Quantum Coherence Effects in Four-level Diamond Atomic System

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A symmetric four-level closed-loop \(\diamond\) type (the diamond structure) atomic system driven by four coherent optical fields is investigated. The system shows rich quantum interference and coherence features. When symmetry of the system is broken, interesting phenomena such as single and double dark resonances appear. As a result, the double electromagnetically induced transparency effect is generated, which will facilitate the implementation of quantum phase gate operation.

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I. INTRODUCTION

Driven by coherent optical fields, atomic system demonstrates abundant interference and coherent effects. Interesting effects such as the coherent population trapping state (CPT)\(^1\), electromagnetically induced transparency (EIT)\(^2,3\), and laser without inversion (LWI)\(^4,5\), have been theoretically and experimentally studied by many research groups in the world\(^6,7,8,9,10,11\). By investigating these coherent effects, the nature of quantum interference and coherence is comprehended further. For example, in the three-level \(\Lambda\) atomic system, the transition from “dark” state to the excited state is canceled via the quantum interference of two-photon resonance transition, makes the optical medium “transparent”, thus the EIT effect is generated\(^1,4\). This EIT effect has many attractive applications in quantum optics, such as the multi-wave mixing\(^12,13,14\), enhancement of nonlinear susceptibilities\(^13,16\). More interestingly, the EIT effect has been found applications in quantum information science, such as the photon information storage and release in an atomic ensemble\(^17\), correlated photon pairs generation\(^8\) and even the entanglement of remote atomic ensembles\(^18\), which form the building blocks of the quantum communication and the quantum computation.

An ideal level structure atomic system with appreciable interference and coherence features will bring great help to implement quantum states operation, and facilitate the information transferring between different systems. The most popular three-level systems in quantum optics are the \(\Lambda\), \(V\) and the \(\Xi\) systems\(^1,7,8\), these atomic systems have the CPT, EIT and LWI effects under different actions of driving fields. More complicated levels structure of atomic system will generate more interesting effects. For example, doubly electromagnetically induced transparency (double EIT) in the double \(\Lambda\) four-level system\(^12,13\). Other multi-level atomic systems, such as the tripod type four-level system\(^20\), the inverted-Y system\(^21\) and the N-type system\(^22,23\), have been explored and found interesting application in the demonstration of quantum logic gate. Looking for an **ideal** atomic system which is rich in quantum interference and coherence effects and easy to demonstrate experimentally is still under way.

Here we are investigating the quantum interference and coherence characteristics of a highly symmetric four-level atomic system, the \(\diamond\) (or the diamond structure) system\(^24,25,26\) driven by four coherent optical fields. This kind of atomic structure has been proposed to study the interaction of laser phase and the steady state\(^25\), and to generate ultra-violet laser gain through the LWI effect\(^4,24\). While our investigation on the system is to seek the application to the implementation of quantum phase gate, an attractive operation based on cross-phase modulation effect or equally the double EIT effect (or double-dark resonance)\(^21\). Inspired by the job of M.D. Lukin et al.\(^6,24\), we generalize their atomic model to a symmetric and closed structure, and discuss several more possible ways of obtaining the multi-dark resonance effect. Under different ways of configuration of the coherent driving fields, the double-dark states are generated, thus the double EIT effect is attained. Due to the symmetric structure of the atomic system, all the coherent driving fields can be assigned to be the probe, trig and couple fields alternately, and the EIT effect on the probe and trig fields simultaneously occur. The transparency of the probe and the trig fields are both controlled by the couple field, they will vary from transparent to opaque

\[\text{FIG. 1: Scheme of a four-level diamond atomic system.}\]
with the strength of the couple field changing. This flexi-
ability of arrangement the coherent driving fields will fa-
cilitate the implementation of quantum phase gate.

This paper is organized as follow: firstly in section II we
use the dressed state method to study the quantum
interference effects of the system driven by four optical
fields; then in section III we solve the master equations
and investigate varying laws of the susceptibility of the
probe and the trig fields, brings up the appreciated dou-
ble EIT effects. Finally in section IV we discuss the po-
tential application to quantum phase gate of the atomic
system and summarize our results.

