HYDROMAGNETICS OF ADVECTIVE ACCRETION FLOWS AROUND BLACK HOLES: REMOVAL OF ANGULAR MOMENTUM BY LARGE-SCALE MAGNETIC STRESSES

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ABSTRACT

We show that the removal of angular momentum is possible in the presence of large-scale magnetic stresses in geometrically thick, advective, sub-Keplerian accretion flows around black holes in steady state, in the complete absence of $\alpha$-viscosity. The efficiency of such an angular momentum transfer could be equivalent to that of $\alpha$-viscosity with $\alpha = 0.01$–0.08. Nevertheless, the required field is well below its equipartition value, leading to a magnetically stable disk flow. This is essentially important in order to describe the hard spectral state of the sources when the flow is non/sub-Keplerian. We show in our simpler 1.5 dimensional, vertically averaged disk model that the larger the vertical-gradient of the azimuthal component of the magnetic field is, the stronger the rate of angular momentum transfer becomes, which in turn may lead to a faster rate of outflowing matter. Finding efficient angular momentum transfer in black hole disks via magnetic stresses alone, is very interesting when the generic origin of $\alpha$-viscosity is still being explored.

Key words: accretion, accretion disks – galaxies: active – ISM: jets and outflows – magnetohydrodynamics (MHD) – X-rays: binaries

1. INTRODUCTION

Blandford & Payne (1982) revealed the possibility of energy and angular momentum removal from a Keplerian accretion disk using the magnetic field lines that extend from the disk surface to large distances. In the framework of infinite conductivity and the self-similar model, they showed that the magnetic stresses can extract the angular momentum from a geometrically thin accretion disk, which helps matter to accrete independently of the presence of viscosity. Furthermore, they argued that such a mechanism is responsible for the observed jets/outflows from accreting sources when magnetic stresses convert a centrifugally driven wind into a collimated jet. It has been argued that the disk matter is outflowing through the outgoing field lines. The time evolution of axisymmetric, weak magnetic fields threading geometrically thin, Keplerian accretion disks with finite conductivity in a specific model framework was investigated by Lubow et al. (1994), however, without considering possible angular momentum transfer by the magnetic field. On the other hand, in the presence of infinite conductivity, the magnetic field, in the same model framework that does not consider the contributions from the magnetic stresses, would be amplified by the accretion of gas until it stopped the accretion (also see Spruit 2013). However, Bisnovatyi-Kogan & Lovelace (2007) and Lovelace et al. (2009) showed that the radially inward flow is possible for plasma-$\beta > 1$ and Prandtl number $\geq 1$, in the stationary channel-type flows, which have small optical depths in the absence of turbulent viscosity and could also exhibit electromagnetic outflows for smaller Prandtl numbers. They showed that the large-scale field keeps drifting inward until a stationary state arises, when the magnetic, centrifugal, and gravitational forces become comparable. This furthermore reveals that the flow velocity profile differs significantly from the Keplerian profile.

The idea of exploring magnetic stress in order to explain astrophysical systems was, in fact, implemented much earlier. For example, the solar wind was understood to have decreased the Sun’s angular momentum through the effect of magnetic stresses (see, e.g., Weber & Davis 1967), the proto-stellar gas clouds might have been contracted by magnetic effects (Mouschovias & Paleologou 1980). In the context of an accretion disk, Ozernoy & Usov (1973) and Blandford (1976) showed that it is possible for the energy to be continuously extracted by electromagnetic torques and twisted field lines. Furthermore, Cao & Spruit (2002) showed, through a linear stability analysis of the accretion disks, that it is possible for the angular momentum to be removed by the magnetic torque exerted by a centrifugally driven wind. The same authors (Cao & Spruit 2013) also discussed that moderately weak fields can cause sufficient angular momentum loss via a magnetic wind to balance outward diffusion in geometrically thin accretion disks. However, plasma-$\beta$ has to be much smaller than unity to explain the tendency for strong flux bundles at the center of disks to stay confined, as seen in numerical simulations. Nevertheless, Ogilvie & Livio (1998) showed the shortcomings of launching an outflow by solving the local vertical structure of a thin accretion disk threaded by a poloidal magnetic field and suggested the existence of an additional source of energy leading to successful launching of the outflow.

However, observationally, outflows/jets are mostly found to be emanating from the disk when it is in a hard state (e.g., Belloni et al. 2000), which is non/sub-Keplerian. Note that jets appear to be highly heterogeneous with velocities ranging from a few tens of million cm s$^{-1}$ to the escape velocity from the disk. Superluminal sources, however, appear to exhibit jet velocity around the speed of light (Miley 1980). The jets are found in disks around stellar mass black hole sources (e.g., GRS 1915+105) as well as supermassive black hole sources (e.g., M87).

Therefore, most of the modern models describing outflows/jets from the accretion disks are based on a sub-Keplerian, advective model when the flow has a significant radial velocity, unlike the Keplerian disk. For example, a class of self-similar, advection dominated solutions was proposed by Narayan & Yi
In a different model framework, Chakrabarti & Titarchuk (1995), Chakrabarti (1999), and Chakrabarti & Manickam (2000) described advective, sub-Keplerian accretion flows in order to explain outflows, quasi-periodic oscillations (QPOs) and spectral states in black hole sources. Furthermore, Mukhopadhyay (2003), Mukhopadhyay & Ghosh (2003), and Rajesh & Mukhopadhyay (2010a) described general advective accretion flows (GAAFs) around black holes and neutron stars and showed the effects of the rotation of the black hole on the solutions. The last authors also explicitly included various cooling effects and showed how the solutions are affected by the cooling properties.

