Regularized Algebraic Nets for

General Covariant QFT on Differentiable Manifolds

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Abstract

Quantum general relativity may be considered as generally covariant QFT on differentiable manifolds, without any a priori metric structure. The kinematically covariance group acts by general diffeomorphisms on the manifold and by automorphisms on the isotonic net of *-algebras encoding the QFT, while the algebra of observables is covariant under the dynamical subgroup of the general diffeomorphism group.

Here, I focus on an algebraic implementation of the dynamical subgroup of dilations. Introducing an small and large scale cutoffs algebraically, their usual a priori conflict with general covariance is avoided. Thereby, a commutant duality between the minimal and maximal algebra is proposed. This allows to extract the modular structure, which is again related to the dilations.

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1 Introduction

Observation procedures represent the abstract kinematical framework for possible preparations of measurements, while the observables encode the kinds of questions one can ask from the physical system. The importance of this distinction was observed by Ekstein [1] about 30 years ago. Since then practically any reasonable attempt for a constructive quantum theory in general, and for a constructive quantum field theory (QFT) in particular, is striving for a consistent implementation of observation procedures and observables.

The covariance group of the observation procedures reflects the general (a priori) redundancy of their mathematical implementation. The more sophisticated the structure of the observation procedures, the smaller the covariance group will be in general. E.g. in [2] the kinematical observation procedures are given by a network of discrete vertices of a specific Riemannian surface embedded in a $3 + 1$-dimensional space-time $M$, whence the covariance group is only that subgroup of Diff($M$) which leaves this structure invariant. In general, it is a difficult question, how much structure might be put on the observation procedures. For the following however, I will just follow a common philosophy of general covariance, to impose as little a priori structure as possible.

In a concrete observation the kinematical covariance will be broken. So in [2] a concrete local observation requires the explicit selection of one of many a priori equivalent vertices, whence it breaks the covariance which holds for the network of vertices as a whole. In the examples of [1] the kinematical covariance was assumed to be broken in a concrete observation by a dynamical interaction with external fields. However, irrespectively of the loss of covariance in a concrete observation, the action of the covariance group may still be well defined on the observation procedures. In any case, the loss of covariance in a concrete observation is related to a specific structure of the state of the physical system.
Let us examine now the consequences of this breaking of general covariance within an algebraic approach to a generally covariant quantum field theory. The first step in this direction was actually already done in [3] and picked up further in [4]. The principle of locality is at the heart of the constructive approach to quantum field theory [5]. Here it is kept in form of the demand that, observation procedures correspond to possible preparations of localized measurements in finite regions. Note that finiteness is a purely topological notion. I do not assume here any priori notion of neither a metric, time, nor even a causal structure. Hence, on different regions there will be no a priori causal relations between observables. It was shown in [3] that, for the net of subalgebras of a Weyl algebra, it is indeed possible to work with a flexible notion of causality rather than with a rigidly given one.

Although in principle it might be possible to construct a net together with its underlying manifold from a partial order via inclusion of the algebras themselves (cf. [7]), we will start in Sect. 2 with a net of *-algebras on a differentiable manifold. On this net, a physical state induces dynamical relations, whence the algebra of observables is covariant under the dynamical subgroup of the general diffeomorphism group. The present examinations emphasize on the dynamical subgroup of dilations. Sect. 3 is devoted to the implementation of a a small and large scale regularization indirectly, thus avoiding the usual direct conflict between cutoffs and general covariance. A new commutant duality between the corresponding minimal and maximal algebra is introduced. In Sect. 4 this duality, together with the isotony property, is used to extract the modular structure. The latter is related to local dilations on the net. Sect. 5 concludes with a brief discussion of some possible implications of the proposed structure for quantum general relativity and a posteriori notions of time and causality.
2 Generally covariant nets of algebras

Let us consider a net on a differentiable manifold $M$, which associates to each open set $O \in M$ a $\ast$-algebra $A(O)$ such that isotony,

$$O_1 \subset O_2 \Rightarrow A(O_1) \subset A(O_2),$$

holds. The following investigations might be seen as an attempt to understand some aspects of quantum field theory (QFT) on differentiable manifolds. This is indeed a very promising approach to quantum general relativity [4]. Selfadjoint elements of $A(O)$ may be interpreted as observation procedures, i.e. possible prescriptions for laboratory measurements in $O$.

