NeAF: Learning Neural Angle Fields for Point Normal Estimation

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Abstract

Normal estimation for unstructured point clouds is an important task in 3D computer vision. Current methods achieve encouraging results by mapping local patches to normal vectors or learning local surface fitting using neural networks. However, these methods are not generalized well to unseen scenarios and are sensitive to parameter settings. To resolve these issues, we propose an implicit function to learn an angle field around the normal of each point in the spherical coordinate system, which is dubbed as Neural Angle Fields (NeAF). Instead of directly predicting the normal of an input point, we predict the angle offset between the ground truth normal and a randomly sampled query normal. This strategy pushes the network to observe more diverse samples, which leads to higher prediction accuracy in a more robust manner. To predict normals from the learned angle fields at inference time, we randomly sample query vectors in a unit spherical space and take the vectors with minimal angle values as the predicted normals. To further leverage the prior learned by NeAF, we propose to refine the predicted normal vectors by minimizing the angle offsets. The experimental results with synthetic data and real scans show significant improvements over the state-of-the-art under widely used benchmarks. Project page: https://lisj575.github.io/NeAF/.

Introduction

Estimating normals for unstructured point clouds is a fundamental task in 3D computer vision. The estimated normal vectors can be leveraged in various downstream applications, such as surface reconstruction (Kazhdan, Bolitho, and Hoppe 2006), registration (Pomerleau et al. 2015), and semantic segmentation (Grilli, Menna, and Remondino 2017). Recently, the learning-based methods have achieved promising results by casting the normal estimation task into a regression problem. They resolved this problem by directly learning a mapping from local patches to normal vectors using neural networks. However, these methods fail to observe adequate samples for accurate normal estimation. This leads to poor generalization ability on unseen scenarios such as large-scale scenes.

As a remedy, state-of-the-art methods introduce to predict the normal of a point by fitting a local geometric surface. They learned point-wise weights for neighboring points to fit the surface, and then predict the normal vector from the fitted surface. However, the geometric surface settings (e.g. the constant order of the polynomial surface) are predefined during the training process, which leads to poor fitting results due to the various complexity of the local point patches. For example, when the geometric surface is more complex than the underlying point patches, it results in overfitting problems. Otherwise, the underfitting leads to over-smoothing details. As a result, the performance of surface fitting methods is dramatically limited in cases with complex topology and geometry.

To resolve these issues, we propose an implicit method to learn the Neural Angle Field (NeAF) of local patches in the spherical coordinate system. Specifically, instead of explicitly predicting the normal from the input point clouds, we design a neural network to implicitly predict the angle offset between the ground truth normal and a randomly sampled query vector. The network observes more diverse samples by learning to model the relationship between vectors from various directions and the target normal, leading to an angle field around the target normal. With a well-learned angle field, the network will have the ability to predict accurate and robust normal vectors.

At inference time, to predict normals from the learned angle fields, we randomly sample query vectors in a unit spherical space and take the vectors with minimal angle offsets as the predicted normals. To further leverage the prior learned by NeAF, we propose a novel learning scheme for refining the predicted normal vectors. Unlike existing learning-based methods that can only predict the normal via one forward pass, our proposed method can further optimize the predicted normal, which leads to more accurate normal estimation.

Our contributions are summarized as follows.

- We propose Neural Angle Field (NeAF) for point normal estimation. Unlike previous methods, our implicit function learns an angle field for each point, which implicitly predicts the angle offset of query vectors.
- Our method can conduct further optimization at inference time by refining the predicted normal vectors.

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### Related Work

#### Normal Estimation

Traditionally, principal component analysis (PCA) (Hoppe et al. 1992) is a simple and efficient method for normal estimation, which is based on constructing a tangent plane from a fixed-scale local neighborhood and analyzes its normal vector. Variants such as truncated Taylor expansion (Jets) (Cazals and Pouget 2005), moving least squares (MLS) (Levin 1998) and multi-scale kernel methods (Aroudj et al. 2017) were proposed to fit more complex local surfaces. However, these methods tend to choose large scales to ensure the robustness of normal estimation. To preserve more details, the approaches (Amenta et al. 1999; Mérigot et al. 2010) were proposed based on analyzing Voronoi cells. In practice, these methods have to tune their parameters to work with different cases, which dramatically limits their application scenarios.

