APPLICATION OF ROBUST OPTIMIZATION FOR A PRODUCT PORTFOLIO PROBLEM USING AN INVASIVE WEED OPTIMIZATION ALGORITHM

ALIREZA GOLI AND HASAN KHADEMI ZARE*
Department of Industrial Engineering
Yazd University, Saffayieh, Yazd, Iran

REZA TAVAKKOLI-MOGHADDAM
School of Industrial Engineering, College of Engineering
University of Tehran, Tehran, Iran

AHMAD SADEGHIEH
Department of Industrial Engineering
Yazd University, Saffayieh, Yazd, Iran

(Communicated by Zhong Wan)

ABSTRACT. Product portfolio optimization (PPO) is a strategic decision for many organizations. There are several technical methods for facilitating this decision. According to the reviewed studies, the implementation of the robust optimization approach and the invasive weed optimization (IWO) algorithm is the research gap in this field. The contribution of this paper is the development of the PPO problem with the help of the robust optimization approach and the multi-objective IWO algorithm. Considering the profit margin uncertainty in real-world investment decisions, the robust optimization approach is used to address this issue. To illustrate the real-world applicability of the model, it is implemented for dairy products of Pegah Golpayegan Company in Iran. The numerical results obtained from the IWO algorithm demonstrate the effectiveness of the proposed algorithm in tracing out the efficiency frontier of the product portfolio. The average risk of efficient frontier solutions in the deterministic model is about 0.4 and for the robust counterpart formulation is at least 0.5 per product. The efficient frontier solutions obtained from robust counterpart formulation demonstrate a more realistic risk level than the deterministic model. The comparisons between CPLEX, IWO and genetic algorithm (GA) shows that the performance of the IWO algorithm is much better than the older algorithms and can be considered as an alternative to algorithms, such as GA in product portfolio optimization problems.

1. Introduction. One of the most important decisions in production and service-based enterprises and organizations is the choice of suitable product portfolio for planning and production over long-term horizons. The term of portfolio refers to a combination of assets that are being invested. Technically, a portfolio includes a set of real and financial assets invested in by an investor, such as stocks, bonds, projects, or even a set of products [8]. According to Cardozo and Smith, given

2010 Mathematics Subject Classification. Primary: 58F15, 58F17; Secondary: 53C35.
Key words and phrases. Product portfolio optimization; Portfolio; Invasive weed optimization; Investment risk; Robust optimization.
the wide scope of portfolio optimization in the field of finance, modern portfolio theory (MPT), which was first introduced by Markowitz [20] is a viable approach for product portfolio planning [5].

Investment markets typically offer a wide range of assets as investment opportunities with varying degrees of quality. However, because of information asymmetry, the identification of high-quality assets is so difficult. An Markowitz mean-variance model help us to find a portfolio such that the risk for a given level of return is minimized [21]. This model has been the subject of many studies. However, solving portfolio selection problems is still a challenge in the field of Financial Engineering. In 2009, Maringer showed that the computational cost of solving this problem increases exponentially with the increase of a number of assets in the portfolio [22]. This highlights the need for better and faster solution methods for the Markowitz mean-variance model. Therefore, many researchers have invested great amounts of effort in providing efficient and fast solution methods for this problem.

One of the main challenges of decision making about investment portfolio selection is the uncertainty of future returns, which contradicts the basic assumption of mathematical programming, which assume all parameters to be deterministic. Making such assumption means ignoring the effect of uncertainty on the quality and validity of solutions. So the cases, in which a parameter may deviate from its nominal value, may lead to the violation of problem constraints and production of suboptimal or even infeasible solutions.

One of the widely accepted ways of dealing with uncertainty in the field of optimization is a methodology called Robust Optimization (RO). Recent years has seen a surge in the popularity of robust optimization because of its high efficiency and practical applicability. In RO, the goal is to find the best solution that is feasible for any realization of uncertainty in a given dataset. RO provides the flexibility required to check the quality of any solutions, rather than just giving one solution based on the worst-case scenario. In addition, the exponential increase in computational complexity with the number of non-deterministic parameters does not disrupt the utility of this methodology [1].

In RO, near-optimal solutions that are highly likely to be feasible are searched. In other words, the objective function is being slightly relaxed in order to guarantee the feasibility of the resulting solution. The notable work in this area of research is the article of Mulvey et al. [25], in which they presented an approach that combined the goal programming formulation with scenarios.

Three fundamental models for RO are based on interval uncertainty, which have been developed by Soyster [31], Ben-Tal and Nemirovski [2], and Bertsimas and Sim [3].

Soyster [31] presented a linear optimization model that provides the best feasible solution, in which each uncertain input parameters can take any value from its interval. However, this approach tends to produce over-conservative solutions, as it allows too much deviation from nominal optimality to ensure the robustness of the solution. Ben-Tal and Nemirovski [2] presented several algorithms for solving convex optimization problems with an ellipsoidal uncertainty set. However, this robust formulation has been derived for the conic quadratic problem, so it cannot be directly used for discrete optimization problems. Bertsimas and Sim [3] introduced a different approach for contorting the level of conservatism. The advantage of this approach is that a linear optimization model will be generated, and therefore it can be applied for discrete optimization problems.
In this study, we use Bertsimas and Sim robust counterpart formulation. The important advantages of this formulation over other two models can be summarized as follows [15]:

1. The model of Soyster is extremely conservative and its objective function values are much worse than those given by the nominal linear optimization model. Hence, this method imposes huge amounts of extra cost and is hardly suitable for solving real problems.

2. The model of Ben-Tal and Nemirovski is a conic quadratic model with a large number of variables and constraints, which is ill-suited for solving discrete optimization models because of its nonlinearity.

The goal of this study is to provide a general approach for robust optimization of product portfolio problem. This goal is pursued by first formulating the problem, then developing an Invasive Weed Optimization (IWO) algorithm for a portfolio selection problem, and finally discussing and analyzing the results.

The rest of this article is organized as follows. A review of the subject literature is provided in section 2. The mathematical model of a product portfolio problem is presented in section 3. Section 4 presents the robust optimization approach developed for dealing with uncertainty in problem parameters. The structure of the IWO algorithm is described in section 5. Section 6 presents the numerical results obtained from IWO, GA and CPLEX. Finally, section 7 concludes the research.

2. Review of literature. In a study conducted by Fernandez and Gomez (2007), they used the Hopfield network and a heuristic method to determine the efficiency frontier for the portfolio optimization problem. They considered the Markowitz portfolio optimization model. The heuristic approach developed in this study was inspired by the transfer function of the Hopfield neural network. At the end, the proposed method was compared with the genetic algorithm (GA), Tabu Search (TS), and simulated annealing (SA) [11].

Fernandes et al. (2012) proposed a new approach for investing in product portfolio with manufacturing flexibility taken into account. They defined the main objective as the reduction of total cost of timely satisfaction of customer demands in the presence of uncertainty in demand values and then optimized the decisions such as product portfolio, required technologies, and the method of procurement of raw materials accordingly. The results of this research show that increasing the manufacturing flexibility affects the product portfolios and their sales prices [12].

Solatikia et al. (2014) presented a mathematical model for portfolio optimization in completely fuzzy conditions. This model had two objectives of maximizing the return and mitigating the risk expressed as return variance. This mathematical model was optimized using a crisp weight method. In the end, the solutions of this model were evaluated and compared with similar instances in previous works [30].

