M-Theory on Orientifolds of $K_3 \times S^1$

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Abstract

We present several Orientifolds of M-Theory on $K_3 \times S^1$ by additional projections with respect to the finite abelian automorphism groups of $K_3$. The resulting models correspond to anomaly free theories in six dimensions. We construct explicit examples which can be interpreted as models with eight, four, two and one vector multiplets and $N = 1$ supersymmetry in six dimensions.
\[M\text{-Theory}\[1, 2, 3, 4, 5, 6, 7, 8, 9\], believed to be a candidate for the unification of all string theories, is at present a focus of attention. At low energies, this theory is represented by the eleven-dimensional supergravity. It was already shown earlier, that the eleven-dimensional supergravity is the strong coupling limit of the ten dimensional type IIA string theory\[1\]. Recent interest in the subject was generated by the fact that the compactification of these theories to ten dimensions on an orientifold\[10, 11, 12\] of \(S^1\) gave rise to the \(E_8 \times E_8\) heterotic string theory in ten dimensions\[4\]. In proving this equivalence, the anomaly cancellations for the ten dimensional \(N = 1\) supersymmetric theories plays an important role\[13\]. This is due to the fact that, in the absence of a complete knowledge of this theory, an explicit construction of the orientifolds is not possible. More recently, compactifications of \(M\)-Theory to six dimensions have also been studied. Its \(T^5/Z_2\) orientifold gave rise to an anomaly free \(N = 2\) supersymmetric theory in six dimensions with 21 tensor multiplets\[5, 6\]. Once again the requirements of the anomaly cancellation\[14\] played a major role in determining the complete spectrum of this theory. It was shown that the fixed points of the torus degenerate into pairs. As a result, the twisted sectors contribute only sixteen extra tensor multiplets, instead of thirty two.

The orientifolds of type II string theories have also been examined \[15, 16, 17\]. In \[17\] construction of a new chiral string theory in six dimensions, through an orientifold compactification of the type IIB strings on \(K_3\), has been presented. It was shown that such a compactification gives rise to an \(N = 1\) supersymmetric theory in six dimensions with anomaly free particle content.

A particle spectrum, consisting of 9 tensor, 8 vector and 20 hypermultiplets and \(N = 1\) supersymmetry was obtained in six dimensions by Sen\[9\] for \(M\)-Theory compactified on an orientifold of \(K_3 \times S^1\). In this case, like in the ten dimensional one, the twisted sector states can not be obtained by a direct \(M\)-Theory calculation. However, Sen was able to determine the spectrum in the twisted sector by comparing the nature of the fixed points for \(K_3 \times S^1\) with those for \(T^5\). It was argued that the physics near
the fixed points in the case of \((K_3 \times S^1)/Z_2\) is identical to that for \(T^5/Z_2\). As a result, the contribution of the twisted sector states are identical in the two cases. This fact will also be utilized in our case below. Since the extra \(Z_2\)’s that we apply act freely and keep the supersymmetry intact, the number of fixed points remains unchanged. Their contribution to the field content also does not change, since they still come as tensor multiplets of the chiral \(N = 2\) algebra due to the supersymmetry preserving nature of the extra projections. This is further confirmed by the fact that we are able to obtain anomaly free combination of fields in all our examples.

In this article, we present new examples of orientifold compactifications of \(M\)-Theory by further orbifolding of the models in \([9]\) with respect to the finite abelian automorphism groups of \(K_3\). In particular, we present several \(N = 1\) supersymmetric examples with different number of vector multiplets. The orbifolds of \(K_3\) for the case of type IIA string compactification was discussed in \([19, 20]\). These orbifolds provided examples of the dual pairs of the heterotic string theories in dimensions six and less with maximal supersymmetry, but with lower rank gauge groups. In this article we focus our attention on the \(Z^k_2\) orbifolds discussed in \([20]\). In our case, we combine one of these \(Z_2\) actions with another operation which changes the sign of the eleven dimensional 3-form fields as well as the eleven-dimensional coordinate \(x^{10}\). As a result we are able to construct several models with \(N = 1\) supersymmetry, but with different number of vector multiplets. Extra \(Z_2\) symmetries, in our case act freely. To avoid new fixed points, we combine these \(Z_2\)’s with the translation symmetries along some of the compactified directions. Consequently, like in \([20]\), our result is strictly valid only in dimensions less than six. The lower dimension spectra can however be seen to arise from the corresponding six dimensional ones, with new anomaly free combinations, in a straightforward way.

