Statistical Entropy of Calabi-Yau Black Holes

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Abstract

We computed the statistical entropy of nonextremal 4D and extremal 5D Calabi-Yau black holes and found exact agreement with the Bekenstein-Hawking entropy. The computation is based on the fact that the near-horizon geometry of equivalent representations contains as a factor the Bañados-Teitelboim-Zanelli black hole and on subsequent use of Strominger’s proposal generalizing the statistical count of microstates of the BTZ black hole due to Carlip.

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1 Introduction

The calculation of the statistical entropy of black holes by reducing the problem to the counting of microstates of the Bañados-Teitelboim-Zanelli (BTZ) black hole has recently attracted the attention of many researchers. A fundamental step was taken by Strominger who realized that the microscopic calculation of the entropy of the BTZ black hole due to Carlip can be performed equally well for any black hole whose near-horizon geometry contains an asymptotically $AdS_3$ region. This proposal is a generalization of a result obtained in where using the fact that some solutions of type IIA can be related via U-duality to 4D (5D) black holes and to the product of $BTZ \times S^2(S^3)$, a microscopical count of the entropy of some black holes was performed. Strominger’s proposal has already been successfully applied to provide a microscopical interpretation for the entropy of several black holes. One of the most important features of this recent approach is that it does not rely on supersymmetry and can be applied to any consistent quantum theory of gravity. This provides the possibility of microscopically calculating the entropy of not only BPS and near-BPS black holes, as in the D-brane approach, but of black holes arbitrarily away from extremality, as shown in . In the light of this advantage it is natural to apply the near-horizon approach to $\mathcal{N}=2$ black holes for which the brane explanation can break down due to loop corrections as opposed to the $\mathcal{N}=4$ and $\mathcal{N}=8$ black holes for which supersymmetry protects the counting.

The study of $\mathcal{N}=2$ black holes was pioneered in the works where some interesting observations were made about the structure of the moduli and where a way to construct explicit solutions was opened. This approach provided the basis of many subsequent investigations where extremal BPS solutions were constructed (see and references therein). Recently a rigorous derivation, accounting for tree-level and loop corrections, of the microscopic entropy of $\mathcal{N}=2$ black holes was given. This latter description relies on the M-brane content of the black holes. The M-theory interpretation of these $\mathcal{N}=2$ black holes has also proved useful in the construction of the nonextremal solutions. The nonextremal ansatz proposed in is based on the fact that $\mathcal{N}=2$ extremal black holes are solutions of M-theory compactification on $CY_3 \times S^1$ and are microscopically represented as three types of M5-branes wrapping four-cycles in $CY_3$ and $S^1$. This suggested that for the nonextremal solution an analogy with nonextremal black holes of toroidally compactified M-theory could also work. Still, the boost parameters characterizing the nonextremal solutions must be subject to certain constraints and explicit solutions were found only for a very restricted class of $\mathcal{N}=2$ string vacua. A refinement of this construction was proposed in where it was shown that the ansatz goes through the equations of motion in the near-horizon region and is valid only to near-BPS saturated black holes. This is a sufficient condition to apply Strominger’s proposal which concentrates on the near-horizon region.

In this paper we apply Strominger’s proposal to provide a microscopic interpretation of the Bekenstein-Hawking entropy of nonextremal 4D and extremal 5D Calabi-Yau black holes in terms of the entropy of the BTZ black hole that enters as a factor in the near-horizon geometry of some of the representations of the considered black holes. Exact agreement is found in both cases.
2 Calabi-Yau Black Holes

2.1 Nonextremal 4D solution

The N=2 supergravity action in D=4 includes in addition to the graviton multiplet, \( n_v \) vector multiplets and \( n_h \) hypermultiplets. The hypermultiplet fields can be consistently taken to be constant leaving one with the following action

\[
S = \frac{1}{32\pi G_4} \int \sqrt{-g} \, d^4x \left[ R - 2g_{AB} \partial^\mu z^A \partial_\mu z^B - \frac{1}{4} F_{\mu\nu}^I (^*G_I)^{\mu\nu} \right], \tag{1}
\]

with the gauge field \( G_{I\mu\nu} \) given by

\[
G_{I\mu\nu} = \text{Re}N_{IJ}F_{J\mu\nu} - \text{Im}N_{IJ}^* F_{J\mu\nu}, \tag{2}
\]

