Analysis of learning obstacles on transformation geometry

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Abstract. This research aims to identify learning obstacles pre-service teacher on transformation geometry. This study is descriptive qualitative research; data were collected from interview sheets and test. There was four problems given to nine pre-service mathematics teachers. The results of the research showed that learning obstacles related to applying the concept, visualizing, principle, understanding of the problem and how to prove. Finally, the writer suggests that the learning process, especially on transformation geometry, needs activities that connect the representation to obtain visualization of the mathematical problems. The knowledge gained during the representation process is used as a tool to grow a mathematical connection, which can eventually be used in the process of constructing a proof.

1. Introduction

Geometry is an integral part of the learning of mathematics. However, the development of learning geometry at this time is less developed. One reason is the difficulty in forming a real construction student carefully and accurately, the notion that to paint geometry requires precision in the measurement and requires a long time, and not infrequently students experiencing obstacles in the process of proving. Meanwhile, the drawing plays an essential role in teaching geometry at school for drawing geometric connection between physical space and theory. If further investigation of the link between the objects with the abstract geometry student obstacles in learning geometry, it will be alleged that there is a problem in teaching geometry at school relates to the formation of abstract concepts. Learn abstract concepts cannot be done only with the transfer of information, but it takes a process of formation of concepts through a series of activities experienced directly by students. The series of abstract concept formation activity of these referred to the process of abstraction [1].

Studying mathematics meant to be also considered branches of mathematics is the science of geometry. Everything in this universe is geometry so that the branches of mathematics through geometry learn about the concepts embodied in the objects that exist in nature through geometric concepts. Thus, assessment of learning geometry must continue to be developed so that each learner can analyse the geometry of objects into a concept of geometry and can construct a geometry knowledge with formal proofs [2].

However, mathematical proofs in geometry material lately become an obstacle that they seem poorly developed. Difficulty in analysing geometric properties are realized in the form of theorems to create a concept widely experienced by the students [2,3].
Proof become a severe matter in determining the school curriculum in every different country. This is what makes reasoning and proof into one of the standard processes of National Council of Teachers of Mathematics (NCTM). It means that in every lesson a teacher must introduce reasoning and proof in each classroom [4]. In traditional learning, mathematical proof is only used as a means to eliminate the doubts of students on the concepts taught. However, the proof is not used as a means of increasing the higher mathematical ability. Just as revealed by Hana [5], that the function of proof and proof are: verification, explanatory, systematization, invention, communication, construction, exploration, and incorporation. Verification of proving and the proof is regarded as the most fundamental functions in the proof because both are products of the process of the development of mathematical thinking very mature. Verification refers to the truth of the statement while explanations provide insight into why this is true.

As for the role played proof in mathematics, namely: 1) to verify that a statement is true, 2) to explain why a statement can be said to be true, 3) to establish communication mathematics, 4) to find or create new math and 5) To make systematic statement in an axiomatic system [6].

According to Hanna [7], there are three main reasons why the ability to prove the need to be improved. First, the proof is significant to learn to explore mathematics. Second, the ability of students in the proof can improve their math skills more broadly, because the proof 'involved in all situations where the conclusion must be reached in the making of decisions to be made', and the third, the difficulties experienced by high school students and college students will affect their ability to perform mathematical proofs on a broader level again, so it is crucial for students to learn mathematical proof on the level of previous education.

2. Methods
This study was conducted to analyze student learning obstacles, especially regarding the difficulty students epistemology regarding materials, both presented in the form of materials or materials in lectures. This research method is descriptive qualitative research that aims to describe obstacles regarding epistemology student learning mathematical proofs related to the subject of transformation geometry. Subjects were nine pre-service mathematics teachers consist of 3 students with high prior knowledge mathematically (S1, S5, S8), three students with medium prior knowledge of the mathematically (S3, S4, S9) and three students with low prior knowledge of mathematically (S2, S6, S7). The beginning of knowledge is based on the acquisition of student achievement index in the previous semester. For students learning obstacles regarding epistemology student at transformations material using five indicators: concept, visualization, principles, understanding of the problem, and mathematical proofs.

