Propagation of strangelets in the Earth’s atmosphere

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Abstract. A new model for the description of strangelets’ behavior in the Earth’s atmosphere is presented. Strangelet fission induced by colliding with air nuclei is included. It is shown that strangelets with certain parameters of initial mass and energy may reach depths near the sea level, which can be examined by ground-based experiments.

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1. Introduction

In a seminal work about two decades ago, Witten proposed that strange quark matter (SQM), the combination of roughly equal number of up, down and strange quarks, might be the true ground state of quantum chromodynamics (QCD) \cite{1}. Later calculations have shown that in spite of the effect of their finite volume, small nuggets of SQM in form of “strangelets” can also be stable as long as the baryon number exceeds a critical value $A_{\text{crit}}$ \cite{2}. On the one hand, various theoretical scenarios have provided chances for strangelet formation \cite{3 4}. They could be produced in highly energetic nuclear collisions \cite{5}, might originate from the collisions of two strange stars \cite{6}, and could also be ejected by supernova explosions \cite{7}. On the other hand, several exotic cosmic ray events have been reported by balloon and mountain experiments \cite{8 9 10}, which are considered to be ideal candidates of strangelets. The ultra-high energy cosmic ray (with energy > $10^{19}$ eV) events could also be the results of extensive air showers of relativistic strangelets accelerated in pulsar magnetospheres \cite{8 11}. Interestingly, one doubly charged event, with charge-mass ratio of $\sim 0.1$, has been detected by the AMS experiment in space \cite{12} and suggested to be strangelet-originated. This idea could be tested in the future AMS02.
Since the existence of stable SQM would have remarkable consequence in cosmology and astrophysics, what is experimentally important is to find out strangelets’ contribution to cosmic ray flux and the mechanism for the propagation of strangelets in the Earth’s atmosphere, both of which are helpful to confirm their existence. Recently, Madsen has estimated the flux of strangelets in cosmic rays incident on the Earth [13]. As for the latter, unfortunately, the necessary interaction input physics is at best poorly known.

Different phenomenological models of strangelet penetration in the atmosphere have been used by several authors to explain exotic cosmic ray events. Wilk et al. conjectured that although the initial mass of strangelets might be quite large, it decreases rapidly due to collisions with air molecules, until the mass reaches a critical value below which the strangelet disintegrates into neutrons [14]. Banerjee et al. provided a quite different scenario in which a strangelet picks up mass from atmospheric atoms [15]. Monreal novelly discussed the issue of strangelet accumulation in the atmosphere [16].

In the present work, we will reinvestigate this issue within the framework of rotating liquid drop model, which is still a phenomenological model. We assume that SQM nuggets produced from any cosmological or astrophysical objects do reach the surface of the atmosphere, and we evaluate their behavior in the atmosphere. We find that strangelets with particular initial baryon number and particular initial Lorentz factor can reach mountain altitudes, even the sea level (∼1000 g cm⁻²).

In the following sections, we will first provide revised results of ground state strangelet calculated from liquid drop model. Then we will investigate the colliding cross section between a liquid strangelet and an air nucleus. Finally we will give numerical results about the propagation of strangelets in the atmosphere. It should be mentioned that, for the sake of simplicity, our calculation is limited to the ordinary (unpaired) strangelets, i.e. the possible color-superconductivity effect of strangelets has not been considered.

2. Ground state properties of strangelets

It was argued that the phenomenal bag model first used by Alcock & Farhi [17] may not suit for strangelets. Nevertheless, at the present level, bag model is still the most effective way to understand the properties of strangelets, such as ground state energy per baryon, charge-to-mass ratio, fissionability, etc.

Within the framework of liquid drop model, He et al. [18] studied ground state properties of strangelets at finite temperature, by minimizing free energy of the system at fixed baryon number. The Coulomb energy was neglected there because the term contributes little to the system energy. In this section, we just go one step further to include Coulomb energy. Although Coulomb energy is negligible in computing $E/A$, it can greatly affect $Z/A$, and hence the fissionability parameters.