Recently, with the development of deep learning, learning-based methods were proposed for normal estimation. Boulch and Marlet (2016) proposed to transform the local patches of point clouds into accumulators in a 2D Hough space and estimate normals from the resulting approximate planes. However, the transformation from 3D to 2D resulted in the loss of the structural information contained in the surface. To preserve a complete local shape, Guerrero et al. (2018) proposed the PCPNet with a multi-scale point cloud normal estimation architecture using PointNet (Qi et al. 2017). Based on this method, Zhou et al. (2020) proposed a new scale selection strategy and extra constraints on the feature. In addition, Zhou et al. (2022a) proposed to introduce additional feature representations to refine the input initial normal. Li et al. (2022b) used the networks to learn hyper surfaces and obtained an improved performance.

Recent studies have achieved encouraging performance on normal estimation by predicting point-wise weights to select neighboring points softly. Lenssen et al. (2020) proposed to use a graph neural network to iteratively generate point-wise weights, and then estimate the normal by fitting a moving least squares plane. Ben-Shabat et al. (2020) used the n-order Jet of weighted points to fit local surfaces. Cao et al. (2021) used a differentiable random sample consensus module to predict normals from a latent tangent plane representation constructed from the neighboring points. To learn better point-wise weights, Zhang et al. (2022) proposed to add direct geometric weight guidance which is constructed by distances between points and the tangent plane. Zhu (2021) proposed to predict an offset to adjust the distribution of clouds and designed a cascaded scale aggrega-
tion (CSA) layer adapting to the scale of the local neighborhood, which achieved state-of-the-art results. Li et al. (2022a) proposed to learn graph convolutional feature representation. Although improvements have been made by predicting point-wise weights and fitting surfaces for normal estimation, these methods still suffer from several inherent problems such as the difficulty in determining the form of the fitted surface and the sensitivity to outliers.

**Neural Implicit Representation in 3D Reconstruction**

Recently, neural implicit representations have gained popularity and are widely used for 3D reconstruction (Zhou et al. 2022; Ma, Liu, and Han 2022; Chen, Liu, and Han 2022; Ma et al. 2022; Li et al. 2022c). These representations were learned by neural networks to map input 3D coordinates to occupancy probabilities (Chen and Zhang 2019; Mescheder et al. 2019) or distance values (Michalkiewicz et al. 2019; Park et al. 2019), which implicitly represent continuous surfaces of shapes and handle complex shape topology. In addition, Neural Radiance Fields (NeRFs) (Mildenhall et al. 2020) implicitly encode the geometry and color information of the scene in the network.

Inspired by neural implicit representations, we propose to utilize the network to implicitly represent the normals of points. Unlike existing learning-based normal estimation methods that focus only on the relationship between local neighborhoods and normal vectors, we use the network to observe the difference between random vectors and the ground truth normal. This provides the network with more prior knowledge by learning a neural angle field, which helps to predict more accurate and robust normals.

**Method**

In this section, we present NeAF, an implicit normal estimation method for point clouds.

**Implicit Angle Functions**

We first define the *implicit angle function* of a given normal vector. The implicit angle function takes an arbitrary query vector in a unit spherical space and a condition $C$ as input and predicts the angle offset between the query vector and the ground truth vector. The corresponding function is defined as follows,

$$ f : C \times \mathbb{R}^3 \rightarrow \mathbb{R}. $$  \hspace{1cm} (1)

Our key idea is to estimate the normal of one point using the implicit angle function that is learned by a neural network. Observing that the normal vector is a local property of a surface, we leverage the local neighborhood of the center point as a condition $C$ indicating the center point, together with the query vector as the input to feed the network, and the output is the angle offset between the ground truth and the query vector. Since we focus on unoriented normal estimation, the angle offset is either the difference from the positive direction or the negative direction of the ground truth, and its value range is $[0, \pi/2]$. The implicit angle function of point $p_i$ can be represented as:

$$ f_\theta(p_i, n^q_i) = \alpha, i \in [1, N]. $$ \hspace{1cm} (2)

where $f_\theta$ denotes the network parameterized by $\theta$, $P_i \in \mathbb{R}^{k \times 3}$ is the condition of point $p_i$, it is the set of $k$ nearest neighbors for the center point $p_i$, $N$ is the number of points on the point cloud, $n^q_i \in \mathbb{R}^3$ is the query vector, and $\alpha \in [0, \pi/2]$ is the angle offset of $n^q_i$ relative to the target normal.

