Encryption dynamics and avalanche parameter for “delayed dynamics”-based cryptosystems

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Abstract

The presented article attempts to characterize the encryption dynamics of delayed dynamics based block ciphers, designed for the encryption of binary data. For such encryption algorithms, the encryption process relies on a coupling dynamics with time delay between different bits in the plaintext (i.e. the “initial” message to be encrypted). Here, the principal dynamics of the encryption process is examined and the Hamming distance is used to quantify the change in ciphertext (i.e. the plaintext after encryption) upon changing a single bit in the plaintext message or slightly perturbing the key used during encryption. More precisely, the previously proposed “encryption via delayed dynamics” (in short: EDDy) algorithm as well as its extended version (termed ExEDDy) are analyzed by means of numerical simulations. As a result it is found that while EDDy exhibits a rather poor performance, ExEDDy performs considerably better and hence constitutes a significant improvement over EDDy. Consequently, the results are contrasted with those obtained for a block cipher that implements the encryption/decryption dynamics by means of reversible cellular automata.

Keywords: cryptography, avalanche parameter, delayed dynamics, reversible cellular automata

1. Introduction

Cryptography (Greek: kryptós = hidden/secret, gráphein = writing) is the science of keeping information secure, e.g. for the purpose of secret communication [1]. In recent years it was realized that elementary, computer-science or physics motivated systems that exhibit a complex behavior allow for the design of cryptosystems, i.e. encryption algorithms (also termed ciphers) and their decryption counterparts. Among those “applied science” encryption algorithms one can distinguish stream ciphers that encrypt a given message one bit (or symbol) at a time [2,3], and block ciphers that operate on blocks of, say, \( N \) bits [4,5,6].

The basic task in cryptography is two-fold: (i) the encryption process, wherein a plaintext message is scrambled using a certain cipher and key. The key specifies the precise way a given plaintext is transformed by means of the cipher. The result of the encryption procedure is a ciphertext message. (ii) the decryption process, where the ciphertext is transformed into the initial plaintext message by use of the corresponding decryption algorithm and a proper key. Note that, depending on the precise cryptography-mode, there is a subtle difference regarding the key: in symmetric-key cryptography, where one deals with a single-key cryptosystem, the same key is used during encryption/decryption. In public-key cryptography, signifying a two-key cryptosystem, the encryption key is public and the decryption key is kept secret. A statement on the security of a cryptographic method is provided by “Kerckhoffs’s principle”. It states that a cryptosystem should be secure if everything about the cryptographic method is known, except the key. Here, secure means that it is difficult or even impossible to obtain the plaintext from the ciphertext. In this regard, a characteristic desirable for any cipher is that a change in plaintext or key, as tiny as it may be, will result in a notable change in the ciphertext. More precise, changing one randomly chosen bit in either, plaintext or key, should invert nearly half of the bits of the ciphertext. This avalanche property was suggested by Feistel in 1973 [7].

Subsequently, only symmetric-key block ciphers designed to process binary plaintext messages will be considered. As regards this, the bulk of the presented article is dedicated to a scrambling scheme that relies on the paradigm of “encryption via delayed dynamics” (termed EDDy), see [5], as well as an extended variant thereof (signified as ExEDDy), see [4]. In Ref. [5] it was shown, how an exemplary initial state evolves with time and found that encoded states at different iteration times of the encryption dynamics are rather uncorrelated. Further, the effect of a slight modification of the key on the ciphertext was illustrated for one plaintext message and concluded that a “nearly” correct guess of the key does not lead to a sufficient overlap between decoded ciphertext and plaintext upon decryption so as to guess the initial message. As a weakness of the method it was mentioned that a change of multiple bits in the plaintext propagates to the ciphertext as a simple superposition of changes due to each changed bit. In Ref. [4] the encryption dynamics was modified to some extend, and it was observed that a minor change in key leads to rather uncorrelated ciphertext messages and that a change of multiple bits in plaintext does not propagate to the ciphertext as a simple superposition of changes due to each changed bit.
The presented article addresses the question of performance of the two ciphers using numerical simulations. Basically speaking, the encryption algorithms are applied to binary plaintexts of \( N = 4 \ldots 2048 \) bits in length and it is checked whether they exhibit the avalanche property. Albeit there are more strict criteria to quantify the security of ciphers, I will solely consider the avalanche property. As it appears, EDDy does not exhibit the avalanche property. Even worse, it also lacks completeness. Completeness requires that for every possible key, every ciphertext bit must depend on all plaintext bits, not only a proper subset of those. Further, I present clear evidence that ExEDDy outperforms EDDy. So as to put the arguments along the plot of the presented article on solid ground, a third, only considered and the results obtained therein are partly reproduced to contrast those for (Ex)EDDy. The encryption algorithms considered here all work in iterative manner. As regards this, Fig. 1 gives a qualitative account of the difference in the respective encryption dynamics.

Note that only few is known about the performance of reversible cellular automata based block ciphers and even less is known on the performance of delayed dynamics based ciphers such as EDDy and ExEDDy. To the best knowledge of the author, the presented article comprises a first attempt to quantify how well the delayed dynamics based ciphers EDDy and ExEDDy perform upon encrypting plaintexts of \( N = 4 \ldots 2018 \) bits in length.

