On one-loop corrections in the Horava-Lifshitz-like QED

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We study the one-loop two point functions of the gauge, scalar and spinor fields for a Horava-Lifshitz-like QED with critical exponent $z = 2$. It turns out that, in certain cases, the dynamical restoration of the Lorentz symmetry at low energies can take place. We also analyze the three point vertex function of the gauge and spinor fields.

I. INTRODUCTION

The Horava-Lifshitz (HL) approach, which is characterized by an asymmetry between space and time coordinates, has aroused great interest because it provides an improvement of the renormalization capabilities of field theories. In this scheme, the equations of motion of relevant models are invariant under the rescaling $x^i \rightarrow bx^i$, $t \rightarrow b^z t$, where $z$, the critical exponent, is a number indicating the ultraviolet behavior of the theory. This procedure may turn to be essential to enable the construction of renormalizable models at scales where quantum gravity aspects cannot be neglected. Different issues related to the HL gravity, including its cosmological features, exact solutions, black holes were considered in a number of papers. However, since the space-time anisotropy breaks Lorentz invariance, to validate a given anisotropic model as physically consistent it is necessary to

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prove that at low energies the Lorentz symmetry is approximately realized. Some studies suggest that this behavior is better achieved in infrared stable models.

It is also worth to point out some examples of studies of the perturbative behavior of the HL-like theories. Some facets of the HL generalizations for the gauge and supersymmetric field theories were presented in [6]. Renormalizability of the HL-like scalar field theory models has been discussed in detail in [7]. The Casimir effect for the HL-like scalar field theory has been considered in [8]. In [9] and [10], the HL modifications of the $CP^{N-1}$ and nonlinear sigma models were respectively studied. Furthermore, the effective potential for various HL models was determined in [11].

In this work, we pursue these investigations by considering a HL generalization of an Abelian gauge theory. For the version of the model containing only scalar and gauge fields, our one-loop calculations, performed at an arbitrary $d$ dimensional space, indicate the emergence of the Lorentz symmetry at low energies unless for some specific values of the space dimension. This conclusion is based on explicit one-loop calculations of the two and three points vertex functions. As we will show, this effect has no universal character and does not happen for a similar model of spinor and gauge fields.

The structure of this work looks as follows. In the section II we present the model, in section III we discuss the one-loop correction to the two point function of the vector field and in the section IV we obtain the one-loop contributions to the two point functions of the scalar and spinor fields. In section V we considered the three point function of the spinor and gauge field. Our conclusions are given in the summary.

**II. AN HL LIKE ABELIAN GAUGE MODEL**

For the sake of the concreteness, we restrict ourselves to the case $z = 2$. In this case, the Lagrangian describing the model we are interested is

$$L = \frac{1}{2} F_{0i} F_{0i} + \frac{a_1^2}{4} F_{ij} \Delta F_{ij} + D_0 \phi (D_0 \phi)^* - a_2^2 D_i D_j \phi (D_i D_j \phi)^* - m^4 \phi \phi^* + \bar{\psi} (i \gamma^0 D_0 + (i a_3 \gamma^i D_i)^2 - m^2) \psi,$$

where $D_{0,i} = \partial_{0,i} - ie A_{0,i}$ is a gauge covariant derivative, with the corresponding gauge transformations being $\phi \to e^{ie\xi} \phi$, $\phi^* \to e^{-ie\xi} \phi^*$, $\psi \to e^{ie\xi} \psi$, $\bar{\psi} \to \bar{\psi} e^{-ie\xi}$, and $A_{0,i} \to A_{0,i} + \partial_{0,i} \xi$. Our metric is $(+ - \ldots -)$, and $\Delta$ denotes the $d$-dimensional Laplacian.
The parameters \( a_i, i = 1, 2, 3 \) were introduced to keep track of the contributions associated to the high derivative terms; they are independent but for simplicity we assume that \( a = a_1 = a_2 = a_3 \). To keep our analysis restricted to the gauge-matter interaction, we do not introduce self-coupling of the matter fields.

