Triangle Cusp and Resonance Interpretations of the $X(2900)$

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We examine whether the LHCb vector $udar{c}ar{s}$ state $X(2900)$ can be interpreted as a triangle cusp effect arising from $D^*K^*$ and $D_s K^*$ interactions. The production amplitude is modelled as a triangle diagram with hadronic final state interactions. A satisfactory fit to the Dalitz plot projection is obtained that leverages the singularities of the triangle diagram without the need for $DK$ resonances. A somewhat better fit is obtained if the final state interactions are strong enough to generate resonances, although the evidence in favour of this scenario is not conclusive.

I. INTRODUCTION

The LHCb collaboration has announced the discovery of a $DK$ enhancement in the reaction $B \rightarrow DDK$ that can be interpreted as Breit-Wigner resonances with parameters [1]:

$$X_0(2866); \, J^P = 0^+, \quad M = 2866.3 \pm 6.5 \pm 2 \text{ MeV}, \quad \Gamma = 57.2 \pm 12.2 \pm 4.1 \text{ MeV}, \quad \text{fit fraction} = 6\%,$$

$$X_1(2904); \, J^P = 1^-, \quad M = 2904.1 \pm 4.8 \pm 1.3 \text{ MeV}, \quad \Gamma = 110.3 \pm 10.7 \pm 4.3 \text{ MeV}, \quad \text{fit fraction} = 31\%.$$ (2)

The discovery adds to a burgeoning list of putative hadrons of unconventional structure that have revitalised hadronic spectroscopy and shed light on the dynamics of QCD in the nonperturbative regime [2].

The $X(2900)$ pair is observed as an enhancement in $D^-K^+$ and is therefore manifestly flavour-exotic, with a minimal quark content of $udar{c}ar{s}$. It is therefore essential to determine the origin of the structure, for example it could be a novel $D^{(*)}K^{(*)}$ (or related) bound state, a compact tetraquark, or a production mechanism effect. Discriminating among interpretations would provide important diagnostic information on the workings of nonperturbative QCD.

In this regard, the related $DK$ system has already provided evidence of ‘unconventional’ dynamics since it is suspected that it strongly influences the enigmatic $D_{s0}(2317)$. An early argument for a $DK$ molecular interpretation is in Ref. [3]. More recently, a lattice gauge computation finds evidence that the $D_{s0}(2317)$ is an isoscalar $DK$ bound state, with hints of $J^P = 1^+$ and $2^+$ states at higher energy [4]. A review can be found in Ref. [5]. Note, however, that the analogy between the $DK$ and $DK$ systems may be misleading. The $DK$ system couples via the strong interaction to $car{s}$, and this is presumably important for $D_{s0}(2317)$ and related states. This coupling is absent for the $DK$ system.

Other flavour-exotic systems have been and gone. The $X(5568)$ claimed by the D0 experiment [6] would have flavour content $subd$. We and others (for example [7, 8]) pointed out the implausibility of explaining the state using any of the standard approaches. Subsequently a number of experiments searched for, but did not find, the the state [9, 12].

A variety of techniques and suggestions concerning the $X(2900)$ pair have appeared recently. Amongst these are the interpretation of the $J^P = 0^+$ enhancement as an isosinglet compact tetraquark that is an analogue of the anticipated $uar{d}bar{b}$ tetraquark [13]. A similar conclusion was reached in Ref. [14], where it was argued that the scalar state is a radially excited tetraquark while the vector state is an orbitally excited tetraquark. Both of these references used effective models with spatially constant interactions to reach their conclusions. Similarly, the author of Ref. [15] finds that a QCD sum rule computation with a scalar-scalar current supports the identification of the scalar signal with a $0^+$ tetraquark. An alternative approach is used in Ref. [16], where a constituent quark model is solved variationally in a Gaussian basis. The authors find an extensive $udar{s}ar{c}$ spectrum, but conclude that the mass spectrum (including four scalars at 2765, 3065, 3152, and 3396 MeV) cannot accommodate $X(2900)$.

