The Next-to-Minimal Coleman-Weinberg Model

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**Abstract**

In the standard model (SM) the condition that the Higgs mass parameter vanishes is stable under radiative corrections and yields a theory that can be renormalized using dimensional regularization. Thus, this model allows to predict the Higgs boson mass. However, it is phenomenologically ruled out in its minimal version. Here, we present a phenomenologically viable, minimal extension which only includes an additional SM singlet and a $U(1)_X$ gauge symmetry.
The particle content of the standard model of elementary particle physics (SM) is minimal
in that it only contains particles that have already been observed plus one Higgs doublet
needed to break the electroweak symmetry. The Lagrangian describing the interactions of
the theory is obtained by forming all gauge-invariant and lorentz-invariant combinations
of fields with dimensions less than four. The coefficients are in general arbitrary factors
which have to be determined by experiment.

In order to obtain a renormalizable theory none of these terms can be omitted as they are
needed to cancel divergent contributions coming from quantum correction. The exception
are terms whose absence enhances the symmetry of the theory. An example is chiral
symmetry in the absence of a tree-level mass term for one or more fermions\[1\].

The potential of the SM contains only one parameter with dimension (mass)^2 [and none
with dimension (mass)], namely the Higgs mass parameter \( \mu^2 \). From this mass term
arises the most sever problem of the SM: the hierarchy problem\[2\]. Since the condition
\( \mu^2 = 0 \) is not protected by any symmetry in the SM\[1\] the large hierarchy between the
electroweak scale and the Planck scale \( \mu^2/M_P \approx 10^{-34} \ll 1 \) can only be achieved by
excessive fine-tuning.

Rather than to explain the smallness of \( \mu^2 \) it is may be conceptually more convincing to
assume \( \mu^2 = 0 \) altogether. The electroweak breaking in such a model can be achieved by
a negative Higgs self-coupling at some low scale \( \Lambda \) due to the renormalization effects of
the gauge couplings. In this model, with one parameter less than the SM ( i.e. \( \mu^2 = 0 \) or
\( \mu^2 \ll \Lambda^2 \)) the Higgs mass (in units of the Higgs vacuum expectation value) is determined
by the gauge and Yukawa couplings. This idea was first introduced by Coleman and Wein-
berg in ref. \[4\] were an upper limit on the Higgs mass (the CW bound) of \( m_h \lesssim 10 \text{ GeV} \)
and implicitly an upper limit on the top quark mass \( m_t \lesssim m_Z \) was established. Unfor-
tunately, both limits are by now in contradiction with experiment\[5\][6]. Nonthless, the
study of models with particular conditions for the Higgs mass parameters is of continued
interest\[7\][8].

In this letter, we will present a simple extension of the CW model that is still phenomeno-
logically viable. We assume that the Higgs mass parameter is generated dynamically as
the vacuum expectation value (VEV) of a singlet field \( S \). The tree-level potential of our
model without mass terms can be written as

\[
V_0 = \frac{\lambda_\phi}{2} (\phi^\dagger \phi)^2 + \frac{\lambda_S}{2} (S^\dagger S)^2 - \lambda_X (\phi^\dagger \phi)(S^\dagger S),
\]

where \( \phi \) denotes the SM Higgs doublet. This potential has an additional U(1)\( _X \) symmetry
that transforms \( S \rightarrow \exp(ia)S \). By promoting this global symmetry to a local symmetry
we introduce a gauge coupling \( g_X \) that can trigger spontaneous symmetry breaking. In
addition, we avoid the existence of a massless goldstone boson.

The \( \beta \) functions are

\[
16\pi^2 \beta_\phi = 16\pi^2 \beta_\phi^{SM} + \lambda_X^2,
\]

\[1\] Only in supersymmetric extensions of the SM can scalar mass terms be absent\[3\].
\[ 16\pi^2 \beta_s = \frac{5\lambda^2}{2} + 2\lambda_X^2 + \frac{3}{8}g_X^4 - 32\pi^2 \lambda_S \gamma_S, \]
\[ 16\pi^2 \beta_X = \lambda_X (3\lambda_\phi + 2\lambda_S - 2\lambda_X) - 16\pi^2 \lambda_X (\gamma_S + \gamma_\phi), \]
\[ 16\pi^2 \beta_{g_X} = \frac{1}{24}g_X^3. \]  

where the anomalous dimensions are \( \gamma_S = 3g_X^2/64\pi^2 \) and \( \gamma_\phi = (3g^2 + 9g^2 - 12h_t^2)/64\pi^2. \)

