A sonic band gap based on the locally resonant phononic plates with stubs

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Abstract. Using the finite element method, we have studied the acoustic properties of a novel phononic crystal (PC) structure constructed by periodically depositing single-layer or two-layer stubs on the surface of a thin homogeneous plate. Numerical results show that the extremely low frequency band gap (BG) of the Lamb waves can be opened by the local resonance (LR) mechanism. We found that the width of such a BG depends strongly on the height and the area of cross section of the stubs. The displacement field distribution of the oscillating modes is given to explain how the coupling of the modes induces the opening of the BG. The physics behind the opening of the LRBG in our phononic structures can be understood by using a simple ‘spring-mass’ model.

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The propagation of an elastic wave in a periodic composite material, called the phononic crystal (PC), has received much attention over the past decade [1]–[7]. Because of the existence of acoustic band gaps (ABGs), the PC’s structure can have many potential applications [4, 6, 7].

The main mechanisms responsible for the ABGs’ creation are based on Bragg scattering and local resonance (LR) [8]–[11]. For the first mechanism, the BG usually falls into the wavelength region of the order of the structural period, but for the second mechanism, a resonant ABG is imposed by the frequency of resonance associated with scattering units and it depends less on the periodicity and symmetry of the structure. Generally, the ABG frequency range based on this mechanism can be almost two orders of magnitude lower than the usual Bragg gap in the frequency region. The LR mechanism was first introduced by Liu et al [8]. They demonstrated the existence of a BG at extremely low frequencies in a three-dimensional (3D) PC. Such behaviors have also been studied by other researchers [9, 10], [12]–[17]. In most of the previous works about LRPCs, the proposed structures are made of LR units with soft materials or with a heavy core coated with soft materials, embedded in a hard matrix, forming a 2D or 3D infinite system, in which the low-frequency ABG for the bulk wave can be obtained. Recently, a 2D LR thin plate structure was studied by Hsu and Wu [9] and Xiao et al [11], and they suggested that an LR ABG of Lamb modes can also be obtained by filling soft rubber in the periodically drilled holes in a thin elastic plate. On the other hand, Wu et al [18] and Pennec et al [19] have reported independently on the elastic behavior of a stubbed plate. The structure they studied is similar to the one we will investigate in the present paper, but we will argue in detail that the opening of the ABG in their model is not due to the LR mechanism. Concerning the LRPC applications, some promising ones associated with low-frequency BG have been suggested in some references [16, 20].

In this paper, we will present a novel LRPC plate and show that, compared with the model suggested in [9, 11], our system can be prepared more easily and the physical behavior behind it can be presented more clearly. As shown schematically in figure 1, the studied system is constructed by depositing the cylinder LR stubs squarely onto the surface of a thin elastic homogenous plate (made of epoxy with thickness \(e\) equal to 0.05\(a\) throughout this study). Two kinds of LR stubs will be considered: a simple one, made of one layer of soft silicone rubber cylinder, and a composite one, made of a lead (Pb) layer on a silicone rubber cylinder. The cylinder stub is described by its thickness \(h = h_1 + h_2\), where \(h_1\) and \(h_2\) are the thicknesses of the silicone rubber and Pb layers, respectively, and by the radius \(r\) of its cross section. For a simple stub, which means only the silicone rubber is to be considered, we will have \(h_2 = 0\). Our calculations are implemented by the finite element method with periodic boundary conditions using Comsol Multiphysics. The material density and the longitudinal and shear wave velocities are \(\rho = 1180\ \text{Kg m}^{-3}\), \(C_l = 2534\ \text{m s}^{-1}\) and \(C_t = 1157\ \text{m s}^{-1}\) for epoxy, \(\rho = 1300\ \text{Kg m}^{-3}\), \(C_l = 24\ \text{m s}^{-1}\) and \(C_t = 6\ \text{m s}^{-1}\) for rubber and \(\rho = 11400\ \text{kg m}^{-3}\), \(C_l = 2160\ \text{m s}^{-1}\) and \(C_t = 860\ \text{m s}^{-1}\) for Pb [9, 13, 21].

