Estimating grouped data models with a binary dependent variable and fixed effect via logit vs OLS: the impact of dropped units

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This letter deals with a very simple issue: if we have grouped data with a binary dependent variable and want to include fixed effects (group specific intercepts) in the specification, is Ordinary Least Squares (OLS) in any way superior to a logit form because the OLS method appears to keep all observations whereas the logit drops all groups which have either all zeros or all ones on the dependent variable? It is shown that OLS averages the estimates for the all zero (and all one) groups, which by definition have all slope coefficients of zero, with the slope coefficients for the groups with a mix of zeros and ones. Thus the correct comparison of OLS to logit is to only look at groups with some variation in the dependent variable. Researchers using OLS are urged to report results both for all groups and for the subset of groups where the dependent variable varies. The interpretation of the difference between these two results depends upon assumptions which cannot be empirically assessed.
1. INTRODUCTION

Many applied researchers include “fixed effects” (unit specific intercepts) to account for unmodeled heterogeneity in grouped data analyses; these fixed effects lead to interesting issues. This is a well worked area when the dependent variable is continuous (Greene 2018, ch. 11.4). The situation is more complicated when the dependent variable is binary, though again the theory is well worked out (Greene 2018, ch. 17.7.3). In particular, the group mean centering solution for estimating a model with fixed effects and a continuous dependent variable does not carry over to non-linear models, such as logit. Obviously a standard logit model with fixed effects (“LOGITFE”) can be estimated; one just adjoins the unit specific dummy variables to the specification. The question is whether one should estimate LOGITFE as compared to rival estimators.

The famous work of results of Neyman and Scott (1948) showed that, as the number of incidental parameters goes to infinity along with the sample size, simply including these incidental parameters leads to biased estimates. In the grouped data case the number of incidental parameters is the number of group specific intercepts. This has put off researchers from using LOGITFE. This became more pronounced when Chamberlain (1980) proposed the conditional logit (“CLOGIT”) model, which conditions out the fixed effects by conditioning on the number of successes (1’s) and failures (0’s) in the group. This conditional approach is what Neyman and Scott proposed in general to solve the incidental parameters problem. Thus CLOGIT produces unbiased estimators in the presence of group specific intercepts.

The gains from using the CLOGIT hinge on the type of data that is being studied, viz., are the groups large or small. Thus we might have groups of size two in a two wave “panel” study; at the other extreme, we often see “time-series-cross-section-data” with well over 30 observations per unit. It is now well known that LOGITFE is essentially unbiased if group size is large enough; simulations show that large enough is say 20 or more (Katz 2001; Greene 2004; Coupé 2005) observations per group. Beck (2018) has recently shown that in such circumstances LOGITFE has similar mean squared error loss to CLOGIT. Thus for data where group sizes are large there is no advantage of CLOGIT over LOGITFE.

CLOGIT also comes with costs; since the fixed effects are conditioned out rather than estimated, marginal effects of covariates cannot be estimated with CLOGIT. These days marginal effects are de rigueur; LOGITFE easily allows for the estimation of marginal effects since the fixed effects are estimated. Thus where group sizes are large (say over 20 or so), LOGITFE should be used instead of CLOGIT.\(^1\) Since this is the type of data considered in the list letter, I only discuss LOGITFE.\(^2\)

FELOGIT appears problematic in that it drop all groups from estimation that are either

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\(^1\)This is not to say that it always or usually is; estimators with even small amounts of bias are often scorned even if they have other advantages. This is not the place to fight this battle even if the outcome should be obvious.

\(^2\)A search of the most recent five years of political science/international relations articles in JSTOR found none using CLOGIT for the very small group size case; about two or three per year use it in the large group case. The search did not turn up any articles using LOGITFE. Most articles with binary dependent variables started with the OLSFE specification.
all failures and/or all successes (“ALLZERO”). In data where successes are rare (say wars or coups) this can lead to the loss of much of the data. The reason why ALLZERO groups are dropped is well laid out in the standard econometric texts already cited. The simple reason is that the fixed effects do a perfect job of predicting outcomes in ALLZERO groups, with the group specific intercept estimator for those groups converging to negative infinity. Similarly, for all success groups the group specific intercept estimator converges to positive infinity and so these groups are dropped from estimation identically to all failure groups; the values of the covariates for either those groups have no effect on the likelihood function. To simplify exposition I focus on all failure groups only, but this leads to no loss of generality; ALLZERO groups include groups with only failure and groups with only success.

