Exclusive photoproduction of Φ on Proton in the quark-diquark model

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Abstract

We present predictions for exclusive photoproduction of Φ-meson on proton at large transfer, where we use a quark-diquark structure model for the proton. Extrapolation from our results to lower transfers is comparable in magnitude with available data in that range. This may support the diquark model in its ability to provide, for that process, an appropriate link between diffractive physics at low transfer and the standard semi-perturbative approach of hard exclusive processes at very large transfer, where the proton recovers its three-quark structure.
I. INTRODUCTION

Up to now, exclusive photoproduction of a vector meson \( V \) \((V = \rho, \omega, \Phi, J/\Psi)\) from proton (reaction (1)) have been measured mainly at very low values of \( t \), \( t \) being the opposite of the squared momentum transfer at the proton vertex.

\[
\gamma + p \rightarrow V + p \tag{1}
\]

In that region (say, \( t \leq 1 \text{ GeV}^2 \)), and in a wide energy range up to HERA energies, the observed characteristics of photoproduction of the lighter mesons are those of a soft diffractive process. As shown by Donnachie and Landshoff [1], this can be well described for the most part in a picture using both Vector Dominance Model (VDM) and Pomeron phenomenology: there, the incoming photon is assumed to convert into a vector meson which afterwards exchanges a soft Pomeron with the proton target. In this respect, one may consider reaction (1) in this small-transfer range as a good testing bench for Pomeron Physics. Indeed, that picture works nicely in case of photoproduction of the lighter mesons \( \rho, \Phi \) and \( \omega \) [2]. However, it fails to reproduce the energy dependence of the cross section for \( J/\Psi \) photoproduction [3].

In a QCD-inspired picture, Pomeron exchange is commonly modelled as the effective exchange of two non-perturbative gluons. Donnachie and Landshoff improved substantially that picture [4]. When applied to photoproduction of light mesons, their two-gluon exchange model leads to results very similar to those provided by the Pomeron-exchange model of the same authors [3]. However, it also fails to describe the energy dependence of \( J/\Psi \) photoproduction, as well as those of virtual photoproductions of \( \rho, \Phi \) and \( J/\Psi \), as observed at HERA [3].

Following the works of Ryskin and of Brodsky et al. [7], this may reveal that QCD perturbative effects enter the game, since a large momentum scale (either the mass of the heavy vector meson produced in case of \( J/\Psi \), or the high \( Q^2 \) of the
virtual photon in case of $\rho$) appears in the reaction. In their approach, Brodsky et al. wrote the amplitude of the process as the product of three terms: an amplitude describing the breaking of the virtual photon into a $q\bar{q}$ pair, the valence $q\bar{q}$ wave function of the vector meson and an amplitude describing a non-perturbative two-gluon structure of the proton. Actually, the latter amplitude plays a crucial role in this approach since the dependence on energy of the cross section directly reflects the small-$x$ behaviour of the gluon momentum distribution in the proton. In this way, one can account for the rapid rise with energy of the cross sections, observed at HERA.

In this paper, we consider (real) photoproduction of $\Phi$ on proton at larger $t$. Since the $\Phi$-meson is a pure $s\bar{s}$ state and the strangeness content of the nucleon wave function is probably small, that process is dominated, at lowest order in QCD, by the two-gluon exchange mechanism and thus provides a unique way to study the latter. This process has been already investigated at moderate $t$ by Laget and Mendez-Galain \cite{8}, using the non-perturbative picture of Donnachie and Landshoff. Their prediction for the differential cross section $d\sigma/dt$ at infinite photon energy exhibits a node at $t = 2.4\text{GeV}^2$, which, in turn, could serve as a test of the model. Unfortunately, the only existing data for $\Phi$-production with real photons or virtual photons correspond to very low values of $t$.

Since higher values of $t$ provide larger momentum scales, it is tempting to apply in that range the semi-perturbative approach of hard exclusive processes developed long ago by Brodsky, Farrar, Lepage (BFL) \cite{9}, and by Chernyak and Zhitnitsky (CZ) \cite{10}. This approach has been described and discussed many times in the literature, and shown to provide a correct order of magnitude of numerous exclusive amplitudes. Indeed, Farrar et al. already applied it to various photoproduction processes \cite{11}.

