A Low-Subpacketization High-Performance MIMO Coded Caching Scheme

MohammadJavad Salehi, Hamidreza Bakhshzad Mahmoodi and Antti Tölli

Centre for Wireless Communications, University of Oulu, 90570 Oulu, Finland
E-mail: {firstname.lastname}@oulu.fi

Abstract—In this paper, we study how coded caching can be efficiently applied to multiple-input multiple-output (MIMO) communications. This is an extension to cache-aided multiple-input single-output (MISO) communications, where it is shown that with an $L$-antenna transmitter and coded caching gain $t$, a cumulative coded caching and spatial multiplexing gain of $t + L$ is achievable. We show that, interestingly, for MIMO setups with $G$-antenna receivers, a coded caching gain larger than MISO setups by a multiplicative factor of $G$ is possible, and the full coded caching and spatial multiplexing gain of $Gt + L$ is also achievable. Furthermore, we propose a novel algorithm for building low-subpacketization, high-performance MIMO coded caching schemes using a large class of existing MISO schemes.

Index Terms—coded caching, MIMO communications, low-subpacketization

I. INTRODUCTION

Wireless communication networks are under mounting pressure to support exponentially increasing volumes of multimedia content [1], as well as to support the imminent emergence of new applications such as wireless immersive viewing [2]. For the efficient delivery of such multimedia content, the work of Maddah-Ali and Niesen in [3] proposed the idea of coded caching as a means of increasing the data rates by exploiting cache content across the network. In a single-stream downlink network of $K$ cache-enabled users who can pre-fetch data into their cache memories, coded caching enables boosting the achievable rate by a multiplicative factor proportional to the cumulative cache size in the entire network. The key to achieving this speedup lies in the multicasting of carefully created codewords to different groups of users, such that each user can use its cache content to remove unwanted parts (i.e., the interference) from the received signal. With a library of $N$ equal-sized files, and given that each user can pre-fetch $M$ files into their cache memory during the so-called placement phase, coded caching enables serving groups of users of size $t + 1$ with any single transmission, where $t := \frac{KM}{N}$ is called the coded caching gain. In other words, with coded caching one can include $t + 1$ independent data streams within a single transmission, and hence, the achievable degrees of freedom (DoF) is increased from one to $t + 1$.

Motivated by the growing importance of multi-antenna wireless communications [4], Shariatpanahi et al. [5], [6] explored the cache-aided multiple-input single-output (MISO) setting, for which it revealed that the same coded caching gain $t$ could be achieved cumulatively with the spatial multiplexing gain. More exactly, for a downlink MISO setup with the transmitter-side multiplexing gain of $L$, the work in [6] developed a method that achieved a DoF of $t + L$, which was later shown in [7] to be optimal under basic assumptions of uncoded cache placement and one-shot linear data delivery. Of course, an even larger cache-aided DoF can be achieved for the same setup using interference-alignment techniques such as in [8]. However, such techniques are overly complex to implement in practice (cf. [3]), and hence, in this paper, we also aim at maximizing the DoF with the underlying assumptions of uncoded cache placement and one-shot linear data delivery.

