Full One-Loop Electroweak Corrections to the
Charged Higgs Decays into a Chargino and Neutralino
in the MSSM

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Abstract

We calculate the full one-loop electroweak (FEW) corrections to $H^{-} \rightarrow \tilde{\chi}^{0}_{i}\tilde{\chi}^{-}_{j}$ in the minimal supersymmetric standard model (MSSM), and compare with the leading order (LO) and the complete leading order (CLO) corrections. We find that the magnitudes of the FEW corrections can be larger than 10% for $H^{-} \rightarrow \tilde{\chi}^{0}_{1}\tilde{\chi}^{-}_{1}$ for $\tan \beta > 30$, and are in general 5% for both $H^{-} \rightarrow \tilde{\chi}^{0}_{1}\tilde{\chi}^{-}_{1}$ and $H^{-} \rightarrow \tilde{\chi}^{0}_{1}\tilde{\chi}^{-}_{2}$. The FEW corrections in general can be larger than the LO and the CLO corrections by about 5% and 3%, respectively. Moreover, the LO and the CLO corrections are negligible small for $\tan \beta < 5$ compared with the FEW corrections, since there are not enhancements from the Yukawa couplings. We also calculate the FEW corrections in the minimal supergravity (mSUGRA) scenario, where the FEW corrections can be larger than the LO and the CLO corrections by more than 20% and 30%, respectively.

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1 Introduction

Beyond the standard model (SM), the supersymmetric (SUSY) [3] [4] extensions of the SM provide a great opportunity to solve some mysterious problems in the SM. The SUSY partners of the SM particles cancel the quadratic divergences in the corrections to the Higgs boson mass, and the hierarchy problem [3] can be solved naturally. If we consider the R-parity conservation as the essential condition, the lightest SUSY particles (LSP) will never decay, and the stability of the LSP provides the most important candidate for the dark matter [5]. The most attractive extension of the SM is the minimal supersymmetric standard model (MSSM) [6]. If we set all the parameters as real in the MSSM, there will be five Higgs bosons [7]: two CP even bosons ($H^0, h^0$), one CP odd boson ($A^0$), and two charged bosons ($H^\pm$). When the Higgs boson of the SM has a mass below 130-140 GeV and the $h^0$ of the MSSM are in the decoupling limit (which means that $H^\pm$ is too heavy anyway to be possibly produced), the lightest neutral Higgs boson may be difficult to be distinguished from the neutral Higgs boson of the SM. But the charged Higgs bosons carry a distinctive signature of the Higgs sector in the MSSM. Therefore, the search for the charged Higgs bosons is very important for probing the Higgs sector of the MSSM, and will be one of the prime objectives of the CERN Large Hadron Collider (LHC) [8, 9].

Current bounds on charged Higgs mass can be obtained at the Tevatron, by studying the top decay $t \rightarrow bH^+$, which already eliminates some region of parameter space [10], whereas the combined LEP experiments gives a low bounds approximately $m_{H^+} > 78.6$ GeV at 95\%CL [11]. In the MSSM, we have $m_{H^\pm} \geq 120$ GeV from the mass bounds from LEP–II for the neutral pseudoscalar $A^0$ of the MSSM ($m_{A^0} \geq 91.9$ GeV) [12].

If the charged Higgs masses could be large enough, there will be many SM and SUSY decay modes. In the MSSM the channels of decay into neutralino and the chargino ($H^\pm \rightarrow \tilde{\chi}_i^{0}\tilde{\chi}_j^{\pm}$) are very important [13], which have been discussed in the Ref. [2], where only the LO corrections were calculated. In this paper, we present the calculations of the FEW corrections, which include the contributions of the one-loop virtual contributions of the (s)leptons and (s)quarks of all the three generations, and all the possible Higgs and gauge bosons, charginos and neutralinos and the real corrections i.e. the real photon emission. In the preparation of this paper, a relevant
work was given in the Ref. [14] as a conference's short report, which doesn't show any detail of the calculation and the numerical results are not complete.

In Sec.2 we define the relevant notations and show the tree-level result. In Sec.3 we present the virtual corrections, including the vertex corrections and the counterterms. In Sec.4 we illustrate the real corrections from the real photon emission using the phase space slicing (PSS) method [15]. In Sec.5 we present the numerical results and conclusion in the low-energy MSSM and the mSUGRA breaking scenario [16].

2 Notations and Tree-level Width

In order to make this paper self-contained, we first present the relevant interaction Lagrangian [2] of the MSSM and the tree level decay width for \( H^+ \tilde{\chi}_i^{-} \tilde{\chi}_j^{0} \) \((i = 1, 2, \ j = 1, 2, 3, 4)\). The Lagrangian is

\[
\mathcal{L}_{H^+ \tilde{\chi}_i^{-} \tilde{\chi}_j^{0}} = -H^+ \tilde{\chi}_i^{-} \left( C_{ij}^L P_L + C_{ij}^R P_R \right) \tilde{\chi}_j^{0} + H.c.,
\]

where,

\[
P_{L,R} = \frac{1}{2} (1 \mp \gamma_5),
\]

\[
C_{ij}^L = \frac{e}{s_W} s_\beta \left( U_{j1}^* N_{i3}^* - \frac{1}{\sqrt{2}} U_{j2}^* (N_{i2}^* + t_W N_{i1}^*) \right),
\]

\[
C_{ij}^R = \frac{e}{s_W} c_\beta \left( V_{j1} N_{i4} + \frac{1}{\sqrt{2}} V_{j2} (N_{i2} + t_W N_{i1}) \right),
\]

for convenience, we take \( s_W = \sin \theta_W, \ c_W = \cos \theta_W, \ t_W = \tan \theta_W, \ s_\beta = \sin \beta, \ c_\beta = \cos \beta \) and \( t_\beta = \tan \beta \).

Here the matrixes \( U, V \) and \( N \) are the chargino and neutralino mixing matrixes, which can diagonalize the corresponding mass matrixes. The chargino mass matrix is

\[
X = \begin{pmatrix}
M & \sqrt{2} m_W s_\beta \\
\sqrt{2} m_W c_\beta & \mu
\end{pmatrix}
\]
and the neutralino mass matrix is

\[
Y = \begin{pmatrix}
M' & 0 & -m_Z s_W c_\beta & m_Z s_W s_\beta \\
0 & M & m_Z c_W c_\beta & -m_Z c_W s_\beta \\
-m_Z s_W c_\beta & m_Z c_W c_\beta & 0 & -\mu \\
m_Z s_W s_\beta & -m_Z c_W s_\beta & -\mu & 0
\end{pmatrix}.
\]  

(4)

The chargino mixing matrixes \((U, V)\) diagonalize the chargino mass matrix

\[
UXV^\dagger = \text{diag} \left( \eta_1 M_{\tilde{\chi}^-_1}, \eta_2 M_{\tilde{\chi}^-_2} \right),
\]  

(5)

and the neutralino mixing matrix \((N)\) diagonalizes the neutralino mass matrix

\[
NYN^\dagger = \text{diag} \left( \epsilon_1 M_{\tilde{\chi}^0_1}, \epsilon_2 M_{\tilde{\chi}^0_2}, \epsilon_3 M_{\tilde{\chi}^0_3}, \epsilon_4 M_{\tilde{\chi}^0_4} \right),
\]  

(6)

where \(\eta_i = \pm 1\) \((i = 1, 2)\) and \(\epsilon_j = \pm 1\) \((j = 1, 2, 3, 4)\), these signs depend on the configuration of the mixing matrixes. If all the elements of the mixing matrixes are real, the mass eigen values above are possibly negative. Therefore, the chargino and the neutralino physical masses are

\[
m_{\tilde{\chi}^-_i} = |M_{\tilde{\chi}^-_i}|, \quad m_{\tilde{\chi}^0_j} = |M_{\tilde{\chi}^0_j}|.
\]  

(7)

From the interaction Lagrangian (1) we can derive the tree-level amplitude as following:

\[
\mathcal{M}_0 = \overline{\mathbf{u}}(p_{\tilde{\chi}^-_j})(C^L_{ij}P_L + C^R_{ij}P_R)v(p_{\tilde{\chi}^0_i}).
\]  

(8)

Then the tree-level decay width is thus given by

\[
\Gamma_0 = \frac{1}{8\pi} \frac{p_{\text{out}}}{m_{H^-}^2} |\mathcal{M}_0|^2,
\]  

(9)

where the momentum value of the outgoing particle

\[
p_{\text{out}} = \frac{1}{2m_{H^-}} \sqrt{(m_{H^-}^2 + m_{\tilde{\chi}^-_j}^2 - m_{\tilde{\chi}^0_i}^2)^2 - 4m_{H^-}m_{\tilde{\chi}^-_j}m_{\tilde{\chi}^0_i}}.
\]  

(10)

For future convenience, we also present here the vertex \(G^+ \tilde{\chi}^-_i \tilde{\chi}^0_j\) to fix the renormalization constant of \(G^-\) and \(H^-\) mixing.

\[
\mathcal{L}_{G^+ \tilde{\chi}^-_i \tilde{\chi}^0_j} = -G^+ \overline{\tilde{\chi}^0_j} \left( D^L_{ij}P_L + D^R_{ij}P_R \right) \tilde{\chi}^-_i + H.c.,
\]  

(11)

where \(D^L_{ij} = -\cot \beta C^L_{ij}, \quad D^R_{ij} = \tan \beta C^R_{ij}\).
3 Virtual Correction

The Feynman diagrams, contributing to the virtual corrections to \( H^- \rightarrow \tilde{\chi}^0_i \tilde{\chi}_j^- \) are shown in Figs.2-8. In the calculation we use the ’t Hooft-Feynman gauge, the dimensional regularization \( (D = 4 - 2\epsilon) \) to regularize the ultraviolet (UV) and infrared (IR) divergences in the virtual loop corrections, and the on-mass-shell scheme for the renormalization [2]. We use the FormCalc program [17] to calculate the amplitudes of the one-loop vertex amplitudes and the self-energy diagrams.

The relevant renormalization constants for the wave function and the fields mixing have the following definitions:

\[
\tilde{\chi}^-_{i0} = \left( \delta_{ik} + \frac{1}{2} \delta Z_{-ik}^L P_L + \frac{1}{2} \delta Z_{-ik}^R P_R \right) \tilde{\chi}^-_k,
\]

\[
\tilde{\chi}^0_{i0} = \left( \delta_{ik} + \frac{1}{2} \delta Z_{0ik}^L P_L + \frac{1}{2} \delta Z_{0ik}^R P_R \right) \tilde{\chi}^0_k,
\]

\[
\begin{pmatrix}
H^- \\
G^-
\end{pmatrix}
= 
\begin{pmatrix}
1 + \frac{1}{2} \delta Z_{H^-} & \frac{1}{2} \delta Z_{H^- G^-} \\
\frac{1}{2} \delta Z_{G^- H^-} & 1 + \frac{1}{2} \delta Z_{H^-}
\end{pmatrix}
\begin{pmatrix}
H^- \\
G^-
\end{pmatrix}.
\]

The renormalization constants for the vertex parameters are defined as

\[
e_0 = \left( 1 + \delta Z_e \right) e, \quad s_{W0} = s_W + \delta s_W, \quad t_{\beta 0} = t_\beta + \delta t_\beta,
\]

\[
U_0 = U + \delta U, \quad V_0 = V + \delta V, \quad N_0 = N + \delta N,
\]

\[
\delta U = \frac{1}{4} \left( \delta Z_{L} - \delta Z_{L}^\dagger \right) U,
\]

\[
\delta V = \frac{1}{4} \left( \delta Z_{R} - \delta Z_{R}^\dagger \right) V,
\]

\[
\delta N = \frac{1}{4} \left( \delta Z_{0} - \delta Z_{0}^\dagger \right) N.
\]

The chargino and the neutralino mixing matrixes renormalization constants cancel the antisymmetric parts of their wave function renormalization constants. Consequently, the chargino
and the neutralino wave function renormalization constants are shifted as,
\[ \delta Z_- \rightarrow \frac{1}{2}(\delta Z_- + \delta Z_+^\dagger), \quad \delta Z_0 \rightarrow \frac{1}{2}(\delta Z_0 + \delta Z_0^\dagger). \]  
\(14\)

Meanwhile, the renormalization for \(\tan \beta\) cancels half of the \(G^- - H^-\) renormalization [2] as
\[ \delta Z_{G-H} \rightarrow \frac{1}{2} \delta Z_{G-H}. \]  
\(15\)

The renormalized virtual amplitudes can be written as
\[ \mathcal{M}_1^V = \mathcal{M}_1^{(v)} + \mathcal{M}_1^{(c)} \]  
\(16\)
including the vertex one-loop contribution \(\mathcal{M}_1^{(v)}\) and the corresponding counterterm \(\mathcal{M}_1^{(c)}\). The vertex part can be derived from the vertex one-loop Feynman diagrams in Fig.2. We list their analytic expressions in Appendix.

