Neutrinos: theory and phenomenology

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Abstract
The theory and phenomenology of neutrinos will be addressed, especially that relating to the observation of neutrino flavor transformations. The current status and implications for future experiments will be discussed with special emphasis on the experiments that will determine the neutrino mass ordering, the dominant flavor content of the neutrino mass eigenstate with the smallest electron neutrino content and the size of CP violation in the neutrino sector. Beyond the neutrino standard model, the evidence for and a possible definitive experiment to confirm or refute the existence of light sterile neutrinos will be briefly discussed.

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(Some figures may appear in colour only in the online journal)

1. Introduction

Fifteen years ago this year, the SuperKamiokande Collaboration presented a talk titled ‘evidence for $\nu_\mu$ oscillations’ at the Neutrino 1998 Conference [1]. This set the particle physics world ‘abuzz’ since, if neutrinos change flavor, it implies that their clocks are ticking and therefore they cannot be traveling at the speed of light. Hence neutrinos have mass.

Fast forward 15 years and the evidence for neutrino flavor conversion is overwhelming. The simplest and only satisfactory description of all the data is that neutrinos have distinct masses and mix. Two distinct baseline ($L$) divided by neutrino energy ($E$) scales have been identified corresponding to two distinct $\delta m^2_{ij} \equiv m^2_i - m^2_j$ for the neutrino mass eigenstates:

\[
\frac{L}{E} = 500 \text{ km GeV}^{-1} \quad \text{and} \quad \delta m^2_{\text{atm}} = 2.4 \times 10^{-3} \text{ eV}^2,
\]

\[
\frac{L}{E} = 15,000 \text{ km GeV}^{-1} \quad \text{and} \quad \delta m^2_{\text{sol}} = 7.6 \times 10^{-5} \text{ eV}^2.
\]

These are known as the atmospheric and solar scales, respectively.

Since it is most likely that the Higgs boson has been discovered at the Large Hadron Collider (LHC), it is natural to ask how the neutrinos couple to the Higgs boson. First, what is ‘mass’ for a fermion? It is a coupling of the right and left components of the field, and for the neutrino this coupling depends on whether the neutrino is a Dirac particle, like all the other fermions in the standard model (SM) or a Majorana fermion, which would make the neutrino unique amongst the particles of the SM. The couplings for both Dirac and Majorana [3] neutrinos are given in the following table:

| Type   | Mass term                                                                 | Coupling to Higgs | # Comp. | Lepton number |
|--------|----------------------------------------------------------------------------|-------------------|---------|---------------|
| Dirac  | $\bar{\nu}_R \nu_L + \bar{\nu}_L \nu_R$                                | $\frac{1}{2} (LH)^2$ | 2       | Conserved     |
| Majorana | $\bar{\nu}_L \nu_L$                                                       | $\bar{\nu}_L H \nu_L$ | 4       | Violated      |

Determining whether the nature of the neutrino is Dirac or Majorana is one of the big unanswered questions in neutrino physics and is being addressed through neutrinoless, double beta decay experiments. Independent of the nature of the neutrino, the partial width of the Higgs decaying to two massive neutrinos is given by

\[
\Gamma_{\text{tree}}(H \rightarrow \nu \bar{\nu}) \approx \left( \frac{m_\nu}{m_\tau} \right)^2 \Gamma(H \rightarrow \tau \bar{\tau}) \approx 10^{-20} \Gamma(H \rightarrow \tau \bar{\tau}).
\]

So not only is this decay invisible, it is impossibly tiny and swamped by other invisible decays of the Higgs, such as $H \rightarrow ZZ \rightarrow 4\nu$!

In seesaw models, where the mass of the neutrinos is naturally very light, it is possible that LHC could see physics beyond the SM, such as right handed heavy neutrinos or...
2. Neutrino masses and mixings

The three known neutrino flavor states, $\nu_e$, $\nu_\mu$, $\nu_\tau$, and the three neutrino mass eigenstates, $\nu_1$, $\nu_2$, $\nu_3$, are related as follows:

$$
\begin{pmatrix}
\nu_e \\
\nu_\mu \\
\nu_\tau
\end{pmatrix}
= 
U
\begin{pmatrix}
\nu_1 \\
\nu_2 \\
\nu_3
\end{pmatrix},
$$

(2)

where the $U$ matrix is unitary and referred to as the Pontecorvo–Maki–Nakagawa–Sakata (PMNS) matrix. The mass eigenstates are labeled such that $|U_{e1}|^2 > |U_{e2}|^2 > |U_{e3}|^2$, which implies that, by definition, the $\nu_e$ component of $\nu_1$ is the atmospheric component of $\nu_2 > \nu_3$ component of $\nu_3$.

