Research Article

On Highly Dimensional Elastic and Nonelastic Interaction between Internal Waves in Straight and Varying Cross-Section Channels

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This manuscript studies the computational solutions of the highly dimensional elastic and nonelastic interaction between internal waves through the fractional nonlinear (4 + 1)-dimensional Fokas equation. This equation is considered as the extension model of the two-dimensional Davey–Stewartson (DS) and Kadomtsev–Petviashvili (KP) equations to a four spatial dimensions equation with time domain. The modified Khater method is employed along the Atangana–Baleanu (AB) derivative operator to construct many novel explicit wave solutions. These solutions explain more physical and dynamical behavior of that kind of the interaction. Moreover, 2D, 3D, contour, and stream plots are demonstrated to explain the detailed dynamical characteristics of these solutions. The novelty of our paper is shown by comparing our results with those obtained in previous published research papers.

1. Introduction

Internal waves are waves that spread inside a stream, with gradients of intensity [1–3]. The surface gravity waves pass along the broad pressure boundary between air and water, while internal waves migrate inside the ocean over gradients of intensity [4–7]. Perturbations of these gradients of intensity are preserved by momentum, which creates a propagating motion [8–11].

Globally, internal waves play a significant role in the ocean, providing nutrients to surface waters that facilitate the growth of phytoplankton, the foundation of the ocean food chain [12–15]. Created primarily by the tide’s interaction with ocean floor and water topography, internal waves may bring the energy from these forces through the entire ocean basins [16, 17]. As internal waves pass through the continental shelf, they interact with the topography, and as the gravity of the surface steepens and splits on the sea, internal waves steep their energy in the shelf and dissipate it [18, 19]. When the internal waves rise, they turn into nonlinear waves of fluid that may assume several forms (e.g., solitons, bores, and boluses), all of which have the potential to bring deep water that has different properties (probably colder, higher in nutrients, lower in oxygen, lower in pH) across the shelf and into shallower waters [20–22].

Depending on the potential of the nonlinear partial differential equation to describe several complicated processes in diverse fields such as physiology, plasma physics, hydrodynamics, fluid mechanics, and optics, numerous precise and computational schemes such as in [23–26] have been developed. Using inspired schemes, computational and technical advances are seen as the basic usefulness of solving these phenomena [27–31]. Such schemes have recently been regarded as simple methods for discovering the different
Formulas of moving wave solutions to these dynamic phenomena [32–34]. However, in the nonlinear partial differential equation (NLPDE), with an integer instruction, several researchers have struggled to extract and formulate certain complex phenomena [35, 36]. The fractional equation is then deemed an appropriate solution to this issue because it includes a nonlocal property that is not NLPDE-based with an integer [37–40].

In this research, we study the nonlinear fractional (4 + 1)-dimensional Fokas model that is mathematically given by [41–43]

\[
4D_\xi^\alpha U_x - U_{xxxx} + U_{yyyy} + 12 U_y U_x + 12 U_{xy} U_y - 6U_{wz} = 0, \quad (0 < \alpha < 1), \tag{1}
\]

where \( U \) is the function of the elastic and nonelastic interaction between internal waves in straight and varying cross-section channels. Implementation of the following AB-derivative definitions on equation (1) with the following wave transformation \( U(x, y, z, \tau, \nu) = (\xi, \zeta) = (((\alpha - 1) \lambda^{\alpha - 1})/B(\alpha) \sum_{n=0}^\infty -((\alpha - 1))^{n} \xi (1 - \alpha \xi) + k_1 x + k_2 y + k_3 z \), where \( k_i \) and \( \lambda (i = 1, 2, 3, 4) \) are arbitrary constants [37–40], while \( B(\alpha) \) is a normalized function, converts the fractional PDE into the next integer order ODE:

\[
a_1 \psi'' + a_2 \psi'''' + a_3 \psi' + a_4 \psi''' = 0, \tag{2}
\]

where \( a_1 = (4k_1 \lambda - 6k_3 k_4), a_2 = (k_1 k_2, a_3 = 12k_4 \), and \( a_4 = -6k_3 k_4 \).

The remaining parts of our research parts are organized as follows: Section 2 employs the modified Khater system [44–49] to provide the nonlinear fractional Fokas model with novel solitonic solutions. Section 3 describes the outcomes and provides the physical description of the sketches seen. This work is concluded in Section 4.

