Analysis of students’ metacognition in solving mathematics problems

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Abstract. Metacognition is a thought about the awareness of thinking itself to plan, organize strategy information, monitor strategies, debugging strategies and assess the results of their abilities. Student activities in the use of their awareness can be used to solve problems. Mathematical problems solving is an important thing in the process of learning mathematics so that complex ways of thinking are needed. This study aims to analyze the level of metacognition of students in Boyolali district in solving problems based on differences in mathematical scores and gender. This study is a qualitative study using written test data and interviews with students and mathematics teachers. The results showed that the level of metacognition of male students who had high mathematical scores was at the level of aware use, the subjects are aware of what they are doing and can be used to solve problems, but the subject cannot solve it well and cannot correct his mistakes. The level of metacognition of female students who have high, medium and low math scores are at the level of tracit use. The student cannot understand the problem at hand and does not know the concept used.

1. Introduction

John Flavell originally coined the term metacognition in the late 1970s to mean “cognition about cognitive phenomena,” or more simply “thinking about thinking” [1 - 2]. Metacognition is a theory of cognition. Metacognition also means knowledge about cognitive abilities possessed and how those abilities can be applied to cognitive processes. Furthermore, metacognition is often associated with personals, task and strategy [3].

The ability to solve problems is seen as a state of mutual influence and complexity between cognition and metacognition. Brown [4] suggested that metacognitive skills or abilities that are essential for any efficient problem solver include the ability to: (1) plan, include estimating results, and scheduling strategies, (2) monitoring, includes testing, revision, and rescheduling of the strategy carried out, and (3) checking, including evaluating the results of implementing a strategy based on efficiency and effectiveness criteria. The similar state that the essential metakognition skill in problem solving is predicting, planning, monitoring, and evaluating [5].

Metacognition is an important aspect of mathematics problems solving [6 - 7]. The role of metacognition in the problem solving is to help the problem solver realize the problem, differentiate the problem, and understand how to achieve the goal or the solution of the problem [8]. Each student
has a different problem solving strategy in solving mathematics problems depending on the level of understanding they have. Thus each student has a different level or level of metacognition in solving mathematical problems. There is no student in the level of reflective use when solving the mathematical problems. This arises a question why it happens. However, it needs to be highlighted that the previous studies use mathematic materials which tend to be deductive[9].

Metacognition in solving mathematical problems in this study is the use of students’ awareness in solving a mathematical question or problem by using their thinking to plan, consider, control, and assess their own cognitive processes and strategies. The following are indicators of student’s level of metacognition in solving mathematics problems[10 - 11]:

- Tacit use (the use of thought without awareness). Planning indicators, namely: students cannot explain what is known (A1), students cannot explain what is asked (A2), and students cannot explain the problem clearly (A3). The monitoring indicators, namely: students do not show any awareness of what is being monitored (AP1) and students are not aware of errors in the concepts and results obtained (AP2). Assessment indicators, namely: students do not do an evaluation or if an evaluation will appear confused or unclear about the results obtained (AL1).

- Aware use (the use of mindfulness). Planning indicators, namely: students have difficulty and confusion because they think of concepts (formulas) and how to calculate what will be done (B1), students only explain part of what is written (B2), and students understand the problem because it can express clearly (B3). Monitoring indicators, namely: students experience confusion because they cannot continue what will be done (BP1), students are aware of concept errors (formulas) and how to calculate but cannot correct them (BP2). Assessment indicators, namely: students do not do an evaluation or if they do an evaluation will appear confused or unclear about the results obtained (AL1) and students do an evaluation but are not sure of the results obtained (BL1).

- Strategic use (the use of strategic thinking). Planning indicators, namely: students understand the problem because it can express clearly (B3), students do not experience difficulty and confusion to find formulas and ways to calculate (D1), and students can explain most of what they write (D2). Monitoring indicators, namely: students are aware of concept errors and how to count (DP1) and students are able to give reasons that support their thinking (DP2). Assessment indicators, namely: students do not do an evaluation or if they do an evaluation will appear confused or unclear about the results obtained (AL1) and students do an evaluation but are not sure of the results obtained (DL1).