The free propagators for the scalar and fermionic fields are

\[
<\phi(k)\phi^*(-k)> = \frac{i}{k_0^2 - a_2 k_4^4 - m^4 + i\epsilon},
\]

\[
<\psi(k)\bar{\psi}(-k)> = \frac{i\gamma^0 k_0 + a\vec{k}^2 + m_2^2}{k_0^2 - (a\vec{k}^2 + m_2^2)^2 + i\epsilon}.
\]

To find the propagator for the vector field we choose to work in the Feynman gauge by adding to \( L_{gf} \) the gauge fixing Lagrangian \( L_{gf} = \frac{1}{2}[-(-a^2\Delta)^{\frac{1}{2}}\partial_0 A_0 - (-a^2\Delta)^{\frac{1}{2}}\partial_i A_i]^2 \), yielding to the free propagators the forms

\[
<A_iA_j> = -\frac{i\delta_{ij}}{k_0^2 - a_2 k_4^4 + i\epsilon}; \quad <A_0A_0> = i\frac{a_2^2 k_0^2}{k_0^2 - a_2 k_4^4 + i\epsilon}.
\]

These expressions will be used to calculate the one-loop contributions in our theory.

**III. ONE-LOOP CORRECTION TO THE VECTOR FIELD PROPAGATOR**

To study the one-loop correction to the gauge field, let us first consider the bosonic sector. It is easy to see that the interaction vertices are

\[
e^2A_0A_0\phi\phi^*; \quad ieA_0(\phi^*\partial_0\phi - \phi\partial_0\phi^*); \quad iea^2(\partial_iA_j)[\phi\partial_i\partial_j\phi^* - \phi^*\partial_i\partial_j\phi] + 2iea^2A_i(\partial_j\phi\partial_i\partial_j\phi^* - \partial_j\phi^*\partial_i\partial_j\phi);
\]

\[
e^2a^2A_iA_j(\phi\partial_i\partial_j\phi^* + \phi^*\partial_i\partial_j\phi) - e^2(A_i\partial_j\phi + A_j\partial_i\phi + (\partial_iA_j)\phi)
\]

\[
\times (A_i\partial_j\phi^* + A_j\partial_i\phi^* + (\partial_iA_j)\phi^*),
\]

yielding to the free propagators the forms
so that, in the Fourier representation, they are given by

\begin{align}
V_3^{(1)} &= e A_0(p) \phi(k) \phi^*(-p - k)(2k_0 + p_0); \\
V_3^{(2)} &= -e^2 A_0(p) \phi(k) \phi^*(-p - k)(2k_j + p_j)((k_i + p_i)(k_j + p_j) + k_i k_j); \\
V_4^{(1)} &= e^2 A_0(p_1) A_0(p_2) \phi(k_1) \phi^*(k_2)(2\pi)^d \delta^3(p_1 + p_2 + k_1 + k_2); \\
V_4^{(2)} &= -e^2 a^2 A_i(p_1) A_j(p_2) \phi(k_1) \phi^*(k_2)(2\pi)^d \delta^3(p_1 + p_2 + k_1 + k_2) \\
& \quad \times [k_1 k_1 + k_2 k_2 - 2\delta_{ij}(k_1 k_2) - k_1 k_{2j} - k_2 k_{1j} - k_{1j} p_2 - k_{2j} p_1] \\
& \quad - \delta_{ij}(k_1 p_2) - \delta_{ij}(k_2 p_1) - \delta_{ij}(p_1 p_2),
\end{align}

where \( p = (p_0, \vec{p}) \), and \( (pk) = \vec{p} \cdot \vec{k} = p_i k_i \). At the one-loop order, there are two types of contributions as indicated in the Fig. 1. Here the wavy line is for the gauge field, and the solid one – for the scalar field.

![FIG. 1: Two-point function of the gauge field in the z = 2 scalar QED](image)

The tadpole graph gives

\[
\Pi_0(p) = -e^2 a^2 A_i(-p) A_j(p) \int \frac{dk_0 d^d k}{(2\pi)^{d+1}} \frac{1}{k_0^2 - a^2 k^2 - m^4} [4k_i k_j + 2\delta_{ij} \vec{k}^2 + \delta_{ij} \vec{p}^2] - e^2 A_0(-p) A_0(p) \int \frac{dk_0 d^d k}{(2\pi)^{d+1}} \frac{1}{k_0^2 - a^2 k^2 - m^4},
\]

where we have omitted the terms vanishing by symmetry reasons. Due to the rotational invariance, we can replace the \( k_i k_j \rightarrow \frac{\delta_{ij} \vec{k}^2}{d} \), so, after integrating in \( k_0 \), we have

\[
\Pi_0(p) = -e^2 a^2 A_i(-p) A_j(p) \int \frac{d^d k}{(2\pi)^d} \frac{1}{\sqrt{a^2 k^2 + m^4}} [\frac{4}{d} + 2] \vec{k}^2 + \vec{p}^2] + e^2 A_0(-p) A_0(p) \int \frac{d^d k}{(2\pi)^d} \frac{1}{\sqrt{a^2 k^2 + m^4}},
\]

