SOLID-FLUID INTERACTION WITH SURFACE-TENSION-DOMINANT CONTACT

Liangwang Ruan*, Jinyuan Liu*, Bo Zhu, Shinjiro Sueda, Bin Wang, Baoquan Chen
(* joint first authors)
Motivation
Motivation

Surface tension

Solid

Fluid
Fluid Simulation

- Eulerian method
  - Solve NS equation on MAC grid
- Implicit surface
  - Level set method
- Surface tension
  - Young-Laplace equation
  - Mean curvature

[Chen et al. 2020]
[Zheng et al. 2006]
Fluid-Solid Coupling

Eulerian fluid & Lagrangian solid

No surface tension
Problems

- Curvature’s precision
- Strong surface tension
- Apply surface tension to the solid

Use Lagrangian surface
Membrane

- Explicit mesh
  - Finer than the grid
- Mass
  - Finite thickness $h$
- Surface Tension
  - Nodal attraction
- Coupling
  - Momentum transfer
Simulation result

\[ \sigma = 72.8 \text{ dyn/cm} \]

\[ \rho_r = 7.9 \text{ g/cm}^3 \]
Our Contributions

• A novel Lagrangian thin membrane representation

• A monolithic coupling framework

• A prediction-correction contact handling scheme
Coupling System
Three-way Coupling

- Rigid
- Fluid
- Membrane

- Velocity constraint
- Buoyancy
- Surface tension
Substance Modeling

Fluid

\[ \frac{1}{\rho} G^T G p \Delta t = G^T u^* \]

Rigid

\[ f_r = [f, \tau] \]

Lagrangian, Newton

\[ M_r \frac{v_{r}^{n+1} - v_r^n}{\Delta t} = \hat{f}_r(q_r^{n+1}, v_r^n) \Delta t \]
Substance Modeling

- **Surface energy**: \( E = \sigma A \)
  - \( \sigma \): surface tension coefficient
  - \( A \): surface area
- **Nodal force**: \( f_{ci,t} = -\sigma \frac{\partial A_t}{\partial x_i} \)
- **Mass**: \( m_s = \rho h A \)
  - \( \rho \): density of the fluid
  - \( h \): membrane thickness
- **Newton equation**: \( M_s \frac{v_{s+1} - v_s}{\Delta t} = f_c \)
Fluid-Membrane

\[
\begin{bmatrix}
\frac{V}{\rho} G^T G & -V G^T W \\
-W^T G V & -\hat{M}_s
\end{bmatrix}
\begin{bmatrix}
\hat{p} \\
v_s^{n+1}
\end{bmatrix}
= \begin{bmatrix}
V G^T u^* \\
-M_s v_s^n - W^T M u^* - f_c \Delta t
\end{bmatrix}
\]

Membrane

Velocity constraint
buoyancy

Fluid
$h = 0.5\Delta x$

$h = \Delta x$

$h = 10\Delta x$

level set
Membrane-Rigid

- Divide membrane into C and F
- **Velocity constraint**: for C
- **Surface tension**: between C and F
Three-way Coupling System

\[
\begin{bmatrix}
\frac{V}{\rho} G^T G & -V G^T W & -V G^T J_r \\
-W^T G V & -\tilde{M}_s & -K_{c,r} \Delta t^2 \\
-J_r^T G V & -K_{a,s} \Delta t^2 & -\tilde{M}_r
\end{bmatrix}
\begin{bmatrix}
\hat{p} \\
v_s^{n+1} \\
v_r^{n+1}
\end{bmatrix}
=
\begin{bmatrix}
V G^T u^* \\
-M_s v_s^n \tilde{f}_c \Delta t - W^T M u^* \\
-M_r v_r^n \tilde{f}_a \Delta t - \tilde{f}_r \Delta t - J_r^T M u^*
\end{bmatrix}
\]
Time Scheme

- Advance position
- Resolve collision
- Correct fluid volume: first solve
- Re-meshing
- Three-way coupling: second solve

\[ T_n \]

1. Advance
2. Resolve collision
3. Compute effective velocity
4. Projection
5. Advection
6. Re-meshing
7. Three-way coupling

\[ T_{n+1} \]
Simulation Results
\[ \sigma = 72.8 \text{ dyn/cm} \]
\[ \rho_r = 0.4 \text{ g/cm}^3 \]
\[ \sigma = 72.8 \text{ dyn/cm} \]

\[ \sigma = 46 \text{ dyn/cm} \]
\( \sigma = 72.8 \text{ dyn/cm} \)

\( \rho_r = 3.8 \text{ g/cm}^3 \)
\[ \sigma = 72.8 \text{ dyn/cm} \]
\[ \rho_r = 6.8 \text{ g/cm}^3 \]
\[ \sigma = 72.8 \text{ dyn/cm} \]
\[ \rho_r = 0.1 \text{ g/cm}^2 \]
\[ \sigma = 72.8 \text{ dyn/cm} \]

\[ \rho_r = 0.4 \text{ g/cm}^3 \]
Summary

• Surface-tension-dominant solid-fluid coupling
  • Membrane representation
  • Three-way coupling
  • Prediction-correction time scheme

• Limitations
  • Large topology change
  • Contact angle
  • Efficiency
Thanks!