Experimental Realization of Nonadiabatic Holonomic Quantum Computation

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Due to the geometric nature, holonomic quantum computation is fault-tolerant against certain types of control errors. Although proposed more than a decade ago, the experimental realization of holonomic quantum computation is still an open challenge. In this Letter, we report the first experimental demonstration of nonadiabatic holonomic quantum computation in a liquid NMR quantum information processor. Two non-commuting one-qubit holonomic gates, rotations about $x$- and $z$-axes, and the two-qubit holonomic CNOT gate are realized by evolving the work qubits and an ancillary qubit nonadiabatically. The successful realizations of these universal elementary gates in nonadiabatic holonomic quantum computation demonstrates the experimental feasibility of this quantum computing paradigm.

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1. Introduction. Holonomic quantum computation (HQC) was first proposed by Zanardi and Rasetti [1]. In their original work, the twisting of eigenspaces of adiabatically varying Hamiltonian was used to manipulate quantum states in a robust manner. Due to the geometric nature, HQC is robust against certain types of control errors. Since control errors are one main obstacle to the realization of quantum computation, HQC has become one promising quantum computing paradigm and attracted increasing interests recently [2, 13].

Early HQC is based on adiabatic evolution, in which states are encoded in degenerate eigenstates of a Hamiltonian, and gates are accomplished by adiabatically varying the Hamiltonian along a loop in the parameter space. Because of the adiabatic requirement, long run-time is naturally required in the parametric control in adiabatic HQC (AHQC). This not only limits the gate speed, but also exposes the system to environment for a long time, and consequently leads to decoherence and reduces the efficiency of AHQC. To overcome these drawbacks in AHQC, nonadiabatic HQC (NHQC) has been pursued, and several NHQC protocols have been proposed [17, 18]. In NHQC, the long run-time requirement is avoided, while still retaining all the robust advantages, making NHQC a very appealing quantum computing paradigm.

In this Letter, we report the first experimental realization of NHQC using a liquid NMR quantum information processor. The NHQC scheme we realize is based on a variant of the recently proposed NHQC scheme in Ref. [18]. In our modified NHQC scheme, decoherence-free subspace is not used and nonadiabatic holonomic evolution is achieved by nonadiabatically evolving the work qubits and an ancillary qubit. To experimentally realize universal quantum computation, nonadiabatic one-qubit holonomic rotation gates about $x$- and $z$-axes and the nonadiabatic two-qubit holonomic CNOT gate are successfully implemented using a three-qubit NMR quantum information processor. These results demonstrate the experimental feasibility of NHQC.

2. Theoretical protocol. We first briefly review the holonomic conditions. Consider an $N$-dimensional quantum system with its Hamiltonian $H(t)$. Assume the state of the system is initially in a $M$-dimensional subspace $S(0)$ spanned by a set of orthonormal basis vectors $\{ |\phi_k(0)\rangle \}_{k=1}^M$. It has been proved that in NHQC the evolution operator is a holonomic matrix acting on $S(0)$ if $|\phi_k(t)\rangle$ satisfy the following conditions,

\begin{equation}
(1) \quad \sum_{k=1}^M |\phi_k(\tau)\rangle \langle \phi_k(\tau)| = \sum_{k=1}^M |\phi_k(0)\rangle \langle \phi_k(0)|,
\end{equation}

\begin{equation}
(2) \quad \langle \phi_k(t)| H(t) |\phi_l(t)\rangle = 0, \quad k, l = 1, \ldots, M.
\end{equation}

where $\tau$ is the evolution period and $|\phi_k(t)\rangle = T \exp(-i \int_0^t H(t_1) dt_1) |\phi_k(0)\rangle$, $T$ being time ordering.

