Mathematical reasoning structure of junior high school students in solving problems based on their working memory capacity

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Abstract. Mathematical reasoning has a clear structure. In mathematical reasoning structure, working memory capacity is considered as something important. This study aimed at knowing the mathematical reasoning structure of junior high school students in solving problems reviewed based on working memory capacity. This research was a qualitative research. This study was carried out in state Junior High School in Makale. The selection of research subjects was done by providing tests to measure students' working memory capacity. The test used was O-SPAN Test and the researcher chose 1 student with high category working memory capacity and 1 student with low category working memory capacity. Then the researcher gave a test to investigate the reasoning structure. Mathematical reasoning structure of research participant with low working memory capacity showed the stage of problematic situation, strategy choice, conclusion could be done. However, for the stage of strategy is implemented, subject implemented the strategy in one way and could not find a maximum result. Subject with low working memory capacity could provide conclusions but could not give verification arguments showing the strategy used was correct or reasonable and could solve the problem. Mathematical reasoning structure of research subject with high working memory capacity fulfilled the four mathematical reasoning structures namely problematic situation, strategy choice, implemented strategy, and conclusion.

1. Introduction
Learning mathematics is inseparable from problem solving process. Problem solving in mathematics is a systematic process. In problem solving process, reasoning is needed [1]. Naming of reasoning process depends on the object being studied. For example, reasoning on mathematical objects is called mathematical reasoning [2]. According to Bottou [3], mathematical reasoning is a thought line or way of thinking used to produce statements and conclusions to solve problems. Whereas Lithner [4] states that mathematical reasoning is a skill in mathematics. Reasoning produces statements and conclusions in problem solving [5-7]. Reasoning does not have to be based on formal logic [8,9] so that students' answers do not have to be correct. However, if students are able to provide reasonable reasons that support the answers, the reasoning process is still justified [10]. The reasoning process will direct to the right information in order to solve the problem. Information obtained from mathematical reasoning is linked to other information that will form a new solution that is suitable with the child's mind. If the
assignment given is in the form of an open problem, it will allow students to choose their own way of providing solutions with mathematical reasoning.

Mathematical reasoning structure in solving problems according to Lithner [11] is done through four steps: 1) problematic situation, namely the description of the problem situation so that it is clear what will be solved. 2) Strategy Choice, namely the selection of strategies used. It can be supported by predictive arguments: arguments that explain why the strategy will be used to solve problems. 3) implemented strategy, which is the step of applying the strategy chosen to solve the problem, which can be supported by verification arguments: arguments that show the strategy used in solving assignments. 4) Conclusion, which is making conclusions. At a glance the steps look the same as the general problem-solving steps. The difference is that the existence of predictive arguments and verification arguments in general problem-solving process is not explicitly started.

Efforts to solve problems both with creative mathematical reasoning, it can be seen that the reasoning process requires the ability to sort and use a lot of information into information that is relevant to the problem [1]. It takes the ability to use the intrinsic properties of mathematics that are relevant to prove the truth. The factor that plays an important role for this is cognitive factors. One cognitive aspect that plays an important role in one's success in learning is working memory. Working memory is a cognitive system that has the task of temporarily storing information to be processed and has the limited capacity. A person's working memory capacity shows how much information can be received, processed and processed in his/her working memory. The greater the capacity is, the more information can be received, processed and selected at one time [12]. It is a common knowledge that every experience leaves an indelible mark on a person’s memory. Unlike a computer, a normal human brain never reaches a point where new experiences can no longer be processed in memory; brain cannot be full [13]. But at the same time, people can be overwhelmed by processing new information to the point that it seems too difficult to understand, too confusing or complicated to be stored in memory. Feelings of being overwhelmed by a lot of new information can occur because of a special type of memory that is usually called working memory [14]. It refers to the relatively small amount of information that can be remembered, cared for, or maintained, technically, maintained in conditions that can be accessed quickly, at one time [15,16]. Therefore a high working memory capacity is needed for this thing.