However, all of the above models were formulated in the framework of Shakura–Sunyaev $\alpha$-viscosity (Shakura & Sunyaev 1973), when the flows are assumed to be embedded with the plasma-$\beta \gg 1$. Hence, the matter transport is assumed to be supported via turbulent viscosity, not by a large-scale magnetic field, unlike that chosen by Blandford & Payne (1982). Nevertheless, Bhattacharya et al. (2010) showed that the transport is also possible in the presence of outflow in a 2.5-dimensional accreting system; it does not matter whether the outflow is magnetic or hydrodynamic. Note that outflows, and even jets, can also be formed in the absence of the magnetic field. This is likely to occur when the flow is radiation trapped and the accretion rate is super-Eddington or super-critical (Lovelace et al. 1994; Fabrika 2004; Begelman et al. 2006; Ghosh & Mukhopadhyay 2009).

The 2.5-dimensional accretion model, proposed by Bhattacharya et al. (2010), will be complete if the effects of the (large-scale) magnetic field is included therein. In that case, one can presumably explain the outflow of matter plunging through the outgoing magnetic field lines more spontaneously, as Blandford & Payne (1982) did in the Keplerian framework.

To the best of our knowledge, so far, there has been no attempt to obtain a self-consistent set of advective disk-outflow coupled accretion solutions in the presence of the large-scale magnetic field, which has a lot of implications on the explanation of the low/hard state of sources. This, however, has been discussed to some extent for circumstellar disks around young stars (see, e.g., Königl & Salmeron 2011), without discussing the detailed solutions of all the dynamical variables. The present work steps forward in order to obtain such a set of solutions for black holes.

In various numerical set ups, magnetohydrodynamic (MHD) simulations of accretion onto magnetized compact objects have already been explored. As examples, some of them considered axisymmetric systems in the presence of magnetospheres (Romanova et al. 2011), and others investigated the advection of matter and the magnetic field in the turbulent/diffusive disks (Dyda et al. 2013) when the field strength decreases due to reconnection and annihilation at a later time. Other groups, explored general relativistic magnetohydrodynamic (GRMHD) simulations of magnetically arrested accretion flows and outflows around black holes, for toroidally and poloidally dominated magnetic fields (Tchekhovskoy et al. 2011). They furthermore demonstrated the possible extraction of net energy from a spinning black hole via the Penrose–Blandford–Znajek mechanism (McKinney et al. 2012). Moreover, there were radiation MHD/GRMHD simulations of accretion and outflows around black holes, exploring three distinct flow phases including the radiatively inefficient phase, which is similar to the flows considered in the present work (Ohsuga & Mineshige 2011; McKinney et al. 2014). Some of the MHD simulations investigated the reasons behind the variability in low angular momentum, underluminous accretion flows in the vicinity of a supermassive black hole (Moscibrodzka et al. 2007). However, all of these works, to the best of our knowledge, considered cases in which any viscosity arises from magnetorotational instability (MRI), leading effectively to the Shakura–Sunyaev $\alpha$ viscosity (Balbus & Hawley 1991).

Here we plan to investigate, semi-analytically, the effects of the large-scale magnetic field, with plasma-$\beta > 1$, on the advective accretion flows, in order to transport matter; however, we are restricted to the simpler 1.5 dimensions. Therefore, we consider the flow variables, averaged over the vertical coordinate, to depend on the radial coordinate only. While the vertical equilibrium assumption corresponds to no vertical component of velocity, we choose the vertical component of the magnetic field to be non-zero. Although, in reality, a non-zero vertical magnetic field induces a vertical motion, in the platform of the present assumption, any vertical motion will be featured as an outward motion. Nevertheless, whether it is a vertical or outward transport, our aim is to furnish the removal of angular momentum from the flow via magnetic stresses, leading to the infall of matter toward black holes.

The plan of the paper is the following. In the next section, we describe the set of MHD/hyrdromagnetic equations at the limit of a very large Reynolds number, as is the case in accretion disks, describing the flow model. This is basically the set of Navier–Stokes equations, but in the presence of magnetic shear stresses (and Lorentz force), the magnetic induction equation, the condition for the absence of magnetic monopole, and finally the conservation of mass. Subsequently, we discuss the numerical solutions of the set of equations in Section 3 and their implications. Finally, we summarize the results along with a discussion in Section 4.

2. MODEL HYDROMAGNETIC EQUATIONS

We describe optically thin, magnetized, viscous, axisymmetric, advective, vertically averaged, steady-state accretion flow, in the pseudo-Newtonian framework with the Mukhopadhyay (2002) potential. The choice of the pseudo-Newtonian framework, for the present purpose, does not hinder any physics, compared to that which would appear in the full general relativistic framework. Hence, the equation of continuity, vertically averaged hyrdromagnetic equations for energy–momentum balance in different directions are given by

$$ \dot{M} = 4\pi x \rho \dot{\varphi}, $$

$$ \vartheta \frac{d\vartheta}{dx} + \frac{1}{\rho} \frac{dP}{dx} - \frac{x^2}{x^3} + F = \frac{1}{4\pi \rho} \left\{ B_z \frac{dB_t}{dx} + s_i \frac{B_z B_i}{h} - \frac{B_z^2}{x} \right\}, $$

where $\vartheta$ is the vertical coordinate, to depend on the radial coordinate only.
\[ \frac{d\lambda}{dx} = \frac{1}{x\rho} \frac{d}{dx}(\chi^2 W_{x0}) \]
+ \frac{x}{4\pi \rho} \left( B_1 \frac{dB_0}{dx} + s_2 B_2 + \frac{B_0 B_2}{x} \right),
\]

where

\[ W_{x0} = \alpha \left( \rho + \rho \vartheta^2 \right), \]
\[ \varrho = \frac{F_0}{x} - \frac{1}{4\pi \rho} \left( B_1 \frac{dB_2}{dx} + s_3 B_3 \right), \]
\[ \frac{dT}{dx} = \frac{\vartheta}{1 - \frac{1}{\Gamma_3}} \left( \frac{dP}{dx} - \frac{\Gamma_1 P d\rho}{\rho dx} \right) \]
\[ = Q^+ - Q^- = Q^+_{\text{vis}} + Q^+_{\text{mag}} - Q^-_{\text{vis}} - Q^-_{\text{mag}} \]

