Mesons of the rho-family in the P-wave of pion-pion scattering

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Abstract

In the approach, based on analyticity and unitarity and assuming an influence of coupled channels, experimental data on the isovector P-wave of ππ scattering have been analyzed to study ρ-like mesons below 1.9 GeV.

1 Introduction

The investigation of vector mesons is an actual problem up to now due to their role in forming the electromagnetic structure of particles and because, e.g., in the ρ-family, only the ρ(770) meson can be deemed to be well understood [1]. The other ρ-like mesons must be either still confirmed in various experiments and analyses or their parameters essentially corrected. For example, the ρ(1250) meson was discussed actively some time ago [2, 3] and it was confirmed relatively recently in the amplitude analysis of the LASS Collaboration [4] and in combined analysis of several processes [5]. However this state is referred to only slightly in the PDG issue [1] (the relevant observations are listed under the ρ(1450)).

On the other hand, the ππ interaction plays a central role in physics of strongly interacting particles and, therefore, it has always been an object of continuous investigation. Let us note only some recent works devoted also to the theoretical study of the isovector P-wave of ππ scattering. First, there are the analyses of available experimental data on the ππ scattering utilizing the Roy equations [6, 7, 8] and the forward dispersion relations [9, 10], in which, e.g., the low-energy parameters of the ππ scattering were obtained. Second, there are the works in which the low-energy parameters are calculated in chiral theories with the linear realization of chiral symmetry [11, 12].

We have used our model-independent method [13] based on the first principles (analyticity and unitarity) directly applied to analysis of experimental data, aiming at studying the ρ-like mesons below 1.9 GeV and obtaining the ππ-scattering length. Unfortunately this method, using essentially a uniformizing variable, is applicable only in the 2-channel case. Here the ππ and ωπ channels are allowed for explicitly (in the threshold range of the latter, one has observed a deviation from elasticity of the P-wave ππ scattering). Influence of other coupled channels is supposed to be taken into account through the background. In order to investigate the coupling of resonances with these other channels, we apply also multichannel Breit–Wigner forms to generate the resonance poles.

The paper is organized as follows. In Section II, we outline the method of the uniformizing variable in applying it to studying the 2-channel ππ scattering and present results of the analysis of the available data [14]–[16] on the isovector P-wave of ππ scattering. Section III is devoted to analysis of the same data using the Breit–Wigner forms. Finally, in Section VI, we summarize and discuss obtained results.
2 Method of the uniformizing variable

Let the \( \pi\pi \)-scattering \( S \)-matrix be determined on the 4-sheeted Riemann surface with the right-hand branch-points at \( 4m_{\pi^+}^2 \) and \( (m_\omega + m_{\pi^0})^2 \) and also with the left-hand one at \( s = 0 \). It is supposed that influence of other branch points can be taken into account through the background. The Riemann-surface sheets are numbered according to the signs of analytic continuations of the channel momenta

\[
k_1 = \frac{1}{2}\sqrt{s - 4m_{\pi^+}^2} \quad \text{and} \quad k_2 = \frac{1}{2}\sqrt{s - (m_\omega + m_{\pi^0})^2}
\]
as follows: \( \text{sign}(\Im k_1, \Im k_2) = +++, --, --, +-- \) correspond to sheets I, II, III, IV, respectively.

The \( S \)-matrix is supposed to be \( S = S_{\text{res}}S_{\text{bg}} \) where \( S_{\text{res}} \) represents resonances and \( S_{\text{bg}} \), the background. In general, an explicit allowance for the \( (m_\omega + m_{\pi^0})^2 \) branch point would permit us to describe transitions between the \( \pi\pi \) and \( \omega\pi \) initial and final states with the help of the only one function \( d(k_1, k_2) \) (the Jost matrix determinant) using the Le Couteur–Newton relations [17]. Unfortunately, data on the process \( \pi\pi \rightarrow \omega\pi \) are absent.

