ARTICLE TYPE

Replacing wakes with streaks in wind turbine arrays

Carlo Cossu*

Summary

Wind turbine wakes negatively impact downwind turbines in wind farms reducing their global efficiency. The reduction of wake-turbine interactions by actuating control on yaw angles and induction factors is an active area of research. In this study, the capability of spanwise-periodic wind turbine arrays with tilted rotors to reduce negative turbine-wakes interaction is investigated by means of large-eddy simulations. It is shown that by means of rotor tilt it is possible to replace turbine far wakes with high-speed streaks where the streamwise velocity exceeds the freestream velocity at hub height. Considering three aligned arrays of wind turbines, it is found that the global power extracted from the wind can be increased by tilting rotors of upwind turbine arrays similarly to what already known for the case of a single row of aligned turbines. It is further shown that global tilt-induced power gains can be significantly increased by operating the tilted turbines at higher induction rates. Power gains can be further increased by increasing the ratio of the rotor diameters and turbine spacing to the boundary layer thickness. All these findings are consistent with those of previous studies where streamwise streaks were artificially forced by means of arrays of wall-mounted roughness elements in order to control canonical boundary layers for drag-reduction purposes.

KEYWORDS:
wind farm control, wake redirection, boundary layer streaks, wind energy

1 INTRODUCTION

In wind farms, turbines impacted by wakes generated by upwind turbines experience significant reductions in the mean available wind power and increased turbulence levels. A significant number of design and control strategies have been proposed to alleviate these negative effects among which great interest has been recently attracted by the approach where the rotor yaw angle is controlled in order to deflect the wake away from downwind turbines. In yawed turbines, indeed, the misalignment of the mean thrust force and wind direction induces a pair of vertically-stacked counter-rotating vortices which increasingly deflect the wake away from the mean wind axis in the horizontal plane. The thrust-wind misalignment reduces the amount of power produced by the yawed turbine but it has been shown that this power loss can be more than compensated by the power gain of downwind turbines induced by the wake deflection.

In complement to yaw control, which is associated to wake deflections in the horizontal plane, it has been recently shown that turbine wakes can be deflected in the vertical direction by acting on the rotor tilt angle and that the power gain in downwind turbines can be larger than the power reduction associated to the tilt of upwind turbines. Best performance was obtained with positive tilt angles for which the wake is deflected towards the ground. Furthermore, power gains obtained by means of tilt were found to be potentially larger than those associated to yaw control because the downwash associated to the positive tilt exploits wind shear by vertically displacing higher-altitude higher-velocity wind towards downwind rotors therefore increasing the available power.

Despite its potential, however, tilt control has been the subject of only a few studies because it is not possible to implement positive tilt angles in turbines with upwind rotors. Positive tilt capabilities can, however be implemented in downwind turbines which are being revisited as
very promising concept because they are resilient in extreme wind situations and are compatible with highly flexible very large blades. Indeed, downwind turbines admit favourable distributions of blade bending loads and benefit from passive yaw control capabilities which are critical in off-grid situations experienced in extreme wind conditions.

Previous investigations of wind farm control by means of tilt have mostly considered the effect of tilting rotors of upwind turbines in single-row aligned turbines configurations. For two-turbines configurations, the best power gains were obtained for positive tilt angles $\varphi \approx 25^\circ$ and, for three-turbines configurations, when both upwind turbines were tilted. A more recent analysis, based on an engineering wind-farm model, has considered global annual power gains (for selected wind roses) of model wind farms where only perimeter turbines were tilted for fixed tilt setting (not depending on the wind direction). It was found that tilt could produce gains of annual power production that were larger for 5MW wind turbines than for 13MW wind turbines. The best gains were obtained for tilt angles smaller than what found for for single-row configurations (actually it was found that a power reduction was experienced for $\varphi \approx 25^\circ$).