when we assume that the variables do not vary significantly in the vertical direction such that \( \partial/\partial z \rightarrow s_i/z \sim s_i/h \), where \( i = 1, 2, 3 \), which is indeed true in the disk flows. Note that \( s_1, s_2, \) and \( s_3 \) are the degrees of scaling for the radial, azimuthal, and vertical components of the magnetic field, respectively. As a consequence, the vertical component of velocity is zero. Here \( M \) is the conserved mass accretion rate and the corresponding equation is the integrated version of the continuity equation, \( \rho \) is the mass density of the flow, \( \vartheta \) is the radial velocity, \( P \) is the total pressure including the magnetic contribution, \( F \) is the force corresponding to the pseudo-Newtonian potential for rotating black holes, \( \lambda \) is the angular momentum per unit mass, \( W_{x0} \) is the viscous shearing stress written following the Shakura–Sunyaev (Shakura & Sunyaev 1973) prescription with appropriate modification (Mukhopadhyay & Ghosh 2003), \( h \sim z \), the half-thickness of the disk, \( \varrho \) is the entropy per unit volume, \( T \) is the (ion) temperature of the flow, \( Q^+ \) and \( Q^- \) are the net rates of energy released and radiated out per unit volume in/from the flow respectively (when \( Q^+_{\text{vis}}, Q^+_{\text{mag}}, Q^-_{\text{vis}}, Q^-_{\text{mag}} \) are the respective contributions from viscous and magnetic parts). We furthermore assume, for the present purpose, the heat radiated out proportional to the released rate with the proportionality constants \((1 - f^m)\) and \((1 - f^n)\), respectively, for viscous and magnetic parts of the radiations. \( \Gamma_1, \Gamma_3 \) indicate the polytropic indices depending on the gas and radiation content in the flow (see, e.g., Rajesh & Mukhopadhyay 2010a for exact expressions) and \( B_0, B_2 \) and \( B_3 \) are the components of magnetic field. Note that, the independent variables \( x \) and \( z \) are the radial and vertical coordinates, respectively, of the flow, expressed in units of \( GM/c^2 \), where \( G \) is the Newton’s gravitation constant, \( M \) is the mass of the black hole, and \( c \) is the speed of light. Accordingly, all of the above variables are made dimensionless, e.g., \( \vartheta \) is expressed in units of \( c \). For any other details, e.g., model for \( Q^+_{\text{vis}}, Q^-_{\text{vis}} \), see the existing literature (Rajesh & Mukhopadhyay 2010a, 2010b), then

\[ Q^+_{\text{vis}} - Q^-_{\text{vis}} = \frac{\alpha_{\text{vis}} \left( P + \vartheta^2 \rho \right) \lambda}{x^2}. \]

We furthermore do not consider the heat generated and absorbed due to the nuclear reactions (Mukhopadhyay & Chakrabarti 2000, 2001). This is to emphasize that all of the variables appearing in the equations are assumed to be their respective vertically averaged quantities.

Hydromagnetic flow equations must be supplemented by (for the present purpose, steady-state) equations of induction and no magnetic monopole, given by

\[ \nabla \times \mathbf{v} \times \mathbf{B} + \nu_m \nabla^2 \mathbf{B} = 0, \]
\[ \frac{d}{dx} (x B_3) + s_3 \frac{B_3}{h} = 0, \]

when \( \mathbf{v} \) and \( \mathbf{B} \) are, respectively, the velocity and magnetic field vectors and \( \nu_m \) is the magnetic diffusivity. On taking the ratio of the orders of the first to the second (diffusive) terms in Equation (7), we obtain \( L |\mathbf{v}|/\nu_m = R_m \), with \( L \) being the order of the length scale of the system. Hence, when the Reynolds number, \( R_m \), is very large, which is the case for accretion disks, the second term (which is associated with the magnetic diffusivity) in Equation (7) can be neglected. However, this term can be rather important inside some localized regions in certain astrophysical systems due to subtle reasons. Nevertheless, for the present purpose, for simplicity, we will neglect this term throughout. Furthermore, because \( \vartheta \) and \( \lambda \) are assumed to be independent of the vertical coordinate, it is easy to check from the radial component of Equation (7) that \( \partial B_i / \partial z \rightarrow 0 \) (and hence \( B_i/h \rightarrow 0 \)). Therefore, the azimuthal and vertical components of Equation (7), at large \( R_m \), respectively, lead to

\[ \frac{d}{dx} \left( \vartheta B_0 - \frac{B_3 \lambda}{x} \right) = 0, \]
\[ \frac{d}{dx} (x B_3) = 0, \]

when the radial component of Equation (7) turns out to be trivial. Subsequently, the Equation (8) reduces to

\[ \frac{d}{dx} (xB_3) = 0. \]

Because of the choice of very large \( R_m \) (ideal MHD), there is a perfect flux freezing in the flow. Therefore, a steady advection of the vertical magnetic flux toward the center may lead to the decrease of \( \beta \), making it close to unity and further smaller, in a pure axisymmetric flow, even if the initial \( \beta \) was high. Hence, at some point, the back reaction of the field will inhibit accretion, depending on the geometry of the field lines. Although the physics of this process is not captured by the equations above and we also do not intend to discuss such physics, we will show in Section 3 the effects of higher magnetic fields, in particular the inner edge (around the critical radius), in the entire flow structure. This is essentially to capture a situation after significant advection done with a certain field geometry.

