V – I characteristics in the vicinity of order-disorder transition in vortex matter

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Abstract

The shape of the $V – I$ characteristics leading to a peak in the differential resistance $r_d = dV/dI$ in the vicinity of the order-disorder transition in NbSe$_2$ is investigated. $r_d$ is large when measured by dc current. However, for a small $I_{ac}$ on a dc bias $r_d$ decreases rapidly with frequency, even at a few Hz, and displays a large out-of-phase signal. In contrast, the ac response increases with frequency in the absence of dc bias. These surprisingly opposite phenomena and the peak in $r_d$ are shown to result from a dynamic coexistence of two vortex matter phases rather than from the commonly assumed plastic depinning.

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Measurement of $V − I$ characteristics or of the differential resistance $r_d = dV/dI$ in superconductors is a very common method to investigate vortex dynamics and to identify various possible pinned and moving phases of the vortex matter. In particular, the specific shape of $V − I$ that leads to a peak in $dV/dI$ has attracted much attention \cite{1-11}. In numerical simulations this peak in $r_d$ is usually ascribed to plastic vortex depinning followed by dynamic ordering of the lattice \cite{8-11}. However, magnetic decorations and SANS studies, which have observed the plastic deformation of the lattice near the depinning followed by the dynamic ordering \cite{12-14}, did not find the predicted peak in $dV/dI$ \cite{12,13}. In addition, in clean systems like NbSe$_2$ the peak in $dV/dI$ is present surprisingly only in the lower part of the peak effect (PE), whereas in the rest of the $H − T$ phase diagram the $V − I$ curves are concave upward with no peak in $dV/dI$ \cite{2,3,15}. Moreover, in the same region where the peak in $r_d$ is observed, a number of anomalous vortex matter properties were recently found, including the striking observation that for an ac current the apparent vortex mobility increases rapidly with frequency \cite{16-20}. Several ideas and models have been proposed in which the ac agitation facilitates the plastic vortex depinning. Yet another important paradox has received little attention: when the ac current is superposed on a dc bias the opposite behavior is observed, i.e., the apparent vortex mobility decreases with frequency, even at frequencies as low as several Hz \cite{3}. None of the models that describe the peak in $r_d$ or the mobility enhancement with frequency \cite{8,11,16} has resolved this apparent paradox.

In this paper we demonstrate that the shape of the $V − I$ curves, the peak in $dV/dI$, and the opposite frequency dependencies stem, instead, from a dynamic coexistence of two vortex matter phases. A highly-pinned metastable disordered phase (DP), generated at the sample edges, anneals into a weakly-pinned equilibrium ordered phase (OP) in the bulk of the sample \cite{17,21,22}. The specific shape of the dc $V − I$ curves results from the fact that most of the sample is in the metastable DP at low currents, but in the OP at high currents. This dynamic transformation results in a peak in $dV/dI$. If measured by a small ac current superposed on a dc bias, we find that the peak in $dV/dI$ decreases with frequency and displays a unique out-of-phase signal due to the slow transformation process. In contrast,
for an ac current with no dc bias only the edges of the sample are contaminated by the DP, resulting in the opposite behavior, i.e., the voltage response grows with the ac frequency.

Transport measurements were carried out on several Fe-doped (200 ppm) NbSe$_2$ crystals in strip-like four-probe configuration in applied field $H \parallel$ c-axis. The data presented here are for a crystal $2 \times 0.4 \times 0.04$ mm$^3$ with $T_c = 5.6$ K, $H_{c2}(4.2$K) = 1 T, and the PE field $H_p(4.2$K) = 0.53 T. Very low contact resistance of $\sim 10$ mΩ was achieved with large current contacts of Au evaporated onto a freshly cleaved surface. By immersing the crystals in liquid He, currents up to 100 mA could be applied with negligible heating. Square wave or sinusoidal $I_{ac}$ was used and the corresponding $V_{ac}$ was measured by a lock-in amplifier.

