Prospects for Direct Detection of Inflationary Gravitational Waves by Next Generation Interferometric Detectors

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Abstract

We study the potential impact of detecting the inflationary gravitational wave background by the future space-based gravitational wave detectors, such as DECIGO and BBO. The signal-to-noise ratio of each experiment is calculated for chaotic/natural/hybrid inflation models by using the precise predictions of the gravitational wave spectrum based on numerical calculations. We investigate the dependence of each inflation model on the reheating temperature which influences the amplitude and shape of the spectrum, and find that the gravitational waves could be detected for chaotic/natural inflation models with high reheating temperature. From the detection of the gravitational waves, a lower bound on the reheating temperature could be obtained. The implications of this lower bound on the reheating temperature for particle physics are also discussed.

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I. INTRODUCTION

Gravitational wave, which is the gravitational counterpart of electromagnetic wave, has eluded detection. Since gravitational waves interact very weakly with matter, the Universe viewed with the gravitational waves appears much transparent than that with photons. Therefore, the detection of the gravitational waves gives us a snapshot of the very early Universe. Since we now already know that the cosmic microwave background (CMB) spectrum has been a powerful tool for cosmology and the analysis of it has revealed the various components of the Universe, it is not difficult to imagine that another new era of cosmology will come after the discovery of gravitational waves. At the time of the discovery of CMB in 1965, who could have imagined the present situation of cosmology. It is important to prepare for the coming age of another precision cosmology.

Among the processes that occurred in the very early Universe, inflation (including reheating) is the most important epoch. Inflation was proposed as the most natural solution to the difficulties of the standard big-bang cosmology, such as the horizon problem and the flatness problem [1]. It also generates primordial gravitational waves (tensor modes) [2] as well as the primordial density fluctuations (scalar modes) [3]. The latter has already been observed as the cosmic microwave background anisotropies by the Cosmic Background Explorer satellite [4] and by the Wilkinson Microwave Anisotropy Probe (WMAP) satellite [5]. The detection of the gravitational waves generated during inflation could determine the energy scale of inflation directly, which is essential to explore the physics behind inflation [6].

In Ref. [7], we calculated for the first time the most precise power spectrum of the inflationary gravitational wave background for several inflation models with perturbative reheating. Our numerical approach has enabled us to obtain the precise amplitude of the spectrum over all frequencies. We compute the spectrum all through the evolution of the Hubble expansion rate, which determines the amplitude of the gravitational waves, by following the dynamics of the inflaton scalar field and its decay into radiation (reheating). We fully take into account of the changes of the effective number of degrees of freedom during the radiation-dominated era which cause the suppression of the spectrum at high frequencies [8]. Therefore, all factors which affect the amplitude of the gravitational wave background spectrum are precisely reflected in our calculation.
In this paper, we aim to forecast the future prospects of the direct detection of the gravitational wave background by next generation satellite detectors, such as the DECi-hertz Interferometer Gravitational wave Observatory (DECIGO) [9, 10] and Big-Bang Observer (BBO) [11]. While the measurement of CMB B-mode polarization, which is the indirect signature of primordial gravitational waves, probes gravitational waves at the present horizon scale, the direct detection observes gravitational waves at a much smaller scale, which would contain unique information about the very early Universe. In particular, we should stress that the shape of the gravitational wave spectrum around the target frequency (∼0.1Hz) is sensitive to the physics of reheating [12]. In Ref. [13], the detectability of the gravitational wave background is estimated taking into account the dependence of the spectrum shape on reheating temperature. Here, we reexamine the detectability by calculating the signal-to-noise ratio (SNR) with the precise prediction of the spectrum amplitude and the specific noise spectra of DECIGO and BBO. We study the dependence of the spectrum on reheating temperature as well as on models of inflation. In addition, we discuss the implications of a possible lower bound of the reheating temperature obtained from the future direct detection by DECIGO/BBO for particle physics.

Note that our investigation is carried out assuming reheating via perturbative decay of the inflaton field, namely we do not consider nonperturbative effects during reheating, called preheating [14, 15]. If the nonperturbative effects are dominant in the reheating process, the picture of reheating and its effect on the gravitational wave background are significantly different from those of perturbative reheating [16–18]. In this paper, since we would like to provide a conservative estimate of the gravitational wave background and its detectability, we only consider perturbative processes which always exist and are directly related with the reheating temperature.

The outline of this paper is as follows. In Sec. III the signal-to-noise ratio expected in future experiments is estimated for four inflation models; chaotic inflation with quadratic and quartic potentials, natural inflation and hybrid inflation. First of all, we present the method to calculate the signal-to-noise ratio and the current specification design of DECIGO and BBO. Then the procedure of the numerical calculation is described briefly, which is used to obtain the amplitude of the spectrum. In Sec. III we calculate the signal-to-noise ratio using the detailed experimental specification and the precise prediction of the spectrum amplitude. The implications of the lower limit of the reheating temperature on particle physics...
physics are also discussed in relation with the gravitino problem in Sec. IV. A summary is given in Sec. V.

