Computation of masses and binding energies of some hadrons and bosons according to the rotating lepton model and the relativistic Newton equation

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Abstract. We compute analytically the masses, binding energies and hamiltonians of gravitationally bound Bohr-type states via the rotating relativistic lepton model which utilizes the de Broglie wavelength equation in conjunction with special relativity and Newton’s relativistic gravitational law. The latter uses the inertial-gravitational masses, rather than the rest masses, of the rotating particles. The model also accounts for the electrostatic charge-induced dipole interactions between a central charged lepton, which is usually a positron, with the rotating relativistic lepton ring. We use three rotating relativistic neutrinos to model baryons, two rotating relativistic neutrinos to model mesons, and a rotating relativistic electron neutrino - positron (or electron) pair to model the $W^\pm$ bosons. It is found that gravitationally bound ground states comprising three relativistic neutrinos have masses in the baryon mass range ($\sim 0.9$ to 1 GeV/$c^2$), while ground states comprising two neutrinos have masses in the meson mass range ($\sim 0.4$ to 0.8 GeV/$c^2$). It is also found that the rest mass values of quarks are in good agreement with the heaviest neutrino mass value of 0.05 eV/$c^2$ and that the mass of $W^\pm$ bosons ($\sim 81$ GeV/$c^2$) corresponds to the mass of a rotating gravitationally confined $e^\pm - \nu_e$ pair. A generalized expression is also derived for the gravitational potential energy of such relativistic Bohr-type structures.

1. Introduction
The potential importance of gravitational interactions between neutrinos was first discussed by Wheeler who actually introduced the term “neutrino geons” [1] long before it was shown that neutrinos have non-zero mass [2, 3].

In a recent book [4] and series of papers [5, 6, 7, 8, 9] we have shown that this is indeed the case for relativistic neutrinos with energies above 100 MeV [4, 5]. The gravitational attraction between such highly relativistic neutrinos becomes stronger than the Coulombic repulsion of two unit electric charges at the same distance [5].

This can be easily shown by just using gravitational rather than rest masses in Newton’s universal gravitational law [4, 5]. The gravitational mass, $m_g$, of a particle with rest mass $m_o$
is equal to its inertial mass, \( m_i \), and the latter equals \( \gamma^3 m_o \) where \( \gamma = (1 - v^2/c^2)^{-1/2} \) is the Lorentz factor.

The equality
\[
m_i = \gamma^3 m_o \tag{1}
\]
was first shown by Einstein in his pioneering 1905 special relativistic paper [10, 11, 12] and this result was shown recently to hold for arbitrary particle motion, including circular motion [4, 5]. In view of equation (1), Newton’s universal gravitational law becomes
\[
F = \frac{G m_1 m_2 \gamma^6}{r^2},
\]
or
\[
F = \frac{G m_1 m_2 \gamma^3 \gamma_2^3}{r^2}.
\]

In the case of eq. (2), \( \gamma \) involves the velocity \( v \) between the two particles, while equation (3) refers to circular motion and the Lorentz factors \( \gamma_1 \) and \( \gamma_2 \) correspond to the velocities, \( v_1 \) and \( v_2 \), of the two particles with respect to the center of rotation [4, 5].

It has been found, surprisingly, that the use of equation (3) in conjunction with the de Broglie wavelength equation in Bohr-type models with gravity as the attractive force describes gravitationally confined fm-size structures with the masses and most of the properties of hadrons [4, 5]. Here we provide a comparative summary of the key results of these studies which have been recently extended to include mixed rotating \( \nu_e-e^\pm \) structures. The latter have been shown to have the properties, such as masses, of \( W^\pm \) and \( Z^0 \) bosons [6, 7].

In the recent works [13, 14, 15, 16] it is argued that hadrons may be viewed as microscopic black holes, where gravitational collapse is prevented by the uncertainty principle. Furthermore, in [17, 18] Bohr-type models are used in order to establish a link between black holes and quantum gravity.