We now begin by describing the \(Z^k_2\) \((k = 1, 2, 3, 4)\) orbifolds presented in \([20]\). The action of the symmetries is represented by \(k\) \(Z_2\) generators, denoted by \(g_i\) \((i = 1, \ldots, k)\). Under these \(Z_2\)’s all the three self-dual two forms and three of the nineteen anti-self-
dual three forms are invariant. On the remaining sixteen anti-self-dual two forms it acts as:

\[ g_1 : \left( (-1)^8, 1^8 \right), \]  
\[ g_2 : \left( (-1)^4, 1^4, (-1)^4, 1^4 \right), \]  
\[ g_3 : \left( (-1)^2, 1^2, (-1)^2, 1^2, (1)^2, (-1)^2, 1^2 \right), \]  
\[ g_4 : \left( -1, 1, -1, 1, -1, 1, -1, 1, -1, 1, -1, 1, -1, 1 \right), \]

with superscripts on 1 denoting the repeated entries. Individually, as will be described below, under any of these \( Z_2 \)'s 34 of the 58 \( K_3 \) moduli fields remain invariant. Similarly, 14 of the 22 two-forms on \( K_3 \) are even under \( Z_2 \)'s and 8 are odd. When more than one of these discrete symmetries is used to mod out the original theory, the resulting spectrum is determined by taking the intersection of the individual ones.

Above discrete symmetries are now used to obtain the untwisted sector of the spectrum when \( M \)-Theory, with low energy spectrum consisting of a graviton \( G_{MN} \) and a third rank antisymmetric tensor \( A_{MNP} \) in eleven dimensions, is compactified on an orientifold of \( K_3 \times S^1 \). For this purpose, the first \( Z_2 \) in equation (1) is combined with an operation \( A_{MNP} \rightarrow -A_{MNP} \) and \( x^{10} \rightarrow -x^{10} \). Here we have used the notation that \( (M, N) = (0, ..., 10) \) and \( (\mu, \nu) = (0, .., 5) \). The rest of the \( g_i \)'s act in the same way as in [20]. We would once again like to point out that projections in [20] with respect to \( Z_2 \)'s in equations (1)-(4) keeps the supersymmetries unbroken. As a result, further orbifolding, by \( g_i \)'s, of the resulting \( N = 1 \) theory also keeps the supersymmetry intact.

We now present the counting of the surviving degrees of freedom when several \( g_i \)'s in equations (1)-(4) are applied on \( G_{MN} \) and \( A_{MNP} \). First, the surviving degrees of freedom for \( G_{MN} \) is determined by examining the local structure of the moduli space. Since the number of supersymmetries for the \( Z_2 \) projections by \( g_i \)'s remain
unchanged, the local structure of the moduli space for the compactifications on the above orbifolds of $K_3$ has the form \[20\]:

\[
\mathcal{M} = \frac{SO(20 - r, 4; R)}{SO(20 - r; R) \times SO(4; R)},
\]

where $r$ is the reduction in the rank of the gauge group from the maximal number 24, which is equal to the one for the toroidal compactification of the heterotic string. Such reductions in the ranks of the gauge group, for the orbifolds of $K_3$ described above, can now be studied in the type IIA theory. We will use this information to determine the number $r$ for the action of the various combinations of $g_i$’s.

All the 24 gauge fields in the $K_3$ compactification of type IIA theory originate in the Ramond-Ramond (R-R) sector. Since for $K_3$, only nonzero Betti numbers are $b_0 = b_4 = 1$ and $b_2 = 22$, one of the gauge fields in six dimensions originates from its ten dimensional counterpart $A_M$. Three-form field components $A_{\mu mn}$ give rise to 22 gauge fields and the dualization of $A_{\mu \nu \rho}$ gives the remaining one. Here $(m, n)$ denote the indices on $K_3$. An observation of the form of $g_i$’s in equations (1)-(4) now gives the values:

(i) $r = 8$ for the action of $g_1$,
(ii) $r = 12$ for the action of $g_1$ and $g_2$ together,
(iii) $r = 14$ for the action of $g_1$, $g_2$ and $g_3$, and finally
(iv) $r = 15$, when all the four $g_i$’s in equations (1)-(4) are applied.

The reduction in the number of gauge fields follows directly by counting the number of 2-forms on $K_3$ that are projected out by the action of $g_i$’s. We will use these informations to determine the invariant components of the metric $G_{MN}$ under compactification. The number of 2-form fields left invariant in the cases (i)-(iv) above are respectively 14, 10, 8 and 7. By subtracting these numbers from the dimension of the coset \[20\], and taking into account the rank-reduction in the last paragraph, we get the number of invariant scalars from the $K_3$ part, originating from the metric $G_{MN}$ in ten dimensions. These are respectively (i) 34, (ii) 22, (iii) 16 and (iv) 13.
We now first construct an orientifold of \( M \)-Theory for the action of symmetry \( g_1 \) and then obtain other models by additional projections with respect to the remaining \( g_i \)'s. As mentioned earlier, to avoid fixed points with respect to these additional \( Z_2 \)'s, one has to combine them with the translations on circles \([19, 20]\) by compactifying further to lower dimensions. However, since the field content for the massless modes does not depend on these shifts, these lower dimensional spectra follow directly from the toroidal compactification of the six dimensional ones. Therefore the effect of these extra \( Z_2 \)'s can be seen directly in terms of a six dimensional spectrum which we now present. In the next four paragraphs, we present the field contents in the cases (i)-(iv) above, starting from the fields \( G_{MN} \) and \( A_{MNP} \) in eleven dimensions. We also show that they are all consistent with the anomaly cancellation requirements.

