Using the developed web-application for solving problems of increased complexity in mathematics

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Abstract. A student is faced with mathematical problems of increased complexity in the process of learning mathematics, in preparation for the unified state exam in Russia. The article presents some elements of the analysis of the results of the unified state examination in mathematics of the profile level, which is passed by school graduates of the Russian Federation, on the example of one of the subjects. The article also describes a web application that is a graphical editor that allows you to work with functions and their graphs. This application can make it easier to solve high-complexity tasks that involve working with functions and their schedules. The work of this application is illustrated by the example of solving mathematical problems with parameters.

1. Introduction
Learning to solve problems of increased complexity that occur when passing the unified state exam in mathematics in the Russian Federation is an important problem of modern school mathematics education. For many years, the mathematics exam in Russian schools in the final 11 grade is mandatory. However, starting in 2015, this exam can be taken in a simplified basic and complicated profile options. The examination work of the profile level consists of two parts, which differ in content, complexity and number of tasks [1].

Tasks of increased complexity (as a rule, these are the last tasks of the proposed option) are solved by students of secondary schools quite rarely due to their complexity and lack of sufficient training [2].

This article will describe the developed web application and show its use as an example of solving problems with the parameter. Using this application makes it easier to solve problems, makes this solution more visual, which allows students to gradually move to an independent solution of these problems. Unlike well-known software products «Maple», «Mathcad», «MATLAB», this application is easy to use and does not require additional material costs. Using the application develops imaginative, visual and spatial thinking.

2. Analysis of the results of the unified state exam in mathematics
To illustrate the existing gaps in the knowledge of Russian schoolchildren, we present an analysis of the results of passing the unified state exam on the example of one of the regions of the Russian Federation – the Chuvash Republic (table 1) [3, 4].

Completing the tasks of part 1 of the examination paper (tasks 1-8) indicates the presence of General mathematical skills necessary for a person in modern society. Tasks in this part test basic computational and logical skills, the ability to analyze information presented on graphs and tables, use the simplest probabilistic and statistical models, and navigate the simplest geometric structures. Part one of the work includes tasks in all the main sections of the course of mathematics: geometry (planimetry and stereometry), algebra, the beginning of mathematical analysis, probability theory and statistics.
Table 1. Results of the unified state exam.

| The number of tasks | Check the skills                                                                 | The level of difficulty of the task | Percentage of students who completed the task |
|---------------------|----------------------------------------------------------------------------------|-----------------------------------|---------------------------------------------|
|                     |                                                                                  | Average, %                        | in the group that didn’t overcome the minimum score (27 points), % | in the group that scored 61-80 points, % | in the group that scored 81-100 points, % |
| 1                   | Be able to use the acquired knowledge and skills in practical activities and everyday life | Basic                             | 96.23                                       | 75.34                                    | 98.12                                       | 99.33                                       |
| 2                   | Be able to use the acquired knowledge and skills in practical activities and everyday life | Basic                             | 91.66                                       | 69.18                                    | 95.33                                       | 97.99                                       |
| 3                   | Be able to perform actions with geometric shapes, coordinates, and vectors       | Basic                             | 97.07                                       | 69.86                                    | 99.61                                       | 99.78                                       |
| 4                   | Be able to build and explore the simplest mathematical models                    | Basic                             | 96.97                                       | 73.97                                    | 99.29                                       | 99.33                                       |
| 5                   | Be able to solve equations and inequalities                                      | Basic                             | 94.20                                       | 43.15                                    | 98.77                                       | 99.33                                       |
| 6                   | Be able to perform actions with geometric shapes, coordinates, and vectors       | Basic                             | 79.82                                       | 10.96                                    | 92.47                                       | 99.11                                       |
| 7                   | Be able to perform actions with functions                                         | Basic                             | 72.53                                       | 10.27                                    | 88.32                                       | 97.32                                       |
| 8                   | Be able to perform actions with geometric shapes, coordinates, and vectors       | Basic                             | 74.64                                       | 10.27                                    | 93.38                                       | 99.11                                       |
| 9                   | Be able to perform calculations and transformations                               | Increased                        | 80.71                                       | 23.29                                    | 95.52                                       | 99.55                                       |
| 10                  | Be able to use the acquired knowledge and skills in practical activities and everyday life | increased | 84.48                                       | 10.27                                    | 97.01                                       | 99.33                                       |
| 11                  | Be able to build and explore the simplest mathematical models                    | Increased                        | 77.32                                       | 6.85                                     | 94.94                                       | 99.33                                       |
| 12                  | Be able to perform actions with functions                                         | Increased                        | 53.83                                       | 1.37                                     | 79.36                                       | 95.30                                       |
| 13                  | Be able to solve equations and inequalities                                      | Increased                        | 46.48                                       | 0.00                                     | 74.01                                       | 96.09                                       |
| 14                  | Be able to perform actions with geometric shapes, coordinates, and vectors       | Increased                        | 11.96                                       | 0.00                                     | 11.23                                       | 58.17                                       |
| 15                  | Be able to solve equations and inequalities                                      | Increased                        | 26.90                                       | 0.68                                     | 36.89                                       | 91.39                                       |
| 16                  | Be able to perform actions with geometric shapes, coordinates, and vectors       | Increased                        | 3.55                                        | 0.00                                     | 0.74                                        | 26.70                                       |
The number of tasks | Check the skills | The level of difficulty of the task | Percentage of students who completed the task |
|-----------------|-----------------|-------------------------------|---------------------------------|
|                 |                 | Average, % in the group that didn’t overcome the minimum score (27 points), % | in the group that scored 61-80 points, % | in the group that scored 81-100 points, % |
| 17              | Be able to use the acquired knowledge and skills in practical activities and everyday life | Increased | 21.95 | 0.00 | 24.62 | 94.41 |
| 18              | Be able to solve equations and inequalities | High | 2.73 | 0.00 | 0.86 | 19.46 |
| 19              | Be able to build and explore the simplest mathematical models | High | 6.81 | 0.00 | 7.24 | 26.40 |

