Resurrecting Minimal Yukawa Sector of SUSY SO(10)

K.S. Babu\textsuperscript{a,*}, Borut Bajc\textsuperscript{b,†} and Shaikh Saad\textsuperscript{a,‡}

\textsuperscript{a}Department of Physics, Oklahoma State University, Stillwater, OK, 74078, USA
\textsuperscript{b}Jožef Stefan Institute, 1000 Ljubljana, Slovenia

Abstract

Supersymmetric SO(10) models with Yukawa coupling matrices involving only a $10_H$ and a $126_H$ of Higgs fields can lead to a predictive and consistent scenario for fermion masses and mixings, including the neutrino sector. However, when coupled minimally to a symmetry breaking sector that includes a $210_H$ and a $126_H$, these models lead either to an unacceptably small neutrino mass scale, or to non-perturbative values of the gauge couplings. Here we show that with the addition of a $54_H$ to the symmetry breaking sector, the successful predictions of these models for fermion masses and mixings can be maintained. The $54_H$ enables a reduction of the $B - L$ symmetry breaking scale to an intermediate value of order $10^{12}$ GeV, consistent with the observed neutrino mass spectrum, while preserving perturbative gauge coupling unification. We obtain an excellent fit to all fermion masses and mixings in this framework. We analyze carefully the prediction of the model for CP violation in neutrino oscillations. Consistency with proton lifetime, however, requires a mini-split SUSY spectrum with the squarks and sleptons having masses of order 100 TeV, accompanied by TeV scale gauginos and Higgsinos. Such a spectrum may arise from pure gravity mediation, which would predict the partial lifetime for the decay $p \rightarrow \pi K^+$ to be an order of magnitude above the current experimental limit.

\textsuperscript{*}E-mail: babu@okstate.edu
\textsuperscript{†}E-mail: borut.bajc@ijs.si
\textsuperscript{‡}E-mail: shaikh.saad@okstate.edu
1 Introduction

Perhaps the most attractive feature of unified models based on $SO(10)$ gauge symmetry is that all fermions of a given family, including the right-handed neutrino, are assembled into a single representation, the $16$–dimensional spinor of $SO(10)$. The Higgs fields that can generate masses for the fermions can be inferred from the fermion bilinears,
\[ 16 \times 16 = 10_s + 120_a + 126_s , \]
where the subscripts $s$ and $a$ denote symmetric and antisymmetric combinations. At least two Higgs fields are needed to generate nontrivial quark and lepton mixings. A minimal Yukawa sector in supersymmetric (SUSY) $SO(10)$, which is the focus of this paper, is thus obtained by the introduction of a $10_H$ and a $\overline{126}_H$ of Higgs fields.\(^1\) In SUSY $SO(10)$ both the $10_H$ and $\overline{126}_H$ are complex chiral superfields, resulting in the Yukawa superpotential:
\[ W_{SO(10)}^{\text{Yukawa}} = 16^T \left( Y_{10} \, 10_H + Y_{126} \overline{126}_H \right) \, 16 . \]

Here $Y_{10}$ and $Y_{126}$ are $3 \times 3$ symmetric matrices in family space. These two matrices will be responsible for generating all of the charged fermion masses and mixings as well as the heavy right-handed and light left-handed neutrino masses and mixings [1]. The mass matrices arising from Eq. (1.2) are displayed in detail in Eqs. (5.4)-(5.8). This simple Yukawa structure has 12 real parameters and 7 phases, which should fit 18 measured quantities – 6 quark masses, 4 quark mixing parameters, 3 charged lepton masses, 2 neutrino mass splittings and 3 leptonic mixing angles. (One Yukawa coupling matrix, say $Y_{126}$, can be made diagonal and real, whence the other would have 6 complex parameters. In addition to the Yukawa couplings, the fermion mass matrices will involve 3 ratios of vacuum expectation values (VEVs), with one ratio remaining complex. This leads to a total of 12 real parameters and 7 phases.) It is remarkable that this minimal setup is able to reproduce the full fermion mass spectrum, including large leptonic mixing angles along with small quark mixing angles [2–12], a nontrivial feat in any quark–lepton unified framework. To its credit, this setup predicted [4,5,7–9,12] a large value for the reactor neutrino mixing angle $\theta_{13}$ very close to the experimental value measured subsequently by the Daya Bay collaboration [13].

A crucial feature of Eq. (1.2) in generating the mass matrices of Eqs. (5.4)-(5.8) is that the Standard Model (SM) singlet in $\overline{126}_H$ acquires a large VEV ($V_R$) of order $10^{12}$.

\(^1\)Choosing any other combinations involving two of these Higgs fields will not lead to viable phenomenology. For example, using two copies of $10_H$ will lead to nonzero quark mixings, but the unviable mass relations $m_\mu^0 = m_\tau^0$ and $m_\mu^0 = m_\nu^0$ will prevail in this case at the GUT scale. If two copies of $\overline{126}_H$ is used instead, unacceptable relations $m_\mu^0 = -3m_\tau^0$ and $m_\mu^0 = -3m_\nu^0$ will emerge. Similarly, combining a $120_H$ with either $10_H$ or $\overline{126}_H$ will also lead to inconsistent phenomenology.
GeV, while its SM doublet components also acquire weak scale VEVs [1]. This is possible only when the light MSSM Higgs fields $H_u$ and $H_d$ emerge at least partially from the Higgs doublet fragments of the $\overline{126}_H$. The simplest choice for generating such mixings among Higgs doublets of $10_H$ and $\overline{126}_H$ is via a renormalizable superpotential coupling involving the $210_H$ field, viz., $W \supset \lambda_1 \overline{126}_H 210_H 10_H$. Thus, a minimal renormalizable symmetry breaking sector is identified: \{10_H + 126_H + 126_H + 210_H\}. Here the 126_H is needed in order to break $SO(10)$ gauge symmetry in the SUSY limit. The \{126_H + 126_H + 210_H\} fields jointly break $SO(10)$ symmetry down to the SM, while the $SU(2)_L$ doublets from these fields along with the doublets from the $10_H$ break the electroweak symmetry at a lower scale. This model has been extensively studied in the literature, and has been referred to as the minimal SUSY $SO(10)$ model.

It has long been recognized [1,14,15] that this minimal SUSY $SO(10)$ model contains all the essential ingredients to be realistic. Furthermore, it was shown in Ref. [16] that this model has the minimal number of parameters among all supersymmetric grand unified theories.\footnote{Here and in the following we will consider only renormalizable models. In fact, any non-renormalizable GUT will have innumerable parameters.} The supersymmetric sector of this model is described by 26 real parameters: $3 + 12 = 15$ real Yukawa couplings and $14 - 3 = 11$ real superpotential parameters from the symmetry breaking sector $W_{SSB}$. There are 7 complex parameters in $W_{SSB}$, of which 3 can be made real. Significant effort has been put into the study of the parameter space of this theory for very good reasons.

## 2 Problems with the minimal model

While the minimal model was found to be very successful in fitting fermion data, it was realized soon thereafter that it faced some hurdles once the symmetry breaking constraints are included [8,17–19]. The overall scale of the right-handed neutrino masses comes out to be $V_R \sim (10^{12} - 10^{13})$ GeV from fits to light neutrino masses, while consistency of symmetry breaking of the model requires $V_R \sim (10^{15} - 10^{16})$ GeV. If $V_R$ takes values in the range of $(10^{12} - 10^{13})$ GeV, then certain colored multiplets from various Higgs fields will have masses of order $V^2_R/M_{GUT} \sim 10^{10}$ GeV, spoiling perturbative gauge coupling unification.

There is an independent issue related to proton decay mediated by the color-triplet Higgsinos of the model. The partial lifetime for the decay $p \rightarrow \pi K^+$ would be shorter than the experimental lower limit of $5.9 \times 10^{33}$ years [20] if all the SUSY particles have masses of order TeV. While this is not a problem in itself, as the SUSY particle masses are currently unknown, a realistic version of SUSY $SO(10)$ should also be compatible with this
constraint. Assuming TeV-scale superparticle masses, it has been proposed that this issue can be overcome by a cancelation mechanism with the inclusion of a $120_H$ [21, 22] into the minimal model. Another possibility that has been suggested to suppress proton decay rate is via non-perturbative (but uncontrollable) suppressions [23] with an asymptotically safe theory in mind [24]. Here we show explicitly the severity of this constraint in the minimal model (without a $120_H$), if all the SUSY particles have masses around a TeV.

Both of these problems, too small a scale for neutrino masses and too short a lifetime for the proton with TeV superparticles, could in principle be solved in a split-SUSY scenario [25–27] with the gaugino–Higgsino masses around 100 TeV and the squark–slepton masses around $10^{13}$ GeV [12]. This would allow for an increase in the light neutrino masses, because the gauge couplings can be kept smaller for higher energies before the bosonic threshold is reached. Simultaneously, $d = 5$ proton decay rate would be highly suppressed, owing to the large sfermion masses. Unfortunately, this solution turns out to have various shortcomings, both theoretical and experimental. First, the Higgs boson mass of 125 GeV discovered after this proposal was made does not allow such a large SUSY breaking scale with small $A$-terms [28, 29]. Second, the reactor neutrino mixing angle, although large, was not large enough in [12]. Third, there may be also issues with stability of the spectrum with such large values of the bosonic masses [30], although this may be an issue of naturalness only.