The present study complements the few previous investigation on tilt control by further considering the effect of tilt applied to spanwise-periodic arrays of wind turbines. In this case a spanwise periodic distribution of counter-rotating quasi-streamwise vortices is forced by the tilted turbines inducing a spanwise-periodic distribution of upwash and downwash flows. Spanwise-periodic distributions of counter-rotating vortices, when immersed in shear flows, are known to induce quasi-streamwise streaks, i.e. streamwise-elongated spanwise-alternating high-speed and low-speed regions which are ubiquitous in transitional and fully developed turbulent shear flows.

The streaks are amplified via the lift-up effect which is a non-modal amplification mechanism which has been exploited as a natural control amplifier for flow control purposes.

Artificially forced streaks have indeed been used to delay transition in laminar boundary layers to reduce pressure drag on idealized car models at high Reynolds numbers to reduce the turbulent friction drag in pipes and to suppress vortex shedding in bluff-body wakes. In this context, spanwise-periodic arrays of tilted wind turbines display a strong similarity to the spanwise periodic arrays of roughness elements used in previous experimental studies of flow control by streaks which, similarly to the tilted turbines, produce spanwise-periodic distributions of wakes and counter-rotating vortices. In these flow-control studies it was found that the in the downstream axis of the roughness the wake; i.e. the downstream region of mean streamwise velocity deficit, was replaced, further downstream, by high-speed streaks, i.e. regions of streamwise velocity excess. We are interested in verifying if a similar effect can be observed in the atmospheric surface layer with forcing given by the tilted rotors of a wind turbines array.

In the present study we will therefore determine if high-speed coherent streaks can be forced by a spanwise-periodic array of wind turbines with tilted rotors and if the total power of model wind farms can be increased by forcing these streaks spanwise-periodic coherent streaks. The effect of changing the tilt angle and the induction factor of the tilted turbines will be also investigated as well as that of increasing the relative size and spacing of the wind turbines with respect to the boundary layer thickness. These effects will be explored by means of large-eddy simulations where wind turbines are modeled by means of the the actuator-disk method.

The paper is organized as follows. The formulation of the problem at hand is introduced in §2 the streaky flow forced by a single spanwise array of turbines is described in §3. The effect of forcing the streaks on the power production of three arrays of wind turbines is presented in §4 where the effect of turbine size, tilt and induction factor are discussed. The main results are summarized and further discussed in §5. Additional details on used numerical methods are provided in Appendix A.

## 2 PROBLEM FORMULATION

We consider the flow developing around a set of wind turbines immersed in a turbulent boundary layer. The turbines are aligned with the mean wind speed at hub height (zero yaw angle). This complex turbulent flow is simulated by means of large-eddy simulations implemented in the Simulator for On/Offshore Wind Farm Applications (SOWFA) where the flow is modeled with the filtered Navier-Stokes equations under the usual Boussinesq approximation for the effects of density variations. Subgrid-scale stresses are modeled with the Smagorinsky model and it is assumed that near the ground the flow adheres to the Monin-Obhukov similarity theory for turbulent boundary layers above rough surfaces by implementing appropriate stress boundary conditions. Slip boundary conditions are enforced at the top plane $z = H$ of the solution domain. Additional details about the used numerical methods and the discretization parameters used in the simulations are provided in Appendix A.

We will limit our analysis to the case of an isothermal flow (neutral boundary layer) driven by a constant pressure gradient neglecting the effect of Coriolis acceleration. The results obtained under these strong assumptions are a reasonable approximation of those that would have been obtained in a neutral atmospheric boundary layer if wind turbines remain confined to the atmospheric surface layer.

Inflow boundary conditions for the simulations are generated by means of ‘precursor’ simulations of the turbulent boundary layer in the absence of wind turbines where periodic boundary conditions are enforced in the horizontal plane with streamwise-spanwise periodicity (the extension of the domain). Once a fully developed statistically stationary regime is attained, the temporal evolution of flow variables on the inflow
plane is stored and then used as inflow boundary condition for the simulations with the wind turbines. In this way, it is possible to expose the turbines to realistic inflow turbulent wind conditions.

