Stability of HDE model with sign-changeable interaction in Brans-Dicke theory

M. Abdollahi Zadeh\(^*\) and A. Sheykhi\(^{1,2}\)†

\(^1\) Physics Department and Biruni Observatory, College of Sciences, Shiraz University, Shiraz 71454, Iran
\(^2\) Research Institute for Astronomy and Astrophysics of Maragha (RIAAM), P.O. Box 55134-441, Maragha, Iran

We consider the Brans-Dicke (BD) theory of gravity and explore the cosmological implications of the sign-changeable interacting holographic dark energy (HDE) model in the background of Friedmann-Robertson-Walker (FRW) universe. As the system’s infrared (IR) cutoff, we choose the future event horizon, the Granda-Oliveros (GO) and the Ricci cutoffs. For each cutoff, we obtain the density parameter, the equation of state (EoS) and the deceleration parameter of the system. In case of future event horizon, we find out that the EoS parameter, \(w_D\), can cross the phantom line, as a result the transition from deceleration to acceleration expansion of the universe can be achieved provided the model parameters are chosen suitably. Then, we investigate the instability of the sign-changeable interacting HDE model against perturbations in BD theory. For this purpose, we study the squared sound speed \(v_s^2\) whose sign determines the stability of the model. When \(v_s^2 < 0\) the model is unstable against perturbation. For future event horizon cutoff, our universe can be stable \((v_s^2 > 0)\) depending on the model parameters. Then, we focus on GO and Ricci cutoffs and find out that although other features of these two cutoffs seem to be consistent with observations, they cannot leads to stable dominated universe, except in special case with GO cutoff. Our studies confirm that for the sign-changeable HDE model in the setup of BD cosmology, the event horizon is the most suitable horizon which can passes all conditions and leads to a stable DE dominated universe.

I. INTRODUCTION

The cosmological observational data from type Ia supernovae (SNIa) \([1-4]\), the Large Scale Structure (LSS) \([5-8]\) and the Cosmic Microwave Background (CMB) anisotropies \([9-11]\), Baryon Acoustic Oscillations (BAO) in the Sloan Sky Digital Survey (SSDS) luminous galaxy sample \([12, 13]\) and Plank data \([14]\), confirm that the observable universe is nearly spatially flat, homogeneous and isotropic at large scale and is experiencing a phase of accelerated expansion in particular in

\(^*\) m.abdollahizadeh@shirazu.ac.ir
\(^†\) asheykhi@shirazu.ac.ir
the redshift $0.45 \leq z \leq 0.9$.

The provenance of this acceleration should be caused due to an un-known energy component with negative pressure which can overcome to gravitation of galaxy and is usually called dark energy (DE). It is nowadays commonly accepted that DE has occupied about $\%73$ of the total energy content of the universe and the rest has been released to dark matter and baryonic matter. On the other side deceleration phase is important for nucleosynthesis as well as for the structure formation. It is important to note that we need a dynamical field in such a way that its dynamics makes at first the deceleration phase in the early time and the acceleration phase in the late time of the universe evolution. This fact has motivated people for investigating dynamical DE models. One of the dramatic candidates for dynamical models is the HDE model which has arisen a lot of attentions. This model is based on the holographic principle that states the number of degrees of freedom of a system scales with its area instead of its volume. The HDE model relates DE density to the large length in the universe, which is usually assumed to be the cosmic horizon. The HDE models have been investigated widely in the literatures.

Scalar-tensor theories of gravity have been widely applied in cosmology. The pioneering study on scalar-tensor theories was done by Brans and Dicke (BD) several decades ago who sought to incorporate Mach’s principle into gravity. In recent years, scalar tensor theories have been reconsidered extensively, because the scalar fields appear in different branches of theoretical physics as a consistency condition. For example, the low energy limit of the string theory leads to introducing a scalar degree of freedom. Since the HDE model have a dynamical behavior, it is more reasonable to consider it in a dynamical framework such as BD cosmology. The BD theory also passed the observational tests in the solar system domain. According to BD theory, the gravitational fields are described by the metric $g_{\mu\nu}$ and a scalar field $\phi$ which is coupled to the gravity via a coupling parameter $\omega$ which is restricted to a very large value. The studies on the HDE model in the framework of BD cosmology have been carried out.

