Quadrupole topological phase and robust corner resonance in Kekulé hexagonal electric circuit

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Abstract

Two-dimensional (2D) quadrupole topological insulators, featured by topologically protected 0D corner modes, have recently attracted tremendous interest in condensed matter and materials physics. Herein, we construct a specific electric circuit made of capacitors and inductors forming a 2D Kekulé hexagonal lattice for quadrupole topological phase and corner modes. Trivial–nontrivial topological phase transition can be controlled by varying capacitance in the circuit, so that distinct topological edge states appear in 1D ribbons and corner states emerge in 0D flakes. We explore the field strength distribution and two-point impedance with respect to excitation frequency, and reveal that the topological corner resonance is robust against size of the LC network and randomness of the capacitors/inductors, a great benefit for experimental detection. Our results enrich the family of designer topoelectrical circuit as a flexible and tunable platform to achieve exotic quantum phases, which may have potential for future telecommunications, signal processing and quantum computing.

1. Introduction

Topological insulators (TIs), with metallic topological surface/edge states that are protected by time-reversal symmetry, have stimulated considerable interest in condensed matter and materials science communities due to their great potential for dissipationless electronic transport and quantum information processing [1–5]. Physically, the Berry phase of TIs can be expressed by a quantized momentum-dependent dipole moment, and the extension to higher-order multipolar moments (such as quadrupole and octopole) through the recursive use of Wilson loop operators leads to a class of new topological phases, also known as higher-order TIs [6–8]. As an example, two-dimensional (2D) quadrupole TIs, or second-order TIs, supports gapped topological edge states and mid-gap topological corner states originating from the nontrivial polarization in bulk crystals. In recent years, tremendous efforts have been made pertaining to the exploration of possible second-order topological phases and other exotic phases in electronic materials, as well as in photonic, mechanical, acoustic and electric systems [9–18].

As a prototype 2D structure, graphene is the first model that has been theoretically predicted to exhibit topological effect [19]. The sp² hybridization in graphene leaves the single p_z orbital of carbon in a hexagonal lattice, giving rise to two linearly dispersive Dirac bands. With the presence of spin–orbit coupling (SOC), the Dirac point opens a gap with nontrivial topology, leading to quantum spin Hall (QSH) effect. However, due to extremely small SOC in carbon, negligible topological gap can be observed [20–22]. To address this, theorists proposed topological gap opening in isotopically strained graphene [23, 24], in which a Kekulé-like hopping texture in hexagonal lattice results in quadrupole phase with topological helical edge states and pseudospin-polarized corner states [25–29]. Recently, γ-graphyne and graphdiyne were demonstrated to exhibit quadrupole topological effect due to the existence of both sp²- and sp³-hybridized carbon that leads to different intracell and intercell hopping [30, 31]. Nevertheless, these
porous materials are normally synthesized via homocoupling reactions in solution or chemical vapor deposition on metal surfaces [32–34], so that the prepared structures either suffer from lack of high quality or have close contact with the underlying metal substrate. In particular, both theoretical and experimental investigations have demonstrated the critical role of substrate in selecting the atomic orbitals and determining the topological properties of the grown 2D structures [35–37]. Consequently, it remains a challenge to find an appropriate candidate and characterize its topological phase in electronic materials.

In this regard, designed electric circuit provides an alternative yet promising physical system to realize these topological phases. By using capacitors and inductors, a variety of lattice models can be simulated, including but not limited to square, hexagonal, Lieb, kagome, pyrochlore and the Su–Schrieffer–Heeger (SSH) models [38–44], so that rich physical phenomena can be achieved, including QSH effect [39], Chern insulators [45], Weyl semimetals [46] and higher-order topological phases [40]. In this work, we construct a specific capacitor–inductor LC network with the Kekulé-like hexagonal lattice and explore the quadrupole topological properties in a systematic way. By correlating the tight-binding model of the lattice with Kirchhoff’s equation of the designed LC network, we reveal topologically trivial-nontrivial phase transition by using different capacitances in the circuit, in which gapped topological edge states arise in one-dimensional (1D) LC ribbons and topological corner states emerge in zero-dimensional (0D) LC flakes. Remarkably, the spatial distribution of field strength and two-point impedance can be effectively controlled by excitation frequency, and the topological corner resonance remains robust with respect to the circuit size and the capacitance/inductance randomness, thus demonstrating great advantages of LC network over electronic materials for experimental verification.

