Abstract

Hadronic systems built from a heavy quark and a cloud of light quarks and massless gluons possess the Isgur-Wise symmetry resulting in certain relations between various form factors. These relations cannot be valid in the entire complex plane. We identify contributions which unavoidably violate strongly the symmetry in the time-like domain near thresholds. A possible connection with the heavy-quark phenomenology is discussed in brief.
1 Introduction

Strong interactions in quark-antiquark systems become simpler if the mass of one of the quarks goes to infinity [1]. In particular, the transition amplitudes of the type $(Q\bar{q}) \rightarrow (Q'\bar{q})$, where $Q$ is a generic notation for a heavy quark with mass $m_Q$, which will be used throughout the paper, and $q$ denotes a light (massless) quark, induced by the vector and axial vector currents $\bar{Q}'\gamma_\mu Q$ and $\bar{Q}'\gamma_\mu \gamma_5 Q$, respectively, are related by a symmetry which takes place even if $m_Q \neq m_{Q'}$, but both masses, $m_Q$ and $m_{Q'}$, are parametrically larger than $\Lambda_{QCD}$. This symmetry is called the Isgur-Wise (IW) symmetry [2]. Moreover, the absolute normalization of the form factors in the small velocity limit is fixed [3].

For finite quark masses the symmetry is, obviously, broken. The parameter which governs the breaking of the IW symmetry can be represented by some positive power of the quantity

$$\epsilon = \frac{M_{B^*}^2 - M_B^2}{M_B^2}. \quad (1)$$

Here $B^*$ and $B$ denote generic vector and pseudoscalar mesons (ground states) with the quark content $(Q\bar{q})$. The quadratic mass difference

$$\delta^2 = M_{B^*}^2 - M_B^2 \quad (2)$$

stays constant in the limit $m_Q \rightarrow \infty$; therefore, one might think that in this limit the heavy quark symmetry (essentially, independence of the strong interactions of the spin orientation of the heavy quark) becomes perfect. While in a large range of momentum transfer this should be the case, there exist kinematic domains where $\epsilon$ does not actually measure the strength of the symmetry breaking. This happens, in particular, when the form factors are considered in the time-like region, near the corresponding thresholds. Under these circumstances the genuine symmetry breaking parameter is $\delta^2/(q^2 - 4M^2)$ rather than $\epsilon$, and relations stemming from the IW symmetry do not hold. As a matter of fact, in certain instances the breaking is even parametrically large. This simple observation will be quantitatively worked out below.

This phenomenon is not unique, of course. Similar behavior is well-known, say, for the isotopic invariance of strong interactions. Amplitudes which are, generally speaking, believed to be isotopically invariant can exhibit very strong deviations from the symmetry predictions provided that a typical energy scale in the process at hand is not large compared to the mass difference $m_d - m_u$. A classical example is the $D\bar{D}$ form factor in the vicinity of $\psi(3.77)$. The imaginary part of this form factor in the resonance is strongly different in the channels $D^+D^-$ and $D^0\bar{D}^0$ due to the fact that the energy release is about 40 MeV – quite comparable to $2M(D^+) - 2M(D^0) \approx 10$

1Historically the observation [3] that the transition form factors are predictable in the small velocity limit for $m_Q \neq m_{Q'}$ served as an initial impetus for the introduction of the IW symmetry [2].
MeV. It is quite typical that the symmetry breaking effects, although large, are
calculable in this case.