Figure 1 shows $V_{ac}$ vs. $I_{ac}$ and $V_{dc}$ vs. $I_{dc}$ in the vicinity of the order-disorder transition in the lower part of the PE at $H = 0.44$ T. The dc curve (solid circles) starts to increase in concave form above 18 mA and then rapidly turns convex. At higher currents linear flux flow behavior is obtained. Although the form of the dc $V - I$ curve may seem rather conventional, we find that in NbSe$_2$ the convex shape is present only in the lower part of the PE, while in the rest of the phase diagram the curves are always concave upward, consistent with previous reports [2,3,15]. $V_{ac}$ vs. $I_{ac}$ measured at various frequencies in Fig. 1 are remarkably different from the dc $V - I$. Even at a frequency as low as 1 Hz the apparent $I_c$ is much lower and the voltage response below 20 mA is strongly enhanced. Furthermore, the apparent vortex mobility increases rapidly with frequency, as noted previously [16–18].

Figure 2 shows the differential resistance $r_d$ measured by superimposing a small $I_{ac}$ (0.1 to 1 mA) on $I_{dc}$, $r_d = V_{ac}/I_{ac}$, along with $r_d = dV/dI$ obtained by numerical differentiation of the dc $V - I$ at $H = 0.31$ T. At this slightly lower field within the PE the dc $V - I$ characteristic (solid curve) is more gradual and turns convex above $\sim 20$ mA. At the inflection point, $dV/dI$ displays a large peak reaching three times the flux flow $r_f$. The remarkable result here is that the ac $r_d$ is significantly different from the dc value [3] and it decreases rapidly with ac frequency. Note that the peak in ac $r_d$ is suppressed to about half of the dc value already at $f$ as low as 3 Hz, indicating the existence of very long characteristic timescales that are even longer than the vortex transit time across the sample, as described
below. None of the microscopic bulk mechanisms, such as plastic depinning or dynamic ordering, can account for such long timescales. Moreover, due to the high sensitivity of lock-in technique, the ac $r_d$ vs. $I_{dc}$ is commonly integrated to derive the full $V - I$ characteristics. The important conclusion from Fig. 2 is that the integral of the ac $r_d$ is not equal to the dc $V - I$.

Figures 1 and 2 display a striking qualitative difference: for a pure $I_{ac}$ the measured $V_{ac}$ increases with frequency, whereas for a small $I_{ac}$ superposed on a larger $I_{dc}$ the resulting $V_{ac}$ decreases with $f$. The lower panel of Fig. 2 shows another puzzling aspect of the data, which is a large out-of-phase component that appears only in the non-linear part of the $V - I$ and is absent in the flux-flow region. An out-of-phase signal is a common feature in ac susceptibility measurements, where the amplitude of the current induced in the sample and the dissipation level depend on the excitation frequency. In transport measurements, in contrast, the amplitude of the current is fixed by the external circuitry and therefore the voltage response, as a rule, is frequency independent and does not show any out-of-phase signal at low frequencies. To the best of our knowledge this is the first published report of an imaginary $r_d$, which further emphasizes the anomalous vortex dynamics in the lower part of the PE. Note that the out-of-phase $r_d$ is non-monotonic with frequency: it vanishes in the limit of high and low $f$ and is largest for the 22 Hz data.

We now discuss the results in view of the recent understanding that the PE reflects a disorder-driven first-order phase transition from a weakly pinned OP (Bragg glass) with a low critical current density $J^{ord}_c$ into a strongly pinned DP with a high $J^{dis}_c$. Local measurements have demonstrated that below the transition, in the lower part of the PE, in the presence of transport current a supercooled metastable DP is formed at the sample edge because of non-uniform vortex penetration through the surface barriers. As the vortex lattice moves across the sample, the metastable DP with a high concentration of dislocations gradually anneals into the dislocation-free OP. We describe the annealing stage of the DP by its local critical current density $J_c(x)$ which has a non-equilibrium excess value $	ilde{J}_c(x) = J_c(x) - J^{ord}_c$ relative to the fully annealed OP. Since in low-
temperature superconductors thermal activation is negligible, the sole annealing mechanism of the metastable DP is through a current-driven displacement that allows rearrangement and disentanglement of the vortices during the motion. We therefore assume, for simplicity, that the relative annealing of \( \tilde{J}_c \) upon displacement by a small \( \Delta x \) is given by \( \Delta x/L_r \), where \( L_r \) is a characteristic relaxation length over which the DP anneals into the OP. Since the lattice flows with velocity \( v \), \( \tilde{J}_c \) at \( x + \Delta x \) and at time \( t + \Delta t = t + \Delta x/v \) is thus described by \( \tilde{J}_c(x + \Delta x, t + \Delta x/v) = \tilde{J}_c(x, t)(1 - \Delta x/L_r) \), which leads to the partial differential equation of the annealing process