or finally

\[
\Pi_0(p) = -\frac{i e^2 m^2}{(4\pi)^{d/2}} \frac{a^2}{m^4} \left[ \frac{1 - d/4}{d/2 - d/4} \right]^{1-d/4} \frac{\Gamma[d/4] \Gamma[1/2 - d/4]}{4 \Gamma[d/2] \sqrt{\pi}} \frac{p^2}{4} \frac{\Gamma[d/4] \Gamma[1/2 + d/4]}{4 \Gamma[d/2] \sqrt{\pi}} A_i(p) A_i(-p) + \frac{4}{d} + 2 \frac{m^2}{a} \frac{\Gamma[-d/4] \Gamma[1/2 + d/4]}{4 \Gamma[d/2] \sqrt{\pi}} A_i(p) A_i(-p) + e^2 \frac{(4\pi)^{d/2} m^2}{d} \frac{a^2}{m^4} \left[ \frac{1 - d/4}{d/2 - d/4} \right]^{d/4} \frac{\Gamma[d/4] \Gamma[1/2 - d/4]}{2 \Gamma[d/2] \sqrt{\pi}} A_0(p) A_0(-p).
\]
It remains to study the “fish” graph which yields three contributions, the first of them corresponds to two external $A_i$ fields, the second of them corresponds to one $A_i$ and one $A_0$ fields, and the third one to two $A_0$ fields:

$$
\Pi_1(p) = e^2 a^4 A_i(p) A_i(-p) \int \frac{dk_0 d^4 k}{(2\pi)^{d+1}} (2k_j + p_j)(2k_m + p_m) \times \left[ (k_i + p_i)(k_j + p_j) + k_ik_j \right] \times \frac{1}{[k_0^2 - a^2 \bar{k}^4 - m^4][(k_0 + p_0)^2 - a^2(\bar{k} + \vec{p})^4 - m^4]}; \quad (11)
$$

$$
\Pi_2(p) = e^2 a^2 A_i(p) A_0(-p) \int \frac{dk_0 d^4 k}{(2\pi)^{d+1}} (2k_j + p_j)(2k_0 + p_0) \times \left[ (k_i + p_i)(k_j + p_j) + k_ik_j \right] \times \frac{1}{[k_0^2 - a^2 \bar{k}^4 - m^4][(k_0 + p_0)^2 - a^2(\bar{k} + \vec{p})^4 - m^4]}. \quad (12)
$$

$$
\Pi_3(p) = e^2 A_0(p) A_0(-p) \int \frac{dk_0 d^4 k}{(2\pi)^{d+1}} (2k_0 + p_0)^2 \times \frac{1}{[k_0^2 - a^2 \bar{k}^4 - m^4][(k_0 + p_0)^2 - a^2(\bar{k} + \vec{p})^4 - m^4]}. \quad (13)
$$

To investigate the restoration of the Lorentz symmetry we expand the above expressions in Taylor series up to the second order at $p = 0$:

$$
\Pi_i(p) \approx \Pi_i(0) + \frac{p_ip_j}{2} \frac{\partial^2 \Pi_i}{\partial p_i \partial p_j} \bigg|_{p=0} + \frac{p_0^2}{2} \frac{\partial^2 \Pi_i}{\partial p_0^2} \bigg|_{p=0} + p_0 p_i \frac{\partial \Pi_i}{\partial p_0 \partial p_i} \bigg|_{p=0}. \quad (14)
$$

It is easily verified that the zeroth order terms, $\Pi_i(0)$ and $\Pi_3(0)$ are, as they should, precisely cancelled by the contributions coming from \(10\). Actually, the sum of the corresponding integrands in \(8, 11\) and \(13\) is a total derivative vanishing upon integration. Notice that, due to the parity symmetry, also $\Pi_2(0) = 0$. We then proceed by explicitly calculating the second order derivative terms (for simplicity, $k \equiv |\vec{k}|$):

$$
\frac{p_0 p_i \partial^2 \Pi_i(0)}{2 \partial p_0 \partial p_i} = e^2 a^4 \int \frac{d^d k}{(2\pi)^d} \left\{ \frac{i k^4[-(4 + d)m^8 + a^2(16 + d)m^4k^4 + 2a^4dk^8]}{d(2 + d)(m^4 + a^2k^4)^{7/2}} A_i p^2 A_i - \frac{i k^4[(28 + 12d + d^2)m^8 + 2a^2(-20 + 6d + d^2)m^4k^4 + a^4(12 + d^2)k^8]}{d(2 + d)(m^4 + a^2k^4)^{7/2}} A_i p_ip_i A_i \right\}. \quad (15)
$$
so that, by performing the integral we get
\[
\frac{p_a p_b}{2} \frac{\partial^2 \Pi_1(0)}{\partial p_a \partial p_b} = \frac{ie^2 m^2}{(4\pi)^{d/2}} \left( \frac{a^2}{m^4} \right)^{1-d/4} \frac{\Gamma[1/2 - d/4] \Gamma[2 + d/4]}{3d\sqrt{\pi} d/[2]} A_i p_i A_i
\]

