Nonlinear Interaction of a 3D Kinetic Alfvén Wave with a Null Point and Turbulence Generation in the Solar Corona

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Received: 30 August 2022 / Accepted: 11 November 2022 / Published online: 25 November 2022 © The Author(s), under exclusive licence to Springer Nature B.V. 2022

Abstract
In the present paper, we have studied nonlinear kinetic Alfvén waves (KAWs) in the vicinity of a null point. We have considered the nonlinearity due to ponderomotive effects associated with KAWs in the solar corona. A 3D model equation representing the dynamics of KAWs is developed in this null point scenario. Using numerical methods, we have solved the model equation for solar coronal parameters. The pseudospectral method and the finite difference method have been applied to tackle spatial integration and temporal evaluation, respectively. The outcome of the simulation demonstrates the formation of localized structures. With the evolution of time, these localized structures become more chaotic. Chaotic (turbulent) structures can efficiently transfer energy. The power spectrum of these turbulent structures shows the Kolmogorov spectral index of nearly $-5/3$ in the inertial range followed by a steeper spectrum of nearly $-3.3$ (in the range of $-2$ to $-4$). These structures also lead to the generation of a current sheet. To understand the physics of our model, we have also done a semi-analytical study for our model equation. Semi-analytical calculations reveal that the current sheet structures have scale sizes of the order of the ion gyro-radius. The relevance of this investigation to the current observations by Parker Solar Probe has also been discussed.

Keywords Solar corona heating--Sun: corona-- turbulence · Wave · Null points

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1. Introduction

A big scientific puzzle of solar physics is how the solar corona is hotter than the photosphere. The explanations to this have been proposed by various mechanisms; however, a concise theory has yet to be developed to explain the qualitative and quantitative description in detail. Energy transport and dissipation of waves coming from the disturbances of photospheric and lower surfaces with the role of shuffling magnetic-field lines are now undoubtedly accepted to be the main resources of much of the energy rise in the solar corona region. In the models proposed by Biermann (1946), Schwarzschild (1948) the convection zone generates acoustic waves and the outward transport of mechanical energy passing through the photosphere meets the need for the non-radiative energy required for heating the corona (Stein and Leibacher, 1974). Turbulence-based coronal heating models (Strauss, 1976; Einaudi et al., 1996; Van Ballegooijen et al., 2011; Dahlburg et al., 2012, 2016) predict that transverse magnetic fluctuations propagating in strong axial magnetic field directions merge with nonlinear effects to produce turbulent energy cascading with Kolmogorov $-5/3$ scale.

The ubiquitous nature of Alfvénic waves has ensured that they remain the focus of increasingly detailed laboratory and space investigations (Gekelman, 1999). Dissipation of magnetic waves plays a crucial role in coronal heating due to the efficient transmission of the Alfvén waves to the solar atmosphere (Hollweg, 1978) and causes the corona to heat (Wentzel, 1974). Alfvén waves are supposed to be involved in coronal heating, but due to propagation through a dispersive medium like the solar corona, they change their angular trajectory and appear as a kinetic Alfvén wave (KAW). The existence of a strong magnetic field background is observed to cascade perpendicular directional energy and cluster the energy fluctuations (Matthaeus and Lamkin, 1986; Matthaeus, Goldstein, and Roberts, 1990). This perpendicular directionality preferred by the KAW on stage plays a crucial role, see for example, Shebalin, Matthaeus, and Montgomery (1983), Carbone, Veltri, and Mangeney (1990), Oughton, Priest, and Matthaeus (1994).

Kinetic Alfvén waves are of great interest because of their essential role in various space and astrophysical plasma phenomena (Wu and Chao, 2004). Mechanisms such as resonance absorption (Ionson, 1978) and phase mixing (Heyvaerts and Priest, 1983) enhance viscous or resistive energy dissipation, but they take too long in the corona. However, the KAW has an electric-field component in the same direction as the background magnetic field. This electric-field component has been shown to interact with plasma particles and stochastically energize them up to the order of 100 eV (equivalent to a temperature rise of $\sim 10^6$ K) within a very small time in accordance with the amplitude of the turbulence present (Mottez, 2012, 2015; Daiffallah, 2022). The power per unit area, which is absorbed by the plasma particles from this electric field, is found to be proportional to the third power of the velocity amplitude and this power is of the order of the energy associated with the turbulent cascade (Malara et al., 2019). This shows that the KAW is at the centre of coronal heating theory, but the generation of turbulence by the KAW needs further study.

The critical step in coronal heating is to recognize a mechanism for converting the magnetic energy to plasma heating. Since classical dissipation coefficients are negligible in the corona because the coronal plasma is low collisional, plasma heating requires the generation of steeper gradients at tiny spatial scales (Génot, Louarn, and Le Quéau, 1999; Tsiklauri, Sakai, and Saito, 2005; Ofman, 2010; Tsiklauri, 2012; Malara et al., 2019). Some numerical simulations have shown how the generation of current sheets occurs, but these models and simulations still need improvement.

Due to the complex magnetic field structures, the topological inhomogeneity of magnetic fields is being sprayed over the turbulent plasma of the solar corona. Measuring the coronal
magnetic field is not an easy task, so the magnetic field emerging from the photosphere is extrapolated to obtain the coronal magnetic-field structures (Van Doorsselaere, Nakariakov, and Verwichte, 2008). Extrapolation of magnetic potential field (Brown and Priest, 2001) provides the existence of locations where the vanishing of the magnetic field and Alfvén speed are found (Dungey, 1958; Sweet, 1958). These specific topological structures, originated due to randomness of the field lines, turning up from the lower solar atmospheres, are known as null points (McLaughlin and Hood, 2005). The study of magnetohydrodynamic (MHD) waves is of fundamental importance in the solar corona. Null points are observed to significantly contribute to driving some high energetic events like solar flares (Shibata and Magara, 2011). MHD wave perturbations and reconnection are omnipresent in the solar corona. These null points are frequently seen around the magnetic reconnection sites, i.e. the null points are common at the reconnection sites. The accumulation of currents near null points has been proposed by McLaughlin and Hood (2004), which increases the importance of the null point in dissipative events by which wave energy can be translated into heat.

