Adaptive Dynamic Surface Control for Active Suspension With Electro-Hydraulic Actuator Parameter Uncertainty and External Disturbance

SHUANG LIU, RUOLAN HAO, DINGXUAN ZHAO, AND ZHIJIAN TIAN

1School of Electrical Engineering, Yanshan University, Qinhuangdao 066004, China
2School of Mechanical Engineering, Yanshan University, Qinhuangdao 066004, China
3XCMG Fire-Fighting Safety Equipment Company Ltd., Xuzhou 221100, China

Corresponding author: Dingxuan Zhao (zdx-yw@ysu.edu.cn)

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ABSTRACT In this paper, a new adaptive dynamic surface control strategy is proposed to deal with the uncertainty parameters of electro-hydraulic actuator and unknown external disturbances of vehicle active suspension system. A dynamic model of nonlinear vehicle active suspension is established. The adaptive parameter estimation laws are designed to estimate the uncertain parameters caused by the change of the physical characteristics of the electro-hydraulic actuator. In other words, the estimation of uncertain parameters in hydraulic system is beneficial to improve the control precision of vehicle. Nonlinear robust feedback signal is introduced to suppress unknown external disturbances. Then, this paper proposes an idea of dynamic surface in order to avoid explosion of complexity that caused by virtual control signal. The dynamic surface function replaces the derivative of virtual control signal in backstepping control, which reduces the computational complexity and improves the riding comfort. The simulation comparison shows that the presented method is effective in improving riding comfort and driving safety.

INDEX TERMS Active suspension, electro-hydraulic actuator, adaptive control, dynamic surface control, explosion of complexity.

I. INTRODUCTION

The notations used in this paper are introduced as follows

- $z_s$, $z_u$: Vertical displacement of vehicle body and vertical displacement of the tire.
- $z_0$: Road excitation.
- $m_s$, $m_u$: Sprung mass and Unsprung mass.
- $F_k$, $F_c$: Nonlinear spring and the damper force.
- $F_t$, $F_b$: Tire stiffness and damping.
- $F_a$: Active control force of active suspension.
- $k_t$, $b_f$: Tire stiffness coefficient and the tire damping coefficient.
- $k$, $k_s$: Stiffness coefficients of the linear and cubic terms.
- $P_L$: Load pressure.

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Suspension system is one of the most significant parts of vehicle. According to the output force, suspension system is divided into passive suspension, active suspension, and semi-active suspension. Different from passive suspension and semi-active suspension, active suspension provides more effective force to enhance the riding comfort, driving safety.
and operational stability [1], [2]. Furthermore, active suspension has been studied by scholars for many years [3]–[6].

However, the vehicle suspension system is a complex nonlinear system [7], [8]. The authors in [9] presented an adaptive sliding mode control technique for the suspension system. With the aim of improving riding comfort, an adaptive backstepping control way was incorporated in [10]. An adaptive robust control method was proposed in [11], which effectively solves the influence of output saturation on suspension system. In [12], a fuzzy adaptive tracking control based on bionic model was proposed. The nonlinear damping was designed to verify the damping characteristics of the bionic reference model. The authors in [13] proposed multi-target control strategies to improve suspension performance, which enhances the riding comfort and riding safety. Two different adaptive backstepping tracking controllers were designed in [14], which improves riding comfort and riding safety in vehicle suspension system. The authors in [15] established an active suspension model and presented an adaptive tracking control strategy.

The above literatures have achieved good achievements. However, the actuators in the above literatures are usually assumed to be ideal force generators, which will limit the application of these methods in practical engineering. Since the electro-hydraulic actuator is smaller and more effective compared with other actuators. Therefore, the electro-hydraulic actuator is usually used as an actuator to generate vibration isolation force in active suspension. The active force is controlled by the voltage of electro-hydraulic actuator. In addition, the voltage is closely related to the density of hydraulic oil, the pressure of oil supply, the effective area of piston, the leakage coefficient of hydraulic cylinder and the elastic stiffness of oil. It will lead to deviation if the dynamics characteristics of electro-hydraulic actuator is neglected in practical engineering application.

Actually, the electro-hydraulic actuator is a highly complex nonlinear system. Recently, the electro-hydraulic actuators have been studied by some scholars in vehicle active suspension system [16]–[18]. The authors in [19] investigated a fault-tolerant control technique for quarter-vehicle active suspension system. The authors in [20] designed an output feedback for active suspension, which enhances the riding comfort. The authors in [21] established hydraulic suspension model and presented a master-slave control method. The authors in [22] established hydraulic active suspension and designed adaptive robust control method, which suppresses the vibration of the vehicle body. The authors in [23] designed a nonlinear backstepping control strategy for electro-hydraulic active suspension, which effectively stabilizes the force transmitted by the vertical motion of the vehicle to passengers. In [24], an adaptive compensation controller was proposed for high-precision motion control of electro-hydraulic servo system, which effectively solves the problems of nonlinearity. The evolutionary algorithm based PID control was proposed to reduce the body acceleration in [25]. The different characteristics of electro-hydraulic actuators in suspension system have been investigated in the above literatures and good control results have been achieved. However, the uncertain parameters of the electro-hydraulic actuator are neglected in most of the above research works. The physical characteristics of servo valve and hydraulic oil can be changed due to different driving conditions. For instance, the hydraulic oil temperature is changed by the continuous operation of vehicles, which will change the density of hydraulic oil. Some parameters in the electro-hydraulic actuator are changed due to the changes of these physical characteristics, which will lead to uncertain parameters in the suspension system. Therefore, it is necessary to study the control strategy considering the uncertain parameters of electro-hydraulic actuator in suspension system.

