Spinelly et. al. [1] have written an interesting paper on composite monopoles, i.e., a global monopole and a magnetic monopole bound together by their gravitational interaction. This paper was followed by two works considering more general cases [2, 3]. Such composite configurations are of interest for several reasons.

One is in connection with the (magnetic) monopole problem. Global monopoles have long range interactions, and their annihilation rate is very efficient. Composite monopoles could share this property, if they remained bound as the global monopoles move to annihilate each other. One could even imagine a scenario in which the global and magnetic monopoles are created independently, with the magnetic monopoles coming from an earlier phase transition. A second transition leads to global monopoles which then capture and drag the magnetic monopoles with them as they go towards a neighbouring defect. Therefore magnetic monopole annihilation could be enhanced with respect to the case of purely magnetic monopoles. If there is no correlation between the signs of the global and magnetic charges, the effect is quite mild, and gives rise to a remnant population of doubly charged, purely magnetic monopoles. On the other hand, if the global and magnetic charges are correlated the effect could be dramatic, strong enough to solve the monopole problem.

Another reason is the spacetime structure around such defects. Pure global monopoles cause a deficit solid angle $\Delta$, plus a small effect due to the core [4]. The mass that can be ascribed to the core turns out to be negative $\Delta$, leading to repulsive interactions. The presence of a magnetic monopole component can in principle change the sign of the core mass to make it positive $\Delta$. Finally, the critical phenomena associated with gravitational collapse of the monopole to form a black hole show an interesting intermediate behaviour between that of the global and magnetic cases [2].

Our comment consists basically of two points. First, some of the results of [1] have a very simple physical interpretation that was somewhat obscured by their choice of units and that we want to clarify. Second, and more important, we want to emphasise that the composite monopole that is bound only gravitationally is almost certainly unstable, whereas the global and magnetic monopole configurations are of interest for several reasons.

Considering first the case where there is only a gravitating global monopole, we see that there are two mass scales in the problem. One is set by the v.e.v of the scalar field, $\eta$, the other by the massive scalar excitations, which have mass $\sqrt{\lambda}\eta$ ($\lambda$ is the quartic coupling). We can use the second one to define a natural length scale, and the first one will define an overall scale for the energy. In other words, in the absence of a magnetic monopole, the rescaling

$$\chi^a \to \eta \chi^a, \quad x_\mu \to \hat{x}_\mu = \frac{x_\mu}{\eta \sqrt{\lambda}},$$

in Lagrangian (7),(10) in reference [1] makes the parameter $\lambda$ disappear from the equations of motion, while the parameter $\eta$ appears only in the adimensional combination $G\eta^2$, which measures the gravitational strength of the global monopole (and, in particular, the deficit solid angle $\Delta = 8\pi G\eta^2$ in the spherically symmetric configuration).

The spherical monopole profile $f(\hat{r})$ is known to be fairly independent of $\Delta$ [2], and we expect this feature to persist in the presence of the magnetic monopole; this independence is observed in ref. [1].

In general, the magnetic monopole would introduce three new mass scales: the v.e.v. of its scalar field, the mass of the scalar excitations and the mass of the vector boson, $e\eta$ ($e$ is the gauge coupling constant). In ref. [1] the first two were chosen to coincide with those of the global monopole, so the only new adimensional parameter is the ratio

$$\frac{\Delta}{1 - \Delta}.$$
of the scalar mass to the vector mass. The parameter $\beta = \lambda/e^2$ measures this ratio (squared). There are no more free parameters in the problem.

We do not expect the presence of the magnetic monopole to alter the global monopole profile $f(r)$ substantially, since there is no direct coupling between them. In particular $f$ should be quite insensitive to the value of $\beta$, if $\Delta$ is kept fixed. The only reason why figs. 3a) and 6c) in ref. [1] appear to give such a dependence is that in [1] the radial coordinate was rescaled using the vector boson mass instead of the scalar mass (a desirable choice in other respects). The difference is just a factor of $\sqrt{\beta}$ in the abscissa and once this is taken into account the curves agree perfectly, as shown in figure 1.

