Statistical Physics Models of Opinion Spread: Theory and Empirical Tests

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We investigate a set of discrete-spin statistical physics models for opinion spread in social networks and explore how suitable they are for predicting real-world belief change on both the individual and group levels. Our studies center on the degree and rate of consensus formation in inhomogeneous models that take into account different individual preferences or set beliefs. We investigate the effects of different social interaction rules (voter, majority, and expert rule), a variety of both artificial and realistic social network structures, different distributions of initial information, dependence on clustering of intrinsic preferences, the effects of having only two choices vs. multiple choices, and different weightings of social information input relative to intrinsic preferences. We find that the many combinations of model inputs results in a smaller, manageable set of consensus formation patterns. We then compare the predictions of these models to empirical social science studies, one done at MIT on 80 individuals living in a dorm during the 2008 presidential election season, and another conducted online by us with 94 participants during the 2016 presidential election primary season. We find that these relatively simple statistical physics-based models contain predictive value on both the individual and group levels. The results underscore the sensitivity of opinion spread to the underlying social network structure and the number of available belief options, and to a somewhat lesser (but still relevant) degree to the social interaction rules.

I. INTRODUCTION

One of the most active areas of application of statistical physics methods to problems outside of physics is to the study of belief spread in social networks (for a review, see [1]). Given a collection of autonomous entities (usually called “agents”), a set of states (corresponding to beliefs or behaviors) which the agents can choose among, and a specified dynamics (deterministic or stochastic or a combination) that determines how the agents update their states as time proceeds, one can study how beliefs propagate through a network depending on the details of the model chosen. While individuals are much more complex than atoms or spins, dynamical processes like opinion spread among large populations are expected to conform to certain statistical rules, and therefore studying statistical physics models of such social phenomena could illuminate their large-scale statistical aspects. Indeed, even very simple models such as the well-studied voter model [2–6] can already provide useful insights. The voter model belongs to a class of models in which agents can assume one of a small number of discrete states, in a fashion similar to Ising or Potts models of magnetic systems. More sophisticated variants include allowing agents to assume a state lying along a continuous range of options (see, for example, [7–9]), analogous to continuous-spin models in statistical physics; models in which the allowed states are represented by vectors rather than scalars [10]; and numerous others.

Although these models have uncovered a wide range of interesting behaviors, many of which are suggestive of phenomena observed in the social and political worlds, skepticism (in both the physics and social science communities) remains as to whether such models can be useful in analyzing, understanding, and predicting social phenomena in the real world. While some studies have compared results of these models to observed phenomena (see, for example, [11][15]), this literature remains relatively small and limited in three related ways. One limitation is that comparisons are typically done between patterns observed in the real world and patterns predicted by a single model. If the model can reproduce the real-world patterns, this is taken as support for its validity. However, similar real-world patterns can be produced by many different mechanisms of belief spread, and only by comparing predictions of several different models can one begin discerning which ones are better than others. A second limitation is that various potentially important model parameters, such as the social interaction rules, the initial conditions, or the network structures, are typically chosen based on a researcher’s best guess about what seems plausible, or by what makes a model analytically tractable, rather than based on some theory of human behavior or by actual measurement of these parameters in the real world. This motivates numerous explorations of the resulting vast parameter spaces without theoretical or empirical

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guidelines for what is actually occurring in the real world. A third limitation is that model predictions are compared with real-world data only on the group level, while it is possible that models that produce reasonable predictions of group-level patterns make poor predictions of an individual’s behavior. For instance, as we show later, dynamics of opinion spread in a group might appear to be similar to that of the voter model, but individuals might actually be using more sophisticated rules such as majority rule, expert rule, or a mix of those. To the extent that one wishes to understand actual human behavior for theoretical or applied reasons (e.g., planning public health interventions), knowing what mechanisms individuals actually use is very important.

The aim of this paper is twofold. Our primary objective is to directly compare results obtained from a large class of simple discrete-spin models of belief spread with data obtained through our own and other, publicly available, longitudinal surveys of small populations on a variety of political, social, and health issues. Of the models investigated, some (e.g., voter or majority rule with no internal fields) have been extensively studied in the literature, some (models with random internal fields) have been studied less extensively, and some (models with spatial clustering of random fields, and models with new types of network structures) are new. Typically one studies one particular model in isolation; in contrast, we compare results from a large variety of Ising or Potts-like models, looking for similarities and differences in behavior for a variety of model choices.

We choose model parameters based on findings from the literature on social learning in humans and other animals [17–19], and from empirical observations of real-world network structures and initial conditions. We determine how sensitive the process of belief spread is to the various model choices considered, and how much insight is gained when models are modified to include real-world features such as community-structured networks and correlations among agent internal preference fields. We use these results to predict both individual belief change and the resulting group-level patterns of belief spread, and compare these predictions with data obtained from measurements in surveys of real-world social networks. In so doing, these results help determine realistic model parameters (e.g., relative strengths of social pressures for conformity vs. intrinsic preferences, or the structure of actual networks used by the groups studied), with the aim of narrowing the set of models for use in predicting results obtained from future surveys.

As noted, we confine our analysis to simple scalar models with a small number of discrete states. Although more sophisticated models would be expected to better capture realistic behaviors, it remains important to determine how well and to what extent the (much more intensively studied) simpler models can not only explain, but also predict, large-scale statistical behaviors of belief spread and related social phenomena. To the extent this can be determined, this type of analysis can hopefully illuminate which behaviors can be understood via simple, robust, and universal model features, and which large-scale phenomena can only be described using more specific, complex, and detailed features. We emphasize that our main interest in this paper is less in the statistical physics behaviors of the various models studied, and more in the applicability of these models to actual social behaviors, as determined by comparison to experimental studies of opinion spread.

A secondary objective is to explore how the addition of inhomogeneities affects the dynamics of these models. Typically, one studies homogeneous models, in which the updating rules are the same for all agents (analogous to non-disordered systems from statistical physics), and the agents themselves have no preferential biases. This will already lead to complicated dynamics, with spatial differences and correlations arising during the evolution from the initial condition. Inhomogeneous models can add quenched disorder in the updating rules themselves, or in internal preferences (site inhomogeneities, modeled as random fields acting at the individual sites). Inclusion of inhomogeneities in different models has led to interesting observations, such as the “zealot effect” [20–23], and, in the case of heterogeneous and partisan voter models [24–30], anomalous dynamics and lowered consensus.

In this paper we will include the effects of site inhomogeneities, and uncover some interesting and perhaps surprising behaviors. Intuition based on the much-studied voter model, and to a lesser extent on majority rule models, leads to the expectation that degree of consensus formation and the rate of approach to the final state depend primarily on the social rule used (i.e., voter rule, majority rule, and so on) and that the effects of varying the number of possible opinion choices or the underlying social network structure have only minor effects. We will see, however, that there is a strong interaction between the number of opinion choices, the network structure, and the social rule employed, especially in the more realistic case when the internal preference fields are clustered.

The plan of the paper is as follows. In Sect. II we introduce the basic features of the statistical physics models that will be used to model opinion/belief spread in social networks. The ingredients include several different homogeneous social interaction rules (voter model, majority rule, and expert rule), a random field term acting on individual sites that corresponds to fixed internal preferences, a variety of different network structures, several possibilities for distribution of initial conditions, dynamical updating procedures, and the number of possible states that each agent can adopt. In Sect. III we compare the results of the different model variations, allowing a grouping into different classes of qualitatively similar behavior. In this way we can extract the effects of the different assumptions that go into the models, and determine which behaviors are sensitive to which assumptions and which are more robust. In Sect. IV we use the models to predict changes in opinions and behaviors of individuals and groups observed in two social science studies. The first is an older study done at MIT on 80 students living in a single dormitory, examining attitudes and behaviors regarding the (then-in-progress) 2008 presidential election, as well as several health-related behaviors. The second is a portion of a longitudinal study conducted online by us using Mechanical Turk to collect data on different beliefs of roughly 100 respondents over a period of a few months during the presidential primary season in early 2016. We conclude in Sect. V by discussing how the results thereby obtained can be used to construct statistical physics models...
of opinion spread that, while retaining a relatively simple structure, can nevertheless display both predictive power and shed illumination on the manner in which people use information from their social environments to construct their individual beliefs.

II. MODELS AND METHODS

A. Preliminaries

In this section we introduce the simplest version of the class of models to be studied; (slightly) more complicated models will be introduced in later sections when modifications become necessary. For now, though, we consider the case where each agent “lives” at one of the nodes $i$ of a graph $\mathcal{G}$, and its state at time $t$ is represented by an Ising spin $\sigma_i(t) = \pm 1$. That is, we begin by allowing only two possible choices for each agent — agree/disagree, Democrat/Republican, and so on — given a single proposition. The “satisfaction” of the agent at site $i$ is measured by a Hamiltonian-like function $H_i$, consisting of both an external, or social, term, and an internal, or intrinsic bias, term; lower values of $H_i$ correspond to higher levels of satisfaction. Specifically,

$$H_i = -h_i^{\text{eff}} \sigma_i = -h_i^{\text{soc}} \sigma_i - h_i \sigma_i. \tag{1}$$

A model of this type belongs to the general class of random field Ising models (RFIM’s), and allows for individual differences and biases among agents. The first term on the RHS of (1) models the overall effect of the social interactions of the agent at $i$, and the second term on the RHS represents the agent’s intrinsic predisposition with respect to the proposition being considered. Each agent will want to minimize his/her value of $H_i$, that is, it wants to align its $\sigma_i$ with its effective field $h_i^{\text{eff}}$. The overall satisfaction in an entire society consisting of $N$ agents is given by $H = \sum_{i=1}^{N} H_i$.

Specific models in this general class are determined by several choices, including specific representations of $h_i^{\text{soc}}$, the distribution from which $h_i$ is chosen, graph structure and connectivity, choice of initial condition, dynamical updating rules (in particular deterministic vs. stochastic), and others which will be discussed below. Perhaps the most straightforward (and often-used) representation of the social term is to treat the system as a ferromagnet, i.e.,

$$H_i = -\sigma_i \sum_{j=1}^{m} J_{ij} \sigma_j - h_i \sigma_i, \tag{2}$$

where $m$ is the number of neighbors of $\sigma_i$ (here a neighbor simply means an agent which directly communicates with $i$, i.e., a node connected to $i$ by an edge in $\mathcal{G}$); the (symmetric, for now) coupling $J_{ij} > 0$ denotes the strength of the interaction between $\sigma_i$ and $\sigma_j$, and the so-called intrinsic bias term $h_i$ acts as an individual field acting solely on $\sigma_i$. Both couplings and fields can vary as their indices change, or they can be uniform throughout a given sample. In either case,

$$H = \sum_{i} H_i = -\sum_{i \sim j} J_{ij} \sigma_i \sigma_j - \sum_{i} h_i \sigma_i, \tag{3}$$

where $i \sim j$ means that site $i$ is connected to site $j$ through an edge in the graph $\mathcal{G}$, i.e., the sum is over all edges in the graph. (In certain cases, such as the complete graph, the coupling term needs to be normalized by a factor depending on the number of nodes in the graph.) When the couplings are uniform ($J_{ij} = J$ independently of edge $(ij)$) (3) corresponds (at zero temperature, as will be discussed below) to the so-called “majority choice” models. However, we will also study other ways of representing the social term, in particular the voter model and expert models, neither of which conforms to the structure of (3).

B. The social interaction term

Perhaps the most crucial choice in specifying a model is the form of the social interaction term: it determines the social interaction rules that people are assumed to use to integrate others’ beliefs in their own belief structure. The possibilities are many, but the most often used (supported at least in part by studies on humans and certain animals \[31\] \[32\]) include aligning one’s beliefs with a majority of contacts, or with a randomly chosen contact, or with a favored person (such as a relative, best friend, or a perceived expert), or with a weighted combination of any of the above. Other rules, including more complicated Bayesian updating models, can be used as well. Which rule an individual actually uses depends on the nature of the issue and her individual predispositions, and can be inferred by asking people about their decision-making process \[33\], by fitting different social learning algorithms to their choice data \[34\], or by conducting laboratory experiments \[35\].

We discuss below three of the most frequently used rules for opinion spread in the statistical physics literature; one of our aims will be to compare the predictions of these different models with each other and with available social science data.
Voter models. These were introduced in the early 1970’s in the mathematical probability literature [2-4, 36] as an interesting statistical model in its own right, but in the past few decades has also been intensively studied as a simple model for opinion spread [1-6, 24-30]. In this model, an agent updates her opinion by choosing one of her neighbors uniformly at random from her connected sites and simply adopts that neighbor’s state (this corresponds to a social learning strategy known as unbiased copying [31]). The probability that the chosen spin updates its state (when $h_i = 0$) is therefore

$$P(\sigma_i^+ = -\sigma_i) = \frac{1}{2m} \sum_{j=1}^{m} |\sigma_i - \sigma_j|$$

where a $+$ superscript denotes the state of a spin immediately after updating and no superscript indicates the state of the spin immediately prior to updating, and $m$ as before is the number of neighbors of $\sigma_i$. In this model, the social field is given simply by

$$h_{i}^{soc} = \sigma_j$$

where $\sigma_j$ is the chosen neighbor.

The voter model has received a large amount of attention in the literature, and is well understood. One result relevant to this paper is its ability to reach consensus (in the absence of all other fields); moreover, because the magnetization is conserved [36], the probability of reaching the uniform $+1$ state is equal to the density of $+1$ spins in the initial configuration.

Majority rule models. In these models, the chosen spin adopts the state of a majority of its neighbors (or else a subset of the neighbors). This is equivalent to the ferromagnetic Ising model given by (2) when its evolution is governed by zero-temperature Glauber dynamics (i.e., the dynamical rule discussed at the end of Sect. II F with $\beta = \infty$). If the internal field $h_i \neq 0$, which will usually be the case, there is no chance of a tie. For those runs in which we set $h_i = 0$, then in the event of a tie a fair coin is flipped to determine the updated value of the chosen spin. This rule corresponds to conformism in the social learning literature [19, 31] and the majority voting rule in political science [37].

