From heliocentrism to epicycles: A commentary on pre-Ptolemaic astronomy

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If one wants to translate the heliocentric picture of planets moving uniformly on circular orbits about the sun to the perspective of a terrestrial observer, using classical (ancient) geometric means only, one is naturally led to the investigation of epicyclic constructions. The announcement of the heliocentric hypothesis by Aristarchos of Samos and the invention of the method of epicycles happened during the 3rd and 2nd centuries BC. The latter developed into the central tool of Hellenistic and Ptolemaic astronomy. In the present literature on the history of astronomy the parallel rise of the heliocentric view and the methods of epicycles is usually considered as a pure contingency. Here I explain why I do not find this view convincing.

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Introduction

Often epicyclic models of planetary motion are considered as contrary to the heliocentric view. This is, however, wrong. If one wants to get an impression of how heliocentric heavenly motions appear to a terrestrial observer, i.e., if one wants to picture the phenomena which follow from a heliocentric ontological assumption, one is naturally lead to the construction of nested cyclical motions by a kind of kinematic inversion. The underlying transformation can easily be developed in the framework of ancient Euclidean geometry; it leads without fail to a set of graphical methods using cycles and epicycles.

Historical evidence for the transition from heliocentrism to epicyclic kinematics is sparse. This seems to be the reason for the deplorable fact that an interdependence of heliocentrism and epicyclic kinematic models is seldom discussed in the literature. Often a connection is even rejected, although with insufficient arguments as will be discussed in section 3 of this paper. In my view this is a surprising and deplorable gap in the interpretation of the sources of Greek astronomy. Although I am neither a historian of astronomy, nor could I present new sources on the origin of the epicyclic approach, the preparation of the present book for Jesper Lützen offers a welcome opportunity to me to develop the simple argument indicated above in written form.¹

In chap. 2 of his beautiful exposition of Greek astronomy Asger Aaboe explains the epicyclic models by a discussion of the kinematic inversion first of the sun’s motion and then for the planets. He emphasizes the importance of this discussion by the following remark:

For a proper appreciation of the Greek planetary models [the epicyclic ones, ES] it is important to recognize that they are more than a mere collection of ad-hoc devices that can reproduce the planets’ apparent behavior. In fact, when appropriately scaled, they turn out to be correct representations of the planets’ motions relative to the earth in distance as well as in direction.

(Aaboe, 2001, p. 82f.. emphasis in the original)

Aaboe’s stops short, however, of drawing conclusions from this observation for the history of the transition from heliocentrism to the method of epicycles. His exposition in this section remains purely didactical; as such it makes good reading in particular for non-experts in ancient astronomical thought.

As the present book is written for broader audience, not particularly for historians of astronomy, it may be useful to remind us of the rough time order of events in Greek astronomy insofar they concern our question by the following timetable.

¹I presented the argument in several talks years ago; among them 2013 at a conference in Aarhus. Jesper made me aware of Aaboe’s appreciation of the kinematic inversion (without using the word) at the Aarhus conference in 2013.
| Time Period          | Astronomy Type                          | Key Figures                                                                 | Source                                                                 |
|---------------------|----------------------------------------|-----------------------------------------------------------------------------|------------------------------------------------------------------------|
| Classical Greek     | Homocentric spheres                    | Eudoxos (ca 390 – ca. 340 BC), Kallippos (ca. 370– 300 BC), Aristotle (384 – 322 BC) | De caelo                                                               |
|                    |                                        |                                                                             |                                                                        |
| Upsurge of Heliocentric |                                      | Aristarchos of Samos (310 – 230 BC) (only indirect sources), Archimedes (∼ 287 – 212 BC), Sand reckoner, Seleukos of Seleukeia (∼ 180 – ∼ 120 BC) | Report by Plutarchos, 46 – 120 AD                                     |
|                    |                                        |                                                                             |                                                                        |
| Rise of Epicyclic   |                                        | Origin unclear, usual picture: rise out of a sudden, about the mid 3rd c. BC. |                                                                        |
|                    |                                        | Investigations of epicycles by Apollonios (∼ 260 – ∼ 190 BC) (source: Almagest). |                                                                        |
|                    |                                        | Late 3rd and 2nd c. BC ff. tool kit for epicyclic planetary models,          |                                                                        |
|                    |                                        | competitive with homocentric spheres.                                       |                                                                        |
|                    |                                        |                                                                             |                                                                        |
| Diverse Planetary  |                                        | Hipparchos of Nicaea (∼ 190 – ∼ 120 BC) (source: Almagest).                |                                                                        |
|                    |                                        | During the following centuries cumulative development of geometric tools for |                                                                        |
|                    |                                        | epicyclic models in order to assimilate diverse “anomalies” in particular   |                                                                        |
|                    |                                        | for the                                                                    |                                                                        |
|                    |                                        | – sun (old),                                                                |                                                                        |
|                    |                                        | – moon model (Hipparchos),                                                  |                                                                        |
|                    |                                        | – planets with two anomalies (source Almagest).                             |                                                                        |
|                    |                                        | Solution finally: decentered epicycles with equant point.                  |                                                                        |
|                    |                                        |                                                                             |                                                                        |
| Ptolemy             |                                        | Ptolemy (∼ 90 – ∼ 168 AD), full epicyclic model in the Almagest.            |                                                                        |
|                    |                                        | Fusion of epicyclic models with homocentric sphere view in Planetary        |                                                                        |
|                    |                                        | Hypotheses. Heliocentrism no longer explicitly mentioned.                  |                                                                        |

Note: Aristarchos’ heliocentrism and the first documented studies of epicyclic model by Apollonios are at most one generation apart, both around mid 3rd. century BC.

The first part of following the paper explains the geometry of the kinematic inversion from a heliocentric to a geocentric view in a systematic perspective, with side remarks to the historical context. For didactical reasons it proceeds stepwise: at first the inversion is considered for the sun (sec. 1.1), then for the interior planets Mercury, Venus (sec. 1.3) and finally for the outer planets Mars, Jupiter Saturn (sec. 1.4). Readers with a good imagination of kinematic constellations can skip these explanations, because they
will grasp immediately how the kinematic inversion of a heliocentric picture of planetary motion on circular orbits to the perspective of a terrestrial observer leads to simple epicyclic constructions. The last two subsections of the first part, however, may be instructive also for them. Sec. 1.5 gives a general survey of how observational data on the motion of the planets, available to Greek astronomers in the 3rd century BC, may be used to adapt the free parameters of these epicyclic constructions and, with it, the underlying heliocentric mechanisms. Although the theory of the moon stood in the center of many of the ancient Greek astronomical investigations, it will not appear in this paper, because its special status among the ancient “planets” makes it less instructive for the differences between a heliocentric and geocentric approach.

The second part of the paper (sec. 2) turns towards the historical perspective. It explains why I do not consider it admissible to neglect the historical relevance of the geometric argument developed in sec. 1 for the development of pre-Ptolemaic Greek astronomy. It is well known that the method of epicyclic motions in Hellenistic Greece originated, and had its first phase of development, in the roughly one and a half centuries between the life of the two main protagonists of the heliocentric view, Aristarchos of Samos (fl. ∼280 BC) and Seleukos of Seleukia (f. ∼150 BC), with Apollonios of Perga (fl. ∼230 BC) being the best known contributor to the theoretical study of epicyclic constellations just in the middle of the period. In my view this cannot be downplayed as a pure coincidence. It rather seems extremely likely that the authors on epicyclic motions of the 3rd and 2nd century BC knew of the relevance of their investigations for the heliocentric picture, and at least some of them were motivated by it. In particular the widely supported claim that neither Aristarchos nor Seleukos made any reasonable attempts for adapting the parameters of their heliocentric geometric pictures to observational data does not seem particularly convincing in the light of what we know on data availability to Greek astronomers in the 3rd and 2nd centuries BC and of what have seen in sec. 1.5.