\[
\frac{\partial \tilde{J}_c(x, t)}{\partial x} + \frac{1}{v} \frac{\partial \tilde{J}_c(x, t)}{\partial t} = -\frac{\tilde{J}_c(x, t)}{L_r(v)},
\]

with a boundary condition at \( x = 0 \), where vortices penetrate into the sample, of \( \tilde{J}_c(0, t) = J_c^{\text{dis}} - J_c^{\text{ord}} \). A key aspect of the annealing process is that the relaxation length \( L_r \) crucially depends on the displacement velocity \( v \). Fast transient measurements \(^{26}\) show that at low velocities \( L_r \) is large, whereas for a potential landscape that is strongly tilted by a large driving force the disentanglement is very rapid, so that empirically \( L_r \approx L_0(v_0/v)^\eta = L_0(V_0/V)^\eta \). Here \( \eta \) is typically in the range of 1 to 3, \( L_0 \), \( v_0 \), and \( V_0 \) are scaling parameters, \( V = vBl \) is the measured voltage drop, \( B \) is the magnetic field, and \( l \) is the distance between the voltage contacts. We now demonstrate that these simple assumptions describe all the essential experimental observations.

We first analyze the time independent behavior. The dc solution of Eq. 1 is \( J_c^{dc}(x) = (J_c^{\text{dis}} - J_c^{\text{ord}}) \exp(-x/L_r) + J_c^{\text{ord}} \), as shown schematically in Fig. 3 inset. By integrating over the width \( W \) we obtain the total critical current of the sample \( I_c = d \int_0^W J_c^{dc}(x)dx \):

\[
I_c(L_r) = (J_c^{\text{dis}} - J_c^{\text{ord}})[1 - e^{-W/L_r}]L_r(V)d + I_c^{\text{ord}},
\]

where \( I_c^{\text{ord}} = J_c^{\text{ord}}Wd \) and \( d \) is the sample thickness. Note that \( I_c \) depends on \( L_r \), which in turn depends on voltage \( V \). This property is central to the described phenomena: Coexistence of the DP and OP results in an inhomogeneous sample and moreover the degree of the inhomogeneity, \( J_c(x) \), changes with vortex velocity. As a result, the total \( I_c \) of the sample is not fixed, but rather changes with voltage.
The dc $V - I$ characteristics can be derived as following. We write for simplicity the $V - I$ of the OP as $V = r_f(I - I_{c\text{ord}})$, where $r_f$ is the flux-flow resistance. Similarly, when the entire sample is in the DP, $V = r_f(I - I_{c\text{dis}})$ with $I_{c\text{dis}} = J_{c\text{dis}} W d$. These two asymptotic $V - I$ solutions are shown by the dashed lines in Fig. 3. Here we have assumed for simplicity that the flux-flow resistance $r_f$ is the same for any of the phases. As a result, when the two phases coexist $V = r_f(I - I_c)$, where $I_c$ is given by Eq. 2 and is voltage dependent through $L_r(V)$. Hence an analytical $I(V)$ relation can be written directly as $I = V/r_f + I_c(L_r(V))$, and the resulting non-linear $V - I$ characteristic is shown by the solid curve in Fig. 3. At very low voltages $L_r$ is larger than the sample width, namely the entire sample is contaminated by the DP, and hence the $V - I$ initially follows the asymptotic dashed line of the DP with $I_c = I_{c\text{dis}}$. At high vortex velocities $L_r$ becomes very short, most of the sample is in the OP and the $V - I$ approaches the asymptotic line of the OP with $I_c = I_{c\text{ord}}$. In the crossover region a specific shape of the curve with an inflection point is obtained alike the experimental dc $V - I$ curves in Figs. 1 and 2. This shape is the result of a continuous decrease of the total $I_c$ of the sample from $I_c = I_{c\text{dis}}$ to $I_c = I_{c\text{ord}}$ with increasing current. The exact curvature in the crossover region depends on the parameters $\eta$, $L_0$, and $V_0$ (see Fig. 3 caption) and may be either gradual or very steep, and may even obtain a negative slope resulting in an S-shaped characteristic as found recently in the lower part of the PE [27].