Now we construct the universal set of NHQC gates. For the nonadiabatic one-qubit holonomic rotation gates, a two-qubit system is used. We choose the logical qubit states as $|0\rangle_L = |0\rangle$, $|1\rangle_L = |1\rangle$. By such a design, all the information of the logical qubit is encoded in the work qubit (the second qubit), and the first qubit acts as an ancillary qubit. We design two types of Hamiltonians, $H_1(\phi_1)$ and $H_2(\phi_2)$, to respectively realize two non-commuting nonadiabatic one-qubit gates,

\begin{equation}
H_1(\phi_1) = \frac{1}{2} (a_1 (X_1 X_2 + Y_1 Y_2) + b_1 (X_1 Y_2 - Y_1 X_2) - a_1 X_1 (I_2 - Z_2) - b_1 Y_1 (I_2 - Z_2)),
\end{equation}

\begin{equation}
H_2(\phi_2) = \frac{1}{2} (a_2 (Y_1 X_2 - X_1 Y_2) - b_2 X_1 (I_2 - Z_2)),
\end{equation}

where $a_1 = J_1 \cos(\phi_1/2)$, $b_1 = J_1 \sin(\phi_1/2)$, $a_2 = J_2 \sin(\phi_2/2)$, $b_2 = J_2 \cos(\phi_2/2)$, $I$ is one-qubit identity matrix, and $X$, $Y$, $Z$ are Pauli matrices. In the basis $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$, the evolution operators $U_1^{\phi_1}(\tau_1)$ and $U_2^{\phi_2}(\tau_2)$ generated by $H_1(\phi_1)$ and $H_2(\phi_2)$ respectively read

\begin{equation}
U_1^{\phi_1}(\tau_1) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & e^{-i\phi_1} \\ 0 & 0 & e^{i\phi_1} & 0 \end{pmatrix},
\end{equation}

\begin{equation}
U_2^{\phi_2}(\tau_2) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & e^{-i\phi_2} & 0 \\ -1 & 0 & 0 & 0 \\ 0 & e^{i\phi_2} & 0 & 0 \end{pmatrix},
\end{equation}

\begin{equation}
U_3^{\phi_3}(\tau_3) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & e^{-i\phi_3} & 0 \\ 0 & e^{i\phi_3} & 0 & 0 \\ 0 & 0 & 0 & e^{i\phi_3} \end{pmatrix},
\end{equation}

\begin{equation}
U_4^{\phi_4}(\tau_4) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & e^{-i\phi_4} & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & e^{i\phi_4} \end{pmatrix}.
\end{equation}
where $J_1 \tau_1 = \pi / \sqrt{2}$ and $J_2 \tau_2 = \pi$. According to Eqs. (3)–(4), it is readily to prove that both conditions (i) and (ii) are satisfied if the state of the two-qubit system is initially in the logical subspace $S^L_2 = \{ |0\rangle_L, |1\rangle_L \}$. So $U_1^\phi (\tau_1)$ and $U_2^\phi (\tau_2)$ are holonomic matrices acting on $S^L_2$. In the basis $\{ |0\rangle_L, |1\rangle_L \}$, $U_1^\phi (\tau_1)$ and $U_2^\phi (\tau_2)$ are respectively equivalent to

$$U_{xz}(\phi_1) = \begin{pmatrix} 0 & e^{-i \phi_1} \\ e^{i \phi_1} & 0 \end{pmatrix}, \quad U_{zz}(\phi_2) = \begin{pmatrix} \cos \phi_2 & i \sin \phi_2 \\ -i \sin \phi_2 & \cos \phi_2 \end{pmatrix}.$$  

Then the one-qubit holonomic rotation gates about $x$- and $z$-axes acting on the space $S^L_2$ can be constructed by using $U_{xz}$ and $U_{zz}$,

$$R^L_z(\theta) = U_{zz}(0)U_{xz}(\frac{\theta}{2}) \rightarrow U_1^\phi (\tau_1)U_{2}^\phi (\tau_2), \quad R^L_z(\phi) = U_{zz}(0)U_{xz}(\frac{\phi}{2}) \rightarrow U_1^\phi (\tau_1)U_{2}^\phi (\tau_2).$$

From the above two gates, an arbitrary one-qubit NHQC operation can be built.