Azari et al. (2015) studied the problem of optimal product portfolio selection in a three-level supply chain where demands for products are predetermined and several products can be manufactured simultaneously. The goal was defined as selecting the product portfolio that maximizes the chain profit without violating the chain constraints. A teaching-learning based optimization (TLBO) algorithm was developed to solve this problem. The numerical results showed that portfolio planning attitude has a significant impact on the profitability of the supply chain [32].
Esfahani et al. (2016) studied the project portfolio optimization based on Markowitz model. This research considered two mathematical models. In the first model, the objective was to maximize the revenue by limiting the risk of the entire project portfolio. In the second model, the objective was to minimize the risk of the portfolio by limiting the total revenue to an optimal level. Finally, these two models were combined into a new mathematical model with the objective of maximizing the weighted sum of risk and revenue of the project portfolio. A harmony search algorithm was developed to solve this mathematical model. The results showed that the proposed algorithm can produce near-optimal solutions [10].

Lejeune and Shen (2016) proposed a probabilistic multi-objective model for financial portfolio optimization with risk exposure. In this model, the risk of investment in each financial market was defined as a probability. The objective was defined as the maximization of the return of the formed portfolio. To solve this model, it was transformed into an integer linear problem. The results demonstrated the effectiveness of this solution approach, compared to exact solution methods, in solving probabilistic multi-objective models [16].

Relich and Pawlewski (2017) studied the problem of product portfolio selection with fuzzy conditions. In this research, a questionnaire was used to inquire about the weight of importance of each product, then linguistic variables were interpreted into fuzzy numbers, and the weight of each product was transformed into a fuzzy number. The objective was to select the products with the highest fuzzy importance with the manufacturing constraints taken into account. After solving the problem, the success of selected product portfolio was evaluated using a BP neural network. The numerical results of this study demonstrated the efficiency of the proposed method and its consistency with reality [16].

Montajabiha et al. (2017) presented a mathematical model for project portfolio optimization under uncertainty. In this mathematical model, the objective was to maximize the compound profit of the project. The budget constraints and dependency relationships of projects were also incorporated into the formulation. A robust optimization approach was used to deal with the uncertainty in the profit of projects. The resulting model was solved using a robust combinatorial algorithm. The results showed that project details can have a deep impact on even the simplest selection of project portfolio under uncertainty [24].

Qu et al. (2017) formulated the problem of multi-objective portfolio optimization with the objective of reducing investment risk and increasing stock return. They then provided a decomposition-based evolutionary algorithm for solving this problem. The computational results demonstrated the capability of the proposed method in finding the best possible stock portfolio in large-scale problems [28].

2.1. Literature of meta-heuristic algorithms for portfolio optimization. A significant portion of research into the subject of portfolio selection is dedicated to the development and use of meta-heuristic algorithms for this purpose. For example, Cura (2009) used the Particle Swarm Optimization (PSO) algorithm to solve a constrained portfolio optimization problem. In that study, this technique was used on the weekly prices of a limited number of stocks in different markets over a five-year period from 1992 to 1997 in order to obtain an efficiency frontier. Finally, it was concluded that this technique works well for portfolio optimization [7].

Lin and Liu (2008) used a Genetic Algorithm (GA) to improve the solution of Portfolio selection problems based on the mean-variance model. In this research,
a two-stage GA was used to solve the multi-objective stock portfolio optimization problem with the objectives of maximizing the returns and minimizing the investment risk with the Markowitz model serving as the fundamental mathematical formulation [17].

Ponsich et al. (2013) reviewed the prominent multi-objective evolutionary algorithms developed for financial portfolio optimization problems. This study found that Non-dominated Sorting Genetic Algorithm (NSGA), Strength Pareto Evolutionary Algorithm (SPEA), and Indicator-Based Evolutionary Algorithm (IBEA) are the algorithms most widely used in the literature for portfolio optimization [27].

Cruz et al. (2014) presented a multi-objective Non-Outranked Ant Colony Optimization (NOACO) for solving portfolio optimization problems. This algorithm determines the Pareto frontier by tracing out user-preferred non-dominated solutions using the structure of ant colony algorithm [6]. Tuba and Bacanin (2015) developed the Upgraded Firefly Algorithm (UFA) for portfolio optimization problems. The stated goal of this work was to improve the quality of solutions of firefly algorithm for the mentioned problem. Ultimately, these modifications resulted in a significant improvement in the results of the firefly algorithm for portfolio optimization problems [34].

Pai and Michel (2017) proposed a hybrid PSO algorithm for the optimization of large portfolios. To achieve better performance, the initial solutions were generated by Sequential Quadratic Programming (SQP) and then improved using the PSO. The results showed that this approach produces better results than the PSO in which initial solutions are generated at random [26]. El-Bizri and Mansour (2017) introduced a Modified Cuckoo Search algorithm (MCS) for portfolio optimization problems. In this work, the basic cuckoo search was modified by adding a number of local optima to the algorithm structure in order to accelerate the search for global optimum [9]. Macedo et al. (2017) presented a hybrid evolutionary algorithm for multi-objective stock portfolio optimization with the risk considered as semi-variance. In this study, the results of NSGA-II and SPEA were first evaluated in terms of quality and then subjected to a technical analysis [18]. The review of the most contributed researchers in the field of portfolio selection shows that only a few researchers have shown interest in using the robust optimization approach for portfolio selection under uncertainty. Also, the utility of new and alternative algorithms such as IWO in product portfolio optimization seems to be low attention. Therefore, the main contribution of this research is to address the research gap in regard to the application of robust optimization approach and IWO algorithm for product portfolio optimization under uncertainty.

3. Product portfolio optimization model. Markowitz expressed the mean-variance mathematical model as a quadratic programming problem with the objective of minimizing the variance of assets for a given expected return [19]. The basic assumption of this model is the risk aversive nature of all investors. In this model, the weight of each asset in the portfolio should be a real and non-negative number and the total weight of all assets should be equal to 1. The product portfolio optimization model which is used in this study is defined based on the Markowitz model. The definition of indexes, parameters, and decision variables is provided below. In this study, the general form of product portfolio optimization model is defined as follows:
Minimize $\lambda \left[ \sum_{i=1}^{N} \sum_{j=1}^{N} z_i x_i z_j x_j \sigma_{ij} \right] - (1 - \lambda) \left[ \sum_{i=1}^{N} z_i x_i \mu_i \right] \quad (1)$

Subject to

\[ \sum_{i=1}^{N} x_i = 1 \quad (2) \]

\[ \sum_{i=1}^{N} z_i = K \quad (3) \]

\[ \varepsilon_i z_i \leq x_i \leq \delta_i z_i \quad \forall \ i = 1, \ldots, n \quad (4) \]

\[ \sum_{i=1}^{N} x_i \mu_i D_i \geq R^* \quad (5) \]

\[ z_i \in [0,1] \quad \forall \ i = 1, \ldots, n \quad (6) \]

\[ x_i \geq 0 \quad \forall \ i = 1, \ldots, n \quad (7) \]

Eq.1 of the mathematical model minimizes the weighted average of risk and return. In this model, $\lambda$ is a tuning parameter that called risk aversion factor and varies in the interval $[0, 1]$; $\lambda = 0$ represents complete concentration of weight on the return, i.e. selection of portfolio with the highest return regardless of risk, and $\lambda = 1$ represents complete concentration on the risk, i.e. selection of portfolio with the lowest risk regardless of return. By adjusting the $\lambda$-value between zero and one, the portfolio can be optimized according to the user’s preferred stance toward risk and return. In other words, higher $\lambda$ values can be used to obtain a more risk-averse portfolio, but with less emphasis on profitability (as a result of reduced $\lambda - 1$) [3].

