Dimension degradation of fractionally spaced super-exponential algorithm for sparse channel equalisation

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Abstract: Fractionally spaced super-exponential (FSSE) algorithm has the disadvantage of computational complexity since it exploits high-order statistics explicitly. The authors propose a dimension degradation technique for FSSE when it is applied to the equalisation of a sparse channel in accordance with the relationship between the coefficients of the cross fourth-order cumulant (CFOC) in FSSE and the channel impulse response. They implement partial updating on the prominent coefficients of the CFOC with all the small coefficients remaining unchanged. The computational complexity of the modified FSSE reduces significantly with an acceptable performance loss, and its performance is validated via numerical simulations.

1 Introduction

We consider reducing the computational dimension of fractionally spaced super-exponential (FSSE) blind equalisers in the case of sparse channel equalisation. Sparse channels are usually encountered in a number of applications [1–5], such as underwater acoustic communications, high-speed code division multiple access communications and data transmission in high-definition television (HDTV), and so on. The impulse response of a sparse channel is characterised by a few significant multipath terms, while the majority taps of which are near zero-valued. A series of literatures exploit the sparse nature of a channel to simplify the complexity of an equaliser.

Sparse implementation of the recursive least square equaliser is proposed for shallow water acoustic communication [1] by ignoring the insignificant taps of the sparse channel. For efficient equalisation of the HDTV terrestrial broadcasting channels, the sparsity of the channel is exploited to simplify the decision feedback equaliser via a tap allocation algorithm, and large reductions in complexity are achieved [6]. From the above literatures, we can conclude that the sparsity of a channel has been used to either accelerate the convergence rate or simplify the computational complexity of an equaliser. However, to the best of our knowledge, there is still no try to simplify an FSSE equaliser of SE type using the sparseness of a channel. According to the fact that the cross fourth-order cumulant (CFOC) in the FSSE is an impulse response estimation of all the equivalent sub-channels, a partially updated approach on the CFOC is proposed by detecting the prominent coefficients in the equaliser. Hence, the dimension of CFOC for calculation during each iteration step degrades significantly.

2 Principle of FSSE algorithm

2.1 Basic principle of SE

Considering the time discrete system model as shown in Fig. 1, the transmitted sequence \( x(n) \) is distorted by the multipath channel \( h \) and additive Gaussian noise \( e(n) \), with \( n \) referring to the discrete-time index. The additive noise \( e(n) \) is assumed to be stationary as well as uncorrelated with \( a(n) \). The received signal \( z(n) \) is filtered by the equaliser \( f \) to recover the transmitted sequence. SE algorithm of Shalvi–Weinstein criterion maximises the magnitude of the complex equaliser output \( z(n) \) [7, 8]

\[ \psi_{p,q} = \text{cum}(z(n); p; z(n); q), \]  

subject to the variance constraint \( \psi_{1,1} = \psi_{1,1}^* \). Here, \( \text{cum}(\cdot) \) represents conjunction, \( \text{cum}(\cdot) \) denotes the operator of the cumulants, and its order is determined by using \( p \) and \( q \).

The composite channel refers to

\[ \zeta = h \otimes f. \]  

where \( \otimes \) denotes the convolution operation. Instead of providing pure time delay \( \Delta \), the composite channel \( \zeta \) for blind equalisation is of the form \( e^{j\varphi \Delta} \), where \( \Delta \) denotes the arbitrary time delay of the channel and equaliser, and \( \varphi \) the confused phase shift caused by the phase blind feature of SE. The aim of SE criterion is to transform the combined channel \( \zeta \) to \( e^{j\varphi \Delta} \) by using the following iterations:

\[ \zeta_1 = \zeta_0, \]  

\[ \zeta_{i+1} = \frac{1}{\|f\|^2} \zeta_i, \]  

where \( \zeta_i \) is the \( i \)th intermediate tap of \( \zeta \), and \( \zeta_{i+1} \) the corresponding \( i \)th tap after normalisation. For \( p + q > 2 \), the ratio between the largest tap and all the other taps would be iteratively increased until the smaller taps vanish due to the exponentiation. Shalvi and Weinstein [7] transform (3) and (4) into the following iteration formula because that the composite impulse response is an unknown vector:

