Energy-Efficient Backscatter-Assisted Coded Cooperative NOMA for B5G Wireless Communications

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Abstract—In this manuscript, we propose an alternating optimization framework to maximize the energy efficiency of a backscatter-enabled cooperative Non-orthogonal multiple access (NOMA) system by optimizing the transmit power of the source, power allocation coefficients (PAC), and power of the relay node under imperfect successive interference cancellation (SIC) decoding. A three-stage low-complexity energy-efficient alternating optimization algorithm is introduced which optimizes the transmit power, PAC, and relay power by considering the quality of service (QoS), power budget, and cooperation constraints. Subsequently, a joint channel coding framework is introduced to enhance the performance of far user which has no direct communication link with the base station (BS) and has bad channel conditions. In the destination node, the far user data is jointly decoded using a Sum-product algorithm (SPA) based joint iterative decoder realized by jointly-designed Quasi-cyclic Low-density parity-check (QC-LDPC) codes. Simulation results evince that the proposed backscatter-enabled cooperative NOMA system outperforms its counterpart by providing an efficient performance in terms of energy efficiency. Also, proposed jointly-designed QC-LDPC codes provide an excellent bit-error-rate (BER) performance by jointly decoding the far user data for considered BSC cooperative NOMA system with only a few decoding iterations.

Index Terms—Beyond fifth-generation (B5G), non-orthogonal multiple access (NOMA), cooperative-NOMA, backscatter communication, imperfect SIC, energy-efficiency.

I. INTRODUCTION

The upcoming beyond fifth-generation (B5G) communication networks have become the focal choice for researchers across academia and industry because of their capability to support billions of communication devices [1]. Furthermore, Internet-of-Things (IoT) has been considered as a new technological revolution by providing services in a broad spectrum of applications including smart homes, smart cities, autonomous vehicles, smart ports, smart hospitals, unmanned aerial vehicles (UAV), and so on [2], [3]. However, this anomalous revolution in next-generation communication systems would require efficient utilization of available resources in terms of spectrum and energy [4], [5]. Furthermore, due to limited energy resources for IoT sensors deployed in mountains, dense forests, radioactive fields, and hidden in appliances and walls of smart buildings, it would be very expensive to replace their batteries over a regular interval of time [6], [7]. In such kinds of scenarios, it is highly desired to find some flexible techniques to enhance the life period of sensors without replacing their batteries [8], [9]. Due to the low energy consumption property of IoT sensor devices, ambient energy harvesting seems to be a promising approach to enhance the life of a sensor node. In this regard, ambient backscatter communication (BSC) has attracted the attention of researchers because of its capability to operate at a very low-power level and harvests energy from existing radio signals available in the surrounding environment [10]. The ambient backscatter modulates and reflects the incoming signals towards different users by utilizing the harvested power from existing radio signals [11]. Furthermore, ambient backscatterers could be very efficient to enhance the coverage and capacity of the network when there is no direct path between base station (BS) and cell-edge distant users [12].

To fulfill the spectrum and energy requirements for next-generation communication systems, Non-orthogonal multiple access (NOMA) and backscatter communication are two emerging technologies for a wide range of applications. It has also been shown that NOMA outperforms its orthogonal-multiple access (OMA) counterpart in terms of sum capacity.
allocation of base BS and roadside units. Authors of [27] communication network by efficiently optimizing the power ratio and time-allocation among NOMA users and relay nodes. Khan et al. [26] investigated a joint optimization problem for the NOMA backscatter-enabled vehicle-to-everything (V2X) system. By optimizing the power-splitting transfer (SWIPT) NOMA network, where the system throughput has been maximized by optimizing the reflection coefficient of ambient backscatter. Further, authors of [25] considered an ambient backscatter communication network have been analyzed by providing closed-form solutions for the OP and ergodic capacity. Nazar et al. [29] have computed the closed-form expressions in terms of bit-error-rate (BER) for NOMA-enabled BSC system. An energy efficiency (EE) maximization problem for the backscatter NOMA system has been solved by optimizing the BS transmit power and reflection coefficient of backscatter in [30]. Authors of [31] have investigated the physical-layer security of backscatter NOMA in terms of security and reliability. They have also computed the analytical solutions for the OP and intercept probability. In [32], the system throughput has been maximized by optimizing the time-allocation between the harvest-then-transmit (HTT) mode and ambient backscatter communication (ABC) mode for a backscatter-enabled cognitive-radio (CR) NOMA network.

A. Technical Literature Review

1) Backscatter-Enabled OMA Communication: Various studies have been conducted by integrating backscatter communication with the OMA protocol. For instance, the achievable rates and closed-form expressions for optimal power allocation using a cooperative backscatter communication network are provided by Guo et al. [17]. Ye et al. investigated the performance of ambient backscatter in terms of outage probability (OP) and proposed an adaptive reflection coefficient that significantly reduces the outage probability [18]. The authors of [19] have computed the closed-form expressions for ambient backscatter which offers a better trade-off between harvested energy and achievable rate under the Rayleigh-fading channel. Wang et al. proposed a resource allocation framework for backscatter communication networks to minimize the energy consumption of mobile users [20]. Authors in [21], proposed a distributed resource allocation framework for channel selection and optimal power allocation for ambient backscatter. An optimal energy detector and expression for symbol-error rate (SER) of ambient backscatter communication network have been provided in [22]. Authors of [23] proposed an optimization framework for throughput maximization which provides an optimal trade-off between the active and sleep states and the reflection coefficient of ambient backscatter. Further, authors in [24] have computed the closed-form expressions in terms of exact and asymptotic outage probability for an ambient backscatter-enabled symbiotic ration NOMA network.

2) Backscatter-Enabled NOMA Communication: The integration of backscatter communication with NOMA has gained significant attention to enhance the throughput, spectral efficiency, coverage, and connectivity of future wireless networks. For instance, authors of [25] considered an ambient backscatter enabled hybrid simultaneously wireless information and power transfer (SWIPT) NOMA network, where the system throughput has been maximized by optimizing the power-splitting ratio and time-allocation among NOMA users and relay nodes. Khan et al. [26] investigated a joint optimization problem for the NOMA backscatter-enabled vehicle-to-everything (V2X) communication network by efficiently optimizing the power allocation of base BS and roadside units. Authors of [27] have computed the closed-form expressions for optimal power allocation of BS and reflection coefficient of backscatter to maximize the sum rate of the backscatter-enabled NOMA network. In [28], the performance of the NOMA-enabled BSC ambient cellular network has been investigated by providing closed-form solutions for the OP and ergodic capacity. Nazar et al. [29] have computed the closed-form expressions in terms of bit-error-rate (BER) for NOMA-enabled BSC system. An energy efficiency (EE) maximization problem for the backscatter NOMA system has been solved by optimizing the BS transmit power and reflection coefficient of backscatter in [30]. Authors of [31] have investigated the physical-layer security of backscatter NOMA in terms of security and reliability. They have also computed the analytical solutions for the OP and intercept probability. In [32], the system throughput has been maximized by optimizing the time-allocation between the harvest-then-transmit (HTT) mode and ambient backscatter communication (ABC) mode for a backscatter-enabled cognitive-radio (CR) NOMA network.

B. Research Motivation and Contributions

In the aforementioned literature [17], [18], [19], [20], [21], [22], [23], [24], the performance of various backscatter-enabled communication networks has been analyzed by providing the analytical expressions in terms of OP, SER, and achievable sum rate of the system. Some of these existing works also attempted to improve the performance of the system by optimizing the reflection coefficient of the backscatter node. However, these works did not consider the NOMA protocol to improve the spectral efficiency and capacity of the system. Besides, cooperation among the users has not been considered. Accordingly, in literature [25], [26], [27], [28], [29], [30], [31], [32], [33], [34], [35], [36], [37], [38], [39], [40], various optimization frameworks have been investigated by integrating the backscatter communication and NOMA to improve the performance of various communication networks.
However, most of these works have considered perfect-SIC decoding which is not practical for the actual implementations of NOMA-enabled communication networks. Besides, cooperation among NOMA users has not been considered to improve the performance of cell-edge NOMA users.