Henceforth, we will also neglect the second term in the parenthesis of Equation (4). The sets of Equations (1)–(5) and (9)–(11) are essentially the modified versions of the set of advective accretion disk equations in the presence of a large-scale magnetic effect, which are otherwise discussed in the literature in absence of it. Generally, in order to understand the hard state of accretion flows around black holes, the flow is
assumed to be purely hydrodynamic with the consideration of a turbulent viscosity arisen due to a weak magnetic field, i.e., MRI (Balbus & Hawley 1991). Although, MRI is a largely accepted idea, so far, in order to explain the origin of turbulence in accretion disks, there are some subtle issues with it (e.g., Mahajan & Krishan 2008; Mukhopadhyay et al. 2011) including its applicability in colder disks. Therefore, transport of matter in disks is much more transparent through magnetic stresses, if the flows are embedded with a large-scale field. Such magnetic stresses are considered here on the right hand side of the radial, azimuthal, and vertical momentum balance equations. In addition, the magnetic heating due to the turbulent viscosity arisen due to a weak magnetic field, however, the other related terms in the momentum balance equations are neglected, again in comparison with the remaining terms in the respective equations, for the purpose of the present work.

Therefore, even in the absence of turbulent viscosity ($\alpha = 0$) and hence viscous stresses, magnetic stresses alone can help in transporting matter in the accretion flows. Such a consideration of large-scale magnetic field, and hence transport via magnetic stress, has not yet been considered for advective accretion flows around black holes, though it has been considered in circumstellar disks around young stars (see Königl & Salmeron 2011 for a recent review).

Questions may arise if the magnetic field with plasma-$\beta > 1$ is adequate enough to describe the infall of matter in order to explain the observation. We will show in Section 2.1 that the large-scale magnetic field, even with a significantly large plasma-$\beta$, can describe advective accretion flows as efficiently as an $\alpha$ does.

### 2.1. Solution Procedure

We have seven equations (excluding the vertical momentum balance equation, which asserts the vertical magnetostatic balance) and seven variables: $\vartheta$, $\lambda$, $P$, $\rho$, $B_x$, $B_\phi$, $B_z$, which we plan to solve along with the vertical magnetostatic balance condition. First, we plan to reduce $d\vartheta/dx$ in terms of other variables and the independent variable $x$ alone (without any other derivatives), given by

$$\frac{d\vartheta}{dx} = \frac{1}{F} \frac{dF}{dx} - \frac{1}{2} \frac{F}{x^2} \left(1 + \frac{1}{\Gamma_1} \right) \left( \frac{\lambda^2}{x^2} - F \right) + \frac{\Gamma_1 - 1}{2 \vartheta \Gamma_1 P} \left( \frac{\alpha_{\nu m} (P + \vartheta^2 \rho) \lambda}{x^2} + \frac{3 f_m |B|^2 \vartheta}{16 \pi x} \right) \left( A \right) \left( B \right) \left( C \right) \left( D \right) \left( E \right) \left( F \right) \left( G \right).$$

As the advective accretion around black holes is necessarily transonic, the flow must pass through a critical radius where

$$\vartheta = \vartheta_c = \frac{2 \Gamma_1 P}{\sqrt{\rho (1 + \Gamma_1)}},$$

when the variables with subscript “$c$” indicate the respective values at that critical radius and the numerator of Equation (13) has to be zero for a continuous solution. At the critical radius, we also prescribe

$$B_{xc} = B_{zc} = B_{xc} = \frac{4 \pi \rho}{\lambda \sqrt{\gamma}}.$$

so that the Alfvén velocity is a fraction of sound speed therein. Although this is a simpler prescription, other choices do not change the picture, being addressed in this work, qualitatively. Note that a steady MHD flow would normally have three critical points—the fast magnetosonic point, the Alfvén point, and the slow magnetosonic point—of which the Alfvén point is not a true critical point (Gammie 1999). The remaining two physically important distinct critical points, corresponding to fast and slow magnetosonic waves, collapse into a single point because of the assumptions made in Equation (15). We typically choose $f_A \sim 10^{-3}$ in our various computations (see the figures). This is to capture a situation when magnetic pressure at the inner edge of the flow is not high enough to hinder the radial infall of the matter, i.e., a situation with a weak back reaction of magnetic fields. Furthermore, from Equation (11), we can write

$$x_c B_{xc} = \text{constant} = C_0 = x B_x$$

which fixes the profile of $B_x$ throughout the flow.

These four conditions, along with the conditions that $\lambda = \lambda_K$ (when $\lambda_K$ being the Keplerian angular momentum per unit mass) and $\vartheta < \ll 1$ at the beginning of the sub-Keplerian flow far away from the black hole, i.e., the outer boundary, and $\vartheta \sim 1$ at $r \sim r_s$ serve as important boundary conditions. Based on all of the conditions, by solving the set of seven coupled differential equations, we can obtain the profiles for all variables including those of $B_x$ and $B_\phi$. Of course, then, one has to supply $M, M, \alpha, \gamma$, and hence $f_{vis}$ and $f_m$ for a flow. See Rajesh & Mukhopadhyay (2010a) for the solution procedure in detail.

### 3. SOLUTIONS

Our main aim is to obtain the solutions of magnetized accretion flows. In other words, the aim is to understand how the large-scale magnetic field (alone) can influence the mass transfer in the accretion process, in particular, in the advective regime. This is important in light of our ignorance of the origin of viscosity in accretion flows, which may have arisen from turbulence, because the molecular viscosity therein is well-known to be insignificant. Hence, our venture here is to investigate whether the large-scale magnetic field, however, with large plasma-$\beta$, can govern the same/similar transport of angular momentum as the well-known $\alpha$-prescription does. Hence, we plan to understand the relative strengths between the
magnetic stress tensors and the viscous stress tensors in order to control advective accretion flows.

Our plan is to explore specifically two situations. (1) Flows with a relatively higher $M$ and, hence, lower $\gamma$, modeled around stellar mass black holes: such flows may or may not form Keplerian accretion disks. (2) Flows with a lower $M$ and, hence, higher $\gamma$, modeled around supermassive black holes: such flows are necessarily hot gas dominated advective (or advection dominated) accretion flows.

