A NOVEL META-HEURISTIC ALGORITHM FOR MULTI-OBJECTIVE DYNAMIC FACILITY LAYOUT PROBLEM

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Abstract. This paper proposes an integrated approach for dynamic facility layout problem considering the material handling equipment (MHE). The objectives of this problem are minimization of the fixed costs of MHE, minimization of material handling cost (MHC) and minimization of machine rearrangement costs (MRC). To be more realistic, MHE fixed costs, MRC and MHC, which might be of different importance to decision maker, are considered separately in three objective functions. An integrated model is proposed which is able to simultaneously select the MHE along with the arranging and re-arranging facilities. The model belongs to the class of multi-objective nonlinear mathematical programming models. Considering the NP-hard nature of the model and inspiring the existing water flow like algorithm, a novel Pareto-based meta-heuristic algorithm called multi-objective water flow like algorithm (MOWFA) is developed to solve the problem. We comprehensively discuss the parameter tuning of the algorithms utilizing Taguchi method. Finally, the performance of the proposed MOWFA is evaluated against two well-known meta-heuristic algorithms called non-dominated sorting genetic algorithm (NSGA-II) and non-dominated ranking genetic algorithm (NRGA). Computational results indicate the greater efficiency of the algorithm compared to the two addressed algorithms for solving the given multi-objective problem.

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1. Introduction

Facility layout problem is concerned with determining the efficient layout of machines, cells, or departments. There are several factors which can directly or indirectly affect the efficiency of a layout, among which the most important factors are: variation on demand, adding or removing a product, changing the manufacturing methods and replacing the equipments. All the factors may affect the patterns of material handling between two machines as well as material handling costs (MHC). According to Tompkins et al. [1], around 20−50 percent of the total operating costs and 15−70 percent of the total manufacturing costs are concerned with MHC. MHC is known as the most important metric to determine the effectiveness of a layout and is often considered as the single objective of facility layout problems. Due to the variations in product demand and changes in product

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mix, MHC fluctuates and often increases. This may cause the current layout to lose its efficiency and make re-layout of facilities necessary.

Changes in the current layout should be investigated carefully since these may result in high machine-rearrangement costs (MRC). The problem of preparing the layout and re-layout of the machines in such a way as to minimize MHC and MRC is called dynamic facility layout problem (DFLP). In DFLP, a planning horizon divided into a number of periods, is assumed. Any changes which results in changing MHC, gives the signal of beginning a new period. The objective of DFLP is to determine a layout in each period of horizon planning so that the sum of MHC and MRC is minimized. Apart from the layout of the machines, material batch size is another factor which can affect MHC. Material batch size can reduce the number of the required trips to transfer materials between two machines. The batch size depends on the capacity of material handling equipment (MHE). There are different MHEs such as forklifts, trucks and automated guided vehicles (AGVs) with different capacity used to transport materials between machines.

Thus, there is a close relation between layout of machines and the selected transporters. The final solution is highly dependent on the given part of the problem; in other words, by selecting the best MHE based on an existing layout, or determining the optimal layout based on pre-specified MHE, a considerable degree of freedom may be lost in obtaining an overall optimal design (Chittratanawat [2]); thus, it is recommended to study the problem of allocating MHE to machines in order to handle the materials and solve the problem of machine layout as an integrated problem. As a result, this paper proposes an integrated approach for DFLP considering MHE. To be more realistic, the regular costs of material handling, the fixed costs of establishing MHE and MRC are considered in three separate objective functions. The main reason of this segregation is the nature of these costs which are significantly important to decision makers. Since the behavior of these objectives are not monotonic, the problem falls into the class of multi-objective optimization problems (MOOP). The proposed MOOP is mathematically modeled as a non-linear integer programming model. The mathematical model belongs to the class of NP-hard problems. Soft computing techniques, specifically evolutionary computations, are usually employed to find near-optimum solutions. This paper introduces a multi-objective water flow like algorithm (MOWFA) as an efficient multi-objective evolutionary algorithm to solve the problem. The performance of MOWFA is compared with two popular algorithms including non-dominated sorting genetic algorithm (NSGA-II) and non-dominated ranking genetic algorithm (NRGA). The rest of the paper is structured as follows: Section 2 gives a brief review of the previous studies; then, the mathematical formulation of the problem is introduced in Section 3. Thereafter, Section 4 describes the proposed methodologies to solve the multi-objective dynamic facility layout problem (MODFLP). The parameter tuning and computational results for the proposed algorithm are presented in Section 5. Finally, the conclusion and ideas for future research are given in Section 7.

2. A BRIEF LITERATURE REVIEW

Layout researches can be categorized into two types of static or dynamic. In static layout, material flow between machines is constant and an optimal layout is designed for a single time period. On the contrary, if the layout is evaluated and modified occasionally with respect to material flow changes, it is called dynamic layout. We give a brief review of the researches regarding the dynamic category. Rosenblatt [3] was the first one who addressed the problem of dynamic facility layout. Rosenblatt [3] developed a procedure based on dynamic programming to determine the optimum design which takes into consideration both MHC and MRC, for multiple periods. In recent years, there have been several efforts to address the DFLP and the research in this area typically hires heuristic and meta-heuristics solution techniques. Urban [4] suggested a heuristic steepest-descent pair-wise interchange procedure combined with the concept of forecast windows similar to CRAFT in order to solve the DFLP. Kouvelis and Kiran [5] proposed a developed dynamic programming to consider the dynamic aspects in automated manufacturing systems. A heuristic tabu search was proposed by Kaku and Mazzola [6] for solving the DFLP. Their heuristic technique was based on initializing a solution to obtain the feasible one. The whole method is divided into two stages: 1. diversifying search and 2. intensifying
the search to identify the best solution among the promising solutions identified in stage one. The experimental results showed that tabu search is effective in providing high value solutions for DFLP.

The genetic algorithm was first employed by Conway and Venkataramanan [7] to solve DFLP. The developed methodology evaluated the appropriateness of GA to generate feasible layouts. Balakrishnan and Cheng [8] proposed an improvement in application of GA procedures to solve DFLP. They adopted a different crossover and mutation operators and also used a new generational replacement strategy to help increase population diversity. The computational study showed that their proposed GA was quite more effective than GA proposed by [7]. Baykasoglu and Gindy [9] denoted the simulated annealing (SA) method to solve the DFLP. Their approach was a straightforward implementation of SA to solve the DFLP. Using the test problems from [8], Baykasoglu and Gindy [9] showed that the SA is more capable than developed GAs to achieve better solutions.

Mckendall and Shang [10] proposed three hybrid ant systems (HAS) for solving the DFLPs. In the first system, a HAS with a pair-wise exchange style was used to improve the solution quality. In the second one named modified heuristic HAS, the pair-wise exchange heuristic was replaced by an SA procedure. In the third heuristic system, a look-ahead/look-back strategy was also used. Furthermore, McKendall et al. [11] offered two SA methods. The first SA heuristic method called SAI, is a direct adaptation of SA for the DFLP. The second SA heuristic method (SAII) is just like the SAI, except that it has an added look-ahead/look-back strategy. Krishnan et al. [12] presented a new tool “Dynamic From Between Chart” to analyze the redesigning layouts. It models the production rate changes using a continuous function.

