Discovery and first models of the quadruply lensed quasar SDSS J1433+6007

Adriano Agnello,† Claudio Grillo,‡ Tucker Jones,§ Tommaso Treu,¶ Mario Bonamigo and Sherry H. Suyu

1 INTRODUCTION

Strong gravitational lensing by galaxies enables the study of distant sources and luminous and dark matter in galaxies over a range of redshifts (see e.g. Courbin, Saha & Schechter 2002, for a review). When the source is a quasar, its multiple images give a wealth of information on: the source central engine and the stellar content of the lens, via microlensing by individual stars (Wambsganss 2001; Kochanek 2004; Braibant et al. 2016; Schechter et al. 2016); sub-structure in the lens, via astrometric and flux-ratio ‘anomalies’ (Mao & Schneider 1998; Dalal & Kochanek 2002; Nierenberg et al. 2014; Xu et al. 2015; Agnello et al. 2017); quasars and their hosts at $z_s \approx 2$ (More et al. 2009; Rusu et al. 2014; Agnello et al. 2016); and from the time-delays between the light curves of different images, cosmological distances and tests of the Lambda cold dark matter concordance cosmology (Refsdal 1964; Kochanek 2003; Paraficz & Hjorth 2009; Suyu et al. 2014).

However, quasar lenses are rare, as they require the precise alignment of a distant source with (at least) a galaxy. Oguri & Marshall (2010, hereafter OM10) estimate that $\approx 0.2$ lensed quasars per degree square, brighter than $i = 21$, should be present in wide-field surveys, with a majority of doubly imaged quasars and $\approx 20$ per cent quadruples. Quasars are rare objects themselves and are less frequent beyond redshifts $z_s \approx 1.8$ (OM10, Ménard et al. 2010), whereas quasar lenses should have source redshifts at $z \approx 2.5 \pm 0.3$, from OM10 models.

Since the first discovery of a lensed quasar (QSO 0957+561, Walsh, Carswell & Weymann 1979) in the radio, about $10^2$ multiply imaged quasars have been discovered so far, with various methods in several optical and radio surveys. Some searches concentrated on bright quasars at redshift $>1$, such as the HST Snapshot Survey (Maoz et al. 1992), the Hamburg Bright Quasar Survey (Wisotzki et al. 1996) and the Magellan survey (Wisotzki et al. 2004, and references therein). The Jodrell Bank-VLA Astrometric Survey (JVAS;
and the lens redshift is $z / \Omega_1 / \Lambda_1$ where necessary we adopt concordance cosmological parameters.

WISE (Wright et al. 2010) magnitudes in the Vega system, and Section 5. In what follows, SDSS magnitudes are in the AB system, in Section 2, the confirmation and follow-up of J1433+0607, at RA = 14:33:22.8, Dec. = +60:07:13.44 (J2000), showed two well-separated blue images on either sides of two red objects, blended in three photometric components by the SDSS pipeline. It received grades 2 and 3 and was initially

in Section 3, and first lens models in Section 4. We conclude in Section 5. In what follows, SDSS magnitudes are in the AB system, WISE (Wright et al. 2010) magnitudes in the Vega system, and where necessary we adopts concordance cosmological parameters $\Omega_m = 0.7$, $\Omega_b = 0.3$, $H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1}$.

2 CANDIDATE SELECTION

Since half of the known quasar lenses in SDSS have extended morphology, due to the presence of the deflector (see Williams et al. 2017, for a discussion), objects with $2 \log L_{\text{star}, i} \leq -11$ or $p_{\textsf{f}_i} > 0.075$, $p_{\textsf{od}_{\text{i}}} < 0.25$, and WISE colours $W_1 - W_2 > 0.55$, $W_2 - W_3 < 1.1 + 1.5(W_1 - W_2 - 1.075)$ were pre-selected. The $i$-band selection is used as a morphological pre-selection, whereas the WISE cuts are an extension of those by Assef et al. (2013) to exclude most quasars at $z < 0.35$ and narrow-line galaxies. Quasar lens targets were then selected based on their catalogue magnitudes, then visually inspected to exclude obvious contaminants, yielding the final candidate sample. Candidates themselves were assigned a grade from I (dubious) to 3 (almost surely a lens), based on visual inspection by two investigators.

