Deceptive Labeling: Hypergames on Graphs for Stealthy Deception

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Abstract—With the increasing sophistication of attacks on cyber-physical systems, deception has emerged as an effective tool to improve system security and safety by obfuscating information or providing misleading information. In this paper, we present a solution to the deceptive game in which a control agent is to satisfy a Boolean objective specified by a co-safe temporal logic formula in the presence of an adversary. The agent intentionally introduces asymmetric information to create payoff misperception. This is done by masking information about the logical formula satisfied given the outcome of the game. We introduce hypergames on graphs as a model to capture the asymmetrical information with one-sided payoff misperception. We present the solution of such a hypergame for synthesizing stealthy deceptive strategies that exploit adversary's misperception. Specifically, deceptive sure winning and deceptive almost-sure winning strategies are developed by reducing the hypergame to a two-player game and one-player stochastic game with reachability objectives. Finally, an illustrative example is introduced to demonstrate the correctness of theory and proposed algorithms.

Index Terms—Formal methods-based control; Linear Temporal logic; games on graphs; hypergame theory.

I. INTRODUCTION

With increasing sophistication of attacks on cyber-physical security, deception has been used to mitigate the information and strategic disadvantages of the defender. In this paper, we consider a class of games where a control agent aims to satisfy a temporal logic formula, which is unrealizable if the adversary knows the exact game. Thus, the agent needs to falsify or obfuscate information to the adversary in order to satisfy its temporal logic specification. We investigate how to synthesize such deceptive strategies given asymmetric information.

Game theory for deception has been investigated extensively using two models, hypergames [1]–[3] and Bayesian games [4], [5]. In a hypergame, the players have different perceptions of their interaction given their information, and higher-order information (information about other’s information). Hypergame analysis has been initially proposed and developed for normal-form games [1], [2]. Gharesifard and Cortés [6], [7] developed H-digraph to monitor how a player’s perception evolves during repeated interactions and to design stealthy deceptive strategy in which the deceivers action does not contradict the perception of the mark. Bakker et al. [8] employ hypergame models to investigate deception by constraint manipulation in continuous systems from the attacker’s perspective.

Besides hypergames, Bayesian games are adopted for designing deceptive strategies. Carrol and Grosu [4] used a signaling game to study honeypot deception. The defender can disguise honeypots as real systems and real systems as honeypots. The attacker can waste additional resources to exploit honeypots. Huang and Zhu [5] used dynamic Bayesian games for active deception in cybernetwork, where the defender has incomplete information about the type of the attacker (legitimate user or adversary) and the attacker also is uncertain about the type of the defender (high-security awareness or low-security awareness). Based on the analysis of Nash equilibrium, these methods enable the prediction of the attacker’s strategy and proactive defense strategy to mitigate losses. Ornak et al. [9] formulate a security game (Stackelberg game) to allocate limited decoy resources in a cybernetwork to mask network configurations from the attacker. This class of deception manipulates the adversary’s perception of the payoffs and thereby causes the adversary to take (or thwart to take) certain actions that aid the objective of the defender. Existing game-theoretic deception describes players’ payoffs quantitatively using rewards/costs.

The specifications of safety-critical systems are often given as temporal logical formulas that express safety, reachability, liveness, and reactivity. To synthesize provably correct systems in dynamic and potentially adversarial environments, reactive synthesis methods are based on the solution concepts of games on graphs, also known as omega-regular games [10]–[12]. In these games, players’ (system and its environment) payoffs are described using linear temporal logic formulas and a labeling function. A player receives a payoff equal to one if the labeling over the outcome (state-sequence) of the game satisfies its temporal logic formula.

We study a class of payoff manipulation in games on graphs where the control agent has full information about the labeling function and falsifies or obfuscates the labeling function perceived by the adversary. These deception techniques are commonly used in cybersecurity (such as honey-X) and defense (such as camouflage) [13], [14]. To synthesize effective deceptive strategies for the agent, we extend the normal-form hypergame models to define hypergames on graphs that capture the adversary’s perceptual game given its information, as well as the agent’s knowledge about the adversary’s perceptual game. We consider both the agent and its adversary are subjective rationalizable [15], that is, the adversary is rational and assumes the agent to act rationally given its subjective view of the game. Since the intelligent
adversary may change its perception by observing the actions taken by the agent, we develop algorithms to synthesize stealthy deceptive strategy for the control agent to satisfy its temporal logic objective surely, or almost surely (with probability one), without contradicting the perception of the adversary until the last moment (formalized later). The solution concepts of hypergames on graphs not only provide provably correct deceptive strategies for the agent but also provide new way to assess the effectiveness of deception and its potential limits. We illustrate the solution concept using an example.