In the finite element method, the size of the space meshing must be adapted to the variation behavior of the solution. The smaller the mesh size that is used, the better the convergence of the computation that can be obtained, but the longer the calculation time that has to be used. So it is important to choose an adaptive mesh that allows us to have sufficiently good convergence in an acceptable calculation time. In our model, since rubber is softer than epoxy and lead, the displacement field will obviously be more important in rubber. Thus, the mesh size in the rubber domain must be finer than that chosen for the epoxy or lead domains (figure 1). For the mesh elements, we chose the default tetrahedral mesh provided by Comsol Multiphysics 3.5a.
Figure 1. Schematic view of the studied PC made of a square lattice of cylinder stubs (a stub of one-layer rubber or a rubber layer with a Pb cap) deposited on a thin epoxy plate. The radius of the stub is \( r \); \( h_1 \) and \( h_2 \) are the thicknesses of the two different layers of the stub. \( h_2 = 0 \) indicates only the one-layer simple stub.

Because of the periodicity of the structure, only one unit cell is considered. We use the stress-free boundary conditions for free surfaces and the periodic boundary conditions for the interfaces between the nearest unit cells, according to the Bloch–Floquet theorem,

\[ u_i(x + a, y + a) = u_i(x, y)e^{-i(k_x a + k_y a)}. \]

where \( i = x, y, z \) and \( k_x \) and \( k_y \) are the Bloch wave vectors. As a test of the convergence, the eigenfrequencies at the point \( (k_x a / 2 = 0.01a, k_y = 0) \) for the system with \( r = 0.03a, h_1 = 0.5a \) and \( h_2 = 0 \) as a function of the number of meshing elements is presented in figure 2, from which it can be seen that good convergence can be obtained when 3000 meshing elements are used.

1. A simple stub: the silicone rubber stub

Figure 2 shows the band structure of the Lamb wave in the case of a simple silicone rubber stub on an epoxy plate. The radius of the cylinder-shaped stub is taken to be \( r = 0.3a, 0.36a, 0.4a \) and \( 0.48a \), for the different maps, respectively, where \( a \) is the lattice constant. The thickness of the stub is \( h = 0.5a \) for all of the structures presented. One can observe that, besides the traditional plate modes, which are the shear, symmetric and antisymmetric Lamb modes, lots of flat modes, which are the resonant modes with energy localized mainly in the soft stub, can be found. Thus, the ABGs, as a result of the coupling of the two kinds of modes mentioned above, appear. The ABGs opened in this system are located about two orders lower than the Bragg scattering mechanism in the frequency region. This means that we are actually in the presence of an LR mechanism. Figure 3 shows that one of the key points of opening such an ABG is to get the low resonant ‘flat band’ to be coupled with the Lamb modes. This means that, to obtain the ABG with the desired frequency position, the position of the ‘lower lying flat band’ should be adjustable by the structure. However, from the figure, it is obvious that the positions of the ‘flat bands’ are independent of the radius of the cylinder \( r \), meaning that, to get a low-frequency ABG, another adjustable parameter, \( h \) (the thickness of the cylinder), of the structure needs to be changed. Before doing this, we would like to give a qualitative estimation of the mode resonant behavior in the stub. Note the fact that the resonant modes are strongly localized.
in the stubs. Their resonant frequency can be roughly estimated by the isolated ‘spring-mass’
model [15, 22]: \( f \propto \sqrt{k/m} \), where \( k \) is the effective elastic constant and \( m \) is the mass of
the stub. Supposing the rubber has an elastic constant \( k_0 \) per unit area \( S_0 \) per unit thickness \( h_0 \), then
the stub with area of cross section \( S \) and thickness \( h \) should have the effective elastic constant
\( (S/S_0)/(h_0/h)k_0 \) (for the simple stub, we have \( h_2 = 0 \)); increasing the cross section of the stub
is a means of increasing the effective elastic constant. But, because at the same time we also
have \( m = \rho Sh \), where \( \rho \) is the mass density of the rubber, the resonant frequency \( f \) in the stub
should be independent of \( r \). On the contrary, based on the above discussion, the position of the
‘flat band’ could be adjusted efficiently by the thickness of the stub. We have to note that, in
figure 3 (\( r = 0.36a \)), we are in the presence of two ABGs separated by a flat band.

In figure 4, we give the band structures of the systems with \( h = 0.1a, 0.2a, 0.4a \) and \( 0.5a \),
respectively. All the systems presented are fixed with \( r = 0.48a \), from which we can see that,
indeed, the ‘flat bands’ are pressed into the low frequency region as \( h \) increases.

