Flavor and CP violating physics from new supersymmetric thresholds

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Treating the MSSM as an effective theory, we study the implications of having dimension five operators in the superpotential for flavor and CP-violating processes, exploiting the linear decoupling of observable effects with respect to the new threshold scale $\Lambda$. We show that the assumption of weak scale supersymmetry, when combined with the stringent limits on electric dipole moments and lepton flavor-violating processes, provides sensitivity to $\Lambda$ as high as $10^{-7}$ to $10^{-9}$ GeV, while the next generation of experiments could directly probe the high-energy scales suggested by neutrino physics.

Weak-scale supersymmetry (SUSY) is a theoretical framework that helps to soften the so-called gauge hierarchy problem by removing the power-like ultraviolet sensitivity of the dimensionful parameters in the Higgs potential. It also has other advantages, notably an improvement in gauge coupling unification and a natural dark matter candidate, which have made it the standard paradigm for physics beyond the Standard Model (SM). However, the simplest scenario -- the minimal supersymmetric standard model (MSSM) -- suffers from a number of well-known tuning problems, due in part to the large array of possible parameters responsible for soft SUSY breaking \cite{1}, and consequently the possibility of catastrophically large flavor and CP violating amplitudes. The absence of new flavor structures and order-one sources of CP-violation in the soft breaking sector, as evidenced respectively by the perfect accord of the observed $\kappa$ and $B$ meson mixing and decay with the predictions of the SM \cite{2} and the null results of electric dipole moment (EDM) searches \cite{3–5}, motivates continuing work on the specifics of SUSY breaking.

In the present Letter we will instead ask, given a solution to the flavor and CP problems in the soft-breaking sector, what sensitivity do we have to new high-scale sources of flavor and CP-violation? Such effects would arise through SUSY-preserving higher-dimensional operators generated at a new threshold $\Lambda \gg M_W$. Such thresholds are indeed expected due to various completions of the MSSM, e.g. via mechanisms for SUSY breaking and mediation, the breaking of flavor symmetries, and moreover via the physics generating neutrino masses and mixings. Intermediate scales are also suggested by the axion solution to the strong CP problem, SUSY leptogenesis scenarios, and more entertainingly as a lowered GUT/string scale arising from large compactification radii of extra dimensions. In contrast to nonuniversal or complex soft-breaking terms, the flavor and CP-violating observables induced by such operators will scale as $(\Lambda m_{\text{susy}})^{-1}$, and thus the constraints on nonminimal flavor or CP translate directly into sensitivity to $\Lambda$ far above the scale of the superpartner masses, $m_{\text{susy}}$.

At dimension five there are several well-known $R$-parity conserving operators associated with neutrino masses, $H_u L H_u L$, and baryon number violation, $U U D E$, $Q Q Q L$ \cite{7}. The constraints on proton decay put severe restrictions on the size of baryon-number violating operators, $\Lambda^5 > 10^{24}$ GeV, where $1/\Lambda$ is the overall normalization scale for these operators. The “super-seesaw” operator $H_u L H_u L$ is a welcome addition to the MSSM superpotential, as it generates Majorana masses and mixing for neutrinos, which imply $\Lambda^5 \sim (10^{14} – 10^{16})$ GeV. Note that in the seesaw scenario, the actual scale of right-handed neutrinos, $M_R$, is lower than $\Lambda^5$, since $\Lambda^5 = Y_{\nu}^2 M_R^{-1}$ with a small $Y_{\nu}$, as is also favored by SUSY leptogenesis.

