Abstract

In the context of $SO(10) \rightarrow SU(5) \times U(1)_\chi$, it is shown how seesaw Dirac neutrinos may be obtained. In this framework, $U(1)$ lepton number is conserved, with which self-interacting dark matter with a light scalar dilepton mediator may be implemented. In addition, $U(1)$ baryon number may be broken to $(-1)^{3B}$, thereby generating a baryon asymmetry of the Universe. The axionic solution to the strong $CP$ problem may also be incorporated.
**Introduction** : In considering $SO(10)$ grand unification, the common approach is to allow an intermediate step with left-right symmetry, i.e. $SU(4) \times SU(2)_L \times SU(2)_R$ or $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{(B-L)/2}$. This has the advantage of forcing into existence the right-handed $SU(2)_R$ lepton doublet $(\nu, l)_R$, so that $\nu_R$ is the Dirac partner of the observed $\nu_L$ which belongs to the $SU(2)_L$ lepton doublet $(\nu, l)_L$. At the same time, $B - L$ becomes a gauge symmetry, and its breaking through an $SU(2)_R$ scalar triplet from the 126 of $SO(10)$ also makes $\nu_R$ massive, realizing thus the canonical seesaw mechanism for a naturally small Majorana $\nu_L$ mass.

Another option [1, 2] is to consider $SO(10) \rightarrow SU(5) \times U(1)_{\chi}$. This is seldom studied because $SU(5)$ is a grand unified symmetry by itself, so $U(1)_{\chi}$ is often thought to be unnecessary and uninteresting. Let the breaking of $SU(5)$ to the standard-model (SM) gauge symmetry $SU(3)_C \times SU(2)_L \times U(1)_Y$ be at the scale $M_U$. If $U(1)_{\chi}$ survives down to a scale much below $M_U$ and not too far above the electroweak scale, there could be important consequences which have been largely overlooked. In fact, whereas the right-handed neutrino $\nu_R$ is a singlet under $SU(5)$, it has a nonzero charge under $U(1)_{\chi}$. The Higgs doublet which connects $u_L$ to $u_R$ also connects $\nu_L$ to $\nu_R$. Hence a Dirac neutrino mass is again obtained and the seesaw mechanism operates as in the left-right case. On the other hand, the detailed phenomenology is very different. Whereas $W^\pm_R$ must exist at the left-right scale $M_R$, it must be heavier than $M_U$ if $SO(10) \rightarrow SU(5) \times U(1)_{\chi}$. The $Z_{\chi}$ gauge boson itself has well-defined couplings to the SM particles. Its existence is routinely searched for at the Large Hadron Collider (LHC), with the present mass limit [3, 4] of about 4.1 TeV, which may be improved [5].

In this paper, new fermions and scalars transforming under $U(1)_{\chi}$ are added to the SM to obtain a number desirable features. With the help of a softly broken $Z_2$ discrete symmetry, naturally light seesaw Dirac neutrinos [6, 7, 8] may be obtained. The resulting Lagrangian
conserves both $B$ and $L$. Further addition of two scalars with $L = -1, -2$ enables the appearance of self-interacting leptonic dark matter $^9$. The analog of leptogenesis (through a heavy singlet Majorana fermion which couples to leptons in the seesaw mechanism) is possible using a heavy singlet Majorana fermion which couples to a scalar diquark and an antiquark, thereby generating the baryon asymmetry of the Universe. A fermion color octet may also be introduced to support a Peccei-Quinn symmetry to obtain an invisible axion for solving the strong $CP$ problem.

**Seesaw Dirac Neutrinos**: The spinorial 16 representation is again chosen for the three families of quarks and leptons and their decompositions shown in Table 1. The necessary

| fermion | $SO(10)$ | $SU(5)$ | $SU(3)_C$ | $SU(2)_L$ | $U(1)_Y$ | $U(1)_X$ | $Z_2$ |
|---------|----------|----------|-----------|-----------|-----------|-----------|-------|
| $d^c$   | 16       | 5*       | 3*        | 1         | 1/3       | 3         | +     |
| $(\nu, e)$ | 16       | 5*       | 1         | 2         | −1/2      | 3         | +     |
| $(u, d)$ | 16       | 10       | 3         | 2         | 1/6       | −1        | +     |
| $u^c$   | 16       | 10       | 3*        | 1         | −2/3      | −1        | +     |
| $e^c$   | 16       | 10       | 1         | 1         | 1         | −1        | +     |
| $\nu^c$ | 16       | 1        | 1         | 1         | 0         | −5        | −     |
| $N$     | 126*     | 1        | 1         | 1         | 0         | 10        | −     |
| $N^c$   | 126      | 1        | 1         | 1         | 0         | −10       | −     |

Table 1: Fermion content of model.

