Eliciting and Distinguishing Between Weak and Incomplete Preferences: Theory, Experiment and Computation

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Abstract

Recovering and distinguishing between the strict-preference, indifference and/or indecisiveness parts of a decision maker’s preferences is a challenging task but also important for testing theory and conducting welfare analysis. This paper contributes towards this goal by reporting on data from a lab experiment on riskless choice that were analyzed with novel theory-guided computational methods. The experiment included both Forced- and Free-Choice treatments. Its primary novelty consisted of allowing all subjects to select multiple alternatives at each menu. Based on a non-parametric goodness-of-fit criterion that we introduce, which generalizes intuitively a widely used pre-existing method to environments of multi-valued choices, each subjects’ decision data were tested against three structured general-choice models that feature maximization of stable but potentially weak and/or incomplete preferences. Nearly 60% of all subjects’ are well-explained by one of these models, typically with a unique model-optimal preference relation per subject. Importantly, preferences usually (80%) had a non-trivial indifference part and, where applicable, a clearly distinct indecisiveness part. The achieved uncoupling of revealed indifference and indecisiveness is documented empirically for the first time.

Keywords:
Weak preferences; incomplete preferences; revealed indifference; revealed indecisiveness; choice correspondences; Jaccard-Houtman-Maks method; goodness-of-fit.

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1 Introduction

The state in which a decision maker is indifferent between two or more choice alternatives plays a prominent role in many domains of economic analysis. As such, it is important to understand what kinds of observable decision environments and data-analytic methods can in principle allow for extracting an individual’s potentially weak preferences and distinguishing between their strict-preference and indifference parts. This is particularly pertinent if one also accepts that the individual in question may be indecisive, with preferences that are transitive but potentially incomplete. In this situation, it is not obvious how such observable decision data could be used to separate indifference from incomparability/indecisiveness.

These problems are important, both from a theoretical and a policy perspective. First, since many economic models of individual, collective or interactive decisions assume that agents have preferences with non-trivial indifference parts, elicitation of the agents’ weak preference relations would allow for testing these models’ descriptive relevance more accurately than if indifferences were assumed away. Additionally, knowing, for example, how many decision makers within a community consider e.g. a tree-planting program to be equally good to the development of a playground and how many of them have a strict preference instead would generally allow the community leader to arrive at a better decision than if all preferences were mistakenly interpreted to be strict. This example is made more concrete in the table below, which shows how eliciting single choices in the presence of indifferences may lead to a Pareto inefficient social outcome while eliciting multiple choices would not:

|               | Anna       | Basel      | Cora       | Majority-rule outcome |
|---------------|------------|------------|------------|-----------------------|
| True preferences | TreePlanting ≻ₐ TreePlanting ≃₀ TreePlanting ≃₁ | TreePlanting ≃₀ PlayGround ≃₂ | TreePlanting ≻₂ | TreePlanting |
| Possible revealed preferences when only single choices allowed | TreePlanting PlayGround | PlayGround | PlayGround |
| Revealed preferences when multiple choices allowed | TreePlanting PlayGround | TreePlanting PlayGround | TreePlanting |

Similarly, understanding when agents are indifferent and when they are indecisive can enable the policy maker to understand whether they should immediately make an active choice that will impact everyone in the group or delay such a choice and aim instead to inform the group’s members towards resolving their indecisiveness, e.g. via targeted information campaigns.

The relevance of indecisiveness as a concept distinct from indifference also manifests itself in large-stake real-world domains such as high-court judicial decision making. Owing to the fact that justices in such courts often have the luxury to choose which cases to hear –and
make a decision on— and which to ignore, it is conceivable that those binary hear/no-hear choices are not random but, instead, driven by some pattern. As a case in point, we cite the work of Hitt (2019) who notes in relation to the US Supreme Court (pp. 6-7) that “the Court could protect the logical consistency and quality of its opinions by ignoring complex and multifaceted cases. [...] Essentially, [the Supreme Court Case Selections Act of 1988] gave the justices almost total freedom to opt not to decide most disputes. As such, whether consciously or not, the modern Supreme Court actively evolved away from decisiveness”. Hitt further argued that such potentially very impactful decisions may have traceable and behaviourally intuitive underlying patterns, noting that “prioritizing consistency means that the Court will leave numerous important questions and conflicts unresolved” because “a desire to produce ‘good’ (consistent) law may induce the justices to prioritize consistency over decisiveness.” Such prioritization of consistency over decisiveness might be viewed by the economic analyst as defining a particular kind of partial preference relation over judicial cases whereby one is preferred to another if and only if the former is less complex/multifaceted and more likely to result in the production of “good”/consistent law if it was taken up by the judge. When confronted with two cases that generate trade-offs across these two dimensions, such a judge would be indecisive between them, with this mental state potentially manifesting itself in delayed hear/no-hear decision. In contrast, if the judge was instead indifferent between such trade-off generating cases, then one would expect both types of cases to be heard in more or less equal frequency, contrary to the legal scholarly arguments cited above.

This paper extends and combines recent developments in choice theory, computational revealed preference analysis and experimental economics in an ambitious attempt to elicit human decision makers’ potentially weak and/or incomplete preferences and, where relevant, to clearly separate their revealed strict preference, indifference and incomparability/indecisiveness components. The paper makes progress against this goal via a combination of methodological and empirical contributions. First, it proposes and reports on an implementation of a lab experimental design which elicits observable behavioural data that could be used to recover an individual’s weak preferences and distinguish between their strict-preference, indifference and incomparability/indecisiveness parts. The main feature of the design is that it allows subjects in both its Forced- and Free-Choice treatments to make multiple choices from every menu of alternatives they see by introducing incentives that help them make such possibly multi-valued choices in a preference-guided way. Subjects in the Free-Choice treatment can additionally avoid/delay making a choice from any menu at a small cost, without receiving any additional information about the alternatives. Subjects in the otherwise standard Forced-Choice treatment on the other hand are always required to choose at least one option. This experimental design was implemented in a lab environment of riskless choice from 50 distinct menus that were derived from 6 pairs of popular £10
gift cards, and data from 273 subjects are reported on. The primary novelty of this design compared to those in existing studies is that it enables the elicitation of multiple choices per subject in both its treatments. Evidently, this novelty is crucial for testing theories of choice that predict multi- and/or empty-valued choice correspondences as the outcome of preference maximization.

The paper’s second methodological contribution consists of analysing these data by applying a novel computational method that is based on combinatorial optimization and allows for a model-rich approach towards recovering an individual’s potentially weak and/or incomplete preferences that pays particular attention to the fact that choices are possibly multi-valued. This method builds on and extends in the direction of multi-valued choice, and in a model-based way, the classic Houtman and Maks (1985) technique that is routinely used in empirical revealed preference tests of the rational choice/utility maximization hypothesis. More specifically, the original Houtman-Maks technique computes the maximal subset of a subject’s dataset that is consistent with rational choice, essentially allowing for an intuitive quantification of the subject’s behavioural proximity with that model. The extension that we use, which we refer to as the Jaccard-Houtman-Maks method, applies this approximation principle simultaneously to the model of (indifference-permitting) rational choice/utility maximization and two additional models of incomplete-preference maximization: (i) undominated choice, whereby an individual chooses the feasible alternative(s) that are not worse than anything else; (ii) dominant choice, whereby they choose the most preferred feasible alternative(s) if and only if those exist and defer otherwise.

Before justifying the addition of Jaccard’s name to those of the method’s original contributors we first note that, consistent with the questions that we raised earlier, all three models predict multi-valued choices under some of their admissible preference orderings, while the latter two also afford or predict two theoretical separations between revealed indifference and incomparability/indecisiveness. By finding which model is closest to explaining each subject’s behaviour, this method effectively allows for recovering –possibly approximately, and with the standard as if qualifications in place– both the individual’s deterministic decision rule and their preferences conditional on that rule. This explains the “model-based” part of the claimed extension. The addition of Jaccard’s name to the method we propose and use is due to the fact that, in order to account for the general multi-valuedness of subjects’ choices and of the model-predicted optimal choices, in this paper we compute the (dis-)similarity between these sets –which intuitively captures the proximity of observed and model-predicted choices– by means of the classic (and widely applied in several other fields) Jaccard (dis-)similarity metric between finite sets of discrete objects (Jaccard, 1901; Marczewski and Steinhaus, 1958; Levandovsky and Winter, 1971). This, in particular, allows one to have a
more nuanced understanding of when a deviation of a subject’s set of chosen alternatives from the model-predicted ones is a “severe” mistake or a relatively “mild” one, contrary to the binary “pass/fail” approach of the straightforward extension of the HM method in this environment.

Using the data collected from an experiment that was designed in the way described previously and analyzing them with the Jaccard-modified Houtman-Maks method outlined a few lines above, the paper makes several novel empirical contributions. First, despite the relatively large number of experimental decisions, 56% of all subjects are well-approximated (specifically, they are no more than 10% away from being perfectly explained) by one of the above three models of preference maximization. Furthermore, and perhaps surprisingly, the model-optimally recovered preferences of nearly 80% of all subjects in this group feature at least one indifference comparison between distinct alternatives. The prevalence of indifference that is suggested by this finding is important and merits further investigation in future studies. As far as the decomposition of preference-maximization types is concerned, 34% and 22% of all subjects are best matched by utility maximization and the two models of incomplete-preference maximization, respectively. In addition, a single optimal preference relation is recovered from 86% of all these subjects, highlighting the strengths of this method in sharp preference identification when applied on relatively rich—if incomplete—data sets and in relation to models that are in principle uniquely identifiable.

Importantly, the preferences elicited from nearly two thirds of all incomplete-preference maximizers document empirically—and for the first time—the distinct theoretical separations between revealed indifference and revealed indecisiveness that are afforded by the two relevant models. Both these separations assume the analyst has access to the kind of multi-valued choice data that we elicit in this paper. Recalling also the motivational remarks earlier in this Introduction, we view this documentation as a significant empirical contribution of the paper because it demonstrates the relevance of theories of incomplete weak preferences, confirms that their predictions can indeed be tested by observable choices, and shows how this can be done. Together, the above findings highlight the potential usefulness of multi-valued choice data, indifference-permitting deterministic models of complete and incomplete preferences, and combinatorial-optimization goodness-of-fit methods of testing them.

The paper’s next empirical contribution—now from analysing the data at the aggregate level—is the documentation of a significant, and nearly identical, negative correlation between subjects’ choice consistency and their average response times. Although seemingly counter-intuitive, this interesting finding is broadly in line with a key prediction of the influential drift-diffusion neuro-economic model (see Ratcliff and McCoon, 2008, Alós-Ferrer et al., 2021 and references therein). The prediction might be interpreted as suggesting that shorter
response times are more likely when the decision maker has a clear preference between the feasible options. Such clarity in turn would be expected to translate into more consistent active choices, which is indeed what we find. In addition, and despite the fact that subjects in our experiment were allowed to—and typically did—make multi-valued instead of single-valued choices, those in the Free-Choice treatment behaved significantly more consistently. This provides a positive robustness check of the main finding in the study by Costa-Gomes, Cueva, Gerasimou, and Tejiščák (2022), which, unlike the present study, was predominantly designed to test for such potential differences in consistency, and did so with single-valued choice data. Finally, we exploit the richness and generality of the collected experimental data by also introducing and discussing the behaviour of new variables/statistics that are specific to multi-valued choice data, such as choice sizes and choice proportions, and analyse them relative to other relevant observables such as response times and choice-consistency indicators.

Although not constructed so as to enable a thorough investigation of such behaviours, we also note that utilizing two important aspects of the experimental design and its implementation allows for inferring that an additional 10% of the analysed subjects could be thought of as exhibiting a systematic satisficing behaviour (Simon, 1956; Caplin, Dean, and Martin, 2011; Reutskaja, Nagel, Camerer, and Rangel, 2011) or a preference for randomization (Agranov and Ortoleva, 2017, 2023; Dwenger, Kübler, and Weizsäcker, 2018; Cettolin and Riedl, 2019; Agranov, Healy, and Nielsen, 2022). Together with the model-based findings mentioned above, this increases the total proportion of behaviourally explainable subjects in our data to 66%. Finally, while the experiment was not designed so as to test for this either, we further investigated the potential presence of a “preference for flexibility” over menus (Kreps, 1979; Dekel, Lipman, and Rustichini, 2001). We did so by entertaining the possibility that some subjects might have “metavisualised” the actual experimental task of choosing potentially many alternatives from each menu, and responded to it by choosing from the collection of all menus that are derivable from that menu. Reassuringly, this analysis did not detect any notable patterns pointing towards a systematic such “metavisualised” preference for flexibility.

2 Theoretical Background

2.1 Three Deterministic Models of Preference-Maximizing Choice

As was mentioned in the Introduction, in the main part of our individual-level analysis we consider three simple general-choice models of deterministic preference maximization that impose a rich structure on observable behaviour:
I. Rational Choice/Utility Maximization.

II. Undominated Choice with Incomplete Preferences.

III. Dominant Choice with Incomplete Preferences.

We focus on these models for several reasons:

1. All three feature stable preferences and predict both single-valued and multi-valued choices under different preference orderings, with multi-valuedness potentially interpretable as revealing indifferences and single-valuedness as revealing strict preferences.

2. They impose strong behavioural restrictions. In particular, all three models satisfy the fundamental Property α or Contraction Consistency (Sen, 1971, 1997) principle, which requires an alternative to be chosen at a menu whenever it is feasible at that menu and also chosen at a larger menu. In addition, the first and third models predict active choices that are actually consistent with the Congruence/Strong Axiom of Revealed Preference principle, which rules out all forms of choice cycles.

