PARTICLES, SUPERPARTICLES AND SUPER YANG–MILLS.

CHRISTOPHER M. HULL

and

JOSE-LUIS VÁZQUEZ-BELLO

Physics Department, Queen Mary and Westfield College,
Mile End Road, London E1 4NS,
UNITED KINGDOM.

ABSTRACT

This paper is concerned with theories describing spinning particles that are formulated in terms of actions possessing either local world-line supersymmetry or local fermionic $kappa$ invariance. These classical actions are obtained by adding a finite number of spinor or vector coordinates to the usual space-time coordinates. Generalizing to superspace leads to corresponding types of ‘spinning superparticle’ theories in which the wave-functions are superfields in some (generally reducible) representation of the Lorentz group. A class of these spinning superparticle actions possesses the same spectrum as ten-dimensional supersymmetric Yang–Mills theory, which it is shown can be formulated in terms of either vector or spinor superfields satisfying supercovariant constraints. The models under consideration include some that were known previously and some new ones.
1. Introduction.

The quantum mechanics of a free superparticle in ten-dimensional space-time is of interest both because of its relationship to ten-dimensional Yang–Mills theory and because it provides a description of the massless sector of the superstring. There are many formulations of ten-dimensional superparticle dynamics, all describing a particle evolving along a world-line in some superspace and possessing a fermionic \(\kappa\) symmetry (this is the local world-line symmetry for which the parameter is a fermionic space-time spinor\([1-4]\)). The original superparticle theory \([5-7]\) was quantized in the light-cone gauge and shown to yield the same spectrum as that of ten-dimensional super Yang–Mills theory. However, this model has defied covariant quantization as there is no satisfactory covariant gauge choice for the \(\kappa\) symmetry. It is generally the case that a manifestly covariant formulation of a quantum theory provides a more elegant and geometric description than one in which manifest covariance is absent. The difficulty in quantising the superparticle in a covariant manner is related to the absence of a gauge field for the \(\kappa\) symmetry in these models which means that the only available gauge conditions are ones involving the superspace coordinates \(x^\mu\) and \(\theta_A\). The gauge choice \(\dot{\theta} = 0\) was proposed in \([8,9]\) but this ran into difficulties \([10,11]\). These stem from the fact that \(\dot{\theta} = 0\) does not constitute a good gauge slice, in the sense that there are field configurations which cannot be transformed to any configuration satisfying the gauge condition by a gauge transformation \([12,13]\). This situation is similar to that for the Nambu-Goto string, for which it is also difficult to find a covariant gauge choice. For the Nambu-Goto string the resolution is to introduce a world-sheet metric, in which case covariant gauges can be defined by imposing conditions on the metric instead of on the coordinates. Similarly, for the superparticle, the resolution is to introduce a gauge field for the \(\kappa\) symmetry, so that covariant gauge choices can be defined by imposing conditions on the gauge field \([1]\). There are several formulations which involve a gauge field \(\psi^A\) for the \(\kappa\) symmetry. The simplest of these, due to Siegel \([1]\), introduces \(\psi^A\) and a momentum conjugate to \(\theta_A\). However, this model has a spectrum which is not
a $N = 1$, $D = 10$ super Yang–Mills multiplet, but is a twisted $N = 2$, $D = 10$ supergravity multiplet with negative norm states [13]. Thus, it is not equivalent to the earlier model [5,7] and is not of direct physical interest since its spectrum has negative norm states. Nevertheless, it serves as an interesting model example since it can be covariantly gauge-fixed by choosing the gauge $\psi = 0$, $e = constant$ where $e$ is the world-line einbein [13,14,15]. A full covariant treatment requires the use of the Batalin–Vilkovisky (BV) procedure [16], which in this case requires an infinite number of ghosts for ghosts; the complete quantum theory was derived in this way in [13].

A modification of this model was proposed in [17,18] and analysed in [19,20]. A full Batalin-Vilkovisky quantization of this model was attempted in [21], but it seemed that there was no solution to the Batalin-Vilkovisky master equation for this model, so that there appeared to be an obstruction to the quantization of this model. However, even if it turned out that this model was quantizable using the Batalin-Vilkovisky approach (for example, if it turned out that the correct ghost structure was different from that proposed in [21]), it seems that this model does not give the required super Yang–Mills spectrum; indeed, it was shown in [22] that, if one simply assumed that a BRST charge existed, enough could be deduced about its structure to calculate the full BRST cohomology of the theory for cohomology classes of low ghost number, and it was found in [22] that a very large class of low ghost-number BRST cohomology classes were in fact trivial.

To get around these problems, it has been suggested that the model should further be modified by adding extra coordinates to the $(x^\mu, \theta_A)$ superspace [22-26]. There are now a number of such models which can be covariantly quantized and which give the super Yang–Mills spectrum. These models fall into two classes, those in which the classical superspace has an infinite number of coordinates [24-32] and those in which it has a finite number [17,22,24]. This paper will be restricted to formulations with a finite number of classical superspace coordinates. There are also a number of other approaches [33,34,35], based on either reformulating the superparticle in terms of twistor variables or using harmonic superspace (which
also involves the introduction of an infinite number of extra variables); these will not be discussed here.

We will consider the quantization of these models and analyse their spectra using several different approaches. We will consider light-cone gauge quantization, a covariant approach in which the constraints are imposed on the wave-functions, and a BRST-type approach. As usual, the first two approaches give the same results for the spectrum as the analysis of the BRST cohomology class with zero ghost number. The full BRST approach requires the use of the Batalin–Vilkovisky formalism and requires an infinite number of ghosts in general. However, for some of these models, there are two types of symmetry, one of which acts on both coordinates and gauge fields and the other of which acts only on the Lagrange multiplier gauge fields, reflecting the fact that the constraints are not all independent. Presumably it is necessary to introduce ghosts corresponding to all of these symmetries to obtain a complete treatment. However, if ghosts are only introduced for the first type of symmetry, only a finite number are needed and it is possible to analyse the full BRST cohomology in this reduced formalism. The methods of [22] can be used to argue that the zero ghost-number BRST cohomology class in this reduced formalism is exactly the same as the zero ghost-number cohomology class in the full formalism with an infinite number of ghosts. Thus to find the zero ghost-number physical states, it is sufficient (and much simpler) to use the reduced formalism. In the cases that have been analysed in full [13], the extra cohomology classes at higher ghost number consist of either a single zero-momentum state, or copies of the zero ghost-number physical states.

Before considering models with space-time supersymmetry we will discuss the way in which particles with spin may be described in terms of a world-line action that incorporates either local world-line supersymmetry or a local world-line kappa symmetry. In section 2 particle actions will be considered that give rise to wave-functions that are in the vector or spinor (or higher-spin) representations of the Lorentz group. The vector particle is obtained by introducing an extra anticommuting coordinate that transforms as a Lorentz vector in such a way that the
resulting theory has $N = 1$ world-line supersymmetry [36-42]. The spinor particle (which is a truncation of the model in [22]) is obtained by introducing an extra anticommuting coordinate that transforms as a Lorentz spinor in such a way that the new theory has a local kappa symmetry. Section 2 is a warm-up for our main objective of constructing covariant actions for superparticles describing the particle content of ten-dimensional super Yang–Mills theory. We will first need to show (in sections 3 and 4) how the $D = 10$ super Yang–Mills multiplet may be described covariantly by either a vector or spinor superfield subject to constraints.

In section 5 we will obtain the superspace generalisation of the two types of spinning particle models considered in section 2. (Other related forms of spinning superparticle have been discussed in [33,43].) These models have wave-functions that are superfields in some non-trivial representation of the Lorentz group. In order to incorporate constraints such as those required to define Yang–Mills superfields in section 4 it is necessary to add certain Lagrange multiplier terms to the superparticle actions. As will be seen in sections 6 and 7 this can be done in a number of ways (some of which are equivalent to models already in the literature and some of which are new). This enhances the local symmetry of the action and upon BRST quantization the zero ghost-number sector of the resulting quantum superparticles is precisely the spectrum of super Yang–Mills theory.

2. Particles, world-line supersymmetry and kappa symmetry.

Before discussing particles moving in superspace, it will be useful to consider spinning particles moving in ordinary $D$-dimensional space-time. A bosonic particle following a world-line $x^\mu(\tau)$ in flat space can be described by

$$S = \int d\tau \left( p_\mu \dot{x}^\mu - \frac{1}{2} \epsilon p^2 \right)$$

(2.1)

where $p_\mu(\tau)$ is the momentum conjugate to $x^\mu(\tau)$, $\dot{x} = dx/d\tau$ and $\epsilon$ is a Lagrange multiplier imposing the constraint $p^2 = 0$. It is invariant under the world-line
reparameterisation invariance

\[ \delta x^\mu = kp^\mu, \quad \delta e = \dot{k}, \quad \delta p_\mu = 0. \quad (2.2) \]

Classically, the world-line reparameterisation invariance can be fixed by choosing the gauge \( e = \text{constant} \) giving a free theory subject to the constraint \( p^2 = 0 \). One way of quantizing this theory is to impose the standard commutation relations

\[ [x^\mu, p_\nu] = i\hbar \delta^\mu_\nu \quad (2.3) \]

and to impose the constraint on the wave-function, \textit{i.e.} to demand that the wave-function \( \Psi(x^\mu) \) satisfies the constraint

\[ p^2 \Psi = 0 \quad (2.4) \]

with \( p_\mu = -i\hbar \partial_\mu \). Thus the wave-function is a scalar field \( \Psi(x^\mu) \) satisfying the Klein-Gordon equation and the bosonic particle is seen to describe a free scalar particle.

These results can also be obtained from a BRST approach. The ghost action for the gauge choice \( e = \text{constant} \) is

\[ \int d\tau \dot{c}\dot{c} \quad (2.5) \]

and the ghost and anti-ghost \( c(\tau), \dot{c}(\tau) \) satisfy the anti-commutation relation

\[ \{c, \dot{c}\} = \hbar \quad (2.6) \]

The BRST charge is

\[ Q = \frac{1}{2} cp^2 \quad (2.7) \]
and the wave-function can be taken to be a function of $x^\mu$ and the ghost $c$:

$$\Phi(x, c) = \Psi(x) + c\Psi_1(x) \quad (2.8)$$

The BRST constraint

$$Q\Phi = 0 \quad (2.9)$$

implies that the ghost-number zero wave-function $\Psi(x)$ satisfies (2.4). Physical states are BRST cohomology classes, corresponding to wave-functions satisfying (2.9) with wave-functions differing by BRST exact pieces identified:

$$\Phi \sim \Phi + Q\Lambda \quad (2.10)$$

Then (2.9) imposes $p^2\Psi = 0$ while (2.10) can be used to gauge away all $\Psi_1$ except those satisfying $p^2\Psi_1 = 0$. Thus the physical states for both ghost number zero and ghost number one are represented by wave-functions $\Psi, \Psi_1$ satisfying the Klein-Gordon equation $p^2\Psi = 0, p^2\Psi_1 = 0$.

Thus the first approach gives the same results as the cohomology of the zero ghost number sector in the BRST approach, but the latter gives a second copy of the same spectrum at ghost number one. This is typical of all of the systems we will encounter in this paper; in each case, the first approach gives the same results as the BRST analysis of zero ghost number physical states; as the first approach is simpler, we will mostly use that in the following models and defer a discussion of the BRST approach to the final section.

