Altschuler and Aronov (AA) have shown that the electron-electron interaction in a weakly-disordered metal suppresses the single-particle density of states (DOS) in the vicinity of the Fermi level \( (E_F) \). According to the AA theory the suppressed DOS exhibits the energy dependence \( \propto \sqrt{|E - E_F|} \) valid for \( |E - E_F| \) smaller than a certain correlation energy \( U_{co} \). Recent experiments have shown that at energies larger than \( U_{co} \), the DOS exhibits a states-conserving dependence on energy, namely, the states removed from near the Fermi level are found at energies above \( U_{co} \) in the energy range of about 3\( U_{co} \). In this work the AA effect is studied beyond the low energy limit theoretically. We consider the AA model in which the electrons interact via the statically screened Coulomb interaction and the modification of the DOS is due to the exchange part of the electron self-energy. We derive the states-conserving DOS heuristically. Namely, we show that the self-energy consists of a diverging part (which we skip on physical grounds) and of the small part of the order of the pair Coulomb energy.

This small part gives the states-conserving DOS which is in qualitative accord with experimental observations at energies above \( U_{co} \) and which reproduces the AA result at energies below \( U_{co} \).

I. INTRODUCTION

In our model \[3, 17\] electrons in the disordered metal interact via the static finite-ranged potential \( V(\vec{r} - \vec{r}') \). If \( V = 0 \), the electrons interact only with the random potential \( V_d(\vec{r}) \), produced by disorder. In such case the electron energies \( E_n \) and wave functions \( \varphi_m \) obey the Schrödinger equation

\[ H \varphi_m(\vec{r}) = E_m \varphi_m(\vec{r}), \]

where \( H = -\hbar^2/2m \Delta + V_d(\vec{r}) \). If we treat the e-e interaction within the first order perturbation theory and consider only the Fock part of the interaction, \( E_m \) is modified to \( \bar{E}_m \) as

\[ \bar{E}_m = E_m + \Sigma^x_m \]

(1)

where \( \Sigma^x_m \) is the Fock first-order self-energy correction:

\[ \Sigma^x_m = - \sum_n f_n \int \frac{d\vec{q}}{(2\pi)^3} V(\vec{q}) | \langle \varphi_m | e^{i\vec{q}\cdot\vec{r}} | \varphi_n \rangle |^2. \]

(2)

Here \( V(\vec{q}) \) is the Fourier transform of \( V(\vec{r} - \vec{r}') \), \( f_n \) is the Fermi function, and \( \sum_n \) is the sum over \( n \) with spin parallel to that of \( m \). Equations (1) and (2) hold if \( \Sigma_m \ll E_m \).

Equations (1) and (2) describe a specific disordered sample. When averaged over many disordered samples, they remain unchanged except that \( E_m \) and \( \Sigma_m \) are the mean values. Most important, the disorder-averaged \( | \langle \varphi_m | e^{i\vec{q}\cdot\vec{r}} | \varphi_n \rangle |^2 \) can be calculated explicitly. For a diffusing electron \[3, 16, 17\]

\[ | \langle \varphi_m | e^{i\vec{q}\cdot\vec{r}} | \varphi_n \rangle |^2 = \frac{1}{\pi \rho(E_n) \Omega} \frac{\hbar D q^2}{(\hbar D q^2)^2 + (E_m - E_n)^2} \]

(3)

where \( D \) is the diffusion coefficient, \( \Omega \) is the volume, and \( \rho(E_n) \) is the DOS for a single spin orientation \( \rho(E_n) \) is often replaced by \( \rho(E_F) \) which is justified for \( E_n \) close to \( E_F \). If we average \( \bar{E}_m \), \( E_m \), and \( \Sigma^x_m \) over all states \( m \) with energies \( E_m = E \), equation (1) can be rewritten as

\[ \bar{E}(E) = E + \Sigma^x(E), \]

(4)

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where
\[ \Sigma^x(E) = - \int_{0}^{E_F} dE' \int \frac{d\tilde{q}}{8\pi^2} V(q) \frac{hDq^2}{(hDq^2)^2 + (E - E')^2}. \]

In the last equation and in all following calculations we assume zero temperature for simplicity.