On the other side, recent observations indicate that the evolution of the two dark components of the universe is not independent and indeed there is a mutual interaction between the Dark matter (DM) and DE, which may solve the coincidence problem. However, the form of this mutual interaction can be written as $Q = 3b^2H(\rho_M + \rho_D)$, where $b^2$ is a coupling constant and $\rho_M$ and $\rho_D$ are the energy density of DM and DE, respectively. Clearly, the sign of this form of interaction term cannot change during the history of the universe. While recent investigations obliviously confirm that the sign of the interaction term may change during the cosmic evolution, in particular in the redshift $0.45 \leq z \leq 0.9$. Wei was the first who suggested a sign-changeable
interaction term in the form \( Q = q(\alpha \dot{\rho} + 3\beta H \rho) \), where \( \alpha \) and \( \beta \) are dimensionless constant and \( q = -1 - \dot{H}/H^2 \), is the deceleration parameter. Obviously, the interaction \( Q \) can change its sign when our universe changes from the deceleration phase \( (q > 0) \) to the acceleration \( (q < 0) \). The investigation on the DE models with sign-changeable interaction term have been carried out in [34].

In the present work, we would like to investigate the HDE model with sign-changeable interaction term in the background of BD theory. First, we study the cosmological implications of this model and then we explore the stability of the model against perturbation by considering the squared sound speed \( v_s^2 = dP/d\rho \) whose sign determines the stability of the model [35]. When \( v_s^2 < 0 \) the model is unstable against perturbation. In the framework of Einstein gravity, instability of DE models have been explored in [36]. While stability of interacting HDE with GO cutoff in BD theory has been discussed in [37], sound instability of nonlinearly interacting ghost dark energy have been studied in [38].

This paper is outlined as follows. In section II, we give a brief review of the interacting HDE model in the context of BD cosmology. In sections III, we study HDE in the framework of BD theory by assuming a sign-changeable interaction term with future horizon as system’s IR cutoff. Sections IV and V also investigate the HDE models in BD cosmology with GO and Ricci cutoffs, respectively. In each cases, we study the evolution of the cosmological parameters as well as the sound stability \( v_s^2 \) of the model. The summary of the results is discussed in the last section.

II. INTERACTING HDE IN BD COSMOLOGY

The action of BD theory is given by

\[
S = \int d^4x \sqrt{g} \left( -\phi R + \frac{\omega}{\phi} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + L_M \right).
\]

By re-defining the scalar field, \( \phi = \phi^2/8\omega \), we can rewrite the above action in the canonical form as [39, 40]

\[
S = \int d^4x \sqrt{g} \left( -\frac{1}{8\omega} \phi^2 R + \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + L_M \right),
\]