With the previous definitions of the renormalization constants, the counterterm Lagrangian for the vertex is
\[ \delta \mathcal{L}_{H^+\tilde{\chi}_j^0}\tilde{\chi}_i^0 = -H^+\bar{\chi}_j^0\left\{ C_{ij}^L \left( \delta Z_e - \frac{\delta S_W}{s_W} + \frac{1}{2} \delta Z_{H^-} - \frac{1}{4} \cot \beta \delta Z_{G^- H^-} \right) \ight. \\
\left. + \frac{1}{4} \sum_{k=1}^{2} C_{ik}^L \left( \delta Z_{-kj}^L + \delta Z_{-jk}^L \right) + \frac{1}{4} \sum_{k=1}^{4} C_{kj}^L \left( \delta Z_{0,ki}^L + \delta Z_{0,ik}^L \right) - \frac{e_s \beta}{\sqrt{2} s_W} U_{j'i'2} \delta t_W N_{i'i'1}' \right\} P_L \\
\left. + \left[ C_{ij}^R \left( \delta Z_e - \frac{\delta S_W}{s_W} + \frac{1}{2} \delta Z_{H^-} + \frac{1}{4} \tan \beta \delta Z_{G^- H^-} \right) + \frac{1}{4} \sum_{k=1}^{2} C_{ik}^R \left( \delta Z_{-kj}^R + \delta Z_{-jk}^R \right) \ight. \\
\left. + \frac{1}{4} \sum_{k=1}^{4} C_{kj}^L \left( \delta Z_{0,ki}^L + \delta Z_{0,ik}^L \right) + \frac{e_c \beta}{\sqrt{2} s_W} V_{i'2} \delta t_W N_{j'i'1}' \right\} \bar{P}_R \tilde{\chi}_j^- . \]  
\(17\)

The counterterm amplitude \(\mathcal{M}_1^{(c)}\) can be explicitly derived from the above Lagrangian. The renormalization of the input parameters \(e, \theta_W, m_Z\) and \(m_W\) follows the conventional on-mass-shell scheme [2]. The other renormalization constants with the on-mass-shell scheme are defined as follows.

The charged Higgs wave function renormalization constant is
\[ \delta Z_{H^-} = -\tilde{R} e \frac{\partial \Sigma_{H^- H^-}}{\partial p^2}(m_{H^-}^2). \]  
\(18\)
The fermion wave function renormalization [2] constants are

\[
\delta Z_{ii}^L = -\tilde{Re}\left\{ \Sigma_{ii}^L(m_i^2) + m_i^2 \left[ \frac{\partial \Sigma_{ii}^L}{\partial p^2}(m_i^2) + \frac{\partial \Sigma_{ii}^R}{\partial p^2}(m_i^2) \right] + m_i \left[ \frac{\partial \Sigma_{ii}^{SL}}{\partial p^2}(m_i^2) + \frac{\partial \Sigma_{ii}^{SR}}{\partial p^2}(m_i^2) \right] \right\},
\]

\[
\delta Z_{ii}^R = -\tilde{Re}\left\{ \Sigma_{ii}^R(m_i^2) + m_i^2 \left[ \frac{\partial \Sigma_{ii}^L}{\partial p^2}(m_i^2) + \frac{\partial \Sigma_{ii}^R}{\partial p^2}(m_i^2) \right] + m_i \left[ \frac{\partial \Sigma_{ii}^{SL}}{\partial p^2}(m_i^2) + \frac{\partial \Sigma_{ii}^{SR}}{\partial p^2}(m_i^2) \right] \right\},
\]

\[
\delta Z_{ij}^L = \frac{2}{m_i^2 - m_j^2} \tilde{Re}\left[ m_j^2 \Sigma_{ij}^L(m_j^2) + m_i m_j \Sigma_{ij}^R(m_j^2) + m_i \Sigma_{ij}^{SL}(m_j^2) + m_j \Sigma_{ij}^{SR}(m_j^2) \right], \quad (i \neq j),
\]

\[
\delta Z_{ij}^R = \frac{2}{m_i^2 - m_j^2} \tilde{Re}\left[ m_j^2 \Sigma_{ij}^R(m_j^2) + m_i m_j \Sigma_{ij}^L(m_j^2) + m_i \Sigma_{ij}^{SR}(m_j^2) + m_j \Sigma_{ij}^{SL}(m_j^2) \right], \quad (i \neq j).
\]

(19)

We use the scheme in Ref. [2] to fix \( G^- - H^- \) mixing renormalization constant,

\[
\delta Z_{G^-H^-} = -\frac{2}{m_W} \tilde{Re}\Sigma_{H^-W^-}(m_{H^-}^2).
\]

(20)

Then the renormalization constants could be derived from the self-energy Feynman diagrams shown in Figs.55. Through the calculation, the UV divergences of the one-loop vertex and the counterterm are

\[
\mathcal{M}_1^{(v)} \Big|_{UV} = \left( -\frac{1}{\epsilon} \right) \left\{ \left[ \frac{\alpha}{16\pi m_W^2 s_\beta^2 s_W^2} \left[ s_\beta^2 (m_Z^2 + 18m_W^2) + \sum_{g=1}^{3} 6m_{u_g}^2 \right] + \frac{\alpha e c_\beta}{\sqrt{2\pi s_W c_W}} N_1 V_{j2} \right] C_{ij}^R P_R 
\]

\[
+ \left[ \frac{\alpha}{16\pi m_W^2 c_\beta^2 s_W^2} \left[ c_\beta^2 (m_Z^2 + 18m_W^2) + \sum_{g=1}^{3} (6m_{d_g}^2 + 2m_{e_g}^2) \right] - \frac{\alpha e s_\beta}{\sqrt{2\pi s_W c_W}} N_1 U_{j2} \right] C_{ij}^L P_L \right\},
\]

(21)

\[
\mathcal{M}_1^{(c)} \Big|_{UV} = \frac{1}{\epsilon} \left\{ \left[ \frac{\alpha}{16\pi m_W^2 s_\beta^2 s_W^2} \left[ s_\beta^2 (m_Z^2 + 18m_W^2) + \sum_{g=1}^{3} 6m_{u_g}^2 \right] + \frac{\alpha e c_\beta}{\sqrt{2\pi s_W c_W}} N_1 V_{j2} \right] C_{ij}^R P_R 
\]

\[
+ \left[ \frac{\alpha}{16\pi m_W^2 c_\beta^2 s_W^2} \left[ c_\beta^2 (m_Z^2 + 18m_W^2) + \sum_{g=1}^{3} (6m_{d_g}^2 + 2m_{e_g}^2) \right] - \frac{\alpha e s_\beta}{\sqrt{2\pi s_W c_W}} N_1 U_{j2} \right] C_{ij}^L P_L \right\},
\]

(22)

where, \( m_{u_g}, m_{d_g} \) and \( m_{l_g} \) represent the mass of the u-type quarks, the d-type quarks and the leptons, and \( g \) is the generation index.

Obviously, the UV divergences of \( \mathcal{M}_1^{(v)} \) and \( \mathcal{M}_1^{(c)} \) can cancel each other, as they must. Then
the renormalized amplitude at one-loop order is UV convergent

$$\mathcal{M}_1^V\big|_{UV} = 0.$$  \hspace{1cm} (23)

Thus the full one-loop virtual correction for the decay width is

$$\Gamma_V = \frac{1}{8\pi} \frac{p_{\text{out}}}{m_{H^-}^2} 2\text{Re}(\mathcal{M}_0\mathcal{M}_1^{V*}),$$  \hspace{1cm} (24)

where the renormalized amplitude $\mathcal{M}_1^V$ is UV finite, but it still contains infrared (IR) divergences, which can be written as:

$$\mathcal{M}_1^V \big|_{IR} = \frac{\alpha}{2\pi} \frac{1}{\epsilon} \left[ -2 + \frac{2x_1x_2 - x_1 - x_2}{x_1 - x_2} \ln \left( \frac{x_1x_2}{x_1x_2 - x_2} \right) \right] \mathcal{M}_0,$$  \hspace{1cm} (25)

where,

$$x_{1,2} = \frac{m_{H^-}^2 - m_{\tilde{\chi}_j}^2 + m_{\tilde{\chi}_0}^2 \pm 2m_{H^-}p_{\text{out}}}{2m_{\tilde{\chi}_0}^2}.$$  \hspace{1cm} (26)

The IR divergences can be cancelled after adding the contributions from the emission of real photons, which will be described in detail in the following section.

## 4 Real Correction

The Feynman diagrams for the real corrections are shown in Fig. 1.

![Feynman Diagrams](image)

Figure 1: The photon emission Feynman diagrams.

The relevant three-body decay width is

$$\Gamma_R = \frac{1}{2\Phi} \int \sum |\mathcal{M}_3|^2 dPS^{(3)},$$  \hspace{1cm} (27)

where $\Phi = m_{H^-}$ is the usual flux factor for the one particle initial state. $dPS^{(3)}$ is the three-body phase space. $\sum |\mathcal{M}_3|^2$ is the squared amplitude averaged over the initial degrees of freedom,
\[ \sum |M_3|^2 = \frac{2e^2}{(p_{\tilde{\chi}^{-}} \cdot p_\gamma)^2} \left[ -m_{\tilde{\chi}^{-}}^2 (p_{\tilde{\chi}^{0}} \cdot p_\gamma) - m_{\tilde{\chi}^{-}}^2 (p_{\tilde{\chi}^{0}} \cdot p_{\tilde{\chi}^{0}}) + (p_{\tilde{\chi}^{-}} \cdot p_\gamma)(p_{\tilde{\chi}^{0}} \cdot p_\gamma) \right] (C_L C_R^\dagger + C_R C_L^\dagger) \]

\[ + \frac{2e^2}{(p_{\tilde{\chi}^{-}} \cdot p_\gamma)^2} m_{\tilde{\chi}^{-}} m_{\tilde{\chi}^{0}} [m_{\tilde{\chi}^{-}}^2 + (p_{\tilde{\chi}^{-}} \cdot p_\gamma)] (C_L C_L^\dagger + C_R C_R^\dagger) + \frac{2e^2}{[(p_{\tilde{\chi}^{-}} \cdot p_\gamma) + (p_{\tilde{\chi}^{0}} \cdot p_\gamma)]^2} 2m_{\tilde{\chi}^{-}}^2 (p_{\tilde{\chi}^{-}} \cdot p_{\tilde{\chi}^{0}}) \]

\[ \times [m_{\tilde{\chi}^{-}}^2 + m_{\tilde{\chi}^{0}}^2 + 2(p_{\tilde{\chi}^{-}} \cdot p_{\tilde{\chi}^{0}}) + (p_{\tilde{\chi}^{-}} \cdot p_\gamma) + (p_{\tilde{\chi}^{0}} \cdot p_\gamma)] [-(p_{\tilde{\chi}^{-}} \cdot p_{\tilde{\chi}^{0}})(C_L C_R^\dagger + C_R C_L^\dagger) \]

\[ + m_{\tilde{\chi}^{-}} m_{\tilde{\chi}^{0}} (C_L C_L^\dagger + C_R C_R^\dagger) + \frac{2e^2}{[(p_{\tilde{\chi}^{-}} \cdot p_\gamma) + (p_{\tilde{\chi}^{0}} \cdot p_\gamma)]^2} 2m_{\tilde{\chi}^{-}}^2 (p_{\tilde{\chi}^{-}} \cdot p_{\tilde{\chi}^{0}}) \]

\[ + 2(p_{\tilde{\chi}^{-}} \cdot p_{\tilde{\chi}^{0}})^2 + 2(p_{\tilde{\chi}^{-}} \cdot p_{\tilde{\chi}^{0}})(p_{\tilde{\chi}^{0}} \cdot p_\gamma) + (p_{\tilde{\chi}^{-}} \cdot p_{\tilde{\chi}^{0}})(p_{\tilde{\chi}^{0}} \cdot p_\gamma) + m_{\tilde{\chi}^{-}}^2 (p_{\tilde{\chi}^{0}} \cdot p_\gamma) \]

\[ - 4m_{\tilde{\chi}^{-}}^2 (p_{\tilde{\chi}^{-}} \cdot p_\gamma)] (C_L C_R^\dagger + C_R C_L^\dagger) + \frac{2e^2}{[(p_{\tilde{\chi}^{-}} \cdot p_\gamma) + (p_{\tilde{\chi}^{0}} \cdot p_\gamma)]^2} (-m_{\tilde{\chi}^{-}} m_{\tilde{\chi}^{0}}) \]

\[ \times [2m_{\tilde{\chi}^{-}}^2 + 2(p_{\tilde{\chi}^{-}} \cdot p_\gamma) + (p_{\tilde{\chi}^{0}} \cdot p_\gamma) + 2(p_{\tilde{\chi}^{-}} \cdot p_{\tilde{\chi}^{0}})] (C_L C_R^\dagger + C_R C_L^\dagger) \]

where \( p_{\tilde{\chi}^{-}} \), \( p_{\tilde{\chi}^{0}} \) and \( p_\gamma \) is the relevant four dimensional momentums.

