A toy model for decoherence in the black hole information problem

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We investigate a plausible route to resolving the black hole information paradox by examining the effects of decoherence on Hawking radiation. In particular, we show that a finite but non-zero rate of decoherence can lead to efficient extraction of information from the evaporating black hole. This effectively pushes the paradox from becoming manifest at the Page time when the black hole has evaporated to half its size, to a timescale solely determined by the rate of decoherence. If this rate is due to a putative interaction with low-energy gravitons, we show that the black hole at this timescale can be expected to be Planck-sized, but notably does not contain an extensive amount of information packed inside. We justify our findings by numerically studying a toy model of stabilizer circuits that can efficiently model black hole evaporation in the presence of decoherence. The latter is found to be well described by effective mean-field like equations of motion for the entanglement, which are further amenable to an explicit solution, and corroborate our findings.

\textbf{Introduction.} — From its inception, Bekenstein\textsuperscript{1} and Hawking’s\textsuperscript{2} results that black holes emit radiation and evaporate away have posed conundrums that challenge our core physical principles and whose resolution remains a fundamental goal in modern physics. In its earliest avatar, it was realized that black hole evaporation is at odds with unitary quantum-mechanical evolution since it predicts a pure state of the black hole universe evolving into a mixed state of incoherent Hawking radiation. In its modern incarnation, it has been reformulated as an information problem by AMPS\textsuperscript{3,4}—while the effective semi-classical description valid at the black hole horizon suggests that Hawking quanta should be nearly maximally entangled with the black hole interior, an information-theoretic calculation by Page\textsuperscript{5} shows that Hawking quanta emitted by an old black hole (evaporated to half its original size), must be maximally entangled with early-time radiation. This contradicts the notion of monogamy of entanglement (or, equivalently, strong subadditivity), a fundamental result which states that a quantum subsystem cannot be maximally entangled with two different subsystems at the same time.

Following AMPS\textsuperscript{3,4}, different schemes have been proposed to resolve this paradox by relaxing one of three core principles—of unitarity of quantum-mechanics, validity of effective field theory in curved geometry, and/or general relativity. These include final state projection\textsuperscript{6}, the ER/EPR proposal\textsuperscript{7}, state-dependent modifications of quantum mechanics\textsuperscript{8,9}, and complexity-theoretic arguments\textsuperscript{10,11}.

Recently, attempts have been made to apply arguments from decoherence in open quantum systems to this problem\textsuperscript{12,13}. Within this “operational” approach, physical “infalling” observers capable of verifying the maximal entanglement between emitted Hawking quanta and the black hole do not have access to certain bath degrees of freedom that decohere the global wavefunction—they effectively reside in a particular branch of the global state, which corresponds to a definite semi-classical geometry; see Fig. 1 (a) for illustration. If this branching due to decoherence is sufficiently rapid, these observers do not witness monogamy violation because quantum entanglement between late-time Hawking quanta and the exterior of the black hole (which includes the bath) is mostly inaccessible to them\textsuperscript{14}. In the context of modern approaches to the information problem, this approach is closely related both to ideas introduced in the alpha-bits work of Refs.\textsuperscript{15,16} and recent AdS/CFT ensemble approaches such as in Refs.\textsuperscript{17–23}.

In this work, we take inspiration from these approaches (and recent developments in the study of entanglement transitions in random circuits, see Refs.\textsuperscript{24–32}) to devise a toy unitary-projective circuit model that emulates black hole evaporation, and encapsulates the decoherence of outgoing Hawking radiation; see Fig. 1 (b) for illustration. Specifically, we consider a system of qubits which models the Hilbert space of the black hole interior and the Hawking radiation. Random Clifford gates, which form a unitary 2-design and thus efficiently capture scrambling in the system, are applied to qubits inside of the black hole horizon which shrinks in time at the rate of $v$ qubits every time step. In the remainder of the system, projective measurements are applied with a certain probability $p$ at each time step. These measurements model decoherence of Hawking radiation by an external bath of, say, vacuum graviton fluctuations.