and \( I, J = 0, 1, \ldots, n_v \). The complex scalar fields \( z^A (A = 1, \ldots, n_v) \) parametrize a special Kähler manifold with metric \( g_{A\bar{B}} = \partial_A \partial_{\bar{B}} K(z, \bar{z}) \), where \( K(z, \bar{z}) \) is the Kähler potential. Both, the gauge field coupling and the Kähler potential are expressed in terms of the holomorphic prepotential \( F(X) \)

\[
e^{-K} = i(\bar{X}^IF_I - X^I\bar{F}_I),
\]

\[
N_{IJ} = \bar{F}_{IJ} + 2i\frac{(\text{Im}F_{IL}X^L)(\text{Im}F_{JM}X^M)}{\text{Im}F_{MN}X^M X^N}, \tag{3}
\]

with \( F_I = \frac{\partial F(X)}{\partial X^I} \) and \( F_{MN} = \frac{\partial^2 F(X)}{\partial X^M \partial X^N} \). The scalar fields \( z^A \) are defined by

\[
z^A = \frac{X^A}{X^0}. \tag{4}
\]

Throughout the paper we will consider the following prepotential

\[
F(X) = \frac{d_{ABC}X^AX^BX^C}{X^0}, \tag{5}
\]

where \( d_{ABC} \) are the topological intersection numbers of the Calabi-Yau manifold. The ansatz discussed in [13, 14] is the following

\[
ds^2 = -e^{-2U} f dt^2 + e^{2U} \left( f^{-1} dr^2 + r^2 d\Omega^2 \right),
\]

\[
e^{2U} = \sqrt{H_0 d_{ABC} H^A H^B H^C},
\]

\[
f = 1 - \frac{\mu}{r},
\]

\[
z^A = iH^A H_0 e^{-2U}, \quad H^A = h^A \left( 1 + \frac{\mu}{r} \sinh^2 \gamma_A \right)
\]

\[
A^0_t = \frac{r\bar{H}_0}{h_0 H_0}, \quad A^0_\varphi = r^2 \cos \theta \, \bar{H}^C \bar{C}^r
\]

\[
\bar{H}^A = h^A \left( 1 + \frac{\mu}{r} \cosh \gamma_A \sinh \gamma_A \right)
\]

\[
H_0 = h_0 \left( 1 + \frac{\mu}{r} \sinh^2 \gamma_0 \right), \quad \bar{H}_0 = h_0 \left( 1 + \frac{\mu}{r} \cosh \gamma_0 \sinh \gamma_0 \right) \tag{6}
\]
where prime denotes derivation respect to $r$. The nonzero components of the gauge field strengths are

$$ F^0_{tr} = \frac{\tilde{H}'_0}{H^2}, \quad F^A_{\varphi \theta} = r^2 \sin \theta \tilde{H}'^A. $$  

As was already said, this ansatz is restricted to the solution of certain conditions on $N_{AB}$ that influence the values of $\gamma$'s [15]. In the approach of [16] the technical restriction on $N_{AB}$ is traded to restricting the solution to the near-horizon region and the final form of the solution describes a near-extremal solution. The Bekenstein-Hawking entropy of this 4D solution is

$$ S = \frac{\pi \mu^2}{G_4} \sqrt{h_0 \cosh^2 \gamma_0 d_{ABC} h^A \cosh^2 \gamma_A h^B \cosh^2 \gamma_B h^C \cosh^2 \gamma_C}, $$

and the asymptotic flatness condition is $h_0 d_{ABC} h^A h^B h^C = 1$.

### 2.2 Extremal 5D solution

The N=2 D=5 supersymmetric Lagrangian describing the coupling of vector multiplets to supergravity is determined by one function which is given by the intersection form on a $CY_3$:

$$ \mathcal{V} = d_{IJK} X^I X^J X^K. $$

The bosonic action is

$$ e^{-1} \mathcal{L} = -\frac{1}{2} R - \frac{1}{4} G_{IJK} F^{I}_{\mu \nu} F^{I \mu \nu, J} - \frac{1}{2} g_{ij} \partial_\mu \phi^i \partial^\mu \phi^j + \frac{e^{-1}}{8} \epsilon^{\mu \nu \rho \sigma \lambda} d_{IJK} F_{\mu \nu}^{I} F_{\rho \sigma}^{J} A_{\lambda}^{K} $$

