Development of smoothed particle hydrodynamics (SPH) method to model the interaction of sand and water during liquefaction with bingham fluid model adaptation

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Abstract. Smoothed Particle Hydrodynamics (SPH) is a mesh-free numerical method that models the movement of particles in the Langrangian method. SPH method treats the domain as discrete particles instead of continuous entities, making it more accurate to model phenomenons that have a big deformation. The purpose of this research is to analyze the interaction between soil and water particles, based on the particle movements, density, pressure, internal forces, and external forces. This research continues the previous research of interaction between fluid-fluid particles in SPH by applying the Bingham fluid model to one type of fluid to represent coarse-grained soil particles. In the future, we hope this research can be further developed into a liquefaction prediction model by quantifying the main factors of liquefaction, such as the amplitude of shear strain, effective stress reduction, and excess pore water pressure.

1. Introduction
Smoothed Particle Hydrodynamics (SPH) is a mesh-free numerical method that models the movement of particles in the Langrangian method. At first, SPH is used in astrophysics [1] to model the movements of astronomical objects. Compared to the grid-based method, SPH is more accurate in modeling phenomenons with big deformation. This happens because the SPH method treats the domain as a discrete particle instead of continuous entities [2], resulting in the more accurate analysis. The base of the SPH method can be explained by its name: “smoothed”, because it approximates the particle properties by averaging the properties of the neighboring particles; “particle”, because it is based on a mesh-free particle theory; and “hydrodynamics” indicates that this method is used for hydrodynamics problems [2].

The purpose of this research is to analyze the interaction between soil and water particles, based on the particle movement, density, pressure, internal forces, and external forces. This research continues the former research [3] that models fluid-fluid particle behavior. Because the former program does not accommodate soil parameters such as the angle of internal friction and cohesion, we incorporated an equation that correlates the non-Newtonian Bingham fluid model with the Mohr-Coulomb failure envelope. By using the said equation, we can use the soil parameters as a direct input to the program, and the program will then treat the soil particles as a Bingham fluid.

However, a thing to be noted is this research has not yet quantified the main factors of liquefaction, such as the amplitude of shear strain, effective stress reduction, and excess pore water pressure. As stated before, this research only covers the analysis of particle movement and its properties. In the future, we hope this research can be further developed into a liquefaction prediction model by quantifying those said factors.
2. Theoretical review

2.1 The concept

The particle's movement can be obtained by calculating the forces applied to the particles. In this research, we categorize forces due to pressure and viscosity as the internal forces, and forces due to surface tension, gravity, and buoyancy as the external forces. Those forces combined would result in particle acceleration, which then can be double derived with respect to time into particle position. The particle's position can then be further analyzed by plotting them into a 2D axis.

The simulations are divided into two parts, which are sensitivity analysis and dimensional analysis. In the sensitivity analysis simulations, different values of initial viscosity, the volume of particles, surface tension, and shear strain rate are tested. The results are then examined to conclude which of the values from four said properties create the most stable model, based on the movements and average density of the particles. Those values then become the input for the dimensional analysis, where the size of the control volume is altered to test if the model stays stable or not.

In each simulation, the initial position of the particles is random, shown in Figure 1. The red dots represent sand particles, and blue dots represent water particles. According to the real-life behavior of saturated sand, the sand particles should settle to the bottom of the control volume, pushing the water particles upwards.

![Figure 1. The initial position of the particles](image)

2.2 SPH method

The kernel function is being used in SPH as the interpolation method, with the function of A(x) in the volume Ω such as:

\[
A(r) = \int_{\Omega} A(r') W(r - r', h) \, dr' \tag{1}
\]

with \(r\) as the particle position, \(W\) as the kernel function within the radius of \(h\). The said function can be written in a numerical form such as:

\[
A(r) = \sum_j A_j \frac{m_j}{\rho_j} W(r - r_j, h) \tag{2}
\]

The governing equation used for this method is the Navier-Stokes equation for incompressible and isothermal fluid, such as:
\[ \rho \frac{du}{dt} = -\nabla p + \mu \nabla^2 u + f \]  

