Induced low-energy effective action in the $6D, \mathcal{N}=(1,0)$ hypermultiplet theory on the vector multiplet background

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Abstract

We consider the six dimensional $\mathcal{N}=(1,0)$ hypermultiplet model coupled to an external field of the Abelian vector multiplet in harmonic superspace approach. Using the superfield proper-time technique we find the divergent part of the effective action and derive the complete finite induced low-energy superfield effective action. This effective action depends on external field and contains in bosonic sector all the powers of the constant Maxwell field strength. The obtained result can be treated as the $6D, \mathcal{N}=(1,0)$ supersymmetric Heisenberg-Euler type effective action.

1 Introduction

The study of the various classical and quantum aspects of $6D$ supersymmetric gauge theories is one of the most interesting and attractive subjects of modern supersymmetric field theory. Such theories have profound links with M-branes, they admit the superfield description, in particular, the $\mathcal{N}=(1,0)$ theories can be formulated in terms of unconstrained harmonic superfields, the $\mathcal{N}=(1,0)$ and $\mathcal{N}=(1,1)$ theories possess the interesting ultraviolet behaviour, in the framework of $6D$ super Yang-Mills theories one can construct the new supersymmetric model called the tensor hierarchy (see e.g. the recent papers [1], [2], [3], [4], [5], [6], [7] and the references therein).

In this letter we consider the hypermultiplet model coupled to external Abelian vector multiplet, where all the fields formulated in $6D, \mathcal{N}=(1,0)$ harmonic superspace, and derive the complete induced low-energy superfield effective action. Such effective action is gauge invariant and depends on space-time constant superfield strengths of $6D$ vector multiplet. As a result we will obtain the new Heisenberg-Euler type superfield effective action.

A computation of the effective action is based on superfield proper-time technique which is a power tool for analysis of the effective actions in the supersymmetric gauge theories (see the
applications of this technique in the various superfield models e.g. in [9], [15], [16]). In the case under consideration the superfield proper-time technique allows us to preserve the manifest 6D, \( \mathcal{N} = (1, 0) \) supersymmetry and gauge invariance on all steps of computations and derive the closed form of the complete low-energy effective action. Calculation of 6D, \( \mathcal{N} = (1, 0) \) supersymmetric effective action has some similarity with one in four-dimensional \( \mathcal{N} = 2 \) theory however, the six-dimensional theory possesses many specific features which should be taken into account at calculations.

The letter is organized as follows. Section 2 is devoted to the main notions of 6D, \( \mathcal{N} = (1, 0) \) harmonic superspace, including the formulations of hypermultiplet and vector multiplet in such a superspace. In section 3 we study a quantum theory of the 6D, \( \mathcal{N} = (1, 0) \) hypermultiplet coupled to 6D, \( \mathcal{N} = (1, 0) \) external Abelian vector multiplet and discuss a definition of induced superfield effective action of the vector multiplet. Section 4 is devoted to calculations of the effective action. It is shown in subsection 4.1 that the effective action is on-shell finite, its divergent part vanishes when the vector multiplet superfield satisfies the classical equation of motion. It is an essential property of 6D, \( \mathcal{N} = (1, 0) \) supersymmetry. For comparison, the corresponding effective actions in 4D supersymmetric theories contain divergences. In subsection 4.2 we consider the calculations of the complete low-energy superfield effective action and obtain the final result (4.28), (4.29). Although the computational technique has some familiar aspects with one in 4D supersymmetric theories, the concrete computations in six dimensional supersymmetric theory contain many specific details. The effective action (4.28), (4.29) can be expanded in power series in superfield strengths and their spinor derivatives allows us to construct the 6D, \( \mathcal{N} = (1, 0) \) on-shell superfield invariants of the vector multiplet. In conclusion we summarize the results obtained.

We want to emphasize that the problems of the effective action in the 6D, \( \mathcal{N} = (1, 0) \) hypermultiplet theory on the vector multiplet background can be considered as the novel type of the external field problems in quantum field theory.

## 2 Basic 6D supersymmetric models in \( \mathcal{N} = (1, 0) \) harmonic superspace

The harmonic 6D, \( \mathcal{N} = (1, 0) \) superspace was introduced in [11], [12], [14]. It is parameterized by the central basis coordinates \( (x^m, \theta^\alpha, u^\pm) \), where harmonics \( u^\pm \) (\( u^+_i = u^{i+1}, u^-_i = 1, (i = 1, 2) \)) belong to the coset R-symmetry of the group \( SU(2)/U(1) \). Also one can introduce the analytical basis \( (\zeta^M_A = \{x^m_A, \theta^{+\alpha}\}, u^\pm, \theta^{-\alpha}) \) by the rule

\[
x^\alpha_A = x^\alpha + i\theta^\gamma \gamma^\alpha \theta^+, \quad \theta^{\pm\alpha} = u^\pm \theta^{\alphai}.
\]

It is notes that the coordinates \( (\zeta^M_A, u^\pm) \) form a subspace closed under the \( \mathcal{N} = (1, 0) \) supersymmetry transformations. The covariant harmonic derivatives form the Lie algebra of \( SU(2) \) group \( [D^{++}, D^{--}] = D^0 \) and in the analytic basis read

\[
D^{\pm\pm} = u^\pm \partial_i^\mp + i\theta^{\alpha} (\gamma^m \partial_m)_{\alpha\beta} \theta^{\pm\beta} + \theta^{\pm\alpha} \partial_\alpha^\mp,
\]

\[
D^0 = u^+ \partial_i^+ - u^- \partial_i^- + \theta^{+\alpha} \partial_\alpha^+ - \theta^{-\alpha} \partial_\alpha^-,
\]

where we have denoted \( \partial_i^\pm = \partial_{\theta^\alpha} \) and \( \partial_\alpha^\pm = \partial_{u^\pm \theta^\alpha} \). Analytic subspace allows us to define the analytical superfields, which satisfy the condition of the Grassmann analyticity \( D_\alpha^+ \phi = 0 \). The spinor derivatives \( D_\alpha^\pm \) in the analytic basis have the form

\[
D_\alpha^+ = \frac{\partial}{\partial \theta^{-\alpha}}, \quad D_\alpha^- = -\frac{\partial}{\partial \theta^{+\alpha}} - 2i\partial_\alpha^\beta \theta^{-\beta}, \quad \{D_\alpha^+, D_\beta^-\} = 2i\partial_\alpha^\beta.
\]
In what follows we also use the following conventions

\[ (D^\pm)^4 = -\frac{1}{4!} \varepsilon^{\alpha\beta\gamma\delta} D^\pm_\alpha D^\pm_\beta D^\pm_\gamma D^\pm_\delta, \quad (D^+)^4(\theta^-)^4 = 1, \quad (D^\pm)^3(\theta^-)^3 = \delta^\pm_\beta, \]

and the other notations from the work [6].