There should not be any a priori relations between observation procedures associated with disjoint regions. In other words, the net $A := \bigcup_{O} A(O)$ has to be free from any relations which exceed its mere definition.

This interpretation allows us to extend the Diff($M$) covariance from the underlying manifold $M$ to the net of algebras, on which Diff($M$) then acts by automorphisms, i.e. each diffeomorphism $\chi \in \text{Diff}(M)$ induces an automorphism $\alpha_{\chi}$ of the observation procedures such that

$$\alpha_{\chi}(A(O)) = A(\chi(O)).$$

The state of a physical system is mathematically described by a positive linear functional $\omega$ on $A$. Given the state $\omega$, one gets via the GNS construction a representation $\pi^{\omega}$ of $A$ by a net of operator algebras on a Hilbert space $\mathcal{H}^{\omega}$ with a cyclic vector $\Omega^{\omega} \in \mathcal{H}^{\omega}$. The GNS representation $(\pi^{\omega}, \mathcal{H}^{\omega}, \Omega^{\omega})$ of any state $\omega$ has a so called folium $\mathcal{F}^{\omega}$, given as the family of those states $\omega_{\rho} := \text{tr}\rho \pi^{\omega}$ which are defined by positive trace class operators $\rho$ on $\mathcal{H}^{\omega}$.

Once a physical state $\omega$ has been specified, one can consider in each algebra $A(O)$ the equivalence relation

$$A \sim B : \iff \omega'(A - B) = 0, \ \forall \omega' \in \mathcal{F}^{\omega}. \quad (2.3)$$
These equivalence relations generate a two-sided ideal $\mathcal{I}^\omega(\mathcal{O}) := \{ A \in \mathcal{A}(\mathcal{O}) | \omega'(A) = 0 \}$ in $\mathcal{A}(\mathcal{O})$. The algebra of observables $\mathcal{A}^\omega_{\text{obs}}(\mathcal{O}) := \pi^\omega(\mathcal{A}(\mathcal{O}))$ may be constructed from the algebra of observation procedures $\mathcal{A}(\mathcal{O})$ by taking the quotient

$$
\mathcal{A}^\omega_{\text{obs}}(\mathcal{O}) := \mathcal{A}(\mathcal{O})/\mathcal{I}^\omega(\mathcal{O}).
$$

Since any diffeomorphism $\chi \in \text{Diff}(M)$ induces an automorphism $\alpha_\chi$ of the observation procedures, one may ask whether, for a given state $\omega$, the action of $\alpha_\chi$ will leave the net $\mathcal{A}^\omega_{\text{obs}} := \bigcup_{\mathcal{O}} \mathcal{A}^\omega_{\text{obs}}(\mathcal{O})$ of observables invariant, with an action of the form

$$
\alpha_\chi(\mathcal{A}^\omega_{\text{obs}}(\mathcal{O})) = \mathcal{A}^\omega_{\text{obs}}(\chi(\mathcal{O})).
$$

In order for this to be possible, the ideal $\mathcal{I}^\omega(\mathcal{O})$ must transform covariantly, i.e. the diffeomorphism $\chi$ must satisfy

$$
\alpha_\chi(\mathcal{I}^\omega(\mathcal{O})) = \mathcal{I}^\omega(\chi(\mathcal{O})).
$$

Hence, the algebra of observables, constructed with respect to the folium $\mathcal{F}^\omega$, does no longer exhibit the kinematical $\text{Diff}(M)$ symmetry of the observation procedures. The symmetry of the observables is dependent on (folium of) the state $\omega$. Therefore, the selection of a folium of states $\mathcal{F}^\omega$, induced by the actual choice of a state $\omega$, results immediately in a breaking of the $\text{Diff}(M)$ symmetry. The resulting effective symmetry group, also briefly called the dynamical group of the state $\omega$, is given by the subgroup of those diffeomorphisms which satisfy the constraint condition (2.6). An automorphisms $\alpha_\chi$ is called dynamical (w.r.t. the given state $\omega$) if it satisfies (2.6).

The remaining dynamical symmetry group, depending on the folium $\mathcal{F}^\omega$ of states related to $\omega$, has two main aspects which we have to examine if we actually want to specify the physically admissible states: Firstly, it is necessary to specify its state dependent automorphic algebraic action on the net of observables. Secondly, one has to find a geometric interpretation for the group and its action on $M$. 