As we show, the entanglement $\gamma$ across the black hole horizon remains area law (much smaller than the number of black hole qubits) in this model and is well approximated as $\gamma \approx v/p$. Then, as long as $\gamma \ll N_{\text{BH}}$, the number of qubits inside the black hole, newly emitted Hawking quanta are maximally entangled with the black hole interior, in agreement with the infalling observer’s expected field theory calculations. In this setup, we find that the paradox only becomes manifest when the black hole has shrunk to a finite size $N_{\text{BH}} \approx v/p$ at which point
Page’s result implies that newly emitted Hawking quanta cannot be maximally entangled with the hole violating expectations from effective field theory. We use dimensional analysis to estimate the rate of decoherence by a putative bath of gravitons, and relate the parameters \( v, p \) to the physical problem of black hole evaporation. Using these results, we argue that this critical black hole size is Planckian, a scale at which quantum gravity is not fully understood, and modifications thereof could resolve the paradox.

**Circuit Model.**—The model we consider is a unitary-projective circuit model of stabilizer states evolved by two-qubit Clifford gates. These Clifford gates form a unitary 2-design which efficiently describe\(^{33,34}\) information scrambling in the black hole—although unitaries implemented by these gates are not Haar-typical with respect to all moments, they are exponentially close (in trace distance) up to the second moment, and reproduce the relevant physics with regards to Page’s theorem/decoupling type arguments\(^{34}\). Concretely, we study a system of \( N \leq 800 \) qubits which, at \( t = 0 \), begin in a random stabilizer state of their Hilbert space generated by application of \( \geq N \) layers of two-qubit Clifford gates arranged in a brickwork fashion. For subsequent times, we assume these \( N \) qubits together model both the black hole interior and its emitted Hawking radiation. The number of qubits \( N_{\text{BH}} \) should be identified with the Bekenstein-Hawking entropy of the black hole, but are not meant to convey any spatial information. At each step, Clifford gates are applied in a brickwork fashion [see Fig. 1 (b)] to the (left) part of the spin chain modeling the interior of the black hole.

Next, since emitted Hawking quanta are radiated outward in all directions and are thus separated by large distances, we assume interactions between them can be neglected. Thus, no unitaries are applied on this part of the spin chain. We model the ejection of Hawking quanta by shrinking the black hole horizon by \( v \) qubits at each time step. Further, we assume that a local background of graviton fluctuations constantly act on and decohere these quanta. For an observer that does not have access to the graviton bath, this effect may be captured by a constant rate of measurement, probability \( p \) every time step, of the qubits representing the Hawking radiation.

Note that steps of the random circuit have physical meaning—\( N_{\text{BH}} \) steps generate a (effectively) Haar-random state inside the black hole horizon; thus, \( N_{\text{BH}} \) time steps should be associated with the scrambling time \( \tau_S \) of an equivalent black hole. In particular, if we denote \( T \) as the depth of the circuit, \( dT/dt \approx N_{\text{BH}}/\tau_S \). In what follows, we will ignore any dependence of \( v, p, T \) on physical time and only invoke it when specifically discussing black hole evaporation. Also, although one may track the density matrix corresponding to the full ensemble of measurement outcomes, it is sufficient to track a specific (randomly chosen) outcome for the purpose of evaluating the mutual information between the black hole interior and the radiated Hawking quanta\(^{35}\). Henceforth we equivalently refer to the above as mutual information across the horizon.

Finally, before we proceed further, let us reiterate the statement of the information paradox in this setting. The time evolution of the state of the above spin system models the evolution of the system accessible to a physical infalling observer. This observer should find near maximal entanglement between a newly emitted quanta of Hawking radiation and the black hole interior, so as to be consistent with their effective field theory calculations at the horizon. The decoupling theorem\(^{5,66}\) states—see Fig. 2 (a) for illustration—that this will be the case as long
as the number of qubits inside the black hole exceeds the mutual information across the horizon by a finite amount. (The entanglement is then exponentially close in this finite amount to being maximal.) Thus, within our scheme, the information paradox does not occur until the mutual information nearly equals $N_{BH}$. Equivalently, the paradox becomes manifest when the mutual information approaches the Page curve (given by the maximal possible entanglement between the black hole and radiated Hawking quanta, min. $[N_{BH}, N - N_{BH}]$) after the halfway point in the evaporation process. Measurements of the Hawking quanta reduce this mutual information and can dramatically delay the onset of the paradox.

