Backstepping Controller Design for Pantograph-Catenary System

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Abstract. This paper proposes an active control method to suppress the contact force fluctuation between pantograph and catenary for high-speed trains with external disturbance. First, fitted by using the non-linear least square method, stiffness of catenary is applied to the Pantograph-Catenary system (PCS). Second, a backstepping control strategy for PCS is developed to decrease fluctuation of the contact force. Theoretical analysis shows that the proposed control strategy has strong robustness and disturbance-attenuation ability. The control performances of the closed-loop systems can be shaped as desired by suitably choosing the design parameters. Finally, simulation studies are presented to validate the proposed scheme.

1. Introduction

High speed trains usually acquire current through the PCS. With the speed of the high-speed trains is further improved, the fluctuation of the contact force between the pantograph and the catenary will be more and more violent. There are two methods to solve this problem. One method is to improve the stiffness of the catenary or increase its tension. The other one is to increase the above-mentioned contact force. However, These two methods will lead to huge investment or severe pantograph catenary wear [1]. In order to decrease the fluctuation of the contact force, an uplift force can be adopted through the lower frame of pantograph. If the uplift force is too large, it will cause too much contact force and eventually cause too serious wear of the slide plate. If the uplift force is too small, it will result in too weak contact force and eventually lead to an unnecessary arc [2]. To overcome the above problem, active control method has been put forward and has been receiving significant attentions [3]. Yamashita et al. developed a low-noise active pantograph, and then applied a PID controller or an impedance control method [4]. However, the complete model of a pantograph mechanism for high-speed rail system is complex so that resulting in tuning difficultly the optimal control gains of PID controller [5]. In [6], J Evans et al. discusses several proposals for pantograph active control, taking into account the feedback of the contact force measure. In [7], Corriga et al. employed a frequency-shaping control approach to improve the pantograph system’s performance. However, the pantograph system was modeled with a constant stiffness to represent the catenary. In [8] the control strategy is based on Extended Kalman Filter technique, used to get a contact force estimation available for control feedback. Wu and Zhang also suggested setting up an active actuator between upper and lower frame of pantograph, and applying fuzzy logic control technology as control algorithm [9]. A three degree of PCS is established, and a controller based on optimal control strategy
is designed in [10]. We can see that the fluctuation of contact force has been significantly improved. However, the contact force has not reached an ideal value, or there is a certain standard deviation with external disturbance.

Combining with backstepping control theory, this paper designs a controller for pantograph catenary. Considering the external disturbance, we can see that the proposed controller has good tracking performance and better robustness. In order to verify the effectiveness of the control method, the latter part also make digital simulation.

2. A model of the PCS
There are many mathematical models of pantograph, such as nonlinear multibody model or simplified mechanical model. But we usually study the control method of PCS, and generally use the lumped-parameters model. There are three reasons for this. First reason, The pantograph adopts the two mass block model, and the simulation results can be compared with the standard. Second reason, the lumped-parameters model is used as the control object. Compared with the multibody model, its dynamic equation is simple and its computation speed is fast. Third reason, nowadays the research of pantograph active control is mainly based on control algorithm, which weakens the influence of pantograph model characteristics on control effect [11]. Therefore, the linear mass model of two degrees of freedom is adopted in this paper [12]. The lumped-parameters model is shown in Figure 1.

![Figure 1. A lumped-parameters model of the Pantograph-Catenary system](image)

The equations of motion of the pantograph-catenary system model shown in Figure 1 can be written as

\[
\begin{align*}
    m_1 \ddot{x}_1 - k_1 (x_2 - x_1) - b_1 (\dot{x}_2 - \dot{x}_1) + k(t) x_1 &= 0 \\
    m_2 \ddot{x}_2 + k_1 (x_2 - x_1) + b_1 (\dot{x}_2 - \dot{x}_1) + b_2 \dot{x}_2 &= u + d(t)
\end{align*}
\]

(1)

where, \( m_1 \) and \( m_2 \) represents the mass of the pantograph head and the quality of the frame, respectively; \( k_1 \) stands for the pantograph head stiffness and frame stiffness; \( b_1 \) represents the damping of the pantograph head and that of frame ; \( x_{1,2} \), \( \dot{x}_{1,2} \) and \( \ddot{x}_{1,2} \) stands for the displacements, velocities and the acceleration, respectively; \( u \) is a controller; \( d(t) \) is an external disturbance signal.

\( k(t) \) is the equivalent stiffness of the catenary. Stiffness of catenary is fitted by using the nonlinear least square method

\[
k(t) = k_0 (1 + \alpha_1 f_1 + \alpha_2 f_2 + \alpha_3 f_1^2 + \alpha_4 f_3^2 + \alpha_5 f_4^2)
\]

(2)

\[
f_1 = \cos\left(\frac{2\pi v}{L_1}t\right), f_2 = \cos\left(\frac{2\pi v}{L_2}t\right), f_3 = \cos\left(\frac{\pi v}{L_4}t\right), f_4 = \cos\left(\frac{\pi v}{L_2}t\right)
\]

(3)
where $k_0$ is average stiffness coefficient within span. $\alpha_{i=4,2,3,4,5}$, $L_{i=4,2}$ are the stiffness variation coefficient of catenary and catenary span, and $V$ is the running speed of the locomotive.

