Study of Linearly Polarized Light Measuring Fiber Diameter

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Abstract. The letter discusses a new way to measure fiber diameters, studies the theory of measuring fiber radius using this way, carries out numerical simulation based on this and provides experiment method as well.

1. Introduction
The ways to measure fiber diameter are contact and non-contact. The contact measurement is to use caliper and micrometer, which is easy to cause damage to fiber and poor in accuracy, as well as can’t finish the real-time measurement during the progress of fiber drawing. There are two kinds of non-contact measurement instruments [1]. One is a device which forward or backward scattered light is used to measure fiber diameter, such measurement device is to work out the fiber radius mainly through analyzing electrode time of spot in forward or backward scattered image; the accuracy of this device is very high, but measurement needs optical collimator, and it is also very difficult to operate. A ray of collimated laser irradiates the measured fiber directly, high-frequency interference spot caused is modulated slowly due to inconsistent light-wave vibration directions, and the superposition of such slow signal makes it difficult to recognize local spot. Meanwhile, as the scattered angel is increasing, the amplitude of spot is becoming smaller and smaller, thus signal-to-noise ratio is becoming lower and lower, which is easy to be affected by interference. Therefore, it is not suitable to be used to measure real-time diameter [2]. The other one is fiber loss and mode field diameter measuring instrument [3]. This way is to measure mode field diameter between fibers, but can’t carry out the real-time measurement during the progress of fiber drawing.
So far, determination of fiber radius is worked out by the known control parameters during making progress, however, if fiber radius is compressed to sub-millimeter magnitude, the above measurement way can’t take effect, thus new calculation or measurement way is necessary. Non-contact way aforesaid seems very inapplicable due to tedious experiment device [4].
The Thesis discusses a new way to measure fiber diameters, studies the theory of measuring fiber radius using this way, carries out numerical simulation based on this and provides experiment method as well.

2. Theoretical Analysis
We shall discuss the isotropic column dielectric body and phase distribution of gauss wave packet scattered amplitude, and obtain the expression of light intensity.
For simplicity, we consider the two dimensional case: such as shown in figure 1. In such case, the wave equation met by the electric field component of the incident beam can be obtained through the following equation:
Where \( \omega \) is the angular frequency, \( \mu, \varepsilon \) are the magnetic permeability and dielectric coefficient in vacuum.

In order to solve the equation (1), we should return to the following anti-fourier Integral [5],

\[
E_i(y, z) = \frac{1}{2\pi} \int_{-\infty}^{\infty} E_i(\alpha) e^{-j(\omega y + \alpha^2 y^2) - j\alpha y} d\alpha
\]  (2)

Where, \( E_i(\alpha) \) is the fourier Integral of luminous source

\[
E_i(\alpha) = \int_{-\infty}^{\infty} E_i(y, -z_0) e^{j\alpha y} dy
\]  (3)

When the luminous source is in \( z = z_0 \), the far field generated by the luminous source is expressed as

\[
E_i(y, -z_0) = E_0 e^{-\beta y^2}
\]  (4)

Where \( \beta = a^2 + jb^2 \), \( 1/a^2 \) is the beam width;

When the origin of wave is assumed as the collimation laser beam, \( \beta = \alpha \) is a real number. In the thesis, imaginary part of \( \beta \) is also considered, insert equation (4) into equation (3) and (2), we can get

\[
E_i(y, z) = \frac{1}{2\pi \sqrt{\pi \beta}} \int_{-\infty}^{\infty} E_i(\alpha) e^{-\alpha^2 y^2 - j(\omega y + \alpha^2 y^2) - j\alpha y} d\alpha
\]  (5)

In order to strictly figure out the integral of the above equation, it is difficult to obtain the analytic solution [6]. But we can get the approximate integral value. It is assumed that \( |\beta\lambda| \) is little, and \( \lambda \) is the wavelength of the free space. Main contribution from the integral in equation (5) is derived from the small area of \( \alpha \). When \( \alpha \) is little, the phase function in the integral expression can be spread into the Taylor series expression of \( \alpha \)

\[
(k_0^2 - \alpha^2)^{1/2} = k_0 - \frac{1}{2} k_0 \frac{\alpha^2}{k_0}
\]  (6)

The integral in equation (5) can be strictly figured out, the following result can be get with the integral formula,
\[
\frac{E_i(y, z)}{E_0} \approx \frac{1}{(1 - jz)^{1/2}} e^{-[\beta z / (1 - jz)]} jk_0(z + z_0) \tag{7}
\]

Where \( z = 2\beta^2 \left(1 / k_0\right) \left( z + z_0 \right) \).