Following the introduction of single- and multi-stream coded caching in [3], [2], many later works in the literature addressed their important scaling and performance issues. For MISO setups, in [10], the authors showed that using optimized beamformers instead of zero-forcing, one could achieve a better rate for communications at the finite signal-to-noise-ratio (SNR) regime. Also in the same paper, two interesting methods for adjusting the DoF and controlling the multiple access channel (MAC) size were introduced for managing the optimized beamformer design complexity. Similarly, the exponentially growing subpacketization issue (i.e., the number of smaller parts each file should be split into) was addressed thoroughly in the literature, for both single-stream (cf. [11]–[13]) and multi-stream (cf. [14]–[18]) setups. Most notably, for multi-stream MISO setups it was shown in [14]–[16] that using signal-level interference cancellation instead of the bit-level approach, one could substantially reduce the subpacketization requirement while achieving the same maximum DoF value of $t + L$. The difference between signal- and bit-level approaches is that with the former, cache-aided interference cancellation is done prior to decoding, while with the latter, it is performed after the received signal is decoded. Of course, as shown in [19], [20], signal-level interference cancellation modestly decreases the achievable rate (with respect to bit-level approaches), especially at the finite-SNR regime. Nevertheless, it does not harm the achievable DoF and yet enables simpler optimized beamformer designs, as discussed in [15].
In this paper, we extend the applicability of coded caching to multiple-input multiple-output (MIMO) communications with multi-stream receivers. We show that MIMO coded caching can be considered as an extension of the shared-cache setting studied in [21], [22]. In shared-cache setups, a number of single-stream users are aided by a smaller number of shared-cache nodes (which can be, e.g., helper nodes) while requesting data from a multi-stream transmitting server. To extend this model to MIMO coded caching, we keep the transmitter in place and replace each shared-cache node with a multi-stream receiver. Then, if the number of attached users to the shared-cache node \( k \) is \( G_k \), we set the receiver-side multiplexing gain of its equivalent multi-stream receiver to be \( G_k \). Such an extension paves the way for using the optimality results in [21], [22] to MIMO coded caching. Specifically, in case all the users in a MIMO setup can receive \( G \) independent streams (which is the case, for example, when they have \( G \) adequately-separated antennas) and \( \eta := \frac{G}{L} \) is an integer, we get the interesting result that the coded caching gain \( t \) can be multiplied by \( G \), and a cumulative single-shot DoF of \( Gt + L \) becomes achievable. We also propose a new algorithm for creating MIMO coded caching schemes, where we elevate baseline linear MISO schemes already available in the literature (with the signal-level cache cancellation approach) to be applicable in MIMO setups while achieving the increased DoF of \( G t + L \). This enables designing low-complexity, low-subpacketization MIMO schemes with full DoF using appropriate baseline MISO schemes such as the one in [15].

In this paper, we use the following notations. For any integer \( J \), we use \([J]\) to denote the set of numbers \( \{1, 2, \cdots, J\} \). Boldface upper- and lower-case letters indicate matrices and vectors, respectively, and calligraphic letters denote sets. Other notations are defined as they are used throughout the text.

II. SYSTEM MODEL

We consider a MIMO communication setup with \( K \) users and a single server, where the server has \( L \) transmit antennas and every user has \( G \) receive antennas. These antenna arrays enable spatial multiplexing gains of \( L \) and \( G \) to be achievable at the transmitter and receiver side, respectively. In other words, if the \( G \times L \) channel matrix from the transmitter to user \( k \in [K] \) is shown by \( \mathbf{H}_k \), the rank of every \( \mathbf{H}_k \) is at least \( G \), and the rank of the cumulative channel matrix, defined by the vertical concatenation of individual channel matrices as

\[
\mathbf{H} = \begin{bmatrix}
\mathbf{H}_1 \\
\vdots \\
\mathbf{H}_K
\end{bmatrix},
\]

is at least \( L \). As a result, the server can deliver at least \( L \) independent data streams to all the users, and each user can receive at least \( G \) independent streams. Let us denote the data stream \( g \in [G] \) at user \( k \in [K] \) with \( s^g_k \). Then, transmit beamformers \( \mathbf{w}_\mathcal{R} \in \mathbb{C}^{L \times 1} \) and receive beamformers \( \mathbf{u}_{k,g} \in \mathbb{C}^{G \times 1} \) can be built such that the zero-forcing set \( \mathcal{R} \) includes \( L - 1 \) streams, and for every \( s^g_k \in \mathcal{R} \) we have

\[
\mathbf{u}^H_{k,g} \mathbf{H}_k \mathbf{w}_\mathcal{R} = 0.
\]

As the size of \( \mathcal{R} \) is \( L - 1 \), each transmit beamformer removes the interference on \( L - 1 \) streams. For simplicity, we define

\[
\mathbf{h}_{k,g} = (\mathbf{u}^H_{k,g} \mathbf{H}_k)^H
\]

as the equivalent channel vector for the stream \( s^g_k \). In this paper, we assume that equivalent channel multipliers (i.e., \( \mathbf{h}_{k,g}^H \mathbf{w}_\mathcal{R} \)) can be estimated at the receivers.

