Restrictions on Information Transfer in Quantum Measurements

S.N. Mayburov
Lebedev Inst. of Physics
Leninsky Prospect 53, Moscow, Russia, 117924
E-mail: mayburov@sci.lebedev.ru

Abstract

Information-theoretical restrictions on the information, transferred in quantum measurements, are regarded for the measurement of quantum object $S$ performed via its interaction with information system $O$. This information restrictions, induced by Heisenberg commutation relations, are derived in the formalism of inference maps in Hilbert space. $O$ restricted states $\xi^O$ are calculated from Shrödinger $S,O$ dynamics and the structure of $O$ observables set (algebra); $O$ decoherence by its environment is also accounted for some $S,O$ systems. It's shown that this principal constraints on the information transfer result in the stochasticity of measurement outcomes; consequently, $\xi^O$ describes the random ‘pointer’ outcomes $q_j$ observed by $O$ in the individual events.

Presented at Symposium on Foundations of Modern Physics
Vienna, June 2007

1 Introduction

Despite its significant achievements, Measurement Theory of quantum mechanics (QM) still contains some open questions concerned with its internal consistency\cite{1,2,3}. The most famous and oldest of them is the State Collapse or Objectification problem, but there are also others, more subtle and less well-known\cite{4,5,6}. In this paper this problem called often the Quantum Measurement problem is studied mainly within the framework of Information Theory. Really, the measurement of some system $S$ includes the transfer of information from $S$ to the information system $O$ (Observer) which processes and memorizes it. In Information Theory, any measuring system (MS) can be regarded as the information transferring channel, which transfers the parameters of $S$ state to $O$. Consequently, the possible restrictions on the information
transfer from $S$ to $O$ can influence the effects observed in the measurements. In our previous paper it was shown that such restrictions are principally important for the whole picture of measurement. In particular, they induce the unavoidable stochasticity in the observed by $O$ outcomes of measurements, which, by the all appearances, coincide with the collapse of measured state$^8$. Our calculations of this effects exploited the formalism of Observable Algebra - $C^*$-algebra which is most general and deep mathematical QM formulation$^4$. However, this formalism is rather complicated and abstract, so in this paper the same calculations are reconsidered by means of standard Schrödinger QM formalism; it permits also to analyze the fundamental QM aspects more simply and straightforwardly. It supposed that the evolution of all objects, including macroscopic ones, like $O$, can be described by the quantum state (density matrix) $\rho(t)$ which obeys to Schrödinger-Liouville equation in arbitrary reference frame (RF).

The information-theoretical approach of system self-description or 'measurement from inside' is applied to the consistent description of information acquisition in quantum measurements$^5$. In Schrödinger QM framework, the formalism of inference maps in Hilbert space$^5$ responds to this approach, it applied for the calculations of information transfer from $S$ to $O$.

The principal features of Schrödinger formalism, essential for our measurement formalism, can be formulated as QM eigenstates ansatz$^7$: for any observable $G$ of quantum finite-dimensional system $\Xi$ there at least two $\Xi$ (pure) states $\rho_i$ for which $\Xi$ possess the different real properties $g_i$ and they are $G$ eigenvalues$^1$. Remind$^4$ that, in general, an arbitrary $\rho$ will be $G$ eigenstate with eigenvalue $g_0$: $G, \rho \to g_0$, iff $\tilde{G}^l = (\tilde{G})^l = g_0^l$ for any natural $l > 0$. Thus, predicts with definiteness the result of $G$ measurements for $\rho_i$ states in the individual events of measurement. All $\Xi$ individual states, i.e. the states in particular event are pure (but can be unknown), yet the statistical (ensemble) states can be also mixed$^6$. As shown below, the set of such $\Xi$ eigenstates $\{\rho_i^G \}$ for various $G$ constitutes for given $\Xi$ the ‘information’ basis, which permits to derive the measurement properties of arbitrary state $\rho'$ from its comparison with $\{\rho_i^G \}$ properties. Overall, we shall argue that together with the systems' self-description formalism of information theory permit to construct the consistent measurement formalism, without inclusion of QM Reduction Postulate. Further details concerning with the inference maps, systems' self-description, etc., can be found elsewhere$^6,8$. Early version of this text was published in$^{12}$.

