Importance of Fermi surface and magnetic interactions for the superconducting dome in electron doped FeSe intercalates
– Supplemental Material –

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FIG. S1. Structural parameters from Ref. 1 (symbols) and interpolation (lines). Calculations were performed for interpolated structures in steps of Δx = 0.01.

I. CRYSTAL STRUCTURES

The study of Li_x(C3N2H10)0.5FeSe requires preparation of model structures as the experimental crystal structure provided in Ref. 1 contains some disorder and is not directly amenable to DFT calculations. First of all, the diaminopropane molecule is placed on an 8g Wyckoff position of the P4212 space group with partial occupancy of 0.093, leading to 0.372 C3N2H10 molecules per FeSe. We simplify this structure by lowering the symmetry to P1 and by picking one of the eight symmetry equivalent C3N2H10 molecules. This leads to the approximate stoichiometry Li_x(C3N2H10)0.5FeSe. This structure is shown in Figure 1 of the main text. Note that Sun et al. show a very similar simplified structure1. The simplification is justified because the diaminopropane molecules are important for fixing the interlayer distance; however, electronically, the highest occupied molecular orbitals are significantly below the Fermi level while the lowest unoccupied molecular orbital is slightly above the Fermi level so that the molecules are not active at the Fermi level. In order to obtain a finely spaced series of crystal structures of Li_x(C3N2H10)0.5FeSe as a function of doping level x, we interpolate the structural data provided in Ref. 1 as shown in Figure S1. This guarantees that we do not incur the well-known difficulties of predicting the chalcogenide z position in iron chalcogenide superconductors. Combining the diaminopropane molecule coordinates given for x = 0.26 with the lattice parameters of Figure S1 would lead to expanded or compressed molecules as a function of doping. This would be unrealistic as the molecules remain approximately neutral over the whole doping range, and their bonds should be rigid. Therefore, we adapt the diaminopropane molecule Wyckoff positions to the changing lattice parameters, keeping all distances and angles within the molecules constant.

We model the doping x as the position in iron chalcogenide superconductors. Combining the diaminopropane molecule Wyckoff positions to the changing lattice parameters, keeping all distances and angles within the molecules constant. We model the doping x by using the virtual crystal approximation for Li, using a nuclear charge between neon and lithium of Z = 2 + x.

II. ELECTRONIC PROPERTIES

In Figure S3, we show the unfolded one iron Fermi surfaces of Li_xFeSe with C3N2H10 molecules for six doping levels. The corresponding Fermi surfaces of Li_x(C3N2H10)0.5FeSe are shown in the main
text. In Figure S2, we show electron densities of $\text{Li}_x\text{(C}_3\text{N}_2\text{H}_{10})$\text{FeSe}$ as a function of doping levels $0.06 \leq x \leq 0.68$. The data are obtained by integrating iron total and orbital resolved densities of states up to the Fermi level.

Figure S4 shows the density of states at the Fermi level $N(E_F)$ for the two series of structures used in this study. (a) Full structure with diaminopropane molecule, (b) $\text{Li}_x\text{FeSe}$ with empty van der Waals gap.

FIG. S5. Superconducting $T_c$ of $\text{Li}_x\text{(C}_3\text{N}_2\text{H}_{10})$\text{FeSe}$ as a function of doping, as reported in Ref. 1.

III. ENERGY MAPPING

The exchange couplings of $\text{Li}_x\text{(C}_3\text{N}_2\text{H}_{10})$\text{FeSe}$ were determined by the energy mapping technique$^{2, 3}$. They are listed in Table S1. $2 \times 2 \times 1$ supercells with $\text{P1}$ symmetry contain eight inequivalent iron sites, allowing for 13 spin configurations with different energies. Two exam-
FIG. S6. Two examples for the energy mapping of Li$_x$(C$_3$N$_2$H$_{10}$)$_{0.5}$FeSe at (a) $x = 0.25$ and (b) $x = 0.55$. In both panels the inset shows an eight iron supercell with the minimum energy spin configuration marked by white for up and black for down. Up to a doping of $x = 0.45$, the stripe-type order is lowest in energy, while from $x = 0.5$, the bicollinear order is stabilized.

TABLE S1. Exchange couplings of Li$_x$(C$_3$N$_2$H$_{10}$)$_{0.5}$FeSe, calculated with GGA and 4 $\times$ 4 $\times$ 4 $k$ points. The errors shown are only the statistical error arising from the energy mapping.

| $x$  | $J_1$ (meV) | $J_2$ (meV) | $J_3$ (meV) |
|------|-------------|-------------|-------------|
| 0.1  | 72.3(2.7)   | 44.1(2.2)   | 4.5(2.0)    |
| 0.15 | 66.7(2.2)   | 43.4(1.8)   | 4.8(1.7)    |
| 0.2  | 59.7(2.1)   | 41.5(1.7)   | 5.1(1.6)    |
| 0.25 | 50.4(2.0)   | 39.0(1.6)   | 5.7(1.5)    |
| 0.3  | 39.5(1.9)   | 35.9(1.6)   | 6.6(1.5)    |
| 0.35 | 28.4(1.7)   | 33.0(1.4)   | 7.6(1.3)    |
| 0.4  | 16.9(1.4)   | 29.6(1.1)   | 8.8(1.0)    |
| 0.45 | 4.9(1.1)    | 25.7(9)     | 10.3(8)     |
| 0.5  | -7.3(1.0)   | 21.6(8)     | 11.2(8)     |
| 0.55 | -20.4(1.1)  | 17.5(9)     | 12.0(9)     |
| 0.6  | -33.1(1.1)  | 13.0(9)     | 12.2(8)     |

FIG. S7. Spin structure factors $S(q)$ at $x = 0.10$ and $x = 0.60$, obtained by Fourier transforming the ground state spin configuration found with iterative minimization.

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IV. ITERATIVE MINIMIZATION METHOD

The iterative minimization method allows the efficient determination of ground state spin configurations of classical spin systems$^{5,6}$. We use this method to obtain the ground states of the classical Heisenberg models with the exchange couplings in Table S1. We Fourier transform the obtained spin configurations to obtain the spin structure factor $S(q)$. The spin structure factors of Li$_x$(C$_3$N$_2$H$_{10}$)$_{0.5}$FeSe for doping values $x = 0.10$ and $x = 0.60$ are shown in Fig. S7. In the doping region of $0.10 \leq x \leq 0.45$, we find peaks at $q = (\pi, 0)$ or $q = (0, \pi)$, indicating stripe AFM order as ground states. On the other hand, for $0.50 \leq x \leq 0.60$, as soon as $J_1$ turns ferromagnetic, we find peaks at $Q = (q, q)$ where $q \sim 0.4\pi$. As it is well known that FeSe also has a substantial bi-quadratic coupling$^7$ which we do not re-determine here, the highly doped Li$_x$(C$_3$N$_2$H$_{10}$)$_{0.5}$FeSe samples will actually be in the bicollinear phase with $q = (\pi/2, \pi/2)$ and magnetically resemble FeTe.
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