In this note we analyse, in the same spirit, the peculiarities of the IW symmetry
breaking in the time-like region, mostly due to anomalous thresholds \[4, 5\]. A
remark on possible phenomenological implications in the so-called molecular quarko-
nium (analog of the molecular charmonium \[3, 4\]) is presented. For definiteness we
concentrate on the diagonal vector current

\[ J_\mu = \bar{Q} \gamma_\mu Q, \]  

although the assertions made below are of a general nature and are applicable, in
principle, to other currents – axial, non-diagonal, etc. For the vector current one
can consider three transition amplitudes:

\[ <B | J_\mu | B >, \quad <B | J_\mu | B^* > \quad \text{and} \quad <B^* | J_\mu | B^* >. \]  

Taking into account current conservation one concludes that the first and the second
transitions are described by one form factor each,

\[ <B(p') | J_\mu | B(p) > = F_+ P_\mu, \]
\[ <B(p') | J_\mu | B^*(p) > = -i f \varepsilon_{\mu\alpha\beta\gamma} p'_\alpha p_\beta \epsilon_\gamma, \]  

while the third amplitude is represented by four form factors,

\[ <B^*(p') | J_\mu | B^*(p) > = F_1(\epsilon\epsilon') P_\mu + F_2[\epsilon_\mu(\epsilon'P) + \epsilon'_\mu(\epsilon P)] \]
\[ + F_3(\epsilon P)(\epsilon' P) P_\mu + F_4[\epsilon_\alpha(\epsilon' P) - \epsilon'_\alpha(\epsilon P)] q^2 M_\ast^2 - q_\mu q_\alpha \]
\[ \quad \quad \quad \quad \quad \quad \quad \quad \quad \frac{M_\ast^2}{q^2 g_{\mu\alpha} - g_{\mu\alpha}} , \]

where

\[ P = p' + p, \quad q = p' - p, \]
\[ M_\ast \] is the \( B^\ast \) mass, \( \epsilon \) and \( \epsilon' \) are the polarization vectors of the initial and final \( B^\ast \)'s.
All six form factors, \( F_+, f, F_1, ..., F_4 \), are functions of \( q^2 \).

The heavy quark symmetry reduces the number of independent functions in the
general decomposition \(5\) and \(6\) to one, in the limit \( m_Q \to \infty \) one obtains \[4\] :

\[ \frac{1}{M} <B | J_\mu | B > = \xi(v' + v)_\mu, \]
\[ \frac{1}{M} <B | J_\mu | B^* > = -i \xi \varepsilon_{\mu\alpha\beta\gamma} v'_\alpha v_\beta \epsilon_\gamma, \]
\[ \frac{1}{M} <B^* | J_\mu | B^* > = \xi\{-(\epsilon\epsilon')(v' + v)_\mu + [\epsilon_\mu(\epsilon'(v' + v)) + \epsilon'_\mu(\epsilon(v' + v))]\}. \]

Here \( v_\mu = p_\mu / M_\ast \), and the function \( \xi \) depends in the IW limit only on the product \( y \)
of four-velocities,

\[ y = v' v. \]  

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Moreover, $\xi(y = 1) = 1$ [3]. Let us draw the reader’s attention to the fact that $F_3$ and $F_4$ are predicted to vanish. Any deviation from these relations will signal a violation of the IW symmetry [4]. An obvious violation occurs above the threshold of the $B\bar{B}$ production but below $B\bar{B}^*$. Indeed, in this domain all form factors have imaginary parts associated with the normal thresholds due to $B\bar{B}$. On the other hand, there is no contribution to the imaginary part from $B\bar{B}^*$ and $B^*\bar{B}^*$. It is quite clear that in the pseudoscalar $B$ meson the spin of the heavy quark $Q$ is rigidly correlated with that of the light cloud. Hence, the spin independence of the heavy quark interaction – raison d’être of the IW symmetry and the origin of eq. (8) – is totally lost. In particular, the “forbidden” function $F_3$ appears. Below we will demonstrate this assertion in a less trivial context of anomalous thresholds. The existence of these thresholds is due to pion exchange. A specific feature which makes their analysis interesting is the fact that they can start parametrically much below the normal thresholds, depending on the interplay between $\delta$ and $m_\pi$, the pion mass. Certainly, in the real world both quantities are such as they are. It is instructive, however, to play with $m_\pi/\delta$ treating this ratio as a free parameter. One can easily change $m_\pi$ by adjusting the light quark mass terms in the QCD lagrangian. We will investigate different regimes allowed for the anomalous thresholds in Sect. 2. Among other things it will be seen that under certain conditions the anomalous threshold starts at a value of $q^2$ which is independent of the heavy quark mass. The corresponding contribution, obviously, has no smooth IW limit in the sense that its dependence on the momentum transfer does not reduce to a $y$ dependence, as prescribed by eq. (9). This particular contribution is not leading at $m_Q \to \infty$. There are other regimes, however, (which also defy the heavy quark symmetry) where we find an enhancement by positive powers of $m_Q$ in the near-threshold domain.

