Study on anti-scanning performance of WFRFT modulation signal

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Abstract. Based on the signal characteristics of the existing weighted fractional fourier transform (WFRFT) communication, this paper studies the equivalent signal to noise ratio (SNR) and bit error rate (BER) of the non-target receiver under the condition of the transformation parameter errors, with the purpose of measuring the anti-parameter scanning characteristics of the single-parameter WFRFT signal. The constellation splitting characteristics of multi-parameter WFRFT signal are further studied. Multi-parameter WFRFT modulation can not only improve the resistance of single-parameter system to parameter scanning detection, but also effectively hide the original modulation mode due to its characteristic of splitting modulation constellation.

1. Introduction

The weighted fractional fourier transform (WFRFT) is a new mathematical tool different from the traditional classical fractional fourier transform (FFT). Shih first proposed FFT in 1995[1], after which its mathematical definition has been continuously extended[2-7].

Literature[8]defined WFRFT of discrete sequence to make it suitable for digital signal processing and introduced it into the communication system, and proposed the framework of WFRFT digital communication system. Compared with the original constellation diagram, the output constellation diagram of single-parameter WFRFT digital communication system has rotation and gauss like distribution characteristics. According to this, literature[8]proposed the possibility of using this feature for secret or anti-interception communication. On this basis, reference is made to the idea of frequency hopping system in literature[9], and a variable parameter WFRFT communication system is proposed to control the parameter changes of WFRFT using pseudo-random sequence, so as to improve the parameter anti-tracking and anti-scanning characteristics of WFRFT system. Literature[10]also uses the gaussian-like statistical characteristics of the single-parameter WFRFT signal to propose a scheme that uses the WFRFT signal as a hidden communication system carrying signals. Literature[11]explains the physical meaning of WFRFT in communication system from another perspective: WFRFT communication system is a hybrid modulation structure of single and multi-carrier system.

Based on literature[8-9], this paper further studies the anti-parametric scanning characteristics of single-parameter WFRFT signal, and gives an approximate formula of equivalent SNR when parameters have errors. In addition, according to the constellation splitting characteristics of multi-parameter WFRFT signal, the causes of constellation splitting of multi-parameter WFRFT signal are analyzed.
2. Digital signal processing based on WFRFT

Let \( X_0(n) \) be any complex number sequence, and \( \{X_0(n), X_1(n), X_2(n), X_3(n)\} \) be 0-3 discrete Fourier transforms of \( X_0(n) \) in which the discrete fourier transform (DFT) adopts the normalized definition form as shown in the formula:

\[
X(k) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} x(n)e^{-\frac{2\pi j n k}{N}}
\]

\[
x(n) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X(k)e^{\frac{2\pi j n k}{N}}
\]

Then its WFRFT definition is:

\[
S_0 = F^\alpha[X_0(n)] = \omega_0(\alpha, V)X_0(n) + \omega_1(\alpha, V)X_1(n) + \omega_2(\alpha, V)X_2(n) + \omega_3(\alpha, V)X_3(n)
\]

Among which, \( X_0 - X \) can be expressed as follows:

\[
X_0(n) = F^\alpha[x(n)] = F^\alpha[X_0(n)] \rightarrow X_0(n) = x(n)
\]

\[
X_1(n) = F^1[x(n)] = F^1[X_0(n)] \rightarrow X_1(n) = X(n)
\]

\[
X_2(n) = F^2[x(n)] = F^2[X_1(n)] \rightarrow X_2(n) = x(-n)
\]

\[
X_3(n) = F^3[x(n)] = F^3[X_2(n)] \rightarrow X_3(n) = X(-n)
\]

Where the weighting coefficient is defined as:

\[
\omega_l(\alpha) = \frac{1}{4} \sum_{n=0}^{3} \exp\left[\frac{-2\pi i}{4} \left[(4m_l + 1)\alpha + (4n_l + 1)\alpha - lk\right]\right] \quad (l = 0, 1, 2, 3)
\]

Formula (4) is controlled by three variables, and the form of weighted coefficients controlled by the nine parameters of \( \alpha, M = (m_0, m_1, m_2, m_3) \) and \( N = (n_0, n_1, n_2, n_3) \), where \( M \) and \( N \) are real vectors. \( V = [M, N] \), when \( V = 0 \), the formula is defined as the single-parameter WFRFT, otherwise, it is defined as the multi-parameter WFRFT. The definition of the extension formula (2) can be obtained:

\[
F^\alpha[X_0(n)] = W(\alpha, V)X = \begin{bmatrix} S_0 \\ S_1 \\ S_2 \\ S_3 \end{bmatrix} = W(\alpha, V)X = \begin{bmatrix} w_0 & w_1 & w_2 & w_3 \\ w_3 & w_0 & w_1 & w_2 \\ w_2 & w_3 & w_0 & w_1 \\ w_1 & w_2 & w_3 & w_0 \end{bmatrix} \begin{bmatrix} X_0 \\ X_1 \\ X_2 \\ X_3 \end{bmatrix} = \begin{bmatrix} w_0X_0 + w_1X_1 + w_2X_2 + w_3X_3 \\ w_3X_0 + w_0X_1 + w_1X_2 + w_2X_3 \\ w_2X_0 + w_3X_1 + w_0X_2 + w_1X_3 \\ w_1X_0 + w_2X_1 + w_3X_2 + w_0X_3 \end{bmatrix}
\]