Lensed quasars are rare among quasars, which, in turn, are rarer than blue galaxies, hence we used a novel outlier selection to mine targets, retaining peculiar objects by excluding more common ones. The improvement in false-positive rejection upon established searches in the SDSS, such as the SQLS, was a requirement for the method to be applicable, in practice, to a wide-field search relying only on photometry, without any UVX or fibre-spectroscopy information. When tested on the SQLS morphologically selected lens candidates of Inada et al. (2012), this procedure recovered 9 of the 10 lenses and excluded half of their 40 false positives.

The method is described elsewhere (Agnello 2017); here we summarize it. Four main classes of ‘common’ objects were defined, roughly corresponding to nearby ($z < 0.75$) quasars, isolated quasars at higher ($z \approx 2$) redshift, blue-cloud galaxies and faint ($W_2 \geq 15$) objects. Each class $k$ was represented by a single Gaussian with mean $\mu_k$ and covariance $C_k$ in a space given by $g - r$, $g - i$, $r - z$, $i - W_1$, $W_1 - W_2$, $W_2 - W_3$, $W_2 - W_3$ and each object $f$ was assigned pseudo-distances defined as $d_i = 0.5(f - \mu_{k_i})^T C^{-1}_{k_i}(f - \mu_{k_i})$. Objects that were ‘far’ enough from the four class centres, based on linear combinations of their $d_i$ values, were retained as targets.

This yielded $\approx 250$ candidates brighter than $i = 20.0$ over the whole SDSS–DR12 footprint, of which $\approx 40$ known quasar lenses or pairs. J1433+0607, at RA = 14:33:22.8, Dec. = +60:07:13.44 (J2000), showed two well-separated blue images on either sides of two red objects, blended in three photometric components by the SDSS pipeline. It received grades 2 and 3 and was initially

\[2 \text{ Here } L_{\text{star}, i} \text{ is the pipeline-estimated probability that an object is a single point-source; details on stellarity and model/psf magnitudes can be found in the SDSS schema browser.} \]

\[3 \text{ The exact numbers depend on the pseudo-distance cuts chosen in this search, and visual-inspection. In this search, the final sample had 253 candidates and 38 known lenses/pairs.} \]
prioritized for follow-up in 2016 June, but observations were weathered out.

3 FOLLOW-UP

Long-slit discovery spectra were obtained on 2017 January 19 and 20 as part of a candidate lens follow-up program (P42−019, PI Grillo), using the Andalucia Faint Object Spectrograph and Camera (ALFOSC) at the 2.5 m Nordic Optical Telescope (NOT) in La Palma (Spain). Standard IRAF routines were used for bias subtraction, flat-field corrections, and wavelength calibration. From ALFOSC BVRi imaging, we obtained the positions and magnitudes subsequently used for lens models. Deeper, high-resolution spectroscopy was obtained with the Echellette Spectrograph and Imager (ESI) at the Keck II telescope on 2017 January 20 (PI Jones), and reduced with ESIREDUX.4

In what follows, quasar images are labelled A, B, C, D in order of expected arrival time (see Fig. 1). ALFOSC pixels measure 0.21 arcsec per side.