II. THE MODEL: TRANSITION ANALYSIS OF
FOUR-LEVEL DIAMOND SYSTEM

The four-level system we are focusing on is shown in
Fig.1 where the four atomic levels form a so-called di-
amond (⊗) or a closed-loop configuration [23, 26]. The
upper state is |c⟩ state, the bottom state is |b⟩ state, and
the two intermediate states are |a⟩ and |d⟩. As shown in
the figure, the four-level system is driven by four
cohort optical fields E1, E2, E3, and E4. These four coherent fields
are coupling the four dipole-allowed transitions of the
atomic system, whose Rabi frequencies are Ωc1, Ωc2, Ωa1,
and Ωa2, respectively. The decay processes are denoted
by decay rates γi (i = 1, 2, 3, 4) in different decay chan-
nels. Within the rotating wave approximation (RWA)
and in the interaction picture, the process of coherently
driving the diamond atomic system can be described by
the following Hamiltonian matrix [1, 2]:

\[
H = -\hbar \begin{pmatrix}
\Delta_{a1} & \Omega_{a1} & \Omega^*_{c1} & 0 \\
\Omega_{a1} & 0 & 0 & \Omega^*_{a2} \\
\Omega^*_{c1} & 0 & \Delta_{a1} + \Delta_{c1} & \Omega^*_{c2} \\
0 & \Omega_{a2} & \Omega^*_{c2} & 0
\end{pmatrix},
\]

(1)

where the sequence of state vectors is \((|a⟩, |b⟩, |c⟩, |d⟩)^T\).
For simplicity, the Rabi frequencies Ωc1 and Ωa1 (i = 1, 2)
are chosen to be real in the following, i.e., Ωc1 = Ω^* c1
and Ωa1 = Ω^* a1. The detunings Δa1 and Δc1 (i = 1, 2)
are defined as Δc1 = ωc1 - ωc1, Δa2 = ωa2 - ωa2, Δa1 = ωa1 - ωa1,
Δb2 = ωb2 - ωb2, and ωi = ωi - ωj, where ωi, (i = a, b, c, d)
denote the frequency of atomic level, ωc1 and ωa1 (i = 1, 2)
are the frequencies of the corresponding coherent fields.
With the Hamiltonian matrix Eq. (1), we can analyze the quantum interference and quantum coherence behaviors of the system under the dressed state picture. Firstly, we choose the driving field \(E_2\) which couples the |c⟩ ↔ |d⟩ transition to be the probe
field, and the left three coherent fields to be coupling and
driving fields. For simplicity, we only consider the situa-
ton of zero detunings \(Δ_k = 0 (k = a1, a2, c1, c2)\), that
is, all the coupling and driving fields are tuned on reso-
ance with the corresponding transitions, respectively.
The four dressed states are:

\[
|D1⟩ = Ω_{a1}| |a⟩ + \sqrt{\frac{y - \sqrt{z}}{z + \sqrt{z}}} \sqrt{\frac{y - \sqrt{z}}{z + \sqrt{z}}} |b⟩ - \frac{\Omega_{c1}}{\sqrt{z - w}} |c⟩ + \frac{\Omega_{a2}}{\sqrt{z + \sqrt{z}}} |d⟩ \tag{2}
\]

\[
|D2⟩ = Ω_{a1}| |a⟩ - \frac{\sqrt{z - \sqrt{y}}}{\sqrt{z + \sqrt{y}}} \sqrt{\frac{y - \sqrt{z}}{z + \sqrt{z}}} |b⟩ + \frac{\Omega_{c1}}{\sqrt{z - w}} |c⟩ + \frac{\Omega_{a2}}{\sqrt{z + \sqrt{z}}} |d⟩ \tag{3}
\]