Therefore, we seek to study the contribution of null points to coronal heating (Roumeliotis and Moore, 1993; McClymont and Craig, 1996; Tsiklauri and Haruki, 2007, 2008). The presence of null points in the solar corona being proportional to the complexity of the magnetic flux distribution is inevitable (Close, Parnell, and Priest, 2004; Longcope and Parnell, 2009). Thus, the encounter of MHD waves and magnetic topology at some point will undoubtedly occur, i.e. MHD wave propagation will interact with coronal null points. The earlier study of MHD wave behavior in the vicinity of magnetic null points often provides critical insights into other areas of plasma physics, including mode conversion, reconnection, and intensification of current (Rickard and Titov, 1996; Galsgaard and Nordlund, 1997; Pontin and Galsgaard, 2007). Different types of plasma waves and turbulence have been found in the vicinity of null points at coronal reconnection sites (Thurgood and McLaughlin, 2013). KAWs with high amplitudes that cause pondermotive nonlinearity have also been investigated, but the effect of the presence of null points has not been studied yet. Kinetic Alfvén waves and magnetic null points interact to redistribute the energy and producing an energy cascade with a steeper power spectrum gradient.

In this paper, we derive equations and perform numerical simulations to investigate the generation of turbulence from the magnetic reconnection sites of the coronal region. The study of the nonlinear interaction of kinetic Alfvén waves in the vicinity of a null point profile has been performed by introducing pondermotive density perturbations.

This paper is organized as follows. Section 2 gives the basic development of the model equations. Section 3 describes the numerical simulation technique used. Further, Section 4 applies a semi-analytical method to describe the physics behind localization, and the scale size of these structures is calculated. Section 5 discusses the summary and conclusions of the present paper.

2. The Model

Consider a 3D kinetic Alfvén wave with wave vector $k_0 = k_{0x} \hat{x} + k_{0y} \hat{y} + k_{0z} \hat{z}$. This wave is propagating in a medium having a background magnetic field along the $z$-axis, i.e. $B_0 \hat{z}$. We assume that there is an additional magnetic field $\delta B$ due to the presence of a null point as $(B_0/L)(x, -y, 0)$. Here, $L$ is the length scale for the variation of the magnetic field. Therefore, the total magnetic field is given as $B = B_0 \hat{z} + (B_0/L)(x \hat{x} - y \hat{y})$, see Figure 1. The electric and magnetic fields can be expressed as $E = -\nabla \phi - (1/c) \partial A_z / \partial t$ and $B_\perp = $
\[ \hat{z} \times \nabla \tilde{A}_z, \] here \( \phi \) is the scalar potential of the field and \( \tilde{A}_z \) is the parallel component of the vector potential. The speed of light is denoted as \( c \).

The ion equation of motion can be written as

\[
\frac{\partial v_i}{\partial t} = -\frac{e}{m_i} \left( \nabla \phi + \frac{1}{c} \frac{\partial \tilde{A}_z}{\partial t} \right) - \frac{e}{m_i c} (v_i \times B_0 + v_i \times \delta B) - \frac{k_B T_i}{n_0 m_i} \nabla n. \tag{1}
\]

Here, \( v_i \) is the ion velocity, \( k_B \) is the Boltzmann constant, \( T_i \) is the ion temperature, \( n_0 \) is the background density, and \( n \) is the modification in the background density. The absolute value of charge for electron and ion is denoted as \( e \) and the mass of ion as \( m_i \).

For the perpendicular component, taking the cross product of Equation 1 with \( \hat{z} \)

\[
\hat{z} \times \frac{\partial v_i}{\partial t} = -\frac{e}{m_i} \left( \hat{z} \times \nabla \phi + \frac{1}{c} \hat{z} \times \frac{\partial \tilde{A}_z}{\partial t} \right) - \frac{e}{m_i c} \hat{z} \times (v_i \times B_0) - \frac{e}{m_i c} \hat{z} \times (v_i \times \delta B) - \frac{k_B T_i}{n_0 m_i} (\hat{z} \times \nabla n).
\]

Taking again the cross product with \( \hat{z} \), which will give us the perpendicular components with reverse of sign, and applying the time derivative on the resulting equation gives,

\[
-\frac{\partial^2 v_{i\perp}}{\partial t^2} = -\frac{e}{m_i} \frac{\partial}{\partial t} (\nabla_{\perp} \phi) - \omega_{ci} \left( \hat{z} \times \frac{\partial v_{i\perp}}{\partial t} \right) - \frac{e v_{iz}}{m_i c} (\hat{z} \times \delta B) +
\]

\[
\frac{k_B T_i}{n_0 m_i} \frac{\partial}{\partial t} (\nabla_{\perp} n).
\]