Because the majority rule doesn’t care about the strength of the majority, when $h_i \neq 0$ and/or temperature is greater than zero Eq. (2) for $h_{i}^{soc}$ must be replaced by

$$h_{i}^{soc} = \text{sgn} \left( \sum_{j=1}^{m} \sigma_j \right)$$

where the sum is over the neighbors of the agent at $i$. In our case, if the number of neighbors is greater than three, we set $m = 3$ and choose three neighbors uniformly at random. In principle this makes it more likely that the system can find states that may otherwise be hidden; however, in our actual runs (detailed in Sect. III below) we find little difference in strongly connected graphs between choosing a subset of neighbors and choosing all neighbors.

Majority rule was studied in the context of opinion dynamics by Krapivsky and Redner [38]. Their rule is slightly different from ours: they select a connected cluster of $M$ spins ($M$ odd, avoiding the possibility of a tie) and all spins then take on the value of the majority at once. Despite this difference, we expect that the general qualitative features of their model and the one considered here are largely the same.

Expert rule models. Here an agent adopts the belief (again assuming $h_i = 0$) of one specific neighbor considered by that agent an expert on the issue (or that otherwise has characteristics that make her/him the favored contact). From a modeling perspective, this category of social learning strategies is similar to the voter model, but differs in implementation in that the influential node for each agent is chosen randomly at the start and is thereafter fixed. Because the favored node is chosen uniformly at random for each agent, there is nothing preventing several individuals from using the same “expert”. Similarly to the voter model, the social field here is simply

$$h_{i}^{soc} = \sigma_e$$

where $\sigma_e$ denotes the state of the chosen expert.

In the simulations that follow, we assumed that participants apply the social interaction rules described above on a random sample of three of their neighbors. In this way we can isolate the effect of a specific network structure from the number of neighbors that agents have in different networks.

C. The internal field term, connection strengths, and the overall effective field

The actual updating of the internal state of a chosen spin depends on the interplay between the social term and the individual preference term governed by $h_i$. These intrinsic predispositions can be measured directly by survey questions (as we will do in
The magnitudes of the couplings measure the relative influence for each agent of the influence of the agent’s social network vs. the pull of her intrinsic tendencies. The ratio between this value and the average value of the connection (i.e., coupling) strengths is of central importance: it measures the relative influence for each agent of the influence of the agent’s social network vs. the pull of her intrinsic tendencies. The magnitudes of the couplings $J_{ij}$ can be measured empirically, for instance through sociometric surveys or by automatically recorded data about social contacts [39][40]. They can also be modeled to represent real-world structures, for example using stochastic block models or small world network algorithms.

While physical models of random-field magnetic systems typically assume a uniformly random spatial distribution of random fields, social networks are very different. In the U.S., for example, the clustering of like-minded individuals into different geographical regions or internet groups is understood to be an important factor in determining election outcomes, among many other political and social events. In order to disentangle the effect of spatial clustering from other factors, we will consider both the cases of a uniformly random distribution and of spatial clustering of intrinsic preferences.

As already noted, not all of the models employed in the literature or analyzed below fit neatly into the simple Hamiltonian-like formulation of (2). For this reason, we have found it convenient to introduce a new parameter $0 \leq \alpha \leq 1$, which is the relative weight between the influence of an agent’s social network and his individual preferences. (In the real world, $\alpha$ will likely vary from agent to agent, but because the $h_i$’s already vary, introducing a variation in $\alpha$ adds an additional layer of complication that is unnecessary at this stage.) A more general formulation than (2) (but which includes it as a special case) for an individual’s overall energy (or, in social science terminology, social-cognitive dissonance) regarding a particular belief is then

$$H_i = -\alpha \sigma_i h_{i}^{soc} - (1 - \alpha) \sigma_i h_i$$

where $\sigma_i$ is individual $i$’s belief state regarding a particular question; $h_{i}^{soc}$ is the social field resulting from $i$’s contacts, calculated using one of the different social learning rules described above; and $h_i$ is $i$’s (time-independent) intrinsic predispositions to particular opinions or behaviors.

### D. Social network structure

Up until now we have focused on the social interaction rules used to update the system at each time point. Of equal importance is the type and connectivity of the network on which the agents reside. Unlike the ordinary statistical mechanics of spin systems, in which networks are usually either finite-dimensional Euclidean lattices or else infinite-range mean-field type structures, we need to examine here a variety of different networks. In Sect. III we will study the updating rules described above on five different network structures that were used in previous studies of belief spread. These include the ring lattice and the fully connected graph, each of which in principle (at least for a sufficiently simple Hamiltonian) enables analytic solutions of a system’s dynamics. In addition, we consider three networks that have been shown to resemble some aspects of real-world social networks: the small world network, and two more networks comprising a number of weakly or strongly interconnected communities, generated by stochastic block models. These are shown in Fig. [1] below.

In the ring lattice (Fig. 1a), each individual is connected to $m$ immediate neighbors. In the small world lattice (Fig. 1b), each node is connected to $m$ immediate neighbors, and also to other nodes with probability $r$ independently for each remaining node. In the fully connected lattice (Fig. 1c), each node is connected to every other node. We compare these more stylized networks with more realistic networks created by stochastic block models. In weakly connected artificial communities (Fig. 1d), individuals are organized in social circles of size $m$; in each circle, one individual is connected to an individual from another circle, leading to at most one connection between any two circles. In strongly connected artificial communities (Fig. 1e), individuals are organized in social circles of size $m$; in each circle, $m/2$ individuals are connected to other circles, leading to multiple connections between any two circles.

For all networks, the average $m$ was chosen to be six. Detailed properties of each network (including two to be introduced in Sect. IV) are given in Table I.

### E. Initial conditions for opinion distributions

An important choice in developing a statistical physics model of opinion spread is determination of type of initial condition: what is the initial frequency distribution of individuals holding different beliefs at the outset? It is also interesting to consider their spatial distribution; here we consider only a uniform distribution of initial beliefs.
We consider both uniform (e.g. 50-50) and nonuniform (e.g. 70-30 and 90-10) distributions of both initial beliefs. We examine combinations of these distributions of beliefs with similar distributions of intrinsic preferences, the latter including uniform (drawn from \(N(0, 1)\), the normal distribution with mean zero and variance one) and nonuniform distributions. This enables us to model situations in which the new belief is rare, and there are few individuals in the population who are intrinsically predisposed to accept it, as well as those in which the new belief is rare, but many in the population are predisposed to accept it. We also consider situations in which intrinsic predispositions are all neutral \((h_i = 0)\).

We start by considering Ising-type models with two possible beliefs, and later turn to Potts-type models in which there are four or five possible beliefs instead of two. These will be introduced and discussed in Sect. III B.
Finally, we need to specify whether information transfer is perfect or noisy and therefore subject to error. We incorporate this by introducing an adjustable parameter $\beta \geq 0$, which as in statistical physics models behaves as an inverse “temperature” of the system. Here $\beta$ represents the reliability of information transfer, or accuracy of an agent’s perception of the beliefs of her neighbors. This reliability can be low (small $\beta$) for issues where social information is not easily observable, or when people’s beliefs about an issue are prone to random fluctuations (e.g., because they are weakly held or not yet well established). The reliability can be higher when it is clear what others are thinking or doing, and when people’s beliefs are strong and well established. For the purpose of simplicity, in all simulations that follow we set $\beta = 100$, which for modeling purposes is effectively the same as zero temperature; that is, each agent has essentially perfect information about his or her neighbors’ opinion states.

Once $\beta$ is determined, Eq. (8) is used to update the states of the agents. We will employ asynchronous updating, in which each agent is assigned its own independent Poisson clock, all with rate $\lambda$. When an agent’s clock rings, it evaluates its local energy $H_i$. If it can lower its energy ($\Delta H_i < 0$) by changing its state, it does so with probability one. If changing its state raises its energy ($\Delta H_i > 0$), it does so with probability $[1 + \exp(\beta \Delta H_i)]^{-1}$. (When the $h_i$’s are chosen from a continuous distribution, the probability that $\Delta H_i = 0$ is zero.)

G. Implementation

We implemented each combination of updating rules, network structures, number of possible beliefs, initial frequency distributions, and initial spatial distributions in a separate simulation. Each simulation was run for at least 30 time steps. At each time step, individuals are activated in random order using their Poisson clocks as described above. The chosen individuals observe their neighbors, as determined by the network structure, and update their beliefs using the updating rule of the particular version of the simulation. At each step, we calculate the consensus level, or the proportion of individuals that hold the most frequent belief. These results are averaged across 100 replications. This was sufficient to achieve stable results under all conditions. We assumed that every individual is subject to changing his or her belief at each time step.

An additional comment about the rate $\lambda$ chosen for the Poisson clocks is in order. The assumed rate $\lambda$ of belief updating determines how many agents consider changing their beliefs during each time period. While the rate of updating does not make a difference in the long run, it matters when trying to predict and explain belief dynamics on the shorter time scales that are typically relevant to the real world. While in statistical physics models many agents change their initial states, at least in the early part of a run, in the real world it is often reasonable to assume that only a relatively small proportion of individuals change their beliefs at any given time. The updating rate is likely to be different for different beliefs and individuals, but it can be estimated from empirical data. We will discuss this further below in Sect. III

III. COMPARISON OF DIFFERENT MODELS

In the previous section we characterized a variety of models of opinion spread, a number of which have previously been discussed in the literature. In this section we compare the results of simulations of these models, with the goal of extracting the effects of different underlying assumptions and determining the sensitivity of certain behaviors to these assumptions. By distilling these results into “universality classes” (roughly speaking) of model behaviors, we simplify the task of comparing theoretical models to experimental data, which will be taken up in the next section.

The first division is between models in which only two choices or opinions are available for the agents (Ising-like models) and those in which there are multiple (in our case, four or five) choices available (Potts-like models). For each class of models, we consider in Eq. (8) the two extremes of $\alpha = 1$ and $\alpha = 0$, where $\alpha = 1$ corresponds to social influence only (no internal preference fields) and $\alpha = 0$ to internal preference only (no social interactions). We then consider patterns for values of $\alpha$ strictly between 0 and 1. Here we make the next division, separating models in which internal preference fields are randomly distributed and those in which they are clustered into “like-minded communities”. Note that this division is irrelevant for situations when $\alpha = 0$ and $\alpha = 1$.

For each value of $\alpha$, we study the dynamics of consensus formation for different initial conditions, social rules, and network structures. We find that there is a crossover at $\alpha = 0.5$; for $\alpha < 0.5$, the qualitative features observed at $\alpha = 0$ dominate the results, and for $\alpha > 0.5$ those observed at $\alpha = 1$ dominate the results. The crossover value of $\alpha$ will depend on the relative strengths of the internal preference fields (determined by the width of their distribution) and that of the social couplings; in our case $\alpha = 0.5$ is the crossover because we took these two strengths to be equal. The most interesting behavior occurs at values of $\alpha$ very close to 0.5, where one finds qualitative behavior different from either extreme. Consequently, we present results for $\alpha = 0.49, \alpha = 0.5$, and $\alpha = 0.51$ in the main text, and relegate results for all other values of $\alpha$ to the supplement.
Given the large number of models investigated, we first provide two overview tables to help guide the reader in navigating the results obtained in this section. Table II provides an overview of main results for two-opinion models, and Table III does the same for multi-opinion models. The tables include a brief summary of main results, which are presented and discussed in detail in Sect. III A and Sect. III B and summarized again in Sect. III C.

A. Two-opinion models

We begin by studying the simplest case of two opinions or beliefs. This is the situation (though usually without the $h_i$'s) most often studied in the statistical physics literature. As noted, in all simulations we set $\beta = 100$, where transmission of information between agents is essentially error-free.

1. Only social interactions: $\alpha = 1$

We first consider the effects of the social term and internal field term separately. Recall from the previous section that the parameter $\alpha$ determines the relative weights of the social term and internal field term (cf. Eq. (8)). By setting $\alpha = 1$, we therefore are examining the social term only. In all cases the two behaviors of interest are the degree of consensus achieved (with 0 denoting a 50-50 split among the population and 1 denoting perfect agreement), and the rate of convergence to the final state. We therefore introduce a consensus function $C(t)$ given by $C(t) = (1/N)|N_1(t) - N_2(t)|$, where $N$ hereafter denotes the total number of agents.

![Consensus vs. time](image)

**FIG. 2**: Consensus vs. time when $\alpha = 1$ for different social interaction rules and different network types, for the two-opinion case. Here consensus is given by $C(t) = (1/N)|N_1(t) - N_2(t)|$, as discussed in the text. The first row uses a well-mixed initial distribution in which each $\sigma_i = \pm 1$ with equal probability (which we call 50-50 in the text), independently of the others. The second row uses the initial condition where $\sigma_i = +1$ with probability 0.7 and $-1$ with probability 0.3 (70-30), and the third row uses the initial condition where $\sigma = +1$ with probability 0.9 and $-1$ with probability 0.1 (90-10). In all cases $N = 90$ and each curve represents an average over 100 runs.