3.1 NOT discovery and follow-up spectra

We used the 1 arcsec-wide long-slit with the #4 grism, covering a wavelength range 3200 < λ < 9600 Å with a dispersion of 3.3 Å pixel$^{-1}$ and resolution $R = 360$. We took two 600 s exposures with the slit aligned north–south, through 14:33:22.8+60:07:13.44, and one (900 s) with east–west alignment, through 14:33:22.8+60:07:14.5. This enabled simultaneous spectroscopy of the two prominent quasar images and the two red objects, respectively. Arc (HeNe, Ar) and flat lamps were used for calibrations. Sky lines around 6000 Å were used to assess the accuracy of wavelength calibration. While flux-calibrated, bandpass-calibrated spectra are useful for studies of variability and continuum spectral shape, lens confirmation requires only that all images correspond to the same source, and hence their spectra have the same redshift, line shape, and flux ratios smoothly varying with wavelength (barring chromatic effects from differential extinction and microlensing). Since bandpass corrections and standard-star calibrations apply the same multiplication to all spectra, and so are not necessary for lens confirmation purposes, we preferred to use the available telescope time to obtain deeper spectroscopic and imaging data once the lens was already confirmed. Following Agnello et al. (2015b), the spectra were modelled as a superposition of multiple Gaussian sources, in order to accurately deblend different traces.

The north–south spectra show three nearly identical traces corresponding to the same $z_s = 2.737 \pm 0.003$ quasar (Fig. 2, top panel), as evaluated on Ly$α$ and C IV. The bright, outer traces correspond to the blue images (A,B) visible in the SDSS. The central, fainter trace is given by a third quasar image C, corresponding to the westmost red object, thus confirming J1433+6007 as a multiply imaged quasar. The east–west spectra (Fig. 2, middle panel) show clear traces corresponding to images C,D and the lens galaxy (G). Images A,B have almost indistinguishable spectra, with uniform flux ratio ≈1.05, whereas image C undergoes substantial extinction bluewards of C III]. There are ≈5 – 10 per cent flux-ratio differences between the continua and emission lines (Fig. 2, bottom panel), which can be explained as an effect of microlensing.

3.2 NOT imaging

Follow-up imaging data with good seeing (≈0.6 arcsec full width at half-maximum at Zenith) were obtained with ALFOSC in B, V, R, i bands, using multiple 60 s exposures per band. Standard PYTHON routines were used for bias subtraction, flat-fielding and coadding. The median coadds and colour-composites are shown in Fig. 1, where Ly$α$ from the quasar images dominates in B band and the lens brightens up in redder bands.

The relative displacements are obtained both from imaging and spectroscopic data. Individual traces in the spectra are well described by Gaussians in the spatial direction, whose run with wavelength can be modelled with uncertainties as low as

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4 Available at http://www2.keck.hawaii.edu/inst/esi/ESIRedux/
Table 1. Image positions, BVRI magnitudes, and model-predicted values of (point-source) magnifications and time-delays. The nominal errors on some magnitudes would be smaller than quoted, but are limited by the accuracy of the ALFOSC zero-points and observing sky conditions.

| Image | $\delta x$ (arcsec) | $\delta y$ (arcsec) | $B$ (mag) | $V$ (mag) | $R$ (mag) | $i$ (mag) | $\mu$ | $t - t_A$ (d) |
|-------|---------------------|---------------------|-----------|-----------|-----------|-----------|-------|--------------|
| A     | 0.00 ± 0.025        | 0.00 ± 0.025        | 20.26 ± 0.04 | 19.78 ± 0.01 | 19.26 ± 0.01 | 19.32 ± 0.01 | 2.53 ± 0.17 | 0.00  |
| B     | −0.070 ± 0.025      | −3.650 ± 0.025      | 20.09 ± 0.03 | 19.63 ± 0.01 | 19.13 ± 0.01 | 19.10 ± 0.01 | 3.51 ± 0.25 | 16.0 ± 5.0 |
| C     | 0.766 ± 0.025       | −2.056 ± 0.025      | 20.50 ± 0.05 | 19.92 ± 0.01 | 19.30 ± 0.01 | 19.14 ± 0.01 | −2.91 ± 0.60 | 26.0 ± 5.0 |
| D     | −2.138 ± 0.050      | −2.132 ± 0.050      | 22.00 ± 0.14 | 21.30 ± 0.02 | 20.63 ± 0.02 | 20.38 ± 0.01 | −0.60 ± 0.10 | 113.0 ± 5.0 |
| G     | −1.152 ± 0.025      | −1.950 ± 0.025      | 21.87 ± 0.15 | 20.69 ± 0.12 | 19.39 ± 0.01 | 18.52 ± 0.01 | −          | −          |

Figure 3. ESI follow-up spectra, with prominent absorption features at $z_1 = 0.407 ± 0.002$, which we associate with $G$. Red dashed lines: Ca HK, G-band $\lambda 4304$, Mg I, Na D absorption at $z_2$. Blue solid lines: quasar emission at $z_s$. The Ca HK complex is visible, albeit at lower signal-to-noise ratio, also in the ALFOSC spectra.

