Extracting the jet azimuthal anisotropy from higher order cumulants

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Abstract

We analyze the method for calculation of a coefficient of jet azimuthal anisotropy without reconstruction of the nuclear reaction plane considering the higher order correlators between the azimuthal position of jet axis and the angles of particles not incorporated in the jet. The reliability of this technique in the real physical situation under LHC conditions is illustrated.

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1 Introduction

Nowadays the strong interest is springing up to the investigations and measurements of azimuthal correlations in ultrarelativistic heavy ion collisions (see, for instance, [1] and references therein). One of the main reasons is that the rescattering and energy loss of hard partons in the azimuthally non-symmetric volume of dense quark-gluon matter, created initially in the nuclear overlap zone in collisions with non-zero impact parameter, can result in the visible azimuthal anisotropy of high-$p_T$ hadrons at RHIC [1, 2, 3, 4].

Recent anisotropic flow data at RHIC [5, 6, 7] can be described well by hydrodynamical models for semi-central collisions and $p_T$ up to $\sim 2$ GeV/c (the elliptic flow coefficient $v_2$ appears to be monotonously growing with increasing $p_T$ [8] in this case), while the majority of microscopical Monte-Carlo models underestimate the flow effects (see however [9]). The saturation and gradual decrease of $v_2$ at relatively large transverse momentum ($p_T \gtrsim 2$ GeV/c), predicted as a signature of strong partonic energy loss in a dense QCD plasma, seem now to be supported by the preliminary data extending up to $p_T \simeq 10$ GeV/c at RHIC. The interpolation between the low-$p_T$ relativistic hydrodynamics region and the high-$p_T$ pQCD-computable region was evaluated in [4].

The initial gluon densities in Pb–Pb reactions at $\sqrt{s_{NN}} = 5.5$ TeV at the Large Hadron Collider (LHC) are expected to be significantly higher than at RHIC, implying stronger partonic energy loss. Moreover, since at LHC energies the inclusive cross section for hard jet production at $E_T \sim 100$ GeV is large enough to study the impact parameter dependence of such processes [10], one can hope to observe the azimuthal anisotropy for hadronic jet itself [11, 12]. In particular, CMS experiment at LHC [13] will be able to provide both the jet reconstruction and adequate measurement of impact parameter of nuclear collision using calorimetric information [14]. In the case of jets, the methodical advantage of azimuthal observables is that one needs to reconstruct only azimuthal position of the jet without measuring its total energy. It can be done more easily and with high accuracy, while the reconstruction of jet energy is a more ambiguous problem [14]. However the measurement of jet production as a function of azimuthal angle requires event-by-event determination of the nuclear event plane based on the anisotropic flow analysis.

The methods for elliptic flow analysis can be generally divided in two categories: two-particle methods suggested and summarized in works [15, 16, 17] and multi-particle methods [18, 19]. In two-particle methods the contribution of non-flow (non-geometric) correlations to the determination of the elliptic flow coefficient $v_2$ is of the order of $1/\sqrt{N_0}$, where $N_0$ is the measured
multiplicity. In multi-particle methods this contribution goes down typically as $1/N_0^{3/4}$, i.e., smaller by a factor of the order of $N_0^{1/4}$. Thus experimental techniques based on higher order cumulant analysis should be able in many cases to allow access to the smaller values of azimuthal particle anisotropy in comparison with two-particle methods, due to automatic elimination of the major non-flow many-particle correlations and the systematic errors originating from azimuthal asymmetry of the detector acceptance. This kind of analysis for particle flow has been already done by the STAR Collaboration at RHIC [20].

In our previous Letter [21] we proposed the method for measurement of jet azimuthal anisotropy coefficients without direct reconstruction of the event plane, and illustrated its reliability in a real experimental situation. This technique is based on the calculation of correlations between the azimuthal position of the jet axis and the angles of particles not incorporated in the jet, the azimuthal distribution of jets being described by the elliptic form. To improve our approach, in the given paper we extend our analysis [21], considering the cumulant expansion [18] of multi-particle azimuthal correlations.