In order to meet the boundary condition, the polar coordinate is changed from the cartesian coordinate, the integration variable is changed into

\[
z = \rho \cos \theta, \quad y = \rho \sin \theta, \quad \alpha = k_0 \sin \gamma \tag{8}
\]

The variable of exponential function in the integrated expression is turned out to be

\[
z(k_0^2 - \alpha^2)^{1/2} + y\alpha = k_0 \rho \cos[\theta - \gamma(\alpha)] \tag{9}
\]

Therefore, the exponential function can be expressed as

\[
e^{-jk_0 \rho \cos[\theta - \gamma(\theta)]} = \sum_{j=-\infty}^{\infty} j^{-n} e^{jn[\theta - \gamma(\theta)]} J_n(k_0 \rho) \tag{10}
\]

Where \( J_n(k_0 \rho) \) is the first kind of bessel function, insert equation (5) into equation (10), we can get

\[
\frac{E_i(y, z)}{E_0} = \sum_{n=-\infty}^{\infty} j^{-n} e^{jn\theta} J_n(k_0 \rho) A_n \tag{11}
\]

Where,

\[
A_n = \frac{1}{2\pi \sqrt{\pi \beta}} \int_{-\infty}^{\infty} e^{-\left(\alpha^2 / 4 \beta^2\right) - j\alpha(k_0^2 - \alpha^2)^{1/2} - jn \gamma(\alpha)} d\alpha \tag{12}
\]

In order to present the wave spread outwards, shape of the fringe field shall be as,

\[
\frac{E_i(\rho, \theta)}{E_0} = \sum_{n=-\infty}^{\infty} j^{-n} e^{jn\theta} H_n^{(2)}(k_0 \rho) B_n \tag{13}
\]

Where \( J_n^{(2)}(k_0 \rho) \) is the second Hankel function, \( B_n \) is the undetermined coefficient determined by the boundary condition.

On the other hand, the light through the optical fiber cannot be neglected [7], and the wave equation in the dielectric circular cylinder is obtained from the following equation (in the polar coordinate)

\[
\frac{\partial^2 E_i}{\partial \rho^2} + \frac{1}{\rho} \frac{\partial E_i}{\partial \rho} + \frac{1}{\rho^2} \frac{\partial^2 E_i}{\partial \theta^2} + k_0^2 e E_i = 0 \tag{14}
\]

Where, \( e \) is the dielectric coefficient of dielectric circular cylinder, let the transmission wave in the cylinder be

\[
\frac{E_i(\rho, \theta)}{E_0} = \sum_{n=-\infty}^{\infty} j^{-n} e^{jn\theta} R_n(\rho) C_n \tag{15}
\]

Insert equation (15) into equation (14), we can get the differential equation related to the bessel function, the result is [8]:

\[
R_n(\rho) = J_n(\sqrt{e}k_0 \rho) \tag{16}
\]
Two conditions must be in succession by using the boundary condition \( \rho = a_0 \) and tangential component of the field to determine two coefficients \( B_n \) and \( C_n \). Therefore, the scattered beam and transmitted light beam are:

\[
\frac{E'_i(\rho, \theta)}{E_0} = \sum_{n=-\infty}^{\infty} j^{-n} e^{j\rho} \left( \frac{J_n(k_0a_0) - J_n(\rho k_0a_0)}{\epsilon J'_n(\rho k_0a_0) - J'_n(\rho k_0a_0)} \right) H_n^{(2)}(k_0\rho) A_n
\]

(17)

\[
\frac{E'_t(\rho, \theta)}{E_0} = \frac{2}{\pi k_0a_0} \sum_{n=-\infty}^{\infty} \frac{J^{-n+1}e^{j\rho}J_n(\rho k_0a_0)A_n}{\epsilon H_n^{(2)}(k_0a_0)J'_n(\rho k_0a_0) - J'_n(\rho k_0a_0)J_n(\rho k_0a_0)}
\]

(18)

When the electric vector is parallel with the optical fiber,

\[
\gamma_m = \frac{n_2 J^m(n_k a_0)}{n_1 J^m(n_k a_0)} \left( \frac{J'_m(n_k a_0)}{J_m(n_k a_0)} \right) - n_1 J'_m(n_k a_0)
\]

(19)

When the magnetic-field vector is parallel with the optical fiber,

\[
\gamma_m = \frac{n_1 J^m(n_k a_0)}{n_1 J^m(n_k a_0)} \left( \frac{J'_m(n_k a_0)}{J_m(n_k a_0)} \right) - n_2 J'_m(n_k a_0)
\]

(20)

Where \( a_0 \) is the Gaussian beam waist radius, \( k_0 \) is wave vector. \( J_m, H_m \) are order Bessel function and Hankel function.

