A New Approach of evaluating the damage in simply-supported reinforced concrete beam by Local mean decomposition (LMD)

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Abstract. How to analyze the nonstationary response signals and obtain vibration characters is extremely important in the vibration-based structural diagnosis methods. In this work, we introduce a more reasonable time-frequency decomposition method termed local mean decomposition (LMD) to instead the widely-used empirical mode decomposition (EMD). By employing the LMD method, one can derive a group of component signals, each of which is more stationary, and then analyze the vibration state and make the assessment of structural damage of a construction or building. We illustrated the effectiveness of LMD by a synthetic data and an experimental data recorded in a simply-supported reinforced concrete beam. Then based on the decomposition results, an elementary method of damage diagnosis was proposed.

1. Introduction
The stability and safety of a construction or a building relies on the performance of its structures. Hence issues of diagnosing or evaluating the structural damage always arouse considerable interest among researchers in the field of civil or architectural Engineering. In recent decades many diagnosis methods have been developed, among which the vibration-based diagnosis methods play an important role. Essentially, the main idea of these methods is about motivating impulse or vibration on the objective structure and collecting the response signals. Then, how to analyze and utilize the received structural response data is the core part of these methods, and therefore a series of data processing methods and tools were introduced and widely used. The classic Fourier transform, and the 2D time-frequency transforms such as short-time Fourier transform [1] and wavelet transform [2] are some examples of possible mathematical tools applicable to the response vibration data.

In most cases, the response vibration signal is nonstationary, and the traditional tools mentioned above are more or less subjected to limitations [3]. Recently a more adaptive method termed empirical mode decomposition (EMD) was introduced and has been used to analyze these response signals [4-5]. It can adaptively decompose the vibration signal into a set of component signals (intrinsic mode functions, IMFs), each of which has a relatively stationary vibration mode and convenient to be analyzed or measured. Analogously in this study, we also introduce a new method in the field of signal processing — the local mean decomposition (LMD) [6] algorithm — to decompose the nonstationary response vibration adaptively and further do the assessment of structure damage detection.

In this abstract, we first briefly introduced the theory of LMD algorithm and illustrated its advantage with synthetic and experimental data respectively. And based on the decomposition results by LMD, we also proposed a preliminary method for assessing the structure damage.
2. Algorithm and theory

2.1. Local mean decomposition

In the EMD scheme, the input nonstationary vibration signal can be decomposed into a group of IMFs iteratively and these iteration procedures are vividly termed as ‘sifting’. In this study, we introduce the LMD algorithm (which was actually inspired by EMD), and decompose the vibration signal in concrete beam using the LMD decomposition scheme. There are two important differences between the above two schemes. The first difference is the way to extract or calculate the mean function and the envelope function within each iteration, which can describe the local attributes of signal. Given a vibration signal \( f(t) \), each two successive extrema can be denoted by \( f(t_i) \) and \( f(t_{i+1}) \). Then the local mean function \( m(t) \) and local envelope function \( a(t) \) in LMD can be defined as follows,

\[
m(t) = \begin{cases} \frac{f(t_i) + f(t_{i+1})}{2} & (t < t_i) \\ \frac{f(t_i) + 2f(t) + f(t_{i+1})}{4} & (t = t_i) \\ \frac{f(t) - f(t_{i+1})}{2} & (t_i < t < t_{i+1}) \\ \frac{f(t) - f(t_i)}{4} & (t = t_i) \end{cases}
\]

(1)

and we illustrate the comparison with that of EMD in Figure 1.

And the second obvious difference is the separating or sifting way within each iteration. In the iteration scheme of LMD, we attempt to derive a product signal of a pure frequency modulated signal and an envelope signal, which was an innovative idea. The iteration process of LMD can be described as follows,

\[
\begin{align*}
h_1(t) &= f(t) - m_1(t) \\
h_2(t) &= s_1(t) - m_2(t) \\
&\vdots \\
h_n(t) &= s_{n-1}(t) - m_n(t)
\end{align*}
\]

(3)

where

\[
\begin{align*}
s_1(t) &= h_1(t) / a_1(t) \\
s_2(t) &= h_2(t) / a_2(t) \\
&\vdots \\
s_n(t) &= h_n(t) / a_n(t)
\end{align*}
\]

(4)

When \( s_n(t) \) is close to an a pure frequency modulated signal, the first product function \( PF_1(t) \) can be represented as \( PF_1(t) = \prod_{i=1}^{n} a_i(t) s_i(t) \). Then we minus the derived \( PF_1(t) \) from original vibration signal and update the input signal by the residual instead \( f(t) = f(t) - PF_1(t) \). After deriving several PFs until the residual meeting the predefined error, the decomposition ends. Then the vibration signal can be represented by the decomposed PFs as

\[
f(t) = \sum_{i=1}^{n} PF_i(t) + \delta_i(t) \]

(5)

We also compare the iteration process shown above with that of EMD in a sketch (see Figure 2).
2.2. Synthetic data example
In this study, we test the LMD method with a synthetic signal (see Fig 3). The PF components generated by the LMD decomposition is shown in Figure 4, while the EMD decomposition results (IMFs) are shown in Figure 5. Note that in Figure 4, the green line for each PF is the envelope, which could be computed by $\int r(t) \, dt$ in equation 4. Obviously, the LMD method can derive sparser results. This will provide the convenience for identification and assessment of the degree of damage in the next step.