(3)

with \( \rho \) as the density, \( \mathbf{u} \) as the vector for velocity, \( p \) as the pressure, \( \mu \) the viscosity, and \( f \) as the total force. With the total force consisting of external and internal forces, the equation then becomes:

\[ \rho \frac{du}{dt} = f_{\text{internal}} + f_{\text{external}} = F \]  

(4)

The internal forces consist of forces due to the pressure and viscosity, while the external forces consist of forces due to gravity, buoyancy, and surface tension [4]. The acceleration of particle \( i \) can be obtained from the following equation:

\[ a_i = \frac{du_i}{dt} = \frac{F_i}{\rho_i} \]  

(5)

Below are the equations used to obtain density, pressure, internal and external forces of particle \( i \):

\[ \rho_i = \sum_j m_j W(r - r_j, h) \]  

(6)

\[ P = \frac{200gH}{\rho \gamma} \left( \left( \frac{\rho}{\rho_0} \right)^\gamma - 1 \right); \gamma = 7 \]  

(7)

\[ f_{\text{pressure}} = -\sum_j p_j \frac{m_j}{\rho_j} \nabla W(r - r_j, h) \]  

(8)

\[ f_{\text{viscosity}} = \sum_j \frac{\mu_i + \mu_j}{2} \left( (\mathbf{u}_j - \mathbf{u}_i) \frac{m_j}{\rho_j} \nabla^2 W(r - r_j, h) \right) \]  

(9)

\[ f_{\text{gravity}} = \rho_i g \]  

(10)

\[ f_{\text{buoyancy}} = b(\rho_i - \rho_0) g \]  

(11)

\[ f_{\text{surface}} = -\sigma \nabla^2 n_i \left( \sum_j \frac{m_j}{\rho_j} W(r - r_j, h) \right) \]  

(12)

2.3 **Equivalent treatment for viscosity coefficient of bingham fluid**

The dynamic characteristics of hydrodynamics fluid can be explained with the equation below:

\[ \tau = \eta_0 \dot{\gamma}^n + \tau_y \]  

(1)

with \( \tau \) as the shear stress, \( \dot{\gamma} \) as the shear strain rate, and \( \eta_0, n, \) dan \( \tau_y \) as the material parameters that vary with the fluid type. With \( n = 1 \) and \( \tau_y > 0 \) to represent the motion of Bingham Fluid, the equation then becomes:

\[ \tau = \eta_0 \dot{\gamma} + \tau_y \]  

(2)

To incorporate the Bingham Fluid constitutive relationship into the SPH model, it is necessary to introduce the concept of equivalent viscosity, obtained from the following equation:

\[ \eta' = \eta_0 + \frac{\tau_y}{\dot{\gamma}} \]  

(3)
with $\eta'$ as the equivalent viscosity, $\tau$ as the shear stress, $\dot{\gamma}$ as the shear strain rate, $\eta_0$ as the initial viscosity, and $\tau_y$ as the yield shear stress. To avoid having an extremely large $\eta'$, the maximum value is defined by the following equation [5]:

$$
\eta' = \begin{cases} 
\eta_0 + \frac{\tau_y}{\dot{\gamma}} & \eta' < \eta_{\text{max}} \\
\eta_{\text{max}} & \eta' > \eta_{\text{max}} 
\end{cases}
$$

(4)

The Bingham fluid model stated above does not directly characterize material properties of soil, so the Mohr-Coulomb equation is incorporated:

$$
\tau_y = \sigma_n \tan \phi + c
$$

(5)

with $\tau_y$ as the shear stress of soil, $\sigma_n$ as the normal stress, $\phi$ as the angle of internal friction, and $c$ is cohesion. The substitution of Mohr-Coulomb into Eq. 15 then creates the following equation:

$$
\eta' = \eta_0 + \frac{\sigma_n \tan \phi + c}{\dot{\gamma}}
$$

(6)

3. Research method

As stated in the last section, this research covers both sensitivity analysis and dimensional analysis. We tested different values of initial viscosity, the volume of particles, surface tension, and shear strain rate in the sensitivity analysis simulation, and we try different sizes of control volume in the dimensional analysis. The simulation results can be depicted as an accurate representation of saturated sand behavior if:

1. The density of the sand is in the range of $1600 \text{ kg/m}^3 - 1800 \text{ kg/m}^3$;
2. The sand particles compact and moves to the bottom of the container;
3. The water particles surge upward.