II. ESTIMATION METHOD OF THE SIGNAL-TO-NOISE RATIO

A. Correlation analysis for detection of a stochastic gravitational wave background

Cosmological gravitational waves are described as tensor perturbations in the Friedmann-Robertson-Walker metric as

\[ ds^2 = -dt^2 + \frac{a^2(t)}{a(t)}(\delta_{ij} + h_{ij})dx^i dx^j, \]

where \( a(t) \) is the scale factor of the Universe. The tensor perturbation \( h_{ij} \) can be expanded into its Fourier components as

\[ h_{ij}(t, x) = \sum_{\lambda=+,-} \frac{d^3k}{(2\pi)^{3/2}} \epsilon_{ij}^\lambda(k) h_k^\lambda(t) e^{i k \cdot x}, \quad (1) \]

where the polarization tensors \( \epsilon_{ij}^{+,\times} \) satisfy symmetric and transverse-traceless conditions and are normalized as \( \sum_{i,j} \epsilon_{ij}^\lambda (\epsilon_{ij}^{\lambda'})^* = 2\delta^{\lambda\lambda'} \). The intensity of a stochastic gravitational wave background is characterized by the dimensionless quantity, \( \Omega_{GW} \equiv (d\rho_{GW}/d\ln k)/\rho_c \), where the critical density of the Universe is defined as \( \rho_c \equiv 3H^2/8\pi G \) with the Hubble expansion rate, \( H = (da/dt)/a \). The energy density of the gravitational waves \( \rho_{GW} \) is given from the 00-component of the stress-energy tensor as

\[ \rho_{GW} = \frac{\langle (\partial_t h_{ij})^2 + (\nabla h_{ij}/a)^2 \rangle}{(64\pi G)}. \]

Then \( \Omega_{GW} \) can be expressed in terms of the Fourier component \( h_k^\lambda \) as

\[ \Omega_{GW} = \frac{1}{12} \left( \frac{k}{aH} \right)^2 \frac{k^3}{\pi^2} \sum_{\lambda} |h_k^\lambda|^2. \quad (2) \]

In future satellite missions like DECIGO and BBO, the analysis of a stochastic gravitational wave background would be performed by taking the cross correlation of the outputs of gravitational wave detectors \[20\text{–}22\]. The signal-to-noise ratio for the correlation analysis is given in terms of an expected (theoretical) form of \( \Omega_{GW}(f) \), and the functions related to the experiment design, such as the noise spectrum \( S_{I,J}(f) \) and the overlap reduction function \( \gamma_{IJ}(f) \) as

\[ [\text{SNR}]^2 = 2 \left( \frac{3H_0^2}{10\pi^2} \right)^2 T_{\text{obs}} \sum_{(I,J)} \int_0^\infty df \frac{|\gamma_{IJ}(f)|^2 \Omega_{GW}^2(f)}{f^6 S_I(f) S_J(f)}, \quad (3) \]

where \( f = k/2\pi \) is the frequency of gravitational waves, \( H_0 \) is the present Hubble expansion rate, \( T_{\text{obs}} \) is the duration of the observation time. The subscripts \( I \) and \( J \) refer to independent signals obtained at each detector, or observables generated by combining the detector signals.
DECIGO is planned to be a Fabry-Perot Michelson interferometer with an arm length of \( L = 1.0 \times 10^3 \text{km} \) \cite{9, 10}. In this case, the SNR is calculated with the noise spectral density of the two interferometers, which are assumed to be the same and given by \cite{20, 21}

\[
S_1(f) = S_2(f) = S_{\text{shot}} + S_{\text{accel}} + S_{\text{rad}},
\]

where the shot noise is given as \( S_{\text{shot}} = 5.29 \times 10^{-42}(1 + f^2/f_c^2)(L/\text{km})^{-2}\text{Hz}^{-1} \), the acceleration noise is \( S_{\text{accel}} = 4.0 \times 10^{-46}(f/\text{Hz})^{-4}(L/\text{km})^{-2}\text{Hz}^{-1} \) and the radiation pressure noise is \( S_{\text{rad}} = 3.6 \times 10^{-51}(f/\text{Hz})^{-4}(1 + f^2/f_c^2)^{-1}\text{Hz}^{-1} \). \(^1\) The cutoff frequency is given by \( f_c = 1/(4\mathcal{F}L) \) with the fineness for the DECIGO detector, \( \mathcal{F} = 10 \).

On the other hand, BBO would adopt a technique called time-delay interferometry, in which new variables \( (I = A, E, T) \) are constructed to cancel the laser frequency noise. The noise transfer functions for the time-delay interferometry variables are given as \cite{24, 25}

\[
S_A(f) = S_E(f) = 8\sin^2(\hat{f}/2)\left[(2 + \cos \hat{f})S_{\text{shot}} + 2(3 + 2\cos \hat{f} + \cos(2\hat{f}))S_{\text{accel}}\right],
\]

\[
S_T(f) = 2[1 + 2\cos \hat{f}]^2[S_{\text{shot}} + 4\sin^2(\hat{f}/2)S_{\text{accel}}].
\]

where \( \hat{f} = 2\pi Lf \). In the case of BBO, the arm length is \( L = 5.0 \times 10^4\text{km} \), and the noise functions are \( S_{\text{shot}} = 2.0 \times 10^{-40}/(L/\text{km})^2\text{Hz}^{-1} \) and \( S_{\text{accel}} = 9.0 \times 10^{-40}/(2\pi f/\text{Hz})^4/(2L/\text{km})^2\text{Hz}^{-1} \).

The overlap reduction function \( \gamma_{IJ}(f) \) can be calculated by taking into account of the relative locations and orientations of the detectors. We use the results of Ref. \cite{20} for FP-DECIGO, and of Ref. \cite{26} for BBO.