3. Results and discussion
Learning obstacles of students to understand the concepts in terms of epistemology in working on the geometry transformation is divided into 5 types, it is in terms of indicators in assessing learning obstacles, namely: a) learning obstacles related to understanding and applying the concept; b) learning obstacles related to visualize the geometry object; c) learning obstacles related to determining principle; d) learning obstacles related to understanding of the problem and e) related to mathematical proofs.

The first learning obstacles are students cannot understanding and applying the concept by the command matter. For examples, on the following question: write the definition of a transformation in the field of V (V is Euclid field). Students do not understand the concept; students cannot mention the definition of transformation. It has happened to students with high, medium and low prior mathematical knowledge. Meanwhile, understanding of concepts is very important to be possessed by students, because there is a strong relationship with mathematical reasoning abilities [8]. S1 and S8 had trouble related the concept, to define the transformation, these students do not write domain/codomain of a function called transformation. S5 can write correctly and complete the definition of transformation. S3 cannot define the transformation correctly; these students mention that the transformation is a bijective function, but do not write domain/codomain of these functions. S4 in defining transformation by merely
mentioning that a transformation is injective functions only. S9 can write the definition of transformation wholly and correctly. Students with low initial knowledge are described as follows: S2 have no difficulty in writing the definition of transformation. S6 and S7 to write the definition of transformation are not complete. Both of these students do not write domain/codomain of the function is the transformation in the field of V. Also, S6 made an error in writing notation.

Second obstacles are related to visualizing the geometry object. The point is that students have obstacles regarding describing the line of the transformation result. Problem is as follows: Draw the line \( g' = M_h(g) \) if \( h = \{(x,y) \mid y = x + 1\} \) and \( g = \{(x, y) \mid y = -x\} \). These obstacles include the inability of students in drawing properly and appropriately. Drawing in geometry very important, because drawing is significantly correlated to problem solving performance in mathematics [9].

S1 is having a problem related to visualized right lines \( g \) and \( h \) so that images reflection created is false images. S5 can visualize by what is known and questioned, but the images are still made without a ruler. S8 can draw lines mirroring the results appropriately. S3 had difficulty in visualizing the line \( h \) so that the results of reflection illustration are wrong. S4 did not use a ruler, but the results are correct reflection depicted. S9 cannot describe all that is known. Obstacles students with low initial knowledge are described as follows: S2’s drawing of reflection results correctly, S6 and S7 cannot describe all that is known so that the reflection nothing, other than that, the two students were not drawing Cartesian coordinates as the first step in drawing a line reflection results.

Third obstacles related to the principles, this obstacle is the difficulty experienced by students in terms solve the problem by defining the principles to be used in solving the problem of transformation. For example, the following problem: Given: T and S isometry. Determine the statements below True or False? Give your reason.

- If \( g \) is a line, then \( g' = (TS)g \) is also a line.
- If \( g \parallel h \) and \( g' = (TS)g \), \( h' = (TS)h \) then \( g' \parallel h' \).
- If \( S \) is a reflection of the \( S \) is involutory.

The obstacles are the inability of students in the mentioned properties of isometry, so it cannot be a member of reasons of the questions in the matter.

Students with high prior knowledge are described as follows: S1 can answer correctly, but the reasons expressed is wrong. S5 can be answered correctly and the reasons for appropriately. S8 can mention the definition of isometry correctly, but cannot answer questions related to the principle of isometry, so that reason given is incorrect. Students with medium prior knowledge are being described as follows: S3 is having difficulty in writing down the definition of isometric and give wrong reasons regarding the statements given. S4 can write for the right reasons, but they are notational wrong. S9 is essential to specify the definition of isometry, giving the wrong reasons. Students with low initial knowledge are described as follows: S2 is essential to specify the definition and the reason given was also incorrect. S6 and S7 wrong in writing down the definition and does not include the reason (no answer).

Fourth obstacles are related to understanding the problem. The obstacles experienced by students are regarding understanding the problem to solve the problem by using the steps in the completion of the write down what is known and asked about the question. For example, on the following question: define a line equation \( g' = M_h(g) \) if \( h = \{(x,y) \mid y = x + 1\} \) and \( g = \{(x, y) \mid y = -x\} \).