In what follows, we analyze in detail the remaining operators allowed in the $R$-parity conserving MSSM at dimension five level \cite{7}. We write the superpotential as

$$W = W_{\text{MSSM}} + \frac{y_h}{\Lambda^5} H_d H_u H_u + \frac{Y_{qq}}{\Lambda^5} (U_i t^A Q_j) E_k L_l$$

$$+ \frac{Y_{ijkl}}{\Lambda^5} (U_i t^A Q_j) (D_k Q_l) + \frac{Y_{ijkl}}{\Lambda^5} (U_i t^A Q_j) (D_k t^A Q_l),$$

(1)

where $y_h$, $Y_{qq}$, $Y_{iqkl}$ and $Y_{ijkl}$ are dimensionless coefficients, the latter three being tensors in flavor space. The parentheses in (1) denote a contraction of colour indices. Note that since we will only consider supersymmetric thresholds, the superfield equations of motion can be used to eliminate all dimension five corrections to the Kähler potential, e.g. $K^{(5)} = c_{q} QU H_d^A$, absorbing them in $W^{(5)}$ and the Yukawa terms, and slightly modifying the soft-breaking sector. A renormalizable realization of (1) can easily be obtained, e.g. the MSSM extended by a singlet $N$ (the NMSSM) or an extra pair of heavy Higgses.

The full Lagrangian descending from (1) is rather cumbersome, and we will focus our attention here on those dimension five operators which are of potential phenomenological interest, specifically those that involve two SM fermions and two sfermions. We then proceed to integrate out the sfermions to obtain operators composed from the SM fields (or more precisely those of a type
II two-Higgs doublet model). We will impose the requirements of flavor triviality and CP conservation in the soft-breaking sector. Thus all dimension ≤ 4 coefficients in the Higgs potential, trilinear terms $M_{ii}$, gaugino masses $M_i$, and the $\mu$-parameter, will be taken real. We will also make the simplifying assumption of universal sfermion masses, denoted $m_{\text{susy}}$, $m_d$, which we will take, along with $\mu$, $M_i$, to be somewhat larger than $M_W$. Deferring the full details [8], we quote the relevant results below:

**Correction to the SM fermion masses:** The SM operators of lowest dimension that are of phenomenological interest are the fermion mass operators. From the diagrams of Fig. 1a, we obtain the following corrections:

\[
\delta(M_e)_{ij} = Y_{klij}^q (M_{u}^{(0)})^*_{kl} \left[ \frac{2 \ln(\Lambda_{q_e}/m_{\text{susy}}) Y_{ij}^{q_e}}{\Lambda_{q_e}^2 m_{\text{susy}}} (A_u^* + \mu \cot \beta) \right]
\]

\[
\delta(M_d)_{ij} = K_{klij}^{q} (M_{u}^{(0)})^*_{kl} \left[ \frac{2 \ln(\Lambda_{q_d}/m_{\text{susy}}) Y_{ij}^{q_d}}{4 \Lambda_{q_d}^2 m_{\text{susy}}} (A_u^* + \mu \cot \beta) \right],
\]

with a similar correction to $M_u$. The notation implies summation over the repeated flavor indices, and we have defined the combination $K^{q} = Y^{q} - 2 Y^{\bar{q}/q} / 3$. $M_{e,d,u}^{(0)}$ denote unperturbed mass matrices arising from dimension four terms in the superpotential. Note that the corrections proportional to $A_u$ directly break SUSY, while those proportional to $\mu$ arise from corrections to the Kähler potential.

**Dipole operators:** At dimension five, dipole operators first arise at two-loop order, as in Fig. 1b. In the charged lepton sector they result in

\[
\mathcal{L}_e = \frac{A_u^* + \mu \cot \beta \alpha_e}{\Lambda_e/m_{\text{susy}}} \left[ Y_{klij}^q \frac{\ln(\Lambda_{q_e}/m_{\text{susy}})}{12 \pi^2 \Lambda_{q_e}} (M_{u}^{(0)})_{kl} (F_{ij}^{e} P_L E_j + (h.c.),
\]

where we treated LR squark mixing as a mass insertion, and used $P_L = \frac{1 - \gamma_5}{2}$ and $F^{\mu}_{ij} = F^{\mu}_{ij} - \sigma^{\mu\nu}$. In the quark sector the corresponding results are more cumbersome due to a large number of possible diagrams.

Jumping an additional dimension, we now consider dimension six four-fermion operators generated by various terms in (1). Two representative diagrams are shown in Fig. 1c,d.

