Fluid induced deformation in porous media – Sensitivity analysis of a poroelastic model

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Abstract. The poromechanical behaviour of granular materials are influenced by the rheological properties and stress state of the injected fluid in addition to the state of the porous media. Fluid injection through a granular continuum generally results in the elastic or plastic deformation of the material which reflects as the change in porosity due to particle rearrangement. This phenomenon results in fluid induced instabilities in the porous media which is commonly observed during CO2 sequestration, oil and hydrocarbon recovery, high pressure grouting, hydraulic fracturing. Solid-fluid interaction in porous media is a fundamental multi-physics problem encountered in civil engineering, petroleum, and mining industries. Injecting fluid into the porous materials results in the deformation of the existing solid skeleton, especially when the flow rate is greater than the ability of the media to permeate the fluid. Deformations can be considered as poroelastic when the storage of reversible elastic energy controls the process. Among the existing poroelastic models which predict the fluid induced deformation in porous media, a coupled non-linear continuum-based model proposed by MacMinn is used in this study. This model takes into account various fluid properties and predicts the material response under different boundary conditions. The efficiency of this model to capture the deformation characteristics of porous media under different flow rate, fluid viscosity, porosity of the media and other geometric parameters will be carried out in this study.

1. Introduction
Geomaterials are generally complex due to the presence of voids, constrictions connecting the voids forming a network, varied particle sizes and shapes. Even with these complex features at microstructural level, the material is assumed to behave as a continuum until the instabilities are formed during large deformation. These instabilities especially arise due to fluid injection through the porous media which has many applications in geological engineering, CO2 sequestration, oil and hydrocarbon recovery and hydraulic fracturing [1, 2]. Improved theories and models for predicting fluid-driven instabilities in porous media are essential in natural geosystems and optimizing the engineering designs.

Fluid injected in porous materials generally causes Saffman-Taylor instability [3], resembling a cavity or a finger-like structure or a fracture. This deformation field and the instability growth in the porous media have not been clearly understood due to multiple phenomena (permeation, solid displacement) taking place simultaneously. Lenormand et al. [4] conducted the first numerical study on fluid flow through porous media and proposed a phase diagram with three domains (viscous fingering, capillary fingering and stable displacement) and validated the same numerically on a simple
two-dimensional network of interconnected capillaries. KGD, PKN, penny-shaped fracture model or radial fracture model and P3D [5] are some of the models used to study the hydro-fracturing in brittle materials like competent rocks. KGD, PKN, and the penny-shaped models are two-dimensional fracture propagation models with the third dimension fixed (usually fracture height) which is a major limitation of the 2D model. The KGD model assumes a 2D plane strain condition in the horizontal plane with a constant fracture height (larger than the fracture length), rectangular vertical cross section, elliptical horizontal cross section, and a constant fracture width, which is independent of fracture height. In the PKN model, fracture height is assumed as constant and independent of fracture length with a 2D plane strain condition in the vertical plane where the fracture has an elliptical shape in both horizontal and vertical plane [6, 7, 8]. In penny shaped fracture model, the growth of fracture is assumed to be axisymmetric and the boundaries of the fracture heights are not bounded from top and bottom [8]. The Pseudo 3D model (P3D model) is regarded as an extension of the KGD or PKN model, which consider the growth of fracture height. P3D model determines the fracture height from the local net fluid pressure, in-situ stress, and toughness of rock by satisfying the local static equilibrium [9]. Linear elastic fracture mechanics (LEFM) have been broadly used to examine the initiation and propagation of fluid-driven fractures in cohesive rocks [10]. Detournay [10] conducted studies to predict the fluid driven fracture propagation in impermeable rocks and proposed solutions for the crack tip, and identified the parameters controlling the fracture growth. For both KGD and penny shaped fractures, Detournay [10] observed that the propagation regime depends only on a dimensionless toughness, \( \kappa \). A poroelastic model on the weak rock with a hole and bi-wing tensile crack was reviewed by Gao et al. [11] to predict the fracture properties. The mathematical model formulated by Gao et al. [11] predicts the theoretical relationship between borehole pressure and injection rate using poroelasticity, lubrication, and LEFM for a steady state of flow. However, studies on fluid injection through highly porous and permeable granular systems are very few.