(16)

By adding the part coming from the tadpole graph, we have that in coordinate space the term
\[
\frac{ie^2 m^2}{(4\pi)^{d/2}} \left( \frac{a^2}{m^4} \right)^{1-d/4} \frac{(10 + d) \Gamma[1/2 - d/4] \Gamma[1 + d/4]}{12d\sqrt{\pi} \Gamma[d/2]} A_i p_ip_i A_i.
\]

(17)

will be generated by the radiative corrections. Besides that,
\[
\frac{p_0^2}{2} \frac{\partial^2 \Pi_2(0)}{\partial p_0^2} = \frac{ie^2}{(4\pi)^{d/2} a} \left( \frac{a^2}{m^4} \right)^{1-d/4} \frac{\Gamma[1 - d/4] \Gamma[3/2 + d/4]}{\Gamma[d/2] 3d\sqrt{\pi}} A_i p_i A_i,
\]

(18)

and, concerning \( \Pi_3 \), whereas \( \frac{\partial^2 \Pi_3(0)}{\partial p_0^2} = 0 \),
\[
\frac{p_a p_b}{2} \frac{\partial^2 \Pi_3(0)}{\partial p_a \partial p_b}
= \frac{ie^2}{(4\pi)^{d/2} a} \left( \frac{a^2}{m^4} \right)^{1-d/4} \frac{\Gamma[1 - d/4] \Gamma[3/2 + d/4]}{\Gamma[d/2] 3d\sqrt{\pi}} A_i p_ip_i A_0.
\]

(19)

The other second order derivatives of \( \Pi_2 \) all vanish due to symmetry reasons. Our results indicate that at low momenta the effective action will be dominated by
\[
S_{\text{low}} = \int dt d^4x [\left( \frac{1}{2} + \alpha \right) F_{0i} F_{0i} + \beta F_{ij} F_{ij}]
\]

(21)

with \( \alpha \) and \( \beta \) fixed as above being given by
\[
\alpha = -\frac{e^2}{(4\pi)^{d/2} a} \left( \frac{a^2}{m^4} \right)^{1-d/4} \frac{\Gamma[1 - d/4] \Gamma[3/2 + d/4]}{\Gamma[d/2] 3d\sqrt{\pi}};
\]

(22)

\[
\beta = \frac{e^2 m^2}{(4\pi)^{d/2}} \left( \frac{a^2}{m^4} \right)^{1-d/4} \frac{(10 + d) \Gamma[1/2 - d/4] \Gamma[d/4]}{48\sqrt{\pi} \Gamma[d/2]}.
\]

(22)

By rescaling \( x, t, A_0 \) and \( A_i \) by the rules \( \partial_0 \rightarrow a_1 \partial_0, \partial_i \rightarrow a_2 \partial_i, A_0 \rightarrow Z_1 A_0, A_i \rightarrow Z_2 A_i \) with \( \left( \frac{1}{2} + \alpha \right) a_1^2 = -2\beta a_2^2 \), one may check that this low-energy effective action can be rewritten in the standard form
\[
S_{\text{low}} = \left( \frac{1}{2} + \alpha \right) (a_1 Z_2)^2 \int dt d^4x F_{\mu\nu} F^{\mu\nu}.
\]

(23)
The above result holds only if \( \frac{(\alpha+1/2)}{\beta} \) is negative which is satisfied if \( d = 4 + 8n \pm r \) with \( n \) an non-negative integer and \( r \in (-2, 2) \).

Therefore, we have showed that within the low-energy limit, and under special relations of parameters of the theory, the usual free Maxwell action is generated. Unlike [9], we have arrived at this result without any restrictions on the background field.