The issue of molecular quarkonium (bound states of the type $B^*\bar{B}^*$) and its possible relation to the violations of the IW symmetry is discussed in Sect. 3.

2 Pion Exchange and Anomalous Thresholds

First of all it is instructive to notice that the form factor of $B$ is free from anomalous thresholds while that of the $B^*$ meson is not. Indeed, let us consider the triangle graphs depicted in Figs. 1a, b. Both graphs can lead to singularities on the physical sheet below the normal thresholds the positions of which depend on the external masses. The existence of such anomalous singularities has been first realized by Karplus, Sommerfield and Wichman [4]. From the standard Landau theory [10] it is not difficult to find the conditions under which the anomalous thresholds may occur.

\footnote{The above reduction of six form factors to one for non-relativistic heavy quarks, i.e. at $|\vec{v}| \ll 1$, is known in the literature for more than fifteen years [3], see also Chapter 4 in the review paper [4].}
Inspecting the dual diagrams associated with Figs. 1a, b we conclude the following:

(i) The graph 1a has an anomalous threshold provided that

\[
\frac{\delta^2}{M + M_*} \leq \mu \leq \delta. \tag{10}
\]

The beginning of the anomalous singularity is at

\[
t_{0BB} = 4M^2 - \left(\frac{\delta^2 - \mu^2}{\mu}\right)^2. \tag{11}
\]

To avoid numerous sub- and superscripts we have introduced the notations \(t \equiv q^2\), \(\mu \equiv m_\pi\), \(M \equiv M_B\) and \(M_* \equiv M_{B^*}\). The position of the anomalous threshold varies from zero at the lower boundary of the interval (10) to the normal threshold \(4M^2\) at the upper boundary.

(ii) The condition on \(\mu\) guaranteeing the existence of an anomalous threshold in graph 1b is:

\[
\frac{\delta^2}{M + M_*} \leq \mu \leq \frac{\delta}{\sqrt{1 + (M/M_*)}}. \tag{12}
\]

The lower and upper boundaries in eq. (12) correspond to the position of the anomalous threshold at \(M_*(M_* + M)\) and \((M + M_*)^2\), respectively.

The general expression for the position of the anomalous threshold in this case is

\[
t_{0BB^*} = M^2 + M_*^2 + \frac{1}{2}(\delta^2 - \mu^2) + \left[\left(\frac{4M_*^2}{\mu^2} - 1\right)(\mu^2 M^2 - \left(\frac{\delta^2 - \mu^2}{2}\right)^2)\right]^{1/2}. \tag{13}
\]

A remarkable feature of eq. (11) is the fact that the position of the anomalous threshold, \(t_{0BB}^\star\), needs not to scale as \(M^2\) in the limit \(m_Q \to \infty\); if \(\mu\) is chosen to lie close to \(M_* - M\) other regimes are quite possible. For instance, if \(\mu = (M_* - M)(1 + \text{const.}M^{-2})\) we find that \(t_{0BB}^\star\) scales like \(M^0\). The momentum transfer dependence then will be expressed by a function of \((M^2/t_{0BB}^\star)(1 - y)\) rather than a function of \(y\), and the IW scaling will be obviously violated. Of course, the above regime is exotic since it requires fine-tuning of the pion mass. One can always fix the pion mass and then proceed to infinitely heavy quarks. This will push \(t_0\) to \(4M^2\) and will restore the \(y\) dependence.