However, we have to point out that, to open an ABG at a sufficiently low frequency, besides
the obtained low-frequency ‘flat bands’, the mode coupling between the resonant band and the
Lamb mode is also very important since an ABG can only be obtained when the ‘partly’ opened
BGs can overlap onto each other. From figures 3 and 4, we find that the resonant ‘flat bands’
can only selectively couple to the plate modes; the key point for the mode coupling is that the
oscillation of the particles should have the same kind of displacement motion at the interface.
A detailed numerical calculation of the elastic displacement distribution of the resonant mode
shows that, in the considered frequency region, there are three kinds of oscillations in the stubs:
the ‘elongation’ mode, the ‘shear’ mode and the ‘breath’ mode. As shown in figure 5, the

Figure 2. Convergence test of the method used in our calculations. The
eigenfrequency of the system with \( r = 0.03a, h_1 = 0.5a \) and \( h_2 = 0 \) at \( \Gamma' \)
\( (k_x/2\pi = 0.01a; k_y = 0) \) is considered. The ratio of the meshing element of
silicone rubber/epoxy to that of silicone rubber/Pb is about 4.
Figure 3. Band structures of the plate with a simple stub (only one rubber layer). The thickness of the stubs is fixed at \( h = h_1 = 0.5a \); the radii \( r = 0.3a, 0.36a, 0.4a \) and \( 0.48a \) are considered. For the first band structure, the points labeled a–g correspond to the wavevector values for which some displacement field will be illustrated in figure 5, and a zoom near the \( \Gamma \) point is given.
Figure 4. Band structure for the PC with simple stub. The thickness of the stubs is chosen to be $h = h_1 = 0.1a$, $0.2a$, $0.4a$ and $0.5a$. All of the structures are obtained by fixing $r = 0.48a$. From this, we can clearly see that the ‘flat bands’ are pressed into the low frequency region as $h$ increases.

Displacement for the ‘elongation’ mode is symmetric in the $x$ and $y$ directions and with nonzero $u_z$ (figure 5(c)). The displacement for the ‘shear’ mode is along the $x$- or $y$-direction (see modes ‘a’ and ‘b’ in figure 5, respectively). However, for the ‘breath’ mode, the displacement is symmetric along the $x$ and $y$ directions and with $u_z$ almost zero. We know that, in the low frequency region, the dominant surface motion of the $S_0$ and $SH$ modes is mainly the in-plane one, so it can couple to the ‘shear’ mode. For the $A_0$ mode, the dominant surface motion is $u_z$, but with slightly small $u_x$, so it can couple with the ‘elongation’ (mode ‘c’ in figure 5) and the ‘shear’ mode at the same time. Lastly, for the ‘breath’ mode (figure 5(d)), no plate mode can couple with it.

Figure 3 also shows that the width of the ABG depends on the radius of the stub $r$. A relatively wide ABG can be obtained in a system with a moderate $r$. This behavior can be understood from the aspect of the strength of the contact between the plate and the stubs. In a system with the stubs having a large $r$, which means that the contact between the stubs and the plate is strong, the energy of the resonant mode will be less localized in the stubs and thus interaction between the near stubs becomes possible. As a consequence, the multi-scattering caused by the periodic structure appears, resulting in a visible adjustment of the ABG width.
Figure 5. The displacement distribution of the modes labeled (a)–(d) in figure 3. It can clearly be seen that modes (a) and (b) have a ‘shear’ oscillation, (c) has an ‘elongation’ oscillation and (d) has a ‘breath’ oscillation. Other points (e)–(g), etc in figure 3 were also checked.

