Determining the Flavour Content of the Low-Energy Solar Neutrino Flux

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Abstract

We study the sensitivity of the HELLAZ and Borexino solar neutrino experiments on discriminating the neutrino species $\nu_e$, $\bar{\nu}_e$, $\nu_{\mu,\tau}$, $\bar{\nu}_{\mu,\tau}$, and $\nu_s$ using the difference in the recoil electron kinetic energy spectra in elastic neutrino-electron scattering. We find that one can observe a non-vanishing $\nu_{\mu,\tau}$ component in the solar neutrino flux, especially when the $\nu_e$ survival probability is low. Also, if the data turn out to be consistent with $\nu_e \leftrightarrow \nu_{\mu,\tau}$ oscillations, a $\bar{\nu}_e$ component can be excluded effectively.
1 Introduction

The flux of solar neutrinos was first measured in the Homestake mine (see [1] and references therein) over thirty years ago. Since then, it was realized that the measured flux was significantly suppressed with respect to theoretical predictions. More recently, a handful of different experiments have also succeeded in measuring the solar neutrino flux [2, 3, 4, 5]. All experiments measure a neutrino flux which is significantly suppressed with respect to the theoretical predictions of the most recent version of the Standard Solar Model (SSM) [6]. This thirty year old problem is what is referred to as the “solar neutrino puzzle.”

There are different types of solutions to the solar neutrino puzzle. At first sight, it appears natural to suspect that the SSM predictions for the solar neutrino flux are slightly off, and/or that the experiments have underestimated their systematic effects, given that detailed models of the Sun and neutrino experiments are highly non-trivial. However, SSM independent analyses of the neutrino data (see [7] for a particularly nice and simple example), together with independent experimental evidence in favour of the SSM [6], seem to indicate that the above solution to the puzzle is strongly disfavoured.

The best solution to the solar neutrino puzzle involves extending the Standard Model of particle physics by assuming that the neutrinos have mass and that they mix, i.e., neutrino mass eigenstates are different from neutrino weak eigenstates. This possibility has become particularly natural in light of the recent strong evidence for $\nu_\mu$ oscillations from the atmospheric neutrino data at SuperKamiokande [8].

Nonetheless, in order to firmly establish that the solution to the solar neutrino puzzle involves physics beyond the Standard Model, it is necessary to come up with SSM independent, robust experimental evidence for, e.g., solar neutrino oscillations. Indeed, these “Smoking Gun” signatures of solar $\nu_e \leftrightarrow \nu_{\text{other}}$ oscillations are among the present goals of the SuperKamiokande experiment, via the measurement of the day-night asymmetry of the solar neutrino data [8] and the recoil electron energy spectrum [10], and the SNO experiment [11], via the measurement of the charged to neutral current ratio, the day-night asymmetry of the data, and the recoil electron kinetic energy spectrum.

Other goals of this and the next generation of neutrino experiments are, if solar neutrino oscillations are established, to determine neutrino oscillation modes and measure masses and mixing angles. The current data allow for
different $\nu_e$ oscillation modes and a handful of disconnected regions in the mass–mixing-angle parameter space (see [12, 13] for two-flavour analyses and also [14] for an extension to the “dark side” of the parameter space).

Experiments dedicated to measuring the flux of low-energy solar neutrinos ($E_\nu = O(100 - 1000)$ keV) are going to be extremely useful, and perhaps crucial, in order to fully solve the solar neutrino puzzle. It was recently shown that future experiments (Borexino [15] and, perhaps, KamLAND [16]) dedicated to measuring the flux of $^7$Be neutrinos (produced by $^7\text{Be} + e^- \rightarrow ^7\text{Li} + \nu_e$ inside the Sun) should be able to establish or exclude the “just-so” solution [12] to the solar neutrino puzzle via the study of the seasonal variations of the neutrino flux [17], and establish or exclude the LOW MSW solution [13] via the study of the zenith angle dependence of their data [18]. Furthermore, the measurement of a sizable $^7$Be neutrino flux would significantly disfavour the SMA MSW [13] solution to the solar neutrino puzzle, especially in the case of $\nu_e \leftrightarrow \nu_s$ oscillations (where $\nu_s$ is a sterile neutrino, i.e., a standard model singlet), and significantly constrain the SSM independent analysis, which require the flux of $^7$Be neutrinos to be virtually absent [0]. Finally, we have shown [19] that, in the advent that the background rates at Borexino and/or KamLAND are exceptionally low, it should be possible to measure a nonzero component of $\nu_{\mu,\tau}$ in the solar neutrino flux by analysing the recoil electron kinetic energy spectrum.

Another exciting possibility is that of measuring the “fundamental” $pp$-neutrinos, which are produced in the interior of the Sun by proton-proton fusion ($p + p \rightarrow ^2\text{H} + e^+ + \nu_e$) in a real time experiment. Future experiments, such as HELLAZ, HERON, LENS, etc (see [20] for an overview) are being designed to do just that. The flux of $pp$-neutrinos is particularly constrained by the photon flux i.e., the Sun’s luminosity, which, of course, is very well measured on the Earth. These lowest energy solar neutrinos ($E_\nu \lesssim 420$ keV) are not only the most abundant ones, but also have the best known flux. Their energy spectrum is also very well known, since it is dictated by the particularly well studied $p + p$ nuclear fusion reaction. Among these proposed experiments, HELLAZ [21] will be able to determine the incoming neutrino energy in an event-by-event basis and have the unique opportunity of studying the solar neutrino spectrum and the recoil electron kinetic energy spectrum separately. Similar to what was shown for $^7$Be neutrinos [19], the authors of [22] showed that HELLAZ may be able to measure a nonzero component of $\nu_{\mu,\tau}$ in the solar $pp$-neutrino flux by analysing the recoil electron kinetic energy spectrum independent of the SSM prediction for the solar
Table 1: Coefficients $A$, $B$ in Eq. 2.1 for different neutrino species.

| species   | $A$        | $B$        |
|-----------|------------|------------|
| $\nu_e$   | $\sin^2 \theta_W + \frac{1}{2}$ | $\sin^2 \theta_W$ |
| $\bar{\nu}_e$ | $\sin^2 \theta_W$ | $\sin^2 \theta_W + \frac{1}{2}$ |
| $\nu_{\mu,\tau}$ | $\sin^2 \theta_W - \frac{1}{2}$ | $\sin^2 \theta_W$ |
| $\bar{\nu}_{\mu,\tau}$ | $\sin^2 \theta_W$ | $\sin^2 \theta_W - \frac{1}{2}$ |

neutrino flux.