2.1. Masses

With this choice of labeling of the neutrino mass eigenstates, the solar oscillations are governed by $\delta m^2_{sol}$ as both $\nu_1$ and $\nu_2$ have a significant $\nu_e$ component, whereas the atmospheric oscillations are governed by $\delta m^2_{atm}$ as $\nu_3$ has a small $\nu_e$ component required by the small $\nu_e$ involvement shown by the results of the SuperKamiokande and Chooz experiments. The mass ordering of $\nu_1$ and $\nu_2$ was determined by matter effects in the interior of the sun by the SNO experiment [2]. Their measurement of the charge current to neutral current ratio of less than one half, for the $^8$B high energy solar neutrinos, implies that the higher mass state has the lower $\nu_e$ component i.e. $m_2^2 > m_1^2$ or $\delta m^2_{21} > 0$.

The atmospheric neutrino mass ordering, $m_3^2 > m_2^2 > m_1^2$, is still to be determined, see figure 1. If $m_3^2 > m_2^2$, the ordering is known as the normal hierarchy (NH), whereas if $m_3^2 < m_2^2$ the ordering is known as the inverted hierarchy (IH). Figure 2 shows the masses as a function of the lightest neutrino mass.

The sum of the masses of the neutrinos satisfies

$$
\sqrt{\delta m^2_{atm}} = 0.05 \text{ eV} < \sum m_{\nu_i} < 0.5 \text{ eV}.
$$

(3)

So the $\sum m_{\nu_i}$ ranges from $10^{-7}$ to $10^{-6}$ times $m_e$, however the mass of the lightest neutrino, $m_{\text{lite}}$, could be very small. If $m_{\text{lite}} \ll \sqrt{\delta m^2_{atm}} \sim 0.01 \text{ eV}^2$, then this is an additional scale to be explained by a theory of neutrino masses and mixings.

2.2. Mixings

The standard representation of PMNS mixing matrix is given as follows:

$$
U_{e2} = \cos \theta_{13} \sin \theta_{12},
U_{\mu 3} = \cos \theta_{13} \sin \theta_{23},
U_{\tau 3} = \sin \theta_{13} e^{-i\phi}
$$

(4)

with all other elements following by unitarity. The square of the elements of the PMNS matrix give the fractional flavor content, e.g. $|U_{e2}|^2$ is the fraction of $\nu_2$ that is $\nu_e$. Figure 3 gives this fraction for all the mass eigenstates.
is used to show how these fractions change as cos δ

unitarity triangle are and the phases are unknown. Thus the size of the sides of this given by

where the magnitude of the elements of

This unique unitarity triangle, \( \left\{ U_{e1}, U_{\mu 2}, U_{\tau 3} \right\} \), is given by

where the \( \approx \) follows from the fact that we know that \( |U_{e3}|^2 \ll 1 \).

Our current knowledge of these mixings angles is approximately as follows:

\[
\sin^2 \theta_{12} \approx \frac{1}{3}, \quad \sin^2 \theta_{23} \approx \frac{1}{3}, \quad \sin^2 \theta_{13} \approx 0.02
\]

where \( 0 \leq \delta < 2\pi \),

which are the values used in this figure. For more precise values see the latest Particle Data Group.

