$D = 5$ Simple Supergravity on $AdS_2 \times S^3$

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Abstract

The Kaluza-Klein spectrum of $D = 5$ simple supergravity compactified on $S^3$ is studied. A classical background solution which preserves maximal supersymmetry is fulfilled by the geometry of $AdS_2 \times S^3$. The physical spectrum of the fluctuations is classified according to $SU(1,1|2) \times SU(2)$ symmetry, which has a very similar structure to that in the case of compactification on $AdS_3 \times S^2$.
1. Introduction  Maldacena’s conjecture \cite{1} is one of the most attractive subjects among recent progress in the non-perturbative string/M-theory. In this paper, we consider $D = 5$ simple supergravity (SUGRA) with 0-brane solution whose near-horizon geometry is the direct product of two-dimensional anti-de-Sitter space and three-sphere ($AdS_2 \times S^3$), and investigate its Kaluza-Klein spectrum. Assuming that the $AdS_2/CFT_1$ correspondence holds \cite{2}-\cite{5}, we show that the spectrum possesses $SU(1,1|2) \times SU(2)$ symmetry. It must be remarked that Gauntlett et.al.\cite{6} has recently evinced the super-Poincaré group of $AdS_2 \times S^3$ SUGRA to be $SU(1,1|2) \times SU(2)$. Our results provide each Kaluza-Klein mode with the precise assignment to the certain representation of $SU(1,1|2) \times SU(2)$.

On the other hand, $D = 5$ simple SUGRA possesses a very similar structure to that of $D = 11$ SUGRA \cite{7}-\cite{10}. In particular, the former theory allows solitonic string (1-brane) and particle (0-brane) solutions that respectively correspond to M5- and M2-brane solutions in the latter \cite{11}. While the near-horizon geometry of the solitonic 1-brane in $D = 5$ simple SUGRA is $AdS_3 \times S^2$, that of the solitonic 0-brane is $AdS_2 \times S^3$. In the previous work \cite{12}, the Kaluza-Klein spectrum in this $AdS_3 \times S^2$ compactification is studied and its $SU(1,1|2)_R \times SU(1,1)_L$ symmetry, which can be regarded as the finite-dimensional subalgebra of chiral $N = (4,0)$ superconformal algebra, is found. Interestingly enough, two different compactifications are endowed with quite similar symmetries, chiral $SU(1,1|2)_R \times SU(1,1)_L$ and single $SU(1,1|2) \times SU(2)$. It originates in the magnetic/electric duality between $AdS_3 \times S^2$ and $AdS_2 \times S^3$ simple SUGRA theories.

2. Field equations  In terms of the metric $g_{MN}$, $U(1)$ gauge field $A_M$, and the spin-3/2 field $\psi_M$, $D = 5$ simple SUGRA is defined by the Lagrangian

$$\begin{align*}
\mathcal{L} &= e_5 \left[ -\frac{1}{4} R - \frac{1}{4} F_{MN} F^{MN} \\
&\quad - \frac{i}{2} \left( \bar{\psi}_M \Gamma^{MNP} D_N \left( \frac{3 \omega - \hat{\omega}}{2} \right) \psi_P + \bar{\psi}_P \Gamma^{MNP} D_M \left( \frac{3 \omega - \hat{\omega}}{2} \right) \psi_N \right) \\
&\quad - \frac{1}{6 \sqrt{3}} e_5^{-1} \epsilon^{MNPQR} F_{MN} F_{PQ} A_R \\
&\quad - \frac{\sqrt{3} i}{8} \psi_M (\Gamma^{MNPQ} + 2 g_M^{[P} g_Q^{N]} \psi_N (F_{PQ} + \hat{F}_{PQ}) \right],
\end{align*}$$

where the notations used here follow those in \cite{8} and the signature is $(+ - - - -)$. Capital Roman letters run from 0 to 4. In the absence of the $\psi_M$ field, Einstein-Maxwell’s equation is shown to be

$$R_{MN} - \frac{1}{2} g_{MN} R = - \left( 2 F_{MP} F^P_N - \frac{1}{2} g_{MN} F^2 \right),$$