\[
|D3⟩ = \frac{\Omega_{a1}}{\sqrt{2(w + y) + 2(wc + 2wy - xy)y'z}} |a⟩ \frac{w + \sqrt{z}}{2w + \sqrt{z}} |b⟩ + \frac{\Omega_{a1}\Omega_{a1}}{\sqrt{w'v'z + 2}} + \frac{\Omega_{a2}(w + \sqrt{z})}{2(w + y) + 2(wc + 2wy - xy)y'z} |d⟩ \tag{4}
\]

\[
|D4⟩ = \frac{\Omega_{a1}}{\sqrt{2(w + y) + 2(wc + 2wy - xy)y'z}} |a⟩ - \frac{w + \sqrt{z}}{2w + \sqrt{z}} |b⟩ + \frac{\Omega_{a1}\Omega_{a1}}{\sqrt{w'v'z + 2}} + \frac{\Omega_{a2}(w + \sqrt{z})}{2(w + y) + 2(wc + 2wy - xy)y'z} |d⟩, \tag{5}
\]

where \(w = Ω_{a2}^2 - Ω_{c1}^2 + Ω_{c2}^2, x = Ω_{c1}^2 - Ω_{c2}^2 - Ω_{a1}^2, \)
\(y = Ω_{c2}^2 + Ω_{c1}^2 + Ω_{a1}^2, \) and \(z = y^2 - 4Ω_{c1}^2Ω_{c2}^2. \) The corre-
sponding eigen-energy are: \(ε_1 = -\sqrt{\frac{x + \sqrt{y}}{2}}, ε_2 = \sqrt{\frac{x + \sqrt{y}}{2}}, ε_3 = -\sqrt{\frac{x + \sqrt{y}}{2}}, \) \(ε_4 = \sqrt{\frac{x + \sqrt{y}}{2}}, \) respectively. By analyzing
these dressed states, we are willing to see that the dia-
mond atomic system contains plenty of quantum inter-
ference and quantum coherence effects. Intuitively, there
are one photon excitation process, two-photon resonance
excitation, and even three-photon resonance excitation
process in this atomic system driven by four coherent
fields, that is, many possible quantum transition paths
exist in the closed-loop system. These different quan-
tum transition paths will interfere with each other, and
result in the interesting effects such as electromagneti-
cally induced transparency, the laser without inversion
and the enhancement of nonlinear refractive index and
etc. As is known to all, the dark state is the heart of
the above quantum interference and coherence effects.
While the dressed states Eq. (2-5) will help us to find
the desired dark states. Obviously all four dressed states
|D1⟩-|D4⟩ are containing all the four bare states elements
|a⟩, |b⟩, |c⟩, |d⟩, indicating the absence of the so-called
“dark”state. In order to acquire the dark state which
is decoupled from the high level state |c⟩ of the prob-
ing transition, we can adjust the three coherent driving
fields. Considering that the atomic structure is a closed-
loop type, a highly symmetric structure, the probe field
can connect to all the four bare states elements through all possible exciting paths, the connection between the high level state $|c\rangle$ and other dressed states will remain. If the symmetry is broken, some dressed states may disconnect from the probe transition, then the dark states will appear. In the following we will break the symmetry of the system by removing one driving field, and inspect the resulted dark states and their interesting results.

There are three coherent optical fields acting on the diamond system except the probe field, we can remove any one of them to reach the aim of symmetry breaking, that is, the closed-loop system will become an open-loop one, and three branches of the $\Diamond$ are left. For convenience, we denote case 1 of removing the driving field $E_3$, case 2 of removing the driving field $E_4$, and case 3 of removing the driving field $E_1$. Firstly, we take case 1 into account, where the $E_3$ field is removed (equally take $\Omega_{a1} \rightarrow 0$), then the dressed states Eq. (2-5) turn into the following form:

$$|D_1\rangle = \frac{1}{\sqrt{\Omega^2_{b2} + \Omega^2_{c2}}}(\Omega_{c1}|b\rangle + \Omega_{c1}|d\rangle),$$