2. Applications

Usage of the modified Khater technique via the concepts of homogeneous equilibrium on equation (2) provides general solutions:

\[
\psi(\xi) = \sum_{i=1}^m a_i K^{\psi(i)} + \sum_{i=1}^m b_i K^{-\psi(i)} + a_0 = a_1 K^{\psi(\xi)} + a_2 K^{2\psi(\xi)} + a_3 K^{-\psi(\xi)}, \tag{3}
\]

where \( a_1, b_1, i = 0, 1, 2, \ldots, a_m \neq 0, \) or \( b_m \neq 0 \). Additionally, \( \psi(\xi) \) is the solution function of \( \psi' = (1/\ln(K)) [\delta + \rho \xi^{\psi(i)} + \kappa K^{-\psi(i)}] \), \( \delta, \rho, \) and \( \kappa \) are arbitrary constants. Using equation (3) through its auxiliary equation in the modified Khater technique's framework gives the following families for the above-mentioned arbitrary constants.

Family I:

\[
a_1 \rightarrow 0, \quad a_2 \rightarrow 0, \quad b_1 \rightarrow \frac{b_4 \delta}{\rho}, \quad a_3 \rightarrow \alpha_4. \tag{4}
\]

Family II:

\[
a_1 \rightarrow \frac{a_1 \delta}{\rho}, \quad a_2 \rightarrow 0, \quad b_1 \rightarrow 0, \quad a_3 \rightarrow \frac{a_2 a_4}{12 \rho^2}, \quad a_4 \rightarrow a_4. \tag{5}
\]

Consequently, the explicit solutions of equation (1) are given in the following forms.

In case of \( \delta^\alpha - 4\rho \xi < 0, \rho \neq 0 \), we obtain

\[
U(\xi)_{\nu 1} = a_0 + \frac{2b_2 \rho \left( \delta \sqrt{4 \rho \xi - \delta^\alpha} \tan \left( \frac{1}{2} \xi \sqrt{4 \rho \xi - \delta^\alpha} \right) - \delta^\alpha + 2 \rho \xi \right)}{\sqrt{\delta - \sqrt{4 \rho \xi - \delta^\alpha} \tan \left( \frac{1}{2} \xi \sqrt{4 \rho \xi - \delta^\alpha} \right)}}. \tag{6}
\]

In case of \( \delta^\alpha - 4\rho \xi > 0, \rho \neq 0 \), we obtain
\[ U(\xi)_{1,3} = a_0 - \frac{2b_3\rho (\delta^2 - 4\rho \kappa \coth((1/2)\xi \sqrt{\delta^2 - 4\rho \kappa}) + \delta^2 - 2\rho \kappa))}{\kappa ((\delta^2 - 4\rho \kappa \coth((1/2)\xi \sqrt{\delta^2 - 4\rho \kappa}) + \delta)^2)}, \]  
\[ U(\xi)_{1,4} = a_0 - \frac{2b_3\rho (\delta^2 - 4\rho \kappa \coth((1/2)\xi \sqrt{\delta^2 - 4\rho \kappa}) + \delta^2 - 2\rho \kappa))}{\kappa ((\delta^2 - 4\rho \kappa \coth((1/2)\xi \sqrt{\delta^2 - 4\rho \kappa}) + \delta)^2)}, \]  
\[ U(\xi)_{1,3} = \frac{a_2(\delta^2 - 4\rho \kappa) \sech^2((1/2)\xi \sqrt{\delta^2 - 4\rho \kappa}) - 4\rho \kappa)}{4\rho^2} + a_0, \]  
\[ U(\xi)_{1,4} = \frac{a_2(\delta^2 - 4\rho \kappa) \csch^2((1/2)\xi \sqrt{\delta^2 - 4\rho \kappa}) - 4\rho \kappa)}{4\rho^2} + a_0. \]  

