A Stochastic Collapse Equation with No New Constants

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Modified versions of the Schrödinger equation have been proposed in order to incorporate the description of measurement processes into the mathematical structure of quantum theory. Typically, these proposals introduce new physical constants, imply small violations of momentum and energy conservation, and restrict the collapse basis to the eigenstates of one particular operator.

The need to introduce new physical constants and the restriction on the collapse basis can be eliminated, and the violations of conservation laws can be eliminated or reduced by using a collapse operator based on two-particle interaction potential energies. The parameters governing the strength, range, and rate of collapse effects can be defined by reference to these interaction energies, insuring that the modified equation reduces to the nonrelativistic Schrödinger equation for stationary states and noninteracting situations. Consistency with conservation laws in individual experiments is maintained because the collapse operator automatically takes into account the small, residual entanglement between the measured system and systems with which it has previously exchanged conserved quantities during interactions. The collapse basis is determined by which interactions take place.

The main assumption made here is that wave function collapse is a real, nonlocal process that is induced by the kinds of individual correlating interactions that constitute measurements. Collapse is achieved by shifting small amounts of amplitude between various components of the wave function until all amplitude is transferred either into or out of the interacting component(s). This idea is implemented by modifying the nonrelativistic Schrödinger equation in the form:

\[ d\psi = \left( -i/\hbar \right) \hat{H} \psi dt, \]  

by adding stochastic nonlinear collapse terms\footnote{gillise@provide.net}

\[ d\psi = \left( -i/\hbar \right) \hat{H} \psi dt + \sum_{j<k} \hat{V}_{jk} \psi d\xi(t) - \frac{1}{2} \sum_{j<k} \hat{V}_{jk} \left( \sum_{m<n} \hat{V}_{mn} \right) \psi dt. \]  

The modified equation is interpreted according to the \textit{Itô} stochastic calculus rules. \( d\xi(t) \) is a white noise Gaussian process whose integral, \( \xi(t) \), is a Wiener process with zero mean. The differentials, \( d\xi(t) \), obey the \textit{Itô} rules, \( d\xi(t) d\xi(t) = dt \), \( d\xi(t) dt = 0 \), and determine the units, \( d\xi = (dt)^{1/2} \). The middle term on the right hand side of equation 2 is based on two-particle interaction potential energies, \( \hat{V}_{jk} \), between system \( j \) and system \( k \). It is primarily responsible for shifting amplitude between components. The third term on the right hand side, \(- \frac{1}{2} \sum_{j<k} \hat{V}_{jk} \left( \sum_{m<n} \hat{V}_{mn} \right) \psi dt\), preserves the norm of the wave function.

The interaction potential, \( \hat{V}_{jk} = \hat{V}_{jk}(r) \), depends only on the separation, \( r \), between systems \( j \) and \( k \), and its magnitude decreases as \( r \) increases. The full stochastic operator, \( \hat{V}_{jk} \), is constructed by subtracting the expectation value, \( \langle \psi | \hat{V}_{jk} | \psi \rangle \), from \( \hat{V}_{jk} \), dividing the result by the sum of the relativistic energies of systems \( j \) and \( k \), and multiplying by \( \sqrt{\gamma_{jk}} \) where \( \gamma_{jk} \) is a parameter based on the rate at which the interaction proceeds:

\[ \hat{V}_{jk} = \sqrt{\gamma_{jk}} (\hat{V}_{jk} - \langle \psi | \hat{V}_{jk} | \psi \rangle) / [(m_j + m_k) c^2]. \]  

The mass terms in the denominator reflect the extent to which the interaction can alter the state of the systems involved, and, thus, the degree of correlation that is established between them. These terms should be understood as representing the \textit{effective} mass; this can be done by aggregating particles in bound states such as atoms, molecules, and other complex structures into single systems with a net charge and a total mass. The square of the speed of light, \( c^2 \), which maintains the dimensional consistency of the equation, is chosen because it is the only nonarbitrary speed. It sets the strength of the stochastic effect of individual interactions, and limits that strength to less than about \( 10^{-3} \). This is due to the nonrelativistic framework in which this proposal is formulated. The relevant potentials are electrostatic, generally with a \( 1/r \) distance dependence.

