OPTIMAL PRODUCTION RUN TIME AND INSPECTION ERRORS IN AN IMPERFECT PRODUCTION SYSTEM WITH WARRANTY

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ABSTRACT. This paper considers an imperfect production system to obtain the optimal production run time and inspection policy. Contrary to the existing literature this model considers that product inspection performs at any arbitrary time of the production cycle and after the inspection, all defective products produced until the end of the production run are fully reworked. Due to some misclassification during inspection, from the inspector’s side two types of inspection errors as Type I and Type II are considered to make the model more realistic rather than existing models. Defective items, found by the inspector, are salvaged at some cost before being shipped. Non-inspected defective items are passed to customers with free minimal repair warranty. The model gives three special cases, where it is found that this model converges over the exiting literature. Some numerical examples along with graphical representations are provided to illustrate the proposed model with comparison with the existing models. Sensitivity analysis of the optimal solution with respect to key parameters of the model has been carried out and the implications are discussed.

1. Introduction. In existing literature, it is considered that production system remains in-control state and products produced by the machinery system are of perfect quality. This assumption may not be true in every production system. In many practical situations, during long-run production system, the machinery system goes through a long-run process, it may shift from in-control state to out-of-control state, where the manufacturing system produces defective/imperfect quality items. These items are reworked at some costs to restore the original quality and the brand image of the company.

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It is reality that the production process anytime can move to out-of-control state and generally the movement from in-control state to out-of-control state is random, which is considered in the literature by a very few researcher (Sarkar et al. [28]). The machine during out-of-control state produces some defective items. In many models, the defective items are reworked (refer to Sarkar and Moon [25]), whereas it is not possible always to rework all items, thus it is not always possible to inspect all items also due to first delivery of products or other reasons. The uninspected items can be sold directly to the market with some warranty price to maintain brand image of the manufacturer. It may happen that the inspected items consists some defective items i.e., the inspection process is not error-free. Misclassification error is an important issue, which cannot be ignored nowadays due to overtime duty, fatigueness, tardiness of human labor (skilled, semi-skilled). This proposed research study fulfills the above mentioned research gaps and considered all together at a time, which is not considered by any research yet.

Many researchers have investigated production models with unreliable machines. Rosenblatt and Lee [16] initially studied the effects of process deterioration on the traditional economic manufacturing quantity (EMQ) model. In their model, they considered that the elapsed time to the out-of-control state is exponentially distributed and concluded that the presence of defective products generates smaller lot sizes than that of the classical economic production quantity (EPQ) inventory model. Porteus [14] discussed an imperfect production process with significant relationship between quality and lot size. as well as obtained an optimal investment for process-quality improvement and setup cost reduction. Harriga and Ben-Daya [9] extended Rosenblatt and Lee's [16] model by assuming a more generalized assumption that an elapsed time, until process shift, is arbitrarily distributed, and provided distribution-based and distribution-free bounds on the optimal cost.

Goyal and Cárdenas-Barrón [8] developed an EPQ model to determine both the optimal lot size and manufacturing process cost in an imperfect production system. Sana, Goyal, and Chaudhuri [18, 19] extended an EMQ model in an imperfect production system in which the defective item are sold at reduce price. Cárdenas-Barrón [1] proposed an inventory model on optimal batch sizing in a multi-stage production system with rework process. Sana [20] developed an imperfect production model to determine the optimal product reliability and production rate to obtain maximum profit. Sana [21] considered an imperfect production system with allowable shortages due to regular preventive maintenance for products sold with free minimal repair warranty. Chung, Widyadana, and Wee [2] considered a deteriorating inventory model with stochastic machine unavailability time and shortage. Hsu and Hsu [10] developed an integrated vendor-buyer inventory model to determine an optimum policy of production, where the vendor's production process is imperfect and produces a certain number of defective items with a known probability density function.

Defective items can be identified by an inspection process, which carries an inspection cost. Chryssolouris and Patel [3] discussed a production process with imperfect items and perfect full inspection process. Salameh and Jaber [17] developed an inventory model with 100% inspection and poor-quality items are sold as a single-batch by the end of the 100% screening process. Wang and Sheu [33] considered a production inventory and product-inspection policy for deteriorating production systems. They did not inspect the first $s$ produced items, but inspect only those items from the $(s + 1)$th till the end of the production run.
Full inspection policy results higher inspection cost and higher expected total cost. To reduce the inspection cost, Wang [35] developed an inspection policy, where inspection was performed at the end of the production run. Using the same concept of Wang [35], Wang and Meng [34] developed another model with offline inspection policy of products. Hu and Zong [11] proposed an extended product inspection policy for a deteriorating production system, where product inspections are performed at any time of a production cycle. There are many situations, where the organization may have more than one objective functions to optimize. Lee [12] determined the optimal production run length and scheduled maintenance inspection policy for a deteriorating system in which the products are sold with free minimal repair warranty. To solve this type of problem, Duffuaa and El-Ghaly [5] developed a multi-objective inventory model using 100% error-free inspection as a means of product control. Sarkar [23] considered a supply chain coordination model with three-stage inspection to ensure perfect quality products, where the products has special features as fixed lifetime.