2 Correlators versus the jet elliptic anisotropy

Let us remind some features of our previous investigation [21]. We start from the essence of techniques [15, 16, 17] for measuring azimuthal elliptic anisotropy of particle distribution, which can be written in the form

$$\frac{dN}{d\varphi} = \frac{N_0}{2\pi} [1 + 2v_2 \cos 2(\varphi - \psi_R)] , \quad N_0 = \int_{-\pi}^{\pi} d\varphi \frac{dN}{d\varphi} .$$

(1)

Knowing the nuclear reaction plane angle $\psi_R$ allows one to determine the coefficient $v_2$ of azimuthal anisotropy of particle flow as an average (over particles) cosine of $2\varphi$:

$$< \cos 2(\varphi - \psi_R) > = \frac{1}{N_0} \int_{-\pi}^{\pi} d\varphi \cos 2(\varphi - \psi_R) \frac{dN}{d\varphi} = v_2 .$$

(2)

However in the case when there are no other correlations of particles except those due to flow (or such other correlations can be neglected), the coefficient of azimuthal anisotropy can be determined using two-particle azimuthal correlator without the event plane angle $\psi_R$:

$$< \cos 2(\varphi_1 - \varphi_2) > = \frac{1}{N_0^2} \int_{-\pi}^{\pi} d\varphi_1 \int_{-\pi}^{\pi} d\varphi_2 \cos 2(\varphi_1 - \varphi_2) \frac{d^2N}{d\varphi_1 d\varphi_2}$$

$$= \frac{1}{N_0^2} \int_{-\pi}^{\pi} d\varphi_1 \int_{-\pi}^{\pi} d\varphi_2 \cos 2((\varphi_1 - \psi_R) - (\varphi_2 - \psi_R)) \frac{dN}{d\varphi_1} \frac{dN}{d\varphi_2} = v_2^2 .$$

(3)
Let us consider now the event with high-\( p_T \) jet (dijet) production, the distribution of jets over azimuthal angle relatively to the reaction plane being described well by the elliptic form \cite{11},

\[
\frac{dN^{jet}}{d\varphi} = \frac{N^{jet}_0}{2\pi} \left[ 1 + 2v^{jet}_2 \cos 2(\varphi - \psi_R) \right], \quad N^{jet}_0 = \int_{-\pi}^{\pi} d\varphi \frac{dN^{jet}}{d\varphi},
\]

(4)

where the coefficient of jet azimuthal anisotropy \( v^{jet}_2 \) is determined as an average over all events cosine of \( 2\varphi \),

\[
\langle \cos 2(\varphi - \psi_R) \rangle_{\text{event}} = \frac{1}{N^{jet}_0} \int_{-\pi}^{\pi} d\varphi \cos 2(\varphi - \psi_R) \frac{dN^{jet}}{d\varphi} = v^{jet}_2.
\]

(5)

One can calculate the correlator between the azimuthal position of jet axis \( \varphi^{jet}_1 \) and the angles of particles, which are not incorporated in the jet(s). The value of this correlator is related to the elliptic coefficients \( v_2 \) and \( v^{jet}_2 \) as

\[
\langle < \cos 2(\varphi^{jet}_1 - \varphi) > \rangle_{\text{event}} = \frac{1}{N^{jet}_0} \int_{-\pi}^{\pi} d\varphi^{jet} \int_{-\pi}^{\pi} d\varphi \cos 2(\varphi^{jet}_1 - \varphi) \frac{dN^{jet}}{d\varphi^{jet}} \frac{dN}{d\varphi} = \frac{1}{N^{jet}_0} \int_{-\pi}^{\pi} d\varphi^{jet} \cos 2(\varphi^{jet}_1 - \psi_R) \frac{dN^{jet}}{d\varphi^{jet}} \frac{dN}{d\varphi} = v^{jet}_2 v_2.
\]

(6)

Using Eq. (3) and intermediate result in Eq. (6) (after averaging over particles \( \cos 2(\varphi^{jet}_1 - \varphi) \) reduces to \( v_2 \cos 2(\varphi^{jet}_1 - \psi_R) \)) we derive the formula for computing absolute value of the coefficient of jet azimuthal anisotropy (without reconstruction of sign of \( v^{jet}_2 \)):

\[
v^{jet}_2 = \left\langle \frac{< \cos 2(\varphi^{jet}_1 - \varphi) >}{\sqrt{< \cos 2(\varphi^{jet}_1 - \varphi) >}} \right\rangle_{\text{event}}.
\]

(7)

This formula does not require the direct determination of reaction plane angle \( \psi_R \). The brackets \( \langle \quad \rangle \) represent the averaging over particles (not incorporated in the jet) in a given event, while the brackets \( \langle \quad \rangle_{\text{event}} \) are the averaging over events.

