ISW–LSS CROSS-CORRELATION IN COUPLED DARK ENERGY MODELS WITH MASSIVE NEUTRINOS

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ABSTRACT

We provide an exhaustive analysis of the Integrated Sachs–Wolfe (ISW) effect in the context of coupled dark energy cosmologies where a component of massive neutrinos is also present. We focus on the effects of both the coupling between dark matter and dark energy and of the neutrino mass on the cross-correlation between galaxy/quasar distributions and ISW effect. We provide a simple expression to appropriately rescale the galaxy bias when comparing different cosmologies. Theoretical predictions of the cross-correlation function are then compared with observational data. We find that, while it is not possible to distinguish among the models at low redshifts, discrepancies between coupled models and ΛCDM increase with z. In spite of this, current data alone does not seem able to distinguish between coupled models and ΛCDM. However, we show that upcoming galaxy surveys will permit tomographic analysis that will allow us to better discriminate among the models. We discuss the effects on cross-correlation measurements of ignoring galaxy bias evolution, b(z), and magnification bias correction and provide fitting formulae for b(z) for the cosmologies considered. We compare three different tomographic schemes and investigate how the expected signal-to-noise ratio, S/N, of the ISW–LSS cross-correlation changes when increasing the number of tomographic bins. The dependence of S/N on the area of the survey and the survey shot noise is also discussed.

Key words: cosmology: miscellaneous – cosmology: observations – cosmology: theory – large-scale structure of universe

1. INTRODUCTION

Several observations made over the recent years, related to a large extension to large-scale structures (LSSs) and anisotropies of the cosmic microwave background (CMB) as well as the magnitude–redshift relation for Type Ia supernovae have given us a convincing picture of the energy and matter density in the universe (Perlmutter et al. 1999; Riess et al. 1998; Spergel et al. 2003; Tegmark et al. 2004; Larson et al. 2011).

Baryonic matter accounts for no more than 30% of the mass in galaxy clusters while the existence of a large clustered component of dark matter (DM) now seems firmly established, although its nature is still unknown. However, they contribute to the total energy density of the universe with only a few percent and about 25%, respectively.

No more than another few percent could be accounted for by massive neutrinos, but only in the most favorable, though unlikely, case. According to Kristiansen et al. (2007; see also Elgarey et al. 2002) the total mass of neutrinos cannot exceed the limit of 1.43 eV (see, however, La Vacca et al. 2009a, 2009b; Kristiansen et al. 2010 for a recent analysis on neutrino mass limits in coupled dark energy models). A very small part (10^{-4}) of the total energy density is due to massless neutrinos and CMB radiation.

The model suggested by observations is only viable if the remaining 75% is ascribed to the so-called dark energy (DE) responsible for the present day cosmic acceleration.

Although strongly indicated by the observations, the existence of DE is even more puzzling than DM. It can be identified with a cosmological constant Λ or with a yet unknown dynamical component with negative pressure. On the other hand, its manifestation can be interpreted as a geometrical property of the gravity on large scales resulting from a failure of general relativity (GR) on those scales (see Copeland et al. 2006 for a review).

Within the context of GR, as an alternative to the cosmological constant, DE is usually described as a scalar field φ, self-interacting through a suitable potential V(φ), or a cosmic fluid with negative pressure (see Peebles & Ratra 2003 and references therein).

Scalar fields naturally arise in particle physics. Furthermore, if they are tracker fields (Steinhardt et al. 1999), fine tunings associated with the small value of the present DE energy density can be significantly alleviated unlike the cosmological constant case.

In addition to self-interaction, a scalar field can in principle be coupled to any other field present in nature. However, in order to drive the cosmic acceleration, its present time mass is expected to be, at least on large scales, m_{φ} \sim H_{0} \sim 10^{-33} \text{ eV} (H_{0} being the present Hubble parameter). Such a tiny mass gives rise to long-range interactions that could be tested with fifth-force type experiments. Couplings to ordinary particles are strongly constrained by such a kind of experiment but limits on the DM coupling are looser (constraints on coupling for specific models were obtained in Maccio' et al. 2004, Amendola & Quercellini 2003, Olières et al. 2005, Lee et al. 2006, Guo et al. 2007, and Mainini & Bonometto 2007 from CMB, N-body simulations, and matter power spectrum analysis).

A possible common origin of DM and DE and/or their direct coupling (Wetterich 1995; Amendola 1999; Gasperini et al. 2002; Bartolo & Pietroni 2000; Chimento et al. 2003; Rhodes et al. 2003; Farrar & Peebles 2004) would ease one of the most critical problems in modern cosmology, the so-called coincidence problem: why did expansion started to accelerate just at the eve of our cosmic epoch, after decelerating during all epochs after inflation? Why do DE and DM have similar densities just now? Because of the coupling DM and DE densities remain at similar values for a long period and the only peculiar feature of the present epoch is the recent overtaking of DM density by DE density.
If present, DM–DE coupling could have a relevant role in the cosmological evolution affecting not only the overall cosmic expansion but also modifying the DM particle dynamics with relevant consequences on the growth of the matter density perturbations in both linear and nonlinear regimes (e.g., on halo density profiles, cluster mass function and its evolution, see Wang & Steinhardt 1998; Mainini et al. 2003a, 2003b; Klypin et al. 2003; Dolag et al. 2004; Maccio’ et al. 2004; Perrotta et al. 2004; Mota & Barrow 2004; Olivares et al. 2006; Nunes et al. 2005; Mota & van de Bruck 2004; Maor & Lahav 2005; Wang 2006; Manera & Mota 2006; Nunes & Mota 2006; Dutta & Maor 2007; Mota et al. 2007; Mainini 2008, 2009; Shaw & Mota 2008; Mota 2008; Baldi et al. 2009; Wintergast & Pettorino 2010; Baldi & Pettorino 2011). LSS is then a powerful probe of DE nature, permitting us to put significant constraints on DE parameters. Constraints often become even more stringent when data from other probes are simultaneously taken into account.

CMB is another powerful probe of DE nature. In principle, by joining anisotropy and polarization data, DE parameters can be significantly constrained. CMB and LSS probe the universe at different epochs and are therefore complementary to each other. Future data from high-resolution CMB experiment such as Planck and LSS surveys (EUCLID, LSST, DES, JDEM, etc.) will allow us to constrain DE to an unprecedented accuracy.

In this paper, we will focus on the Integrated Sachs–Wolfe (ISW) effect (Sachs & Wolfe 1967). The ISW effect is a secondary anisotropy of the CMB and a direct signature of DE. The effect arises when a photon from the last scattering surface passes through a time-dependent gravitational potential changing its energy so that additional temperature anisotropies are generated. Decay of gravitational potentials may occur through cosmic curvature, in the presence of DE or in alternative gravity models.

Assuming GR is the correct theory of gravity and that the universe is spatially flat, large-scale gravitational potentials do not evolve significantly in the matter era. Cosmic acceleration, however, causes the gravitational potentials to decay making the ISW effect highly sensitive to the presence of DE.

Though difficult to detect directly in the CMB, the ISW signal can be measured by cross-correlating the CMB with tracers of LSS and has recently been detected using Wilkinson Microwave Anisotropy Probe (WMAP) data of CMB in combination with several LSS surveys at the ~3σ–4σ confidence level providing independent evidence for the existence of the DE (see Giannantonio et al. 2008a; Xia et al. 2009 and references therein).