The effect of wind turbines on the flow is accounted for by means of the the actuator disk model (ADM) which has been shown to correctly reproduce the main characteristics of turbines wakes except in the wake formation region. In the chosen ADM approach the forces exerted by turbines blades on the fluid are averaged over the whole rotor disk. Following previous investigations, the total force exerted by each turbine on the fluid is assumed to be 

\[ F = -C_T \rho U_D^2 A e_{\perp} / 2, \]

where \( C_T \) is the disk-based thrust coefficient, \( e_{\perp} \) is the unit vector aligned with the rotor axis, \( U_D \) is wind velocity component along \( e_{\perp} \), averaged over the rotor surface of area \( A = \pi D^2 / 4 \) and \( D \) is the rotor diameter. The force is assumed to be uniformly distributed over the rotor surface and the effects of wake rotation are neglected.

Turbines are assumed to always operate in Region II. The power produced by each turbine is \( P = C_T \rho (U^3 / 2) \) where \( C_T = \chi^D \) with the coefficient \( \chi = 0.9 \) accounting with the power lost by wing-tip vortices. The conventional thrust and power coefficients \( C_T, C_P \) which are based on the mean incoming velocity \( U_\infty \) far upstream of the turbine can be expressed in terms of \( C_T \) using the one-dimensional momentum theory for ideal turbines, as \( C_T = 16C_T/(C_T + 4)^2 \) and \( C_P = 64C_T/(C_T + 4)^3 \). The optimal Betz value maximizing the power output of an isolated ideal turbine is obtained for \( C_T = 2 \).

3 | FORCING STREAKS BY TILTING ROTORS

Simulation setting. We first consider the effect of tilt on the wakes of a (single) spanwise-periodic array of wind turbines. The flow is simulated in a domain extending 1 km in the vertical direction (which corresponds to the boundary layer thickness \( H \)) and 6 km x 0.5 km (i.e., 6 \( H \) x H/2) in the streamwise and spanwise direction respectively. The relatively short lateral extension of the domain allows for the analysis of ‘pure’ coherent structures generated by tilted rotors excluding their interactions with boundary layer large-scale motions which have larger spanwise extension (typically in the range \( H \sim 3H \)). The lateral domain extension corresponds to the spacing \( \lambda \) of the turbines of the array. Actuator disk models (\( D = 126 \) m, hub height \( z_h = 89 \) m) are based on the NREL 5-MW turbine model. The chosen ratio \( \lambda / D = 4 \) is equal to the one used in previous investigations of streak generation by arrays of roughness elements of diameter \( D \). Turbines are located at \( x_h = 500 \) m, \( (\approx 4D) \) downstream of the inflow boundary, their wake is simulated up to \( \approx 40D \) and they operate at constant \( C_T = 1.5 \) corresponding to \( C_T = 0.8 \) and \( C_P = 0.577 \) values consistent with those observed in real wind farms.

The precursor simulation is run in the chosen domain with periodic boundary conditions allowing the complete development of the turbulent boundary layer in the absence of wind turbines with the applied constant pressure gradient \( dP/dx = -0.5 \times 10^{-3} \) Pa/m. The mean incoming wind velocity at hub height is \( U_0 = 13 \) m/s. The velocity and pressure fields on the inflow (west) plane are stored and used to rerun the simulation in the presence of the wind turbines which are operated starting from \( t_0 = 20000 \) s. Statistics are accumulated starting from \( t = 24000 \) s when the wakes are well developed, up to \( t = 40000 \) s. First, a reference case is run where the rotor has the usual small negative tilt \( \varphi = -5^\circ \) enforced to prevent any impact of the blades on the tower. Then, the runs are repeated with same parameters except for the rotor tilt angle \( \varphi \).

