Optimization of natural frequencies of large-scale two-stage raft system

LV Zhiqiang1,2,3, HE Lin1,2, SHUAI Changgeng1,2,4
1 Institute of Noise and Vibration, Naval University of Engineering, Wuhan 430033, P.R.China
2 National Key Laboratory on Ship Vibration and Noise, Wuhan 430033, P.R.China
Email: 3 lazy2002y@sina.com, 4 chgshuai@163.com

Abstract: A multi-parameter optimization method for the natural frequencies of two-stage raft system is presented in this paper. The method can limit the natural frequencies in a relatively narrow band by quantitatively adjusting the natural frequencies of the system, which helps to avoid resonances and isolate vibration effectively. This method is of great significance in ship noise reduction engineering.

1. Introduction
Vibration isolation device is one of the most effective approaches to control mechanical noise of ship. With increasing noise reduction demand, complicated two-stage vibration isolation and large-scale two-stage raft that can bear tens of power equipment [1-3] are developed and applied gradually to replace early-used single-stage vibration isolation method. The issue of optimal design of parameters of two-stage vibration isolation system has been presented in many literatures. The method for optimizing intermediate mass of the two-stage vibration isolation system is discussed and the minimal mass ratio that can be selected under certain vibration isolation conditions is provided in reference [4]. The relationship between response of the vibration isolation system and design parameters is researched in reference [6] and [7] by mode superposition method and finite element method. Admittance principle is used to probe into the optimized layout scheme for vibration isolators in the multi-stage vibration isolation system in reference [8]. In reference [9] variation-tolerant polyhedron method and perturbation method are applied to optimize the stiffness parameter in a multi-stage vibration isolation system. In practice, there is 6(n+1) degrees of freedom and the same amount of natural frequencies in a large-scale two-stage floating raft vibration isolation system (n is the number of equipment under vibration isolation). To avoid resonance excitation and achieve effective vibration isolation, multiple parameters influencing the performance of the vibration isolation system must be optimized at the same time to make the first order and the highest order of natural frequencies fall within the selected frequency zone. The zone is determined by common speed of the machine, stability of the vibration isolation system and requirement for vibration attenuation, which is actually an important theoretical difficulty in the design of the large-scale two-stage floating raft vibration isolation system. A multi-parameter optimization method for natural frequencies of two-stage vibration isolation system is developed in this paper that can limit the natural frequencies of vibration isolation system in a relatively narrow band so as to realize the small intermediate mass and the optimal effect of vibration isolation. The method is of great guiding significance for engineering design of ship noise reduction.
2. Analysis of design parameters’ effect on system natural frequencies

K and M mean the total stiffness matrix and total inertia matrix of the vibration isolation system respectively. According to the theory of vibration, natural frequencies $\omega$ of the system and corresponding dominant mode of vibration $X$ agree with the following equation

$$[K - \omega^2 M]X = 0$$ (1)

The system dominant mode of vibration $X$ at order $i$ and corresponding natural frequency $\omega_i$ satisfy

$$[K - \omega_i^2 M]X_i = 0$$ (2)

If the system has $n$ degrees of freedom, then the above equation is an $n$-dimension algebraic equation set, and left multiply equation(2) by $X_i^T$

$$X_i^T [K - \omega_i^2 M]X_i = 0$$ (3)

Generally, $K$ and $M$ are $n$-order symmetrical matrixes, which are positive definite matrixes if there is no stiffness movement in the system. It is then drawn that

$$K^T = K, M^T = M$$

$$[K - \omega_i^2 M]^T = [K^T - \omega_i^2 M^T] = [K - \omega_i^2 M]$$

$$\{X_i^T [K - \omega_i^2 M]\}^T = [K - \omega_i^2 M]X_i = 0$$ (4)

Thus

$$X_i^T [K - \omega_i^2 M] = 0$$ (5)