We have used the ion cyclotron frequency \( \omega_{ci} = eB/m_i c \), for the second term at the RHS, using Equation 1 in the above equation

\[
\left( \frac{\partial^2}{\partial t^2} + \omega_{ci}^2 \right) v_{i\perp} = -\frac{e}{B} \omega_{ci} \frac{\partial}{\partial t} (\nabla_{\perp} \phi) - \frac{e}{B} \omega_{ci}^2 (\hat{z} \times \nabla_{\perp} \phi) -
\]

\[
\omega_{ci} \frac{k_B T_i}{n_0 m_i} (\hat{z} \times \nabla_{\perp} n) - \frac{k_B T_i}{n_0 m_i} \frac{\partial}{\partial t} (\nabla_{\perp} n) + \frac{e \omega_{ci}}{m_i c} v_{iz} B + \frac{e v_{iz}}{m_i c} (\hat{z} \times \delta B). \tag{2}
\]
Taking a dot product with $\nabla_\perp$ in Equation 2 and taking the ion continuity equation into consideration:

$$
\left( \frac{\partial^2}{\partial t^2} + \omega_{ci} \right) \frac{n_i}{n_0} = -\frac{c}{B} \omega_{ci} (\nabla_\perp^2 \phi) - \frac{k_B T_i}{n_0 m_i} (\nabla_\perp^2 n_i) - \frac{2e\delta B}{m_i c} \omega_{ci} (\nabla_\perp v_i).
$$

(3)

So, from modifying Equation 3 one can get the following form:

$$
\left( \frac{\partial^2}{\partial t^2} + \omega_{ci} \right) \left( \frac{\partial^2}{\partial t^2} - c^2 \frac{\partial^2}{\partial z^2} \right) n + \frac{en_0}{m_i} \frac{\partial^2}{\partial t^2} (\nabla_\perp^2 \phi) + \frac{k_B T_i}{m_i} (\nabla_\perp^2 n) = 0.
$$

(4)

Now for electrons, the expression for the equation of motion can be written as

$$
\frac{\partial v_e}{\partial t} = -\frac{e}{m_e} E - \frac{e}{m_e c} (v_e \times B) - \frac{k_B T_e}{n_0 m_e} \nabla n.
$$

After taking the $z$-component, the equation of motion for electrons can be written as

$$
\frac{\partial v_{ez}}{\partial t} = -\frac{e}{m_e} E_z - \frac{e}{m_e c} (v_e \times B)_z - (v_e \times \delta B) - \frac{k_B T_e}{n_0 m_e} \partial n / \partial z.
$$

(5)

Since $B$ is in the $z$-direction, the cross product term will vanish and Equation 5 in terms of potential fields is

$$
-\frac{m_e}{e} \frac{\partial (-n_0 e v_{ez})}{\partial t} = n_0 e \left( \frac{\partial \phi}{\partial z} + \frac{1}{c} \frac{\partial \tilde{A}_z}{\partial t} \right) - k_B T_e \frac{\partial n}{\partial z}.
$$

Using the value of current density $j_z = -n_0 e v_{ez}$ in the above equation, one can get the equation in the $z$-direction as

$$
\frac{\partial \phi}{\partial z} + \frac{1}{c} \frac{\partial \tilde{A}_z}{\partial t} = -\frac{m_e}{n_0 e^2} \frac{\partial (j_z)}{\partial t} + \frac{k_B T_e}{n_0 e} \frac{\partial n}{\partial z}.
$$

(6)

Again taking the derivative of Equation 6 in the $z$-direction.

$$
\frac{\partial^2 \phi}{\partial z^2} + \frac{1}{c} \frac{\partial^2 \tilde{A}_z}{\partial z \partial t} = -\frac{m_e}{n_0 e^2} \nabla \cdot (\nabla (j_z)) + \frac{k_B T_e}{n_0 e} \frac{\partial^2 n}{\partial z^2}.
$$

(7)

From the electron continuity equation and $\frac{\partial n}{\partial t} + \nabla \cdot (n v_e) = 0$ using Ampere’s law, $j_z = -\frac{c}{4\pi} \nabla_\parallel^2 A_z$ we get:

$$
\frac{\partial n}{\partial t} + \frac{c}{4\pi e} \nabla_\parallel (\nabla_\perp^2 \cdot \tilde{A}_\parallel) = 0,
$$

$$
\frac{\partial^2 n}{\partial t^2} + \frac{c}{4\pi e} \nabla_\parallel \cdot \frac{\partial (\nabla_\perp^2 \cdot \tilde{A}_\parallel)}{\partial t} = 0.
$$

(8)

Using Equations 7, 8, and 4, the dynamical equation of the KAW in terms of the vector potential is written as

$$
(1 - \lambda_e^2 \nabla_\perp^2) \frac{\partial^2 \tilde{A}_z}{\partial t^2} = v_A^2 \left[ 1 - \rho_e^2 \nabla_\perp^2 - \frac{2 \delta B}{B_0} \right] \frac{\partial^2 \tilde{A}_z}{\partial z^2}.
$$

(9)
Here $\rho_s$ is the ion gyro radius defined as $\rho_s \approx c_s/\omega_{ci}$, $\omega_{ci} = eB_0/m_i c$ is the ion gyro frequency, the ion sound speed is given as $c_s = (k_B(T_e + T_i)/m_i)^{1/2}$, and $\lambda_e = (m_e c^2/4\pi n_0 e^2)^{1/2}$ is the collision-less electron skin depth (inertial length). In the above equation $v_A = (B_0^2/4\pi n_0 e^2)^{1/2}$ is the Alfvén speed. Due to the density modification, the Alfvén speed takes the following form:

$$v_A^2 = \frac{B_0^2}{4\pi(n_0 + \delta n)e^2} = \frac{B_0^2}{4\pi n_0 e^2 (1 + \frac{\delta n}{n_0})} \approx v_A^2 \left(1 - \frac{\delta n}{n_0}\right).$$