In order to design the controller conveniently, we take the following state variables.

$$z(t) = [z_1, z_2, z_3, z_4]^T = [x_1, \dot{x}_1, x_2, \dot{x}_2]^T$$

Taking into account external disturbance, the formula (1) can be written as a state space expression.

$$\begin{cases}
\dot{z}(t) = A(t)z(t) + Bu(t) + d(t) \\
y(t) = C(t)z(t)
\end{cases}$$

where $y(t)$ is the contact force and

$$A(t) = \begin{bmatrix}
0 & 1 & 0 & 0 \\
-k_1 + k(t) & -b_1 & k_1 & b_1 \\
m_1 & m_1 & m_1 & m_1 \\
0 & 0 & 0 & 1 \\
k_1 & b_1 & -k_1 & -b_1 + b_2 \\
m_2 & m_2 & m_2 & m_2 
\end{bmatrix}$$

$$B = \begin{bmatrix}
0 & 0 & 0 & \frac{1}{m_2} 
\end{bmatrix}^T$$

$$d(t) = \begin{bmatrix}
0 & 0 & 0 & \delta(t) 
\end{bmatrix}^T$$

$$C(t) = \begin{bmatrix}
k(t) & 0 & 0 & 0 
\end{bmatrix}$$

So, we will combine the state space expression and backstepping method to design controller.

3. Controller design

We use backstepping control theory to regulate the contact force. Our aim is to adjust the contact force be kept constant. In order to achieve this goal, we must assume that all the state variables are available for backstepping control. Because our goal is to suppress the fluctuation of the contact force, so we can get a reference signal

$$z_{1r} = \frac{F_c}{k(t)}$$

where $F_c$ is an ideal contact force.

In this purpose, we will build the control law by the three following steps:

Step 1: Let the first tracking error

$$e_1 = z_1 - z_{1r}$$

and considering the Lyapunov function positive definite

$$V_1 = \frac{1}{2} e_1^2$$

Its time derivative is

$$\dot{V}_1 = e_1 \dot{e}_1 = e_1 (z_\dot{1} - z_{1r}) = e_1 (z_2 - z_{1r})$$

we found that $z_2$ is the virtual control, so there will be an error $e_2$

$$e_2 = z_2 - \beta_1$$

where $\beta_1$ is the virtual control stabilization function to be tracked by

$$\beta_1 = -a_1 e_1 + \dot{z}_{1r}$$

where $a_1$ is positive real. So, deriving $V_1$ with respect to time yields
\[ \dot{V}_1 = e_1(e_2 - a_1 e_1 + \dot{z}_{i_1} - \dot{z}_i) \]
\[ = -a_1 e_1^2 + e_1 e_2 \]  

(16)

Step 2: Introducing the modified Lyapunov function

\[ V_2 = V_1 + \frac{1}{2} e_2^2 \]  

(17)

Its time derivative can be calculated by

\[ \dot{V}_2 = \dot{V}_1 + e_2 \dot{e}_2 \]
\[ = -a_1 e_1^2 + e_1 e_2 + e_2 (\dot{z}_2 - \dot{\beta}_2) \]
\[ = -a_1 e_1^2 + e_1 e_2 + e_2 (-\frac{k_1 + k(t)}{m_i} z_i - \frac{b_1}{m_i} z_2 + \frac{b_1}{m_i} z_3 + \frac{b_1}{m_i} z_4 - \dot{\beta}_1) \]  

(18)

we found that \( z_4 \) is the virtual control, so there will be an error \( e_3 \)

\[ e_3 = z_4 - \beta_2 \]  

(19)

\( \beta_2 \) is the virtual control stabilization function to be tracked by

\[ \beta_2 = \frac{m_i}{b_1} (\dot{\beta}_1 - \frac{k_1}{m_i} z_3 + \frac{b_1}{m_i} z_2 + \frac{k_1 + k(t)}{m_i} z_1 - e_1 - a_2 e_2) \]  

(20)

where \( a_2 \) is positive real. So, deriving \( V_2 \) with respect to time yields

\[ \dot{V}_2 = -a_1 e_1^2 + e_1 e_2 + e_2 (\dot{z}_2 - \dot{\beta}_2) \]
\[ = -a_1 e_1^2 + e_1 e_2 + e_2 \left( \frac{m_i}{b_1} \left( \dot{\beta}_1 - \frac{k_1}{m_i} z_3 + \frac{b_1}{m_i} z_2 + \frac{k_1 + k(t)}{m_i} z_1 - e_1 - a_2 e_2 \right) - \dot{\beta}_1 \right) \]
\[ = -a_1 e_1^2 - a_2 e_2^2 + \frac{b_1}{m_i} e_2 e_3 + e_2 \left( \frac{k_1}{m_i} z_1 + \frac{b_1}{m_i} z_2 - \frac{k_1 + k(t)}{m_i} z_3 - \frac{b_1 + b_2}{m_i} z_4 + \frac{1}{m_i} u + \delta(t) - \dot{\beta}_2 \right) \]  