Due to averaging over disorder the unperturbed DOS (per spin) reads \( \rho_0(E) = (m/2)^{3/2} \sqrt{E/\pi^2\hbar^3} \), as for the free electrons. The perturbed DOS versus \( E \), \( dn/dE \equiv \rho(E) \), can be expressed \[3, 16, 17\] from equation (4) as
\[ \rho(E) = \rho_0(E) \frac{1}{1 + d\Sigma^x(E)/dE} \sum_{E'} \frac{1}{1 + d\Sigma^x(E)/dE}. \]

where the right hand side holds for \( \rho_0(E) \approx \rho_0(E_F) \). Note that the perturbed DOS, \( \rho(E) \), is expressed as a function of \( E \) rather than of \( \tilde{E} \). This approximation is valid within the first order perturbation theory \[3, 16, 17\].

It is customary to change the integral \( \int_{0}^{E_F} dE' \) in equation (5) as \( \int_{-\infty}^{\infty} dE' \). This infinite band approximation is justified for weak interaction. Then, substituting \( E \) by variable \( \varepsilon = E - E_F \), one can rewrite equation (5) as \[16\]
\[ \Sigma^x(\varepsilon) = - \int_{\varepsilon}^{\infty} d\varepsilon' \frac{d\Sigma^x(\varepsilon')}{d\varepsilon'}. \]

where \[3, 16, 17\]
\[ \frac{d\Sigma^x(\varepsilon)}{d\varepsilon} = \int \frac{d\tilde{q}}{8\pi^2} V(q) \frac{hDq^2}{(hDq^2)^2 + \varepsilon^2}. \]

The AA effect was studied \[3, 16, 17\] for \( V(q) \) so small that \( d\Sigma^x(\varepsilon)/d\varepsilon \ll 1 \). Then \( \rho(E) \approx \rho_0(E_F)[1 - d\Sigma^x(\varepsilon)/d\varepsilon] \), or
\[ \rho(\varepsilon) \approx \rho_0(0)[1 - d\Sigma^x(\varepsilon)/d\varepsilon]. \]

In the simplest model \[3, 16, 17\] with static screening
\[ V(q) = \frac{e^2}{\varepsilon_{\infty}(q^2 + k_s^2)}, \]

where \( k_s = \sqrt{e^2/\rho_0(E_F)/\varepsilon_{\infty}} \) is the reciprocal screening length (the factor of 2 is due to the spin degeneracy), and \( \varepsilon_{\infty} \) is the high-frequency permittivity of the metal.

From equations (8) and (9) one obtains the result of Altshuler and Aronov, \[11, 17\]
\[ \rho(E) = \rho(E_F) + \frac{1}{4 \sqrt{2} \pi \hbar D (m/2)^{3/2}} |\varepsilon|^{1/2}, \]

where \( \rho(E_F) \) is the DOS at the Fermi level and the second term on the right hand side is the AA interaction correction. In Refs. \[11, 17\] the integral in equation (8) was calculated assuming \( V(q) \approx V(0) \). For \( V(q) = V(0) \) the integral diverges in the upper limit, therefore, the upper limit was restricted to \( q_{max} = \sqrt{e|\varepsilon|/\hbar D} \). Since \( V(q) \approx V(0) \) only for \( q \lesssim k_s \), the obtained result [equation (11)] holds only for \( |\varepsilon| \lesssim \hbar D k_s^2 \).

Within this approach the term \( \rho(E_F) \) remains undetermined, it is usually determined experimentally \[14, 15\].

In the following text we present an alternative derivation which is not restricted to the low-energy limit. At low energies our derivation will reproduce equation (11) and also determine explicitly the term \( \rho(E_F) \). However, our major goal is to go beyond the low energy limit and to derive the DOS which conserves the states similarly as in the experiment \[12\].

In Sect. 2 we show that the DOS given by equations (9), (8), and (10) does not conserve the states. In Sect. 3. we identify why this is so and present a heuristic derivation of the states-conserving DOS. Comparison with experiment is presented in Sects. 3 and 4. Finally, in Sect. 5 we interpret the AA effect with conservation of states in terms of coupling between the interaction (10) and matrix element (3).

II. THE STATES CONSERVATION PROBLEM

Figure 1 shows schematically the typical experimental output \[12\]. At energies below \( U_{co} \) the data show the AA singularity described by the \( |\varepsilon|^{1/2} \) law. All states repelled from the AA singularity are found at energies above \( U_{co} \) in the range of about 3\( U_{co} \). This local conservation of states should be distinguished from the conservation of states in strongly correlated disordered systems where the states repelled by interaction are transferred far away from the Fermi level \[18\]. In the latter case one cannot use the approximation of the infinitely wide band which on the contrary has no effect if conservation of states takes place locally near the Fermi level. Whenever we speak about the conservation of states, we have in mind the local conservation of states similar to that in figure 1.