The IR singularities arise from the phase space integration for the real soft photon emission, which can be conveniently isolated by slicing the space into two regions defined by suitable cut-off \( \delta_s \), according to whether the energy of the emitted photon is soft, i.e. \( E_\gamma \leq \delta_s m_H^{-}/2 \), or not. Correspondingly, the three-body decay width can be written into two parts as following.

\[ \frac{1}{2\Phi} \int \sum |M_3|^2 dPS^{(3)} = \frac{1}{2\Phi} \int_{soft} \sum |M_3|^2 dPS^{(3)} + \frac{1}{2\Phi} \int_{hard} \sum |M_3|^2 dPS^{(3)}, \quad (28) \]

where the corresponding parts are \( \Gamma_{soft} \) and \( \Gamma_{hard} \) respectively.

The hard part \( \Gamma_{hard} \) is IR finite and can be numerically calculated using the Cuba program \cite{18}. The IR divergences only live in the soft part \( \Gamma_{soft} \). Using the eikonal approximation, the soft part can be factorized into an IR factor, multiplied by the tree-level decay width.

\[ \Gamma_{soft} = \delta_{IR} \Gamma_0, \quad (29) \]
where

\[
\delta_{IR} = \frac{\alpha}{2\pi} \left\{ \frac{1}{\epsilon} \left[ 2 - \frac{2x_1x_2 - x_1 - x_2}{x_1 - x_2} \ln \left( \frac{x_1x_2 - x_2}{x_1x_2 - x_1} \right) \right] + \ln \left( \frac{4\pi\mu^2}{\delta_s m^2_{H^-}} \right) - \gamma_E \right\}
\times \left[ 2 - \frac{2x_1x_2 - x_1 - x_2}{x_1 - x_2} \ln \left( \frac{x_1x_2 - x_2}{x_1x_2 - x_1} \right) \right] + 2 + \frac{2x_1x_2 - x_1 - x_2}{x_1 - x_2}
\times \left[ \ln \left( \frac{x_1x_2 - x_2}{x_1x_2 - x_1} \right) - 2Li_2 \left( \frac{2(x_1 - x_2)}{x_1x_2 - x_2} \right) - \frac{1}{2} \ln^2 \left( \frac{x_1x_2 - x_2}{x_1x_2 - x_1} \right) \right].
\]

(30)

From Eq. (25) and Eq. (30) we can see that the IR divergences in \( \Gamma_V \) and \( \Gamma_R \) can be cancelled. Finally, summing up the tree-level, the virtual and the real corrections, the decay width of \( H^- \rightarrow \tilde{\chi}^-\tilde{\chi}^0 \), including the FEW corrections, is

\[
\Gamma = \Gamma_0 + \Gamma_V + \Gamma_R.
\]

(31)

5 Numerical Results

We now present some numerical results of two charged Higgs decay modes: \( H^- \rightarrow \tilde{\chi}^-\tilde{\chi}^0 \) (mode 1) and \( H^- \rightarrow \tilde{\chi}^1\tilde{\chi}^0 \) (mode 2). The SM input parameters are chosen as follows [19],

\[
\begin{align*}
    m_Z &= 91.1875 GeV, \quad m_W = 80.45 GeV, \quad \alpha_{EW} = 1/137, \\
    m_e &= 0.51 MeV, \quad m_\mu = 105.658 MeV, \quad m_\tau = 1.777 GeV, \\
    m_u &= 53.8 MeV, \quad m_c = 1.5 GeV, \quad m_t = 178 GeV, \\
    m_d &= 53.8 MeV, \quad m_s = 150 MeV, \quad m_b = 4.7 GeV.
\end{align*}
\]

For the phenomenological MSSM parameters, we choose all the parameters are real and use the suspect program [1] to calculate the particle spectrum. Our calculations are mainly based on the relevant inputs as following unless specified:

\[
\begin{align*}
    m_{H^-} &= 250 GeV, \quad m_{\tilde{\chi}_1^-} = 100 GeV, \quad m_{\tilde{\chi}_2^-} = 300 GeV, \quad m_{\tilde{\chi}_1^0} = 60 GeV, \\
    m_Q = m_{\tilde{u}} = m_{\tilde{d}} = m_{\tilde{e}} = m_{\tilde{\mu}} = A_t = A_b = A_\tau = M_{SUSY} = 200 GeV.
\end{align*}
\]

The above input parameters are consistent with all the existing experiment data [19]. Note that the inputs are the same as the ones in Ref. [2]. But we vary \( \tan \beta, m_{H^-}, m_{\tilde{\chi}_1^-} \) and \( m_{\tilde{\chi}_1^0} \) to examine their effects on the decay widths.
Fig. 9 presents the dependence of the FEW corrected decay width on the arbitrary soft cut-off scale $\delta_s$, introduced in the PSS method. As $\delta_s$ varies from $10^{-1}$ to $10^{-10}$, the uncertainty of the decay width is below $\pm 0.02\%$. However, where $\delta_s$ is near 0.1, the decay width has much uncertainty, which comes from the breaking of the PSS method. Meanwhile, when $\delta_s$ is very small ($< 10^{-10}$), the calculation also becomes unreliable as shown in Ref. [15]. Nevertheless, when $\delta_s$ lies between $10^{-7}$ and $10^{-2}$, the decay width, varying only $\pm 0.002\%$, is almost unchanged. Therefore we set the soft cut-off scale $\delta_s$ as $10^{-5}$ through our numerical calculations.

Fig. 10 shows the tree-level decay width and the LO, the CLO and the FEW corrected decay width as the functions of $\tan \beta$. As $\tan \beta \geq 4$, the LO corrections increases the tree-level decay width for decay mode 1 and slightly decreases it for decay mode 2.

Fig. 11 shows the LO, the CLO and the FEW corrections as the functions of $\tan \beta$. As $\tan \beta$ ranges between 2 and 50, all the corrections keep increasing or decreasing with the increasing of $\tan \beta$ for decay modes 1 and 2, respectively. When $\tan \beta < 5$, the LO corrections is small and at the order of $1\%$. When $\tan \beta > 36$, both the magnitudes of the LO corrections can be larger than $10\%$ for decay modes 1 and 2. Also we can see that the curves for the LO corrections is almost the same as ones in Ref. [2]. Comparing with the LO corrections, the FEW and CLO corrections are larger than them by about $6\%$ and $3\%$, respectively, and we find that the LO corrections is no longer important as $\tan \beta < 5$, since when $\tan \beta$ is small, the quark mass dependent term in the $\tilde{\chi}\tilde{q}\bar{q}$ vertex will not be the largest and the mass-independent terms are important, thus the contributions from the first and second generation quarks should also be significant, as shown in the figure.

Fig. 12 shows the LO, the CLO and the FEW corrections for $\tan \beta = 4$ as the functions of $m_{H^-}$. These corrections are not sensitive to $m_{H^-}$. As $m_{H^-}$ ranges between 250GeV and 600GeV, the corrections almost do not change. In general, the FEW corrections are larger than the LO and the CLO corrections by about $6\%$ and $2\%$, respectively. There are many dips on the curves, which come from the singularities at the threshold points, for example, respective ones of which on the LO and the CLO corrections curves are shown as following:

\[
m_{H^-}(392.1 GeV) = m_{\tilde{t}_1}(183.4 GeV) + m_{\tilde{b}_1}(208.7 GeV),
\]
\[
m_{H^-}(404.2 GeV) = m_{\tilde{t}_1}(183.4 GeV) + m_{\tilde{b}_2}(220.8 GeV),
\]
Moreover, there are also more little dips appearing on the curves of the FEW corrections that come from the singularities of other loop Feynman diagrams.

Fig.13 is the same case as Fig.12 except \( \tan \beta = 30 \). The corrections are also not sensitive to \( m_{H^-} \). The FEW corrections are generally larger than the LO and the CLO corrections by about 5% and 2%, respectively. The respective dips on the LO and the CLO corrections curves, arising from the singularities at the threshold points, are

\[
\begin{align*}
    m_{H^-}(378.1 GeV) &= m_{\tilde{t}_1}(196.1 GeV) + m_{\tilde{b}_2}(182.0 GeV), \\
    m_{H^-}(440.2 GeV) &= m_{\tilde{t}_1}(196.1 GeV) + m_{\tilde{b}_2}(244.1 GeV), \\
    m_{H^-}(502.0 GeV) &= m_{\tilde{t}_2}(320.0 GeV) + m_{\tilde{b}_2}(182.0 GeV), \\
    m_{H^-}(564.1 GeV) &= m_{\tilde{t}_2}(320.0 GeV) + m_{\tilde{b}_2}(244.1 GeV).
\end{align*}
\]

Moreover, there are also more little dips on the curves of the FEW corrections. In comparison, the results of the LO corrections shown in Figs.12 and 13 are almost the same as Ref. [2], but we find another dip with \( m_{H^-} = 564.1 GeV \) shown in Fig.13 which does not exist in corresponding curves in Ref. [2].

Fig.14 presents the LO, the CLO and the FEW corrections to \( H^- \rightarrow \tilde{\chi}^- \tilde{\chi}^0 \) as the functions of \( m_{\tilde{\chi}^0_1} \) for \( \tan \beta = 4 \) and 30. The LO corrections is less than 1% for \( \tan \beta = 4 \) and 6% for \( \tan \beta = 30 \). In general for both above cases the CLO corrections are larger than the LO corrections by about 3%, and the FEW corrections are larger than the LO corrections by about 5%.

Fig.15 presents the LO, the CLO and the FEW corrections to \( H^- \rightarrow \tilde{\chi}^- \tilde{\chi}^0 \) as the functions of \( m_{\tilde{\chi}^0_1} \). Here, we choose the same parameter space as above except \( m_{\tilde{\chi}^-_1} = 128 \) GeV. When \( \tan \beta = 4 \), the LO and the CLO corrections almost do not change, and the FEW corrections slightly decrease with the increasing of \( m_{\tilde{\chi}^0_1} \). When \( \tan \beta = 30 \), all three corrections increase with the increasing of \( m_{\tilde{\chi}^0_1} \). In general, the CLO corrections are larger than the LO corrections by about 3%, and the FEW corrections are larger than the LO corrections by about 5%.

In the following calculations, the MSSM parameters are constrained within the mSUGRA [16], in which there are only five free input parameters, i.e. \( m_{1/2}, m_0, A_0, \tan \beta \) and sign of \( \mu \), where \( m_{1/2}, m_0, A_0 \) are the universal gaugino mass, scalar mass at GUT scale and the trilinear soft breaking parameter in the superpotential terms, respectively.
Fig. 16 presents the LO, the CLO and the FEW corrections as the functions of $m_0$ ranging between 0 and 1000 GeV, assuming $m_{1/2} = 200$ GeV, $A_0 = 0$ and $\tan \beta = 10$. We find that in both decay modes the LO and the CLO corrections only slightly change as $m_0$ varies except in decay mode 2 when $\mu < 0$. In decay mode 1, the FEW corrections can be larger than the LO and the CLO corrections by 10% and 15% when $m_0 > 600$ GeV, respectively. In decay mode 2, the FEW corrections can be larger than the LO corrections by 20%, and than the CLO corrections by 15% when $m_0 > 400$ GeV for $\mu > 0$, and however the magnitudes of the FEW corrections vary significantly, which drops below 5% when $m_0$ ranges between 450 – 700 GeV for $\mu < 0$.

Fig. 17 presents the LO, the CLO and the FEW corrections as the functions of $A_0$, assuming $m_{1/2} = 200$ GeV, $m_0 = 200$ GeV and $\tan \beta = 10$. The LO and the CLO corrections have the similar trend as the FEW corrections. In general, the FEW corrections can be larger than the LO and the CLO corrections by about 15% and 10% in decay mode 1, respectively. In decay mode 2, for $\mu > 0$ the FEW corrections can in general be larger than the LO and the CLO corrections by about 10% and 5%, respectively. For $\mu < 0$, the magnitudes of the FEW corrections are larger than the ones of the LO and the CLO corrections by about 28% and 30% at most, respectively.

