Kaon production in heavy-ion collisions and maximum mass of neutron stars

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We determine an ‘empirical’ kaon dispersion relation by analysing and fitting recent experimental data on kaon production in heavy-ion collisions. We then investigate its effects on hadronic equation of state at high densities and on neutron star properties. We find that the maximum mass of neutron stars can be lowered by about 0.4\(M_\odot\), once kaon condensation as constrained by our empirical dispersion relation is introduced. We emphasize the growing interplay between hadron physics, relativistic heavy-ion physics and the physics of compact objects in astrophysics.

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There is currently growing interplay between the physics of hadrons (especially the properties of hadrons in dense matter which might reflect spontaneous chiral symmetry breaking and its restoration), the physics of relativistic heavy-ion collisions (from which one might extract hadron properties in dense matter), and the physics of compact objects in astrophysics (which needs as inputs the information gained from the first two fields). A notable example is the kaon \((K\) and \(\bar{K}\)), which, being a Goldstone boson with strangeness, plays a special role in all of the three fields mentioned.

Ever since the pioneering work of Kaplan and Nelson \[1\] on the possibility of kaon condensation in nuclear matter, much theoretical effort has been devoted to the study of kaon properties in dense matter. Brown \et al. \[2\] have carried out a detailed study of free-space and in-medium kaon-nucleon scattering using chiral perturbation theory. Yuba \et al. \[3\] have extended the chiral perturbation calculation to include the coupled-channel effects which are important for the \(K^-\) meson \[4\]. Another type of study, which is based on the extension of the Walecka mean-field model from SU(2) to SU(3), was pursued by Schaffner \et al. \[5\] and Knorren \et al. \[6\]. Although quantitatively results from these different models are not identical, qualitatively, a consistent picture has emerged; namely, in nuclear matter the \(K^+\) feels a weak repulsive potential, whereas the \(K^-\) feels a strong attractive potential.

Measurements of kaon spectra and flow have been carried out in heavy-ion collisions at SIS (1-2 AGeV), AGS (10 AGeV), and SPS (200 AGeV) energies \[7\]. By comparing transport model predictions with experimental data, one can learn not only the global reaction dynamics, but more importantly, the kaon properties in dense matter. Of special interest is kaon production in heavy-ion collisions at SIS energies, as it has been shown that particle production at subthreshold energies is sensitive to its properties in dense matter \[8\]. Recently, high quality data concerning \(K^+\) and \(K^-\) production in heavy-ion collisions at SIS energies have been published by the KaoS collaboration at GSI \[9\]. The KaoS data show that the \(K^-\) yield at \(1.8\) AGeV (projectile nucleus kinetic energy in the laboratory frame) agrees roughly with the \(K^+\) yield at \(1.0\) AGeV. This is a nontrivial observation. These beam energies were purposely chosen, such that the Q-values for \(NN \rightarrow NK\) and \(NN \rightarrow N\bar{K}\) are both about -230 MeV. Near their respective production thresholds, the cross section for the \(K^-\) production in proton-proton interactions is one to two orders of magnitude smaller than that for \(K^+\) production \[10\]. In addition, antikaons are strongly absorbed in heavy-ion collisions, which should further reduce the \(K^-\) yield. The KaoS results of \(K^-/K^+ \sim 1\) indicate thus the importance of kaon medium effects which act oppositely on \(K^+\) and \(K^-\) production in nuclear medium.

Studies of neutron star properties also have a long history. A recent compilation by Thorsett quoted by Brown \[11\] shows that well-measured neutron star masses are all less than 1.5\(M_\odot\). On the other hand, most of the theoretical calculations based on conventional nuclear equations of state (EOS) predict a maximum neutron state mass above 2\(M_\odot\). The EOS can, therefore, be substantially softened without running into contradiction with observation. Various scenarios have been proposed that can lead to a soft EOS, including the high-order self-interactions of the vector field \[12\], the possibility of kaon condensation \[13\], the existence of hyperons \[14\], and the transition to quark matter \[15\]. All these possibilities need to be examined against the available empirical information from, e.g., relativistic heavy-ion collisions.