**Numerical Results.**— Some results for different choices of (fixed-in-time) $v, p$ are presented in Fig. 1 (c). The results exhibit a simple trend: mutual information across the horizon grows to approximately the value $\sim v/p$ provided the system is large enough, that is, $N_{BH} \gg v/p$; eventually, the mutual information tracks the Page curve. We now derive a mean-field equation of motion for the entanglement growth in this system, that correctly captures the steady state result $\gamma \approx v/p$ and well approximates the dynamical results.

**Equation of motion for the mutual information.**— To describe entanglement growth, it is first important to visit the decoupling theorem and its adaptation to the present setting; see alternatively, Ref. 30 for a similar discussion. Consider two coupled systems $A$ and $B$ initialized in a joint pure state. Assume further $A(B)$ has $\gamma$ bits of (mutual) information about $B(A)$. We can perform a unitary transformation that acts on $B$ and distills the $\gamma$ maximally entangled bits in $B$ into a subsystem $\tilde{B}$. By monogamy, the rest of $B$ is now unentangled with $A$ and $\tilde{B}$. The central question we wish to answer is this—how much information does measuring $N_m$ qubits in $A$ yield about $\tilde{B}$ (and thus $B$)? Assuming system $A + \tilde{B}$ is in some Haar-typical state, the decoupling theorem states, that if qubits $N_m + \gamma < N_{A} - N_{m}$, then the qubits $N_m$ in fact have exponentially little information about the $\gamma$ entangled qubits in $\tilde{B}$. This result will be used repeatedly below to formulate dynamics of the mutual information.

We now discuss the rules for entanglement change $\Delta \gamma(T)$ as a function of the current entanglement $\gamma(T)$, the bits to be ejected in this time step $v(T)$, the present number of black hole qubits $N_{BH}(T)$, unique qubits $q(T)$ measured in this time step, and $Q(T)$, which counts all unique qubits measured at times before the present time $T$. See Fig. 2 for an illustration of these quantities.

Two processes occur that change the mutual information: ejection of qubits from the black hole, and measurement of qubits outside the black hole. Note that measurement eliminates the measured qubit from further participating in the dynamics. In what follows, we will make the assumption that all these processes keep the wavefunction of the system in a Haar-typical state of appropriate dimensionality (accounting for the entanglement across the horizon, and the qubits removed by measurement)—thus the decoupling theorem will always apply on an appropriately defined set of qubits in the system. In Fig. 2, this corresponds to the qubits drawn with some form of patterning.

During ejection, if $N_{BH} - v > v + \gamma$, the decoupling theorem implies that $v$ qubits to be ejected are monogamously entangled with the rest of the black hole interior. Thus, each such ejected qubit contributes an increase $\Delta \gamma \approx +1$. If the former condition is not satisfied, then the ejected qubits contain little to no information about the qubits inside the black hole. In the extreme case when they are maximally entangled with the black hole exterior, their ejection results in a reduction of entanglement $\Delta \gamma = -1$ per qubit. The latter occurs when $\gamma(T)$ approaches its maximal value $N_{BH}(T)$. Assuming $v(T)$ is a small number, one may capture the above approximately by

$$\Delta \gamma_{ej} = v \cdot \text{sgn}(N_{BH} - \gamma - 2v)$$

where $\text{sgn}(x)$ is the sign function.

During measurement, if $q + \gamma < N - N_{BH} - Q - q$, then the measured qubits reveal (exponentially) close to nothing about the black hole’s internal structure. As a result, mutual information across the horizon is changed only if the former condition is violated. In that event, we make a mean field-like assumption that the information $\gamma$ is distributed equally among the $N - N_{BH} - Q$ external qubits that have not already been measured. Subsequently, each measured qubit decreases entanglement across the horizon by an amount $\gamma / (N - N_{BH} - Q)$. Thus,

$$\Delta \gamma_{me} = -q \cdot \frac{\gamma}{N - N_{BH} - Q} \cdot \Theta(2q + \gamma - N + N_{BH} + Q)$$

where $\Theta$ is the Heaviside function. The change of entanglement then is given by the sum of the two