3.1. Accretion around Stellar Mass Black Holes

We choose $M = 10M_\odot$ and $M = 0.1M_{\text{Edd}}$, when $M_\odot$ and $M_{\text{Edd}}$ are the solar mass and the Eddington accretion rate respectively. However, this choice of $M$ does not necessarily imply the flow to be purely Keplerian, rather advective, which is indeed, in general, under consideration. Such flows have temperatures $T \gtrsim 10^9\, \text{K}$ and $\rho \gtrsim 10^{-7}\, \text{gm/cc}$ (Sinha et al. 2009; Rajesh & Mukhopadhyay 2010a), which were extensively explored in the context of the formation of shock in hot accretion disks and subsequent outflows and observed spectral states (Chakrabarti & Titarchuk 1995; Chakrabarti 1996). A relation of $\gamma$ (and $M$) with the ratio of the gas and radiation content of the flow and the corresponding variations have been discussed by Mukhopadhyay & Dutta (2012). Therefore, following previous authors, and for the convenience of comparison of the previous results without magnetic fields, we choose $\gamma = 1.335$ along with the intermediate $f_{\text{vis}}$ and $f_{\text{mr}}$. Note that while the results depend on the sign of $s_2$, they do not depend on $s_1$ and $s_3$.

Figure 1 compares accretion flows (1) in the presence of the large-scale magnetic field, but in the absence of $\alpha$-viscosity (magnetic flow) and (2) in the presence of viscosity, but in the absence of large-scale magnetic field (viscous flow). Here we consider $B_\phi$ to have the same direction as that of $\lambda$. It shows that the large-scale magnetic field $\sim 10^{-5} - 10^{-3}$ G, with its distribution in the inner edge of the flow defined by Equation (15), is adequately able to transport angular momentum, as viscous flows do with $\alpha = 0.017$ and 0.012, respectively, for nonrotating and rotating black holes. Figures 1 (a) and (b) show that the disk sizes are the same for the respective viscous and magnetic sub-Keplerian flows with the above mentioned respective $\alpha$-s. However, the transport of angular momentum takes place faster in magnetic flows, where the flows become quasi-spherical at larger radii than at the radii they become in the respective viscous flows. Away from the black hole, $B_\phi$ increases, which implies that the matter is prone to outflow from the outer region through the magnetic field lines extended in the outward direction. In a self-consistent model, including the flow variation in the vertical direction, the above features should have appeared due to the increasing magnetic field along with the vertical coordinate. Such a model is planned to be developed in the future. In the present 1.5 dimensional magnetic flow model, when the magnetic stresses play the main role in removing angular momentum and hence overcoming the centrifugal barrier, as matter advances toward the black hole, the magnetic field decreases. Note that, as shown in Figures 1(b) and (d), $\lambda$ and $|B_\phi|$ both decrease toward the black hole: to overcome large $\lambda$, the flow needs a large $|B_\phi|$ and vice versa—they are the self-consistent solutions to each other. Note furthermore that the negative sign in $B_\phi$ in Figure 1(d) is due to our choice of decreasing $B_\phi$ with increasing $z$, i.e., negative $s_2$, in the outer edge of the flow (which results in the same trend of the flow almost throughout, except in the very inner region). Figure 1(c) shows that our chosen regime of magnetic flows, allowing a steady infall of matter, corresponds to a relatively high plasma-$\beta$ (actually the inverse of $\beta$ is shown). This furthermore renders lower magnetic pressures in respective flows compared to their maximum allowed values based on the virial theorem/equipartition principle. As discussed in previous works (e.g., Mukhopadhyay 2003), a strong centrifugal effect is depicted in the Mach number profiles in Figure 1(a) (featured as slowing down the matter at around $x = 20 - 25$) in the high angular momentum flows around nonrotating black holes, compared to the low angular momentum flows around rotating black holes.

Now we hypothesize that $B_\phi$ increases with increasing $z$, i.e., positive $s_2$, at the outer edge of the flow (which results in the same trend of the flow almost throughout, except for the very inner region). In this case, the right hand side of Equation (3), for a magnetic flow, completely flips the sign compared to the negative $s_2$ case. Figure 2 shows that as the matter falls in, the toroidal component of the magnetic field slows down the azimuthal motion of matter faster, making $\lambda = 0$ and, subsequently, inverting the orientation of $\lambda$. This is effectively due to the change in signs of magnetic stress components: $B_x B_y$ and $B_x B_\phi$. Figure 2(b) shows the variation of the magnitude of $\lambda$, as the matter falls in. $\lambda$ is positive far away from the black hole, but it is negative close to the black hole. The location around $\lambda = 0$ reveals a trough-like region in the flow. Hence, on either side of $\lambda = 0$, there is a stronger centrifugal barrier that stores matter around $\lambda = 0$ (due to the competition between radial and azimuthal flows). This region is prone to kick the matter out, producing outflows. Hence, if $B_\phi$ increases with $z$ in the flow to start with, then as matter advances toward the black hole, a “potential well” forms to produce outflows. Note however, the very close to a rotating black hole, matter will be under the influence of the black hole completely and hence $\lambda$ cannot have an opposite sign with respect to that of the black hole. Therefore, this solution is not valid very close to the rotating black hole. Indeed, the pseudo-Newtonian description is not applicable very close to the black hole. Nevertheless, the above solution implies the possibility of having such an origin of outflows in a magnetized accretion flow in the presence of a finite conductivity (when the field is not frozen with the matter, when the term associated with magnetic diffusivity in the induction equation is retained). Note that the plasma-$\beta > 1$ is maintained throughout the flows.

