Analysis of the laser noise propagation mechanism on the laser interferometer gravitational wave antenna

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Abstract. The propagation mechanisms of noise imposed on the light used to illuminate complex optical systems, from the noise source to the signal readout, can be very complicated, such as gravitational wave detectors. It is very important to understand these mechanisms both qualitatively and quantitatively, in order to effectively suppress the noise contribution to the interferometer readout. In this article, a method for the systematic treatment of the noise propagation mechanisms, and a way to analyze a noise contributions in complex optical systems, is described.

1. Introduction

First generation gravitational wave (GW) detectors, initial-LIGO¹, VIRGO², GEO600³ and TAMA300⁴ are currently being commissioned and some of them are operating with antenna sensitivities close to the design. All of these antennas employ a variant of a Michelson interferometer together with other optical systems, such as a mode cleaner, before and/or after the main interferometer. The optical configuration of TAMA300 is shown in Figure 1. The light from the stabilized laser is phase modulated at radio frequency (RF), producing phase modulation sidebands around the carrier frequency. The light then illuminates the GW antenna system, which is comprised of a mode cleaner and a main interferometer. The main interferometer is an asymmetric Michelson interferometer with Fabry-Perot arm cavities. The intentional asymmetry in the Michelson interferometer is referred to as Schnupp asymmetry, which allows the Michelson to leak the RF sidebands at the “dark port”. These RF sidebands and the phase modulated carrier light, which is generated by the differential change of the arm cavity lengths, beat against each other on the photo-detector, the resulting photo-current is demodulated with the same RF signal to produce an interferometer output signal. In other words, the RF sidebands leaked to the dark port serve as a phase reference in the photo-detection process.

Based on this length-sensing scheme, in general, the variations in the phase and the amplitude of the illuminating electric fields can contribute to the interferometer output signal as noise, in connection with the coupling factors such as interferometer asymmetry, imbalances and imperfections. Therefore both the original level of the variations, and the coupling factors...
Figure 1. The schematic diagram of the TAMA interferometer. The laser light passes through the mode-cleaning triangular cavity before illuminating the main interferometer. The light is RF phase modulated with the electro-optic modulator (EOM), producing modulation sidebands. The RF sidebands, shown in green, also pass through the mode cleaner and circulate only inside the power-recycling cavity, then leak to the dark port. The carrier and the RF sidebands at the dark port are photodetected, then demodulated to produce an interferometer output signal.

have to be carefully controlled to suppress this noise sources. Nevertheless, recent noise hunting efforts on the GW antennas show that the noise contributions from the variations of the input light, especially of the RF oscillator, is at a level competing with other noise sources. As the GW antennas have multiple, complex optical systems, the process of noise propagation becomes very complicated. In this article a novel framework, which can trace the noise propagation by multiple optical systems intuitively, is proposed. The framework will help the understanding of the laser field noise behavior, and would be an helpful tool for noise hunting in the existing detectors, and also for designing of next generation GW antennas.

2. The noise transfer mechanism
Noises that are imposed on the ideal state of the laser light can appear as a noise on the GW readout. This is understood by the idea that the electric field is transformed in some way by optical systems such as the mode cleaner or the main interferometer. As for the GW detectors, there are multiple optical systems that the laser fields pass through, so this makes the noise propagation mechanism more and more complicated. In addition, there are a number of asymmetry and imperfection parameters which couple the transfer functions of the electric fields, so the situation was hard to understand clearly and totally. Under such circumstances, an idea to unthread the intricate situation was newly suggested, whose essential points are the followings:

• Decomposition of the light field into the noise bases
• Matrix expressions for the transfer characteristics of the optical systems.

Using this framework, the contribution of the noises can be projected on the noise curve of the interferometer, with well-defined coupling factors. In the following, the above two key points are described more in detail.

