QCD DYNAMICS FROM M-THEORY

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The field theories on the surface of non-supersymmetric D-brane constructions are identified. By moving to M-theory a semi-classical, strong coupling expansion to the IR non-supersymmetric gauge dynamics is obtained. The solution is consistent with the formation of a quark condensate but there is evidence that in moving to strong coupling scalar degrees of freedom have not decoupled.

1 D-Brane Constructions of Field Theories

A large literature has grown up on engineering supersymmetric field theories using D-brane constructions in type IIA string theory (a comprehensive review and list of references is to be found in ref 2). The essential ingredient is that the massless string modes of strings ending on the surfaces of D-branes correspond to gauge fields living on the D-branes’ surfaces, plus superpartners. In addition the D-branes break half the supersymmetries of the background space-time (the precise generators depending upon their orientation). In this section we will use these properties in perturbative string theory to construct field theories in 4D with N=2, N=1 and N=0 supersymmetries. Whilst a perturbative identification between the string states in the type IIA theory and the UV field theory may be made, the IIA picture provides no information about the strongly coupled IR dynamics of the theory. Such information would correspond to short distance structure of the branes, but due to the strongly coupled nature of the core of NS5 branes, the precise structure of an NS5 D4 junction is unclear. In section 2 by moving to M-theory a strong coupling expansion will be made allowing some understanding of strong coupling.

The basic brane configuration from which we start corresponds to \( SU(N) \) N=2 SQCD with \( F \) quark matter flavours. It is given from left to right by the branes

\[
\begin{array}{|c|c|c|c|c|c|c|c|}
\hline
\text{NS5} & 1 & - & - & - & \bullet & \bullet & \bullet \\
\hline
\text{D4} & N & - & \bullet & \bullet & -j & \bullet & \bullet \\
\hline
\text{NS5'} & 1 & - & - & - & \bullet & \bullet & \bullet \\
\hline
\text{D4'} & F & - & \bullet & \bullet & - & \bullet & \bullet \\
\hline
\end{array}
\] (1)
\( R^4 \) is the space \( x^0 - x^3 \). A dash \( - \) represents a direction along a brane’s world volume while a dot \( \bullet \) is transverse. For the special case of the D4-branes’ \( x^6 \) direction, where the world volume is a finite interval corresponding to their suspension between two NS5 branes at different values of \( x^6 \), we use the symbol \([-\,].\) The field theory exists in the world volume of the D4 branes on scales much greater than the \( L_6 \) distance between the NS5 branes. The fourth space like direction of the D4-branes generates the coupling of the gauge group in the effective 4D theory. The semi-infinite D4’ brane is responsible for providing the quark flavours.

The \( U(1)_R \) and \( SU(2)_R \) symmetries of the N=2 field theory are manifest in the brane picture. They correspond to isometries of the configuration; an SO(2) in the \( x^4, x^5 \) directions and an SO(3) in the \( x^7, x^8, x^9 \) directions.

N=2 supersymmetry may be broken in the configuration by rotations of the branes away from this configuration of maximal symmetry. The positions of the branes in these configurations break supersymmetry and hence we expect there to be supersymmetry breaking parameters introduced in the low energy field theory lagrangian. These parameters must be the supersymmetry breaking vevs of fields in the string theory since at tree level there are no supersymmetry breaking parameters. The vevs occur as parameters because the fluctuations of those fields are being neglected in the field theory; such fields are spurions. They have a natural interpretation in the brane configuration. The fields are those describing the positions of the branes and their fluctuations are neglected because the infinite branes are very massive. If though we choose to include these fields in the field theory description they occur subject to the stringent constraints of N=2 supersymmetry. The spurions whose vevs correspond to the supersymmetry breaking parameters must be the auxiliary fields of N=2 multiplets. This constraint is sufficient to identify the spurions.

- The NS5 branes may be rotated in the \( x^4, x^5, x^7, x^8, x^9 \) space (rotations from the N=2 configuration into the \( x^6 \) direction cause the NS5 branes to cross, changing the topology of the configuration in such a way that it can no longer be easily identified with a field theory). These rotations correspond to components of the spurion fields occurring as vector fields in the prepotential of the N=2 theory as \( \mathcal{F} = (S_1 + iS_2)A^2 \). This is the unique way in which a spurion may be introduced compatible with the N=2 supersymmetry. The scalar spurion vevs generate the gauge coupling \( \tau \). When we allow the auxiliary fields of the spurions to be non-zero we obtain the tree level masses.
\[- \frac{N_c}{8\pi^2} \text{Im} \left( (F_1^* + iF_2^*) \frac{\psi_A^* \psi_A}{i} + (F_1 + iF_2) \lambda^\alpha \lambda^\alpha + i\sqrt{2}(D_1 + iD_2) \frac{\psi_A^* \lambda^\alpha}{i} \right) \]

\[- \frac{N_c}{4\pi^2} \text{Im}(s_1 + is_2) \left( (|F_1|^2 + D_1^2/2) \text{Im}(a^a)^2 + (|F_2|^2 + D_2^2/2) \text{Re}(a^a)^2 \right) \]

\[+ (F_1F_2^* + F_1^*F_2 + D_1D_2) \text{Im}(a^a) \text{Re}(a^a)) \]