Higgs scalars for fermion masses belong to the 10 representation, as shown in Table 2.

| scalar | $SO(10)$ | $SU(5)$ | $SU(3)_C$ | $SU(2)_L$ | $U(1)_Y$ | $U(1)_X$ | $Z_2$ |
|--------|----------|----------|-----------|-----------|-----------|-----------|-------|
| $(\phi_1^0, \phi_1^\pm)$ | 10       | 5*       | 1         | 2         | −1/2      | −2        | +     |
| $(\phi_2^0, \phi_2^+) $ | 10       | 5        | 1         | 2         | 1/2       | 2         | +     |
| $(\eta^+, \eta^0)$ | 144      | 5        | 1         | 2         | 1/2       | 7         | −     |
| $\sigma$ | 16       | 1        | 1         | 1         | 0         | −5        | +     |

Table 2: Scalar content of model.

New fermions $N, N^c$ belonging to $126^*, 126$ respectively are added per family, as well as a Higgs doublet from 144 and a singlet from 16. Note that their $Q_\chi$ charges are fixed by
the $SO(10)$ representations from which they come. It should also be clear that incomplete $SO(10)$ and $SU(5)$ multiplets are considered here (which is the case for all realistic grand unified models). An important $Z_2$ discrete symmetry is imposed so that $\nu^c, N, N^c$ and $\eta$ are odd, and the other fields are even. Since $\Phi_1^\dagger$ transforms exactly like $\Phi_2$, the linear combination

$$\Phi = \frac{(v_1 \Phi_1^\dagger + v_2 \Phi_2) / \sqrt{v_1^2 + v_2^2}}\,$$

is the analog of the standard-model Higgs doublet, where $\langle \phi_{1,2}^0 \rangle = v_{1,2}$. The $Z_2$ symmetry is respected by all dimension-four terms of the Lagrangian. It will be broken softly by the dimension-three trilinear term $\mu \sigma \Phi_1^\dagger \eta$ as well as spontaneously by $\langle \eta^0 \rangle = v_3$. The $4 \times 4$ neutrino mass matrix spanning $(\nu, \nu^c, N, N^c)$ is then given by

$$M_\nu = \begin{pmatrix} 0 & 0 & 0 & f_\eta v_3 \\ 0 & 0 & f_\sigma u & 0 \\ 0 & f_\sigma u & 0 & m_N \\ f_\eta v_3 & 0 & m_N & 0 \end{pmatrix},$$

where $u = \langle \sigma \rangle$ which breaks $U(1)_\chi$. The above mass matrix generates a seesaw Dirac neutrino with $m_\nu = f_\eta f_\sigma v_3 u / m_N$, which is naturally small.

**Scalar Sector:** The scalar potential consisting of $\Phi, \eta, \text{and } \sigma$ is given by

$$V = \mu_\Phi^2 \Phi^\dagger \Phi + \mu_\eta^2 \eta^\dagger \eta + \mu_\sigma^2 \sigma^\dagger \sigma + [\mu \sigma \Phi_1^\dagger \eta + H.c.]$$

$$+ \frac{1}{2} \lambda_\Phi (\Phi^\dagger \Phi)^2 + \frac{1}{2} \lambda_\eta (\eta^\dagger \eta)^2 + \frac{1}{2} \lambda_\sigma (\sigma^\dagger \sigma)^2$$

$$+ \lambda_{\Phi \eta} (\Phi^\dagger \Phi)(\eta^\dagger \eta) + \lambda_{\Phi \sigma} (\Phi^\dagger \Phi)(\sigma^\dagger \sigma) + \lambda_{\eta \sigma} (\eta^\dagger \eta)(\sigma^\dagger \sigma). \quad \text{(2)}$$

