Robust multiculturality emerges from layered social influence

Federico Battiston,1 Vincenzo Nicosia,1 Vito Latora,1 and Maxi San Miguel2
1School of Mathematical Sciences, Queen Mary University of London, London E1 4NS, United Kingdom
2IFISC (CSIC-UIB), E-07071 Palma de Mallorca, Spain
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Why is our society multicultural? Based on the two mechanisms of homophily and social influence, the classical model for the dissemination of cultures proposed by Axelrod predicts the existence of a fragmented regime where different cultures can coexist in a social network. However, in such model the multicultural regime is achievable only when a high number of cultural traits is present, and is not robust against cultural drift, i.e. the spontaneous mutations of agents’ traits. In real systems, social influence is inherently organised in layers, meaning that individuals tend to diversify their connections according to the topic on which they interact. In this work we show that the observed persistence of multiculturality in real-world social systems is a natural consequence of the layered organisation of social influence. We find that the critical number of cultural traits that separates the monocultural and the multicultural regimes depends on the redundancy of pairwise connections across layers. Surprisingly, for low values of structural redundancy the system is always in a multicultural state, independently on the number of traits, and is robust to the presence of cultural drift. Moreover, we show that layered social influence allows the coexistence of different levels of consensus on different topics. The insight obtained from simulations on synthetic graphs are confirmed by the analysis of two real-world social networks, where the multicultural regime persists even for a very small number of cultural traits, suggesting that the layered organisation of social interactions might indeed be at the heart of multicultural societies.

Our understanding of societies, and social dynamics more in general, is increasingly taking advantage of concepts, models and methods from statistical and computational physics [1–4]. Within this framework, the validation of sociological theories relies on the construction of appropriate mechanistic agent-based models [5] which can be tested against existing and upcoming large data sets.

The existence of multiculturality and group boundaries is a well-established feature of social systems [1–5] and much effort has been devoted into explaining possible mechanisms able to reproduce such empirical finding. In particular, cultural diversity appears as a striking phenomenon, supposedly in contrast with the widely acknowledged principle of social influence [6]. According to such mechanism, changes in the cultural traits of an agent are influenced by the acquaintances and friends of that agent facilitating the local convergence towards a set of common cultural traits and promoting homogeneity across the populations. For instance, homogenous absorbing states are typical of imitative dynamics where the state of the individuals is described through a scalar variable, as for the voter model [7] in finite size populations.

An interesting solution to reconcile social influence and the emergence of cultural diversity (polarization) commonly observed in human societies was suggested by Robert Axelrod, who proposed in 1997 a simple agent-based mechanistic model of dissemination of cultural traits [8]. In the original model the state of individuals is not described through a scalar variable, but each agent is endowed with a set of cultural features $F$, each of them taking one of a number of different cultural traits $q$. Based on pairwise interactions among agents, together with social influence the model mimics another important social principle, known as homophily [9–11], i.e., the tendency of individuals to connect and interact preferentially with similar ones. As a result of pairwise interactions, imitation still occurs but is limited to the update of one feature at a time. Homophily and social influence acting together constitute a self-reinforcing dynamics leading to local homogenization. However, a main result of the analysis of Axelrod was that in spite of local convergence, global polarization was possible: agents become more similar by local interactions, but cleavages among different cultural groups are created so that these groups do not longer interact. The overall result is the possible emergence of a globally polarized or multicultural state with coexistence of different cultural groups. A quantitative analysis of the model [12] unveiled a non-equilibrium phase transition at a critical number of cultural traits $q_c$ from a mono-cultural phase (global culture) to a polarized or multicultural phase, where several groups with different cultural traits survive.

However, it was found that the multicultural phase is achievable only for a high initial number of traits $q$ in the population. In addition, the multicultural phase is not stable under cultural drift, meaning that the spontaneous tendency of agents to modify their cultural traits independently of their environment drives the system towards a monocultural state [13–14]. Different mechanisms have been invoked to account for robust cultural diversity, such as the plasticity of the social relations among agents, so that the social networks coevolves with the dynamics.
of the cultural state \cite{15}. Other alternative proposals focus on modifications of the form of the local interactions, for instance based on assimilation-contrast theory \cite{16}, also known as bounded confidence \cite{17, 19}. It has been shown how integrating metric cultural states \cite{20, 21}, as opposed to nominal states, the bounded confidence mechanism can lead to robust cultural diversity \cite{22}. Another type of modification \cite{23} addresses the nature of social influence in line with other models of social contagion \cite{24, 27}: the dyadic interpersonal influence of the original model is replaced by a mechanism in which an agent modifies its cultural state depending on the state of all agents in her neighborhood. Finally, the mechanism of social differentiation in which agents tend to increase cultural differences has also been analyzed in \cite{28, 29}.

