An experimental study on distributed damage detection algorithms for structural health monitoring

Madhuka Jayawardhana, Xinqun Zhu and Ranjith Liyanapathirana
School of Engineering, University of Western Sydney, Penrith South DC, NSW 1797, Australia
E-mail: m.jayawardhana@uws.edu.au, xinqun.zhu@uws.edu.au, ranjith@ieee.org

Abstract. Distributed structural damage detection has become the subject of many recent studies in Structural Health Monitoring (SHM). Development of smart sensor nodes has facilitated the growth of this concept enabling decentralized data processing capabilities of nodes whose sole responsibility once was acquisition of data. An experimental study has been carried out on a two span reinforced concrete slab in this paper. Different crack damages are created by the static loads and the impact tests that are carried out on the slab. Two damage detection and localization methods, one based on Auto Correlation Function-Cross Correlation Function (ACF-CCF) and the other on Auto Regressive (AR) time series model are used to detect damage from measured responses. The results from the two methods are compared in order to determine which method has been more effective and reliable in determining the damage to the concrete structure.

1. Introduction
With the advancements of wireless communication technologies and smart devices, structural health monitoring is no longer a concept limited only to theory and research. Wireless Sensor Networks (WSNs) have made the deployment of SHM systems practically realisable and manageable. These actual applications have brought the attention of engineers and researchers to many new areas and issues related to SHM. Distributed structural damage detection is one such new area with high potential for development. In the light of development of sensor nodes as intelligent devices, distributed computing strategy for structural damage detection has proved to significantly reduce the power consumption of the system while giving more robust and accurate results with increased efficiency.

Several Time-Series based methods are available in the literature for SHM. Many of these are primarily based on the Auto Regressive (AR) model. AR (Sohn et al., 2000; Fugate et al., 2001), AR-ARX (Auto Regressive with exogenous input) (Sohn and Furrer, 2001; Lynch et al., 2003; Lynch et al., 2004; Lei et al., 2003) and ARMA (Auto Regressive Moving Average) (Nair et al., 2005) are some. The main concept behind these methods is that if the structure is damaged, the prediction model developed from the undamaged response data time series will not be able to reproduce the new data series obtained from the unknown state of the structure. Time-series computations can be time consuming and complex in implementation.

The Damage Location Assurance Criterion (DLAC) (Messina et al., 1998; Clayton et al., 2005) is a correlation based method which uses modal frequencies. It correlates modal frequency change caused by possible structural damages between the actual structural response data and those synthesized from an analytical model of the structure to identify and localize damage.
Although it is energy efficient, relying on structural model is an undesirable property in this method as accurate models of actual complex structures can be extremely difficult to produce. In this study a correlation based method and a statistical time-series method have been tested on actual structural response data obtained by a series of tests carried out on a two-span reinforced slab structure. The correlation based approach, called ACF-CCF method (Liu et al., 2009) is a recently developed method. The AR-ARX time-series approach (Sohn and Farrar, 2001; Lynch et al., 2003; Lynch et al., 2004; Lei et al., 2003) has been used for detecting damage and localizing for some time now. In the remainder of this paper we will first be discussing the implementation of the two algorithms in detail and the experimental set up used to obtain the structural data. We will then present the damage detection results obtained through the implementation of both algorithms and compare their accuracy.

2. Damage detection algorithms
2.1. ACF-CCF method
This damage detection and identification method uses the autocorrelation function (ACF) of the measured signal of each node to detect damage and the cross-correlation function (CCF) of node pairs to locate damage. It is based on the premise that if the structure is damaged, there would be a difference in the ACF/CCF coefficients of reference and damaged structures. The choice of ACF/CCF has been due to its sensitivity to damage and its robustness to input and environmental changes. The functionality of this strategy has two levels. In the first level the damage detection takes place. This is carried out in each sensor node independently. The second level is carried out only if damage is detected. In this level the sensor nodes work in pairs to perform damage localization. To determine if the ACF/CCF coefficients calculated from new data are statistically different from reference ACF/CCF data, the X-bar control chart method (Montgomery, 1996) is used in this algorithm. The implementation of the ACF-CCF method is done in two stages; The Offline (Initialization) stage and the Online stage.

Offline stage: This stage initializes the algorithm by setting up references to which the online data can be compared with. The data for this stage has to be obtained when the structure is in a healthy state.