The users request files from a library \( \mathcal{F} \) of \( N \) files, each with size \( F \) bits. Each user has a cache memory of size \( MF \) bits. For notational simplicity, we use a normalized data unit words, if the cache memory is shown by \( \mathcal{R} \). This can be done, for example, using downlink precoded pilots.

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where we have used $z_{k,g}(i) = u_{k,g}^H z_k(i)$. The goal is to design cache placement and transmission vectors such that for every $s_k^g \in S(i)$, $X_k^g(i)$ can be decoded interference-free from $y_k(i)$. As mentioned earlier, the performance metric is the number of simultaneously delivered independent data streams within each transmission, and hence, we want to maximize $|S(i)|$ for every time interval. The maximum value of $|S(i)|$ for a MISO setup is shown to be $t + L$. In this paper, we show that an increased DoF value of $Gt + L$ is achievable in a MIMO setup with a low subpacketization complexity.

### III. Signal-level Interference Cancellation

The MISO coded caching scheme in [14] introduced a novel cache-aided interference cancellation approach, different than [6], to reduce the subpacketization without altering the maximum achievable DoF. This approach, which we refer to as the signal-level approach, removes the interference using the cache contents before the signal is decoded at the receiver. This is in contrast with the classical bit-level approach, where the cache-aided interference removal is done after decoding the received signal. Many other papers in the literature, including the one in [15], have also used the signal-level approach to reduce the required subpacketization. It is worth mentioning that, although we consider only the subpacketization issue here, the signal-level approach also has other benefits such as enabling much simpler optimized beamformer designs, as demonstrated in [15].

The MIMO coded caching scheme proposed in this paper can be built using any baseline MISO scheme with the signal-level interference cancellation approach. Interestingly though, with minor modifications, any bit-level MISO coded caching scheme can be transformed into an equivalent signal-level MISO scheme, without any DoF loss. The key to this transformation is to decouple the codewords and use a separate beamformer for every data element being sent. The following example enlightens the difference between the signal- and bit-level approaches, and shows how a bit-level scheme can be transformed to an equivalent signal-level scheme.

#### Example 1.
Consider a small MISO setup with $K = 3$ users where the coded caching gain is $t = 1$ and the spatial multiplexing gain of $L = 2$ can be achieved at the transmitter. Let us consider the bit-level coded caching scheme in [6] as the baseline. This scheme requires each file $W \in F$ to be split into three subpackets $W_p, p \in [3]$. The user $k, k \in [3]$, stores subpackets $W_k$ of every file $W$ in its cache memory; i.e., the cache contents of user $k$ are $\{W_k ; \forall W \in F\}$. Then, during the delivery phase, if the files requested by users 1-3 are shown by $A, B$, and $C$, respectively, we transmit

$$x = (A_2 + B_1)w_3 + (A_3 + C_1)w_2 + (B_3 + C_2)w_1,$$

where $\oplus$ represents the bit-wise XOR operation, and $w_k$ is the zero-forcing vector for user $k$ (i.e., $h_k^H w_k = 0$, where $h_k$ is the channel vector from the transmitter to user $k$). Without loss of generality, let us consider the decoding process at user one. Following the zero-force beamforming definition, this user receives

$$y_1 = (A_2 + B_1)h_1^H w_3 + (A_3 + C_1)h_1^H w_2 + z_1,$$

where $z_1$ represents the AWGN at user one. The next step is to decode both $(A_2 + B_1)$ and $(A_3 + C_1)$ from $y_1$, using, for example, a successive interference cancellation (SIC) receiver. Finally, after decoding, user one has to use its cache contents to remove $B_1$ and $C_1$ from the resulting terms, to get $A_2$ and $A_3$ interference-free. As the cache-aided interference cancellation is done after the received signal is decoded at the receiver, we have a bit-level coded caching scheme here.