5
If we consider the dynamical group as an inertial, and therefore global, manifestation of dynamically ascertainable properties of observables, then its (local) action should be correlated with (global) operations on the whole net of observables. This implies that at least some of the dynamical automorphisms $\alpha_x$ are not inner. (For the case of causal nets of algebras it was actually already shown in [9] that, under some additional assumptions, the automorphisms of the algebras are in general not inner.)

Note that one might consider instead of the net of observables $A^\omega_{obs}(\mathcal{O})$ the net of associated von Neumann algebras $R^\omega_{obs}(\mathcal{O})$, which can be defined even for unbounded $A^\omega_{obs}(\mathcal{O})$, if we take from the modulus of the von Neumann closure $(A^\omega_{obs}(\mathcal{O}))''$ all its spectral projections [3]. Then the isotony (2.1) induces a likewise isotony of the net $R^\omega_{obs} := \bigcup_{\mathcal{O}} R^\omega_{obs}(\mathcal{O})$ of von Neumann algebras.

### 3 Algebraic small and large scale regularization

In the following I want to exhibit a possibility to introduce both, small and large scale cutoff regularizations on the net of von Neumann algebras. This essentially exploits a local partial ordering on the net, which is induced by the isotony property.

Let us now make use of the given $(C^\infty)$ topological structure of $M$ and choose at point $x \in M$ a topological basis of nonzero open sets $\mathcal{O}^x_s \ni x$, parametrized by a real parameter $s$ with $0 < s < \infty$, such that

\[ s_1 < s_2 \iff \mathcal{O}^x_{s_1} \subset \mathcal{O}^x_{s_2} \tag{3.1} \]

and

\[ s \to 0 \iff \mathcal{O}^x_s \to \emptyset. \tag{3.2} \]

Let us now further restrict the parameter $s$ such that $0 < s_{\text{min},x} < s < s_{\text{max},x} < \infty$ and assume

\[ s_{\text{min},x} = s_{\text{min}}, \quad s_{\text{max},x} = s_{\text{max}} \quad \forall x \in M. \tag{3.3} \]
Then, for each \( x \in M \), open sets \( \mathcal{O}_s^x \) with \( s \in [s_{\min}, s_{\max}] \) generate local cobordisms between \( \partial \mathcal{O}_{s_{\min}}^x \) and \( \partial \mathcal{O}_{s_{\max}}^x \), and the isotony property implies that

\[
\mathcal{R}^\omega_{\text{obs}}(\mathcal{O}_{s_{\min}}^x) \subset \mathcal{R}^\omega_{\text{obs}}(\mathcal{O}_s^x) \subset \mathcal{R}^\omega_{\text{obs}}(\mathcal{O}_{s_{\max}}^x). \tag{3.4}
\]

The key step is now to impose a commutant duality relation between the inductive limits given by the minimal and maximal algebras,

\[
\mathcal{R}^\omega_{\text{obs}}(\mathcal{O}_{s_{\min}}^x) = \left( \mathcal{R}^\omega_{\text{obs}}(\mathcal{O}_{s_{\max}}^x) \right)', \tag{3.5}
\]

where \( \mathcal{R}' \) denotes the commutant of \( \mathcal{R} \) within some \( \mathcal{R}_{\text{max}} \supset \mathcal{R} \). Then the bicommutant theorem \( (\mathcal{R}'' = \mathcal{R}) \) implies that likewise also

\[
\mathcal{R}^\omega_{\text{obs}}(\mathcal{O}_{s_{\max}}^x) = \left( \mathcal{R}^\omega_{\text{obs}}(\mathcal{O}_{s_{\min}}^x) \right)', \tag{3.6}
\]

If one now demands that all maximal (or all minimal) algebras are isomorphic to each other, independently of the choice of \( x \) and the open set \( \mathcal{O}_{s_{\max}}^x \) (resp. \( \mathcal{O}_{s_{\min}}^x \)), then by (3.5) (resp. (3.6)) also all minimal (resp. maximal) algebras are isomorphic to each other. I then denote the universal minimal resp. maximal algebra as \( \mathcal{R}^\omega_{\text{min}} \) and \( \mathcal{R}^\omega_{\text{max}} \) respectively. In the following the commutant will always been taken within \( \mathcal{R}^\omega_{\text{max}} \). Then, the duality (3.5) implies that \( \mathcal{R}^\omega_{\text{min}} \) is Abelian.