3 Proposed solution to the problems

It is clear that the minimal SUSY $SO(10)$ model consisting of \{10$_H$ + $\overline{126}_H + 126_H + 210_H$\} Higgs fields needs to be extended, in view the problems it faces. One avenue is to add a $120_H$ to the theory in which case the fermion mass fits of the model will be significantly affected. Consistency has been shown in this case [21, 22, 31] (or in the non-supersymmetric case recently [32]). An alternative, which we discuss in this paper, is to modify the symmetry breaking sector without affecting the Yukawa sector. This is achieved by adding a 54$_H$, without the need for a $120_H$. Thus the proposed model has the Higgs content of \{10$_H$ + $\overline{126}_H + 126_H + 210_H + 54_H$\}.

One can compare the number of parameters that are introduced by the addition of a 54$_H$ with that for a $120_H$. Since 54$_H$ has no Yukawa couplings to the fermions in 16, the only new parameters are the 6 new complex couplings that appear in $W_{SSB}$. After removing one phase by field redefinition of 54$_H$, we arrive at $26 + 12 - 1 = 37$ real parameters, keeping the model still minimal among the renormalizable versions [16]. In contrast, adding a $120_H$

\footnote{It has to be said however, that in the fit of Ref. [12] only the upper experimental limit has been put on $\theta_{13}$, so a new $\chi^2$ fit with the measured value of $\theta_{13}$ may well give a better and acceptable value.}
would introduce 3 complex Yukawa couplings and 9 new complex couplings in $W_{SSB}$, leading to $26 + 6 + 18 - 1 = 49$ real parameters.

The advantage of introducing a $54_H$ is that the symmetry breaking can proceed in three steps that is consistent with gauge coupling unification:

$$\begin{align*}
SO(10) & \xrightarrow{(54_H),(210_H)} SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1)_{B-L} \\
& \xrightarrow{(126_H),(10^H)} SU(3)_c \times SU(2)_L \times U(1)_Y \\
& \xrightarrow{(10_H)} SU(3)_c \times U(1)_{em}
\end{align*}$$

(3.1)

The first step of symmetry breaking occurs at $M_{GUT} \approx 10^{16}$ GeV, which leaves the rank of $SO(10)$ intact. The next step occurs around $V_R \sim 10^{12}$ GeV where the $U(1)_{B-L}$ symmetry gets spontaneously broken. The $54_H$ allows to have such an intermediate scale without any light remnants, apart from a SM singlet pair. The renormalization group running of the gauge couplings does not change except for a small threshold correction, which we found to be well under control.

It is easy to see why with the addition of $54_H$ an intermediate scale $V_R$ for $B - L$ breaking is possible without any fields carrying SM quantum numbers surviving to $V_R$. Note that the superpotential for the $210_H$ alone, viz., $W_{210_H} = \frac{1}{2} m_1 210^2_H + \lambda_1 210^3_H$ allows for the breaking $SO(10) \rightarrow SU(5) \times U(1)$, while the superpotential for $54_H$ alone, viz., $W_{54_H} = m_5 54^2_H + \lambda_5 54^3_H$ breaks $SO(10)$ down to $SU(4)_c \times SU(2)_L \times SU(2)_R$. The joint effect of $210_H$ and $54_H$ is to break $SO(10)$ down to $SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1)_{B-L}$. The cross coupling in the superpotential, viz., $\lambda_{10} 54_H 210^2_H$, ensures that all of the would-be Goldstone bosons will acquire masses. We have explicitly verified these statements by examining the full Higgs spectrum with the $54_H$ and with an intermediate scale $V_R \sim 10^{12}$ GeV. Such a solution is not permitted in the case of using $\{\overline{126}_H + 126_H + 210_H\}$ for GUT symmetry breaking due to the intricate nature of $\overline{126}_H - 126_H - 210_H$ cross coupling, which on the one hand should induce $SU(5)$-breaking VEV in $210_H$, and on the other hand should also supply masses for the would-be Goldstone bosons.

The problem of rapid proton decay via $d = 5$ operators can be kept under control only with a mini-split SUSY scenario. We will show that a SUSY spectrum with TeV scale gauginos and Higgsinos and 100 TeV squark and sleptons would suppress the most dangerous mode sufficiently to be in accord with the experimental limit. Such a mini-split SUSY spectrum is compatible with pure gravity mediated SUSY breaking [33,34], which we shall elaborate on. Within such a framework, the partial lifetime for the decay $p \rightarrow \nu K^+$ is found to be about an order of magnitude above the present experimental lower limit.

We perform a new fit to fermion data, taking into account the two new ingredients of this proposed scenario: a mini-split SUSY case, and the extra intermediate $U(1)_{B-L}$ scale.
We have found that an excellent fit to all light fermion masses and mixings is possible in this scenario.

4 The Higgs potential and the vacuum structure

The supersymmetric Higgs sector of $SO(10)$ involving $\{10_H + \overline{126}_H + 126_H + 210_H + 54_H\}$ has been studied in Ref. [35]. We shall follow closely the notation of that paper. The most general renormalizable Higgs superpotential is:

$$W_{SO(10)}^{SSB} = \frac{1}{2} m_1 210_H^2 + m_2 \overline{126}_H 126_H + m_3 10_H^2 + \frac{1}{2} m_5 54_H^2$$

$$+ \lambda_1 210_H^3 + \lambda_2 210_H \overline{126}_H 126_H + \lambda_3 126_H 10_H 210_H + \lambda_4 \overline{126}_H 10_H 210_H$$

$$+ \lambda_8 54_H^3 + \lambda_{10} 54_H 210_H^2 + \lambda_{11} 54_H 126_H^2 + \lambda_{12} 54_H \overline{126}_H^2 + \lambda_{13} 54_H 10_H^2$$  \hspace{1cm} (4.1)

By denoting the vacuum expectation values in the Pati-Salam $SU(4)_c \times SU(2)_L \times SU(2)_R$ notation as

$$\langle (1, 1, 1)_{210} \rangle = V_1, \quad \langle (15, 1, 1)_{210} \rangle = V_2, \quad \langle (15, 1, 13)_{210} \rangle = V_3$$  \hspace{1cm} (4.2)

$$\langle (1, 1, 1)_{54} \rangle = V_E, \quad \langle (10, 1, 13)_{126} \rangle = V_R, \quad \langle (10, 1, 3)_{126} \rangle = \overline{V}_R$$  \hspace{1cm} (4.3)

we get in the limit $|V_R| = |\overline{V}_R| << V_{1,2,3,E}$ for the vacuum expectation values

$$\frac{V_1}{V_2} = \frac{1}{2 \sqrt{3}} \left( \frac{V_3}{V_2} \right)^2$$  \hspace{1cm} (4.4)

$$\frac{m_1}{V_2} = -\frac{\lambda_1}{5 \sqrt{2}} \left( 3 + \left( \frac{V_3}{V_2} \right)^2 \right)$$  \hspace{1cm} (4.5)

$$\frac{V_E}{V_2} = \frac{\lambda_1}{\lambda_{10} \sqrt{30}} \left( -2 + \left( \frac{V_3}{V_2} \right)^2 \right)$$  \hspace{1cm} (4.6)

$$\frac{m_2}{V_2} = -\frac{\lambda_2}{120} \left( 6 \sqrt{2} + 12 \left( \frac{V_3}{V_2} \right) + \sqrt{2} \left( \frac{V_3}{V_2} \right)^2 \right)$$  \hspace{1cm} (4.7)

$$\frac{m_5}{V_2} = \frac{1}{20 \sqrt{2} \lambda_1 \lambda_{10}} \left( -5 \lambda_1^3 - 2 \lambda_1^2 \lambda_8 \left( \frac{V_3}{V_2} \right)^2 - \left( -20 \lambda_1^3 + 4 \lambda_1^2 \lambda_8 \right) \right)$$  \hspace{1cm} (4.8)

which spontaneously break $SO(10)$ down to $SM \times U(1)_{B-L}$, if $V_R = \overline{V}_R$ is ignored. This symmetry breaks further down to the SM once a nonzero $V_R$ is generated. Eq. (4.7) is the condition necessary for inducing a nonzero $V_R$.

The Higgs spectrum of this theory has been worked out in Ref. [35]. It is easy to verify for this spectrum that with $V_R \ll V_{1,2,3,E}$, there are no light states in the theory.

\footnote{The missing masses and couplings are connected to $45_H$ and $120_H$ used in Ref. [35], which are, however, not part of our model.}

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except for two SM singlet fields $\sigma + \bar{\sigma}$ carrying $B-L$ charges of $\pm 2$. These fields acquire VEVs at an intermediate scale, breaking the $B-L$ symmetry and thereby generating heavy right-handed neutrino masses.

The relations in Eqs. (4.4)-(4.8) will be used in our numerical computations of proton decay amplitude.