In this paper we extend the analysis done in [19], and study the flavour composition of the flux of $pp$ and $^7$Be neutrinos using the recoil electron kinetic energy spectrum. In particular we will address the capability of future low-energy solar-neutrino experiments to see evidence for $\nu_{\mu,\tau}$ coming from the Sun, and, in light of such evidence, exclude more “exotic” oscillation scenarios, such as $\nu_e \leftrightarrow \nu_s$ or $\nu_e \leftrightarrow \bar{\nu}_{\text{any}}$ oscillations.

Our presentation is organised as follows: Sec. 2 describes the flavour-dependent recoil kinetic energy distribution of events at Borexino and HELLAZ. Sec. 3 presents the technique for determining the presence of $\nu_{\mu,\tau}$ coming from the Sun, independent of the SSM prediction for the neutrino flux. We present simulations for both Borexino and HELLAZ and show how such a determination can be improved once we take the SSM prediction for the neutrino flux into account. Sec. 4 describes how the same procedure can be used to exclude the presence of antineutrinos or sterile neutrinos in the solar neutrino flux. In Sec. 5, we conclude.

2 Recoil Electron Kinetic Energy Spectrum

In this section, we discuss the differences in the recoil electron kinetic energy spectra among different neutrino species. Low-energy solar neutrinos are detected via $\nu + e^- \rightarrow \nu + e^-$ elastic scattering in the experiments which will be considered here. By “$\nu$” in the previous sentence, one actually means any of $\nu_e$, $\nu_{\mu}$, $\nu_{\tau}$, $\bar{\nu}_e$, $\bar{\nu}_{\mu}$, or $\bar{\nu}_{\tau}$. Because $\nu_{\mu}$ and $\nu_{\tau}$ are indistinguishable as far as the reaction above is concerned, we will refer to both as $\nu_{\mu}$.

The kinetic energy distribution of the recoil electrons, for a given incoming
neutrino energy $E_\nu$ is very well known and given by \[23\]

$$
\frac{d\sigma(T, E_\nu)}{dT} = 2G_F^2 m_e \left[ A^2 + B^2 \left( 1 - \frac{T}{E_\nu} \right)^2 - AB \frac{m_e T}{(E_\nu)^2} \right],
$$

(2.1)

where $m_e$ is the electron mass, $T$ is the kinetic energy of the recoil electron, and $G_F$ is the Fermi constant. The parameters $A$ and $B$ are given in Table 1. The sign difference in the term $1/2$ is a consequence of the presence (absence) of $W$-boson exchange and the interchange of $A$ and $B$ between neutrino and anti-neutrino cases is a consequence of the “handedness” of the weak interactions. Eq. (2.1) is a tree-level expression, but higher order corrections are known to be very small [24], especially for the neutrino energies of interest, and will be neglected throughout.

Borexino (under construction) is an ultra-pure liquid scintillator tank which detects the scintillating light produced by the recoil electron absorbed by the medium. For more details see [15, 17]. It is sensitive to recoil electron kinetic energies greater than 250 keV, and is therefore sensitive to the (almost) monochromatic $^7$Be neutrinos with $E_\nu = 862$ keV. The expected resolution for the kinetic energy measurement varies from roughly 12%, for $T = T_{\text{min}} = 0.25$ MeV, to 7% for $T = T_{\text{max}} = 0.66$ MeV [15]. They expect 53 events/day in the SSM (BP95) together with 19 background events/day with the anticipated radiopurity of the scintillator of $10^{-16}$ g/g for U/Th, $10^{-18}$ g/g for $^{40}$K, and $^{14}$C/$^{12}$C = $10^{-18}$ and no radon diffusion. It is remarkable, however, that the München group of Borexino achieved a radiopurity for an organic liquid (Phenyl-ortho-xylylethane) better than $1 \times 10^{-17}$ g/g [25]; this is an upper bound on the contamination, limited by the sensitivity of the neutron activation measurement and hence the actual radiopurity may be even better. In this paper, we ignore the background to the $^7$Be solar neutrino signal at Borexino. This is probably an overoptimistic assumption, but could be realised in future upgrades given the above-mentioned achievement.

HELLAZ (proposed) is a large time projection chamber (TPC) filled with roughly 2000 m$^3$ of cool helium gas ($\sim 6$ tons at 5 atmos, 77 K), which serves as the target for $\nu$-e scattering. The recoil electron propagates in the gas medium before being absorbed, leaving a track of ionization electrons. These are then collected, yielding information about the kinetic energy and the flight direction of the recoil electron. HELLAZ is sensitive to recoil kinetic energies greater than $\sim 50$ keV, and can therefore “see” most of the $pp$-neutrino spectrum. Most importantly, since not only the recoil kinetic
energy of scattered electrons is measured but also their direction, it is possible to reconstruct the incoming neutrino energy, given that the position of the Sun in the sky is known, via the simple kinematic relation

\[ T = m_e \frac{2 \cos^2 \theta}{(1 + m_e/E_\nu)^2 - \cos^2 \theta}, \]  

(2.2)

where \( \theta \) is the recoil electron scattering angle with respect to the incoming neutrino direction in the laboratory frame. Incidentally, from Eq. (2.2) it is very easy to compute the maximum value of the recoil electron kinetic energy, \( T_{\text{max}} = T(\theta = 0) = E_\nu/(1 + m_e/(2E_\nu)) \). HELLAZ expects to measure the recoil electron kinetic energy with a resolution which varies roughly from 2% to 4% and the incoming neutrino energy with a resolution which varies between 5% and 12% [21]. They expect around 7 events/day from \( pp \) neutrinos in the SSM with negligible background. The major sources of background at HELLAZ are radioactive impurities from \( ^{232}\text{Th} \) and \( ^{238}\text{U} \) in the structure of the TPC. However, because of the detector’s total event reconstruction capabilities (including directional information), very efficient background rejection schemes are possible (see [21] and references therein for further information).

