The effect of carrier density gradients on magnetotransport data measured in Hall bar geometry

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We have measured magnetotransport of the two-dimensional electron gas in a Hall bar geometry in the presence of small carrier density gradients. We find that the longitudinal resistances measured at both sides of the Hall bar interchange by reversing the polarity of the magnetic field. We offer a simple explanation for this effect and discuss implications for extracting conductivity flow diagrams of the integer quantum Hall effect.

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I. INTRODUCTION

The primary technique for probing the two-dimensional electron gas in semiconductor heterostructures is magnetotransport. In practice, this is realized by passing a constant current $I$ through a Hall bar, i.e. a rectangular sample with several contact pads for sensing the longitudinal voltage $V_{xx}$ and the transverse or Hall voltage $V_{xy}$. Hall bars are routinely prepared from semiconductor wafers by photo-lithographic techniques. Typically, the channel length is in the order of 2000 $\mu$m, with the distance between the longitudinal voltage contacts $L \sim 500\, \mu$m and the channel width $W$ a factor 2-5 smaller. For a homogeneous sample three voltage contacts are sufficient to determine the longitudinal and transverse resistances, $R_{xx} = V_{xx}/I$ and $R_{xy} = V_{xy}/I$. However, in the presence of macroscopic sample inhomogeneities, i.e. inhomogeneities on a scale much larger than the typical microscopic length scales of the electron gas, the resulting resistances present average values. If the length scale of the inhomogeneities is comparable to the sample size, information about the inhomogeneities can be obtained by placing additional contacts on the sample. An example of a macroscopic inhomogeneity is a spatial variation in the carrier density, such as a small gradient along the channel direction. Such gradients are not uncommon in Hall bars and arise directly from the growth process. Besides, the spatial variation might depend on the cooling procedure.

The influence of macroscopic sample inhomogeneities, geometrical effects and contacts on magnetotransport data taken from Hall bars has been investigated by a number of authors. For example, von Klitzing and Ebert investigated a Hall bar with a fairly large carrier density variation ($\sim 10\%$). They reported differences in the longitudinal resistances measured on both sides of the Hall bar, as well as a strong dependence of the amplitude of the Shubnikov-de Haas oscillations on the magnetic field polarity. Geometrical effects of Hall bars, such as the influence of the channel width, the position of contacts, etc., have been investigated in detail by Haug, who ob-

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{hall_bar_diagram.png}
\caption{Schematic picture of a Hall bar. $L$ denotes the distance between the longitudinal voltage contacts and $W$ the channel width.}
\end{figure}
TABLE I: Transport parameters of Hall bars, prepared from different semiconductor structures, before and after illumination: 

| System          | Sample          | \( L \times W \) (\( \mu \)m)² | \( n_e \) (10¹¹ cm⁻²) | \( \Delta n_e/n_e \) | \( \mu \) (cm²/Vs) |
|-----------------|-----------------|-------------------------------|------------------------|---------------------|-------------------|
| GaAs/AlGaAs QW  | #659 before illum. | 1260 \times 1000              | 4.7                    | 0.017               | 220 000           |
|                 | #659 after illum. | 1260 \times 1000              | 6.1                    | 0.0025              | 300 000           |
| GaInAs/AlGaAs QW| 31232-#3 before illum. | 387 \times 75                | 1.8                    | 0.014               | 19 000            |
|                 | 31232-#3 after illum. | 387 \times 75                | 3.6                    | 0.006               | 25 000            |
|                 | 31232-#2         | 387 \times 75                | 2.2                    | 0.018               | 25 000            |
| GaInAs/InP HS   | #2              | 1070 \times 650              | 2.2                    | 0.016               | 16 000            |

In this paper we present magnetotransport data obtained from various quantum wells and heterostructures. All samples showed the interchange of \( R_{xx}^b \) and \( R_{xx}^l \) upon field reversal (except one heterostructure where contact misalignment was dominant). We offer a simple explanation for this phenomenon, namely a small carrier gradient along the channel direction of the Hall bar. To the best of our knowledge, these specific observations and the corresponding explanation, have not been reported in literature before. Our results are important for the study of conductivity flow diagrams of the quantum Hall effect. They show that complications may arise when critical conductivities are extracted in the standard fashion from experimentally obtained resistivities on the plateau-plateau transitions.