Now, let us consider the two point function for the vector field in the spinor QED. Here, the vertices are

\[
V_1 = ie\bar{\psi}\gamma^0 A_0 \psi, \quad V_2 = ie\alpha^2 \bar{\psi}\gamma^j (A_i \partial_j + A_j \partial_i) \psi = 2ie\alpha^2 \bar{\psi} A^i \partial_i \psi, \\
V_3 = ie\alpha^2 \bar{\psi}\gamma^j (\partial_j A_i) \psi = ie\alpha^2 \bar{\psi} (\partial_i A^i + \frac{1}{2} \gamma^{ij} F_{ij}) \psi, \quad V_4 = e^2 a^2 \bar{\psi} A^i A_i \psi. \tag{24}
\]

In the momentum space they look like

\[
V_1 = ie\alpha^2 \bar{\psi}(k)\gamma^0 A_0(p) \psi(-p - k), \quad V_2 = -2e\alpha^2 (p_i + k_i) \bar{\psi}(k) A^i(p) \psi(-p - k); \\
V_3 = a\alpha^2 p_i \bar{\psi}(k)\gamma^j A_j(p) \psi(-p - k), \\
V_4 = e^2 a^2 \bar{\psi}(k_1) A^i(p_1) A_i(p_2) \psi(k_2)(2\pi)^d+1 \delta(k_1 + k_2 + p_1 + p_2). \tag{25}
\]

There are two corresponding Feynman graphs depicted at Fig. 2.

![Feynman graphs](image)

FIG. 2: Two-point function of the gauge field in the \( z = 2 \) spinor QED

Here the dashed line corresponds to fermions. It is easy to see that the tadpole graph gives the following contribution (the only one involving the \( V_4 \) vertex); after a Wick rotation (by the rule \( k_0 \rightarrow ik_{0E} \)):

\[
\Pi_4(p) = -ie^2 A_i(-p) A^i(p) \int \frac{dk_{0E} d^d k}{(2\pi)^{d+1}} \frac{\text{tr}[(\gamma^0 k_{0E} + a\vec{k}^2 + m^2)]}{k_{0E}^2 + (a\vec{k}^2 + m^2)^2}. \tag{26}
\]

By symmetry reasons, and taking into account that \( \text{tr} \mathbf{1} = D \), with \( D \) being the dimension of the gamma matrices, we can rewrite this expression as

\[
\Pi_4(p) = -ie^2 DA_i(-p) A^i(p) \int \frac{dk_{0E} d^d k}{(2\pi)^{d+1}} \frac{a\vec{k}^2 + m^2}{k_{0E}^2 + (a\vec{k}^2 + m^2)^2}. \tag{27}
\]
As \( \int \frac{d\kappa_0}{\kappa_0^2 + A_0^2} = \frac{\pi}{A} \), we arrive at the complete cancellation of the factors \( a\vec{k}^2 + m^2 \) in the numerator and in the denominator. Thus, \( \Pi_4(p) \propto \int \frac{dk}{(2\pi)^d} \), but such ”integral of a constant” vanishes in the dimensional regularization. It remains to analyze the second graph. Its contribution looks like

\[
\Pi_5(p) = -\frac{e^2}{2} \text{tr} \int \frac{dk_0dk d\kappa}{(2\pi)^{d+1}} \left( \gamma^0 A_0(-p) + 2a^2 A_i(-p)k_i + a^2 p_i \gamma^i \gamma^j A_j(-p) \right) \\
\times \frac{\gamma^0 k_0 + a\vec{k}^2 + m^2}{-k_0^2 + (a\vec{k}^2 + m^2)^2} \\
\times \left( \gamma^0 A_0(p) + 2a^2 A_i(p)(k_i + p_i) - a^2 p_k \gamma^k \gamma^i A_i(p) \right) \frac{\gamma^0 (k_0 + p_0) + a(\vec{k} + \vec{p})^2 + m^2}{-(k_0 + p_0)^2 + (a(\vec{k} + \vec{p})^2 + m^2)^2}.
\]

After long but straightforward calculations, it turns out to be that both \( F_{0i}F_{0i} \) and \( F_{ij}F_{ij} \) parts of this contribution identically vanish in an arbitrary space dimension. Furthermore, we found that in \( 2 + 1 \) dimensions, up to one loop order there is no contribution to a Chern-Simons-like term. This result is strictly dependent on the absence of low order spatial derivative terms in the starting Lagrangian while otherwise, the Maxwell (and Chern-Simons) terms are generated.

It is worth to point out that in a study performed in \[13\] for a HL-like extended spinor QED, in the five-dimensional case, it was also obtained the absence of one-loop correction to the two point photon function. The basic reason why these contributions vanish is the fact that, as it happens in the usual nonrelativistic field theory, the contribution from a closed fermionic loop can be decomposed into a sum of two terms each of them having poles only in the lower half plane of the \( k_0 \) variable. Nonetheless, our action for the spinor QED differs from that one considered in \[13\].

