Simple GUTs

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ABSTRACT
Together with some positive features, such as the constrained hypercharge assignments, grandunification models typically have some unwelcome aspects, mostly associated with the rich and peculiarly selected matter content. I discuss some ideas which might help to interpret (part of) this apparent complexity arising in grandunification in terms of simple structures. I also point out that the conventional criteria used to establish fine tuning in grandunified models should be modified if these models are viewed as effective low-energy descriptions of a more fundamental theory.

1 Introduction

In spite of its remarkable phenomenological success, the Standard Model (SM) is not believed to provide a fundamental theory of particle physics. This expectation originates for many of us not only from the fact that SM does not incorporate gravity, and should therefore become inadequate at some very high scale\(^1\), but also because of its rather complex structure. It is probably worth emphasizing, at least for the benefit of the students who attended Quarks96 and might read the proceedings, that we have nothing (from the conceptual viewpoint) to assure us that nature should be describable in terms of simple laws. Still, most of us do expect this simplicity, probably extrapolating from the history of physics, which has proceeded through a

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\(^2\)At this high scale, possibly the Planck mass \(M_P \sim 10^{19}\) GeV, one must after all even consider the possibility that the usual tools of particle physics might lose meaning.
series of steps of deeper understanding and simplification (such as the description of
the baryon spectrum in terms of the quark model). From the point of view of this
expected simplicity, the $SM$ is quite unsatisfactory, since it leaves unanswered several
questions; in particular,

(Qa) Particle physics is described by a gauge theory with the peculiar gauge group
$G_{SM} \equiv SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$.

(Qb) The corresponding three coupling constants, $\alpha_s$, $\alpha_2$, and $\alpha_Y$, are free parameters
of the model.

(Qc) A peculiar bunch of IRREPs (irreducible representations) of $SU(3)_c \otimes SU(2)_L$
hosts the quarks and leptons.

(Qd) The hypercharge assignments to the quark and lepton IRREPs are arbitrary.

(Qe) The Yukawa couplings are arbitrary.

(Qf) The entries of the Cabibbo-Kobayashi-Maskawa matrix are arbitrary.

(Qg) Each of the quarks and leptons of the model is present in triplicate copy (family
structure).

(Qh) A peculiar (bunch of) IRREP(s) hosts the Higgs particles.

(Qi) The parameters of the Higgs potential, which determine the Higgs mass(es) and
all the aspects of symmetry breaking, are arbitrary.

(Qj) The anomaly cancellation is a (apparently accidental) result of the structure of
the (arbitrarily selected) matter content of the model.

GUTs [models with a (grand)unified description of particle interactions] have been
investigated primarily because, as illustrated in the discussion of $SO(10)$ GUTs given
in the next section, they address/simplify (Qa)-(Qf) and, in some cases, (Qj), and
are therefore good candidates for the description of particle physics if the trend of
incremental simplifications of this description is to continue. However, it should be
noted that not only GUTs bring no improvement in relation to (Qg), but they actually
increase the complexity associated to (Qh) and (Qi). Therefore, from the “aesthetic”
viewpoint, GUTs have merits and faults (with the merits outnumbering, but not
necessarily outweighing, the faults).

Phenomenological encouragement for the GUT idea comes from the observed low-
energy values of $\alpha_s$, $\alpha_2$, and $\alpha_Y$, which appear to be arranged just as needed for
unification. Indeed, (although the simplest GUT candidate, minimal $SU(5)$, does
not pass this test) there are several examples of GUTs which reproduce these data
on the coupling constants while being consistent with the present experimental
lower limit on proton decay, \( \tau_{p \rightarrow e^+ \pi^0} \geq 9 \cdot 10^{32} \) years. One important feature of phenomenologically consistent GUTs is that they must involve at least one extra scale, besides the unification scale \( M_X \), at which the RGEs (renormalization group equations) of the SM couplings are modified. In fact, the recent precise determination of the gauge coupling constants at the scale \( M_Z \) has allowed to show that, if only SM particles contribute to the RGEs between \( M_Z \) and \( M_X \), the three running coupling constants of \( SM \) meet at three different points and only the meeting point of \( \alpha_2(\mu) \) and \( \alpha_s(\mu) \) corresponds to a value of the scale \( \mu \) sufficiently high to comply with the experimental lower limit on proton decay. Equivalently, one can describe this situation by stating that if only SM particles contribute to the RGEs between \( M_Z \) and \( M_X \), a one-step unification would require low-energy values of the couplings that are inconsistent with experimental data.

