Study of the stator geometry for a Moineau pump

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Abstract. Moineau pumps are volumetric pumps with progressive cavities and have an eccentric rotor towards the rotation axis. The eccentric rotor performs a rotational movement towards a stator, usually made of an elastomer moulded in a cylindrical tube. Producing the pump involves knowing the cross-section of the helical rotor model, as well as knowledge of the stator shape as a complex surface, and of the enwrapping of the rotor lobes in their relative movement, in relation to the stator. An analysis for the trilobed helical pump case is proposed, in which, by defining the rotor shape, the stator model can be formed. The two objects, the rotor and the stator are in reciprocal enwrapping movement, as a result of the revolving without sliding movement (rolling) of two circular centrodes, defined in relation to their front sections. The helical surfaces of the rotor and stator are cylindrical helical surfaces.

1. Introduction
Current fluid extraction technologies involve multiple pumping technologies, the most common being: pumping with piston pumps, centrifugal pumping, pumping with hydraulic pumps with jet, respectively helical pumping [1 - 3].

Helical pumps with a progressive cavity, whose operation is based on Moineau’s principle, involve the formation of cavities by inserting a rotor with a simple helical outer surface inside a stator with a double helical inner surface. At the beginning of the rotor movement, the cavities move from the suction to the compression, thus leading to a continuous flow [4].

Usually, helical pumps have a stator with (z+1) teeth, so with (z+1) beginnings, and a rotor with z helical flute. In this case, the ratio between the length of the stator pitch and the length of the rotor pitch became (z+1)/z [5 - 7].

The study of the active surfaces of Moineau pumps [8 - 10] can be done based on the fundamental theorems of the enwrapping surfaces, Olivier or Gohman [11,19] as well as on the basis of the complementary theorems: „minimum distance method” [12, 13] or „the method of the substitute circles family” [12, 14, 15, 20].

The problem proposed for the study can be solved based on the fundamental theorems of enveloping surfaces: Olivier or Gohman. Also, complementary theorems can be used: the „minimum distance” method [16], the method of generated plane trajectories [17].

This paper proposes approaching the problem of stator profiling based on the „family of the substitute circles” theorem [18], applied to conjugate surfaces associated with the rolling centrodes.

The method of the „substituting circles family” theory, is based on a definition of the substitute circles family of a plane profile.
Definition: a profile associated with a centrode, can be associated with a circles family with the centers on the associated centrode and also tangent to the profile.

The „substitute circles family” theorem is defined as follows: „the enveloping of a profile associated with a couple of rolling centrodes is defined by the transpose, in the rolling movement of the centrodes, of the substituting circles family of the given profile” [12].

The trilobed profile of the rotor - cylindrical worm, associated with a cylindrical axode, with the radius \( r_{p1} \) - the rolling circle radius in the cross-section of the rotor, is presented in figure 1.

![Figure 1. Rotor profile of the trilobed pump](image)

In figure 1, the rolling centrodes are shown:
- \( C_1 \) - centrode associated with the cross-section of the rotor;
- \( C_2 \) - centrode associated with the cross profile of the pump stator.

2. The method of „the substitute circles family” [18]
The kinematics process of generating the pump stator profile (in fact, the die in which the future stator of the pump is injected), is based on the rolling of the two centrodes: \( C_1 \) - associated with the trilobed rotor; \( C_2 \) - centrode associated with the pump stator. Constructively, the pump rotor executes a planetary movement, around the \( O_1 \) axis, figure 1, thus, physically, the pump stator is fixed.

The reference systems are defined:
- \( x_1y_1 \) - fixed system, initially superimposed on the rotor axis; in the figure, the axis of the rotor passes through the \( O \)-point;
- \( x_0y_0 \) - fixed system, superimposed on the rotor axis, the axis passing through \( O_1 \)-point;
- \( XY \) - mobile reference system, associated with the rotor in a rotational movement around its own axis, \( C_1 \) centrode;
- \( \xi\eta \) - mobile system, in the planetary movement of the rotor (the apparent rotational movement of the \( C_2 \) centrode).