We begin by studying the simplest case of two opinions or beliefs. This is the situation (though usually without the $h_i$'s) most often studied in the statistical physics literature. As noted, in all simulations we set $\beta = 100$, where transmission of information between agents is essentially error-free.
| Spatial distribution of initial opinions | Section | Weight of social influence | Frequency distribution of initial opinions | Alignment of internal field distribution with initial conditions | Figure/row | General consensus dynamics | Effects of social interaction rules | Effects of network structures |
|----------------------------------------|---------|---------------------------|--------------------------------------------|-------------------------------------------------|------------|--------------------------|---------------------------------|---------------------------------|
| mixed | III.A.1 | \( \alpha = \frac{1}{5} \) | 50-50 | not relevant | 2/1 | Rate of convergence and final level of \( C \) depend on network structure and social interaction rule. | Majority and voter often reach full \( C \). Expert reaches full \( C \) only on complete graph. Voter is slowest to converge. | Strong interaction between network structure and both majority and expert rule. Weak interaction between network structure and voter rule. |
| | | | | | | | | Similar patterns, but faster convergence. | |
| mixed | III.A.2 | \( \alpha = \frac{1}{5} \) | 50-50 | aligned | | \( C \) lowest for \( 0 < \alpha < 0.49 \), highest for \( \alpha > 0.51 \). | No effects of rules and networks. | No effects of rules and networks. |
| | | | | | | \( C \) first drops, then rises. | | |
| mixed | III.A.3a | \( 0 < \alpha < 1 \) | 50-50 | aligned | 4-6/1 | \( C \) highest for \( 0 < \alpha < 0.49 \), lowest for \( \alpha > 0.51 \). | For \( \alpha < 0.5 \), majority rule strongest in countering internal field preferences. Majority rule leads to highest \( C \) and shows fastest convergence, expert lowest \( C \). | Complete graph most likely, weakly connected community least likely to reach full \( C \). |
| | | | | | | \( C \) first drops, then rises. | | |
| clustered | III.A.3b | \( \alpha = 0 \) & \( \alpha = \frac{1}{10} \) | both | both | 8-10/1 | Results the same as for mixed spatial distribution. | | |
| clustered | III.A.3c | \( 0 < \alpha < 1 \) | 50-50 | aligned | 8-10/2 & 4 | Similar to mixed spatial distribution, but lower \( C \) and slower convergence. | Majority typically leads to highest \( C \), expert to lowest \( C \); but in weakly connected communities with \( \alpha < 0.5 \) majority achieves lowest \( C \). | Compared to mixed case, network effects more pronounced, especially at low values of \( \alpha \). |

**TABLE II:** Overview of main results for two-opinion models. As noted in the text, a symmetric initial condition distribution for multi-opinion models means that the distribution of initial opinions is symmetric about zero. “Alignment of internal field distribution with initial conditions” means the following: if the initial condition is 80% of agents in state \( +1 \) and 20% in state \( -1 \) and the internal field distribution is the same, we say they are aligned. For the same initial distribution, if 20% of internal fields favor \( +1 \) and 80% favor \( -1 \), we say they are not aligned; and similarly for other distributions. Note that this is purely a global criterion — internal fields and initial conditions are always assigned independently of each other. See Sect. III.A for more detailed presentation and discussion of all results.
| Spatial distribution of initial opinions | Section | Weight of social influence | Frequency distribution of initial opinions | Alignment of internal field distribution with initial conditions | Figure/row | General consensus dynamics | Effects of social interaction rules | Effects of network structures |
|----------------------------------------|---------|---------------------------|-----------------------------------------|-------------------------------------------------|-------------|----------------------------|-------------------------------|----------------------------------|
| mixed                                  | III.B.1 | α = 1                     | symmetric                                | not relevant                                     | 12-13/1     | For 4 opinions, results similar to 2-opinion case. For 5 opinions, results different for $C_4$ (majority achieves lowest $C_4$), and similar to 2-opinion case for $C_3$. For both C1 and C3, results similar to 2-opinion case. | | |
|                                       | III.B.2 | α = 0                     | symmetric                                | aligned                                         | 14-15/1     | $C_4$ similar to 2-opinion case. $C_3$ behaves slightly differently. | | |
|                                       |         |                           | asymmetric                               | aligned                                         | 14-15/4     | | | |
|                                       |         |                           |                                            | not aligned                                      | 14-15/2,3,5 | | | |
|                                       | III.B.3a| 0 < α < 1                 | symmetric                                | aligned                                         | 16-17/1     | Up to α = 0.5, results similar to α = 0, differences more pronounced with $C_4$. From α > 0.5, results similar to α = 1. Results for 4- and 5-opinion case similar. | For α < 0.5, few effects of rules. For α > 0.5: majority reaches highest $C$, expert generally lowest. | Differences due to network structure emerge only for α > 0.5: $C$ highest for complete graph, lowest for weakly connected community. |
|                                       |         |                           | asymmetric                               | aligned                                         | 16-17/4     | | | |
|                                       |         |                           |                                            | not aligned                                      | 16-17/2,3,5 | | | |
| clustered                              | III.B.3b| α = 1 & α = 0              | both                                    | both                                            | 18-19/1     | Results the same as for mixed spatial distribution. | Results similar as for mixed spatial distribution. | Differences due to network structure emerge from α = 0.5, more pronounced for $C_4$. |
|                                       |         |                           | symmetric                                | aligned                                         | 18-19/4     | Up to α = 0.49, results similar to α = 0, differences more pronounced with $C_4$. From α > 0.5, results similar to α = 1. Results for 4- and 5-opinion case similar; compared to 2-opinion case, beliefs easier to change and $C$ higher. | | |
|                                       |         |                           | asymmetric                               | not aligned                                      | 18-19/2,3,5 | | | |

TABLE III: Overview of main results for multi-opinion models. See Sect. III B for more detailed presentation and discussion of all results.
number of agents, and \( N_1(t) \) \((N_2(t))\) denotes the total number of agents in state 1(2) at time \( t \). (In physics terms, the consensus function is simply the absolute value of the magnetization per spin.)

In Fig. 2, we show the results of simulations for the three different social rules with three (well-mixed) initial conditions, 50-50, 70-30, and 90-10. Because most of the interesting behavior occurs at relatively short times, all of the figures shown in this section focus on these short times, typically up to \( t = 30 \). Because of this, a number of curves shown have not yet reached a final consensus value. Runs taken out to \( t = 900 \), where all curves converge to a final consensus value, are shown in the supplement (Figs. S1) \[41\]. The long-time behavior of these models will also be discussed in what follows.

For an initial distribution of 50-50, we see that network structure is less important for voter rule dynamics than for majority or expert rule. This is hardly surprising, given that the basis for updating an agent’s state in the voter rule chooses a single spin at random from the agent’s connections, so that the details of network structure are not very important — all that matters is the number of connections of each agent.

This is less true for expert rule, where the fully connected lattice (i.e., complete graph) in particular shows a much stronger tendency than other network structures to reach a high degree of consensus. Recall that in expert rule, each agent’s reference node for updating remains fixed. As a result, given that in the complete graph every agent is connected to every other, there’s a much greater chance (compared to locally connected networks) that “experts” will be shared among multiple agents. In contrast, in locally connected networks, only a handful of spins can share the same expert, so the potential for disagreement is greater.

In locally connected graphs, where geometry plays a role, expert rule can display a peculiar phenomenon absent in the other two dynamical rules. Because experts are chosen at the beginning for each agent and are thereafter fixed, sufficiently large graphs will inevitably display “closed expert loops”: A’s expert is B, B’s expert is C, and so on to Z, whose expert is A. A closed loop can be as small as 2: A and B are each other’s experts. A closed loop will end in a final state of everyone within the loop agreeing, but which state that will be depends on the initial conditions and the dynamical realization. Hence locally connected graphs will expert rule will display a large degree of metastability with many 1-spin-flip stable states, thereby lowering the overall consensus obtained.

The majority rule exhibits perhaps the most interesting dynamics. Consensus converges toward one for all networks except weak community, which settles at a relatively low rate of consensus (cf. Figs. S1 in the supplement \[41\]). Recall that in the weak community structure, all agents are clustered into small self-reinforcing networks which we call “social circles”; each with only a single connection to an agent in another circle. Because of the very weak coupling of each social circle to agents outside the circle, with a 50-50 initial condition a given social circle is roughly equally likely to “freeze” into either the \(+1\) or \(−1\) state. That is, there is a large degree of metastability for majority rule that is absent in, say, the voter model. This tendency of majority rule dynamics in which the consensus freezes at relatively low values for certain networks, particularly the weak community structure, will be seen to persist in many of the different situations considered in this section. For other lattices, one expects convergence to full consensus, which is indeed observed to occur. In fact, this result can be proved for the complete graph (Appendix, Sect. \[VIA\]).

The above discussion focused on a distribution of 50-50. At the (almost) opposite end, i.e., an initial distribution of 90-10 (third row of Fig. 2), all network structures for both voter and majority rule dynamics reach full consensus relatively quickly (the voter model as usual displaying the slowest convergence). Expert rule as usual achieves lower consensus; here only the complete graph achieves full consensus, while all others achieve a final consensus value of roughly 0.8. These behaviors are again understood by invoking the same reasoning as that used above. The remaining case is the intermediate one of 70-30 (second row of Fig. 2); here the behavior is also intermediate between 50-50 and 90-10. It appears that the behavior of all of the dynamical rules and networks smoothly varies as the initial ratios of opinion vary from 50-50 to 90-10 (and beyond).

2. **Only internal fields: \( \alpha = 0 \)**

We now turn to the case \( \alpha = 0 \), where social interactions have been turned off. This case is exactly soluble analytically, so simulations are not needed (except as a consistency check). Because the social term is absent, the graph on which the agents sit is irrelevant; all that matters are the initial conditions.

When \( \alpha = 0 \), the Hamiltonian is simply \( \mathcal{H} = -\sum h_i \sigma_i \). The zero-temperature process for the case of two possible opinions can then be modeled as a 4-state Markov chain. Let \( N_{11}(t) \) denote the number of agents in state 1 at time \( t \) and with \( h_i = +1 \) (i.e., they prefer option 1); \( N_{12}(t) \) is the number of agents in state 1 at time \( t \) but with \( h_i = -1 \) (they prefer option 2); and similarly for \( N_{21}(t) \) and \( N_{22}(t) \). The \( h_i \)’s, which are fixed in time, are uniformly distributed Bernoulli random variables, and the initial states \( N_{11}(0), N_{12}(0) \), etc., are also uniformly distributed according to some initial distribution. Of course, \( N_{11}(t) + N_{12}(t) + N_{21}(t) + N_{22}(t) = N \), independent of time.

If an agent is in the state \( \{11\} \) or \( \{22\} \) at any time, it thereafter remains in that state. If it’s in the state \( \{21\} \), then when its Poisson clock rings it changes its state from \(-1\) to \(+1\) with probability \( p \); similarly, if it’s in the state \( \{12\} \), it changes its state from \(+1\) to \(-1\) with probability \( q \).
The dynamical equations for the chain are therefore (in a discrete time formulation)

\[
\begin{bmatrix}
N_{11}(t) \\
N_{12}(t) \\
N_{21}(t) \\
N_{22}(t)
\end{bmatrix}
= \begin{bmatrix}
1 & 0 & p/N & 0 \\
0 & 1 - q/N & 0 & 0 \\
0 & 0 & 1 - p/N & 0 \\
0 & q/N & 0 & 1
\end{bmatrix}
\begin{bmatrix}
N_{11}(t-1) \\
N_{12}(t-1) \\
N_{21}(t-1) \\
N_{22}(t-1)
\end{bmatrix}.
\]  

(9)

Letting \( \mathbf{N}(t) \) denote the 4-component state vector at time \( t \) and \( \mathbf{A} \) the \( 4 \times 4 \) Markov transition matrix, Eq. (9) can be written compactly as \( \mathbf{N}(t) = \mathbf{A} \mathbf{N}(t-1) \); it follows that \( \mathbf{N}(t) = \mathbf{A}^t \mathbf{N}(0) \). This set of equations is easily solved by diagonalizing \( \mathbf{A} \); rescaling \( t \) by \( Nt \) and letting \( N \to \infty \), the solution is

\[
\begin{align*}
n_{11}(t) &= n_{11}(0) + n_{21}(0) \left( 1 - e^{-pt} \right) \\
n_{12}(t) &= n_{12}(0) e^{-qt} \\
n_{21}(t) &= n_{21}(0) e^{-pt} \\
n_{22}(t) &= n_{22}(0) + n_{12}(0) \left( 1 - e^{-qt} \right).
\end{align*}
\]  

(10)

The consensus function is \( C(t) = (1/N) |N_{11}(t) + N_{12}(t) - N_{21}(t) - N_{22}(t)| \). At zero temperature (\( p = q = 1 \)), when the initial beliefs have a 50-50 split and the internal fields are also 50-50 (but independently of the initial beliefs), then \( n_{ij}(0) = .25 \) independently of \( i \) and \( j \), and \( C(t) = 0 \) for all time. Regardless of the distribution of initial beliefs, if the internal fields have the same distribution, the consensus function will be constant in time. Specifically, suppose that the probability of an agent having an initial belief of \(+1\) is \( r \) and having an initial belief of \(-1\) is \( 1 - r \). Suppose also that the probability of an agent having \( h_i = +1 \) is again \( r \), with a probability \( 1 - r \) of having \( h_i = -1 \). As always, initial beliefs and internal fields are assigned to the agents independently. This gives \( n_{11}(0) = r^2 \), \( n_{12}(0) = n_{21}(0) = r(1 - r) \), and \( n_{22}(0) = (1 - r)^2 \). Inserting these values into (10) yields \( C(t) = 2r - 1 \), independent of time.

The interesting case is when the two differ. For example, suppose that the initial belief distribution has a 10-90 split, but the internal field distribution is 90-10. This case corresponds to an initially rare belief, but one that suits the internal preferences of a majority of the population. For this particular case, \( n_{11}(0) = .09N \), \( n_{12}(0) = .01N \), \( n_{21}(0) = .81N \), and \( n_{22}(0) = .09N \). In this case, the consensus drops from an initial value of 0.8 to 0 at \( t = \ln 2 \) as agents change their initial beliefs, and then rises again to 0.8. The resulting consensus function appears in Fig. 3.

