Some remarks on the angular momenta of galaxies, their clusters and superclusters

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Abstract

We discuss the relation between angular momenta and masses of galaxy structures based on the Li model of the universe with global rotation. In our previous paper (Godłowski et al 2002) it was shown that the model predicts the presence of a minimum in this relation. In the present paper we discuss observational evidence allowing us to verify this relation. We check these theoretical predictions analysing Tully’s galaxy groups. We find null angular momentum \( J = 0 \) for the masses corresponding to mass of galaxy groups and non-vanishing angular momenta for other galaxies structures. The comparing of alignment in different galactic structures are consistent with obtained theoretical relation \( J(M) \) if we interpret the growing alignment as the increasing angular momenta of galaxies in the large scale.
1 Introduction

The empirical relation between the angular momentum and mass of celestial bodies was investigated for a long time. [1, 2, 3, 4]. Usually this relation is presented as $J \sim M^{5/3}$. There are several methods of explanation single relation $J \propto M^{5/3}$. Muradyan explained this relation in terms of the Ambarzumian’s superdense cosmogony [5]. Mackrossan involved thermodynamical consideration for its explanation [6]. Wesson (1983) argued that this is a consequence of self similarity of Newtonian problem applied to rotating gravitionally bound systems [2]. The considerations connected with this relation were involved in showing its possible important role in the unification of gravitation and particle physics [7], as well as in constructing the new universal constant [8]. Later on Catalan and Theuns explained the relation $J \sim M^{5/3}$ in the tidal torque model [9]. Sistero incorporated the rotational velocity of the Universe [10]. A similar approach was presented by Carrasco, Roth & Serrano, who explained this relation as a consequence of mechanical equilibrium between the gravitational and rotational energy[3].

Already it was proposed by Li more general relation $J(M)$ [11]. It predict different relation for low magnitude of masses than simple $J \sim M^{5/3}$. Only for high masses simply relation between the angular momentum and mass of celestial bodies $J \sim M^{5/3}$ is satisfies. The crucial point of his paper [11] is the explanation this relation for galaxies as a result of the influence of the global rotation of the Universe on their formation.

The problem of the rotation of the whole Universe attracted the attention of several scientists since [12]. It was shown that the reported value of rotation is too big when compared with the CMB anisotropy. Silk [13] pointed out that the dynamical effects of a general rotation of the Universe
are presently unimportant, contrary to the early Universe for which angular velocity $\Omega \geq 10^{-13}\text{rad/yr}$. He stressed that now the period of rotation must be greater than the Hubble time, which is a simple consequence of the CMB isotropy. Barrow, Juszkiewicz and Sonoda also addressed this question [14]. They showed that the cosmic vorticity depends strongly on the cosmological models and assumptions connected with linearisation of homogeneous, anistropic cosmological models over the isotropic Friedmann Universe. For a flat universe the obtained the value $\omega/H_0 = 2 \cdot 10^{-5}$.

It is interesting to confront the generalized relation obtained from the Li model with the observations. In the previous paper [15] it was shown that the Li relation between the angular momentum and the mass of the celestial bodies posses a minimum. In the present paper we discuss observational evidence allowing us to verify this relation. The angular momentum of a structure is the sum of spins of their components and its own angular momentum. There is no clear empirical evidence that galaxy groups and cluster rotate. Therefore, the study of galaxy alignment (i.e. ordering of galaxy axes or its normal vectors to galaxy planes) in a large structure can be useful for testing, if the primordial correlation of galaxy spins can be still observed in present-day structures. in order to check Li relation we need to calculate alignment for different size structures.

The observations indicate that different galactic structures have different total angular momentum. We consider pair of galaxies, compact galaxy groups, groups of galaxies and galaxy clusters as well as superclusters. For two first and two last size ranges are taken from the literature. For this paper we analyse the alignment of galaxies in the galaxy groups. As a sample 18 Tully’s groups of galaxies [16] is taken.