II. PRELIMINARIES

We begin with some background on the games on graphs. In this game, a control agent interacts with an adversary. We refer to the agent as player 1 (P1, pronoun ‘he’) and the adversary as player 2 (P2, pronoun ‘she’). A game on graph consists of an arena and payoffs of players defined by temporal logic formulas. In this work, we restrict to turn-based, deterministic games on graphs and co-safe Linear Temporal Logic (LTL) formulas.

**Game arena:** A turn-based, deterministic game arena consists of a tuple \( G = (S, A, T, \mathcal{AP}, L) \) where

- \( S = S_1 \cup S_2 \) is a finite set of states partitioned into P1’s states \( S_1 \) and P2’s states \( S_2 \);
- \( A = A_1 \cup A_2 \) is the set of actions where \( A_1 \) (resp., \( A_2 \)) is the set of actions for P1 (resp., P2);
- \( T : (S_1 \times A_1) \cup (S_2 \times A_2) \rightarrow S \) is a deterministic transition function that maps a state-action pair to a next state. If \( T(s, a) \) is defined, we say action \( a \) is enabled at state \( s \).
- \( \mathcal{AP} \) is the set of atomic propositions.
- \( L : S \rightarrow 2^{\mathcal{AP}} \) is the labeling function that maps each state \( s \in S \) to a subset \( L(s) \subseteq \mathcal{AP} \) of atomic propositions which evaluate to true at \( s \).

A path \( \rho = s_0s_1 \ldots \) is a state sequence such that for any \( i \geq 0 \), there exists \( a \in A_1 \cup A_2 \), \( T(s_i, a) = s_{i+1} \). A path \( \rho = s_0s_1 \ldots \) can be mapped to a word in \( 2^{\mathcal{AP}} \), \( w = L(s_0)L(s_1) \ldots \), which is evaluated against logical formulas.

**Payoffs:** The set of atomic propositions and the labeling function together define the players’ payoffs using linear temporal logic formulas. An LTL formula over \( \mathcal{AP} \) is inductively defined as follows:

\[
\varphi := \top | \bot | p | \neg \varphi | \varphi \land \psi | \varphi \cup \psi | \exists \varphi | \varphi \U \psi,
\]

where \( \top, \bot \) are universally true and false, respectively, \( p \in \mathcal{AP} \) is an atomic proposition, \( \exists \) is a temporal operator called the “next” operator. \( U \) is a temporal operator called the “until” operator. The operators \( \Diamond \) (read as eventually) and \( \Box \) (read as always) are defined by \( \Diamond \varphi = \top \cup \varphi \) and \( \Box \varphi = \neg \Diamond \neg \varphi \). For details about the syntax and semantics of LTL, the readers are referred to [16]. Given a word \( w \in (2^{\mathcal{AP}})^\omega \), if it satisfies an LTL formula \( \varphi \), then we write \( w \models \varphi \).

In this work, we restrict the objectives of P1 to a subclass of LTL called syntactically co-safe LTL (scLTL) [17]. The advantage of restricting to scLTL is that the specification can be represented equivalently by Deterministic Finite-State Automaton (DFA). A DFA of scLTL formula \( \varphi \) is a tuple \( A = (Q, \Sigma, \delta, \iota, F) \) which includes a finite set \( Q \) of states, a finite set \( \Sigma = 2^{\mathcal{AP}} \) of symbols, a deterministic transition function \( \delta : Q \times \Sigma \rightarrow Q \), a unique initial state \( \iota \), and a set \( F \) of final states. The transition function is recursively extended as \( \delta(q, aw) = \delta(\delta(q, a), w) \) for given \( a \in \Sigma \) and \( w \in \Sigma^* \). A word \( w \) is accepting if and only if \( \delta(q, u) \in F \) and \( w \) is a prefix of \( w_i \), i.e., \( w = uv \) for \( u \in \Sigma^* \) and \( v \in \Sigma^\omega \). A word \( w \) is accepting only if \( w \models \varphi \).

**Zero-sum Game on a Graph:** Given a game arena \( G \) and the scLTL specification \( \varphi_i \) of P1, a zero-sum game on a graph is a tuple, \( \mathcal{G} = (G, \varphi_1) \). For a path (outcome of the game) \( \rho \in S^\omega \), if the labeling \( L(\rho) \) satisfies \( \varphi_1 \), then the path is winning for P1. Otherwise, it is winning for P2.

