Disordered extra dimensions

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Abstract. A very large extra dimension may contain many localized branes. We discuss the possibility of formulating such models as a spin system where each spin indicates the supersymmetry (SUSY) direction preserved by the corresponding brane. In the evolution of the universe, the extra dimensions might have ended up as a vacuum made of patches with different orientations of the spins, responsible for the observed breaking of SUSY. We discuss the limit where the separation of these patches is by very thin defects, described as localization of gravitino masses.

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1. Introduction

It might be useful, in order to account for the complexity of the world, to embed the observed world in a higher-dimensional space. This allows, by proper engineering, to give a geometric origin for the low-energy features of models. Extra dimensions have thus been introduced about a century ago by Nordström et al [1]. Such a program has then been pursued with a renewal of interest in the last few decades (see, for example, [2]–[7]). Of particular interest for us, there are many attempts to engineer supersymmetry (SUSY) breaking mechanisms, classify them and
obtain experimental predictions. In these studies, the importance of extra dimensions depends on the relative sizes of the SUSY breaking scale \( M_{\text{SUSY}} \) and the compactification one \( M_C \). For \( M_{\text{SUSY}} \ll M_C \), one can restrict the analysis to the four-dimensional (4D) effective theory (for a review, see, for example, [8]). There, a knowledge of the data of the extra dimensions allows one to ‘understand’ the fields content and their interactions. In contrast, for \( M_{\text{SUSY}} \gg M_C \), the analysis should be performed in the higher-dimensional theory. We are interested in the latter case.

A way of breaking SUSY with extra dimensions is the Scherk–Schwarz mechanism [9], where different higher-dimensional SUSY are conserved at different points. For instance, in 5D models compactified on an interval, \( \mathcal{N} = 2 \), SUSY is preserved in the bulk, while at each of the two boundaries a different \( \mathcal{N} = 1 \) supercharge survives, leading to SUSY fully broken in the 4D effective theory (see, for example, [2, 7], [10]–[14]). Here, we shall be interested in a straightforward generalization of such a scenario, where instead of two branes at the boundaries, one deals with many branes at different points of the extra dimension(s).

Assumptions about the size, shape and content of extra dimensions turn in fact into assumptions about the very early history of the universe. As the extra dimensions are expected to have a small volume, their evolution can be thought to be short, ending before nucleosynthesis to avoid unobserved variations of fundamental constants. Even if, for such a small size, all parts are causally related, we want to argue that in the presence of branes the homogenization of this space might not have been efficient enough. In an alternative to the usual scenario of a symmetric internal space, we assume that the cooling has been very fast, maybe in a non-adiabatic way, and, as a consequence, the internal space was not driven all to a single ground state. Instead, we will assume that a situation similar to the domains of ferromagnets might arise.

In section 2, we describe the scenario of branes represented as a disordered system of spins living in the internal dimensional space. This is to be contrasted with the usual wisdom of considering symmetric internal spaces where, in a brane-world framework, the disorder consists in the presence of a single anti-brane as the source of SUSY breaking. In the disordered extra dimension set-up discussed here, the ‘defects’ separating different supersymmetric sub-spaces play a major role, as they are responsible for the breaking of SUSY. Section 3 discusses the case of one extra dimension and illustrates how a ‘domain wall’ can be approximated as a localization of a gravitino mass making further computations simpler. In fact, the effects of localized gravitino masses are well studied in one extra dimension for the case of an interval with two boundaries[15]–[17], or including many branes[18]. In contrast, the case of localized mass in six dimensions has not yet been discussed to our knowledge, and it will be treated in section 4. Most of the content of these two sections is original material that can be read independently of the rest. We end the paper with some conclusions.

2. The world as a lattice and the branes as spins

As for the observable ones, the properties of the extra dimensions, such as geometry and topology, could be determined by the very early history of the universe. Contrary to the large observed spacetime, very little is known about the evolution of the (small volume) internal space; some geometrical parameters have to be frozen before nucleosynthesis as their values are associated with those of fundamental constants, whose variations are very constrained. The interaction between different parts of the extra dimensions can be achieved very quickly.
($t < 10^{-13}$ s), as all parts can be causally connected. The possibility that some degrees of freedom in the internal space are at a finite and sizable temperature is not excluded.

The history of the internal space is assumed to proceed through three steps: (i) spacetime is nucleated, (ii) diverse bubbles are created in the extra dimensions where the branes are in excited states preserving no SUSY and (iii) while the ‘temperature’ decreases, the branes move to minimize their energy and different patches have branes that are ‘pointing’ towards a different SUSY. The internal space finishes frozen in a (meta-stable) state made of ‘domains’ described by different ground states.

The branes can be located at arbitrary points in the extra dimensions. Patches in well-defined supersymmetric ground states have branes positioned as long molecules of a liquid crystal in a nematic phase. One can also consider the branes on a regular lattice; the inter-brane spacing is then fixed by some kind of van der Waals forces [19] or by the equilibrium between forces due to a combination of electric and dyonic charged branes [20, 21]. Such mechanisms appeal to physics at scales of order or smaller than the inter-brane separation, which needs the knowledge of the field content of the fundamental theory. The existence of such a possibility to locate the branes at fixed position would imply the non-supersymmetric vacuum to be (meta)stable, and also stabilizes the size of the extra dimension [19]. As we stress again in the conclusions, achieving the stability of non-supersymmetric configurations is not obvious and remains an open issue in string theory.

For each brane $i$, we associate a vector, we will denote as the spin $\vec{S}_i$, corresponding to the central charge in the $\mathcal{N} = 2$ super-algebra, which describes the direction of the SUSY preserved by the brane. The spins $\vec{S}_i$, with unit norm $|\vec{S}_i|^2 = 1$, live in a two-dimensional space spanned by the unitary orthogonal basis vectors $(\vec{e}_X, \vec{e}_Y)$:

$$\vec{S}_i = \cos \theta_i \vec{e}_X + \sin \theta_i \vec{e}_Y$$

and we will define $\vec{e}_Z = \vec{e}_X \wedge \vec{e}_Y$.

