Applicability of the model of particles drift for the study of the disperse phase transfer in a gas flow

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Abstract. Two different approaches are currently available for the analysis of the behavior of the solid particles in flows. These are termed Eulerian and Langrangian. In the Lagrangian method on the trajectories of the individual size fractions are evaluated by solving time dependent ordinary differential equations. In the Eulerian approach, partial differential equations for the conservation of mass and momentum are written for each of the particles fractions, which are solved together with the equation of the liquid flow. The particles drift model is based on the algebraic slip velocity approach. In this approach the relative velocities between particles and liquid are evaluated by consideration of the forces acting on the particles. The analysis of results enable to conclude that if $\frac{1}{Kn} < 1$, $Re < 1$ using model of particles drift gives the error that is not exceeding 1%.

1. Introduction

Multiphase flows may be encountered in various forms in industrial practice, as for example, transient flows with a transition from pure liquid to a vapour flow as a result of external heating, separated flows (i.e. stratified flows, slug flows, or film flows), and dispersed two-phase flows where one phase is present in the form of particles, droplets or bubbles in a continuous carrier phase (i.e. gas or liquid). Such dispersed two-phase flows are encountered in numerous technical and industrial processes, as for example in particle technology, chemical engineering, and biotechnology.

For the description of properties of polyphase flows two methods based on the way of Lagrange and Euler [1] are used. Within approach of Lagrange equations of motion of the separate particles, considered as the material points, in the form of the second Newton's laws in which right members there are forces operating on a particle in a stream are written out. Despite apparent simplicity of the description of driving of particles within approach of Lagrange, this method possesses, at least, two essential shortcomings. The first of them is bound to computing difficulties, bound to need to solve huge number of equations of motion for an assembly of particles. So for the description of space movement N of particles it is required to solve 6N equations. The problem becomes even more composite if there is a need of model operation of driving of particles taking into account their interaction. The second problem is bound to difficulty of the accounting of stochastic nature of particles driving in a stream with turbulence. The approaches applied now based on application of a
Monte-Carlo method [2] demand carrying out the whole series of calculations, so that the result of their averaging had the objective character.

The equation of motion for small particles in a viscous quiescent fluid (i.e. for small particle Reynolds-numbers, which is also referred to as Stokes flow) goes back to the pioneering work of Basset [3], Boussinesq [4] and Oseen [5]. Therefore, the equation of motion is mostly referred to as BBO-equation. Numerous publications deal with the extension of the BBO equation for turbulent flows. The thesis of Tchen [6] was probably the first study on particle motion in turbulent flows based on the BBO equation. A rigorous derivation of the equation of motion for small particles in non-uniform flow has been performed by Maxey and Riley [7]. Inter-particle collisions, the method of the determination of the diffusion coefficient of the solid phases as well as the influence of the physical characteristics and velocity of the fluid on the bubble dynamics were investigated in the paper [8, 9, 10]. Effects of interaction of phases, stochastic nature of driving of a big assembly of particles can be considered within approach of Euler, in compliance with which the polyphase environment is considered as set multi-speed continuums (a carrying agent and various fractions of particles). For each of these continuums equations of motion in the form of Euler, and also equations of mass conservation of each considered continuums are written.

In case of particles with a small inertance this approach can be replaced with the models based on the concept of a drift of a disperse phase concerning a carrying agent. Thus the speed of a disperse phase is defined in the assumption of a smallness of inertial terms or, otherwise, dynamic balance of forces operating on particles. Thus, there is no need to solve the complete differential equations of driving, and it is enough to consider the equation of dynamic balance of forces.

Owing to the simplicity and profitability models of particles drift were widely adopted in engineering practice. Nevertheless, possibility of their application has to be defined not by intuitive estimates and simplicity reasons but by the corresponding quantitative assessments.

Both approaches of Lagrange and Euler are based on the assumption of a smallness of particle sizes $d_p$ in comparison with the reference size of area of a current $L$ for which within computing hydrodynamics it is convenient to take the size of a finite difference grid. Thus, the ratio has to be carried out:

$$Kn_p = d_p / L << 1.$$  \hspace{1cm} (1)

In case of violation of conditions (1) within approach of Lagrange the particle cannot be taken for the material point, and it is necessary to investigate a picture of a flow of a particle. Non-performance of a condition (1) within approach of Euler is equivalent to impossibility to apply model interpenetrating continuums.

2. Mathematical model

The paper considers some volume of liquid corresponding to a finite difference cell. Carrying agent velocity in this volume $\bar{v}_l$ can be considered independent of coordinates. Entering the frame moving with a velocity $\bar{v}_l$, the carrying agent will be fixed in the frame.

At model operation of driving of a disperse phase in a finite difference cell it will be proceed from the following assumptions:

- driving of a particle is defined by Archimedes force and resisting strength;
- the particle of a disperse phase with initial velocity $\bar{v}_0$ gets to some volume of based liquid;
- the vector of velocity of a particle in an initial instant is parallel to a vector of the acceleration caused by Archimedes force;
- particles of a disperse phase are assumed as spherical;
- interaction between particles is not considered.