**Semileptonic operators:** Integrating out gauginos and sfermions as in Fig. 1c, we find the following semileptonic operators, sourced by $Q_{LLE}$,

\[
\mathcal{L}_{qq} = \frac{1}{2 \Lambda_{q_e} m_{\text{susy}} 16 \pi^2} \left[ \frac{8 Y_{ij}^{q_e} \bar{U}_i Q_j \bar{U}_k L_l}{3} + (h.c.),
\]

where $m_{\text{susy}}^{-1}$ denotes a combination of superpartner masses folded with a loop function $F$: $m_{\text{susy}}^{-1} = M_{3} m_{\text{susy}}^{-2} F(M_{3}^2 / m_{\text{susy}}^2)$, and $F(a) = 2 - \ln(a) / 2 + \ln(2)$ with $F(1) = 1$ (see [10] for the unequal mass case). In (4) we have retained only the gluino-squark contribution, which is expected to dominate unless there are additional hierarchies between the masses of sleptons and squarks.

**Four-quark operators:** Integrating out gluinos and squarks as in Fig. 1c, we arrive at the following four-quark effective operators:

\[
\mathcal{L}_{qq} = \frac{1}{2 \Lambda_{q_e} m_{\text{susy}} 16 \pi^2} \left[ \frac{8 Y_{ij}^{q_e} \bar{U}_i Q_j \bar{U}_k L_l}{3} + (h.c.),
\]

where the summation over flavor is carried out exactly as in (1). The largest down-type $\Delta F = 2$ operator arises instead from Fig. 1d,

\[
\mathcal{L}_{dd} = \frac{1}{2 \Lambda_{q_d} m_{\text{susy}} 16 \pi^2} \left[ \frac{8 Y_{ij}^{q_d} \bar{D}_i D_j \bar{D}_k L_l}{3} + (h.c.),
\]

which inevitably contains additional Yukawa suppression originating from the Higgsino-fermion-sfermion vertices. Here $m_{\text{susy}}$ is a combination of SUSY masses as in (4) and (5) with $M_3$ replaced by $\mu$.

We will now turn to the phenomenological consequences and the sensitivity to $\Lambda^e$ and $\Lambda^{q_d}$ in various experimental channels. Of course, one of the most important issues is the flavour structure of the new couplings constants, $Y^{q_e}$, $Y^{q_d}$, and $Y^{q_{u,d}}$. We will assume that these coefficients are of order one, and do not factorize: $Y^{q_e} \neq Y^{q_d}$. With this assumption, we should first determine the natural scale for $\Lambda$ such that the corrections to SM fermion masses do not exceed their measured values.

**Particle masses and $\theta$ term:** Taking $(M_{u,A} A_{u})_{kl} = (M_{u,A} A_{u})_{333} \sim m_t A_t \sim 175 \text{ GeV} \times 300 \text{ GeV}$ in (2), and assuming a maximal $Y_{3311}^{q_e} \sim 0(1)$, we arrive at the esti-
mate,
\[ \Delta m_c \sim \frac{3m_A Y_{ee}^{\text{SM}} \ln(\Lambda^{\text{qg}}/m_{\text{susy}})}{8\pi^2 \Lambda^{\text{qg}}} \sim 1\text{MeV} \frac{10^7\text{GeV}}{\Lambda^{\text{qg}}}. \]  
Eq. (7) clearly implies that the natural scale for new physics encoded in the semileptonic operators in the superpotential is \( \Lambda^{\text{qg}} \sim 10^7 \text{ GeV} \), while the corresponding scale in the quark sector is slightly lower.