The tetraquark interpretation for $udar{s}ar{c}$ is motivated in part by the analogy with $uar{d}bar{b}$, for which there is evidence from lattice QCD for a bound state [17–21]. The analogy is questionable considering that the $X(2900)$ states, being far above threshold, are not bound states. In this context it is noteworthy (and has had hardly been discussed in recent literature) that the same lattice QCD calculation which finds a bound $bar{b}uar{d}$ system does not find binding in $udar{s}ar{c}$ [22]. Similarly there is no evidence for bound $udar{s}ar{c}$ states in early quark model calculations [23], or a recent QCD sum rules study [24]. If the $X(2900)$ states have a tetraquark nature, they may be very different from their bound $uar{d}bar{b}$ analogues.

In the molecular model, we note an early prediction for an isoscalar $D^*K^*$ state, with $J^P = 0^+$, and with mass and decay width comparable to the observed $X_0$ [25]. The model, which is based on the vector hidden gauge formalism, was recently refined in response to the LHCb discovery, and the authors predict partner states with $J^P = 1^+$ and
2\textsuperscript{+}, but do not propose and interpretation for the observed 1\textsuperscript{−} state \textit{X}\textsubscript{1} \cite{20}. Liu \textit{et al.} use an effective Lagrangian that couples heavy quark fields and light mesons to compute binding energies of possible \textit{DDK}, \textit{D\textsuperscript{*}K}, \textit{DK\textsuperscript{*}} and \textit{D\textsuperscript{*}K\textsuperscript{*}} molecules \cite{22}. They argue that \textit{X}\textsubscript{0} can be interpreted as an isoscalar \textit{D\textsuperscript{*}K\textsuperscript{*}} molecule, but find no viable explanation for \textit{X}\textsubscript{1}. By contrast He and Chen \cite{21}, with a similar model, reach a different conclusion, favouring an isovector interpretation for the \textit{X}\textsubscript{0} as a \textit{D\textsuperscript{*}K\textsuperscript{*}} molecule, and proposing that \textit{X}\textsubscript{1} is a virtual state – also isovector – from the \textit{D\textsubscript{1}K} interaction. Hu \textit{et al.} \cite{24} argue that \textit{X}\textsubscript{0} is an isoscalar \textit{D\textsuperscript{*}K\textsuperscript{*}} molecule and, using heavy-quark symmetry, they predict several partner states.

Other papers consider the possible role for both molecular and diquark degrees of freedom. Refs. \cite{28,29} both advocate \textit{X}\textsubscript{0} as an isoscalar \textit{D\textsuperscript{*}K\textsuperscript{*}} molecule, and whereas Ref. \cite{30} does not propose an explanation for \textit{X}\textsubscript{1}, Ref. \cite{28} argues that \textit{X}\textsubscript{1} is a P-wave diquark-antidiquark state. Finally, production and decay of the \textit{X}(2900) pair was studied in Ref. \cite{31}.

We do not offer a detailed analysis of these recent results; we note, rather, that many of the conclusions are based on methods that have not been carefully validated in the multiquark sector. It is therefore important to continue to explore options for the LHCb signal. Here we examine the possibility that a triangle cusp can give rise to enhancement in FSI. 

As described in the next section, we go beyond these initial observations by making a detailed fit to the mass distribution, by incorporating nonperturbative final state interactions (FSI), and by successfully allowing for the broadening of the peak when realistic \textit{K\textsuperscript{*}} and \textit{D\textsubscript{1}} widths are taken into account. As described in the next section, we go beyond these initial observations by making a detailed fit to the mass distribution, by incorporating nonperturbative final state interactions (FSI), and by successfully allowing for the measured \textit{K\textsuperscript{*}} and \textit{D\textsubscript{1}} widths. We find that it is possible to fit the mass distribution very well with the \textit{D\textsuperscript{*}K\textsuperscript{*}} – \textit{D\textsubscript{1}} cusp effect. An even better fit is obtained when \textit{D\textsubscript{1}} \textit{K\textsuperscript{*}} and strong final state interactions are considered.