In the above mentioned papers, authors considers error-free inspection process to detect the imperfect items. But it is quite unrealistic that during machinery or human inspection process error may not occur. Raouf, Jain, and Sathe [15] suggested that during inspection two types of human error may occur, one is Type I error (classifying a non-defective item as defective) and another is Type II error (classifying a defective item as non-defective). Duffuaa and Khan [6] extended Raouf, Jain, and Sathe’s [15] inspection process for the cases of six types of misclassification errors. Wang [32] developed an inventory model with two types of inspection errors in order to facilitate the adaptation of economic inspection/disposition model to real world applications. Darwish and Ben-Daya [4] proposed a production-inventory model with the effect of imperfect production processes, preventive maintenance, and inspection errors.

Wang, Dohi, and Tsai [36] considered an inventory model to purchase lot size under a partial inspection policy over commonly used policy for both full and no inspection. Duffuaa and El-Ghaly [7] extended their [5] model by developing more realistic multi-objective optimization model that integrates measurement errors in inspection system. Most recently, Sarkar and Saren [29] considered a product-inspection policy for an imperfect production system with inspection errors and warranty cost. To reduce the inspection cost, they considered the inspection policy at the end of the production cycle and the non-inspected defective items are shifted to the market with warranty.

In this proposed model, an effort has been made to obtain optimal production run time and inspection policy for an imperfect production system under an extended inspection policy. Production process is subject to a random deterioration from an in-control state to an out-of-control state with Weibull distribution. Type I and Type II inspection errors are considered. Defective items are identified through product inspection policy and salvaged with a fixed cost before being shipped. A free minimal warranty for customer is provided for the non-inspected item. Rest of the paper is designed as follows: In Section 2.1., the problem is defined and Section 2.2. gives the notation and Section 2.3 considers the assumptions of the model. Section 3 describes the mathematical model. In Section 3.1., some special cases are discussed. Numerical experiments are given in Section 4 and sensitivity analysis is given in Section 4.1. Finally, conclusions are made in Section 5.
2. Problem definition, notation, and assumptions. This section contains problem definition, notation and assumptions.

2.1. Problem definition. An imperfect production system for a single-type of item is considered. Production starts from in-control state and after a period of operation, the production system may shift to out-of-control state until the end of the production-run. At each state, $\theta_1$ and $\theta_2$ represent the percentage of the number of defective items during in-control state and out-of-control state, respectively with $\theta_1 < \theta_2$. The elapsed time until the production system shifts to the out-of-control state is denoted by $X$, which follows an exponential distribution with $f(x)$ as probability density function, $F(x)$ as distribution function, and $F(=1-F(x))$ as survival function. The failure rate function of the random variable $X$ is defined as $\phi(x) = f(x)/F(x)$. After completion of a lot, the system is inspected with fixed cost $\eta$ to obtain the information about the state of the system. If the system is in out-of-control state, after completion of the production cycle, the production system brought back to the in-control state with an additional restoration cost $r$. To detect the defective items produced in a produced lot, a product inspection policy is carried out at a fixed cost $C_I$. The inspection time is considered as negligible. Product inspection policy starts from $(pu_1t)$th item to $(pu_2t)$th item, and the defective items from those inspected will be salvaged at some fixed cost $C_s$ before being shipped. After completion of inspection, all produced products during production time $u_2t$ to the end of production are reworked without inspections. During inspection, due to misclassification an inspector classified some non-defective items as defective with a fixed rate $m_1$ and classified some defective items as non-defective with a fixed rate $m_2$. The non-inspected defective items are taken as salvageable and those items are sent to the market with post sale (warranty) cost $C_w$ with the assumption $C_I + C_s < C_w$.

