Reflectivity simulation by using transfer matrix method

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Abstract. Efficiency of optoelectronic devices can be enhanced by using Distributed Bragg Reflector (DBR). Distributed Bragg Reflector (DBR) is a multipair layered structure of high and low refractive indices. High reflectivity of DBR is due to constructive interference of reflected light from interfacial layers of DBR. In the present paper, reflectivity of DBR composed of II-VI compound semiconductor is simulated using Transfer Matrix Method (TMM). In this paper, reflectivity of 4, 8, 12, 16 and 20 periods for the incidence angle of 45° is simulated. Also, the effect of interchange of constituent layers in DBR stack, on reflectivity is simulated.

1. Introduction

Distributed Bragg Reflector (DBR) is a periodic structure of repeating layers. In case of DBR, the bandwidth of reflection is determined by contrast of refractive index (RI). Lower the difference in RI of constituent layers, narrower is the stop band [1]. Stop band is the band of wavelength which restrict incident light to pass into DBR stack.

It is expected that the difference in RI should be considerable while constructing the DBR structure. Due to difference in RI of the constituent layers, DBR act as highly reflecting mirror. It is reported that the efficiency of optoelectronic devices can be enhanced by using DBR mirror. Therefore, currently DBR structures are popularly used in component of various optoelectronic devices such as vertical-cavity surface emitting lasers (VCSELs), Fabray Perot modulators and resonant cavity light emitting diodes (RCLEDs) [2-6].

Reflectivity is one of the essential parameters which characterize the DBR. The reflectivity of DBR depends on the parameters such as periods of structure (N), difference in refractive indices (Δn), angle of incidence (θi) etc. Main aim of this paper is to simulate various parameters which effects on DBR reflectivity. Transfer Matrix Method (TMM) is one of the most efficient and widely used method for the analysis of DBR. In the present paper the reflectivity of DBR is simulated by TMM.

When polychromatic light is made incident on DBR, different wavelengths get reflected from the interface of DBR layers and interfere constructively. As a result, reflectivity of DBR enhances. It is
reported that, DBR reflectivity is very high compared to the metallic mirrors [6]. Interference of light which gives reflection for a single period is shown in figure 1.

Figure. 1 Reflectance and transmittance in single period system

For high reflectivity around a central wavelength $\lambda_{\text{Bragg}}$, the thickness of the layer should be

$$t_i = \frac{\lambda_{\text{Bragg}}}{4n_i}.$$  \hspace{1cm} (1)

Where $n_i$ is the corresponding RI.

The reflectivity (R) of DBR at Bragg wavelength ($\lambda_{\text{Bragg}}$) with N quarter-wave pairs is given by equation 2 [7,8]

$$R_{\text{DBR}} = \left[ \frac{1 - \left( \frac{n_H}{n_L} \right)^{2N}}{1 + \left( \frac{n_H}{n_L} \right)^{2N}} \right]^{2N}.$$  \hspace{1cm} (2)

Where $n_s, n_H, n_L, n_0, N$ are substrate RI, high RI, low RI, RI of incident medium and periods of DBR respectively.

2. **Simulation**

2.1. **Transfer Matrix Method (TMM)**

Simulation is one of the useful techniques as often employed to characterize DBR within short period of time and at low cost. The TMM is the simplest method to simulate the effect of various parameters of DBR reflectivity.

For the layers in DBR having RI ($n_i$) and thickness ($t_i$) as shown in figure 2 the characteristic matrix is given by

$$M_i = \begin{bmatrix}
\cos \frac{2\pi n_i t_i}{\lambda} & \frac{i \sin \left( \frac{2\pi n_i t_i}{\lambda} \right)}{n_i} \\
in_i \sin \frac{2\pi n_i t_i}{\lambda} & \cos \frac{2\pi n_i t_i}{\lambda}
\end{bmatrix}.$$  \hspace{1cm} (3)

This equation (3) is equivalent to the impedance matrix in the transmission line theory [9].