IV. TWO POINT FUNCTIONS OF THE MATTER FIELDS

We can now calculate the two point function of the matter field. First, let us consider the scalar QED given by \[1\]. The vertices are given again by \[7\], and the Feynman diagrams are depicted at Fig. 3.

It is clear that within the framework of the dimensional regularization, the tadpole Feynman graph identically vanishes. The fish graph, after some simple arrangements, yields the
FIG. 3: Two-point function of the scalar field

following contribution from the $< A_i A_j >$ propagator

$$J_1(p) = e^{2}a^4\phi^*(-p)\phi(p) \int \frac{dk_0d^d k}{(2\pi)^{d+1}} \frac{1}{[k_0^2 - a^2k^4][(k_0+p_0)^2 - a^2(k+p)^4 - m^4]} \times (k_j + 2p_j)(k_l + 2p_l)[p_ip_j + (k_i + p_i)(k_j + p_j)][p_ip_l + (k_i + p_i)(k_l + p_l)], \quad (29)$$

and the following contribution from the $< A_0 A_0 >$ propagator:

$$J_2(p) = e^{2}a^4\phi^*(-p)\phi(p) \int \frac{dk_0d^d k}{(2\pi)^{d+1}} \frac{\vec{k}^2}{[k_0^2 - a^2k^4][(k_0+p_0)^2 - a^2(k+p)^4 - m^4]} \times (2p_0 + k_0)(-2p_0 - k_0). \quad (30)$$

After integration, we arrive at the expression for the small momenta two point function of the scalar field:

$$S_{sc} = \frac{e^{2}}{2} \int dt d^d x \bar{\phi} \left[ - \frac{(a^2(d + 2) + 2)m^d\Gamma\left(-\frac{d}{4} - 1\right)}{2^{\frac{d}{2}}\pi^{d/2}a^{\frac{d}{2} + 1}\Gamma\left(\frac{d}{2}\right)} - \frac{(d(a^2(d + 2) - d - 18) - 128)m^{d-4}\Gamma\left(-\frac{d}{4} - 2\right)}{2^{\frac{d}{2}}\pi^{d/2}a^{\frac{d}{2} + 1}\Gamma\left(\frac{d}{2}\right)} \partial_0^2 \right. \left. \right] \Delta \phi. \quad (31)$$

We conclude that for small momenta this action can be presented as

$$S_{sc} = \frac{e^{2}}{2} \int dt d^d x \bar{\phi} \left[ \alpha_1 + \alpha_2\partial_0^2 + \alpha_3\Delta \right] \phi, \quad (32)$$

where $\alpha_1, \alpha_2, \alpha_3$ are constants whose values can be read from the expression (31). Therefore, under some rescaling, the Lorentz symmetry is again restored at the low-energy limit for special relations of the parameters of the theory yielding $\alpha_3 = -\alpha_2$. We note that if $d$ is odd, this correction is finite.

The spinor QED can be treated in a similar way. In this case, we have the Feynman diagrams depicted at Fig. 4.
The explicit form of the one-loop two point function of the spinor field therefore is

\[ \omega = \int \left( -e^2 \bar{\psi} \gamma^0 < \psi \bar{\psi} > \gamma^0 < A_0 A_0 > \psi \right. \]

- \[ e^2 a^2 \bar{\psi} \gamma^j \gamma^i < \psi \bar{\psi} > \gamma^k \gamma^j \psi < \partial_i \partial_j \partial_k A_i > \]

- \[ 2e^2 a^2 \bar{\psi} < \partial_i \psi \bar{\psi} > \gamma^k \gamma^j \psi < A^i \partial_k A_i > -2e^2 a^2 \bar{\psi} \gamma^k \gamma^j \psi < \partial_k A_i A^i > < \psi \bar{\psi} > \partial_i \psi \]

- \[ 8e^2 a^2 \bar{\psi} < \partial_i \psi \bar{\psi} > \partial_k \psi < A^i A^k > \right) . \tag{33} \]

The explicit form of the one-loop two point function of the spinor field therefore is

\[ \Sigma_\psi = e^2 \int \frac{d^d k}{(2\pi)^d+1} \left[ -\bar{\psi}(-p) \gamma^0 (p_0 + k_0)^2 - (a \vec{k}^2 + m^2)^2 \right] \frac{a^2 (\vec{p} + \vec{k})^2}{(p_0 + k_0)^2 - a^2 (\vec{p} + \vec{k})^4} \psi(p) \]

+ \[ a^2 \bar{\psi}(-p) \gamma^j \gamma^i \frac{\gamma^0 k_0 + a \vec{k}^2 + m^2}{k_0^2 - (a \vec{k}^2 + m^2)^2} \gamma^k \gamma^l \psi(p)(p_i + k_i)(p_k + k_k) \delta_{j} \delta_{l} \frac{1}{(p_0 + k_0)^2 - a^2 (\vec{p} + \vec{k})^4} \]