2.2. Notation. In this model, following notation are used.

| Decision variables | Description |
|--------------------|-------------|
| $t$                | production-run length (unit time) |
| $u_1$              | first non-inspected fraction in a batch |
|                    | $(0 \leq u_1 \leq 1)$ (units) |
| $u_2$              | second non-inspected fraction in a batch |
|                    | $(0 \leq u_2 \leq 1)$ (units) |

| Parameters | Description |
|-----------|-------------|
| $d$       | annual demand per unit time (units/unit time) |
| $p$       | production rate per unit time (units/unit time) |
| $k$       | setup cost for each production-run per setup ($/setup$) |
| $h$       | unit inventory holding cost of a product per unit time ($/unit/unit time$) |
| $C_m$     | labor cost for production per item ($/unit$) |
| $\theta_1$| percentage of defective items produced in in-control state |
| $\theta_2$| percentage of defective items produced in out-of-control state, $\theta_2 > \theta_1$ |
| $X$       | random variable during the elapsed time of system in in-control state |
• Parameters

| Description                                      |
|------------------------------------------------|
| $F(x)$                                          |
| $\bar{F}(x)$ survival function of $X$ i.e., $\bar{F}(x) = 1 - F(x)$ |
| $f(x)$ probability density function of $X$      |
| $E(X)$ mean lifetime of $X$                     |
| $\eta$ inspection cost to check the process for determining the state of the system ($) |
| $r$ restoration cost to transfer the process to in-control |
| $C_I$ unit inspection cost ($/unit)             |
| $C_s$ salvaged cost per defective lot after inspection ($/defective lot) |
| $C_w$ post sale (warranty) cost for non-inspected defective lots ($/non-inspected defective lot) |
| $C_a$ cost of falsely accepting defective lot   ($/defective lot) |
| $C_r$ cost of falsely rejecting non-defective lot ($/non-defective lot) |
| $m_1$ random variable representing Type I error |
| $m_2$ random variable representing Type II error |
| $C_{d_i}$ defective cost includes the costs of defective lot ($/defective lot); $i = 1, 2$ |
| $C(t, u_1, u_2)$ expected total cost per item ($/unit)$ |

2.3. Assumptions. The following assumption are made to develop the model.

1. The production starts from in-control state for a single-type of products. After sometime, the production system shifts to out-of-control state until the end of the production-run and starts to produce defective items.

2. It is assumed that the production is always greater than demand, i.e., $(p > d)$, there is no shortage in this model.

3. This model assumes $\theta_1 < \theta_2$ i.e., the probability of number of defective items in in-control state is less than the probability or number of defective items in out-of-control state. It indicates that the production of defective items during in-control state is less than the production of defective items during out-of-control state.

4. The product inspection policy is performed to detect the defective items. These items are salvaged with some cost $C_s$.

5. Two types of errors i.e., Type I and Type II errors are introduced during product inspection policy and two separate production batch interval is considered as non-inspection production batch.

3. Model formulation. The inventory level starts with $p - d$ rate and depletes with a rate $-d$, where production rate $(p) >$ demand rate $(d) > 0$. The total produced items are $pt$ during cycle time $t$ and the time duration of a production cycle is $pt/d$. The production cost per product is $C_m$ and the inventory holding cost per unit per unit time is $h$. Thus, the maximum inventory is $(p - d)t$ and the holding cost is $\frac{1}{2}h(pt/d)(p - d)t$. Hence, the holding cost per item is $\frac{h(p-d)t}{2d}$. Setup cost for each production-run is $k$. Theretofore, setup cost per item is $\frac{k}{pt}$ and hence,
the system inspection cost per item is $\frac{C}{m}$. If the system moves to out-of-control, then $r$ is the fixed cost to transfer the system back to the in-control state. Therefore, the restoration cost per unit item is $rF(t)$. The number of defective items in the time interval $[0, u_1t]$ (say $N_1(t)$) are

$$N_1(t) = \begin{cases} \theta_1pX + \theta_2p(u_1t - X), & \text{if } X < u_1t \\ \theta_1pu_1t, & \text{if } X \geq u_1t \end{cases}$$

Thus, the expected value of $N_1(t)$ is

$$E[N_1(t)] = \int_0^{u_1t} \left[ \theta_1pX + \theta_2p(u_1t - X) \right] f(x)dx + \int_{u_1t}^{\infty} \theta_1pu_1tf(x)dx$$

$$= \theta_2pu_1t - (\theta_2 - \theta_1)p \int_0^{u_1t} F(x)dx$$

(See Appendix A for the derivation of $E[N_1(t)]$)

Similarly, the expected number of defective items in the time interval $[0, u_2t]$ is

$$E[N_2(t)] = \theta_2pu_2t - (\theta_2 - \theta_1)p \int_0^{u_2t} F(x)dx$$

Hence, the expected number of defective items in the time interval $[u_1t, u_2t]$ is

$$E[N_2(t)] - E[N_1(t)] = \theta_2p(u_2 - u_1)t - (\theta_2 - \theta_1)p \int_{u_1t}^{u_2t} F(x)dx$$

