Direct numerical simulations of stratified open channel flows

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Abstract. We carry out numerical simulations of wall-bounded stably stratified flows. We mainly focus on how stratification affects the near-wall turbulence at moderate Reynolds numbers, i.e. \( Re_\tau = 360 \). A set of fully-resolved open channel flow simulations is performed, where a stable stratification has been introduced through a negative heat flux at the lower wall. In agreement with previous studies, it is found that turbulence cannot be sustained for \( h/L \) values higher than 1.2, where \( L \) is the so-called Monin-Obukhov length and \( h \) is the height of the open channel. For smaller values, buoyancy does not re-laminarize the flow, but nevertheless affects the wall turbulence. Near-wall streaks are weakly affected by stratification, whereas the outer modes are increasingly damped as we move away from the wall. A decomposition of the wall-normal velocity is proposed in order to separate the gravity wave and turbulent flow fields. This method has been tested both for open channel and full channel flows. Gravity waves are likely to develop and to dominate close to the upper boundary (centerline for full channel). However, their intensity is weaker in the open channel, possibly due to the upper boundary condition. Moreover, the presence of internal gravity waves can also be deduced from a correlation analysis, which reveals (together with spanwise spectra) a narrowing of the outer structures as the stratification is increased.

Introduction

Stably stratified boundary layers have been studied for a long time and are still subject of current research (e.g. Nieuwstadt, 2005; Flores & Riley, 2010; García-Villalba & del Álamo, 2011). In order to understand how heat, momentum, moisture and pollutants are exchanged with the earth surface, the study of the atmospheric boundary layers is crucial. An important property of such a flow is its stability: buoyancy forces, due both to humidity and temperature, are present and they actively interact with the flow. During daytime, positive heat fluxes develop at the ground and lead to convective motions. On the other hand, during night time and/or in polar regions, negative fluxes (cooling) are prevalent, and the flow is usually stably stratified. How turbulence is affected by a stable stratification, its suppression for very high stability as well as the influence of internal gravity waves and their interaction with the underlying turbulence, are not fully resolved issues. Nieuwstadt (2005) and Flores & Riley (2010) carried out Direct Numerical Simulations (DNSs) in order to study the turbulence collapse in open-channel cases when a constant heat-flux is forced at the lower wall. Armenio & Sarkar (2002) considered Large-Eddy Simulations (LESs) of a full channel at very high stability, where the stratification was imposed by a constant temperature difference in the vertical direction. The authors found that turbulence remains very active close to the wall, even for very strong stratification, whereas...
in the center of the channel wave motions were dominating. Similar results were also recently obtained by García-Villalba & del Álamo (2011), who addressed the problem through DNSs of full channels. Nevertheless, they could not reach continuously turbulent states at such strong stratification as studied by Armenio & Sarkar (2002). As they point out, the turbulence collapse is highly dependent on box sizes, and further investigations on this subject would need boxes large enough to fit both laminar and turbulent patches. Which parameter should be used to determine whether turbulence is suppressed by stable stratification is still disputed: Nieuwstadt (2005) uses the gradient Richardson number provided by the stability analysis \( \text{Ri} < 0.25 \), Flores & Riley (2010) suggest \( L/l^+ \approx 10^2 \) as a criterion, whereas García-Villalba & del Álamo (2011) quantifies how close the flow is to relaminarization through the Nusselt number \( \text{Nu} \). García-Villalba & del Álamo (2011) also investigated the structures which develop in statistically quasi-stationary limits, finding an intermediate region where the Monin-Obukhov theory seems to apply well. On the other hand, near-wall structures were found to rather scale in viscous units and to be weakly affected by the stable stratification.

In this work, we extend some of these studies, mainly focusing on the statistically steady regimes. We address them through a set of both fully-resolved open channel DNSs and full channel LESs, where a stable stratification is introduced through either a cooling at the lower wall or a constant temperature difference between the upper and the lower walls. Flow structures are studied using correlations analysis, both in the horizontal plane and along the vertical direction. Moreover, in order to better quantify and characterize the gravity wave activity, a new decomposition able to separate the background turbulence from the wave part is developed.