- Reflective use (the use of reflective thinking). Planning indicators, namely: students know the methods used to solve problems (E2), students are able to explain the strategies used to solve problems (E3), students understand problems well because they can identify important information in problems (F1), and students can explain what is written on the answer sheet (F2). Monitoring indicators, namely: students are able to apply the same strategy to other problems (EP3) and students are aware of the concept mistakes made and can correct them (FP1). Assessment indicators, namely: students evaluate each step made and believe in the results obtained (FL1).

Gender is a term of difference between the male and female who have an innate nature and cultural formation, including differences in solving problems [12]. Gender differences do not play a role in learning success [13], but there are differences in the ability to solve mathematics problems between male and female students [14]. The male prefer problems solving using estimation strategies while female students prefer problems solving using algorithmic strategies. Female students using algorithmic strategies and male students using estimation strategies show the cognitive strategies used in solving a problem. On the other hand, metacognitive contributes to gender based difference in learning achievement [15]. There is difference in male and female students’ metacognitive skill, in which female students have higher metacognitive skill than male students [16 - 20]. Other researches show that significant difference in male and female students’ metacognitive skill, however, with male students’ metacognitive skill higher than that of female students[21 - 22]. And the other researches show that there is no difference in male and female students’ metacognitive skill [23]. This study aims
to analyze the level of metacognition of students in terms of gender differences in solving mathematical problems. The level of metacognition students who have high, medium and low math scores are at the level of tracit use, aware use, strategic use or reflective use.

2. Methods

This research used the qualitative descriptive method. The subjects of this study were students of grade X in MAN 1 Boyolali and mathematics teachers in grade X. The data collection method is snowball sampling. The selected subjects consisted of 6 students. The main data in this study were written test results and interviews with students, besides supporting data were the results of interviews with supporting teachers. Researchers take data from teachers who are able to grade X mathematics about class conditions and the delivery of material, it aims to find out the teacher concerned has provided knowledge about metacognition.

3. Results and Discussion

Data retrieval begins with observation on teaching activities in the class (KBM) and then conducting interview on the condition of the class. The data obtained by researchers at the time of observation are the teacher concerned does not provide direction or insight about metacognition, does not perform the stages of metacognition, and the teacher only provides subject matter according to the module used. From the results of interviews with instructor teachers, it was found that there were a variety of student conditions that had high, medium and low mathematical abilities. Most students have low ability in doing mathematics, the results of many tests below the average value. In addition, the teacher's method of conducting learning is by lecturing and then conducting tests to determine students' abilities. The way the teacher evaluates is that if the student answers according to the steps that have been explained and the answer is correct, then he has understood what is conveyed by the teacher and has a high ability for the teacher concerned. From some data obtained from the teacher, it can be concluded that the student’s mathematics ability is still low and the level of student metacognition has not yet appeared and is still relatively low because the teacher has not accustomed learning to find out the student’s metacognition.

Mathematics problem-solving test carried out in grade X in MAN 1 Boyolali were 36 students. The test to determine mathematics problem-solving skill. In scoring the test description to facilitate the analysis of items, the range of scores for each item must be the same [24]. Based on the results of the written test data classified mathematical score criteria as follows: (1) high score ≥ 68; (2) 33 < medium score < 68; and (3) Low score ≤ 33[25]. The categories of students in the written test obtained include Table 1.

| Table 1. Categories of score and gender | Gender |
|----------------------------------------|--------|
|                                        | Male   | Female |
| High                                   | 4      | 3      |
| Medium                                 | 4      | 15     |
| Low                                    | 3      | 7      |

The results obtained from 6 subjects who will be analyzed mathematics problem-solving skill based on gender gender according to differences in mathematics scores. Below are the description of 6 students’ answers and the analysis of their metacognition levels.
Figure 1. The answer of Subject 1 (S1) as a male student with high mathematics scores

The following are the results of the interviews with S1.