The issue we would like to concentrate on is whether the shapes of the recoil electron kinetic energy distributions for different (anti)neutrino species are statistically different at Borexino and HELLAZ. With this in mind, Fig. 1 depicts the normalised distribution of events at HELLAZ (left) and Borexino (right). In the case of Borexino, the data is binned into ten kinetic energy bins, between 250 keV and 650 keV. In the case of HELLAZ, the data is binned into \( 4 \times 21 \) bins in \( E_\nu \times T \). The bins have a width of 50 keV in the \( E_\nu \) direction and central values of 245, 295, 345 and 395 keV, while in the \( T \) direction they have a width of 10 keV in the range from 50 to 260 keV. The bin sizes have been chosen such that they are roughly the same as the resolution of both detectors. In order to integrate over the incoming neutrino

*In addition to \( pp \)-neutrinos, HELLAZ is also sensitive to \( ^7\text{Be} \) neutrinos, as well as the \( pep \)-neutrinos and the neutrinos coming from the CNO-cycle. \( ^7\text{Be} \) neutrinos can be clearly separated from \( pp \)-neutrinos, while the number of \( pep \) and CNO-cycle neutrino generated events is expected to be less than 10% that of \( pp \)-neutrinos. Borexino is sensitive to, in addition to \( ^7\text{Be} \) neutrinos (with \( E_\nu = 862 \) keV), a fraction of the \( pep \) and the CNO-cycle neutrinos, which produce approximately 10% as many events as \( ^7\text{Be} \) neutrinos. We assume throughout, for simplicity, that only \( pp \) (\( ^7\text{Be} \)) neutrinos are detected at HELLAZ (Borexino).
energy at HELLAZ, the (normalised) BP98 pp-spectrum presented at [26] was used.

Figure 1: Normalised recoil electron kinetic energy distributions, for each of the 4 neutrino energy bins (see text) at HELLAZ (left) and for $^7$Be neutrinos (right).

Many important features of the recoil electron kinetic distributions are worthwhile to point out. First of all, it is quite clear that the spectrum produced by $\bar{\nu}_e$-$e$ is much steeper than all the other ones. Second, the $\nu_e$ and $\nu_\mu$ generated spectra have opposite slopes when the neutrino energy is small enough, while their shapes start to look more and more similar as the neutrino energy increases. Finally, the spectra produced by $\nu_\mu$-$e$ and $\bar{\nu}_\mu$-$e$ scattering are extremely similar, especially at very low energies.

All of these features can be readily understood from Eq. (2.1). First, it is convenient to write the expression for the normalised recoil electron kinetic

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†Some of these features were pointed out in [27].
energy distributions,

\[
\frac{d\bar{\sigma}}{dy} = N \left[ \frac{A}{B} + \frac{B}{A} (1 - y)^2 - \frac{m_e}{E_\nu} y \right],
\]  

(2.3)

where \( y = T/E_\nu \), and \( N \) is a normalisation constant, such that \( \int \frac{d\sigma}{dT} \) from \( T_{\text{min}} \) to \( T_{\text{max}} \) equals unity.

For \( \nu_e \), \( \frac{A}{B} \sim 3 \), while \( \frac{B}{A} \sim 1/3 \). For \( \bar{\nu}_e \) the situation is reversed, while for both \( \nu_\mu \) and \( \bar{\nu}_\mu \), \( \frac{A}{B} \sim \frac{B}{A} \sim -1 \).\(^4\) Note, curiously enough, that the reason for the similarity between the \( \nu_\mu \) and \( \bar{\nu}_\mu \) cases is simply due to the accidental fact that \( \sin^2 \theta_W \) is close to \( 1/4 \). This similarity is even more pronounced at very low energies, when the \( \frac{m_e}{E_\nu} \) term dominates over the \( \frac{A}{B} \) and \( \frac{B}{A} \) terms.

Keeping in mind that \( 0.59 \lesssim \frac{m_e}{E_\nu} \lesssim 2.3 \) in the neutrino energy range of interest and \( 0 < y \leq (1 + \frac{m_e}{2E_\nu})^{-1} < 1 \), one may write approximate expressions

\[
\left( \frac{d\bar{\sigma}}{dy} \right)_{\nu_e} \propto 3 \left( 1 - \frac{m_e}{E_\nu} y \right),
\]  

(2.4)

\[
\left( \frac{d\bar{\sigma}}{dy} \right)_{\bar{\nu}_e} \propto 3(1 - y)^2 - \frac{m_e}{E_\nu} y,
\]  

(2.5)

\[
\left( \frac{d\bar{\sigma}}{dy} \right)_{\nu_\mu} \propto \left( \frac{d\bar{\sigma}}{dy} \right)_{\bar{\nu}_\mu} \propto 1 + (1 - y)^2 + \frac{m_e}{E_\nu} y.
\]  

(2.6)

In the limit \( \frac{m_e}{E_\nu} \ll 1 \), all three distributions are quite different (see, e.g., Fig. 1(B) in [19]). The \( \nu_e \) case is roughly flat, the \( \bar{\nu}_e \) case ranges from 3 at \( y = 0 \) to 1 at \( y = 1 \) and the \( \nu_\mu, \bar{\nu}_\mu \) case ranges from \( 3/2 \) at \( y = 0 \) to \( 3/4 \) at \( y = 1 \).

For \( \frac{m_e}{E_\nu} \gtrsim 1 \), things are slightly more complicated, but still easy to understand. For example, the slope of the distributions for small values of \( y \) are, up to normalisation factors, \( -\frac{m_e}{E_\nu} \), \( -\left( 6 + \frac{m_e}{E_\nu} \right) \) and \( \left( \frac{m_e}{E_\nu} - 2 \right) \) for \( \nu_e, \bar{\nu}_e \) and \( \nu_\mu, \bar{\nu}_\mu \) respectively. It is then easy to note that the \( \bar{\nu}_e \) slope is significantly more negative than the other two, and that, in the case of \( \nu_\mu, \bar{\nu}_\mu \), the slope is actually positive if \( E_\nu \) is small enough. This is indeed what one observes in Fig.\(^4\).