where \( R \) and \( \phi \) are the scalar curvature and the BD scalar field, respectively. Also, \( \omega \) stands for the generic dimensionless parameter of the BD theory and \( L_M \) is the Lagrangian of the matter. The term \( \phi^2 R \), which is the non-minimal coupling term, is replaced with the Einstein-Hilbert term \( R/G \), in such a way that \( G_{\text{eff}}^{-1} = 2\pi \phi^2/\omega \), where \( G_{\text{eff}} \) is the effective gravitational constant. We
consider a FRW universe which is described by the line element
\[ ds^2 = dt^2 - a(t)^2 \left( \frac{dr^2}{1 - kr^2} + r^2 d\Omega^2 \right). \] (3)
where \( a(t) \) is the scale factor, and \( k \) is the curvature parameter with \( k = -1, 0, 1 \) corresponding to open, flat, and closed universes, respectively. Varying action (2) yields the following field equations
\[ \frac{3}{4\omega}\phi^2 \left( H^2 + \frac{k}{a^2} \right) - \frac{1}{2}\dot{\phi}^2 + \frac{3}{2\omega}H\dot{\phi}\phi = \rho_M + \rho_D, \] (4)
\[ \frac{-1}{4\omega}\phi^2 \left( \frac{\ddot{a}}{a} + H^2 + \frac{k}{a^2} \right) - \frac{1}{\omega}H\dot{\phi}\phi - \frac{1}{2\omega}\ddot{\phi}\phi - \frac{1}{2} \left( 1 + \frac{1}{\omega} \right) \dot{\phi}^2 = p_D, \] (5)
\[ \ddot{\phi} + 3H\dot{\phi} - \frac{3}{2\omega} \left( \frac{\ddot{a}}{a} + H^2 + \frac{k}{a^2} \right) \phi = 0, \] (6)
where \( H = \dot{a}/a \) is the Hubble parameter, \( \rho_D \) and \( p_D \) are, respectively, the energy density and pressure of dark energy. There are not compelling reason for choice of \( \phi \), thus we assume the BD field as \( \phi = \phi_0 a^\alpha(t) \), which leads to the following relations
\[ \frac{\dot{\phi}}{\phi} = \alpha\dot{H}, \quad \frac{\ddot{\phi}}{\phi} = \alpha^2 H^2 + \alpha\dot{H}, \quad \frac{\ddot{\phi}}{\phi} = \left( \alpha + \frac{\dot{H}}{H^2} \right) H. \] (7)
We further assume the energy density of the HDE can be written as
\[ \rho_D = \frac{3c^2\phi^2}{4\omega L^2}, \] (8)
where \( \phi^2 = \omega/2\pi G_{\text{eff}} \). In the limiting case where \( G_{\text{eff}} \) reduces to \( G \), the energy density (8) reduces to the energy density of HDE in standard cosmology,
\[ \rho_D = \frac{3c^2}{8\pi GL^2} = \frac{3c^2m_p^2}{L^2}. \] (9)
If we define the critical energy density as
\[ \rho_{\text{cr}} = \frac{3\phi^2H^2}{4\omega}, \] (10)
then the dimensionless density parameters can be written
\[ \Omega_M = \frac{\rho_M}{\rho_{\text{cr}}} = \frac{4\omega\rho_M}{3\phi^2H^2}, \] (11)
\[ \Omega_k = \frac{\rho_k}{\rho_{\text{cr}}} = \frac{k}{H^2a^2}; \] (12)
\[ \Omega_D = \frac{\rho_D}{\rho_{\text{cr}}} = \frac{4\omega\rho_D}{3\phi^2H^2}. \] (13)
For the FRW universe filled with DE and DM, with mutual interaction, the semi-conservation equations are as follow
\[ \dot{\rho}_D + 3H\rho_D(1 + w_D) = -Q, \] (14)
\[ \dot{\rho}_M + 3H\rho_M = Q, \] (15)
where $Q$ is the interaction term which we assume has the form $Q = 3b^2qH(\rho_M + \rho_D)$, where $b^2$ is a coupling constant and $q$ is the deceleration parameter,

$$q = -\frac{\ddot{a}}{aH^2} = -1 - \frac{\dot{H}}{H^2}. \quad (16)$$

Combining Eqs. (7) and (10) with Friedmann Eq. (4), we arrive at

$$\rho_{cr} + \rho_k = \rho_M + \rho_D + \rho_\phi, \quad (17)$$

where we have defined

$$\rho_\phi \equiv \frac{1}{2}\alpha H^2 \phi^2 \left( \alpha - \frac{3}{\omega} \right). \quad (18)$$

Dividing Eq. (17) by $\rho_{cr}$, this equation can be rewritten as

$$\Omega_m + \Omega_D + \Omega_\phi = 1 + \Omega_k, \quad (19)$$

where

$$\Omega_\phi = \frac{\rho_\phi}{\rho_{cr}} = -2\alpha \left( 1 - \frac{\alpha\omega}{3} \right). \quad (20)$$

Next, we introduce the ratio of the energy densities, $r$, which can be written

$$r = \frac{\Omega_m}{\Omega_D} = -1 + \frac{1}{\Omega_D} \left[ 1 + \Omega_k + 2\alpha \left( 1 - \frac{\alpha\omega}{3} \right) \right]. \quad (21)$$