This furthermore motivates us to check for such a possibility in viscous flows with viscosities $\alpha = 0.08$ and 0.056, respectively, for nonrotating and rotating black holes. As shown in Figures 2(a) and (b), the angular momentum profiles in the respective magnetic and viscous flows appear to be similar, which furthermore makes the respective radial velocity profiles similar, unlike the previous cases, as shown in Figure 1, with the positive $\lambda$ throughout. In the previous cases, the magnetic stresses are able to remove the angular momentum faster than the viscous stresses, in particular, at a large distance from a Schwarzschild black hole, which is clearly understood from Figures 1(a) and (b). Although the same is true for a Kerr black hole, because the disk angular momentum itself is lower there, it does not effectively create any impact on the Mach number profiles. However, due to the choice of larger $\alpha$, in the counterrotating cases, viscous stresses appear to be almost
equivalent to the magnetic stresses and hence the radial velocity profiles in either of the respective flows appear similar.

Let us now understand, in more detail, how the various components of magnetic stress are responsible for inflow and/or angular momentum transfer therein. Figures 3(a) and (b) show the variations of various components of the magnetic field as functions of the radial coordinate, around Schwarzschild and Kerr black holes, which are responsible for the various components of the magnetic stress tensor. The profiles of field components and their magnitudes are partly dependent on their prescription given by Equation (15). Figure 4(a) shows that the stress tensor component $B_x B_z$ around a Schwarzschild black hole increases almost throughout as matter advances toward the black hole. This implies that the flow is prone to outflow through the field lines, which indirectly helps to remove the angular momentum, which furthermore renders its infall toward the black hole. However, very close to the black hole, $B_x B_z$ decreases, because outflow is indeed not possible in the near vicinity of the back hole, in particular, once the matter passes through the (inner) sonic point. By this radius, the flow angular momentum becomes very small which practically does not affect the infall. The magnitude of $B_x B_z$ decreases until the inner region of the accretion flow, implying that the matter is spiralling out and hence removing the angular momentum. A larger $|B_x B_z|$ at a larger radius implies the requirement of the removal of larger $\lambda$ therein. This automatically emerges from the self-consistent solutions of the set of equations. Nevertheless, close to the black hole, this effect reverses, rendering infall. Finally, the magnitude of $B_x B_z$ increases at a large and a small distances from the black hole (except around the transition radius), which helps infall, in the same way as the Shakura–Sunyaev viscous stress would do with the increase of matter pressure. However, at the intermediate zone, the transfer of angular momentum through $B_x B_z$ reverses and a part of the matter outflows. At the Keplerian to sub-Keplerian transition zone, flow/disk thickness increases, which effectively kicks the matter vertically, showing a decrease of $B_x B_z$. Most of these features remain unchanged for the flow around a rotating black hole, as shown in Figure 4(b). However, as a rotating black hole renders a stronger/efficient outflow/jet, here, except at the inner zone, $B_x B_z$ decreases throughout, which helps to transfer the angular momentum inward and kick the matter outward. Nevertheless, such a flow does not exhibit a high $\lambda$ either so
that does not necessarily require an increasing $B_z B_0$ to remove $\lambda$. Figures 4(c) and (d) furthermore confirm that the above properties are invariant for the cases of $B_0$ increased with increasing $z$. The only difference here is that $B_x B_0$ and $B_z B_0$ have opposite signs with respect to the previous cases, when the components of magnetic field considered here are their respective averaged values. Note that all the components of magnetic stress tensor as functions of radius are determined by the associated components of magnetic field. The components of magnetic field are, however, determined by solving the underlying set of equations self-consistently with their prescription at the inner edge of the flow for the regime of interest.

Figure 5 compares three cases of magnetic flows. (1) A counter rotating disk throughout, when $B_0$ decreases with $z$ almost throughout (solid line). (2) A disk having $B_0$ increasing with $z$ almost throughout, which is corotating far away, but counterrotating close to the black hole (dotted line). (3) A disk having $B_0$ increasing with $z$ almost throughout, which is counterrotating far away, but corotating close to the black hole (dashed line). It is important to note in the latter two cases that the increasing $B_0$ with disk height induces the change of the disk’s handedness during the infall of matter. However, the profiles of Mach number and $\beta$ practically appear similar in all of the three cases.

In Figure 6, we compare the disk hydromagnetics between the magnetic flows with high and low magnetic fields. As expected, a flow with the higher magnetic field transports the angular momentum much faster, leading to a smaller sub-Keplerian flow. In other words, in the presence of a higher magnetic field, when the magnetic stresses are stronger, the Keplerian flow (when $K_\text{ll} = 1$) as well as the boundary between the Keplerian and sub-Keplerian flows are able to advance toward the black hole, shrinking the sub-Keplerian zone because of efficient angular momentum transfer. Figure 6(c) shows that at a given radius the magnetic pressure, and hence the Alfvén speed, is much larger in a flow with the higher magnetic field (but still $\beta > 1$). Naturally, such a high field magnetic flow would be equivalent to a viscous flow with much larger $\alpha$, compared to the cases shown in Figure 1. An even higher magnetic field in the inner region would hinder any infall due to backreactions. Interestingly, in the radii close to the black hole, the sign of $B_0$ becomes distinctly more opposite than the outer region in the high magnetic field case. However, by this radius, the required amount of angular momentum has
already been transferred outward in order to advance the matter close to the black hole and hence the change in the sign of $B_z$ does not create any physical impact on the flow. If we vary the conditions chosen in Equation (15), e.g., assume the components of magnetic field to be inequal, the qualitative picture remains unchanged—the magnetic stress in the presence of a large-scale magnetic field could adequately transfer angular momentum. However, it is very important to

Figure 3. Components of magnetic field: $B_x$ (solid line), $B_0$ but normalized by 30 (dotted line), $B_z$ (dashed line), for (a) Schwarzschild magnetic flow of Figure 1, (b) Kerr magnetic flow of Figure 1.

