ION COLLISIONAL TRANSPORT COEFFICIENTS IN THE SOLAR WIND AT 1 au

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ABSTRACT

Proton and alpha particle collisional transport coefficients (isotropization, relative deceleration frequencies, and heating rates) at 1 au are quantified using the Wind/Solar Wind Experiment data. In agreement with previous studies, the ion–ion Coulomb collisions are generally important for slow solar wind streams and tend to reduce the temperature anisotropies, the differential streaming, and the differences between the proton and alpha particle temperatures. In slow solar wind streams the Coulomb collisions between protons and alpha particles are important for the overall proton energetics, as well as the relative deceleration between the two species. It is also shown that ion temperature anisotropies and differential streaming need to be taken into account for evaluation of the collisional transport coefficients.

Key words: solar wind

1. INTRODUCTION

The solar wind plasma is weakly collisional and far from thermal equilibrium; the observed particle velocity distribution functions exhibit important temperature anisotropies and differential streaming between different populations (Hellinger et al. 2006; Marsch 2006). While the solar wind electrons are relatively strongly collisional, ions are essentially collisionless except in slow solar wind streams (Neugebauer 1976; Marsch & Goldstein 1983; Livi & Marsch 1986). However, many ion properties in the solar wind are relatively well ordered by the ion collisionality measured through an estimated collisional age (Kasper et al. 2008). On the other hand, because many solar wind properties are correlated (Hellinger & Trávníček 2014), the role of collisions in the solar wind is not clear.

Transport in weakly collisional plasmas may strongly deviate from theoretical predictions obtained for collision-dominated plasmas (Spitzer & Härm 1953; Braginskii 1965, pp. 205–311). Even for the relatively strongly collisional electrons such predictions (derived as a perturbation of thermal equilibrium) generally fail (Landi et al. 2014). The collisional energy and momentum transport coefficients can be calculated by taking moments of the collisional operator assuming a particular form of particle distribution functions (Kogan 1961, pp. 153–161; Lehner 1967; Barakat & Schunk 1981; Hernandez & Marsch 1985). For drifting bi-Maxwellian velocity distribution functions, these transport coefficients can be derived in a closed form involving generalized double-hypergeometric functions (Hellinger & Trávníček 2009).

For modeling of the solar wind, simplified versions of the transport coefficients that neglect the temperature anisotropy and/or the differential streaming are often used (Hernandez et al. 1987; Echim et al. 2011; Maruca et al. 2013; Cranmer 2014; Tracy et al. 2015). Observed important ion temperature anisotropies and differential streaming make such approaches questionable (Matteini et al. 2012). In this paper we quantify the collisional energy and momentum transport coefficients in the solar wind using Wind/Solar Wind Experiment (SWE) observations at 1 au. This paper is organized as follows: Section 2 gives an overview of theoretical collisional transport coefficients for drifting bi-Maxwellian velocity distribution functions; in this section the collisional isotropization, relative deceleration, and heating frequencies are defined. Section 3 presents an analysis of the Wind/SWE data for proton and alpha particles. The different collision frequencies are evaluated and compared with each other. The error due to neglecting the temperature anisotropy and/or the differential streaming is also estimated. Section 4 summarizes and discusses the results.

2. TRANSPORT COEFFICIENTS

Here we assume a homogeneous plasma consisting of species with bi-Maxwellian velocity distribution functions \( f_s \) drifting along the magnetic field (subscripts \( s \) and \( t \) denote different species and subscripts || and \( \perp \) denote directions with respect to the ambient magnetic field):

\[
\bar{f}_s(v_{\parallel}, v_{\perp}) = \frac{n_s}{(2\pi)^{3/2}v_{\parallel}^3v_{\perp}^2} \exp\left(-\frac{v_{\parallel}^2}{2T_{\parallel}} - \frac{v_{\perp}^2}{2T_{\perp}}\right) \tag{1}
\]

where \( n_s \) is the species number density; \( v_{\parallel} = (k_BT_{\parallel}/m_s)^{1/2} \) and \( v_{\perp} = (k_BT_{\perp}/m_s)^{1/2} \) are the parallel and perpendicular thermal velocities corresponding to the parallel and perpendicular temperatures, \( T_{\parallel} \) and \( T_{\perp} \), respectively; \( k_B \) is the Boltzmann constant; \( m_s \) is the mass; and \( v_{\parallel} \) is the parallel drift velocity. For these distribution functions, the collisional transport coefficients for the parallel and perpendicular temperatures may be given as (Hellinger & Trávníček 2009)