The rate parameter for each interacting pair of systems, \( \gamma_{jk} \), is defined as the ratio of the magnitude of the rate of change of potential energy to the maximum possible change in potential energy expected during the interaction. In order to insure compliance with the Born probability rule these measures are restricted to the interacting components of the two systems, \( j \) and \( k \). These components are picked out by multiplying the wave function \( \psi \) by the ratio, \( \sqrt{\gamma_{jk} / \langle \psi | \hat{V}_{jk} | \psi \rangle} \):

\[ \psi_{jk} = \sqrt{\gamma_{jk} / \langle \psi | \hat{V}_{jk} | \psi \rangle} \psi. \]  

\[ \gamma_{jk} = \left| \frac{\int dt (\psi_{jk}^* \hat{V}_{jk} \psi_{jk})}{\int (\psi_{jk}^* \hat{V}_{jk} \max) \psi_{jk}} \right|. \]  

The numerator in equation 5 can be computed as the magnitude of the rate of change of kinetic energy in the
center-of-mass frame of the two interacting components:

\[
\| \int d/dt(\psi^*_j \psi_k \hat{V}_{jk}) \| = \| (i\hbar) \int \left[ \psi^*_j \psi_k \left( \hat{V}_{jk} \cdot \left( -\nabla^2 \psi_{jk} + \frac{\hat{V}^2_{jk}}{2m_j} + \frac{\hat{V}^2_{jk}}{2m_k} \right) \right) \right] dt \|.
\]

The integrals are over the entire configuration space, as are all integrals in this work, unless otherwise noted. The denominator in equation 5 depends on the specific configuration of the wave function and the sign of the potential. For positive potentials it is the sum of the potential energy and the radial component of the kinetic energy; for negative potentials it is the norm of the effective potential of the lowest available energy state:

\[
\| \langle \psi_j^* | \hat{V}_{\text{max}} | \psi_j \rangle \| \text{pos} = \int \psi_j^* \left( \hat{V}_{jk} \psi_j - \frac{\hbar^2}{\mu_{jk}} \partial^2 \psi_{jk}/\partial r^2 \right) dt,
\]

\[
\| \langle \psi_j^* | \hat{V}_{\text{max}} | \psi_j \rangle \| \text{neg} = \| \hat{V}_{jk} + \hat{L}_{jk}^2/r^2 \|.
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\]
value of $\hat{V}_{jk}$ over $I$ as:

$$\overline{V}^I_{jk} = \frac{\langle I|V_{jk}|I \rangle}{\langle I|I \rangle} \quad (13)$$

Label the integral of $\psi^*\psi$ over the interacting branch as $\mu^*\mu$, and that of the orthogonal branch, $\nu^*\nu = 1 - \mu^*\mu$. The average of $\hat{V}_{jk}$ over the entire wave function is then: $\langle \hat{V}_{jk} \rangle = \mu^*\mu \overline{V}^I_{jk}$. The average values of $(\hat{V}_{jk} - \langle \hat{V}_{jk} \rangle)$ across the interacting component and noninteracting components are:

$$(\text{int}) : \overline{V}^I_{jk}(1 - \mu^*\mu) = \nu^*\nu \overline{V}^I_{jk}; \quad (\text{nonint}) : -\mu^*\mu \overline{V}^I_{jk}. \quad (14)$$

The total value of $\hat{V}_{jk}$ over each of these components is then:

$$\nu^*\nu \langle I|I \rangle \left[ \psi^* \overline{V}^I_{jk} \frac{\sqrt{g_{jk}}}{(m_j + m_k)c^2} \psi \right] = \left( \mu^*\mu \nu^*\nu \right) \left[ \psi^* \overline{V}^I_{jk} \frac{\sqrt{g_{jk}}}{(m_j + m_k)c^2} \psi \right]; \quad (15)$$

$$- \mu^*\mu \langle O|O \rangle \left[ \psi^* \overline{V}^I_{jk} \frac{\sqrt{g_{jk}}}{(m_j + m_k)c^2} \psi \right] = - \left( \mu^*\mu \nu^*\nu \right) \left[ \psi^* \overline{V}^I_{jk} \frac{\sqrt{g_{jk}}}{(m_j + m_k)c^2} \psi \right].$$