The position of the anomalous threshold in the complex plane, although interesting by itself, says nothing about the relative weight of the anomalous singularities. \textit{A priori} it is conceivable that they totally decouple at \(m_Q \to \infty\). Whether or not they actually decouple depends on the behavior of the pion constant.

A brief reflection shows that the leading \(M\) dependence of the pion coupling to the heavy mesons should be such that after proceeding to non-relativistic normalization \(M\) should disappear altogether \([7, 11]\). A relation between the \(B^*B^*\) and
\(B^*B\pi\) vertices can be readily obtained by combining the heavy quark and chiral symmetries \([12, 13, 14]\). In the relativistic normalization one gets

\[
B^*B\pi : \frac{4g}{f_\pi} M(\epsilon k) + O(M^0), \tag{14}
\]

\[
B^*B^*\pi : -i\frac{2g}{f_\pi} \varepsilon_{\alpha\beta\gamma\delta}(p + p')_{\alpha} k_{\beta}\epsilon'_{\gamma}\epsilon_{\delta}, \tag{15}
\]

where \(p\) , \(p'\) and \(k = p - p'\) are the momenta of the initial and final heavy mesons and the pion, \(g\) is a dimensionless coupling constant of order 1 \([12, 13, 14]\), and \(f_\pi\) is the pion decay constant. It is easy to check, for instance, that with eq. (14) the \(D^* \to D\pi\) width does not contain the heavy meson mass, in full accordance with the arguments of refs. \([7, 11]\).

We pause here to make an important remark. The fact that the \(B^*B\pi\) vertex is proportional to \(M\) (in the relativistic normalization) is a consequence of the standard assumptions of the effective heavy quark theory \([15, 16]\). Under certain kinematic conditions when the IW symmetry is broken in the sense discussed in this note, the \(BB^*\pi\) vertex may deviate from the standard scaling law exhibited in eq. (14). In other words, this vertex may be proportional to another power of \(M\) for trivial kinematic reasons which will become clear shortly. For the time being let us stick, however, to eq. (14), with \(g = O(1)\).

It is important that the discontinuity of the amplitude at the anomalous cut is determined by the triangular graphs of Fig. 1 with all three internal particles on mass shell \([4, 10]\). Therefore, the pion coupling constant which enters all our formulæ is the on-mass-shell coupling.

Since our purpose is mostly illustrative it is reasonable to limit our analysis to the anomalous singularities of graph 1a . The singularity of this graph can, in principle, be arbitrary close to \(t = 0\) while that of Fig. 1b necessarily lies higher than \(2M^2\) (see eq. (13)) and, moreover, can be pushed to the unphysical sheet provided that \(\mu\) is chosen larger than \(\delta(1 + (M/M_\ast)^{-1/2})\).

The contribution of the triangle graph depicted on Fig. 1a can be written as follows:

\[
A_{\mu} = 16i\tilde{g}^2 \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2 - \mu^2} \frac{(k\epsilon)(k\epsilon')(p + p' + 2k)_\mu}{((p + k)^2 - M^2)((p' + k)^2 - M^2)}, \tag{16}
\]

where according to eq. (14):

\[
\tilde{g} = \frac{gM}{f_\pi}. \tag{17}
\]

The anomalous cuts can be easily evaluated using Cutkosky \([4]\) rules, yielding the imaginary parts \(\rho_i(t)\) of the form factors \(F_i(t)\),

\[
\rho_1(t) = \frac{\tilde{g}^2(4M_\ast^2\mu^2 - \delta^4 - \mu^2(2\delta^2 + \mu^2 + t))(4M_\ast^2 - 2\delta^2 - 2\mu^2 - t)}{\sqrt{t}(4M_\ast^2 - t)^{5/2}}, \tag{18}
\]
\[
\rho_2(t) = -\tilde{g}^2 \frac{\left(4M^2\mu^2 - \delta^2 - \mu^2(2\delta^2 + \mu^2 + t)(2\delta^2 + 2\mu^2)\right)}{\sqrt{t(4M^2 - t)^{5/2}}},
\]
\[
\rho_3(t) = -2M^2 \frac{\partial}{\partial t} \rho_1(t),
\]
\[
\rho_4 = 0.
\]