The minimum of $V$ satisfies the conditions

$$0 = \mu_\Phi^2 + \lambda_\Phi v^2 + \lambda_{\Phi \eta} v_3^2 + \lambda_{\Phi \sigma} u^2 + \mu v_3 u / v, \quad \text{(3)}$$

$$0 = \mu_\eta^2 + \lambda_\eta v_3^2 + \lambda_{\Phi \eta} v^2 + \lambda_{\eta \sigma} u^2 + \mu v u / v_3, \quad \text{(4)}$$

$$0 = \mu_\sigma^2 + \lambda_\sigma u^2 + \lambda_{\Phi \sigma} v^2 + \lambda_{\eta \sigma} v_3^2 + \mu v v_3 / u. \quad \text{(5)}$$

Assuming that $u >> v >> v_3$, the solutions to the above are

$$u^2 \simeq -\frac{\mu_\sigma^2}{\lambda_\sigma}, \quad v^2 \simeq -\frac{(\mu_\Phi^2 + \lambda_{\Phi \sigma} u^2)}{\lambda_\Phi}, \quad v_3 \simeq -\frac{\mu v u}{(\mu_\eta^2 + \lambda_{\eta \sigma} u^2)}. \quad \text{(6)}$$
The $3 \times 3$ mass-squared matrix spanning $\sqrt{2}[\text{Im}(\phi^0), \text{Im}(\eta^0), \text{Im}(\sigma)]$ is given by

$$M^2_I = -\mu \begin{pmatrix} v_3 u/v & -u & -v_3 \\ -u & vu/v_3 & v \\ -v_3 & v & v_3 u/v \\ \end{pmatrix},$$

which has two zero eigenvalues and one massive eigenstate with

$$m^2_{\eta_I} = -\mu \left( \frac{v_3 u}{v} + \frac{vu}{v_3} + \frac{v_3 u}{u} \right) \approx -\frac{\mu vu}{v_3} \approx \mu^2_{\eta_I} + \lambda_{\eta \sigma} u^2.$$

The $3 \times 3$ mass-squared matrix spanning $\sqrt{2}[\text{Re}(\phi^0), \text{Re}(\eta^0), \text{Re}(\sigma)]$ is given by

$$M^2_R = 2 \begin{pmatrix} \lambda_\phi v^2 & \lambda_{\phi \eta} v_3 v & \lambda_{\phi \sigma} vu \\ \lambda_{\phi \eta} v_3 v & \lambda_\eta v^2_3 & \lambda_{\eta \sigma} v_3 u \\ \lambda_{\phi \sigma} vu & \lambda_{\eta \sigma} v_3 u & \lambda_\sigma u^2 \\ \end{pmatrix} - \mu \begin{pmatrix} v_3 u/v & -u & -v_3 \\ -u & vu/v_3 & -v \\ -v_3 & -v & v_3 u/v \\ \end{pmatrix},$$

which is approximately diagonal with

$$m^2_{\phi_R} \approx 2\lambda_\phi v^2, \quad m^2_{\eta_R} \approx m^2_{\eta_I}, \quad m^2_{\sigma_R} \approx 2\lambda_\sigma u^2.$$

Note that the $\phi_R - \sigma_R$ mixing (with the assumption that $\phi_R$ is much lighter than $\sigma_R$) is roughly $\lambda_{\phi \sigma} vu/\lambda_\sigma u^2$ which is naturally suppressed by $v/u$. As for $\eta_R$, its mass is dominated by $-\mu vu/v_3$, and its mixing with $\phi_R$ and $\sigma_R$ is suppressed by $v_3/v$ and $v_3/u$ respectively. This justifies the diagonal approximation assumed here.

As a numerical example, let $u = 10 \text{ TeV}$, $v_3 = 10 \text{ GeV}$, then $m_\nu = 0.1 \text{ eV}$ is obtained for $f_\eta = f_\sigma = 0.1$ and $m_N = 10^{13} \text{ GeV}$. The soft $Z_2$ breaking parameter $\mu$ is then 6 GeV for $m_{\eta_I} = 1 \text{ TeV}$. 