2 Model of Quantum Measurements

Here our measurement model will be described and some aspects of QM Measurements Theory, essential for our approach, discussed at semiqualitative level. In our model MS consists of the studied system $S$, detector $D$ and the information system $O$, which memorizes and process the information about MS current state. The effects of $D$, $O$ decoherence by their environment aren’t of primary importance in our theory, they will be regarded briefly in the final part of the paper. As in many other models of measurement$^{1,10}$, $S$ is taken to be the particle with the spin $\frac{1}{2}$ and the measurement of its projection $S_z$ is regarded. Its $u,d$ eigenstates denoted $|s_{1,2}\rangle$, the measured $S$ pure state is: $\psi_S = a_1|s_1\rangle + a_2|s_2\rangle$. For the comparison, the incoming $u,d$ 'test' mixture with the same $\hat{S}_z$ should be regarded also. This is $S$ ensemble described by the gemenge$^1$ $W^S = \{|s_i\rangle, P_i\}$, where $P_i = |a_i|^2$ are the probabilities of individual state $|s_i\rangle$ in this ensemble, its statistical state described by the density matrix:

$$\rho^m_S = \sum_i |a_i|^2 |s_i\rangle \langle s_i|$$ (1)

Analogously to $S$ state, $D$ state in $O$ RF is supposedly described by Dirac vector $|D\rangle$ in two-dimensional Hilbert space $\mathcal{H}_D$. Its basis constitutes $|D_{1,2}\rangle$ eigenstates of $Q$ 'pointer' observable
with eigenvalues $q_{1,2}$. The initial $D$ state is: $|D_0\rangle = \frac{1}{\sqrt{2}}(|D_1\rangle + |D_2\rangle)$. $S$, $D$ interaction $\hat{H}_{S,D}$ starts at $t_0$ and finishes effectively at some $t_1$; for Zurek Hamiltonian $\hat{H}_{S,D}$ with suitable parameters it would result in $S$, $D$ entangled final state $\rho_{S,D}$ or the corresponding state vector:

$$\Psi_{S,D} = \sum a_i |s_i\rangle |D_i\rangle$$

(2)

relative to $O$ RF. Hence the measurement of $S$ eigenstate $|s_{1,2}\rangle$ results in $\Psi^{1,2} = |s_{1,2}\rangle |D_{1,2}\rangle$, this is factorized $S$, $D$ state. In case $a_{1,2} \neq 0$, $D$ also possess the quantum state $R_D$, but it can’t be completely factorized from the entangled $S$, $D$ state $\Psi_{S,D}$. If to neglect this entanglement, $D$ statistical state coincides with MS partial state, obtained by tracing out $S$ degrees of freedom from $S$, $D$ state. It will be argued that the situation with individual $D$ state ansatz is more subtle and ambiguous, but in our calculations no particular $R_D$ ansatz is used. It turns out that $\hat{Q} = |a_1|^2 - |a_2|^2$, so $D$ performs $S_z$ measurement of first kind$^1$. At $t > t_1$ $D$, $O$ interaction starts and finishes at some $t_2$, during this interval, some information about $D$ state is transferred to $O$. In this chapter $O$ isn’t described consistently as the quantum object, it will be done in chap. 3. It assumed arbitrarily that the acquisition of information by $O$ doesn’t violate QM laws. It’s supposedly true for human observer also, below some terms characteristic for human perception will be used in illustrative purposes.

In Information Theory, the signal induced by the measured state and registrated by $O$ in event $n$ is characterized by the information pattern (IP) $J(n) = \{e_1, ..., e_l\}$, this is the array of numerical parameters, which represent the complete signal description available for $O$$^6$. The difference between two individual states for $O$ is reflected by the difference of their IPs$^6$; if some $e_i$ are the same for all studied states, they can be omitted in $J$. In general, the set of all possible $J$ constitutes the independent ‘information space’, which describes $O$ recognition of measured states or signals$^7$. In quantum case, some states parameters can be uncertain, below it will be shown that it’s unimportant for the corresponding IPs and only definite $e_i$ should be regarded.