The chief aim of this paper is to determine, from the recent KaoS data on kaon production, together with previous analysis of nucleon flow, kaon flow and dilepton spectra \[16\], an ‘empirical’ kaon dispersion relation in dense matter. We will show that these data are consistent with the scenario that the \(K^+\) feels a weak repulsive potential and the \(K^-\) a strong attractive potential, as predicted by the chiral perturbation calculation. We then study the effects of this empirical dispersion relation on the possibility of kaon condensation and on the neutron star properties. We find that \(K^-\) condensation happens at about 3\(\rho_0\), and the maximum mass of neutron stars is lowered by about 0.4\(M_\odot\) once the kaon condensation is introduced. These values change by about 20\% when different nuclear equations
of state are used (see Ref. [22] for a detailed discussion).

We use the relativistic transport model for the description of heavy-ion collisions and for the calculation of kaon production [9]. The nucleon dynamics is governed by the chiral Lagrangian recently developed by Furnstahl, Tang and Serot [23], which is derived using dimensional analysis, naturalness arguments, and provides a very good description of nuclear matter and finite nuclei. Furthermore, a recent analysis [21] showed that this model reproduces nicely the nucleon flow [18] and dilepton spectra [20] in heavy-ion collisions, indicating that its extrapolation to $2-3\rho_0$ is consistent with empirical information. We will thus use this model as our basis for the determination of kaon dispersion relation and for the analysis of neutron star properties.

In the mean-field approximation, the energy density for the general case of asymmetric nuclear matter is given by

$$
\varepsilon_N = \frac{2}{(2\pi)^3} \int_0^{Kf_P} dk \sqrt{k^2 + m_{N}^{*2}} + \frac{2}{(2\pi)^3} \int_0^{Kf_N} dk \sqrt{k^2 + m_{N}^{*2}} + \frac{1}{2\rho} W^2 - \frac{1}{2\rho} R^2 + \frac{1}{2S} \Phi^2 + \frac{S'}{4C_V} d^2 \left\{ \left(1 - \frac{\Phi}{S'} \right)^{-4/d} \left[ \frac{1}{d} \ln \left(1 - \frac{\Phi}{S'} \right) - \frac{1}{4} \right] + \frac{1}{4} \right\} - \frac{\xi}{24} W^4 - \frac{\eta}{2C_V S} \Phi W^2. \tag{1}
$$

The nucleon effective mass $m_{N}^{*}$ is related to its scalar field $\Phi$ by $m_{N}^{*} = m_N - \Phi$. $W$ and $R$ are the isospin-even and isospin-odd vector potentials, respectively. The last three terms give the self-interactions of the scalar field, the vector field, and the coupling between them. The meaning and values of various parameters in Eq. (1) can be found in [23]. In this work, we use the parameter set T1 listed in Table 1 of [23].

![FIG. 1. Proton and π− transverse mass spectra in central Ni+Ni collisions at 1.06 AGeV.](image-url)
From Eq. (1), we can derive a relativistic transport model for heavy-ion collisions. At SIS energies, the colliding system consists mainly of nucleons, delta resonances, and pions. While medium effects on pions are neglected, nucleons and delta resonances propagate in a common mean-field potential according to the Hamilton equation of motion,

\[
\frac{dx}{dt} = \frac{p^*}{E^*}, \quad \frac{dp}{dt} = -\nabla_x (E^* + W),
\]

where \( E^* = \sqrt{p^2 + m^2} \). These particles also undergo stochastic two-body collisions \(^1\). In Fig. 1 we compare our results for proton and pion transverse mass spectra in central Ni+Ni collisions with experimental data from the FOPI collaboration \(^2\). The nice agreement with the data provides further support to the use of the chiral Lagrangian of \(^2\) in the present analysis.