Q : How do you understand the questions?
S1 : I read the questions and then made a mathematical form, ma'am.
Q : Why do you change shapes into mathematical models?
S1 : To make it easier to count, Ma'am, the method you give is also like that.
Q : Why did you eliminate the "y" variable first?
S1 : Because it's easier to get rid of the "y" variable.
Q : Why are you looking for the "y" variable?
S1 : Because what orange is asking about is the variable "y" Ma'am.
Q : How do you correct?
S1 : I re-examine the answer correctly.

Based on Figure 1, the results of S1 answer indicated that the students understand what to look for, but do not know the error he wrote. S1 still look confused in its completion, the students complete it and have gotten what is sought but is still looking for other variables so it is not clear from the student's work. Otherwise, the results of interviews with S1, information was obtained that S1 still found it difficult to explain what was written and the student was not sure of what was written, but the students had already corrected his work.

Figure 2. The answer of Subject 2 (S2) as a female student with high mathematics scores
The following are the results of the interviews with S2.

Q : How can you understand the problem?
S2 : I read it many times, but I didn't really understand what it meant.

Q : Why do you get such an equation?
S2 : I only followed Mom's story, then I subtracted 5 because 5 years ago.

Q : How do you go about working on that problem?
S2 : I am also not sure about my answer, but I tried to do it by turning the numbers, then I put it in the equation and got the answer, then I added 2 because for the next 2 years.

Q : How do you correct?
S2 : Because I'm also not sure whether it's right or wrong. I just checked the count.

Based on Figure 2, the results of S2 answer indicated that the students know the form of the questions but does not understand the problem. Students experience errors but do not fix them. Otherwise, the results of interviews with S2, information was obtained that the S2 were not sure of what they wrote. Students have difficulty in solving problems and only write down their knowledge. Students do not know the work is right or wrong. Because it only twists the numbers. The student evaluates and tries to improve it even though the results of the settlement are not sure whether it is right or wrong.

![Figure 3](image.png)

**Figure 3.** The answer of Subject 3 (S3) as male student with moderate mathematics scores

The following are the results of the interviews with S3.

Q : How do you understand the problem?
S3 : I read it over and over again.

Q : Why do you do it like that?
S3 : I did not know I wanted to pay attention to ma'am, then I wrote according to the problem.

Q : What are your steps for working on that problem?
S3 : Just add the numbers, ma'am

Q : How do you correct with your work like that?
S3 : No, I corrected Ma'am, because I didn't know.

Based on Figure 3, the results of S3 answer indicated that the students have difficulty in understanding the questions and do not know the concept of working on the problem. Otherwise, the results of interviews with S3, information was obtained that S3 did not understand the questions given. The steps taken are not aware that it is important to write down what he knows and do not know the work is right or wrong. The student did not re-evaluate because he was doubtful from the beginning of the work.
The following are the results of the interviews with S4.

Q: How can you understand the problem?
S4: I imagine that I woke up rectangle.
Q: Why do you make such equations?
S4: I only tried \( x + y = 44 \) because the problem was number 44, then \( x - y = 6 \) because the problem was number 6.
Q: How do you go about answering after the problem?
S4: I just eliminated ma'am to get \( x \) and \( y \), that will be the answer later.
Q: How do you correct your work correctly?
S4: No, I corrected.

Based on Figure 4, the results of S4 answer indicated that students have difficulty in understanding the questions and do not know the concept of working on the problem. Otherwise, the results of interviews with S4, information was obtained that S4 was skeptical about the questions given. Students try to understand with their own concepts and reasoning and the student believes that what is done is right. In fact students make a mistake and the answer is wrong. The student did not do an evaluation because they felt they were sure.

The following are the results of the interviews with S5.