3. They are defined in terms of (and hence in principle allow the analyst to recover) a single preference relation, thereby making the welfare-relevant parts of the analysis unambiguous.

4. All three models are uniquely identifiable. That is, if a decision maker’s observable behaviour is perfectly compatible with one of these models and the available data are sufficiently rich (as is the case in our experiment), then there is a unique preference relation with which that model explains the individual’s behaviour.

5. They are sufficiently computationally tractable to allow for the behaviourally intuitive optimization-based goodness-of-fit test that we describe below.

6. In light also of the preceding discussion about the structure and possible interpretation of the experimental design, the three models predict the kinds of active-choice and deferring behaviour that one might expect to observe in our data.

To state the models formally we first define a decision maker’s choice dataset $D = (A_i, C(A_i))_{i=1}^k$ on a finite grand choice set $X$ to be a collection of pairs that comprise a non-empty menu $A_i \subseteq X$ and a possibly empty set of alternatives that were chosen at this menu when the decision maker was presented with it. Thus, $\emptyset \subseteq C(A_i) \subseteq A_i$ holds for all $i \leq k$. Dataset $D$ is explainable by rational choice/utility maximization if there exists a complete and transitive preference relation $\succsim$ on $X$ such that, for all $i \leq k$,

$$C(A_i) = \{x \in A_i : x \succsim y \text{ for all } y \in A_i\}. \quad (1)$$
If, instead, (1) is true for all $i \leq k$ with respect to a reflexive and transitive but incomplete preference relation, then $D$ is explainable by the model of dominant choice with incomplete preferences. In that case we have

$$C(A) \neq \emptyset \iff \text{there is } x \in A \text{ such that } x \succeq y \text{ for all } y \in A,$$

$$C(A) = \emptyset \iff \text{for all } x \in A \text{ there is } y \in A \text{ such that } x \nless y.$$

Finally, $D$ is explainable by the model of undominated choice with incomplete preferences if there is a reflexive, transitive and incomplete preference relation $\succeq$ whose asymmetric part is $>',$ such that, for all $i \leq k,$

$$C(A_i) = \{ x \in A_i : y \not\succ x \text{ for all } y \in A_i \}.$$ (2)

The model of rational choice/utility maximization was characterized by Richter (1966) in a general environment that encompasses the one within which we are operating here. The model of undominated choice with incomplete preferences was, to the best of our knowledge, introduced by Schmeidler (1969) in a general equilibrium setting and was analyzed choice-theoretically under a variety of decision environments and preference structures by, most notably, Schwartz (1976), Bossert, Sprumont, and Suzumura (2005), Eliaz and Ok (2006), Bossert and Suzumura (2010) and Stoye (2015). Dominant choice with incomplete preferences was studied theoretically in Gerasimou (2018a) and empirically in Costa-Gomes, Cueva, Gerasimou, and Tejisˇc´ak (2022), with the latter analysis limited to an environment of single-valued non-forced choice experimental data. Those data, in particular, do not allow for testing the model of undominated choice or distinguishing between indifference and indecisiveness in either of the two models of choice with incomplete preferences, which is a primary goal of this paper.

The two models of incomplete-preference maximization are logically distinct. Moreover, if we replace the term “incomplete” with “possibly incomplete” in their respective statements, then these models generalize rational choice in different ways. The first does so by relaxing

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1This model is implicitly also used in stochastic-dominance applications of portfolio efficiency, along the lines studied in Levy (2016), Linton, Post, and Whang (2014) and Arvanitis, Scaillet, and Topaloglou (2023), for example.

2A related pre-dating study is Dean (2008), which proposed a class of decision-avoidance models that focused on explaining the increasing prevalence of status quo bias -a phenomenon that is distinct from choice deferral- as menu size increases. When the decision problem contains no natural status quo and no dominant alternative, these models predict a compensatory decision process via which active choices -generally not fully consistent- are always made. Dominant choice with incomplete preferences on the other hand focuses on decision problems without a natural status quo option and features a non-compensatory decision process whereby the decision maker makes (fully consistent) active choices when and only when a most preferred alternative exists.
active-choice consistency while retaining the decisiveness (non-emptiness) assumption that requires \( C(A_i) \neq \emptyset \) for all \( i \leq k \). The second model does so by relaxing the decisiveness assumption while retaining active-choice consistency.

### 2.2 Distinguishing Between Indifference and Incomparability/Indecisiveness

For a decision maker with incomplete preferences who is also indifferent between some alternatives, a non-trivial question that emerges naturally is how one might use observable behavioural data in conjunction with some model in order to separate those pairs of alternatives between which the agent is indifferent from those where the agent is indecisive/unable to compare. Eliaz and Ok (2006) were the first to raise and provide an answer to this question. Taking the model of undominated choice as their primitive, the authors focused on and characterized the special case where the model’s rationalizing incomplete preference relation \( \succeq \) is “regular” in the sense that whenever \( x \nsubseteq y \) and \( y \nsubseteq x \) are both true, then there is \( z \in X \) such that either \( x \nsubseteq z, z \nsubseteq x \) and \( y \succ z \) or \( z \succ y \), or \( y \nsubseteq z, z \nsubseteq y \) and \( x \succ z \) or \( z \succ x \) (Table 1 shows how the number of indifference-permitting regular incomplete preorders varies when \( |X| \in \{3, 4, 5, 6, 7\} \)). The authors’ proposed distinction can then be summarized as follows:

An agent whose incomplete preferences are captured by a regular preorder and who maximizes these preferences according to the undominated-choice model is revealed to be:

- **indifferent** between \( x \) and \( y \) only if \( [x, y \in A, y \in C(A)] \Rightarrow x \in C(A) \);
- **indecisive** between \( x \) and \( y \) only if \( x \in C(A), y \in A \setminus C(A), y \in C(B), x \in B \) for distinct menus \( A \) and \( B \).

In words, the agent is indifferent only if the two options are either chosen or rejected together when both are feasible, while they are indecisive only if one is chosen over the other in some menu and the latter is chosen in the presence of the former in another menu. Importantly, the revealed indifference relation here is transitive, whereas the revealed indecisiveness one is not (see also Mandler, 2009). Also importantly, although this choice-reversal-based distinction between the two notions is intuitive, it is not robust. Indeed, any behaviour that is compatible with such an indifference-permitting instance of that model is observationally equivalent to the same instance of the model where the decision maker is simply indecisive between any two alternatives that are not ranked by strict preference (see also Theorem 1 in Bossert et al., 2005 and Theorem 3.3 in Bossert and Suzumura, 2010).

More recently, Gerasimou (2018a) noted a distinct and robust behavioural separation between indifference and indecisiveness that is afforded by the dominant-choice model. This can be summarized as follows:
Table 1: Enumeration of regular incomplete preorders that allow for non-trivial indifferences.

| $|X|$ | All indifference-permitting incomplete preorders | Regular indifference-permitting incomplete preorders | %  |
|-----|-------------------------------------------------|---------------------------------------------------|-----|
| 3   | 3                                               | 0                                                 | 0.00%|
| 4   | 85                                              | 54                                                | 63.53%|
| 5   | 2,290                                           | 1,705                                             | 74.45%|
| 6   | 75,541                                          | 60,455                                            | 80.03%|
| 7   | 3,363,129                                       | 2,799,615                                         | 83.24%|

Source: Output from constraint-satisfaction problems written in the Essence’ language and solved by the MINION solver (Gent, Jefferson, and Miguel, 2006) using the Savile Row modelling assistant (https://savilerow.cs.st-andrews.ac.uk/).

A decision maker whose incomplete preferences are captured by a preorder and who maximizes these preferences according to the dominant-choice model is revealed to be:

indifferent between $x$ and $y$ if and only if $[x, y \in A, y \in C(A)] \Rightarrow x \in C(A)$;

indecisive between $x$ and $y$ if and only if $x, y \in A \Rightarrow x, y \not\in C(A)$.

In words, the agent is indifferent iff both options are either chosen or rejected together when both are feasible, and they are indecisive between these options iff neither is ever chosen in the presence of the other. This distinction is obviously robust and not subject to alternative interpretations because reducing a relation’s indifferences to incomparabilities in this case generates different active-choice and deferring behaviour.

2.3 Measuring a Choice Correspondence’s Proximity to A Model: The $\text{Jaccard}$-Houtman-Maks Index

Recall that a dataset $\mathcal{D}$ consists of a finite collection of observations $(A, C(A))$, where $\emptyset \neq A \subseteq X$ and $\emptyset \subseteq C(A) \subseteq A$. Denote by $\mathbf{D}$ the collection of all such datasets. In the important special case where $|C(A)| = 1$ is true for every pair $(A, C(A))$ in $\mathcal{D} \in \mathbf{D}$ one can measure the proximity of $\mathcal{D}$ to the model of utility maximization with strict preferences with the classic HM index of rationality due to Houtman and Maks (1985)$^3$, defined by the function $\text{HM} : \mathbf{D} \rightarrow \mathbb{N}$ such that

$$\text{HM}(\mathcal{D}) := \min_{\succ \in \mathcal{P}} \{(A, C(A)) \in \mathcal{D} : C(A) \neq C_{\succ}(A)\},$$

(3)

where $\mathcal{P}$ is the set of strict linear orders on $X$ and $C_{\succ}(A)$ the unique $\succ$-maximizer at menu $A$. In words, $\text{HM}$ associates with each dataset a natural number that captures the smallest number of choices that need to be dropped or modified in that dataset in order for it to be

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$^3$An axiomatization of this index in a rich dataset environment is given in Apesteguia and Ballester (2015).
rationalizable by utility maximization under some strict preference relation.

Now suppose $|C(A)| \geq 1$ for every $(A, C(A))$ in $\mathcal{D}$. Replacing in (3) the set $\mathcal{P}$ with the collection of complete preorders on $X$—denoted by $\mathcal{R}$—and $C_{\succ}(A)$ with $C_{\succ}(A)$ for $\succ \in \mathcal{R}$ provides a direct translation of the HM rationality index with strict preferences to its weak-preference counterpart. There is, however, a conceptual subtlety that is lost in this translation: while there is a unique way in which a single-valued choice $C(A)$ could differ from the $\succ$-optimal choice $C_{\succ}(A)$ (i.e., either the two coincide or they do not), there are many ways in which a multi-valued choice $C(A)$ can differ from the $\succ$-optimal choice $C_{\succ}(A)$. We illustrate this with the following example that presents three hypothetical subjects’ choices at the same menu:

$$\succsim : \quad a \sim b \succ c \succ d$$

$$A = \{a, b, c\}$$

$$C_{\succsim}(A) = \{a, b\}$$

$$C_1(A) = \{a\}$$

$$C_2(A) = \{b, c\}$$

$$C_3(A) = \{a, b\}$$

Here, the $\succsim$-optimal choices at $A$ are in the set $\{a, b\}$ and coincide with the choices made by subject 3. Those made by subjects 1 and 2 on the other hand both deviate from the optimal choice. However, there is a natural sense in which the former subject’s choice is “closer” to being optimal than that of subject 2: it includes one of the two $\succsim$-best alternatives at the menu, whereas the choice of subject 2 contains neither. In the language of mistakes, one might label the first subject’s as a “partially” while that of the second subject as a “fully” mistaken decision. The straightforward extension of the HM index presented above does not account for this nuance and “penalizes” both $C_1(A)$ and $C_2(A)$ in the same way.

For this reason, instead of generalizing (3) to the multi-valued choice case in what appears to be a counter-intuitively punitive way, we propose a generalization that explicitly accounts for the content-similarity of sets $C(A)$ and $C_{\succsim}(A)$ across all pairs $(A, C(A)) \in \mathcal{D}$ and every weak preference relation $\succsim \in \mathcal{R}$. More specifically, to assess how dissimilar is the observed choice $C(A)$ to the choice $C_{\succsim}(A)$ that is optimal under some $\succsim \in \mathcal{R}$ we use the classic Jaccard dissimilarity metric (Jaccard, 1901), whereby

$$J(C(A), C_{\succsim}(A)) := \frac{|C(A) \cup C_{\succsim}(A)| - |C(A) \cap C_{\succsim}(A)|}{|C(A) \cup C_{\succsim}(A)|}$$

$$= 1 - \frac{|C(A) \cap C_{\succsim}(A)|}{|C(A) \cup C_{\succsim}(A)|}$$

$$\in [0, 1].$$
This function, which is widely applied in the life sciences and elsewhere, is a proper metric (Marczewski and Steinhaus, 1958; Levandowsky and Winter, 1971; Gerasimou, 2024) and identifies the dissimilarity between two sets as the proportion of their non-common elements relative to the total number of distinct elements between them. With this definition in place, we can now introduce the Jaccard-Houtman-Maks (JHM) generalization of (3) for the case of possibly multi-valued choice data as the function $JHM : D \rightarrow \mathbb{Q}_+$ where

$$JHM(D) := \min_{\zeta \in \mathbb{R}} \sum_{(A, C(A)) \in D} J(C(A), C_{\zeta}(A))$$

(6)

In words, $JHM$ differs from the cruder multi-valued choice extension of $HM$ in that it distinguishes between –and penalizes– “partially” and “fully” mistaken decisions at a menu according to the Jaccard similarity between each set of actual choices and the corresponding set of model-optimal choices at every menu. As such, one readily observes that $JHM(D) \leq HM(D)$ for all $D \in D$, and $JHM \equiv HM$ in the subdomain of $D$ where $|C(A)| = 1$ for all $(A, C(A)) \in D$.

Applied to the example given above, $JHM$ assigns scores $\frac{1}{2}$, 1 and 0 to the first, second and third subject, respectively, whereas the “binary” or “pass/fail” extension of $HM$ assigns scores 1, 1 and 0 instead. Our analysis in Sections 4 and 5 builds on this index. Appendix B reports the main results of Section 5 when the “binary” $HM$ extension is used instead.