Cross-correlation then provides a powerful method to discriminate among different DE models and, in particular, to detect a possible interaction between DE and DM other than investigate the clustering properties of DE on large scales. If present, DM–DE coupling changes both the scaling of the DM energy density and the growth rate of matter perturbations yielding a significant evolution of the metric potentials even in the matter era.

The aim of this paper is to provide an exhaustive analysis of the ISW effect in the context of the so-called coupled DE cosmologies (Amendola 2000) mainly focusing on the effects of the DM–DE coupling on the cross-correlation between galaxy/quasar distributions and the ISW effect.

Such models can be motivated in the context of scalar–tensor theories of gravity or describe the low-energy limit of a more fundamental theory beyond the standard model of particle physics, e.g., string theory.

The models that we aim to investigate differ from the standard ΛCDM in three different aspects: (1) DE is a self-interacting scalar field φ rather than a cosmological constant Λ. We shall consider a class of self-interaction potentials V(φ) admitting tracker solutions. (2) A linear DM–DE coupling is allowed. (3) We allow neutrinos to be massive.

The effects of massive neutrinos in cosmology have been studied thoroughly for many years (for a review, see Lesgourgues & Pastor 2006). Cosmological observations are mostly sensitive to the sum of neutrino masses, Mν. Currently, the strongest upper limit on the neutrino mass scale comes from cosmology. One of the effects of massive neutrinos is to induce a small decay of the gravitational potentials during both matter and DE domination so that, in principle, the ISW effect would also provide information on their mass. Furthermore, as recently outlined in La Vacca et al. (2009a, 2009b) and Kristiansen et al. (2010), the effects that massive neutrinos have on the angular power spectrum of the CMB anisotropies, Cℓ, and matter power spectrum, P(k), are almost opposite to those of the DM–DE coupling, resulting in a strong degeneracy between the coupling strength β and Mν. A recent analysis by means of the Monte Carlo Markov Chain method has shown that a cosmology with significant Mν and β is statistically preferred to one with no coupling and almost massless neutrinos. Further, when priors on the neutrino mass from Earth-based neutrino mass experiments (Heidelberg–Moscow neutrinoless double β-decay, KATRIN tritium β-decay) are added to the analysis, a 5σ–6σ detection of a DM–DE coupling is found.

The plan of the paper is as follows: in Section 2 we describe our model while the ISW effect theory is reviewed in Section 3, where we also discuss how the ISW signal depends on the main parameters of the model. In Section 4, we discuss galaxy bias and magnification bias. Comparison between theoretical prediction and observation data is presented in Section 5 while a tomographic analysis is performed in Section 6. Section 7 is devoted to the conclusions.

2. THE MODEL

We assume a spatially flat Friedmann–Robertson–Walker (FRW) background with metric ds2 = a2(η)(−dt2 + dx,2 + dx,3) (η is the conformal time) filled by baryons, photons, neutrinos, DM, and a component of DE which will be ascribed to a scalar field φ self-interacting through a potential V(φ). In the following, the indexes b, c, v, and φ denote baryons, cold DM, massive neutrinos, and DE, respectively. Photons and massless neutrinos will be referred to as radiation and denoted by r.

In addition to self-interaction, we also consider a possible interaction between the scalar field and DM. Here we give only the equations for baryons, DM, and DE, with the equations for the other components the usual ones (see, e.g., Ma & Bertschinger 1995).

The Friedmann equation for the scale factor a, the continuity equations for baryons and DM, and the evolution equation for

3 http://www.rosr.esa.int/index.php?project=planck
4 http://www.euclid-imaging.net/
5 http://www.lsst.org/lsst
6 https://www.darkenergysurvey.org/
7 http://jdem.gsfc.nasa.gov/
8 http://www-ik.fzk.de/katrin/publications/documents/Design Report2004-12Jan2005.pdf
the scalar field read
\[ H^2 = \frac{8\pi}{3} G (\rho_b + \rho_c + \rho_\phi) a^2 \] (1)
\[ \dot{\rho}_b + 3 H \rho_b = 0 \] (2)
\[ \dot{\rho}_c + 3 H \rho_c = -C \dot{\phi} \rho_c \] (3)
\[ \ddot{\phi} + 2 H \dot{\phi} + a^2 V(\phi) = C a^2 \rho_c, \] (4)
where the overdot denotes the derivative with respect to \( \eta \), \( H = \dot{a} / a \), \( \rho_i \) is the energy density of the component \( i = b, c, \nu, \phi, r \), and the constant \( C \) parameterizes the DM–DE coupling strength.

Working in the conformal Newtonian gauge the metric of a perturbed flat FRW universe takes the form
\[ ds^2 = a^2(\tau) \left[ -(1 + 2\Phi) d\tau^2 + (1 - 2\Psi) dx^i dx_i \right], \] (5)
where \( \Phi \) plays the role of the Newtonian potential, \( \Psi \) is the Newtonian spatial curvature, \( |\Phi|, |\Psi| \ll 1 \), and
\[ \Phi = -\frac{3 H^2}{2 k^2} \sum_i \left[ \Omega_i \delta_i + \frac{3 H^2}{k^2} (1 + w_i) \Omega_i \theta_i \right] \] (6)
\[ \Psi = \Phi - \sum_i \frac{9 H^2}{2 k^2} (1 + w_i) \Omega_i \sigma_i, \] (7)
where \( \Omega_i = \rho_i / \rho_c, \delta_i = \delta \rho_i / \rho_i, \theta_i, \sigma_i \), and \( w_i = p_i / \rho_i \) are the density parameter, density contrast, four-velocity divergence, shear, and state parameter of the component \( i = b, c, \nu, \phi, r \), respectively; they allow tracker solutions for any \( \alpha \) and \( \Lambda \) are assigned, the present time DE density parameter \( \Omega_a(\alpha) \) is uniquely defined.

As we are interested in the cross-correlation between the ISW effect and galaxy distributions, the above equations can be simplified. At late time radiation and massive neutrinos can be neglected so that no shear stresses are present and \( \Phi = \Psi \). Furthermore, the cross-correlation signal comes from scales well within the horizon, \( \sim 100–200 \) Mpc, so that the second term in Equation (6) can be neglected.

The above equations then reduce to the usual Poisson equation for the gravitation potential,
\[ \Phi = -\frac{3 H^2}{2 k^2} \left[ \Omega_c \delta_c + \Omega_\nu \delta_\nu + \Omega_\phi \delta_\phi \right], \] (13)
a modified Jeans equation for DM and the usual one for baryons,
\[ \ddot{\delta}_b + (H - C \dot{\phi}) \dot{\delta}_b = \frac{3}{2} H^2 \left[ (1 + \frac{4}{3} \beta^2) \Omega_\nu \delta_\nu + \Omega_\phi \delta_\phi + \Omega_\phi \delta_\phi \right] \]
\[ \ddot{\delta}_\phi + H \dot{\delta}_\phi = \frac{3}{2} H^2 \left[ \Omega_c \delta_c + \Omega_\nu \delta_\nu + \Omega_\phi \delta_\phi \right] = 0, \] (14)
and a Poisson-like equation for the scalar field perturbation:
\[ \ddot{\delta}_\phi = \frac{H^2}{k^2} \left[ \Omega_c \delta_c + \Omega_\nu \delta_\nu + \Omega_\phi \delta_\phi \right], \] (15)
where we have defined the dimensionless coupling parameter
\[ \beta = \sqrt{3/16\pi m_p C} \] (m_p \( = G^{-1/2} \) is the Planck mass).