In this work we propose an alternative mechanism to account for robust multiculturality that naturally brings together social influence and the nature of the social network of interactions with no need of introducing any additional dynamical feature. In real societies the relationships among individuals are inherently layered, i.e. individuals tend to interact with different neighbors on different topics. Hence, interactions in social systems are better described in terms of multiplex networks \cite{30, 31}, i.e., networks composed of many layers, each one of them describing a particular type of interaction, with a wide range of overlap between the links at the different layers as found in multiplex empirical data \cite{32, 33}. We model cultural dissemination associating each cultural feature to a different layer of a multiplex network. Social influence becomes now a layered mechanism in which an agent is allowed to imitate only a subset of all the cultural features of its neighbors, namely those for which there exists a link on the corresponding interaction layer. The consequences of layered social influence are strongly dependent on the structural overlap among different layers, so that the overlap becomes the control parameter for the nonequilibrium transition in the system. We remark that the multiplex topology has been already successfully applied in other types of social dynamics, such as opinion formation, giving rise to novel critical behavior \cite{37, 38}.

We find that layered social influence in synthetic and empirical multiplex networks with heterogeneous layers easily leads to a global state of cultural diversity. This state exists for any number of cultural traits provided that the interaction patterns into the population are sufficiently layered, i.e. the value of the structural overlap is below a critical point. We remark that this is a qualitative shift in the behavior of the system, only achievable in multiplex networks, and it is in agreement with empirical evidence of fragmented states even in social contexts with a limited number of available cultural choices \(q\). Moreover, differently from the fragmented states found by Axelrod, such new multicultural state is robust against cultural drift, thus providing an explanation for the persistence of multiculturality we experience in real-world society. In addition, we find novel phases of cultural diversity in which a global culture for a number of cultural features coexist with polarization in other features, a situation reminiscent of the so-called chimera states \cite{39, 41}. More in general, we observe that different levels of homogeneity may be achieved on different cultural features, depending on the heterogeneity of the structure of interactions across the layers. Finally, we investigate two layered social networks from the real-world, showing how considering the nature of the different interactions promotes cultural fragmentation in the population.

I. MODEL

We propose to take into account the multilayer structure of real social interactions by modelling layered social influence between pairs of individuals, as shown in Fig. 1. We describe the specific pattern of social influence at each cultural level \(f\) by mean of a multilayer network, i.e. a set of \(F\) adjacency matrices \(A[f] = \{a_{ij}^f\}\), \(f = 1, \ldots, F\), where \(a_{ij}^f = 1\) if agents \(i\) and \(j\) influence one another on topic \(f\) \cite{30, 32}.

Given two individuals \(i\) and \(j\), the normalized total number of connections between them is measured by the so-called edge overlap, \(o_{ij} = \frac{1}{F} \sum_{f=0}^F a_{ij}^f\) which is a number ranging in the interval \([0,1]\) \cite{32}. We can then define the structural overlap \(o\) of a multilayer network as the average number of links among connected individuals:

\[
o = \frac{\sum_i \sum_{j \neq i} o_{ij} \Theta(o_{ij})}{F \sum_i \sum_{j \neq i} \Theta(o_{ij})},
\]

where \(\Theta(x)\) is the Heaviside function, i.e. \(\Theta(x) = 1\) if \(x > 0\), and \(1/F \leq o \leq 1\). When \(o \approx 1\) the layered nature of social influence will be negligible. Indeed, if the peculiar nature of the links is neglected, the corresponding aggregated network of social interactions (formally defined at the end of this Section) will be similar to the interaction networks relative to each feature. Conversely, when \(o \approx 1/F\) interactions are extremely diversified across the different topics. As a consequence, the aggregated network will be very different from the interactions occurring at each layer, hinting at possible different outcomes between the layered model and the original model on the aggregated network, to which we refer as the classical case.