In Ref. [13] it was shown how one can obtain the multichannel resonance representations by poles and zeros on the Riemann surface with the help of the formulae, expressing analytic continuations of the matrix elements, describing the coupled processes, to unphysical sheets in terms of those on sheet I. It is convenient to start from resonance zeros on sheet I. Then in the 2-channel \( \pi\pi \) scattering, we have three types of resonances: (a) described by a pair of complex conjugate zeros in the \( S \)-matrix element on sheet I and by a pair of conjugate shifted zeros on sheet IV; (b) described by a pair of conjugate zeros on sheet III and by a pair of conjugate shifted zeros on sheet IV; (c) which correspond to a pair of conjugate zeros on sheet I, a pair on sheet III and two pairs of conjugate zeros on sheet IV. The poles on sheet II, III and IV are situated in the same energy points as the corresponding zeros on sheet I, IV and III, respectively. Note that the size of shift of zeros on sheet IV relative to the ones on sheets I and III is determined by the strength of coupling of the channels (here \( \pi\pi \) and \( \omega\pi \)). The cluster kind is related to the nature of resonance.

With the help of the uniformizing variable\(^1\)

\[
v = \frac{(m_\omega + m_{\pi^0})/2}{\sqrt{s - 4m_{\pi^+}^2 + m_{\pi^+}^2} + \sqrt{s - (m_\omega + m_{\pi^0})^2}} \frac{\sqrt{s - (m_\omega + m_{\pi^0})^2}}{\sqrt{s - ((m_\omega + m_{\pi^0})/2)^2 - m_{\pi^+}^2}}, \tag{1}
\]

the considered 4-sheeted Riemann surface is mapped onto the \( v \)-plane, divided into two parts by the unit circle centered at the origin. Sheets I (II), III (IV) are mapped onto the exterior (interior) of the unit disk on the upper and lower \( v \)-half-plane, respectively. The physical region extends from the point \( i \) on the imaginary axis (\( \pi\pi \) threshold) along the unit circle clockwise in the 1st quadrant to the point 1 on the real axis (\( \omega\pi \) threshold) and then along the real axis to the point \( b = \sqrt{(m_\omega + m_{\pi^0} + 2m_{\pi^+})/(m_\omega + m_{\pi^0} - 2m_{\pi^+})} \) into which \( s = \infty \).

\(^1\)The analogous uniformizing variable has been used, e.g., in Ref. [18] in studying the forward elastic \( p\bar{p} \) scattering amplitude and in Ref. [19] in the combined analysis of data on processes \( \pi\pi \rightarrow \pi\pi, K\bar{K} \) in the channel with \( I^GJ^{PC} = 0^{++} \).
is mapped on the $v$-plane. The intervals $(-\infty, -b], [-b^{-1}, b^{-1}], [b, \infty)$ on the real axis are the images of the corresponding edges of the left-hand cut of the $\pi\pi$-scattering amplitude. The (a) resonance is represented in $S(\pi\pi \to \pi\pi)$ by two pairs of poles on the images of sheets II and III, symmetric to each other with respect to the imaginary axis, and by zeros, symmetric to these poles with respect to the unit circle. Note that the symmetry of the zeros and poles with respect to the imaginary axis appears due to the real analyticity of the $S$-matrix, and the symmetry of the poles and zeros with respect to the unit circle ensures a realization of the known experimental fact that the $\pi\pi$ interaction is practically elastic up to a vicinity of the $\omega\pi^0$ threshold.

The resonance part of $S$-matrix $S_{\text{res}}$ becomes a one-valued function on the $v$-plane and, in the $\pi\pi$ channel, it is expressed through the $d(v)$-function as follows:

$$S_{\text{res}} = \frac{d(-v^{-1})}{d(v)}$$

(2)

where $d(v)$ represents the contribution of resonances, described by one of three types of the pole clusters in the 2-channel case, i.e.,

$$d = v^{-M} \prod_{n=1}^{M} (1 - v_n \alpha)(1 + v_n v)$$

(3)

with $M$ the number of pairs of the conjugate zeros.

The background part $S_{\text{bg}}$ is taken in the form:

$$S_{\text{bg}} = \exp \left[ 2i \left( \sqrt{\frac{s - 4m^2}{s}} \right)^3 \left( \alpha_1 + \alpha_2 \frac{s - s_1}{s} \theta(s - s_1) + \alpha_3 \frac{s - s_2}{s} \theta(s - s_2) \right) \right]$$

(4)

where $\alpha_i = a_i + ib_i$, $s_1$ is the threshold of $4\pi$ channel noticeable in the $\rho$-like meson decays, $s_2$ is the threshold of $\rho 2\pi$ channel. Due to allowing for the left-hand branch-point at $s = 0$ in the uniformizing variable (1), $a_1 = b_1 = 0$. Furthermore, $b_2 = 0$ is an experimental fact.