In order to understand the frequency dependence of the differential resistance shown in Fig. 2 and in more detail in Fig. 4a, we solve Eq. 1 for a small periodic velocity perturbation $v = v_{dc} + v_{ac} e^{i\omega t}$ caused by applied current $I_{dc} + I_{ac} e^{i\omega t}$. In this case $L_r(v) = L_{dc} + (dL_r/dv)v_{ac} e^{i\omega t}$. Following a simple calculation and keeping only the linear terms in $v_{ac}$, we obtain $J_c(x, t) = J_{c\text{dc}}(x) + J_{c\text{ac}}(x) e^{i\omega t}$, where $J_{c\text{ac}}(x) = (J_{c\text{dis}} - J_{c\text{ord}})(dL_r/dv)(v_{dc} v_{ac}/L_{dc}^2)e^{-x/L_{dc}} (1 - e^{-i\omega x/v_{dc}})/i\omega$. Note that $J_{c\text{ac}}(x)$ is negative because of a negative $dL_r/dv$, reflecting the fact that the system becomes more ordered with increasing $v$. In the limit of low frequency, $J_c(x, t)$ slowly varies between two extreme dc solutions determined by $L_{dc} \pm (dL_r/dv)v_{ac}$, as shown in Fig. 3 inset. At the minimum value of the current, $I_{dc} - I_{ac}$, the vortex velocity is lowest, $L_r$ is largest, and hence $J_c(x)$ is highest. At
the maximum of the current, $I_{dc} + I_{ac}$, $L_r$ is smallest and the sample is ‘cleanest’, resulting in a large enhancement in vortex velocity. Hence the large $r_d$ at low $f$ arises from the varying contamination of the sample which significantly amplifies the voltage response. However, modification of the sample contamination is a slow process because the only mechanism of enhancement of the local disorder in the bulk is by transporting a more disordered lattice from the edge of the sample. This results in characteristic timescales comparable with the vortex transit time across the sample $\tau_t = W/v$ (or even longer, see below).

To obtain the full frequency dependence of $r_d$ we calculate $I_{ac} = d \int_0^W J_{ac}^c(x)dx$, and by noting that $V_{ac} = r_f(I_{ac} - I_{ac}^c)$, we find $r_d = V_{ac}/I_{ac} = (1/r_f + I_{ac}^c/V_{ac})^{-1}$. This result shows that the enhancement of $r_d$ results from the fact that $I_{ac}^c$ is negative, and depending on the $L_r(v)$ parameters, $r_d$ can become infinite and even negative for the case of S-shaped $V – I$ curves. The full analytical expression for $r_d$ is too extensive to be presented here. Figure 4b therefore shows the calculated real and imaginary parts of $r_d/r_f$ vs. frequency at the operating point in Fig. 3. At low frequencies the in-phase $r_d$ is large and it decreases towards $r_f$ when $f$ approaches the transit frequency $f_t = 1/\tau_t$. We can understand this behavior as follows. As indicated by the arrows in Fig. 3, at low frequencies $r_d$ is given by $dV/dI$ which can be very large depending on the specific shape of the dc $V – I$ curve. However, the surprising result here is that at higher frequencies, in contrast to the common belief, the experimental $r_d = V_{ac}/I_{ac}$ does not measure the true $dV/dI$. At high $f$ the local degree of disorder $J_c(x,t)$ cannot adjust to the rapid variations and therefore $J_c(x,t)$ is fixed at $J_{dc}^c(x)$, and $J_{ac}^c(x)$ and $I_{ac}^c$ vanish. As a result, at high frequency the ac signal, instead of following the dc curve, follows a trajectory shown by the dotted line in Fig. 3. This line is the $V – I$ characteristic of a sample with a fixed $I_c = I_{dc}^c$, $V = r_f(I - I_{dc}^c)$, resulting in $r_d = r_f$, as indeed observed experimentally in Figs. 2 and 4a.