Of course the beautiful, but too simple assumption of uniform circular movement was an obstacle for adapting kinematically inverted epicyclic models of heliocentric planetary motions to the data. The ensuing separation of a pragmatic use of epicyclic models from its heliocentric background is shortly discussed in sec. 2.2. We learn from Ptolemy’s later report in the *Almagest* and Neugebauer’s magisterial work on ancient astronomy (Neugebauer, 1975) that the difficulties of designing quantitatively satisfactory epicyclic models of planetary motions increased about the middle of the 2nd century BC and culminated in a critical review by Hipparchos of Nicea, the towering contemporary of Seleukos. Hipparchos, also an impressive observational astronomer, seems to have been a sceptic with regard to theoretical speculations which did not lead to quantitative models living up to the latest precision data of the time. This may have contributed to sever the pragmatic treatment of epicyclic models from its former link to the heliocentric view even further. At the time of Ptolemy (fl. ∼120 AD) the link was close to forgotten, perhaps even consciously suppressed because of ideological reasons.

In the final discussion (sec. 3) it will be explained why I find the present neglect of the role of heliocentric considerations on the trajectory of pre-Ptolemaic astronomy neither convincing nor fruitful.
1 Epicyclic models from kinematic inversion (systematic introduction)

1.1 Kinematic inversion of the earth’s motion about the sun

Aristarchos and other supporters of the heliocentric view had to address the question, how heliocentric motions (the imputed “true” ones) express themselves as “apparent” ones for a terrestrial observer. Until the times of Hipparchos to whom the first chord tables are attributed heliocentric thinkers had to rely mainly on geometrical methods, later called “synthetic” in contrast to trigonometric methods (or even algebraic/analytic ones) which were not available at the time. Lacking detailed historical sources, we still can look for appropriate geometrical constructions that may be used for converting heliocentric motions into those seen by a terrestrial observer. In this section this will be done in a systematic way; in sec. 2 it will then be discussed why this matters for our historical understanding of Hellenistic astronomy.

The simplest case and first step to do is, of course, to translate the motion of the earth ♁ (terra) about the sun ⊙ (soles). To do so one needs a geometric imagination of the “true motion” which can be represented in a Euclidean plane by a similar picture in the strict geometrical sense (see figure 1, left, for the motion of the earth about the sun). Here and in the following symbols labelled by an asterisk like S*, T* denote (moving) points in the plane representing the true objects, here the sun and the earth. The corresponding symbols without asterisk in the figure on the right represent a geometrical image of the apparent motion as seen from the earth T. Here, of course, S is the moving point. Because the practically “infinite” distance to the vernal point ♃ (aries), the ecliptic locus marking the position of the sun at the spring equinox, the broken lines (indicating the direction from S* to ♃, respectively from T to ♃) are parallel; for the connecting lines T*S* and TS the same holds, including orientation if one wants. Thus the respective angles are equal, \( \lambda^*(t) = \lambda(t) \) (a simple exercise in similarity geometry). The translation results in a motion of the sun S along the ecliptic, which is kinematically similar to the motion of the earth T* about S* in the following sense:

- Angles at a given time are equal, \( \lambda(t) = \lambda^*(t) \), and proportions are conserved, i.e., the ratios of distances in the figure on the right (the picture for a terrestrial observer) are equal to those of the corresponding distances on the left (similar to the true motion).

- In particular uniformity of circular motions and their periods are conserved. For the case of disuniformity similarity still holds in the sense of \( \lambda(t) = \lambda^*(t) \) (see fig. 2).

- The role of T* and S* is being inverted in the pair S and T.

For those accepting a heliocentric hypothesis (perhaps even sharing the heliocentric ontological conviction) such a geometrical consideration explains the apparent motion of

\(^2\)Here and in the following the attribute true motion is used in an Aristarchian heliocentric perspective; in the following it will no longer be relativized by using of quotation marks.

\(^3\)A more detailed picture can be found in (Aaboe, 2001, fig. 8).
the sun in the course of the year through the ecliptic circle, independent of the question of quantitative precision. Moreover this simple example makes clear in which sense we encounter here a “kinematic inversion” from true to apparent motions.

![Figure 1: Kinematic inversion for the motion of earth .observe and sun ⊙. Left: heliocentric motion of T∗ about S∗. Right: inverted motion as seen from an observer at T.](image)

But a warning is appropriate: A kinematically inverted picture of the earth’s uniform circular motion about the sun does not do justice to the unevenness of the sun’s motion in the course of the year (the “sun anomaly”). The sun anomaly had long been known in Greek science, probably already in the Miletian period. It was well documented at the time of Eudoxos (Szabó, 1992, p. 295) and accounted for in the homocentric sphere model by Kallippos about the middle of the 4th century BC by an ad hoc introduction of additional spheres. In the above form the heliocentric picture could not yet compete with the refined homocentric sphere approach of Eudoxos and Kallippos. But also here an ad hoc method, even a simpler one than in the homocentric model, may be used to mend the deficiency. It consists of a displacement of the true sun from the center of the earth’s motion which is still assumed as uniform and circular with regard to a center O∗ ≠ S∗. A similar decentration is then obtained for the apparent motion of the observable sun S which moves uniformly with respect to a center O such that ΔTSO ∼ ΔS∗T∗O∗ (orientation of edges changed) (figure 2). This results in an eccentric circular motion of which we know that it was studied by Apollonios. For the planetary motion this type of correction implies a simple eccentric deferent model (figure 7).

The uniform motion of S about T in fig. 1 reflects the motion of the so-called mean sun. Although Greek astronomer knew well that uniform motion is not what one sees

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4 Kallippos is reported to have used the data (95, 92, 89, 90) for the length of the seasons, counted from the vernal equinox (Dreyer, 1953, p. 106)
at the sky, one can still work with it as an idealization arising from averaging. The observational position of the sun deviates from the mean sun; it was called the true sun (in a second, phenomenological meaning). For disambiguating symbols, one may prefer to introduce different letters for the mean sun $S_m$ and the observationally “true” sun which we will denote by $S_{⊙}$, where necessary.

1.2 Side-remark on the order of the planets

In the homocentric system of spheres the order of the arrangement of the planets was a conventional question and changed with different authors. A heliocentric view, on the other hand, will immediately see a qualitative difference of the kinematic appearance between planets with orbital radius smaller than that of the earth and those of larger radius. Those of the first group stay kinematically close to the sun, seen from the earth, i.e. they never can form an angle $\alpha > 90^\circ$ (a right angle $R$ in early Greek terminology), the latter will take on any angular distance to the sun, with the extreme positions of opposition ($\alpha = 180^\circ = 2R$) and conjunction ($\alpha = 0$) to the sun. The planets of the first group (Mercury ♂ and Venus ♀) are called the “inferior planets” by historians of astronomy who try to avoid the heliocentric connotation of “inner”. The second group (Mars ♂, Jupiter ♃, Saturn ♄) are then called the “superior planets”,

Figure 2: Sun anomaly represented by decentered sun. Left: Earth $T^*$ moving uniformly on a circle about the center $O^*$ with an eccentric sun $S^*$. Right: Kinematic inversion with exchange of the role of $T$ and $S$. Here $\Delta OTS \sim \Delta O^*S^*T^*$ with corresponding sides parallel: $OT \parallel O^*S^*$, $TS \parallel S^*T^*$, $SO \parallel T^*O^*$. $S$ moves uniformly about $O$, while for an observer in $T$ its motion is non-uniform, i.e., the sun’s ecliptical longitude $\lambda(t)$ no longer progresses proportional to the time.
avoiding the qualification as “outer” ones.\(^5\) The anomalies of the motion on the ecliptic (in particular the maximal elongations of the inferior planets from the sun, and for the superior planets from its mean position) suffices for establishing an order of increasing orbital radii, already on the basis of rough data on the change of planetary positions over the course of the year. Although the terminology \textit{inferior, superior} is not to be found in Ptolemy’s \textit{Almagest}, he made a clear distinction between the two groups and proposed a definite order of the “spheres” of the planets and the sun indicated by (Book IX, 1):

\[
♀ ♀ ⊙ ♂ X ♄
\] (1)

This is the heliocentrically “correct” order if one keeps the kinematic inversion of the role of sun and earth in mind. Ptolemy \textit{gave no systematic reason} for this order, although he could have given one by the order of the respective radii of deferents. He rather preferred to give a slightly confused \textit{historical justification} for the placing of Venus and Mercury below the sun.\(^6\) All this looks very much like a hidden (forgotten or suppressed) influence of an input of heliocentrism into the tradition of planetary theory in Hellenistic times.\(^7\)