Figure 4. Components of magnetic stress: $B_x B_0$ (solid line), $B_x B_z$ (dotted line), and $B_z B_0$ (dashed line), for (a) the Schwarzschild magnetic flow of Figure 1, (b) the Kerr magnetic flow of Figure 1, (c) the Schwarzschild magnetic flow of Figure 2, and (d) the Kerr magnetic flow of Figure 2.
note that if the strength of the magnetic field and the corresponding value at the inner edge around the sonic radius would have been even higher, above a certain value, then the infall would no longer be possible. This is similar to the situation in which above a certain value of \( \lambda \) at the sonic radius in a given flow, the infall is no longer possible (see Rajesh & Mukhopadhyay 2010a).

If the flow is gas pressure dominated with larger \( \gamma \), then all the above basic features remain the same. Be it radiation or gas dominated, large scale magnetic stresses, yet \( \beta > 1 \), can transport angular momentum as efficiently as the \( \alpha \)-prescription does. Nevertheless, below we discuss the effects of large scale magnetic field in the gas pressure dominated flows around a supermassive black hole.

### 3.2. Gas Dominated Accretion around Supermassive Black Holes

The supermassive black hole at the center of our galaxy, Sgr A*, presumably exhibits a gas pressure dominated, advection dominated, accretion flow. This motivates us to undertake this case, when we choose \( M = 10^7 M_\odot \) and \( M = 10^{-4} M_{\text{Edd}} \), and hence an appropriate \( \gamma = 1.55 \). We furthermore choose \( f_{\text{vis}} = f_m = 0.95 \), when strongly advective matter hardly has a chance to radiate photons. However, such a flow may also arise around a stellar mass black hole, e.g., microquasars.

The basic features in Figure 7 are similar as those for the stellar mass cases, as shown in Figure 1. However, due to the gas dominance, and hence lower angular momenta, the Mach number profiles practically do not have any centrifugal barrier. Such flows are hotter, with \( T \gtrsim 10^{11} \) K, and more quasi-spherical, compared to the radiation dominated flows, when a very small part of the dissipated heat can be radiated away. However, the most significant difference in these flows lies in their low magnetic fields, compared to those discussed in Section 3.1. This is due to the largeness of black hole masses in these flows, which leads to a much larger size of sub-Keplerian flows, when the dimensional flow size scales as \( M \). As a result, due to the law of equipartition, the magnetic field decreases significantly compared to the cases of stellar mass black hole, as shown in Figure 7(d) (as compared to those shown in Figure 1(d)).

The magnetic flows around nonrotating and rotating, both the black holes, have their viscous counterparts with respective \( \alpha = 0.011 \) and 0.0092. This furthermore confirms that even in gas dominated flows, the large-scale magnetic field is able to transfer the angular momentum as efficiently as the \( \alpha \)-prescription does with the most plausible values of \( \alpha \).
Hypothesizing the increasing $B_0$ with increasing $z$ at the outer edge of the sub-Keplerian flow, we obtain the same results as those for stellar mass black hole accretion flows described above, except at much lower magnetic fields. Like the stellar mass cases, here also a “potential well” forms that is featured in Figure 8(b), rendering the systems to have a zone for producing outflows. We do not repeat the detailed properties of it. Figures 8(a) and (b) also show the viscous flows resembling magnetic flows with $\alpha = 0.075$ and 0.07, respectively, for nonrotating and rotating black holes.

3.3. Dependence on $s_1$, $s_2$, $s_3$

As defined in Section 2, $s_1$, $s_2$, $s_3$ parametrize the scaling of the variations of $B_\alpha$, $B_0$, $B_\gamma$, respectively, in the vertical direction. This has been considered because of our averaging the flows in the vertical direction, while the variation of magnetic field in the vertical direction has not been neglected, as it has not been for $P$. Interestingly, the solutions practically do not depend on the choices of $s_1$ and $s_3$. However, with the increase of the magnitude of $s_2$, which implies the increasing change of the magnetic field with the vertical coordinate, the size of sub-Keplerian flow decreases. This is because the stronger the vertical variation of the magnetic field is, and hence, the larger the change of the magnetic stresses in the vertical direction is, on average, the faster the infall of matter is. This also argues for the faster rate of throwing the matter via outflows, when the outflows, in a more self-consistent 2.5 dimensional flow, are expected to plunge out via the magnetic field lines in the vertical direction. Hence, with the increase of magnetic field in the vertical direction, the system becomes more prone to outflow matter. Subsequently, a faster rate of outflow renders a faster removal of angular momentum and hence a faster rate of infall. As a result, the flow could remain Keplerian with the aid of adequate mechanisms of angular momentum transfer, until a further inner region of the flow. Hence, the boundary between the Keplerian and sub-Keplerian flows advances toward the black hole.

3.4. Interconnection between Advection and Magnetic Field

Because the current $J$ in the conducting fluid with conductivity $\sigma$ and electric field $E$ is known to be $J = \sigma(E + v \times B)$, Faraday’s law of induction in the steady-state for axisymmetric accretion disks considered here, as given by Equation (7), can be recalled as

$$\nabla \times \frac{J}{\sigma} = \nabla \times (v \times B) = 0.$$  (17)
For a flow with very large $R_{in}$ (when $\tau_{in} \propto \sigma^{-1} << 1$), the $z$-component of Equation (17), averaged in $z$ and integrated over $\phi$, is given by
\[
\int \partial B_z \, x \, d\phi = \text{constant} = C,
\] (18)
when the constant can be identified as
\[
C = d/dt \left( \int B_z \, ds_{\phi z} \right) = d\Phi/dt,
\]
where $ds_{\phi z}$ is the elementary surface area in the disk plane and $\Phi$ is the magnetic flux. Therefore, Equation (18) fixes the relation between advection and $B_z$, and hence the magnetic flux in the accretion flow. This also can be understood by recasting Faraday’s law of induction into
\[
\nabla \times \left( E + \frac{\partial A}{\partial t} \right) = 0, \quad \text{when } B = \nabla \times A,
\] (19)
which furthermore argues for
\[
E = - \nabla V - \frac{\partial A}{\partial t} - C,
\] (20)
when $V$ is the Coulomb potential and $C$ is a constant vector. Hence, for a steady axisymmetric accretion flow
\[
E_\phi = \frac{J_\phi}{\sigma} + x\partial B_z = -C_\phi.
\] (21)
Therefore, for $R_{in} \gg 1$, $x\partial B_z$ is conserved. The constant $C_\phi$ or $C$ can be fixed from a given boundary condition. In reality, however, the flow is not expected to be purely axisymmetric and, hence, $C_\phi$ or $C$ can also be mimicked as the contribution from non-axisymmetry. Earlier Lubow et al. (1994) discussed a model of geometrically thin accretion flows in the presence of the weak magnetic field, but assuming $C_\phi = 0$, which is not true in general.