A number of consistency checks support the identification. Switching on any one of the six independent real supersymmetry breakings in the field theory leaves the same massless spectrum in the field theory as in the brane picture when any one of the six independent rotations of the NS5 brane is performed. The field theory and brane configurations possess the same sub-manifold of N=1 supersymmetric configurations.

- Supersymmetry may also be broken by forcing the D4 and D4’ branes to lie at angles to each other. With the introduction of matter fields in the field theory a single extra spurion field is introduced associated with the quark mass. The only possibility is to promote the mass to an N=2 vector multiplet associated with $U(1)_B$. Switching on its auxiliary field vevs induce the tree level supersymmetry breaking operators

\[2\text{Re}(F_3q\bar{q}) + D_M \left( |q|^2 - |\bar{q}|^2 \right) \]  (3)

Again a number of consistency checks support the identification of these field theory breakings with the angles between D4 branes. There are three independent real parameters in both the field theory and the brane picture. The scalar masses in the field theory break $SU(2)_R$ but leave two $U(1)_R$ symmetries of the supersymmetric theory intact. The scalar masses may always be brought to diagonal form by an $SU(2)_R$ transformation that mixes $q$ and $\bar{q}^*$. In the resulting basis there is an unbroken $U(1)$ subgroup of $SU(2)_R$. In the brane picture the D4’ branes lie at an angle in the $x^6 - x^9$ directions breaking the $SU(2)_R$ symmetry but leaving two $U(1)_R$ symmetries unbroken.

The NS5 branes may be rotated in such a way as to preserve $N = 1$ supersymmetry (corresponding to, for example, setting $F_2 = iF_1$ in the field theory) and the resulting configuration with the adjoint matter field decoupled is given by
Finally we note that for generic angles of the NS5 branes supersymmetry is completely broken and the gaugino is massive. Although the scalar masses we can generate at tree level by deformations of the brane construction are always unbounded and force the theory to a higgs branch, in the full theory we expect radiative masses for the scalars of order the supersymmetry breaking scale. Thus we expect that if a large gaugino mass is introduced the theory in the IR is non-supersymmetric QCD. Note that the construction does not constitute a phenomenological model of QCD because the gravitons of the theory (which we can make arbitrarily weakly coupled relative to the gauge dynamics) still live in a 10D space-time. These constructions are therefore only mathematical tools for studying gauge theories.

2 Strong Coupling From M-Theory

To attempt to understand the strong coupling behaviour of the field theories constructed above we move to M-theory. Type IIA string theory with coupling $g_s$ is 11 dimensional M-theory compactified on a circle of radius, $R \sim g_s$, which in the IR is described by weakly coupled 11D SUGRA. In M-theory NS5 branes and D4 branes become aspects of a single M5 brane wrapped in places on the compact dimension. The junctions between these objects may thus be smoothly described by a minimal area embedding of the M5 brane. Increasing the M-theory compactification radius from zero allows the study of the string theory with increased coupling at the string scale. It is therefore possible to make a strong coupling expansion to the field theory. That is to smoothly deform the field theory of interest to a theory with the same global symmetries and parameters but which is fundamentally a theory of strongly interacting strings in the $R \to \infty$ limit. For intermediate $R$ the theory has Kaluza Klein states in addition to those of the field theory. We hope that by making this transition between smoothly related theories there is no phase transition and that the two theories lie in the same universality class. This technique has been used to derive the existence of gaugino condensation in N=1 super Yang Mills theory and Seiberg’s duality in the theories with flavour amongst other results. For these supersymmetric theories the holomorphic and BPS properties of states in the theory contribute to making the motion to strong...
coupling smooth. For non-supersymmetric theories we have no such protection 
but we can hope to look for a consistent picture free of transitions.