Noted with \( r_{p1} \), respectively, \( r_{p2} \) - are the radii of the two circular centrodes.
The rolling movement of the $C_1$ centrode, which apparently rotates without sliding on the $C_2$ centrode (of the stator) imposes the condition:

$$R_{\varphi_1} \cdot \varphi_1 = R_{\varphi_2} \cdot \varphi_2,$$

(1)

with $\varphi_1$ și $\varphi_2$ the size of the rotation angles in the movement of the rotor ($\varphi_1$) and „apparently“ of the stator, in reality the planetary movement of the rotor, of angle $\varphi_2$.

The stator has $(z+1)$ teeth (specifically, 4 teeth). From the condition of the circular pitch equality of the „teeth” on the two centrodes, the equality results:

$$R_{\varphi_1} \cdot \frac{2\pi}{3} = \frac{2\pi}{4} \cdot R_{\varphi_2},$$

(2)

so, it results:

$$R_{\varphi_1} = \frac{3}{4} \cdot R_{\varphi_2}.$$

(3)

For the rotor profile, profile composed of arc:

- $AB$ - arc with the center in $C_1$ (with radius „$r$”);
- $BC$ - arc with the center in $O_2$ (with radius „$R$”);
- $CD$ - arc with the center in the $C_1$ centrode (with radius „$r$”), principled, the parametric expression is accepted:

$$\begin{align*}
S &= \sum; \quad X = X(v) ; \\
    &\quad Y = Y(v),
\end{align*}$$

(4)

with $v$ - variable parameter.

The family of the „substituting circles” of the $BC$ arc is defined by the current circle of radius $r_i$, in contact with the $\Sigma$ profile, in the $M_i$ point, figure 1.

In this way, the parametric equations of the circle, with the origin in $M_i$, on the $C_1$ centrode, of radius $R_{\varphi_1}$, has the equations, in the $XY$-system:

$$\begin{align*}
C_i: \quad X &= -R_{\varphi_1} \cdot \cos \varphi_i - r_i \cdot \cos \beta_i ; \\
    &\quad Y = R_{\varphi_1} \cdot \sin \varphi_i + r_i \cdot \sin \beta_i ,
\end{align*}$$

(5)

for the coordinates of the current point $M_i$ on the circle of radius $r_i$ - circle of the substitute circles family.

At this stage, the size of the parameters $\beta_i$ and $r_i$ have not been defined yet.

The condition that the $C_i$ circle (5) is tangent to the $\Sigma$ profile (4), involves the accomplishment of the conditions:

- The common point condition, meaning from (4) and (5), results:

$$\begin{align*}
X(\nu) &= -R_{\varphi_1} \cdot \cos \varphi_1 - r_i \cdot \cos \beta_1 ; \\
    &\quad Y(\nu) = R_{\varphi_1} \cdot \sin \varphi_1 + r_i \cdot \sin \beta_1,
\end{align*}$$

(6)

- The tangency condition of the two curves (4) and (6), with $\varphi_1$ - director parameter:
\[ \dot{X}_v = \frac{d}{d\beta_i} \left[ -R_{\rho_i} \cdot \cos \varphi_i - r_i \cdot \cos \beta_i \right]; \]
\[ \dot{Y}_v = \frac{d}{d\beta_i} \left[ R_{\rho_i} \cdot \sin \varphi_i + r_i \cdot \sin \beta_i \right], \] (7)

or

\[ \dot{X}_v = r_i \cdot \sin \beta_i; \]
\[ \dot{Y}_v = r_i \cdot \cos \beta_i, \] (8)

The equations assembly (7) and (8) allows the determination of the enwrapping condition specific to the method, in principle, a dependence between the parameters \( \varphi_i \) and \( v \), as well as definitions for \( r_i \), respectively \( \beta_i \), depending on the \( M_i \)-point position on the rolling circle, \( C_i \).

From equations (8), by division, it results:
\[ \tan \beta_i = \frac{\dot{X}_v}{\dot{Y}_v}, \] (9)

where \( \dot{X}_v \) and \( \dot{Y}_v \) are the partial derivatives of the parametric equations of the \( \Sigma \) curve (4).

