Lateral and torsional vibration analysis of composite shaft

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Abstract. In Energy Industries reliability of rotating equipment’s plays a key role in overall plant reliability and safer operation, in fact maintaining these equipment’s in larger scale is a challenging task. The magnitude of vibration in rotating equipment’s is high when the rotor excitation force coincides with the natural frequency. This paper deals with the comparison study on existing steel shaft over laminated composite shaft’s lateral and torsional critical speed, which aims to optimize the natural frequency. The rotor system model represented here is undamped, without disc and simply supported with bearing for lateral vibration analysis and considering two discs at ends for torsional vibration analysis. The analysis report represents the comparison of both steel and laminated composite shaft, which features with Campbell diagram.

1. Introduction
Industries are striving to improve the asset life cycle with established maintenance and reliability methodologies and to further extension of this goal, industry are also investing on adapting latest technologies and novel materials for their applications. There have been various studies in past years which proves in adapting novel composite materials as a compliment for conventional metals. When focusing on rotating equipment failure’s which are majorly due to inefficiency in maintenance practice because of larger number of equipment’s in field and it also seen that most of their failure’s modes are intensified by vibration, which can be reduced to greater extend during design and manufacturing phase by using novel composite material as a shaft for rotating equipment. Additionally, composite material features with internal damping properties, better strength to weight ratio and can be tailor made according to the suitable applications, which in turn fosters the maintenance efficiency.

The analytical governing equation to calculate the natural frequency for the lateral vibration analysis of rotor have been explained by Tochio Yamamoto [1]. M. Ku [2] studied a simply supported rotor system without disc and validate the numerical results of lateral vibration rotor dynamic characteristic’s with previously published papers. The finite element method used in the work is C0-class Timoshenko beam, which is compare with other similar work to prove the improvised accuracy of the numerical model. Since, the paper deals with simple rotor system, it provides better insights to understand the concepts of rotor dynamic characteristics. Moreover, it features with study of whirl speeds (natural frequencies) & its associated modes for the wide range of speed and represented in Campbell diagram. C.Sharvankumar [3] studied two rotor sytem’s critical speed considering gyroscopic effects and compared the analysis results Numerical finite element method using Matlab and FE tool (Ansys). R. Sino [4] and other author’s carried out dynamic analysis of rotating composite shaft using homogenized finite element beam model to evaluate natural frequency and
instability of the composite rotor and compared the results obtained by other models namely equivalent beam modulus theory (EMBT), modified EMBT and layer wise beam theory (LBT). Moreover, they further investigated the influence of laminate parameters effect on composite rotor and particularly examined the effect of lamina stacking sequence on natural frequencies and instability of the carbon/epoxy composite shaft.

Several studies have been performed to investigate the effect of torsional load in the rotor, as it is critical to understand the torsional load effect on the natural frequency in rotor dynamics. An empirical formula for calculating the torsional natural frequency for the rotor system was derived by Rajiv Tiwari [5]. Jinguang Zhang [6] focused on torsional stiffness study of composite shaft with effect to stacking sequence of lamina. He compared the results with analytical model, FEA and experiment which elaborates the stacking sequence effect on torsional stiffness.

This paper aims to study the lateral and torsional vibration analysis of rotor system parameters namely critical speed and mode shapes for both conventional steel shaft and carbon epoxy hollow composite shaft and results the benefits of composite shaft.

2. Theoretical Formulation
Theoretical formulation for Lateral and Torsional vibration analysis of steel shaft’s analytical model is discussed in below sub section, which is further used to validate the results obtained from Ansys.

2.1 Lateral Vibration
The governing equations of motion for lateral vibration is given in below Equation (1).

\[
m\ddot{x} = F_{1x} + F_{2x} \\
m\ddot{y} = F_{1y} + F_{2y} \\
I\dddot{\theta}_x - I_p \omega^2 \dot{\theta}_y = M_{y_c} = -F_{1y}l_1 + F_{2y}l_2 \\
I\dddot{\theta}_y - I_p \omega^2 \dot{\theta}_x = M_{x_c} = F_{1x}l_1 - F_{2x}l_2
\]

(1)

The reaction forces and moments are shown in below Equation (2).

\[
F_{1x} = -k'_1x_1 \\
F_{2x} = -k'_2x_2 \\
F_{1y} = -k'_1y_1 \\
F_{2y} = -k'_2y_2 \\
M_{y_c} = \left(-k'_1x_1l_1\right) - \left(-k'_2x_2l_2\right) \\
M_{x_c} = \left(-k'_1y_1l_1\right) + \left(-k'_2y_2l_2\right)
\]

(2)