Numerical Scheme

The governing equations of the system are the incompressible Navier-Stokes equation within the Boussinesq approximation. Including also the terms related to sub-grid stress modeling, the mathematical model reads

\[
\begin{align*}
\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} &= -\frac{\partial p}{\partial x_i} + \frac{1}{Re} \nabla^2 u_i + Ri \delta_{ij} + \frac{\partial r_{ij}^{\text{LES}}}{\partial x_j}, \\
\frac{\partial u_i}{\partial x_i} &= 0,
\end{align*}
\]

where the temperature \( \theta \) satisfies an advection-diffusion equation:

\[
\begin{align*}
\frac{\partial \theta_i}{\partial t} + u_j \frac{\partial \theta_i}{\partial x_j} &= \frac{1}{RePr} \nabla^2 \theta_i + \frac{\partial q_{ij}^{\text{LES}}}{\partial x_j}.
\end{align*}
\]

Here, \( u,v \) and \( w \) are the velocity along the streamwise, wall-normal and spanwise directions, respectively. Note that when LES is considered, the physical quantities \( u,v,w \) and \( \theta \) must be regarded as the filtered counterparts. The Reynolds number, Richardson number and Prandtl number are here defined as:

\[
Re = \frac{u_L L}{\nu}, \quad Ri = \frac{g \alpha \theta_{ref} h}{u_L^2}, \quad Pr = \frac{\nu}{\kappa},
\]

where \( \nu \) and \( \kappa \) are the momentum and thermal diffusivity respectively, \( g \) is the acceleration due to gravity and \( \alpha \) is the thermal expansion coefficient. The reference temperature \( \theta_{ref} \) is chosen to be the temperature difference between the upper and lower walls. These equations are discretized using a pseudo-spectral method, assuming periodicity and Fourier expansions in the wall-parallel plane, whereas Chebyshev polynomials are used in the wall-normal direction (Chevalier et al.,
For the open channel, the upper boundary condition, namely \( v = \partial u / \partial y = \partial w / \partial y = 0 \), allows us to use half the number of Chebyshev polynomials, either the symmetric or anti-symmetric ones. This method yields a better distribution of the collocation points which avoids the clustering at the free-slip surface, reducing the wall-normal resolution as well as the computational time (Deusebio, 2010).

When LES is considered, the dynamic Smagorinsky model (Germano et al., 1991) has been used in order to estimate the deviatoric part of \( \tau_{ij}^{LES} \), as in the simulation of Armenio & Sarkar (2002). The sub-grid heat fluxes were similarly modelled using an eddy-diffusivity which was deduced from the eddy-viscosity by applying a constant turbulent Prandtl number \( Pr_t = 0.6 \). The deviatoric part of the SGS terms and the sub-grid scalar fluxes are therefore assumed to be aligned to the strain rate and to the mean scalar gradient, respectively.

In table 1, the different simulation cases are summarized. Slightly different setups were adopted in order to meet the reference cases: while the stratification was introduced through a negative heat-flux at the wall in the open channel simulations, a constant temperature difference between the lower and the upper wall was prescribed in the full channel. Moreover, whereas all the open channel simulations were always started from a neutral flow, in the full channel cases the stratification was progressively increased among the runs, as done by Armenio & Sarkar (2002) and García-Villalba & del Álamo (2011). In order to compare with the works by Nieuwstadt (2005) and Flores & Riley (2010), the stratification in the open channel flows is quantified by the non-dimensional inverse of the Monin-Obukhov scale:

\[
\frac{h}{L} = -\frac{g\alpha q_0}{u_f^2}. \tag{5}
\]

All the simulations have been run for a sufficiently long time for the flow to reach an almost statistically stationary state. Whereas the velocity field adjusts relatively quickly when stratification is introduced, the temperature field converges extremely slowly and long times are therefore required to achieve the same heat flux at the upper and lower walls.

