Optimization of the internal damping rubber elements of the roller wheel of tracked system

S A Korostelev¹ and A F Verbilov²

¹ Department of land transport-technological systems, Polzunov Altai State Technical University, Barnaul, Altai Region, Russia
² Firearms and Technical Training Department, Barnaul Law Institute of the Ministry of Internal Affairs of Russia, Barnaul, Altai Region, Russia

E-mail: korsan73@mail.ru

Abstract. The article presents the formulation and solution of the problem of optimization of the internal damping rubber elements of the roller wheel of tracked system. The optimal design search is based on the results of calculating the stress-strain state of the rubber element caused by the assembly of the roller and operational loads. As an objective function, the specific energy of deformation of the rubber was chosen. The geometric parameters of the rubber element of the roller wheel in the initial state and after installation in the structure are taken as variable parameters. Explicit and functional constraints in the form of inequations are imposed on the variable parameters. As a result of solving the optimal design problem, the shape and design parameters of the internal damping rubber elements of the track roller of the tracked system were obtained, which made it possible to reduce the specific strain energy during radial loading by 1.85 times

1. Introduction

Expanding the scope of tracked vehicles, increasing the requirements for the durability of elements of a tracked mover while increasing specific loads and the desire to reduce metal consumption [1, 2] forces engineers to use new solutions in the design of track rollers.

The increase in the speeds of the high-speed tracked vehicles and the energy saturation of agricultural tractors led to an increase in the dynamic loads acting on the elements of the tracked mover. The high level of dynamic stresses affects the strength, reliability and durability of the elements of the tracked system. Reducing dynamic loads affecting the elements of the tracked mover is an important engineering problem [1, 3 - 5].

2. Features of the use of rubber elements in the constructions of roller wheels

One of the possible designs of the roller wheel, which allows to reduce dynamic loads, is a construction, in which the elastic connection between the hub and the metal bandage is carried out using internal damping rubber elements [1, 2, 6].

In roller wheels with internal rubber elements, rubber works in favorable conditions [2]. Namely, most of it is protected from aggressive environments, rubber elements perceive compression and shear deformations, rubber is protected from direct exposure to foreign objects penetrating between the track and the roller wheel. All this allows to significantly increase the durability of the rubber element. Another important advantage of designs with internal rubber elements is their maintainability. In such case, both the rubber elements and the bandage are the least durable, the replacement of which during the repair of the caterpillar mover, for some designs, is possible without dismantling the entire roller.

The geometric shape of the power rubber elements determines their characteristics and, first of all, elastic and compensation properties. Suffice it to say that the stiffness in compression, torsion or shear, the values of permissible deformations of individual types of elements of the same overall size, can
differ by an order of magnitude. The geometric shape has a significant effect on the damping ability of rubber elements.

When designing power rubber elements of a tracked system, one of the main issues is the question of their durability. The existing variety of design of power rubber elements does not allow focusing only on experimental research data, since their implementation are expensive and time-consuming.

To assess the durability of the rubber elements of the roller wheel of the tracked system, it is advisable to use calculation methods based on the application of the criteria for fatigue strength of rubber. Fatigue strength criteria are formed taking into account the influence of the stress-strain state of rubber elements under static and dynamic loads, temperature, physical and mechanical properties of specific rubber grades.

The process of destruction of the internal damping rubber elements of the roller wheel is subject to the general laws characteristic of all materials: the process of volumetric destruction proceeds in two stages and is accompanied by a period of accumulation of damage and subsequent global destruction of the element; the destruction process is unidirectional and statistical in nature. The destruction of rubber elements during cyclic deformation is accompanied by self-heating, which is due to the viscoelastic properties of the material. Hysteresis phenomena during cyclic deformation of rubber elements in combination with low thermal conductivity require taking into account the effect of temperature on their fracture process.

Power rubber elements subjected to intense cyclic deformation are destroyed by three main damage mechanisms [7]: thermomechanical failure, fatigue failure and mixed failure.

In the case of fatigue failure during the entire period of operation of the rubber element, its temperature remains at a level significantly less than value for a given rubber grade. Fatigue failure begins with the appearance of fatigue microcracks due to the influence of cyclic deformation and subsequently turn into main cracks until the complete destruction of the element.