As clearly visible from the above equations and widely discussed in Amendola (2000), coupling affects the dynamics of DM particles. As a consequence, baryons and DM develop a bias \( b^* \), i.e., \( \delta_b = b^\ast \delta_{dm} \). Note that this bias, whose origin is ascribed to the coupling, is something completely different from the galaxy bias due to hydrodynamical effects discussed in the subsequent sections.

It is also worth mentioning that, unlike the uncoupled case, in the presence of coupling, the universe goes through an evolutionary phase named \( \phi \)-matter dominated era (\( \phi \)MDE) just after matter–radiation equivalence. In this period, the scalar field \( \phi \) behaves as stiff matter (\( \rho_\phi / \rho_\phi = 1 \)) having a non-negligible kinetic energy that dominates over the potential one. After this stage, the usual matter era follows before entering in the accelerated regime with a final De Sitter attractor. Note also that because of the \( \phi \)MDE and the non-usual scaling of the DM energy density, i.e., \( \rho_\phi \propto a^{-3/\alpha - 1} \), after equivalence the background expansion law will differ from the usual \( a \propto \eta^2 \).

2.1. Potential

We shall consider the Ratra–Peebles (RP; Ratra & Peebles 1988) and SUGRA self-interaction potentials (Brax et al. 2000),
\[ V(\phi) = \frac{\Lambda^{\alpha + 4}}{\phi^\alpha} \] RP \hspace{1cm} (16)
and
\[ V(\phi) = \frac{\Lambda^{\alpha + 4}}{\phi^\alpha} e^{4 \frac{\phi^2}{m_p^2}} \] SUGRA, \hspace{1cm} (17)
respectively; they allow tracker solutions for any \( \alpha > 0 \). For both potentials, once \( \alpha \) and \( A \) are assigned, the present time DE density parameter \( \Omega_a(\alpha) \) is uniquely defined.

Limits on these models without coupling between DE and CDM have been studied in La Vacca & Kristiansen (2009). They find that only quite small \( \lambda = \log A / \text{GeV} \) are allowed. In the SUGRA case, in particular, only \( \lambda \lesssim -3.5 \) is allowed. Such small values are well below the range motivated by particle physics. Therefore, the physical appeal of the SUGRA potential is spoiled.

Let us however outline that, when the \( \beta \) degree of freedom is opened, \( \Lambda \) values as large as 30 GeV become allowed, at the 1\sigma level, while at the 2\sigma level, no significant constraint on the energy scale \( \Lambda \) remains. Even for the RP potential, for which a limit \( \lambda \lesssim -8.5 \) is held, in the absence of coupling, values of \( \lambda \sim -2 \) become allowed (La Vacca et al. 2009a, 2009b).

In the absence of DM–DE coupling, RP yields quite a slowly varying \( w_\phi(\alpha) = p_\phi / \rho_\phi \) state parameter. On the contrary,
SUGRA yields a fast varying $u_\phi$. Although coupling causes a $u_\phi$ behavior significantly different from the uncoupled case, one could again consider these potentials as examples of rapidly or slowly varying $u_\phi$.

The results shown in the next two sections are qualitatively the same for RP and SUGRA models. We will show them only for SUGRA cosmologies while the results for RP models will be shown only when comparing the theoretical predictions with observational data and dealing with redshift tomography.

3. ISW EFFECT

The ISW effect arises when CMB photons from the last scattering surface pass through a time-dependent gravitational potential changing its energy so that additional temperature anisotropies are generated. The ISW temperature fluctuation, $\Delta T_{\text{ISW}}$, is given by

$$\Delta T_{\text{ISW}} = T \int e^{-\tau} (\Phi + \Psi) d\eta,$$

where $T$ is the CMB temperature and $e^{-\tau}$ is the visibility function of the photons.

As outlined in previous section, we will deal with scales within the horizon and redshifts such that radiation and any anisotropic stress can be neglected ($\Phi = \Psi$). In the matter era and in the absence of DM–DE coupling ($\beta = 0$), the Poisson equation reads

$$\Phi = \frac{3 H^2}{2 k^2} \Omega_m \delta_m$$

($\Omega_m$ and $\delta_m \propto a \propto \eta^2$ are the total matter density parameter and density contrast, respectively) from which one can appreciate that the gravitational potential stays constant, $\Phi = 0$, and no ISW effect arises. However, when DE starts to dominate the cosmic expansion, $\Phi$ is no longer constant and the ISW effect generates secondary anisotropies in the CMB.

On the other hand, as explained in the previous section, DM–DE coupling affects both the background and density perturbation evolution, resulting in a variation of $\Phi$ even during the matter domination.

Figure 1 shows how $\beta$ and $m_\gamma$ affect the time evolution of the sum $\Phi + \Psi$, the time derivative of which forms the source of the ISW effect. Evolution of the gravitational potentials is obtained by a modified version of the public code CMBFAST integrating the fully relativistic equations and taking into account the contributions of all of the components, i.e., photons, DM, baryons, neutrinos, and DE.

Note how $\beta$ and $m_\gamma$ affect $\Phi + \Psi$ in opposite fashion.

Performing a measurement of the ISW effect is, however, a difficult task because of its small signal compared with that of primary anisotropies (~10 times larger). Furthermore, while on small scales the small differences in temperature tend to cancel out, the large scales, from which most of the ISW effect contributions come from, are strongly affected by the cosmic variance.

The problem can be overcome by cross-correlating the ISW anisotropies with some tracers of the matter density, e.g., astrophysical objects like galaxies.

The observed galaxy density contrast in the direction $\hat{n}_1$ is

$$\delta_{\text{gal}}(\hat{n}_1) = \int b(z) \frac{dN}{dz}(z) \delta_m(\hat{n}_1, z) dz,$$

where $dN/dz$ is the normalized selection function of the galaxy survey and $b(z)$ is the galaxy bias, which will be discussed in the next section, relating the galaxy density contrast to the inhomogeneities in the mass distribution, $\delta_m = h \delta_m$. Since $\delta_m$ is related to the gravitational potential through the Poisson equation, the observed galaxy density will be correlated with the ISW anisotropies in the nearby direction $\hat{n}_2$:

$$\Delta T_{\text{ISW}}(\hat{n}_2) = 2T \int e^{-\tau(z)} \frac{d\Phi}{dz}(\hat{n}_2, z) dz.$$