The idea for investigating the stability of any DE model comes from the perturbation theory. For this purpose, we assume a small perturbation in the background energy density. We are interested in checking whether the perturbation grows with time or it will collapse. In the linear perturbation theory, the perturbed energy density of the background can be written as

$$\rho(t, x) = \rho(t) + \delta\rho(t, x), \quad (22)$$

where $\rho(t)$ is unperturbed background energy density. The energy conservation equation ($\nabla_\mu T^{\mu\nu} = 0$) yields

$$\delta\ddot{\rho} = v_s^2 \nabla^2 \delta\rho(t, x), \quad (23)$$

where $v_s^2 = dP/d\rho$ is the squared of the sound speed. There are two kinds of solutions for Eq. (23). In the first case where $v_s^2 > 0$, Eq. (23) becomes an ordinary wave equation which have a wave solution in the form $\delta\rho = \delta\rho_0 e^{-i\omega t + i\vec{k} \cdot \vec{x}}$. Clearly, in this case the density perturbations propagates with time and the system is stable. In the second case where $v_s^2 < 0$, the frequency
of the oscillations becomes pure imaginary and the density perturbations will grow with time as
\( \delta \rho = \delta \rho_0 e^{\omega t + ik \cdot \vec{x}}. \) This implies a possible emergency of instabilities in the background. Therefore, the sign of \( v_s^2 \) plays a crucial role in determining the stability of DE model. If \( v_s^2 < 0 \) (\( v_s^2 > 0 \)) it means that we have the classical instability (stability) of a given perturbation. The quantity \( v_s^2 \) for the FRW universe is given by
\[
v_s^2 = \frac{\dot{P}}{\dot{\rho}} = \frac{\dot{P_Dw_D + \rho_Dw_D}}{\dot{\rho_D}(1 + r) + \rho_D r'},
\]
where \( P = P_D \) is the pressure of DE and \( \rho = \rho_M + \rho_D \) is the total energy density of DE and DM.

### III. SIGN-CHANGEABLE HDE IN BD THEORY WITH FUTURE HORIZON CUTOFF

At the beginning, we consider the future event horizon as system’s IR cutoff, which is defined as
\[
L = R_h = a(t) \int_t^{\infty} \frac{dt}{a(t)},
\]
from which we can get
\[
\dot{R}_h = HR_h - 1,
\]
and hence from Eqs. (8) and (13) we can obtain
\[
\rho_D = \frac{3c^2 \phi^2}{4\omega R_h^2},
\]
\[
\Omega_D = \frac{\rho_D}{\rho_{cr}} = \frac{c^2}{H^2 R_h^2}.
\]
Taking the time derivative of the energy density \( \rho_D \) in Eq. (27) and simultaneously using Eqs. (7), (26) and (28) we arrive at
\[
\dot{\rho}_D = 2H \rho_D \left( \alpha - 1 + \frac{\sqrt{\Omega_D}}{c} \right).
\]
Next, we determine the EoS parameter with inserting Eq. (29) in semi-conservation law (14). We find
\[
w_D = -1 - b^2 q(1 + r) - \frac{2}{3} \left( \alpha - 1 + \frac{\sqrt{\Omega_D}}{c} \right).
\]
FIG. 1: Evolution of $\Omega_D$ versus redshift parameter $z$ for the sign-changeable interacting HDE with future event horizon as IR cutoff in BD cosmology. Here, we have taken $\Omega_D^0 = 0.73$, $\alpha = 10^{-4}$, $\omega = 10^4$ and $b^2 = 0.1$.

FIG. 2: Evolution of $w_D$ versus redshift parameter $z$ for the sign-changeable interacting HDE with future event horizon as IR cutoff in BD cosmology. Here, we have taken $\alpha = 10^{-4}$, $\omega = 10^4$ and $b^2 = 0.1$ as the initial condition.

The deceleration parameter $q$ can be obtained by dividing Eq. (5) by $H^2$ and using Eqs. (7), (16) and (28),

$$q = \frac{1}{2\alpha + 2} \left[2\alpha^2(\omega + 2) + 2\alpha + 1 + 3\Omega_D w_D\right].$$

(31)

Substituting $\omega_D$ from (30) in above relation, we reach

$$q = \frac{c(1 + 2\alpha)\gamma_0 - c(2\alpha c + 2\sqrt{\Omega_D})\Omega_D}{2c\gamma_0 + 3b^2c(1 + r)\Omega_D},$$

(32)

where $\gamma_0 = 1 - 2\omega\alpha^2/3 - 2\alpha$. Taking the time derivative of Eq. (28) and using Eq. (26) as well as
FIG. 3: Evolution of the deceleration parameter $q$ against redshift parameter $z$ for the sign-changeable interacting HDE with future event horizon as IR cutoff in BD cosmology. Here, we have taken $\alpha = 10^{-4}$, $\omega = 10^4$ and $b^2 = .1$ as the initial condition.