It is clear that the potential is flat in the direction \( \tan^2 \beta_0 \equiv \lambda_X/\lambda_\phi \) if

\[ \lambda_\phi \lambda_S = \lambda_X^2. \]  

In this case the VEV is determined by quantum corrections. Let us assume that \( \lambda_\phi \lambda_S - \lambda_X^2 > 0 \) at the Planck scale \( M_P \approx 10^{19} \). At a scale \( \Lambda < M_P \) the effective coupling are obtained by solving the renormalization group equations (RGEs) in eq. 2. We find that \( \lambda_\phi \) will converge for small \( \Lambda \) to its infrared (IR) fixed point, \( \lambda_X \) [assumed to be small] will only change very slowly with \( \Lambda \), and \( \lambda_S \) will continue to decrease for small \( \Lambda \) and sufficiently large \( g_X \). Thus, at some scale \( \Lambda_c < M_P \) eq. 3 will be satisfied.

In fig. 1 we present contours for which eq. 3 is satisfied (a) in the \( \alpha_X(M_P) - \ln \Lambda \) plane and (b) in the \( \lambda_S(M_P) - \ln \Lambda \) plane. We fix \( \lambda_S(M_P) = 0.1, \lambda_\phi(M_P) = 1, \) and \( \alpha_X(M_P) = 0.06 \) whenever these parameters are not varied. Furthermore, we set \( m_t = 176 \) GeV in all plots. If there are two scales \( \Lambda \) for which eq. 3 is satisfied then the larger (smaller) one corresponds to a minimum (maximum). This implies that there is an upper (lower) limit for \( \lambda_S (\alpha_X) \) as a function of \( \alpha_X (\lambda_S) \) for which the potential can have a minimum at the electroweak scale.
Without any mass terms in the potential it is convenient to transform to polar coordinates \( \phi \to \sin \beta r \) and \( S \to \cos \beta r \) where \( \tan \beta = \phi / S \). At the scale \( \Lambda_c \) the RG improved potential can be written as

\[
V_{\text{RG}} = r^4 \left[ \bar{v}_0 + \frac{\beta_r}{2} \ln \left( \frac{r^2}{\Lambda_c^2} \right) \right],
\]

with \( \beta_r = \beta \phi \sin^4 \beta + \beta_S \cos^4 \beta - \beta_X \sin^2 2\beta / 2 \) and \( \bar{v}_0 = \lambda_1 \cos^4 \beta (\tan^2 \beta - \tan^2 \beta_0)^2 / 2 \).

The Higgs mass matrix is given by

\[
M_{ij} = \left. \frac{dV_{\text{RG}}}{2d\phi_i d\phi_j} \right|_{r,\beta}, \quad v_1, v_2 = \phi, S,
\]

with \( r \) and \( \beta \) obtained by solving the minimum conditions \( dV/dr = 0 \) and \( dV/d\beta = 0 \). The Higgs mass eigenvalues and mixing angles are

\[
(m_{H/h})^{\text{RG}} = \frac{1}{2} \left( \text{tr}M^2 \pm \sqrt{\text{tr}^2M^2 - 4\det M^2} \right),
\]

\[
\sin 2\alpha^{\text{RG}} = \frac{2\mathcal{M}^2_{\phi S}}{\text{tr}M^2}
\]

Unfortunately, the minimization of the potential yields rather complicated expressions. The calculation can be simplified significantly by using the tree-level relation \( \alpha = \beta = \beta_0 \). This is justified by the observation that any mixing effect of the radial degree of freedom \( r \) with the remaining degree of freedom on the mass eigenvalues will only be of second order and can be neglected. This means that the lightest (heavies) mass eigenvalue \( m_h \) (\( m_H \)) is only determined by \( V_{\text{RG}}(V_0) \). Thus, we can derive the approximate RG improved Higgs mass

\[
\frac{dV_{\text{RG}}}{dr} = 2\beta_r r^3 \left[ \ln \left( \frac{r^2}{\Lambda_c^2} \right) + \frac{1}{2} + \frac{\bar{v}_0}{\beta_r} \right] = 0,
\]

\[
(m_{H/h})^{\text{app}}_{\text{RG}} = \frac{d^2V_{\text{RG}}}{2d^2r} = 3\beta_r r^2 \left[ \ln \left( \frac{r^2}{\Lambda_c^2} \right) + \frac{7}{6} + \frac{\bar{v}_0}{\beta_r} \right] = 2\beta_r r^2 = \frac{3\alpha_x^2}{8\sqrt{2}G_\mu}\cot^2 \beta \left[ 1 + O(\lambda_X/\lambda_\phi) \right].
\]