In conclusion, we have calculated the FEW corrections to the charged Higgs decays into a neutralino and chargino in the MSSM, and compared with the LO and the CLO corrections. We first calculate the corrections in the phenomenological MSSM. The magnitudes of the FEW corrections can be larger than 10% for $H^- \rightarrow \tilde{\chi}_1^0 \tilde{\chi}_1^- \tilde{\chi}_1^-$ for $\tan \beta > 30$, and are in general 5% for both decay modes. The FEW corrections in general can be larger than the LO and the CLO corrections by about 5% and 3%, respectively. Moreover, the LO and the CLO corrections are negligible small for $\tan \beta < 5$ compared with the FEW corrections, since there are not enhancements from the Yukawa couplings. We have also calculated the FEW corrections in the mSUGRA scenario, where the FEW corrections can be larger than the LO and the CLO corrections by more than 20% and 30%, respectively. Thus the FEW corrections may be detectable for the future high precision experiments for Higgs physics if supersymmetry exists.
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where,

\[ M_1^{(v)} = \sum_{i=1}^{35} (f_1^i F_1 + f_2^i F_2), \]  

(32)

In this appendix, we list the explicit expressions for the vertex one-loop amplitude. The vertex one-loop amplitudes are expanded with two Dirac matrix elements \(^{20}\) with 35 coefficients for each of them, corresponding to the 35 Feynman diagrams in Fig.\(^2\).

\[ F_1 = \overline{\psi}(p_{\chi_j^-}) P_R \psi(p_{\chi_j^0}), \quad F_2 = \overline{\psi}(p_{\chi_j^-}) P_L \psi(p_{\chi_j^0}). \]  

(33)

In our paper, we use the Passarino-Veltman integrals, which are defined in Ref.\(^{20}\). For simplicity, we define the notations as following:

\[ c_{(0,1,2)}^1 = C_{(0,1,2)}(m_{\chi_j^2}, m_{\chi_j^0}, m_{\chi_{e1}^2}, m_{\chi_{e1}^0}, m_{\chi_{n1}^2'}, m_{\chi_{n1}^0'}), \quad b_0^1 = B_0(m_{\chi_j^0}', m_{\chi_{e1}^0}', m_{\chi_{n1}^0'}), \]

\[ c_{(0,1,2)}^2 = C_{(0,1,2)}(m_{\chi_j^2}, m_{\chi_j^0}, m_{\chi_{e1}^2}, m_{\chi_{e1}^0}, m_{\chi_{n1}^2'}, m_{\chi_{n1}^0'}), \quad b_0^2 = B_0(m_{\chi_j^0}', m_{\chi_{e1}^0}', m_{\chi_{n1}^0'}), \]

\[ c_{(0,1,2)}^3 = C_{(0,1,2)}(m_{\chi_j^2}, m_{\chi_j^0}, m_{\chi_{e1}^2}, m_{\chi_{e1}^0}, m_{\chi_{n1}^2'}, m_{\chi_{n1}^0'}), \quad b_0^3 = B_0(m_{\chi_j^0}', m_{\chi_{e1}^0}', m_{\chi_{n1}^0'}), \]

\[ c_{(0,1,2)}^4 = C_{(0,1,2)}(m_{\chi_j^2}, m_{\chi_j^0}, m_{\chi_{e1}^2}, m_{\chi_{e1}^0}, m_{\chi_{n1}^2'}, m_{\chi_{n1}^0'}), \quad b_0^4 = B_0(m_{\chi_j^0}', m_{\chi_{e1}^0}', m_{\chi_{n1}^0'}), \]

\[ \text{Appendix} \]

In this appendix, we list the explicit expressions for the vertex one-loop amplitude. The vertex one-loop amplitudes are expanded with two Dirac matrix elements \(^{20}\) with 35 coefficients for each of them, corresponding to the 35 Feynman diagrams in Fig.\(^2\).
\[ c_{(0,1,2)}^5 = C_{(0,1,2)}(m_{\tilde{\chi}_j}^2, m_{\tilde{\chi}_i}^2, m_{H^+}^2, m_{\tilde{\chi}_{11}}^2, m_{\tilde{\chi}_1^0}^2), \quad b_{0}^5 = B_0(m_{\tilde{\chi}_i^0}^2, m_{H^+}, m_{\tilde{\chi}_1^0}^2) \]

\[ c_{(0,1,2)}^6 = C_{(0,1,2)}(m_{\tilde{\chi}_j}^2, m_{\tilde{\chi}_i}^2, m_{H^+}^2, m_{\tilde{\chi}_{11}}^2, m_{W^0}^2, m_{\tilde{\chi}_1^0}^2), \quad b_{0}^6 = B_0(m_{\tilde{\chi}_i^0}^2, m_{W^0}, m_{\tilde{\chi}_1^0}^2) \]

\[ c_{(0,1,2)}^7 = C_{(0,1,2)}(m_{\tilde{\chi}_j}^2, m_{\tilde{\chi}_i}^2, m_{H^+}^2, m_{\tilde{\chi}_{11}}^2, m_{\tilde{\tau}_{l91}}^2, m_{\tilde{\tau}_{191}}^2), \quad b_{0}^7 = B_0(m_{\tilde{\tau}_{l91}}^2, 0, m_{\tilde{\tau}_{191}}^2) \]

\[ c_{(0,1,2)}^8 = C_{(0,1,2)}(m_{H^+}^2, m_{\tilde{\chi}_j}^2, m_{\tilde{\chi}_i}^2, m_{\tilde{\tau}_{l91}}^2, m_{\tilde{\tau}_{e_{91}}^2}, m_{\tilde{\tau}_{e_{91}}^2}), \quad b_{0}^8 = B_0(m_{\tilde{\tau}_{l91}}^2, m_{\tilde{\tau}_{e_{91}}^2}, m_{\tilde{\tau}_{e_{91}}^2}) \]

\[ c_{(0,1,2)}^9 = C_{(0,1,2)}(m_{H^+}^2, m_{\tilde{\chi}_j}^2, m_{\tilde{\chi}_i}^2, m_{\tilde{\tau}_{l91}}^2, m_{\tilde{\tau}_{g_{21}}^2}, m_{\tilde{\tau}_{h_{91}}^2}^2, m_{\tilde{\tau}_{s_{919}}^2}^2), \quad b_{0}^9 = B_0(m_{\tilde{\tau}_{l91}}^2, m_{\tilde{\tau}_{g_{21}}^2}, m_{\tilde{\tau}_{h_{91}}^2}^2) \]

\[ c_{(0,1,2)}^{10} = C_{(0,1,2)}(m_{H^+}^2, m_{\tilde{\chi}_j}^2, m_{\tilde{\chi}_i}^2, m_{\tilde{\tau}_{l91}}^2, m_{\tilde{\tau}_{g_{21}}^2}, m_{\tilde{\tau}_{d_{919}}^2}^2, m_{\tilde{\tau}_{d_{s_{919}}^2}}^2), \quad b_{0}^{10} = B_0(m_{\tilde{\tau}_{l91}}^2, m_{\tilde{\tau}_{g_{21}}^2}, m_{\tilde{\tau}_{d_{919}}^2}) \]

\[ c_{(0,1,2)}^{11} = C_{(0,1,2)}(m_{\tilde{\chi}_j}^2, m_{\tilde{\chi}_i}^2, m_{H^+}^2, m_{\tilde{\chi}_{11}}^2, m_{\tilde{\chi}_1^0}^2, m_{H^+}^2) \]

\[ c_{(0,1,2)}^{12} = C_{(0,1,2)}(m_{\tilde{\chi}_j}^2, m_{\tilde{\chi}_i}^2, m_{H^+}^2, m_{h_0}^2, m_{\tilde{\chi}_1^0}^2, m_{H^+}^2) \]

\[ c_{(0,1,2)}^{13} = C_{(0,1,2)}(m_{\tilde{\chi}_j}^2, m_{\tilde{\chi}_i}^2, m_{\tilde{\chi}_{11}}^2, m_{\tilde{\chi}_1^0}^2, m_{H^+}^2, m_{H^+}^2) \]

\[ c_{(0,1,2)}^{14} = C_{(0,1,2)}(m_{\tilde{\chi}_j}^2, m_{\tilde{\chi}_i}^2, m_{H^+}^2, m_{H^0}^2, m_{\tilde{\chi}_1^0}^2, m_{H^+}^2) \]

\[ c_{(0,1,2)}^{15} = C_{(0,1,2)}(m_{\tilde{\chi}_j}^2, m_{\tilde{\chi}_i}^2, m_{\tilde{\chi}_{11}}^2, m_{\tilde{\chi}_1^0}^2, m_{H^0}^2, m_{H^0}^2) \]

\[ c_{(0,1,2)}^{16} = C_{(0,1,2)}(m_{\tilde{\chi}_j}^2, m_{\tilde{\chi}_i}^2, m_{H^+}^2, m_{h_0}^2, m_{\tilde{\chi}_1^0}^2, m_{H^0}^2) \]

\[ c_{(0,1,2)}^{17} = C_{(0,1,2)}(m_{\tilde{\chi}_j}^2, m_{\tilde{\chi}_i}^2, m_{H^+}^2, m_{W^0}^2, m_{\tilde{\chi}_1^0}^2, m_{W^0}^2) \]

\[ c_{(0,1,2)}^{18} = C_{(0,1,2)}(m_{\tilde{\chi}_j}^2, m_{\tilde{\chi}_i}^2, m_{H^+}^2, m_{H^0}^2, m_{\tilde{\chi}_1^0}^2, m_{W^0}^2) \]

\[ c_{(0,1,2)}^{19} = C_{(0,1,2)}(m_{\tilde{\chi}_j}^2, m_{\tilde{\chi}_i}^2, m_{H^+}^2, m_{W^0}^2, m_{H^0}^2, m_{W^0}^2) \]

\[ c_{(0,1,2)}^{16} = C_{(0,1,2)}(m_{\tilde{\chi}_j}^2, m_{\tilde{\chi}_i}^2, m_{H^+}^2, m_{h_0}^2, m_{\tilde{\chi}_1^0}^2, m_{H^0}^2) \]
\[ c_{0,1,2}^{20} = C_{0,1,2}(m_{\chi_j}^2, m_{\chi_i}^2, m_{H^-}, m_{A_0}^2, m_{\chi_{c1}}^2, m_W^2) \]

\[ c_{0,1,2}^{21} = C_{0,1,2}(m_{\chi_j}^2, m_{\chi_i}^2, m_{H^-}, m_{\epsilon_{g1}}^2, m_{\epsilon_{g1}}^2) \]

\[ c_{0,1,2}^{22} = C_{0,1,2}(m_{\chi_j}^2, m_{\chi_i}^2, m_{H^-}, m_{\epsilon_{g1}}^2, m_{\epsilon_{g1}}^2, m_{\epsilon_{g1}}^2) \]

\[ c_{0,1,2}^{23} = C_{0,1,2}(m_{\chi_j}^2, m_{\chi_i}^2, m_{H^-}, m_{\epsilon_{g1}}^2, m_{\epsilon_{g1}}^2, m_{\epsilon_{g1}}^2) \]

\[ c_{0,1,2}^{24} = C_{0,1,2}(m_{\chi_j}^2, m_{\chi_i}^2, m_{H^-}, m_{\epsilon_{g1}}^2, m_{\epsilon_{g1}}^2, m_{\epsilon_{g1}}^2) \]

\[ c_{0,1,2}^{25} = C_{0,1,2}(m_{\chi_j}^2, m_{\chi_i}^2, m_{H^-}, m_{\epsilon_{g1}}^2, m_{\epsilon_{g1}}^2, m_{\epsilon_{g1}}^2) \]

\[ c_{0,1,2}^{26} = C_{0,1,2}(m_{\chi_j}^2, m_{\chi_i}^2, m_{H^-}, m_{\epsilon_{g1}}^2, m_{\epsilon_{g1}}^2, m_{\epsilon_{g1}}^2) \]

\[ c_{0,1,2}^{27} = C_{0,1,2}(m_{\chi_j}^2, m_{\chi_i}^2, m_{H^-}, m_{\epsilon_{g1}}^2, m_{\epsilon_{g1}}^2, m_{\epsilon_{g1}}^2) \]

\[ c_{0,1,2}^{28} = C_{0,1,2}(m_{\chi_j}^2, m_{\chi_i}^2, m_{H^-}, m_{\epsilon_{g1}}^2, m_{\epsilon_{g1}}^2, m_{\epsilon_{g1}}^2) \]

\[ c_{0,1,2}^{29} = C_{0,1,2}(m_{\chi_j}^2, m_{\chi_i}^2, m_{H^-}, m_{\epsilon_{g1}}^2, m_{\epsilon_{g1}}^2, m_{\epsilon_{g1}}^2) \]

\[ c_{0,1,2}^{30} = C_{0,1,2}(m_{\chi_j}^2, m_{\chi_i}^2, m_{H^-}, m_{\epsilon_{g1}}^2, m_{\epsilon_{g1}}^2, m_{\epsilon_{g1}}^2) \]

\[ c_{0,1,2}^{31} = C_{0,1,2}(m_{\chi_j}^2, m_{\chi_i}^2, m_{H^-}, m_{\epsilon_{g1}}^2, m_{\epsilon_{g1}}^2, m_{\epsilon_{g1}}^2) \]

\[ c_{0,1,2}^{32} = C_{0,1,2}(m_{\chi_j}^2, m_{\chi_i}^2, m_{H^-}, m_{\epsilon_{g1}}^2, m_{\epsilon_{g1}}^2, m_{\epsilon_{g1}}^2) \]

\[ c_{0,1,2}^{33} = C_{0,1,2}(m_{\chi_j}^2, m_{\chi_i}^2, m_{H^-}, m_{\epsilon_{g1}}^2, m_{\epsilon_{g1}}^2, m_{\epsilon_{g1}}^2) \]

\[ c_{0,1,2}^{34} = C_{0,1,2}(m_{\chi_j}^2, m_{\chi_i}^2, m_{H^-}, m_{\epsilon_{g1}}^2, m_{\epsilon_{g1}}^2, m_{\epsilon_{g1}}^2) \]
\[c_{(0,1,2)}^{35} = C_{(0,1,2)}(m_{\chi_i}^2, m_{\chi_i}^2, m_{\chi_i}^2, m_{\chi_i}^2, m_{\chi_i}^2, m_{\chi_i}^2), \quad b_0^{35} = B_0(m_{\chi_i}^2, m_{\chi_i}^2, m_{\chi_i}^2)\]