For multilayer of N periods, characteristic matrix M becomes
\[ M = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \]  

Therefore, reflectance coefficient is calculated by [10-13]

\[ \text{Reflectance coefficient}(r) = \begin{bmatrix} M_{21} \\ M_{11} \end{bmatrix} \]  

And hence the reflectivity \( R = |r|^2 \)  

In the present investigation, due to relatively large difference in RI (\( \Delta n = 0.4 \)) of ZnSe (2.6123) and ZnTe (3.063), these materials are the good combination for high reflectivity DBR. Therefore, in the present paper reflectivity of ZnTe/ZnSe based DBR is simulated. For Simulation MATLAB R2013 version is used.

2.1.1. Effect of period on reflectivity

Reflectivity of DBR is one of the factors which influence the DBR efficiency. Light incident on DBR basically comes through the surrounding air medium. Hence the RI of the surrounding is considered to be 1.

In order to obtain high reflectivity, the DBR structure is simulated by alternately considering multilayer stack composed of high RI (\( n_H \)) and a low RI (\( n_L \)) materials at angle of incidence 45°.

The thickness of ZnTe/ZnSe stack satisfying the condition of 1/4 of bragg wavelength (\( \lambda_{\text{Bragg}} = 570 \text{ nm} \)) is selected. Bragg wavelength is the wavelength at which maximum amount of incident light get reflected. The wavelength of the incident light is simulated at each wavelength band at intervals of 1 nm from \( \lambda = 200 \text{ nm} \) to 800 nm. From this simulation we obtained the Reflectance (%) spectrum as a function of wavelength.

Figure 3 shows the simulation result. It shows that as the number of periods are increased, reflectivity increases.

Our simulation result indicates that the reflectivity increases as the number of periods increase. However, from nature of stop band for 4 and 8 periods it is clear that small amount of light gets transmitted for period of 4 and 8 as compared to 12,16 and 20 periods. Centre of stop band theoretically should be 570 nm [14]. However, our simulation shows centre of stop band to be at 554 nm. Reflectivity obtained from simulation for 4, 8,12,16,20 periods is listed in table1.
TABLE 1  Periods of DBR and reflectivity

| Sr. No. | Period | Reflectivity |
|---------|--------|--------------|
| 1       | 4      | 57.6%        |
| 2       | 8      | 86.8%        |
| 3       | 12     | 96.44%       |
| 4       | 16     | 99.07%       |
| 5       | 20     | 99.76%       |

2.1.2. Effect of order of layer on reflectivity

In the study of DBR the effect of order of layers in period is also important. This decides the ratio of RI [15-17]. In the present simulation work we have kept the RI of substrate and surrounding medium fixed. Figure 4 and 5 shows reflectivity curves for the DBR with different order of the layers. It is found that the reflectivity is 99.76% for ZnTe/ZnSe order, with RI ratio \( \frac{n_H}{n_L} = 1.1713 \) and the reflectivity found 99.16% for the RI ratio \( \frac{n_L}{n_H} = 0.8528 \) for ZnSe/ZnTe order.

**Figure 4** Reflectivity vs wavelength for 20 periods with angle of incidence 45° for ZnTe/ZnSe order

**Figure 5** Reflectivity vs. wavelength for 20 periods with angle of incidence 45° for ZnSe/ZnTe order
This result indicates that the first layer on the substrate should be of low RI and the top layer of the DBR stack should be of high RI to achieve maximum reflectivity. If the ratio of RI is high, the reflectivity obtained will be higher. Therefore, the RI ratio of layers in the stack has a significant effect on the peak reflectivity.

3. Conclusion

In the present paper reflectivity of ZnTe/ZnSe Distributed Bragg Reflector (DBR) is simulated by Transfer Matrix Method (TMM). It has been observed that as the number of periods increase the reflectivity also gets increased. It is observed that at 4 and 8 periods the light is transmitted through the stack but after 12 periods the transmission of light is not observed. The reflectivity gets saturated after 20 periods. The centre of stop band is found to be shifted to 554 nm. This shift may be due to approximation done in optical admittance of DBR while constructing TMM matrix. The order of the layers affects the reflectivity of DBR. Our simulation results suggest that ZnTe/ZnSe are the suitable materials for DBR.

4. References

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