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\( \phi \) meson mass and decay width in nuclear matter and nuclei

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The mass and decay width of the $\phi$ meson in cold nuclear matter are computed in an effective Lagrangian approach. The medium dependence of these properties are obtained by evaluating kaon–antikaon loop contributions to the $\phi$ self-energy, employing the medium-modified kaon masses, calculated using the quark-meson coupling model. The loop integral is regularized with a dipole form factor, and the sensitivity of the results to the choice of cutoff mass in the form factor is investigated. At normal nuclear matter density we find a downward shift of the $\phi$ mass by a few percent, while the decay width is enhanced by an order of magnitude. For a large variation of the cutoff parameter, the results for the $\phi$ mass and the decay width turn out to vary very little. Our results support results in the literature which suggest that one should observe a small downward mass shift and a large broadening of the decay width. In order to explore the possibility of studying the binding and absorption of $\phi$ mesons in nuclear, we also present the single-particle binding energies and half-widths of $\phi$-nucleus bound states for some selected nuclei.

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evidence for a substantial mass shift. For example, the KEK-E325 collaboration [32] reported a mass reduction of 3.4% and an in-medium decay width of \( \approx 14.5 \text{ MeV} \) at normal nuclear matter density. The latter disagrees with the SPring8 [18] result, which reported a large in-medium \( \phi N \) cross section leading to a decay width of 35 MeV. But this 35 MeV is in close agreement with the two JLab CLAS collaboration measurements reported in Refs. [33] and [34].

In an attempt to clarify the situation, the CLAS collaboration at JLab [35] performed new measurements of nuclear transparency ratios, and estimated in-medium widths in the range of 23–100 MeV. These values overlap with that of the SPring8 measurement [18]. More recently, the ANKE-COSY collaboration [36] has measured the \( \phi \) meson production from proton-induced reactions on various nuclear targets. The comparison of data with model calculations suggests an in-medium \( \phi \) width of \( \approx 50 \text{ MeV} \). This result is consistent with that of SPring8 [18], as well as the one deduced from CLAS at JLab [35]. However, the value is clearly larger than that of the KEK-E325 collaboration [32].

From the discussions above, it is obvious that the search for evidence of a light vector meson mass shift is indeed complicated. It certainly requires further experimental efforts to understand better the changes of \( \phi \) properties in a nuclear medium. For example, the J-PARC E16 collaboration [37] intends to perform a more systematic study for the mass shift of vector mesons with higher statistics. Furthermore, the E29 collaboration at J-PARC has recently put forward a proposal [38, 39] to study the in-medium mass modification of \( \phi \) via the possible formation of the \( \phi \)-nuclear bound states [26], using the primary reaction \( p \rightarrow \phi \phi \). Finally, there is a proposal at JLab, following the 12 GeV upgrade, to study the binding of \( \phi \) (and \( \eta \)) to \( ^4\text{He} \) [40].

On the theoretical side, various authors predict a downward shift of the in-medium \( \phi \) meson mass and a broadening of the decay width. The possible decrease of the light vector meson masses in a nuclear medium was first predicted by Brown and Rho [41]. Thereafter, many theoretical investigations have been conducted, some of them focused on the self-energies of the \( \phi \) due to the kaon–anti-kaon loop. Ko et al. [42] used a density-dependent kaon mass determined from chiral perturbation theory and found that at normal nuclear matter density, \( \rho_0 \), the \( \phi \) mass decreases very little, by at most 2%, and the width \( \Gamma_\phi \approx 25 \text{ MeV} \) and broadens drastically for large densities. Hatsuda and Lee calculated the in-medium \( \phi \) mass based on QCD sum rule approach [43, 44], and predicted a decrease of 1.5%–3% at normal nuclear matter density. Other investigations also predict a large broadening of the \( \phi \) width: Ref. [45] reports a negative mass shift of \( <1 \% \) and a decay width of 45 MeV at \( \rho_0 \); Ref. [46] predicts a decay width of 22 MeV but does not report a result on the mass shift; and Ref. [47] gives a rather small negative mass shift of \( \approx 0.81\% \) and a decay width of 30 MeV. More recently, Ref. [48] reported a downward mass shift of \( \approx 2\% \) and a large broadening width of 45 MeV; and finally, in Ref. [49], extending the work of Refs. [46, 47], the authors reported a negative mass shift of 3.4% and a large decay width of 70 MeV at \( \rho_0 \). The reason for these differences may lie in the different approaches used to estimate the kaon–anti-kaon loop contributions for the \( \phi \) self-energy.