Consequently, based on the above discussion, it is highly desired to investigate the backscatter-enabled cooperative NOMA system assuming no direct link between BS and far user under imperfect-SIC decoding at the receiver. To the best of our knowledge, an optimization framework that simultaneously optimizes the transmit power of the source, power allocation coefficients (PAC) of NOMA users, and power of the relay node to maximize the energy efficiency (EE) under the imperfect-SIC decoding for the backscatter-enabled cooperative NOMA system has not been investigated yet. To address this gap, we aim to propose a new optimization framework that simultaneously optimizes the transmit power of the source, PAC, and power of the relay node to maximize the EE by considering the QoS, power budgets, and cooperation constraints under the imperfect-SIC decoding for BSC-enabled cooperative NOMA system.

Moreover, in cooperative NOMA networks, the relay node exploits the SIC decoding principle to estimate the far user’s data. However, if the relay node fails to decode the data for the far user and a decoding error occurs, then this error will propagate and the performance of both the relay and destination node would be compromised. Hence, to avoid this chain process of error propagation, efficient error-correction codes can be utilized at the relay and destination nodes [41], [42], [43]. In this regard, we propose a channel-coding framework to improve the performance of the far user of the considered backscatter-enabled cooperative NOMA system. The main contributions of this manuscript are summarized as follows:

- An energy-efficient backscatter-assisted cooperative NOMA system has been considered to realize the future NOMA-enabled wireless communication networks under the imperfect-SIC decoding at the receiver, where it is assumed that there is no direct link communication between the BS and far user due to the presence of large obstructions. Moreover, the performance of the considered system is further improved, in terms of energy-efficiency, by considering the cooperation among NOMA users.
- Further, a low-complexity energy-efficient alternating optimization framework has been proposed to maximize the energy-efficiency of the considered backscatter-enabled cooperative NOMA system by optimizing the transmit power of the source, PAC, and power of relay node under the quality-of-service (QoS), cooperation, and power-budget constraints.
- Moreover, to tackle the considered non-linear fractional optimization problem, we first exploit the successive convex approximation (SCA) which transforms the considered problem into a tractable concave-convex fractional programming (CCFP) problem with low complexity. Then, we propose an alternating optimization algorithm that decouples the considered optimization problem into three sub-problems, where the optimal solution for each sub-problem is computed based on the Dinkelbach algorithm and Lagrange dual theory followed by the sub-gradient method. Numerical results evince that the proposed alternating optimization algorithm exhibits an efficient energy-efficiency performance by providing convergence within a few iterations.

- In addition, to further improve the performance of the NOMA far user, a channel-coding framework has been proposed to jointly decode the far user’s data received from the backscatter-assisted source and relay nodes based on the Sum-product algorithm (SPA) aided iterative decoder realized by jointly-designed QC-LDPC codes. Simulation results evince that the proposed jointly-designed QC-LDPC codes provide an efficient error-correction performance by jointly decoding the far user data with only a few decoding iterations.

The remaining part of this manuscript is arranged as follows: In Section II, the proposed system model of BSC cooperative NOMA network and problem formulation for EE maximization are provided. The optimal solutions of formulated EE maximization problem are given in Section III. In Section IV, jointly designed QC-LDPC codes based on CBSEC for BSC cooperative NOMA system are presented. Numerical simulation results and discussions are given in Section V, and finally, the conclusion of this manuscript is presented in Section VI.

II. SYSTEM MODEL AND PROBLEM FORMULATION

A backscatter-enabled cooperative NOMA network considered in this work is depicted in Fig. 1, where a base station is supposed to serve two NOMA users assuming Rayleigh-fading transmission. Moreover, the considered cooperative NOMA system also contains a backscatter tag that modulates and reflects the superimposed data coming from the source node towards both near and far users being served in a downlink scenario. Further, it is assumed that all of the nodes (i.e., BS, near user, and far user) involved in considered BSC cooperative NOMA system are equipped with a single antenna terminal. We assume that the perfect channel state information (CSI) is available at the source node. Based on the NOMA protocol, the user closest to the BS, denoted as $U_n$, has better channel condition, whereas the far user with bad channel condition is denoted as $U_f$. We also assume that there is no direct communication link between BS and $U_f$, where a backscatter tag (BST) is utilized to reflect signal towards $U_f$. Further, we assume the imperfect SIC decoding at the receiver for the proposed BSC cooperative NOMA network. In the first time frame, BS broadcasts the superimposed data towards $U_n$, and BST. The BST modulates and reflects the superimposed data coming from BS towards $U_n$ and $U_f$. Based on the SIC principle [7], [27], the $U_n$ first decodes the $U_f$ data and then subtracts this decoded data from the received signal to decode its own information. Consequently, the superimposed signals received at $U_n$ and $U_f$ during the first time slot are given as follows:

$$y_{n,1} = \sqrt{g_{s,n}} \left( \sqrt{P_{\xi,n} x_n} + \sqrt{P_{\xi,f} x_f} \right) + \sqrt{g_{s,b} g_{b,n} v} \left( \sqrt{P_{\xi,n} x_n} + \sqrt{P_{\xi,f} x_f} \right) m_b + w_{n,1},$$

(1)
where $P$ represents the transmit power of BS, $\xi_n$ and $\xi_f$ denote the power allocation coefficients for near user and far user, respectively. $x_n$ and $x_f$ are unit variance data symbols for $U_n$ and $U_f$ transmitted from the BS, and $m_b$ is the signal added by backscatter tag such that $E[|m_b|^2] = 1$, where $\psi_i$ denotes the reflection coefficient of BST in the first time frame. $w_{n,1}$ and $w_{f,1}$ denote the additive-white Gaussian noises (AWGN) with zero mean and variance $\sigma^2$ at $U_n$ and $U_f$, respectively. Further, $g_{s,n}$, $g_{s,f}$, and $g_{b,n}$ represent the channel gains from BS to $U_n$, BS to BST, and BST to $U_n$, respectively. Similarly, $h_{b,f}$ is the channel gain from BST to $U_f$ in first time slot.

In the second time slot, the $U_n$ acts as a relay and broadcasts the $U_f$ signal. Thus, the signal received at $U_f$ is given as:

$$y_{f,2} = \sqrt{P_v h_{n,b,f} x_f} + \sqrt{P_v h_{n,b,f} g_b \psi_i m_b} + w_{f,2},$$

where $P_v$ denotes the transmit power of relay node, $w_{f,2}$ is the AWGN noise at $U_f$ with zero mean and variance $\sigma^2$, and $\psi_i$ represents the reflection coefficient of BST in the second time frame. Further, $h_{n,b}, h_{b,f}$ and $h_{f,n}$ are the channel gains from $U_n$ to BST, BST to $U_f$, and $U_n$ to $U_f$, respectively. Therefore, the desired signal to interference plus noise ratio (SINR) at $U_n$ to decode the $U_f$ signal is given as follows:

$$\gamma_{nf} = \frac{P \xi_f (g_{s,n} + g_{s,f} g_b \psi_i)}{P \xi_n (g_{s,n} + g_{s,f} g_b \psi_i) + \sigma^2},$$

where the corresponding data rate at $U_n$ to decode the $U_f$ data in first time frame can be expressed as $R_{nf} = \frac{1}{2} W \log_2 (1 + \gamma_{nf})$. $W$ denotes the bandwidth allocated. Further, the SINR at $U_n$ to decode its data $x_n$ in the first time frame can be expressed as

$$\gamma_1 = \frac{P \xi_n (g_{s,n} + g_{s,b} g_b \psi_i)}{P \xi_f (g_{s,n} + g_{s,b} g_b \psi_i) + \sigma^2},$$

where $\eta$ denotes the imperfect SIC parameter which occurs due to the fraction of residual component of $U_f$ signal [27]. Thus, the corresponding data rate of $U_n$ to decode its own signal can be expressed as $R_1 = \frac{1}{2} W \log_2 (1 + \gamma_1)$. Further, the received SINR at $U_f$ to decode its signal during the first time frame can be given as