Let’s regard first the measurement of $S$ eigenstate $|s_{1,2}\rangle$, in that case, $O$ supposedly perceives $|D_{1,2}\rangle$ state in event $n$ as IP:

$$J(n) = J^{D}_{1,2} = e_1 = q_{1,2}$$

For final $|D_{1,2}\rangle$ states their $Q$ eigenvalues $q_{1,2}$, which describe $D$ pointer position are $D$ real properties$^1$, corresponding to the orthogonal projectors $P^{D}_{1,2}$. Hence for $O$ the difference between this $D$ states is the objective or Boolean Difference$^3$ (BD). It means that this states difference is equivalent to the distinction between the logical operands $Yes/No$, or that’s the same between the values $1/0$ of some discrete parameter $L_g$ available for $O$. Note that in QM all measurable parameters are related to the observables which represented by Hermitian Operators on $\mathcal{H}$ (or POV in general formalism). For example, for $|D_{1,2}\rangle$ the parameter $L_g = 1/0$ is the eigenvalue of projector $P^D_1$. It will be argued that such strict correspondence reduces the number of feasible parameters, which can be applied for the discrimination of quantum states.

Now let’s regard the possible measurement outcomes when $a_{1,2} \neq 0$, i.e. $\psi_s$ is $|s_i\rangle$ superposition. The standard or ‘Pedestrian’ Interpretation$^2$ (PI) of QM claims that without the inclusion of Reduction Postulate into QM formalism, $O$ should perceive $S$, $D$ entangled state $\Psi_{S,D}$ as the superposition of IPs induced by $|D_{1,2}\rangle$, i.e. the simultaneous coexistence of two IP $J^D_1$ and $J^D_2$. This hypothesis is the essence of famous ‘Schrödinger Cat’ Paradox$^3$. More realistically, one can expect at least that IP $J^s$, induced by $\Psi_{S,D}$, would differ for $O$ from $J_{1,2}^D$. Yet the situation isn’t so simple and doesn’t favor such prompt jump to the conclusions. Really, given PI implications are correct, $O$ should distinguish in a single event $\Psi_{S,D}$, i.e. the corresponding $D$ state $R_D$ from each $|D_{1,2}\rangle$. Hence the relation of corresponding $O$ IPs should be characterized by BD, i.e. for $\Psi_{S,D}$ the corresponding $J^s \neq J^D_{1,2}$. Hence it should be at least one measurable $D$
parameter $G^D$ which value $g_0$ for $\Psi_{S,D}$ is different from its values $g_{1,2}$ for $|D_{1,2}\rangle$. To verify this hypothesis for $\Psi_{S,D}$ and $|D_{1,2}\rangle$, one should check the set (algebra) of $D$ PV observables \{G^D\} as possible candidates (the role of joint $S$, $D$ observables regarded below). The simple check shows that no $D$ observable also can satisfy to this demands; POV generalization of this ansatz will be discussed below, but it doesn’t change this conclusion. Really, suppose that such $G^D$ - Hermitian operator exists, then it follows:

$$G^D\Psi_{S,D} = a_1|s_1\rangle G^D|D_1\rangle + a_2|s_2\rangle G^D|D_2\rangle = g_0\Psi_{S,D} \tag{3}$$

As easy to see, for $D$ observables such equality fulfilled only for $G^D = I$. It can be shown that any nontrivial $G^D$ with such properties can respond only to the nonlinear operator on $\mathcal{H}_D$, hence the observation of such difference is incompatible with standard QM formalism. Consequently, it’s impossible for $O$ to distinguish $|D_i\rangle$ from $R_D$ i.e. from $\Psi_{S,D}$ of (2) in a single event. Meanwhile, for $S$ ensemble one can expect that the correct $\bar{Q}$ is obtained by $O$ in $S$ detection, to fulfill this condition, $O$ should observe the stochastic $q_{1,2}$ outcomes with probabilities $P_{1,2}$. This considerations put doubts on the necessity of independent Reduction Postulate in QM, the similar hypothesis was proposed first by Wigner\(^9\) from the considerations of quantum measurements and consequent information acquisition by human observer. To give more arguments in favor of our theory, it’s instructive to consider in detail the possible influence of quantum measurements and consequent information acquisition by human observer. To give suitable $D$, $O$ Hamiltonian $H_{D,O}$ one can obtain at $t > t_2$:

$$\Psi_{S,D,O} = \sum a_i|s_i\rangle |D_i\rangle |O_i\rangle$$

As easy to see, $D$ states only double $S$ states for this set-up, so $D$ can be dropped for the simplicity. In such scheme $S$ directly interacts with $O$ by means of Hamiltonian $H_{S,O}$, resulting in the final state $\rho_{MS}$ and corresponding state vector:

$$\Psi_{MS} = \sum a_i|s_i\rangle |O_i\rangle \tag{4}$$

relative to some external RF $O'$. Our aim is to find find the relation between this state and the information acquired by $O$, which is quite intricate problem. In Information Theory, the measurement of parameters of arbitrary system $S'$ by an information system $O'$ is the mapping of $S'$ states set $N_S$ to the set $N_O$ of $O'$ internal states\(^6\). In general case, which is generic for QM, the information acquisition by $O'$ can be described by the formalism of systems’ self-description\(^7\).