0.125 pixels = 25 mas. Uncertainties from imaging-only data, though nominally smaller, are dominated by systematics from different noise realizations. Residuals between imaging data and model, mostly due to faint features and PSF mismatch, are within the noise level.

Table 1 gives the positions of the four images (A,B,C,D) and deflector (G), relative to image A, and BVRI magnitudes. Images C and D are substantially reddened, and blending between D and G is significant in $B$ band.

3.3 Keck-ESI follow-up

ESI spectra of G were taken in echelle mode, with 1.0 arcsec slit-width oriented east–west and resolution $R = 4900$, roughly constant to within 10 per cent over the whole spectral range. The total integration time was 45 min split into three exposures of 900 seconds each. Bias and dark subtraction, flat-field correction, sky subtraction, and spectral extraction are performed directly by ESIREDX. The combined 1D spectrum of G and D, shown in Fig. 3, has distinctive absorption features at $z_1 = 0.407 ± 0.002$ (Ca HK, G band, H $\beta$, Mg I/Fe complex, Na D), which we associate with the lens galaxy. The same features could be seen in the ALFOSC discovery spectra, although not as clearly.

We measured the lens velocity dispersion by fitting a Gaussian profile to the Ca K line, yielding $\sigma_V = (216 ± 55) \text{ km s}^{-1}$. This corresponds to the integrated velocity dispersion within a 1 arcsec wide spectroscopic aperture centred on the lens galaxy G, which is partly blended with image D due to ∼1 arcsec seeing.

Table 2. Lens model parameters: best fit (first column) and 68 per cent confidence intervals, marginalized over other parameters. Tight (and expected) degeneracies among parameters are present, given in equation (2), except for the combination $\theta_{E_{1}}$, which corresponds to the Einstein radius. Angles $\phi_l$, $\phi_s$, are positive counter-clockwise from West.

|        | $\theta_{E_{1}}$ (arcsec) | $q$ | $\phi_l$ (rad) | $\phi_s$ (rad) | $\gamma_s$ |
|--------|--------------------------|-----|----------------|----------------|------------|
| best   | 1.80 ± 0.05              | 0.50 | 0.18           | 1.12           | 0.10       |
| 68 per cent low | 1.70 ± 0.05 | 0.43 | 0.13           | 0.96           | 0.08       |
| 68 per cent high | 1.90 ± 0.05 | 0.58 | 0.25           | 1.22           | 0.13       |

4 LENS MODELS

From the relative displacements in Table 1, we fit a simple lens model to obtain the enclosed (2D) mass and predicted magnifications and time-delays. We do a conjugate-point analysis using GLEEs (Suyu & Halkola 2010; Suyu et al. 2012), adopting 25 mas uncertainties on the positions of A,B,C and G, and 50 mas uncertainties on the position of image D. We do not fit to the flux-ratios, as, in general, they can be significantly affected by differential extinction, microlensing, and time-delays, and we can expect this to happen also in this case from the ratio of spectra shown in Fig. 2.

The surface density of the lens is described by

$$\kappa(x, y) = \frac{\Sigma(x, y)}{\Sigma_{cr}} = \frac{b}{2\sqrt{X^2/(1 + e)^2 + Y^2/(1 - e)^2}}$$

(Kassiola & Kovner 1993), along the principal axes of G, where $\Sigma_{cr} = c^2 D_s/4\pi G D_A D_t$ factors the dependence on angular-diameter distances. The coordinates $(X, Y)$ are rotated from the standard coordinates $(\delta x = -\cos(\text{Dec.})\delta RA, \delta y = \text{Dec.})$ by an angle $\phi_t$. The Einstein radius is defined as the geometric mean of the major and minor axes of the critical curve, $\theta_{E} = 2b\sqrt{q/(1 + q)}$.