Variation of natural frequencies $\omega_i$ with system structural parameter $S$ can be indicated by its partial derivative for $S$. Seek partial derivative of equation (3) for $S$ and put it into equation (5) to obtain

$$\frac{\partial \omega_i}{\partial S} = \frac{1}{2} \left[ \omega_i^{-1} X_i^T \frac{\partial K}{\partial S} X_i - \omega_i X_i^T \frac{\partial M}{\partial S} X_i \right]$$ (6)

If the vector of vibration mode is normalized, then $X_i^T M X_i = 1$ and the above equation turns to

$$\frac{\partial \omega_i}{\partial S} = \frac{1}{2} \left[ \omega_i^{-1} X_i^T \frac{\partial K}{\partial S} X_i - \omega_i X_i^T \frac{\partial M}{\partial S} X_i \right]$$ (7)

Equation (7) is the variation rate of frequency $\omega_i$ with system structural parameter $S$. The variation rate of $\omega_i$ with $S$ can be attained by seeking for partial derivative of $K$ and $M$ for $S$ and putting into $\omega_i$ and $X_i$ obtained from equation (2). The purpose of research in this paper is how to reasonably change system structural parameter that is selected preliminarily when first-order natural frequency $\omega_1$ is smaller than the minimal natural frequency $\omega_1^*$ specified and the $n$-order natural frequency $\omega_n$ is larger than the maximal natural frequency $\omega_n^*$ specified so as to effectively increase $\omega_1$ while decrease $\omega_n$ to limit them in a narrow band for the sake of the optimal effect of vibration isolation.
If there are \( m \) variable parameters in the vibration isolation system: 
\[ P = \{ S \} = 1 \ldots m \}, \]
where variable domain of \( S_i \) is \( (S_{1i}, S_{2i}) \), could be parameters such as mass, rotational inertia, stiffness and arrangement position of vibration isolators. Substitute \( \omega \) and \( \omega_n \) obtained from equation (2) into equation (7)
\[
\begin{align*}
\frac{\partial \omega}{\partial S_i} &= \frac{1}{2} \left[ a_1^{-1} X_i^T C K X_i - a_2^{-1} X_i^T C M X_i \right] \\
\frac{\partial \omega_n}{\partial S_i} &= \frac{1}{2} \left[ a_1^{-1} X_n^T C K X_n - a_2^{-1} X_n^T C M X_n \right]
\end{align*}
\]
According to equation (8) and equation (9), the variable parameter set can be divided into two subsets:
\[
A_1 = \left\{ S_i | \frac{\partial \omega}{\partial S_i} \geq 0 \right\}; \quad A_2 = \left\{ S_i | \frac{\partial \omega}{\partial S_i} < 0 \right\}
\]
\[
B_1 = \left\{ S_i | \frac{\partial \omega_n}{\partial S_i} \geq 0 \right\}; \quad B_2 = \left\{ S_i | \frac{\partial \omega_n}{\partial S_i} < 0 \right\}
\]
Where \( i, j, k \) and \( l \) are subscript variables, which are all smaller than \( m \); the sets \( A_1, A_2, B_1 \) and \( B_2 \) satisfy \( A_1 = B_1 \cup B_2 \).

According to the definitions of sets \( A_1, A_2, B_1 \) and \( B_2 \) and physical significances of derivatives:

1. Increase of \( S_i \in A_1 \) (except for the \( S_i \) making \( \frac{\partial \omega}{\partial S_i} = 0 \)) or decrease of \( S_i \in A_2 \) may increase \( \omega \). If \( S_i \) satisfies \( \left| \frac{\partial \omega}{\partial S_i} \right| = \max \left\{ \left| \frac{\partial \omega}{\partial S_j} \right| \right\} (i = 1, 2, \ldots, m) \), then change of \( S_i \) has the maximal effect on increase of \( \omega \).

