The Current State of Coherent States*

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Abstract

The original canonical coherent states could be defined in several ways. As applications for other sets of coherent states arose, the rules of definition were correspondingly changed. Among such rule changes were a change of group and relaxation of the analytic nature of the labels. Recent developments have done away with the group connections altogether and thereby allowed sets of coherent states to be defined that are temporally stable for a wide variety of dynamical systems including the hydrogen atom. This article outlines some of the current trends in the definitions and properties of present-day coherent states.

Introduction

The modern reincarnation of what are now often called canonical coherent states began in 1960 [1] (with a mathematical-physics application to define coherent state path integrals), in 1961 [2] (with a thorough mathematical study), and in 1963 [3] (with a physics application central to the new theory of quantum optics). Over the years, generalizations of the original family of canonical coherent states have been introduced based largely on mathematical or possibly mathematical-physics grounds. These generalizations have frequently involved one or another of the mathematical properties of the

*Contribution to the 7th ICSSUR Conference, June 2001.
canonical coherent states and its elevation to the central concept in defining new sets of coherent states. As examples, we cite group-defined coherent states \cite{1,2,3}, annihilation-operator-eigenstate defined coherent states \cite{4}, and minimum-uncertainty-state defined coherent states \cite{5}. Such generalizations typically lead to new sets of coherent states alright, but (apart perhaps from the group-defined coherent states) such rules for generating new sets of coherent states have always seemed to the present author to be overly mathematical and rather divorced from any specific physics. After all, what is the physics involved in choosing annihilation-operator eigenstates or in choosing minimum uncertainty states? What would be so wrong in choosing states for which the minimum uncertainty product was exceeded by a factor of three, for example?

These views have in recent years prompted the author to seek other generalizations of the canonical coherent states often with specific physical criteria chosen as the key factor involved in defining and obtaining such generalizations. Although other prescriptions exist, we shall, in the interests of brevity and consistency, pursue just one path among many in our discussion of new sets of coherent states.

A few introductory remarks are useful: For convenience, we denote each of the coherent states by $|l\rangle \in \mathcal{H}$, $|l\rangle \neq 0$, where $l = (l^1, l^2, \ldots, l^L) \in \mathcal{L}$, $l^j \in \mathbb{R}$, denotes an $L$-dimensional (real) label lying in a label space $\mathcal{L}$ which locally is topologically equivalent to $\mathbb{R}^L$. This latter property means that we can identify continuous functions on $\mathcal{L}$. It is often useful to regard $l$ as a classical variable in a classical (phase) space $\mathcal{L}$. Although we shall not generally do so, it is often useful to group some or all of the real parameters by pairs and to form complex parameters. Throughout, we choose units so that $\hbar = 1$.

With these remarks as background, we start with what we regard as the basic minimum properties for any set of states to be called a set of coherent states:

1. **Continuity of Labeling:** The map from the label space $\mathcal{L}$ into the Hilbert space $\mathcal{H}$ is strongly continuous.

   **Comment:** Specifically, this condition requires that the expression $\| |l'\rangle - |l\rangle \| \to 0$ whenever $l' \to l$ in $\mathcal{L}$. This condition is equivalent to the joint continuity of the coherent state overlap function, $\langle l'' | l' \rangle$, in its two arguments.
2. Resolution of Unity: A positive measure $\mu(l)$ on $\mathcal{L}$ exists such that the unit operator $1$ admits the representation

$$1 = \int_{\mathcal{L}} |l\rangle \langle l| \, d\mu(l),$$

where $|l\rangle \langle l|$ denotes the rank-one operator that takes an arbitrary vector $|\psi\rangle$ into a multiple (namely $\langle l|\psi\rangle$) of the vector $|l\rangle$.