By isotony and (3.4) together with (3.3), the mere existence of \( \mathcal{R}^\omega_{\text{min}} \) resp. \( \mathcal{R}^\omega_{\text{max}} \) fixes already a common size (as measured by the parameter \( s \)) of all sets \( \mathcal{O}_{s_{\min}}^x \) resp. \( \mathcal{O}_{s_{\max}}^x \) independently of \( x \in M \). So in this case \( s_{\min} \) and \( s_{\max} \) really denote an universal small resp. large scale cutoff. Note that, in the context of Sect. 2, the universality assumption (3.3) is indeed nontrivial, because local diffeomorphisms consistent with the structure above must preserve \( s_{\min}, s_{\max}, \) and the monotony of the ordered set \( [s_{\min}, s_{\max}] \). The number \( s \in [s_{\min}, s_{\max}] \) parametrizes the partial order of the net of algebras spanned between the inductive limits \( \mathcal{R}^\omega_{\text{min}} \) and \( \mathcal{R}^\omega_{\text{max}} \).

Although in local QFT usually the supports of an algebra and its commutant are not at all
related, it might be nevertheless instructive to consider the case where the algebras satisfy

$$\left(\mathcal{R}_\text{obs}^\omega(O_s^x)\right)' \subset \mathcal{R}_\text{obs}^\omega(O_s^x).$$  \hspace{1cm} (3.7)$$

Then, with the center of $\mathcal{R}_\text{obs}^\omega(O_x^s)$ defined as $Z(\mathcal{R}_\text{obs}^\omega(O_x^s)) := \mathcal{R}_\text{obs}^\omega(O_x^s) \cap \left(\mathcal{R}_\text{obs}^\omega(O_x^s)\right)'$, one obtains $Z\left(\mathcal{R}_\text{obs}^\omega(O_x^s)\right) = \left(\mathcal{R}_\text{obs}^\omega(O_x^s)\right)' = Z\left(\left(\mathcal{R}_\text{obs}^\omega(O_x^s)\right)\right)'$, and especially $Z\left(\mathcal{R}_\text{max}^\omega\right) = \mathcal{R}_\text{min}^\omega = Z\left(\mathcal{R}_\text{min}^\omega\right)$. So, for a pair of commutant dual algebras satisfying Eq. (3.7), the smaller one is always Abelian, namely it is the center of the bigger one. With (3.7), the isotony of the net implies the existence of an algebra $Z^\omega$ which is maximal Abelian, in other words commutant selfdual, satisfying $Z^\omega = (Z^\omega)' = Z(Z^\omega)$. This algebra is given explicitly via the Abelian net of all centers, $Z^\omega := \bigcup O Z(\mathcal{R}_\text{obs}^\omega(O))$. $Z^\omega$, located on an underlying set $O_{s_z}^x$ of intermediate size s.th. $s_{\text{min}} < s_z < s_{\text{max}}$, separates the small Abelian algebras $\mathcal{R}_\text{obs}^\omega(O_z^s) = Z\left(\mathcal{R}_\text{obs}^\omega(O_z^s)\right)'$, with $s \leq s_z$, from larger non-Abelian algebras $\mathcal{R}_\text{obs}^\omega(O_z^s) = Z\left(\left(\mathcal{R}_\text{obs}^\omega(O_z^s)\right)\right)'$, with $s > s_z$.

For a net subject to (3.7), its lower end is Abelian, whence observations on small regions with $s \leq s_z$ are expected to be rather classical. Nevertheless, for increasing size $s > s_z$, there might well exist a non-trivial quantum (field) theory (in [9] it was shown that, for causal nets, the algebras of QFT are not Abelian and not finite-dimensional). For quantum general relativity there might indeed be a kinetic substructure [10]. Classical elementary constituents of the latter naturally span an Abelian algebra. It is interesting in this context that the Abelian part of the loop algebra of quantum general relativity provides indeed the classical spectrum [11, 12].