**II. EXPERIMENTAL**

Magnetotransport experiments have been performed on different semiconductor structures: high-mobility (\( \mu \sim 30\,0000 \) cm²/Vs) GaAs/AlGaAs quantum wells and low-mobility (\( \mu \sim 20\,000 \) cm²/Vs) GaInAs/AlInAs quantum wells, as well as InGaAs/InP heterostructures. The electron densities for all our samples were fairly low (\( n_e = 1.8-6.1 \times 10^{11} \) cm⁻²), such that all samples showed distinct quantum Hall features within our available magnetic field range (\( B \leq 8 \) T). Mobilities and electron densities of the samples are listed in Table I. In some cases the electron density was increased by illumination with a LED at \( T = 4.2 \) K. Samples were selected such that no carrier relaxation occurred during the measurements. Throughout this paper we label the voltage contacts on the Hall bar as sketched in Fig. 1, where \( V_{xx} \) refers to contacts 3-4 and \( V_{xx}^{b} \) to 5-6. The experiments were performed in an adsorption-pump operated \(^3\)He cryostat, equipped with a superconducting magnet \( (B_{\text{max}} = 8 \) T). The longitudinal and transverse resistance were measured simultaneously, using standard lock-in techniques at a frequency of 13 Hz. The excitation current ranged from 5 to 50 nA. The data presented here were all taken at \( T = 0.4 \) K.

**III. MAGNETOTRANSPORT RESULTS: ANTISYMMETRY IN \( \mathbf{R}_{xx} \)**

In Fig. 2 we show the Hall resistance between 3 and 8 T, before illumination, of the GaAs/AlGaAs quantum well (sample #659) measured at the left (3-5) and right (4-6) contact pairs across the Hall bar. The left and right Hall resistance traces, \( R_{xy}^l = R_{xy}^{35} = (V_3 - V_5)/I \) and \( R_{xy}^r = R_{xy}^{46} = (V_4 - V_6)/I \), were measured during the same field sweep. Upon reversing the magnetic field, \( R_{xy}^l \) and \( R_{xy}^r \) stay identical, except for the change of sign as it should: \( R_{xy}^{l,r}(B) = -R_{xy}^{l,r}(-B) \). This implies that the contribution of \( R_{xx} \) to the Hall resistance is absent and that misalignment of Hall voltage contacts is negligible. The plateau-plateau (PP) transitions measured...
Fig. 3 shows that $\rho_{xx} = 3$ i quantum Hall regime at higher fields. For a homogeneous resistance peaks associated with the PP transitions in the dinal resistance when measured on both sides of the Hall vice versa to within 1%. We conclude that the longitu-
density difference $\Delta n_e/n_e = \Delta B/B = 0.017$. For simplicity, we will assume, in the remainder of the paper, that the carrier gradient along the channel is constant.

Next we present in Fig. 3 the longitudinal resistances $R_{xx}^i = R_{xx}^{34} = (V_3 - V_4)/I$ and $R_{xx}^b = R_{xx}^{56} = (V_5 - V_6)/I$ of sample #659, measured at top and bottom sides of the Hall bar. Data are taken for field up $(B \uparrow)$ and down $(B \downarrow)$ directions ($|B| \leq 8$ T). The data show the familiar Shubnikov-de Haas oscillations at low fields and the resistance peaks associated with the PP transitions in the quantum Hall regime at higher fields. For a homogeneous Hall bar we expect $R_{xx}^i = R_{xx}^b$, which is clearly not the case here. Instead we find a large difference in the peak values $R_{xx}^i$ and $R_{xx}^b$, which amounts up to 50% for the $i = 3 \rightarrow 2$ transition. Moreover, a close inspection of Fig. 3 shows that $R_{xx}^i$ for $B \uparrow$ equals $R_{xx}^b$ for $B \downarrow$ and vice versa to within 1%. We conclude that the longitudinal resistance when measured on both sides of the Hall bar shows a remarkable “antisymmetry”:

$$R_{xx}(B) = R_{xx}^b(-B).$$

After illumination, the GaAs/AlGaAs quantum well becomes more homogenous as expected. Magnetotransport data near the $i = 6 \rightarrow 5$ PP transition are shown in Fig. 4. The carrier density increases from $4.7 \times 10^{11}$ cm$^{-2}$, to $6.1 \times 10^{11}$ cm$^{-2}$, while the carrier difference decreases to $\Delta n_e/n_e = 0.0025$. The longitudinal resistance still re-
mains antisymmetric, but the effect is now much smaller and amounts to only 20% for the $i = 6 \rightarrow 5$ PP transition.

Magnetotransport data for the GaAs/AlGaAs quantum well are shown in Figs. 5 and 6. Again we show the data before illumination ($n_e = 1.8 \times 10^{11}$ cm$^{-2}$). From Fig. 5 we extract $\Delta n_e/n_e = 0.014$. The anti-
symmetry effect (Fig. 5) is not as pronounced as for the GaAs/AlGaAs quantum well, but nevertheless signif-
ificant. After illumination ($n_e = 3.6 \times 10^{11}$ cm$^{-2}$) $\Delta n_e/n_e = 0.006$ (Table I) and the antisymmetry ef-
fect is reduced. Magnetotransport measurements on a second Hall bar, prepared from the same wafer, with $n_e = 2.2 \times 10^{11}$ cm$^{-2}$ show a comparable $\Delta n_e/n_e \sim 0.018$ (see Table I) and the corresponding antisymmetry effect.

Finally, we mention, that we have also investigated an InGaAs/InP heterostructure. This sample with $n_e = 2.2 \times 10^{11}$ cm$^{-2}$ and $\mu = 16000$ cm$^2$/Vs has been used previously for magnetotransport studies of the plateau-
plateau and plateau-insulator transitions in the quantum

![Fig. 2: Hall resistance as a function of magnetic field for the GaAs/AlGaAs quantum well (#659 - no illumination) near plateaux $i = 2 - 6$ at $T = 0.4$ K. The solid and dashed lines show data taken at the right and left Hall contact pairs.](image)

![Fig. 3: Longitudinal resistivity as a function of magnetic field ($B \uparrow$ and $B \downarrow$) for a GaAs/AlGaAs quantum well (#659 - no illumination). The solid and dashed lines show data taken at contact pairs located at the top and bottom side of the Hall bar.](image)
Hall regime (Refs. [4,5,6,7]). The Hall data show that it has a fairly large carrier density difference $\Delta n_e/n_e \sim 0.016$. Although $R_{xy}^t$ and $R_{xy}^l$ differ considerably the data are not antisymmetric, thus $R_{xy}^l(B) \neq R_{xy}^t(-B)$. This is the result of an additional “misalignment” of the voltage contacts due to intrinsic inhomogeneities (i.e. different effective values of $L$).

IV. THE EFFECT OF A CARRIER DENSITY GRADIENT

In the following we show that the observed asymmetry effect can be accounted for by a small carrier density gradient in the sample. Let us assume that the carrier density gradient points from the left current contact to the right one. In this case, $R_{xy}^l$ and $R_{xy}^r$ are slightly different. We define

$$\Delta R_{xy} = R_{xy}^l - R_{xy}^r.$$  \hfill (2)

and for the difference in the longitudinal resistances measured at both sides of the Hall bar

$$\Delta R_{xx} = R_{xx}^l - R_{xx}^r.$$  \hfill (3)

For a perfect Hall bar $\Delta R_{xy} = 0$ and $\Delta R_{xx} = 0$. Since

$$\Delta R_{xy} = \frac{(V_3 - V_5) - (V_4 - V_6)}{I} = \frac{(V_3 - V_4) - (V_5 - V_6)}{I} = \Delta R_{xx},$$  \hfill (4)

a finite $\Delta R_{xy}$ immediately implies that there is a difference between the longitudinal resistances measured at the top and bottom side of the Hall bar, $\Delta R_{xx} \neq 0$. 