2. Model and methodology

We first explicitly illustrate the correspondence between tight-binding model and LC analogy of 2D Kekulé hexagonal lattice. For this lattice, the low-energy Hamiltonian can be expressed as [23, 25, 29],

$$\hat{H} = \varepsilon_0 \sum_i \hat{c}_i^\dagger \hat{c}_i + t_1 \sum_{\langle ij \rangle} \hat{c}_i^\dagger \hat{c}_j + t_2 \sum_{\langle ij \rangle} \hat{c}_i^\dagger \hat{c}_j,$$

where  $\hat{c}_i^\dagger (\hat{c}_i)$ represents the creation (annihilation) operator for a particle on site $i$. The first term is the on-site potential, and the second term is the intracell nearest-neighbor hopping from site $i$ to site $j$ with hopping parameter $t_1$. The third term describes the intercell hopping process with intercell hopping parameter $t_2$, similar to the SSH model [23, 24]. Then, the Hamiltonian in momentum space ($k$ space) can be obtained by applying a Fourier transformation to equation (1),

$$\hat{H} = \sum_{k \in BZ} \hat{\Psi}_k^\dagger \hat{H}_k \hat{\Psi}_k,$$

where $\hat{\Psi}_k = (\hat{\psi}_{1k}, \hat{\psi}_{2k}, \hat{\psi}_{3k}, \hat{\psi}_{4k}, \hat{\psi}_{5k}, \hat{\psi}_{6k})$ as there are six atomic orbitals within one unit cell in the lattice [29]. The $6 \times 6$ matrix $\hat{H}_k$ can be written as,

$$\hat{H}_k = \begin{pmatrix}
\varepsilon_0 & h_{12} & h_{13} & h_{14} & h_{15} & h_{16} \\
h_{12}^* & \varepsilon_0 & h_{23} & h_{24} & h_{25} & h_{26} \\
h_{13}^* & h_{23}^* & \varepsilon_0 & h_{34} & h_{35} & h_{36} \\
h_{14}^* & h_{24}^* & h_{34}^* & \varepsilon_0 & h_{45} & h_{46} \\
h_{15}^* & h_{25}^* & h_{35}^* & h_{45}^* & \varepsilon_0 & h_{56} \\
h_{16} & h_{26} & h_{36} & h_{46} & h_{56}^* & \varepsilon_0 \\
\end{pmatrix} \quad (3)$$

with

$$h_{13} = h_{15} = h_{24} = h_{26} = h_{35} = h_{46} = 0$$
$$h_{12} = h_{14}^* = t_1 e^{\frac{\sqrt{h_{12}}}{\sqrt{h_{12}}}}$$
$$h_{14} = -t_2 e^{\frac{\sqrt{h_{14}}}{\sqrt{h_{14}}}}$$
$$h_{16} = h_{34} = -t_1 e^{\frac{\sqrt{h_{16}}}{\sqrt{h_{16}}}}$$
$$h_{23} = h_{56} = -t_1 e^{\frac{\sqrt{h_{23}}}{\sqrt{h_{23}}}}$$
$$h_{25} = -t_2 e^{\frac{\sqrt{h_{25}}}{\sqrt{h_{25}}}}$$
$$h_{36} = -t_2 e^{\frac{\sqrt{h_{36}}}{\sqrt{h_{36}}}}.$$  