We shall be interested in modifications of the particle action that give wave-functions transforming in some non-trivial representation of the Lorentz group, such as the spinor or vector representation. All of the methods we shall discuss for giving spin to the wave-function involve adding extra coordinates and momenta to the $(x^\mu, p_\nu)$ phase space.
Consider first the addition to the bosonic target space of an anticommuting vector coordinate $\lambda^\mu$, so that the particle world-line is $x^\mu(\tau), \lambda^\mu(\tau)$. Suppose that the gauge-fixed particle lives in a phase-space $(x^\mu, p_\nu, \lambda^\mu)$ with commutation relations given by (2.3) and

$$\{\lambda^\mu, \lambda^\nu\} = \hbar \eta^{\mu\nu} \quad (2.11)$$

so that $\lambda$ is a self-conjugate variable. A maximally anticommuting set of the grassmann variables consists of $^* \lfloor D/2 \rfloor$ independent linear combinations of the $\lambda^\mu$, which we shall denote by $\lambda^\alpha, \alpha = 1, \ldots, \lfloor D/2 \rfloor$. The (ghost-number zero) wavefunction can be taken to be $\Phi(x^\mu, \lambda^\alpha)$, and the expansion

$$\Phi(x^\mu, \lambda^\alpha) = \psi_0 + \psi_1^\alpha \lambda_\alpha + \psi_2^{\alpha\beta} \lambda_\alpha \lambda_\beta + \psi_3^{\alpha\beta\gamma} \lambda_\alpha \lambda_\beta \lambda_\gamma + \cdots + \psi_{\lfloor D/2 \rfloor}^{\alpha_1 \cdots \alpha_{\lfloor D/2 \rfloor}} \lambda_{\alpha_1} \cdots \lambda_{\alpha_{\lfloor D/2 \rfloor}} \quad (2.12)$$

gives a set of $2^{\lfloor D/2 \rfloor}$ component fields $\psi_0(x), \psi_1^\alpha(x), \ldots, \psi_{\lfloor D/2 \rfloor}^{\alpha_1 \cdots \alpha_{\lfloor D/2 \rfloor}}(x)$ which can be combined into an object $\psi_A(\tau), (A = 1, \ldots, 2^{\lfloor D/2 \rfloor})$ with $2^{\lfloor D/2 \rfloor}$ components. Then $p_\mu$ and the remaining $D - \lfloor D/2 \rfloor$ components of $\lambda_\mu$ can be represented as differential operators acting on the space with coordinates $(x^\mu, \lambda^\alpha)$. As a result, $\lambda_\mu$ becomes a linear operator acting on $\psi_A$ which can be represented as a matrix $\Gamma_{\mu A}^B$. Then (2.11) implies that the matrices $\Gamma_{\mu A}^B$ can be normalised so as to satisfy the Clifford algebra

$$\{\Gamma_\mu, \Gamma_\nu\} = 2\eta_{\mu\nu} \quad (2.13)$$

so that the $\Gamma_{\mu A}^B$ are gamma matrices for the Lorentz group $SO(D - 1, 1)$. The operators

$$L_{\mu\nu} = i(x_\mu p_\nu - x_\nu p_\mu) + \frac{1}{4} \{\lambda_\mu, \lambda_\nu\} \quad (2.14)$$

generate the $SO(D - 1, 1)$ Lorentz algebra and $\psi_A$ transforms as a spinor under this. Thus introducing $\lambda^\mu$ has led to a spinor wave-function as required.

* $\lfloor D/2 \rfloor$ denotes the integer part of $D/2$. 

7
The Dirac equation for this wave-function is $p^\mu \Gamma_{\mu A}^{B} \psi_B = 0$ or $p^\mu \lambda_\mu \Phi = 0$, and this would arise if the classical particle were subject to the constraint $p^\mu \lambda_\mu = 0$. Squaring this constraint gives $p^2 = 0$ and so this is needed to close the algebra of (first-class) constraints. This motivates the classical action

$$S = \int d\tau \left( p_\mu \dot{x}^\mu + \frac{i}{2} \lambda^\mu \dot{\lambda}_\mu - \frac{1}{2} e p^2 + i \chi p_\mu \lambda^\mu \right)$$

(2.15)

where $e, \chi$ are lagrange multipliers imposing the constraints

$$p^\mu \lambda_\mu = 0, \quad p^2 = 0.$$

(2.16)

This is in fact the action for a superparticle with local world-line supersymmetry and $\chi$ is the world-line gravitino [36,40]. The gauge choice $e = \text{constant}, \chi = 0$ leads to the free theory with commutation relations (2.3) and (2.11), subject to constraints corresponding to (2.16). A BRST analysis leads to a wave-function $\Psi = \Phi(x^\mu, \lambda_\alpha) + \gamma$ where $\gamma$ denotes ghost-dependent terms and the BRST constraint $Q \Psi = 0$ imposes the constraints

$$p^\mu \lambda_\mu \Phi = 0, \quad p^2 \Phi = 0$$

(2.17)

so that the ghost-number zero physical states correspond precisely to a Dirac particle. For further details of this construction, see [36,40].

An alternative way to obtain a spinor wave-function is to introduce an extra spinor coordinate $\phi^A$, together with its conjugate momentum $\hat{\phi}_A$, so that the phase space is $(x, p, \phi, \hat{\phi})$. We choose the commutation relations given by (2.3) and

$$\{ \phi^A, \hat{\phi}_B \} = \hbar \delta^A_B$$

(2.18)

if $\phi, \hat{\phi}$ are grassmann-odd, or

$$[ \phi^A, \hat{\phi}_B ] = i \hbar \delta^A_B$$

(2.19)

if they are commuting. Here $\hat{\phi}_A$ transforms as the conjugate spinor representation,
so \( \hat{\phi}_A \phi^A \) is a Lorentz singlet. Then the wave-function can be taken to be

\[
\Psi(x^\mu, \phi^A) = \psi_0(x) + \psi_{1A}(x) \phi^A + \psi_{2AB}(x) \phi^A \phi^B + \ldots \tag{2.20}
\]

and we see that \( \psi_{1A}(x) \) is a spinor wave-function. The constraint \((N-1)\Psi = 0\) where the number operator is \(N = \phi^A \hat{\phi}_A\) implies \(\Psi = \psi_{1A}(x) \phi^A\), so that only the spinor wave-function survives. Further imposing

\[
p^2 \Psi = 0, \quad p^\mu (\Gamma_\mu)_A^B \hat{\phi}_B \Psi = 0 \tag{2.21}
\]

implies that \(\psi_{1A}\) satisfies the Dirac equation

\[
p^\mu (\Gamma_\mu)_A^B \psi_{1B} = 0 \tag{2.22}
\]

These constraints arise from the classical action

\[
S = S_1 + S_2 \tag{2.23}
\]

where

\[
S_1 = \int d\tau \left( p_\mu \dot{x}^\mu + i \hat{\phi}_A \dot{\phi}^A - \frac{1}{2} e p^2 + i \chi p_\mu \Gamma^\mu \hat{\phi} \right) \tag{2.24}
\]

and \(e, \chi^A, \lambda\) are Lagrange multipliers imposing constraints. In particular, \(\chi^A\) is a gauge field for a kappa-type symmetry

\[
\delta \phi = \dot{p} \kappa, \quad \delta \chi = \dot{\kappa} - i \lambda \kappa, \quad \delta x^\mu = i \dot{\phi} \Gamma^\mu \kappa \tag{2.26}
\]

The action (2.24) becomes Siegel’s original superparticle [1] after a field redefinition if \(\phi\) is anticommuting (identifying \(\phi^A \sim \theta^A\)) so we see that a spinning (Dirac) particle is obtained by adding the Lagrange multiplier term (2.25) to the Siegel superparticle (2.24). The action (2.24) has an \(N = 2\) twisted space-time supersymmetry [13] which is completely broken by the term (2.25).
An alternative action (generalisations of which will be relevant later) is given by replacing (2.25) with
\[ S_2 = \frac{1}{2} \int d\tau \lambda^{AB} \dot{\phi}_A \dot{\phi}_B \] (2.27)
leading to the constraint \( \dot{\phi}_A \dot{\phi}_B \Psi = 0 \), implying \( \Psi = \psi_0 + \psi_1 A \phi^A \). This leads to a spectrum that is reducible, described by a scalar wave-function \( \psi_0 \) satisfying the Klein-Gordon equation and a spinor wave-function \( \psi_1 A \) satisfying the Dirac equation.

A variation of this is to consider
\[ S_2 = \frac{1}{2} \int d\tau \lambda^{AB} d_A d_B \] (2.28)
instead of (2.27), where
\[ d_A = \dot{\phi}_A + p_\mu (\Gamma^\mu)_{AB} \phi^B \] (2.29)
If \( \phi \) is anticommuting then (identifying \( \phi^A \sim \theta^A \) and making a field redefinition) this becomes the modified superparticle given in [17,18]. This leads to the wavefunction (2.20) satisfying the constraint
\[ d_A d_B \Psi = 0 \] (2.30)
and this implies that \( \Psi \) must be constant, representing a zero-momentum state [22]. This constraint breaks the twisted \( N = 2 \) supersymmetry down to \( N = 1 \). (An alternative treatment is given in [19,20].)

To summarise, we have seen that a spinor wave-function can be obtained either from a spinning particle (2.15) with local world-line supersymmetry, or from a particle action (2.23)-(2.25) with local kappa symmetry (2.26). In the latter case, the extra coordinate \( \phi \) can be either anticommuting (in which case the action is closely related to Siegel’s action [1]) or commuting. We now turn to the construction of particles with wavefunctions that are other non-trivial representations of
the Lorentz group. One approach is to generalise the model (2.23)-(2.25) with 
\( N = 1 \) local world-line supersymmetry to one with \( N \) extended local world-line 
supersymmetry. In one version of this model [42,44], there are \( N \) anticommuting 
vectors \( \lambda^\mu_i, \; (i = 1, \ldots, N) \) and the corresponding wave-function is a multi-spinor 
\( \Psi_{A_1A_2\ldots A_N} \) [42,44].

Another approach is to modify the model (2.23)-(2.25) by replacing the spinor 
variables \( \phi, \hat{\phi} \) by a coordinate \( \phi_R \) in some representation \( R \) of the Lorentz group 
with a conjugate momentum \( \hat{\phi}_{\bar{R}} \) in a conjugate representation \( \bar{R} \) (so that \( R \otimes \bar{R} \) contains a Lorentz singlet). In this way, expanding the wave-function \( \Phi(x, \phi) \) in \( \phi \) gives \( \Phi = \psi_0 + \psi_1 \cdot \phi + \ldots \) where \( \psi_1(x) \) is a wave-function in the \( \bar{R} \) representation. 
Imposing the constraint \( (N - 1)\Phi = 0 \) where \( N \) is the number operator \( \phi \cdot \bar{\phi} \) 
then gives \( \Phi = \psi_1 \cdot \phi \) as required. One then imposes \( p^2 \Phi = 0 \), possibly together 
with other \( p \)-dependent constraints, generalising the constraint \( p\hat{\phi}\Phi = 0 \) of the 
model (2.23)-(2.25). For example, consider the case in which \( R \) is the vector 
representation, so that the phase space has coordinates \( x^\mu, p_\mu, \phi_\mu, \hat{\phi}_\mu \). We shall 
suppose that \( \phi, \hat{\phi} \) are commuting; all of the following analysis also holds if they 
are anticommuting, with only some changes in some of the signs. An appropriate 
action is \( S = S_1 + S_2 \) with

\[
S_1 = \int d\tau \left( p_\mu \dot{x}^\mu + \hat{\phi}^\mu \dot{\phi}_\mu - \frac{1}{2} e p^2 - \chi p_\mu \hat{\phi}^\mu \right), \quad S_2 = \int d\tau \lambda (N - 1) \quad (2.31)
\]

where \( e, \chi, \lambda \) are lagrange multipliers imposing the classical constraints \( p^2 = 0 \), 
\( p_\mu \hat{\phi}^\mu = 0 \) and \( N = 1 \). Here \( \chi \) is a gauge field for a symmetry corresponding to the 
kappa-type symmetry (2.26):

\[
\delta \phi_\mu = p_\mu \kappa, \quad \delta \chi = \dot{\kappa} + \lambda \kappa, \quad \delta x^\mu = \hat{\phi}^\mu \kappa \quad (2.32)
\]

Note that this is closely related to the world-line diffeomorphism symmetry (2.2). 
Quantization leads to a wave-function \( \Phi(x, \phi) \) satisfying the constraints

\[
p^2 \Phi = 0, \quad p_\mu \hat{\phi}^\mu \Phi = 0, \quad (N - 1) \Phi = 0 \quad (2.33)
\]
and identified under the gauge transformation

$$\Phi \sim \Phi + p_\mu \Lambda^\mu$$  \hspace{1cm} (2.34)

This leads to a vector wave-function $\psi_1^\mu$ satisfying

$$p^2 \psi_1^\mu = 0, \quad p_\mu \psi_1^\mu = 0$$  \hspace{1cm} (2.35)

and identified under the gauge transformations

$$\psi_1^\mu \sim \psi_1^\mu + p^\mu \lambda$$  \hspace{1cm} (2.36)

which clearly corresponds to a vector gauge potential in the Lorentz gauge with $D - 2$ independent degrees of freedom. It is straightforward to modify this to obtain a covariant vector particle in a general gauge. The appropriate action is (2.23) with

$$S_1 = \int d\tau \left( p_\mu \dot{x}^{\mu} + \frac{i}{2} \dot{\phi}^{\mu} \phi_{,\mu} - \chi^{\nu} (\eta_{\mu\nu} p^2 - p_\mu p_\nu) \dot{\phi}^{\mu} \right) \quad S_2 = \int d\tau \lambda (N - 1),$$  \hspace{1cm} (2.37)

leading to a wave-function $\psi_1^\mu$ satisfying

$$p^\mu F_{\mu\nu} = 0, \quad F_{\mu\nu} = p_\mu \psi_1^\nu - p_\nu \psi_1^\mu$$  \hspace{1cm} (2.38)

and identified under (2.36).
3. Light-cone super Yang–Mills theory.

In the next section we will obtain the constraints satisfied by the superfield of ten-dimensional super Yang–Mills theory which will establish the constraints to be satisfied by the superparticle wavefunctions in the following sections. Our method will make use of the light-cone gauge description of the physical states [45] which we will review in this section.