So the effects of $\hat{V}_{jk}$ on the interacting and noninteracting branches are equal and opposite, and the process generated by the stochastic operator can be viewed as an unbiased random walk of the integrated value, $\mu^*\mu$, between the end points, 0 and 1.

With a large enough number of interactions all amplitudes will be transferred either into or out of each interacting branch. The probability that simultaneous interactions in distinct branches could frustrate the collapse is vanishingly small since these interactions would have to be almost perfectly synchronized in both time and interaction strength over a walk of more than $10^6$ steps. The possibility of collapse to a single point is eliminated by the very short duration and relative weakness of the collapse operator relative to the Hamiltonian.

To establish conformity with the Born rule it is necessary to show that the probability that the squared amplitude, $\mu^*\mu$, of an interacting branch is eventually increased to 1 is $\mu^*\mu$. The total change in the squared amplitude occurring at any time can be represented by using equation $[15]$ and summing over all of the interactions taking place in the interacting branch:

$$\pm \left( \mu^*\mu \nu^*\nu \right) \sum_{jk} \left[ \psi^* \overline{V}^I_{jk} \frac{\sqrt{g_{jk}}}{(m_j + m_k)c^2} \psi \right] d\xi^*(t) + d\xi(t). \quad (16)$$

Each individual term integrates to a value less than $10^{-3}$ over the course of the interaction. The stochastic process, $\xi(t)$, is continuous, and it has a variance of $t_2 - t_1$ over a time period from $t_1$ to $t_2$. By taking time increments, $dt$, that are sufficiently small one can insure that the sum in equation $[16]$ is less than 1. Since $\mu^*\mu \nu^*\nu$ is updated continuously the total value of the expression in equation $[16]$ is less than or equal to the difference between $\mu^*\mu$ and either end point (0 or 1). Designate this value as $\delta$. Label the probability of $\mu^*\mu$ eventually reaching 1 as $Pr(\mu^*\mu)$; so $Pr(0) = 0$, $Pr(1) = 1$. Since the step size, $\delta$, is less than $\mu^*\mu \nu^*\nu$ it is less than or equal to the distance to either end point. Since steps in either direction are equally likely

$$Pr(\mu^*\mu) = \frac{1}{2}[Pr(\mu^*\mu - \delta)] + \frac{1}{2}[Pr(\mu^*\mu + \delta)] \quad (17)$$

for all values of $\mu^*\mu$ and $\delta$. Therefore, $Pr(p)$ is linear, and the Born rule follows. This insures consistency with relativity at the level of observation.

### II. CONSERVATION LAWS

After an extended period of interaction nearly all systems are entangled to some extent. [16, 17] Conserved quantities are shared among these systems. The branching processes referred to above are brought about by conservative interactions that split both interacting systems into distinct parts. The interaction alters the distribution of conserved quantities between the systems, but it does not alter the total amount of those quantities in either branch. A photon passing through a beam-splitter provides a simple example of this point. The exchange of momentum between the reflected branch and the beam-splitter changes the state of the reflecting component of the beam-splitter as well as that of the photon. There is no resulting difference in the total momentum of the two distinct branches.