In heavy-ion collisions at SIS energies, kaons can be produced from pion-baryon and baryon-baryon collisions. For the former we use cross sections obtained in the resonance model by Tsushima et al. \(^25\). For the latter the cross sections obtained in the one-boson-exchange model of Ref. \(^26\) are used. Both models describe the available experimental data very well. For antikaon production from pion-baryon collisions we use the parameterization proposed by Sibirtsev et al. \(^27\). For baryon-baryon collisions, we use a somewhat different parameterization, which describes the experimental data better, than Ref. \(^27\). In addition, the antikaon can also be produced from strangeness-exchange processes such as \( \pi Y \rightarrow \bar{K}N \) where \( Y \) is a hyperon. The cross sections for these processes are obtained from the inverse ones, \( K\bar{N} \rightarrow \pi Y \), by the detailed-balance relation. The latter cross sections, together with the \( K\bar{N} \) elastic and absorption cross sections, are parameterized based on the available experimental data \(^1\). The details about elementary cross sections, the transport model, and the neutron star calculation will be given elsewhere \(^28\).

From the chiral Lagrangian the kaon and antikaon in-medium energies can be written as \(^2\)

\[
\omega_K = \left[ m_K^2 + k^2 - a_K \rho_S + (b_K \rho)^2 \right]^{1/2} + b_K \rho
\]

\[
\omega_{\bar{K}} = \left[ m_{\bar{K}}^2 + k^2 - a_{\bar{K}} \rho_S + (b_{\bar{K}} \rho)^2 \right]^{1/2} - b_{\bar{K}} \rho
\]

where \( b_K = 3/(8f_\pi^2) \approx 0.333 \text{ GeVfm}^3 \), \( a_K \) and \( a_{\bar{K}} \) are two parameters that determine the strength of the attractive scalar potential for kaon and antikaon, respectively. If one considers only the Kaplan-Nelson term, then \( a_K = a_{\bar{K}} = \Sigma_{KN}/f_\pi^2 \). In the same order, there is also the range term which acts differently on kaon and antikaon, and leads to different scalar attractions. Since the exact value of \( \Sigma_{KN} \) and the size of the higher-order corrections are still under intensive debate, we take the point of view that \( a_{K,\bar{K}} \) can be treated as free parameters and try to constrain them from the experimental observables. Since the \( KN \) interaction is relatively weak, impulse approximation should be reasonable at low densities. This provides some constraints on \( a_K \). We find that \( a_K \approx 0.22 \text{ GeV}^2\text{fm}^3 \), corresponding to \( \Sigma_{KN} \approx 400 \text{ MeV} \), gives a repulsive \( K^+ \) potential of about 20 MeV at normal nuclear matter density, slightly smaller than the 25 MeV found in \(^29\). We will show later that this value also gives a good fit to the \( K^+ \) spectra in heavy-ion collisions.

From the chiral Lagrangian we can also derive equations of motion for kaons \(^30\)

\[
\frac{dx}{dt} = \frac{p^*}{\omega_{K,\bar{K}} \mp b_K \rho}, \quad \frac{dp}{dt} = -\nabla_x \omega_{K,\bar{K}}.
\]

The minus and the plus sign correspond to kaon and antikaon, respectively.

For \( K^+ \) and \( K^- \) production in heavy-ion collisions, we consider two scenarios; namely, with and without kaon medium effects. As mentioned, we use \( a_K \approx 0.22 \text{ GeV}^2\text{fm}^3 \) for \( K^+ \). For \( K^- \), we adjust \( a_{\bar{K}} \) such that we achieve a good fit to the experimental \( K^- \) spectra. We find \( a_{\bar{K}} \approx 0.45 \text{ GeV}^2\text{fm}^3 \), which leads to a \( K^- \) potential of about 110 MeV at normal nuclear matter density. This is somewhat smaller, in magnitude, than the ‘best’ value of \( -200 \pm 20 \) MeV extracted from kaonic atoms \(^31\). The latter value, however, depends sensitively on the extrapolation procedure from the surface of nuclei to their interiors \(^31\). On the other hand, kaon production in heavy-ion collisions and neutron star calculations are sensitive chiefly to higher densities (2-3\( \rho_0 \)), where our value should be more relevant.
FIG. 2. $K^+$ (upper window), $K^-$ (middle window), and $K^+/K^-$ (lower window) kinetic energy spectra in Ni+Ni collisions.