$$|D_2\rangle = \frac{1}{\sqrt{\Omega^2_{b2} + \Omega^2_{c1}}}(-\Omega_{c1}|b\rangle + \Omega_{c1}|d\rangle),$$

$$|D_3\rangle = \frac{1}{\sqrt{2}}(|b\rangle + |d\rangle),$$

$$|D_4\rangle = \frac{1}{\sqrt{2}}(-|b\rangle + |d\rangle),$$

where the strength of the left driving fields $E_1, E_4$ satisfy the condition: $\Omega_{c1} \neq \Omega_{a2}$; while for situation $\Omega_{c1} = \Omega_{a2}$, the dressed states Eq. (2-5) become degenerated:

$$|D_{1,3}\rangle = \frac{1}{\sqrt{2}}(|b\rangle + |d\rangle),$$

$$|D_{2,4}\rangle = \frac{1}{\sqrt{2}}(-|b\rangle + |d\rangle),$$

The above results Eq. (6) and Eq. (7) show an interesting feature: the four dressed states $|D_{1,2,3,4}\rangle$ do not have the upper level state element $|c\rangle$ state of the probing transition. Thus the two states $|D_{2,4}\rangle$ form the desired dark states. Now the probing field exciting the $|c\rangle \Leftrightarrow |d\rangle$ transition is coupling with the two dark states $|D_{2,4}\rangle$ and form the double dark resonances, quantum interference will occur for these two transition paths. More interestingly, when the strength of the two driving fields are equal: $\Omega_{c1} = \Omega_{a2}$, the four dress states $|D_{1,2,3,4}\rangle$ will degenerate into two states as shown in Eq. (7), thus the two dark states of Eq. (6) merge into one dark state, and the interaction of dark states will disappear. This particular feature will bring up interesting phenomenon, which will be discussed in the following. While all the four dressed states contain the bare state $|b\rangle$ component, thus the probe field excites a two-photon excitation recitation: $|c\rangle \Leftrightarrow |b\rangle$. The transition picture is shown in Fig. 2.

For finding other dark states resonance, we now discuss the second way of symmetry broken of the diamond structure atomic system, the case 2. Instead of removing the driving field $E_3$, we now withdraw the field $E_4$, equally take $\Omega_{a2} \rightarrow 0$. As a result, the dressed states Eq. (2-5) become:

$$|D_1\rangle = \frac{1}{\sqrt{2(\Omega^2_{b1} + \Omega^2_{c1})}}(\Omega_{c1}|b\rangle - \Omega_{a1}|c\rangle) + \frac{1}{\sqrt{2}}|d\rangle,$$

$$|D_2\rangle = \frac{1}{\sqrt{2(\Omega^2_{b1} + \Omega^2_{c1})}}(-\Omega_{c1}|b\rangle + \Omega_{a1}|c\rangle) + \frac{1}{\sqrt{2}}|d\rangle,$$

$$|D_3\rangle = \frac{1}{\sqrt{2}}|a\rangle + \frac{1}{\sqrt{2(\Omega^2_{b1} + \Omega^2_{c1})}}(\Omega_{a1}|b\rangle + \Omega_{c1}|c\rangle),$$

$$|D_4\rangle = \frac{1}{\sqrt{2}}|a\rangle - \frac{1}{\sqrt{2(\Omega^2_{b1} + \Omega^2_{c1})}}(\Omega_{a1}|b\rangle + \Omega_{c1}|c\rangle).$$