**Comment:** If $|l\rangle = 0$ for some $l$, these vectors would make no contribution to the resolution of unity, and so we have already assumed that $|l\rangle \neq 0$, i.e., $\langle l|l\rangle > 0$. If $d\mu(l) = 0$ for a set of nonzero measure, then these vectors would also not contribute to the resolution of unity. Hence, there is no loss of generality to require that $\mu(l)$ is a strictly positive measure (up to sets of measure zero). In addition, it is often useful to assume that $\mu(l)$ is scaled (or rescaled, if necessary) so that $\langle l|l\rangle = 1$ for all $l \in \mathcal{L}$. If $\langle l|l\rangle = 1$, then it follows that $|l\rangle \langle l|$ is a one-dimensional projection operator. (Ideally, $\mu(l)$ should be a countably additive measure, but a finitely additive measure is generally sufficient, which is a distinction for positive measures that may arise when an infinite number of degrees of freedom are involved.)

**Remark:** The two postulates about coherent states above were proposed in substantially this form nearly forty years ago [9], even before such states were called “coherent states”. With very few exceptions, all states that have been so named have fulfilled these two postulates and for purposes of the present article we shall require that these two postulates hold. [For a recent study of a case where a resolution of unity (Postulate 2) fails to hold, see [10].]

The generality of the first two postulates, and their mathematical specificity as well, has been done deliberately so that a vast catalog of sets of coherent states implicitly exists; ideally, it is the analysis of a specific physical problem which, whenever possible, puts on additional physical restrictions that singles out a subset of coherent-state sets—or even a single coherent-state set—tied to the specified physical problem.

A useful analogy to the present point of view lies in the mathematical concept of a set of orthonormal functions. Initially, one can define the properties that make a set of functions an acceptable set of orthonormal functions, i.e., completeness, orthogonality, and normalization. Finally, one can introduce criteria to select some sets or even one set of orthonormal functions relevant to some specific physical problem.
Remark: The philosophy of defining coherent states expressed here is, of course, just one of many possible choices. Others are free to choose alternative definitions, although it naturally diminishes the utility of the phrase when it is used too widely. The ultimate value of a definite rule of definition stems from its usefulness in applications; and applications generally arise for specific and concrete systems.

We now turn our attention to picking out suitable sets or even a single set of coherent states by adopting certain physical criteria rather than imposing selected mathematical requirements as discussed in the previous section.

Temporal Stability

For our first additional property we shall study time evolution as dictated by a specific Hamiltonian operator $\mathcal{H}$. The evolution of any coherent state $|l\rangle$ may always be captured by the relation

$$e^{-i\mathcal{H}t} |l\rangle \equiv |l,t\rangle,$$

a definition that imposes no restriction whatsoever. However, we can ask for much more. Let us first restrict attention to normalized coherent states, $\langle l|l \rangle = 1$, for all $l \in \mathcal{L}$. Then we may ask that the following condition holds:

3. **Temporal Stability**: The time evolution of any coherent state always remains a coherent state. In symbols,

$$e^{-i\mathcal{H}t} |l\rangle = |l(t)\rangle$$

for all $l \in \mathcal{L}$ and all $t \in \mathbb{R}$, where $l(0) = l$.\(^1\)

Comment: In order to avoid any time-dependent scale factors, it has been useful to first assume that all coherent states are normalized. While the set of coherent states satisfies temporal stability, the same cannot be said for the temporal evolution of a general state $e^{-i\mathcal{H}t} |\psi\rangle \equiv |\psi, t\rangle$. Nevertheless,\(^1\)

\(^1\)Although temporal stability refers to the quantum evolution of the coherent states, there is nonetheless an induced classical dynamics inherent in this concept that realizes the label-space map $l \rightarrow l(t)$ for each $l \in \mathcal{L}$. We shall touch on this classical dynamics below.
in a coherent state representation that enjoys temporal stability, *dynamics becomes kinematics*. In other words,

\[ \langle l|\psi, t \rangle \equiv \langle l|e^{-itH}|\psi \rangle \equiv \langle l(-t)|\psi \rangle, \]

namely, the dynamical evolution of an arbitrary state \( \psi(l) \equiv \langle l|\psi \rangle \) in the coherent state representation simply amounts to a “reshuffling of the labels”, \( \psi(l, t) \equiv \psi(l(-t)) \).