Q: How can you understand the problem?
S5: I rewrote the problem that Ma'am knew.
Q: How do you go about getting an equation like that?
S5: I paired the variables x and y according to the problem, then I counted.
Q: How do you know that your way is correct?
S5: I get that answer because the coefficient is worth 1 all.
Q: Are you correcting again?
S5: No ma'am.

Based on Figure 5, the results of S5 answer indicated that students do not understand the questions given, but students write down what they know. The student does not complete with steps or concepts, only writes what is in the problems. Otherwise, the results of interviews with S5, information was obtained that S5 was confused and did not know the answers to be written so students blank the answers. Students also have no plans to understand or rethink the problem.

Figure 6. The answer of Subject 6 (S6) as a male student with low mathematics scores

The following are the results of the interviews with S6.
Q: How can you understand the problem?
S6: I don't understand the problem.
Q: How do you go about doing that?
S6: I only remember what the teacher explained about x and y, then I made an equation.
Q: How do you know that your way is true?
S6: I don't know, I'm confused about what to write to answer it.
Q: Did you try to correct your work from the problem?
S6: No, because I am still doubtful about my steps.

Based on Figure 6, the results of S6 answer indicated that the students do not understand the questions given. The students write what is known and do not know the method used. The student did not complete the answer from what he wrote. Otherwise, the results of interviews with S6 obtained information that S6 do not understand the questions given and there is no picture of what he wrote. The student cannot complete it because he does not know the concept used. The student did not conduct an evaluation in the settlement because finding answers that were still in doubt.

Male students who have high scores are at the level of aware use. This level, subjects are aware of what they are doing and can be used to solve problems, but the subject cannot solve it well and cannot correct his mistakes. At the level of aware use, students use their thoughts in solving problems and realize what they are thinking [11]. Indicators of this level are: expressing why and how the thought is used, experiencing confusion when reading problems because they have not come up with an idea of was read, making a decision against a specific reason, being aware of weaknesses, knowing what they don't know, understanding the problem which is solved, mastering the mathematical concepts that underlie the problem. While students in.

Addition, female students who have high, medium and low math scores are at the level of tracit use. The student cannot understand the problem at hand and does not know the concept used. At the level of tracit use that students make decisions without thinking about the decisions themselves [13]. At the level of tracit use that students use their thinking in completing but tend not to realize why that thought is used [26]. Indicators of this level are: giving explanations or answers that are uncertain (just
answering), not knowing that what is said is not meaningful, not aware of any mistakes or weaknesses, solving problems by just trying, not knowing what they don't know, giving answers which is inconsistent, has weaknesses in mastering the material and analyzing the problem.

4. Conclusion
Male students who have high scores are at the level of aware use. At this level, subjects are aware of what they are doing and can be used to solve problems, but the subject cannot solve it well and cannot correct his mistakes. In addition, female students who have high, medium and low math scores are at the level of tracit use. The students cannot understand with their problem and also do not know the concept that are used to solve it.