The equations assembly that allows the identification of the substitute circles family and the specific enwrapping condition is:

\[ X(v) = -R_{\rho_i} \cdot \cos \varphi_i - r_i \cdot \cos \beta_i; \]
\[ Y(v) = R_{\rho_i} \cdot \sin \varphi_i + r_i \cdot \sin \beta_i; \]
\[ \dot{X}_v = r_i \cdot \sin \beta_i; \]
\[ \dot{Y}_v = r_i \cdot \cos \beta_i, \] (10)

The enwrapping condition is deduced from the equality:
\[ \frac{\dot{X}_v}{\dot{Y}_v} = \frac{Y(v) - R_{\rho_i} \cdot \sin \varphi_i}{-X(v) + R_{\rho_i} \cdot \cos \varphi_i}, \] (11)

or

\[ X(v) \cdot \dot{X}_v + Y(v) \cdot \dot{Y}_v = \left( \dot{X}_v \cdot \cos \varphi_i - \dot{Y}_v \cdot \sin \varphi_i \right) \cdot R_{\rho_i}; \]
\[ \dot{X}_v \cdot \cos \varphi_i - \dot{Y}_v \cdot \sin \varphi_i = \frac{X(v) \cdot \dot{X}_v + Y(v) \cdot \dot{Y}_v}{R_{\rho_i}}, \] (12)

representing a dependence between the \( v \) parameter of the generated profile and \( \varphi_i \) - the movement parameter, in essence, the specific enwrapping condition.

The circle radius of the substituting circles family, from (10) is:
\[ r_i = \sqrt{\left( X(v) + R_{\rho_i} \cdot \cos \varphi_i \right)^2 + \left( Y(v) - R_{\rho_i} \cdot \sin \varphi_i \right)^2}. \] (13)

It is observed that the stator profiles corresponding to the \( AB \) and \( CD \) arcs are identical with the rotor profile (radius circle \( r \)), Figure 2, in the points on the stator centrode corresponding to the \( P_A \), respectively \( P_B \) arcs:
for \( \varphi_1 = \frac{\pi}{6} \) results \( \varphi_2 = \frac{\pi}{6} \cdot \frac{R_{p2}}{R_{p1}} \), see (1) and (2). (14)

According to the „substituting circles family“ theorem, the circles family transposed on the \( C_2 \) centrode (of the stator), in the movements assembly:

- The rotor rotation around the \( z_1 \)-axis, of the fixed system \( x_1y_1z_1 \):

\[
x_1 = \omega_1^T (\varphi_1) \cdot X,
\]

where, the \( X \)-matrix is:

\[
X = \begin{bmatrix} X(v) \\ Y(v) \\ 0 \end{bmatrix},
\]

(16)

- The mobile system rotation associated with the stator (planetary movement):

\[
x_0 = \omega_0^T (\varphi_2) \cdot \xi;
\]

(17)

- The relative position of the system \( x_1y_1z_1 \) and \( x_0y_0z_0 \):

\[
x_0 = x_1 + A, \text{ with the matrix } A = \begin{bmatrix} A \\ 0 \\ 0 \end{bmatrix}.
\]

(18)

So the transposition movement of the substituting circles family on the \( C_2 \) centrode is:
\[ \xi = \omega \left( \phi_2 \right) \cdot \left[ \omega^T \left( \phi_1 \right) \cdot \begin{bmatrix} X \\ Y \\ 0 \end{bmatrix} + \begin{bmatrix} A \\ 0 \end{bmatrix} \right] \tag{19} \]

In the movement (18), the substituting circles family of the BC profile of the rotor, transposes on the \( C_2 \) centrode:

\[
\begin{bmatrix} \xi \\ \eta \\ \zeta \end{bmatrix} = \begin{bmatrix} \cos \phi_2 & \sin \phi_2 & 0 \\ -\sin \phi_2 & \cos \phi_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \cos \phi_1 & -\sin \phi_1 & 0 \\ \sin \phi_1 & \cos \phi_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} -R_{\eta} \cdot \cos \phi_1 - r_i \cdot \cos \beta_i \\ R_{\eta} \cdot \sin \phi_1 + r_i \cdot \sin \beta_i \\ 0 \end{bmatrix} + \begin{bmatrix} A \\ 0 \end{bmatrix} \tag{20} \]

