Numerical simulation and experimental investigation of a novel Scotch yoke for a Gifford-McMahon cryocooler

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Abstract. A novel Gifford-McMahon (GM) cycle, called an asymmetric GM cycle, is proposed. In an asymmetric GM cycle, the displacer moves slowly when it is adjacent to the upper dead point but moves rapidly when it is adjacent to the lower dead point. Therefore, the expansion process is longer, while the discharging process is shorter than in a conventional GM cycle. Meanwhile, the duration of the charging process can be kept the same as that in a conventional GM cycle. Accordingly, the phase shift between the pressure and the displacement can be improved, and the mass flow rate into the expansion space can also be increased. Therefore, the P-V work and the cooling capacity can be increased. To implement the GM cycle, a novel Scotch yoke was invented. In the Scotch yoke, there is a concave part at the upper center of the slide groove and a convex part at the lower center of the slide groove. The effect of the Scotch yoke has been confirmed by numerical simulation and experimental investigation. With a conventional Scotch yoke, the cooling capacity was 44 W at 37.4 K at the first stage and 1.0 W at 3.94 K at the second stage. With a novel Scotch yoke, the cooling capacity was 44 W at 35.9 K at the first stage and 1.0 W at 3.96 K at the second stage. The cooling capacity at the first stage at 40 K was improved by about 10%, from 51.5 W to 57.3 W.

1. Introduction
Since 1990, the efficiency of 4 K GM cryocoolers has been continuously improved by optimizing the operation parameter [1-2] and valve timing [3]. 4 K GM cryocoolers have been widely used for cooling superconducting magnets, such as magnets in MRI systems. A large amount of power in an MRI system is consumed by the cryocooler. Especially at night, more than half of the power is consumed by the cryocooler since no diagnosis is performed. To reduce the power consumption, a high-efficiency cold head was developed and the input power was reduced by about 30% compared to a conventional 1 W 4 K GM cryocooler [4-6]. In order to improve the efficiency of the cold head, several novel and effective concepts were proposed and investigated. One novel concept was to modify the Scotch yoke to carry out a GM cycle [7-8]. The configuration, numerical simulation, and experimental investigation with the novel Scotch yoke are reported in this paper.

2. Conventional GM cycle and Scotch yoke
In a GM cryocooler, a Scotch yoke is used to transfer valve motor rotation to reciprocating displacer motion. In a conventional 4 K GM cryocooler, the Scotch yoke has a rectangle slide groove as shown...
in figure 1a. With this conventional type Scotch yoke, the displacement of the displacer is close to a sine wave. In general, to maximize the area of the P-V work diagram for the expansion space, a rapid increase in pressure is required when the displacer is at the bottom of the cylinder and a sudden decrease when the displacer is at the top of the cylinder. Efforts to achieve this result in rapid surges of gas through the system can cause a serious decline in regenerator performance. A solution to the problem was to force the displacer to move in a discontinuous manner so that there are lengthy pauses at each end of the stroke [9-10]. However, in this case, since the total duration of a cycle is fixed, the mass flow rate into the expansion space decreases because of shorter charging and discharging processes. Therefore, the actual cooling capacity decreases. Furthermore, it is difficult to carry out such a cycle with a Scotch yoke.

3. Asymmetric GM cycle and novel Scotch yoke
To overcome the above dilemma, a novel GM cycle, which is called asymmetric GM cycle, was proposed [7]. In an asymmetric GM cycle, the displacer moves slowly when it is adjacent to the upper dead point but moves rapidly when it is adjacent to the lower dead point. Because the displacer moves slowly when it is adjacent to the upper dead point, gas can be expanded more thoroughly. In contrast, the displacer moves rapidly when it is adjacent to the lower dead point. During the discharging process, gas is discharged out of the expansion space. Meanwhile, the remaining gas exchanges heat with the wall of the expansion space. The temperature of the gas discharged from the expansion space increases as the process goes on. The impact on cooling performance is relatively small even though gas is more rapidly discharged when the displacer is adjacent to the lower dead point. Accordingly, in an asymmetric GM cycle, the duration of the charging process can be kept almost the same as that in a conventional GM cycle. Therefore, the phase shift between the pressure and the displacement is improved, and the mass flow rate into the expansion space is increased. Accordingly, the P-V work and the cooling capacity can be increased.