We obtained that there is no alignment of the Tully’s groups. We
interprete this as a zero value of total angular momentum. Taking into account that the \( J \neq 0 \) for the other sizes we find that all five groups fits well to generalized Li relation. It seems that there is simple relation \( J(M) \) for the whole mass range while the global rotation (even it is controversial) allows us to obtain the \( J(M) \) relation which well fit to data.

This paper is organised in the following manner. In Section 2 for the completence we present theoretical considerations, while Section 3 gives the main results together with a discussion of the previous studies. Section 4 concludes the paper.

2 Theoretical Considerations

In homogeneous and isotropic models, the Universe with matter may not only expand, but also rotate relative to the local gyroscope. The motion of the matter can be described by the Raychaudhuri equation. This is a relation between the scalar equation \( \Theta \), the rotation tensor \( \omega_{ab} \) and the shear tensor \( \sigma_{ab} \) [11]. The perfect fluid has the stress-energy tensor: \( T_{ab} = (\rho + p)u_{a}u_{b} + pg_{ab} \), where \( \rho \) is the mass density and \( p \) is the pressure. The Raychaudhuri equation can be written as:

\[
-\nabla_{a}A^{a} + \dot{\Theta} + \frac{1}{3} \Theta^{2} + 2(\sigma^{2} - \omega^{2}) = \frac{-4\pi G}{c^2} (\rho + 3p),
\]

where : \( A^{a} = u^{b}\nabla_{b}u^{a} \) is the acceleration vector, while \( \omega^{2} \equiv \omega_{ab}\omega^{ab}/2 \) and \( \sigma^{2} \equiv \sigma_{ab}\sigma^{ab}/2 \) are the scalar of rotation and shear respectively.

It has been shown that a spatially homogeneous, rotating and expanding Universe filled with a perfect fluid must have non-vanishing shear [17].

Because \( \sigma \) falls off more rapidly than the rotation \( \omega \) as the Universe expands, it is reasonable to consider such generalization of the Friedman
equation in which only the "centrifugal" term is present, i.e.
\[
\frac{\dot{a}^2}{2} + \frac{\omega^2 a^2}{2} - \frac{4\pi G a^2}{3c^2} \epsilon = -\frac{k c^2}{2},
\] (2)
where \( \epsilon = \rho c^2 \), \( k \) is the curvature constant, \( a \) a scalar factor and \( \dot{a} \equiv \frac{d}{dt} \).

Equation (2) should be completed with the principle of the conservation of energy-momentum tensor and angular momentum:
\[
\dot{\epsilon} = -(\epsilon + p) \Theta, \quad \Theta \equiv 3 \frac{\dot{a}}{a}
\] (3)
\[
\frac{p + \epsilon}{c^2} a^5 \omega = J
\] (4)
From that we can observe that if \( p = 0 \) (dust), then \( \rho \propto a^{-3} \) and \( \omega \propto a^{-2} \), while in general \( \sigma \) falls as \( a^{-3} \) [18]. The law of momentum conservation should be satisfied for each kind of matter, and consequently angular velocity of the Universe will evolve according to different laws in different epochs. Before the decoupling \( (z = 1000) \), matter and radiation interact, but after decoupling dust and radiation evolve separately and they have their own angular velocities \( \omega_d \) and \( \omega_r \). Parameters \( \omega \) and \( \rho \) can be written as \( \omega = \omega_0(1 + z)^2, \rho = \rho_{do}(1 + z)^3 + \rho_{ro}(1 + z)^4 \), with the latter being the total mass density of matter and radiation.