The rearrangement of the bulk disorder always lags the external ac drive, giving rise to a pronounced imaginary $r_d$ component (Fig. 4b) that has a maximum at intermediate frequencies. The qualitative agreement between the results of our simplified calculations in Fig. 4b and the experimental data in Fig. 4a is remarkable, including the frequency scale:
The experimental transit frequency \( f_t = \frac{V_{dc}}{BWl} \), marked by the arrow in Fig. 4a, was obtained by a direct measurement of \( V_{dc} \), and the maximum of the out-of-phase \( r_d \) occurs at about \( 0.1f_t \), in excellent agreement with the calculated behavior \( \[28\] \).

Finally, we comment briefly on the response to a large \( I_{ac} \) with no \( I_{dc} \) as shown in Fig. 1. In contrast to the previous situation, here all the metastable DP exits and reenters the sample during every ac cycle, as shown previously \( [17] \). The detailed analysis of this case is much more complicated since \( J_c(x,t) \) changes significantly during the ac cycle and is different for the entering and exiting edges, resulting in highly nonlinear behavior that cannot be treated analytically. Qualitatively, however, the maximum depth to which the DP penetrates the sample from each edge during the corresponding half period of the ac cycle is \( x_{ac}^{d} = \frac{v}{2f} \), where \( v \) is the average vortex velocity during the half cycle, while the central part of the sample remains ordered. Therefore the fraction of the sample occupied by the DP decreases with \( f \) as \( x_{ac}^{d}/W \propto 1/f \). Consequently, the volume of the OP increases with frequency, the integrated \( I_c \) decreases, and \( V_{ac} \) increases. At sufficiently high \( f \), practically the entire sample becomes ordered so that the experimental \( V_{ac} - I_{ac} \) in Fig. 1 follows the asymptotic dashed OP curve in Fig. 3. Thus a high-frequency measurement of \( V_{ac} - I_{ac} \) is a useful method to reduce edge contamination and to approximate the true \( V - I \) of the OP.

In summary, the process of edge contamination by the metastable disordered phase in the vicinity of the order-disorder transition is shown to explain the convex shape of the \( V - I \) characteristics in the lower part of the PE, the large difference between \( V_{dc} - I_{dc} \) and \( V_{ac} - I_{ac} \) curves that grows with frequency, the peak in the differential resistance and its rapid suppression with frequency, the out-of-phase signal in ac transport measurements, and the opposite frequency dependence in the different cases of ac drive.

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REFERENCES

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[1] R. Wordenweber and P. H. Kes, Phys. Rev. B 34, 494 (1986).

[2] S. Bhattacharya and M. J. Higgins, Phys. Rev. Lett. 70, 2617 (1993).

[3] S. Bhattacharya and M. J. Higgins, Phys. Rev. B 52, 64 (1995); M. J. Higgins, and S.
Bhattacharya, Physica C 257, 232 (1996).

[4] F. Garten et al., Phys. Rev. B 51, 1318 (1995).

[5] M.C. Hellerqvist et al., Phys. Rev. Lett. 76, 4022 (1996).

[6] J.M.E. Geers et al., Phys. Rev. B 63, 094511 (2001).