![FIG. 3: Time evolution of the consensus function when the initial distribution of beliefs is 10-90 but internal preference fields are 90-10. The dashed blue line is the analytic solution, using Eqs. (10) and initial conditions described in the text, and the solid red line is the result of numerical simulations.](image)

3. Social interactions and internal fields: \( 0 < \alpha < 1 \)

a. Uniform mixing of internal preferences  Now that we have a feel for the behavior of the various choices of the social term and the intrinsic field separately, we turn to the combined model, whose Hamiltonian is given by (3). In this part, the internal fields \( h_i \) are assigned randomly and independently at each site, but now using a normal distribution \( N(\mu, 1) \), i.e., a Gaussian with mean \( \mu \) and variance one. We choose different means depending on the case we’re interested in studying: in all cases we have chosen \( \mu \) as follows. If the individual preferences have probability \( r \) of being \(+1\) and \( 1 - r \) of being \(-1\), then two different values
of $\mu$ are studied: one is chosen such that a fraction $r$ of individual preferences have $h_i > 0$ (i.e., internal preferences statistically aligned with initial beliefs), and the other has a fraction $r$ with $h_i < 0$ (internal preferences statistically opposed to initial beliefs). So for the case where the initial preferences are 50-50, only one distribution for the $h_i$’s is needed, namely $N(0, 1)$. For the case 70-30, we use $N(\pm 0.5245, 1)$, and for 90-10, we use $N(\pm 1.8215, 1)$.

Final consensus values using long run times are presented in the supplement (Figs. S1). What one sees there is that for $\alpha \lesssim 0.49$, all curves are essentially the same as for $\alpha = 0$ (social effects are negligible). Similarly, when $\alpha \gtrsim 0.51$, the behavior is essentially unchanged from $\alpha = 1$ (individual preference effects are negligible). The interesting region is that close to 0.5, where both effects compete, and we focus on this region here.

We begin with $\alpha = 0.49$ (slightly greater weight for internal preferences), shown in Fig. 4.

**FIG. 4:** Consensus vs. time for $\alpha = 0.49$, for two-opinion case. The first row uses with an initial distribution of 50-50 and $h_i$ drawn from $N(0, 1)$. The second row uses the initial condition 70-30 and $h_i$ drawn from $N(0.5425, 1)$, while the third row uses the same initial condition but with the $h_i$ drawn from $N(-0.5425, 1)$. The fourth row uses the initial condition 90-10 and $h_i$ drawn from $N(1.2815, 1)$, while the fifth row uses the same initial condition but with the $h_i$ drawn from $N(-1.2815, 1)$. In all cases $N = 90$ and each curve represents an average over 100 runs. In this and all subsequent figures in this section, the legend is the same as that in Fig. 2.

For 50-50, we see that the voter and expert models manage to achieve some small amount of consensus, as opposed to $\alpha = 0$; but the social effects are not sufficient to overcome the strong effect of the internal preferences, and consensus does not rise above 0.2 for either. Majority rule behaves differently, and the effects of social network are more important. As before, the greatest degree of consensus is achieved on the complete graph, where it rises to roughly 1/2.

For the cases of 70-30 and 90-10, the social effects of all three social interaction rules strongly change the behavior from $\alpha = 0$,
achieving full consensus for 90-10 (regardless of the $h_i$ distribution). For 70-30, the dependence on social rule is stronger: voter and expert rise to 0.6 (from 0.4 when $\alpha = 0$) when the $h_i$ are drawn from $N(0.5245, 1)$ (i.e., the more common initial belief is also aligned with internal preferences of a majority of the population), and this doesn’t change for the voter rule when the $h_i$ are drawn from $N(-0.5245, 1)$ (the more common initial belief is opposed to the internal preferences of a majority of the population). The expert rule, however, achieves a slightly lower degree of consensus (roughly 1/2) for all but the complete graph, which as discussed above is expected to achieve higher consensus than other networks for expert rule. As usual, majority rule presents the most interesting case, with stronger (but not too strong, yet) dependence on network structure and a significantly greater degree of consensus, clustering around 0.8.

![FIG. 5: Consensus vs. time for $\alpha = 0.5$, for two-opinion case. The first row uses an initial distribution of 50-50 and $h_i$ drawn from $N(0, 1)$. The second row uses the initial condition 70-30 and $h_i$ drawn from $N(0.5425, 1)$, while the third row uses the same initial condition but with the $h_i$ drawn from $N(-0.5425, 1)$. The fourth row uses the initial condition 90-10 and $h_i$ drawn from $N(1.2815, 1)$, while the fifth row uses the same initial condition but with the $h_i$ drawn from $N(-1.2815, 1)$. In all cases $N = 90$ and each curve represents an average over 100 runs. In this and all subsequent figures in this section, the legend is the same as that in Fig. 2. Turning to $\alpha = 0.5$ (equal weights for social interactions and internal preferences) in Fig. 5, we see a marked change in behavior. For 50-50 initial beliefs, the voter and expert rules don’t change much, but majority rule now displays a wide range of behavior depending on network structure, achieving a large degree of consensus ($\approx 0.8$) for the complete graph, and differing degrees of consensus for other lattices: $\approx 0.6$ for strong community, and $\approx 0.4$ for the remaining three (the ring lattice and weak community network don’t converge to this final value until longer times than shown in the graph (cf. Figs. S1 in the supplement) [41]. For both 70-30 and 90-10, when the initial conditions are aligned with the internal preferences (positive
means for the Gaussian distributions of the $h_i$'s), the behavior, not surprisingly, is little changed from that when $\alpha = 0.49$. For 90-10, when the $h_i$'s are drawn from $N(-1.2815, 1)$, the behavior is again only slightly changed from that at $\alpha = 0.49$, with most of the change being a slightly greater differentiation of behavior due to network structure in the majority and expert rule cases. The interesting case is the intermediate one of 70-30 initial condition and 30-70 initial preferences ($h_i$'s drawn from $N(-0.5425, 1)$), shown in the bottom row. In particular, the majority rule case shows clear differences for different network types, and a slower degree of convergence.

Finally, we turn to $\alpha = 0.51$ in Fig. 6. For all cases, it is seen that the results are already nearly indistinguishable from the case $\alpha = 1$: the internal preferences have small to negligible effect, and only social effects matter. (This can of course change if one allows the variance of the distribution of the $h_i$'s to increase, but that would presumably only rescale the values of $\alpha$ at which the various types of behaviors are seen.) Particularly notable is the crossover in behavior from $\alpha = 0.5$ to $\alpha = 0.51$. At

\[
\alpha = 0.5, \text{ the curves for 70-30 initial distribution and } h_i \text{ drawn from } N(-0.5245, 1), \text{ and 90-10/N(-1.2815, 1) still show an initial dip: agents starting with an opinion differing from their internal preference are changing their state, but at a rate now dependent on social interaction rule and (especially for majority rule) network type. The fraction of agents doing so is also dependent on social interaction rule and network type. For } \alpha = 0.51, \text{ this effect appears to have vanished completely for 90-10 and largely}
\]

FIG. 6: Consensus vs. time for $\alpha = 0.51$, for two-opinion case. The first row uses an initial distribution of 50-50 and $h_i$ drawn from $N(0, 1)$. The second row uses the initial condition 70-30 and $h_i$ drawn from $N(0.5425, 1)$, while the third row uses the same initial condition but with the $h_i$ drawn from $N(-0.5425, 1)$. The fourth row uses the initial condition 90-10 and $h_i$ drawn from $N(1.2815, 1)$, while the fifth row uses the same initial condition but with the $h_i$ drawn from $N(-1.2815, 1)$. In all cases $N = 90$ and each curve represents an average over 100 runs. In this and all subsequent figures in this section, the legend is the same as that in Fig. 2.
for 70-30. In the latter case, while differing internal preferences prevent full consensus from being reached on weak community and ring lattice structures, the mechanism now appears to be agents starting out with the majority opinion simply not changing their state due to the increased weight of the social interaction vs. the internal field.

As was the case for $\alpha = 1$, all networks for $\alpha = 0.51$ eventually reach full consensus for voter rule and all but the weak community network reach full consensus for majority rule, while the final consensus value is more lattice-dependent for expert rule. For $\alpha = 0.49$ and $\alpha = 0.5$, Figs. 4 and 5 show that most cases have already achieved final consensus values at relatively short times: it appears that when internal fields have an effect, they not only change final consensus values but also greatly speed up convergence to the final value. Figures displaying behavior for all of these cases at very long times appear in the supplement (Figs. S1) [41].

b. Clustering of internal preferences

In the real world, internal preferences or beliefs are not uniformly mixed. A well-known example is the distribution of political worldviews in the United States; for a variety of reasons, populations are increasingly clustered into regions of (mostly) like-minded individuals. The advent of the internet has intensified this into a further non-geographical segregation of increasingly disconnected segments of the population holding differing ideologies and utilizing different sources and avenues of information. Because our focus here is on relatively simple statistical physics models, we do not attempt to model this in detail, but in this section we do explore the effects of geometric clustering of internal preferences (the $h_i$ fields). We will continue using a well-mixed random distribution of initial beliefs. (One could also consider clustering of initial beliefs, both correlated and anti-correlated with the internal preference clustering. We do not consider these two cases here, but will return to them in future work.)

Three cases will be unaffected. The first is $\alpha = 0$, because in this limit social fields are absent and so all agents react independently of each other to their own internal field; as a result, the presence of clustering in the distribution of internal preference fields does not affect the dynamics. The second unaffected case is $\alpha = 1$, because the only change between the clustered and unclustered models is in the distribution of the $h_i$'s, and their effects are absent when $\alpha = 1$. The third case is the complete graph for any $\alpha$: given that on this graph every agent is connected to every other, there is no difference between uniform mixing and clustering. Therefore, the discussion in the remainder of this section excludes the complete graph and focuses only on the four remaining network structures for $0 < \alpha < 1$.

There are many possible ways to model clustering of the $h_i$'s; we adopt the following procedure. For specificity, consider the case where 70% of the $h_i$'s are positive and 30% are negative. We first subdivide each network into smaller contiguous blocks: for the community lattices, each block is the community of six agents shown in Fig. 1. For the ring lattice and small world network, we likewise partition the graph into contiguous sequences of seven agents. Then for each block an agent (the ‘target spin’) is chosen uniformly at random from the nodes within the block and assigned an $h_i$ value chosen from $\mathcal{N}(0.5425, 1)$. One then assigns the target spin’s value of $h_i$ (both magnitude and sign.) to every spin in its block.

This procedure models a simplified (but not completely unrealistic) population which is divided into perfectly homogeneous neighborhoods (or counties, or states), but with the shared internal preference and its intensity varying from neighborhood to neighborhood. Specific realizations of well-mixed initial conditions coupled with clustered internal preference fields are shown in Fig. 7.

FIG. 7: Specific realizations of initial conditions and $h_i$ distributions for the ring lattice (top) and weak community network (bottom). A. A realization of a 50-50 division; B. A realization of 70-30 (initial condition) and 30-70 ($h_i$ assignments); C. A realization of 90-10 (initial conditions) and 10-90 ($h_i$ assignments). For each realization initial conditions are shown on the left and internal preference fields are shown on the right. Red represents $\sigma_i = +1$ and blue $\sigma_i = -1$. Initial conditions are well-mixed, i.e., each spin is chosen from the appropriate distribution independently of all others. For the figures showing internal preference fields, red represents $h_i > 0$ and blue $h_i < 0$; magnitudes are not shown.
Because clustering strongly enhances the effects of internal preferences in these models, one already sees that different network structures (particularly weak community) begin to affect the dynamics differently for $\alpha$ as low as 0.2 (See Figs. S2 in the supplement [41]); recall that in the unclustered case, differences from $\alpha = 0$ became substantial at $\alpha \approx 0.49$. One might then have expected the effects of clustering to persist well above $\alpha = 0.5$, but surprisingly this does not appear to be the case. Fig. 8 shows results for $\alpha = 0.51$ and, as in the unclustered case, the results are largely the same as for $\alpha = 1$.

![Consensus vs. time for $\alpha = 0.51$ with clustered internal preferences, for two-opinion case.](image)

**FIG. 8:** Consensus vs. time for $\alpha = 0.51$ with clustered internal preferences, for two-opinion case. The presentation of results is the same as in Fig. [5]

When $\alpha = 0.49$ (Fig. [5]), the effects of internal field are weighted slightly more than social effects. Not surprisingly, the effects of clustering lead to a more pronounced dependence on network structure than in the corresponding unclustered case.

In particular, the weak community structure splits off from the others, particularly in the important case of majority rule, and (except for the 90/10 initial conditions, 90/10 internal fields case) remains at a low consensus value throughout. The effect is most interesting when the initial distribution of beliefs is opposite to the distribution of internal fields in the 70-30 and 90-10 cases. For these situations, in most communities the initial belief of most agents runs counter to their internal preferences. Because the connection to other communities is weak, majority rule prefers keeping the state of belief constant.

For completeness, we show results for $\alpha = 0.5$ in Fig. [10] although the behavior is similar to that of $\alpha = 0.49$. One can see a rounding of the minimum in the 70-30/30-70 and 90-10/10-90 majority rule plots, but otherwise the qualitative features are much the same as before. As in the unclustered case, the sharp transition in behavior occurs between $\alpha = 0.5$ and $\alpha = 0.51$. At $\alpha = 0.5$, the effects of the internal field are still substantial, but these largely vanish at $\alpha = 0.51$, as discussed above.

For the clustered runs, we examined the cases with both $N = 90$, as before, and $N = 450$, to ensure a sufficient number
of blocks. We found that the only difference was that larger $N$ led (unsurprisingly) to slower convergence times, but otherwise the general qualitative behavior and final convergence values were unaffected. Runs with $N = 450$ are presented in the supplement (Figs. S3) \[41\].