Rezazadeh et al. [13] applied an extended particle swarm algorithm (PSO) for the DFLP. Balakrishnan and Cheng [14] studied the performance of algorithms in static and rolling horizons, under forecast uncertainty for the DFLP. Sahin and Turkbey [15] also suggested a novel hybrid meta-heuristic algorithm based on the SA approach supplemented with a tabu list. Mekendall and Liu [16] proposed three tabu search (TS) heuristics for DFLP. The first heuristic was a simple TS heuristic. The second heuristic added diversification and intensification strategies to the first one, while the third one was a probabilistic TS heuristic. Chen [17] proposed a new encoding and decoding scheme for solution representation within ant colony algorithm framework. It revealed the significant impact of solution representation on the efficiency of heuristics in terms of computational time.

It is only recently that researchers have been proposing multi-objective approaches for DFLP. Chen and Rogers [18] were the first ones who proposed a multi-objective dynamic facility layout model to search about several features of the facility layout problems such as time and distance-based objective as well as the adjacency-based objective. They applied a meta-heuristic optimization algorithm called ant colony optimization to solve the MODFLP. Their results indicated this heuristic technique provides the DFLP with a practical decision support tool.

Jolai et al. [19] consider a multi-objective DFLP with unequal fixed size departments and pick up/drop off locations. Their objectives were to minimize the MHC and the MRC and maximize the total adjacency and distance rate between facilities. They implemented a multi-objective PSO algorithm to solve the problem. Emami and Nookabadi [20] proposed a model in which both quantitative and qualitative analyses of dynamic facility layout problems were simultaneously taken into account. In their model, the re-arrangement and material handling costs have been considered as two distinct functions. Similar to the qualitative objective, the adjacency-based objective also aims at maximizing the adjacency scores of the facilities.

Most of the previous studies have focused only on minimizing the sum of MHC and MRC as the most important performance criteria; however, the transporters type used to move the materials between the machines has not been taken into consideration. On the other hand, in the researches available in the literature, the importance of MHC and MRC are identical while they may be of different importance to decision makers. To eliminate these types of shortcomings, this research takes the transporters type into account and proposes a multi-objective model to capture the costs separately. Furthermore, a novel powerful meta-heuristic algorithm is devised to optimize the proposed multi-objective problem. All of these contribute to make our approach more realistic and applicable for solving the layout problem of manufacturing systems.
3. Mathematical model

In this section, the integrated problem is formulated as a multi-objective non-linear integer programming model to minimize the sum of MHC, MHE fixed costs and MRC. In order to help a better understanding of the model presented in the paper, the assumptions, parameters and the decision variables are first defined as follows:

The problem is formulated under the following assumptions:

1. The material flow between machines is dynamic and predetermined.
2. There is a potential set of MHE with predefined fixed costs.
3. The sizes of the machines and locations are equal.
4. The distances between locations are known in advance.

The parameters and indices are:

- \( N \) The number of machines/locations;
- \( T \) The number of periods in the planning horizon;
- \( TR \) The number of available transporters;
- \( i, j, k, l \) Index of machines/locations;
- \( t \) Index of time periods;
- \( tr \) Index of transporters;
- \( A_{i,j,l} \) The cost of shifting machine \( i \) from location \( j \) to \( l \) in period \( t \);
- \( C_{t}^{i,j,k} \) The material transporting cost between machine \( i \) and machine \( k \) in period \( t \) by transporter \( tr \);
- \( m_{t}^{tr} \) The maximum number of vehicles \( tr \) available in period \( t \);
- \( cap^{tr} \) Transport capacity of transporter \( tr \);
- \( R_{t,i,k} \) 1 if any material is transported between machine \( i \) and machine \( k \) in period \( t \) 0 otherwise;
- \( FIT_{tr} \) Fixed cost of establishing of transporter \( tr \);
- \( AT_{t}^{tr} \) The available time of transporter \( tr \) in period \( t \);
- \( Time_{t,i,k}^{tr} \) The time is required to complete a tour from machine \( i \) to machine \( k \) by transporter \( tr \) in period \( t \);
- \( AvgT_{t,i,k}^{tr} \) The average time is required to complete a tour from location \( j \) to location \( l \) by transporter \( tr \);
- \( F_{t,i,j,l} \) The material flow from machine \( i \) to machine \( j \) in period \( t \);
- \( D_{t,j,l} \) The distance between location \( j \) and location \( l \) in period \( t \).

The decision variables are:

- \( X_{t,i,j} \) 1 if facility \( i \) is allocated to location \( j \) in period \( t \)
- 0 otherwise;
- \( Y_{t,i,k}^{tr} \) 1 if transporter \( tr \) is selected to handle material from machine \( i \) to machine \( k \) in period \( t \)
- 0 otherwise;

The proposed multi-objective dynamic facility layout problem is now formulated as a non-linear integer programming model. This formulation is an extension of the model presented by McKendall et al. [10]:

\[
\text{Min } Z_1 = \sum_{i=1}^{N} \sum_{j=1}^{N} \sum_{k=1}^{N} \sum_{l=1}^{N} \sum_{t=1}^{T} \sum_{tr=1}^{TR} \left( \frac{F_{t,i,k}^{tr}}{cap^{tr}} \right) \times D_{t,j,l} \times C_{t,i,k}^{tr} \times X_{t,i,j} \times X_{t,k,l} \times Y_{t,i,k}^{tr}
\]

\[
\text{Min } Z_2 = \sum_{i=1}^{N} \sum_{j=1}^{N} \sum_{l=1}^{N} \sum_{t=2}^{T} X_{(t-1)ij} \times X_{tl} \times A_{t,i,j,l}
\]
Min \( Z_3 = \sum_{t=1}^{T} \sum_{i=1}^{N-1} \sum_{k=i+1}^{N} \sum_{tr=1}^{Tr} Y_{t,i,k}^{tr} \times Ftr \) \hspace{1cm} (3.3)

\[ \sum_{j=1}^{N} X_{ti,j} = 1, \forall i = 1, 2, \ldots, N, \forall t = 1, 2, \ldots, T \] \hspace{1cm} (3.4)

\[ \sum_{i=1}^{N} X_{tij} = 1, \forall j = 1, 2, \ldots, N, \forall t = 1, 2, \ldots, T \] \hspace{1cm} (3.5)

\[ \sum_{tr=1}^{TR} Y_{t,i,k}^{tr} = R_{t,i,k}, \forall i = 1, 2, \ldots, N - 1; \forall k = i + 1, 2, \ldots, N; \forall t = 1, 2, \ldots, T \] \hspace{1cm} (3.6)