The nontrivial two-qubit NHQC gate we realize is the nonadiabatic holonomic CNOT gate. A three-qubit system is used to implement this gate. $|100\rangle$, $|010\rangle$, $|110\rangle$ and $|111\rangle$ are encoded as $|0\rangle_L$, $|01\rangle_L$, $|10\rangle_L$ and $|11\rangle_L$. We see that all the information of the logical two-qubit state is encoded in the two work qubits (the second qubit and the third qubit), and the first qubit acts as an ancillary qubit. The Hamiltonian $H_3$ for realizing the CNOT gate can be expressed as

$$H_3 = \frac{J_3}{4} (X_1 (I_2 - Z_2) X_3 + Y_1 (I_2 - Z_2) Y_3 - X_1 (J_2 - Z_2) (I_3 - Z_3)).$$

Letting the evolution time satisfy the condition $J_3 \tau_3 = \pi / \sqrt{2}$, the evolution operator in the basis $\{ |000\rangle, |001\rangle, |010\rangle, |011\rangle, |100\rangle, |101\rangle, |110\rangle, |111\rangle \}$ reads

$$U_3(\tau_3) = \text{Diag}[1, 1, 1, -1, 1, 1, X].$$

According to Eqs. (11) and (12), we can prove that both conditions (i) and (ii) are satisfied if the state of the three-qubit system is initially in the logical subspace $S^L_3 = \{ |00\rangle_L, |01\rangle_L, |10\rangle_L, |11\rangle_L \}$. So $U_3(\tau_3)$ is a holonomic matrix acting on $S^L_3$. In the basis $\{ |00\rangle_L, |01\rangle_L, |10\rangle_L, |11\rangle_L \}$, $U_3(\tau_3)$ is equivalent to the nonadiabatic holonomic CNOT gate.

As the Hamiltonians $H_1(\phi_1)$, $H_2(\phi_2)$ and $H_3$ are time-independent, their holonomic evolution operators can be respectively written as

$$U_1^\phi (\tau_1) = \Pi_{i=1}^{N_1} U_1^\phi (\Delta t), \quad U_2^\phi (\tau_2) = \Pi_{i=1}^{N_2} U_2^\phi (\Delta t), \quad U_3(\tau_3) = \Pi_{i=1}^{N_3} U_3(\Delta t),$$

where $\Delta t, i \in \{1, 2, 3\}$ is small time interval and its value is $\tau_i / N_i$, $N_i$ being the number of the time steps of the holonomic evolution. By using a modification of the Trotter formula which is correct up to $(\Delta t)^2$ [10], the short time evolutions respectively read

$$U_1^\phi (\Delta t) = e^{-i \Delta t H_1(\phi)} \approx T_1^\phi (\Delta t),$$

$$U_2^\phi (\Delta t) = e^{-i \Delta t H_2(\phi)} \approx T_2^\phi (\Delta t),$$

$$U_3(\Delta t) = e^{-i \Delta t H_3} \approx T_3(\Delta t).$$

Here $T_1^\phi (\Delta t)$, $T_2^\phi (\Delta t)$ and $T_3(\Delta t)$ can be realized by a combination of radio-frequency pulses and evolutions of the J-coupling constants between the neighboring qubits in NMR technique [21–23]. According to Eqs. (13)–(15), the nonadiabatic holonomic gates $R^L_z(\theta)$, $R^L_z(\phi)$ and $U^L_{\text{cnot}}$ can be realized by

$$R^L_z(\theta) \rightarrow \Pi_{i=1}^{N_1} T_1(\Delta t) \Pi_{i=1}^{N_1} T_1^{-\frac{\theta}{2}} (\Delta t),$$

$$R^L_z(\phi) \rightarrow U_2^\phi (\tau_2) \Pi_{i=1}^{N_2} T_2^{-\frac{\phi}{2}} (\Delta t),$$

$$U^L_{\text{cnot}} \rightarrow \Pi_{i=1}^{N_3} T_3(\Delta t).$$

Notably, in Eq. (20), $U_2^\phi (\tau_2)$ can be implemented directly with no approximations, thus it reads $U_2^\phi (\tau_2) = e^{i \frac{\Delta t}{2} X_1 (I_2 - X_1 Z_2)}$.