The remainder of the presented article is organized as follows. In section 2 the considered cryptosystems are introduced and illustrated in more detail. Section 3 contains the numerical results obtained via simulations for plaintext of different size and section 4 concludes with a summary. A more elaborate summary of the presented article is available at the papercore database.

2. Encryption algorithms

Dynamical rules that describe the evolution of simple system often induce complex behavior with characteristics that are desirable for an encryption scheme. In this regard, the remainder of this section describes three cryptographic methods that rely on iterative dynamic procedures, exhibiting complex behavior, used to scramble a binary input message so as to hide its information content.

2.1. EDDy: encryption via delayed dynamics

Based on the paradigm of delayed dynamics, a block cipher for encryption of binary data, here termed EDDy (as in “encryption with delayed dynamics”), was proposed. EDDy is a symmetric-key cryptosystem that works in an iterative manner and as such it requires the following input: (i) a binary plaintext message \( m = s(0) \), consisting of a sequence of \( N \) bits \( s_i(0) \) (taking values \( \pm 1 \)), and (ii) a key \( K = (P, \tau, T) \), used during the encryption/decryption process. The component \( P \) signifies a permutation obtained from the sequence \( (0, 1, \ldots, N - 1) \), \( \tau \) represents a delay-time vector of length \( N \), where \( \tau_i \in [1, \tau_{\text{max}}] \) is an integer delay time associated with bit \( i \) and \( T \) denotes the number of iterations carried out by the encryption algorithm. Note that the key required by EDDy is rather large: if we aim to encrypt a \( N \)-bit plaintext message and agree to use an integral data type that requires \( s \) bits (bear in mind that the common C/C++ data type \( \text{int} \) has 32 bits). Now, from the point of view of an adversary that plans a cryptographic attack on EDDy, assume \( \tau_{\text{max}} \) is known. If the attack merely consists in guessing a key, then there are as much as \( (N!)^{\tau_{\text{max}}} \) possible choices for a pair of \( P \) and \( \tau \), as well as a guess for \( T \); left. Hence, the key space of EDDy is enormous. For comparison: the data encryption standard (DES) which is restricted to a blocksize of 64 bits employs a 56 bit key. For fixed \( N = 64 \) and \( \tau_{\text{max}} = 10 \), there are as much as \( (N!)^{\tau_{\text{max}}} \approx 2^{540} \) keys for the EDDy scheme.

Based on the ingredients above, the encryption dynamics read:

\[
\begin{align*}
    s_i(t) &= -s_{P_i}(\text{max}(0, t - \tau_i)) \\
    s_i(t) &= -s_{P_i}(\text{max}(0, t - \tau_i)) + \sum_{j \neq i} s_j(t) - s_{P_j}(\text{max}(0, t - \tau_j))
\end{align*}
\]

Therein, \( P_i \) and \( \tau_i \) are the \( i \)-th elements of \( P \) and \( \tau \), respectively. One iteration of the cipher consists in updating each bit once. The ciphertext \( c = s(T) \) is obtained when the exact number of iterations \( T \), predefined in the key, are carried out. Decryption is performed by using the same key \( K \) and applying the reverse dynamics \( s_i(t) = -s_{P_i}(\text{max}(0, t - \tau_i)) \), where \( P_i = k \). So as to facilitate a proper decryption, the sender needs to communicate a sequence of at least \( \tau_{\text{max}} + 1 \) successive encoded states, i.e. the “full” ciphertext \( c_f \), to the receiver. An exemplary encryption procedure for a plaintext of 8-bit in length is shown in Fig. 2a. In effect, the dynamics defined by the updating rule Eq. 1 induces a linear dynamics. The state of a particular bit, say bit \( i \), is related to
namics since an arbitrary bit at time step
As opposed to the EDDy scheme, this causes a nonlinear dy-
therein,
plex dynamics and might be interpreted as discrete structures
2.3. ReCA: encryption via reversible cellular automata
Figure 2: Illustration of the encryption process for a 8-bit plaintext message
| (a) EDDy cipher: m = 00101010 |
| (b) ExEDDy cipher: |
| (c) ReCA cipher: |

the state of exactly one other bit with integer identifier \( j = P_j \)
at some preceding time.

2.2. ExEDDy: extended variant of encryption via delayed dy-
namics

The extended variant of the EDDy cipher uses the same input information as above, i.e. a binary plaintext message \( m = s(0) \)
and a key \( K = (P, r, T) \). Using the same notation as above, the
ExEDDy cipher reads

\[ s_i(t + 1) = s_{p_i}(\max(0, t - \tau_i)) \cdot \theta \left( \sum_{i=0}^{N-1} s_i(t) \right). \]  (2)

Therein, \( \theta(\cdot) \) is a step function satisfying

\[ \theta(x) = \begin{cases} +1, & \text{if } x > 0, \\ -1, & \text{if } x \leq 0. \end{cases} \]   (3)

As opposed to the EDDy scheme, this causes a nonlinear dy-
namics since an arbitrary bit at time step \( t \) (explicitly) depends
on all \( N \) bits at time step \( t-1 \). Fig.2(b) illustrates an exemplary
encryption procedure for the ExEDDy cipher.

2.3. ReCA: encryption via reversible cellular automata

Cellular automata (CA) are abstract computing models that
work on a discrete space-time background. They exhibit com-
plex dynamics and might be interpreted as discrete structures
that approximate differential equations [11]. As such, CA high-
light the close connection between the realms of computer sci-
ence and physics [12 13 14]. Further, there is a possibility
to use CA as cryptosystems for the purpose of data encryption
[2 4].