5 The Yukawa sector and GUT threshold corrections

The renormalizable Yukawa superpotential of the model is given in Eq. (1.2). After the GUT symmetry breaking parametrized by the VEVs in Eqs. (4.4)-(4.7) the only light states (compared to $M_{GUT}$) are the SM superfields and the SM singlets $\sigma, \bar{\sigma}$ which for small $V_R$ are purely from $126_H, \bar{126}_H$ that do not mix with other singlets. As a result, after integrating out all the heavy states with mass $M_{GUT}$, the superpotential below the GUT scale will have the form:

$$W = W_{Yukawa} + W_{Higgs}$$

with

$$W_{Yukawa} = \frac{1}{2} Y_{ij}^R \nu_i^c \nu_j^c \sigma + Y_{ij} \nu_i^c L_j H_u + Y_{ij}^D \nu_i^c Q_j H_d + Y_{ij}^U e_i^c L_j H_u + Y_{ij}^D d_i^c Q_j H_d$$

$$W_{Higgs} = \mu H_u H_d + \frac{V^2_R}{M_{GUT}} \sigma \bar{\sigma} - \frac{(\sigma \bar{\sigma})^2}{2M_{GUT}}.$$

Here the mass term for $\sigma$ field and the quartic coupling are determined in terms of the fundamental parameters of Eq. (4.1), but their actual forms are not so relevant for our analysis. After minimizing the potential we get $|\langle \sigma \rangle| = |\langle \bar{\sigma} \rangle| = V_R$. The Majorana Yukawa coupling matrix $Y_R$ obeys the boundary condition $Y_R(M_{GUT}) = Y_{126}(M_{GUT})$.

At $M_{GUT}$ we have the following SO(10) relations:

$$v_u Y_U = v_{10}^{10} Y_{10} + v_{126}^{126} Y_{126}$$

$$v_d Y_D = v_{10}^{10} Y_{10} + v_{126}^{126} Y_{126}$$

$$v_d Y_E = v_{10}^{10} Y_{10} - 3v_{126}^{126} Y_{126}$$

$$v_u Y_{\nu_d} = v_{10}^{10} Y_{10} - 3v_{126}^{126} Y_{126}$$

$$M_N = v_i Y_{126} - (v_u Y_{\nu_d})^T (V_R Y_{126})^{-1} (v_u Y_{\nu_d}).$$

Here $v_u$ and $v_d$ are the VEVs of the MSSM fields $H_u$ and $H_d$, and we define as usual $\tan \beta = v_u/v_d$. While the light neutrino masses receive contributions from type-I seesaw as well as type-II seesaw, the magnitude of the latter turns out to be small. This is because the weak triplet(s) in the model have masses of order the GUT scale. In our analysis we keep only the type-I contribution to neutrino masses.
Since \( M_{\nu_R} \) and \( V_R \) are less than \( M_{GUT} \), we have corrections to the 1-step RGE running from \( M_Z \) to \( M_{GUT} \). To analyze these effects, we choose without loss of generality a basis where \( Y_{126} \) is real and diagonal. The corrections to the Yukawa couplings due to the intermediate scale threshold is obtained by integrating the RGE equations, Eqs. (C.1)-(C.5) of Appendix C (denoting the diagonal entries of \( Y_{126} \) as \( Y_{d126} \)):

\[
Y'_{\nu_D}^{ij} = Y_{\nu_D}^{ij} - \frac{1}{(4\pi)^2} \sum_{k=1}^{3} \left[ 3 \left( Y_{\nu_D} Y_{\nu_D}^\dagger \right)^{ik} \left( Y_{\nu_D} \right)^{kj} \log \left( \frac{M_{GUT}}{\max \left( \left| V_R (Y_{d126}^d)^i \right|, \left| V_R (Y_{d126}^d)^k \right| \right)} \right) \right. \\
+ Y_{\nu_D}^{ij} \left( Y_{\nu_D} Y_{\nu_D}^\dagger \right)^{kk} \log \left( \frac{M_{GUT}}{\max \left( \left| V_R (Y_{d126}^d)^i \right|, \left| V_R (Y_{d126}^d)^k \right| \right)} \right) \\
- \frac{1}{(4\pi)^2} \left[ -\frac{3}{2} g_B^2 Y_{\nu_D}^{ij} \log \left( \frac{M_{GUT}}{g_B V_R} \right) \left| \left( Y_{d126}^d \right)^i \right|^2 \left( Y_{\nu_D} \right)^{ij} \log \left( \frac{M_{GUT}}{\max \left( \left| V_R (Y_{d126}^d)^i \right|, \left| V_R^2 \right|/M_{GUT} \right)} \right) \right] \\
\left(5.9\right) \\
\]

\[
Y'_{U}^{ij} = Y_{U}^{ij} - \frac{1}{(4\pi)^2} \sum_{k=1}^{3} \left( Y_{E} Y_{\nu_D}^\dagger \right)^{ik} \left( Y_{\nu_D} \right)^{kj} \log \left( \frac{M_{GUT}}{\max \left( \left| V_R (Y_{d126}^d)^i \right|, \left| V_R (Y_{d126}^d)^k \right| \right)} \right) - \frac{3}{2} g_B^2 Y_{E}^{ij} \log \left( \frac{M_{GUT}}{g_B V_R} \right) \\
\left(5.10\right) \\
\]

\[
Y'_{D}^{ij} = Y_{D}^{ij} - \frac{1}{(4\pi)^2} Y_{U}^{ij} \left( \sum_{k=1}^{3} \left( Y_{\nu_D} Y_{\nu_D}^\dagger \right)^{kk} \log \left( \frac{M_{GUT}}{\max \left( \left| V_R (Y_{d126}^d)^i \right|, \left| V_R (Y_{d126}^d)^k \right| \right)} \right) - \frac{1}{6} g_B^2 \log \left( \frac{M_{GUT}}{g_B V_R} \right) \right) \\
\left(5.11\right) \\
\]

\[
Y'_{E}^{ij} = Y_{E}^{ij} - \frac{1}{(4\pi)^2} Y_{D}^{ij} \left( -\frac{1}{6} g_B^2 \log \left( \frac{M_{GUT}}{g_B V_R} \right) \right) \\
\left(5.12\right) \\
\]

where \( Y'_{U,D,E} \) are the numerical values one would get by solving the RGEs from \( M_Z \) to \( M_{GUT} \) without intermediate states. On the other hand, the numerical value of \( M_N \) with \( Y_{\nu_D} \) replaced by \( Y'_{\nu_D} \) is obtained by solving the RGE from \( M_Z \) to the intermediate scale. We shall use these equations as the first step in obtaining fit to fermion masses and mixings. This would determine, to a very good approximation, the intermediate spectrum as well as the scale \( V_R \) from the fermion fit. One can in principle improve the fermion fit by using this intermediate spectrum and using the exact one-loop RGE for crossing the threshold as given in Eqs. (C.1)-(C.5) of Appendix C. However, our numerical analysis performed in the next section shows that these threshold effects are very small, hence we do not repeat the process.
6 Fit to the fermion masses and mixings

We rewrite Eqs. (5.4) - (5.8) as:

\[ Y_D = H + F \]  
\[ Y_U = r(H + sF) \]  
\[ Y_E = H - 3F \]  
\[ Y_{\nu D} = r(H - 3sF) \]  
\[ M_N = -(v_u Y_{\nu D})^T (c_R F)^{-1} (v_u Y_{\nu D}) \],

where we have defined

\[ r = \frac{v_u^{10}}{v_d^{10} \tan \beta}, \quad s = \frac{v_u^{126}}{v_d^{126} v_u^{10}}, \quad c_R = V_R \frac{v_d}{v_d^{126}}, \]

\[ Y_{10} = \frac{v_d}{v_d^{10}} H, \quad Y_{126} = \frac{v_d}{v_d^{126}} F. \]

As noted before, this Yukawa sector has 12 real parameters and 7 phases to fit 18 measured quantities. Without loss of generality one can go to a basis where the symmetric matrix \( F \) is real and diagonal, whence \( H \) becomes a general complex symmetric matrix. The parameters \( r \) and \( c_R \) can be made real, whilst \( s \) will remain a complex parameter.

To fit the fermion masses and mixings we perform a \( \chi^2 \)-analysis, with the pull and the \( \chi^2 \)-function defined as:

\[ P_i = \frac{O_{i, \text{th}} - E_{i, \text{exp}}}{\sigma_i}, \quad \chi^2 = \sum_i P_i^2, \]

where \( \sigma_i \) represent experimental 1\( \sigma \) uncertainty and \( O_{i, \text{th}} \) and \( E_{i, \text{exp}} \) represent theoretical prediction, experimental central value and pull of observable \( i \). We perform the fit at the GUT scale, which we take to be \( M_{\text{GUT}} = 2 \times 10^{16} \) GeV.