Perhaps the simplest GUTs meeting the minimum requirement of agreement with the data on the coupling constants and with the experimental limit on proton decay are some \( SO(10) \) models, which are reviewed in the next section. They naturally predict a two-scale breaking to \( G_{SM} \); in fact, a typical \( SO(10) \) breaking chain is given by

\[
SO(10) \xrightarrow{M_X} G' \xrightarrow{M_R} G_{SM} \xrightarrow{M_Z} SU(3)_c \otimes U(1)_{e.m.}
\]

Importantly, in \( SO(10) \) the hypercharge \( Y \) is the combination of two generators belonging to the Cartan,

\[
Y = T_{3R} + \frac{B - L}{2}
\]

where \( B - L \) and \( T_{3R} \) belong respectively to the \( SU(4)_{PS} \) (the \( SU(4) \) containing \( SU(3)_c \) and \( U(1)_{B-L} \), which was first considered by Pati and Salam) and the \( SU(2)_R \) subgroups of \( SO(10) \). This leads to a high unification point \( M_X \) if the intermediate symmetry group \( G' \) contains \( SU(2)_R \) and/or \( SU(4)_{PS} \), since then, between \( M_Z \) and \( M_R \), the Abelian evolution of \( \alpha_Y \) (predicted by \( SM \)) is replaced by the non-Abelian one of either component of \( Y \). \( M_X \) is connected with the masses of the lepto-quarks that can mediate proton decay, and this \( SO(10) \) mechanism for a higher unification point turns out to be useful in allowing to meet the condition

\[
M_X \geq 3.2 \cdot 10^{15} \text{ GeV}
\]

which is necessary for agreement with the present experimental limit on proton decay.

Although (relatively) simple GUTs, such as \( SO(10) \), can work, they are affected by the hierarchy problem, and this causes many to prefer SUSY (supersymmetric) GUTs. Also model building in the SUSY GUT case has little difficulties reaching the minimum requirement of agreement with the data on the coupling constants and the experimental limit on proton decay; SUSY \( SU(5) \) GUTs already meet this minimum requirement. These models typically predict one-step breaking of \( SU(5) \) to \( G_{SM} \) at
the scale $M_X$, but the RGEs are already modified at a much lower scale, $M_{susy}$, where SUSY breaking occurs

$$\text{SUSY}SU(5) \xrightarrow{M_X} \text{SUSY}G_{SM} \xrightarrow{M_{susy}} G_{SM} \xrightarrow{M_Z} SU(3)_c \otimes U(1)_{e.m.}$$

(4)

In this paper (but not necessarily elsewhere) I take the position that for the GUTs (whether they are SUSY or not) the aesthetic advantages and the consistency with the observed low-energy values of the gauge coupling constants outweigh the damage done in regard to (Qh) and (Qi). This motivates me to look for possible ways to associate hidden simplicities to the apparently complicated GUT structures affecting (Qh) and (Qi) (and (Qg)); the reader is warned of the fact that the resulting discussion is accordingly quite speculative.

2 SO(10) Models as GUT Prototypes

In this section I illustrate some of the aspects of GUTs that I mentioned in the Introduction, using as an example the GUTs based on the SO(10) group[4]. In these models the electromagnetic, weak, and strong interactions are unified in an SO(10) gauge interaction characterized by one gauge coupling. SO(10) GUTs also reduce the amount of arbitrariness which characterizes the SM in regard to the content of leptons and quarks; in fact, they accommodate in one IRREP, the 16-dimensional spinorial representation, the fifteen known left-handed fermions of a generation plus a new particle, whose quantum numbers are the same as those of the not-yet-discovered $\nu^c_L$. Within the 16 (here and in the following I refer to the IRREPs of a group by their dimension only), quarks and leptons are also automatically assigned hypercharges in agreement with the observed quantization; these hypercharge assignments are instead a free input of SM. Moreover, the presence of $\nu^c_L$ can lead to a mass matrix, for one generation of neutrinos, of the form

$$(\nu^c_R \nu^c_L) \left( \begin{array}{cc} 0 & \frac{m_D}{M} \\ \frac{m_D}{2} & M \end{array} \right) \left( \begin{array}{c} \nu^c_L \\ \nu^c_L \end{array} \right) + h.c.,$$