4 Modular structure and dilations

If we consider the small and large scale cutoffs as introduced above, it should be clear that only those regions $[2.1]$ of size $s \in [s_{\text{min}}, s_{\text{max}}]$ are admissible for measurement. The commutant duality between $\mathcal{R}_\text{min}^\omega$ and $\mathcal{R}_\text{max}^\omega$ inevitably yields large scale correlations in the structure of any physical state $\omega$ on any admissible region $O_z^x$ of measurement at $x$. Let us assume here
that \( \omega \) is properly correlated, i.e. the GNS vector \( \Omega^\omega \) is already cyclic under \( \mathcal{R}^\omega_{\text{min}} \). Then, by duality, it is separating for \( \mathcal{R}^\omega_{\text{max}} = \mathcal{R}^\omega_{\text{min}}' \). Furthermore \( \Omega^\omega \) is also cyclic under \( \mathcal{R}^\omega_{\text{max}} \), and hence separating for \( \mathcal{R}^\omega_{\text{min}} \).

So \( \Omega^\omega \) is a cyclic and separating vector for \( \mathcal{R}^\omega_{\text{min}} \) and \( \mathcal{R}^\omega_{\text{max}} \), and by isotony also for any local von Neumann algebra \( \mathcal{R}^\omega_{\text{obs}}(\mathcal{O}_s^x) \).

As a further consequence, on any region \( \mathcal{O}_s^x \), the Tomita operator \( S \) and its conjugate \( F \) can be defined densely by

\[
SA^\omega := A^\omega \Omega^\omega \quad \text{for} \quad A \in \mathcal{R}^\omega_{\text{obs}}(\mathcal{O}_s^x) \tag{4.1}
\]

\[
FB^\omega := B^\omega \Omega^\omega \quad \text{for} \quad B \in \mathcal{R}^\omega_{\text{obs}}(\mathcal{O}_s^x)' \tag{4.2}
\]

The closed Tomita operator \( S \) has a polar decomposition

\[
S = J \Delta^{1/2}, \tag{4.3}
\]

where \( J \) is antiunitary and \( \Delta := FS \) is the self-adjoint, positive modular operator. The Tomita-Takesaki theorem [12] provides us with a one-parameter group of state dependent automorphisms \( \alpha_t^\omega \) on \( \mathcal{R}^\omega_{\text{obs}}(\mathcal{O}_s^x) \), defined by

\[
\alpha_t^\omega(A) = \Delta^{-it} A \Delta^{it}, \quad \text{for} \quad A \in \mathcal{R}^\omega_{\text{max}}. \tag{4.4}
\]

So, as a consequence of commutant duality and isotony assumed above, we obtain here a strongly continuous unitary implementation of the modular group of \( \omega \), which is defined by the 1-parameter family of automorphisms (4.4), given as conjugate action of operators \( e^{-it \ln \Delta} \), \( t \in \mathbb{R} \). By (4.4) the modular group, for a state \( \omega \) on the net of von Neumann algebras, defined by \( \mathcal{R}^\omega_{\text{max}} \), might be considered as a 1-parameter subgroup of the dynamical group. Note that, with Eq. (4.2), in general, the modular operator \( \Delta \) is not located on \( \mathcal{O}_s^x \). Therefore, in general, the modular automorphisms (4.4) are not inner. It is known (see e.g. [13]) that the modular automorphisms act as inner automorphisms, iff the von Neumann algebra \( \mathcal{R}^\omega_{\text{obs}}(\mathcal{O}_s^x) \) generated
by ω contains only semifinite factors, i.e. factors of type I and II. In this case ω is a semifinite trace.

Above we considered concrete von Neumann algebras $\mathcal{R}_\text{obs}^\omega(O_x)$, which are in fact operator representations of an abstract von Neumann algebra $\mathcal{R}$ on a GNS Hilbert space $\mathcal{H}^\omega$ w.r.t. a faithful normal state ω. In general, different faithful normal states generate different concrete von Neumann algebras and different modular automorphism groups of the same abstract von Neumann algebra.

The outer modular automorphisms form the cohomology group $\text{Out}\mathcal{R} := \text{Aut}\mathcal{R}/\text{Inn}\mathcal{R}$ of modular automorphisms modulo inner modular automorphisms. This group is characteristic for the types of factors contained in von Neumann algebra $\mathcal{R}$ (cf. [14]). Per definition $\text{Out}\mathcal{R}$ is trivial for inner automorphisms. Factors of type $\text{III}_1$ yield $\text{Out}\mathcal{R} = \mathbb{R}$.