With this definition, the target normal of $p_i$ is implicitly represented as the zero level set of the implicit angle function, which is denoted as $n^t$, i.e. $f_\theta(p_i, n^t) = 0$.

**Training for Implicit Angle Functions**

To learn the parameters $\theta$ of the neural network $f_\theta$, we first randomly sample query vectors $n^q_j, j = 1, \ldots, M$ on the unit sphere. For a given point $p_i, i = 1, \ldots, N$, we use the K-Nearest-Neighbors algorithm to sample the nearest neighbors $P_j$ as the local patch centered at $p_i$. Next, the network takes the local patch $P_j$ and query vector $n^q_j$ as inputs and predicts the angle offsets between the ground truth normal $n^g_j$ and $n^q_j$. We minimize the difference between the predicted angle offset $f_\theta(P_j, n^q_j)$ and ground truth angle offset $\alpha_{ij}^g$ by the following $L1$ loss,

$$ L = \frac{1}{NM} \sum_{i=1}^{N} \sum_{j=1}^{M} |f_\theta(P_j, n^q_j) - \alpha_{ij}^g|, $$ \hspace{1cm} (3)

and the $\alpha_{ij}^g$ is computed by,

$$ \alpha_{ij}^g = \arcsin(||n^g_j \times n^t_i||), $$ \hspace{1cm} (4)

where $\arcsin(\cdot)$ represents the arcsine function, $|| \cdot ||$ is the L2-norm, and $\times$ is cross product.

After convergence, the network learns an implicit angle field for each point. In Figure 3, we visualize the optimizing
Figure 3: The visualization of learning an implicit angle field. Each point on the unit sphere represents a query vector, and the color map indicates their angle offsets to the ground truth normal represented as the black line. After convergence, an angle field centered on the ground truth normal is formed.

process of the angle field around a single point, where the well-learned angle field can predict correct angle offsets on the whole spherical surface.

The angle field implicitly represents the target vector as the zero level-set of the learned angle field. At inference time, to predict the target vector from the learned field, we introduce a gradient descent based optimization to explore the prior learned by the angle field. An intuitive implementation is to directly optimize some randomly sampled vectors which cover the whole spherical space until their angle offsets approach zero, but it increases the computational burden to optimize a large number of vectors during inference.

To resolve this issue, we propose to first predict a few coarse normals and then optimize these predicted coarse normals by a gradient decent based optimization algorithm.

**Inference of Angle Fields**

**Coarse Normal Prediction.** To predict coarse normals at inference time, we discretize the continuous unit spherical space into sections, which are the different query directions. Then a certain density of points is sampled from the discrete sections to form a query vector set $S = \{n_k^q|k = 1, 2, ..., m\}$ to cover $m$ different directions. If the query vectors in $S$ are infinitely dense to cover the whole spherical space, there is a vector $n^c_s \in S$ that satisfies $f_\theta(P_i^t, n^o) = 0$, then $n^o$ can be viewed as the approximated direction of $n^c_s$.

However, it is impossible to cover the full spherical space during testing since the discrete query vectors approximate the continuous spherical space and it is also time-consuming to infer highly dense query vectors. Therefore, we propose to predict a few coarse normals from the learned angle fields first.

In practice, given a test point $p_i^t$, the trained network $f_\theta$ takes the local neighborhood $P_i^t$ around $p_i^t$ and the query vectors in $S$ as input, and predicts the angle offsets $A = \{\alpha_{ik}|k = 1, 2, ..., m\}$, that is $f_\theta(P_i^t, n_k^q) = \alpha_{ik}$. We select the query vectors $\{n_k^q|s = 1, ..., l\}$ from $S$ with the minimum $l$ angle offsets as the predicted coarse normals. The coarse normals will be further refined in the following.

**Coarse Normal Refinement.** The analysis of normal prediction by the learned angle fields shows that the random sampling of discrete spherical points cannot perfectly cover the entire continuous spherical space, thus the forward pass prediction cannot make full use of the learned angle fields.