Given the equality between branches strict compliance with conservation laws in individual experiments can be established by showing that equation $[2]$ implies that the (normalized) change in the relevant quantities within each branch is zero. This will be demonstrated exactly for momentum and angular momentum. A generally similar result will be shown for energy. The apparent violations of energy conservation that are sometimes attributed to the narrowing of the measured system’s wave function are compensated for by correlated changes in entangled systems. However, when there is an exchange of conserved quantities between systems the standard nonrelativistic accounting does not accurately track energy changes associated with relativistic mass corrections, radiation, and small contributions from antiparticles. Since equation $[2]$ accommodates only the usual nonrelativistic types of energy it implies small deviations from strict conservation in these cases. These deviations closely parallel the discrepancies entailed by ordinary Schrödinger evolution. In stationary states and noninteracting situations equation $[2]$ reduces to the Schrödinger equation, and there are no deviations or discrepancies regarding changes in
energy. This proposal is based on the premise that collapse occurs in precisely those situations in which the discrepancies occur. It is possible, therefore, that these deviations are an artifact of the nonrelativistic formulation of the theory, and that they could be eliminated by an extension of this proposal that fully accounts for relativistic effects.

Following the derivation of equation\(^\text{11}\) above and abbreviating \(\sum_{j<k} \hat{V}_{jk} \equiv \hat{V}\), the change induced by an operator, \(\hat{Q}\), at each point in configuration space is:

\[
d(\psi^* \hat{Q} \psi) = \left[ (\hat{H} \hat{Q} \psi^*) - \psi^*(\hat{Q} \hat{H} \psi) \right] dt
- \psi^*(\frac{1}{2} \hat{V} \hat{Q} \psi) dt + \psi^* \hat{V} \hat{Q} \psi dt d\xi^* + \psi^* \hat{Q} \psi dt d\xi.
\]

(18)

The three quantities that are of interest here all commute with the Hamiltonian. For momentum and angular momentum this is insured by the restriction to two-particle, distance-dependent potentials, and it is obviously true for energy. So these quantities are conserved across the wave function, and since the Hamiltonian operates independently on each branch it also conserves the quantities within each branch. Therefore, to determine whether equation\(^\text{23}\) respects the relevant conservation laws, we need only to assess the effect of the stochastic operator:

\[
d(\psi^* \hat{Q} \psi)_{\text{coll}} = -\frac{\hbar}{2} \psi^* [\hat{V}^2 \hat{Q} + \hat{Q} \hat{V}^2] \psi dt
+ \psi^* [\hat{V} \hat{Q} \hat{V}] \psi dt
+ \psi^* \hat{Q} \psi d\xi^* + \psi^* \hat{Q} \psi d\xi.
\]

(19)

What will be shown here is that the change in these quantities induced by the collapse equation is directly proportional to the change that is induced in the squared amplitude at that point:

\[
d(\psi^* \hat{Q} \psi) / (\psi^* \hat{Q} \psi) = d(\psi^* \psi) / (\psi^* \psi).
\]

(20)

It is because the change in amplitude at each point in configuration space affects every subsystem, all of which have been shaped by previous interactions, that the collapse process is able to maintain strict conservation.

The change in \(\psi^* \psi\) due to the stochastic operator was shown by equation\(^\text{11}\) to be

\[
d(\psi^* \psi) = (\psi^* \hat{V} \psi) (d\xi^* + d\xi).
\]

(21)

From equation\(^\text{19}\) this can be done by demonstrating that

\[
\hat{Q} \hat{V} \psi = \hat{V} \hat{Q} \psi
\]

(22)

at each point in configuration space.

The momentum and angular momentum operators are:

\[
\hat{P} \equiv -\hbar \sum_j \hat{\mathbf{V}}_j;
\]

\[
\hat{L} = -i \hbar \sum_j \mathbf{w}_j \times \hat{\mathbf{V}}_j.
\]

(23)

The only spatial dependence in the collapse operator, \(\hat{V}\), is in the terms representing the conservative interaction potentials, \(\hat{V}_{jk}\). Since \(\hat{V}_j \hat{V}_{jk} = -\hat{V}_k \hat{V}_{jk}\), we get

\[
P(\hat{V}_{jk} \psi) = (-i\hbar) \left[ (\hat{\mathbf{V}}_j \hat{V}_{jk}) \psi + \hat{\mathbf{V}}_{jk} \hat{V}_j \psi \right]
+ (\hat{V}_k \hat{V}_{jk} \psi) \psi
+ (\hat{V}_k \hat{V}_{jk} \psi) \psi
= (-i\hbar) \left[ \hat{\mathbf{V}}_j \hat{V}_{jk} \psi + \hat{\mathbf{V}}_{jk} \hat{V}_j \psi \right]
= \hat{V}_{jk} (\hat{\mathbf{P}} \psi)
\]