**Figure 1.** Comparison of the mean estimate (red line) and the envelope estimate (green line) in LMD (a) and EMD (b).

**Figure 2.** The sketch of the iteration process of the LMD and EMD method.

**Figure 3.** A time-varying synthetic vibration signal.
Figure 4. The PFs of the synthetic signal generated by the LMD decomposition.

Figure 5. The IMFs of the synthetic signal generated by the EMD decomposition.

2.3. Damage measurement based on the decomposition

We can use the $\ell_2$-norm to define the state of each decomposed IMF by EMD. So here we also employ the $\ell_2$-norm to describe the energy of each PF component by LMD, and denote the $k$th PF component by $E_{kE}$ as

$$E_{kE} = \sqrt{\int_0^t |PF_k|^2(t) dt} = \|PF_k\|_2.$$  

(6)

In practical calculation for the assessment of damage, we can record the response vibration data in the early-service period (e.g. the structure part just start serving or has just been constructed), and make regular diagnosis and record the response vibration signals in the decades-long service period. It should be noted that the same impulse power is needed. As to the recorded response signal of each service period, we could use the LMD method to decompose and compute the corresponding $E_{kE}$. Then we can compare each $E_{kE}$ with that of initial recorded vibration data. Then we can realize the state of the target structure of the building. Because that the decomposition by LMD is sparser than that of EMD [7], we can use less parameter to describe the degree of damage. And meanwhile the description by PFs using the LMD method could be more accurate, since in some literature it was pointed the decomposition by PFs using the LMD is more meaningful and precise [8].
3. Experimental data processing

In this abstract, we present an experiment of decomposing the vibration data in simply-supported reinforced concrete beam by LMD. The schematic of reinforced concrete beam is shown in Figure 6 and the detailed parameters are shown in Table 1. We excited an impact on one side of the concrete with hammer using same force each time, and collected the vibration on the other side with acceleration sensors. And then the concentrated load were pressed on the mid-span with increasing 10kN force of loading each time. And the response vibration signals in loading force state of 0kN (i.e. the initial state), 10kN, 20kN and 30kN are shown in Figure 7 (a) ~ (d) respectively.

As to the response vibration data in the preseted loading force state, i.e. 0kN, 10kN, 20kN and 30kN, we decompose these signals using the LMD and EMD method respectively. Here we show two of decomposition results (the initial state and in loading force state of 30kN) in Figure 9 and Figure 10.

| Experimental Parameter | Value       |
|------------------------|-------------|
| length                 | 1000mm      |
| net span               | 900mm       |
| section size           | 100mm×150mm |
| grade of concrete      | C20         |
| tension zone           | 2φ10        |
| compression zone       | 2φ6         |
| stirrup ratio          | φ6@100      |

Table 1. Experimental parameters of reinforced concrete beam

Figure 6. The schematic of reinforced concrete beam.

Figure 7. The response vibration data in loading force state of 10kN (b), 20kN(c), 30kN (d). The response vibration of the initial state is shown in (a).

Figure 8. The IMFs of the experimental data (in the initial state, see Figure 7a) decomposed by EMD. The residual signal is shown at the bottom.
and we also show a decomposition result by EMD in Figure 8. The LMD method can derive less components, which means that the decomposition is sparser.

**Figure 9.** The PFs of the vibration data (in the initial state, see Figure 7a) decomposed by LMD. The residual signal is shown at the bottom.

**Figure 10.** The PFs of the vibration data (in loading force state of 30kN, see Figure 7d) decomposed by LMD. The residual signal is shown at the bottom.

Then the $\ell_2$-norm for each component can be calculated by the equation 6, and in this study we simply compared the $E_{PF}^{PP}$ of the first five PFs ($k=1$–5) in the different loading force state by LMD decomposition (shown in Figure 11). And in Figure 11, there is a general decrease of energy with the decomposition process. And when there is a loading force on simply-supported reinforced concrete beam, the $E_{PF}^{PP}$ of the second vibration mode (2nd PF) has an obvious increase. It should be noted that during the decomposition in loading force state of 30kN, the energy of the fifth PF turns out to rise, which appears as a different trend. This may be viewed as a criterion of the existence of structure damage.

**Figure 11.** The derived $E_{PF}^{PP}$ of five PFs of response data in different loading force state by the LMD method.

4. Conclusions

In this study, we introduced a new approach for analyzing the nonstationary response vibration data in the construction structures, and the corresponding application for damage detection and diagnose is preliminarily discussed. Compared with the widely used EMD method, the LMD method can decompose the vibration signal more sparsely, and the derived components are more meaningful. One can process these decomposed components further, and assess the degree of damage of structures to be detected. We tested
the performance of the LMD method with a synthetic data and an experimental data recorded in a simply-supported reinforced concrete beam. And based the decomposition results, an elementary estimate of damage was presented. However, we recognized that the LMD method is still based on the empirical induction and the effectiveness of the LMD method would be based on some specific conditions, and thus all above-mentioned requires more detailed work in the following research.

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