In order to transform this scheme into an equivalent signal-level scheme, we need to simply decouple the codewords (i.e., the XOR terms) and use a separate beamformer for every single data element. This means, instead of the transmission vector $x$, we transmit

$$\bar{x} = A_2v_3 + B_1w_2^1 + A_3w_2^1 + C_1w_2^2 + B_3w_2^1 + C_2w_2^2,$$

where the superscripts are used to distinguish two beamformers with the same zero-forcing set (but different powers). After the transmission of $\bar{x}$, user one receives

$$\bar{y}_1 = A_2h_1^T w_3^1 + B_1h_1^T w_3^2 + A_3h_1^T w_3^1 + C_1h_1^T w_3^2 + z_1.$$  

Then, in order to get its required data elements, i.e., $A_2$ and $A_3$, user one has to first build $B_1h_1^T w_3^2$ and $C_1h_1^T w_3^2$ using its cache contents and the acquired channel multiplier information, and remove these interference terms from the received signal before decoding it (otherwise, these interference terms will degrade the achievable rate severely). As the cache-aided interference cancellation is now done before the decoding process, this second scheme is using a signal-level approach.

For better clarification, the encoding and decoding processes for the proposed bit- and signal-level schemes in Example 1 are shown in Figure 1. Note that in this figure, the decoding process for both schemes is shown only for user one. With the same procedure outlined in Example 1, we can transform any MISO scheme with the bit-level approach to another one with the signal-level approach. Hence, although our MIMO coded caching scheme requires a baseline signal-level MISO scheme, this condition does not in fact limit its applicability. Nevertheless, throughout the rest of this paper, we use the scheme in [15], due to its lower subpacketization, as our baseline scheme.

![Figure 1: Bit- and Signal-Level Schemes in Example 1](image-url)
IV. MIMO Coded Caching

A. The Elevation Procedure

As stated earlier, our new algorithm for MIMO coded caching works through elevation of a baseline signal-level MISO scheme. In a nutshell, this elevation procedure works as follows: for a given $K$-user MIMO network with coded caching gain $t$ and spatial multiplexing gains of $L$ and $G$ at the transmitter and receivers, we first consider a virtual MISO network with the same number of $K$ users and caching gain $t$, but with the spatial multiplexing gain of $\eta = \frac{L}{G}$ at the transmitter. Then, we design cache placement and transmission vectors for the virtual network following one of the available signal-level MISO schemes in the literature. Finally, cache contents for users in the real network are set to be the same as the virtual network, but transmission vectors of the virtual network are stretched to support multi-stream receivers and the increased DoF.

The stretching process of the transmission vectors of the virtual network is straightforward. Assume the zero-forcing beamformers for the virtual network are shown with $w_T$, where $T$ denotes the set of users where the data is zero-forced. Then, a transmission vector $x$ of the virtual network is a linear combination of $t + \eta$ terms $W^q_p(k)w_T(k)$, where $W^q_p(k)$ denotes a subpacket of the file $W(k)$ requested by user $k$ (the subpacket index is clarified by $p$ and $q$), and $w_T(k)$ is the beamformer vector associated with $W^q_p(k)$. Now, for stretching, for every term $W^q_p(k)w_T(k)$ in $x$, we first split the subpacket $W^q_p(k)$ into $G$ smaller parts $W^q_p,g(k)$, $g \in [G]$, and then replace the term with

$$W^q_p(k)w_T(k) \rightarrow W^q_p,1(k)w_{R_1} + \cdots + W^q_p,G(k)w_{R_G},$$

(11)

where the zero-forcing sets $R_g$ for the real MIMO network are built as

$$R_g = \bigcup_{k \in T(k)} \{s^1_k, \ldots, s^G_k\} \cup \{s^1_k, \ldots, s^G_k\} \setminus \{s^G_k\}. \quad (12)$$