The conservation of angular momentum of a structure relative to the gyroscopic frames in dust epochs gives:
\[
J = k M^{5/3} - l M,
\] (5)
where \( k = \frac{2}{5} \left( \frac{3}{4\pi\rho_{do}} \right)^{2/3} \omega_0 \), \( \rho_{do} \) is the density of (dust) matter in the present epoch, and \( l = \beta r_f^2 (1 + z_f)^2 \omega_0 \), \( r_f \) is the radius of protostructure, and \( \beta \) is a parameter determined by the distribution of the mass in it. In this model we assumed that dark matter is collisionless and we considered only barionic matter.
In [11] the present value of the angular velocity of the Universe is estimated. A suitable value for $k$ parameter is assumed as 0.4 (in CGS Units). Taking $\rho_d = 1.88 \cdot 10^{-29} \Omega h^2 g cm^{-3}$ and $h = 0.75$, $\Omega = 0.01$ [19] for rich clusters of galaxies, see also [20, 21], we obtain $\omega_0 \simeq 6 \cdot 10^{-21} rad s^{-1} \simeq 2 \cdot 10^{-13} rad yr^{-1}$

In our previous paper [15] we show that relation (5) exhibits a clear minimum (see Fig.1). The value of mass of such structure is:

$$M_{\text{min}} = \left( \frac{3l}{5k} \right)^{3/2} = 1.95 r_f^3 (1 + z_f)^3 \rho_d,$$

and it does not depend on the present value of $\omega_0$ rotation of the Universe. We should note that when $|J| = 0$, the corresponding mass $M_0 = \left( \frac{3}{2} \right)^{3/2} \approx 2.15 M_{\text{min}}$. It is easy to observed that for less and more masive structures considered model predict $|J(M)| \neq 0$. It is interesting that from the observational point of view it is possible to observe just the cases when $|J| \approx 0$. It means that Li model can be verified by observations.

There is a large number of scenario of galaxy cluster formation (see for example [22, 23, 24]). We assumed that galaxy clusters are formed through the collapse of the protostructures [25, 26]. For a protostructure with a diameter of $60 Mpc$ and $z_f = 6$, we obtain $M_0 \approx 5 \cdot 10^{13} M_\odot$ as an estimate of dust mass for a formed structure with vanishing angular momentum. This gives the total mass of a structure of the order of $10^{14} - 10^{15} M_\odot$, which is a typical mass of a small galaxy cluster. The Fig. 1 shows dependence of $J(M)$ in that case.

3 Results and Discussion

The masses of the order $10^{14} - 10^{15} M_\odot$, for which the expected total angular momenta vanishes, are typical masses of the galaxy groups. Therefore,
Figure 1: The relation between angular momentum $J$ (in CGS units divided by $10^{60}$) and $M$ ($M_\odot$) of the astronomical object for the protostructure with a diameter of $60\, Mpc$ (solid line). Note that in neighbourhood of minimum $J(M)$ relation Li approximated by $J \propto M^{5/3}$ (dashes line).
we study the alignment of galaxies with the individual Tully’s group in the
LSC [16]. So for the present study we chose 18 Tully’s galaxy groups, each
containing more than 40 galaxies (see also [27]). We investigated the galaxy
alignment in each group because the existence of alignment is interpreted
as existence of non-vanishing angular momentum.

In our analysis, we used the supergalactic coordinate system \((L, B, P)\)
with the basic great circle (‘meridian’) chosen to pass through the LSC
centre in the Virgo cluster [28, 29, 30]. For any galaxy we consider two pa-
rameters: the galactic position angle \(p\) and the inclination angle \(i\). With
the use of these angles two orientation angles are determined: \(\delta\) - angle
between the normal to the galaxy and the LSC plane, and \(\eta\) - angle be-
tween the projection of this normal on the LSC plane and the direction
toward the LSC centre (seen from the Earth). The distributions of the
”supergalactic position angles” \(P\), as well as two angles \(\delta\) and \(\eta\) can be ana-
lyzed using statistical tests briefly summarized in the Appendix. A detailed
description of our method can be found in [29, 30].