Figure 3: List of the 8 possible configurations \( s_{i-1}(t), s_i(t), s_{i+1}(t) \) for a radius 1
CA, that might serve as input to the local update rule \( f \).
The configurations are interpreted as 3 digit binary numbers (bits with value 1 are shown as nonfilled
squares) and sorted in decreasing fashion from left to right. The 8 solitary cells below cell \( i \) indicate the state
\( s_i(t+1) = f[s_{i-1}(t), s_i(t), s_{i+1}(t + 1)] \). The
respective 8 bit sequence fully specifies the update rule. In Wolfram’s notation, the integer representation of the latter sequence is used as a name \( R \) for the rule
(here: \( R = 150 \), which represents the most simple finite difference variant of the
heat equation in modulo 2 arithmetic [11]).

The most basic CA [13] consist of a circular array \( s \) of, say,
\( N \) cells that can take values \( s_i \in \{0, 1\} \) for all \( i \in \{0, \ldots, N-1\} \).
The evolution of CA proceeds in discrete time steps \( t \), where
the values of the individual components \( s_i(t) \) are updated syn-
chronously according to a certain local update rule. So as to
evolve state \( s_i(t) \rightarrow s_i(t + 1) \), the local update rules \( f \) con-
sidered in the presented article take into account the state of
cell \( i \) as well as the states of those cells that are located within
a neighborhood of radius \( r = 2 \) enclosing \( i \) at time \( t \), hence
\( s_i(t + 1) = f[s_{i-2}(t), \ldots, s_{i+2}(t)] \) (note that the array of cells is
circular, i.e. cells 0 and \( N-1 \) are adjacent). Considering a neigh-
borhood of radius \( r \), there are an over all \( 2^{2r+1} \) differ-
ent input configurations to \( f \). Hence, the local rule for a one-
dimensional radius 2 CA is specified by a \( 2^5 = 32 \)-digit binary
number. In turn, there are \( 2^{25} \) different CA rules for the speci-
fied setup. A more simple example of a local rule for \( r = 1 \)
CA is shown in Fig.3. Therein, also the name convention for
CA rules is detailed. Note that in general, CA are not time re-
versal symmetric, a feature one would request in order to set up
a convenient dynamics for encryption as well as its reverse dy-
namics for decryption [6]. Nevertheless, such basic CA might
already be used as stream ciphers [2 14]. It is important to note
that there is a possibility to transform any (non-reversible) rule
into a time reversal invariant rule. For a CA with a binary state
alphabet and \( r = 2 \) neighborhood, the recipe to achieve this sim-
ply reads: \( s_i(t + 1) = (f[s_{i-2}(t), \ldots, s_{i+2}(t)] - s_i(t - 1)) \mod 2 \)
[11]. Note that the modified rule is second order in time, i.e. so
as to start the dynamics of the CA one has to provide a configu-
ration \( s(t=0) \) and its predecessor \( s(-1) \). The intriguing feature
of time reversal invariance is: a sequence of configurations can
be obtained in reverse order by means of the same rule \( f[\cdot] \),
by reversing the last two configurations. In effect, this is the core
idea of encryption via reversible CA (ReCA) [6].

In order to use ReCA for the purpose of data encryption,
one has to supply two ingredients: (i) the input data, where
\( s(0) = m \), and some random initial data (rid) taken as \( s(-1) \) so as
to be able to start the ReCA dynamics. (ii) a key that gov-
erns the encryption dynamics, given by an updating rule that
specifies the evolution of the state \( s(t) \rightarrow s(t + 1) \) and a num-
ber \( T \) of iteration steps, after which the ciphertext is obtained.
Note that for such a ReCA cipher, the full ciphertext consists of
two sequences: \( s(T) \) which is identified with the ciphertext
c, and the final encrypted data (fd) $s(T + 1)$. An exemplary encryption process using a ReCA algorithm is illustrated in Fig. 2(c). Decryption using ReCA is particularly simple: upon setting $s'(0) = s(T)$, $s'(-1) = s(T + 1)$, and using the same key as during encryption, it will hold that $s'(T) = m$. The configuration $s'(T + 1)$ contains the random initial data and as such, it is not of interest after decryption. However, it might be checked as an additional degree of freedom: upon using a particular key, a given plaintext message can be scrambled into numerous ciphertexts, depending on the random initial data. So as to realize secure communication using this approach, there are some subtleties concerning the communication of the final encrypted data, see Ref. [6]. However, the focus here is on the principal dynamics of the encryption schemes.

The subsequent section presents the results of the numerical simulations and attempts to give some more intuition on the principle dynamics on which the considered encryption algorithms are based.

3. Results

In the presented section, the encryption process, wherein a plaintext message $m$ is scrambled using a certain cipher $F$ (either EDDy, ExEDDy or ReCA) and key $K$, is put under scrutiny. The result of the encryption procedure is a ciphertext message $c$, and the process might be written as $c = F_K(m)$. In order to probe the avalanche criterion (the precise definition of the avalanche criterion is given below in Subsect. 3.2) for a given cipher $F$, a three step procedure is adequate: Obtain a random key $K$ (valid for $F$), then (i) generate a random binary plaintext message, $N$ bits in length, and obtain the corresponding ciphertext $c$, (ii) flip one bit, say bit $m_i$, of the plaintext message to obtain a perturbed plaintext $m'$ and obtain the corresponding ciphertext $c'$, (iii) compute the Hamming distance $d_H(c, c')$ between the two ciphertexts, defined as the number of differences between the components of $c$ and $c'$. For binary sequences $c$ and $c'$ of $N$ bit length and using Boolean notation, the Hamming distance may be written as

$$d_H(c, c') = \#_1(c \oplus c').$$

(4)

Therein $\oplus$ denotes the logical xor-operation, and $\#_1(c)$ signifies the number of occurrences of the digit 1 in the binary representation of $c$.