3 Experimental Design

3.1 General Remarks

The experiment was conducted between September 2019 and 2021 at the St Andrews Experimental Economics Lab with a total of 282 subjects (more details in Table 2). In the main part subjects were presented sequentially with a series of 50 menus that consisted of 3, 4 or 5 pairs of gift cards, each drawn from a grand choice set of 6 such pairs. Each gift card was worth £10, thereby making each choice alternative in each menu valued at £20. All gift cards came from popular UK and/or international brands: two supermarkets, two coffee shops, one bookshop, and a gift card that enabled dining at one of nine restaurants. All 6 cards in these 6 pairs could be redeemed in at least one venue in the local town centre (subjects were explicitly informed about this), with the restaurant gift card being redeemable in 3 such venues. Each of the 6 gift cards appeared in exactly 2 of the 6 pairs, once showing up as the first item in the pair and once as the second (see Appendix A for more details).

There are several reasons why the experimental choice alternatives were selected to be gift-card pairs, and those ones in particular. First, gift cards can be thought of as restricted...
forms of cash that can be used for consumption, informally traded or, indeed, gifted. As such, they are intrinsically valuable. Second, these particular gift cards were issued by some of the most popular leisure and grocery destinations for the local student population, and were all within a short walking distance from each other. Hence, all were expected to be desirable to everyone in this experiment’s subject pool. Finally, presenting the choice alternatives as gift card pairs rather than as separate gift cards is realistic (many online retailers invite consumers to choose their “gift card bundle”) and could potentially lead to some relatively hard decisions.

Out of the 63 non-empty menus that are derivable from this set of 6 alternatives, subjects were presented with the 15 binary, 20 ternary and 15 quaternary menus. Each of the 50 distinct menus was presented once and the order of menu presentation was randomized and differed between subjects. This choice domain is therefore rich, heterogeneous and symmetric in the sense that all alternatives are feasible in exactly the same number of menus (see also Appendix B for more details on this). Furthermore, the total number of 50 decisions coincides with the one used in experiments on budget-constrained choice from Arrow-Debreu securities such as Choi, Fisman, Gale, and Kariv (2007) and Halevy, Persitz, and Zrill (2018). Unlike these studies, the choice objects in this experiment were riskless and presented naturalistically, while subjects did not operate under a budget constraint.

The experiment’s computer interface was programmed in Qualtrics and executed in full-screen mode on a web browser that prevented subjects from exiting the interface without the experimenter’s intervention. Subject recruitment was done with ORSEE (Greiner, 2015). All menus appeared as unnumbered vertical choice lists. The “I’m not choosing now” option in the Non-Forced-Choice treatment was always the last item. Because subjects often had to scroll down to find and select that option, this positioning means that it was often physically harder for them to avoid/delay choice. There were a total of 141 participants in each of the two treatments. The 9 subjects who always deferred or always chose everything were excluded from the analysis because their choice behaviour is completely uninformative. Every subject received both their gift-card and cash rewards, as explained below. A nine-question understanding quiz preceded the main part of the experiment in each of the two treatments. Subjects could not proceed to the main part of the experiment until they answered all questions correctly.

3.2 Forced-Choice Treatment

At the beginning of the experiment subjects in both treatments were allocated a monetary endowment of $I = £2.40$. When a menu was shown to subjects in this treatment, they were asked to choose one or more items from that menu. Subjects knew that one menu would be
picked at random for them at the end of the experiment. They also knew that they would be rewarded with an element of their randomly selected menu, and that the decision they made at that menu during the main part of the experiment would be reminded to them before they are asked to make their final, payoff-relevant decision there. Once subjects were past a menu during the main part of the experiment they never saw it again unless that menu later turned out to be their randomly selected one. No additional information about the choice alternatives was provided at any point.

If a subject in this treatment chose one or more—but not all—gift-card pairs from their randomly selected menu during the main part of the experiment, and also chose something from those previously selected options if that menu was determined to be their payoff-relevant one, then they received that item as an in-kind reward and $I$ as a cash reward. If, instead, at that point they chose something that was not among their previously chosen options at that menu, then they received that item and $I_{\text{rev}} = £1.20 < I$. Finally, if they had chosen everything at that menu originally, then they received their original endowment $I$ and a randomly selected element of that menu.

### 3.3 Free-Choice Treatment

Free-/Non-Forced-Choice subjects were asked to either choose one or more of the alternatives at each menu or to delay/avoid making such an active choice by selecting “I’m not choosing now”. What happens here if a subject makes one or more active choices from their randomly selected menu during the main part of the experiment coincides with what happens in the Forced-Choice treatment in the respective cases. But if a subject here had delayed choice at that menu originally, they are asked to choose an item now. In this case they receive this item together with an amount $I_{\text{def}} = £2.10$ that lies strictly between the full initial endowment $I$ and the cash reward associated with a choice reversal, $I_{\text{rev}}$.

### 3.4 Payment

As soon as subjects finished all tasks, their randomly selected menu showed up on their screens, together with the reminder of the decision they had made at this menu. As an additional incentive for subjects to make deliberated and non-rushed decisions, they were told from the beginning that no participant would be able to receive their rewards and leave the lab in the first 50 minutes of the session. The experimenter (this author) went to each subject’s desk once they were finished (and after this threshold was exceeded), asked them about their final choice at this menu, and later gave them their cash and gift card rewards accordingly. Subjects who had chosen everything at their randomly selected menu were
Table 2: Summary information on the two experimental treatments.

|                          | Free-Choice treatment | Forced-Choice treatment |
|--------------------------|-----------------------|-------------------------|
| Original sample          | 141                   | 141                     |
| Subjects excluded because they always chose everything | 1 | 3 |
| Subjects excluded because they always deferred | 5 | N/A |
| Subjects excluded - total | 6 | 3 |
| Subjects after all exclusions | 135 | 138 |
| Choice objects           | 6 pairs of £10 gift cards (£20 total value) | | |
| Number of menus & decisions | 50 | |
| Reward frequency         | Every subject         |                         |
| Location                 | St Andrews lab        |                         |
| Dates when the experimental sessions were conducted | 17–20 Sept 2019 | 29 Jan 2020 (62) |
|                          |                       | 22 Sep 2021 (79)        |

in turn invited to the experimenter’s desk and, after the appropriate numerical range was specified on the random-number generating website https://random.org, and the way in which the numbers in this range were mapped to the relevant gift card pairs was agreed, a random number was generated to determine the pair they would be rewarded with. The total cost from payments to experimental subjects was approximately £6,270.

3.5 Discussion

Denoting by $A$ a subject’s payoff-relevant randomly selected menu, and by $C^1(A)$, $C^2(A)$ the choices they made at that menu during the main and final parts, respectively, Table 3(a) summarizes the incentive structure in the Forced-Choice treatment. Under the assumption of a utility-maximizing decision maker with possibly weak preferences, $C^1(\cdot)$ denotes their nonempty- but possibly multi-valued choice correspondence. That is, for a Forced-Choice subject $C^1(\cdot)$ satisfies $\emptyset \neq C^1(M) \subseteq M$ for every menu $M$. By contrast, $C^2(A)$ assigns a single chosen alternative to the subject’s randomly selected menu $A$.

By the very definition of indifference, a utility-maximizing subject who operates under the Forced-Choice design and is indifferent between all feasible alternatives at a menu is also indifferent between choosing all these alternatives and any sub-collection thereof. Hence, such a subject has no strict incentive to misreport their indifference. Conversely, however, they will choose everything in a menu only if they are indifferent between all alternatives at that menu because, by definition of indifference, this individual does not care which alternative they will get or whether this is decided by someone else or randomly (see also Danan (2010) for a formal elaboration of this point). Thus, it is weakly dominant for this
Table 3: Incentives in the Forced- and Free-Choice treatments are structured around the subjects’ 1st and 2nd decisions at their randomly selected menu (denoted here by \(A\)).

| 1st decision (main stage) | 2nd decision (payoff stage) | Choice reward | Cash reward | Possible interpretation |
|--------------------------|-----------------------------|---------------|-------------|-------------------------|
| \(C^1(A) = A\)          | none possible               | random \(a \in A\) | initial endowment \(I\) | Revealed total indifference is costless & responded to literally |
| \(C^1(A) \subset A\)    | \(C^2(A) \in C^1(A)\)     | \(C^2(A)\)   | \(I\)       | Stable revealed preference is costless |
| \(C^1(A) \subset A\)    | \(C^2(A) \notin C^1(A)\)   | \(C^2(A)\)   | \(I_r < I\)  | Unstable revealed preference is costly |

| (b) Free-Choice Treatment |
|---------------------------|
| \(C^1(A) = A\)           | none possible               | random \(a \in A\) | initial endowment \(I\) | Revealed total indifference is costless & responded to literally |
| \(\emptyset \neq C^1(A) \neq A\) | \(C^2(A) \in C^1(A)\)     | \(C^2(A)\)   | \(I\)       | Stable revealed preference is costless |
| \(\emptyset \neq C^1(A) \neq A\) | \(C^2(A) \notin C^1(A)\)   | \(C^2(A)\)   | \(I_r < I\)  | Unstable revealed preference is costly, and more so than revealed indecisiveness |
| \(C^1(A) = \emptyset\)   | \(C^2(A) \in A\)          | \(C^2(A)\)   | \(I_d : I_r < I_d < I\) | Revealed indecisiveness |

subject to choose everything feasible in a menu whenever they are indifferent between all items contained in it.

At the same time, if a utility-maximizing subject has one or more optimal alternative(s) at the menu, and these are strictly better than something else feasible, then choosing everything is dis-incentivized by the design because doing so comes with the risk of potentially receiving an inferior alternative as a reward. The design instead incentivizes such a subject to select those and only those options they prefer the most, as this enables them to choose one of these superior options at the end and to also receive their full cash endowment. For a utility-maximizing agent, therefore, the design in this treatment novelty combines incentive-compatibility for truthful preference revelation in the sense of Azrieli, Chambers, and Healy (2018) with a standard interpretation of multi-valued choice that is articulated in Kreps (2012, p.2) as follows: 
"The story is that the consumer chooses one element of \(A\). Nonetheless, we think of \(c(A)\) as a subset of \(A\), not a member or element of \(A\). This allows for the possibility that the consumer is happy with any one of the several elements of \(A\), in which case \(c(A)\) lists all those elements. When she makes a definite choice of a single element, say \(x\), out of \(A\)—when she says in effect, ‘I want \(x\) and nothing else’—we write \(c(A) = \{x\}\), or the singleton set consisting of the single element \(\{x\}\). But if she says, ‘I would be happy with either \(x\) or \(y\), then \(c(A) = \{x, y\}\).”

The choice correspondence \(C^1(\cdot)\) for a Free-Choice subject on the other hand is possibly empty- and multi-valued; hence, it satisfies \(\emptyset \subseteq C^1(A) \subseteq A\) for every menu \(A\). As in the
Forced-Choice treatment, however, $C^2(\cdot)$ assigns a single chosen alternative to the relevant subject’s randomly selected menu. In this treatment too, moreover, a utility-maximizing Free-Choice subject is (weakly) incentivized to choose their most preferred option(s) at every menu, with the opportunity to defer at a cost being irrelevant to them. Now suppose again that the subject is not a utility maximizer but has incomplete preferences and cannot compare any two of the feasible options at a menu. Then, depending on the individual’s subjective perception of the decision’s importance, they may either opt to choose everything and end up with something at random or opt instead to incur the relatively small cost $I - I_{def}$ to delay making an active choice at that menu themselves. The former kind of behaviour could be seen as analogous to recent findings suggesting that decision makers prefer to randomize when they are faced with a difficult decision repeatedly (Agranov and Ortoleva, 2017; Dwenger, Kübler, and Weizsäcker, 2018). Importantly though, it could also be compatible with the model of undominated choice with incomplete preferences –discussed in Section 4– whenever every feasible option is incomparable to all others. By contrast, deferring at a small cost could be seen as a manifestation of indecisiveness-driven deferral (Tversky and Shafir, 1992; Danan and Ziegelmeyer, 2006; Costa-Gomes, Cueva, Gerasimou, and Tejiščák, 2022) and may potentially be compatible with the model of dominant choice with incomplete preferences that was discussed in the previous section.

### 3.6 A Potential Criticism

It is worth commenting, finally, on the discrepancy in the way that the design deals with situations where $C(A) = A$ or $\emptyset \neq C(A) \subset A$ at the payoff-relevant menu $A$. Recall that in the former case subjects receive their reward from $C(A)$ at random and a full cash reward alongside it, whereas in the latter case they can themselves choose anything from $A$, possibly at the cost of receiving the reduced cash reward if their two choices at $A$ exhibit a reversal. An alternative design choice here would be to break this dichotomy by applying the uniform randomization rule at $C(A)$ in both situations. This would have been a perfectly legitimate way to proceed in our environment of multi-valued choice. Introducing this dichotomy here was driven by the motivation to have a design that remained comparable to the one in Costa-Gomes, Cueva, Gerasimou, and Tejiščák (2022). The one proposed here achieves this goal because it does indeed reduce to the one proposed by these authors in the special case where $C(A)$ is required to be single-valued. With the alternative design mentioned above there would be no role for the reduced cash payment $I_{rev}$ that is associated with a choice reversal. The presence of this additional payment parameter, however, and its lower value compared to that of the full-payment parameter $I$, is potentially useful in instilling a mindset towards preference-guided choices by subjects. On the other hand, the unified choice-reward
rule where randomization is applied on any non-empty $C(A)$ is immune to the possibility of subjects’ choosing dominated alternatives, which in theory is left open by the dichotomous rule in the current design. While this theoretical possibility can indeed not be ruled out, it is worth keeping in mind that a subject who includes dominated alternatives in their first-round choice $C(A) \subset A$ not only does not benefit from doing so but, in fact, spends additional time and effort –thereby incurring a cost– in the process.