The two-point angular cross-correlation function (CCF) and auto-correlation function (ACF) in the harmonic space are then defined as

$$C_{\text{ISW-gal}}(\theta) = \langle \Delta T(\hat{n}_1) \delta_{\text{gal}}(\hat{n}_2) \rangle$$

and

$$C_{\text{gal-gal}}(\theta) = \langle \delta_{\text{gal}}(\hat{n}_1) \delta_{\text{gal}}(\hat{n}_2) \rangle$$

where $\theta = |\hat{n}_1 - \hat{n}_2|$, $P_l$ are the Legendre polynomials, and the cross- and auto-correlation power spectra are given by

$$C_l^{\text{ISW-gal}} = 4\pi \int \frac{dk}{k} \Delta^2(k) I_l^{\text{ISW}}(k) I_l^{\text{gal}}(k)$$

and

$$C_l^{\text{gal-gal}} = 4\pi \int \frac{dk}{k} \Delta^2(k) [I_l^{\text{gal}}(k)]^2,$$

where $\Delta^2$ is the primordial power spectrum of scalar perturbations and the integrands $I_l^{\text{ISW}}$ and $I_l^{\text{gal}}$ are

$$I_l^{\text{ISW}}(k) = 2T \int e^{-\tau(z)} \frac{d\Phi}{dz} j_l[k \chi(z)] dz,$$

and

$$I_l^{\text{gal}}(k) = \int b(z) \frac{dN}{dz}(z) \delta_m(k, z) j_l[k \chi(z)] dz$$

(Here $j_l(x)$ are the spherical Bessel functions and $\chi$ is the comoving distance).
In the following we use our modified CMBFAST code to calculate the theoretical CCF and ACF. In order to better understand the effects of $\Lambda$, $\beta$, and $m_\nu$ on them, we start to compute the ISW–matter CCF and ACF, $C^{ISW-m}(\theta)$ and $C^{m-m}(\theta)$, and power spectra, $C^{ISW-m}_{l}$ and $C^{m-m}_{l}$, the values of which are obtained similar to Equations (22) and (23) by replacing $\delta_{gal}(\hat{n}_1)$ and $I^{gal}_{l}(k)$ with

$$\delta_{m}(\hat{n}_1) = \int \frac{dN}{dz}(z)\delta_{m}(\hat{n}_1, z)dz,$$  

$$I^{m}_{l}(k) = \int \frac{dN}{dz}(z)\delta_{m}(k, z)j_{l}[k\chi(z)]dz,$$

where the only difference from $\delta_{gal}$ and $I^{gal}_{l}$ is the bias factor.

Here, we model $dN/dz$ as a narrow Gaussian centered at two different redshifts, $z = 0.3$ and $z = 3$. This will permit us to obtain some information about the time evolution of the correlations. Realistic selection functions will be considered in the next sections. Cosmological parameters are assumed to be the same as in the WMAP 5 year best-fit $\Lambda$CDM model (Komatsu et al. 2009).

### 3.1. Dependence on $\Lambda$, $\beta$, and $m_\nu$

Let us consider first the uncoupled case ($\beta = 0$) and no massive neutrinos. Figure 2 shows the redshift evolution of the gravitational potentials (left panel) and the source of the ISW effect $\Phi + \Psi$ (right panel) for different values of $\Lambda$. The dependence on $\Lambda$ of the ISW–matter correlations $C^{ISW-m}_{l}$ and $C^{ISW-m}(\theta)$ is then displayed in Figure 3 at two different redshifts, $z = 0.3$ and $z = 3$. Models are the uncoupled SUGRA with massless neutrinos.

![Figure 2](image1.png)

**Figure 2.** Redshift evolution of $\Phi + \Psi$ (left) and its time derivative (right) for different values of $\Lambda$ in uncoupled SUGRA with massless neutrinos.

![Figure 3](image2.png)

**Figure 3.** ISW–matter cross-correlation power spectra (top) and functions (bottom). Their dependence on $\Lambda$ is shown at $z = 0.3$ (left) and $z = 3$ (right). Models are the uncoupled SUGRA with massless neutrinos.
redshifts, $z = 0.3$ and $z = 3$. When increasing $\Lambda$, both $C_1^{\text{ISW} - m}$ and $C_1^{\text{ISW} - m}(\theta)$ show opposite behaviors at low and high redshifts. This reflects the behavior of $\dot{\Phi} + \dot{\Psi}$.

In the presence of coupling one can distinguish between two different behaviors for small and large $\Lambda$. This is shown in Figures 4 and 5 for $\beta = 0.1$. In the first case, the evolution of the gravitational potentials and the cross-correlation signal are almost independent of $\Lambda$. It can be understood by noticing that for small $\Lambda$, $\phi_{\text{MDE}}$ is very long and the usual tracker solution is (almost) never reached. In this phase, coupling terms in the field equations dominate so that the tracker solution is almost independent from $\Lambda$. When increasing $\Lambda$, $\phi_{\text{MDE}}$ becomes shorter and the behavior resembles that of the uncoupled case. The transition between the two above regimes occurs around $\Lambda = 1 \text{ GeV}$.

Dependence on $\beta$ is shown in Figures 6 and 7. Again, the behavior of the cross-correlation reflects that of the ISW source. However, while coupling can have opposite effects on the cross-correlation at different redshifts, i.e., it can increase or decrease the signal, massive neutrinos always decrease the signal. It is shown in Figure 8 which displays the behavior of $C_1^{\text{ISW} - m}$ as a function of $m_\nu$ at two different redshifts. Cross-correlation signal always decreases with increasing neutrino mass.

### 4. Galaxy Bias and Magnification Bias

The galaxy bias $b$ can, in general, evolve both in redshift or as a function of the scale. However, on the large scales of interest for the ISW effect, the bias is usually assumed to be linear, spatially constant, and only redshift dependent.
Figure 6. Redshift evolution of $\Phi + \Psi$ (left) and its time derivative (right) for different values of $\beta$ in SUGRA models with massless neutrinos.

Figure 7. ISW–matter cross-correlation power spectra (top) and functions (bottom). Their dependence on $\beta$ is shown at $z = 0.3$ (left) and $z = 3$ (right). Models are the SUGRA models with massless neutrinos.

Figure 8. Effect of massive neutrinos on ISW–matter cross-correlation.

i.e., $\delta_{\text{gal}} = b(z)\delta_m$. This assumption is fully consistent with results from numerical simulations, redshift surveys, and semi-analytic calculation in the context of the so-called halo model (see Blanton et al. 1999; Percival et al. 2007).

However, given a galaxy selection function $dN/dz$ picked at a certain redshift $\bar{z}$, the bias can be approximated with an appropriate constant. In this case it will be

$$C_{\text{gal–gal}} = b^2 C_{\text{m–m}}.$$  \hspace{1cm} (28)

Within the above approximation, given a particular survey and assuming a cosmological model, the bias is usually estimated by fitting the theoretical matter–matter correlation function, $C_{\text{m–m}}$, for the assumed cosmology, to the observed galaxy–galaxy correlation function, $C_{\text{gal–gal}}$. 
Biases have been estimated for different surveys by several authors assuming the WMAP best-fit ΛCDM cosmology (see Boughn & Crittenden 2002, 2004; Giannantonio et al. 2006; Myers et al. 2006; Rassat et al. 2007; Blake et al. 2007). Since we are considering cosmological models different from a ΛCDM, we need to appropriately rescale those biases to each of our models. Note, however, how the estimation of $b$ in Equation (28) depends on the normalization of the power spectrum in $C_{m-m}$ (see Equation (23)). For a fixed normalization, taking into account Equation (28), biases will be rescaled according to

$$b^2_{\text{model}} = b^2_{\Lambda\text{CDM}} \left( \frac{C_{m-m}}{C_{m-m}_{\Lambda\text{CDM}}} \right)$$  \hspace{1cm} (29)$$

where ( ) indicates the average on the angular scales $\theta$ of interest.