3. **Experimental procedures and results.** Figures (a) and (b) respectively illustrate the implementations of the nonadiabatic one-qubit and two-qubit holonomic gates. The diethyl-fluoromalonate dissolved in $d_6$ acetone is used as the NMR quantum processor. $^{13}$C, $^{19}$F and $^1$H nuclear spins respectively act as the ancillary qubit and the two work qubits. Both in the one-qubit and two-qubit cases, the ancillary qubit stays in states $|1\rangle$ before and after the nonadiabatic holonomic evolutions. The input states (output states) of the work qubits are denoted as
Experiments, the number of iterations are chosen to be 3, 2 and 3, respectively. We prepare the initial state \(|\psi_{in}\rangle\) and the whole state of the NMR processor is initialized in the pseudopure state \(|\psi_{in}\rangle\) and \(\rho_{in} = |\psi_{in}\rangle\langle\psi_{in}|\) for the one-qubit and two-qubit cases respectively.

We here realize the following four one-qubit NHQC gates, \(R_x^A(\pi/2), R_y^A(\pi), R_y^A(\pi/2), R_x^A(\pi)\) and the NHQC CNOT gate \(U_{CNOT}^A\). In order to demonstrate we can implement one-qubit NHQC gates on both \(^{19}\text{F}\) and \(^{1}\text{H}\), \(R_x^A(\pi/2)\) and \(R_y^A(\pi/2)\) are implemented on \(^{19}\text{F}\) and \(R_x^A(\pi)\) and \(R_y^A(\pi)\) are implemented on \(^{1}\text{H}\). In our experiments, the number of iterations are chosen to be \(N_1 = 3, N_2 = 2\) and \(N_3 = 2\). We prepare the initial states using the cat-state method \([27, 29]\). For the one-qubit gates, we prepare the work qubit in \(\rho_{in}^A\) and the ancillary qubit in \(|1\rangle\langle1|\). Without loss of generality, the spectator work qubit is prepared in \(|0\rangle\langle0|\). Specifically, the NMR processor is initialized in the pseudopure states \(|\psi_{in}\rangle\otimes\rho_{in}^A\otimes|0\rangle\langle0|\) (for \(R_x^A(\pi/2)\) and \(R_y^A(\pi/2)\)) or \(|\psi_{in}\rangle\otimes|0\rangle\langle0|\otimes\rho_{in}^A\otimes|1\rangle\langle1|\) (for \(R_x^A(\pi)\) and \(R_y^A(\pi)\)). For the CNOT gate, the ancillary qubit is also prepared in \(|1\rangle\langle1|\) and the whole state of the NMR processor is \(|\psi_{in}\rangle\otimes\rho_{in}^B\). In terms of the deviation matrices \([30]\), the input states \(\rho_{in}^A\) and \(\rho_{in}^B\) are prepared in each of the following sets

\[
\rho_{in}^B \in \{IX, IY, IZ, XI, XX, XY, XZ, YI, YX, YY, YZ, ZI, ZX, ZY, ZZ\}.
\]