In case of \( \rho \kappa > 0, \kappa \neq 0, \rho \neq 0, \) and \( \delta = 0, \) we obtain

\[ U(\xi)_{1,5} = a_0 + \frac{b_3\rho \cot^2((\xi \sqrt{\rho \kappa}))}{\kappa}, \]  
\[ U(\xi)_{1,6} = a_0 + \frac{b_3\rho \tan^2((\xi \sqrt{\rho \kappa}))}{\kappa}, \]  
\[ U(\xi)_{1,5} = \frac{a_2\kappa \tan^2((\xi \sqrt{\rho \kappa}))}{\rho} + a_0, \]  
\[ U(\xi)_{1,6} = \frac{a_2\kappa \cot^2((\xi \sqrt{\rho \kappa}))}{\rho} + a_0. \]

In case of \( \rho \kappa < 0, \kappa \neq 0, \rho \neq 0, \) and \( \delta = 0, \) we obtain

\[ U(\xi)_{1,7} = a_0 + \frac{b_3\rho \cot^2((\xi \sqrt{\rho \kappa}))}{\kappa}, \]  
\[ U(\xi)_{1,8} = a_0 + \frac{b_3\rho \tan^2((\xi \sqrt{\rho \kappa}))}{\kappa}, \]  
\[ U(\xi)_{1,7} = \frac{a_2\kappa \tan^2((\xi \sqrt{\rho \kappa}))}{\rho} + a_0, \]  
\[ U(\xi)_{1,8} = \frac{a_2\kappa \cot^2((\xi \sqrt{\rho \kappa}))}{\rho} + a_0. \]

In case of \( \delta = 0 \) and \( \kappa = -\rho, \) we obtain

\[ U(\xi)_{1,9} = a_0 + b_2\tanh^2((\xi \kappa)), \]  
\[ U(\xi)_{1,9} = a_2\coth^2((\xi \kappa)) + a_0. \]

In case of \( \delta = (\kappa/2) = \kappa \) and \( \rho = 0, \) we obtain

\[ U(\xi)_{1,10} = a_0 + \frac{b_2\rho \xi^2}{2(e^{\xi^2} - 2)^2}. \]

In case of \( \delta = \rho = \kappa \) and \( \kappa = 0, \) we obtain

\[ U(\xi)_{1,10} = \frac{2a_2\kappa (\delta^2 - 2)(2\rho \kappa (\delta^2 + 2) - \delta^3 \xi)}{\delta^4 \xi^2 \rho} + a_0. \]

3. Results and Discussion

This section shows our obtained solutions and their novelty. Also, we compare our obtained solutions with those of previously published articles to show the similarity and...
difference between our and their solutions. Our discussion is divided into three main parts, which are the used analytical method, obtained solutions, and figure interpretation:

(1) The used computational scheme:
The modified Khater method have been used for the first time for applying to the fractional nonlinear (4+1)-dimensional Fokas equation. This modified method is considered as one of the most general analytical schemes in this field; especially, it covers more than twelve recent analytical schemes [50].

(2) The obtained solutions:
This part gives a comparison between our obtained solutions and those obtained in previously accepted papers. In [41–43] by Wan-Jun Zhang and Tie-Cheng Xia, Ruoxia Yao, Yali Shen, and Zhibin Li, and Wei Li and Yinping Liu, respectively, who applied the Hirota bilinear method, the bilinear form, and Hirota method, receptively to a fractional nonlinear (4 + 1)-dimensional Fokas equation, many distinct types of solutions for these fractional nonlinear models were obtained. All our obtained solutions of the investigated model are new and different from those obtained in [41–43].

(3) The figures interpretation:
We have represented some of our obtained solutions in three distinct types of figures (3D, 2D, and contour plots) to explain kink, antikink, periodic, and singular shapes to illustrate the perspective view of the solution, the wave propagation pattern of the wave along x-axis, and the overhead view of the solution for the following values of the parameters:

Figure 1: Solitary wave solutions equation (7) in three, two, and contour plots.
Figure 2: Solitary wave solutions equation (9) in three, two, and contour plots.

Figure 3: Continued.
Figure 3: Solitary wave solutions equation (13) in three, two, and contour plots.

