Modeling of real particles

Z C Liang
College of Electronic and Optical Engineering, Nanjing University of Posts and Telecommunications, Nanjing, 210046, China

E-mail: zcliang@njupt.edu.cn

Abstract. Real physics is a physical theory based on real particles. Real particles are elastic particles that have mass and volume, that can spin and deform. Real particles have three independent motion modes of translation, rotation and vibration. The three mode energies form a Cartesian energy space. The energy space has six phases and three zones. The three zones represent the states of gas, liquid and solid. Energy space is quantized by three quanta of translation, rotation and vibration. The ensemble statistics of real particles gives the partition function of objects, and establishes the thermodynamic relations and equations of real physics. Particles in real space form fields, and a complete set of particle field equations is derived through vector calculus. The particle field theory reveals the origin of gravitation and electromagnetic forces, and realizes the unification of interactions. This article introduces the theoretical framework of real physics and shows the main achievements of its application. The emphasis is to expound the thoughts and approaches of physical modeling, and to give consistent explanation to the fundamental problems of physics.

1. Introduction
What are particles? This is the fundamental problem of physics. Classical physics is a magnificent building based on the ideal model of point particles. Point particle is a geometric concept with discrete feature, and its typical representatives are point mass and point charge [1]. Quantum theory is a pillar of modern physics, and its core assumption is the model of wave particles. Wave particles are figment with both wave and particle features [2]. Quantum field theory regards particles as the excited states of fields, and its typical representative is the photon with neither mass nor charge [3].

A common feature of point particles and wave particles is that they allow different particles to intersect or penetrate each other in space. The penetration of particles will lead to density singularity. Common sense tells us that physical singularity does not exist in reality. Real physics must eliminate spatial and temporal singularities in principle. Therefore, physics should look for particle models other than points and waves.

Another feature of point and wave particles is that they do not contain interactions themselves. In classical physics, point particles are granted with mass or charge to characterize the gravitational or electromagnetic interaction. In quantum mechanics, the interaction of wave particles is provided by potential energy. Granting mass, charge and potential to particles is a priori hypothesis, and its validity is verified only by experiments. Unfortunately, the origin of gravitational and electromagnetic forces eventually became a tough problem in physics. In order to achieve the unification of physics, we should look for a particle model self-containing fundamental interactions.

1 To whom correspondence should be addressed.
Common sense tells us that particles in reality are three-dimensional objects with mass, volume and elasticity. Point is a geometric concept that has no physical realistic content. There are good reasons to believe that the point and wave particles are the approximation of elastic particles. Four years ago, the author put forward an elastic particle model and established a real physical theory [4]. Elastic particles, or body particles [5,6,7], entity particles[8], are three-dimensional object with mass, volume and elasticity. Volume repulsion is a fundamental interaction between elastic particles. The real physics is essentially the statistical theory of elastic particles. The author has developed a particle field theory based on mass and momentum statistics [7,8], an object states theory based on energy statistics [5], and a thermodynamic theory based on ensemble statistics [6]. This article introduces the theoretical framework and the main achievements of real physics. The focus is to expound the thoughts and approaches of physical modeling, and to give consistent explanation to the fundamental problems of physics.

2. Theoretical framework

2.1. Physical modeling principles

2.1.1. Reality. Physical models must be based on authenticity. Mathematical theories are abstract symbolic logic systems whose content can be unrelated to the real world. Physical theories are the mappings of real world and must be based on authenticity. Objects in reality are entities with mass, volume and elasticity. According to the principle of reality, mass, volume and elasticity are indispensable attributes for real particles. Point or wave particles are abstractive models that lose the essential feature of real particles.

2.1.2. Simplicity. Physical model must be simple. Simplicity is a belief in nature, is the foundation for physical unity. Point particle is a simple model lacking authenticity, while wave particle is a complex model lacking simplicity. Real particles are defined as three-dimensional objects with only mass, volume and elasticity, which has both authenticity and simplicity.

2.1.3. Sufficiency. The physical model should contain enough elements to derive the object property in completeness. Ideal models are not sufficient. Physics uses mathematics for logical reasoning. Mathematics cannot deduce what is not included in the physical model. Point and wave particles lack sufficiency and need additional assumptions (e.g. various charges) to describe interactions. Real particles possess mass, volume and elasticity, which contain the interaction of volume repulsion. The Newton gravity theory assumes the vacuum between celestial bodies, so it is impossible to find the dark matter within the classical physics. To understand the nature of dark matter, the vacuum hypothesis must be abandoned. The real space model assumes that physical space is full of elastic particles, which is the right way to reveal the nature of gravity and dark matter.

2.1.4. Consistency. Self consistency and mutual consistency are the basic requirements for physical theories. The self consistency is guaranteed by the tight of mathematical tools, and the mutual consistency is guaranteed by the simplicity and sufficiency of the physical models. Both theories of point and wave particles might be self consistent, but classical physics and modern physics are inconsistent with each other. The elastic physical model is simple and sufficient. Establishing a mathematical model based on elastic particle is the guarantee for the consistency of real physics.

2.1.5. Objectivity. Natural laws are independent of human consciousness. Real physics must exclude subjective factors and be expressed by the mathematical structure of pure objectivity. The subjective factors hiding in current physics mainly come from the process of measurement. Compared with classical physics, modern physics blurs the boundary between objective and subjective, resulting in
confusion between existence and consciousness. Establishing a mathematical model independent of measurement process is the key to the success of real physics.

2.2. Physical models
The real physics is different from both classical physics and modern physics. The following concepts, axioms and theorems constitute the basic model of real physics [5-8].

2.2.1. Real particle model. (1) Real particle. Real particles are elastic particles that have mass and volume, that can spin and deform. Such particle is also termed body particle in contrast to the point particle[5,6,7]. (2) Object structure (Axiom 1). Any object is composed of elastic particles with nesting structure. (3) Primary particle: A primary particle is indivisible elastic particle that has constant mass. (4) Primary particle (Axiom 2). There are only two types of primary particles of different mass: proton and electron. (5) Mass conservation theorem. Primary particles cannot be destroyed. The masses of proton and electron are conserved.

2.2.2. Real space and real time. (1) Real space (Axiom 3). Space is the room in which the objects exist and move. Real space is continuous, uniform, three-dimensional, and full of elastic particles. (2) Real time (Axiom 4). Time is a progression with which the objects exist and move. Real time is continuous, uniform and unidirectional, which follows the law of causality. (3) Motion. Motion is a process that the state of object changes with time in space. (4) Volume. The object volume is the space required for the motion of its internal particles. (5) Volume repulsion theorem. Elastic particle has invariable mass but variable volume. The volumes of different particles do not intersect in real space at the same time.

2.2.3. Objectivity principle. (1) Measurement. Physical measurement or observation is a subjective process. The choice of reference point (origin) is subjective, and the choice of metrics (scales) is subjective. (2) Objectivity axiom (Axiom 5). Physics is realistic and physical laws are objective. Real physics must be independent of the measurement. It requires that physical formulations are origin irrelevance and scale irrelevance. (4) Real physics. Real physics is a systematic theory based on real particle, real space, real time and objectivity principle.

2.3. Mathematical models

2.3.1. Real quantity. All physical quantities are real quantities defined in the real number domain. A real quantity \( \mathbf{x} \) can be expressed in the real form
\[
\mathbf{x} = x_s \cdot \mathbf{\bar{x}}; \quad x_s > 0.
\]  
(1)
The factor \( x_s \) is termed scale and factor \( \mathbf{\bar{x}} \) is termed digit. The scale is a scalar. The digit can be scalar, vector or tensor, depending on the real quantity. The scale is the identifier and the metric of physical quantity. As metrics, scales are positive real variables and play the role of quantum. The physical quantity \( \mathbf{x} \) is objective and absolute quantity, the scale \( x_s \) is subjective factor, and the digit \( \mathbf{\bar{x}} \) is relative factor. The definition of real quantity clearly distinguishes objective and subjective factors in real physics.