2.1. Noise expression and the decomposition into bases
For the first generation GW antennas, a single phase modulation is used for an interferometer control of longitudinal degrees of freedom. So there are only three optical fields in an ideal case; the carrier and the upper/lower sidebands for the RF phase modulation as electric fields, which illuminate the interferometer system. However, in practice, unwanted noises have to be included in the input fields. Among possible contaminations of the input fields, the intrinsic noise sources
Figure 2. The schematic diagram for the propagation of the noise state. The pure input noise state (a) is transformed to a different noise state (b) by passing through the mode cleaner with some imperfections, then transformed again into yet another noise state (c) by the main interferometer, with asymmetries and imperfections. When the system comprises multiple optical systems like GW antenna, the whole view of the propagation becomes very difficult to characterize.

are the laser noise and the oscillator noise. This aspect is comprehensible if these noises are translated to noise sidebands in an optical spectrum, which is reviewed and summarized in reference [5]. In this picture, the laser noise (frequency and intensity noise) and the oscillator (phase and amplitude noise) can be expressed with nine (or seven for the oscillator noise) optical fields. This set of nine fields, defined as the “noise state”, is incident on the optical system and experiences its response by passing through it. In more detail, each of the nine fields experiences different responses from the optical systems depending on its optical frequencies. In the mode cleaner, for example, the amplitudes and relative phases of the nine fields are transformed in a different but rigorous manner, according to the transfer functions defined as

$$H_{MC}(\Omega_i) = \frac{t_1 t_2 e^{-iL_{MC}\Omega_i/c}}{1 - r_1 r_2 r_3 e^{-2iL_{MC}\Omega_i/c}} \approx \frac{(1 - R)e^{-iL_{MC}\Omega_i/c}}{1 - Re^{-2iL_{MC}\Omega_i/c}},$$

where, $r_1(t_1), r_2(t_2)$ and $r_3$ are the amplitude reflectivities (transmissivities) of the input coupler, output coupler, and end mirror respectively. $L_{MC}$ is the mode cleaner cavity length and $l_{MC}(\ll L_{MC})$ is the separation between the input and output couplers. $\Omega_i$ is the optical frequency of the electric fields, and $c$ is a speed of light. Supposing $r_3 = 1$ and $r_1 r_2 = R$ and no loss, the expression can be simplified as shown above.

The input noise state experiences this kind of transformation in each optical system; the situation is shown in Figure 2 for the case of TAMA300. The issue is now what is the best way to deal with this intricately transformed set of sidebands, which look like randomly distributed fields, in a comprehensible manner. In order to characterize the arbitrary noise state, the idea of decomposing the noises into some bases was introduced. Suppose the carrier is always considered as a phase reference, then the rest of eight fields, the set of sidebands including RF modulation sidebands can be regarded as a superposition of the simple, well-defined sets of sidebands (noise base). As a noise bases, the laser and the oscillator phase and amplitude noises were chosen together with two kinds of RF modulation. Total eight kinds of noise bases are adopted in this
a noise frequency, and be unity for simplicity, $\Omega$ is the RF modulation frequency with modulation depth of $\Gamma$, $\omega$ itself, remains. On the other hand, if there is finite $dl$, fields in the last line correspond to noise base OPN-PM, OAN-PM, OAN-PM and OAN-AM as the phase reference. Then, the noise state is decomposed to the noise bases defined in Table 1. The carrier and the phase modulation sidebands as shown above. The four sets of fields is then normalized with the carrier response function in order to maintain the carrier as the ideal operation point by $\pi dl/\lambda$, only the A-component (OPN-PM), which is an input noise itself, remains. On the other hand, if there is finite $dl$ offset in mode cleaner, small amount of

### Table 1

| Modulation coupling          | Phase Modulation | Amplitude Modulation |
|------------------------------|------------------|----------------------|
| Laser frequency noise        | LFN-PM (FP)      | LFN-AM (FA)          |
| Laser intensity noise        | LIN-PM (IP)      | LIN-AM (IA)          |
| Oscillator phase noise       | OPN-PM (PP)      | OPN-AM (PA)          |
| Oscillator amplitude noise   | OAN-PM (AP)      | OAN-AM (AA)          |

framework, which are listed in Table 1. Using this noise bases, an arbitrary noise state can be decomposed to eight noise bases by an elementary calculation using complex numbers. In order to see the actual decomposition, consider the simple case of noise transformation by a mode cleaner as an example. Suppose there is only the oscillator phase noise as an input noise coupled with RF phase modulation, and the mode cleaner is locked with a microscopic offset from the ideal operation point by $dl$. After passing through the mode cleaner, the expressions for each of the nine fields are given as follows.