The formula (7) can be generalized by introducing as weights the particle momenta,

\[
v^{jet}_{2(p)} = \left\langle \frac{< p_T(\varphi^{jet}_1 - \varphi) >}{\sqrt{< p_T(\varphi^{jet}_1 - \varphi) >}} \right\rangle_{\text{event}}.
\]

(8)

In this case the brackets \( \langle \quad \rangle \) denote the averaging over angles and transverse momenta of particles. The other modification of (8),

\[
v^{jet}_{2(E)} = \left\langle \frac{< E(\varphi^{jet}_1 - \varphi) >}{\sqrt{< E(\varphi^{jet}_1 - \varphi) >}} \right\rangle_{\text{event}},
\]

(9)

\( (E_i(\varphi_i) \) being energy deposit in a calorimetric segment \( i \) of position \( \varphi_i \)) allows one using calorimetric measurements (9) for the determination of jet azimuthal anisotropy.

\footnote{The other possibility is to fix the azimuthal position of a leading particle in the jet. In this case calculating azimuthal correlations can provide the information about azimuthal anisotropy of high-\( p_T \) particle spectrum.}


3 Higher order correlators

The main advantage of the higher order cumulant analysis is in the fact that, as argued in Ref. [18], if flow is larger than non-flow correlations, the contribution of the latter to $v_2$ extracted from higher order correlators is suppressed\(^2\) by powers of particle multiplicity $N_0$ in an event.

Thus, for example, the fourth order cumulant for elliptic particle flow is defined as [18]

$$
c_2[4] \equiv \langle \cos 2(\varphi_1 + \varphi_2 - \varphi_3 - \varphi_4) \rangle - \langle \cos 2(\varphi_1 - \varphi_3) \rangle - \langle \cos 2(\varphi_2 - \varphi_4) \rangle - \langle \cos 2(\varphi_1 - \varphi_4) \rangle - \langle \cos 2(\varphi_2 - \varphi_3) \rangle, \quad (10)
$$

and in the case of existing only correlations with the reaction plane (i.e. the factorization of multi-particle distributions is held as in Eq. (3)) is equal to

$$
c_2[4] = -v_2^4. \quad (11)
$$

If now one defines the coefficient $v_2$ of azimuthal anisotropy through two-particle correlator

$$
v_2 = \sqrt{\langle \cos 2(\varphi_1 - \varphi_2) \rangle}, \quad (12)
$$

then the contribution of non-flow correlations, as argued in [18], is of order $1/\sqrt{N_0}$. While their contribution to $v_2$, extracted from the fourth order correlator

$$
v_2 = (-c_2[4])^{1/4}, \quad (13)
$$

scales as $1/N_0^{3/4}$, i.e. it is suppressed by an extra factor of $1/N_0^{1/4}$. Corresponding data analysis based on Eq. (10) and result (11) has been already carried out at STAR [20].

Now using the derivation of Eq. (7) and the result (11) it is straightforward to obtain the formula for calculation (measurement) of coefficient of jet azimuthal anisotropy through the higher order correlator, which is less sensitive to non-flow correlations:

$$
v_2^{jet}[4] = \left\langle \frac{1}{\sqrt{(-c_2[4])^{3/4}}} \left[ -\langle \cos 2(\varphi_{jet} + \varphi_1 - \varphi_2 - \varphi_3) \rangle + \langle \cos 2(\varphi_{jet} - \varphi_2) \rangle - \langle \cos 2(\varphi_1 - \varphi_3) \rangle + \langle \cos 2(\varphi_{jet} - \varphi_3) \rangle - \langle \cos 2(\varphi_1 - \varphi_2) \rangle \right] \right\rangle_{event}. \quad (14)
$$

Here we stress once more that in the case of existing only correlations with the reaction plane, Eq. (14) together with Eq. (7) transforms into identity. The formula just derived can be, as in