However, Equation (29) should be generalized when magnification bias effect due to gravitational lensing is non-negligible. Gravitational lensing by intervening matter changes the observed galaxy number density $\hat{\rho}_{\text{gal}}$, leading a correction term $\delta_m$ being added to the intrinsic galaxy fluctuation $\delta_{\text{gal}}$, $\hat{\rho}_{\text{gal}} = \rho_{\text{gal}} + \delta_m$.

With this correction, the observed ACF becomes

$$C_{\text{gal-gal}} = C_{\text{gal-gal}} + 2C_{\text{gal-\mu}} + C_{\mu-\mu} = b^2 C_{m-m} + 2bc_{m-\mu} + c_{\mu-\mu},$$  \hspace{1cm} (30)$$

where $C_{x-y} = \langle \delta_x \delta_y \rangle$ and the rescaled bias will then be the solution of

$$b_{\text{model}}^2 (C_{m-m}_{\text{model}}) + 2b_{\text{model}} (C_{m-\mu}_{\text{model}}) + (C_{\mu-\mu}_{\text{model}}) - \langle \hat{C}_{\text{gal-gal}} \rangle_{\Lambda\text{CDM}} = 0.$$  \hspace{1cm} (31)$$

Auto- and cross-correlations corrected for magnification bias are obtained considering in Equation (25) the function (Ho et al. 2008)

$$f(z) = b(z) \frac{dN}{dz} \delta_m(k, z) + \int_{z}^{\infty} d\zeta W(z, \zeta) (\alpha(\zeta) - 1) \frac{dN}{d\zeta},$$  \hspace{1cm} (32)$$

where $\alpha(\zeta)$ is the slope of the number counts of galaxy number density as a function of the flux, $N(> F) \propto F^{-\alpha}$. It depends on the choice of galaxy sample and is redshift dependent. The lensing window function (in a flat universe) is

$$W(z, \zeta) = k^2 \Phi(k, z) \frac{\chi(\zeta) - \chi(z)}{\chi(\zeta)} \chi(z).$$

Magnification bias increases with redshift and could be important when dealing with deep survey, e.g., quasars. This is shown in Figure 9, which compares the effect of the magnification bias on the ISW-gal correlation at $z = 1.5$ and $z = 3$. A detailed analysis on how magnification bias affects ACF and CCF can be found in LoVerde et al. (2007), Hui et al. (2007), LoVerde et al. (2008), and Hui et al. (2008).

5. COMPARISON TO OBSERVATIONAL DATA

Investigations of CMB–LSS correlations were made in a recent series of works that rely on WMAP data and a variety of LSS probes (Nolta et al. 2004; Afshordi et al. 2004; Cabrè et al. 2006, 2007; Rassat et al. 2007; Raccanelli et al. 2008; Ho et al. 2008; Giannantonio et al. 2008a; Xia et al. 2009). There is an overall agreement among different groups in finding evidence for a positive ISW effect at the $\sim 3\sigma - 4\sigma$ confidence level. A substantial agreement between the observed cross-correlations and the expected signal arising from the ISW effect in the WMAP best-fit ΛCDM cosmology was also found. Different DE models were also considered in Oliveares et al. (2008), Schäfer (2008), and Giannantonio et al. (2008b).

In a recent work (Giannantonio et al. 2008a) a combined analysis of the ISW effect was performed by cross-correlating the CMB map provided by the WMAP collaboration with all the relevant large-scale data sets and modeling their covariance properties with different methods.

In this section, we compare our theoretical predictions based on the models described above with the measurements made in Giannantonio et al. (2008a) by considering six different galaxy catalogs: the optical Sloan Digital Sky Survey (SDSS), the infrared Two Micron All-Sky Survey (2MASS), the X-ray catalog from the High Energy Astrophysical Observatory (HEAO), and the radio galaxy catalog from the NRAO VLA Sky Survey (NVSS). In addition, given the high quality of the SDSS data, some further subsamples were extracted from it, consisting of luminous red galaxies (LRGs) and quasars (QSOs).

As shown in Figure 10 their redshift distributions, $dN/dz$, span a redshift range $0 < z < 2.5$. In order of increasing mean redshift of the sample we have: 2MASS, SDSS galaxies, LRG, NVSS, HEAO, and QSO (for details on the catalogs see Giannantonio et al. 2008a and Ho et al. 2008).

Constraints from CMB, SN Ia, and LSS on cosmologies with coupling and massive neutrinos have been obtained in La Vacca et al. (2009b) and Kristiansen et al. (2010) by means of the Monte Carlo Markov Chain technique. Best-fit parameters from
their analysis will be used in this and in the next section when dealing with redshift tomography. Parameters are summarized in Table 1 where \( \omega_{0b,c} \) are the physical baryon and cold dark matter density parameters, \( \omega_{b,c} = \Omega_{0b,c} h^2 \), where \( h \) is the dimensionless Hubble parameter, \( \tau \) is the optical depth to reionization, \( n_s \) is the scalar spectral index, \( \beta \) is the coupling parameter between DM and DE. In the following we will use \( \Lambda = 10^{-6} \text{GeV} \) and \( \Omega = 1 \text{GeV} \) for RP and SUGRA, respectively, which correspond to the—for the three splitting schemes described above—1\( \sigma \) limits. Best-fit parameters for coupling and neutrino mass are approximately the same for both models, i.e., \( \beta \sim 0.1 \) and \( \Omega_{\nu} = M_{\nu}/h^293.14 \text{eV} \sim 0.01 \).

For each catalog and model, biases are shown in Table 2 and calculated according to Equation (29) using for \( b_{\Lambda CDM} \) the values given in Giannantonio et al. (2008a). Given the low mean redshifts of the catalogs we neglect the magnification bias effect which amounts to a few percent only in the case of quasars. It will be, however, considered in the next section when dealing with redshift tomography and higher \( z \). We will also discuss how well to approximate \( b \) with a constant is.

For each catalog, we then determine the expected CCFs for our models.

Comparison with observational data is shown in Figure 11 for SUGRA and RP models. The predictions for \( \Lambda \text{CDM} \) is also displayed. Note that, because of known contamination from the Sunyaev–Zeldovich effect in the 2MASS data (Afshordi et al. 2004), the four smallest angle bins should be disregarded. While it is not possible to distinguish among the models at low redshifts, discrepancies between coupled models and \( \Lambda \text{CDM} \) increase with \( z \) even though the RP and SUGRA models remain indistinguishable. In spite of this, however, current data alone does not seem able to discriminate between coupled models and \( \Lambda \text{CDM} \).

### 6. REDSHIFT TOMOGRAPHY

As already outlined, unlike uncoupled DE models with massless neutrinos, both coupling and massive neutrinos cause the gravitational potentials to evolve even in the matter-dominated epoch. Therefore, a detection of a non-vanishing ISW effect signal at such high redshifts would rule out a vast class of DE models indicating a possible interaction in the dark sector. Upcoming galaxy surveys will cover a large redshift range. One goal will be to use the photometric redshifts of the galaxies to split the survey into multiple redshift bins, allowing for tomographic analysis.