**II. TRIANGLE FINAL STATE INTERACTION MODELS**

**A. Production**

The LHCb collaboration found that the angular distribution (in \textit{DDK}) provides strong evidence for a 1\textsuperscript{−} Breit-Wigner component in their amplitude model. The fit was further improved by including the \textit{X}\textsubscript{0} (2866) Breit-Wigner,
although we note that its contribution to the angular distribution is negligible. As such, we suspect that the effect of the $X_0$ can be mimicked by background or other dynamics and that it will prove to be unnecessary. Because of this we focus attention on the vector $X_1(2904)$ in the following.

The process we consider is illustrated in Fig. 1. In general we consider three intermediate mesons labelled $a$, $b$, and $c$. Meson combinations that dominate the process should be (i) colour enhanced, (ii) S-wave where possible, and (iii) permit the triangle to go nearly on-shell. Possible combinations are shown as $D_s^{(*)}D^{(*)}K^{(*)}$ in the figure. Experimentally, the largest branching ratios of the $B^+$ are to states accessible via colour-enhanced transitions such as $D_s^+D$ (at order one percent) so we focus on these. However, one expects a series of “$D_s^*$” states to contribute, with those near $m(B^+) = m(D_s) + m(D)$ dominating the sum. We account for this by using an effective $D_s^*$ meson of spin-parity $1^-$ and mass of 3 GeV in the following. (We will also examine the effect of changing this mass.)

We note in passing that the model of Ref. [48] for $X_0(2866)$ relies on intermediate states that arise only from colour-suppressed transitions, specifically $\chi_{c1}K^*$, where $\chi_{c1}$ is either $\chi_{c1}(3872)$, $\chi_{c1}(4140)$ or $\chi_{c1}(4274)$. It is questionable whether colour-suppressed transitions could produce prominent triangle enhancements over a background of colour-enhanced processes. For comparison we note that, experimentally, $B(B^+ \to X(3872)K^+) < 2.6 \times 10^{-4}$, whereas colour-favoured transitions are typically at the percent level.

![FIG. 1: Production Model for the $X(2900)$.](image)

As indicated in Fig. 1, mesons $b$ and $c$ will be taken to have flavour $\bar{D}K$. Clearly, choosing $bc = \bar{D}^*K^*$ and $\bar{D}_1K$ will be important to creating reaction strength near 2900 MeV. For completeness we shall expand this collection to include related channels which can also couple to $1^-$, specifically $\bar{D}K|_F$, $\bar{D}^*K^*|_F$, $\bar{D}_1K|_S$, and $\bar{D}_1K^*|_S$, where the partial wave is indicated with a subscript. (We do not include $\bar{D}^*K|_P$, $\bar{D}K^*|_P$ for reasons discussed later.) In practice we will find that an excellent fit to the experimental data can be obtained with only a subset of these channels.

The next step is to model the vertices in the triangle diagram. The electroweak vertex will be written in terms of the heavy quark effective field, with the transition $B \to D^{(*)}$ represented by the Isgur-Wise function $\xi(w)$ with $\xi \sim (2/(1 + w))^2$ and the orbital transition $B \to D_1$ represented by the Isgur-Wise function $\tau(w)$. The variable $w$ is defined as $p_B \cdot p_D/(m_B m_D)$ and the function $\tau$ will be approximated by $(2/(1 + w))^{6/5}$. Decay constants, CKM matrix elements, etc will be absorbed into coupling strengths (to be defined shortly) and will henceforth be suppressed. We remark that dependence on the form of the electroweak vertex can be readily absorbed into model coupling parameters.

The upper vertex represents a strong decay and we have chosen to model it with the well-known ‘3P0’ model of hadronic transitions [51]. Thus, for example, the transition $D_s^+ \to \bar{D}K$ is approximately proportional to $x \exp(-x^2/12)$, where $x = p/\beta_{3P0}$ and $\beta_{3P0}$ is a universal width parameter describing simple harmonic oscillator wavefunctions for the mesons. Although this form is typical, we believe that the exponential decay is not physical for large momentum and have replaced this functional form with

$$x \frac{1}{1 + x^2/12}$$

though, in practice, this makes little difference. Once again, the strength of the vertex will be absorbed into a global coupling.