This has led to a somewhat paradoxical situation, with applied researchers choosing to use a linear probability model with unit specific intercepts (“OLSFE”). OLSFE does not drop all failure or all success units and so appears to work with all the data. I use the word paradoxical because logit is surely the model of choice with cross-sectional binary dependent variable data, but OLSFE seems to have that status with grouped binary dependent variable data.

The argument for LOGITFE vs OLSFE at least partly turns on what appears to be the loss of the ALLZERO groups in LOGITFE, which can radically change the properties of the data set. The purpose of this letter is to elucidate this effect. This letter deals only with grouped data where group size is large enough (say 20 or more) so that LOGITFE produces essentially unbiased and efficient estimators, allowing me to only compare LOGITFE and OLSFE.

Researchers are aware of this issue, but they often seem to take as either a robustness issue or as a reason to pursue a non-obvious line of attack. This can be seen in two recent examples in very prominent journals. Wright, Frantz and Geddes (2103) examines the link between oil wealth and autocratic regime survival using a country-year design with a binary dependent variable denoting whether a regime survived a given year. The article notes that fixed effects are generally included in models similar to theirs in order to deal with unobserved unit heterogeneity. Wright, Frantz and Geddes (2103, p. 294) goes on to say “[T]his strategy, however, drops [between 26 and 64] countries from the analysis that do not experience [various types of] regime change.... Dropping countries that do not experience regime change may bias estimates downward by selecting only those where regime change has occurred in the sample period, particularly if those stable political systems have high oil wealth. Below, we investigate the possibility that this restriction on the sample induces selection bias [by dropping fixed effects from the logit model].” The results differ a lot but is that due to the change in sample or the dropping of fixed effects; if unobserved unit

3 The largest number of cases dropped due to groups with no successes that I know is is the Green, Kim and Yoon (2001) fixed effects analysis of Militarized Interstate Disputes, where 93% of the data does not enter the likelihood function.

4 There may be other reasons why OLSFE is common, such as it is easy to read off the marginal effects, which are just the estimated parameters; for LOGITFE there is an additional (trivial) step. Casual conversation with those using OLSFE believe that the results are the same as with LOGITFE, and are obtained at a lower (intellectual) cost. This letter is dealing with the question of whether the two methods are as similar as believed.
heterogeneity is important, dropping fixed effects from the model does not seem ideal.

In a second important example, Besley and Reynal-Querol (2011) studies whether democracies provides more educated leaders by estimating models, for example, of the probability of a leader having a graduate degree in a large number of countries, where the specification includes country fixed effects. This article provides both OLS estimates of the LPMFE specification and CLOGIT estimates of the LOGITFE specification. Besley and Reynal-Querol (2011, p. 559) estimates the LMPFE specification (without justification) and then only mentions using CLOGIT (LOGITFE) as a robustness check. “[In the CLOGIT (LOGITFE) specification], we estimate a conditional logit model to recognize the discrete nature of the lefthand-side variable. The core finding of [the LPMFE model] remains.” This is clearly correct if we only care about the sign and significance of a coefficient, but, as we shall see, the difference between the two estimations is not trivial, albeit not enormous.

To keep this clear we need a bit of notation. Section 3 then shows the consequence of including the ALLZERO groups in the OLS estimation and the difference between OLSFE and LOGITFE as a function of dropping those group. Section 4 reanalyzes one Besley and Reynal-Querol (2011) result to show the practical import of the the analytic results. The conclusion discusses some interpretative issues in these differences and indicates when one might prefer the logit or OLS estimates of the marginal effects.

2. NOTATION

Let \( y_{g,i} \) be a binary dependent variable with the exogenous covariates being \( x_{g,i} \), where \( g \) indexes groups and \( i \) indexes particular units in a group. It simplifies notation to assume that all groups are of the same size, and dropping this one extra subscript has no consequences for the argument: let this be group size be \( N \), with \( G \) being the number of groups. Let the number of covariates be \( k \). \( \alpha_g \) refers to the fixed effect for group \( g \), that is the group specific intercept.