**Conserved Baryon and Lepton Numbers with Self-Interacting Dark Matter**: Because of the $U(1)_X$ assignments of the particle content of this model, the resulting Lagrangian conserves both baryon number $B$ and lepton number $L$ even after the symmetry breaking of $U(1)_X$ by $u, v, v_3$ and $SU(2)_L \times U(1)_Y$ by $v, v_3$. As usual, the quarks have $B = 1/3, L = 0$ and the leptons (including the Dirac neutrinos) have $B = 0, L = 1$. This means that if a new particle is added, it may be assigned $B$ and $L$ numbers appropriately, according to its assumed
interactions with the known quarks and leptons \cite{10, 11}. These assignments lie outside $U(1)_\chi$, hence $Q_\chi$ is now not a marker of dark matter, as in previous studies \cite{1, 2}.

| scalar | SO(10) | SU(5) | SU(3)_C | SU(2)_L | U(1)_Y | U(1)_\chi | L |
|--------|--------|-------|---------|---------|--------|------------|---|
| $\zeta$ | 126    | 1     | 1       | 1       | 0      | -10        | -2 |
| $\rho$  | 16     | 1     | 1       | 1       | 0      | -5         | -1 |

Table 3: Leptonic scalars for self-interacting dark matter.

For example, consider the scalar singlet $\zeta \sim (1, -10)$ from the 126 of SO(10). It has the allowed Yukawa coupling $\zeta^* \nu^c \nu^c$. In conventional models, $\zeta$ is assumed to have a vacuum expectation value, thereby breaking $U(1)_\chi$ and giving $\nu^c$ a large Majorana mass, breaking thus also lepton number $L$. Here it may be assumed instead that $L$ is conserved and $\zeta$ has $L = -2$. Note that $U(1)_\chi$ is broken instead by $\sigma$ which may be assigned $L = 0$, together with $L = \pm 1$ for $N, N^c$, and $L = 0$ for $\Phi$ and $\eta$.

With $\zeta$ as a scalar dilepton which couples only to the Dirac neutrinos, it is then a simple step to consider a scalar singlet $\rho \sim (1, -5)$ from the 16 of SO(10) with $L = -1$ so that it can be self-interacting dark-matter \cite{12} with $\zeta$ as its light mediator, as proposed recently \cite{9}. Since $\zeta$ decays only to two neutrinos, it does not disrupt the cosmic microwave background (CMB) from its enhanced production at late times due to the Sommerfeld effect. It removes an important objection \cite{13, 14} to models where the light mediator decays to electrons and photons, usually through Higgs mixing, which is forbidden here by $L$ conservation.

**Baryogenesis**: Since lepton number is strictly conserved, the usual mechanism of generating the baryon asymmetry of the Universe through leptogenesis is not possible. However, the analog process of having a heavy Majorana fermion $\psi$ decaying to $B = \pm 1$ final states \cite{13} may be implemented with the addition of two scalar diquarks $h_{1,2}$, as shown in Table 4.

The allowed couplings involving the new particles are

$$h_{2}ud, \quad h_{2}^*u^c d^c, \quad h_{1}d^c \psi, \quad h_{1}^*h_{2}\sigma.$$  \hspace{1cm} (11)
### Table 4: New particles for baryogenesis.

| scalar/fermion | $SO(10)$ | $SU(5)$ | $SU(3)_C$ | $SU(2)_L$ | $U(1)_Y$ | $U(1)_X$ | $B$ |
|----------------|----------|---------|----------|----------|----------|----------|-----|
| $h_1$ (scalar) | $16^*$   | 5       | 3        | 1        | $-1/3$  | $-3$    | $-2/3$ |
| $h_2$ (scalar) | 10       | 5       | 3        | 1        | $-1/3$  | 2        | $-2/3$ |
| $\psi$ (fermion) | 45       | 24      | 1        | 1        | 0        | 0        | 1   |