It is quite obvious that the above expressions badly violate the heavy-quark symmetry, since \(\rho_1 \neq -\rho_2\) and, moreover, \(\rho_3 \neq 0\). The above equations should be supplemented by the statement that the anomalous part does not vanish in the limit \(M \to \infty\). It is convenient to carry out the analysis of the absolute normalization separately in two distinct cases:

(i) The case in which \(\delta\) and \(\mu\) are fixed and thus the anomalous threshold \(t_0^{BB}\) is near the normal one. This is referred to as the relativistic case;

(ii) the case in which \(\delta\) stays fixed but \(\mu\) decreases at least as \(M^{-1}\) and thus the anomalous threshold is far below the normal one. This is referred to as the non-relativistic case.

It should be noted that in the case considered here the anomalous singularities are due to hadronic intermediate states and not to quarks, as in the case of formfactors of heavy quarkonia, considered in refs. [18, 19].

2.1 Relativistic case

If \(\delta\) and \(\mu\) are fixed the anomalous threshold stays, according to eq. (11), in the vicinity of the normal threshold, i.e. the relevant variable is \(u = 4M^2 - t\). Keeping only the leading in \(M\) terms we get

\[
\rho_1(u) = \tilde{g}^2 \frac{2\delta^2 + 2\mu^2 - u(\delta^4 + 2\delta^2\mu^2 + \mu^2(\mu^2 - u))}{2M^2 u^{5/2}},
\]
\[
\rho_2(u) = \tilde{g}^2 \frac{(\delta^2 + \mu^2)(\delta^4 + 2\delta^2\mu^2 + \mu^2(\mu^2 - u))}{M^2 u^{5/2}},
\]
\[
\rho_3(u) = -\tilde{g}^2 \frac{M^4}{2M^2 u^{7/2}} \{\mu^2 u^2 - 3u(\delta^4 + 4\delta^2\mu^2 + 3\mu^4) + 10(\delta^2 + \mu^2)^3\}
\]

The anomalous cut in the \(u\) plane extends from \((\delta^2 + \mu^2)^2/\mu^2\) to \(4\delta^2\). If \(u = O(\delta^2)\) and \(\tilde{g} = O(M)\) the discontinuities are large rather than small,

\[
\rho_1(u), \rho_2(u) \propto M, \rho_3(u) \propto M^3,
\]

leading to a huge violation of the heavy-quark symmetry in the near-threshold domain (by the near-threshold domain we mean an interval of \(t\) centered at \(4M^2\) whose length is \(O(\delta^2)\).) Furthermore, it is quite trivial to do the dispersion integral with the discontinuities given in eqs. (22-24). With

\[
F^\text{an}_i \equiv \frac{1}{\pi} \int_{t_0}^{4M^2} \rho_i(t')dt'/t' - t
\]
one finds outside the near-threshold domain (at $t = O(M^2)$)

$$F_1^{an} \sim F_2^{an} \sim \frac{1}{M}, \quad F_3^{an} \sim M. \quad (27)$$

Specifically we obtain

$$F_3^{an}(t = 0) = \tilde{g}^2 \frac{(\delta^2 - \mu^2)^3}{64M_s\delta^5} = \frac{g^2(\delta^2 - \mu^2)^3M_s}{4\delta^5f_\pi^2}. \quad (28)$$

The expression (28), taken at its face value, would signal a parametrically large symmetry violation even at $t = 0$. We hasten to add that the anomalous terms we have calculated should be (better to say, are expected to be) cancelled by normal contributions of the hadronic graphs if one considers the form factors $F_i$ outside the near-threshold domain.