4 Aggregate-Level Analysis

4.1 Choice Sizes

We start with a key new variable of interest, which we refer to as the subjects’ choice size, and which is defined simply as the number of gift-card pairs that were chosen at each menu. This variable ranges between 0 and 4 in the Non-Forced-Choice treatment, and between 1 and 4 in the Forced-Choice treatment. However, because choice sizes 1 and 2 (as well as 0 in the former treatment) are always feasible, whereas 3 and 4 are not, we adjust their relative frequencies accordingly. Specifically, Figure 1 presents the distributions of menu-size adjusted choice sizes in the two treatments, which are derived once their absolute frequencies are divided by the total number of menus where these choice sizes might be observed. For example, the denominators here are $50 \times N$ and $15 \times N$ for choice sizes 1 and 4, respectively, where $N$ is the total number of subjects in the relevant treatment.

This comparison shows that the modal adjusted choice size was 1 in both treatments, at a rate of just over 50%, and was followed by 2 and 3. The deferral rate in the Non-Forced-Choice treatment, defined as the relative frequency of a zero choice size, is 7.4%. This is slightly higher than the 6.9% choose-everything rate, which is obtained by weighted-averaging the relative frequencies where $n$ out of $n$ items were chosen, for $n = 2, 3, 4$ (Table 4). The latter rate is also lower than the corresponding one in the Forced-Choice treatment (8.5%), and the difference is statistically significant ($p < 0.001$; two-sided Fisher’s exact test). This difference is consistent with the intuition that, in the absence of the possibility to delay choice when faced with a potentially difficult decision, subjects are more likely to choose everything that is available in the menu, either as a result of following a more general decision rule or, in the context of our experimental design, possibly due to a preference for randomization. Finally, the rate at which subjects chose all 4 alternatives was very low in both treatments, at 3.2% and 2.5% in the Forced- and Free-Choice treatments, respectively.

Table 4 elaborates further on this theme by also showing how the relative frequencies of different choice sizes vary with the number of alternatives at different menus, and by also
Figure 1: Relative frequencies of the different choice sizes, adjusted for feasibility.

Table 4: Choice sizes and the corresponding relative frequencies and average response times at menus of different sizes.

|                      | Alternatives chosen in the Forced-Choice treatment | Alternatives chosen in the Free-Choice treatment |
|----------------------|---------------------------------------------------|--------------------------------------------------|
|                      | (average response times, in seconds, in parenthesis) | (average response times, in seconds, in parenthesis) |
|                      | 0       | 1       | 2       | 3       | 4       | 0       | 1       | 2       | 3       | 4       |
| Binary menus         |         |         |         |         |         | 11.95%  | 74.52%  | 13.53%  |         |         |
| Ternary menus        | –       | 83.77%  | 16.23%  | –       | –       | 7.31%   | 47.25%  | 6.56%   | –       | –       |
| Quaternary menus     | –       | –       | 28.50%  | 41.01%  | 37.50%  | 8.27%   | 10.06%  | 9.93%   | 27.02%  | 2.47%   |


presenting the corresponding average response times. These conditional relative frequencies are uniformly higher in the Forced-Choice treatment for choice sizes 1 and 2 in binary and ternary menus. In addition, proportionally more subjects in that treatment chose 2, 3 or 4 gift-card pairs in quaternary menus too. As far as deferrals in the Free-Choice treatment are concerned, these were more likely in binary menus (≈ 12%) than in ternary (≈ 7%) or quaternary menus (≈ 4%).

4.2 Response Times

Notably, the average response time in both treatments was always lowest (≈ 6 secs) when subjects chose a single pair of gift cards, while it increased monotonically (and statistically significantly) in similar ways as the size of choices and menus increased (Figure 2). These novel empirical facts might be seen as intuitive evidence suggesting that the subjects’ decision was easier at menus where they chose a single gift-card pair, perhaps because that was their clearly preferred one. The latter explanation would be in line with the one often mentioned in the literature of binary forced-choice decisions via sequential-sampling processes such as the drift-diffusion model and extensions (Ratcliff and McKoon, 2008; Alós-Ferrer et al., 2021), although the novelty of identifying a possibly structure for the evolution of this process in free- and non-binary choices would remain.

However, the findings are also consistent with an alternative interpretation whereby the time was shorter in those cases because subjects only had to move the mouse cursor on their computer to check a single choice box on their screen, thereby mechanically resulting in a shorter response time than when multiple items were selected from relatively larger menus. This argument is less relevant in the case of deferral decisions, however, where the average response time increases monotonically from (approximately) 6 to 10 to 13 secs when the menu includes 2, 3 and 4 alternatives, respectively. This suggests that the decision to defer was not generally based on some menu-irrelevant strategy (e.g. to reach the end of the experiment quickly), but was influenced instead by the relevant menu’s composition and, presumably, the subjects’ preferences at that menu. Finally, there is no significant difference in the distributions of response times in the subjects’ active choices across treatments (p = 0.612; two-sided Mann-Whitney U test).

4.3 Choice Proportions

Next, we define a subject’s choice proportion at a menu as the number of chosen alternatives divided by the number of feasible alternatives at that menu. The distributions of average choice proportions across the two treatments are shown in Figure 3 (top panel) and are sig-
Figure 2: Response times are positively correlated with menu and choice size in both treatments.

(a) Forced-Choice treatment

(b) Free-Choice treatment

Note: The violin plots are thicker (thinner) in regions with more (fewer) observations, and also show the conditional average response times and 95% confidence intervals per menu/choice size.
Figure 3: Distributions of the average choice proportions and menus where subjects chose everything or deferred.

(a) Forced-Choice treatment

(b) Free-Choice treatment

Notes: Unless otherwise noted, all p-values below are from two-sided Mann-Whitney U tests. The distribution of average choice proportions (top panel) is significantly different between treatments, both when deferrals are included (p = 0.006) and when they are not (p < 0.001; figure not shown). The per subject distribution of menus where everything was chosen (middle panel) is not significantly different between treatments (p = 0.127), although the rate at which subjects exhibited such behaviour is higher in the Forced-Choice treatment (8.4% vs 6.9%; p < 0.001; two-sided Fisher’s exact test), and so is the proportion of subjects who did so at least once (71% vs 60%; p = 0.057; two-sided Fisher’s exact test). See also main text. In the Non-Forced-Choice treatment the distribution of the number of menus where everything was chosen is not significantly different from the corresponding distribution where subjects deferred (bottom right panel; p = 0.498).
significantly different (higher in the Forced-Choice treatment) both when deferrals are included and when they are not \( (p = 0.006 \) and \( p < 0.001 \), respectively; two-sided Mann-Whitney U tests). The mean/median choice proportions across all subjects in the Forced- and Non-Forced-Choice treatments were 0.54/0.5 and 0.49/0.5, respectively. That is, subjects in both treatments tended to choose around half of the feasible alternatives, on average.

Additionally, 69 subjects (51.1%) in the Free-Choice treatment deferred at least once, with deferrals per subject ranging between 1 and 42 (Figure 3-(b); bottom panel), and with a mean/median occurrence of 7.28/5 menus. Moreover, 81 subjects in this treatment (60%) chose all feasible gift-card pairs at least once, with such occurrences per subject ranging between 1 and 38 menus (Figure 3-(b); middle panel), and with a mean/median occurrence of 3.42/1 menus. By contrast, in the Forced-Choice treatment there were 98 subjects (71%) who chose everything at least once, with occurrences per subject ranging between 1 and 45 (Figure 3-(a); middle panel), and with a mean/median of 4.22/2 menus. The difference in the two proportions is borderline (in)significant at the 5\% level \( (p = 0.058; \) two-sided Fisher’s exact test). These findings lend further support to the possibility that, unable to delay their choice when faced with a hard decision, subjects are more likely to choose everything.

### 4.4 Choice Consistency

For this analysis—which is summarized in Table 5—we compute in \( \text{Prest} \) and then compare the Jaccard-Houtman-Maks (JHM) scores of subjects’ active choices while accounting for the possibility of non-trivial indifferences. That is, we compare subjects’ single- and/or multi-valued choices—ignoring any deferral decisions—that are compatible with indifference-permitting utility maximization. First, we focus on the distributions of the JHM scores across the two treatments. Their mean (5.38 v 4.43), median (4.96 v 3.50) and standard deviation (4.02 v 3.75) are uniformly and significantly \( (p = 0.045) \) lower in the Free-Choice than in the Forced-Choice treatment. Thus, the number of partially or fully “mistaken”—from the point of view of utility maximization—active choices is significantly smaller for subjects who were not asked to always choose some gift-card pair(s). This is also reflected in our second test for treatment effects in active-choice consistency, where we compare the proportions of subjects who made perfectly (JHM score of 0) or approximately (JHM score less than or equal to 1, 2, 3, 4 or 5) consistent active choices in the two treatments. There were approximately 4\% and 13\% perfectly consistent subjects in the Forced- and Free-Choice treatments, respectively \( (p = 0.007) \). The pattern is similar for approximately consistent subjects, with 14.5\% v 26\% making up to one fully mistaken choice \( (p = 0.023) \), 27.5\% v 37\% up to two \( (p = 0.092) \), 36\% v 50\% up to three \( (p = 0.028) \), 43\% v 54\% up to four \( (p = 0.069) \) and 51\% v 59\% up to five \( (p = 0.181) \).
Table 5: Active-choice consistency in the two treatments.

|                  | Forced Choice | Free Choice | p-value |
|------------------|---------------|-------------|---------|
| Mean JHM         | 5.38          | 4.43        | 0.045   |
| Median JHM       | 4.96          | 3.50        |         |
| St.Dev JHM       | 4.02          | 3.75        |         |
| JHM = 0          | 5             | 17          | 0.007   |
| JHM ≤ 1          | 20            | 35          | 0.023   |
| JHM ≤ 2          | 38            | 51          | 0.092   |
| JHM ≤ 3          | 50            | 67          | 0.028   |
| JHM ≤ 4          | 59            | 73          | 0.069   |
| JHM ≤ 5          | 70            | 80          | 0.181   |
| N                | 138           | 135         |         |

Note: All p-values are from two-sided Mann-Whitney (1st) and Fisher’s exact (2nd to 7th) tests.

To account for the fact that deferring at a menu can never decrease a decision maker’s active-choice consistency we also carry out subject-specific simulations to find how likely it would be for a possibly deferring subject to attain their Jaccard-Houtman-Maks score given that they made their active choices only at those particular menus where they did so. More specifically, in line with Selten (1991), Beatty and Crawford (2011) and Costa-Gomes, Cueva, Gerasimou, and Tejiščák (2022), we compute in each treatment the Selten measure of predictive success of utility maximization on subjects’ active choices. We do so as follows:

(i) for every experimental subject, create 10,000 datasets from as many artificial subjects who were restricted to make uniform-random active choices only at those menus where the experimental subject did so;

(ii) find the proportion of such subject-specific simulated datasets within this block that have a zero Jaccard-Houtman-Maks score when utility maximization under both strict and weak preferences are accounted for. Then, from the proportion, \( p_i \), of perfectly consistent human subjects in treatment \( i \), subtract the average proportion, \( a_i \), of artificial subjects who are also perfectly consistent.

The closer the difference \( m_i := p_i - a_i \) is to 1, the higher the proportion of subjects whose active choices were consistent with utility maximization, and the more likely it is that this could not have happened randomly. Low but positive values on the other hand could arise either because (i) relatively many experimental subjects were consistent but this could probably be due to chance; or (ii) because relatively few subjects were consistent and consistency in this environment was unlikely to occur by chance.

It turns out that the latter case applies to the data from both our treatments, where we
In line with the comparisons presented in Table 5, however, one may extend this analysis further to the case of approximate active-choice compliance with utility maximization. Taking as our approximation threshold the JHM distance score of 5 that corresponds to 10% of the maximum value that this score can take, we find that a similar gap remains in the predictive “approximate” success of utility maximization between treatments:

\[
\begin{align*}
    m_{FC}^{10\%} &= p_{FC}^{10\%} - a_{FC}^{10\%} \approx 0.507 - 0 = 0.507 \\
    m_{NFC}^{10\%} &= p_{NFC}^{10\%} - a_{NFC}^{10\%} \approx 0.592 - 0.074 = 0.585
\end{align*}
\]

The results from all tests that are presented in this subsection document a clear negative effect that the act of forcing choice exerts on subjects’ choice consistency. This complements in some important ways the respective finding that was originally documented in Costa-Gomes, Cueva, Gerasimou, and Tejiščák (2022). In particular, the results here show that forced choices are less consistent even when subjects have to decide from nearly twice as many menus and even when the experimental design allows them to make multi-valued choices that in turn enables the analyst to test for the possibility that their behaviour is well-approximated by utility maximization with or without indifferences.