Streaks formation and wake reversal. From Fig. 1 showing the time-averaged streamwise velocity field in a longitudinal vertical slice through the rotor axis, it can be verified that a positive rotor tilt induces the deflection of the wake towards the ground strongly reducing its streamwise extent when compared to the reference case. The wake-shortening effect is also clearly visible in Fig. 2 representing the time-averaged streamwise velocity field in the horizontal plane at hub height for the reference case and for increasing tilt angles. From Fig. 2 it can also be seen that for sufficiently large tilt angles the wakes are not only shortened but replaced by high-velocity regions (high-speed streaks) where the mean streamwise velocity is higher than the mean free-stream velocity at hub height (\( U_0 = 13 \) m/s). The process by which wakes are replaced by high-speed streaks can be appreciated in Fig. 3 where the time-averaged velocity fields of the reference and positive tilt \( \varphi = 30^\circ \) cases are shown in the cross-stream planes situated \( 7D \) and \( 20D \) downstream of the turbine, respectively. The two-counter rotating vortices
produced by the positive rotor tilt are clearly visible just as is the downwash they induce downwind of the rotor which produces the wake deflection towards the ground and its replacement by the high-speed streak further downstream.

From Figs. 2 and 3 it can be seen that in the middle and far-wake regions not only the low-speed region (wake) is replaced by high-speed streaks but low-speed fluid is repositioned laterally in the streamwise corridors between the turbines (recall that a single spanwise wavelength is shown in those figures). Also, the spanwise size of the wakes and of the high-speed streaks, which are of the order of the rotor diameter $D$ in the near wake region increase towards $\approx \lambda/2$ (half the turbine separation) in the far wake region, as can also be appreciated from Fig. 2. These features are similar to those observed when streaks are forced by roughness elements in canonical laminar and turbulent flat-plate boundary layers, where high-speed streaks emerge downwind of the roughness elements and low-speed streaks replace the high-speed regions in the corridors.

**Direct and indirect effects of tilt on wakes.** From Fig. 2c it can be seen that the downwash associated to the counter-rotating vortices generated by the tilted rotors decays downstream, just as observed in flat-plate boundary layers. However, in the present case the rotor tilt has two separate important effects on the wake in what concerns streamwise velocities. The first, direct, effect of tilt is the reduction of the initial
velocity deficit in the wake associated to the reduction of the streamwise component of the thrust vector. This reduction, which induces reductions of extracted wind power, increases with tilt angles as can be appreciated in Fig. 4(a). The second, indirect, effect is that the forced counter-rotating vortices redistribute momentum in the vertical direction increasing speeds downstream of the rotor where higher-located higher-speed fluid is displaced downwards (this is the famous lift-up effect\textsuperscript{[13]}. The respective weight of those two effects can be appreciated by examining Fig. 4(b) where are reported the streamwise evolutions of the \( \Delta U \) differences between the mean streamwise velocity in the wake of tilted rotors and that of the reference wake. The initial positive \( \Delta U \) (observed for the smallest distances from the rotor) is a result of the direct tilt effect; if only this effect was at play, one would observe only decaying \( \Delta U(x-x_h) \) curves starting from different initial values, just as in the case of a reduction of the turbine thrust coefficient. However, the indirect effect, associated to lift-up, induces the transient growth of \( \Delta U(x-x_h) \). It is this latter effect that makes possible the replacement of the wake by high-speed streaks where the streamwise velocity is higher than the incoming mean flow speed at hub height.

Fig. 4(b) also reveals that the tilt-induced maximum absolute gain in streamwise velocity with respect to the reference case is obtained roughly four diameters, i.e. one spanwise wavelength \( \lambda = 4D \) downstream of the turbine and is relevant up to \( \approx 8D = 2\lambda \) downstream. The best streamwise spacing of turbines with tilt control will therefore be a compromise between having a large power gain induced by the tilt (close spacing) and allowing sufficient wake recovery (larger spacing).