We just remind here that in this formalism, the amplitude of a given exclusive
process is obtained by a convolution formula of relativistic hadron wave functions with elementary hard scattering amplitudes involving the valence quarks and antiquarks of the hadrons taking part in the reaction. When using the baryon wave functions derived from QCD sum rules by (CZ), this method provides a correct order of magnitude for many exclusive amplitudes involving baryons. However, there exist important subasymptotic helicity-flip effects that do not fit the above picture where any spin effect is to be described by the so-called helicity conservation rule [12]. While this rule should be valid asymptotically, it appears to be inconsistent with most experimental data at intermediate energies. To cure this failure for processes involving baryons, a quark-diquark structure of baryons has been proposed [13]. In that alternative picture, two of the three quarks of a baryon are clustering together in a diquark structure. In the subasymptotic region, diquarks are supposed to act as quasi-elementaries constituents having direct couplings with photons and gluons. Helicity-flips are then caused by vector diquarks. On the other hand, diquarks should asymptotically dissolve into quarks, restoring in this way the usual three-quark picture of baryons. The diquark hypothesis provides natural explanations for many phenomena that are otherwise difficult to describe by standard models [14]. In the following, we apply that picture in the framework of the (BFL) scheme, as a first semi-perturbative calculation of the process under study, from moderate to large values of $t$.

Another point of theoretical interest in the study of elastic $\Phi$-photoproduction is found in the structure of the amplitude of the underlying hard scattering process. Indeed, that amplitude exhibits singularities coming from on-shell quark lines. Farrar et al. [15] have shown that in fact, any exclusive photoproduction process is, to leading twist, insensitive to long distance physics and do not require Sudakov resummation. The propagator singularities are integrable and their presence does not affect the validity of the hard scattering approach. The appearance of an
imaginary part of the amplitude at leading order in $\alpha_s$ is thus considered as a non
trivial prediction of perturbative QCD.

Studies of $J/\Psi$ or $\eta_c$ photoproduction are of course of the same interest as that
of $\Phi$ photoproduction, since the charm content of the proton is probably negligible
too, or even inexistent. However, on one hand, the high value of the c-quark mass is
a source of computational complications, and, in the other hand, more complicated
graphs are involved in case of $\eta_c$ production. So, we leave these two processes for
future investigations.

In Section 2 a short description of the quark-diquark model for exclusive pro-
cesses involving the proton is presented. The details of calculation of the hard
scattering amplitude for $\Phi$-photoproduction is given in Section 3. In Section 4 we
give our numerical results and concluding remarks.

II. THE QUARK-DIQUARK MODEL

Let us first notice that, formally, the amplitudes of the two processes $\gamma + p \rightarrow
V + p$ and $V \rightarrow p + \bar{p} + \gamma$ are just related by crossing. Two of us already studied
the decay process $J/\Psi \rightarrow p + \bar{p} + \gamma$, using the quark-diquark model structure of
the proton. So, we largely refer to our previous paper \cite{15} for notations.

The formalism we are using below is the same as that of (BFL), except that the
three-body structure of the proton is replaced by a two-body one. To lowest order
in QCD, the photoproduction of $\Phi$ on proton is thus described by the generic graph
of Fig. 1. The corresponding amplitude is obtained here too from a convolu-
tion formula

\[ T = K \int [dx][dx'][dy] \frac{\alpha_s^2}{g^2 G^2} T_{\mu\nu} \varepsilon^\alpha(\phi) \varepsilon^\beta(\gamma) I^{\mu\nu} \]

where $[dx] = \delta(1 - x_1 - x_2)dx_1dx_2$, $[dx'] = \delta(1 - x'_1 - x'_2)dx'_1dx'_2$, $[dy] =
\delta(1 - y_1 - y_2)dy_1dy_2$. Here, collinearity of the constituents with the parent hadron
is assumed : \( x_1 = x \) (resp. \( x'_1 = x' \)) is the four-momentum fraction of the quark inside the ingo0ing (resp. outgoing) proton, and \( x_2 = 1 - x \) (resp. \( x'_2 = 1 - x' \)) that of the accompanying diquark ; \( y_1 = y \) (resp. \( y_2 = 1 - y \)) is the four-momentum fraction of the strange quark (resp. that of the strange antiquark) inside the \( \Phi \)-meson. The tensor \( T_{\mu \nu \alpha \sigma} \) is the amplitude for the subprocess \( gg\gamma \rightarrow \Phi \) with two space-like gluons having four momenta \( g = xp - x'p' \) and \( G = (1 - x)p - (1 - x')p' \) respectively, \( p \) being the four-momentum of the ingoing proton and \( p' \) that of the outgoing proton ; \( \varepsilon'^{\alpha}_\Phi \) and \( \varepsilon'^{\sigma}_\gamma \) are polarization vectors for the \( \Phi \) and for the photon respectively ; \( I^{\mu \nu} \) is a tensor amplitude describing the two-gluon scattering by a quark-diquark system ; \( K \) is the overall normalization factor :

\[
K = \sqrt{4\pi \alpha_s} \frac{4\pi^2 f_\Phi}{9\sqrt{6}} C \tag{3}
\]

with the color factor \( C = -2/(3\sqrt{3}) \) and the \( \Phi \) decay constant \( f_\Phi \sim 150\text{MeV} \).