In the light-cone gauge $SO(9, 1)$ representations are decomposed into representations of a manifest $SO(8)$ little group. An $SO(9, 1)$ vector $A_\mu$ ($\mu = 0, 1, \ldots, 9$) decomposes into an $SO(8)$ vector $A_i$ ($i = 1, \ldots, 8$) and two $SO(8)$ singlets $A_-, A_+$ ($A_\pm = A_0 \pm A_9$). A 16-component Weyl spinor of $SO(9, 1)$, $\psi^A$, ($A = 1, \ldots, 16$) decomposes into two 8-component $SO(8)$ spinors $\Psi^a$, $\Psi^{\dot{a}}$ ($a = 1, \ldots, 8$ and $\dot{a} = 1, \ldots, 8$ label the inequivalent spinor representations of $SO(8)$). The $SO(8)$ gamma matrices $\gamma^{i}_{ab}$ satisfy

$$\gamma^{i}_{ab} \gamma^{j}_{bb} + \gamma^{j}_{ab} \gamma^{i}_{bb} = 2 \delta^{ij} \delta_{ab}$$

and we define

$$\gamma^{i}_{ij \ldots k} = \gamma^{i} \gamma^{j} \ldots \gamma^{k}.$$  \hspace{1cm} (3.1)

The Yang–Mills multiplet consists of a gauge potential $A_\mu$ and a Majorana-Weyl spinor $\lambda^A$ taking values in the Lie algebra of the gauge group. In the light-cone gauge only the physical degrees of freedom remain and these consist of the 8 transverse degrees of freedom $A_i$ of the Yang–Mills field and the 8 spinor degrees of freedom $\lambda^a$ (the Dirac equation $\partial \lambda = 0$ eliminates 8 of the 16 components of $\lambda^A$ in the light-cone gauge). Expanding an unconstrained superfield $\Phi(x^\mu, \theta_A)$ gives $2^{16}$ component fields, which is clearly too large to describe the $8 + 8$ components we require, so the Yang–Mills multiplet must be described by a constrained superfield.

* The $SO(8)$ indices may be raised and lowered trivially using the metric $\delta_{ij}$ and the charge conjugation matrices $C_{ab} = \delta_{ab}$ and $C_{\dot{a}\dot{b}} = \delta_{\dot{a}\dot{b}}$. 

13
In the next section we shall find $SO(9, 1)$-covariant constraints that lead to the correct spectrum, but for now we shall content ourselves with the $SO(8)$ covariance of the light-cone gauge. As a first step the superspace will be restricted to $(x^\mu, \theta^a)$, i.e., the $SO(8)$-covariant constraint $\partial \Phi / \partial \theta^a = 0$ will be imposed on any superfield, $\Phi$. There are still $2^8$ component fields in $\Phi$ so further constraints must be imposed.

One possible approach to imposing further constraints is to abandon manifest $SO(8)$ invariance and decompose the coordinates into representations of a $U(4)$ subgroup [46,47]. Thus $\theta^a$ is decomposed into $\theta^\alpha$, $\bar{\theta}_\alpha$ which transform as a 4 and $\bar{4}$, respectively. Then imposing the extra constraints $\partial \Phi / \partial \bar{\theta}_\alpha = 0$ gives a superfield $\Phi(x^\mu, \theta^\alpha)$ which is unconstrained in the reduced superspace with coordinates $(x^\mu, \theta^\alpha)$. Following Siegel [48], we shall refer to this as the euphoric formalism. The superfield $\Phi(x^\mu, \theta^\alpha)$ has eight $(1 + 6 + 1)$ bosonic and eight $(4 + \bar{4})$ fermionic components and so gives the correct spectrum for super Yang–Mills in terms of $U(4)$-covariant states.

Finding $SO(8)$-covariant constraints is more tricky but was solved in [45] by choosing $\Phi$ to be either an $SO(8)$ vector superfield $\Psi^i(x^\mu, \theta^\dot{a})$, or a spinor superfield $\Psi_a(x^\mu, \theta^\dot{a})$. The vector superfield is taken to satisfy the linear constraint

$$ (\gamma^i \gamma^j - 8 \delta^{ij}) \dot{a} \dot{b} D^{\dot{b}} \Psi^j = 0, \quad (3.3) $$

where the $SO(8)$ supercovariant derivative is

$$ D_{\dot{a}} = \frac{\partial}{\partial \theta^a} + p^+ \theta_{\dot{a}}, \quad (3.4) $$

and $p^+$ is the component of the 10-momentum $p^\mu = (p^+, p^-, p^i)$, which is set to a constant in the light-cone gauge. Acting on (3.3) with another $D_{\dot{a}}$, gives the quadratic constraint

$$ \frac{1}{8p^+} D^{\dot{a}} (\gamma^{ij})_{\dot{a} \dot{b}} D^{\dot{b}} \Psi^k = \delta^{ij} \Psi^i - \delta^{ik} \Psi^j, \quad (3.5) $$

which was shown in [45] to be fully equivalent to the linear constraint (3.3). It was also shown in [45] that the constraint (3.3) (or (3.5)) leaves precisely the 8 bosonic and 8 fermionic degrees of freedom of light-cone gauge super Yang–Mills theory.
The spinor superfield satisfies the linear constraint [45]

\[(\gamma^{ijk})_{ab} D^b \Psi_a = 0.\] (3.6)

Again, this is equivalent to a quadratic constraint

\[D_{[a} D_{b]} \Psi_c = 2 p^+ \gamma^{cd} \Psi_d,\] (3.7)

where \(\gamma^{cd}_{ab} = \frac{1}{28} (\gamma^{ij})_{ab} (\gamma^{ij})^{cd}\). The spinor and vector superfields are related by

\[(\gamma^i)_{ab} \Psi_b = \frac{1}{8} D_a \Psi^i.\]

4. Covariant constraints for super Yang–Mills theory

We will now present \(SO(9,1)\)-covariant superfield formulations of super Yang–Mills which reduce to the \(SO(8)\)-covariant ones of the last section in the lightcone gauge. We use a superspace with coordinates \((x^\mu, \theta^A)^*\) and supercovariant derivatives

\[D^A = \frac{\partial}{\partial \theta_A} + \dot{\theta}^A \theta_B.\] (4.1)

We will now show that the \(SO(8)\) vector superfield \(\Psi_i(x^\mu, \theta^\alpha)\) satisfying (3.3) may be obtained from an \(SO(9,1)\) vector superfield \(\Psi_\nu(x^\mu, \theta_A)\) together with the linear constraints

\[p^2 \Psi_\mu = 0, \quad (\dot{\theta}^A D^B) \Psi_\mu = 0, \quad p^\mu \Psi_\mu = 0,\] (4.2)

and

\[D^A \Psi_\mu = \frac{1}{7}(\Gamma^{\mu\nu})^A_B D^B \Psi_\nu,\] (4.3)

where \(\Gamma_{\mu\nu\ldots\rho} = \Gamma_{[\mu} \Gamma_\nu \ldots \Gamma_\rho]\). The first constraint in (4.2) implies the momentum is null and the remaining constraints will be analysed in a Lorentz frame in which the momentum is \(p^\mu = (p^+, 0, 0, 0)\), i.e., \(p^- = 0, p^i = 0\).

* Upper and lower \(SO(9,1)\) Weyl spinor indices \(A = 1, 2, \ldots, 16\) are used to distinguish chirality, so that \(\Phi^A\) and \(\theta_A\) \((A = 1, 2, \ldots, 16)\) have opposite chirality. We use a Majorana representation in which the gamma matrices, \(\Gamma_{\mu AB}\) and \(\Gamma^{\mu AB}\), are real and symmetric and satisfy \(\Gamma_{\mu AC} \Gamma_{\nu CB} + \Gamma_{\nu AC} \Gamma_{\mu CB} = 2 \delta^B_A \delta_{\mu\nu}\).
It will be convenient to define $m^\mu = p^\mu / p^+$ and introduce a null vector $n^\mu$ such that $m^2 = 0$, $n^2 = 0$ and $m \cdot n = 1$, and then choose a representation of the gamma matrices such that the projectors $\slashed{m} \slashed{n}$ and $\slashed{n} \slashed{m}$ are diagonal. The Lorentz transformations preserving the null vectors $m^\mu, n^\mu$ form an $SO(8)$ transverse group and any chiral $SO(9, 1)$ spinor, $\chi^A$ or $\psi_A$, may be decomposed into two inequivalent $SO(8)$ spinors (distinguished by dotted and undotted indices, $\dot{\alpha}$ and $\alpha$) as

$$
\psi_A = \begin{pmatrix} \psi_a \\ \psi_{\dot{\alpha}} \end{pmatrix}, \quad \chi^A = \begin{pmatrix} \chi_a \\ \chi_{\dot{\alpha}} \end{pmatrix}.
$$

(4.4)

In this basis, the matrices $m \cdot \Gamma$ and $n \cdot \Gamma$ take the form

$$
(\slashed{m})_{AB} = \begin{pmatrix} 0 & 0 \\ -\delta_{ab} & 0 \end{pmatrix}, \quad (\slashed{n})^{AB} = \begin{pmatrix} 0 & \delta_{\dot{\alpha} b} \\ 0 & 0 \end{pmatrix},
$$

(4.5)

Spinors $\chi^A, \psi_A$ can be decomposed as follows

$$
(\slashed{m})_{AB} \chi^B \rightarrow -\chi_a, \quad (\slashed{n})^{AB} \chi^B \rightarrow \chi_{\dot{\alpha}},
$$

(4.6)

$$
(\slashed{m})_{AB} \psi_B \rightarrow \psi_a, \quad (\slashed{n})^{AB} \psi_B \rightarrow -\psi_{\dot{\alpha}}.
$$

(4.7)

It follows that $\slashed{p}_{AB} D^B \rightarrow -p^+ \partial / \partial \theta a$ and the second constraint in (4.2) becomes

$$
\frac{\partial}{\partial \theta a} \Psi_\mu = 0,
$$

(4.8)

while $p^\mu \Psi_\mu = p^+ \Psi_+ = 0$ implies that $\Psi_+ = 0$ if $p^+ \neq 0$. We are then left with superfields $\Psi_-(x^\mu, \theta \dot{\alpha})$ and $\Psi_+(x^\mu, \theta \dot{\alpha})$ satisfying (4.3), which implies

$$
D^\dot{\alpha} \Psi_+ = 0.
$$

(4.9)

Using $D_a = \partial / \partial \theta \dot{\alpha} + p^+ \theta a$, we find that (4.8) implies $D^\dot{\alpha} D^\dot{\beta} \Psi_- = 2p^+ \delta^{\dot{\alpha} \dot{\beta}} \Psi_- = 0$.
and hence $\Psi_- = 0$, if $p^+ \neq 0$. This leaves a superfield $\Psi_i(x^\mu, \theta^\da)$ satisfying

$$D^\da \Psi^i = \frac{1}{7}(\gamma^{ij})^\da D^b \Psi_j,$$  \hspace{1cm} (4.9)$$

which is precisely the super Yang–Mills constraint (3.3). Thus the covariant constraints (4.2) and (4.3) give a covariant superfield formulation of super Yang–Mills.

Similarly, the quadratic constraint (3.5) is recovered from the following covariant constraints

$$p^2 \Psi_\mu = 0, \quad (\dot{p}_{AB} D^B) \Psi_\mu = 0, \quad p^\mu \Psi_\mu = 0,$$  \hspace{1cm} (4.10)$$

and

$$D^A D^B \Psi_\mu + 8(\Gamma_\mu)^{C[A} (\Gamma^\nu \dot{\phi})_{B]} \Psi_\nu = 0.$$  \hspace{1cm} (4.11)$$

To see that this is the correct quadratic covariant formulation of super Yang–Mills, we follow a null frame analysis similar to the above and set $p^\mu = (p^+, 0, \vec{0})$. The last equation in (4.10) implies that $\Psi_+ = 0$ while $\dot{p}_{AB} D^B \Psi_\mu = 0$ implies that $\Psi_\mu = \Psi_\mu(x, \theta^\da)$. We are then left with superfields $\Psi_-(x^\mu, \theta^\da)$ and $\Psi_i = \Psi_i(x^\mu, \theta^\da)$ satisfying (4.11). This constraint implies $\Psi_- = 0$ (provided that $p^+ \neq 0$) leaving a superfield $\Psi_i$ satisfying

$$D^\da D^b \Psi^i = 8p^+ (\gamma^i)^{c[i} (\gamma^j)^{j]} \Psi_j$$  \hspace{1cm} (4.12)$$

which is equivalent to (3.5).