There are input properties that stays the same and used in all of the simulations, stated below:

| Table 1. Material Properties |
|-----------------------------|
| **Soil**                    | **Water**          |
| Density                     | 1600              | 997             |
| Water content               | 15 %              |                 |
| Cohesion                    | 0 kg/cm$^2$       |                 |
| Internal friction angle     | 30 $^\circ$       |                 |

| Table 2. Numerical Properties |
|------------------------------|
| **Soil**                     | **Water**          |
| Gravitational acceleration   | 9.81               | 9.81            |
| Coefficient of restitution   | 1                  | 1               |
| Buoyancy                     | 0                  | 0               |
| Number of particles          | 1720               | 303             |
| Spacing between particles    | 6.25x10$^{-3}$     | 6.25x10$^{-3}$  |
| Number of particles inside a| 30                 | 30              |
| Size of control volume       | 0.1 x 0.1 x 0.1    | 0.1 x 0.1 x 0.1 |
| dt (s)                       | 0.002              | 0.002           |
| Volume of particles (m$^3$)  | 0.0005             | 0.0001          |
The soil and water properties are chosen based on the common properties of sand [6] and the experimental research on granular soil [7]. Since this research is using the same program as the fluid-fluid modeling done by Rachman, 2018, the numerical properties are assumed to be the same.

The coefficient of restitution is set to 1 to model perfectly elastic collision, and the buoyancy is set to 0 to not model the air particles. The value of properties defined above is then used as the input for the program. However, the previous research does not accommodate the adaptation of the Bingham fluid model, hence further modification is needed. Codes for calculating the equivalent viscosity (Eq. 18) are incorporated into the program.

4. Results

4.1 Sensitivity analysis

4.1.1 Variation of initial viscosity

For the first sensitivity analysis simulation, we vary the value of the initial viscosity. Below are the parameters used as the input, followed by the results.

Table 3. Parameters used for sensitivity analysis of initial viscosity

| Parameter                             | Soil       | Water      |
|---------------------------------------|------------|------------|
| Number of particles                   | 1720       | 303        |
| Spacing between particles (m)         | 6.25x10⁻³  | 6.25x10⁻³  |
| Number of particles inside a single kernel | 30         | 30         |
| Size of control volume (m³)           | 0.1 x 0.1 x 0.1 | 0.1 x 0.1 x 0.1 |
| dt (s)                                | 0.002      | 0.002      |
| Volume of particles (m³)              | 0.0005     | 0.0001     |
| Density (kg/m³)                       | 1600       | 997        |
| Initial Viscosity (Pa.s)              | 1; 5; 10   | 0.000891   |
| Surface tension (N/m)                 | 0          | 0          |

Table 4. Results for sensitivity analysis of initial viscosity

| Time       | η₀ = 1 Pa.s | η₀ = 5 Pa.s | η₀ = 10 Pa.s |
|------------|-------------|-------------|--------------|
| 0.002 s    | ![Image](image1.png) | ![Image](image2.png) | ![Image](image3.png) |
| 0.2 s      | ![Image](image4.png) | ![Image](image5.png) | ![Image](image6.png) |
| 0.6 s      | ![Image](image7.png) | ![Image](image8.png) | ![Image](image9.png) |
In the three simulations above, the sand particles settle to the bottom and the water particles surge upwards, creating a separate layer of both sand and water. However, it can be seen that by increasing the value of initial viscosity, the particles move slower. At one second into the simulation, only the simulation with \( \eta_0 = 1 \text{ Pa.s} \) shows stable particle movement.

The average density of the first simulation shows stability at 0.7 seconds, meanwhile, the two other simulations’ average density is still fluctuating until the end. The graph shows that the average density is around 1700 kg/m\(^3\), meaning that it accurately represents the actual density of saturated sand (1600 kg/m\(^3\) – 1800 kg/m\(^3\)).