A randomized strategy for player \( i, i \in \{1, 2\} \) is a function \( \pi_i : S^*S_i \rightarrow D(A_i) \), where \( D(A_i) \) is the set of discrete probability distributions over \( A_i \). A strategy is deterministic if \( \pi_i(\rho) \) is a Dirac delta function. A strategy is sure winning (resp., almost-sure winning) for player \( i \) from an initial state if and only if by committing to this strategy, no matter which strategy the adversary commits to, the path resulting from their interaction is winning for player \( i \) (resp., with probability one). Let \( \Pi_i \) be the set of strategies of player \( i \). A pair \( (\pi_1, \pi_2) \) of strategies is a strategy profile.

To solve the game \( \mathcal{G} \) where \( \varphi_1 \) is an scLTL formula, we construct a product game, defined as follows.

**Definition 1 (Product game).** Given an arena \( G = (S, A, T, \mathcal{AP}, L) \), let \( A = (Q, \Sigma, \delta, \iota, F) \) be a DFA equivalent to an scLTL formula \( \varphi_1 \), the product game is a tuple:

\[
G \otimes A = (S \times Q, A = A_1 \cup A_2, \Delta, (s_0, q_0), S \times F),
\]

where

- \( S \times Q \) is a set of states partitioned into P1’s states \( S_1 \times Q \) and P2’s states \( S_2 \times Q \);
- \( \Delta : (S_1 \times Q \times A_1) \cup (S_2 \times Q \times A_2) \rightarrow S \times Q \) is a deterministic transition function that maps a game state \( (s, q) \) and an action \( a \) to a next state \( (s', q') \) where \( s' = T(s, a) \) and \( q' = \delta(q, L(s')) \);
- \( (s_0, q_0) \in S \times Q \) where \( q_0 = \delta(\iota, L(s_0)) \) is the initial state of the product game.

- \( S \times F \subseteq S \times Q \) is a set of final states.

By the product construction, a product state sequence \( \rho = (s_0, q_0)(s_1, q_1) \ldots \) is winning for P1 if there exists \( i \geq 0 \), \( q_i \in F \). Otherwise it is winning for P2. In other words, P1 can win by reaching the set \( S \times F \) in the product game. P2 can win by always avoiding \( S \times F \). Thus, the product game is a reachability game for P1 and a safety game for P2.

A turn-based reachability game \( G \otimes A \) is determined. That is, we can partition the product game states into two sets, \( W_{in1}, W_{in2} \), which satisfy \( W_{in1} \cup W_{in2} = S \times Q \) and \( W_{in1} \cap W_{in2} = \emptyset \). For each state \( (s, q) \in W_{in1} \), player i has a memoryless, deterministic, sure winning strategy \( \pi_i : S \times Q \rightarrow A_i \), for \( i = 1, 2 \). The winning strategies can be computed by first computing the winning region using Zielonka’s recursive algorithm [18].
Slightly abusing the notation, we denote $G = (G, \varphi_1)$ to be $G = G \otimes A$ where $A$ is the DFA for formula $\varphi_1$.

III. GAME ON GRAPH WITH LABELING MISPERCEPTION

In this section, we investigate the solution concepts of the games on graphs with labeling misperception using the hypergame model. A hypergame allows us to model players who play different games in their minds.

**Definition 2 (Hypergame [1]).** A level-1 two-player hypergame is a pair $HG^1 = \langle G_1, G_2 \rangle$, where $G_1, G_2$ are games perceived by players P1 and P2, respectively. A level-2 two-player hypergame is a pair $HG^2 = \langle HG^1, G_2 \rangle$, where P1 perceives the interaction as a level-1 hypergame and P2 perceives the interaction as game $G_2$.

In general, it is possible to define a level-$k$ hypergame for $k \geq 2$ (see [2] for details). However, we find that a level-2 hypergame is sufficient to model the game between P1 and P2 that we consider in this paper. We refer to the game perceived by player $i$ as the perceptual game of player $i$. With this notation, we introduce a class of hypergames in which P2 has a labeling function different from that of P1.

**Definition 3 (Hypergame on Graph with Labeling Misperception).** Given P2’s perceived labeling function $L_2 : S \rightarrow 2^{AP}$, a level-2 hypergame between P1 and P2, wherein P1’s labeling function is $L_1 = L$ and he knows $L_2$, is the tuple $HG^2 = \langle HG^1, G_2 \rangle$, where $HG^1 = \langle G_1, G_2 \rangle$ is a level-1 hypergame with $G_1 = \langle G_1 = \langle S, A, T, AP, L_1 = L \rangle, \varphi_1 \rangle$, and $G_2 = \langle G_2 = \langle S, A, T, AP, L_2 \neq L \rangle, \varphi_1 \rangle$.