In the present article we report results for the \( \phi \) mass shift and decay width in nuclear matter, taking into account the medium dependence of the \( K \) and \( \bar{K} \) masses. The latter are included by an explicit calculation based upon the quark-meson coupling (QMC) model [50, 51]. The QMC model is a quark-based model of finite nuclei and nuclear matter, and has been very successful in describing the nuclear matter saturation properties, hadron properties in nuclear medium, as well as the properties of finite nuclei [52] and hypernuclei [53]—for a comprehensive review of the QMC model, see Ref. [54].

The paper is organized as follows. In Sec. 2 we present the effective Lagrangian used to calculate the \( \phi \)-meson self-energy in vacuum, and give explicit expressions for its real and imaginary parts. Since the in-medium properties of the \( \phi \) are dependent on the kaon and anti-kaon masses in a nuclear medium calculated within the QMC model, we briefly review this model in Sec. 3, and provide the necessary detail to understand the dressing of the kaons in nuclear medium. In Sec. 4 we calculate the \( \phi \)-meson self-energy in nuclear matter and report the in-medium \( \phi \)-meson mass and decay width, as well as the binding energies and widths of selected \( \phi \)-nuclear bound states. Finally, conclusions and perspectives are given in Sec. 5.

2. \( \phi \) meson self-energy in vacuum

We use the effective Lagrangian of Refs. [42, 55] to compute the \( \phi \) self-energy: the interaction Lagrangian \( \mathcal{L}_{\text{int}} \) involves \( \phi K \bar{K} \) and \( \phi \bar{\phi} K \bar{K} \) couplings dictated by a local gauge symmetry principle:

\[
\mathcal{L}_{\text{int}} = \mathcal{L}_{\phi K \bar{K}} + \mathcal{L}_{\phi \bar{\phi} K \bar{K}}.
\]

(1)

where

\[
\mathcal{L}_{\phi K \bar{K}} = ig_\phi \phi \mu \left[ K^\dagger \left( \partial_\mu - M_\phi \right) K - \left( \partial_\mu K^\dagger K \right) \right],
\]

(2)

and

\[
\mathcal{L}_{\phi \bar{\phi} K \bar{K}} = g_\phi^2 \phi \mu \phi \left( K^\dagger K - \bar{K}^\dagger \bar{K} \right) K.
\]

(3)

We use the convention:

\[
K = \left( \begin{array}{c} K^+ \\ K^0 \end{array} \right), \quad \bar{K} = \left( \begin{array}{c} K^- \\ K^0 \end{array} \right).
\]

(4)

We note that the use of the effective interaction Lagrangian of Eq. (1) without the term given in Eq. (3) may be considered as being motivated by the hidden gauge approach in which there are no four-point vertices, such as Eq. (3), that involve two pseudoscalar mesons and two vector mesons [56, 57]. This is in contrast to the approach of using the minimal substitution to introduce vector mesons as gauge particles where such four-point vertices do appear. However, these two methods have been shown to be consistent if both the vector and axial vector mesons are included [58–61]. Therefore, we present results with and without such an interaction. We consider first the contribution from the \( \phi K \bar{K} \) coupling given by Eq. (2) to the scalar part of the \( \phi \) self-energy, \( \Pi_\phi(p) \); Fig. 1 depicts this contribution. For a \( \phi \) meson at rest, it is given by

\[
i \Pi_\phi(p) = -\frac{8}{3} g_\phi^2 \int \frac{d^4q}{(2\pi)^3} q^2 D_K(q) D_K(q-p),
\]