4.1.2 Variation of particles volume

For the second sensitivity analysis simulation, we vary the particle volume. We choose three different volumes for both the soil and water particles. Below are the parameters used as the input, followed by the results.

| Parameter                              | Soil        | Water       |
|----------------------------------------|-------------|-------------|
| Number of particles                    | 1720        | 303         |
| Spacing between particles (m)          | \(6.25 \times 10^{-3}\) | \(6.25 \times 10^{-3}\) |
| Number of particles inside a single kernel | 30        | 30          |
| Size of control volume (m\(^3\))      | 0.1 x 0.1 x 0.1 | 0.1 x 0.1 x 0.1 |
| \(dt\) (s)                            | 0.002       | 0.002       |
|                                         | 0.0005;     | 0.0001;     |
| Volume of particles (m\(^3\))         | 0.000420;   | 0.000074;   |
|                                         | 0.005       | 0.001       |
| Density (kg/m\(^3\))                  | 1600        | 997         |
| Initial Viscosity (Pa.s)               | 1           | 0.000891    |
| Surface tension (N/m)                  | 0           | 0           |
Figure 3. The average density for sensitivity analysis of particles volume

Table 6. Results for sensitivity analysis of particles volume

| Time  | Soil | Water     | Soil | Water     | Soil | Water     |
|-------|------|-----------|------|-----------|------|-----------|
| 0.002 s | 0.0005 m³ | 0.0001 m³ | 0.000420 m³ | 0.000072 m³ | 0.005 m³ | 0.001 m³ |
| 0.2 s    | 0.0005 m³ | 0.0001 m³ | 0.000420 m³ | 0.000072 m³ | 0.005 m³ | 0.001 m³ |
| 0.6 s    | 0.0005 m³ | 0.0001 m³ | 0.000420 m³ | 0.000072 m³ | 0.005 m³ | 0.001 m³ |
| 1 s      | 0.0005 m³ | 0.0001 m³ | 0.000420 m³ | 0.000072 m³ | 0.005 m³ | 0.001 m³ |

Visually, the first and second simulation represents the sand and water interaction quite accurately. The difference is the surface of sand settled for around 7.5 – 10 cm in the second simulation. In the third simulation, the particles immediately exploded for around 500 times of the control volume size. This concludes that the volume of particles is highly sensitive to the results because the volume directly correlates to the internal forces.

Due to the extremely large value of average density obtained from the third simulation, the graph only shows the average density of the first and second simulation. The graph shows that the average
density is around 1750 kg/m³, meaning that it accurately represents the actual density of saturated sand (1600 kg/m³ – 1800 kg/m³).

4.1.3 Variation of surface tension

For the third sensitivity analysis simulation, we vary the value of surface tension. Below are the parameters used as the input, followed by the results.

Table 7. Parameters used for sensitivity analysis of surface tension

| Parameter                     | Soil  | Water |
|-------------------------------|-------|-------|
| Number of particles           | 1720  | 303   |
| Spacing between particles (m) | 6.25x10⁻³ | 6.25x10⁻³ |
| Number of particles inside a single kernel | 30 | 30 |
| Size of control volume (m³)   | 0.1 x 0.1 x 0.1 | 0.1 x 0.1 x 0.1 |
| dt (s)                        | 0.002 | 0.002 |
| Volume of particles (m³)      | 0.0005 | 0.0001 |
| Density (kg/m³)               | 1600  | 997   |
| Initial Viscosity (Pa.s)      | 1     | 0.000891 |
| Surface tension (N/m)         | 0; 0.0756; 1 | 0; 0.0756; 1 |

Table 8. Results for sensitivity analysis of surface tension

| Time  | σ = 0 N/m | σ = 0.0756 N/m | σ = 1 N/m |
|-------|-----------|---------------|----------|
| 0.002 s | ![Image](image1.png) | ![Image](image2.png) | ![Image](image3.png) |
| 0.2 s   | ![Image](image4.png) | ![Image](image5.png) | ![Image](image6.png) |
| 0.6 s   | ![Image](image7.png) | ![Image](image8.png) | ![Image](image9.png) |
| 1 s     | ![Image](image10.png) | ![Image](image11.png) | ![Image](image12.png) |
Figure 4. The average density for sensitivity analysis of surface tension

There is no significant difference between the first and second simulations. Both of them show the sand particles settling and water particles moving upwards. However, when the surface tension is increased to 1 N/m, the particles move sporadically and did not reach stability until the end of the simulation. The average density obtained is not much different than the variation of volume simulation, which is around 1750 kg/m³.