Let us see how we can explicitly implement temporal stability. For convenience, we restrict attention to Hamiltonians with a discrete, nondegenerate spectrum and energy levels of the form \( 0 = E_0 < E_1 < E_2 < \cdots \). It follows that \( \lim_{n \to \infty} E_n = E^* \), and cases where \( E^* = \infty \) and \( E^* < \infty \) are both of interest. We set \( e_n \equiv E_n/\omega \), for some convenient choice of \( \omega \), to generate a sequence of dimensionless energy levels. If \( E^* < \infty \), we can, without loss of generality, choose \( \omega = E^* \) so that \( \lim_{n \to \infty} e_n = 1 \). Furthermore, we let \( |n\rangle \), \( n = 0, 1, 2, \ldots \), be energy eigenvalues for \( H \), such that

\[ H|n\rangle = E_n|n\rangle = \omega e_n|n\rangle. \]

We then define (see [11]) coherent states associated with this system by the expression

\[ |J, \gamma\rangle \equiv N(J)^{-1/2} \sum_{n=0}^{\infty} \frac{J^{n/2}e^{-ie_n\gamma}}{\sqrt{\rho_n}} |n\rangle, \]

where \( 0 \leq J < J^* \leq \infty \) and \( -\infty < \gamma < \infty \), expressed with the aid of a set of positive weight factors \{\( \rho_n \}\}, with \( \rho_0 \equiv 1 \) for convenience. Here normalization is achieved by setting

\[ N(J) = \sum_{n=0}^{\infty} \frac{J^n}{\rho_n}, \]

and where \( J^* \equiv \lim \inf_{n \to \infty} [\rho_n]^{1/n} \) denotes the radius of convergence of this series. We note first that

\[ e^{-itH}|J, \gamma\rangle \equiv N(J)^{-1/2} \sum_{n=0}^{\infty} \frac{J^{n/2}e^{-ie_n\gamma-\omega e_n t}}{\sqrt{\rho_n}} |n\rangle 
= |J, \gamma + \omega t\rangle \]
whatever the choice of the weight factors $\{\rho_n\}$. Thus by a careful choice of the phase factor we have ensured temporal stability.

Let us next discuss the freedom in the choice of the factors $\{\rho_n\}$ so that the coherent states fulfill Property 2 dealing with the resolution of unity. To that end, we assume there exists a nonnegative weight function $\rho(u)$, $\rho(u) \geq 0$, $0 \leq u < U \leq \infty$, with the property that

$$
\rho_n \equiv \int_0^U u^n \rho(u) \, du ; \quad \rho_0 = 1 .
$$

Next we observe that

$$
\int |J, \gamma\rangle \langle J, \gamma| \, d\nu(\gamma) \equiv \lim_{\Gamma \to \infty} (2\Gamma)^{-1} \int_{-\Gamma}^{\Gamma} |J, \gamma\rangle \langle J, \gamma| \, d\gamma
$$

$$
= N(J)^{-1} \sum_{n=0}^{\infty} \frac{J^n}{\rho_n} |n\rangle \langle n| .
$$

Finally, if we introduce $k(J) \equiv N(J) \rho(J)$ and $U \equiv J^*$, then we find that

$$
\int |J, \gamma\rangle \langle J, \gamma| \, d\mu(J, \gamma) \equiv \int_0^U k(J) \, dJ \int d\nu(\gamma) |J, \gamma\rangle \langle J, \gamma|
$$

$$
= \sum_{n=0}^{\infty} \frac{|n\rangle \langle n|}{\rho_n} \int_0^U J^n \rho(J) \, dJ
$$

$$
= \sum_{n=0}^{\infty} |n\rangle \langle n| \equiv \mathbf{1} .
$$

As a result of this analysis, we learn that there are a vast number of coherent state sets, all of which fulfill temporal stability for a single Hamiltonian, and which are distinguished from each other by the presence of different weight factor sets $\{\rho_n\}$.

We now seek an additional physical criterion that picks out a single set of weights $\{\rho_n\}$ for a given Hamiltonian, thereby reducing the vast family of coherent states down to a single set.