What is critical for this article is that \( G \) is fixed; asymptotics are in terms of \( N \). This surely holds for a very common type of data seen in comparative politics: time-series–cross-section data. Here \( G \) is the number of countries of other units being compared, and \( N \) (often denoted \( T \)) is the number of periods (usually years) that a unit is observed. Note that \( G \) may be large, but it is fixed; even if we compare, say, 5000 US counties, the number of counties is not growing infinitely large. In this data \( N \) is often (but not) always reasonably large (say 20 or more). What is critical is that in asymptotic thought experiments \( \lim_{N \to \infty} \frac{G}{N} = 0 \) and that there are no asymptotics in a fixed (perhaps large) \( G \). But the time-series-cross-section structure of the data is not at all critical here; what is critical is that asymptotics are in \( N \) and not \( G \) and these asymptotics yield the stated limit. This would hold if people are divided into ethnic groups with the number of people studied in each group being sufficiently large. It would not hold for, say, a two or short wave panel, nor would it hold for educational studies where the group is a classroom.

The LOGITFE model is

\[
P(y_{g,i} = 1) = \frac{1}{1 + e^{-(x_{g,i} \beta + \alpha_g)}}
\]  

(1)
where \( \alpha_i \) is the group specific intercept. The OLSFE model is

\[
y_{g,i} = x_{g,i}\beta + \alpha_g + \epsilon_{g,i}
\]  

and \( P(y_{g,i} = 1) \) is estimated in the obvious way. For both OLS and logit we can then estimate \( \frac{\partial P}{\partial x_{g,i}} \), the marginal effect of the covariates on \( P(y = 1) \).

There may be some groups where every member of the group has \( y = 0 \) (“ALLZERO” groups).\(^5\) A common reason for having groups with all zeros is that events \( (y = 1) \) may be rare; think of whether a country has a civil war in a year, with many countries having no civil war ever. This letter deals with the issue of the consequences of such groups, and it is shown that that the consequences, not surprisingly, increase as the number of ALLZERO groups increases.

3. DIFFERENCES BETWEEN WHAT IS ESTIMATED WITH LPMFE AND LOGITFE

As noted, any of the methods used to estimate a LOGITFE specification drop the ALLZERO groups. The LPMFE model estimated by OLS does use information on all the groups. To see the consequences of this, note that the OLS estimate of \( \beta \) is a weighted average of the estimates in the ALLZERO and the other (“NOTALLZERO”) groups.\(^6\) For the ALLZERO groups, \( y_{g,i} = 0 \) so the OLS estimate of \( \beta \) for those data is zero.\(^7\) Thus the OLS estimate of \( \beta \) in the full data is a weighted average of the OLS estimate of \( \beta \) in the NOTALLZERO groups and \( mathbf{0} \) where the weights depend non-linearly on the covariates in the two groups.

For the OLS computations, it is simplest to work with group mean centered data to avoid putting the group intercepts in the specification. Let \( \bar{X} \) and \( \bar{y} \) be the group mean centered data and let \( \bar{X}_0 \) be the covariate matrix for the ALLZERO groups with \( \bar{X}_1 \) being the corresponding matrix for the NOTALLZERO groups with \( \bar{y}_1 \) being the group mean centered vector of observations on \( y \) for the NOTALLZERO (and obviously the corresponding vector for the ALLZERO groups is \( \bar{y}_0 = 0 \). The subscript 01 refers to the complete data.

Thus the OLS estimate of \( \beta_{01} \) for the entire data set are given by

\[
\hat{\beta}_{01} = (\bar{X}_1'\bar{X}_1 + \bar{X}_0'\bar{X}_0)^{-1}(\bar{X}_1'\bar{y}_1)
\]  

\(^5\)Groups with only successes or a mixture of ALLZERO and all success groups yield identical results as shall be shown in the next section. I therefore refer to ALLZERO groups as including all success groups, without loss of generality. While groups with all failures seem more common in political science, there is no reason we cannot observe a mis of all failure and all success groups.

\(^6\)Any estimator can be seen as a combination of estimators for subgroups of data; this is a long standing idea in econometrics, with perhaps the best known and long standing example being the the Chow (1960) test of the equality of regression lines in two subsets of data. The calculations here are even simpler because of the nature of the dependent variable in the ALLZERO group.

