11}$ and $K_7/K_\nu \sim 2.6 \cdot 10^{-7}$. So the quadrupole and r-mode gravitational radiation torque will significantly exceed nonlinear electrodynamics torque coupled with the terms $\sim \omega^5$ and $\sim \omega^7$ in spin-down equation. For another parameter set the opposite case takes place. If the pulsar distortion and ellipticity is two orders of magnitude lower ($\epsilon \sim 10^{-8}$), and the pulsar field is stronger $B_p \sim 10^{15}G$ then $K_5/K_\nu \sim 12.6$ and $K_7/K_\nu \sim 25.6$. However, it should be noted that the rapidly rotating pulsars with such a strong field have not been observed yet. Nevertheless, the theoretical models assuming the blizzars as the sources of Fast Radio Burst [45] do not eliminate the possibility of such a strong electromagnetic fields for the rapidly rotating pulsar. Therefore obtained ratio between the torques seems very exotic but still can not be completely discarded.

5 Conclusion

In this work, we have studied vacuum nonlinear electrodynamics influence on rapidly rotating pulsar radiation in parameterized post-Maxwellian electrodynamics. In assumption of flat space-time the analytical description of radiation intensity (31) was obtained. Despite on the fact that the expression for the intensity is quite complicating for analysis some new features of pulsar’s radiation have been obtained. For instance, it was shown that for the rapidly rotating pulsar vacuum nonlinear electrodynamics corrections observation is optimal only for certain relations between the inclination angle $\theta_0$ of the magnetic dipole moment to the rotation axis and the angular velocity $\omega$. Such relation plays a role of the contrast for nonlinear corrections on total pulsar radiation background. It follows that enhancing of vacuum nonlinear electrodynamics influence on pulsar radiation requires not only increasing magnetic field but also needs compliance of conditions marked on Fig.1 to ensure the best possible contrast for the nonlinear corrections.

The obtained radiation intensity was used to estimate pulsar spin-down. In this framework, for description in terms of the torque functions the power-low expansion of the intensity (31) was carried out (35)-(37) with the decomposition coefficients listed in Table 1. This provided an opportunity to compare nonlinear electrodynamics torque with the the weak mechanisms of the energy dissipation, for instance with gravitational waves radiation. For such compar-
ison the most realistic scenarios of gravitational radiation by isolated pulsar were selected – quadrupole gravitational radiation and r-modes radiation. The quantitative comparison has shown that for the common rapidly rotating pulsar, gravitational radiation torques significantly exceed non-linear electrodynamics torques coupled with the terms of same \( \sigma \) powers in spin-down equation. This result can be explained by low surface magnetic field \( B_s < 10^{11} G \) specific for most of rapidly rotating pulsar’s population. Implementation of similar estimates for the compact object possessing stronger magnetic field (hypothetical blitzar) \( B_s \sim 10^{15} G \) shows the possibility of the opposite case when vacuum non-linear electrodynamics torques exceed gravitational torque and play more significant role in spin-down equation under certain conditions.