We include external shear, with amplitude $\gamma_s$ and angle $\phi_s$, and allow all parameters to vary freely, including the lens flattening $q = (1 - e)/(1 + e)$ and position angle $\phi_t$. The models then have seven parameters ($b, \phi_t, q, \phi_s, \gamma_s$, and the source position) and eight constraints with uncertainties (the relative displacements of the four image positions). The lens centre, nominally adding two free parameters, is already well constrained within (at most) 0.05 arcsec by the ALFOSC cutout models (Table 1).

The results are summarized in Tables 1 and 2, and Fig. 4. The Einstein radius is robustly determined to $\theta_{E} = (1.80 ± 0.10)$ arcsec, very close to half the A–B separation and independent of other inferred quantities. Flux-ratios from the best-fitting model are comparable to those measured in $i$ band, accounting for differential extinction whose existence in lensing is well established (e.g. Falco et al. 1999; Mediavilla et al. 2005; Elfdštörr et al. 2006; Agnello et al. 2017). The predicted delays $t_{D} - t_{A} > 100$ d, $t_{C} - t_{A} \approx 25$ d can be accurately measured in one or two seasons of high-cadence monitoring.
with the above values, we then obtain \( \sigma \) for massive ellipticals (Treu et al. 2005). A direct comparison between the estimate in equation (3) and direct measurements expected for massive ellipticals (Treu et al. 2005) is unexpected for quasar lenses (Keeton, Kochanek & Seljak 1997; Holder & Schechter 2003), and, in fact, multipole contributions can, in general, be enough to appreciably affect flux ratios (Xu et al. 2015; Hsueh et al. 2016; Gilman et al. 2017).

Monopole–quadrupole degeneracies (e.g. Kochanek 2006) are present, in particular,

\[
\begin{align*}
\gamma_1 &\approx 0.10 + 0.35(q - 0.5), \\
\phi_1 &\approx 1.1 - 1.5(b - 2.5), \\
\gamma_s &\approx 0.10 + 0.35(\phi_l - 0.18),
\end{align*}
\]

for this system. These may be broken with: observations of the line-of-sight environment, to characterize external contributions to the deflections; and higher resolution imaging, of both the lensed quasar host and of G, to disentangle shear and lens flattening. Significant quadrupole, in the form of external shear or ellipticity, is not unexpected in quasar lenses (Keeton, Kochanek & Seljak 1997; Holder & Schechter 2003), and, in fact, multipole contributions can, in general, be enough to appreciably affect flux ratios (Xu et al. 2015; Hsueh et al. 2016; Gilman et al. 2017).

The velocity dispersion \( \sigma \) of stars in the lens is a useful observable that can be estimated from the lens model itself. A direct measurement of \( \sigma \) and its comparison with predictions from lensing can be used to measure cosmological distances and to constrain the dark matter density profile of the lens (Treu & Koopmans 2004; Grillo, Lombardi & Bertin 2008; Paraficz & Hjorth 2009; Suyu et al. 2014; Sonnenfeld et al. 2015; Jee et al. 2016). In the singular isothermal sphere (SIS) limit \( q \to 1 \), \( \sigma \) depends weakly on location and is well approximated by

\[
\sigma_{\text{los}}^2 = \frac{c^2 \theta_e D_s}{4 \pi D_L^2}.
\]

With the above values, we then obtain \( \sigma = (290 \pm 9) \, \text{km} \, \text{s}^{-1} \), as expected for massive ellipticals (Treu et al. 2005). A direct comparison between the estimate in equation (3) and direct measurements will need dynamical models that encompass asphericity and inclination effects (Barnabè et al. 2011), as well as a more accurate measurement of \( \sigma \) from deeper spectra with a robust subtraction of the quasar contamination.