2. Decrease of \( S_i \in B_1 \) (except for the \( S_i \) making \( \frac{\partial \omega}{\partial S_i} = 0 \)) or increase of \( S_i \in B_2 \) may decrease \( \omega_n \). If \( S_i \) satisfies \( \left| \frac{\partial \omega}{\partial S_i} \right| = \max \left\{ \left| \frac{\partial \omega}{\partial S_j} \right| \right\} (i = 1, 2, \ldots, m) \), then change of \( S_i \) has the maximal effect on decrease of \( \omega_n \).

3. Method of optimal design for parameters
When optimizing the natural frequencies of two-stage floating raft vibration isolation system, it is expected that the increase of \( \omega \) will not cause increase of \( \omega_n \) while decrease of \( \omega_n \) will not cause decrease of \( \omega \). Therefore, the best method is that: increase parameters in the difference set \( E_1 = A_1 - B_1 \) or decrease those in the difference set \( E_2 = A_2 - B_2 \) to increase \( \omega \); and increase parameters in the difference set \( D_1 = B_1 - A_1 \) to decrease \( \omega_n \). However, it is possible that increasing \( \omega \) causes inevitable increase of \( \omega_n \) and decreasing \( \omega_n \) causes inevitable decrease of \( \omega \). In this case, a balanced method should be used to decrease \( \omega_n \) and increase \( \omega \), i.e. the parameters being able to cause significant decrease of \( \omega_n \) and little decrease of \( \omega \) are selected to decrease \( \omega_n \), and the loss caused by \( \omega \) can be compensated by the parameters being able to cause substantial increase of \( \omega \) and little increase of \( \omega_n \). In this way, \( \omega_n \) is decreased while \( \omega \) is increased.

To summarize, the optimal design of frequencies of the two-stage floating raft vibration isolation system can be achieved by the following steps:
(1) Firstly decide initial parameters of the vibration system, the set of variable parameters 
\[ P = \left\{ S_i \mid i = 1, \cdots, m \right\} \]
and variation domains of variable parameters.

(2) Obtain total stiffness matrix and inertia matrix of the system.

(3) According to initial parameters, obtain natural frequencies \( \omega_i \) and \( \omega_n \) as well as corresponding system feature vectors \( X_i \) and \( X_n \). If \( \omega_i > \omega_i^* \) and \( \omega_n \leq \omega_n^* \), the existing parameters would be a set of optimized parameters, otherwise turn to step (4).

(4) Seek \( \frac{\partial M}{\partial S_i} \) and \( \frac{\partial K}{\partial S_i} \) for every \( S_i \in P \) and put the results into equation (8) and equation (9) to calculate \( \frac{\partial \omega_i}{\partial S_i} \) and \( \frac{\partial \omega_n}{\partial S_i} \).

(5) According to the calculation result of step (4), divide the variable parameter set \( P \) into two subsets

\[
A_i = \left\{ S_i \mid \frac{\partial \omega_i}{\partial S_i} \geq 0 \right\}; \quad A_i = \left\{ S_i \mid \frac{\partial \omega_i}{\partial S_i} < 0 \right\}
\]

\[
B_i = \left\{ S_i \mid \frac{\partial \omega_i}{\partial S_i} \geq 0 \right\}; \quad B_i = \left\{ S_i \mid \frac{\partial \omega_i}{\partial S_i} < 0 \right\}
\]

and determine the difference set

\[
E_i = A_i - B_i; \quad E_i = A_i - D_i; \quad D_i = B_i - A_i; \quad D_i = B_i - A_i
\]

(6) Discuss the result in three conditions:

i) If \( E_i, E_i, D_i \), and \( D_i \) are non-empty sets, then increase valid parameters in the sets \( E_i \) and \( D_i \) while decrease valid parameters in the sets \( E_i \) and \( D_i \) to form a new set of structural parameters and turn to step (1).

