1. Introduction

As an important part of CNC machine, cutting tools will be aging in the long-term operation. The reliability of cutting tools influences the whole manufacturing effectiveness and stability of equipment. The malfunction of machine tools may result in the halt of the whole production and bring about tremendous financial losses. For example, in the case of complex installations such as automobile assembly lines, it can be as high as $20,000 per minute. With an accurate estimate of tool lifetime, worn tools can be changed in time to reduce waste product and tools costs noticeably. It is even possible to guarantee a certain surface quality [1,2]. Effective and reliable condition monitoring methods and procedures are seen as a means to achieve high availability and reduce unscheduled production shutdowns [3].

Tool failure and lifetime is judged by its wear measurement described in several standards (ISO3685, ISO8688 and ANSI/ASME B94.55M) [4]. Due to the constraints of high cost, discontinuity and susceptibility to operational environment, the direct methods are restricted to a very narrow range of applications and need to be improved. In indirect methods, the wear state can be estimated by the cutting force, torque, temperature, acoustic emission and vibration [5,6]. For individual condition, tool wear is a gradual process without emergencies and the wear state can be estimated by the cutting force, torque, temperature, acoustic emission and vibration. We can identify the working condition and predict the future trend of the tool through this kind of signals, which are known as the data-driven approaches. These approaches attempt to derive models directly from collected history data or performance data instead of building models based on failure mechanisms [7].

The traditional reliability theory emphasizes that the system performance is either satisfactory or unsatisfactory randomly. Randomness is just one respect of the uncertainty and the system state transition process is continuous, which means there is neither totally success state nor failure state, but a state between success and failure. Thus the fuzziness which is another respect of the uncertainty can’t be ignored. It is well known that the binary state model for the reliability of components or the system is too simplistic and it’s difficult to
propose a precise criterion. As the single evaluation threshold is not easy to determine, the paper puts forward the concept of fuzzy threshold to solve this problem. A perspective methodology is proposed to model the behavior of the system/component using the theory and methods of fuzzy sets.

The rest of the paper is organized as follows: the methodology of state space model and fuzzy reliability modeling are described in section 2 and section 3 respectively. Section 4 provides a case study of the approach including the experiment content and modeling. The fuzzy reliability is calculated based on a pre-set fuzzy threshold, and the optimal replacement time of tools is analyzed based on the relevant decision model. Application results are also discussed in this section. Finally, conclusions are drawn in section 5.

2. The state space model

The state space model is a time domain method and it adopts the concept of state variable. A state space model is composed of an observation equation and a transition equation. The observation equation describes the change rule of dynamic system state from a previous moment to the current and the transition equation describes the relationship between the observed value and the state of the system [8].

A number of facts show that a non-stationary time series can be decomposed into trend component, stochastic perturbation component, seasonal component, and irregular component, that is:

\[ Y(t) = T(t) + C(t) + S(t) + I(t) \]  

(1)

The system’s deterioration, represented by the trend of the deterioration measure, is the component featured by low frequency. In this research, an integrated random walk (IRW) model is adopted to model the low frequency trend, which is represented by the following state-space model [9]:

\[ x(t+1) = Fx(t) + G\eta(t) \]

\[ y(t+1) = Hx(t+1) \]  

(2)

Where \( x(t) \) is a state vector that represents the deterioration state \( x(t) = [u(t) \ \beta(t)]^T \), which includes two terms: level \( u(t) \) (the current deterioration level) and slope \( \beta(t) \) (current deterioration rate). \( \eta(t) \) is an white noise vector, \( H \) is an observation coefficient matrix, \( F \) is a transition matrix, and \( G \) is an input matrix. \( F, G, \) and \( H \) take the following matrix elements for the IRW model:

\[ F = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, \ G = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \ H = [1,0]. \]

To obtain the trend component from a structural time series, a double interval-smoothing algorithm is employed: forward fixed interval smoothing and a backward smoothing algorithm. The degradation state of a system can be evaluated by one or more parameters which are related to the degradation process. The measured value of future time \( y(t+l) \) is a random variable and can be regarded as the mean of the measured parameter. \( f(y(t+l) | t) \) is the probability density function of the measured value. The mean and variance can be obtained from (4), (5) and (6).