Students S1 is having trouble determining what is known and troubleshooting procedures are still wrong. S5 can understand the problem, find out what is known and asked, can solve the problem by the settlement procedures. S8 can understand the problem and solve it according to the procedure. Students with prior knowledge are being described as follows: S3 can understand the problem and solve the problem with proper procedures, but there are errors in arithmetic operations. S4 can understand the problem and solve the problem according to the procedure. S9 can not understand the problem and can not solve it. Students with low prior knowledge are described as follows: S2 can understand the problem but can not finish with the correct procedure. S6 and S7 are not able to understand the problems and did not finish.
The fifth obstacles related to mathematical proving. This obstacle is the difficulty experienced by students in constructing a proof of the geometry problem. For example, the following problem: prove that the reflection on the line $g$ is an isometry. Students with high prior knowledge are described as follows: S1 can construct the proofs correctly, but there is still incorrect notation. S5 can construct the proofs properly. S8 can construct proofs in part, at the end of the part that is wrong in giving reasons. Students with prior knowledge are being described as follows: S3 and S4 obstacles for constructing proofs, cannot use the existing definition. S9 cannot start constructing proofs. Students with low prior knowledge are described as follows: S2 cannot use a definition for constructing proofs; S6 and S7 begin constructing the proof about be proved, difficulty in starting the construction of the proofs and not be able to use the definition for constructing proofs.

Some research showed that mathematical proofs are essential. Activities considered difficult for students to learn and teachers to teach include justification or proof [10]. Research studies conducted Dryfus showed that students always fail to look at the adequacy of the proofs because student is too often asked to prove things that are obvious to them [11]. Students also fail to distinguish between the different forms of mathematical reasoning such as heuristic or argument, explanation or proofs. A significant gap in the research literature is still at least mindset students "because it looks right" instead of "because he worked on issues" for the argument that believed.

The research result Knuth showed that teachers recognize the many roles of the play proof in mathematics, in learning the role of proofs should not be abandoned, and the proof as a tool for learning mathematics [6]. The results also show that many teachers still have limitations in determining the nature of the proof used in the study of mathematics.

Hinze and Reiss research results showed that some students found with some of the answers wrong though ideas about the solution are proving correct. It occurs in the empirical argument. Many errors occur experienced by students is related to aspects of the structure of proofs [12]. Most of the students interviewed were mostly already know that the argument does not form empirical proofs. Study interviews in this study also showed that the three aspects of methodological knowledge of significant proof when assessing the proof. It seems that on this aspect conclusion chain is not problematic, because the proofs are right mostly depicted as real. However, some cases it was not clear whether the students understand every step of the proof compiled. The problem with this aspect of the scheme of proof, in particular, an inductive argument, often trained using inductive argument in elementary school. Students have difficulty bridging the gap between empirical arguments to formal arguments. This is confirmed by a study by Lin showed that students a different problem that is wrong or improper argument transformation in improving formal arguments were observed [12].

Mariotti study revealed the geometry construction is an integral part of the experience of students who should be organized [13]. Results of the study Mariotti showed that if the geometry is just a pencil and paper geometry theory perspective, it is difficult to understand. When students draw on paper students can only focus on the images being constructed and can not manipulate it.

4. Conclusions and implications

Here are the conclusions obtained in this study. There are five kinds of obstacle related to the students regarding epistemology on transformations, namely a) obstacle related to understanding and applying the concept; b) related to visualizing the geometry object; c) related to determining principle; d) related to understand the problem and e) related to mathematical proofs. Finally, the writer suggests that the learning process, especially on transformation geometry, needs activities that connect the representation to obtain visualization of the mathematical problems. The knowledge gained during the representation process is used as a tool to grow a mathematical connection, which can eventually be used in the process of constructing a proof. Proving the mathematical ability of students is still relatively low, especially for students with prior knowledge of medium and low. It is necessary to apply a model of learning that guides students in constructing proofs.
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