We also ask how a subject’s choice consistency, as captured by the above Jaccard-Houtman-Maks index, is related to their response times. Figure 4 (top panel) shows that there is a significant—and very similar in size, with a Spearman coefficient of 0.35 and 0.45—positive correlation in both the forced- and free-choice treatments between subjects’ active-choice Jaccard-Houtman-Maks indices and their average response times. That is, choice consistency is negatively correlated with the time it takes for subjects to make their active choices. This interesting finding is broadly consistent with the important prediction of the drift-diffusion neuro-economic model\(^4\) whereby shorter response times are more likely when a clearly preferred alternative exists, which would imply in turn more consistent active-choice behaviour. The novelty here in this respect is that such a finding is documented in a rich dataset that comprises both binary and non-binary menus. Another possible explanation for this finding is that subjects with a higher cognitive ability (an unmeasured variable in this study) are both more consistent and faster than subjects with a lower cognitive ability. A distinct possible explanation is that spending more time before deciding is more characteristic of people who are prone to second thoughts and therefore more likely to be involved in choice reversals when presented with a series of decision problems. Conversely, it is also

\(^4\)See Ratcliff and McKoon (2008), Alós-Ferrer, Fehr, and Netzer (2021) and references therein.
Figure 4: Choice consistency is negatively correlated with average response times in both treatments.

(a) Forced-Choice treatment

(b) Free-Choice treatment

Notes: $R$ is the Spearman coefficient and $p$ is the $p$-value. Shaded areas indicate 95% confidence intervals. There is no significant difference in the average active-choice response times between treatments ($p = 0.550$; 2-sided Mann-Whitney test).
possible that subjects who had neither a stable preference relation over the choice alternatives nor a clear decision rule ended up spending more time on average at each menu, but without this extra time ultimately alleviating the effects of their ambivalence.

Figure 5: Choice consistency is negatively correlated with the average choice size.

![Graph showing correlation between choice consistency and average choice size]

Note: $R$ is the Spearman correlation coefficient and $p$ is the $p$-value. Shaded areas indicate 95% confidence intervals.

Figure 5 further shows that there is also a significant negative correlation between subjects’ active-choice consistency and their average choice sizes in each of the two treatments, with this relationship being more than twice as pronounced for Free-Choice subjects ($R = 0.44$ vs $R = 0.19$ on Jaccard-Houtman-Maks index). In line with our preceding findings and discussion, an intuitive explanation for this difference is that, unlike Free-Choice subjects, their Forced-Choice counterparts could not defer at menus where they might perhaps have wished to do so, opting instead for more alternatives per menu. But while delaying choice when confronted with a difficult problem safeguards the consistency of one’s behaviour, choosing more (possibly all) alternatives could do the opposite because it opens up more possibilities for choice reversals/cycles to emerge.

Some additional support to this explanation, finally, is obtained by also comparing the active-choice consistency of Free-Choice subjects who deferred at least once to the consistency of those who did not. More specifically, 14 of the 69 (20%) deferring subjects were perfectly consistent in this sense, while 3 of the 66 (4.5%) non-deferring ones were so (equivalently, 14 out of the 17 perfectly consistent in this treatment deferred at least once). The difference between these two proportions is significant ($p = 0.008$; two-sided Fisher’s exact test). Moreover, the two groups had an average/median Jaccard-Houtman-Maks score of 3.58/2.33 and 5.31/5.08, respectively, and the difference in the two distributions is significant too ($p = 0.003$; two-sided Mann-Whitney $U$ test). In fact, as shown in Figure 6, deferring subjects are uniformly more consistent than non-deferring ones in the sense that the distribution of their JHM scores first-order stochastically dominates the corresponding distribution of the latter subjects. That is, for every number less than or equal to $n$, and for every
n, non-deferring subjects were more likely to be strictly more than n active choices away from utility maximization. This within-treatment effect further highlights the important mediating role of deferrals for consistency.

5 Individual-Level Analysis

5.1 Non-Parametric Model Fitting

We now turn to the tests of our possibly multi- and/or empty-valued experimental choice data for compatibility with each of the three models introduced in Section 2. These tests were done by applying a model-based extension of the classic Houtman and Maks (1985) that was used in the aggregate-level analysis, and extending it to our generally multi- and/or empty-valued general-choice data. Specifically, we ask what is the smallest number of decisions that need to be dropped/changed in a subject’s dataset in order for it to be compatible with that model under some of its instances/admissible preference orderings. We refer to this number as the model’s distance score. These scores were computed using a method that was originally introduced in Costa-Gomes, Cueva, Gerasimou, and Tejiščák (2016, 2022).

5 We use the term “distance score” rather than “distance” partly because the underlying function is not a proper metric and partly to highlight the ranking aspect of this JHM-based goodness-of-fit approach.

6 The 2016 working-paper version of that study indirectly constructed choice correspondences by augmenting the subjects’ actual choices with their survey responses in indifference-vs-preference questions that followed binary menus only. Before constructing the subjects’ indifference classes from these data, the survey responses were analysed for consistency, and responses that eventually led to intransitive indifferences were discarded. Whenever this was the case, the subjects’ actual choices were interpreted as revealing strict
and which, since Gerasimou and Tejiščák (2018), has been extended considerably and made freely available online in the open-source desktop application Prest.7

More specifically, a brute-force algorithm was implemented for these combinatorial-optimization computations. This involved the production of all choice datasets that are generated by all possible instances of every model, and comparing each such dataset against every subject’s own dataset in order to find the model(s) and instance(s) that are closest to it in the minimum distance-score sense. Therefore, this method detects perfect as well as approximate model fits, and in the latter case it quantifies the approximation in an intuitive way that may be interpreted as the number of “mistakes” made by an agent who might be portrayed as following a particular decision rule.8

We stress that despite the vast numbers of weak orders (4,683), partial orders (130,023) and, especially, incomplete preorders (209,527) that are defined on a set of 6 alternatives (OEIS, 2021), the above tool makes such an exact computation possible very quickly—in less than 12 seconds—for all subjects in each treatment, for each of the three models. We also stress that, although this tool lies within the rapidly expanding realm of artificial-intelligence technologies, broadly interpreted, it builds on combinatorial-optimization methods to arrive at exact solutions to a clearly defined and behaviourally intuitive problem. In particular, it does not feature so-called “deep-learning” decision algorithms that build on neural-network structures which may be unclear/uninterpretable or prone to over-fitting. Finally, although the brute-force algorithm is linear in the number of subjects but exponential in the number of alternatives, the model-rich distance-score method itself is scalable and can be extended to analyse datasets that are derived from much larger sets of alternatives by employing powerful open-source constraint solvers.

Table 6 and Figure 7 present summaries of this goodness-of-fit analysis for both Forced- and Free-Choice subjects. In line with standard practices in empirical revealed preference analysis whereby one also wishes to understand the extent to which a certain behaviour or perfect/approximate model fit could have been generated randomly (Bronars, 1987), we

preferences, regardless of the survey response.

7With retention of its original focus on the model of Rational Choice/Utility Maximization with strict preferences, the HM approach has been extended in other intuitive directions by Apesteguia and Ballester (2015) and Dean and Martin (2016). However, although the “Swaps” measure of Apesteguia and Ballester (2015) that pertains to general choice environments is also readily computable in Prest, it is currently unclear how it should be computed when subjects choose multiple items from the same menu, or how it should be extended in models that generalise utility maximization.

8For other approximations that have been proposed or applied recently for/at distinct classes of models and choice environments we refer the reader to Choi, Fisman, Gale, and Kariv (2007); Echenique, Lee, and Shum (2011); Choi, Kariv, Müller, and Silverman (2014); Apesteguia and Ballester (2015); Dean and Martin (2016); Bonacida and Martin (2021); Dembo, Kariv, Polisson, and Quah (2021); Fudenberg, Liang, and Gao (2023); Cherchye, Demuynck, Lanier, and Rock (2023); Echenique, Imai, and Saito (2023) and references therein.
also performed our model analysis on 10,000 simulated datasets of artificial uniform-random behaving subjects under both a Forced- and a Free-Choice configuration (details on the simulations’ structure are available in Appendix C). The JHM distance-score distributions of simulated subjects are juxtaposed in the relevant graphs of Figure 7 to those of the experimental subjects for each of the three models, while the minimum values from the simulated-subject JHM distributions are also presented in Table 6.

Table 6: Classification of subjects who are perfectly/approximately explainable by one of the three models under the Jaccard-Houtman-Maks goodness-of-fit method.

|                                | Utility Maximization | Undominated Choice with Incomplete Preferences | Dominant Choice with Incomplete Preferences | All              |
|--------------------------------|----------------------|-----------------------------------------------|--------------------------------------------|------------------|
| Subjects with score = 0        | 5 (4%)               | 0                                             | 0                                          | 5 (4%)           |
| Subjects with score ≤ 5        | 61 (44%)             | 16 (11.5%)                                    | 0                                          | 77 (56%)         |
| Mean/median best score (≤ 5)   | 2.01/1.83            | 3.56/4                                        | –                                          | 2.34/2           |
| Mean / median best-model       | 1.23/1               | 1.19/1                                        | –                                          | 1.22/1           |
| preference orderings (score ≤ 5)|                     |                                               |                                             |                  |
| Minimum score in simulations   | 14.87                | 13.92                                         | –                                          |                  |

For human subjects, Rational Choice often tied with one, but never both, of the other two models. The classification presented in Table 6 broke ties in favour of that model whenever a subject’s JHM distance score was produced by Rational Choice and another model (see the notes of Table 6 for more details). Under this tie-breaking assumption, and in line also with model-approximation methods used in the above-cited studies, the two exhibits show the number, proportion and relative-frequency distributions of distinct subjects that are on average within 10% (equivalently, within 5 decisions) away from being explainable perfectly by an instance of some model, separately for each of the two experimental treatments. We chose this conservative approximation range for two factors: (i) simulations suggest that a JHM distance score of 5 for any of the three models is extremely unlikely to occur randomly in this decision environment (see Table 6 and Figure 7); (ii) for subjects with a JHM score not greater than 5 there is typically only one best-matching preference ordering that explains
Figure 7: Distributions of all subjects’ Jaccard-Houtman-Maks distance scores for each of the three models, and the corresponding scores from simulations.

(a) Forced-Choice treatment

(b) Free-Choice treatment
their behaviour under the respective subject-optimal model.

The total number (proportion) of Forced-Choice subjects with a perfect or 10%-approximate model fit was 5 (4%; all under Rational Choice) and 77 (56%), respectively, with 61 (44%) and 16 (11.5%) of those in the latter group classified under Rational Choice and Undominated Choice with Incomplete Preferences. Furthermore, the total number (%) of Free-Choice subjects with a perfect or 10%-approximate fit was 10 (7%) and 74 (55%), respectively. Thirty (22%), 7 (5%) and 37 (27%) of all subjects in this treatment were categorized under Rational Choice, Undominated and Dominant Choice with Incomplete Preferences, respectively. In addition, 3 and 7 of these subjects were perfectly compatible with the first and third model, respectively, with one subject among the former three and five among the latter seven revealing indifferences in their weakly ordered and weakly preordered preferences.

The model fit is slightly better on average in the Free-Choice treatment than in the Forced-Choice one (mean/median best JHM scores: 2.05/2 vs 2.34/2), although the distributions are not significantly different. Similarly, and in line with the results from the consistency analysis presented in the previous section, the proportion of subjects who were best-matched by one of the two models of consistent active choices (i.e. Rational Choice and Dominant Choice with Incomplete Preferences) is higher in the Free-Choice treatment (49% vs 44%), while the proportion of subjects best-matched by the third model (Undominated Choice with Incomplete Preferences) that predicts generally inconsistent active choices is lower in that treatment (5% vs 11.5%). Moreover, when Rational Choice was not the best-matching model, its distance score was on average 1.4 and 5.7 units higher in the Forced- and Free-Choice treatments, respectively. Finally, the highest JHM distance score of 5 for experimental subjects in this approximate model-fitting analysis is far below the corresponding minimum scores of simulated random-behaving subjects, which ranged between 14 and 21.

Figure 8: All 37 subjects that were best-matched by Dominant Choice with Incomplete Preferences were unlikely to have achieved this classification at random given the menus where they deferred.
Prompted by the greater permissiveness of behaviour in the Free-Choice treatment, moreover, for the 37 subjects who are best-explained by Dominant Choice with Incomplete Preferences we also conducted a new robustness check. By analogy to the way in which the predictive success of Rational Choice on Free-Choice subjects’ active-choice consistency was evaluated in Section 4.4, we generated another set of 10,000 random-behaving simulated subjects for each of the 37 human experimental participants who were best-matched by that model, this time holding fixed in each such subject’s simulations block the menus at which the relevant human participant deferred. This allows for evaluating the predictive success of the Dominant-Choice model’s 10%-approximation range in this decision environment. Figure 8 shows the histogram with JHM distance-score differences for this model between the 2.5th percentile value in each subject-specific 10k-simulations block and the relevant subject’s actual score. For all 37 subjects the difference is positive and typically much greater than 10 score units (i.e. active-choice or deferral decisions). This analysis suggests that Dominant Choice with Incomplete Preferences is not excessively permissive in this environment, and that its good fit is useful towards explaining behaviour and eliciting preferences.

5.2 Recovery of Strict Preferences, Indifference and Indecisiveness

For the vast majority of all subjects that are approximately matched by a model there is a unique weak/strict and complete/incomplete preference ordering generating their minimum JHM distance score under the respective optimal model, with the mean/median number of such orderings being 1.22/1 and 1.08/1 in the Forced- and Free-Choice treatments, respectively. Importantly, for 60 of 77 (78%) and 59 of 74 (80%) subjects in the Forced- and Free-Choice treatments, respectively, the recovered optimal preference relation(s) has –or, in the case of Undominated Choice with Incomplete Preferences, might be interpreted as having– a non-trivial indifference component. This highlights the potential usefulness of eliciting indifferences for revealed preference and welfare analysis.

The multi- and/or empty-valued experimental choice data and the non-parametric optimization goodness-of-fit method that were presented in the previous (sub)sections jointly allow us to test for the potential presence of both these theory-guided distinctions for the first time. Additionally, going beyond the boundaries of the above two theoretical studies which assumed that the analyst observes active choices/deferrals at all menus that can be derived from the grand choice set, this method allows us to carry out this test and recover the individual’s strict preferences and, where relevant, indifferences and incomparabilities from an incomplete collection of menus that comprises only 50 out of the 63 possible ones.