\(^2\)This can be essential under data analysis with not high enough multiplicity of particles in an event.
Sect. 2, generalized for calorimetric measurements of energy flows:

\[ v_{\text{jet}}^2[4] = \frac{1}{(c_{2(E)}[4])^{3/4}} \left[ - E_1(\varphi_1) E_2(\varphi_2) E_3(\varphi_3) \cos 2(\varphi_{\text{jet}} + \varphi_1 - \varphi_2 - \varphi_3) > 
+ < E_2(\varphi_2) \cos 2(\varphi_{\text{jet}} - \varphi_2) > < E_1(\varphi_1) E_3(\varphi_3) \cos 2(\varphi_1 - \varphi_3) > 
+ < E_3(\varphi_3) \cos 2(\varphi_{\text{jet}} - \varphi_3) > < E_1(\varphi_1) E_2(\varphi_2) \cos 2(\varphi_1 - \varphi_2) > \right]_{\text{event}}, \]

where

\[ c_{2(E)}[4] = < E_1(\varphi_1) E_2(\varphi_2) E_3(\varphi_3) E_4(\varphi_4) \cos 2(\varphi_1 + \varphi_2 - \varphi_3 - \varphi_4) > 
- < E_1(\varphi_1) E_3(\varphi_3) \cos 2(\varphi_1 - \varphi_3) > < E_2(\varphi_2) E_4(\varphi_4) \cos 2(\varphi_2 - \varphi_4) > 
- < E_1(\varphi_1) E_4(\varphi_4) \cos 2(\varphi_1 - \varphi_4) > < E_2(\varphi_2) E_3(\varphi_3) \cos 2(\varphi_2 - \varphi_3) > . \] (16)

In the case when the azimuthal position of jet axis correlates not only with the reaction plane, one can try to improve this technique using the multiple correlators of another form: averaging over not all events but selecting some their sub-events. For instance, one can consider sub-events 1 and 2, when jets are produced with the rapidity \( y > 0 \) and \( y < 0 \). Then calculating correlator

\[ c_{2}^{\text{jet}}[4] = \frac{1}{\sqrt{< \cos 2(\varphi_1 - \varphi_2) > < \cos 2(\phi_1 - \phi_2) >}} \left[ < \cos 2(\varphi_{\text{jet}} - \varphi + \phi_{\text{jet}} - \phi) > 
+ < \cos 2(\varphi_{\text{jet}} - \varphi - \phi_{\text{jet}} + \phi) > 
- < \cos 2(\varphi_{\text{jet}} - \varphi) > < \cos 2(\phi_{\text{jet}} - \phi) > \right]_{\text{sub-event 1, 2}}, \] (17)

we obtain that, if there are flow particle correlations only and the distribution of jets over azimuthal angles is described by the elliptic form (4) in every sub-event, it is equal to

\[ c_{2}^{\text{jet}}[4] = v_{2}^{\text{jet}}(y > 0) v_{2}^{\text{jet}}(y < 0). \] (18)

In Eq. (17) the angles \( \varphi \) are defined as the azimuthal angles of particles and jets in sub-event with \( y > 0 \), and \( \phi \) — in sub-event with \( y < 0 \). Correspondingly the brackets \( < > \) represent the averaging over particles in sub-events 1, 2, while the brackets \( < >_{\text{sub-event 1, 2}} \) are the averaging over these sub-events. The generalization of Eq. (17) for calorimetric measurements of energy flow is obvious (similar to Eqs. (9) and (15)). We do not write also this result specially as the examples of utilizing six- and other higher order correlators.
4 Non-flow correlations

Here we discuss the influence of non-flow correlations\textsuperscript{3} on the $v_{2}^{\text{jet}}$ determination. There are various sources of such correlations, among which minijet production \cite{22}, global momentum conservation \cite{23, 24}, resonance decays (in which the decay products are correlated), final state Coulomb, strong or quantum interactions \cite{25}. We restrict our consideration to two-particle correlations only. It will be enough to illustrate the advantage in using higher order cumulants. In this case multi-particle distributions are not factorized again and instead of Eq. (3) we have

$$c_{2}[2] \equiv <\cos 2(\varphi_1 - \varphi_2) > = \frac{1}{N_{2}} \int_{-\pi}^{\pi} d\varphi_1 \int_{-\pi}^{\pi} d\varphi_2 \cos 2(\varphi_1 - \varphi_2) \left[ \frac{dN}{d\varphi_1} \frac{dN}{d\varphi_2} + \frac{dN_{\text{cor}}}{d\varphi_1 d\varphi_2} \right]$$

