Transmutation of Scale Dependence into Truncation Uncertainty via RG-Improvement of the $R(s)$ Series

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Abstract

The arbitrariness in how the logarithm is defined within the QCD series for the inclusive electroproduction cross-section is shown to affect the summation to all orders in $\alpha_s$ of leading and successively-subleading logarithms within that perturbative series, even though such summations largely eliminate the residual dependence of the original series on the arbitrary renormalization scale $\mu$. However, given that the original (unimproved) series is known to third-order in $\alpha_s(\mu)$, this logarithm ambiguity is shown not to enter the optimally improved summation-of-logarithms series until the term fourth-order in $\alpha_s(s)$, where $s$ is the physical center-of-mass energy squared. Consequently, the ambiguity in how the logarithm is defined is absorbable in the uncertainty associated with truncating the original perturbative series after its calculationally known terms.
The renormalization-scale ($\mu$) dependence of a perturbative series for a physical process necessarily arises as part of the process by which order-by-order infinities are parametrised and excised. However, such scale dependence within the known terms of a series is widely regarded to be a reflection of the next-order uncertainty of that series. This folkloric assertion appears reasonable insofar as the series taken to all orders of perturbation theory must be independent of the unphysical scale $\mu$. Indeed, the invariance of physical processes under changes in $\mu$ is the underlying justification for the renormalization-group equation.

However, it is one thing to realize that $\mu$-dependence of a physical series disappears as the number of series terms increases, but quite another to say that the $\mu$-dependence exhibited at a given order of perturbation theory is indicative of truncation uncertainty associated with our ignorance of the next-order term. In the present note, we demonstrate that this stronger assertion is indeed correct within the context of $\overline{\text{MS}}$ QCD corrections to the inclusive electroproduction cross-section series $R(s)$. Specifically, we demonstrate how renormalization-group (RG) improvement of the terms within the known perturbative series for this process replaces $\mu$-dependence with a new ambiguity contingent upon how the perturbation-theory logarithm is defined, and that this ambiguity can be back-converted into arbitrariness in any RG-optimal choice for the scale $\mu$. We are able to show, however, that this new ambiguity does not enter the unimproved $\mu^2 = s$ perturbative series until the (RG-inaccessible) unknown next-order term, reflective of the truncation uncertainty of the original series.

Radiative corrections to the inclusive electroproduction cross-section are scaled by a perturbative QCD series ($S$):

$$R(s) \equiv \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} = 3 \sum_f Q_f^2 S.$$  \hspace{1cm} (1)

Neglecting quark masses, the perturbative series within Eq. (1) is of the form

$$S = 1 + \sum_{n=1}^{\infty} a^n(\mu) \sum_{m=0}^{n-1} T_{n,m} L^m.$$  \hspace{1cm} (2)
where the $n^{th}$ power of the QCD couplant $x(\mu) \equiv \alpha_s(\mu)/\pi$ is multiplied by a degree $n - 1$ polynomial in the logarithm $L \equiv \log(\mu^2/s)$. The parameter $\mu$ is the arbitrary renormalization scale that enters as a consequence of the perturbative renormalization procedure. The coefficients $T_{n,0}$ have been calculated explicitly in the chiral limit from \overline{MS} perturbation theory out to $n = 3$ \cite{1,2},

\begin{align*}
    n_f = 3 : & \quad T_{1,0} = 1, \quad T_{2,0} = 1.63982, \quad T_{3,0} = -10.2839, \\
    n_f = 4 : & \quad T_{1,0} = 1, \quad T_{2,0} = 1.52453, \quad T_{3,0} = -11.6856, \\
    n_f = 5 : & \quad T_{1,0} = 1, \quad T_{2,0} = 1.40924, \quad T_{3,0} = -12.8046, \quad (3)
\end{align*}

and for $n \leq 3$, the coefficients $T_{n,m}$ ($1 \leq m \leq n - 1$) can easily be obtained from the process-appropriate RG equation \cite{1,4}:

\begin{align*}
    T_{2,1} = \beta_0, \quad T_{3,1} = 2\beta_0 T_{2,0} + \beta_1, \quad T_{3,2} = \beta_0^2, \quad (4)
\end{align*}

where

\begin{align*}
    \beta_0 = (11 - 2n_f/3)/4, \quad \beta_1 = (102 - 38n_f/3)/16, \\
    \beta_2 = (2857/2 - 5033n_f/18 + 325n_f^2/54)/64. \quad (5)
\end{align*}