(i) The projection with respect to \( g_1 \) leaves the components \( G_{\mu\nu} \) and \( G_{(10)(10)} \) of \( G_{MN} \) invariant, and also gives 34 scalar fields from the local moduli in \( K_3 \). In addition, taking into account that \( A_{MNP} \) is odd under this \( Z_2 \) and \( x^{10} \) changes sign, we find that the component \( A_{\mu\nu(10)} \) remains invariant and gives rise to an antisymmetric tensor field in six dimensions. We also get 8 gauge fields \( A_{\mu mn} \) from the eight 2-forms of \( K_3 \) which are odd under \( g_1 \). 14 more scalars arise from even 2-form components \( A_{mn(10)} \) on \( K_3 \). Together, these give us the graviton, an antisymmetric tensor, 8 vectors and 49 scalars in the untwisted sector\([9]\). Combining these with the states arising from the twisted sectors, namely 8 tensor and 8 hypermultiplets, we find the following final spectrum. (a) 9 tensor multiplets (b) 1 graviton multiplet (c) 8 vector multiplets and (d) 20 hypermultiplets. These are precisely the combination needed for the anomaly cancellatin in \( N = 1 \) supersymmetric theories\([14, 9]\).

(ii) For this case the spectrum in the untwisted sector is obtained by taking intersection with respect to the projections in (i), as described in the last paragraph, with those with respect to \( g_2 \). The surviving degrees of freedom from \( G_{MN} \) now consists of \( G_{\mu\nu} \), \( G_{(10)(10)} \) and 22 local moduli fields. Among the \( A_{MNP} \) components, which survive this projection are now (a) \( A_{\mu\nu(10)} \), (b) 4 of the eight vectors \( A_{\mu mn} \) in the
last paragraph and (c) 10 of the fourteen scalars $A_{mn(10)}$. Together these provide a graviton, an antisymmetric tensor, 4 vectors and 33 scalars in the untwisted sector. Combining these, once again, with the fields from the twisted sectors, namely the 8 tensor and 8 hypermultiplets we get the full field content for this theory as: (a) 9 tensor multiplets, (b) a gravity multiplet, (c) 4 vector multiplets and (d) 16 hypermultiplets. This is once again an anomaly free spectrum$[14]$. As stated earlier, $g_2$ acts freely only when it is combined with a half-shift along one of the circles of compactification. For this purpose, one has to consider this orientifold only in dimensions five. However, since the field content for the massless fields does not depend on the shift, the resulting five dimensional fields can easily be seen to arise from the compactification of the above massless spectrum in six dimensions on a circle. This is a new anomaly free combination, other than the one discussed in $[9]$.

(iii) The effect of the actions of $g_2$ and $g_3$ on the orientifold of $g_1$ in (i) can be studied exactly in the same way as in (ii) above. The massless fields from the untwisted sector now consist of a graviton, 1 antisymmetric tensor, 2 vector fields and 25 scalar fields. Once again, combining these with the 8 tensor and 8 hypermultiplets of the twisted sector gives rise to 9 tensor multiplets, a graviton multiplet, 2 vector multiplets and 14 hypermultiplets. This is an anomaly free field content. As in the last paragraph, the two $g_i$’s have to be combined with appropriate shifts along the compactified directions. As a result, this time the model has a valid interpretation only in four dimensions. However once again, the resulting four dimensional fields can be seen to originate from a six dimensional theory with the above anomaly free field content through a simple toroidal compactification.

(iv) Finally, when all the four $g_i$’s are applied together, with $g_1$ acting as an orientifold as above, we have from the untwisted sector, a graviton, 1 antisymmetric tensor, 1 vector field and 21 scalars. Combining them with the twisted sector states, i.e. 8 tensor and 8 hypermultiplets, we get another anomaly free field content: 9 tensor multiplets, 1 gravity multiplet, 1 vector multiplet and 13 hypermultiplets.
To conclude, we have presented new anomaly free combinations in six dimensions that arise from the $K_3 \times S^1$ orientifold of $M$-Theory by additional projections with respect to the abelian automorphism groups of $K_3$. One of the key ingredients in constructing these $N = 1$ supersymmetric theories has been the fact that the number of supersymmetries do not reduce by the above actions of the automorphisms of $K_3$. We have also avoided the presence of any new twisted sector states by combining the extra $Z_2$ projections with the shifts along the directions on which the six dimensional theory is compactified. Another way to avoid additional fixed points may be by combining these additional $Z_2$’s with the shifts along the central charges of the supersymmetry algebra$^2$, We will then have a genuinely six dimensional theory with the same field contents as mentioned in this paper. One may also directly study the contributions from all the additional fixed points, which have been avoided here, and see if the anomaly cancellation condition is maintained.
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