We consider strangelet as gas of $u,d,s$ quarks, their antiquarks, and gluons confined in an MIT bag model. The grand potential of the system $\Omega = \sum_i \Omega_i + BV$, where $B$ is
the bag constant, $V$ is the volume, and the grand potential of species $i$ is
\[ \Omega_i = \pm T \int_0^\infty dk \rho_i(k) \log(1 \pm \exp(-\sqrt{k^2 + m_i^2} - \mu_i)/T)). \]

(1)

In the above equation, “±” denotes for Fermions/Bosons, $\mu$ is the chemical potential, $\rho(k)$ denotes the density of states, which is given by
\[ \rho(k) = \frac{1}{2\pi^2} k^2 V + f_S \left( \frac{m}{k} \right) kS + f_C \left( \frac{m}{k} \right) C, \]

(2)

where $S (= 4\pi R^2$ for a sphere) is the surface area, $C (= 8\pi R$ for a sphere) is the curvature. The surface and curvature term for quarks are $\int_S (\frac{m}{k}) S + \int_C (\frac{m}{k}) C$, respectively; for gluons, $\int_S = 0, \int_C = -1/(6\pi^2)$.

The free energy of the system is given by
\[ F = \sum_i (\Omega_i + N_i \mu_i) + E_{\text{coul}} + BV, \]

(3)

in which the term of Coulomb energy $E_{\text{coul}} = (3/5)\alpha Z^2/R$ if electric charge is uniformly distributed in the sphere, where $\alpha$ is the fine structure constant and $Z$ is the total electric charge. Since Coulomb energy is taken into account, the chemical potential of up quark and down quark (strange quark) are no longer identical.

By minimizing $F$, the chemical potentials, charge to mass ratio and energy per baryon at any given baryon number and temperature can be calculated. Taking strange quark mass $m_s = 150$ MeV and bag constant $B = (145\text{ MeV})^4$, we get the following fitting values for the parameters. At zero temperature,
\[ M_{\text{str}}/A = (314.6(4) A^{-0.532(4)} + 875.9(1)) \text{ MeV}, \]

(4)

therefore, the minimum baryon number for stability ($M/A < 930$ MeV) is $A_{\text{crit}} = 27$.

The surface energy is
\[ E_S/A^{2/3} = (69.0(2) A^{-0.466(3)} + 77.9(1)) \text{ MeV}, \]

(5)

and the rescaled radius is
\[ r_0 = R/A^{1/3} = (0.124(1) A^{-0.445(3)} + 0.941(1)) \text{ fm}, \]

(6)

The numbers in parenthesis following each value indicate just the fitting uncertainties of the value in the last digit. The charge-to-mass ratio with respect to the equivalent baryon number is shown in Fig. (1).

As for a strangelet at excitation states to de-excite, $\gamma$–ray emission, hadron emission and fission into small parts should be under consideration. $\gamma$–ray emission and meson emission do not change the baryon number a little. According to CEFT model \[19\], baryon evaporation is suppressed in term of meson evaporation due to much smaller probability to simultaneously form two quark-antiquark pairs than one pair. Banerjee \textit{et al.} \[20\] and Sumiyoshi \textit{et al.} \[21\] has calculated meson and baryon evaporation rate of QGP ($\mu_q = 0$), respectively. As for a strangelet, in which the quarks
have non-zero chemical potentials, the numerical results are as follows. The energy loss rate caused by baryon evaporation is

\[ \frac{dE}{dt}_{\text{baryon}} = -6.23 \times 10^{19} A^{2/3} T^2 \exp(-999.9/T) \text{ MeV sec}^{-1}, \]

while the energy loss rate caused by meson evaporation is

\[ \frac{dE}{dt}_{\text{meson}} = -1.12 \times 10^{20} A^{2/3} T^2 \exp(-381.1/T) \text{ MeV sec}^{-1}. \]

Therefore, in the de-excitation process the only effective way to change the baryon number of the strangelet is the fission. Note that a strangelet at excited states will rapidly release its energy in about \(10^{-18} \sim 10^{-15}\) s, which is negligible compared with colliding time intervals.

3. Fission of strangelets by colliding with air nuclei

Now we investigate the stability of strangelets using the rotating liquid drop model. In the case of non-rotating systems, the relation between the nature of the stationary points and the stability of a system is simple, a maximum in one or more degrees of freedom indicates instability. However, the case for rotating systems is more subtle.

Consider a configuration of a rotating incompressible uniformly charged fluid endowed with a surface tension. The effective potential energy is given by

\[ E = E_S + E_C + E_R, \]

where \(E_S\) is the surface energy, \(E_C\) the electrostatic energy, and \(E_R\) the rotational energy. We neglect the curvature energy because it contributes little to the issue we consider.
We may write the deformation energy, measured with respect to the energy of the sphere, in the following dimensionless form, familiar in the literature of nuclear fission,

\[
\xi = \frac{E - E^{(0)}}{E_S^{(0)}} = (B_S - 1) + 2x(B_C - 1) + y(B_R - 1).
\] (10)

Here \(B_S = E_S/E_S^{(0)}, \ B_C = E_C/E_C^{(0)}, \ B_R = E_R/E_R^{(0)},\) and the two dimensionless parameters \(x\) and \(y\) specify the ratios of electrostatic and rotational energies of the sphere to the surface energy of the sphere, which are defined as \(x \equiv E_C^{(0)}/2E_S^{(0)}\) and \(y \equiv E_R^{(0)}/E_S^{(0)}\). According to our calculations, we found that the fissionability parameter \(x\) of strangelets varies from 0.001 to 0.030, which is much smaller than normal nuclei. Therefore, we take \(x = 0\) for simplicity in the following calculations.