To further improve the accuracy of the predicted normals at inference time, we propose a novel learning scheme for refining the predicted normal vectors by a gradient descent based optimization algorithm. Unlike existing learning-based methods that can only predict normals via a single forward pass during inference, our proposed method can fully leverage the prior learned by NeAF to adjust the predicted normals, and can further optimize the predicted normal at inference time, thus leading to a more accurate estimation.

Specifically, the proposed NeAF represents the relationship between the target normal and an arbitrary vector in the neural network by measuring the angle offset. Thus, we can adopt further refinement given the predicted coarse normals by minimizing the angle offsets of these normals.

At inference time, we utilize the network $f_\theta$ with fixed parameters to output an angle offset $\alpha_{is}$ for the coarse normal $n_s^c$, and then update $n_s^c$ to make $\alpha_{is}$ closer to zero following the formula,

$$n_s^c = \arg \min_{n_s^c} \mathcal{L}_1(f_\theta(P_i^t, n_s^c) - 0),$$

where $\mathcal{L}_1$ is the $L1$ loss, and $P_i^t$ is the local patch for the testing point $p_i^t$.

After coarse normal refinement, $n_s^c$ is optimized to approximate the real zero level-set of the learned angle field.

To ensure the robustness of the results, we propose to average these refined normals $n_s^c$, $s \in [1, l]$ as the final estimation $n_{pred}$ by the following formula,

$$n_{pred} = \frac{1}{l} \sum_{s \in [1, l]} n_s^c.$$
| Methods | Density | Noise | average |
|---------|---------|-------|---------|
|         | Stripes | Gradients | No noise | Low Noise | Med Noise | High Noise |         |
| Jets (Cazals and Pouget 2005) | 13.39 | 13.13 | 12.23 | 12.84 | 18.33 | 27.68 | 16.29 |
| PCA (Hoppe et al. 1992) | 13.66 | 12.81 | 12.29 | 12.87 | 18.38 | 27.5 | 16.25 |
| HoughCNN (Bouch et al. 2016) | 12.47 | 11.02 | 10.23 | 11.62 | 22.66 | 33.39 | 16.90 |
| PCPNet (Guerrero et al. 2018) | 11.74 | 13.42 | 9.66 | 11.46 | 18.26 | 22.8 | 14.56 |
| Nesti-Net (Ben-Shabat et al. 2019) | 8.47 | 9.00 | 6.99 | 10.11 | 17.63 | 22.28 | 12.41 |
| IterNet (Lenssen et al. 2020) | 7.73 | 7.51 | 6.72 | 9.95 | 17.18 | 21.96 | 11.84 |
| DeepFit (Ben-Shabat et al. 2020) | 7.31 | 7.92 | 6.51 | 9.21 | 16.72 | 23.12 | 11.8 |
| Refine-Net (Zhou et al. 2022a) | 6.61 | 7.02 | 6.27 | 9.18 | 16.59 | 22.57 | 11.37 |
| AdaFit (Zhu et al. 2021) | 6.04 | 5.90 | 5.19 | 9.05 | 16.44 | 21.94 | 10.76 |
| NeAF (Ours) | 4.89 | 4.88 | 4.20 | 9.25 | 16.35 | 21.74 | 10.22 |

Table 1: Comparison of the angle RMSE with the state-of-the-art methods on PCPNet dataset.

We perform a visual comparison with the PCPNet (Guerrero et al. 2018), DeepFit (Ben-Shabat et al. 2020), RefineNet (Zhou et al. 2022a), and AdaFit (Zhu et al. 2021) in Figure 5, where the numbers represent the angle RMSE between the predicted normals and the real normals. The color of the shape indicates errors, and the closer to yellow the larger the error, the closer to blue the smaller the error. The results show that our method outperforms others with more details, and can better handle complex regions such as sharp corners and sharp edges.
Evaluation on real scanned dataset

We employ the model pretrained on PCPNet dataset to report our results on the scanned dataset.

SceneNN dataset. The SceneNN (Hua et al. 2016) dataset provides indoor scenes in the form of reconstructed meshes. We randomly sample 1 million points on the mesh of the shape to obtain a point cloud, and calculate the normal of each point. We randomly select 40% of all points to calculate RMSE. The results of the baseline methods are obtained by testing on the dataset using the models provided by (Ben-Shabat et al. 2020) and (Zhu et al. 2021) under the same experimental conditions. The angle RMSE comparison between NeAF, AdaFit (Zhu et al. 2021) and DeepFit (Ben-Shabat et al. 2020) is given in Table 2, and the qualitative results are shown in Figure 6. The numerical and visual comparisons show that NeAF achieves the best performance.