It is instructive to check how the above cancellation works by directly computing the graphs of Fig. 1 (this is not a realistic computation of the form factors for many reasons, of course, but just an exercise allowing one to see the restoration of the symmetry). If $t$ is not especially close to the threshold, the diagram 1a is not singled out. One must consider all possible triangle graphs, both for $B^*$ and $B$, with the intermediate states containing $BB$, $BB^*$ and $B^*B^*$. Adding them together, we observe that all symmetry-violating terms with positive powers of $M$ cancel, if we start with formfactors satisfying eq. (8).

### 2.2 Non-relativistic limit

Let us consider another limiting case, when $\mu$ is only slightly larger than $M_s - M$, so that $\mu$ scales as $M^{-1}$. Then the anomalous threshold begins far below the normal one, i.e. $t_0 \ll 4M^2$.

We start with

$$\mu = \frac{\delta^2}{2M\gamma} \quad (29)$$

where $\gamma$ is a fixed number close to 1 but slightly less than one. Then

$$t_0 \approx (1 - \gamma^2)4M^2. \quad (30)$$

Neglecting $t$ compared to $M^2$ we get for the anomalous discontinuities near the point $t = t_0$ the leading terms:

$$\rho_1(t) = \frac{\tilde{g}^2\delta^4}{8\gamma^2M_s^2\sqrt{t}} (1 - \gamma^2), \quad (31)$$

$$\rho_2(t) = -\frac{\tilde{g}^2\delta^6}{16\gamma^2M_s^6\sqrt{t}} (1 - \gamma^2), \quad (32)$$

$$\rho_3(t) = \frac{\tilde{g}^2\delta^4}{8\gamma^2M_s^3\sqrt{t}} (1 - \gamma^2). \quad (33)$$
The anomalous contribution depends on $t$ through $t/[4(1-\gamma^2)M_*^2]$, and the form factors scale in the following way

\[ F_1^{an} \sim F_3^{an} \sim M_*^{-2}, \quad F_2^{an} \sim M_*^{-4}. \]  

(34)

In other words, the anomalous contribution decouples in the limit $m_Q \to \infty$.

Of more interest is the case when $\gamma \to 1$ for $M \to \infty$. Specifically, let us consider

\[ \mu = (M_* - M)(1 + \frac{x}{M^2}), \]  

(35)

where $x$ is a parameter of order $\delta^2$. Then $t_{BB}^B \approx 8x = O(M^0)$. Under this choice – when $M + \mu - M_* \equiv E = O(M^{-3})$ – it is reasonable to turn to a model according to which $B^{*}$ is a non-relativistic bound state of $B\pi$, or, at least, such a four-quark (molecular) component is present in $B^*$ with a certain prabability.

To make the situation more graphic, assume, at first, that $B^*$ completely reduces to a loosely bound system of $B\pi$, analogous to the deuteron. Clearly the spin symmetry between $B^{*}$ and $B$ is maximally violated in this case, since the pseudoscalar meson, the would-be partner of the $B^*$, does not look like this bound $B^*$ at all. Moreover, in this case one would expect that the heavy-quark symmetry is violated not only in the near-threshold domain, but everywhere in the complex plane. The question is how this comes out formally.

The answer to this question is rather obvious. If $B^*$ is like the deuteron, its coupling constant $g$ is rigidly fixed in terms of the binding energy $E$ (see e.g. [20] for an S-wave bound state) and turns out to be much larger than that given in eq. (17). Before we proceed to derive the relation between $E$ and $\tilde{g}$ let us quote the expressions for the anomalous cuts in this limit,

\[ \rho_1(t) = \frac{-\tilde{g}^2\delta^4}{32M_*^5\sqrt{t}}(t - 8x), \]  

(36)

\[ \rho_2(t) = \frac{\tilde{g}^2\delta^6}{64M_*^7\sqrt{t}}(t - 8x), \]  