With formulas (2)–(4), we have analyzed data [14]–[16] for the inelasticity parameter ($\eta$) and phase shift of the $\pi\pi$-scattering amplitude ($\delta$): $S(\pi\pi \to \pi\pi) = \eta \exp(2i\delta)$. First, we considered the data of Refs. [14, 15] introducing three ($\rho(770)$, $\rho(1250 - 1580)$ and $\rho(1550 - 1780)$) and four (the indicated ones plus $\rho(1860 - 1910)$) resonances.

From a variety of possible resonance representations by pole-clusters, the analysis selects the following one to be the most relevant: the $\rho(770)$ is described by the cluster of type (a) and the other resonances by type (b). Description in both assumed cases is satisfactory: $\chi^2$/NDF is 204.795/(146−15) = 1.563 (three resonances) and 197.936/(146−19) = 1.558 (four resonances). The background parameters are: $a_2 = 0.0039$, $a_3 = 0.0253$, and $b_3 = -0.0337$ for the case of three resonances and $a_2 = -0.00065$, $a_3 = 0.00432$, and $b_3 = 0.0001$ for four resonances. When calculating $\chi^2$ for the inelasticity parameter, three points of data [15] at 990, 1506 and 1825 MeV have been omitted in both cases as giving the anomalously big

2Other authors also have used the parameterizations with the Jost functions at analyzing the $S$-wave $\pi\pi$ scattering in the one-channel approach [20] and in the two-channel one [21]. In latter work, the uniformizing variable $k_2$ has been used and the $\pi\pi$-threshold branch-point has been neglected, therefore, their approach cannot be employed near by the $\pi\pi$ threshold.
contribution to $\chi^2$. When calculating $\chi^2$ for the phase shift, three points of data [15] at 1841, 1804 and 1882 MeV have been omitted in the case of three resonances and those at 765, 1643 and 1804 MeV in the case of four resonances.

Let us show the obtained pole clusters on the lower $\sqrt{s}$-half-plane in Table 1 (it is clear that there are also complex-conjugate poles on the upper half-plane).

| Three resonances | Sheet | II     | III       | IV          |
|------------------|-------|--------|-----------|-------------|
| $\rho(770)$      | $\sqrt{s_r}$, MeV | 767 - i(74.8) | 786.5 - i(64.1) |
| $\rho(1250)$     | $\sqrt{s_r}$, MeV  | 1249.9 - i(156.5) | 1249.6 - i(153.1) |
| $\rho(1600)$     | $\sqrt{s_r}$, MeV  | 1576 - i(141.7) | 1575.6 - i(78.4) |

| Four resonances  | Sheet  | II     | III       | IV          |
|------------------|--------|--------|-----------|-------------|
| $\rho(770)$      | $\sqrt{s_r}$, MeV | 767.2 - i(74.8) | 785.2 - i(64.1) |
| $\rho(1250)$     | $\sqrt{s_r}$, MeV  | 1249.5 - i(156.4) | 1249.7 - i(152.2) |
| $\rho(1600)$     | $\sqrt{s_r}$, MeV  | 1578.7 - i(142.5) | 1578.2 - i(77.1) |
| $\rho(1900)$     | $\sqrt{s_r}$, MeV  | 1871.5 - i(95.7) | 1894.9 - i(98.2) |

On figures 1, we demonstrate results from our fitting to data [14, 15]. The dashed curve is for the three-resonance description and the solid one for the four-resonance case.

![Graph](image1.png)

Figure 1: The phase shift of amplitude (the left part) and module of matrix element (the right part) of the $P$-wave $\pi\pi$-scattering. The dashed curve – 3-resonance description; the solid curve – 4-resonance one. The data are from Refs. [14, 15].

Though $\chi^2$/NDF is practically the same in both cases, consideration of the obtained parameters and energy dependence of the fitted quantities suggests that the fourth resonance $\rho(1900)$ is desired. Therefore, in the 4-resonance picture, we have also carried out an analysis of data [16] jointly with the already considered data [14, 15]. In Ref.[16], results of two analyses are cited: one uses the $s$-channel helicity amplitude when extracting the $\pi\pi$-scattering amplitude on the $\pi$-exchange pole; in the other, the $t$-channel one is used instead. Therefore, we have taken both analyses as independent. There are given the data for the phase shift of...
amplitude below the $K\bar{K}$ threshold. Comparing these data with the ones of Refs.[14, 15], one can see that the points of the former lie systematically by $1^0-5^0$ higher than the ones of the latter, except for two points of Ref.[14] at 710 and 730 MeV, which lie by about $2^0$ higher than the corresponding points of Ref.[16] and which are omitted in the subsequent analyses.