\[ \gamma' = (H^H H)^{-1} H^H g, \]  

\[ \gamma'' = \frac{1}{\sqrt{f^H H^H y}} y', \]  

where \( [\cdot]^H \) denotes conjugate transpose, \( H \) is the channel convolution matrix with elements

\[ H_{ij} = h(i-j), \]  

and \( g \) is a vector of size \( M \times 1 \) with components
\[ g_i = c_i^\text{FS}(\text{FS})^\text{FS} \]  

Until now \( H \) and \( g \) are still unknown components, and their substitutions will be given in the following discussions.

### 2.2 Equivalent multi-channel model of FSSE

Oversampling the received data with respect to the input data rate, the model of the fractionally spaced equaliser is equivalent to the multi-channel model of time diversity as shown in Fig. 2. The communication system can be viewed as a single input multiple output system with \( L \) sub-channels and sub-equalisers, where \( L \) is the oversampling rate, \( \theta^i(n) \) denotes the transmitted symbols in the \( i \)th sub-channel, \( h^i(n) \) represents the impulse response with the length of \( N_l \), \( \varepsilon^i(n) \) is the additive Gaussian noise and \( z(n) \) denotes the composite output.

Consider the discrete signal at the receiver of each sub-channel

\[ x^i(n) = N_l - 1 \sum_{k=0}^{N_l-1} h^i(k)u(n-k) + \varepsilon^i(n), \]  

(9)

where

\[ x^i(n) = x(nL + i), \]  

(10)

and other variables \( y^i(n), h^i(n), \theta^i(n), f^i(n) \) are defined similarly. The output of each sub-equaliser can be expressed as

\[ y^i(n) = N_l - 1 \sum_{k=0}^{N_l-1} f^i(k)v^i(n-k), \]  

(11)

where \( N_l \) is the length of the sub-equaliser.

Defining \( f \) as the equaliser vector containing the multiple sub-equalisers and \( f \) the corresponding equaliser before normalisation, we obtain the updating process of the fractionally spaced SE as shown below, which is similar to the iteration procedure defined in (5) and (6):

\[ \tilde{f} = (\tilde{H}^i\tilde{H})^{-1}\tilde{H}^i\tilde{g}, \]  

\[ f = \frac{1}{\sqrt{f^T\tilde{H}^i\tilde{H}f}}\tilde{f}, \]  

where \( \tilde{H} = [H^0, H^1, ..., H^{L-1}]^T \). The elements of of \( H^k \) for negative \( l \) satisfy

\[ H^k_l = h^k(i-j) = h^{L+i}(i-j-1) \]  

(14)

Notice that the channel impulse response in (12) and (13) is an unknown vector, \( \tilde{H}^i\tilde{H} \) and \( \tilde{H}^i\tilde{g} \) should be replaced with measurable cumulants. Under the condition of transmitting i.i.d. source signal \( a^i(n) \) and by choosing \( p = 2, q = 1 \), the channel \( h^i(n) \) can be expressed by using the auto-correlation matrix of the received samples \( x^i(n) \) and the CFOC between the output \( z(n) \) and the input \( x^i(n) \).

\[ \tilde{f} = \tilde{R}^2\tilde{V}, \]  

\[ f = \frac{\tilde{f}}{\sqrt{\tilde{f}^T\tilde{R}\tilde{f}}}, \]  

(16)

where \( \tilde{R} \) a \( N_f \times N_f \) (\( N_f = L \times N_l \)) correlation matrix of the oversampled receiving signals \( x^i(n) \), which can be expressed as

\[ \tilde{R} = \begin{bmatrix} R^{0,0} & R^{0,1} & \cdots & R^{0,L-1} \\ R^{1,0} & R^{1,1} & \cdots & R^{1,L-1} \\ \vdots & \vdots & \ddots & \vdots \\ R^{L-1,0} & R^{L-1,1} & \cdots & R^{L-1,L-1} \end{bmatrix}. \]  