The remaining vertex (the solid circle) represents the final state interactions in the $\bar{D}K$ system, which will be discussed in the next section. For now we denote the relevant portion as $F_L \cdot Y_{LM}$. 
Finally, we use nonrelativistic kinematics as all the mesons are predominantly moving slowly in the $B^+$ rest frame. The resulting expression for the triangle diagram is

$$
\Delta_{aa}(s_{DK}) = \int \frac{d^3 q}{(2\pi)^3} F_{ew}(q + k/2) F_{ew}(3k/4 - q/2) F_{L\alpha}(q) Y_{L\alpha,M\alpha}(\hat{q}) \cdot \left[ m_B - m_{b}^\alpha - m_{c}^\alpha - q^2 / (2\mu_{bc}) + i\Gamma_{\alpha}^\alpha / 2 + i\Gamma_c^\alpha / 2 \right]^{-1} \cdot \left[ m_B - E_D - m_b^\alpha - m_c^\alpha - (q + k/2)^2 / (2m_B^2) - (q - k/2)^2 / (2m_B^2) + i\Gamma_B^\alpha / 2 + i\Gamma_c^\alpha / 2 \right]^{-1}.
$$

The notation “$a\alpha$” indicates that the diagram depends on the meson $a$ and the channel index $\alpha$, which contains a meson pair $bc$. We have allowed for an angular momentum $L_{\alpha}$ (normally 0 or 1) in the final state interaction vertex. Widths of the mesons $a$, $b$, and $c$ are also accounted for as they can have a strong attenuation effect on the amplitude. Finally, $k$ is the momentum of the outgoing $D$ meson while its energy is $E_D$; these quantities can be used to obtain the Mandelstam variable, $s_{DK}$.

B. Final State Interactions

We have argued that four channels are naturally relevant to the description of $\bar{D}K$ production in $B$ decay and have chosen to use one-pion-exchange as a guide in modelling the final state interactions among these channels. Thus interactions that carry $D \to D$, $K \to K$, and $D_1 \to D$ are disallowed. In addition, the vertex $\bar{D}_1 D \pi$ is S-wave and thus is somewhat unusual in modelling of this sort. Its phenomenology has been discussed in the context of the LHCb pentaquark states in Ref. [47]

Although this description of the final state interactions can be carried to completion, we find it expedient to simplify and generalize it somewhat. Thus we replace the central, tensor, and vector interactions of the one-pion-exchange scenario with regulated separable interactions that are described by universal form factors and channel couplings, as follows:

$$
\langle pLM; \alpha|V|p'LM'; \alpha' \rangle = \lambda_{\alpha\alpha'} Y_{LM}(\hat{p}) F_L(p) \cdot Y_{L'M'}(\hat{p}') F_{L'}(p').
$$

The hadronic form factor is modelled as

$$
F_L(x) = \frac{x^{L/2}}{1 + x^2}, \quad x = \frac{p}{\beta},
$$

where $\beta$ is a universal scale and the numerator implements the expected angular momentum barriers in hadronic decays and interactions.

In this way we arrive at the interaction potential matrix shown in Tab. I. We have chosen to neglect the low mass $D^*K$ and $\bar{D}K^*$ channels as these do not couple strongly to the rest of the system and are not expected to contribute strongly to the dynamics at 2000 MeV. Furthermore, we do not include $\bar{D}K$ as a driving (triangle) amplitude since this subamplitude is only expected to contribute at the left edge of phase space. Lastly, one anticipates that the couplings will have magnitudes that follow the pattern $C_1 > C_2 > C_3$ because the angular momenta in the relevant channels increase in this pattern.