$$= v_{2}^{2} \frac{1 + v_{\text{cor}}^{2}/v_{2}^{2}}{1 + \Delta}, \quad (19)$$

where

$$N_{2} = \int_{-\pi}^{\pi} d\varphi_1 \int_{-\pi}^{\pi} d\varphi_2 \left[ \frac{dN}{d\varphi_1} \frac{dN}{d\varphi_2} + \frac{dN_{\text{cor}}}{d\varphi_1 d\varphi_2} \right],$$

$$\Delta = \frac{1}{N_{0}^{2}} \int_{-\pi}^{\pi} d\varphi_1 \int_{-\pi}^{\pi} d\varphi_2 \frac{dN_{\text{cor}}}{d\varphi_1 d\varphi_2},$$

$$v_{\text{cor}}^{2} = \frac{1}{N_{0}^{2}} \int_{-\pi}^{\pi} d\varphi_1 \int_{-\pi}^{\pi} d\varphi_2 \cos 2(\varphi_1 - \varphi_2) \frac{dN_{\text{cor}}}{d\varphi_1 d\varphi_2}. \quad (20)$$

Eq. (6) remains unchangeable and result (7) transforms into

$$v_{2}^{\text{jet}}[2] \equiv \left< \frac{<\cos 2(\varphi_{\text{jet}} - \varphi) >}{\sqrt{<\cos 2(\varphi_1 - \varphi_2) >}} \right>_{\text{event}} = v_{2}^{\text{jet}} \sqrt{\frac{1 + \Delta}{1 + v_{\text{cor}}^{2}/v_{2}^{2}}}. \quad (21)$$

After some algebra the fourth order cumulant (10) reduces to

$$c_{2}[4] = v_{2}^{4} \frac{1 + 4v_{\text{cor}}^{2}/v_{2}^{2} + 2v_{\text{cor}}^{2}/v_{2}^{4} + 2v_{\text{cor}}^{2}/v_{2}^{2} + v_{\text{cor}}^{4}/v_{2}^{4}}{1 + 6\Delta + 3\Delta^{2}}$$

$$-2v_{2}^{4} \left( \frac{1 + v_{\text{cor}}^{2}/v_{2}^{2}}{1 + \Delta} \right)^{2}, \quad (22)$$

where

$$v_{\text{cor}}^{4} = \frac{1}{N_{0}^{4}} \int_{-\pi}^{\pi} d\varphi_1 \int_{-\pi}^{\pi} d\varphi_2 \cos 2(\varphi_1 - \psi_{R} + \varphi_2 - \psi_{R}) \frac{dN_{\text{cor}}}{d\varphi_1 d\varphi_2},$$

$$v_{\text{cor}}^{4} = \frac{1}{N_{0}^{4}} \int_{-\pi}^{\pi} d\varphi_1 \int_{-\pi}^{\pi} d\varphi_2 \int_{-\pi}^{\pi} d\varphi_3 \int_{-\pi}^{\pi} d\varphi_4 \cos 2(\varphi_1 + \varphi_2 - \varphi_3 - \varphi_4) \frac{dN_{\text{cor}}}{d\varphi_1 d\varphi_2 d\varphi_3 d\varphi_4},$$

\textsuperscript{3}See also Appendix of work \cite{18} and Ref. \cite{22}
\[ v_{cor}^- = \frac{1}{N_0} \int_{-\pi}^{\pi} d\varphi_1 \int_{-\pi}^{\pi} d\varphi_2 \int_{-\pi}^{\pi} d\varphi_3 \int_{-\pi}^{\pi} d\varphi_4 \cos(\varphi_1 + \varphi_2 - \varphi_3 - \varphi_4) \frac{dN_{cor}}{d\varphi_1 d\varphi_3} \frac{dN_{cor}}{d\varphi_2 d\varphi_4} \]

\[ = (v_{cor}^-)^2 - \left( \frac{1}{N_0} \int_{-\pi}^{\pi} d\varphi_1 \int_{-\pi}^{\pi} d\varphi_3 \sin(2(\varphi_1 - \varphi_3)) \frac{dN_{cor}}{d\varphi_1 d\varphi_3} \right)^2 \]

\[ = (v_{cor}^-)^2 \text{ if } \frac{dN_{cor}}{d\varphi_1 d\varphi_2} \text{ is an even function of angular difference } (\varphi_1 - \varphi_2). \]

