Axial Vector Couplings of the Nucleon in Chiral Quark Model

Incorporating $U(1)_A$ Anomaly Effects

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Abstract

Renormalization of the axial vector currents due to Goldstone loops is studied in a simple extension of Manohar - Georgi chiral quark model which incorporates $U(1)_A$ anomaly effects. The polarized strange quark sea in the polarized nucleon results from different renormalization of the flavor singlet and octet currents and is in reasonable agreement with the experiment.

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The chiral quark model of Manohar and Georgi [1] offers an explanation of why the nonrelativistic quark (NRQ) model works so well for hadrons made up of light quarks. The electroweak properties of constituent quarks such as axial vector couplings and magnetic moments, however, remain as undetermined coefficients in nonlinear chiral Lagrangian. Using the above model in the chiral limit, Weinberg [2] argued some time ago that to the leading order in the large $N_c$ expansion ($N_c$ is the number of colors), constituent quarks behave like bare Dirac particles, i.e., with isospin triplet axial vector coupling $g_A = 1$ and anomalous magnetic moment $\kappa = 0$. The next to leading order corrections have also been estimated by sum rules or Feynman diagram calculations in the nonlinear $\sigma$ model or the linear $\sigma$ model using the $\sigma$ field as an effective regulator [3–6].

What about the flavor singlet axial vector coupling $g_A^0$ of the quark? Particular interest in this problem arises when analyzing the EMC [7] and more recent SMC [8] and SLAC [9] measurements of the nucleon spin structure functions. The original EMC result combined with data on nucleon and hyperon $\beta$ decays suggested that quarks carry only a small fraction of the nucleon spin and the strange quark sea is substantially polarized in the direction opposite to the nucleon spin, thus in conflict with the simple picture of the NRQ model. Recent experiment results have reduced the polarization effect of the strange quark sea but increased its statistical significance. As emphasized by Kaplan and Manohar [10], upon understanding this discrepancy, the key is to remember that the constituent quarks of the quark model are not the same things as the current quarks of QCD. Thus, when going from the current to constituent quarks the axial vector currents are subject to renormalization due to spontaneously broken chiral symmetry in QCD. Furthermore, this renormalization in the picture of chiral quark model arises from loops of Goldstone bosons. If the singlet and octet axial vector currents are renormalized differently, it is possible to have a polarized strange quark sea in the polarized nucleon. In this letter we will study the whole three axial vector couplings $G_A^{3,8,0}$ of the nucleon (i.e., the 3rd and 8th components of the octet and the singlet) in a simple extension of chiral quark model. We will see that a polarized strange quark sea so obtained can indeed be in reasonable agreement with experiment.
Any theoretical approach trying to give a reasonable value for the flavor singlet coupling $G_A^0$ is faced with the difficulty of how to incorporate the $U(1)_A$ anomaly effects. In the sector of pure pseudoscalar nonet it has generally been agreed how to do this in a chiral and large $N_C$ expansion though controversies still persist \[11,12\]. With the inclusion of quarks, i.e., in chiral quark model, simply adding the anomaly term to chiral Lagrangian would double-count the anomaly effects because the anomaly is still hidden in the quark integral measure when we quantize quark fields. Fortunately there is a way to avoid the dilemma. The key point is that one may use a $U(1)_A$-neutral quark field \[13\]. This is possible because the $U(1)_A$ transformation of a quark field is just a chiral analog of the ordinary phase redefinition. For example, if the quark field $\psi$ and the flavor singlet pseudoscalar $\eta_0$ transform under $U(1)_A$ as $\psi \rightarrow \psi' = \exp(i\omega\gamma_5)\psi$ and $\eta_0 \rightarrow \eta_0' = \eta_0 - \omega f_0\sqrt{6}$, one may define the $U(1)_A$-neutral quark field to be $Q = \exp(i\eta_0\gamma_5\sqrt{6}f_0)\psi$. The quark integral measure in terms of $Q$ is also neutral and no longer generates $U(1)_A$ anomaly so that one can now unambiguously include the anomaly terms in the chiral lagrangian for pseudoscalars, $L_{ps}$. For our purpose here the relevant point is that the $\eta_0$ gets a large mass which is nonzero even in the chiral limit. Symmetry considerations dictate the form of chiral Lagrangian at the lowest order in chiral and large $N_C$ expansion

$$
\mathcal{L} = \bar{Q}(i\hat{\partial} - M)Q + \bar{Q}(\bar{\psi} + g_A A\gamma_5 + g_A^0 A^0\gamma_5)Q \\
- Q(\hat{\xi}m\hat{\xi}\xi P_- + \hat{\xi}^\dagger m\hat{\xi}^\dagger(\hat{\xi}\hat{\xi})^\dagger P_+)Q + L_{ps},
$$