[7] Z. L. Xiao et al., Phys. Rev. Lett. 85, 3265 (2000).

[8] A. E. Koshelev and V. M. Vinokur, Phys. Rev. Lett. 73, 3580 (1994).

[9] S. Ryu et al., Phys. Rev. Lett. 77, 5114 (1996).

[10] C. J. Olson, C. Reichhardt, and F. Nori, Phys. Rev. Lett. 81, 3757 (1998); A. B. Kolton
et al., ibid. 83, 3061 (1999).

[11] M. C. Marchetti et al., Phys. Rev. Lett. 85, 1104 (2000).

[12] U. Yaron et al., Nature 376, 753 (1995).

[13] A. Duarte et al., Phys. Rev. B 53, 11336 (1996).

[14] F. Pardo et al., Phys. Rev. Lett. 78, 4633 (1997).

[15] Z. L. Xiao et al., Phys. Rev. B 65, 094511 (2002).

[16] W. Henderson, E. Y. Andrei, and M. J. Higgins, Phys. Rev. Lett. 81, 2352 (1998).
[17] Y. Paltiel et al., Nature 403, 398 (2000).

[18] V. Metlushko et al., cond-mat/9804121, 1998.

[19] S. N. Gordeev et al., Nature 385, 324 (1997).

[20] W. K. Kwok et al., Physica C 293, 111 (1997).

[21] M. Marchevsky, M. J. Higgins, and S. Bhattacharya, Nature 409, 591 (2001); Phys. Rev. Lett. 88, 087002 (2002).

[22] D. Giller et al., Phys. Rev. Lett. 84, 3698 (2000); C. J. van der Beek et al., Phys. Rev. Lett. 84, 4196 (2000).

[23] Y. Paltiel et al., Phys. Rev. Lett. 85, 3712 (2000).

[24] N. Avraham et al., Nature 411, 451 (2001).

[25] G. Ravikumar et al., Phys. Rev. B 63, 024505 (2001).

[26] W. Henderson et al., Phys. Rev. Lett. 77, 2077 (1996).

[27] A.A. Zhukov et al., Phys. Rev. B 61, R886 (2000).

[28] Note that in Fig. 4 the characteristic timescale is ten times longer than the transit time. This is one of the interesting consequences of Eq. 2 and arises from the fact that for a full rearrangement of the $J_c(x)$ distribution from minimum to maximum (see Fig. 3 inset) the vortices have to cross the sample several times, depending on $L_r(v)$ and the operating point on the $V-I$ curve.
FIGURE CAPTIONS

Fig. 1. $V_{ac} - I_{ac}$ characteristics at various frequencies and $V_{dc} - I_{dc}$ ($\bullet$). The voltage response increases with ac frequency.

Fig. 2. In-phase and out-of-phase differential resistance $r_d$ at dc ($\bullet$) and various ac frequencies (left axis), and dc $V - I$ characteristic (right axis). In-phase $r_d$ decreases with frequency, while the out-of-phase $r_d$ is maximal at intermediate frequencies.

Fig. 3. Theoretical dc $V - I$ characteristic (solid line) with $I_c^{ord} = 2$ mA, $I_c^{dis} = 18$ mA, $r_f = 2$ mΩ, $W = 400$ µm, and $L_r(V) = L_0(V_0/V)^\eta$ with $\eta = 2$, $L_0 = 200$ µm, and $V_0 = 30$ µV. The frequency dependence of the differential resistance at the operating point ($\bullet$) is shown in Fig. 4b. Inset: Schematic sample geometry and $J_c(x)$ across the sample width. $J_c(x)$ decreases with increasing current.

Fig. 4. (a) Frequency dependence of the in-phase ($\bullet$) and out-of-phase ($\circ$) $r_d$ at $I_{dc} = 22$ mA near the peak of $r_d$ in Fig. 2. (b) Calculated $r_d/r_f$ vs. $f/f_t$ at the operating point in Fig. 3.
Fig. 1, Paltiel et al.
Fig. 2, Paltiel et al.
Fig. 4, Paltiel et al.