Semantic3D dataset. The Semantic3D (Hackel et al. 2017) dataset provides 30 non-overlapping outdoor scenes acquired with the Terrestrial Laser Scanner in the form of point clouds. This dataset does not provide reconstructed meshes, so the ground truth normals are not available, and we mainly report visual comparisons on this dataset. The comparison between NeAF and baseline methods is shown in Figure 7. The results show that the proposed method can estimate normals at sharp edges more accurately than baseline methods.

Surface Reconstruction

As an important property of local surfaces, normals are often used as the input for 3D surface reconstruction tasks, and accurate normal vectors play a key role in Poisson reconstruction (Kazhdan, Bolitho, and Hoppe 2006). We use the estimated normals for surface reconstruction, and the comparison with the baseline methods is shown in Figure 8. The results show that the normals estimated by NeAF are more accurate, which helps to reconstruct surfaces with higher accuracy.

Table 2: Normal RMSE of NeAF, AdaFit (Zhu et al. 2021) and DeepFit (Ben-Shabat et al. 2020) on the SceneNN dataset.
Figure 8: The comparison of the Poisson surface reconstruction using the estimated normals from NeAF, DeepFit (Benshadat et al. 2020), and AdaFit (Zhu et al. 2021).

Ablation Studies

Framework design. We demonstrate the effectiveness of each design of our framework in Table 3. We first skip predicting the coarse normals and instead directly refine the random vectors the same number of times. As shown by “No CNP”, the performance degenerates dramatically. We also predict normals without coarse normal refinement, and the result demonstrates that the coarse normal refinement can effectively improve the accuracy of the predicted normals in all settings as shown by “No CNR”. We replace averaging coarse normals with selecting the coarse normals with the minimum angle offsets after refinement as shown by “Min” and find a drop in performance.

Density of query vectors. We explore the effect of the sampling number $M$ of query vectors on the angle field learned by $f_{\theta}$. We report the performance of different $M = \{2.5k, 5k, 7.5k, 10k\}$ in Table 4. The query vector set that is too sparse (“2.5k”) cannot provide enough sample vectors for the network to learn the angle offsets. We found “5k” is a proper trade-off.

Number of coarse normals. In Table 5, we conduct experiments on the PCPNet dataset to explore how the coarse normal number $l$ affects coarse normal refinement at the inference time. We report the performance of different $l = \{1, 5, 10, 20, 50\}$, where we find that optimizing a single coarse normal leads to the degradation of results, and the best accuracy is achieved for the first time with 10 coarse normals. Optimizing more coarse normals requires longer time and more memory without improving the accuracy.

Conclusion

In this paper, we proposed NeAF to estimate point normals implicitly. We randomly sample the query vectors in a unit spherical space, estimate their angle offsets to ground truth normals, and output the query vector with the smallest angle offset as the estimated normal. To fully leverage the prior learned by NeAF, we refine the predicted normal vector by minimizing the estimated angle offset for more accurate normal estimation. NeAF achieves state-of-the-art performance on the synthetic dataset PCPNet and exhibits good generalization on real scans in SceneNN and Semantic3D. Furthermore, the promising results in the surface reconstruction task with normals estimated by NeAF justify our effectiveness in real applications.

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References

Amenta; et al. 1999. Surface reconstruction by Voronoi filtering. *Discrete & Computational Geometry*, 22(4): 481–504.

Aroujdu, S.; Seemann, P.; Langguth, F.; Guth, S.; and Gosele, M. 2017. Visibility-consistent thin surface reconstruction using multi-scale kernels. *ACM Transactions on Graphics (TOG)*, 36(6): 1–13.

Ben-Shabat, Y.; et al. 2019. Nesti-Net: Normal estimation for unstructured 3D point clouds using convolutional neural networks. In *Proceedings of the IEEE/CVF Conference on Computer Vision and Pattern Recognition*, 10112–10120.

Ben-Shabat, Y.; et al. 2020. DeepFit: 3D surface fitting via neural network weighted least squares. In *European Conference on Computer Vision*, 20–34. Springer.