The cultural profile of each agent is described by a feature vector of \(F\) integer variables \(\{s_1, \ldots, s_F\}\). Each feature \(f\), with \(f = 1, \ldots, F\), takes one of \(q\) possible traits, \(s_f = 0, 1, \ldots, q - 1\). Because of the layered structure of interactions, when an agent interacts with a neighbor it only considers the subset of features where the two are connected. According to the principle of homophily, the probability of a social interaction between two connected agents \(i\) and \(j\) is assumed to be proportional to their
FIG. 1. Social influence is inherently layered. Social systems are multiplex networks, in which each layer describes a different type of relations or interactions. Usually, the individuals tend to differentiate their social patterns, choosing recipients according to the subject of the interactions and viceversa. As an example, the couple of agents $i$ and $j$ discusses sports and politics, the couple $j$ and $k$ discusses sports and religion, while $m$ and $k$ can only discuss politics. In classical models of social influence such as the Axelrod model, the cultural profile of each individual is a vector of different features (sport, religion, politics), shown as a stack of cylinders, and each feature can take one of a series of different traits, represented as the colors of the cylinders. However, the peculiar nature of each interaction is neglected and all connections are treated together. Consequently, interactions at any level can potentially impact (and thus modify) any cultural trait (a). For instance, the existence of a link between nodes $i$ and $j$, and the different traits of the pair on the feature “religion”, suggest that this bond is still active. According to social influence, one of the two agents will eventually absorb the religious trait of the other one, making the pair equal on all the three features. Conversely, we propose a layered cultural influence model where the structure of the relationships pertaining to different social spheres is preserved and explicitly taken into account, assuming that each layered interaction can affect the corresponding cultural features (b). Agents $i$ and $j$ are not linked at the religious layer. As a consequence, since the two individuals already have the same sport and politics traits, the interaction between $i$ and $j$ is frozen. Indeed, the two individuals already reached consensus at all levels where they are linked, and imitation can not concern religious beliefs, since religion is not discussed by them.

layered cultural overlap, which we define as:

$$\omega_{ij} = \frac{1}{\sum_{f=1}^{F} a_{ij}^{[f]}} \sum_{f=1}^{F} a_{ij}^{[f]} \delta(s_f(i), s_f(j)). \tag{2}$$

where $\delta(s_f(i), s_f(j))$ is the Kronecker’s delta function. Notice that the cultural overlap of two agents is proportional to the number of shared traits on the layers in which the two agents are directly connected. When two agents interact, their cultural profiles are updated according to layered social influence. In practice, when agent $i$ interacts with one of its neighbors $j$, imitation occurs, and $j$ aligns one of its cultural features to that of $i$, choosing only among cultural features in which $i$ and $j$ interact, i.e., only those features for which a link between $i$ and $j$ exist in the corresponding layer.

The dynamics of the model proceeds in epochs. During each epoch we consider each of the $N$ agents in a random order, and perform the following steps:

1. Consider agent $i$, chosen uniformly at random.
2. Consider a neighbor $j$ of $i$, chosen uniformly at random among all the neighbours of $i$ (nodes with a link to $i$ on at least one layer).
3. Let $i$ and $j$ interact with probability $\omega_{ij}$ in Eq. 2.
4. If the interaction takes place, choose at random a feature $f$ such that $a_{ij}^{[f]} = 1$ and set $s_{f(j)} = s_{f(i)}$.

The dynamics stops when all pairs of connected agents $i$ and $j$ have either $\omega_{ij} = 0$ or $\omega_{ij} = o_{ij}$. This implies the existence and stability of interacting pairs of agents who only share a limited number of cultural features, a realistic property observed in real systems but not achievable in the classical model.

In fact, if we neglect the layered structure of interactions, we are implicitly assuming that any link between two individuals potentially allows for an exchange of traits over any feature. In that case, the structure of interactions in the systems can be described by the corresponding aggregated network, i.e. the single-layer graph $A = \{a_{ij}\}$ associated to the original multiplex such that $a_{ij} = 1$ if $i$ and $j$ have a link in at least one of the $F$ layers. We note that the aggregate network does not lead to any discrepancy with the structure of the original multilayer networks for the dynamics of cultural diffusion when $o = 1$.