Eq.2 states that the sum of the ratios of investment in different products should be equal to 1. This equation ensures that the desired return will be realized. Eq.3 ensures that $K$ number of products will be included in the portfolio. Eq.4 specifies the minimum and maximum capital required for each selected product, and Eq.5 ensures that the minimum investment profit will be achieved. Finally, Eqs.6-7 specify the types of decision variables.

4. Robust optimization approach. Bertsimas and Sim have introduced a general formulation for robust optimization. In this section, the formulation of Bertsimas and Sim for linear optimization problems is explained where the objective function is of minimization type and there are uncertainty factors in both objective function and constraints. Then, the model used in the present study and how the formulation of Bertsimas and Sim robust counterpart formulation is described. Let each constraint coefficient $\tilde{a}_{ij}, i \in \{1, 2, \ldots, m\}, j \in \{1, 2, \ldots, n\}$ as an independent random variable with symmetric but unknown distribution $\tilde{a}_{ij}$, which takes values in the interval $[a_{ij} - \hat{a}_{ij}, a_{ij} + \hat{a}_{ij}]$ where $\hat{a}_{ij}$ represents the deviation from the nominal coefficient $a_{ij}$. Each objective function coefficient is modeled as $\tilde{c}_j, j \in \{1, 2, \ldots, n\}$, which takes values in the interval $[c_j - \hat{c}_j, c_j + \hat{c}_j]$, where $\hat{c}_j$ represents the deviation from the nominal coefficient $c_j$. To formulate the robust counterpart of the problem, $\Gamma_i$ (for uncertain constraints) and $\Gamma_0$ (for objective
function with uncertain coefficient) are defined. Thus, our goal is to find the optimal solution in situations with the effect of $\Gamma_0$ and $\Gamma_i$ [33].

In general, higher $\Gamma_0$ values raise the level of conservatism in the objective function. On this basis, the robust counterpart formulation of the aforementioned nominal linear optimization problem is derived as follows Bertsimas and Sim [3]. It should be noted that in this formulation it has been assumed that all $c_j$ and $a_{ij}$ are uncertain.

$$\min \sum_{j=1}^{n} c_j x_j + z_0 \Gamma_0 + \sum_{j=1}^{n} r_{0j}$$

Subject to

$$\left( \sum_{j=1}^{n} a_{ij} x_j \right) + z_i \Gamma_i + \left( \sum_{j=1}^{n} r_{ij} \right) \leq b_i \quad \forall i \in \{1, 2, \ldots, m\}$$

$$z_0 + r_{0j} \geq \hat{c}_j y_j \quad \forall j \in \{1, 2, \ldots, n\}$$

$$z_i + r_{ij} \geq \hat{a}_{ij} y_j \quad \forall i \in \{1, 2, \ldots, m\}, j \in \{1, 2, \ldots, n\}$$

$$-y_j \leq x_j \leq y_j \quad \forall j \in \{1, 2, \ldots, n\}$$

$$l_j \leq x_j \leq u_j \quad \forall j \in \{1, 2, \ldots, n\}$$

$$r_{0j}, r_{ij} \geq 0 \quad \forall i \in \{1, 2, \ldots, m\}, j \in \{1, 2, \ldots, n\}$$

$$y_j \geq 0 \quad \forall j \in \{1, 2, \ldots, n\}$$

$$z_0, z_i \geq 0 \quad \forall i \in \{1, 2, \ldots, m\}$$

The variables added to the robust counterpart model $(z_i, y_j, r_{ij}, z_0, r_{0j})$ can be used to adjust the robustness of the solution and the degree of protection provided by the model. In the mathematical model of this study, the profit margin of each product depends on several factors, including fixed and variable costs, intra-organizational performance, inflation, and the performance of competitors. Therefore, it is not possible to assume a fixed and definite value for $\hat{\mu}_i$. The uncertainty of this parameter should be incorporated into the mathematical model. As mentioned, the robust optimization approach is used to deal with uncertainty. Given the uncertain of the $\hat{\mu}_i$, the robust approach of Bertsimas and Sim [4] is used to investigate the robust counterpart formulation of the problem. By using the Bertsimas and Sim approach, the parameter $\hat{\mu}_i$ is modeled as uncertainty interval $[\mu_i - \hat{\mu}_i, \mu_i + \hat{\mu}_i]$, where each non-deterministic $\hat{\mu}_i$ is bounded in a symmetric limited interval centered at $\hat{\mu}_i = \rho \mu_i$, where $\mu_i$ represents the estimated return (profit margin) of each product, $\hat{\mu}_i$ is the rate of fluctuation in the return of each product, and $\rho > 0$ is the level of uncertainty.

Accordingly, the final form of the robust model developed based on the robust approach of Bertsimas and Sim is as Eq.17-25.
Minimize $\lambda \left[ \sum_{i=1}^{N} \sum_{j=1}^{N} z_i x_i z_j x_j \sigma_{ij} \right] - (1 - \lambda) \left[ \sum_{i=1}^{N} z_i x_i \mu_i + \gamma_0 \Gamma_0 + \sum_{i=1}^{N} r_{0i} \right] \tag{17}$

Subject to

$$\sum_{i=1}^{N} x_i = 1 \tag{18}$$

$$\sum_{i=1}^{N} z_i = K \tag{19}$$

$$\varepsilon_i z_i \leq x_i \leq \delta_i z_i \quad \forall \ i \in \{1, \ldots, n\} \tag{20}$$

$$\sum_{i=1}^{N} x_i \mu_i \ D_i - \gamma_1 \Gamma_0 - \sum_{j \in J} P_j \geq R^* \tag{21}$$

$$\gamma_1 + P_j \geq \tilde{\mu}_i \ D_i x_j \quad \forall j \in \{1, \ldots, n\} \tag{22}$$

$$\gamma_0 + r_{0i} \geq \tilde{\mu}_i x_i \quad \forall i \in \{1, \ldots, n\} \tag{23}$$

$$\gamma_0, \gamma_1, r_{0i}, P_j, x_j \geq 0 \quad \forall i \in \{1, \ldots, n\} \tag{24}$$

$$z_i \in \{0, 1\} \quad \forall i \in \{1, \ldots, n\} \tag{25}$$

The parameter $\Gamma_0$ controls the level of robustness in the objective function. Thus, the goal is to find the optimal solution in situations with the different value of $\Gamma_0$. In general, higher values of $\Gamma_0$ raise the level of conservatism in the objective function. The positive variables $r_{0i}$, $\gamma_0$, $\gamma_1$, and $P_j$ are used to adjust the robustness of the solution in the model. In the next sections, IWO algorithm that is used to solve this PPO model is discussed.

5. IWO algorithm. Introduced in 2006 by Mehrabian and Lucas [23], IWO is a numerical optimization algorithm inspired by the growth and dissemination of weeds. Weeds are wild plants whose fast and aggressive growths pose a serious threat to human-cultivated crops. Weeds are very stable and quite adaptable to environmental changes. Thus, the properties of weeds can serve as a source of inspiration for the development of robust optimization problems.

5.1. Optimization based on weed behavior. The IWO algorithm takes inspiration from the natural steps that a weed species takes to dominate an environment. These steps are briefly described in the following:

step1: Spread of seeds in the target environment

step2: Growth of seeds depending on desirability (reproduction) and environmental dispersal

step3: Survival of weeds with higher desirability (competitive elimination)

step4: Continuation of the process until reaching the best desirability
The corresponding steps in the IWO algorithm are described in the following.

**Step 1: Generation of a random initial population and evaluation of their objective function values**

In this step, each weed produces a certain number of seeds. The number of seeds that a weed is allowed to produce has a linear relationship with its own fitness and the maximum and minimum fitness of the weed colony. In other words, the number of seeds produced by a weed species depends on how well adapted it is to its environment. Hence, the best solution among the current population produces the greatest number of new solutions, and the worst solution will be the source of the lowest number of new solutions. The number of solutions changes linearly between these two limits.