A strikingly high naturalness scale emerges from consideration of the effective shift of \( \theta \) due to the mass corrections (2). Assuming uncorrelated phases between \( Y_{\nuq} \) and the eigenvalues of \( Y_u \) and \( Y_d \), we find,
\[ \Delta \theta \sim \frac{\ln m_d}{m_d} \sim \frac{K_{\text{qg}}^{\text{SM}} m_A \ln(\Lambda^{\text{qg}}/m_{\text{susy}})}{4\pi^2 m_d \Lambda^{\text{qg}}} \sim 10^7 \text{ GeV} \frac{\Lambda^{\text{qg}}}{\Lambda^{\text{Y}}} \]  
Eq. (8) translates directly to an extremely strong bound on \( \Lambda^{\text{qg}} \) in scenarios where \( \theta \sim 0 \) is engineered by hand, either by using discrete symmetries at high energies [11] or by imposing an approximate global \( U(1) \) symmetry at tree level to ensure \( m_u(0) = 0 \). In these cases, the experimental bound on the neutron EDM, \( |d_n| < 6 \times 10^{-26} \text{e cm} \) [5] (soon to be updated [6]), combined with standard estimates for \( d_n(\theta) \) [12] implies remarkable sensitivity to scales \( \Lambda^{\text{qg}} \sim 10^{17} \text{ GeV} \). Future progress in EDM searches (both for neutrons and heavy atoms) can bring this up to the Planck scale and beyond. In contrast, no constraints from (8) ensue within the axion scenario.

**Electric dipole moments from four-fermion operators:** Electric dipole moments (EDMs) of neutrons and heavy atoms and molecules are the primary probes for sources of flavor-neutral CP violation [12]. In addition to \( d_n \), the strongest constraints on CP-violating parameters arise from the atomic EDMs of thallium, \( |d_{\text{Tl}}| < 9 \times 10^{-25} \text{e cm} \) [3], and mercury, \( |d_{\text{Hg}}| < 2 \times 10^{-28} \text{e cm} \) [4].

Assuming that \( \theta \) is removed by an appropriate symmetry, EDMs are mediated by higher-dimensional operators and both (4) and (5) are capable of inducing atomic/nuclear EDMs if the overall coefficients contain an extra phase relative to the quark masses. Restricting Eq. (4) to the first generation, we find the following CP-odd operators (with real \( m_c, m_u \)):
\[ \mathcal{L}_{CP} = -\frac{\alpha_N \text{Im} Y_{ee}^{\text{SM}}}{6\pi \Lambda^{\text{qg}} m_{\text{susy}}} [(\bar{u}u)\bar{e}e\bar{\gamma}5e + (\bar{u}u\bar{e}e\bar{\gamma}5u)\bar{e}e] \]  
Accounting for QCD running from the SUSY scale to 1GeV, and using the hadronic matrix elements over nucleon states, \( \langle N|\bar{u}u + dd |2N] \approx 4NN \) and \( \langle n|\bar{u}u\bar{e}e|n] \approx -0.4(m_N/m_u)\bar{u}u\bar{e}e \), we determine the induced corrections to the CP-odd electron-nucleon Lagrangian, \( \mathcal{L} = CsN\bar{e}e\bar{\gamma}5e + C_P N\bar{e}e\bar{\gamma}5e\bar{e}e \),
\[ C_S \sim \frac{2 \times 10^{-4}}{1\text{GeV} \times \Lambda^{\text{qg}}} \quad C_P \sim \frac{4 \times 10^{-3}}{1\text{GeV} \times \Lambda^{\text{qg}}} \]  
using maximal \( \text{Im} Y_{ee}^{\text{SM}} \) and taking \( m_{\text{susy}} = 300 \text{ GeV} \).

Comparing (10) to the limits on \( C_S \) and \( C_P \) deduced from the TI and Hg EDM bounds [12], we obtain the following sensitivity,
\[ \Lambda^{\text{qg}} \gtrsim 3 \times 10^8 \text{ GeV} \quad \text{from TI EDM} \]  
\[ \Lambda^{\text{qg}} \gtrsim 1.5 \times 10^8 \text{ GeV} \quad \text{from Hg EDM} \]  
\[ \Lambda^{\text{qg}} \gtrsim 3 \times 10^7 \text{ GeV} \quad \text{from Hg EDM}. \]  
The last relation results from sensitivity to the CP violating operators \( (\bar{d}\gamma_5 d)(\bar{u}u) \) from (5), leading to the Schiff nuclear moment and the Hg EDM. These are remarkably large scales, and indeed not far below the scales suggested by neutrino physics. In fact, the next generation of atomic/molecular EDM experiments [13] may reach sensitivities sufficient to push \( \Lambda^{\text{qg}} \) into regions close to the suggested scale of right-handed neutrinos.