II. RESULTS

In this Section we show how the presence of layered interactions, which mimic the multilayer structure of real-world societies [39, 35], can explain empirical observations on the presence of multiculturalism in real systems. In order to study the impact of layered interactions on culture diffusion, we model the structure of a social net-
work by tuning its level of structural overlap $o$. We considered multilayer networks with $F$ layers, each of them being an Erdős-Rényi graph with the same number of links such that $\langle k[f] \rangle = 1/\sum_{i,j} a_{ij}^{[f]} = 4 \forall f = 1, \ldots, F$. In the case of $F$ identical layers, the structural overlap is maximum and we have $o = 1$. Starting with this configuration, we then assign to each edge of each layer a probability $p$ to be rewired at random. Since each edge is rewired independently, we can express the structural overlap $o$ has a function of $p$ (see Materials and Methods). In the larger, the larger the value of $p$, the lower the value of $o$, with $o = 1$ for $p = 0$ and $o = 1/F$ for $p = 1$, and the denser the corresponding aggregated network.

In Fig. 2(a) we plot the size of the largest cultural component $S$ as a function of the number of cultural traits $q$ and for different values of the structural overlap $o$, both for the classical Axelrod model (a), which runs on the aggregate graph associated to the multiplex network, and for the layered social influence model (b). Two individuals are considered to be part of the same cultural component if they are connected on at least one layer and have equal cultural traits for all $F$ features. In the classical model, a lower value of overlap by making the aggregated network denser is favouring globalisation, i.e. $S \approx 1$, increasing the critical number of cultural traits $q_c(o)$ at which multicritularity appears, i.e. $S \approx 0$.

For $o = 1$ (i.e., when all the layers are identical) the classical and layered model are undistinguishable, and multicritularity can only be achieved for large values of $q$. For instance, in the population considered in Fig. 2(a), multicritularity can be achieved only for $q > 140$. In the layered social influence model, where an agent can absorb the cultural trait of his neighbor only if they are connected instead, $q_c(o)$ becomes smaller as $o$ decreases, until it vanishes at critical value $o_c(q,[r_1,...,r_4]) = 1$, whose value decreases with the overlap, and goes to 0 for $o$ smaller than a critical value. This means that, for sufficiently small values of the overlap, multicritularity is always achieved independently from the values of $q$. Under the presence of drift $r$, the critical overlap sustaining globalisation depends both on $q$ and $r$. Noticeably, the polarised phase obtained for $o < o_c(q,r = 0) \approx 2/F$ is stable under the presence of noise. This is shown in (b), where we plot with different dashes the critical lines relative to four increasing values of noise $r = r_1, \ldots, r_4$. Conversely, the multicultural states on the classical model are not stable in presence of drift. The two projections of the phase diagram for layered social influence are reported in (c) and (d). Transitions from globalisation to multicritularity are steep both as a function of the number of cultural traits and the overlap.

FIG. 2. Modeling classical versus layered social influence. The size of the largest cultural component $S$ in the classical (a) and in the layered (b) social influence model is shown as function of the number of cultural traits $q$, and for multiplex networks with $N = 625$ nodes, $F = 10$ layers, $\langle k \rangle = 4$, and a tuneable structural overlap $o$. In the case of classical social influence, a decrease in $o$ increase the density of the aggregated networks, hence increasing the value of $q_c(a)$. Conversely, for layered social influence, the critical number of cultural traits $q_c(a)$ separating globalisation (red region) from multicritularity (blue region) decreases with the overlap, and goes to 0 for $o$ smaller than a critical value. This means that, for sufficiently small values of the overlap, multicritularity is always achieved independently from the values of $q$. Under the presence of drift $r$, the critical overlap sustaining globalisation depends both on $q$ and $r$. Noticeably, the polarised phase obtained for $o < o_c(q,r = 0) \approx 2/F$ is stable under the presence of noise. This is shown in (b), where we plot with different dashes the critical lines relative to four increasing values of noise $r = r_1, \ldots, r_4$. Conversely, the multicultural states on the classical model are not stable under the presence of drift. The two projections of the phase diagram for layered social influence are reported in (c) and (d). Transitions from globalisation to multicritularity are steep both as a function of the number of cultural traits and the overlap.
position of the critical line separating globalisation from fragmentation under four increasing values of noise, namely \( r_1 = 3 \times 10^{-5} \), \( r_2 = 5 \times 10^{-5} \), \( r_3 = 6 \times 10^{-5} \) and \( r_4 = 7 \times 10^{-5} \). We note that the, given a number of cultural traits \( q \), the critical value of overlap needed to sustain globalisation depends on the amount of noise in the system, i.e. \( o_c = o_c(q, r) \). In analogy to the classical case, for \( o > o_c(q, r) \) the multicultural region appearing at high values of \( q \) is unstable under cultural drift. Conversely, the polarised phase obtained for \( o < o_c(q, r) \) is not affected by the presence of noise and allows to explain the persistence of multicultural states that we observe in real-world societies. Noticeably, in the limit \( q \to 2 \) the value of \( o_c(q, r) \) appears to be independent of \( r \) and approaching the value of \( 2/F \). Conversely, in the classical model shown in (a) all multicultural states with \( q < 300 \) are destroyed already for a drift as small as \( r_3 = 6 \times 10^{-5} \), no matter the value of the structural overlap. We remark that the instability of multicultural states is meant under the introduction of a moderate level of drift. Conversely, if drift is too high, the system enters a noisy dynamical state, describing an unrealistic society where spontaneous mutation accounts for most of the cultural chances and the effect of social influence widely neglected.