**Step 2: Fitness-based reproduction and updating the standard deviation**

Depending on the species, weeds may reproduce with or without the use of sexual cells. Reproduction is performed using seeds. In the reproduction, pollen grains fertilize the seeds, resulting in the creation of seeds in the parent weed. The seeds are then scattered by wind, water, animals, etc. (seed dispersal). Under the right conditions and when an opportunity arises, the scattered seeds start to germinate and grow. They continue to interact and compete with nearby plants until reaching maturity. In the final stage of life, they transform into flowering plants and produce seeds.

**Competitive elimination**

In nature, a weed species that is unable to reproduce in competitive scales will be destroyed. Therefore, there should be a simulated competition between weeds to limit the maximum number of weeds in the colony. For this purpose, after generating the seeds around the weeds, only a predetermined number ($P_{\text{max}}$) of weeds and seeds will be transferred to the next generation. Plants that survive this stage will again reproduce and the above steps will be repeated from the beginning. Hence, the solutions produced in each iteration will be fitter than the ones obtained in previous iterations. This mechanism also gives a chance of reproduction to the fewer fit plants, as their seeds have a good chance of survival if they are fit to the colony. This algorithm stops when the number of iterations reaches a predetermined maximum. The maximum number of weeds can be the same as the initial population size, in which case one of the algorithm parameters will be eliminated.

**Checking the stop conditions**

Like in other meta-heuristic algorithms, stop conditions of IWO can be defined as one of the following.

- **Acceptable quality of solution:** Suppose that the total cost of a company is 1,000 units, and we seek to reduce this cost to 800, which is reasonable enough from the management’s point of view. This goal is set irrespective of the fact that 800 units may be a sub-optimal cost, and the optimization process may be able to eventually produce even better objective function values.

- **A predetermined elapsed time/number of iterations:** In this case, the algorithm ends after running for a certain amount of time or when a predetermined number of iterations is reached. For example, the algorithm can be set to stop at iteration 100 regardless of the solution quality achieved.

- **Elapsed time/number of iterations without noticeable improvement in solution:** Algorithm can be set to stop after failing to achieve a certain degree of improvement in objective function value in the certain number of subsequent iterations. For example, we can set an algorithm to end after $n = b - a$ number of
subsequent iterations (from iteration a to b) without success to make the desired level of improvement. To better understand the structure of the algorithm, its pseudocode is illustrated in Figure 1.

1. Generate a random population of nPop0 solutions (W)
2. For iter = 1 to the maximum number of generations (MaxIt)
   a. Evaluate the objective function for each individual in W;
   b. Compute maximum and minimum fitness in the colony;
   c. For each individual $w_\in W$
      i. Compute the number of seeds of $w$, corresponding to its fitness;
      ii. Randomly distribute the generated seeds over the search space with the normal distribution around the parent plant ($w$);
      iii. Add generated seeds to the solution set, W;
   d. If ($|W| = N > P_{max}$)
      i. Sort the population W in descending order of their fitness;
      ii. Truncate population of weeds with smaller fitness until $N = P_{max}$;
3. Next iter;

Figure 1. Pseudocode of IWO algorithm

5.2. Solution representation in meta-heuristic algorithms. In this study, the solution of the problem is a product portfolio that is represented using an encoding system with values ranging in the interval [0,1]. The solution representation considered for this purpose is a vector with 2N cells (where N is the total number of available products). The first part of this vector displays the products to be included in the portfolio. In this section, cells with the value that is greater than 0.5 signify the inclusion of corresponding product into the portfolio. The second part of the vector displays the percentage of investment in each product, i.e. the portion of the available investment capital that must be directed toward the corresponding product (if that product is to be included in the portfolio). For example, a solution string for $N = 5$ and $K=2$ products can be demonstrated in Figure 2.

| 0.39 | 0.23 | 0.94 | 0.24 | 0.66 | 0.27 | 0.16 | 0.25 | 0.64 | 0.75 |

Figure 2. An example of solution representation

In Figure 2, the 3rd and 5th products are selected to be included in the portfolio. According to the second part of this solution, 25% of the available capital should be invested in product 3 and the remaining 75% should be invested in product 5. However, this solution may not satisfy some of the constraints of the developed
5.3. Constraints of the developed mathematical model in the meta-heuristic algorithm. In the course of designing an algorithm for solving an optimization problem, most attention should be paid to the problem constraints and its feasible spaces [13, 14]. The product portfolio optimization problem considered in this study includes the following three constraints.

A. The sum of investments in all products should be equal to 1. This constraint applies to the second part of the solution representation code. After selecting the products to be included in the portfolio, the corresponding values in the second part of the solution string will be checked. If the sum of these values is not equal to 1, they will be normalized such that this condition is met.

B. The investment rate in each product should agree with user-defined limitations. An investor may want, to limit the minimum or maximum amount of investment in certain options. For example, an investor may be unwilling to invest more than half of his capital in a certain product, or may not want to invest less than a certain amount in any single product in order to avoid extra exchange costs. To satisfy this constraint, the value of each cell in the second part of the solution string is bounded to the lower and upper investment limits defined for that product.

C. The portfolio should include a specified integer number of products. Models, in this study, portfolio is limited to the number of products in the portfolio by a specified integer number. This is a reasonable addition to the model as investors tend to specify the number of assets to be included in their portfolios. The addition of integer constraint to the portfolio optimization model results in a combinatorial quadratic integer programming problem, which is NP-hard and cannot be solved by exact solution methods in large-scale instances.

Following the approach of Seyedhosseini et al. [29], in this study, the constraint on the number of products in the portfolio is incorporated into the structure of the meta-heuristic algorithm.

To apply this constraint to the number of products to be selected, let $Q$ be the set of products included in the solution vector $x$, and $K^* = \sum_{i=1}^{n} Z_i$ be the number of products in the set $Q$. With $K$ assumed as the desired number of assets in the portfolio, if $K^* < K$, then a number of products should be added to $Q$, and if $K^* \geq K$, then a number of products should be removed from $Q$ until $K^* = K$. To decide which products should be added to or removed from $Q$, the relative effect of each product on the fitness function ($c_i$) will be measured. The product with relatively greater effect on ($c_i$) are in higher priority in addition to $Q$, and conversely, the product with relatively smaller effect on ($c_i$) are in higher priority for removal from $Q$. The fitness function ($c_i$) is calculated using the Eqs. 26-30.

$$\theta_i = 1 + (1 - \lambda) \mu_i \quad i = 1, 2, \ldots, N$$  \hspace{1cm} (26)

$$\rho_i = 1 + \lambda \frac{\sum_{j=1}^{n} \sigma_{ij}}{N} \quad i = 1, 2, \ldots, N$$  \hspace{1cm} (27)

$$\eta = -1 \times \min \{0, \theta_1, \ldots, \theta_n\}$$  \hspace{1cm} (28)

$$\psi = -1 \times \min \{0, \rho_1, \ldots, \rho_n\}$$  \hspace{1cm} (29)
\[ c_i = \frac{\theta_i + \eta}{\rho_i + \psi}, \quad i = 1, 2, \ldots, N \]  \quad (30)

Eqs. 26-27 are used to prevent particular states such as \((1 - \lambda) \mu_i < -1\) or \(\lambda \sum_{j=1}^{n} \sigma_{ij} < -1\). Thus, when \(K^* > K\) the product with the lowest \(c\) value will be removed from \(Q\), and when \(K^* < K\) the product with the highest \(c\) value will be added to this set. Eqs. 28-29 are used to calculate \(c_i\) in Eq. 30.