(5)

where $m_D$ is a Dirac mass, which can be conjectured to be of the order of the other masses of the fermions in the given generation, and $M$ is a Majorana mass for the right-handed neutrinos. If $m_D \ll M$, this mass matrix has the two eigenvalues

$$m_{\nu_1} \sim \frac{m_D^2}{M}, \quad m_{\nu_2} \sim M,$$

(6)

leading to the relation $m_{\nu_1} \ll m_D$ as observed experimentally.

Interestingly, the choice of SO(10) as the grandunification group also leads to the absence of triangle anomalies, due to the fact that in SO(10) it is not possible to construct a cubic invariant with the adjoint representation, which describes the gauge
bosons. In the SM, and in some GUTs, there is an accidental (ad hoc) cancellation among different (nonvanishing) contributions to the anomaly.

Up to this point I have only discussed positive features of SO(10) GUTs (and ignored the annoying family triplication, which is left unmodified by the move from SM to SO(10), and in SO(10) is understood to imply the need for three copies of the 16). This has been the case because I have not yet discussed the Higgs sector. I shall do this next. It will put in evidence several puzzles, while allowing me to point out that indeed, as needed for the phenomenological considerations mentioned in the Introduction, in SO(10) GUTs it is quite natural to have a two-step breaking to SM.

In discussing the Higgs sector it is useful to classify the components of the smallest IRREPs of the group that are invariant under $G_{SM}$. The IRREPs 10, 120, 320 contain no $G_{SM}$ singlet. The 16 and the 126 contain one $G_{SM}$ singlet each, and in both cases the little group of the singlet is $SU(5)$. The 45 contains two independent $G_{SM}$ singlets (i.e., it contains a two dimensional space of $G_{SM}$ singlets), one with little group $SU(3)_c \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L} \times D$ (D stands for the left-right discrete symmetry [5] which interchanges $SU(2)_L$ and $SU(2)_R$) and the other with little group $SU(4)_{PS} \otimes SU(2)_L \otimes U(1)_{T_{3R}}$. The 54 contains one $G_{SM}$ singlets, with little group $SU(4)_{PS} \otimes SU(2)_L \otimes SU(2)_R \times D$. The 144 contains one $G_{SM}$ singlet, with little group $SU(3)_c \otimes SU(2)_L \otimes SU(2)_R \otimes D$.

From these properties of the smallest IRREPs of SO(10) one can easily see that the typical pattern of SSB of SO(10) to $G_{SM}$ has two steps; indeed, with the exception of the singlet in the 144, the little group of all the above mentioned $G_{SM}$ singlets is larger than $G_{SM}$.

Actually, either for phenomenological or for technical reasons, some of the above mentioned $G_{SM}$ singlets, cannot be used for the first SSB step. The use of the $G_{SM}$ singlets in the 16, 126 and 144 representations would lead to the result that, like in SU(5) GUTs, the unification of the $G_{SM}$ coupling constants is inconsistent with the low-energy data on these couplings. (In the cases of the 16 and the 126 the SSB of SO(10) to $G_{SM}$ occurs in two steps, but the unification of the $G_{SM}$ couplings occurs in one step, since the intermediate symmetry group in such cases is the simple group $SU(5)$.) Concerning the 45 representation, one can show that the only non-trivial positive definite invariant with degree $\leq 4$ (as necessary in order to have a renormalizable potential) that can be constructed with the 45 is not extremized along any of the two $G_{SM}$ singlets of the 45. Moreover, the $G_{SM}$ singlet of the 210 which has little group $SU(3)_c \otimes SU(2)_L \otimes U(1)_{T_{3R}} \otimes U(1)_{B-L}$ is not a good candidate for the first SSB step because of the above mentioned need for an intermediate symmetry group containing either SU(4)$_{PS}$ or SU(2)$_R$.