During product screening process, the inspectors make Type I and Type II errors. They classify some non-defective item as defective item i.e., $\theta = 1$ during product screening process. The inspectors make Type I and Type II errors. Hence, the expected number of defective items in the time interval $[0, u_1t]$ is

$$E[N_1(t)] = \theta_2p(u_2 - u_1)t - (\theta_2 - \theta_1)p \int_{u_1t}^{u_2t} F(x)dx$$

In this case, the inspector accepts some defective items, thus, two cases arise.

**Case I.** $X < u_1t$

During $[0, X]$, inspector accepts $\theta_1pX$ defective items in which falsely accepted defective items are $\theta_1pXm_2$ with some fixed cost $C_A$ per unit and falsely rejected non-defective items are $(1 - \theta_1)pXm_1$ with some fixed cost $C_R$ per unit. Therefore, the defective cost in $[0, X]$ is $C_Rp(1 - \theta_1)Xm_1 + C_A\theta_1pX(1 - m_2)$.

On the other hand, during $[X, u_1t]$ inspector accepts $\theta_2p(u_1t - X)$ defective items, in which falsely accepted defective items are $\theta_2p(u_1t - X)m_2$ and falsely rejected non-defective items are $(1 - \theta_2)p(u_1t - X)m_1$.

Therefore, the defective cost in $[X, u_1t]$ is $C_Rp(1 - \theta_2)(u_1t - X)m_1 + C_A\theta_2p(u_1t - X)(1 - m_2)$.

Hence, the defective cost in $[0, u_1t]$ under the condition $X < u_1t$ is $C_Rp[(1 - \theta_1)X + (1 - \theta_2)(u_1t - X)]m_1 + C_Ap[\theta_1X + \theta_2(u_1t - X)](1 - m_2)$.

**Case II.** $X \geq u_1t$

In this case, the inspector accepts $\theta_1pu_1t$ defective items in which falsely accepted defective items are $\theta_1pu_1tm_2$ and falsely rejected non-defective items are $(1 - \theta_1)pu_1t$.

Therefore, the defective cost in $[0, u_1t]$ under the condition $X \geq u_1t$ is $C_R(1 - \theta_1)pu_1tm_1 + C_A\theta_1pu_1t(1 - m_2)$. 
Therefore, total defective cost is
\[
C_d = \begin{cases} 
C_R p[(1 - \theta_1)X + (1 - \theta_2)(u_1 t - X)]m_1 \\
+ C_A p[\theta_1 X + \theta_2 (u_1 t - X)](1 - m_2), \text{ if } X < u_1 t \\
C_R (1 - \theta_1)pu_1 t m_1 + C_A \theta_1 pu_1 t (1 - m_2), \text{ if } X \geq u_1 t 
\end{cases}
\]
The expected value of defective cost within the time interval \([0, u_1 t]\) is
\[
E[C_{d1}] = p(\theta_1 - \theta_2)\{C_A (1 - E[m_2]) - C_R E[m_1]\} \int_0^{u_1 t} F(x)dx \\
+ pu_1 t \{C_A \theta_2 (1 - E[m_2]) + C_R (1 - \theta_2) E[m_1]\}
\]
(See Appendix A for the derivation of \(E[C_{d1}]\))