The importance of working memory capacity in solving problems can be seen from the role of working memory in selecting relevant information that is used to solve problems from a collection of information that is owned quite a lot. In an effort to solve problems with creative mathematical reasoning, students need relevant information [1]. Thus, large working memory capacity helps individuals to direct their attention to relevant information more easily than individuals with small working memory capacity [17, 18]. Working memory capacity has an effect on reducing errors in retrieving information that is not needed in problem solving [19], so working memory capacity is considered important in problem solving [20]. Based on the background, this study aims at determining the mathematical reasoning structure of junior high school students in solving problems reviewed based on working memory capacity.

2. Research Method
This research was a qualitative research. The study was conducted at state Junior High School 1 Makale. The research subjects were determined using o-span test (Operation span test) to select subject with high memory capacity (HM) and subject with low memory capacity (LM). The instrument to measure the working memory capacity was Operation span test with the numbers to be recalled. Computer-based tests were used with automatic timing, where each number that must be remembered and the operations that must be counted were displayed for 4 seconds and at the end of the process. Subjects were given 10 seconds to write down all the numbers that must be remembered. For more details, an example of o-span test can be seen in [12]. If subjects could answer the operation questions correctly and remember all given numbers, the total score was 100. Subjects were categorized to have high working memory capacity if the score was more than 55 on o-span test and they were categorized to have a low working memory capacity if the score was less than or equal to 55 on o-span test. To control the condition of the two subjects, the researcher selected two subjects with the same sex and those who had relatively similar mathematical abilities. To find out the mathematical reasoning structure, both subjects were given a
mathematical reasoning test. The test was a problem about how to install rectangular carpets with a size of 200 cm x 310 cm on the floor of a rectangular hotel room with a size of 11 m x 15 m and determine the maximum number of carpets that could be installed to cover the floor with the condition that the carpets were not cut and not overlapped. The question given was in the form of open problem so that there would be many ways to solve it. Cardboard paper was also provided for subjects if needed to solve problems. In addition, the interview guidelines used by researchers were used to reveal the structure of mathematical reasoning when the subject completed a mathematical reasoning test. The questions written in the interview guide are based on the steps of a mathematical reasoning structure, namely the problematic situation, choice strategy, implemented strategy, and conclusion. The results of mathematical reasoning tests and the results of task-based interviews were then analyzed to get conclusions.

3. Results and Discussion
From the test of working memory capacity two subjects were chose. They were LM with low working memory capacity of 48 and HM subject with high working memory capacity of 85. Both were students and had relatively similar mathematical abilities namely 75 for LM and 82 for HM. The analysis of mathematical reasoning structure in problem solving which was based on the result of mathematical reasoning tests done by the subjects and the result of the task-based interviews are described as follows.

3.1. Description of Subject’s Mathematical Reasoning Structure Based on Low Working Memory Capacity (LM)

Problematic Situation: The following are the results of LM’s interview regarding the problematic situation

Q: Do you understand the questions?
A: Yes
Q: Do you think that the problem in the questions can be solved using a formula?
A: No,
Q: Why?
A: I don’t know what formula to be used, ma’am
Q: Do you know the solution directly after reading the problem?
A: No, I don’t. I have to think about how to solve it first Mom
Q: Have you ever solved the similar question before?
A: Never.

LM understood that the problem was an open problem and it could not be solved only by using a formula. LM needed time to think about how to solve it. The task was a new problem for LM, LM had never been solved the similar problem and worked on before. So although LM was able to understand the problem well, she did not know how to solve it directly.

Strategy Choice: The following shows the results of the LM interview related to Strategy Choice

Q: What strategy did you use to solve the problem?
A: by drawing
Q: Have you used this strategy to solve the problem?
A: Not yet
Q: How did you come up with this strategy?
A: Because I've seen people carpeting at home
Q: Are you sure that this strategy can be used to solve the problem?
A: yes
Q: Why?
A: Because by drawing, I can see which parts are not covered with carpet
Q: So what are you going to draw?
A: The shape of the floor and the shape of the carpet

LM stated that there were several strategies that could be done to solve the problem. According to LM, the way to solve the problem could be by drawing and dividing the floor area with the carpet area.
However, the strategy chosen by LM to solve the problem was by drawing. Drawing strategy was a strategy that had never been used by LM to solve problems. LM could think of this strategy and choose it to solve the problem because LM had ever seen the process of installing carpet on the floor of her house. LM also believed that the strategy could solve the problem given because according to her the drawing strategy could show which parts of the floor that had not been covered with carpet. LM was able to provide predictive arguments which were logical reasonings that the strategy chosen could solve the problem. The LM’s reason to chose the drawing strategy was because according to LM installing carpet on the floor did not have a formula, unless what being asked were the floor area and carpet area. It could use a rectangular area.

