Theoretical Aspects of $CP$ Violation in Hyperon Decays

G. Valencia

Department of Physics and Astronomy, Iowa State University, Ames, IA 50011

Abstract

I review the estimate of the $CP$ violating asymmetry $A(\Lambda^0)$ within the standard model. I then review the estimate of the upper bound on this asymmetry that arises from measurements of $CP$ violation in kaon decays.

One of the systems where it is possible to search for $CP$ violation is in the non-leptonic decay of hyperons. Of particular interest for the upcoming experiment E871 is the asymmetry $A(\Lambda^0)$ \cite{1}. E871 expects to reach a sensitivity of $10^{-4}$ for the sum of asymmetries $A(\Lambda^0) + A(\Xi^-)$ \cite{2}. Unfortunately, the calculation of these asymmetries is plagued by theoretical uncertainties in the estimate of the hadronic matrix elements involved. Nevertheless, a conservative study of these asymmetries within the minimal standard model has shown that $A(\Lambda^0)$ is likely to occur at the level of a few times $10^{-5}$ \cite{3}. Similarly, a recent estimate has indicated that current constraints from measurements of $CP$ violation in kaon decays do not preclude $A(\Lambda^0)$ from reaching values of order a few times $10^{-4}$ beyond the standard model \cite{4}. In view of this, the potential results of E871 are very exciting.

The general framework to discuss $A(\Lambda^0)$ can be found for example in Ref. \cite{1}. With the experimentally known values for the strong rescattering phases and the moduli of the weak decay amplitudes, we can write:

$$A(\Lambda^0) \approx 0.13 \sin(\phi^p_1 - \phi^s_1) + 0.001 \sin(\phi^p_1 - \phi^s_3) - 0.0024 \sin(\phi^p_3 - \phi^s_1)$$

(1)

In the case of the minimal standard model, the effective weak Hamiltonian in the notation of Buras \cite{5} is,

$$H^{SM}_W = \frac{G_F}{\sqrt{2}} V^*_{ud} V_{us} \sum_i \left( z_i(\mu) - \frac{V^*_{td} V_{ts}}{V^*_{ud} V_{us}} y_i(\mu) \right) Q_i(\mu) + \text{h. c.}$$

(2)

The calculation of the weak phases would proceed by evaluating the hadronic matrix elements of the four-quark operators in Eq. \cite{2} to obtain real and imaginary parts for the amplitudes, schematically:

$$\langle p\pi | H^{eff}_w | \Lambda^0 \rangle \left| \ell \right> = \text{Re} M^\ell + i \text{Im} M^\ell.$$  

(3)

At present, however, we do not know how to compute the matrix elements so we cannot actually implement this calculation.
For a simple estimate, we can take the real part of the matrix elements from experiment (assuming that the measured amplitudes are real, that is, that $CP$ violation is small), and compute the imaginary parts in vacuum saturation. This approach provides a conservative estimate for the weak phases because the model calculation of the real part of the amplitudes is smaller than the experimental value. Nevertheless, the numbers should be viewed with great caution.

In the standard model, use of vacuum saturation to estimate the matrix elements gives $\phi_s^1 \approx -3y_6\text{Im}\tau$ and $\phi_p^1 \approx -0.3y_6\text{Im}\tau$. Using $y_6 \approx -0.08$ and $\text{Im}\tau = A^2\lambda^4\eta \leq 0.001$ we find [4]:

$$A(\Lambda^0) \approx 3 \times 10^{-5}$$

(4)

Other models of $CP$ violation contain additional short distance operators with $CP$ violating phases. In Table [4] we list all the four quark operators of dimension six that change strangeness by one unit [4].

| Operator | Ref. | $|\Delta S| = 1$ |
|----------|------|----------------|
| $O^{(1,1)}_{qq}$ | $O^{(1,1)}$ | $\frac{i}{2}d_L\gamma_Ls_L(\bar{u}_L\gamma_Lu_L + d_L\gamma_Ld_L)$ |
| $O^{(8,1)}_{qq}$ | $O^{(8,1)}$ | $\frac{i}{2}d_L\lambda^a\gamma_Ls_L(\bar{u}_L\lambda^a\gamma_Lu_L + d_L\lambda^a\gamma_Ld_L)$ |
| $O^{(1,3)}_{qq}$ | $O^{(1,3)}$ | $\frac{i}{2}(2\bar{u}_L\gamma_Ls_Ld_L\gamma_Lu_L - \bar{u}_L\gamma_Lu_Ld_L\gamma_Ls_L) + d_L\gamma_Lu_Ld_L\gamma_Ls_L + d_L\lambda^a\gamma_Lu_Ld_L\lambda^a\gamma_Ls_L) \frac{i}{2}$ |
| $O^{(8,3)}_{qq}$ | $O^{(8,3)}$ | $\frac{i}{2}(2\bar{u}_L\lambda^a\gamma_Ls_Ld_L\lambda^a\gamma_Lu_L - \bar{u}_L\lambda^a\gamma_Lu_Ld_L\lambda^a\gamma_Ls_L) + d_L\lambda^a\gamma_Lu_Ld_L\lambda^a\gamma_Ls_L$ |

Table 1: Dimension six $|\Delta S| = 1$ four-quark operators.