\[
\left(\frac{dT_{\parallel}}{dt}\right)_c = -\nu_T S_{\parallel} (T_{\parallel} - T_{\perp}) + T_{\perp} \sum_{i=1}^{N_s} \nu_i^{(0)} H_{s\perp} \tag{2}
\]

\[
\left(\frac{dT_{\perp}}{dt}\right)_c = 2\nu_T (T_{\parallel} - T_{\perp}) + T_{\parallel} \sum_{i=1}^{N_s} \nu_i^{(0)} H_{s\parallel} \tag{3}
\]

where \( \nu_T \) is the (intraspecies) isotropization frequency (Kogan 1961, pp. 153–161)

\[
\nu_T = \frac{q_s^4 n_s \ln A_{ss}}{30\pi^{3/2} e_0^2 m_s^2 v_{\parallel}^3} \gamma F(2, 3/2, 7/2, 1 - A_s) \tag{4}
\]

and \( \gamma F \) is the standard (Gauss) hypergeometric function. Here \( e_0 \) denotes the electric permittivity, \( q_s \) is the charge,
\( A_s = T_{\perp}/T_{||} \) is the temperature anisotropy, and \( \ln \Lambda_{st} \) is the Coulomb logarithm.

The energy transfer between the different species is quantified by the perpendicular and parallel heating rates \( \nu_{Hs\perp} \) and \( \nu_{Hs\parallel} \), which may be expressed analytically as

\[
\nu_{Hs\perp} = \frac{v_{st}}{A_{st}} \left[ m_s \left( \frac{T_{\perp}}{T_{\parallel}} - 1 \right) F_{\perp}^{(st)} + \frac{F_{\perp}^{(st)}}{2 + \frac{v_{st}^2}{v_{st\perp}^2}} \right]
\]

\[
\nu_{Hs\parallel} = \nu_{st} \left[ \frac{m_s}{m_t} \frac{T_{\parallel}}{T_{\perp}} - 1 \right] \left( F_{\parallel}^{(st)} - 2 \left( F_{\parallel}^{(st)} - F_{\parallel}^{(st)} \right) 
\]

\[
+ \frac{v_{st}^2}{2v_{st\parallel}^2} F_{\parallel}^{(st)} \right].
\]

Here

\[
v_{st\perp} = \sqrt{\frac{v_{s\perp}^2 + v_{t\perp}^2}{2}} \quad \text{and} \quad v_{st\parallel} = \sqrt{\frac{v_{s\parallel}^2 + v_{t\parallel}^2}{2}}
\]

are combined parallel and perpendicular thermal velocities, respectively; \( v_{st} = v_s - v_t \) is the relative velocity between the two species; \( m_{st} = m_s m_t / (m_s + m_t) \) is a combined mass;

\[
A_{st} = \frac{v_{st\perp}^2}{v_{st\parallel}^2} = \frac{m_s T_{\perp} + m_t T_{\parallel}}{m_t T_{\perp} + m_s T_{\parallel}}
\]

is a combined temperature anisotropy; and

\[
\nu_{st} = \frac{q_s^2 q_t^2 n_{st}}{12 \pi^2 \epsilon_0^2 m_s m_t v_{st\parallel}^2} \ln \Lambda_{st}
\]

is a collision frequency of species \( s \) on species \( t \) (\( \ln \Lambda_{st} \) being the corresponding Coulomb logarithm). Finally, \( F_{\alpha\beta}^{(st)} \) are defined through generalized double hypergeometric or Kampé de Fériet functions (Exton 1976)

\[
F_{11}^{(st)} = e^{-\frac{x^2}{2}} \left( a, b \right)_{c, d} \left( x, y \right)_{a+n} = \sum_{n,k=0}^{\infty} \frac{(a+n)(b)(c+n+k) \cdot n! k!}{k!}
\]

These functions can be represented as double series

\[
F_{11}^{(st)} = \frac{\Gamma(c) \Gamma(a+n)}{\Gamma(a) \Gamma(c-a)} \times \int_0^1 \left( 1 - t \right)^{c-a-1} e^{-\frac{t}{\frac{T_{st}}{T}} d} dt
\]

that may be used for numerical evaluation.