Another property of layered social influence is that it allows for different levels of consensus on single topics to coexist, possibly resulting in the emergence of globalisation on some topics, i.e. \( S[f] \approx 1 \) for some features \( f \), and the persistence of fragmentation in some other ones, \( S[f] \approx 0 \) for some other \( f \). We call this regime feature-level consensus, or topical consensus. Two individuals \( i \) and \( j \) are considered to be part of the same topical component \( f \) if they are connected on that layer, i.e. \( a_{ij}^f = 1 \), and share the same cultural trait on feature \( f \), i.e. \( s_{f(i)} = s_{f(j)} \). We remark that this realistic phenomenon can not be achieved with the classical version of the model, where at the absorbing state the largest component on each layer is as large as the one computed taking into account all topics simultaneously.

In Fig. 3, we show how feature-level consensus may be possible only as an effect of the difference in the number of social interactions related to the different features, i.e. different layers’ densities. Let us consider two sets of layers. The first set is made of five equal layers with average degree \( \langle k[f] \rangle = 8 \), whereas the second is made of five equal layers with average degree \( \langle k[f] \rangle = 4 \). When all edges of the layers in the second set also belong to those in the first set, the structural overlap in the system is maximised, \( o = o_{\text{max}} \). Conversely, when no edges in the low-density layers are also present in the high-density ones the overall will be minimum, \( o = o_{\text{min}} \), and the system behaves as two isolated five-layer classical models with different average degree. Hence, because of the dif-

FIG. 3. Global and feature-level consensus. A toy social system with three cultural features (a). With classical social influence (top), such system is still far from the equilibrium. Conversely, with layered social influence (bottom), the system is already in the absorbing state, and the largest cultural components at each layer differ from the global one. In (b) we show the phase diagram of the global largest cultural component of the system for classical (top) and layered (bottom) social influence as a function \( q \) and for different values of the overlap \( o \) in networks with \( N = 625 \), \( F = 10 \) and five low-density layers ((\( k \)) = 4) and five high-density layers ((\( k \)) = 8). In the classical case, the system reach a state where either globalization (red region) or fragmentation (blue region) is achieved for all features. In the layered case, instead, when \( o \) is smaller than a critical value \( o_2 \) , it is possible to have globalisation in the five high-density layers together with multiculturality in the five low-density ones (orange region). In (c), for the layered case, we plot the values of \( S \) as a function of \( q \) for one low-density and one high-density layer, and for three different values of the overlap, namely \( o_{\text{min}}, o_1 \) and \( o_2 \). In (d), for the same values of overlap, we show the average and the standard deviation of the largest component \( S[f] \) over all layers.
For $a = a_{\text{min}}$, the critical values separating globalisations from multiculturalities for the low-density and high-density layers are different. While both sets of layers are globalised for $q < 60$ and fragmented for $q > 120$, in the range $60 < q < 120$ globalisation is achieved only in the high-density layers, for which $S^{\{l\}} \approx 1$, and the low-density ones are instead polarised, $S^{\{l\}} \approx 0$. Strikingly, such property of the system is not peculiar of the case $a = a_{\text{min}}$, but it is preserved for a finite range of values of structural overlap of to a critical value $o_2$. Such region is highlighted in orange in the diagram shown in Fig. 3(b, bottom), and separates the globalised region (red) from the fragmented one (blue) when the value of overlap is sufficiently low.