In its framework, $O'$ considered as the subsystem of larger system $\Xi_T = S', O'$ with the states set $N_T$. This approach gives the most fundamental and mathematically self-consistent description of the information transfer in arbitrary $S'$ measurement called ’measurement from inside’. In this case, the information acquired by $O'$ about the surrounding objects and $O'$ itself described by $O'$ internal state $R_O$ called also $\Xi_T$ restricted state or restriction. For given $\Xi_T$ system $R_O$ is
defined by the inference map $M_O$ of $\Xi_T$ state to $N_O$ set, in general, $M_O$ should be derived from the first QM principles. The internal $O^I$ state $R_O$ corresponds to $O^I$ internal degrees of freedom, which, in principle, can 'record' the incoming information. For example, if $O^I$ is the atom, $R_O$ would describe the state of its electron shells, which are the functions of the relative coordinates of electrons and nucleus. The important property of inference map $M_O$ is formulated by Breuer Theorem: if for two arbitrary $\Xi_T$ states $\Gamma, \Gamma'$ their restricted states $R, R'$ coincide, then for $O^I$ this $\Xi_T$ states are indistinguishable, and for any nontrivial $S', O^I$ at least one such pair of states exist. In classical case, the origin of this effect is obvious: $O^I$ has less degrees of freedom than $\Xi_T$ and hence can't discriminate all possible $\Xi_T$ states. In quantum case, the entanglement and nonlocality play the additional important role in this effect and make its description more complicated. Despite that $R_O$ are incomplete $\Xi_T$ states, they are the real physical states for $O^I$ observer - 'the states in their own right', as Breuer characterizes them. as will be argued, in this approach the information acquired by $O$ also expressed by IP $J$, main features of IPs described in chap. 2, conserved also for $R_O$ states.

The mapping relations between $S', O^I, \Xi_T$ states are applicable to MS measurement model which can be also treated as 'measurement from inside'. However, Schrödinger QM dynamics by itself doesn't permit to derive the inference map $M_O(\Xi_T \rightarrow O^I)$ unambiguously, it needs more detailed development of formalism described below. Breuer attempted to avoid this ambiguity phenomenologically, assuming that for arbitrary $\Xi_T$ its restricted state is equal to the partial trace of $\Xi_T$ individual state over $S'$, i.e. $R_O$ is $\Xi_T$ partial state on $O^I$. In our set-up for MS pure state $\Psi_{MS}$ of (3) it gives:

$$R^B_O = Tr_s \rho_{MS} = \sum |a_i|^2 |O_i\rangle\langle O_i|$$

Plainly, such ansatz excludes beforehand any kind of stochastic $R_O$ behavior, and this is natural for Schrödinger formalism. For MS mixed ensemble, induced by incoming $W^s$ ensemble of (1), the individual MS state differs from event to event:

$$\xi^{MS}(n) = |O_i\rangle\langle O_i| |s_l\rangle\langle s_l|$$

where the frequencies of random $l(n)$ appearance in given event $n$ are stipulated by the probabilistic distribution $P_1 = |a_i|^2$. In this approach $O$ restricted state for this mixed ensemble is also stochastic: in a given event

$$R^{mix}_O(n) = \xi^O_1...\xi^O_2,$$

where $\xi^O_i = |O_i\rangle\langle O_i|$ appears with the corresponding probability $P_1$, so that the ensemble of $O$ states described by the gemenge $W^{mix}_O = \{\xi^O_i P_1\}$. $R^{mix}_O(n)$ differs formally from $R^B_O$ in any event $n$, hence for the restricted $O$ individual states the main condition of cited theorem is violated. From that Breuer concluded that $O$ can discriminate the individual pure/mixed MS states 'from inside', therefore, $O$ can discriminate the individual pure and mixed $S$ states, it supposedly means that the collapse of pure state doesn't occur.