While, in the extra dimensions discussed here, the branes appear as localized spins at some points, in other smaller dimensions they are wrapping cycles intersecting at angles that define the associated spin. In treating the branes as a spin system, one can have (i) long-range interactions (this is the case for toroidal constructions in string models); (ii) the values for the spins take discrete values. Because of these, there would be no fluctuation destruction of long-range order following the Mermin–Wagner–Berezinskii theorem, and SUSY ordering is expected in all dimensions. Moreover, the spin system is supposed to be at zero temperature nowadays. Unless stated otherwise, we will always implicitly assume the spins to take discrete values but we shall consider finite range spin–exchange interactions.

Working at an effective description, we can take the Hamiltonian of this system to have the very simple form:

$$\mathcal{H} = \sum_{i,j} J_{ij} |\vec{S}_i \wedge \vec{S}_j|^p + \sum_i a_i |\vec{S}_i \wedge \partial_t \vec{S}_i|^q - \sum_i \vec{H}_i \cdot \vec{S}_i,$$

$$= \sum_{i,j} J_{ij} |\sin (\theta_j - \theta_i)|^p + \sum_i a_i |\partial_t \theta_i|^q - \sum_i b_i \cos(\theta_i - \theta_i^{(H)}),$$

where we have used $\vec{H}_i = b_i (\cos \theta_i^{(H)} \vec{e}_X + \sin \theta_i^{(H)} \vec{e}_Y)$. Here, $\{p, q\}$ are exponents that depend on the microscopic theory and we take $p = q = 2$. 

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Let us discuss each term:

- The first term tends to align all the spins, giving a supersymmetric ground state with all the branes preserving the same $\mathcal{N} = 1$ SUSY. The interaction strength $J_{ij}$ is positive with strength and action range depending on the microscopic model.

- The second term is included to take into account that the rotation of the spin requires a rotation of the brane, which costs a nonzero amount of energy. This is because the SUSY preserved by the brane is associated with its orientation in other smaller extra dimensions. The rigidity $a_i$ could be related to the brane tension, and it need not be the same at each point.

- The third term describes the effect of the background fields. It will determine the nontrivial vacuum structure as it leads to constraints that pull the branes towards a specific direction. This can contain, for example, the effect of the presence of specific boundary conditions on the internal space (as orientifold objects).

It is the last term of equation (2.1) that parametrizes the source of SUSY breaking. Our aim then would be to find simple forms of $\vec{H}_i$ that, on the one hand, lead to a minimum of the Hamiltonian with broken SUSY and, on the other hand, can a posteriori be understood at the level of a microscopic fundamental theory.

The system of $N$ spins with constrained boundaries can be thought to evolve during the early time from the initial conditions as

$$\frac{\partial \vec{H}_i}{\partial t} = \vec{F}_i(\vec{H}_j, \vec{S}_j) \quad \text{with} \quad \vec{H}_0(t) = \vec{H}_0, \quad \vec{H}_N(t) = \vec{H}_N,$$

where the exact form of $\vec{F}_i$ requires the knowledge of the microscopic theory. A simple phenomenological approximate of such an equation could take the form

$$\vec{F}_i(\vec{H}_j, \vec{S}_j) = \sum_j \alpha_{ij} \vec{H}_j \wedge \vec{H}_i + \sum_j \beta_{ij} \vec{S}_j \wedge \vec{H}_i$$

that tends to bring the vacuum to aligned spins but for the imposed non-supersymmetric boundary conditions. With appropriate $\alpha_{ij}, \beta_{ij}$, one can achieve a background made of domains preserving different SUSY.

As stated above, the problem of SUSY breaking is now formulated as the problem of obtaining appropriate forms of the constraints $\vec{H}_i$ and studying their properties as the phase space and correlation function in the corresponding spin model.

Examples of constraints that allow us to obtain ground states with domains where the spins point towards different directions can be constructed by having either a strong localized force or a weak long-range force, in both cases opposing the effect of the first term of (2.1). We will illustrate this through examples for one extra dimension.

The first option can be realized as

$$\vec{H}_i = \sum_j b_j \exp \left[ -\frac{(y_i - Y_j)^2}{\Delta} \right] (f_j(y)\vec{e}_X + g_j(y)\vec{e}_Y), \quad b_j \gg J_{ij},$$

where $f_j$ and $g_j$ are slowly varying functions that force the spins to change directions around the point $Y_j$. For example, if the points $Y_j = 0, \pi R$ are the boundaries of a compact dimension, we can take

$$\vec{H}_i = b_0 \exp \left[ -\frac{(y_i)^2}{\Delta} \right] \vec{e}_X + b_\pi \exp \left[ -\frac{(y_i - \pi R)^2}{\Delta} \right] \vec{e}_Y.$$

Another example is used in the next section.
The second option can also be obtained from the interplay of weak and long-range forces. For instance, the aligning force is taken to be effective only between nearest neighbors, while it parametrizes an anti-alignment force at long range:

$$\vec{H}_i = \sum_{j \neq i} \frac{b}{(y_j - y_i)^\alpha} \vec{S}_j,$$

where $\alpha$ is a positive integer. We take $J_{ij}$ to be a nearest-neighbor interaction, and $b$ as positive and small, $b \ll J$; we see that the energy of the system is lowered for spins aligned in domains, but a large amount of such spins generates an interaction that tends to flip the spins. The net result is the formation of domains with different alignments separated by transition regions.

In this picture of disordered extra dimension, the SUSY breaking is localized in the ‘defects’ separating different supersymmetric sub-spaces. This is the subject of the following sections. We will assume that the fundamental length scale $\kappa$, the inter-brane separation $d_i$ and the compactification scale $R$ are well separated $\kappa \ll d_i \ll R$. The first inequality allows us to neglect quantum geometry effects and the second to get a large number of branes. When needed for the purpose of explicit computations, we will also use a flat metric.

3. A defect in one extra dimension

We consider the spacetime extended with a fifth dimension with the coordinate $y \in [0, \pi R]$. Along this direction, $N+1$ branes are located at the points $y = y_i, i = 0 \ldots N$ with $y_0 = 0$, $y_N = \pi R$ and $y_n < y_{n+1}$.