### B. Numerical calculation for the spectrum of the gravitational wave background

We briefly describe the method we used to obtain the theoretical prediction for the amplitude of the spectrum \( \Omega_{GW} \) in our previous work (For details, see Ref. \cite{22}). We numerically solve the evolution equation for gravitational waves, which is derived from the perturbed Einstein equation under the Friedmann-Robertson-Walker metric,

\[
\ddot{h}_k^\lambda + 3H\dot{h}_k^\lambda + \frac{k^2}{a^2}h_k^\lambda = 0,
\]

\(^1\) A major improvement of the sensitivity is under consideration.
where the over dot describes the time derivative. The initial condition is randomly taken from the Bunch-Davis vacuum, of which variance is given as

\[ |h_k^2| = \frac{16\pi}{2ka^2m_{\text{Pl}}^2}, \]  

(8)

where \( m_{\text{Pl}} = G^{-1/2} \) denotes the Planck mass.

The important point of our evaluation of the spectrum is that we compute the evolution of the gravitational waves with following all through the history of the cosmic expansion from inflation to the present epoch. It allows us to calculate the spectrum with no use of the slow-roll approximation, which overestimates the amplitude of the spectrum at the direct detection scale in some inflation models. The spectrum, which is Taylor-expanded around the CMB scale, can overestimate the amplitude by \( 10^{-20\%} \). The numerical approach also enables us to precisely evaluate the effect of the changes in the relativistic degrees of freedom during the radiation-dominated era.

If we assume reheating is proceeded by perturbative decay of the inflaton field into light fermions, all the processes from inflation to the end of reheating can be calculated by simultaneously solving the following equations,

\[
\ddot{\phi} + (3H + \Gamma)\dot{\phi} + \frac{\partial V(\phi)}{\partial \phi} = 0, \tag{9}
\]

\[
\dot{\rho}_r + 4H\rho_r = \Gamma \rho_\phi, \tag{10}
\]

\[
H^2 = \frac{8\pi}{3m_{\text{Pl}}^2}(\rho_\phi + \rho_r), \tag{11}
\]

where \( \Gamma \) is the decay rate of the scalar field into radiation, \( V(\phi) \) is the potential of the scalar field, \( \rho_r \) is the energy density of the radiation, and the energy density of the scalar field is given as \( \rho_\phi = \dot{\phi}^2/2 + V(\phi) \).

During inflation, the Hubble expansion rate is determined by the dynamics of the scalar field \( \phi \) which drives inflation. The Universe enters a reheating phase after inflation, and the scalar field oscillates at the bottom of the potential decaying into radiation. During this phase, the Universe evolves like a matter-dominated Universe in most of the inflation models, and turns into a radiation-dominated era after it ends. The exception is, for example, the case where the potential has a \( \lambda \phi^4 \) shape at its bottom. In this case, the Universe behaves as a radiation-dominated Universe and connects to the subsequent radiation-dominated era with no change in the Hubble expansion rate.
If the matter-dominated reheating phase exists before the radiation-dominated era, the inflationary gravitational wave spectrum is suppressed at high frequencies \[12,13\]. The suppression is seen on the modes which enter the horizon during reheating, since the matter-dominated phase induces frequency dependence of \(f^{-2}\) on the spectrum while a radiation-dominated era gives a flat spectrum \(\propto f^0\). The characteristic frequency, where the change of the frequency dependence from \(f^{-2}\) to \(f^0\) arises, is given in terms of the temperature of the Universe at the end of reheating as \[27\]

\[f_{RH} \simeq 0.3 \left( \frac{T_{RH}}{10^7 \text{GeV}} \right) \left( \frac{g_{s,\text{RH}}}{220} \right)^{1/2} \left( \frac{g_{s,s,\text{RH}}}{220} \right)^{-1/3} \text{Hz,} \tag{12}\]

where \(g_s\) and \(g_{s,s}\) are the effective number of relativistic degrees of freedom contributing to the radiation density and the entropy density. The subscript "RH" denotes the value at the end of reheating. We take \(g_{s,\text{RH}}\) to be \(\sim 220\), which includes degrees of particles in minimal supersymmetric standard model. Since both \(g_s\) and \(g_{s,s}\) take the same value before electron-positron annihilation, \(\sim 0.1\text{MeV}\), where the temperature of neutrinos becomes lower than that of photons, \(g_{s,s,\text{RH}}\) is also taken to be 220. In the case of the perturbative decay, the reheating temperature \(T_{RH}\) is related to the decay rate \(\Gamma\) as \[28\]

\[T_{RH} \simeq g_{s,\text{RH}}^{-1/(4)} \left( \frac{45}{8\pi^3} \right)^{1/4} (m_{\text{Pl}}\Gamma)^{1/2}. \tag{13}\]

Note that the above expressions are valid only for the case where the mass of the interacting fermion is much smaller than the mass of the inflaton field and also the coupling constant is sufficiently small. Otherwise, parametric resonance between the two interacting fields may give nonperturbative creation of particles and dramatically shorten the time scale of reheating. We do not deal with such nonperturbative particle production during reheating, called preheating \[14\]. If the nonperturbative growth is sufficient to reheat the Universe, the \(f^{-2}\) dependence would not arise on the inflationary gravitational wave background and it may rather be important to look at gravitational waves originating from large inhomogeneities in the matter field during the nonperturbative stage \[16,18\]. Such gravitational waves typically have very high frequency beyond the sensitivity bandwidth of DECIGO and BBO, but some models predict an infra-red tail which overlaps or exceeds the inflationary gravitational wave background at detectable frequency. However, in this paper, our focus is on the effect of reheating on the inflationary gravitational wave background and consider only the case of reheating with perturbative fermionic decay. For reheating via fermionic
decay, the rapid particle production is suppressed by Pauli blocking, while for reheating via bosonic decay could be easily completed by the exponential growth of the number of particles. Although the effect of the nonperturbative decay can still be important for the fermionic case [15] and could change the time scale of reheating, our investigation, after all, turns out to be only the weak coupling case, where the nonperturbative effects are negligibly small. The detailed analysis of the effects of nonperturbative processes like preheating on the inflationary gravitational wave background will be left to a future work.