The previous considerations lead to four scenarios, in which the first steps of breaking are:
The type-I $SO(10)$ models require that an appropriate vector \[ (Ia) \] in the space of $G_{SM}$ singlets of the 210 acquires a v.e.v. (vacuum expectation value) at the GUT scale. Analogously the type-II model requires that the $G_{SM}$ singlet of the 54 acquires a v.e.v. at the GUT scale.

An appealing\[10\] possibility for the completion of the models of type-Ia,b,c and type-II is the one of realizing the second SSB step, at a scale $M_R$, with the $G_{SM}$ singlet of the 126 $\oplus$ 126 representation, and the third SSB step with a combination of the $SU(3)_c \otimes SU(2)_L \otimes SU(2)_R \times D$, such a way to avoid the unwanted relation $m_t = m_b$ \[10\]. Through the see-saw mechanism, the scale $M_R$ is related to the masses of the (almost) left-handed neutrinos.

The type-Ia,b,c and type-II models have been studied in the Refs.\[7, 8, 9, 10, 11, 12, 13, 14\], and they have been found to be consistent with the unification of couplings and the experimental bound on the proton lifetime, although in the case of the type-II model the consistence with the experimental bound on the proton lifetime is only marginal\[10\]. Within the see-saw mechanism, one also finds that these models predict masses for the (almost) left-handed neutrinos in an interesting range.

This phenomenology depends however on the values of the parameters of the Higgs potential, which are free inputs of the model. Most importantly, as mentioned above, the parameters of the Higgs potential must be chosen so that the desired SSB pattern is realized. Although this does not involve a particular fine tuning\[10, 11, 12, 13\], it does introduce an element of undesirable arbitrariness in the models. Similarly, the “matter ingredients” of the models (e.g., in the type-I models, $16 \oplus 16 \oplus 16$ for the fermions plus $10 \oplus 10 \oplus 126 \oplus 126 \oplus 210$ for the Higgs bosons) is selected with the only constraint of reproducing observation (i.e. the matter content is not constrained by any requirement of internal consistency of the models).

\footnote{Notably, whereas the vectors used for the type-Ia and type-Ib models correspond to maximal little groups, the vector used for the type-Ic model, which can be freely chosen within a two-dimensional space, corresponds to a little group which is not maximal. The fact that there exist\[7, 8, 9, 10\] Higgs potentials constructed with the 210 that have absolute minimum appropriate for the type-Ic case, i.e. along a direction which does not correspond to a maximal little group, provides one of the few known counter examples of Michelle’s conjecture.}
3 RG Naturalness of Higgs Parameters

One way to render a GUT more predictive would be the discovery of a dynamical mechanism (quasi) fixing the values of the parameters of the Higgs potential at the GUT scale $M_X$. In this section I discuss one such mechanism which might be available when looking at the GUT as an effective low-energy description of a more fundamental theory (possibly including gravity). From this effective theory viewpoint the GUT becomes relevant for the description of particle physics at some scale $M^*$ higher than $M_X$, and one could investigate the RGE running of the parameters of the Higgs potential between $M^*$ and $M_X$. In certain circumstances the running can push the parameters toward certain ranges of values, and this, besides having implications for the mass spectrum, might determine the SSB. Obviously, in such a scenario the IR structure of the RGEs would be important. For example, to clarify the concept, let me assume that the RGEs have an IR fixed point and the running between $M^*$ and $M_X$ is extremely fast. Under these ideal conditions, the value of the Higgs parameters at the GUT scale would “necessarily” (unless the input parameters at the scale $M^*$ are fine-tuned to avoid that) be within a small neighborhood of the IR fixed point. The SSB would be then essentially determined to be the one corresponding to the values taken by the Higgs parameters at the IR fixed point, so in this sense it would be dynamically determined. Clearly one should not expect to find this ideal scenario in physically relevant contexts. In realistic cases there might or might not be an IR fixed point and the running between $M^*$ and $M_X$ might or might not be fast enough to drive the parameters close to their IR destination. Concerning the speed of the running it must be noted that this speed can receive important contributions from the many particles that become effectively massless at $M_X$, and indeed recent studies[15] have found examples of very fast running above $M_X$. This RG running will not always be driven by IR fixed points, but one can expect the running to be characterized by some IR features and this can affect the likelihood for the Higgs parameters to have given values at the GUT scale $M_X$ in correspondence to given input values at the scale $M^*$. Importantly, the IR features of the RGEs will likely be related to symmetries, so that they might favor values of the Higgs parameters that correspond to one type of SSB (residual symmetry) of the model. (This is clarified in the discussion of some examples in Subsections 3.2 and 3.3.) One can therefore use the investigation of a candidate GUT from this RG viewpoint to establish whether the phenomenological SSB pattern (the SSB pattern ultimately consistent with $SM$) is natural in that GUT.