We must therefore minimal area embed an M5 brane in the background 
11D space with metric

\[ ds^2 = \sum_{i,j=0}^{9} \eta_{ij} dx^i dx^j + R^2 (dx^{10})^2 \]  

(5)

3+1 dimensions of the M5 brane are flat (the space the field theory lives in) and we must concentrate on embedding the remaining two dimensions of its surface 
in the space \( \vec{X} = x^4 - x^{10} \). Classically a minimal area embedding corresponds 
to when the energy momentum tensor of the two dimensional theory on the 
surface of the brane vanishes

\[ T_{zz} = g_{ij} \partial_z X^i \partial_z X^j = 0 \]  

(6)

We proceed by guessing solutions with the topology that we require. A solution 
is (with \( v = x^4 + ix^5, \ w = x^7 + ix^8 \) and \( t = \exp(x^6 + i x^{10}/R) \))

\[ v = z + \frac{\xi}{z}, \quad w = \frac{\xi}{z} + \frac{1}{z}, \quad t = \kappa z^N/(z - m)^F, \]  

(7)

subject to the constraint

\[ \eta \xi + \xi \lambda = 0 \]  

(8)

The curve has two \( U(1) \) symmetries associated with rotations in the \( v \) and \( w \) planes which are broken by the parameters of the curve. The symmetries 
may be restored by assigning the parameters spurious charges

\[
\begin{align*}
\begin{array}{cccccccc}
\text{ } & v & w & t & z & m & \eta & \xi & \lambda & \kappa \\
U(1)_v & 2 & 0 & 0 & 2 & 4 & 0 & 2 & 2 & 2(F-N) \\
U(1)_w & 0 & 2 & 0 & 0 & 0 & 0 & 2 & -2 & 0
\end{array}
\end{align*}
\]  

(9)

These symmetries are just the \( U(1)_R \) symmetries of the field theory and may be used to identify the parameters with field theory parameters in the 
limit \( R \to 0 \).

Thus for example we find the curve describing an \( N=1 \) \( SU(N) \) field theory 
with \( F \) quark flavours

\[ v = z, \quad w = z^{N} m^{F-N}/(z - m)^F \]  

(10)
Viewing the curve asymptotically and as $\Lambda \to 0$
\[ z \to \infty \quad w = 0 \quad t = v^{N-F}m^{F-N} \]
\[ z \to 0 \quad v = 0 \quad t = \left( \frac{1}{w} \right)^N \Lambda_b^m m^{F-N} \quad (11) \]

The $U(1)_v$ and $U(1)_w$ symmetries (allowing $m$ to transform spuriously but not $\Lambda$) are broken to $Z_N$ and $Z_{N-F}$ discrete subgroups as is the case for the $U(1)_R$ symmetries of the field theory. The $N=1$ theory behaves like supersymmetric Yang Mills theory below the matter field mass scale and dynamically generates a gaugino condensate. The theory has $N$ degenerate vacua associated with the spontaneous breaking of the low energy $Z_N$ symmetry. In the curve this corresponds to the $N$ curves in which $\Lambda^b_0 = \Lambda^b_0 \exp(2\pi i n)$. In the UV these curves can be made equivalent by a $Z_N$ transformation.

We may now switch on the parameter $\epsilon$ and break supersymmetry. The curve becomes
\[ v = z + \frac{\epsilon}{z} \quad w = \frac{\Lambda^b_0/N}{m^{F/N}}, \quad t = z^N m^{F-N} / (z-m)^F, \]
\[ x^9 = 4\epsilon^{1/2} Re \ln z \quad (12) \]

In the $R \to 0$ limit the D4 branes lie in the $x^6$ and $x^9$ directions and the NS5 brane lying in the $w$ direction has been rotated in a non-supersymmetric fashion into the $v$ direction. We generically expect the field theory to have the supersymmetry breaking terms
\[ D \left( |q|^2 - |\tilde{q}|^2 \right) + m_\lambda \lambda \lambda \quad (13) \]

We may identify the parameter $\epsilon$ with field theory parameters from its symmetry charges. Requiring that the brane configuration retains, asymptotically, the $Z_F$ symmetry remnant of $U(1)_A$, which is left unbroken by these mass terms, forces
\[ \epsilon = (m^N \Lambda^b_0)^{1/N} \quad (14) \]

Including the gaugino mass breaks the $Z_N$ symmetry of the curve and the $N$ vacua of the SQCD theory are no longer equivalent. This should be compared with the field theory where, when the SQCD theory is perturbed by a small gaugino mass, the dynamical superpotential $\Lambda^3$ implies a potential of the form
\[ V = \Lambda^3 |F| \simeq \Lambda^3 m_\lambda \cos [\theta_{\text{vac}}] \quad (15) \]
where $\theta_{\text{vac}}$ is the phase of the SQCD vacuum. The potential splits the degeneracy between the $N$ vacua. The brane picture is consistent with that behaviour.
It is possible to take the decoupling limit for the gaugino mass by taking \( \epsilon \to \infty \). \( m \) is the only R-charged parameter remaining and there is therefore nothing that can play the role of either a gaugino mass or condensate fitting the assumption that the gaugino has been decoupled. We must define a new strong scale parameter below the gaugino mass \( \Sigma = m^{N/F} \Lambda^{b_0/F} \). The decoupled curve is

\[
v = z + \left( \frac{m \Sigma}{z} \right)^{F/N}, \quad t = z^N m^{F-N}/(z-m)^F, \quad x^9 = 4(m \Sigma)^{F/2N} \text{Re} \ln z\]

(16)

The curve has a \( Z_F \) symmetry that is broken by \( m \) and \( \Sigma \) that transform spuriously with charges +2 and -2 respectively. That is \( m \) has the charge of a quark mass parameter and \( \Sigma \) the charge of a quark condensate. This suggests the nice interpretation that the gaugino has decoupled and the scalars radiatively with it, leaving a theory that is non-supersymmetric QCD in the IR with a quark condensate.

