Influence of zonal flows on unstable drift modes in ETG turbulence

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Abstract
The linear instability of the electron temperature gradient (ETG) driven modes in the presence of zonal flows is investigated. Random and deterministic cos-like profiles of the zonal flow are considered. It is shown that the presence of shearing by zonal flows reduces the instability growth rates. The sufficiently strong random shearing can fully stabilize the linear instability of ETG drift modes.

1. Introduction
The electron temperature gradient (ETG) driven mode [1–3] is often considered as a possible candidate for the explanation of electron thermal transport through internal transport barriers when ion temperature gradient (ITG) turbulent fluctuations are suppressed by $E \times B$ shear flow. At the same time, small scale ETG fluctuations have the typical spatial scale of the order of electron gyroradius $\rho_e$ and are less susceptible to quenching by shearing [4–6]. It is widely thought that drift-wave-type turbulence can excite zonal flows which are associated with azimuthally symmetric band-like shear flows that depend only on the radial coordinate [7, 8]. Zonal flows play a crucial role in regulating drift-wave turbulence and transport in tokamaks. It is now quite clear that zonal flows are generated by modulational instability of drift waves [9–13]. For ETG driven turbulence, the excitation of zonal flow was considered in [14] (the case of broad turbulent spectrum) and [15] (the four-wave coupling scheme).

Effects of $E \times B$ velocity shear on turbulence and transport in magnetic confinement devices have been extensively studied [16]. It is well known that the presence of shear flow gives rise not only to instability of the sheared layer (Kelvin–Helmholtz instability) but also to stabilization of other instabilities (ITG driven modes, resistive interchange modes etc) [7, 17–19, 20]. Up to now, the stabilizing effect on the linear instabilities has been considered mostly for the case of mean smooth flows. Effects of zonal flow suppression on drift waves in ITG turbulence based on a quasilinear theory have been studied in a recent paper [21]. Note that zonal flow shearing differs from that for mean flow shearing on account of the complexity of the flow pattern.
contrast to smooth, static mean flows, the zonal flow patterns can be expected to have finite correlation time and complex, possibly random, spatial structure [7, 22, 23].

In this work we consider the influence of zonal flow with a random velocity profile on the linear stability of ETG drift modes. We derive a linear differential equation including the reaction of the zonal flow on ETG modes. The resulting stochastic model can be analyzed in the Bourrett approximation. We show that the presence of random shearing by zonal flows strongly affects the linear stability of ETG modes and has a stabilizing effect. The shearing reduces the linear growth rates. If the root mean square amplitude of zonal flow exceeds some critical value, the linear instability of ETG modes is suppressed for all poloidal wave numbers \( k_y \).

2. Basic equations

Assuming a slab two-dimensional geometry, charge quasineutrality and the adiabatic ion response, we consider the following simplified model describing curvature driven ETG turbulence and including viscosity and thermal diffusivity [5]:

\[
\left( \frac{\partial}{\partial t} + v_E \cdot \nabla \right) \left( \phi - \Delta_\perp \phi \right) + \frac{\partial}{\partial y} \left( \phi + p \right) + \nu \Delta^2 \phi = 0, \tag{1}
\]

\[
\left( \frac{\partial}{\partial t} + v_E \cdot \nabla \right) p - r \frac{\partial}{\partial y} \phi - \chi \Delta p = 0, \tag{2}
\]

where \( v_E = [\hat{z} \times \nabla \phi] \) is the \( E \times B \) drift velocity, \( \phi \) and \( P \) are the normalized electrostatic potential and plasma pressure, respectively, and \( \{A, B\} = \partial_x A \partial_y B - \partial_y A \partial_x B \) is the Jakobian. The variables in equations (1) and (2) have been rescaled as follows:

\[
r = \frac{\epsilon_B \epsilon_{si}}{\epsilon_{ei}^2},
\]

\[
\phi = \frac{1}{\epsilon_{ei} T_i}, \quad \quad P = \frac{\epsilon_B P}{\epsilon_{ei}^2 P_{i0}},
\]

\[
x = \frac{x'}{\rho_c \sqrt{\tau}}, \quad \quad y = \frac{y'}{\rho_c \sqrt{\tau}}, \quad \quad t = \epsilon_{si} \omega_B t',
\]