Given the arenas $G_1$ and $G_2$, using Def. [1] we obtain an explicit graphical representation of $G_1$ and $G_2$. The solutions of two games; $G_1$ and $G_2$, yield different partitions of the product state space $S \times Q$. We denote these partitions as the pair $\langle \text{Win}_1, \text{Win}_2 \rangle$, where $\text{Win}_1$ (resp., $\text{Win}_2$) represents the winning region of P1 (resp., P2) in $G_k$. A pair of winning strategies is denoted $\pi_1^k : \text{Win}_1^k \rightarrow A_1$ and $\pi_2^k : \text{Win}_2^k \rightarrow A_2$ for P1 and P2 respectively in the game $G_k$.

Such deceptive labeling is commonly used in cybersecurity. For instance, consider a cyber network in which the attacker’s objective is to compromise a critical host and the defender’s objective is to prevent this. In such a scenario, the defender may deploy a set of decoys that attacker cannot distinguish from the critical host. We identify this as defender inducing a labeling misperception in the attacker.

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1Two labeling functions, say $L_1$ and $L_2$, are equal if and only if $L_1(s) = L_2(s)$ for all $s \in S$.

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A. Problem Formulation

Our goal is to compute a strategy for P1 to satisfy $\varphi_1$. To this end, we generalize the solution concept of subjective rationalizability in hypergames from normal-form games in [15] to the hypergame model in Def. [3].

To facilitate this extension, we introduce the utility function for P1 as $u_1 : S^+ \times \Pi_1 \times \Pi_2 \rightarrow R$ such that $u_1(\rho, \pi_1, \pi_2)$ is the probability of satisfying the specification $\varphi_1$ given that players follow the strategy profile $\langle \pi_1, \pi_2 \rangle$ for a given history $\rho \in S^+$. The utility function for P2 is $u_2(\rho, \pi_1, \pi_2) = 1 - u_1(\rho, \pi_1, \pi_2)$. Given that different players may have different views of the utility functions due to different labeling functions, we denote $u_i^k$ the utility function of player $i$ perceived by player $j$.

**Definition 4 (Subjective Rationalizability).** Given a level-2 hypergame $HG^2 = \langle HG^1, G_2 \rangle$ and the history $\rho$, strategy $\pi_i^k$ (resp., $\pi_j^k$) is subjective rationalizable for P2 if and only if for all $\pi_i \in \Pi_i$, we have $u_i^k(\rho, \pi_i^k, \pi_j^k) \geq u_i^k(\rho, \pi_i, \pi_j^k)$, where $(i, j) \in \{(1, 2), (2, 1)\}$. The strategy $\pi_1^k$ is subjective rationalizable for P1 if and only if it satisfies, $\forall \pi_1 \in \Pi_1$, $u_1^k(\rho, \pi_1^k, \pi_2^k) \geq u_1^k(\rho, \pi_1, \pi_2^k)$, where $\pi_2^k$ is subjective rationalizable for player 2.

In words, a strategy is called subjectively rationalizable for player $i$ if it is the best response in that player’s perceptual game to some strategy of player $j$, which, for player $j$, is the best response to player $i$ in player $j$’s subjective view of player $i$’s perceptual game. To solve for P1’s subjective rationalizable strategy in $HG^2$, we construct the following transition system, which only exists in P1’s mind. This transition system allows P1 to keep track of history, his progress in satisfying the LTL specification, as well as P2’s subjective view of the evolution of the game.

**Definition 5.** Given a game arena $G$, P1’s labeling function $L_1$, P2’s perceived labeling function $L_2$, and a specification DFA $A$ for the scLTL formula $\varphi_1$, a hypergame transition system (HTS) is a tuple,

$HTS = \langle S \times Q \times Q, A, \Delta, (s_0, q_0, p_0) \rangle$

where 1) the transition function $\Delta$ is defined as follows: given $(s, q, p), (s', q', p') \in S \times Q \times Q$, $\Delta((s, q, p), a) = (s', q', p')$ for some $a \in A$ if and only if $s' = T(s, a)$ and $q' = \delta(q, L_1(s'))$ and $p' = \delta(p, L_2(s'))$; 2) the initial state is $(s_0, q_0, p_0)$ where $s_0$ is the initial state in the game arena, $q_0 = \delta(t, L_1(s_0))$, and $p_0 = \delta(t, L_2(s_0))$.

