ABOUT THE $Q^2$ DEPENDENCE OF THE MEASURED ASYMMETRY $A_1(x)$

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We propose the new approach for taking into account the $Q^2$ dependence of measured asymmetry $A_1$. This approach is based on the similarity of the $Q^2$ behaviour and the shape of the spin-dependent structure function $g_1(x, Q^2)$ and spin averaged structure function $F_3(x, Q^2)$. The analysis is applied on the SMC and E154 experimental data.

An experimental study of the nucleon spin structure is realized by measuring of the asymmetry $A_1(x, Q^2) = g_1(x, Q^2)/F_1(x, Q^2)$. The most known theoretical predictions on spin dependent structure function $g_1(x, Q^2)$ of the nucleon were done by Bjorken and Ellis and Jaffe for the so called first moment value $\Gamma_1 = \int_0^1 g_1(x)dx$.

Studying the properties of $g_1(x, Q^2)$ and the calculation of the $\Gamma_1$ value require the knowledge of structure function $g_1$ at the same $Q^2$ in the hole $x$ range. Experimentally asymmetry $A_1$ is measuring at different values of $Q^2$ for different $x$ bins. An accuracy of the modern experiments allows to analyze data in the assumption that asymmetry $A_1(x, Q^2)$ is $Q^2$ independent (structure functions $g_1$ and $F_1$ have the same $Q^2$ dependence)

$$A_1(x, Q^2) = A_1(x) \quad (1)$$

But the precise checking of the Bjorken and Ellis - Jaffe sum rules requires considering the $Q^2$ dependence of $A_1$ or $g_1$. Moreover, the assumption (1) that asymmetry $A_1(x, Q^2)$ is $Q^2$ independent does not follow from the theory. On the contrary, the behaviour of $F_1$ and $g_1$ as a functions of $Q^2$ is expected to be different due to the difference between polarized and unpolarized splitting functions.

There are several approaches (see and its references) how to take into account the $Q^2$ dependence of $A_1$. They are based on different approximate solutions of the DGLAP equations. Some of them have been used already by Spin Muon Collaboration (SMC) and E154 Collaboration in the last analyses of experimental data (see and, respectively).
In this article we suggest to use another idea which is based on the observation that the splitting functions of the DGLAP equations for the SF $g_1$ and $F_3$ and the shapes of the SF themselves are the similar in a wide $x$ range and, thus, the $Q^2$ dependence of them has to be close as a consequence. Our approach for $Q^2$-dependence of $A_1$ are very simple (see eq.(2)) and leads to the results, which are very similar to ones based on the DGLAP evolution.

To demonstrate the validity of the observation, we note that the r.h.s. of DGLAP equations for NS parts of $g_1$ and $F_3$ is the same (at least in first two orders of the perturbative QCD) and differs from $F_1$ already in the first subleading order. For the singlet part of $g_1$ and for $F_3$ the difference between perturbatively calculated splitting functions is also negligible (see [13,15]). This observation allows us to conclude the function

$$A_1^*(x) = \frac{g_1(x, Q^2)}{F_3(x, Q^2)}$$

should be practically $Q^2$ independent and the asymmetry $A_1$ at some $Q^2$ can be defined than as:

$$A_1(x_i, Q^2) = \frac{F_3(x_i, Q^2)}{F_3(x_i, Q_1^2)} \cdot \frac{F_1(x_i, Q^2)}{F_1(x, Q^2)} \cdot A_1(x_i, Q_1^2), \quad (2)$$

where $x_i (Q_1^2)$ means an experimentally measured value of $x (Q^2)$.

To apply the proposed approach we use the SMC[4] and E154 Collaboration[5] data. To use relation (2) we parametrize CCFR data on $F_2(x, Q^2)$ and $xF_3(x, Q^2)$[10] in the same form as NMC fit of the structure function $F_2(x, Q^2)$[11] (see [13]). To obtain structure function $F_1(x, Q^2)$ we also take the parametrization of the CCFR data on $F_2(x, Q^2)$[10] and the SLAC parametrization of $R(x, Q^2)$[12] and use relation:

$$F_1(x, Q^2) = \frac{F_2(x, Q^2)}{2x(1 + R(x, Q^2))} \cdot (1 + \frac{4M^2x^2}{Q^2}), \quad (3)$$

We use in Eq.(3) parametrizations of CCFR data[10] for both SF $xF_3(x, Q^2)$ and $F_2(x, Q^2)$ to avoid systematical uncertainties and nucleon correlation in nuclei.

The SF $g_1(x, Q^2)$ is calculated using the asymmetry $A_1(Q^2)$ as

$$g_1(x, Q^2) = A_1(x, Q^2) \cdot F_1(x, Q^2), \quad (4)$$

where spin average SF $F_1$ has been calculated using NMC parametrization of $F_2(x, Q^2)$[11]. The results are presented in Fig. 1 and Fig. 2 for E154 and
SMC data, respectively. Our results are in an excellent agreement with the calculations which are based on direct DGLAP evolution.

Figure 1. The structure function $xg_1^p(x, Q^2)$ evolved to $Q^2 = 5\text{GeV}^2$ using our eq.(2), DGLAP NLO evolution and the assumption that $g_1^n/F_1^n$ is $Q^2$ independent. Last two sets are from 6.

Figure 2. The structure function $xg_1^n(x, Q^2)$ evolved to $Q^2 = 10\text{GeV}^2$ using our eq.(2), the assumption that $g_1^n/F_1^n$ is $Q^2$ independent, and DGLAP NLO evolution according to the analysis 9.

To make another comparison with the theory we have calculated also the first moment value of the structure function $g_1$ at different $Q^2$. Using eq.(2), we recompute the SMC measured asymmetry of the proton and deuteron and E154 one of neutron at $Q^2 = 100\text{ GeV}^2$, 30 GeV$^2$, $Q^2 = 10\text{ GeV}^2$ and 3 GeV$^2$.
and get the value of \( \int g_1(x)dx \) through the measured \( x \) ranges. To obtain the first moment values \( \Gamma_1^{p(d)} \) we use an original estimations of SMC and E154 for unmeasured regions. As the last step we calculate the difference \( \Gamma_1^p - \Gamma_1^n \) (for SMC proton and deuteron data \( \Gamma_1^p - \Gamma_1^n = 2\Gamma_1^p - 2\Gamma_1^n/(1 - 1.5 \cdot \omega_D) \) where \( \omega_D = 0.05 \)). In Table 1 we present the results for the mean values of \( \Gamma_1^p - \Gamma_1^n \), because the errors coincide with the errors of original analyses\(^a\). The value of \( \Gamma_1^p - \Gamma_1^n \) at \( Q^2 = 10GeV^2 \) obtained by direct DGLAP evolution are taken from article\(^4\).

Let us now present the main results, which are following from the Table 1 and the Figures.

- The results are in excelent agreement with \( g_1(x,Q^2) \) data of SMC and E154 Collaborations, based on direct DGLAP evolution.
- Our method allows to test of the Bjorken sum rule in a simple way with a good accuracy. Obtained results on the \( \Gamma_1^p - \Gamma_1^n \) show that used experimental data well confirm the Bjorken sum rule prediction.

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\(^a\)The theoretical predictions computed in \(^1\) to the third order in the QCD \( \alpha_s \).
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