If there is no rotation \((y = 0)\), the ground state is a sphere and the saddle shape is the configuration of two tangent spheres. With increased rotation, the ground state sphere is flatten into an axially symmetric (Hiskes) shapes, and the saddle varies to the so-called Pik-Pichak shapes. If \(y\) is even larger, the ground state will convert to a triaxial (Beringer-Knox) shape, which is quite close in appearance to the Pik-Pichak saddle. In fact, there exist a critical value \(y_{crit}\) above which the fission barrier vanishes, which implies a maximum rotation for stability.

Cohen et al. [22] has calculated the ground state and fission barrier energy measured with respect to the energy of the sphere,

\[
\xi_{ground} = y(-0.056 + 0.049y - 1.358y^2 + 0.946y^3),
\] (11)

\[
\xi_{barrier} = 0.280 - 0.778y + 0.622y^2 - 0.105y^3.
\] (12)

For \(x = 0, \ y_{crit} = 0.79.\)

Now we consider a cosmic ray strangelet incident in the Earth’s atmosphere. The center-of-mass energy \(E_{cm} = (M_{str}^2 + M_{air}^2 + 2\gamma M_{str} M_{air})^{1/2} - M_{str} - M_{air},\) where \(M_{air} = 14M_0\) \((M_0: \text{approximately the proton mass})\) is the mass of the air nucleus, \(M_{str}\) and \(\gamma\) is the mass and the Lorentz factor of the strangelet, respectively. There exists a nonzero colliding cross section as long as the strangelet has enough kinetic energy to overcome the Coulomb barrier \(E_v\) between the strangelet and the air nucleus, i.e. if \(E_{cm} > E_v,\) they will have a chance to “fusion” into a compound strangelet. In other words, the strangelet “absorbs” the air nucleus.

After fusion of the two, the excitation energy \(E_e = E_{cm} - E_v - E_R^{(0)} - E_{ground}.\) According to rotating liquid drop model, if the projectile strangelet is energetic enough, \(E_e\) will be higher than the fission barrier \(E_b,\) the newly formed compound strangelet will fission into two smaller strangelets which have nearly equal baryon numbers.

However, with the increasing kinetic energy of projectile strangelet, the effect of rotation should no longer be neglected, since if the fission barrier of rotating compound system approaches zero, no compound strangelet will form. It is reasonable to suppose the interaction time scale in this case is much shorter than the former.

The geometric cross section for contact is \(\sigma_{geo} = \pi(r_0 A_{str}^{1/3} + 1.12A_{air}^{1/3})^2,\) which is used by some earlier studies for crude calculations. However, the cross section is not
always $\sigma_{geo}$ for different baryon numbers and different velocities. The cross section for close collision can be put as

$$\sigma_{col} = \sigma_{geo}(1 - Z_{str} Z_{air} e^2 / (R_{str} + R_{air}) E_{cm}),$$

and the cross section for fusion is

$$\sigma_{cri} = 2\pi y_{crit} I_0 E_z^{(0)} (M_{str}^2 + M_{air}^2 + 2\gamma M_{str} M_{air}) / ((\gamma^2 - 1) M_{str}^2 M_{air}).$$

Fig. (2) gives the critical value of velocity for fusion and fission as a function of baryon number. The relation between the impact parameter $b$ in a collision between the two masses and the velocity of the strangelet is shown in Fig. (3).

4. Propagation of strangelets in the atmosphere

We consider strangelets with zero zenith angle of trajectory in this work, and the influence of gravity force is neglected because of its small contribution to the issue. Therefore, strangelet-air collision and ionization effect are what we mainly concern.

In our model, there exists a critical velocity of the strangelet in the strangelet-air collisions below which the air nucleus will be fused with the strangelet, and above which some mass will be stripped from the strangelet. The model is based on the analogical result in nuclear collisions, i.e. the linear momentum transfer between the strangelet and the air nucleus reaches a maximum around some critical energy.