2.3. The neutrino unitarity triangle

The orthogonality of the rows and columns of the PMNS mixing matrix, gives six unitarity relationships, that can be shown as triangles in the complex plane. However, only one of these triangles does not involve the \( \tau \)-neutrino which is experimentally challenging in both detection and production. This unique unitarity triangle, \( \left\{ U_{e1}, U_{\mu 2}, U_{\tau 3} \right\} \), is given by

where the magnitude of the elements of \( U \) are approximately given by

\[
|U_{\mu 1}| \approx \sqrt{\frac{1}{6}} \quad |U_{e1}| \approx \sqrt{\frac{2}{3}} \quad |U_{\mu 2}| \approx \frac{1}{3} \quad |U_{e2}| \approx \frac{1}{3}
\]

\[
\times |U_{\mu 3}| \approx \sqrt{\frac{1}{6}} \quad |U_{e3}| \approx \frac{1}{6}
\]

and the phases are unknown. Thus the size of the sides of this unitarity triangle are

\[
|U_{\mu 1}| |U_{e1}| \approx 0.1 - 0.4, \quad |U_{\mu 2}| |U_{e2}| \approx 0.2 - 0.4,
\]

with \( |U_{\mu 3}| |U_{e3}| \approx 0.08 - 0.12 \),

\[
\text{CPT} \Rightarrow \text{invariant } \delta \leftrightarrow -\delta
\]

Figure 3. The flavor content of the neutrino mass eigenstates [4]. Copyright 2004 by the American Physical Society. The width of the lines is used to show how these fractions change as cos \( \delta \). Of course, this figure must be the same for neutrinos and anti-neutrinos, if CPT is conserved.

Alternatively, we can write

\[
\sin^2 \theta_{13} \equiv |U_{e3}|^2, \quad \sin^2 \theta_{12} \equiv \frac{|U_{e2}|^2}{1 - |U_{e2}|^2} \approx |U_{e2}|^2,
\]

\[
\sin^2 \theta_{23} \equiv \frac{|U_{e3}|^2}{1 - |U_{e3}|^2} \approx |U_{e3}|^2,
\]

separately. Current experiments do not allow this determination. To separate these two elements one needs, for example, a \( \nu_\mu \) disappearance experiment at the solar \( L/E \approx 15\,000 \text{ km GeV}^{-1} \). A \( \nu_\mu \) beam to a detector in geosynchronous orbit is one possibility, but at the current time this is science fiction. Without imposing unitarity, the knowledge of some of the elements of the PMNS matrix is poor. For example, all of our information on \( U_{e1} \) comes entirely from imposing unitarity!

Figure 4. The neutrino unitarity triangle [6] for the first two rows of the PMNS matrix. The sides are in the approximate ratio of 3:3:1 and twice the area of this triangle is the Jarlskog invariant [7], which determines the size of CP violation.

2.4. Leptons versus quarks

The lepton and quark mixing matrices are very different:

\[
U_{\text{PMNS}} \sim \begin{pmatrix} 0.8 & 0.5 & 0.2 \\ 0.4 & 0.6 & 0.7 \\ 0.4 & 0.6 & 0.7 \end{pmatrix}
\]

\[
V_{\text{CKM}} \sim \begin{pmatrix} \approx 1 & 0.2 & 0.001 \\ 0.2 & \approx 1 & 0.01 \\ 0.001 & 0.01 & \approx 1 \end{pmatrix}.
\]

The CKM mixing matrix is approximately the identity matrix plus small (Cabibbo) corrections whereas the lepton matrix
could be some special matrix, bimaximal or tribimaximal, plus small (Cabbibo) corrections. This way of thinking has led to a number of testable relationships between the mixing angles [8], such as

\[ \theta_{13} \approx \theta_c / \sqrt{2}, \]
\[ \theta_{12} = \theta_c + \theta_{13} \cos \delta, \quad \text{where } \theta_c = 45^\circ, 35^\circ \text{ or } 32^\circ. \quad (9) \]
\[ \theta_{23} = 45^\circ \pm \kappa \theta_{13} \cos \delta, \quad \text{where } \kappa = \sqrt{2} \text{ or } -1 / \sqrt{2}. \]

Although, these models are not completely compelling, the relationships they produce are worth testing as maybe we will make progress in understanding this exceedingly challenging problem. Much like the Rutherford–Bohr atom lead to a more complete understanding of the atom with the discovery of quantum mechanics.

3. Neutrino phenomenology

In this section, I will address some important topics in neutrino phenomenology related to disappearance and appearance experiments.