\( x', y' \) and \( t' \) being the original physical coordinates (with \( x' \) the poloidal and \( y' \) the radial coordinate),

\[
\epsilon_{si} = \frac{\rho_c \sqrt{\tau}}{L_{n}}, \quad \quad \epsilon_B = \frac{\rho_c \sqrt{\tau}}{L_{B}}, \quad \quad \epsilon_{se} = \frac{\rho_c \sqrt{\tau}}{L_{P}},
\]

where \( L_n, L_B \) and \( L_P \) are the background gradient scales for the density, magnetic field and pressure, respectively, \( \rho_c \) is the electron gyroradius, and \( \tau = T_i / T_e \). The effect of zonal flow can be included by assuming the \( E \times B \) drift velocity of

\[
v_E = v(x, t) \hat{y} + [\hat{z} \times \nabla \phi], \tag{3}
\]

where the first term accounts for zonal flow. Consideration of the interaction of drift waves and zonal flow in the framework of a fully self-consistent nonlinear model would yield the self-regulating system (in the spirit of phenomenological models such as prey + predator for drift turbulence + zonal flow [7, 26]), but here we are interested in the influence of the zonal flow on the linear instability of ETG drift modes. We neglect the back reaction of the drift
modes on the zonal flow and assume that the profile \( v(x, t) \) is fixed. Neglecting nonlinear terms in equations (1) and (2), taking into account equation (3) and representing

\[
\varphi(x, y, t) = \Phi(x) \exp(ik_y y - i\omega t),
\]

\[
p(x, y, t) = P(x) \exp(ik_y y - i\omega t),
\]

one can obtain

\[
(-i\omega + ik_y v) \left( 1 + k_y^2 \frac{d^2}{dx^2} \right) \Phi + ik_y (\Phi + P) + v \left( k_y^4 + \frac{d^4}{dx^4} - 2k_y^2 \frac{d^2}{dx^2} \right) \Phi = 0,
\]

(4)

\[
(-i\omega + ik_y v) P - ik_y r \Phi + \chi \left( -k_y^2 + \frac{d^2}{dx^2} \right) P = 0.
\]

(5)

In the inviscid limit and the absence of zonal flow, after taking \( \Phi(x), P(x) \sim \exp(ik_x x) \), equations (4) and (5) give the dispersion relation for ETG modes

\[
\omega_{1,2} = \frac{k_y}{2(k^2 + 1)} \left( 1 \pm \sqrt{1 - 4r(k^2 + 1)} \right),
\]

(6)

where \( k^2 = k_x^2 + k_y^2 \), and the plus sign describes the drift waves dispersion, while the minus sign corresponds to the dispersion of the convective cells. Equation (6) predicts instability with the growth rate

\[
\gamma = \frac{k_y}{2(k^2 + 1)} \sqrt{4r(k^2 + 1) - 1},
\]

(7)

if \( 4r(k^2 + 1) \geq 1 \).

3. ETG drift modes in the presence of zonal flow

We consider the case when the correlation time of zonal flow is large enough and the zonal flow can be regarded as steady (no dependence on time) [22]. We assume that the profile of zonal flow \( v(x) \) is the random function and \( v(x) \) is a zero mean, \( \langle v(x) \rangle = 0 \), real homogeneous Gaussian process with correlation function

\[
\langle v(x)v(x') \rangle = D(x - x'),
\]

(8)

where \( \langle \cdots \rangle \) means statistical averaging. It is assumed that the intensity of the noise is small, \( D(x) \ll 1 \). When neglecting the \( v^2 \) term, and in the inviscid limit, equations (4) and (5) reduce to the equation

\[
\omega^2 \left( \frac{d^2}{dx^2} - k_y^2 - 1 \right) \Phi + \omega \left( k_y - 2k_y v \left( \frac{d^2}{dx^2} - k_y^2 - 1 \right) \right) \Phi - k_y^2 (v + r) \Phi = 0,
\]

(9)

which after the Fourier transforming (\( d/dx \rightarrow -iq \)) can be rewritten as

\[
G_0^{-1}(p) \Phi(p) + \int k_y [k_y - 2\omega(q^2 + k_y^2 + 1)]v(q) \Phi(\omega, q, k_y) \delta(q - q_1 - q_2) dq_1 dq_2 = 0,
\]

(10)

where \( p \equiv (\omega, q, k_y) \) and

\[
G_0^{-1}(p) = \omega^2 (q^2 + k_y^2 + 1) - \omega k_y + k_y^2 r.
\]

(11)

To avoid the cumbersome expressions, we write equation (10) in the symbolic form

\[
G_0^{-1}(p) \Phi(p) + \gamma(p, p_1, p_2) v(p_1) \Phi(p_2) = 0,
\]