From the form of the above dressed states $|D_i\rangle (i = 1, 2, 3, 4)$, it's clear that they are all coupled with the state $|c\rangle$, thus the dark state does not show up at the moment. If we take the two coherent driving field $E_1$ and $E_3$ equally interacting on the atomic system, that is, $\Omega_{c1} = \Omega_{a1}$, then the dressed state Eq. (8) become a much simple form: $|D_1\rangle = \frac{1}{\sqrt{2}}(|b\rangle - |c\rangle) + \frac{1}{\sqrt{2}}|d\rangle$, $|D_2\rangle = \frac{1}{\sqrt{2}}(|b\rangle + |c\rangle) + \frac{1}{\sqrt{2}}|d\rangle$, $|D_3\rangle = \frac{1}{\sqrt{2}}|a\rangle + \frac{1}{\sqrt{2}}(|b\rangle + |c\rangle)$, and $|D_4\rangle = \frac{1}{\sqrt{2}}|a\rangle - \frac{1}{\sqrt{2}}(|b\rangle + |c\rangle)$. The four dressed states are containing the transition $|c\rangle \Leftrightarrow |b\rangle$, corresponding the two-photon resonances transition, and states $|D_{1,2}\rangle$ include transition $|d\rangle \Leftrightarrow |b\rangle$, corresponding a three-photon resonance transition. These transitions interact with each other, results in abundant quantum interference effects, the dressed states transition picture is shown in Fig. 3.
Fig. 4. This kind of situation is quite the same as that discussed in Ref. [24]. If we further discuss the situation in the limit of vanishing perturbation of the coherent driving fields $\Omega_1 \to 0$ (or $\Omega_3 \to 0$), the dressed states Eq. (8) will turn out to be the single dark resonance (or double dark resonances). Thus the quantum interference effects induced by dark resonance (or interacting dark resonances) will appear.

Likewise, in the symmetry broken case 3, the coherent field $E_1$ is removed, then the diamond atomic system is driven by the field $E_{3,4}$ and the probe field $E_2$. As a result, the dressed states Eq. (9) now become:

$$\langle D_1 \rangle = \frac{1}{\sqrt{2(\Omega_2^2 + \Omega_3^2)}}(-\Omega_2|a\rangle + \Omega_3|d\rangle) - \frac{1}{\sqrt{2}}|c\rangle,$$

$$\langle D_2 \rangle = \frac{1}{\sqrt{2(\Omega_2^2 + \Omega_3^2)}}(-\Omega_2|a\rangle + \Omega_3|d\rangle) + \frac{1}{\sqrt{2}}|c\rangle,$$

$$\langle D_3 \rangle = \frac{1}{\sqrt{2(\Omega_2^2 + \Omega_3^2)}}(-\Omega_2|a\rangle + \Omega_3|d\rangle) + \frac{1}{\sqrt{2}}|b\rangle,$$

$$\langle D_4 \rangle = \frac{1}{\sqrt{2(\Omega_2^2 + \Omega_3^2)}}(-\Omega_2|a\rangle + \Omega_3|d\rangle) - \frac{1}{\sqrt{2}}|b\rangle.$$  

Obviously, the dressed states $|D_{3,4}\rangle$ are decoupled from the excited state $|c\rangle$, and form the desired dark states. As is discussed in the symmetry broken case 2, here the two-photon resonance transition $|a\rangle \leftrightarrow |d\rangle$ and the three-photon resonance transition $|c\rangle \leftrightarrow |a\rangle$ are involved in the dressed states. The dressed states transition picture is drawn as Fig. 4.

The above dressed states analysis has shown the variety quantum interference and coherent effects in the system. However, it is just a kind of qualitative study of the system, for we don’t take the detuning of each coherent fields into account, neither the decay rates, and these two factors are also important that they will modify the coherent characters of the system. In order to learn more subtle behaviors of the atomic system, further investigations on the variation law of level-population terms, probe susceptibility terms and multi-photon resonance excitation terms are needed. In the next section, we will study the master equations and the corresponding solutions to the system, and reveal the interesting quantum interference and coherence effects.

III. MASTER EQUATIONS AND SOLUTIONS OF THE $\diamond$ SYSTEM

We denote the density matrix for the atomic system by $\rho$, the master equation for the density matrix $\rho$ is:

$$\frac{\partial}{\partial t} \rho = -\frac{i}{\hbar}[H, \rho] + \mathcal{L}\rho,$$

where the first term on the right side is the coherent driving part, the corresponding Hamiltonian $H$ takes the form of Eq. (11). The second term is the relaxation part, i.e. the spontaneous decay process. In the present four-level $\diamond$ atomic system, there are four decay channels, from upper levels decay to lower levels, as shown in Fig. 1.