B. Multi-opinion models

Many statistical physics studies of opinion spread focus on the Ising case of two opinions or beliefs \[1\]; models of this type are potentially applicable to situations where choices are binary. However, in many situations, multiple choices are available; and even when the choice is binary (or effectively binary, as in U.S. presidential elections), there can be multiple shades of opinion. We therefore also studied models in which both 4 and 5 states are available for every agent. In the case of 5 states these are $-1$ (state 1), $-0.5$ (state 2), 0 (state 3), 0.5 (state 4), and 1 (state 5); the 4-state case removes the option $\sigma_i = 0$ (so that $\sigma_i = 1/2$ is now state 3 and $\sigma_i = 1$ is now state 4). Depending on the situation, these states can be interpreted as different strengths of opinions on one side or the other of a binary issue, or else as different beliefs altogether when multiple options are available.

We begin by discussing the 5-state case. As in our treatment of two-state models, we study five different possible cases of initial conditions and internal field distributions. These are chosen for their correspondence to real-world-type situations, and are not intended to be (and generally are not) merely a 5-state extension of the 2-state situations covered in Sect. \[III A\].
FIG. 10: Consensus vs. time for $\alpha = 0.5$ with clustered internal preferences, for two-opinion case. The presentation of results is the same as in Fig. 5.

different cases to be studied are:

(a) The frequency distribution of initial beliefs is symmetric about zero, with probabilities of 0.1 for $-1$, 0.2 for $-0.5$, 0.4 for 0, 0.2 for $+0.5$, and 0.1 for $+1$, while the $h_i$’s are chosen from a normal distribution $N(0, 1)$ also symmetric about zero. We will refer to this case as symmetric/symmetric. To simplify the remaining discussion, we refer to this distribution of initial beliefs as 10-20-40-20-10.

(b) The distribution of initial beliefs is again 10-20-40-20-10, and the $h_i$’s are chosen from $N(0.5425, 1)$ (symmetric/positive).

(c) Initial beliefs skew toward the positive, with a distribution of 10-20-20-40-10, and the $h_i$’s are again chosen from $N(0, 1)$ (positive/symmetric).

(d) The initial belief distribution is again 10-20-20-40-10, and the $h_i$’s are chosen from $N(0.5425, 1)$ (positive/positive).

(e) The initial belief distribution is again 10-20-20-40-10, and the $h_i$’s are chosen from $N(-0.5425, 1)$ (positive/negative).

For the case of four possible opinions, the same five categories will be used, but now “symmetric initial conditions” indicates a distribution of 20-30-30-20, and “positively skewed” indicates 20-20-20-40.
Turning to social interaction rules, voter and expert rule models are unchanged from the two-state case. However, because there are now five beliefs rather than two, majority rule is replaced by “plurality rule” (although we continue to refer to it as majority rule): an agent updating its state takes on the value of a plurality of its neighbors.

Fig. 11 shows typical realizations of unclustered initial beliefs and clustered internal fields for the cases of symmetric/symmetric, positive/negative, and positive/positive.

Finally, we need to determine how to measure consensus. There are many possible ways to do this, but perhaps the simplest three are:

- \( C_1(t) = (1/N)|N_1(t) + N_2(t) - N_4(t) - N_5(t)| \) (unweighted consensus);
- \( C_2(t) = (1/N)|N_1(t) + (1/2)N_2(t) - (1/2)N_4(t) - N_5(t)| \) (weighted consensus);
- \( C_3(t) = (1/N) \max_i N_i(t) \) (maximum frequency consensus)

where \( \max_i N_i(t) \) refers to the number of agents in state \( i \), \( i \) being the state having the largest number of adherents at time \( t \). The first two consensus functions assume that the middle option represents indifference or no opinion, while \( C_3(t) \) assumes that each option represents a different opinion. In all of our runs, we have found that the difference between using \( C_1(t) \) and \( C_2(t) \) is insignificant, so hereafter we comment on results only for \( C_1(t) \) and \( C_3(t) \).

1. Social interactions only: \( \alpha = 1 \)

We begin as usual with the case \( \alpha = 1 \). For this case internal fields are effectively turned off, so we now have only two remaining possibilities: initial distribution symmetric (10-20-40-20-10), and skewed (10-20-20-40-10). Results are shown in Fig. 12.

Starting with \( C_1(t) \) and comparing to Fig. 2 at first glance it seems that consensus is more difficult to achieve. The relative behaviors of the five network types is much the same as before, but final consensus values for an initial symmetric distribution of beliefs are, in almost all cases, significantly lower (in particular the cases in which full, or close to full, consensus was achieved in the 2-opinion case). However, this conclusion is misleading, as can be seen by using the third consensus function (right side of Fig. 12). We see for both types of initial condition that final consensus values are much the same as in the 2-state case. In particular, for majority rule, several lattices converge to full consensus, as in the 2-opinion case. At first this seems inconsistent with results for \( C_1(t) \). This is resolved as follows: for all runs (corresponding to different realizations of initial condition), all agents settle into the same state. But the state differs depending on initial conditions. While choosing the uniform initial condition 20-20-20-20-20 — not shown here — the final state depends also on the realization of the initial condition. Runs whose final state is any of 1, 2, 4, or 5 contribute 1 to \( C_1(t) \); but those whose final state is 3 contribute zero. Comparing the left and right sides of Figs. 12 we see that in the case of symmetric initial conditions most runs end up in state 3; this can be verified by comparing with the behavior of \( C_1(t) \) in the 4-state case (left side of Fig. 13).
We next consider $\alpha = 0$, where network structure is irrelevant. Short-time dynamical behavior will, of course, depend on both the initial state and the distribution of the $h_i$’s. Final consensus, however, depends only on the distribution of the $h_i$’s. Results for all five cases (symmetric/symmetric, symmetric/positive, etc.) are shown in Fig. 14 for the case of five beliefs and Fig. 15 for the case of four beliefs.

When the $h_i$’s are chosen from $N(0, 1)$, one expects $C_1(t) = 0$, which is observed. (In fact, $C_1(t)$ has a value slightly above 0 for any finite $N$ due to fluctuations in any $h_i$ realization. For $N = 90$, this value is slightly below 0.1, as expected for fluctuations of order $\sqrt{N}$. We have run tests using larger $N$ and found that $C(t)$ tends to 0 as $N$ increases, as expected.) $C_3(t)$, however, shows a different behavior, converging to a value slightly above 0.2. This corresponds to the fraction of agents whose $h_i$ value is closer to 0 than to $\pm 1/2$. The fact that the value is slightly above 0.2 is again due to fluctuations; in the limit $N \to \infty$, this will converge to very slightly under 0.2.

Consensus function 1 ($C_1(t)$) behaves qualitatively the same as in the two-opinion case in all situations. Consensus function 3 ($C_3(t)$), however, behaves slightly differently. When the $h_i$’s are distributed symmetrically about zero (rows 1 and 3 of Fig. 14), $C_3(t)$ converges to 0.4, which is the same value as that for $C_1(t)$, but for a different reason: $C_1(t)$ takes on the value 0.4 because roughly 60% of the agents take on the values $1/2$ or 1, while roughly 20% take on the values $-1/2$ or $-1$, with the remaining 20% taking on the value 0. On the other hand, $C_3(t)$ takes on the value 0.4 because slightly greater than 40% of all agents have internal fields larger than 0.75, forcing the agents to take on state 5 ($\sigma_i = +1$).
FIG. 14: Consensus vs. time for five-opinion case when $\alpha = 0$ for different initial conditions and internal field distributions. The first row shows results for symmetric/symmetric, second row symmetric/positive, third row positive/symmetric, fourth row positive/positive, and fifth row positive/negative cases. Left figure shows results using $C_1(t)$, and the right uses $C_3(t)$. In all cases $N = 90$ and each curve represents an average over 100 runs.

The 4-state case can be understood similarly. Once again we expect $C_3(t)$ to settle close to 0.4, the approximate fraction of agents in what is now state 4 (again, $\sigma_i = +1$). However, the symmetric distribution yields a higher value of consensus than in the 5-state case, because now roughly 0.27 of the weight under the normal distribution curve lies between 0 and 0.75 (and similarly between 0 and -0.75); so, depending on fluctuations, either state 2 ($\sigma_i = -1/2$) or state 3 ($\sigma_i = +1/2$) will take on the highest weight, with roughly 30% of all agents.

3. Social interactions and internal fields: $0 < \alpha < 1$

a. Uniform mixing of internal preferences We turn now to intermediate values of $\alpha$, focusing mostly on the 5-state case. Beginning with the case of unclustered — i.e., well-mixed — internal fields, for both $C_1(t)$ and $C_3(t)$ differences with the $\alpha = 0$ result emerge only (of the cases examined) above $\alpha = 0.5$, a significant departure from the behavior in the 2-opinion case. (This will not be true, however, for the case when the $h_i$’s are clustered, as discussed below.) For $\alpha = 0.5$ and below, behavior in all cases is mostly similar to that at $\alpha = 0$, except that final consensus values are somewhat higher, particularly for majority rule (Fig. 17); for $\alpha \geq 0.51$, behavior is similar to that at $\alpha = 1$. Interestingly, differences due to network structure don’t emerge until $\alpha = 0.51$ and higher (Fig. 16).

While these results hold for both consensus functions, the difference between final consensus values for $\alpha = 0$ and $\alpha = 0.5$ is more striking with $C_1(t)$. Results for all $\alpha$ can be found in the supplement [41], Figs. S5.

b. Clustering of internal preferences The situation is much the same when the internal field distributions are clustered, except now differences in network structure begin to manifest themselves at $\alpha = 0.5$; this is expected given that clustering enhances the effect of network structure.

Again, this effect is more pronounced for $C_1(t)$. And as in the unclustered case, differences in behavior at $\alpha = 0.51$ (Fig. 19) and $\alpha = 1$ are minor.
FIG. 15: Consensus vs. time for four-opinion case when $\alpha = 0$ for different initial conditions and internal field distributions. The first row shows results for symmetric/symmetric, second row symmetric/positive, third row positive/symmetric, fourth row positive/positive, and fifth row positive/negative cases. Left figure shows results using $C_1(t)$, and the right uses $C_3(t)$. In all cases $N = 90$ and each curve represents an average over 100 runs.

Behavior of the 4-state case for intermediate values of $\alpha$ is much the same as that of the 5-state case; plots for all $\alpha$ are shown in the supplement [41], Figs. S4. The interesting conclusion to be drawn from this discussion is that availability of more choices might lower overall consensus by a small amount, but also tends to erase differences between network structures unless the social component becomes relatively strong compared to internal preferences.

C. Summary of model comparisons

The results displayed in this section demonstrate that patterns of belief dynamics depend on all of the factors examined: social interaction rules, internal fields, social network structures, and initial frequency and spatial distributions of beliefs and fields. The patterns studied include both the final level of consensus and the rate at which this consensus is achieved (the latter also scaling with population size).

While each of these factors is important, we have also found that belief dynamics are sensitive to certain features and insensitive to others. For example, strongly connected networks, such as the complete graph and the strong community structure, behave similarly in most cases, while (relatively) weakly connected networks, such as Euclidean and weak community, behave differently from strongly connected ones. The details of the connections (e.g., fully connected vs. strong community) were seen to matter less than the degree of connectivity itself. In this sense the various detailed models can be grouped, roughly speaking, into universality classes, where behavior is similar between models as long as they share a basic fundamental characteristic (in the example used here, strong vs. weak connectivity).

In particular, the weakly connected community network was least likely to achieve consensus in most of our simulations. This structure assumes that people are organized in social circles, and that each social circle is connected to another circle through a single person. For example, this kind of network would correspond to a situation where people mostly discuss an issue within their own family, but occasionally individuals communicate with members of different families. This kind of network is most
FIG. 16: Consensus vs. time when $\alpha = 0.51$ for different initial conditions and well-mixed internal field distribution, for five-opinion case. The first row shows results for symmetric/symmetric, second row symmetric/positive, third row positive/symmetric, fourth row positive/positive, and fifth row positive/negative cases. Left figure shows results using $C_1(t)$, and the right uses $C_3(t)$. In all cases $N = 90$ and each curve represents an average over 100 runs.

likely to maintain diversity of opinions (that is, to not achieve a consensus).

On the other end of the spectrum are fully connected networks which almost always achieve consensus. Our strongly connected communities behave similarly to fully connected networks, although they are somewhat slower to achieve consensus. In these structures, social circles are connected through several members, enabling relatively fast spread of beliefs. Their performance is often similar to that of the small-world networks, a popular formalism for describing a situation in which most people are connected to local neighbors, but some long-range links exist. Finally, the ring lattice, which has exclusively local connections, is similar in performance to the weakly connected networks.

Which type of network will be relevant in a particular real-world situation will depend on the belief in question. Some beliefs are discussed mostly in close-knit circles of friends and family, such as those about politics, racial relations, or health. For such beliefs, the assumption of weakly connected communities will often be appropriate. Other beliefs, such as those about best travel destinations, favorite dishes, or fashion choices, are often discussed with a much wider range of contacts. For these beliefs, one can assume more densely connected networks, sometimes resembling fully connected networks.

All three of the social interaction rules we studied exhibited different patterns of belief spread. While voter rule, as it has been shown before, almost always converges to full consensus regardless of the network structure, majority rule in weakly connected communities often does not. However, in better connected network structures majority rule is typically the fastest to reach high levels of consensus. Expert rule is least flexible and often does not allow for consensus, except when networks are fully connected.

It also matters how many different beliefs or positions about an issue are available. This property interacts with the clustering of internal fields. When there are only 2 beliefs about an issue, clustering matters a lot: when beliefs are clustered within "friendly" internal fields, it is difficult or impossible to overturn the beliefs within a local grouping and therefore more difficult to achieve consensus in the larger society as a whole. However, when there are more than 2 beliefs, consensus is easier to achieve even with clustering, because differences between beliefs are smaller and so belief change is somewhat easier.
FIG. 17: Consensus vs. time when $\alpha = 0.5$ for different initial conditions and well-mixed internal field distributions, for five-opinion case. The first row shows results for symmetric/symmetric, second row symmetric/positive, third row positive/symmetric, fourth row positive/positive, and fifth row positive/negative cases. Left figure shows results using $C_1(t)$, and the right uses $C_3(t)$. In all cases $N = 90$ and each curve represents an average over 100 runs.