Several numerical methods such as lattice Boltzmann method coupled with discrete element method (LBM-DEM) [12], discrete element method coupled with computational fluid dynamics (DEM-CFD) [1], meshless methods applied to computational fluid dynamics (CFD) [13] are available to study the flow through porous media. These methods are computationally expensive and relatively new areas of research. Research hitherto generally focused on instabilities such as fracturing in rocks and fingering in porous geomaterials [1, 2, 3, 4, 6, 8], while these predictive models are usually too simplistic resulting in failure to capture the realistic response under appropriate boundary conditions. A visco-elastic continuum model based on the theory of poroelasticity and Darcy’s law was proposed by MacMinn et al. [6] to predict the deformation characteristics of porous materials. The spatial and temporal variation of porosity during fluid injection is quantified using this model. In this study, the efficacy of this model under different sets of parameters such as injection fluid properties (flow rate and viscosity) and properties of the porous media will be studied in detail. A parametric study is conducted to capture the fluid flow within a porous media and the impact of the two phases (fluid and grain) on each other.

2. Poroelastic model

Porous materials are found extensively in nature such as in living organisms (bones, flesh) as well as non-living materials (soils, rocks) and also in synthetic materials like polyurethane foams. These porous materials consist of voids/pores, constrictions, fissures, cracks, cavities, channels of various shapes and sizes which are randomly distributed over the entire volume. Since it is impossible to quantify the geometry and orientation for each of these components, for modelling purpose, it was assumed that properties are homogenized. To model the physical-mechanical behavior of porous geomaterials during fluid injection, a simplified poroelastic theory is assumed as compared to the complex plasticity formulations primarily due to the multiple operative mechanism during fluid flow. During mechanical or hydraulic loading, the poroelastic media are subjected to stresses and strains [4,5,6]. The fundamental governing equations used to analyze the behavior of porous media includes Darcy’s law, conservation of mass, continuity equation and effective stress principle [6]. Effective stress principle has applications in modeling partially or fully saturated geomaterials.
Recently, MacMinn et al. [6] proposed a coupled visco-elastic continuum model to predict the deformation characteristics of porous granular media during fluid injection. The variation in the porosity ($\phi$) and particle displacement ($U_r$) when the fluid is injected at a constant flow rate ($Q$) is quantified for a system of saturated, and incompressible polyacrylamide hydrogel particles packed between two transparent discs of radius ($b$), 105 mm. The viscous fluid entering through the injection port of radius ($a$), 2.5 mm spreads radially outward and exits through the spacer at the outer edge causes a pressure gradient across the packing, resulting in particle displacement which causes a cavity-like instability near the injection port (Figure 1). The coupling of particle displacement and fluid pressure subsequently reaches steady state resulting in a balance of elastic stress of the particles and pressure gradient of the fluid. The model assumed that the packing is homogeneous, the deformation is axisymmetric, fluid and hydrogel particles are incompressible, permeability of the medium is constant ($k$), initial porosity ($\phi^*$) is uniform and the elastic stress in the particles are isotropic. The cavity formed after injection of fluid reaches a steady state where the maximum cavity area irrespective of the repetition of the experimental cycle will be constant and bounded with a cavity wall.

![Figure 1. Geometry and boundary conditions used in [6]](image)

According to MacMinn et al. [6], the conservation of mass in Eulerian description under axisymmetric conditions in radial coordinate system is given as,

$$\frac{\partial \phi}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r} (r \phi v^w) = 0 \quad (1)$$

$$\phi v^w = \frac{Q}{2\pi rh} - (1 - \phi) v^s \quad (2)$$

where $\phi$ is the porosity of the porous media, $v^w(r, t)$ and $v^s(r, t)$ are the velocity of fluid and solid respectively as a function of space and time, $Q$ is the injection rate/flow rate, and $h$ is the gap between two transparent sheets as shown in figure 1. According to Darcy’s law, the relative velocity between the particles and fluid flow is given as,

$$\phi(v^s - v^w) = \frac{k}{\mu} \frac{\partial p}{\partial r} \quad (3)$$

where $\mu$ is the viscosity of the injected fluid, $k$ is the permeability of the porous media and $p$ is the fluid pressure. Based on this model’s assumptions, the elastic stress in the particles are isotropic. At steady state, the fluid pressure is balanced by the effective stress of the particles, and hence under poromechanical equilibrium both fluid pressure ($p$) gradient and effective stress ($\sigma^e$) gradient are equal. By solving the above equations (Eq.1, Eq.2 and Eq.3), a nonlinear conservation law for porosity, $\phi(t)$ is obtained as given in equation 4.

