Numerical simulation of plasma flows in curved coaxial ducts with longitudinal magnetic field

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Abstract. The results of numerical researches of plasma flows in coaxial plasma accelerators ducts of a various geometry with an external longitudinal magnetic field are presented. The non-steady-state two-dimensional MHD-model of relaxation of transonic super-Alfven flows with acceleration is considered. The paper focuses on the cumulative influence of a duct geometry and longitudinal magnetic field on flow characteristics.

1. Introduction

1.1. Plasma accelerators. History and applications

Physical processes in plasma are diverse and have a wide range of effective technological applications. The design and research of plasma accelerators, in which electromagnetic energy transforms into kinetic energy of plasma fluxes emerging from them, is the major direction in the area of plasma processes using. The idea of plasma acceleration in crossed electric and magnetic fields belongs to Alexey Morozov [1]. The successful development of these devices was launched in the 1950s in USSR under Lev Artsimovich [2] and in Gersh Budker’s laboratory. The development, research and design of different varieties of plasma accelerators over the following decades were mainly conducted by the same Alexey Morozov [3].

On the one hand, these include powerful high-current quasi-stationary plasma accelerators (QSPA) (figure 1), which demonstrated record velocities and energies of plasma fluxes emerging from them [4]. Its thrust is commensurate with rocket engine thrust, and, in terms of its technological applications, these devices can be used as electric jets for air and space flights [5]. They have a number of advantages over liquid jets working on chemical fuel, but require a powerful and compact source of electric power. On another the hand – low-power small stationary plasma thrusters (SPT) (figure 2) [6] with a long lifetime, which actively apply for orbital corrections, stabilization and maneuvers of artificial satellites.

Plasma accelerators scope of application is not limited to cosmic and transport directions. They also can be used as a magnetoplasma compressor, which is used to generate high-temperature plasma and...
for high-energy plasma fluxes injection into magnetic traps developed at the issue of thermonuclear fusion. Finally, these devices can be used for effective processing and surface properties modification of various materials.

1.2. **Longitudinal magnetic field**

A flow acceleration mechanism in plasma accelerator ducts is based on an interaction between an electric discharge current passing through plasma with own transverse magnetic field generated by an electric current flowing into the plant. The further technological development of the plasma acceleration scheme is the imposition of a longitudinal magnetic field induced by conductors external to the accelerator.

![Figure 1. Quasi-stationary plasma accelerator](image1)

![Figure 2. Stationary plasma thruster](image2)

The existence of a longitudinal magnetic field complicated the picture of plasma motion in the accelerator duct. Depending on the value of an external magnetic field, in terms of a classification, there are two regimes of plasma flows significantly varying on their characteristics [7]: super-Alfven and sub-Alfven (in relation to Alfven velocity corresponding to a longitudinal magnetic field) in a sufficiently weak and strong longitudinal magnetic field, respectively. In an accelerator duct at the same time there may be areas of super-Alfven and sub-Alfven flows.

1.3. **Mathematical modelling and its role in plasma processes research**

Mathematical modelling of plasma processes and numerical solution of corresponding mathematical problems play an important role in the successful developments and researches of plasma accelerators. This approach allows to determine basic qualitative regularities in properties of these processes, and quantitative results provide an opportunity to reduce costs of expensive experiments.

Plasma flows in plasma accelerator ducts in own transverse magnetic field have been well understood in the previous studies [8]. In [9] it has been demonstrated that, in terms of acceleration effectiveness, ducts with a curved central electrode are preferable to accelerators with a curved external electrode. Plasma flows with an external longitudinal magnetic field are less studied. The first steps in this field lied in the two-dimensional calculations of flows in the duct with the weakly curved central electrode [10] (see also [11]). The paper [12] is focused on the researches of ionization and radiation transport processes in plasma accelerator ducts. The two-dimensional MHD-model of flows in plasma accelerators as in injectors for magnetic traps is considered in [13].

1.4. **Objective of the work**

The objective of the present work is mathematical modelling and numerical simulation of accelerating plasma flows in QSPA-type plasma accelerator ducts of a various geometry formed by markedly curved electrodes with a longitudinal magnetic field.
2. Mathematical model

2.1. Object of simulation
The accelerator nozzle-type duct is formed by two coaxial electrodes connected to a capacitor bank (figure 3).

Plasma flowing in the duct completes the electrical circuit, and the electric current with a density \( j \) arise. It interacts with own transverse azimuthal magnetic field \( H_\perp \) and creates Ampere’s force \( F_j = (1/c) j \times H_\perp \), which causes plasma acceleration along the duct axis (figure 4).