References

1. D. L. Burke et al. Phys. Rev. Lett. 79, 1626 (1997)
2. V. I. Denisov, I. P. Denisova, S. I. Svertilov, Theoretical and Mathematical Physics 135, 720 (2003)
3. G. O. Schellstede, V. Perlick, C. Lämmerzahl, Phys. Rev. D, 92, 025039 (2015)
4. F. D. Valle et al., Eur. Phys. J. C, 76, 24 (2016)
5. G.V. Dunne, Eur. Phys. J. D 55, 327 (2009).
6. http://www.eli-beams.eu/
7. G. Mourou, T. Tajima, Optics & Photonics News 22, 47 (2011)
8. http://www.hzdr.de/db/Cms?pOid=35325&nId=3214
9. V. I. Denisov, I. P. Denisova, Optics and Spectroscopy 90, 928 (2001)
10. A. Paredes, D. Novoa, D. Tommasini, Phys. Rev. Lett. 109, 253903 (2012)
11. V. I. Denisov, I. P. Denisova, Theoretical and Mathematical Physics, 129, 1421 (2001)
12. J. Schwinger, Phys. Rev., 82, 664 (1951)
13. S.L. Adler, Ann. Phys., 67, 599 (1971)
14. P. A. Vshivtseva, V. I. Denisov, I. P. Denisova, P. Doklady Physics, 47, 798 (2002)
15. V. B. Berestetskii, L. P Pitaevskii, E.M. Lifshitz, Quantum Electrodynamics, (Pergamon Press, Oxford, UK, 1982)
16. V. I. Denisov, I. P. Denisova, S.I.Svertilov, Doklady Physics, 46, 705 (2001)
17. J.Y. Kim, Journal of Cosmology and Astroparticle Physics, 10, 056 (2011)
18. V. I. Denisov, Theoretical and Mathematical Physics, 132, 1071, (2002)
19. M. G. Baring, A. K. Harding, ApJ, 547, 929 (2001)
20. M. Born, L. Infeld, Proc. Roy. Soc., A144, 425 (1934)
21. M. Born, L. Infeld, Proc. Roy. Soc., A147, 522 (1934)
22. J. B. Kogut, D. K. Sinclair, Phys. Rev. D 73, 114508, (2006)
23. E. S. Fradkin, A. A. Tseytlin, Phys. Lett. B 163, 123 (1985)
24. S. Cecotti, S.Ferrara, Phys. Lett. B, 187, 335 (1987)
25. B. Zwiebach, A First Course in String Theory, (Cambridge University Press., 2004)
26. P. Gaete, J. Helayë-Neto, Eur. Phys. J. C, 74, 3182 (2014)
27. Z. Bilanicka, I. Bialynicki-Birula, Phys. Rev. D 2, 2341 (1970)
28. B. Hoffmann, L. Infeld, Phys. Rev. 51, 765, (1937)
29. W. Heisenberg, H. Euler, Z. Phys., 26, 714 (1936)
30. R. R. Wilson, Phys. Rev. 90, 720, (1953)
31. C. Bula et al. Phys. Rev. Lett. 76, 3116 (1996)
32. Wei-Tou Ni, Physics Letters A 379, 1297 (2015)
33. Wei-Tou Ni, Hsien-Hao Mei, Shan-Jyun Wu, Modern Physics Letters A 28, 1340013 (2013)
34. P. Soffitta et al., Exp. Astron. 36, 523, (2013)
35. V.R. Khililov, Electrons in Strong Electromagnetic Fields: An Advanced Classical and Quantum Treatment, (Gordon and Breach Science Pub New York, Netherlands, 1996)
36. V. I. Denisov, I. P. Denisova, Doklady Physics, 46, 377, (2001)
37. V. I. Denisov, I. P. Denisova, Optics and Spectroscopy 90, 282 (2001)
38. V.I. Denisov, V.A Sokolov, M.I. Vasili’ev, Phys. Rev. D, 90, (2014)
39. I.P. Denisova, M. Dalal, J. Math. Phys., 38, 5820 (1997)
40. V.I. Denisov, I. P. Denisova, V.A Sokolov, Theoretical and Mathematical Physics, 172, 1321 (2012)
41. J. Mathews,R.L. Walker, Mathematical methods of physics (2nd ed), New York: W. A. Benjamin, 1970
42. K.V. Zhukovski, Moscow University Physics Bulletin, 70, 93, (2015)
43. K.V. Zhukovski, Moscow University Physics Bulletin, 71, 237, (2016)
44. R.N. Manchester et al., ATNF pulsar catalog (Manchester, 2005)
45. H. Falcke, L. Rezzolla, Astronomy & Astrophysics, 562, A137 (2014)
46. A. Loeb, Y. Shvartzvald, D. Maoz, MNRASL 439, L46 (2014)
47. C. Palomba, Astron. Astrophys., 354, 163 (2000)
48. J.P. Ostriker, J.E. Gunn, ApJ, 157, 1395 (1969)
49. R.N. Manchester, J.H. Taylor, Pulsars, (San Francisco: W. H. Freeman, 1977)
50. L.D. Landau; E.M. Lifshitz,. The Classical Theory of Fields. V ol. 2 (4th ed.),(Butterworth-Heinemann,1975)
51. N. Stergioulas, J. A.Font, Phys. Rev. Lett., 86, 1148, (2001)
52. B. J. Owen et al. Phys. Rev. D 58, 084020 (1998)
53. Alford M. G. and Schwenzer K., MNRAS 446, 3631-3641, 2015.
54. M. G. Alford, K. Schwenzer, ApJ, 26, 781, (2014)