\[ Y_{t,i,k}^{tr} = Y_{t,k,i}^{tr}, \forall i, k = 1, 2, \ldots, N; \forall t = 1, 2, \ldots, T; \forall tr = 1, 2, \ldots, TR \] \hspace{1cm} (3.7)

\[ \sum_{i=1}^{N-1} \sum_{k=i+1}^{N} Y_{t,i,k}^{tr} \leq m_{t}^{tr}, \forall tr = 1, 2, \ldots, TR; \forall t = 1, 2, \ldots, T \] \hspace{1cm} (3.8)

\[ \sum_{i=1}^{N} \sum_{k=1}^{N} \sum_{l=1}^{N} F_{t,i,k}^{cap} \times Y_{t,i,k}^{tr} \times Time_{t,i,k}^{tr} \leq AT_{t}^{tr}, \forall tr = 1, 2, \ldots, TR; \forall t = 1, 2, \ldots, T \] \hspace{1cm} (3.9)

\[ \text{Time}_{t,i,k}^{tr} = \sum_{j=1}^{N} \sum_{l=1}^{N} X_{t,i,j} \times X_{t,k,l} \times AvgT_{j,l}^{tr}, \forall i = 1, 2, \ldots, N, \forall k = 1, 2, \ldots, N, \forall t = 1, 2, \ldots, TR; \forall tr = 1, 2, \ldots, T \] \hspace{1cm} (3.10)

\[ X_{t,i,j} \in \{0, 1\}, \forall i, j = 1, 2, \ldots, N, \forall t = 1, 2, \ldots, T \] \hspace{1cm} (3.11)

\[ Y_{t,i,k}^{tr} \in \{0, 1\}, \forall i, k = 1, 2, \ldots, N, \forall tr = 1, 2, \ldots, TR, \forall t = 2, \ldots, T. \] \hspace{1cm} (3.12)

The objective function (3.1) of this model is to minimize the total MHC. Objective function (3.2) minimizes the MRC and objective function (3.3) minimizes the fixed cost of the MHE. Constraints (3.4) ensure that each facility should be located in one position while constraints (3.5) ensure that in each position only one facility should be allocated. Set of constraints (3.6) state that for any pair of machines, if the materials are handled, a transporter must be assigned. Set of constraints (3.7) guarantee that one type of transporter is assigned to each pair of machines. Set of constraints (3.8) control the number of available vehicles in each period. Constraints (3.9) restrict the functions allocated a transporter to its available time. Constraints (3.10) control the time required to complete a tour between two machines. Constraints (3.11) and (3.12) restrict decision variables. This integer binary programming model is an extension to the classic DFLP and correspondingly is a complex combinatorial optimization problem. Next section presents an efficient algorithm to solve the proposed problem.
4. Solution methodology

On the earth’s surface, gravitation force drives water flows to constantly move to lower altitudes. A flow will split into multiple sub-flows while moving from higher altitudes to lower ones on a rugged terrain. The mass and velocity are two main characteristics of a flow which determine the flow’s momentum. When a number of water flows move to the same position, they will be merged into a single water flow with higher momentum. Governed by gravity and driven by fluid momentum, flows can run to higher levels or run over bumps to navigate various terrains. Water flowing will cease and stagnate at lowest local or global depression, when the momentum left cannot drive out the water out of the hollow.

Inspiring this behavior of water flow, Yang and Wang [21] proposed a new meta-heuristic optimization algorithm, called water flow-like algorithm (WFA), for solving NP-hard combinational optimization problems. Tran and Ng [22] proposed a water-flow algorithm to solve scheduling problem. They applied the algorithm to a maltose syrup production problem, and illustrated it’s ability for solving problems in practical applications. Wu et al. [23] and Chang and Wu [24] used the WFA logic and developed a heuristic algorithm for solving the cell formation problem. Their computational results showed that the proposed algorithm has performed better than other benchmarking approaches in terms of both solution effectiveness and efficiency. Recently, Tran and Ng [25] used a hybrid WFA for solving multi-objective flexible flow shop scheduling problem with limited buffers. The performance of the proposed algorithm was investigated by randomly generated test problems. The computational results proved the effectiveness and efficiency of the WFA.

In WFA, the solution space of a problem is mapped as a geographical terrain and the objective function value is considered as the altitude of a water flow while each water flow represent a solution agent. Water splitting and moving to a lower position can be considered as a process of searching for the optimum status. A flow with larger momentum will generate more sub-flows and its surrounding space will be better explored. The mass and velocity of each flow are distributed to its sub-flows so that high quality sub-flows are of higher proportion of the mass and velocity of the addressed flow. When more than two flows move to the same location, they will merge into a single flow. Mass and velocity are then amassed to create an aggregated flow. The new aggregated flow is able to reinforce the searching process around its location. Furthermore, to escape local optima, some water flows may evaporate and return to the terrain by precipitation.

4.1. Multi-objective water flow-like algorithm

In this paper, we have developed a multi-objective version of WFA for discrete optimization problems. Our multi-objective water flow like algorithm (MOWFA) is based on the Pareto approach. The MOWFA consists of five main operations: (3.1) flow splitting and moving; (3.2) flow ranking; (3.3) flow merging based on similarity coefficient method; (3.4) flow evaporation; and (3.5) heuristic precipitation. Physics quantities such as mass, velocity, fluid momentum, energy, and gravitational acceleration are also used as basic parameters to construct this algorithm. In the proposed MOWFA, the flows are ranked based on the two key concepts of non-dominated sorting and crowding distance.

In MOWFA, solution agents are considered as water flows, solution space is mapped to the geographical terrain and the objective function value is considered by the altitude of a water flow. Searching the solution space is modeled as the terrain traversed by the flows. Initially, a cloud, representing an iteration, randomly produces a set of flows into different positions of the ground. The flows are ranked based on non-dominated sorting and crowding distance metrics. Then, based on the rank, each flow is given a momentum. The flows with higher momentums are allowed to generate more sub-flows. Then any tow flows at the same location (sub-flows), are merged into a single flow with higher momentum. In each iteration, the momentum of all flows is decreasing. The flows with zeros momentum will be evaporated and then are returned to the ground by precipitation. The way of encoding the flows, flow splitting and moving, flow merging operation, flow evaporation operation and
precipitation used in the MOWFA are described later. To give a better understanding of MOWFA, Figure 1 shows its Pseudo-code.

**4.2. A fast non-dominated sorting approach**

First, the flows are sorted based on non-domination relationship. The non-domination is an individual factor and is said to dominate the other one if its objective function is not worse than the other and at least, one of its objective functions is better than the other one. Figure 2 shows a graphical representation of the fast non-dominated sorting when minimizing the both objectives is desired. Individuals such as $x_1$, $x_4$, $x_6$ and $x_7$ are assigned ranks as rank 1 since there is no individual superiority to them with respect to $f_1(x)$ and $f_2(x)$. After elimination of the individuals classified as rank 1, individuals with rank 2 are selected, and this process is repeated until all individuals are classified. The non-dominated sorting approach will require at most $O(MN^2)$ computations, where $M$ is the number of objectives and $N$ is the number of solutions in the population [26].