The form of the series (2) is unaffected by a redefinition of the logarithm. If we redefine the logarithm to be

\begin{align*}
    L_k = \log(k\mu^2/s) = L - \log(k), \quad (6)
\end{align*}

the series (2) becomes

\begin{align*}
    S = 1 + \sum_{n=1}^{\infty} x^n(\mu) \sum_{m=0}^{n-1} \tilde{T}_{n,m}(L_k)^m, \quad (7)
\end{align*}

where

\begin{align*}
    \tilde{T}_{1,0} = T_{1,0} (= 1), \quad \tilde{T}_{2,0} = T_{2,0} - T_{2,1} \log(k), \quad \tilde{T}_{2,1} = T_{2,1},
\end{align*}

\footnote{Although the form of \cite{1} suggests that the singlet contributions proportional to $x^3 \left( \sum_f Q_f \right)^2$ \cite{3} have been omitted, such singlet contributions have been absorbed into the values for $T_{3,0}$.}
\[ T_{3,0} = T_{3,0} - T_{3,1} \log(k) + T_{3,2} \log^2(k), \]
\[ \hat{T}_{3,1} = T_{3,1} - 2T_{3,2} \log(k), \quad \hat{T}_{3,2} = T_{3,2}. \]  
(8)

Thus the redefinition (6) of the logarithm is compensated trivially by appropriate shifts in the coefficients \( T_{m,n} \), such that the series (2) and (7) agree order-by-order in the expansion parameter \( x(\mu) \). Note that incorporation of the redefined logarithm \( L_k (6) \) does not entail any compensating change in the renormalization scale \( \mu \); the argument of \( x(\mu) \) has not been altered.

In refs. [6, 7], it is shown that truncations of the series (2) can be optimally RG-improved through inclusion of all terms in that series accessible via the RG-equation, an approach suggested first (to our knowledge) by Maxwell [8]. Given the current determination of the \( \overline{MS} \) QCD \( \beta \)-function to four orders in \( \alpha_s \) [9], one can show that knowledge of \( T_{1,0} \) is sufficient to determine all leading-logarithm coefficients \( T_{n,n-1} \); knowledge of \( T_{2,0} \) is sufficient to determine all next-to-leading logarithm coefficients \( T_{n,n-2} \); and knowledge of \( T_{3,0} \) is sufficient to determine all two-from-leading logarithm coefficients \( T_{n,n-3} \) [6]. Thus one can obtain all-orders summations of sequentially-subleading logarithms within the series (2):

\[
S \rightarrow S_\Sigma = 1 + x(\mu) \sum_{n=1}^{\infty} T_{n,n-1} (x(\mu)L)^{n-1}
+ x^2(\mu) \sum_{n=2}^{\infty} T_{n,n-2} (x(\mu)L)^{n-2}
+ x^3(\mu) \sum_{n=3}^{\infty} T_{n,n-3} (x(\mu)L)^{n-3} + ...
= 1 + x(\mu)S_1 (x(\mu)L) + x^2(\mu)S_2 (x(\mu)L) + x^3(\mu)S_3 (x(\mu)L) + ...
\]
(9)