3.1. Neutrino disappearance experiments

For neutrino disappearance experiments, the vacuum oscillation probability for \( \nu_a = (\nu_e, \nu_\mu, \nu_\tau) \) can be written as [9]

\[ P(\nu_a \to \nu_a) = 1 - 4|U_{a1}|^2|U_{a2}|^2 \sin^2 \Delta_{21} - 4|U_{a3}|^2(1 - |U_{a3}|^2) \times |r_a \sin^2 \Delta_{31} + (1 - r_a) \sin^2 \Delta_{32}|, \quad (10) \]

where \( \Delta_{ij} = \frac{\delta m_{ij}^2}{4E} \) and \( r_a = \frac{|U_{a1}|^2}{|U_{a1}|^2 + 4|U_{a3}|^2}. \)

Near the atmospheric first oscillation minimum, \( \Delta_{31} \approx \Delta_{32} \approx \pi / 2 \), this can be approximated by

\[ P(\nu_a \to \nu_a) = 1 - \sin^2 2\theta_{aa} \sin^2 \frac{\delta m_{aa}^2 L}{4E} + O(\Delta_{31}^2), \quad (11) \]

where \( \delta m_{aa}^2 \equiv r_a |\delta m_{31}^2| + (1 - r_a)|\delta m_{32}^2| \) and \( \sin^2 2\theta_{aa} = 4|U_{a3}|^2(1 - |U_{a3}|^2). \)

Any other choice for the effective \( \delta m^2 \), other than \( \delta m_{aa}^2 \), induces a \( O(\Delta_{31}) \) term in equation (11) and since \( \Delta_{31} \approx 1/20 \) this reduces the accuracy of the approximation from 0.3 to 5%, a significant change.

Until your uncertainty on your measurement of \( P \) is less than \( O(\Delta_{31}^2) \sim 0.003 \) then

- three flavor effects are invisible,
- the effective \( \delta m^2 \) measured is \( \delta m_{aa}^2 = r_a |\delta m_{31}^2| + (1 - r_a)|\delta m_{32}^2| \), the \( \nu_a \) average of \( |\delta m_{31}^2| \) and \( |\delta m_{32}^2| \),
- and the effective \( \sin^2 2\theta \) is \( \sin^2 2\theta_{aa} = 4|U_{a3}|^2(1 - |U_{a3}|^2). \)

So the MINOS, T2K and NOvA \( \nu_\mu \) disappearance experiments [2] all measure

\[ \delta m_{\mu \mu}^2 = \frac{|U_{\mu 1}|^2 |\delta m_{31}^2| + |U_{\mu 2}|^2 |\delta m_{32}^2|}{|U_{\mu 1}|^2 + |U_{\mu 2}|^2} \]

2 Matter effects are very small in the \( \nu_a \) disappearance channel at these baselines.
Figure 6. The left panel shows the two components $P_{\text{atm}}$ and $P_{\text{sol}}$ in matter for the normal and inverted hierarchies for $\sin^2 2\theta_{13} = 0.04$ and a baseline of 1200 km. The right panel shows the total probability including the interference term between the two components for various values of the CP phase $\delta$ for the neutrino. Notice that the coherent sum of two amplitudes shows a rich structure depending on the hierarchy and value of CP phase. These curves can also be interpreted as anti-neutrino probabilities if one interchanges the hierarchy and the values of the CP phase.

Figure 7. The left panel is the bi-probability plot for the T2K/HyperK experiment showing the correlation between neutrino and antineutrino $\nu_\mu \rightarrow \nu_e$ probabilities. The matter effect is small but non-negligible for T2K/HyperK. Whereas the left panel is for the NO$\nu$A experiment where the matter effect is three times larger.

3.2. Neutrino appearance experiments

Genuine three flavor effects, like CP violation, can be observed in long baseline $\nu_\mu \rightarrow \nu_e$ appearance experiments or in one of its CP or T conjugate channels. That is, in one of following transitions

\[
\begin{align*}
\nu_\mu &\rightarrow \nu_e \\
\bar{\nu}_\mu &\rightarrow \bar{\nu}_e \\
\mathrm{CP} &\quad \leftrightarrow \\
\nu_e &\rightarrow \nu_\mu \\
\bar{\nu}_e &\rightarrow \bar{\nu}_\mu \\
\mathrm{T} &\quad \leftrightarrow \\
\end{align*}
\]

Processes across the diagonal are related by CPT. The first row will be explored in very powerful conventional beams, T2K [12], NO$\nu$A [13], Superbeams, HyperK [14], LBNE [15] and ESSnuSB [16], whereas the second row could be explored in Nu-Factories [17] or Beta Beams [18].