\[ x(t+1 \Delta t | t) = F^l x(t | t), \ l = 1,2,3, \ldots, n \]  

(3)

\[ y(t+1 \Delta t | t) = H x(t+1 \Delta t | t) \]  

(4)

\[ P(t+1 \Delta t | t) = F^l P(t+1 \Delta t | t) F^T + \sum_{j=0}^{n-l-1} P_j F^j Q F^T - \]  

(5)

\[ \text{Var}(y(t+1 \Delta t | t)) = \sigma^2 [1 + HP(t+1 \Delta t | t) H^T] \]  

(6)

Where \( P \) is a covariance matrix of the prediction error, and \( Q \) is a covariance noise variance ratio (NVR) matrix.

3. Fuzzy reliability modeling

3.1 Motivation for fuzzy reliability model

According to classic reliability modeling, the states of component/system are assumed to be binary. The binary state assumption implies that the success and failure of a component can be precisely determined with respect to a threshold value. The binary state model for the reliability of components or the system is too simplistic and does not capture the reality for most systems which can have many levels of performance.

According to the fuzzy set theory, the component's success and failure are treated as fuzzy events. For any given value of the substitute characteristic \( y \), the component exhibits a certain degree of success \( \mu_y(y) \) as shown in Figure 1. Since it is difficult or unrealistic to use a single threshold value to divide success from failure, fuzzy set theory is applied to deal with the fuzzy nature of the success and failure definition [10].

\[ \text{Fig. 1. The fuzzy reliability model} \]

The success and failure are treated as fuzzy events, which contains many elements or substitute characteristic values \( y \) exhibiting different degrees of success or failure. As a natural extension of the traditional binary state reliability evaluation, the fuzzy reliability of a component can be evaluated as the probability of the fuzzy event of success, which is uniquely determined by its membership function. Thus we define fuzzy reliability as:

\[ R = P[\text{success}] = \int \mu_y(y) dF(y) = E[\mu_y(y)] \]  

(7)
Where $F(y)$ is the cumulative distribution function of the substitute characteristic variable $y$.

### 3.2 Fuzzy reliability evaluation

It's difficult to choose a single threshold value as the reliability evaluation criterion, so a fuzzy set of discourse domain of the measured value are employed to represent failure state in this paper. At the beginning of the degradation, the membership function value is 1. At a moment (suppose the measured degradation value is $y_{pca}$), the membership value begins to decrease as the degradation process goes on, at last the membership function value comes to zero (suppose the measured degradation value is $y_{pca}$ at this moment). Then a fuzzy zone forms between the fuzzy distribution and degradation curve and locates between $y_{pca}$ and $y_{pca}$, as shown in Figure 2. For a given membership function $u_g(y)$, the failure probability of the degradation process can be expressed by the conditional probability, and the probability value calculated by the definition of the probability of fuzzy random events equals the degree of reliability when combining the probability density function with the membership function $[11, 12]$. The degree of reliability at time $t + l \Delta t$ is

$$R(t + l \cdot \Delta t | t) = \int u_g(y) f(y(t + l \cdot \Delta t | t))dy$$

(8)

Where $R(t + l \cdot \Delta t | t)$ is the fuzzy reliability at future time $t + l \cdot \Delta t$.

The failure probability at time $t + l \Delta t$ is

$$F(t + l \cdot \Delta t | t) = 1 - R(t + l \cdot \Delta t | t) = 1 - \int u_g(y) f(y(t + l \cdot \Delta t | t))dy$$

(9)

Where $F(t + l \cdot \Delta t | t)$ is the fuzzy failure probability at future time $t + l \cdot \Delta t$.