**The Action Identity**

Let us return to the appropriate label map $l \to l(t)$ for the set of coherent states under discussion. Specifically, the appropriate map in the present case
is clearly given by \((J, \gamma) \rightarrow (J, \gamma + \omega t)\). This temporal evolution is the most general solution of the two equations of motion

\[
\dot{\gamma} = \omega, \quad \dot{J} = 0,
\]

which in turn arise, for example, from the “classical action functional”

\[
I = \int [J \dot{\gamma} - \omega J] \, dt
\]
as the relevant Euler-Lagrange equations. In point of fact, other action functionals would work just as well, say, for instance,

\[
I' = \int [J^3 \dot{\gamma} - \omega J^3] \, dt.
\]

However, there is an additional sense in which \(I\) is preferred since in that case \(J\) and \(\gamma\) can be said to be classical canonical coordinates; this interpretation is not supported by using \(I'\) (or any other such form). Let us accept the physical notion that \(J\) and \(\gamma\) should represent classical canonical coordinates and thus \(I\) corresponds to the appropriate classical action.

It is a longstanding proposal \[4, 12\] that there is just one action principle in physics, and that in particular, the classical action principle is just the quantum action principle applied to a restricted set of Hilbert space vectors. We can illustrate this proposal as follows: Let

\[
I_Q = \int [i \langle \psi(t) | (d/dt) | \psi(t) \rangle - \langle \psi(t) | H | \psi(t) \rangle] \, dt
\]
denote the usual quantum action functional. Extremizing this functional over all bra vectors \(\langle \psi(t) |\) leads to Schrödinger’s equation

\[
i(d/dt) | \psi(t) \rangle = H | \psi(t) \rangle.
\]

Let us ask the question, however, what is the result if we extremize the quantum action functional over a limited set of vectors such as those in a set of coherent states. For example, consider states of the form

\[
|p, q\rangle \equiv e^{-iqP} e^{ipQ} |0\rangle,
\]

where \([Q, P] = i\mathbb{I}\) and \(|0\rangle\), say, is a unit vector which satisfies \((Q+iP)|0\rangle = 0\). It is then straightforward to show that

\[
I_Q = \int [i \langle p(t), q(t) | (d/dt) | p(t), q(t) \rangle - \langle p(t), q(t) | H | p(t), q(t) \rangle] \, dt
\]

\[
= \int [p(t) \dot{q}(t) - H(p(t), q(t))] \, dt,
\]
where \( H(p, q) \equiv \langle p, q | \mathcal{H} | p, q \rangle \) is a classical Hamiltonian symbol associated with the quantum Hamiltonian \( \mathcal{H} \). Clearly, extremal variation of \( I_Q \) within the limited set of coherent states, i.e., for general functions \( p(t) \) and \( q(t) \), leads to traditional classical equations of motion for the canonical variables \( p \) and \( q \). In this interpretation, classical dynamics is what remains of quantum dynamics when the latter is subject to a sufficiently large class of constraints that restrict possible variations. Stated otherwise, classical dynamics is quantum dynamics restricted to the only quantum degrees of freedom that may possibly be varied at a macroscopic level, namely, the mean position and the mean momentum (or velocity).

The foregoing discussion can be applied to the problem at hand as follows: If we seriously wish to identify the variables \( J, \gamma \) of the coherent states \( |J, \gamma\rangle \) as canonical coordinates, then it is necessary that

\[
I = \int [J \dot{\gamma} - \omega J] dt = \int [i\langle J, \gamma | (d/dt) |J, \gamma\rangle - \langle J, \gamma | \mathcal{H} | J, \gamma\rangle] dt.
\]

Consequently, we are led to the next, and last, postulate, namely

4. **Action Identity:** To ensure that the variables \( J \) and \( \gamma \) correspond to physical canonical coordinates, we require that

\[
\langle J, \gamma | \mathcal{H} | J, \gamma\rangle = \omega J.
\]

**Comment:** As easily seen, this last condition is equivalent to requiring that \( i\langle J, \gamma | d | J, \gamma\rangle = J d\gamma \). The action identity is a strong requirement, and we next show that it will uniquely specify the weight factors \( \{\rho_n\} \) for a given Hamiltonian \( \mathcal{H} \).