In ref. [7], \( S_1, S_2 \) and \( S_3 \) are calculated explicitly from their generating coefficients \( T_{1,0}, T_{2,0}, T_{3,0} \) by solving successive differential equations obtained by inserting Eq. (9) into the RG
equation. The solutions obtained in ref. [7] are

\[ S_1(xL) = \frac{1}{(1 - \beta_0 xL)} \] (10)

\[ S_2(xL) = \frac{T_{2,0} - \frac{\beta_1}{\beta_0} \log (1 - \beta_0 xL)}{(1 - \beta_0 xL)^2} \] (11)

\[ S_3(xL) = \left( \frac{\beta_1^2}{\beta_0^2} - \frac{\beta_2}{\beta_0} \right) / (1 - \beta_0 xL)^2 \]

\[ + \frac{T_{3,0} - \left( \frac{\beta_1^2}{\beta_0^2} - \frac{\beta_2}{\beta_0} \right) - \frac{\beta_1}{\beta_0} \left( 2T_{2,0} + \frac{\beta_1}{\beta_0} \right) \log(1 - \beta_0 xL) + \frac{\beta_1^2}{\beta_0^2} \log^2(1 - \beta_0 xL)}{(1 - \beta_0 xL)^3} \] (12)

Since the infinite series (9) is by construction independent of the renormalization scale \( \mu \), it is not surprising that truncations of the RG-improved series (9) exhibit far less dependence on the renormalization scale \( \mu \) than corresponding truncations of the original series (2), which necessarily reflect residual renormalization-scale dependence [6, 7]. For example, if the series (2) is truncated after its known \( \mathcal{O}(x^3) \) contributions, one finds for \( n_f = 5 \) and \( \sqrt{s} = 15 \) GeV that this truncated unimproved series \( S_3^{(3)} \) increases from 1.0525 to 1.0540 as \( \mu \) varies from \( \sqrt{s}/2 \) to \( 2\sqrt{s} \), given 4-loop evolution of \( \alpha_s(\mu) \) from \( \alpha_s(M_z) = 0.11800 \) [7].

By contrast, a corresponding truncation \( S_3^{(3)} \Sigma \) of the optimally RG-improved series (9) after its \( x^3(\mu)S_3(x(\mu)L) \) term eliminates almost all such residual scale dependence; i.e., \( S_3^{(3)} \Sigma \) stays between 1.0537 and 1.0538 over the same range of \( \mu \) [7]. Since Eqs. (2) and (9) are identical when \( L = 0 \), one can argue that such optimal RG-improvement favours the physical choice \( \mu = \sqrt{s} \) for the renormalization scale occurring in the original series (2).

However, this elimination of scale dependence appears to come at a price. Suppose one redefines the logarithm via Eq. (6) and then sums successively-subleading logarithms in the series (7). One finds that this new summation-of-logarithms series differs from the series (9), even though the unimproved series (2) and (7) are order-by-order equivalent. The new
summation-of-logarithms series is

\[ S \to S'_\Sigma = 1 + x(\mu) \tilde{S}_1(x(\mu)L_k) + x^2(\mu) \tilde{S}_2(x(\mu)L_k) \]
\[ + x^3(\mu) \tilde{S}_3(x(\mu)L_k) + \ldots \]  
(13)

where

\[ \tilde{S}_1(xL_k) = \frac{1}{(1 - \beta_0 xL_k)} \]  
(14)
\[ \tilde{S}_2(xL_k) = \frac{\bar{T}_{2,0} - \frac{\beta_1}{\beta_0} \log (1 - \beta_0 xL_k)}{(1 - \beta_0 xL_k)^2}, \]  
(15)
\[ \tilde{S}_3(xL_k) = \left( \frac{\beta_1}{\beta_0} - \frac{\beta_2}{\beta_0} \right) / (1 - \beta_0 xL_k)^2 
+ \frac{\{ \bar{T}_{3,0} - (\frac{\beta_1}{\beta_0} - \frac{\beta_2}{\beta_0}) - \frac{\beta_1}{\beta_0} (2\bar{T}_{2,0} + \frac{\beta_1}{\beta_0}) \log(1 - \beta_0 xL_k) + \frac{\beta_2}{\beta_0} \log^2(1 - \beta_0 xL_k) \}}{(1 - \beta_0 xL_k)^3}. \]  
(16)