After substituting of the above equation’s with the geometrical relationships

\[
x_1 = x - l_1\theta_y; x_2 = x + l_2\theta_y; y_1 = y + l_1\theta_x; y_2 = y - l_2\theta_x
\]

The equations of motion can be written in a matrix form as shown below equation (3).

\[
\begin{bmatrix}
m & 0 & 0 & 0 \\
0 & m & 0 & 0 \\
0 & 0 & I & 0 \\
0 & 0 & 0 & I
\end{bmatrix} \begin{bmatrix}
x \\
y \\
\theta_x \\
\theta_y
\end{bmatrix} = \begin{bmatrix}
0 \\
0 \\
0 \\
-I_p\omega^2
\end{bmatrix}
\]

\[
\begin{bmatrix}
x' \\
y' \\
\theta_x' \\
\theta_y'
\end{bmatrix} = \begin{bmatrix}
k'_{xx} + k'_{xx} & 0 & k'_{x1}l_2 - k'_{x2}l_1 & 0 \\
0 & k'_{yy} + k'_{yy} & 0 & k'_{y1}l_2 - k'_{y2}l_1 \\
k'_{x1}l_1 - k'_{x2}l_2 & 0 & k'_{xx}l_1 + k'_{xx}l_2 & 0 \\
k'_{y1}l_1 - k'_{y2}l_2 & 0 & 0 & k'_{yy}l_1 + k'_{yy}l_2
\end{bmatrix} \begin{bmatrix}
x \\
y \\
\theta_x \\
\theta_y
\end{bmatrix}
\]

(3)

For the analytical calculation, the above equation is used to calculate natural frequency for lateral vibration analysis rotor model.
2.2 Torsional Vibration
The empirical formula to calculate the torsional natural frequency is shown in below equation (4).

\[ \omega_{nf} = \sqrt{ \frac{(I_{p1} + I_{p2})K_t}{I_{p1}I_{p2}}} \]  

(4)

3. Rotor Model
Rotor model for lateral vibration and torsional vibration is modelled using Ansys. The below sections discuss about the parameters involved in designing the rotor system for steel and composite shaft.

3.1. Lateral Vibration
The purpose of rotor system model is to calculate lateral critical speed, this system is undamped and simply supported by bearings. For simplicity, the rotor is modelled without any disc for both steel and composite shaft.

3.1.1 Steel shaft. The steel shaft for the rotor is modelled in Ansys, the below table 1 & table 2 shows the geometry parameters and material properties of the shaft.

| Table 1. Material properties of Structural Steel. |
|-----------------------------------------------|
| Specifications | Parameters            |
| Material       | Structural Steel     |
| Density        | 7850 kg m\(^{-3}\)   |
| E              | 200 GPa              |
| v              | 0.3                  |
| K              | 166.67 GPa           |
| G              | 76.923 GPa           |

| Table 2. Geometry dimensions of the rotor. |
|-------------------------------------------|
| Specifications | Parameters |
| Diameter       | 0.1016 m   |
| Length         | 1.27 m     |

Figure 1 shows the model of rotor system with simple supported bearings, the bearing stiffness value \( K_{11}=K_{22}=1.7513 \times 10^7 \text{ N/m} \) is used for model analysis.
3.1.2 Composite shaft. The Composite hollow shaft shares the same geometry dimensions and bearing stiffness value except inner diameter, which depends on number of lamina layers and lamina thickness. The below table 3 shows the composite material specification for the rotor and table 4 shows the lamina specification used in the shaft.

**Table 3. Material properties of Carbon Epoxy.**

| Specifications | Parameters |
|----------------|------------|
| Material       | Carbon Epoxy |
| Density        | 1540 kg/m³ |
| $E_{11}$       | 209 G Pa   |
| $E_{22} = E_{33}$ | 9.45 GPa |
| $\nu_{12}$    | 0.27       |
| $G_{12}$       | 5.5 GPa    |

**Table 4. Composite Lamina specification for the shaft.**

| Specifications           | Parameters |
|--------------------------|------------|
| Number of layers         | 10         |
| Layer thickness          | 1.321 mm   |
| Wall thickness           | 13.21 mm   |
| Fibre orientation angle  | $[0°/90°/0°/90°/0°]$ |

Figure 2 shows the design of composite shaft simply supported by bearings with same bearing stiffness values as used in steel shaft rotor system.
3.2 Torsional Vibration

Torsional vibration analysis is conducted to obtain the torsional critical speed for the steel and composite shaft. The same rotor system model of steel and composite shaft utilized for this analysis. The steel and composite shaft at both ends is added with disc as shown in figure 3 to depict rotating equipment’s rotor, which in real scenario deals with various polar mass moment of inertia across the system.