In accord with figure 1 and Ref. \[12\], the conservation of states for the AA model reviewed in Sect. 1 reads
\[ \int_{0}^{\infty} d\varepsilon [\rho(\varepsilon) - \rho_0(0)] = 0. \]

Inserting equation (9) into the conservation law (12) we find that the conservation of states is fulfilled only if
\[ \int_{0}^{\infty} d\varepsilon \frac{d\Sigma^x(\varepsilon)}{d\varepsilon} = 0. \]
However, equation (13) is not fulfilled because \( d\Sigma^x(\epsilon) / d\epsilon \) is positive for any \( \epsilon \) [see equation (8)]. This means that the model of Sect. 1 does not conserve the states.

Furthermore, integral \( \int_0^\infty d\epsilon \ d\Sigma^x(\epsilon) / d\epsilon \) not only fails to fulfill equation (13) but even diverges in the upper limit (see the next section). This means that also the self-energy (7) diverges which is another problem, in addition to the states conservation problem. In principle, the divergence could be eliminated by considering the energy band of finite width, however, the self-energy would then depend on the band width which is also not sound (for weak interaction we expect \( \Sigma^x \lesssim \epsilon^2 / 4\pi\varepsilon_{co}k_s^{-1} \) independently on the band width). We will see that the divergent self-energy is closely related to the conservation of states problem.

To conserve the states and to obtain a finite self-energy, one has to identify the limitations of the model and to modify it properly. First, the diffusive approximation (3) holds only at small energies. Second, the interaction (10) and diffusive matrix element (3) are mutually independent while in reality they should affect each other.

Concerning the first point, the diffusive approximation (3) holds for the eigen-states which are correlated in time and space [3, 16, 17]. Therefore, equation (3) is valid only if \( E_m - E_n \lesssim h/\tau \) and \( q \lesssim 1/\sqrt{\epsilon} \), where \( \tau \) is the elastic scattering time and \( \sqrt{\epsilon} \) is the Fermi velocity. In addition, the e-e interaction with spatial range \( \sim 1/k_s \) introduces the correlation time \( \tau_{eo} \sim k_s^{-2} / D \) which is usually longer than \( \tau \) and gives rise to the correlation energy \( U_{co} \sim hDk_s^2 \). This restricts the validity of equation (3) to even smaller \( |E_m - E_n| \), say to \( E_m - E_n \lesssim U_{co} \). For \( |E_m - E_n| > U_{co} \) the matrix element (3) has to be modified by e-e interaction and to depend on \( U_{co} \). Due to the decaying correlation between \( m \) and \( n \), it should decay with increase of \( |E_m - E_n| \) faster than now.

Second, in a realistic self-consistent model the interaction (10) has to be modified to the form which depends on the energy difference \( |E_m - E_n| \) and diffusion coefficient \( D \). From figure 1 it follows that the conservation of states (equation 12) can be fulfilled only if \( d\Sigma^x(\epsilon) / d\epsilon \) in equation (13) changes the sign at energy \( \epsilon = U_{co} \). Indeed, one can see from equation (8) that such sign change can arise only from the sign change of the interaction \( V \).

### III. THE STATES-CONSERVING DOS AND SELF-ENERGY

The question is how to modify equation (8) to be valid also beyond the low energy limit. A direct self-consistent solution of the problem would be difficult. Therefore, as a first approach, we develop a simple heuristic theory. Inserting for \( V(q) \) the equation (10), using substitutions \( a = \sqrt{\epsilon / hDk_s^2} \) and \( x = q/k_s \), and performing a simple algebra, we rewrite equation (8) as

\[
\frac{d\Sigma^x(\epsilon)}{d\epsilon} = \frac{1}{\pi\hbar Dk_s^2} \frac{\epsilon^2}{4\pi\varepsilon_{co}k_s^{-1}} \left[ 1 + \left( \frac{\epsilon^2}{2\hbar^2 D^2k_s^4} \right)^{-1} \right] \\
\times \frac{2}{\pi} \int_0^\infty dx \left( \frac{1}{1 + x^2} - \frac{1}{1 + (\frac{\epsilon}{2\hbar^2 Dk_s^2})^2} + \frac{x^2}{1 + (\frac{\epsilon}{2\hbar^2 Dk_s^2})^4} \right) .
\]

(14)

The right hand side of the last equation is composed of three terms. We will show that the only modification we need is the omission of the third term.

