SCALAR PERTURBATIONS IN DGP BRANEWORLD COSMOLOGY

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We solve for the behaviour of cosmological perturbations in the Dvali-Gabadadze-Porrati (DGP) braneworld model using a new numerical method. Unlike some other approaches in the literature, our method uses no approximations other than linear theory and is valid on large scales. We examine the behaviour of late-universe density perturbations for both the self-accelerating and normal branches of DGP cosmology.

1 Introduction

The Dvali-Gabadadze-Porrati (DGP) model postulates that we live in a 4-dimensional hypersurface in a 5-dimensional Minkowski bulk. General Relativity (GR) is recovered at small scales (smaller than the crossover scale $r_c$) due to the inclusion of an induced gravity term in the action.

This model has two distinct classes of cosmological solutions. One of them exhibits accelerated expansion at late times without the need to include any exotic cosmological fluids, such as dark energy, or any brane tension that acts as an effective 4-dimensional cosmological constant. Hence, this branch of solutions is called “self-accelerating”. To explain the observed acceleration we require $r_c \sim H_0^{-1}$, where $H_0$ is the current value of the Hubble parameter. It is expected that structure formation will help to distinguish the self-accelerating DGP universe from dark energy models based on 4-dimensional GR. This is because the growth of cosmological perturbations is very sensitive to the existence of an extra dimension. A full 5-dimensional treatment is required to model these perturbations, which is why obtaining observational predictions for the behaviour of fluctuations in the DGP model is technically challenging.

Several authors have considered the problem of the dynamics of perturbations in the DGP model, but they have all relied on some sort of approximation or simplifying ansatz. One example of this is the quasi-static (QS) approximation scheme, which solves the perturbative equations of motion by focussing on the extreme subhorizon regime.

In this paper we present a complete numerical analysis of the evolution of scalar perturbations in the DGP model. Mathematically, the problem involves the solution of a partial differential equation in the bulk coupled to an ordinary differential equation on the brane. A numerical method for dealing with such systems has previously been developed for cosmological perturbations in the Randall-Sundrum (RS) model. However, the DGP problem is more complicated than the RS case due to a non-local boundary condition on the bulk field. Hence, the algorithm used in this paper represents a significant generalization of the one used in the previous case.

As alluded to above, there is another “normal” branch of solutions in the DGP model. We cannot explain the late time accelerated expansion of the Universe using the normal branch without including an effective cosmological constant induced by the brane tension $\sigma$. However, by allowing a non-zero $\sigma$, normal branch solutions can mimic dark energy models with the equation of state $w$ smaller than $-1$. Unlike 4-dimensional models that realize $w < -1$ by the introduction of a phantom field, the normal branch of DGP cosmology is ghost-free. This unique feature is what motivates us to numerically study the perturbations of the normal branch of DGP cosmology.

*Based on the work done in collaboration with Kazuya Koyama, Sanjeev S. Seahra and Fabio P. Silva.
2 Background solution

In the DGP model the brane dynamics is governed by

\[ H = \dot{a} = \frac{1}{2r_c} \left[ \epsilon + \sqrt{1 + \frac{4}{3} \kappa_4^2 r_c^2 (\rho + \sigma)} \right], \quad (1) \]

\[ \frac{d\rho}{dt} = -3(1 + w)\rho H, \quad (2) \]

where \( a(t) \) is the scale factor of the brane universe, normalized to unity today, a dot denotes the derivative with respect to the proper time along the brane, \( t \), and \( r_c = \kappa_5^2 / 2\kappa_4^2 \). We assume that the stress-energy tensor of the brane matter is of the perfect fluid form. Note that, as in GR, the stress-energy tensor is conserved. The \( \epsilon = \pm 1 \) parameter reflects the fact that when we impose the \( \mathbb{Z}_2 \) symmetry across the brane, we have two choices for the half of the bulk manifold we discard. When \( \epsilon = +1 \), we have that \( H r_c \approx 1 \) when the density of brane matter is small, \(|\rho + \sigma| \ll \kappa^{-2} r_c^{-2} \). This implies a late-time accelerating universe, which is why the \( \epsilon = +1 \) case is called the self-accelerating branch and the \( \epsilon = -1 \) case is called the normal branch.

3 Master equations governing perturbations

In this section we describe the formulae governing scalar perturbations in the late-time matter dominated universe. We take the matter content of the brane to be a dust fluid (i.e., cold dark matter), \( w = 0 \), therefore we have \( \rho \propto a^{-3} \).

It can be shown that the dynamics of all the scalar perturbations of the bulk geometry can be derived from a single scalar bulk degree of freedom. After Fourier decomposition, we find that the mode amplitude \( \Omega = \Omega(u, v) \) of this master field obeys

\[ 0 = \frac{\partial^2 \Omega}{\partial u \partial v} - \frac{3}{2v} \frac{\partial \Omega}{\partial u} + \frac{k^2 r_c^2}{4v^2} \Omega, \quad (3) \]

where \( u \) and \( v \) are dimensionless null coordinates.