$$\gamma_2 = \frac{P \xi_f (g_{s,b} h_{b,f} \psi_i)}{P \xi_n (g_{s,b} h_{b,f} \psi_i) + \sigma^2},$$

where the corresponding data rate at $U_f$ to decode its signal is expressed as $R_2 = \frac{1}{2} W \log_2 (1 + \gamma_2)$. Similarly, the SINR at $U_f$ to decode its signal in the second time slot of the BSC-enabled cooperative NOMA system can be expressed as:

$$\gamma_3 = \frac{P \xi_f (h_{n,f} + h_{n,b} h_{b,f} \psi_j)}{\sigma^2},$$

where the corresponding data rate of $\gamma_3$ is given as $R_3 = \frac{1}{2} W \log_2 (1 + \gamma_3)$. Further, if the source and relay nodes are realized by two independent codebooks, then the channels of $BS - U_f$ through BST in the first time slot, $U_n - U_f$ plus $U_n - U_f$ through BST in the second time slot, can be considered as a set of parallel channels. Thus, the resultant maximum rate achieved at the far user can be expressed as follows [44], [45]:

$$R_2 + R_3 = \frac{W}{2} (\log_2 (1 + \gamma_2) + \log_2 (1 + \gamma_3)),$$

Moreover, considering decode-and-forward protocol with $U_n$ successfully decodes the $U_f$ signal, the end-to-end maximum rate achieved at $U_f$ is expressed as $\min\{R_{nf}, (R_2 + R_3)\}$ [45], [46]. However, the near $U_n$ can only decode the $U_f$ signal successfully if the rate of $U_f$ at $U_n$ is greater than or equal to the resultant maximum rate achieved at the $U_f$. Thus, to achieve successful cooperation from $U_n$, we consider that the rate of the far user at near user, $R_{nf}$, must satisfy the following condition as [47]:

$$\frac{W}{2} \log_2 (1 + \gamma_{nf}) \geq \frac{W}{2} (\log_2 (1 + \gamma_2) + \log_2 (1 + \gamma_3)),$$
Similarly, to fulfill the minimum QoS requirement, we consider that the achievable rates for $U_n$ and $U_f$ must satisfy the following conditions as:

$$\frac{W}{2} \log_2(1 + \gamma_1) \geq R_{\min},$$  \hspace{1cm} (10)

$$\frac{W}{2} \log_2(1 + \gamma_2 + \log_2(1 + \gamma_3)) \geq R_{\min},$$  \hspace{1cm} (11)

where $R_{\min}$ represents the minimum data rate to meet the QoS requirement of $U_n$ and $U_f$ for the considered system. If $P_c$ denotes the circuit power of the system, then the total power consumption $P_T$ can be expressed as follows:

$$P_T = P(\xi_n + \xi_f) + P_r + P_c,$$  \hspace{1cm} (12)

Next, the main objective of this proposed work is to maximize the EE of the backscatter-enabled cooperative NOMA system under imperfect SIC decoding at the receiver. The total EE of the considered system can be expressed as follows:

$$EE = \frac{R_{\text{sum}}}{P_T},$$  \hspace{1cm} (13)

where $R_{\text{sum}}$ represents the total sum-rate of BSC cooperative NOMA network and can be expressed as $R_{\text{sum}} = R_1 + (R_2 + R_3)$ provided that $(R_2 + R_3) \leq R_{nf}$. Further, a mathematical expression for a joint EE maximization problem (P) is given as follows:

$$(P) \max_{(P,\xi_n,\xi_f,P_t)} R_{\text{sum}},$$  \hspace{1cm} s.t. C1: $\frac{W}{2} \log_2(1 + \gamma_1) \geq R_{\min},$

$$C2: \frac{W}{2} \log_2(1 + \gamma_2 + \log_2(1 + \gamma_3)) \geq R_{\min},$$

$$C3: \frac{W}{2} \log_2(1 + \gamma_{nf}) \geq \frac{W}{2} \log_2(1 + \gamma_2 + \log_2(1 + \gamma_3)),$$

$$C4: 0 \leq P(\xi_n + \xi_f) \leq P_t,$$

$$C5: 0 \leq P_r \leq P_r(\text{max}),$$

$$C6: \xi_n + \xi_f \leq 1.$$  \hspace{1cm} (14)

where $P_t$ and $P_r(\text{max})$ denote the total power budget of source and relay node, respectively. Constraints C1, C2, and C3, obtained from (9), (10), and (12), ensure the successful cooperation and QoS demand for NOMA users. Constraints C4 and C5 limit the transmit power based on the total power budget of the source and relay node, respectively. Finally, constraint C6 restricts the PAC values of $U_n$ and $U_f$ within the practical range.

Further, the optimization problem defined by (14) has non-linear fractional form and it is very difficult to solve. Therefore, we utilize successive convex approximation (SCA) [48] which transforms the considered problem into a tractable concave-convex fractional programming (CCFP) problem with low complexity. The SCA approximation gives the following lower bound:

$$\Phi \log_2(\gamma) + \Psi \leq \log_2(1 + \gamma) \ orall \gamma \geq 0,$$  \hspace{1cm} (15)

where $\Phi = \frac{\gamma_o}{1 + \gamma_o}, \Psi = \log_2(1 + \gamma_o) - \frac{\gamma_o}{1 + \gamma_o} \log_2(\gamma_o)$ are the approximation constants and the bound becomes tight at $\gamma = \gamma_o$ for all for all $\gamma \geq 0$.

Based on the SCA approximation given in (15), the sum-rate of considered system can be expressed as follows:

$$\sum_{n=1}^{3} \frac{W}{2} (\Phi_n \log_2(\gamma_n) + \Psi_n) \leq \tilde{R}_{\text{sum}},$$  \hspace{1cm} (16)

where $\Phi_n = \frac{\gamma_{n_o}}{1 + \gamma_{n_o}}, \Psi_n = \log_2(1 + \gamma_{n_o}) - \frac{\gamma_{n_o}}{1 + \gamma_{n_o}} \log_2(\gamma_{n_o})$, and the above lower-bound becomes tight at $\gamma_n = \gamma_{n_o}$. Hence, the updated EE maximization problem in (14) can be reformulated as follows:

$$(P1) \max_{(P,\xi_n,\xi_f,P_t)} \tilde{R}_{\text{sum}},$$  \hspace{1cm} s.t. C1: $\gamma_1 \geq \frac{2R_{\text{min}} - \Phi_1 W}{\Psi_1 W},$

$$C2: \gamma_2 \Phi_2 + \gamma_3 \Phi_3 \geq \frac{2R_{\text{min}} - \Psi_2 W - \Psi_3 W}{W},$$

$$C3: \gamma_{nf} \Phi_{nf} - \Psi_2 - \Psi_3 \geq \gamma_{nf} \Phi_2 + \Phi_3 - \Phi_{nf},$$

$$C4: 0 \leq P(\xi_n + \xi_f) \leq P_t,$$

$$C5: 0 \leq P_r \leq P_r(\text{max}),$$

$$C6: \xi_n + \xi_f \leq 1.$$  \hspace{1cm} (17)

III. ENERGY EFFICIENCY MAXIMIZATION SOLUTION

The concave-convex fractional optimization problem given in (17) is a non-convex problem. Thus, it is very difficult to find a global optimal solution due to the coupled variables, i.e., $P, \xi_n, \xi_f$, and $P_r$. Therefore, a sub-optimal solution could be found by using an alternating optimization algorithm which solves the problem in three stages as: i) In first stage, transmit power $P$ is computed for the fixed values of $\xi_n, \xi_f$, and $P_r$; ii) The PAC, $\xi_n, \xi_f$, are computed in second stage for given values of $P^* \text{ and } P_r$; iii) Finally, in third stage, relay power $P_r$ is computed for given $P^*, \xi_n^*, \text{ and } \xi_f^*.$