We also define the mean heating rate \( \nu_{Hs}^{(0)} \) as

\[
\nu_{Hs}^{(0)} = \frac{1}{3} \frac{T_{||}}{T_s} \nu_{Hs\parallel} + \frac{2}{3} \frac{T_{\perp}}{T_s} \nu_{Hs\perp}
\]

where \( T_s = (2T_{\perp} + T_{||})/3 \) is the mean temperature of the species.

For the relative deceleration between species \( s \) and \( t \) through Coulomb collisions, one gets (Hellinger & Trávníček 2009)

\[
\frac{d \nu_{st}}{dt} = -\nu_{V} \nu_{st}
\]

where the deceleration frequency \( \nu_{V} \) may be given as

\[
\nu_{V} = \frac{q_s^2 q_t^2 n_{st}}{24 \pi^2 \epsilon_0^2 m_s m_t v_{st\parallel}^2} \ln \Lambda_{st} F_{11}^{(st)}
\]

where \( n_{st} = (n_s m_s + n_t m_t) / (m_s + m_t) \) is a combined number density.

3. COLLISIONAL TRANSPORT COEFFICIENTS

Here we use fitted data from the two Faraday Cup instruments in the SWE on the Wind spacecraft. Wind is a rotating spacecraft with a spin-axis perpendicular to the ecliptic plane and a period of three seconds. A Faraday Cup is an energy/charge instrument with a large, conical field of view that measures the current produced by particles within a given energy window. Proton and alpha particle properties, number densities, and parallel and perpendicular temperatures are obtained using a nonlinear least-squares fitting of data to a theoretical model assuming bi-Maxwellian proton and alpha particle distribution functions and the magnetic field direction obtained from 3 s measurements provided by the Magnetic Field Investigation on the Wind spacecraft. In this paper we use a large statistical data set (about 4 million data points) from 1995 to 2012 (Kasper et al. 2008; Maruca et al. 2012). We use only the data when the Wind spacecraft was situated in the solar wind at about 1 au. (Portions of time when it was inside the magnetosphere before 2004 were removed from the data set.)

For the fitted proton and alpha particle parameters we calculate the different transport coefficients (given in Section 2) approximating the ion–ion Coulomb logarithm by Tracy et al. (2015)

\[
\ln \Lambda_{st} = 29.9 - \ln \left[ \frac{q_s q_t (m_s + m_t)}{e^3 (m_s \tilde{T}_t + m_t \tilde{T}_s)} \left( \frac{n_s q_s^2}{\tilde{T}_t} + \frac{n_t q_t^2}{\tilde{T}_s} \right)^{1/2} \right]
\]

where the temperature anisotropies and differential streaming are neglected. In this expression, \( \tilde{T} \) stands for the temperature of species \( s \) in electronvolts and \( e \) is the proton charge. For numerical evaluation of the generalized double-hypergeometric functions we use the integral representation (Equation 13).

We start with the proton-alpha particle deceleration frequency \( \nu_{V}^{(0)} \) (hereafter we drop the superscript) with respect to the characteristic transit/expansion time \( t_{\varphi} = R/v_{\text{sw}} \), where \( R \) is the radial distance from the Sun (being 1 au here) and \( v_{\text{sw}} \) is the solar wind velocity. The product of a collisional transport coefficient such as \( \nu_{V} \) and the expansion time \( t_{\varphi} \) may be used as a proxy for the collisional age (Kasper et al. 2008), which is defined as the integral value over a relevant time interval \( (t_0, t) \)
of a given collisional frequency \( \nu A_c = \int_0^\infty \nu (t') dt' \) (Salem et al. 2003; Chhiber et al. 2016); here we prefer to interpret \( \nu A_c \) (and other such products) as a way to compare the two local characteristic times.

The left panel of Figure 1 shows the distribution of data in the space \((v_{sw}, \nu_A t_e)\). This distribution (and all the following ones) was obtained by calculating the number of data points in each bin and dividing that by the bin size (Maruca et al. 2012); the results were then globally renormalized to have the maximum value of the distribution equal to 1. We obtain the well-known result that slower streams are typically more collisional. It is interesting to compare the relative deceleration frequency \( \nu \) and the (proton-proton) isotropization frequency \( \nu_T \). The right panel of Figure 1 shows the distribution of data in the space \((\nu_T t_e, \nu_A t_e)\) (see Hellinger & Trávníček 2014). The two collisional frequencies are almost proportional to each other as expected. Note that the deceleration frequency \( \nu A \) is noticeably larger than the proton isotropization one \( \nu_T \) (Matteini et al. 2012).