However, we find that the structure of MS observables set (algebra) together with Schrödinger dynamics permit to calculate MS restriction to $O$ unambiguously, and the obtained results contradict to Breuer conclusion. Consider the measurement of $S$ eigenstate $|s_i\rangle$, it produces MS individual $\xi^{MS}$ states, which restriction are $\xi^O_{i,2}$ states with eigenvalues $q^O_{1,2}$. One can expect that $O$ identifies this states as IP:

$$J = J^O_{1,2} = q^O_{1,2}$$

Really, the difference between $\xi^O_i$ states is boolean (classical), because their mutual relation expressed as: $|O_i\rangle|O_j\rangle = \delta_{ij},$ and according to Segal theorem, it corresponds to the relation
between the classical discrete states defined on $Q_O$ axe$^4$. In addition, in QM formalism $\xi_i^O$ eigenvalues $q_i^O$ are $O$ real properties, so this hypothesis seems to be well founded. The proposed correspondence $\xi_i^O \to J_i^O$ is quite important for our theory, because it establishes the connection (mapping) between some $O$ quantum states and classical IP set \{\textit{J}\}. Generally speaking, this set is the principally different entity from any space of dynamical states, both quantum and classical, since its elements \textit{J} describe the results of $O$ identification of incoming signals$^7$.

Now we shall use the obtained IP ansatz for the consideration of state collapse, in particular, we shall reconsider Breuer conclusions. Note that the formal difference of two restricted states doesn’t mean automatically that their difference will be detected by $O$. Such difference is the necessary but not sufficient condition, there should be also some particular effect available for $O$ observation, which indicate this difference. For $\xi_i^O$ this are the eigenvalues $q_i^O$ of observable $Q_O$, resulting in BD of $\xi_i^O$ for $O$ and described by IP $J_i^O$. Analogously, one should explore whether some discriminating effect observed by $O$ can indicate $R_O$ and $\xi_i^O$ difference. The check of this hypothesis can be performed analogously to the ansatz described by formulae (3), but for the completeness of our proof it’s instructive to use the alternative operator methods. Suppose that $R_O \neq \xi_{1,2}^O$ for $O$ in BD sense, so that their relation should be expressed also by some $O$ parameter $g$ with the values $g_0 \neq g_{1,2}$, correspondingly. In QM formalism, such parameter $g$, if it exists, should correspond to some $O$ observable $G^O$. It should be such $O$ PV observable $G^O$, for which $R_O, \xi_i^O$ are its eigenstates with projectors $P^R, P_i^O$ and eigenvalues $g_0, g_i$, such that $g_0 \neq g_i$. From Spectral Theorem$^3$ an arbitrary Hermitian operator $G^O$, for which $R_O, \xi_i^O$ are eigenstates, admits the orthogonal decomposition:

$$G^O = G' + g_0 P^R + g_1 P_{1}^O + g_2 P_{2}^O \tag{7}$$

here $G'$ is an arbitrary operator for which $G' P_{i}^O = 0, G' P^R = 0$; it should be also $P^R P_{i}^O = 0$. But $P_{i}^O$ constitute the complete algebra of projectors in $\mathcal{H}_O$ corresponding to the orthogonal unit decomposition: $\sum P_{i}^O = I$, and so:

$$P^R = P^R I = P^R \sum P_{i}^O = \sum P^R P_{i}^O = 0, \tag{8}$$

Hence $R_O$ can’t possess the independent projector in $\mathcal{H}_O$ and such nontrivial $G^O$ doesn’t exist. From the correspondence between the states and their projectors follows that $R_O^B$ of (5) isn’t proper ansatz for $\Psi_{MS}$ restriction $R_O$, the only solution is to accept that in any event $P^R = P_{i}^O$ and correspondingly:

$$R_O = \xi_{1}^O \; \text{or} \; \xi_{2}^O. \tag{9}$$

i.e. it coincides with $R_{O}^{mix}$ as the individual state.