The last assumption enables to investigate driving of a single particle only.

Within these assumptions the equation of motion of a single particle can be presented in the following form [1 – 9]:

2
\[
\frac{d\vec{v}_p}{dt} = -\frac{3}{4} \frac{\rho}{\rho_p} C_D d_p^{-1} |\vec{v}_p^*| \vec{v}_p + \frac{\rho_p - \rho}{\rho_p} g
, \quad (2)
\]

where \( \vec{g} \) is the free fall acceleration, \( d_p \) - diameter of a disperse phase, \( \rho, \rho_p \) - density of a bearing and disperse phase \( \vec{v}_p \) - the velocity of a disperse phase, drag coefficient \( C_D \) is function of a relative of Reynolds \( Re = \frac{\rho |\vec{v}_p| d_p}{\mu} \).

For small Reynolds numbers \( Re < 0.5 \) viscous effects dominate and no separation is observed. Therefore, an analytic solution for the drag coefficient is possible as proposed by Stokes. In the transition region \( (0.5 < Re < 1000) \) inertial effects become of increasing importance. Above a Reynolds number of about 24 the flow around the particle begins to separate. Initially this separation is symmetric. It becomes unstable and periodic above \( Re \approx 130 \). Above \( Re \approx 1000 \) the drag coefficient remains almost constant up to the critical Reynolds number, since the wake size and structure is not considerably changing. This regime is referred to as Newton-regime. At the critical Reynolds number \( Re > 2 \cdot 10^5 \) a drastic decrease of the drag coefficient is observed, being caused by the transition from a laminar to a turbulent boundary layer around the particle. This results in a decrease of the particle wake. In the super-critical region \( Re > 4 \cdot 10^5 \) the drag coefficient again increases continuously. The reference drag curve can be approximated patch dependences of the form:

\[
C_D = C' Re^n , \quad (3)
\]

where values of coefficients are given in a formula (3) in Table 1.

The next step is defining the displacement velocity of a particle within drift model. In case of a gravitational sedimentation of particles the equation of dynamic balance of forces can be presented as follows:

\[
\frac{3}{4} C_D \frac{\rho}{\rho_p} d_p^{-1} |\vec{v}_s^*| \vec{v}_s + \frac{\rho_p - \rho}{\rho} g = 0 , \quad (4)
\]

where \( \vec{v}_s \) - the velocity of a stationary deposition of a particle (sedimentation velocity).

| Reynolds number Re | \(< 1\) | \(1 < Re < 10\) | \(10 < Re < 800\) | \(800 < Re \leq 2 \cdot 10^5\) |
|-------------------|------|----------------|----------------|----------------|
| \( C' \)         | 24   | 26.3         | 12.3         | 0.44          |
| \( n \)          | -1   | -0.8        | -0.5        | 0             |

At the driving of particles described by the law of resistance of Stokes speed of sedimentation it can be defined as:

\[
\vec{v}_s = \frac{\rho_p - \rho}{18 \mu} \vec{g} d^2 , \quad (5)
\]

In the transitional area the velocity of sedimentation will be defined as follows:

\[
\vec{v}_s = \left[ \frac{4}{3} (C')^{-1} (\rho_p - \rho) \right] \frac{\rho_p g}{\rho \mu^{1/3}} d_p^{2/3} \quad \text{for} \quad 1 < Re < 10 , \quad (6)
\]

\[
\vec{v}_s = \left[ \frac{4}{3} (C')^{-1} (\rho_p - \rho) \right] \frac{\rho_p g}{\rho \mu^{1/3}} d_p^{2/3} \quad \text{for} \quad 10 < Re < 800 . \quad (7)
\]

When driving particle the velocity of sedimentation will be equal in the Newtonian mode:

\[
\vec{v}_s = \sqrt{\frac{4}{3} \rho_p \frac{C_D}{\rho} g d_p} . \quad (8)
\]
Velocity of sedimentation can be considered as the scale characteristic characterizing displacement velocity of a particle in the resisting environment. Other such scale characteristics include time of a sedimentation $t_s$ and sedimentation length $l_s$. These scales can be defined as follows:

$$ t_s = \frac{\rho v_s}{\rho - \rho_p} g, \quad l_s = v_s t_s. $$ (9)

For the further analysis it is convenient to present equations of motion in the dimensionless look. For this purpose we will enter the dimensionless velocities, coordinates and time:

$$ \varphi = \frac{v_s}{v_r}, \quad \xi = \frac{x}{l_s}, \quad \tau = \frac{t}{t_s}. $$ (10)

Thus, particle driving within model of a drift will be described by the following dependences:

$$ \tau = \varphi = \varphi_0, \quad \xi = \xi_0 = \tau. $$ (11)

In need of the accounting of inertial properties of a particle it is necessary to solve differential equations of driving which in the dimensionless form are presented as follows:

$$ \frac{d\varphi}{d\tau} = -n \varphi^{1/n} + 1, \quad \frac{d\xi}{d\tau} = \varphi, $$ (12)

where $n$ - an exponent at a Reynolds number in resistance law.