### Table 1

| Parameter                | RP   | SUGRA |
|--------------------------|------|-------|
| \( 10^3 \omega_b \)     | 2.260 ± 0.061 | 2.260 ± 0.065 |
| \( \omega_c \)           | 0.1039 ± 0.0062 | 0.1042 ± 0.0084 |
| \( \tau \)               | 0.087 ± 0.016 | 0.088 ± 0.017 |
| \( M_c \) (eV) (95\% C.L.) | < 1.13 | < 1.17 |
| \( \beta \) (95\% C.L.)  | < 0.17 | < 0.18 |
| \( \log_{10}(A/\text{GeV}) \) (95\% C.L.) | < -4.2 | < 6.3 |
| \( n_s \)               | 0.969 ± 0.015 | 0.970 ± 0.018 |
| \( \ln(10^{10}A_s) \)    | 3.055 ± 0.040 | 3.057 ± 0.041 |
| \( H_0 \) (km s\(^{-1}\) Mpc\(^{-1}\)) | 71.8 ± 2.5 | 71.9 ± 2.7 |

**Note.** Only upper limits on \( M_c \), \( \beta \), and \( \Lambda \) are shown.

Following the procedure of Hu & Scranton (2004), given a galaxy distribution, \( n(z) = dN/dz \), the galaxies can be divided into photometric bins, labeled with index \( i \):

\[
  n(z) = \sum_i n_i(z).
\]

Assuming a distribution \( n(z) \) of the standard form

\[
  n(z) = \frac{\beta}{\Gamma(z_{i+1}/z_0)} \frac{z^{m-1}}{z_{i+1}^{m+1}} \exp \left[ -\frac{z}{z_0} \right] \tag{33}
\]

and that the photometric redshift errors are Gaussian distributed with an rms fluctuation \( \sigma(z) \), the resulting photometric redshift distributions are given by

\[
  n_i(z) = \frac{1}{2} n(z) \left[ \text{erfc} \left( \frac{z_{i+1} - z}{\sqrt{2} \sigma(z)} \right) - \text{erfc} \left( \frac{z_i - z}{\sqrt{2} \sigma(z)} \right) \right].
\]

We now propose to study how tomographic analysis is affected when considering different splitting schemes. This will permit us to single out the optimal splitting choices which guarantee a signal-to-noise ratio, S/N, high enough to distinguish among different DE models. As we are interested in high \( z \), we model the overall distribution \( n(z) \) according to Equation (33) with \( m = 2 \), \( \beta = 2.2 \), and \( z_0 = 1.62 \). These values provide a good fit of the quasar distribution from SDSS DR6 considered in the previous section (Xia et al. 2009). We assume the shape of such a distribution to be approximately the same as for that expected from future surveys.

Further, we assume \( \sigma(z) = 0.03(1+z) \) as expected from future experiments, and consider three different splitting schemes, each with five bins in the redshift range from \( z = 0.75 \) to \( z = 4 \).

1. **Bins equally spaced in \( z \), with \( \Delta z = z_{i} - z_{i-1} = 0.65 \).**
2. **Same number of galaxies, \( \Delta z_0 \), in each bin.**
3. **Bins sizes increasing proportionally to the photometric error, \( \Delta z \propto \sigma(z) \).**

The three splitting schemes are shown in Figure 12. The thick line is the overall quasar distribution while the other curves are the true (spectroscopic) distributions that correspond to the divisions (vertical lines) in photo-z space.

We also take into account the magnification bias effect which can be important at higher \( z \). In a recent analysis by Ho et al. (2008), the slope of the quasar counts, \( \alpha \), entering in Equation (32), was found to be redshift dependent. They found \( \alpha = 0.82 \) in the photometric redshift range \( 0.65 < z_{\text{photo}} < 1.45 \) and \( \alpha = 0.9 \) for \( 1.45 < z_{\text{photo}} < 2 \). For simplicity, we assume a constant slope \( \alpha = 0.9 \) as in Xia et al. (2009).
Figure 11. Observed CCF of six different galaxy catalogs. The curves are the theoretical predictions for the best-fit $\Lambda$CDM, SUGRA, and RP cosmologies (see the text).

Figure 12. Splitting schemes (1) (left panel), (2) (middle panel), and (3) (right panel) described in the text.
Table 3: Effects of Ignoring Galaxy Bias Evolution and Magnification Bias on the S/N for \( \Lambda \)CDM

| Quasar Bias | Magnification Bias | S/N |
|-------------|--------------------|-----|
| \( b = b(\bar{z}) \) | No | 4.56 |
| \( b = b(z) \) | No | 4.31 |
| \( b = b(\bar{z}) \) | Yes | 4.61 |
| \( b = b(z) \) | Yes | 4.38 |

Note. \( \bar{z} \) is the mean redshift of quasar distribution used (see the text).

6.1. Dependence on Galaxy Bias Evolution and Magnification Bias

Before comparing the different models we discuss the effects of ignoring magnification bias and quasar bias evolution. This is done for the \( \Lambda \)CDM cosmology. Results, however, are valid for RP and SUGRA as well.

Note that implications of galaxy bias evolution on ISW measurements and parameter estimation have been considered in pioneering works by Schäfer (2009) and Schäfer et al. (2009).

For the quasar bias evolution in \( \Lambda \)CDM, we use the empirical formula derived by Croon et al. (2004):

\[
b(z) = 0.53 + 0.289(1 + z)^2,
\]

which provides a good fit of the recent observational findings by Xia et al. (2009).

We first consider the overall quasar distribution and calculate the expected S/N of the cross-correlation in the following cases: constant quasar bias \( b = b(\bar{z}) \) (where \( \bar{z} \) is the mean redshift of the survey) and \( b = b(z) \) as given by Equation (34). For each of them, the S/N is obtained by neglecting or considering the magnification bias correction. Results are summarized in Table 3.

For Gaussian fields, the expected S/N is given by (see, e.g., Cooray 2002)

\[
(S/N)^2 = f_{\text{sky}} \sum_{l_{\text{max}}}^l (2l + 1) \left( C_l^{ISW} \right)^2 \left( C_l^{\text{gal}} \right)^2
\]

\[
\times \left( C_l^{ISW-\text{gal}} \right)^2 + \left( C_l^{ISW} + C_l^{N_{\text{ISW}}} \right) \left( C_l^{\text{gal}} + C_l^{N_{\text{gal}}} \right),
\]

where \( C_l^{N_{\text{ISW}}} = C_l^{T} + C_l^{\text{det}} \) is the noise contribution to the ISW (\( C_l^{T} \) and \( C_l^{\text{det}} \) being the total anisotropy contribution and any detector noise contribution, negligible on the scales we are interested in) while \( C_l^{N_{\text{gal}}} = 1/N \), the shot noise associated with the galaxy/quasar catalog (\( N \) is the surface density of galaxies/quasars per steradians). Here, \( f_{\text{sky}} \) is the fraction of sky common to both CMB and galaxy/quasar survey maps.

Note that Equation (35) is strictly true only in the case of Gaussian fields and full-sky coverage, \( f_{\text{sky}} = 1 \). In this section, we assume that to be the case other than a negligible \( C_l^{N_{\text{gal}}} \).

These assumptions will be relaxed in the following. For partial sky coverage, one can, in first approximation, multiply for \( f_{\text{sky}} \) the values of S/N here presented.

The value of the lowest multipole, \( l_{\text{min}} \), can be approximately set to \( l_{\text{min}} = \pi/2 f_{\text{sky}} \) in order to account for the loss of low-multipole modes. Although a significant part of the ISW signal comes from lower multipoles, we set \( l_{\text{min}} = 10 \) in order to avoid effects of gauge correction on very large scales recently discussed by Yoo et al. (2009) even if this implies a reduction of S/N. The maximum multipole, \( l_{\text{max}} \), is set to \( l_{\text{max}} = 1000 \). However, we will show later that the contribution to the S/N from \( l > 400 \) multipoles is negligible.

