Berry curvature and dynamics of a magnetic bubble

Wataru Koshibae 1,3 and Naoto Nagaosa 1,2

1 RIKEN Center for Emergent Matter Science (CEMS), Wako, Saitama 351-0198, Japan
2 Department of Applied Physics, The University of Tokyo, 7-3-1, Hongo, Bunkyo-ku, Tokyo 113-8656, Japan
3 Author to whom any correspondence should be addressed.
E-mail: wataru@riken.jp

Keywords: skyrmion, magnetic bubble, antiskyrmion

Supplementary material for this article is available online

Abstract

Magnetic bubbles have been the subject of intensive studies aiming to investigate their applications to memory devices. A bubble can be regarded as the closed domain wall and is characterized by the winding number of the in-plane components or the skyrmion number $N_{sk}$, which are related to the number of Bloch lines (BLs). For the magnetic bubbles without BLs, the Thiele equation assuming no internal distortion describes the center-of-mass motion of the bubbles very well. For the magnetic bubbles with BLs, on the other hand, their dynamics is affected seriously by that of BLs along the domain wall. Here we show theoretically, that the distribution of the Berry curvature $b_z$, i.e., the solid angle formed by the magnetization vectors, in the bubble plays the key role in the dynamics of a bubble with $N_{sk} = 0$ in a dipolar magnet. In this case, the integral of $b_z$ over the space is zero, while the nonuniform distribution of $b_z$ and associated Magnus force induce several nontrivial coupled dynamics of the internal deformation and center-of-mass motion as explicitly demonstrated by numerical simulations of Landau–Lifshitz–Gilbert equation. These findings give an alternative view and will pave a new route to design the bubble dynamics.

Magnetic textures in ferromagnets play vital roles in determining their physical properties. Domain walls (DWs) and bubbles are the representative examples of magnetic textures, and their dynamics is a keen issue from the viewpoints of both fundamental science and applications [1, 2]. The DWs are classified into Bloch wall and Néel wall, where the direction of the in-plane magnetization at the wall is parallel or perpendicular to the wall itself, respectively. From the viewpoint of the dipolar energy, Bloch wall is preferable since the magnetic charge $-\nabla \cdot M$ (where $M$ is the magnetization) is zero. However, it often happens that the Bloch lines (BLs) are inserted into the Bloch wall. At the BL, the local Néel wall is introduced to separate the two Bloch wall regions of opposite magnetic directions.

When the DW forms a cylinder, it becomes a magnetic bubble. In this case, BL has a topological meaning because it is related to the skyrmion number $N_{sk}$ of the bubble [3, 4]. Let $N_{win}$ be the vorticity of the bubble defined as the winding number of the in-plane components of the magnetization direction along the wall. Let $\Phi(\phi)$ be the angle of the in-plane magnetization along the wall measured from the $x$-axis, i.e., $M_x = M \cos \Phi(\phi)$, $M_y = M \sin \Phi(\phi)$ ($M = |M|$) at the direction angle $\phi$. (See figure 1(b) for the definitions.) Then the winding number $N_{win}$ is defined as

$$N_{win} = \int_0^{2\pi} \frac{d\phi}{2\pi} \frac{\partial \Phi(\phi)}{\partial \phi} = [\Delta \Phi].$$

On the other hand, the skyrmion number $N_{sk}$ is defined by the solid angle subtended by the magnetization directions $n = M/|M|$, i.e.,
Here \( \Phi \) is the gyromagnetic ratio and \( \alpha \) is the Gilbert damping constant. Starting with the LLG equation, Thiele [6–8] derived an equation of motion for the DWs, and the magnetic bubble motion is expressed by the equation,

\[
m \frac{d\mathbf{v}}{dt} + \mathbf{G} \times (\mathbf{v} - \mathbf{v}_0) + \mathbf{D}(\alpha \mathbf{v}_d - \beta \mathbf{v}_f) = -\nabla U.
\]  

(4)