FIG. 5: Hall resistance as a function of magnetic field for the GaInAs/AlInAs quantum well (21232-#3 - no illumination) near plateaux $i = 1 - 3$ at $T = 0.4$ K. The solid and dashed lines show data taken at the right and left Hall contact pairs.

FIG. 6: Longitudinal resistivity as a function of magnetic field ($B \uparrow$ and $B \downarrow$) for the GaInAs/AlInAs quantum well (21232-#3 - no illumination) at $T = 0.4$ K. The solid and dashed lines show data taken at contact pairs located at the top and bottom side of the Hall bar.
Because the Hall voltage is an odd function of the magnetic field, reversing the polarity changes the sign of the \(\Delta R_{xy}\) and \(\Delta R_{xx}\):

\[
\Delta R_{xy}(-B) = -\Delta R_{xy}(B), \quad (5)
\]

and

\[
\Delta R_{xx}(-B) = -\Delta R_{xx}(B). \quad (6)
\]

Thus \(\Delta R_{xx}\) is an antisymmetric function, which holds under the conditions of eq. (1). Eq. (4) may be directly verified by comparing the differences \(\Delta R_{xx}(B)\) and \(\Delta R_{xx}(B)\) calculated from the measured data (compare for instance \(\Delta R_{xx} = (L/W)\Delta \rho_{xx}\) in Fig. 4b, with \(\Delta R_{xy}\) in Fig. 4a)).

V. PLATEAU-PLATEAU TRANSITIONS

The effect of a carrier density gradient on the magnetotransport data as described here is in fact generally applicable to any 2D and 3D system, as follows from Eq. (1). However, the effect can be dramatically large for 2D quantum Hall samples, because of the steep slope of \(R_{xy}(B)\) at the plateau-plateau (PP) transitions. Consequently, our findings yield important constraints on the construction of the conductivity flow diagrams of the PP transitions. In the following paragraphs, a formal treatment of magnetotransport at the PP transition in the presence of a small carrier density gradient is presented.

We start from the usual equations for the Hall bar geometry, which tell us that a uniform current density \(j_x\) results from an applied electric field \(E_x\) along the channel direction \((x)\):

\[
E_x = \rho_0 j_x, \quad E_y = \rho_H j_x. \quad (7)
\]

Here \(j_y = 0\) and \(\rho_0\) and \(\rho_H\) are the longitudinal and Hall resistivity for a perfectly homogeneous sample. In a simple model for sample inhomogeneities we assume that the critical field \(B^*\) or filling fraction \(\nu^*\) for the PP transition varies linearly in \(x\) due to a gradient in the carrier density. At the PP transition the Hall resistivity slopes from one Hall plateau to the other, while the longitudinal resistivity forms a peak, so that \(|\partial \rho_H / \partial B| > |\partial \rho_0 / \partial B|\).

As such, an \(x\)-dependence of the local filling factor \(\nu(x)\) therefore mainly affects the electric field component:

\[
E_x = \rho_0 j_x, \quad E_y = (\rho_H + \alpha x) j_x, \quad \alpha = \frac{\partial \rho_H}{\partial \nu} \frac{\partial \nu}{\partial x}. \quad (8)
\]

This result, however, violates an important condition for having a stationary state, i.e. the electric field must be rotation free \(\nabla \times E = 0\). To satisfy this condition we proceed by inserting a \(y\)-dependent current density \(j_x \rightarrow j_x(1 + \beta y)\), where \(\beta = \alpha / \rho_0\). Working to linear order in the coordinates \(x\) and \(y\) we can write

\[
E_x = (\rho_0 + \rho_0 \beta y) j_x, \quad E_y = (\rho_H + \alpha x + \rho_H \beta y) j_x. \quad (9)
\]