![FIG. 2. Processes of black hole evaporation, with bits organized to facilitate the application of the decoupling theorem.](image_url)
contributions—

$$\Delta \gamma (T) = \gamma (T+1) - \gamma (T) = \Delta \gamma_{\text{eq}} (T) + \Delta \gamma_{\text{me}} (T) \quad (3)$$



\begin{array}{|c|c|c|c|}
\hline
\text{Time} & \text{Black Hole} & \text{Random Circuit} \\
\hline
\text{Black Hole DOF} & S_{\text{BH}} = 4\pi M^2 & \text{Bits inside horizon, } N_{\text{BH}} \\
\text{Scrambling Time} & t_{\text{S}} = \kappa M & \text{Circuit depth } N_{\text{BH}} \\
\text{Evaporation Rate} & \frac{d\gamma}{dT} = \frac{\gamma}{\sqrt{x}} & v \equiv \frac{dN_{\text{BH}}}{dT} \sim N_{\text{BH}}^{-1} \\
\text{Measurement Rate} & \frac{dp}{dT} = \alpha \left( \frac{1}{\sqrt{\pi M}} \right)^x & p \equiv \frac{dN_{\text{BH}}}{dT} \sim N_{\text{BH}}^{(x+1)/2} \\
\hline
\end{array}

TABLE I. A mapping between the black hole evaporation problem and the stabilizer circuit model of Fig. 1 (b). The equivalence of the scrambling time of the black hole, and that of the random circuit is the constitutive relation tying the flow of time in the physical problem to circuit depth; it asserts \( \frac{dT}{dt} = \frac{dN_{\text{BH}}}{dt} \). The remaining relations are found by simple algebraic manipulations and well-established black hole physics.

The above equations are easily solved in the situation where the entanglement entropy remains small compared to the size of the black hole, \( N_{\text{BH}} \). In this case, \( \Delta \gamma_{\text{eq}} \approx v \)—thus, each ejected qubit is well entangled with the black hole interior (as is necessary for consistency with effective field theory). Moreover, we can safely neglect \( q, \gamma \) in favor of factors that scale with the number of total qubits \( N \) in the Heaviside function in Eq. (2); the latter is then naturally satisfied, and we find \( \Delta \gamma_{\text{me}} \approx \gamma \cdot q/ (N - N_{\text{BH}} - Q) \approx \gamma \cdot p \), where \( p \) is just the probability with which each qubit is measured at each time step \( T \). Thus, assuming area law entanglement, the equations simplify remarkably to a straightforward detailed balance equation

$$\frac{d\gamma}{dT} = v - \gamma \cdot p \; ; \; \gamma \text{area law} = \frac{v}{p} \quad (4)$$

where we also note the steady state solution which matches nicely with our numerical results, see Fig. 1 (c).

Note that the solution is naturally stable to perturbations corresponding to blips of enhanced/lowered rate of measurement of Hawking quanta—these perturbations are damped over a circuit depth \( \sim 1/p \). When \( v, p \) are time dependent, as in the case of black hole evaporation, for the solution to remain instantaneously valid, we require the rate of change \( \left| \frac{d(dv/p)}{dT} \right| \ll p \) or \( \left| \frac{d(v/p)}{dT} \right| \ll v \).

**Implications for black hole evaporation.**— We first discuss the mapping of the circuit model to black hole evaporation as enumerated in Table I; below we use natural units \( h = c = G = 1 \). The circuit depth \( T \) is the natural analog of physical time \( t \). The Bekenstein-Hawking entropy \( S_{\text{BH}} = 4\pi M^2 \) enumerates black hole degrees of freedom (DOF); thus it must be given by the \( N_{\text{BH}} \) qubits inside the horizon in the circuit model. The black hole scrambling time \( t_{\text{S}} \sim \mathcal{O}(M \log M) \leq \kappa M \) (where we have subsumed the weak log-dependence on \( M \) into a sufficiently large constant \( \kappa \)) corresponds to a circuit of depth \( N_{\text{BH}} \) which completely scrambles (transform into Haar-random state) any initial state of the \( N_{\text{BH}} \) black hole qubits. Note that the equivalence of the scrambling time in the two models gives physical meaning to the circuit depth; in particular, it asserts in the continuum limit the relation

$$\frac{dT}{dt} \approx N_{\text{BH}} t_{\text{S}} \approx \sqrt{N_{\text{BH}}/4\pi^2} \quad (5)$$