A set of front-1 individuals is called Pareto-optimal front.

If two solutions (flows) fall into a same front, a new metric called “crowding distance” is used to evaluate the individuals. The crowding distance is a measure of how close an individual is to its neighbors. The basic idea behind the crowding distance is finding the Euclidean distance between each individual in a front based on their $m$ objectives in $m$-dimensional hyperspace.

Then, the standard operators of the WFA are performed on the flows to enhance the current flows and generate new flows as the next generation. Furthermore, we include a heuristic precipitation process in the algorithm to raise the solution exploitation capability of the search process. The details of the other operations of the MOWFA for the MODFLP are described in the following subsections.
4.3. Encoding the flows

Initially, a set of flows as initial random solutions are randomly distributed on some positions on the ground. As mentioned earlier, flows are mapped as solutions and must be a representative of decision variables. We present each flow by two sub-matrices each of which corresponds to a special area of decision making. The first sub-matrix, which is related to the way machines are placed in positions, is presented by a $1 \times NT$ vector in which $N$ is the number of machines and $T$ is the number of periods. The cells $1-T$ of sub-matrix 1 correspond to machine layout in period one, cells $T+1-2T$ correspond to machine layout in period two and so on. The value inside each cell denotes the machine number and the cell’s rank represents the location of the machine. For example, Figure 3 shows a solution with two periods of time in which during period one, machine 2 is placed in location 1, machine 4 is placed in position 2, machine 1 is placed in position 3, machine 5 is placed in position 4 and machine 3 is placed in position 5. The second sub-matrix is related to assigning the transporters to machines in which the type of selected transporter is pointed to. For instance, in typical solution of Figure 3, the transporter type 3 is responsible for handling the materials between machines 1 and 2 in period 1.

4.4. Flow splitting and moving

Driven by flow momentum, the flows start to move to new locations and explore the solution space for better solutions. A flow with higher momentum generates more streams of sub-flows than one with less momentum.
The number of sub-flows split from a flow can be computed by equation (4.1),

\[ n_i = \min \left\{ \bar{n}, \text{int} \left( \frac{M_i V_i}{T} \right) \right\} \]  

(4.1)

where \( \bar{n} \) is an upper bound imposed on the number of sub-flows, \( M_i \) represent the mass and \( V_i \) represent the velocity of the flow “i”. When the momentum of a flow is lower than a predefined base momentum \( T \), no splitting happens and the flow moves as a single stream to the neighboring location. Furthermore, if a flow has a zero velocity, it will stagnate at its location. The locations of the split sub-flows are derived from the neighboring locations of the original flow. In reality, a flow movement is a search procedure from the current location to a neighboring one. The design of the flow-moving operation is problem-dependent. For more efficiency of proposed MOWFA, we have designed two neighborhood strategies which are capable of keeping the feasibility of solutions.

In neighborhood strategy type 1, after randomly selecting a period, two different cells of sub-matrix 1 and two different cells of sub-matrix 2 are selected and swapped (as is depicted in Fig. 4). In neighborhood strategy type 2, two periods are randomly selected and all the data related to the machine layout and transporter allocation of these periods are exchanged.

As previously mentioned, the mass of flow \( i \) is distributed to its sub-flows. For this purpose, the sub-flows of flow \( i \) are ranked based on non-dominated sorting and crowding distance metric. The mass of sub-flow \( j \) distributed by flow \( i \), \( U_{ij} \), can be obtained from equation (4.2).

\[ U_{ij} = \left( \frac{n_i + 1 - \text{rank}_j}{\sum_{r=1}^{n_i} r} \right) \times M_i \]  

(4.2)

where \( \text{rank}_j \) represents the rank of sub-flow \( j \) with respect to the other sub-flows. For instance, if flow \( i \) splits to 5 sub-flows, the mass of rank 1 sub-flow is obtained from equation (4.3).

\[ U_{ij} = \left( \frac{5 + 1 - 1}{15} \right) \times M_i = \frac{5}{15} \times M_i. \]  

(4.3)
Also, velocity of sub-flow \( j \) is computed by equation (4.4).

\[
\mu_{i,j} = \begin{cases} 
\sqrt{V_i^2 + 2g\delta_{i,j}} & \text{if } V_i^2 + 2g\delta_{i,j} > 0 \\
0 & \text{otherwise}
\end{cases} 
\]

(4.4)

\[
\delta_{i,j} = \text{mean lim}_k \delta_k^{i,j}
\]

where “\( g \)” is the gravitational acceleration and \( \delta_k^{i,j} \) represents the altitude drop from flow \( i \) to its sub-flow \( j \) in objective function \( k \) which means the improvement of \( k \)th objective function when moving from current solution \( i \) to neighborhood solution \( j \). The value of \( \delta_k^{i,j} \) for each of minimization and maximization objective functions is computed separately (using Eq. (4.5)) and then is averaged,

\[
\delta_k^{i,j} = \begin{cases} 
f_k^i - f_k^{i,j}, & \text{for minimization} \\
f_k^{i,j} - f_k^i, & \text{for maximization}
\end{cases}
\]

(4.5)

where \( f_k^i, f_k^{i,j} \) represents the value of objective function \( k \) of flow \( i \) and \( f_k^{i,j} \) represents the value of \( k \)th objective function of sub-flow \( j \) distributed by flow \( i \).

**4.5. Flow-merging operation**

When more than two flows arrive at the same location, they will merge into a single flow with bigger momentum. Whether the location of two flows are the same or not is investigated by our proposed criterion called similarity coefficient (SC). If the SC between two flows is greater than a predefined threshold, their locations are supposed to be identical and the merging takes place. Figure 5 is proposed to find the SC between two flows \( a \) and \( b \):

\[
SL_{ab} = \sum_{i=1}^{N} \sum_{t=1}^{T} \frac{\partial (X_{ia}, X_{ib})}{N \times T}
\]

If \( SL_{ab} = 1 \)

\[
SC_{ab} = \sum_{i=1}^{N} \sum_{t=1}^{T} \frac{\partial (Y_{iak}, Y_{ikb})}{N \times T}
\]

Else

\( SC_{ab} = 0 \)

End If

/ \( SL_{ab} \) is the rate of similarity in terms of layout of machines /
/ \( X_{ia} \) and \( X_{ib} \) are the location of machine \( i \) in period \( t \) in the flows \( a \) and \( b \) respectively /
/ \( N \) is the number of machines /
/ \( T \) is the number of periods /
/ \( SC_{ab} \) is the rate of similarity coefficient /
/ \( Y_{iak}, Y_{ikb} \) are selected transporter between machine \( i \) and \( k \) in the period \( t \) in the flows \( a \) and \( b \) respectively /