(24)

Therefore,

\[
d(\psi^* \hat{P} \psi) = (\psi^* \hat{V} \hat{P} \psi) (d\xi^* + d\xi),
\]

(25)

and momentum is strictly conserved by the collapse equation\(^\text{11}\).

An explicit expansion of the x-component of angular momentum for systems, \(j\), and \(k\), gives:

\[
[\hat{L}_x(w_j) + \hat{L}_x(w_k)](\hat{V}_{jk} \psi)
= -i\hbar \left[ \hat{V}_j \hat{V}_{jk} \psi + \hat{V}_{jk} \hat{V}_j \psi \right]
+ \hat{V}_{jk} \hat{L}_x \psi.
\]

(26)

Since:

\[
\left( \partial \hat{V}_{jk} / \partial z_j + \partial \hat{V}_{jk} / \partial z_k \right) = 0;
\]

\[
\left( \partial \hat{V}_{jk} / \partial y_j + \partial \hat{V}_{jk} / \partial y_k \right) = 0;
\]

we get:

\[
\hat{L}_x(\hat{V}_{jk} \psi) = \hat{V}_{jk} (\hat{L}_x \psi)
\]

(28)

with similar relationships for the y and z components. So, the stochastic operator also strictly conserves orbital angular momentum at each point.

To evaluate the effect of the stochastic operator on energy we need to examine the relationship between \(\hat{H} \hat{V}_{jk}\) and \(\hat{V}_{jk} \hat{H}\). Since it is clear that, at each point in configuration space, \(\hat{V} \hat{V} = \hat{V} \hat{V}\), the only deviations from perfect conservation that can arise are those involving the kinetic energy terms. The action of the kinetic energy operator on the stochastic operator is:

\[
(\frac{\hbar^2}{2m} \nabla^2) \hat{V}_{jk} = \hat{V}_{jk} (-\frac{\hbar^2}{2m} \nabla_{\hat{V}}^2 + \nabla_{\hat{V}}^2)
+ (\frac{\hbar^2}{2m}) \{ \nabla_{\hat{V}}^2 \hat{V}_{jk} + \nabla_{\hat{V}} \hat{V}_{jk} \}
+ 2(\nabla_{\hat{V}} \hat{V}_{jk} \cdot \nabla_{\hat{V}} \hat{V}_{jk} \cdot \nabla_{\hat{V}} \hat{V}_{jk})
\]

(29)

Applying this to equation\(^\text{19}\) we get:

\[
d(\psi^* \hat{H} \hat{V} \psi) = \psi^* \hat{V} \hat{H} \psi (d\xi^* + d\xi)
- (\frac{\hbar^2}{2m}) \psi^* \left[ \sum_{j<k} \left( \nabla_{\hat{V}}^2 \hat{V}_{jk} + \nabla_{\hat{V}} \hat{V}_{jk} \right) \right]
\]

\[
- 2(\nabla_{\hat{V}} \hat{V}_{jk} \cdot \nabla_{\hat{V}} \hat{V}_{jk} \cdot \nabla_{\hat{V}} \hat{V}_{jk})
\]

\[
+ (\frac{\hbar^2}{2m}) \psi^* (\nabla \hat{V} \cdot \nabla \hat{V}) dt.
\]

(30)

\(^4\) Although the strict conservation of momentum allows the center of mass to evolve in a unitary manner, this does not interfere with the nonunitary collapse process, because the description of this evolution requires that the c-o-m is decoupled from the rest of the system.
The first line gives the desired proportionality between $d(\psi^* \hat{H} \psi)_C$ and $d(\psi^* \psi)_C$. This term highlights the usually small amount of entanglement between the measured system and those systems (such as the “preparation apparatus”) which is so often overlooked in discussions of the way in which measurements affect energy conservation.