Here \((X, Y)\) is the centre-of-mass coordinate of the bubble, \(\mathbf{v}_j = (X, Y)\) is its velocity and \(\mathbf{v}_j\) represents the spin current by the spin wave or the spin polarized electron current in the metallic system. Recent studies [9, 10] have revealed that the mass \(m\) of the skyrmion emerges with the deformation of the magnetic texture compared with the static solution. The gyrovector \(\mathbf{G} = (4\pi N_\mathbf{s})\mathbf{e}_z\) (\(\mathbf{e}_z\): unit vector along \(z\)-direction) relates the skew motion of the bubble and the bubble topology. The dyadic \(\mathbf{D}\) is composed of the dimensionless diagonal components being order of unity, \(\beta\) represents the coefficient of the non-adiabatic effect, \(U\) is the potential and \(-\nabla U\) is the force acting on the magnetic bubble, e.g., those from the boundary, gradient of the magnetic field, and the impurities.

The gradient of the magnetic field drives the magnetic bubble motion. However, just after turnoff of the driving force, motion of bubble which is not always understood by an inertia effect is often observed. To explain this interesting bubble motion beyond the Thiele equation (equation (4)), the BLs’ dynamics induced mechanism has been studied [1, 2, 11–17]. The generalized Thiele equation has been formulated by Slonczewski [17] to describe the motion of the DW of generalized shape and the effect of the gyrovector density. However, to solve Slonczewski equations is very difficult in general, and the discussion in these past is mostly in the high-\(Q\) limit where \(Q\) is the quality factor defined as \(Q = K/2\pi M^2\) (in CGS units) with uniaxial anisotropy \(K\). In this limit, the BL is considered to be the one-dimensional topological defects on the two-dimensional DW and the line dynamics on the cylinder plays a crucial role for the bubble motion.

In many magnets [18], the DWs and the BLs have substantial thickness and width compared with the bubble sizes. In those cases, the dynamics of the magnetic texture with spatial extent becomes an important issue to discuss the bubble motion. To characterize the local magnetic texture, we define the Berry curvature.
\[ b_y(x, y) = \frac{1}{2} n \cdot \left( \frac{\partial n}{\partial x} \times \frac{\partial n}{\partial y} \right) \]  

(This \( b_y \) is the integrand of equation (2), and also important for the transport properties in the metallic magnetic systems \([3]\).) Because \( G = 2 \iint b_y d^2 r \cdot e_y, b_y \) is the origin of the Magnus force (see equation (4)). Therefore, a local Magnus force is naturally expected by the local gyrovector \( 2b_y e_y \). In the present paper, we focus on the spatial distribution of \( b_y \), which is more basic quantity and directly linked to the local Magnus force acting on each part of the magnetic bubble.

**Results**

**Model and Simulation**

The model Hamiltonian for the dipolar magnets defined on the two-dimensional square lattice is given by

\[ \mathcal{H} = -J \sum_{r} n_r \cdot \left( n_{r+e_x} + n_{r+e_y} \right) \]

\[ + I_{dip} \sum_{r,r'} \frac{1}{|r-r'|^3} \left\{ n_r \cdot n_{r'} - \frac{3}{|r-r'|^2} \left[ (r-r') \cdot n_r \right] \left[ (r-r') \cdot n_{r'} \right] \right\} \]

\[ - K \sum_{r} \langle n_{z,r} \rangle^2 - H \sum_{r} n_{z,r} \]

where, \( I_{dip} \) and \( K \) represent the dipole interaction and the uniaxial anisotropy, respectively. The wavenumber \( q \) of the helix structure is given by \( q \equiv I_{dip}/J \) in the present case \([19]\). The evolution in magnetic texture is experimentally observed \([18]\), i.e., the helix ground state is seen at zero magnetic field and the magnetic bubbles emerge as the magnetic field is increased.

Using this model Hamiltonian equation (6), we solve the LLG equation (equation (3)) numerically and study the real-time dynamics of the skyrmion, the antiskyrmion and the type-II magnetic bubbles (see figure 1). We use \( 1/(\gamma J) \) for the unit of time \( t \). Typically \( J \sim 10^{-3} \text{ eV} \) and the unit \( 1/(\gamma J) \) becomes \( \sim 0.7 \text{ ps} \) for \( \gamma = g_{\mu_B}/\hbar \) (\( g_{\gamma} \): electron spin g-factor, \( \mu_B \): Bohr magneton).