**Strategy is implemented:** The following are the results of the LM interview about Strategy is implemented:

- **Q:** How did you solve the problem?
- **A:** I drew the shape of the floor and the shape of the carpet that was used to cover the floor
- **Q:** Can you explain how you came up with a solution?
- **A:** At first, I drew the shape of the floor with its dimensions, then I drew the carpet in the floor.
- **Q:** What do you consider in drawing a rug in a floor drawing?
- **A:** I adjusted the size of the floor according to the size of the carpet

LM implemented strategies according to what she chose to solve the problem. However, LM could not solve the problem completely. LM was able to implement drawing strategy, but LM had difficulty in providing verification arguments that the strategy chosen could solve the problem. LM did not believe that the answer she gave was correct or reasonable one. LM steps to solve the problem were sketching a rectangle with a length of 1100 cm and a width of 1500 with a scale of 1: 100 as a floor. Then LM drew a rectangular sketch of 310 cm x 200 cm (scale 1: 100) for the carpet in the floor drawing. The picture created was incorrect because the size of 15 m was made shorter than the size of 11 m. LM drew a rectangle (representing a carpet) of the same size as many as 20 rectangles in the same position, leaving a space of 260 cm at the bottom and 100 cm at the right. LM did it because the carpet installation process was like that. After that LM stopped drawing and stated that 20 was the maximum number needed to cover the floor. After being investigated, LM emptied some parts of the floor because she thought there was no carpet that had the right size to cover the floor that had not been filled. LM had a difficulty in giving different answers to solve the problem, so the answer in Figure 1 was the only answer given by LM. Furthermore, LM did not believe that the answer she gave was correct and there was no underlying reason that the strategy she used was the right one to solve the problem. Thus, it could be said that LM could not provide a verification argument for the strategy she had chosen and implemented.

**Conclusion.** After solving the problem given, LM drew a conclusion from the answer by writing down the number of carpets needed to cover the hotel floor by 20 carpets.

The following are the results of LM’s interview about conclusion:

- **A:** So, how much carpet is needed to cover the floor?
- **Q:** 20
- **A:** Is this the maximum required amount?
- **Q:** yes ma’am
- **A:** Are there any answers other than 20?
- **Q:** No ma’am
The image sketch as the answer from LM is shown in Figure 1.

![Image of a grid with measurements](image1)

**Figure 1. LM answer**

3.2. *Description of Subject’s Mathematical Reasoning Structure Based on High Working Memory Capacity (HM)*

**Problematic Situation:** The following is a snippet of HM’s interview regarding the problematic situation:

- **Q:** Do you understand the questions?
  **A:** Yes ma’am
- **Q:** Do you think that the problem in the questions can be solved using a formula?
  **A:** No ma’am
- **Q:** Why?
  **A:** because what must be answered is the number of carpets, so it is not clear what formula can be used
- **Q:** Do you know the solution directly after reading the problem?
  **A:** No, I don’t. I imagine that the rugs have to be arranged one by one to cover the floor
- **Q:** Have you ever finished the problem like this?
  **A:** Never.

Subject with high working memory capacity (HM) understood the task that was a problematic situation. The reason was that the problem given could be well understood, even though the problem was a new problem for her and she had never been solved it before. HM stated that after reading the problem, HM was able to imagine that the carpet had to be installed one by one to cover the floor but the task given was not clear yet how to implement it. The problem could not be solved by using a formula because according to HM there was no clear formula that could be used to determine the number of carpets.