The action identity asserts, for all \( J, 0 \leq J < J^* \), that

\[
\sum_{n=0}^{\infty} \frac{e_n J^n}{\rho_n} = J \sum_{n=0}^{\infty} \frac{J^n}{\rho_n}.
\]

Equating like powers of \( J \), we are led to the condition \( e_n/\rho_n = 1/\rho_{n-1} \), or \( \rho_n = e_n \rho_{n-1} \). Choosing \( \rho_0 = 1 \) (as already noted), we find that

\[
\rho_n \equiv e_n \cdot e_{n-1} \cdot \cdots e_1 = \prod_{l=1}^{n} e_l.
\]

The final result is, therefore, the set of coherent states introduced by Gazeau and Klauder [13].
It is instructive to apply the final coherent-state prescription to a familiar example, namely, to the harmonic oscillator. In that case, $E_n = \omega n$, or $e_n = n$, and so $\rho_n = n!$. If we let $|z| \equiv J^{1/2}$ and set $z \equiv |z|e^{-i\gamma}$, then we find that

$$|J, \gamma\rangle \equiv |z\rangle = e^{-\frac{1}{2}|z|^2} \sum_{n=0}^{\infty} \frac{z^n}{\sqrt{n!}} |n\rangle.$$ 

Observe that in the present case it suffices that $\pi \leq \gamma < \pi$ to achieve the needed orthogonality. Reassuringly, therefore, we have been able to deduce from our several postulates that the canonical coherent states are the unique family of coherent states associated with the harmonic oscillator dynamics.

### Application to Hydrogen-like Spectrum

Finding coherent states for the bound state portion of the hydrogen atom has been a long-standing problem. Surely, various proposals for such coherent states have been made (see, e.g., [14]), and just as surely they generally differ one from another. One means to gauge such proposals is how well they do in the semi-classical regime, namely, what is the spread in the energy levels for highly excited systems. Ideally, one would prefer that the spread decreases as the excitation level rises so that more nearly classical-like behavior is obtained. A measure of the spread is provided by the variance, and therefore it is appropriate to focus on the variance in the proposed coherent states. While the full hydrogen atom has been treated elsewhere, we content ourselves here with a simple one-dimensional model which serves to illustrate the principles involved in a clearer fashion.

We now turn our attention to a one-dimensional model problem with the hydrogen-like spectrum

$$E_n = \omega \left[1 - 1/(n+1)^2\right].$$

In this case

$$\rho_n = \prod_{l=1}^{n} \frac{(l^2 + 2l)}{(l+1)^2} = \frac{1}{2} \left(\frac{n+2}{n+1}\right).$$
and thus the coherent states in question are defined by

\[ |J, \gamma\rangle = N(J)^{-1/2} \sum_{n=0}^{\infty} \sqrt{\left(\frac{2n + 2}{n + 2}\right)} J^{n/2} e^{-i\gamma[1-(n+1)^{-2}]} |n\rangle. \]

Here

\[ N(J) = \sum_{n=0}^{\infty} \left(\frac{2n + 2}{n + 2}\right) J^n = \frac{2}{1 - J} + \frac{2}{J^2} [J + \ln(1 - J)], \]

provided that \(0 \leq J < J^* = 1\). As in the general case, these states clearly exhibit temporal stability, i.e.,

\[ e^{-i\mathcal{H}t} |J, \gamma\rangle = |J, \gamma + \omega t\rangle. \]

**Variance**

By design, of course, these states fulfill the condition

\[ \langle J, \gamma| \mathcal{H} |J, \gamma\rangle = \omega J. \]

A question of particular interest, however, refers to the variance of the energy in each of the given coherent states since this quantity serves to indicate how well the energy is peaked about its mean value in the coherent state \(|J, \gamma\rangle\).