All three integrals in equation (14) can be calculated analytically. Introducing notations \( U_{co} = 2hDk_s^2 \) and \( U_i = \epsilon^2 / 4\pi\varepsilon_{co}k_s^{-1} \), we obtain

\[
\frac{d\Sigma^x(\epsilon)}{d\epsilon} = \frac{d\Sigma_A^x(\epsilon)}{d\epsilon} + \frac{d\Sigma_B^x(\epsilon)}{d\epsilon} ,
\]

(15)

where

\[
\frac{d\Sigma_A^x(\epsilon)}{d\epsilon} = \frac{2}{\pi U_{co}} \left[ 1 + 4 \left( \frac{\epsilon}{U_{co}} \right)^2 \right]^{-1} \left[ 1 - \sqrt{\frac{\epsilon}{U_{co}}} \right] ,
\]

(16)

corresponds to the first two terms in equation (14) and

\[
\frac{d\Sigma_B^x(\epsilon)}{d\epsilon} = \frac{2}{\pi U_{co}} \left[ 1 + 4 \left( \frac{\epsilon}{U_{co}} \right)^2 \right]^{-1} \left( \frac{2}{U_{co}} \right)^{3/2} ,
\]

(17)

corresponds to the third term.

Using equations (16) and (17) we find that

\[
\int_0^\infty \frac{d\Sigma_A^x(\epsilon)}{d\epsilon} = 0 , \quad \int_\epsilon^\infty \frac{d\Sigma_B^x(\epsilon')}{d\epsilon'} = \infty ,
\]

(18)

where the second integral diverges because in the upper limit

\[
\frac{d\Sigma_B^x(\epsilon)}{d\epsilon} \propto \epsilon^{-1/2} .
\]

We can now discuss the self-energy. Inserting equation (15) into the equation (7) we obtain

\[
\Sigma^x(\epsilon) = - \int_\epsilon^\infty d\epsilon' \frac{d\Sigma_A^x(\epsilon')}{d\epsilon'} - \int_\epsilon^\infty d\epsilon' \frac{d\Sigma_B^x(\epsilon')}{d\epsilon'} ,
\]

(19)

where \( d\Sigma_A^x(\epsilon) / d\epsilon \) and \( d\Sigma_B^x(\epsilon) / d\epsilon \) are given by equations (16) and (17). Due to one of equations (13) the self-energy (19) diverges which is not a sound result. A physically sound self-energy should be finite and of the order of \( U_i \) (in our weak-interaction case). This can only be achieved if we omit in equation (19) the integral \( \int_\epsilon^\infty d\epsilon' \) \( d\Sigma_B^x(\epsilon') / d\epsilon' \), or in other words, if we set \( d\Sigma_B^x(\epsilon') / d\epsilon' = 0 \). In fact, we will see soon that the choice \( d\Sigma_B^x(\epsilon) / d\epsilon = 0 \) is the only way how to ensure both the finite self-energy and conservation of states. Due to this choice equation (19) reduces to

\[
\Sigma^x(\epsilon) = \frac{\epsilon}{|\epsilon|} \frac{2}{\pi} U_i \left\{ \frac{\pi}{4} + \frac{1}{8} \ln \left[ \frac{1 + 2\sqrt{\frac{|\epsilon|}{U_{co}}} + 2\frac{|\epsilon|}{U_{co}}}{1 - 2\sqrt{\frac{|\epsilon|}{U_{co}}} + 2\frac{|\epsilon|}{U_{co}}} \right] \right\} \\
- \frac{1}{4} \arctan \left( \frac{2 - \sqrt{\frac{|\epsilon|}{U_{co}}}}{\frac{3}{4} \arctan \left( \frac{1 - 2\sqrt{\frac{|\epsilon|}{U_{co}}} + 2\frac{|\epsilon|}{U_{co}}} \right) + 2\frac{|\epsilon|}{U_{co}}} \right) .
\]

(20)
As expected, the self-energy (20) is finite and smaller than \( U_i \). It starts from zero value at \( \epsilon = 0 \), reaches the peak value \( \pm 0.16U_i \) at \( \epsilon = \pm U_{co} \), and eventually shows logarithmic decay to zero for \( |\epsilon| \gg U_{co} \).