(i) Data collected from the healthy structure subjected to various environmental conditions is standardized as \( \bar{x}_i = \frac{x_i - \mu_{x_i}}{\sigma_{x_i}} \) where \( \mu_{x_i} \) and \( \sigma_{x_i} \) are the mean and the standard deviation of the \( i^{th} \) sample of reference data, respectively. Hereafter \( \bar{x}_i \) will be represented by \( x_i \) for convenience.

(ii) Reference ACF is calculated with standardized data:

\[ ACF_x^i(m) = \sum_{n=1}^{M-m} x_i(n) * x_i(n + m) \]  

where, \( M \) is the length of \( \{x_i\} \) and \( m = 0, 1, ..., M - 1 \).

(iii) Novelty Index \( (NI^x) \) is calculated using \( ACF_x^i \) and its mean \( \bar{ACF}^x \):

\[ NI^x_i = 1 - \text{corrcoef}(ACF_x^i, \bar{ACF}^x) \]  

where, \( \text{corrcoef}(x, y) = \frac{\sum_{k=1}^{M}(x(k) - \mu_x)(y(k) - \mu_y)}{\|x - \mu_x\| \|y - \mu_y\|} \)

(iv) Upper-Control-Limit (UCL\( ^x \)) and Lower-Control-Limit (LCL\( ^x \))are calculated:

\[ UCL^x = \mu_{NI^x} + \gamma \sigma_{NI^x} \quad \& \quad LCL^x = \mu_{NI^x} - \gamma \sigma_{NI^x} \]  

where, \( \mu_{NI^x}, \sigma_{NI^x} \) are the mean and standard deviation of \( NI^x \), and \( \gamma = 3 \) is chosen corresponding to an interval of 99.7% confidence in a normal distribution.
X-bar control chart (Montgomery, 1996) of Novelty Index with $UCL_x$ and $LCL_x$ is drawn. The above reference database is established for each sensor node initialization. This can be calculated offline and stored in each node.

Similarly, reference CCF database is calculated for each node pair, but using the two sets of measured reference data from the two nodes in the pair. The two nodes in each pair are designated, one as master node and the other as slave node.

Online stage: In this stage the current data ($Y$) of the structures that are in use are measured and the Novelty Index ($NI^y$) is calculated similar to the reference stage but with current data. Using $UCL_x$ and $LCL_x$ it is determined if the $ACF^y$ is different from $ACF^x$. That is if $NI^y$ exceeds $UCL_x$ it is called an outlier and damage is detected. To decrease the rate of false alarms, it is defined that unless three consecutive data sets indicate damage as per above, damage is not detected.

In the case of damage, the $CCF^y$ is calculated between the node pair on which the damage indication is given. The master node of the pair is in charge of this calculation. Synchronization between the nodes of a pair has to be carried out prior to calculating the $CCF^y$. The statistical difference between the $CCF^x$ and $CCF^y$ is determined similarly as the ACF comparison. With these results the master node is able to determine in which area the damage has occurred. (Liu et al., 2009)

### 2.2. AR-ARX Method

Auto Regressive-Auto-Regressive with exogenous input (AR-ARX) is a statistical time series approach for detection and localization of damage. It is based on the premise that the statistical prediction model developed from the time series measurement data of the undamaged (reference) data would not be able to reproduce or predict the newly obtained time series, if the current structure is damaged. The prediction model used here is the Auto Regressive (AR) model. In the second part of the algorithm the same model is used with an exogenous input for reasons that will be discussed subsequently. The algorithm is described as follows;

(i) The collected undamaged structural response data is normalized at each node. Assuming that this response is stationary, an AR model is fitted to these data:

$$x_k = \sum_{i=1}^{p} \phi_i^x x_{k-i} + e_k^x$$

where, $p$ is the order of the model and $x_{k-i}$ stands for the $p$ previous responses. $\phi_i^x$ are the AR coefficients of the previous responses and $e_k^x$ is the residual error, which is the Damage Sensitive Feature (DSF) in this case.

The residual error of an AR model is influenced by the operational variability of the structure causing inaccuracies in damage detection. Therefore in order to avoid the effect of operational variability and to obtain the residual error resulting from the structural damage, an AR model with exogenous input (ARX) is introduced.