where $R$ is the scalar curvature, $F^{I}_{\mu \nu} = 2 \partial_\mu A^{I}_\nu$ is the Maxwell field-strength tensor and $e = \sqrt{-g}$ is the determinant of the F"{u}nfbein $e^\mu_\nu$. The fields $X^I = X^I(\phi)$ are the special coordinates satisfying

$$ X^I X_I = 1, \quad d_{IJK} X^I X^J X^K = 1 $$

where, $X_I$, the dual coordinate is defined by,

$$ X_I = d_{IJK} X^J X^K. $$

The moduli-dependent gauge coupling metric is related to the prepotential via the relation

$$ G_{IJK} = -\frac{1}{2} \frac{\partial}{\partial X^I} \frac{\partial}{\partial X^J} (\ln \mathcal{V}) \big|_{\mathcal{V}=1} $$

The metric $g_{ij}$ is given by

$$ g_{ij} = G_{IJK} \partial_i X^I \partial_j X^J \big|_{\mathcal{V}=1}; \quad (\partial_i \equiv \frac{\partial}{\partial \phi^i}) $$

Here we will consider the static spherically symmetric BPS black hole solution of N=2 supergravity in D=5 [19] all other solutions are particular cases of this one. In [19]

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1 For the complete action, including the fermionic part, see for example [18]
an electrically charged BPS solutions of supergravity coupled to an arbitrary number of vector multiplets was constructed. The metric is of the form

$$ds^2 = -e^{-4U}dt^2 + e^{2U}(dr^2 + d\Omega_3^2). \quad (15)$$

Again our main interest is in the form of the solution in the near-horizon region. Here the general principle established in [12] that allows one to compute the area of the horizons of \(N=2\) extremal black holes as an extremum of the central charge is enough to obtain the near-horizon geometry of the solution. Namely

$$e^{2U}|_{\text{hor}} = \frac{1}{3} X^A H_A|_{\text{hor}} = \frac{1}{3} X^A|_{\text{hor}} q_A = \frac{Z_0}{3r^2}, \quad (16)$$

here \(Z = q_A X^A\) is the central charge of the superalgebra and \(Z_0\) its value at the horizon. This way, the near-horizon geometry is

$$ds^2 = -(1 + \frac{Z_0}{3r^2})^{-2}dt^2 + (1 + \frac{Z_0}{3r^2})(dr^2 + r^2 d\Omega^2_3) \quad (17)$$

If the value of the moduli at the horizon can be consistently kept fixed throughout the spacetime, as is the case in some 4D solutions, then the obtained metric is the double-extreme black hole solution [18]. The Bekenstein-Hawking entropy is

$$S = \frac{\pi^2}{2G_5} \left( \frac{Z_0}{3} \right)^{\frac{3}{2}} \quad (18)$$

### 3 Statistical Entropy

To relate the Bekenstein-Hawking entropy of the black holes reviewed in the previous section to the counting based in the BTZ black hole entropy it is necessary to show that in an equivalent representation the near-horizon geometries contain a BTZ black hole as a factor. For the nonextremal 4D and extremal 5D black hole we consider here the corresponding representations are as in [4], for the 4D black hole we find a 5D representation whose near horizon region takes the form \(BTZ \times S^2\) and for the 5D black hole \(BTZ \times S^3\).

#### 3.1 Statistical entropy of 4D black holes

To relate the 4D Calabi-Yau black hole of the previous section to the BTZ it is convenient to start with the following 5D metric

$$ds_5^2 = \left( h_0 d_{ABC} H^A H^B H^C \right)^{-1} (-dt^2 + dy^2 + \frac{\mu}{r} (\cosh \gamma_0 dt + \sinh \gamma_0 dy)^2)$$

$$+ \left( h_0 d_{ABC} H^A H^B H^C \right)^{2/3} \left( \frac{dr^2}{1 - \frac{\mu}{r}} + r^2 d\Omega_2^2 \right) \quad (19)$$