The corresponding relaxation operator describing the decay process is:

$$\mathcal{L}\rho = \mathcal{L}_{ca}\rho + \mathcal{L}_{cd}\rho + \mathcal{L}_{ab}\rho + \mathcal{L}_{db}\rho = \frac{\gamma_1}{2}(|a\rangle\langle c|c\rangle\rho|a\rangle - |c\rangle\langle c|\rho|c\rangle - |a\rangle\langle a|\rho|a\rangle) + \frac{\gamma_2}{2}(|d\rangle\langle d|d\rangle\rho|d\rangle - |d\rangle\langle d|\rho|d\rangle) + \frac{\gamma_3}{2}(|b\rangle\langle b|b\rangle\rho|b\rangle - |b\rangle\langle b|\rho|b\rangle).$$

With the Hamiltonian Eq. (11) and Eq. (11), we have the motional equations for the atomic density matrix elements:

$$\dot{\rho}_{aa} = -\gamma_3\rho_{aa} + \gamma_1\rho_{cc} + i\Omega_1(\rho_{aa} - \rho_{bb}) + i\Omega_3(\rho_{aa} - \rho_{cc}),$$

$$\dot{\rho}_{ab} = \gamma_1\rho_{aa} + \gamma_3\rho_{bb} + i\Omega_1(\rho_{bb} - \rho_{aa}) + i\Omega_3(\rho_{bb} - \rho_{cc}),$$

$$\dot{\rho}_{cc} = -\gamma_1\rho_{cc} - \gamma_3\rho_{bb} + i\Omega_1(\rho_{cc} - \rho_{aa}) + i\Omega_3(\rho_{cc} - \rho_{bb}),$$

$$\dot{\rho}_{bb} = \gamma_3\rho_{aa} - \gamma_1\rho_{cc} + i\Omega_1(\rho_{bb} - \rho_{cc}) + i\Omega_3(\rho_{bb} - \rho_{aa}),$$

$$\dot{\rho}_{ac} = -i[\Omega_1\gamma_1 + \frac{1}{2}(\gamma_1 + \gamma_2 + \gamma_3)]\rho_{ac} + i\Omega_1(\rho_{cc} - \rho_{aa}),$$

$$\dot{\rho}_{ad} = i[\Omega_3\gamma_1 - \frac{1}{2}(\gamma_1 + \gamma_2 + \gamma_3)]\rho_{ad} + i\Omega_1(\rho_{cc} - \rho_{aa}),$$

$$\dot{\rho}_{bc} = -i[\Omega_1\gamma_1 + \frac{1}{2}(\gamma_1 + \gamma_2 + \gamma_3)]\rho_{bc} - i\Omega_1\rho_{ac} + i\Omega_1\rho_{ab},$$

$$\dot{\rho}_{bd} = i[\Omega_3\gamma_1 - \frac{1}{2}(\gamma_1 + \gamma_2 + \gamma_3)]\rho_{bd} - i\Omega_1\rho_{ad} + i\Omega_1\rho_{ab},$$

$$\dot{\rho}_{cd} = -i[\Omega_1\gamma_1 + \frac{1}{2}(\gamma_1 + \gamma_2 + \gamma_3)]\rho_{cd} + i\Omega_1\rho_{ad} + i\Omega_1\rho_{ab},$$

where Eq. (13) expresses the conservation of probability for the closed diamond system.
According to the dressed states analysis, we also discuss three kinds of symmetry broken situations here, that is, the closed-loop structure of the system will become an open one, by removing one of the three driving optical fields. Firstly we discuss case 1, where the field $E_3$ is removed, that is, $\Omega_{a1} = 0$. For convenience, the left three coherent fields are denoted as the probe field $E_2$, the trig(signal) field $E_1$ and the couple field $E_4$, respectively. The solutions to the master equations Eq. (12) are shown in Fig. 5.