The relative proton–alpha particle velocity in the solar wind tends to decrease with the radial distance at a rate similar to that of the Alfvén velocity \( v_A \) (Marsch et al. 1982; Verscharen et al. 2015). It is therefore interesting to compare the collisional rate \( d\eta /dR \) and the decrease rate \( dv_A /dR \); we investigate the ratio of the two rates:

\[
\eta = \frac{\nu_A v_A}{v_{sw} d\eta /dR}. \tag{18}
\]

For the evaluation of \( \eta \), we assume that the ion number densities decrease as \( R^{-2} \), and the magnetic field follows the Parker spiral with the angle 45° at 1 au. The obtained results are shown in Figure 2; the left panel shows the distribution of data in the space \((v_{sw}, \eta)\), whereas the right panels show the distribution in \((\eta /v_{sw}), \eta)\). Figure 2 indicates that Coulomb collisions between protons and alpha particles may be sufficient to decelerate the two species with respect to each other at a rate comparable to the decrease rate of \( v_A \) in some slow solar wind streams.

For the alpha particle isotropization frequency \( \nu_T \), one expects much smaller values than for the proton frequency, owing to the smaller alpha particle abundance (and typically larger temperatures). The observations, indeed, show that \( \nu_T \) is about an order of magnitude smaller than (and almost proportional to) \( \nu_T \).

We now look at the heating rates due to the Coulomb collisions between the protons and alpha particles. Figure 3 shows the data distribution in \((v_{sw}, v_H^{(p+\perp)} t_e)\) (top left panel), \((\nu_T t_e, v_H^{(p+\perp)} t_e)\) (top right panel), \((v_{sw}, v_H^{(p+\perp)} t_e)\) (middle left panel), \((\nu_T t_e, v_H^{(p+\perp)} t_e)\) (middle right panel), \((v_{sw}, v_H^{(p+\perp)} t_e)\) (bottom left panel), and \((\nu_T t_e, v_H^{(p+\perp)} t_e)\) (bottom right panel).

Figure 4 shows the data distribution in \((v_{sw}, v_H^{(p+\perp)} t_e)\) (top left panel), \((\nu_T t_e, v_H^{(p+\perp)} t_e)\) (top right panel), \((v_{sw}, v_H^{(p+\perp)} t_e)\) (middle left panel), \((\nu_T t_e, v_H^{(p+\perp)} t_e)\) (middle right panel), \((v_{sw}, v_H^{(p+\perp)} t_e)\) (bottom left panel), and \((\nu_T t_e, v_H^{(p+\perp)} t_e)\) (bottom right panel).
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Figure 4. Alpha particle heating rates through collisions with protons. Color scale plots of the data distributions \( \nu_{H \alpha}^{(\text{pp},T_s)}, t_e \) (top left panel), \( \nu_{H \alpha}^{(\text{pp},T_s)}, t_e \) (top right panel), \( \nu_{H \alpha}^{(\text{pp},T_s), l_T} \) (middle left panel), \( \nu_{H \alpha}^{(\text{pp},T_s), l_T} \) (middle right panel), \( \nu_{H \alpha}^{\text{pp}, l_T} \) (bottom left panel), and \( \nu_{H \alpha}^{\text{pp}, l_T} \) (bottom right panel).

Figure 4 indicates that alpha particles (in slower, more collisional streams) are often cooled through collisions with protons; the energy stored in the proton–alpha particle differential velocity (as well as a part of the alpha particle thermal energy) most likely goes to protons. The cases when alpha particles are heated typically correspond to the cases when protons and alpha particles have comparable temperatures. In these cases, the differential streaming energies are split between protons and alpha particles. Alpha particles are sometimes heated in either parallel or perpendicular directions; this typically happens in the cases of parallel or perpendicular alpha particle temperature anisotropy. Alpha particle collisions with protons also tend to reduce the alpha particle temperature anisotropy.