Semileptonic operators involving heavy quark superfields are in turn strongly constrained via two-loop corrections (3) to the dipole amplitudes. The bound on \( d_{\text{Tl}} \) implies \( |d_e| \lesssim 1.6 \times 10^{-23} \text{e cm} \), which for maximal \( \text{Im} Y_{ee}^{\text{SM}} \) implies:
\[ \Lambda^{\text{qg}} \gtrsim 1.3 \times 10^8 \text{GeV}. \]  
Results analogous to (3) apply for the quark EDMs and color EDMs, furnishing a similar sensitivity to \( \Lambda^{\text{qg}} \).

**Lepton flavour violation:** Searches for lepton-flavour violation (LFV), such as \( \mu \rightarrow e\gamma \) decay, and \( \mu \rightarrow e \) conversion in nuclei, have resulted in stringent upper bounds on the corresponding branching ratio, \( \text{Br}(\mu \rightarrow e\gamma) < 1.2 \times 10^{-11} \) [14], and the rate of conversion normalized on capture rate, \( R(\mu \rightarrow e^- \text{ on Ti}) < 4.3 \times 10^{-12} \) [15], with further improvement anticipated. The latter bound implies a particularly high sensitivity to the semileptonic operators in (1). The conversion is mediated by \( \bar{u}u\bar{e}e\bar{\gamma}5u \) and \( \bar{u}u\bar{e}e\bar{\gamma}5u \), and involves the same matrix elements as \( C_S \). Using bounds on such scalar operators derived elsewhere (see e.g. [16]), we conclude that \( \mu \rightarrow e \) conversion probes energy scales as high as
\[ \Lambda^{\text{qg}} \gtrsim 1 \times 10^{9} \text{GeV} \quad \text{from } \mu^- \rightarrow e^- \text{ on Ti}. \]  
The constraint on \( \mu \rightarrow e\gamma \) probes similar, but slightly lower, scales as it requires a two-loop diagram as in Fig. 1b. Disregarding an \( O(1) \) factor between (11) and (15), we conclude that searches for EDMs and LFV probe these extensions of the MSSM up to comparable energy scales of \( \sim 10^8 \) GeV.

**Hadronic flavor constraints:** Often, the most constraining piece of experimental information comes from the contribution of new physics to the mixing of neutral mesons, \( K \) and \( B \). However, in the present case, there is necessarily a significant loop and Yukawa suppression arising from (6), and the sensitivity is correspondingly weakened. Taking \( \Delta m_K \text{exp} \approx 3.5 \times 10^{-6} \text{eV} \) [17], we find \( \Lambda^{\text{qg}} \gtrsim (\tan \beta/50) \times 200 \text{ GeV} \) [8]. \( \Delta m_B \) exhibits a similar sensitivity, while \( \epsilon_K \) is about three orders of magnitude more sensitive, but still well below the
sensitivity to $\Lambda$ (GeV)

| operator | sensitivity to $\Lambda$ (GeV) | source |
|----------|-------------------------------|--------|
| $Y_{331}^{e}$ | $\sim 10^7$ | naturalness of $m_e$ |
| Im($Y_{331}^{e}$) | $\sim 10^{17}$ | naturalness of $\theta$, $d_n$ |
| Im($Y_{411}^{e}$) | $10^{-7} - 10^9$ | Tl, Hg EDMs |
| $Y_{112}^{e}$, $Y_{121}^{q}$ | $10^{-7} - 10^{-8}$ | $\mu \to e$ conversion |
| Im($Y_{qq}^{e}$) | $10^{-7} - 10^{-8}$ | Hg EDM |
| Im($y_{1h}$) | $10^{-7} - 10^{-8}$ | $d_e$ from Tl EDM |

TABLE I: Sensitivity to the threshold scale. The naturalness bound on Im($Y_{qq}^{e}$) doesn’t apply to the axionic solution of the strong CP problem, the best sensitivity to Im($y_{1h}$) is achieved at maximal $\tan \beta$, and the Hg EDM constraint on Im($Y_{qq}^{e}$) applies when at least one pair of quarks belongs to the 1st generation.

scales probed by EDMs and LFV. In contrast, it is clear that these observables provide much better sensitivity to SUSY dimension-six operators, which impose no additional suppression factors. Denoting the corresponding scale as $\Lambda'$, we find $\Lambda' \gtrsim 8 \times 10^6$ GeV, while $\epsilon_K$ is sensitive to scales $\sim 10^8$ GeV.