The spinor constraints can be obtained similarly. The spinor superfield $\Psi_a(x^\mu, \theta^\da)$ must come from an $SO(9, 1)$ spinor superfield $\Psi_A(x^\mu, \theta_A)$. We impose the linear constraints

$$p^2 \Psi_A = 0, \quad (\dot{p}_{AC} D^C) \Psi_B = 0, \quad \dot{p}^{AB} \Psi_B = 0,$$  \hspace{1cm} (4.13)$$

and

$$(\Gamma^{\mu\nu\rho\sigma})^{\mu}_{A} B D^A \Psi_B = 0.$$  \hspace{1cm} (4.14)$$

where $\Gamma^{\mu\nu\rho\sigma} = \Gamma^{[\mu} \Gamma^{\nu} \Gamma^{\rho} \Gamma^{\sigma]}$. The first constraint in (4.13) implies the momentum is
null so that the remaining ones can be solved in the Lorentz frame in which $p^\mu = (p^+, 0, \vec{0})$. The second constraint in (4.13) again implies that $\Psi_B = \Psi_B(x^\mu, \theta^a)$. The last constraint in (4.13) becomes $p^+ \Psi_a = 0$ and hence $\Psi_\dot{a} = 0$ (provided that $p^+ \neq 0$), leaving a superfield $\Psi_a$ satisfying

$$(\gamma^{ijk})^{\dot{a} \dot{b}} D_\dot{b} \Psi_a = 0,$$  \hspace{2cm} (4.15)

which is precisely the light-cone Yang–Mills constraint (3.6).

Finally, the quadratic spinor constraint (3.7) can be obtained from

$$p^2 \Psi_A = 0, \quad (\dot{\Psi}_{AC} D^C) \Psi_B = 0, \quad \dot{p}^{AB} \Psi_B = 0,$$  \hspace{2cm} (4.16)

and

$$D^A D^B \Psi_C + 8(\Gamma^\mu)^D[A (\Gamma_\mu \dot{p})^B] C \Psi_D = 0.$$  \hspace{2cm} (4.17)

The second constraint in (4.16) implies that $\Psi_B = \Psi_B(x^\mu, \theta^a)$, if $p^+ \neq 0$. The last constraint in (4.16) becomes $p^+ \Psi_\dot{a} = 0$ and hence $\Psi_\dot{a} = 0$ while the former expresses that the momentum is null. We are then left with a superfield $\Psi_a$ satisfying

$$D^\dot{a} D^b \Psi_c - 8p^+(\gamma^{ij})^{\dot{a} \dot{b}} (\gamma_{ij})_{cd} \Psi^d = 0.$$  \hspace{2cm} (4.18)

which is (3.7).
5. Covariant superparticle actions.

The remaining sections of this paper are aimed at generalizing the particle actions of section 2 to incorporate space-time supersymmetry in such a manner as to reproduce covariant super Yang–Mills wave-functions. This section will be concerned with the superspace generalization of section 2 ignoring the issue of the Yang–Mills constraints, which will be dealt with in the next two sections.

5.1. The original superparticle.

To begin with it will be useful to review the superparticle of [5,7] even though there appears to be no way of quantizing it in a manifestly covariant manner. It is defined by the action

\[ S_0 = \int d\tau \left[ p_\mu (\dot{x}^\mu - i\bar{\theta}\Gamma^\mu \dot{\theta}) - \frac{1}{2} \epsilon p^2 \right], \quad (5.1) \]

where \( \dot{\theta} = d\theta/d\tau \). This describes a particle with world-line parametrized by \( \tau \) moving through a 10-dimensional \( N = 1 \) superspace with coordinates \( (x^\mu, \theta_A) \). The momentum of the particle is \( p^\mu \) and \( \epsilon \) is a world-line einbein. The action \( S_{BSC} \) is invariant under a 10-dimensional super-Poincaré symmetry

\[ \delta \theta = \epsilon, \quad \delta x^\mu = i\epsilon \Gamma^\mu \theta, \quad (5.2) \]

(where \( \epsilon_A \) is a constant Grassmann parameter) and it is invariant also under world line-reparametrizations together with the local kappa symmetry

\[ \delta \theta = \dot{\phi} \kappa, \quad \delta \epsilon = 4i\kappa \dot{\theta} + \dot{\xi}, \]

\[ \delta x^\mu = i\theta \Gamma^\mu \dot{\kappa} + \xi p^\mu, \quad \delta p^\mu = 0. \quad (5.3) \]

The Grassmann spinor \( \kappa_A \) parametrizes the local fermionic symmetry, while \( \xi \) parametrizes a linear combination of world-line diffeomorphisms and a ‘trivial’
local symmetry.

These symmetries can be fixed by going to the light-cone gauge, in which the reparametrization invariance is used to set $e$ to be a constant and the fermionic symmetry is used to impose the condition $\gamma^+ \theta = 0$, eliminating half of the components of $\theta_A$. Finally, the condition $x^+ = p^+ \tau + x_0^+$ is imposed for some constants $p^+, x_0^+$, and the constraint $p^2 = 0$ is solved to give $p^- = p^i p^i / p^+$. 

The light-cone action is

$$S_{lc} = \int d\tau \left( p^i \dot{x}^i - \frac{1}{2} p^i p^i + \frac{1}{2} i \theta^a \dot{\theta}^a \right),$$

(5.4)

(where factors of $p^+$ have been absorbed into redefinitions of the fields) which is a free action. In an operator approach it is quantized by imposing the (anti-) commutation relations

$$[x^i, p^j] = -i \delta^{ij}, \quad \{\theta^a, \theta^b\} = 2 \delta^{ab}. \quad (5.5)$$

It is convenient to use a euphoric ($U(4)$) basis, in which $\theta^a$ is written in terms of $\theta^\alpha$ ($\alpha = 1, \ldots, 4$) and its complex conjugate $\bar{\theta}^\alpha$, so that the kinetic term $\frac{1}{2} i \theta^a \dot{\theta}^a$ becomes $i \bar{\theta}^\alpha \dot{\theta}^\alpha$ and the anti-commutation relations become

$$\{\bar{\theta}^\alpha, \theta^\beta\} = \delta^\alpha_\beta. \quad (5.6)$$

The wave-function is a function of a maximal commuting subset of the phase space variables, and these can be taken to be $(p^i, p^+, \theta^\alpha)$. It is then an unconstrained function $\Psi(p^\mu, \theta^\alpha)$ satisfying $p^2 \Psi = 0$. Expanding $\Psi$ in $\theta^\alpha$ gives the eight bosonic and eight fermionic component fields of super Yang–Mills. Since this theory cannot be covariantly quantized we now turn to consider other superparticle actions which lead to the covariant constraints described in the last section.

* A ‘trivial’ symmetry is one under which all fields transform into equations of motion, so that the symmetry does not eliminate on-shell degrees of freedom. Any action $S(\phi^i)$ dependent on fields $\phi^i$ will automatically be invariant under local transformations of the form $\delta \phi^i = \lambda J^i(\phi) \delta S / \delta \phi^i$ (with local parameter $\lambda$) provided $J^i$ is (graded) anti-symmetric. The corresponding Noether current vanishes on-shell [49,50].

† Recall that in the light-cone gauge, $p^- = p^i p^i / p^+$. 

---

---
5.2. Twisted superparticles

In [1,2], Siegel proposed the following modified superparticle action:

\[ S_t = \int d\tau \left[ p_\mu (\dot{x}^\mu - i\bar{\theta} \Gamma^\mu \dot{\theta}) + i\dot{\theta} + \frac{1}{2} e p^2 + i\psi \dot{p} d \right] , \tag{5.7} \]

where \( d \) is introduced so that \( \theta \) has a conjugate momentum \( \dot{\theta}^A = d^A - \dot{p}^{AB} \theta_B \), while \( \psi \) is a gauge field for the kappa symmetry (which we shall sometimes refer to as the \( B \) symmetry)

\[
\begin{align*}
\delta \psi &= \kappa, & \delta \theta &= \dot{p} \kappa, & \delta d &= 2p^2 \kappa, \\
\delta x^\mu &= -i\kappa \dot{p} \Gamma^\mu \theta, & \delta e &= 4i\dot{\theta} \kappa. \tag{5.8}
\end{align*}
\]

The action is also invariant under world-line reparameterisations (the \( A \) symmetry) and the \( E \) and \( F \) symmetries given by

\[
\delta \psi = \dot{p} \eta, \quad \delta e = -2i\dot{\eta}. \tag{5.9}
\]

and

\[ \delta \psi^A = \omega d^A \tag{5.10} \]

The action (5.7) was shown in [13] to be invariant under an \( N = 2 \) twisted space-time super-Poincaré symmetry, which includes two supercharges \( Q_1, Q_2 \) which are both Majorana-Weyl spinors in \( D = 10 \) satisfying

\[
\{Q_1, Q_1\} = \dot{\rho}, \quad \{Q_2, Q_2\} = -\dot{\rho}, \quad \{Q_1, Q_2\} = 0. \tag{5.11}
\]

As one might suspect from the opposite signs of the anticommutators \( \{Q_1, Q_1\} \) and \( \{Q_2, Q_2\} \) the spectrum of the theory has negative norm states. The physical states of the theory were shown in [13] to be represented by a scalar superfield wave-function \( \Phi(x^\mu, \theta_A) \) satisfying the constraints

\[
p^2 \Phi = 0, \quad \dot{p} d\Phi = 0 \tag{5.12}
\]

where \( \dot{\theta} \) is represented by \( \dot{\theta}^A = i\hbar \partial / \partial \theta_A \) so that \( d^A = i\hbar \partial / \partial \theta_A - \dot{p}^{AB} \theta_B \).
The light-cone gauge action is [13]

\[ S_{lc} = \int d\tau \left( p_i \dot{x}^i - \frac{1}{2} \tilde{p}^i \tilde{p}^i + i \dot{\theta}^a \theta^a \right), \]  

(5.13)

where factors of \( p^+ \) have been absorbed into field redefinitions and

\[ \pi^a = d^a - p^+ \theta^a. \]  

(5.14)

The corresponding spectrum contains \( 2^8 \) states and is \emph{not} equivalent to the \( N = 1 \) superparticle with action (5.1), which has a spectrum of \( 2^4 \) states [51]. Defining the \( SO(8) \) spinors \( \theta^1 = \frac{1}{2}(\theta + \tilde{\theta}), \theta^2 = \frac{1}{2}(\theta - \tilde{\theta}) \), the light-cone action becomes

\[ S_{lc} = \int d\tau \left( p_i \dot{x}^i - \frac{1}{2} \tilde{p}^i \tilde{p}^i + i \theta^1_1 \dot{\theta}^1_1 - i \theta^2_2 \dot{\theta}^2_2 \right). \]  

(5.15)

and the relative minus sign between the two fermion kinetic terms implies that half of the physical states must have negative norm.

A further modification of this theory, proposed in [18,17], is to add the term

\[ S'_t = \frac{1}{2} \int d\tau \chi_{AB} d^A d^B \]  

(5.16)

involving a Lagrange multiplier \( \chi_{AB} \) to \( S_t \) to give \( S'_t = S_t + S' \). This leads to a scalar wave-function satisfying (5.12) together with

\[ d^A d^B \Phi = 0 \]  

(5.17)

This constraint implies that either the wave-function vanishes, or that the momentum \( p \) is zero [13], so that the only physical states are zero-momentum ground states.*

---

* Note that although these constraints would be expected to emerge from the BRST constraints for the ghost-number zero physical states in any covariant BRST analysis, there has not yet been a complete covariant BRST analysis of this model and it was argued in [21] that such an analysis may not be possible, in the sense that there may not be any solution of the Batalin-Vilkovisky master equation for this system.
5.3. Spinning twisted superparticles

In section 2 it was seen that the usual bosonic particle had a scalar wavefunction, but by adding appropriate world-line degrees of freedom it became possible to obtain theories with wave-functions describing spin. We now wish to generalise this by adding extra degrees of freedom to the twisted superparticle action (5.7) so as to obtain wave-functions which are superfields with spin. As in section 2 there will be two ways of doing this, one of which leads to an extra world-line supersymmetry and one of which leads to an extra kappa-type symmetry.

We first generalise the world-line supersymmetric action (2.15) and consider the action

\[ S^{(1)}_{st} = S_t - i \int d\tau \left( \frac{1}{2} \lambda^\mu \dot{\lambda}^\mu + \chi \rho \lambda^\mu \right) \]  

(5.18)

with an extra anticommuting vector coordinate \( \lambda^\mu(\tau) \), where \( S_t \) is given by (5.7). The action (5.18) is invariant under the local kappa symmetry

\[ \delta \theta = \dot{\kappa}, \quad \delta \psi = \dot{\kappa}, \quad \delta e = 4i\dot{\theta} \kappa \]
\[ \delta x^\mu = i\theta \gamma^\mu \delta \theta + id \gamma^\mu \kappa, \quad \delta d = 2p^2 \kappa, \]  

(5.19)

with spinor parameter \( \kappa^A(\tau) \) and the local world-line supersymmetry

\[ \delta x^\mu = i\epsilon \lambda^\mu, \quad \delta \chi = \dot{\epsilon}, \quad \delta \lambda^\mu = \epsilon p^\mu \]  

(5.20)

with parameter \( \epsilon(\tau) \).