For this tight-binding model, we construct an LC circuit network with lumped elements to simulate the physical behaviors. As displayed in figure 1(a), each node (corresponding to the lattice site) is connected to
the ground via an inductor with inductance $L$. Two types of capacitors with capacitance $C_1$ and $C_2$ are inserted in the circuit, so that each unit cell contains six nodes with distinct intracellular hopping ($C_1$) and intercell hopping ($C_2$). The dynamic properties of this circuit are described by the Kirchhoff’s law as,

$$\frac{d}{dt}I_a = \sum_b C_{ab} \frac{d^2}{dt^2}(U_a - U_b) + \frac{U_a}{L} = 0 \quad (5)$$

where $I_a$ is the current from node $a$, $C_{ab}$ is the capacitance between node $a$ and the adjacent node $b$, and $U_i$ is the voltage at node $i$ ($i = a, b$). When an ac field with dynamic factor $e^{i\omega t}$ is introduced and a Fourier transformation is applied to equation (5), the equation can be rewritten as,

$$\sum_b i\omega C_{ab}(U_{ab} - U_{bk} \cdot e^{i\rho_{ab}}) + \frac{U_{ab}}{\omega L} = 0 \quad (6)$$

where $i$ is the imaginary unit and $\rho_{ab}$ is the vector from node $a$ to the adjacent node $b$ in real space. Then, the equation of motion in $k$ space can be written as,

$$\frac{1}{\omega^2} U_k = M_k U_k \quad (7)$$

where $U_k = (U_{1k}, U_{2k}, U_{3k}, U_{4k}, U_{5k}, U_{6k})^T$ with $T$ the transpose symbol. The matrix $M_k$ is expressed as,

$$M_k = L \times \begin{pmatrix} m_0 & m_{12} & m_{13} & m_{14} & m_{15} & m_{16} \\ m_{12} & m_0 & m_{23} & m_{24} & m_{25} & m_{26} \\ m_{13} & m_{23} & m_0 & m_{34} & m_{35} & m_{36} \\ m_{14} & m_{24} & m_{34} & m_0 & m_{45} & m_{46} \\ m_{15} & m_{25} & m_{35} & m_{45} & m_0 & m_{56} \\ m_{16} & m_{26} & m_{36} & m_{46} & m_{56} & m_0 \end{pmatrix} \quad (8)$$

with

$$m_0 = 2C_1 + C_2$$
$$m_{13} = m_{15} = m_{24} = m_{26} = m_{35} = m_{46} = 0$$
$$m_{12} = m_{45}^* = -C_1 e^{i\frac{\sqrt{7}\pi}{3}}$$
$$m_{14} = -C_2 e^{i\frac{\sqrt{7}\pi}{2}}$$
$$m_{16} = m_{34} = -C_1 e^{i\frac{\sqrt{7}\pi}{4}}$$
$$m_{23} = m_{46}^* = -C_1 e^{i\frac{\sqrt{7}\pi}{3}}$$
$$m_{25} = -C_2 e^{i\frac{\sqrt{7}\pi}{2}}$$
$$m_{36} = -C_2 e^{i\frac{\sqrt{7}\pi}{3}} \quad (9)$$

By comparing equations (3) and (8), we can easily identify that the dynamic matrix of LC network $M_k$ has exactly the same form as that of the tight-binding Hamiltonian, suggesting that it is feasible to use LC network to realize the electronic and topological properties encoded in the 2D Kekulé hexagonal lattice. It should be noted that although $M_k$ has the eigenvalue $1/\omega^2$ instead of $\omega^2$ (which usually corresponds to the energy), we can still obtain the topological nontrivial band structures because they are mainly influenced by the eigenvectors.

### 3. Results and discussion

#### 3.1. Topological band structures for 2D and 1D LC networks

The calculated band structures for an infinite 2D LC network for $C_1 > C_2$, $C_1 = C_2$ and $C_1 < C_2$ are shown in figure 1(b). As a generic constant coefficient, the inductance $L$ is set to 1 in our calculations. Obviously, with $C_1 > C_2$, there are six bands in the spectra with a finite gap between the upper three and lower three bands. A clear band evolution can be observed as we increase $C_2$ relative to $C_1$ (see more details in figure S1 in supplementary material (https://stacks.iop.org/NJP/00/000000/mmedia)), and at a critical point with $C_1 = C_2$, the six bands evolve into four with the formation of a Dirac point. Actually, with $C_1 = C_2$ the Kekulé hexagonal lattice evolves into lattice of ideal graphene, except that the Dirac point of graphene at $K$
Figure 1. (a) Top (upper panel) and side (lower panel) views of the electric circuit made of capacitors and inductors forming a 2D Kekulé hexagonal lattice. Two types of capacitors with \( C_1 \) and \( C_2 \) are shown, with each node grounded by an inductor \( L \). The unit cell is also indicated. (b) The calculated band structures for the circuit with \( C_1 > C_2 \), \( C_1 = C_2 \) and \( C_1 < C_2 \). Here, ‘±’ indicates parity of the eigenvectors.