We can now informally state our problem: Is it possible for P1 to achieve his objective with probability one without making P2 aware of her potential misperception? To answer this question, we consider the following assumption.

**Assumption 1.** P2 knows that $L_2 \neq L_1$ if she observes that P1’s action is not subjectively rationalizable in P2’s perceptual game.

**Definition 6 (Stealthy deceptive winning strategy).** A strategy $\pi_1 : S \times Q \times Q \rightarrow D(A_1)$ defined on the HTS is stealthy
deceptive (sure/almost-sure) winning in the hypergame $HG^2$ (in Def. 3) if the following two conditions are satisfied:

- Stealthy: For any $(s, q, p) \in S_1 \times Q \times Q$, if $(s, q) \notin \text{Win}_1^1$, then $\pi_1((s, q, p), a) > 0$ only if $a$ is subjective rationalizable in $G_2$.

- Winning: By following $\pi_1$, for any subjective rationalizable strategy $\pi_2$ of P2, for any path $\rho \in (S \times Q \times Q)^*$ induced by $(\pi_1, \pi_2)$, $\rho$ will visit some state in $\text{Win}_1^1 \times Q$.

Problem 1. Given a hypergame on graph $HG_2$ in Def. 3 and Assumption 1 how to compute a stealthy deceptive winning strategy and the stealthy deceptive winning region from which this strategy is defined?

B. Synthesis of a stealthy deceptive sure winning strategy

For P1’s strategy to be stealthy, he needs to follow a strategy $\pi_1^2$ that is subjective rationalizable for P2 until reaching the winning region $\text{Win}_1^1$. At the same time, P2 will follow a subjective rationalizable strategy $\pi_2^2$. However, for games on graphs, such a strategy profile $(\pi_1^2, \pi_2^2)$ is not unique. To encompass all possible subjective rationalizable strategies, we employ the notion of permissible strategy [19].

Definition 7 (Rephrased from Def. 2.4 and 2.5 of [19]). For player $i \in \{1, 2\}$, a strategy $\pi_i : S \times Q \to 2^{A_i}$ is permissible if for any other winning strategy $\pi_i' : S \times Q \to 2^{A_i}$ (resp., a randomized strategy $\pi_i^r : S \times Q \to D(A_i)$), it holds that $\pi$ subsumes $\pi'$, i.e., $\pi(s, q) \subseteq \pi(s, q)$ (resp., $\pi_i^r(s, q), a) > 0$ implies $a \in \pi_i(s, q)$). A permissible strategy is maximal if there is no other strategy that subsumes it.

Given P2’s perceptual game $G_2 = G_2 \otimes A = (S \times Q, A, A_2, (s_0, p_0), s \times F)$, a maximally permissible strategies exists and is denoted $\pi_2^2 : \text{Win}_2^2 \cap (S_2 \times Q) \to 2^{A_2}$ such that $\pi_2^2(s, p) = \{a \mid \Delta_2((s, p), a) \in \text{Win}_2^2\}$.

In words, if P2 starts in $\text{Win}_2^2$ and selects actions according to $\pi_2^2(s, p)$, then the game state is ensured to stay within her perceptual winning region $\text{Win}_2^2$. For P2’s perceptual winning region $\text{Win}_2^2$ of P1, a winning strategy $\pi_2^1 : \text{Win}_2^1 \cap (S_1 \times Q) \to 2^{A_1}$ is defined for P1, such that

$$\pi_2^1(s, p) = \{a \mid \Delta_2((s, p), a) \in \text{Win}_2^1\}$$

In words, an action of P1 is perceived to be rational by P2 if it allows P1 to stay within $\text{Win}_2^1$.

Proposition 1. Given HTS $= (S \times Q \times Q, A, A_2, (s_0, q_0, p_0))$, P2’s maximally permissible winning strategy $\pi_2^2 : S \times Q \to 2^{A_2}$ and P1’s permissible winning strategy $\pi_2^1 : S \times Q \to 2^{A_1}$ perceived by P2. P1 has a stealthy deceptive sure winning strategy if and only if he has a sure winning strategy in the following reachability game:

$$\tilde{HG} = (S \times Q \times Q, A, A, \Delta, (s_0, q_0, p_0), \text{Win}_1^1 \times Q)$$

where $\Delta$ is obtained from $\Delta$ by restricting both players’ actions as follows: For a given state $(s, q, p) \in S \times Q \times Q$ and action $a \in A$,

- Case I: $(s, p) \in \text{Win}_2^2$ and $(s, q) \notin \text{Win}_1^1$,

$$\tilde{\Delta}((s, q, p), a) = \begin{cases} \Delta((s, q, p), a) & \text{if } s \in S_1, \\ \Delta((s, q, p), a) & \text{if } s \in S_2 \text{ and } a \in \pi_2^2(s, p), \\ \uparrow & \text{if } s \in S_2 \text{ and } a \notin \pi_2^2(s, p). \end{cases}$$

where $\uparrow$ means that the transition is undefined.