In other words, instead of a single data stream $W^q_p(k)$ for user $k$ in the virtual network, we consider $G$ parallel streams $W^q_p,g(k)$ for user $k$ in the real network. Moreover, if the zero-forcing set for $W^q_p(k)$ in the virtual network is $T(k)$, the interference from $W^q_p,g(k)$ in the real network should be zero-forced at:

1) Every stream sent to every user $\bar{k} \in T(k)$. As $|T(k)| = \eta - 1$ and each stream in the virtual network is transformed into $G$ streams in the real network, this includes $G \times (\eta - 1)$ streams in total;
2) Every other stream sent to user $k$. This includes $G - 1$ streams in total.

As a result, the interference from $W^q_p,g(k)$ must be zero-forced on a total number of

$$G(\eta - 1) + G - 1 = \eta G - 1 = L - 1 \quad (13)$$

Note that the virtual network has a MISO setup, and hence, each user can receive only one data stream at a time. Moreover, virtual beamformer vectors $w_T$ are defined only to clarify the transmission (and decoding) process in the virtual network, and hence, it is not necessary to calculate their exact values at any step.

B. A Clarifying Example

Let us consider a small network with $K = 6$ users and caching gain $t = 1$, where the spatial multiplexing gains of $L = 4$ and $G = 2$ are achievable at the transmitter and receivers, respectively. Using MISO schemes, the maximum achievable DoF for this network is $L + t = 5$. However, with the proposed elevation algorithm, the achievable DoF can be increased to $Gt + L = 6$. As mentioned earlier, without any loss of generality, we assume the reduced-subpacketization scheme in [15] is chosen as the baseline MISO scheme.

The first step is to find proper parameters for the virtual MISO network. Clearly, the virtual network has the same number of $K = 6$ users and caching gain $t = 1$, but the spatial multiplexing gain of $\eta = \frac{L}{G} = 2$ at the transmitter. Cache placement in the virtual network directly follows that of [15], i.e., every file $W \in F$ is first split into $K = 6$ equal-sized packets denoted by $W_p, p \in [6]$, and then, each packet is split into $t + \eta = 3$ smaller subpackets denoted by $W^q_p, q \in [3]$. Finally, every user $k$ caches $W^q_p, k$ for every $W \in F$ and $q \in [3]$. For example, user one caches $\{W^q_1, W^q_2, W^q_3\}$ for every file $W$ in the library.

For simplicity, let us use $W(1) \equiv A$, $W(2) \equiv B$, ..., and $W(6) \equiv F$ (note that it is still possible for some users to request the same file). Using the coded caching scheme in [15] for the virtual network, the first and second transmission vectors (corresponding to the transmissions in the first and second time intervals) are built as

$$x(1) = A^1_2w_3 + B^1_1w_3 + C_1w_2,$$
$$x(2) = A^2_3w_4 + C^2_2w_4 + D^2_1w_3, \quad (14)$$

where the brackets for the zero-forcing sets are ignored for notational simplicity. For better clarification, graphical representations for $x(1)$ and $x(2)$ are provided in Figure 2. These representations follow the same principles as in [15], i.e., a black cell means (every subpacket of) the respective data part

Note that in this case, multiple receive antennas can be used for achieving an additional beamforming gain (cf. [10]). However, they do not result in any increase in the DoF value.
is available in the cache memory, and a gray cell indicates that (a sub)packet of (the) respective part is sent during the transmission.