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The equation of motion for the bosonic fields reads

\[ F^{MN} = \frac{1}{2\sqrt{3}} e^{-1} \epsilon^{NPQRS} F_{PQ} F_{RS}. \] (2)

The above equation of motion enjoys Freund-Rubin-like solutions \[13\] that preserve the maximal number of the supersymmetries after three-dimensional compactification. One of these such solutions over which we will be considering the Kaluza-Klein spectrum possesses the geometry of \( AdS_2 \times S^3 \), namely,

\[ \hat{R}_{\mu\nu\rho\sigma} = \frac{4}{3} f^2 \left[ \hat{g}_{\mu\rho} \hat{g}_{\nu\sigma} - \hat{g}_{\mu\sigma} \hat{g}_{\nu\rho} \right], \]

\[ \hat{R}_{\mu\nu pq} = -\frac{1}{3} f^2 \left[ \hat{g}_{mp} \hat{g}_{nq} - \hat{g}_{mq} \hat{g}_{np} \right], \] (3)

\[ \hat{g}_{\mu m} = \hat{F}^{\mu m} = 0, \quad \hat{F}_{\mu\nu} = f \hat{e}_{2\epsilon_{\mu\nu}}, \quad \hat{F}_{mn} = 0, \]

where Greek letters are used for 0 or 1, small Roman letters for 2, 3 or 4, and \( ^{o} \) means the background. Let us set the free parameter \( f \) to be \( \sqrt{3} \).

3. Kaluza-Klein spectrum: bosonic modes The spectrum of small variations around the above \( AdS_2 \times S^3 \) background is derived from Einstein-Maxwell’s equation (2). We docket the small variations of the bosonic fields, \( g_{MN} \) and \( A_M \), like

\[ \delta g^{MN} = h^{MN}, \quad \delta A_M = a_M. \] (4)

After fixing the gauge and diffeomorphism degrees of freedom in the way that

\[ a_m;^m = 0, \quad h^{\mu m}_{\nu m} = 0, \quad h^{\mu m}_{;m} = 0, \quad h^m_m = 0, \] (5)

the equation of motion for the bosonic fields reads

\[ -\frac{1}{2} \left( h^\lambda_{\mu;\nu;\lambda} + h^\lambda_{\nu;\mu;\lambda} - h^\lambda_{\lambda;\mu;\nu} - h^\lambda_{\mu\nu;\lambda} \right) = 0, \]

\[ +4 h_{\mu\nu} - 5 \hat{g}_{\mu\nu} h^\lambda_{\lambda} + \frac{1}{2} \hat{g}_{\mu\nu}(h^{\mu\nu}_{;;\mu\nu} + h_{;;m}^{mn} - h^\lambda_{\lambda;\mu\nu}) + 4\sqrt{3} \left\{ \hat{e}_{2\epsilon^{\lambda}} a_{[\lambda;\nu]} + \hat{e}_{2\epsilon^{\lambda}} a_{[\lambda;\mu]} \right\} - 2\sqrt{3} \hat{g}_{\mu\nu} \hat{e}_{2\epsilon^{\lambda}} a_{\sigma;\lambda} = 0, \] (6)

\[ -\frac{1}{2} (h^\lambda_{\mu,\lambda} + h^r_{m;\mu;\nu} - h^\lambda_{\lambda;\mu;\nu} - h^m_{\mu;\lambda}) - 3 h_{\mu\nu} + 4\sqrt{3} \hat{e}_{2\epsilon^{\lambda}} a_{[\lambda;\mu]} = 0, \] (7)

\[ -\frac{1}{2} (h^r_{m;\mu;\nu} + h^r_{n;\mu;\nu} - h^\lambda_{\lambda;\mu;\nu} - h^m_{\mu;\lambda}) - 2 h_{mn} + \hat{g}_{mn} h^\lambda_{\lambda} + \frac{1}{2} \hat{g}_{mn}(h^{\mu\nu}_{;\mu\nu} + h_{;,m}^{mn} - h^\lambda_{\lambda;\mu\nu}) - 2\sqrt{3} \hat{g}_{mn} \hat{e}_{2\epsilon^{\lambda}} a_{\sigma;\lambda} = 0, \] (8)

