Experimental selection of young’s modulus according the structure of concrete

A A Varlamov¹, S Y Tverskoi²
¹Department of Design and Building Structures, Nosov Magnitogorsk State Technical University, 38, Lenin Ave, Magnitogorsk 455000, Russia
²Design Institute ”Magnitogorskgrazhdanproekt”, 79, K. Marksa Ave, Magnitogorsk 455028, Russia

E-mail: mgrp@mgn.ru

Abstract: The article provides an overview of the dependencies connecting structure of concrete with young's modulus. The developed method experimental evaluation of the known dependencies. In the studies used samples with different number and different size of crushed stone. Shows the compositions of the used concretes. According to the results of the tests were obtained the theoretical diagrams of the concrete. The paper presents the results of experimental studies of various compositions of concrete. Part of the dependency of the young's modulus was checked for consistency with experimental data. Proposed practical methodology for the evaluation of the concrete structure. According to the results of the comparison proposed a theoretical model for the estimation of the young's modulus for concrete structures.

1. Introduction

The modulus of elasticity of the material is one of its most important characteristics. The elastic characteristic is important when construction and the design and examination of buildings and structures. Direct determination of such characteristics is quite challenging. About the existence of communication of strength and deformation characteristics of concrete and its structure is known by results of researches of many scientists [1-6]. The proposed work binds the concrete structure with its modulus of elasticity. The evaluation of this communication can significantly facilitate the determination of the elastic modulus. Ultimately, this will significantly improve the quality of the design and operation of structures.

2. Models are considered for the study

In previous works [7-11] based on the analysis of various structural models of concrete received a number of theoretical dependencies of the elastic behavior of concrete. All the obtained model was determined by two parameters: the ratio of filler to the volume of the entire sample and the ratio of elastic moduli of filler and matrix in the sample. The shape and distribution of filler within the sample, according to the models depended on the form of the obtained expressions. Various forms of the derived expressions is shown below:

The dependence obtained on the basis of a flat model of concrete with to bringing filler in the cross section to trapezium:
\[ \frac{E_M}{E_p} = 1 - \frac{c}{h} + 2(c - l) \left[ 2h + \left( \frac{E_F}{E_M} - 1 \right) \left( \frac{d + b}{a} \right) h \right]^{-1} + l \left[ h + \left( \frac{E_F}{E_M} - 1 \right) \frac{hb}{a} \right]^{-1} \]  

(1)

The dependence obtained on the basis of a flat model of concrete under uniform distribution of filler material in section the model:

\[ \frac{E_M}{E_p} = \left[ 1 + \left( \frac{E_F}{E_M} - 1 \right) \frac{b}{a} \right]^{-1} \]  

(2)

On the basis of flat models with spherical filler:

\[ \frac{E_M}{E_p} = 1 - \frac{c}{h} - \frac{c}{h} \left[ 1 + \pi \frac{b}{4a} \left( \frac{E_F}{E_M} - 1 \right) \right]^{-1} \]  

(3)

Based on the three-dimensional model with a spherical filler and a cast of the distribution of filler material in cross-section to an ellipse when

\[ k < 1 \rightarrow \frac{E_M}{E_p} = \frac{\pi k}{2} - \frac{k^2}{2} \ln \left( \frac{1 - k^2}{k} \right)^{1/2} \]  

(4)

when

\[ k > 1 \rightarrow \frac{E_M}{E_p} = \frac{\pi k}{2} - \frac{2k^2}{(1 - k^2)^{1/2}} \arctan \frac{k - 1}{(1 - k^2)^{1/2}}. \]  

(5)

where \( k = a \left( E_F - E_M \right) \left( bE_M \right)^{-1} \).

Based on the three-dimensional model with cast filler to a rectangular prism:

\[ \frac{E_M}{E_p} = \left[ 1 + \frac{V_F}{V_p} \left( \frac{E_F}{E_M} - 1 \right) \right]^{-1} \]  

(6)

On the basis of a three-dimensional model for a spherical filler cast to by a rectangular prism:

\[ \frac{E_M}{E_p} = \left[ 1 + \frac{\pi b^2}{6a^2} \left( \frac{E_F}{E_M} - 1 \right) \right]^{-1}. \]  

(7)