2.3.2. Real time. The real time is defined as
\[
t = t_s \cdot \mathbf{T}; \quad t_s > 0, \quad \mathbf{T} = 0,1,2,\ldots,k,k + 1,\ldots
\]  
(2)
where \( t_s \) is the scale of time (time quantum), and \( \mathbf{T} \) is natural numbers. \( \mathbf{T} = 0 \) is starting time (time origin), \( \mathbf{T} = k \) is any time, and \( \mathbf{T} = k + 1 \) is next time. Then, any real quantity \( \mathbf{x} \) can be expressed by a set of discrete time sequence
\[
\mathbf{x}(\mathbf{T}) = \langle \mathbf{x}(\mathbf{T}) | \mathbf{T} \in \mathbf{N} \rangle,
\]  
(3)
where \( \mathbf{N} \) is the set of natural numbers, and \( \langle \cdot \rangle \) represents an ordered time sequence. The real time is unidirectional and does not have inverse symmetry. The law of causality is explicitly included in the definition of time.
2.3.3. Real space. Let $O$-XYZ be a laboratory reference frame (laboratory coordinates), where $O$ is an arbitrary spatial reference point (space origin). Let $P$ be any point in space, then the location of $P$ can be expressed by a position vector
\[ r = r_s \cdot \mathbf{r} = \overrightarrow{OP} = (x, y, z), \] (4)
where $r_s$ is the scale of length (space quantum). The length of the position vector is
\[ r = |r| = |\overrightarrow{OP}| = \sqrt{x^2 + y^2 + z^2}. \] (5)
Both the position vector and the length of position vector depend on the space origin.

2.3.4. Synchronization. Synchronization is a protocol to set a starting time for all space positions. A signal $t = 0$ is sent from the space origin $O$ with a communication speed $c$. When the observer at $P(r)$ receives the signal, then the time is set to
\[ t_0 = r/c = (r_s/c) \cdot \xi_0 = t_s \cdot \xi_0; \quad t_s = r_s/c, \xi_0 = \mathbf{r}. \] (6)
After synchronization, the spatial position at any time can be expressed by $P(r, t)$, where $r$ and $t$ are mutually independent parameters.

2.3.5. Movement. Let $P_i$ be any particle with the position vector $\mathbf{r}_i = \overrightarrow{OP}_i = (x_i, y_i, z_i)$, then the movement of $P_i$ can be expressed by a time sequence of position vectors
\[ \mathbf{r}_i(t) = (\mathbf{r}_i(t)) \mid t \in \mathbb{N}. \] (7)
If an object is comprised of $N$-particles, the movement of the object can be expressed by
\[ \text{Movement} = \{\mathbf{r}_i(t) \mid i = 1, 2, 3, \ldots, N\}. \] (8)

2.3.6. Displacement. The displacement of a particle is defined at any time as
\[ \Delta \mathbf{r}_i(k) = \mathbf{r}_i(k + 1) - \mathbf{r}_i(k) = r_s \cdot \Delta \mathbf{R}_i(k); \quad \Delta \mathbf{R}_i(k) = \mathbf{R}_i(k + 1) - \mathbf{R}_i(k). \] (9)
The displacement is origin independent because
\[ \Delta \mathbf{r}_i(k) = \overrightarrow{OP}_i(k + 1) - \overrightarrow{OP}_i(k) = \overrightarrow{P_i(k + 1)} - \overrightarrow{P_i(k)} = \Delta \mathbf{R}_i(k). \] (10)

2.3.7. Volume repulsion. The spacing between two particles at any time is origin independent, because
\[ \mathbf{r}_{ij}(k) = \mathbf{r}_j(k) - \mathbf{r}_i(k) = \overrightarrow{OP}_j - \overrightarrow{OP}_i = \overrightarrow{P_jP_i}, \quad i \neq j. \] (11)
The length of spacing is expressed by
\[ r_{ij} = |\overrightarrow{P_jP_i}| = \sqrt{\mathbf{r}_{ij} \cdot \mathbf{r}_{ij}} > 0. \] (12)
The constraint $r_{ij} > 0$ is a mathematical expression for the theorem of volume repulsion.

2.3.8. Origin irrelevance. To make the formulation independent of the space origin, the position vectors cannot directly appear in the physical equations. To make the formulation independent of the time origin, the physical quantities should be defined at any time.

2.3.9. Scale irrelevance. The physical formulation should be independent of the scales (subjective factors). Therefore, the physical equation and the digital equation must be equivalent, namely
\[ z = R(x, y) = z_s \cdot \tilde{z} ; \quad \tilde{z} = R(\tilde{x}, \tilde{y}). \] (13)

2.4. Operation rules
Scale irrelevance requires that physical quantities satisfy the following operation rules.

(1) Addition and subtraction.
\[ z = x \pm y = x_s \cdot \tilde{x} \pm y_s \cdot \tilde{y} = x_s \cdot (\tilde{x} \pm \tilde{y}) = z_s \cdot \tilde{z}; \] (14)
\[ z_s = x_s = y_s, \quad \tilde{z} = \tilde{x} \pm \tilde{y}. \]

(2) Multiplication.
\[ z = x \cdot y = (x_s \cdot \tilde{x}) \cdot (y_s \cdot \tilde{y}) = (x_s \cdot y_s) \cdot (\tilde{x} \cdot \tilde{y}) = z_s \cdot \tilde{z}; \] (15)
\[ z_s = x_s \cdot y_s, \quad \tilde{z} = \tilde{x} \cdot \tilde{y}. \]

(3) Division.
\[
z = \frac{y}{x} = \frac{y_s \cdot \tilde{y}}{x_s \cdot \tilde{x}} = \frac{y_s}{x_s} \cdot \frac{\tilde{y}}{\tilde{x}} = z_s \cdot \tilde{z}; \quad z_s = \frac{y_s}{x_s}, \quad \tilde{z} = \frac{\tilde{y}}{\tilde{x}}.
\]

(4) Difference.
\[
\Delta x_k = x_{k+1} - x_k = x_s \cdot (\tilde{x}_{k+1} - \tilde{x}_k) = x_s \cdot \Delta \tilde{x}_k; \quad \Delta \tilde{x}_k = \tilde{x}_{k+1} - \tilde{x}_k.
\]

(5) Integral.
\[
z(n) = \sum_{k=1}^{n} \left[ y(x_k) \cdot \Delta x_k \right]_{\Delta x_k = x_s} = \left( y_s \cdot x_s \right) \cdot \sum_{k=1}^{n} \tilde{y}(x_k) = z_s \cdot \tilde{z}; \quad z_s = y_s \cdot x_s, \quad \tilde{z}(n) = \sum_{k=1}^{n} \tilde{y}(x_k).
\]

(6) Difference quotient.
\[
\frac{dy}{dx} = \frac{(\Delta y)}{(\Delta x)}_{\Delta x=1} = \frac{y_s}{x_s} \cdot \frac{(\Delta \tilde{y})}{(\Delta \tilde{x})}_{\Delta \tilde{x}=1} = \frac{y_s}{x_s} \cdot \Delta \tilde{y}.
\]

(7) Differential quotient.
\[
\frac{dy}{dx} = \frac{(\Delta y)}{(\Delta x)}_{\Delta x=0} = \frac{y_s}{x_s} \cdot \frac{(\Delta \tilde{y})}{(\Delta \tilde{x})}_{\Delta \tilde{x}=0} = \frac{y_s}{x_s} \cdot \frac{d\tilde{y}}{d\tilde{x}}.
\]

(8) Others. The exponential, logarithmic and trigonometric operations demand \( x_s = 1 \).
\[
e^x = e^{ex} = (e^x)^x = e^x, \\
\ln x = \ln(x_s \cdot \tilde{x}) = \ln x_s + \ln \tilde{x} = \ln x, \\
sin x = \sin(x_s \cdot \tilde{x}) = \sin \tilde{x}.
\]

Please note that the notation of difference quotient borrows from the differential quotient in current literatures. The difference quotient has no limit operation of the differential quotient. The mathematical structure based on real physical model is called real mathematics. We emphasized that real mathematics encompasses the basic conception of modern physics. The scale of real quantity is actually the quantum, and the objectivity principle is actually the principle of relativity.

2.5. Object structure

According to the physical model, any object can be expressed as a set of primary particles (electron \( \mathbb{E} \) and proton \( \mathbb{P} \)) [5]. For example, a neutron is the set of electron and proton, \( \mathbb{N} = \{ \mathbb{P}, \mathbb{E} \} \). Atoms can be expressed as
\[
\text{Atom}(a, b, c) = \left\{ \mathbb{E}^a, \{ \mathbb{P}^b, \mathbb{N}^c \} \right\},
\]

where \( a \) is the number of electrons outside the atomic nucleus \( \{ \mathbb{P}^b, \mathbb{N}^c \} \). \( b \) is the number of protons and \( c \) is the number of neutrons. Electrons, protons, neutrons, atoms and molecules are all elastic particles.