point is now folded to \( \Gamma \). With further increased \( C_2 \ (C_1 < C_2) \), the bulk gap reopens. As such, the capacitance controlled gap closing and reopening suggests band inversion in the spectra. To see this, we calculated ‘parity’ of the eigenvectors. Here, the parity is the eigenvalue of \( \pi \) rotation over the \( z \) axis at a specific \( k \) point for the \( n \)th energy band, which corresponds to the phase of static voltage in LC circuit. Due to the \( C_{6v} \) point group symmetry, we only need to compare one \( \Gamma \) and one \( M \) point, as the three \( M \) points are equivalent for the four inversion invariant points in hexagonal lattice. Indeed, band inversion happens at \( \Gamma \) and \( M \) points (see figure 1(b)), suggesting trivial-nontrivial topological phase transition.

Resembling the SSH model, topological dipole moment should exist in the Kekulé hexagonal lattice with \( t_1 < t_2 \) in equation (1), and a finite quadrupole appears [29]. As there are two pairs of doubly degenerate states at \( \Gamma \) due to the \( C_{6v} \) point group symmetry, we can regard the two components in the pair of degenerate states as pseudospin because they can transform to each other under mirror reflection, and also they are time reversal, similar to real spins [29]. By introducing this concept, topological helical edge states and pseudospin-polarized corner states may arise in the LC circuit. To demonstrate this, we consider a semi-infinite 1D LC network with a ribbon-like structure, which is finite along \( x \) direction and infinite along \( y \) direction (figure 2(a)). It should be noted that for electric circuit with open boundaries (1D ribbon and 0D flake), the grounding at the open edge is required to be different from the bulk to make the admittance matrix \( \mathcal{J} \) consistent with the Hamiltonian (see details in supplementary material).

By diagonalizing the corresponding dynamic matrix \( M_k \), we can obtain the band structures of 1D LC ribbon. With \( C_1 > C_2 \), the band structures show a bulk gap with no edge states (see figure S2 in supplementary material), suggesting a topological trivial phase, in accordance with the results on 2D network. With \( C_1 < C_2 \), there is a pair of edge-state bands residing within the bulk gap of the circuit, as shown in figure 2(b). These edge states are spin-polarized with opposite pseudospin at the same \( K \) point, which are located separately at the two edges of the ribbon, and have been demonstrated to be helical and can be characterized by the \( Z_2 \) invariant [29]. It should be also noted that a small gap exist at the crossing point of the two edge bands, in which the gap size can be efficiently controlled by varying the difference...
between $C_1$ and $C_2$, as well as the specific geometry of the ribbon (figures S3 and S4). The gapped topological edge states indicate the existence of topological corner states, as we shall see later.