Here, \( \alpha_x = g_X^2 / 4\pi \) and \( G_\mu = (1.16639 \pm 0.00002) \times 10^{-5} \text{ GeV}^{-2} \) is the fermi-constant. Furthermore, we find that the Higgs vacuum expectation value \( r = \exp(-1/4)\Lambda_c \) is exponentially sensitive to all the input parameters. However, a prediction of \( r \) in terms of \( \lambda_i(M_P) \) \( (i = \phi, S, X) \) is not of interest for us but rather the value of the Higgs masses in units of \( r \).

The renormalization group approach is very well suited to evolve the effective theory over a large energy range and it provides in general very simple and transparent formulae. Nonetheless, an independent check via a more complete calculation is desirable. Here, we will use the one-loop effective potential given in dimensional reduction by

\[
V_1 = \frac{r^4}{64\pi^2} \sum_{\phi} N_\phi \overline{m}_\phi \left( \ln \overline{m}_\phi^2 - \frac{3}{2} - \Delta + \ln \frac{r^2}{\Lambda_c^2} \right).
\]
Here, $N_\phi$ denotes the number of degrees of freedom (with a $-$ sign for fermions), $\overline{m}_\phi$ stands for the masses of all the particles $\phi$ in units of $r$ and $\Delta$ parameterizes the regularized divergences. The one-loop divergences in eq. 9 are canceled by the divergences of the bare quantities. In a modified minimal subtraction scheme ($\overline{MS}$) the renormalized quantities are obtained from the bare quantities by subtracting only the pieces proportional to $\Delta$. Thus, we have to interpret the fields and couplings as bare quantities (with superscript 0) which are split into renormalized quantities (without superscript) plus counter-term, i.e.

$$\lambda_i \rightarrow \lambda_i^0 = \lambda_i + \beta_\lambda_i \Delta,$$

$$\Phi \rightarrow \Phi^0 = \Phi \left(1 + \frac{1}{2} \gamma_\Phi \Delta\right).$$

(10)

Here, $i = \phi, X, S$ and $\Phi = \phi, S$. Thus, the renormalized one-loop effective potential is obtained from eq. 1 by interpreting all couplings and fields as $\overline{MS}$ quantities and by setting $\Delta = 0$ in eq. 9. Note that we have removed all the one-loop divergences in dimensional regularization[12] without requiring a mass counter-term. The situation would be different had we used a cut-off scheme [see e.g. ref. [7]]. Clearly, if the cut-off is physical then a mass term of the order of the cut-off will be generated via radiative corrections in a non-supersymmetric model. However, if the only scale is $M_P$ and there is no new physics [except gravity which presently can not be consistently combined with a quantum field theory] then our results based on dimensional regularization will not be invalidated because it does not include a cut-off.

The minimization of the full one-loop potential is more complicated and can only be done numerically. The mass matrix is obtained from eq. 9 by replacing $V_{RG}$ with $V_{1L} = V_0 + V_1$. 

Figure 2: comparison of (a) $m_h$ and (b) $\sin \alpha$ and $\sin \beta$ as a function of $\lambda_X$ using different approximations.
Figure 3: Contours of constant (a) $m_h$, (b) constant $m_H$, (c) constant $\cot \alpha$ and (d) constant $m_B$ in the $\alpha_X-\lambda_X$ plane. In the shaded region the potential has maximum rather than a minimum and the region above the dashed curve is ruled out by non-observation of the process $Z \rightarrow hf \bar{f}$. 
A convenient analytic result can again be obtained by neglecting mixing effects. Following the procedure of eq. 7–8 we obtain

\[(m^2_h)_{1L}^{app} = \frac{r^2}{16\pi^2} \sum_\phi m^4 \phi, \tag{11}\]

We can easily check that the expressions for \((m^2_h)_{RG}^{app}\) and \((m^2_h)_{1L}^{app}\) are equivalent (they still differ numerically since in general the place of the minimum is different, i.e. : \(\beta_{RG} \neq \beta_{1L}\)).