Figure 2 shows the time evolution in the magnetic texture of the system which initially involves skyrmion and antiskyrmion, i.e., in figure 2(a), the left (right) one is the skyrmion (antiskyrmion) being topologically the same as the magnetic bubble shown in figure 1(a) (b)). (See also the supplementary movie.) A parameter set, \( \{ J = 1, I_{dip} = 0.09, K = 0.6, H = 0.026 \} \) and the disc-shaped system with a diameter of 150 lattice spacings (17,663 sites are involved) in free boundary condition are used. In the initial state (see figure 2(a)), skyrmion and antiskyrmion are arranged nearby the left and right edges, respectively. From the system edges, the skyrmion and the antiskyrmion are affected by the confining force. The confining potential induces the Magnus force parallel to the edge \([20–23]\) which is well described by the Thiele equation, and hence the skyrmion and antiskyrmion move upward direction along the system edge (see figures 2(a) \( \rightarrow \) (b)). As a result, the skyrmion and antiskyrmion collide and are merged into a type-II magnetic bubble (see figures 2(b) \( \rightarrow \) (c)). Note that in this dynamics, the skyrmion number \( N_{sk} \) is conserved in total. The resulting type-II magnetic bubble in figure 2(c) is topologically the same as the bubble shown in figure 1(c), i.e., the left (right) half of the DW structure is the same as that of figure 1(a) (b)). Later on, the type-II magnetic bubble shows the bounce dynamics at the edge of the system for a while like a pinball in a bowl and eventually, the bubble is stabilized at the center of the system.

It is noted that the Thiele equation (equation (4)) is not enough to understand the dynamics of type-II bubble observed in our numerical simulation. Instead, the domain structure of the distribution of \( b_y \), and the Magnus effect acting on each domain induced by both the external force and internal force are the key to analyze the complex behavior of type-II bubble with the zero net \( b_y \). For example, Thiele equation (equation (4)) predicts that the motion of the type-II bubble is expected to be parallel to the force because \( N_{sk} = 0 \) and Magnus effect is absent. The bounce back from the upper edge shown in figures 2(c) \( \rightarrow \) (d) \( \rightarrow \) (e) is apparently consistent with Thiele equation (equation (4)). However, it passes through the center of the system, and moves to the lower edge of the system against the confining force from the lower edge, and again shows the bounce behavior at the lower edge as shown in figures 2(e) \( \rightarrow \) (f) \( \rightarrow \) (g). This behavior is beyond the description of Thiele equation (equation (4)), and indicates that the internal force plays the essential role on the center-of-mass motion.

To investigate the dynamics in more details, we set the initial configuration of type-II magnetic bubble as shown in figure 3(b). (See also the supplementary movie.) To prepare this, we first stabilize a magnetic skyrmion at the center of the system (figure 3(a)), and apply an operation \( n_y \mapsto -n_y \) to the magnetic texture on the right-half of the system. The spatial distribution of \( b_y \) is topologically the same as the bubble shown in figure 2(c) where the singly connected \( b_y > 0 \) (\( b_y < 0 \)) segment at the DW is on the right-half (left-half) of the magnetic bubble. The spatial distribution of \( b_y \) represents the mixed winding segments with Bloch and Néel wall structures, where the latter reverses the direction of the winding. The initial state shown in figure 3(b) is not stable, and hence
induces the relaxation dynamics shown in figures 3(b)–(h). In the early stage, the positive (negative) $b_z$ segment is arranged in right (left) side of the type-II magnetic bubble, and the center-of-mass moves in the upper direction as seen in figures 3(c)→(d). As getting closer to the upper edge (see figures 3(d)→(e)), the negative $b_z$ segment emerges inside the positive one and the fourfold pattern appears. Succeedingly, the negative $b_z$ segment on the left disappears to result in the twofold pattern as shown in figures 3(e)→(f). (See also the supplementary movie). This process eventually results in the exchange of the arrangement of $b_z$, i.e., the positive (negative) $b_z$ segment is
in left (right) side of the bubble. When this exchange of \( b_z \) segments is achieved, the bubble makes a turn and moves in the lower direction (see figures 3(f)\textendash}(g)). Such bounce behavior is seen during the relaxation dynamics, and finally, the bubble is stabilized at the center of the system with the fourfold pattern of \( b_z \) (figure 3(h)). Note that the singly connected \( b_z > 0 \) and \( b_z < 0 \) segments are on the right and left hand sides to the moving direction of the center-of-mass.