**Strategy Choice:** The results of the interview with HM regarding Strategy Choice are as follows:

- **Q:** How many strategies do you think can be used to solve the problem?
  **A:** many ways
- **Q:** Explain it
  **A:** arranging like a puzzle, dividing the area, or drawing,
- **Q:** What strategy did you use to solve the problem?
  **A:** I put them together like a puzzle
- **Q:** Have you used this strategy to solve the other problem?
  **A:** Never
- **Q:** How did you come up with this strategy?
  **A:** For me, the problem is like a puzzle game
- **Q:** Are you sure that this strategy can be used to solve the problem?
  **A:** yes
- **Q:** Why?
  **A:** because I think it's very easy

HM was able to choose the strategy used to solve problems. HM stated that there were many ways to solve the problem, some of which were by compiling like a puzzle, dividing the area, or drawing. To
solve this problem, HM chose a strategy to install the carpets by putting together like a puzzle. HM could give predictive arguments that chose the strategy because she believed that the strategy could solve the problem. According to her the strategy will make her easier to find answers to the problems given. This is based on the experience she had. HM claimed she often play puzzles before. Thus, even though HM had never solved the problem like that before by using the strategy of compiling the puzzle, HM thought of the strategy because according to her the problem given was like a puzzle game.

**Strategy is implemented:** The strategy implementation carried out by HM is known through interviews as follows:

**Q:** How did you solve the problem?
**A:** I made the puzzle like this (cut the paper into small rectangles)

**Q:** how much should you make?
**A:** I don’t know yet, I’ll just make a lot of it first, later when it’s installed, I’ll know how much is needed

**Q:** and then?
**A:** And here is the board, (draw a rectangle under the paper and give the measurements on each side)

**Q:** So how do you find the solution?
**A:** I put together the puzzle on this paper, (while putting on the paper). If everything was installed, I just counted how many were installed so I knew the number of carpets

**Q:** how do you install it so that you get the minimum quantity?
**A:** I put it flat and stand up

**Q:** is it okay to be like that?
**A:** it doesn’t have any rules how to install it

**Q:** Does the puzzle cover all the boards?
**A:** no

HM could implement the strategy properly. HM solved the problem by considering that the carpets were puzzle pieces and the floor was the puzzle board. The settlement step taken by HM was by making puzzle pieces first, namely by cutting rectangular paper with size of 2 cm x 3.1 cm as shown in Figure 2. HM did not determine the number of puzzle pieces she made. According to her the number of puzzle pieces she needed would be determined by the number of puzzles to be installed. Then she drew a rectangle on paper with a size of 11 cm x 15 cm as the puzzle board like Figure 3. Then HM arranged the puzzle pieces on the puzzle board that she had made, and she calculated the installed puzzle. HM arranged the puzzle several times and counted the installed puzzles (see Figure 4). According to her these were done to get the maximum number. HM obtained the maximum number by installing puzzles in a vertical and horizontal way (like Figure 5). The number of carpets that could be installed by HM were 23 and according to her, it was the maximum number.

![Figure 2. HM’s Puzzle pieces](image1)

![Figure 3. HM’s Puzzle Board](image2)
HM gave predictive arguments that she believed the strategy she chose and implemented could solve the problem. The reason was after doing the installation of the puzzle pieces in different ways, HM could find the number of carpets that could cover the floor even though the carpets could not cover the floor completely because of the requirement in question which stated that the carpets could not be cut.

For the final step, HM could provide a conclusion by stating that 23 carpets were the maximum number that could be installed on the floor.

A: How much carpet is needed to cover the floor?
Q: 23
A: Is this the maximum amount needed?
Q: yes
A: is there any other answer besides 23?
Q: no ma’am

The sketch of carpets installation according to HM can be seen in Figure 6.

The mathematical reasoning structure of students with research subjects with high working memory capacity shows the four mathematical reasoning structures, namely problematic situation, choice strategy, implemented strategy, and conclusion. This is supported by research conducted by Bresgy which states that there is a strong relationship between working memory and learning and skills [21]. Various other empirical studies [22] also show that working memory is also important in math skills in children; so that children with high working memory capacities are able to think of appropriate solutions to the problems given.
4. Conclusion

The mathematical reasoning structure of research subjects with low working memory capacity shows that at the problematic situation stage, the subject considers that the task given is a problem where the solution is not clear, the strategy choice stage: the subject chooses a drawing strategy that will be used to solve the problem and provides predictive arguments. For strategy implementation, the subject draws the carpet and arranges it in the same pattern but cannot provide a verification argument. Conclusion is carried out by the subject, namely being able to provide answers according to the results they get. The mathematical reasoning structure of students with research subjects with high working memory capacity shows the four mathematical reasoning structures, namely the problematic situation: the subject can understand problems that are open problems, a strategy choice where the subject can choose strategies, namely composing like puzzles to solve problems and being able to provide predictive arguments, implemented strategy, namely the subject can implement the strategy he chooses to find answers to the problems given and provide verification arguments, Conclusion, namely the subject can provide answers to the problems given.