I observe that, besides leading to the possibility of an increased predictivity (in the sense clarified above) of the SSB pattern, viewing GUTs as effective low-energy descriptions of a more fundamental theory, with the associated RG implications, requires a modification of the conventional tests of the naturalness of a GUT. These conventional tests typically assign a “low grade” to GUTs in which a fine tuning of the Higgs parameters is needed for a phenomenological SSB pattern; however, the
effective-theory viewpoint on GUTs would require to check whether the phenomenological SSB pattern corresponds to fine tuning of the Higgs parameters at the scale $M^*$. It is plausible that a scenario requiring no fine tuning of the Higgs parameters at $M^*$ might correspond via the RG running (for example in presence of an appropriate infra-red fixed point) to a narrow region (apparent fine tuning) of the Higgs parameter space at $M_X$, where the SSB is decided. On the other hand, it is also plausible that a SSB pattern corresponding to a significant portion of the Higgs parameter space actually requires some level of fine tuning at $M^*$ (for example, the considered portion of Higgs parameter space might be “disfavored” by the RG running).

Concerning the scale $M^*$ at which the GUT becomes relevant as an effective low-energy theory, it should be noticed that, while any scenario with $M^* > M_X$ is plausible, the present (however limited) understanding of physics beyond the GUT scale $M_X$ suggests that $M^*$ could be within a few orders of magnitude of the Planck scale $M_P$. In fact, it is reasonable to expect that beyond the GUT there is a theory incorporating gravity (a quantum gravity), and $M_P$ is the scale believed to characterize this more fundamental theory.

It is also important to realize that the type of RG naturalness that I am requiring for GUTs is really a minimal requirement once the GUT is seen as an effective low-energy description of a more fundamental theory. In order to get a consistent GUT from this viewpoint one would also want that “nothing goes wrong” in going from the scale $M^*$ to the scale $M_X$. For example, SSB should not occur “prematurely” at a scale $\mu_{SSB}$ such that $M_X < \mu_{SSB} < M^*$, and the running of the masses involved in the RGEs should be taken into account. In relation to this point, it is interesting that the investigation of the RG naturalness of the parameters of the Higgs potential might ultimately help understanding also the emergence of the GUT scale. At present this scale is just a phenomenological input of a GUT, resulting from the observed (low-energy) values of the $G_{SM}$ coupling constants, but it would be interesting to see it emerging as a scale within the GUT. By studying the RGEs for the parameters of the Higgs potential one might find such a scale; for example, assuming not-too-special initial conditions for the parameters at the scale $M^*$, one might find that the running of the parameters is such that SSB of the GUT occurs typically in the neighborhood of a certain scale (hopefully a phenomenologically reasonable one).

I also want to stress that one could consider additional consistency/naturalness conditions in order to render the GUT consistent with a working cosmological (early universe evolution) scenario. Such conditions should be properly formulated in the language of finite temperature field theory, and should take into account the fact that (contrary to the expectations often expressed in the literature) the dependence of couplings on the renormalization scale is different from their temperature dependence.