For the sake of consistency of the model, we have neglected the masses of the constituents as well as those of the parent hadrons, whenever possible. This led us to use the relativistic form of the \( \Phi \) wave function. Depending on the helicity \( h \) of the \( \Phi \), it is given by

\[
\Psi_\Phi = \frac{f_\Phi}{\sqrt{24}} \frac{1}{\sqrt{3}} \sum_{\text{color}} \sum \langle s\bar{s} \rangle \begin{cases} \mathcal{P} \phi_L(y) & \text{for } h = 0 \\ \mathcal{P} \phi_T^{(h)}(y) & \text{for } h = \pm 1 \end{cases} \tag{4}
\]

where \( \phi_L(y) \) and \( \phi_T(y) \) are normalized \( y \)-distributions for, respectively, a longitudinally and a transversally polarized meson.

In that approximation, and due to the particular structure of the amplitude of the subprocess (odd number of \( \gamma \)-matrices), it appears then that only longitudinal \( \Phi \) are produced.
The tensor amplitude $T_{\ldots}$ in (2) is thus simply obtained from the amplitude of $3\gamma \to e^+e^-$ with massless electrons, by removing the coupling constant, using appropriate four-momenta and making the substitution

$$V_{e^+\bar{e}^-} \to P$$

The wave functions of mesons have been derived by (CZ) from QCD sum rules technics. We here use their longitudinal-$\Phi$ wave function

$$\phi_L(y) = 6y(1-y) \{ y(1-y) + 0.8 \}$$

that can be found in [10], p. 259.

Allowing for both scalar ($S$) and vector ($V$) diquarks, a quark-diquark proton state corresponding to a proton helicity “up” or “down” takes on the general form:

$$|p^{\uparrow\downarrow} > \sim - f_s \left[ 2\phi_1(x) + \phi_3(x) \right] S(ud) \ u^{\uparrow\downarrow} \pm f_v \left[ \phi_2(x) \left\{ \sqrt{2} \ V_\pm(uu) \ u^{\downarrow\uparrow} - 2 V_\pm(uu) \ d^{\uparrow\downarrow} + \phi_3(x) \left\{ \sqrt{2} \ V_0(uu) \ d^{\downarrow\uparrow} - V_0(uu) \ u^{\downarrow\uparrow} \right\} \right]$$

where $V_h(q_1q_2)$ is an isovector-(pseudo)vector diquark state made of two quarks having flavors $q_1$ and $q_2$, $h = 0, \pm 1$ being its helicity; $S(ud)$ is the isoscalar-scalar diquark state.

The $\phi_i(x)$ are normalized wave functions, and $f_s$ and $f_v$ are normalization constants that may be chosen unequal to allow for various admixtures of scalar and vector components. Expressions of diquark-gluon couplings have been given in refs [17]. In the space-like channel ($g + D \to D'$) these expressions are, using obvious notations and omitting color factors as well as coupling constants:

$$\langle S' S \rangle_\mu = F_s \ (D_\mu + D'_\mu)$$

for a pair of scalar diquarks and
\[(V'_{h'}V_h)_\mu = -F_1 (D_\mu + D'_{\mu})\varepsilon_{D'}^{h'}\cdot\varepsilon^{h}_D + F_2 \left\{ (D_\mu\varepsilon_{D'}^{h'})\varepsilon^{h}_D + (D'_\mu\varepsilon^{h}_D)\varepsilon_{D'}^{h'} \right\} \]
\[ - F_3 (\varepsilon_{D'}^{h'}\cdot D)(\varepsilon^{h}_D\cdot D')(D_\mu + D'_\mu) \]
\[(9)\]

for a pair of vector diquarks \(1\)

The \(F\) above are the diquark form factors depending on \(Q^2 = -G^2 = -(D - D')^2\). A possible parametrization, aiming at describing the natural evolution of the diquark model into the usual three-quark picture, has been proposed by the authors of ref [17]. It has the following form :

\[F_s(Q^2) = \chi \frac{Q_0^2}{Q^2 + Q_0^2}, F_1(Q^2) = \chi \left( \frac{Q_1^2}{Q^2 + Q_1^2} \right)^2\]
\[F_2(Q^2) = (1 + k_\nu)F'_1(Q^2), F_3(Q^2) = \frac{Q^2 F_1(Q^2)}{(Q^2 + Q_1^2)^2}\]
\[(10)\]

with

\[
\chi = \begin{cases} 
\alpha_s(Q^2)/\alpha_s(Q_0^2) & \text{for } Q^2 \geq Q_0^2 \\
1 & \text{for } Q^2 \leq Q_0^2
\end{cases}
\]
\[(11)\]