In order to solve the problems above, the physical characteristics of electro-hydraulic actuator is considered and the nonlinear electro-hydraulic actuator model is established in this paper. Through the above analysis, adaptive control methods [26]–[29] are usually used to deal with uncertain parameters in nonlinear systems. The adaptive parameter estimation laws are designed to approximate the uncertain parameters in electro-hydraulic actuator system. This article designs a nonlinear robust feedback control to suppress the nonlinear disturbances in practice. The order of the system model is higher due to the nonlinear electro-hydraulic actuator established in this paper. Since virtual control signals lead to explosion of complexity, which means that the dynamic characteristics and the stability of the hydraulic suspension system will be significantly reduced. Then the vehicle performance such as acceleration of vehicle body, suspension working space and dynamic load ratio will be deteriorated. To solve the above problems, this paper designs a new dynamic surface control method [30], [31]. The dynamic surface function replaces the derivative of virtual control signal, which effectively solves the impact of explosion of complexity in the suspension system and improves the control accuracy. The simulation results show that the controller designed in this paper is superior than passive suspension (PASSIVE) and adaptive backstepping controller (QLF) in improving driving safety, operation stability and riding comfort.

The contributions of this paper can be summarized as follows.

1) An adaptive dynamic surface control is proposed for active suspension with electro-hydraulic actuator, which effectively improves the vehicle performance. As compared with [13], [14], the nonlinear electro-hydraulic actuator is given in this paper, and the control voltage with physical meaning is given, which has powerful guiding significance for engineering application.

2) As compared with [17], [22], the uncertain parameters are considered in the electro-hydraulic actuator, which is not only beneficial to the precise control of vehicle active suspension system but also good for practical application in the future and the improvement of vehicle performance.
The content of this paper is arranged as follows. Part 2: A nonlinear active suspension with electro-hydraulic actuator model is established. Then, the performance evaluation indexes are given in suspension system. Part 3: The controller is given and the stability is proved by Lyapunov function. Finally, the stability of zero dynamic system is analyzed. Part 4: The effectiveness of the controller is illustrated by simulation. Part 5: Summary of the work done in this paper.

II. PROBLEM FORMULATION

A. NONLINEAR HYDRAULIC ACTIVE SUSPENSION MODEL

The nonlinear suspension model is established as shown in Fig. 1.

![Vehicle suspension model](image)

FIGURE 1. Vehicle suspension model.

The dynamic equations of vehicle active suspension are defined as follows:

\[ \begin{cases} m_s \ddot{z}_s + F_c + F_k = F_u \\ m_u \ddot{z}_u - F_c - F_k + F_t + F_b = F_u \end{cases} \]  \tag{1}

\[ F_t, F_b, F_k, F_c \] can be expressed as

\[ \begin{align*} F_t &= k_t (z_a - z_0) \\ F_b &= b_t (z_a - z_0) \\ F_c &= b_c (z_a - \dot{z}_a) \\ F_k &= k (z_s - z_a) + k (z_s - z_a)^3 \end{align*} \]

\( F_u \) is produced by electro-hydraulic system through oil supply. Therefore, the electro-hydraulic actuator is established. The force of the suspension system is supplied by the hydraulic device. The dynamic equation of electro-hydraulic actuator is established [17]:

\[ \begin{align*} F_u &= A P_L \\ P_L &= \frac{4be}{V_t} \left[ C_d \omega \dot{z}_c c_1 - A (\dot{z}_s - \dot{z}_a) - C_p P_L \right] \end{align*} \]  \tag{2}

respectively. \( z_s \) can be controlled by the input voltage \( u \).

The dynamics equation of servo valve can be approximated as

\[ \dot{z}_v = \frac{1}{\tau} (-z_v + u) \]  \tag{3}

According to [18], we can get \( z_v + u = 0 \) then, \( u = z_v \).