For example, if one truncates both unimproved series (2) and (7) after only two terms and then notes [Eq. (8)] that \( T_{1,0} = \bar{T}_{1,0} = 1 \), one finds that corresponding truncations of the summation-of-leading logarithms series are inequivalent because of their differing logarithms:

\[ S^{(1)}_{\Sigma} = 1 + x(\mu) / (1 - \beta_0 x(\mu)L), \]  
(17)
\[ S^{(1)}_{\Sigma'} = 1 + x(\mu) / (1 - \beta_0 x(\mu)L_k). \]  
(18)

If both expressions are comparably free of residual \( \mu \)-dependence, we see that the \( \mu \)-dependence from the corresponding truncation of the unimproved series,

\[ S^{(1)} = 1 + x(\mu), \]  
(19)
appears to be replaced by the \( k \)-dependence in Eq. (18), a reflection of the arbitrariness in how the logarithm is defined. Moreover, the (essentially \( \mu \)-independent) RG-improved series (13) coincides with the unimproved series (7) at \( L_k = 0 \), corresponding in this latter series
to a preferred value for $\mu$ of $\mu = \sqrt{s/k}$, where $k$ is arbitrary.\(^2\) Thus the ambiguity in the renormalization-scale $\mu$ chosen for the unimproved series is replaced by a corresponding ambiguity in how the logarithms are to be defined in the improved series prior to their summation.

In actual fact, such $k$-dependence can be subsumed entirely into the truncation uncertainty of the Eq. (19) when $\mu$ is chosen to be the external physical scale $\sqrt{s}$. Thus, we argue that the $k$-dependence of Eq. (18) is really a reflection of the error implicit in truncating the series (2) after its first two terms (19). To see this, consider first the explicit $\mu$-dependence of the one loop running couplant ($\mu^2 dx/d\mu^2 = -\beta_0 x^2$) about its RG-invariant reference value $x(\sqrt{s})$:

$$x(\mu) = x(\sqrt{s})/ (1 + \beta_0 x(\sqrt{s}) \log(\mu^2/s)) \quad (20)$$

If we substitute Eq. (20) into Eq. (18), we find that

$$S^{(1)}_\Sigma = 1 + x(\sqrt{s})/ (1 - \beta_0 x(\sqrt{s}) \log(k)) = 1 + x(\sqrt{s}) + x^2(\sqrt{s}) \beta_0 \log(k) + ... \quad (21)$$

Note that this expression is totally independent of the renormalization scale $\mu$. Moreover, its dependence upon $\log(k)$ [the logarithm ambiguity] occurs only in the next order of $x(\sqrt{s})$, i.e., within a term not determined by the RG equation. Consequently, Eq. (21) is fully consistent with the unimproved expression (19) when $\mu = \sqrt{s}$, regardless of $k$.

Surprisingly, we find that the transfer all $k$-dependence to the first post-truncation order (as well as the absence of any $\mu$-dependence in the first post-truncation order as well as in previous orders) is upheld when the next two subsequent orders of perturbation theory are taken into account. Suppose one wishes to make an optimal RG-improvement of the series $S$ [Eq. (7)] truncated after all of its known terms:

$$S^{(3)} = 1 + x(\mu) + (\widetilde{T}_{2,0} + \widetilde{T}_{2,1} L_k) x^2(\mu)$$