A. Efficient Transmit Power Allocation for BS

The EE maximization problem given in (17) can be simplified to an efficient power allocation problem for the given values of PAC and $P_r$. Thus, the subproblem to optimize the transmit power of BS can be expressed as follows:

$$\max_{P} \ EE = \max_{P} \frac{\tilde{R}_{\text{sum}}}{P_T},$$  \hspace{1cm} s.t. C1 – C4.  \hspace{1cm} (18)

The objective function in (18) is still non-convex. Thus, we exploit the parameter transformation based on the Dinkelbach method in order to reduce the complexity of the solution. Consider the maximum energy efficiency of the system given as follows [49]:

$$\gamma_{EE} \ EE = \max_{P} \frac{\tilde{R}_{\text{sum}}}{P_T},$$  \hspace{1cm} (19)

Please note that the problem in (19) is a non-linear concave-convex fractional programming problem which can be transformed into an equal parameterized non-fractional subtractive form given as [49], [50]:

$$\max_{P} \ \tilde{R}_{\text{sum}} - \gamma_{EE} P_T,$$  \hspace{1cm} s.t. C1 – C4.  \hspace{1cm} (20)
where $\gamma_{EE}$ denotes a scaling parameter for $P_T$. Consider a function given as:

$$F(\gamma_{EE}) = \max_{P} \left\{ \tilde{R}_{sum} - \gamma_{EE} P_T \right\},$$

(21)

where $F(\gamma_{EE})$ results in a negative quantity when $\gamma_{EE}$ approaches to $\infty$ and positive quantity while $\gamma_{EE}$ approaches to $-\infty$. Hence, $F(\gamma_{EE})$ is an affine function with respect to $\gamma_{EE}$ [50]. Consequently, solving the optimization problem in (18) is analogous to determine the maximum energy efficiency $\gamma^*_E$. Further, $\gamma^*_E$ can be achieved if and only if [50]:

$$F(\gamma^*_E) = \max_{P} \left\{ \tilde{R}_{sum} - \gamma^*_E P_T \right\} = 0,$$

(22)

Based on the SCA approximation, the sum-rate in (20) can be expressed as follows:

$$\tilde{R}_{sum} = \frac{W}{2} \left\{ \Phi_1 \log_2 \left( \frac{P\theta}{P\Theta + n} \right) + \Psi_1 \right\}$$

$$+ \left( \Phi_2 \log_2 \left( \frac{PT}{P\Delta + n} \right) + \Psi_2 \right)$$

$$+ \left( \Phi_3 \log_2 \left( \frac{Pr\varphi}{n} \right) + \Psi_3 \right),$$

(23)

where $\theta = \xi_n(g_n + g_kb, b, n, \psi_i), \Theta = \xi_f(g_n + g_kb, b, n, \psi_i)$, and $\sigma^2 = n, \Gamma = \xi_f(g_n + g_kb, b, n, \psi_i), \Delta = \xi_n(g_n + g_kb, b, n, \psi_i)$, and $\varphi = h_{0,f} + h_{0,b}b, b, n, \psi_i$.

Next, we exploit the Lagrange dual method and sub-gradient method to compute the sub-optimal solution of optimization problem defined in (20). The Lagrangian function of considered optimization problem in (20) can be formulated as follows:

$$L(P, \lambda) = \frac{W}{2} \left\{ \Phi_1 \log_2 \left( \frac{P\theta}{P\Theta + n} \right) + \Psi_1 \right\}$$

$$+ \left( \Phi_2 \log_2 \left( \frac{PT}{P\Delta + n} \right) + \Psi_2 \right)$$

$$+ \left( \Phi_3 \log_2 \left( \frac{Pr\varphi}{n} \right) + \Psi_3 \right) - \gamma_{EE} \left( P(\xi_n + \xi_f) + P_P + P_c \right)$$

$$+ \lambda_1 \left\{ P\theta - \left( \frac{2R_{\min} - \Psi_{nf} W}{\Psi_{nf} W} \right)(P\Theta + n) \right\}$$

$$+ \lambda_2 \left\{ PT - \left( \frac{2R_{\min} - \Psi_{nf} W}{\Psi_{nf} W} \right)(P\Delta + n) \right\}$$

$$+ \lambda_3 \left\{ \left( \Phi_{nf} \log_2 \left( \frac{Pc}{Pb + n} \right) - \Phi_2 \log_2 \left( \frac{PT}{P\Delta + n} \right) \right) - \left( 2\omega - \Psi_2 - \Psi_{nf} \right) \right\}$$

$$+ \lambda_4 \left\{ P_T - P_P \right\},$$

(24)

where $\lambda = \{\lambda_1, \lambda_2, \lambda_3, \lambda_4\}$ are the Lagrange multipliers for the considered optimization subproblem, $\omega = \frac{1}{2} \left( \Phi_3 \log_2 \left( \frac{Pr\varphi}{n} \right) + \Psi_3 \right), \epsilon = \xi_f(g_n + g_kb, b, n, \psi_i), \delta = \xi_n(g_n + g_kb, b, n, \psi_i)$, and $\Pi = \xi_n + \xi_f$.

Further, the Karush-Kuhn-Tucker (KKT) conditions have been exploited to find the sub-optimal solution in terms of the efficient transmit power allocation for BS. Based on the KKT method, we can write as follows:

$$\frac{\partial L(P, \lambda)}{\partial P} |_{P=P^*} = 0,$$

(25)

After taking the partial derivative of Lagrangian function $L(P, \lambda)$ in (25) with respect to $P$, the above equation can be written as follows:

$$\phi + \frac{n\Phi_1}{\Psi_{nf}} + \frac{n\Phi_2}{\Psi_{nf}} + \frac{P\alpha\lambda_3 + \beta\lambda_3}{\Psi_{nf}} + \frac{\Delta\lambda_3}{\Psi_{nf}} + \zeta = 0,$$

(26)

where $\phi = \Theta_2\ln(2), \tau = n\ln(2), \phi_1 = \Delta_2\ln(2), \Delta = n\Delta\Phi_4 - n\delta\Phi_2, \beta = n^2\Phi_4 - n^2\Phi_2, N_1 = 2\ln(2), N_2 = n\ln(2)\delta + n\ln(2)\Delta$, and $N_3 = n^22\ln(2)$. After applying some mathematical computations on Eq. (26), we obtain the following quintic polynomial given as follows:

$$P^* = (P^5\lambda_1 + P^4\lambda_1 + P^3\lambda_3 + P^2\lambda_2 + P\lambda_1 + \lambda),$$

(27)

where $\lambda_1 = (N_2\phi_1\lambda_1 + N_1\tau\phi_1\lambda_1 + N_1\tau\phi_1\lambda_1)$ and $\lambda = (N_1\lambda_1\Phi_1 + N_1\lambda_1\Phi_2 + \tau\lambda_1\Phi_3)$. The values of $\lambda_1, \lambda_2, \lambda_3, \text{and} \zeta$ are given in Eqs. (28), (29), (30), and (31), respectively. Moreover, the optimal solution in terms of $P$ can be easily computed by employing the built-in functions provided in MATLAB and Mathatica solvers.

$$\lambda_1(l+1) = \left[ \lambda_1(l) + \mu(l) \right] \times \left\{ \left( \xi_n P(g_n + g_kb, b, n, \psi_i) \right) \left( 2 - \frac{2R_{\min} - \Psi_{nf} W}{\Psi_{nf} W} \right) \right\}^{+},$$

$$\lambda_2(l+1) = \left[ \lambda_2(l) + \mu(l) \times \left\{ \left( \frac{(P\xi_f g_n b, b, f, \psi_i) W}{\Psi_{nf}} \right) \times (\sigma^2)^{\Phi_3} \right\} \times \left( \xi_f (P_{nf} g_n b, b, f, \psi_i) + \sigma^2)^{\Phi_3} \right\} \right.$$

$$\left. \left( \frac{2R_{\min} - \Psi_{nf} W}{\Psi_{nf} W} \right) \right\}^{+} \left( (\sigma^2)^{\Phi_3} \times \left( \frac{P_{nf} g_n b, b, f, \psi_i}{\sigma^2} \right)^{\Phi_3} \right)^{+},$$

(32)

$$\lambda_3(l+1) = \left[ \lambda_3(l) + \mu(l) \times \left\{ \left( 2 - \frac{2R_{\min} - \Psi_{nf} W}{\Psi_{nf} W} \right) \times \left( \frac{P_{nf} g_n b, b, f, \psi_i}{\sigma^2} \right)^{\Phi_3} \right\} \times \left( \frac{P_{nf} g_n b, b, f, \psi_i}{\sigma^2} \right)^{\Phi_3} \right.$$

$$\left. \left( \frac{2R_{\min} - \Psi_{nf} W}{\Psi_{nf} W} \right) \right\}^{+} \left( (\sigma^2)^{\Phi_3} \times \left( \frac{P_{nf} g_n b, b, f, \psi_i}{\sigma^2} \right)^{\Phi_3} \right)^{+},$$

(33)
where $l$ is the iteration index and $\mu(l)$ denotes the step size of the sub-gradient method. Note that an appropriate step size is required for the convergence of the algorithm.