On quantization, \( \lambda^\mu \) satisfies the Clifford algebra (2.11) and the wave-function can be taken to be a scalar function \( \Phi(x^\mu, \theta_A, \lambda^\alpha) \) depending on \([D/2]\) of the \( \lambda \)'s, \( \lambda^\alpha, \alpha = 1, \ldots, [D/2] \). The wave-function has the expansion

\[ \Phi(x^\mu, \theta_A, \lambda^\alpha) = \psi_0 + \psi_1^\alpha \lambda_\alpha + \psi_2^{\alpha\beta} \lambda_\alpha \lambda_\beta + \psi_3^{\alpha\beta\gamma} \lambda_\alpha \lambda_\beta \lambda_\gamma + \ldots \]  

(5.21)

giving a set of \( 2^{[D/2]} \) component fields \( \psi_0(x, \theta), \psi_1^\alpha(x, \theta), \ldots, \psi_{[D/2]}^{\alpha_1\ldots\alpha_{[D/2]}}(x, \theta) \) which can be combined into an object \( \psi_A(x, \theta), (A = 1, \ldots, 2^{[D/2]}) \) with \( 2^{[D/2]} \) components.
Then $p_\mu, \hat{\theta}, d$ and the remaining $D-[D/2]$ components of $\lambda_\mu$ can be represented as differential operators acting on the space with coordinates $(x^\mu, \lambda^\alpha)$. In particular, $\lambda_\mu$ becomes a linear operator acting on $\psi_A$ which can be represented as a matrix $\hat{\Gamma}_{\mu A}^B$. Equation (2.11) implies that these matrices can be normalised so as to satisfy the Clifford algebra (2.13) and the $\hat{\Gamma}_{\mu A}^B$ are related to the usual $SO(D-1,1)$ gamma matrices, $\Gamma_{\mu A}^B$, by a similarity transformation. A basis can then be chosen in which they are identified so that $\hat{\Gamma} = \Gamma$ in which case $\psi_A(x, \theta)$ transforms as a spinor under the action of the modified Lorentz generators $L'_{\mu \nu} = L_{\mu \nu} + \frac{1}{4} [\lambda_\mu, \lambda_\nu]$, where $L_{\mu \nu}$ are the standard Lorentz generators. The wave-function $\psi$ satisfies the constraints

$$\hat{p}\psi = 0, \quad p^2 \psi = 0, \quad (\hat{p}d)\psi = 0 \quad (5.22)$$

The spectrum is then represented by a constrained spinor superfield; however, this corresponds to a reducible representation of supersymmetry. Multi-spinor wavefunctions $\psi_{A_1 A_2 \ldots A_N}$ can be obtained by adding $N$ variables $\lambda_\mu^i, i = 1, \ldots N$, so as to obtain $N$ extended local world-line supersymmetry.

Next we consider the generalisation of the particle action with local kappa symmetry defined by (2.23),(2.24) and (2.25) to the superparticle action

$$S^{(2)}_{st} = S_t + S_1 + S_2 \quad (5.23)$$

where

$$S_1 = \int d\tau \left( i \dot{\phi}_A^{\dot{A}} - i \chi p_\mu \Gamma^\mu \dot{\phi} \right), \quad (5.24)$$

$$S_2 = \int d\tau \lambda (N - 1), \quad (5.25)$$

with extra spinor coordinates $\phi^{\dot{A}}$, $\dot{\phi}_A$. The action is invariant under the usual kappa symmetry given by (5.8) with all other fields invariant, and a new kappa-type symmetry given by (2.26) with all other fields invariant. Instead of (5.25) we could
have equally well used (2.27) which would impose the constraint $\hat{\phi}_A \hat{\phi}_B = 0$, as this gives rise to the same spectrum.

The analysis of the physical states is similar to that of section 2. The wavefunction can be taken to be a function $\Psi(x^\mu, \theta_A, \phi^A)$ and the constraint $(N-1)\Psi = 0$ implies that $\Psi = \psi_A \phi^A$ for some spinor superfield $\psi_A(x, \theta)$. The remaining constraints are

$$p^2 \psi = 0, \quad (\phi d) \psi = 0, \quad \hat{p} \psi = 0.$$  \hspace{1cm} (5.26)

Again this gives a reducible representation of supersymmetry.

Also as before, this can be generalised to obtain a wave-function in any representation $R$ of the Lorentz group by adding a momentum $\hat{\phi}$ that transforms according to the $R$ representation, together with a conjugate coordinate $\phi$. In particular, for the vector representation, we consider the action (5.23) with

$$S_1 = \int d\tau \left( \hat{\phi}_\mu \hat{\phi}^\mu - \chi p_\mu \hat{\phi}^\mu \right),$$  \hspace{1cm} (5.27)

and $S_2$ is given by (5.25) with $N = \phi^\mu \hat{\phi}_\mu$. The extra vector coordinates $\phi^\mu, \hat{\phi}_\mu$ are taken to be commuting. The action is invariant under the standard kappa symmetry (5.8) together with the ‘kappa/diffeomorphism’ symmetry (2.32). This leads to physical states described by a vector superfield $\psi^\mu(x, \theta)$ satisfying the constraints

$$p^2 \psi^\mu = 0, \quad p_\mu \psi^\mu = 0, \quad \hat{p} d \psi^\mu = 0$$  \hspace{1cm} (5.28)

and identified modulo the gauge transformations

$$\psi^\mu \sim \psi^\mu + p^\mu \lambda$$  \hspace{1cm} (5.29)

for arbitrary superfields $\lambda(x, \theta)$. This corresponds to a super-gauge connection in Lorentz gauge and corresponds to a reducible multiplet in general.
To summarise, there are a number of ways of constructing superparticle actions that give rise to wave-functions that are spinor or vector superfields satisfying kinematic constraints, and corresponding to reducible multiplets. It was seen in section 4 that super Yang–Mills theory in 10 dimensions is described by precisely such wave-functions subject to certain extra super-covariant constraints. Superparticle theories with spectra coinciding with that of super Yang–Mills can therefore be constructed by adding Lagrange multiplier terms to these superparticle actions, leading to these extra constraints. The remainder of this paper is devoted to a detailed analysis of such models.

6. Covariant superparticles with spinor super wave-functions.

In this section we shall obtain actions that describe a superparticle with a spinor super wave-function satisfying either the quadratic constraint (3.7) or the linear one (3.6). We shall start with models with extra spinor variables which have kappa symmetries (models of the first and second type) and then consider models with extra vector coordinates (models of the first ilk) which have world-line supersymmetry. The models with the quadratic constraint of the first type and the first ilk were originally presented in [22] and [24], respectively.

6.1. First type (quadratic constraint).

It was shown in the last section that the model (5.23)-(5.25) in which an extra spinor coordinate $\phi^A$ was introduced, together with its conjugate momentum $\hat{\phi}_A$ and a momentum $\hat{\theta}^A$ conjugate to $\theta_A$, gives a spinor wave-function superfield subject to the constraints (5.26) (recall that $\hat{\theta}^A = d^A - \hat{\phi}^{AB} \theta_B$). To describe super Yang–Mills we wish to impose the extra constraint (3.7) which can be done by adding to the action (5.23)-(5.25) the term

$$S_3 = \int d\tau \left[ \frac{1}{2} d\chi d + 2\hat{\phi} \Gamma^\mu \chi \Gamma_\mu \hat{p} \phi \right]$$

(6.1)

involving a Lagrange multiplier $\chi_{AB} = -\chi_{BA}$. The alternative action in which the constraint $N = 1$ imposed by (2.25) is replaced by the constraint $\hat{\phi} \hat{\phi} = 0$ that
comes from the term (2.27) (as in [22]) gives a very similar model so here we shall simply review the model of [22]. The total action is the sum of a free action

\[ S_0 = \int d\tau \left[ p_\mu \dot{x}^\mu + i\dot{\theta} + i\dot{\phi} \right], \tag{6.2} \]

plus the term

\[ S' = \int d\tau \left[ -\frac{1}{2}e p^2 + i\psi \dot{p} + i\Lambda \dot{\phi} + \frac{1}{2} \phi \Gamma \dot{\phi} + \frac{1}{2} d\chi d + 2\dot{\phi} \Gamma^\mu \chi \Gamma_{\mu} \phi \right], \tag{6.3} \]

where \( e, \psi^A, \chi_{AB} = -\chi_{BA}, \Lambda_A \) and \( \Gamma^{AB} = -\Gamma^{BA} \) are Lagrange multipliers imposing the first class constraints

\[
\begin{align*}
p^2 &= 0, \\
\dot{p} &= 0, \\
\dot{\phi} &= 0, \\
\dot{\phi}_A \dot{\phi}_B &= 0, \\
d^A d^B &- 8(\dot{\phi} \Gamma^\mu)[A(\Gamma^\mu \phi)]^B = 0,
\end{align*}
\]

and are also gauge fields for corresponding local symmetries.

The action is invariant under the global space-time supersymmetry transformations,

\[
\delta \theta = \epsilon, \quad \delta x^\mu = i\epsilon \Gamma^\mu \theta, \tag{6.5}
\]

(where \( \epsilon_A \) is a constant Grassmann parameter) and a number of local symmetries, which generalize ones found for the earlier superparticle actions. The symmetries divide into two kinds [20]. Symmetries of the ‘first kind’ are those under which a gauge field transforms into the derivative of a gauge parameter. These include world-line reparameterizations which, when combined with a trivial symmetry, gives the \( A \) transformations

\[
\delta x^\mu = \xi \rho^\mu, \quad \delta \epsilon = \dot{\xi}, \tag{6.6}
\]

the other fields being inert. There are also two fermionic symmetries of the first

\footnote{Spinor indices are supressed and we use a matrix notation, so that \( d\dot{\theta} = d^A \dot{\theta}_A \), \( \theta \Gamma^\mu \dot{\theta} = \theta_A (\Gamma^\mu)^{AB} \theta_B \), \( d\chi d = d^A \chi_{AB} d^B \), \( \phi \Gamma^\mu \chi \Gamma_{\mu} \phi = \phi_A (\Gamma^\mu)^{AB} \chi_{BC} (\Gamma^\mu)^{CD} \phi_D \phi^E \), etc.}
kind, $B$ and $B'$, with fermionic spinor parameters $\kappa^A(\tau)$ and $\zeta_A(\tau)$,

$$\delta\theta = \dot{\rho}\kappa, \quad \delta\psi = \dot{k}, \quad \delta x^\mu = i\theta\Gamma^\mu\dot{\rho}\kappa + i\dot{\theta}\Gamma^\mu\kappa, \quad \delta e = 4i\dot{\theta}\kappa, \quad (6.7)$$

$$\delta\phi = \zeta, \quad \delta\Lambda = \dot{\zeta}, \quad \delta x^\mu = i\hat{\phi}\Gamma^\mu\zeta, \quad \delta e = 4\hat{\phi}\Gamma^\mu\chi\Gamma_\mu\zeta. \quad (6.8)$$

Finally, there are further bosonic symmetries of the first kind associated with the gauge fields $\chi, \Upsilon$ (the $C$ and $C'$ symmetries) defined by

$$\delta\theta = -i\rho d, \quad \delta d = -2i\rho d, \quad \delta\chi = \dot{\rho} + 2i(\chi\hat{\rho} - \rho\hat{\chi}),$$

$$\delta x^\mu = i\theta\Gamma^\mu\delta\theta - 2i\hat{\phi}\Gamma^\mu\rho\Gamma_\nu\Gamma^\nu\phi, \quad \delta e = 4\psi\rho d + 4\Lambda\Gamma_\mu\rho\Gamma_\mu\hat{\phi},$$

$$\delta\phi = 2i\Gamma_\mu\rho\Gamma^\mu\hat{\phi}, \quad \delta\phi = -2i\hat{\phi}\Gamma_\mu\rho\Gamma_\mu\hat{\phi},$$

$$\delta\Upsilon = 2i(\Upsilon\rho\Gamma_\mu\rho\Gamma^\mu + \Gamma_\mu\rho\Gamma^\mu\rho\Upsilon), \quad (6.9)$$

$$\delta\Upsilon = \dot{\psi} - 2i\Gamma^\mu\chi\Gamma_\mu\hat{\phi} + 2iv\hat{\phi}\Gamma_\mu\chi\Gamma^\mu, \quad \delta\phi = iv\hat{\phi}, \quad (6.10)$$

where $\rho_{AB} = -\rho_{BA}$ and $v^{AB} = -v^{BA}$ are bosonic bispinor parameters.