To assess the fatigue endurance of rubbers from mechanical stress under complex loading conditions, when the sample is subjected to both dynamic and static deformations, dependencies based on the energy criterion are used. The study of the dependence of the fatigue life of rubber using the energy criterion was considered in [7 - 9]. In these works, on the basis of experimental data, it is proved that the specific strain energy is the most accurate and universal criterion for fatigue strength.

Thus, in order to assess the workability of the internal damping rubber elements of the roller wheel of a particular design, it is necessary to know their stress-strain state caused by the assembly of the studied unit as well as the stresses, deformations, and temperatures occurring in the body of the rubber element during subsequent operation. In addition, it is necessary to know the stiffness characteristics of the rubber elements and be able to evaluate their influence on the dynamic loads occurring in the elements of the tracked system in order to determine the feasibility of using rubber elements.

3. Theoretical approach for assessing the stress-strain state of the inner damping rubber element of the roller wheel

The phenomenological approach is used to describe the elastic properties of elastomers. It allows to describe the dependences between strains and stresses observed in experiments[10–11].

Nonlinear relationships between stresses and strains in rubber are described using the elastic potential, which is a function of the invariants of the Cauchy strain tensor. The simplest form of elastic potential is the Treloar potential [10, 11]:

$$ W = \frac{1}{2} \mu (I_1 - 3), $$

where $\mu$ is the shear modulus of rubber.

The Treloar potential allows to describe the properties of a perfectly elastic material in the medium strain region. In this work, the elastic properties of rubber are described by the Treloar elastic potential. Given the dependence (1), the potential energy functional of the system for an incompressible material is written as [12]

$$ II = \int_\Omega \left[ \frac{\mu}{2} (I_1 - 3) + p(\tau - 1) \right] dv - \oint_\partial \bar{F} \hat{u} ds, $$

where $I_1$ is the first invariant of the Cauchy Green deformation tensor.
where \( s \) is the surface of the undeformed volume; \( \nu, p \) is the hydrostatic pressure function; \( \vec{F}, \vec{u} \) respectively are the vector of forces and the vector of displacements at the boundary of the area where external forces are specified.

Internal damping rubber and rubber-metal elements of track roller wheel are bodies of revolution. In this regard, the problem of the stress-strain state is solved in a cylindrical coordinate system.

During assembly, rubber experiences large deformations, therefore, the algorithm for calculating rubber elements is based on the basic relations of the nonlinear theory of elasticity of an incompressible material, namely, the theory of superposition of small elastic deformations on equilibrium finite ones.

In accordance with the algorithm of the method proposed in the works of E. Lavendell the continuous deformation of the rubber element is replaced by a step-by-step procedure. At each step of solving the nonlinear problem, linearized relations of the nonlinear theory of elasticity are applied. Thus, at each step, the linear problem is solved, but the stress-strain state that occurs in the rubber element as a result of deformation at the previous deformation steps is taken into account.

The variation in the increment of potential energy at each step of deformation is determined by the equation [12]:

\[
\delta (\Delta \Pi) = \delta \left( \int_{V} W dv - \frac{1}{2} \int_{s} \vec{F} : \Delta \vec{u} ds \right),
\]

where \( W = \frac{1}{2} \eta \left[ \nu \left( \frac{p - 1}{3} \right) \right] \) : \( W \) is the Treloar elastic potential; \( \eta \) is the small parameter; \( \Delta \vec{F}, \Delta \vec{u} \) are accordingly, the increment of the force vector and the displacement vector at the boundary of the area where external forces are specified.

The above equations make it possible to determine the stress-strain state of rubber elements both caused by assembly and during secondary loading caused by subsequent operation.

4. Example of the stress-strain state assessment of the internal damping rubber element of the roller wheel

In figure 1 a design variant of the roller wheel [12] with internal cushioning in which the rubber elements operate in shear is presented. Roller wheels with ring rubber elements have a large radial and low axial flexibility, which reduces dynamic loads in the area of contact with the treadmill of the link and eliminates the shift of the bandage during movement of tracked vehicle.

When assembling the roller, the rubber elements are preliminarily compressed in the axial direction, which increases their fatigue strength, and the uniform load of the entire cross section of the ring during shearing, combined with its simple configuration, significantly reduces the probability of the stress concentrators. The design of the roller wheels of this group allows additional tightening of the rubber elements during operation. In addition, structurally, it is possible to ensure the limitation of radial deformation, which allows protecting the rubber elements from overload.