\[ a^{\mu;\lambda}_{;M} - 4 a^{\mu\nu} - \frac{\sqrt{3}}{2} \hat{e}_{2\epsilon^{\sigma}} h^\lambda_{\lambda;\sigma} = 0, \] (9)

\[ a^{m;\lambda}_{;M} + 2 a^m + 4 \hat{e}_{2\epsilon} a_{\phi;\nu} + \sqrt{3} \hat{e}_{2\epsilon} h^m_{\sigma;\lambda} = 0. \] (10)
To diagonalize the mass matrices derived from the above equation, we adopt the spherical harmonics on $S^3$ with the rank-0, 1 and 2. To begin with, let us summarize the properties of three-dimensional spherical harmonics. Because the isometry group of $S^3$ is $SO(4)$, eigenfunctions of the three-dimensional Laplacian $\Delta = \nabla^m \nabla_m$, and the three-dimensional spherical harmonics, simultaneously belong to a certain representation of $SO(4)$. On the other hand, $SO(4)$ is decomposed into a direct product of two $SU(2)$’s, namely $SU(2) \times SU(2)$. Therefore, we are able to use spins of two $SU(2)$’s to classify the Kaluza-Klein spectrum. Let $\vec{J}$ ($\vec{\bar{J}}$) denote the Casimir operator of $SU(2)$ ($SU(2)$) with eigenvalues $j (j + 1)$ for $j, \bar{j} = 0, 1/2, 1, 3/2, \cdots$. $\Phi (\bar{\Phi})$ is the representation of $SU(2)$ ($SU(2)$) with spin $j$ ($\bar{j}$). In general, $j$ and $\bar{j}$ can take different half-integer values. The difference $|j - \bar{j}|$ is called the rank of the spherical harmonic. This rank corresponds to that of the tensor structure not on $AdS_2$ but on $S^3$. Spherical harmonics with rank-0, 1, and 2 are explicitly constructed as follows.

1. rank-0 harmonics

First, for scalar functions on $S^3$, one can confirm the equality among $\vec{J}^2$, $\vec{\bar{J}}^2$, and $\Delta$

$$- \Delta = 4 \vec{J}^2 = 4 \vec{\bar{J}}^2. \quad (11)$$

It is shown that a product $Y^{(k)} = \Phi^{(k/2)} \bar{\Phi}^{(k/2)}$ behaves as a scalar on $S^3$ and is an eigenfunction of $\Delta$. In fact, its eigenvalue for $\Delta$ is given by $-k(k+2) = -4j(j+1) = -4\bar{j}(\bar{j}+1)$, where $k = 2j = 2\bar{j} = 0, 1, 2, \cdots$.

2. rank-1 harmonics

In a similar way to the case for rank-0 harmonics, one can prove that $Y_m^{(k, \pm)} = \Phi^{(k+1/2)} \bar{\Phi}^{(k-1/2)}$ are vector functions on $S^3$ and, simultaneously, eigenfunctions of the Laplacian with the eigenvalue $-\Delta = k(k+2) - 1$ for $k = 1, 2, 3, \cdots$. Introducing the rotational derivative for vector fields $(\text{rot } v)^m = \varepsilon_{mnp} v^p; n$, we should remark that

$$\text{rot } Y_m^{(k, \pm)} = \pm (k+1) Y_m^{(k, \pm)}. \quad (12)$$

3. rank-2 harmonics

$Y_m^{(k, \pm)} = \Phi^{(k+3/2)} \bar{\Phi}^{(k-3/2)}$ are rank-2 tensors on $S^3$ and are eigenfunctions of $\Delta$ with the eigenvalue $-\Delta = k(k+2) - 2$ for $k = 2, 3, 4, \cdots$.