The results of these statistical analyses are given in Tables 1-3. The
tables show the results of statistical tests \((\chi^2\) test, Fourier test and auto-
correlation test). Table 1 and 2 contains the results of the investigations
of galaxy plane orientations. It follows that a weak alignment of galaxy
planes is observed. However, the real alignment is very small, because a
part of these positive-signal detections is due to the observational effect in
determining galaxy axies in the Tully catalogue [27]. Moreover, when we
analysed spiral galaxies only, we found no group exhibiting clear evidence
for alignment existence. The analysis of the distribution of galaxy position
angles (Table 3) shows non-randomness in one group only. So we conclude
that in the case of the Tully’s group we do not find any galaxy alignment.
It should be noted that (because of the small number of object) we repeated the derivations for different values of \( n \) bins (see Appendix), but no significant differences were observed.

There is no clear empirical evidence that galaxy groups and cluster rotate. However, some evidence connected with the motion of their components around the mass centre in very poor groups can be observed. As a result, it can be accepted that the angular momentum of a galaxy group or cluster is connected mainly with the galaxy spins. Even when taking into account the eventual orbital motion of galaxies around the common mass centre, it can be easily seen that more numerous structures exhibit larger angular momentum. Because of that reason, we assumed that total angular momentum of the galactic groups and clusters consists from the sum of spins of their components only. For masses of structure close to \( M_0 \) we do not find evidence that angular momenta \( J(M) \neq 0 \), which is in agreement with predictions of the considered Li model.

The less massive structures are compact groups and galaxy pairs. In compact groups of galaxies, member galaxies rotate along elongated orbits around the gravitational centre of the group [31] which obviously add some angular momentum to the total angular momentum of the system.

The study of paired galaxies showed that the angular momentum in these systems becomes mainly in the orbital motion of galaxies [32, 33, 34]. It showed that for the structures with masses smaller then \( M_0 \) there are some indicators of the non-vanishing angular momentum.

The study of galaxy orientation, which substitutes the investigation of the galaxy spin distribution, yields different results. Nevertheless, it is clear that in isolated Abell clusters of galaxies only the dominant brightest cluster members exhibit the sign of alignment [35, 36, 37]. However, in
Table 1: Test for isotropy of the orientations of galaxy plane. The distribution of the angle $\delta$ of galaxies (Tully’s group)

| group | $N$  | $\chi^2$ | $C$ | $P(\Delta_1)$ | $\Delta_{11}$ | $\sigma$ |
|-------|-----|---------|-----|----------------|--------------|--------|
| 11    | 626 | 62.8    | 9.50| .000           | -.237       | .058   |
| 12    | 332 | 25.7    | -11.48| .299          | -.125       | .080   |
| 13    | 128 | 29.7    | -6.94| .891          | .002        | .129   |
| 14    | 426 | 24.0    | 3.52 | .154          | -.120       | .071   |
| 15    | 130 | 13.1    | -1.88| .737          | .004        | .128   |
| 17    | 80  | 13.8    | -5.50| .569          | -.100       | .164   |
| 21    | 248 | 14.2    | 0.24 | .065          | -.035       | .093   |
| 22    | 126 | 7.0     | 0.68 | .496          | .098        | .130   |
| 23    | 100 | 17.0    | -1.82| .607          | .140        | .146   |
| 31    | 210 | 33.7    | 15.98| .000          | .137        | .101   |
| 41    | 192 | 22.8    | 6.54 | .020          | -.004       | .106   |
| 42    | 230 | 24.7    | -3.78| .320          | .056        | .096   |
| 44    | 80  | 26.7    | 7.95 | .024          | -.200       | .164   |
| 51    | 228 | 29.6    | -0.41| .009          | .093        | .097   |
| 52    | 172 | 21.6    | 3.04 | .005          | -.203       | .112   |
| 53    | 260 | 13.3    | -3.29| .492          | -.055       | .091   |
| 61    | 258 | 19.9    | -3.13| .825          | -.049       | .091   |
| 64    | 102 | 28.7    | -7.54| .113          | .080        | .145   |
Table 2: Test for isotropy of the orientations of galaxy plane. The distribution of the angle $\eta$ of galaxies (Tully’s group)