- Case II: $(s, p) \in \text{Win}_1^1$ and $(s, q) \notin \text{Win}_1^1$,

$$\tilde{\Delta}((s, q, p), a) = \begin{cases} \Delta((s, q, p), a) & \text{if } s \in S_1 \text{ and } a \notin \pi_2^1(s, p), \\ \Delta((s, q, p), a) & \text{if } s \in S_2. \end{cases}$$

The winning condition is defined by $\text{Win}_1^1 \times Q$—that is, P1 wins if he reaches the set $\text{Win}_1^1 \times Q$.

Proof (Sketch). Before reaching the set $\text{Win}_1^1 \times Q$, at any state $(s, q, p)$, if $(s, p)$ is perceived winning by P2, then P2 will select actions from her maximally permissible winning strategy, $\pi_2^2$. Meanwhile, P1’s actions are restricted to P1’s permissible strategy given P2’s perceptual game $G_2$. Thus, P2 will not know that a misperception exists, until P1 reaches $\text{Win}_1^1$. If P1 wins the game $HG$, then he is ensured to reach $\text{Win}_1^1 \times Q$. After reaching the set, P1 can follow the true winning strategy $\pi_1^1$, defined for $\text{Win}_1^1$.

C. Synthesis of a deceptive almost-sure winning strategy

In synthesizing the winning strategy for P1, we employed the formulation of two-player zero-sum game, where we assumed that P2 actively selects actions to play against P1’s objective. However, P2 cannot construct this hypergame transition system and thus may make “mistakes” due to her lack of information about the true game. To see this, let us consider the winning strategy for P2 in the reachability game $HG_2$, $\pi_2^2 : S \times Q \times Q \to 2^{A_2}$. For P2 to exercise the second strategy, P2 should know the value of $q$ in the tuple $(s, q, p)$, which means that P2 should have a knowledge about $L_1$. This contradicts our assumption. Next, we consider a realistic assumption on the policy of P2.

Assumption 2. For a P2 state $(s, q, p)$ in the HTS, if $(s, p) \in \text{Win}_2^2$, then P2 selects every action from her maximally permissible winning strategy $\pi_2^2(s, p)$ with a non-zero probability. Otherwise, when $(s, p) \in \text{Win}_2^2$, P2 selects any enabled action at $s$ with a non-zero probability.

With Assumption 2 we reduce the synthesis of stealthy deceptive winning strategy for P1 to qualitative planning in a one-player stochastic game with reachability objective, defined as follows,

$$HG_M = (V = V_1 \cup V_P, A_1, P, v_0, F = \text{Win}_1^1 \times Q),$$

where the states are partitioned into two subsets: $V_1 = S_1 \times Q \times Q$ are a set of P1’s states and $V_P = S_2 \times Q \times Q$ are a set of probabilistic state. The transition function is partially

\[2\text{At a P1's state } v \in V_1, \text{ P1 chooses an action } a \in A_1 \text{ and reaches the next state deterministically. At a probabilistic state } v, \text{ a successor state is chosen according to a probabilistic distribution defined by } P(v).\]
Case I-1: \((s, p) \in \text{Win}_1^2\), for any action \(a \in A_1\) enabled from \(s\), \(P((s', q', p'))(s, q, p, a) = 1\) where \((s', q', p') = \Delta((s, q, p), a)\).

Case I-2: \((s, p) \in \text{Win}_2^1\), for any action \(a \in \pi_2^1((s, p))\), \(P((s', q', p'))(s, q, p, a) = 1\) where \((s', q', p') = \Delta((s, q, p), a)\).

At a state \((s, q, p) \in V_p\), we distinguish two cases:

Case II-1: \((s, p) \in \text{Win}_2^1\), then for any action \(a \in \pi_2^1((s, p))\), \(P((s', q', p'))(s, q, p) > 0\) where \((s', q', p') = \Delta((s, q, p), a)\).