(ii) In ARX, the relationship between the measured response $x_k$ and the AR model residual error $e_k^x$ is computed:

$$x_k = \sum_{i=1}^{a} \alpha_i x_{k-i} + \sum_{j=1}^{b} \beta_j e_{k-j} + e_k^x$$

where, $a$ and $b$ are model orders and $e_k^x$ is the residual error after fitting the ARX($a,b$) which is the new DSF unaffected by operational state. $\alpha_i$ and $\beta_j$ are the coefficients of past measurements and the residual error of past measurements, respectively. The AR and ARX model orders, $p, a$ and $b$ are determined by exploring the autocorrelation function of the model residual errors (Sohn et al. 2001, Lynch et al. 2003).
(iii) The structural response data in the unknown state is collected, normalized and fit to an AR model with order \( p \) similarly as above:

\[
y_k = \sum_{i=1}^{p} \phi_i^y y_{k-i} + e^y_k
\]

(iv) The signal segment \( x_k \) from the reference database which is closest to the new signal \( y_k \) is chosen by minimizing the following difference of AR coefficients:

\[
\text{Difference} = \sum_{i=1}^{p} (\phi_i^x - \phi_i^y)^2
\]

This is performed in order to select the reference signal which is recorded under operational conditions closest to the newly obtained signal. If there is no damage to the structure and the operational conditions are close, the selected reference AR model will closely approximate the measured signal. If there is damage, even the closest AR model of the database will not approximate the measured response.

(v) Equation (5) is used to determine the residual error \( \epsilon^y_k \) of the ARX model of the new response \( y_k \), by substituting \( y_k \) and the corresponding residual error \( e^y_k \) as follows:

\[
\epsilon^y_k = y_k - \sum_{i=1}^{a} \alpha_i y_{k-i} - \sum_{j=1}^{b} \beta_j e^y_{k-j}
\]

(vi) The ratio of standard deviations of the residual errors of undamaged and unknown state of the structure is defined as the DSF. This ratio is monitored for structural anomalies.

\[
DSF = \frac{\sigma(\epsilon^y)}{\sigma(\epsilon^x)}
\]

Another technique of detecting damage with the AR-ARX method is by testing the null hypothesis \( H_0 : \sigma^2(\epsilon_x) = \sigma^2(\epsilon_y) \) against the one sided alternative \( H_1 : \sigma^2(\epsilon_x) < \sigma^2(\epsilon_y) \) of the variance ratio \( \sigma^2(\epsilon^y)/\sigma^2(\epsilon^x) \) which follows the F-distribution (Sohn and Farrar 2001).

\[
F = \frac{\sigma^2(\epsilon^y)}{\sigma^2(\epsilon^x)}
\]

The Degree of Freedom (DOF) of this F-distribution are \( n_x - 1 \) and \( n_y - 1 \) where \( n_x \) and \( n_y \) are the numbers of samples of \( \epsilon_x \) and \( \epsilon_y \), respectively. The null hypothesis \( H_0 \) is rejected when the F-statistic in equation (14) exceed the upper \( 100 * \alpha \) percentile of the F-distribution.

In standard deviation ratio DSF, the ratio value reaches a maximum near the actual damage localization. In the F-statistic technique, the number of rejections of the null hypothesis is at a maximum near the damage location. (Sohn and Farrar, 2001; Lynch et al., 2003; Lynch et al., 2004; Lei et al., 2003)

3. Experimental set-up
These tests have been conducted on a two-span reinforced concrete slab of dimensions 6400 mm 800 mm 100 mm. The spans are 3000 mm with a 200 mm overhang at each end (Figure 1). It was supported by wooden planks placed over three steel UB sections.

In the experiment the slab has been continuously loaded with an incremental load with the goal of creating crack damage. A four-point loading was used at the middle of each span as
shown in Figure 1. The loading system is also connected to the slab supports in order to reduce the effect of the system on the supports. Twelve loading levels were performed on the structure while increasing the maximum loading level. Table 1 gives the static loads on the two spans recorded using two load cells while measuring the displacements and monitoring crack locations and lengths. The deflection under the static load was measured by four displacement transducers located at the middle of each span. The dynamic loading test was conducted using a 5.4 kg impact hammer and three sets of measurements with the nine accelerometers evenly distributed along the slab in each set were obtained as dynamic responses. The sensor and impact locations are shown in Figure 2. Data of length 4096 has been acquired at a sampling rate of 500 Hz from all the channels including the impulse load, using a data acquisition system.