\(^7\)All group intercepts in the ALLZERO group are also zero, and the fit appears to be perfect, with the estimated group specific intercepts being zero. It is necessary to assume some variation in the covariates within the ALLZERO groups so the model is identified. Note that for groups with only successes, the estimate of \( \beta \) is still zero, and the fit is still perfect, although the estimated intercepts are now one. That is why it is not necessary to separate all failure and all success groups.
whereas the corresponding estimate for the NOTALLZERO groups ($\beta_1$) is given by

$$\hat{\beta}_1 = (\tilde{X}_1'\tilde{X}_1)^{-1}(\tilde{X}_1'\tilde{y}_1).$$

(4)

We can also compare the variance covariance matrix of the two estimates. For the entire data set this matrix is

$$(\tilde{X}_1'\tilde{X}_1 + \tilde{X}_0'\tilde{X}_0)^{-1}\hat{\sigma}_{01}^2$$

(5)

whereas the corresponding estimate for the NOTALLZERO groups ($\beta_1$) is given by

$$(\tilde{X}_1'\tilde{X}_1)^{-1}\hat{\sigma}_1^2$$

(6)

where $\hat{\sigma}_{01}^2$ and $\hat{\sigma}_1^2$ refer to estimates of the standard error of the regression in the full and restricted data sets respectively.

It is immediately obvious that the two equations only differ by the $\tilde{X}_0'\tilde{X}_0$ portion of the $X'X$ matrix that is being inverted. Alternatively, it is obvious that the OLS estimates for all the data is a weighted average of 0 and the $\beta_1$; $\beta_{01}$ shrinks $\beta_1$ towards 0. The amount of shrinkage is a somewhat complicated function that depends on the relative scale of $\tilde{X}_0'\tilde{X}_0$ and $\tilde{X}_1'\tilde{X}_1$. As the proportion of ALLZERO groups goes up, $\beta_{01}$ goes to 0, but the path may not always be monotonic for all components of $\beta_{01}$.

The variance covariance matrix of the estimates has two components which move in different directions as we move from the entire data set to the NOTALLZERO data set. The estimated $\sigma^2$ will get smaller, since we are eliminating non-homogenous cases; however the $\tilde{X}_1'\tilde{X}_1$ matrix in the NOTALLZERO data will also be smaller in scale than the corresponding $\tilde{X}_0'\tilde{X}_0$ matrix used to estimate the variance covariance matrix of of $\beta_{01}$. Note however that the estimated standard error of the regression will be limited in how much it changes since the variance of $\tilde{y}$ is limited by it being a binary variable; the $\tilde{X}'\tilde{X}$ matrix is not similarly limited by any scaling, and so could shrink considerably as the ALLZERO cases are dropped. Usually, the estimated standard errors of $\hat{\beta}_1$ will be smaller than the corresponding estimates for $\beta_{01}$. The change in $\hat{\beta}$ and its estimated standard error offset, and so we usually see smaller impacts of dropping the ALLZERO groups on the $t$-ratio associated with $\beta_{01}$ as compared to $\beta_1$. This smaller change in $t$-ratio may be one reason that authors are content to conclude that the substantive results from LOGITFE are similar to those of LPMFE. But we should go beyond simply inquiring as to the sign of a coefficient and whether its “significance” is beyond some standard threshold to actually looking at coefficients.

It is very simple to see what is going on by looking at the scalar $x$ case, where once again $\tilde{y}$ and $\tilde{x}$ have been group mean centered. The OLS estimate of $\beta_{01}$ for the entire data set is given by

$$\hat{\beta}_{01} = \frac{\sum_{\text{NOTALLZERO}} \tilde{x}_{g,i}\tilde{y}_{g,i}}{\sum_{\text{ALLDATA}} \tilde{x}_{g,i}^2}$$

(7)

whereas the corresponding estimate for the NOTALLZERO groups ($\beta_1$) is given by

$$\hat{\beta}_1 = \frac{\sum_{\text{NOTALLZERO}} \tilde{x}_{g,i}\tilde{y}_{g,i}}{\sum_{\text{NOTALLZERO}} \tilde{x}_{g,i}^2}.$$  

(8)
These two equations differ only by an extra $\sum_{\text{ALLZERO}} \tilde{x}_{g,i}^2$ in the denominator of Equation 7; this extra term is the sum of squares and so non-negative, so $\hat{\beta}_{01} < \hat{\beta}_1$. The standard error for $\hat{\beta}_{01}$ for the entire data set is given by

$$\sqrt{\frac{\sigma_{01}^2}{\sum_{\text{All Data}} \tilde{x}_{g,i}^2}}$$

whereas the corresponding standard error for the NOTALLZERO groups ($\hat{\beta}_1$) is given by

$$\sqrt{\frac{\sigma_1^2}{\sum_{\text{NOTALLZERO}} \tilde{x}_{g,i}^2}}$$

where again the extra summation terms in the denominator must be positive.