Now we discuss the DOS. From equations (15) and (9)
\[
\rho(\epsilon) = \frac{1}{\rho_0(0)} = 1 - \frac{d\Sigma_A^x(\epsilon)}{d\epsilon} - \frac{d\Sigma_B^x(\epsilon)}{d\epsilon}.
\]

We have already mentioned the conservation of states problem. To conserve the states, equation (21) has to fulfill equation (12). So it conserves the states only if
\[
\int_0^\infty \frac{d\Sigma_A^x(\epsilon)}{d\epsilon} + \int_0^\infty \frac{d\Sigma_B^x(\epsilon)}{d\epsilon} = 0,
\]

which is not the case due to the second of equations (18). Evidently, the term \( d\Sigma_B^x(\epsilon)/d\epsilon \) is not sound due to its behavior at large energies, in accord with the fact that equation (5) is not valid at large energies. On the other hand, the first of equations (18) implies that the term \( d\Sigma_A^x(\epsilon)/d\epsilon \) is sound in the sense that it conserves the states. Motivated by equations (18) and (22), we set in equation (21) \( d\Sigma_B^x(\epsilon)/d\epsilon \approx 0 \) and we obtain the DOS \( \rho(\epsilon)/\rho_0(0) = 1 - d\Sigma_A^x(\epsilon)/d\epsilon \) which conserves the states. In final form
\[
\rho(\epsilon)/\rho_0(0) = 1 - \frac{2}{\pi} \frac{U_i}{U_{co}} \left[ 1 + \frac{4|\epsilon|^2}{U_{co}^2} \right]^{-1} \left[ 1 - \sqrt{\frac{|\epsilon|}{U_{co}}} \right].
\]

The DOS expression (23) is plotted in a full line in panel \( a \) of figure 2. The full curve exhibits the AA singularity at energies \( |\epsilon| \lesssim U_{co} \). Due to the conservation of states, the states removed from the AA singularity are found at energies \( |\epsilon| > U_{co} \). About one half of states pile up between \( |\epsilon| = U_{co} \) and \( |\epsilon| = 10U_{co} \), a further one third (not shown) piles up between \( |\epsilon| = 10U_{co} \) and \( |\epsilon| = 20U_{co} \).

For comparison, panels \( b, c, d, \) and \( e \) show the experimental data for various disordered metals, the experimental data of panel \( b \) are shown also in panel \( a \) (the dotted-dashed line). The origin of all these data is specified in the next section. Here we point out the following.

First, all experimental curves exhibit the AA singularity accompanied by pile up of states above \( U_{co} \). This is a sign of the local conservation of states (Fig. 1). Second, the states-conserving DOS (23) (the full curve in panel \( a \)) captures the main experimental features in the sense that it describes the AA singularity quantitatively (at \( T = 0 \)) and mimics the observed pile up of states qualitatively. Quantitatively, the width of the pile up of states in region of the experiment is much smaller (five to eight times) than in our theory. This is likely due to the fact that we ignore the second order interaction effects which can be important for large \( |\epsilon| \). Moreover, we rely on the constant diffusion coefficient \( D \) (Sect. 1) while in reality \( D \) should decrease with increase of \( |\epsilon| \) and even turn to zero due to the Anderson localization. Finally, experimental data are affected by finite \( T \) while we assume \( T = 0 \).

Equation (23) not only conserves the states but in the limit \( |\epsilon|^2 \ll U_{co}^2/4 \) gives the formula
\[
\rho(\epsilon)/\rho_0(0) = 1 - \frac{2}{\pi} \frac{U_i}{U_{co}} \frac{2}{\pi} \frac{U_i}{U_{co}} \sqrt{|\epsilon|}/U_{co},
\]

which reproduces the AA result (11) and in addition expresses \( \rho(E_F) \) as \( \rho(E_F) = \rho_0(E_F)[1 - (2/\pi)(U_i/U_{co})] \). In figure 2, equation (24) is plotted in a dashed line.