Table 1. Loading stages and damage scenarios.

| Loading stage | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 |
|---------------|---|---|---|---|---|---|---|---|---|----|----|----|----|
| P1(kN)        | 0 | 3 | 6 | 12| 18| 18| 18| 18| 25| 32 | 35 | 38 |
| P2(kN)        | 0 | 0 | 0 | 0 | 0 | 3 | 6 | 12| 18| 25 | 25 | 35 | 38 |
| Damage scenario | No damage zone | One damage zone | Two damage zone | Three damage zone |
4. Results and discussion
The ACF-CCF algorithm and the AR-ARX algorithm as described in section 2 were implemented in Matlab. Actual structural response data obtained from the above set of tests was used as input. For convenience of illustration, we present here the output resulting from only one set of sensors from the above experimental set-up. We have chosen the sensor set 2, which consists of responses from the nine sensors located in the center row of the structure (Figure 2). For each case, the test is repeated six times resulting in six data sample sets.

4.1. ACF-CCF
Figure 3 shows the plots of NIs of each sensor in set 2, the undamaged structural data (reference data) given in a continuous line and the unknown state of the structure—given in a dashed line. The two horizontal dashed lines are the Upper Control Limit (UCL) and Lower Control Limit (LCL) computed from the reference NI data. This plot is obtained from data of loading stage 3 (Table 1) which is from the One damage zone. In Figure 3, the NI values of Sensors 7 and 8 are above the UCL which indicates potential damage in the structure. Sensors being numbered from right to left along the length of the slab, Sensors 7 and 8 are located in the middle area of the left span. From Figure 4, which shows the experimental records of crack patterns in the structure, it is apparent that the first damage zone is in the middle of left span which carried the only load at this stage.

Figure 5 shows the localization results of the above damage scenario. Although damage localization is performed only in the sensor pairs where at least one sensor has detected damage, we have displayed the data of all the sensor pairs for comparison purposes. Since the sensor set is short of one to pair-off perfectly, the eighth sensor is paired with both seventh and ninth sensors separately, making 5 pairs of sensors. Similar to Figure 3, the NIs of undamaged state of all the sensor pairs in Figure 5 are quite similar to that of the reference state except for sensors 7 and 8. Therefore we can conclude that the damage location is between sensors 7 and 8.

The same applies to Figure 6, where damage is detected in sensors 2, 6, 7 and 8. This plot
resulted from loading stage 8 of Set 2 sensors. Comparing with experimental records of Figure 4 it can be explained that since stage 8 is in the Three damage zone, damage occurs in the middle of both left and right spans as well as at the mid support of slab. Damage indicated sensors in the simulated results are in these areas of damage showing that the algorithm was able to
identify the three damages of the structure.

The next online stage of the algorithm for loading stage 8 gives the damage localization plot given in Figure 7. This plot shows that the online NIs of sensor pairs 1 and 2, 3 and 4, 7 and 8, 9 and 8 are above the UCL. We can conclude from these results that damages exist between sensors 1 and 2, 3 and 4, 7, 8 and 9. According to the experimental records in Figure 4 we can verify that the results obtained above are quite accurate. ACF-CCF results will be compared with AR-ARX results in section 4.3.

4.2. AR-ARX

Tables 2 and 3 show the damage detection and localization results of the AR-ARX method. For illustration purposes only the results of 4 loading stages are presented. The loading stages are chosen from each of the three damage zones and the undamaged case.

The results in Table 2 do not show significant increases of the DSF except in the case of Three damage zone. That is, according to experimental records as per Figure 4, the structure is damaged from One damage zone through to Three damage zone near sensors 7, 8 and 9. But in AR-ARX results of Table 2, no significant increase of the DSF can be noticed in either of those 3 sensors in One and Two damage zones. In Three damage zone, a noticeable increase has occurred. In fact, in the Three damage zone significant increases of DSF can be seen in sensors 1, 3, 5, 6, 8 and 9, which correspond to the experimental records of three damage locations. But such an increase does not appear in other sensors.

In Table 3, null hypothesis rejections out of the hypothesis tests performed illustrated. Since six tests were performed in each loading case, the number of hypothesis tests performed here is also six. Therefore in order to reject the null hypothesis in the final result, at least three tests out of six has to be rejected. In Table 3, the Three damage zone gives successful results rejecting null hypothesis in sensors 1, 5, 6, 8 and 9 which matches with the experimental records giving damage indication and location. In Two damage zone of the table, sensor 4 indicates

Figure 6. Damage detection of Set 2 sensors, loading stage 8.
damage matching the experimental records of middle damage area of the RC structure, but fails to indicate the damage in sensors 7, 8 or 9. There is a false indication of potential damage in sensor 6 in One damage done. From the results of Tables 2 and 3, we can conclude that the second DSF - the F-statistic has been more successful in detecting damage than the standard deviation ratio used in our study.