4 \quad FORCING HIGH-SPEED STREAKS TO INCREASE GLOBAL EFFICIENCY

4.1 \quad Influence on tilt angle and thrust coefficient on power gains

Simulation setting. We now consider the potential power gains that can be obtained by tilting rotors of upwind turbines in multiple-array configurations. Preliminary tests show that, similarly to the case of two turbines,\textsuperscript{[4]} moderate global power gains can be obtained with two spanwise arrays of wind turbines by tilting the rotors in the upwind array. A more recent study has shown that higher power gains could be obtained in the case of three turbines aligned in a single row, when both the upwind and the middle arrays rotor are tilted.\textsuperscript{[8,9]} New simulations are therefore performed with three arrays of ADM models of NREL 5-MW turbines with 4D spanwise spacing in each array and corresponding turbines wind-aligned. A 7D streamwise spacing of the turbine arrays is considered (as in previous studies of tilt control in two and three-turbines settings\textsuperscript{[13,14]}). This value is large enough to attain sufficient absolute values of the mean streamwise velocity in the wake of upwind turbines (see Fig. 5), but it does not exceed by too large amount the value \( \approx 4 - 6D \) where the extra velocity recovery due to the tilt is maximum (see Fig. 4).

Simulations are run in a domain with the same height (1 km) and pressure gradient as in \textsuperscript{[13]} but with a different 3km x 3km horizontal extension which permits large-scale motions to develop in the turbulent boundary layer\textsuperscript{[13,14]} allowing for reliable statistics of turbines power production. The usual precursor simulation is run and used to generate the inflow boundary conditions that are used for the simulation in the presence of the turbines. The presence of a large-scale coherent boundary layer low-speed streak can indeed be clearly discerned in Fig. 5. Simulations are then performed in the presence of the 3 aligned arrays of 6 turbines which can be accommodated in the simulation domain with the chosen streamwise and spanwise spacing (see Fig. 5). The upwind array is situated 4D downstream of the inflow boundary where the mean incoming wind velocity at hub height is \( U_0 \approx 13 m/s \). Statistics are accumulated from \( t=24000s \) (more than one hour after the turbines are switched on at \( t=20000s \)) to \( t=30000s \).

First, the reference case is simulated with all turbines operating in the same conditions (\( \varphi = -5^\circ \), \( C_f = 1.5 \)). For this case, the usual situation where the wake of upwind turbines strongly reduces the mean wind seen by the aligned downwind turbines is observed, as shown in Figs. 5(b)
FIGURE 5 Instantaneous (top panels a, b, c) and time-averaged (bottom panels d, e, f) streamwise velocity field in the horizontal plane at hub height $z_h = 89\, \text{m}$ for the precursor simulation (left panels a, d), the reference case (middle panels b, e) and the case where rotors of the upwind and middle arrays are tilted by $\varphi = 30^\circ$ (right panels c, f). The same color scale is used in all panels. All turbines have $D = 126\, \text{m}$ and are operated at $C'_T = 1.5$. The flow is from the west (left to right). The signature of a persistent large-scale boundary layer low-speed streaks is clearly discernible near $y \approx 1600$ in all panels.

and Fig. 5(e). The runs are then repeated with all conditions unchanged except for the rotor tilt of the upwind and middle turbine arrays. Preliminary computations (not shown) indicate that the best power gains are obtained when the rotors of the upwind and the middle arrays are both tilted. Results are shown for the case where they are tilted by the same angle $\varphi$, for selected values of $\varphi$.

Influence of tilt control angle at fixed thrust coefficient. Instantaneous and mean streamwise velocity fields are reported in Fig. 5 in the plane at hub height for the precursor simulation, the reference case and the case with $30^\circ$ tilt of the rotors of the upwind and middle turbine

FIGURE 6 Time-averaged vertical velocity field in the horizontal plane at hub height $z_h$ for the reference case (panel a) and the case where rotors of the upwind and middle arrays are tilted by $\varphi = 30^\circ$ (panel b). The same color scale is used all panels. Turbines have $D = 126\, \text{m}$ and are operated at $C'_T = 1.5$. 

arrays. Mean vertical velocity fields at hub height are reported in Fig. 6. From these figures it can be seen that, as already observed in the case of three aligned turbines and two aligned arrays of roughness elements in a plane channel the effects of rotor tilt in the two upwind arrays are almost additive resulting in stronger vertical downwards velocities (downwash) and wake recoveries downwind of the middle array of turbines than downwind of the most upwind array.