For pure MS ensemble the expectation value $\tilde{Q}_l^O$, for any natural $l$ can be calculated without the use of Reduction Postulate from Graham-Hartle theorem$^{11}$, based on quite loose assumptions. To reproduce this $\tilde{Q}_l^O$ values, $O$ should observe the collapse of pure MS state to one of $q_i^O$ at random with probability $P_l = |a_l|^2$, i.e. the ensemble of $O$ states described by the genejence $W^O = \{\xi_i^O, P_l\}$. It induces the corresponding $O$ IP ensemble $Z^O = \{J_i^O, P_l\}$. Eventually, the inference map $M_O$ for $\Psi_{MS} \to R_O$ restriction is stochastic, we don’t present here $M_O$ ansatz in the analytical form, which can be easily derived from the previous calculations but is rather tedious. Our studies show that POV generalization of standard QM PV observables don’t change our conclusions. The reason of it is that POV parameters respond to the nonorthogonal unit decomposition, yet BD can exist only between mutually orthogonal states, hence it can be shown that there is no $O$ POV observable which responds to BD between $R_O$ and $\xi_i^O$. In the regarded case, only the parameters corresponding to nonlinear operators can establish BD between this states.
Thus, our assumption about the role of information constraints in state collapse is proved formally, here we shall discuss in more qualitative terms the physical mechanism of such stochasticity in the measurements, because it presents the significant interest for the study of QM foundations. Remind that in QM formalism two kinds of uncertainties exist: suppose that for some state $\rho$ the value $\tilde{g}$ of observable $G$ lays in the interval $g_{\text{min}} \leq \tilde{g} \leq g_{\text{max}}$. Then, depending on $\rho$, it can be either the stochastic value, i.e. objectively $\tilde{g} = g_i$ with some probability $P^i_i$, if $\rho$ is a mixed state, or it can be truly uncertain (fuzzy) value $\tilde{g}$ for pure $\rho$. The difference between this two states revealed by 'interference term' (IT) observable $s$, which demonstrate the presence of $\tilde{g}$ superposition. For MS entangled states no $O$ observables are sensitive to it, it can be only joint $S,O$ observables $B^{MS}$. As the example, consider the symmetric IT for MS:

$$B = |O_1\rangle\langle O_2|s_1\rangle\langle s_2| + j.c. \quad (10)$$

Being measured by external RF $O'$ via its interaction with $S$, $O$, it gives $\tilde{B} = 0$ for any $|s_i\rangle$ incoming mixture, but $\tilde{B} \neq 0$ for entangled MS states of (4). For example, for the incoming symmetric $S$ state $\psi^S_1$ with $a_{1,2} = \frac{1}{\sqrt{2}}$, the corresponding $\Psi^S_{MS}$ is $B$ eigenstate with eigenvalue $b_1 = 1$. However, $B$ value can’t be directly measured by $O$ 'from inside', at least simultaneously with $S_2$, because they don’t commute\(^5\). In addition, when $S,O$ interaction finishes, $S$ can become free particle again, and so the joint $S,O$ observables can become unavailable for $O$ in a short time. From the same reasons the whole set of IT observables $\{B^{MS}\}$ is unavailable for $O$ during $S_2$ measurement, but only some $O$ internal observables. Note that, in general, the pure/mixed MS states with the same $Q_O$ can be discriminated only statistically, since their distributions of $B$ values (or other $B^{MS}$) overlap. Namely, for $\Psi^S_{MS}$ the probability $P_B(b_{1,2}) = .5$ for such mixture, so its $b$ distribution intersects largely with $b$ distribution for $B$ eigenstate $\Psi^S_{MS}$. Consequently, even $O'$ can’t discriminate the pure/mixed MS states in a single event, but only statistically for MS ensemble with $N \to \infty$. Overall, it follows from this analysis that no IT observables are available for $O$, and so one can expect that $O$ can’t discriminate the pure and mixed $O$ states, for which $q^O$ is 'smeared' inside the same uncertainty interval. Yet we know that $P^O_{\text{mix}}$ ensemble $W^O_{\text{mix}}$ can reproduce the same $Q^O_f$ values as $\Psi^O_{MS}$ ensemble. Because of this reasons, $R_O$ ensemble $W^O$ should coincide with the mixed ensemble $W^O_{\text{mix}}$. Consequently, for $\Psi^O_{MS}$ the genuine $q_0$ uncertainty can be released in $O$ RF only in the form of $q_0$ randomness inside its uncertainty interval $\{q^O_1,q^O_2\}$. Roughly speaking, $\Psi^O_{MS}$ induces $O$ signal, which in each event put some income into the resulting $Q^O_f$ value, so that their ensemble should produce the proper $Q^O_f$ values. But it turns out that such signal can be only stochastic, no ensemble of identical signals with demanded properties exist. It seems that our results constitute the kind of no-go theorem for the observation of $|s_i\rangle$ superpositions by $O$.