The simplest basic 6D, \( \mathcal{N} = (1, 0) \) supersymmetric models are ones of hypermultiplet and vector multiplet.

The 6D, \( \mathcal{N} = (1, 0) \) hypermultiplet is described by the superfields \( q^i(x, \theta), i = 1, 2 \), and their conjugate \( \bar{q}_i(x, \theta) = (q^i)^\dagger \), under the constraint

\[ D^i_\alpha q^j_\beta(\xi, \bar{\nu}) = 0. \]

The superfield \( q^i(x, \theta) \) has a short expansion \( q^i(z) = f^i(x) + \theta^{\alpha i} \psi_\alpha(x) + \ldots \), with a doublet of massless scalars \( f^i \) and the spinor \( \psi_\alpha \) fields. Thus, the 6D, \( \mathcal{N} = (1, 0) \) hypermultiplet has 4 bosonic and 4 fermionic real degrees of freedom.

One can use the analytic superfields in harmonic superspace to construct the off-shell Lagrangian formulation of the 6D, \( \mathcal{N} = (1, 0) \) hypermultiplet. In this case the 6D, \( \mathcal{N} = (1, 0) \) hypermultiplet is described by an unconstrained analytic superfield \( q^+_A(\xi, \bar{\nu}) \)

\[ D^+_\alpha q^+_A(\xi, \bar{\nu}) = 0. \]

The analytic superfield \( q^+_A(\xi, \bar{\nu}) \) satisfies the reality condition \( \bar{q}^+_A = \varepsilon_{AB} q^+_B \), where the Pauli-Gürsey index \( A = 1, 2 \) is a lowered and raised by the matrices \( \varepsilon_{AB}, \varepsilon^{AB} \).

The classical model of the 6D, \( \mathcal{N} = (1, 0) \) hypermultiplet is described by the action

\[ S_q = -\frac{1}{2} \int d\xi d^4\theta q^+_A D^{++} q^+_A, \]

where \( d(\xi d^4\theta) = d^6 x d^4\theta^+ \) is the analytic superspace integration measure. The corresponding equations of motion follows from the action (2.8) and have the form

\[ D^{++} q^+_A(\xi, \bar{\nu}) = 0. \]

In principle the formulation above allows us to write down the most general hypermultiplet self-couplings in the form of the arbitrary potential \( \mathcal{L}^{(+)}(q^+, \bar{q}^+) \) [10].

The off-shell 6D, \( \mathcal{N} = (1, 0) \) vector multiplet is realized in conventional superspace in the following way\(^3\). First of all one can introduce the gauge-covariant derivatives \( D_M = D_M + A_M \), where the flat derivatives \( D_M = (D_a, D^i_\alpha) \) obeying the anti-commutation relations (2.3) and the superfields \( A_M \) are the gauge connection taking the values in the Lie algebra of the gauge group. The gauge-covariant derivatives satisfy the algebra

\[ \{D^i_\alpha, D^j_\beta\} = -2i \varepsilon^{ij} D_{\alpha\beta}, \quad [D^i_\gamma, D_{\alpha\beta}] = -2i \varepsilon_{\alpha\beta\gamma\delta} W^{i\delta}, \]

\[ [D_a, D_b] = F_{ab}, \]

\(^3\)See the details in [11], [12], [13] [14].
where $W^\alpha$ is the superfield strength of the anti-Hermitian superfield gauge potential. In this paper we consider the interaction of the hypermultiplet with background Abelian vector multiplet.

One can solve the constraints (2.10) and (2.11) in the framework of the harmonic superspace. The integrability condition $\{\mathcal{D}_\alpha^+, \mathcal{D}_\beta^+\} = 0$ allows us to express the spinor covariant derivatives in the form $\mathcal{D}_\alpha^+ = e^{-ib}D_\alpha^+ e^{ib}$, where $b(z, u)$ is a some Lie-algebra valued harmonic superfield of zero harmonic $U(1)$ charge. In the $\lambda$-frame, the spinor covariant derivatives $\mathcal{D}_\alpha^+$ coincide with the flat ones, $\mathcal{D}_\alpha^+ = D_\alpha^+ = \frac{\partial}{\partial \theta^\alpha}$. In this case the harmonic covariant derivatives acquire the connection $\mathcal{V}^{++}$,

$$\mathcal{D}^{++} = D^{++} + \mathcal{V}^{++}, \tag{2.12}$$

which is an unconstrained analytic potential of the theory. The component expansion of $\mathcal{V}^{++}(\zeta, u)$ in the Wess-Zumino gauge

$$\mathcal{V}^{++}_{WZ} = \theta^+ \theta^+ \theta^+ A_{\alpha\beta}(x_A) + (\theta^+)^3 \lambda^- \theta^+ (x_A) + 3(\theta^+)^4 \mathcal{V}^{--}(x_A), \tag{2.13}$$

involves the physical fields $A_{\alpha\beta} = (\gamma^m)_{\alpha\beta} A_m$ and $\lambda^\alpha$ and the auxiliary ones which are collected in the superfield $\mathcal{V}^{--}$.