Boulch, A.; et al. 2016. Deep learning for robust normal estimation in unstructured point clouds. In *Computer Graphics Forum*, volume 35, 281–290. Wiley Online Library.

Cao, J.; Zhu, H.; Bai, Y.; Zhou, J.; Pan, J.; and Su, Z. 2021. Latent tangent space representation for normal estimation. *IEEE Transactions on Industrial Electronics*, 69(1): 921–929.

Cazals, F.; and Pouget, M. 2005. Estimating differential quantities using polynomial fitting of osculating jets. *Computer Aided Geometric Design*, 22(2): 121–146.

Chen, C.; Liu, Y.-S.; and Han, Z. 2022. Latent partition implicit with surface codes for 3D representation. In *European Conference on Computer Vision*, 322–343. Springer.

Chen, Z.; and Zhang, H. 2019. Learning implicit fields for generative shape modeling. In *Proceedings of the IEEE/CVF Conference on Computer Vision and Pattern Recognition*, 5939–5948.

Grilli, E.; Menna, F.; and Remondino, F. 2017. A review of point clouds segmentation and classification algorithms. *The International Archives of Photogrammetry, Remote Sensing and Spatial Information Sciences*, 42: 339.

Guerrero, P.; et al. 2018. PCPNet: Learning Local Shape Properties from Raw Point Clouds. *Computer Graphics Forum*, 37(2): 75–85.

Hackel, T.; Savinov, N.; Ladicky, L.; Wegner, J. D.; Schindler, K.; and Pollefeys, M. 2017. SEMANTIC3D.NET: A new large-scale point cloud classification benchmark. In *ISPRS Annals of the Photogrammetry, Remote Sensing and Spatial Information Sciences*, volume IV-1-W1, 91–98.

Hoppe, H.; DeRose, T.; Duchamp, T.; McDonald, J.; and Stuetzle, W. 1992. Surface reconstruction from unorganized points. In *Proceedings of the 19th annual conference on Computer graphics and interactive techniques*, 71–78.

Hua, B.-S.; Pham, Q.-H.; Nguyen, D. T.; Tran, M.-K.; Yu, L.-F.; and Yeung, S.-K. 2016. Scenenn: A scene meshes dataset with annotations. In *2016 fourth international conference on 3D vision (3DV)*, 92–101. Ieee.

Huang, X.; Liang, Z.; Zhou, X.; Xie, Y.; Guibas, L. J.; and Huang, Q. 2019. Learning transformation synchronization. In *Proceedings of the IEEE/CVF Conference on Computer Vision and Pattern Recognition*, 8082–8091.

Kazhdan, M.; Bolitho, M.; and Hoppe, H. 2006. Poisson surface reconstruction. In *Proceedings of the fourth Eurographics symposium on Geometry processing*, volume 7, 61–70.

Lensen, J. E.; et al. 2020. Deep iterative surface normal estimation. In *Proceedings of the IEEE/CVF Conference on Computer Vision and Pattern Recognition*, 11247–11256.

Levin, D. 1998. The approximation power of moving least-squares. *Mathematics of Computation*, 67(224): 1517–1531.

Li, K.; Zhao, M.; Wu, H.; Yan, D.-M.; Shen, Z.; Wang, F.-Y.; and Xiong, G. 2022a. GraphFit: Learning Multi-scale Graph-Convolutional Representation for Point Cloud Normal Estimation. In *European Conference on Computer Vision*, 651–667. Springer.

Li, Q.; Liu, Y.-S.; Cheng, J.-S.; Wang, C.; Fang, Y.; and Han, Z. 2022b. HSurf-Net: Normal Estimation for 3D Point Clouds by Learning Hyper Surfaces. *Advances in Neural Information Processing Systems (NeurIPS)*.

Li, T.; Wen, X.; Liu, Y.-S.; Su, H.; and Han, Z. 2022c. Learning deep implicit functions for 3D shapes with dynamic code clouds. In *Proceedings of the IEEE/CVF Conference on Computer Vision and Pattern Recognition*, 12840–12850.

Ma, B.; Liu, Y.-S.; and Han, Z. 2022. Reconstructing surfaces for sparse point clouds with on-surface priors. In *Proceedings of the IEEE/CVF Conference on Computer Vision and Pattern Recognition*, 6315–6325.