1.3 Kinematic inversion of the motion of inferior planets

The kinematic inversion of the motion of planets \(P^*\) with true orbit inside the path of the earth \(T^*\) (Mercury \(♀\) and Venus \(♀\)) is nearly as simple as that for the earth itself, if one assumes concentric circular true orbits of \(P^*\) and \(T^*\). The apparent motion of such a planet is given by a point \(P\) moving on a circle about a center, in the terminology of epicyclic models called the \textit{deferent} \(D\) (figure 3). The geometrical motion resides on parallel and similar triangles with directed sides. The deferent moves with the apparent position \(S\) of the sun, constructed in the first step (figure 1). Because corresponding connecting lines are parallel, \(TD \parallel T^*S^*\), \(DP \parallel S^*P^*\) and \(TP \parallel T^*S^*\) the kinematic picture (fig. 3 right) is again similar to the true geometric constellation (fig. 3 left). In particular the ratios formed by the radii \(r_P = |S^*P_i^*|\), \(r_T = |S^*T^*|\) and the radii \(r_1 = |TD|\), \(r_2 = |DP_i|\) of the inverted picture are equal:

\[
r_P : r_T = r_2 : r_1
\]

\(^5\)The terminology “inner” and “outer” planets is used in (Neugebauer, 1975)

\(^6\)But concerning the spheres of Venus and Mercury, we see that they are placed below the sun’s by the more ancient astronomers, but by some of their successors these too are placed above [the sun’s], for the reason that the sun has never been obscured by them [Venus and Mercury] either.” (Ptolemaios, 1998, p. 419)

\(^7\)This impression is strengthened, against the intention of the author, by O. Neugebauer’s discussion of the question in his \textit{History of Mathematical Astronomy.} He over and over states that heliocentric investigations could not have been of much influence on the development of Greek planetary theory because, according to him, no quantitative conclusion could be drawn from them (see sec. 3). But the information he gives on the sources seem to speak against this picture. In an extended presentation of source material diverse different arrangements of the planets in Greek and Mesopotamian astronomy (Neugebauer, 1975, IV C 2.1, §2.1) he comments Ptolemy’s order (1). He remarks that Cicero (106–45 BC), Vitruvius (1st c. BC), Pliny (23–79 AD), and Plutarch (45–125 AD) “took it for granted” (Neugebauer, 1975, p. 691). With other words, the order of planets stabilized to the one indicated by a heliocentric view in the time after Hipparchos. This is no definitive proof of a heliocentric influence on Greek planetary theory but adds to the circumstantial evidence for it.
The description of the apparent motion of interior planets (under the assumption of concentric true orbits) leads necessarily to a simple epicyclic presentation. For heliocentrist such epicycles are, of course, not “real” but only constructions devised to “save” (express) the phenomena, i.e., they are phenomenological models in the modern terminology.

Figure 3: Kinematic inversion for inferior planets. Left: Motion of an inferior planet \( P_i \) and the earth \( T \) about the sun \( S \). Right: Apparent motion of the planet \( P_i \) seen from a terrestrial observe at \( T \). \( P_i \) moves on an epicycle with center \( D \) identical to the position of the apparent sun \( (D = S) \). The similarity of triangles \( \Delta T P_i S \sim \Delta T P_i S \) is due to the parallelity of corresponding sides. \( \lambda(t) \) is the time dependent ecliptic longitude of the planet.

1.4 Superior planets, qualitative kinematics

After the first kinematic inversion of the motion of the sun as above, a superior planet \( P_s \) will run on an epicycle with larger radius than that of the deferent \( (D_1 = S) \) (figure 4, middle). Mathematically this is no problem. Basically this is all one needs for a geometrical representation of the superior planet’s motion seen from the earth. For a graphical (or later quantitative) evaluation it may be a disadvantage that the deferent \( D_1 = S \) has a smaller period (and thus greater angular velocity) than the motion of \( P_s \) on its epicycle. The constellation can just as well be understood as a circular motion of \( P_s \) about a variable eccenter \( D_1 \) moving on a circle with center \( T \).

This seems puzzling and one may look for an equivalent kinematic representation which is closer in structure to the one for the inferior planets. A similarity consideration of moving triangles allows to transform the kinematics of \( P_{s1} \) about \( T \) (fig. 4, middle) into one given by an epicyclic motion with deferent radius \(|TD_2|\) larger than the radius...
of the epicycle. Figure 4, right, shows how this can be achieved by a second kinematic inversion by which the roles of deferent and epicycle are exchanged.

In this figure the following parallelisms are demanded to hold at each moment of the motion:

\[ D_1 P_s \parallel S^* P_s^* \parallel TD_2, \quad D_1 T_1 \parallel S^* T^* \parallel P_s D_2, \quad T^* P_s^* \parallel TP_s \]

Then the following moving triangles are similar:\(^8\)

\[ \Delta T_1 D_1 P_s \text{ (middle)} \sim \Delta T^* S^* P_s^* \text{ (left)} \sim \Delta P_s D_2 T \text{ (right)} \]

If one keeps the similarity ratios constant during the motion, \( D_2 \) lies on a circle about \( T \) and \( P_s \) on a (moving) circle about \( D_2 \). Note also the direction of edges. Expressed in modern terminology, the orientation of the triangle on the right has been changed with regard to the other ones (left and middle).

Kinematically the configuration of fig. 4, right, implies an apparent motion of \( P_s \), satisfying the following conditions:

(i) \( D_2 \) moves on a circle about \( T \) similar to the motion of \( P_s^* \) about \( S^* \) (with the same orientation and angular velocity).

(ii) \( P_s \) moves on a circle about \( D_2 \) similar to the motion of \( T^* \) about \( S^* \).

(iii) The planet is in opposition to the sun (i.e. \( P_s^*, T^*, S^* \) collinear) for \( T, P_s, D_2 \) collinear; i.e., in the “middle” of the retrograde motion of the planet on the “lower” arc of the epicycle.

Using the language of epicycles, the (superior) planet \( P_s \) moves on an epicycle about the deferent \( D_2 \); but after the second kinematic inversion the epicyclic model gives a rather “surreal” picture of the real geometric constellations (e.g. by item (iii)). A typical retrogradation loop in the apparent motion of superior planet \( P_s \) is generated by passing through the epicycle once; in this time its mean position marked by \( D_2 \) has progressed along the ecliptic; this motion is called a *synodic cycle* of the planet. In the course of such a cycle specific *synodic events* play a distinguished role for observation and theory: the opposition or conjunction to the sun, a (forward, respectively backward) stationary point in the loop of retrogradation, or the day of first or last visibility.

The directed line \( D_2 P_s \) (deferent — planet) is parallel (including direction) to \( T^* S^* \) and thus to \( TD_1 \), the direction of the mean sun (because of \( D_1 = S \)). Condition (i) tells us then that the epicyclic motion of the planet mirrors the motion of the earth about the sun respectively, after the first kinematic inversion, the motion of the mean sun. This property was still demanded by the construction in Ptolemy’s *Almagest*; but there it appeared as an *unexplained and surprising feature* inbuilt into the formal rules of epicycle construction (cf. section 3). Similarly condition (ii) means that the motion of the deferent mirrors the averaged (mean) motion of the planet about the sun. *This*

\(^8\)In vector notation with similarity factor \( \lambda \), Def/construction: \( \overrightarrow{TD} = \lambda \overrightarrow{S^* P_s^*} \), \( \overrightarrow{DP} = \lambda \overrightarrow{T^* S^*} \); thus \( \overrightarrow{TP} = \overrightarrow{TD} + \overrightarrow{DP} = \lambda (\overrightarrow{S^* P_s^*} + \overrightarrow{T^* S^*}) = \lambda \overrightarrow{T^* P_s^*} \). This can, of course, been expressed in terms of classical geometry like in the main text.
Figure 4: Kinematic inversion of the motion of the earth $T^*$ and a superior planet $P^*_s$ about the sun $S^*$. Left: True motion about $S^*$. Middle: First kinematic inversion like for inferior planets with deferent identical to the mean sun $D_1 = S_m$ (cf. fig. 3). This leads to a larger radius of the epicycle than for the deferent. Right: Second kinematic inversion with interchanged radii. The deferent $D_2$ moves now on the larger circle, the planet $P_s$ as seen from an observer on $T$ on an epicycle of smaller radius. Note the similarity of triangles, in particular $\Delta T^* S^* P^*_s \sim \Delta P_s D_2 T$ (see text). Here motion of the deferent $D_2$ reflects the motion of $P^*_s$ about $S^*$, while the epicyclic motion of $P_s$ about $D_2$ reflects the motion of the earth $T^*$ about the sun $S^*$. (Ecliptic longitude $\lambda(t)$ in this picture is being charted to the West.)
feature could not even be expressed in the framework of Ptolemaos’ reduced epicyclic framework, although \( TD_2 \) was miraculously interpreted as as a representative for some kind of mean distance to the planet. His epicyclic method was reduced in the sense that the heliocentric origin of the construction was no longer part of contemporary knowledge; at least as regards the parts which were transmitted to later centuries (see section 2).