Equations of motion of a particle (12) become isolated by the following starting conditions:

$$ \tau = 0: \varphi = \varphi_0, \quad \xi = 0. $$ (13)

The solution of a set of equations (12) with starting conditions (13) depends on resistance law. Let us consider at first driving of particles in a mode of resistance of Stokes: $n = -1$. Integration of the equations (12) with conditions (13) allows determining the velocity and particle movement for various instants. In the dimensionless look these dependences have an appearance:

$$ \varphi = 1 + (\varphi_0 - 1) e^{-\tau}, \quad \xi = \tau + (\varphi_0 - 1) [1 - e^{-\tau}]. $$ (14)

Movement of the particle moving in a mode of resistance of Stokes differs from the speed of a drift and the movement caused by a drift on the following values:

$$ \Delta \varphi = (\varphi_0 - 1) e^{-\tau}, \quad \Delta \xi = (\varphi_0 - 1) [1 - e^{-\tau}]. $$ (15)

Time for which the particle will move on the reference size of area of current $L$, within model of a drift will be defined as:

$$ \Delta \tau = \frac{L}{l_s} = \lambda = \frac{18}{Kn_p Re_s}, $$ (16)

where $Re_s = \rho v_s d_p / \mu$ - a sedimentation Reynolds number, $Kn_p = d_p / L$ – Knudsen number. Thus, the relative accuracy in the particle shift, given by model of a drift can be written down as:

$$ \frac{\Delta \xi}{\lambda} = \frac{Kn_p Re_s}{18} \left( \varphi_0 - 1 \right) \left[ 1 - e^{\left( \frac{18}{Kn_p Re_s} \right) \left( \varphi_0 - 1 \right)} \right]. $$ (17)

The model of a drift of particles can be applied if $Kn_p = \Delta \xi / \lambda << 1$.

Limits of applicability of a formula (17) are defined by limits of applicability of the Stokes law of resistance:

$$ Re_s \leq 1, \quad Re \varphi_0 \leq 1. $$ (18)

At the Newtonian mode of resistance change of speed and particle coordinate over time, and also the relative accuracy of model of a drift allows finding the solution of equations of motion of a particle:

$$ \varphi = \frac{(1 + \varphi_0) - (1 - \varphi_0) e^{2\tau}}{(1 + \varphi_0) + (1 - \varphi_0) e^{2\tau}}, $$ (19)
\[ \xi_{\tau} = \ln \left( \frac{1 - \phi_0 + (1 + \phi_0) \exp \left( \frac{3}{2} C_D Kn_p \right)}{2} \right) - 2\tau, \quad (20) \]

\[ \frac{\Delta_{\xi}}{\lambda} = \frac{4}{3} C^*_D Kn_1 \cdot \ln \left( \frac{1 - \phi_0 + (1 + \phi_0) \exp \left( \frac{3}{2} C_D Kn_p \right)}{2} \right) - 2. \quad (21) \]

Definition of the law of motion of a particle and definition of an error of model of a drift in the transitional area in an analytical form is, unfortunately, impossible. Therefore, for definition of characteristics of driving it is necessary to use numerical methods.

**Figure 1.** The relative error of the model drift \( Re < 1 \)

**Figure 2.** The relative error of the model drift \( 1 < Re < 10 \)

**Figure 3.** The relative error of the model drift \( 10 < Re < 800 \)

**Figure 4.** The relative error of the model drift \( 800 < Re \)

In Figures 1-4 lines of equal relative accuracies \( Kn_{\tau} = const \) for various values \( Re \) and \( \phi_0 \), and also various laws of resistance are presented. The analysis of these drawings allows to come to
conclusion that at reasonably chosen parameters of a grid ($Kn_p < 0.1$) usage of model of a drift of a particle at $Re < 1$ gives the error which is not exceeding 1%.

With increasing of a Reynolds number and initial velocity of a particle the relative accuracy increases in the transitional area. So, at $Re = 800$ with increase from $\varphi_0$ 2 to 5 $Kn$ increases from 1% up to 10%. At $Re > 800$ the size of the relative accuracy depends only on $\varphi_0$ size.

Conclusions
Theoretical analysis of the particle motion is carried out and the model of particle drift in the flow is proposed. When calculating disperse multi-phase flows with fine particle (for example dust), the relaxation time for the particles will be much smaller than the flow time scale. If disperse phase are at low volume fractions, it is unnecessary to solve momentum equations for the particle phase separately, as it can be assumed that the particles are always at their terminal velocity.

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