To carry out the above processes, a novel Scotch yoke with a curved slide groove was invented [7-8]. Figure 1b shows the schematic diagram of the novel Scotch yoke that illustrates the principal. Compared to a conventional Scotch yoke, there is a concave part at the upper side of the slide groove and a convex part at the lower side.
4. Displacer movement

In a GM cryocooler, through a crank and roller bearing, a Scotch yoke is used to transfer valve motor rotation to reciprocating displacer motion. When the roller bearing comes in contact with the concave part of the upper side and moves upward, the Scotch yoke does not move in the vertical direction. The displacer in a GM cryocooler stays at the upper dead point. In contrast, when the roller bearing comes in contact with the convex part of the lower side moves downward in the slide groove, the Scotch yoke moves faster than in a conventional GM cryocooler.

Next, the principal operation of a displacer using this novel Scotch yoke is described. As shown in figure 2, at the starting point, for example, an angle of 5.5° after the supply valve opened, the roller bearing is positioned at the boundary between the lower horizontal side and the convex part in the slide groove at the right side. When the crank rotates an angle of $\theta$ from this state, the roller bearing is forced to move the Scotch yoke downward. With this movement, the roller bearing moves inside the slide groove to the left while in contact with the convex part. Here, attention is drawn to the movement speed of the Scotch yoke, which is equivalent to the movement speed of the displacer. The convex part projects in the upper direction relative to the lower horizontal side. The speed of the Scotch yoke is greater when the roller bearing is in contact with the convex part than when the roller bearing is in contact with the lower horizontal side. When the displacer reaches the lower dead point, the roller bearing is positioned at the top of the convex part. When the roller bearing passes this position, the moving direction of the Scotch yoke is reversed. That is, the Scotch yoke moves upward when the roller bearing goes past the position corresponding to the lower dead point. The roller bearing shifts to come in contact with the concave part during the period between the lower dead point and a crank angle of $\theta$ past the lower dead point. The crank further rotates, so that the roller bearing moves inside the slide groove while engaging with the left upper horizontal side. With this movement of the roller bearing, the displacer moves upward. The speed of the Scotch yoke is greater when the roller bearing is in contact with the concave part than when the roller bearing is in contact with the left upper horizontal side.

Next, movement of the roller bearing at the time when the roller bearing is engaged with the concave part and moves upward in the slide groove, is described. The concave part has a shape.

![Figure 2. Movement and displacement of the displacers in a GM cryocooler.](image-url)
depressed relative to the upper horizontal side. The concave part is formed to prevent the Scotch yoke from moving in the vertical direction while the roller bearing is engaged with the concave part and moves upward in the slide groove. When the roller bearing is engaged with the upper horizontal side, the roller bearing forces the Scotch yoke and the displacer to move upward. However, the concave part is formed in the Scotch yoke, and the concave part is depressed relative to the upper horizontal side. Accordingly, even when the roller bearing moves upward with the rotation of the crank, the roller bearing enters the concave part to prevent the Scotch yoke from moving in the vertical direction, so the Scotch yoke becomes stationary. Theoretically, the concave part has such a shape to prevent the Scotch yoke from moving in the vertical direction while the roller bearing is engaged with the concave part. When the roller bearing passes by the position corresponding to the upper dead point of the displacer with the rotation of the crank, the moving direction of the Scotch yoke is reversed. The Scotch yoke moves downward when the roller bearing goes past the position corresponding to the upper dead point. The roller bearing remains in contact with the concave part during the period between the upper dead point and a crank angle of $\theta$ past the upper dead point. The roller bearing moves inside the slide groove to the right from this position to engage with the lower right horizontal side, so that the Scotch yoke moves downward. With this movement of the Scotch yoke, the displacer moves downward.

5. Effect of a novel Scotch yoke

Next, effects produced by providing the concave and convex parts in the Scotch yoke are described. In a GM cryocooler, slightly before the upper dead point is reached, the exhaust valve is opened to expand the refrigerant gas to produce a cooling effect. The exhaust valve is opened at a crank angle before the Scotch yoke reaches the upper dead point. This opening of the exhaust valve causes the refrigerant gas to expand producing a cooling effect. Here, it is assumed that the first-stage and second-stage displacers move fast around the upper dead point. In this case, because of the pressure drop through the regenerators, the expansion process is not long enough. After the expansion process, the pressure in the expansion spaces is higher than the low pressure, and the gas in the expansion spaces cannot be thoroughly expanded reducing the cooling. In contrast, the first-stage and second-stage displacers are paused for a predetermined period of time at the upper dead point. Therefore, the expansion process is extended, the pressure in the expansion space is much closer to the low pressure and the gas in the expansion space can be thoroughly expanded. Also, the asymmetric drive results in the displacer moving more slowly and there is less pressure drop in the regenerator after the inlet valve closes. Therefore, by providing the concave part in the Scotch yoke, it is possible to increase the ideal cooling capacity of the GM cryocooler.