$$\frac{\partial \phi}{\partial t} + \frac{1}{rr'} \frac{\partial}{\partial rr'} \left[ \alpha(\phi - \phi^*) - r'(1 - \phi) \frac{\partial \sigma^e}{\partial r'} \right] = 0 \quad (4)$$
where \( t' \) and \( r' \) are the time and radial position scaled with poroelastic timescale \( \theta = \frac{\mu b^2}{Kk} \) and radius of the outer boundary \( b \) respectively. The model assumes a dissipation of viscous force due to the rearrangement of the particles, effective stress of the particles is linear with the rate of change of porosity and an effective viscosity \( \eta \). \( \sigma' \) is the effective stress on the particles scaled with bulk modulus \( K \). The model consists of two dimensionless parameters, \( \alpha = \frac{\mu Q}{2\pi rh} \) and \( \beta = \frac{\eta k}{\mu b^2} \). \( \alpha \) signify the pressure gradient and \( \beta \) considers the effective viscosity of the particle wall. Effective stress of the particles dissipates, (2) outer boundary is open to flow of the injected fluid and restrain the particle movement.

\[
(1 - \theta^*) \sigma' = \beta \frac{\partial \theta}{\partial t'} + (\theta - \theta^*) \left[ (\theta - \theta^*) \frac{1}{(1 - \theta^*)} \right]^{\frac{1}{2}} \tag{5}
\]

The steady-state radial displacement of the particles is solved using Eq. (6) and the porosity field is obtained by solving coupled partial differential equations Eq. (4) and Eq. (5).

### 3. Model predictions

The coupled partial differential equations as discussed above are used to determine the variation of porosity and particle displacement in porous media during fluid injection under different set of variables. The dimensionless constants \( \alpha \) and \( \beta \) which are based on the packing and fluid properties respectively were obtained by validating the model proposed by MacMinn et al. [6] and solving the poroelastic continuum model (Eq. 4 to Eq. 6). A suite of variables such as (1) fluid injection rate – \( Q \), (2) initial porosity – \( \theta^* \), (3) thickness or the gap between the disks – \( h \) and (4) dimensionless viscosity constant – \( \beta \) were identified to study the effect of these variables (Table 1) on the poroelastic model predictions.

#### Table 1. Variable parameters.

| Unit          | Notation | Range of variation |
|---------------|----------|--------------------|
| Fluid injection rate | mL/min   | Q                  | 4 - 30 |
| Initial porosity       | \( \theta^* \) | 0.2 - 0.8          |
| Thickness or gap between the disks | mm | h                  | 2 - 25 |
| Viscosity constant | - \( \beta \) | 2 - 10             |

#### 3.1 Fluid injection rate

To study the effect of fluid injection rate on the change in porosity and radial displacement of particles, three different injection rates (4 mL/min, 16 mL/min and 30 mL/min) are used in this exercise while keeping the initial porosity of the porous medium (0.51), viscosity of the injection fluid (0.012 Pas) and other geometric parameters (\( h = 1.4 \) mm and \( b = 105 \) mm) constant. The variation of normalized porosity (ratio of instantaneous porosity to initial porosity) and normalized radial displacement (ratio of radial displacement to the radius of the particle packing) of particles across the entire radius of the specimen with increase in fluid injection rate is shown in figure 2 and 3 respectively. The maximum radial displacement of the particles and the porosity increases with increase in injection rate. The percentage of porosity change varies from 2% to 2.7% as the flow rate increases from 4 to 30 mL/min respectively. Further, the normalized porosity and radial displacement decreases with increase in radius from the injection port. The increase in cavity area due to increase in the flow rate is shown in figure 4a and 4b. The maximum cavity areas obtained from three different simulations were shown as points fitted with a polynomial curve (Figure 4a).
3.2 Initial porosity
Porosity or void fraction is defined as the ratio of volume of voids to the total volume. In order to study the effect of initial porosity on the deformation characteristics, three different porosities were chosen. The porous granular material is packed at loose, medium, and dense state having initial porosities 0.8, 0.5 and 0.2 respectively. The fluid injection rate (16 mL/min), viscosity of the fluid (0.012 Pas) and other boundary conditions ($h = 1.4$ mm and $b = 105$ mm) are kept constant while quantifying the effect of initial porosity. Since the porosity is directly related to permeability and bulk modulus of the porous media, the dimensionless constant ($\alpha$) of the model will be subjected to change as porosity variations. The variation of normalized porosity and radial displacement due to initial packing is given in figure 5 and 6 respectively when the steady state is reached. It was observed that highly porous material undergoes large radial displacement compared to less porous material i.e., maximum radial displacement of the particles also proportionally increases with the initial porosity (Figure 6). Further, the particle displacement decreases with increase in radial distance from the injection port and initial porosity. With increase in initial porosity, the change in normalized porosity decreases (Figure 5) to about 8%.