For a given plaintext of length $N$, the results are then averaged over different plaintext bits $m$ and keys $K$. In this regard, for $N \leq 10$ and for a particular key, it is feasible to consider the full input alphabet $2^N$ consisting of $2^N$ distinct plaintext messages. Here, a number of 200 different keys was taken into account in order to compute averages $(d_H(c, c'))_{m,k}$. For $N > 10$, tuple consisting of a plaintext and a key were sampled uniformly at random from among all possible choices. Further, the “equiprobable ensemble”, featuring binary messages with probability $p = 1/2$ for bits in state 1, was considered. In this case, a number of 400 plaintext/key pairs was used so as to compute averages.

In the remainder of the presented section, the dynamics of the encryption process is considered. In this respect, in the first subsection evidence is collected on whether plaintext and ciphertext blocks, obtained via the different encryption dynamics, are statistically independent. For this purpose, a $n$-block frequency test (note that the frequency test carried out above resembles the monobit frequency test suggested by NIST [15] to check the randomness of binary sequences. Here, instead of monobits, bytes are considered.) is employed, a compression test is performed, and a measure of (structural) complexity for sequences of encoded states is considered. The respective subsection is closed by a simple graphical correlation analysis regarding successive encoded states. The subsequent subsection explains and quantifies the avalanche parameter for the considered ciphers. Therefore, the evolution of the plaintext is considered and the stationary behavior of the avalanche parameter is examined. The presented section concludes with a note on completeness for the different encryption schemes.

3.1. Dynamics of the encryption processes for the considered ciphers

A given cryptosystem provides perfect secrecy, if the plaintext and ciphertext blocks processed by the cipher are statistically independent [1]. Here, the principle dynamics of the encryption process is considered by monitoring the evolution of plaintext messages during the encryption process. Therein, the aim of the presented subsection is to employ various tests on the three ciphers introduced above, to check if the sequence of encoded configurations exhibits obvious statistical regularities. To perform the necessary tests, a large number of up to $T = 10^4$ iteration steps are used with the respective ciphers. Albeit this might be infeasible for any practical cryptographic application, it is nevertheless useful as far as the principle dynamics of the algorithms is considered. Further, so as to contrast the results on the three ciphers above, a 1d CA with $r = 1$ neighborhood that implements Wolfram rule 30 (denoted as R30) is considered. R30 generates effectively random bit sequences and might be used as a stream cipher for cryptographic purposes, see Refs. [2][14]. As such, it has already passed a series of statistical tests [16] and is solely considered for the purpose of comparison.

$n$-block frequency test for configurations during encryption. At first, the statistics for a sequence $s(t)$ of encrypted binary configurations, obtained during the encryption process, is put under scrutiny. The basic question is whether the resulting statistics is compatible with that for sequences of effectively random configurations. A proper tool to check this is a $n$-block frequency test. This test can be cast into the following three step procedure: (i) slice the $N$ bit input configuration $s(i)$ for a given step $t$ into a number of $M = [N/n]$ $n$-bit blocks $b_i \in \mathbb{M}_n$, and discard all remaining bits. Then, follow the “evolution” of the input configuration up to $t_0 + \Delta t$ iteration steps, where the configurations obtained during the first $t_0 = N$ iteration steps are skipped (the value of $t_0$ is picked rather arbitrarily, it is meant to eliminate possible effects of the initial “short-time” dynamics). Represent each $n$-bit block $b_i$ by means of its integer decimal value \text{Id}[b_i] \in \{0, \ldots, 2^n - 1\}$, and accumulate the resulting integers in a histogram $b_i$ having $2^n$ bins with id $i \in \{0, \ldots, 2^n - 1\}$. The bins then contain the observed frequencies $h_i$ associated to the
random sequences, it is also of interest to look for structural regularities. As regards this, here, an oversimplified compression test using the convenient Unix-tool gzip is performed. As a prerequisite for the test, for each of the three encryption algorithms and the R30 dynamics, a number of 100 plaintext messages (128 bits in length) is evolved for $10^3$ iteration steps. Again, R30 is considered for comparison only. The evolution of each plaintext is stored in a single file. Prior to compression, the individual files have a size of, say, $B_0$ bits. The gzip data-compression tool uses the Lempel-Ziv (LZ77) algorithm \cite{18}, so as to reduce the size of the supplied data. Loosely speaking, LZ77 works by finding sequences of data that are repeated and exploits those “patterns” to perform the compression process. After compression via gzip, the size of a given file is reduced to $B_{\text{gzip}}$. Then, the compression factor $\kappa = B_{\text{gzip}}/B_0$ for each file is computed and its mean and standard error are listed in Tab.\ref{table:results}. As it appears, $\langle \kappa \rangle$ is approximately equal for the ExEDDY, ReCA and R30 data, while it assumes a slightly smaller value for the EDDY data. This indicates that, compared to the other encryption schemes, EDDY exhibits a larger degree of structural regularity.