The object is a set of elastic particles, and can be expressed as
\[
\text{Object} = \{ P_i \mid i = 1, 2, 3, \ldots, N \}.
\]

There \( P_i \) is base-particle, \( N \) is the number of base-particles. Object has a spatial nested structure that consists of particles at different levels. Nesting means that one level particle comprises all particles of the next levels. The object structure can be expressed by a general model as [5]
\[
\text{Top particle} \supseteq \text{Meso particle} \supseteq \text{Base particle} \supseteq \text{Sub particle}.
\]

The top-particle is the object under study and the base-particle is the basic statistical unit. Top-particle, meso-particle, base-particle and sub-particle are all relative concepts. For example, water is the top-particle when studying the property of water. If the water molecule \( \text{H}_2\text{O} \) is regarded as base-particles, then the atoms of hydrogen and oxygen are sub-particles, and the molecular clusters are meso-particles.

2.6. Particle states

An elastic particle has three spatial states: position, posture and profile. The position indicates the spatial location, the posture indicates the spatial orientation, and the profile indicates the outline of the
The translation energy is the and the change of The change of position and profile of 2.7 corresponding to the eigenvalues are where is a three.

\[ \mathbf{r}_c = \mathbf{OP}_c = (x_c, y_c, z_c) = \sum_{j=1}^{N_0} \left( \frac{m_j}{M} \right) \mathbf{r}_j, \quad M = \sum_{j=1}^{N_0} M_j. \]

\( \mathbf{r}_c \) is a three-dimensional vector. A reference frame taking \( \mathbf{P}_c \) as the origin is called center-of-mass coordinates.

The posture and profile of the base-particle are described by an inertia matrix. The inertia matrix of base-particle in the laboratory coordinates is

\[ I = \begin{pmatrix} I_{11} & -I_{12} & -I_{13} \\ -I_{12} & I_{22} & -I_{23} \\ -I_{13} & -I_{23} & I_{33} \end{pmatrix}, \quad I_s = M_s r_s^2, \]

where \( I_s \) is the inertia quantum and \( M_s \) is the mass quantum. The elements of inertia matrix are [1]

\[ I_{11} = \sum_{j=1}^{N_0} M_j (y_j^2 + z_j^2), \quad I_{22} = \sum_{j=1}^{N_0} M_j (x_j^2 + z_j^2), \quad I_{33} = \sum_{j=1}^{N_0} M_j (x_j^2 + y_j^2), \]

\[ I_{12} = I_{21} = \sum_{j=1}^{N_0} M_j x_j y_j, \quad I_{13} = I_{31} = \sum_{j=1}^{N_0} M_j x_j z_j, \quad I_{23} = I_{32} = \sum_{j=1}^{N_0} M_j y_j z_j. \]

The inertia matrix is a real symmetric matrix. It has three real eigenvalues \( I_{1}, I_{2}, I_{3} \). The eigenvectors corresponding to the eigenvalues are \( \mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3 \). The profile of base-particle is represented by eigenvalues \( I_c = (I_{1}, I_{2}, I_{3}) \) of the inertia matrix. The posture of the base-particle is represented by the angles \( \theta_c = (\theta_1, \theta_2, \theta_3) \) between the eigenvectors and the axes of laboratory coordinates.

2.7. Motion and energy

Motion is a phenomenon that the spatial state of particles changes with time. Let the position, posture and profile of a base-particle \( P_l \) at any time \((\hat{t} = k)\) be \( \mathbf{r}_l = (x_l, y_l, z_l), \mathbf{\theta}_l = (\theta_{1l}, \theta_{2l}, \theta_{3l}) \) and \( I_l = (I_{1l}, I_{2l}, I_{3l}) \). The corresponding position, posture and profile of \( P_l \) at next time \((\hat{t} = k + 1)\) be \( \mathbf{r}'_l = (x'_l, y'_l, z'_l), \mathbf{\theta}'_l = (\theta'_{1l}, \theta'_{2l}, \theta'_{3l}) \) and \( I'_l = (I'_{1l}, I'_{2l}, I'_{3l}) \). Then the changes of spatial states are

\[ \Delta \mathbf{r}_l(k) = \mathbf{r}'_l(k + 1) - \mathbf{r}_l(k) = (\Delta x_l, \Delta y_l, \Delta z_l), \]

\[ \Delta \mathbf{\theta}_l(k) = \mathbf{\theta}'_l(k + 1) - \mathbf{\theta}_l(k) = (\Delta \theta_{1l}, \Delta \theta_{2l}, \Delta \theta_{3l}), \]

\[ \Delta I_l(k) = I'_l(k + 1) - I_l(k) = (\Delta I_{1l}, \Delta I_{2l}, \Delta I_{3l}). \]

The change of position \( \Delta \mathbf{r}_l(k) \) is called translation, the change of posture \( \Delta \mathbf{\theta}_l(k) \) is called rotation, and the change of profile \( \Delta I_l(k) \) is called vibration as it is caused by profile deformation. Obviously, the translation, rotation and vibration are origin irrelevant [5].

The translation velocity of the particle at any time is defined as

\[ \mathbf{u}_l(k) = \frac{\Delta \mathbf{r}_l(k)}{t_s} = \left( \frac{r_{x}}{t_s}, \frac{r_{y}}{t_s}, \frac{r_{z}}{t_s} \right) \cdot \Delta \mathbf{\hat{r}}_l(k) = u_s \cdot \mathbf{\hat{u}}_l(k) = (u_{1l}, u_{2l}, u_{3l}); \]

\[ u_s = \frac{r_s}{t_s}, \quad \mathbf{\hat{u}}_l(k) = \Delta \mathbf{\hat{r}}_l(k) = (\Delta \mathbf{\hat{x}}_l, \Delta \mathbf{\hat{y}}_l, \Delta \mathbf{\hat{z}}_l). \]

The translation momentum is

\[ \mathbf{p}_l = p_s \cdot \mathbf{\hat{p}}_l = (p_{1l}, p_{2l}, p_{3l}); \quad p_s = M_s u_s, \quad p_{\alpha l} = M_l u_{\alpha l}, \quad \alpha = 1, 2, 3. \]

(31)

The translation energy is

\[ K_l = \frac{1}{2} \mathbf{p}_l \cdot \mathbf{u}_l = \sum_{\alpha=1}^{3} K_{l\alpha}, \quad K_{l\alpha} = \frac{p_{\alpha l}^2}{2M_l}, \quad K_s = p_s u_s = M_s u_s^2. \]
There $K_s$ is the translation quantum. Then we have the total translation energy of the object

$$K = \sum_{i=1}^{N} K_i = \sum_{\alpha=1}^{3} K_\alpha, \quad K_\alpha = \sum_{i=1}^{N} K_{\alpha i}.$$ 

The rotation velocity of the particle at any time is defined as

$\omega_i(k) \equiv \Delta \theta_i(k)/t_s = (\theta_i(t)/t_s) \cdot \Delta \dot{\theta}_i(k) = \omega_s \cdot \ddot{\theta}_i(k) = (\omega_{i1}, \omega_{i2}, \omega_{i3})$;

$$\omega_s = \theta_i/t_s = 1/t_s, \quad \ddot{\theta}_i(k) = \Delta \dot{\theta}_i(k) = (\Delta \dot{\theta}_{i1}, \Delta \dot{\theta}_{i2}, \Delta \dot{\theta}_{i3}).$$

The rotation momentum is

$$s_i = s_s \cdot \theta_i = (s_{i1}, s_{i2}, s_{i3}); \quad s_s = l_s \omega_s, \quad s_{\alpha i} = l_i \omega_{\alpha i}, \quad \alpha = 1, 2, 3.$$ 

The rotation energy is

$$L_i = \frac{1}{2} s_i \cdot \omega_i = \sum_{\alpha=1}^{3} L_{\alpha i} \cdot \omega_{\alpha i} = \frac{s_{\alpha i}^2}{2 I_{\alpha i}}; \quad L_s = \omega_s s_s = l_s \omega_s^2.$$ 

There $L_s$ is the rotation quantum. Then we have the total rotation energy of the object

$$L = \sum_{i=1}^{N} L_i = \sum_{\alpha=1}^{3} L_{\alpha} \cdot \omega_{\alpha} = \sum_{i=1}^{N} L_{\alpha i}.$$ 