| TABLE I: Final State Interaction Model |
|---------------------------------------|
| $\lambda (\bar{D}K)_{P}$ | $D^*K^+|P$ | $D_1K|S$ | $D_1K^+|S$ | $\bar{D}K|P$ |
| $D^*K^+|P$ | $C_3$ | $C_2$ | $C_2$ | $C_2$ |
| $D_1K|S$ | 0 | $C_1$ | 0 |
| $D_1K^+|S$ | $C_1$ | 0 |
| $\bar{D}K|P$ | 0 |

Note in particular that we do not find it necessary or useful to follow the usual procedure of computing the one-pion exchange matrix elements in terms of central, vector or tensor potentials. Instead we use pion-exchange as a guide to identify any zeroes in the matrix, and then parametrise remaining couplings of a similar type with a common parameter – in particular, we use $C_1$, $C_2$ and $C_3$ for $\bar{D}_1 K^{(*)} \to \bar{D}_1 K^{(*)}$, $\bar{D}_1 K^{(*)} \to \bar{D}^{(*)} K^{(*)}$ and $\bar{D}^{(*)} K^{(*)} \to \bar{D}^{(*)} K^{(*)}$, respectively. The justification for this simplification is that, apart from $\bar{D}K$, all channels are production channels, and each of these has an associated coupling constant (denoted as $g_{\alpha\alpha}$ below) which is fit to data. Since the number of non-zero entries in the FSI matrix is comparable to the number of fit parameters, relative numerical factors in the matrix turn out to be unimportant.
For the same reason, we also do not need to consider all possible spectroscopic assignments for our channels. While most have unique assignments, the $D^* K^*$ channel includes several sub-channels $^3P_1$, $^3P_1$ and $^5P_1$. A full treatment of the interaction potential due to pion-exchange requires all of these channels to be included. But since each additional channel comes with its own fit parameter (the production coupling constant), the inclusion of all these channels does not ultimately further constrain the results. Hence for the sake of simplicity, we consider a single $D^* K^*|p\rangle$ channel.

Table I reveals the importance of the $D^* K^*$ channel, as this is the only channel that couples directly to the $\bar{D}K$ discovery mode. We also see that the $D_1$ mesons are (possibly) important since they contribute the only S-wave interactions in the FSI system. A variety of coupling strengths will be examined in the following, with the conclusion that the particular choices for the couplings $C_i$ do not affect the final fit qualities very much. This is likely because each channel (apart from $\bar{D}K$) is also a production channel and therefore has a coupling that can offset changes to the FSI potential, and because the triangle singularities are capable of describing the data (as will be shortly demonstrated).

Nonperturbative final state interactions are obtained by solving the Bethe-Heitler equation, $T = V + V GT$, using standard techniques. In this case a reduced T-matrix is employed that is defined via

$$\langle pLM; a|T|p'L'M'; a'\rangle \equiv Y_{LM}(\hat{p})F_L(\hat{p}) \cdot t_{aa'}(p,p') \cdot Y_{L'M'}^{a'}(\hat{p'})F_L'(\hat{p'}).$$

We are now prepared to write the amplitude model taking into account the triangle production diagram and the full final state interactions represented by Eqs. (4) and (6), and Table I. The result is

$$A = \frac{g_{bg}}{m_B} + \sum_{a \alpha(a'c)} g_{aa} \Delta a\alpha (s_{DK}) \cdot t_{a\alpha DK}(s_{DK}) \cdot Y^{a}_{L'M'}(k_{KD})F_{L'}(k_{KD}).$$

Notice that different triangle diagrams can drive the coupled channel system and that these are given couplings $g_{aa}$ which are to be fit to the data. A background scattering term, $g_{bg}$, has been included in the amplitude model. This term is taken to be a fixed complex constant, as we are – perhaps foolishly – heavily biased against overly elaborate background models.

### III. FIT RESULTS

Our objective is to fit the $\bar{D}K$ invariant mass distribution obtained by the LHCb collaboration. We further simplify by following the collaboration’s lead and restricting the projection to $m(D^+D^-) > 4.0$ GeV, thereby removing many charmonia/BB resonances, which simplifies the amplitude model. Of course, fitting a single projection misses much of the information available in the full Dalitz plot – eventually fitting the amplitude model to the full data set would be very interesting.