the fact that $\dot{\Omega}_D = \Omega'_D H$, we can obtain the equation of motion for $\Omega_D$ as

$$\Omega'_D = 2\Omega_D \left( q + \sqrt{\frac{\Omega_D}{c}} \right),$$

(33)

where the dot and the prime indicate differentiation with respect to the cosmic time and $x = \ln a$, respectively. Substituting $q$ from relation (32) in Eq. (33), we can Setting $n = 0$ ($\omega \to \infty$), which is the limiting case of Einstein gravity [43], the obtained results in Eqs. (30), (32) and (33) reduce to their respective expressions in Einstein gravity [45].

FIG. 4: Evolution of the squared of sound speed $v_s^2$ against redshift parameter $z$ for the sign-changeable interacting HDE with Future cutoff in BD cosmology. Here, we have taken $c^2 = 1$, $b^2 = 0.1$ and $\omega = 10^4$ in the left panel and $\alpha = 10^{-4}$, $\omega = 10^4$ and $c^2 = 1$ in the right panel, as the initial condition, respectively

Substituting Eqs. (21), (29) and (30) in Eq. (24), we can obtain the explicit expression for $v_s^2$. Since this expression is too long, we shall not present it here, instead we study the evolution of $v_s^2$ via
FIG. 5: Evolution of the squared of sound speed $v_s^2$ against redshift parameter $z$ for the sign-changeable interacting HDE with Future cutoff in BD cosmology. Here, we have taken $\alpha = 10^{-4}$, $\omega = 10^4$ and $b^2 = 0.1$ in the left panel and $\alpha = .003$, $b^2 = 0.1$ and $c^2 = 1$ in the right panel, as the initial condition, respectively.

figures. To illustrate the cosmological consequences of the HDE with sign-changeable interaction term in BD cosmology, we also plot the density parameter $\Omega_D$, and the EoS parameter $w_D$ and the deceleration parameter $q$ and the squared sound speed $v_s^2$ in Figs. 1-5. In Fig. 1, as we expect, it is seen that in early time of universe ($1 + z \to \infty$) we have $\Omega_D \to 0$, while at the late time where ($1 + z \to 0$), we have $\Omega_D \to 1$. From Fig. 2 one can clearly see that for $c \lesssim 1$ the EoS parameter $w_D$ can cross the phantom line and when $c \geq 1$, we always have $w_D > -1$. As it is obvious from Fig. 3 for all values of $c$, the deceleration parameter $q$ transits from deceleration ($q > 0$) in the early time to acceleration ($q < 0$) in the last time around $z \approx 0.6$. The evolution of $v_s^2$ versus $z$ for the different parameters $\alpha$, $b^2$, $c$ and $\omega$ are plotted in Figs. 4 and 5. Graphical analysis of $v_s^2$ shows that in Fig. 4 (left panel) the sign-changeable HDE in BD theory could be stable for suitable values of $\alpha$. Also, in Fig. 5 by increasing $c$, the squared sound speed, $v_s^2$, is positive which implies that the sign-changeable interacting HDE with the future cutoff in BD cosmology can be stable. Since at the present time, our Universe is experiencing a phase of accelerated expansion, thus we need to find a model which respects the stability condition around the present time. Obviously our present model passes all conditions.

IV. SIGN-CHANGEABLE HDE WITH GO CUTTOFF IN BD THEORY

The energy density of HDE in BD theory with GO cutoff, is given by

$$\rho_D = \frac{3\dot{\phi}^2}{4\omega} (\gamma H^2 + \beta \dot{H}).$$  (34)
Dividing Eq. (4) by $H^2$ and using Eq. (7) as well as the above relation, we obtain
\[
\frac{\dot{H}}{H^2} = \frac{1 - 2\omega\alpha^2/3 + 2\alpha}{\beta(1 + r)} - \frac{\gamma}{\beta}.
\] (35)

From Eq. (16), it follows that
\[
q = -1 + \frac{\gamma}{\beta} - \frac{1 - 2\omega\alpha^2/3 + 2\alpha}{\beta(1 + r)}.
\] (36)