Since we do not know the energy dependence of the remarked deviations of points, we have supposed a constant systematic error that must be determined in the combined analysis of data. We have obtained a satisfactory description with $\chi^2/NDF = 284.777/(186 − 19) = 1.705$ and with the indicated systematic error equal to $−1.9^0$.

Let us indicate (Table 2) the obtained resonance zero positions on the right-hand $v$-half-plane (there are also zeros symmetric to the indicated ones with respect to the imaginary axis).

| Resonance $\rho$(770) | $v_1 = 1.044245 + 0.215405i$ | $v_2 = 0.926647 − 0.178156i$ |
|----------------------|-----------------------------|-----------------------------|
| $\rho$(1250)         | $v_3 = 1.238348 − 0.032333i$ | $v_4 = 0.806223 − 0.021908i$ |
| $\rho$(1600)         | $v_5 = 1.282453 − 0.007478i$ | $v_6 = 1.282453 − 0.007478i$ |
| $\rho$(1900)         | $v_7 = 1.305758 − 0.005171i$ | $v_8 = 0.766563 − 0.003215i$ |

The background parameters are: $a_2 = 0.00599$, $a_3 = 0.02585$ and $b_3 = −0.00498$.

On figures 2, we demonstrate energy dependences of the analyzed quantities compared with all used experimental data [14]–[16].

![Figure 2](image.png)

Figure 2: The phase shift of amplitude and module of matrix element of the $P$-wave $\pi\pi$-scattering. The data from Refs [14]–[16].

In Table 3, we show the obtained pole clusters on the lower $\sqrt{s}$-half-plane.

Masses and widths of the obtained $\rho$-states can be calculated from the pole positions on sheet II for resonances of type (a) and on sheet IV for resonances of type (b). If the resonance part of the amplitude reads as

$$T_{res} = \frac{\sqrt{s} \Gamma_{el}}{m_{res}^2 − s − i\sqrt{s} \Gamma_{tot}},$$
Table 3: Pole clusters for the $\rho$-like resonances.

| Sheet | II | III | IV |
|-------|----|-----|----|
| $\rho(770)$ | $\sqrt{s_r}$, MeV | $767.1 - i(73.3)$ | $783 - i(65.8)$ | $1249.5 - i(144.6)$ |
| $\rho(1250)$ | $\sqrt{s_r}$, MeV | $1251.2 - i(152.2)$ | $1579.7 - i(75)$ |
| $\rho(1600)$ | $\sqrt{s_r}$, MeV | $1584.6 - i(141.3)$ | $1579.7 - i(75)$ |
| $\rho(1900)$ | $\sqrt{s_r}$, MeV | $1872.3 - i(96.6)$ | $1895.5 - i(95)$ |

we obtain for the masses and total widths, respectively, the following values (in the MeV units):

- for $\rho(770)$: 770.6 and 146.6;
- for $\rho(1250)$: 1257.8 and 289.2;
- for $\rho(1600)$: 1581.5 and 150;
- for $\rho(1900)$: 1897.9 and 190.

3 The Breit–Wigner analysis of $P$-wave $\pi\pi$ scattering

In various works [1], it was shown that the $\rho$-like resonances obtained in the previous section have also other considerable decay channels in addition to those considered explicitly above. It was observed that the $\rho(1450)$ (and/or $\rho(1250)$?) have also the decay modes: $\eta\rho^0 (<4\%)$, $4\pi$ (seen), and $\phi\pi (<1\%)$. The $\rho(1700)$ has also the large branching into the $4\pi$ (large), $\rho2\pi$ (dominant), and $\eta\rho^0$ (seen) channels.