Now we discuss the relation between the center-of-mass motion and distribution of \( b_z \). During the relaxation dynamics, the size of the magnetic bubble shrinks (see the difference in the bubble size between figures 3(c) and (h)). In other words, the internal shrinking force to the center of the bubble is acting on the DW at the perimeter. The Thiele equation generalized to the local region tells us that the Magnus effect appears on the magnetic texture perpendicular to the acting internal or external force when \( b_z \neq 0 \), which explains the bubble dynamics described above. In the early stage (see figures 3(c)\textendash}(d)), the positive (negative) \( b_z \) segment is arranged in right (left) side of the type-II magnetic bubble. Therefore, the positive and negative \( b_z \) segments have the drift velocity in the same upper direction due to the Magnus effect by the internal shrinking force as seen in figures 3(c)\textendash}(d). As approaching to the upper edge of the system, the effect of the confining force in the lower direction from the edge becomes strong. In the case figures 3(d)\textendash}(e), the Magnus force due to the confining force appears in opposite direction, i.e., it is left (right) for the positive (negative) \( b_z \) segment of the type-II magnetic bubble. This leads to the change in the arrangement of \( b_z \) from figures 3(d) to 3(f). Note that during the change in the arrangement of \( b_z \), the fourfold pattern appears once (see the supplementary movie). The separation and merge of the \( b_z \) segments determines the dynamics of BLs in the type-II magnetic bubble. In the fourfold pattern of \( b_z \), the net Magnus effect vanishes due to the cancellation between four segments, and hence the meta-stable final configuration shown in figure 3(h) is achieved at the center of the sample.

Next, we examine the bubble dynamics starting with the fourfold pattern as shown in figures 4(a) and (b). (See also the supplementary movie.) Note that it is similar to that in figure 3(h), but the bubble size is the same as that in figure 3(a). Namely, we apply the operations, \( (n_x, n_y) \rightarrow (-n_y, n_x) \) for \( \phi/4 < \phi < 3\pi/4 \), \( (n_x, n_y) \rightarrow (-n_y, -n_x) \) for \( 5\pi/4 < \phi < 7\pi/4 \), and \( (n_x, n_y) \rightarrow (-n_x, -n_y) \) for \( |\phi| < \pi/4 \) (mod 2\( \pi \)) to the magnetic skyrmion shown in figure 3(a).

Since the size of the bubble is larger than that of the meta-stable state, the shrinking force causes the Magnus force in the circumferential (\( \phi \)) direction, and the sign of \( b_z \) determines its direction, i.e., it is clockwise (counterclockwise) direction for \( b_z < 0 \) (\( b_z > 0 \)). This results in the elongation of the bubble as shown in figure 4(c) in the early stage of the relaxation dynamics. Along this elongation, the rotation in magnetic texture occurs. This is because the \( b_z > 0 \) segment of the DW costs an energy due to the magnetic dipolar interaction, and this asymmetry between the \( b_z > 0 \) and \( b_z < 0 \) segments of the DW drives the rotation dynamics. It is noted
that the shrinking relaxation dynamics in this case does not show any center-of-mass motion in sharp contrast to that in figure 3.

Discussion

In many cases, the bubble dynamics has been well described by the Thiele equation (equation (4)), and the skyrmion number which characterizes the topology of the magnetic texture is a key parameter. Another important concept is the BL dynamics, i.e., dynamics of the one-dimensional topological defects on the two-dimensional DW influences the bubble motion. Here we examine the dynamics of the Berry curvature $b_z$. Our numerical results clearly show that the more detailed information, i.e., the spatial distribution of the Berry curvature $b_z$ is the important factor. The local Magnus effect induced by both the external and internal forces acting on $b_z$ modulates the distribution of $b_z$ and also the center-of-mass motion. The results shown here are not reduced to the previous studies even in the high-$Q$ limit. In reality, many magnets show that the widths of the DWs and the BLs are comparable with the bubble sizes [18]. In those magnets, the dynamics originated from the Berry curvature $b_z$ is naturally expected. For example, optically excited bubble dynamics reported in [24] clearly shows the curvature induced $b_z$ and Magnus force under the spin current of magnons.