Upon compactification over the compact direction \(y\) one obtains the metric of the nonextremal ansatz [4]. This metric is the direct generalization to the Calabi-Yau case of the toroidal compactification of three M5-branes that intersect orthogonally over a common string that carries momentum [17]. As a solution of M-theory this metric must have an 11D lifting, a nice fact noted in [4] is that the 6D part of the 11D M-theory
solution decouples and then one can consider the 5D part as a solution of compactification on either a $T^6$ or a $CY_3$. To see that this is indeed true it suffices to check that upon compactification over the 6D part (following the notation of [17]) it amounts to

$$(F_1 F_2 F_3)^{-4} (F_2 F_3)^2 (F_1 F_3)^2 (F_1 F_2)^2 = 1$$

as a factor of the 5D metric. The near-horizon region is $r \to 0$ or more exactly

$$\frac{\mu h_\alpha \sinh^2 \gamma_\alpha}{r} >> 1$$

for any $\alpha = 0, A$. In this region, defining

$$l = 2(h_0 d_{ABC} p^A p^B p^C)^{1/3}$$

with $p^A = \mu h^A \sinh^2 \gamma_A$, we have

$$d s_5^2 = 2 r \left(-d t^2 + \frac{\mu}{r} (\cosh \gamma_0 dt + \sinh \gamma_0 d y)^2\right) + \frac{l^2}{4 r^2 (1 - \frac{\mu}{r})} d r^2 + \frac{l^2}{4} d \Omega_2^2,$$

where it is explicitly shown that the near-horizon region includes a factor $S^2$. To check that the remaining three-dimensional part is a BTZ black hole, it is useful to make the following change of variables which follows [4] and [7]

$$\tau = \frac{l}{R} t, \quad \phi = \frac{1}{R} y \quad \rho^2 = \frac{2 R^2}{l} (r + \mu \sinh^2 \gamma_0),$$

where $R$ is the compactification radius of $y$. In these variables the 5D metric is:

$$d s_5^2 = d s_{BTZ}^2 + \frac{l^2}{4} d \Omega_2^2$$

$$d s_{BTZ} = -N^2 d \tau^2 + \rho^2 (d \phi + N_\phi d \tau)^2 + N^{-2} d \rho^2$$

$$N^2 = \frac{\rho^2}{l^2} + \frac{\mu^2 R^4 \sinh^2 2 \gamma_0}{\rho^2 l^4} - \frac{2 \mu R^2 \cosh 2 \gamma_0}{l^3}$$

$$N_\phi = \frac{\mu R^2 \sinh 2 \gamma_0}{\rho^2 l^2}$$

The geometry of the BTZ black hole is described in [1] from where we find that the mass and the angular momentum are

$$M_{BTZ} = \frac{2 \mu R^2 \cosh 2 \gamma_0}{l^3}$$

$$J_{BTZ} = \frac{\mu R^2 \sinh 2 \gamma_0}{4 G_3 l^2}.$$  \hspace{1cm} (25)

To relate the 3D Newton’s constant to the 4D one we can follow for example [4, 9], where the only ingredients needed are the fact that the Ricci scalar in $BTZ \times S^2$ decompose as the sum of the Ricci scalar of each factor and that the 5D action can be written in terms of the 4D Newton’s constant. The final result is

$$G_3 = G_4 \frac{2 R}{l^2}$$
To provide a statistical explanation for the entropy of black holes one always needs a central charge. The relevant central charge for the BTZ black hole was initially found in [20] where it was obtained as the central charge of the Virasoro algebra of the conformal field theory generated by the algebra of diffeomorphisms of asymptotically $AdS_3$ spaces. As was already pointed in [9], to move from a BTZ geometry to an asymptotically $AdS_3$ one needs to go to the region of large $r$ and therefore moves away from the initial near-horizon geometry. A more appropriate approach is the one of [21] where the same central charge is obtained from the algebra of global charges and it can be found for any value of the radius. In either approach the central charge is

$$c = \frac{3l}{2G_3}. \quad (27)$$

The zero modes of the Virasoro generators are related to the mass and angular momentum of the BTZ black hole as (see second reference in [1])

$$M_{BTZ} = \frac{8G_3}{l}(L_0 + \bar{L}_0)$$
$$J_{BTZ} = L_0 - \bar{L}_0. \quad (28)$$

Now using Cardy’s formula for the degeneration of states in a conformal field theory of given central charge an oscillator levels one finds

$$S = 2\pi \left( \sqrt{\frac{cHR}{6}} + \sqrt{\frac{cHL}{6}} \right)$$
$$= \frac{\pi}{4G_3} \left( \sqrt{l(lM_{BTZ} + 8G_3J_{BTZ})} + \sqrt{l(lM_{BTZ} - 8G_3J_{BTZ})} \right), \quad (29)$$

