Cross-talk minimising stabilizers

Kaila C. S. Hall and Daniel K. L. Oi

SUPA Department of Physics, University of Strathclyde, Glasgow, G4 0NG, United Kingdom

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Verification and characterisation of quantum states are crucial for the implementation of quantum information processing, especially for many-body systems such as cluster states in optical lattices. In theory, it is simple to estimate the distance of a state with a target cluster state by measurement of a set of suitable stabilizer operators. However, experimental non-idealities can lead to complications, in particular cross-talk in single site addressing and measurement. By making a suitable choice of stabilizer operator sets we may be able to reduce, but not eliminate, these cross-talk errors. The degree of cross-talk mitigation depends on the geometry of the cluster state and subsets of cross-talk free stabilizers can be generated for certain shapes using a simple algorithm.

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I. INTRODUCTION

Cluster states, highly entangled many-body systems of qubits, are a resource for quantum information processing through a sequence of local measurements and feedback [1][2] and their generation is a highly active area of research [3][4]. A leading candidate for measurement-based quantum computation is an array of atoms trapped by an optical lattice and recent experiments have demonstrated individual qubit addressing [10].

In addition to the experimental challenge of generating cluster states, it is especially difficult to verify and characterise such a many-body quantum system [11][12] where quantum tomography [13] is infeasible. More efficient ways of determining key characteristics are required, for example entanglement can be detected using an entanglement witness [14]. It is also important to know how close the actual state produced is to the desired one. For cluster states, theoretically it is a simple matter to compare the expectation values of the set of stabilizer operators that describe the desired cluster state with the measured results [1][2][15].

Measuring such quantities may not be straightforward however. Typically, the spacing of atoms in an optical lattice is of the same order as the wavelength of light used to individually address them which may lead to cross-talk [10]. Such cross-talk can cause errors in the measured operators hence reducing the accuracy of the distance estimation. Composite or compensation pulse sequences, as carried out in NMR could be used to correct for systematic errors, but it may be desirable to avoid the complexity and overhead this introduces [16].

Instead of using the standard (or canonical) set of stabilizers to characterize a cluster state, we instead seek to find alternative descriptions that can reduce the problem of cross-talk. We achieve this by finding some stabilizers that eliminate the need to perform local addressing as well as others that minimize the number of neighbouring measurements in different bases. We find such cross-talk minimizing sets for a range of different cluster state geometries, in particular square and triangular lattices.

A. Cluster States

A cluster state is a many body quantum system defined as the simultaneous +1 eigenstate of a set $\mathcal{S}$ of commuting stabilizer operators $\hat{S}^a$ [1],

$$\hat{S}^a|\psi\rangle = +|\psi\rangle, \forall \hat{S}^a \in \mathcal{S}. \tag{1}$$

For $n$ qubits, we require only $n$ linearly independent stabilizers to uniquely define a state, an exponential reduction compared to the description of an arbitrary pure quantum state. The standard cluster state description uses stabilizer operators (entangling observables) $\hat{S}^a$ on a regular lattice of the form,

$$\hat{S}^a = X^a \bigotimes_{N(a)} Z^b, \tag{2}$$

where a Pauli $X$ operator acts on qubit $a$ and $Z$ acts on the set $b$ of neighbouring qubits to $a$, i.e. those sharing an edge with $a$ in the associated graph [17]. We shall be primarily interested in square or triangular lattices, reflecting the cluster states easily created in optical lattices. The set of stabilizer operators describing a cluster state is not unique and we exploit this in order to generate stabilizer operators with reduced cross-talk.

B. Fidelity of Cluster States

Experimentally, determining the closeness of the actual state to the desired state is an important issue. One measure of closeness between the ideal pure state $|\psi\rangle$ and the actual (mixed) state, $\rho$, is the fidelity that is defined as

$$F = \sqrt{\langle\psi|\rho|\psi\rangle} \tag{3}$$

Note some author’s define the fidelity as $F' = F^2$ [18].