We find that a total of 16 subjects (5 in the Free-Choice treatment) out of the 23 who were best-matched by the Undominated-Choice model revealed “regular” incomplete prefer-
Figure 9: A subject’s complete weak preferences, optimally recovered by the model of Rational Choice.

\[
\begin{align*}
C &\sim D \\
B &\sim E \\
A &
\end{align*}
\]

Figure 10: A subject’s incomplete preferences, optimally recovered by the Undominated-Choice model, with or without indifferences (right and left, respectively).

\[
\begin{align*}
E &
\end{align*}
\]

\[
\begin{align*}
F &\quad A &\quad D &\quad B &\quad C
\end{align*}
\]

\[
\begin{align*}
E &
\end{align*}
\]

\[
\begin{align*}
D &\quad A \sim F &\quad B &\quad C
\end{align*}
\]

Figure 11: A subject’s incomplete weak preferences, optimally recovered by the Dominant-Choice model.

\[
\begin{align*}
B &\sim D \sim E \\
C &\quad A &\quad F
\end{align*}
\]
ences that afford the Eliaz-Ok distinction between indifference and indecisiveness. Figure 10 illustrates one such example, with the graph on the right depicting the revealed incomplete weak preference relation and the graph on the left the observationally equivalent revealed incomplete strict preference relation. We also find that 30 out of the total 37 Free-Choice subjects for whom the Dominant-Choice model provided the best fit (including the 5 subjects for whom this fit was perfect) revealed incomplete preferences with a non-degenerate indifference part. Figure 11 depicts such an example.

Although indifference is generically non-existent for a large class of the benchmark Bewley (2002) class of incomplete preference relations under uncertainty that are defined on a space of bundles of continuous commodities or uncertain acts (Gerasimou, 2018b), this is very much not the case when such preferences are over finitely many discrete options, as in the choice environment analysed in this paper. Indeed, the preceding analysis suggests that out of all 60 (22% of the total) subjects that the two models of incomplete-preference maximization explain optimally in our sample across the two treatments, 36 (63%) exhibited behaviour that allows for recovering a transitive strict preference relation together with a transitive indifference relation and a generally intransitive incomparability/indecisiveness relation.

5.3 Cross-Validation Test of Model-Fitting Analysis

We finally performed a cross-validation exercise to assess the out-of-sample robustness of the model-fit estimates that were reported earlier. More specifically, we split each subject’s data into the first and second half (recall that each subject had a randomly different menu-presentation order) and conducted the same goodness-of-fit analysis separately for each half. We then counted the subjects for whom the same model was found to be optimal in each of the first and second 25 decisions, as well as is all 50.

In the Forced-Choice treatment, 50 of the 77 subjects (65%) were best-matched by the same model –possibly in different modes– in both halves and overall. From the remaining 17 subjects, moreover, 14 were best-matched by Undominated Choice with Incomplete Preferences in the first half and by Rational Choice in the second. This suggests the possibility that some subjects may have learned their preferences and behaved increasingly more rationally during the experiment.

In the Free-Choice treatment on the other hand, where all three models have meaningful explanatory power and therefore more possibilities arise, things are less straightforward. More specifically, 15 of the 74 subjects in this treatment (20%) were best-matched by the
same model both in the first and second half and overall. The learning hypothesis that was suggested above is relevant here too, with 17 subjects (23%) that were best-matched by one of the two models of incomplete-preference maximization in the first half being optimally described by utility maximization in the second half. Another 8 subjects (11%) went in the opposite direction, while 7 (9%) were as if they alternated between the two models of choice with incomplete preferences across the two halves.

In summary, this cross-validation exercise shows that a sizeable proportion of subjects who are within 5 decisions (10%) away from being perfectly explainable by one of the three models are best-matched by the same model throughout, particularly in the Forced-Choice treatment, while for another considerable proportion it is possible that preference learning may have contributed to a better fit of Rational Choice in the second half.

5.4 Satisficing

Simon (1956) famously coined the term *satisficing* to describe resource-constrained agents who, instead of searching through all available alternatives in order to find the best, only search until they find one that meets an acceptability threshold. Two recent forced-choice studies that tested the satisficing hypothesis in economics are Reutskaja, Nagel, Camerer, and Rangel (2011) and Caplin, Dean, and Martin (2011). The former found weak evidence for such behaviour in choice from lists of food snacks under intense time pressure, whereas the latter found strong evidence in choice from lists of monetary amounts that were described verbally through a series of additions and subtractions, without (or with limited) time pressure. The fact that all 50 menus in both treatments of our experiment were presented vertically as unnumbered lists allows us to add to this literature by complementing the preceding model-based analysis with a test for satisficing in the present time-unconstrained decision environment of multi-valued choice over pairs of gift cards.

To this end, we first conduct a relatively narrow but potentially informative test of satisficing by focusing on the frequency with which subjects opted to choose only the first alternative that appeared in the menus they saw, and then asking whether this frequency can be viewed as being above and beyond what might be reasonably interpretable differently. Specifically, we find that 5 and 4 subjects (3.3%) in the Forced- and Free-Choice treatments, respectively, who were not classified as approximately explainable by one of the three deterministic models discussed earlier chose only the first option at frequencies that strictly exceeded the 97.5% cut-off values of 0.28 and 0.29 that are derived from the relevant simulations. These numbers rise to 27 and 20 (17%), respectively, if the non-randomness criterion is retained but the model-classification requirement is dropped.
In addition, we compute and study the average position of each subject’s chosen item(s) in the 50 menus’ list orderings because, intuitively, an unusually low average value of this metric could also be indicative of satisficing behaviour for the subject in question. We find that for an additional 7 and 5 (4.4%) subjects in the Forced- and Free-Choice treatments, respectively, who were not classified as approximately explainable by one of the three deterministic models that were discussed above, the average positions of their chosen item(s) in the 50 menus’ list orderings were strictly below the 2.5% percentile cut-off value of 1.84 that is derived from simulated uniform-random forced choices. These two distinct analyses therefore suggest that a total of 21 subjects (7.7%) who were not model-classified in Table 6 might be thought of as exhibiting a systematic satisficing behaviour.

Figure 12: Forced-Choice subjects are more likely to select items that are higher up in the menu list.
(a) Forced-Choice treatment
(b) Free-Choice treatment

Interestingly, a cross-treatment comparison of how the average positions of all chosen alternatives are distributed (see Figure 12) indicates that Forced-Choice subjects may be significantly more likely to choose items that appear higher up in the menu list ($p = 0.009$; two-sided Mann-Whitney $U$ test). This finding could be explained intuitively as follows: for individuals who are either insufficiently motivated to make 50 careful decisions over 2–4 pairs of gift cards in an experimental lab or find the task to be cognitively challenging, being unable to avoid/defer the decision at a small cost could make the use of a non-compensatory decision heuristic such as satisficing more likely.

5.5 Preference for Randomization

Agranov and Ortoleva (2017, 2023), Dwenger, Kübler, and Weizsäcker (2018), Cettolin and Riedl (2019) and Agranov, Healy, and Nielsen (2022), among others, have recently documented a preference for randomization in repeated choices from binary menus of money lotteries. Such a preference refers to subjects’ frequent tendency to change their choices

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9Ong and Qiu (2023) report such a preference in an ultimatum-bargaining setting where receivers are
from one occurrence of a menu to the next, and their willingness to even incur a small cost in order to have their choice determined randomly. In addition to the indifference-permitting goodness-of-fit analysis of the three models that was presented earlier, the aspect of our experimental design whereby subjects are rewarded with a random alternative at a menu if they chose all alternatives at that menu allows us to test for the existence of a similar preference for randomization in our data, even without any choice repetitions. Being another by-product of the experiment’s features, however, this additional test is limited in its scope because the experimental design was not constructed to detect all possible manifestations of preference for randomization in multi-valued choice environments.

As with our analysis of satisficing, to proceed with this investigation we regard a subject as potentially exhibiting such a preference if: (i) they are not approximately explainable by one of the three deterministic models of preference maximization; and (ii) the number of menus were they chose everything strictly exceeds the 97.5% simulations-based cut-off values of 14 and 11 menus in the Forced- and Free-Choice treatments, respectively. Both criteria are satisfied by 3 and 4 subjects (2.5%) in the two treatments, and these rise to 6 and 9 (5.5%) when the first criterion is dropped.\(^{10}\) In addition, none of these 7 subjects belongs to the “satisficing” category that was defined above. This finding adds to the existing and growing literature on preference for randomization by showing that such a preference could potentially manifest itself in binary as well as non-binary menus of riskless alternatives, even when the choice-deferral option is feasible and acts as another obvious way for an individual to deal with a difficult decision, and even in non-repeated-choice environments.

5.6 Menu Meta-Visualization and Menu Preferences for Flexibility

Subjects in our experiment were asked to make choices from menus. However, because the design specifically allowed them to choose multiple items at each menu, an analyst might be tempted to view the resulting choices as choices over menus instead. The argument here is that subjects who are presented with menu \(A\) and choose the elements of some \(C(A) \subseteq A\) might be thought of as choosing the menu \(C(A)\) from the “metavisualised” collection of feasible menus \(\{A' : A' \subseteq A\}\).\(^{11}\) Such a view is contrary to the experiment’s motivation, design and the way in which each choice problem was presented to subjects in the experimental interface. Furthermore, it effectively assumes that participants are endowed with sufficiently rich memory and other cognitive resources to be able to mentally translate

\(^{10}\) These estimates are conservative and may be better seen as lower bounds. The reason is that the experimental design only allows subjects who might have exhibited such a systematic preference for randomization to reveal it only when they were faced with difficulty deciding between all feasible alternatives at a given menu, not from a proper subset thereof.

\(^{11}\) In the free-choice treatment only, this collection also includes the empty subset of \(A\).
each of the 50 distinct menus $A$ into a collection of 4, 8 or 16 (when $|A| = 2, 3, 4$, respectively) weak sub-menus of $A$ and to form preferences over those. With these caveats in mind, and to make our analysis as comprehensive as possible, we now re-examine our data through the lens of this distinct point of view.

The leading theories of menu preferences in the literature portray decision makers as exhibiting either a preference for flexibility (Kreps, 1979; Dekel, Lipman, and Rustichini, 2001) or a preference for commitment (Gul and Pesendorfer, 2001, 2005). The former preference is manifested at a menu $A$ if $B$ is preferred to $A$ whenever $B \supset A$. By contrast, a preference for commitment in the Gul-Pesendorfer theory of temptation and self-control is formalized with the Set Betweeness axiom whereby “$A$ is preferred to $B$” implies “$A$ is preferred to $A \cup B$ is preferred to $B$”.

Under the postulated metavisualization and the associated cognitive-richness and mental-preference hypotheses, we proceed now to treating the subjects’ choices over alternatives as choices over menus. Because the choice alternatives and menus in our experiment do not seem to invite any temptation and self-control trade-offs, our aim here is to test for any systematic preferences for flexibility at the individual subject level. We acknowledge from the outset, however, that this analysis, while targeting the hallmark of the respective decision theories cited above, do not constitute a comprehensive test thereof. The reason is that these models rely on several additional axioms on preferences and assumptions on the structure of the preference domain which go beyond the riskless choice environment of our experiment.

Figure 13: Subjects’ cumulative densities of flexibility violations at the 50 menus under the menu meta-visualisation hypothesis.

However, our basic analysis at the aggregate level (see, in particular, Table 4), readily

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12Toussaert (2018) reports on an experimental design that was explicitly constructed to test for the presence of such preferences.
suggests that few -if any- subjects in our sample of eligible subjects are likely to exhibit a systematic preference for flexibility.\textsuperscript{13} This point is validated in Figure 13, which depicts the subjects’ cumulative densities of flexibility violations. More specifically, 5 and 12 are the smallest numbers of menus where this principle was violated, in each case by a single subject, while the means/medians in the two treatments stand at 45.8/48 (Forced-Choice) and 46.6/49 (Non-Forced Choice). Therefore, in line with the caveats expressed at the beginning of this subsection, looking at our experimental data through this lens does not seem to add any notable new insights into subjects’ behaviour. This provides some further reassurance toward the validity of the results presented earlier in this section.

5.7 Related Literature

The core of this paper’s experimental design extends in the multi-valued choice direction the design of Costa-Gomes, Cueva, Gerasimou, and Tejiščák (2022). That study also reported on forced- and non-forced choice treatments but only elicited single-valued choices. This made it impossible to raise and answer the novel questions that this paper is mainly concerned with. Although a pre-publication version of that paper from 2016 had proposed a method of multi-valued choice construction, that method is considerably more restrictive than the one advanced here. In particular, contrary to this paper’s unrestricted elicitation of subjects’ multi-valued choices, that method used survey data at binary menus in order to augment with (constructed) transitive indifference relations the subjects’ single-valued choices at binary as well as non-binary menus. In work that followed the above-mentioned study but which predates the present one, Bouacida (2018) proposed a different design that elicits multi-valued choices directly, by paying subjects a fixed extra amount for every additional alternative they choose from a menu of real-effort tasks, assigning them one of these tasks at random as a reward.

