MCMC approach for shield tunnel grouting layer estimation using ground penetrating radar

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Abstract. Ground penetrating radar (GPR) has been suggested as an effective tool for evaluating the grouting layer behind shield tunnel linings, yet the estimation of the grouting layer thickness is usually difficult. In this paper, we propose a probabilistic inversion method to evaluate the grouting layer using GPR images. This method uses a sliding window along the GPR scan axis combined with a Markov chain Monte Carlo (MCMC) simulation with Bayesian inversion to infer the grouting layer thickness together with the relative permittivity and electric conductivity. We illustrate this approach using a synthetic GPR experiment that simulates grouting layer detection in a shield tunnel along the longitude direction. A sliding window with a width of 0.2 m is used to estimate the model parameters and is moved along the scan axis with a step size of 0.2 m after each inversion. The results demonstrate successful estimation of the grouting layer thickness and its relative permittivity and electric conductivity by the proposed method. Moreover, this method is capable of quantifying uncertainties in the inversion results.

1. Introduction

In shield tunnel construction, the backfill grout between tunnel lining and surrounding ground is essential for the structural integrity of the tunnel structure and plays an important role in ground settlement control [1]. Although the quality of grouting is usually measured by the grouting quantity and pressure, the distribution of the grouting layer can be more important because estimating if the gap between tunnel lining and surrounding ground is filled properly is more reliable.

Among the various methods used to evaluate the grouting layer distribution, ground penetrating radar (GPR) is widely accepted as a nondestructive technique that uses high-frequency electromagnetic (EM) waves to detect anomalies buried underground (Figure 1(a)) [2-5]. However, the interpretation of GPR data is generally difficult because (i) the steel rebars in the concrete lining cause strong scattering effects of EM waves, and (ii) the grout induces considerable attenuation of GPR signals. This interference makes it difficult to recognize the grout–soil interface in the GPR image (Figure 1(b)) and increases the complexity of GPR data processing.

Efforts have been made to enhance GPR data to better estimate the grouting layer. Liu et al. proposed a wave-equation redatuming method to enhance target reflections by eliminating diffractive scattering from steel rebars in the GPR profile [6]. Zhao et al. presented a Maxwell curl equation datuming approach that redefined the reference to a deeper horizon to improve the quality of GPR images.
beneath the tunnel lining [7]. Feng et al. used cross-correlation attribute analysis to suppress background noise and highlight reflections from deep interfaces [8]. Xie et al. applied Karhunen–Loeve (K–L) filtering to GPR image processing for a better discrimination of the grout–soil interface [9, 10]. In addition to these data processing techniques, Liu et al. used the common midpoint (CMP) measurement to suppress random noise in GPR data for an accurate estimation of the grouting layer distribution [11].

In this paper, we present a novel probabilistic inversion method taking advantage of Bayes’ theorem and Markov chain Monte Carlo (MCMC) sampling to estimate the grouting layer thickness together with the relative permittivity and electric conductivity from GPR images. The proposed method is first introduced, followed by a synthetic example that is used to illustrate this method, before presenting the main results and conclusion.

Figure 1. (a) GPR detection of shield tunnel grouting layer, and (b) a typical GPR image

2. Methodology

The GPR detection of a shield tunnel grouting layer is depicted in Figure 1(a). Antennae are placed on the surface of tunnel lining and moved along the longitude direction of the tunnel, transmitting EM waves into the ground and receiving reflected waves from the subsurface. The GPR detection process can be described using the following equation:

\[ d = G(m) + e \]  

(1)

where, \( d \) is the measured GPR data; \( G(m) \) is the set of Maxwell’s equations that govern the propagation of EM waves in the underground media; \( m \) denotes the model parameters, including the thickness, relative permittivity, and electric conductivity of the grouting layer; and \( e \) signifies the measurement error.

In view of Bayes’ law, the model parameters can be derived using the following relation:

\[ p(m \mid d) \propto p(m)L(m \mid d) \]  

(2)

where, \( p(m) \) and \( p(m \mid d) \) describe the prior and posterior distribution of the model parameters, respectively, and \( L(m \mid d) \) is the likelihood function used to measure the distance between the observed and simulated GPR data.

Although the prior distribution and likelihood function can be defined in a straightforward manner, the posterior distribution of model parameters is usually difficult to derive analytically. Therefore, we resort to MCMC simulation to explore the posterior distribution [12]; it uses a Markov chain to generate samples from the proposed distribution and decide whether to move to the new state using the Metropolis rule [13]:

\[ p_{acc} = \min[1, \frac{p(m_i)}{p(m_{i-1})}] \]  

(3)

where, \( p(m_i) \) and \( p(m_{i-1}) \) are the probability of the proposed and current state, respectively, and \( p_{acc} \) is the acceptance probability. After many iterations, the Markov chain converges to the target distribution, which is the posterior distribution of the model parameters.