To get the GUT scale values of the observables in the charged fermion sector we take the central values at the \( M_Z \) scale from Table-1 of Ref. [36]. For neutrino observables, the low energy values are taken from Ref. [37]. For reader’s convenience, we collect all these low energy values in Table I. With these inputs, we do the RGE running of the Yukawa couplings [38,39], the CKM parameters [40] and the effective couplings of the neutrino 5D operator, \( \kappa \) [41–43] within the SM up to the scale \( \mu = 1 \) TeV. Proton decay constraints on the model requires a mini-split SUSY scenario, wherein the gauginos and Higgsinos are taken to be at the TeV scale in order to satisfy gauge coupling unification constraint as well the dark matter constraints. Proton decay constraints lead to a lower limit on the squark and slepton masses of \( O(100) \) TeV within the model. For the fit purpose, we fix the squark and slepton mass scale to be 100 TeV. In order to correctly take into account the...
### Table I: Values of observables in the charged fermion sector at $M_Z$ scale, taken from Ref. [36]. Here experimental central values with associated 1 $\sigma$ uncertainties are quoted. The masses of the charged fermions are given by the relations $m_i = \nu y_i$, with $\nu = 174.104$ GeV.

For neutrino observables, the low energy values are taken from Ref. [37].

For neutrino observables, the low energy values are taken from Ref. [37].

The values of the observables resulting from the RGE running (the input values) and also the best fit values are presented in Table II. The best fit parameters corresponding to $\tan \beta = 10$ are found to be:

$$r = 8.730612, \quad s = 3.397119 \times 10^{-1} + 1.039406 \times 10^{-2} i, \quad c_R = 3.187055 \times 10^{14} \text{ GeV}. \quad \quad (6.9)$$

$$H = \begin{pmatrix}
8.242157 \times 10^{-5} - 9.797007 \times 10^{-5} i & -5.245430 \times 10^{-4} + 4.580976 \times 10^{-4} i & 2.334517 \times 10^{-3} + 1.273798 \times 10^{-3} i \\
-5.245430 \times 10^{-4} + 4.580976 \times 10^{-4} i & 2.117891 \times 10^{-3} - 2.129217 \times 10^{-3} i & -1.111526 \times 10^{-2} - 7.542882 \times 10^{-3} i \\
2.334517 \times 10^{-3} + 1.273798 \times 10^{-3} i & -1.111526 \times 10^{-2} - 7.542882 \times 10^{-3} i & -1.649528 \times 10^{-2} + 5.153891 \times 10^{-2} i
\end{pmatrix}, \quad \quad (6.10)
We shall use these best fit parameters in our evaluation of proton lifetime, discussed in the next section.

Table II: Best fit values of the observables corresponding to the type-I dominance seesaw scenarios is presented. This best fit corresponds to total $\chi^2 = 4$. The values of the observables at the GUT scale are obtained by RGE evolution as explained detail in the text. We have chosen $M_{GUT} = 2 \times 10^{16}$ GeV and we have fixed $\tan \beta = 10$ for this numerical analysis. For the associated $1\sigma$ uncertainties of the observables at the GUT scale, we keep the same percentage uncertainty with respect to the central value of each quantity as that at the $M_Z$ scale. For the charged lepton Yukawa couplings at the GUT scale, a relative uncertainty of 1% is assumed in order to take into account the theoretical uncertainties arising for example from threshold effects. As explained in the text, the RGE running of the two mass squared differences in the neutrino sector are performed from the low scale to the intermediate scale. The parameters corresponding to this best fit are presented in Eqs. (6.9) - (6.11).
Prediction of the model for neutrino CP violation

Corresponding to the best fit, the predicted quantities in our model are presented in Table III. We note that the best fit value of the CP violating parameter $\delta_{CP}$ for neutrino oscillations is $17^0$, which is not however a firm prediction. In Fig. 1, we show the variation of this phase corresponding to the case of $\chi^2 < 20$, which should all be acceptable. For example, if $\delta_{CP}$ is measured to be $3\pi/2$, the model is still acceptable, with a $\chi^2$ per degree of freedom around 0.9.

| Quantity | Predicted Value |
|----------|-----------------|
| $\{m_1, m_2, m_3\}$ (in eV) | $\{3.32 \times 10^{-3}, 9.89 \times 10^{-3}, 5.42 \times 10^{-2}\}$ |
| $\{\delta_{PMNS}, \alpha_{21}^{PMNS}, \alpha_{31}^{PMNS}\}$ | $\{17.0^0, 344.13^0, 337.45^0\}$ |
| $\{m_{\cos}, m_\beta, m_{\beta\beta}\}$ (in eV) | $\{6.74 \times 10^{-2}, 6.47 \times 10^{-3}, 6.11 \times 10^{-3}\}$ |
| $\{M_1, M_2, M_3\}$ (in GeV) | $\{1.29 \times 10^{10}, 6.25 \times 10^{11}, 4.13 \times 10^{12}\}$ |

Table III: Predictions corresponding to the best fit values presented in Table II for type-I dominance seesaw scenario. $m_i$ are the light neutrino masses, $M_i$ are the right handed neutrino masses, $\alpha_{21,31}$ are the Majorana phases following the PDG parametrization, $m_\cos = \sum_i m_i$, $m_\beta = \sum_i |U_{ei}|^2 m_i$ is the effective mass parameter for beta-decay and $m_{\beta\beta} = |\sum_i U_{e\alpha}^2 m_i|$ is the effective mass parameter for neutrinoless double beta decay.

Figure 1: Variation of the Dirac type CP violating phase $\delta_{CP}$ in the neutrino sector with respect to the model parameters. For this plot we restrict ourselves to the case $\chi^2 < 20$. 
7 Proton decay calculation

At this point we can estimate the proton decay rate or, better, we can determine the minimal allowed value of the sfermion mass (assumed here for simplicity to be universal) from the proton decay constraint. We assume that these rates are dominated by wino exchange and take as a benchmark the value of its mass to be \( m_{\text{wino}} = 1 \text{ TeV} \). Different fits are possible and the resulting sfermions mass scale \( m_S \) depends very much on that.

To compute proton decay rate, we define the relevant amplitude functions \( A_{1bcp}^\nu \) as [47]:

\[
A_{1bcp}^\nu = \frac{M_{T11}^{-1}}{N_d^2} \left( \hat{A}_{1bcp}^\nu[H, H] + x\hat{A}_{1bcp}^\nu[F, F] + y\hat{A}_{1bcp}^\nu[H, F] + z\hat{A}_{1bcp}^\nu[F, H] \right),
\]

where (denoting \( Y^{(1)}, Y^{(2)} = H \) or \( F \))

\[
\hat{A}_{1bcp}^\nu[Y^{(1)}, Y^{(2)}] = \sum_{\ell=1}^{3} Y_{1\ell}^{(1)} Y_{2\ell}^{(2)} V_{ib} V_{2c} U_{\ell \rho} - \sum_{\ell=1}^{3} Y_{2\ell}^{(1)} Y_{1\ell}^{(2)} V_{ib} V_{2c} U_{\ell \rho} - \sum_{\ell=1}^{3} Y_{1\ell}^{(1)} Y_{2\ell}^{(2)} V_{ib} V_{2c} U_{\ell \rho}
\]

\[
+ \sum_{\ell=1}^{3} Y_{1\ell}^{(1)} Y_{3\ell}^{(2)} V_{ib} V_{3c} U_{\ell \rho} - \sum_{\ell=1}^{3} Y_{2\ell}^{(1)} Y_{3\ell}^{(2)} V_{ib} V_{3c} U_{\ell \rho} - \sum_{\ell=1}^{3} Y_{3\ell}^{(1)} Y_{1\ell}^{(2)} V_{ib} V_{2c} U_{\ell \rho}
\]

\[
- \sum_{\ell=1}^{3} Y_{1\ell}^{(1)} Y_{3\ell}^{(2)} V_{ib} V_{3c} U_{\ell \rho} + \sum_{\ell=1}^{3} Y_{2\ell}^{(1)} Y_{3\ell}^{(2)} V_{ib} V_{3c} U_{\ell \rho} + \sum_{\ell=1}^{3} Y_{3\ell}^{(1)} Y_{2\ell}^{(2)} V_{ib} V_{3c} U_{\ell \rho},
\]

and the \( x, y, z \) parameters are defined as [48]:

\[
x = \left( \sqrt{3} \right)^2 \frac{M_{T31}^2}{M_{T11}^2}, \quad y = \left( \sqrt{3} \right) \frac{M_{T12}^-}{M_{T11}^-}, \quad z = \left( \sqrt{3} \right) \frac{M_{T11}^-}{M_{T11}}.
\]

Here, \( M_T \) is the mass matrix for the color triplets which mediate the \( d = 5 \) proton decay given in Eq. (A.10), and \( q_2 \) is the component of MSSM field \( H_d \) arising from \( \mathbf{10}_H \) given in Eq. (A.7) of Appendix A (the normalization factor \( N_d \) is also defined in Appendix A). The supergraphs generating these amplitudes are shown in Fig. 2.