To include explicitly influence of several interested channels and to obtain information about couplings with these channels only on the basis of analysis of the $\pi\pi$-scattering data, we have used 5-channel Breit–Wigner forms in constructing the Jost matrix determinant $d(k_1, \cdots, k_5)$, to generate the resonance poles and zeros in the Le Couteur–Newton relation:

$$S_{res} = \frac{d(-k_1, \cdots, k_5)}{d(k_1, \cdots, k_5)}.$$ (5)

In eq.(5) $k_1$, $k_2$, $k_3$, $k_4$ and $k_5$ are the $\pi\pi$-, $\pi^+\pi^-2\pi^0$-, $2(\pi^+\pi^-)$-, $\eta2\pi^-$- and $\omega\pi^0$-channel momenta, respectively. The $d$-function is taken as $d = d_{res}d_{bg}$ with $d_{res}$, describing resonance contributions, and $d_{bg}$, the background.

The Breit–Wigner form for the resonance part of the $d$-function is assumed as

$$d_{res}(s) = \prod_r \left[ M_r^2 - s - i \sum_{j=1}^{5} \rho_{rj}^2 R_{rj} f_{rj}^2 \right],$$ (6)

where $\rho_{rj} = k_j(s)/k_j(M_r^2)$, $f_{rj}^2/M_r$ is the partial width of resonance with mass $M_r$, $R_{rj}$ is a Blatt–Weisskopf barrier factor [22] conditioned by the resonance spins. For the vector particle this factor have the form:

$$R_{rj} = \frac{1 + \frac{1}{4}(\sqrt{M_r^2 - 4m_j^2 r_{rj}})^2}{1 + \frac{1}{4}(\sqrt{s - 4m_j^2 r_{rj}})^2}.$$ (7)
with radius $r_{rj} = 0.705$ fermi for all resonances in all channels as a result of our analysis. Furthermore, we have assumed that the widths of resonance decays to $\pi^+\pi^-2\pi^0$ and $2(\pi^+\pi^-)$ channels are related with each other by relation: $f_{r2} = f_{r3}/\sqrt{2}$.

The background part $d_{bg}$ is

$$d_{bg} = \exp \left[-i \left(\frac{s - 4m_{\rho0}^2}{s}\right)^3 \left(\alpha_1 + \alpha_2 \frac{s - s_1}{s} \theta(s - s_1)\right)\right]$$

where $\alpha_i = a_i + ib_i$, $s_1$ is the threshold of $\rho 2\pi$ channel (it is clear that $a_2$ and $b_2$ take into account also influence of other channels opened at higher energies than the $\rho 2\pi$ threshold); $b_1$ is taken to be zero.

With formulas (5)–(8) we have carried out the analysis, just as in the previous section, both with three resonances and with the four ones. We have obtained the same reasonable description in both cases: the total $\chi^2/NDF = 316.206/(186−17) = 1.871$ for three resonances and equals $315.254/(186−22) = 1.922$ for four resonances. Let us show the obtained values of resonance parameters for the second case in Table 4. The systematic error of data [16], discussed in the previous section, is equal to $−2.031^0$ in this case. When calculating $\chi^2$ for the inelasticity parameter, three points of data [15] at 990, 1030 and 1825 MeV have been omitted as giving the anomalously big contribution to $\chi^2$. When calculating $\chi^2$ for the phase shift, three points of data [16] have been omitted: one at 790 MeV from the $s$-channel analysis, and two at 790 and 850 MeV from the $t$-channel one. For the background we find: $a_1 = −0.001 ± 0.002$, $a_2 = −0.1065 ± 0.012$, and $b_2 = 0.00715 ± 0.016$.

| Resonance | $\rho(770)$ | $\rho(1250)$ | $\rho(1600)$ | $\rho(1900)$ |
|-----------|-------------|-------------|-------------|-------------|
| $\rho_3$  | $777.57±0.33$ | $1247.8±15.9$ | $1580±4.7$ | $1908.3±38.3$ |
| $\rho_2$  | $343.5±0.75$  | $75.9±7.5$  | $247±5.3$  | $46.4±16$  |
| $\rho_1$  | $25.8±5.9$    | $175.8±40.7$| $236.8±8.7$| $138$       |
| $\rho_{12}$| $36.5±8.4$   | $248.7±57.5$| $334.9±12.3$| $195$       |
| $\rho_{13}$| $316$        | $176±112$   | $159±32$   | $29.7$      |
| $\rho_{14}$| $316$        | $162±116$   | $106.5±40.6$| $34$        |
| $\rho_{15}$| $316$        | $>124$      | $>168$     | $>32$       |

$\rho_{12}$ is taken to be zero.