Now, using the stretching mechanism in (11), the stretched version of \( \mathbf{x}(1) \) can be written as

\[
\mathbf{x}(1) = A_1^{1,1} \mathbf{w}_{s_1, s_1} + A_1^{1,2} \mathbf{w}_{s_1, s_2} + A_1^{2,1} \mathbf{w}_{s_2, s_1} + A_1^{2,2} \mathbf{w}_{s_2, s_2} + B_1^{1,1} \mathbf{w}_{s_1, s_1} + B_1^{1,2} \mathbf{w}_{s_1, s_2} + B_1^{2,1} \mathbf{w}_{s_2, s_1} + B_1^{2,2} \mathbf{w}_{s_2, s_2} + C_1^{1,1} \mathbf{w}_{s_1, s_1} + C_1^{1,2} \mathbf{w}_{s_1, s_2} + C_1^{2,1} \mathbf{w}_{s_2, s_1} + C_1^{2,2} \mathbf{w}_{s_2, s_2} + D_1^{1,1} \mathbf{w}_{s_1, s_1} + D_1^{1,2} \mathbf{w}_{s_1, s_2} + D_1^{2,1} \mathbf{w}_{s_2, s_1} + D_1^{2,2} \mathbf{w}_{s_2, s_2}
\]

where \( I_1(i) \) is the interference term at user one at the first time interval, defined as

\[
I_1(i) = B_1^{1,1} \mathbf{w}_{s_1, s_1} + B_1^{1,2} \mathbf{w}_{s_1, s_2} + B_1^{2,1} \mathbf{w}_{s_2, s_1} + B_1^{2,2} \mathbf{w}_{s_2, s_2} + C_1^{1,1} \mathbf{w}_{s_1, s_1} + C_1^{1,2} \mathbf{w}_{s_1, s_2} + C_1^{2,1} \mathbf{w}_{s_2, s_1} + C_1^{2,2} \mathbf{w}_{s_2, s_2} + D_1^{1,1} \mathbf{w}_{s_1, s_1} + D_1^{1,2} \mathbf{w}_{s_1, s_2} + D_1^{2,1} \mathbf{w}_{s_2, s_1} + D_1^{2,2} \mathbf{w}_{s_2, s_2}.
\]

The next step is to multiply the received signal \( y_1(1) \) by receive beamformers \( \mathbf{u}_{1,1} \) and \( \mathbf{u}_{1,2} \), in order to extract \( s_1 \) and \( s_2 \), respectively. After multiplication by \( \mathbf{u}_{1,1} \), we get

\[
y_1(1) = \mathbf{u}_{1,1}^H y_1(1) = \mathbf{h}_{1,1}^H \mathbf{x}(1) + z_{1,1}(1) = A_2^{1,1} \mathbf{h}_{1,1}^H \mathbf{w}_{s_1, s_1} + A_2^{1,2} \mathbf{h}_{1,1}^H \mathbf{w}_{s_1, s_2} + A_2^{2,1} \mathbf{h}_{1,1}^H \mathbf{w}_{s_2, s_1} + A_2^{2,2} \mathbf{h}_{1,1}^H \mathbf{w}_{s_2, s_2} + B_2^{1,1} \mathbf{h}_{1,1}^H \mathbf{w}_{s_1, s_1} + B_2^{1,2} \mathbf{h}_{1,1}^H \mathbf{w}_{s_1, s_2} + B_2^{2,1} \mathbf{h}_{1,1}^H \mathbf{w}_{s_2, s_1} + B_2^{2,2} \mathbf{h}_{1,1}^H \mathbf{w}_{s_2, s_2} + C_2^{1,1} \mathbf{h}_{1,1}^H \mathbf{w}_{s_1, s_1} + C_2^{1,2} \mathbf{h}_{1,1}^H \mathbf{w}_{s_1, s_2} + C_2^{2,1} \mathbf{h}_{1,1}^H \mathbf{w}_{s_2, s_1} + C_2^{2,2} \mathbf{h}_{1,1}^H \mathbf{w}_{s_2, s_2} + D_2^{1,1} \mathbf{h}_{1,1}^H \mathbf{w}_{s_1, s_1} + D_2^{1,2} \mathbf{h}_{1,1}^H \mathbf{w}_{s_1, s_2} + D_2^{2,1} \mathbf{h}_{1,1}^H \mathbf{w}_{s_2, s_1} + D_2^{2,2} \mathbf{h}_{1,1}^H \mathbf{w}_{s_2, s_2} + E_1^{1,1} \mathbf{h}_{1,1}^H \mathbf{w}_{s_1, s_1} + E_1^{1,2} \mathbf{h}_{1,1}^H \mathbf{w}_{s_1, s_2} + E_1^{2,1} \mathbf{h}_{1,1}^H \mathbf{w}_{s_2, s_1} + E_1^{2,2} \mathbf{h}_{1,1}^H \mathbf{w}_{s_2, s_2} + F_1^{1,1} \mathbf{h}_{1,1}^H \mathbf{w}_{s_1, s_1} + F_1^{1,2} \mathbf{h}_{1,1}^H \mathbf{w}_{s_1, s_2} + F_1^{2,1} \mathbf{h}_{1,1}^H \mathbf{w}_{s_2, s_1} + F_1^{2,2} \mathbf{h}_{1,1}^H \mathbf{w}_{s_2, s_2},
\]