An oversimplified compression test for the sequences of encoded states. Besides checking whether the sequence $s(t)$ of encrypted configurations exhibits the same statistics as truly

integer numbers $i$. For effectively random sequences one would expect to find each integer id with probability $2^{-n}$, hence the expected frequencies are simply given by $e_i = N_{\text{samp}} \times 2^{-n}$ wherein $N_{\text{samp}} = M \Delta t$. Now, if the encrypted configurations exhibit the statistics of random binary sequences, the histogram $[h_i]$ should be “flat” with each bin having the same expected frequency. Consequently the null hypothesis reads: “The sample follows a uniform distribution”. (ii) Probe the $\chi^2$ statistics, and, so as to check the null-hypothesis compute the associated $p$-value, see Ref.\cite{17}. (iii) Finally, reject the hypothesis if $p \leq 0.01$. I.e., if $p > 0.01$, the sample follows the assumption that the histogram is flat. The $\chi^2$-test described in steps (ii) and (iii) above was carried out for $N = 128$-bit input sequences regarding four different dynamics given by the EDDY, ExEDDY, and ReCA cipher, as well as the R30 dynamics. In the analysis, $\Delta t = 10^3$ and blocks of $n = 8$ bits in length, i.e. bytes, were considered. The results of the analysis are listed in Tab.\ref{table:results} and probability density functions (pdfs) for the ratio of the observed/expected frequencies related to the four different dynamics are shown in Fig.\ref{fig:results}. To summarize the findings: while the configurations obtained using the ExEDDY, ReCA, and R30 dynamics exhibit statistics compatible with those of random sequences, the computed $p$-values clearly suggest that EDDY gives rise to seemingly non-random sequences of configurations during encryption.

### Table 1: From left to right: Type of dynamics, $\chi^2/\text{ dof}$ and $p$-values, resulting from the $n$-block frequency analysis, compression factor $\kappa$ obtained using a simple data compression test, and minimal number $N_{\text{ps}}$ of pair substitution steps needed to transform a sequence of bit states into a constant sequence. For the latter quantity, the values in the braces list the standard deviation.

|                  | $\chi^2/\text{dof}$ | $p_{n=8}$ | $\kappa$  | $N_{\text{ps}}$ |
|------------------|---------------------|-----------|-----------|-----------------|
| EDDY             | (6.24, 10^{-397})   | 0.1378(4) | 74(35)    |                 |
| ExEDDY           | (0.87, 0.93)        | 0.1714(7) | 218(34)   |                 |
| ReCA             | (0.99, 0.53)        | 0.17365(9) | 217(13)   |                 |
| R30              | (0.87, 0.94)        | 0.17378(2) | 225(4)    |                 |
only. Such a sequence has zero information entropy

$$H(x) = -\sum_{i=0}^{N-1} n_i \log_2 \left( \frac{n_i}{N} \right), \quad (5)$$

wherein $n_i$ specifies the observed frequency of symbol $a_i$ in the sequence $x$ of length $N_i$, and is called a \textit{constant} sequence. Here, as a measure of algorithmic randomness associated to the initial sequence $x_0$, the minimal number of pair substitution steps $N_{ps}$, needed to transform $x_0$ to a constant sequence with $H(x_{N_{ps}}) = 0$, is considered. In order to facilitate intuition, consider the 6 bit sequence $x = 110010$. An iterative application of the pair substitution step on $x$ yields the result:

| step $i$ | $x_i$    | $H(x_i)$ | $e_{mfp}$ | $a_q$ |
|---------|---------|----------|-----------|-------|
| 0       | 110010  | 1.0      | 10        | 2     |
| 1       | 1202    | 1.5      | 02        | 3     |
| 2       | 123     | 1.58     | 23        | 4     |
| 3       | 14      | 1.0      | 14        | 5     |
| 4       | 5       | 0.0      |           |       |

I.e., after $N_{ps} = 4$ steps, the NSRPS algorithm terminates for the constant sequence $x_4 = 5$. For the slightly different sequence $y = 101010$ (also with initial entropy $H(y_0) = 1$), the NSRPS algorithm terminates after a single pair substitution step, where the constant sequence reads $y_1 = 222$. One may now conclude that sequence $x$ exhibits a higher degree of randomness than sequence $y$, since the NSRPS algorithm requires more elementary pair substitution steps in order to arrive at a constant sequence. This is in accord with intuition, since, in contrast to sequence $x$, $y$ shows a regular pattern.