In order to effectively select graduates to continue their education in higher education institutions with high requirements for the level of mathematical training of applicants [5,6], the tasks of the second part test knowledge at the level of requirements that are traditionally presented by universities with a profile exam in mathematics. The last two tasks of the second part are intended for competitive selection to universities with the highest requirements for mathematical training of applicants.

The system for evaluating tasks with a detailed response is based on the following principles:

1. There are different methods of the deployed solution and its records. The main requirement is that the solution must be mathematically literate, and the author’s reasoning must be clear from it. Otherwise (method, form of recording), the decision can be arbitrary. The completeness and validity of the arguments are evaluated regardless of the chosen solution method. At the same time, the graduate’s progress in solving the problem is evaluated, and not shortcomings in comparison with the "reference" solution.

2. It is possible to use mathematical facts without evidence and references in solving the problems contained in textbooks and manuals recommended for use in the implementation of state-accredited educational programs of secondary General education without evidence and references.

The last two tasks of the exam paper are positioned by the compilers as the most complex and focused on students with a high level of mathematical training. Analyzing the results, we can conclude that the problem with the parameter number 18 is one of the most problematic for schoolchildren. Accordingly, a relatively small number of students started to solve this problem.

For a deeper understanding of the condition and possible solution to the problem with the parameter, you need to visualize it, and track the behavior of the function with the parameter when it changes in dynamics. To do this, you must use software packages that allow you to work with parametrically defined functions.

3. Illustration of a web application

You can use MathLab, Mathcad, and Maple to visualize the solution of problems with parameters. These are specialized mathematical packages for professionals, but they are difficult for ordinary, "non-advanced" users to master. Among the many online services, Google Graph and WolframAlpha [7, 8] are popular and frequently used at the moment.

If we consider the principles of their operation, then in the case of Google Graph, everything is quite simple: we enter a function in the search engine, and the service provides a graph, while no interactivity is provided here. The app WolframAlpha does not have this drawback. WolframAlpha can also build graphs, but it has an interactivity that is not present in Google Graph. In order to use it, you need to purchase a paid premium version, which is problematic for the student or student.
Therefore, the task was set to create a web application that should be as simple as possible for the user, even without the experience and skills of using a computer, with an emphasis on cross-platform. This means that the program must work stably on any device and, at the same time, look good, adapting to different device screens.

Web pages were generated and user interaction with the interface was programmed. After analyzing the existing programming languages, their prospects for the future and the possibility of creating a web application focused on cross-platform, we decided on the JavaScript programming language.

The distinctive features of the created software product that provide a certain novelty and originality of development are as follows [9]:

1. The application works on almost all modern platforms that support the browser.
2. Created an adaptive interface, suitable for different screen sizes.
3. Easy and convenient to use.
4. Visual representation of function changes depending on the parameter using dynamic graphics.
5. Used modern web technologies.