The deformation of the particle at any time is defined as

$$\varepsilon_i(k) \equiv \Delta l_i/l_i = (\Delta l_{i1}, \Delta l_{i2}, \Delta l_{i3})/l_i = (\varepsilon_{i1}, \varepsilon_{i2}, \varepsilon_{i3})$$;

$$l_i = l_{i1} + l_{i2} + l_{i3}; \quad \varepsilon_s = 1, \quad \varepsilon_{\alpha i} = \Delta \epsilon_{\alpha i}/l_i, \quad \alpha = 1, 2, 3.$$ 

(38)

If the principal elastic modulus of the particle are $Y_{i1}, Y_{i2}, Y_{i3}$, then the elastic energy density of the particle is

$$F_i = \sum_{\alpha=1}^{3} \left( \frac{1}{2} Y_{i\alpha} \varepsilon_{\alpha i}^2 \right) = \sum_{\alpha=1}^{3} \left( \frac{\chi_{\alpha i}^2}{2 Y_{i\alpha}} \right); \quad F_s = Y_s \varepsilon_s^2 = Y_s = \chi_s.$$ 

There $\chi_{\alpha i}$ is the principal stress. If the volume of the particle is $V_i$, then the vibration energy of the particle is

$$H_i = V_i \cdot F_i = \sum_{\alpha=1}^{3} H_{i\alpha} \cdot \varepsilon_{\alpha i}, \quad H_{i\alpha} = \frac{V_i \chi_{i\alpha}^2}{2 Y_{i\alpha}}; \quad H_s = F_s V_s = Y_s V_s.$$ 

There $H_s$ is the vibration quantum. Then we have the total vibration energy of the object

$$H = \sum_{i=1}^{N} H_i = \sum_{\alpha=1}^{3} H_{\alpha} \cdot \varepsilon_{\alpha}, \quad H_{\alpha} = \sum_{i=1}^{N} H_{\alpha i}.$$ 

Translation, rotation and vibration are three independent modes of motion in particle center-of-mass coordinates. Each mode has three energy terms and each particle has nine energy terms. For an object with $N$ base-particles, there are a total of $9N$ degrees of freedom of motion.

2.8. Energy quanta

The state of an object can be represented by the three mode energies, namely

$$H = H_i \cdot H_s > 0; \quad H_i = H_{i\alpha} > 0, \quad H_s = Y_s V_s.$$ 

(42a)

$$L = L_i \cdot L_s > 0; \quad L_i = l_i \omega_i^2.$$ 

(43a)

$$K = K_i \cdot K_s > 0; \quad K_i = M_i \omega_i^2.$$ 

(44a)

By introducing vibration constant $h$, rotation constant $l$ and translation constant $k$, the energy quanta can also be expressed by the vibration intensity $\nu$, rotation intensity $\zeta$ and translation intensity $T$, namely

$$H_s = H/N = h \nu, \quad \nu = H_s/h = H/(Nh).$$

(42b)

$$L_s = L/N = l \zeta, \quad \zeta = L_s/l = L/(Nl).$$

(43b)

$$K_s = K/N = kT, \quad T = K_s/k = K/(Nk).$$ 

(44b)
In the international system of units (SI system), the vibration intensity \( \nu \) is the frequency with the unit hertz (Hz), and the vibration constant is Planck constant \( h = 6.626069 \times 10^{-34} \text{ J} \cdot \text{Hz}^{-1} \). The rotation intensity \( z \) is the magnetic induction with the unit tesla (T), and the rotation constant is Bohr magneton \( l = 9.2740095 \times 10^{-24} \text{ J} \cdot \text{T}^{-1} \). The translation intensity \( T \) is the thermodynamic temperature with the unit kelvin (K), and the translation constant is Boltzmann constant \( k = 1.3806506 \times 10^{-23} \text{ J} \cdot \text{K}^{-1} \).

### 2.9. Energy space

The energy space is a Cartesian space constructed by the three mode energies [5]. The definition and the basic relations of energy space are listed in table 1.

| Zone (index \( x \)) | Gas (index \( h \)) | Solid (index \( l \)) | Liquid (index \( k \)) |
|----------------------|-------------------|-------------------|-------------------|
| Zone definition \( \mathbb{E}^x \) | \( \mathbb{E}^h = (L^h, K^h, H^h) \) | \( \mathbb{E}^l = (L^l, L^l, L^l) \) | \( \mathbb{E}^k = (H^k, L^k, K^k) \) |
| Ahead energy | \( L^h \) | \( K^l \) | \( H^k \) |
| Back energy | \( K^h \) | \( H^l \) | \( L^k \) |
| Major energy | \( H^h \) | \( L^l \) | \( K^k \) |
| Ahead parameter \( a^x \) | \( a^h = L^h / H^h \) | \( a^l = K^l / L^l \) | \( a^k = H^k / K^k \) |
| Back parameter \( b^x \) | \( b^h = K^h / H^h \) | \( b^l = H^l / L^l \) | \( b^k = L^k / K^k \) |
| Entire energy \( E^x \) | \( E^h = L^h + K^h \) | \( E^l = K^l + L^l \) | \( E^k = H^k + L^k \) |
| Thermal energy \( Q^x \) | \( Q^h = K^h + H^h \) | \( Q^l = H^l + L^l \) | \( Q^k = L^k + K^k \) |
| Potential energy \( J^x \) | \( J^h = L^h - H^h \) | \( J^l = K^l - L^l \) | \( J^k = H^k - L^k \) |
| Chemical energy \( G^x \) | \( G^h = K^h - L^h \) | \( G^l = H^l - K^l \) | \( G^k = L^k - H^k \) |
| Vibration factor \( \mu_1^x \) | \( \mu_1^h = E^h / H^h \) | \( \mu_1^l = E^l / H^l \) | \( \mu_1^k = E^k / H^k \) |
| Rotation factor \( \mu_2^x \) | \( \mu_2^h = E^h / L^h \) | \( \mu_2^l = E^l / L^l \) | \( \mu_2^k = E^k / L^k \) |
| Translation factor \( \mu_3^x \) | \( \mu_3^h = E^h / K^h \) | \( \mu_3^l = E^l / K^l \) | \( \mu_3^k = E^k / K^k \) |
| State vector \( E^x \) | \( E^h = (H^h, L^h, K^h) \) | \( E^l = (H^l, L^l, K^l) \) | \( E^k = (H^k, L^k, K^k) \) |
| Entire energy \( E^x \) | \( \sqrt{(H^h)^2 + (L^h)^2 + (K^h)^2} \) | \( \sqrt{(H^l)^2 + (L^l)^2 + (K^l)^2} \) | \( \sqrt{(H^k)^2 + (L^k)^2 + (K^k)^2} \) |
| Equilibrium condition | \( H^h = \sqrt{2L^h K^h} \) | \( L^l = \sqrt{2K^l H^l} \) | \( K^k = \sqrt{2H^k L^k} \) |

In table 1, \( H^x, L^x, K^x \) are called motion energy, \( E^x, Q^x, J^x, G^x \) the auxiliary energy, \( a^x, b^x \) the order parameter, and \( \mu_1^x, \mu_2^x, \mu_3^x \) the mode factor.

As shown in figure 1, the energy space is confined in the first octant (+,+,+) of the Cartesian space. In energy space, 3-dimensional region (body) is noted by symbol B[\( * \)], 2-dimensional region (surface) is noted by symbol S[\( * \)], 1-dimensional region (line) is noted by symbol L[\( * \)], and 0-dimensional region (point) is noted by symbol P[\( * \)]. Asterisk * gives the condition or code that the body, surface, line, and point satisfy. Figure 1 and the following tables exhibit the structure of the energy space.
Figure 1. (a) Structure of energy space. (b) Equilibrium surfaces in energy space.