For the scalar case it is obvious that including the ALLO groups shrinks $\hat{\beta}_1$ towards zero, where the amount of shrinkage depends on how many ALLZERO groups there are and the variation of the centered $x$’s in those groups. The estimated standard error of $\beta_1$ also gets smaller (in general), since the larger denominator due to $\sum_{\text{ALLZERO}} \tilde{x}_{g,i}^2$ will almost always offset the increase in the estimate of the standard error of the regression due to the greater heterogeneity of $y$ of the full data set. This again leads to offsetting effects in changing $t$-ratios.

4. EXAMPLES

If readers need convincing of the mathematics, one example should do. Here I reanalyze the Besley and Reynal-Querol (2011) results cited previously since the article is important and the replication data were provided by the author. It is easy to compare the LPMFE and LOGITFE results of the two estimate for the effect of democracy on whether a leader had a graduate degrees. These results are presented in Table 1 of the original article, with Column 1 being the LPMFE model and Column 3 being the LOGITFE model. The regression results are based on 1146 country-year observations; the logit results lose 190 of those because in some countries no leader ever had a graduate degree. I present in Table 1 re-analyses of a slightly simpler specification here, so that this letter can focus on the dropped cases issues and use LOGITFE instead of CLOGIT; when comparable, the results here are similar to those of the original article.

With all data the effect of the democracy dummy on the (linear probability) of a leader having a graduate degree is 26% (with a standard error of 4.3%; restricting the sample to countries with at least one leader having a graduate degrees increases this coefficient to 29.5% (with a small increase in the standard error); this is a 14% increase in the estimate coefficient when 17% of the data are dropped. The LOGITFE, which automatically drops the ALLZERO groups, show the average marginal effect of a country being a democracy
| OLSFE/All | OLSFE/No ALL0 | LOGITFE | OLSFE/ALL0 only |
|---|---|---|---|
| Democracy | \( \hat{\beta} \) | SE | \( \hat{\beta} \) | SE | \( \hat{\beta} \) | SE | \( \hat{\beta} \) |
| \( \hat{\beta} \) | 0.260 | 0.043 | 0.295 | 0.046 | 1.748 | 0.269 | 0.000 |
| Av. Marg. Effect (Dem) | 0.297 | 0.040 |
| N | 1146 | 956 | 956 | 190 |

Table 1: LPMFE and LOGITFE estimates of effect of a democracy dummy variable on probability a leader has a graduate degree in a given country and year. Data as in Besley and Reynal-Querol; the full data set has 197 distinct countries possibly observed from 1848–2004, though few countries have complete data over that period. LOG(GDP per capita) is also included in the specification, as are country (but not year) dummy variables. Full regression results and replication data and Stata code are available at the dataverse for this paper.

on either not having or having a leader with a graduate degree of 29.7%, almost exactly the corresponding LPMFE estimated effect dropping the ALLZERO groups. For those who doubt the algebra of the previous section, I also report the regression results including only the ALLZERO groups; the estimated coefficients is, of course, zero (to 17 decimal places).

5. CONCLUSION

The takeaway from this article is fairly simple. Researchers often require fixed effects specifications to treat unmodeled heterogeneity which is correlated with the covariates. Such researchers often either choose LOGIT or OLS without justification, or present the results of both. While in many cases both LOGIT and OLS yield the same sign and crossing of the \( p < .05 \) level, we have seen that the appropriate comparison for LOGIT is regression dropping groups that do not vary on the dependent variable.

One can make a case that either estimate is correct, with the choice between them being based on theoretical ideas that have no empirical referent. LPMFE results using all groups, which average zero with the LPMFE on the restricted data set, make sense in that the marginal effect of the covariates on \( y \) could be thought of as being zero in the ALLZERO groups. After all, if \( P(y_{g,i} = 1) = 0 \mid g \in \text{ALL}_0 \), then then marginal effect of any \( x \) in the ALLZERO groups is indeed zero. Alternatively, we can think of this as a meaningless exercise, since some change in an \( x \) in an ALLZERO group member will change a failure to a success and thus its marginal effect cannot be zero. Researchers can report both numbers and their interpretation; what is clear is that researchers must understand the difference between the two estimates, and understand how to compare LOGITFE and LPMFE results. And clearly researchers should not naively compare OLSFE estimates using all the data with LOGITFE estimates which are only for the NOTALL0 groups. In any event, the notion that OLSFE is superior to LOGITFE because the former does not drop the ALLZERO, is clearly incorrect, and the OLSFE uses the ALLZERO group data in a very artificial manner unless
one believes the untestable assumption that the effect of the covariates in the ALLZERO groups is really zero.

6. REFERENCES

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