We assume that the evolution of the universe has ended in a non-supersymmetric vacuum. The branes orientation varies when going from one boundary to the opposite boundary, such that the distribution of the associated spins has the form of a localized kink. Obviously, nontrivial boundary conditions are needed such that at different regions the (spin) branes have to point in different directions. An abrupt separation of domains would be costly in spin-exchange energy. In fact, with the combination of boundary conditions, spin exchange and long-range interactions, the minimization of the total energy will induce a kink with a certain thickness localized around a point $Y$, as for ferromagnets. As translation invariance is broken by both the presence of boundaries and the conditions imposed to break SUSY, the minimization of the total energy will fix the value of $Y$ (i.e. gives masses to the collective modes of the soliton). Because we are not interested here in this issue of fixing the moduli, we choose a final configuration and illustrate how it can be parametrized as the result of an applied ‘magnetic field’ $H_i$ on the spin system. Our aim will be to show how our order parameter, the gravitino mass, is related to the final spin configuration.

We will first show how a kink localized at a single position $Y$ can be described as if the branes are embedded in a background field $\vec{H}_i$. We consider the constraint:

$$\vec{H}_i = b \exp \left[ -\frac{(y_i - Y)^2}{\Delta^2} \right] \left( \cos \left( \frac{Y}{\Delta} \right) \vec{e}_X + \sin \left( \frac{Y}{\Delta} \right) \vec{e}_Y \right),$$

which means that at the boundary $y = 0$, the spins are forced to point in the direction $\vec{e}_X$, while at $y = \pi R$, the spins are forced to point along $\vec{e}_Y$, as well as

$$J_{ij} = J[\delta_{j,i+1} + \delta_{j,i-1}], \quad a_i = a.$$
The Hamiltonian is given by
\begin{equation}
\mathcal{H} = \sum_i J \sin^2 (\theta_i - \theta_{i+1}) + \sum_i a |\partial_i \theta_i|^2 - \sum_i b \exp \left[ - \frac{(y_i - Y)^2}{\Delta} \right] \cos \left( \theta_i - \frac{y_i}{\Delta} \right). \tag{3.3}
\end{equation}

The minimum of the potential is obtained for
\begin{equation}
\sin 2(\theta_i - \theta_{i+1}) = -\frac{b}{J} \exp \left[ - \frac{(y_i - Y)^2}{\Delta} \right] \sin \left( \theta_i - \frac{y_i}{\Delta} \right). \tag{3.4}
\end{equation}

Taking \( b \gg J \), we obtain the following limits.

In the vicinity of \( Y \), \( y_i \sim Y \), we can approximate
\begin{equation}
\sin \left( \theta_i - \frac{y_i}{\Delta} \right) = -\frac{J}{b} \sin 2(\theta_i - \theta_{i+1}) \rightarrow 0, \quad \text{i.e.} \quad \theta_i \rightarrow \frac{y_i}{\Delta}, \tag{3.5}
\end{equation}

while elsewhere:
\begin{equation}
\sin 2(\theta_i - \theta_{i+1}) \rightarrow 0, \quad \text{i.e.} \quad \theta_i \rightarrow \theta_{i+1}. \tag{3.6}
\end{equation}

This means that we have two patches of aligned spins, separated by a region of size of order \( 2\Delta' \).

In this transition region, the spins rotate by an angle of order \( 2\Delta'/\Delta \), which if it is not a multiple of \( \pi \), implies that the two patches preserve two different SUSYS. It is only this quantity that will be relevant for our purpose. Of course, one can compute the energy carried by the interface between the two domains, and study the process of bubbles nucleation, the size of the interface or the processes of homogenization, all well-known issues.

We are interested in describing here the effect of the simplest such defect in a five-dimensional supergravity. The total action is given by the sum of a bulk and a brane component:
\begin{equation}
S = \int_0^{2\pi R} dy \int d^4x \left[ \frac{1}{2} \mathcal{L}_{\text{BULK}} + \sum_{i=0}^{N} \mathcal{L}_i \delta(y - y_i) \right]. \tag{3.7}
\end{equation}

The brane \( n \) will be characterized by the SUSY it preserves, which is correlated with the choice of the couplings to the bulk operators, in particular the gravitino. The non-vanishing set of such operators \( \Phi_{\text{even}} \) is determined as those being even under a \( \mathbb{Z}_2 \) action at the point \( y = y_i \):
\begin{equation}
\Phi_{\text{even}}(y_i + y) = \mathcal{P}_i \Phi_{\text{even}}(y_i - y) = \Phi_{\text{even}}(y_i - y). \tag{3.8}
\end{equation}

The operators might themselves be made of products of even numbers of odd fields. The SUSY preserved by the brane \( i \), associated with the ‘spin’ \( \vec{S}_i \), can be read from the gravitino components \( \psi'^{\mu+}_i \) that couple with it, while it breaks the orthogonal SUSY direction associated with \( \psi'^{\mu-}_i \). We can choose a basis \((\psi^{\mu+}_i, \psi^{\mu-}_i)\) for the \( \mathcal{N} = 2 \) gravitino in terms of two-component spinors and define
\begin{equation}
\begin{align*}
\psi^{\mu+}_i &= \cos 2\theta_i \psi^{\mu+}_1 + \sin 2\theta_i \psi^{\mu+}_2, \\
\psi^{\mu-}_i &= -\sin 2\theta_i \psi^{\mu+}_1 + \cos 2\theta_i \psi^{\mu+}_2, \\
\psi^{\nu+}_i &= \sin 2\theta_i \psi^{\nu+}_1 + \cos 2\theta_i \psi^{\nu+}_2, \\
\psi^{\nu-}_i &= \cos 2\theta_i \psi^{\nu+}_1 - \sin 2\theta_i \psi^{\nu+}_2.
\end{align*} \tag{3.9}
\end{equation}

Without loss of generality, we will take \( \theta_0 = 0 \). The spin \( \vec{S}_i \) of the chain makes an angle \( \theta_i \) with \( \vec{S}_0 \).
We shall now consider the case of a single domain wall separating two phases. The generalization to more than one domain is straightforward. At leading order, we shall consider the brane-world volumes to be supersymmetric. The whole SUSY breaking is then concentrated in the transition interval \([y_n, y_{n+1}]\) of length \(d_n\). It is encoded in the wave function of the gravitino which interpolates between the two values \(\psi_{\mu_+}^n\) and \(\psi_{\mu_+}^{n+1}\). This variation is associated with a gravitino mass \(M_n(\theta_n, d_n)\). We take a configuration where

- all spins \(S_i, 0 \leq i \leq n\), are aligned with \(\vec{S}_0\); thus \(\theta_0 = \cdots = \theta_n = 0\). We will denote this as phase A.
- all spins \(S_i, n + 1 \leq i \leq N\), are aligned with \(\vec{S}_N\); thus \(\theta_{n+1} = \cdots = \theta_N = 2\theta\). We will denote this as phase B.