+ \[ 2a^2 \bar{\psi}(-p) \frac{\gamma^0 k_0 + a \vec{k}^2 + m^2}{k_0^2 - (a \vec{k}^2 + m^2)^2} \gamma^k \gamma^j \psi(p)(p_k + k_k) \delta_{i} \frac{1}{(p_0 + k_0)^2 - a^2 (\vec{p} + \vec{k})^4} \]

+ \[ 2a^2 \bar{\psi}(-p) \frac{p_i(p_k + k_k) \delta_{i} \gamma^j}{(p_0 + k_0)^2 - a^2 (\vec{p} + \vec{k})^4} \left[ \frac{1}{k_0^2 - (a \vec{k}^2 + m^2)^2} \right] \gamma^0 k_0 + a \vec{k}^2 + m^2 \psi(p) \]

+ \[ 8a^2 \bar{\psi}(-p) k_i p_i \delta^{ik} \gamma^0 k_0 + a \vec{k}^2 + m^2 \left[ \frac{1}{k_0^2 - (a \vec{k}^2 + m^2)^2} \right] \frac{1}{(p_0 + k_0)^2 - a^2 (\vec{p} + \vec{k})^4} \psi(p) \]. \tag{34} \]

The superficial degree of divergence for the fish diagram is \( \omega = d - 2 \). Therefore, for \( d = 3 \), the two point function is linearly divergent. Actually, by symmetry reasons, the divergence is at most logarithmic.

As before, to verify the possible dynamical restoration of the Lorentz symmetry, we may explicitly calculate the above integral keeping only the zero and first orders in the external momentum \( p_0, p_i \). The result for the general space-time dimension, for \( a = 1 \), looks like

\[ \Sigma(p) = -ie^2 \frac{d - 3}{1(d/2)} m^{d-4} \csc\left(\frac{d\pi}{2}\right) \Gamma\left(\frac{d - 2}{4}\right) \gamma^0 \partial_0 + \frac{m^2}{2} + O(\partial^2) \psi. \]
In particular, at $d = 3$ we arrive at zero result, hence in this case there is no generation of Dirac action and thus no dynamical restoration of the Lorentz symmetry. Also, for any even space dimension more or equal than four, $d = 2k \geq 4$, we have a divergent contribution which is in accord with the result found for $d = 4$ in [13] (note nevertheless that in [13] other gauge was used).

V. THE TRILINEAR VERTEX IN THE MODIFIED SPINOR QED

To proceed with our one-loop analysis we will now study low momentum corrections to the three-point function $\langle \bar{\psi} A \psi \rangle$. That study is based in the Feynman diagram showed at Fig. 5.

FIG. 5: Three-point function in the $z = 2$ spinor QED

There are twelve contributions to it. However, following our general prescription, we restrict this study to the usual terms $\bar{\psi} \gamma^0 A_0 i \psi$, and therefore disregard all dependence on the external momenta. Thus, we have only six parts to consider, they are

\[
T_1(p_1, p_2) = e^3 \int \frac{d^d k k_0}{(2\pi)^{d+1}} \bar{\psi}(p_2) \gamma^0 G(k) \gamma^0 A_0(p) G(k + p) \gamma^0 \psi(p_1)
\times < A_0(-k + p_2) A_0(k - p_2) >;
\]

\[
T_2(p_1, p_2) = 2a^2 e^3 \int \frac{d^d k k_0}{(2\pi)^{d+1}} \bar{\psi}(p_2) \gamma^0 G(k) A_i(p)(k_i + p_i) G(k + p) \gamma^0 \psi(p_1)
\times < A_0(-k + p_2) A_0(k - p_2) >;
\]

\[
T_3(p_1, p_2) = -a^4 e^3 \int \frac{d^d k k_0}{(2\pi)^{d+1}} \bar{\psi}(p_2) \gamma^i \gamma^j G(k) \gamma^0 A_0(p) G(k + p) \gamma^k \gamma^l \psi(p_1)
\times (k_i - p_2 i)(k_k - p_2 k) < A_j(-k + p_2) A_i(k - p_2) >;
\]