Interestingly, on a more global level we also observe that similar aggregate trends can result from populations using different social learning rules and having different underlying network structure and spatial distribution of beliefs. This underlines the importance of understanding individual-level social behavior, which can often diverge from group trends and tends to be more sensitive to model details.

Importantly, some of the most realistic assumptions we investigated, such as clustered beliefs and internal fields, community-based network structures, and majority or expert rules, particularly when coupled with less nuanced views about issues (being either for an against an idea, with no other options), are also the least likely to achieve perfect consensus. These results resonate with the real-world observation that complete consensus about any particular issue is rare, which is in contrast with theoretical findings in many stylized statistical physics models, where consensus often develops [1].

Taken together, results presented in this section suggests that accurate modeling of real-world patterns requires attending to several properties of real-world societies, in particular the number of available options, spatial and frequency distributions of internal predispositions, initial conditions, social interaction rules, and network structures. To avoid parametric explosion, one should ideally determine the values of as many of these parameters as possible at the outset by empirical measurement in a particular society, or from theoretical understanding of human behavior, rather than determine them separately for each study or application. We demonstrate this approach in the next section.

IV. MODELING BELIEF DYNAMICS IN REAL-WORLD SOCIETIES

In the previous section we studied the behavior of simple discrete-spin models with a focus on the degree of consensus obtained and the rate of convergence to the final state. We examined the sensitivity of these behaviors with respect to several model features, the most important being network type, rule for social interaction, number of available options, and the relative
FIG. 18: Consensus vs. time when $\alpha = 0.5$ for different initial conditions and clustered internal field distributions, for five-opinion case. The first row shows results for symmetric/symmetric, second row symmetric/positive, third row positive/symmetric, fourth row positive/positive, and fifth row positive/negative. Left figure shows results using $C_1(t)$, and the right uses $C_3(t)$. In all cases $N = 90$ and each curve represents an average over 100 runs.

strengths of social interaction vs. intrinsic preferences. We found several broad categories of behavior, with variations depending on different subsets of these features.

With this in hand, we turn now to two fairly detailed studies on opinion (or belief, or behavior) spread within two different kinds of social networks. The first of these is a data set on 80 students and student tutors living in an MIT dormitory in 2008 and 2009. The second is a longitudinal online study conducted in four waves over a period of two months in 2016, with each wave two weeks apart, and with participants recruited from the crowdsourcing service Mechanical Turk. We begin with the MIT study.

### A. Modeling belief change in the MIT Social Evolution Project

#### 1. Procedures

In 2008 and 2009, a study was conducted on 80 students and student tutors who lived in an undergraduate dormitory at MIT; details of the study and its results can be found in [42] and at [http://realitycommons.media.mit.edu/socialevolution.html](http://realitycommons.media.mit.edu/socialevolution.html). The data consisted of a number of successive measurements of the participants’ network characteristics, as well as their beliefs and behaviors. The data can be divided into three categories: type of social network, evolution of political opinions, and evolution of health-related behaviors.

**Networks.** These were determined in two ways:

1. **Objective networks** were determined through proximities of individuals as captured by Bluetooth signals indicating that the cell phones of two individuals were within 10 meters of each other. These data were additionally weighted by the
FIG. 19: Consensus vs. time when $\alpha = 0.51$ for different initial conditions and clustered internal field distribution, for five-opinion case. The first row shows results for symmetric/symmetric, second row symmetric/positive, third row positive/symmetric, fourth row positive/positive, and fifth row positive/negative cases. Left figure shows results using $C_1(t)$, and the right uses $C_3(t)$. In all cases $N = 90$ and each curve represents an average over 100 runs.

likelihood that the two individuals were on the same floor, as captured by scanning nearby WiFi access points. For these we use data for the months October-December 2008, and March-June 2009.

2. Subjective networks were determined by collecting socioemetic data from individuals about their contacts and the nature of their relationship with each contact (close friend, political discussant, someone they socialize with twice per week, and/or social media contact). For these we use data for the months October-December 2008, and March-May 2009.

Political opinions. These were collected in surveys conducted twice before the 2008 presidential election (September and October 2008) and once afterward (November 2008). Survey questions focused on likelihood to vote, preferred candidate, overall interest in politics, preferred party, and political orientation as of September 2008.

Health-related behaviors. Participants reported the number of salads they ate per week, portions of fruits and vegetables per week, number of days per week when they did at least 20 minutes of aerobic exercise, number of sport activities per week, and overall healthiness of diet. These data were collected in surveys conducted in September, October, and December of 2008, and March, April, and June of 2009.

The objective and subjective networks are shown in Fig. 20. Edges in objective empirical networks are undirected and, as indicated above, weighted by the frequency of two individuals being within 10 meters of each other and on the same floor in a given month. Edges in subjective empirical networks are directed and weighted by the strength of the relationship reported by one participant for another: close friend (edge value = 1), a person they socialize with at least 2 times per week (edge value = .8), a person with whom they discuss politics (edge value = 1 for political issues and 0.5 for health issues), a person with whom they shared all tagged Facebook photos (edge value = 0.5), or a person with whom they shared blog, live journal, or Twitter activities (edge value = 0.5). The edges change somewhat at each time point, reflecting the development of social relationships in this particular society over time. Detailed characteristics of these networks can be found in Table I in Sect. II.
FIG. 20: Empirically derived network structures investigated in the MIT study: a. objectively measured network; b. subjectively measured network. Characteristics of these and other networks examined in this paper appear in Table I.

Table IV contains details of survey questions, the possible responses to each question, and the way responses were coded for modeling purposes. The number of possible beliefs and behaviors differed depending on the issue. Note that for political questions the number of possible responses was larger before the presidential election (September and October 2008) than after the election (November 2008). To account for this and enable predictions over time, we recoded all answers in September and October as follows: likelihood to vote ranged from -1 (including answers ‘definitely will vote’ and ‘most likely will vote’) to 1 (including answers ‘don’t know’, ‘most likely will not vote’, ‘definitely will not vote’). Preferred candidate response values ranged from -1 (including ‘definitely Obama’ or ‘probably’ Obama), to 0 (‘other candidate’ or ‘undecided’ or in November, ‘did not vote’), to 1 (including ‘definitely McCain’ and ‘probably McCain’).

| Question topic and possible responses (recoded to values -1 to 1) |
|-------------------|-----------------|-----------------|-----------------|
| Likelihood to vote | -1=Voted, 1=Did not vote |
| Preferred candidate | -1=Obama, 0=Other candidate or did not vote, 1=McCain |
| Political orientation | -1=Extremely liberal, -.66=Moderately liberal, -.33=Slightly liberal, 0=Moderate, .33=Slightly conservative, .66=Moderately conservative, 1=Extremely conservative |
| Preferred party | -1=Strong Democrat, -.66=Democrat, -.33=Not very strong Democrat, 0=Neither, .33=Not very strong Republican, .66=Republican, 1=Strong Republican |
| Salads per week | -1=6, -.66=5, -.33=4, 0=3, .33=2, .66=1, 1=0 times per week |
| Fruits and vegetables per day | -1=7, -.66=6, -.33=5, 0=3-4, .33=2, .66=1, 1=0 times per day |
| Aerobic activity per week | -1=6, -.66=5, -.33=4, 0=3, .33=2, .66=1, 1=0 times per week |
| Sport activity per week | -1=7, -.66=6, -.33=5, 0=3-4, .33=2, .66=1, 1=0 times per week |
| Maintaining healthy diet | -1=Very healthy, -.66=Healthy, -.33=Average, .33=Below average, .66=Unhealthy, 1=Very unhealthy |

TABLE IV: Questions included in the survey.

Initial distributions of individual beliefs and/or behaviors were determined from participants’ answers in time step 1 (September 2008) to questions about politics (likelihood to vote and preferred candidate) and health (consumption of salads, fruits and vegetables, and engagement in aerobic activities and sports). Distributions of intrinsic predispositions were determined from
participants’ answers to questions that reflected their broader views about politics (for likelihood to vote: interest in politics; for preferred candidate: average of preferred party and political orientation) and health (trying to maintain a healthy diet).

We next discuss modeling of social dynamics. We assume that participants used one of the three social learning rules discussed above: voter, majority, or expert/best friend rule. Because we do not have empirical measures of the rules that participants actually used, we will compare predictions based on each of the three rules to see which one predicts the empirically observed belief dynamics best. Both voter and majority rule are straightforwardly applied: voter rule entails taking a social signal from a randomly chosen member of one’s network contacts, while majority rule takes as a social signal the belief or behavior of the majority (or plurality if there are several options) of one’s contacts. For expert/best friend rule, we used for each individual a contact whose opinion one might consider to be particularly relevant for the decision under question. So for political beliefs, an expert is a person that is either a close friend or a person that the individual discusses politics with. For health-related behaviors, an expert is one’s close friend, who is presumably both similar to and attuned to the individual’s own dietary and physical activity needs and preferences.

All three social rules assume that people first select a random sample from their contacts, weighted by frequency of contact (for objective networks) or closeness of contact (for subjective networks), and then apply the particular rule on that sample. The sample size is set to three or to the overall number of contacts, whichever is lower. We compare predictions of models based on empirically measured networks with three artificial networks that are often used in statistical physics literature to mimic the real-world networks: complete graph or fully connected network, ring lattice, and small-world network. This comparison will enable us to evaluate how well these artificial networks represent the real-world society studied here.

As in Sect. III, we then use the Hamiltonian (8) to model the time evolution of opinion distribution throughout the networks. Modeling proceeded as in Sect. III, at each time step, an individual is chosen with probability given by a Poisson distribution with rate $\lambda$ of updating. If chosen, the individual determines his or her current social-cognitive dissonance $\mathcal{H}$ according to (8), and (if possible) changes his or her belief/behavior to that which would best reduce the dissonance. As before, inverse temperature is fixed at $\beta = 100$.

2. Results

We begin by examining parameter values determined to produce the best predictions of October 2008 beliefs and behaviors, based on participants’ answers in September 2008. The model has two free parameters: the rate of updating $\lambda$, and the weight $\alpha$ assigned to social information. We determined the value of these two parameters for each combination of network structure and updating rule, using grid search in steps of 0.1 from 0 to 1. We selected the combination of parameter values that produced the best predictions (in terms of average percentage of correct individual predictions) of beliefs at the second time point (October) from beliefs at the first time point (September). To avoid overfitting given the small number of data points for each participant, we fixed the values of $\lambda$ and $\alpha$ at their average value across all networks and used these values to predict beliefs and behaviors for all future time points and for all participants.

These parameter values are presented in Table V. The updating rates ranged, not surprisingly, from relatively low (for aerobic exercise) to relatively high (for a presidential candidate). Also unsurprisingly, the weight of social information is higher than the weight of internal preferences for all questions except preferred presidential candidate, where intrinsic political orientation might be expected to play a stronger role than social interactions. It is particularly interesting that the highest weight given to social interaction corresponds to an individual’s likelihood to vote, where one might expect social pressure could play an important role in a close-knit community. The next highest weight for social interaction corresponds to sports played per week; given that many sports are group activities, this is probably unsurprising (but does provide an independent check on the utility of the models).

| Question                  | Rate of updating $\lambda$ | Weight on social information $\alpha$ |
|---------------------------|----------------------------|-------------------------------------|
| Likelihood to vote        | .06                        | .88                                 |
| Preferred candidate       | .19                        | .38                                 |
| Salads per week           | .30                        | .73                                 |
| Fruits and vegetables per day | .19                        | .53                                 |
| Aerobic exercise per week | .09                        | .69                                 |
| Sports per week           | .21                        | .82                                 |

TABLE V: Values of parameters $\lambda$ and $\alpha$, fitted by grid search for the combination that produced the best prediction of answers at the second time point (October 2008) based on answers at the first time point (September 2008), in the MIT study. These average parameter values were then used to predict participants’ answers at the third time point and beyond.

We now turn to model predictions for political questions. Fig. 21 shows predictions for changes in the level of consensus over time.
FIG. 21: Empirically observed and modeled level of consensus, for political questions about likelihood to vote and the preferred candidate in the 2008 presidential election, in the MIT study, using different assumptions about social learning rules (columns) and underlying network structures (lines, see legend).

However, level of consensus alone is not sufficient: even if it is predicted well, it could be that the models predict convergence on the candidate who receives a minority of the vote. Group-level patterns of consensus formation will look the same whether the group converges on one particular belief, or on the completely opposite belief. The exact belief that the society is converging on makes little difference in abstract models, but is enormously significant if the model is going to be useful in the real world. We therefore complement Fig. 21 with Fig. 22, which shows the predicted and actual proportion of the different possible responses.

FIG. 22: Empirically observed and modeled proportion of different responses for political questions about likelihood to vote and preferred candidate, in the MIT study. Likelihood to vote is shown in the first row, with upper lines showing response for will vote/voted and lower lines showing response for will not/did not vote). Second row shows preferred candidate in the 2008 election with upper lines showing preference for Obama and lower lines preference for McCain.

Finally, Table VI shows measures of group and individual prediction accuracy for the different models, along with a control trial of randomly predicted responses. Importantly, all models outperform the results that would be achieved by simply randomly guessing participants’ answers (Table VI, last row). In this table, deviations from the group average were calculated as the square root of the average (across all time periods but the first one, and across 50 replications) of the squared difference between the actual and predicted group average (RMSD). Here the group average is the average response across participants (e.g., for preferred candidate, Obama is assigned a value of $-1$, McCain a value of $+1$, and other candidate/don’t know is assigned a
For the individual answers in Table VI, the percentage of individuals for which each model predicted a correct answer was computed, and was again averaged across all time periods but the first one, and across 50 replications. The root mean square deviation was again calculated as the square root of the average of the squared difference between the actual and predicted answers of each individual, again across time and replications.