For proton decay calculation we work in the up-quark mass matrix diagonal basis and \( V = V_{CKM}, U = U_{PMNS} \), with low energy values,

\[
\sin \theta_{12}^l = 0.22460, \sin \theta_{23}^l = 0.04193, \sin \theta_{13}^l = 0.00351, \delta^l = 1.21409 \quad (7.4)
\]

\[
\sin^2 \theta_{12}^l = 0.30004, \sin^2 \theta_{23}^l = 0.40987, \sin^2 \theta_{13}^l = 0.02297, \delta^l = 0. \quad (7.5)
\]

\(^1\)One can easily transform the result for other values of this mass, knowing that \( \tau_p \propto 1/m_{\text{wino}}^2 \).
The rates for the dominant proton decay processes in our model are given by:

$$
\Gamma(p \rightarrow \bar{\nu}K^+) = \frac{(m_p^2 - m_K^2)^2 R_L^2}{32\pi m_p^3} \left( \frac{g_2^2}{16\pi^2} 2f(m_a, m_b)A_S \right)^2 
\times \sum_{\rho=1}^3 |A_{21\rho}^\nu(K^+|(us)_{Ld_L}|p) + A_{112\rho}^\nu(K^+|(ud)_{Ls_L}|p)|^2, 
$$

$$
\Gamma(p \rightarrow \bar{\nu}\pi^+) = \frac{(m_p^2 - m_\pi^2)^2 R_L^2}{32\pi m_p^3} \left( \frac{g_2^2}{16\pi^2} 2f(m_a, m_b)A_S \right)^2 
\times \sum_{\rho=1}^3 |A_{111\rho}^\nu(\pi^+|(ud)_{Ld_L}|p)|^2.
$$

The nuclear matrix elements appearing in Eqs. (7.6)-(7.7) are given in Appendix A, Eq. (A.14). The values of the dressing functions $f(a, b)$ are given by:

$$
f(a, b) = \begin{cases} 
1/2 & , \quad a = b = 1 \\
1-b+b \log b & , \quad a = 1 \neq b \\
1-a+a \log a & , \quad b = 1 \neq a \\
\frac{1}{(a-1)^2} & , \quad a = b \neq 1 \\
-1+a-a \log a & , \quad a = b \neq 1 \\
\frac{1}{a-b} \left( \frac{a}{a-1} \log a - \frac{b}{b-1} \log b \right) & , \quad 1 \neq a \neq b \neq 1 
\end{cases}
$$

We define

$$
(f_{UE})_{ij} = f \left( (m_i^U)^2, (m_j^E)^2 \right) 
$$

$$
(f_{UD})_{ij} = f \left( (m_i^U)^2, (m_j^D)^2 \right),
$$
which we take for simplicity to be universal:

$$m_{1,2,3} = \frac{m_{\text{susy}}}{m_{\text{wino}}}.$$  \hfill (7.12)

It is tempting to maximize the proton lifetime with respect to the parameters \((x, y, z)\) appearing in Eq. (7.1), as was done in Ref. [48]. However, we found that maximizing proton lifetime with respect to \((x, y, z)\) does not maximize it with the full set of model parameters. In particular, \(M_{T11}^{-1}/N_d^2\) appears in the amplitude, which should be included in the maximization process. The constraints we obtain are much stronger than the ones quoted in Ref. [48].

To get the constraints on the SUSY scale from proton decay we followed the following procedure:

- We first set \(\lambda_1 = 1\) and \(V_3 = M_{\text{GUT}} = 2 \times 10^{16}\) GeV. Then we randomly chose the perturbative values of the real input parameters \(\lambda_2, \lambda_3, \lambda_{10}\), and of the complex input parameters \(\lambda_4, \lambda_{11}, \lambda_{12}, \lambda_{13}\). Finally we chose complex \(V_2\) of the same order as \(V_3\). The equations of motion determine then \(V_1, V_E, m_1\) and \(m_2\) through (4.4)-(4.7), while the doublet-triplet fine-tuning determines \(m_3\) and eq. (4.8) the parameter \(m_5\) as a function of \(\lambda_5\), which does not enter in any other quantity and is thus free. We demanded that the choice of parameters reproduce \(r\) and \(s\) (with at most 10% error) given in Eq. (6.9);

- We took the Yukawa couplings from Eqs. (6.10)-(6.11) and calculated the proton decay amplitude.

- With parameters so chosen, we calculated the minimal \(m_S\) (sfermion mass) which satisfies all bounds for proton decay.

By taking \(\tau_p(p \to K^+\bar{\nu}) \geq 5.9 \times 10^{33}\) yrs. and \(\tau_p(p \to \nu\pi^+) \geq 3.9 \times 10^{32}\) yrs [20], we got the histogram probability vs minimal \(m_S\) (in GeV) shown in Fig. 3 for 11737 fits. The minimal value among all fits obtained is \(m_S = 238\) TeV, for the inputs \(|V_R| = |\overline{V}_R| = 10^{12}\) GeV and

$$m_1/M_{\text{GUT}} = 0.408879 - 0.0444793i$$  \hfill (7.13)

$$m_2/M_{\text{GUT}} = -0.0446538 - 0.0166942i$$  \hfill (7.14)

$$m_3/M_{\text{GUT}} = -1.22975 - 1.46081i$$  \hfill (7.15)

\begin{align*}
\lambda_1 &= 1, \quad \lambda_{10} = 0.742887 \\
\lambda_2 &= 0.966662, \quad \lambda_{11} = 0.213946 + 0.241336i \hfill (7.16)
\end{align*}
while $\lambda_8$ can be chosen arbitrarily with $m_5$ given by eq. (4.8).

For these choices we get

$$V_1/M_{GUT} = -0.351264 - 0.24144i$$ (7.20)  
$$V_2/M_{GUT} = -0.558131 + 0.38363i$$ (7.21)  
$$V_3/M_{GUT} = 1.$$ (7.22)  
$$v_E/M_{GUT} = -0.0247119 - 0.394114i$$ (7.23)  

and

$$r = 7.85956, \quad s = 0.315991 - 0.0045605i$$ (7.24)

which is within 10% of the original $r,s$ from (6.9).

We thus conclude that with a mini-split SUSY spectrum, the model is compatible with proton lifetime constraints.

8 Embedding of the model in pure gravity mediation

As shown by the proton decay analysis, the model requires a heavy SUSY particle spectrum, with lighter gauginos and Higgsinos. The latter will provide a dark matter candidate if the lightest of these fermions mass is of order TeV. While the squark and slepton masses can be varied at will without affecting gauge coupling unification (as they belong to complete
multiplets of GUT), the same is not true for the gauginos and to some extend the Higgsinos. Their masses being at the TeV scale would preserve gauge coupling unification as in the MSSM.

Such a mini-split SUSY spectrum may have a natural origin in pure gravity mediated SUSY breaking \cite{33,34}. In this scheme, all mass parameters, including the $\mu$ term for the MSSM Higgsino mass, arise from SUSY breaking mediated by gravity. Gaugino masses are zero at tree-level, but are induced by anomaly mediated contributions, as well as by Higgsino threshold effects. Thus, these models have naturally light gauginos. The mass spectrum of the gauginos is given by \cite{34}:

\begin{align}
M_1 &= \frac{33}{5} \frac{g_1^2}{16\pi^2} (m_{3/2} + \frac{1}{11} L) \\
M_2 &= \frac{g_2^2}{16\pi^2} (m_{3/2} + L) \\
M_3 &= -3 \frac{g_3^2}{16\pi^2} m_{3/2},
\end{align}

(8.1)

where

\[ L = \mu_H \sin 2\beta \frac{m_A^2}{|\mu_H|^2 - m_A^2} \ln \frac{|\mu_H|^2}{m_A^2}. \]

(8.2)

Here terms proportional to $L$ arise from Higgsino threshold. The $\mu$ term and the $B\mu$ term in this scheme have the form

\begin{align}
\mu_H &= cm_{3/2} \\
B\mu_H &= cm_{3/2}^2 + c' |F_X|^2 \frac{1}{M_{Pl}^2}.
\end{align}

(8.3)

where $c, c'$ are order one coefficients. Thus the Higgsino threshold corrections to the gaugino masses are of the same order as the anomaly mediated corrections. Consequently, the mass ratios $M_2/M_3$ is not exactly predicted.

It should be noted that the neutral Wino is the lightest SUSY particle in this scheme, and can serve as the dark matter. For a thermal dark matter, relic abundance would require the Wino mass $M_2$ to be near 2.7 TeV. However, in these models, Wino dark matter can be produced non-thermally, via the decay of the gravitino, in which case the 2.7 TeV mass constraint does not hold. In fact, the preferred range for Wino dark matter is below 1 TeV \cite{33}. The Wino cannot be below about 200 GeV, as that would modify BBN significantly.

With Wino mass in the range 200 GeV to 1 TeV, the gluino can have mass of order $(3-20)$ TeV. The squark and slepton masses are of order $(100-300)$ TeV. This is precisely the spectrum preferred in the model from proton decay constraints. Within this scenario, the squark and slepton masses cannot be much above 300 TeV, as opposed to general split-SUSY models. This leads to a prediction for the partial lifetime for $p \rightarrow \bar{\nu}K^+$ within the
model: It cannot exceed a factor of 10 compared to the current lower limit of $5.9 \times 10^{33}$ yrs. Thus the model is testable in proton decay searches. In addition, the Wino LSP and its charged partners may be observable at colliders. Furthermore, flavor violation arising from SUSY particle exchange such as $\mu \rightarrow e\gamma$ are highly suppressed in this scenario, owing to the large masses of the sleptons and squarks.