![Figure 1. The design of the track roller with internal shock absorption](image-url)

The structural parameters of the internal damping rubber elements of the roller wheel before and after assembly, together with the loads acting on the roller during operation, determine their stress-strain state. In turn, the stress-strain state of the rubber elements in combination with the loading modes of the roller wheels determines the durability of the rubber elements.
The solution to the problem of determining the stress-strain state after assembly caused by compression in the axial direction is carried out in a cylindrical coordinate system, an axisymmetric problem is considered, the rubber element section is discretized by isoparametric finite elements with eight nodes and non-linear approximation of displacements. Subsequent loading by radial force, the problem is considered in a cylindrical coordinate system and volumetric isoparametric finite elements with twenty nodes are used to solve it.

As a result of the calculation, the distribution fields of the components of strain tensors, stresses, and specific strain energy over the volume of the rubber element of internal depreciation were obtained (Figures 2-4).

**Figure 2.** Cross-section of the rubber element of the track roller:
- a - mesh of finite elements in a free state;
- b - finite element mesh after compression in the axial direction;
- c - finite element mesh after applying a radial load to the support bandage

**Figure 3.** Distribution of normal stresses over the cross section of a rubber element:
- a - \( \sigma_z \);
- b - \( \sigma_r \);
- c - \( \sigma_\theta \)

Figure 4 presents patterns of the distribution of shear stresses caused by assembly deformations, and the specific strain energy caused by subsequent loading by radial force.

The maximum values of shear stresses caused by the assembly for the studied design are valid in areas 1 and 4, 2 and 3 (Figure 4a) and are 0.696 MPa. During secondary loading with the radial force of maximum values, the specific strain energy reaches in areas 1 and 4, 2 and 3 (Figures 4b, 4c). In area 1, its value reaches 100 kJ / m³, and in areas 3 and 4 it exceeds 76 kJ / m³.
Thus, the tangential stresses reach their maximum values both during assembly and during secondary loading in the same regions, i.e. at the extreme contact points of the rubber element with the surfaces of the rim and the hub of the support roller, therefore, the beginning of fatigue failure for elements of this design will be observed in these areas.

5. Proposal approach of optimal design of rubber elements of roller wheels to increase their durability

The formulation of the problem of optimal design of the internal damping rubber element on the example of the roller wheel shown in figure 1 will be considered. As the objective function, the maximum value of the specific strain energy during secondary loading with radial force will be chosen.

Variational parameters \( \mathbf{x} \) are: the coordinates of the nodal points of the cross section, which determine the shape of the rubber element and geometric dimensions prior to assembly in the initial undeformed state (Figure 3, b); the value of compression acting on the rubber elements in the axial direction after assembling the roller.

The mathematical formulation of the optimization problem has the form: to find a vector \( \mathbf{x} \) that minimizes the objective function

\[
Z = \min_{\mathbf{x}} \left[ \max_{\Omega} (W) \right],
\]

where \( \Omega \) is the cross section zone of the element with a variational boundary \( \Gamma \).

Constraints on variational parameters (geometric) are set by the system of inequations:

\[
x_j^\text{min} \leq x_j \leq x_j^\text{max}, \quad (j=1, 2, \ldots, m),
\]

where \( x_j^\text{min}, x_j^\text{max} \) are the limits of variation of the components of the vector \( \mathbf{x} \); \( m \) – the number of independent parameters.

Functional constraints are set by the system of inequations:

\[
F_z - [F_z] > 0;
\]

\[
\max_{\Omega} ([\tau_{rz}] - [\tau_{rz}]) < 0;
\]

\[
K_{z_{\text{min}}} \leq K_z \leq K_{z_{\text{max}}};
\]

\[
K_{z_{\text{min}}} \leq K_z \leq K_{z_{\text{max}}},
\]

5. Proposal approach of optimal design of rubber elements of roller wheels to increase their durability

Thus, in the problems of optimal design of the internal damping rubber elements of the roller wheels of caterpillar systems, the objective function and functional constraints imposed on the variational parameters are not set analytically, but in the form of an algorithm [13]. To solve such problems, optimization methods related to direct search methods are used [14–16]. In this paper, to solve...
optimization problems, the complex Box method is used [14, 15]. This approach is a modification of the Nelder–Mead simplex method [16], which allows one to take into account the constraints on various parameters, both explicitly and functional limitations.