After reheating ends, the Universe enters a radiation-dominated phase. Including the effects of $g_*(T)$ and $g_{ss}(T)$, the Hubble expansion rate can be written as [29]

$$H^2 = H_0^2 \left[ \left( \frac{g_*(T)}{g_{*,0}} \right) \left( \frac{g_{ss}(T)}{g_{ss,0}} \right)^{-4/3} \Omega_\gamma a^{-4} + \Omega_m a^{-3} + \Omega_\Lambda \right],$$ \hspace{1cm} (14)

where 0 denotes values at the present time, which are given by summing the contributions from photons and neutrinos: $g_{*,0} = 3.36$ and $g_{ss,0} = 3.90$. The change of the Hubble expansion rate affects the evolution of the inflationary gravitational waves and causes damping at the frequency where the direct detection experiments are targeting. The suppression is about $(g_*(T = 10^7 \text{GeV})/g_{*,0})(g_{ss}(T = 10^7 \text{GeV})/g_{ss,0})^{-4/3} = (220/3.36)(220/3.90)^{-4/3} \sim 0.3$.

We calculate the spectrum of the gravitational waves by numerically solving the evolution of each mode with Eq. (7). At the same time, we follow the evolution of the Hubble expansion, which is calculated by Eqs. (9), (10) and (11) during the inflation and reheating phase, and by Eq. (14) after reheating. In this paper, the energy density of radiation is taken to be $\Omega_\gamma h^2 = 4.15 \times 10^{-5}$ and the other cosmological parameters are given by the maximum likelihood values from the combined constraints of the WMAP 7 yr, BAO, and supernova data [30]: matter density $\Omega_m h^2 = 0.1344$, cosmological constant density $\Omega_\Lambda = 0.728$, amplitude of curvature perturbations $\Delta_k^2 = 2.45 \times 10^{-9}$, and the Hubble parameter $h = 0.702$.

III. PREDICTIONS FOR THE DETECTABILITY IN FUTURE EXPERIMENTS

A. Signal-to-Noise Ratio

Now we forecast the detectability of the inflationary gravitational wave background with DECIGO/BBO for several inflation models: chaotic inflation with a quadratic and quartic
potential, natural inflation \[31, 32\], and hybrid inflation \[33\]. The potentials are respectively given by,

**quadratic**: \[ V = \frac{1}{2}m^2\phi^2, \] (15)

**quartic**: \[ V = \frac{1}{4}\lambda\phi^4, \] (16)

**natural**: \[ V = \Lambda^4 \left[ 1 \pm \cos \left( \frac{N\phi}{f} \right) \right], \] (17)

**hybrid**: \[ V = \frac{1}{4\lambda}(M^2 - \lambda\sigma^2)^2 + \frac{1}{2}m^2\phi^2 + \frac{1}{2}g^2\phi^2\sigma^2. \] (18)

The spectra of these four models and the sensitivity curves of the DECIGO and BBO experiments are shown in Fig. 1. The normalization of the scalar perturbations \( \Delta^2_R = 2.45 \times 10^{-9} \) fixes the value of the potential parameters. It gives \( m = 1.72 \times 10^{13}\text{GeV} \) for the quadratic potential, \( \lambda = 1.54 \times 10^{-13}\text{GeV} \) for the quartic potential, \( \Lambda = 2.04 \times 10^{16}\text{GeV} \) for the natural inflation model with \( N = 1 \) and \( f = 2 m_{\text{Pl}} \), \( \Lambda = 1.33 \times 10^{16}\text{GeV} \) for the natural inflation model with \( N = 1 \) and \( f = m_{\text{Pl}} \), and \( M = 1.45 \times 10^{16}\text{GeV} \) for the hybrid inflation model with \( \lambda = 1, g = 8 \times 10^{-4} \), and \( m = 2.5 \times 10^{-7} m_{\text{Pl}} \). \(^2\) The parameter values are searched numerically by adjusting the resulting Hubble expansion rate to be \( H_0 = 100 h \text{km/s/Mpc} \). Note that these parameters also slightly depend on the reheating temperature, since it is related to the length of inflation. The above values are obtained when \( T_{\text{RH}} = 10^7\text{GeV} \).

In Fig. 2, we show the SNR for cross-correlation analysis expected with a 10-year observation by DECIGO and BBO. The SNR decreases significantly when the reheating temperature \( T_{\text{RH}} \) is lower than \( 10^7\text{GeV} \), since the suppression due to the presence of the reheating phase reduces the amplitude of the spectrum at the target frequency of the direct detection experiments. The only exception is the case of the quartic potential, in which the Hubble expansion rate behaves as a radiation-dominated Universe during the reheating phase and the suppression does not arise.