Figure 4: Solitary wave solutions equation (16) in three, two, and contour plots.
\[ a_0 = -2, b_2 = 1, \delta = 5, k_1 = -1, k_2 = 3, k_3 = 4, k_4 = \frac{33}{2}, \lambda = -6, \rho = 2, w = -\frac{2}{3}, y = 1, z = 2, \kappa = 3a_2 = 7, \]
\[ &a_0 = 6, \delta = 3, k_1 = 5, k_2 = 3, k_3 = 4, k_4 = \frac{33}{2}, \lambda = 9, \rho = 1, w = -\frac{2}{3}, y = 1, z = 2, \kappa = 2 & & a_0 = -2, b_2 = 1, \]
\[ \delta = 0, k_1 = -1, k_2 = 3, k_3 = 4, k_4 = \frac{33}{2}, \lambda = -6, \rho = -3, w = -\frac{2}{3}, y = 1, z = 2, \kappa = 3 & & a_0 = -2, \delta = 0, \]
\[ \kappa = 2, k_1 = -1, k_2 = 3, k_3 = 4, k_4 = \frac{33}{2}, \lambda = -6, w = -\frac{2}{3}, y = 1, z = 2 \] \hspace{1cm} (23)

4. Conclusion

This research paper has successfully investigated the nonlinear fractional nonlinear (4 + 1)-dimensional Fokas model via the modified Khater method that has used the Atangana–Baleanu derivative operator to convert the fractional form of the studied model to a nonlinear ordinary differential equation with an integer order. Many distinct exact traveling and solitary wave solutions have been obtained. These solutions have been illustrated via various sketches (Figures 1–4) that explain more novel properties of the considered fractional models. The accuracy and novelty of our obtained solutions have been explained. The powerfulness and effectiveness of the used techniques are also explained and verified.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare no conflicts of interest.

Authors’ Contributions

Mostafa M. A. Khater and Qiang Zheng are responsible for the analytical simulation. Haiyong Qin and Raghda A. M. Attia are responsible for the final editing and revision of the whole paper.

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References

[1] A. S. J. Wyatt, J. J. Leichter, L. T. Toth, T. Miyajima, R. B. Aronson, and T. Nagata, “Heat accumulation on coral reefs mitigated by internal waves,” Nature Geoscience, vol. 13, no. 1, pp. 28–34, 2020.
[2] C. Chen, W. Liu, and C. Bi, ”A two-grid characteristic finite volume element method for semilinear advection-dominated diffusion equations,” Numerical Methods for Partial Differential Equations, vol. 29, no. 5, pp. 1543–1562, 2013.
[3] C. Chen and X. Zhao, “A posteriori error estimate for finite volume element method of the parabolic equations,” Numerical Methods for Partial Differential Equations, vol. 33, no. 1, pp. 259–275, 2017.
[4] E. C. Reid, T. M. DeCarlo, A. L. Cohen et al., ”Internal waves influence the thermal and nutrient environment on a shallow coral reef,” Limnology and Oceanography, vol. 64, no. 5, pp. 1949–1965, 2019.
[5] C. Chen, K. Li, Y. Chen, and Y. Huang, ”Two-grid finite element methods combined with Crank-Nicolson scheme for nonlinear Sobolev equations,” Advances in Computational Mathematics, vol. 45, no. 2, pp. 611–630, 2019.
[6] C. Chen, H. Liu, X. Zheng, and H. Wang, ”A two-grid MMOC finite element method for nonlinear variable-order time-fractional mobile/immobile advection-diffusion equations,” Computers & Mathematics with Applications, vol. 79, no. 9, pp. 2771–2783, 2020.
[7] X. Zhang, Y. Wu, and L. Caccetta, ”Nonlocal fractional order differential equations with changing-sign singular perturbation,” Applied Mathematical Modelling, vol. 39, no. 21, pp. 6543–6552, 2015.
[8] L.-A. Coulston, D. Lecoanet, B. Favier, and M. Le Bars, ”The energy flux spectrum of internal waves generated by turbulent convection,” Journal of Fluid Mechanics, vol. 854, 2018.
[9] X. Zhang, L. Liu, and Y. Wu, ”Multiple positive solutions of a singular fractional differential equation with negatively perturbed term,” Mathematical and Computer Modelling, vol. 55, no. 3–4, pp. 1263–1274, 2012.
[10] X. Zhang, L. Liu, Y. Wu, and Y. Cui, ”A sufficient and necessary condition of existence of blow-up radial solutions for a k-hessian equation with a nonlinear operator,” Nonlinear Analysis: Modelling and Control, vol. 25, no. 1, pp. 126–143, 2020.
[11] X. Zhang, J. Xu, J. Jiang, Y. Wu, and Y. Cui, ”The convergence analysis and uniqueness of blow-up solutions for a Dirichlet problem of the general k-hessian equations,” Applied Mathematics Letters, vol. 102, Article ID 106124, 2020.
[12] B. Qiu, T. Nakano, S. Chen, and P. Klein, ”Submesoscale transition from geostrophic flows to internal waves in the northwestern pacific upper ocean,” Nature Communications, vol. 8, no. 1, pp. 1-10, 2017.
[13] X. Zhang, J. Jiang, Y. Wu, and Y. Cui, ”The existence and nonexistence of entire large solutions for a quasilinear Schrödinger elliptic system by dual approach,” Applied Mathematics Letters, vol. 100, Article ID 106018, 2020.
[14] X. Zhang, Y. Wu, and L. Caccetta, ”Nonlocal fractional order differential equations with changing-sign singular
perturbation," Applied Mathematical Modelling, vol. 39, no. 21, pp. 6543–6552, 2015.