Note that, in Table 4, we do not show the errors for $f_{42}$, $f_{43}$, $f_{44}$, and $f_{45}$, because insufficiency of data does not permit us to fix reliably these parameters. On figure 3, we demonstrate results from our fitting to data [14]-[16]. We may conclude that whereas the model-independent analysis testifies somehow in favour of existence of the $\rho(1900)$, the Breit–Wigner approach cannot verify this result.

We have also calculated the isovector $P$-wave length of $\pi\pi$ scattering, $a_1^\pi$. Its value is shown in Table 5 in comparison with the ones known from various evaluations in the local [12] and non-local [11] Nambu–Jona-Lasinio (NJL) model and from the ones with the use of Roy’s equations [9, 8, 7] (in the second work one apply also the chiral perturbation theory (ChPT) to construct a precise $\pi\pi$-scattering amplitude at $s^{1/2} \leq 0.8$ GeV).

### 4 Conclusions

The reasonable description of all the accessible experimental data on the isovector $P$-wave of $\pi\pi$ scattering for the inelasticity parameter ($\eta$) and phase shift of amplitude ($\delta$) [14]-[16]
Figure 3: The phase shift of amplitude and module of matrix element of the \( P \)-wave \( \pi \pi \)-scattering. The curves show result of fitting to data [14]–[16] using the Breit-Wigner form.

Table 5: Comparison of values of the \( \pi \pi \) scattering length \( a_1 \) from various approaches.

| \( a_1 \times 10^{-3} m_{\pi}^{-3} \) | References | Remarks |
|--------------------------------|------------|---------|
| 33.98 ± 2.03                 | This paper | Breit–Wigner analysis |
| 34                           | [12]       | Local NJL model     |
| 37                           | [11]       | Non-local NJL model |
| 37.9 ± 0.5                   | [8]        | Roy equations using ChPT |
| 38.4 ± 0.8                   | [9]        | Roy equations |
| 39.6 ± 2.4                   | [7]        | Roy equations |

have been obtained up to 1.88 GeV based on the first principles (analyticity and unitarity) directly applied to analysis of the data. Analysis has been carried out in the model-independent approach using the uniformizing variable (here the satisfactory description is obtained: \( \chi^2/NDF = 1.558 \)) and applying multichannel Breit–Wigner forms to generate the resonance poles and zeros in the \( S \)-matrix (\( \chi^2/NDF = 1.922 \)). The aim of analysis (except for obtaining a unified formula for the \( P \)-wave \( \pi \pi \) scattering amplitude in the whole of investigated energy range) was to study the \( \rho \)-like mesons below 1.9 GeV and to obtain the \( P \)-wave \( \pi \pi \)-scattering length.

For the \( \rho(770) \), the obtained value of mass is a little bit smaller in the model-independent approach (770.6 MeV) and a little bigger in the Breit–Wigner (777.57 ± 0.33 MeV) one than the averaged one (775.5 ± 0.4 MeV) cited in the PDG tables [1], however, it also occurs in analysis of some reactions [1]. The obtained value of the total width in the first case (146.6) coincides with the averaged PDG one (146.4 ± 1.1 MeV) and it is a little bit bigger in the second case (≈ 154.3 MeV) than the averaged PDG value, however, this is encountered also in other analyses [1]. Note that predicted widths of the \( \rho(770) \) decays to the \( 4\pi \)-modes are significantly larger than, e.g., the ones evaluated in the chiral model of some mesons based on the hidden local symmetry added with the anomalous terms [23].

The second \( \rho \)-like meson has the mass 1257.8 MeV in the 1st analysis and 1247.8 ± 15.9 in the 2nd one. This differs significantly from the mass (1459 ± 11) of the 2nd \( \rho \)-like meson cited in the PDG tables [1]. We told already in Introduction that the \( \rho(1250) \) meson was discussed...
actively some time ago [2, 3], and next the evidence for it was obtained in some analyses [4, 5]. To the point, if this state is interpreted as the first radial excitation of the $1^+1^{--}$ state, then it lies down well on the corresponding linear trajectory with a universal slope on the $(n, M^2)$ [24] (n is the radial quantum number of the $q\bar{q}$ state), while the meson with mass $M = 1450$ MeV turns out to be considerably higher than this trajectory.