Finally, we express explicitly equation (21):
\[
\rho(\epsilon)/\rho_0(0) = 1 - \frac{2}{\pi} \frac{U_i}{U_{co}} \left[ 1 + \frac{4|\epsilon|^2}{U_{co}^2} \right]^{-1} \left[ 1 - \sqrt{\frac{|\epsilon|}{U_{co}}} \right] \times \left[ 1 - \sqrt{\frac{|\epsilon|}{U_{co}}} + \frac{1}{\sqrt{2}} \left( \frac{2 |\epsilon|}{U_{co}} \right)^{3/2} \right].
\]

In figure 2, the DOS expression (25) is shown in a dotted line, a similar dependence was found in work (19). It shows no pile up of states as it does not conserve them. It also deviates too early from the AA singularity because it reduces to the low energy limit (24) for \( |\epsilon| \ll U_{co}/2 \) rather than for \( |\epsilon|^2 \ll U_{co}^2/4 \). The former limit is more restrictive and this restriction
is due to the term \(d\Sigma_{I_b}(\epsilon)/d\epsilon\) in equation (23). This is another reason for choice \(d\Sigma_{I_b}(\epsilon)/d\epsilon \equiv 0\).

IV. INSIGHT INTO EXPERIMENT

The experimental data in figure 2 originate from the tunneling spectroscopy experiments \([8–10, 12]\). In these experiments the electron current \((I)\) is driven through the metal-insulator-metal (MIM) tunnel junction composed of the clean metal and disordered metal of interest. The output is the differential conductance \(G(\epsilon) \equiv dI(\epsilon)/dV\) in dependence on voltage \(V\). At low temperatures \(G(\epsilon)\) can be expressed as \(G(\epsilon)/G_0(\epsilon) \sim \rho(\epsilon)/\rho_0(\epsilon)\), where \(\epsilon = -eV\) and \(G_0(\epsilon)\) is the differential conductance that is independent on the interaction \([12]\). In reality \(G_0(\epsilon)\) is a parabolic function which has to be specified \([12]\) if one studies \(G(\epsilon)\) at large energies.

To our knowledge, experiment \([12]\) is the only experiment which studied the AA effect with intention to observe the pile up of states. In that work \(G(\epsilon)\) was measured for the disordered \(La_{0.28}Sr_{1.72}Mn_2O_2\) metal at 4.2 K and \(\rho(\epsilon)/\rho_0\) was determined from equation \(G(\epsilon)/G_0(\epsilon) \sim \rho(\epsilon)/\rho_0(\epsilon)\) after specifying \(G_0(\epsilon)\). The resulting \(\rho(\epsilon)/\rho_0\) data are those presented in panel \(c\) of figure 2; these data represent the first observation \([12]\) of the local conservation of states in the weakly disordered metal. Now we show that such local conservation of states was present (unnoticed) also in other experiments.

The \(\rho(\epsilon)\) dependence for the AlPdMn alloy, shown in panel \(b\) of figure 2, has been determined (by us) from the \(G(\epsilon)\) data measured in work \([10]\). Figure 3 shows the \(G(\epsilon)\) curves measured \([10]\) at 5 K and 25 K, normalized by \(G(\epsilon)\) at 5 K. The only difference between these curves is a pronounced AA singularity at 5 K and almost no AA singularity at 25 K, otherwise both curves coincide. Thus, the \(G(\epsilon)\) curve at 25 K can play the role of \(G_0(\epsilon)\) and we can determine \(\rho(\epsilon)/\rho_0(\epsilon)\) at 5 K as \(\rho(\epsilon)/\rho_0(\epsilon) \simeq G(\epsilon)/G_0(\epsilon)\). The resulting \(\rho(\epsilon)\) is shown in figure 3; the same result was used in panel \(b\) of figure 2. The \(G(\epsilon)\) curve at 5 K exhibits the peaks which we have marked by arrows. These peaks are due to the pile up of states repelled from the AA singularity.

The \(\rho(\epsilon)\) dependencies for \(In\) and \(Tl_{0.7}Te_{0.3}\), shown in panels \(c\) and \(d\) of figure 2 were extracted by us from the \(G(\epsilon)\) spectra measured in works \([8, 9]\). The spectra exhibit the AA singularity together with features due to the pile up of states. We have extracted from the spectra the parabolic \(G_0(\epsilon)\) background and we have obtained \(\rho(\epsilon)/\rho_0(\epsilon)\).

V. DISCUSSION, ALTERNATIVE VIEW ON THEORY

Our major result is the states-conserving DOS given by equation (23). At low energies equation (23) reproduces the original AA result \([11]\) and determines the term \(\rho(E_F)\). Most important, at large energies it provides the conservation of states, manifested by pile up of states above the correlation energy \(U_{cor}\). This pile up of states is qualitatively similar to that observed experimentally \([12]\). We have shown that such pile up of states feature was present also in other experiments \([8–10]\).