Based on the three-dimensional model with bringing the sphere to the ellipsoid:

\[ \frac{E_M}{E_p} = \frac{\rho}{\sqrt{1 + 2\rho}}, \text{ where } \rho = \frac{2V_F E_M}{3V_p (E_F - E_M)} \]  

(8)

The basis of the three-dimensional model with he alteration of the spherical Filler on multiple ellipsoids:

\[ \frac{E_M}{E_p} = \frac{\rho}{\sqrt{1 + 2\rho}}, \text{ where } \rho = \frac{2A_p E_M}{A_{F,\text{max}} (E_F - E_M)} \]  

(9)

In the above dependencies \( E_p, E_F \) and \( E_M \) – the modulus of elasticity of standard prism, filler and matrix (mortar part of concrete); \( V_p \) and \( V_F \) the volume of the prism and the filler; \( c \) and \( h \) - are the
width and the height of the prism; \( d \) – is the diameter of the filler; \( a \) and \( b \) – the number of the placeholder by width and height of the prism; \( A_p \) and \( A_{b, \text{max}} \) – the cross-sectional area of the prism and the area of the filler in the cross section of the prism.

The analysis of the received dependences shows that under uniform distribution of filler in the models obtained almost the same ratio, independent of the shape of filler – dependencies (2), (3), (5), (6).

3. Research methodology
Gravel and sand for a day soaked in water. Prior to mixing the water was poured. Crushed stone was placed on a cloth and wiping. Pre-determined density of sand, gravel and water by standard methods – see table.1. Characteristics of the obtained mixtures are shown in table.2

The difference between the theoretical and the actual weight of samples made up for the 1st of the composition 1.6 %, for the 2nd composition of 2.6 %. The samples were hardened under normal conditions, 2 weeks, and then in the laboratory until stabilization of the weight. Stabilization of weight and shrinkage occurred within 90 days of curing. Tests were carried out according to standard methods in the age of 4 months.

Pre-tested cubic samples. The modulus of elasticity was determined after prior exposure of the sample under load of 50% breaking. The exposure was 10 minutes. After aging the load is removed. The measurements were performed during repeated loading. Tests were carried out on a 200-ton press. The measurements were carried out with the help of digital indicators with division 0.001 mm. Values of the coefficients of variation series less than 6 %. The results are shown in the table.3. The table shows average values.

4. The results of experimental data processing
Processing of the experimental data allowed to suggest the following correlation chart of concrete

\[
\varepsilon = \varepsilon_{ol} + \varepsilon_{pl} = \frac{\sigma}{E} + \exp\left(\frac{\ln\varepsilon_{pl\text{max}}}{\varepsilon_{el\text{max}}}\right)
\]

where \( \frac{\varepsilon_{ol}}{\varepsilon_{el\text{max}}} = \frac{R_o}{E}, \varepsilon_{pl\text{max}} = \varepsilon_{p} - \varepsilon_{el\text{max}} \).

| Series | Density of crushed stone, g/cm³ | Density of sand, g/cm³ | Density of cement, g/cm³ |
|--------|--------------------------------|------------------------|--------------------------|
| 1      | 2.75                           | 2.50                   | 3.10                     |
| 2      | 2.28                           | 2.50                   | 3.10                     |

Table 1. Characteristics of raw materials.

| Composition | \( \frac{W}{C} \) | Concrete density, g/cm³ | The relative volume of solution | The relative volume of filler |
|-------------|-------------------|-------------------------|-------------------------------|-----------------------------|
|             | wet | dry |                           |                              |                            |
| Characteristics of mixtures of the first series of experiments |
| 1       | 0.34 | 2.18 | 2.15 | 1.00 | 0 |
| 2       | 0.34 | 2.37 | 2.24 | 0.67 | 0.33 |
| 3       | 0.34 | 2.39 | 2.29 | 0.58 | 0.42 |
| 4       | 0.34 | 2.40 | 2.34 | 0.56 | 0.44 |
| 5       | 0.34 | 2.41 | 2.33 | 0.55 | 0.45 |
| Characteristics of the mixtures of the second series of experiments |
| 1       | 0.36 | 2.18 | 2.24 | 1.00 | 0 |
| 2       | 0.36 | 2.24 | 2.30 | 0.6  | 0.4 |
| 3       | 0.36 | 2.24 | 2.43 | 0.55 | 0.45 |
| 4       | 0.36 | 2.25 | 2.44 | 0.50 | 0.50 |
To verify was chosen as the third model -(3) (based on the analysis on the plane) and the fifth model- (5) (on the basis of the three-dimensional model) model. The selected model is most easy to work. When a well-known data structure in the cross section of a concrete sample - modules are determined in the third model. At a known volumetric indicators of the equations by selected models in accordance with the conducted experiments.