In Table I we present the relation between the value of \( \Omega_{GW} \) at the detection frequency, \( f = 0.2\text{Hz} \), and the tensor-to-scalar ratio, which is usually evaluated at the CMB scale \( k_{\text{CMB}} = 0.002\text{Mpc} \) as

\[
\begin{align*}
\Delta^2_R(k_{\text{CMB}}) & \equiv r \Delta^2_R(k_{\text{CMB}}) \simeq 16\epsilon,
\end{align*}
\] (19)

\(^2\) Although there is much freedom in the choice of the parameters for hybrid inflation, here we take one example where the parameter \( M \) corresponds to grand unification scale.
FIG. 1: Spectra of the gravitational wave background for different inflation models, shown with the sensitivity curves of DECIGO (dotted) and BBO (solid). The spectra are calculated assuming $T_{RH} = 10^7\text{GeV}$. The cases of $T_{RH} = 10^6\text{GeV}$ and $10^8\text{GeV}$ are also plotted assuming the quadratic potential model. Note that the spectrum lines mean the time-averaged value of $\Omega_{GW}$.

where the slow-roll parameter is defined as $\epsilon \equiv m_{\phi}^2/(16\pi)(V'/V)|_{k_{\text{CMB}}=aH}$. The reheating temperature is set as $T_{RH} = 10^9\text{GeV}$ which is so high that the suppression does not arise at the detection frequency. As is clear from the comparison between $m_{\phi}^2$ and $\lambda\phi^4$ model, the amplitude of the gravitational wave at the direct detection scale is not proportional to the tensor-to-scalar ratio $r$ because the tilt of the spectrum $n_T \simeq -2\epsilon$ and the higher order terms of the Taylor-expansion becomes important when connecting the two different scales [34]. This is prominent in models which predict larger $r$.

For DECIGO, the inflationary gravitational background could be detected with $\text{SNR} \geq 3$ if the inflation model is chaotic inflation with $T_{RH} \gtrsim 10^7\text{GeV}$. For BBO, the inflationary gravitational background could be detected with $\text{SNR} \geq 5$ if the inflation model is chaotic inflation with $T_{RH} \gtrsim 2 \times 10^6\text{GeV}$ or natural inflation with $f \gtrsim m_{\text{Pl}}$ and $T_{RH} \gtrsim 10^7\text{GeV}$. Therefore, from the contraposition, if DECIGO does not detect the inflationary gravitational wave background, the chaotic inflation model will be excluded unless the reheating temperature is lower than $10^7\text{GeV}$. The same argument holds for BBO except that it can apply to natural inflation.
FIG. 2: Signal-to-noise ratio vs reheating temperature calculated for the four different inflation models. The upper panel shows the SNR for DECIGO and the lower panel is for BBO. The gray region shows the 1σ uncertainty in the normalization $\Delta^2_R = (2.441^{+0.088}_{-0.092}) \times 10^{-9}$ from WMAP 7 yr, which is used to determine the energy scale of inflation.

| Model       | $r$      | $\Omega_{GW}$   | SNR (BBO) | SNR (DECIGO) |
|-------------|----------|-----------------|-----------|---------------|
| $m^2\phi^2$ | 0.144    | $1.12 \times 10^{-16}$ | 13.3      | 3.02          |
| $\lambda\phi^4$ | 0.262    | $8.75 \times 10^{-17}$ | 10.5      | 2.40          |
| Natural ($f = 2m_{\text{Pl}}$) | 0.108    | $1.01 \times 10^{-16}$ | 12.0      | 2.72          |
| Natural ($f = m_{\text{Pl}}$) | 0.0406   | $5.70 \times 10^{-17}$ | 6.79      | 1.53          |
| Hybrid      | 0.0104   | $2.44 \times 10^{-17}$ | 2.90      | 0.646         |

TABLE I: The tensor-to-scalar ratio $r$, the amplitude of the gravitational wave $\Omega_{GW}$ at $f = 0.2\text{Hz}$, and the signal-to-noise ratio in DECIGO and BBO for each inflation model. The reheating temperature is set as $T_{RH} = 10^9\text{GeV}$. 
FIG. 3: Parameter dependence of the signal-to-noise ratio for the natural inflation model.

FIG. 4: Parameter dependencies of the signal-to-noise ratio for the hybrid inflation model. The stars show the fiducial point, at which parameters are set as $\lambda = 1$, $g = 8 \times 10^{-4}$, and $m = 2.5 \times 10^{-7} m_{\text{Pl}}$. The dotted line in the top panel shows $y = 1$. The shaded region means $\sigma \neq 0$ when the observable scale (the present Hubble horizon) exits the horizon.
B. Parameter Dependence

We also investigate the parameter dependence of the SNR, shown in Figs. 3 and 4 by setting the reheating temperature to be $T_{RH} = 10^9$ GeV. For the natural inflation model, the amplitude of the gravitational waves becomes larger as $f$ increase. This can be interpreted as follows: Since we use the normalization of the scalar perturbation $\Delta^2_R = 2.45 \times 10^{-9}$, the value of $\epsilon$ determines the amplitude of the gravitational waves at the CMB scale, as we know $\Delta^2_h = 16\epsilon \Delta^2_R$ from Eq. (19). 3

In the case of the natural inflation model with $N = 1$, $\epsilon$ is given as 35

$$\epsilon = \frac{1}{16\pi} \left( \frac{m_{Pl}}{f} \right)^2 \left[ \frac{\sin(x)}{1 + \cos(x)} \right]^2,$$