For the purpose of the illustration, we can take the extra dimension to be flat; thus \(M_n(\theta, d_n) = \theta/d_n\). The gravitino wave function associated with the SUSY preserved on the left side of the defect is given by

\[
\psi_{\mu_1}(y) = \begin{cases} 
1, & \text{for } y \in [0, y_n], \\
\cos \left(\frac{y - y_n}{d_n} 2\theta\right), & \text{for } y \in [y_n, y_{n+1}], \\
\cos 2\theta, & \text{for } y \in [y_{n+1}, \pi R],
\end{cases}
\]  
(3.10)

while the orthogonal one is given by

\[
\psi_{\mu_2}(y) = \begin{cases} 
0, & \text{for } y \in [0, y_n], \\
\sin \left(\frac{y - y_n}{d_n} 2\theta\right), & \text{for } y \in [y_n, y_{n+1}], \\
\sin 2\theta, & \text{for } y \in [y_{n+1}, \pi R]
\end{cases}
\]  
(3.11)

such that the right side of the defect preserves the combination: \(\cos 2\theta \psi_{\mu_1} + \sin 2\theta \psi_{\mu_2}\).

For a bulk observer outside the domain \([y_n, y_{n+1}]\), in the limit \(d_n \ll \pi R\) the breaking of SUSY can be accounted for by a variation of the bulk field (here the gravitino’s) wave function between the points \(y_n\) and \(y_{n+1}\). We would like to describe the wave function outside the defect in the limit where the latter can be considered as point-like, i.e. \(d_n \to 0\).

In this limit, we describe the 5D gravitino by two wave functions: a continuous one \(\Psi_c\) that couples with the defect with a mass \(M_n\) and a discontinuous one \(\Psi_D\) that does not couple. We can find the respective values building these functions in the interval \([y_n, y_{n+1}]\):

\[
\Psi_{\mu_1}^{cn}(y) = c_\theta \psi_{\mu_1}^n(y) + s_\theta \psi_{\mu_2}^n(y),
\]  
(3.12)

\[
\Psi_{\mu_1}^{dn}(y) = s_\theta \psi_{\mu_1}^n(y) - c_\theta \psi_{\mu_2}^n(y),
\]  
(3.13)

where

\[
c_\theta = \cos \theta, \quad s_\theta = \sin \theta.
\]  
(3.14)

In the limit \(d_n \to 0\), \(y_n = y_{n+1} = Y_n\), the gravitino component that couples with the defect domain wall at \(y_n = y_{n+1} = Y_n\) is given by the even wave function value:

\[
\Psi_{\mu_1}^{cn}(Y_n^-) = \Psi_{\mu_1}^{cn}(y_n) = c_\theta = \Psi_{\mu_1}^{cn}(Y_n^{+}) = \Psi_{\mu_1}^{cn}(Y_n^+),
\]  
(3.15)
while the orthogonal component
\[
\Psi^D_\mu(y_n) = s_0 = -\Psi^D_\mu(y_{n+1})
\] (3.16)
is odd and corresponds to the gravitino component that does not couple with the defect.

We can now build, in this limit, an effective wave function:
\[
\Psi^C_\mu(y) = \begin{cases} 
  c_\theta, & \text{for } y \in [0, Y_n], \\
  s_\theta, & \text{for } y \in [Y_n, \pi R]
\end{cases}
\] (3.17)

orthogonal to
\[
\Psi^D_\mu(y) = \begin{cases} 
  c_\theta, & \text{for } y \in [0, Y_n], \\
  -s_\theta, & \text{for } y \in [Y_n, \pi R]
\end{cases}
\] (3.18)

The breaking of SUSY by defect can now be described as due to a localized mass term \(M_n\). Such a localized mass gives rise to the equations of motion for the gravitinos \(\Psi^{I_n}_\mu\) (we assume \(e^5_5 = 1\)):
\[
\begin{align*}
\partial_5 \Psi^I_\mu + m_{3/2}^I \Psi^C_\mu &= 2M_n \Psi^C_\mu \delta(Y_n), \\
\partial_5 \Psi^C_\mu - m_{3/2}^I \Psi^D_\mu &= 0,
\end{align*}
\] (3.19)

where we have used the 4D equation of motion for gravitinos of mass \(m_{3/2}^I\):
\[
\epsilon^{\mu \nu \rho \lambda} \sigma_\nu \partial_\rho \Psi^I_\lambda = -2m_{3/2}^I \sigma^{\mu \nu} \Psi^I_\nu, \quad I = C, D.
\] (3.20)

It can be clearly seen from equations (3.19) that while \(\Psi^C_\mu\) is a continuous field, \(\Psi^D_\mu\) has a jump at the point \(y = Y_n\), its first derivative being proportional to a Dirac \(\delta\) distribution. We can then identify the gravitino mass as
\[
M_n = \kappa^{-1} \tan \theta_{n+1} = \kappa^{-1} (\vec{S}_{n+1} \wedge \vec{S}_n) \cdot \vec{e}_Z / (\vec{S}_{n+1} \cdot \vec{S}_n),
\] (3.21)

Given the knowledge of the localized gravitino mass in 5D, we can use this result to derive the 4D gravitino mass,
\[
m_{3/2} = \frac{1}{\pi R} \arctan \left( \frac{(\vec{S}_{n+1} \wedge \vec{S}_n) \cdot \vec{e}_Z}{(\vec{S}_{n+1} \cdot \vec{S}_n)} \right).
\] (3.22)

As an illustration of this simple case, let us consider \(\kappa \lesssim d_n \sim \text{TeV}^{-1}\). The observable sector lives on a 4-brane extended between the two points \(y_n\) and \(y_{n+1}\), which is part of a large extra dimension of size \(\pi R\) responsible for the hierarchy between the string and the Planck length. We can use the previous example to see that the resulting gravitino mass is \(\theta_{n+1}/\pi R\). In the case of a system of brane–anti-brane, \(\theta_{n+1} = \pi/2\) leads to \(m_{3/2} = 1/2R\). As explained in [18] for the case of an explicit localized \(F\)-term, the breaking cannot be compensated for by opposite twists in other parts of the extra dimension.