4.1.4 Variation of shear strain rate

For the last sensitivity analysis simulation, we vary the shear strain rate. Below are the parameters used as the input, followed by the results.

Table 9. Parameters used for sensitivity analysis of shear strain rate

| Parameter                              | Soil   | Water  |
|----------------------------------------|--------|--------|
| Number of particles                    | 1720   | 303    |
| Spacing between particles (m)          | 6.25x10⁻³ | 6.25x10⁻³ |
| Number of particles inside a single kernel | 30     | 30     |
| Size of control volume (m³)            | 0.1 x 0.1 x 0.1 | 0.1 x 0.1 x 0.1 |
| dt (s)                                 | 0.002  | 0.002  |
| Volume of particles (m³)               | 0.0005 | 0.0001 |
| Density (kg/m³)                        | 1600   | 997    |
| Initial Viscosity (Pa.s)               | 1      | 0.000891 |
| Surface tension (N/m)                  | 0      | 0      |
| Shear strain rate (s⁻¹)                | 0.7; 5; 10 | 0.7; 5; 10 |

Table 10. Results for sensitivity analysis of shear strain rate

| Time (s) | $\dot{\gamma} = 0.7$ s⁻¹ | $\dot{\gamma} = 0.5$ s⁻¹ | $\dot{\gamma} = 10$ s⁻¹ |
|----------|----------------------------|---------------------------|--------------------------|
Visually, by varying the shear strain rate, there is little to no difference in particle position. It can be concluded that the shear strain rate does not have a great effect on the acceleration of the particles. However, the average equivalent viscosity is affected by the shear strain rate. The graph shows that by increasing the value of the shear strain rate, the value of equivalent viscosity decreases.

### 4.2 Dimensional analysis

#### 4.2.1 0.1 x 0.1 x 0.1 m

Based on the sensitivity analysis, we chose the parameters which best represent the three success criteria. Below are the parameters and the results.

| Table 11. Parameters used for dimensional analysis (0.1 x 0.1 x 0.1 m) |
|---------------------------------------------------------------|
| Parameter                                      | Soil     | Water     |
| Number of particles               | 1720     | 303       |
| Spacing between particles (m)       | 6.25x10^{-3} | 6.25x10^{-3} |
| Number of particles inside a single kernel | 30       | 30        |
| Size of control volume (m³)         | 0.1 x 0.1 x 0.1 | 0.1 x 0.1 x 0.1 |
| dt (s)                           | 0.002    | 0.002     |
| Volume of particles (m³)           | 0.0005   | 0.0001    |
| Density (kg/m³)                    | 1600     | 997       |
| Initial Viscosity (Pa.s)           | 1        | 0.000891  |
| Surface tension (N/m)              | 0        | 0         |
| Shear strain rate (s⁻¹)            | 0.7      | 0.7       |
In the first few iterations, the sand particles move towards the bottom of the control volume and the water particles are pushed upwards. The water layer is visible at 0.04 s, and the acceleration of the particles decreased at 0.08 s. At 1 s, the particles have reached almost zero acceleration. The average density obtained from this simulation is around 1700 kg/m$^3$.

4.2.2 0.01 x 0.01 x 0.01 m

We decrease the size of the control volume by ten folds, using the parameters stated above.

**Table 13. Parameters used for dimensional analysis (0.01 x 0.01 x 0.01 m)**

| Parameter                          | Soil          | Water         |
|------------------------------------|---------------|---------------|
| Number of particles                | 1720          | 303           |
| Spacing between particles (m)      | 6.25x10$^{-4}$| 6.25x10$^{-4}$|
| Number of particles inside a single kernel | 30            | 30            |
| Size of control volume (m$^3$)     | 0.01 x 0.01 x 0.01 | 0.01 x 0.01 x 0.01 |
| dt (s)                             | 0.0002; 0.0003 | 0.0002; 0.0003 |
| Volume of particles (m$^3$)        | 4.2x10$^{-7}$  | 7.4x10$^{-8}$ |
| Density (kg/m$^3$)                 | 1600          | 997           |
| Initial Viscosity (Pa.s)           | 1             | 0.000891      |
| Surface tension (N/m)              | 0             | 0             |
| Shear strain rate (s$^{-1}$)       | 0.7           | 0.7           |
Table 14. Results of dimensional analysis (0.1 x 0.1 x 0.1 m)

| dt = 0.0002 | dt = 0.0003 |
|-------------|-------------|
| 0.0002 s    | 0.0003 s    |
| 0.02 s      | 0.03 s      |
| 0.04 s      | 0.06 s      |
| 0.1 s       | 0.15 s      |