It may be shown (e.g., by direct computation) that for the hydrogen-like model under discussion the variance

\[
 v(J) \equiv \langle J, \gamma| \mathcal{H}^2 |J, \gamma\rangle - \langle J, \gamma| \mathcal{H} |J, \gamma\rangle^2 \leq \left(\frac{3\omega^2}{4}\right) J (1 - J).
\]

It is noteworthy that the variance vanishes not only for \(J = 0\) but for \(J = 1\) as well. This fact implies that the state \(|J, \gamma\rangle\) is peaked in its energy values about its mean value when \(J \approx 0\) and when \(J \approx 1\).

We now proceed to discuss the variance in a more general fashion.

**Variances for more General Systems**

Let us discuss the variance for rather general systems for which \(J^* = 1\). This analysis leads to further information about the hydrogen-like model as well as many other examples.
In the general case, the energy variance is defined by
\[ v(J) = \langle J, \gamma \mid \mathcal{H}^2 \mid J, \gamma \rangle - \langle J, \gamma \mid \mathcal{H} \mid J, \gamma \rangle^2 \]
\[ = \frac{\Sigma e_n^2 J^n / \rho_n}{\Sigma J^n / \rho_n} - \left( \frac{\Sigma e_n J^n / \rho_n}{\Sigma J^n / \rho_n} \right)^2 \]
\[ = \frac{1}{2} \frac{\Sigma_{n,m} (e_n - e_m)^2 J^{n+m} / \rho_n \rho_m}{\Sigma_{n,m} J^{n+m} / \rho_n \rho_m} . \]

Let us examine \( v(J) \) at the two extremes \( J \approx 0 \) and \( J \approx 1 \).

First, for \( J \approx 0 \), we readily see that
\[ v(J) = e_1 J + O(J^2) . \]
In short,
\[ v(J) \propto J \]
near \( J = 0 \).

For \( J \approx 1 \) the analysis is somewhat more involved. We note that \( v(J) \)
may be written as
\[ v(J) = \frac{1}{2} \frac{\Sigma J^n / \rho_n \Sigma (\delta_n - \delta_m)^2 J^n / \rho_n}{\Sigma J^n / \rho_n \Sigma J^n / \rho_n} , \]
where \( \delta_n \equiv 1 - e_n \). Observe that \( \delta_n \to 0 \) as \( n \to \infty \). For the moment we
assume even more, namely, that \( \Sigma \delta_n^2 < \infty \). Since large \( n \) values dominate
the \( n \)-sums in the numerator and the denominator, then near \( J = 1 \) it suffices
to consider
\[ v(J) = \frac{1}{2} \frac{\Sigma \delta_m^2 J^n / \rho_m \Sigma J^n / \rho_n}{\Sigma J^n / \rho_m \Sigma J^n / \rho_n} \]
\[ = (1 - J) \left( \frac{1}{2} \rho_m \Sigma \delta_m^2 / \rho_m \right) + O([1 - J]^2) . \]
Roughly speaking, if \( \delta_m^2 \propto m^{-\tau} \), for large \( m \), \( 1 < \tau \), then we have shown to
leading order that
\[ v(J) \propto (1 - J) \]
near \( J = 1 \). On the other hand, if \( \delta_m^2 \propto m^{-\tau} \), for large \( m \), \( 0 < \tau < 1 \), it
follows to leading order that
\[ v(J) \propto (1 - J)^\tau \]
near $J = 1$.

Finally, we learn that the vanishing of the variance for large quantum numbers, i.e., when $J \approx 1$ is a rather general phenomena, given the choice of coherent states to which we have been led in the present article. This fact would seem to confirm their utility in semi-classical analyses rather generally.

**Related work**

Several other papers have recently appeared dealing with topics raised in this article, and the interested reader may wish to consult them directly. In [15] temporally stable coherent states are developed for the infinite square well and for the Pöschl-Teller potential. A review of various attempts to develop coherent states in general and hydrogen atom coherent states in particular is given in [16].

**Acknowledgements**

I take this opportunity to thank several colleagues who have been recently involved with the author in one way or another regarding the subject of coherent states. These collaborators are: J.-P. Antoine, B. Bodmann, J.-P. Gazeau, P. Monceau, K.A. Penson, J.-M. Sixdeniers, S.V. Shabanov, and G. Watson.

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