![Figure 3. Schematic view of coaxial plasma accelerator](image)

![Figure 4. Scheme of coaxial plasma accelerator. Section by plane \( \varphi = \text{const} \)](image)

It is assumed that a longitudinal magnetic field \( H_l \) induced by external conductors can exist in the system. In the interaction with the discharge current \( j \) the longitudinal magnetic field causes plasma rotation in the transverse direction of the duct with the force \( F_\perp = (1/c) j \times H_l \).

So the object of the simulation in the present paper is plasma flow in plasma accelerator ducts in the external longitudinal magnetic field.

2.2. Formulation of the problem
In the present paper, plasma is considered as a continuous medium consisting of ions and electrons with common macro-parameters. Plasma flux injected into the accelerator is axisymmetric \( (\partial/\partial \varphi = 0) \). Two-dimensional problems therefore are formulated in the duct section by the plane \( \varphi = \text{const} \) in a cylindrical coordinate system \( (r, \varphi, z) \).

Plasma with a specified density \( \rho_0 \), temperature \( T_0 \) and pressure \( p_0 \) enters the duct through its input section \( (z = 0) \) without rotation \( \nu_{\varphi 0} = 0 \). The discharge current in the input section of the duct has strictly a radial direction, therefore azimuthal magnetic field at the duct input is set as \( H_{\varphi 0} = r_0 H_0 / r \), where \( H_0 = 2J/c r_0 \) – a characteristic value of the magnetic field intensity in the input section, \( J \) – a value of the full electric current flowing through the central electrode, \( r_0 \) – a characteristic radius of the duct. The longitudinal magnetic field can be determined as a constant value \( H_{l0} \) in the duct input. At the exit of the duct \( (z = 1) \) the flow is supersonic (in relation to fast magnetosonic velocity), so there are no boundary conditions.

The initial conditions of the problem can be quite arbitrary as long as they provide acceleration of plasma, because steady-state flows are of particular interest.

The electrodes \( r_1(z) \) and \( r_2(z) \) forming the accelerator duct are assumed to be solid \( \nu_n = 0 \) and equipotential \( H_n = 0 \), where \( n \) – a normal line to the electrodes surface.
2.3. Method of calculation

For the numerical solution of the problem, it is convenient to use a dimensionless form of variables. The units of measurement are the dimensional quantities participating in the formulation of the problem. In order to get rid of curved boundaries of the calculation area, new dimensional coordinates \( (z, y) \), which transform the duct area to a square (figure 5), have been introduced:

\[
z = z, \quad r = (1 - y)r_1(z) + yr_2(z)
\]

(1)

\[\begin{array}{c}
\frac{\partial}{\partial t} \rho \mathbf{r} + \frac{\partial}{\partial z} \rho \mathbf{v}_z \mathbf{r} + \frac{\partial}{\partial y} \rho \mathbf{v}_y \mathbf{r} = 0 \\
\frac{\partial}{\partial t} \rho \mathbf{v}_z \mathbf{r} + \frac{\partial}{\partial z} \left( \rho \mathbf{v}_z^2 + P - H_z^2 \right) \mathbf{r} + \frac{\partial}{\partial y} \left( \rho \mathbf{v}_y \mathbf{v}_z - H_z \mathbf{v}_y \right) \mathbf{r} = 0 \\
\frac{\partial}{\partial t} \rho \mathbf{v}_y \mathbf{r} + \frac{\partial}{\partial z} \left( \rho \mathbf{v}_y \mathbf{v}_z - H_z \mathbf{v}_y \right) \mathbf{r} + \frac{\partial}{\partial y} \left( \rho \mathbf{v}_y^2 - P - H_y^2 \right) \mathbf{r} = 0 \\
\frac{\partial}{\partial t} \rho E \mathbf{r} + \frac{\partial}{\partial z} \left( E \mathbf{v}_z - H_z \mathbf{v}_y \right) \mathbf{r} + \frac{\partial}{\partial y} \left( E \mathbf{v}_y - H_y \mathbf{v}_y \right) \mathbf{r} = 0 \\
\frac{\partial}{\partial z} H_z \mathbf{r} + \frac{\partial}{\partial y} E \mathbf{r} = 0
\end{array}
\]

(2)

\[p = (\gamma - 1)\rho \mathbf{e} = \frac{\beta}{2} \rho T, \quad P = p + \frac{H_z^2 + H_y^2 + H_e^2}{2},\]

где \( \mathbf{v}_z = \mathbf{v}_y, \quad H_z = H_z - H_z \mathbf{r}_z, \quad E = E_e = H_z \mathbf{v}_y - H_y \mathbf{v}_y, \quad R(z) = r_1(z) + yr_2(z).\]