As the incoming neutrino energy increases, the distributions generated by \( \nu_e \) and \( \nu_\mu, \bar{\nu}_\mu \) look more and more similar. One hint of this behaviour is

\(^4\)In this case, the normalisation constant \( N \) is negative.

\(^5\)The fact that there is a sign difference between \( g_L \) and \( g_R \) for muon-type (anti)neutrinos is irrelevant, since these coefficients either appear as squares or as the product \( g_L g_R \).
that the slope of the $\nu_e$ case increases (decreases in absolute value), while the slope of the $\nu_\mu, \bar{\nu}_\mu$ decreases. One can easily estimate that for $0.9 \text{ MeV} \lesssim E_\nu \lesssim 1.0 \text{ MeV}$ the shapes of the $\nu_e$ and $\nu_\mu$ induced recoil kinetic energy distributions are most similar. Indeed, for $^7\text{Be}$ neutrino energies, one can already note that the difference between the $\nu_e$ and the $\nu_\mu$ cases is similar to the difference between the $\nu_\mu$ and the $\bar{\nu}_\mu$ cases.

3 Measuring a $\nu_{\mu,\tau}$ Component in the Solar Neutrino Flux

In this section, we address the question whether the shapes of the recoil electron kinetic energy distributions presented in Sec. 2 are statistically different at Borexino or HELLAZ. In the affirmative case, there is hope that one may be sensitive to a “contamination” of other neutrino types in the solar neutrino flux by analysing the shape of the recoil kinetic energy spectrum. We consider this an “appearance experiment” of the “wrong” types of neutrinos from the Sun. In this section we will only consider the case of $\nu_e \leftrightarrow \nu_\mu$ oscillations.

In the advent of neutrino oscillations, a mixture of different neutrino weak eigenstates reaches the Earth. Given an electron-type neutrino survival probability $P_{ee}$, a fraction $P_{ee}$ of all the neutrinos arriving at the detector are $\nu_e$, while a fraction $1 - P_{ee}$ are $\nu_\mu$. The recoil electron kinetic energy distribution will, therefore, be given by

$$\frac{d\sigma(T, E_\nu)}{dT} = P_{ee} \times \left( \frac{d\sigma(T, E_\nu)}{dT} \right)_{\nu_e} + (1 - P_{ee}) \times \left( \frac{d\sigma(T, E_\nu)}{dT} \right)_{\nu_\mu}.$$  (3.7)

Note that, in general, $P_{ee}$ is a function of the neutrino oscillation parameters (the mass-squared differences of the neutrino mass eigenstates and the neutrino mixing angles) and the neutrino energy.

We simulate “data” at Borexino and HELLAZ for different values of $P_{ee}$. We use the distributions presented in Sec. 2, while the flux of $pp$ and $^7\text{Be}$ neutrinos are taken from the SSM \[7\]. In the case of Borexino, the energy dependence of $P_{ee}$ is irrelevant, given the monochromatic nature of $^7\text{Be}$ neutrinos. In the case of HELLAZ, we assume that $P_{ee}$ is constant inside each individual neutrino energy bin. Following the central idea presented in \[10\], we perform a $\chi^2$ fit to the “data” using a linear combination of $\nu_e$-e
scattering and $\nu_\mu$-$e$ scattering with arbitrary coefficients,

$$C_e \times \left( \frac{d\sigma(T, E_\nu)}{dT} \right)_{\nu_e} + C_\mu \times \left( \frac{d\sigma(T, E_\nu)}{dT} \right)_{\nu_\mu},$$

(3.8)

i.e., we perform a two parameter ($C_e$ and $C_\mu$) fit to the data. This measurement procedure is independent of the SSM prediction for the neutrino flux. Therefore, if a nonzero coefficient of the $\nu_\mu$-$e$ scattering distribution is measured, one can claim to have detected evidence for neutrinos other than $\nu_e$ coming from the Sun. This “appearance” result certainly qualifies as a smoking gun signature for neutrino oscillations.

Fig. 2 (long, thin error bars) depicts the measured value of $1 - P_{ee} = \frac{C_\mu}{C_e + C_\mu}$ in each of the neutrino energy bins defined in Sec. 2 as a function of the input value of $P_{ee}$, for 5 years of simulated HELLAZ data. As was mentioned in the previous paragraph, the relevant information one should obtain from the plot is if the measured value of $1 - P_{ee} \propto C_\mu$ is statistically different from zero.

Fig. 3 (right, long, thin error bars) depicts the measured value of $1 - P_{ee}$ as a function of the input value of $P_{ee}$, for two years of Borexino running. This is just a repetition of Fig. 2(A) in [19]. Fig. 3 (left, long, thin error bars) depicts the result obtained at HELLAZ if all energy bins are used in the “data” analysis. This result is only meaningful if $P_{ee}$ is roughly constant for neutrino energies ranging from 220 keV to 420 keV. This happens to be the case for most of the currently preferred regions of the two-neutrino oscillation parameter space, especially LMA, LOW and VAC solutions (see, e.g., [12]). Clearly, the significance of the measurement is better than the one obtained for individual energy bins (Fig. 2).

Next, the same analysis as above is repeated, except that the SSM prediction for the solar neutrino flux is included in the $\chi^2$ analysis. An uncertainty of 20% (5%) was assumed for the $^7$Be ($^{13}$C) neutrino flux. The theoretical error was considered Gaussian for simplicity.\footnote{In [19], a different variable, $P = 1 - P_{ee}$, was used. Both results are, of course, equivalent.} Note that the uncertainties assumed

\footnote{For the SMA solution, there is a sharp drop in $P_{ee}$ at $E_\nu \approx 0.4$ MeV, and the three lower bins can be combined without any problem. At HELLAZ, this will show up in the data, as the $E_\nu$ spectrum differs from the expected $pp$-neutrino spectrum, and hence is not a concern.}

\footnote{This procedure follows the one used in [17]. The readers are referred to this article for details.}
Figure 2: The “measured” value of $1 - P_{ee}$ as a function of the input value of $P_{ee}$ for each of the 4 neutrino energy bins (see text), after 5 years of HELLAZ running. The long, thin error bars correspond to model-independent analyses based only on the electron recoil energy spectrum shape, while the short, thick ones correspond to analyses which include the SSM prediction for the solar $pp$-neutrino flux, with an (inflated) uncertainty of 5%.