Inserting (25) and (26) we obtain the following entropy

$$S = \frac{\pi \mu^2}{G_4} \sqrt{h_0 \cosh^2 \gamma_0 d_{ABC} h^A \sinh^2 \gamma_A h^B \sinh^2 \gamma_B h^C \sinh^2 \gamma_C} \quad (30)$$

which coincides with the geometrical entropy of the nonextremal 4D Calabi-Yau black hole [3] for any value of $\gamma_0$ and for large values of $\gamma_I$ which means, in physical terms, for any electric charge and for large magnetic charges. In the brane picture this is the so called dilute gas regime $\gamma_I >> 1$ [22]. This way one finds that in this limit the Bekenstein-Hawking entropy of this black hole can be given a statistical interpretation in terms of the degrees of freedom associated with the conformal theory of the BTZ black hole.

### 3.2 Statistical entropy of 5D black holes

The ideology to solve this problem for the extremal N=2 D=5 black holes is exactly the same as for the 4D black holes and therefore most of the details are omitted. Before writing a 6D metric which upon compactification on one compact dimension gives the 5D black hole metric ([17]) let us consider the candidates obtained from analogy with toroidal compactifications of M-theory. The 5D black hole in toroidal compactifications of M-theory can be obtained as compactification of two M-theory configurations: three orthogonally intersecting M2-branes and a M2-brane orthogonally intersecting a M5-brane.
(see [17] and [23] for a detailed analysis). The technical reason for which the three M2-brane picture can not be used to obtain the statistical entropy of the three-charge 5D black hole is that it is not necessary to introduce a boost between time and a compact coordinate. This boost is what determine the $AdS_3$ geometry that we need to be able to apply Strominger’s proposal. To justify the use of the $2 \perp 5$ picture we still need to prove that the 5D part that lifts the 6D metric to a solution of M-theory decouples as was the case in the 4D case. The metric of the configuration of a M2 intersecting a M5 with a boost along the common string is [17, 23]

$$ds_{11}^2 = T^{2/3} F^{1/3}(-K^{-1} f dt^2 + K \dot{y}_1^2) + T^{-1/3} F^{-2/3} (f^{-1} dr^2 + r^2 d\Omega_3^2)$$

$$+ T^{2/3} F^{-2/3} dy_2^2 + T^{-1/3} F^{1/3} (dy_3^2 + dy_4^2 + dy_5^2),$$

with $f = 1 - \mu^2/r^2$ and $T^{-1}, F^{-1}$ harmonic functions. It can be seen that in the limit we are interested in, all charges equal (17), the 5D part describe by \((y_2, y_3, y_4, y_5, y_6)\) decouples, in fact this part decouples for any 5D black hole having two equal charges \((F = T)\). Thus we conclude that the 6D metric that upon compactification gives the 5D black hole (17) can be lifted as a solution of M-theory. In the conclusions we will comment of what may be the relation of this 6D solution to the actual compactification of M-theory on $CY_3$ we are considering. The 6D metric which upon compactification over the compact direction $y$ gives the 5D black hole metric (17) is

$$ds_6^2 = T \left(-dt^2 + dy^2 + \mu^2/r^2 (cosh \sigma dt + sinh \sigma dy)^2\right)$$

$$+ T^{-1} \left(\frac{dr^2}{f} + r^2 d\Omega_3^2\right).$$

(32)

It is still necessary to put

$$T^{-1} = 1 + \frac{Z_0}{3r^2},$$

(33)

and take the limit: $\mu^2 \to 0, \sigma \to \infty$ with $\mu^2 \sinh^2 \sigma \to Z_0/3$. Performing the change of variable

$$R^2 = \frac{r^2}{l^2} (r^2 + \mu^2 \sinh^2 \sigma) \quad \phi = \frac{y}{R} \quad \tau = \frac{l}{R} t,$$

(34)

with $R$ the compactification radius and taking $l = (Z_0/3)^{1/2}$ one find that the 6D metric can be written as

$$ds_6^2 = ds_{BTZ} + \frac{l^2}{4} d\Omega_3^2$$

$$ds_{BTZ} = -N^2 d\tau^2 + \rho^2 (d\phi + N_\phi d\tau)^2 + N^{-2} d\rho^2$$

$$N^2 = \frac{\rho^2}{l^2} + \frac{\mu^4 R^4 \sinh^2 2\sigma}{4\rho^2 l^6} - \frac{\mu^2 R^2 \cosh 2\sigma}{l^4}$$

$$N_\phi = \frac{\mu^2 R^2 \sinh 2\sigma}{2\rho^2 l^3}.$$

(35)