6. Numerical results. To evaluate the performance of the proposed mathematical model and IWO algorithm, they were implemented for the products of Pegah Golpayegan Company. Pegah Golpayegan is a dairy factory operating as a subsidiary of Iran Dairy Industries Co. This factory was founded in 1973 by the purchase of machinery from Bulgaria and was completed and launched in 1983. Following the initiation of a development plan in 1991, in 1994 the company used the technology purchased from a German company called ALPEMA to launch a fermented white cheese production line with 10-ton hourly milk processing capacity and 200-ton daily production capacity. The development continued with the purchase and launch of new machinery for production and packaging of processed cheese, Doogh (a yogurt-based beverage) and yoghurt. In 2006, the company diversified its production by introducing sterilized products and Feta cheese based on Ultra Filtration (UF) technology. At present, this company produces more than 80 different products. Here, the information related to these products is used to determine the efficiency frontier of portfolio selection problem based on risk and return for the products of this company. Because of the high number of products, the exact solution of the mathematical model cannot be obtained with optimization software. Therefore, IWO algorithm is used as an alternative approximation technique. Given the uncertainty in the return of products, we will also discuss and compare the results of the deterministic and robust approach to modeling.

The algorithms used in this study were coded in MATLAB R2016 and run on a system with CORE-I5 CPU and 5G-RAM.

6.1. Input parameters. Input parameters were set as described in the following.

Risk aversion factor (\(\lambda\)): As explained in Section 3, the algorithm uses the risk aversion factor, which is a value between 0 and 1, to trace out the efficiency frontier. In this study, the efficiency frontier is plotted by changing this factor by 0.1 unit per iteration. This allows us to obtain 10 points at the efficiency frontier, which is sufficient for accurate comparison.

Lower and upper bounds of investment (\(\varepsilon_k\) and \(\delta_k\)): Any preference in regard to the manner of investment in a certain product can be added to the model as the lower and upper bounds of investment in that product. In this study, for all products, the lower and upper bounds of investment were set to 0.001 and 0.999, respectively.

Product portfolio size (\(K\)): This parameter specifies the number of products to be included in the portfolio. For the better examination of the product portfolio optimization problem, the problem was solved once with \(K\) set to 10 (as small-scale problem instance) and another time with \(K\) set to 50 (as large-scale problem instance).

Protection level (\(\Gamma_0\)): The protection levels of objective function (\(\Gamma_0\)) were set to \(N/2\).

Uncertainty level (\(\rho\)): The problem was solved for uncertainty levels of \(\rho = 0.2\),...
\( \rho = 0.4 \) and \( \rho = 0.6 \). The profit margin is calculated as \( \hat{\mu}_i = \rho \mu_i \) and it explained in section 4.

6.2. **Parameters of the meta-heuristic algorithm.** Suitable values for the parameters of the IWO algorithm were determined by trial and error. For this purpose, IWO algorithm was run with a series of preliminary parameter settings and then the results were examined to identify the best setting. The summary of results obtained from this trial and error (the best value obtained for each parameter) are listed in Table 1.

| Parameter                                      | Symbol         | Optimal Value |
|-----------------------------------------------|----------------|---------------|
| Max iteration                                 | Maxit          | 100           |
| Number of first population                    | Npop0          | 50            |
| Maximum population size                       | Pmax           | 100           |
| Minimum seed                                  | Smin           | 20            |
| Maximum seed                                  | Smax           | 50            |
| Reducing power of standard deviation (pu)     | \( N \)        | 0.01          |
| Initial standard deviation                    | Sigma_final    | 0.5           |
| Final standard deviation                      | Sigma_final    | 0.1           |

6.3. **Implementation results for \( K = 10 \).** With portfolio size (\( K \)) set to 10, investment risks and returns and the objective function value, which is a linear combination of risk and return, were calculated for different risk aversion factors. The results obtained by solving the model with deterministic assumptions are presented in Table 4 in the appendix. Here, deterministic assumption means setting each uncertain parameter equal to the mean of its corresponding uncertainty interval (its nominal value). Tables 5 to 7 in the appendix present the results obtained by solving the mathematical model for different levels of uncertainty using IWO algorithm.

As shown in Tables 4 to 7, the use of higher risk aversion factors results in lower investment portfolio risk. This is because increasing the risk aversion factor increases the effect of variance and decreases the effect of return on the objective function value. Thus, during its search for optimum, algorithm puts more emphasis on achieving lower risk, which results in the discovery of less risky portfolios. Increasing the risk aversion factor also decreases the return of the portfolio by reducing the effect of return on the objective function value. As a result, the algorithm puts lesser emphasis on return and converges to low return portfolios.

This clearly demonstrates the conflict between risk and return of the portfolio. In other words, reducing the risk of investment requires a degree of tolerance to lower returns and seeking higher return requires taking higher risks.

The decision in this regard can be facilitated by providing an efficiency frontier to enable the decision maker to assess different risk-return tradeoffs and choose the tradeoff that yields the desired level of return at a tolerable level of risk. The efficiency frontiers obtained from the deterministic and robust model of the studied company are plotted in Figure 3.
Figure 3. Efficiency frontier of portfolio selection problem with 10 products

As shown in Figure 3, in the deterministic state, the efficiency frontier is at the highest position, but in the robust state, it is markedly lower. In other words, with the uncertainties taken into account, the investor needs to tolerate higher risks to achieve the same level of return. This is because of fluctuations in the return of each product. The authors believe that the robust formulation leads to a more realistic investment efficiency frontier, as the fluctuations in the return of products in a given time horizon are not constant and have their own volatility, which imposes more risk on the investor.

6.4. Implementation results for $K = 50$. The process followed for the model with 10 products was repeated for 50 products. The results obtained from the deterministic model and from the robust model with different uncertainty levels are presented in Table 8 to 11 in the appendix.

The results show that as the risk aversion factor increases, algorithm focuses more on the investment risk and less on the investment return, which results in achieving a lower risk at the expanse of return. The efficiency frontiers obtained from the deterministic and robust model with 50 products are presented in Figure 4.

The frontiers obtained from the robust models in Figure 4 are positioned below the deterministic model. The expected return that can be achieved at a certain level of risk decreases with the fluctuations in the uncertain parameter. Since uncertainty in the return of products is an inevitable reality, the investment efficiency frontier obtained based on robustness levels of up to 50% can be considered to be more realistic.

Since the objective function of the IWO algorithm is a linear combination of risk and return with dependence on risk aversion factor, its values are also a function of robustness level. Figures 5 and 6 present the summary of results regarding the effect of robustness level on the objective function value at different risk aversion states.

Figures 3 and 4 show that regardless of portfolio size ($K$), objective function value increases with the increase of risk aversion factor. When risk aversion factor
is low, the expected return has a higher importance weight, which results in a lower objective function value. On the contrary, as risk aversion factor increases, the importance weight of expected return decreases and that of risk increases. As a result, the final solution provides lower returns and objective function value shifts from negative values to positive values.

Also, as the robustness level increases from 0 to 50%, the objective function value increases. This is true for both small-scale problem (with 10 products) and the large-scale problem (with 50 product). As argued in the first section, increasing the robustness level increases the risk that must be taken for a given level of return.
to be achieved. This affects the entire objective function and causes the robust model to yield a higher objective function value than its deterministic counterpart.

6.5. **IWO efficiency.** In this section, the performance of the IWO algorithm is examined. This performance should be made in comparison with an exact solution method for small-scale problems and other meta-heuristic algorithms for large-scale problems. Therefore, the results of IWO algorithm is compared with the results of CPLEX solver of GAMS software and the genetic algorithm (GA), one of the most well-known meta-heuristic algorithms.