In Fig. 5 results for a repeated application of the pair substitution process on 128 individual bit strings, each of which is $10^3$ bits in length, are shown. The bit strings are obtained as follows: a 128 bit plaintext is prepared and subsequently encrypted using $10^3$ encryption steps. Each individual bit of the plaintext is monitored during the encryption process so as to yield a bit sequence that is $10^3$ bits in length. More precise, the figure illustrates the evolution of the parameter $C_i = L_i \times H(x_i)$ for each of the 128 individual bit strings (each of which initially is $10^3$ symbols in length), wherein $L_i$ signifies the length of the sequence at iteration step $i$. Results are shown for the EDDy and ExEDDy ciphers, only. Upon execution of the NSRPS algorithm, the parameter value $C_i = 0$ might be used as a stopping criterion to detect whether a constant sequence is obtained \cite{21}. Here, the application of the NSRPS algorithm to each bit string sheds some light on the “internal” dynamics of the encryption schemes. Consider, e.g., the EDDy cipher: as evident from Fig. 5 there are some bit strings that transform to constant ones almost immediately, while there are others that last for quite a large number of iteration steps. The reason for this has an intuitive appeal: during an encryption process using EDDy, the dynamics of a given bit is only coupled to those bits that are located along the same cycle in the cycle decomposition of the index-permutation $P$ that forms part of the key (see discussion below). Hence, the encryption dynamics of EDDy for a plaintext of a given size can be decomposed into independent dynamics taking place on the set of cycles in the cycle-decomposition of $P$. Depending on the particular delay-times that characterize the bits in such a cycle, the respective dynamics might turn out to be rather regular. This can most easily be seen for a fixed point $p_i = i$ of the permutation $P$: once the dynamics $s_i(t) = -s_i(\max(0, t - \tau_i))$ is kicked off, the state of the respective bit changes with period $\tau_i$. For the simple case
where \( \tau_i = 1 \), the sequence of the state of that particular bit reads just 01010101 ... (given that \( s(0) = 0 \)), and the NSRPS algorithm will terminate after a single step. Similarly, a delay time \( \tau_i = 2 \) might yield the sequence 00110011 ... keeping the NSRPS algorithm busy for 3 steps. Somewhat “less regular” patterns are found for cycles that are larger in length. Consequently, a widespread distribution of the pair substitution steps \( N_p \), needed to obtain a constant sequence, is observed. The average number of such pair substitution steps, taking into account 200 different 128 bit plaintexts, that where evolved for \( 10^3 \) iteration steps each, is listed in Tab. 1. The results for the ExEDDy cipher are completely different: the fates of the individual bit sequences upon application of the NSRPS algorithm appear to be less diverse, see Fig. 5. In comparison to the results obtained for EDDy, this is manifested by a smaller standard deviation associated to \( \langle N_p \rangle \), as listed in Tab. 1. The results for the ReCA cryptosystem and R30 dynamics are listed in Tab. 1 also. They look similar to those obtained for ExEDDY and are thus not shown.

**Graphical analysis of correlations between successive encoded states.** One may further wonder whether the sequence \( s(t) \) obtained during encryption exhibits correlations with respect to, e.g., subsequent time steps. In order to check this, it is most simple to label a configuration at a given time step \( t \) by means of its integer decimal representation \( \text{Id}[s(t)] \) and to plot \( \text{Id}[s(t + \Delta t)] \) as function of \( \text{Id}[s(t)] \). Such a graphical correlation analysis for the evolution of one random 16 bit plaintext message is illustrated in Fig 6(a) and (b), following the encryption dynamics of the EDDy and ExEDDy ciphers, respectively. Therein, the correlation between subsequent configurations, i.e. \( \Delta t = 1 \), was considered. As evident from the figure, EDDY exhibits a statistically rather regular behavior, which might be attributed to its linear dynamics, whereas ExEDDY and ReCA exhibit no obvious regularities for \( \Delta t = 1 \) (the results for the ReCA dynamics look similar to those of ExEDDY and are not shown). Further, the regular pattern generated by EDDY suggests a rather short cycle length for its dynamics.

### 3.2. Quantifying the avalanche parameter for the considered ciphers

The avalanche criterion would require to check whether \( \langle d_H(c, c^\prime) \rangle_{m,K} \approx 1/2 \) for each \( i \in \{0, \ldots, N-1\} \). Here, the bits are thought of as equivalent entities, hence, in the remainder of the presented section, only one flip for each plaintext is attempted. This means, step (ii) above yields a modified plaintext \( m^\prime \) (where the flipped bit is chosen uniformly at random), for which \( c^\prime = F_K(m^\prime) \) and it is only checked whether \( \langle d_H(c, c^\prime) \rangle_{m,K} \) agrees with the desired value 1/2 within errorbars. If the latter requirement is satisfied, the respective cipher is said to exhibit the avalanche property. Subsequently, the Hammingdistance, averaged over different plaintext messages \( m \) and keys \( K \), i.e. \( \langle d_H(c, c^\prime) \rangle_{m,K} \), is referred to as avalanche parameter.