The numerator in Eq. (14) is rewritten in the following form:

\[
Num \equiv - < \cos 2(\varphi_{jet} + \varphi_1 - \varphi_2 - \varphi_3) > \\
+ < \cos 2(\varphi_{jet} - \varphi_2) > < \cos 2(\varphi_1 - \varphi_3) > + < \cos 2(\varphi_{jet} - \varphi_3) > < \cos 2(\varphi_1 - \varphi_2) > \\
= - v_2^3 \cos 2(\varphi_{jet} - \psi_R) \left( 1 + \frac{2 v_{cor}^- / v_2^2 + v_{cor}^+ / v_2^2}{1 + 3\Delta} \right) \\
+ 2v_2^3 \cos 2(\varphi_{jet} - \psi_R) \left( 1 + \frac{v_{cor}^- / v_2^2}{1 + \Delta} \right) + SIN, \quad (24)
\]

where the terms \( SIN \) are proportional to \( \sin 2(\varphi_{jet} - \psi_R) \) and vanishing after averaging over \( \varphi_{jet} \).

At the first glance it is hard to see the advantage in using higher order cumulants from Eqs. (21), (22) and (24). However, it is reasonable to suppose that the contribution of two-particle correlations to the normalization factor \( N_2 \) is small, \( \Delta \ll 1 \), while their “second Fourier harmonic” \( v_{cor}^- \) can be of the order of \( v_2^2 \). Then all direct two-particle correlations \( v_{cor}^- \) are automatically canceled out\(^4\) from Eqs. (22) and (24) in first order in \( \Delta \), but are survived in Eq. (21). The non-direct two-particle correlations \( v_{cor}^+ \), \( v_{cor}^{++} \) are survived. But they are suppressed in the comparison with the direct correlations \( v_{cor}^- \) (contributing to \( v_{2}^{jet}[2] \)) due to the fact that \( \frac{dN_{cor}}{d\varphi_1 d\varphi_2} \) is an even function of angular difference \( (\varphi_1 - \varphi_2) \) only in most physically interesting cases \([22]\). Moreover, for the small-angle \( \delta \)-like correlations \( (\frac{dN_{cor}}{d\varphi_1 d\varphi_2} \sim \exp(-\frac{(\varphi_1 - \varphi_2)^2}{2\sigma^2}), \sigma \to 0) \) and for the large-angle oscillating ones \( (\frac{dN_{cor}}{d\varphi_1 d\varphi_2} \sim \cos 2(\varphi_1 - \varphi_2)) \) the non-direct correlations \( v_{cor}^+, v_{cor}^{++} \) are equal to zero. Then

\[ v_2^{jet}[4] = \left\langle \frac{Num}{(-c_2[4])^{3/4}} \right\rangle_{\text{event}} \approx v_2^{jet} \quad (25)\]

in this case, while

\[ v_2^{jet}[2] \approx v_2^{jet} \sqrt{\frac{1}{1 + v_{cor}^- / v_2^2}}. \quad (26) \]

Thus Eqs. (25) and (26) demonstrate the better accuracy of higher order cumulants explicitly.

\(^4\)This is one of the main advantage of the cumulant expansion.
5 Discussion

In order to illustrate the applicability of the method presented with regard for the real physical situation, we consider the following model (see Ref. [21] for details).

The model

The initial jet distributions in a nucleon-nucleon sub-collision at \( \sqrt{s} = 5.5 \) TeV have been generated using PYTHIA\_5.7 [26]. We simulated the rescattering and energy loss of jets in gluon-dominated plasma, created initially in the nuclear overlap zone in Pb–Pb collisions at different impact parameters. For details of this approach one can refer to Refs. [10, 11]. Essential for our consideration here is that in non-central collisions the azimuthal distribution of jets is approximated well by the elliptic form (4). In the model the coefficient of jet azimuthal anisotropy increases almost linearly with the growth of impact parameter \( b \) and becomes maximum at \( b \sim 1.2R_A \), where \( R_A \) is the nucleus radius. After that \( v_2^{jet} \) drops rapidly with increasing \( b \): this is the domain of impact parameter values, where the effect of decreasing energy loss due to reducing effective transverse size of the dense zone and initial energy density of the medium is crucial and not compensated anymore by stronger non-symmetry of the volume. The kinematical cuts on jet transverse energy and rapidity has been applied: \( E_T^{jet} > 100 \) GeV and \( |y^{jet}| < 1.5 \). After this the dijet event is superimposed on the Pb–Pb event containing anisotropic flow.