$$
\hat{\xi} = \exp(i\pi^a\lambda^a/2f), \quad \hat{\xi} = \exp(i\eta_0/\sqrt{6}f_0),
$$

$$
P_\pm = \frac{1}{2}(1 \pm \gamma_5), \quad m = \text{diag}(0, 0, m_s)
$$

$$
V_\mu = \frac{i}{2}(\hat{\xi}\partial_\mu\hat{\xi}^\dagger + \hat{\xi}^\dagger\partial_\mu\hat{\xi}), \quad A_\mu = \frac{i}{2}(\hat{\xi}\partial_\mu\hat{\xi}^\dagger - \hat{\xi}^\dagger\partial_\mu\hat{\xi}),
$$

$$
A_0^\mu = \frac{i}{2}(\hat{\xi}\partial_\mu\hat{\xi}^\dagger - \hat{\xi}^\dagger\partial_\mu\hat{\xi}) = \frac{1}{\sqrt{6}f_0}\partial_\mu(\hat{\xi}\eta_0).
$$

Several remarks are in order. The second term arises from spontaneous chiral symmetry breaking, giving quark a constituent mass $M$. The explicit $SU(3)$ breaking is induced by the $m$ term which also gives rise to symmetry breaking terms in $L_{ps}$. We have ignored the current mass of u and d quarks. The remaining $g_A$ and $g_A^0$ terms account for the rule that
in effective field theory we must include all terms that are allowed by symmetry and of the same order by chiral power counting. Compared to the Manohar - Georgi chiral Lagrangian the above Lagrangian contains an additional term proportional to $g_0^A$. The singlet and octet axial vector couplings of quark, $g_0^A$, $g_A$ are free parameters in chiral quark model. As the Weinberg’s argument for $g_A = 1$ in large $N_c$ limit was later challenged and there were indications that $g_A = 1$ might not be a necessary result of large $N_c$ QCD [14–16], we would like to take $g_A$ as free and calculate its renormalization effects arising from chiral loops which are believed to be one of the important contributions at the next order. In the case of $g_0^A$ the situation is more obscure. There is no analogous sum rule to constrain it as used for $g_A$, so we set it free as well.

We are now ready to compute $G_{A}^{0,3,8}$, which are defined by the nucleon matrix elements of the QCD current $j^a_\mu = \bar{q}_\mu \gamma_\mu \frac{\lambda^a}{2} q$ with $\lambda_a$ ($a = 1 - 8$) the Gell-Mann matrices and $\chi^0 = \sqrt{\frac{2}{3}} \text{diag}(1, 1, 1)$,

\[
\begin{align*}
< N(p + q) | j_{\mu_5}^0 | N(p) > &= \frac{1}{\sqrt{6}} \bar{u}(p + q) [\gamma_\mu G_{A}^{0}(q^2) + q_\mu H_{A}^{0}(q^2)] \gamma_5 u(p), \\
< N(p + q) | j_{\mu_5}^3 | N(p) > &= \frac{1}{2} \bar{u}(p + q) [\gamma_\mu G_{A}^{3}(q^2) + q_\mu H_{A}^{3}(q^2)] \gamma_5 \tau^3 u(p), \\
< N(p + q) | j_{\mu_5}^8 | N(p) > &= \frac{1}{2\sqrt{3}} \bar{u}(p + q) [\gamma_\mu G_{A}^{8}(q^2) + q_\mu H_{A}^{8}(q^2)] \gamma_5 u(p), \\
G_{A}^{0,3,8} &= G_{A}^{0,3,8}(0).
\end{align*}
\]

Note that for $G_{A}^{0}$ we have to specify its renormalization scale $\mu \leq \Lambda_\chi_\text{SB}$ (chiral symmetry breaking scale). In the spirit of effective field theory, at scale $\mu \leq \Lambda_\chi_\text{SB}$ we may make the appropriate substitution $j_{\mu_5}^a \to J_{\mu_5}^a$, where $J_{\mu_5}^a$ is derived from chiral Lagrangian,

\[
\begin{align*}
J_{\mu_5}^a &= \frac{1}{4} \bar{Q} \gamma_\mu (\hat{\xi} \lambda^a \hat{\xi} - \hat{\xi}^\dagger \lambda^a \hat{\xi}) Q + \frac{g_A^0}{4} \bar{Q} \gamma_\mu \gamma_5 (\hat{\xi} \lambda^a \hat{\xi}^\dagger + \hat{\xi}^\dagger \lambda^a \hat{\xi}) Q + \text{(meson terms)}, \\
J_{\mu_5}^0 &= \frac{g_A^0}{\sqrt{6}} \bar{Q} \gamma_\mu \gamma_5 Q + \text{(meson terms)}.
\end{align*}
\]