Assuming a large Majorana mass for $\psi$ which breaks $B$ to $(-1)^B$, it may now decay to both $h_1^* \bar{d}^c$ ($B = 1$) and $h_1 d^c$ ($B = -1$). The subsequent decay of $h_1$ to $u^c d^c$ through $h_1 - h_2$ mixing from $\langle \sigma \rangle$ establishes a baryon asymmetry in analogy to the case of a lepton asymmetry if $\nu^c$ were a heavy Majorana fermion. The one-loop vertex [16] and self-energy [17] diagrams which contribute to the $CP$ asymmetry and thus the $B$ asymmetry are depicted in Fig. 1. They are completely analogous to those of leptogenesis where $\psi_{1,2}$ are replaced by $\nu_{1,2}^c$, the scalar diquark $h$ by $\phi^+$, and $d^c$ by $l$. However, since the Yukawa interactions $hd^c\phi_{1,2}$ are not constrained by lepton masses as in $\phi^+ l\nu_{1,2}$, the desirable asymmetry is easily obtained. Note of course that $m_h << m_{\psi_1} < m_{\psi_2}$ is assumed. The resulting $B$ asymmetry is converted to a $B - L$ asymmetry through the spheralons, again in analogy to what happens in leptogenesis.

A possible scenario is high-scale baryogenesis with $m_{\psi_1} \sim 10^{13}$ GeV. Instead of three families of leptons in leptogenesis, just one set of scalar diquarks $h_{1,2}$ is needed. The $CP$ asymmetry generated by the decay of $\psi_1$ assuming that $\psi_2$ is much heavier is given by

$$
\epsilon = -\frac{3}{16\pi} \frac{m_{\psi_1}}{m_{\psi_2}} \frac{Im[(f_1 f_2^*)^2]}{|f_1|^2},
$$

where the $\psi_1$ decay rate is $\Gamma_1 = |f_1|^2 m_{\psi_1}/8\pi$. Consider the parameter $K = \Gamma_1/H(T = m_{\psi_1})$. 7
where the Hubble parameter is $H = 1.66 \sqrt{g_*} (T^2/M_{Pl})$, as a measure of the deviation from equilibrium. If $K << 1$, which means $|f_1| << 0.02$, then the baryon asymmetry is of order $\epsilon/g_*$. Setting this to $10^{-10}$, and assuming $m_{\psi_2}/m_{\psi_1} = 6$, then $|f_2| = 10^{-3}$ if the relative phase between $f_1$ and $f_2$ is of order 1.

**Axionic dark matter** : To obtain an axionic solution to the strong $CP$ problem, a colored fermion is needed which has an anomalous Peccei-Quinn charge. Instead of the usual quark triplet, a fermion color octet, such as the gluino of supersymmetry, may be used [18]. In a nonsupersymmetric context, it may just be any fermion color octet [19] unrelated to the gluon. Here it is called $\Omega$ and it obtains a large Majorana mass through the interaction $S^* \Omega \Omega$, so that the dynamical phase of $S$ becomes the invisible axion which is a component of dark matter.

**Concluding remarks** : In the context of $U(1)_\chi$, new light is shed on some of the outstanding problems in particle physics and astroparticle physics. It is shown how naturally light Dirac neutrinos may be obtained in a seesaw mechanism which is usually reserved for considering Majorana neutrinos. With light Dirac neutrinos, an elegant solution to an important problem in self-interacting dark matter may also be solved. The light scalar mediator here is a dilepton and decays only to two neutrinos, so it does not disrupt the cosmic microwave background at late times. With the conservation of lepton number, the possibility of breaking baryon number $B$ to baryon parity, i.e. $(-1)^{3B}$, allows baryogenesis to occur, in analogy to leptogenesis, from the decay of a heavy Majorana fermion carrying $B = 1$. To explain strong $CP$ conservation, a Majorana fermion color octet with anomalous Peccei-Quinn charge is

| scalar/fermion | $SO(10)$ | $SU(5)$ | $SU(3)_C$ | $SU(2)_L$ | $U(1)_Y$ | $U(1)_\chi$ | $PQ$ |
|----------------|----------|---------|-----------|-----------|----------|-------------|-----|
| $\Omega$ (fermion) | 45       | 24      | 8         | 1         | 0        | 0           | 1   |
| $S$ (scalar)     | 54       | 24      | 1         | 1         | 0        | 0           | 2   |
postulated. It acquires a large mass through its coupling to a singlet scalar, the dynamical
phase of which becomes the invisible axion and contributes to dark matter.

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