When fusion happens, the velocity of the strangelet drops to

$$\gamma' = (\gamma M_{str} + M_{air}) / (M_{str}^2 + 2\gamma M_{str} M_{air})^{1/2}$$

Figure 2. Critical value of strangelet velocity. The upper curve represents the fission threshold of a non-rotating strangelet, and the lower curve represents the collision threshold.
after each collision, and particularly if $E_e > E_b$, the strangelet will fission into two smaller strangelets with equal baryon number $0.5(A_{str} + A_{air})$ and probably equal longitudinal velocity.

When the strangelet is more energetic, no compound strangelet will form. If the experimental law in nuclear collision is adaptable in this case, the velocity of the strangelet is assumed to drop to

$$
\gamma' = \gamma - (\gamma_{cri} - \gamma'_{cri}),
$$

where $\gamma_{cri}$ is the critical gamma factor for fusion, and $\gamma'_{cri}$ can be deduced from Eq. (15) given $\gamma = \gamma_{cri}$. The value of $\gamma_{cri}$ can be found by solving the equation $\sigma_{cri} = 0.5 \sigma_{col}$, i.e. the watershed of fusion-dominated and stripping-dominated collisions. It should also be mentioned that, we assume new strangelet will have a baryon number of $(A_{str} - A_{air})$ after each collision in numerical calculations. Indeed, at the present level, the mass and energy spectrum of decay products in this range are quite uncertain. Although our supposition is somewhat crude, it nevertheless tells some important information.

In addition to the effect of colliding with air nuclei, the issue of the energy loss of the strangelet through ionization of surrounding media can not be avoided, which is described by the Bethe-Bloch stopping power formula [23],

$$
dE/dx = -0.153\beta^{-2}Z_{str}^2(\ln(\gamma^2 - 1) - \beta^2 + 9.39) \text{ MeV}/(\text{g cm}^{-2})
$$

If $v < v_0 Z_{str}^{2/3}$, where $v_0 = 2.2 \times 10^8 \text{ cm/s}$ is the speed of electron in the first Bohr orbit, the effective charge $Z_{str} \rightarrow (v/v_0)Z_{str}^{1/3}$ due to the effect of electron capture [24].

**Figure 3.** A collision diagram of square of the impact parameter $b$ (times $\pi$) versus the velocity of a strangelet with $A_{str} = 100$, for the bombardment of an air nucleus by the strangelet.
The following figures show our numerical results. In Fig.(4), we present the mass evolvement of strangelets with $\gamma_0 = 10^3$ as a function of atmospheric depth. It is obvious that the larger the $\gamma_0$ and $A_0$, the more possible the strangelet reach the sea level. In Fig.(5), we show the distribution of particular final ($x = 1036$ g cm$^{-2}$) baryon number of strangelets as a function of initial baryon number and gamma factor. The upper serried oblique lines correspond to products of fission (lower energy), and the lower transverse lines correspond to products of stripping (higher energy). We find that if the initial strangelets have $A_0 \geq 3000$ or $\gamma_0 \geq 140$, they will have a chance to be detected by ground-based experiments. In Fig.(6), we give the final Lorentz factor as a function of initial baryon number.

![Figure 4.](image)

**Figure 4.** Mass evolvement of strangelets with $A_0 = 1000, 3000, 5000, 10000$ and $\gamma_0 = 10^3$ as a function of atmospheric depth.

5. Conclusions

From above discussion, it is reasonable to make a conclusion that strangelets with large mass and energy have the chance to penetrate the atmosphere to reach the sea level. Our model gives a lower limit of initial baryon number, that is, $A_0 \geq 3000$ or $\gamma_0 \geq 140$. Relevant flux for ordinary strangelets is unclear yet, but as predicted by Madsen [13], it is about $1 \sim 10$ per sqm year sterad, so there is a possibility for ground-based experiments to detect them. Madsen [13] predicts a flux of strangelets with a velocity spectrum (an event with $\gamma = 140$ could be very unlikely there) in a model where strangelets originate only by merging of binary strange stars. However, the $\gamma$ factor of strangelets produced in other ways (e.g., ejected and accelerated in pulsar’s magnetospheres [11, 25, 26]) may
Figure 5. Final mass distribution of strangelets ($x = 1036 \text{ g cm}^{-2}$) as a function of initial baryon number and lorentz factor.

Figure 6. Final lorentz factor ($x = 1036 \text{ g cm}^{-2}$) as a function of initial baryon number with $\gamma_0 = 200, 400, 600, 800, 1000$.

be greater than 140, and the possibility of an event with higher $\gamma$ could then not be ruled out yet.
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