The EoS parameter $w_D$ of the HDE in BD theory is given
\[
w_D = \frac{1}{3\left[\gamma - \beta(1 + q)\right]} \left[(2q - 1) - 4\alpha - 4\alpha^2 + 2\alpha(1 + q) - 2\alpha^2\omega\right].
\] (37)

Dividing Eq. (5) by $H^2$ and using of relation (34), after inserting Eq. (36) in Eq. (37), we arrive at
\[
w_D = \frac{6 + (9\beta - 6\gamma)(1 + r) + 6\alpha \left[3 + 2\beta(1 + r) - \gamma(1 + r)\right] - 4\alpha^2\omega + 2\alpha^2 [6 - 2\omega + 3\beta(1 + r)(2 + \omega)]}{3\beta[-3 + 2\alpha(-3 + \alpha\omega)]}.
\] (38)

Using relation $\Omega_D = \rho_D/\rho_{cr}$, and Eq. (34), we get
\[
\Omega_D = \gamma + \beta \frac{\dot{H}}{H^2}.
\] (39)

Taking the derivative of this relation respect to the cosmic time $t$ and using Eq. (35), we find
\[
\dot{\Omega}_D = -\dot{r} \left(1 - \frac{2\omega\alpha^2}{3} + 2\alpha\right) \frac{1}{(1 + r)^2},
\] (40)

where $\dot{\Omega}_D = H\dot{\Omega}_D$, and $\dot{r}$ can be obtained using $r = \rho_m/\rho_D$ as well as the conservation equations,
\[
\dot{r} = 3H \left[b^2q(1 + r)^2 + \omega D r\right].
\] (41)

When $\alpha = 0$, Eqs. (36), (38) and (40) reduce to their respective expressions in flat standard cosmology [45]. Computing $\dot{w}_D$ and using Eq. (41), after replacing in relation (24), we can investigate the squared speed of sound $v_s^2$. Again, for the economic reason, we do not bring the explicit expression of $v_s^2$, instead we focus on its behaviour via figures.

The behavior of $\Omega_D$ against redshift parameter for HDE with GO cutoff and in the setup of BD theory has been plotted in Fig. 6. We find that at the late time where the DE dominates we have $\Omega_D \rightarrow 1$, while $\Omega_D \rightarrow 0$ at the early time. The graphical behavior of the EoS parameter, which is given in Eq. (37), also plotted in Fig. 7, showing that for $\gamma = 0.8$ the EoS parameter can cross the phantom line. The behavior of the deceleration parameter $q$ has also plotted in Fig. 8 which indicates that our Universe has a phase transition from deceleration to an acceleration.
FIG. 6: Evolution of $\Omega_D$ versus $1 + z$ parameter for the sign-changeable interacting HDE with GO cutoff in BD cosmology. Here, we have taken $\Omega_D(z = 0) = 0.73, \alpha = .003, \omega = 10^4, b^2 = 0.1$ and $\gamma = 1.2$ as the initial conditions.

FIG. 7: Evolution of $w_D$ versus $z$ for HDE with GO cutoff. In the left panel we have taken $\alpha = 10^{-4}, \omega = 10^4, b^2 = .1$ and $\gamma = 1.2$ and in the right panel $\alpha = 10^{-4}, \omega = 10^4, b^2 = 0.1$ and $\gamma = 0.8$ as the initial condition, respectively.

The behaviour of $v_s^2$ is plotted against $z$ in Fig. 9 (left panel) for different values of the coupling parameter $b^2$ and (right panel) for different values of $\omega$. From these figure we see that increasing $b^2$, leads to more instability against perturbations. Also, the squared sound speed is studied in Fig. 10 for different values of $\beta$ by assuming $\gamma = 1.2$ (left panel) and $\gamma = 0.8$ (right panel), which reveals that for $\gamma > 1$ this model is instable, whereas we can obtain an stable universe by taking $\gamma < 1$ ($\gamma = 0.8$).
We have taken $\alpha = 10^{-4}$, $\omega = 10^4$, $b^2 = 0.1$ and $\gamma = 1.2$ as the initial condition.