\(^2\)VE is grateful to P. Lepage for pointing out this ambiguity, which is also discussed in ref. [10].
As before, one can express the running couplant \( x(\mu) \) in Eq. (22) in terms of the physical reference value \( x(\sqrt{s}) \). In Eq. (5.14) of ref. [11], summation of logarithm techniques are employed to expand any RG-invariant effective couplant \( x(p) \) in powers of the running couplant \( x(\mu) \). A straightforward inversion of this expression with \( p = \sqrt{s} \) yields the following three leading terms:

\[
\begin{align*}
  x^{(3)}(\mu) & = x(\sqrt{s}) \left[ \frac{1}{1 + \beta_0 x(\sqrt{s}) L} \right] \\
  & - x^2(\sqrt{s}) \left[ \frac{\beta_1 \log[1 + \beta_0 x(\sqrt{s}) L]}{\beta_0 [1 + \beta_0 x(\sqrt{s}) L]^2} \right] \\
  & + x^3(\sqrt{s}) \left[ \frac{(\beta_1^2 - \beta_2 \beta_0) \beta_0 x(\sqrt{s}) L - \beta_1^2 \{ \log [1 + \beta_0 x(\sqrt{s}) L] - \log^2 [1 + \beta_0 x(\sqrt{s}) L] \}}{\beta_0^2 [1 + \beta_0 x(\sqrt{s}) L]^3} \right],
\end{align*}
\]

(23)

where \( L \equiv \log(\mu^2/s) \), as before. If one substitutes Eq. (23) for \( x(\mu) \) everywhere it appears within the RG-improvement of Eq. (22),

\[
S^{(3)}_\Sigma = 1 + x(\mu) \tilde{S}_1 (x(\mu) L_k) + x^2(\mu) \tilde{S}_2 (x(\mu) L_k) \\
+ x^3(\mu) \tilde{S}_3 (x(\mu) L_k),
\]

(24)

one obtains the following power series expansion in \( x(\sqrt{s}) \) via Eqs. (8) and (14-16):

\[
S^{(3)}_\Sigma = 1 + x(\sqrt{s}) + T_{2,0} x^2(\sqrt{s}) + T_{3,0} x^3(\sqrt{s}) \\
+ x^4(\sqrt{s}) \left[ 2 \beta_0^3 \log^2(k) - \left( 6 \beta_0^2 T_{2,0} + 5 \beta_1 \beta_0 \right) \log(k) \right] \\
+ 6 \beta_0 T_{3,0} + 4 \beta_1 T_{2,0} + 2 \beta_2 \right] \log(k) / 2 \\
+ x^5(\sqrt{s}) \left[ A(k) + B(k) \log(\mu^2/s) \right] + O(x^6),
\]

(25)

\[
A(k) = \left[ 3 \beta_0^4 \log^2 k - \left( 8 T_{2,0} \beta_0^3 + \frac{26}{3} \beta_0^2 \beta_1 \right) \log k \right]
\]

8
\[
B(k) = -3\beta_0^2 \log^2 k + \left(3\beta_1^2 + 6\beta_0 \beta_1 T_{2,0} + 2\beta_2 \beta_0 \right) \log k \\
- 3\beta_1 T_{3,0} - 2\beta_2 T_{2,0}.
\] (26)

The power series (25) is consistent with Eq. (22), the \(O(x^3)\) truncation of the unimproved series (7) evaluated at \(\mu = \sqrt{s}\). The \(k\)-ambiguity (i.e. the arbitrariness in how the logarithm is defined) is entirely absorbed in the not-yet-known next order of perturbation theory; indeed this \(O(x^4(\sqrt{s}))\) contribution is zero if \(k\) is chosen equal to one. Moreover, this contribution does not exhibit any renormalization scale dependence, which is not seen to arise until the \(O(x^5(\sqrt{s}))\) contribution to Eq. (25). In other words, neither the \(k\)-ambiguity nor any \(\mu\)-dependence occurs in terms that should be determined by the RG equation. The parameter \(k\) does not occur until the first RG-inaccessible order, and the parameter \(\mu\) does not occur until the second RG-inaccessible order.