Let us introduce the non-analytic harmonic connection $\mathcal{V}^{--}(z, u)$. The superfield $\mathcal{V}^{--}(z, u)$ is determined in terms of $\mathcal{V}^{++}$ uniquely as a solution of the zero-curvature condition [10], which in the Abelian case is reduced to

$$D^{++}\mathcal{V}^{--} - D^{--}\mathcal{V}^{++} = 0. \tag{2.14}$$

The gauge transformations of the connection $\mathcal{V}^{--}$ has the form $\delta \mathcal{V}^{--} = -\mathcal{D}^{--} \Lambda$, where the gauge parameter $\Lambda$ is an analytic anti-Hermitian superfield. The decomposition of the superfield $\mathcal{V}^{--}$ in terms of the component fields [5] reads

$$\mathcal{V}^{--} = \theta^- \theta^+ \mathcal{V}^{++}(x_A, \theta^+) + (\theta^+) \mathcal{V}^{++}(x_A, \theta^+) + (\theta^-)^4 \mathcal{V}^{++}(x_A, \theta^+). \tag{2.15}$$

The components of superfields $\mathcal{V}^{--}_{\alpha\beta}, \mathcal{V}^{--}_{\alpha\beta}$ and $\mathcal{V}^{++}$ are discussed in details in the [5]. For our aims it is useful to write down only the components of $\mathcal{V}^{++}$

$$\begin{align*}
\mathcal{V}^{++} &= Y^{++} + \theta^+ \chi^{+}_{\alpha} + \theta^+ \theta^+ \Omega_{\alpha\beta} + (\theta^+)^3 \rho^- \alpha + (\theta^+)^4 \pi^{(-2)}, \\
\chi^{+}_{\alpha} &= i \mathcal{D}_{\alpha\beta} \lambda^\beta, \quad \epsilon^{\alpha\beta\gamma} \Omega_{\alpha\beta} = -2i \mathcal{D}^{\alpha\beta} Y^{++} - 2D^{[\alpha\beta}(\gamma_{4} F_{\gamma]} - \frac{1}{4} \{\lambda^{+\alpha}, \lambda^{-\beta}\}), \\
\rho^- \alpha &= 2i \mathcal{D}^{\alpha\beta} \chi^\beta, \quad \pi^{(-2)} = \mathcal{D}^{\alpha\beta} \mathcal{D}_{\alpha\beta} Y^{--} - \frac{1}{2} \{\lambda^{+\alpha}, \mathcal{D}_{\alpha\beta} \lambda^{-\beta}\}. 
\end{align*} \tag{2.16}$$

With the help of the connection $\mathcal{V}^{--}$ we construct the spinor and the vector superfield connections $\mathcal{A}^-_{\alpha} = -D_{\alpha}^{++} \mathcal{V}^{--}, \quad \mathcal{A}_{\alpha\beta} = \frac{i}{2} D_\alpha^{++} D_\beta^{++} \mathcal{V}^{--}$ and determine the field strength (in the $\lambda$-frame)

$$W^{\alpha+} = -\frac{1}{4} (D^+)^3 \mathcal{V}^{--}, \tag{2.17}$$

The Bianchi identities lead to relations

$$D_{\alpha}^{++} W^{-\alpha} = D_{\alpha}^{--} W^{\alpha+}, \quad D_{\alpha}^{\pm} F_{ab} = i D_{[a}(\gamma_{b]} \epsilon_{\alpha\beta} W_{\pm} \epsilon_{\beta}. \tag{2.18}$$
The vector superfield strength, $F^\beta_\alpha = (\gamma_{ab})^{\beta}_\alpha F^{ab}$, is defined as follows

$$F^\beta_\alpha = (D_\alpha W^{+\beta} - D^{+\alpha} W^{-\beta}) = 2N^\beta_\alpha. \quad (2.19)$$

The other useful consequences of the Bianchi identities are

$$D^{+\beta} W^{+\alpha} = \frac{1}{4} \delta^\beta_\alpha Y^{++}, \quad Y^{++} = -(D^+) V^{--}, \quad D^{++} Y^{++} = 0, \quad (2.20)$$

$$W^{-\alpha} = D^{-\alpha} W^{+\alpha}, \quad D^{-\alpha} Y^{++} = 2D^{+\alpha} W^{+\alpha}, \quad D^{++} W^{+\alpha} = 0. \quad (2.21)$$

These relations define the superfield $Y^{++}$ which will be used further.

The superfield action of $6D, \mathcal{N} = (1, 0)$ SYM theory has the form \[11, 12, 13, 14\]

$$S_{SYM} = \frac{1}{f^2} \sum_{n=1}^{\infty} \frac{(-1)^n}{n} \text{tr} \int d^{14}z du_1 \ldots du_n \frac{V^{++}(z, u_1) \ldots V^{++}(z, u_n)}{(u_1^+ u_2^+) \ldots (u_n^+ u_1^+)}, \quad (2.22)$$

where $f$ is the dimensional coupling constant ($[f] = -1$). The corresponding equations of motion read

$$Y^{++} = (D^+) V^{--} = 0. \quad (2.23)$$

The connection between component fields of $W^{+\alpha}$ and $V^{++}$ is caused by the zero-curvature condition (2.14) and the definition (2.17).

### 3 Superfield effective action

The effective action $\Gamma[V^{++}]$, induced by hypermultiplet matter, is defined by

$$e^{i\Gamma[V^{++}]} = \int Dq^+ D\bar{q}^+ \exp \left( -i \int d\zeta (-4) \bar{q}^+ D^{++} q^+ \right). \quad (3.1)$$

The expression (3.1) yields

$$\Gamma[V^{++}] = i \text{Tr} \ln D^{++} = -i \text{Tr} \ln G^{(1,1)}. \quad (3.2)$$

Here $G^{(1,1)}(\zeta_1, u_1|\zeta_2, u_2) = \langle \bar{q}^+(\zeta_1, u_1) q^+(\zeta_2, u_2) \rangle$ is the superfield Green function in the $\tau$-frame. This Green function is analytic with respect to both arguments and satisfies the equation

$$D^{++}_1 G^{(1,1)}(1|2) = \delta^{(3,1)}_A(1|2). \quad (3.3)$$

Here $\delta^{(3,1)}_A(1|2)$ is the appropriate covariantly analytic delta-function

$$\delta^{(q_4, -q)}_A = (D^{+}_2)^4 \delta^{14}(z_1 - z_2) \delta^{(q_4, -q)}(u_1, u_2). \quad (3.4)$$