So, effectively we have

\[
< N | j_{\mu_5}^a | N > = < N | J_{\mu_5}^a | N > = < N | J_{\mu_5}^a | N >.
\]

In the above second equality we have included renormalization effects from chiral loops in the coefficients of $J_{\mu_5}^a$ while the matrix elements themselves are to be evaluated at tree
level in specific quark models. Since $SU(3)$ is explicitly broken $J_{\mu 5}^0$ and $J_{\mu 5}^8$ mix under renormalization so that $a'$ involves components besides $a$. Since isospin is conserved, $J_{\mu 5}^3$ is renormalized multiplicatively. We use the NRQ model to evaluate the matrix elements, so $\bar{J}_{\mu 5}^a$ involves only $U$ and $D$ quarks,

$$\bar{J}_{\mu 5}^0 = \frac{a_0}{\sqrt{6}}(\bar{U}\gamma_\mu\gamma_5 U + \bar{D}\gamma_\mu\gamma_5 D),$$

$$\bar{J}_{\mu 5}^3 = \frac{a_3}{\sqrt{2}}(\bar{U}\gamma_\mu\gamma_5 U - \bar{D}\gamma_\mu\gamma_5 D),$$

$$\bar{J}_{\mu 5}^8 = \frac{a_8}{2\sqrt{3}}(\bar{U}\gamma_\mu\gamma_5 U + \bar{D}\gamma_\mu\gamma_5 D),$$

(5)

where $a_{0,3,8}$ are effective axial vector couplings of quark in chiral quark model and related to observables $G_A^{0,3,8}$ by

$$G_A^0 = a_0, \quad G_A^3 = \frac{5}{3}a_3, \quad G_A^8 = a_8. \quad (6)$$

Explicit calculation of Feynman diagrams in Fig. 1 shows that $a_{0,3,8}$ have the following structure,

$$a_0 = g_A^0(1 - A),$$

$$a_3 = g_A(1 - A) - 2B^\pi + B^K,$$

$$a_8 = g_A(1 - A) - 3B^K.$$  

(7)

Instead of writing down the lengthy formulae for $A$ and $B^{\pi(K)}$, we emphasize the following features. $A$ is a sum of terms contributed by the whole pseudoscalar nonet, while $B^{\pi(K)}$ only receives contributions from $\pi^\pm$ ( $K^\pm$, $K^0$ and $\bar{K}^0$). The $\eta'$ contributes to $a_{0,3,8}$ in the same way as the Goldstone octet does. This is because we have actually treated $\eta'$ as if it were a Goldstone boson. Although it is guided by large $N_c$ arguments, numerical analysis will tell us whether it is a good approximation. The singlet and nonsinglet couplings are renormalized differently even in the chiral limit and with $g_A = g_A^\Lambda$. This difference arises because Fig. 1(c) and (d) appear only in the nonsinglet channel. Physically it is responsible for the polarized strange quark sea in the polarized nucleon,

$$\Delta S = \frac{a_0 - a_8}{3} = \frac{1}{3}(g_A^0 - g_A)(1 - A) + B^K. \quad (8)$$

5
The splitting between $a_3$ and $a_8$ is due to explicit $SU(3)$ breaking, so in the $SU(3)$ limit we should have $a_3 = a_8$. Indeed, using the 'experiment' value $a_3 = 0.75$ and $a_8 = 0.6$ we estimate that $SU(3)$ breaking effects are within 30%.