| angle | group | $N$ | $\chi^2$ | $C$ | $P(\Delta_1)$ | $\Delta_{11}$ | $\sigma$ |
|-------|-------|-----|----------|-----|----------------|---------------|---------|
| 11    | 626   | 60.0| 5.96     | .000| 0.304          | .057          |         |
| 12    | 332   | 28.5| 7.56     | .001| -.069          | .078          |         |
| 13    | 128   | 25.6| 2.78     | .079| 0.242          | .125          |         |
| 14    | 426   | 27.4| 6.63     | .090| 0.055          | .069          |         |
| 15    | 130   | 22.9| 2.51     | .764| 0.036          | .124          |         |
| 17    | 80    | 13.1| -5.97    | .470| -.059          | .158          |         |
| 21    | 248   | 26.9| -3.55    | .054| 0.058          | .090          |         |
| 22    | 126   | 11.7| 5.57     | .177| 0.194          | .126          |         |
| 23    | 100   | 20.2| -0.10    | .081| -.164          | .141          |         |
| 31    | 210   | 24.0| 0.43     | .046| 0.194          | .098          |         |
| 41    | 192   | 27.2| 10.22    | .001| 0.300          | .102          |         |
| 42    | 230   | 15.9| 3.30     | .033| 0.036          | .093          |         |
| 44    | 80    | 20.4| -3.05    | .226| 0.148          | .158          |         |
| 51    | 228   | 30.6| -5.37    | .042| 0.218          | .094          |         |
| 52    | 172   | 38.1| 12.29    | .001| 0.212          | .108          |         |
| 53    | 260   | 12.2| -8.28    | .816| 0.008          | .088          |         |
| 61    | 258   | 23.7| -12.56   | .549| 0.091          | .088          |         |
| 64    | 102   | 50.1| -3.53    | .002| 0.402          | .140          |         |
Table 3: Test for isotropy of the distribution of supergalactic position angles

| angle | group | \( N \) | \( \chi^2 \) | \( C \) | \( P(\Delta_1) \) | \( \Delta_{11} \) | \( \sigma \) |
|-------|-------|--------|--------|-----|-----------------|----------|------|
| 11    | 185   | 22.7   | -11.42 | .728 | 0.081           | .104     |
| 12    | 106   | 17.6   | -1.57  | .198 | -.083           | .137     |
| 13    | 50    | 13.4   | -2.48  | .714 | -.056           | .200     |
| 14    | 133   | 14.9   | -1.05  | .990 | -.011           | .123     |
| 15    | 48    | 11.3   | -0.75  | .185 | -.006           | .204     |
| 17    | 22    | 10.7   | -5.64  | .727 | -.152           | .302     |
| 21    | 85    | 13.5   | -2.84  | .878 | -.058           | .153     |
| 22    | 43    | 16.0   | -1.98  | .910 | -.089           | .216     |
| 23    | 33    | 12.3   | 0.27   | .230 | -.337           | .246     |
| 31    | 63    | 20.1   | 1.00   | .729 | 0.081           | .178     |
| 41    | 54    | 20.0   | 10.67  | .595 | -.112           | .192     |
| 42    | 71    | 19.0   | 1.51   | .124 | -.254           | .168     |
| 44    | 25    | 18.9   | 1.64   | .367 | 0.161           | .283     |
| 51    | 69    | 23.1   | 3.00   | .576 | -.176           | .170     |
| 52    | 50    | 14.8   | -0.32  | .631 | 0.154           | .200     |
| 53    | 88    | 17.5   | 2.82   | .080 | 0.243           | .151     |
| 61    | 85    | 27.9   | 6.48   | .007 | -.116           | .153     |
| 64    | 31    | 18.4   | 2.10   | .295 | -.397           | .254     |
very rich galaxy clusters, such as A754 [38], or A1656 [39, 40, 41] the non-random distribution of galaxies has been observed.

