Identification of Parameters using iterative Bi-parabolic Target Functions

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The Levenberg Marquardt algorithm \cite{1} is a common choice for identification problems. While the construction of the residuals and therefore the target function is quite simple, the results are often superior. This changes if the final target consists out of several target values which affects each other to a not unique solution. The specification of so called border curves or error bands will improve the identification process. New ways to use this border curves and to construct a new target function will be shown.

\section{Motivation}

The choice of a suitable material model and the following identification of material parameters are important subjects to build up an efficient simulation of real components via FEM. Therefore, the simulation curve has to be compared to experimental data points. Some given error bands or a known corridor of suitable solutions for the curves are developed with the free parameter $\frac{1}{2} \sum_{i=1}^{n} r_i^2(x)$ be the least square sum of residuals $r_i(x) = s(x,t_i) - e_i$ of experimental data points $e_i$ and simulation points $s(x,t_i)$ of a material parameter set $x$ and step time $t_i$. A corridor is given by two border functions $u(t_j)$ and $o(t_j)$ for a time step $t_j$ with the relation $u(t_j) \leq e_j \leq o(t_j)$ which implies, that $o(t_j)$ is the upper and $u(t_j)$ the lower border function. At each time step $t_k$ exists a parabola like in Fig. 1a). The following task is the modelling of a suitable alternative target function to exchange the residuals $r_i(x)$ to a new corridor-driven formulation. The function formulation of the new target function should be differ in the inner corridor between $u_j$ and $o_j$ and the outer regions. While the outer formulation is more penalizing to the distribution and reproduction in any medium, provided the original work is properly cited.

\section{The Bi-parabolic Target Function}

Let $f(x) = \frac{1}{2} \sum_{i=1}^{n} r_i^2(x)$ be the least square sum of residuals $r_i(x) = s(x,t_i) - e_i$ of experimental data points $e_i$ and simulation points $s(x,t_i)$ of a material parameter set $x$ and step time $t_i$. A corridor is given by two border functions $u(t_j)$ and $o(t_j)$ for a time step $t_j$ with the relation $u(t_j) \leq e_j \leq o(t_j)$ which implies, that $o(t_j)$ is the upper and $u(t_j)$ the lower border function. At each time step $t_k$ exists a parabola like in Fig. 1a). The following task is the modelling of a suitable alternative target function to exchange the residuals $r_i(x)$ to a new corridor-driven formulation. The function formulation of the new target function should be differ in the inner corridor between $u_j$ and $o_j$ and the outer regions. While the outer formulation is more penalizing to the distribution and reproduction in any medium, provided the original work is properly cited.

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\begin{figure}[h]
\centering
\includegraphics[width=\linewidth]{fig1.png}
\caption{a) Parabolas at different time steps, b) Asymmetric set of parabolas, c) Iterative structure of asymmetric parabolas}
\end{figure}

While the symmetrical approach expects the experimental data points as the means between the $u(t_j)$ and $o(t_j)$, a more general formulation is an asymmetrical formulation. The two parabolic functions $q_1$ and $q_2$ are replaced by a set $\tilde{q}_1$ and $\tilde{q}_2$ for the upper region and $\tilde{q}_{1\star}$ and $\tilde{q}_{2\star}$ for the lower region like in Fig. 1b).

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This leads to a system with four independent parameters $p_k$ with the prior used conditions. A disadvantage of this method is the not comparability between both sides because the transition of the simulation values over each border point is independent of each other. To ensure the comparison, a function value formulation with $q_1(g(t_i)) = \eta_1(\psi(t_i))$ is used to fix the function values each time. As an another idea, a slope value formulation with $q'(g(t_i)) = -\eta_1(\psi(t_i))$ is established to fix the absolute slope at the border points. Both formulations will reduce the amount of free parameters to three. To ensure also an additional iterative process (Fig 1c)), the following system of linear residuals is developed:

$$r^{j+1}(\hat{y}, t_i) = \begin{bmatrix}
X_{\hat{y} \geq \gamma^{j+1}(t_i)} \cdot \sqrt{p_2^{j+1} - p_1^{j+1}(\gamma^{j+1}(t_i) - \hat{y})} \\
X_{\hat{y} = 0} \cdot \sqrt{p_1^{j+1} \hat{y}} \\
X_{\hat{y} \in (\gamma^{j+1}(t_i), 0)} \cdot \sqrt{p_1^{j+1} \frac{\gamma^{j+1}(t_i)}{\gamma^{j+1}(t_i)} \hat{y}} \\
X_{\hat{y} < \gamma^{j+1}(t_i)} \cdot \sqrt{p_3^{j+1} (\hat{y} - \gamma^{j+1}(t_i)) + \frac{p_3^{j+1} \gamma^{j+1}(t_i)^2}{2 \gamma^{j+1}(t_i)^2}} \\
X_{\hat{y} < \gamma^{j+1}(t_i)} \cdot \sqrt{p_1^{j+1} \frac{\gamma^{j+1}(t_i)}{\gamma^{j+1}(t_i)} (1 - \frac{p_3^{j+1} \gamma^{j+1}(t_i)^2}{2 \gamma^{j+1}(t_i)^2})}
\end{bmatrix} \quad \text{with} \quad p_1^{j+1} = p_1^j \left[ \frac{\gamma^{j+1}}{\gamma^j} \right]^2 \quad p_2^{j+1} \geq p_1^{j+1} \left[ \frac{\gamma^{j+1}}{\gamma^j} \right]^2 \quad p_3^{j+1} \geq p_1^{j+1} \left[ \frac{\gamma^{j+1}}{\gamma^j} \right]^2.$$

### 3 Example

The method was implemented to generate an alternative target value of an identification process. Therefore, a simultaneous identification of the eight MORPH [2] material parameters with biaxial and uniaxial simulations [3] is used. While a pure identification without the bi-parabolic approach leads to a poor biaxial fitting, an iterative approach is used in comparison. The method was implemented to generate an alternative target value of an identification process. Therefore, a simultaneous identification to ensure a concurrent adjustment. This is superior to a typical penalty method, which will not be important which is given by industrial requirements or error information. This is also a possibility to support a simultaneous identification with an additional worsening because of the hardening of the developed border curves.

### 4 Conclusions

The shown method is an alternative to a normal identification process. In contrast to a pure fitting is the corridor more important which is given by industrial requirements or error information. This is also a possibility to support a simultaneous identification to ensure a concurrent adjustment. This is superior to a typical penalty method, which will not be $C^1$-continuous and can not be used as a linear residual. The development of the iterative bi-parabolic system can be directed by user defined rules. Further investigations should deliver automatic conditions.

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