If the matrix form is developed (19), it is reached at the analytical forms:

\[
\xi = -R_{\eta} \cdot r_i \cdot \cos (\phi_i - \beta_i) + A \cdot \cos \phi_2 - r_i \cdot \sin (\phi_i - \beta_i) \cdot \sin \phi_2; \\
\eta = -R_{\eta} \cdot r_i \cdot \cos (\phi_i - \beta_i) + A \cdot \sin \phi_2 - r_i \cdot \sin (\phi_i - \beta_i) \cdot \cos \phi_2, \tag{21} \]

or, finally, the substituting circles family of the rotor profile BC on the \( C_2 \) centrode, has the main form:

\[
\begin{align*}
\xi &= -R_{\eta} \cdot \cos \phi_2 - r_i \cdot \cos (\phi_i - \beta_i) - A \cdot \cos \phi_2; \\
\eta &= R_{\eta} \cdot \sin \phi_2 - r_i \cdot \sin (\phi_i - \beta_i) - A \cdot \sin \phi_2, \tag{22} \end{align*}
\]

with definitions from (9) and (13):

\[
\varphi_2 = \frac{R_{\eta}}{R_{\eta}} \cdot \varphi_1; \\
\tan \beta_i = \frac{\dot{X}}{\dot{Y}}; \\
r_i = \sqrt{\left( \dot{X}(v) + R_{\eta} \cdot \cos \phi_1 \right)^2 + \left( \dot{Y}(v) - R_{\eta} \cdot \sin \phi_1 \right)^2}, \tag{23} \]

and the enwrapping condition:

\[
\dot{X}_i \cdot \cos \phi_1 - \dot{Y}_i \cdot \sin \phi_1 = \frac{X(v) \cdot \dot{X}_i + Y(v) \cdot \dot{Y}_i}{R_{\eta}}, \text{ see (12).} \]

3. Applications for a trilobed profile circle arc

Figure 3 shows the substituting circles family associated with the rotor profile, circles with centers on centrode \( C_f \) and also the circles family transposed on the centrode \( C_2 \), the stator, in the planetary movement of the pump rotor. The stator profiling for the BC arc involves customizing the profile shape (3). Thus, in correlation with figure 4, the profile equations are defined:

\[
\begin{align*}
X &= -e + R \cdot \cos v; \\
Y &= R \cdot \sin v, \tag{24} \end{align*}
\]

with \( e \) and \( R \) constructive sizes and \( v \) variable parameter between the limits:
\[ -v_{\text{max}} \leq v \leq v_{\text{max}}, \text{with } v_{\text{max}} = \arcsin \left( \frac{R_{r_p} \cdot \sin 60^\circ}{R + r} \right). \]  

\[ (25) \]

\( \text{Figure 3. The circles families on the } C_1 \text{ centrode of} \)
\( \text{the pump rotor and transposed on } C_2. \)

Equations (10) become:
\[ -e + R \cdot \cos v = -R_{r_p} \cdot \cos \phi_1 - r_i \cdot \cos \beta_i; \]
\[ R \cdot \sin v = -R_{r_p} \cdot \sin \phi_1 + r_i \cdot \sin \beta_i. \]  

\[ (26) \]

The enwrapping condition (12) can be expressed, in this case, by:
\[ \dot{X}_v = -R \cdot \sin v; \]
\[ \dot{Y}_v = R \cdot \cos v. \]  

\[ (27) \]

The enveloping condition (12) become:
\[-R \cdot \sin \cdot \cos \phi_1 - R \cdot \cos \cdot \sin \eta_1 = \frac{-(e + R \cdot \cos \cdot \sin \eta) \cdot \sin \phi + R \cdot \sin \cdot \cos \eta}{R_{p_1}}.\] (28)

The condition (28) is available for values $\nu = \phi, \eta_{\text{max}}$, for profile`s zone, Figure 4, $PC$ and $\nu_{\text{max}}$ from (25).