From the level population Fig.5 it’s clear that the four bare level-states are divided into two groups, group 1 comprises state $|a\rangle$ and state $|c\rangle$, group 2 comprises state $|b\rangle$ and state $|d\rangle$. State population in the same group are nearly equal, which matches well with the dressed states analysis result Eq. (6). Varying with the probe field detuning $\Delta_2$, all four state-population come to peaks or troughs, revealing the situation of population transferring between the four bare states, as well as the absorption of the coherent driving field and the probe fields.

In order to learn more details of optical fields interacting with the atomic system, we investigate the probe field’s absorption and dispersion features vary with the detuning $\Delta$, the result is shown in the Fig. 6. Comparing the Fig.5 and the Fig.6 it’s easy to find that the level population and the absorption of the probe field come to the same peaks and troughs at the same values of independent variable detuning $\Delta$, which demonstrates that the variation of the absorption of the probe field results in the perturbation of level-population.

And the dispersion and absorption features of the probe field deserve special attention. From Fig.6 it’s clear that the normal and anomalous dispersion alternatively appear varying with the detuning $\Delta$, considering the behavior of the absorption part together, the probe field goes through a three-windows electromagnetically induced transparency (EIT) process. For investigating the EIT effect further, we decrease the strength of the couple field and study the resulted variation of the probe field. When the strength of the couple field decreases, the depth and width of the middle transparency window become smaller than those in Fig. 5. When the strength of the couple field equals to that of the trig field, that is, $\Omega_{a2} = \Omega_{c1} = 10$, the probe field lose the transparency character and turns out to be a absorption peak, and the anomalous dispersion curve turns to a normal one, as shown in Fig. 7a. However, if we continue to decrease the strength of the couple field, the middle transparency window appears once again, and the two sigmoidal transparency windows become less transparency, as is shown in Fig. 7b, where the couple field is much weaker than the trig field: $\Omega_{a2} = 3$. From the dressed states analysis Eq. (6), we know that the two dark states $|D_2\rangle$ and $|D_4\rangle$ will become degenerate for condition $\Omega_{c1} = \Omega_{a2}$, thus the on resonance EIT feature of the probe field will vanish.

Another interesting feature of the dark resonances of
the system comes from the absorption and dispersion behaviors of the trig field $E_1$, which is represented by the imaginary and real part of the $\rho_{ca}$ transition term, as is shown in Fig.8(A). It’s obvious that the trig field becomes transparency on resonance, thus the trig field also has the EIT effect. While the couple field $E_4$ goes through a steady absorption process, as is shown in Fig.8(B). If the strength of couple field is equal to that of trig field $E_1$, the EIT effect for the trig field will also disappear. Combining the above double-EIT behaviors of the probe field $E_2$ and the trig field $E_1$, this system will be useful on realizing the quantum phase gate $^{20,21}$, which is very important in quantum information science.

The quantum interference will be revealed more clearly if we investigate the two-photon resonance excitation term $\rho_{cb}$, which is plotted in Fig.9. Obviously, the absorption behaviors of the two-photon resonance transition are identical to those of the probe transition, except that in the negative detuning region, the absorption character of the two-photon excitation become negative ones, which means gain characteristic. And if we adjust the strength of couple field $E_4$ equal to that of trig field $E_1$, the two absorption peaks on both sides of zero detuning will merge together and cancel each other, indicates that the two-photon resonance excitation disappear on resonance, which also confirm the above dressed states analysis.