In fact the interpretation is not so clear cut\[14,15\]. To expose a problem with the strong coupling expansion let us investigate the fate of Seiberg’s N=1 SQCD duality when supersymmetry is broken. The duality is exhibited by the N=1 curve\[14\]. If we make the transformations

\[
m = M^{N/F-N} \Lambda^{-b_0/F-N} \\
\Lambda^{b_0} = \tilde{\Lambda}^{-\tilde{b}_0} \\
z = \frac{M^{F/N} \Lambda^{-b_0/F-N}}{z'} \]

(17)

the curve (10) is transformed to

\[
v = \frac{M^{F/\tilde{N}} \tilde{\Lambda}^{\tilde{b}_0/\tilde{N}}}{z'}, \quad w = z', \quad t = \frac{M^{F-N} z' \tilde{N}}{(M-z')^F} \]

(18)

where \( \tilde{N} = F - N \) and \( \tilde{b}_0 = (3\tilde{N} - F) \).

In the limit \( \tilde{\Lambda} \to 0 \) at fixed \( M \) this curve degenerates to the IIA configuration describing a \( SU(F-N) \) gauge theory with \( F \) quark flavours of mass \( M \). This theory is Seiberg’s dual SQCD theory. Note that in terms of the electric curve this configuration is obtained when \( \Lambda \to \infty \) with \( m \) scaled to zero appropriately to keep \( M \) fixed. The duality of the field theory is a strong-weak duality.

In fact this dual nature of the curve persists with supersymmetry breaking. For example, in the decoupled limit the transformations

\[
m = M^{N/F-N} \Sigma^{-F/F-N} \]

would
\[ \Sigma \equiv \bar{\Sigma}^{-1} \] (19)
\[ z' \equiv \frac{m^F F^{-N} \Sigma^{-F/N}}{z} \]
give the dual curve
\[ v = \bar{z}' + \frac{(M\bar{\Sigma})^{F/2}}{2}, \quad t = z'^{\bar{N}} M^{F-N} / (M - z')^F, \]
\[ x^9 = -4(M\bar{\Sigma})^{F/2\bar{N}} \text{Re} \ln z' \] (20)

Again the dual picture emerges from the M-theory curve in the limit where \( \Sigma \to \infty \) and \( m \to 0 \). Surprisingly the M-theory seems to be telling us that for massless quarks there is a duality symmetry between an \( SU(N) \) gauge theory with \( F \) quarks and an \( SU(F - N) \) gauge theory with \( F \) quarks. Presumably some components of the dual meson of SQCD also survive though we have been cavalier in the above discussion as to the boundary conditions at infinity of the semi-infinite D4s.

The theory on the branes surface is clearly not QCD. QCD can not have a flavour dependent duality such as the curve suggests because the theory has no higgs branch with which to correctly reduce the gauge group as dual quark flavours are made massive. What has gone wrong? We suggest two possibilities. One is that as the strong coupling scale is taken to infinity the radiative masses of the scalars will only be of order \( \Lambda \) and they can not be considered decoupled (in fact the dual quarks are weakly coupled so their radiative masses may be small). The other possible problem is that the curve for some value of \( m_\lambda \) has stopped being the true vacuum of the theory and become a local minimum at a first order phase transition. Some other curve would then describe the true QCD physics.

It is amusing to speculate on whether the curve’s duality in the limit where the gaugino is decoupled can be interpreted as a duality of non-supersymmetric theories with light (fine tuned) scalars. In fact the anomaly matching conditions and flows with the introduction of quark mass terms continue to hold in the field theory and its dual without a gaugino. Duality may therefore be the result of light scalars rather than supersymmetry (although supersymmetry is the only natural way to stabilize massless scalars against the hierarchy problem).

The failure of the scalar fields to decouple even in the absence of supersymmetry is awkward for the discussion of true QCD dynamics. Ideally we would like to decouple the scalars at tree level and not rely on radiative effects. Unfortunately the deformations of the brane configuration do not appear to correspond to a suitable scalar mass term. The best we could hope to achieve is
to switch on masses of the form of (3) but these masses are always unbounded
and trigger a higgs branch of the theory. We do not know of anyway to give all
scalars a positive tree level mass and so their decoupling remains frustratingly
elusive.

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