$$
\begin{bmatrix}
E_{U+} \\
E_{USB} \\
E_{U-} \\
E_{C+} \\
E_{CR} \\
E_{C-} \\
E_{L+} \\
E_{LSB} \\
E_{L-}
\end{bmatrix}
= E_0
\begin{bmatrix}
-\Gamma a_0/4 \cdot H_{MC}(\Omega + \omega) \\
\Gamma/2 \cdot H_{MC}(\Omega) \\
-\Gamma a_0/4 \cdot H_{MC}(\Omega - \omega) \\
0 \\
0 \\
\Gamma a_0/4 \cdot H_{MC}(-\Omega + \omega) \\
\Gamma/2 \cdot H_{MC}(-\Omega) \\
\Gamma a_0/4 \cdot H_{MC}(-\Omega - \omega)
\end{bmatrix}
\rightarrow
\begin{bmatrix}
-\Gamma a_0/4 \cdot H_{MC}(\Omega + \omega)/H_{MC}(0) \\
\Gamma/2 \cdot H_{MC}(\Omega)/H_{MC}(0) \\
-\Gamma a_0/4 \cdot H_{MC}(\Omega - \omega)/H_{MC}(0) \\
0 \\
1 \\
0 \\
\Gamma a_0/4 \cdot H_{MC}(-\Omega + \omega)/H_{MC}(0) \\
\Gamma/2 \cdot H_{MC}(-\Omega)/H_{MC}(0) \\
\Gamma a_0/4 \cdot H_{MC}(-\Omega - \omega)/H_{MC}(0)
\end{bmatrix}
$$

$$
\begin{bmatrix}
0 \\
\Gamma/2 \\
0 \\
0 \\
1 \\
0 \\
i \Gamma/2 \\
0 \\
0
\end{bmatrix}
\begin{bmatrix}
A \\
0 \\
0 \\
0 \\
1 \\
i \\
0 \\
0 \\
1
\end{bmatrix}
\begin{bmatrix}
-1 \\
0 \\
0 \\
0 \\
i \\
0 \\
0 \\
0 \\
i
\end{bmatrix}
\begin{bmatrix}
1 \\
0 \\
0 \\
0 \\
i \\
0 \\
0 \\
0 \\
i
\end{bmatrix}
\begin{bmatrix}
N_{FP} \\
N_{IP} \\
N_{FA} \\
N_{IA} \\
N_{PP} \\
N_{AP} \\
N_{PA} \\
N_{AA}
\end{bmatrix}
\equiv \mathbf{N}
$$

The notations are basically same with that of reference [5]; a carrier amplitude $E_0$ is set to be unity for simplicity, $\Omega$ is the RF modulation frequency with modulation depth of $\Gamma$, $\omega$ is a noise frequency, and $a_0$ is an amplitude of the phase noise. The transformed set of electric fields is then normalized with the carrier response function in order to maintain the carrier as the phase reference. Then, the noise state is decomposed to the noise bases defined in Table 1, the carrier and the phase modulation sidebands as shown above. The four sets of fields in the last line correspond to noise base OPN-PM, OAN-PM, OAN-PM and OAN-AM respectively, and A, B, C and D are coupling coefficients for each noise base given as $A = (1 - R)/(1 - R e^{-2iMC\omega/c})$, $B = 0$, $C = \sin(2\pi dl/\lambda)(1 - e^{-2iMC\omega/c})/(1 - R e^{-2iMC\omega/c})^2$, $D = 0$. If the mode cleaner is ideal ($dl = 0$), only the A-component (OPN-PM), which is an input noise itself, remains. On the other hand, if there is finite $dl$ offset in mode cleaner, small amount of
C-component (OPN-AM) is created, in other words, the OPN-PN is scattered to the OPN-AM. Now the noise state transformed by an imperfect mode cleaner is expressed by a superposition of the noise components using noise bases, which simply corresponds to a base change. Then this decomposition gives a noise vector \( \mathbf{N} \), whose elements are noise components. The advantageous points of this base selection are:

- All of the bases has physical meaning as noises
- The bases can be clearly categorized whether they will be noise at the signal readout port after the demodulation

With regard to the first point, there is only RF phase modulation on the incident fields, however, it can be converted to an amplitude modulation component after passing through an optical system, such as an offset-locked optical cavity. So the noises caused by amplitude modulation have to be considered. As for the second point, among the eight kinds of noise in our noise bases, only the OPN-AM is coupled to the GW readout port signal. At the dark port, the output fields of the interferometer are detected and then demodulated to give the GW signal. In this process, the demodulation is performed so as to maximize the response to differential motion of the arm cavities, which is referred to as a quadrature-phase demodulation. Meanwhile, the LIN-AM and OAN-AM components become noise at the dark port when they are demodulated in-phase, and the other five noise components do not contribute to the noise on the demodulated signal at all, which can be confirmed by a simple calculation.