\(k_\nu\) being the anomalous magnetic moment of the vector diquark. The value \(k_\nu=1\) is commonly assumed. The above-defined evolutionary picture also induces one to use, for the sake of consistency, coupling constants of the running form and to set \(\alpha_s^2 = \alpha_s(-g^2)\alpha_s(-G^2)\), with \(-g^2 = txx'\), yet restricting \(\alpha_s\) to some maximum value \(c_1\). Let us remind that setting the factor \(\chi\) in diquark form factors provides the correct power of \(\alpha_s\)'s in amplitudes at large transfer.

\[1\] We do not consider here a possible mixed coupling involving both scalar and vector diquarks, as it is commonly expected to give a small contribution.
In [16], we used such a parametrization to fit the proton magnetic form factor \( G_M \) in the space-like region. Modelling the momentum fraction distributions by a wave function of asymptotic form, i.e. taking

\[
\phi_1(x) = \phi_2(x) = \phi_3(x) = \phi_{as}(x) = 20x(1-x)^3
\]

a quite good fit were obtained with the following values of parameters :

\[
f_s = 40 \text{ MeV}, \quad f_v = 96 \text{ MeV}, \quad Q_1^2 = 2 \text{ GeV}^2, \quad Q_0^2 = 2.3 \text{ GeV}^2
\]

for \( c_1 = 0.3 \)

where, as said above, \( c_1 \) is the maximum allowed value of the running coupling constant \( \alpha_s \). Given that success, we used the same parametrization for the present calculation.

**III. THE HARD-SCATTERING SUB-AMPLITUDES**

As already mentionned, according to the model here used, longitudinal \( \Phi \) are preferentially produced. There are then, a priori, eight dominant helicity amplitudes describing the process, which we denote by \( T^{\lambda\lambda\Lambda} \) where \( \lambda \lambda' \) and \( \Lambda \) are the helicities of, respectively, the ingoing proton, the outgoing proton and the real incident photon. Thanks to parity and rotational invariance, that number in fact reduces to four and we have :

\[
T^{\downarrow\downarrow\downarrow} = -T^{\uparrow\uparrow\uparrow} \quad T^{\downarrow\downarrow\uparrow} = -T^{\uparrow\uparrow\downarrow}
\]

\[
T^{\downarrow\uparrow\downarrow} = T^{\uparrow\downarrow\uparrow} \quad T^{\downarrow\uparrow\uparrow} = T^{\uparrow\downarrow\downarrow}
\]

To be more specific, let us now concentrate on the calculation of the amplitude \( T^{\uparrow\uparrow\uparrow} \). To compute the amplitudes, we have chosen for convenience the polarization states of the particles according to a “t-channel helicity-coupling scheme” [18]. In
that scheme, the photon helicities represent photon spin projections on the “vertex plane” defined by the Φ and photon four-momenta, and the photon polarization vectors $\varepsilon^{(\pm)}_{\gamma}$ are then perpendicular to that plane. Similarly, the helicities of the protons are projections of their respective spins on the vertex plane defined by their two four-momenta.

From eq. (2) we thus obtain

$$T_{\uparrow\uparrow} = K\frac{\sqrt{2t}}{t^2} \int [dx][dx'][dy] \frac{\alpha_s^2}{x(1-x)x'(1-x') } \phi_L(y) T_{\uparrow\uparrow}$$

where

$$T_{\uparrow\uparrow} = 12\sqrt{\frac{t^2}{\mu_v^2}} F_2(Q^2)(1-x)(1-x') \cot(\theta/2) \times$$

$$\frac{1}{y(1-y)} \left\{ \frac{(1-y)A}{d'} + \frac{yB}{d} + \frac{y(1-y)C\sin^2(\theta/2)}{dd'} \right\}$$

$s$ being the center-of-mass energy squared, $\theta$ the Φ emission angle relative to the incident photon direction in the center-of-mass frame ($\sin^2(\theta/2) = t/s$), $Q^2 \sim t(1-x)(1-x')$, $\mu_v$ the diquark mass usually taken equal to 600 MeV; $d$ and $d'$ are the s-quark propagator factors:

$$d = xx' \sin^2(\theta/2) + y(x \cos^2(\theta/2) - x') - i\epsilon$$

$$d' = (1-x)(1-x') \sin^2(\theta/2) + (1-y)((1-x) \cos^2(\theta/2) - (1-x')) - i\epsilon$$

where, following the usual prescription, $\epsilon \to 0^+$. Finally, $A$, $B$ and $C$ are given by
\begin{align*}
A &= \phi_3(x')\phi_2(x)((1 - x')\sin^2(\theta/2) + (1 - y)\cos^2(\theta/2)) \\
&\quad - \phi_2(x')\phi_3(x)((1 - x)\sin^2(\theta/2) - (1 - y)) \\