On the other hand, by providing the concave and convex parts in the Scotch yoke, the displacer moves rapidly when it is adjacent to the lower dead point. During the discharging process, gas is discharged out of the expansion space. Meanwhile, the remaining gas exchanges heat with the wall of the expansion space. The temperature of the gas discharging from the expansion space increases as the process goes on. Therefore, the impact on heat transfer performance is relatively small even though gas is more rapidly discharged at the vicinity of the lower dead point. Accordingly, in an asymmetric GM cycle, the duration of the charging process can be kept almost the same as that in a conventional GM cycle. Therefore, the phase shift between the pressure and the displacement can be improved, and the mass flow rate into the expansion space can also be increased. Therefore, the P-V work and the cooling capacity can be increased.

6. Simulation

To understand the mechanism of the novel Scotch yoke, a numerical simulation for a two-stage 4K GM cryocooler was done by using the method introduced in reference [4-5]. The only difference is the displacer movement.

With a conventional Scotch yoke, the displacement of the first stage displacer is,
\[ x_{el} = S_{el} + 0.5 \times S_{el} \times \left\{ 1 - \cos \left( (\alpha - \varphi_{el}) \times \frac{\pi}{180} \right) \right\} \]

where \( S_{el} \) is the length of the dead space and \( S_{el} \) is the stroke of the first stage displacer, \( \varphi_{el} \) is the delay angle of the displacer motion to the starting angle of the supply process.

With a novel Scotch yoke, in principle, the displacement of the first stage displacer is,

\[
x_{el} = \begin{cases} 
S_{el} + 0.5 \times S_{el} \times \left\{ 2.0 - \cos \left( (\alpha - \varphi_{el}) \times \frac{\pi}{180} \right) - \cos \left( \frac{\pi}{180} \theta \right) \right\} & 0 < \alpha \leq \beta \\
S_{el} + S_{el} \times \left\{ 1.0 - \cos \left( (\alpha - \varphi_{el}) \times \frac{\pi}{180} \right) \right\} & \beta < \alpha \leq \gamma \\
S_{el} + 0.5 \times S_{el} \times \left\{ 2.0 - \cos \left( (\alpha - \varphi_{el}) \times \frac{\pi}{180} \right) - \cos \left( \frac{\pi}{180} \theta \right) \right\} & \gamma < \alpha \leq 180 + \beta \\
S_{el} + S_{el} \times \left\{ 2.0 - \cos \left( (\alpha - \varphi_{el}) \times \frac{\pi}{180} \right) + \cos \left( \frac{\pi}{180} \theta \right) \right\} & 180 + \beta < \alpha \leq 360 
\end{cases}
\]

where \( \theta \) is the angle from the center of the Scotch yoke to the edge of the concave part, \( \beta \) and \( \gamma \) are,

\[ \beta = \varphi_{el} - \theta \]
\[ \gamma = \varphi_{el} + \theta \]

The displacement of the second stage displacer and the length variation of the compression space can be calculated similarly by referring to reference [4-5].

For a typical calculation, the high and low pressures are 2.00 MPa and 0.55 MPa, respectively. The cold head is operated at 1.0 Hz. The inner diameters of the first and the second stage cylinder are 82 mm and 35 mm, respectively. The stroke of the displacer is 25 mm. The first stage regenerator is filled with \#150 phosphorous bronze screens. The second stage regenerator is filled with bismuth and HoCu2 spheres.

The displacements of the displacer and the P-V work diagrams in the expansion spaces are shown in figure 2 and figure 3. As shown in figure 2, near the lower dead point, the slope of the displacement curve with a novel Scotch yoke is larger than that with a conventional Scotch yoke. On the other hand, near the upper dead point, the slope of the displacement curve with a novel Scotch yoke is almost zero.

**Figure 3.** P-V diagrams of the expansion spaces in a GM cryocooler. (a) The first stage and (b) the second stage.

As shown in figure 3, the area of the P-V work diagrams with a novel Scotch yoke is larger than
that with a conventional Scotch yoke.