Table 2. Zones and phases of energy space.

| Gas zone \(B[H]\) | Solid zone \(B[L]\) | Liquid zone \(B[K]\) |
|-------------------|-------------------|-------------------|
| \(B[G^2_H]\) \(L < K < H\) | \(B[G^2_L]\) \(K < L < H\) | \(B[G^2_K]\) \(H < K < L\) |
| \(B[G^2_L]\) \(K < L < H\) | \(B[G^2_K]\) \(H < K < L\) | \(B[G^2_K]\) \(L < H < K\) |

Table 3. Phase interfaces of energy space.

| Name               | Symbol | Condition                      |
|--------------------|--------|--------------------------------|
| J-type interface   | \(S[J^h_0]\) | \(J^h = L^h - H^h = 0\)          |
|                    | \(S[J^l_0]\) | \(J^l = K^l - L^l = 0\)          |
|                    | \(S[J^k_0]\) | \(J^k = H^k - K^k = 0\)          |
| G-type interface   | \(S[G^h_0]\) | \(G^h = K^h - L^h = 0\)          |
|                    | \(S[G^l_0]\) | \(G^l = H^l - K^l = 0\)          |
|                    | \(S[G^k_0]\) | \(G^k = L^k - H^k = 0\)          |

Table 4. Definitions and properties of order parameters.

| Zone    | Ahead parameter | Back parameter | Equilibrium condition | Range                  |
|---------|-----------------|----------------|-----------------------|------------------------|
| \(B[H]\) | \(a^h = L^h/H^h\) | \(b^h = K^h/H^h\) | \(2a^h b^h = 1\) | \(1/2 \leq (a^h, b^h) \leq 1\) |
| \(B[L]\) | \(a^l = K^l/L^l\) | \(b^l = H^l/L^l\) | \(2a^l b^l = 1\) | \(1/2 \leq (a^l, b^l) \leq 1\) |
| \(B[K]\) | \(a^k = H^k/K^k\) | \(b^k = L^k/K^k\) | \(2a^k b^k = 1\) | \(1/2 \leq (a^k, b^k) \leq 1\) |

Table 5. Equilibrium surfaces in energy space.

| Name               | Symbol | Condition                      |
|--------------------|--------|--------------------------------|
| Vibration surface  | \(S[H]\) | \(H = \sqrt{2LK}\)          |
| Rotation surface   | \(S[L]\) | \(L = \sqrt{2KH}\)          |
| Translation surface| \(S[K]\) | \(K = \sqrt{2HL}\)          |
The main results of real physics are showed in this section. Please refer to literatures [5-8] for detailed derivation.

3.1. Integer theorem of energy

The energy digits in energy space must be integer, which is called integer theorem of energy [5]. Therefore, the order parameters must be rational numbers. The digital expressions of the state vector, the motion energy and the auxiliary energy are listed in table 7. It is seen that the digital state can be completely determined as long as the particle number (N) and the order parameter (a or b) are known.

| Zone (index x) | Gas (index h) | Solid (index l) | Liquid (index k) |
|----------------|---------------|----------------|-----------------|
| State vector $E^x$ | $E^h = H_s \bar{E}^h$ | $E^l = L_s \bar{E}^l$ | $E^k = K_s \bar{E}^k$ |
| Energy quantum | $H_s = Y_s Y_s = \hbar v$ | $L_s = l_s \omega_s^2 = l z$ | $K_s = M_s u_s^2 = k T$ |
| Digit of state vector $\bar{E}^x$ | $\bar{E}^h = \bar{H}^h + \bar{L}^h + \bar{R}^h \bar{k}$ | $\bar{E}^l = \bar{H}^l + \bar{L}^l + \bar{R}^l \bar{k}$ | $\bar{E}^k = N a^h < N$ |
| Vibration energy $\bar{H}^x$ | $\bar{H}^h = N$ | $\bar{H}^l = N b^l < N$ | $\bar{H}^k = N a^h < N$ |
| Rotation energy $\bar{L}^x$ | $\bar{L}^h = N a^h < N$ | $\bar{L}^l = N b^l < N$ | $\bar{L}^k = N a^k < N$ |
| Translation energy $\bar{R}^x$ | $\bar{R}^h = N b^h < N$ | $\bar{R}^l = N a^l < N$ | $\bar{R}^k = N$ |
| Entire energy $\bar{E}^x$ | $\bar{E}^h = N (a^h + b^h)$ | $\bar{E}^l = N (a^l + b^l)$ | $\bar{E}^k = N (a^k + b^k)$ |
| Thermal energy $\bar{Q}^x$ | $\bar{Q}^h = N (b^h + 1)$ | $\bar{Q}^l = N (b^l + 1)$ | $\bar{Q}^k = N (b^k + 1)$ |
| Potential energy $\bar{J}^x$ | $\bar{J}^h = N (a^h - 1)$ | $\bar{J}^l = N (a^l - 1)$ | $\bar{J}^k = N (a^k - 1)$ |
| Chemical energy $\bar{G}^x$ | $\bar{G}^h = N (b^h - a^h)$ | $\bar{G}^l = N (b^l - a^l)$ | $\bar{G}^k = N (b^k - a^k)$ |

3.2. Equations of state

The entire energy of an object can be decomposed according to mode, volume, particle number and cluster number [5]. Table 8 lists several decomposition formulas of entire energy. Make all decomposition formulas equal and give the equations of state. The response functions can be calculated according to the equations of state [6].

| Zone (index x) | Gas (index h) | Solid (index l) | Liquid (index k) |
|----------------|---------------|----------------|-----------------|
| Energy quantum | $H_s = \hbar v$ | $L_s = l z$ | $K_s = k T$ |
| Mode decomposition | $E^h = \mu_s^x H^h$ | $E^l = \mu_s^l L^l$ | $E^k = \mu_k^x K^k$ |
| Volume decomposition | $E^h = q_s^x V^h$ | $E^l = q_s^l V^l$ | $E^k = q_s^k V^k$ |
| Particle decomposition | $E^h = \mu_s^x N H_s$ | $E^l = \mu_s^x N L_s$ | $E^k = \mu_k^x N K_s$ |
3.3. Phase transition

There are two types of phase interfaces in energy space, which represent continuous (G-type) and discontinuous (J-type) phase transitions [5,6]. Table 9 gives the conditions and parameters of the J-type phase transition. The energy at the J-type interface is discontinuous. The jump of potential energy represents the latent energy, and the jump of order parameter is constant 1/2.

Table 9. The discontinuity at the zone interfaces (J-type phase transition)

| Zone interface | Gas-Solid $S[J^h]$ | Solid-liquid $S[J^l]$ | Liquid-Gas $S[J^k]$ |
|----------------|---------------------|----------------------|----------------------|
| Equilibrium condition | $H^h = L^h = 2K^h$ | $L^l = K^l = 2H^l$ | $K^k = H^k = 2L^k$ |
| Order parameter | $\Delta a^{hl} = -1/2$ | $\Delta a^{lk} = -1/2$ | $\Delta a^{kh} = -1/2$ |
| Entire energy | $\Delta E^{hl} = 3(K^l - K^h)$ | $\Delta E^{lk} = 3(H^k - H^l)$ | $\Delta E^{kh} = 3(L^h - L^k)$ |
| Chemical energy | $\Delta G^{hl} = K^l + K^h$ | $\Delta G^{lk} = H^k + H^l$ | $\Delta G^{kh} = L^h + L^k$ |
| Thermal energy | $\Delta Q^{hl} = 4K^l - 3K^h$ | $\Delta Q^{lk} = 4H^k - 3H^l$ | $\Delta Q^{kh} = 4L^h - 3L^k$ |
| Potential energy | $\Delta J^{hl} = -K^l$ | $\Delta J^{lk} = -H^k$ | $\Delta J^{kh} = -L^h$ |

3.4. Ensemble statistics

The principle of ensemble statistics of real particles is similar to that of computer tomography. The statistical ensemble is a set of particle configurations in three-dimensional space, which is accurately described by a cluster matrix [6]. Ensemble statistics is a bridge between real space and energy space. Table 10 lists the main statistical functions of elastic particle system.

Table 10. Statistical functions of elastic particle system.