From Table 3, it follows that cross-correlation measurements are more affected by errors when ignoring the quasar bias evolution rather than the magnification bias correction. In fact, in this last case, an error of \( \sim 1/6 \) on the S/N is obtained while the error rises to \( \sim 7\% \) if the quasar bias is approximated by a constant value, \( b = b(\bar{z}) \) (\( \bar{z} \) being the mean redshift of the quasar distribution) and it is \( \sim 4\% \) if both quasar bias evolution and magnification bias are neglected.

We now turn to tomography. According to Equation (35), the S/Ns, \( S/N_i \), in the \( i \)-th tomographic bin are calculated for each of the cases listed in Table 4. Figure 13 shows the errors \( \Delta S/N_i \) relative to the case (e) for each bin of the three splitting schemes described above. Unlike the overall quasar distribution, in each bin distributions are very narrow and the quasar bias can be approximated by a constant, \( b_i = b(\bar{z}_i) \) (\( \bar{z}_i \) being the mean redshift of the \( i \)-th bin), leading to only a minor error \( \lesssim 2\% \) on \( S/N_i \) (case (c)). On the other hand, ignoring magnification bias correction might be critical at high \( z \) \((\Delta S/N_i)/S/N_i \sim 7.5\% \), case (b)) and if a constant bias, \( b = b(\bar{z}) \), is used for all bins, the error can reach \( \sim 10\% \) at higher redshifts.

However, since photometric redshift errors cause the bins to overlap and magnification of the galaxies at \( z_i \) probes structures at \( z < z_i \), cross-correlation measurements at high \( z \) are quite correlated with those at low redshifts. Taking into account such correlations, the net accumulated S/N for measurements from all the bins up to \( z_{\text{max}} \) is given by

\[
(S/N(\bar{z}_{\text{max}}))^2 = \sum_{z_i z_j < \bar{z}_{\text{max}}} \sum_{l_{\text{min}}}^{l_{\text{max}}} C_l^{ISW-\text{gal}} [\text{Cov}_{l}]_{ij} C_l^{ISW-\text{gal}},
\]

where

\[
[\text{Cov}_{l}]_{ij} = \frac{C_l^{ISW-\text{gal}} C_l^{ISW-\text{gal}} + (C_l^{ISW} + C_l^{N_{\text{ISW}}})(C_l^{\text{gal}} + C_l^{N_{\text{gal}}})}{f_{\text{sky}}(2l + 1)^{-1}},
\]

The relative error on \( S/N(\bar{z}_{\text{max}}) \) is shown in Figure 14 for the same cases as in Figure 13. In all cases, despite the significant differences in \( S/N_i \) in high \( z \) bins, errors are always \( < 2\% \). This is understood because most of the cumulative S/N comes from low redshifts. However, in case (e), errors considerably increase if the shot noise associated with the catalog is non-negligible. Figure 15 compares the relative errors on \( S/N(\bar{z}_{\text{max}}) \) (top panel) and \( S/N(\bar{z}_{\text{max}}) \) (bottom panel) in the case of negligible shot noise.
and assuming an SDSS DR6-like survey with a number density of quasars, \( N_q \), of \( \sim 120 \text{deg}^{-2} \). In this last case, the error on the cumulative S/N ranges from 20% to 40%, depending on the number of bins considered while the error on S/N\( _i \) can also reach 50% in bins at higher redshift.

6.2. Model Comparison

In the previous section we have shown that when dealing with narrow tomographic bins, we can approximate the galaxy bias in each bin with a constant, committing an error of no more than a few percent.

Then, after calculating the cross-correlation power spectra for the best-fit \( \Lambda \)CDM, SUGRA, and RP models we apply Equation (31) to each bin of the three splitting schemes. The rescaled biases \( b(\bar{z})_{\text{RP}} \) and \( b(\bar{z})_{\text{SUGRA}} \) are then fitted with an expression similar to Equation (34), obtaining

\[
b(z) = 0.54 + 0.291(1 + z)^2
\]

for RP and

\[
b(z) = 0.54 + 0.287(1 + z)^2
\]

for SUGRA. These expressions, which will be used in the rest of this paper, are valid up to \( z = 4 \) leading a galaxy bias evolution only slightly different from that of \( \Lambda \)CDM.

In Figure 16, we show the cross-correlation power spectra for the splitting (1). Lower frames of each panel display the ratio between the SUGRA and RP spectra and the \( \Lambda \)CDM spectrum. Mean redshifts of the true bin distributions are also indicated. A similar redshift evolution could be obtained by considering (2) or (3). As clearly visible from the figure, a better discrimination among the models is expected at higher redshifts.
Figure 16. Splitting scheme (1) (see the text) is shown in the top panel on the left. Other panels show the cross-correlation signal in the five bins considered. The lower frames of each panel display the ratio between the SUGRA and RP spectra and the $\Lambda$CDM spectrum.

Figure 17. Expected ISW–LSS cross-correlation signal-to-noise ratio, $S/N$, for the different splitting schemes considered in the text as a function of the mean redshift of the bins for the best-fit $\Lambda$CDM, RP, and SUGRA cosmologies. The expected $S/N_1$ and $S/N(z_{\text{max}})$ for the different splitting schemes are shown in Figures 17 and 18, respectively, for the cosmologies considered. Despite the same qualitative behavior for all the models, higher $S/N$s are expected in the SUGRA case, RP being between SUGRA and $\Lambda$CDM. Horizontal lines in Figure 18 indicate the $S/N$ obtained by using the full distribution.

For each splitting scheme, in Figure 19, we plot the ratio between the RP and SUGRA $S/N_1$ and $S/N(z_{\text{max}})$ values and those expected in the $\Lambda$CDM case. The first thing to note is
the overlapping, for both RP and SUGRA, of the three curves corresponding to the different splittings indicating that the three schemes considered perform equally well in discriminating among the models. A better discrimination is however achieved looking at high redshifts.

In Figure 20, we investigate how $S/N$ changes when the number of the bins increases (left panel) and whether a greater number of bins could permit us to better discriminate between models (right panel). Results are shown for splitting scheme (3) with 2, 3, 5, and 10 bins in the RP model. Similar results are obtained in the other cases. The $S/N$ in the case of no splitting is also shown. More bins, in principle, would permit us to have a more detailed description of the redshift evolution of the ISW effect. However, as clearly visible in the left panel, the $S/N$ decreases with increasing bin number. In the right panel, the RP model is compared to $\Lambda$CDM. Even though at high redshifts tomography permits us to distinguish among the models better than using the full distribution, the figure shows that increasing the number of the bins from 2 to 10 would permit only a minor improvement in discriminating between the models.