(37)

\[ \rho_3(t) = \frac{\tilde{g}^2\delta^4}{32M_*^3 t^{3/2}}(t + 8x). \]  

(38)

If we had substituted $\tilde{g} = O(M)$ we would have got at $t = 0$

\[ F_1^{an}(0) \sim 1/M_*^2 \quad F_2^{an}(0) \sim 1/M_*^5 \quad F_3^{an}(0) \sim 1/M_* \]  

(39)

Now, let us derive the actual scaling law for $\tilde{g}$ in this scenario of a loosely bound state. Consider the non-relativistic contribution of diagrammm 1a , by doing the corresponding manipulations in eq. (16). We put the pion on mass shell, i.e. take only the imaginary part of its propagator, which just singles out the anomalous contribution. We then replace the zero components of the occurring Lorentz-vectors by their corresponding nonrelativistic expressions:

\[ p_0 = M_* + \vec{p}^2/(2M_*), \quad k_0 = \mu + \vec{k}^2/(2\mu) \]  

(40)
The pion is nonrelativistic, since the binding energy and hence the virtual momenta are small as compared to the pion mass according to (34). It is most convenient to work in the Breit-frame, where $\vec{p} = -\vec{p}' = \vec{q}/2$. We also neglect the pion mass $\mu$ against the heavy mass $M_*$. This yields for the zero component of the current:

$$\Delta_{\text{nonrel}} = \frac{i\tilde{g}^2\mu}{16M_*(2\pi)^3} \int d^3k \frac{(\vec{k} \cdot \vec{\epsilon} - \mu/(2M_*)\vec{q} \cdot \vec{\epsilon})(\vec{\epsilon}' \cdot \vec{k} - \mu/(2M_*)\vec{\epsilon} \cdot \vec{q})}{(\vec{k} - \mu/(2M_*)\vec{q})^2 + \alpha^2(\vec{k} + \mu/(2M_*)\vec{q})^2 + \alpha^2}$$  (41)

where $\alpha = \sqrt{2\mu E}$ with the binding energy

$$E = M + \mu - M_*$$  (42)

We compare this expression to the form factor of the $B^*$-meson, which is assumed to be a bound state of a $B$-meson and a pion. We evaluate it in the impulse approximation with the momentum space wave function

$$\tilde{\psi}(\vec{k}) = N_\alpha f(k/\alpha)\vec{k}/(k^2 + \alpha^2)$$  (43)

with $k = |\vec{k}|$. The unphysical pole at $k^2 = -\alpha^2$ reflects the exponential tail of the pion cloud.

For the normalization constant we obtain

$$1/N_\alpha^2 = 4\pi \alpha \int \frac{f(y)^2y^4}{(y^2 + 1)^2} dy,$$  (44)

i.e. $N_\alpha^2 \sim C/\alpha$. The impulse approximation for the zero component of the vector form factor yields:

$$\Delta^{i.a.} = \frac{2M_*}{2\pi^3} \int d^3k\vec{e}' \cdot \tilde{\psi}(\vec{k} - \mu/(2M_*)\vec{q}) \tilde{\psi}^*(\vec{k} + \mu/(2M_*)\vec{q}) \cdot \vec{\epsilon}$$  (45)

A comparison of (41) and (45) shows, that the function $f(k/\alpha)$ in (43) corresponds to some momentum cutoff in (41), and that the two expressions coincide, if the coupling constant is given by

$$\tilde{g}^2 = \frac{CM_*^2}{\mu\alpha}$$  (46)

where the constant $C$ depends on the momentum cutoff. Notice that the relation between $\tilde{g}$ and $E$ is different from that discussed in [20] due to the fact that the resonance we consider is P-wave.