2. Mathematical models

2.1. Baryons

For the case of baryons the model comprises three rotating neutrinos or antineutrinos [4, 5] and the equation of motion for circular ground state orbits is [4, 5]
\[
\gamma m_o \frac{v^2}{r} = \frac{G m_2 \gamma^6}{\sqrt{3} r^2},
\]
coupled with the de Broglie wavelength equation
\[
\lambda = \frac{\hbar}{\gamma m_o v}.
\]

Assuming \( \lambda = r \) one obtains [4, 5]
\[
v \approx c \quad ; \quad \gamma = 3^{1/12} \left( m_P/m_o \right)^{1/3} = 7.163 \times 10^9 \quad ; \quad r = \hbar/\gamma m_o c = 0.63 \times 10^{-15} \text{ m},
\]
where \( m_P = (\hbar c/G)^{1/2} \) is the Planck mass. From (5) and (6) it follows that the mass, \( m_B \), of the confined baryon-type state is
\[
m_B = 3^{13/12} \left( m_P m_o^3 \right)^{1/3}.
\]

Exact agreement of eq. (7) with the neutron mass \( m_B = 939.565 \text{ MeV}/c^2 \) [20, 21]) is obtained for \( m_o = 0.04372 \) [4, 5]. This is in excellent agreement with the current best experimental estimates of the heaviest neutrino mass of \( 0.051 \pm 0.01 \text{ eV}/c^2 \) [2] and with the currently recommended value of \( 0.048 \text{ eV}/c^2 \) [19].
2.2. Mesons

In this case the model comprises a neutrino-antineutrino pair and equation (4) becomes

\[ \gamma m_o \frac{v^2}{r} = \frac{Gm_o^2 \gamma^6}{4r^2}. \]  

(8)

Solving this equation in conjunction with the de Broglie equation (5), one obtains

\[ v \approx c \quad \gamma = 2^{1/3}(m_{Pl}/m_o)^{1/3} = 8.234 \times 10^9 \quad r = \hbar/\gamma m_oc = 0.548 \text{ fm}. \]  

(9)

From (8) and (9) it follows that the mass, \( m_M \), of the confined meson-type state is

\[ m_M = 2^{4/3}(m_{Pl}m_o^2)^{1/3} = 2(2m_{Pl}m_o^2)^{1/3}. \]  

(10)

For \( m_o = 0.04372 \text{ eV}/c^2 \) [4, 5] one obtains \( m_M = 720 \text{ MeV}/c^2 \), which is in reasonable qualitative agreement with the mass of \( \rho \) mesons, i.e. 775 \( \text{MeV}/c^2 \) [20, 21].

Note that this problem has been already treated in reference [4] pp. 78-80. Due to an error of a factor of 2 in eq. 6.32, the first factor given for eq. (10) is \( 2^{7/6} \) instead of the current \( 2^{4/3} \).

It has been shown recently [6] that equation (10) can be generalized for a rotating pair of particles with arbitrary rest masses \( m_{1,o} \) and \( m_{2,o} \). In this case equation (10) takes the form

\[ m_M = 2^{4/3}(m_{Pl}m_{1,o}m_{2,o})^{1/3} = 2(2m_{Pl}m_{1,o}m_{2,o})^{1/3}. \]  

(11)

It is worth noting that, as long as the centripetal force exerted on all the rotating particles is the same, the following equation is always found to be valid [6, 7]:

\[ \gamma_{1m_{1,o}} = \gamma_{2m_{2,o}}, \]  

(12)

which follows directly from the application of eq. (8) to each particle separately.

2.2.1. Bosons

Equation (11) has been also derived recently for the mass a rotating relativistic electron-neutrino pair, yielding [6]

\[ m = 2m_W = 2^{1/3}(m_{Pl}m_em_o)^{1/3} = 2(81.74) \text{ GeV}/c^2 \]  

(13)

Remarkably \( m_W (= 81.74 \text{ GeV}/c^2) \) corresponds within 1.3% to the experimental mass of the \( W^\pm \) boson (80.42 \text{ GeV}/c^2). The suggestion that the \( W^\pm \) boson appears to be a relativistic \( \nu_e - \bar{\nu}_e \) pair decaying to two fragments of equal mass \( \gamma_{\nu_em_e} = \gamma_{\nu}m_{\nu} \), is not too surprising if one recalls that the main decay products of the \( W^\pm \) bosons are electrons, positrons and neutrinos [20, 21].

The same approach has been used recently [6] to compute the masses of the \( Z^0 \) and of the \( H \) bosons. The former is modeled as a rotational \( e^+ - e^- - \nu_e \) structure, and it yields

\[ m = 2m_Z = 2^{1/2}(m_{Pl}m_em_\nu)^{1/3} = 2(91.75 \text{ GeV}/c^2). \]  

(14)

Remarkably the thus computed \( m_Z \) value corresponds within 1% to the experimental value of 91.19 \text{ GeV}/c^2 [7].