The value of $S^{\{l\}}$ for low-density and a high-density layers is also shown in Fig. 2(c), as well as the average value of the largest cultural components $S^{\{l\}}$ and its standard deviation. The standard deviation $\sigma(S^{\{l\}})$, approximately 0 for both the globalised and fragmented regions, takes values close to 0.5 in the mixed orange region, where consensus only exists on some layers.

We remark that higher heterogeneity in the structure of the layers produces even richer and more diverse patterns of consensus across the different cultural features. A typical case is when the activity of the nodes on the layers is heterogenous [43], i.e. when not all agents are involved in discussion with other individuals on all topics, preventing the spread of a cultural traits across the whole population for some given features. This is the case of many real-world systems, where the structure of interactions is often inherently layered and diverse levels of activity occur at the different layers (see Materials and Methods). As a test-case, we study the dynamics of our model with layered social influence on two multilayer social systems, namely the network of Indonesian terrorists [32] and the collaboration network of the Pierre Augier observatory [34] (see Methods for details).

In Fig. 4 we plot for both systems, as a function of $q$, the size of the largest cultural components found in the model with layered social influence (dashed lines) compared with those obtained with the classical model simulated on the corresponding aggregated networks (solid lines). For the terrorists network (panel (a)), a giant component of the order of the number of nodes appear for small $q$ even in the layered case, while for $q \approx 20$ fragmentation appears on all features. The classical model predicts instead $q_c \approx 50$. For the Pierre-Augier collaboration network (panel (b)), in the classical model we have $S \approx 1/N$ only for $q > 1000$. Conversely, the layered model predicts multiculturality already for $q = 2$, producing a qualitative different result. Such striking difference between the layered and classical dynamics is due to the relatively low level of structural overlap and to the heterogeneity in the activity and density of the layers of the multiplex. Such qualitative different behaviors highlights the importance of the underlying structure of social interactions to understand cultural patterns in real-world societies.

III. DISCUSSION

Reconciling the phenomenon of social influence and imitation with the empirical evidence of multiculturality at a global scale is a long-debated problem. The model for the dissemination of culture introduced by Axelrod in 1997 [8] has proven to be able to produce socially diversified states in spite of the existence of a locally polarising rule. However, such multicultural states occur only for a very high number of cultural traits $q$ and are not robust against cultural drift. In spite of several proposals [15, 22, 23, 25, 29], explaining the emergence and persistence of multicultural states that we experience in our life still appears as an open problem. In our work we considered agents having different interaction patterns according to the different cultural features. This is a
property of many social systems, where individuals usually choose the recipients of their discussion according to the content of their messages and vice versa. By constraining the imitation of the cultural traits to the cultural features where two individuals are actually linked, we naturally introduce the notion of layered social influence, inherently connected to the average structural overlap $o$ of a social system. Such definition implies the existence of stable states where connected agents might not have completely equal or different cultural profiles, but instead share just a limited number of features.

A main finding is that a robust multicultural regime emerges as a natural consequence of taking into account the actual structure of social interactions, which are inherently layered. Interestingly, when the structural overlap among the layers is small, i.e. when connections are not too much redundant across layers, only multicultural states will be allowed, independently of the actual number of possible cultural traits, suggesting a qualitative shift in the behavior of the system. This finding is also in agreement with the empirical evidence of socially fragmented societies even under a limited number of cultural choices. Moreover, differently from the multicultural regime predicted when the layering of social influence is not taken into account, and achievable only for a large number of traits, such novel multicultural regime is robust to the presence of cultural drift, providing a potential explanation to the persistence of global diversity over time.

Another interesting aspect of layered social influence is the ability to explain the existence of a novel regime where globalisation appears only on a limited number of cultural features, while the remaining ones are characterised by multiculturality. The coexistence of different levels of multiculturality in different aspects of a cultural profile is in fact another typical property observed in human societies.