A lot of work has been done in the study of the alignment of galaxies in superclusters (for earlier work, see the review of Djorgovski [42]. The results are ambiguous, but several independent investigations claimed the existence of a weak alignment of galaxies in respect to the supercluster’s main plane. This was the case of the Local Supercluster [28, 43, 29, 30], Hercules Supercluster [44], Coma/A1367 [39, 40, 41, 45], and the Perseus Supercluster [46, 47, 48]. In the latter supercluster, the weak alignment has been observed also when data taken from scans, i.e. without a personal bias [49], were involved instead of the data derived from visual measurements. The distribution of real spins was random, but the sample considered was small [50]. The above-mentioned studies were restricted to the high-density regions of superclusters. From the above considerations it follows that the alignment of galaxies in Abell clusters is much weaker than in the case of superclusters.

Summarising our results, there is an evidence of non-vanishing angular momentum in both small and larger structures, but not for galaxy groups. Our results also show that during analysing our sample of the galaxy groups, the most prominent possible alignment of galaxy plane orientation, we obtain for the Virgo cluster itself, the most massive substructure studied by us. This results are consistent with predictions of the discussed Li model.

4 Conclusions

In the present paper we discuss the relation between angular momentum and mass of the galaxy structures obtained from Li model. The classical relation $J \propto M^{5/3}$ implies a monotonical increase of angular momentum
with the mass. We show that this simple relation is not confirmed by observations. In the Li model simple relation $J \propto M^{5/3}$ is valid only for larger masses, while the model predicts the presence of a minimum in the relation between angular momenta and masses of galaxy structures. It is interesting that the zero of $J(M)$ relation can be tested observationally. We demonstrate that with reasonable assumptions the mass $M_0$ for which $|J| = 0$, corresponds to galaxy group mass.

Analysing Tully’s groups of galaxies we do not find any evidence for galaxy alignment for that groups. Our results - evidence for vanishing angular momentum for the galaxy groups and indicators of non-vanishing angular momentum for less and more massive structures is consistent with predictions of the discussed model.

Our general conclusion is that Li model in which celestial bodies acquire angular momenta during their formation from the global rotation of the Universe, gives us correct predictions. This for example explain why for galaxy groups, contrary to the more massive structures, no alignment of galaxies is dedected.

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Appendix. Statistical methods applied

In order to check the distribution of galaxy orientation angles ($\delta, \eta$) and position angles $p$, we tested whether the respective distribution of the $\delta, \eta$ or $p$ angles is isotropic. We applied statistical tests originally introduced (for that problem) by [51, 52], but modified by us as compared to the original version (see the detailed description in [29, 30]). Below, a short summary is presented of the tests considered here (not always explicitly): the $\chi^2$-test, the Fourier test and the auto-correlation test. In all of these tests, the entire range of the $\theta$ angle (where for $\theta$ one can put $\delta + \Pi/2$, $\eta$ or $p$ respectively) is divided into $n$ bins, which in the $\chi^2$ test gives $n-1$ degrees of freedom. During the analysis, we used $n = 18$ bins of equal width. Let $N$ denote the total number of galaxies in the considered cluster, and $N_k$ - the number of galaxies with orientations within the $k$-th angular bin. Moreover, $N_0$ - denotes the average number of galaxies per bin and, finally, $N_{0,k}$ - the expected number of galaxies in the $k$-th bin. The $\chi^2$-test of the distribution yields the critical value 27.6 (at the significance level $\alpha = 0.05$) for 17 degrees of freedom:

\[
\chi^2 = \sum_{k=1}^{n} \frac{(N_k - N_{0,k})^2}{N_{0,k}}.
\] (A1)