ii) If all above difference sets are empty sets, then the abovementioned balanced method should be used to adjust variable parameters. To increase \( \omega_i \), increase parameters in the set \( A_i \) that are able to make \( \frac{\partial \omega_i}{\partial S_i} \) larger and \( \frac{\partial \omega_n}{\partial S_i} \) smaller or decrease parameters in the set \( A_i \) that are able to make \( \frac{\partial \omega_i}{\partial S_i} \) larger and \( \frac{\partial \omega_n}{\partial S_i} \) smaller. Similarly, to decrease \( \omega_n \), decrease the parameters in the set \( B_i (= A_i) \) that are able to make \( \frac{\partial \omega_n}{\partial S_i} \) larger and \( \frac{\partial \omega_i}{\partial S_i} \) smaller or increase the parameters in the set \( B_i (= A_i) \) that are able to make \( \frac{\partial \omega_n}{\partial S_i} \) larger and \( \frac{\partial \omega_i}{\partial S_i} \) smaller to form a new set of initial parameters and then turn to step (1).

iii) If there are both empty sets and non-empty sets among \( E_i, E_i, D_i \), and \( D_i \), combine the above two methods to form a new set of initial parameters and then turn to step (1).

An numerical implementation and solution can be applied to complete the above mentioned optimal design steps by properly programming.

4. Numerical Analysis
Optimized frequencies of the system model for two-stage vibration isolation illustrated in Figure 1 are analyzed and verified. To simplify the analysis, it is only considered the movement in two-dimensional space. In Figure 1, all elastic supports are arranged symmetrically and the global coordinate O-X1X2 of \( M_1 \) is in accordance with its centroid coordinate. Local coordinate \( O^1-X_1-X_2^1 \) of each elastic support is consistent with the direction of elastic axis and total coordinate of mass \( M_2 \) is consistent with its centroid and center-of-mass coordinates and on the same direction of inertia axis. Coordinate of the center of mass of mass \( M_1 \) in its global coordinate is \((a, b)\); global coordinate of point \( O^1 \) is \((S_1, S_2)\) and coordinates of points \( O^1 \) and \( O^5 \) in global coordinate of mass \( M_2 \) are \((S_1, S_2)\) and \((S_1, S_2)\).
respectively. Coordinates of points $O_2$, $O_4$ and $O_6$ can be derived by symmetry.

![Diagram of two-stage vibration isolation system]

**Figure 1.** Illustration of two-stage vibration isolation system.

Given that $M_1$ is an engine with revolving speed of 1000rpm and a basic unbalanced disturbance of 16.6Hz. To attain effective vibration isolation, the minimal natural frequency of the vibration isolation system is limited at $f_L=3.5$Hz and the maximal natural frequency is $f_H=15$Hz taking the stability and weight of the device into account. According to reference [4] it can be obtained $M_2=0.25M_1$.

Given $M_1=100$kg, then $M_2=25$kg, $K_1=91728$N·m$^{-1}$, $K_2=114660$N·m$^{-1}$. The above parameters are determined by the optimization method in reference [4] and regarded invariable parameters in system design. The optimization method stated in last section is applied to optimize the system frequency. Firstly determine initial parameters of the system.

Given $J_1=21.59$kg·m$^2$, $J_2=3.64$kg·m$^2$, $K_3=10^5$N·m$^{-1}$, $K_4=1.2 \times 10^5$N·m$^{-1}$, $S_1=0.6$m, $S_2=0.4$m, $S_3=0.65$m, $S_4=0.1$m where $J_1$ is rotational inertia of the engine, which is an invariable parameter. $J_2$, $K_3$, $K_4$ and $S_i (i=1\cdots5)$ are variable parameters, i.e.

$$P = \{J_2, K_3, K_4, S_i \mid i = 1\cdots5\}$$  \hspace{1cm} (15)

The second step is to obtain inertia and stiffness matrix of the system by the method provided in reference [2]. Equation of system free motion is

$$M \ddot{X} + KX = 0$$  \hspace{1cm} (16)

where $M$ and $K$ are overall inertia and stiffness matrixes of the two-stage vibration isolation system illustrated in Figure 1. By seeking derivation of each variable parameter obtains the following matrix

$$\frac{\partial M}{\partial J_2}, \frac{\partial K}{\partial J_2}, \frac{\partial K}{\partial K_3}, \frac{\partial K}{\partial K_4}, \frac{\partial K}{\partial S_1}, \frac{\partial K}{\partial S_2}, \frac{\partial K}{\partial S_3}, \frac{\partial K}{\partial S_4}$$  \hspace{1cm} (17)