Here are inflated with respect to the ones quoted in the SSM calculations [6] (9% and 1%, respectively), in order to render the results very conservative. Since the rates are very high both at Borexino and HELLAZ, the error bars are dominated by the uncertainties in the fluxes and hence they can be approximately scaled according to the assigned flux uncertainties.

The results are presented in Figs. 2 and 3 (short, thick error bars). The significance of the measured value of $1 - P_{ee}$ improves significantly, especially at HELLAZ, because of the small assigned uncertainty on the $pp$-neutrino flux. After five years of HELLAZ running, for example, one should be able to determine a 1-sigma-away-from-zero $\nu_\mu$ component in the $pp$-neutrino
Figure 3: The “measured” value of $1 - P_{ee}$ as a function of the input value of $P_{ee}$, after 5 years of HELLAZ (left) and 2 years of Borexino running (right). $P_{ee}$ is assumed to be constant for $E_\nu = 220–420$ keV in the case of HELLAZ. The long, thin error bars correspond to model-independent analyses based only on the electron recoil energy spectrum shape, while the short, thick ones correspond to analyses which include the SSM prediction for the solar $pp$ ($^7$Be) neutrino flux, with an (inflated) uncertainty of 5% (20%).

flux even for $P_{ee} \sim 0.9$. It is also noteworthy that in the case of the SMA MSW solution to the solar neutrino puzzle $P_{ee} \sim 0$ for $^7$Be neutrinos, in which case a 4-sigma-away-from-zero evidence for $\nu_\mu$ in the solar neutrino flux can be established in only two years of Borexino running! It is important to emphasise that using SSM predictions for the solar neutrino flux is a reasonable thing to do, especially for $pp$-neutrinos. As mentioned before, the flux of $pp$-neutrinos is very well known because it is tightly related to the flux of light coming from the Sun. It is, therefore, the neutrino flux which is least sensitive to detailed modelling of the Sun’s innards.

Some comments are in order. First, only statistical uncertainties were considered, and there are no background events in our “data.” As discussed in Sec. 2, the assumption of a negligible background rate seems less than realistic at Borexino, but may be possible in future upgrades. It may, however, be a fair assumption in the case of HELLAZ. If the real experimental data contains a sizable number of background events, it is necessary to either
subtract the background in a bin-by-bin basis or to somehow model the recoil kinetic energy distribution produced by background events. Analysing either of these procedures, however, is beyond the scope of this paper.

Second, the analysis which does not include the SSM flux predictions is completely model-independent (the only assumption being the electron recoil spectrum as predicted by the standard electroweak theory), while the one which includes the SSM flux predictions is model-dependent. Obviously, one obtains a much better determination of $P_{ee}$ with the additional input of the SSM flux predictions. For establishing the “wrong neutrino component” in the solar neutrino flux as a smoking gun signature of the solar neutrino oscillations, the former approach is desired. However, for the purpose of determining the oscillation parameters, the energy dependence of the survival probability, and excluding other neutrino oscillation modes, such as $\nu_e \leftrightarrow \nu_s$ or $\nu_e \leftrightarrow \bar{\nu}_{e,\mu,\tau}$ (as will be discussed in Sec. 4), it is reasonable to include the SSM predictions in the analysis.

Finally, we point out that the results we obtained for HELLAZ are similar to the ones obtained by J. Séguinot et al [22]. Indeed, we chose neutrino energy bins at HELLAZ which coincide with the ones used in [22]. They also perform two different analyses of their simulated data, one which is independent of the SSM prediction for the solar neutrino flux, and one which assumes the SSM prediction for the flux. However, their data analysis procedure is somewhat different, and they do not take the theoretical uncertainty of the solar neutrino flux prediction into account.

4 Testing for the $\nu_e \leftrightarrow \nu_s$ or $\bar{\nu}_{e,\mu,\tau}$ Hypotheses

Although it is most natural to assume that electron-type neutrinos oscillate into some linear combination of muon-type and tau-type neutrinos, there is a logical possibility that electron-type neutrinos might oscillate into standard model singlet sterile neutrinos [28], or, perhaps, into antineutrinos of all flavours* (see [27] and references therein). In this section, we will address the issue of excluding these solar neutrino oscillation modes if the data collected at Borexino and HELLAZ are consistent with $\nu_e \leftrightarrow \nu_\mu$ oscillations.

One can already address these “exotic” oscillation modes with the current experimental data. The flux of electron-type anti-neutrinos from the Sun

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*The original neutrino oscillation paper by Bruno Pontecorvo [29] did, after all, consider $\nu_e \leftrightarrow \bar{\nu}_e$!
is particularly constrained by the SuperKamiokande and the LSD experiments [30]: the 95% CL SuperKamiokande upper bound on the flux of $\bar{\nu}_e$ from the Sun with energies $\gtrsim 6.5$ MeV is $\Phi_{\bar{\nu}_e} < 1.8 \times 10^5$ cm$^{-2}$ s$^{-1}$, or $\Phi_{\bar{\nu}_e}/\Phi_{8\text{BSSM}}^* = 0.035$, where $\Phi_{8\text{BSSM}}^*$ is the SSM prediction for the $8\text{B}$ neutrino flux. KamLAND, being a dedicated detector for $\bar{\nu}_e$, will improve this bound further down to 0.1% of the SSM flux above reactor anti-neutrino energies ($E_{\nu} \gtrsim 8$ MeV) after one year of running [16]. There are, however, scenarios in which, for energies below the SuperKamiokande threshold, the $\nu_e \leftrightarrow \bar{\nu}_e$ mixing is quite large ([27] and references therein). Such a possibility can only be addressed by low-energy solar neutrino experiments.

Below, we discuss the exclusion of electron-type neutrino oscillations into sterile neutrinos or into one of the antineutrino types separately at Borexino and HELLAZ experiments.

4.1 $\nu_e \leftrightarrow \nu_s$

The $\nu_e \leftrightarrow \nu_s$ oscillation mode is allowed by the analysis of current solar neutrino data [13], even though in the case of the MSW solutions to the solar neutrino puzzle, only the equivalent of the SMA MSW solution exists at the 99% confidence level [13]. It is curious to note that, in the case of atmospheric neutrinos, the $\nu_e \leftrightarrow \nu_s$ hypothesis is currently somewhat disfavoured [31].