Like in four-dimensional case \[15\] and \[16\] we will act by the operator $(D^{--}_1)^2$ on both sides of (3.3)

$$D^{++}_1 (D^{--}_1)^2 G^{(1,1)}(1|2) = (D^{--}_1)^2 \delta^{(3,1)}_A(1|2) = 2D^{++}_1 (D^{+}_2)^4 \frac{\delta^{14}(z_1 - z_2)}{(u_1^+ u_2^+)^3}. \quad (3.5)$$
Now, since the equation $D^{++} f^{-|q|} = 0$ has only the trivial solution $f^{-|q|} = 0$, after the action of the operator $(\mathcal{D}_+^+)^4$ we obtain:

$$(\mathcal{D}_+^+)^4(\mathcal{D}_-^-)^2 G^{(1,1)}(1|2) = -8 \Box G^{(1,1)}(1|2) = 2(\mathcal{D}_+^+)^4(\mathcal{D}_2^+)^4 \frac{\delta^{14}(z_1 - z_2)}{(u_1^+ u_2^+)^3}. \quad (3.6)$$

Thus we obtain

$$\Box G^{(1,1)}(1|2) = -\frac{1}{4}(\mathcal{D}_1^+)^4(\mathcal{D}_2^+)^4 \frac{\delta^{14}(z_1 - z_2)}{(u_1^+ u_2^+)^3}, \quad (3.7)$$

where $1/(u_1^+ u_2^+)^3$ is a special harmonic distribution. In Eq. (3.7) the operator $\Box$ is the covariantly analytic d’Alembertian ($[\mathcal{D}_+^+, \Box] = 0$) which arises when $(\mathcal{D}_+^+)^4(\mathcal{D}_-^-)^2$ acts on the analytical superfield and has the form

$$\Box = \frac{1}{8} (\mathcal{D}_+^+)^4(\mathcal{D}_-^-)^2 = \mathcal{D}_a \mathcal{D}^a + W^+ \mathcal{D}_-^- + Y^{++} \mathcal{D}_-^- - \frac{1}{4}(\mathcal{D}_-^- W^+). \quad (3.8)$$

The operator $\Box$ acts on the space of covariantly analytic superfields. Let us introduce a new second-order operator $\Delta$,

$$\Delta = \Box - W^- \mathcal{D}_+^+ \quad (3.9)$$

which coincides with $\Box$ on the space of covariantly analytic superfields$^4$. We have to note that the Green function, $G^{(1,1)}(1|2)$, is analytic with respect to both arguments thus we obtain

$$G^{(1,1)}(1|2) = -\frac{1}{4\Delta} (\mathcal{D}_1^+)^4(\mathcal{D}_2^+)^4 \frac{\delta^{14}(z_1 - z_2)}{(u_1^+ u_2^+)^3}. \quad (3.10)$$

Like in four- and five-dimensional cases [15], [16] one can obtain the useful identity

$$(\mathcal{D}_1^+)^4(\mathcal{D}_2^+)^4 \frac{\delta^{14}(z_1 - z_2)}{(u_1^+ u_2^+)^3} = (\mathcal{D}_1^+)^4 \left\{ (u_1^+ u_2^+)(\mathcal{D}_-^-)^4 - (u_1^- u_2^-)\mathcal{D}_-^- \right\} \delta^{14}(z_1 - z_2). \quad (3.11)$$

Here we have introduced the notation

$$\Omega^- = i\mathcal{D}^\alpha \mathcal{D}_-^- + 4W^- \mathcal{D}_+^+ + (\mathcal{D}_-^- W^+) \quad (3.12)$$

This identity is used later for computing the effective action.

The definition (3.2) of the one-loop effective action is purely formal. The actual evaluation of the effective action can be done in various ways (see e.g. [15], [16]). Further we will follow [16] and use the relation

$$\Gamma[V^{++}] = \Gamma_{g=0} + \int_0^1 dg \partial_g \Gamma(gV) = -i \int_0^1 dg \text{Tr} (V^{++} G^{(1,1)}(gV))$$

$$= -i \int d\zeta_1^{(-4)} du_1 V^{++} \int_0^1 dg G^{(1,1)}(1|2)|_{2=1}, \quad (3.13)$$

where $G^{(1,1)}(gV)$ means the Green function depending on the superfield $gV^{++}$.

$^4$Note that in $6D, \mathcal{N} = (1, 0)$ hypermultiplet theory, the operator (3.9) differs from the analogical operator in $4D, \mathcal{N} = 2$ hypermultiplet theory.
4 Calculation of the effective action

Let us discuss a generic scheme of the calculations. We substitute the Green function (3.10) to the effective action (3.13) and one-loop effective action takes the form

$$\Gamma[V++] = \frac{i}{4} \int d\zeta_1^{(-4)} du_1 V^{++} \int_0^1 dg \frac{1}{\Delta_1} (D_1^+)^4 (D_2^+)^4 \frac{\delta^{14}(z_1 - z_2)}{(u_1^+ u_2^+)^4} \bigg|_{z=1}. \quad (4.1)$$

4.1 Divergent part of the effective action

In the framework of the proper-time technique, the inverse operator $\frac{1}{\Delta}$ is defined as follows

$$-\frac{1}{\Delta} = \int_0^\infty d(is) e^{i s \Delta}. \quad (4.2)$$

To avoid the divergences on the intermediate steps it is necessary to introduce a regularization. We will use a variant of dimensional regularization (so called $\omega$-regularization) accommodative for regularization of the proper-time integral (see e.g. [9]). The $\omega$-regularized version of the relation (4.2) is

$$-(\frac{1}{\Delta})_{\text{reg}} = \int_0^\infty d(is)(is\mu^2)^\omega e^{i s \Delta}, \quad (4.3)$$

where $\omega$ tends to zero after renormalization and $\mu$ is an arbitrary parameter of mass dimension.