In Fig. 7(a) the mean total power $P$ produced by the 18 turbines, when the rotors of 12 of them are tilted is compared to the mean power $P_{Ref}$ produced in the reference condition. From this figure it is seen that the effect of increasing the positive rotor tilt $\varphi$ is to decrease the mean power extracted by the most upwind array of turbines (because of the reduction of the normal momentum flux trough the tilted rotor) and to increase that of the last (most downwind) array of turbines (because of the increase of the mean streamwise velocity on the rotor). A milder variation is observed for the power extracted by the middle array (where the two contrasting effects are at play). Overall, the beneficial effects overcome the detrimental effect of the tilt resulting in a small total power increase with respect to the reference case, which is maximal for $\varphi \approx 25^\circ - 30^\circ$.

**Influence of an increased induction in tilted turbines.** We note that an increase of $C_T$ from 1.5 to 3 ($C_T$ increased from 0.79 to 0.98) in the tilted turbines results in an increased vertical component of the thrust enhancing the streamwise vortices at the cost of a only moderate reduction of the produced power ($C_T$ is reduced from 0.577 to 0.56). The simulations with tilted rotors have been therefore repeated by operating the tilted rotors at $C_T = 3$ while leaving the turbines with non-tilted rotor at the nominal $C_T = 1.5$. The results, reported in Fig. 7(b), show that operation of the tilted rotors at $C_T = 3$ leads to a substantial increase of the tilt-induced power gain, with an almost tripled maximum power gain obtained near $\varphi = 30^\circ$. This indicates that there certainly is room for further enhancement of tilt-induced power gains by optimizing $\varphi - C_T$ combinations in wind farm operation. We leave such an optimization for future study.

### 4.2 Influence of the relative rotor size on power gains

The computation of optimal perturbations of canonical turbulent wall-bounded flows indicates that the largest energy amplifications of coherent streamwise streaks can be attained when the spanwise spacing of the streaks and of the vortices used to force them is of a few boundary layer thicknesses requiring roughness elements with diameters of the order of the boundary layer thickness. The results reported in the previous sections, obtained with actuator disks based on the NREL 5-MW turbines correspond to a ratio $D/H = 0.126$ which is an order of magnitude smaller than optimal ratios. Indeed optimal $D/H = O(1)$ ratios can not be considered in the present setting where the boundary layer thickness coincides with the vertical extension of the solution domain where (horizontal) slip boundary conditions are enforced. However, even if relatively far from the values of optimal spanwise spacing, a moderate increase of the amplification of the streaks issued from quasi-streamwise vortices of given energy can be expected if the $D/H$ ratio is, even moderately, increased.

We therefore consider two reasonably larger ratios $D/H = 0.18$ (approximately corresponding to the DTU 10-MW turbine model with $D = 178\,m$ immersed in a 1km-thick boundary layer) and $D/H = 0.36$ (obtained by further halving the boundary layer thickness or doubling the turbine diameter) for three arrays of turbines keeping constant to 4 $D$ and 7 $D$ their relative spanwise and streamwise spacing. Additional simulations are therefore performed with 4x3 turbines with $D/H = 0.180$ in the 3km x 3km x 1km domain considered in §4.1 and 2x3 turbines...
FIGURE 8 Effect of turbine size relative to the boundary layer thickness on the streamwise velocity in the horizontal plane at hub height for (a) the previously considered model 5MW turbines in the 1km-thick boundary layer (\(D/H = 0.126\)), (b) the intermediate ratio \(D/H = 0.18\), and (c) the largest ratio \(D/H = 0.36\). Upwind and middle arrays turbines are operated with \(\phi = 30^\circ\) and \(C'_T = 3\). Turbines of the most downwind (eastward) array are operated at reference parameters values. The mean wind is from the west (left to right). Same color scale in all panels.

in an additional 3km x 6km x 1km domain as shown in Fig. 8. Building on the previous findings, the same tilt angle is enforced on all rotors of the upwind and middle array turbines which are operated with \(C'_T = 3\), while the most downwind array is operated at reference values.