Similarly, the expected defective cost in the time interval \([0, u_2 t]\) is
\[
E[C_{d2}] = pu_2 t \{C_A \theta_2 (1 - E[m_2]) + C_R (1 - \theta_2) E[m_1]\} \\
+ p(\theta_1 - \theta_2) \{C_A (1 - E[m_2]) - C_R E[m_1]\} \int_0^{u_2 t} F(x)dx
\]
Hence, the expected defective cost within the interval \([u_1 t, u_2 t]\) is
\[
E[C_{d2}] - E[C_{d1}] = p(u_2 - u_1) t \{C_A \theta_2 (1 - E[m_2]) + C_R (1 - \theta_2) E[m_1]\} \\
+ p(\theta_1 - \theta_2) \{C_A (1 - E[m_2]) - C_R E[m_1]\} \int_{u_1 t}^{u_2 t} F(x)dx
\]
Therefore, the expected total warranty, salvage, defective, and rework cost with lot size \(pt\) under the inspection policy \((u_1, u_2)\) is
\[
C_w E[N_1(t)] + C_s \{E[N_2(t)] - E[N_1(t)]\} + E[C_{d2}] - E[C_{d1}] + C_r pt(1 - u_2)
\]
Now, the expected total cost per item i.e., \(C(t, u_1, u_2)\) is the addition of manufacturing cost, holding cost, setup cost, process inspection cost, restoration cost, product inspection cost, warranty cost, salvage cost, defective cost, and rework cost as follows:
\[
C(t, u_1, u_2) = C_m + \frac{h(p - d)t}{2d} + \frac{K + r F(t)}{pt} + (C_t + C_s \theta_2)(u_2 - u_1) + C_w \theta_2 u_1 \\
+ C_r (1 - u_2) + (u_2 - u_1) \{C_A (1 - E[m_2]) \theta_2 + C_R E[m_1] (1 - \theta_2)\} \\
+ \frac{(\theta_1 - \theta_2)}{t} \{C_s + C_A (1 - E[m_2]) - C_R E[m_1]\} \int_{u_1 t}^{u_2 t} F(x)dx \\
+ \frac{C_w (\theta_1 - \theta_2)}{t} \int_0^{u_1 t} F(x)dx
\]
where \(K = k + \eta\).

The objective is to obtain the optimum value of \(t, u_1\), and \(u_2\) such that \(C(t, u_1, u_2)\) is minimum. The optimization of this model is done by numerical example. This model is the extension of several models which are discussed in the next section.
3.1. Some special cases. Case I. If \( u_2 = 1 \) and \( C_r = C_s \), then this model converges to Sarkar and Saren’s [29] model. In this case, the cost function is

\[
C(t, u_1) = C_m + \frac{h(p-d)t}{2d} + \frac{K + rF(t)}{pt} + C_I(1-u) + C_w\theta_2u \\
+ (1-u)\left[C_s\theta_2 + C_A(1-E[m_2])\theta_2 + C_R E[m_1](1-\theta_2)\right] \\
+ \frac{(\theta_1-\theta_2)}{t} \left\{C_s + C_A(1-E[m_2]) - C_R E[m_1]\right\} \int_{u_1}^t F(x)dx \\
+ \frac{C_w(\theta_1-\theta_2)}{t} \int_0^{u_1} F(x)dx
\]

which is the same cost function of Sarkar and Saren’s [29] model.

Case II. If \( C_A = C_R = 0 \), and \( C_s = C_r \), then the model becomes Hu and Zong’s [11] model. In this case, the cost function is

\[
C(t, u_1, u_2) = C_m + \frac{h(p-d)t}{2d} + \frac{K + rF(t)}{pt} + (C_I + C_r\theta_2)(u_2-u_1) + C_w\theta_2u_1 \\
+ C_r(u_2-u_1) + \frac{(\theta_1-\theta_2)}{t} \left\{C_w - C_r\right\} \int_0^{u_1} F(x)dx + C_r \int_0^{u_2} F(x)dx
\]

which is the same cost function with Hu and Zong’s [11] model.

Case III. If \( u_2 = 1 \), \( C_A = C_R = 0 \), and \( C_s = C_r \), then the model becomes Wang’s [35] model. In this case, the cost function is

\[
C(t, u_1, u_2) = C_m + \frac{h(p-d)t}{2d} + \frac{K + rF(t)}{pt} + (C_I + C_r\theta_2)(1-u) + C_w\theta_2u \\
+ C_r(1-u) + \frac{(\theta_1-\theta_2)}{t} \left\{C_w - C_r\right\} \int_0^{u_1} F(x)dx + C_r \int_0^t F(x)dx
\]

which is the same cost function with Wang’s [35] model.

In the next part, the numerical comparison of these three models with our model is discussed.

4. Numerical experiments. This section presents some numerical examples to demonstrate the present model and the results are compared with the existing models. Here, it is considered that the production system follows a Weibull distribution \( F(x) = 1 - e^{-(x/\alpha)^\beta} \), where \( \alpha > 0 \) is scale parameter and \( \beta \geq 1 \) is shape parameter and inspection errors (Type I and Type II) follow the uniform distribution with

\[
f(m_1) = \begin{cases} 
25, & 0 \leq m_1 \leq 0.04 \text{ i.e., } E[m_1] = 0.02 \\
0, & \text{otherwise}
\end{cases}
\]

and

\[
f(m_2) = \begin{cases} 
25, & 0 \leq m_2 \leq 0.04 \text{ i.e., } E[m_2] = 0.02 \\
0, & \text{otherwise}
\end{cases}
\]

The parametric values are taken as \( K = k + \eta = 20000 \) setup, \( p = 1200 \) units/day, \( d = 350 \) units/day, \( h = 0.5 \) unit/day, \( C_m = 0.3 \), \( C_w = 8 \) non-inspected defective lot, \( C_s = 2.6 \) defective lot, \( C_r = 3 \) defective lot, \( C_I = 1.2 \) per defective lot,
$r = 1500, C_A = 0.3/ \text{defective lot}, C_R = 0.1/ \text{defective lot}, F(t) = 1 - e^{-(t/0.5)^2}, F(x) = 1 - e^{-(x/0.5)^2}$ (See Figure 1 and Figure 2).