Table 3. Tests results.

| Compositions Indicators | 1 series | 2 series |
|-------------------------|----------|----------|
|                         | 1        | 2        | 3        | 4        | 5        | 1        | 2        | 3        | 4        |
| $R$, MIIa               | 31.6     | 31.6     | 34.7     | 34.1     | 35.2     | 34.0     | 35.1     | 31.6     | 37.4     |
| $R_b$, MIIa             | 25.6     | 30.7     | 29.8     | 28.1     | 29.3     | 27.6     | 29.2     | 27.3     | 30.9     |
| $E_b$, MIIa             | 23.9     | 30.4     | 32.1     | 32.8     | 33.1     | 25.1     | 32.1     | 32.8     | 33.5     |
| $\varepsilon_R \cdot 10^3$ | 190     | 220     | 230     | 210      | 235      | 205      | 212     | 220     | 235      |
| $\Theta = V_f/N_p$      | 0        | 0.334    | 0.42     | 0.44     | 0.454    | 0        | 0.4     | 0.454    | 0.498    |

Verification of theoretical models. To verify was chosen as the third model -(3) (based on the analysis on the plane) and the fifth model- (5) (on the basis of the three-dimensional model) model. The selected model is most easy to work. When a well-known data structure in the cross section of a concrete sample - modules are determined in the third model. At a known volumetric indicators of the equations by selected models in accordance with the conducted experiments.

Table 4. The results of calculations of modules according to a third formula (3) according to test results of prisms of the first series.

| The system of equations | 1+2 | 1+3 | 1+4 | 1+5 | 2+3 | 2+4 | 2+5 | 3+4 | 3+5 | 4+5 | The average value | The scatter of data, % |
|-------------------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-------------------|-----------------------|
| Full $E_M$, MIIa        | 23.9 | 23.9 | 23.9 | 23.9 | 21.6 | 23.2 | 24.3 | 17.5 | 19.9 |      | 25.8             | 22.8                  | 36.0                  |
|                         | 46.7 | 46.3 | 47.4 | 47.0 | 55.3 | 48.9 | 45.4 | 67.6 | 57.4 |      | 43.6             | 50.6                  | 47.0                  |
| No 3rd series $E_M$, MIIa | 23.9 | -   | 23.9 | 23.9 | -   | 23.2 | 24.3 | -   | -   |      | 25.8             | 24.2                  | 10.7                  |
|                         | 46.7 | -   | 47.4 | 47.0 | -   | 48.9 | 45.4 | -   | -   |      | 43.6             | 46.5                  | 11.4                  |

Table 5. The results of calculations of modules according to a third formula (3) according to test results of prisms of the second series.

| The system of equations | 1+2 | 1+3 | 1+4 | 1+5 | 2+3 | 2+4 | 2+5 | 3+4 | 3+5 | The average value | The scatter of data, % |
|-------------------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-------------------|-----------------------|
| $E_M$, MIIa             | 25.1 | 25.1 | 25.1 | 27.2 | 26.7 | 26.2 | 25.9 |      | 8.1              | 10.3                  |
| $E_F$, MIIa             | 44.8 | 43.8 | 43.6 | 40.4 | 41.4 | 42.0 | 42.7 |      |                 |                       |

Table 6. The results of calculations of modules according to a fifth formula (5) according to test results of prisms of the first series.

| The system of equations | 1+2 | 1+3 | 1+4 | 1+5 | 2+3 | 2+4 | 2+5 | 3+4 | 3+5 | 4+5 | The average value | The scatter of data, % |
|-------------------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-------------------|-----------------------|
| Full $E_M$, MIIa        | 23.9 | 23.9 | 23.9 | 23.0 | 23.8 | 22.8 | 22.8 | 17.7 | 19.6 | 23.9 | 22.7             | 22.8                  |
|                         | 43.2 | 43.3 | 44.0 | 44.0 | 43.6 | 45.5 | 45.4 | 51.9 | 49.4 | 44.1 | 45.4             | 19.2                  |
| No 3rd series $E_M$, MIIa | 23.9 | -   | 23.9 | 23.0 | -   | 22.8 | 22.8 | -   | -   | 23.9 | 23.4             | 4.70                  |
|                         | 43.2 | -   | 44.0 | 44.0 | -   | 45.5 | 45.4 | -   | -   | 44.1 | 44.4             | 5.20                  |
Table 7. The results of calculations of modules according to a fifth formula (5) according to test results of prisms of the second series.