**Evolution of the Hammingdistance upon modification of the plaintext.** Regarding the EDDY cipher, the Hammingdistance \( \langle d_H(c, c^\prime) \rangle_{m,K} \), interpreted as a function of the iteration steps \( t \) carried out by the encryption dynamics, fluctuates around a value of \( N^{-1} \) for all considered plaintext lengths \( N \). Fig. 7 illustrates the results at \( N = 32 \), where the distribution of the distance-values \( \langle d_H(c, c^\prime) \rangle_{m,K} \), collected over an appropriate number of iteration steps, follows a Gaussian distribution with mean value \( \mu(d_H) \approx 0.031 \) and width \( \sigma(d_H) \approx 0.001 \) (not shown). The argument for such a small mean value is quite plausible: Consider, e.g., a 6 bit plaintext, encrypted by using a key containing the index-permutation \( P = (3, 4, 2, 0, 5, 1) \). The respective cycle decomposition reads \((2)(3, 0)(4, 5, 1)\). As a result, the ciphertext-bit \( c_5 \) is only affected by a finite number of plaintext-bits, namely bits \( m_4 \) and \( m_1 \). Now, if the value of a single bit in the plaintext is changed, say \( m_5 = -m_5 \), the difference in the evolution of the altered plaintext will be confined to the cycle \((3, 4, 1)\) only. This is illustrated in Fig. 8(a), where the difference between two configurations \( c \) and \( c^\prime \), i.e. \( c \oplus c^\prime \), during an encryption process using EDDY is monitored. Therein, the encryption processes was started at two plaintexts that differed only in the value of the center bit. In the analysis, if the flipped bit is chosen uniformly at random, the length of the corresponding cycle is best fit by the expression \( \ell_c = 0.49(1)(N - 0.9(2))^{1.003(1)} \approx N/2 \) (where the probability mass function for cycles with length \( \ell \) behaves as \( n(\ell) \approx \ell^{-1} \), measured for permutations \( P \) of length \( N = 128 \), not shown). Now, the length \( \ell_c \) of a particular cycle sets an upper bound for the Hammingdistance, giving rise to the restriction \( \langle d_H(c, c^\prime) \rangle_{m,K} \approx 1/2 \). Apparently, since the difference \( c \oplus c^\prime \) typically stems from only a small number of bits contained in the respective cycle (see Fig. 8(a)), it appears that on average \( \langle d_H(c, c^\prime) \rangle_{m,K} \approx N^{-1} \) for all \( N \) considered (see Fig. 9 and discussion below).

As evident from Fig. 7 the behavior of the ExEDDY cipher is
the perturbed time steps a change of one bit in plaintext leads to a distance where for a finding compares well to the observation reported in Ref. [6], the result dynamics for further plaintext lengths qualitatively similar results (not shown). Hence, after fit using a functional form during the initial stage of the encryption dynamics. Indeed, a between two ciphertexts Figure 8: Exemplary plots that illustrate the evolution of the di

dynamics one can expect an approximate linear increase of configurations that initially di

ference between two ciphertexts Figure 8: Exemplary plots that illustrate the evolution of the di

dynamics one can expect an approximate linear increase of configurations that initially di

ference (cf. rules 30 and 60 for elementary cellular automata [22]). To

ning rule implemented by the CA and might also be anisotropic upper bound, the actual velocity depends on the precise updat-

ing time scales by means of which a change is transmit-

ed only in the value of the centered bit. Nonfilled squares signify

ered only in the value of the center bit. Considering

ared plaintext bit-lengths

the focus is on the EDDy ExEDDy ciphers only.

As regards (i), the avalanche parameter for the EDDy cipher (averaged over the iteration steps $t$) shows the clear scaling behavior \[ |(d_{\text{H}}(c,c'))_{m,K}| \propto N^{-1} \] for the full range of the considered plaintext bit-lengths $N$, see Fig. 9. This is in accord with the observation above, where the evolution of the Hammingdistance during an encryption process was considered. The behavior of the ExEDDy cipher is strikingly different: the value of the avalanche parameter, averaged over iteration steps $t \geq 200$, is only slightly smaller than 1/2, see Fig. 9. E.g., the numerical value \[ |(d_{\text{H}}(c,c'))_{m,K}| = 0.482(5) \] attained at $N = 32$ is only 3.5 standard deviations below the desired value. Given the apparent similarity of the two encryption schemes, this is a considerable improvement over EDDy. The ReCA based cipher appears to yield the best results: while there are slight deviations from the desired value for small plaintext sizes $N < 9$ (e.g., at $N = 8$ the numerical value \[ |(d_{\text{H}}(c,c'))_{m,K}| = 0.496(2) \] of the avalanche parameter is 2 standard deviations below 1/2 the value of the avalanche parameter is compatible with $1/2$ for all values $N = \ldots$ of 19 (38) iterations of the encryption dynamics was recommend.

Stationary behavior of the avalanche parameter. So as to quantify the stationary behavior of the avalanche parameter, it is useful to distinguish the kind of modification imposed on the encryption process. In this regard, the presented analysis distinguishes between (i) a modification of the plaintext, where a single bit of the plaintext is changed (the bit is chosen uniformly at random), and (ii) a modification of the key. In the latter case, the focus is on the EDDy/ExEDDy ciphers only.

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are about 2 (averaged over the data for one randomly chosen delay-time yields an avalanche parameter

... swapping two random entries of the index-permutation, the only slightly below 1

... state. The solid horizontal line indicates a value of $\langle d_{\text{ff}}(c,c') \rangle_{m,K} = 1/2$.

$9 \ldots 64$ (e.g., at $N = 64$ one finds $\langle d_{\text{ff}}(c,c') \rangle_{m,K} = 0.498(3)$).

The results for the ReCA cipher where averaged over iteration steps $t \geq 350$.

Considering a modification of the key and focusing on the ExEDDy and EDDy cryptosystems, a further distinction is drawn between altering the permutation $P$ and the delay-time vector $\tau$. Regarding the former modification, two randomly chosen entries are swapped, while in the latter case a single bit is chosen uniformly at random and the associated delay time is redrawn from the interval $[1, \tau_{\text{max}}]$. In both cases, the results for the EDDy cipher are best fit by powerlaw functions of the form $\langle d_{\text{ff}}(c,c') \rangle_{m,K} \propto (N + \Delta N)^a$, where $\Delta N \approx 265$ and $a = -1.29(6)$ for a change in $P$, see Fig. [10] and $\Delta N \approx 65$ and $a = -1.09(3)$ for a change in $\tau$, see inset of Fig. [10]. Consequently, in the limit of large $N$, the effect of a minimal change in the plaintext on the ciphertext is negligible. The precise numerical values of $\langle d_{\text{ff}}(c,c') \rangle_{m,K}$ in the range $N = 24 \ldots 2048$ are about $\pm 2 - 5$ standard deviations below $1/2$.