The difference of mathematical reasoning structure of the two subjects is the different selection of strategies and the implementation of strategies. Subject with low working memory capacity is only able to think of one position in compiling a rectangle that represents the carpet. She is unable to think of various positions to obtain the maximum result. She also cannot give verification arguments. This may happen because the working memory capacity that is not high prevents subject in processing a lot of information and sorting it according to what is needed in solving problems to get a maximum result. Subject also had a difficulty in giving reasons related to solving the problem given due to lack of information that could be recalled, linked and understood by the subject.

Subject with high working memory capacity, in addition to being able to choose strategies that are flexible and effective, can also implement strategies using puzzle strategies in various ways, namely changing the position of the rectangles representing the carpets so that maximum results are obtained, and she can provide predictive and verification reasons of the strategies used. This may happen because she has a high working memory capacity, so she can choose and process much information at one time. Similar research needs to be done with broader subjects to get stronger results. Based on the results of this study, it is necessary to think of ways how to train students with low working memory capacity in doing their mathematical reasoning structure in solving problems so that they are more optimal or think of ways to increase students’ working memory capacity.

References

[1] Palengka I, Juniati D and Abadi 2019 J. of Phys.: Conf. Series 1417 012055
[2] Aineamani B 2018. How Learners Communicate Their Mathematics Reasoning in Mathematics Discourse. ICME-13 Monographs 65
[3] Léon B 2014 Mach Learn. 94 133
[4] Jonsson B, Kulaksiz Y C, and Lithner J 2016 Int. J. of Math. Educ. in Sci. and Tech., 47 1206
[5] Norqvist M 2019 J. of Math.1 Behav. 40 104
[6] Jonsson B, Kulaksiz Y C, and Lithner J 2016 Int. J. of Math. Educ. in Sci. and Tech. 47 1206
[7] Lithner J 2015 Learning Mathematics by Creative or Imitative Reasoning. Selected Regular Lectures from the 12th International Congress on Mathematical Education 487
[8] Bergqvist, T and Lithner J 2012 Mathematical reasoning in teachers’ presentations. The Journal of Mathematical Behavior 31 252
[9] Jonsson B, Norqvist M, Liljekvist Y, and Lithner J 2014 The J. of Math. Behav. 36 20
[10] Lithner J 2017 ZDM 49 937
[11] Lithner J 2008 Educ. Stud. in Math. 67 255
[12] Juniati D and Budayasa I K 2020 J. for the Educ. of Gift. Young Sci. 8 271
[13] Cowan N 2005 Working Memory Capacity (New York: Psychology Press)
[14] Baddeley A D 2000 Sci. 4 417
[15] Oberauer K, Farrell S, Jarrold C and Lewandowsky S 2016 Psychol. Bulletin 142 758
[16] Conway A R A, Kane M J and Engle R W 2003 *Trends in Cognitive Sci.* 7 547
[17] de Fockert J W, Rees G, Frith C D and Lavie N 2001 *Sci.* 291 1803
[18] Süß H M, Oberauer K, Schulze R, and Wilhelm O 2005 *Psychol. Bulletin* 131 61
[19] Wiley J and Jarosz A F 2012 *How Working Memory Capacity Affects Problem Solving. Psychology of Learning and Motivation* 185
[20] Fyfe E R, Eisenband M L, Hunt K M and Alibali M W 2019 *J. of Comm. Disorders* 77 17
[21] Bresgi L, Alexander D L M, and Seabi J 2017 *Int. J. of Educ. Res.* 81 1
[22] Holmes J, Adams J, and Hamilton C 2008 *European J. of Cognitive Psychol.* 20 272