We will now concentrate on calculating the divergent part of the effective action (4.1). In the regularization scheme under consideration, the divergences mean the pole terms of the form $\frac{1}{\omega}$. Taking into account the relation (4.3) and the relations (3.7) one gets

$$\Gamma_{\text{div.part}} = -\frac{i}{4} \int du_1 d\zeta_1^{(-4)} V^{++}(1) \int_0^\infty d(is)(is\mu^2)^\omega e^{i s \Delta_1} (D_1^+)^4 (D_2^+)^4 \frac{\delta^{14}(z_1 - z_2)}{(u_1^+ u_2^+)^4} \bigg|_{\text{div.part}}. \quad (4.4)$$

In the expression (4.4) we used the explicit form of the operator $\Delta$ (3.9) and omit the additional term $W^{-\alpha} D_{\alpha}^+$, thus it immediately coincide with the $\Box$ (3.8). We use the momentum representation of the delta function, $\delta^{14}(z_1 - z_2) = \delta^0(x_1 - x_2) \delta^4(\theta_1^+ - \theta_2^+) \delta^4(\theta_1^- - \theta_2^-)$,

$$1 \delta^{(14)}(z_1 - z_2) = \int \frac{d^6p}{(2\pi)^6} e^{ip_a \rho^a} \delta^{(8)}(\rho^{\alpha \pm}) I(z, z') \quad (4.5)$$

where

$$\rho^\alpha = (x_1 - x_2)^\alpha - 2i(\theta_1^+ - \theta_2^+) \gamma^\alpha \theta_1^- \quad \rho^{\alpha \pm} = (\theta_1^\pm - \theta_2^\pm)^\alpha \quad (4.6)$$

The parallel displacement propagator $I(z, z')$ is required only beyond the one-loop approximation and not required for actual one-loop calculations. Moving the exponential to the left through the differential operators, $e^{i s \Box}\delta^{14}(z_1 - z_2)$ becomes in the coincidence limit

$$\int \frac{d^6p}{(2\pi)^6} e^{ip_a\gamma^\alpha \theta_1^-} \delta^{(8)}(\theta_1^- - \theta_1^-)|_{1=2}. \quad (\text{X's being defined by } X_a = D_a + ip_a \text{ and } X_a^- = D_a^- + 2p_{\alpha \beta} \rho^{-\beta}).$$

Note also that the term $p_{\alpha \beta} \rho^{-\beta}$ in $X_a^-$ vanishes in the coincidence limit since there are no $D_\alpha^+$ operators required to kill $(\theta_1^+ - \theta_2^+)$. 


By expanding the \( e^{i \xi \Omega_1(X)} \) in the (4.4) and leaving only the terms relating to divergences one gets
\[
e^{i \xi \Omega_1(u_1^+ u_2^+)} (D_1^-)^4 (D_1^-)^4 \delta^8( \theta_1 - \theta_2) \Big|_{1=2} = -\int_0^\infty \frac{d(is)}{(is)^3} (is)^2 \omega e^{-i sm^2} \{ i s Y^{++} + \frac{(is)^2}{2} \Omega, Y^{++} \}.
\]
(4.7)

Here we have introduced the infrared regulator \( m^2 \) which is a natural element of the calculations of effective action in massless gauge theories\(^8\). By calculating the proper-time integral and extracting the pole terms one gets for the right hand side of the above expression
\[
\frac{1}{\omega} m^2 Y^{++} - \frac{1}{2\omega} \Box Y^{++} - \frac{1}{2\omega} W^{\alpha} D^- Y^{++}.
\]
(4.8)

One can see that the divergent part of the effective action (4.4) is proportional to \( Y^{++} \) and it cancel if we assume the 'on-shell' condition (classical equation of motion) \( Y^{++} = 0 \) (2.23) for the background fields. As a result the on-shell induced effective action in 6D, \( \mathcal{N} = (1, 0) \) theory is ultraviolet finite. It is a important distinctive property of six-dimensional supersymmetric model. In four dimensions the analogous non-supersymmetric models and \( \mathcal{N} = 1, \mathcal{N} = 2 \) supersymmetric models are on-shell divergent.

### 4.2 Low-energy effective action

To find the complete low-energy effective Lagrangian we should fix the appropriating background superfields. It was shown in [5], [6] that such a background is given by the covariantly constant vector background without the auxiliary fields ('on-shell' background)
\[
Y^{++} = 0, \quad D_a W^{\pm \alpha} = 0, \quad D_{\pm} F_{a b} = 0.
\]
(4.9)

Thus, in the first equation of (4.9) we assume that background fields solve the classical equations of motion. Two other equations in (4.9) mean the covariant space-time independence. For the 'on-shell' background under consideration the operators \( \Delta \) and \( \Omega^{--} \) take a simple form and depends only on the background fields \( W^{+\alpha} \) and \( D^- a W^{+\beta} \). Since the form of the effective Lagrangian is defined by the coefficients of these operators we can conclude that the low-energy effective Lagrangian should have the following general form
\[
\mathcal{L}^{(+4)}_{\text{eff}} = \mathcal{L}^{(+4)}_{\text{eff}}(W^{+\alpha}, D^{+}_{\alpha} W^{\pm \beta}).
\]
(4.10)

The aim of this paper is to derive the complete low-energy effective Lagrangian.

We use the proper time representation (4.2) for the inverse operator \( \Delta_1 \) and act to the delta-function. It should be noted that the operator \( \Delta_1 \) and the covariant derivative \( D_1^- \) does not commute even for the background under constrains (4.9), indeed, \([\Delta, D^+_a] = -N^{a}_{\beta} D^+_b \). This is a crucial differences of the theory under consideration in comparison with 4D, \( \mathcal{N} = 2 \)

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\(^8\)The 4D, \( \mathcal{N} = 2 \) vector multiplet contains the scalar which in principle can serve as the infrared regulator. In 6D, \( \mathcal{N} = (1, 0) \) gauge theory such a scalar is absent. However, such a scalar is present in 6D tensor hierarchy (see e.g. [5] and the references therein). Therefore one can expect that in this case there will be no necessary to introduce the special infrared regulator (see some attempt to study the effective action in the hypermultiplet theory on background of tensor hierarchy in [6]).
hypermultiplet theory where the corresponding operators commute on corresponding on-shell background. Substituting (3.11) into the effective action (4.1) we obtain

$$
\Gamma[V^{++}] = -\frac{i}{4} \int d\zeta_1^{(-4)} du_1 V^{++} \int_0^1 dg \int_0^\infty d(is) e^{is\Delta_1}(D^+_1)^4(D^+_2)^4 \frac{\delta^{14}(z_1 - z_2)}{((u_1^+ u_2^+)^3} |_{2=1}
$$

$$
= -\frac{i}{4} \int d\zeta_1^{(-4)} du_1 V^{++} \int_0^1 dg \int_0^\infty d(is) (e^{-isN}D^+_1)^4 \left\{ (u_1^+ u_2^+)(D^+_1)^4 - (u_1^- u_2^+)(D^-_1)^4 \right\} e^{is\Delta_1}\delta^{14}(z_1 - z_2) |_{2=1}.
$$

(4.11)

The divergences of the effective action have been found in the previous subsection. These divergences are proportional to $Y^{++}$, therefore they vanish on the background under consideration. Hence the effective Lagrangian is on-shell finite.