In vacuum, the probability for $\nu_\mu \rightarrow \nu_e$ is derived as follows [19]:

\[
P(\nu_\mu \rightarrow \nu_e) = |U_{\mu e}|^2 + |U_{\mu 2}|^2 + |U_{\mu 3}|^2 - 2|U_{\mu e} U_{\mu 2}^*| \cos \delta - 2|U_{\mu e} U_{\mu 3}^*| \sin \delta - 2|U_{\mu 2} U_{\mu 3}^*| \sin \Delta_{21}.
\]

where $\sqrt{P_{\text{atm}}} = 2|U_{\mu 3}||U_{\mu 1}| \sin \Delta_{31} = \sin \theta_{23} \sin 2\theta_{13} \sin \Delta_{31}$ and $\sqrt{P_{\text{sol}}} \approx \cos \theta_{23} \sin 2\theta_{13} \sin \Delta_{21}$.

Note, $\sqrt{P_{\text{atm}}}$ and $\sqrt{P_{\text{sol}}}$ are just the two flavor oscillation amplitudes at the atmospheric and solar scales, respectively.
For anti-neutrinos $\delta$ must be replaced with $-\delta$ and the interference term changes
\[2\sqrt{P_{\text{atm}}}\sqrt{P_{\text{sol}}} \cos(\Delta_{32} + \delta) \Rightarrow 2\sqrt{P_{\text{atm}}}\sqrt{P_{\text{sol}}} \cos(\Delta_{32} - \delta).\]

Expanding $\cos(\Delta_{32} \pm \delta)$, one has a CP conserving part
\[2\sqrt{P_{\text{atm}}}\sqrt{P_{\text{sol}}} \cos \Delta_{32} \cos \delta \quad (16)\]

and the CP violating part, where $- (\pm)$ sign is for the neutrino (anti-neutrino) channel,
\[\pm 2\sqrt{P_{\text{atm}}}\sqrt{P_{\text{sol}}} \sin \Delta_{32} \sin \delta \quad (17)\]

where $J = \sin \delta \sin 2\theta_{13} \cos \theta_{13} \sin 2\theta_{12} \sin 2\theta_{23}$
\[\sin \Delta_{31} \sin \Delta_{32} \sin \Delta_{21} \quad (17)\]

In matter, the two flavor amplitudes, $\sqrt{P_{\text{atm}}}$ and $\sqrt{P_{\text{sol}}}$, are modified as follows:
\[\sqrt{P_{\text{atm}}} \Rightarrow \sin \theta_{23} \sin \theta_{13} \frac{\sin(\Delta_{31} - aL)}{(\Delta_{31} - aL)} \Delta_{31}, \]
\[\sqrt{P_{\text{sol}}} \Rightarrow \cos \theta_{23} \sin \theta_{13} \frac{\sin(\Delta_{21})}{(\Delta_{21})} \Delta_{21}, \quad (18)\]

where $a = \pm G_F N_c / \sqrt{2} \approx (\rho \gamma / 1.3 \text{ g cm}^{-3}) (4000 \text{ km})^{-1}$

and the sign is positive for neutrinos and negative for anti-neutrinos. This change follows since in both the (31) and (21) sectors the product $[\delta m^2 \sin 2\theta]$ is approximately independent of matter effects. Figure 6 shows the $\nu_e$ appearance probability as a function of the energy for a distance of 1200 km. In figure 7 the bi-probability plots for both T2K [12] (as well as the future possible HyperK [14]) and NOvA [13] experiments. It is possible that these two experiments will determine the mass ordering, and give a hint of CP violation in the neutrino sector with sufficient statistics.

The critical value of $\sin^2 \theta_{13} \sin \theta_{13}$ at which the bi-probability ellipses for the NH and the IH separate is given by [20]
\[(\tan \theta_{23} \sin 2\theta_{13})_{\text{crit}} = \left[ \frac{\Delta_{31}^2 \sin 2\theta_{12}}{1 - \Delta_{31} \cot \Delta_{31}} \right] \frac{\delta m_{23}^2 / (aL)}{\delta m_{31}^2 / (aL)} \approx 2.3 \frac{\delta m_{23}^2}{\delta m_{31}^2} / (aL) \quad (19)\]

For the NOvA experiment, this corresponds to
\[\tan^2 \theta_{23} \sin^2 2\theta_{13} \_{\text{crit}} = 0.13. \quad (20)\]

The difference of these probabilities can be used to determine the CP violation phase and the mass hierarchy.