* kaila.hall@strath.ac.uk

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Reconstructing $\rho$ through full quantum state tomography [13] and then calculating $F$ using Eq. (3) becomes infeasible with more than a few qubits due to the exponential number of parameters. However a lower bound on $F$ can be achieved by using certain measurements that are linear in $n$ [15]. By defining the following operator

$$\hat{S} = \frac{1}{2} \left[ \left( \sum_{a=1}^{n} \hat{S}^a \right) - (n - 2) \mathbb{1} \right],$$

a lower bound of fidelity is given by the expectation of this operator with $\rho$

$$F^2(\rho, \psi) \geq \langle \hat{S} \rangle_{\rho}. \quad (5)$$

Hence we only require $n$ expectation values of $\{\hat{S}^a\}$ in order to estimate the right hand side of Eq. (5).

C. Measurement and Cross-Talk

The above method requires that the expectation values of the stabilizer operators be determined. One could try to measure the stabilizer operator observables directly but this is an entangled measurement and difficult to perform in practice. Alternatively, one can synthesize the measurement value from separate measurements of the Pauli operator on each qubit and multiplying the results gained during each instance of the experiment. Averaging over many runs gives an estimate of the expectation value. To obtain the expectation values for all the standard stabilizer operators (Eq. 2) for a cluster state on a square lattice, it is sufficient to perform two measurement patterns as indicated in Fig. 1.

Experimentally, such patterns may be problematic. In the case of optical lattices the natural qubit measurement basis is $Z$, other measurements directions are produced by unitary rotations before a $Z$ measurement. These rotations can either be applied simultaneously to all qubits, or individually with the aid of an addressing laser beam. However, the beams have waist sizes on the order of the lattice spacing, hence neighbouring qubits may pick up unwanted evolutions. This can in principle be ameliorated by the use of composite pulses [16] but this adds additional complexity and is undesirable. A simple mitigation would be find measurements that would reduce or eliminate the degree of crosstalk. This can be achieved by exploiting the non-uniqueness of stabilizer sets describing a given cluster state.

II. STABILIZER OPERATOR SETS

A. Equivalent sets of Stabilizer Operators

To uniquely define a cluster state we need only specify $n$ linearly independent stabilizer operators, this choice of $n$ operators is not unique. We use this fact to our advantage in creating an equivalent set of stabilizer operators with a reduced amount of cross-talk. Two sets of stabilizers specify the same state $|\psi\rangle$ if $\hat{S}^a$ is related to $\hat{S}'^a$ by a non-singular binary matrix $m = (m_{jk})$ with $m_{jk} = 0$ or 1,

$$\hat{S}' = \prod_{k=1}^{n} (\hat{S}^k)^{m_{jk}}, \quad (6)$$

this construction of the new equivalent set also allows for reconstruction of the canonical set $S$ found using Eq. (2).

Example:

Fig. 2 shows a $3 \times 2$ cluster state with the following
that this does not have to be a parameter operator requiring only one type of operator. Note that this measurement with no cross-talk we could globally rotate all the qubits in the lattice by $\frac{\pi}{2}$ so the $X$ operators can be measured and locally address the $Z$ operators to rotate them back, the locally addressed qubits are far enough away from each other that this will not cause any cross-talk errors. (b) is an example of a homogeneous CTF as it only contains one type of operator this can be achieved simply by globally rotating all the qubits in the lattice.

An equivalent set of stabilizer operators for this cluster state can be specified by the non-singular matrix $m$

$$m = \begin{pmatrix}
1 & 1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 1
\end{pmatrix}$$

which corresponds to the new set

$$s_{12} = X_1Z_1Z_2X_2Z_3Z_4Z_5Z_6$$

$$s_{34} = Z_1Z_2X_3Z_4Z_5Z_6$$

$$s_{56} = X_1Z_1Z_2Z_3Z_4Z_5Z_6$$

$$s_6 = 1_{12}1_{3}1_{4}1_{5}1_{6}$$

this set still stabilizes the cluster state shown in Fig. 2 conversely the canonical set can be specified from these operators.