### Results

The collapse of turbulence in ground-cooled open channel flows is first investigated. Results are in agreement with the finding by Nieuwstadt (2005) and Flores & Riley (2010). The temporal evolution of the turbulent kinetic energy shows that turbulence is completely suppressed by the stable stratification for \( h/L \) values higher than 1.2, and the flow relaminarizes. On the other hand, values lower than 1.2 allow for a continuously turbulent state which is, however, affected by buoyancy. In the latter case, the turbulent kinetic energy first decreases, due to a temporary collapse of turbulence at the wall, and then increases to a value which is the same as for the unstratified case.

| Case  | SGS model | Resolution | Box Size | \( Re_t \) | \( Ri \) | \( h/L \) |
|-------|-----------|------------|----------|-------------|--------|----------|
| OCH0  | No        | 768 x 129 x 768 | 8\( \pi h \times h \times 4\pi h \) | 360 | 0 | 0 |
| OCH1  | No        | 768 x 129 x 768 | 8\( \pi h \times h \times 4\pi h \) | 360 | 55 | 0.71 |
| OCH2  | No        | 768 x 129 x 768 | 8\( \pi h \times h \times 4\pi h \) | 360 | 113 | 1.2 |
| OCH3  | No        | 768 x 129 x 768 | 8\( \pi h \times h \times 4\pi h \) | 360 | 167 | 1.5 |
| CH0   | Smag.     | 64 x 97 x 64   | 4\( \pi h \times 2h \times 2\pi h \) | 180 | 0 | 0 |
| CH1   | Smag.     | 64 x 97 x 64   | 4\( \pi h \times 2h \times 2\pi h \) | 180 | 44  | 0.85 |
| CH2   | Smag.     | 64 x 97 x 64   | 4\( \pi h \times 2h \times 2\pi h \) | 180 | 87  | 1.44 |

Table 1. Summary of the simulations.

(2007)
In Fig. 1, the mean velocity profile and the mean temperature profile are shown for unstratified and stratified cases. Close to the wall the mean velocity is not affected by the stratification. This is not surprising, reflecting the fact that the pressure gradient, and therefore the wall-shear stress, is the same in the two cases. In the outer region, however significant differences can be observed. As the stratification is increased, the velocity profile steepens progressively and, especially very close to the upper boundary, attains a parabolic (laminar) shape. This is related to the fact that the turbulent wall-normal momentum transport, $u'v'$, becomes less efficient due

![Figure 1. Mean profiles. a) streamwise velocity; b) temperature. —— baseflow; …… $h/L = 0.71$; — $h/L = 1.20$.](image1)

![Figure 2. Root mean square profiles. a) streamwise velocity; b) wall-normal velocity; c) spanwise velocity; d) temperature. —— baseflow; …… $h/L = 0.71$; — $h/L = 1.20$.](image2)
to reduced vertical motions. A similar conclusion can also be drawn from the temperature profile, which approaches a linear dependence, i.e. the one found in the laminar case. Also one point statistics (Fig. 2) fall on top of each other in the near-wall region, departing more and more as we move far from the wall: significant decreases of the fluctuations are observed close to the upper boundary for all the three velocity components. Interestingly, close to the upper boundary temperature fluctuations show non-monotonic behavior with increasing stratification, due to the combined effect of turbulence reduction and of the consequent increase of temperature gradients.

Two point lateral spectra
In Fig. 3 the lateral (spanwise) pre-multiplied spectra are shown, defined as

$$\phi_{ii}(k_z, y) = E_{ii}(k_z) \cdot k_z,$$

where $E_{ii}$ is the Fourier transform of the auto-correlation $B_{ii}(y, r) = \langle u_i(x, y, z)u_i(x, y, z + r) \rangle_x / u_{i,\text{rms}}(y)$. The spectra in Fig. 3 are plotted as functions of $y^+$ and $\lambda^+$. In the unstratified case, the footprint of near-wall streaks, which scales in viscous units, can be recognized as well as the outer structures which scale in outer units. The streak spacing in the $B_{uu}$ spectra appears to be roughly $\lambda^+ = 120$, which agrees with previous simulations and experiments of wall-bounded flows (Jiménez, 1998). Moreover, it can be noted that the spacing deduced by the $B_{vv}$ spectra is roughly half of the one found in $B_{uu}$, as it has already been observed by Kim et al. (1987) and Jiménez (1998). When we turn to the stratified lateral pre-multiplied spectra some important observations can be made. First of all, the structures very close to the wall seem to be only slightly influenced by stratification. In spite of the fact that the stratification is largest there (where the temperature gradient is largest), the shape of the spectra close to the wall does not change significantly when compared to the unstratified case. This is true both for the $uu$ and $vv$ spectra. However, significant differences arise in the outer region where it is found that structures become significantly narrower as the stratification is introduced. This is particularly evident in the spanwise spectra $uu$ where the energy maximum, corresponding to the outer structures, goes from $\lambda^+ \approx 750$ down to $\lambda^+ \approx 280$. Note that the normalized spectra do not tell us whether the peak-shift is due to a damping of the largest structures or to an intensification of the narrower structures. However, the unnormalized counterpart of Fig. 3 shows that energy concentrates more and more towards smaller scales when compared to the unstratified cases. Large structures are damped and the smaller ones enhanced. The inhibition of vertical motion not only favors thinner structures in the vertical direction, but also narrower structures in the spanwise direction. On the other hand, it can be noted from the longitudinal
spectra (not shown) that the length scales in the streamwise direction do not seem to be affected by stratification.