For designing the controller, state variables are defined as:

\[ x_1 = z_s, \quad x_2 = \dot{z}_s, \quad x_3 = z_u, \quad x_4 = \dot{z}_u, \quad x_5 = P_L \]

The dynamic equations of suspension system and electro-hydraulic actuator are written as:

\[ \begin{align*} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= \theta_1 (-F_c - F_k + A x_5) + d \\ \dot{x}_3 &= x_4 \\ \dot{x}_4 &= \frac{1}{m_u} (F_c + F_k - F_t - F_b - A x_5) \\ \dot{x}_5 &= \theta_2 \text{sign} (P_s - \text{sign} (u) x_5) \sqrt{|P_s - \text{sign} (u) x_5|u} + \theta_3 (x_4 - x_2) - \theta_4 x_5 \end{align*} \]  \tag{4}

where \( F = -F_c - F_k \theta_1 = 1/m_s \) is an uncertain parameter determined by the number of passengers or the quality of the loaded goods. \( \theta_2 = \gamma, \theta_3 = \alpha \theta \) and \( \theta_4 = \beta \) are uncertain parameters in electro-hydraulic actuator systems, which can be changed by vehicle running state and physical characteristics of electro-hydraulic actuator, where \( \alpha = \frac{4be}{V_t}, \beta = \alpha C_p, \gamma = \frac{\alpha C_p}{\Delta p} \). \( d \) is an assumed uncertain nonlinear term. Suppose that there is a positive constant \( \delta_d \) that satisfying \( |d| \leq \delta_d \). To sum up, (4) is the nonlinear hydraulic active suspension model.

B. SUSPENSION PERFORMANCE INDEXES

The performance indexes of vehicle active suspension mainly involve the points as:

1) RIDING COMFORT

The meaning of riding comfort is to reduce the body vibration in vertical direction, so that people are not affected by the vibration caused by road roughness. The vertical acceleration mainly affects the riding comfort, so it is necessary to reduce the vertical acceleration.

2) SUSPENSION WORKING SPACE LIMITED

The limit of suspension working space denotes the limit of suspension mechanical structure, which is the relative displacement of unsprung mass and sprung mass. For improving the service life of the vehicle, the suspension working space should be within its physical limits. The requirements are described as follows:

\[ |z_s - z_a| \leq z_{\text{max}} \]  \tag{5}

3) RIDING SAFETY

Riding safety refers to the contact between the road and the wheel. The dynamic load ratio of tire should be less than 1, which can ensure the safety of vehicle during driving.

\[ \frac{F_t + F_b}{(m_s + m_u) g} < 1 \]  \tag{6}

However, these performance requirements are usually conflicting. This paper designs a reasonable controller to enhance the performance of the vehicle suspension.
III. ADAPTIVE DYNAMIC SURFACE CONTROL METHOD

An adaptive control technique based on dynamic surface (ADSC) is proposed to adjust suspension system. The control scheme of the proposed method is shown as Fig.2.

![Figure 2. Control scheme of the proposed ADSC controller.](image)

A. ADSC CONTROLLER DESIGN

The system errors are defined as follows:

\[ z_1 = x_1 - y_d \] (7)

where \( z_1 \) represents the tracking error; \( y_d \) is the reference trajectory.

\[ z_2 = x_2 - \alpha_1 \] (8)

\[ z_3 = x_3 - \alpha_2 \] (9)

where \( z_2 \) and \( z_3 \) are the state errors. \( \alpha_1 \) and \( \alpha_2 \) are the stable virtual control signals. The virtual control signal in the first step is designed as follow:

\[ \beta_1 = -k_1 z_1 + \dot{y}_d \] (10)

where \( k_1 \) is a positive constant. For avoiding explosion of complexity, \( \beta_1 \) pass a first-order low-pass filter to generate a new variable \( \alpha_1 \). The derivative of the virtual control is estimated by the dynamic surface function [30], where \( \tau_1 \) is the time constant.

\[ \tau_1 \dot{\alpha}_1 + \alpha_1 = \beta_1, \quad \alpha_1 (0) = \beta_1 (0) \]

Then, the dynamic surface function is defined as \( S_1 = \alpha_1 - \beta_1 \), we can conclude that:

\[ \dot{\alpha}_1 = \frac{\beta_1 - \alpha_1}{\tau_1} = \frac{S_1}{\tau_1} \] (11)

The dynamic equation of \( S_1 \) is

\[ \dot{S}_1 = \dot{\alpha}_1 - \beta_1 = -\frac{S_1}{\tau_1} + \varepsilon_1 \] (12)

\[ \varepsilon_1 = -\dot{\beta}_1 = -\frac{\partial \beta_1}{\partial x_1} x_1 - \frac{\partial \beta_1}{\partial y_d} y_d - \frac{\partial \beta_1}{\partial y_d} y_d \] (13)

\( \varepsilon_1 \) is continuous and bounded, and its maximum value is defined as \( M_1, \ varepsilon_2 = x_2 - \alpha_1 \) is the state error in the second step. A virtual filter function \( \beta_2 \) is introduced to avoid the problem of the explosion of complexity. Similarly, we can get

\[ \tau_2 \dot{\alpha}_2 + \alpha_2 = \beta_2, \quad \alpha_2 (0) = \beta_2 (0) \]

Then, the dynamic surface function is defined as \( S_2 = \alpha_2 - \beta_2 \), where \( \tau_2 \) is the time constant. Then,

\[ \dot{\alpha}_2 = \frac{\beta_2 - \alpha_2}{\tau_2} = -\frac{S_2}{\tau_2} \] (14)