In the case of $\nu_e \leftrightarrow \nu_s$ oscillations, one expects the recoil electron kinetic energy spectrum to be exactly the same as the one generated by $\nu_e$-$e$ scattering, since $\nu_s$ do not interact with electrons. The only effect of the neutrino oscillations would be to suppress the expected number of events, i.e., the hypothesis of $\nu_e \leftrightarrow \nu_s$ oscillations is identical to assuming that the solar neutrino flux is, somehow, suppressed. Therefore, we attempt to fit the “data” simulated according to Eq. (3.7) in Sec. 3 (remember that the “data” is consistent with $\nu_e \leftrightarrow \nu_\mu$ oscillations) to the trial function Eq. (3.8), where the piece which corresponds to $C_\mu$ vanishes identically. This is a one parameter $\chi^2$ fit to $C_e$. Note that the only discrimination against $\nu_s$ is the recoil energy spectrum, because the rate can be always fitted with the free parameter $C_e$. The inclusion of the SSM flux prediction does not help in excluding the $\nu_e \rightarrow \nu_s$ oscillation because the free parameter $C_e$ makes the predicted flux irrelevant.\footnote{One can “discover” $\nu_s$ by observing a nearly vanishing rate for $7\text{Be}$ neutrinos. In the case of $\nu_e \leftrightarrow \nu_{\mu,\tau}$ oscillations, neutral-current scattering guarantees at least (when $P_{ee} = 0$)}
almost completely oscillated to $\nu_s$ for the (only available) SMA solution: $P_{ee} = 0.009^{+0.244}_{-0.005} \, [2]$.

Fig. 4 depicts the value of $\chi^2$ obtained when one attempts to fit the “data” to the sterile neutrino hypothesis, for 5 years of HELLAZ running (left) and 2 years of Borexino running (right). In the case of HELLAZ, we have assumed that $P_{ee}$ is constant over the entire $pp$-neutrino energy range. The value of $\chi^2$ is determined using the philosophy employed in [12], and should be compared to $N_{\text{bins}} - 1$ ($N_{\text{bins}}$ is the number of “data” bins). After 5 years of HELLAZ running, one should be able to exclude sterile neutrinos coming form the Sun at more than 99.9% confidence level (CL) if all electron-type neutrino have turned into muon-type neutrinos ($P_{ee} = 0$). After 2 years of Borexino running, the sterile neutrino hypothesis is only ruled out, at best, at the 89% CL. The explanation for this is the fact that the recoil electron kinetic energy spectra are very different when one compares the $\nu_e$-e and the $\nu_\mu$-e scattering cases at very low energies, i.e., $pp$-neutrinos, and similar at $O$(MeV) energies, i.e., $^7$Be neutrinos, as discussed in Sec. 2. The exclusion CL decreases with increasing $P_{ee}$, and sterile neutrinos are excluded after 5 years of HELLAZ running only at the 77% CL for $P_{ee} = 0.4$.

4.2 $\nu_e \leftrightarrow \bar{\nu}$, Model-independent Fit

In the case of $\nu_e \leftrightarrow \bar{\nu}_{e,\mu}$ oscillations, we perform a two parameter fit to the “data” simulated as in Sec. 3 to a linear combination of the $\nu_e$-e and $\bar{\nu}_{e,\mu}$-e scattering recoil kinetic energy distributions. Fig. 5 depicts the value of $\chi^2$ obtained when such a fit is performed, for 5 years of Borexino and HELLAZ running. The value of $\chi^2$ is to be compared to $N_{\text{bins}} - 2$ to determine exclusion confidence levels. As advertised in Sec. 2, the $\nu_\mu$-e and $\bar{\nu}_\mu$-e scattering cases produce almost identical recoil kinetic energy spectra, and are almost undistinguishable at HELLAZ. At Borexino, however, the difference between $\nu_\mu$-e and $\bar{\nu}_\mu$-e scattering is similar to the difference between the $\nu_e$-e and $\nu_\mu$-e cases, as mentioned in Sec. 2 (see Fig. 1), and some discrimination seems possible. Furthermore, upon close inspection, one should note that the shape of the distribution produced due to $\nu_e$-e scattering is more similar to the $\nu_\mu$-e case than the $\bar{\nu}_\mu$-e. Therefore, any $\bar{\nu}_\mu$ component in the trial function makes 21% of the SSM rate. For this purpose, one should rely on the SSM flux prediction.

\footnote{The situation does improve, of course, if more events are collected at Borexino. After 5 years of Borexino running, for example, one can exclude sterile neutrinos for $P_{ee} \lesssim 0.1$ at more than 95% CL.}
Figure 4: Minimum $\chi^2$ values as a function of the input value of $P_{ee}$, obtained when one tries to fit the “data” (which is consistent with $\nu_e \leftrightarrow \nu_\mu$ oscillations and $P_{ee} = P_{ee(\text{input})}$) with a $\nu_e + \nu_\mu$ distribution (see text), for 5 years of HELLAZ (left) and 2 years of Borexino running (right). The dotted lines indicate the 95%, 99% and 99.9% exclusion confidence levels.

the value of $\chi^2$ larger, i.e., the minimum of $\chi^2$ is obtained when the coefficient of the $\bar{\nu}_\mu$ component is zero. This is exactly what happens in the case of $\nu_e \leftrightarrow \bar{\nu}_e$ oscillations, at both experiments. Any $\bar{\nu}_e$ component in the flux makes the agreement between the theoretical function and the “data” worse, and again the best value of $\chi^2$ is obtained when the coefficient of the $\bar{\nu}_e$ scattering distribution is zero.

One can see from Fig. 4 that, after 5 years of HELLAZ data, $\bar{\nu}_e$ coming from the Sun can be ruled out at more than 95% CL if $P_{ee} \lesssim 0.2$, while $\nu_e \leftrightarrow \bar{\nu}_\mu$ oscillations are not constrained at all, even for $P_{ee} = 0$. After 5 years of Borexino data, both $\nu_e \leftrightarrow \bar{\nu}_\mu$ and $\nu_e \leftrightarrow \bar{\nu}_e$ oscillations are ruled out at more than 95% CL if $P_{ee} \lesssim 0.1$.