Due to the existence of reversibility, the results \( X_0(n) \) after signal processing are obtained by WFRFT of order \(-\alpha\).

\[
X_0(n) = F^{-\alpha}[S_0(n)] = \omega_0(-\alpha, V)S_0(n) + \omega_1(-\alpha, V)S_1(n) + \omega_2(-\alpha, V)S_2(n) + \omega_3(-\alpha, V)S_3(n)
\]

Figure 1. Block diagram of digital signal processing System modulated by WFRFT.

Although the sequence WFRFT defined by formula (2)-(4) is similar to the continuous WFRFT of Shih in form, it is not a discrete algorithm of continuous WFRFT, and the definition is only for any complex sequence. According to literature[8] and [9], in the digital communication system model shown in the figure 1, MPSK and other discrete signals that can be projected onto the complex plane can be
adopted. The output signal has a power spectrum consistent with the input signal, which is fully applicable to current communication systems without the need for additional devices.

3. Anti-scan characteristics of single parameter WFRFT digital signal modulation system

Literature[8-9] studied the constellation diagrams of BPSK, QPSK and 16QAM signals after WFRFT, and found that they all obey the gaussian distribution in time-frequency domain. The statistical characteristics of signals are studied mainly because statistical analysis methods based on high-order cumulants are important means for blind signal detection and modulation recognition[12]. The premise of this study is that the basic information of non-target receiver pair transmitting signals is unknown. Another important criterion to measure the anti-interception performance of WFRFT signal is the possibility and cost of obtaining the correct demodulation parameters by means of parameter scanning when the non-destination receiver is known to use the single-parameter WFRFT digital signal modulation and the specific conversion parameters are unknown.

Assuming that the signal is evenly distributed on the complex plane, when QPSK modulation is adopted, the schematic diagram of the complex plane of the signal after single-parameter WFRFT transformation is shown in figure 2:

![Figure 2](image2.png)

Figure 2. Complex plane distribution of single-parameter WFRFT signal.

If the error of demodulation time parameter $\alpha$ is set as $\Delta \alpha$, it can be understood as demodulation of the original signal after WFRFT with parameter of $\Delta \alpha$. The following figure shows the bit error rate curve at the time of parameter error. Respectively represent the bit error rate curve of $\Delta \alpha$, the illegal party obviously cannot demodulate the correct signal.

![Figure 3](image3.png)

Figure 3. SNR and BNR curves at different $\Delta \alpha$ values.

4. Constellation fission of multi-parameter WFRFT signal processing

When, the multi-parameter 4-WFRFT-modulated communication system is shown in figure 1. When $V=0$, the multi-parameter digital modulation signal processing will degenerate into a single parameter, and the multi-parameter will improve the anti-parameter scanning ability of the processed signal, because the increase in parameters makes it more difficult to scan and detect a certain parameter, and it is almost impossible to obtain nine parameters at the same time.
As a generalization of single-parameter modulation, multi-parameter modulation has all the properties of single-parameter modulation. In addition, the introduction of vector V makes some new changes to the processed signal. As shown in the figure, after the QPSK modulation signal passes through the multi-parameter WFRFT, the original constellation points will not only rotate and diffuse, but also split. It makes the transformed signal look more like a 16QAM signal that has rotated and added noise.

Obviously, this "constellation splitting" characteristic is related to the introduction of parameters M and N. The function of M is reflected in the compression and expansion of parameter range, which changes the WFRFT period, that is, changes the distance over the time-frequency domain parameter. The function of N is mainly reflected in the rotation of the eigenvector. Since each weighted coefficient is the sum of four complex exponentials, and each complex exponential corresponds to one component in M and N, the influence of each complex exponential on the weighted coefficient and even the weighted sequence is different.

Therefore, in the information sequence, even the symbols on the same constellation point will change differently after transformation due to their different positions in the sequence. For each constellation, there are four different trends. With the further change of parameter parameters, sub-constellation points separated from different constellation points will overlap and even overlap with each other, which will make the transformed signal constellation map more complex and thus harder to detect and crack.

Figure 4. Diagram of constellation splitting of multi-parameter system.

5. Conclusion
Based on the study of the anti-parameter scanning performance of WFRFT signal, this paper analyzes the effect of the conversion parameter error on the SNR of the receiver, and verifies the correctness of the empirical formula of equivalent SNR of the non-target receiver. This paper also finds that the adoption of multi-parameter WFRFT can make the signal constellation diagram undergo special changes of constellation splitting. Multi-parameter WFRFT signal has many conversion parameters, and its constellation splitting is complicated. Multi-parameter WFRFT can hide the original modulation mode more effectively and resist the scanning of transform parameters by non-target receiver, which is a digital signal modulation processing mode with broad application prospects.

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