B &= \phi_2(x')\phi_3(x)(x'\sin^2(\theta/2) + y\cos^2(\theta/2)) \\
&\quad - \phi_3(x')\phi_2(x)(x\sin^2(\theta/2) - y) \\

C &= \phi_2(x')\phi_3(x) \{(y(1 - x) - x(1 - x))\cos^2(\theta/2) + x'(1 - y) - x'(1 - x')\} \\
&\quad + \phi_3(x')\phi_2(x) \{(x(1 - y) - x(1 - x))\cos^2(\theta/2) + y(1 - x') - x'(1 - x')\}
\end{align*}

From eq. (17), it is clear that the kernel eq. (15) has singularities within the domain of integration, since the real parts of \(d\) and \(d'\) have zeroes in \(x\) respectively located at

\begin{align*}
z_0 &= \frac{x'y}{y\cos^2(\theta/2) + x'\sin^2(\theta/2)} \leq 1 \\

\text{and}
\end{align*}

\begin{align*}
z_1 &= 1 - \frac{(1 - x')(1 - y)}{(1 - y)\cos^2(\theta/2) + (1 - x')\sin^2(\theta/2)} \leq 1
\end{align*}

These singularities correspond to one or the two exchanged s-quarks going on-shell in the graph of Fig 1. It is important to notice that when \(x' = y\) the two zeroes coincide and are both equal to \(x'\) (or \(y\)). The one-pole terms (\(\sim \frac{1}{d}\) or \(\sim \frac{1}{d'}\)) can be treated readily, using the general formula

\begin{align*}
\frac{1}{u - i\epsilon} &= \mathcal{P}\left(\frac{1}{u}\right) + i\pi\delta(u) \quad (20)
\end{align*}

where \(\mathcal{P}\) denotes the principal value. The two-pole term \(\sim \frac{1}{dd'}\) corresponds to the graph where the photon line is sandwiched between the two gluon lines. Setting \(r = \Re(d), r' = \Re(d')\) (\(\Re\) means real part), one gets
\[
\frac{1}{dd'} = \mathcal{P}\left(\frac{1}{r}\right) \mathcal{P}\left(\frac{1}{r'}\right) - \pi^2 \delta(r) \delta(r') + 
\]

\[
i\pi \left\{ \mathcal{P}\left(\frac{1}{r}\right) \delta(r') + \mathcal{P}\left(\frac{1}{r'}\right) \delta(r) \right\} \tag{21}
\]

It appears that the product of two delta-functions, which is apparently the most singular term, leads in fact to a null contribution. This is to be imputed to the fact that the amplitude of a fermion-antifermion-vector-meson vertex is zero when all particles are massless.

Let us first consider the imaginary part of the full amplitude. It has the general form

\[
C_1(x, x', y)\delta(r) + C_2(x, x', y)\delta(r') + C_3(x, x', y) \left\{ \mathcal{P}\left(\frac{1}{r}\right) \delta(r') + \mathcal{P}\left(\frac{1}{r'}\right) \delta(r) \right\} \tag{22}
\]

that can be trivially integrated by hand over the variable \(x\), yielding an expression of the form:

\[
\frac{C_1(z_0, x', y)}{\alpha} + \frac{C_2(z_1, x', y)}{\alpha'} + \]

\[
\frac{1}{\alpha'} C_3(x_1, x', y) \mathcal{P}\left(\frac{1}{r}\right)_{x = z_1} + \frac{1}{\alpha} C_3(z_0, x', y) \mathcal{P}\left(\frac{1}{r'}\right)_{x = z_0} \tag{23}
\]

where \(\alpha = x' \sin^2(\theta/2) + y \cos^2(\theta/2)\) and \(\alpha' = (1 - x') \sin^2(\theta/2) + (1 - y) \cos^2(\theta/2)\).

One must be cautious with the two last terms as they are source of difficulties in the subsequent (numerical) integrations, as now explained. Since

\[
\frac{1}{\alpha'} \mathcal{P}\left(\frac{1}{r}\right)_{x = z_1} = \frac{1}{\alpha} \mathcal{P}\left(\frac{1}{r'}\right)_{x = z_0} = \frac{4}{\sin^2\theta} \mathcal{P}\left(\frac{1}{(x' - y)^2}\right) \tag{24}
\]

a double-pole-like term \(\sim 1/(x' - y)^2\) appears. However, that “singularity” is tempered by zeroes of the factors \(C_3\) when \(x' = y\) \((C_3 \propto (x' - y))\). These zeroes, which are of degree one, are reminiscence of the already mentioned fact that the amplitude of a fermion-antifermion-vector-meson vertex is zero when all particles are massless; and, precisely, the two exchanged s-quarks that are coupled to the
real photon in the corresponding “singular” Feynman graph get both massless when \( x = x' = y \). Consequently, the (seemingly) double-pole reduces to a simple-pole:

\[
(x' - y) \mathcal{P} \left( \frac{1}{(x' - y)^2} \right) \rightarrow \mathcal{P} \left( \frac{1}{x' - y} \right)
\]