By expanding the bosonic fields $a_M$ and $h_{MN}$ in terms of the above spherical harmonics, the field equation (6)-(10) is separated into three sets with rank-0, 1 and 2. The $SO(4)$-charge of each bosonic mode is inherited from the used spherical harmonic with a certain rank. The results are summarized in the following.
1. rank-0
The eigenvalues of the two-dimensional d’Alembertian $\square$ on $AdS_2$ for $h_{mn}^{\cdot \cdot}; h_{\mu\nu}^{\cdot \cdot}$, $\hat{\eta}^{\lambda\sigma} a_{\sigma, \lambda}$ and $h^{\lambda}_{\cdot \cdot}$ are

$$\lambda^2 = -\square = k^2 - 2k \quad \text{for} \quad k = 2, 3, 4, \cdots \quad (13)$$

and

$$\lambda^2 = k^2 + 6k + 8 \quad \text{for} \quad k = 0, 1, 2, \cdots . \quad (14)$$

The reason why the tower (13) begins from $k = 2$ is because at $k = 0$ the mass-matrix is degenerated, and at $k = 1$ the eigenvalue corresponding to this tower is inconsistent with the tensor structure of $h_{mn}^{\cdot \cdot}$ which is the origin of $h_{mn}; h_{\mu\nu}^{\cdot \cdot}$. The $SO(4)$-charges corresponding to both of the above two branches prove identically

$$(j, \bar{j}) = (\frac{k}{2}, \frac{k}{2}). \quad (15)$$

2. rank-1
Let us consider the mass-matrices on $\hat{\eta}^{\lambda\sigma} h_{m\lambda, \sigma}$ and $a_m$. One can immediately see that the mass matrices take different forms for $Y^{(k, +)}_m$ or $Y^{(k, -)}_m$. The eigenvalues for $Y^{(k, +)}_m$ are

$$\lambda^2 = k^2 + 8k + 15 \quad \text{or} \quad k^2 - 1 \quad (16)$$

with the same $SO(4)$ charge

$$(j, \bar{j}) = (\frac{k+1}{2}, \frac{k-1}{2}) \quad (17)$$

for $k = 1, 2, 3, \cdots$. The eigenvalues for $Y^{(k, -)}_m$ are

$$\lambda^2 = k^2 - 4k + 3 \quad \text{for} \quad k = 2, 3, 4 \cdots \quad (18)$$

and

$$\lambda^2 = k^2 + 4k + 3 \quad \text{for} \quad k = 1, 2, 3 \cdots \quad (19)$$

with the $SO(4)$-charges

$$(j, \bar{j}) = (\frac{k-1}{2}, \frac{k+1}{2}). \quad (20)$$

We should remark that the eigenvalue (18) for $k = 1$ is missing. At $k = 1$, this mode is massless and degenerates into that of $k^2 - 1$ ($k = 1$). However, the dynamical degree of freedom for massless scalars in the present case is merely one, and the three-dimensional chirality such that $j - \bar{j} < 0$ is not consistent with the two-dimensional parity of the scalar fields. Therefore, we drop that mode.
3. rank-2

On the rank-2 field $h_{mn}$, the d’Alembertian has the eigenvalue

$$\lambda^2 = k^2 + 2k \quad \text{for} \quad k = 2, 3, 4 \cdots,$$

(21)

for both

$$(j, \bar{j}) = (\frac{k+2}{2}, \frac{k+2}{2}).$$

(22)

4. Kaluza-Klein spectrum: fermionic modes

The fermionic fluctuation around the $AdS_2 \times S^3$ background, $\psi_M = 0$, is labeled simply by $\psi_M$ itself. Its linearized equation of motion is drawn from the Lagrangian $[I]$:

$$i \Gamma^{MNP} \psi_{P, N} - \frac{\sqrt{3}}{4} i (\Gamma^{MNPQ} + 2 \hat{g}^{[P} \hat{g}^{Q]} N) \psi_N \hat{F}_{PQ} = 0.$$

(23)

To analyze the Kaluza-Klein spectrum from (23), we utilize rank-1/2 spherical harmonic $\xi^m$ on $S^3$ which is an eigenspinor of the equation

$$\Gamma^m \nabla_n \xi^m + \Gamma^m \kappa \xi^m = \kappa \xi^m.$$

(24)

Once such rank-1/2 spherical harmonic $\xi^m$ and eigenvalue $\kappa$ are found, the field equation (23) is reduced to the following two-dimensional Dirac equation for $\psi^m$,

$$\gamma^5 \gamma^\mu \psi^m_{;\mu} + \left( \kappa - \frac{1}{2} \right) \psi^m = 0.$$

(25)

Therefore, the fermionic spectrum is obtained by solving the eigenvalue problem (24).