9 Discussion and conclusion

In this paper we have resurrected the minimal Yukawa sector of SUSY $SO(10)$, which explains the entirety of fermion masses and mixings in terms of two symmetric Yukawa matrices. While the fermion masses and mixing angles were known to fit well with such a structure arising from a $10_H$ and a $\overline{126}_H$ of Higgs fields, once they are coupled minimally to a $\{126_H + 210_H\}$ Higgs sector, the model becomes inconsistent. This is because of the tight nature of symmetry breaking and fermion mass generation. The scale of right-handed neutrino masses is required from light neutrino mass spectrum to be $V_R \sim 10^{12}$ GeV, while for such values of $V_R$, the masses of several colored chiral superfields become of order $V_R^2/M_{GUT} \sim 10^{10}$ GeV. This causes the gauge couplings to become non-perturbative before reaching the GUT scale, causing inconsistencies.

Our proposal to fix this problem is to introduce a $54_H$ that has no Yukawa couplings to the SM fermions. This enables a reduction of the $B-L$ symmetry breaking scale to $V_R \sim 10^{12}$ GeV, without causing any of the chiral superfields to remain light (except for a pair of SM singlets needed for $B-L$ symmetry breaking). The supersymmetric sector of such a theory has a relatively small number of parameters, and the predictions arising from the Yukawa sector now become reliable and consistent.

We have performed a fermion fit in this scenario including threshold effects from $V_R$ to $M_{GUT}$. In addition, proton decay constraints require the SUSY spectrum to be mini-split, with the gauginos and Higgsinos having masses of order TeV and the squarks and sleptons of order 100 TeV. In our new fit to fermion data we have incorporated the mini-split SUSY spectrum as well. Such a scenario has a natural embedding in pure gravity mediation. In this rendition proton lifetime for decay into $\pi K^+$ cannot exceed another order of magnitude compared to the present lower limit of $5.9 \times 10^{33}$ yrs. The model can also be tested at colliders with the discovery of Wino LSP along with its charged partners.
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Appendices

A Details of proton decay calculation

Here we provide more details on the proton decay calculation.

Corresponding to the solution of the stationary conditions Eqs. (4.4) -(4.8), the $4 \times 4$ Higgs doublet mass matrix is given by

$$
M_D = \begin{pmatrix}
    m_3 + \sqrt{\frac{3}{5}} V_E \lambda_{13} & \frac{(\sqrt{2}V_2-V_3)\lambda_3}{2\sqrt{5}} & -\frac{(\sqrt{2}V_2-V_3)\lambda_3}{2\sqrt{5}} & -\frac{\lambda_4 V_R}{\sqrt{5}} \\
    \frac{(\sqrt{2}V_2-V_3)\lambda_3}{2\sqrt{5}} & 2V_2^2 + 16V_2V_3 + \sqrt{2}V_3^2 & \frac{(V_2^2 - 2V_3^2)\lambda_1\lambda_{12}}{15\sqrt{2}V_2\lambda_R} & 0 \\
    -\lambda_3 V_R \frac{\sqrt{5}}{\sqrt{5}} & \frac{(V_2^2 - 2V_3^2)\lambda_1\lambda_{11}}{15\sqrt{2}V_2\lambda_R} & \frac{(2\sqrt{2}V_2^2 + 8V_2V_3 + \sqrt{2}V_3^2)\lambda_2}{120V_2} & \frac{\lambda_3 V_R}{10} \\
    \frac{\lambda_4 V_R}{\sqrt{5}} & 0 & \frac{\lambda_3 V_R}{10} & m_{44}
\end{pmatrix},
$$

(A.1)

where

$$
m_{44} = \frac{\lambda_1}{10V_2} \left( 2\sqrt{2}V_2^2 + 5V_2V_3 - \sqrt{2}V_3^2 \right) - \frac{1}{4} \sqrt{\frac{3}{5}} \lambda_{10} V_E.
$$

(A.2)

The matrix in Eq. (A.1) is written in a basis where the row vector

$$\{H_{(1,2,2)}^{(1,2,-1/2)}, \Delta_{(15,2,2)}^{(1,2,-1/2)}, \Delta_{(15,2,2)}^{(1,2,-1/2)}, \Phi_{(6,2,2)}^{(1,2,-1/2)}\}$$

multiplies the matrix from the left and the column vector

$$\{H_{(1,2,2)}^{(1,2,1/2)}, \Delta_{(15,2,2)}^{(1,2,1/2)}, \Delta_{(15,2,2)}^{(1,2,1/2)}, \Phi_{(6,2,2)}^{(1,2,1/2)}\}^T$$

multiplies from the right. Here we have used the decomposition under $SU(4)_c \times SU(2)_L \times SU(2)_R$ as subscripts and the decomposition under SM as superscripts.
The determinant of the Higgs doublet mass matrix should be near zero. This condition is achieved by a fine-tuning, which we use to determine \( m_3 \). Note that, in the limit \( V_R = 0 \), the Higgs doublet from the \( 210_H \) decouples from the rest, hence only the upper \( 3 \times 3 \) block is relevant to an excellent approximation. The MSSM Higgs superfields \( H_u \) and \( H_d \) are mixtures of the doublets coming from \( 10_H, 126_H \) and \( \overline{126}_H \) multiplets denoted as

\[
H_u = N_u(H_u^{10} + p_2 H_u^{126} + p_3 H_u^{\overline{126}}),
\]

\[
H_d = N_d(H_d^{10} + q_2 H_d^{126} + q_3 H_d^{\overline{126}}).
\]

The normalization factors are defined as \( N_u = \frac{1}{\sqrt{1 + |p_2|^2 + |p_3|^2}} \) and \( N_d = \frac{1}{\sqrt{1 + |q_2|^2 + |q_3|^2}} \). The expressions for \( p_i \) and \( q_i \) can be found in a straightforward way from the left and the right eigenvectors corresponding to the zero eigenvalue of Eq. (A.1):

\[
p_2^* = \frac{6\sqrt{3}\lambda_{10}V_2}{\lambda_2^2\lambda_{10}^2} \left( 4V_2^2 + 6\sqrt{2}V_4V_2^2 - 6V_3V_2 - \sqrt{2}V_3^3 + 4\lambda_1\lambda_2\lambda_3 \left( 2V_2 + \sqrt{2}V_3 \right) \left( 2V_2^2 - V_3^2 \right) \right),
\]

\[
p_3^* = -\frac{6\sqrt{3}\lambda_{10}V_2}{\lambda_2^2\lambda_{10}^2} \left( 4V_2^2 + 24\sqrt{2}V_4V_2^2 + 68V_3V_2^2 + 12\sqrt{2}V_3^2V_2 + V_4^3 \right) - 16\lambda_1^2\lambda_2\lambda_3 \left( V_2^2 - V_3^2 \right)^2,
\]

\[
q_2^* = \frac{6\sqrt{3}\lambda_{10}V_2}{\lambda_2^2\lambda_{10}^2} \left( 4V_2^2 + 6\sqrt{2}V_4V_2^2 - 6V_3V_2 - \sqrt{2}V_3^3 + 4\lambda_1\lambda_2\lambda_3 \left( 2V_2 + \sqrt{2}V_3 \right) \left( 2V_2^2 - V_3^2 \right) \right),
\]

\[
q_3^* = -\frac{6\sqrt{3}\lambda_{10}V_2}{\lambda_2^2\lambda_{10}^2} \left( 4V_2^2 + 24\sqrt{2}V_4V_2^2 + 68V_3V_2^2 + 12\sqrt{2}V_3^2V_2 + V_4^3 \right) - 16\lambda_1^2\lambda_2\lambda_3 \left( V_2^2 - V_3^2 \right)^2,
\]

With these definitions, the parameters \( r \) and \( s \) of Eq. (6.6) which are determined from the fermion mass fit can be expressed in terms of the superpotential parameters:

\[
r = \frac{N_u}{N_d}, \quad s^* = \frac{p_3}{q_2}.
\]

In our numerical scan for proton decay amplitude, we impose the constraints on \( r \) and \( s \) from fermion mass fit in conjunction with Eq. (A.9).