1.5 First quantitative adaptation to observational data and a provisional resumé

While the historical knowledge on Mesopotamian astronomical data and algorithms can now rely on detailed source material (Neugebauer, 1975), this is not the case for classical and Hellenistic Greek astronomy. This may have contributed to the general view that numerical evaluation of geometrical models using data of Mesopotamian astronomy did not start effectively before Hipparchos, who was also the central figure for introducing sexagesimal numerics and trigonometric tables into Greek applied mathematics (Jones, 1990, 1991). We know, however, that theoretical questions which are important for the quantitative adaptation of epicyclic models to observational data were already topical at the time of Apollonios. It is thus not convincing to assume that astronomer-mathematicians of the late 3rd c. BC have not undertaken attempts for such quantitative adaptations of the models.

Regarding observational data on planetary motion in pre-Hipparchian times, Greek astronomers could start from a rudimentary knowledge of the number count of synodic cycles \( s \) and the associated number \( r \) of revolutions of a planet in the ecliptic during a given number \( N \) of solar years. They were recorded by Mesopotamian and/or Egyptian scribes for long periods of the planets.\(^9\) This suffices for a first quantitative adaptation of the epicyclic models to the observations by considerations which were later refined by Hipparchos. Later they found entrance into the corpus of the Almagest. The procedure is explained in terms of Mesopotamian sexagesimal calculations in (Aaboe, 2001, p. 79ff.). The same method can just as well be expressed in terms of the classical Greek concept of angle quantities and the calculus of numerical ratios, i.e., with the methods available already during the 3rd c. BC.

In the 4th c. B.C. the arc of the regular dodecagon, also called a zodion \( z \) (lat. signum, i.e., sign of the zodiac) was sometimes used as a basic angle unit (\( 1z = 30° \)).\(^{10}\) With a set of data \((r, s, N)\) as above for a superior planet \( P_s \), \( r \) number of ecliptic revolutions, \( s \) of synodic cycles in \( N \) years, the mean angular progression \( \omega_1 \) of \( P \) per day \( (d) \) in the ecliptic (in later terminology the angular velocity in units \( zd^{-1} \)) is given by the ratio

\[
\omega_1 = \frac{12r}{N \cdot Y} \quad \text{with } Y = \text{number of days per year (usually } 365\frac{1}{4} \).
\]

Similarly for the progression \( \omega_2 \) (angular velocity in units \( zd^{-1} \)) of the deferent \( D_2 \), the

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\(^9\)A “long period” is the time between two events in which the planet stands at the same place in the ecliptic in equal synodic constellations. A “synodic constellation” may be specified by any of the distinguished synodic events.

\(^{10}\)The zodion was used by Autolykos (4th c. B.C.) (Hultsch, 1885, pp. 92f.), cf. (Szabó, 1994, p. 99), also (Neugebauer, 1975, p. 593).
“mean motion” of the planet is

\[
\omega_2 = \frac{12s}{N \cdot Y}.
\]

As the progression of \( P_s \) on the epicycle is measured against a synodic reference line, e.g. the one defining the conjunction of the planet with the mean sun, the progression \( \omega \) of \( P_s \) with regard to a fixed reference point of the ecliptic, e.g. aries \( \text{♈} \), is given by

\[
\omega = \omega_1 + \omega_2 \quad \text{(fig. 5)}.
\]

**Figure 5:** Progression of angles in epicyclic model: The deferent \( D \) progresses by angle \( \omega_1 \) with regard to a fixed reference direction on the ecliptic (here aries \( \text{♈} \)). The planet \( P \) moves on an epicycle with angle \( \omega_2 \) in relation to the radial direction of the deferent. The angle \( \omega \) of the planet \( P \) with regard to \( \text{♈} \) (and its progression) is \( \omega = \omega_1 + \omega_2 \).

For an outer planet \( \omega \) is the progression of the mean sun, which is 1 round angle per year, or \( \frac{12}{Y} \) in terms of zodiacs per day. The heliocentric hypothesis leads thus to the constraint \( \frac{12}{Y} = \frac{12(r+s)}{N \cdot Y} \), i.e.,

\[
r + s = N \quad \text{for superior planets.} \tag{2}
\]

Moreover, it is an immediate consequence of the heliocentric assumption that

\[
r = N \quad \text{for inferior planets.} \tag{3}
\]
These are structural predictions of the heliocentric hypothesis, which could easily be stated with the knowledge of Greek astronomy of the 4th century BC. Due to the lack of sources we cannot say which Mesopotamian data were available to Greek astronomers at this time, but it is clear that (2), (3) are consistent with Mesopotamian observations and also with the later Greek ones. Such data were contained in Babylonian ("goal year) texts of the 4th c. BC.\(^{11}\) Much later they were reproduced in the *Almagest*, vol. IX.3, by Ptolemy who ascribed them to Hipparchos. It seems likely that such simple data sets were already known to Greek astronomers of the 3rd c. BC. In the following table Ptolemy’s data \((N, r, s)\) are given in simplified form, rounded to integer numbers. The corresponding values for \(\omega_1, \omega_2\) in \(z/d\) (zodion per day) have been added in the Greek style as continued fractions up to order 2 \(\left[a_0; a_1, a_2\right]\) in decimal fractions \(\left(^{{°}}/d\right)\) and in total revolutions per year.

|       | N  | r  | s  | \(\omega_1 (z/d)\) | \(\omega_2 (z/d)\) | \(\left(\omega_1, \omega_2\right) \quad \left(^{{°}}/d\right)\) | \(\left(\omega_1, \omega_2\right)\) rev./y |
|-------|----|----|----|---------------------|---------------------|---------------------------------------------|---------------------------------------------|
| ♂ Mercury | 46 | 46 | 145 | [0; 30,2] | [0; 9,1] | (0.986, 3.107) | (1, 3.15) |
| ♀ Venus | 8  | 8  | 5   | [0; 30,2] | [0; 48,1] | (0.986, 0.616) | (1, 0.625) |
| ♁ Mars | 79 | 42 | 37  | [0; 57,3] | [0; 64,1] | (0.524, 0.462) | (0.532, 0.468) |
| ♄ Jupiter | 71 | 6  | 65  | [0; 360,5] | [0; 33,4] | (0.083, 0.902) | (0.085, 0.915) |
| ♅ Saturn | 59 | 2  | 57  | [0; 897,1] | [0; 31,1] | (0.033, 0.952) | (0.034, 0.966) |

Obviously the identities (2), (3) are satisfied.\(^{12}\)

For Aristarchos, or any of his contemporaries and successors studying the heliocentric hypothesis, the question arose how the motion of celestial bodies appears to a terrestrial observer. The answer for the mean motion of the sun and for the planets must have led a geometry of the time, who posed the question, to an insight boiling down to the kinematic inversion of secs. 1.1, 1.3, 1.4. On a qualitative level the retrograde motions of planets are well explained by kinematically inverting a heliocentric picture of the motion of the earth and the planets, already under the most simple (and in a first approach ontologically appealing) assumption of uniform motion of the planets including the earth, on concentric circles about the sun. And even quantitative predictions (2) and (3) could be derived from the heliocentric assumption translated to the corresponding epicyclic kinematics. In Mesopotamian astronomy the corresponding relations were known to hold by empirical reasons;\(^{13}\) in the later Ptolemaic codification they appeared as formal rules without reason.