The dynamic equation of \( S_2 \) is

\[ \dot{S}_2 = \dot{\alpha}_2 - \beta_2 = -\frac{S_2}{\tau_2} + \varepsilon_2 \] (15)

\[ \varepsilon_2 = -\dot{\beta}_2 = -\frac{\partial \beta_2}{\partial x_2} x_2 - \frac{\partial \beta_2}{\partial y_d} y_d - \frac{\partial \beta_2}{\partial y_d} y_d \] (16)

\( \varepsilon_2 \) is continuous and bounded, and its maximum value is defined as \( M_2 \)

\[ \beta_2 = \beta_2 \theta_1 \] (17)

\[ \beta_{2 \theta} = \frac{1}{\theta_1} \left( \frac{S_1}{\tau_1} - z_1 - k_2 z_2 \right) - F \frac{1}{A} \] (18)

where \( \beta_{2 \theta} \) is a nonlinear robust feedback control signal which can suppress the effects of various uncertain disturbances in the model. \( \beta_{2 \theta} \) satisfies the following conditions [11]. \( k_2 \) is a positive constant.

\[ \begin{cases} \dot{z}_2 [\hat{\theta}_1 \beta_{2 \theta} A - \hat{\theta}_1 (Ax_5 + F) + d] \leq \Delta_1 \\ z_2 \beta_{2 \theta} \hat{\theta}_1 \leq 0 \end{cases} \] (19)

\( \beta_{2 \theta} = -\frac{h_1^2}{\min_{\Delta_1} z_2} \) where \( h_1 = \| \theta_1_{\max} - \theta_1_{\min} \| / \| F + Ax_5 \| + \| d \| \) is an arbitrarily small positive constant.

\( z_3 = x_5 - \alpha_2 \) is the state error. \( k_3 \) is a positive constant. The control voltage is given as follows.

\[ u = -\left( \frac{\theta_3 (x_4 - x_2) - \theta_4 x_5 + \frac{\theta_2}{\theta_3} \theta_1 x_2 + k_3 z_3}{\theta_2 \sign (P - \sign (u) x_5) \sqrt{|P - \sign (u) x_5|}} \right) \] (20)

The adaptive law for uncertain parameters is defined as follows:

\[ \begin{cases} \dot{\theta}_1 = \text{Proj}_{\theta_1} (r_1 (F + Ax_5) z_2) \\ \dot{\theta}_2 = \text{Proj}_{\theta_2} (r_2 \sign (P - \sign (u) x_5) \sqrt{|P - \sign (u) x_5|}) \\ \dot{\theta}_3 = \text{Proj}_{\theta_3} (r_3 (x_4 - x_2) z_3) \\ \dot{\theta}_4 = \text{Proj}_{\theta_4} (-r_4 x_3 z_3) \end{cases} \] (21)

where \( \dot{\theta}_i \) is the estimate of uncertain parameter \( \theta_i \), \( r_i \) is positive constant, \( i = 1 \sim 4 \).

By designing an appropriate adaptive law \( \text{Proj}_{\dot{\theta}_i} (\Theta) \), the uncertain parameters in suspension system will be estimated online, and then the controller designed can finally adapt to the influence of parameter uncertainty.

The projection map is defined as follows:

\[ \text{Proj}_{\dot{\theta}_i} (\Theta) = \begin{cases} 0, & \text{if } \dot{\theta}_i \geq \theta_{i_{\max}}, \text{ and } \Theta > 0 \\ 0, & \text{if } \dot{\theta}_i \leq \theta_{i_{\min}}, \text{ and } \Theta < 0 \\ \Theta, & \text{otherwise} \end{cases} \] (22)
Theorem 1: The parameter estimation error is defined as 
\[ \hat{\theta}_i = \hat{\theta}_i - \theta_i \ (i = 1 \sim 4). \] Besides, the bounded of estimation error \( \hat{\theta}_i \) is 
\[ [0, \max \{|\theta_i - \theta_{i\min}|, |\theta_i - \theta_{i\max}|\}] \]
The following requirements should be satisfied [11]:
1. \( \hat{\theta}_i \) is bounded, \( \theta_{i\min} \leq \theta_i \leq \theta_{i\max} \).
2. \( \hat{\theta}_i (\hat{\theta}_i - \theta_i) \leq 0 \)

Proof:

a. The uncertain parameters \( \hat{\theta}_i \) is bounded.
   i. If \( \theta_i = 0 \), then \( \hat{\theta}_i \) keeps the current value unchanged because \( \hat{\theta}_i = \Theta_i = 0 \);
   ii. If \( \theta_i > 0 \), then \( \hat{\theta}_i \) monotonously increase to \( \theta_{i\max} \); \( \hat{\theta}_i \) is maintained.
   iii. Similarly, if \( \theta_i < 0 \), then \( \hat{\theta}_i \) monotonously reduces to \( \theta_{i\min} \); \( \hat{\theta}_i \) is maintained.