### B. Construction of cross-talk-free stabilizer operators

The cross-talk in the stabilizer operators comes from having to measure two different operators, namely $X$ and $Z$, on adjacent qubits. To counter this we have two options: either we split up the $X$ and $Z$ operators so they are no longer applied to adjacent qubits, or we find stabilizer operators requiring only one type of operator. Note that this does not have to be a $Z$ operator as we can globally rotate all the qubits in the lattice without cross-talk. There are potentially various kinds of cross-talk free (CTF) stabilizer operators (Fig. 3) each with different properties.

We can immediately rule out the first idea of separating the $X$ and $Z$ operators by examining how the stabilizer operators are constructed from Eq. (2). If we consider a one dimensional line of $n$ qubits and we would like to have the $X$ operator as far away as possible from the $Z$ operator so the stabilizer operator would look like Eq. (10)

$$X_1\mathbb{I}_2\mathbb{I}_3\cdots\mathbb{I}_{n-2}\mathbb{I}_{n-1}Z_n,$$

To create this pattern we follow the steps shown

**Step 1** $X_1Z_2\mathbb{I}_3\cdots\mathbb{I}_n$

**Step 2** $X_1\mathbb{I}_2X_3Z_4\mathbb{I}_5\cdots\mathbb{I}_n$

$$\vdots$$

**Step n_{even}** $X_1\mathbb{I}_2\mathbb{I}_3\cdots\mathbb{I}_{n-2}X_{n-1}Z_n$

**Step n_{odd}** $X_1\mathbb{I}_1X_3\cdots\mathbb{I}_{n-1}X_n$

When $n$ is odd there is no possible way to eliminate the $Z$ operator on qubit $n$ without performing a trivial operation that just reassigns the $XZ$ pairing somewhere else. However when $n$ is even we find no $Z$ operators which leads nicely to our second idea of using only one type of operator in our stabilizer operators.

As we have just seen it is possible to eliminate all the $Z$ operators in our stabilizer operator, let us now look to see if we can also eliminate all the $X$ operators. When we apply Eq. (2) to each qubit in the system there will only be one $X$ operator applied to each qubit over the whole set of operators this means that we cannot eliminate the $X$ operators as it is only possible to multiply the $X$ operator with a $Z$ or an $I$ operator. Given that the only way to find CTF stabilizer operators is with a single type of operator we have to look for those with only $X$ operators. We will call these homogeneous cross-talk free (HCTF) stabilizer operators.
Ideally, we would want all the stabilizers that describe a cluster state to be CTF. As we have shown above we can only generate HCTF stabilizers. A set of only HCTF stabilizers would only be able to uniquely specify an eigenstate of $X$ of each qubit. We are thus forced to include stabilizers that have cross-talk, we will see how to choose sets that minimize this.

### III. MINIMIZING CROSS-TALK

To completely define our cluster state using stabilizer operators we must be able to recreate the canonical set defined by Eq. (2). It is clear that our set of HCTF stabilizer operators cannot do this alone as there is no way to create $Z$ Pauli operators from $X$ and $\hat{1}$ operators and so we must include some non-CTF stabilizer operators. This means there will be some cross-talk in the system, but if we are intelligent about our choice of non-CTF stabilizer operators we can see that this can be brought in at a minimum. Here we define a cross-talk penalty $P_{CT}$ that shows how many pairs of $XZ$ Pauli operators share an edge in any stabilizer operator. Initially we define the following

$$A = \sum_{jk} a_{jk} E_{jk}, \quad F_X^a = \sum_j x_j^a E_j, \quad F_Z^a = \sum_k z_k^a E_k, \quad F_{XZ}^a = \sum_j x_j^a z_k^a E_{jk},$$

(11)

where $A$ is the adjacency matrix of the cluster state with $A_{jk} = 1$ when qubits $j$ and $k$ share an edge and $A_{jk} = 0$ otherwise, and $E_{jk}$ is a basis matrix with $E_{jk} = \delta_{jm}\delta_{kn}$. For each stabilizer $S^a$, the vector $x_j^a$ specifies the position ($x_j = 1$, otherwise 0) of $X$ operators, similarly $z_k^a$ for the $Z$ operators. The $E_j$ and $E_k$ are both basis vectors with the $j$th and $k$th element as 1 otherwise 0, and $F_{XZ}^a$ is the outer product of $F_X^a$ and $F_Z^a$.