An estimate of the relative radii \(r_1 = |TD|, r_2 = |DP|\) of the representing circles from empirical data was an important step for the adaptation of heliocentric and epicyclic models to the data. It could be achieved in specific cases by chord-calculations without the recourse to general chord tables on the observational basis of maximal elongations. The latter are easier to observe than stationary points, at least for inferior planets, cf. *Almagest*, IX.2.

\(^{11}\)(Neugebauer, 1975, p. 151),(Gray and Steele, 2008).

\(^{12}\)For a discussion of the Mesopotamian background of these data see (Jones and Duke, 2005). See also (Aaboe, 2001, p. 79f.) for the corresponding values for Venus and Mars in sexagesimal notation.

\(^{13}\)(Neugebauer, 1983, p. 312) calls relations (2), (3) “empirical fact(s)” and at another place as “well known in Babylonian astronomy” (Neugebauer, 1975, p. 389).
Figure 6: Maximal elongation $\beta$ of a planet $P$ on epicycle about $D$, seen from earth $T$.

If $\beta$ is the maximal elongation angle between a planet $P$ and the deferent $D$ (the mean sun for inferior planets and the mean planet for the superior ones), $r_1$ the radius of the deferent circle and $r_2$ the radius of the epicycle radius, one finds (fig. 6)\(^{14}\)

$$r_2 = r_1 \sin \beta, \quad \text{or} \quad 2r_2 = r_1 \text{ch} 2\beta \quad \text{in terms of chords \(ch\ x\).}$$

For the inferior planets (and if one is interested primarily in epicyclic modelling) one may like to choose $r_1 = 1$ (radius of earth orbit as unit), or $r_1 = 60^p$ ($p$ for partes) in Ptolemaic units. For the heliocentric geometry of a superior planet one better sets $r_2 = 1$, respectively $60^p$, (i.e., the kinematically inverted radius of the earth’s orbit is taken as unit) and to solve for $r_1$, the radius of the planet’s orbit.

For interior planets the maximal elongation angle $\beta$ as evening or as morning stars has been observed quite early by Greek astronomers. For Mercury Ptolemy reports data taken in 262 BC (\textit{Almagest}, IX.7) with a mean of $\beta \approx 25^\circ$, while two data taken by himself (or his group) (\textit{Almagest}, IX.9) have a mean $\beta \approx 23^\circ$. In pre-Hipparchian times, when chord tables were not yet established, one could approximate this elongation by half the angle of a regular octagon ($\beta = 22.5^\circ$). Then (4) leads to $r_2 : r_1 \approx 0.38 \sim 23^p$.\(^{15}\) Ptolemy reports elongation data for Venus taken by Timocharis in 272 BC (\textit{Almagest}, IX.10).\(^{16}\)

\(^{14}\)This presupposes, of course, centered deferent circles; in case of eccentricity modifications have to be made.

\(^{15}\)Ptolemy’s own value for Mercury is $r_1 = 22^\circ 30'$ (\textit{Almagest} IX.10).
He translated them into the angle to the mean sun, with mean $\beta \approx 42.5^\circ$. In pre-Hipparchian considerations this could be described roughly as half the angle of the regular quadrangle ($\beta \approx 45^\circ$), which implies $r_2 : r_1 \approx 0.71 \sim 42^p.17$

Observations of the maximal elongation of superior planets from their mean position play no role for Ptolemy’s approach; this may be the reason that they are not recorded by him. But once one has the picture of the second kinematic inversion of superior planets in mind (fig. 4, right), it is clear how to observe it: One may, e.g., start from the opposition of the planet (the position of $P_s$ collinear with $T$ and $D$ on the lower arc of the epicycle) at ecliptic longitude $\lambda_0$. Then one observes the retrograde motion of $P_s$ and continues to follow the path of the planet until, after the time $\Delta t$ (in days), it reaches the longitude $\lambda_1$, where its motion is now prograde and in tune with the progression of the mean planet ($D_2$ in our notation). Then the maximal elongation $\beta$ has been reached. With $\Delta \lambda = \lambda_0 - \lambda_1$ and the progression of $D_2$ given by $\omega_1$ and $\Delta t$ it is

$$\beta = \Delta \lambda + \omega_1 \Delta t. \quad (5)$$

One expects that for large outer radii (Jupiter and Saturn) $\Delta t$ is roughly a quarter of a year; while for a planet with orbit not much outside the earth’s orbit (i.e., Mars) $D_2$ has considerably progressed during this time, and $\Delta t$ may be considerably longer than a quarter year. Lacking historical records on such data, the application of this method is here being checked on “simulated” data. With just one data set of 2022 for each outer planet (thus less balanced than taking mean values of $\beta$ starting from different positions of the opposition on the ecliptic) we find (rounded to half degrees): 18 for Mars $\beta \approx 42^\circ$, for Jupiter $\beta \approx 11^\circ$ for Saturn $\beta \approx 6^\circ$

In pre-Hipparchian times one would approximate chords of twice these angles by regular $n$-polygons, with $n = 4$ for Mars ($\beta \approx 45^\circ$ like for Venus, but with reciprocal proportion), $n = 16$ for Jupiter ($\beta \approx 11.25^\circ$), and $n = 30$ for Saturn ($\beta \approx 6^\circ$). The corresponding values for the radii of the planets expressed in terms of the radius of the earth orbit are given in the following table; added are the proportions $r_2 : r_1$ in partes and the corresponding Ptolemaic values; in the last column the modern mean distances of the planets to the sun are given in astronomical units $AU$. 

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16 van der Waerden (1984, p. 130) conjectures that the data taken by Timocharis, like those of other astronomers discussed in his paper, were intended to determine the parameters of a heliocentric planetary model.

17 In his refined model Ptolemy arrived at $r_2 = 43^\circ 10'$. 

18 Opposition of Mars Dec 08, 2022, of Jupiter Sep 26, 2022, and of Saturn Aug 14, 2022. Ecliptical longitudes at opposition: Mars 52.03°, Jupiter 339.64°, Saturn 297.79°; $\Delta \lambda$ for Mars $-26.25^\circ$, for Jupiter $4.2^\circ$, for Saturn $3.04^\circ$ after $\Delta t = 130$ days for Mars, 90 days for Jupiter and Saturn. For Saturn the angular velocity is so small that a naked eye observation of the exact day of maximal elongation is impossible. But the progression of longitude between $\Delta t = 90$ and 100 is just $+0.23^\circ$ and $10\omega_1 = 0.33$, which leads to a negligible difference in (5). Ephemeris data 2022 from internet resource https://www.mpanchang.com/planets/ephemeris/. This (Vedic) astrological website uses modern ephemeris calculations; it is useful for our purpose because it displays ecliptical longitudes and ecliptical angular velocities of all planets for arbitrary times. Those who prefer could even “simulate” the elongation observations for any date in the 3rd century BC.
For the simplest possible approach to heliocentrically founded epicyclic astronomy the
quantitative results are surprisingly close to the values of Ptolemy’s much more refined
model and the modern values of the mean radii of the planetary orbits. It should be
added that all the regular \( n \)-gons appearing here (\( n = 4, 8, 16, 30 \)) are constructible in
the Euclidean style, i.e., with ruler and compass.

But it must have been clear that planets do not move as uniformly as such an averaged
picture suggests. If one goes a bit deeper into quantitative detail, two types of deviations
from uniformity become visible:

(i) The retrogradation loops and the angles \( \beta \) of maximal elongation do not cover
always the same arc but change with the position of the deferent in the ecliptic
(so-called “solar anomaly”).

(ii) The mean motion (modelled by the deferent) deviates from uniform circular motion
also for superior planets, similar to the sun’s anomaly during the course of the year
(“zodiacal anomaly”).