(25)

In order to manage this fact in a cautious way, we have proceeded as follows. First, we split the products of wave functions \( \phi \) into symmetrical and antisymmetrical parts:

\[
S_{23}(x, x') = \frac{1}{2} \{ \phi_2(x')\phi_3(x) + \phi_3(x')\phi_2(x) \}
\]

(26)

\[
A_{23}(x, x') = \frac{1}{2} \{ \phi_2(x')\phi_3(x) - \phi_3(x')\phi_2(x) \}
\]

Of course, this operation is useful only when \( \phi_2 \neq \phi_3 \) and is thus inoperant for the symmetrical parametrization used in this paper. We just present it here for further applications. The imaginary part of the factor in brackets in formula (16) may then be rewritten as

\[
\pi \delta(x - z_0) \{ S_{23}(x_0, x')S_0 + A_{23}(x_0, x')A_0 \} +
\]

(27)

\[
\pi \delta(x - z_1) \{ S_{23}(x_1, x')S_1 + A_{23}(x_1, x')A_1 \}
\]

In order to maximally reduce the effect of the pseudo-pole \( \propto 1/(x' - y) \), we further made the shift \( x' = y + (x' - y) \) and the appropriate simplifications in all coefficients \( S \) and \( A \). We then arrived at the more manageable expressions:

\[
S_0 = \frac{y}{\alpha^2} \left\{ x'(x' - y) \sin^4(\theta/2) + y \alpha(1 + \cos^2(\theta/2)) + (1 - y) \left[ 2x'(2x' - 1) + \cos^2(\theta/2) \left[ (y - x')(2x' - 1) - 2y^2 + y \sin^2(\theta/2) \right] \right] \right. +
\]

(28)

\[
\left. - \frac{(1 - y)}{\cos^2(\theta/2)} \left[ (x' + y)(2x' - 1) + 2y^2 \right] +
\right. y^2(1 - y)(1 - 2y) \left( \frac{1 + \cos^4(\theta/2)}{\cos^2(\theta/2)} \right) \mathcal{P} \left( \frac{1}{x' - y} \right) \}
\]
\[ A_0 = \sin^2(\theta/2) \frac{y}{\alpha^2} \left\{ x' y + \sin^2(\theta/2)(x' - y)^2 + x' - y + y(1 - y) \cos^2(\theta/2) + \right. \\
- \left. \frac{1}{\cos^2(\theta/2)} (1 - y)(x' + y) - y^2(1 - y) \frac{1 + \cos^2(\theta/2)}{\cos^2(\theta/2)} \mathcal{P} \left( \frac{1}{x' - y} \right) \right\} \] (29)

The other factors \( S_1 \) and \( A_1 \) are obtained respectively from \( S_0 \) and \( -A_0 \) by the simple replacement \( x' \to 1 - x', \ y \to 1 - y \).

Let us now turn to the real part of the amplitude. It appears that different terms of the same (large) magnitude compensate each other in the domain of integration. To cure that new difficulty which causes numerical uncertainties, we decided to put all the expression on the same denominator \( rr' \) and, again, to introduce symmetrical and antisymmetrical combinations of wave functions so that the real part of the factor in brackets in (16) takes on the form

\[ \{ S_{23}(x, x') S' + A_{23}(x, x') A' \} \mathcal{P} \left( \frac{1}{r} \right) \mathcal{P} \left( \frac{1}{r'} \right) \] (30)

Then, we set \( u = x' - x, \ v = y - x' \), and rewrote the coefficients \( S' \) and \( A' \) as polynomials in \( u \) and \( v \). We thus got

\[ S' = v^3 H_3 + v^2 H_2 + v H_1 + H_0 \] (31)
\[ H_3 = -4u \cos^4(\theta/2) + 2\sin^2(\theta/2)(1 + \cos^2(\theta/2))(1 - 2x') \]

\[ H_2 = 2u(1 - 2x')(1 + 2\cos^4(\theta/2)) + \sin^2(\theta/2)(1 + \cos^2(\theta/2))(8x'(1 - x') - 1) \]

\[ H_1 = u^2 \sin^4(\theta/2)(1 - 2x') + 2u \sin^4(\theta/2)x'(1 - x') + 2u \sin^2(\theta/2)(1 - 4x'(1 - x')) + 2u(6x'(1 - x') - 1) - 2\sin^2(\theta/2)(1 + \cos^2(\theta/2))x'(1 - x')(1 - 2x') \]

\[ H_0 = ux'(1 - x') \left\{(2u - 2x' + 1) \sin^4(\theta/2) - 2(1 - 2x') \right\} \]

and

\[ \mathcal{A}' = \sin^4(\theta/2) \left\{-2v^3 + v^2(1 - 2u - 2x') + v(2x'(1 - x') - u^2) + ux'(1 - x') \right\} \]