The constraint on advection, which arose in Equation (21), is clearly visible in the velocity profiles in Figure 1(a) with respect to the variations of the vertical component of magnetic field shown in Figure 3. For the magnetic flow around a Schwarzschild black hole, $\theta$ first increases steadily with the decrease of $B_z$ at larger radii and, subsequently, matter tends to slow down due to the centrifugal effect (with the relative
decrease of infall rate $d\dot{M}/dx$ with the increase of $B_z$ until $x = 10$. Finally, matter plunges into the black hole steadily with a sharp decrease of $B_z$. For a rotating black hole, however, $\dot{\vartheta}$ steadily increases with the steady decrease of $B_z$ almost throughout. Nevertheless, very close to the black hole, $B_z$ slightly increases due to the spin effect of the black hole, which is decreasing $d\dot{\vartheta}/dx$ slightly. This hints that the power of the black hole’s spin plunges the matter out.

4. DISCUSSION AND SUMMARY

We have discussed the power of the large-scale magnetic field in advective accretion flows around black holes in order to transport angular momentum, enabling the infall of matter. In a simpler Keplerian, self-similar model framework, such an investigation had been initiated by Blandford & Payne (1982) long ago, and in the cases of circumstellar disks around young stars (e.g., Königl & Salmeron 2011), such an approach has been explored. However, it remained unexplored, to the best of our knowledge, in the advective accretion disk around black holes, when it may exhibit a hard spectral state, until this work.

Note that, often, only hard spectral states of disks are associated with the outflows/jets.

We have found that flows with plasma-$\beta > 1$ exhibit adequate magnetic transport—as efficient as the $\alpha$-viscosity with $\alpha = 0.08$ would do. This is interesting as the origin of $\alpha$ (and the corresponding instability and turbulence) is itself not well understood. The maximum required large-scale magnetic field is a few factor times $10^5$ G in a disk around $10M_\odot$ black holes and $\sim 10$ G in a disk around $10^7M_\odot$ supermassive black holes. The presence of such a field, in particular, for a stellar mass black hole disk when the binary companion supplying mass is a Sun-like star with the magnetic field on average $1$ G, may be understood, if the field is approximately frozen with the disk fluids (or if the supplied fluids from the companion star remain approximately frozen with the magnetic field) or if the disk fluids exhibit large Reynolds numbers. Indeed, all of the present computations are done with the limit of a large Reynolds number, as is really the case in accretion flows, such that the term associated with the magnetic diffusivity in the induction equation can be neglected. The size of a disk around supermassive black holes is proportionately larger compared to that around a stellar mass black hole. Hence, from the

Figure 8. Same as in Figure 7, except $s_2 = 0.5$, when $\lambda_1 = -2.8$ and $-1.62$ for nonrotating and rotating magnetic flows, respectively, and $\alpha = 0.075$, $\lambda_2 = -2.65$ and $\alpha = 0.07$, $\lambda_2 = -1.6$ for nonrotating and rotating viscous flows respectively.
equipartition theory, the magnetic field is indeed expected to be decreased here compared to that around stellar mass black holes.

Is there any observational support for the existence of such a magnetic field, as required for the magnetic accretion flows discussed here? Interestingly, the polarization measurements in the hard state of Cyg X-1 imply that it should have an at least 10 mG at the base of the jet and hence in the underlying accretion disk. Also, the magnetic field in the inner region of accretion disks around more than a dozen black holes has been found to be very high, based on a model relating the observed kinetic power of relativistic jet to the magnetic field of accretion disks (Piotrovich et al. 2014).

Different components of magnetic stress tensors have different roles: $B_i B_j$ controls the infall in the disk plane, whereas $B_i B_j$ renders the flow to spiral outward and, hence, outflow. Moreover, $B_i B_j$ helps to kick the matter out vertically. The larger the field strength, the larger the power of magnetic stresses is. Interestingly, the magnitude of the magnetic field decreases as the steady-state matter advances toward the black hole. This is primarily because $B_i B_j$ (and also $B_i B_j$ for a rotating black hole) decreases inward almost entirely in order to induce outflow. This furthermore reveals a decreasing $|B_i|$ as the output of self-consistent solutions of the coupled set of equations, which is reflected in the $|B|$ profile.

In the present computations, we have assumed the flow to be vertically averaged without allowing any vertical component of the flow velocity (but keeping all of the components of the magnetic field). The most self-consistent approach, in order to understand vertical transport of matter through the magnetic effects, which in turn leads to the radial infall of the rest of the matter, is considering the flow to be moving in the vertical direction from the disk plane as well. Such an attempt, in the absence of magnetic effects, was made earlier by Bhattacharya et al. (2010) in the model framework of coupled disk-outflow systems. In such a framework, the authors furthermore showed that the outflow power of the correlated disk-outflow systems increases with the increasing spin of black holes. Our future goal is now to combine that model with the model of the present work so that the coupled disk-outflow systems can be investigated more self-consistently and rigorously, when the magnetic field plays an indispensable role in order to generate vertical flux in the three-dimensional flows.

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