Substitute initial parameters of the system into computer program to obtain the variation rate of natural frequencies at every order as well as minimum and maximum natural frequencies of variable parameters.
\[ f = \{3.19; 3.5; 7.46; 11; 14.9; 24.4\} \]

\[ \frac{\partial f_1}{\partial J_2} = -3.98 \times 10^{-3}; \quad \frac{\partial f_1}{\partial K_3} = 4.25 \times 10^{-6}; \]

\[ \frac{\partial f_1}{\partial K_4} = 0.84 \times 10^{-9}; \quad \frac{\partial f_1}{\partial S_1} = 2.25; \]

\[ \frac{\partial f_6}{\partial K_2} = -3.18; \quad \frac{\partial f_6}{\partial S_4} = 1.51; \]

\[ \frac{\partial f_6}{\partial S_3} = -0.22 \]

\[ \frac{\partial f_6}{\partial S_4} = -3.23; \quad \frac{\partial f_6}{\partial K_3} = 1.26 \times 10^{-6}; \]

\[ \frac{\partial f_6}{\partial K_4} = 2.18 \times 10^{-6}; \quad \frac{\partial f_6}{\partial S_1} = 17.45; \]

\[ \frac{\partial f_6}{\partial S_2} = 0.06; \quad \frac{\partial f_6}{\partial S_3} = 20.55; \]

\[ \frac{\partial f_6}{\partial S_4} = 4.16 \]

where \( f_1 = 3.19 \text{ Hz} \) and \( f_6 = 24.4 \text{ Hz} \).

It is revealed that both minimal natural frequency and maximal natural frequency do not satisfy specified design requirements. Among which, \( f_1 \) is smaller than the limited value by 8.86% and \( f_6 \) is larger than its limited value by 62.7%. obviously it is \( f_6 \) that mostly needs to be decreased. Observing from derivatives of \( f_1 \) and \( f_6 \) to each variable parameter, increasing \( J_2 \) would decrease \( f_6 \) considerably while with little effect on \( f_1 \) and decreasing \( S_3 \) would decrease \( f_6 \) in the most effective manner but decrease \( f_1 \) as well, which could be compensated by decreasing \( S_2 \). In addition, decreasing \( S_4 \) can decrease \( f_6 \) while increase \( f_1 \). Therefore, following parameters are reselected according to relationship between derivative and function increment after estimation.

\[
J_2 = 8 \text{ kg} \cdot \text{m}^2 \\
S_2 = 0.2 \text{ m} \\
S_3 = 0.5 \text{ m} \\
S_4 = 0.05 \text{ m}
\]

With other parameters unchanged, substitute corrected parameters into the program to obtain system natural frequencies with the new parameters are:

\[ f = \{3.49; 3.5; 3.77; 10.98; 14.8; 14.84\} \]

The result of calculation with new parameters reveals that error between the minimum natural frequency and the limited value is smaller than 0.1% and the maximum natural frequency is smaller than the limited value. Therefore, two-stage vibration isolation system designed with corrected parameters can satisfy the requirement of small intermediate mass and narrow distribution of frequencies.

It is discussed under the condition of \( a = 0 \) and \( b = 0 \). Actually, the system designed with the corrected parameters of \( a = 10 \text{ cm} \) and \( b = 10 \text{ cm} \) can still satisfy the requirements.