However, when we consider individual clusters the number of galaxies involved may be small in some cases, and the $\chi^2$ test will not necessarily work well (e.g. the $\chi^2$ test requires the expected number of data per bin to equal at least 7; see, however, [53, 54]. As a check, in a few cases we repeated the derivations for different values of $n$, but no significant differences appeared. However, the main statistical test used in the present paper is the Fourier test. In the Fourier test the actual distribution $N_k$ is approximated as:
\( N_k = N_{0,k}(1 + \Delta_{11} \cos 2\theta_k + \Delta_{21} \sin 2\theta_k) \), \hspace{1cm} \text{(A2)}

(we take into account only the first Fourier mode). We obtain the following expression for the coefficients \( \Delta_{ij} \) \((i, j = 1, 2)\):

\[
\Delta_{1j} = \frac{\sum_{k=1}^{n} (N_k - N_{0,k}) \cos 2J\theta_k}{\sum_{k=1}^{n} N_{0,k} \cos^2 2J\theta_k} \hspace{1cm} \text{(A3)}
\]

\[
\Delta_{2j} = \frac{\sum_{k=1}^{n} (N_k - N_{0,k}) \sin 2J\theta_k}{\sum_{k=1}^{n} N_{0,k} \sin^2 2J\theta_k} \hspace{1cm} \text{(B4)}
\]

with the standard deviation

\[
\sigma(\Delta_{11}) = \left( \sum_{k=1}^{n} N_{0,k} \cos^2 2\theta_k \right)^{-1/2} \approx \left( \frac{2}{nN_0} \right)^{1/2} \hspace{1cm} \text{(A5a)}
\]

\[
\sigma(\Delta_{21}) = \left( \sum_{k=1}^{n} N_{0,k} \sin^2 2\theta_k \right)^{-1/2} \approx \left( \frac{2}{nN_0} \right)^{1/2} \hspace{1cm} \text{(A5b)}
\]

where \(N_0\) is the average of all \(N_{0,k}\). However, we should note that we could formally replace the symbol \(\approx\) with \(=\) only in the cases where all \(N_{0,k}\) are equal (for example, in the cases when we tested the isotropy of the distribution of the position angle). The probability that the amplitude:

\[
\Delta_1 = (\Delta_{11}^2 + \Delta_{21}^2)^{1/2} \hspace{1cm} \text{(A6)}
\]

is greater than a certain chosen value is given by the formula:

\[
P(>\Delta_1) = \exp \left( -\frac{n}{4}N_0\Delta_1^2 \right) \hspace{1cm} \text{(A7)}
\]
while the standard deviation of this amplitude is

\[ \sigma(\Delta_1) = \left( \frac{2}{nN_0} \right)^{1/2} \]  \hspace{1cm} (A8)

From the value of \( \Delta_{11} \) one can deduce the direction of the departure from isotropy. If \( \Delta_{11} < 0 \), then, for \( \theta = \delta + \pi/2 \), an excess of galaxies with rotation axes parallel to the LSC plane is present. For \( \Delta_{11} > 0 \) the rotation axes tend to be perpendicular to the LSC plane. Similarly, while analysing the distribution of the position angles of galaxies (\( \theta \equiv p \)), if \( \Delta_{11} < 0 \), an excess of galaxies with position angles parallel to the plane of the coordinate system (i.e. normal to the galaxy plane is perpendicular to the plane of the coordinate system) is present. For \( \Delta_{11} > 0 \), the position angles of galaxy are perpendicular to the plane of the coordinate system.

The auto-correlation test quantifies the correlations between the galactic numbers in adjoining angular bins. The correlation function is defined as:

\[ C = \sum_{k=1}^{n} \frac{(N_k - N_{0,k})(N_{k+1} - N_{0,k+1})}{[N_{0,k}N_{0,k+1}]^{1/2}} \]  \hspace{1cm} (A9)

In the case of an isotropic distribution we expected \( C = 0 \) with the standard deviation:

\[ \sigma(C) = n^{1/2} \]  \hspace{1cm} (A10)