The dimensionless parameter \( \beta = \frac{8\pi p_0}{H_0^2} \) traditionally participates in modelling of plasma processes. It determines, how many times a characteristic magnetic pressure of a problem differs from a gas-dynamic pressure. Let us introduce also the concept of a velocity along a trajectory \( \mathbf{v}_l = \sqrt{v_z^2 + v_y^2}.\)

The problems are solved numerically until steady-state flow regimes would be achieved. As a numerical method of a solution of the non-steady-state two-dimensional MHD-problems, FCT algorithm [14] with a fully multidimensional flux limiter [15] was selected.
3. Numerical results

In the present paper the results of numerical researches of plasma acceleration in coaxial ducts with a longitudinal magnetic field in the two-dimensional formulation of the problem are presented. The main focus is on the cumulative influence of a duct geometry and longitudinal magnetic field on parameters and properties of transonic super-Alfven flows with acceleration in a sufficiently weak longitudinal magnetic field. The studies are a continuation and development of the previous works [10] with calculations in a longitudinal magnetic field in accelerator ducts of a relatively smooth geometry realized in the experiments on QSPA in 1990s [4].

Numerical results, as in [9], are set out in two basic series differing by electrodes geometry. In them $\beta = 0.05$, which corresponds to a sufficiently strong azimuthal magnetic field and electric current, and $H_{z0} = 0.3$ – the value close to the upper bound of the longitudinal field range acceptable to transonic super-Alfven flows.

A steady-state flow in a duct is characterized by distributions of plasma parameters presented on figures 6 and 7 by solid lines of levels. The influence of a longitudinal magnetic field can be seen clearly in the comparison with the results of similar calculations without it, which are presented by dashed lines.

3.1. Plasma accelerators with curved central electrode

We consider the plasma accelerator with the following configuration:

$$0 < z < 1, \quad r_1(z) = 0.55 - (z - 0.5)^2, \quad r_2(z) = 0.8$$  \hspace{1cm} (3)

![Figure 6. Plasma parameters distributions (a, b, c) and tubes of flow trajectories (d) in the duct with the curved central electrode](image_url)
The longitudinal magnetic field pushes plasma up to the external electrode, resulting in increasing of plasma flow density (figure 6a), and hence, of its temperature in all area of the duct. The longitudinal field deflects electric current lines towards the duct input section and turns them counterclockwise from the radial direction (figure 6b).

Plasma flux acceleration is more intensively near the curved central electrode (figure 6c) by increasing an input value of an azimuthal magnetic field [9]. The longitudinal magnetic field slightly reduces acceleration properties of the duct: the input plasma velocity increases, but the output velocity decreases, because a part of accumulated electromagnetic energy transforms into energy of plasma rotation in the transverse direction of the duct.

Flow tubes are limited by trajectories, which are also level lines of a longitudinal magnetic field. The medium tube even without a longitudinal field is asymmetrical relative to the duct minimal section (figure 6d). This result is followed from the curviness of the central electrode and plasma motion from left to right. The longitudinal field strengthens the trajectories deviation to the external electrode.

3.2. Plasma accelerators with curved external electrode
We consider the plasma accelerator with the following configuration:

$$0 < z < 1, \quad r_1(z) = 0.3, \quad r_2(z) = 0.55 + (z - 0.5)^2$$

Figure 7. Plasma parameters distributions (a, b, c) and tubes of flow trajectories (d) in the duct with the curved external electrode

In this geometry, plasma is pushed up to the external duct electrode to such an extent that a zone of flux compression appears and then grows in the input and central area of the duct: plasma density rises to the maximum value which is higher than the input value (figure 7a). This result has been already
The present work has been considered to determine the effects of the input value of an azimuthal magnetic field (figure 7c). In so doing, the longitudinal magnetic field, as in the previous case 3.1, in general, reduces acceleration properties of the duct.

Trajectories and thin flow tubes formed by them have a tendency to «uncurling» in the output area of the duct (figure 7d). They deviate down from those symmetrical relative to the minimum section. The longitudinal field seeks to restore the mentioned symmetry.

4. Conclusions

In the present work the non-steady-state two-dimensional MHD-model of relaxation of transonic super-Alfen flows with acceleration in plasma accelerator ducts of a various geometry with an external longitudinal magnetic field has been considered and implemented in calculations.

It has been determined that a longitudinal magnetic field in both of duct configurations leads to the following effects:

- It pushes plasma up to an external electrode. In terms of technical applications, this result can be used to reduce an amount of plasma flux artificially injected through an external electrode to solve a «anode crisis» problem in QSPA;
- It deflects an electric current towards a duct input and turns its lines counterclockwise from the radial axis neutralizing at a specific polarity an influence of Hall effect negative for acceleration;
- It slightly reduces acceleration properties of a duct and causes a flux rotation in its transverse direction.

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