4. A localized defect in two dimensions

In this section, we will illustrate the case of supergravity with two extra dimensions, i.e. in six dimensions. The defects can be either a 1D curve or a point. The latter can appear in the spin system as the zero size limit of a vortex. The gravitino wave functions can be taken
in the absence of SUSY breaking as a holomorphic (or anti-holomorphic) function of the complex coordinate \( z = x^5 + ix^6 \) describing the two extra dimensions. When a gravitino mass \( m_0 \) localized at the point \( z_0 \) is included for the component \( \psi_{\mu_1} \), it appears as a flux in the circulation of the gravitino wave function \( \psi_{\mu_2} \), of the form

\[
\oint_{\partial S} \psi_{\mu_2} dz = -im_1 \int_S \psi_{\mu_1} dx^5 dx^6 = -2im_0 \psi_{\mu_1}(z_0),
\]

where \( S \) is a surface containing the point \( z_0 \) and having as boundary \( \partial S \), while \( m_1 \) is the bulk mass appearing in the equation of motion of \( \psi_{\mu_1} \). In this section, we will derive the resulting lightest 4D gravitino mass.

The two extra dimensions are taken compactified on the orbifold \( T^2/\mathbb{Z}_2 \) parametrized by the coordinates \((x^5, x^6)\). The torus \( T^2 \) coordinates obey \((x^5, x^6) \equiv (x^5 + 2\pi m R_5, x^6 + 2\pi n R_6)\), \((m, n) \in \mathbb{Z}\), and the orbifold is obtained through the identification \((x^5, x^6) \equiv -(x^5, x^6)\). There are four fixed points of this action at \((0, 0)\), \((\pi R_5, 0)\), \((0, \pi R_6)\) and \((\pi R_5, \pi R_6)\). We will consider the simplest case with a single defect located at the origin \((x^5, x^6) = (0, 0)\).

The bulk Lagrangian volume must describe the 6D supergravity. The supermultiplets of supergravity in six dimensions in its minimal form are the sechsbein \( e^a_\mu \), the gravitino \( \Psi_m \), a real scalar field \( \Phi \), a fermion \( X \) and the Kalb–Ramond two-form denoted by \( B_{MN} \) that gives rise to the three-form \( H = 3\partial_B \Phi \). The action of supergravity in the volume is \( N = 2 \) supersymmetric as it preserves eight supercharges. Our study focuses on the gravitino. Its standard kinetic term reads

\[
\mathcal{L}_{\text{kin}} = -ie_6M_6^2\overline{\Psi}_M\Gamma^{MNP}D_N\Psi_P,
\]

where \( M_6 = \kappa^{-1} \) is the fundamental Planck mass in six dimensions and \( E_6 \) the sechsbein determinant. It is useful to express this in two-component spinor notation:

\[
\begin{align*}
\mathcal{L}_{\text{kin}} &= \kappa^{-2}e_6 \left[ \frac{1}{2} \epsilon^{\mu
u\rho} \left( \overline{\Psi}_{\mu_1} \overline{\sigma}_\nu D_\rho \psi_{\rho_1} + \overline{\Psi}_{\mu_2} \overline{\sigma}_\nu D_\rho \psi_{\rho_2} \right) + \psi_{\mu_1} \sigma^{\mu\nu} \left( D_\nu + iD_\rho \right) \psi_{\rho_1} \\
&- \psi_{\mu_2} \sigma^{\mu\nu} \left( D_\nu + iD_\rho \right) \psi_{\rho_1} - \left( \psi_{\rho_1} + i\psi_{\rho_2} \right) \sigma^{\mu\nu} D_\rho \psi_{\rho_2} + \left( \psi_{\rho_2} + i\psi_{\rho_1} \right) \sigma^{\mu\nu} D_\rho \psi_{\rho_1} \\
&- \psi_{\mu_1} \sigma^{\mu\nu} D_\nu \left( \psi_{\rho_1} + i\psi_{\rho_2} \right) + \psi_{\mu_2} \sigma^{\mu\nu} D_\nu \left( \psi_{\rho_1} + i\psi_{\rho_2} \right) \right] + \text{h.c.}
\end{align*}
\]

To define this theory in the orbifold \( T^2/\mathbb{Z}_2 \), we must impose parity of various fields under the action of the symmetry \( \mathbb{Z}_2 \) in a manner consistent with the action of supergravity and SUSY transformations. Expressing \( \Psi_m \) and \( X \) in two-component spinor notation: (i) the fields \( e_\mu^i \), \( e_\mu^i \), \( B_{\mu\nu} \), \( \Phi \), \( \psi_{\mu_1} \) and \( \psi_{\mu_2} \) and \( \chi_1 \) are taken even under \( \mathbb{Z}_2 \) and (ii) the fields \( e_\mu^i \), \( e_\mu^i \), \( B_{\mu\nu} \), \( \psi_{\mu_1} \) \( \psi_{\mu_2} \), \( \psi_{i1} \) and \( \chi_2 \) are odd under the \( \mathbb{Z}_2 \) action. Here the indices \( i \) and \( j \) denote coordinates of the extra dimensions: \( i, j \in \{ 5, 6 \} \).