(20)

where we define $x \equiv \phi/f$. The field value can be written in terms of the e-folding number, $N \equiv \ln(a_{end}/a)$, as 36

$$\sin\left(\frac{x}{2}\right) = \sin\left(\frac{x_{end}}{2}\right) \exp\left( -\frac{m_{Pl}^2}{16\pi f^2 N} \right).$$

(21)

The end of inflation $x_{end}$ is defined at the point where the slow-roll condition is violated, $\epsilon = 1$, which gives

$$\cos(x_{end}) = \frac{1 - 16\pi(f/m_{Pl})^2}{1 + 16\pi(f/m_{Pl})^2}.$$  

(22)

Combining Eqs. (20), (21), and (22), we find that $\epsilon$ is written as $\epsilon = 1/[(1 + \chi) \exp(2N/\chi) - \chi]$ where $\chi \equiv 16\pi f^2/m_{Pl}^2$. This is an increase function of $\chi$, which implies that $\Omega_{GW}$ increases as $f$ increases.

In the standard picture of hybrid inflation that the $\phi$ field determines the evolution of the Universe during inflation, the potential can be recast as $V = \Lambda [1 + (\phi/\mu)^2]$ for the inflation stage, where we define $\Lambda \equiv M^2/(4\lambda)$ and $\mu \equiv M^2/(m\sqrt{2}\lambda)$ 37. Then the slow-roll parameter is given as

$$\epsilon = \frac{1}{4\pi} \left( \frac{m_{Pl}}{\mu} \right)^2 \frac{y^2}{[1 + y^2]^2}.$$  

(23)

The field value is given in terms of the e-folding number 38,

$$y = \sqrt{W_0 \left\{ y_{end}^2 \exp\left[ y_{end}^2 + \frac{\mathcal{N}}{2\pi} \left( \frac{m_{Pl}}{\mu} \right)^2 \right] \right\}},$$

(24)

3 Although this relation does not hold at scales well below the CMB scale, the amplitude at the direct detection scale is a monotonically increasing function of $\epsilon$ for the natural and hybrid inflation models, which predict relatively small slow-roll parameters.
where we define \( y = \phi / \mu \) and \( W_0(x) \) is the principal branch of the Lambert function, which satisfies \( x = W_0(x)e^{W_0(x)} \). In the case of hybrid inflation, inflation ends with the waterfall field \( \sigma \) rolling down when \( \phi_{\text{end}} = M/g \), which gives

\[
y_{\text{end}} = \frac{m\sqrt{2\lambda}}{Mg}.
\]  

(25)

The potential looks more like quadratic with increasing \( m \), which is the mass of the \( \phi \) field. When \( y > 1 \), the inflation dynamics becomes the same as the case of the quadratic potential. So, we focus our interest on the case of \( y < 1 \). The value of \( m \) affects both \( \mu \) and \( y \), which respectively decreases and increases with increasing \( m \). From Eqs. (23) (24) and (25), we find \( \epsilon \) becomes larger with decreasing \( \mu \) and increasing \( y \) for \( 0 < y < 1 \). Hence \( \Omega_{GW} \) increases as \( m \) increases as seen in the top panel of Fig. 4.

The parameter \( \lambda \) determines the potential minimum of the \( \sigma \) field, \( \sigma_{\text{min}} = M/\sqrt{\lambda} \). Since observables are determined by the dynamics of the \( \phi \) field in the standard hybrid inflation, the value of \( \lambda \) has little effect on the amplitude of the gravitational waves as shown in the middle panel of Fig. 4.

The parameter \( g \) determines the value of \( \phi_{\text{end}} = M/g \), where the waterfall field \( \sigma \) starts to roll down. The value of \( y \) is affected through \( y_{\text{end}} = \phi_{\text{end}}/\mu \) which becomes smaller as \( g \) increases. Thus, the slow-roll parameter \( \epsilon \), which is an increase function of \( y \) for \( 0 < y < 1 \), becomes smaller with increasing \( g \). Hence, \( \text{SNR} \propto \Omega_{GW} \) decreases as \( g \) becomes larger in Fig. 4.

For too large \( \lambda \) or small \( g \), the condition \( m^2/\phi_{\text{end}}^2 = m^2M^2/g^2 \ll M^4/\lambda \) is not satisfied, which results in that the waterfall field \( \sigma \) starts to roll down slowly before inflation is dominated by the vacuum energy of the \( \phi \) field. The shaded region in the figure means \( \sigma \neq 0 \) when the observable scale (the present Hubble horizon) exits the horizon, which is not the standard behavior of the hybrid inflation model.

IV. IMPLICATIONS OF THE LOWER LIMIT OF THE REHEATING TEMPERATURE FOR PARTICLE PHYSICS

In previous sections, we found that if inflation is the chaotic type and its reheating temperature is higher than \( 10^7 \) GeV, then the inflationary stochastic gravitational wave background would be detected by future space-based interferometric detectors like DECIGO
or BBO. Unfortunately we have a small chance to determine the exact value of the reheating temperature from the detection. The detection only allows us to set a lower bound on the reheating temperature. However, this lower bound $10^7$ GeV provides useful and unique information of the very early Universe. In this section, as an example, we consider the implications of the lower bound of the reheating temperature for the nature of gravitino production in the early Universe.