(12)
where
\[ \gamma(p, p_1, p_2) = \kappa(p_1, p_2)\delta(p - p_1 - p_2), \]
and integration over repeated indices is assumed, as usual. Introducing a source \( \eta(p) \) in the right-hand side of equation (12) and taking the functional derivative \( \delta/\delta \eta(p') \), we have
\[ G_0^{-1}(p)G(p, p') + \gamma(p, p_1, p_2)v(p_1)G(p_2, p') = \delta(p - p'), \]
where \( G(p, p') = \delta\Phi(p)/\delta \eta(p') \) is the Green function, and we have taken into account that \( \delta \eta(p)/\delta \eta(p') = \delta(p - p') \). The function \( G_0(p, p') \) is a free (i.e. in the absence of the noise \( v \)) Green function. Representing the Green function as a sum of the average and fluctuating parts
\[ G(p, p') = \langle G(p, p') \rangle + \tilde{G}(p, p'), \]
substituting it into equation (14) and averaging, one can obtain
\[ G_0^{-1}(p)\langle G(p, p') \rangle + \gamma(p, p_1, p_2)v(p_1)\langle G(p_2, p') \rangle = \delta(p - p'). \]
Subtracting equation (16) from equation (14), we get
\[ G_0^{-1}(p)\tilde{G}(p, p') + \langle v(p_1, p_2)v(p_1)\tilde{G}(p_2, p') \rangle = 0. \]
In the Bourrett approximation [24], which is justified when the intensity of the noise is small enough, we can neglect the term \( v\tilde{G} - \langle v\tilde{G} \rangle \) and get for the fluctuating part of the Green function
\[ \tilde{G}(p, p') = -G_0(p)\gamma(p, p_1, p_2)v(p_1)\langle G(p_2, p') \rangle. \]
Inserting this expression into equation (16), we obtain
\[ G_0^{-1}(p)\langle G(p, p') \rangle - \gamma(p, p_1, p_2)\gamma(p_1', p_2')G_0(p_2)v(p_1)\langle G(p_2', p') \rangle = \delta(p - p'). \]
As follows from equation (8), in the wave number domain the correlator has the form \( \langle v(p)v(p') \rangle = I(p)\delta(p + p') \). Due to homogeneity of the random process, the Green function in equation (19) has the structure \( \langle G(p, p') \rangle = G(p)\delta(p - p') \). Making use of this and equation (13), one can perform some integrations in equation (19) and finally we get
\[ G(p) = \frac{1}{G_0^{-1}(p) - \kappa(p - p_1, p_1)\kappa(p_1 - p, p)G_0(p_1)I(p - p_1)} \]
The poles of the Green function \( G(p) \) determine the spectrum of elementary excitations and the corresponding dispersion relation is
\[ G_0^{-1}(p) - \kappa(p - p_1, p_1)\kappa(p_1 - p, p)G_0(p_1)I(p - p_1) = 0. \]
In the absence of zonal flow we have \( G_0^{-1}(p) = 0 \) and recover the previous result (6) with \( k_z = q \). In what follows we consider the case when the profile of the zonal flow is described by the random function \( v(x) \) which has the form
\[ v(x) = v_0 \cos(q_0 x + \vartheta), \]
where the random amplitude \( v_0 \) is a zero mean, a normally distributed value with variance \( \sigma^2 \), and the random phase \( \vartheta \) is uniformly distributed between 0 and \( 2\pi \). The correlation function (8) of such a process is \( D(x) = (\sigma^2/2) \cos(q_0 x) \) or, in the wave number domain,
\[ I(q) = \frac{\sigma^2}{4} [\delta(q - q_0) + \delta(q + q_0)]. \]
In this case the noise has an infinite correlation length and is concentrated at the wave number $q_0$ so that the characteristic scale length of the zonal flow is of the order $\sim q_0^{-1}$. As follows from equation (7), in the absence of zonal flow the maximum growth rate for the fixed $k_y$ is achieved at $k_y = 0$. Thus, to simplify calculations, we will consider the influence of zonal flow on the most unstable modes and put $q = 0$. Then, substituting equation (23) into equation (21) one can obtain the dispersion relation