Anisotropic flow was generated using the simple hydrodynamical Monte-Carlo code [27, 21] giving hadron (charged and neutral pion, kaon and proton) spectrum as a superposition of the thermal distribution and collective flow. To be definite, we fixed the following “freeze-out” parameters: the temperature \( T_f = 140 \) MeV, the collective longitudinal rapidity \( Y_L^{max} = 3 \) and the collective transverse rapidity \( Y_T^{max} = 1 \). We set the Poisson multiplicity distribution and took into account the impact parameter dependence of multiplicity in a simple way, just suggesting that the mean multiplicity of particles is proportional to the nuclear overlap function. We also suggested [21] that the spatial ellipticity of the “freeze-out” region is directly related to the initial spatial ellipticity of the nuclear overlap zone. Such “scaling” allows one to avoid using additional parameters and, at the same time, results in the elliptic anisotropy of particle and energy flow due to the dependence of effective transverse size of the “freeze-out” region on the azimuthal angle of a “hadronic liquid” element. Obtained in such a way azimuthal distribution of particles is described well by the elliptic form (1) for the domain of reasonable impact parameter values.

To be specific, we consider the geometry of CMS detector [13] at LHC. The central ("bar-
rel”) part of the CMS calorimetric system covers the pseudo-rapidity region $|\eta| < 1.5$, the segmentation of electromagnetic and hadron calorimeters being $\Delta \eta \times \Delta \phi = 0.0174 \times 0.0174$ and $\Delta \eta \times \Delta \phi = 0.0872 \times 0.0872$ respectively [13]. In order to reproduce roughly the experimental conditions (not including real detector effects, but just assuming calorimeter hermeticity), we applied Eqs. (9) and (15) to the energy deposition $E_i(\varphi_i)$ of generated particles, integrated over the rapidity in 72 segments (according to the number of segments in the hadron calorimeter: $72 \times 0.0872 = 2\pi; i = 1, \ldots, 72$) covering full azimuth.

Note that in the CMS heavy ion physics program, the modified sliding window-type jet finding algorithm has been developed to search for “jet-like” clusters above the average energy, and to subtract the background from the underlying event [14]. Strictly speaking, after jet extraction the background energy deposition in the calorimetric cells should be redefined and can appear to be not exactly equal to the initially generated one. However we neglect this effect here. In a real experimental situation, in order to avoid the influence of jet contribution on the particle flow, one can consider jets and particles incorporated in the energy flow analysis in different rapidity regions.

**Numerical results**

We have found [21] that the accuracy of $v^{jet}_2$ determination from Eq. (9) is close to 100% for semi-central ($b \lesssim R_A$) collision and becomes significantly worse in very peripheral collision ($b \sim 2R_A$), wherein decreasing multiplicity and azimuthal anisotropy of the event results in relatively large fluctuations of energy deposition in each segment.

In the given paper we test the efficiency of the higher order correlator (15) and have found, at first, that the results for $v^{jet}_2$ obtained from Eqs. (9) and (15) are practically the same. This is explained by the fact that our simple Monte-Carlo event generator gives the elliptic anisotropy of energy flow, correlated with the reaction plane, but no correlations between energy deposition in the calorimeter segments. We can introduce such correlations at the calorimetric level “by hand”, simply assuming that the probability of finding the energy $E_i$ in a segment $i$ and the energy $E_j$ in a segment $j$ is proportional to $E_iE_j(1 + c_{ij})$, where the “correlation strength” $c_{ij}$ may be, for example, proportional to $\delta_{ij}$ (the short-range $\delta$-like correlations) or $\cos 2(\varphi_i - \varphi_j)$ (the long-range oscillating correlations). In this case we became convinced that the higher order cumulant (15) was almost independent of such correlations (as it was shown in Sect. 4), while the result of calculation (9) changed, closely following the formula (26) corrected by autocorrelation terms which are non-vanishing in finite summation [18].

We have also found at the calorimetric level that, taking into account the effect of possible detector inefficiency (i.e. that the particles and jets are not detected in a “blind” azimuthal
sector of size $\alpha$, the accuracy of $v_2^{jet}$ determination appears to be less than 50% at $\alpha \gtrsim 30^\circ$ in our model calculation without correlations and at $b \geq R_A$, whichever algorithm (9) or (15) we used.