For the present field $\mathbf{B} = B_{0x}\hat{x} + B_{0y}\hat{y} + B_{0z}\hat{z}$ where, $B_{0x} = xB_0/L$, $B_{0y} = -yB_0/L$, $B_{0z} = B_0$ we use the modification

$$\frac{\partial}{\partial z} = \frac{(B_{0z})}{B_0} \mathbf{\nabla} = \frac{B}{B_0} = \frac{1}{B_0} (B_{0x}\hat{x} + B_{0y}\hat{y} + B_{0z}\hat{z}) \cdot \mathbf{\nabla} = \frac{B_{0x}}{B_0} \frac{\partial}{\partial x} + \frac{B_{0y}}{B_0} \frac{\partial}{\partial y} + \frac{\partial}{\partial z}.$$

Furthermore, using the above modification in Equation 9

$$\left(\frac{\partial}{\partial z}\right) \frac{\partial}{\partial z} = \left(\frac{B_{0x}}{B_0}\frac{\partial}{\partial x} + \frac{B_{0y}}{B_0}\frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right) \left(\frac{B_{0x}}{B_0}\frac{\partial}{\partial x} + \frac{B_{0y}}{B_0}\frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)$$

$$\left(\frac{\partial}{\partial z}\right) \frac{\partial}{\partial z} = \left(\frac{B_{0x}}{B_0}\right)^2 \frac{\partial^2}{\partial x^2} + 2 \left(\frac{B_{0x}}{B_0}\frac{B_{0y}}{B_0}\right) \frac{\partial^2}{\partial x \partial y} + 2 \left(\frac{B_{0y}}{B_0}\right)^2 \frac{\partial^2}{\partial y^2}$$

Rewrite Equation 9 using the above modification and neglecting the higher-order terms:

$$\left(1 - \lambda^2_e \nabla^2\right) \frac{\partial^2 \tilde{A}_z}{\partial t^2} - v_A^2 \left(1 - \rho_s^2 \nabla^2 - \frac{\delta n}{n_0}\right) \left[\left(\frac{B_{0x}}{B_0}\right)^2 \frac{\partial^2}{\partial x^2} + 2 \left(\frac{B_{0x}}{B_0}\frac{B_{0y}}{B_0}\right) \frac{\partial^2}{\partial x \partial y} + 2 \left(\frac{B_{0y}}{B_0}\right)^2 \frac{\partial^2}{\partial y^2}\right] \tilde{A}_z = 0.$$ (10)

Next, we assume the envelope solution for the vector potential in the above Equation 10 in the following form $\tilde{A}_z = A_z(x, y, z, t) e^{i(k_{0x}x + k_{0y}y + k_{0z}z - \omega_0 t)}$. If the effect of field modification due to the null point is not present, then Equation 10 with the envelope solution gives the usual dispersion relation for KAWs as:

$$\omega^2 = \frac{k_{0z}^2 v_A^2 (1 + \rho_s^2 k_{0z}^2)}{(1 + \lambda^2_e k_{0z}^2)}.$$

Now, substituting the envelope solution into our fully fledged nonlinear Equation 10, we get the following expression.

$$-2\omega_0 (1 + \lambda^2_e k_{0z}^2) \frac{\partial A_z}{\partial t} + 2ik_0 (\lambda^2_e \omega_0^2 - v_A^2 \rho_s^2 k_{0z}^2) \frac{\partial A_z}{\partial x} + 2ik_0 (\lambda^2_e \omega_0^2 -$$
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The fluctuation in density is given using the fluid model by Yadav and Sharma (2014)

\[ \frac{\delta n}{n_0} = \left( \frac{\rho_s^2 k_{0z}^2}{v_A^2 \rho_s^2 k_{0z}^2} \right) \left( \frac{B_0}{B_0} \right)^2 k_{0x}^2 + \left( \frac{B_0}{B_0} \right)^2 k_{0y}^2 + 2 \left( \frac{B_0}{B_0} \right) \left( \frac{B_0}{B_0} \right) k_{0x} k_{0z} + 2 \left( \frac{B_0}{B_0} \right) \left( \frac{B_0}{B_0} \right) k_{0y} k_{0z} \right] A_z = 0. \tag{11} \]

The following normalizing parameters are used to make Equation 11 dimensionless

\[ x_n = y_n = \frac{2(\lambda_s^2 \omega_0^2 - v_A^2 \rho_s^2 k_{0z}^2)}{v_A^2 \rho_s^2 k_{0z}^2 k_{0\perp}}, \quad t_n = \frac{2 \omega_0(1 + \lambda_s^2 k_{0z}^2)}{v_A^2 \rho_s^2 k_{0z}^2 k_{0\perp}}, \quad \delta n/n_0 = |A_z|^2. \]

Therefore, we obtain the dimensionless nonlinear dynamical equation as follows:

\[ -i \frac{\partial A_z}{\partial t} + i \frac{\partial A_z}{\partial x} + i \frac{\partial A_z}{\partial y} + i \frac{\partial A_z}{\partial z} + C_1 \frac{\partial^2 A_z}{\partial x^2} + C_1 \frac{\partial^2 A_z}{\partial y^2} - C_2 \frac{\partial^2 A_z}{\partial z^2} - C_3 \frac{\partial^2 A_z}{\partial t \partial z} \]

\[ -C_4 \frac{\partial^2 A_z}{\partial x \partial z} - C_4 \frac{\partial^2 A_z}{\partial y \partial z} + i C_5 \frac{\partial^3 A_z}{\partial t \partial^2 y} + i C_5 \frac{\partial^3 A_z}{\partial t \partial^2 z} + i C_6 \frac{\partial^3 A_z}{\partial x \partial^2 z} + i C_6 \frac{\partial^3 A_z}{\partial y \partial^2 z} + i C_7 \frac{\partial^3 A_z}{\partial x \partial^2 z} + i C_7 \frac{\partial^3 A_z}{\partial y \partial^2 z} + C_8 \frac{\partial^4 A_z}{\partial^2 z \partial^2 x} + C_8 \frac{\partial^4 A_z}{\partial^2 z \partial^2 y} - |A_z|^2 - \frac{(1 + \rho_s^2 k_{0z}^2)}{k_{0z}^2 \rho_s^2 k_{0\perp}} \]

\[ \left[ \left( \frac{B_0}{B_0} \right)^2 k_{0x}^2 + \left( \frac{B_0}{B_0} \right)^2 k_{0y}^2 + 2 \left( \frac{B_0}{B_0} \right) \left( \frac{B_0}{B_0} \right) k_{0x} k_{0z} + 2 \left( \frac{B_0}{B_0} \right) \right] A_z = 0. \tag{12} \]