(17)

The elements in (11) are calculated as follows:

\[ R^{k,l} = \sum_{n=0}^{L-1} x^i(n-k)x^j(n-l). \]  

(18)

where \( C_{ij} \) represents the second-order cumulant of the \( a(k), i \) and \( j \) denote the row and column index of the element in \( R \), and \( k \) and \( l \) represent the index of the oversampled channel. The \( N_f \times 1 \) dimensional fourth-order joint cumulants vector \( V \) in (9) can be expressed with

\[ V = [v^{0}\cdots v^{N_f}, v^{1}\cdots v^{N_f}, \cdots, v^{N_f-1}\cdots v^{N_f}]^T. \]  

(19)

The element of which is also a vector of \( N_f \times 1 \) dimension

\[ v^k = [v^{i,k}_0, v^{i,k}_1, \ldots, v^{i,k}_{N_f-1}]^T. \]  

(20)

The element in (13) is based on the equaliser output \( z(k) \) and input \( \tilde{x}^i(k) \), the \( i \)th element of \( V \) can be written as

\[ v^k = \frac{\text{cum}(z(n-\delta); p; z^*(n-\delta); q; \tilde{x}^i(n-l))}{C^a_{2,2}}. \]  

(21)

where \( C^a_{2,2} \) represents the fourth-order cumulant of \( a(k) \) with

\[ C^a_{2,2} = \text{cum}(a(k); 2; a(k), 2), \]  

(22)

and \( \delta \) represents the position of initialisation spike of FSSE sub-equaliser. According to Fig. 2, the output of the equaliser is given by

\[ z(n) = \frac{1}{L-1} \sum_{k=0}^{L-1} f^k(n)X^k(n). \]  

(23)

with

\[ f^k = [f^k_0, f^k_1, \ldots, f^k_{N_f-1}]^T. \]  

(24)

and

\[ X^k = [x^k(n), x^k(n-1), \ldots, x^k(n-N_f+1)]^T. \]  

(25)

The iteration procedure of FSSE consists of (15)-(23). The FSSE algorithm usually converges in four to five iterations.
3 Sub-channel identification using FSSE

As shown in [7], channel identification is achieved by the estimation of CFOC for SE algorithm. The relationship of the sub-channels and the element of vector $V$ in FSSE are shown in [9]:

$$
\tilde{v}_k^i = h^{k\delta}\tilde{y}(\delta - i),
$$

(26)

where parameter $\delta$ is an arbitrary time delay set in the equaliser, and the estimated channel impulse response migrates with the same time delay. The vector $V$ contains the reversed conjugated impulse response of all sub-channels:

$$
V = [h^{0\delta}(\delta), h^{0\delta}(\delta - 1), \ldots, h^{0\delta}(\delta - N_f + 1), h^{1\delta}(\delta), h^{1\delta}(\delta - 1), \ldots, h^{1\delta}(\delta - N_f + 1), \ldots, h^{L-i\delta}(\delta), h^{L-i\delta}(\delta - 1), \ldots, h^{L-i\delta}(\delta - N_f + 1)]^T.
$$

(27)

The elements of each sub-channel with arbitrary time delay $\delta$ can be observed in each sub-vector $\tilde{v}_k^i$:

$$
\tilde{v}_k^i = [\tilde{v}_k^0(\delta), \tilde{v}_k^0(\delta - 1), \ldots, \tilde{v}_k^0(\delta - N_f + 1)]^T.
$$

(28)

The length of each sub-equaliser $N_i$ must be greater than the length of each sub-channel $N_f$.

The important conclusion can be drawn from (28) is that a fractionally sparse channel will lead to a sparse cross-cumulant vector $V$. The relationship illustrated in (28) is the basis of the partial update cumulant, which will be discussed in the next section. During the convergence process of the FSSE, some prominent elements in $V$ can be selected for update in the next iterations, while the small element remains unchanged. Hence, the dimension of $V$ being necessary to calculate degrades significantly after such processing.