The results for the $K^+$ and $K^-$ kinetic energy spectra are shown Fig. 2. The solid and dotted histograms give the results with and without kaon medium effects, respectively. The open circles are the experimental data from the KaoS collaboration [10]. For the $K^+$, it is seen that the results with kaon medium effects are in good agreement with the data, while those without kaon medium effects slightly overestimate the data. We note that the kaon feels a slightly repulsive potential; thus the inclusion of the kaon medium effects reduces the kaon yield. For the $K^-$, it is seen that without medium effects, our results are about a factor 3-4 below the experimental data. With the inclusion of the medium effects which reduces the antikaon production threshold, the $K^-$ yield increases by about a factor of 3 and our results are in good agreement with the data. This is similar to the findings of Cassing et al. [32]. For both $K^+$ and $K^-$, the differences between the two scenarios are most pronounced at low kinetic energies. The experimental data at these momenta will be very useful in discriminating the two scenarios.

The effects of kaon and antikaon mean-field potentials can be more clearly seen by looking at their ratio as a function of the kinetic energy, which is shown in the lower window of Fig. 2. Without kaon medium effects, the $K^+/K^-$ ratio decreases from about 7 at low kinetic energies to about 1 at high kinetic energies, which is in complete disagreement with the data. Since the antikaon absorption cross section by nucleons becomes large at low momentum, low-momentum antikaons are more strongly absorbed than high-momentum ones. This makes the $K^+/K^-$ ratio increase with decreasing kinetic energies. When medium effects are included, we find that the $K^+/K^-$ ratio is almost unity in the entire kinetic energy region, which agrees very well with the data. The shapes of the $K^+$ and $K^-$ spectra change in opposite ways in the presence of their mean-field potential. Kaons are 'pushed' to high momenta by the repulsive potential, while antikaons are 'pulled' to low momenta. The good description of the $K^+/K^-$ ratio, together with the fit to the kaon flow in heavy-ion collisions [19,30], gives us confidence that our in-medium kaon and antikaon dispersion relations are reasonable.
We take the antikaon dispersion relation constrained by the heavy-ion data as empirical indication of an attractive antikaon potential in dense matter. We combine this with the energy density of Eq. (1) for nuclear matter, and calculate neutron star properties. We find that around $3\rho_0$, the effective mass of the $K^-$ drops below the chemical potential of the electron, indicating the onset of kaon condensation. After this the $K^-$ density increases rapidly, leading to a large proton fraction in the neutron star. The results for neutron star mass as function of central density is shown in Fig. 1, where the solid and dotted curves give the results with and without $K^-$, respectively. It is seen that the maximum mass of the neutron stars is reduced by about $0.4M_\odot$ with the introduction of the kaon condensation. Both the critical density for kaon condensation and the amount of lowering in the maximum neutron star mass change by about 20% when different nuclear equations of state are used. The exact value of the maximum neutron star mass, on the other hand, depends more sensitively on the particular nuclear EOS used, as discussed in [13].

It should be mentioned that alternative explanations for lowering the maximum neutron star mass have been proposed. Of particular interest is the introduction of $\Sigma^-$ hyperons in [15]. This is a complementary rather than competing scenario, since the $\Sigma^-$-particle–neutron-hole state has a $p$-wave coupling to the $K^-$. A unification of the scenarios can be achieved by introducing the ‘kaesobar’, a linear combination of $K^-$ and $\Sigma^-$-particle–neutron-hole [33], but the results of the present work will not be strongly modified.

In summary, we studied $K^+$ and $K^-$ production in Ni+Ni collisions at 1-2 AGeV, based on the relativistic transport model including the strangeness degrees of freedom. We found that the recent experimental data from the KaoS collaboration are consistent with the predictions of chiral perturbation theory that the $K^+$ feels a weak repulsive potential and the $K^-$ feels a strong attractive potential in the nuclear medium. Using the kaon in-medium properties constrained by heavy-ion data, we have studied the possibility of kaon condensation and its effects on neutron star properties. The critical density for kaon condensation was found to be about $3\rho_0$, and the maximum mass of neutron stars was found to be reduced by about $0.4M_\odot$ once kaon condensation is introduced. We have emphasized the growing interdependence of hadron physics, relativistic heavy-ion physics and the physics of compact stars in...
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