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References
[1] Flavell J H 1979 Metacognition and cognitive monitoring: A new area of cognitive-developmental inquiry American Psychologist 34 pp. 906-911
[2] Huitt W 1997 Metacognition Educational Psychology Interactive (Valdosta, GA: Valdosta State University)
[3] Schraw G and Moshman D 1995 Metacognitive theories Educational Psychology Review 7 pp. 351-371
[4] Panaoura A and Philippou G 2001 Young Pupils’ Metacognitive Abilities in Mathematics.European Research in Mathematics Education III
[5] Lucangeli D and Cornoldi C 1997 Mathematics and metacognition : what is the nature of the relationship? Mathematical Cognition 3 pp. 121-139
[6] Hartman H J 1998 Metacognition in Teaching and Learning: an Introduction Science International Journal of Learning and cognition 26 pp. 1-3
[7] Chimuma L L and Johnson I D 2016 Assessing Students’ Use of Metacognition during Mathematical Problem Solving Using Smartpens Educational Research: Theory & Practice 28(1) pp. 22-36
[8] Siagian M V, Saragih S and Sinaga B 2019 Development of Learning Materials Oriented on Problem – Based Learning Model to Improve Students’ Mathematical Problem Solving Ability and Metacognition Ability International Electronic Journal of Mathematics Education 14 (2) pp. 331 - 340
[9] Kuzle A 2013 Patterns of metacognitive behavior during mathematics problem-solving in dynamic geometry environment International Electronic Journal of Mathematics Education 8 pp. 20-40
[10] Agustina L M and Trineke J M 2013 Identifikasi tingkat metakognisi siswa dalam memecahkan masalah matematika berdasarkan perbedaan perbedaan skor matematika MATHEdunesa 2
[11] Schwartz R and Perkins D 1989 Teaching Thinking-Issues and Approaches (Pacific Grove, CA: Midwest Publications)
[12] Laurens T 2010 Valid Student Metacognition Gap and Reliability Journal of Education and Teaching 17(2) pp. 201-210
[13] Sudia M 2015 Profil Siswa SMP dalam memecahkan masalah terbuka ditinjau dari perbedaan gender Jurnal pendidikan dan pembelajaran 22(1)
[14] Hightower M W 2003 The Boy-Turn in Research on Gender and Education Review of Educational Research 73 pp. 471-498
[15] Zhu Z 2007 Gender Differences in Mathematical Problem Solving Patterns: A review of Literature International Education Journal 8(2) pp. 187-203
[16] Yerdelen-Damar S and Peşman H 2013 Relations of gender and socioeconomic status to physics through metacognition and self-efficacy *The Journal of Educational Research* **106**(4) pp. 280–289

[17] Carr M and Jessup D L 1997 Gender differences in first-grade mathematics strategy use: Social and metacognitive influences *Journal of Educational Psychology* **89**(2) p. 318

[18] Liliana Cand Lavinia H 2011 Gender differences in metacognitive skills: A study of the 8th grade pupils in Romania *Procedia-Social and Behavioral Sciences* **29** pp. 396–401

[19] Zimmerman B J and Martinez-Pons M 1990 Student differences in self-regulated learning: Relating grade, sex, and giftedness to self-efficacy and strategy use *Journal of Educational Psychology* **82**(1) p. 51

[20] Nurmaliah C 2009 Analisis keterampilan metakognisi siswa SMP Negeri di Kota Malang berdasarkan kemampuan awal, tingkat kelas dan jenis kelamin *Jurnal Biologi Edukasi* **1**(2) pp. 18–21

[21] Ramdiah S 2013 Pengaruh strategi pembelajaran PQ4R terhadap keterampilan metakognitif dan hasil belajar biologi siswa putra dan putri kelas XI SMA di Kota Banjarmasin *Proceeding Biology Education Conference: Biology, Science, Environment, and Learning* **10** pp. 96–102

[22] Chen S, Huang C C and Chou T L 2016 The effect of metacognitive scaffolds on low achievers’ laboratory learning *International Journal of Science and Mathematics Education* **14**(2) pp. 281–296

[23] Suherman D P, Purwianingsih and Diana S 2018 The analysis of self-efficacy and metacognitive and its relation with academic performance of high school students based on gender on genetics concept *Assimilation: Indonesian Journal of Biology Education* **1**(1) pp. 14–20

[24] Nunaki J H, Damopolii I, Kandowangko N Y and Nusantari E 2019 The Effectiveness of Inquiry-based Learning to Train the Students’ Metacognitive Skills Based on Gender Differences *International Journal of Instruction* **12** pp. 505 - 516

[25] Budiyono 2017 *Pengantar Metodologi Penelitian Pendidikan* (Surakarta: UNS Press)

[26] Arikunto S 2009 *Dasar-Dasar Evaluasi Pendidikan* (Jakarta : Bumi Aksara)

[27] Gregory G H 2005 *Differentiating instruction with style: aligning teacher and learner intelligences for maximum achievement* (California: Corwin Press)