### TABLE VI: Group and individual-level prediction accuracy of different models, for political questions in the MIT study. As a control, we include results that would be obtained by random choice of answers (last row).

|                      | Deviation of group predictions from group average* | % correctly predicted individual answers** |
|----------------------|--------------------------------------------------|------------------------------------------|
|                      | Likelihood to vote | Preferred candidate | Likelihood to vote | Preferred candidate |
| Voter rule           |                    |                    |                  |
| Fully connected network | 0.21              | 0.05              | 0.66             | 0.79               |
| Ring lattice         | 0.22               | 0.04              | 0.67             | 0.80               |
| Small world          | 0.22               | 0.05              | 0.66             | 0.80               |
| Empirical network-objective | 0.10          | 0.08              | 0.68             | 0.79               |
| Empirical network-subjective | 0.21          | 0.05              | 0.68             | 0.80               |
| Majority rule        |                    |                    |                  |
| Fully connected network | 0.31              | 0.04              | 0.68             | 0.79               |
| Ring lattice         | 0.28               | 0.04              | 0.68             | 0.80               |
| Small world          | 0.26               | 0.03              | 0.69             | 0.79               |
| Empirical network-objective | 0.15          | 0.05              | 0.79             | 0.79               |
| Empirical network-subjective | 0.26          | 0.05              | 0.73             | 0.80               |
| Expert rule          |                    |                    |                  |
| Fully connected network | 0.19              | 0.05              | 0.65             | 0.79               |
| Ring lattice         | 0.20               | 0.05              | 0.65             | 0.79               |
| Small world          | 0.22               | 0.04              | 0.68             | 0.80               |
| Empirical network-objective | 0.14          | 0.05              | 0.78             | 0.78               |
| Empirical network-subjective | 0.20          | 0.06              | 0.73             | 0.80               |
| Random choice        | 0.40               | 0.59              | 0.56             | 0.47               |

*Calculated as Root Mean Square Deviations, $RMSD = \sqrt{\text{mean}((\text{true group average} - \text{predicted group average})^2)}$. Lower value is better. **Higher value is better.

Analyzing the data in Table VI we first consider predictions for aggregate patterns on the group level, as typically studied in standard statistical physics models. For likelihood to vote, the voter and expert rules do roughly equally well in predicting group-level patterns; majority rule does more poorly (cf. Table VI first two columns). Regarding network structures, the objectively measured network structure enabled somewhat better predictions than subjectively measured networks. The objectively measured empirical network also enabled better predictions than any of the synthetic networks (full, ring lattice, and small world). For the question about preferred candidate, all rules work roughly equally well; it is possible that the participants used some mixture of these rules to
update their opinions. More likely, however, this result follows from the low value of \( \alpha \) empirically obtained for this question — internal preferences carry far greater weight than social interactions (at least in this particular example) in deciding whom to vote for.

We turn now to data and predictions for health-related behaviors. Unlike the political questions which asked for discrete either/or answers (will/will not vote, Obama/McCain), the health-related questions attempted to capture discrete points along a continuous spectrum related to the frequency of each individual’s health-related behaviors. Consequently, there were seven or eight possible responses to each question (see Table IV).

The predicted and empirically observed level of consensus (using consensus function \( C_3(t) \) defined in Sect. III B) are shown in Fig. 23.

FIG. 23: Empirically observed and modeled level of consensus, for questions regarding health-related behaviors in the MIT study, again using different assumptions about social learning rules (columns) and underlying network structures (lines, see legend).

Because the responses to health-related questions were points along a continuous spectrum, Fig. 24 provides information about average empirical and predicted responses (as opposed to the relative frequency of different answers, as in Fig. 22).

Finally, Table VII provides information on the deviations from empirical values rather than the percentage of correct predictions (as in Table VI). Note that for health-related questions there are data from six time points (as opposed to three time points for political questions).

Figs. 23 and 24 show that all three social interaction rules have roughly similarly accuracy in predicting group trends, with objectively measured empirical networks providing better predictions than subjective ones, and both empirical networks providing better predictions than synthetic ones. This is also reflected in the left-hand side of Table VII.

On the individual level, the most accurate predictions again corresponded to the objectively measured empirical network (Table VII, right-hand side). The fact that in all cases the objectively measured network outperformed the subjectively measured one suggests that people’s own reports about their network connections do not include all relevant connections, which are recorded by automated measurements. Finally, all models performed better than simple random guessing.

Given that the results for both political and health-related questions point to the objective network as the most realistic, it makes
FIG. 24: Empirically observed and modeled proportion of different responses for health-related behaviors in the MIT study.

sense to compare the three social interaction rules by focusing on the objective network only; this removes the noise generated by modeling these rules on less realistic networks. Table VI then indicates that for both group and individual predictions on political opinions, majority and expert rule do equally well, and both do significantly better than voter rule. This is encouraging, given that voter rule (without individual fields) was formulated as a tractable analytical model but perhaps should not be expected to function well in a realistic setting; one might expect that few people will take on the opinion of a randomly selected contact. The fact that majority and expert rule indeed do a better job at prediction is a validation that the relatively simple statistical physics approach studied in this paper does capture some aspects of opinion or belief spread in the real world. It further provides evidence for the plausible conjecture that in formulating beliefs, people rely on a combination of internal set preferences, opinion of a majority of their neighbors, and the opinion of an “expert” (defined broadly as either a real expert or a trusted friend or associate).

For health-related behaviors, and again looking at the objective network only, one finds some interesting differences from the political sphere. Table VII indicates that all three social interaction rules give similar results — except in the case of sports at the individual level, where majority rule performs significantly better than the other two. This is quite plausible, because many sports activities require multiple participants, so one might expect majority rule to do better than its competitors. For the other behaviors, where all three social rules perform equally well, the results may be construed to point toward the conclusion that for many of these behaviors — say, the number of salads an individual eats per week — social interactions play a smaller role.

B. Modeling belief change in a longitudinal survey study of the US public

1. Procedures

For the second comparison of models to data, we conducted a longitudinal study in four waves, each wave two weeks apart, with participants recruited from the crowdsourcing service Mechanical Turk. Two hundred respondents participated in the first wave, and 94 completed all four waves of study. The study was conducted in a relatively turbulent political period in the United States, with the first two waves conducted just before the first 2016 presidential primary (Iowa, February 1), and the other two
TABLE VII: Group and individual-level prediction accuracy of different models, for health-related questions in the MIT study. As a control, we again include results that would be obtained by random choice of answers (last row).

| Deviation of group predictions from group average* | Average deviation of individual predictions from individual answers** |
|--------------------------------------------------|---------------------------------------------------------------|
|         | Salads | Fruit & veg | Aerobic | Sports | Salads | Fruit & veg | Aerobic | Sports |
| Voter rule |         |           |         |        |         |           |         |        |
| Fully connected network | 0.17 | 0.11 | 0.16 | 0.14 | 0.64 | 0.51 | 0.72 | 0.70 |
| Ring lattice | 0.15 | 0.09 | 0.17 | 0.15 | 0.64 | 0.51 | 0.71 | 0.70 |
| Small world | 0.15 | 0.13 | 0.16 | 0.14 | 0.64 | 0.53 | 0.71 | 0.69 |
| Empirical network-objective | 0.14 | 0.08 | 0.12 | 0.12 | 0.62 | 0.48 | 0.56 | 0.64 |
| Empirical network-subjective | 0.15 | 0.10 | 0.12 | 0.12 | 0.67 | 0.51 | 0.69 | 0.68 |

| Majority rule |         |           |         |        |         |           |         |        |
| Fully connected network | 0.22 | 0.12 | 0.17 | 0.19 | 0.62 | 0.48 | 0.69 | 0.61 |
| Ring lattice | 0.18 | 0.11 | 0.14 | 0.12 | 0.63 | 0.50 | 0.67 | 0.63 |
| Small world | 0.21 | 0.10 | 0.17 | 0.14 | 0.65 | 0.48 | 0.67 | 0.62 |
| Empirical network-objective | 0.18 | 0.09 | 0.12 | 0.12 | 0.59 | 0.47 | 0.58 | 0.58 |
| Empirical network-subjective | 0.22 | 0.10 | 0.13 | 0.13 | 0.66 | 0.48 | 0.66 | 0.62 |

| Expert rule |         |           |         |        |         |           |         |        |
| Fully connected network | 0.19 | 0.10 | 0.16 | 0.15 | 0.65 | 0.51 | 0.71 | 0.69 |
| Ring lattice | 0.15 | 0.10 | 0.14 | 0.14 | 0.65 | 0.51 | 0.69 | 0.68 |
| Small world | 0.14 | 0.09 | 0.17 | 0.12 | 0.64 | 0.51 | 0.71 | 0.68 |
| Empirical network-objective | 0.14 | 0.10 | 0.12 | 0.13 | 0.60 | 0.51 | 0.59 | 0.66 |
| Empirical network-subjective | 0.15 | 0.10 | 0.12 | 0.15 | 0.69 | 0.51 | 0.64 | 0.71 |

| Random choice | 0.28 | 0.32 | 0.37 | 0.58 | 0.79 | 0.79 | 0.91 | 1.01 |

*Calculated as Root Mean Square Deviations, RMSD = \( \sqrt{\text{mean}((\text{true group average} - \text{predicted group average})^2)} \). **Calculated as RMSE = \( \sqrt{\text{mean}((\text{true individual answers} - \text{predicted individual answers})^2)} \). Lower values are better.

TABLE VII: Group and individual-level prediction accuracy of different models, for health-related questions in the MIT study. As a control, we again include results that would be obtained by random choice of answers (last row).

afterward. In each wave, participants were asked about several topics. The evolution of beliefs was similar for many of the questions, so to avoid repetition we present here only a subset of the questions and the results pertaining to them; these illustrate well the different observed trends. The questions reported here were taken from existing national surveys to enable comparisons of our sample with the general population, and covered the topics of guns, terrorism, and vaccinations; their exact wording, possible responses, and the coding of responses appear in Table VIII. A full description with the complete set of questions and trends will appear elsewhere [43].

Determination of the distribution of initial beliefs was performed similarly to the methods used in the MIT study, described in Sect. IV A. Determination of the social signal received by each participant necessarily differed, given that (unlike in the MIT study) the members of the respondents’ social network were generally not respondents themselves; we therefore needed to rely on the participants’ perceptions of those beliefs.

Respondents were asked to report on the perceived beliefs, on each of the issues, of up to three of their social contacts with whom they discussed these issues in the two weeks prior to the survey. These social contacts could differ from wave to wave. On average, each participant reported about 5.8 contacts across the four study waves, for a total of 548 different contacts. Information was also collected on additional characteristics of these social contacts, including their perceived expertise about the issues, their preferred social learning rules, other sources of information they use, rules for network updating, and several psychological traits. For voter rule, we then used the perceived belief of a random contact; for majority rule, the perceived belief of a plurality of a participant’s contacts; and for expert rule, the perceived belief of the contact determined by the participant to have the greatest expertise. Details will be reported elsewhere [43]. Finally, we collected each participant’s intrinsic inclination toward certain beliefs (i.e., their \( h_i \)), operationalized as beliefs corresponding to one’s particular political orientation (ranging from extreme left to extreme right).

The collected data enabled an empirical determination of the initial frequency distribution of beliefs and the intrinsic predis-
TABLE VIII: Questions included in the Mechanical Turk survey.

| Question | Responses |
|----------|-----------|
| Guns     | In general, do you feel that the laws covering the sale of firearms should be made more strict, less strict, or kept as they are now? |
|          | -1=Much more strict, -0.5=Somewhat more strict, 0=Kept as they are now, 0.5=Somewhat less strict, 1=Much less strict |
| Terrorism| How worried are you that there will soon be another terrorist attack in the United States? |
|          | -1=Not at all worried, -0.33=Not too worried, 0.33=Somewhat worried, 1=Very worried |
| Vaccination| Thinking about childhood diseases, such as measles, mumps, rubella and polio, which statement comes closest to your view? |
|          | -1=All healthy children should be required to be vaccinated, 0=Parents should be able to decide NOT to vaccinate their children in some circumstances, 1=Parents should always be able to decide NOT to vaccinate their children |

positions of participants. The idea, as in the MIT study, was then to use the survey responses in different waves to determine the change in beliefs about different issues, creating a benchmark against which we can judge the success of different modeling setups. We compared these data to predicted patterns of belief dynamics obtained assuming the three different social learning rules using different network structures, shown in Fig. 25, two of which are empirical versions of the artificially created community-based networks in the previous section. For the weakly connected empirical communities, the participants are embedded in their actual social circles and are also connected to one other participant outside their circle; that participant is chosen based on the political orientations of both participants, with the probability of a link between participants with different political orientations determined from the empirical data. For the strongly connected empirical communities, each participant is now connected to at least two other participants. For comparison, we also used the fully connected network, ring lattice, and small world network.

To avoid overfitting, we used a different procedure from that used in the MIT study to determine the parameters $\lambda$ (rate of belief updating) and $\alpha$ (relative weights between social information and intrinsic preferences). Rather than using first-wave data to determine these parameters separately for each question, we instead used the average values from the previous study (presented in Table V); consequently, $\lambda$ was set to 0.17 and $\alpha$ to 0.67 for all participants and all questions. This procedure was meant to test how well parameters estimated for one group translate to a wholly different group of participants.
As in the MIT study, the model starts from the initial belief distribution measured in the first wave, and develops predicted belief trajectories at future time points using predicted beliefs in each subsequent wave. For each combination of factors (five network structures and three social learning rules), we ran 30 replications lasting 160 time steps, which was enough to achieve stable results.

2. Results

Fig. 26 shows empirical trends of belief change related to the three issues across the four study waves, with predictions obtained using Eq. 8 in three different network structures.