The third $\rho$-like meson turns out to have the mass 1580 MeV rather than 1720 MeV cited in the PDG tables [1]. Note that in a number of previous analyses of some reactions one has also obtained the resonance with mass near 1580 MeV [1]. However, some time ago it was shown that the 1600-MeV region contains in fact two $\rho$-like mesons. This was made on the basis of investigation of the consistency of the $2\pi$ and $4\pi$ electromagnetic form factors and the $\pi\pi$-scattering length [25] and as a result of combined analysis of data on the $2\pi$ and $4\pi$ final states in the $e^+e^-$ annihilation and photoproduction [26]. We assume this possibility, i.e., that in the energy range 1200–1800 MeV there are three $\rho$-like mesons, but for the final conclusion the combined analysis of several processes with which the investigated resonances are appreciably coupled have to be performed. Note also a rather big obtained coupling of these $\rho$-like mesons with the $4\pi$ channels.

Finally, as to the $\rho(1900)$, in this energy region there are practically no data on the $P$-wave of $\pi\pi$ scattering. The model-independent analysis, maybe, somehow testifies in favour of existence of this state, whereas the Breit–Wigner one gives the same description with and without the $\rho(1900)$. For more definite conclusion about this state, the $P$-wave $\pi\pi$ scattering data above 1.88 GeV are needed. Furthermore, the combined analysis of coupled processes should be carried out.

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References

[1] W.-M. Yao et al. (PDG), J. Phys. G 33, 1 (2006).
[2] N.M. Budnev, V.M. Budnev, and V.V. Serebryakov, Phys. Lett. B 70, 365 (1977).
[3] S.B. Gerasimov and A.B. Govorkov, Z. Phys. C 13, 43 (1982).
[4] D. Aston et al., Nucl. Phis. Proc. Suppl. B21, 105 (1991).
[5] T.S. Belozerova and V.K. Henner, Phys. Elem. Part. Atom. Nucl. 29, part 1, 148 (1998).
[6] I. Caprini, G. Colangelo, J. Gasser, and H. Leutwyler, Phys. Rev. D 68, 074006 (2003).
[7] R. Kamiński, L. Leśniak, and B. Loiseau, Phys. Lett. B 551, 241 (2003).
[8] I. Caprini, G. Colangelo, and H. Leutwyler, Int. J. Mod. Phys. A 21, 954 (2006).
[9] J.R. Peláez and F.J. Yaduráin, Phys. Rev. D 71, 074016 (2005).
[10] R. Kaminski, J.R. Pelaez, and F.J. Yndurain, Phys. Rev. D 74, 014001 (2006); ibid., 079903 (2006), Erratum.

[11] A.A. Osipov, A.E. Radzhabov, and M.K. Volkov, hep-ph/0603130.

[12] V. Bernard, A.A. Osipov, and U.G. Meissner, Phys. Lett. B 285, 119 (1992).

[13] D. Krupa, V.A. Meshcheryakov, and Yu.S. Surovtsev, Nuovo Cimento A 109, 281 (1996).

[14] S.D. Protopopescu et al., Phys. Rev. D 7, 1279 (1973).

[15] B. Hyams et al., Nucl. Phys. B 64, 134 (1973).

[16] P. Estabrooks and A.D. Martin, Nucl. Phys. B 79, 301 (1974).

[17] K.J. Le Couteur, Proc. Roy. Soc. A 256, 115 (1960); R.G. Newton, J. Math. Phys. 2, 188 (1961).

[18] B.V. Bykovsky, V.A. Meshcheryakov, and D.V. Meshcheryakov, Yad. Fiz. 53, 257 (1990).

[19] Yu.S. Surovtsev, D. Krupa, and M. Nagy, Eur. Phys. J. A 15, 409 (2002).

[20] J. Bohacik and H. Kühnelt, Phys. Rev. D 21, 1342 (1980).

[21] D. Morgan and M.R. Pennington, Phys. Rev. D 48, 1185 (1993).

[22] J. Blatt and V. Weisskopf, *Theoretical nuclear physics*, Wiley, N.Y., 1952.

[23] N.N. Achasov and A.A. Kozhevnikov, Phys. Rev. D 71, 034015 (2005).

[24] A.V. Anisovich, V.V. Anisovich, and A.V. Sarantsev, Phys. Rev. D 62, 051502 (2000).

[25] C. Erkal and M.G. Olsson, Z. Phys. C 31, 615 (1986).

[26] A. Donnachie and H. Mirzaie, Z. Phys. C 33, 407 (1987).