In the case of thermal equilibrium states, corresponding to factors of type $\text{III}_1$ (see [12]), there is a distinguished 1-parameter group of outer modular automorphisms, which is a subgroup of the dynamical group.

Looking for a geometric interpretation for this subgroup, parametrized by $\mathbb{R}$, it should not be a coincidence that our partial order defined above is parametrized by open intervals (namely $[s_{\text{min}}, s_{\text{max}}]$ for the full net and, in the case of (3.7), $[s_z, s_{\text{max}}]$ for the non-Abelian part), and hence diffeomorphically likewise by $\mathbb{R}$. This way, dilations of the open sets $O^x_s$ within the open interval may give a geometrical meaning to the 1-parameter group of outer modular automorphisms of thermal equilibrium states. Even more, this might provide a perspective for understanding the thermal time hypothesis of [15]. Indeed, a local equilibrium state might be characterized as a KMS state (see [12, 16]) over the algebra of observables on a (suitably defined) double cone, whence the 1-parameter modular group in the KMS condition might be related to the time evolution. Note that, for double cones, a partial order can be related to the split property of the algebras (see also [4]).
5 Discussion

A geometric action of the modular group might be obtained by relating the thermal time to the geometric notion of dilations of the open sets. For any $x \in M$, the parameter $s$ measures the extension of the sets $O^x_s$. As accessability regions for a local measurement in $M$, these sets naturally increase with time. Hence it is natural to suggest (at least for $s > s_z$) that the parameter $s$ might be related to a (thermal) time $t$ such that, for any set $O^x_s$, $s > s_{\text{min}}$, we have $t < s$ within the set and $t = s$ on the boundary $\partial O^x_s$.

For the ultralocal case ($s_{\text{min}} \to 0$, without UV cutoff), in [17] a construction of the causal structure for a space-time from the corresponding net of operator algebras was given. Let us consider here the (a priori given) underlying manifold $M$ of the net. Locally around any point $x \in M$ one may induce open double cones as the pullback of the standard double cone, which in fact is the conformal model of Minkowski space. These open double cones then carry natural notions of time and causality, which are preserved under dilations. Therefore it seems natural to introduce locally around any $x \in M$ a causal structure and time by specializing the open sets to be open double cones $K^x_s$ located at $x$, with time-like extension $2s$ between the ultimate past event $p$ and the ultimate future event $q$ involved in any measurement in $K^x_s$ at $x$ (time $s$ between $p$ and $x$, and likewise between $x$ and $q$). Since the open double cones form a basis for the local topology of $M$, we might indeed consider equivalently the net of algebras located on open sets

$$O^x_s := K^x_s.$$ (5.1)

Although some (moderate form of) locality might be indeed an indispensable principle within any reasonable theory of observations, it is nevertheless an important but difficult question, under which consistency conditions a local notion of time and causality might be extended from nonzero local environments of individual points to global regions. This is of course also related to the non-trivial open question, how open neighbourhoods of different points $x_1 \neq x_2$
should be related consistently. Although, in a radical attempt to avoid some part of these difficulties, one might try to replace the notion of points and local regions by more abstract algebraic concepts, a final answer to these questions has not yet been found. At least it seems natural that, on manifolds with no a priori causal structure, the net should satisfy a disjoint compatibility condition,

\[ \mathcal{O}_1 \cap \mathcal{O}_2 = \emptyset \implies [\mathcal{A}(\mathcal{O}_1), \mathcal{A}(\mathcal{O}_2)] = 0. \]  

(5.2)

This condition is e.g. also satisfied for Borchers algebras. Of course the inverse of (5.2) is not true in general.

Moreover, it is not yet clear whether (3.7) is not a too strange condition (although we should of course be aware that the feature of quantum general relativity might indeed be very different from that of usual QFT). (3.7) makes sense, if further investigation are able to proof the existence of an Abelian substructure under reasonable conditions.

Note however that only the commutant duality (3.5), but not (3.7), was essential for the extraction of the modular group. If we assume the presence of factors of type III$_1$ in our von Neumann algebras (in the case of (3.7) for a size $s > s_z$), or likewise the existence of local equilibrium states, the choices for time and causality, made above on the basis of the partial order and related dilations are apparently natural.

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