The defect is located at the fixed point of the orbifold \((x^5, x^6) = (0, 0)\); therefore only the operators even under the \( \mathbb{Z}_2 \) action couple with it. We are interested in the case where a constant localized 4D gravitino mass:

\[
\mathcal{L}_{\text{mass}} = -e_4 \delta(x^5) \delta(x^6) \left( M_0 \psi_{\mu_1} \sigma^{\mu\nu} \psi_{\rho_1} + \text{h.c.} \right)
\]

is present, as well as new bi-linear terms that mix the 4D gravitino \( \psi_{\mu_1} \) with the internal dimensional components \( \psi_{\rho_2} \) and \( \psi_{\rho_2} \). The constant \( M_0 \) is proportional to the value \( W_0 \) of the localized superpotential: \( M_0 = \sqrt{g_{\mathbb{Z}_2}} W_0 \). A necessary step is gauge fixing. A possible choice is the unitary gauge where the terms bi-linear mixing the 4D gravitino \( \psi_\mu \) fields with \( \psi_5 \) and \( \psi_6 \)
are absent, so that the part of the Lagrangian that describes the bi-linear terms for the gravitino is given by

\[ \mathcal{L}_{K_{+m}} = \kappa^{-2} \left[ \frac{1}{2} \epsilon^{\mu
u\lambda\sigma} \left( \psi_{\mu1} \sigma_{\nu} \partial_{\lambda} \psi_{\lambda1} + \bar{\psi}_{\mu2} \sigma_{\nu} \partial_{\lambda} \psi_{\lambda2} \right) + 2 \psi_{\mu1} \sigma^{\mu \nu} (\partial_{\xi} + i \partial_{\eta}) \psi_{\nu2} \right] - \delta(x^5) \delta(x^6) M_0 \psi_{\mu1} \sigma^{\mu \nu} \psi_{\nu1} + \text{h.c.} \]  

(4.5)

To study the properties of the gravitino there are two approaches: one can study its equations of motion and boundary conditions as done in the 1D case or we can study the theory reduced to four dimensions. In this section, we follow the second method.

First, we Fourier expand the gravitinos \( \psi_{\mu1}(x^\mu, x^5, x^6) \) and \( \psi_{\mu2}(x^\mu, x^5, x^6) \), taking into account their parities under the \( \mathbb{Z}_2 \) action:

\[ \psi_{\mu1}(x^\mu, x^5, x^6) = \frac{\kappa}{\sqrt{\pi^2 R_5 R_6}} \left[ \frac{1}{2} \psi^0_{\mu1}(x^\mu) + \sum_{p, q \in Y} \psi_{p, q}^{\mu1}(x^\mu) \cos \left( \frac{p x^5}{R_5} + \frac{q x^6}{R_6} \right) \right], \]

\[ \psi_{\mu2}(x^\mu, x^5, x^6) = \frac{\kappa}{\sqrt{\pi^2 R_5 R_6}} \sum_{p, q \in Y} \psi_{p, q}^{\mu2}(x^\mu) \sin \left( \frac{p x^5}{R_5} + \frac{q x^6}{R_6} \right) \]  

(4.6)

with the sum over \( Y \) defined as \( \sum_{p, q \in Y} = \sum_{p = -\infty}^{p = +\infty} \sum_{q = -\infty}^{q = +\infty} \left[ \sum_{q = -1}^{q = +1} \right]_{p = 0} \). When plugged in (4.5), it gives

\[ \mathcal{L}_{K_{+m}} = \frac{1}{2} \epsilon^{\mu
u\lambda\sigma} \left[ \psi^{0}_{\mu1} \sigma_{\nu} \partial_{\lambda} \psi^{0}_{\lambda1} + \sum_{p, q \in Y} \psi_{p, q}^{\mu1} \sigma_{\nu} \partial_{\lambda} \psi_{\lambda1}^{p, q} + \sum_{p, q \in Y} \psi_{p, q}^{\mu2} \sigma_{\nu} \partial_{\lambda} \psi_{\lambda2}^{p, q} \right] \]

\[ - \frac{M_0 \kappa^2}{\pi^2 R_5 R_6} \left[ \frac{1}{2} \psi^0_{\mu1} + \sum_{k, l \in Y} \psi_{k, l} \right] \sigma^{\mu \nu} \left[ \frac{1}{2} \psi_{\nu1} + \sum_{p, q \in Y} \psi_{p, q} \right] \]

\[ + 2 \sum_{p, q \in Y} \psi_{p, q}^{\mu1} \sigma^{\mu \nu} \left( \frac{p}{R_5} + \frac{q}{R_6} \right) \psi_{\nu2} + \text{h.c.} \]  

(4.7)

Note that the phases in the masses that appear in the Lagrangian have no physical consequences, because the phases of the masses of Kaluza–Klein \( \frac{p}{R_5} + \frac{q}{R_6} \) can be eliminated by the redefinition of fields \( \psi_{\mu1}^{p, q} \). A phase in localized masses \( M_0 \) may also be eliminated by a redefinition of the fields \( \psi_{\mu1}^{0} \) and \( \psi_{\nu1}^{p, q} \). We can then do the following substitutions:

\[ \frac{p}{R_5} + \frac{q}{R_6} \rightarrow \sqrt{\frac{p^2}{R_5^2} + \frac{q^2}{R_6^2}} = m_{p, q}. \]  

(4.8)

with a change of basis \( \psi_{\mu1}^{\pm} = \frac{1}{\sqrt{2}} \left[ \psi_{\mu1}^{p, q} + \psi_{\mu2}^{p, q} \right] \) and \( \psi_{\mu1}^{-} = \frac{1}{\sqrt{2}} \left[ \psi_{\mu1}^{p, q} - \psi_{\mu2}^{p, q} \right] \).