There are also local symmetries of the 'second kind' that act only on the gauge fields. These are the $E$ and $E'$ symmetries with fermionic parameters $\eta_A$ and $\omega^A$

$$\delta\psi = \dot{\rho}\eta, \quad \delta\psi = \dot{\rho}\omega, \quad \delta\Lambda = \dot{\rho}\omega, \quad \delta e = -2i\dot{\eta}d - 2i\dot{\phi}\omega, \quad (6.11)$$

and the $F, F'$ and $G'$ symmetries with parameters $\sigma^A_B, u^B_A$ and $\Sigma^{ABC}$, respectively, which are given by

$$\delta\psi = \sigma d, \quad \delta\chi = i(\rho\sigma - \sigma^t\rho), \quad \delta\Lambda = u\hat{\phi} - 2(\Gamma_\mu\sigma\Gamma^\mu + 2\sigma^t)\hat{\phi}, \quad \delta e = 4i\hat{\phi}(2\sigma + \Gamma_\mu\sigma^t\Gamma^\mu)\phi, \quad (6.12)$$

where $\Sigma^{ABC} = -\Sigma^{BAC}$, $\Sigma^{[ABC]} = 0$ and the transpose of $\sigma^A_B$ is $(\sigma^t)^A_B$. The presence of the symmetries of the second kind reflects ambiguities in the definition of the $A$, $B$, $B'$, $C$ and $C'$ symmetries and relations between the corresponding constraints.
The covariant quantization of this superparticle was discussed in [22] in the
gauge $e = 1$ with the other gauge fields set to zero. Covariant quantization requires
the methods of Batalin and Vilkovisky [16] since the gauge algebra only closes on
shell, and requires an infinite number of ghost fields since the symmetries are
infinitely reducible. The classical gauge-fixed action takes the form

$$S_{\text{fixed}} = \int d\tau \left[ p_\mu \dot{x}^\mu - \frac{1}{2}p^2 + i\hat{\theta}\dot{\theta} + i\hat{\phi}\dot{\phi} \right]$$

leading to the commutation relations

$$[x^\mu, p^\nu] = -i\delta_{\mu\nu} , \quad \{d^A, \theta^B\} = \delta^A_B , \quad \{\hat{\phi}_A, \phi^B\} = \delta_A^B . \quad (6.14)$$

The wave-function is taken to be a function of $p^\mu, \theta^A, \phi^A$ together with an
infinite set of ghost coordinates. The spectrum is then found by seeking the BRST
cohomology classes, using the BRST operator that follows from the Batalin and
Vilkovisky procedure.

We now turn to the BRST cohomology of the model. The full analysis requires
the complete solution of the BV master equation with an infinite number of ghost
fields. However, as explained in [13,22], there is also a ‘small formalism’ which
only involves the finite number of ghosts that are required for the symmetries of
the first kind (i.e. those that act on the coordinates as well as the gauge fields). It
was shown in [22] that the BRST cohomology of the full formalism at zero ghost
number is the same as that for the BRST cohomology calculated with the small
formalism, and for this reason we shall restrict ourselves to physical states of zero
ghost number in this paper. A state of zero ghost number corresponds to a ghost
independent wave-function $\Phi = \Phi(p, \theta, \phi)$. It was shown in [22] that in this case
$Q\Phi = 0$ implies the constraints

$$p^2\Phi = 0 , \quad \dot{p}d\Phi = 0 , \quad \dot{p}\hat{\phi}\Phi = 0 , \quad \hat{\phi}_A\hat{\phi}_B\Phi = 0 , \quad [d^A d^B - 8(\hat{\phi}\Gamma^\mu)^{\left[A}(\Gamma^\mu\hat{\phi})^{\left[B]\right]}\Phi = 0 . \quad (6.15)$$

29
Expanding the wave-function in powers of $\phi$,

$$
\Phi(p, \theta, \phi) = \Psi_0(p, \theta) + \phi^A \Psi_A(p, \theta) + \frac{1}{2} \phi^A \phi^B \Psi_{AB}(p, \theta) + \ldots,
$$

(6.16)

the constraint $\hat{\phi}_A \hat{\phi}_B \Phi = 0$ implies that only $\Psi_0$ and $\Psi_A$ are non-vanishing. The constraint $\left[ d^A d^B - 8 (\hat{\phi} \Gamma^\mu)^{[A} (\Gamma_{\mu} \hat{\phi})^{B]} \right] \Phi = 0$ implies that $\Psi_0$ satisfies $d^A d^B \Psi_0 = 0$ and this implies that $\Psi_0$ is trivial (unless $p^\mu = 0$). Thus the only non-trivial part of the wave-function is $\Psi_A$, and (6.15) implies that it satisfies precisely the covariant constraints (4.16), (4.17), which lead to the Yang–Mills spectrum. Note that the constraints (6.15) are precisely what one expects from taking the classical constraints (6.4), and requiring that the corresponding operators annihilate the wave-function.

The $SO(8)$ light-cone gauge formalism of section 3 may be recovered directly by fixing the gauge in the covariant action by setting $e = 1$ and the other gauge fields to zero and imposing $x^+ = p^+ \tau + x_0^+$, $\gamma^+ \theta = 0$ and $\gamma^+ \phi = 0$. The light-cone action is given by $S_{lc} = S_{0lc} + S'_{lc}$, where

$$
S_{0lc} = \int d\tau \left[ p^i \dot{x}^i - \frac{1}{2} p^i p^i + i \hat{\theta}^a \dot{\hat{\theta}}^a + i \hat{\phi}^a \dot{\hat{\phi}}^a \right],
$$

(6.17)

and

$$
S'_{lc} = \frac{1}{2} \int d\tau \left[ \chi_{\dot{a} \dot{b}} \left( \dot{d}^a \dot{d}^b + 2 p^+ \gamma_{cd} \phi^c \phi_d \right) + \Upsilon^{ab} \hat{\phi}_a \hat{\phi}_b \right],
$$

(6.18)

where $\chi_{\dot{a} \dot{b}}$, $\Upsilon^{ab}$ are Lagrange multipliers imposing the remaining constraints

$$
d^a \dot{d}^b + 2 p^+ \gamma_{cd} \phi^c \phi_d = 0, \quad \hat{\phi}_a \hat{\phi}_b = 0,
$$

(6.19)

and are gauge fields for a local $SO(8) \times SO(8)$ symmetry.
6.2. Second type (linear constraint).

In this section, a superparticle theory with a spinor wave-function satisfying the linear constraint (3.6) will be considered. We start with the superparticle action (5.23)-(5.25) formulated in an extended superspace with coordinates \( (x^\mu, \theta_A, \phi^A) \) where \( \theta_A \) and \( \phi^A \) are anticommuting Majorana-Weyl spinors and add a lagrange multiplier term imposing the constraint \( d^A(\Gamma^{\mu\nu\rho\sigma})_B A \hat{\phi}_B = 0 \), which then leads to the condition (3.6) on the wave-function. In fact, to close the constraint algebra and obtain a gauge invariant superparticle action, it is necessary to include a lagrange multiplier term imposing the extra constraint \( \hat{\phi} \hat{\phi} = 0 \).

The new superparticle action is then given by the sum of

\[
S_0 = \int d\tau \left[ p_\mu \dot{x}^\mu + i\hat{\theta} \dot{\theta} + i\hat{\phi} \dot{\phi} \right],
\]

and

\[
S'' = \int d\tau \left[ -\frac{1}{2} e p^2 + i\psi \hat{p} d + i\phi \hat{p} \hat{\phi} + i\Lambda_{\mu\nu\rho\sigma} d \Gamma^{\mu\nu\rho\sigma} \hat{\phi} - i\beta (\hat{\phi} \hat{\phi} - 1) + \frac{1}{2} \hat{\phi} \omega \hat{\phi} \right],
\]

where, as usual, \( p_\mu \) is the momentum conjugate to the space-time coordinate \( x^\mu \), \( d^A \) is a spinor introduced so that the Grassmann coordinate \( \theta \) has a conjugate momentum \( \hat{\theta}^A = d^A - \hat{\theta}^{AB} \theta_B \), \( \phi^A \) is a new spinor coordinate and \( \hat{\phi}_A \) is its conjugate momentum. The fields \( e, \psi^A, \varphi_A, \Lambda_{\mu\nu\rho\sigma}, \beta \) and \( \omega^{AB} \) are all Lagrange multipliers (which are gauge fields for corresponding local symmetries) imposing the following constraints

\[
\begin{align*}
p^2 &= 0, & \hat{\phi} \hat{d} &= 0, & \hat{\phi} \hat{\phi} &= 0 \\
\hat{\phi}_A \hat{\phi}_B &= 0, & \phi^A \hat{\phi}_A - 1 &= 0, & d^A(\Gamma^{\mu\nu\rho\sigma})_B A \hat{\phi}_B &= 0
\end{align*}
\]  

(6.22)

World-line reparameterization, when combined with a trivial symmetry, gives the \( A \) symmetry of (6.6). There are two fermionic symmetries of the first kind, \( B \)
and \( B' \) with fermionic spinor parameters \( \kappa^A(\tau) \) and \( \zeta_A(\tau) \) given by

\[
\delta \psi^A = \dot{\kappa}^A, \quad \delta \theta = \dot{\theta}_{AB}\kappa^B, \quad \delta e = 4i\dot{\theta}_{A}\kappa^A, \\
\delta x^\mu = i\dot{d}^A(\Gamma^\mu)_{AB}\kappa^B + i\dot{\theta}_A(\Gamma^\mu)^{AC}\dot{\phi}_{CB}\kappa^B,
\]

(6.23)

and

\[
\delta \varphi_A = \dot{\zeta}_A + \beta \zeta_A, \quad \delta \phi^A = \zeta_B\dot{\phi}^{BA}, \quad \delta x^\mu = i\dot{\phi}_A(\Gamma^\mu)^{AB}\zeta_B
\]

(6.24)

where \( \zeta_A \) is a spinor parameter. The bosonic symmetries associated with the gauge fields \( \beta \) and \( \omega^{AB} \) (the \( C \) and \( C' \) symmetries) are defined by

\[
\delta \beta = \dot{\eta}, \quad \delta \hat{\phi}_A = \eta\dot{\phi}_A, \quad \delta \phi^A = -\eta \phi^A, \\
\delta \omega^{AB} = -2\eta\omega^{AB}, \quad \delta \Lambda_{\mu\nu\rho\sigma} = \eta \Lambda_{\mu\nu\rho\sigma}, \quad \delta \varphi_A = -\eta \varphi_A,
\]

(6.25)

and

\[
\delta \omega^{AB} = \dot{\Upsilon}^{AB} + 2\beta\Upsilon^{AB}, \quad \delta \phi^A = i\Upsilon^{AB}\dot{\phi}_B,
\]

(6.26)

where \( \eta \) is a bosonic parameter and \( \Upsilon^{AB} = -\Upsilon^{BA} \) is a bosonic bispinor parameter. There is also a tensor symmetry associated with the gauge field \( \Lambda_{\mu\nu\rho\sigma} \) (\( F \) symmetry) with bosonic parameter \( \Sigma_{\mu\nu\rho\sigma} \) and given by

\[
\delta \Lambda_{\mu\nu\rho\sigma} = \dot{\Sigma}_{\mu\nu\rho\sigma} + \beta \Sigma_{\mu\nu\rho\sigma}, \quad \delta d^A = -2\dot{\phi}_B\Sigma^B_{\mu\nu\rho\sigma}C\phi^A, \\
\delta \theta_A = -\dot{\phi}_B\Sigma^B_A, \quad \delta \phi^A = d^B\Sigma^A_{\mu\nu\rho\sigma}B\phi^B, \quad \delta e = 4i\dot{\phi}_B\Sigma^B_A\psi^A, \\
\delta x^\mu = i\dot{\phi}_B\Sigma^B_A(\Gamma^\mu)^{AC}\theta_C, \quad \delta \omega^{AB} = 4i\Sigma^A_{\mu\nu\rho\sigma}C\phi^B_{CD}\Lambda^D_B.
\]

(6.27)

The covariant quantization of this model (in the gauge \( e = 1 \) with other gauge fields vanishing) gives the classical gauge-fixed action

\[
S_{\text{fixed}} = \int d\tau \left[ p_\mu \dot{x}^\mu - \frac{1}{2} \dot{p}^2 + i\dot{\theta} + i\dot{\phi} \right].
\]

(6.28)

Once again, a complete treatment of the infinite number of ghosts requires the solution of the BV master equation, which is not undertaken here and we shall only
consider the ghost number zero cohomology class. This is given by the wavefunction 
\[ \Phi = \Phi(x, \theta, \phi), \] 
and \( Q \Phi = 0 \) implies that
\[ p^2 \Phi = 0, \quad \hat{p} d \Phi = 0, \quad \hat{p} \phi \Phi = 0, \]
\[ \hat{\phi}_A \hat{\phi}_B \Phi = 0, \quad (\phi^A \phi_A - 1) \Phi = 0, \quad d^A (\Gamma^{\mu \nu \rho \sigma})^B_A \hat{\phi}_B \Phi = 0. \] (6.29)

Expanding the wavefunction in powers of \( \phi \)
\[ \Phi(x, \theta, \phi) = \Psi_0(x, \theta) + \phi^A \Psi_A(x, \theta) + \frac{1}{2} \phi^A \phi^B \Psi_{AB}(x, \theta) + \ldots, \] (6.30)
the constraint \( \hat{\phi}_A \hat{\phi}_B \Phi = 0 \) implies that only \( \Psi_0 \) and \( \Psi_A \) are non-vanishing, while \( (\phi \hat{\phi} - 1) \Phi = 0 \) implies that \( \Psi_0 \) is trivial. Hence, the only non-trivial part of the wavefunction is \( \Psi_A \), and (6.29) implies that it satisfies precisely the covariant constraints (4.13) and (4.14), which lead to the super Yang–Mills spectrum.