In this work, we use a sliding window for forward computing of GPR waves. As depicted in Figure 2(b), we place a narrow window (red box) on the full model and build a window model with four
layers, including air, concrete lining, grout, and soil. Five parameters of this model are subject to estimation: relative permittivity ($\varepsilon_r$) and electric conductivity ($\sigma$) of the grout, and layer thicknesses ($x_1$, $x_2$, and $x_3$) of the grouting layer on the left, middle, and right side of the window model. Once the window model is defined, we use the gprMax simulator [14], a FDTD solver of Maxwell’s equations, to compute GPR waves in the forward model output.

Figure 2. Sliding window inversion method: (a) full model, and (b) window model

3. Synthetic example

In this section, we use a synthetic example to demonstrate our inversion method. The full model shown in Figure 2(a) is taken as the reference model. GPR waves are simulated using the gprMax software by placing the 900 MHz antennae on the lining surface and moving them from 0 to 1 m along the horizontal direction. A GPR trace is collected every 0.01 m, and a total of 101 GPR traces are obtained as the measurement data. Furthermore, we add 5% Gaussian noise to the data to simulate measurement error. After this, we create a window model (Figure 2(b)) with a window width of 0.2 m. The window model is first used to invert the parameters of the full model from 0 to 0.2 m, then moved along the scan axis to infer parameters of other parts of the full model. In this example, the window step size is set to 0.2 m, and five windows are used to invert all parts of the full model.

As an illustration, we discuss the inversion results of the window model from 0.4 to 0.6 m. Table 1 shows the true values of the model parameters as well as the prior distribution of each parameter. Given that there is no prior knowledge before GPR measurement, a uniform distribution is used.

| Parameter                      | True value | Unit | Lower bound | Upper bound | Distribution   |
|-------------------------------|------------|------|-------------|-------------|----------------|
| Relative permittivity ($\varepsilon_r$) | 25         | -    | 20          | 30          | Uniform        |
| Electric conductivity ($\sigma$) | 0.01       | S/m  | 0.001       | 0.1         | Log-uniform    |
| Layer thickness 1 ($x_1$)     | 0.237      | m    | 0           | 0.4         | Uniform        |
| Layer thickness 2 ($x_2$)     | 0.182      | m    | 0           | 0.4         | Uniform        |
| Layer thickness 3 ($x_3$)     | 0.179      | m    | 0           | 0.4         | Uniform        |

We use the DREAM algorithm to explore the posterior distribution of model parameters [15]. This algorithm is an adaptive MCMC algorithm and was designed to accelerate the convergence of model parameters. We run eight Markov chains in parallel and plot the values of the log-likelihood and weighted root mean square error (wRMSE), shown in Figure 3, to monitor the evolution of model parameters. The increase in log-likelihood and decrease in wRMSE indicate that the simulated model is getting closer to the true model, and the distance between simulated and measured data is becoming smaller. After approximately 60000 iterations, the log-likelihood and wRMSE reach steady state and we consider the model parameters to have converged to the posterior distribution.
Figure 3. Evolution of Markov chains: (a) log-likelihood, and (b) weighted RMSE
After convergence, we take the last 50% of parameters of the eight chains to form the posterior distribution. Figure 4 shows the posterior distribution of the five model parameters. The red cross denotes the true value, the orange circle signifies the posterior mean value, and the black triangle depicts the maximum a-posterior (MAP) value. Except for the electric conductivity, the true values of relative permittivity and grouting layer thicknesses are well in the range of their posterior distributions and close to the posterior mean and MAP values.

Figure 4. Posterior distributions of model parameters: (a) relative permittivity, (b) electric conductivity, and (c)–(e) grouting layer thicknesses

Figure 5. Layer thickness estimation of the window model using (a) posterior mean, and (b) MAP parameters
As shown in Figure 5, we reconstruct the grouting layer thickness of the window model using the posterior mean and MAP parameters. The MAP estimation can better represent the true layer thickness values marked with red crosses than the posterior mean estimation. Measurement and simulated

Figure 6. Measurement and simulated GPR waveform data
waveform data are also plotted in Figure 6. It is obvious that the MAP model fits the measurement data very well.

After the inversion of parameters for all window models is accomplished, we can summarize all inverted parameters and plot the grouting layer thickness of the full model, shown in Figure 7. This method successfully reconstructed the grouting layer thickness, with the MAP parameters very close to the true values marked with red crosses.

![Figure 7](image_url)

**Figure 7.** Grouting layer thickness estimation of the full model using (a) posterior mean, and (b) MAP parameters

### 4. Conclusions

In this paper, we developed a probabilistic inversion method for GPR waveform data to estimate the grouting layer behind shield tunnel linings. This method takes the advantage of Bayesian inversion and MCMC simulation to search for the posterior distribution of the grouting layer thickness together with the relative permittivity and electric conductivity values. Instead of inferring parameters of the full model, we use a sliding window strategy to reduce the number of model parameters and save computational budget. In the synthetic example, this method successfully estimates all model parameters, and quantifies the uncertainty of each parameter.

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