It is also worth noting the following, that sum of the neutrino and anti-neutrino probabilities at oscillation maximum (OM) can be directly compared to the value of $\sin^2 2\theta_{13}$ measured by the reactor disappearance experiments
\[\langle P(\nu_e) \rangle = 0.09\]

\[P(\nu_e) = 0.09 \quad \text{or} \quad \delta = \pi/2 \quad \text{or} \quad \delta = 3\pi/2\]

\[\text{Normal Mass Hierarchy}\]

\[\text{Inverted Mass Hierarchy}\]

\[L = 1300 \text{ km}, \quad <E> = 3.2 \text{ GeV}\]

\[\text{Figure 8. The biprobability plot for the LBNE experiment at the same } L/E \text{ as the NOvA experiment [21]. Notice how widely the normal (blue) and the inverted (red) hierarchies are separated here.}\]

\[\text{In vacuum, the larger this asymmetry the easier it will be to see CP violation.}\]

\[\text{For the NOvA experiment, this corresponds to}\]
\[\text{the two hierarchies satisfies the following relationship:}\]
\[\langle \sin \delta \rangle_{\text{NH}} - \langle \sin \delta \rangle_{\text{IH}} = 2(\tan \theta_{23} \sin 2\theta_{13}) / (\tan \theta_{23} \sin 2\theta_{13})_{\text{crit}} \]
\[\approx 0.57 \tan \theta_{23}, \quad \text{NOvA,}\]
\[1.7 \tan \theta_{23}, \quad \text{T2K/HyperK.}\]

\[\text{For the NOvA experiment, this corresponds to}\]
\[\text{the two hierarchies satisfies the following relationship:}\]
\[\langle \sin \delta \rangle_{\text{NH}} - \langle \sin \delta \rangle_{\text{IH}} = 2(\tan \theta_{23} \sin 2\theta_{13}) / (\tan \theta_{23} \sin 2\theta_{13})_{\text{crit}} \]
\[\approx 0.57 \tan \theta_{23}, \quad \text{NOvA,}\]
\[1.7 \tan \theta_{23}, \quad \text{T2K/HyperK.}\]
Figure 9. The neutrino asymmetry as defined in equation (22) as a function of $\sin^2 2\theta_{13}$, at the first OM [6] (left panel) and at the second OM (right panel) in vacuum. At current measured value of $\sin^2 2\theta_{13} = 0.090$, the asymmetries are $A = 0.3 \sin \delta$ for the first OM and $A = 0.75 \sin \delta$ for the second OM.

\begin{align*}
\nu_e-\text{Appearance} & \quad P(\nu_\mu \rightarrow \nu_e) \quad \text{and} \quad P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e) \\
\nu_e-\text{Disappearance} & \quad P(\bar{\nu}_e \rightarrow \bar{\nu}_e) \\
\nu_\mu-\text{Disappearance} & \quad P(\nu_\mu \rightarrow \nu_\mu)
\end{align*}

Figure 10. The allowed regions in the $\sin^2 \theta_{13} \cdot \sin^2 \theta_{23}$ plane for the three different types of experiment. The red line is the exact solution assuming that the input values are $\sin^2 \theta_{23} = 0.45$, $\sin^2 \theta_{13} = 0.022$ and $\delta = 70^\circ$. The marks on this red line indicate values of $\delta$ that are $10^\circ$ apart and the corner is $\delta = 90^\circ$. The blue lines are the 1, 2, 3 $\sigma$ allowed regions assuming reasonable uncertainties on the measurements.

which implies that $P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e)$ is between $\frac{1}{2}$ and 2 times $P(\nu_\mu \rightarrow \nu_e)$. Whereas at the second OM, the vacuum asymmetry is

$$A \approx 0.75 \sin \delta \quad \text{at} \quad \Delta_{31} = 3\pi/2,$$

which implies that $P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e)$ is between $\frac{1}{7}$ and seven times $P(\nu_\mu \rightarrow \nu_e)$, see figure 9. So that experiments at the second OM, like ESSnuSB [16], have a significantly larger difference between the neutrino and anti-neutrino channels.