b. \( \hat{\theta}_i (\hat{\theta}_i - \theta_i) < 0 \)
   i. If \( \hat{\theta}_i \geq \theta_{i\max} \) and \( \theta_i > 0 \), then \( \hat{\theta}_i = 0 \), thus \( \hat{\theta}_i (\hat{\theta}_i - \theta_i) = -\theta_i (\hat{\theta}_i - \theta_i) \theta_i < 0 \);
   ii. If \( \hat{\theta}_i \leq \theta_{i\min} \) and \( \theta_i < 0 \), then \( \hat{\theta}_i = 0 \); \( \hat{\theta}_i (\hat{\theta}_i - \theta_i) = -\theta_i (\hat{\theta}_i - \theta_i) \theta_i < 0 \);
   iii. Otherwise, \( \hat{\theta}_i = \theta_i \) and \( \hat{\theta}_i (\hat{\theta}_i - \theta_i) = 0 \), \( \hat{\theta}_i (\hat{\theta}_i - \theta_i) \leq 0 \);

B. STABILITY ANALYSIS

The first Lyapunov function is selected as follows:
\[ V_1 = \frac{1}{2} \dot{z}_1^2 + \frac{1}{2} S_1^2 \]
The derivation of the above equation can be obtained as follows:
\[ \dot{V}_1 = z_1 \dot{z}_1 + S_1 \dot{S}_1 \]
Further, we can conclude that:
\[ \dot{V}_1 = z_1 (z_2 + S_1 + \beta_1 - \dot{y}_d) + S_1 \left( -\frac{S_1}{t_1} + \varepsilon_1 \right) \]
Then, we can obtain:
\[ \dot{V}_1 = -k_1 z_1^2 + z_1 z_2 + z_1 S_1 - \frac{S_1^2}{t_1} + S_1 \varepsilon_1 \]
From Yang’s inequality, we can get:
\[ |S_1 \varepsilon_1| \leq \frac{1}{2} S_1^2 + \frac{\sigma_1}{2} \leq \frac{1}{2} S_1^2 + \frac{\sigma_1}{2}, \quad \sigma_1 > 0 \]
\[ z_1 S_1 \leq \frac{1}{4} S_1^2 \]
From above, we can get:
\[ \dot{V}_1 \leq - (k_1 - 1) z_1^2 - \left( \frac{1}{t_1} - \frac{1}{4} - \frac{1}{2\sigma_1} M_1^2 \right) S_1^2 + z_1 z_2 + \frac{\sigma_1}{2} \]  
(29)