[15] X. Zhang, L. Liu, and Y. Wu, "Multiple positive solutions of a singular fractional differential equation with negatively perturbed term," Mathematical and Computer Modelling, vol. 55, no. 3–4, pp. 1263–1274, 2012.

[16] L.-A. Couston, D. Lecocanet, B. Favier, and M. Le Bars, "Order out of chaos: slowly reversing mean flows emerge from turbulently generated internal waves," Physical Review Letters, vol. 120, no. 24, Article ID 244505, 2018.

[17] P. Chen, X. Zhang, and Y. Li, "Existence and approximate controllability of fractional evolution equations with nonlocal conditions via resolvent operators," Fractional Calculus and Applied Analysis, vol. 23, no. 1, pp. 286–291, 2020.

[18] R. Barkan, K. B. Winters, and J. C. McWilliams, "Stimulated imbalance and the enhancement of eddy kinetic energy dissipation by internal waves," Journal of Physical Oceanography, vol. 47, no. 1, pp. 181–198, 2017.

[19] P. Chen, X. Zhang, and Y. Li, "Approximate controllability of non-autonomous evolution system with nonlocal conditions," Journal of Dynamical and Control Systems, vol. 26, no. 1, pp. 1–16, 2020.

[20] O. Kurkina, E. Rouvinskaya, T. Talipova, and T. Soomere, "Propagation regimes and populations of internal waves in the Mediterranean Sea basin," Estuarine, Coastal and Shelf Science, vol. 185, pp. 44–54, 2017.

[21] D. Bouffard, R. E. Zdorovennov, G. E. Zdorovennova, N. Pasche, A. Wüest, and A. Y. Terzhevik, "Ice-covered lake Omega: effects of radiation on convection and internal waves," Hydrobiologia, vol. 780, no. 1, pp. 21–36, 2016.

[22] P. Chen, X. Zhang, X. Zhang, and Y. Li, "A blowup alternative result for fractional nonautonomous evolution equation of Volterra type," Communications on Pure & Applied Analysis, vol. 17, no. 5, pp. 1975–1992, 2018.

[23] H. Kim, R. Sakhivel, A. Debbouche, and D. F. M. Torres, "Traveling wave solutions of some important Wick-type fractional stochastic nonlinear partial differential equations," Chaos, Solitons & Fractals, vol. 131, p. 109542, 2020.

[24] A. R. Seadawy, D. Lu, and M. M. A. Khater, "Bifurcations of solitary wave solutions for Dodd-Bullough-Mikhailov equation and coupled Higgs equation and their applications," Advances in Differential Equations, vol. 2020, no. 1, pp. 1–12, 2020.

[25] M. Elbrolosy and A. Elmandough, "Bifurcation and new traveling wave solutions for (2 + 1)-dimensional nonlinear Nizhnik–Novikov–Veselov dynamical equation," The European Physical Journal Plus, vol. 135, no. 6, p. 533, 2020.

[26] W. Zhu, Y. Xia, Y. Bai et al., "Bifurcations and exact traveling wave solutions of Gerdjikov-Ivanov equation with perturbation terms," Advances in Differential Equations, vol. 25, no. 5/6, pp. 279–314, 2020.