We also define some abbreviations for the frequent combinations as following:

\[\lambda_{ab}^1 = (c_W N_{a2} - s_W N_{a1})(s_\alpha N_{b3} + c_\alpha N_{b4}) + (c_W N_{b2} - s_W N_{b1})(s_\alpha N_{a3} + c_\alpha N_{a4})\]

\[\lambda_{ab}^2 = (c_W N_{a2} - s_W N_{a1})(c_\alpha N_{b3} - s_\alpha N_{b4}) + (c_W N_{b2} - s_W N_{b1})(c_\alpha N_{a3} - s_\alpha N_{a4})\]

\[\lambda_{ab}^3 = (c_W N_{a2} - s_W N_{a1})(c_\beta N_{b4} - s_\beta N_{b3}) + (c_W N_{b2} - s_W N_{b1})(c_\beta N_{a4} - s_\beta N_{a3})\]

\[\lambda_{ab}^4 = (c_W N_{a2} - s_W N_{a1})(c_\beta N_{b3} + s_\beta N_{b4}) + (c_W N_{b2} - s_W N_{b1})(c_\beta N_{a3} + s_\beta N_{a4})\]

\[\lambda_{gs}^8 = c_\beta m_W R_{s_{a2}} \tilde{g}^* (s_W N_{i1} + c_W N_{i2}) - c_W m_{e_g} R_{s_{a2}} \tilde{g}^* N_{i3}\]

\[\lambda_{gs}^9 = 2m_W s_\beta R_{s_{a1}} \tilde{g}^* V_{j1} - \sqrt{2} m_{u_g} R_{s_{a2}} \tilde{g}^* V_{j2}\]

\[\lambda_{gs}^{10} = 2c_\beta m_W U_{j1}^* R_{s_{a1}} \tilde{g}^* - \sqrt{2} m_{d_g} U_{j2}^* R_{s_{a2}} \tilde{g}^*\]

\[\lambda_{gs}^{21} = (m_{s_\beta} - t_\beta m_{e_g}^2) R_{s_{a1}} \tilde{g}^* - (\mu + t_\beta A_{\epsilon g}^*) m_{e_g} R_{s_{a2}} \tilde{g}^*\]

\[\lambda_{jc}^{25} = s_\omega \delta_{c,j} - U_{c_1} U_{j1}^* - \frac{1}{2} U_{c_2} U_{j2}^*\]

\[\lambda_{cn}^{26} = U_{c_1}^* N_{n2} + U_{c_2}^* N_{n3}/\sqrt{2}\]

\[\gamma_{ab}^1 = s_\alpha U_{a2} V_{b1} - c_\alpha U_{a1} V_{b2}, \quad \gamma_{ab}^2 = c_\alpha U_{a2} V_{b1} + s_\alpha U_{b1} V_{a2}\]

\[\gamma_{ab}^3 = s_\beta U_{a2} V_{b1} + c_\beta U_{a1} V_{b2}, \quad \gamma_{ab}^4 = s_\beta U_{a1} V_{b2} - c_\beta U_{a2} V_{b1}\]

\[\gamma_{gs}^8 = -2 U_{j1}^* R_{s_{a1}} \tilde{g}^* + \sqrt{2} m_{e_g} U_{j2}^* R_{s_{a2}} \tilde{g}^*/(c_\beta m_W)\]
\[ \mathcal{Y}_{g_s}^9 = 4m_W s_\beta s_W R_{s_2}^{R_s} N_{i_1} - 3c_W m_{u_9} R_{s_1}^{R_s} N_{i_4} \]

\[ \mathcal{Y}_{g_s}^{10} = (2c_\beta m_W s_W R_{s_2}^{R_s} N_{i_1}^* + 3c_W m_{d_9} R_{s_1}^{R_s} N_{i_3}^*) m_{d_9} \]

\[ \mathcal{Y}_{c_1}^{25} = s_W^2 \delta_{c,j} - V_{j1} V_{c_1} - \frac{1}{2} V_{j2} V_{c_2} \]

\[ \mathcal{Y}_{c_1}^{26} = V_{c_1}^* N_{n_2} - V_{c_2}^* N_{n_4} / \sqrt{2} \]

\[ \mathcal{Z}_{g_s}^8 = 2c_\beta m_W s_W R_{s_2}^{R_s} N_{i_1} + c_W m_{e_9} R_{s_1}^{R_s} N_{i_3}^* \]

\[ \mathcal{Z}_{g_s}^9 = m_W s_\beta R_{s_1}^{R_s} (s_W N_{i_1} + 3c_W N_{i_2}) + 3c_W m_{u_9} R_{s_2}^{R_s} N_{i_4}^* \]

\[ \mathcal{Z}_{g_s}^{10} = c_\beta m_W R_{s_1}^{R_s} (-s_W N_{i_1} + 3c_W N_{i_2}) - 3c_W m_{d_9} R_{s_2}^{R_s} N_{i_3}^* \]

\[ \mathcal{Z}_{ab}^{25} = N_{a_3} N_{b_3}^* - N_{a_4} N_{b_4}^* \]

\[ \mathcal{W}_{nc}^1 = (m_{\chi_c} C_{nc}^{L} + m_{\chi_0} C_{nc}^{R}), \quad \mathcal{W}_{nc}^2 = (m_{\chi_0} C_{nc}^{L} + m_{\chi_c} C_{nc}^{R}) \]

\[ \mathcal{W}_{nc}^3 = (m_{\chi_c} D_{nc}^{L} + m_{\chi_0} D_{nc}^{R}), \quad \mathcal{W}_{nc}^4 = (m_{\chi_0} D_{nc}^{L} + m_{\chi_c} D_{nc}^{R}) \]

\[ \mathcal{W}_{c_1 c_2}^5 = (m_{\chi_{c_1}} \chi^1 + m_{\chi_{c_2}} \chi^{1*}), \quad \mathcal{W}_{c_1 c_2}^6 = (m_{\chi_{c_1}} \chi^2 + m_{\chi_{c_2}} \chi^{2*}) \]

\[ \mathcal{W}_{c_1 c_2}^7 = (m_{\chi_{c_1}} \chi^3 - m_{\chi_{c_2}} \chi^{3*}), \quad \mathcal{W}_{c_1 c_2}^8 = (m_{\chi_{0_1}} \chi^1 - m_{\chi_{0_2}} \chi^{1*}) \]

\[ \mathcal{W}_{g_1 g_2 s_1}^9 = \sqrt{2} t_\beta m_{d_{g_1}}^2 U_{j_1}^{R_s} R_{s_1}^{R_s} s_\beta, \quad \mathcal{W}_{g_1 g_2 s_1}^{10} = \sqrt{2} V_{j_2}^{R_s} m_{u_{g_1}}^2 R_{s_1}^{R_s} \]

\[ T^1 = T^2 = T^3 = T^4 = \alpha / (8 \sqrt{2} c_W \pi s_W^2), \quad T^5 = T^6 = \alpha / (4 \pi e^2) \]

\[ T^7 = \alpha e_{\beta} (c_W N_{i_2}^* - s_W N_{i_1}^*) / (8 m_W c_W \pi s_W^3), \quad T^8 = \alpha e_{\beta} / (16 c_\beta c_W m_W^2 \pi s_W^3) \]
\[ T^9 = \alpha e K_{g2g1} K^*_{g2g1} / (16c_W m^3_W \pi s^2_\beta \beta^3 s^3_W t_\beta), \quad T^{10} = \alpha e K_{g1g2} K^*_{g1g2} / (16c_W m^3_W \pi s^2_\beta s^3_W) \]

\[ T^{11} = -\alpha m_W [c_\beta s_\alpha + (2c^2_W) + s_\beta - \alpha] / (8c_W \pi s_W^2) \]

\[ T^{12} = -\alpha m_W [c_\beta s_\alpha + (2c^2_W) + s_\beta - \alpha] / (4\sqrt{2} \pi s_W^2) \]

\[ T^{13} = \alpha m_W [c_\beta - (2c_\beta + \beta)/(2c^2_W)] / (8c_W \pi s_W^2) \]

\[ T^{14} = \alpha m_W [c_\beta - (2c_\beta + \beta)/(2c^2_W)] / (4\sqrt{2} \pi s_W^2) \]

\[ T^{15} = \alpha m_W [c_\beta - (2c_\beta + \beta)/(2c^2_W)] / (16c_W \pi s_W^2) \]

\[ T^{16} = \alpha m_W (c_\beta - (2c_\beta + \beta)/(2c^2_W)) / (8\sqrt{2} \pi s_W^2) \]

\[ T^{17} = \alpha m_W [s_\beta - (2c_\beta + \beta)/(2c^2_W)] / (16c_W \pi s_W^2) \]

\[ T^{18} = \alpha m_W [s_\beta - (2c_\beta + \beta)/(2c^2_W)] / (8\sqrt{2} \pi s_W^2) \]

\[ T^{19} = \alpha m_W / (16c_W \pi s_W^2), \quad T^{20} = \alpha m_W / (8\sqrt{2} \pi s_W^2) \]

\[ T^{21} = \alpha e / (16c_W m_W \pi s_W^3), \quad T^{22} = \alpha e / (8\sqrt{2} c^2_\beta c_W m^3_W \pi s^3_W) \]

\[ T^{23} = \alpha e K_{g2g1} K^*_{g2g1} / (16c_W m^3_W \pi s^2_\beta s^3_W t_\beta), \quad T^{24} = \alpha e K_{g1g2} K^*_{g1g2} / (16c_W m^3_W \pi s^2_\beta s^3_W) \]

\[ T^{25} = \alpha / (4c^2_W \pi s_W^2), \quad T^{26} = \alpha / (2\pi s_W^2) \]

\[ T^{27} = \alpha e c_\beta - \alpha / (16\pi c_W s_W^2), \quad T^{28} = \alpha e c_\beta - \alpha / (8\sqrt{2} \pi s_W^3) \]

\[ T^{29} = \alpha e s_\beta - \alpha / (8\sqrt{2} \pi s_W^3), \quad T^{30} = \alpha e / (8\sqrt{2} \pi s_W^3) \]
\[ T^{31} = -\alpha/(4\pi), \quad T^{32} = \alpha c_2 w / (8\pi c_2^2 s_w^2) \]

\[ T^{33} = \alpha e c_{\beta - \alpha} / (16 c_w \pi s_w^2), \quad T^{34} = \alpha e s_{\beta - \alpha} / (16 c_w \pi s_w^2) \]

\[ T^{35} = \alpha e / (16 c_w \pi s_w^2) \]