### B. Optimization for Power Allocation Coefficients

We compute the power allocation coefficients $\xi_n$ and $\xi_f$ for considered BSC-enabled cooperative NOMA system. For the given values of $P^*$ and $P_r$, the EE maximization optimization problem given in (17) can be expressed as follows:

$$
\max_{(\xi_n, \xi_f)} \frac{\tilde{R}_{\text{sum}}}{P_T} = \max_{(\xi_n, \xi_f)} \tilde{R}_{\text{sum}} - \gamma_{EE} P_T, \quad \text{s.t.} \quad C1 - C4, C6. \tag{36}
$$

Further, the sum-rate in (36) can be expressed as follows

$$
\tilde{R}_{\text{sum}} = \frac{W}{2} \left\{ \left( \Phi_1 \log_2 \left( \frac{\xi_n \theta_1}{\xi_f \Theta_1} + n \right) + \Psi_1 \right) + \left( \Phi_2 \log_2 \left( \frac{\xi_f \Gamma_1}{\xi_n \Gamma_1 + n} \right) + \Psi_2 \right) + \left( \Phi_3 \log_2 \left( \frac{P_r \varphi}{n} \right) + \Psi_3 \right) \right\}, \tag{37}
$$

where $\theta_1 = P(g_s, n + g_s, b g_b, n \psi_1)$, $\Theta_1 = P(\xi_f, g_s, n + g_s, b g_b, n \psi_1)$, $\Gamma_1 = P(g_s, b g_b, n \psi_1)$.

As, the objective function of non-convex optimization problem in (36) is an affine function with respect to $\gamma_{EE}$ [50]. Thus, we exploit the KKT condition to find the sub-optimal solution of power allocation coefficients. However, for the sake of simplicity, we omit the similar derivation steps adopted in the transmit power allocation subproblem. Consequently, by employing KKT conditions and updating the corresponding Lagrange multipliers using the sub-gradient method, the optimal values of $\xi_n$ and $\xi_f$ can be computed as follows:

$$
\xi_n^* = \left( \xi_n^4 \pi_4 + \xi_n^3 \pi_3 + \xi_n^2 \pi_2 + \xi_n \pi + \pi = 0 \right), \tag{38}
$$

$$
\xi_f^* = 1 - \xi_n^*, \tag{39}
$$

where $\pi = N_4 \Phi_1 \tau_1$, $\pi_4 = \phi_2 \phi_5 N_1 \xi_1$, $\phi_2 = \Gamma_1 2 \ln(2)$, $\xi_1 = \Gamma_1 \phi_2 (2 \phi_2 + 2 \theta_1)$, $\phi_3 = (\Gamma_1 \phi_2 (2 \phi_2 + 2 \theta_1) (2 \phi_2 + 2 \theta_1))$, $\alpha_1 = \Gamma_1 \phi_2 (2 \phi_2 + 2 \theta_1)$, $\beta_1 = n 2 \theta_1 (\Phi_2 \Gamma_1 - \Phi_{nf} \theta_1)$, $\Phi_{nf} = \frac{\psi}{\xi_n \Gamma_1 + n}$, $\psi$ is the iteration index and $\pi_2$ and $\pi_3$ are given in Eqns. (40), (41), (42), and (43), respectively, where $\gamma_1, \gamma_2, \gamma_3$, and $\gamma_5$ are the Lagrange multiplier associated with constraints C1, C2, C3, C4, and C6, respectively. Moreover, the quartic polynomial given in (38) can be easily solved using any conventional solver to find the optimal value of $\xi_n^*$. Finally, once the value of $\xi_n^*$ is in hand, $\xi_f^*$ can be computed using Eq. (39).

$$
\pi_1 = N_1 N_2 \Gamma_1 + N_2 \Phi_1 \phi_2 + \Phi_1 \phi_4 + N_2 \pi_1 \gamma_3 - N_2 \Phi_2 \Gamma_1, \tag{40}
$$

$$
\pi_2 = N_1 N_2 \phi_2 \xi_1 + N_1 \psi_4 \xi_1 + \Phi_1 \phi_2 \phi_4 + \Phi_1 \phi_3 \tau + N_1 \tau \alpha_1 \gamma_3 + N_1 \phi_2 \beta_1 \gamma_3 - N_1 \Phi_2 \Gamma_1 \phi_4, \tag{41}
$$

$$
\pi_3 = \phi_2 \phi_4 N_1 \xi_1 + \phi_3 \tau N_1 \xi_1 + \Phi_1 \phi_2 \phi_3 + N_1 \phi_2 \alpha_1 \gamma_3 + N_1 \Phi_2 \Gamma_1 \phi_3, \tag{42}
$$

$$
\xi_1 = \xi_1 \theta_1 - \gamma_2 \pi_1 \frac{2 \Gamma_{\text{sum}} - 2 \varphi_{\text{EE}}}{\phi_2} \Gamma_1 - \gamma_4 P - \gamma_5 - \gamma_{EE} P, \tag{43}
$$

### C. Efficient Power Optimization for Relay Node

In this section, we compute the optimal transmit power for relay node, $P_r$, in the second time slot of the considered BSC-enabled cooperative NOMA system. For the given values of $P^*$, $\xi_n^*$, and $\xi_f^*$, the EE maximization optimization problem can be written as follows:

$$
\max_{P_r} \frac{\tilde{R}_{\text{sum}}}{P_T} = \max_{P_r} \tilde{R}_{\text{sum}} - \gamma_{EE} P_T, \quad \text{s.t.} \quad C2, C3, C5. \tag{44}
$$

Next, the sum-rate given in (44) can be written as follows:

$$
\tilde{R}_{\text{sum}} = \frac{W}{2} \left\{ \left( \Phi_1 \log_2 \left( \frac{\xi_n \theta_1}{\xi_f \Theta_1} + n \right) + \Psi_1 \right) + \left( \Phi_2 \log_2 \left( \frac{\xi_f \Gamma_1}{\xi_n \Gamma_1 + n} \right) + \Psi_2 \right) + \left( \Phi_3 \log_2 \left( \frac{P_r \varphi}{n} \right) + \Psi_3 \right) \right\}, \tag{45}
$$

Further, the KKT conditions can be exploited to compute the optimal value of $P_r$. The Lagrangian function of considered optimization problem in (44) can be formulated as follows:

$$
(\Phi_1, \tilde{R}_{\text{sum}})^* = \frac{1}{2} \left\{ \left( \Phi_1 \log_2 \left( \frac{\xi_n \theta_1}{\xi_f \Theta_1} + n \right) + \Psi_1 \right) + \left( \Phi_2 \log_2 \left( \frac{\xi_f \Gamma_1}{\xi_n \Gamma_1 + n} \right) + \Psi_2 \right) + \left( \Phi_3 \log_2 \left( \frac{P_r \varphi}{n} \right) + \Psi_3 \right) \right\} \tag{46}
$$

where $\Phi = \{\Gamma_1 + \gamma_2 + \gamma_3 + \gamma_5\}$ are the Lagrange multipliers, $\omega_f = \frac{1}{2} (\Phi_2 \log_2 \left( \frac{\xi_f \Gamma_1}{\xi_n \Gamma_1 + n} \right) + \Psi_2)$, and $\omega_{nf} = \frac{1}{2} (\Phi_3 \log_2 \left( \frac{\xi_n \theta_1}{\xi_f \Theta_1} + n \right) + \Psi_3)$. Next, based on KKT conditions, we can write as follows:

$$
\frac{\partial L(P_r, \Phi)}{\partial P_r} |_{P_r = P_r^*} = \frac{\Phi_3}{\xi_n N_1} - \zeta_2 = 0, \tag{47}
$$

where $\zeta_2 = (\Theta_3 + \gamma_2 + \gamma_{EE} - \Theta_1) \phi$. After some mathematical computation, the closed-form expression for the optimal transmit power of relay node can be obtained as follows:

$$
P_r^* = \left[ \frac{\Phi_3}{\xi_n N_1} \right]^+, \tag{48}
$$

where $[x]^+ = \max[0, x]$. 