The mass matrix for the color triplets Higgsinos mediating \( d = 5 \) proton decay is given by

\[
M_T = \begin{pmatrix}
\frac{(m_3 + 2\sqrt{2}\lambda_1\lambda_3)}{\sqrt{\lambda}} & \frac{(2V_2^2-V_3^2)\lambda_3}{\sqrt{V_2^2+V_4^2}} & \frac{(2V_2^2-V_3^2)\lambda_4}{\sqrt{V_2^2+V_4^2}} & -\frac{(2V_2^2+V_4^2)\lambda_4}{\sqrt{V_2^2+V_4^2}} & -\frac{\lambda_4 V_4}{\sqrt{V_4}} \\
\frac{(2V_2^2-V_3^2)\lambda_3}{\sqrt{V_2^2+V_4^2}} & \frac{(2V_2^2-V_3^2)\lambda_3}{\sqrt{V_2^2+V_4^2}} & \frac{(2V_2^2-V_3^2)\lambda_4}{\sqrt{V_2^2+V_4^2}} & -\frac{(2V_2^2+V_4^2)\lambda_4}{\sqrt{V_2^2+V_4^2}} & -\frac{\lambda_4 V_4}{\sqrt{V_4}} & 0 \\
\frac{(2V_2^2-V_3^2)\lambda_3}{\sqrt{V_2^2+V_4^2}} & \frac{(2V_2^2-V_3^2)\lambda_3}{\sqrt{V_2^2+V_4^2}} & \frac{(2V_2^2-V_3^2)\lambda_4}{\sqrt{V_2^2+V_4^2}} & -\frac{(2V_2^2+V_4^2)\lambda_4}{\sqrt{V_2^2+V_4^2}} & -\frac{\lambda_4 V_4}{\sqrt{V_4}} & 0 \\
0 & 0 & 0 & \frac{\lambda_4 V_4}{\sqrt{V_4}} & \frac{\lambda_4 V_4}{\sqrt{V_4}} & \frac{\lambda_4 V_4}{\sqrt{V_4}} & \frac{\lambda_4 V_4}{\sqrt{V_4}} & -\frac{\lambda_4 V_4}{\sqrt{V_4}} & -\frac{\lambda_4 V_4}{\sqrt{V_4}} & \frac{\lambda_4 V_4}{\sqrt{V_4}} & \frac{\lambda_4 V_4}{\sqrt{V_4}} & 0 \\
\frac{(2V_2^2-V_3^2)\lambda_3}{\sqrt{V_2^2+V_4^2}} & \frac{(2V_2^2-V_3^2)\lambda_3}{\sqrt{V_2^2+V_4^2}} & \frac{(2V_2^2-V_3^2)\lambda_4}{\sqrt{V_2^2+V_4^2}} & -\frac{(2V_2^2+V_4^2)\lambda_4}{\sqrt{V_2^2+V_4^2}} & -\frac{\lambda_4 V_4}{\sqrt{V_4}} & 0 \\
\frac{(2V_2^2-V_3^2)\lambda_3}{\sqrt{V_2^2+V_4^2}} & \frac{(2V_2^2-V_3^2)\lambda_3}{\sqrt{V_2^2+V_4^2}} & \frac{(2V_2^2-V_3^2)\lambda_4}{\sqrt{V_2^2+V_4^2}} & -\frac{(2V_2^2+V_4^2)\lambda_4}{\sqrt{V_2^2+V_4^2}} & -\frac{\lambda_4 V_4}{\sqrt{V_4}} & 0 \\
\frac{(2V_2^2-V_3^2)\lambda_3}{\sqrt{V_2^2+V_4^2}} & \frac{(2V_2^2-V_3^2)\lambda_3}{\sqrt{V_2^2+V_4^2}} & \frac{(2V_2^2-V_3^2)\lambda_4}{\sqrt{V_2^2+V_4^2}} & -\frac{(2V_2^2+V_4^2)\lambda_4}{\sqrt{V_2^2+V_4^2}} & -\frac{\lambda_4 V_4}{\sqrt{V_4}} & 0 \\
\frac{(2V_2^2-V_3^2)\lambda_3}{\sqrt{V_2^2+V_4^2}} & \frac{(2V_2^2-V_3^2)\lambda_3}{\sqrt{V_2^2+V_4^2}} & \frac{(2V_2^2-V_3^2)\lambda_4}{\sqrt{V_2^2+V_4^2}} & -\frac{(2V_2^2+V_4^2)\lambda_4}{\sqrt{V_2^2+V_4^2}} & -\frac{\lambda_4 V_4}{\sqrt{V_4}} & 0 \\
\frac{(2V_2^2-V_3^2)\lambda_3}{\sqrt{V_2^2+V_4^2}} & \frac{(2V_2^2-V_3^2)\lambda_3}{\sqrt{V_2^2+V_4^2}} & \frac{(2V_2^2-V_3^2)\lambda_4}{\sqrt{V_2^2+V_4^2}} & -\frac{(2V_2^2+V_4^2)\lambda_4}{\sqrt{V_2^2+V_4^2}} & -\frac{\lambda_4 V_4}{\sqrt{V_4}} & 0
\end{pmatrix}
\]
where
\[
m_{55} = -\frac{\lambda_1}{60V_2} \left( 8\sqrt{2}V_2^2 - 40V_2V_3 + \sqrt{2}V_3^2 \right) + \frac{\lambda_{10}}{2\sqrt{15}} V_E. \tag{A.11}
\]

The matrix in Eq. (A.10) is written in a basis where the row vector
\[
\{ H_{(6,1,1)}^{(3,1,1/3)}, \overline{\Delta}_{(6,1,1)}^{(3,1,1/3)}, \Delta_{(6,1,1)}^{(3,1,1/3)}, \overline{\Delta}_{(10,1,1)}^{(3,1,1/3)}, \Phi_{(15,1,1/3)}^{(3,1,1/3)} \}
\]
multiplies the matrix from the left and the column vector
\[
\{ H_{(6,1,1)}^{(3,1,-1/3)}, \overline{\Delta}_{(6,1,1)}^{(3,1,-1/3)}, \Delta_{(6,1,1)}^{(3,1,-1/3)}, \overline{\Delta}_{(10,1,1)}^{(3,1,-1/3)}, \Phi_{(15,1,1/3)}^{(3,1,-1/3)} \}^T
\]
multiplies from the right.

In the limit \( V_R = 0 \), only the upper 4 \times 4 block of Eq. (A.10) is relevant. We thus work in the approximation that 210\(_H\) contribution decouples. Note that the decoupled color triplets from 210\(_H\) do not lead to \( d = 5 \) proton decay.

The relevant nuclear matrix elements for proton decay calculation in our model are [49]:
\[
\langle K^+|(us)_{L}d_{L}|p\rangle = 0.041 \text{ GeV}^2, \tag{A.12}
\]
\[
\langle K^+|(ud)_{L}s_{L}|p\rangle = 0.139 \text{ GeV}^2, \tag{A.13}
\]
\[
\langle \pi^+|(ud)_{L}d_{L}|p\rangle = 0.189 \text{ GeV}^2. \tag{A.14}
\]

We use as input \( m_p = 0.94 \text{ GeV} \), \( m_K = 0.494 \text{ GeV} \), \( m_\pi = 0.139 \text{ GeV} \) \( R_L = 1.25 \), \( A_S = 6.54 \), \( m_{\text{wino}} = 1 \text{ TeV} \), and \( g_2(m_{\text{susy}}) = 0.6518 \).

## B RGEs from TeV scale to SUSY scale

In this Appendix we collect the relevant RGEs for split-SUSY relevant for evolution from 1 TeV to the squark and slepton mass scale (\( m_\tilde{t} \)) of order 100 TeV. The 2-loop renormalization-group equations for the gauge couplings are given by [25]:
\[
(4\pi)^2 \frac{d}{dt} g_i = g_i^3 b_i + \frac{g_i^3}{(4\pi)^2} \left[ \sum_{j=1}^{3} B_{ij} g_j^2 - \sum_{\alpha=u,d,e} d_{d} \text{Tr}(g^\alpha y^{\alpha}) \right. \\
- \left. \left. d_W \left( \bar{g}_u^2 + \bar{g}_d^2 \right) - d_B \left( \bar{g}_u^2 + \bar{g}_d^2 \right) \right] , \tag{B.1}
\]
where \( t = \ln \tilde{\mu} \) with \( \tilde{\mu} \) being the renormalization scale. Here the convention used is \( g_1^2 = (5/3) g^2 \). Eq. (B.1) is scheme-independent up to the two-loop order.

In the effective theory below \( m_\tilde{t} \), the \( \beta \)-function coefficients are
\[
b = \begin{pmatrix} 9/2 & -7/6 & -5 \end{pmatrix}, \quad B = \begin{pmatrix} 104/25 & 18/5 & 44/5 \\ 6/5 & 106/3 & 12 \\ 11/10 & 9/2 & 22 \end{pmatrix} , \tag{B.2}
\]
The renormalization-group equations for the gaugino couplings are:

\[
d^u = \left( \frac{17}{10}, \frac{3}{2}, \frac{3}{2}, \frac{3}{2} \right), \quad d^d = \left( \frac{1}{2}, \frac{3}{2}, \frac{3}{2}, \frac{3}{2} \right), \quad d^e = \left( \frac{3}{2}, \frac{1}{2}, 0 \right),
\]

\[
d^W = \left( \frac{9}{20}, \frac{11}{4}, 0 \right), \quad d^B = \left( \frac{3}{20}, \frac{1}{4}, 0 \right).
\]

(B.3) \hspace{1cm} (B.4)

For the Yukawa coupling evolution to one loop order, one has

\[
(4\pi)^2 \frac{d}{dt} y^u = y^u \left( -3 \sum_{i=1}^{3} c_i^u g_i^2 + \frac{3}{2} y^u y^u - \frac{3}{2} y^{d\dagger} y^d + T \right),
\]

\[
(4\pi)^2 \frac{d}{dt} y^d = y^d \left( -3 \sum_{i=1}^{3} c_i^d g_i^2 + \frac{3}{2} y^u y^u + \frac{3}{2} y^{d\dagger} y^d + T \right),
\]

\[
(4\pi)^2 \frac{d}{dt} y^e = y^e \left( -3 \sum_{i=1}^{3} c_i^e g_i^2 + \frac{3}{2} y^{e\dagger} y^e + T \right),
\]