Similar conclusions can also be drawn from flow visualizations. In Fig. 4, the streamwise velocity field in a $y - z$ plane is shown both for a stratified and unstratified case. Buoyancy forces and stable stratification mainly affect the outer region, where they damp structures that would extend throughout the whole domain otherwise. These structures penetrate the buffer region from well above and they correspond to the so-called global modes identified in several works, e.g., García-Villalba & del Álamo (2011), Hoyas & Jiménez (2006) and Örlü & Schlatter (2011). In Fig. 5, the wall-normal velocity integral length scale in spectral space, defined as:

$$\overline{L_y}(k_x, k_z) = \int_0^h \int_0^h \frac{\text{Re}(\hat{v}(k_x, k_z, y)\hat{v}(k_x, k_z, \tilde{y}))}{v_{\text{rms}}(y)v_{\text{rms}}(\tilde{y})} \, d\tilde{y} \, dy$$  \hspace{1cm} (7)

is displayed. Flores & Jiménez (2006) used this quantity in order to characterize structures well correlated in the wall-normal direction for smooth and rough walled unstratified flows. This measure was computed for several open channel flow fields, averaged and then, due to the rather noisy behaviour, top-hat filtered. In Fig. 5, both stratified ($h/L = 1.2$) and unstratified cases are shown. First, it can be noted that the magnitude of $L_y$ decreases as the stratification is increased, due to the inhibition of the wall-normal motions. For the unstratified cases the most correlated modes are rather elongated structures in the streamwise direction, i.e., modes with $k_x$ small. These structures can still be seen when stratification is introduced, however they tend to move to higher $k_z$, corresponding to the fact that they become more narrow, as also shown in Fig. 3. More interestingly, a new peak also appears which is located at a rather small $k_z$. It is likely that this peak is associated with gravity waves, i.e. motions which are expected to have a higher degree of coherence in the wall-normal direction. The streamwise wave length of these modes, $\lambda_x \approx 2$, agrees with the finding of García-Villalba & del Álamo (2011).

**Gravity waves**

In order to detect gravity wave activity, time series of individual Fourier coefficients were collected. We here propose and apply a decomposition of the flow field which is able to separate the turbulent part from the wave part, allowing for a characterization of the different contributions separately. With the aim of testing the novel procedure, LES of channel flows were therefore carried out with conditions similar to the ones used by Armenio & Sarkar (2002) and García-Villalba & del Álamo (2011) (Table 1), who observed strong gravity wave activity in the central region of a full channel.
For any given Fourier mode, it is in general possible to decompose the wall-normal velocity in a **in-** and **out-** phase component with respect to the temperature. For small amplitude gravity waves, a linear analysis predicts a 90-degree phase-shift between the wall-normal velocity and the active scalar (temperature) and therefore a solely **out-**phase component should be expected. Using complex operators, the in-phase and out-phase components can be defined as:

\[
\begin{align*}
    v_{IP}(k_x, k_z, y, t) &= \text{Re}(\hat{v}(k_x, k_z, y, t)\hat{\Theta}^*(k_x, k_z, y, t)), \\
    v_{OP}(k_x, k_z, y, t) &= \text{Re}(\hat{v}(k_x, k_z, y, t)i\hat{\Theta}^*(k_x, k_z, y, t)),
\end{align*}
\]