With the above abbreviations, we present the coefficients as following:

\[ f_1^1 = T^1 \sum_{c_i=1}^{2} \sum_{n_i=1}^{4} [b_{0i}^0 y_{c_i j}^1, x_{n_i 1}^1, \psi_{n_i 1}^1, n_i, c_{n_i 1}^1 + c_2^1 (m_{x_j}^1, \psi_{n_i 1}^1, x_{j 1}^1, \psi_{n_i 1}^1, n_i, c_{n_i 1}^1) + m_{x_j}^1, \psi_{n_i 1}^1, x_{j 1}^1, \psi_{n_i 1}^1, n_i, c_{n_i 1}^1) + m_{x_j}^1, \psi_{n_i 1}^1, x_{j 1}^1, \psi_{n_i 1}^1, n_i, c_{n_i 1}^1) + c_0 \phi_{n_i 1}^1, \psi_{n_i 1}^1, x_{j 1}^1, \psi_{n_i 1}^1, n_i, c_{n_i 1}^1) \]

\[ = f_1^1 (y_{c_i j}^1, y_{j 1}^1, x_{n_i 1}^1, x_{j 1}^1, \psi_{n_i 1}^1, \psi_{n_i 1}^1, n_i, c_{n_i 1}^1, \psi_{n_i 1}^1, n_i, c_{n_i 1}^1) \]

\[ f_2^1 = f_1^1 (y_{c_i j}^2, y_{j 1}^2, x_{n_i 1}^2, x_{j 1}^2, \psi_{n_i 1}^2, \psi_{n_i 1}^2, n_i, c_{n_i 1}^2, \psi_{n_i 1}^2, n_i, c_{n_i 1}^2) \]

\[ f_1^2 = f_1^2 (y_{c_i j}^3, y_{j 1}^3, x_{n_i 1}^3, x_{j 1}^3, \psi_{n_i 1}^3, \psi_{n_i 1}^3, n_i, c_{n_i 1}^3, \psi_{n_i 1}^3, n_i, c_{n_i 1}^3) \]

\[ f_2^2 = f_1^2 (y_{c_i j}^4, y_{j 1}^4, x_{n_i 1}^4, x_{j 1}^4, \psi_{n_i 1}^4, \psi_{n_i 1}^4, n_i, c_{n_i 1}^4, \psi_{n_i 1}^4, n_i, c_{n_i 1}^4) \]

\[ f_1^3 = f_1^3 (y_{c_i j}^5, y_{j 1}^5, x_{n_i 1}^5, x_{j 1}^5, \psi_{n_i 1}^5, \psi_{n_i 1}^5, n_i, c_{n_i 1}^5, \psi_{n_i 1}^5, n_i, c_{n_i 1}^5) \]

\[ f_2^3 = f_1^3 (y_{c_i j}^6, y_{j 1}^6, x_{n_i 1}^6, x_{j 1}^6, \psi_{n_i 1}^6, \psi_{n_i 1}^6, n_i, c_{n_i 1}^6, \psi_{n_i 1}^6, n_i, c_{n_i 1}^6) \]

\[ f_1^4 = f_1^4 (y_{c_i j}^7, y_{j 1}^7, x_{n_i 1}^7, x_{j 1}^7, \psi_{n_i 1}^7, \psi_{n_i 1}^7, n_i, c_{n_i 1}^7, \psi_{n_i 1}^7, n_i, c_{n_i 1}^7) \]
\( \mathcal{Y}_{c_{1j}} \rightarrow \mathcal{Y}_{c_{1j}}, \mathcal{Y}_{j_{c_1}} \rightarrow \mathcal{Y}_{j_{c_1}} \)

\[ f^4_2 = f^4_1(\mathcal{Y}_{c_{1j}} \rightarrow \mathcal{Y}_{j_{c_1}}, \mathcal{X}_{\in_1} \rightarrow \mathcal{X}_{\in_1}, C^L_{n_{1c_1}} \rightarrow C^L_{n_{1c_1}}, \mathcal{W}_{n_{1c_1}} \leftarrow \mathcal{W}_{n_{1c_1}}^2) \]

\[ f^5_1 = f^5_1(T^4 \rightarrow T^5, b_0^4 \rightarrow b_0^5, c_{(0,1,2)}^4 \rightarrow c_{(0,1,2)}^5, \mathcal{m}_{w_0}^0 \rightarrow \mathcal{m}_{w_0}^5, \mathcal{X}_{\in_1}^4 \rightarrow C^L_{i_{c_1}}, \mathcal{X}_{\in_1}^4 \rightarrow -C^R_{i_{c_1}}, \]

\[ \mathcal{Y}_{c_{1j}} \rightarrow C^R_{n_{1j}}, \mathcal{Y}_{j_{c_1}} \rightarrow -C^L_{n_{1j}}, \mathcal{W}_{n_{1c_1}} \leftarrow \mathcal{W}_{n_{1c_1}}^2 \) 

\[ f^5_2 = f^5_1(L \leftrightarrow R, \mathcal{W}_{n_{1c_1}} \leftarrow \mathcal{W}_{n_{1c_1}}^2) \]

\[ f^6_1 = f^6_1(T^5 \rightarrow T^6, b_0^5 \rightarrow b_0^6, c_{(0,1,2)}^5 \rightarrow c_{(0,1,2)}^6, C^L_{i_{c_1}} \rightarrow -C^L_{i_{c_1}}, C^L_{i_{c_1}} \rightarrow -C^L_{i_{c_1}}) \]

\[ f^6_2 = f^6_1(L \leftrightarrow R) \]

\[ f^7_1 = T^7 \sum_{g_1=1}^{3} \mathcal{m}^2_{v_{g_1}} \mathcal{m}^9_{\chi_i} \left[ V_{j_1}(c_0^7 + c_2^7) + c_1^7 \mathcal{m}_{\chi_j} - U_{j_2}^*/(\sqrt{2}c_\beta \mathcal{m}_w) \right] \]

\[ f^7_2 = T^7 \sum_{g_1=1}^{3} \mathcal{m}^2_{v_{g_1}} \left( \frac{U_{j_2}^*}{\sqrt{2}c_\beta \mathcal{m}_w} (b_0^7 + m_{\chi_2}^9 - c_1^7)^2 + m_{\chi_j}^9 + c_0^7 \mathcal{m}^9_{v_{g_1}} \right) \]

\[ f^8_1 = T^8 \sum_{g_1=1}^{3} \sum_{g_1=1}^{2} \left( c_0^8 + c_2^8 \right) \mathcal{m}_{\chi_j}^9 m_{\chi_{g_1}}^9 \mathcal{Y}_{g_1 s_1}^8 \mathcal{X}_{g_1 s_1}^8 - \mathcal{m}_{\chi_j}^9 m_{\chi_{g_1}}^9 \mathcal{Y}_{g_1 s_1}^8 \mathcal{X}_{g_1 s_1}^8 \]

\[ f^8_2 = T^8 \sum_{g_1=1}^{3} \sum_{g_1=1}^{2} \left( b_0^8 + c_2^8 \mathcal{m}^9_{\chi_j} + m_{\chi_2}^9 \right) \mathcal{m}_{\chi_{g_1}}^9 \mathcal{Y}_{g_1 s_1}^8 \mathcal{X}_{g_1 s_1}^8 - c_1^8 \mathcal{m}_{\chi_{g_1}}^9 \mathcal{Y}_{g_1 s_1}^8 \mathcal{X}_{g_1 s_1}^8 \]

\[ f^9_1 = \sum_{g_1=1}^{3} \sum_{g_2=1}^{2} \sum_{g_1=1}^{3} \sum_{g_1=2}^{2} T^9 \left[ \mathcal{m}_{u_{g_2}}^9 \mathcal{Y}_{g_2 s_1}^9 \mathcal{Y}_{g_2 s_1}^9 \right] \left( b_0^9 + c_0^9 \left( \mathcal{m}^2_{u_{g_2}} + t^2_{\beta} \mathcal{m}^2_{u_{g_1}} \right) + c_1^9 \mathcal{m}_{H_2}^9 + c_2^9 \mathcal{m}_{\chi_i}^9 \right) \]

\[ - t^2_{\beta} \mathcal{W}_{g_{1g_2 s_1}} \mathcal{m}_{\chi_j}^9 \left( \mathcal{m}_{u_{g_2}}^9 \mathcal{Y}_{g_2 s_1}^9 \left( c_0^9 + c_1^9 / s^2_{\beta} \right) + c_2^9 \mathcal{m}_{\chi_j}^9 \mathcal{Z}_{g_2 s_1}^9 \right) \]

\[ + \mathcal{m}_{\chi_i}^9 \mathcal{Z}_{g_2 s_1}^9 \mathcal{Z}_{g_2 s_1}^9 \left( c_0^9 \mathcal{m}_{u_{g_2}}^9 + \left( c_1^9 + c_2^9 \right) \left( \mathcal{m}_{u_{g_2}}^9 + t^2_{\beta} \mathcal{m}_{u_{g_1}}^9 \right) \right) \]
\[ f_2^0 = \sum_{g_1=1}^{3} \sum_{g_2=1}^{3} \sum_{s_1=1}^{2} T^9 [W_{g_1 g_2 s_1}^0 \mathcal{Z}_{g_2 s_1}^0 t_\beta^2 (b_0^g + c_0^g m_{u g_2}^2 / s_{\beta}^2 + c_2^g m_{\chi_i}^2 + c_1^g m_{H^-}^2)
\]
\[ + W_{g_1 g_2 s_1}^0 \gamma_{g_2 s_1}^0 m_{\chi_i} \mathcal{W}_{u g_2} (c_0^g + c_1^g)^2 / s_{\beta}^2 + c_0^g m_{\chi_i}^2) - \lambda_{g_2 s_1}^0 m_{\chi_j} (c_0^g m_{\chi_i}^2 + c_1^g m_{u g_2}^2 \gamma_{g_2 s_1}^0)
\]
\[ + ((c_0^g + c_1^g) m_{u g_2}^2 + c_0^g m_{d g_1} t_\beta^2) \mathcal{Z}_{g_2 s_1}^0 ]
\]
\[ f_1^{10} = \sum_{g_1=1}^{3} \sum_{g_2=1}^{3} \sum_{s_1=1}^{2} T^{10} [W_{g_1 g_2 s_1}^{10} \mathcal{Z}_{g_2 s_1}^{10} (b_0^g + m_{u g_1}^2 + t_\beta^2 m_{u g_2}^2) c_0^1 + m_{\chi_j}^2 c_0^2 + m_{H^-}^2 c_1^{10})
\]
\[ - \lambda_{g_2 s_1}^{10} \mathcal{Z}_{g_2 s_1}^{10} m_{\chi_j}^2 ((c_0^1 + c_1^2) t_\beta^2 (m_{u g_1}^2 + t_\beta^2 m_{u g_2}^2) + c_0^1 m_{u g_1}^2)
\]
\[ + \gamma_{g_2 s_1}^{10} m_{\chi_i}^2 ((c_0^1 + c_1^2) + (c_0^1 + c_1^2) W_{g_1 g_2 s_1}^{10} - \lambda_{g_2 s_1}^{10} c_0^1 m_{\chi_j}^2 t_\beta^3)]
\]
\[ f_2^{10} = \sum_{g_1=1}^{3} \sum_{g_2=1}^{3} \sum_{s_1=1}^{2} T^{10} [\lambda_{g_2 s_1}^{10} \gamma_{g_2 s_1}^{10} (t_\beta^2 (b_0^g + c_0^1 m_{H^-}^2 + c_2^0 m_{\chi_j}^2) + (1 + t_\beta^2) m_{u g_1}^2 c_0^{10})
\]
\[ - W_{g_1 g_2 s_1}^{10} \gamma_{g_2 s_1}^{10} m_{\chi_j}^2 ((1 + t_\beta^2) c_0^1 + c_1^2) + c_0^1 t_\beta^2 \mathcal{Z}_{g_2 s_1}^{10} m_{\chi_i}^2 (\lambda_{g_2 s_1}^{10} t_\beta (c_0^1 m_{u g_1}^2
\]
\[ + c_1^0 (m_{u g_1}^2 + t_\beta^2 m_{u g_2}^2)) - c_2^1 m_{\chi_j}^2 W_{g_1 g_2 s_1}^{10} ]
\]
\[ f_1^{11} = T^{11} \sum_{n_1=1}^{4} [c_1^{11} m_{\chi_j}^0 \lambda_{m_1}^0 C_{n_1 j}^L + c_0^{11} \lambda_{m_1}^0 W_{n_1 j}^2 + c_1^{11} (m_{\chi_j}^0 \lambda_{m_1}^0 C_{n_1 j})]
\]
\[ f_2 = f_1^{11} (C_{n_1 j}^L \leftrightarrow C_{n_1 j}^R, \lambda_{m_1}^0 \leftrightarrow \lambda_{m_1}^1, W_{n_1 j}^2 \rightarrow W_{n_1 j}^1]
\]
\[ f_1^{12} = T^{12} \sum_{c_1=1}^{2} [c_1^{12} m_{\chi_j}^0 \lambda_{j c_1}^1 C_{c_1}^L + c_0^{12} W_{c_1}^5 C_{c_1}^R + c_2^{12} (m_{\chi_j}^0 \gamma_{j c_1}^1 C_{c_1}^R - m_{\chi_i}^0 \gamma_{j c_1}^L C_{c_1}^L)]
\]
\[ f_2^{12} = f_1^{12} (C_{c_1}^L \leftrightarrow C_{c_1}^R, \gamma_{c_1 j}^1 \leftrightarrow \gamma_{c_1 j}^1, W_{c_1 j}^5 \rightarrow W_{c_1 j}^5]
\]
\[ f_1^{13} = T^{13} \sum_{n_1=1}^{4} [c_1^{13} m_{\chi_j}^0 \lambda_{m_1}^2 C_{n_1 j}^L + c_0^{13} \lambda_{m_1}^2 W_{n_1 j}^2 + c_1^{13} (m_{\chi_j}^0 \lambda_{m_1}^2 C_{n_1 j} + m_{\chi_i}^0 \lambda_{m_1}^2 C_{n_1 j})]
\]
\[ f_2^{13} = f_1^{13} (C_{n_1 j}^L \leftrightarrow C_{n_1 j}^R, \lambda_{m_1}^2 \leftrightarrow \lambda_{m_1}^3, W_{n_1 j}^2 \rightarrow W_{n_1 j}^1]
\]
\[ f_1^{14} = T_1^{14} \sum_{c_1=1}^{2} [c_1^{14} m_{\bar{\chi}_j} \gamma_{j c_1}^{2*} C_{ic_1}^R + c_0^{14} \mathcal{W}_{c_1}^5 C_{ic_1}^R + c_2^{14} (m_{\bar{\chi}_j} \gamma_{j c_1}^{2*} C_{ic_1}^R - m_{\bar{\chi}_j} \gamma_{c_1}^{2} C_{ic_1}^L)] \]