The degree of reliability of each interval $\Delta t$ can be calculated by the parameter $y(t + l \cdot \Delta t)$ and $u_g(y)$ when the time interval $l \cdot \Delta t$ is fixed, and the degree of reliability of each interval $\Delta t$ forms a time sequence, and the conditional probability at time $t + l \Delta t$ is:

$$R(t + l \cdot \Delta t | t) = \prod_{i=l}^{t} R(i | survival to t)$$

(10)

### 4. Case study

A test is carried out in the OKUMA vertical three axis milling machine at the molding tool graduate school of Dalian University of Technology. The test tool is 7792VXD cow nose cutter, and the diameter, overhanging length and blade number is 32 mm, 200 mm and 3 respectively. Spindle speed, cutting depth and feeding speed is 400 mm/min, 0.4 mm and 1000 r/min, respectively. The acoustic emission signal and the force signal are measured at the same time. The acoustic emission (AE) signal is collected every 10 seconds. The sampling frequency is 2048 kHz and the sampling length for the data set is 512000.

Both amplitude and distribution of the acoustic emission signals change along with tools’ state from sharp to worn. Some features are salient and closely related to the wear, but others are not. The acoustic emission signals are decomposed into 64 bands by wavelet packet (WP) transform, and the WP energy of different sampling time is calculated and normalized. The maximum WP energy is used to estimate the tool wear and its change is shown in Figure 4 with a non-monotonically increasing trend.
Fig. 4 Normalized WP energy change of the maximum energy

Figure 5 presents the multi-step ahead forecast results at $t = 1650s$. The deterioration trend and two 95% confidence bands are exposed. The output of 30-step ahead forecast at $t=1650s$ is shown in Figure 6.

Assuming the variability of domain $y$ obeys a normal distribution, and then the probability density function of the normal distribution is:

$$f(y) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(y-\mu)^2}{2\sigma^2}}$$  \hspace{1cm} (11)

Where $\mu$ and $\sigma^2$ are the mean and the variance of the normal distribution.

When the membership function of fuzzy event obeys a descending half-normal distribution, the membership function of the descending half-normal distribution, the fuzzy reliability:

$$R = \int_{a}^{\infty} u_y(y)f(y)dy = \Phi\left(\frac{d_2-u}{\sigma}\right) - \frac{1}{\sqrt{2\pi}} e^{-\frac{(d_2-u)^2}{2\sigma^2}}$$

\hspace{1cm} (12)

Where,

$$u_y(y) = \begin{cases} 1, & \text{if } y \leq a \\ \frac{\exp[-k(y-a)^2]}{\sigma\sqrt{2k\sigma^2+1}}, & \text{if } y > a \end{cases}$$

$$b_1 = \frac{a-u}{\sigma\sqrt{2k\sigma^2+1}}$$

$$b_2 = 2k\sigma^2 + \frac{\mu^2}{\sigma^2}(2k\sigma^2+1)$$

When the membership function of fuzzy event obeys a descending half trapezoidal distribution, the membership function of the descending half trapezoidal distribution, the fuzzy reliability:

$$R = \int_{a}^{\infty} u_y(y)f(y)dy = \Phi\left(\frac{d_2-u}{\sigma}\right) - \frac{1}{\sqrt{2\pi}} e^{-\frac{(d_2-u)^2}{2\sigma^2}} \left(\frac{\left|\alpha_2-y\right|}{\alpha_2-a_2} - a \right)$$

\hspace{1cm} (13)

Where,

$$u_y(y) = \begin{cases} 1, & \text{if } y \leq a_1 \\ \frac{a_2-a}{a_2-a_1}, & \text{if } a_1 \leq y \leq a_2 \\ 0, & \text{if } y > a \end{cases}$$

When the membership function of fuzzy event obeys a descending half rectangular distribution, the membership function of the descending half rectangular distribution, the fuzzy reliability:

$$R = \int_{a}^{\infty} u_y(y)f(y)dy = \Phi\left(\frac{d_2-u}{\sigma}\right)$$

\hspace{1cm} (14)

Where,

$$u_y(x) = \begin{cases} 1, & \text{if } y \leq a \\ 0, & \text{if } y > a \end{cases}$$
In formula (11)-(14), $\Phi$ denotes the cumulative distribution function of the standard normal distribution and $\mu$ and $\sigma^2$ are the mean and the variance. Parameter $a$, $a_1$, $a_2$, and $k$ are the constant of the membership functions, while $b_1$ and $b_2$ are substitute variables.