2. A composite stub: a silicone rubber stub with Pb cap

In analyzing the case of the simple stub, we can expect that the frequency of the ‘flat band’ can also be changed by an additional heavy mass on top of the stub (Pb is used as a cap in our system), which means that a composite stub can be used to achieve such a purpose. In this case, because the stiffness of the capped Pb is far greater than that of rubber, the Pb cap can be treated as a ‘stiff’ body, the effective elastic constant of the stub is determined mainly by the rubber layer and the only effect of adding the cap is just a change in the total mass of the resonator. So, the ‘flat band’ can also be pressed into the low frequency region by increasing only the thickness of Pb $h_2$ (by increasing the mass of the stub, in fact) or by increasing only the thickness of rubber $h_1$ (by increasing the mass and decreasing the effective elastic constant at the same time). In figure 6, we give the band structures for the system with a fixed stub thickness ($h = h_1 + h_2 = 0.1a + 0.3a$), but for $r = 0.3a, 0.36a, 0.4a$ and $0.48a$, respectively. From this, we can see that a relatively wide ABG appears for the system with $r = 0.48a$. In figure 7, we give the band structures for the stubs with fixed $r$ and $h_1$ ($r = 0.3a, h_1 = 0.1a$), but for $h_2 = 0.1a, 0.2a, 0.3a$ and $0.4a$, respectively. From this, we can clearly see the trend of the ‘flat band’ being pushed into the low-frequency region as $h_2$ increases.

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From the above discussion, we see that the system with a composite stub can be viewed as a special case of the system with a simple stub: changing the height of the Pb cap can change the total mass of the resonator but not the effective ‘spring constant’, leading to only a shift in the position of the resonant frequency (flat band). To adjust the width of the LR ABG in this case, we have to change the coupling strength of the Lamb and resonant modes by changing the radius of the rubber layer.

It is worth noting that, recently, as mentioned in the introduction, Wu et al [18] and Pennec et al [19] have reported independently on the elastic behavior of a stubbed plate. The structure they studied is similar to the one presented in this paper, but, instead of using the very soft rubber stub, the structure was constructed with a hard stub on a thin plate (aluminum and steel, respectively). We argue here that the mechanism of opening the ABG in their models and in

Figure 6. The same as figure 3, but for the composite stubs. For all of this system, the thickness of the rubber and the Pb layer is fixed at $h_1 = 0.1a$ and $h_2 = 0.3a$, respectively. We see that a relatively wide ABG around $\omega a/c_t = 0.04$ is obtained for $r = 0.48a$.

To show that the effect of adding the cap is just a change in the total mass of the resonator, we have also given in figure 7 (the case $h_2 = 0.3a$) the result for a stub capped by steel [19] (instead of Pb but with the same total mass as the Pb cap). One can see that, although the elastic constant of the cap is different, there is very good overlap of both band structures, which means that the capped material, whether Pb or steel, can be treated as a ‘stiff’ body and just plays the role of mass.
ours is similar in some sense: both of them are a result of the coupling between the Lamb mode and the stub mode. However, the difference between them is also obvious. The key difference is that, compared to Wu and Pennec’s models, the coupling between the Lamb and stub modes in our model is very weak because of the weak ‘link’ between the plate and the stub, leading to strong localization (and a large quality factor ($Q$)) of the mode in the stub. However, in Wu and Pennec’s models, the coupling between the Lamb and stub modes is strong, leading to a small $Q$, which means a wide resonant frequency spectrum of the stub mode. As a result, in our model, the ABG can be obtained even by a single resonant coupling, but in Wu and Pennec’s models, the resonant coupling is very strong, a Bragg scattering procedure, which means that a periodic structure has to be used to obtain an ABG. Another issue is that an ABG based on LR is usually narrower than one based on the Bragg scattering procedure, which can also be understood by the same discussion as above: a small (large) $Q$ factor means a sharp (wide) frequency spectrum and leads to a narrow (wide) ABG when mode coupling in the plate and stub appears.
Finally, compared with the results reported by Xiao et al [11], the main differences between our model and structure and those described in this reference lie in the PC shape and in the numerical method used. Both of the obtained results can be supported by the ‘spring-mass’ model. Indeed, in [11], the obtained results showed, using the plane wave expansion method, how we can open and enlarge the flexural vibration BG based on the LR mechanism by modulating some physical parameters. This structure is composed of embedded resonant units in a plate. In our LR stubbed plate, which can be fabricated more simply, the physics of the LR can be understood more easily because we can separately change the effective ‘spring’ constant and the total mass.

To conclude, in this work, we have studied the mechanism of opening the LR ABG in a stubbed plate with a simple one-layer soft stub and a two-layer composite stub. To obtain this kind of ABG in the structure, we need not only the low-frequency flat resonant mode, which can be obtained by adjusting the thickness and cross section of the soft stub or by adding a heavy cap on top of the soft stub, but also the mode coupling between the localized mode in the stub and the Lamb mode in the plate. The opening of a very low frequency ABG gives way to a new approach for dealing with the so-called acoustic metamaterials.

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