The second Lyapunov function is selected as:
\[ V_2 = V_1 + \frac{1}{2} S_2^2 + \frac{1}{2} \dot{z}_2^2 \]
(30)
The derivative of \( V_2 \) is given
\[ \dot{V}_2 = \dot{V}_1 + S_2 \dot{S}_2 + z_2 \dot{z}_2 \]
(31)
\[ V_2 = \dot{V}_1 + z_2 (\theta_1 F + \theta_1 A x_5 - \dot{\alpha}_1 + d) \]
(32)
Then we can get
\[ \dot{V}_2 = \dot{V}_1 + S_2 (\dot{\alpha}_2 - \dot{\beta}_2) + H \]
(33)
Let
\[ H = z_2 (\theta_1 F + \theta_1 A (z_3 + \beta_2 + S_2) - \dot{\alpha}_1 + d) \]
(34)
Then,
\[ |S_2 \varepsilon_2| \leq \frac{1}{2\sigma_2} S_2^2 \quad \leq \frac{1}{2\sigma_2} S_2^2 + \frac{\sigma_2}{2}, \quad \sigma_2 > 0 \]
(35)
\[ z_2 S_2 \leq \frac{z_2}{4} S_2^2 \]
(36)
Then, we can get:
\[ \dot{V}_2 \leq - (k_1 - 1) z_1^2 + \left( \frac{1}{t_1} - \frac{1}{4} - \frac{1}{2\sigma_1} M_1^2 \right) S_1^2 \]
\[ + \left( \frac{1}{t_2} - \frac{1}{4} \theta_{1\max} A - \frac{1}{2\sigma_2} M_2^2 \right) S_2^2 - (k_2 - \theta_{1\max}) z_2^2 \]
\[ + \frac{\sigma_1}{2} S_1^2 + \frac{\sigma_2}{2} + \dot{\theta}_1 A z_2 z_3 + S_2 \left( \dot{\theta}_1 \dot{\beta}_2 - \dot{\alpha}_1 (A x_5 + d) \right) \]
From (19) \( z_2 \left( \dot{\theta}_1 \dot{\beta}_2 - \dot{\alpha}_1 (A x_5 + d) \right) \leq \Delta_1 \)
Then,
\[ \dot{V}_2 \leq - (k_1 - 1) z_1^2 + \left( \frac{1}{t_1} - \frac{1}{4} - \frac{1}{2\sigma_1} M_1^2 \right) S_1^2 \]
\[ + \left( \frac{1}{t_2} - \frac{1}{4} \theta_{1\max} A - \frac{1}{2\sigma_2} M_2^2 \right) S_2^2 - (k_2 - \theta_{1\max}) z_2^2 \]
\[ + \frac{\sigma_1}{2} + \frac{\sigma_2}{2} + \dot{\theta}_1 A z_2 z_3 + \Delta_1 \]
when the disturbance \( d = 0 \), \( V_{2a} \) is defined as follows:
\[ V_{2a} = V_2 + \frac{1}{2} r_1^{-1} \dot{\theta}_1^2 \]
(37)
The derivative of \( V_{2a} \) is given
\[ \dot{V}_{2a} = \dot{V}_1 + S_2 \dot{S}_2 + z_2 \dot{z}_2 + r_1^{-1} \dot{\dot{\theta}_1} \dot{\theta}_1 \]
\[ \dot{V}_{2a} = \dot{V}_1 + S_2 (\dot{\alpha}_2 - \dot{\beta}_2) + z_2 (\theta_1 F + \theta_1 A x_5 - \dot{\alpha}_1) + r_1^{-1} \dot{\dot{\theta}_1} \dot{\theta}_1 \]
(38)
Then, we have
\[
\dot{V}_{2a} = \dot{V}_1 + S_2 (\dot{\theta}_2 - \dot{\theta}_3) + r_1^{-1} \dot{\theta}_1 \dot{\theta}_1 + z_2 (\theta_1 F + \theta_1 A (z_3 + \beta_2 + S_2) - \dot{\alpha}_1) 
\]
(39)
\[
\dot{V}_{2a} \leq - (k_1 - 1) z_2^2 - \left( \frac{1}{\tau_1} - \frac{1}{4} - \frac{1}{2\sigma_1} M_1^2 \right) S_1^2 + z_2 (\theta_1 F + \theta_1 A (z_3 + \beta_2 + S_2) - \dot{\alpha}_1) 
\]
\[
+ \frac{\sigma_1}{2} + S_2 \dot{z}_2 - \frac{S_2^2}{\tau_2} + r_1^{-1} \dot{\theta}_1 \dot{\theta}_1 
\]
(40)
Let
\[
H = z_2 (\theta_1 F + \theta_1 A (z_3 + \beta_2 + S_2) - \dot{\alpha}_1 + d) 
\]
when the disturbance \( d = 0 \).
Then, we have
\[
H = \dot{\theta}_1 A z_2 z_3 + \dot{\theta}_1 A z_2 S_2 - z_1 z_2 - k_2 z_2^2 
\]
\[
+ z_2 \left( \dot{\theta}_1 A \dot{\theta}_2 z_3 + \dot{\theta}_1 F + \dot{\theta}_1 (F + A x_5) \right) 
\]
(41)
Then, we can get:
\[
\dot{V}_{2a} \leq - (k_1 - 1) z_2^2 - \left( \frac{1}{\tau_1} - \frac{1}{4} - \frac{1}{2\sigma_1} M_1^2 \right) S_1^2 
\]
\[
- \left( \frac{1}{\tau_2} - \frac{1}{4} \theta_{\text{max}} A - \frac{1}{2\sigma_2} M_2^2 \right) S_2^2 
\]
\[
- (k_2 - \theta_{\text{max}}) z_2^2 + \frac{\sigma_1}{2} + \frac{\sigma_2}{2} + \dot{\theta}_1 A z_2 z_3 
\]
\[
+ r_1^{-1} \dot{\theta}_1 \dot{\theta}_1 + \dot{\theta}_1 A \dot{\theta}_2 z_3 - \dot{\theta}_1 (F + A x_5) z_2 
\]
According to (19) and Theorem 1, we can get
\[
\dot{V}_{2a} \leq - (k_1 - 1) z_2^2 - \left( \frac{1}{\tau_1} - \frac{1}{4} - \frac{1}{2\sigma_1} M_1^2 \right) S_1^2 
\]
\[
- \left( \frac{1}{\tau_2} - \frac{1}{4} \theta_{\text{max}} A - \frac{1}{2\sigma_2} M_2^2 \right) S_2^2 
\]
\[
- (k_2 - \theta_{\text{max}}) z_2^2 + \frac{\sigma_1}{2} + \frac{\sigma_2}{2} + \dot{\theta}_1 A z_2 z_3 
\]
\[
+ r_1^{-1} \dot{\theta}_1 \dot{\theta}_1 + \dot{\theta}_1 A \dot{\theta}_2 z_3 - \dot{\theta}_1 (F + A x_5) z_2 
\]
\[
\text{The Lyapunov function is selected as:}
\]
\[
V_3 = V_{2a} + \frac{1}{2} z_3^2 + \frac{1}{2} r_2^{-1} \theta_2^2 + \frac{1}{2} r_3^{-1} \theta_3^2 + \frac{1}{2} r_4^{-1} \theta_4^2 
\]
(42)
\[
\text{The derivative of (43) is given}
\]
\[
\dot{V}_3 = \dot{V}_{2a} + z_3 \dot{z}_3 + r_2^{-1} \dot{\theta}_2 \dot{\theta}_2 + r_3^{-1} \dot{\theta}_3 \dot{\theta}_3 + r_4^{-1} \dot{\theta}_4 \dot{\theta}_4 
\]
Then, we have
\[
\dot{V}_3 \leq - (k_1 - 1) z_2^2 - \left( \frac{1}{\tau_1} - \frac{1}{4} - \frac{1}{2\sigma_1} M_1^2 \right) S_1^2 
\]
\[
- \left( \frac{1}{\tau_2} - \frac{1}{4} \theta_{\text{max}} A - \frac{1}{2\sigma_2} M_2^2 \right) S_2^2 
\]
\[
- (k_2 - \theta_{\text{max}}) z_2^2 + \frac{\sigma_1}{2} + \frac{\sigma_2}{2} + \dot{\theta}_1 A z_2 z_3 
\]
\[
+ r_1^{-1} \dot{\theta}_1 \dot{\theta}_1 + \dot{\theta}_1 A \dot{\theta}_2 z_3 - \dot{\theta}_1 (F + A x_5) z_2 
\]
\[
\text{when the controller parameters satisfy}
\]
\[
k_1 > 1, \tau_1 < \frac{4\sigma_1}{\sigma_1 + 2M_1^2}, \tau_2 < \frac{4\sigma_2}{\sigma_2 + 2M_2^2}, k_2 > \theta_{\text{max}}, 
\]
\[
C_1 = \min \left\{ \frac{k_1 - 1}{\tau_1} - \frac{1}{2\sigma_1} M_1^2, \frac{1}{\tau_2} - \frac{1}{4} \theta_{\text{max}} A - \frac{1}{2\sigma_2} M_2^2 \right\} 
\]
\[
B_1 = \frac{\sigma_1}{2} + \frac{\sigma_2}{2}, \text{then we have}
\]
\[
\dot{V}_3 \leq - C_1 V_3 + B_1 
\]
Then, we can obtain
\[
V_3 (t) \leq \left( V_3 (0) - \frac{B_1}{C_1} \right) e^{-C_1 t} + \frac{B_1}{C_1} 
\]
(46)
From (46) we can conclude that the system is stability.