This trick is supposed to moderate both the above-mentionned cancellations and the double pole \( \sim 1/(rr') \). At this point, let us notice that when \( v \to 0 \), one has \( 1/(rr') \sim 1/(u^2x'(1 - x')) \) whereas the numerators of the amplitudes behave like \( ux'(1 - x') \) so that the double-pole in \( u \) turns into a simple-pole (with no end-point singularities). Similarly, when \( u \to 0 \), then \( 1/(rr') \sim 1/(v^2x'(1 - x')) \) while the numerators of the amplitudes are \( \sim vx'(1 - x') \) so that the amplitude is only \( \sim 1/v \).

As for end-point singularities due to the denominator \( x(1 - x)x'(1 - x')y(1 - y) \) coming from gluon and s-quark propagators, they cause no problem since the above denominator is cancelled by an equivalent factor contained in the product of hadron wave functions. We assume that this cancellation of end-point singularities is sufficient, namely, that no extra Sudakov form factor that could a priori suppress more drastically those singularities is to be implemented. The problem of possible Sudakov suppression of end-point singularities is beyond the scope of the
present work. It has been discussed in different contexts by various authors to whom we refer the reader [19]. For our calculations, given the uncertainties in the parametrization of the diquark model in its present form, we think it senseless to introduce an additional complication with a Sudakov factor, the precise form of which for that model and for the process here studied is yet unknown anyway. Nevertheless, we here apply a kind of suppression of end-point singularities by cutting off the dangerous growth of coupling constants $\alpha_s(-g^2)$ and $\alpha_s(-G^2)$ in the end-points region by means of the parameter $c_1$ (see above).

All amplitudes have been treated in the same way. Moreover, to prevent additional instabilities in our numerical evaluations, we modeled principal values by the approximate form

$$P \left( \frac{1}{z} \right) \sim \frac{z}{\epsilon^2 + z^2} \quad (34)$$

with $\epsilon \ll 1$. For instance, in real parts we made the substitution

$$P \left( \frac{1}{rr'} \right) \sim \frac{r}{\epsilon^2_1 + r^2} \frac{r'}{\epsilon^2_2 + r'^2} \quad (35)$$

Our numerical results are presented in the next section.

**IV. RESULTS AND CONCLUSIONS**

We spent a lot of time in searching the best method of integration, and checking the stability of our computations. Given the rather simple dependence on $y$ of the integrant, we tried to integrate by hand over $y$ first. This is not a good method because it induces spurious singularities in the subsequent integration over $x$ and $x'$ through denominators like $x \cos^2(\theta/2) - x'$ which does has a zero in the integration domain. We also tried a numerical integration in the complex $y$-plane, but this also led to intractable instabilities. So, we finally adopted the strategy described
in Section 3. We then varied the parameters \( \epsilon \) down to a value as small as \( 10^{-9} \) and even cross-checked our results using two different programs of integration, namely a Gaussian quadrature method (RGAUSS) and a Monte-Carlo method (VEGAS). It appears that the real parts of amplitudes are the most intractable: the less the value of \( \epsilon \), the greater should be the number of calls of the integrant. However, a value of \( \epsilon \) between \( 10^{-4} \) and \( 10^{-5} \) seems to provide the best stability. By chance, for the particular parametrization here used, the contributions of real parts appear to be much less than those of imaginary parts in the whole kinematical range we have investigated, by a factor of a few per cent or less. On the other hand, the results obtained for the imaginary parts are very stable, being much less sensitive to the value of \( \epsilon \).

Thus, the results we are presenting now take account of imaginary parts of amplitudes only.

In Fig. 2, is shown the momentum transfer distribution we obtain in the range \( 2 \text{ GeV}^2 - 10 \text{ GeV}^2 \) and for two values, 5 GeV (CEBAF) and 70 GeV (HERA), of the proton-photon invariant mass \( W \). One sees that this distribution is almost independent of \( W \). On the other hand, as expected, the distribution exhibits a kind of power-law fall-off, like \( t^{-5.5} \) around 3 GeV\(^2\) and like \( t^{-6.5} \) around 9 GeV\(^2\). Actually, we have found that the very simple form

\[
F(t) = \frac{A}{t^5(1 + Bt + Ct^2)}
\]  

(36)

with \( A = 94.5 \text{ nb GeV}^{10}, B = -0.113 \text{ GeV}^{-2}, C = 0.043 \text{ GeV}^{-4} \), provides an excellent fit of our results from \( t = 2 \text{ GeV}^2 \) up to values of \( t \) as large as 15 GeV\(^2\). It may be useful in a generator program for a simulation of the process. Integrating this form between 2 GeV\(^2\) and 10 GeV\(^2\) yields a cross-section of about 1.5 nb.