The terms in the braces in (4.11) are considered as follows. First term in the braces was analysed in previous subsection and it was shown that it is proportional to $Y^{++}$ which is equal to zero on the background under consideration. Last term in the braces vanishes since there is no enough number of $D$-factors to annihilate the delta-function of anticommuting variables with the help of relation

$$
(D^+_1)^4(D^-_1)^4\delta^8(\theta - \theta')|_{\theta=\theta'} = 1.
$$

(4.12)

Second term in the braces in (4.11) contains the operator $\Omega^{--}$ (3.12). One can show that only first term in this operator gives rise to the effective action. Indeed, the term $D^-_a W^{-\alpha}$ in this operator is equal to $Y^{--}$ which vanishes on the 'on-shell' background.

Taking into account the properties of $\Delta$ described, the functional (4.11) can be rewritten in the form:

$$
\Gamma[V^{++}] = -\frac{i}{4} \int d\zeta_1^{(-4)} du_1 V^{++} \int_0^1 dg \Omega^{--} \int_0^\infty d(is)(e^{-isN}D^+_1)^4 e^{is\Delta_1}\delta^{14}(z_1 - z_2) |_{2=1}.
$$

(4.13)

A further simplification occurs in the case of a covariantly constant vector multiplet (4.9). Then, the first-order operator appearing in $\Delta$,

$$
Y = W^{+\alpha}D^-_\alpha - W^{-\alpha}D^+_\alpha
$$

(4.14)

turns out to commute with the vector covariant derivative $D_\alpha$, that allows us to represent $e^{is\Delta}$ in factorized form $e^{isY}e^{isD^aD_\alpha}$. This allows us to build the calculation of the heat kernel in the form

$$
K(z_1, z_2|s) = e^{isY}e^{isD^\alpha D_\alpha}\delta^{14}(z_1 - z_2) = e^{isY}\tilde{K}(z_1, z_2|s).
$$

(4.15)

The reduced heat kernel $\tilde{K}(z_1, z_2|s)$ can now be evaluated in the same way by generalizing the Schwinger construction

$$
\tilde{K}(z_1, z_2|s) = \frac{i}{(4\pi)^3} \det^{1/2} \frac{\sinh(sF)}{sF} e^{\frac{i}{F}Fcoth(sF)\rho_+} (\rho^+)^4(\rho^-)^4 I(z_1, z_2),
$$

(4.16)

Here we follow fourth paper in [16], where the analogous consideration in $4D, \mathcal{N} = 2$ hypermultiplet theory has been carried out.
where the determinant is computed with respect to the Lorentz indices. To compute the kernel $K(z_1, z_2|s)$ we need to evaluate the action of the $\exp(is\Upsilon)$ on the $\tilde{K}(z_1, z_2|s)$. The formal result reads

$$K(z_1, z_2|s) = \frac{i}{(4\pi is)^3} \det \frac{1}{2} \left( \frac{sF}{\sinh(sF)} \right) e^{\frac{i}{4}\rho^\gamma(s)(F \cosh sF)_{\alpha\beta}\rho^\beta(s)\rho^4(s)\rho^{-4}(s)I(z_1, z_2|s)},$$

where we have denoted, $\rho^A = (\rho^\alpha, \rho^{\alpha+}, \rho^{\alpha-})$,

$$\rho^A(s) = e^{is\Upsilon}\rho^A e^{-is\Upsilon}, \quad I(z, z'|s) = e^{is\Upsilon}I(z, z').$$

Using the formula $e^A B e^{-A} = B + [A, B] + \ldots$ and our constraints on the background (4.9) we obtain\footnote{Here we use $D_{\alpha}^\beta \rho^{\beta-} = \delta_{\alpha}^\beta$ and $D_{\alpha}^\beta \rho^{\beta+} = -\delta_{\alpha}^\beta$.}

$$\rho^{\alpha+}(s) = \rho^{\alpha+} - W^{\beta+} N_{\beta}^\alpha, \quad \rho^{\alpha-}(s) = \rho^{\alpha-} - W^{\beta-} N_{\beta}^\alpha,$$

$$\rho^\alpha(s) = \rho^\alpha - 2 \int_0^s dt W^-(t) \gamma^\alpha \rho^+(t), \quad W^{\alpha-}(s) = W^{\beta-}(e^{isN})_{\beta}^\alpha.$$

Here we have used the notation (2.19) and $N_{\alpha}^\beta = (\frac{e^{isN}}{s})_{\alpha}^\beta$. We need not the explicit expression for $I(z, z'|s)$ but it is easy to check by differentiating over the proper time $s$ the identity

$$I(z_1, z_2|s) = \exp \left[ \int_0^s dt \Sigma(z_1, z_2|t) \right] I(z_1, z_2), \quad (4.21)$$

$$\Sigma(z_1, z_2|t) = e^{it\Upsilon}\Sigma(z_1, z_2)e^{-it\Upsilon}, \quad (4.22)$$

and $\Sigma(z_1, z_2)$ is defined by

$$(W^{+\alpha} D_{-\alpha}^- - W^{-\alpha} D_{-\alpha}^+)I(z_1, z_2) = \Sigma(z_1, z_2)I(z_1, z_2). \quad (4.23)$$

Now let us return to the calculation of the effective action (4.13). According to the (4.9) we obtain for the $\Omega^{--}$ (3.9) in the (4.13)

$$\Gamma[V^{++}] = -\frac{i}{4} \int d\zeta^{(-4)} du_1 V^{++} \int_0^\infty d(is)$$

$${\times} \int \left[ d(\exp iD_{\alpha}^{\beta+} D_{-\alpha}^- + 4W^{-\alpha} D_{-\alpha}^-)(e^{isN} D_{-\alpha}^+)K(z_1, z_2|s) \right]_{2=1}.$$