Many models of supersymmetry breaking, in the context of either supergravity or superstring theory, predict the presence of scalar fields with Planck-suppressed couplings and masses around or heavier than the weak scale. These fields are generically called moduli. The coherent oscillation of the modulus soon dominates the Universe, and the late decay of the modulus results in very low reheating temperature, upsetting the success of the big-bang nucleosynthesis (BBN) [called the moduli problem [39]] unless the modulus is ultraheavy: $M_X \gtrsim 100$ TeV.

Gravitino is the fermionic superpartner of graviton and has Planck-suppressed interaction. The gravitino, once produced, decays with a very long lifetime if it is unstable. The gravitino may cause several problems in cosmology. For example, if the gravitino is unstable (if its mass is $M_{3/2} \sim 100$ GeV $- 100$ TeV), the decay products of the gravitino would destroy the primordial light elements by photodissociation and hadrodissociation, and thus spoil the success of BBN (called the gravitino problem [40]). Hence, the yield of the gravitinos $Y_{3/2} = n_{3/2}/s$ (s is the entropy density) should be constrained: $Y_{3/2} < Y_{3/2}^{BBN}$, where the value of $Y_{3/2}^{BBN}$ depends on the gravitino mass. According to the recent analysis [41, 42], $Y_{3/2}^{BBN} \sim 10^{-10}$ for $M_{3/2} \sim 1$ TeV and $Y_{3/2}^{BBN} \sim 10^{-15} - 10^{-13}$ for $M_{3/2} \sim 10 - 100$ TeV. On the other hand, if the gravitino is stable (for $M_{3/2} \lesssim 1$ GeV), the gravitinos can be cold/warm dark matter. Hence the abundance of the gravitinos $\Omega_{3/2}h^2$ should also be bounded in order not to exceed the dark matter abundance $\Omega_{dm}h^2 \simeq 0.134$ which is precisely determined by the WMAP [30].

Recently, it has been found that gravitinos are produced not only by the thermal scatterings at the reheating [43] but also by the decay of heavy scalar fields (for example, inflaton and moduli) [44]. Therefore, the "ultraheavy moduli solution" to the moduli problem may cause instead a new gravitino problem by the moduli decay.

In the following, we consider the implication of the possible lower bound of the reheating temperature on gravitino cosmology in the context of chaotic/natural inflation. Then the
nonthermal production of the gravitinos is only due to the moduli decay \[45\] since the vacuum expectation value of the inflaton is vanishing for chaotic/natural inflation (with \(Z_2\) symmetry) \[46\]. A similar consideration (without moduli decay) is given in \[12\].

### A. Unstable Gravitino

Firstly we consider the case of unstable gravitinos. The yield of the gravitinos produced by the thermal scatterings at the reheating temperature \(T_{RH}\) is estimated by \[41, 43\]

\[ Y^T_{3/2} \simeq 1.4 \times 10^{-12} \left( \frac{T_{RH}}{10^{10}\text{GeV}} \right)^3, \]  

(26)

Thus if the reheating temperature is found to be high \((T_{RH} \gtrsim 10^7 \text{ GeV})\), it would immediately imply that large gravitino mass \(M_{3/2} \gtrsim 10 \text{ TeV}\) is favored \[42\].

The yield of the gravitinos by the \(X\) decay is given by \[44, 45\]

\[ Y^X_{3/2} \simeq \frac{1}{192\pi} \sqrt{\frac{90}{\pi^2 g_*(T_X) T_X M_{Pl}}} \frac{d_{3/2}^2 M_X^2}{T_X M_{Pl}}, \]  

(27)

where \(M_{Pl} = m_{Pl}/\sqrt{8\pi}\) is the reduced Planck mass and \(d_{3/2} \simeq \langle |X| \rangle / M_{Pl}\) is related to the partial decay rate of the process \(X \rightarrow \psi_{3/2} + \psi_{3/2}\) as

\[ \Gamma_{3/2} = \frac{d_{3/2}^2 M_X^3}{288\pi M_{Pl}^2}. \]  

(28)

\(T_X\) is the reheating temperature by \(X\) decay with the decay rate \(\Gamma_X \simeq M_X^3/(8\pi M_{Pl}^2)\), \(^5\) and is given by

\[ T_X = \left( \frac{90}{\pi^2 g_*(T_X)} \right)^{1/4} \sqrt{\Gamma_X M_{Pl}} = 5.8 \times 10^4 \text{GeV} \left( \frac{M_X}{10^{10}\text{GeV}} \right)^{3/2}, \]  

(29)

where \(g_*(T_X)\) is the effective relativistic degrees of freedom at \(T = T_X\) and we use \(g_*(T \gtrsim 1 \text{TeV}) \simeq 220\). Then Eq. (27) becomes

\[ Y^X_{3/2} \simeq 2.4 \times 10^{-7} \left( \frac{M_X}{10^{10}\text{GeV}} \right)^{1/2} d_{3/2}^2 \]  

(30)

\(^4\) The thermal production of gravitinos at \(T_X\) is a factor of \(\rho_X/\rho_\phi\) smaller than that at \(T_{RH}\) and hence may be negligible.