| Zone | Gas | Solid | Liquid |
|------|-----|-------|--------|
| Energy quantum | $H_x = H/N = Y \omega_z^2 = h\nu$ | $L_x = L/N = L_x \omega_z^2 = l\zeta$ | $K_x = K/N = M_x \omega_z^2 = kT$ |
| Partition function | $Z^h = \prod_{n=1}^{N} \left( \frac{\varphi_n^{h}}{\varphi_n^{h}} \right)^{\varphi_n^{h}}$ | $Z^l = \prod_{n=1}^{N} \left( \frac{\varphi_n^{l}}{\varphi_n^{l}} \right)^{\varphi_n^{l}}$ | $Z^k = \prod_{n=1}^{N} \left( \frac{\varphi_n^{k}}{\varphi_n^{k}} \right)^{\varphi_n^{k}}$ |
| Vibration energy | $\bar{R}^h = -\sum (c_n^h \cdot \ln \varphi_n^{h})$ | $\bar{R}^l = -\sum (c_n^l \cdot \ln \varphi_n^{l})$ | $\bar{R}^k = -\sum (c_n^k \cdot \ln \varphi_n^{k})$ |
| Rotation energy | $\bar{L}^h = -\sum (c_n^h \cdot \ln \varphi_n^{h})$ | $\bar{L}^l = -\sum (c_n^l \cdot \ln \varphi_n^{l})$ | $\bar{L}^k = -\sum (c_n^k \cdot \ln \varphi_n^{k})$ |
| Translation energy | $\bar{R}^h = -\sum (c_n^h \cdot \ln \varphi_n^{h})$ | $\bar{R}^l = -\sum (c_n^l \cdot \ln \varphi_n^{l})$ | $\bar{R}^k = -\sum (c_n^k \cdot \ln \varphi_n^{k})$ |
| Order parameter | $a^h = \frac{3}{4} \cdot \ln (2\pi \bar{c}_z)$ | $a^l = \frac{3}{4} \cdot \ln (2\pi \bar{c}_z)$ | $a^k = \frac{3}{4} \cdot \ln (2\pi \bar{c}_z)$ |
| Particle correlation | $\ln \varphi_z = \sum (\rho_n^z \cdot \ln \varphi_z)$ | $\ln \varphi_z = \sum (\rho_n^z \cdot \ln \varphi_z)$ | $\ln \varphi_z = \sum (\rho_n^z \cdot \ln \varphi_z)$ |
| Equilibrium condition | $\sum \rho_n^h (\varphi_n^h + \varphi_n^l + \varphi_n^k) = 1$ | $\sum \rho_n^l (\varphi_n^l + \varphi_n^l + \varphi_n^k) = 1$ | $\sum \rho_n^k (\varphi_n^k + \varphi_n^l + \varphi_n^k) = 1$ |
3.5. Equations of energy

According to the ensemble statistical theory [6], the energy relations and equations of the liquid are derived and showed in table 11. The energy equation gives the expression of energy difference, which is similar to the fundamental thermodynamic relations. We find that $-H$ is the Helmholtz free energy and $-L$ is the grand potential, after compared with current thermodynamics.

### Table 11. Energy relations and equations of liquid.

| Energy          | Relation            | Equation                                                  |
|-----------------|---------------------|-----------------------------------------------------------|
| Vibration energy| $H = J + K = L - G = Q - U$ | $dH = SdT + q_2dV - \gamma dC$                           |
| Rotation energy | $L = H + G = Q - K$, $L = q_2V$ | $dL = SdT + q_2dV + Cdy$                                   |
| Translation energy| $K = H - J = Q - L = U - G$ | $dK = TdS - q_2dV - Cdy$                                  |
| Thermal energy  | $Q = L + K = H + U$, $Q = ST$ | $dQ = TdS + Vdq_2 - Cdy$                                  |
| Chemical energy | $G = L - H = U - K$, $G = C\gamma$ | $dG = -SdT + Vdq_2 + \gamma dC$                        |
| Internal energy | $U = K + G = K + L - H = L - J$ | $dU = TdS - q_2dV + \gamma dC$                        |
| Enthalpy        | $Y = Q + G = K + 2L - H$ | $dY = TdS + Vdq_2 + \gamma dC$                        |
| Zero            | $0 = Q - Q = L - L = G - G$ | $0 = SdT - Vdq_2 + Cdy$                                  |

3.6. Equations of particle fields

The density fields of mass and momentum can be obtained by particle statistics in finite closed space. Potential fields are constructed by the density fields. The spatial derivatives of the potential fields give the action fields. The coupling of the action fields and the density fields give the force fields. Table 12 contains a complete set of difference equations, which describe the evolution of particle fields in three-dimensional flat space. The results show that the gravitational field and electromagnetic field are unified in the particle field [7,8].

### Table 12. The quantities and equations of the particle fields.

| Name             | Definition and Equation | Scale                      |
|------------------|-------------------------|----------------------------|
| Mass density     | $\rho = \rho(x, t)$     | $\rho_s = M_s V_s^{-1}$    |
| Momentum density | $j(x, t) = \rho u$      | $j_s = M_s u_s V_s^{-1}$    |
| Velocity         | $u(x, t) = j/\rho$      | $u_s = c$                  |
| Mass conservation| $\frac{\partial \rho}{\partial t} + \nabla \cdot j = 0$ | $\rho_s t_s^{-1} = M_s V_s^{-1} t_s^{-1}$ |
| Mass potential   | $\Phi(x, t) = -\frac{1}{4\pi \rho_s} \iiint \frac{\rho(x', t)}{|x - x'|} \, dx'$ | $\Phi_s = J_s M_s^{-1} = u_s^2$ |
| Momentum potential| $A(x, t) = \alpha_s \frac{1}{4\pi} \iiint \frac{j(x', t)}{|x - x'|} \, dx'$ | $A_s = K_s \rho_s^{-1} = u_s$ |
| Total potential energy| $J(t) = \iiint \rho(x', t) \Phi(x', t) \, dx'$ | $J_s = M_s u_s^2$ |
| Total translation energy| $K(t) = \iiint j(x', t) \cdot A(x', t) \, dx'$ | $K_s = M_s u_s^2$ |
| Medium constant  | $\varphi = \vec{\varphi} \cdot \varphi_s$, $\varphi = 4\pi$ | $\varphi_s = M_s r_s^{-3} = r_s M_s^{-1}$ |
| Dynamic constant | $a = \vec{a} \cdot \alpha_s$, $\vec{a} = (4\pi)^{-1}$ | $\alpha_s = A_s r_s^{-3}$ |

12
Boundary constraint
\[ B(S) = -\frac{\alpha_s}{4\pi} \int_D f(x', t') \cdot \frac{dS'}{r} = 0 \quad \alpha_s r_s = \tau_s^{-1} \]

Gradient field
\[ G = -\nabla \phi = -\frac{1}{4\pi \varphi_s} \int_D \rho(x', t') r \cdot d\mathbf{x}' / r^3 \quad G_s r_s^{-1} = \tau_s^{-2} \]

Curl of the gradient field
\[ \nabla \times G = -\nabla^2 \phi \equiv 0 \quad G_s r_s^{-1} = \tau_s^{-2} \]

Divergence of the gradient field
\[ \nabla \cdot G = -\nabla^2 \phi = -\rho / \varphi_s \quad G_s r_s^{-1} = \tau_s^{-2} \]

Poisson equation of gradient field
\[ \nabla^2 G = -\nabla \rho / \varphi_s \quad G_s r_s^{-2} = \tau_s^{-1} \]

Divergence field, Wave equation
\[ D = \nabla \cdot A = \frac{1}{u_s^2} \frac{\partial \Phi}{\partial t} \quad D_s A_s r_s^{-1} = \tau_s^{-1} \]

Wave potentials
\[ \Phi_w = -W(\xi), \quad A_w = (\kappa / \omega) W(\xi) \quad \Phi_s = u_s^2 A_s = u_s \]

Wave parameter
\[ \xi = \kappa \cdot x - \omega t \quad \xi_s = 1 \]

Gradient of the divergence field
\[ \nabla D = \nabla (\nabla \cdot A) = -\frac{1}{u_s^2} \frac{\partial G}{\partial t} \quad D_s r_s^{-1} = \tau_s^{-1} \]

Curl field
\[ \mathcal{C} = \nabla \times A = \frac{\alpha_s}{4\pi} \int_D \left( f(x', t') \times r \right) / r^3 \cdot d\mathbf{x}' \quad \mathcal{C}_s = A_s r_s^{-1} = \tau_s^{-1} \]

Divergence of the curl field
\[ \nabla \cdot \mathcal{C} = \nabla \cdot (\nabla \times A) \equiv 0 \quad \mathcal{C}_s r_s^{-1} = \tau_s^{-1} \]

Curl of the curl field
\[ \nabla \times \mathcal{C} = \alpha_s j - \frac{1}{u_s^2} \frac{\partial G}{\partial t} \quad \mathcal{C}_s r_s^{-2} = \tau_s^{-2} \]

Poisson equation of momentum potential
\[ \nabla^2 \mathcal{A} = -\alpha_s j \quad A_s r_s^{-2} = \tau_s^{-1} \]

Poisson equation of curl field
\[ \nabla^2 \mathcal{C} = -\alpha_s \nabla \times j \quad \mathcal{C}_s r_s^{-2} = \tau_s^{-1} \]

Density of gradient force
\[ f_G = \rho G = -\rho \nabla \phi \quad f_s = \rho_s A_s = \rho_s r_s^{-2} \]

Density of divergence force
\[ f_D = D j = \rho D u \quad f_s = j_s A_s = \rho_s r_s^{-2} \]

Density of curl force
\[ f_C = C \times j = \rho C \times u \quad f_s = j_s C_s = \rho_s r_s^{-2} \]

Density of entire force
\[ f = \rho a = \rho \left( \frac{d\mathbf{u}}{dt} + (\mathbf{u} \cdot \nabla) \mathbf{u} \right) \quad a_s = f_s r_s^{-1} = \tau_s^{-2} \]

4. Physical fundamentals

Real physics is a formal logic system based on the axioms of real particle, real space, real time and objectivity principle. It can derive the known and unknown physical laws, and gives consistent explanations and conclusions to the fundamental problems of physics.