$$c_1 \omega^4 - c_2 \omega^3 + c_3 \omega^2 - c_4 \omega + c_5 = 0,$$

Figures 1(a) shows the growth rate $\gamma$ of the ETG driven mode as a function of the root mean square zonal flow amplitude $\sigma$ for different values of the poloidal wave number $k_y$. The characteristic wave number of zonal flow used in the calculations is $q_0 = 0$. The parameter $r$ has been fixed at $r = 0.25$ so that, as follows from equation (7), in the absence of random shearing there is a linear instability for all $k_y \neq 0$. It is seen that the growth rate decreases with increasing $\sigma$ and vanishes above some value of $\sigma$ which depends on $k_y$. Thus, zonal flow shearing stabilizes the instability of ETG modes. In figure 2(b) we plot the dependence of the growth rate on the poloidal wave number $k_y$ for different values of the root mean square zonal flow amplitude $\sigma$. In the absence of zonal flow ($\sigma = 0$) the growth rate $\gamma$ increases with increasing $k_y$ and saturates at the level $\gamma_{\text{max}} = \sqrt{r}$. The presence of shearing ($\sigma \neq 0$) changes the situation drastically. The reduced growth rate initially increases as a function of $k_y$ from some $k_{y,1}$ and then decreases, becoming zero at some $k_{y,2}$. For not too large $\sigma$, the linear instability is restricted to the region $k_{y,1} < k_y < k_{y,2}$. The lower $k_{y,1}$ and upper $k_{y,2}$ boundary
values increase and decrease, respectively, as the root mean square zonal flow amplitude $\sigma$ increases and above some critical value $\sigma_{cr} \sim 0.36$ (for $r = 0.25$) the instability of ETG modes is suppressed for all poloidal wave numbers $k_y$. An estimate for the critical value $\sigma_{cr}$ can be obtained from equation (24). The growth rate remains zero as $k_y \to \infty$ (the most dangerous case) if $\sigma = \sigma_{cr}$. From equation (24) one can see that $\omega$ scales as $k_y^{-1}$ as $k_y \to \infty$. Then, we can get the estimate $\sigma_{cr} = \sqrt{2r}$. This theoretical prediction for the dependence of the critical value of the root mean square zonal flow amplitude on $r$ is in very good agreement with numerical results.

Note that the recent studies [25, 26] have indicated that the statistics in the strongly nonlinear regime can be far from Gaussian. This is connected to the formation of coherent structures and intermittency phenomenon. The justification for Gaussian statistics in our case can be based on the assumption that the mean square amplitude of zonal flow is small enough (and, in particular, the Bourrett approximation is valid) so that the strongly nonlinear effects and intermittency can be neglected.

Next, we consider the case when the zonal flow profile is deterministic and has the form $v(x) = v_0 \cos(qx)$. In addition, we include the effects of viscosity and thermal diffusivity. Then, equations (4) and (5) can be rewritten as an eigenvalue problem:

\begin{equation}
\begin{pmatrix}
\hat{A}^{-1} \hat{B} & k_y \hat{A}^{-1} \\
-rk_y \hat{I} & k_y \hat{C}
\end{pmatrix}
\begin{pmatrix}
\Phi \\
p
\end{pmatrix}
= \omega
\begin{pmatrix}
\Phi \\
p
\end{pmatrix},
\end{equation}

(30)

where

\begin{align*}
\hat{A} &= 1 + k_y^2 - \frac{d^2}{dx^2}, \\
\hat{B} &= 1 + v_0 \cos(qx) \hat{A} - \frac{v}{k_y} \hat{D}, \\
\hat{D} &= k_y^4 - 2k_y^2 \frac{d^2}{dx^2} + \frac{d^4}{dx^4}, \\
\hat{C} &= v_0 \cos(qx) - i\chi k_y + \frac{\chi}{k_y} \frac{d^2}{dx^2}.
\end{align*}

Employing a finite differencing approximation, we numerically solved the eigenvalue problem (30). The dissipative coefficients have been fixed at $\nu = \chi = 0.01$. The instability growth rate is plotted in figure 2 for two values of the zonal flow amplitude $v_0$ and different values of $q$ (for $r = 0.5$). It is seen that the presence of zonal flow reduces the growth rate though the stabilizing effect manifests itself not so sharply as in the case of random shearing.
4. Conclusion

In conclusion, we have investigated the influence of zonal flows on the linear instability of ETG driven modes. Random and deterministic cos-like profiles of the zonal flow have been considered. For the random profile of zonal flow, we have obtained in the Bourrett approximation the dispersion relation for ETG modes in the presence of shearing. We have shown that the presence of random shearing caused by zonal flow has a strong stabilizing effect on the ETG driven mode destabilized by the temperature and pressure gradients. The shear of the flow reduces the linear instability growth rates. If the mean square amplitude of zonal flow exceeds some critical value, the linear instability of ETG modes is suppressed for all poloidal wave numbers $k_y$.

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