Finally, before proceeding to the discussion of two GUTs whose RG naturalness analysis could be quite interesting, I would like to point the attention of the reader toward other ideas expressed in the literature which are somewhat connected with the RG naturalness that I am advocating. First of all, it should be noticed that this RG
naturalness is closely related to the ideas on the role of the IR structure of the RGEs in particle physics discussed in the Refs.\[13, 16, 17\]. Actually, the observation\[17\] that all known physics is consistent with the idea that the relevant (low-energy) parameters are strongly influenced by the IR structure of RGEs is of encouragement for the hope that the RG naturalness that I am advocating might prove useful in analyzing candidate GUTs. The reader will also notice that there are certain analogies between the RG techniques useful to investigate RG naturalness and those used in studies of stability\[18\] and “finite (SUSY) GUTs”\[19\]. Concerning stability analyses of the type in Ref.\[18\], it is worth emphasizing that the concept of stability lives fully at the GUT scale, and therefore does not involve the interpretation of the GUT as an effective low-energy description of a more fundamental theory. Concerning the “finite GUTs”, I observe that some of them, those which correspond to an IR fixed point of the RGEs of the Higgs parameters, should be expected to appear very RG natural from the point of view advocated in this paper; however, dedicated analyses are necessary since often the literature on “finite GUTs” has not paid much attention to SSB and certain parameters of the Higgs potential.

3.1 A nonSUSY Case

In order to render more explicit the idea of RG naturalness of the parameters of the Higgs potential I want to discuss briefly the Higgs potentials of two GUTs, focusing for simplicity on the first step of SSB at the scale $M_X$. In this subsection I consider a nonSUSY case, the one of the type-I $SO(10)$ models reviewed in Sec.2, the next subsection will be devoted to a SUSY case.

In the type-I $SO(10)$ GUTs the first step of SSB involves the Higgs of the 210-dimensional IRREP. The most general\[7, 8, 9, 10, 11, 12\] (renormalizable) Higgs potential which can be constructed with the 210 can be written as

$$V(\Phi) = A ||(\Phi\Phi)_{45}|| + B ||(\Phi\Phi)_{54}|| + C ||(\Phi\Phi)_{210}|| + D \sqrt{||\Phi||} \ ((\Phi\Phi)_{210} \times \Phi) \ ,$$  \hspace{1cm} (7)

where $(R'R'')_R$ stands for the component in the $R$ IRREP of the product of the two IRREPS $R'$ and $R''$.

As discussed in Refs.\[1, 8, 9, 11, 12\], many different directions can be obtained as the minimum of this Higgs potential upon appropriate choices of the values of the Higgs parameters $A$, $B$, $C$, and $D$. Most importantly, different values of the Higgs parameters, by leading to a different direction for the minimum, correspond to different groups of intermediate symmetry (the group of invariance of the vector acquiring a v.e.v. at the scale $M_X$), and in particular some of the possible groups of intermediate symmetry are not phenomenologically interesting since they do not contain $G_{SM}$. Following a conventional approach to GUT model building one would search for regions of $A$, $B$, $C$, $D$ parameter space corresponding to phenomenologically interesting SSB; for example, some conditions on the parameters $A$, $B$, $C$, $D$ necessary
for the realization of the scenarios of type I-a,b,c where derived in Refs. [7, 8, 9, 10, 11, 12]. The analysis of the \( RG \) \textit{naturalness} (in the sense discussed above) of these models would require the study of the RGEs of the parameters \( A, B, C, D \), and the identification of the regions of \( A, B, C, D \) parameter space which are \textit{favored} by the running. For example, the reader can immediately see the substantial difference between the (ideal) case in which one finds a strongly attractive fixed point in a region of \( A, B, C, D \) parameter space corresponding to the type-Ia scenario and the (ideal) case in which one finds a similarly attractive fixed point in a region of \( A, B, C, D \) parameter space corresponding to a phenomenologically inconsistent SSB. In general, even when there are no IR fixed points, one can expect that regions of \( A, B, C, D \) parameter space corresponding to certain symmetries will be preferred by the infrared structure of the RGEs with respect to regions of \( A, B, C, D \) parameter space corresponding to other symmetries.

The investigation of the relevant RGEs is left for future work.