Using the definitions in Eq. (11) we define $B^a$ by taking the Hadamard product (⊙) of $A$ with $F_{XZ}^a$

$$B^a = A \circ (F_{XZ}^a),$$

(12)

The Hadamard product or entry wise product $[19]$ is formed by $B_{jk} = A_{jk}(F_{XZ}^a)_{jk}$. The cross-talk penalty $P_{CT}$ for a stabilizer is now

$$P_{CT}(S^a) = Tr[(B^a)^T B^a] = \sum_{jk} (x_j^a)^2 (z_k^a)^2 A_{jk}^2$$

(13)

where $A_{jk}, x_j, z_j = 0,1$.

If we take a $3 \times 3$ cluster state as an example, the $P_{CT} = 24$ if we were to individually measure each of the canonical stabilizer operators, whereas if we use a set of stabilizer operators that include the HCTF set and a subset of choice CT stabilizer operators (Fig.4) then the $P_{CT} = 13$ which is a big improvement.

This model can be extended to incorporate more complicated definitions of how the cross-talk interferes with the system. For example if qubits are connected by an edge but the physical distance between them is greater than the range of the cross-talk it would not be included in the $P_{CT}$ (Fig.5).

### IV. CROSS-TALK-FREE STABILIZERS

In arbitrary shaped cluster states it is hard to find HCTF stabilizer operators, this problem is similar to that of tiling problems which are non local and NP complete [20-22]. Given these difficulties for the general problem we have identified, for simple shapes, patterns and observations starting with the simplest example of a square lattice. This then leads on to fixed width triangular lattices that share many similarities with the squares.

Though we cannot create HCTF stabilizer operators for all shapes of lattice, in certain cases it is possible. In particular we specify how many HCTF stabilizer operators can be found in general for square, rectangular and fixed width triangular lattices, and we present algorithms for generating HCTF stabilizer operators for constant width lattices.
A graph state of a modified cluster state. By changing the connectivity of the graph state we can see how the physical distance between the qubits is important when considering the impact of cross-talk. In (a) qubits 2 and 3 are joined but the distance between them is greater than the range of cross-talk, this means when we apply a stabilizer operator to qubit 2 (in (b)) the cross-talk between qubits 2 and 3 is not included, this is achieved by modifying the adjacency matrix (c). This model could be further extended by allowing the elements of $A$ to have a value between 0 and 1 to get a more accurate value for the impact of the cross-talk, in this case we would take the square root of the value between 0 and 1 as $A_{jk}$.

**FIG. 5. (Colour online)** A graph state of a modified cluster state. By changing the connectivity of the graph state we can see how the physical distance between the qubits is important when considering the impact of cross-talk. In (a) qubits 2 and 3 are joined but the distance between them is greater than the range of cross-talk, this means when we apply a stabilizer operator to qubit 2 (in (b)) the cross-talk between qubits 2 and 3 is not included, this is achieved by modifying the adjacency matrix (c). This model could be further extended by allowing the elements of $A$ to have a value between 0 and 1 to get a more accurate value for the impact of the cross-talk, in this case we would take the square root of the value between 0 and 1 as $A_{jk}$.

**A. Shapes of lattices that allow HCTF**

It is possible to create cluster states in many shapes with different kinds of connectivity, however not all these shapes and connectivity of lattices allow for non-trivial HCTF stabilizer operators (Fig. 6). This is due to the number of edges connecting each of the nodes in the cluster state, a node with an odd number of edges cannot be surrounded by stabilizer operators as this will lead to $Z$ Pauli operators that cannot be cancelled.