We estimate the contribution of these operators to $A(\Lambda^0)$ in vacuum saturation taking the real part of the amplitudes from experiment as before [4]. The effective Hamiltonian now reads:

$$H_{eff} = H_W^{SM} + \frac{g^2}{\Lambda^2} \left( \sum_i \lambda_i O_i^{\text{new}} + \text{h. c.} \right)$$

(5)
The new operators violate $\mathcal{C}\mathcal{P}$ if the coupling $\lambda_i$ has an imaginary part. In this case they also contribute to $\mathcal{C}\mathcal{P}$ violation in kaon decays. For direct $\mathcal{C}\mathcal{P}$ violation we write:

$$\frac{\epsilon'}{\epsilon} = \left(\frac{\epsilon'}{\epsilon}\right)_6 \left(1 - \Omega_{SM} - \Omega_{NEW}\right).$$

(6)

We require that $\Omega_{NEW} \leq 1$ to place bounds on the parity violating $\mathcal{C}\mathcal{P}$ violating phases using vacuum saturation to estimate the matrix elements.

In general, $\epsilon'$ provides tighter constraints on new $\mathcal{C}\mathcal{P}$ violating interactions that does $\epsilon$. Nevertheless, it is necessary to consider constraints from $\epsilon$ because the ones that arise from $\epsilon'$ do not apply to parity conserving operators that do not contribute to the decay $K^0 \to \pi\pi$. Each of the new operators contributes to $\epsilon$:

$$|\epsilon_i| \approx \frac{1}{\sqrt{2}} \frac{|\text{Im}M_{12}|_i}{\Delta m_k}. \quad \text{(7)}$$

We require that $|\epsilon_i| \leq |\epsilon|_{\text{exp}}$ to place bounds on the parity conserving $\mathcal{C}\mathcal{P}$ violating phases using the model of Ref. [7] and of Ref. [8].

The bounds on the weak phases are presented in Table 2 [4]. The blank entries indicate that there is no bound because the particular operator does not contribute to that amplitude.

The bounds on the $p$-wave phases arise from the contributions of the operator to $\epsilon$, and are weaker than the bounds on the $s$-wave phases that arise from the contributions to $\epsilon'$. We illustrate separately the bounds on each parity and isospin amplitude because it is possible to construct operators with definite parity and isospin.

There are also two-quark operators that can contribute to the processes under consideration [6]:

$$\mathcal{O}_{dG} = (\bar{q}\sigma_{\mu\nu}\lambda^a d)\phi G_{\mu\nu}^a. \quad \text{(8)}$$

Following the procedure used for the four-quark operators but taking the matrix elements from MIT bag model calculations this time [8], we find [4]:

$$A(\Lambda_0^0) \leq \begin{cases} 
3 \times 10^{-4} & \text{Parity conserving operator} \\
6 \times 10^{-5} & \text{Parity violating operator}
\end{cases} \quad \text{(9)}$$

To summarize, the minimal standard model predicts that $A(\Lambda_0^0)$ is of the order of a few times $10^{-5}$ [3]. The effect of $\mathcal{C}\mathcal{P}$ violating operators beyond the standard model is constrained by the $\mathcal{C}\mathcal{P}$ violation observed in kaon decays. Most of all the possible dimension six operators would naturally induce contributions to $A(\Lambda_0^0)$ at the $10^{-5}$ level, making them indistinguishable from the minimal standard model (as long as precise calculations of the matrix elements are not available), and inaccessible to the search to be conducted by E871. However, there are certain operators, $\mathcal{O}_{qq}^{(1)}$ and $\mathcal{O}_{dG}$ that could induce an asymmetry $A(\Lambda_0^0)$ as large as a few times $10^{-4}$.

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Table 2: Bounds on the phases that enter $A(\Lambda_0)$.

| Operator | $\phi_1^p \times 10^5$ | $\phi_1^s \times 10^5$ | $\phi_3^p \times 10^5$ | $\phi_3^s \times 10^5$ |
|----------|-----------------|-----------------|-----------------|-----------------|
| $O^{(1)}_{qq}$ | 2.9 | -10 | -- | -- |
| $O^{(8)}_{qq}$ | 9.0 | -10 | -- | -- |
| $O^{(1,3)}_{qq}$ | 10 | -10 | -- | -- |
| $O^{(8,3)}_{qq}$ | -24 | -10 | -- | -- |
| $O^{(1)}_{dd}$ | 5.3 | -10 | 400 | -16 |
| $O^{(1)}_{ud}$ | -3.7 | -10 | 400 | -16 |
| $O^{(8)}_{dd}$ | 5.3 | -10 | 400 | -16 |
| $O^{(8)}_{ud}$ | 14 | -10 | 400 | -16 |
| $O^{(1)}_{qu}$ | 14 | -9.3 | 400 | -15 |
| $O^{(8)}_{qu}$ | -24 | -10 | 400 | -16 |
| $O^{(1)}_{qd}$ | 9 | 22 | -- | -- |
| $O^{(1)}_{qd}$ | 5.4 | 2.8 | -400 | -15 |
| $O^{(8)}_{qd}$ | 5.3 | -10 | 400 | -16 |
| $O^{(1)}_{qsq}$ | 16 | -14 | 400 | -15 |
| $O^{(8)}_{qsq}$ | 24 | -2.8 | -400 | 15 |
| $O^{(1)}_{qq}$ | -74 | 4.2 | 400 | -15 |
| $O^{(8)}_{qq}$ | 14 | -9.3 | 400 | -15 |
| $O^{(1)}_{qs}$ | 29 | 22 | -- | -- |
| $O^{(8)}_{qs}$ | 29 | 22 | -- | -- |

References

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