The mass and angular momentum of this solution are

$$M_{BTZ} = \frac{\mu^2 R^2 \cosh 2\sigma}{l^4}$$

$$J_{BTZ} = \frac{\mu^2 R^2 \sinh 2\sigma}{8G_3 l^3}.$$
The relation between the Newton’s constants is
\[
\frac{1}{G_3} = \frac{1}{G_5} \frac{\pi l^3}{R}. \tag{37}
\]
Using (36), (37) and (29) the statistical entropy is
\[
S = \frac{\pi^2}{2G_5} \mu \cosh \sigma, \tag{38}
\]
which coincides with the entropy (18) in the limit of large \(\sigma\), which is indeed needed to obtain the extremal limit.

4 Conclusions

Here it has been shown that the Bekenstein-Hawking entropy of Calabi-Yau black holes can be given a statistical interpretation using Strominger’s proposal. For the 4D black hole the nonextremal case was considered for an ansatz that is more general than the actual black hole metric. As shown in [13, 10] some other conditions must be included that restrict the ansatz. Exact agreement was found in the ”dilute gas” regime or more precisely for large values of the magnetic charges and arbitrary values of the electric charge. In the 5D case the extremal solution was treated using the general form of the near-horizon metric presented in [13] and based on the beautiful results of [12]. Hopefully for the nonextremal 5D black holes a counting similar to the one carried here for the 4D could be performed, still some work is needed to consistently find the explicit form of the 5D nonextremal Calabi-Yau black holes. In the present work one step has been made in this direction noting that the M-brane picture that is consistent with the counting of the microstate selects the microscopic content of the black hole as a bound state of M2 and M5-branes and not that of only M2-branes. It could also be conjectured that a twelve-dimensional theory may be of importance in the construction of nonextremal 5D black holes since in the counting use was made of a theory of that has a \(S^1\) factor and upon compactification on it yields the 5D black hole. The 5D black hole in the picture used in this paper seems to be a solution of a twelve-dimensional theory compactified on a \(CY_3 \times S^1\). Another sensible picture is that of M-theory compactified on a \(CY_3\) that can effectively be described as having a \(S^1\) factor.

Since \(N=2\) black holes receive quantum corrections it is worth explaining to what extent this microscopic counting could include them. Unfortunately a rigorous statistical explanation of the quantum corrections as presented in [14] does not seem to be available in this picture. Still, it is worth noting that certain quantum corrections can be included in the approach used in this paper. Namely those that preserve the polynomial degree of three of the prepotential. One example of this type of quantum corrections was presented in [24] and has the form
\[
d_{ABC}H^AH^BH^C = H^1H^2H^3 + a(H^3)^3. \tag{39}
\]
The metric for this quantum corrected solution is
\[
g_{00}^2 = 4(h_0 + q_0) \left( (h_1 + \frac{p_1}{r})(h_2 + \frac{p_2}{r})(h_3 + \frac{p_3}{r}) + a(h_3 + \frac{p_3}{r})^3 \right), \tag{40}
\]
redefining \(l\) in (21) one can consistently include this quantum correction to the geometric entropy in the statistical description presented here.
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References

[1] M. Bañados, C. Teitelboim and J. Zanelli, The Black Hole in Three Dimensional Space Time, Phys. Rev. Lett. 69 (1992) 1849, [hep-th/9204099].
M. Bañados, M. Henneaux, C. Teitelboim and J. Zanelli, Geometry of the (2+1) Black Hole, Phys. Rev. D48 (1993) 1506, [gr-qc/9302012].

[2] A. Strominger, Black hole entropy from near horizon microstates, [hep-th/9712251].

[3] S. Carlip, The Statistical Mechanics of the (2+1)-Dimensional Black Hole, Phys. Rev. D51 (1995) 632, [gr-qc/9409052]. The Statistical Mechanics of the Three-Dimensional Euclidean Black Hole, Phys. Rev. D55 (1997) 878, [gr-qc/9606043]. Statistical Mechanics and Black Hole Thermodynamics, Nucl. Phys. Proc. Suppl. 57 (1997) 8, [gr-qc/9702017].