(21)

Step 3: Introducing the Lyapunov function

\[ V = V_2 + \frac{1}{2} e_3^2 \]  

(22)

Its time derivative can be calculated by

\[ \dot{V} = \dot{V}_2 + e_2 \dot{e}_2 = \dot{V}_2 + e_3 (\dot{z}_4 - \dot{\beta}_3) \]
\[ = -a_1 e_1^2 - a_2 e_2^2 + \frac{b_1}{m_i} e_2 e_3 + e_2 \left( \frac{k_1}{m_i} z_1 + \frac{b_1}{m_i} z_2 - \frac{k_1 + k(t)}{m_i} z_3 - \frac{b_1 + b_2}{m_i} z_4 + \frac{1}{m_i} u + \delta(t) - \dot{\beta}_2 \right) \]  

(23)

At this point, we find that the controller has appeared. The controller is as follows

\[ u = m_2 (\dot{\beta}_1 + \frac{b_1 + b_2}{m_2} z_4 + \frac{k_1}{m_2} z_3 - \frac{b_1}{m_2} z_2 - \frac{k_1}{m_2} z_1 - \frac{b_1}{m_2} e_2 - (a_2 + c) e_3) \]  

(24)

where \( a_2 \) and \( c \) is positive real. So, deriving \( V \) with respect to time yields

\[ \dot{V} = -a_1 e_1^2 - a_2 e_2^2 - a_2 e_2^2 - ce_3^2 + \delta(t)e_3 \]  

(25)

For convenience, here we make the order \( \Delta \)

\[ \Delta = -ce_3^2 + \delta(t)e_3 \]  

(26)

\[ \Delta \leq -ce_3^2 + k \| e_3 \| = -ce_3^2 + b \| e_3 \| \frac{k}{b} \]  

(27)

where \( k \) and \( b \) is positive real, \( k \) satisfaction \( \| \delta(t) \| \leq k \), So
\[
\Delta \leq -ce_i^2 + \frac{1}{2} b^2 e_i^2 + \frac{k^2}{2b^2}
\]

Then \( \dot{V} \) satisfaction

\[
\dot{V} \leq -a_t e_i^2 - a_s e_s^2 - a_e e_e^2 + \frac{k^2}{2b^2}
\]

when \( k \) satisfaction \( k \ll b \)

\[
[z_i - z_{ie}]_{t \to \infty} = 0
\]

4. Simulation results

In order to prove the performance of the proposed controller, numerical simulation has been carried out. The model parameters with the following values:

Catenary: \( k_0 = 3600 N/m; L_4 = 65m; L_5 = 9m; \alpha_1 = 0.4665; \alpha_2 = 0.0832; \alpha_3 = 0.2603; \alpha_4 = -0.2801; \alpha_5 = -0.3364 \)

Pantograph: \( m_1 = 8kg; k_1 = 10000 N/m; b_1 = 120Ns/m; m_2 = 12kg; b_2 = 30Ns/m \)

Ideal contact force: \( F_c = 100N \)

The first simulation takes into account the train in 300km/h, and the external signal \( s(t) = \sin(2 \pi t) \). The first simulation results are found Figure 2. We will eventually find that the contact force will reach a constant value 100N. The second simulation takes into account the train in 100m/s, and the external signal \( s(t) = 10\sin(2 \pi t) \). The second simulation results are found Figures(3-4). It is also found that the contact force is almost constant 100N. Comparing with [10], the backstepping control method proposed in this paper greatly decreases the fluctuation of contact force under the speed of 400km/h and external disturbance. The simulation results can prove a good tracking effect of contact force at high speed and a strong robustness against interference. Therefore, the effectiveness of the backstepping method is verified.

![Figure 2. Contact force when the speed is 300km/h](image)

![Figure 3. Equivalent stiffness of catenary](image)

5. Conclusion

This paper presents a method for suppressing the contact force fluctuation between pantograph and catenary for high-speed trains using a backstepping active control technology. Theoretical analysis shows that the proposed control strategy has strong robustness and disturbance-attenuation ability. The control performances of the closed-loop systems can be shaped as desired by suitably choosing the design parameters. The simulation results show that: The first simulation, under the speed of 300km/h and external disturbance, the contact force can approximately reach the ideal value. The second simulation, the backstepping control method proposed in this paper greatly decreases the fluctuation of
contact force under the speed of 400km/h and external disturbance. Therefore, the controller proposed in this paper has good tracking performance and a better robustness.

![Diagram of contact force](image)

**Figure 4.** Contact force when the speed is 400km/h

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