5 DISCUSSION AND CONCLUSIONS

Lensed quasars are rare, and quadruply lensed quasars are even rarer. Assembling homogeneous sets of quasar lenses, with a selection function that can be well characterized and with the capability of discovering ‘new’ quads, is paramount to different lines of investigation (microlensing, structure lensing, cosmography) that require samples of \( \approx 40 \) lenses each, with suitable ancillary data. The advent of wide-field surveys helps overcome the rarity of these systems, provided lens searches can be performed on photometric data and recover systems to a good completeness level, spanning source redshifts \( z_s > 2 \), where most lenses are expected to lie.

We have found a new, quadruply lensed quasar at RA = 14:33:22.8, Dec. = +60:07:14.5, via an outlier-selection procedure applied to the SDSS–DR12 photometric footprint (detailed by Agnello 2017). Similar to other recent, wide-field photometric searches, this did not rely on previous spectroscopic or UV excess information. This approach enables the discovery of systems with sources at higher redshifts than typically probed, and with appreciable differential reddening by the lens galaxy. This aspect is particularly relevant if, for example, microlensing or substructure studies are concerned, since one needs to ensure that the samples themselves are not biased against systems with significant departures from the simplest image and flux-ratio configurations.

Discovery NOT–ALFOSC data confirmed this system as a lens with \( z_l = 2.737 \) and four images on a fold configuration. The lens redshift is \( z_l = 0.407 \) from follow-up Keck-ESI spectroscopy, which together with the lens model results with \( \theta_E = (1.80 \pm 0.10) \) arcsec predicts a velocity dispersion \( \sigma = (290 \pm 9) \, \text{km} \, \text{s}^{-1} \). The velocity dispersion measured from ESI data, within a 1 arcsec aperture in l arcsec seeing conditions, is \( \sigma_{\text{los}} = (216 \pm 55) \, \text{km} \, \text{s}^{-1} \), but deeper data in better seeing are needed, to accurately correct for contamination by the quasar images (C,D) and model aperture effects. Saddle-point images C,D are significantly reddened by the lens galaxy, while the A/B flux ratios agree with predictions by the lens model within \( \pm 0.1 \) mag.

This lens is then relevant to multiple applications. Microlensing is evident in the differential magnification of lines and continua, and it can yield a determination of the stellar mass fraction in a galaxy at \( z_s \approx 0.4 \) if deeper spectroscopic data are obtained. The expected time-delays (Table 1) can be measured accurately with high-cadence campaigns spanning one or two monitoring seasons, making this system an ideal lens for a cosmographic sample. With current data, significant monopole-quadrupole degeneracies arise in the lens model, but they can be broken with deeper and higher resolution follow-up. This, in turn, will reduce the systematic errors in the lens macro-model, with applications to the study of dark matter density profiles, time-delay cosmography, microlensing, and detection of substructure.

ACKNOWLEDGEMENTS

CG and MB acknowledge support by VILLUM FONDEN Young Investigator Programme through grant no. 10123. TJ acknowledges support provided by NASA through Program # HST-HF2-51359 through a grant from the Space Telescope Science Institute, which is...
operated by the Association of Universities for Research in Astronomy, Inc., under NASA contract NAS 5-26555. TT acknowledges support by the Packard Foundation through a Packard Research Fellowship and by the National Science Foundation through grant AST-1450141. SHS gratefully acknowledges support from the Max Planck Society through the Max Planck Research Group.

The data presented here were obtained in part with ALFOSC, which is provided by the Instituto de Astrofísica de Andalucía (IAA) under a joint agreement with the University of Copenhagen and NOTSA. We thank R.T. Rasmussen and T. Pursimo for support at the NOT.

Some of the data presented herein were obtained at the W.M. Keck Observatory, which is operated as a scientific partnership among the California Institute of Technology, the University of California and the National Aeronautics, and Space Administration. The Observatory was made possible by the generous financial support of the W.M. Keck Foundation. The authors wish to recognize and acknowledge the very significant cultural role and reverence that the summit of Maunakea has always had within the indigenous Hawaiian community. We are most fortunate to have the opportunity to conduct observations from this mountain.

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