Comment on *Increased mortality is predicted of Inachis io larvae caused by Bt-maize pollen in European farmland*, Ecological Modelling, 2013

Francesco Camastra\textsuperscript{a}, Angelo Ciaramella\textsuperscript{a}, Antonino Staiano\textsuperscript{a,*}

\textsuperscript{a}Department of Science and Technology, University of Naples Parthenope, Centro Direzionale Isola C4, 80143 Naples, Italy

Abstract

In this comment, we analyze the mathematical model by Holst et al that estimates the increased mortality of *Inachis io* larvae caused by Bt-maize pollen in European farmland. In particular, we focus our attention on the part of Holst et al’s work that models the Bt-maize pollen loss on plant leaves by a differential equation. We provide the exact solution of the differential equation, showing that the behaviour of the solution, when the time goes to infinity, is different from what claimed in Holst et al’s work.

*Keywords:* Inachis io; Bt-maize pollen; pollen density in plant leaves; inhomogeneous linear differential equation of first order

1. Introduction

Nowadays, the cultivation of Genetically Modified Plants (GMP) is very widespread in the world, in particular in America and Asia. On the other
side, the debate in the scientific community and in the public opinion about the GMP effects is becoming harder and harder (Sanvido et al., 2011). To this purpose, recently appeared some mathematical models for estimating the effects of Bt-maize (Perry et al., 2010, 2012; Holst et al., 2013), on non-target Lepidoptera. In this comment, we analyze the mathematical model by Holst et al (Holst et al., 2013) that estimates the increased mortality of *Inachis io* larvae caused by Bt-maize pollen in farmaland of Northern Europe. In particular, we focus our attention on the part of Holst et al’s work that models the Bt-maize pollen exposure of *Inachis io* larvae. The work is organized as follows: In Section 2 the Bt-maize pollen exposure of *Inachis io* larvae model is discussed; finally, some conclusions are drawn in Section 3.

2. Bt-maize Pollen exposure of *Inachis io* larvae

In their model, Holst et al represents the pollen exposure in terms of pollen concentration in the environment. In the specific case of *Inachis io* larvae pollen exposure is expressed in terms of average pollen density, measured in cm$^{-2}$, on the leaves of the food plant. Pollen deposition rate follows a parabolic curve through the pollination period (Kawashima et al., 2004). The integral of the curve provides the accumulated deposition. However, pollen deposited on food plant leaves will be lost after deposition. Holst et al model pollen loss on plant leaves by means of the following differential equation:

$$\frac{dN}{dt} = at^2 + bt + c - \epsilon N,$$  \hspace{1cm} (1)
where $N$ is pollen density (expressed in cm$^{-2}$), $t$ is time (measured in days), $\epsilon$ is loss rate, i.e., relative amount of pollen lost per day, and $a, b, c$ are the coefficients of the parabola.

Holst et al declare that the equation (1) was integrated and re-parameterized to predict average pollen density on leaves ($N$) and show, in figure 2 of the manuscript, that the average pollen density have a maximum at $N_{\text{peak}}$ and goes to zero when the time $t \to \infty$.

Having said that, we observe that the equation (1) is an inhomogeneous linear differential equation of first order (Korn and Korn, 2000) and can be solved exactly. To this purpose, we recall that a inhomogeneous linear differential equation of first order has the following form:

$$\frac{dN}{dt} = u(t)N + v(t),$$  \hspace{1cm} (2)

and has as general solution:

$$N(t) = e^{\int_{t_0}^{t} u(x)dx} [K + \int_{\beta}^{t} v(x)e^{-\int_{x}^{t} u(s)ds} dx],$$  \hspace{1cm} (3)

where $K \in \mathbb{R}$ and $\alpha, \beta$ are values that have to be chosen in a properly way that for convenience in our case we put to zero. Since in the case of equation (1) it is

$$u(t) = -\epsilon, \quad v(t) = at^2 + bt + c;$$  \hspace{1cm} (4)

the solution of the differential equation (1) is:

$$N(t) = e^{-\epsilon t}[K + \int_{0}^{t} (ax^2 + bx + c)e^{\epsilon x} dx]$$

$$= Ke^{-\epsilon t} + e^{-\epsilon t}\left[\frac{a(2 - 2x\epsilon + x^2\epsilon^2) + \epsilon(c\epsilon + b(-1 + x\epsilon))}{\epsilon^3}\right]_{0}^{t}$$

$$= \frac{a}{\epsilon}t^2 + \left(\frac{-2a}{\epsilon^2} + \frac{b}{\epsilon}\right)t + \left(\frac{2a}{\epsilon^3} + \frac{c}{\epsilon} - \frac{b}{\epsilon^2}\right) + (K - \frac{2a - b\epsilon + c\epsilon^2}{\epsilon^3})e^{-\epsilon t}(5)$$
where $K \in \mathbb{R}$.

It is immediate to verify that the limit of pollen density $N(t)$, when $t \to \infty$ is not 0, as shown in figure 2 of Holst et al’s manuscript, but:

$$\lim_{t \to \infty} N(t) = \begin{cases} \frac{c}{\varepsilon} & \text{if } a = b = 0 \\ \infty & \text{otherwise} \end{cases}$$

(6)

The degenerate case $a = b = 0$ has no practical interest, since the parabola $v(t) = at^2 + bt + c$ reduces to a constant line $v(t) = c$. In all other cases, pollen density $N(t)$ goes to infinity when time $t \to \infty$. Since it is clearly non acceptable, this means that the differential equation (1) cannot model correctly pollen density on the food plant. Therefore the theoretical soundness of Holst et al’s model is mined remarkably, in our opinion.

3. Conclusion

In this paper, we have discussed the mathematical model by Holst et al that estimates the increased mortality of *Inachis io* larvae caused by Bt-maize pollen in farmaland of Northern Europe. We have paid particular attention on the differential equation that Holst et al use to model the Bt-maize pollen loss, providing its exact solution. We showed that the behaviour of the solution, when the time goes to infinity, is different from what claimed in Holst et al’s work. Since the differential equation does not model correctly the pollen density on the food plant, the theoretical soundness of Holst et al’s work seems to be mined remarkably.
References

O. Sanvido, J. Romeis, F. Bigler, Environmental change challenges decision-making during post-market environmental monitoring of transgenic crops, Transgenic Research 20 (2011) 1191–1201.

J. N. Perry, Y. Devos, S. Arpaia, D. Bartsch, A. Gathmann, R. S. Hails, J. Kiss, K. Lheureux, B. Manachini, S. Mestdagh, G. Neemann, F. Ortego, J. Schiemann, J. B. Sweet, A mathematical model of exposure of non-target Lepidoptera to Bt-maize pollen expressing Cry1Ab within Europe, Proceedings of the Royal Society B, Biological Sciences 277 (2010) 1417–1425.

J. N. Perry, Y. Devos, S. Arpaia, D. Bartsch, C. Ehlert, A. Gathmann, R. S. Hails, N. B. Hendriksen, J. Kiss, A. Messean, S. Mestdagh, G. Neemann, M. Nuti, J. B. Sweet, C. C. Tebbe, Estimating the effects of Cry1F Bt-maize pollen on non-target Lepidoptera using a mathematical model of exposure, Journal of Applied Ecology 49 (2012) 29–37.

N. Holst, A. Lang, G. Lovei, M. Otto, Increased mortality is predicted of Inachis io larvae caused by Bt-maize pollen in European farmland, Ecological Modelling 250 (2013) 126–133.

S. Kawashima, K. Matsuo, M. Du, Y. Takahashi, S. Inoue, S. Yonemura, An algorithm for estimating potential deposition of corn pollen for environmental assessment, Environmental Biosafety Research 3 (2004) 197–207.

G. A. Korn, T. A. Korn, Mathematical Handbook for Scientists and Engineers, Dover Publications, 2000.