The numerical comparison in fig. 2 reveals an excellent agreement of the various methods under investigation here. We present the prediction of \(m_h\), \(\sin \alpha\) and \(\sin \beta\) using a RG and one-loop effective potential approach and we compare \(m_h\) obtained by diagonalizing the \(2 \times 2\) mass matrix with the second derivative in radial direction (denoted by subscript \(app\); of course in this approximation \(\alpha \equiv \beta\)). Note that for values of \(\lambda_X < 0.002\) the one-loop mass corresponding to the field parallel to the VEV becomes larger than the tree-level mass corresponding to the field orthogonal to the VEV. Thus, for \(\lambda_X \gtrsim 0.002\) the mass-eigenstate \(h\) is predominantly the CP-even, neutral component of the SM Higgs doublet with \((m_h/r)^2 \propto \lambda_X^{-1}\). On the other hand, for \(\lambda_X \lesssim 0.002\) the mass-eigenstate \(h\) is predominantly the CP-even component of the SM Higgs singlet, \(S\), with constant mass \(m_h\). However, \((m_h)^{app}\) is defined as the one-loop mass of the Higgs boson in radial direction and will continue to rise with decreasing \(\lambda_X\) even if \(\lambda_X < 0.002\).

We will now determine the phenomenologically allowed region in parameter space. We assume that there is no new physics below \(M_p\) which implies that the SM Higgs self coupling is very close to its IR fixed point (we chose \(\lambda_\phi = 1\)). The Higgs singlet self coupling is determined by fixing the scale of electroweak symmetry breaking (i.e. \(\lambda_S = \lambda_X^2 \lambda_\phi\)). We note that the singlet couples to the SM particles only through mixing in the Higgs sector via \(\lambda_X\). This allows for a detection of the Higgs bosons via \(Z \rightarrow hf\bar{f}\) and \(Z \rightarrow Hf\bar{f}\). The branching fractions of \(h\) and \(H\) into fermions and gauge bosons are the same as in the SM except for the possible decay \(H \rightarrow hh\). However, the decay widths \(\Gamma(Z \rightarrow hf\bar{f})\) and \(\Gamma(Z \rightarrow Hf\bar{f})\) are suppressed with respect to the SM Higgs production by a factor \(\sin^2 \alpha\) and \(\cos^2 \alpha\), respectively.

In fig. 3 we present the particle spectrum of our model in the \(\alpha - \lambda_X\) plane in the RG approximation. The phenomenologically interesting quantities are the two CP-even Higgs masses [fig. 3 (a) and (b)], the corresponding mixing angle \(\cot \alpha\) [fig. 3 (c)] and the mass of the \(U(1)_X\) gauge boson \(m_B\) [fig. 3 (d)]. The analysis of LEP data in ref. [13] for the two Higgs doublet model is directly applicable to our case by replacing \(\sin^2(\beta - \alpha)\) in favor of \(\sin^2 \alpha\). The resulting limits are indicated in fig. 3. The area above the dashed curve is ruled out by non-discovery of a Higgs boson at LEP [13].

To summarize, we have investigated a model for spontaneous electro-weak symmetry breaking without mass parameters. In the SM this scenario is ruled out by the large top quark mass and we have demonstrated that it is still possible in models with an extended Higgs sector. The model under investigation here contains only an additional Higgs singlet and an additional \(U(1)_X\) gauge symmetry needed to break the symmetry and to absorb the related goldstone boson. The main idea of the model is that the generation of the
mass term via the Coleman-Weinberg mechanism occurs in a SM singlet sector and it is communicated to the SM via mixing in the Higgs sector parameterized by $\lambda_X$. The symmetry breaking is triggered by the $U(1)_X$ gauge symmetry. Note that there is not Yukawa-type coupling of fermions to the Higgs singlet $S$ that could spoil this mechanism. The breaking of $SU(2)_L \times U(1)_Y$ gauge symmetry in our model is not triggered by the Higgs self-coupling $\lambda_\phi$ turning negative at low scales as is the case in the CW model, but in the conventional fashion by a negative Higgs squared mass term $m_\phi^2 = -\lambda_X \langle S \rangle^2$. As a result, we find that the lightest Higgs boson mass is unconstrained as long as this mixing with the SM Higgs boson is small ($\lambda_X \lesssim 0.01$. The lower limit of the mass of the $U(1)_X$ gauge boson is $m_B \gtrsim 250$ GeV. The upper bound of the SM like Higgs boson is essentially the SM IR fixed point obtained in ref. [14] possibly enhanced by as much as 20 GeV due to mixing effects for large $\lambda_X$ and large $\alpha_X$.

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