**Figure 5.** Determining the similarity coefficient between two flows \( a \) and \( b \).
In Figure 5, \( \partial(A, B) \) is the similarity between two especial bits given by equation (4.6).

\[
\partial(A, B) = \begin{cases} 
1 & \text{if } A = B \\
0 & \text{otherwise.}
\end{cases}
\]  

(4.6)

According to Figure 5, the SC between two flows is zero if there is only a small difference in machine layout design. This is because the layout of machines has a great impact on the objective functions. If \( SC_{ab} \) exceeds a predefined threshold, the flows \( a \) and \( b \) are recognized as identical flows and merged. Suppose that the flows \( a \) and \( b \) are sharing the same location, then both flows \( a \) and \( b \) will be removed and the new flow will be generated based on the characteristics presented in equations (4.7)–(4.8).

\[
M_{\text{new}} = M_a + M_b
\]

(4.7)

\[
V_{\text{new}} = \frac{V_a M_a + V_b M_b}{M_a + M_b}
\]

(4.8)

The new generated flow is quite similar to flow \( a \) and \( b \) in terms of sub-matrix 1. But, since sub-matrix 2 (transporter allocation) of flows \( a \) and \( b \) may be different, the structure of the merged flow in sub-matrix 2 (transporter allocation) is followed by the flow with higher rank. Therefore, by reducing the number of agents which are representative of similar positions, MOWFA avoids doing redundant searches.

4.6. Flow evaporation operation

It is natural that flows of water evaporate after possible movement and return to the ground through precipitation. In MOWFA, water evaporation and precipitation mechanism help the algorithm to escape from local optima. If a flow falls into a local optimum, it will be stagnated and so loss the capability of moving, merging or splitting. To overcome this problem and release the flow from the local optima, the trapped flow is forced to evaporate into the atmosphere.

The proposed MOWFA establishes a velocity-based evaporation according to the flows with smaller velocities so that they will evaporate more speedily than flows with larger velocities. The formulation of the velocity-based evaporation proposed by [28], is presented in equations (4.9)–(4.10).

\[
M_i = (1 - \rho_i) M_i
\]

(4.9)

\[
\rho_i = \begin{cases} 
1, & \text{if } \mu_{i,j} = 0 \\
0, & \text{if } \mu_{i,j} \frac{V_i}{V} \geq 1 \\
1 - \mu_{i,j} \frac{V_i}{V}, & \text{if } 0 \leq \mu_{i,j} \frac{V_i}{V} \leq 1.
\end{cases}
\]

(4.10)

4.7. Heuristic precipitation

After a number of iterations, the evaporated water will return to the ground by precipitation operator. In this paper, a seasonal rainfall is applied periodically in which the precipitation takes place when the number of evaporated flows reaches to a predefined level. In order to enhance the algorithm’s diversity, the location of returned flow is deviated far away from the location of flow before evaporation. In this way, the bits of flow are arranged inversely (Fig. 6).

Finally, all the initial flows, all the generated sub-flows and the flows generated via precipitation are gathered and form a new population. Then in this population, the members are ranked by non-dominated sorting and crowding distance criteria. By eliminating the overflow members, a population is constructed as initial population of next generation. The process continues to be performed until the stopping criteria are satisfied.
4.8. Stopping criterion of the algorithms

For stopping MOWFA, first we define the set of $m$ iterations as a round. Then, in every iteration, the best value of every objective function is determined. If, the mean change of all the objective functions between two successive rounds remains constant within 0.95% confidence interval, the algorithm is terminated.

5. Computational experiments

In order to verify and evaluate the results obtained by MOWFA some computational experiments are conducted and the performance of MOWFA is compared with one of the NSGA-II and NRGA algorithms. To do this, a set of test problems are randomly generated. There are 4 different problem sizes. The first contains problems with 6 machines in 5 periods, second contains problems with 6 machines in 10 periods, third contains problems with 15 machines in 5 periods and the fourth contains problems with 15 machines in 10 periods. For each problem size, we generate eight instances. Note that throughout this paper all calculations were performed a PC with Intel Core2 Duo 2.26 GHz CPU and 2 GB RAM. Moreover, algorithms were coded in MATLAB software (Version 7.10.0.499, R2010a).

5.1. Multi-objective performance metrics

In multi-objective optimization algorithms, convergence to the Pareto optimal front and the maintenance of a diverse set of solutions are two objectives. There is no single metric to evaluate the performance of the algorithms to satisfy these objectives. In this paper, five metrics of spacing, mean ideal distance, diversity, number of found solutions in Pareto front and number of function evaluations have been used to measure the algorithms’ convergence and diversity.

5.1.1. Spacing metric

Zitzler [29] suggested spacing metric that calculates the relative distance between consecutive solutions in the non-dominated set obtained as equation (5.1).

$$S = \sqrt{\frac{1}{n_{os} - 1} \sum_{i=1}^{n_{os}} (d_{i} - \bar{d})^2}$$

where $d_{i}$ is the sum of the differences in objective function values between solution $i$ and its two nearest neighbors for each objective and $\bar{d}$ is the average of all $d_{i}$’s. When the solutions are nearly uniformly spaced, the spacing measure will be small. Thus, an algorithm with a smaller spacing is preferred.
5.1.2. Mean ideal distance (MID)

This metric proposed by Zitzler and Thiele [30] measures the convergence rate of Pareto fronts to an ideal point (0, 0),

\[
MID = \frac{1}{NOS} \sum_{i=1}^{NOS} c_i
\]  

(5.2)

where \( c_i = \sqrt{\sum_{j=1}^{J} f_{ji}^2} \) and \( f_{ji} \) is the \( j \)th objective function of \( i \)th solution. The low value of MID means that the solutions in Pareto front are high-quality solutions.

5.1.3. Diversity metric

This metric, proposed by Zitzler Thiele [30], evaluate scatter of solutions in the Pareto front. As a rule, whatever the scale is bigger, better.

\[
D = \sqrt{\sum_{j=1}^{J} \left( \max_i f_{ji} - \min_i f_{ji} \right)^2}.
\]  

(5.3)

5.1.4. Number of found solutions in Pareto front (NOS)

Count the number of the solutions in Pareto optimal front. Thus, more solutions in Pareto frontier imply better performance of the algorithm [31].

5.1.5. Number of function evaluations (NOF)

In order to give a general perspective to the readers about the algorithms’ speed, the number of function evaluations is considered as a performance metric. The speed of running the algorithms to find near optimum solutions is one of the most important indexes in order to evaluate the algorithms.

5.2. Tuning algorithm’s parameters

The value of the parameters significantly affects the quality of the algorithms. Algorithms cannot reach to the appropriate final solutions if their parameters are not adjusted properly. In this section, we investigate the behavior of proposed MOWFA in different levels of parameters and determine the best level of the parameters.

The full factorial design of experiment is a conventional statistical method used for tuning the parameters, but this method is not always well-organized because its calculations are increasingly complex when the number of parameters is high. Therefore, in this paper, to limit the number of the experiments, a practical experimental design technique, known as Taguchi method, is used. Taguchi method is a fractional factorial experiment that is proposed by Taguchi as an efficient alternative for full factorial experiments.