4.3. **ACF-CCF vs AR-ARX**

In our implementation of the two methods with the use of experimental data from the RC structure, the ACF-CCF method was successful in identifying and locating damage. Identifying and localizing of one damage and three damages were illustrated in the previous sections. Even in the Three damage zone case, ACF-CCF was able to distinctly identify and localize damage. But in the AR-ARX implementation, the results showed only some damage occurrences. In one occasion a false damage indication was given. These shortcomings of the AR-ARX method could be a result of the low number of samples that we have used in this implementation, as the availability of samples in each test case was limited to six. AR-ARX method has been used

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**Figure 7.** Damage localization of Set 2 sensor pairs, loading stage 8.

**Table 2.** Ratio of standard deviations ($\sigma(\epsilon^y)/\sigma(\epsilon^x)$).

| Damage zones   | S1  | S2  | S3  | S4  | S5  | S6  | S7  | S8  | S9  |
|----------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| None           | 0.9952 | 1.1143 | 1.1025 | 1.0859 | 1.0122 | 1.0473 | 1.0605 | 1.0616 | 1.0667 |
| One damage     | 0.7984 | 1.0217 | 0.9136 | 1.0803 | 1.071 | 1.2543 | 0.9216 | 0.8936 | 0.8956 |
| Two damages    | 0.814 | 0.9914 | 0.9573 | 1.174 | 0.9159 | 1.1211 | 0.8778 | 0.8697 | 0.9059 |
| Three damages  | 1.4728 | 1.026 | 1.234 | 0.7779 | 1.3457 | 1.4011 | 1.1664 | 1.2676 | 1.4437 |
Table 3. Null hypothesis test: \( H_0 : \sigma^2(\epsilon_x) = \sigma^2(\epsilon_y) \) against \( H_1 : \sigma^2(\epsilon_x) < \sigma^2(\epsilon_y) \)

| Damage zones     | S1 | S2 | S3 | S4 | S5 | S6 | S7 | S8 | S9 |
|------------------|----|----|----|----|----|----|----|----|----|
| None             | 0  | 0  | 0  | 1  | 0  | 0  | 0  | 0  | 0  |
| One damage       | 0  | 0  | 0  | 1  | 0  | 3  | 0  | 0  | 0  |
| Two damages      | 0  | 0  | 0  | 3  | 0  | 2  | 0  | 0  | 0  |
| Three damages    | 6  | 0  | 2  | 0  | 5  | 5  | 0  | 4  | 6  |

successfully in literature to detect and locate damage. Nevertheless this proves a limitation of the AR-ARX method as it requires more data to detect and locate damage as opposed to the ACF-CCF method.

The descriptions of the two damage detection algorithms differentiate the two in numerous ways. Firstly, in the AR-ARX method each sensor node gathers and processes its sensor data independently without sharing with the neighbouring nodes, whereas in the ACF-CCF method the second half of the method communicates within node pairs. The ability of AR-ARX to provide accurate damage location is limited because of its inability to incorporate the available spatial information. Even though this method does not share sensor information between neighbours, much energy is spent on the transmission of AR coefficients to the base station in order to retrieve the corresponding ARX coefficients. However, the inter-nodal communication in ACF-CCF can be justified because it occurs only after a damage has been detected in the structure. The computation of the AR and ARX models in AR-ARX method is quite complex compared with the ACF and CCF functions of the ACF-CCF method, and also it is time consuming. This was observed during the implementation of the algorithms where the execution of the AR-ARX method in the matlab code took more than four times the time taken by ACF-CCF. Both algorithms do not rely on the structural model, which is a desirable feature and both use time series sensor data directly to compute the DSF of the method.

5. Conclusion

In this paper a comparison between a correlation based distributed damage detection method and a statistical damage detection method based on time series models has been presented. Measurement data from an experimental study carried out on a two-span concrete slab has been used to verify these algorithms. The results show that in this study the ACF-CCF method proves to be a better damage detection and localization method than the AR-ARX method. The NI value of the ACF-CCF method could be a good indicator of the damage in concrete slab structures making this method applicable and effective in wireless sensor network based structural health monitoring. Further study is needed to test the applicability of this algorithm in various structures and to develop the embedded algorithm for wireless sensor units.

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