When the membership function obeys the descending half-normal distribution ($a = 0.32, k = 50$), the descending half trapezoidal distribution ($a = 0.32, a_0 = 0.6$), the descending half rectangular distribution ($a = 0.4$) respectively, the corresponding membership function are shown in Figure 7.

When the membership function obeys the descending half rectangular distribution ($a = 0.4$), the threshold value is 0.4.

The conditional reliability curves which obey three membership function distribution are shown in Figure 8, as we can see from the figures, the number of prediction steps increase while the reliability decreases gradually, but the reliability degradation processes of the three membership function are different.

3.4 Decision models

Preventive maintenance means to use the method of the statistical methods, condition monitoring and other means to determine the appropriate maintenance programs to reduce the harm caused by unexpected failures before the system fail. Condition based on maintenance (CBM) is a kind of preventive maintenance by monitoring to control system status and to identify problems and take appropriate measures before some failures occur.

By the means of CBM, we can prevent the incidence of serious failure, reduce the failure rate, save the costs of maintenance, narrow the scope of maintenance and reduce the workload of maintenance, improve the utility rate of system. According to the reliability calculated, we can decide whether to replace the tool based on the following decision model[7].

$$\text{cost per unit time} = \frac{\text{expected cost over the current lifecycle}}{\text{expected current lifecycle}}$$

$$\text{expected cost over the current lifecycle} = c_r (1 - P(t+l)) + c_r P(t+l) \quad (16)$$

$$P(t+l) = \sum_{i=1}^{l} (1 - P(t+i)) P(t+i) + \sum_{i=1}^{l} (1 - P(t+i)) P(t+i) \quad (17)$$

$$\text{expected current lifecycle}$$
$$= \sum_{i=1}^{l} (1 - P(t+i)) P(t+i) + \sum_{i=1}^{l} (1 - P(t+i)) P(t+i) \quad (18)$$

Where, $t+l$ denote the preventive replacement time of the inductor given that condition data has been collected up to time $t$. ($a = 0.4$) $c_r$ denote the cost of a preventive replacement, $c_r$ denote the failure replacement cost, $P(t+l)$ denote the probability of the feature signal first exceeding the threshold in the period $(t+i-1, t+i)$, where $(t+i)$ is an update point and $i \geq 1$, $P(t+l)$ denote the probability of the feature signal first exceeding the threshold in the time interval $(t, t+i)$.

Suppose $t = 1650s$, $c_f = 75000$, $c_p = 15000$, as is shown in Figure 9, expected cost per unit time changes under different membership function. Based on all of the history data before $t = 1650s$, the lowest point of the curve is the theoretical optimal replacement time. The model outputs also depend on the value of $c_r$ and the value of $c_p$. With the value of $c_r$ increase or the value of $c_p$ decrease, the optimal replacement time $l$ decrease. Suppose $r = \frac{c_r}{c_p}$, $c_r = 15000$, $t = 1650s$ and the membership function obeys the descending half-normal distribution, the expected cost per unit time with different values of the parameter $r$ are shown in Figure 10. And the figure shows the expected cost increase with the corresponding $r$. When $r = 1$, the cost of replacement equals the cost of failure, and there is no optimal replacement time, and preventive replacement does not make sense at this time.
5. Conclusion

The paper proposed the state space model to estimate the reliability of cutting tools. As the single evaluation threshold isn’t easy to determine, the paper puts forward the concept of fuzzy threshold to solve the problem. The system state is treated as fuzzy event which is uniquely characterized by membership functions defined over the substitute characteristic variable. The wavelet packet (WP) energy extracted from the acoustic emission signal was selected as the feature to estimate the tool state. Multi-step ahead forecasts at different forecast origins are discussed. The deterioration tendency and the corresponding reliability at a specific time were predicted and calculated respectively. The positive results showed that the model was feasible and effective which will have a broad prospect in improving the ability to improve equipment safety, and reduce maintenance.

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