Even if the $\nu_e \leftrightarrow \bar{\nu}$ hypothesis cannot be ruled out at some reasonable CL, one may still be able to place upper limits on the flux of anti-neutrinos coming from the Sun. In the case of $\nu_e \leftrightarrow \bar{\nu}_e$ oscillations, it is straightforward to place upper bounds on the flux of electron-type antineutrinos at

\footnote{We only allow nonnegative coefficients of the distribution functions in the fits, for obvious reasons.}
both HELLAZ and Borexino. The 95% CL upper bounds on the $\bar{\nu}_e$ flux are depicted in Fig. 5. Of course, for $P_{ee} \lesssim 0.2 \ (0.1)$ at HELLAZ (Borexino) the upper bound on the flux is meaningless, since the hypothesis of $\bar{\nu}_e$ is already ruled out at more than 95% CL. Note that the upper bounds on the antineutrino fluxes are normalised by the SSM prediction for the $pp$-neutrino flux $\Phi_{SSM}^{pp} = 5.94 \times 10^{10} \text{ cm}^{-2}\text{s}^{-1}$ for the HELLAZ result, and the SSM prediction for the $^7$Be neutrino flux $\Phi_{SSM}^{^7\text{Be}} = 4.8 \times 10^9 \text{ cm}^{-2}\text{s}^{-1}$ for the Borexino result. For comparison, the 95% CL SuperKamiokande upper bound is 3.5% of the SSM flux, while KamLAND will improve it to 0.1% after one year of running. However, both of them are only for the $^8\text{B}$ neutrinos.

Both the HELLAZ and (especially) the Borexino limits obtained from 5 years of “data” are competitive with the SuperKamiokande limit for lower energy neutrinos.

In the case of $\nu_e \leftrightarrow \bar{\nu}_\mu$ oscillations the situation is more ambiguous, especially at HELLAZ.\footnote{The same is true at Borexino if one assumes the SSM prediction of the total neutrino flux, as will be described later.}

Figure 5: Minimum $\chi^2$ values as a function of the input value of $P_{ee}$, obtained when fitting the “data” with a $\nu_e + \bar{\nu}$ distribution (see text). The fit does not include the SSM prediction for the solar neutrino flux and is for 5 years of HELLAZ (left) and Borexino running (right). The dotted lines indicate the 95%, 99% and 99.9% exclusion confidence levels.
Figure 6: Upper limit on the flux of electron-type antineutrinos after 5 years of HELLAZ (left) and Borexino (right) running. The upper limits are normalised by Standard Solar Model (SSM) prediction for the $pp ({}^{7}\text{Be})$ neutrino flux at HELLAZ (Borexino).

but in some cases (especially for small values of $P_{ee}$) a zero $\bar{\nu}_\mu$ flux is ruled out at more than 95% CL. In such cases, it seems that the reasonable thing to do is to measure the antineutrino flux, not determine upper limits! The only exception to this is the case $P_{ee} = 1$, when the data looks exactly like the SSM prediction, without neutrino oscillations. Indeed, one can not only set upper limits on the antineutrino fluxes, but should also set limits to the $\nu_\mu$ flux. Such limits are presented in Table 2.

It is worthwhile to comment that the information contained in Figs. 5, 6, and in Table 2 is also valid for the case of any unknown source of solar

Table 2: Model-independent 95% CL upper limits on the flux of solar muon-type neutrinos and antineutrinos, when the data after 5 years of HELLAZ/Borexino running is consistent with SSM predictions.

| Experiment | $\Phi_{\nu_\mu}/\Phi_{SSM}$ | $\Phi_{\bar{\nu}_\mu}/\Phi_{SSM}$ |
|------------|----------------------------|---------------------------------|
| HELLAZ     | 1.18                       | 1.44                            |
| Borexino   | 1.62                       | 3.77                            |
antineutrinos of the electron and the muon-types, not only neutrino oscillations. This is because our "data" was analysed assuming that the total flux of solar neutrinos is unknown. We emphasise that $P_{ee}$ is the survival probability of electron neutrinos assuming that they oscillate into active neutrinos, i.e., $\nu_e \leftrightarrow \nu_\mu$ oscillations.

### 4.3 $\nu_e \rightarrow \bar{\nu}$, SSM-dependent Fit

Next, the same analysis can be repeated assuming that the solar neutrino flux is known within theoretical errors. Again, the value of $\chi^2$ is computed and compared with $N_{\text{bins}} - 2 + 1$ (the $-2$ corresponds to the two coefficients that are varied during the minimisation procedure and the $+1$ corresponds to the solar neutrino flux constraint). Fig. 6 depicts the minimised values of $\chi^2$ obtained with 5 years of HELLAZ (left) and Borexino (right) "data." The theoretical uncertainty on the $pp$ ($^7$Be) neutrino flux was taken to be $2\%$ ($20\%$); we inflated the theoretical errors by roughly a factor of two from those in BP98.

A comparison between Figs. 5 and 6 reveals that the exclusion confidence levels increase, sometimes significantly. For example, after 5 years of HELLAZ one can exclude $\nu_e \leftrightarrow \bar{\nu}_e$ oscillations for virtually all values of $P_{ee}$ at more than 99.9\% CL. This is mostly because the $\bar{\nu}_e$ has a total cross section which is significantly larger than $\nu_\mu$, and the oscillation $\nu_e \leftrightarrow \bar{\nu}_e$ cannot account for the large suppression in the event rate in the "data" (due to $\nu_e \leftrightarrow \nu_\mu$). Even the elusive $\nu_e \leftrightarrow \bar{\nu}_\mu$ case can be excluded at Borexino at more than 95\% CL for $P_{ee} \lesssim 0.2$. Note that at HELLAZ the ability to discriminate between $\nu_\mu$ and $\bar{\nu}_\mu$ is still quite limited. It is worthwhile to comment that, unlike in the case of model-independent fits in Sec. 4.2, the minimum value of $\chi^2$ is in general obtained for a nonzero coefficient of the $\bar{\nu}$-$e$ scattering distribution. The reason for this is that, even though the shape of the $\bar{\nu}$-$e$ scattering recoil electron kinetic energy distribution is "more wrong," the contribution to the overall cross section is smaller than the $\nu_e$-$e$ scattering case, and therefore one obtains values of the solar neutrino flux which are closer to the theoretical ones by having a finite $\bar{\nu}$ component, decreasing the value of $\chi^2$.