Figure 7. Average density of dimensional analysis (0.01 x 0.01 x 0.01 m)

By using a timestep of 0.0002 s, the acceleration of the particles is close to zero. Until the end of the simulation, the sand and water particles have not created a separate layer. However, in the next
simulation where the timestep is 0.0003 s, the particles moved in a way that looks like floating around the control volume. Both of the simulations do not represent the liquefaction phenomenon.

4.2.3 1 x 1 x 1 m
We also tested the stable parameters in a control volume that is ten times bigger than the original size. Below are the parameters used, followed by the results.

Table 15. Parameters used for dimensional analysis (1 x 1 x 1 m)

| Parameter                                      | Soil   | Water  |
|------------------------------------------------|--------|--------|
| Number of particles                            | 1720   | 303    |
| Spacing between particles (m)                  | 6.25x10^{-2} | 6.25x10^{-1} |
| Number of particles inside a single kernel     | 30     | 30     |
| Size of control volume (m^3)                   | 1 x 1 x 1 | 1 x 1 x 1 |
| dt (s)                                         | 0.01   | 0.01   |
| Volume of particles (m^3)                      | 0.42   | 0.074  |
| Density (kg/m^3)                               | 1600   | 997    |
| Initial Viscosity (Pa.s)                       | 1      | 0.000891|
| Surface tension (N/m)                          | 0      | 0      |
| Shear strain rate (s^{-1})                     | 0.7    | 0.7    |

Table 16. Result of dimensional analysis (1 x 1 x 1 m)

Figure 8. The average density of dimensional analysis (1 x 1 x 1 m)
By increasing the size of the control volume to ten times its actual size, the sand and water start separating at the first second. However, until the end of the simulation, the water particles still floats around the control volume. At the 1.5 s into the simulation, the average density spikes up from 1400 kg/m$^3$ to 2400 kg/m$^3$. The average density is too high to be regarded as saturated sand. This simulation does not represent the liquefaction phenomenon.

5. Conclusion and Future Work

In the previous research that models the fluid-fluid interaction, the viscosity is a property that is carried by each particle which is obtained by calculating the average viscosity of two interacting particles. In this research, one of the fluid types is modeled as non-Newtonian Bingham fluid with their equivalent viscosity to accurately model sand when liquefaction happens. The equivalent viscosity equation is obtained by correlating the equation of the Bingham fluid and Mohr-Coulomb failure envelope. The results prove that adding codes to the program to accommodate the equivalent viscosity equation creates an accurate model of sand and water interaction during liquefaction.

Based on the density, pressure, viscosity, internal and external forces analysis, below are the parameters which create the result that best represents the phenomenon.

| Parameter                          | Soil   | Water            |
|------------------------------------|--------|------------------|
| Number of particles                | 1720   | 303              |
| Spacing between particles (m)      | 6.25x10$^{-3}$ | 6.25x10$^{-3}$ |
| Number of particles inside a single kernel | 30     | 30               |
| Size of control volume (m$^3$)     | 0.1 x 0.1 x 0.1 | 0.1 x 0.1 x 0.1 |
| dt (s)                             | 0.002  | 0.002            |
| Volume of particles (m$^3$)        | 0.0005 | 0.0001           |
| Density (kg/m$^3$)                 | 1600   | 997              |
| Initial Viscosity (Pa.s)           | 1      | 0.000891         |
| Surface tension (N/m)              | 0      | 0                |
| Shear strain rate (s$^{-1}$)       | 0.7    | 0.7              |

By applying the parameters mentioned above, the behavior of the particles represents liquefaction best. The sand settles to the bottom of the control volume and the water surges upwards, creating a separate layer above the sand. The water layer forms at 0.8 seconds into the simulation.

In the future, it is best to improve the void area near the container by using the ghost particle method. Further works also needed to be done to quantify the main factors of liquefaction, such as the amplitude of shear strain, effective stress reduction, and excess pore water pressure to create a better liquefaction prediction.

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