![Fig. 26: Dynamics of consensus in the Mechanical Turk study: model predictions assuming different underlying network structures and social learning rules, compared with the actual answers.](image)

Fig. 27 shows the predicted and actual average answers (computed similarly to the MIT study) to each question over time. Model predictions assuming different network structures and social learning rules can be seen from these figures to correspond reasonably well to empirical trends for many but not all of the characteristics; in particular, the data showed that during the course of this study, beliefs regarding vaccinations remained roughly constant, but the models predicted change over time, with voter model doing the best. This is a possible indication that by the time of the survey, intrinsic beliefs about vaccinations were already fairly strong and no longer subject to modification through social interaction.

More formally, we can consider as before errors of predictions on both the group and individual levels (Table IX). The group level indicators were variable, and none of the different network structures or social interaction rules did clearly better than the others on average. The individual level indicators are more telling. The two empirical network structures (weak and strong communities) did roughly the same, and significantly better than the three synthetic networks (full, ring, and small world). This suggests that there is a value in the approach of, whenever possible, using empirically derived network structures rather than trying to approximate them using artificial networks. Restricting attention to the community networks, we again observe that on average the majority and expert rules do better than voter rule, and all do better than random prediction.
V. DISCUSSION

There is now a large and continually growing literature dealing with the application of statistical physics models to opinion or belief spread in social networks. Beginning with the homogeneous two-state voter model on simple Euclidean lattices, these models have increased in variety, complexity and sophistication. It has consequently become difficult to evaluate similarities and differences in behavior of these many approaches, as well as their relative merits in explaining and predicting belief propagation in actual human communities. While comparisons to empirical data, whether gleaned from elections or from controlled social studies, have appeared in the literature, they remain relatively rare compared to the proliferation of new models and approaches.

In this paper we studied the properties and predictive values of a well-studied class of models — those in which agents are presented with a small number of available options, and who interact on various networks. These discrete-spin models are probably the simplest of the many already investigated, and their study constitutes the largest subset of the literature of physics-based social science models. The large majority of these studies, though, are of homogeneous models — that is, where every agent is assigned the same Hamiltonian-like function. (This is not the same as mean-field models, in which fluctuations are ignored and every agent finds him- or herself in the same environment as everyone else. These form a subset of the homogeneous models referred to here.) To inject more realism into such models, we have added a random-field term to account for the ingrained tendencies of each individual, at the time of study, toward certain beliefs. Of course, there are other ways of accomplishing this, but adding a random-field term is a simple first step.

As discussed in Sect. II, even within this limited class of models there are many possible variations. There are numerous possibilities for the social interaction term, internal field distribution, initial conditions, social network structure, rate of updating, relative weights of social information vs. intrinsic preferences, and so on. There is also the question of deterministic vs. stochastic updating of beliefs. In Sect. III we confined ourselves to three social interaction rules: voter rule, majority rule, and expert rule. The internal preference field for each agent was independently chosen from a simple normal distribution centered at zero and whose variance was set at the same magnitude as the strength of the social interaction term. We confined ourselves to (effectively) zero-temperature dynamics, which corresponds to each agent having perfect information about the opinion or belief states of his/her neighbors. Even with these restrictions, there remained a large number of models to study and compare, including varying the number of possible belief states, social network structure, distribution of initial conditions, relative weight of social information vs. internal preferences, and whether the intrinsic belief states of agents were independent random variables or clustered into adjacent regions depending on the network geometry. For each possibility we then investigated the final consensus state of the network and the rate at which the final state was reached.

In so doing, we were able to determine which behaviors were sensitive to which model inputs. The major classifications separated two-state (Ising-like) vs. multiple-state (Potts-like) models, relative weights of social information vs. intrinsic prefer-
TABLE IX: Group and individual-level prediction accuracy of different models for the Mechanical Turk study. As a control, we include results that would be obtained by random choice of answers (last row).

|                   | Deviation of group predictions from group average* | Average deviation of individual predictions from individual answers** |
|-------------------|-----------------------------------------------|---------------------------------------------------------------|
|                   | Guns  | Terrorism | Vaccines | Guns  | Terrorism | Vaccines |
| Voter rule        |       |           |          |       |           |          |
| Fully connected network | 0.12  | 0.10      | 0.06     | 0.80  | 0.74      | 0.97     |
| Ring lattice      | 0.12  | 0.04      | 0.06     | 0.81  | 0.61      | 0.87     |
| Small world       | 0.08  | 0.08      | 0.05     | 0.66  | 0.70      | 0.91     |
| Weakly connected comm. | 0.16  | 0.05      | 0.01     | 0.67  | 0.52      | 0.66     |
| Strongly connected comm. | 0.05  | 0.02      | 0.05     | 0.66  | 0.58      | 0.74     |
| Majority rule     |       |           |          |       |           |          |
| Fully connected network | 0.13  | 0.02      | 0.10     | 0.79  | 0.75      | 0.95     |
| Ring lattice      | 0.01  | 0.00      | 0.15     | 0.73  | 0.67      | 0.78     |
| Small world       | 0.08  | 0.07      | 0.10     | 0.74  | 0.67      | 0.84     |
| Weakly connected comm. | 0.12  | 0.04      | 0.04     | 0.56  | 0.52      | 0.69     |
| Strongly connected comm. | 0.02  | 0.05      | 0.08     | 0.55  | 0.56      | 0.61     |
| Expert rule       |       |           |          |       |           |          |
| Fully connected network | 0.10  | 0.07      | 0.06     | 0.81  | 0.67      | 0.87     |
| Ring lattice      | 0.03  | 0.05      | 0.06     | 0.75  | 0.70      | 0.81     |
| Small world       | 0.02  | 0.08      | 0.00     | 0.71  | 0.70      | 0.83     |
| Weakly connected comm. | 0.08  | 0.03      | 0.01     | 0.59  | 0.58      | 0.53     |
| Strongly connected comm. | 0.07  | 0.01      | 0.02     | 0.55  | 0.51      | 0.53     |
| Random choice     | 0.30  | 0.06      | 0.38     | 0.86  | 0.79      | 1.07     |

*Calculated as Root Mean Square Deviations, \( \text{RMSD} = \sqrt{\text{mean}((\text{true group average} - \text{predicted group average})^2)} \). **Calculated as \( \text{RMSD} = \sqrt{\text{mean}((\text{true individual answers} - \text{predicted individual answers})^2)} \). Lower values are better.

ences, and clustering vs. random distribution of individual preferences. In all cases, the two extremes of behavior were at \( \alpha = 0 \) (internal preference only, no social interactions) and \( \alpha = 1 \) (social interactions only, no internal preference fields). While results depended quantitatively on \( \alpha \), qualitative behavior in all cases was similar to \( \alpha = 0 \) for models with \( \alpha < 0.5 \) and to \( \alpha = 1 \) for models with \( \alpha > 0.5 \). In most cases, more complicated crossover behavior was seen as \( \alpha \) varied from (roughly) \( \alpha = 0.45 \) to \( \alpha = 0.55 \), with the most rapid crossover behavior occurring in the narrow range \( 0.49 \leq \alpha \leq 0.51 \).

Among the three social interaction possibilities, the voter model was unsurprisingly the least sensitive to network structure and initial conditions, and generally the slowest to arrive at the final consensus value; but typically it would ultimately achieve high consensus values. For majority and expert rule, final consensus values were much more sensitive to both network structure and initial condition distribution. Our comparison to real-world data confirms that this sensitivity to network structure is indeed an important factor in determining the degree of consensus.

Another important factor was the number of possible beliefs, which we found interacts strongly with the clustering of internal fields. With only two possible beliefs, clustering can make a large difference, making it more difficult to achieve consensus in the entire network. But when there many possible belief states, consensus is easier to achieve even with clustering. The reasons for this behavior are discussed in Sect. III C.

Perhaps one of the most important conclusions arising from the studies in Sect. III is that the most realistic network structures, social learning rules, and spatial distribution of beliefs (independently determined by surveys and other social science studies) are those least likely to achieve high levels of consensus. This corresponds to real-world behavior and contrasts to those statistical physics models which achieve high levels of consensus. A summary of other conclusions arising from comparisons of different models can be found in Sect. III C.
Finally, in Sect. [IV] we compared these models to two studies done almost a decade apart. The first was a study of 80 students in an MIT dormitory during the presidential election of 2008, and the second an online longitudinal study with 94 participants done by us during the presidential election of 2016. The MIT study looked at both political beliefs and health-related behaviors, while the longitudinal study focused on both politics and science-related issues.

Comparing the data obtained in each of these studies with predictions made by the various discrete-spin models, we found that models using objectively determined network structures did a better job of capturing the data than those using artificial networks (Euclidean lattices, complete graphs, small world, ring lattices, and others). Moreover, majority and expert rule generally did much better than voter rule, with the precise nature of the question determining whether majority or expert rule had more predictive value. Perhaps the two most important conclusions were first, that discrete spin models do have real predictive value despite their simplicity, and even in the worst cases do better than control studies using random prediction; and second, that when one looks at globally averaged behaviors one finds similar aggregate trends that are not terribly sensitive to different social learning rules, underlying network structures, and spatial distribution of beliefs. In contrast, individual-level trends are often very sensitive to these factors. This underlines the importance of understanding opinion formation at the individual level, and supports the importance of including a random-field or similar term to capture some aspects of this behavior. In future studies, we will examine these terms more closely, and investigate whether the inclusion of noise (positive-temperature models), in which an individual’s perceptions of others’ beliefs may contain errors, does a better job of capturing real-world phenomena. We will also investigate the dynamics exhibited by populations of agents that use a mixture of different social interaction rules, to approximate the heterogeneity observed in the real world.

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VI. APPENDIX

A. Proof that majority rule leads to full consensus on the complete graph

Below we present an informal argument (but one which is easily made rigorous) that majority rule on a complete graph will always achieve full consensus at zero temperature, given any initial distribution and $N$ large. We first discuss the case where all neighbors contribute to the determination of the majority, and then to the case of three randomly chosen neighbors, as used in our numerical simulations.

Consider first any distribution other than 50-50, and where all spins (except the one chosen for updating) contribute to a determination of the majority. Suppose, for example, there is an initial probability of 0.6 assigned to each agent of being in the $+1$ state, and a probability 0.4 of being in the $-1$ state. Then the initial state will have a ratio of 3:2 of $+1$ spins to $-1$ spins, with fluctuations from that ratio of order $\sqrt{N}$. So for $N$ already greater than 20, the probability of more spins being $-1$ than $+1$ in the initial state is already extremely small.

Now we start the dynamics. No matter which agent is picked, the majority of her neighbors will be $+1$. So whatever her initial state, she has the state $+1$ afterward. But this increases the dominance of $+1$ spins, so the same argument continues to apply for each successive agent when her Poisson clock rings. Therefore the final state must be $+1$ with probability rapidly approaching 1 as $N \to \infty$. (This result is consistent with the famous Condorcet Jury Theorem [37] in political science, where this behavior is typically modeled by a cumulative binomial distribution.)

What about when the initial distribution is 50-50? Here, one can also prove that the system achieves complete consensus in any single run — but half the time the final state is uniformly $+1$ and half the time it’s uniformly $-1$. The reasoning is similar to that above, with the added ingredient that the initial state will have $\sqrt{N}$ fluctuations that lead to a small excess of either $+1$ or $-1$ spins with equal probability. But this excess is sufficient that every dynamical realization starting from the initial state will then lead to the uniform final state fully predetermined by the initial excess of plus or minus spins. The only situation where the dynamical realization determines whether the final state is all-plus or all-minus is that where the initial spin configuration has exactly half $+1$ spins and half $-1$ spins, plus or minus a single spin. In this case, the initial dynamics will quickly lead to a small excess of either $+1$ or $-1$ spins with equal probability, again leading to full consensus, but with the final state now determined by the dynamics rather than the initial condition. In any case, the probability of occurrence of such an initial state goes to zero as $N^{-1/2}$.

This set of arguments, using the usual Curie-Weiss Hamiltonian rather than majority rule (both of which result in identical dynamics at zero temperature), leads to a conclusion significantly stronger than that of the system simply achieving full consensus (i.e., a uniform final state). The stronger conclusion is that the final state is completely predetermined by the initial spin configuration (i.e., all dynamical realizations lead to the same final state) with probability approaching one as the number of spins $N \to \infty$ [44].

Simulations demonstrating this behavior for an initial 50-50 distribution on the complete graph are shown in Fig. 28. As predicted, full consensus is always achieved, with half the outcomes in the uniform $+1$ state and half in the uniform $-1$ state.

![FIG. 28: Individual runs using majority rule dynamics on the complete graph ($N = 90$) with an initial distribution of 50-50.](image-url)

For the case when only three agents are chosen randomly to determine the majority, the arguments become more detailed, but the conclusion is the same. An easy way to see why this should be so is to consider the dynamical stable states of the system. A dynamically stable state is a fixed point of the dynamics: once in that spin configuration, no realization of the dynamics can lead to a different state. For the complete graph, the only dynamically stable states under majority rule (and also voter rule, but not expert rule — cf. Figs. S1 in the supplement [41]) are all agents adopting the state $+1$ or all adopting $-1$. The main difference between majority rule using all neighbors vs. three randomly chosen neighbors is that the former case leads to a final state completely determined by the initial spin configuration, while in the latter case the dynamical realization also plays a role in determining the final state. (For a general discussion of the competing roles of initial conditions and dynamical realizations in determining final states, see [44–55].)

Importantly, these arguments and results do not hold for locally connected networks: for example, in the weak community structure, any community (i.e., social circle) will be dynamically stable if all its agents share the same state, regardless of what’s happening outside the community. Therefore, if $N_c$ is the number of communities in the entire system, there are already $2^{N_c}$ such states, which serves as a lower bound for the number of metastable (i.e., nonuniform but dynamically stable) states under majority rule.

Of course, when the initial condition is extremely skewed, high or full consensus will also be achieved for most other types of networks.