4 FSSE with partially updated cumulant (PUC-FSSE)

To simplify the computational complexity of the original FSSE operating in sparse channel scenarios, a technique with partially updated cumulant is shown in Fig. 3. During iterating process of simplified FSSE, the original FSSE is performed during the first iteration, and all elements of the cross-cumulant are calculated. However, only prominent coefficients of the cumulant are calculated from the second iteration, with all the other small elements remaining unchanged. Therefore, during the limited number of iterations of the FSSE, the computational complexity of the FSSE reduces greatly due to the sparse channel impulse response. We term this technique as equaliser with partially updated cumulant, and the corresponding FSSE as PUC-FSSE.

The detailed steps to implement the PUC-FSSE are illustrated as follows [8–10]:

**Step 1: First iteration**

i. Each sub-equaliser of the FSSE is initialised with a spike at time delay $\delta$, with all the other coefficients setting as zeros.

ii. Set the number of the iteration as $N_f$.

iii. The auto correlation matrix $R$ is calculated using (17) and (18), and it is not necessary to calculate $R$ during the next iterations.

iv. Calculate all the elements of the CFOC $V$ using (20) and (21).

v. The equaliser is achieved using (15) and normalised using (16).

vi. The output of the equaliser is calculated using (23).

**Step 2: Prominent coefficients selection from the fourth-order joint cumulant**

This step is performed before the second iteration as shown in Fig. 3.

i. The modulus of the elements of $V$ is calculated.

ii. A threshold for prominent element selection is set as

$$
|v_i| > \eta_0
$$

(29)

The method to determine $\eta_0$ is as follows:

$$
\eta_0 = \frac{\rho_0}{100}\max\{|v_i|\}
$$

(30)

where $\max\{|\cdot|\}$ denotes getting the maximum value.

**Step 3: Partial update of the CFOC**

i. Calculate the prominent coefficients using (21)

$$
\tilde{v}_k^i = [v_k^0, v_k^1, \ldots, v_k^{N_f-1}, \ldots, v_k^0(\delta - N_f + 1)]^T.
$$

(31)

where $\tilde{v}_k^i$ with a bar refers to the prominent coefficient, and other elements without a bar denote the unchanged elements.

ii. Update the prominent elements in (31).

Implement (15), (16) and (23) to obtain the new equaliser output.

**Step 4: Implementation of the next iterations**

i. If the loop number does not reach $N_f$, calculate the sparse vector $V$ and the fractionally equaliser weight for the next iteration.

ii. If the loop number reaches $N_f$, we will finish the iteration process.

5 Numerical simulations

The simulation conditions are listed in Table 1. The simulations aim to illustrate the performances of FSSE and PUC-FSSE from three aspects.

i. The performance of FSSE without partial cumulant update;
ii. The performance of FSSE with partial cumulant update;
iii. The performance of the PUC-FSSE with different oversampling rate $L$.

5.1 Numerical simulation I

An oversampled sparse channel with four prominent taps are shown in Table 2, and two sub-channels are considered here.

The estimated cumulant by the PUC-FFSE with an oversampling rate of two is shown in Fig. 4. It can be seen that the modulus of the sub-channels is identical to the channel coefficients in reverse order. Fig. 5 shows the curves of the mean square error (MSE) of the FSSE and PUC-FSSE. The MSE of the FSSE is defined as

$$\zeta = \frac{1}{N_s} \sum_{k=0}^{N_s-1} (r - |z(k)|^2)(r - |z(k)|^2)$$

(32)

with

$$\gamma = \frac{E(|a(k)|^4)}{E(|a(k)|^2)}$$

(33)

where $N_s$ is the sample number for iteration process. The MSE of PUC-FSSE with threshold 0.05 increased about 0.5 dB in comparison with the original FSSE, and an 5 dB increase was observed for PCU-FSSE with threshold 0.1. The elements number of the updated cumulants is 7 corresponding to threshold 0.05, and 6 corresponding to threshold 0.05, respectively. As a compromise between computational complexity and MSE performance, 0.05 is an appropriate threshold for PCU-FSSE. The constellations of the recovered signal of FSSE and PCU-FSSE (threshold 0.05) are shown in Fig. 6. We can see that the eye diagram of PCU-FSSE is sufficiently wide open to cascade a decision-directed least mean square (DD-LMS) algorithm for quantisation decision.