When $\alpha = 10^{-4}$, $\gamma = 1.2$, $\beta = .5$ and $\omega = 10^4$ in the left panel and $\alpha = .003$, $b^2 = 0.1$, $\gamma = 1.2$ and $\beta = 0.5$ in the right panel, as the initial condition, respectively.

V. SIGN-CHANGEABLE HDE IN BD THEORY WITH RICCI CUTOFF

In this section we choose the Ricci scalar $R$ as IR cutoff [46], which is given for the flat FRW Universe as

$$R = 6(\dot{H} + 2H^2).$$

Thus, the HDE density is written as

$$\rho_D = \frac{3c^2 \phi^2}{4\omega} \left( \dot{H} + 2H^2 \right).$$
FIG. 10: Evolution of the squared of sound speed $v_s^2$ versus $z$ for the sign-changeable interacting HDE with GO cutoff in BD cosmology. When $b^2 = 0.1$, $\alpha = 10^{-4}$, $\gamma = 1.2$, and $\omega = 10^4$ in the left panel and $\alpha = 10^{-4}$, $\omega = 10^4$, $b^2 = 0.1$ and $\gamma = 0.8$ in the right panel, as the initial condition, respectively.

Following the method of the previous section we can find deceleration parameter $q$ by dividing Eq.(4) by $H^2$ and using relation (43). We find

$$\dot{H} H^2 = -2 + \frac{1}{3 + \frac{2\alpha}{\omega}} + \frac{2\alpha c^2 (1 + r)}{r}.$$  \hfill (44)

Substituting Eq.(44) in relation (16), we have

FIG. 11: Evolution of $\Omega_D$ versus redshift parameter $z$ for HDE with Ricci cutoff in the BD cosmology when $\Omega_D(z = 0) = .73, \alpha = .003, c^2 = .8$ and $b^2 = .1$.

$$q = 1 - \frac{1 - 2\omega \alpha^2 / 3 + 2\alpha}{c^2 (1 + r)}.$$  \hfill (45)
FIG. 12: The evolution of $w_D$ versus redshift parameter $z$ for HDE with Ricci cutoff in the BD cosmology when $\alpha = .003$, $c^2 = .8$ and $b^2 = .1$.

FIG. 13: The evolution of the deceleration parameter $q$ against redshift parameter $z$ for HDE with Ricci cutoff in the BD cosmology when $\alpha = .003$, $c^2 = .8$ and $b^2 = 0.1$.

The EoS parameter $w_D$ can be obtained by dividing Eq. (5) by $H^2$ and using relation (43). We find

$$w_D = \frac{-2q + 1 + 4\alpha + 2[\alpha^2 - \alpha(1 + q)] + 2\alpha^2(1 + \omega)}{3c^2(1 - q)}. \tag{46}$$

Substituting $q$ from (45) in Eq. (46) we reach

$$w_D = \frac{3c^2(1 + r)[-1 + 2\alpha^2(2 + \omega) - (1 + 2\alpha)] - 2(1 + \alpha)[2\alpha(-3 + \alpha\omega) - 3]}{3c^2[2\alpha(-3 + \alpha\omega) - 3]}. \tag{47}$$

Taking the time derivative of relation $\Omega_D = \rho_D/\rho_{cr}$ with respect to the cosmic time $t$ and using Eqs. (43) and (44), we can find

$$\dot{\Omega}_D = -\frac{\dot{r}(1 - 2\omega\alpha^2/3 + 2\alpha)}{(1 + r)^2}. \tag{48}$$
In what follows, we shall study the squared speed of sound $v_s^2$ by computing $\dot{w}_D$ and replacing in relation (24).

**FIG. 14:** Evolution of the squared of sound speed $v_s^2$ versus redshift parameter $z$ for the sign-changeable interacting HDE with Ricci cutoff in BD cosmology. When $b^2 = .01$, $c^2 = 1$ and $\omega = 10^4$ in the left panel and $\alpha = 10^{-4}$, $\omega = 10^4$ and $b^2 = .01$ in the right panel, as the initial condition, respectively.

**FIG. 15:** Evolution of the squared of sound speed $v_s^2$ versus redshift parameter $z$ for the sign-changeable interacting HDE with Ricci cutoff in BD cosmology. Here, we have taken $b^2 = .1$, $c^2 = .8$ and $\alpha = .003$ in the left panel and $\alpha = 10^{-4}$, $\omega = 10^4$ and $c^2 = .8$ in the right panel, as the initial condition, respectively.