While for the second and the third kind of symmetry broken situation, where dark states resonance will also appear, the same analysis method can be applied, that is, by investigating the level-population, the absorption and dispersion of the probing transition, and the two-photon or three-photon transition terms, the quantum interference and quantum coherence features of the four-level atomic system interacting with multi-optical fields will be revealed clearly. Here the absorption and dispersion behaviors of the probe and trig fields transition terms are shown in Fig.10 and Fig.11 respectively. Let’s discuss the second symmetry broken case a little further, where the diamond atomic system is driven by three coherent optical fields $E_1$, $E_2$ and $E_3$. For convenience, we denote the field $E_1$ as the couple field, $E_2$ the probe field and $E_3$ the trig field. In the Fig.10 the absorption and dispersion characters of the three fields are drawn for two kinds of different coupling strength. On the left row, the coupling strength ($\Omega_{c1} = 5$) is much stronger than that of the probe one ($\Omega_{c2} = 1/10$), and interestingly the EIT effects for the probe field and the trig field appear at the same time(figure a, b); but if the strength of the couple field decreases (on the right row of Fig.11), the depth of the transparency windows for the probe and the trig fields will become smaller, and the transparency characters of both fields will vanish when the strength of the couple field takes $\Omega_{c1} = 1$ (figure a’ and b’), thus the

FIG. 8: Trig(figure A) and couple(figure B) transition terms $\rho_{ca}$ and $\rho_{cb}$ vary with field detuning $\Delta$. The parameters are the same as those in Fig.5. The solid line is the real part of $\rho_{cb}$; the dash is the imaginary part of $\rho_{cb}$.

FIG. 9: Two-photon resonance excitation term $\rho_{cb}$ varies with probe field detuning $\Delta$. The parameters are the same as those in Fig.5. The solid line is the real part of $\rho_{cb}$; the dash is the imaginary part of $\rho_{cb}$.

FIG. 10: For the 2nd symmetry broken case, the probe term $\rho_{cd}$ (figure a and a’), trig transition term $\rho_{cb}$ (figure b & b’) vary with field detuning $\Delta$, respectively. The parameters for situation in the left column (figure a, b) are: $\Omega_{c1} = 1/10, \Omega_{c2} = 0, \Omega_{c3} = 5, \Omega_{c4} = 1/10$, all the decay rates take $\gamma_i = 1(i = 1, 2, 3, 4)$, all detunings take zero value except the probing one. We decrease the strength of the couple field to $\Omega_{c1} = 1$ for situation on the right column (figure a’, b’), other parameters are remained unchanged as the left column.
two-qubits all-optical quantum phase gate (QPG). When the two qubits are defined as two polarized optical pulses: a probe light pulse and a trigger one, the phase gate is implemented by the cross-phase-modulation between the two pulses. If these two optical pulses are able to go through an enhancement of large nonlinearity simultaneously, then the XPM process, the quantum phase gate operation will be achieved. Thus the double-EIT effects of the probe and trigger optical fields is necessary to realize the QPG \cite{19, 20, 21, 22, 23}. The \( \Diamond \) system we are discussing here is a new type of multi-level EIT system differs from the systems studied in the above Ref. \cite{19, 20, 21, 22, 23}, though the detail of implementing the QPG operation by this \( \Diamond \) system remains to further study, it is confirmable to say that the diamond system will be useful to quantum information science. Comparing with all the other atomic structures in the above references, the \( \Diamond \) system is a kind of more subtle multi-level system, and it has several advantages. Firstly, the diamond system is a very general system, the atomic levels and the four coherent optical driving fields are nearly free of restriction, so it would be easier to realize experimentally. Secondly, there are many ways of arranging the roles of the probe field and the trigger field, that is, all four fields are able to assign to be the probe and the trigger field. Finally, we are aware that an experiment of coherently controlling the \( \Diamond \) system in cold \(^{87} \text{Rb} \) has been carried out recently \cite{27}, which demonstrates the feasibility of implementing the quantum phase gate operation. This part of contents are worth to study in the future.

In summary, we have explored the rich quantum interference and coherence effects of the \( \Diamond \) system driven by four coherent optical fields, and reveal the origin of the attractive effects such as electromagnetically induced transparency. By comparing to other type of atomic system, we demonstrate that the diamond system is much more accessible experimentally, which promises charming applications in quantum optics, nonlinear optics and quantum information science.

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