![Figure 4. Profile`s zone PC.](image)

**4. Numerical application**

Table 1 shows the stator profile coordinates, conjugated to the rotor profile with the elements:

- $R_{p_1} = \frac{3}{4} \cdot R_{p_2}$; $R_{p_1} = 30$ mm;
- $r = 8.5934$ mm; $R = 35$ mm;
- $\tan \beta_\nu = \frac{\dot{X}_\nu}{\dot{Y}_\nu} = \frac{R \cdot \sin \nu}{R \cdot \cos \nu}$;
- $r_1$ is calculated from (13) for the concrete case, the $PC$ profile`s zone, Figure 4.

| $\nu$ [°] | $\phi$ [°] | $r_1$ [mm] | $X$ [mm] | $Y$ [mm] | $\xi$ [mm] | $\eta$ [mm] |
|-----------|-------------|-------------|---------|---------|----------|----------|
| -0.638485 | -0.82       | 1.682743    | -21.895 | -20.8593| -17.4858 | -10.24    |
| -0.510788 | -0.44       | 8.775366    | -19.4674| -17.1103| -26.0494 | -1.39569  |
| -0.383091 | -0.29       | 12.09141    | -17.537 | -13.0826| -30.3894 | 1.010818  |
| -0.255394 | -0.18       | 13.83301    | -16.1353| -8.84193| -33.0344 | 1.410554  |
| -0.127697 | -0.09       | 14.72357    | -15.285 | -4.45726| -34.5188 | 0.897423  |
| 0         | 0.00        | 15          | -15     | 0       | -35      | 0         |
| 0.1276969 | 0.09        | 14.72357    | -15.285 | 4.457255| -34.5181 | -0.89742  |
| 0.2553938 | 0.18        | 13.83301    | -16.1353| 8.841927| -33.0344 | -1.41055  |
| 0.3830908 | 0.29        | 12.09141    | -17.537 | 13.0826 | -30.3894 | -1.01082  |
| 0.5107877 | 0.44        | 8.775366    | -19.4674| 17.11026| -26.0494 | 1.395691  |
| 0.6384846 | 0.82        | 1.682743    | -21.895 | 20.85927| -17.4858 | 10.24002  |
Figure 5 shows the front profile of the rotor pump stator with multiple cavities.

![Figure 5. Front profile of the stator z=4 lobes.](image)

5. Conclusions
In order to helical pumps to have increased efficiency in exploitation, they are required to have a specific geometric profile for the rotors, which allows a flow section as large as possible, but also a gearing line as short as possible, to obtain an increased flow and a constant velocity. This leads to a high volumetric efficiency, which also leads to an increase in adiabatic efficiency, because the power losses are smaller during the compression process, through internal recirculation. In this context, constant pitch helical surfaces generating becomes a problem with multiple technical applications, the helical surfaces being usually generated by milling with the help of rotary tools bordered by revolution peripheral surfaces.

Thus, in the present paper, the theorem of the substituting circles family and the specific enwrapping condition were used to determine the envelopes of some curves belonging to the helical surface by a repetitive solution. A set of „parallel circles” of the primary peripheral surface of the future side mill tools generating of the helical surface was defined, starting from the stator profiling in circle arc.

The method of the „substituting circles family” allows a rigorous approach to the determination of the pump stator enveloping in the relative movement of the trilobed rotor (planetary movement).

The stages necessary to be completed, such as an analysis for the type of considered pump, were successively passed.

The numerical application presented proves the ability of the method to approach such a technical problem.

The presented methodology can be an exhortation for other similar applications in the field, too.

Acknowledgments
This work was supported by a grant of the Romanian National Authority for Scientific Research and Innovation, CNCS-UEFISCDI, project number PN-III-P1-1.2-PCCDI-2017-0446 - TFI PMAIAA, within PNCDI III.

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