In the new basis \( \psi_{\mu1}^{\pm}, \psi_{\mu1}^{-}, \psi_{\mu2}^{\pm}, \psi_{\mu2}^{-} \) the bi-linear terms can be expressed as

\[ \mathcal{L}_{K_{+m}} = \frac{1}{2} \sum_{\mu} \epsilon^{\mu
u\lambda\sigma} \psi_{\mu1}^{\sigma} \partial_{\nu} \psi_{\lambda1} - \sum_{\mu} \psi_{\mu1}^{\sigma} M_{3/2} \psi_{\nu1}^{\sigma} + \text{h.c.} \]  

(4.9)

and the gravitino mass matrix takes the form

\[ M_{3/2} = \begin{pmatrix} m_0 & m_0 & m_0 & m_0 \\ m_0 & m_0 - \delta_{k/l} \delta_{m, q} m_{p, q} & m_0 & m_0 + \delta_{k/l} \delta_{m, q} m_{p, q} \\ m_0 & m_0 & m_0 & m_0 - \delta_{k/l} \delta_{m, q} m_{p, q} \\ m_0 & m_0 + \delta_{k/l} \delta_{m, q} m_{p, q} & m_0 & m_0 \end{pmatrix}, \quad m_0 = \frac{M_0 \kappa^2}{2 \pi^2 R_5 R_6}. \]  

(4.10)
We will now diagonalize the mass matrix and obtain the eigenvalues and eigenvectors. We denote by $\Psi_m$ the eigenvector associated with the eigenvalue $m$. It can be written in the above basis (4.9) as $\Psi_m = (\psi^{0}_{m}, \psi^{p}_{m}, \psi^{q}_{m})$. With these notations, the equations that define the vectors and eigenvalues of the mass matrix $M_{3/2} \Psi_m = m \Psi_m$ take the form

$$m_0 \begin{bmatrix} \psi^{0}_{m} + \sum_{p,q \in Y} \psi^{p,q}_{m+} + \sum_{p,q \in Y} \psi^{p,q}_{m-} \\ m_0 \psi^{0}_{m} + \sum_{p,q \in Y} \psi^{p,q}_{m+} + \sum_{p,q \in Y} \psi^{p,q}_{m-} \\ m_0 \psi^{0}_{m} + \sum_{p,q \in Y} \psi^{p,q}_{m+} + \sum_{p,q \in Y} \psi^{p,q}_{m-} \end{bmatrix} = m \psi^{0}_{m},$$

$$m_0 \begin{bmatrix} \psi^{0}_{m} + \sum_{p,q \in Y} \psi^{p,q}_{m+} + \sum_{p,q \in Y} \psi^{p,q}_{m-} \\ -m_0 \psi^{k,l}_{m+} = m \psi^{k,l}_{m+}, \\ m_0 \psi^{0}_{m} + \sum_{p,q \in Y} \psi^{p,q}_{m+} + \sum_{p,q \in Y} \psi^{p,q}_{m-} \end{bmatrix} + m_0 \psi^{k,l}_{m-} = m \psi^{k,l}_{m-}. \tag{4.11}$$

Some straightforward algebra leads then to the eigenvalues equation:

$$\sum_{p,q=-\infty}^{+\infty} m_0 \frac{m_0^2 - p_5^2 R_5^2 - q_6^2 R_6^2}{m_0^2 - p_5^2 R_5^2 - q_6^2 R_6^2} = \frac{1}{m}. \tag{4.12}$$

We note that the double infinite sum in this equation has a logarithmic divergence. A regularization procedure is needed and leads to a result dependent on the ultraviolet cutoff. A ‘truncation’ of the sum leads to the lowest eigenvalue gravitino mass:

$$\frac{1}{m_0} \approx -\pi R_5 R_6 \ln \left( \Lambda^2 R^2 \right) + \frac{1}{m_0^2}, \tag{4.13}$$

when taking $R \approx R_5 \approx R_6$. Retaining only the dominant terms in $M_0^2 \kappa^4 \ln(\Lambda R) / R_5 R_6$, we obtain:

$$\frac{1}{m} \approx \frac{1}{m_0} + \frac{M_0^2 \kappa^2}{\pi} \ln \left( \Lambda R \right). \tag{4.14}$$

We see that for a small size of the extra dimensions ($\Lambda \sim 1$), we recover the effective 4D result $M_{3/2} \sim m_0$. On the other hand, for very large radius (typically $\ln(\Lambda R) > 2\pi$), we can instead have

$$M_{3/2} \sim \frac{m_0}{2\pi m_0^2 R^2 \ln \left( \Lambda R \right)}, \tag{4.15}$$

which is reduced compared to $m_0$.

We describe now the wave functions for eigenstates of the gravitinos. According to (4.6) the eigenvectors of the mass matrix $M_{3/2}$ (we have denoted $\psi_{m\mu}$) can be written as

$$\psi_{m\mu_1}(x^\mu, x^5, x^6) = \frac{\kappa N e^{i\delta}}{\sqrt{\pi^2 R_5 R_6}} \left[ \frac{1}{\sqrt{2}} + \sum_{p,q \in Y} \frac{\sqrt{2m^2 \cos \left( \frac{p x^5}{R_5} + \frac{q x^6}{R_6} \right)}}{m^2 - p_5^2 R_5^2 - q_6^2 R_6^2} \right] \chi_{m\mu}(x^\mu), \tag{4.16}$$

$$\psi_{m\mu_2}(x^\mu, x^5, x^6) = -\frac{\kappa N m \sqrt{2}}{\sqrt{\pi^2 R_5 R_6}} \sum_{p,q \in Y} \frac{\sqrt{p_5^2 R_5^2 + q_6^2 R_6^2}}{m^2 - p_5^2 R_5^2 - q_6^2 R_6^2} e^{i\pi p-q} \sin \left( \frac{p x^5}{R_5} + \frac{q x^6}{R_6} \right) \chi_{m\mu}(x^\mu). \tag{4.17}$$

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In these expressions, the field $\chi_{m\mu}$ is a spinor that does not depend on extra dimensions $(x^5, x^6)$. It is a massive spin-3/2 state with mass $m$ as given by equation (4.12). The phases $e^{i\beta}$ and $e^{i\alpha p,q}$ are given by

$$e^{i\beta} = \sqrt{\frac{|M_0|}{M_0}}, \quad e^{i\alpha p,q} = \sqrt{\frac{p^2}{R_5^2} + \frac{q^2}{R_6^2}} e^{-i\beta}. \quad (4.18)$$

The normalization constant $N$ can be determined by imposing the unitary standard eigenvector $\Psi_m = \Psi^0_m, \psi_{m1}^{p,q}, \psi_{m2}^{p,q}$.

$$N = \left[ \sum_{p,q=\pm \infty}^{+\infty} \frac{2m^2}{(m^2 - \frac{p^2}{R_5^2} - \frac{q^2}{R_6^2})^2} - \frac{m}{m_0} \right]^{-1}. \quad (4.19)$$

One can check explicitly that the wave functions (4.16) and (4.17) are solutions of the equations of motion of the gravitinos in six dimensions.