Again the light-cone gauge equations of section 3 can be obtained directly from the light-cone gauge action \( S_{lc} = S_{0lc} + S'_{lc} \), where
\[ S_{0lc} = \int d\tau \left[ p^i \dot{x}^i - \frac{1}{2} p^i p^i + i \hat{\theta}^a \hat{\phi}_a + i \hat{\phi}^a \hat{\phi}_a \right], \] (6.31)
and
\[ S'_{lc} = \int d\tau \left[ \Upsilon^{ab} \hat{\phi}_a \hat{\phi}_b + \beta (\phi \hat{\phi} - 1) + \Lambda_{ijk} d^a (\gamma^{ijk})^b_a \hat{\phi}_b \right]. \] (6.32)
The Lagrange multipliers \( \Upsilon^{ab}, \beta \) and \( \Lambda_{ijk} \) impose the required constraints.

6.3. First ilk (extra vector coordinates).

The action (5.18) with world-line supersymmetry leads to a wave-function \( \Phi(x, \theta, \lambda_\alpha) \) and expanding this in \( \lambda_\alpha \) gives \( 2^{[D/2]} \) superfields which can be assembled into a spinor wave-function \( \Psi(x, \theta)_A (A = 1, \ldots, 2^{[D/2]} \) satisfying the constraints (5.22). The addition of the Lagrange multiplier term,
\[ S = \frac{1}{720} \int d\tau \Upsilon^{\mu \nu \rho} \left( d\Gamma_{\mu \nu \rho} d + 4 p_{[\mu} \lambda_{\nu} \lambda_{\rho]} \right), \] (6.33)
leads to the constraint \( d\Gamma_{\mu \nu \rho} d + 4 p_{[\mu} \lambda_{\nu} \lambda_{\rho]} = 0 \). Imposing this on the wave function gives (3.7) when rewritten in terms of \( \Psi_A \) so that the spectrum of Yang–Mills is
again obtained. This is the Lagrangian formulation of the first ilk superparticle of [24]. The full Batalin-Vilkovisky quantization of this model was given in [25,26], confirming that its spectrum is indeed that of the Yang–Mills supermultiplet.

7. Superparticles with vector super wave-functions.

In this section, we shall consider superparticle theories which are formulated in an extended superspace with coordinates \((x^\mu, \theta_A, \phi^\mu)\) where \(\theta_A\) is an anticommuting Majorana-Weyl spinor, and \(\phi^\mu\) is a new commuting vector coordinate. The model defined by (5.23), (5.27) was shown to lead to a vector wave-function satisfying the constraints (5.28) and identified modulo (5.29). We now wish to modify this to obtain models with wave-functions satisfying the extra constraints (3.5) or (3.3). These will be referred to as models of the third and fourth type, respectively.

7.1. THIRD TYPE (QUADRATIC CONSTRAINT)

As in the first type of model discussed in the last section, there are two versions of this model, one with the constraint \(N = 1\) and one with the constraint \(\hat{\phi}\hat{\phi} = 0\). Here we shall just consider the latter version, which leads to a vector wavefunction satisfying the quadratic constraint (3.5). The new superparticle action is given by

\[
S_0 = \int d\tau \left[ p_\mu \dot{x}^\mu + i \dot{\theta}^A + \dot{\phi}^\mu \phi^\mu \right], \tag{7.1}
\]

plus the term

\[
S_1 = \int d\tau \left[ -\frac{1}{2} ep^2 + i \psi \phi d + \frac{1}{2} d\chi d + 2i \phi^\mu \chi \phi^\nu \phi^\nu - \omega p^\mu \phi^\mu + \frac{1}{2} \phi^\mu \rho^{(\mu\nu)} \phi_\nu \right], \tag{7.2}
\]

where we use the convenient notation

\[
p^{AB}_\mu = p^{\rho \Gamma^{AB}}_\mu \rho_\nu. \tag{7.3}
\]

The fields \(e, \psi^A, \chi_{AB} = -\chi_{BA}\), \(\rho_{\mu\nu} = \rho_{\nu\mu}\) and \(\omega\) are all Lagrange multipliers.
imposing the following constraints

\[ \begin{align*}
p^2 &= 0, & \hat{p} \hat{d} &= 0, & p^\mu \hat{\phi}_\mu &= 0, \\
\hat{\phi}_\mu \hat{\phi}_\nu &= 0, & d^A d^B + 4 \hat{\phi}_\mu (\hat{p}^\mu)^{AB} \phi^\nu &= 0,
\end{align*} \] (7.4)

and are also gauge fields for their corresponding symmetries.

The action (7.1)-(7.2) is invariant under the global space-time supersymmetry transformations

\[ \begin{align*}
\delta \theta &= \epsilon, & \delta x^\mu &= i \epsilon \Gamma^\mu \theta, \\
\delta \phi^\mu &= \mu^\mu, & \delta e &= 4 i \theta^A \kappa^A, \\
\delta x^\mu &= i d^A (\Gamma^\mu)^{AB} \kappa^B + i \theta^A (\Gamma^\mu)^{AC} \hat{p}^B \kappa^B.
\end{align*} \] (7.5)

where \( \epsilon \) is a Grassmann parameter, together with a number of local symmetries of the first-kind. World-line reparameterization, when combined with a trivial symmetry gives the \( \mathcal{A} \) transformations, (6.6). The fermionic symmetry, \( \mathcal{B} \), with fermionic spinor parameter \( \kappa^A(\tau) \), is given by

\[ \begin{align*}
\delta \psi^A &= \kappa^A, & \delta \theta^A &= \hat{p}^{AB} \kappa^B, & \delta e &= 4 i \dot{\theta}^A \kappa^A, \\
\delta x^\mu &= i d^A (\Gamma^\mu)^{AB} \kappa^B + i \theta^A (\Gamma^\mu)^{AC} \hat{p}^B \kappa^B.
\end{align*} \] (7.6)

and the bosonic symmetries associated with the gauge fields \( \omega \) and \( \chi \) (the \( \mathcal{C} \) and \( \mathcal{C}' \) symmetries, respectively) are defined by

\[ \begin{align*}
\delta \omega &= \dot{\zeta}, & \delta \phi^\mu &= \zeta p^\mu, & \delta x^\mu &= \kappa \phi^\mu, & \delta e &= 4 i \zeta (\Gamma^\mu)^{AB} \lambda_{AB} \phi^\mu, \\
\delta \chi_{AB} &= \dot{\lambda}_{AB} + 4 i \lambda_{AC} \hat{p}^{CD} \chi_{DB}, & \delta x^\mu &= -d^A (\Gamma^\mu)^{BC} \dot{\theta}_C - 2 i \phi_\sigma (\Gamma_{\sigma\nu})^{AB} \phi^\nu, \\
\delta e &= -4 d^A \lambda_{AB} \psi^B - 4 i \phi_\mu (\Gamma^\mu)^{AB} \lambda_{AB} \omega,
\end{align*} \] (7.7)

and

\[ \begin{align*}
\delta d^A &= 2 i \hat{p}^{AB} \lambda_{BC} d^C, & \delta p^{(\mu\nu)} &= -4 i (\hat{p}_{(\mu})^{\sigma \rho} (\Gamma_{\sigma\rho})^{AB} \lambda_{AB} \rho^{\nu)C}, \\
\delta \phi^\mu &= -2 i \lambda_{AB} (\hat{p}^\mu)^{AB} \phi^\nu, & \delta \phi_\mu &= 2 i \phi_\nu \lambda_{AB} (\hat{p}^\nu)^{AB}, \\
\delta \theta^A &= -i d^B \lambda_{BA}, & \delta \chi_{AB} &= \dot{\lambda}_{AB} + 4 i \lambda_{AC} \hat{p}^{CD} \chi_{DB}, \\
\delta x^\mu &= -d^A (\Gamma^\mu)^{BC} \dot{\theta}_C - 2 i \phi_\sigma (\Gamma_{\sigma\nu})^{AB} \phi^\nu, & \delta e &= -4 d^A \lambda_{AB} \psi^B - 4 i \phi_\mu (\Gamma^\mu)^{AB} \lambda_{AB} \omega,
\end{align*} \] (7.8)

where \( \lambda_{AB} = -\lambda_{BA} \) is a bosonic bispinor parameter. There is also a tensor sym-
metry associated with the gauge field $\rho_{\mu \nu}$ (the $E$ symmetry) and given by

$$\delta \rho^{(\mu \nu)} = \dot{u}^{(\mu \nu)} + 4 i u^{(\mu \sigma} \chi_{AB} (p_\sigma^{\nu)}) AB , \quad \delta \phi^\mu = -u^{(\mu \nu)} \hat{\phi}_\nu,$$  \hspace{1cm} (7.9)

where $u_{\mu \nu} = u_{\nu \mu}$ is a tensor parameter.

The classical gauge-fixed action (in the gauge with $e = 1$ and other gauge fields vanishing) here takes the form

$$S = \int d\tau \left[ p_\mu \dot{x}^\mu - \frac{1}{2} p^2 + i \dot{\theta} \theta + \dot{\phi}_\mu \phi^\mu \right].$$  \hspace{1cm} (7.10)

The wave-function $\Phi = \Phi(x, \theta_A, \phi^\mu)$ is a function of $x^\mu$, $\theta_A$, and $\phi^\mu$ (again we are considering only the zero-ghost sector). The constraint $Q \Phi = 0$ implies

$$p^2 \Phi = 0, \quad p^i \Phi = 0, \quad p_\mu \phi^{\mu} \Phi = 0,$$

$$\dot{\phi}_\mu \phi^{\mu} \Phi = 0, \quad [dA dB + 4 \hat{\phi}_\mu (p^{\mu} \phi^\nu)] \Phi = 0.$$  \hspace{1cm} (7.11)

Expanding the wavefunction in powers of $\phi$

$$\Phi(x, \theta_A, \phi^{\mu}) = \Psi_0(x, \theta_A) + \phi^{\mu} \Psi_\mu(x, \theta_A) + \frac{1}{2} \phi^{\mu} \phi^{\nu} \Psi_{\mu \nu}(x, \theta_A) + \ldots,$$  \hspace{1cm} (7.12)

the constraint $\dot{\phi}_\mu \phi^{\mu} \Phi = 0$ implies that only $\Psi_0$ and $\Psi_\mu$ are non vanishing. The condition $(dd + 4 \phi^{\mu} p^{\mu} \phi^{\nu}) \Phi = 0$ implies $dd \Psi_0 = 0$, and hence that $\Psi_0$ is trivial (if $p^{\mu} \neq 0$). Thus, the only non-vanishing part of the wave-function is $\Psi_\mu$, and (7.11) implies that it satisfies precisely the covariant constraints (4.10) and (4.11) which lead to the physical spectrum of the super Yang–Mills theory.