\[ f_2^{14} = f_1^{14} (C_{ic_1}^L \leftrightarrow C_{ic_1}^R, \gamma_{c_1 j}^{2*} \leftrightarrow \gamma_{j c_1}^{2*}, \mathcal{W}_{c_1 j}^6 \rightarrow \mathcal{W}_{c_1 j}^6) \]

\[ f_1^{15} = T_1^{15} \sum_{n_{11}=1}^{4} [c_1^{15} m_{\bar{\chi}_j} \mathcal{X}_{n_{11}}^{1} D_{n_{11}}^L + c_0^{15} \mathcal{X}_{n_{11}}^{1} \mathcal{W}_{n_{11}}^5 + c_2^{15} (m_{\bar{\chi}_j} \mathcal{X}_{n_{11}}^{1} D_{n_{11}}^L - m_{\bar{\chi}_j} \mathcal{X}_{n_{11}}^{1} D_{n_{11}}^R)] \]

\[ f_2^{15} = f_1^{15} (D_{n_{11}}^L \leftrightarrow D_{n_{11}}^R, \mathcal{X}_{n_{11}}^{1} \leftrightarrow \mathcal{X}_{n_{11}}^{1*}, \mathcal{W}_{n_{11}}^3 \rightarrow \mathcal{W}_{n_{11}}^3) \]

\[ f_1^{16} = T_1^{16} \sum_{c_{11}=1}^{2} [c_1^{16} m_{\bar{\chi}_j} \gamma_{j c_{11}}^{1*} D_{c_{11}}^L + c_0^{16} \gamma_{c_{11} j}^{1*} D_{c_{11}}^R + c_2^{16} (m_{\bar{\chi}_j} \gamma_{j c_{11}}^{1*} D_{c_{11}}^L - m_{\bar{\chi}_j} \gamma_{c_{11} j}^{1} D_{c_{11}}^R)] \]

\[ f_2^{16} = f_1^{16} (D_{c_{11}}^L \leftrightarrow D_{c_{11}}^R, \gamma_{c_{11} j}^{1*} \leftrightarrow \gamma_{j c_{11}}^{1*}, \mathcal{W}_{c_{11}}^5 \rightarrow \mathcal{W}_{c_{11}}^5) \]

\[ f_1^{17} = T_1^{17} \sum_{n_{11}=1}^{4} [c_1^{17} m_{\bar{\chi}_j} \mathcal{X}_{n_{11}}^{2} D_{n_{11}}^L + c_0^{17} \mathcal{X}_{n_{11}}^{2} \mathcal{W}_{n_{11}}^5 + c_2^{17} (m_{\bar{\chi}_j} \mathcal{X}_{n_{11}}^{2} D_{n_{11}}^L - m_{\bar{\chi}_j} \mathcal{X}_{n_{11}}^{2} D_{n_{11}}^R)] \]

\[ f_2^{17} = f_1^{17} (D_{n_{11}}^L \leftrightarrow D_{n_{11}}^R, \mathcal{X}_{n_{11}}^{2} \leftrightarrow \mathcal{X}_{n_{11}}^{2*}, \mathcal{W}_{n_{11}}^3 \rightarrow \mathcal{W}_{n_{11}}^3) \]

\[ f_1^{18} = T_1^{18} \sum_{c_{11}=1}^{2} [c_1^{18} m_{\bar{\chi}_j} \gamma_{j c_{11}}^{2*} D_{c_{11}}^L + c_0^{18} \gamma_{c_{11} j}^{2*} D_{c_{11}}^R + c_2^{18} (m_{\bar{\chi}_j} \gamma_{j c_{11}}^{2*} D_{c_{11}}^L - m_{\bar{\chi}_j} \gamma_{c_{11} j}^{2} D_{c_{11}}^R)] \]

\[ f_2^{18} = f_1^{18} (D_{c_{11}}^L \leftrightarrow D_{c_{11}}^R, \gamma_{c_{11} j}^{2*} \leftrightarrow \gamma_{j c_{11}}^{2*}, \mathcal{W}_{c_{11}}^6 \rightarrow \mathcal{W}_{c_{11}}^6) \]

\[ f_1^{19} = T_1^{19} \sum_{n_{11}=1}^{4} [c_1^{19} m_{\bar{\chi}_j} \mathcal{X}_{n_{11}}^{3} D_{n_{11}}^L + c_0^{19} \mathcal{X}_{n_{11}}^{3} \mathcal{W}_{n_{11}}^5 + c_2^{19} (m_{\bar{\chi}_j} \mathcal{X}_{n_{11}}^{3} D_{n_{11}}^L + m_{\bar{\chi}_j} \mathcal{X}_{n_{11}}^{3*} D_{n_{11}}^R)] \]

\[ f_2^{19} = -f_1^{19} (D_{n_{11}}^L \leftrightarrow D_{n_{11}}^R, \mathcal{X}_{n_{11}}^{3} \leftrightarrow \mathcal{X}_{n_{11}}^{3*}, \mathcal{W}_{n_{11}}^3 \rightarrow \mathcal{W}_{n_{11}}^3) \]

\[ f_1^{20} = T_1^{20} \sum_{c_{11}=1}^{2} [c_1^{20} m_{\bar{\chi}_j} \gamma_{j c_{11}}^{3*} D_{c_{11}}^L + c_0^{20} \gamma_{c_{11} j}^{3*} D_{c_{11}}^R + c_2^{20} (m_{\bar{\chi}_j} \gamma_{j c_{11}}^{3*} D_{c_{11}}^L + m_{\bar{\chi}_j} \gamma_{c_{11} j}^{3} D_{c_{11}}^R)] \]

\[ f_2^{20} = -f_1^{20} (D_{c_{11}}^L \leftrightarrow D_{c_{11}}^R, \gamma_{c_{11} j}^{3*} \leftrightarrow \gamma_{j c_{11}}^{3*}, \mathcal{W}_{c_{11}}^6 \rightarrow \mathcal{W}_{c_{11}}^6) \]
\[ f_{21} = T^{21} \sum_{g_1=1}^{3} \sum_{s_1=1}^{2} (c_0^{21} + c_1^{21} + c_2^{22}) m_{\chi} \gamma_{s_1} P^{21}_{g_1 s_1} \]

\[ f_{22} = -T^{21} \sum_{g_1=1}^{3} \sum_{s_1=1}^{2} c_2^{22} m_{\chi} \gamma_{s_1} P^{21}_{g_1 s_1} \]

\[ f_{22} = T^{22} \sum_{g_1=1}^{3} \sum_{s_1=1}^{2} [c_1^{22} m_{\chi} m_{e_{g_1}} U_{j_2} \phi_{g_1 s_1} + (c_0^{22} m_{e_{g_1}} \gamma_{g_1 s_1} + c_2^{22} m_{\chi} \gamma_{g_1 s_1}) \gamma_{g_1 s_1} V_1 c_{\beta} m_{W} ] \]

\[ f_{21}^{22} = T^{22} \sum_{g_1=1}^{3} \sum_{s_1=1}^{2} \frac{c_1^{22} m_{\chi} m_{e_{g_1}} U_{j_2} \phi_{g_1 s_1} \bar{\gamma}_{s_1} + (c_0^{22} m_{e_{g_1}} \gamma_{g_1 s_1} + c_2^{22} m_{\chi} \gamma_{g_1 s_1}) \gamma_{g_1 s_1} V_1 \sqrt{2} c_{\beta} m_{W} }{2} \]

\[ f_{22} = T^{22} \sum_{g_1=1}^{3} \sum_{s_1=1}^{2} [c_1^{22} m_{\chi} m_{e_{g_1}} U_{j_2} \phi_{g_1 s_1} \bar{\gamma}_{s_1} + (c_0^{22} m_{e_{g_1}} \gamma_{g_1 s_1} + c_2^{22} m_{\chi} \gamma_{g_1 s_1}) \gamma_{g_1 s_1} V_1 \sqrt{2} c_{\beta} m_{W} ] \]
\[ f_{2}^{26} = f_{1}^{26}(L \leftrightarrow R, \mathcal{W}_{n_{1}c_{1}}^{1} \leftrightarrow W_{n_{1}c_{1}}^{2}, \mathcal{X}_{j_{n_{1}}^{26}} \leftrightarrow \mathcal{Y}_{j_{n_{1}}^{26}}^{26}, \mathcal{X}_{c_{1}i}^{26} \leftrightarrow \mathcal{Y}_{j_{n_{1}}^{26+}}^{26}) \]

\[ f_{1}^{27} = T_{n_{1}^{27}}^{27} \sum_{n_{1}=1}^{4} [-b_{0}^{27} C_{n_{1}j}^{R} Z_{m_{1}j}^{25} + c_{1}^{27} m_{\chi_{j} L}^{j} C_{n_{1}j}^{L}(m_{\chi_{j}^{0}} Z_{m_{1}j}^{25} - m_{\chi_{j}^{0}} Z_{m_{1}j}^{25}) \]

\[ + c_{2}^{27}(m_{H}^{2} C_{n_{1}j}^{R} Z_{m_{1}j}^{25} - m_{\chi_{j}^{0}} Z_{m_{1}j}^{25} W_{m_{1}j}^{25}) \]

\[ + c_{0}^{27}(m_{\chi_{j}^{0}} Z_{m_{1}j}^{25} W_{m_{1}j}^{1} - m_{\chi_{j}^{0}} Z_{m_{1}j}^{25} W_{m_{1}j}^{2})] \]

\[ f_{2}^{27} = f_{2}^{27}(L \leftrightarrow R, Z_{m_{1}j}^{25} \leftrightarrow -Z_{m_{1}j}^{25}, W_{m_{1}j}^{1} \leftrightarrow W_{m_{1}j}^{2}) \]

\[ f_{1}^{28} = T_{c_{1}^{28}}^{28} \sum_{c_{1}=1}^{2} [(c_{0}^{28} m_{H}^{2} - c_{2}^{28} m_{H}^{2}) Y_{c_{1,j}}^{1} \mathcal{X}_{c_{1}i}^{26} + (c_{2}^{28} - c_{0}^{28})(m_{\chi_{j}^{0}} \mathcal{X}_{c_{1}i}^{26} W_{j_{c_{1}}^{5}}^{5}) \]

\[ - m_{\chi_{j}^{0}} Y_{c_{1,j}}^{5} + c_{2}^{28} m_{\chi_{j}^{0}} Y_{j_{c_{1}}^{1*}}^{1*}(m_{\chi_{j}^{0}} \mathcal{X}_{c_{1}i}^{26} + m_{\chi_{j}^{0}} Y_{j_{c_{1}}^{1}}^{1}) \]

\[ f_{2}^{28} = f_{1}^{28}(Y_{c_{1,j}}^{1} \leftrightarrow Y_{j_{c_{1}}^{1*}}^{1*}, \mathcal{X}_{c_{1}i}^{26} \leftrightarrow Y_{j_{c_{1}}^{2}}^{26}) \]

\[ f_{1}^{29} = f_{1}^{28}(h^{0} \rightarrow H^{0}, T_{c_{1}^{29}}^{29} \rightarrow T_{c_{1}^{29}}^{29}, b_{0}^{0} \rightarrow b_{0}^{0}, c_{0}^{28} \rightarrow c_{0}^{29}, c_{0}^{29} \rightarrow c_{0}^{29}, Y_{c_{1,j}}^{1} \rightarrow Y_{c_{1,j}}^{2}, Y_{c_{1,j}}^{1} \rightarrow Y_{c_{1,j}}^{2}) \]

\[ Y_{c_{1,j}}^{1*} \rightarrow Y_{c_{1,j}}^{2*}, W_{5}^{5} \rightarrow W_{5}^{6}) \]

\[ f_{2}^{29} = f_{2}^{28}(h^{0} \rightarrow H^{0}, T_{c_{1}^{29}}^{29} \rightarrow T_{c_{1}^{29}}^{29}, b_{0}^{0} \rightarrow b_{0}^{0}, c_{0}^{28} \rightarrow c_{0}^{29}, c_{0}^{29} \rightarrow c_{0}^{29}, Y_{c_{1,j}}^{1} \rightarrow Y_{c_{1,j}}^{2}, Y_{c_{1,j}}^{1*} \rightarrow Y_{c_{1,j}}^{2*}, W_{5}^{5} \rightarrow W_{5}^{6}) \]