\[
T_4(p_1, p_2) = -4a^4 e^3 \int \frac{d^d k k_0}{(2\pi)^{d+1}} \bar{\psi}(p_2) G(k) \gamma^0 A_0(p) G(k + p) \gamma^j \gamma^j (k - p_2)_i
\times < A_j(-k + p_2) A_i(k - p_2) > k^j \psi(p_1);
\]
\[ T_5(p_1, p_2) = -8a^6e^3 \int \frac{d^4kd\phi}{(2\pi)^{d+1}} \bar{\psi}(p_2)G(k)A_l(p)(k_1 + p_1)G(k + p)\gamma^i\gamma^j(k - p_2); \]
\[ T_6(p_1, p_2) = -2a^6e^3 \int \frac{d^4kd\phi}{(2\pi)^{d+1}} \bar{\psi}(p_2)\gamma^i\gamma^jG(k)A^r(p)(k_r + p_r)G(k + p) \times \gamma^k\gamma^l \psi(p_1) < A_j(-k + p_2)A_l(k - p_2) > (-k_i + p_2i)(k_1 - p_2). \] (36)

Here \( G(k) = < \psi(-k)\bar{\psi}(k) > \) is the spinor propagator, and \( p = -(p_1 + p_2) \) is the momentum carried by the external gauge field. By considering now that the propagators and vertices do not depend on the external momenta and taking into account that all propagators are even with respect to \( \vec{k} \), we find that only \( T_1, T_3 \) and \( T_4 \) survive. We rest with

\[ T_1(p_1, p_2) = e^3 \int \frac{d^4kd\phi}{(2\pi)^{d+1}} \bar{\psi}(p_2)\gamma^0G(k)\gamma^0A_0(p)G(k)\gamma^0\psi(p_1) \times \]
\[ \times < A_0(-k)A_0(k) >; \]
\[ T_3(p_1, p_2) = a^4e^3 \int \frac{d^4kd\phi}{(2\pi)^{d+1}} \bar{\psi}(p_2)\gamma^i\gamma^jG(k)\gamma^0A_0(p)G(k)\gamma^k\gamma^l \psi(p_1) \times \]
\[ \times k_i k_k < A_j(-k)A_l(k) >; \]
\[ T_4(p_1, p_2) = -2a^4e^3 \int \frac{d^4kd\phi}{(2\pi)^{d+1}} \bar{\psi}(p_2)G(k)\gamma^0A_0(p)G(k)\gamma^i\gamma^j k_i \times \]
\[ \times < A_j(-k)A_i(k) > k^l \psi(p_1). \] (37)

Now, let us put \( a = 1 \) for simplicity henceforth. Proceeding with calculations in an arbitrary space-time dimension \( d \), we obtain the following sum of these three expressions:

\[ T = -ie^3(d - 3)(d - 2) m^{d-4} \csc(\frac{d\pi}{2}) \pi^{1-d/2} 2^{-2-3d/2} \bar{\psi}\gamma^0 A_0 \psi. \] (38)

Therefore, it is easy to see that the contribution to the three-point function vanishes for \( d = 3 \). We see that there is no contribution involving any components of \( A_{0,i} \), what agrees with the gauge invariance of the model.

VI. SUMMARY

In this paper we calculated explicitly the low momenta contributions for the two point function of the gauge field within \( z = 2 \) spinor and scalar QED. Unlike \cite{9} where the scalar \( z = 2 \) QED in \( 2 + 1 \) dimensions has been discussed with a proper time method, we used the Feynman diagram approach. Also, we obtained explicitly the numerical factors accompanying the \( F_{0i}F_{0i} \) and \( F_{ij}F_{ij} \) contributions. We showed that the Maxwell term naturally
emerges, therefore the Lorentz symmetry is dynamically restored. Our studies differ from [9] since we, first, considered generic $d$, second, did not impose any restrictions on the gauge field.

We have also studied the two point function of the scalar and gauge fields in the $z = 2$ QED. We found that for the scalar QED, the terms of second order both in time and in space derivatives are generated as quantum corrections in the purely gauge and scalar sectors, so, after some rescaling (however, different for the gauge and scalar sectors), the restoration of the Lorentz symmetry was made explicit.

For the spinor QED, the kinetic term for the gauge field does not receive any corrections. Besides this, we calculated the two point function for the spinor field and showed that there is no terms of zero and first orders in derivatives for $d = 3$, and therefore, there is no possibility to achieve the dynamical restoration of the Lorentz symmetry in the one-loop order. We also found the three-point vertex function with results compatibles with the corrections for the two point function. Actually, for generic space dimension, we found that there are no renormalization of the charge and of the gauge field strength $A_{0,i}$; there is only a wave function renormalization of the spinor field $\psi$.

Apparently, to find the fermion anomalous magnetic moment one should consider also higher momentum corrections to the two and three point functions. We expect to do this in a forthcoming paper.

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