6.3. Sky Coverage and Shot Noise

Up to now, we have considered the ideal case of Gaussian fields, full sky coverage, and negligible shot noise. However, after cutting out our galaxy from the analysis, future CMB and galaxy maps are expected to cover, at best, a sky fraction $f_{\text{sky}} = 0.7–0.8$. In this case different multipoles are no longer independent and Equations (35)–(37) only provide approximate estimations and a more rigorous analysis taking into account effective survey geometry is needed (Cabre' et al. 2007; Hivon et al. 2002; Xia et al. 2011). It has been however shown that, under the above approximations, a better estimation can be obtained by binning the power spectra data in bins of appropriate size $\Delta l$ making the bins independent. In this case, Equations (35) and (36) are increased by a multiplicative factor of $\Delta l$. Cabre' et al. (2007) found that $\Delta l = 20, 16, 8, 1$ works well for $f_{\text{sky}} = 0.1, 0.2, 0.4, 0.8$.

In Figures 21 and 22 we show some results for the cumulative $S/N$ from all the bins of splitting scheme (3). Results are the same for the other schemes.

The left panel of Figure 21 shows the contribution to the cumulative $S/N$ from each multipole while the cumulative $S/N$ up to $l = l_{\text{max}}$ is displayed in the right panel. The dependence on $f_{\text{sky}}$ and $\Delta l$ has been removed. As clearly visible, most of the cross-correlation signal comes from lower multipoles and contributions from $l > 400$ are negligible.

In Figure 22, cumulative $S/N$ contour levels are plotted in the plane $f_{\text{sky}} - N/N_{\text{SDSS}}$, where $N_{\text{SDSS}}$ is the quasar number density for an SDSS DR6-like survey. Given that such a survey cover $\sim 20\%$ of the sky, future experiments covering a sky fraction $f_{\text{sky}} = 0.8$ will increase the cumulative $S/N$ by a factor of $\sim 3$ if the shot noise is reduced by 1/10 and by a factor of $\sim 3.5–4$ in the case $N = 100N_{\text{SDSS}}$. No significant improvement is obtained by further reducing the shot noise. For $f_{\text{sky}} = 0.8$ and negligible shot noise, increasing $S/N$ in the $i$th tomographic bin can range from a factor of 4 (low $z$ bins) up to 10 (high $z$ bins). This is shown in Figure 23.
7. CONCLUSIONS

In this work we have investigated ISW–LSS cross-correlation in coupled DE models with massive neutrinos. The presence of a coupling between DM and DE as well as massive neutrinos change both the background and matter perturbation evolutions yielding, unlike in the \( \Lambda \) CDM case, a time variation of the gravitational potentials even during the matter domination. A significant ISW signal is thus expected also at high redshifts.

First, we have investigated the dependence on the energetic scale, \( \Lambda \), of the DE potential, the coupling strength \( \beta \), and the neutrino mass \( m_\nu \). We considered first the uncoupled case (\( \beta = 0 \)) and massless neutrinos. We found that, when increasing \( \Lambda \), both \( C_{l}^{ISW} \) and \( C_{l}^{ISW-m}(\theta) \) show opposite behaviors at low and high redshifts. This in fact reflects the behavior of \( \Phi + \Psi \).

In the presence of coupling one can distinguish between two different behaviors for small and large \( \Lambda \). In the first case, the
evolution of the gravitational potentials and the cross-correlation signal are almost independent from $\Lambda$. It can be understood by noting that for small $\Lambda$, coupling terms in the DE field equations dominate so that its solution is almost independent of $\Lambda$. When increasing $\Lambda$, the behavior resembles that of the uncoupled case. Dependence on $\beta$ was also investigated and, again, the behavior of the cross-correlation at different redshifts reflects that of the ISW source. However, while coupling can affect $C_{\text{SW} \rightarrow \text{m}}$ (and $C_{\text{SW} \rightarrow \text{m}}(\theta)$) in opposite fashion at high and low redshifts, massive neutrinos always decrease the cross-correlation signal.

Second, we have provided a simple expression, Equation (29), which permits us to appropriately rescale the galaxy bias when comparing different cosmologies once the bias of a particular model, e.g., $\Lambda$CDM, is known and the normalization of the power spectrum in each model is fixed. We also give a generalized version of Equation (29) in the case when the magnification bias effect due to gravitational lensing is non-negligible (see Equation (31)).

Then, we compare the theoretical prediction on the cross-correlation function for our models with the observational data obtained for six different galaxy catalogs by Giannantonio et al. (2008a). We found that, while it is not possible to distinguish among the models at low redshifts, discrepancies between coupled models and $\Lambda$CDM increase with $z$ even though the RP and SUGRA models remain indistinguishable. In spite of this, however, current data alone does not seem able to discriminate between coupled models and $\Lambda$CDM.

Finally, we studied the redshift tomography. Upcoming galaxy surveys will cover a large redshift range, also providing photometric redshifts of the galaxies with high accuracy. This will permit us to split a survey into multiple photometric redshift bins, allowing for tomographic analysis. Here, we are interested in studying how a tomographic analysis of the ISW–LSS cross-correlation is affected when considering different splitting schemes and assuming photometric redshift errors as expected from future experiments. As we are interested in high redshifts, where our models, unlike the $\Lambda$CDM case, are expected to provide a significant ISW effect signal, the ISW effect was cross-correlated with quasars. The quasar distribution was thus split into tomographic bins according to three different schemes: (1) bins equally spaced in $z$; (2) the same number of galaxies in each bin; and (3) bin sizes increasing proportionally to the photometric error. Cross-correlation was then calculated in each bin. Our tomographic study was based on an $S/N$ analysis.

We started our discussion by investigating the effect on $S/N$ where the quasar bias evolution and magnification bias correction for an ideal survey are ignored. We found that, if the overall quasar distribution is used, cross-correlation measurements are more affected by errors when ignoring the quasar bias evolution ($\Delta S/N/S/N \sim 7\%$) rather than when ignoring the magnification bias correction ($\Delta S/N/S/N \sim 1.6\%$).

However, when dealing with tomography the error on $S/N_i$ (i indicating the $i$th bin) never overcome $\sim 2.5\%$ if the quasar bias in each bin is approximated with an appropriate constant, but it can reach $\sim 7.5\%$, at high redshifts, when magnification bias is ignored. Errors on the cumulative $S/N$, however, always stay below the $2\%$. On the other hand, errors can increase up to $50\%$ if the shot noise associated with the quasar survey is set to the current values.

We then used tomographic analysis in order to compare different cosmologies. We found that the above splitting schemes perform equally well in discriminating among the models. A better discrimination is however achieved looking at high redshifts.

We also investigated how the expected $S/N$ of the cross-correlation changes when increasing the number of bins and whether a greater number of bins could permit us to better discriminate between models. Even though more bins would allow us to have more information on the redshift evolution of the ISW effect, the $S/N$ decreases with increasing bin numbers. As a consequence, although tomography, at high redshifts, would permit us to distinguish among the models better than using the full distribution, when comparing our models to $\Lambda$CDM it was shown that increasing the number of the bins from 2 to 10 would permit only a minor improvement in the discrimination.

Finally, we showed that future wide-field surveys ($f_{\text{sky}} \sim 0.8$) can increase the cumulative $S/N$ of the cross-correlation by a factor of $\sim 3$ ($3.5-4$) if the current shot noise is reduced by 1/10 (1/100) while the $S/N$ of the single bins can increase by up to a factor of 10 at high redshift.

Our $S/N$ analysis suggests a discrimination power of future ISW–LSS cross-correlation measurements able to distinguish among different cosmologies. However, in order to assess the discrimination, more rigorous analysis in terms of the Fisher Matrix and Monte Carlo Markov Chain are needed. They are currently under investigation and left for future works.

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