The polar mass moment of inertia for disc 1 and disc 2 about the Z-axis are 0.0873 kg-m² and 2.069 kg-m² respectively as shown in table 5. Considering the disc are thin and rigid for the system.
Table 5. Specification of discs added to rotor system for torsional vibration analysis.

| Disc no. | Diameter (m) | Polar mass moment of inertia (kg m²) |
|----------|--------------|-------------------------------------|
| 1        | 0.3048       | 0.873                               |
| 2        | 0.4064       | 2.069                               |

4. Results and Discussion

Critical speed caused by natural frequency in lateral vibration and torsional vibration rotor system for steel shaft and composite shaft is discussed in this section.

4.1 Lateral Vibration Analysis

To calculate the critical speed associated with the lateral vibration of the rotor, first steel shaft’s critical speed from the M. Ku [2] paper is considered as reference.

The derived analytical governing equation of motion is used to obtain the critical speeds, which resulted with two critical speeds. The analytical model and reference paper results are further used to validate the results and procedure of modal analysis module in Ansys workbench.

Ansys model resulted with three critical speeds for wide speed range as shown in figure 4 and the same is validated with other two results. Since the comparison results of steel shaft are fine, the same Ansys modal analysis procedure is utilized to obtain the result for composite shaft and the comparison of critical speed for steel and composite shaft is shown in table 6.

Composite shaft for the same speed range resulted with only two critical speeds as shown in figure 5, also a notable absence of critical speed between 5000 rpm to 19000 rpm shows a major advantage for considering the composite shaft in rotating equipment application.

![Figure 4. Campbell diagram of Steel shaft.](image-url)
Figure 5. Campbell diagram of Composite shaft.

Table 6. Lateral Critical Speed results from Campbell diagram for steel shaft and composite shaft.

| Mode | Whirl Direction | Critical speed of steel shaft from reference paper [2] | Analytical Critical speed of Steel shaft | Critical Speed of Steel shaft from Ansys | Critical Speed of Composite shaft from Ansys |
|------|-----------------|------------------------------------------------------|-----------------------------------------|----------------------------------------|-------------------------------------------|
|      |                 | Rpm                                                  | Rpm                                     | Rpm                                    | Rpm                                       |
| 1    | FW              | 4960                                                 | 5250                                    | 4926.4                                 | 4550                                      |
| 2    | FW              | 10500                                                | 10504                                   | 10453                                  | 19841                                     |
| 3    | FW              | -                                                    | -                                      | 21447                                  |                                            |

4.2 Torsional Vibration Analysis

To investigate the torsional critical speed influenced by the torsional load of rotor system, there are few assumption and consideration are been made. First, the bearing is considered as friction less and the discs polar moment of inertia is applied to both steel shaft and composite shaft rotor model.

The empirical formula [5] to calculate torsional natural frequency of the system is used to obtain the analytical results of the steel shaft rotor system, which is further validated with the Ansys modal analysis model. Figure 6 shows the torsional mode of the steel shaft obtained due to torsional natural frequency at 159.74 Hz. Similarly, the composite shaft rotor system is analyzed and shows a drastic variation of torsional natural frequency, which is obtained at 744.32 Hz as shown in figure 8 with the associated mode shape.

Additionally, figure 7 and figure 9 displays the associated torsional critical speed in the Campbell plot for steel and composite shaft respectively. Also, table 7 compares the torsional critical speed results for steel and composite shaft.
Figure 6. Steel shaft mode shape at torsional natural frequency.

Figure 7. Torsional Critical Speed of Steel shaft.

Figure 8. Composite shaft mode shape at torsional natural frequency.
Figure 9. Torsional Critical Speed of Composite shaft.

Table 7. Torsional natural frequency of steel shaft & composite shaft.

| Mode | Analytical Critical speed of Steel shaft Rpm | Critical Speed of Steel shaft from Ansys Rpm | Critical Speed of Composite shaft from Ansys Rpm |
|------|-------------------------------------------|-------------------------------------------|-----------------------------------------------|
| 1    | 9701                                      | 9584                                      | 44659                                         |

5. Conclusion

Composite material shows better performance in rotor dynamics study when compared to conventional steel, the results proved here may add value to engineers in considering composite shaft in design stage to certain applications of rotating equipment’s.

The number of Lateral critical speeds in composite shaft are less when compared to steel shaft for the same range of rotational speed, composite shaft exhibits absence of critical speed between 5000 to 19000 rpm whereas steel shaft’s exhibits second critical speed around 10000 rpm. In other case, torsional critical speed of composite shaft exhibits a way far when compared to the steel shaft. Future studies of composite shaft will be incorporated with unbalance response and stability analysis.

6. References

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