D. The Proposed Algorithm and Its Complexity Analysis

In this section, we propose a low-complexity energy-efficient alternating optimization algorithm to maximize the energy-efficiency of the considered backscatter-enabled cooperative NOMA system by optimizing the transmit power $P$, PAC $\xi_n$, $\xi_f$, and relay power $P_r$. In stage 1, the optimal transmit power $P^*$ is computed for the given values of $\xi_n$, $\xi_f$, and $P_r$. With $P^*$ in hand, the optimal values of PAC $\xi^*_n$, $\xi^*_f$ are computed in stage 2. Finally, for given values of $P^*$, $\xi^*_n$, $\xi^*_f$, the closed-form expression for the relay power $P^*_r$ is computed in stage 3. Moreover, the Lagrange dual variables are iteratively updated in each stage until convergence is achieved.

Further, we analyze the computational complexity of the proposed alternating optimization algorithm and Exhaustive-search algorithm [51] and presented in Table I. Let $J_1$, $J_2$, and $J_3$ denote the number of iterations required for Stage 1, Stage 2, and Stage 3, respectively. If $I$ represents the number of iterations required for the convergence of overall algorithm, then the complexity of Algorithm 1 can be computed as $O[I(J_1 + J_2 + J_3)]$. Moreover, for the complexity of Exhaustive-search algorithm, let $\alpha_{\text{max}}$, $P_{\text{max}}$, and $P_{r(\text{max})}$ denote the maximum power-allocation coefficient (PAC), maximum source power, and maximum power of the relay node, respectively. If $K$ and $\tau$ denote the number of NOMA users and the step size, respectively, then the complexity of Exhaustive-search algorithm can be computed as $O[(\alpha_{\text{max}})K + (P_{\text{max}})K + (P_{r(\text{max})})K]$. Please note that the performance of the Exhaustive-search increases by decreasing the value of its step size $\tau$, however, its computational complexity further increases. In addition, the computational complexity exponentially increases by increasing the number of users in the network. Based on the performance analysis (please refer to Fig. 7 in Section V), it is revealed that the proposed Algorithm 1 is more practical that provides a better trade-off between the complexity and performance of the considered system.

| Algorithm                  | Complexity                                                                 | Optimality                  |
|----------------------------|----------------------------------------------------------------------------|-----------------------------|
| Exhaustive-search Algorithm| $O[(\alpha_{\text{max}})K + (P_{\text{max}})K + (P_{r(\text{max})})K]$ | High complexity - Impractical |
| Proposed Algorithm 1       | $O[2(J_1 + J_2 + J_3)]$                                                   | Low complexity - Practical   |

```
Algorithm 1: Proposed Alternating Optimization Algorithm for Considered BST-Assisted Cooperative NOMA

Initialization: Initialize step sizes, iteration index, $j = j_1, j_2, j_3 = 1$, and dual variables

while not converge do
    Stage 1: Compute transmit power $P$, for the fixed values of PAC and $P_r$
    while not converge do
        for $j_1 = 1 : J_1$ do
            Compute $\gamma_{EE}(j_1) = \frac{R_{\text{sum}}(j_1)}{P_r}$
            Update $\lambda_1, \lambda_2, \lambda_3, \lambda_4$ using Eqs. (32)-(35)
            Compute optimal transmit power $P^*$ using Eq. (27)
        end
    endwhile
    Stage 2: With optimal $P^*$ and fixed $P_r$, compute PAC
    while not converge do
        for $j_2 = 1 : J_2$ do
            Compute $\gamma_{EE}(j_2)$
            Update Lagrange dual variables
            Compute optimal value of $\xi^*_n$ using Eq. (38)
            Compute optimal value of $\xi^*_f$ using Eq. (39)
        end
    endwhile
    Stage 3: With optimal $P^*$ and PAC in hand, compute $P_r$
    while not converge do
        for $j_3 = 1 : J_3$ do
            Compute $\gamma_{EE}(j_3)$
            Update Lagrange dual variables
            Compute the optimal $P^*_r$ using Eq. (48)
        end
    endwhile
    Return $P^*$, $\xi^*_n$, $\xi^*_f$, $P^*_r$
```

IV. JOINTLY-DESIGNED QC-LDPC CODES FOR BSC COOPERATIVE NOMA

In this section, we proposed a class of jointly-designed QC-LDPC codes to enhance the error-correction performance of the far user. More specifically, at the source node, two parity-check matrices $H^{(1)}$ and $H^{(2)}$ would be utilized to encode the $U_n$ and $U_f$ signals, respectively, where the relay node is realized by a parity-check matrix $H^{(3)}$ to decode the incoming superimposed signal from the BS. In the destination node, the signals are alternatively received from the backscatter-aided source and $U_n$ relay node by a matched filter. The received signals are multiplexed, and finally decoded by SPA-based joint iterative decoder [41], where decoder utilizes a parity-check matrix $H$, a joint combination of $H^{(1)}$, $H^{(2)}$, and $H^{(3)}$, to jointly decode the $U_f$ signal.

A. Basic Preliminaries

In this Section, we first provide some basic preliminaries and fundamental concepts for balanced incomplete block design (BIBD) [52], [53] and CBSEC to facilitate the readability of readers.

**Definition 1:** A design can be represented by a pair $(\mathcal{R}, \mathcal{B})$, where $\mathcal{R}$ is total number of varieties hold by a set, and $\mathcal{B}$ consists of subsets (non-empty), also called blocks, of $\mathcal{R}$. For any three positive integers $\Omega$, $f$, and $\chi$, suppose that
Ω > f ≥ 2. A \((Ω, f, χ)\)-BIBD, defined a pair \((R, B)\), must hold the properties given below:

1. \(|R| = Ω,\)
2. there are maximum \(f\) number of varieties hold by each subset of \(B\), and
3. if there are exactly \(χ\) blocks in which each pair of the elements appears.

Definition 2: A pair \((R, B)\) is called balanced sampling plans excluding contiguous units with \(R = Ω\) and \(B\) denotes the nonempty blocks of \(R\), such that no elements pair \((x, y)\) appears in any subset of \(B\) if \(x − y = ±1, \ldots, ±Ω \pmod{Ω}\), however, any other pair of elements participates exactly in \(χ\) subsets, where \(Ξ\) denotes a natural number.

Next, the notation \((Ω, f, χ; Ξ)\)-BSEC will be used to denote a balanced sampling plan excluding contiguous units. Let \(Z_Ω\) be a cyclic additive group of order \(Ω\) and \((R, B)\) a \((Ω, f, χ; Ξ)\)-BSEC. A design \((R, B)\) is called cyclic if \(Z_Ω\) is an automorphism of \((Ω, f, χ; Ξ)\)-BSEC. We use the notation \((Ω, f, χ; Ξ)\)-CBSEC to denote a cyclic BSEC [54], [55].