Our derivation was heuristic. We have found that the AA self-energy consists of a diverging part and of the small part of size \(\sim e^2/\varepsilon_{\infty}k_s^{-1}\) (equation 20). We have removed the diverging part (on physical grounds) by omitting the term \(d\Sigma_{I_b}(\epsilon)/d\epsilon\) and the remaining part has produced the states-conserving DOS which we have searched for.
the conservation of states is inherent to the AA model.

We want to finish by providing an alternative view on our derivation. In the AA model the matrix element \( \langle 1 \rangle \) and interaction \( \langle 10 \rangle \) were independent, however, in reality they should affect each other. We show that just this happens in our model due to the omission of term \( d\Sigma^x_B(e)/de \).

We first repeat in a slightly different form our major results. We take equation (14) and omit the third term on the right hand side (the third term is \( d\Sigma^x_B(e)/de \)). We get

\[
\frac{d\Sigma^x(e)}{de} = \frac{1}{8\pi^2} \left( \begin{array}{c} 1 + \frac{e^2}{\hbar^2 D^2 k_0^4} \end{array} \right)^{-1} 4\pi \int_0^\infty \mathrm{d}q \frac{e^2}{\epsilon_{\infty}(q^2 + k_0^2)} (\hbar Dq)^2 + e^2 \left[ 1 - \frac{e^2}{\hbar^2 D^2 k_0^4} \right] \left( 1 - \sqrt{\frac{\epsilon}{U_{co}}} \right), \\
\]

where the right hand side recalls the final result (16). If we compare equation (26) with the original equation (8), we see that the diffusive approximation (3) is modified as

\[
| \langle \varphi_m | e^{i\hat{q}\hat{q}^*} | \varphi_n \rangle |^2 = \frac{1}{\pi \rho \Omega (\hbar Dq)^2 + e^2} \left[ 1 + \frac{e^2}{\hbar^2 D^2 k_0^4} \right]^{-1} \\
\]

and the interaction (10) as

\[
V(q, e) = \frac{e^2}{\epsilon_{\infty}(q^2 + k_0^2)} \left[ 1 - \frac{e^2}{\hbar^2 D^2 k_0^4} \right].
\]

Thus, the matrix element and interaction are no longer independent. First, equation (27) contains the factor \( (1 + 4\epsilon^2/U_{co}^2)^{-1} \) which has the Lorentzian shape with spread \( U_{co} = 2\hbar Dk_0^2 \). Due to this factor, expression (27) decreases with increase of \( |\epsilon| \) faster than the diffusive approximation (3). This is in accord with expectation (Sect. 2) that the interaction will suppress correlation between the states with \( |E_m - E_n| > U_{co} \). Second, interaction \( e^2/\epsilon_{\infty}(q^2 + k_0^2) \) is modified by factor \( 1 - k_0^2 e^2/\hbar^2 D^2 k_0^4 q^2 \) which depends on energy \( \epsilon \equiv E_m - E_n \) and which changes the sign of the interaction for \( |\epsilon| > \hbar Dk_0 q \). This is again in accord with argument (Sect. 2) that the conservation of states is only possible if the interaction changes the sign for \( |\epsilon| \sim U_{co} \).

The energy dependence in equation (28) should not be confused with dynamic screening. If we replace \( V(q) = e^2/\epsilon_{\infty}(q^2 + k_0^2) \) by the real part of \( V(q, \omega) = e^2/[q^2 \epsilon(q, \omega)] \) where \( \epsilon(q, \omega) = \epsilon_{\infty} \left( 1 + k_0^2 \frac{Dq^2}{\hbar^2} \right) \), the interaction remains positive.

Finally, note that the interaction (28) is closely related to the AA effect [equations (11) and (24)]. Indeed, the AA equation (11) is obtained as it stands if we skip in equation (26) the factor \( (1 + e^2/\hbar^2 D^2 k_0^4)^{-1} \) but keep the factor \( 1 - k_0^2 e^2/\hbar^2 D^2 k_0^4 q^2 \). Owing to this factor it is not necessary to introduce the upper cutoff used in the original derivation [see the discussion of equation (11)].

Acknowledgement

This work was supported by the APVV project No. APVV-0560-14 and by grant VEGA 20200014. We thank Richard Hubina for useful discussions and for help with equations (14), (16), and (17).

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