We now turn to the composite monopole’s stability. To understand the nature of the problem, consider the simpler case of the interaction between a magnetic monopole and a conical singularity (such as would be produced, for example, by an infinitely thin, straight, idealised cosmic string). In the plane perpendicular to the string, the metric is flat and has no effect on test masses other than lensing. But this is not true of gravitating masses and charges, whose long–range electric or magnetic fields can detect the presence of the singularity. The net effect turns out to be that masses are attracted to the conical singularity and charges (electric and magnetic) are repelled [6]. In the case of a magnetic charge, the convergence of magnetic field lines behind the singularity is responsible for the repulsion.

Now consider the three-dimensional version, a magnetically charged point particle in a spherically symmetric space–time with a deficit solid angle concentrated in a point-like singularity. Without loss of generality the charge can be assumed to move in the equatorial plane ($\theta = \pi/2$).

This is a reasonable approximation to the interaction between a global monopole and a magnetic monopole that are well separated. But then the arguments of [6] apply, and we expect the monopoles to repel. Moreover, in this case, even the gravitational interaction is repulsive, as the “mass” of the core of the global monopole is negative. An explicit calculation of the interaction between a pointlike electrically charged particle and a global monopole (including the effect of the core) confirms that it is repulsive [7]; we expect this result to hold when considering a magnetic charge and a global monopole.

We conclude that the cosmological capture of a magnetic monopole by a global monopole seems quite improbable (we have neglected the effect of the finite–size cores in this argument, but since the monopoles only interact gravitationally we do not expect our conclusions to change qualitatively). While this is disappointing, it is not necessarily a problem for the composite monopole since in that case the cores are superposed, the magnetic field lines are evenly distributed with spherical symmetry and thus there is no net repulsion between the two cores.

The problem arises when one considers the stability of the composite monopole to angular perturbations. This is an important consideration because, in flat space, global monopoles have a zero mode which allows the redistribution of gradient energy density in an axisymmetric way, without any cost in energy [8]. Following the notation in [1], where the spherical global monopole is given by:

$$\chi^a = f(r) \frac{x^a}{r}, \quad (2)$$

purely angular deformations with axial symmetry can be described using the following ansatz [8]

$$\chi^1 = f(r) \sin \bar{\theta}(r, \theta) \cos \varphi;$$

$$\chi^2 = f(r) \sin \bar{\theta}(r, \theta) \sin \varphi;$$

$$\chi^3 = f(r) \cos \bar{\theta}(r, \theta).$$

(3)
Note that $\bar{\theta} = \theta$ corresponds to the unperturbed monopole.

As shown by Goldhaber \[8\], in flat space, configurations of the form

$$\tan \left( \frac{\bar{\theta}}{2} \right) = \tan \left( \frac{\theta}{2} \right) e^{\xi}, \quad \xi = \text{const},$$

where $\xi$ is an arbitrary constant, are degenerate in energy with the spherical monopole. For any given $r$, $\xi \gg 1$ corresponds to concentrating the gradient energy in an arbitrarily small region around the north-pole with no cost in energy.

The extra tension created at the north pole drags the core of the global monopole upwards, restoring spherical symmetry \[9, 11, 12\]. But consider its effect on the magnetic monopole: a larger gradient energy density in the global monopole causes a redistribution of the deficit angle “density”, with the larger contribution now coming from the north pole. But then magnetic field lines are pushed together there, causing the magnetic core to move downwards, i.e. in the opposite direction to the global monopole core. Once the cores are separated, they are expected to repel, so an attractive interaction between the cores seems necessary for the composite object to survive.

To summarise, the stability of composite monopoles can by no means be taken for granted and is expected to depend quite sensitively on the details of the interaction between the two constituents. Any attempt to consider composite monopoles in an astrophysical or cosmological context must address this problem.

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