Now we apply this idea to the case of bubble dynamics under the magnetic field gradient. Because in the core of the magnetic bubble, the magnetic moments are antiparallel to the applied magnetic field perpendicular to the thin magnet, the magnetic bubble moves from higher to lower magnetic field region. The driving force is parallel to $-\nabla H$. This force causes the Magnus effect on the DW of the bubble, and prefers the twofold pattern of $b_z$ where the $b_z < 0$ ($b_z > 0$) segment is on the left (right) hand side to $-\nabla H$. This effect modulates the distribution of $b_z$. In addition, the bubble size is increased as it moves to the lower $H$ region, which causes the Magnus effect opposite to the direction of $-\nabla H$ and hinders the center-of-mass motion.

In the previous studies [1, 2], the magnetic field gradient driven bubble motion was extensively investigated. In particular, it was observed that the velocity of the magnetic field driven type-II magnetic bubble is smaller than that of skyrmion (normal bubble). Our theory explains the experiments described in [2] in a consistent way. We expect that the design of $b_z$ distribution and associated Magnus effect will open a new avenue to the bubble dynamics in the future.

Acknowledgments

This work was supported by Grant-in-Aids for Scientific Research (Nos. 24360036, 24540387, 24224009, 15H03553, 15H05853) from the Ministry of Education, Culture, Sports, Science and Technology (MEXT) of Japan, and ImPACT Program of Council for Science, Technology and Innovation (Cabinet Office, Government of Japan).

References

[1] Chikazumi S 2009 Physics of Ferromagnetism Int. Series of Monographs on Physics 94 (Oxford: Oxford University Press)
[2] Malozemoff A P and Slonczewski J C 1979 Magnetic Domain Walls in Bubble Materials (New York: Academic)
[3] Nagaosa N and Tokura Y 2013 Nat. Nanotechnol. 8 899
[4] Skyrme T H R 1962 Nucl. Phys. 31 356
[5] Koshibae W and Nagaosa N 2016 Nat. Commun. 7 10542
[6] Thiele A A 1973 Phys. Rev. Lett. 30 230
[7] Thiele A A 1974 J. Appl. Phys. 45 377
[8] Thiele A A 1976 J. Appl. Phys. 47 2759
[9] Schütte C, Iwasaki J, Rosch A and Nagaosa N 2014 Phys. Rev. B 90 174434
[10] Büttner F et al 2015 Nat. Phys. 11 225
[11] Slonczewski J C 1974 J. Appl. Phys. 45 2705
[12] Malozemoff A P and Slonczewski J C 1975 IEEE Transactions on Magnetics Mag-11 1091
[13] Malozemoff A P, Slonczewski J C and De Luca J C 1976 AIP Conf. Proc. 2958
[14] Argyle B E, Maekawa S, Dekker P and Slonczewski J C 1976 AIP Conf. Proc. 34 131
[15] Maekawa S and Dekker P 1976 AIP Conf. Proc. 34 148
[16] Malozemoff A P and De Luca J C 1978 J. Appl. Phys. 49 1844
[17] Slonczewski J C 1979 J. Magn. Magn. Mat. 12 108
[18] Yu X Z, Mostovoy M, Tokunaga Y, Zhang W, Kimoto K, Matsumi Y, Kaneko Y, Nagaosa N and Tokura Y 2012 Proc. Natl Acad. Sci. 109 8856
[19] Garel T and Doniach S 1982 Phys. Rev. B 26 325
[20] Sampaio J, Cros V, Rohart S, Thiaville A and Fort A 2013 Nat. Nanotechnol. 8 839
[21] Iwasaki J, Koshibae W and Nagaosa N 2014 Nano Lett. 14 4432
[22] Koshiba W, Kaneko Y, Iwasaki J, Kawasaki M, Tokura Y and Nagaosa N 2015 Jpn. J. Appl. Phys. 54 053001
[23] Iwasaki J, Mochizuki M and Nagaosa N 2013 Nature Commun. 4 1463
[24] Ogawa N, Koshibae W, Beekman A J, Nagaosa N, Kubota M, Kawasaki M and Tokura Y 2015 Proc. Natl Acad. Sci. 112 8977