Ma, B.; Liu, Y.-S.; Zwicker, M.; and Han, Z. 2022. Surface reconstruction from point clouds by learning predictive context priors. In *Proceedings of the IEEE/CVF Conference on Computer Vision and Pattern Recognition*, 6326–6337.

Mérigot; et al. 2010. Voronoi-based curvature and feature estimation from point clouds. *IEEE Transactions on Visualization and Computer Graphics*, 17(6): 743–756.

Mescheder, L.; Oechsle, M.; Niemeyer, M.; Nowozin, S.; and Geiger, A. 2019. Occupancy networks: Learning 3D reconstruction in function space. In *Proceedings of the IEEE/CVF Conference on Computer Vision and Pattern Recognition*, 4460–4470.

Metzer, G.; Hanocka, R.; Zorin, D.; Giryes, R.; Panazzo, D.; and Cohen-Or, D. 2021. Orienting point clouds with dipole propagation. *ACM Transactions on Graphics (TOG)*, 40(4): 1–14.

Michalkiewicz, M.; Pontes, J. K.; Jack, D.; Baktashmotlagh, M.; and Eriksson, A. 2019. Implicit surface representations as layers in neural networks. In *Proceedings of the IEEE/CVF International Conference on Computer Vision*, 4743–4752.

Mildenhall, B.; Srinivasan, P. P.; Tancik, M.; Barron, J. T.; Ramamoorthi, R.; and Ng, R. 2020. NeRF: Representing scenes as neural radiance fields for view synthesis. In *European Conference on Computer Vision*, 405–421. Springer.
Mullen, P.; De Goes, F.; Desbrun, M.; Cohen-Steiner, D.; and Alliez, P. 2010. Signing the unsigned: Robust surface reconstruction from raw pointsets. In *Computer Graphics Forum*, volume 29, 1733–1741. Wiley Online Library.

Muller, M. E. 1959. A note on a method for generating points uniformly on n-dimensional spheres. *Communications of the ACM*, 2(4): 19–20.

Park, J. J.; Florence, P.; Straub, J.; Newcombe, R.; and Lovegrove, S. 2019. DeepSDF: Learning continuous signed distance functions for shape representation. In *Proceedings of the IEEE/CVF Conference on Computer Vision and Pattern Recognition*, 165–174.

Pomerleau, F.; Colas, F.; Siegwart, R.; et al. 2015. A review of point cloud registration algorithms for mobile robotics. *Foundations and Trends® in Robotics*, 4(1): 1–104.

Qi, C. R.; et al. 2017. PointNet: Deep learning on point sets for 3d classification and segmentation. In *Proceedings of the IEEE Conference on Computer Vision and Pattern Recognition*, 652–660.

Wu, S.; Huang, H.; Gong, M.; Zwicker, M.; and Cohen-Or, D. 2015. Deep points consolidation. *ACM Transactions on Graphics (ToG)*, 34(6): 1–13.

Zhang, J.; Cao, J.-J.; Zhu, H.-R.; Yan, D.-M.; and Liu, X.-P. 2022. Geometry Guided Deep Surface Normal Estimation. *Computer-Aided Design*, 142: 103119.

Zhou, H.; Chen, H.; Zhang, Y.; Wei, M.; Xie, H.; Wang, J.; Lu, T.; Qin, J.; and Zhang, X.-P. 2022a. Refine-Net: Normal Refinement Neural Network for Noisy Point Clouds. *IEEE Transactions on Pattern Analysis & Machine Intelligence*, 1–1.

Zhou, J.; Ma, B.; Yu-Shen, L.; Yi, F.; and Zhizhong, H. 2022b. Learning Consistency-Aware Unsigned Distance Functions Progressively from Raw Point Clouds. In *Advances in Neural Information Processing Systems (NeurIPS)*.

Zhou, J.; et al. 2020. Normal estimation for 3D point clouds via local plane constraint and multi-scale selection. *Computer-Aided Design*, 129: 102916.

Zhu, R.; Liu, Y.; Dong, Z.; Wang, Y.; Jiang, T.; Wang, W.; and Yang, B. 2021. AdaFit: Rethinking Learning-based Normal Estimation on Point Clouds. In *Proceedings of the IEEE/CVF International Conference on Computer Vision*, 6118–6127.