Our approach is to adopt reasonable values for the form factor parameters, specifically

$$\beta_{3P0} = 500 \text{ MeV}, \quad \beta = 700 \text{ MeV}.$$  

Furthermore, we choose to fix the final state interaction model according to different criteria. We report on three models in the following. These models are meant to represent weak final state interactions, moderate interactions, and interactions just strong enough to generate resonances (see Tab. II).

| case      | $C_1$ | $C_2$ | $C_3$ |
|-----------|-------|-------|-------|
| weak      | -3    | 3     | 2     |
| moderate  | -30   | 17    | 7     |
| strong    | -60   | 17    | 7     |

With this approach, the only free parameters are the couplings $g_{aa}$ and the background term $g_{bg}$, giving a total of $2N - 1$ parameters in an $N$-channel system.

In our experience it is too easy – and far too common – to fit an elaborate model to data and then ascribe physical reality to features of the model that are not warranted. We have therefore taken a conservative approach to the fit in which we progressively expand the model size, starting with the minimum number of channels required, $D^* K^*$ and $\bar{D}K$. In this case the only relevant coupling is $C_3$. The result of the three-parameter fit for the moderate and strong cases are shown as the blue line in Fig. 2; this fit has a chi-squared per degree of freedom of 2.39. It is evident that
the $X(2900)$ peak is reasonably well described, while the chief failing of the model is in describing the high $m(\bar{D}K)$ distribution.

We next expand the model to three channels, $\bar{D}^*K^*$, $\bar{D}_1K$, and $\bar{D}K$, with the new channel selected because its threshold is close to that of $\bar{D}^*K$, and couples to it via one-pion exchange. In this case the relevant couplings are $C_2$ and $C_3$ and there are five parameters to be fit. The result for the moderate and strong cases is displayed as the purple line in Fig. 2 and has $\chi^2$/dof = 1.48. Evidently the substantial improvement in the fit quality is due to the better description of the high mass region.

The Dalitz plot reported in Ref. [1] contains a band of strength near $m(\bar{D}K) = 3.2$ GeV that appears as the weak enhancement seen near 3.2 GeV in Fig. 2. While this enhancement is not terribly significant, and it passed unmentioned in the LHCb analysis, it is too tempting for theorists to pass up. We therefore consider a four-channel model that adds the $\bar{D}_1K^*$ mode. This will naively add a triangle cusp near $m(D_1) + m(K^*) = 3315$ MeV that could prove useful in describing the high mass region. The resulting fit for the strong FSI case is shown as the green line in Fig. 2. Once again, the fit quality has improved, with $\chi^2$/dof = 1.21, largely due to a better fit through the high mass region.

At this stage we choose to stop expanding the amplitude model because fits with more parameters will likely lose physical significance. Instead we proceed by analysing the fits that have been obtained to aid in their interpretation. The first task is to examine model sensitivity by varying parameters and refitting for the four-channel case. Since all the resulting fits are very similar to that shown in Fig. 2, we only report the chi-squared per degree of freedom as a measure of the resulting differences (in Table III). The first three entries in the table refer to the FSI models of Table II. We see that the strong case is marginally preferred. The remaining rows implement specific changes to the strong FSI or production models. Some of the changes that are induced can be substantial, but, as the chi-squared shows, all changes can be almost completely absorbed into the free couplings of the amplitude model. As expected, changing the mass of meson $a$ (“$D^*_s$”) as indicated in the final two rows has a substantial effect on the global strength of the amplitude; however, this change can be countered by a commensurate increase in the channel couplings.