3.2. Topological corner state and impedance

To explore the topological corner states in the LC circuit, we construct a finite 0D flake comprising $10 \times 10$ unit cells (figure 3(a)) and calculate the discrete energy levels. As there are six levels (nodes) in each unit cell, a total of 600 energy levels will be present for the flake. In figure 3(b), we show the energy levels around level index of 300. Clearly, two eigenstates with eigenvalue of $1/\omega^2 \sim 0$ can be observed, which should correspond to the corner modes. We compute the field strength of all cells on the flake, in which the field strength of each unit cell is defined as, $U_{ij} = \sum_q |U_{ijq}|$, with $q = 1, 2, 3, 4, 5, 6$. The calculated results are shown in figures 3(c)–(f). With an excitation frequency corresponding to the index of 300, the field strength is localized at the lower-left and upper-right corners of the flake (see figure 3(c)), confirming the two topological corner states with $1/\omega^2 \sim 0$. By calculating the field strength distribution of 1D ribbon and 0D flake with a range of $C_2/C_1$ values, we are able to confirm that the corner state is induced by the bulk quadrupole polarization but not the edge dipole polarization (see figure S5 for details). When the frequency is tuned away from $1/\omega^2 \sim 0$ (level index = 310), the field strength becomes localized on two edges of the flake (figure 3(d)), suggesting that it has entered the topological edge-state region. Then, with the frequency further away (level index = 550), the field strength are spread over the whole flake (figure 3(e)), which means that it becomes bulk mode. Ultimately, with the level index of 600, the field strength becomes mostly localized in the center region. Thus, by tuning the excitation frequency, we are able to control the field strength distribution of the LC circuit.

Besides the field strength, we further probe the two-point impedance of the LC flake to take a deep insight into the topological corner modes. With Fourier transformation, Kirchhoff’s current law equation (5) can be rewritten as,

$$I_a(\omega) = \sum_b J_{ab}(\omega) U_b(\omega),$$

with

$$J_{ab}(\omega) = i\omega \left[ C_{ab} + \delta_{ab} \left( \sum_c C_{ac} - \frac{1}{\omega^2 L} \right) \right],$$

where the matrix $J(\omega) = [J_{ab}(\omega)]$ is the circuit Laplacian. By diagonalizing the matrix, we can obtain the eigenvalue $j_n$ and the eigenvector $|\psi_n>. The two-point impedance is given by [15, 40, 47],

$$Z_{ab} = \frac{U_a - U_b}{I_{ab}} = \sum_n \frac{|\psi_{na} - \psi_{nb}|^2}{j_n}.$$

By considering the circuit Laplacian and formula (12), we are able to determine the resonant frequency, which is given by the zero of the main diagonal elements in the matrix, i.e., $\omega_0 = 1/\sqrt{L(2C_1 + C_2)}$. At $\omega_0$, the impedance will manifest itself as a huge resonance peak, and there should be no divergence because the size of system is finite [40]. To confirm this, we calculate the spatial distribution of two-point impedance for the flake, with results displayed in figure 4. Indeed, when the excitation frequency is set to the resonant frequency $\omega_0$, the maximum impedance is strongly localized on the resonant corner sites (left-down and right-top corners), whether the position of fixed node is taken at the center or edge of the flake (figures 4(a) and (b)). When the fixed node is taken at the resonant corners, the impedance is generally large for the
Figure 3. (a) Schematic of a finite 0D LC flake with $n_x \times n_y$ unit cells. (b) The calculated discrete energy levels of an LC flake having $10 \times 10$ unit cells with $C_1 = 1$, $C_2 = 2$, where two corner states can be identified. (c)–(f) Field strength distribution of the flake with level index of 300 (c), 310 (d), 550 (e) and 600 (f), respectively.

Figure 4. (a)–(d) Spatial distribution of the two-point impedance for an LC flake of $10 \times 10$ unit cells, with the fixed node located in the center of the flake (a), at the middle point of the edge (b), at the left-down corner (c) and right-down corner (d), respectively. The excitation frequency $\omega$ was set to the resonant frequency $\omega_0$. (e)–(h) The same as (a)–(d) with the excitation frequency $\omega$ set to a value in the edge-state region.

whole flake (figure 4(c)). When taken at the other two corners (left-top and right-down), we can still observe the prevailing distribution of impedance at the resonant corners. The emergence of resonant modes provides an excellent sign for the topological phase, a great advantage of experimental detection. On the other hand, when the excitation frequency is set to the edge-state region, the impedance on the edge is obviously smaller than that in the bulk and no resonance can be observed (figures 4(e)–(h)). Hence, external excitation frequency offers an efficient and practical way to manipulate the status of the LC circuit.