4.1. Matter, motion and energy

The essence of matter is a fundamental problem of physics. The basic assumption of real physics is the particle nature of matter. Both object and field are made up of real particles. Real particles are 3-dimensional objects with only mass, volume and elasticity. Two kinds of primary particles (protons and electrons) are enough to constitute all real matter.

Mass is an essential attribute of objects, volume and energy are the motion feature of objects. Motion produces energy, and energy determines volume. The motion of particles is the reason why objects have energy and volume. The mass of the particles cannot be destroyed, and the mass and energy cannot be converted. Real physics adheres to the principle of mass or particles conservation in classical physics, and abandons the assumption of mass-energy conversion in modern physics. Einstein's mass-energy relation is interpreted as scale relation in real physics.

The motion of particles has three independent modes: translation, rotation and vibration. Three
modes of motion energy constitute a Cartesian energy space. The energy in energy space is quantized. The object state can be analyzed and explained in detail by mean of energy space.

The absence of dark matter in current physics is rooted in the vacuum hypothesis. The real physics holds that the physical space is full of elastic particles. The so-called vacuum is a space full of electrons. Electrons are dark matter in the cosmic background. The theory based on real space and elastic particles perfectly explains the origin of gravitation and electromagnetism, thus realizing the unification of fundamental interactions.

4.2. Fields and interactions

Real physics clarifies the relationship between fields and particles. The origin of the fields is particles. Field is the statistical effect of a huge number of elastic particles. The universe is full of particles, and there is no vacuum without particles. The cosmic background is the electron field, and the common solids and fluids are atomic or molecular fields. The electrons in cosmic background are the dark matter.

Under the constraints of mass conservation and volume repulsion, a complete set of particle field equations is derived (table 12). Particle fields include density field, potential field, action field and force field. Density field includes mass density and momentum density. Potential field includes mass potential (scalar potential) and momentum potential (vector potential). Action field includes gradient field, divergence field and curl field. The force field includes gradient force, divergence force and curl force. As showed in table 13, there are definite conversion relationships between gravitational field and the electromagnetic field [7]. It is seen that electronic mass is equivalent to electric charge and electronic momentum is equivalent to electric current. Therefore, the gravitational field and electromagnetic field are unified in the particle field.

The particle field includes three basic interactions: gradient force, divergence force and curl force. Gradient force is equivalent to gravitational force and electrostatic force, and the attraction or repulsion between particles depends on the sign of gradient field. Both weak and strong forces are the combined effects of three basic interactions. All interactions essentially come from the constraints of mass conservation and volume repulsion. Particles are the medium of interaction. Electrons are the medium that transmits gravitational force and electromagnetic force. The interaction is localized and there is no action at a distance. The action of electronic field propagates at the speed of light, while the action of molecular field propagates at the speed of sound.

| Electromagnetic field | Relationship | Conversion coefficient |
|-----------------------|--------------|------------------------|
| Vacuum permittivity   | \( \varepsilon_s = \theta \varphi_s \) | \( \theta = 7.4261454 \times 10^{-21} \text{C}^2\text{kg}^{-2} \) |
| Vacuum permeability   | \( \mu_s = \theta^{-1} \alpha_s \) | \( \theta^{-1} = 1.3465936 \times 10^{20} \text{C}^2\text{kg}^{-2} \) |
| Charge density        | \( \rho_e = \beta \theta \rho \) | \( \beta \theta = -4.2222312 \times 10^{-32} \text{Ckg}^{-1} \) |
| Current density       | \( j_e = \beta \theta j \) | \( \beta \theta = -4.2222312 \times 10^{-32} \text{Ckg}^{-1} \) |
| Electric potential    | \( \Phi_e = \beta \Phi \) | \( \beta = -5.6856296 \times 10^{-12} \text{kgC}^{-1} \) |
| Magnetic potential    | \( A_e = \beta A \) | \( \beta = -5.6856296 \times 10^{-12} \text{kgC}^{-1} \) |
| Electric field        | \( E_e = \beta G \) | \( \beta = -5.6856296 \times 10^{-12} \text{kgC}^{-1} \) |
| Magnetic induction    | \( B_e = \beta C \) | \( \beta = -5.6856296 \times 10^{-12} \text{kgC}^{-1} \) |

4.3. Laws of physics

The conclusions of real particle theory include the empirical laws of physics. The density field is the mathematical solution of the particle field equations. In principle, the evolution of the particle field can be predicted once the initial condition of the density field is given. The particle field equations
(table 12) include the laws of gravitation and the laws of electromagnetism. Both attractive and repulsive forces are predicted by the particle field theory.

A general equation of motion is derived from the particle field equations

\[ f = \rho a = \rho \left[ \frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} \right] = \rho(G + Du + C \times \mathbf{u}), \]

where \( a \) is the acceleration. The term \( \frac{\partial \mathbf{u}}{\partial t} \) is linear acceleration and the term \( (\mathbf{u} \cdot \nabla) \mathbf{u} \) is curvilinear acceleration. The first equation \( f = \rho a \) is the Newtonian second law, and the last one is the Navier-Stokes equation with known forces.

The zero-th law of thermodynamics is a conclusion of the equilibrium state theory. There are three equilibrium surfaces in the energy space, among them the translation surface represents the thermal equilibrium, and the equilibrium parameter is temperature \( T \). The third law of thermodynamics is included in the theorem of motion persistence [5]. The non-stopping motion of particles ensures the positive-definiteness of the translation energy, that is \( K = NkT > 0 \). It means that the thermodynamic temperature \( T \) cannot reach to zero. The first and second laws of thermodynamics are included in the energy equations in table 11. The internal energy difference \( dU \) includes three terms, which are the contributions of rotation energy \( L = q_z V \), thermal energy \( Q = ST \) and chemical energy \( G = Cy \). There \( q_z \) is the density of rotation energy, \( S \) is entropy and \( y \) is chemical potential

\[ S = (b + 1)Nk, \quad y = (1 - ab^{-1})kT. \] (46)

It is seen that the laws of thermodynamics are logical conclusions of real physics.

4.4. Essence of relativity

The soul of relativity principle is the thought of physical unification. The principle of special relativity requires that the laws of mechanics and electromagnetism be valid in all inertial frames. Its goal is to pursue the unification of mechanics and electrodynamics. The principle of general relativity requires that the form of physical laws remain unchanged in all reference frames. Its aim is to pursue the unification of gravitational and electromagnetic forces. The principle of relativity requires that physical equations are covariant under coordinate transformation.

In fact, the choice of coordinate frames is a subjective behavior of the observer. The change of origin is the translation of reference point, and the change of coordinates is the conversion of metrics. The intention of covariance is to eliminate the arbitrariness of selecting coordinate frame. However, the covariance principle does not really exclude the influence of measurement, because it does not include the invariance of space and time translation. Real physics chooses special space and time reference points, thus completely getting rid of the restriction of coordinate frames.

Objectivity principle is the development of relativity principle, which is actually a principle of universality. Origin irrelevance indicates that physical laws are applicable to any place and any time. Scale irrelevance indicates that physical laws are applicable to any scale. For examples, there is no absolute reference frame, and there is no difference between macro physics and micro physics. Objects obey the universal laws of motion, regardless of their size, weight, speed and energy.

4.5. Origin of quantization

The quantization of modern physics originates from digitization. Physical study inevitably involves numerical computation. The digitization of physical quantities needs the aid of coordinate system, and coordinate conversion leads to the principle of relativity. Therefore, quantization is closely related to the principle of relativity.