We next turn attention to dependence on the assumed strength of the final state interactions. We refit the four-channel model with the weak FSI case as a test of this dependence, obtaining the results shown in Fig. 3. With $\chi^2$/dof = 1.4, it is evident that the weak FSI case describes the data quite well, although it has a slightly poorer fit through
TABLE III: Model Robustness Evaluation

| variation        | $\chi^2$/dof |
|------------------|--------------|
| weak FSI         | 1.36         |
| moderate FSI     | 1.44         |
| strong FSI       | 1.21         |
| $\beta = 500$ MeV| 1.32         |
| replace EW vertex $\tau$ with $\xi$ | 1.22 |
| $m(D^*_{s}) = 2500$ MeV | 1.37 |
| $m(D^*_s) = 2112$ MeV | 1.25 |

The enhancement peak and the high mass region. Because the weak case generates little final state interactions, we conclude that triangle cusps are capable of describing the $\bar{D}K$ mass distribution.

Alternatively, the interactions in the strong FSI case are sufficiently strong to generate resonances, which occur at 2849 $-i23$ MeV and 3173 $-i236$ MeV. These have structure dominated by $\bar{D}^*K^* - \bar{D}_1K$ and $\bar{D}_1K^*$ respectively. Since the strong FSI case has a better fit quality, these observations may be regarded as marginal evidence in favour of exotic (i.e., resonance-forming) dynamics in the $\bar{D}K$ system. Thus a more detailed fit to the full LHCb data set should help clarify the issue and would be interesting to pursue.

It is tempting to interpret the previous observations as evidence for exotic resonances, but as we have noted, non-exotic interpretations are also feasible. An indication of the lack of robustness of the exotic interpretation is obtained by setting $g_{D^*D^*K^*}$ and $g_{D^*D^*_1K}$ to zero and replotting while leaving the other parameters fixed. The result is a very large amplitude, which must be cancelled against the other subamplitudes. This is an indication that the different subamplitudes are acting as a basis in the fit to the data, rather than physically motivated entities. In contrast, performing the same experiment in the moderate FSI case yields subamplitude of reasonable size that are readily interpretable. For this reason, we hesitate to make proclamations about the interpretation of the $X(2900)$, other than to note that simple triangle cusps can explain the mass projection, and strong final state interactions can improve the fit.
IV. CONCLUSIONS

A model of $D\bar{D}K$ production from $B^+$ decay has been developed that combines a triangle diagram production mechanism with nonperturbative final state interactions in the $\bar{D}K$ channel. The final state interactions were modeled with a separable potential with a structure motivated by one-pion-exchange phenomenology. We remark that the experimental widths of the $K^*$ and $D_1$ mesons have been used in the formalism. The reasonably narrow peak seen in Fig. 2 is thus at odds with the broader peaks described in Ref. [48].

The results of a variety of fits to the $\bar{D}K$ invariant mass distribution yield strong evidence that the structure at 2900 MeV can be interpreted as a triangle cusp due to the $\bar{D}^*K^*$ intermediate state, possibly enhanced by the $\bar{D}_1K$ mode. Unlike tetraquark models, this scenario does not imply a proliferation of partner states since, as we have argued, the $\bar{D}^*K^*$ and $\bar{D}_1K$ channels are uniquely important from the perspective of pion exchange.

Our model permits the generation of molecular states with flavour $ud\bar{s}s$ and these are marginally preferred by the fits, with one such FSI generating poles at 2849 - $i23$ MeV and 3173 - $i236$ MeV. Nevertheless, we regard the evidence as weak, primarily because the strong FSI case yields large cancelling amplitudes, which raises the possibility that the good fit is due to intrinsic variability in the model as opposed to the quality of the underlying physics. Performing a fit to the full LHCb data set will provide vital further information that might be able to pin down the model characteristics. It will also be interesting to describe the final state interactions with a one-pion-exchange formalism, as this will help to constrain the model. Finally, the question of the $X_0(2866)$ needs to be addressed: does the model developed here obviate the need for this component, or is it required by the full dataset?

Finally, we remark that other production and decay modes of the $X(2900)$ states are possible. The experimental observation or otherwise of these modes can sharply discriminate among their possible interpretations. We explore these ideas in a forthcoming paper [52].

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