**Figure 1.** Plot of expected total cost $C(u_1, u_2 | t^* = 2.19)$

**Figure 2.** Plot of expected total cost $C(t | u_1^* = 0.00132, u_2^* = 0.01023)$

| Table 1. Summary of numerical results |
|---------------------------------------|
| This model | Sarkar and Saren [29] | Hu and Zong [11] | Wang [35] |
| $(t^*, u_1^*, u_2^*)$ | $(t^*, u^*)$ | $(t^*, u_1^*, u_2^*)$ | $(t^*, u^*)$ |
| $C(t^*, u_1^*, u_2^*)$ | $C(t^*, u^*)$ | $C(t^*, u_1^*, u_2^*)$ | $C(t^*, u^*)$ |
| (2.04, 0.058, 0.251) | (1.85, 0.064) | (2.05, 0.062, 0.239) | (1.83, 0.069) |
| $8.48$ | $8.90$ | $8.49$ | $8.97$ |

Sarkar and Saren [29] considered inspection at the end of the production process, whereas this model considers inspection at the arbitrary time of the production process. From Table 1, one can find out that expected total cost per item for this
model is $C(t^*, u_1^*, u_2^*) = 8.48$. If Sarkar and Saren’s [29] model is applied, then total cost is $C(t^*, u^*) = 8.90$ with a percentage of cost penalty (PCP) which is computed as $100 \times [C(t^*, u_1^*, u_2^*) - C(t^*, u^*)]/C(t^*, u^*) = 4.71\%$.

Hu and Zong [11] considered a extended inspection policy, where inspection process was performed at the middle of the production process, but they did not consider the inspection error. This model considers same extended inspection policy with more realistic assumption as two types of inspection errors. Table 1 shows that expected total cost per item for this model is $C(t^*, u_1^*, u_2^*) = 8.48$. If Hu and Zong’s [11] model is applied, then total cost is $C(t^*, u_1^*, u_2^*) = 8.49$.

Wang [35] considered an inspection policy, where inspection process was performed at the end of the production process and the inspected items was salvaged. Non-inspected items were sold at market with free minimal warranty. But they did not consider inspection error. From Table 1, one can find that expected total cost per item for this model is $C(t^*, u_1^*, u_2^*) = 8.48$. If Wang’s [35] model is applied, then total cost is $C(t^*, u^*) = 8.97$ with a percentage of cost penalty (PCP) which is computed as $100 \times [C(t^*, u_1^*, u_2^*) - C(t^*, u^*)]/C(t^*, u^*) = 5.46\%$.

4.1. Sensitivity analysis. In this section, the effects of changes in major cost parameters such as $C_h, C_s, C_w, C_r, C_i$ and $r$ on optimal solution are studied. The results are presented in Table 2. The effect of major cost parameters on the optimal solution are shown in Figure (3-8).

From Table 2, the following observations are made:

* Table 2 indicates that increasing value of $C_h$ increases the total cost per item $C(t^*, u_1^*, u_2^*)$. Holding cost can be reduced by reducing the production run length $t^*$. Reducing value of production run length will reduce the total produced items as well as total holding cost and the total cost per item. Therefore, the production run length decreases as holding cost increases.

* From Table 2, one can observe if $C_w$ increases, then the total cost per item $C(t^*, u_1^*, u_2^*)$ increases. Increasing value of $C_w$ implies more warranty cost, which results in greater total cost per item. Therefore, manufacturer have to decrease the failure rate of the items in the warranty period to avoid the greater warranty cost.

* From Table 2, one can find out if salvage cost $C_s$ increases, then total cost per item $C(t^*, u_1^*, u_2^*)$ increases. Increasing value of $C_s$ implies more defective items, which implies more items to inspect, i.e., more inspection cost as well as greater total cost. To reduce the total salvage cost as well as the total cost per item, manufacturer has to reduce the total inspected batch size, i.e., $u_2^*$. Therefore, increasing value of $C_s$ decreases $u_2^*$.