Thus, for $\mu$ scaling like (35) we get

$$\tilde{g}^2 \propto M_*^5.$$  (47)

It is rather straightforward to check that the form factors within this scenario will have nothing to do with eq. (8), as expected.
A more realistic scenario will be to say that there is an admixture of the \( B\pi \) bound state in \( B^* \). If the amplitude of this component scales as \( M^{-3/2} \), we have \( \tilde{g} = O(M) \) and return back to eq. (39).

The scale for the \( t \)-dependence of the anomalous contributions is now however \( t_0^{BB} \), which does not scale with \( M^* \) and hence we have for that case a – not very spectacular – violation of IW symmetry, which might be remarked as a spike near \( t = 0 \) in the derivatives of \( F_i(t) \).

The question “what is the actual admixture of the molecular \( B\pi \) state in \( B^*? \)” is a dynamical one, and cannot be solved on the basis of essentially kinematical arguments presented above. The answer depends on the intricacies of the large distance dynamics and can be a non-trivial function of \( E \). While intuitively it is clear that the overlap is less than 1, it needs not be as small as \( M^{-3/2} \). Then \( \tilde{g} \) will not scale as prescribed by eq. (17). If the overlap is larger than \( O(M^{-1/2}) \), the spin symmetry violation in the entire complex plane will survive in the limit \( M \to \infty \).

We should mention that in the chiral limit \( \mu = 0 \) there is no anomalous threshold on the physical sheet. In that case the \( B^* \) is unstable, but its width vanishes as \( g^2 \delta^2/(f_\pi^2 M^3) \).

3 Molecular Quarkonium

We would like to remind that the well-forgotten issue of the molecular quarkonium – loosely bound states in the systems \( B\bar{B}, B\bar{B}^*, B^*\bar{B}, B^*\bar{B}^* \) is relevant to the discussion of the heavy-quark symmetry in the complex plane. If the long-range forces binding these “molecules” are independent of the heavy quark spins, i.e. eq. (8) holds, it is easy to check that the ratio of yields in these channels is 1:4:7. This is the ratio stemming from eq. (8) after squaring these expressions and averaging over the spacial orientations of the momenta. The question is how the symmetry is realized, if at all, in the “molecular” domain. (The “molecules” can be either bound states or resonances above threshold.)

The answer depends on the relation between the spin splitting \( \delta^2/(2M) \) and the molecular-level splittings. For the lowest molecular levels the latter are expected to be parametrically larger than \( \delta^2/(2M) \). Then the spin symmetry will be reflected in an (approximate) triple degeneracy of the levels. On average (summing over the triplet of the degenerate levels) all symmetry relations, including 1:4:7 for the ratio of yields, will be fullfilled, but at exactly the position of each individual resonance they are maximally violated. On the other hand, if there are molecular states very close to the threshold, \( M_{\text{res}} - 2M_* \leq \delta^2/(2M) \), it can well happen that such resonances exist, say, only in the \( B^*\bar{B}^* \) channel and are absent in \( B\bar{B} \) (as is the case in charmonium). Then the form factors near the resonances will not be symmetric.
4 Concluding Remarks

We have investigated the implications of anomalous thresholds of heavy meson formfactors due to pion exchange. For the actual values of the pion mass and the quadratic mass splitting of the pseudoscalar and vector heavy mesons we have found violations of the IW symmetry near the production threshold of pairs of these mesons. These violations are not only the trivial ones, due to the finite quadratic mass splitting, but singularities of formfactors, which are predicted to vanish become parametrically large, i.e. increase with the heavy quark mass $m_Q$. The relation of these violations of the IW symmetry near $t = 4m_Q^2$ with the molecular quarkonium is stressed.

If the pion mass is treated as free parameter, anomalous thresholds might approach the region $t = 0$ arbitrarily close. The quantitative importance of such ”low-lying” singularities depends on the degree of admixture of the molecular $B\pi$ state in the $B^*$-meson.

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Figures

Figure 1. Diagrams with pion exchange contributing to the $B^*$ formfactor. Note that there is also a diagram corresponding to Fig. 1b with the internal $B$- and $B^*$-lines interchanged.