### 3.2 A SUSY Case

In order to illustrate the idea of \( RG \) \textit{naturalness} also in the context of SUSY GUTs, in this subsection I want to discuss briefly the Higgs potential of the simplest such model: minimal SUSY\( SU(5) \). While nonSUSY \( SU(5) \) GUTs are essentially ruled out experimentally (in particular, they predict one-step unification), and therefore the \( SO(10) \) models discussed in the previous subsection and in Sec.2 provide a somewhat minimal (simple-group based) phenomenologically interesting nonSUSY GUT scenario, on the SUSY side even the \( SU(5) \) case (while being affected by several technical difficulties\[20\], which are better handled by some of its extensions\[21\]) can be made consistent with the low-energy data on the coupling constants and the present experimental lower limit on proton decay. It is therefore reasonable to use the minimal SUSY\( SU(5) \) model, in which the chiral field \( \Sigma \) involved in the first step of SSB is described by the 24-dimensional (adjoint) IRREP, to illustrate the idea of \( RG \) \textit{naturalness} in the context of SUSY GUTs.

The Higgs potential relevant for the first step of SSB can be written (see, \textit{e.g.}, Ref.[22]) as

\[
V = V_{\text{susy}} + V_{\text{soft}} ,
\]

where \( V_{\text{susy}} \) is the supersymmetric part of the potential, which can be expressed in terms of the superpotential \( W \),

\[
W(\Sigma) = \frac{\mu}{2} \text{Tr}\Sigma^2 + \frac{b}{3} \text{Tr}\Sigma^3 ,
\]

as

\[
V_{\text{susy}}(\Sigma) = \left| \frac{\partial W}{\partial \Sigma_i} \right|^2 + \text{“Dterms”} ,
\]
while $V_{soft}$ contains the terms breaking SUSY “softly” and can be written as:

$$V_{soft}(\Sigma) = \frac{m_o^2}{2} |\Sigma_i|^2 + m_o \Sigma_i \frac{\partial W}{\partial \Sigma_i} + m_o (A - 3) W + \text{h.c.} .$$

(11)

The supersymmetric part of the potential has several degenerate minima, corresponding to directions with little group $SU(5)$, $SU(4) \otimes U(1)$, or $G_{SM}$. The degeneracy is removed by the soft terms; in particular, in the phenomenologically relevant case $m_o/\mu < < 1$ one can show that for $A < 3$ the minimum is in a direction with little group $G_{SM}$ (SSB), whereas for $A > 3$ the minimum is in a direction with little group $SU(5)$ (unbroken symmetry).

Within this SUSY model the analysis of the RG naturalness of the phenomenological SSB to $G_{SM}$ requires the study of the RGEs of the parameters of $V$ in order to establish whether it is likely/plausible that $A < 3$ at $M_X$. Ideally, one would like to find a strongly attractive infra-red fixed point corresponding to $A < 3$.

4 A Simple GUT Matter Content?

As illustrated by the review of SO(10) GUTs in Sec.2, GUTs typically involve a remarkably complex matter content. Most notably, the Higgs sector consists of several carefully selected IRREPs of the GUT group, and, like in the $SM$, the fermionic sector of leptons and quarks is arbitrary and is plagued by the family triplication.

This complexity might well be telling us that the GUT idea needs drastic revisions; however, in this paper I take the point of view that the complexity of the matter content might be only apparent. I therefore want to mention a few appealing scenarios in which this complexity might arise from a fundamental simplicity.

For continuity with the line of argument advocated in the previous section, let me start by mentioning the possibility that as an effective low-energy description of a more fundamental theory, the (effective) matter content of the GUT at the scale $M_X$ might be fixed by the RG running. It is in fact plausible that some IRREPs tend to get heavy masses via RG running, whereas the masses of other IRREPs (the ones relevant for symmetry breaking and low-energy phenomenology) might tend to be light (i.e. order $M_X$ or less). This type of running of masses (or other parameters) might even be responsible for the cancellation of anomalies at low energies; indeed, the RG running is known to be easily driven by symmetries.

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4 The reader should notice that I put the emphasis on the (apparently) ad hoc selection of the Higgs representations, rather than the size of the representations involved. For example, the Higgs content of the SO(10) GUTs reviewed in Sec.2 is unappealing to some authors primarily because of the size of some of the IRREPs involved, while I am most puzzled by the fact that among the available IRREPs only a carefully selected bunch is to be involved (in order to achieve agreement with phenomenology).