* The effect of rework cost $r$ on total cost $C(t^*, u_1^*, u_2^*)$ is clear from Table 2. Increasing value of $r$ indicates more rework cost as well as greater total cost per item. Therefore, rework cost can be decrease by reducing imperfect items. By decreasing the production run time, one can reduce the total number of imperfect items and higher rework cost. Therefore, the production run length $t^*$ decreases as $r$ increases.

* When the inspection cost $C_l$ increases, the total cost per item $C(t^*, u_1^*, u_2^*)$ increases. To reduce the total inspection cost, the manufacturer has to reduce the inspected fraction batch $u_2^*$. Therefore, $u_2^*$ decreases as $C_l$ increases.

* When the restoration cost $r$ increases, the expected total cost per item $C(t^*, u_1^*, u_2^*)$ also increases. Therefore, to avoid the frequent restorations, the
production run length should be longer.

**Figure 3.** Impact of holding cost ($C_h$) on expected total cost $C(t^*, u_1^*, u_2^*)$

**Figure 4.** Impact of salvage cost ($C_s$) on expected total cost $C(t^*, u_1^*, u_2^*)$

**Figure 5.** Impact of warranty cost ($C_w$) on expected total cost $C(t^*, u_1^*, u_2^*)$
Figure 6. Impact of rework cost ($C_r$) on expected total cost $C(t^*, u_1^*, u_2^*)$

Figure 7. Impact of inspection cost ($C_I$) on expected total cost $C(t^*, u_1^*, u_2^*)$

Figure 8. Impact of restoration cost ($r$) on expected total cost $C(t^*, u_1^*, u_2^*)$
Table 2. Impact of key parameters on optimal solution

| $C_h$ | $t^*$ | $u_1^*$ | $u_2^*$ | $C(\cdot)$ |
|-------|-------|---------|---------|-------------|
| 0.1   | 4.56  | 0.026   | 0.112   | 7.12        |
| 0.3   | 2.64  | 0.045   | 0.194   | 7.92        |
| 0.5   | 2.04  | 0.058   | 0.251   | 8.48        |
| 0.7   | 1.72  | 0.068   | 0.296   | 8.93        |
| 0.9   | 1.52  | 0.078   | 0.336   | 9.32        |
| 1.1   | 1.37  | 0.086   | 0.372   | 9.68        |

| $C_s$ | $t^*$ | $u_1^*$ | $u_2^*$ | $C(\cdot)$ |
|-------|-------|---------|---------|-------------|
| 2.0   | 1.98  | 0.032   | 0.382   | 8.41        |
| 2.2   | 2.01  | 0.042   | 0.317   | 8.44        |
| 2.4   | 2.03  | 0.050   | 0.278   | 8.46        |
| 2.6   | 2.04  | 0.058   | 0.251   | 8.48        |
| 2.8   | 2.05  | 0.065   | 0.229   | 8.49        |
| 3.0   | 2.06  | 0.072   | 0.211   | 8.51        |
| 3.2   | 2.07  | 0.079   | 0.196   | 8.52        |

| $C_w$ | $t^*$ | $u_1^*$ | $u_2^*$ | $C(\cdot)$ |
|-------|-------|---------|---------|-------------|
| 6.4   | 2.014 | 0.123   | 0.254   | 8.446       |
| 6.8   | 2.024 | 0.105   | 0.253   | 8.457       |
| 7.2   | 2.031 | 0.089   | 0.252   | 8.466       |
| 7.6   | 2.037 | 0.073   | 0.251   | 8.473       |
| 8.0   | 2.042 | 0.058   | 0.251   | 8.479       |
| 8.4   | 2.045 | 0.041   | 0.250   | 8.483       |
| 8.8   | 2.047 | 0.018   | 0.249   | 8.486       |

| $C_r$ | $t^*$ | $u_1^*$ | $u_2^*$ | $C(\cdot)$ |
|-------|-------|---------|---------|-------------|
| 120   | 1.94  | 0.061   | 0.264   | 8.35        |
| 130   | 1.97  | 0.060   | 0.259   | 8.40        |
| 140   | 2.01  | 0.059   | 0.255   | 8.44        |
| 1500  | 2.04  | 0.058   | 0.251   | 8.48        |
| 1600  | 2.07  | 0.057   | 0.246   | 8.52        |
| 1700  | 2.11  | 0.056   | 0.243   | 8.56        |
| 1800  | 2.14  | 0.055   | 0.239   | 8.60        |