The expressions for the different collisional transport coefficients are rather complex. It is therefore interesting to test whether it is necessary to take into account the temperature anisotropy or the differential streaming. We estimated the importance of these parameters assuming isotropic populations (with the mean temperature \( T_s = (T_p + 2T_a)/3 \)) and/or neglecting the differential ion streaming (i.e., setting \( v_{\alpha \text{drift}} = 0 \)) for the present data set. The relative error in the isostropization frequency \( \nu_T \) is small (\( \lesssim 10\% \)) when ignoring the temperature anisotropy for protons and alpha particles. However, the relative error in the relative deceleration frequency \( \nu_V \) can be quite large (\( \lesssim 40\% \)) when ignoring the temperature anisotropy and/or the differential streaming. In the case of the (total) collisional heating rates \( \nu_{H \alpha} \), the difference between the full (drifting and anisotropic) version and that which neglects the temperature anisotropy and/or the differential streaming, \( \Delta \nu_{H \alpha} \), could be of the order of 0.1/\( t_e \) for protons and 1/\( t_e \) for alpha particles (the parallel and perpendicular heating rates typically have somewhat larger errors). These values are comparable to the maximum (absolute) values of the heating rates (see Figures 3 and 4).

4. DISCUSSION

We quantified proton and alpha particle collisional transport coefficients at 1 au using the Wind/SWE data. In agreement with previous studies, our results show that ion–ion Coulomb collisions are generally important for slow solar wind streams; they tend to reduce the differences between the ion mean velocities and temperatures, as well as the ion temperature anisotropies. The different collisional frequencies have typically disparate values leading to different collisional ages. The two isostropization frequencies and the relative deceleration frequency are nearly proportional each to other, so one expects that the corresponding collisional times will be also proportional. On the other hand, the heating rates are not simply related to these frequencies, and, moreover, these heating rates have positive and negative values.

The observations indicate that the relative proton–alpha particle collisional deceleration is more efficient than the proton isostropization (and the alpha particle isostropization); on the other hand, proton–alpha particle collisions also tend to reduce both the proton and alpha particle temperature anisotropies. (In a relatively small number of cases, the proton–alpha particle collisions enhance the ion temperature anisotropy.) In some slow solar wind streams, the Coulomb collisions are sufficiently strong to reduce the differential velocity between proton and alpha particles at a pace that is comparable to that of the decrease rate of the Alfvén velocity.

Protons are typically heated through collisions with alpha particles, whereas alpha particles are very often cooled; this is a consequence of the temperature difference between the two species: alpha particles are typically hotter than protons. When protons and alpha particles have comparable temperatures, both the species are typically heated at the expense of the relative proton–alpha particle velocity. Our results indicate that the proton heating through collisions with alpha particles in slow solar wind streams reaches values \( \Delta T_p \sim 0.2 T_p / t_e \) at 1 au. This value is an important fraction of the needed average proton heating rate estimated from the Helios observations for an average slow solar wind \( \Delta T_p / dt \text{CGL} \sim 0.45 T_p / t_e \) (Hellinger et al. 2013). In the fast solar wind the needed proton heating rate has similar values \( \sim 0.32 T_p / t_e \), but the collisional proton heating rates are much smaller. Coulomb collisions with alpha particles are not energetically important for protons in the fast solar wind at 1 au.

The collisional interaction of alpha particles with protons may be important in slow solar wind streams. Similarly, Tracy et al. (2015) show that Coulomb collision with protons may be also important for other minor ions. It is possible that the (summed effect of the) interaction between protons and other minor ions may have a nonnegligible heating effect on protons; the abundances of the other minor ions are much lower than the abundance of alpha particles, but together they have about 1% and are about mass-proportionally hotter than alpha particles.

Our results show that the different parallel and perpendicular temperatures and differential velocities between the ion species need to be taken into account for the collisional transport
coefficients, especially in slow/collisional solar wind streams. For the given data set of the Wind/SWE observations at 1 au, neglecting the ion temperature anisotropy and/or the differential streaming leads to relatively large errors. Consequently, it is important to use the general, anisotropic, and drifting approximation for the collisional transport coefficients, particularly for modeling slow solar wind streams where the induced error cumulates. The present results have (at best) theoretical uncertainties of the order of 1/lnΛ. Equation (17) gives the 4%–5% uncertainties for the ion–ion interaction. Equation (17) is simplified; the temperature anisotropy and the differential streaming are neglected. It would be interesting to extend Equation (17) to include these effect, but the Coulomb logarithm is only weakly/logarithmically dependent on the plasma parameters, so we do not expect significant changes. On the other hand, our model assumes bi-Maxwellian particle velocity distribution functions drifting with respect each other along the ambient magnetic field. If the ion distribution functions in the solar wind strongly depart from this model, these collisional transport coefficients are likely not applicable and further work is needed. For instance, the solar wind protons often consist of two populations (Marsch 2006); such a velocity distribution function can to some extent be modeled as a superposition of two drifting bi-Maxwellian velocity distribution functions and the present model can be directly applied.

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