The rank-1/2 spherical harmonic $\xi^m$ can be expanded by the product of rank-1 harmonics and Killing spinors on $S^3$, i.e. the separation of the total angular momentum into the orbital and the internal ones. By proceeding this expansion with the help of the gauge fixing $\Gamma_M \psi^M = 0$, we see that the eigenvalue $\kappa$ can take one of the values

$$\kappa = -k - \frac{5}{2}, \quad (j, \bar{j}) = (\frac{k+2}{2}, \frac{k-1}{2}) \quad \text{for} \quad k = 1, 2, 3 \cdots,$$

(26)

$$\kappa = k - \frac{1}{2}, \quad (j, \bar{j}) = (\frac{k-2}{2}, \frac{k+1}{2}) \quad \text{for} \quad k = 2, 3, 4 \cdots,$$

(27)

$$\kappa = -k - \frac{1}{2}, \quad (j, \bar{j}) = (\frac{k}{2}, \frac{k-1}{2}) \quad \text{for} \quad k = 1, 2, 3 \cdots,$$

(28)

$$\kappa = k + \frac{3}{2}, \quad (j, \bar{j}) = (\frac{k}{2}, \frac{k+1}{2}) \quad \text{for} \quad k = 1, 2, 3 \cdots.$$  

(29)

Several remarks are in order. First, for the two towers (26) and (27), the gamma-traceless condition, $\Gamma_m \xi^m = 0$, which means $|j - \bar{j}| = 3/2$, is satisfied while it is not satisfied for the
other two. Secondly, the eigenstate for (27) at $k = 1$ is absent because one can show that for that tower, $\xi^m = 0$ at $k = 1$ by using its explicit form.

The way to assign the $SO(4)$-charge to each mode indicated in (26)-(29) may need some explanation. To see this, one should utilize the above mentioned fact that $\xi^m$ can be expanded by the product of $Y_m^{(k,\pm)}$ with $SO(4)$-charge $(k/2 \pm 1/2, k/2 \mp 1/2)$ and Killing spinors with $(1/2, 0)$. Let us note here that the Killing spinors with $(1/2, 0)$ possess the negative chirality for $\mathbb{R}^4$ in which $S^3$ is embedded. In summary, the rank-1/2 spherical harmonic $\xi^m$ has one of the $SO(4)$ charges in the following decomposition.

\[
\left(\frac{k+1}{2}, \frac{k-1}{2}\right) \otimes \left(\frac{1}{2}, 0\right) \oplus \left(\frac{k-1}{2}, \frac{k+1}{2}\right) \otimes \left(\frac{1}{2}, 0\right) = \left(\frac{k+2}{2}, \frac{k-2}{2}\right) \oplus \left(\frac{k+2}{2}, \frac{k+1}{2}\right) \oplus \left(\frac{k-2}{2}, \frac{k+1}{2}\right) \oplus \left(\frac{k}{2}, \frac{k+1}{2}\right),
\]

where the representation as $\left(\frac{k+1}{2}, \frac{k-2}{2}\right)$ starts from $k = 2$ while the others from $k = 1$. The irreducible decomposition (30) gives the charges in (26)-(29).

### 5. Symmetry of the spectrum

Let us classify the acquired Kaluza-Klein spectrum by a certain symmetry group. By considering the super-Poincaré group consisting of the isometry and the supercharges, the symmetry of the modes is shown to be a superalgebra $SU(1,1|2) \times SU(2)$ \cite{3}. We should remark that the bosonic part of $SU(1,1|2) \times SU(2)$ is $SU(1, 1) \times SU(2) \times SU(2) \cong SO(1,2) \times SO(4)$, which is nothing but the product of the isometry group of $AdS_2$ and that of $S^3$. Moreover, let us note that $SU(1,1|2)$ is a finite-dimensional subalgebra of $N = 4$ super-Virasoro algebra. The oscillator representations \cite{15}, which is very helpful to visualize the representations, of $SU(1,1|2)$ has already appeared in the analysis of the Kaluza-Klein spectrum on $AdS_3 \times S^3$ and also on $AdS_3 \times S^2$ \cite{16}-\cite{20}.