(5)

where

\[
D_K(q) = \left( q^2 - m_K^2 + i\epsilon \right)^{-1}
\]

is the kaon propagator; \( p = (p^0 = m_\phi, \vec{0}) \) is the \( \phi \) meson four-momentum vector, with \( m_\phi \) the \( \phi \) meson mass; \( m_K (= m_{\bar{K}}) \) is the kaon mass. When \( m_\phi < 2m_K \) the self-energy \( \Pi_\phi(p) \) is real. However, when \( m_\phi > 2m_K \), which is the case here, \( \Pi_\phi(p) \) acquires an imaginary part. The mass of the \( \phi \) is determined from the real part of \( \Pi_\phi(p) \).
$m_{\phi}^2 = \left( m_{\phi}^0 \right)^2 + \Re \Pi_{\phi}(m_{\phi}^2)$,\hspace{1cm} (6)

with $m_{\phi}^0$ being the bare mass of the $\phi$ and

$$\Re \Pi_{\phi} = - \frac{2}{3} g_{\phi}^2 \rho \int \frac{d^2q}{(2\pi)^3} \frac{1}{q^2} \frac{1}{E_K(E_K^2 - m_{\phi}^2/4)}.\hspace{1cm} (7)$$

Here $\rho$ denotes the Principal Value part of the integral Eq. (5) and $E_K = (q^2 + m_{\phi}^2)^{1/2}$. The decay width of $\phi$ to a $KK$ pair is given in terms of the imaginary part of $\Pi_{\phi}(\rho)$

$$\Im \Pi_{\phi} = - \frac{g_{\phi}^2}{24\pi} m_{\phi}^2 \left(1 - \frac{4m_{\phi}^2}{m_{\phi}^2} \right)^{3/2},\hspace{1cm} (8)$$

as

$$\Gamma_{\phi} = \frac{1}{m_{\phi}} \Im \Pi_{\phi} = \frac{g_{\phi}^2}{24\pi} m_{\phi} \left(1 - \frac{4m_{\phi}^2}{m_{\phi}^2} \right)^{3/2}.\hspace{1cm} (9)$$

The integral in Eq. (7) is divergent and needs regularization; we use a phenomenological form factor, with a cutoff parameter $\Lambda_K$, as in Ref. [62]. The coupling constant $g_{\phi}$ is determined by the experimental width of $\phi$ in vacuum [63]. For the $\phi$ mass, $m_{\phi}$, we use its experimental value: $m_{\phi}^{exp} = 1019.461$ MeV [63]. For the kaon mass $m_K$, there is a small ambiguity since $m_{K^*} \neq m_{K^0}$, as a result of charge symmetry breaking and electromagnetic interactions. The experimental values for the $K^*$ and $K^0$ meson masses in vacuum are $m_{K^*}^{exp} = 493.677$ MeV and $m_{K^0}^{exp} = 497.611$ MeV, respectively [63]. For definitiveness we use the average of $m_{K^*}^{exp}$ and $m_{K^0}^{exp}$ as the value of $m_K$ in vacuum. The effect of this tiny mass ambiguity in the in-medium kaon (antikaon) properties is negligible. Then, we get the coupling $g_{\phi} = 4.539$, and can fix the bare mass $m_{\phi}^0$.

3. The quark-meson coupling model and the in-medium kaon mass

Essential to our results for the in-medium $\phi$ mass, $m_{\phi}^*$, and decay width, $\Gamma_{\phi}^*$, at finite baryon density $\rho_B = \rho_p + \rho_n$ (sum of the proton and neutron densities), is the in-medium kaon mass, $m_K^*$, which is driven by the interactions of the kaon with the nuclear medium—we denote with an asterisk an in-medium quantity. The in-medium kaon mass is calculated in the QMC model. This model has been successfully applied to investigate the properties of infinite nuclear matter and finite nuclei. Here we briefly present the necessary details needed to understand our results. For a more in depth discussion of the model see Refs. [4,50,54] and references therein.