Table 2 represents the results of IWO, GA, and CPLEX for ten small-scale problems. In these problems, the risk aversion factor is assumed to be equal to 0.5 and $\rho = 0.5$. Also, a large scale problem is generated and optimized with IWO and GA under different risk aversion factors. The results are presented in Table 3. It should be noted that for these comparisons, we used input parameters which are described in section 6.1 with $\rho = 0.5$ and $K = 50$. However, the execution time is limited to 3600 seconds in each solution method to find the best possible solution in a reasonable time.

In Table 2, Z and T represent the objective function value and execution time in seconds respectively. GAP is used to show the percent of the difference between the approximation and exact solution method and is calculated by Eq.31.

$$GAP = \left| \frac{Z_{Metaheuristic} - Z_{CPLEX}}{Z_{CPLEX}} \right| \times 100 \quad (31)$$

As shown in Table 2, the GAP of GA in different problems has increased by about 7%, while the IWO algorithm has the maximum error of 3.4%. The average error of GA is 2.1% and the IWO algorithm error is 1.1%. This indicates that the IWO algorithm is able to search the solution space better and could provide better results than the GA. Due to the complexity of the problem and the NP-hardness of the research problem, the CPLEX execution time has increased significantly. So
Table 2. Computational results of CPLEX, GA and IWO in the small-scale problems

| Problem | K   | CPLEX | GA | IWO |
|---------|-----|-------|----|-----|
| P1      | 2   | 0.678 |
| P2      | 4   | -1.112|
| P3      | 6   | -1.487|
| P4      | 8   | -1.992|
| P5      | 10  | -2.786|
| P6      | 12  | -3.785|
| P7      | 14  | -5.211|
| P8      | 16  | -8.633|
| P9      | 18  | -      |
| P10     | 20  | -      |

Table 3 shows the results for different risk aversion factor. When the risk aversion decreases the return value and risk value increases. In such a situation, the algorithm

As shown in Figure 7, and since the execution time for solving in CPLEX is exponential, so it is not recommended to use this method to solve the large-scale problems. Therefore, in Table 3, the results of GA and IWO in large-scale problems are compared.
Table 3. Computational results of GA and IWO in the large-scale problems

| Risk aversion factor | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1 |
|----------------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|---|
| 8.24                | 7.39| 5.99| 4.89| 4.98| 1.53| 1.24| 0.83| 0.82| 0.33| 0.21|
| 38.09               | 15.3| 7.9 | 5.39| 5.18| 2.16| 2.07| 1.59| 1.6  | 1.59| 1.58|
| 10.02               | 8.52| 7.36| 6.62| 6.45| 3.1 | 2.94| 2.37| 2.35| 1.87| 1.78|
| 37.1                | 14.58| 7.75| 5.5 | 5.01| 2.03| 1.95| 1.6  | 1.53| 1.65| 1.54|

will perform better if the risk increase is less than the other algorithm and the return increases are more than another one. To better understand this, the return and risk in each of the risk aversion factor for GA and IWO are presented in Figure 8.

![Figure 8. GA and IWO portfolio efficient frontier](image)

By reviewing the portfolio effective frontiers, the IWO algorithm has performed better than the GA algorithm in all cases. The reason for this is that, at a consistent level of risk, the IWO algorithm has reported higher return than the GA algorithm. As a result, it can be shown that the performance of the IWO algorithm is much better than the older algorithms and can be considered as an alternative to algorithms such as GA in product portfolio optimization problems.

7. **Conclusion.** In this study, a robust product portfolio optimization problem was formulated and solved with the IWO algorithm. With the risk defined as the joint variance of products, the robust counterpart mathematical model was formulated with portfolio risk minimization and portfolio return maximization objectives. A risk aversion factor was used to establish a relationship between portfolio return and risk. Given the dependence of returns of products on many factors, it is often difficult and sometimes impossible to obtain a deterministic and constant estimation of returns. Therefore, the return of each product was defined as an interval with a level of uncertainty and the robust optimization approach was utilized to address this uncertainty. With the robust counterpart formulation of Bertsimas and Sim, a new mathematical model was developed which was capable of dealing with the uncertainty of return.

In order to solve this problem, an IWO algorithm was developed and coded in MATLAB software. This algorithm was then used to solve an illustrative problem instance at three uncertainty levels. The numerical results demonstrated the ability
of the IWO algorithm to solve the portfolio optimization problem in the shortest possible time. Comparison of results obtained in deterministic and robust states revealed that using the robust optimization approach to ensure the incorporation of uncertainty results in a more realistic investment efficiency frontier with higher risk than the deterministic model, as the variations and fluctuations of uncertain product returns will also be incorporated into the analysis. Given that uncertainty of returns is an undeniable part of real-world investment decisions, the investment efficiency frontiers produced by the proposed robust optimization model is expected to be closer to reality.

Future studies are suggested to further examine the performance of IWO and other novel metaheuristic algorithms in solving the developed mathematical model and to use accelerated exact solution methods to evaluate and validate the results of this study.

8. Appendix.

Table 4. Summarized results of IWO algorithm for the deterministic portfolio selection problem with 10 products

| Risk aversion factor | Total return | Total risk | Objective function Value | Execution time |
|----------------------|--------------|------------|--------------------------|----------------|
| 0                    | 1.29630      | 0.69760    | -56.99680                | 0.95800        |
| 0.1                  | 1.19926      | 0.60310    | -50.19030                | 1.39730        |
| 0.2                  | 1.17887      | 0.49650    | -40.04700                | 1.05580        |
| 0.3                  | 1.08299      | 0.46870    | -32.99290                | 0.95450        |
| 0.4                  | 1.06606      | 0.44740    | -26.26950                | 1.00600        |
| 0.5                  | 1.02268      | 0.42300    | -18.88370                | 0.79310        |
| 0.6                  | 0.96123      | 0.43990    | -11.05700                | 0.96800        |
| 0.7                  | 0.87897      | 0.43960    | -4.22660                 | 0.85980        |
| 0.8                  | 0.87608      | 0.43850    | 2.91500                  | 1.02450        |
| 0.9                  | 0.80853      | 0.42070    | 8.71360                  | 0.92530        |
| 1                    | 0.80043      | 0.39650    | 14.14370                 | 1.03380        |
| Average              | 1.01558      | 0.48098    | -19.33584                | 0.99783        |