Where,

\[ C_1 = \frac{\lambda_s^2 \omega_0^2 - v_A^2 \rho_s^2 k_{0z}^2}{v_A^2 \rho_s^2 k_{0z}^2 k_{0\perp} x_n^2}, \quad C_2 = \frac{(1 + \rho_s^2 k_{0z}^2)}{\rho_s^2 k_{0z}^2 k_{0\perp}^2}, \quad C_3 = \frac{4 \omega_0 \lambda_s^2}{v_A^2 \rho_s^2 k_{0z}^2 k_{0\perp}^2}, \quad C_4 = \frac{4}{k_{0z}^2 k_{0\perp}^2 x_n z_n}, \quad C_5 = \frac{2 \omega_0 \lambda_s^2}{v_A^2 \rho_s^2 k_{0z}^2 k_{0\perp}^2 x_n}, \quad C_6 = \frac{2}{k_{0z}^2 k_{0\perp}^2 x_n^2 z_n}, \quad C_7 = \frac{2}{k_{0z}^2 k_{0\perp}^2 x_n^2 z_n}, \quad C_8 = \frac{1}{k_{0z}^2 k_{0\perp}^2 x_n^2 z_n}. \]
Equation 12 is used to model the evolution of nonlinear (coherent) structure and turbulence generation by the interaction of nonlinear KAWs in the neighborhood of the null point. We have taken a simple profile of the magnetic field near the null point. Substituting the profile of magnetic null point $B = B_0/L \cdot (x, -y, 0)$ in Equation 12, we have the final equation as

$$\begin{align*}
-\frac{i}{\tau} \frac{\partial A_z}{\partial t} &+ i \frac{\partial A_z}{\partial x} + i \frac{\partial A_z}{\partial y} - i \frac{\partial A_z}{\partial z} + C_1 \frac{\partial^2 A_z}{\partial x^2} + C_1 \frac{\partial^2 A_z}{\partial y^2} - C_2 \frac{\partial^2 A_z}{\partial z^2} - C_3 \frac{\partial^2 A_z}{\partial t \partial z} \\
-C_4 \frac{\partial^2 A_x}{\partial x \partial z} &- C_4 \frac{\partial^2 A_y}{\partial y \partial z} + i C_5 \frac{\partial^3 A_z}{\partial t \partial^2 x} + i C_5 \frac{\partial^3 A_z}{\partial t \partial^2 y} + i C_6 \frac{\partial^3 A_z}{\partial x \partial^2 z} + i C_6 \frac{\partial^3 A_z}{\partial y \partial^2 z} + \\
C_8 \frac{\partial^4 A_z}{\partial^2 z^2 \partial^2 x} &+ C_8 \frac{\partial^4 A_z}{\partial^2 z^2 \partial^2 y} - |A_z|^2 - \frac{(1 + \rho_1^2k_0^2)}{k_0^2 \rho_1^2 k_0^2} \left[ \left( \frac{x}{L} \right)^2 k_0^2 + \left( \frac{-y}{L} \right)^2 k_0^2 - \left( \frac{y}{L} \right) \left( \frac{x}{L} \right) k_{0y} k_{0x} \right] A_z = 0.
\end{align*}$$

(13)

3. Numerical Simulation and Result

Here we solve Equation 12 in a periodic spatial domain $[2\pi/\alpha_x] \times [2\pi/\alpha_y] \times [2\pi/\alpha_z]$ with grid points $(128 \times 128 \times 128)$. We have applied the pseudo-spectral method for space integration and the finite difference method with the predictor-corrector scheme to study the temporal evolution. In the simulation, we take $\alpha_x = \alpha_y = \alpha_z = 0.2$. These are the wave numbers of the perturbation. Our present study considers the initial condition (Biskamp and Welter, 1989) for the simulation as

$$A_z(x, y) = \cos (2x + 2.3) + \cos (y + 4.1).$$

(14)

We have used the typical solar corona plasma parameters as: $B_0 \approx 32$ G, $n_0 \approx 10^8$ cm$^{-3}$, $T_i \approx 235$ eV, $T_e \approx 37$ eV (Chameaux, Passot, and Sulem (1997)). We have further calculated the following parameters:

$$v_A \approx 6.98 \times 10^8 \text{ cm s}^{-1}, \quad \omega_{ci} \approx 3 \times 10^5 \text{ rad s}^{-1}, \quad \omega_{ci} \approx 1.32 \times 10^7 \text{ rad s}^{-1}, \quad \omega_e \approx 5 \times 10^8 \text{ rad s}^{-1}, \quad \omega_0 = 1.5 \times 10^5 \text{ rad s}^{-1}, \quad k_0 = 3.9 \times 10^{-4} \text{ cm}^{-1}, \quad k_{01} = 9.5 \times 10^{-3} \text{ cm}^{-1}, \quad k_{01}, \lambda_e = 0.2.$$

For these parameters, one can find $x_n = y_n = 66$ cm $\approx \lambda_e$, $z_n = 1.6 \times 10^4$ cm $\approx 1/k_0$ and $t_n = 39$ ms $\approx 1/\omega_0$.