5. Conclusions. This paper developed an imperfect production process under the presence of Type I and Type II errors. The out-of-control probability of the system as well as Type I and Type II inspection errors were considered as random variable with known probability density function. Variable inspection interval was considered. Inspected items were salvaged with some fixed cost before selling. Non-inspected items were sold at market with post-sale warranty. This model minimized the total cost per item by obtaining the production run time and inspection policy. The numerical comparison between other models has been conducted to prove the impact of inspection errors on the optimum solutions. Finally, the sensitivity analysis on the optimal solution with respect to key parameters were studied to illustrate the model and some managerial insights were provided. This model can be extended for items having linear increasing demand, price, and advertising-dependent demand or power-demand. This study can be extended further by considering stochastic demand (see for instance Pal, Sana, and Chaudhuri [13]). One another extension can be considered by allowing process maintenance during a production run (see for instance Sana [21]). The model can be extended by considering safety stock and random machine breakdown Sarkar et al. [30]. If defective items are also considered for it, then it would be more perfect with random breakdown with two maintenance repairs as corrective and preventive Sarkar et al. [31]. Throughout this proposed model, the demand was considered as constant, hence it would be an interesting research to assume stochastic demand within imperfect environment with inflation and time-value of money Sarkar et al. [31], Sarkar and Moon [25]. By reducing system failure rate or increasing system reliability Sarkar et al. [24], Sarkar [22], the model can be extended further where the quality improvement of products
can be easily considered as a major extension Sarkar and Moon [26], Sarkar et al. [24], Sarkar et al. [27]. Considering skill level of inspectors would be another major extension of this field.

**Appendix A.** The derivation of $E[N_1(t)]$

\[
E[N_1(t)] = \int_0^{u_1t} \left[ \theta_1 p X + \theta_2 p(u_1 t - X) \right] f(x) dx + \int_{u_1t}^{\infty} \theta_1 p u_1 t f(x) dx
\]

\[
= \theta_1 p \left[ -u_1 t F(u_1(t)) + \int_0^{u_1t} F(x) dx \right] + \theta_2 p u_1 t \left[ 1 - F(u_1(t)) \right]
\]

\[
- \theta_2 p \left[ -u_1 t F(u_1(t)) + \int_0^{u_1t} F(x) dx \right] + \theta_1 p u_1 t F(u_1(t))
\]

\[
= \theta_2 p u_1 t - (\theta_2 - \theta_1) p \int_0^{u_1t} F(x) dx
\]

The derivation of $E[C_{di}]$

\[
E[C_{di}] = \left\{ C_R[(1 - \theta_1) X + (1 - \theta_2)(u_1 t - X)] p E[m_1] + C_A[(1 - \theta_2)(u_1 t - X)] p(1 - E[m_2]) \right\} \int_0^{u_1t} f(x) dx
\]

\[
+ \left\{ C_R(1 - \theta_1) pu_1 t E[m_1] + C_A \theta_1 pu_1 t (1 - E[m_2]) \right\} \int_{u_1t}^{\infty} f(x) dx
\]

\[
= p(\theta_1 - \theta_2) \left\{ C_A(1 - E[m_2]) - C_R E[m_1] \right\} \int_0^{u_1t} x f(x) dx
\]

\[
+ \left\{ C_R p(1 - \theta_2) u_1 t E[m_1] + C_A p \theta_2 u_1 t (1 - E[m_2]) \right\} \int_0^{u_1t} f(x) dx
\]

\[
+ \left\{ C_R(1 - \theta_1) pu_1 t E[m_1] + C_A \theta_1 pu_1 t (1 - E[m_2]) \right\} \int_{u_1t}^{\infty} f(x) dx
\]

\[
= p(\theta_1 - \theta_2) \left\{ C_A(1 - E[m_2]) - C_R E[m_1] \right\} \left[ -u_1 t F(u_1(t)) + \int_0^{u_1t} F(x) dx \right]
\]

\[
+ \left\{ C_R p(1 - \theta_2) u_1 t E[m_1] + C_A p \theta_2 u_1 t (1 - E[m_2]) \right\} \left[ 1 - F(u_1(t)) \right]
\]

\[
+ \left\{ C_R(1 - \theta_1) pu_1 t E[m_1] + C_A \theta_1 pu_1 t (1 - E[m_2]) \right\} F(u_1(t))
\]

\[
= p(\theta_1 - \theta_2) \left\{ C_A(1 - E[m_2]) - C_R E[m_1] \right\} \int_0^{u_1t} F(x) dx
\]

\[
+ pu_1 t \left\{ C_A \theta_2 (1 - E[m_2]) + C_R (1 - \theta_2) E[m_1] \right\}
\]

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