In order to apply the Taguchi method, the levels of the factors should first be determined. The initial levels of factors are shown in Table 1. The presented factors include the actual names along with their brief names.

In the Taguchi method, the values of quality characteristics obtained through the experiments are transferred into a measure called signal to noise (S/N) ratio. The frame of this ratio is different for each response type. equation (5.4) formulates S/N for a larger-the-better response type, where \( y_i \) is the \( i \)th observed value of the response (quality characteristic) and \( n \) is the number of observations in a trial.

\[
\frac{S}{N} = -10 \log \left( \frac{1}{n} \sum_{i=1}^{n} y_i^2 \right).
\]  

(5.4)

Multi-objective coefficient of variation (MOCV), proposed by Rahmati et al. (2013), is considered as the response for the experiments.

\[
MOCV = \frac{MID}{\text{Diversity}}.
\]  

(5.5)
Table 1. Algorithm parameter ranges along with their levels.

| Solving methodology | Parameter | Description | Level 1 | Level 2 | Level 3 | Level 4 |
|---------------------|-----------|-------------|---------|---------|---------|---------|
| NSGA-II             | Popsize   | Initial pop size | 30     | 70     | 140     | 200     |
|                     | $P_c$     | Percent of cross over | 0.7    | 0.8    | 0.85    | 0.9     |
|                     | $P_m$     | Percent of mutation | 0.1    | 0.15   | 0.17    | 0.2     |
|                     | Round     | Number of generations | 200   | 300    | 400     | 500     |
| NRGA                | Popsize   | Initial pop size | 50     | 100    | 150     | 200     |
|                     | $P_c$     | Percent of cross over | 0.7    | 0.8    | 0.85    | 0.9     |
|                     | $P_m$     | Percent of mutation | 0.1    | 0.15   | 0.2     | 0.3     |
|                     | Round     | Number of generations | 200   | 350    | 500     | 600     |
| MOWFA               | Popsize   | Number of initial flows | 30     | 50     | 70      | 90      |
|                     | $M_0$     | Initial mass of original low | 20    | 30     | 40      | 50      |
|                     | $V_0$     | Initial velocity of original low | 10    | 20     | 30      | 30      |
|                     | Round     | Number of Iterations | 100   | 200    | 300     | 400     |

Table 2. The orthogonal array and computational results to tune NSGA-II and NRGA.

| Exp No. | P1 | P2 | P3 | Iteration | MOCV1 | MOCV2 | MOCV3 | S/N   | MOCV1 | MOCV2 | MOCV3 | S/N   |
|---------|----|----|----|-----------|-------|-------|-------|-------|-------|-------|-------|-------|
| 1       | 1  | 1  | 1  | 1         | 1.02  | 1.29  | 1.69  | -2.68 | 1.27  | 1.55  | 2.06  | -4.41 |
| 2       | 1  | 2  | 2  | 2         | 0.66  | 0.49  | 0.42  | 5.47  | 0.99  | 0.85  | 0.74  | 1.25  |
| 3       | 1  | 3  | 3  | 3         | 0.72  | 0.84  | 0.77  | 2.15  | 0.90  | 1.03  | 0.96  | 0.33  |
| 4       | 1  | 4  | 4  | 4         | 0.87  | 0.98  | 0.82  | 0.99  | 1.05  | 1.20  | 1.00  | -0.71 |
| 5       | 2  | 1  | 2  | 3         | 0.55  | 0.58  | 0.60  | 4.75  | 0.67  | 0.71  | 0.77  | 2.86  |
| 6       | 2  | 2  | 1  | 4         | 1.11  | 0.69  | 0.60  | 1.62  | 1.33  | 0.86  | 0.74  | -0.10 |
| 7       | 2  | 3  | 4  | 1         | 0.89  | 0.82  | 0.71  | 1.83  | 1.08  | 1.00  | 0.90  | 0.03  |
| 8       | 2  | 4  | 3  | 2         | 1.11  | 0.85  | 0.95  | 0.21  | 0.79  | 0.89  | 0.74  | 1.84  |
| 9       | 3  | 1  | 3  | 4         | 0.39  | 0.41  | 0.43  | 7.79  | 0.49  | 0.50  | 0.55  | 5.78  |
| 10      | 3  | 2  | 4  | 3         | 0.52  | 0.74  | 0.60  | 4.07  | 0.63  | 0.89  | 0.73  | 2.42  |
| 11      | 3  | 3  | 1  | 2         | 0.68  | 0.59  | 0.71  | 3.58  | 0.83  | 0.74  | 0.89  | 1.66  |
| 12      | 3  | 4  | 2  | 1         | 0.68  | 0.52  | 0.84  | 3.17  | 0.84  | 0.64  | 1.01  | 1.43  |
| 13      | 4  | 1  | 4  | 2         | 0.68  | 0.62  | 0.59  | 4.00  | 0.86  | 0.75  | 0.75  | 2.08  |
| 14      | 4  | 2  | 3  | 1         | 0.35  | 0.47  | 0.52  | 6.89  | 0.47  | 0.52  | 0.65  | 5.17  |
| 15      | 4  | 3  | 2  | 4         | 0.57  | 0.59  | 0.66  | 4.32  | 0.71  | 0.71  | 0.80  | 2.60  |
| 16      | 4  | 4  | 1  | 3         | 0.61  | 0.62  | 0.72  | 3.70  | 0.72  | 0.77  | 0.92  | 1.87  |

As mentioned earlier, in Pareto-based algorithms, two main goals including acceptable convergence and diversity are considered. Since, MID measures the convergence rate of the algorithm and diversity measures, the diversification in Pareto front. MOCV is a comprehensive combination of major metrics which is used as a single response in the Taguchi method. Tables 2 and 3 summarize the experimental results of NSGA-II, NRGA and MOWFA. Regarding equations (5.4) and (5.5) these tables present S/N and MOCV as well.

Figures 7 and 9 shows how the index values of S/N are changing at different levels of the algorithms. Levels where the index S/N has reached the maximum are selected as the optimal levels. Optimal parameter levels of the algorithms are highlighted in Table 1.