Thus, if one implements RG-improvement on Eq. (22), which is just the series (7) truncated after calculationally known terms, by summing leading and two subsequently subleading sets of logarithms to all orders of perturbation theory, one finds that the arbitrariness in how the logarithm in the series (7) is defined does not enter the RG-improved expression (25) until the first post-truncation order of \(x(\sqrt{s})\), a term sensitive to the not-yet-known coefficient \(T_{4,0}\). Consequently, the ambiguity in how the logarithm is defined can be absorbed in the truncation-uncertainty of the series. This uncertainty is decoupled from any residual dependence on the renormalization scale parameter \(\mu\). The RG-improved expression (25) is seen to retain renormalization-scale independence even to this first post-truncation order.

In the truncation of a conventional perturbative series such as Eq. (2), minimization of the residual scale dependence is known to be of value in extracting information about higher order terms. Such an approach has, for example, been employed in the extraction of \(\alpha_s\) from deep inelastic scattering structure functions [12]. Pertinent to our present analysis,
the minimization of residual scale dependence has also proved useful in obtaining estimates of the (as-yet-uncalculated) series (2) coefficient $T_{4,0}$ \cite{13}, estimates which appear to be corroborated by the $\mathcal{O}(n_f^2)$ contributions to the absorptive parts of the five-loop vector-current vacuum polarization function \cite{14} used to construct $R(s)$. The values of log($k$) that respectively correspond to ref. \cite{13}'s predicted values for $T_{4,0}$ [-128 ($n_f = 3$), -112 ($n_f = 4$), -97 ($n_f = 5$)] are log $k = \{-1.76, 1.41, 4.51\}$ for $n_f = 3$, log $k = \{-2.03, 1.26, 4.82\}$ for $n_f = 4$, and log $k = \{-2.38, 1.13, 5.10\}$ for $n_f = 5$.

Within Eq. (25), one might similarly speculate that minimization of the sensitivity of the $x^4(\sqrt{s})$ coefficient to changes in $k$ might also serve to predict $T_{4,0}$, or at least its approximate magnitude, consistent with Stevenson’s Principle of Minimal Sensitivity \cite{15}. If one optimizes Eq. (25)'s $x^4(\sqrt{s})$ coefficient with respect to log($k$), one obtains a quadratic equation whose solution yields two real optimization points. The larger of these points of minimal sensitivity to $k$ yields values with the same sign and approximately double the magnitude of ref. \cite{13} predictions for $T_{4,0}$. However, values of log($k$) for which $A(k) = 0$, eliminating truncation uncertainty at $\mathcal{O}(x^5)$ for $\mu = \sqrt{s}$, respectively result in the predictions $T_{4,0} = \{-117, -126, -133\}$ for $n_f = \{3, 4, 5\}$—values in reasonable agreement with the ref. \cite{13} estimates. It is particularly of interest that such $T_{4,0}$ estimates were obtained via the Adler function in the Euclidean momentum region \cite{13}, and that they nevertheless appear to be corroborated by the Minkowski region approach delineated above. Moreover, different choices of renormalization scale do not have a significant effect on the values of log($k$) that eliminate the full $\mathcal{O}(x^5)$ truncation uncertainty.

To summarize, residual renormalization-scale dependence of truncations of the perturbative series (7) [which is term-by-term equivalent to the original series (2)] no longer occurs within the optimally RG-improved series obtained through all-orders summation of leading and successively-subleading logarithms \cite{7}. However, such RG-improved results now depend on how the logarithm is defined, \textit{i.e.}, the parameter $k$ in Eq. (6), even though truncations
of the unimproved series (7) are independent of this choice. Moreover, a clear correspondence \( \mu = \sqrt{s/k} \) exists between the “k-ambiguity” of the improved series and the scale \( \mu \)-dependence of the unimproved series. We have shown here that the ambiguity in how the logarithm is defined is ultimately a reflection of the uncertainty deriving from truncation of the series itself. Thus, residual renormalization-scale dependence is demonstrably a reflection of the uncertainty associated with the unknown next-order term.

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