A change of the progression of synodic events (e.g. the forward stationary point) in the
ecliptic was known to the Mesopotamian astronomers; they accounted for it by changing
the arithmetical rules for the progression of two consecutive events in different parts of
the ecliptic.\(^{19}\) In their way the Mesopotamian astronomers thus knew of irregularities
 corresponding to the effect (ii). In the epicyclic model arising from kinematic inversion
of the heliocentric picture both effects can be taken into account only \textit{separately}, and ad
hoc, by relaxing the condition that the terrestrial observer \( T \) is placed at the center \( C \)
of the deferent circle, e.g., by an eccentric model like in figure 7. In any case the maximal
error in longitude of an eccentric epicyclic model stayed than at the order of magnitude
of \( 10^\circ \) for Mars.\(^{20}\)

The geometry of kinematic inversion is of course valid independently of assuming
uniform circular motion; an example has been given above for the simple eccentric model,
fig. 2. Applied to the Kepler kinematic of elliptical orbits it may help the modern
reader to understand and to analyse the ad hoc moves of ancient astronomers to bring
their epicyclic models closer to the observed phenomena. An approximation of Kepler
kinematics by a heliocentric equant model for the orbits of outer planets and a simplified
uniform circular motion of the earth would, e.g., boil down to Ptolemy’s equant model
after the kinematic inversion (see end of sec. 3).

\(^{19}\)The change was implemented roughly by a step function in the Mesopotamian system \( A \) and more
subtly by a piecewise linear (zig-zag) function in system \( B \) (Neugebauer, 1975; van der Waerden,
1966),(Aaboe, 2001, p. 42ff.).

\(^{20}\)(Evans, 1984; Rawlins, 1987).
Figure 7: Simply eccentric deferent model arising from kinematic inversion of the orbit of an inferior planet moving about a decentered true sun (cf. fig. 2). The deferent $D$ moves uniformly on a circle about $O$, but is seen with non-uniformly changing ecliptic longitude $\lambda$ for a terrestrial observer at $T$ (“ecliptic anomaly” for the motion of the mean planet).

2 Another look at pre-Ptolemaic epicyclic theory (historical argument)

2.1 Heliocentrism and early work with epicyclic models (Aristarchos to Seleukos)

The argument developed above strongly suggests that the method of epicycles was born during the 3rd century BC as a consequence of the researches of Greek astronomer-mathematicians who investigated the heliocentric hypothesis. As the sources on heliocentrism during the Hellenistic period are extremely sparse, we can neither be completely sure about this, nor ascribe this achievement to a specific person or school with any acceptable certainty. It is a pity that we have extremely thin documentary evidence on the work of Aristarchos who would be an obvious candidate for such insights. In any case, he is generally accepted as the first “serious” (i.e., mathematically literate) proponent of the heliocentric view in Hellenistic times. He had good reasons for such a view on the basis of his estimates for the comparative sizes and distances of the earth, moon and sun, even though they are far away from later, more precise, estimates (Aristarchos, 1913).\(^{21}\)

\(^{21}\)His only transmitted work is (Aristarchos, 1913). The most reliable information of him having seriously defended the heliocentric hypothesis stems from Archimedes speaking about Aristarchos: “His hypotheses are that the fixed stars and the sun remain unmoved, that the earth revolves about the sun on the circumference of a circle, the sun lying in the middle of the orbit, and that the sphere of the fixed stars, situated about the same centre as the sun, is so great that the circle in which
We do know that half a century later Apollonios of Perga investigated epicyclic kinematics. According to Ptolemy, Apollonios studied the equivalence between a simple eccentric circular motion and an adapted concentric model with one epicycle (Almagest XII.1); moreover he gave a geometric characterization of the stationary points of a retrogradation loop for an epicyclic kinematic model (of the type shown in fig. 4, right). He is thus often considered as the inventor of the methods of epicycles. Such an attribution is highly problematic, however, because the source gives only evidence for him being an “auctor ante quem”, an author up to whom the method of epicycles has been introduced. Be this as it may, Ptolemy’s report is clear evidence that the method of epicyclic models including eccentric decentration of circular motion was in use during the late 3rd century BC already. Moreover it shows that the study of kinematic equivalence of different models between heavenly motions was a topic for mathematicians at that time.

What was the motivation for such investigations? Ptolemy remained silent on this question, and Apollonios’ work is not preserved. But for describing the phenomenology of the heliocentric point of view the question analysed by Apollonios makes perfect sense: The simplest method for producing a non-uniform motion of the apparent sun $S$ is to assume an eccentric dislocation of the true sun $S^*$ from the circle’s center $A^*$. Then the kinematic inversion of the earth’s true motion results in a motion of the apparent sun $S_C$ on an eccentric circle with regard to the earth $T$, which deviates from the uniform motion of the mean sun $S_m$ (fig. 2). As this is a disturbing feature, one may be inclined to ask whether the generated motion may be substituted by a construction using the kinematic elements provided by the first kinematic inversion, i.e. an epicyclic motion about the mean sun $S_m$. From this perspective, Apollonios’ equivalence theorem appears no longer as a purely theoretical exercise. It also gives an answer to a question arising naturally in an attempt for bringing the kinematic inversion of the earth’s motion into agreement with the observation of the sun’s yearly course through the ecliptic.

His study of the stationary points of retrograde motions in an epicyclic model indicate a strong interest in the quantitative adaptation of the latter to observational data. In the light of Archimedes’ clear and respectful reference to Aristarchos it would seem strange to assume that Apollonios, only a generation later, did not know about the heliocentric hypothesis. And a geometer like him would surely be aware that the motion of planets and the sun are given by epicyclic constructions for a terrestrial observer. We do not know whether Apollonios considered the heliocentric hypothesis as realistic, but a mathematical investigation of the latter could be done independent of whether one shared the ontological perspective of Aristarchos. His investigation of eccentric motions shows that Apollonios was well aware of the fruitfulness of considering kinematically equivalent constructions. It would be plausible to ascribe the detection of the second kinematic inversion for the motion of outer planets to him.

It is difficult to specify when Mesopotamian planetary tables became known to Greek

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he supposes the earth to revolve bears such a proportion to the distance of the fixed stars as the centre of the sphere bears to its surface.” (Archimedes, 1987, p. 222 (409)). Another testimony for Aristarchos’ heliocentric view is Plutarchos De facie in orbe Lunae, quoted in (van der Waerden, 1987, p. 526).

For example in (Kragh, 2007; Evans, 1984; North, 2001; Pedersen, 1974/2011).
astronomers, although it seems to have taken place during 3rd/2nd c. BC.\textsuperscript{23} We know from Ptolemy that at this time observational planetary data were already taken by Greek astronomers with quite some precision (\textit{Timocharis} in the year 272 BC, see above). It may be that the Babylonian astronomical tables (ephemerides) were considered a challenge to Greek astronomy already at the time of Apollonios; at least it is clear that the problem of a quantitative adaption of the epicyclic or homocentric spherical geometrical models to more or less realistic data was posed and had to be attacked already from the 3rd century BC onward.

Thus the claim that this challenge was \textit{not} accepted by the supporters of the heliocentric hypothesis appears rather outlandish. Just to the contrary, the close systematic connection between exploring the visible phenomena at the sky on the basis of a heliocentric assumption with the study of epicyclic constructions \textit{and} the latters’ rise during the 3rd century BC makes it highly plausible that the method of epicycles arose in the context of researches aiming at drawing not only qualitative, but also quantitative consequences from the heliocentric hypothesis. In contrast to what is usually stated in the history of Hellenistic astronomy (see sec. 3) it \textit{was possible} for Greek astronomers of the 3rd and 2nd centuries BC to adapt their kinematic models arising from the heliocentric picture to observational data (sec. 1.5) and to check their reliability with the the course of time. Due to the lack of sources we cannot, of course, prove without doubt that \textit{they did so}.