5.3. Algorithm’s evaluation

In this section, we present results of numerical experiments for the proposed algorithm, compared to those for NSGA-II and NRGA using random generated test problems. Since the meta-heuristic algorithms are naturally stochastic, each instance of the problem is replicated 45 times and the averaged results are reported. The results
Table 3. The orthogonal array and computational results to tune MOWFA.

| Exp No. | Popsiz ety | M₀ | V₀ | Round | MOCV₁ | MOCV₂ | MOCV₃ | S/N  |
|---------|------------|----|----|-------|-------|-------|-------|------|
| 1       | 1          | 1  | 1  | 1     | 1.35  | 0.99  | 1.15  | −1.38|
| 2       | 1          | 2  | 2  | 2     | 1.13  | 1.08  | 0.90  | −0.36|
| 3       | 1          | 3  | 3  | 3     | 0.44  | 0.29  | 0.16  | 10.03|
| 4       | 1          | 4  | 4  | 4     | 1.24  | 1.07  | 0.91  | −0.69|
| 5       | 2          | 1  | 2  | 3     | 0.55  | 0.59  | 0.74  | 3.99 |
| 6       | 2          | 2  | 1  | 4     | 0.44  | 0.57  | 0.41  | 6.40 |
| 7       | 2          | 3  | 4  | 1     | 1.11  | 1.33  | 0.98  | −1.21|
| 8       | 2          | 4  | 3  | 2     | 0.30  | 0.19  | 0.13  | 13.23|
| 9       | 3          | 1  | 3  | 4     | 0.22  | 0.21  | 0.04  | 15.04|
| 10      | 3          | 2  | 4  | 3     | 0.34  | 0.25  | 0.17  | 11.69|
| 11      | 3          | 3  | 1  | 2     | 0.89  | 0.98  | 1.01  | 0.35 |
| 12      | 3          | 4  | 2  | 1     | 1.02  | 1.09  | 1.12  | −0.64|
| 13      | 4          | 1  | 4  | 2     | 0.50  | 0.52  | 0.58  | 5.46 |
| 14      | 4          | 2  | 3  | 1     | 0.09  | 0.16  | 0.17  | 16.80|
| 15      | 4          | 3  | 2  | 4     | 0.50  | 0.49  | 0.65  | 5.15 |
| 16      | 4          | 4  | 1  | 3     | 0.95  | 1.04  | 1.06  | −0.15|

Figure 7. The mean S/N ratio plot for each level of the factors for NRGA.

are shown in Appendix A where obtained values of five performance measures are given. Figure 10 shows these results graphically. Figure 10a shows the algorithms’ performances using the spacing metric. Since the standard spacing is less, it can be concluded that the NRGA algorithm has the worst performance among the algorithms. MOWFA and NSGA-II are approximately performing similarly.

Figure 10b compares MID metric of the algorithms. It is clear that in almost all instances the MOWFA algorithm performs better in finding high quality solutions. As shown in Figure 10c, in terms of Diversity metric, MOWFA gives better values compared with the two other algorithms for the most of the test problems. Also, it is clear that the performance of NSGA-II and NRGA is at the same level. As shown in Figure 10d, based on NOS metric, the performances of all the algorithm are approximately equivalent; however, as the problem size increases, the quality of NRGA is decreasing.
Figure 8. The mean $S/N$ ratio plot for each level of the factors for NSGA-II.

Figure 9. The mean $S/N$ ratio plot for each level of the factors for MOWFA.

Figure 10e illustrates the superiority of the proposed MOWFA algorithm in comparison with NSGA-II and NRGA in terms of NOF. According to Figure 10e, for small-sized problems, the MOWFA performs similar to other algorithms, while for the large-sized problems, MOWFA considerably outperforms than the two other algorithms, especially than NSGA-II.

Figure 11 is a graphical representation of the obtained Pareto front for problems 1, 9, 17 and 25. As shown in Figure 11, the solutions from MOWFA are of the best values of all objective functions which in turn is a good evidence for the effectiveness MOWFA in terms of MID.

In order to statistically verify the results shown in Figure 10, we used the analysis of variance (ANOVA) test and statistically compared algorithms according to each of metrics separately. In this case, the value of each metric is transformed to a normalized performance measure named relative percentage deviation (RPD),
Table 4. ANOVA test.

| Metric's name | $F$-value | $P$-value | Test results |
|---------------|-----------|-----------|--------------|
| Spacing       | 22.82     | 8.63e–09  | Null hypothesis is rejected |
| MID           | 86.28     | 6.47e–22  | Null hypothesis is rejected |
| Diversity     | 13.45     | 7.37e–05  | Null hypothesis is rejected |
| NOS           | 29.97     | 8.98e–11  | Null hypothesis is rejected |
| NOF           | 38.58     | 6.30e–13  | Null hypothesis is rejected |

Figure 10. Comparison of the proposed algorithm according to spacing metric MID metric, diversity metric, metric NOS and NOF metric.

obtained by the following formula:

$$RPD(i, j, k) = \frac{Alg_{sol}(i, j, k) - \min_{sol}(j, k)}{\min_{sol}(j, k)} \times 100.$$ (5.6)

In equation (28), $Alg_{sol}(i, j, k)$ shows the value of performance measure $j$ in problem number $k$ that has been obtained by algorithm $i$ and $\min_{sol}(j, k)$ is the best value of performance measure $j$ between all algorithms for
Figure 11. Visual presentation of obtained Pareto-front four test problem 1, 9, 17 and 25.

problem number \( k \). The ANOVA results are shown in Table 4, in which rejecting the Null hypothesis indicates that there is significant difference between algorithms.

Based on the outputs of ANOVA test, it is clear that the algorithms have significant differences in terms of all performance metrics which necessitate the use of Tukey test to statistically rank algorithms. The results of the 95% Tukey simultaneous confidence intervals are shown in Figure 12. The results prove well performance of the algorithms based on the mean and Tukey intervals. As Figure 12, we can say that NRGA has the lowest spacing performance among all the algorithms. But, the efficiency of the MOWFA and NSGA-II is at the same quality. In terms of MID metric, the MOWFA algorithm is dramatically better than two other algorithms. Also NSGA-II can perform better than NRGA. Better performance of the MOWFA in terms of Diversity metric can be shown in Figure 12. Based on NOS index the MOWFA and NSGA-II are statistically at the same level. Based on Figure 12 we can prove that MOWFA is the most preferable technique in terms of the required NOF.
6. Conclusion

This paper explained how to develop a more realistic mathematical model for dynamic facility layout problem. Such models can be applied in manufacturing systems with a variety of MHE. In addition to minimizing MHC and MRC, reducing the fixed/fixing cost of the MHE by selecting a safer MHE was also considered as the other objective of the proposed model. To solve the problem, a new meta-heuristic algorithm inspired from the existing water flow like algorithm called MOWFA was proposed. Various test problems were designed to evaluate the performance of the algorithm in comparison with two well-known multi-objective evolutionary algorithms called NSGA-II and NRGA. The experimental results indicated that our proposed algorithm outperforms both NSGA-II and NRGA and it is also able to improve the quality of the solutions. The MID measure showed that the MOWFA could achieve better the objective-function values, especially in large-sized problems. Also, the MOWFA was so faster than the two other algorithms; this can show its applicability for the large-sized real-world problems. The presented model was still open for incorporating other features, such as transporter failure, machine breakdown and random processing time. Another clue for a future research is to consider the proposed algorithm on other types of combinatorial optimization problems. As another direction, it could be interesting to work on some other meta-heuristics and compare them with the given algorithms.
APPENDIX A.