Table 5. Summarized results of IWO algorithm for the robust portfolio selection problem with 10 products ($\rho = 0.2$)

| Risk aversion factor | Total return | Total risk | Objective function Value | Execution time |
|----------------------|--------------|------------|--------------------------|----------------|
| 0                    | 1.21507      | 0.66178    | -51.84080                | 0.97237        |
| 0.1                  | 1.20126      | 0.62716    | -45.59330                | 1.44621        |
| 0.2                  | 1.09569      | 0.51388    | -35.16300                | 1.07660        |
| 0.3                  | 1.09358      | 0.49401    | -26.80290                | 1.07190        |
| 0.4                  | 1.05358      | 0.49130    | -19.26790                | 1.10600        |
| 0.5                  | 1.01167      | 0.49102    | -15.62370                | 0.88859        |
| 0.6                  | 0.94073      | 0.48330    | -8.05990                 | 1.06238        |
| 0.7                  | 0.90638      | 0.47537    | 0.99940                  | 0.94625        |
| 0.8                  | 0.86803      | 0.46300    | 9.80500                  | 1.15400        |
| 0.9                  | 0.80883      | 0.45101    | 15.94460                 | 1.22232        |
| 1                    | 0.73322      | 0.40602    | 19.17160                 | 1.06134        |
| Average              | 0.99891      | 0.50535    | -14.22099                | 1.09163        |
Table 6. Summarized results of IWO algorithm for the robust portfolio selection problem with 10 products ($\rho = 0.4$)

| Risk aversion factor | Total return | Total risk | Objective function Value | Execution time |
|----------------------|--------------|------------|--------------------------|----------------|
| 0                    | 1.26527      | 0.69400    | -50.32180                | 1.05357        |
| 0.1                  | 1.20462      | 0.64976    | -45.06090                | 1.68514        |
| 0.2                  | 1.13801      | 0.59878    | -36.04860                | 1.13963        |
| 0.3                  | 1.07096      | 0.55574    | -24.82890                | 1.01938        |
| 0.4                  | 0.99564      | 0.53676    | -18.00080                | 1.27179        |
| 0.5                  | 0.93429      | 0.53060    | -9.88430                 | 1.01640        |
| 0.6                  | 0.84920      | 0.51546    | -5.03810                 | 1.10240        |
| 0.7                  | 0.75594      | 0.48834    | 10.17900                 | 1.23596        |
| 0.8                  | 0.72847      | 0.39742    | 16.83960                 | 1.04254        |
| 0.9                  | 0.72423      | 0.39170    | 20.17070                 | 1.20169        |
| Average              | 0.95513      | 0.52804    | -12.96634                | 1.16659        |

Table 7. Summarized results of IWO algorithm for the robust portfolio selection problem with 10 products ($\rho = 0.6$)

| Risk aversion factor | Total return | Total risk | Objective function Value | Execution time |
|----------------------|--------------|------------|--------------------------|----------------|
| 0                    | 1.22255      | 0.76641    | -45.06180                | 1.24743        |
| 0.1                  | 1.18229      | 0.72025    | -37.85050                | 1.72660        |
| 0.2                  | 1.09349      | 0.72213    | -32.92260                | 1.47810        |
| 0.3                  | 0.99892      | 0.64398    | -19.87430                | 1.36637        |
| 0.4                  | 0.96053      | 0.64205    | -14.73980                | 1.26848        |
| 0.5                  | 0.92569      | 0.57386    | -7.35830                 | 1.01640        |
| 0.6                  | 0.90423      | 0.57049    | 3.98354                  | 1.06400        |
| 0.7                  | 0.85309      | 0.56688    | 1.54040                  | 1.10240        |
| 0.8                  | 0.78659      | 0.45533    | 16.30550                 | 1.21371        |
| 0.9                  | 0.75671      | 0.43173    | 25.85560                 | 1.27669        |
| 1                    | 0.75242      | 0.41958    | 28.11682                 | 1.45525        |
| Average              | 0.94877      | 0.59206    | -6.58231                 | 1.29231        |

Table 8. Summarized results of IWO algorithm for the deterministic portfolio selection problem with 50 products

| Risk aversion factor | Total return | Total risk | Objective function Value | Execution time |
|----------------------|--------------|------------|--------------------------|----------------|
| 0                    | 1.4327       | 0.6733     | -32.3022                 | 0.9202         |
| 0.1                  | 1.3333       | 0.6593     | -29.6012                 | 1.0732         |
| 0.2                  | 1.2003       | 0.5725     | -23.4824                 | 1.0453         |
| 0.3                  | 1.1757       | 0.5570     | -19.8822                 | 1.1412         |
| 0.4                  | 1.1636       | 0.5561     | -15.9330                 | 1.1919         |
| 0.5                  | 1.1184       | 0.5254     | -11.2173                 | 0.7735         |
| 0.6                  | 1.0364       | 0.4975     | -6.4951                  | 0.9450         |
| 0.7                  | 1.0268       | 0.4714     | -2.2812                  | 0.7081         |
| 0.8                  | 0.9291       | 0.4580     | 1.6748                   | 1.0726         |
| 0.9                  | 0.9080       | 0.4226     | 4.6130                   | 1.0580         |
| 1                    | 0.8963       | 0.4052     | 7.7444                   | 0.9811         |
| Average              | 1.1110       | 0.5271     | -11.5602                 | 0.9924         |
Table 9. Summarized results of IWO algorithm for the robust portfolio selection problem with 50 products ($\rho = 0.2$)

| Risk aversion factor | Total return | Total risk | Objective function Value | Execution time |
|----------------------|--------------|------------|--------------------------|---------------|
| 0                    | 1.4913       | 0.7941     | -29.2069                 | 1.1059        |
| 0.1                  | 1.4243       | 0.7584     | -25.6067                 | 1.1246        |
| 0.2                  | 1.3192       | 0.6295     | -19.4184                 | 1.1503        |
| 0.3                  | 1.1959       | 0.5930     | -14.7174                 | 1.2152        |
| 0.4                  | 1.1603       | 0.5929     | -12.8385                 | 1.3426        |
| 0.5                  | 1.1007       | 0.5813     | -4.1561                  | 1.0164        |
| 0.6                  | 1.0427       | 0.5296     | 1.6449                   | 1.0640        |
| 0.7                  | 0.9459       | 0.4873     | 3.8428                   | 1.1024        |
| 0.8                  | 0.9456       | 0.4772     | 6.3238                   | 1.1423        |
| 0.9                  | 0.9126       | 0.4655     | 10.6742                  | 1.2695        |
| 1                    | 0.9099       | 0.4435     | 10.7638                  | 1.1976        |
| Average              | 1.1317       | 0.5775     | -6.6086                  | 1.1573        |

Table 10. Summarized results of IWO algorithm for the robust portfolio selection problem with 50 products ($\rho = 0.4$)

| Risk aversion factor | Total return | Total risk | Objective function Value | Execution time |
|----------------------|--------------|------------|--------------------------|---------------|
| 0                    | 1.4954       | 0.8165     | -27.1532                 | 1.0138        |
| 0.1                  | 1.3815       | 0.7981     | -25.3369                 | 1.1246        |
| 0.2                  | 1.2973       | 0.7045     | -18.3760                 | 1.1505        |
| 0.3                  | 1.1837       | 0.6121     | -13.7562                 | 1.2547        |
| 0.4                  | 1.1388       | 0.6095     | -9.9320                  | 1.2114        |
| 0.5                  | 1.0932       | 0.5819     | -3.1832                  | 1.0164        |
| 0.6                  | 0.9935       | 0.5793     | -0.4791                  | 1.0640        |
| 0.7                  | 0.9866       | 0.5776     | 2.8418                   | 1.1024        |
| 0.8                  | 0.9594       | 0.5133     | 10.7808                  | 1.3515        |
| 0.9                  | 0.9504       | 0.5106     | 12.6253                  | 1.2808        |
| 1                    | 0.8817       | 0.4496     | 11.7754                  | 1.0439        |
| Average              | 1.1238       | 0.6139     | -5.4721                  | 1.1467        |

Table 11. Summarized results of IWO algorithm for the robust portfolio selection problem with 50 products ($\rho = 0.6$)

| Risk aversion factor | Total return | Total risk | Objective function Value | Execution time |
|----------------------|--------------|------------|--------------------------|---------------|
| 0                    | 1.4189       | 0.8471     | -28.1382                 | 1.0433        |
| 0.1                  | 1.3362       | 0.7674     | -28.4752                 | 1.1246        |
| 0.2                  | 1.2420       | 0.7352     | -19.2514                 | 1.3819        |
| 0.3                  | 1.1947       | 0.7170     | -12.8606                 | 1.3707        |
| 0.4                  | 1.1767       | 0.7040     | -8.8690                  | 1.3882        |
| 0.5                  | 1.1766       | 0.6931     | -2.0909                  | 1.0164        |
| 0.6                  | 1.1491       | 0.6668     | 1.5295                   | 1.0640        |
| 0.7                  | 1.0411       | 0.6338     | 2.3588                   | 1.1024        |
| 0.8                  | 1.0095       | 0.5640     | 6.8008                   | 1.1792        |
| 0.9                  | 0.9435       | 0.5015     | 11.6470                  | 1.2315        |
| 1                    | 0.9186       | 0.4455     | 16.9694                  | 1.6090        |
| Average              | 1.1461       | 0.6614     | -4.8527                  | 1.2288        |

REFERENCES

[1] A. Ben-Tal and A. Nemirovski, Selected topics in robust convex optimization, *Mathematical Programming, 112* (2008), 125–158.