The output states \(\rho_{out}^A\) and \(\rho_{out}^B\) are determined by quantum state tomography (QST) \([31]\). To measure the sameness of the theoretical output state \(\rho_{th}\) and the experimental output state \(\rho_{out}\), the attenuated and unattenuated state fidelities \([32, 33]\), which are respectively defined as

\[
\text{Tr}((\rho_{out}\rho_{th})/\sqrt{\text{Tr}(\rho_{th}\rho_{th})\text{Tr}(\rho_{in}\rho_{in}))}
\]

and

\[
\text{Tr}((\rho_{out}\rho_{th})/\sqrt{\text{Tr}(\rho_{out}\rho_{out})\text{Tr}(\rho_{th}\rho_{th}))}
\]

are used. The attenuated fidelity takes into account the signal loss, while
the unattenuated fidelity ignores certain errors due to the signal loss and quantifies how similar in direction $\rho_{\text{out}}$ and $\rho_{\text{th}}$ are \cite{32,33}. The average experimental attenuated fidelities are 60.7\%, 61.8\%, 86.1\%, 77.7\%, 47.9\% for the output states of $R_z^1(\pi/2)$, $R_x^1(\pi)$, $R_y^1(\pi/2)$, $R_z^1(\pi)$ and $U_{\text{cnot}}^L$ respectively, while their average experimental unattenuated fidelities are 97.6\%, 97.3\%, 97.9\%, 95.7\% and 93.12\% respectively. These numbers are in-line with the results of other experiments done using diethylfluoromalonate \cite{34}. The differences between the attenuated and unattenuated fidelities are consistent with the signal loss rates measured in our experiments (see Supporting Material). Figure 2 shows the unattenuated output state fidelities in our experiments. Figure 3 shows the unattenuated and unattenuated fidelities are 98\% and 98\% respectively. These numbers are in-line with the results of other experiments done using diethylfluoromalonate \cite{34}, which take into account the signal loss, are 70.5\%, 71.3\%, 89.5\%, 83.3\% and 51.2\% for the $R_z^1(\pi/2)$, $R_x^1(\pi)$, $R_y^1(\pi/2)$ and $R_z^1(\pi)$ and $U_{\text{cnot}}^L$ respectively. The deviations between $\chi_{\text{th}}$ and $\chi_{\exp}$ are mainly caused by overall loss of signal. To see the sameness of theoretical and experimental quantum processes when ignoring certain errors due to signal loss, we use the unattenuated $\chi$ fidelity defined as $\text{Tr}(\rho_{\text{out}}^\dagger \chi_{\text{th}})/\sqrt{\text{Tr}(\chi_{\exp} \chi_{\text{th}}^\dagger)\text{Tr}(\chi_{\text{th}} \chi_{\exp}^\dagger)}$ \cite{33,36,37}. The unattenuated experimental $\chi$ fidelities of the gates $R_z^1(\pi/2)$, $R_x^1(\pi)$, $R_y^1(\pi/2)$, $R_z^1(\pi)$ and $U_{\text{cnot}}^L$ are 95.9\%, 95.9\%, 96.1\%, 96.3\% and 91.43\%, respectively. It is interesting to note that the Trotter approximations in Eqs. (16), (17) and (18) give very good approximations to the exact evolution and the theoretical $\chi$ fidelities are 99.2\%, 98.6\%, 99.2\%, 97.4\% and 98.7\% for $R_z^1(\pi/2)$, $R_x^1(\pi)$, $R_y^1(\pi/2)$, $R_z^1(\pi)$ and $U_{\text{cnot}}^L$ respectively.

Quantum process tomography (QPT) \cite{35} is used to quantitatively describe the implementation of the NHQC gates. According to QPT, each quantum process is characterized by a $\chi$ matrix. For a given input state $\rho_{\text{in}}$, the output state is expressed as $\rho_{\text{out}} = \sum e_k |i\rangle \langle i | \rho_{\text{in}} |k\rangle \langle k|$, where $e_k$ belongs to an operation basis set. The elements of the operator basis set for the one-qubit and two-qubit cases can be respectively chosen as

$$e_k \in \{I, X, -iY, Z\}, k = 1, ..., 4,$$

$$e_k \in \{II, IX, -iY, IZ, XI, XX, -iXY, XZ, -iYI, -iYX, -iYY, -iYZ, ZI, ZX, -iZY, ZZ\},$$

$k = 1, ..., 16$.