The electric field is expressed as the following equation by the amplitude \( u \):

\[
E = jn_k \hat{v}
\]

(E is parallel with the),

(21)

\[
E = -jmu \hat{r} - \frac{\partial u}{\partial \theta}
\]

(E is perpendicular to the optical fiber),

(22)

Where \( \hat{v} \) is the unit vector under the rectangular coordinate system \((x, y)\). For these two conditions, the complex amplitude is in proportion with square of the electric field intensity [9],

\[
|E|^2 \propto |u|^2
\]

(23)

In order to figure out the field intensity in the computer, the infinite summation function in equation (17) must be converted to the finitude summation function. With order number \( m \) increase, the summation function tends towards 0. For a great \( z \), the bessel function (not in coefficient \( A_m \) and \( \gamma_m \)) in equation (17) can be substituted by the following equation [10]:

\[
J_m(z) \to \left( \frac{2}{\pi} \right)^{1/2} \cos(z - m\pi/2 - \pi/4)
\]

(24)
\[ Y_m(z) \rightarrow \left( \frac{2}{\pi} \right)^{1/2} \sin(z-m\pi/2-\pi/4) \tag{25} \]

The light intensity passing through the optical fiber is superposition of scattered beam and transmitted beam \([11]\), we can get the light intensity expression:

\[ I(\theta, r) = \left| \int_0^\theta \sum_n \exp(-im\theta) \mu^m A_m(\omega_0, k_0) \times \left[ J_n(n_r k_r r) \right] - H_m^{(1)}(n_r k_r r) \gamma_m(k_0, r, n) \right|^2 \tag{26} \]

We shall establish a model with the mathematical software to simulate the infection of various parameters in the light intensity curve.

3. Numerical Simulation and test system

We can observe from equation (26) that after the optical fiber refractive index and optical fiber radius are determined, the light intensity is only the function of scattering angle \(\theta\). We can draw the relation curve between the scattering angle and light intensity through the mathematical software. For an assigned optical fiber radius, the light intensity corresponding to the scattering angle extreme value is regular. The minimal value and maximal value alternatively distribute and correspond to a set of scattering angle extreme value. Different optical fiber radius shall correspond to different extreme value of scattering angle. Curve shown in Fig. 2 is the light intensity curve when the radius is respectively \(r=0.6\mu m\) and \(r=0.3\mu m\).

![Figure 2. Light intensity curve when the radius, r, is 0.3µm and 0.6µm, respectively.](image)

Let radius varies in a certain range, calculate the scattering angle corresponding to the light intensity extreme value within certain radius span with the mathematical software through equation 3-26, repeat the operation, you can draw a radius~angle relation curve.

Devices include fiber test system composed of laser, polarizer, condenser, magnifier, shadow shield and receiving screen and image acquisition and data analysis system composed of CCD camera and computer \([12]\), shown in Fig. 3. Laser, polarizer, condenser, magnifier, shadow shield, receiving screen and CCD camera are fixed and installed in order within the shell. There is central gap left in the middle of shadow shield. Pre-position of measured fiber is set in the hollow of shell. Semiconductor monochromatic laser is used; laser beam sent out by it becomes polarized light through polarizer; condenser and magnifier are adjusted to make distance between two lens become the sum of focal distance of two lens; location of shadow shield is adjusted to make central gap level to align measured fire; CCD camera is adjusted to make lens to align receiving screen; CCD camera and computer are connected with data wire. CCD camera adopts interference diffraction fringe map on receiving screen; the interference diffraction fringe map is transmitted to computer through data wire; computer reads
strong maximum of interference diffraction fringe map and comparison of computer memory database; diameter value of fiber measured is output; measurement results is displayed by indicator finally. CCD camera takes photos in high frequency and transmits the map to the computer continuously, and the computer treat data in real time and outputs measurement result; real-time, rapid and accurate measurement is finished during the progress of fiber drawing.

4. Conclusion

We discuss a kind of new method in measuring the diameter of the fiber based on the forward scattering method, further study the theory on measuring the optical fiber radius with the method, and establish a set of integrated database on the foundation. For the tapered fiber with unknown radius, at first, measure the scattering intensity, draw the strength distribution curve, and then you can find out the optical fiber radius in the relation storage in accordance with the angle corresponding to the maximal and minimal value in the curve. The method is rapid and convenient in measuring the radius of the fiber, and can be taken as a real time method in measuring the fiber diameter.

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