2.2. Matrix expression of the transfer characteristics of the optical systems

This transformation of the noise state occurs sequentially for each optical system, however, the primary goal is to understand noise behavior at the GW signal readout port. Therefore the problem is simplified to the characterization of the OPN-AM component at the dark port, which is the final destination before the photo-detection. The situation becomes comprehensible if one simplifies the whole noise propagation process to a combination of simple transfer elements each of which correspond to one optical system. The eight noise components are transformed by the imperfect or asymmetric optical system, so each optical systems can in general be characterized by an \( 8 \times 8 \) matrix. Accordingly, the whole noise propagation process is expressed by the formulation

\[
\mathbf{N}' = \prod_{i=1}^{n} \mathbf{X}_i \cdot \mathbf{N},
\]

where, \( \mathbf{X}_i \) is the scattering matrix corresponding to \( i \)-th optical system, and \( \mathbf{N} \) and \( \mathbf{N}' \) are the initial and final noise vectors. Each matrix element is a frequency dependent transfer function, corresponding to the scattering coefficient from the input noise to the output noise. Furthermore, these transfer functions are complicated functions of the coupling factors, asymmetries and imperfections of the interferometer such as an offset lock of the cavities, a mismatch of the modulation frequency and FSR of the cavity, and an imbalance of the optical characteristics between arm cavities. Analytical expressions for each matrix elements will appear in a separate article. For the simple case of the mode cleaner, which was introduced in a previous section, the

\[\text{On the contrary, the demodulation phase which gives a minimum signal for the differential motion of the arm cavities is called as in-phase.}\]
expression for $X_{MC}$ is given as

$$X_{MC} = \begin{bmatrix}
A & 0 & 0 & E & 0 & 0 & 0 & 0 \\
E & A & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & E & A & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & E & A & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & A & 0 & C & 0 \\
0 & 0 & 0 & 0 & C & 0 & A & 0 \\
0 & 0 & 0 & 0 & C & 0 & A & 0
\end{bmatrix} \rightarrow \begin{bmatrix}
U & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & U & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & U & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & U & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & U & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & U & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & U & 0
\end{bmatrix}.$$  

The matrix elements $A$ and $C$ were given previously and $E = \sin(2\pi dl/\lambda)R(1-e^{-2iL_{MC}\omega/c}(1-Re^{-2iL_{MC}\omega/c})^2$. When the mode cleaner operates at the exact resonant point ($dl = 0$), the $X_{MC}$ corresponds to a diagonal matrix, with a diagonal element $U = 1/(1-2iR_{MC}\omega/(1-R)/c)$ which has a low-pass feature at the cavity pole frequency of the mode cleaner. Once the scattering matrices are characterized, the noise propagation become fully understandable within the assumed situations. The complexity caused by the multiple scattering of the noise state is now consolidated in the product of the scattering matrices.

### 3. Discussion

In this article, asymmetries and imperfections in the longitudinal degrees of freedom are considered, however, the transverse degrees of freedom are also important for the noise propagation. If the light fields inside the interferometer are scattered to higher-order transverse mode, such as the TEM$_{10}$ mode for example, the beat signals between these TEM$_{10}$ modes can also be demodulated at a signal readout port, producing a noise contribution, even if there is no unwanted asymmetries or imperfections in the longitudinal degrees of freedom inside the interferometer. This mechanism may put more severe requirements on alignment control of the interferometer optics or on the figure error of the optics, including the thermal lensing effect. This issue will be discussed more closely in a separate article.

### 4. Conclusion

A new method to manipulate the complicated laser noise propagation mechanism was suggested. The points of the framework are 1) decomposition of the noise state to the noise bases, 2) reduction of the behavior of the optical system to a matrix expression of the transfer functions. Using this framework, the laser noises, including the oscillator noises, can be treated with much clarity, good prospects and sometimes with the intuitive picture. This provides a solid foundation for analyzing the noise behavior of the existing GW detectors and also for designing next generation GW antennas.

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