Using coordinates to digitize physical quantities is not the unique solution. The digitizing scheme without coordinates is to express the physical quantity in the real form (formula 1). The objectivity principle that replaces the relativity principle is the origin irrelevance and the scale irrelevance. The origin irrelevance demands that the physical quantity is independent of the coordinate origin. The scale irrelevance demands that the physical relation and the digital relation are equivalent (equation 13). Scale is the metric and identifier of a real quantity. As metric, scale is actually the quantum. The real physics actually includes the principles of quantization and relativity.
The integralization of energy originates from the dispersion of particles. Real particles have three motion modes of vibration, rotation and translation. The three mode energies are $H, L, K$, and the corresponding scales are vibration quantum $H = Y_s V_s = h\nu$, rotation quantum $L = I_s \omega^2 = I_\ell$ and translation quantum $K = M_s u_s^2 = kT$. The energy scale is the statistic average of the total number of particles ($N$), so the energy digits ($\bar{H}, \bar{L}, \bar{R}$) must be integer. Vibration quantum ($H = h\nu$) is known to quantum theory. If the speed of light is taken as the scale of velocity ($u_s = c$), then the translation quantum ($K = M_c c^2$) is the relativistic mass-energy relation. The discovery of rotation quantum ($L = I_\ell \omega^2$) is one of the important achievements of real physics.

Since vibration energy is equal to mechanical energy ($H = J + K$), the wave functions determined by Hamiltonian operator represent the vibration of particles. According to this point of view, the wave functions in real space can be explained as the classical probability of the elastic particle profile. Different eigenstates represent different modes of vibration. This is the classical interpretation of quantum theory by real physics.

5. Scientific dialectics

5.1. Physics and mathematics

Physics is a natural theory expressed by mathematics. The core of physics lies in physical model rather than mathematical formulation. Physical model is the abstraction of object structure, and mathematical theory is the thinking tool serving physical model. Mathematical modeling and physical modeling are two different concepts. Physical models can be expressed by different mathematical structures. Mathematical models should accurately describe physical models, and physical principles must make strict restriction on mathematical models. In order to establish causality, for example, the symmetry of time inversion must be broken in the mathematical structure. In order to eliminate the subjective influence, we must strictly distinguish the subjective and objective factors in the mathematical structure.

Physical models are premised on reality while mathematical models are aimed at consistency. Tight mathematics can deduce everything that a physical model allows, including true and untrue conclusions. However, mathematics cannot deduce what is not included in the physical model. For example, spatial singularities are inevitable in current physics because the point and wave particle models have the property of spatial penetration. Dark matter cannot be discovered because it is forbidden by the vacuum hypothesis of current physics. Real particles have volume repulsion, so the possibility of black holes is excluded. Real space contains real particles, so real physics is bound to uncover the secret of dark matter.

5.2. Simplicity and complexity

The world is complex but the physical principles are simple. Nature in various forms runs in an orderly manner according to universal laws. Firstly, the nature of matter is simple. The nature of matter is particles, not waves. The nature of fields is also particles. Secondly, the composition of objects is simple. There are only two kinds of primary particles in nature. All objects are made up of protons and electrons. Thirdly, the form of motion is simple. There are only three independent modes of motion for real particles. The energy space describes the motion state of the object in completeness. Fourthly, the real mathematics is simple. The theoretical core is particle statistics, the basic structure is set theory, and calculus is a bridge from discrete statistics to continuous analysis.

5.3. Finiteness and infinity

There is no singularity in real world. Real objects are finite, and infinity is an abstraction of mathematics. Finiteness is a consistent principle of real physics. Firstly, the physical model stipulates that the mass, volume and number of particles are limited, which excludes the infinity of particle density in principle. Secondly, the real mathematics prescribes that the scales are positive and real variables, which excludes any mathematical singularity in logic.
5.4. Continuity and discreteness
The nature of matter is discrete. The real particles are separated. Any object is made up of separate particles. The nature of the fields is discontinuous, it is the statistical average effect of a large number of elastic particles.

The motion of particles is continuous. Motion can be described by continuous mathematics or discrete mathematics. Discrete descriptions have natural advantages because of the discreteness of particles. The approach of real physics is to express the motion quantities by discrete time sequence. The study of particle fields and thermodynamics starts from particle statistics and then transits to continuous cases. Calculus is the important tool for realizing the transition from discrete statistics to continuity analysis.

The motion energies of an object are quantized in energy space. The energy quantization comes from the particle statistics, as the energy quanta are the average motion (vibration, rotation and translation) energies of elastic particles. For the system of fewer particles, the statistical fluctuation is larger and the quantum effect is obvious. The statistical fluctuation of a large number of particle systems is small and the quantum effect can be neglected.

5.5. Randomness and determinacy
Real physics is essentially the statistical theory of elastic particles. Both field and object are finite systems composed of separate particles, and its macroscopic properties are determined by microscopic statistics of particles. The particle field theory is based on mass and momentum statistics, the object state theory is based on energy statistics, and the thermodynamic theory is based on the ensemble statistics. On the one hand, the nature of real physics is random, because the motion state of individual particles cannot be completely determined. On the other hand, the laws of real physics are deterministic, because the statistical laws of a large number of particles are inevitable.

5.6. Relativity and absoluteness
Relativity is a popular concept in daily life, such as good and bad, beauty and ugliness. The concept of relativity involves the evaluation of things. Different standpoints and standards on the same thing may lead to different evaluation results. Here, the thing under evaluation is objective and absolute, the standard of evaluation is subjective, and the result of evaluation is relative. This can be called the principle of popular relativity.

The principle of objectivity is no other than the principle of popular relativity. An important development is that we find a mathematical formulation for popular relativity. Measuring an object with different metrics will lead to different numerical results. Here, the measured object is objective and absolute, the employed scale is subjective factor and the obtained digit is relative factor. The real form of physical quantity is exactly a mathematical model of popular relativity.

5.7. Subjectivity and Objectivity
Physical laws are objective and do not depend on mankind consciousness. Subjective factors must be excluded from the physical model and mathematical structure. As the spatial and temporal reference points are subjective, the physical equation must be origin irrelevant. As the metrics of physical quantity are subjective, the physics equations must be scale irrelevant. Origin irrelevance indicates that physical laws are applicable to any time and any place, and scale irrelevance indicates that physical laws are applicable to any scale. Objectivity principle is essentially the principle of unification of physics.

6. Conclusions
Reality, simplicity, sufficiency, consistency and objectivity are the basic principles of physical modeling. Different from the modeling of pure mathematics, reality is the first priority of physical modeling. The over-idealization of point and wave particles is the main reason for the inconsistency of fundamental interactions. The serious consequence of the vacuum hypothesis is the loss of dark matter.
The subjective factors introduced by the measurement process blur the boundaries between physics and mathematics, objectivity and subjectivity, existence and consciousness. The excessive dependence of physics on mathematics has even led to scientific superstition.

Real physics is a realistic theory based on elastic particles, real space, real time and objectivity principle. The elastic particle model is a modification to the point particles in classical physics and wave particles in modern physics. The real space model is a modification to the vacuum assumption in classical and modern physics. The real time model is a faithful expression of causality. The objectivity principle is a comprehensive principle of relativity and quantum theory.

The breakthrough of real physics lies in a deep inspection of reality and objectivity. It gives birth to the basic ideas of real physics, such as the conceptions of scale, relativity and quantization, then the physical models of elastic particles, real space and real time. The real physics leads to the discovery of laws of motion and interaction. The principle of real physics is simple, the logic is consistent, and its conclusions are of general significance. The achievements of real physics make us firmly believe that nature really operates under simple and unified ways.

7. References
[1] Jin S N and Ma Y L 2002 *Theoretical Mechanics* (Beijing: Higher Education Press)  
[2] Zeng J Y 2018 *Quantum Mechanics* (Beijing: Science Press)  
[3] Li L F 2015 Quantum Field Theory (Beijing: Science Press)  
[4] Liang Z C 2015 *Physical Principles of Finite Particle System* (Wuhan: Scientific Research Publishing). DOI:10.13140/RG.2.1.2409.8004  
[5] Liang Z C 2019 Motion, energy and state of body particle system *Theoretical Physics*, 4 66-84. DOI:10.22606/tp.2019.42003  
[6] Liang Z C 2019 Cluster ensemble statistics of body particle system *New Horizons in Mathematical Physics* 3 53-73. DOI:10.22606/nhmp.2019.32002  
[7] Liang Z C 2019 The origin of gravitation and electromagnetism *Theoretical Physics*, 4 85-102. DOI:10.22606/tp.2019.42004  
[8] Liang Z C 2018 Essence of light: particle, field and interaction *Proc. SPIE*. 10755 1075501-10755014 Photonic Fiber and Crystal Devices: Advances in Materials and Innovations in Device Applications XII (San Diego, USA, 28 August, 2018). DOI:10.1117/12.2316422.

Acknowledgements
Author would like to acknowledge the financial support from NSFC (No.61775102).