We consider nuclear matter in its rest frame, where all the scalar and vector mean field potentials, which are responsible for the nuclear many-body interactions, are constants in Hartree approximation. The Dirac equations for the quarks and antiquarks ($q = u$ or $d$, and $s$) in a hadron bag in nuclear matter at the position $\vec{r} = (t, \vec{r})$ (with $|\vec{r}| \leq R_h^*$, the in medium bag radius) are given by [4,54]:

$$[\not\! \partial - m_q^* \mp \gamma^0 V_+] (\psi_q) = 0, \hspace{1cm} (\psi_u) = 0, \hspace{1cm} (\psi_d) = 0, \hspace{1cm} (\psi_s) = 0,$$ \hspace{1cm} (10)

where $m_q^* = m_q - V_+^q$ and $V_+ = V_+^u = 1/2 V_+^u$. Here we neglect the Coulomb force, and assume SU(2) symmetry for the light quarks ($m_u = m_d = m_s$). The constant mean-field potentials in nuclear matter are defined by $V_+^q \equiv g_{\sigma}^q \sigma$, $V_0^q \equiv g_{\omega}^q \omega$, and $V_+^\pi \equiv g_{\pi}^q \delta b$, where $b$ is the time component of the $\rho$ mean field, with $g_{\sigma}^q$, $g_{\omega}^q$, and $g_{\pi}^q$, the corresponding quark-meson coupling constants. Note that $V_+^\pi \propto (\rho_p - \rho_n) = 0$ in symmetric nuclear matter, although this is not true in a nucleus where the Coulomb force may induce an asymmetry between the proton and neutron distributions even in a nucleus with the same number of protons and neutrons, resulting in $V_+^\pi \propto (\rho_p - \rho_n) \neq 0$ at a given position in a nucleus.

The normalized, static solution for the ground-state quarks or antiquarks with flavor $f$ in the hadron $h$ may be written as $\psi_f(x) = \psi_f(\tilde{r})/R^*_K \psi_f(\tilde{r})$, where $R^*_K$ and $\psi_f(\tilde{r})$ are the normalization factor and the corresponding spin and spatial part of the wave function, respectively. The in-medium bag radius $R^*_K$ of hadron $h$ is determined through the stability condition for the mass of the hadron against the variation of the bag radius [50,54]—see Eq. (15) below. The eigenenergies in units of $1/R^*_K$ are given by

$$\epsilon^*_q = \Omega^*_q \mp R^*_h V_+, \hspace{1cm} (11)$$

$$\epsilon^*_d = \Omega^*_q \mp R^*_h V_-, \hspace{1cm} (12)$$

$$\epsilon_s = \epsilon_3 = \epsilon_2. \hspace{1cm} (13)$$

Recall that $V_+^q = 0$, as explained earlier. The in-medium hadron mass, $m_h^*$, is calculated by

$$m_h^* = \sum_{j=q,\bar{q},s} n_j \Omega_j^* - \frac{2n_\omega}{3} + \frac{4\pi}{3} R^*_h^3 B, \hspace{1cm} (14)$$

where

$$\Omega_j^* = \Omega_j^* = [x_j^2 + (R^*_h m_j^*)^2]^{1/2} \quad \text{with} \quad \Omega_j^* = \Omega_j^* = [x_j^2 + (R^*_h m_j^*)^2]^{1/2}, \quad x_j, s$ being the lowest bag eigenfrequencies; and $n_q(n_\bar{q}), n_s$ are the quark (antiquark) numbers for the quark flavors $q$ and $s$, respectively. The MIT bag quantities, $z_h, B$, and $g_{\pi}$ are the parameters for the sum of the c.m. and gluon fluctuation effects, bag constant, lowest eigenvalues for the quarks $q$ or $s$ respectively, and the corresponding current quark masses. The parameters $z_h (z_h)$ and $B$ are fixed by fitting the nucleon (hadron) mass in free space.