We also find the following boundary condition for \( \Omega \):

\[ (\partial_y \Omega)_b = -\frac{\epsilon \gamma_1}{2H} \Omega_b + \frac{9\epsilon \gamma_3}{4} \Omega_b - \frac{3(\epsilon \gamma_3 k^2 + \gamma_4 H^2 a^2)}{4H a^2} \Omega_b + \frac{3\epsilon r_c \kappa_4^2 \rho a^3 \gamma_4}{2k^2} \Delta; \quad (4) \]

the following equation of motion for the density contrast of the cold dark matter, \( \Delta \):

\[ \ddot{\Delta} + 2H \dot{\Delta} - \frac{1}{2} \kappa_4^2 \rho \dot{\gamma}_2 \Delta = -\frac{\epsilon \gamma_4 k^4}{4a^5} \Omega_b; \quad (5) \]

and the following expressions for the two metric potentials \( \Phi \) and \( \Psi \):

\[ \Phi = +\frac{\kappa_4^2 \rho a^2 \gamma_1}{2k^2} \Delta + \frac{\epsilon \gamma_1}{4r_c a} \dot{\Omega}_b - \frac{\epsilon (k^2 + 3H^2 a^2) \gamma_1}{12H r_c a^3} \Omega_b, \quad (6) \]

\[ \Psi = -\frac{\kappa_4^2 \rho a^2 \gamma_2}{2k^2} \Delta + \frac{\epsilon \gamma_1}{4H r_c a} \dot{\Omega}_b - \frac{3\epsilon H \gamma_4}{4a} \dot{\Omega}_b + \frac{\epsilon (k^2 r_c \gamma_4 + H a^2 \gamma_2)}{4r_c a^3} \Omega_b; \quad (7) \]

where \( \Omega_b = \Omega_b(t) = \Omega(u_b(t), v_b(t)) \) and \( (\partial_y \Omega)_b \) are the values of the bulk field and its normal derivative, respectively, evaluated at the brane. In these expressions, the dimensionless \( \gamma \)-factors are functions of \( H \) and \( \dot{H} \).

The bulk wave equation (3), boundary condition (4) and (5) are the equations we must solve. Once we know \( \Delta \) and \( \Omega \) the metric perturbations \( \Phi \) and \( \Psi \) can be obtained by differentiation.
Figure 1: Linear growth factor and alternate gravitational potentials $\Phi_{\pm}$ from simulations and the QS approximation in the self-accelerating branch (left) and normal branch with $\Omega_{rc} = 0.05$ (right). We normalize $g(a)$ and $\Phi_-$ to unity at early times. For comparison, we also show the relevant results for the concordance $\Lambda$CDM model with $\Omega_m = 0.26$ and $\Omega_\Lambda = 0.74$.

4 Scalar perturbations in the self-accelerating universe

In this section, we concentrate on the $w = \sigma = 0$ and $\epsilon = +1$ DGP scenario as a model for the late-time accelerating universe. By examining probes of the expansion history,\cite{12} it has been found that

$$\Omega_{rc} = \frac{1}{4H_0^2r_c^2} = 0.15 \pm 0.02,$$

at 95% confidence. We use the best fit value for $\Omega_{rc}$ in our simulations.

For all plots in this paper, we select the bulk field to be zero and the brane field non-zero initially. We have also simulated several different choices of initial data, such as the bulk field being constant along the initial null hypersurface, and have found that the simulation results remain the same as long as the initial time is early enough. This is analogous to what happens in the RS case.\cite{7}

In a previous work,\cite{5} a ‘quasi-static’ (QS) approximation was developed to describe the behaviour of DGP perturbations whilst well inside the cosmological horizon, $k \gg H_0a$, and with physical wavelengths much less than the crossover scale, $a \ll kr_c$. Here, we compare the QS approximation to our simulations to determine just how large $k$ must be for it to be valid.

In Fig. 1 (left), we compare simulation results versus the QS approximation for the linear growth factor $g(a) = \Delta(a)/a$ and the alternate gravitational potentials $\Phi_{\pm} = \frac{1}{2}(\Phi \pm \Psi)$. We see that the simulation results are consistent with the QS approximation for $k \gtrsim 10^{-2} h$ Mpc$^{-1}$. On larger scales, the potential $\Phi_-$, which determines the integrated Sach-Wolfe (ISW) effect, shows more suppression than the QS prediction.
5 Scalar perturbations in the DGP normal branch

We now turn our attention to the behaviour of density perturbations in the normal branch of the DGP model. Unlike the $\epsilon = +1$ case, this branch does not naturally have a late time accelerating phase. So, in order to be made consistent with observations, we must allow for the brane to have a nonzero tension that acts as an effective 4-dimensional cosmological constant (we call this the $\Lambda$DGP model). Assuming that the matter sector is CDM-dominated, the Friedmann equation for this scenario follows from the general form \((1)\) with $\epsilon = -1$ and $w = 0$. The background dynamics has been compared to observations of $H(z)$, \(12\) and the following parameter values were found:

$$\Omega_m = \frac{\kappa_4^2 \rho_0}{3 H_0^2} = 0.23 \pm 0.04, \quad \Omega_{rc} = \frac{1}{4 H_0^2 r_c^2} \leq 0.05,$$

(9)

at 95% confidence. Here, $\rho_0$ is the present day CDM density. Note that the observationally preferred value of $\Omega_{rc}$ is zero. Since the DGP model goes over to GR in this limit, this implies that $\Lambda$CDM gives a better fit to the data than $\Lambda$DGP. We will assume the best fit value of 0.23 for $\Omega_m$.

In Fig. 1 (right), we compare the results of our simulations to the QS approximation and $\Lambda$CDM in the case $\Omega_{rc} = 0.05$. As in \(4\) we find that the simulation results are fairly insensitive to initial conditions provided that the initial data surface is set far enough into the past. In contrast to the self-accelerating case, we find that the linear growth factor and $\Phi_-$ potential are generally larger than in the $\Lambda$CDM case. The general trend is for $\Phi_-$ to become larger on small scales. We also notice that the QS approximation seems to provide a very good match to the simulation results for $\Delta$ on all scales. Finally, as in the self-accelerating case, we see that the QS approximation provides reasonably accurate results (with errors $\lesssim 5\%$) on scales $k \gtrsim 0.01 \, h \, \text{Mpc}^{-1}$.

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