Hence

\[
E_x = (\rho_0 + \alpha y) j_x, \quad E_y = (\rho_H + \alpha x + \frac{\partial \rho_H}{\partial \nu} y) j_x. \quad (10)
\]

are the appropriate equations for the PP transition. Notice that the stationary state condition \(\nabla \times E = 0\) and charge conservation \(\nabla \cdot j = 0\) are satisfied.

The longitudinal resistance at the top and bottom and the Hall resistance at the left and right contacts of the Hall bar are given by

\[
R_{xx}^l = \frac{V_{xx}^l}{I_x} = \frac{L}{W}(\rho_0 + \alpha \frac{W}{2}), \quad (11a)
\]

\[
R_{xx}^b = \frac{V_{xx}^b}{I_x} = \frac{L}{W}(\rho_0 - \alpha \frac{W}{2}), \quad (11b)
\]

\[
R_{xy}^l = \frac{V_{xy}^l}{I_x} = (\rho_H - \frac{L}{2}), \quad (12a)
\]

\[
R_{xy}^b = \frac{V_{xy}^b}{I_x} = (\rho_H + \frac{L}{2}), \quad (12b)
\]

where we take zero coordinates \((x, y)\) in the center of the Hall bar. When \(B \rightarrow -B\), \(\rho_H\) and \(\alpha\) change sign, but \(\rho_0\) remains unchanged. The results therefore explain the observed asymmetry at the PP transition, \(R_{xx}^*(B^*) = R_{xx}^*(B^*)\). Notice that with this specific form of \(\alpha\), eq. (12) can be regarded as a Taylor expansion of a local Hall resistance \(\rho_H\) at values \(x = \pm L/2\).

VI. DISCUSSION

Eqs. (11)-(12) show that the probed Hall resistance \(R_{xy}\) is a local resistance, and that the longitudinal resistance \(R_{xx}\) is not identical under field reversal. This gives rise to complications in extracting the conductivity components \(\sigma_{xx} = \rho_{xx}/(\rho_{xx}^2 + \rho_{xy}^2)\) and \(\sigma_{xy} = \rho_{xy}/(\rho_{xx}^2 + \rho_{xy}^2)\) of the PP transitions. As an example we show in Fig. 4b the \(\sigma_{xx} \sigma_{xy}\) conductance plane with data for the \(i = 4 \rightarrow 3\) and \(3 \rightarrow 2\) PP transitions of the GaAs/AlGaAs quantum well (before illumination). By choosing different combinations of \(R_{xx}^l = R_{xx}^b\) and \(R_{xy}^l = R_{xy}^b\) or \(R_{xy}^l\) or \(R_{xy}^b\) we obtain four different \(\sigma_{xx}, \sigma_{xy}\) curves for one field direction (for the reverse field direction identical curves are obtained, but the \(R_{xx}, R_{xy}\) labelling is different). Clearly, all curves show strong deviations from the “ideal” semicircle relation \(\sigma_{xx}^2 + (\sigma_{xy} - n\sigma^*)^2 = (\sigma^*)^2\) with \(\sigma^* = e^2/2h\) and \(n = 7\) and 5 for the \(i = 4 \rightarrow 3\) and \(3 \rightarrow 2\) PP transitions, respectively. After illumination, the density increases and the sample becomes more homogeneous as can be concluded from Fig. 5, where we show \(\sigma_{xx}, \sigma_{xy}\) for the \(i = 6 \rightarrow 5\) PP transition. After averaging the different curves, \(\sigma_{xx}, \sigma_{xy}\) follows quite close the semicircle relation with \(n = 11\).