Subsequently, we briefly describe the construction of \((Ω, f, χ; π)\)-CBSEC based on perfect and hooked Langford sequences [56]. Suppose that \(A \subseteq Ω\). Let \(ΔA = \{a_i − a_j \pmod{Ω}; a_i, a_j \in A, a_i \neq a_j\}\). Let \([x, y]\) be a set of all integers \(r\) such that \(x ≤ r ≤ y\), and \(\chi_{[x, y]}\) represents a multi-set which contains each element of \([x, y]\) \(χ\) times. From literature [55], we can obtain the following Lemma given as follows:

**Lemma 1:** Suppose that there exist \(f\)-subsets (non-empty) \(A_1, A_2, \ldots, A_w\) of \(Z_Ω\) such that the multi-set union \(\bigcup_{i=1}^w ΔA_i = χ_{[1, Ω − 1]}\), then there exists a \((Ω, f, χ; Ξ)\)-CBSEC.

The nonempty \(k\)-subsets \(A_1, A_2, \ldots, A_w\) of \(Z_Ω\) are called the base blocks of \((Ω, f, χ; Ξ)\)-BSEC. **Lemma 1** enables us to construct \((Ω, f, χ; Ξ)\)-CBSEC based on perfect and hooked Langford sequences [56].

**Definition 3:** A sequence \(E = (e_1, e_2, \ldots, e_{2w})\) of \(2w\) elements is called a Langford sequence (order \(w\)) with defect \(V\) under all of the properties given as:

i. for each \(s \in \{V, V + 1, \ldots, V + w − 1\}\) there exist exactly two elements \(e_i, e_j \in E\) such that \(e_i = e_j, s\), and

ii. if \(e_i = e_j = s\) with \(i < j\), then \(j − i\) is an empty matrix.

**Definition 4:** A sequence of \(2w + 1\) elements, \(E = (e_1, e_2, \ldots, e_{2w+1}, 0, e_{2w+1})\), is called a hooked Langford sequence with order \(w\) and defect \(V\) under all of the properties given as follows:

i. for each \(s \in \{V, V + 1, \ldots, V + w − 1\}\) there exist exactly two elements \(e_i, e_j \in E\) such that \(e_i = e_j, s\), and

ii. if \(e_i = e_j = s\) with \(i < j\), then \(j − i\) is an empty matrix.

The following two lemmas enable us to construct \((Ω, f, χ; Ξ)\)-CBSEC based on Langford sequences given as follows [56]:

**Lemma 2:** Let \(w, V \equiv 0, 1, 1, 0, 0, 3, 0 \pmod{4, 2}\) with \(w ≥ 2V − 1\). Thus, the interval \([V, V + 3w − 1]\) can be divided into triplets or sets as \(\{x_i, y_i, z_i\}, 1 ≤ i ≤ w\), such that \(x_i + y_i = z_i\).

**Lemma 3:** Let \(w, V \equiv 2, 0, 1, 0, 2, 1, 3, 1 \pmod{4, 2}\) such that \(w(w − 2V + 1) + 2 ≥ 0\). Then \([V, V + 3w − 1]\) \(\{V + 3w − 1\}\) can be divided into sets \(\{x_i, y_i, z_i\}, 1 ≤ i ≤ w\), such that \(x_i + y_i = z_i\).

Based on above discussion, Langford sequences can be used to construct \((Ω, f, χ; Ξ)\)-CBSEC of order \(Ω\) using the steps given as follows:

1. build pairs \((i, j)\), where \(e_i = e_j \in E\);
2. (ii) change each pair \((i, j)\) into sets of triplets as \(\{x_i, y_i, z_i\}\) such that \(x_i + y_i = z_i, 1 ≤ i ≤ w\), where \(x_i = j − i, y_i = i + V + w − 1\), and \(z_i = j + V + w − 1\);
3. (iii) the base blocks \(\{0, x_i, z_i\}\) can be constructed using each triplet;
4. (iv) add 1 to each base block set \(\{0, x_i, z_i\} \pmod{Ω}\) to construct a \((Ω, f, χ; Ξ)\)-CBSEC.

**B. CBSEC-Based QC-LDPC Codes**

Based on above discussion, we utilize the \((Ω, f, χ; Ξ)\)-CBSEC construction to design a length-4 cycles free QC-LDPC codes. Consider a basic matrix \(E(1)\) given as follows:

\[
E^{(1)} = \begin{bmatrix} E_1 & E_2 \\ \hline P_{1,1} & P_{1,2} & \cdots & P_{1,w} \\ P_{2,1} & P_{2,2} & \cdots & P_{2,w} \end{bmatrix},
\]

where the base matrix \(E(1)\) consists of two rows blocks \(E_1\) and \(E_2\). Both of the row blocks contain \(f × f\) circulant permutation matrices \(P_{1,i}\) and \(P_{2,i}\), respectively, \(1 ≤ i ≤ w\), constructed using \((Ω, f, χ; Ξ)\)-CBSEC. More specifically, the \(f × f\) circulant matrices \(P_{1,i}\) are \(1 ≤ i ≤ w\), appearing in the first row block \(E(1)\), are constructed using the base blocks of \((Ω, f, χ; Ξ)\)-CBSEC, where the base block denotes the first row of \(P_{1,i}\), \(1 ≤ i ≤ w\), while all of the other rows are obtained from cyclic-shift operation. Moreover, the \(f × f\) circulant matrices \(P_{2,i}\) are \(1 ≤ i ≤ w\), participating in second row block are constructed from negative base blocks of \((Ω, f, χ; Ξ)\)-CBSEC (mod \(Ω\)) [52]. Hence, the base matrix \(E(1)\) in (49) gives a length-4 cycles free QC-LDPC code, where the minimum distance of designed code is lower bounded by \(2f + 1\) and rate is equal to \((w−2)/w\) [57].

Furthermore, let \(GF(ς)\) is a finite field, where \(ς\) denotes the total number of varieties (elements) exist in field \(GF(ς)\). For each non-zero element \(φ_{11}^ς\) of \(GF(ς)\), we construct a \((ς−1)\)-tuple \(η_ς(φ_{11}^ς) = (η_0, η_1, \ldots, η_{ς−2})\), \(0 ≤ s < ς − 1\), based on the binary field \(GF(s = 2)\), where \(φ_{11}\) is a primitive element of finite field \(GF(ς)\). All of the elements of \(η_ς\) are zero except the \(s\)th entry \(µ_s = 1\). Moreover, if \(φ_{11}^ς = 0\), the binary \((ς−1)\)-tuple is represented by an all-zero vector as \(η_ς(φ_{11}^ς) = 0 = (0, 0, \ldots, 0)\).

Furthermore, for any element \(ψ_{11}φ_{11}\) over \(GF(ς)\), the binary \((ς−1)\)-tuple of \(ψ_{11}φ_{11}\) over \(GF(ς)\) can be obtained from the cyclic-shift of binary \((ς−1)\)-tuple \(η_ς(ψ_{11}φ_{11})\) of any field element \(ψ_{11}\). Thus, a \((ς−1) \times (ς−1)\) binary circular permutation matrix, \(Z_ς(ψ_{11})\), can be constructed based on the \((ς−1)\)-tuples of \(ψ_{11}, ψ_{11}φ_{11}, φ_{11}^2ψ_{11}, \ldots, φ_{11}^{ς−2}ψ_{11}\). However, the binary matrix dispersion \(Z_ς(ψ_{11} = 0)\) returns a \((ς−1) × (ς−1)\) all-zero matrix over \(GF(ς = 2)\).