The design proposed herein differs from both these pre-existing ones and, in our view, materially so. Specifically, like Bouacida (2018) but unlike Costa-Gomes, Cueva, Gerasimou, and Tejiščák (2016), we allow subjects to choose multiple alternatives at every menu directly, thereby avoiding the imposition of a priori consistency constraints on the collected multi-valued choice data. Yet, like Costa-Gomes, Cueva, Gerasimou, and Tejiščák (2016) and unlike Bouacida (2018), we penalize inconsistent choices towards incentivizing subjects to reveal their true preferences. Furthermore, Costa-Gomes, Cueva, Gerasimou, and Tejiščák (2016) implemented their design on all menus derived from a set of 5 headsets and with a

\textsuperscript{13}We note that a total of 4 subjects in the original sample could indeed be thought of as showing such a consistent preference for flexibility throughout the experiment by always choosing all alternatives at every menu. Those subjects were excluded (see Table 2) from the subsequent analysis because their behaviour was completely uninformative about their preferences over gift-card pairs.
1-out-of-4 subject reward frequency\textsuperscript{14}, while Bouacida (2018) implemented his on all menus derived from a set of four real-effort tasks where all subjects were rewarded. In addition, there are significant differences in the choice alternatives, number of choice problems, subjects’ reward frequency, and most importantly, in the data-analytic methods, research questions and conclusions of these papers.

Turning to the pre-existing literature on the elicitation of incomplete preferences, this has mainly revolved around choice over binary menus of lotteries and/or uncertain acts, and is reviewed in detail in Costa-Gomes, Cueva, Gerasimou, and Tejiščák (2022). More summarily here: to identify incompleteness in such environments those studies have used choice-deferral alongside preference-for-flexibility models (Danan and Ziegelmeyer, 2006); partially incentivized methods of imprecise-preference revelation from menu lists (see Cubitt, Navarro-Martinez, and Starmer (2015) and references therein); and incentivized methods where incompleteness can potentially be revealed via certain patterns of randomized choice (Cettolin and Riedl, 2019; Agranov and Ortoleva, 2023). In general choice environments over durable goods on the other hand, \textit{strict} incomplete preferences were documented in the non-forced-choice treatment of Costa-Gomes, Cueva, Gerasimou, and Tejiščák (2022) via costly choice deferrals and restricted application of the model-based Houtman-Maks method to the dominant-choice model with \textit{strict} incomplete preferences. By allowing for multi-valued choices in both Forced- and Free-Choice treatments, and by proposing and building the analysis upon the novel Jaccard-similarity modified variation of the Houtman-Maks method for multi-valued choice data, the design proposed in the present paper significantly improves upon Costa-Gomes et al. (2022) by enabling the analyst to test the hypothesis of of incomplete-preference maximization in the most general choice environments possible. Applied in these data, and based on the distinct choice-theoretic separations of these relations that were proposed in Eliaz and Ok (2006) and Gerasimou (2018a), the method allowed to document empirically –and for the first time– the uncoupling of revealed indifference and incomparability/incompleteness that is predicted by these models.

6 Concluding Remarks

This paper proposes and implements theory-guided methods of data collection and analysis that aim to contribute towards eliciting a decision maker’s possibly weak or incomplete choices. A more complete list of the main differences between this experiment and those of that study is the following: (i) It allows for potentially multi-valued choices vs requires necessarily single-valued ones; (ii) Features 6 vs 5 choice alternatives (gift-card bundles vs headphones); (iii) Presents 50 decisions vs 26; (iv) Includes 273 subjects in the sample vs 161 & 121; (v) Gives choice rewards to every subject vs 1 out of 4; (vi) No provision of any information about the options in the randomly selected menu before the 2nd decision vs provision of such information in the 1st experiment of the earlier study.
preferences and, where relevant, towards distinguishing between their indifference and indecisiveness parts. On the data-collection side, the paper contributes an incentivized experimental design with multi-valued forced- and free-/non-forced choice treatments. On the data-analytic side, it deploys a model-rich combinatorial-optimization method that allows for recovering an individual’s possibly weak or incomplete preferences from their multi-valued choices in a model-optimal way. This method is a novel extension of the celebrated Houtman and Maks (1985) approach that is fine-tuned to the generally multi-valued nature of the choice data by accounting for the (Jaccard) similarity between the observed and model-optimal choices, and penalizing a subject’s proximity score relative to a model accordingly. We showed how this method is fruitfully applicable on the general model of rational choice with potential indifferences, as well as to two models of incomplete-preference maximization that allow for –and distinguish between– indifference and/or incompleteness. The method is obviously general, however, and in the future can be applied on other models of bounded rationality that predict multi-valued choices.

Despite the relatively large number of decisions, the behaviour of 56% of all subjects in our sample is either perfectly or approximately matched by some simple but richly structured deterministic model of complete or incomplete preference maximization, with uniquely recovered and indifference-exhibiting preferences in the vast majority of cases. Rational choice accounts for the behaviour of over 60% of all subjects in this group (34% of the total), while the two models of incomplete-preference maximization together account for the remaining subjects. Importantly, the optimal preference relation that is recovered conditional on a subject’s best-matching model typically features a non-trivial indifference relation, and therefore highlights the importance of accounting for indifferences in revealed-preference analyses. Finally, exploiting two relevant aspects of the experimental design allows for inferring that a further 10% of all analysed subjects could be thought of as engaging in systematic satisficing behaviour or displaying a robust preference for randomization. This increases to 66% the proportion of individuals whose behaviour can be categorized as following a specific pattern.

Our analysis points to the usefulness of multi-valued and indifference-/incomparability-permitting choice experiments. It also highlights the importance of methodologically pluralistic methods that allow for recovering preferences and/or choice rules by comparing observed behavioural with the predictions made by utility maximization as well as by models of bounded-rational choice or decision heuristics.
References

Agranov, M., P. J. Healy, and K. Nielsen (2022): “Stable Randomization,” Economic Journal, forthcoming.

Agranov, M. and P. Ortoleva (2017): “Stochastic Choice and Preferences for Randomization,” Journal of Political Economy, 125, 40–68.

——— (2023): “Ranges of Preferences and Randomization,” Review of Economics and Statistics, forthcoming.

Alós-Ferrer, C., E. Fehr, and N. Netzer (2021): “Time Will Tell: Recovering Preferences When Choices Are Noisy,” Journal of Political Economy, 129, 1828–1877.

Apesteguia, J. and M. Ballester (2015): “A Measure of Rationality and Welfare,” Journal of Political Economy, 1278–1310.

Arvanitis, S., O. Scaillet, and N. Topaloglou (2023): “Spanning Analysis of Stock Market Anomalies Under Prospect Stochastic Dominance,” Management Science, 70, 6002–6025.

Azrieli, Y., C. P. Chambers, and P. J. Healy (2018): “Incentives in Experiments: A Theoretical Analysis,” Journal of Political Economy, 126, 1472–1503.

Beatty, T. K. M. and I. A. Crawford (2011): “How Demanding is the Revealed Preference Approach to Demand,” American Economic Review, 101, 2782–2795.

Bewley, T. F. (2002): “Knightian Decision Theory. Part I,” Decisions in Economics and Finance, 25, 79–110.

Bossert, W., Y. Sprumont, and K. Suzumura (2005): “Maximal-Element Rationalizability,” Theory and Decision, 58, 325–350.

Bossert, W. and K. Suzumura (2010): Consistency, Choice and Rationality, Cambridge: Harvard University Press.

Bouacida, E. (2018): “Identifying Choice Correspondences,” Working Paper.

Bouacida, E. and D. Martin (2021): “Predictive Power in Behavioral Welfare Economics,” Journal of the European Economic Association, 19, 1556–1591.

Bronars, S. G. (1987): “The Power of Non-parametric Tests of Preference Maximization,” Econometrica, 55, 693–698.

Caplin, A., M. Dean, and D. Martin (2011): “Search and Satisficing,” American Economic Review, 101, 2899–2922.
Cettolin, E. and A. Riedl (2019): “Revealed Preferences Under Uncertainty: Incomplete Preferences and Preferences for Randomization,” *Journal of Economic Theory*, 181, 547–585.

Cherchye, L., T. Demuynck, J. Lanier, and B. D. Rock (2023): “Are Consumers (Approximately) Rational? Shifting the Burden of Proof,” *Review of Economics and Statistics*, forthcoming.

Choi, S., R. Fisman, D. Gale, and S. Kariv (2007): “Consistency and Heterogeneity of Individual Behavior under Uncertainty,” *American Economic Review*, 97, 1921–1938.

Choi, S., S. Kariv, W. Müller, and D. Silverman (2014): “Who is (More) Rational?” *American Economic Review*, 104, 1518–1550.

Costa-Gomes, M., C. Cueva, G. Gerasimou, and M. Tejiščák (2016): “Choice, Deferral and Consistency,” *University of St Andrews Working Paper 1416*, available at https://georgiosgerasimou.com/_static/paper-CDC-2016.12.26.pdf. Version under “revise & resubmit” for *Econometrica* between June 2017 & June 2019.

——— (2022): “Choice, Deferral and Consistency,” *Quantitative Economics*, 13, 1297–1318.

Cubitt, R., D. Navarro-Martinez, and C. Starmer (2015): “On Preference Imprecision,” *Journal of Risk and Uncertainty*, 50, 1–34.

Danan, E. (2010): “Randomization vs Selection: How to Choose in the Absence of Preference?” *Management Science*, 56, 503–518.

Danan, E. and A. Ziegelmeyer (2006): “Are Preferences Complete? An Experimental Measurement of Indecisiveness Under Risk,” *Working Paper*.

Dean, M. (2008): “Status Quo Bias in Large and Small Choice Sets,” *Working Paper*.

Dean, M. and D. Martin (2016): “Measuring Rationality with the Minimum Cost of Revealed Preference Violations,” *Review of Economics and Statistics*, 98, 524–534.

Dekel, E., B. L. Lipman, and A. Rustichini (2001): “Representing Preferences with a Unique Subjective State Space,” *Econometrica*, 69, 891–934.

Dembo, A., S. Kariv, M. Polisson, and J. K.-H. Quah (2021): “Ever Since Allais,” *Working Paper*.

Dwenger, N., D. Kübler, and G. Weizsäcker (2018): “Flipping a Coin: Evidence from University Applications,” *Journal of Public Economics*, 167, 240–250.

Echenique, F., T. Imai, and K. Saito (2023): “Approximate Expected Utility Rationalization,” *Journal of the European Economic Association*, 21, 1821–1864.
Echenique, F., S. Lee, and M. Shum (2011): “The Money Pump as a Measure of Revealed Preference Violations,” *Journal of Political Economy*, 119, 1201–1223.

Eliaz, K. and E. A. Ok (2006): “Indifference or Indecisiveness? Choice-theoretic Foundations of Incomplete Preferences,” *Games and Economic Behavior*, 56, 61–86.

Fudenberg, D., A. Liang, and W. Gao (2023): “How Flexible is that Functional Form? Quantifying the Restrictiveness of Theories,” *Review of Economics and Statistics*, forthcoming.

Gent, I. P., C. Jefferson, and I. Miguel (2006): “Minion: A Fast, Scalable, Constraint Solver,” in *Proceedings of the 17th European Conference on Artificial Intelligence (ECAI 2006)*, available at https://constraintmodelling.org/minion/download/.

Gerasimou, G. (2018a): “Indecisiveness, Undesirability and Overload Revealed Through Rational Choice Deferral,” *Economic Journal*, 128, 2450–2479.

——— (2018b): “On the Indifference Relation in Bewley Preferences,” *Economics Letters*, 164, 24–26.

——— (2024): “Characterization of the Jaccard Dissimilarity Metric and a Generalization,” *Discrete Applied Mathematics*, 355, 57–61.

Gerasimou, G. and M. Tejiščák (2018): “Prest: Open-Source Software for Computational Revealed Preference Analysis,” *Journal of Open Source Software*, 3, https://doi.org/10.21105/joss.01015, available at https://prestsoftware.com.

Greiner, B. (2015): “Subject Pool Recruitment Procedures: Organizing Experiments with ORSEE,” *Journal of the Economic Science Association*, 1, 114–125.

Gul, F. and W. Pesendorfer (2001): “Temptation and Self-Control,” *Econometrica*, 69, 1403–1435.

——— (2005): “The Simple Theory of Temptation and Self-Control,” *Working Paper*.

Halevy, Y., D. Persitz, and L. Zrill (2018): “Parametric Recoverability of Preferences,” *Journal of Political Economy*, 126, 1558–1593.

Hitt, M. P. (2019): *Inconsistency and Indecision in the United States Supreme Court*, Ann Arbor: Michigan University Press.

Houtman, M. and J. A. Maks (1985): “Determining All Maximal Data Subsets Consistent with Revealed Preference,” *Kwantitatieve Methoden*, 19, 89–104.

Jaccard, P. (1901): “Étude Comparative de la Distribution Florale dans une Portion des Alpes et des Jura,” *Bulletin de la Société Vaudoise des Sciences Naturelles*, 37, 547–579.
Kreps, D. M. (1979): “A Representation Theorem for ‘Preference for Flexibility’,” *Econometrica*, 47, 565–577.

——— (2012): *Microeconomic Foundations I: Choice and Competitive Markets*, Princeton: Princeton University Press.

Levandowsky, M. and D. Winter (1971): “Distance Between Sets,” *Nature*, 234, 34–35.

Levandowsky, M. and D. Winter (1971): “Distance Between Sets,” *Nature*, 234, 34–35.

Levy, H. (2016): *Stochastic Dominance: Investment Decision Making under Uncertainty*, Heidelberg: Springer, 3rd ed.

Linton, O., T. Post, and Y. Whang (2014): “Testing for the stochastic dominance efficiency of a given portfolio,” *The Econometrics Journal*, 17, S59–S74.

Mandler, M. (2009): “Indifference and Incompleteness Distinguished by Rational Trade,” *Games and Economic Behavior*, 67, 300–314.

Marczewski, E. and H. Steinhaus (1958): “On a Certain Distance of Sets and the Corresponding Distance of Functions,” *Colloquium Mathematicum*, 6, 319–327.