\[ f_{1}^{30} = -f_{1}^{29}(H^{0} \rightarrow A^{0}, T_{c_{1}^{30}}^{30} \rightarrow T_{c_{1}^{30}}^{30}, b_{0}^{0} \rightarrow b_{0}^{0}, c_{0}^{29} \rightarrow c_{0}^{29}, Y_{c_{1,j}}^{1} \rightarrow Y_{c_{1,j}}^{2}, Y_{c_{1,j}}^{1*} \rightarrow -Y_{c_{1,j}}^{2*}, W_{5}^{5} \rightarrow W_{5}^{6}) \]

\[ f_{2}^{30} = -f_{2}^{29}(H^{0} \rightarrow A^{0}, T_{c_{1}^{30}}^{30} \rightarrow T_{c_{1}^{30}}^{30}, b_{0}^{0} \rightarrow b_{0}^{0}, c_{0}^{29} \rightarrow c_{0}^{29}, Y_{c_{1,j}}^{1} \rightarrow Y_{c_{1,j}}^{2}, Y_{c_{1,j}}^{1*} \rightarrow -Y_{c_{1,j}}^{2*}, W_{5}^{5} \rightarrow W_{5}^{6}) \]
\[
\mathcal{Y}_{c_{ij}}^{1*} \to -\mathcal{Y}_{c_{ij}}^{2*}, \quad \mathcal{W}^5 \to \mathcal{W}^6
\]

\[
f_{31}^1 = T^{31}[(c_3^{31} + 2c_6^{31} + 2c_1^{31})(m_H^2 - m_{\tilde{\chi}_j}^2 - m_{\chi_i}^2)C_{ij}^R + (b_0^{31} + 2c_2^{31}m_H^2)C_{ij}^R - 2c_1^{31}m_{\tilde{\chi}_j}m_{\chi_i}C_{ij}^L]
\]

\[
f_{31}^2 = f_{31}^1 (L \leftrightarrow R)
\]

\[
f_{31}^2 = T^{32} \sum_{c_1=1}^2 [b_0^{32} \mathcal{Y}_{c_{ij}}^{1*}C_{ic_1}^R + c_1^{32}(m_{\tilde{\chi}_j}^{c_{ij}}\chi_{c_{ij}}^R\mathcal{W}_{ic_1}^2 + (2m_H^2 - m_{\tilde{\chi}_j}^2 - m_{\chi_i}^2)\mathcal{Y}_{c_{ij}}^{1*}C_{ic_1}^R + m_{\tilde{\chi}_j}m_{\chi_i}^0\chi_{c_{ij}}^{25}C_{ic_1}^L]
\]

\[
+ c_0^{32}(2m_{\tilde{\chi}_j}^{c_{ij}}\chi_{c_{ij}}^R\mathcal{W}_{ic_1}^2 - 2m_{\chi_i}^0\chi_{c_{ij}}^R\mathcal{W}_{ic_1}^l + (2m_H^2 + m_{\chi_i}^2)\mathcal{Y}_{c_{ij}}^{1*}C_{ic_1}^R)
\]

\[
+ c_2^{32}(m_{\tilde{\chi}_j}^{c_{ij}}\chi_{c_{ij}}^R\mathcal{W}_{ic_1}^2 + 3m_H^2 - m_{\chi_i}^0\chi_{c_{ij}}^{25}C_{ic_1}^R - m_{\chi_i}^0\chi_{c_{ij}}^{25}\mathcal{W}_{ic_1}^l)
\]

\[
f_{32}^2 = f_{32}^1 (L \leftrightarrow R, \mathcal{W}_{ic_1}^l \leftrightarrow \mathcal{W}_{ic_1}^2, \chi_{c_{ij}}^{25} \leftrightarrow \chi_{c_{ij}}^{25})
\]

\[
f_{33}^1 = -f_{33}^1 (T^{32} \to T^{33}, b_0^{32} \to b_0^{33}, c_0^{32}(0,1,2) \to c_0^{33}(0,1,2), \mathcal{W}_{ic_1}^l \to \mathcal{W}_{ic_1}^{8*}, \mathcal{W}_{ic_1}^2 \to \mathcal{W}_{ic_1}^{8*}, \mathcal{Y}_{c_{ij}}^{1*} \to \mathcal{Y}_{c_{ij}}^{2*})
\]

\[
\mathcal{Y}_{c_{ij}}^{25} \to \mathcal{Y}_{j_{in_1}}^{26*}, \chi_{c_{ij}}^{25} \to \chi_{j_{in_1}}^{26*}, C_{ic_1}^L \to \chi_{c_{in_1}}^{1*}, C_{ic_1}^R \to \chi_{c_{in_1}}^{1*}
\]

\[
f_{32}^2 = f_{32}^1 (\chi_{c_{in_1}}^{1*} \leftrightarrow \chi_{c_{in_1}}^{1*}, \chi_{j_{in_1}}^{26*} \leftrightarrow \chi_{j_{in_1}}^{26*})
\]

\[
f_{33}^1 = f_{33}^1 (T^{33} \to T^{34}, b_0^{33} \to b_0^{34}, c_0^{33}(0,1,2) \to c_0^{33}(0,1,2), \chi_{c_{in_1}}^{1*} \to \chi_{c_{in_1}}^{1*}, \chi_{c_{in_1}}^{1*} \to \chi_{c_{in_1}}^{1*})
\]

\[
f_{32}^2 = f_{32}^1 (\chi_{c_{in_1}}^{1*} \leftrightarrow \chi_{c_{in_1}}^{1*}, \chi_{j_{in_1}}^{26*} \leftrightarrow \chi_{j_{in_1}}^{26*})
\]

\[
f_{33}^1 = f_{33}^1 (T^{34} \to T^{35}, b_0^{34} \to b_0^{35}, c_0^{34}(0,1,2) \to c_0^{35}(0,1,2), \chi_{c_{in_1}}^{3*} \to \chi_{c_{in_1}}^{3*}, \chi_{c_{in_1}}^{3*} \to \chi_{c_{in_1}}^{3*})
\]

\[
f_{32}^2 = f_{32}^1 (\chi_{c_{in_1}}^{3*} \leftrightarrow \chi_{c_{in_1}}^{3*}, \chi_{j_{in_1}}^{3*} \leftrightarrow \chi_{j_{in_1}}^{3*})
\]

\[
f_{33}^1 = f_{33}^1 (T^{35} \to T^{36}, b_0^{35} \to b_0^{36}, c_0^{35}(0,1,2) \to c_0^{36}(0,1,2), \chi_{c_{in_1}}^{2*} \to \chi_{c_{in_1}}^{2*}, \chi_{c_{in_1}}^{2*} \to \chi_{c_{in_1}}^{2*})
\]

\[
f_{32}^2 = f_{32}^1 (\chi_{c_{in_1}}^{2*} \leftrightarrow \chi_{c_{in_1}}^{2*}, \chi_{j_{in_1}}^{2*} \leftrightarrow \chi_{j_{in_1}}^{2*})
\]
Figure 2: The vertex one-loop Feynman diagrams for $H^- \rightarrow \tilde{\chi}^- \tilde{\chi}^0$. $F_1 F_2 S$: $\tilde{\chi}^0 \tilde{\chi}^- \phi^0, \tilde{\chi}^- \tilde{\chi}^0 \phi^-$, $f' f \bar{f} f$; $S_1 S_2 F$: $\phi_B^0 H^- \tilde{\chi}^-, \phi_T^0 G^- \tilde{\chi}^-, H^- \phi_B^0 \tilde{\chi}^0, G^- \phi_T^0 \tilde{\chi}^0, f' f \bar{f} f$; $F_1 F_2 V$: $\tilde{\chi}^- \tilde{\chi}^0 Z, \tilde{\chi}^0 \tilde{\chi}^- W$; $S V F$: $H^- Z \tilde{\chi}^0, \phi_T^0 W^- \tilde{\chi}^-; V S F$: $\gamma H^- \tilde{\chi}^-, Z H^- \tilde{\chi}^-, W^- \phi_T^0 \tilde{\chi}^0, \phi^0 = \{h^0, H^0, A^0, G^0\}, \phi_B^0 = \{h^0, H^0\}, \phi_T^0 = \{h^0, H^0, A^0\}$, $\phi^- = \{H^-, G^-\}$

Figure 3: The $\gamma \gamma$ and $\gamma Z$ self-energy Feynman diagrams. $S_1 S_2$: $H^- H^+, G^- G^+, \tilde{e} \bar{e}, \tilde{u} \bar{u}, \tilde{d} \bar{d}$; $S$: $\phi^-, \tilde{e}, \tilde{u}, \tilde{d}; F_1 F_2$: $e \bar{e}, u \bar{u}, d \bar{d}, \tilde{e} \bar{e} \tilde{e} \tilde{e}, \tilde{u} \bar{u} \tilde{u} \tilde{u}, \tilde{d} \bar{d} \tilde{d} \tilde{d}$; $V_1 V_2$: $W^- W^+; V$: $W^-; V S$: $W^- G^+, W^+ G^-$. 
Figure 4: The $H^-W^-$ self-energy Feynman diagrams. $S_1S_2$: $H^-\phi^0_B, G^-\phi^0_B, \tilde{e}\tilde{\nu}, \tilde{d}\tilde{u}$; $F_1F_2$: $e\bar{\nu}, \tilde{d}\tilde{u}, \tilde{\chi}^0, \tilde{\chi}^0; VS$: $W^-\phi^0_B$.

Figure 5: The $A^0Z$ self-energy Feynman diagrams. $S_1S_2$: $A^0\phi^0_B, G^0\phi^0_B, \tilde{e}\tilde{e}, \tilde{u}\tilde{u}, \tilde{d}\tilde{d}$; $F_1F_2$: $e\bar{\nu}, u\bar{u}, d\bar{d}, \tilde{\chi}^0, \tilde{\chi}^0, \tilde{\chi}^0, \tilde{\chi}^0; VS$: $Z\phi^0_B$.

Figure 6: The $H^-H^-$ self-energy Feynman diagrams. $S_1S_2$: $H^-\phi^0_B, G^-\phi^0_T, \tilde{e}\tilde{\nu}, \tilde{d}\tilde{u}$; $S$: $\phi^-, \phi^0, \tilde{c}, \tilde{\nu}, \tilde{u}, \tilde{d}$; $F_1F_2$: $e\bar{\nu}, d\tilde{u}, \tilde{\chi}^0, \tilde{\chi}^0; V$: $\gamma, Z, W^-; VS$: $\gamma H^-, ZH^-, W^-\phi^0_T$. 

Figure 7: The $\tilde{\chi}^-\tilde{\chi}^-$ self-energy Feynman diagrams. $SF$: $\phi^0\tilde{\chi}^-, \phi^-\tilde{\chi}^0$, $\tilde{\nu}e$, $\tilde{\nu}\bar{e}$, $\tilde{l}d$, $\tilde{d}u$; $VF$: $\gamma\tilde{\chi}^-, Z\tilde{\chi}^-, W^-\tilde{\chi}^0$.

Figure 8: The $\tilde{\chi}^0\tilde{\chi}^0$ self-energy Feynman diagrams. $SF$: $\phi^0\tilde{\chi}^0$, $\phi^-\tilde{\chi}^+$, $\phi^+\tilde{\chi}^-$, $\tilde{\nu}\bar{e}$, $\tilde{e}\bar{e}$, $\tilde{l}\tilde{l}$, $\tilde{l}\bar{l}$; $VF$: $Z\tilde{\chi}^0$, $W^-\tilde{\chi}^+$, $W^+\tilde{\chi}^-$.

Figure 9: The dependence of the decay width on the soft photon cut-off scale.
Figure 10: The decay width as the functions of $\tan \beta$.

Figure 11: The relative correction as the functions of $\tan \beta$. 
$\delta \Gamma / \Gamma_0 [%]$

Figure 12: The relative correction as the functions of $m_{H^-}$ for $\tan \beta = 4$.

$\delta \Gamma / \Gamma_0 [%]$

Figure 13: The relative correction as the functions of $m_{H^-}$ for $\tan \beta = 30$. 

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Figure 14: The relative correction as the functions of $m_{\tilde{\chi}_1}$ for $H^- \rightarrow \tilde{\chi}_1 \tilde{\chi}_1^0$.

Figure 15: The relative correction as the functions of $m_{\tilde{\chi}_1}$ for $H^- \rightarrow \tilde{\chi}_1 \tilde{\chi}_1^0$. 
Figure 16: The relative correction as the functions of $m_0$. 
Figure 17: The relative correction as the functions of $A_0$. 