Assuming that this SUGRA is coupled to a one-dimensional conformal field theory at the boundary of $AdS_2$, we can translate each Kaluza-Klein mode into a conformal weight $h$ by means of the formula in \cite{14, 21}

\[
h = \frac{1 + \sqrt{1 + \lambda^2}}{2}. \tag{31}
\]

Hence we can make Table 1.
On the other hand, we can carry on the same mapping for the fermionic modes by using
the asymptotic form of $\psi^m$ determined by (25),
\[
h = \frac{1}{2} \left| \kappa - \frac{1}{2} \right| + \frac{1}{2},
\]
(32)

The table for the fermionic modes turns out as Table 2.

| $\kappa$ | $h$ | $(j, \bar{j})$ | $j - \bar{j}$ | Fig. |
|----------|-----|----------------|----------------|------|
| $k + \frac{3}{2}$ (k ≥ 1) | $\frac{k+\frac{3}{2}}{2}$ | $(\frac{k}{2}, \frac{k+1}{2})$ | $-\frac{1}{2}$ | 1 |
| $-k - \frac{1}{2}$ (k ≥ 1) | $\frac{k+\frac{1}{2}}{2}$ | $(\frac{k}{2}, \frac{k-1}{2})$ | $\frac{1}{2}$ | 2 |
| $k - \frac{1}{2}$ (k ≥ 2) | $\frac{k+\frac{1}{2}}{2}$ | $(\frac{k-2}{2}, \frac{k+1}{2})$ | $-\frac{3}{2}$ | 3 |
| $-k - \frac{5}{2}$ (k ≥ 1) | $\frac{k+\frac{5}{2}}{2}$ | $(\frac{k+2}{2}, \frac{k-1}{2})$ | $\frac{3}{2}$ | 4 |

Table 2: Fermionic modes.

Finally we can fit these results into representations of $SU(1,1|2) \times SU(2)$. In [13], we
dealt with a chiral $SU(1,1|2)_R \times SU(1,1)_L$ symmetry. Upon replacing the $SU(1,1)_L$ by
$SU(2)$, we confirm that all Kaluza-Klein modes fall into four supermultiplets of $SU(1,1|2) \times
SU(2)$ in a similar manner to the case of $AdS_3 \times S^2$. Every spectrum obeying this single
$SU(1,1|2) \times SU(2)$ symmetry is summarized in Figs. 1-4. In those four figures, one of
the two $SU(2)$-charges, namely $\bar{j}$ is fixed at a certain value. For example, the fields with
the same $\bar{j} = k/2$ and $j - \bar{j} = 0, -1/2, -1$ are gathered in Fig. 1. In each figure,
one can notice a multiplet-shortening very similar to that for the chiral primary fields in
two-dimensional $N = 4$ superconformal theories. The shortest supermultiplet appears as a
doubleton in Fig. 3 (k = 1). The massless multiplets exist in Figs. 1, 2, 3 (k = 2).
6. Discussions  From the explicit calculation of the Kaluza-Klein spectrum of the \(AdS_2 \times S^3\) simple SUGRA, we have seen that the two different compactifications, \(AdS_3 \times S^2\) and \(AdS_2 \times S^3\), are characterized by closely related superalgebras, \(SU(1,1|2)_R \times SU(1,1)_L\) and \(SU(1,1|2) \times SU(2)\), respectively, each of which allows short representations. The similarity between the spectra of \(AdS_3 \times S^2\) and \(AdS_2 \times S^3\) is not accidental but owing to the magnetic/electric duality for the solitonic 1-brane and the 0-brane in \(D = 5\) simple SUGRA. Einstein-Maxwell’s equation (2) allows two kinds of solitonic objects. One is the solitonic 1-brane solution as follows. The metric with a solitonic 1-brane is