The QPT $\chi$ matrix is calculated using the output states via the technique described in Ref. \cite{19}. The experimental $\chi$ for one-qubit and two-qubit gates are shown in Figs. 4 and 5 respectively. We use $\chi$ fidelities to evaluate the performance of NHQC gates. The attenuated $\chi$ fidelities $|\text{Tr}(\chi_{\exp} \chi_{\text{th}}^\dagger)|$ \cite{33}, which take into account the signal loss, are 70.5\%, 71.3\%, 89.5\%, 83.3\% and 51.2\% for the $R_z^1(\pi/2)$, $R_x^1(\pi)$, $R_y^1(\pi/2)$, $R_z^1(\pi)$ and $U_{\text{cnot}}^L$ respectively. The deviations between $\chi_{\text{th}}$ and $\chi_{\exp}$ are mainly caused by overall loss of signal. To see the sameness of theoretical and experimental quantum processes when ignoring certain errors due to signal loss, we use the unattenuated $\chi$ fidelity defined as $\text{Tr}(\rho_{\text{out}}^\dagger \chi_{\text{th}})/\sqrt{\text{Tr}(\chi_{\exp} \chi_{\text{th}}^\dagger)\text{Tr}(\chi_{\text{th}} \chi_{\exp}^\dagger)}$ \cite{33,36,37}. The unattenuated experimental $\chi$ fidelities of the gates $R_z^1(\pi/2)$, $R_x^1(\pi)$, $R_y^1(\pi/2)$, $R_z^1(\pi)$ and $U_{\text{cnot}}^L$ are 95.9\%, 95.9\%, 96.1\%, 96.3\% and 91.43\%, respectively. It is interesting to note that the Trotter approximations in Eqs. (16), (17) and (18) give very good approximations to the exact evolution and the theoretical $\chi$ fidelities are 99.2\%, 98.6\%, 99.2\%, 97.4\% and 98.7\% for $R_z^1(\pi/2)$, $R_x^1(\pi)$, $R_y^1(\pi/2)$, $R_z^1(\pi)$ and $U_{\text{cnot}}^L$ respectively.

**FIG. 4:** (color online) The QPT $\chi$ matrices of one-qubit hollonomic gates $R_z^1(\frac{\pi}{2})$, $R_x^1(\pi)$, $R_y^1(\frac{\pi}{2})$ and $R_z^1(\pi)$. The (a) and (c) columns are the real parts and imaginary parts of the theoretical $\chi$ matrices, respectively. The (b) and (d) columns are the real parts and imaginary parts of the experimental $\chi$ matrices, respectively. The numbers in the $x$- and $y$- axes refer to the operators in the operator basis set \{I, X, -iY, Z\}.

**FIG. 5:** (color online) The QPT $\chi$ matrices of $U_{\text{cnot}}^L$. (a) and (c) are the real part and imaginary part of the theoretical $\chi$ matrix, respectively. (b) and (d) are the real part and imaginary part of the experimental $\chi$ matrix, respectively. The numbers 1 to 16 in the $x$- and $y$- axes refer to the operators in the operator basis set \{II, IX, -iY, IZ, XI, XX, -iXY, XZ, -iYI, -iYX, -iYY, -iYZ, ZI, ZX, -iZY, ZZ\}.

4. Summary. As a proof of principle, we experimentally implemented NHQC via a NMR quantum information
processor using a variant version of the scheme proposed in Ref. [18]. In our experiments, one-qubit nonadiabatic holonomic gates and two-qubit holonomic CNOT gate, which compose a universal set of NHQC gates, are implemented by using an ancillary qubit which provides the additional dimension needed in the holonomic evolution. This is the first experimental demonstration of NHQC, which is a step towards fault-tolerant quantum computing. The successful realizations of these universal elementary gates in NHQC demonstrate the feasibility of implementing NHQC using present experimental techniques.

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