Case II-2: \((s, p) \in \text{Win}_2^2\), then for any action \(a \in A_2\) enabled from \(s\), \(P((s', q', p'))(s, q, p) > 0\) where \((s', q', p') = \Delta((s, q, p), a)\).

It is noted that only the support of \(P((s, q, p), a)\) is known but not the exact probability distribution. The partial knowledge of the transition probability function gives us a graph of the underlying one-player stochastic game, also known as an Markov Decision Process (MDP). The almost-sure stealthy deceptive winning strategy for P1 is to ensure, with probability one, a state in \(\text{Win}_1^2 \times Q\) can be reached. Next, we describe Algorithm [11] to solve the almost-sure stealthy and deceptive winning strategy for P1.

Algorithm 1 Computation of the almost-sure winning region and strategy for P1 in the one-player stochastic game.

| Inputs: | \(\mathcal{HG}_M = (S = V_1 \cup V_p, A_1, P, \mathcal{F})\). |
| Outputs: | \(X_k, \{Y_i\}\). |
| 1. | \(X_0 = V, Y_0 = \mathcal{F}, k \leftarrow 0\), |
| 2. | while True do |
| 3. | \(i \leftarrow 0\), |
| 4. | while True do |
| 5. | \(Y_{i+1} = \text{Pre}(Y_i, X_k) \cup Y_i\) |
| 6. | if \(Y_i = Y_{i+1}\) then |
| 7. | Break. |
| 8. | \(i \leftarrow i + 1\) |
| 9. | if \(Y_i = X_k\) then |
| 10. | Break. |
| 11. | \(X_{k+1} = Y_i, k \leftarrow k + 1\), |

The algorithm uses a function \(\text{Pre}\) defined as follows.

\[\text{Pre}(v, X) = \{v' \in V_1 \mid \exists a \in A_1, P(v|v', a) = 1\} \cup \{v' \in V_p \mid P(v|v') > 0 \implies v \in X\}\]  \hspace{1cm} (1)

and \(\text{Pre}(Y, X) = \bigcup_{Y \subseteq Y} \text{Pre}(v, X)\).

Intuitively, the set \(\text{Pre}(Y, X)\) includes any state starting from which P1 can ensure to reach the set \(Y\) with a positive probability, while staying in \(X\) with probability one. The following result is readily obtained by construction.

Proposition 2. The fix-point \(X^* = X_k = X_{k+1}\) is the almost-sure winning region for P1 in the one-player stochastic game \(\mathcal{HG}_M\).

Given the fixed point \(X^*\), let \(Y_0, Y_1, \ldots, Y_k\) be a sequence of states computed using \(X = X^*\) in the inner loop, we can extract P1’s deceptive almost-sure winning strategy \(\pi_1\) as follows. For each \(v \in V \setminus V_{-1}, i > 0, \pi_1(v, a) = 1\) if \(P(Y_{i-1} | v, a) = 1\). After reaching \(\mathcal{F}\), P1 follows his sure winning strategy in \(\text{Win}_1^1\).

Theorem 1. Under Assumption [2] the almost-sure winning strategy for P1 in the one-player stochastic game \(\mathcal{HG}_M\) is stealthy deceptive winning strategy for P1 in the hypergame \(\mathcal{HG}^2\).

Proof. The action of P1 before reaching \(\mathcal{F}\) are restricted to subjective rationalizable actions in P2’s perceptual game. As long as P2 randomizes all subjective rationalizable actions given the history in her perceptual game, P1’s almost-sure winning strategy in \(\mathcal{HG}_M\) ensures to reach \(\text{Win}_1^1 \times Q\) with probability one. Since the set \(\text{Win}_1^1\) is the true winning region for P1, then once a state in \(\text{Win}_1^1 \times Q\) is reached, P1 can follow his sure winning strategy in the game \(G_1\) to ensure the co-safe specification is satisfied.

The solutions of deceptive strategies are based on solving multiple games (two-player zero-sum, turn-based games and one player stochastic games). The space/time complexity is linear in the size of the product game for solving the deceptive sure winning strategy, and polynomial for solving the deceptive almost-sure winning strategy.

IV. AN ILLUSTRATIVE EXAMPLE

We illustrate the proposed method using an example. In this game graph, there are two players, P1 (cycle player) and P2 (square player). As the names suggest, at a cycle state, P1 can take an action to transit to another state. At a square state, P2 takes an action. Given the transitions are deterministic, we omit the action set and use the edges of the graph to refer to players’ actions.