Table 1 shows the simulation results of the P-V work and the cooling capacity. The cooling capacity at the first stage at 40 K was improved from 55.3 W to 58.9 W and the second stage cooling capacity at 4.2 K was improved from 1.63 W to 1.73 W. The first stage cooling performance increases as the P-V work increases. At the second stage, the cooling capacity improvement is small although the P-V work increases about 1 W because the second stage performance is strongly dependent on real gas effect [11-13] and regenerator efficiency.

|                | Cooling capacity | P-V Work | Mass flow rate |
|----------------|------------------|----------|----------------|
|                | 1st stage at 40 K (W) | 2nd stage at 4.2 K (W) | 1st stage (W) | 2nd stage (W) | 1st stage (g/s) | 2nd stage (g/s) |
| Conventional   | 55.3             | 1.63     | 91.9           | 19.7          | 7.1            | 2.8            |
| Asymmetric     | 58.9             | 1.73     | 96.8           | 20.7          | 7.3            | 3.1            |

Table 1. Simulation results of cooling performance.

7. Experimental Results

According to the simulation results, several novel Scotch yokes were designed, built and tested. To make the roller bearing move smoothly, a transition slope is connected between the curved part and horizontal side as shown in figure 4. Also, the concave part and the convex part were designed to be a part of circles with a common center. The difference of the radius between the concave part and the convex part was designed to be equal to the radius of the roller bearing.

![Figure 4. Photos of Scotch yokes. (a) A conventional Scotch yoke and (b) a novel experimental one.](image)

The cooling capacity of a 4 K GM cryocooler with a novel experimental Scotch yoke and a conventional Scotch yoke is shown in table 2. The compressor is an F-70 compressor from Sumitomo Heavy Industries, Ltd. The cold head is operated at 1.0 Hz, and the compressor is operated at 50 Hz. The static charging pressure is 1.38 MPa. The inner diameters of the first and the second stage cylinder are 82 mm and 35 mm, respectively. The stroke of the displacer is 25 mm. The first stage regenerator is filled with #150 phosphorous bronze screens. The second stage regenerator is filled with bismuth, HoCu$_2$ and Gd$_2$O$_2$S (GOS) spheres.

As shown in table 2, with a conventional Scotch yoke, the temperatures were 37.4 K at the first stage and 3.94 K at the second stage with 44 W and 1.0 W heat load, simultaneously, and 22.1 K and 2.99 K without heat load. With a novel experimental Scotch yoke, the temperatures were 35.9 K at the first stage and 3.96 K at the second stage with 44 W and 1.0 W heat load, simultaneously, and 22.3 K and 3.00 K without heat load. Although the minimum temperatures were almost the same, the cooling capacity at the first stage at 40 K was improved by about 10 % from 51.5 W to 57.3 W. The first stage cooling performance increases as the P-V work increases. At the second stage, the cooling capacity slightly decreased because of a lower first stage temperature which resulted in a smaller amount of gas flowing into the second stage expansion space. The pressure difference between the high and low
pressure at the compressor side was 1.58 MPa with a conventional Scotch yoke and 1.52 MPa with a novel one, respectively.

### Table 2. Experimental results of cooling performance.

|                      | Temperature without heat load (K) | Temperature with 44 W / 1.0 W heat load (K) | Estimated cooling capacity (W) | Pressure at compressor side (MPa) |
|----------------------|-----------------------------------|---------------------------------------------|--------------------------------|----------------------------------|
|                      | 1st  | 2nd  | 1st  | 2nd  | 1st at 40 K | 2nd at 4.2 K | Ph  | Pl  | dP   |
| Conventional         | 22.1 | 2.99 | 37.4 | 3.94 | 51.5  | 1.27        | 2.05 | 0.47 | 1.58 |
| Asymmetric           | 22.3 | 3.00 | 35.9 | 3.96 | 57.3  | 1.25        | 2.05 | 0.53 | 1.52 |

### 8. Conclusions

A novel GM cycle, called an asymmetric GM cycle, is proposed. In the GM cycle, the expansion process is longer, while the discharging process is shorter than in a conventional GM cycle. To carry out the GM cycle, a novel Scotch yoke was invented. In the Scotch yoke, there is a concave part at the upper center of the slide groove and a convex part at the lower center of the slide groove. The effect of the Scotch yoke has been confirmed by numerical simulation and experimental investigation. The cooling capacity at the first stage at 40 K was improved by about 10% from 51.5 W to 57.3 W.

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