The additive wake-shortening effect induced by the rotor tilts is seen to operate similarly for all the considered \(D/H\) ratios, as the relative spanwise and streamwise turbine spacing have been kept constant. As anticipated, however, the mean power gain are increased for larger \(D/H\) ratios, as reported in Fig. 9 up to a maximum observed power gain exceeding 40% for the largest considered value of \(D/H\). Despite their very encouraging nature, however, the latter results should be taken with care because for \(D/H \gtrsim 0.2\), turbines are no more confined into the logarithmic region of the considered pressure-driven boundary layer which has a structure similar to that of the atmospheric surface layer. In this case, streak amplifications cease to be ‘universal’ but depend on the particular structure of the flow in the outer layer. Additional work is therefore needed to confirm and extend the results obtained for the largest \(D/H\) ratio in more realistic atmospheric boundary layer settings.

FIGURE 9 Effect of turbine diameter to boundary layer ratio \(D/H\) on the variation of the total produced power \(P\) with respect to the one produced in the reference case \(P_{\text{ref}}\) for selected tilt angles \(\phi\) when the tilted turbines are operated at \(C'_T = 3\).

5 | CONCLUSION

This main goals of this study were to determine if rotor tilt in spanwise-periodic arrays of wind turbines could be used to replace turbine (low speed) wakes with high-speed streaks and determine the global power gains that could be obtained by forcing these streaks in order to enhance the power production of downwind turbines.

It is found that spanwise-periodic arrays of wind turbines with tilted rotors can be effectively used to force spanwise-periodic distributions of quasi-streamwise vortices which, for sufficiently large positive tilt angles, are able to ‘reverse’ the wakes by replacing them with high-speed streaks. The observed wake-reversal is similar to that documented in previous investigations where a spanwise-periodic array of cylindrical roughness elements was used to force streaks in canonical flat-plate boundary layers. However, contrary to these related previous results, while streamwise velocities higher than the mean incoming freestream velocity at hub height can be found in the far wake, the maximum of the mean streamwise velocity recovery induced by the rotor tilt is found to be localized at edge the near-wake region (\(\approx 3 - 4D\) downwind of the rotor).
The analysis of the power production of three aligned spanwise-periodic arrays of wind turbines confirms that the total mean power can be increased by tilting the rotors in the front (upwind) and the middle turbine arrays, with best performances obtained when $\varphi \approx 25^\circ - 30^\circ$ for the considered configurations. For these parameters, the increase of the power produced by the turbines in the downwind array outperforms the power loss experienced by turbines with tilted rotors. The relative power gains obtained with NREL 5-MW type turbines operated at the usual $C_T^* = 1.5$ (i.e. $C_T = 0.79$ and $C_P = 0.577$) are similar to those found for three turbines in a row by including, unlike in this study, the effects of wake rotation, radial distribution of the aerodynamics forces on the blades, Coriolis acceleration and the capping inversion. This demonstrates the robustness of the mechanisms underlying power gains obtained by means of rotor tilts.

It was then verified if further power gains could be obtained by increasing the thrust coefficient of the tilted turbines. Such an increase would not lead to excessive mechanical loads on the turbines because tilt does reduce the horizontal component of the thrust force converting it into a (positive, upward) lift. It is found that operating the tilted turbines at $C_T^* = 3$ (i.e. $C_T = 0.98$ and $C_P = 0.56$) instead of $C_T^* = 1.5$, tilt-induced power gains can be tripled. It is believed that additional power gain improvements can be achieved by means of a systematic optimization of $\varphi - C_T^*$ distributions for tilted turbines that we leave for future study.