Fig. 1 is presented to illustrate the improvement due to the fourth order cumulant method in the determination of jet azimuthal anisotropy $v_2^{jet}$ depending on the ratio $\bar{v}_{cor}/\bar{v}_2^2$, where $\bar{v}_2$ is the coefficient of elliptic azimuthal anisotropy of energy flow defined here as

$$\bar{v}_2 = \frac{1}{2} \frac{E_{max(i)} - E_{min(i)}}{E_{max(i)} + E_{min(i)}},$$

(27)

and $E_{max(i)}$ and $E_{min(i)}$ are the maximum and minimum energy deposit in a segment respectively ($i = 1, ..., 72$). The coefficient $\bar{v}_{cor}$ determines the “correlation strength” at the calorimetric level, $c_{ij} = 72\bar{v}_{cor}\delta_{ij}$ for short-range correlations 5 (the similar result is obtained for long-range correlations with $c_{ij} = 2\bar{v}_{cor}\cos 2(\varphi_i - \varphi_j)$). The plots show the $b$-dependence of the “theoretical” value of $v_2^{jet}$ (calculated including collisional and radiative energy loss when the reaction plane angle is known in each event), and the $v_2^{jet}$ determined by the methods (9) and (15) for the three ratios $\bar{v}_{cor}/\bar{v}_2^2 = 0, 0.01, 0.1$. We used two values of the input parameter, the number of charged particles per unit rapidity at $y = 0$ in central Pb–Pb collisions: $dN^+/dy = 3000$ (Fig.1a) and 6000 (Fig.1b).

One can see that improvement due to the fourth order cumulant method (result of (15) is independent of $\bar{v}_{cor}/\bar{v}_2^2$ and coincides with the result of (9) for $\bar{v}_{cor}/\bar{v}_2^2 = 0$) is pronounced for more peripheral collisions, smaller particle multiplicities and larger “correlation strengths”.

\footnote{Here one should note that the majority of sources of non-flow correlations mentioned above is effective at small angles between particles. In our case these correlations can be partially smoothed out after summing particle energies over the azimuthal angles in a calorimeter segment of finite size ($\sim 5^\circ$). This can result in the smaller value of the ratio $\bar{v}_{cor}/\bar{v}_2^2$ in comparison with the ratio $v_{cor}/v_2^2$ (and, as consequence, in a somewhat less improvement due to the higher order method at the calorimetric level (15) in comparison with the particle level (14)). We still have no any adequate Monte-Carlo generator for particle flow effects at LHC including the physical model for correlations. Thus we can not estimate the real value of $\bar{v}_{cor}/\bar{v}_2^2$ (true benefit from the higher order method) and just treat it here as a phenomenological parameter.}
6 Conclusions

In the present paper we have analyzed the method for measurements of jet azimuthal anisotropy coefficients without reconstruction of the event plane considering the higher order correlators between the azimuthal position of the jet axis and the angles of particles not incorporated in the jet. The method is generalized by introducing as weights the particle momenta or the energy deposit in the calorimeter segments. In the latter case, we have illustrated its reliability in the real physical situation under LHC conditions. Introducing in the model correlations between energy deposits in the calorimeter segments does not practically change the accuracy of the method using fourth order cumulant calculations (15), while the result obtained with second order correlator (9) is dependent significantly of the “strength” of such correlations. The advantage of the higher order cumulant analysis is pronounced for more peripheral collisions and smaller particle multiplicities.

To summarize, we believe that the present technique may be useful investigating azimuthal anisotropy of jets and high-\(p_T\) particles in heavy ion collisions at RHIC and LHC.

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Figure 1: The impact parameter dependence of “theoretical” value of $v_{\text{jet}}^2$ including collisional and radiative energy loss (solid curve), and $v_{\text{jet}}^2$ determined by the method (9) for $\bar{v}_{\text{cor}}^2/\bar{v}_2^2 = 0$ (dashed curve), 0.01 (dotted curve) and 0.1 (dash-dotted curve). The result obtained using the fourth order cumulant method (15) coincides with the dashed curve. $dN^{\pm}/dy(y = 0, b = 0) = 3000$ (a) and 6000 (b).