It’s well known that the decoherence of pure states by its environment $E$ is the important effect in quantum measurements\(^1,2,10\), we find yet that $O$ decoherence by $E$ doesn’t play the principal role in our theory. However, its account stabilizes the described collapse mechanism additionally and defines unambiguously the preferred basis $PB$ of $O$ stable final states $\{\xi^O\}$ exploited here. Really, for the specially chosen Hamiltonian of $O,E$ interaction\(^10\), one obtains that MS,$E$ final state is:

$$\Psi^{MS,E} = \sum a|s_i\rangle|O_i\rangle \prod_j |E^j_i\rangle$$

where $E^j$ are $E$ elements, $N_E$ is $E^j$ total number. As easy to demonstrate, if an arbitrary $O$ pure state $\Psi_O$ is produced, it will also qecohere in a very short time into the analogous $|O_i\rangle$ combinations, entangled with $E$, so that $O$ can practically percept only $|O_i\rangle$ final states. Such
O PB can be derived from other more subtle arguments, yet O decoherence even for quite small $N_E \sim 2 \div 4$ already defines it effectively.

We conclude that standard Schrödinger QM formalism together with the theory of quantum systems’ self-description permit to obtain the collapse of the measured pure state without implementation of independent Reduction Postulate into QM axiomatics. As was shown, in this approach the main source of stochasticity is the principal constraint on the transfer of specific information in $S \rightarrow O$ information channel. This information, unavailable for $O$, characterizes the purity of $S$ state because of it, $O$ can’t discriminate the pure/mixed $S$ states. As the result of this information incompleteness, the stochasticity of measurement outcomes appear, which is the analog of fundamental ‘white noise’. In addition, the formalism of systems’ self-description permits to resolve also the old problem of Heisenberg cut in quantum measurements, by the inclusion of the information system into quantum formalism properly and on equal terms with other MS elements. Of course, the most exciting and controversial question is whether this theory is applicable to the observations made by human observer $O$, in particular, whether IP $J$ describes the true $O$ ‘impressions’ about their outcomes? This is open problem, but since our theory is based on standard QM premises, and at the microscopic level the human brain should obey QM laws, we believe that the answer can be positive. Note that in our theory the brain or any other processor plays only the passive role of signal receiver, the real effect of information loss, essential for collapse, occurs ‘on the way’, when the quantum signal passes through the information channel. The interesting feature of this theory is that the same MS state can be stochastic in $O$ RF, but evolve linearly in $O'$ RF. In particular, $\Psi_{MS}$ restriction to $O$ in $O$ RF is stochastic state $R_O$ of (9), yet in $O'$ RF $O$ partial state is $R_{O'}$ of (4), i.e. is the 'weak superposition'. The detailed explanation of this effect is given by the unitarily nonequivalent representations admitted in Algebraic QM. Here we notice only that $O$ and $O'$ deal with different sets of MS observables, and so the transformation of MS state between them can be nonunitary. Obtained results agree well with our calculation in $C^*$ Algebras formalism, in that approach the inference map $M_O$ is the operator restriction of MS observable algebra to $O$ (sub)algebra.

References

[1] P.Busch, P.Lahti, P.Mittelstaedt, Quantum Theory of Measurements, (Springer-Verlag, Berlin,1996), pp. 8–26
[2] W.D'Espagnat Found Phys. 20 (1990) 1157
[3] J.M.Jauch 'Foundations of Quantum Mechanics' (Adison-Wesly, Reading, 1968), pp. 85-116
[4] (1972) G.Emch, 'Algebraic Methods in Statistical Physics and Quantum Mechanics', (Wiley,N-Y,1972)
[5] T.Breuer Synthese 107 (1996) 1
[6] P.Mittelstaedt 'Interpretation of Quantum Mechanics and Quantum Measurement Problem’, (Oxford Press, Oxford,1998) pp. 67–109
[7] K.Svozil 'Randomness and undecidability in Physics', (World Scientific, Singapour,1993) pp. 46–87
[8] S.Mayburov, Information-Theoretical Restrictions in Quantum Measurements, quant-ph/0506065;
[9] E.Wigner, 'Scientist speculates',(Heinemann, London,1961), pp 47–59
[10] W.Zurek Phys. Rev. D26 (1982) 1862
[11] J.B.Hartle Amer. J. Phys. 36 (1968) 704
[12] S.Mayburov Int. J. Quant. Inf. 5 (2007) 279