It is additionally shown that further substantial tilt-induced power gains are obtained with ratios $D/H$ and $\lambda/H$ of turbines diameter and spanwise-spacing to boundary-layer thickness larger than those of the NREL 5-MW turbines immersed in a 1 km-thick boundary layer ($D/H = 0.126, \lambda/H = 0.5$). This is consistent with previous results showing that maximum amplifications of coherent large-scale streaks in turbulent boundary layers are obtained for $\lambda/H$ ratios of a few units. Further investigations are, however, needed to confirm and extend these results in the $D/H = O(1)$ regime where realistic atmospheric boundary layer profiles and the effect of Coriolis acceleration must be taken into due account. This high-$D/H$ regime is not only of interest for futuristic very-large turbines but also for current-generation turbines operating in shallow atmospheric boundary layers, such as the nocturnal boundary layer. In the latter case, furthermore, the higher vertical velocity gradients, promoting more efficient streak amplification, coupled with reduced turbulent levels in the incoming flow, associated to poorer wake recovery in the reference case, might lead to dramatic power gains. This is the subject of current intense research effort.

Additional investigations are also needed to explore the benefits of tilt control in deep turbine arrays of large wind farms and for a complete range of wind directions. Configurations with peripheral tilted turbines acting on a wind-farm have been very recently investigated for constant-tilt zero-yaw and constant $C_P$ operation mode. It would be very interesting to extend such type of investigations using optimized $\varphi - C_T^*$ combinations possibly complemented with yaw control to optimally target the forced high-speed streaks to downstream turbines.

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APPENDIX

A METHODS

The standard numerical schemes and parameters implemented in SOWFA\(^{35}\) and built on standard OpenFOAM (release 2.4.x) solvers are used to solve the filtered Navier-Stokes equations with Boussinesq fluid model and Smagorinsky\(^{36}\) modeling for the subgrid scale motions. The PIMPLE scheme is used for time advancement. Schumann’s stress boundary conditions\(^{38}\) are enforced at the near-ground horizontal boundary.

Numerical simulations of the considered turbulent boundary layers have been performed in \(L_x \times L_y \times H\) numerical domains of (vertical) height \(H\), (streamwise) length \(L_x\) and width \(L_y\). Three domains have been considered. Domains D1 (6km x 0.5km x 1km) and D2 (3km x 3km x 1km), both discretized with 15m x 14m x 14m cells, have been used for the simulations implying actuator disks with the dimensions of the NREL 5-MW (\(D/H = 0.126\)) and DTU 10-MW (\(D/H = 0.18\)) turbines. Domain D3 (6km x 3km x 1km), discretized with 21.4m x 20 x 14m cells, has been used for the simulations implying actuator disks with \(D/H = 0.36\). The solutions are advanced with \(\Delta t = 0.8\) s time steps which satisfy the CFL <0.45 constraint and maintain reasonable the amount of data stored in precursor simulations.

The original actuator disk (ADM) turbine model implemented in SOWFA, which includes wake rotation effects as well as the blade-derived radial dependence of the forces acting on the fluid\(^{39}\), has been modified to implement the in-house ADMC model used in the present study by: (a) keeping the same discretization points on the disk but distributing the body force uniformly in the radial direction, (b) setting the body force magnitude \(F = -C_{T'} \rho U_0^2 A e / 2\) removing its dependence on the turbine controller, (c) removing body force components parallel to the rotor plane (inducing wake rotation). In this way, the turbine response only depends on \(C_{T'}\) and not on the turbine controller settings, simplifying the interpretation of the results. The Gaussian projection of the control-point-discretized body forces with a smoothing parameter \(\varepsilon\) is left unchanged with \(\varepsilon = 20m\) for simulations in domains D1 and D2 and \(\varepsilon = 30m\) for simulations performed in domain D3.