Two-loop contributions to $b \to s\gamma$ (as in Fig. 1b) are not Yukawa suppressed and, with the current precision $\Delta B r(B \to X_s\gamma) \sim 10^{-4}$ [17], are somewhat more sensitive. We find $\Lambda'' \gtrsim 10^3 - 10^4$ GeV (for $Y_{331}^{e} \sim 10^6$), still well below the sensitivity in other channels.

Constraints on the Higgs operator: The high sensitivity to QULE and QUQD arises primarily because they can flip the light fermion chirality without Yukawa suppression. It would then come as no surprise if $H_u H_d H_u H_d$ were to have little implication for $C P$ and flavor-violating observables; the operator will of course provide corrections to the sfermion and neutralino mass matrices, and can induce $C P$-odd mixing between $A$ and $h$, $H$, but these effects do not lead to high sensitivity to $\Lambda_h$.

Remarkably enough, it turns out that EDMs do exhibit a high sensitivity to $H_u H_d H_u H_d$ at large $\tan \beta$ through corrections to the Higgs potential, and in particular the effective shift of the $m_{12}^2$ parameter,

$$m_{12}^2 H_u H_d \to (m_{12}^2)_{eff} H_u H_d \equiv \left( m_{12}^2 + \frac{\mu y h_{\nu}^2 M_{SM}}{\Lambda_h} \right) H_u H_d.$$  \hspace{1cm} (16)

Crucially, a complex phase in $(m_{12}^2)_{eff}$, due to Im($y_{1h}$), is enhanced at large $\tan \beta$ because $m_{12}^2 \simeq m_{31}^2 / \tan \beta$. The resulting phase affects the one-loop SUSY EDM diagrams (see e.g. [18]):

$$d_e = \frac{e m_e \tan \beta}{16 \pi^2 m_{suy}^2} \left( \frac{5 \nu^2_2}{24} + \frac{9 \nu^2_1}{24} \right) \sin \left[ \text{Arg} \frac{\mu M_{2}}{(m_{12}^2)_{eff}} \right].$$  \hspace{1cm} (17)

Expanding to leading order in $1/\Lambda_h$, using (16), and imposing the present limit on $d_e$ discussed earlier, one finds impressive sensitivity for large $\tan \beta$,

$$\Lambda_h \gtrsim 2 \times 10^7 \text{ GeV} \left( \frac{\tan \beta}{50} \right)^2 \left( \frac{300 \text{ GeV}}{m_{suy}} \right) \left( \frac{300 \text{ GeV}}{m_A} \right)^2.$$  \hspace{1cm} (18)

In conclusion, we have examined new flavor and $C P$ violating effects mediated by dimension five superpotential operators, and shown that the sensitivity to these operators extends far beyond the weak scale (as summarized in Table 1). The semileptonic operators that mediate flavor violation in the leptonic sector and/or break $C P$ could be detectable even if the scale of new physics is as high as $10^6$ GeV, and well above the naturalness scale. Our results can be translated into constraints on $C P$ and flavor violation in specific models leading to (1), e.g. the NMSSM or the MSSM with an extra pair of Higgses. Moreover, the sensitivity quoted in (11) and (15) is robust, having a mild dependence on the SUSY threshold. Finally, since these effects decouple linearly, an increase in sensitivity by just two orders of magnitude would already start probing scales relevant for neutrino physics. Our results motivate further searches for EDMs and LFV in the SUSY framework even if the soft-breaking sector provides no new sources, as happens e.g. in models with low scale SUSY breaking.