The computational complexity of FSSE and PCU-FSSE ($n_0 = 0.5$) is compared in Table 3. The values of the parameters in Table 3 are $N_f = 60$, $N_s = 10,000$, $N_c = 10$, and $L_s = 7$. The results in Table 3 illustrate that the PCU-FSSE algorithm achieves a 50% reduction in the number of complex multiplication in comparison with that of the original FSSE.

### Table 1 Conditions of two numerical simulation

| Simulation conditions | Simulation I | Simulation II |
|-----------------------|--------------|---------------|
| transmitted signal $a(k)$ | QPSK | QPSK |
| signal-to-noise ratio (SNR) | 25dB | 25dB |
| dimension of the equaliser $N_f$ | $30 \times 2 = 60$ | $45 \times 4 = 180$ |
| oversampling rate $L$ | 2 | 4 |
| iteration number of FSSE $N_I$ | 10 | 10 |
| time delay of the equaliser $\delta$ | 15 | 15 |
| the length of data frame $N_s$ | 10,000 | 10,000 |
| the threshold for PUC-FSSE $\rho_0$ | 0.05/0.1 | 0.05/0.1 |

### Table 2 Coefficients of an oversampled four-ray channel

| Sub-channels | Ray 1 | Ray 2 | Ray 3 | Ray 4 |
|--------------|-------|-------|-------|-------|
| $h^{(0)}$ | $h^{(0)}(0) = 1$ | $h^{(0)}(0) = 0.6e^{j0.4}$ | $h^{(0)}(5) = 0.28e^{j1.4}$ | $h^{(0)}(7) = 0.07e^{j2.2}$ |
| $h^{(1)}$ | $h^{(1)}(0) = 1$ | $h^{(1)}(3) = -0.5e^{j1.6}$ | $h^{(1)}(7) = 0.33e^{j0.2}$ | $h^{(1)}(9) = 0.03e^{j0.2}$ |

![Fig. 4](image1.png)

Fig. 4 Modulus of the cross fourth-order cumulant

![Fig. 5](image2.png)

Fig. 5 MSE curves of the FSSE and PUC-FSSE

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Conclusions

FSSE equaliser has been investigated in the case of equalising a sparse channel. Compared to the original FSSE, the FSSE with partially updated cumulant is computationally simpler at the expense of some acceptable performance loss. The constellation of the output signal of the simplified equaliser is wide enough to switch to a DD-LMS tracking mode. The PCU-FSSE exhibits good MSE performance and implement efficiency simultaneously.

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Table 3

| Items for calculation | FSSE | PCU-FSSE |
|-----------------------|------|----------|
| auto correlation $R$  | $NN_s$ | unchanged |
| joint cumulant $V$    | $9NN_sN_c$ | $9LN_s(N_s−1) + NN_s$ |
| equaliser vector $f$  | $N_f^2 + N_f$ | unchanged |
| equaliser normalisation $\tilde{f}$ | $3N_f$ | unchanged |
| output convolution $z(n)$ | $NN_sN_c$ | unchanged |
| number of complex multiplication | 95,616,240 | 47,886,240 |

Fig. 6 Constellation of the recovered signal
(a) FSSE, (b) PCU-FSSE

Table 3: Computational complexity comparison between FSSE and PCU-FSSE

| Items for calculation | FSSE | PCU-FSSE |
|-----------------------|------|----------|
| auto correlation $R$  | $NN_s$ | unchanged |
| joint cumulant $V$    | $9NN_sN_c$ | $9LN_s(N_s−1) + NN_s$ |
| equaliser vector $f$  | $N_f^2 + N_f$ | unchanged |
| equaliser normalisation $\tilde{f}$ | $3N_f$ | unchanged |
| output convolution $z(n)$ | $NN_sN_c$ | unchanged |
| number of complex multiplication | 95,616,240 | 47,886,240 |

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