(8)

where \((\cdot)^*\) stands for the complex conjugate. The second order momentum \(v_{rms}^2\) can be obtained through Parceval’s relation equally in spectral space or in physical space, and, using the fact that **in-** and **out-**phase components are perpendicular to each other, it can be split in the two contributions:

\[
v_{rms}^2 = \int \int v_{IP}(k_x, k_z, y)\bar{v}_{IP}(k_x, k_z, y) dk_x dk_z + \int \int v_{OP}(k_x, k_z, y)\bar{v}_{OP}(k_x, k_z, y) dk_x dk_z
\]

(9)

In Fig. 6, the different contributions are plotted for a stratified full and open channel case. Unstratified cases (not shown) reveal a very similar behaviour to Fig. 6 close to the wall. However, in the central region significant differences arise. The peak of \(v_{rms}\) in fig 6a) at
the centerline was already found both by Armenio & Sarkar (2002) and Garcia-Villalba & del Álamo (2011) who related it to gravity wave activity. The proposed decomposition provides support for this hypothesis, showing that the main contribution comes from the out of phase component, e.g. the one where gravity waves should be found. Nevertheless, it turns out that in this central region turbulence is weaker but still active, accounting for 10% of the magnitude of $v_{rms}$. Note that in the outer regions of unstratified cases, the two components contribute with the same amount to the total variance, showing significant differences with Fig. 6. When we turn to open channel cases, an increase of the relative contribution of the out-phase component can clearly be seen close to the upper boundary. However, the difference is not as large as in the case of full channel flows, possibly due to the boundary condition which forces both $v$ and $\theta$ to zero. A similar conclusion can also be drawn from the probability density function of the angle between the wall-normal velocity and the temperature, which is shown in Fig. 7. Interestingly, the flow can be divided into three regions: close to the wall and at the upper boundary (centerline for the full channel cases) the phase-shift approaches roughly $\pi/2$, whereas in between these two regions a broader peak is attained around $-\pi$, consistent with Komori et al. (1983) and McBean & Miyake (1972). Note that the peak in the outer region attains a value of 0.26 for the full channel and around 0.14 for the open channel, indicating that there is a reduced gravity wave activity in the open channel as compared to the full channel.

![Figure 7](image)

**Figure 7.** Magnitude-weighted PDFs of the phase shift between the Fourier coefficients of temperature and wall-normal velocity. *a*) full channel flows (CH2); *b*) open channel flows (OCH2). Color from 0 (blue) to 0.14 (red)

**Conclusions**

DNSs and LESs of wall-bounded turbulent flows at high stratification have been carried out for both open channel and full channel flows at moderate $Re_\tau = 180,360$. Following previous studies (Nieuwstadt, 2005; Flores & Riley, 2010), such flows can be regarded as a model for the atmospheric boundary layer. First of all, the turbulence collapse has been analyzed. In agreement with previous works (Nieuwstadt, 2005; Flores & Riley, 2010) for the considered $Re_\tau$, the relaminarization of the flow occurs when the ratio of the channel height with the Monin-Obukhov length is around 1.2. However, as pointed out by Flores & Riley (2010), the ratio at which re-laminarization is observed depends on the Reynolds number and the condition cannot be used as a general criterion for the turbulence collapse.

We have analyzed the structures that develop in this stably stratified regime. The structures close to the wall seem to be unaffected by the presence of the stratification and near-wall structures for the unstratified and stratified cases fall on top of each other. As we move further from the wall, the stratification starts to play an important role and the flow structures
significantly differ. Outer layer structures become narrower as the stratification increases and the comparison between the unstratified and the stratified case with $h/L = 1.2$ shows a ratio between their spanwise width of about 3. Nevertheless, their streamwise length is hardly changed.

In order to characterize wave-like structures, a decomposition able to separate turbulent and wave contribution has been proposed. In agreement with the findings of García-Villalba & del Álamo (2011), wave-like motions are mainly expected in the center of the channel, and they localize in spectral space around wavelength $\lambda_x \approx 2 - 3$, $\lambda_z \approx \infty$, i.e. structures which are extremely long in the spanwise direction. Gravity waves were observed in open channel cases as well, however their magnitude attained smaller values, possibly due to the presence of the boundary condition. Wall-normal correlation analysis has been shown to be able to lead to similar results, showing both the appearance of gravity waves as well as the narrowing of the outer wall-normal-well-correlated structures as the stratification was increased.

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