(B.5) \hspace{1cm} (B.6) \hspace{1cm} (B.7)

where,

\[
T = \text{Tr} \left( 3y^u y^u + 3y^{d\dagger} y^d + y^{e\dagger} y^e \right) + \frac{3}{2} \left( \tilde{g}^2_u + \tilde{g}^2_d \right) + \frac{1}{2} \left( \tilde{g}^2_u + \tilde{g}^2_u \right)
\]

\[
c^u = \left( \frac{17}{60}, \frac{3}{4}, \frac{3}{3} \right), \quad c^d = \left( \frac{1}{12}, \frac{3}{4}, \frac{1}{3} \right), \quad c^e = \left( \frac{3}{4}, \frac{3}{4}, \frac{1}{0} \right).
\]

(B.8) \hspace{1cm} (B.9)

The renormalization-group equations for the gaugino couplings are:

\[
(4\pi)^2 \frac{d}{dt} \tilde{g}_u = -3 \tilde{g}_u \sum_{i=1}^{3} C_i g_i^2 + \frac{5}{4} \tilde{g}_u^3 - \frac{1}{2} \tilde{g}_u \tilde{g}_d^2 + \frac{1}{4} \tilde{g}_u \tilde{g}_u^2 + \tilde{g}_u \tilde{g}_d \tilde{g}_u^2 + \tilde{g}_u T
\]

\[
(4\pi)^2 \frac{d}{dt} \tilde{g}_u^3 = -3 \tilde{g}_u \sum_{i=1}^{3} C_i g_i^2 + \frac{5}{4} \tilde{g}_u^3 + \frac{3}{2} \tilde{g}_u \tilde{g}_d^2 + \frac{3}{4} \tilde{g}_d \tilde{g}_u^2 + 3 \tilde{g}_d \tilde{g}_u \tilde{g}_u + \tilde{g}_u T
\]

(B.10) \hspace{1cm} (B.11)

\[
(4\pi)^2 \frac{d}{dt} \tilde{g}_d = -3 \tilde{g}_d \sum_{i=1}^{3} C_i g_i^2 + \frac{5}{4} \tilde{g}_d^3 - \frac{1}{2} \tilde{g}_d \tilde{g}_u^2 + \frac{1}{4} \tilde{g}_u \tilde{g}_u^2 + \tilde{g}_u \tilde{g}_d \tilde{g}_d^2 + \tilde{g}_u \tilde{g}_d \tilde{g}_d^2 + \tilde{g}_d \tilde{g}_d \tilde{g}_d + \tilde{g}_d T
\]

\[
(4\pi)^2 \frac{d}{dt} \tilde{g}_d^3 = -3 \tilde{g}_d \sum_{i=1}^{3} C_i g_i^2 + \frac{5}{4} \tilde{g}_d^3 + \frac{3}{2} \tilde{g}_d \tilde{g}_u^2 + \frac{3}{4} \tilde{g}_u \tilde{g}_d^2 + 3 \tilde{g}_u \tilde{g}_d \tilde{g}_d + \tilde{g}_d \tilde{g}_d \tilde{g}_d + \tilde{g}_d T
\]

(B.12) \hspace{1cm} (B.13)

with \( C = \left( \frac{3}{20}, \frac{11}{4}, 0 \right), \quad C' = \left( \frac{3}{20}, \frac{3}{4}, 0 \right). \)

(B.14)

The one-loop RGE for the Higgs quartic coupling (which appears in the RGE for the \( d = 5 \) neutrino mass operator) is given by

\[
(4\pi)^2 \frac{d}{dt} \lambda = 12 \lambda^2 + \lambda \left[ -9 \left( \frac{g_1^2}{5} + g_2^2 \right) + 6 \left( \tilde{g}_u^2 + \tilde{g}_d^2 \right) + 2 \left( \tilde{g}_u^2 + \tilde{g}_d^2 \right) \\
+ 4 \text{Tr} (3y^{u\dagger} y^u + 3y^{d\dagger} y^d + y^{e\dagger} y^e) \right] + \frac{9}{2} \left( \frac{g_1^2}{2} + \frac{3g_1^2}{50} + \frac{g_2^2}{5} \right) \\
- 5 \left( \tilde{g}_u^4 + \tilde{g}_d^4 \right) - 2 \tilde{g}_u \tilde{g}_d \tilde{g}_u \tilde{g}_d - \left( \tilde{g}_u^2 + \tilde{g}_d^2 \right)^2 - 2 \left( \tilde{g}_u \tilde{g}_u + \tilde{g}_d \tilde{g}_d \right)^2 \\
- 4 \text{Tr} \left[ 3(y^{u\dagger} y^u)^2 + 3(y^{d\dagger} y^d)^2 + (y^{e\dagger} y^e)^2 \right].
\]

(B.15)
We have extended the one-loop RGE for the \( d = 5 \) neutrino mass operator to the case of split-SUSY:

\[
16\pi^2 \beta_\kappa = -\frac{3}{2} \left[ \kappa \left( Y_e^\dagger Y_e \right) + \left( Y_e^\dagger Y_e \right)^T \kappa \right] + \lambda \kappa - 3 g_2^2 \kappa \\
+ 2 \left[ \text{Tr} \left( 3Y_u^\dagger Y_u + 3Y_d^\dagger Y_d + Y_e^\dagger Y_e \right) + \frac{3}{2} \left( \tilde{g}_u^2 + \tilde{g}_d^2 \right) + \frac{1}{2} \left( \tilde{g}_u^2 + \tilde{g}_d^2 \right) \right] \kappa \tag{B.16}
\]

The neutrino mass matrix is given by \( \mathcal{M}_\nu = -\frac{\sigma^2}{2} \kappa \), with \( \sigma \simeq 174 \text{ GeV} \).

### C RGEs from \( V_R \) to \( M_{GUT} \)

Between \( V_R \) and \( M_{GUT} \) we have \( SM \times U(1)_{B-L} \). Due to the presence of the extra \( U(1) \) gauge boson, the RGEs of the Yukawa couplings get modified and are relevant for our study. Furthermore, the right-handed neutrinos have masses of order \( V_R \), and they will contribute to the evolution of parameters above \( V_R \). Here we provide the new RGEs including these effects. The coupling of \( \nu^c \) with a SM singlet field \( \bar{\sigma} \) is defined to be \( W \supset Y_\nu \nu^c \bar{\sigma} \).

\[
16\pi^2 \frac{dY_D}{dt} = Y_D \left[ 3Y_D^\dagger Y_D + Y_U^\dagger Y_U + \text{Tr} \left( Y_E^\dagger Y_E \right) + 3\text{Tr} \left( Y_D^\dagger Y_D \right) - \frac{7}{15} g_1^2 - 3g_2^2 - \frac{16}{3} g_3^2 - \frac{1}{6} g_B^2 \right] \tag{C.1}
\]

\[
16\pi^2 \frac{dY_U}{dt} = Y_U \left[ 3Y_U^\dagger Y_U + Y_D^\dagger Y_D + \text{Tr} \left( Y_{\nu_D}^\dagger Y_{\nu_D} \right) + 3\text{Tr} \left( Y_U^\dagger Y_U \right) - \frac{13}{15} g_1^2 - 3g_2^2 - \frac{16}{3} g_3^2 - \frac{1}{6} g_B^2 \right] \tag{C.2}
\]

\[
16\pi^2 \frac{dY_E}{dt} = Y_E \left[ 3Y_E^\dagger Y_E + Y_{\nu_D}^\dagger Y_{\nu_D} + 3\text{Tr} \left( Y_D^\dagger Y_D \right) - \frac{9}{5} g_1^2 - 3g_2^2 - \frac{3}{2} g_B^2 \right] \tag{C.3}
\]

\[
16\pi^2 \frac{dY_{\nu_D}}{dt} = Y_{\nu_D} \left[ 3Y_{\nu_D}^\dagger Y_{\nu_D} + Y_E^\dagger Y_E + \text{Tr} \left( Y_{\nu_D}^\dagger Y_{\nu_D} \right) + 3\text{Tr} \left( Y_U^\dagger Y_U \right) + Y_R^\dagger Y_R - \frac{3}{5} g_1^2 - 3g_2^2 - \frac{3}{2} g_B^2 \right] \tag{C.4}
\]

\[
16\pi^2 \frac{dY_R}{dt} = Y_R \left( Y_{\nu_D}^\dagger Y_{\nu_D} + Y_R^\dagger Y_R \right) + \left( Y_{\nu_D} Y_{\nu_D}^* + Y_R Y_R \right) Y_R + Y_R \left[ \text{Tr} \left( Y_R^\dagger Y_R \right) - \frac{9}{4} g_B^2 \right] \tag{C.5}
\]

where \( g_B \) is the \( B-L \) gauge coupling and \( Y_R(M_{GUT}) = Y_{126}(M_{GUT}) \). The gauge coupling \( g_B \) evolves as \( 16\pi^2 d g_B / dt = 9g_B^3 \), while the other three gauge couplings evolve as in MSSM.
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