Let us consider the second term in the braces. Firstly we have to note that $D_{\alpha}^-(e^{-isN} D_{-\alpha}^+) K(z_1, z_2|s) \big|_{2=1} \sim (W^+)^3$ and the connection (2.21) between $W_{-\alpha}$ and $W_{+\alpha}$. Integrating by parts $D^{--}$ from $W_{-\alpha}$ and using the analyticity of the field $V^{++}$ and zero-curvature condition (2.14) in Abelian case one can show that this term vanish. Schematically it has the form

$$\int d\zeta^{(-4)} du V^{++} W^{-}(W^+)^3 \sim \int d\zeta^{(-4)} du V^{++} D^{--}(W^+)^4 \sim \int d\zeta^{(-4)} du V^{-+} D^{++}(W^+)^4 = 0.$$
Now we integrate by parts in the first term of the (4.24) keeping in mind our restriction on the background (4.9) (see the similar analysis in the work [6]). After that we have

\[ \Gamma[V^{++}] = -\frac{3i}{2} \int d\zeta^{(-4)} \int_0^1 dg \int_0^\infty d(is) W^{\alpha+} D_{1\alpha}^- (e^{-isN} D_1^+)^4 K(z_1, z_2|s) \bigg|_{s=1} \]  \hspace{1cm} (4.25)

Then we act by the operators \( D_\alpha^- \) and \((e^{-isN} D_+)^4\) on the kernel (4.17). In the limit of coincidence, these derivatives act only on the two-point function \(\rho^{++}(s)\) and \(\rho^{-4}(s)\). According to (4.19) and (2.19) we have

\[ (e^{-isN} D_1^+)^4 \rho^{++}(s) \bigg|_{s=1} = 1, \]

\[ W^{\alpha+} D_{1\alpha}^- \rho^{++}(s) \bigg|_{s=1} = \frac{1}{6} \varepsilon_{\alpha\beta\gamma\delta} \varepsilon^{\alpha'\beta'\gamma'\delta'} (W^+ e^{isN})_{\alpha'}^\alpha (W^+ \mathcal{N})_{\beta'}^\beta (W^+ \mathcal{N})_{\gamma'}^\gamma (W^+ \mathcal{N})_{\delta'}^\delta \]

\[ = (W^+)^4 \frac{d}{d(is)} ((is)^4 \det \mathcal{N}(is)) \bigg|_{s=1}. \]  \hspace{1cm} (4.27)

We recall that all gauge fields, except \(V^{++}\) in the (4.13), linearly depend on the \(g\). We make the change, \(isg \to s\), in the proper time integral (4.25) and then integrate over \(g\). As a result we obtain

\[ \Gamma[V^{++}] = \frac{1}{(4\pi)^3} \int d\zeta^{(-4)} du (W^+)^4 \xi(F, N), \]  \hspace{1cm} (4.28)

\[ \xi(F, N) = \frac{1}{2} \int_0^\infty ds e^{-sm^2} \frac{d}{ds} \left( s^4 \det \left( \frac{e^{sN} - 1}{sN} \right) \right) \det \left( \frac{sF}{\sin sF} \right). \]  \hspace{1cm} (4.29)

This is the final expression for complete low-energy effective action in the theory under consideration. One can show that the integrand in (4.28) is analytic superfield under the on-shell condition \(Y^{++} = 0\). The effective action (4.28), (4.29) is manifestly gauge invariant and manifestly \(\mathcal{N} = (1, 0)\) supersymmetric by construction. Expanding the effective action in power of the superspace strengths and integrating over proper time, we will get the effective action as an expansion in power of on-shell gauge and supersymmetric invariants. In certain sense the effective action obtained can be considered as the generating functional of on-shell \(\mathcal{N} = (1, 0)\) supersymmetric invariants.

As an example to illustrate the general situation we derive several such invariants on the base of the expressions (4.28), (4.29). Decomposition of the determinants in (4.29) up to the forth order over \(s\) has the form.

\[ \det \left( \frac{sF}{\sin sF} \right) = 1 + \frac{s^2}{12} \text{tr} F^2 + \frac{s^4}{288} (\text{tr} F^2)^2 + \frac{s^4}{360} \text{tr} F^4 + \ldots, \]  \hspace{1cm} (4.30)

\[ \det \left( \frac{e^{sN} - 1}{sN} \right) = 1 + \frac{s^2}{24} \text{tr} N^2 - \frac{s^4}{2880} \text{tr} N^4 + \frac{s^4}{1152} (\text{tr} N^2)^2 + \ldots, \]  \hspace{1cm} (4.31)

---

8One of the basic differences of the effective action in the 6D, \(\mathcal{N} = (1, 0)\) hypermultiplet theory with one in the 4D, \(\mathcal{N} = 2\) hypermultiplet theory is the derivative with respect proper-time in the integrand (4.28).

9See first of the identities (2.20).

10We remind that the effective action is obtained under on-shell condition \(Y^{++} = 0\).
where
\[
\text{tr} F^2 = \frac{1}{2} \text{tr} N^2, \quad \text{tr} F^4 = -\frac{1}{4} \text{tr} N^4 + \frac{3}{16} (\text{tr} N^2)^2 \quad (4.32)
\]

Then we substitute (4.30) and (4.31) into \( \xi(F, N) \)
\[
\xi = 2 \int_0^\infty ds e^{-sm^2} \left( 1 + \frac{5s^2}{48} \text{tr} N^2 + \frac{11s^4}{1920} (\text{tr} N^2)^2 - \frac{s^4}{720} \text{tr} N^4 + \ldots \right) .
\]

Integrating over proper time \( s \) and using the expression
\[
\text{tr} N^2 = D^+_\alpha D^+_{\beta} W^{-\alpha} W^{-\beta},
\]
\[
\text{tr} N^4 = \frac{1}{2} (\text{tr} N^2)^2 - 4(D^+)^4(W^-)^4
\]

we finally have
\[
\Gamma[V^{++}] = \frac{1}{32\pi^4 m^2} \int d\zeta^{-4} du (W^+)^4 \left( 1 + \frac{5}{24 m^2} D^+_\alpha D^+_{\beta} W^{-\alpha} W^{-\beta} \right.
\]
\[
+ \frac{29}{240 m^2} (D^+ D^+_{\alpha} W^{-\alpha} W^{-\beta})^2 - \frac{1}{15 m^2} (D^+)^4 (W^-)^4 + \ldots \right) .
\]