Based on above discussion, substituting non-zero elements \(ψ_{11}\) of base matrix \(E(1)\) in (49) by \((ς−1) \times (ς−1)\) binary dispersion matrix \(Z_ς(ψ_{11})\) and zero-elements by their \((ς−1) \times (ς−1)\) all-zero matrices \(Z_ς(0)\), We obtain a \(2f × wf\)
array $\mathbf{H}^{(1)}_b$ given as follows:

\[
\mathbf{H}^{(1)}_b = \begin{bmatrix}
Z(0, 0) & Z(0, 1) & \cdots & Z(0, fw - 1) \\
Z(1, 0) & Z(1, 1) & \cdots & Z(1, kw - 1) \\
\vdots & \vdots & \ddots & \vdots \\
Z(2f - 1, 0) & Z(2f - 1, 1) & \cdots & Z(2f - 1, wf - 1)
\end{bmatrix}
\]

(50)

where each $Z(i, j)$ in (50) denotes a $\varsigma \times \varsigma$ matrix over binary field $GF(\varsigma = 2)$, for $0 \leq i < 2f$, $0 \leq j < wf$. Thus, the array $\mathbf{H}^{(1)}_b$ returns a $2f \varsigma \times wf \varsigma$ binary matrix over $GF(\varsigma = 2)$. Moreover, the binary matrix $\mathbf{H}^{(1)}_b$ also satisfies the RC-constraint [57]. Thus, the null space of $\mathbf{H}^{(1)}_b$ gives a class of binary length-4 cycles free QC-LDPC codes.

C. CBSEC-Based Joint Design of QC-LDPC Codes

Here, a joint framework of QC-LDPC codes is provided using the CBSEC-based construction of QC-LDPC codes provided in Section IV-B. Suppose that the null space of three parity-check matrices $\mathbf{H}^{(1)}_{J_1 \times N}$, $\mathbf{H}^{(2)}_{J_2 \times N}$, and $\mathbf{H}^{(3)}_{J_3 \times N}$, realizing the source and relay nodes, give three QC-LDPC codes $C^{(1)}(N, J_1)$, $C^{(2)}(N, J_2)$, and $C^{(3)}(N, J_3)$, respectively. Moreover, $\mathbf{H}^{(1)}_{J_1 \times N}$, $\mathbf{H}^{(2)}_{J_2 \times N}$, and $\mathbf{H}^{(3)}_{J_3 \times N}$ are obtained from $(\Omega, f, \chi; \Xi)$-CBSEC construction of QC-LDPC codes provided in (50). Thus, the joint construction of parity-check matrix $\mathbf{H}_{(J_1 + J_2 + J_3) \times (N + J_3)}$ for the joint decoding of $U_f$ signal can be expressed as follows

\[
\mathbf{H}_{(J_1 + J_2 + J_3) \times (N + J_3)} = \begin{bmatrix}
\mathbf{H}^{(1)}_{J_1 \times N} & 0_{J_1 \times J_3} \\
\mathbf{H}^{(2)}_{J_2 \times N} & 0_{J_2 \times J_3} \\
\mathbf{H}^{(3)}_{J_3 \times N} & \mathbf{H}^{(3)}_{J_3 \times J_3}
\end{bmatrix}
\]

(51)

where $J_1 = \lambda_2(\varsigma - 1)$, $J_2 = \lambda_2(\varsigma - 1)$, $J_3 = \lambda_3(\varsigma - 1)$, and $N = \mu_3(\varsigma - 1)$, for $1 \leq \lambda_1, \lambda_2, \lambda_3 \leq 2f$ and $1 \leq \mu_3 \leq wf$. Finally, at the destination node, the joint parity-check matrix $\mathbf{H}_{(J_1 + J_2 + J_3) \times (N + J_3)}$ is used to realize the SPA-based joint iterative decoder which decodes the corrupted streams of $U_f$ data revived in two distinct time frames for considered BSC-enabled cooperative NOMA system.

V. NUMERICAL RESULTS AND DISCUSSION

To evaluate the performance of the proposed BSC-enabled cooperative NOMA system, the simulation results have been presented in terms of energy efficiency under various performance simulation parameters. For comparison, the performance of the proposed cooperative NOMA framework with backscatter tag (WBST) is compared with the cooperative NOMA network with no backscatter tag (NBST), as well as Exhaustive-search algorithm. Based on [39], [40], we adopt the simulation parameters within the practical range. The path loss $L$ at distance $d$ is modeled as $L(d) = \eta_0 (\frac{d}{d_0})^{-\mu}$, where $\eta_0$ denotes the path loss at reference distance $d_0$, $\mu$ is the path loss exponent and $d$ denotes the distance between two wireless nodes of the considered system. Also, $\eta_0 = -30$ dB, $d_0 = 1(m)$, and $\mu = 4$. Further, Rayleigh-fading transmission is assumed in the presence of AWGN noise with noise power $\sigma^2 = -114$ dBm. Further, Monte Carlo simulations, under $10^4$ channel realizations, have been exploited to obtain the simulation results. Unless specified, the simulation parameters adopted for considered systems are summarized in Table II.

The effect of total available transmit power of source node $P_t$ on the energy efficiency of the considered system under different values of $\eta$ has been depicted in Fig. 2. It can be observed that initially, the energy efficiency of the system increases by increasing the value of $P_t$. However, after a certain point, a further increment in the value of $P_t$ has no impact on the performance of the system, and the energy efficiency becomes constant. The reason for this trend is that the transmit power becomes efficient to meet QoS requirement and allocated power remains unchanged with further increase in the value of $P_t$. Furthermore, the simulation results demonstrate that the proposed WBST system outperforms its NBST competitor under different values of $\eta$. 

| Parameters                              | Values |
|----------------------------------------|--------|
| Channel realizations                   | $10^4$ |
| Bandwidth ($W$)                        | 1 MHz  |
| BS – $U_n$ distance                    | $d_{a,n} = 20m$ |
| BS – $U_f$ distance                    | $d_{a,b} = 20m$ |
| $U_n$ – $U_f$ distance                 | $d_{n,f} = 5m$ |
| $BST$ – $U_n$ distance                 | $d_{b,n} = 10m$ |
| $BST$ – $U_f$ distance                 | $d_{b,f} = 10m$ |
| Imperfect SIC parameter ($\eta$)       | 0.1 – 0.4 |
| Path-loss exponent ($\mu$)             | 4      |
| Transmit power budget                  | 30 dBm |
| Minimum data rate for QoS ($R_{\min}$) | 0.5 bits/sec |
| Relay power budget                     | 10 dBm |
| Circuit power ($P_c$)                  | 0.001 W |
| Noise power ($\sigma^2$)               | -114 dBm |
| Fast-fading                            | Rayleigh-fading |

Fig. 2. The effect of maximum available transmit power of source on the energy efficiency under different values of $\eta$. 

TABLE II 
SIMULATION PARAMETERS
Fig. 3. The impact of maximum available power of relay node on the energy efficiency of the system under different values of $\eta$.

Fig. 4. The impact of increasing $P_c$ on energy efficiency under different values of $\eta$.

Fig. 5. The impact of increasing $\eta$ on energy efficiency under different values of $P_c$.

Fig. 6. The convergence of energy efficiency for different number of iterations.

Fig. 7. The convergence behavior of the considered BSC cooperative NOMA system under different values of $\eta$.

Fig. 8. The comparative performance analysis of the proposed WBST framework and its Exhaustive-search competitor for increasing values of imperfect-SIC parameter $\eta$.

Fig. 9. The convergence of energy efficiency for different number of iterations.

Fig. 10. The impact of imperfect SIC parameter $\eta$ on the energy efficiency of considered BSC-enabled cooperative NOMA system.
maximization problem was decoupled into three sub-problems to find the optimal solutions which finally yields an energy-efficient low-complexity alternating optimization algorithm that requires only a few iterations for convergence. The simulation results evince that the proposed optimization framework outperforms its NBST and Exhaustive-search counterparts by providing an efficient trade-off in terms of computational complexity and energy-efficiency performance. Furthermore, a new class of jointly designed QC-LDPC codes has been proposed that provides an efficient error-correction performance for considered BSC-enabled coded-cooperative NOMA system. Consequently, efficient channel coding techniques could play a crucial role to tackle the chain process of error propagation due to imperfect SIC decoding for NOMA-enabled next-generation communication systems.

VI. CONCLUSION AND REMARKS

A novel alternating optimization framework has been proposed to enhance the energy efficiency of considered BSC-enabled cooperative NOMA system. More specifically, the energy-efficiency of the system has been maximized by optimizing the transmit power of the source, power allocation coefficients, and power of the relay node under the imperfect SIC decoding at the receiver. The proposed energy-efficiency
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