[27] C. Park, M. M. Khater, R. A. Attia, W. Alharbi, and S. S. Alodhaibi, "An explicit plethora of solution for the fractional nonlinear model of the low–pass electrical transmission lines via Atangana–Baleanu derivative operator," Alexandria Engineering Journal, vol. 59, no. 3, pp. 1205–1214, 2020.

[28] C. Yue, M. M. Khater, R. A. Attia, and D. Lu, "The plethora of explicit solutions of the fractional KS equation through liquid–gas bubbles mix under the thermodynamic conditions via Atangana–Baleanu derivative operator," Advances in Difference Equations, vol. 2020, no. 1, pp. 1–12, 2020.

[29] M. M. Khater, R. A. Attia, and D. Lu, "Computational and numerical simulations for the fractional nonlinear Kolmogorov–Petrovskii–Piskunov (FKP) equation," Physica Scripta, vol. 95, no. 5, Article ID 055213, 2020.

[30] A.-H. Abdel-Aty, M. M. A. Khater, R. A. M. Attia, M. Abdel-Aty, and H. Eleuch, "On the new explicit solutions of the fractional nonlinear space-time nuclear model," Fractals, vol. 28, no. 8, Article ID 2040035, 2020.

[31] W.-J. Zhang and T.-C. Xia, "Solitary wave, M-lump and localized interaction solutions to the (4 + 1)-dimensional Fokas equation," Physica Scripta, vol. 95, no. 4, Article ID 045217, 2020.

[32] R. Yao, Y. Shen, and Z. Li, "Lump solutions and bilinear Bäcklund transformation for the (4 + 1)-dimensional Fokas equation," Mathematical Sciences, vol. 59, no. 3, pp. 1205–1214, 2020.

[33] Y. Li and Y. Liu, "To construct lumps, breathers and interaction solutions of arbitrary higher-order for a (4 + 1)-dimensional Fokas equation," Modern Physics Letters B, vol. 34, no. 21, Article ID 2050221, 2020.

[34] R. A. M. Attia, D. Lu, T. Ak, and M. M. A. Khater, "Optical wave solutions of the higher-order nonlinear Schrödinger equation with the non-Kerr nonlinear term via modified perturbation," Applied Mathematical Modelling, vol. 39, no. 21, pp. 6543–6552, 2015.
Khater method,” *Modern Physics Letters B*, vol. 34, no. 5, Article ID 2050044, 2020.

[45] H. Qin, R. A. Attia, M. Khater, and D. Lu, "Ample soliton waves for the crystal lattice formation of the conformable time-fractional (N + 1) Sinh-Gordon equation by the modified Khater method and the Painlevé property," *Journal of Intelligent & Fuzzy Systems*, vol. 38, no. 3, pp. 2745–2752, 2020.

[46] M. M. Khater, C. Park, A.-H. Abdel-Aty, R. A. Attia, and D. Lu, "On new computational and numerical solutions of the modified Zakharov–Kuznetsov equation arising in electrical engineering," *Alexandria Engineering Journal*, vol. 59, no. 3, pp. 1099–1105, 2020.

[47] C. Yue, D. Lu, M. M. Khater, A.-H. Abdel-Aty, W. Alharbi, and R. A. Attia, "On explicit wave solutions of the fractional nonlinear DSW system via the modified Khater method," *Fractals*, 2020.

[48] A. T. Ali, M. M. A. Khater, R. A. M. Attia, A.-H. Abdel-Aty, and D. Lu, "Abundant numerical and analytical solutions of the generalized formula of Hirota-Satsuma coupled KdV system," *Chaos, Solitons & Fractals*, vol. 131, Article ID 109473, 2020.

[49] M. M. Khater, C. Park, D. Lu, and R. A. Attia, "Analytical, semi-analytical, and numerical solutions for the Cahn–Allen equation," *Advances in Difference Equations*, vol. 2020, no. 1, pp. 1–12, 2020.

[50] M. M. A. Khater, R. A. M. Attia, and D. Lu, "Explicit lump solitary wave of certain interesting (3 + 1)-dimensional waves in physics via some recent traveling wave methods," *Entropy*, vol. 21, no. 4, p. 397, 2019.