Also shown for comparison in the same figure is the fit of low-\( t \) experimental data provided by the ZEUS Collaboration [20]. The observed distribution has an
exponential fall-off $\sim \exp(-bt)$ with $b \sim 7.3 \text{ GeV}^{-2}$ for $<W> = 70 \text{ GeV}$, in agreement with the expectation of a diffractive character of the process in that range.

The comparison in Fig. 2 is indeed encouraging for the diquark model since a direct extrapolation from our results towards lower values of $t$ nicely compare in magnitude with the above-mentioned data. Let us remind here that according to the well-known asymptotic constituent counting-rule, one expects a change of the observed exponential fall-off of the $t$-distribution at low $t$ into a power one as $t$ is increasing. The diquark model is just supposed to account for that transition. One may thus consider the diquark model as a good candidate in describing future data at larger $t$, most probably with more refined wave functions and diquark form factors, thus establishing, at least for that kind of process, a link between diffractive physics that holds at low $t$, and the semi-perturbative approach of hard exclusive processes one expects to hold at very large $t$ (the proton recovering there its three-quark structure).

Fig. 3 shows $s^7d\sigma/dt$ as a function of $\cos(\theta)$ where $s = W^2$. In the pure three-quark picture of the proton, that distribution is predicted to be independent of $s$ at large $s$, provided $\alpha_s$, the strong coupling constant, is taken as a constant. Obviously, that scaling law does not hold here. Essentially, this is because we are using an evolutionary picture of the diquark structure where the $\alpha_s$'s are expressed in running forms. It can be easily checked that one recovers the expected scaling law, if one takes both the $\alpha_s$'s and the factor $\chi$ in diquark form factors as constants. However, numerical computations show that this works better for $W \geq 10 \text{ GeV}$. In this respect, $W = 5 \text{ GeV}$ is not an asymptotic value.

Such deviations of the diquark model predictions from the asymptotic scaling law have also been obtained by Kroll and co-workers in their recent calculation on photoproduction of $K$ and $K^*$ mesons off proton, using the diquark model too.
with a similar parametrization \[21\].

In our numerical calculations, we also tried two other parametrizations of the quark-diquark structure of the proton. The first one is that of Kroll et al. in \[22\]. We found that the corresponding contribution of imaginary parts of amplitudes alone yields a \(t\)-distribution that is larger than that of Fig. 2 by more than one order of magnitude, which seems to rule out that parametrization since the corresponding rates look too high, especially at low \(t\). The second parametrization has a quark-diquark wave function derived from the asymmetrical three-quark proton wave function obtained by King and Sachrajda from QCD sum rules \[23\]. It is described in \[16\]. This time, the contribution of imaginary parts of amplitudes is much less than that obtained from the asymptotic wave function. On the other hand, real parts seem now to contribute much more. But, as said before, the latter are difficult to evaluate properly because of instabilities in numerical computations. So, we cannot draw any conclusion at the present time about that parametrization. At least, this shows that the study of \(\Phi\)-photoproduction at intermediate \(t\) would allow one to discriminate between various models \[24\]. Unfortunately, exclusive photoproduction processes are yet largely unexplored at large transfers though they possess in that range an undoubted physics potential. We hope very much to dispose, in a not too far future, of new data from CEBAF or HERA, at least at intermediate transfers.
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[24] One may wonder at this point whether our calculation is sensitive or not to the choice of the $\Phi$ wave function. As results from our numerical computations, replacing the (CZ) wave function by the asymptotic one ($6y(1-y)$) increases the rates by about 40%.
FIGURES

FIG. 1. A typical diagram for $\gamma + p \rightarrow \Phi + p$ in the quark-diquark picture

FIG. 2. Diquark-model predictions for $d\sigma/dt$ vs $t$; solid line: $W = 5$ GeV; dashed line: $W = 70$ GeV. Dash-dotted line: fit of low-$t$ data at $<W> = 70$ GeV [20].

FIG. 3. Diquark-model predictions for $s^7d\sigma/dt$ vs $\cos\theta_{cm}$; solid line: $W = 5$ GeV; dotted line: $W = 20$ GeV; dashed line: $W = 70$ GeV.
Fig. 1: A typical diagram for $\gamma + p \rightarrow \Phi + p$ in the quark-diquark picture
Figure 2