**FIG. 6. (Colour online)** Graphical representation that it is not possible to construct non-trivial HCTF stabilizers for any shaped/connectivity of lattice. [a] shows a square connectivity lattice with extra $Z$ operators that do not cancel due to the shape of the overall lattice. [b] shows a triangular connectivity lattice again with extra $Z$ operators that do not cancel due to the shape of the overall lattice.

**B. HCTF stabilizer operators in fixed width lattices**

When considering square lattices with square connectivity and $n \times n$ qubits we find there are $n$ linearly independent HCTF stabilizer operators. This is due to the construction of the HCTF stabilizer operators, if we approach the lattice row by row and apply a single stabilizer operator in the first row, there are $n$ possible places for this stabilizer to start, then by considering the lattice one row at a time we see that each of these cases leads to an independent HCTF stabilizer operator (Fig. 7). It is clear that they are each linearly independent as they do not share any qubits in the initial row. These linearly independent operators are the building blocks for the overall HCTF stabilizer operators, all other HTCFs are made up of combinations of these (Fig. 7).

**FIG. 7. (Colour online)** The three linearly independent HCTF stabilizer operators in a $3 \times 3$ square lattice created by applying a single stabilizer operator individually to each of the qubits in the initial row are shown in (a) (b) and (c). Where as (d) shows a HCTF stabilizer operator that is not in the canonical set that is constructed using (a) and (c).

Leading on from the square lattices with square connectivity we find that it is possible to extend lattices beyond just $n \times n$ qubits and still create a set of HCTF stabilizer operators, the shape of these extended lattices is restricted to the form $(km + (k - 1)) \times (lm + (l - 1))$ (Fig. 8).

Fixed width triangular connectivity lattices also have no extra degrees of freedom so we can find a similar deterministic algorithm as we did for the squares by considering the lattice from an fixed initial pattern on the first row of the lattice (algorithm 2). In this case we find $n$ HCTF stabilizer operators for a lattice of width $n$ qubits (Fig. 9). The HCTFs again appear as squashed square shapes this allows all the nodes in the graph to have an even number of edges meaning it is possible for all the $Z$ operators to cancel out.

**C. Algorithm for finding HCTF stabilizer operators**

As we have already discussed we can find HCTF stabilizer operators by starting from a pattern of stabilizer operators in the initial row of the cluster state, this allows us to create an algorithm to find any HCTF stabilizer operators where the initial row has been defined.

**V. TRIANGLE TRIANGULAR CONNECTIVITY LATTICES**

Due to the changing degrees of freedom in a triangle triangular connectivity lattice our previous approach to finding a deterministic algorithm does not work, and so we considered the lattices individually using a brute
The canonical set of HCTF can be found by first applying stabilizer operators to all $r$ qubits along the edges of the lattice, then completing the pattern internally to eliminate the $Z$ operators. The second HCTF stabilizer operator is found by applying stabilizer operators to the qubits along the edges avoiding the corner qubits, $r = 1$ and $r = r$. The next HCTF avoids qubits $r = 1, r = 2$ and $r = r, r = r - 1$, this pattern is repeated until the last HCTF stabilizer operator where the stabilizer operator is applied to the central qubit ($r = \frac{r+1}{2}$ for $r$ odd) or qubits ($\frac{r}{2}$ and $\frac{r+2}{2}$ for $r$ even) (Fig. 10).

**Algorithm 1:** Algorithm to form HCTF stabilizer operators in a rectangular lattice given an initial first row. $a^c_r$ denotes the qubit in row $r$, column $c$. The number of qubits in the initial row is $m$, we add additional 0 elements at the start and end of the initial row and a dummy row that sits above our initial row to ensure the equation holds. The program finds the configuration of the $X$ operators in each row of the HCTF stabilizer operator and shows how many rows is necessary to complete the HCTF.