Further, before proceeding to solve the present equation, we cross-checked the accuracy of the code. Accuracy has been confirmed by setting up the modified nonlinear Schrodinger equation (MNLS) and using the conservation of the plasmon number, i.e. $\Sigma_k |A_z|^2$. It remained constant up to an accuracy of $10^{-5}$ during the computation. After validating the reliability of the code, we updated it to solve the equation of the dynamical model. Now, the model equation is solved using numerical techniques.

The spatial evolution of the vector potential $A_z$ of KAWs in the $x-y$ plane at different times is shown in Figure 2, at fixed $z$. As time evolves the profile is modified. Initially, coherent structures are formed and with time evolution we can see the evolving chaotic structures and with further increasing time, from $t = 30$ to $t = 60$, these structures become more random and irregular showing the turbulence generation and evolution in the reconnection site. The evolution of these structures confirms an indication of turbulence.

In Figure 3, the contour plot of the magnetic flux and the current density are shown. The spatial evolution of current density $j = -\nabla^2 A_z$ has been studied with the time evolution.
As time evolves, we see that the proper $X-O$ structures get distorted, and we get a fully chaotic pattern with increasing amplitude as the temporal evolution occurs. The presence of these chaotic structures indicates the generation and evolution of the turbulence.

Furthermore, in Figure 4, the spectrum plot of the normalized field intensity associated with KAWs in the solar corona is plotted. The dark red and green lines are not fits but are used for reference purposes. We find the Kolmogorov $-5/3$ features in the inertial range and after the breakpoint, it shows the steepening of the spectra plot (in the range of $-2$ to $-4$) due to the nonlinear interaction of the KAW and the null point. Observations from the Parker Solar Probe (PSP) mission are giving a vital contribution to our research area, although until now only observations in the range of 0.17 AU to 0.25 AU are being reported. A study of the data obtained by PSP shows that the power spectra are steepening in the range of $-3.2$ to $-5.8$, as reported by Huang et al. (2022). As PSP continues to follow its
The variation of $|A_z|^2$ with perpendicular wave number averaging over all parallel wave numbers for different values of $z$.

path, it will get closer to the solar corona and continue to provide us with more data. We can further proceed to generalize our model to different parameters, which can be tested and will improve our understanding according to the additional new observations. The Alfvén speed is density dependent and in the case of a non-homogeneous density, we see the variation of the Alfvén speed while propagating from a lower to a higher density medium. This dependency of the Alfvén speed over the density causes the Alfvén waves to refract. This wave refraction also causes the magnetic fluctuations power spectrum steepening in the higher-density regions, and the heating channeled to these regions from the surrounding lower-density plasma (Ofman, 2010). The variation of the Alfvén speed around a null point can also have a possible contribution to this steepening and subsequent heating phenomenon.

The interactions between the localized structures and the coronal particles are random and frequent in nature. The stochastic interaction, also known as Fermi acceleration of the second kind, has been associated with particle acceleration and heating phenomena (Vlahos and Isliker, 2018). To further elucidate the proper mechanism of the thermal tail that improves the velocity distribution function as a result of turbulence, Sharma and Kumar (2010) present the Fokker–Planck diffusive formalism. This formalism describes the evolution of the velocity distribution function due to the interaction of KAWs and the coronal particles. The recurrent interaction between plasma particles and the localized intense fields can be studied by using the quasi-linear diffusion equation, which is given by

$$\frac{\partial f}{\partial t} = \frac{\partial}{\partial v} \left( D(v) \frac{\partial f}{\partial v} \right).$$

Here, the diffusion coefficient is $D(v)$ and $f(t, v)$ is the velocity distribution function.

The distribution function can be treated as time-independent if the observation time $t_{\text{obs}}$ is longer than the characteristic time taken by the ponder-motive nonlinearity in settling up. In this case, the distribution function is proposed as $f(v) \propto v^{2-\eta}$. The spectral index $[\eta]$ greatly affects the behavior of the velocity distribution function. Assuming a quasi-steady state, the distribution function for the localized structures, as per Sharma and Kumar (2010),
takes the form \( f(v) \propto v^{-1.3} \), where \( \eta = 3.3 \) from the simulation result. This enhances the thermal tail segment of the distribution function and paves the way for thermal heating of the particles.

For intense phase of localized field structures, the quasi-linear theory is inadequate and the fractional diffusion approach should be taken into consideration. In this approach, the fractional diffusion equation describes the interaction of coronal particles with the given intense localized fields (Bian and Browning, 2008). The distribution function \( f(v, t) \), in this scenario is given as \( f(v) \sim v^{-(1+\mu)} \). The simulation’s output provides the spectral index value as \( (\mu = 3.3) \). Consequently, the distribution function is provided by \( f(v) \sim v^{-(4.4)} \). As a result, the thermal tail of the energetic electrons becomes strengthened, which causes the corona to heat.

So, we anticipate the strengthening of the thermal tail of the energetic electrons and subsequent coronal heating and particle acceleration due to this interaction between the localized structures and coronal particles.