Ong, Q. and J. Qiu (2023): “Paying for Randomization and Indecisiveness,” *Journal of Risk and Uncertainty*, 67, 45–72.

Online Encyclopedia of Integer Sequences (2021): “Entries A000670, A001035 and A000798,” available at [https://oeis.org](https://oeis.org).

R Core Team (2021): *R: A Language and Environment for Statistical Computing*, R Foundation for Statistical Computing, Vienna, Austria.

Ratcliff, R. and G. McKoon (2008): “The Diffusion Decision Model: Theory and Data for Two-Choice Decision Tasks,” *Neural Computation*, 20, 873–922.

Reutskaja, E., R. Nagel, C. F. Camerer, and A. Rangel (2011): “Search Dynamics in Consumer Choice under Time Pressure: An Eye-Tracking Study,” *American Economic Review*, 101, 900–926.

Richter, M. K. (1966): “Revealed Preference Theory,” *Econometrica*, 34, 635–645.

RStudio Team (2020): *RStudio: Integrated Development Environment for R*, RStudio, PBC., Boston, MA.

Schmeidler, D. (1969): “Competitive Equilibria in Markets with a Continuum of Traders and Incomplete Preferences,” *Econometrica*, 37, 578–585.
SCHWARTZ, T. (1976): “Choice Functions, “Rationality” Conditions, and Variations of the Weak Axiom of Revealed Preference,” *Journal of Economic Theory*, 13, 414–427.

SELten, R. (1991): “Properties of a Measure of Predictive Success,” *Mathematical Social Sciences*, 21, 153–167.

SEN, A. (1971): “Choice Functions and Revealed Preference,” *Review of Economic Studies*, 38, 307–317.

——— (1997): “Maximization and the Act of Choice,” *Econometrica*, 65, 745–779.

SIMON, H. A. (1956): “Rational Choice and the Structure of the Environment,” *Psychological Review*, 63, 129–138.

STOYE, J. (2015): “Choice Theory when Agents Can Randomize,” *Journal of Economic Theory*, 155, 131–151.

TOUSSAERT, S. (2018): “Eliciting Temptation and Self-Control Through Menu Choices: A Lab Experiment,” *Econometrica*, 86, 859–889.

TVERSKY, A. and E. SHAfIR (1992): “Choice under Conflict: The Dynamics of Deferred Decision,” *Psychological Science*, 3, 358–361.
Appendices to
“When Towards Eliciting Weak or Incomplete Preferences in the Lab: A Model-Rich Approach”

A  Experiment Details

A.1  Instructions: Non-Forced-Choice Treatment (verbatim)

Welcome and thank you for your participation!

At the beginning of the experiment you will be automatically allocated £2.40.

During the main phase you will be presented with 50 menus of pairs of gift cards, one menu at a time. (Note: all gift cards come from well-known brands and each one is worth £10.)

For every gift card there is at least one branch in St Andrews town centre (on Market St. or South St.) where that gift card can be used instead of cash for making purchases.

At each of these 50 menus, you will be asked to choose one or more of the available pairs of gift cards, or to choose “I’m not choosing now”.

Once you have left a menu during that phase you will not be able to go back to it and change your decision.

At the end of the experiment one of the 50 menus will be randomly selected for you. Each menu is equally likely to be selected.

You will win a pair of gift cards (worth a total of £20) from your randomly selected menu.

First, you will be reminded which pair(s) of gift cards you chose at that menu, or if you chose “I’m not choosing now”.

Following that, your pair of gift cards and cash rewards will be determined as follows:

1. If you had previously chosen one or more—but not all—pairs from your randomly selected menu, then you will be asked to choose a single pair at this time:
   - If you now choose a pair among those previously chosen, you will win that pair and also receive the £2.40 that you were initially allocated.
   - If you now choose a pair not among those previously chosen, you will win that pair and also receive £1.20 from the amount that you were initially allocated.

2. If you had previously chosen all pairs from your randomly selected menu, then you will not be able to choose a pair now and the experimenter will choose one for you at random. You will win that pair and also receive £2.40 that you were initially allocated.

3. If you had previously chosen “I’m not choosing now” at your randomly selected menu, then you will be asked to choose a single pair now. You will win that pair and also receive £2.10 from the amount that you were initially allocated.

(Note: the decisions you made at all menus other than your randomly selected one will not affect your gift card and cash rewards.)

After at least fifty minutes have passed since the experiment’s start, the experimenter will start giving the gift card and cash rewards to participants who have finished.

No participant will be given their rewards before such time, regardless of how early they finish.
A.2 Instructions: Forced-Choice Treatment (verbatim)

Welcome and thank you for your participation!

At the beginning of the experiment you will be automatically allocated £2.40.

During the main phase you will be presented with 50 menus of pairs of gift cards, one menu at a time. (Note: all gift cards come from well-known brands and each one is worth £10.)

For every gift card there is at least one branch in St Andrews town centre (on Market St. or South St.) where that gift card can be used instead of cash for making purchases.

At each of these 50 menus, you will be asked to choose one or more of the available pairs of gift cards.

Once you have left a menu during that phase you will not be able to go back to it and change your decision.

At the end of the experiment one of the 50 menus will be randomly selected for you. Each menu is equally likely to be selected.

You will win a pair of gift cards (worth a total of £20) from your randomly selected menu.

First, you will be reminded which pair(s) of gift cards you chose at that menu.

Following that, your pair of gift cards and cash rewards will be determined as follows:

1. If you had previously chosen one or more –but not all– pairs from your randomly selected menu, then you will be asked to choose a single pair at this time:
   - If you now choose a pair among those previously chosen, you will win that pair and also receive the £2.40 that you were initially allocated.
   - If you now choose a pair not among those previously chosen, you will win that pair and also receive £1.20 from the amount that you were initially allocated.

2. If you had previously chosen all pairs from your randomly selected menu, then you will not be able to choose a pair now and the experimenter will choose one for you at random. You will win that pair and also receive the £2.40 that you were initially allocated.

(Note: the decisions you made at all menus other than your randomly selected one will not affect your gift card and cash rewards.)

After at least fifty minutes have passed since the experiment’s start, the experimenter will start giving the gift card and cash rewards to participants who have finished.

No participant will be given their rewards before such time, regardless of how early they finish.
A.3 The six pairs of £10 gift cards used in the experiment

A.4 Example decision problem shown in the main part*

Please choose one or more of the gift card pairs that are shown in this menu, or select “I’m not choosing now”:

☐ Waterstones Gift Card

☐ TESCO Giftcard

☐ I’m not choosing now

* This example is taken from the Non-Forced-Choice treatment. Presentation in the Forced-Choice treatment is identical except that “I’m not choosing now” is unavailable and the instruction reads “Please choose one of more gift cards from this menu”. 
Example randomly selected menu shown at the end

Your randomly selected menu is shown below:

You chose

at that menu.

Please wait for the experimenter to come to your desk.

Your rewards will be determined as described in your instructions.
Table 7: Classification of subjects who are perfectly/approximately explainable by some model under the multi-valued choice extensions of HM that follow (3) with $\succeq$ and $\mathcal{R}/\mathcal{Q}$ replacing $\succ$ and $\mathcal{P}$.

|                  | Utility Maximization | Undominated Choice with Incomplete Preferences | Dominant Choice with Incomplete Preferences | All   |
|------------------|----------------------|-----------------------------------------------|--------------------------------------------|-------|
| Forced-Choice treatment ($N = 138$) |                      |                                               |                                            |       |
| % of subjects with score = 0 | 3.62%                | 0%                                            | 0%                                         | 3.62% |
| % of subjects with score $\leq$ 5 | 33.33%               | 2.17%                                         | 0%                                         | 35.50%|
| % of subjects with score $\leq$ 10 | 46.38%               | 8.70%                                         | 0%                                         | 55.07%|
| Mean/median best score ($\leq$ 10) | 4.06/4               | 7.25/8                                        | –                                          | 4.56/4|
| Minimum score in simulations | 25                   | 23                                            | –                                          | 23    |
| Mean/median best-model preference orderings ($\leq$ 10) | 1.17/1               | 1.00/1                                        | –                                          | 1.15/1|
| Non-Forced-Choice treatment ($N = 135$) |                      |                                               |                                            |       |
| % of subjects with score = 0 | 2.22%                | 0%                                            | 5.19%                                      | 7.41% |
| % of subjects with score $\leq$ 5 | 18.51%               | 2.96%                                         | 21.48%                                     | 42.96%|
| % of subjects with score $\leq$ 10 | 24.44%               | 5.19%                                         | 28.15%                                     | 57.78%|
| Mean/median best score ($\leq$ 10) | 4.15/3               | 5.57/5                                        | 3.39/3                                     | 3.91/3|
| Minimum score in simulations | 27                   | 26                                            | 18                                         | 18    |
| Mean/median best-model preference orderings ($\leq$ 10) | 1.00/1               | 1.29/1                                        | 1.08/1                                     | 1.06/1|

Notes: Model-score ties were always broken in favor of Rational Choice/Utility Maximization (no other ties emerged). When Rational Choice/Utility Maximization was not a subject’s optimal model, the difference between this and the optimal model’s extended HM score was on average 4.2 and 6.5 in the Forced- and Non-Forced-Choice treatments, respectively.
C Motivation for Strongly Symmetric Menu Collections

Recall that subjects were presented with the 50 menus that comprised all those with two (15), three (20) and four (15) alternatives. This excludes all singleton menus and all those with 5 or 6 alternatives. Obtaining decision data from the full collection of menus may be desirable at some levels (e.g. because they provide additional information) but undesirable at others (e.g. because choice fatigue could negatively affect decision quality) or even impractical (for example, when the grand choice set comprises 6, 7 or 8 alternatives the full collection includes 63, 127 or 255 menus). In situations such as this the researcher may be inclined to limit the menus seen by subjects to some manageable number. How should this be done?

We argue that the collection of menus presented to subjects should satisfy strong symmetry in the sense that the distribution of menu sizes where an alternative is feasible is the same for every alternative. This requirement in turn implies the weak symmetry condition whereby all alternatives are feasible at some menu the same number of times. For example, a collection that comprises menus $P = \{x, y, z\}$, $Q = \{w, z\}$ and $R = \{x, y, w\}$ satisfies weak but not strong symmetry because, although each alternative appears twice in the dataset, $x$ and $y$ appear only in ternary menus while $w$ and $z$ do so in one binary and one ternary menu instead. By contrast, a collection consisting of $P = \{x, y, z\}$, $R = \{x, y, w\}$, $S = \{w, y, z\}$ and $T = \{w, x, z\}$ satisfies strong symmetry, with each alternative appearing three times in as many menus of the same size.

The motivation for the strong symmetry requirement, which is indeed satisfied in our experimental dataset, is intuitive: if the menu-size distributions differ across alternatives, this means that at least one of them is feasible in at least one larger/smaller menu compared to at least one other alternative. This could then pave the way for a potential bias in favour of or against that option in the ensuing analysis. In the first menu collection above, for example, suppose $x$ is (uniquely) chosen at $P$, $z$ at $Q$ and $w$ at $R$. Clearly, this is a revealed-preference cycle. Moreover, removing any of the three observations from this dataset breaks that cycle. Which one should be removed? One might be tempted to keep the choice at $Q$ as the potentially more accurate of the three because it is derived from a binary menu. But doing so and removing $P$, for example, would amount to giving a possibly unfair (dis)advantage to alternative $z$ ($x$). Indeed, $z$ ($x$) would then appear first (last) in the inferred revealed preference ordering even though $x$ ($z$) would have been second (third) if $Q$ had been removed instead. Yet even though it could be that $x$ would continue to be chosen over $z$ also at the binary menu $\{x, z\}$, the (symmetry-breaking) fact that this observation is unavailable works against that alternative.
D Choice Probabilities in Simulations

The probability of an alternative being chosen at a menu under multi-valued choice simulations is interpreted here as the probability that this alternative belongs to the chosen submenu of that menu. Assuming Forced Choice first, and considering an arbitrary menu $A$ with $k$ alternatives, every non-empty submenu $A' \subseteq A$ is equally likely to be chosen, and is therefore chosen with probability $\frac{1}{2^{k-1}}$. Since each of the $k$ feasible alternatives belongs to exactly $\frac{2^k}{2}$ of these submenus, it follows that each of them is chosen in the above sense with probability $\frac{2^k}{k(k-1)}$. Under Non-Forced Choice now, since some non-empty submenu of $A$ is chosen with probability $\frac{k}{k+1}$ because choice is deferred with probability $\frac{1}{k+1}$, the corresponding probability for each of the $k$ active-choice alternatives is adjusted accordingly. This results to the probabilities described in Table 8 and Figure 15.

Table 8: Choice/deferral probabilities at a menu with $k$ alternatives under multi-valued choice simulations.

|                       | Probability of each non-empty submenu being chosen | Probability of each active-choice alternative being chosen | Probability of deferral |
|-----------------------|-----------------------------------------------------|----------------------------------------------------------|-------------------------|
| Forced-Choice simulations | $\frac{1}{2^{k-1}}$                              | $\frac{1}{2} \frac{2^k}{k}$                             | NA                      |
| Non-Forced-Choice simulations | $\frac{1}{2^{k-1} k}$                          | $\frac{1}{2} \frac{2^k}{k} \frac{1}{k+1}$              | $\frac{1}{k+1}$        |

Figure 15: Choice/deferral probabilities under multi-valued choice simulations at various menu sizes.
(a) Every feasible alternative is chosen with $\approx 0.5$ probability as menu size increases, while deferral becomes less likely.

(b) The probability of choosing any non-empty submenu decreases as menu size increases.