**Data:**
- initial row $a_r = \{0_0, 0_1, \ldots, 1_m, 0_{m+1}\}$
- dummy row $a_{r-1} = \{0_0, \ldots, 0_{m+1}\}$

**Result:** HCTF Stabilizer Operator
- rownumber = 3;

**while** Number of $X$ operators in the current row $\neq 0$ **do**
  - $a^c_{r+1} = a^c_r + a^{c+1}_r + a^{c+1}_{r-1} \mod 2$;
  - for $c = 2 \ldots m$;
  - Print $a^c_{r+1}$ from $c = 2 \ldots m$;
  - Count $X$ operators in the row;
  - $a_{r+1} = \{0, a^c_{r+1}, a^{c+1}_{r+1}, \ldots, a^{m+1}_{r+1}, 0\}$;
- rownumber = rownumber + 1;
**end**

**Algorithm 2:** Algorithm to form HCTF stabilizer operators in a fixed width triangular lattice given an initial row. $b^c_r$ denotes the qubit in row $r$, column $c$. The number of qubits in the initial row is $m$, we add additional 0 elements at the start and end of the initial row and a dummy row that sits above our initial row to ensure the equation holds. The program finds the configuration of the $X$ operators in each row of the HCTF stabilizer operator and shows how many rows is necessary to complete the HCTF.

**Data:**
- initial row $a_r = \{0_0, 0_1, \ldots, 1_m, 0_{m+1}\}$
- dummy row $a_{r-1} = \{0_0, \ldots, 0_{m+1}\}$

**Result:** HCTF Stabilizer Operator
- rownumber = 3;

**while** Number of $X$ operators in the current row $\neq 0$ **do**
  - $b^c_{r+1} = b^c_r + b^{c+1}_r + b^{c+1}_{r-1} + b^{c+2}_r \mod 2$;
  - for $c = 2 \ldots m$;
  - Print $b^c_{r+1}$ from $c = 2 \ldots m$;
  - Count $X$ operators in the row;
  - $b_{r+1} = \{0, b^c_{r+1}, b^{c+1}_{r+1}, \ldots, b^{m+1}_{r+1}, 0\}$;
- rownumber = rownumber + 1;
**end**
it is important that we also consider the cost of these improvements, in the initial set up it takes two different patterns of measurement to construct all the stabilizer operators of our system, these measurements are heavily affected by cross-talk reducing the reliability of the result. However when we introduce our improved set of measurements we find that there is more than two different patterns of measurements needed to construct all the stabilizer operators of our system (Fig. 11). As each pattern is measured many times to build up good statistics, there may be a trade-off between fewer patterns and more measurements per pattern, or more patterns with reduced cross-talk but worse statistics.

VI. CONCLUSION

By verifying our cluster state using stabilizer operators we come across problems such as cross-talk in the physical measurement process. By adapting the measurements we perform on the system we can reduce these affects to give a more realistic value to our measurement.

Exploring different shaped lattices we find simple algorithms that produce sets of linearly independent HCTF stabilizer operators for lattices where the connectivity and number of qubits remain constant on each row. Given that it is not possible to find a complete set of HCTF stabilizer operators to describe our cluster state we consider how best to choose from the non-CTF stabilizer operators to reduce the overall affect the cross-talk has on the system, and introduce a rating system to compare the stabilizer operators.

We could look at different types of lattices such as hexagons but here we run into the same problems as the triangle shaped triangular lattices. The connectivity and the number of qubits change in every row no matter how the hexagonal lattice is constructed, this changes the degree of freedom each time we add a new row meaning it is not possible to find a deterministic algorithm.

Now we have an algorithm to form HCTF stabilizer operators and a rating system to determine the best set of modified stabilizer operators for a rectangular lattice.

To get around the problem of cross-talk in a physical sense we could construct lattices differently so that the atoms that are connected by an edge are physically far apart so that when the active rotation is performed by the addressing beam the pairs of $X$ and $Z$ operators in the stabilizer operator no longer feel the cross-talk (Fig. 12a). There are a couple of things to note about this idea, firstly it is important to take into consideration the complexity of the entangling operations as the more complex the shape the harder it is to create. It is also important to consider how many measurements will be needed to reconstruct the stabilizer operator expectation values in the newly shaped lattice.