4. Semi-analytical Model

To understand the physical mechanism of the coherent structures and current sheet generation, we have solved the model equation by the semi-analytical method. Since the numerical method solution covers a wide domain of space and time, we have developed a steady-state model for simplification. In this situation, from Equation 11 we obtain the following form,

\[
(\omega_0^2 \lambda^2_k - k_0 v^2_A \rho_s^2) \frac{\partial^2 A_z}{\partial x^2} + (\omega_0^2 \lambda^2_k - k_0 v^2_A \rho_s^2) \frac{\partial^2 A_z}{\partial y^2} - 2k_0 v^2_A (1 + \rho_s^2 k_0^2)
\]

\[
\frac{\partial A_z}{\partial z} - v^2_A \rho_s^2 k^2_{0,\perp} k_0 \frac{\delta n}{n_0} A_z - v^2_A (1 + k_0^2 \rho_s^2) \left[ \frac{B_0}{B} \right]^2 k_{0,\parallel}^2 + \left( \frac{B_0}{B} \right)^2 k_{0,\perp}^2
\]

\[
2 \left( \frac{B_0}{B} \right) \left( \frac{B_0}{B} \right) k_{0,\parallel} k_{0,\perp} \right] A_z = 0.
\]

Here we have assumed the variation of the vector potential \( A_z \) in terms of the eikonal equation (Akhmanov, Sukhorukov, and Khokhlov, 1968) as

\[
A_z = A_0(x, y, z) e^{i k_0 S(x, y, z)}.
\]

Substituting this assumed solution in Equation 16 and comparing the real and imaginary parts, we get

\[
\gamma \left( \frac{\partial^2 A_0}{\partial x^2} + \frac{\partial^2 A_0}{\partial y^2} \right) - \gamma k_0^2 A_0 \left( \frac{\partial^2 S}{\partial x^2} + \frac{\partial^2 S}{\partial y^2} \right) + 2 \left( \frac{1 + k^2_{0,\parallel} \rho_s^2}{k^2_{0,\perp} \rho_s^2} \right)
\]

\[
A_0 \frac{\partial S}{\partial z} - \frac{\delta n}{n_0} A_0 - \left( \frac{1 + k^2_{0,\parallel} \rho_s^2}{k^2_{0,\perp} \rho_s^2} \right) \left[ \frac{B_0}{B} \right]^2 k_{0,\parallel}^2 + \left( \frac{B_0}{B} \right)^2 k_{0,\perp}^2 +
\]

\[
2 \left( \frac{B_0}{B} \right) \left( \frac{B_0}{B} \right) k_{0,\parallel} k_{0,\perp} \right] A_0 = 0.
\]

\[
2 \gamma \frac{\partial S}{\partial x} + 2 \gamma \frac{\partial S}{\partial y} - 2 \left( \frac{1 + k^2_{0,\parallel} \rho_s^2}{k^2_{0,\perp} \rho_s^2} \right) \frac{\partial A_0}{\partial z} = 0.
\]
Here,

$$\gamma = \frac{(\omega_0^2 \lambda_e^2 - k_0^2 v_A^2 \rho_s^2)}{k_{01}^2 \rho_s^2 k_{00}^2 v_A^2}.$$  

Further, we have assumed the solution to Equations 17 and 18 in the form of a Gaussian profile as follows

$$A_0 = \frac{A_0^2}{f_1(z) f_2(z)} e^{-\frac{x^2}{2 \beta_1(z)} + \frac{y^2}{2 \beta_2(z)}},$$

$$S = \beta_1(z) \frac{x^2}{2} + \beta_2(z) \frac{y^2}{2}.$$  

Substituting the above assumed solution in Equation 18, we obtain

$$\beta_1 = \frac{v_A^2 (1 + k_{01}^2 \rho_s^2)}{(\omega_0^2 \lambda_e^2 - k_0^2 v_A^2 \rho_s^2)} \frac{1}{f_1} \frac{\partial f_1}{\partial z},$$

$$\beta_2 = \frac{v_A^2 (1 + k_{01}^2 \rho_s^2)}{(\omega_0^2 \lambda_e^2 - k_0^2 v_A^2 \rho_s^2)} \frac{1}{f_2} \frac{\partial f_2}{\partial z}.$$  

Here, $f_1$, $f_2$ are the beam width parameters of the wave, $r_{01}$ and $r_{02}$ are the scale size, and $\beta_1$ and $\beta_2$ are slowly varying functions of $z$. Using Equation 19–20 in Equation 17 along with the paraxial approximation, i.e., $x \ll r_{01} f_1$, $y \ll r_{02} f_2$ and equating the coefficients of $x^2$ and $y^2$ on both sides, we obtain the dimensionless differential equations for beam width parameter as

$$\frac{\partial^2 f_1}{\partial Z^2} = R_d \frac{(\omega_0^2 \lambda_e^2 - k_0^2 v_A^2 \rho_s^2)}{(1 + \rho_s^2 k_{01}^2)} \left( \frac{4(\omega_0^2 \lambda_e^2 - v_A^2 \rho_s^2 k_{01}^2)}{v_A^2 k_{00}^2 (1 + \rho_s^2 k_{01}^2)} \frac{1}{r_{01}^4 f_1^3} - \frac{k_{02}^2}{v_A^2 k_{00}^2 L^2} \right) f_1 - \frac{\rho_s^2 k_{01}^2}{(1 + \rho_s^2 k_{01}^2) r_{01}^4 f_1^3 f_2^2} \left( \alpha A_{01}^2 \right),$$

$$\frac{\partial^2 f_2}{\partial Z^2} = R_d \frac{(\omega_0^2 \lambda_e^2 - k_0^2 v_A^2 \rho_s^2)}{(1 + \rho_s^2 k_{01}^2)} \left( \frac{4(\omega_0^2 \lambda_e^2 - v_A^2 \rho_s^2 k_{01}^2)}{v_A^2 k_{00}^2 (1 + \rho_s^2 k_{01}^2)} \frac{1}{r_{02}^4 f_2^3} - \frac{k_{02}^2}{v_A^2 k_{00}^2 L^2} \right) f_2 - \frac{\rho_s^2 k_{01}^2}{(1 + \rho_s^2 k_{01}^2) r_{02}^4 f_2^3 f_1^2} \left( \alpha A_{02}^2 \right).$$

Here $Z = z/R_d$ and $R_d = k_0 v_A r_{01}^2$.  