For ExEDDy, however, results are completely different: upon swapping two random entries of the index-permutation, the avalanche parameter (averaged over iteration steps $t \geq 200$) is only slightly below $1/2$, see Figs. [10]. As regards this, the numerical values of $\langle d_{\text{ff}}(c,c') \rangle_{m,K}$ in the range $N = 24 \ldots 2048$ are about $\pm 2 - 5$ standard deviations below $1/2$. Further, changing one randomly chosen delay-time yields an avalanche parameter (averaged over the data for $t \geq 200$) clearly smaller than $1/2$, see inset of Fig. [10]. In conclusion, a modification of plaintext or index permutation leads to quite similar numerical values for the associated avalanche parameters. In comparison, a modification of a randomly chosen delay time results in somewhat smaller values of the avalanche parameter.

3.3. A note on completeness

As pointed out above, the encryption dynamics of EDDy for a plaintext of a given size can be decomposed into independent dynamics taking place on the set of cycles in the cycle-decomposition of $P$. As a result, the dynamics of a given bit is only coupled to those bits that are located in the same cycle of the cycle decomposition. This means that typically, a given ciphertext bit depends on a few plaintext bits only. Thus, the linear dynamics provided by EDDy lacks completeness (as explained in the introduction). The situation is different for ExEDDy: since each bit depends on the sum of states of all bits in the previous time step, completeness is trivially satisfied. Also, if the dynamics of the ReCA encryption algorithm is iterated for $t_{\text{min}} \approx N/2$ time-steps, each bit in ciphertext might be affected by a given bit in the plaintext message. Note that this argument was established from a statistical point of view (as discussed above), the precise value of $t_{\text{min}}$ depends on the particular key used during the encryption process. Further, bear in mind that, as discussed above, a lower bound on the respective time scale for a $r = 2$ CA is given by $t = N/4$. Hence, ReCA exhibits completeness after a certain (key dependent) minimal number of time-steps.

4. Conclusions

In the presented article, the encryption process of three different cryptosystems for block-encryption, i.e. EDDy, ExEDDy (an extended variant of EDDy) and ReCA, is put under scrutiny. While EDDy and ExEDDy are based on the paradigm of delayed dynamics, ReCA relies on the updating rules provided by reversible cellular automata. Starting from an initial plaintext message, all three cryptographic schemes iteratively process the plaintext, yielding a sequence of encoded configurations until, finally, the ciphertext is obtained. Upon analysis of the statistical properties related to the sequence of encoded configurations, it was found that the EDDy cipher gives rise to a statistically rather regular behavior: the sequence of encoded configurations does not exhibit the properties expected for effectively random sequences. For the purpose of comparison, effectively random binary sequences produced by a certain 1d CA, referred to as R30, were considered. Further, EDDy does not satisfy the avalanche criterion (neither for a modification of plaintext nor a modification of the key) and it lacks completeness. In contrast to this, the ExEDDy cipher clearly outperforms EDDy in all test considered. The statistical properties of the sequence of encoded configurations are in good agreement with those of the ReCA cipher and the R30 dynamics. ExEDDy does not strictly satisfy the avalanche criterion but performs reasonably well. Further, it features completeness. Given the apparent similarity of the algorithmic procedures that underlie
both delayed-dynamics based ciphers, ExEDDy constitutes a striking improvement over EDDY. Finally, the statistical properties of the sequences of encoded configurations that are obtained during an encryption process using the ReCA cipher are in excellent agreement with those obtained for the R30 dynamics. From the analysis of plaintexts with bit-size \( N = 4 \ldots 64 \) it is obvious that ReCA satisfies the avalanche criterion (in support of Ref. [2]), where 32 and 64-bit plaintexts are considered only). Further, the ReCA cipher shows completeness after a key-dependent minimal number of iteration steps.

Another issue of interest from a point of view of application is the size of the full ciphertext, relative to the plaintext. As regards this, the three cryptosystems considered here lead to data expansion. As pointed out in subsect. 2.1 the key space of the delayed dynamics based cryptosystems increases with the maximal delay time as \( \tau = (\tau_{\text{max}} + 1) \times N \) and hence depends on the parameter \( \tau_{\text{max}} \) as part of the key. In contrast to the delayed dynamics ciphers, the size of the full ciphertext for the ReCA cryptosystem is always \( 2 \times N \). To the knowledge of the author, the cryptographic schemes discussed in the presented article are of academic interest. The respective encryption dynamics are governed by rather basic model dynamics that, without doubt, are interesting in their own right since they give rise to a very complex behavior and eventually exhibit properties that can be exploited so as to set up a working encryption algorithm. However, so as to implement a full cryptographic method that allows for encryption and decryption of data, a rather large key is necessary and the ciphertext may turn out to be considerably larger in size than the plaintext (also known as data expansion). These effects somehow limit the practicability of the discussed cryptographic schemes.

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