When different experimental setups it is important to take into account the possibility of vacancies in our system and incomplete measurement [23]. In this circumstance it becomes important to think about which measurement result to assign to the vacant result, this is dependent upon the number of Pauli operators in the stabilizer operator. For example, say we have a $3 \times 3$ qubit square connectivity lattice then then canonical set
FIG. 12. (Colour online) Rearranged lattice for cluster state generation. The qubits in (a) that share an edge have been physically moved so that the distance between them is greater than the cross-talk range making these newly positioned individual stabilizer operators cross-talk free. (b) and (c) show the checkerboard pattern required to reconstruct the stabilizer operator expectation values, these patterns are obviously not cross-talk free. However this is still an improvement on the checkerboard patterns in Fig. 1 as $P_{CT} = 16$. The entangling operations require shifts along various lattice vectors.

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[1] R. Raussendorf, D. E. Browne, and H. J. Briegel, Phys. Rev. A 68, 022312 (2003).
[2] R. Raussendorf and H. J. Briegel, Phys. Rev. Lett. 86, 5188 (2001).
[3] P. Dong, Z.-Y. Xue, M. Yang, and Z.-L. Cao, Phys. Rev. A 73, 033818 (2006).
[4] H. Wunderlich, C. Wunderlich, K. Singer, and F. Schmidt-Kaler, Phys. Rev. A 79, 052324 (2009).
[5] X.-W. Wang and G.-J. Yang, Optics Communications 281, 5282 (2008).
[6] M. Cramer, B. A., N. Fabbri, L. Fallani, C. Fort, S. Rosi, F. Caruso, M. Inguscio, and M. B. Plenio, Nature Communications 4 (2013).
[7] O. Mandel, M. Greiner, A. Widera, T. Rom, T. W. Hänsch, and I. Bloch, Nature 425, 937 (2003).
[8] P. Walther, K. J. Resch, T. Rudolph, E. Schenck, H. Weinfurter, V. Vedral, M. Aspelmeyer, and A. Zeilinger, Nature 434, 169 (2005).
[9] K. L. Brown, C. Horsman, V. Kendon, and W. J. Munro, Phys. Rev. A 85, 052305 (2012).
[10] C. Weitenberg, M. Endres, J. F. Shelby, M. Cheneau, P. Schauz, T. Fukuhara, I. Bloch, and S. Kuhr, Nature 471, 319 (2011).
[11] M. Paris and J. Reháček, eds., Quantum State Estimation, Lecture Notes in Physics, Vol. 649 (Springer Berlin Heidelberg, 2004).
[12] D. Gottesman, in Group22: Proceedings of the XXII International Colloquium on Group Theoretical Methods in Physics, edited by S. P. Corney, R. Delbourgo, and P. D. Jarvis (Cambridge, MA, International Press, 1999) pp. 32–43, available from arXiv:quant-ph/9807006v1.
[13] K. Vogel and H. Risken, Phys. Rev. A 40, 2847 (1989).
[14] B. M. Terhal, Physics Letters A 271, 319 (2000).
[15] R. D. Somma, J. Chiaverini, and D. J. Berkeland, Phys. Rev. A 74, 052302 (2006).
[16] H. K. Cummins, G. Llewellyn, and J. A. Jones, Phys. Rev. A 67, 042308 (2003).
[17] M. B. Elliott, B. Eastin, and C. M. Caves, Journal of Physics A: Mathematical and Theoretical (2010).
[18] R. Jozsa, Journal of Modern Optics 41, 2315 (1994).
[19] C. Davis, Numerische Mathematik 4, 343 (1982).
[20] R. Berger, Memoirs of the American Mathematical Society 66, 72 (1966).
[21] R. M. Robinson, Inventiones mathematicae 12, 177 (1971).
[22] H. R. Lewis, 2013 IEEE 54th Annual Symposium on Foundations of Computer Science 0, 35 (1978).
[23] K. C. S. Hall and D. K. L. Ot, “Bell inequality violation in the presence of vacancies and incomplete measurements,” (2014), arXiv:1412.7502.