\[
ds_5^2 = H_1^{-1} (-dt^2 + dy^2) + H_1^2 (dr^2 + r^2 d\Omega_2^2), \quad H_1 = 1 + \frac{Q_1}{r},
\]

where \((t,y)\) is the coordinate of the world-sheet, \(r\) is the distance from the soliton, and \(Q_1\) is a constant. The non-zero field strength of the gauge field is

\[
F^{ij} = -\sqrt{3} \varepsilon^{ijk} H_1^{-4} \partial_k H_1,
\]

where both indices \(i\) and \(j\) denote transverse directions, \(i.e.\) not \(t\) or \(y\). 

From the form of the field strength, this soliton corresponds to a magnetic monopole. It is worthwhile to mention that the near-horizon geometry is nothing but \(AdS_3 \times S^2\). The other one is the solitonic 0-brane solution. The metric and the field strength in this configuration are

\[
ds_5^2 = -H_0^{-2} dt^2 + H_0 (dr^2 + r^2 d\Omega_3^2), \quad F^{\mu\nu} = (\delta_\mu^t \delta_\nu^r - \delta_\mu^r \delta_\nu^t) \cdot \sqrt{3} \frac{Q_0}{r^3 H_0},
\]

\[
H_0 = 1 + \frac{Q_0}{r^2},
\]

where \(t\) parameterizes the world-line, \(r\) is the distance from the 0-brane, and \(Q_0\) is the charge of the solitonic 0-brane. One can see that the gauge field corresponds to that induced by an electric charge and that the near-horizon geometry turns out to be \(AdS_2 \times S^3\). Therefore, the similarity of the spectra can be regarded as a reminiscent of the five-dimensional magnetic/electric duality.

On the other hand, as is discussed in [3], the \(SU(1,1)\) isometry of \(AdS_2\) can be enlarged to a single infinite-dimensional Virasoro symmetry in the boundary similarly to that in the case of \(AdS_3/CFT_2\) [22]. This single (super-)Virasoro algebra has been explicitly constructed in the study of conformal mechanics that describes the behavior of a test particle near the horizon of a four-dimensional extreme Reissner-Nordström black hole [2]. It would be interesting to explore the relation between the conformal mechanics and \(AdS_2 \times S^3\) simple SUGRA.
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Figure Captions

Fig.1  Multiplet of boundary fields with $j - \bar{j} = 0, -\frac{1}{2}, -1$.

Fig.2  Multiplet of boundary fields with $j - \bar{j} = 1, \frac{1}{2}, 0$.

Fig.3  Multiplet of boundary fields with $j - \bar{j} = -1, -\frac{3}{2}, -2$.

Fig.4  Multiplet of boundary fields with $j - \bar{j} = 2, \frac{3}{2}, 1$. 
\[ j - \tilde{j} = -1 \quad \frac{k+2}{2}, \frac{k-2}{2} \]
\[ j - \tilde{j} = -\frac{1}{2} \quad \frac{k+1}{2}, \frac{k-1}{2} \]
\[ j - \tilde{j} = 0 \quad \frac{k}{2}, \frac{k}{2} \]
\[ j - \tilde{j} = 1 \quad \frac{k+2}{2}, \frac{k-2}{2} \]

\[ \tilde{j} = \frac{k+2}{2} \quad (k \geq 2) \]

**Fig. 1**

\[ j - \tilde{j} = 0 \quad \frac{k+2}{2}, \frac{k-2}{2} \]
\[ j - \tilde{j} = \frac{1}{2} \quad \frac{k+1}{2}, \frac{k-1}{2} \]
\[ j - \tilde{j} = 1 \quad \frac{k}{2}, \frac{k}{2} \]

\[ \tilde{j} = \frac{k+2}{2} \quad (k \geq 2) \]

**Fig. 2**
\[ j = \frac{k+2}{2} \quad (k \geq 1) \]

**Fig. 3**

\[ j = \frac{k+4}{2} \quad (k \geq 4) \]

**Fig. 4**