Plutarchos informs us about another astronomer of Hellenistic time, Seleukos of Seleukeia (Mesopotamia), \textasciitilde{} 180 – \textasciitilde{} 120 BC, who probably knew about both, the Mesopotamian and the Greek traditions. In his remark, Plutarchos’ distinguishes between Aristarchos who \textit{supported the hypothesis} (\textit{ὑποτιθέμενος μόνον}) of heliocentrism, while Seleukos \textit{declared it to be true}, or \textit{demonstrated it} (\textit{καὶ ἀποφεινόμενος}).\textsuperscript{24} It has been discussed by different authors what this linguistic difference may mean, but without convincing results.\textsuperscript{25} On the other hand it is clear that Seleukos, writing a century after Aristarchos, was in a much better position to draw quantitative consequences of the heliocentric picture than the initiator of the hypothesis. It is possible that he was able to compare some of its results with ephemeride predictions of the Mesopotamian systems \textit{A} or \textit{B}. This presupposed more complicated chord-calculations than needed for the first quantitative determination of parameters discussed in section 1.5, at least for a series of selected values. But why should one not assume that the calculational skills necessary for the preparation and application of chord tables attributed to Hipparchos were shared by other members of the community of his time?

As long as we are lacking direct sources on heliocentrism during the 3rd and 2nd centuries BC it is recommendable to complement the indirect source evidence with investigations of systematic relationships between the heliocentric view and the epicyclic method. This relationship is so close that we can state the following:

\textsuperscript{23}Transmission of mathematical astronomy, chronologically, (\ldots) seems more or less with the transmission of astrology, probably not much before 200 B.C.” (Neugebauer, 1975, p. 613ff.).

\textsuperscript{24}\textit{Platonicæ questiones} 1006C, here cited from (Heath, 1913, p. 515), compare (van der Waerden, 1987, p. 528).

\textsuperscript{25}(van der Waerden, 1987; Russo, 2004).
Epicyclic constructions were a topic for mathematician-astronomers of the 3rd century, in particular Apollonius; but this does not signify that Apollonius invented the method of epicycles.

It is likely that the epicyclic method arose as a result of attempts at translating heliocentric planetary motions into the apparent kinematic seen from the earth. We do not know when this happened, but we can say that it took place between Aristarchos and Apollonius (boundaries included, with preference for the first one).

In the 3rd and 2nd centuries BC Greek astronomy was characterized by two different competing geometric programs: the traditional homocentric sphere approach of Eudoxos and Kallypos, in which the central position of the earth was a fundamental assumption, and a new one built on the method of epicyclic models which had its background and origin in a kinematic inversion of the heliocentric perspective of Aristarchos. In the latter case the central position of the earth in the epicyclic constructions played here only a perspectival, not an ontological fundamental role.

Eccentric deferent models (figure 7) were known and studied already at the time of Apollonius. They were explored as tools for generating deviations from uniform motion of the planets in the sky. Although standing in tension with an “orthodox” heliocentric view of planetary motion, they may have been considered for some while as an acceptable modification of it.

Mesopotamian counts of long periods, enriched by observations of Greek astronomers of the 3rd and 2nd centuries BC, added by simple chord calculations sufficed for a first adaption of epicyclic models to observational data, including a realistic interpretation of the proportion of radii as relative distances of the planets from the sun. It was thus possible to derive basic quantitative predictions from the heliocentric view of planetary motions. It is possible that at the time of Seleukos more complex comparisons with the discrete time series for planetary ephemerides derived in the Mesopotamian systems were performed.

In his challenging book Russo pushed the thesis to the extreme that Seleukos was convinced of a dynamical underpinning of the heliocentric view by considerations of the contemporary research on the origin of tides (Russo, 2004, sec. 10.12). I agree that it may very well be that dynamical considerations played an important supporting role for the heliocentric perspective in the time between Aristarchos and Seleukos. But the increasing importance of eccentric modifications of epicyclic models in astronomy must have contributed to undermining its plausibility. In case some supporter of heliocentrism had played with the idea of, e.g., some kind of “counter-sun” as a true center of the dynamics (in analogy to the “counter-earth” postulated by some Pythagorean thinkers in early Greek science) in order to make eccentric epicyclic models compatible with heliocentrism, the increasing precision knowledge about different types of eccentricities must have aggravated the problems for the paradigm.
2.2 Separation of epicycles from a heliocentric background (Hipparchos to Ptolemy)

These problems became manifest in the work of Hipparchos in the middle of the 2nd century BC. Closer inspection of planetary data by Greek astronomers showed that for different superior planets the deferents have to be decentered differently, which was a problem for a dynamical interpretation of the heliocentric hypothesis. Even worse, with rising observational precision the existence of two anomalies for the motion of superior planets became apparent:

- uneven motion of $D_2$ in the ecliptic (in later terms, due to Kepler’s second law)
- and uneven retrograde arcs for $P_s$ (due to Kepler’s first law)

The two effects were not reducible to one eccentric mounting of the deferent. Ptolemy describes the critical evaluation of the planetary theories of the 2nd century BC by Hipparchos as follows (Almagest. IX.2):

...he [Hipparchos] thought that one must not only show that each planet has a twofold anomaly, or that each planet has retrograde arcs which are not constant, and are of such and such sizes (whereas the other astronomers had constructed their geometrical proofs on the basis of a single unvarying anomaly and retrograde arc); nor that these anomalies can in fact be represented either by means of eccentric circles or by circles concentric with the ecliptic and carrying epicycles, or even by combining both, the ecliptic anomaly being of such and such a size, and the synodic anomaly of such and such ... (Ptolemaios, 1998, p. 421, translation Toomer)

This is clear evidence of the existence of quantitative epicyclic models with simple eccentricity in the 2nd c. BC. and their invalidation by Hipparchos on the basis of observational data most of which were inherited from the Mesopotamian astronomers. Ptolemy’s text continues:

...(for these representations have been employed by almost all those who tried to exhibit the uniform circular motion by means of the so-called ‘Aeon-tables’, but their attempts were faulty and at the same time lacked proofs: some of them did not achieve their object at all, the others only to a limited extent); ...(ibid., p. 122)

This is an illuminating remark. It states explicitly that in the time of Hipparchos the quantitative evaluation of epicyclic models was not restricted to sporadic, specifically selected data of interest, but also to the preparation of tables of long duration.\(^{26}\)

Even if epicyclic models were initially motivated by the heliocentric hypothesis, once they were introduced they could be used pragmatically without subscribing to the commitment to heliocentrism. This was apparently so for Hipparchos, an astronomer-mathematician in contrast to Aristarchos and Apollonios who were mathematician-astronomers. It is well known that he developed numerical methods for astronomy in

\(^{26}\)“Aeon-tables” in Toomer’s translation, “perpetual tables” in (Neugebauer, 1975, p. 789), “Tafeln für ewige Zeiten” in the German translation by K. Manitius, 1963.
terms of Mesopotamian sexagesimal numbers. With such methods he established a successful moon model and a sun model, while he did not propose an alternative model of his own for the planets (in the present sense) (North, 2001).

After the invalidation of simple eccentric deferent models by Hipparchos, perhaps even already before it, “serious” astronomers may have looked for a better fit of epicyclic models to the observational data without bothering about ontological questions, a late Hellenistic version of scientific positivism. The remark by Ptolemy, quoted above, about the combination of eccentric deferents (for one anomaly) and an additional epicycle (for the other anomaly) seems to hint in this direction. A further influx of Mesopotamian data and calculational methods, accompanying astrological practices during the 2nd/1st centuries BC (Neugebauer, 1975, pp. 608, 613) must have led to increasing difficulties for epicyclic kinematics, and with it for heliocentrism. In the time between Hipparchos and Ptolemy (roughly the 1st centuries BC and AD) a diversity of kinematic ad hoc models for different planets were developed.27

A solution of the double anomaly problem for epicyclic kinematics was finally found by introducing a separate point $E$ relative to which the motion of the deferent $D$ on a circle with center $O$ appears uniform, while the observations is made from another decentered point $T$ (earth). The center of uniformity $E$ is called the equant (figure 8) (Almagest, IX.6ff). Then two different perigees arise: direction of $TO$ (about which the smallest retrogradation arc is observed) and the direction of $EO$ in which the angular velocity of the deferent is smallest. Observational evidence of Mars may have led to a collinear placement of $E,O,T$ and $O$ in the midpoint of $ET$.28 This led to a satisfying kinematic model for the outer planets. The historical origin of the equant construction is unclear; most authors praise Ptolemy for it, but there are indications of an earlier use of the method of an eccentric deferent circle with equant (Duke, 2005) and before him (van der Waerden, 1961). Observations of the strong “solar anomaly” for Mars (varying retrograde arcs) seem to predate Ptolemy considerably, as is the case for large parts of the observational data collected in his famous fixed star catalogue.29