Table A.1. Multi-objective performance measures obtained for each algorithm.

| Spacing | MID       | Diversity |
|---------|-----------|-----------|
|         | MOWFA     | NRGA      | NSGA-II   | MOWFA     | NRGA      | NSGA-II   |
| P1      | 4364      | 4311      | 2517      | 89170     | 107245    | 85720     | 102056    | 93258     | 102825    |
| P2      | 2928      | 3125      | 2985      | 63720     | 99720     | 70301     | 92867     | 95507     | 65781     |
| P3      | 2778      | 3874      | 3223      | 68162     | 84268     | 82671     | 92266     | 92721     | 57835     |
| P4      | 2660      | 4712      | 3135      | 77740     | 99636     | 97115     | 89727     | 47798     | 97404     |
| P5      | 4184      | 4282      | 3536      | 65359     | 88359     | 112199    | 78184     | 33551     | 91797     | 77049     |
| P6      | 3600      | 3768      | 3087      | 64243     | 96525     | 82667     | 62843     | 67231     | 58281     |
| P7      | 2659      | 5385      | 3829      | 56215     | 82645     | 57337     | 37288     | 66708     | 101320    |
| P8      | 2800      | 3240      | 3817      | 76389     | 99850     | 91132     | 108879    | 88495     | 65230     |
| P9      | 2969      | 3539      | 3150      | 108094    | 164423    | 108605    | 102685    | 85674     | 55295     |
| P10     | 3016      | 4193      | 2081      | 111321    | 171307    | 149924    | 133687    | 68388     | 81878     |
| P11     | 2679      | 4327      | 2245      | 128346    | 154520    | 125884    | 120610    | 50642     | 64082     |
| P12     | 3092      | 4056      | 3164      | 114660    | 137378    | 162938    | 68065     | 73147     | 72590     |
| P13     | 2675      | 3846      | 3386      | 111383    | 165054    | 147971    | 146458    | 136729    | 122052    |
| P14     | 3241      | 4713      | 2785      | 105834    | 146264    | 151001    | 123827    | 112637    | 63483     |
| P15     | 2832      | 3749      | 2221      | 128319    | 178727    | 171941    | 99148     | 80979     | 86176     |
| P16     | 2979      | 3782      | 4342      | 123021    | 183561    | 136524    | 107005    | 85610     | 79217     |
| P17     | 5074      | 6149      | 5935      | 15901     | 190025    | 176880    | 75429     | 96786     | 90547     |
| P18     | 5118      | 8359      | 7322      | 151633    | 221223    | 221693    | 86343     | 105678    | 105563    |
| P19     | 5180      | 7611      | 5524      | 205728    | 236400    | 196257    | 98305     | 83073     | 53921     |
| P20     | 6250      | 6197      | 6741      | 179172    | 235829    | 198488    | 121580    | 88626     | 81865     |
| P21     | 7035      | 8531      | 7195      | 178895    | 251686    | 218866    | 113301    | 76071     | 88086     |
| P22     | 6201      | 8311      | 6767      | 189820    | 251138    | 231416    | 158760    | 102014    | 90023     |
| P23     | 6792      | 6251      | 5966      | 186711    | 211750    | 179490    | 156124    | 117459    | 96406     |
| P24     | 7787      | 8538      | 5369      | 179168    | 236281    | 204928    | 165063    | 84770     | 146450    |
| P25     | 8693      | 9091      | 8774      | 235733    | 347238    | 300896    | 164189    | 145473    | 90855     |
| P26     | 10368     | 10928     | 9718      | 230223    | 354887    | 311530    | 179110    | 99619     | 147893    |
| P27     | 8966      | 11440     | 9838      | 253287    | 416590    | 307895    | 184045    | 132195    | 116752    |
| P28     | 8391      | 11718     | 9819      | 239873    | 364283    | 340194    | 185933    | 132107    | 123084    |
| P29     | 9207      | 11063     | 8729      | 254977    | 394400    | 352149    | 184908    | 164806    | 93441     |
| P30     | 9209      | 10271     | 8087      | 263560    | 395843    | 304959    | 195043    | 126198    | 134747    |
| P31     | 10641     | 12461     | 9246      | 269918    | 359465    | 328036    | 195429    | 136831    | 124167    |
| P32     | 9729      | 11624     | 7705      | 273595    | 357401    | 396082    | 200425    | 140932    | 146759    |
### NoS vs. NOF* 10^2

|     | MOWFA | NRGA | NSGA-II | MOWFA | NRGA | NSGA-II |
|-----|-------|------|---------|-------|------|---------|
| P1  | 12    | 7    | 14      | 23075 | 20833| 22916   |
| P2  | 13    | 10   | 19      | 24491 | 19287| 23747   |
| P3  | 12    | 9    | 16      | 18743 | 20912| 26011   |
| P4  | 9     | 15   | 11      | 18335 | 23206| 24098   |
| P5  | 9     | 10   | 14      | 24308 | 24101| 19158   |
| P6  | 10    | 13   | 15      | 25613 | 18224| 21047   |
| P7  | 14    | 12   | 14      | 20355 | 21585| 25784   |
| P8  | 11    | 11   | 10      | 21191 | 19215| 21807   |
| P9  | 15    | 9    | 19      | 35907 | 62173| 61021   |
| P10 | 18    | 16   | 11      | 32382 | 59457| 68730   |
| P11 | 14    | 8    | 14      | 30102 | 37164| 66658   |
| P12 | 10    | 13   | 17      | 39461 | 65003| 59471   |
| P13 | 16    | 8    | 16      | 47504 | 41041| 50179   |
| P14 | 17    | 13   | 23      | 39940 | 36675| 46817   |
| P15 | 11    | 13   | 12      | 33215 | 60239| 46830   |
| P16 | 14    | 11   | 16      | 36061 | 33964| 68700   |
| P17 | 18    | 16   | 16      | 90888 | 102456|130278 |
| P18 | 11    | 14   | 19      | 107114|128744|155434  |
| P19 | 12    | 9    | 17      | 104367|114636|131606  |
| P20 | 14    | 9    | 19      | 89960 | 100856|148224  |
| P21 | 22    | 9    | 16      | 92478 | 116691|172430  |
| P22 | 14    | 17   | 22      | 91947 | 127904|158870  |
| P23 | 19    | 17   | 20      | 91275 | 105495|156189  |
| P24 | 13    | 17   | 19      | 103861|110974|145701  |
| P25 | 22    | 20   | 28      | 448618|541867|757899  |
| P26 | 20    | 18   | 24      | 384950|545010|739484  |
| P27 | 21    | 22   | 32      | 439349|483017|719259  |
| P28 | 23    | 18   | 29      | 402728|489441|655397  |
| P29 | 22    | 17   | 27      | 395391|478866|716287  |
| P30 | 20    | 19   | 27      | 383603|416241|735609  |
| P31 | 22    | 17   | 33      | 400575|411374|843477  |
| P32 | 25    | 21   | 31      | 414777|482183|862108  |

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