\(^5\) We hereby have fixed the order one coefficient.
Therefore, from the success of the BBN ($Y_{3/2} < Y_{3/2}^{\text{BBN}}$), the upper bound on the moduli mass is found: \(^{6}\)

\[
M_X \lesssim 2 \times 10^{-3}\text{GeV} d_{3/2}^{-4} \left( \frac{Y_{3/2}^{\text{BBN}}}{10^{-13}} \right)^2.
\]  

(31)

**B. Stable Gravitino**

Next, we consider the case of stable gravitinos. The present-day abundance of the gravitinos produced by the thermal scatterings at the reheating is given by \(^{[43]}\)

\[
\Omega_{3/2} h^2 \simeq 0.27 \left( \frac{T_{\text{RH}}}{10^8\text{GeV}} \right) \left( \frac{M_{3/2}}{1\text{GeV}} \right)^{-1},
\]  

(32)

where we have set the gluino mass $M_{\tilde{g}} = 1\text{TeV}$. From Eq. \(^{[30]}\), the abundance of the gravitinos produced by the $X$ decay is given by

\[
\Omega_{3/2} h^2 = \frac{M_{3/2} Y_{3/2} X}{\rho_{cr} s_0} \simeq 6.8 \times 10^1 d_{3/2} \left( \frac{M_X}{10^{10}\text{GeV}} \right)^{1/2} \left( \frac{M_{3/2}}{1\text{GeV}} \right),
\]  

(33)

where $\rho_{cr}$ is the present critical density of the Universe and $s_0$ is the present entropy density. The total abundance should satisfy the bound $\Omega_{3/2} h^2 = \Omega_{3/2}^{\text{TH}} h^2 + \Omega_{3/2}^{X} h^2 \leq \Omega_{dm} h^2 \simeq 0.134$.

Therefore, if $T_{\text{RH}} > T_{\text{gw}}^{\text{RH}}$ from gravitational wave experiments, then, using Eq. \(^{[32]}\) from $\Omega_{3/2}^{\text{TH}} h^2 < \Omega_{dm} h^2$, we find the lower bound on the gravitino mass:

\[
M_{3/2} > 0.21\text{GeV} \left( \frac{T_{\text{gw}}^{\text{RH}}}{10^7\text{GeV}} \right) \left( \frac{\Omega_{dm} h^2}{0.134} \right)^{-1}.
\]  

(34)

Moreover, from the geometric mean,

\[
\Omega_{dm} h^2 \geq \Omega_{3/2} h^2 = \Omega_{3/2}^{\text{TH}} h^2 + \Omega_{3/2}^{X} h^2 \geq 2 \sqrt{\Omega_{3/2}^{\text{TH}} \Omega_{3/2}^{X} h^2} \simeq 8.6 \left( \frac{T_{\text{RH}}^{\text{TH}}}{10^8\text{GeV}} \right)^{1/2} \left( \frac{M_X}{10^{10}\text{GeV}} \right)^{1/4} d_{3/2} \left( \frac{\Omega_{dm} h^2}{0.134} \right)^{1/2},
\]  

(35)

we obtain the upper bound on the moduli mass:

\[
M_X < 6 \times 10^4\text{GeV} \left( \frac{T_{\text{gw}}^{\text{RH}}}{10^7\text{GeV}} \right)^{-2} \left( \frac{\Omega_{dm} h^2}{0.134} \right)^4 d_{3/2}^{-4}.
\]  

(36)

\(^{6}\) Here we have neglected the effect of dilution by possible entropy productions by moduli decay. The effect would decrease the thermal yield by the dilution factor $F^{-1}$ and would weaken the bound on $M_X$ by $F^2 \^{[12]}$. 

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V. SUMMARY

The direct detection of the inflationary gravitational wave background by the next generation space-based gravitational wave missions may enable us to explore the early Universe more deeply than current observations. In this paper, the detectability of the inflationary gravitational wave background in the future experiments, DECIGO and BBO, has been estimated using our precise predictions for the spectrum. We have considered several inflation models and have taken into account the effect of the reheating temperature which determines the frequency where the signature of reheating arises on the gravitational wave spectrum.

We have found that DECIGO could detect the inflationary gravitational background with \( \text{SNR} \geq 3 \), for the chaotic inflation model with \( T_{\text{RH}} \gtrsim 10^7 \text{GeV} \). The higher sensitivity of BBO makes possible a detection with \( \text{SNR} \geq 5 \), for the chaotic inflation model with \( T_{\text{RH}} \gtrsim 2 \times 10^6 \text{GeV} \) or the natural inflation model with \( f \gtrsim m_{\text{Pl}} \) and \( T_{\text{RH}} \gtrsim 10^7 \text{GeV} \). This means, conversely, that non detection of the inflationary gravitational wave background by DECIGO would exclude the chaotic inflation model unless the reheating temperature is lower than \( 10^7 \text{GeV} \). BBO would exclude it unless \( T_{\text{RH}} \lesssim 2 \times 10^6 \text{GeV} \) and further exclude the natural inflation model with \( f \gtrsim m_{\text{Pl}} \) unless \( T_{\text{RH}} \lesssim 10^7 \text{GeV} \).

We have also discussed the implications of the possible lower bound of the reheating temperature on gravitino cosmology in the context of chaotic/natural inflation. Taking into account of both thermal and nonthermal production of gravitinos, we find that from the lower bound on the reheating temperature we could obtain a lower bound on the gravitino mass and an upper bound on the moduli mass. These bounds may provide information regarding the gravitino mass and the moduli mass complementary to collider experiments. Thus, in future, the direct detection of the inflationary gravitational wave background could play a crucial role in probing not only the history of the early Universe but also particle physics.

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