where \( \mathbf{h}_{1,1} \) is the interference vector at user one, by using the stretching mechanism in (11), during the stretching process for the original MIMO network is replaced by the virtual MISO network has the same number of users and coded caching gain \( t \), but the transmitter-side multiplexing gain is reduced to \( \eta = \frac{L}{\gamma} \). So, each transmission vector in the virtual network delivers data to \( t + \eta \) users simultaneously.
parallel streams, and hence, the DoF is increased to \(Gt + L\). As discussed in [21], this DoF value is also optimal among any linear coded caching scheme with uncoded cache placement and single-shot data delivery.

The subpacketization value of the resulting MIMO scheme depends on the subpacketization requirement of the baseline MISO scheme. If the subpacketization of the baseline scheme is \(f(K, t, L)\), where \(K\), \(t\), and \(L\) represent the user count, the coded caching gain, and the transmitter-side multiplexing gain, respectively, the resulting MIMO scheme from the proposed elevation mechanism will have the subpacketization of

\[
G \times f(K, t, \eta) .
\]

(23)

This is because the subpacketization requirement for the virtual network is \(f(K, t, \eta)\), and during the elevation process, each subpacket is further split into \(G\) smaller parts. So, for example, if the low-subpacketization scheme in [15] is selected as the baseline MISO scheme, the subpacketization requirement would be

\[
G \times K(t + \eta) = K(Gt + L) .
\]

(24)

However, the scheme in [15] is applicable to the virtual network only if \(\eta \geq t\), or \(G \geq Gt\). On the other hand, if we select the original MISO scheme in [9] as the baseline, the resulting MIMO scheme would have the subpacketization requirement of

\[
G \times \binom{K}{t} \left( \frac{K - t - 1}{\eta - 1} \right),
\]

(25)

and the scheme would be applicable regardless of \(t\), \(L\), and \(G\) values as long as \(\eta\) is an integer. In summary, depending on the selection of the baseline scheme, the resulting MIMO scheme can benefit from a very low subpacketization complexity. The subpacketization growth with the user count can even be linear, if the baseline scheme is chosen as the one in [15].

V. CONCLUSION AND FUTURE WORK

In this paper, we investigated how coded caching schemes can be applied to multi-input multi-output (MIMO) communication setups. We introduced a novel mechanism to elevate any multi-input single-output (MISO) scheme in the literature to be applicable to MIMO settings. The elevation mechanism keeps the properties of the baseline scheme (e.g., the low subpacketization requirement) while achieving an increased coded caching gain. We also used results from the shared-cache model to claim the optimality of the achievable degrees of freedom (DoF) of the resulting scheme among the class of linear, single-shot caching schemes with uncoded data placement. Overall, the proposed elevation mechanism allows designing efficient, low-complexity coded caching schemes for any MIMO network.

The discussions in this paper were focused on communications in the high-SNR regime where the DoF metric is an appropriate performance indicator. However, a thorough analysis of the performance at the finite-SNR regime is due for later work. Also, investigating the applicability of the proposed scheme under dynamic network conditions where users can freely join/leave the network is part of the ongoing research.

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