![Figure 2. Variation of porosity field for different injection rate.](image1)

![Figure 3. Radial particle displacement for different injection rate.](image2)

![Figure 4. (a) Plot for maximum cavity area against the injection rate and (b) Cavity area with approximate radius for different injection rate.](image3)
3.3 Thickness of packing

Increasing the thickness between two discs results in increasing the stiffness of the packing which will affect the pressure gradient across the radius of the specimen. Further the thickness of the packing is indirectly proportional to the dimensionless constant $\alpha$. A study is conducted for three different thicknesses ($h = 2$ mm, $10$ mm and $25$ mm) and the variation of porosity across the radius is quantified. The variation of normalized porosity across the radius decreases with increase in thickness of packing as shown in figure 7. The percentage of variation in porosity ranges from 0.7% to 1.4%. Further the zone of influence of the particle displacement decreases with increase in thickness of the packing as shown in figure 8 while the maximum radial particle displacement is equal for all thicknesses.

3.4 Viscosity constant

Viscosity is the measure of resistance to deformation of a fluid at a given injection rate. Viscosity constant ($\beta$) directly depends on the ratio of effective viscosity to viscosity of the fluid. The injection rate (16 mL/min), thickness (1.4 mm), and initial porosity of packing (0.51) are kept constant for this set of predictions to study the effect of viscosity constant on the deformation characteristics. Three different $\beta$ values (2, 6 and 10) were used for this prediction exercise. The variation in the normalized porosity along the radius increases with $\beta$. The percentage of variation in porosity ranges from 1.2% to 3% (Figure 9). Radial displacement of the particles decreases from cavity wall to outer boundary, from 14.7 mm to zero (Figure 10).
4. Discussion

The numerical implementation of the coupled continuum model suggested by MacMinn et al. [6] using a multiphysics software package COMSOL with a partial differential equation (PDE) interface was carried out effectively. Furthermore, a parametric study of injection rate, packing thickness, initial porosity, and \( \beta \) was performed using this model by varying \( \alpha \) and \( \beta \) accordingly.

The study of injection rate shows that the rate of change of porosity increases with an increase in the injection rate. The maximum cavity area also increases with the injection rate. For higher injection rate, pressure at the cavity wall is directly proportional to \( Q \), which causes an increased pressure gradient in the radial direction from the cavity wall to the outer boundary, which eventually results in a higher displacement of particles. Particles near the vicinity of the cavity wall displace more than the particles near the outer perimeter, and this results in an area of compaction zone. Size of the compaction zone increases with the increase in injection rate.

\[ \text{Initial porosity determines the degree of packing of the porous medium. Under loose state, the voids/pores will be more in the porous medium resulting in higher porosity. The study with varying initial porosity results in a decrease in normalized porosity. With increase in initial porosity, the change in normalized porosity decreases. This is due to the fact that highly porous material will allow the fluid to permeate more without much change in overall porosity as compared to less porous material. Further the compaction zone near the cavity decreases with increase in initial porosity. Therefore, radial displacement of the particle between the cavity wall and the outer boundary decreases with initial porosity.} \]

\[ \text{When the thickness of the packing increases, more particles will be present near the injection port or number of particles/ voids which take part in the dissipation of injected fluid pressure increases, which results in less variation of the porosity across the radius of the packing. The compaction zone near the cavity wall also decreases with an increase in thickness.} \]

\[ \text{When the fluid's viscosity decreases, } \beta \text{ increases, which further increases the variation of normalized porosity. For a less viscous fluid, the pressure applied across the particles will be less, resulting in minor variations of porosity and particle displacement. The compaction zone increases with an increase in the viscosity of the fluid.} \]

\[ \text{The viscoelastic continuum model proposed by MacMinn et al. [6] predicts the porosity variation and radial displacement of the particle for a porous media packed between two glass discs, when fluid is injected through the center. From the experiments conducted by MacMinn et al. [6], radial displacement of the particles of the system agrees relatively well with the continuum model but does not capture the porosity field smoothly and satisfactorily. The model proposed by MacMinn et al. [6]} \]
predicts the porosity and displacement field for cavity like instability and does not predicts the behavior of fingering or fracturing.

5. Conclusion

Porosity along with displacement field of the particles in a porous media subjected to fluid injection is studied using a viscoelastic coupled continuum model. Continuum equations suggested by MacMinn et al. [6] are validated using a suitable multiphysics simulation software. The observations made from the parametric study conducted to capture the fluid flow within a porous media are as follows.

- Both rate of change of porosity and the particle displacement increases with increase in injection rate
- With increase in initial porosity, the change in normalized porosity decreases and radial displacement of the particles increases
- Both rate of change of porosity and the particle displacement decreases with increase in thickness of packing
- With increase in viscosity constant $\beta$, the change in normalized porosity increases and radial displacement of the particles decreases

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