The light-cone action is $S_{0lc} + S'_{lc}$ with

$$S_{0lc} = \int d\tau \left[ p_i \dot{x}_i - \frac{1}{2} p^2 \right] + i \dot{\theta}_\alpha \hat{\theta}^{\alpha} + \dot{\phi}_i \phi^i,$$  \hspace{1cm} (7.13)

and

$$S'_{lc} = \frac{1}{2} \int d\tau \left[ \rho^{(ij)} \phi^i \phi^j + \chi_{ab} \left( 4 p^a \phi^b - 4 p^+ \phi^i (\gamma^i_{(j}) \hat{a}^b \phi^{j)} \right) \right],$$  \hspace{1cm} (7.14)

where $\rho_{ij}$ and $\chi_{ab}$ are Lagrange multipliers that impose the quadratic constraints. Applying these constraints to the light-cone wave function leads directly to the conditions (3.5).
7.2. Fourth type (linear constraint)

A superparticle theory with a vector wave-function satisfying the linear constraint (3.3) is obtained from the action (5.23), (5.27) formulated in the superspace with coordinates \((x^\mu, \theta_A, \phi^\mu)\) together with a Lagrange multiplier term that imposes the constraint \(C^A_\mu = 0\), where

\[
C^A_\mu = \dot{\phi}^\mu d^A - \frac{1}{2} \dot{\phi}_\nu (\Gamma^\nu_\mu)^A_B d^B. \tag{7.15}
\]

The complete superparticle action is then given by \(S_0 + S'\) where,

\[
S_0 = \int d\tau \left[ p_\mu \dot{x}^\mu + i \dot{\theta} + \dot{\phi}^\mu \phi^\mu \right], \tag{7.16}
\]

and

\[
S' = \int d\tau \left[ -\frac{1}{2} ep^2 + i \phi d + \lambda p^\mu \phi^\mu + \frac{1}{2} \phi^\nu \phi^\nu + \beta (\phi^\mu \phi^\mu - 1) + \Upsilon^\mu_A C^A_\mu \right]. \tag{7.17}
\]

The action is again invariant under \(A\) transformations as well as fermionic symmetries associated with the gauge fields \(\psi^A, \Upsilon^\mu_A\) (the \(B\) and \(B'\) symmetries respectively) which are defined by

\[
\begin{align*}
\delta \psi^A &= \kappa^A, \\
\delta \theta_A &= \hat{\phi}_{AB} \kappa^B, \\
\delta e &= 4i \dot{\theta} \kappa^A, \\
\delta x^\mu &= i d^A (\Gamma^\mu_\nu)^B_A \phi^\nu + i \theta_A (\Gamma^\mu_\nu)^A_C \phi^\nu C^A_B, \\
\delta d^A &= -2i [\phi^\mu B^A - \frac{1}{2} \phi^\nu B^A \phi^\nu] \phi^B. \\
\end{align*} \tag{7.18}
\]

\[
\begin{align*}
\delta \Upsilon^\mu_A &= \chi^\mu_A + \beta \chi^\mu_A, \\
\delta \phi^\mu &= -\chi^\mu_A A^A d^A + \frac{1}{2} \chi^\nu_A B^A \Gamma^\mu_\nu d^A, \\
\delta \theta_A &= -i \chi^\mu_A \phi^\mu + \frac{1}{2} i \chi^\nu_A \phi^\nu \Gamma^\mu_\nu, \\
\delta x^\mu &= -\left[ \chi^\nu_A \phi^\nu - \frac{1}{2} \chi^\sigma_A \phi^\sigma \Gamma^\nu_\sigma \right] (\Gamma^\mu_\theta)^A, \\
\delta d^A &= -2i [\chi^\nu_B \phi^\nu A^B - \frac{1}{2} \chi^\nu_B \phi^\nu B^A \phi^\nu] \phi^B, \\
\delta e &= -4i [\chi^\mu_A \phi^\mu - \frac{1}{2} \chi^\nu_A \phi^\nu C^A_B] \phi^A. \\
\end{align*} \tag{7.19}
\]

The bosonic symmetries associated with the gauge fields \(\lambda\) and \(\beta\) (\(C\) and \(C'\) sym-
The wave-function is a function of $p^\mu$, $\theta_A$ and $\phi^\mu$ together with the infinite set of ghost coordinates. Once again the zero ghost-number wavefunction $\Phi = \Phi(p, \theta, \phi)$ satisfying $Q\Phi = 0$ satisfies constraints that arise from the variation of the Lagrange multipliers in the action.

\begin{align}
\dot{p}^\mu \Phi &= 0, \\
\dot{p}d \Phi &= 0, \\
p^\mu \hat{\phi}_\mu \Phi &= 0, \\
\hat{\phi}_\mu \hat{\phi}_\nu \Phi &= 0, \\
C_\mu^A \Phi &= 0, \\
(\phi^\mu \hat{\phi}_\mu - 1)\Phi &= 0.
\end{align}

(7.24)

Expanding the wavefunction in powers of $\phi$

\begin{align}
\Phi(p, \theta, \phi) &= \Psi_0(p, \theta) + \phi^\mu \Psi_\mu(p, \theta) + \frac{1}{2} \phi^\mu \phi^\nu \Psi_{\mu\nu}(p, \theta) + \ldots,
\end{align}

(7.25)

the constraint $\hat{\phi}_\mu \hat{\phi}_\nu \Phi = 0$ implies that only $\Psi_0$ and $\Psi_\mu$ are non-vanishing. The constraint $(\phi \hat{\phi} - 1)\Phi = 0$ makes $\Psi_0$ trivial. Hence, the only non-trivial part of the wavefunction is $\Phi_\mu$, and (7.24) implies that it satisfies precisely the covariant constraints (4.2)-(4.3). We have thus shown that the BRST cohomology class with no ghost dependence gives the physical spectrum of the super Yang–Mills theory.
The light-cone action (in which \(x^+ = p^+ \tau + x_0^+\), \(\gamma^+ \theta = 0\) and \(\phi^+ = 0\)) is given by \(S_{0lc} + S'_{lc}\) with

\[
S_{lc} = \int d\tau \left[ p^i \dot{x}^i - \frac{1}{2} p^i p^i + i \hat{\theta}^a \dot{\hat{a}}^a + \hat{\phi}^i \dot{\hat{\phi}}^i \right],
\]

(7.26)

and

\[
S'_{lc} = \frac{1}{2} \int d\tau \left[ \omega^{ij} \dot{\phi}^i \dot{\phi}^j + \beta (\phi^i \dot{\phi}^i - 1) + \Upsilon^{i\dot{a}} C^{i\dot{a}} \right],
\]

(7.27)

where \(\omega^{ij}\), \(\beta\) and \(\Upsilon^{i\dot{a}}\) are Lagrange multipliers imposing constraints that lead to the linear condition (3.3) on the light-cone gauge vector wave function.

Acknowledgements:

The early parts of this work were done in collaboration with Michael Green and we would like to thank him for many helpful discussions and comments.

REFERENCES

1. W. Siegel, Class. Quantum Grav. 2 (1985) L95.

2. W. Siegel, Nucl. Phys. B263 (1985) 93.

3. W. Siegel, Phys. Lett. B205 (1988) 257.

4. W. Siegel, in Unified String Theories, M.B. Green and D. J. Gross eds. (World Scientific, Singapore 1986).

5. R. Casalbuoni, Phys. Lett. B293 (1976) 49; Nuovo Cim. A33 (1976) 389.

6. P. G. O. Freund, unpublished, as quoted in A. Ferber, Nucl. Phys. B132 (1978) 55.

7. L. Brink and J. H. Schwarz, Phys. Lett. B100 (1981) 310.

8. U. Lindström, M. Roček, W. Siegel, P. van Nieuwenhuizen and A. E. van de Ven, Phys. Lett. B224 (1989) 285.
9. W. Siegel, *Proc. of the 1989 Texas A + M String Workshop*.

10. J. Fisch and Henneaux, Bruxells Preprint ULB TH2-89-04.

11. F. Bastianelli, W. Delius and E. Laenen, Phys. Lett. **B229** (1989) 223.

12. M. B. Green and C. M. Hull, *The Covariant Quantization of the Superparticle*, in the ‘Proceedings of the 1989 International Workshop on Superstrings’, edited by R. Arnowitt, M.J. Duff and C.N. Pope, (World Scientific, Singapore, 1990).

13. M. B. Green and C. M. Hull, Nucl. Phys. **B344** (1990) 115.

14. R. E. Kallosh and M. A. Rahmanov, Phys. Lett. **B209** (1988) 233.

15. E. Nissimov, S. Pacheva and S. Solomon, Nucl. Phys. **B296** (1988) 462; Nucl. Phys. **B297** (1988) 349.

16. I. A. Batalin and G. A. Vilkovisky, Phys. Lett. **B102** (1981) 27; Phys. Rev. **D28** (1983) 2567.

17. W. Siegel, Phys. Lett. **B203** (1988) 79.

18. I. A. Batalin, R. E. Kallosh and A. Van Proeyen, *Symmetries of Superparticles and Superstring Actions*, in Quantum Gravity, eds. M. A. Markov, V. A. Berezin and F. P. Frolov (World Scientific, 1988).

19. A. Miković and W. Siegel, Phys. Lett. **B209** (1988) 47.

20. R. E. Kallosh, A. Van Proeyen and W. Troost, Phys. Lett. **B212** (1988) 428.

21. U. Lindström, M. Roček, W. Siegel, P. van Nieuwenhuizen and A. E. van de Ven, J. Math. Phys. **31** (1990) 1761.

22. M. B. Green and C. M. Hull, Mod. Phys. Lett. **A18** (1990) 1399.

23. L. Brink, in *Physics and Mathematics of Strings*, eds. L. Brink, D. Friedan and A. M. Polyakov, (World Scientific, 1990).

24. A. Miković, M. Roček, W. Siegel, A. E. van de Ven, P. van Nieuwenhuizen and J. P. Yamron, Phys. Lett. **B235** (1990) 106.
25. F. Eβler, E. Laenen, W. Siegel and J. P. Yamron, Phys. Lett. B254 (1991) 411.
26. F. Eβler, M. Hatsuda, E. Laenen, W. Siegel, J. P. Yamron, T. Kimura and A. Miković, Nucl. Phys. B364 (1991) 67.
27. E. A. Bergshoeff and R. E. Kallosh, Nucl. Phys. B333 (1990) 605.
28. E. A. Bergshoeff and R. E. Kallosh, Phys. Lett. B240 (1990) 105.
29. E. A. Bergshoeff, R. E. Kallosh and A. Van Proeyen, Phys. Lett. B251 (1990) 128.
30. R. E. Kallosh, Phys. Lett. B251 (1990) 134.
31. M. Huq, Int. J. Mod. Phys. A7 (1992) 4053.
32. J. L. Vázquez-Bello, Intern. J. Mod. Phys. A19 (1992) 4583.
33. D. P. Sorokin, V. I. Tkach and D. V. Volkov, Mod. Phys. Lett. A4 (1989) 901; D. P. Sorokin, V. I. Tkach and D. V. Volkov and A. A. Zheltukhin, Phys. Lett. B216 (1989) 302; A. I. Gumenchuk and D. P. Sorokin, Sov. J. Nucl. Phys. 51 (1990) 350; F. Delduc and E. Sokatchev, Preprint PAR-LPTE-91-14 (1991); E. A. Ivanov and A. A. Kapustnikov, Preprint IC-90-425 (1990); Phys. Lett. B267 (1991) 175.
34. A. Galperin and E. Sokatchev, Phys.Rev.D46 714, (1992); F. Delduc, A. Galperin, and E. Sokatchev, Nucl.Phys. B368 143, (1992); F. Delduc, A. Galperin, P. Howe and E. Sokatchev, Phys. Rev. D47 (1993) 578.
35. N. Berkovits, Stony Brook preprint ITP-SB-92-42; King’s College preprints KCL-TH-92-6, KCL-TH-93-3; Nucl. Phys. B379 96, (1992); Nucl. Phys. B358 169, (1991); Nucl. Phys. B350 193, (1991).
36. L. Brink, S. Deser, B. Zumino, P. DiVecchia and P. S. Howe, Phys. Lett. B64 (1976) 435.
37. F.A. Berezin and M.S. Marinov, Ann. Phys. 104 (1977) 336.
38. L. Brink, P. DiVecchia and P. S. Howe, Nucl. Phys. B118 (1977) 76.
39. C. Galvao and T. Teitelboim, J. Math. Phys. 21 (1980) 1863.

40. P. S. Howe, S. Penati, M. Pernici and P. K. Townsend, Phys. Lett. B215 (1988) 555.

41. V. D. Gershun and V. I. Tkach, JETP Lett. 29 (1979) 320.

42. W. Siegel, Intern. J. Mod. Phys. A3 (1988) 2713.

43. S. Aoyama, J. Kowalski-Glikman, J.W. van Holten and J. Lukierski, Phys. Lett. B201 (1988) 487; Phys. Lett. B216 (1989) 133; J. Kowalski-Glikman, Phys. Lett. B202 (1988) 343; E. Bergshoeff and J. Van Holten, Phys. Lett. B226 (1989) 93; J. Kowalski-Glikman and J. Lukierski, Mod. Phys. Lett. A4 2437, (1989).

44. P. S. Howe, S. Penati, M. Pernici and P. K. Townsend, Class. Quantum Grav. 6 (1989) 1125.

45. L. Brink, M. B. Green and J. H. Schwarz, Nucl. Phys. B223 (1983) 125.

46. L. Brink, O. Lindgren and E. W. Nilsson, Nucl. Phys. B212 (1983) 401.

47. S. Mandelstam, Nucl. Phys. B213 (1983) 149.

48. W. Siegel, Introduction to String Field Theory, (World Scientific, 1988).

49. L. J. Romans, Nucl. Phys. B281 (1987) 639.

50. P. K. Townsend, in Superunification and Extra Dimensions, eds. R. D’Auria and P. Fré, (World Scientific, 1986).

51. T. J. Allen, Mod. Phys. Lett. A2 (1987) 209.