This expression allows us to write down the following gauge and (1, 0) supersymmetric invariants
\[
I_1 = (W^+)^4, \quad I_2 = (W^+)^4 D^+_{\alpha} D^+_{\beta} W^{-\alpha} W^{-\beta}, \quad (4.37)
\]
\[
I_3 = (W^+)^4 (D^+_{\alpha} D^+_{\beta} W^{-\alpha} W^{-\beta})^2, \quad I_4 = (W^+)^4 (D^+)^4 (W^-)^4. \quad (4.38)
\]

The last invariant, \( I_4 \), corresponds to the term \( (W^+)^4(W^-)^4 \) in the harmonic 6D, \( N = (1, 0) \) superspace in the central basis or \( \sim (W^i)^8 \) in conventional one. It is clear that the similar procedure allows us, in principle, to obtain any term in expansion of the effective action in power series in superfield strengths and their spinor derivatives.

It is useful to compare the invariants (4.37), (4.38) with the on-shell invariants which were obtained in the recent paper [7] (see also [8]). In the nomenclature of the work [7] the invariants (4.37), (4.38) have the following canonical dimensions \( d(I_1) = 8, d(I_2) = 12, d(I_3) = 16, d(I_4) = 16 \), where \( d(I_i) \) is the dimension of the invariant \( I_i, i = 1, 2, 3, 4 \).\(^{11}\) The invariant \( I_1 \) coincides with the corresponding \( d = 8 \) invariant in the paper [7]. The similar invariant appeared also as a leading low-energy contribution to effective action in the 6D model of hypermultiplet coupled to tensor hierarchy [6]. The invariants with the dimensions \( d = 12, 16 \) are not considered in [7]. However, there is the invariant of the dimension \( d = 10 \) in [7] which is absent in our case. We will show that such an invariant vanishes in the case of covariantly constant background (4.9).

In analytic basis the \( d = 10 \) invariant has the structure
\[
S^{(10)} = \int d\zeta^{-4} du \varepsilon_{\alpha\beta\gamma\delta} (D^+)^4 (W^{\alpha+\beta} W^{-\gamma} W^{+\delta}) .
\]

However in our case of covariantly constant on-shell background (4.9), the \( d = 10 \) invariant (4.39) equals to zero. First of all we note that according to (2.20) and first condition in (4.9)
\[
D^+_{\alpha} W^{+\beta} = \frac{1}{4} \delta_{\alpha}^{\beta} Y^{++} = 0 .
\]

\(^{11}\)Following to the work [7] we suppose \( d \) as a mass dimension of corresponding component 6D Lagrangian.
Now one uses constrains $D^\pm_a F^{ab} = 0$, which is also a part of background conditions (4.9). Hence
\[ 2D^\pm_\alpha N^\delta_\gamma = D^\pm_\alpha F^\delta = (\gamma^{ab})_\gamma D^\pm_\alpha F^{ab} = 0. \tag{4.41} \]
As a result we have in the action (4.39)
\[ (D^+)^{\underline{3}}(W^{+\alpha}W^{-\beta}W^{+\gamma}W^{-\delta}) \sim \varepsilon^{\rho\sigma\tau\kappa}D^+_\rho D^+_\sigma (N^{\beta}_\tau N^{\delta}_\kappa)W^{+\alpha}W^{+\gamma} = 0. \tag{4.42} \]

Thus we see the reason why the $d = 10$ invariant (4.39) is absent in our approach. The background conditions (4.9) contain not only on-shell condition $Y^{++} = 0$ but also the condition of space-time constancy of the background. Namely this condition forbids $d = 10$ invariant. It is worth emphasizing that the last condition is crucial to obtain the closed Heisenberg-Euler type effective action. However, if to set a goal to derive the on-shell invariants from the effective action, we can relax the background conditions (4.9) eliminating the space-time constancy conditions. In this case the effective action can be constructed in form of expansion in derivatives of the superspace strengths and we expect that all possible on-shell invariants will be obtained.

5 Conclusion

Let us briefly summarize the main results. We have considered a problem of the induced effective action in the $6D, \mathcal{N} = (1,0)$ hypermultiplet theory coupled to an external field of vector multiplet. The theory is formulated in six dimensional $(1,0)$ harmonic superspace in terms of an unconstrained analytic hypermultiplet superfield in the external superfield corresponding to an Abelian vector multiplet. The effective action is computed in the framework of superfield proper-time technique which allows us to preserve a manifest gauge invariance and $\mathcal{N} = (1,0)$ supersymmetry. It was shown that the effective action under consideration is on-shell finite.

To calculate the low-energy effective action it is sufficient to consider a special background (4.9). We have developed a generic procedure for calculating the effective action on such a background and found the complete effective action (4.28). As we pointed out the divergences are absent on-shell (2.23) and therefore the complete low-energy effective action (4.28) is automatically finite.

Note that the theory under consideration is anomalous [17]. Therefore, in principle, the total effective action can contain, besides above result, the additional effective action generating the anomaly. Since the anomaly is stipulated by divergences and the divergences are absent on the background under consideration, one can expect that such an additional action is also absent in our case. However, even if this additional action exists on the given background, this will be independent contribution to the total effective action (4.28) and its calculation is a separate problem.

We expect that the obtained results have relation to the problem of the effective action of a single isolated D5-brane [3]. However, to calculate the effective action for such a model we should study a quantum vector/tensor + hypermultiplet system. Of course, such a problem requires special consideration. Another aspect, which is essential for finding the effective action of D5-brane is that the calculations should be carried out on a conformally broken phase of the $6D$ non-Abelian supersymmetric gauge theory (see definition of this phase e.g. in [1]).
Nevertheless, we hope that the methods, developed in this paper, can be used to analyze the general problem of the effective action of the D5-brane.

The methods and results of the present work can be generalized in the following directions: (i) calculation of the low-energy effective action beyond the leading approximation, (ii) calculation of the effective action in a non-Abelian theory in the broken phase, (iii) calculation of the effective action of the quantum vector/tensor+hypermultiplet system.

We announce with deep sorrow that our friend and co-author Nicolay Pletnev passed away suddenly when the paper was practically finalized.

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