5. Engineering Verification
The above method has been successfully applied to the design of a two-stage vibration isolation of a large-unit floating raft for a new ship in China. As shown in Figure 2, the system consists of three units, middle raft body and elastic elements such as upper and lower vibration isolators, involving 24 orders of model vibration modes. The middle raft body is composed of three sub-raft bodies. There are a total of 30 upper and 36 lower vibration absorbers. The units have a vibration grade of 120dB. The vibration isolation system is accepted to achieve a fall of 45dB, thus optimal design of the system need to avoid resonance and minimize the distribution range of modal frequencies under all modes of vibration. As arrangement form of the equipment and installation form of its interfaces are fixed during the design of input conditions, the optimization can only be made through structural design adjustment of the floating raft body by controlling its models to best match with system vibration, which is an important factor to meet design objectives.

\[ \text{Figure 2. Principle sketch of two-stage floating raft vibration isolation system for large unit.} \]

The frequency range of 24-order modes obtained with initial design parameters is 1.06-67.8Hz, which cannot meet design requirements. According to the variation rate of natural frequencies at all orders and system minimal and maximal natural frequency with variable parameters, low-order mode of the raft body is influenced directly by the connecting rods stiffness of three sub-raft bodies and proportional relationship of mass among three sub-raft bodies. As for adjustment of connecting rods stiffness, alloy steel can be used to improve their elastic modulus; structural forms of connecting rods and their connecting seats are properly selected to improve their sectional inertia torque; strength and locating precision of connecting bolts are increased to improve the reliability of connection. As for adjustment of the sub-raft body inertia parameter, structural form of the steel frame is reasonably designed to maximize its stiffness and weight ratio; weight ratio between raft body and internally poured epoxy resin concrete is optimized to increase internal damping while control mass distribution of the sub-raft bodies. Through optimal design and quantitative adjustment, the width of system modal vibration frequencies decrease from the initial design 66.79Hz (1.06Hz-67.85Hz) to 35.8Hz (5.6Hz-41.4Hz), reducing by 46% and with vibration level difference of 45dB (10Hz-8kHz). This satisfies the requirements of design objectives.

6. Conclusion
It is obvious from the theoretical analysis and numerical analysis that the optimization method presented in the paper is valid for optimal design of natural frequencies of the two-stage vibration isolation system. The method provides the specific means to improve design performances of the system in explicit and quantitative manner, which avoids blindness in design modification and appears favorable engineering guiding significance. In addition, analysis in the paper indicates that the minimal mass ratio obtained from simple model is applicable to systems with multiple degrees of
freedom as well.

References
[1] F. F. Vane 1958 *A guide for the selection and application of mountings for shipboard equipments* Revised David Taylor model basin report 880
[2] J. C. Snowdon 1979 Vibration isolation: Use and characterization *The Journal of the Acoustical Society of America* 68(5) 1245-1274
[3] He Lin 2006 Development of Submarine Acoustic Stealth Technology *Ship Science and Technology* 28(2) 9-17
[4] He Lin 1984 Optimization Analysis of Two-stage Vibration Isolation System *Collected Paper of Academic Symposium on Dynamics of Naval Vessels* Academic Committee on Marine Engine of Chinese Society of Naval Architects and Marine Engineers (CSNAME) 119-126
[5] Thomson W T and Dahleh M D 2005 *Theory of Vibration with Applications* Beijing: Tsinghua University Press
[6] Hua Hongxing, Shi Yinming and Qu Zuqing 1999 Analysis of Frequency Response Sensitivity of Floating Raft System *Shipbuilding of China* 40(3) 92-97
[7] Wang Guozhi and Li Liangbi 2002 Study on Influencing Factors over Dynamic Characteristics of Ship Floating Raft System *Shipbuilding of China* 43(1) 43-51
[8] Du Kui, Wu Xianjun and Cheng Guangli 2005 Research of Best Arrangement Scheme of Vibration Isolators in Floating Raft Vibration Isolation System *Journal of Naval University of Engineering* 17(2) 92-94
[9] Yu Zhi, Shen Rongyin and Yan Jikuan 1995 Research on Parameters Optimization of Multi-stage Vibration Isolation System *Noise and Vibration Control* (5) 14-19