4. The generalized intrinsic degeneracy

Let us assume for the moment we known all the parameters governing neutrino oscillation except for $\sin^2 \theta_{23}$, $\sin^2 \theta_{13}$ and $\delta$ and we will use three different neutrino experiments to determine these parameters [22]:

- $\nu_\mu \rightarrow \nu_e$ appearance experiments in both the neutrino and antineutrino channels: i.e. $P(\nu_\mu \rightarrow \nu_e)$ and $P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e)$.

In the $\sin^2 \theta_{13} \cdot \sin^2 \theta_{23}$ plane, these measurements constrain you to a line labeled by the values of $\delta$. See red line in the left panel of figure 10.

- $\bar{\nu}_e \rightarrow \bar{\nu}_e$ disappearance experiments: $P(\bar{\nu}_e \rightarrow \bar{\nu}_e)$, this measurement determines $\sin^2 \theta_{13}$ independently of the other variables. Middle panel of figure 10.

- $\nu_\mu \rightarrow \nu_\mu$ disappearance experiments: $P(\nu_\mu \rightarrow \nu_\mu)$ this measurement determines the combination of parameters $4 \cos^2 \theta_{13} \sin^2 \theta_{23}(1 - \cos^2 \theta_{13} \sin^2 \theta_{23})$. Right panel of figure 10.

Also shown in figure 10 is the allowed region for pseudo-experiments which illustrates the allowed region in the $\sin^2 \theta_{13} \cdot \sin^2 \theta_{23}$ for each of these different types of experiment.

Notice the difficult in determining $\sin^2 \theta_{23} \approx 1/2$ and the value of $\delta \approx \pi/2$. These degeneracies can be solved by information from a sufficient broad neutrino energy spectrum.
5. Beyond the neutrino standard model

There are tensions in the νSM as follows:

- LSND. 3.8σ evidence for anti-νe appearance.
- MiniBooNE. 3.8σ combined evidence for νe and anti-νe appearance.
- Reactor. 3.0σ evidence for anti-νe disappearance.
- Gallium. 2.7σ evidence for νe disappearance.

This data can be interpreted as the effects of one or more additional sterile neutrinos with a $\delta m^2 \sim 1\text{eV}^2$. However, there is also tensions with this extended model between the appearance and disappearance data [24]. There are a number of experiments that are taking data or are planned to address these anomalies. These include

- reactor and source experiments looking at the $L/\E$ depends for $\bar{\nu}_e \rightarrow \bar{\nu}_e$ and $\nu_e \rightarrow \bar{\nu}_e$;
- $\nu_\mu \rightarrow \nu_\mu$ disappearance experiments with both near and far detectors;
- $\nu_\mu \rightarrow \bar{\nu}_e$ or $\nu_e \rightarrow \nu_\mu$ appearance experiments.

One of the more ambitious experiments is NuSTORM [25], which stores muons in a racetrack shaped ring providing a source of νe with essentially no contamination from other neutrino flavors. Such an experiment could exclude the LSND allowed region at about 10σ and would also be a useful source for measuring neutrino cross sections as the neutrino flux can be calculated from the decaying muon flux with high precision. Such an experiment is a first step on the way to a Neutrino Factory [17] and maybe a Muon Collider [26] in the future.

6. Conclusions

If neutrinos are Majorana in nature and CP violation is observed in neutrino oscillation then the credibility of leptogenesis will be greatly enhanced. Neutrino oscillation experiments can not only measure CP violation but can also determine whether the atmospheric mass hierarchy is normal or inverted and can determine whether the $\nu_\mu$ flavor content is more or less than the $\nu_e$ content for the neutrino mass eigenstate with the smallest amount of $\nu_\tau$. The precise measurement of the neutrino mixing and mass parameters will allow us to test the various models predicting these parameters and may lead to a more complete understanding of this notoriously difficult physics problem. If the mass of the lightest neutrino, is significantly smaller than the square root of the solar $\delta m^2$ then there is a new scale in particle physics that needs to be explained. Finally, neutrinos have surprised us in the past and are expected to do so in the future. Where are these surprises? Are there light sterile neutrinos? Do neutrinos decay? What is the size of non-standard interactions? Will LHC find new physics related to neutrino mass? Only the results from further experiments will provide us the answer to these most important questions!