Choosing the same set of parameters, we start our analysis by plotting the behavior of all cosmological parameters such as $\Omega_D$, $w_D$ and $q$ in Figs. 11, 12 and 13. The main results of this figures are as follows. 

(i) At late time, the EoS parameter cannot cross the phantom line and we have always $w_D > -1$, $q$ is in acceleration phase and $\Omega_D \to 1$. 

(ii) At early time $w_D$ increases with increasing the redshift parameter, $q$ is in deceleration phase and $\Omega_D \to 0$. Having the squared sound speed at hand, we can discuss the stability of this model against perturbations. In summary,
for this model ($\Omega_D$, $w_D$ and $q$) seem to be consistent with observations. Besides, Figs. 14 and 15 indicate that $v_s^2$ remains negative which shows a sign of instability.

VI. CLOSING REMARKS

In this paper, we have explored the role of the sign-changeable interacting HDE with three infrared (IR) cutoffs, including the future event horizon, the GO and the Ricci cutoffs in the framework of BD cosmology. At first, we used the future event horizon as system’s IR cutoff to describe the dynamics of the FRW Universe by calculating the cosmological parameters such as the density parameter, the EoS parameter, the deceleration parameter and the squared sound speed. By choosing the set of parameters as $\alpha = 10^{-4}$, $\omega = 10^4$ and $b^2 = 0.1$, as the initial condition, we found out that the density parameter can fill universe by DE in the long future. We also observed that for $c \lesssim 1$ the EoS parameter $w_D$ can cross the phantom line and when $c \geq 1$, we have always $w_D > -1$. Besides, for all values of $c$, the deceleration parameter $q$ transits from deceleration ($q > 0$) in the early time to acceleration ($q < 0$) in the last time. We plotted the evolution of $v_s^2$ versus $z$ for the different parameters $\alpha$, $b^2$, $c$ and $\omega$ in Figs. 4 and 5. Graphical analysis of $v_s^2$ shows that in Fig. 4 (left panel) for $\alpha < .007$ and in (right panel) for all values of $b^2$ our model can be stable. Also, in Fig. 5 (left panel) by increasing $c$, and in (right panel) for all of values $\omega$, the sign-changeable interacting HDE with the future cutoff in BD cosmology can lead to a stable DE dominated universe.

Furthermore, we have focused on the GO cutoff and observed that by suitable selection for the model parameters and for all values of $\beta$, at the late time where the DE dominates we have $\Omega_D \rightarrow 1$ which have plotted in Fig. 6. Also, we plotted $w_D$ versus $z$ and see that in Fig. 7 (left panel) for $\gamma > 1$ we cannot cross phantom while for $\gamma < 1$ for different values of $\beta$ we have $w_D < -1$. The behavior of $v_s^2$ is also plotted in Figs. 9 and 10. In Fig. 9 we observed that for different values of $b^2$ (left panel) and $\omega$ (right panel) this model does not allow a stable DE dominated universe. In Fig. 10 (left panel) by taking the parameters as $\alpha = 10^{-4}$, $\omega = 10^4$ and $b^2 = 0.1$ we see if $\gamma > 1$, for different values of $\beta$, $v_s^2$ cannot be stable while for $\gamma < 1$ (right panel) (e.g. $\gamma = 0.8$) our model is stable. Finally, we studied these cosmological parameter when the system’s IR cutoff is the Ricci cutoff in Figs. 11-15. We see three parameters, $\Omega_D$, $w_D$ and $q$ are consistent with observations, however, $w_D$ cannot cross the phantom line for different values of $\omega$. Finally, in Figs. 14 and 15 the evolution of $v_s^2$ is plotted. Obviously, we see that for all different values of $\alpha$, $c^2$, $\omega$ and $b^2$, the squared sound speed $v_s^2$ remains negative which shows a sign of instability for the HDE in BD
theory with Ricci cutoff.

In conclusion, our studies show that for the sign-changeable HDE model in the setup of BD cosmology, among the above three IR cutoffs, the event horizon is the most suitable horizon which can passes all conditions and leads to a stable DE dominated universe.

Acknowledgments

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