The Hawking result for the evaporation rate can be translated to the number of qubits ejected \( v \equiv dN_{\text{BH}}/dT \sim N_{\text{BH}}^{-1} \) in a brickwork time step using preceding relations. To determine the measurement rate, we consider the following minimal setting for decoherence—imagine a Schwarzchild black hole in four dimensional flat space at zero temperature wherein Hawking quanta is decohered only by vacuum graviton fluctuations. There are two relevant energy scales for such a process—the Hawking temperature \( T_H = 1/(8\pi M) \), and the Planck temperature. The decoherence rate \( d\gamma/dt \) is thus some undetermined function of \( T_H \) in natural units. Assume then \( \frac{d\gamma}{dT} = \alpha \left( \frac{1}{\sqrt{\pi M}} \right)^x \) with an exponent \( x \sim \mathcal{O}(1) \) constant. The probability of measurement per time step in the random circuit is then given by \( p \equiv d\gamma/dT \sim (N_{\text{BH}})^{-(x+1)/2} \). Finally, a naive Fermi’s Golden Rule calculation suggests \( d\gamma/dT \sim T_H^x \) or \( x = 2 \)—the dependence comes from the square of the coupling proportional to the temperature \( T_H \) of Hawking radiation to low-energy gravitons, which in turn have an intrinsic density of states independent of \( T_H \); see also the result of Ref. 13.

Given Table I, and for \( x < 3 \) (valid for the naive estimate above), we find the ratio \( v/p \ll N_{\text{BH}} \), and the stability condition \( \left| \frac{d(dv/p)}{dT} \right| \ll v \) is satisfied for large enough black holes. In fact, the two conditions amount to the same relation barring an \( \mathcal{O}(1) \) constant. Thus, we anticipate for evaporating black holes, Eq. (4) applies and a local continuous-in-time decoherence of Hawking radiation is enough to keep entanglement instantaneously tracking the value \( v/p \) until it shrinks to a critical size \( N_{\text{BH}}^c \) at which point the mutual information across the horizon saturates its internal degrees of freedom. Until this critical limit is reached, the entanglement remains below the Page curve, and as per Eq. (1), emitted Hawking quanta are maximally entangled with the black hole, thus avoiding monogamy violation. Beyond this limit, the black hole is evaporating faster than information can be extracted from it, and newly emitted Hawking quanta appear to be maximally entangled with the exterior, signalling the onset of the information paradox. However, as mentioned above, this critical black hole has a size \( N_{\text{BH}}^c \) that is Planckian; in particular

$$N_{\text{BH}}^c = \left( \frac{16\pi^{3/2}}{\sqrt{a}} \right)^2 \left( \frac{4\sqrt{\pi}}{3} \right)^{x-1} b^{2x-2} \quad (6)$$
in terms of undetermined $O(1)$ constants $a, b$, and $x$; note that the scrambling time in fact drops out of the consideration completely, marked by an absence of $\kappa$.

**Discussion.**— As mentioned above, Eq. (6) suggests that the black hole information paradox becomes an issue only when the black hole is Planck-sized. At this point, the paradox may be resolved involving currently unexplored Planck-scale physics, where a myriad of possibilities may occur—for instance, the black hole may simply stop evaporating\(^{38}\) (importantly, this remnant black hole need only store information within the limit of the Bekenstein bound). It is however, important to remind ourselves that this result has been obtained strictly under the assumption that there exists a sensible distinction between the Hilbert space corresponding to Hawking quanta and the decohering environment, and that physical observers do not have access to this environment. The validity of these assumptions is not by any means obvious. However, our work provides a strong impetus to consider further exploration into such ideas.

While the applicability of this work to black hole evaporation is highly speculative, we anticipate these results could be used to motivate experiments on noisy intermediate-scale quantum computers\(^{39}\) and other artificial quantum systems\(^{40–42}\) to specifically study quantum entanglement dynamics and benchmark their progress. The physics we explore here is most clearly seen in spin chains much larger in size than those than can be simulated via exact diagonalization, but which do not require scaling to the thermodynamic limit.

**Note Added.**— During the completion of this work, we learned of upcoming work\(^{43}\) that could be synergistic with the ideas in this work.

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