[4] K. Sfetsos and K. Skenderis, Microscopic derivation of the Bekenstein-Hawking entropy formula for nonextremal black holes, [hep-th/9711138].

[5] S. Hyun, U-duality between Three and Higher Dimensional Black Holes, [hep-th/9704003].
H. J. Boonstra, B. Peeters and K. Skenderis, Duality and Asymptotic Geometries, Phys. Lett. B411 (1997) 59, [hep-th/9706192]. Branes and anti-de Sitter Spacetimes, [hep-th/9801076].

[6] D. Birmingham, String Theory Formulation of anti-de Sitter Black Holes, [hep-th/9801143].

[7] V. Balasubramanian and F. Larsen, Near Horizon Geometry and Black Holes in Four Dimensions, [hep-th/9802198].

[8] E. Teo, Statistical entropy of charged two-dimensional black holes, [hep-th/9803064].

[9] M. Z. Iofa and L. A. Pando Zayas, Statistical Entropy of Magnetic Black Holes from Near-Horizon Geometry, [hep-th/9803083].

[10] N. Kaloper, Entropy Count for Extremal Three-Dimensional Black Strings, [hep-th/9804062].

[11] G. Lopes Cardoso, Charged Heterotic Black-Holes in Four and Two Dimensions, [hep-th/9804064].

[12] S. Ferrara, R. Kallosh and A. Strominger, N=2 Extremal Black Holes, Phys. Rev. D52 (1995) 5412, [hep-th/9508072].
S. Ferrara and R. Kallosh, Supersymmetry and attractors, Phys. Rev. D54 (1996) 1514, [hep-th/9602136]. Universality of Supersymmetric Attractors, Phys. Rev. D54 (1996) 1525, [hep-th/9603090].
[13] K. Berndt, D. Lust and W. Sabra, Stationary Solutions of N=2 Supergravity, Nucl. Phys. B510 (1998) 247, hep-th/9705169.

[14] J. Maldacena, A. Strominger and E. Witten, Black Hole Entropy in M-theory, hep-th/9711053; C. Vafa, Black Holes and Calabi-Yau Threefolds, hep-th/9711067.

[15] D. Kastor and K. Z. Win, Non-extreme Calabi-Yau Black Holes, Phys. Lett. B411 (1997) 33, hep-th/9705090.

[16] K. Behrndt, M. Cvetic and W. Sabra, The Entropy of Near-Extreme N=2 Black Holes, hep-th/9712221.

[17] M. Cvetic and A. A. Tseytlin, Non-Extreme Black Holes from Non-Extreme Intersecting M-branes, Nucl. Phys. B478 (1996) 181, hep-th/9606033.

[18] A. H. Chamseddine, S. Ferrara, G. W. Gibbons and R. Kallosh, Enhancement of Supersymmetry Near a 5D Black Hole Horizon, Phys. Rev. D55 (1997) 3647, hep-th/9610157.

[19] W. A. Sabra, General BPS Black Holes in Five Dimensions, Mod. Phys. Lett A13 (1998) 239, hep-th/9708103; A. H. Chamseddine and W. A. Sabra, Metrics Admitting Killing Spinors in Five Dimensions, hep-th/9801161.

[20] J. D. Brown and M. Henneaux, On the Poisson Brackets of Differentiable generators in classical field theory, J. Math. Phys. 27 (1986) 489; Central Charges in the Canonical Realization of Asymptotic Symmetries: An example from Three-Dimensional Gravity, Commun. Math. Phys.104 (1986) 207.

[21] M. Bañados, Global charges in Chern-Simons theory and the 2+1 black hole, Phys. Rev. D52 (1995) 5816, hep-th/9405171; M. Bañados, T. Brotz and M. Ortiz, Boundary dynamics and the statistical mechanics of the 2+1 dimensional black hole, hep-th/9802076.

[22] J. Maldacena and A. Strominger, Black Hole Grey Body Factors and D-Brane Spectroscopy, Phys. Rev. D55 (1997) 861, hep-th/9609026.

[23] A.A. Tseytlin, Harmonic Superpositions of M-branes, Nucl. Phys. B475 (1996) 149, hep-th/9604033.

[24] K. Behrndt, Quantum Corrections for D=4 Black Holes and D=5 strings, Phys. Lett. B396 (1996) 77, hep-th/9610232.