**C. THE STABILITY OF ZERO DYNAMICS**

In the design process of the controller, the controller should satisfy the tracking requirements for the body displacement, but it is necessary to consider the unsprung mass, so the proof of zero dynamic stability is given.

Let \( z_1 = 0, \ z_2 = 0 \).
Then, we can conclude that
\[
\beta_2 = \frac{F_c + F_k}{A} 
\]
According to \( z_3 = 0 \) then,
\[
A x_5 = F_c + F_k 
\]
(47)
Then we can obtain
\[
X = A_1 X + B Z 
\]
(48)
where \( X = \begin{bmatrix} x_3 \\ x_4 \end{bmatrix}, A_1 = \begin{bmatrix} 0 & -\frac{1}{m_o} & 1 & 0 \\ -\frac{k_s}{m_o} & -\frac{b_s}{m_o} & 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 & 0 \end{bmatrix}, \)
\[
Z = \begin{bmatrix} \dot{x}_0 \\ \dot{x}_4 \end{bmatrix} \]
\( A_1 \) is Hurwitz’s. Therefore, the zero dynamics of the system is asymptotically stability.

The reference trajectory \( y_d \) is chosen as (49), \( T_r \) is a preset time.
\[
y_d = \begin{cases} a_0 + a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4, & t < T_r \\ 0, & t \geq T_r \end{cases} 
\]
(49)
In this paper, \( T_r = 1 \), where the reference trajectory \( y_d \) satisfies the following conditions:
\[
y_d (0) = x_1 (0), \ y_d (0) = x_2 (0) 
\]
\[
y_d (T_r) = \dot{y}_d (T_r) = \ddot{y}_d (T_r) = 0 
\]
(50)
We assume the initial value of the nonlinear system as \( x_1 (0) = 0.03, x_2 (0) = 0, x_3 (0) = 0, x_4 (0) = 0 \). According to (49) and (50), we can calculate the reference trajectory \( y_d \).
\[
y_d = 0.03 - 0.18 t^2 + 0.24 t^3 - 0.09 t^4 
\]

**IV. SIMULATION INVESTIGATION AND DISCUSSION**

In this paper, dynamic surface control is considered, which effectively solves the explosion of complexity problem. The simulation comparisons are analyzed with MATLAB in this part. The ADSC proposed in this paper is compared with the passive suspension and the adaptive backstepping algorithm QLF [13], [14] without considering the dynamic surface
The root mean square (RMS) values of suspension performance indexes are calculated under different conditions. The parameters of the suspension system are shown as follow [13], [17].

\[
m_s = 600\text{kg}, \quad m_a = 60\text{kg}, \quad k_s = 1000\text{Ns/m}, \quad k_t = 2 \times 10^5\text{N/m}, \quad b_f = 1000\text{Ns/m}, \quad k = 18000\text{N/m}, \quad \alpha = 4.151 \times 10^5\text{N/m}^5, \quad \beta = 1\text{s}^{-1}, \quad \gamma = 1.545 \times 10^9\text{N/m}^{5/2}, \quad A = 3.35 \times 10^{-4}\text{m}^2, \quad P_s = 10342500\text{Pa}
\]