For the current quark masses relevant for this study, we use ($m_{u,d}, m_s$) = (5.250) MeV, where these values were used in Refs. [4,54] and many studies made in the standard version of the QMC model. Since the effects of the current-quark mass values on the final results are very small, we use the same values as those used in the past, so that we can compare and discuss the results with those obtained previously. The bag radius of the nucleon in vacuum is taken to be $R_N = 0.8$ fm, and the parameter $z_N$, simulating the zero-point and c.m. energy, is obtained $z_N = 3.295$. For the kaon, the values in vacuum calculated here are $(R_K, z_K) = (0.574$ fm, 3.295). The bag constant calculated for the present study is $B = (170$ MeV)$^4$. The quark-meson coupling constants, which are determined so as to reproduce the saturation properties of symmetric nuclear matter—the binding energy per nucleon of 15.7 MeV at $\rho_0 = 0.15$ fm$^{-3}$—are $(g_{\sigma}^u, g_{\omega}^u, g_{\pi}^u) = (5.69, 2.72, 9.33)$. In addition, the incompressibility obtained is $K = 279.3$ MeV. The $\sigma$ coupling at the nucleon level, which is not trivial, is related by $g_{\sigma} = g_{\omega}^u = 3g_{\sigma}^u S_N(\sigma = 0) = 3 \times 5.69 \times 0.483 = 8.23$ [4,54], where

$$S_N(\sigma) = \int d^3 r \bar{\psi}_q(\tilde{r}) \gamma_\sigma \psi_q(\tilde{r}), \hspace{1cm} (16)$$
with the ground state light-quark wave functions evaluated self-consistently in-medium.

The resulting in-medium kaon (Lorentz-scalar) mass, calculated via Eqs. (14) and (15), is shown in Fig. 2, with the parameters fixed by the nuclear matter saturation properties. The kaon effective mass at normal nuclear matter density $\rho_0 = 0.15 \text{ fm}^{-3}$ decreases by about 13%. This is a little larger than the 10% decrease used in Ref. [42]. Note that, the isoscalar-vector $\omega$-mean-field potentials arise both for the kaon and antikaon. However, they have opposite signs and cancel each other (or they can be eliminated by a variable shift) in the calculation of the $\phi$ self-energy, and therefore we do not show here—see Ref. [4] for details.

4. $\phi$ meson in matter

The in-medium $\phi$ mass is calculated by solving Eq. (6) by replacing $m_K$ by $m_K^*$ and $m_\phi$ by $m_\phi^*$, and the width is obtained by using the solutions in Eq. (9). We regularize the associated loop integral with a dipole form factor using a cutoff mass parameter $\Lambda_K$. In principle, this parameter may be determined phenomenologically using, for example, a quark model—see Ref. [62] for more details. However, for simplicity we keep it free and vary its value over a wide interval, namely 1000–3000 MeV.

In Table 1, we present the values for $m_\phi^*$ and $\Gamma_\phi^*$ at normal nuclear matter density $\rho_0$. A negative kaon mass shift of 13% induces only $\approx 2\%$ downward mass shift of the $\phi$. On the other hand, $\Gamma_\phi^*$ is very sensitive to the change in the kaon mass; at $\rho_B = \rho_0$, the broadening of the $\phi$ becomes an order of magnitude larger than its vacuum value and it increases rapidly with increasing nuclear density, up to a factor of $\sim 20$ enhancement for the largest nuclear matter density treated, $\rho_B = 3\rho_0$. This can be seen in Fig. 3, where we plot $m_\phi^*$ and $\Gamma_\phi^*$ as a function of the ratio $\rho_B/\rho_0$. The effect of the in-medium kaon mass change gives a negative shift of the $\phi$ meson mass. However, even for the largest value of density treated in this study, the downward mass shift is only a few percent for all values of the cutoff parameter $\Lambda_K$. For $m_\phi^*$ at normal nuclear matter density, the average downward mass shift is 1.8% with a 0.7% standard deviation from the averaged value, while $\Gamma_\phi^*$ broadens in average by a factor of 10 with a 0.7% standard deviation from the average.