We do not know when and by whom the equant method was invented; so we cannot exclude that it may have been designed originally in a heliocentric framework for modelling the non-uniform “true” motions of the planets (in particular for Mars) and was translated by kinematic inversion to the geocentric model later published by Ptolemy.30 If so, such researches probably have added to the irritations for the ontological hypothesis of helio-

27For the study of and the confusion about how to deal with the “two anomalies” of planets one may consult (Neugebauer, 1975, 801ff.). Additional historical evidence for crude and indecisive modifications of eccentric models with different perigees is given in Plinius Historia Naturalis, book II sec. xvi.
28A systematic reconstruction is given in (Evans, 1984).
29See the detailed historical study (Graßhoff, 1990), a short description in (Graßhoff, 2014).
30Rawlings even assumes that this was the case not only for the orbit of the outer planets but for all of them including the earth. This would imply not only an equant construction for the deferent, but also a decentered epicycle with equant, surpassing the precision of Ptolemy’s model by far (Rawlins, 1987). This appears quite far-fetched; Rawlins’ conjecture seems to be the result of an ex-post approximation of the Kepler dynamics by decentered circular orbits about the sun with symmetric equant. The historical arguments given in the paper are quite meager.
centrism. Moreover, the determination of parameters and the check of empirical adequacy could be done just as well, and from the pragmatic point of view even more directly in the geocentric version of the equant approach. For practising astronomers without philosophical ambition the heliocentric origin of the approach may have appeared more and more as ballast which could just as well be dropped, as long as dynamical explanations in mathematical form were out of reach.\textsuperscript{31}

With the development of calculational tools the question of heliocentrism versus geocentrism lost its practical importance even more because of the kinematic equivalence of the pictures.\textsuperscript{32} For Copernicus, however, the equant approach became a stumbling stone which he had to overcome for his attempt to reverse the second kinematic inversion and to reconstruct a heliocentric picture as he understood it. But his leads over to a different story, namely the replacement of the equant by additional epicycles and the long story leading finally to Kepler’s detection of elliptic kinematics.

\textsuperscript{31}Russo disagrees with this estimation, see end of sec. 2.1, and so does V. Blasjø (personal communication).

\textsuperscript{32}For early medieval Indian astronomy, in particular for Aryabatha (5th c. AD) it was apparently a superfluous question (Subramanian, 1994, 1998), (Plofker, 2009, p. 111ff). This does not contradict van der Waerden’s interpretation of Aryabatha’s identification of the apogees of Venus and the true sun as a trace of a heliocentric origin (van der Waerden, 1987, p. 532). This view fits well to the evidence of pre-Ptolemaic Hellenistic influence on Aryabatha’s astronomical scheme collected by (Duke, 2005).
Figure 8: Epicyclic motion of $P$ with eccentric deferent $D$ and equant $E$. $D$ moves non-uniformly on a circle about $O$ decentered from the earth $T$; but the angle $\omega_1 = \angle \wedge ED$ progresses uniformly (constant angular velocity). In principle $E, O, T$ need not be collinear, but by empirical reasons (for ancient astronomers), they are (even with $OE = OT$). $\lambda(t)$ is the ecliptic longitude angle of the planet for an observer at $T$. 
3 Discussion and final remarks

In large parts of the literature on pre-Ptolemaic astronomy the heliocentric approach of Aristarchos and successors is attributed only a marginal role. It seems to be a widely shared consensus that ancient heliocentrism was a speculative play of ideas only, without a noticeable effect on the course of Greek astronomy. Neugebauer argues: “Without the accumulation of a vast store of empirical data and without a serious methodology for their analysis the idea of heliocentricity was only a useless play on words” (Neugebauer, 1975, p. 698). In a similar vain H. Kragh sees a contrast between the ideas of a mathematician (Aristarchos) against the more serious and empirically founded investigations of astronomers like Hipparchos and Ptolemy (Kragh, 2007, p. 27). Exceptions of this consensus view come from outsiders only, like (van der Waerden, 1987; Rawlins, 1987) and more recently (Russo, 2004).

The argument presented here is that any mathematician-astronomer who is interested in observations and has only classical geometrical tools at his disposal cannot but introduce epicyclic constructions. This argument is so simple that it should have been considered in the literature long since. The lack of sources is surely a problem, but we have similar effects in other parts of ancient history of science, e.g. in the field of pre-Euclidean mathematics. Nobody would claim that demonstrative mathematics started only with Euclid (and perhaps Eudoxos), because pre-Euclidean or pre-Eudoxian sources on Greek mathematics do not exist and its contributions can be reconstructed only tentatively from later fragmentary reports and indirect traces in the transmitted corpus of knowledge.

In the case of those parts of pre-Ptolemaian astronomy which were not taken up in the later main corpus of texts this seems to be handled differently. We are even confronted with trivially circular arguments of the form:

It seems unlikely that Aristarchus developed any specific planetary theory because our sources are silent on this topic; … (Neugebauer, 1975, p. 692)

From the perspective developed in the present paper this appears highly surprising. It seems much more likely that planetary theories based on the heliocentric view were formulated and had a long range influence through their contributions to the epicyclic method. But why was this no longer explicitly acknowledged in the later source literature? Probably this was the result of two mutually enforcing causes. One was the increasing irritation of the complicated ad-hoc modifications of epicyclic models necessary to cope with the increasing precision of the empirical knowledge on planetary motion. This has been discussed above (sec. 2). The other was apparently of a more ideological nature: Heliocentrism was considered as a heterodox and even heretic perspective. This seems to have led to the exclusion of an open acknowledgement of heliocentric convictions or influences.33

Once our eyes are opened, we may start to see a lot of traces of heliocentrism in the literature on the epicyclic method like, e.g., the grouping of the planets into inferior and inferior

33In (Russo, 2004) the cultural degeneration in the Roman Imperial period of post-Hellenistic antiquity, and ideological factor are seen as the predominant, if not the only cause.
superior ones and their succession, the role of the mean sun in epicyclic constructions, the realistic interpretation of distances in the epicyclic picture etc. We cannot judge, whether Ptolemy still knew about the heliocentric background to the epicyclic method and preferred not to write about it, or whether at his time (and in his position) the knowledge was already suppressed. But some of his remarks may indicate at least a remnant of the recollection of a more systematic origin of the epicyclic method than he could, or wanted to, explain. In the passage cited above on principles of planetary motions in which he discussed Hipparchos’ contributions he continued:

The point of the above remarks was not to boast. Rather, if we are at any point compelled by the nature of our subject to use a procedure not in strict accordance with theory (...); or [...] to make some basic assumptions which we arrived at not from some readily apparent principle, but from a long period of trial and application, ...we know too that assumptions made without proof, provided only that they are found to be in agreement with the phenomena, could not have been found without some careful methodological procedure; even if it is difficult to explain how one came to conceive them (...) we know, finally, that some variety in the type of hypotheses associated with the circles [...] cannot plausibly be considered strange or contrary to reason; ... (Ptolemaios, 1998, p. 395f., emph. ES)

This is a cryptic remark hinting vaguely at some methodological rationality behind the principles or assumptions which were often introduced by Ptolemy relatively ad hoc for his final model. It is open to interpretation. I tend to interpret it as a possible indirect hint at a heliocentric rationality behind the construction of the final epicyclic model of planetary motion in the Almagest.

Of course, all this deserves more historical scrutiny. But a precondition for this is an acceptance at least of the possibility, if not of a high plausibility, that heliocentrism has had the potential to act as an effective component in the rise of epicyclic models, i.e., in a crucial phase of Greek astronomy.

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34http://www2.math.uni-wuppertal.de/~fritzsch/kf_latx.html
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