Again, one may set upper limits on the antineutrino flux. As before, there is some ambiguity with regard to setting upper limits for the $\bar{\nu}_\mu$ flux, because

\[\text{This procedure follows the one used in [17]. The readers are referred to this article for details.}\]
Figure 7: Minimum $\chi^2$ values as a function of the input value of $P_{ee}$, obtained when fitting the “data” with a $\nu_e + \bar{\nu}$ distribution (see text). The fit assumes the SSM prediction for the solar neutrino flux with an (inflated) uncertainty of 2% (20%) for $pp$ ($^7$Be) neutrinos, for 5 years of HELLAZ (left) and Borexino running (right). The dotted lines indicate the 95%, 99% and 99.9% exclusion confidence levels.

for almost all values of $P_{ee} \neq 1$ at both experiments a zero flux is excluded at more than 95% CL. On the other hand, the $\nu_e \leftrightarrow \bar{\nu}_e$ oscillation hypothesis is almost completely ruled out by HELLAZ and the upper limits obtained at Borexino are not much better than the ones depicted in Fig. 6. For this reason, the equivalent of Fig. 6 in the case at hand is not presented.

Table 3 contains the obtained upper limits on the (anti)neutrino fluxes when $P_{ee} = 1$, i.e., when the data agrees with the predictions of the SSM. Unlike the case of a free total flux analysis, the results presented in Table 3 assume that the total neutrino flux of neutrinos to be detected at HELLAZ and Borexino is the one predicted by the SSM, i.e., there is no “room” for other, yet unknown, low-energy solar neutrino sources. For this reason, of course, the bounds obtained are (in some cases) much more stringent.

Finally, as argued before, we emphasise that fixing the value of the solar neutrino flux to its SSM value is a reasonable thing to do, especially for $pp$-neutrinos. In these “exclusion analyses” such a procedure is even more natural, especially if one keeps in mind that a theoretical hypothesis, i.e.,
Table 3: 95% CL Upper limits on the flux of solar muon-type neutrinos and antineutrinos when the data after 5 years of HELLAZ/Borexino running is consistent with SSM predictions, assuming that the total $pp$ ($^7$Be) neutrino flux is the one predicted by the SSM, with 2% (20%) (inflated) uncertainty.

| Experiment | $\Phi_{\nu_\mu}/\Phi_{SSM}$ | $\Phi_{\bar{\nu}_\mu}/\Phi_{SSM}$ | $\Phi_{\nu_e}/\Phi_{SSM}$ |
|------------|-----------------------------|---------------------------------|-----------------|
| HELLAZ     | 0.14                        | 0.14                            | 0.13             |
| Borexino   | 0.66                        | 0.68                            | 0.058            |

$\nu_e \leftrightarrow \nu_\mu$ oscillations plus the SSM computed values for the solar neutrino flux, has been “confirmed experimentally.”

5 Conclusions

In order to unambiguously solve the solar neutrino puzzle, and to establish the oscillations of solar neutrinos (if they occur), clear “smoking gun” signatures are required. Such signatures include a large day-night effect, anomalous seasonal variations, or an obvious distortion of the neutrino energy spectrum. Another unambiguous signature is a discrepancy between the number of charged current and neutral current events at SNO, which can be viewed as an “appearance” experiment of $\nu_{\mu,\tau}$. However, SNO can look for this “appearance” signature only for $^8$B neutrinos with $E_\nu \gtrsim 6.5$ MeV and hence similar studies for lower energy neutrinos such as $^7$Be and $pp$ neutrinos, which are less sensitive to details of the solar model, are important.

We have argued in this paper that a careful analysis of the recoil kinetic energy spectrum at Borexino and HELLAZ serves as another “smoking gun” signature, in the sense that one may be able to infer, independent of the SSM prediction for the solar neutrino flux, the existence of $\nu_{\mu,\tau}$ coming from the Sun. It is worthwhile to emphasise that this is different from distortions in the incoming neutrino energy spectrum. In our case we are describing an “appearance” experiment, while the analysis of the neutrino energy spectrum is a (energy dependent) “disappearance” experiment.

It is important to point out that, in our simple simulations, no background events were included. While this is probably an oversimplification in the case of Borexino, it may well be a good approximation for HELLAZ. Moreover future upgrades of Borexino may reduce the background further according
to recent encouraging progress [25]. One should keep in mind that, even if the background rates are significant, the procedure we described may still be useful if the background can be successfully dealt with (one should not underestimate the ability and creativity of experimental physicists!).

We have also included in the analysis the SSM prediction of the flux of solar neutrinos. While the results obtained in this manner are model-dependent (they are not “smoking gun” signatures of neutrino oscillations), we found them very useful. This is a reasonable thing to do especially for \( pp \)-neutrinos, whose flux is constrained well by the solar luminosity. This additional input makes the measurement of the oscillation probability more precise.

Finally, we have argued that, if the data collected at Borexino and HELLAZ is consistent with \( \nu_e \leftrightarrow \nu_{\mu,\tau} \) oscillations, one can try to exclude other neutrino oscillation modes (\( \nu_e \leftrightarrow \nu_s \) and \( \nu_e \leftrightarrow \bar{\nu}_{e,\mu,\tau} \)) using the same procedure or, at least, to set upper limits on the flux of solar antineutrinos. Again we considered the possibility of constraining the solar neutrino flux to the SSM predicted value. The main result we obtained is that \( \nu_e \leftrightarrow \bar{\nu}_e \) oscillations can, in general, be excluded, while the \( \nu_e \leftrightarrow \bar{\nu}_{\mu,\tau} \) case is much more elusive. Nonetheless, Borexino should be able to exclude \( \nu_e \leftrightarrow \bar{\nu}_{\mu,\tau} \) oscillations if the SMA MSW solution to the solar neutrino puzzle happens to be the correct one.

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