Next, we present predictions for single-particle energies and half widths for $\phi$-nucleus bound states for several selected nuclei. We solve the Schrödinger equation for a complex $\phi$-nucleus scalar potential determined by a local-density approximation using the $\phi$ mass shift and decay width in nuclear matter. This amounts to using the following for the complex $\phi$-nucleus ($A$) potential

$$V_{\phi A}(r) = \Delta m_\phi^* (\rho_B(r)) - \frac{(i/2)}{\Gamma_\phi^*(\rho_B(r))},$$  

where $\Delta m_\phi^*(\rho_B(r)) \equiv m_\phi^*(\rho_B(r)) - m_\phi$, $r$ is the distance from the center of the nucleus and $\rho_B(r)$ is the density profile of the given nucleus, which we calculate in the QMC model. Table 2 shows the results for the real and imaginary parts of the single-particle energies $E = E - (i/2)\Gamma^2$ in $^4$He, $^{12}$C and $^{208}$Pb. We present results with and without the imaginary (absorptive) part of the $\phi$-nucleus potential $V_{\phi A}(r)$. One sees that $\phi$ is not bound to $^4$He when the imaginary part of the potential is included. For larger nuclei, the $\phi$ does bind but while the binding is substantial the energy levels are quite broad; the half widths being roughly the same size as the central values of the real parts.

To conclude and for completeness, we show the impact of adding the $\phi\phi K\bar{K}$ interaction of Eq. (3) on the in-medium $\phi$ mass.

| $A$  | $\rho_B/\rho_0$ (GeV) | $\Gamma_{\phi A}$ (MeV) | $\rho_B/\rho_0$ (GeV) | $\Gamma_{\phi A}$ (MeV) |
|------|----------------------|-------------------------|----------------------|-------------------------|
| $^4$He | $0.15$  | 0.131 | $0.15$ | 0.131 |
| $^12$C | $0.15$  | 0.131 | $0.15$ | 0.131 |
| $^{208}$Pb | $0.15$ | 0.131 | $0.15$ | 0.131 |

Fig. 2. In-medium kaon mass $m_K^*$.

Fig. 3. In-medium $\phi$ mass (upper panel) and width (lower panel) for three values of the cutoff parameter $\Lambda_K$.
and width. Fig. 4 presents the results. We have used the notation that $\xi = 1(0)$ means that this interaction is (not) included in the calculation of the $\phi$ self-energy. One still gets a downward shift of the in-medium $\phi$ mass when $\xi = 1$, although the absolute value is slightly different from $\xi = 0$. The in-medium width is not very sensitive to this interaction.

5. Conclusions and perspectives

We have calculated the $\phi$ meson mass and width in nuclear matter within an effective Lagrangian approach up to three times of normal nuclear matter density. Essential to our results are the in-medium kaon masses, which are calculated in the quark-meson coupling (QMC) model, where the scalar and vector meson mean fields couple directly to the light $u$ and $d$ quarks (antiquarks) in the $K$ ($\bar{K}$) mesons.

At normal nuclear matter density, allowing for a very large variation of the cutoff parameter $\Lambda_K$, although we have found a sizable negative mass shift of 13% in the kaon mass, this induces only a few percent (1.8% on average) downward shift of the $\phi$ meson mass. On the other hand, it induces an order-of-magnitude broadening of the decay width.

Given the nuclear matter results, we have used a local density approximation to infer the position dependent attractive complex scalar potential, $V_S(p_\phi(r)) = \Delta m^2 \phi^2(r) - (i/2)\Gamma^2(r)\phi^2(r)$, in a finite nucleus. This allowed us to study the binding and absorption of a number of $\phi$-nuclear systems, given the nuclear density profiles, $\rho_N(r)$, also calculated using the QMC model. While the results found in this study show that one should expect the $\phi$ meson to be bound in all but the lightest nuclei, the broadening of these energy levels, which is comparable to the amount of binding, may introduce challenges in observing such states experimentally.

In the present study, we have focused on the $\phi$ self-energy in medium due to the medium modified kaon–antikaon loop. However, more study of gluonic color forces is needed on binding of the $\phi$-meson to a nucleus.

As a possible extension of this work, we note that the medium effects on the $\phi$ meson may lead to some enhancement of the strangeness content of the bound nucleon, with consequences, for example, for dark matter detection.

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