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References

[1] Kajita T (Super-Kamiokande Collaboration) 1999 Nucl. Phys. Proc. Suppl. 77 123
[2] Ahmed S N et al (Super-Kamiokande Collaboration) 1999 Phys. Rev. Lett. 82 2644
[3] Weinberg S 1979 Phys. Rev. Lett. 43 1566
[4] Mena O and Parke S J 2004 Phys. Rev. D 69 117301
[5] Farzan Y and Smirnov A Y 2002 Phys. Rev. D 65 115001
[6] Nunokawa H, Parke S J and Valle J W F 2005 Phys. Rev. D 72 053009
[7] Carlskog C 1985 Phys. Rev. Lett. 55 1039
[8] Ramond P and Raidal M 2004 Phys. Rev. Lett. 93 161801
[9] Ramond P 2005 Int. J. Mod. Phys. A 20 1234
[10] Minakata H and Smirnov A Y 2004 Phys. Rev. D 70 073009
[11] Minakata H and Smirnov A Y 2006 Annu. Rev. Nucl. Part. Sci. 56 569
[12] King S F and Luhn C 2013 Rep. Prog. Phys. 76 056201
[13] Nunokawa H, Parke S J and Zukanovich Funchal R 2005 Phys. Rev. D 72 013009
[14] Minakata H, Nunokawa H, Parke S J and Zukanovich Funchal R 2006 Phys. Rev. D 74 053008
[15] Minakata H, Nunokawa H, Parke S J and Zukanovich Funchal R 2007 Phys. Rev. D 76 053004
[16] Minakata H, Nunokawa H, Parke S J and Zukanovich Funchal R 2007 Phys. Rev. D 76 079901 (erratum)
[17] Parke S J, Minakata H, Nunokawa H and Funchal R Z 2009 Nucl. Phys. Proc. Suppl. 188 115
[18] Qian X, Dwyer D A, McKeown R D, Vogel P, Wang W and Zhang C 2013 Phys. Rev. D 87 033005
[19] Itow Y et al (T2K Collaboration) 2001 arXiv:hep-ex/0106019
[20] Ayres D et al 2002 arXiv:hep-ex/0210005
[21] Abe K et al 2011 arXiv:1109.3262 [hep-ex]
[22] Adams C et al (LBNE Collaboration) 2013 arXiv:1307.7335 [hep-ex]
[23] Baussean E, Dracos M, Ekelof T, Martinez E F, Ohman H and Vassilopoulos N 2012 arXiv:1212.5048 [hep-ex]
[24] Apollonio M et al (IDS-NF Collaboration) 2012 Nucl. Phys. Proc. Suppl. 229–232 515
[25] Burt G, Dexter A C, Chance A, Payet J, De Melo Mendonca T, Hansen C, Wildner E H M and Benedetto E et al 2011 Conf. Proc. C 110904 2535
[26] Cervera A, Donini A, Gavela M B, Gomez-Cadenas J J, Hernandez P, Mena O and Rigolin S 2000 Nucl. Phys. B 579 17
[27] Cervera A, Donini A, Gavela M B, Gomez-Cadenas J J, Hernandez P, Mena O and Rigolin S 2001 Nucl. Phys. B 593 731 (erratum)
[28] Mena O and Parke S J 2004 Phys. Rev. D 70 093011
[21] Nunokawa H 2012 private communication
[22] Minakata H and Parke S J 2013 Phys. Rev. D 87 113005
    Coloma P, Minakata H and Parke S J 2013 in preparation
[23] Dick K, Freund M, Lindner M and Romanino A 1999 Nucl. Phys. B 562 29
[24] Kopp J, Machado P A N, Maltoni M and Schwetz T 2013
    J. High Energy Phys. JHEP05(2013)050
[25] Adey D et al 2013 arXiv:1305.1419 [physics.acc-ph]
[26] Alexahin Y et al 2013 arXiv:1308.2143 [hep-ph]