To quantify our discussion we regularize as usual ultraviolet divergences by the cutoff $\Lambda_{\chi SB} = 4\pi f$, where $f = 84$ MeV is the decay constant of the Goldstone bosons in the chiral limit. The relevant input is, $M = 350$ MeV, $m_s = 150$ MeV, $m_\pi = 135$ MeV, $m_K = 492$ MeV, $m_\eta = 547$ MeV, $m_{\eta'} = 958$ MeV, $\theta = -20^\circ$ ($\eta - \eta'$ mixing angle), $f_\pi = f_\eta = f_{\eta'} = 130/\sqrt{2}$ MeV, $f_K = 160/\sqrt{2}$ MeV. Our discussion does not depend on the details of the input. Then $G_{A}^{0,3,8}$ are functions of $g_A$ and $g_{A}^{0}$. As they are sensitive to $g_A$ we choose a number for $g_A$ so that we may get a not-too-bad number for $G_{A}^{3}$ in a reasonable range of $g_{A}^{0}$. The result with $g_A = 1.13$ is plotted in Fig. 2. We see that the global pattern of $G_{A}^{0,3,8}$ with a positive $g_{A}^{0}$ is in reasonable agreement with experiment. For example, at $g_A = g_{A}^{0} = 1.13$, we have $G_{A}^{3} = 1.23 , G_{A}^{8} = 0.45 , G_{A}^{0} = 0.16$ and $\Delta S = -0.10$. Considering the simplicity of our working Lagrangian this is encouraging. Especially, an important part of the strange quark polarization can indeed be attributed to different renormalization of the singlet and octet axial vector currents by chiral loops. Although it is possible to fit all of $G_{A}^{0,3,8}$ to experiment, this requires a large negative value for $g_{A}^{0}$. (For example, $G_{A}^{0} = 0.3 , G_{A}^{3} = 1.25$ and $G_{A}^{8} = 0.6$ using $g_A = 1.16 , g_{A}^{0} = -2.14$.) A negative sign for the ratio $\zeta = g_{A}^{0}/g_A$ was also favored by a recent work based on a quantum-mechanical analysis in chiral quark model [17]. But we still think this is not very natural because it is hard to believe that the next order correction in a good perturbative expansion would change the value of $G_{A}^{0}$ from $-2$ or $-1$ to $0.3$.

We should mention a weak point in our discussion which seems to deserve further study. The corrections to $G_{A}^{0,3,8}$ computed here are dominated by chiral loops of the Goldstone octet. The contribution of the $\eta'$ is less important even in the singlet channel while intuitively one expects that the $\eta'$ should couple strongly to the singlet channel. This occurs because we have actually treat $\eta'$ as a Goldstone boson. We guess that if we put in somehow the strong coupling between $\eta'$ and the singlet current we would have a better expansion of
the experimental values of $G_{A}^{0,3,8}$. This is not totally impossible. An additional diagram like Fig. 1(c) or (d) would produce some "B" term which partially cancels the "A" term in $a_0$. Indeed a similar cancellation does occur in the octet coupling $a_{3,8}$. If the guess is really correct we can start with a smaller $g_{A}^{0}$ but end up with larger $G_{A}^{0}$ (hence a more reliable perturbative treatment in the singlet channel) and $G_{A}^{3,8}$, thus in closer agreement with experiment. Inversely, this may imply that some higher order terms in large $N_C$ expansion of chiral Lagrangian are probably important. The other point not touched upon in this letter concerns the running property of the singlet axial vector current in the region $\mu < \Lambda_{\chi_{SB}}$. According to the analysis in Ref. [1] strong interactions in this region are much weakened, compared to the naive extrapolation from the perturbative region of QCD. If we consider our computed $G_{A}^{0}$ to be evaluated at $\mu \approx M$ and mimic the running in the intermediate region $\Lambda_{\chi_{SB}} > \mu > M$ by simply using a "scaled-down" QCD (just as we mimic technicolor by using a "scaled-up" QCD) we estimate $G_{A}^{0}(\mu = \Lambda_{\chi_{SB}}) \approx 0.98 G_{A}^{0}(\mu = M)$ [18].

Finally we argue that $g_{A}$ appearing in the chiral Lagrangian depends in some sense on the number of light flavors involved. Usually we work with two flavors (u and d) when we determine from $g_{A}$ the coupling $G_{A}^{3}$ as measured in the neutron $\beta$ decay. To determine couplings of other components, $G_{A}^{0}$ and $G_{A}^{8}$, we surely have to include the strange quark. But we should obtain the same $G_{A}^{3}$ whether we work with two or three flavors. To the leading order, the relation between $g_{A}$ and $G_{A}^{3}$ is unchanged, e.g. as in the NRQ model. Its next order corrections arising from chiral loops are basically determined by quadratic Casimir operators of the flavor symmetry group in the symmetry limit. Then one way out is that $g_{A}$ also depends on the number of flavors involved. This can be understood in the language of effective field theory. When we are working with two flavors we have already integrated out the strange quark, $K$ and $\eta$ fields and inserted their renormalization effects directly into $g_{A}$ which is a parameter in chiral Lagrangian with two flavors. This explains why we required a larger value of $g_{A}$ than usual to fit $G_{A}^{3}$.

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**Figure Caption**

Fig. 1 Feynman diagrams contributing to renormalization of axial vector currents. Thick lines and thin lines represent quarks and pseudoscalars respectively. Solid circles represent insertion of current.

Fig. 2 The computed couplings $\frac{3}{5}G_A^3$, $G_A^8$ and $G_A^0$ (upper, middle and lower curves) of the nucleon are shown as functions of $g_A^0$ at $g_A = 1.13$. 
Fig. 1 Xiaoyuan Li, Phys. Lett. B
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