The apparent deviations from perfect energy conservation are represented on the middle and last lines. They are relevant only when the interacting component of the wave function is selected since $\nabla \psi \approx 0$ over the noninteracting component. They can be compared to the first two relativistic corrections to the kinetic energy formula:

$$\frac{mc^2}{\sqrt{1-(v/c)^2}} - mc^2 = mc^2 \left[ \frac{1}{2} \frac{v^2}{c^2} + \frac{3}{2} \left( \frac{v^2}{c^2} \right)^2 + \ldots \right]$$

$$= KE_{nr} \left[ 1 + \frac{3}{2} \frac{KE_{nr}}{mc^2} + \frac{9}{8} \left( \frac{KE_{nr}}{mc^2} \right)^2 + \ldots \right],$$

where $KE_{nr}$ is the nonrelativistic kinetic energy. Other adjustments to the nonrelativistic energy accounting include radiation equal to approximately $KE_{nr} \frac{v KE_{nr}}{mc^2}$, and anti-particle content which contributes a fraction of about $(KE_{nr}/mc^2)^2$ to localized wave packets, $\psi^* \psi$.\[18\]

The individual terms from the middle lines of equation \[30\] can be rewritten as:

$$\left( \frac{i \hbar}{2m} \right) \left[ \psi^* (\nabla_j^2 \hat{V}_{jk} \psi) + 2 \psi^* (\nabla_j \hat{V}_{jk} \cdot \nabla_j \psi) \right] \ast \left( \sqrt{\gamma_{jk} d\xi} \frac{v KE_{nr}}{(m_j+m_k)c^2} \right).$$

The conversion of potential to kinetic energy during an interaction according to the Schrödinger equation is:

$$\left( \frac{i \hbar}{2m} \right) \left[ \psi^* (\nabla_j^2 \hat{V}_{jk} \psi) + 2 \psi^* (\nabla_j \hat{V}_{jk} \cdot \nabla_j \psi) \right] \ast dt. \ [33]$$

Comparison of equations \[32\] and \[33\] illustrates that the deviations implied by equation \[2\] arise in exactly the same situations in which the conventional theory fails to account for all of the changes in kinetic energy associated with the lowest order relativistic corrections.

To determine the ratio of the deviation implied by equation \[32\] to the change in kinetic energy described by equation \[33\] we can use the time defined by the expression $\frac{v KE_{nr}}{(m_j+m_k)c^2}$ (since the integral of $\sqrt{\gamma_{jk} d\xi}$ is of order 1), and compare it to the characteristic interaction time from equation \[9\] $dt_{int} = \frac{\Delta \nabla_{jk}}{\Delta \nabla_{jk} E_{nr}}$. This yields a ratio of $\frac{KE_{nr}}{(m_j+m_k)c^2}$, closely matching the first order correction from equation \[31\].

The term on the last line of equation \[30\] is reduced from the first order deviation by another factor of $\frac{KE_{nr}}{(m_j+m_k)c^2}$, similar to the second order relativistic correction for kinetic energy. The fact that it is strictly positive might raise some concerns, but these can be alleviated by noting that it is of the same magnitude as the contributions from anti-particle amplitudes generated during interactions, and several orders of magnitude smaller than radiative effects. It should also be noted that a relativistic characterization of interaction energies would include more than just scalar potentials.

The close qualitative and quantitative parallels between the deviations implied by equation \[2\] and the discrepancies entailed by the nonrelativistic Schrödinger equation suggest that the apparent violations of strict energy conservation are an artifact of the nonrelativistic formulation presented here. It is possible, therefore, that an extension of this collapse proposal that includes relativistic effects could be fully consistent with the major conservation laws.

Experimental tests of this proposal would require very highly sensitive measurements of possible nonlinear effects of correlated interactions. A relativistic extension that included a description of photons and higher interaction energies might offer additional experimental opportunities.

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