In the above Equations 21 and 22, the first term is due to the finite transverse size of the KAW structures and leads to the spreading effect, the second term is due to the null point, which is responsible for convergence of the structures, and the last term is due to the density modification in the background, which is responsible for convergence of the structures (both in $x$ and $y$). The critical value of the scale size can be found by solving Equations 21 and 22 for a given power value. For an initially plane wavefront, we assume the following initial conditions $f_1 = f_2 = 1$ at $z = 0$ and $\partial f_1/\partial z = \partial f_2/\partial z = 0$ at $z = 0$

$$1 - \frac{v_A^2 (1 + \rho_s^2 k_{01}^2) k_{00}^2 k_{01}^4}{4(\omega_0^2 \lambda_e^2 - v_A^2 \rho_s^2 k_{01}^2) L^2} \frac{\rho_s^2 k_{01}^2 v_A^2 k_{00}^2 (\alpha A_{01}^2) r_{01}^2}{4(\omega_0^2 \lambda_e^2 - v_A^2 \rho_s^2 k_{00}^2)} = 0,$$
As it is clear from Equations 23 and 24, in the absence of KAW, i.e. when the power of KAW ($\alpha A_{0}^{2}$) is zero, the scale size of the localized KAW structures is quadratic in $r_{01}^2$ and $r_{02}^2$, respectively, with a valid root of the equation as

$$r_{01} = \left(\frac{4(\omega_{0}^{2}\lambda_{e}^{2} - v_{A}^{2}\rho_{s}^{2}k_{0x}^{2})L^{2}}{(1 + \rho_{s}^{2}k_{0x}^{2}v_{A}^{2}v_{s}^{2})}\right)^{\frac{1}{2}}, \quad r_{02} = \left(\frac{4(\omega_{0}^{2}\lambda_{e}^{2} - v_{A}^{2}\rho_{s}^{2}k_{0x}^{2})L^{2}}{(1 + \rho_{s}^{2}k_{0x}^{2}v_{s}^{2}v_{A}^{2})}\right)^{\frac{1}{2}}.$$  

Magnetic fluxes are related to current density by Ampere’s law. So, we have studied the evolution of magnetic flux and current density. To calculate the size of the scale of the current sheets along the x and y directions, if the relation can be used as $j = -\nabla A_{0}$, where $A_{0} = \frac{A_{0}^{2}}{f_{1}/f_{2}} \exp\left(-\frac{x^{2}}{r_{01}^{2}/f_{1}} - \frac{y^{2}}{r_{02}^{2}/f_{2}}\right)$. The expression is further simplified to give the scale size of current sheet along the x-direction as the order of $\rho_{s}/\sqrt{3}$. A similar expression can be found for the y-component as $\rho_{s}/\sqrt{3}$. The kinetic Alfvén wave appears at an intermediate $\beta$ (ratio of thermal pressure to the magnetic pressure) regime when $1 > \beta > m_{e}/m_{i}$. In the case of the kinetic Alfvén wave, the thermal velocity is more than the Alfvén velocity and the pressure gradient term is accountable for the parallel electric field. Due to this term, the ratio of thermal acoustic velocity and the gyrofrequency i.e. the ion gyroradius, becomes an attributing length scale in the case of the kinetic Alfvén wave (Stasiewicz et al., 2000). Further, if we take into account the power of KAW ($\alpha A_{0}^{2}$), then for any finite power the solution to Equations 21 and 22 with assumed initial values of $f_{1}$ and $f_{2}$ will give some different value of current density. So, the KAW with different finite power will give a varying scale size. Consequently, this demonstrates that the scale size of coherent structures and current sheet dimensions depends on the KAW power as well.

5. Conclusion

The present work demonstrates the generation of localized structures when KAW propagates in an environment where null points are present (as considered here). The model equation to represent a 3-D kinetic Alfvén wave interacting with the null point using the nonlinear density perturbation has been developed and numerically solved. The formation of the localized structure of the magnetic field provides a pathway for energy transfer from a larger scale to a smaller spatial scale. Small-scale localized structure formation shows the coupling of Alfvén waves with null points. This localization of the wave and the resulting turbulence leads to energy transfer between large and small scales. The generated turbulence spectrum shows that for the energy spectrum of the power in the inertial region for $k_{0} \rho_{s} < 1$, we have Kolmogorov scaling nearly $k^{-5/3}$, but after the inertial range for $k_{0}\rho_{s} > 1$ the slope starts to change and gets steeper, having power spectra in the range $k^{-2}$ to $k^{-4}$. The steepening assures us that the energy transfer rate from one scale to the smaller scales get increased in the temporally evolved turbulence. Therefore, the signature of turbulence is apparent in the present power spectrum at the reconnection site. This emergence of turbulence ensures the enhancement of the energy-transfer rate. We have also presented the energetic particle distribution function by using this turbulence with the help of the Fokker-Planck equation and velocity-space diffusion coefficient. The semi-analytical study has also been done, which gives us an estimate of the scale size of coherent structures formed. The current sheet size
turns out to be of the order of the ion gyro radius but it gets affected by the power of the KAW significantly. Therefore, we find that the localized magnetic structure and resulting turbulence are the outcomes of this KAW interaction, near the null point and possibly responsible for coronal heating.

Acknowledgments This research is supported by the Council for Scientific & Industrial Research (CSIR), New Delhi, India.

Author contributions Garima Patel wrote the main manuscript text and prepared Figures 1–4. All authors reviewed the manuscript.

Declarations

Competing interests The authors declare no competing interests.

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