Let’s demonstrate how the web application works using the example of the following task.

Example. Let’s find all the values of parameter $a$ for each of which the equation

$$\frac{x^2 + x + a}{x^2 - 2x + a^2 + 6a} = 0$$

has exactly two different roots.

The problem with a parameter (number 18) allows for a wide variety of solutions. The most common ones are:

- a purely algebraic solution;
- a solution method based on the construction and research of a geometric model of this problem;
- a functional method in which there can be both algebraic and geometric moments, but the basic one is the study of a certain function. Often the graphical method is more visual, but it is not applicable to every task [10, 11]. In addition, the specific text of the decision may well contain elements of each of the three methods listed.

In this example, let’s consider functions $y = x^2 + x + a$ whose graphs are the set of parabolas with vertices at points $\left( -\frac{1}{2}, a - \frac{1}{4} \right)$ and functions $y = x^2 - 2x + a^2 + 6a$ whose graphs are the set of parabolas with vertices at points $\left( 1, a^2 + 6a - 1 \right)$. If the system

$$\begin{align*}
y &= x^2 - 2x + a^2 + 6a, \\
y &= x^2 + x + a, \\
x^2 + x + a &= 0
\end{align*}$$

has a solution, then parabolas intersect and have at least one common point on the abscissus axis (common root).

For this task, the graphical method is the selection method. However, at the initial stages of preparation for solving such complex problems, it is quite difficult to visualize the graphs of the given function at different parameter values. This can be implemented using the computer program in question.

A clear illustration can be demonstrated using a developed web application that allows, unlike most similar programs, to consider several functions that depend on a parameter, as well as several mutually independent parameters, which, in general, goes beyond the complexity of exam tasks with a parameter. This application is also cross-platform oriented and implemented in the JavaScript programming language (Fig. 1 a, b). The program allows you to set the necessary functions of a single variable and plot them, dynamically changing the parameter value in the specified range with a specified step. You can stop, slow down and repeat dynamic changes to function graphs depending on the parameter. This allows you to specify the required intervals for the parameter according to the task condition. Figure 1
shows the program interface in dynamics. Based on the observed behavior of graphs, the possible value of the parameter is determined and its exact value is checked.

Using this application is also very useful for solving other mathematical problems that require plotting functions.

![Program interface](image1)

![Graph of a function](image2)

**Figure 1.** Program interface.

### 4. Conclusion

Thus, when preparing students to solve difficult tasks, it is necessary to take into account the following features:

1. It is important to choose the most rational solution method depending on the problem condition.
2. Solving the problem in several ways, if possible, allows you to assess the complexity of a particular solution and choose the optimal one. At the same time, it allows you to check the level of mastering mathematical skills.
3. Visualization using special programs, attracting Web resources helps the student in understanding the problem and determining the direction of its solution.

Using specialized programs to visualize the process of solving problems really facilitates the learning process.
References

[1] Yashchenko I V, Semenov A V and Chernyaeva M A 2020 Guidelines students by individual organization preparation for the unified state exam in 2020. Mathematics (profile) (Moscow: Federal Institute for pedagogical measurements) p 14

[2] Grinshpon Ya S and Podstrigich A G 2015 TSPU Bulliten 8(161) 48-52

[3] Mikishanina E A and Volodina E V 2019 Analysis of the results of the Unified state exam and the Main state exam in the Chuvash Republic in 2019: didactic and statistical aspects 85-97

[4] Mikishanina E A and Volodina E V 2019 Mathematical models and their applications 107-21

[5] Suharev L A and Kochugaev P N 2015 Integration of education 19 4 66-70

[6] Stepkina M A and Baigusheva I A 2016 Teacher of the 21st century 4 211-9

[7] Dane Cameron A 2013 Software Engineer Learns HTML5, JavaScript and jQuery (Cisdal publishing) p 256

[8] WolframAlpha: Computational Intelligence Retrieved from: http://www.wolframalpha.com

[9] Volodina E V, Kitaev A V and Yakovlev S V 2015 Mathematical models and their applications 17 58-62

[10] Volodina E V, Ilina I I and Timofeeva N N 2016 Bulletin of the Northern (Arctic) Federal University. Series: Natural Sciences 1 97-103

[11] Volodina E V and Ilina I I 2019 State and prospects of it education development. Collection of reports and scientific articles of the all-Russian scientific and practical conference pp 382-7