Consider the following labeling function \(L_1(k) = 0\) for \(k \in \{0, 1, 2, 3, 4, 6, 7\}\), and \(L_1(5) = A\). P2’s labeling function \(L_2(k) = 0\) for \(k \in \{0, 1, 3, 4, 5, 6, 7\}\) and \(L_2(2) = A\). P1’s co-safe specification is \(\diamond A\). Using Zielonka’s algorithm [18], given the reachability objective, we can solve \(G_1\) and \(G_2\) for the same P1’s task and different labeling functions perceived by P1 and P2. The solutions give rise to 1) \(\text{Win}_1^1 = \{5, 6, 7\}\). The remainder are \(\text{Win}_1^2\). 2) \(\text{Win}_2^1 = \{2, 3\}\). The remainder are \(\text{Win}_2^2\).

![Fig. 1. A reachability game. The red (resp. blue) nodes are P1’s winning region \(\text{Win}_1^1\) in \(G_1\) (resp. \(\text{Win}_1^1\) in \(G_1\)).](image_url)

![Fig. 2. The DFA for \(\varphi = \diamond A\).](image_url)

Given the DFA for \(\diamond A\) shown in Fig. [2] we construct HTS, \(\mathcal{HG}\), and \(\mathcal{HG}_M\), shown in Fig. [3]. In this figure, the red,
dashed edges correspond to actions that are not subjective rationalizable in P2’s perceptual game and thus removed to obtain $\overline{HG}$ and $\overline{HG}_M$. For example, at state $(3, 0, 0)$, P2 thinks that it is irrational for P1 to reach $(4, 0, 0)$ instead of $(2, 0, 1)$ given P2 misperceives the labels of states and thinks that P1 needs to reach state 2.

![Graph](image)

Fig. 3. A graph representing HTS, $\overline{HG}$, and $\overline{HG}_M$. The blue and dashed dot edges are probabilistic choices in one-player stochastic game $\overline{HG}_M$ and deterministic choices in two-player reachability game $\overline{HG}$. The red and dashed edges are not subjectively rationalizable for P2. Unreachable states in $\overline{HG}$ and $\overline{HG}_M$ are drawn dashed.

In the reachability game $\overline{HG}$, we calculate the stealthy deceptive sure winning region for P1, which includes $\{(5, 1, 0), (6, 1, 0), (7, 1, 0), (4, 1, 0), (4, 0, 0)\}$. This means that P1 can satisfy his objective deceptive from states $(4, 5, 6, 7)$—that is, one state more than the game where P2 does not have misperception. Due to P2’s misperception, P2 will not select to go to state 3 from state 4—making the state 4 deceptive winning for P1.

The deceptive sure winning region does not include $(1, 0, 0)$. This is because at state $(1, 0, 0)$, P2 can choose to reach $(0, 0, 0)$ if she knows $\overline{HG}$. With the analysis from Sec. III:C there is a chance that P2 selects edge $(1, 4)$. The dash dot edges colored in blue in Fig. 3 correspond to the probabilistic choices of P2. As a result, we have the one-player stochastic game. Using Alg. 1 we compute 1) $Y_0 = W\delta_1 \times Q = \{(5, 1, 0), (6, 1, 0), (7, 1, 0)\}$, (here we omitted unreachable states) 2) $Y_1 = \{(4, 1, 0), (4, 0, 0)\} \cup Y_0$. 3) $Y_2 = \{(0, 0, 0)\} \cup Y_1, Y_3 = \{(0, 0, 0)\} \cup Y_2$. Because $Y_4 = Y_3$. The inner loop of Alg. 1 ends. Because now all reachable states in $X_3$ are in $Y_3$. We have $X_0 = X_2$ and the outer loop of Alg. 1 ends. Thus, the deceptive almost-sure winning region includes all states of the game. At state 0, P1’s strategy is to reach state 1. When at some point P2 selects to reach 4, P1 is to reach 5, and then follows his sure winning strategy in $G_1$.

V. CONCLUSION AND DISCUSSIONS

This paper presents a theory of hypergame for synthesizing stealthy deceptive strategies with temporal logic specifications. We have shown that different from the games with complete information where the sure winning and almost-sure winning region overlap, the deceptive sure winning and almost-sure winning regions are different when one player has incomplete or incorrect information. However, this framework assumes that P2 can only detect the asymmetric information when P1 deviates from subjective rationalizable strategy in P2’s perceptual game. This may not be realistic in practice. For example, in decoy-based deception, P2 may learn the label after interacting with the decoy. Future work will investigate adaptive deception for cybersecurity against learning adversary.

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