The controller parameters are

\[
\theta_1^{\text{max}} = 1/500, \quad \theta_1^{\text{min}} = 1/700, \quad \theta_2^{\text{max}} = 1.5459 \times 10^5, \quad \theta_2^{\text{min}} = 1.545 \times 10^5, \quad \theta_3^{\text{max}} = 1.39 \times 10^{10}, \quad \theta_3^{\text{min}} = 1.38 \times 10^{10}, \quad \omega_4^{\text{max}} = 1.3, \quad \omega_4^{\text{min}} = 0.7, \quad r_1 = 0.01, \quad r_2 = 0.01, \quad r_3 = 0.01, \quad r_4 = 0.01, \quad k_1 = 1, \quad k_2 = 30, \quad k_3 = 30, \quad r_1 = 0.001, \quad r_2 = 0.001.
\]

The initial value of the parameter not given is set to zero. Fig.3 is the curve of reference trajectory \(y_d\). We calculate the RMS of suspension performance indexes as shown in Table 1.

**TABLE 1.** RMS value of suspension performance index.

| System                  | ADSC | QLF   | PASSIVE |
|-------------------------|------|-------|---------|
| Acceleration (m/s²)     | 0.059| 0.111 | 0.445   |
| Suspension working space (m) | 0.007| 0.006 | 0.010   |
| Dynamic load ratio      | 0.010| 0.016 | 0.042   |

a. Bump road excitation

\[z_0 = \begin{cases} h_0 \left(1 - \cos \left(\frac{10}{3} \pi t\right)\right) & 0 \leq t \leq 0.6 \\ 0 & \text{otherwise} \end{cases}\]

where \(h_0 = 0.025\text{m}\)

The acceleration contrast curve is shown in Fig.4. According to Table 1, the vertical acceleration in this paper is lowered by 86.7% compared to passive suspension. The explosion of complexity complicates the controller design and reduces the riding comfort in suspension system. For solving this problem, the dynamic surface control idea is introduced. The ADSC method decreases the vertical acceleration by 46.8% compared with the QLF method. Moreover, the suspension working space in this paper is reduced by 30% compared with the passive suspension and the suspension working space is shown as Fig.5. The proposed method in this paper is beneficial to reduce the suspension working space. From Fig.6, we can calculate that the dynamic load ratio in this paper is reduced by 76.2% compared with passive suspension, the dynamic load ratio is reduced by 37.5%.
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FIGURE 7. The estimate of uncertain parameter $\theta_1$.

FIGURE 8. The estimate of uncertain parameter.

FIGURE 9. The estimate of uncertain parameter $\theta_3$.

FIGURE 10. The estimate of uncertain parameter $\theta_4$.

Moreover, the uncertain parameters are estimated by adaptive laws in hydraulic servo valve system, which are shown as Fig.8, Fig.9, Fig.10.

However, the electro-hydraulic actuator introduced increases the order of the suspension system. In this part, the comparison results are given. From the comparison results, we can conclude that the dynamic characteristics of suspension system are obviously reduced when higher order suspension system occurs explosion of complexity. Then, the performance of riding comfort and riding safety are getting worse. The method proposed in this paper effectively improves the riding comfort, driving safety and the service life of vehicle.

V. CONCLUSION

The nonlinear vehicle suspension model was established in this paper. This article was different from the literatures that considered force generating devices to be ideal forms. In this paper, the electro-hydraulic actuator model was considered in the establishment of the suspension model. Moreover, the adaptive estimation law estimated the uncertain parameters in the electro-hydraulic actuator. The nonlinear robust feedback control was adopted to solve the unknown external disturbance. For the purpose of solving explosion of complexity problem, the idea of dynamic surface control was designed. The dynamic surface control effectively solved the impact of explosion of complexity on vehicle suspension system. From the simulation comparison we can conclude that the proposed method effectively enhanced the riding comfort, operation stability and vehicle driving safety. Therefore, the performance of vehicle suspension was significantly improved. In the future research, the control scheme will be extended to saturation of servo in vehicle suspension system. Furthermore, we will consider the problem of uncertainty in multi-vehicles case [29], [32].

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SHUANG LIU received the Ph.D. degree in engineering from Yanshan University. His main research interests include dynamic analysis and control of complex electromechanical systems, engineering robot, and artificial intelligence systems.

RUOLAN HAO is currently pursuing the Ph.D. degree in control science and engineering with Yanshan University. Her main research interests include robot control, dynamic analysis and control of complex electromechanical systems, engineering robot, and artificial intelligence systems.

DINGXUAN ZHAO is currently a Distinguished Professor of the Chang Jiang Scholars Program. He is also a Professor with the School of Mechanical Engineering, Yanshan University. His main research interests include engineering robotics, dynamics, and simulation of complex mechanical systems.

ZHJIAN TIAN received the Ph.D. degree. He is currently a Professor Status High Level Engineer with XCMG Fire-Fighting Safety Equipment Company Ltd., Xuzhou. His main research interests include vehicle structure design and electro hydraulic servo control.