5 For GUTs in which (like the $SM$ and unlike SO(10) GUTs) the absence of anomaly results from an “accidental” cancellation among the anomaly contributions of the different IRREPs involved, one has even more reasons to be puzzled.
The viability of such a mechanism of selection of the matter content could be tested in simplified/familiar contexts; for example, one could study the RG equations between $M_P$ and $M_X$ in a model obtained by adding one extra IRREP of Higgses to the type-I SO(10) GUT, and check whether the extra IRREP tends to decouple from the rest.

If not resulting from RG running, the complexity of the matter content might even be the result of compositeness, in line with the most honored tradition of particle physics. I point to the attention of the reader the investigations reported in Ref.[24] (and references therein) which consider the possibility of preonic GUTs.

Another hypothesis which has been gaining some momentum in the literature on low-dimensional Quantum-Gravity toy models is that Quantum Gravity might be quite selective concerning the type of matter “it likes to deal with”, i.e. the requirement of overall consistency of Quantum Gravity might fix the matter content. Results pointing (however faintly) in this direction can be found for example in certain studies of discretized two-dimensional Quantum Gravities[25], and studies of the Dirac quantization of certain two-dimensional Quantum Gravities in the continuum[26].

In general it is clear that, in trying to make sense of the matter content of a GUT, it would be useful to observe group theoretical properties that single out the IRREPs corresponding to that matter content. Just to give an example of what I mean with “group theoretical properties”, I observe that the following SO(10) relations

\[
\begin{align*}
16 \otimes 16 &= 10_S \oplus 120_A \oplus 126_S \\
\ol{16} \otimes \ol{16} &= 10_S \oplus 120_A \oplus \ol{126}_S \\
\ol{16} \otimes 16 &= 1 \oplus 45 \oplus 210 \\
\ol{16} \otimes 16 \otimes 16 &= 16 \oplus 16 \oplus 16 \oplus 144 \oplus 144 \oplus 560 \oplus 560 \oplus 1200 \oplus 1440 
\end{align*}
\]

(12)
come quite close to singling out the matter content of the type-I SO(10) GUTs. One could then examine similar group theoretical properties, and try to figure out their origin. For example, an observation such as [(12)] might motivate related work on preonic GUTs, and from the preonic viewpoint one might be even more intrigued by the observation that the fermions (16’s) would be contained in the product of an odd number of preons ($\ol{16} \otimes 16 \otimes 16$), while the bosons (Higgses of $10 \oplus 126 \oplus \ol{126} \oplus 210$) would be contained in the product of an even number of preons ($(16 \otimes 16) \oplus (\ol{16} \otimes \ol{16}) \oplus (16 \otimes 16)$).

5 Closing Remarks

Perhaps the only robust concept discussed in this paper is the one concerning the way in which the conventional tests of the naturalness of a GUT need to be modified if the GUT is seen as a low-energy effective description of a more fundamental theory.
On a more speculative side, I also articulated the hope that the correct GUT (if
there is one) could be such that its SSB pattern is essentially predicted (in the sense
of the \textit{RG naturalness} I discussed) rather than being a free input; this would fit well
the general trend of increased predictivity at each new stage of our understanding of
particle physics.

I have also looked at the complexity of the matter content of GUTs, and explored
the possibility that this might be an apparent complexity, hiding a fundamental sim-
plicity. In Sec.4 I have speculated on a few appealing candidates for this simplicity;
however, it is reasonable to expect that real progress in this direction will require
dramatic new developments.

The ideas discussed in this paper have been developing over a long period of time,
and there are several colleagues I am indebted to. Subsec.3.3 was added only after
the start of preliminary discussions with B. Allanach and O. Philipsen, ultimately
leading to the collaboration\cite{27}. The suitability of the minimal SUSYSU(5) model
for a \textit{RG naturalness} analysis emerged in the context of those discussions. I would
also like to thank G. Ross, for exposing me to his powerful arguments on the possible
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his vantage point my first GUTsy steps\cite{8} a few years ago.

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