| The system of equations | 1+2 | 1+3 | 1+4 | 2+3 | 2+4 | 3+4 | The average value | The scatter of data, % |
|-------------------------|-----|-----|-----|-----|-----|-----|-------------------|------------------------|
| $E_M$, MPa              | 25.1| 25.1| 25.1| 15.4| 26.4| 25.7| 23.8              | 46.2                   |
| $E_F$, MPa              | 42.6| 42.1| 42.0| 57.1| 40.7| 41.4| 44.3              | 37.0                   |

5. Analysis of the obtained results

Compare the results of the first series of experiments. As can be seen the scatter of data obtained by the third model, twice exceeds the scatter of the results obtained by the fifth model. However, the average values of the modulus of elasticity of mortar parts in the third and fifth formulas are practically identical and only differ by 10.8%. for the modulus of elasticity of the filler. If you don't count the samples from the solution, the scatter of the results of determination of modules remains virtually unchanged, accordingly, for the third model. 37.6...45.3 %, for the fifth model 17.8...to 28.5 %. If do not consider a third series of experiments, the results of which differ from the parallel experiments, the scatter of data is reduced to 4.7...11.4 %.

For the second series of experiments the opposite is true: the third model gives the scatter of data is three times smaller than the fifth model. A distinction average in series between the third and fifth models is for the mortar part of 8.4% for filler 3.7%.

In General, the results of determination of modulus of elasticity on the third and fifth models are similar. You can define modules and the third and fifth models. A few better spread of performance is as the third model. The choice of model will mainly depend on the source data. With known volumetric content of filler more than simple fifth model. When you define a module on textural indicators is preferable to the third mode.

For example, we present the results of determination of the volume content of filler on the texture of the third experience of the first series of samples. One cube in this series was cut to pieces. Received prismatic specimens of size 24x24x100 mm. Figure 1 shows photographs of four sides of one of the prisms.

![Figure 1. Photos of side surfaces of the prism.](image_url)

The volume of filler was determined for the three longitudinal sections on each side. As a result of processing of results of measurements of the volume content of aggregate in the concrete was received 0.424±0.016 that no more than 5% different from its content in the used concrete.

References
[1] Karpenko N I, Sokolov B S and Radaikin O V 2013 Analysis and improvement of curvilinear...
diagrams of deformation of concrete for the calculation of reinforced concrete structures on the deformation scheme Industrial and civil construction 1 25–30

[2] Kolchunov V I, Yakovenko N A and Klyuev N V 2013 Method physical models for resistance of concrete Industrial and civil construction 12 51–55

[3] Bondarenko V M 2002 Dialectics of mechanics of reinforced concrete Concrete and reinforced concrete 1 24–27

[4] Bondarenko V M and Rimshin V I 2015 Dissipative theory of the power of resistance concrete (Moscow: Student) p 111

[5] Skramtaiv B G and Leszczynski M Y 1964 Strength test of concrete samples and products buildings (Moscow:Stroyizdat) p 234

[6] Erofeev V T, Al D S and Mishunyaeva O A 2016 Ways to improve the durability and reliability of concrete structures Safety building Fund 1 13–19

[7] Varlamov A A 2012 Overall energy approach to the estimation of deformations of concrete Concrete and reinforced concrete 3 27–30

[8] Varlamov A A 2014 Model the elastic behavior of the concrete Izvestiya KSASU 3 (29) pp 19–26

[9] Varlamov A A 2014 The evaluation of the modulus of elasticity of concrete (Magnitogorsk: Publishing house Magnitogorsk. GOS. tehn. UN-TA im. G. I. Nosova) 2 pp 65–70

[10] Varlamov A A 2016 Design of the chart behavior of concrete Concrete and reinforced concrete 1 pp 6–8

[11] Varlamov A A 2016 On the new structure diagram of the concrete (Magnitogorsk: Publishing house Magnitogorsk. GOS. tehn. UN-TA im. G. I. Nosova) 2 11–14