The divergence in the tree level computation (4.22) of the gravitino brane mass arises here because of the $\delta(x_5)\delta(x_6)$ singularity due to the zero brane thickness limit. It shows that the field theory considered here is not a valid description of the physics in the UV as the internal structure of the brane cannot be neglected. This behavior is well known, it has been encountered in [22] (see also [23]), and it was shown that in a fundamental theory, as in string models, it is finite, regularized by an effective thickness [22, 24].

The sensitivity to cutoff scale of the theory can be interpreted as a classical running of the mass parameter between the cut-off and the compactification scales, and it can be re-summed. It was studied in 6D models with orbifold compactifications as a tree level renormalization of brane coupling constants [25]. While these properties have been discussed for particles of spin-0 and -1/2, here we can generalize this phenomenon for a spin-3/2 gravitino with brane localized masses. The logarithmic divergence in (4.14) can be absorbed by defining a bare coupling $M_0$ replaced by the renormalized coupling $M_0^{\text{ren}}$:

$$\frac{1}{M_0} = \frac{1}{M_0^{\text{ren}}} - \frac{M_0\kappa^4}{2\pi^3 R_5 R_6} \ln (\Lambda R), \quad (4.20)$$

which implies the following running for the gravitino brane mass:

$$\mu \frac{d}{d\mu} M_0(\mu) = \frac{\kappa^4}{2\pi^3 R_5 R_6} [M_0(\mu)]^3. \quad (4.21)$$

This has the solution:

$$M_0^2(\mu) = \frac{M_0^2(\mu_0)}{1 - \frac{\kappa^4}{\pi^3 R_5 R_6} M_0^2(\mu_0) \ln \left( \frac{\mu}{\mu_0} \right)}. \quad (4.22)$$

If $M_0$ is positive then equation (4.22) shows that the mass $M_0$ increases in the UV and would reach a Landau singularity at $\mu = \mu_0 \exp[\pi^3 R_6/\kappa^4 M_0^2(\mu_0)]$.

**5. Conclusions**

Our interest in this work is the situation where spontaneous SUSY breaking is described in a higher-dimensional theory. The presence of many localized objects (branes) coupled with bulk...
fields forces on the latter specific boundary conditions. When the bulk wave functions have to interpolate between the different boundary conditions, as in the Scherk–Schwarz mechanism, this leads to SUSY breaking. We try to formulate these multiple boundary conditions as a system of spins forced to point in peculiar directions by a constraining field \( \vec{H}_i \). The amount of SUSY breaking is measured by the departure from the total alignment of all the spins. The problem of building a particular configuration comes back to finding the appropriate field \( \vec{H}_i \) and then a microscopic realization of it.

The relevance of this picture requires that the fundamental scale, \( \kappa \sim M_s^{-1} \), the branes separation distance \( d_i \) and the compactification radius \( R \sim M_c^{-1} \) be separated as \( \kappa \ll d_i \ll R \). A non-exhaustive set of examples is given by

- The very large extra dimensions as introduced by Arkani-Hamed et al [3] responsible for the weakness of strength of 4D gravitational interactions; these have compactification scales as low as \( M_c \sim 10^{-4} - 10^7 \) eV. Along these, we can suppose the presence of 3-branes separated by typically \( d_i \sim \text{TeV}^{-1} \) distances. The observable world lives on some of these 3-branes or on higher branes stretched between them.

- The so-called ‘large volume’ scenario [5], with a fundamental scale in the intermediate energies \( M_s \sim 10^{11} \) GeV, with electroweak scale compactification radius \( M_c \sim M_w \), where the branes are separated by distances smaller than \( \text{TeV}^{-1} \).

Our formulation aims to study the phase space of a number of branes located at well-separated points in extra dimensions, to discuss the ‘landscape’ of configurations that break SUSY, with an estimate of its size. Given such a configuration, one needs to explain the origin of the constraint \( \vec{H}_i \), as well as information about the microscopic details at scales below the inter-brane distance which is relevant for many phenomenological issues.

Finally, we would like to comment on the possibility of embedding such a scenario, with discrete values for the spins \( \vec{S}_i \), in a string theory framework. In the very early history of the internal space, one can imagine that some cycles shrink to a very small size. If a cycle carries some (quantized) Ramond–Ramond flux, this might give birth to a (stack of) D-brane(s) at this point (see, for example, [26]). Different shrinking cycles can be located at points separated by potential barriers (due to wrapping factors, for example), which make them potential wells where D-branes are located. The latter are driven there in order to minimize their energy contribution through the wrapping effect [30]. A construction of compactifications with both branes and anti-branes exists (see, for example, [27]). However, there is the issue of the stability of the configuration, either by annihilation between the brane and anti-brane, or due to decompactification pushing them infinitely away from each other. Often, studies of non-supersymmetric branes in toroidal compactifications ignore the instability problem, having as another aim to try to find exact conformal field theory descriptions of systems of brane–anti-branes. Most recent studies have concentrated on trying to find a meta-stable configuration of a single anti-brane (negatively charged) in a background with positive-charged Ramond–Ramond flux (see [28]). Assuming that one can explain why the anti-brane appears at that point and then account correctly for the backreaction on the background, it remains to be checked that the anti-brane will not annihilate too quickly with part of the flux, as discussed in [29]. Building meta-stable string backgrounds with broken SUSY, even with a minimal number of branes, remains an interesting open issue.

The main purpose here is to try instead to have a description where the whole complex system is parametrized by an effective Hamiltonian as a spin system in a bottom-up approach to
the brane models construction. The branes play here a role similar to the one of atoms in solid state physics, and some of the machinery of spin systems could be applied.

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