In general, agreement with experimental values can be further improved by considering the Coulombic charge-charge, charge-induced dipole and induced dipole-induced dipole forces between positrons, electrons and electrically polarized neutrinos [6, 7].
3. Relativistic gravitational potential energy

For rotational structures with relativistic velocities comprising a number \( N \) of rotating particles the following analysis can be used for the computation of the relativistic gravitational potential energy of the particles.

We start from the generalized form of equation (4), namely

\[
F_j = \gamma_j m_{o,j} \frac{v_j^2}{r} = \frac{G m_{o,i} m_{o,j} \gamma_i^3 \gamma_j^3}{A_{ij} r^2} \tag{15}
\]

where \( i \) and \( j \) are integers up to \( N \), \( A_{ij} \) is a constant depending on \( N \) and on the particular structure geometry. For example for the three neutrino model it is \( A = \sqrt{3} \), (eq. (4)) and for mesons it is \( A = 4 \), (eq. (8)).

The angular momentum of the particle, \( m_{o,j} L_j \), is given by

\[
L_j = \gamma_j m_{o,j} v_j r. \tag{16}
\]

Recalling that \( L_j \) is conserved and assuming that \( L_j = n_j \hbar \), where \( n_j \) is an integer we find

\[
\frac{\gamma_j m_{o,j}}{n_j} = \frac{\gamma_i m_{o,i}}{n_i}. \tag{17}
\]

Equations (15) and (17) together with the approximation \( v \approx c \), imply

\[
\left(\frac{2A_{ij}}{r_{s,j}}\right)^{1/5} r = \gamma_j^3 \left(\frac{n_i}{n_j}\right)^{3/5} \left(\frac{m_{o,j}}{m_{o,i}}\right)^{2/5}, \tag{18}
\]

where \( r_{s,j} \) is the Swarzschild radius of \( m_{o,j} \). Replacing in the first of equations (15) \( \gamma_j \) via equation (18) we find

\[
F = C_{ij} m_{o,j} c^2 \frac{1}{r^{4/5}} \quad ; \quad C_{ij} = \left(\frac{2A_{ij}}{r_{s,j}}\right)^{1/5} \left(\frac{n_i}{n_j}\right)^{3/5} \left(\frac{m_{o,i}}{m_{o,j}}\right)^{2/5} \tag{19}
\]

Hence

\[
V_G(r) - V_G(r_{min}) = \int_{r_{min}}^{r} F dr' = -5C_{ij} m_{o,j} c^2 \left(r^{1/5} - r_{min}^{1/5}\right). \tag{20}
\]

Since \( r_s \) and \( r_{min} \) are of the order of \( 10^{-58} \) to \( 10^{-63} \) m for \( \nu_e \) and \( \nu_e \) respectively, it follows that the last term in parenthesis is negligible and thus using (18), it follows that

\[
V_G(r) = -5\gamma_j m_{o,j} c^2. \tag{21}
\]

This is an important result, already obtained in [4] for the three-neutrino problem and here shown to be valid for arbitrary elementary particle and geometry as long as \( \gamma >> 1 \), thus \( v \rightarrow c \). We note that it also holds for bosons, such as the \( W^\pm \) bosons [6] where it leads to

\[
V_G(r) = -5\gamma_e m_e c^2 = -5m_Z c^2 \tag{22}
\]

where \( m_Z \) is the boson mass. However in [6], where the \( Z^0 \) boson mass \( m_Z \) was calculated within \( \sim 1\% \) without any adjustable parameters, due to an error in eq. (38) the result reached for \( V_G(r) \) in eq. (40) is different from (22) and incorrect, affecting also the ensuing numerical results of section 5 of [6]. Thus the correct equation (38) in ref. [6] is the above equation (22) of the present paper.
In conjunction with the fact that the total kinetic energy of a $N$ particle structure is

$$T = \sum_i (\gamma_i - 1)m_o,i c^2$$  \hspace{1cm} (23)$$
equation (21) shows that the total Hamiltonian, $\mathcal{H}(= V_G + T + RE)$, where $RE$ is the rest energy of the leptons, can be is negative only for $N < 5$.

This implies, that only up to five leptons can participate to the rotating lepton ring of these Bohr-type structures and that pentaquarks, if they exist, are extremely unstable.

4. Conclusions
The methodology based on the relativistic Newton equation coupled with the de Broglie wavelength equation allows for the computation of the masses of hadrons and bosons with very good ($\sim 1\%$) accuracy.

The gravitational potential energy $V_G(r)$ is the same for all rotating hadrons and bosons (eq. 21) which supports the conclusion that the strong force can be modeled efficiently as a relativistic gravity via the relativistic Newton equation.

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