3.3. Robustness of topological corner resonance
At last, we investigate effects of the size of LC circuit as well as the randomness of capacitors/inductors on the topological corner modes. We first construct a smaller LC flake with $6 \times 6$ unit cells and compare the corner impedance with that of the bigger LC flake with $10 \times 10$ unit cells. In figures 5(a)–(c), we present the corner impedance of the bigger flake as a function of the ratio of excitation frequency and resonant...
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Figure 5. Corner impedance $|Z|$ as a function of $\omega/\omega_0$ for an LC flake of $10 \times 10$ unit cells, with $C_2/C_1 = 2$ for (a), $C_2/C_1 = 3$ for (b), and $C_2/C_1 = 5$ for (c). $\omega_0$ is the resonant frequency and the impedance of the resonant peak is highlighted for (b) and (c). (d)–(f) Same as (a)–(c) for an LC flake of $6 \times 6$ unit cells. (g)–(i) Same as (d)–(f) with 5% of randomness.

frequency ($\omega/\omega_0$). With $C_2/C_1 = 2$, the resonant peak with $\omega/\omega_0$ has a high corner impedance $|Z|$ of 500 Ω, with several side peaks of 150–300 Ω (figure 5(a)). With increased $C_2/C_1$, the impedance of the resonant peak becomes huge, which reaches $10^4$ and $10^6$ Ω with $C_2/C_1 = 3$ and $C_2/C_1 = 5$, respectively (see figures 5(b) and (c)), indicating a good signature for the experimental observation. When the flake size is reduced to $6 \times 6$ unit cells, one can see that the resonant peak remains quite similar to that of the larger flake (figure 5(d)), which also soars to $10^4$ and $10^5$ Ω with $C_2/C_1 = 5$ and $C_2/C_1 = 10$ respectively (figures 5(e) and (f)). It should also be noted that for flakes of even smaller size, the resonance peak becomes split (see figure S6 in the supplementary material), for which the critical size for the split of the peak depends on the value of $C_2/C_1$. Similar to previous studies [40], we find that the finite size effect renders no divergence of impedance at the resonant frequency, demonstrating that the corner resonance is robust against size of the circuit.

To explore the effects of capacitor/inductor randomness, we change $C_i$ and $L_i$ to $C_i(1+\alpha_i)$ and $L_i(1+\beta_i)$, with $\alpha_i$ and $\beta_i$ uniformly distributed random variables ranging from $-\delta$ to $\delta$. Here, we set $\delta$ to 0.05, corresponding to 5% of randomness in the LC circuit. As shown in figures 5(g)–(i), the resonant peak remains similar to that without randomness, suggesting that the corner resonance are not affected by local disorders in the circuit. It is interesting to notice that all other side peaks (as noises) are strictly restricted, especially for the circuit with larger $C_2/C_1$ (figures 5(h) and (i)). Besides the bulk, we also explored the capacitor/inductor randomness in the edge and corner. It is found that the field strength distribution for the edge/corner states is sensitive for the randomness around the edge/corner, but the corner impedance resonance remains regardless of randomness distribution (see figure S7). This is remarkable, because the pronounced topological corner resonance, which is robust against size and randomness of the circuit, can provide a valuable signal for characterization of quadrupole topological phase in LC circuits.

4. Conclusion

In summary, we have demonstrated the realization of quadrupole topological phase in Kekulé hexagonal electric circuit by using simple capacitors and inductors. The trivial-nontrivial topological phase transition can be controlled by varying the intracell and intercell capacitance, so that gapped topological edge states appear in 1D LC ribbons and topological corner states emerge in 0D LC flakes. Currently, it remains a challenge to characterize the topological edge/corner states in 2D electronic materials due to the weak intensity, such as by angle-resolved photoemission spectroscopy or scanning tunneling microscopy [48]. Our designed LC circuit exhibits huge corner resonant peak, which is robust against size of the circuit and randomness of capacitors/inductors, a great advantage for experimental observation. With high flexibility and tunability, topological electric circuit may not only provide an ideal platform to achieve various
quantum phenomena by using classical systems, but also have great potential applications in telecommunications, sensing and microwave engineering.

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