In Figure 5 the finite element model of the cross section of the rubber element of the internal depreciation of the track roller is presented. The structural parameters of that model were obtained as a result of solving the optimal design problem (4–6) as applied to the structure shown in Figure 1. The coordinates of the nodal points of the finite element model located on the inner and outer diameters of the rubber ring were variable. As a result of solving the optimal design problem, the maximum value of the specific strain energy caused by secondary loading by radial force is reduced by 1.49 times compared with the initial design (Figures 4b and 5c). In this case, the tangential stresses caused by the assembly decreased by 1.24 times (Figures 4a and 5b).

![Figure 5](image-url)

**Figure 5.** The initial state of the section (a), shear stresses after assembly (b), specific strain energy (c), caused by secondary loading of the radial force

During the search for structural parameters of the section, the finite-element model of which is presented in Figure 6, the coordinates of the nodes located on the end surface of the rubber ring were additionally taken as variable parameters of the optimal design problem (4–6), which allowed us to reduce the maximum value of the specific strain energy caused by secondary loading by radial force compared to the original design by 1.85 times (Figure 4b and 6c). In this case, the tangential stresses caused by the assembly decreased by 1.35 times (Figure 4a and 6b).

![Figure 6](image-url)

**Figure 6.** The initial state of the cross section (a), the tangential stress after assembly \( \tau_{zz} \) (b), the specific strain energy (c), caused by the secondary loading of the radial force

6. **Conclusions**

As a result of solving the optimal design problem, the shape and design parameters of the internal damping rubber element of the roller wheel of the tracked system were calculated, which made it possible to reduce the specific strain energy under radial loading by 1.85 times. In this case, the shear stresses caused by the assembly decreased by 1.35 times.
References

[1] Platonov V F 1973 Dinamika i nadezhnost' gusenichnogo dvizhitelya [Dynamics and reliability of tracked propeller] (Moscow. Mashinostroenie) p 232

[2] Sharipov V M 2009 Konstruirovanie i raschet traktorov [Design and calculations of tractors] (Moscow. Mashinostroenie) p 752

[3] Campanelli M Shabana A A Choi J H 1998 Chain Vibration and Dynamic Stress in Three-Dimensional Multibody Tracked Vehicles Multibody System Dynamics 2(3): 277-316

[4] Choi J H, Lee H C and Shabana A A 1998 Spatial Dynamics of Multibody Tracked Vehicles Part I: Spatial Equations of Motion Vehicle System Dynamics 29(1):27-49.

[5] Lee H C, Choi J H and Shabana A A 1998 Spatial Dynamics of Multibody Tracked Vehicles Part II: Contact Forces and Simulation Results Vehicle System Dynamics 29(2):113-137

[6] Korostelev S A, Verbilov A F and Kovalev V V 2013 Influence of elastic characteristics of the road wheels of tracked system on their dynamic loading Izvestia of the Samara Research Academy of Sciences 15(2):515-518

[7] Richter E E, Berezin I J 2009 Energy fracture criterion to estimate the fatigue resistance of constructions with elastoplastic Vestnik Yuzhno-Ural'skogo Gosudarstvennogo Universiteta. Seriya Mashinostroenie (11):73-78

[8] Hromov M K 1983 On pattern of variations of the in fatigue endurance of rubber Kauchuk I Rezina (6):29-38

[9] Zhang J, Xue F, Wang Y, Zhang X, Han S 2018 Strain energy-based rubber fatigue life prediction under the influence of temperature Royal Society Open Science 5(10):180951

[10] Treloar L R G 1975 The Physics of Rubber Elasticity, (Clarendon Press, Oxford) p 310

[11] Green A E, Adkins J E 1960 Large Elastic Deformations and Non-Linear Continuum Mechanics (Clarendon Press, Oxford) p 348

[12] Korostelev S A 2015 Definition the stress-strain state of rubber elements of internal shock absorbers of the road wheels of caterpillar tracks Izvestia of the Samara Research Center of the Russian Academy of Sciences 17(4):793-798

[13] Zhao J, Li Q, Shen X 2008 Finite Element Analysis and Structure Optimization for Improving the Fatigue Life of Rubber Mounts Journal of Macromolecular Science Part A: Pure and Applied Chemistry (6): 479-484

[14] Bunday B 1984 Basic Optimization Methods (London: Edward Arnold, Bedford Square)

[15] Box M J 1965 A New Method of Constrained Optimization and a Comparison with Other Methods Computer Journal (8): 42-52

[16] Nelder J A, Mead R 1965 A Simplex Method for Function Minimization Computer Journal (7): 308-313