[2] A. Ben-Tal, L. El Ghaoui and A. Nemirovski, *Robust Optimization*, Princeton University Press, 2009.

[3] D. Bertsimas and M. Sim, The price of robustness, *Operations Research, 52* (2004), 35–53.
[4] D. Bertsimas and D. Pachamanova, Robust multiperiod portfolio management in the presence of transaction costs, Computers & Operations Research, 35 (2008), 3–17.
[5] R. N. Cardozo and D. K. Smith Jr, Applying financial portfolio theory to product portfolio decisions: An empirical study, The Journal of Marketing, (1983), 110–119.
[6] L. Cruz, E. Fernandez, C. Gomez, G. Rivera and F. Perez, Many-objective portfolio optimization of interdependent projects with a priori incorporation of decision-maker preferences, Applied Mathematics & Information Sciences, 8 (2014), 1517–1526.
[7] T. Cura, Particle swarm optimization approach to portfolio optimization, Nonlinear Analysis: Real World Applications, 10 (2009), 2396–2406.
[8] L. Czaplewski, R. Bax, M. Clokie, M. Dawson, H. Fairhead, V.A. Fischetti, S. Foster, B.F. Gilmore, R.E. Hancock and D. Harper, Alternatives to antibiotic pipeline portfolio review, The Lancet Infectious Diseases, 16 (2016), 239–251.
[9] S. El-Bizri and N. Mansour, Metaheuristics for Portfolio Optimization, in: International Conference in Swarm Intelligence, Springer, (2017), 77–84.
[10] H. N. Esfahani, M. h. Sobhiyah and V. R. Yousefi, Project portfolio selection via harmony search algorithm and modern portfolio theory, Procedia - Social and Behavioral Sciences, 226 (2016), 51–58.
[11] A. Fernández and S. Gómez, Portfolio selection using neural networks, Computers & Operations Research, 34 (2007), 1177–1191.
[12] R. Fernandes, J. B. Gouveia and C. Pinho, Product mix strategy and manufacturing flexibility, Journal of Manufacturing Systems, 31 (2012), 301–311.
[13] A. Goli and S. M. R. Davoodi, Coordination policy for production and delivery scheduling in the closed loop supply chain, Production Engineering, 1 (2018), 1–11.
[14] A. Goli, A. Azami and A. Jabbarzadeh, Accelerated cuckoo optimization algorithm for capacitated vehicle routing problem in competitive conditions, International Journal of Artificial Intelligence, 16 (2018), 88–112.
[15] N. Gulpinar and E. Çanakoğlu, Robust portfolio selection problem under temperature uncertainty, European Journal of Operational Research, 256 (2017), 500–523.
[16] M. A. Lejeune and S. Shen, Multi-objective probabilistically constrained programs with variable risk: Models for multi-portfolio financial optimization, European Journal of Operational Research, 252 (2016), 522–539.
[17] C.-C. Lin and Y.-T. Liu, Genetic algorithms for portfolio selection problems with minimum transaction lots, European Journal of Operational Research, 185 (2008), 393–404.
[18] L. L. Macedo, P. Godinho and M. J. Alves, Mean-semivariance portfolio optimization with multiobjective evolutionary algorithms and technical analysis rules, Expert Systems with Applications, 79 (2017), 33–43.
[19] H. Markowitz, Portfolio selection, The Journal of Finance, 7 (1952), 77–91.
[20] H. Markowitz, Portfolio Selection: Efficient Diversification of Investments, Cowles Foundation Monograph, No. 16, John Wiley & Sons, 1959.
[21] H. M. Markowitz and G. P. Todd, Mean-variance Analysis in Portfolio Choice and Capital Markets, John Wiley & Sons, 2000.
[22] D. Maringer and P. Parpas, Global optimization of higher order moments in portfolio selection, Journal of Global Optimization, 43 (2009), 219–230.
[23] A. R. Mehrabian and C. Lucas, A novel numerical optimization algorithm inspired from weed colonization, Ecological Informatics, 1 (2006), 355–366.
[24] M. Montajabih, A. Afsar-Nadjafi and B. Afshar-Nadjafi, A robust algorithm for project portfolio selection problem using real options valuation, International Journal of Managing Projects in Business, 10 (2017), 386–403.
[25] J. M. Mulvey, R. J. Vanderbei and S. A. Zenios, Robust optimization of large-scale systems, Operations Research, 43 (1995), 264–281.
[26] G. V. Pai and T. Michel, Metaheuristic optimization of constrained large portfolios using hybrid particle swarm optimization, International Journal of Applied Metaheuristic Computing, 8 (2017), 1–23.
[27] A. Ponsich, A. L. Jaimes and C. A. C. Coello, A survey on multiobjective evolutionary algorithms for the solution of the portfolio optimization problem and other finance and economics applications, IEEE Transactions on Evolutionary Computation, 17 (2013), 321–344.
[28] B. Qu, Q. Zhou, J. Xiao, J. Liang and P. Suganthan, Large-scale portfolio optimization using multiobjective evolutionary algorithms and preselection methods, Mathematical Problems in Engineering, 17 (2017), 1–14.
[29] S. M. Seyedhosseini, M. J. Esfahani and M. Ghaffari, A novel hybrid algorithm based on a harmony search and artificial bee colony for solving a portfolio optimization problem using a mean-semi variance approach, *Journal of Central South University*, 23 (2016), 181–188.

[30] F. Solatikia, E. Kiliç and G. W. Weber, Fuzzy optimization for portfolio selection based on embedding theorem in fuzzy normed linear spaces, *Organizacija*, 47 (2014), 90–97.

[31] A. L. Soyster, Convex programming with set-inclusive constraints and applications to inexact linear programming, *Operations Research*, 21 (1973), 1154–1157.

[32] M. A. Takami, R. Sheikh and S. S. Sana, Product portfolio optimisation using teaching learning-based optimisation algorithm: a new approach in supply chain management, *International Journal of Systems Science: Operations & Logistics*, 3 (2015), 236–246.

[33] E. B. Tirkolaee, A. Goli, M. Bakhsh and I. Mahdavi, A robust multi-trip vehicle routing problem of perishable products with intermediate depots and time windows, *Numerical Algebra, Control & Optimization*, 7 (2017), 417–433.

[34] M. Tuba and N. Bacanin, Upgraded firefly algorithm for portfolio optimization problem, in: *Computer Modelling and Simulation, 2014 UKSim-AMSS 16th International Conference on*, IEEE, (2014), 113–118.

Received February 2018; 1st revision June 2018; final revision August 2018.

E-mail address: A.goli@stu.yazd.ac.ir
E-mail address: Hkhademiz@yazd.ac.ir
E-mail address: tavakoli@ut.ac.ir
E-mail address: sadeghieh@yazd.ac.ir