In conclusion, the model of layered social influence proposed in this paper, provides a mechanism leading to robust multiculturality, revealing the important role played by a multilayer organisation of social interactions in avoiding globalisation, and pointing out the effect of heterogeneity of layer densities in the emergence of partially globalised regimes. While previous proposals to account for robust multiculturality invoke different forms of interactions among agents in an aggregated social network, we provide here an alternative mechanism not based in the form of the interaction, but on the structural properties of the social network. From this new perspective, the relevant control parameter of polarization-globalization transitions is no longer the number of cultural traits per feature, but a structural property, namely the overlap parameter accounting for the heterogeneity of social links among the different layers of the social network. The relevance of the structural properties of multilayered social interactions, illustrated here for cultural dissemination, is far reaching, and should be taken into account to reconsider a number of results based on models of social interactions among agents in aggregated networks.

IV. MATERIALS AND METHODS

Constructing synthetic multilayer social networks. In this section we focus on the model to produce synthetic multilayer networks with different values of structural overlap $o$. Let us consider a multiplex network with $F$ layers, each with the same number of links $K^{[f]}$, so that we have $K^{[f]}F$ links in total. We then consider the aggregated network obtained by collapsing all the layers. If all the layers are identical, the aggregated graph will have $K = K^{[f]}F$ edges. Hence $o = \frac{1}{F} \delta o_m = 1$ for $p = 0$. When $p \neq 0$ and links are rewired independently at each layer, the number of edges in the aggregated networks increases. If we neglect the probability that two links of different layers are rewired in the same position, the number of edges in the aggregated network becomes equal to

$$K = \sum_{m=1}^{F} \binom{F}{m} F^{2m} M^{F-m} K^{[f]} + \sum_{m=0}^{F} \binom{F}{m} (1-p)^m p^{F-m} K^{[f]} (F-m)$$

However the total number of connections in the multiplex remains $K^{[f]}F$. Hence

$$o \approx \frac{1}{\sum_{m=0}^{F} \binom{K}{m} (1-p)^m p^{F-m} (1 - \delta o_m + F - m)}$$

higher values of rewiring correspond to lower values of overlap and vice versa, with the limiting cases of $o = 0$ for $p = 0$ and $o = 1/F$ for $p = 1$. Given the average degree $\langle k^{[f]} \rangle$, the average number of neighbours $\langle k \rangle$ in the aggregated network can be obtained as $\langle k \rangle = \langle k^{[f]} \rangle / o$. Hence, the higher the $p$, the lower the $o$ and the higher the density of the aggregated network. The results shown in Fig.2 have been obtained for a system with $N = 25^2$ agents. Each interaction layer $f$ has average degree $\langle k^{[f]} \rangle = \frac{1}{2N} \sum_{i,j \neq i} o^{[f]}_{ij} = 4$. In Fig.3 we consider layers with different average degrees $\langle k^{[f]} \rangle = \frac{1}{2N} \sum_{i,j \neq i} o^{[f]}_{ij}$. Five of them are equal networks with $\langle k^{[f]} \rangle = 4$, the other five are equal networks with $\langle k^{[f]} \rangle = 8$. We consider different combinations of the two sets of layers such in a way to span the range of values of structural overlap $o \in [o_{\text{min}}, o_{\text{max}}]$. When $o = o_{\text{max}}$, all edges in the emptier layers are also present in the denser ones. Conversely, when $o = o_{\text{min}}$ no edge of the emptier layer is shared with the denser ones, and the system behaves as two five-layer decoupled classical models.

Data sets. In Fig.5 we apply the classical and layered social influence models to two real-world multilayer networks with layered interaction patterns and different activities of the node and layers. A node $i$ is defined as active on layer $f$, i.e. $b^{[f]}_{i} = 1$, if $k^{[f]}_{i} > 0$. Otherwise it is consider inactive, i.e. $b^{[f]}_{i} = 1$. The first dataset is the network of Indonesian terrorists presented in [32].
which has $F = 3$ layers, corresponding to trust, common operations and exchanged communication across 78 individuals, most of which are active on all layers, and has a structural overlap $o \approx 0.4817$, with moderate difference in the layers’ activity. The second data set is the multi-layer collaboration network of the Pierre-Augier observatory [34], which consists of $F = 16$ layers. To allow, at least potentially, complete globalisation in the system, we selected the subset of individuals who belong to the giant component of the aggregated network. Such system includes 475 individuals. The system has a structural overlap of $o \approx 0.069$ and high heterogeneity in the activity of its members.

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