Temperature driven \(\sigma_{xx}, \sigma_{xy}\) flow diagrams are an important experimental test of the two-parameter renormalization group theory of the quantum Hall effect.
In the scaling theory of the quantum Hall effect, the critical resistivity \( \rho_0(B^*) \) (or critical conductivity \( \sigma_0(B^*) = \sigma^* \)) at the PP transition is a constant when \( T \to 0 \). The presence of a carrier density gradient, therefore, brings about a second complication in analysing magnetotransport data, as the parameter \( \alpha \) varies with temperature through \( \partial \rho_H/\partial \nu \), namely a strong temperature variation of \( R_{xy}^{t,b}(B^*) \). The temperature dependence of the measured critical resistance at the PP transition, eq. \( 11 \), can be written as

\[
R_{xx}^{t,b}(B^*, T) \propto (1 \pm \text{const} \cdot \alpha(T))
\]  

Notice that when \( B \to -B \) the amplitudes change according to \( R_{xx}^{t}(B^*, T) = R_{xx}^{b}(-B^*, T) \). Corresponding changes in amplitude occur in the conductivities \( \sigma_{xx}(B^*) \) evaluated from \( R_{xx}^{t} \) or \( R_{xx}^{b} \). This feature of density gradients has been noticed in InGaAs/InP heterostructures. The strong temperature variation of \( \sigma_{xx}(B^*) \) is removed at low temperatures when averaging the data sets obtained for both directions of the magnetic field. Thus reversing the magnetic field is of paramount importance in determining the proper temperature variation of the conductivity peak height \( \sigma_{xx}(B^*) \). The temperature variation of \( \sigma_{xx}(B^*) \), reported in the literature at several places, might thus well be the result of not averaging the magnetotransport data over up and down magnetic field directions. This suggests that the occurrence of a “non-universal” critical conductivity can be explained without the need to evoke percolation models with macroscopic fluctuations of the local filling factor.

At this point it is important to stress that the experimental data reported in Figs. 7 and 8 are taken on a GaAs/AlGaAs quantum well. In general, in GaAs based quantum wells and heterostructures the dominant scattering mechanism is long-ranged due to remote impurity doping. This complicates the observation of universal scaling laws in the quantum Hall effect, as the scaling behavior is pushed to very low temperatures, where it is difficult to access experimentally. At the “elevated” temperatures used here thermal smearing due to the Fermi-Dirac distribution results in transport properties semi-classical in nature. The observation of a near “ideal” semicircle relation (Fig. 8) in our GaAs/AlGaAs quantum well is therefore not a signature of scaling.

The analysis of the PP transition has been made for Hall bars with a constant density gradient along the channel direction. In a real sample the situation may be more complex (gradients in \( x \)-\( y \) directions, non-linear density variations). In addition, intrinsic sample inhomogeneities can also lead to Hall potential contact misalignment, which in general gives rise to a contribution from \( R_{xx} \) to the \( R_{xy} \) data. In Ref. 2 a novel analytical procedure was presented to disentangle the universal quantum critical aspects of the magnetotransport data and sample dependent aspects, such as density gradients and contact misalignment. The methodology is based on decomposing the measured \( \rho_{ij} \) in symmetric and anti-

![Figure 7](image1.png)

**FIG. 7:** The longitudinal conductance \( \sigma_{xx} \) as a function of the Hall conductance \( \sigma_{xy} \) for the \( i = 4 \to 3 \) and \( 3 \to 2 \) PP transitions of the GaAs/AlGaAs quantum well before illumination \((T = 0.4 \text{ K})\). The four different lines are obtained by using all possible combinations of \( \rho_{xx} \) and \( R_{xy} \).

![Figure 8](image2.png)

**FIG. 8:** (a) The longitudinal conductance \( \sigma_{xx} \) as a function of the Hall conductance \( \sigma_{xy} \) for the \( i = 5 \to 6 \) PP transition of the GaAs/AlGaAs quantum well after illumination \((T = 0.4 \text{ K})\). The four different lines are obtained by using all possible combinations of \( \rho_{xx} \) and \( R_{xy} \). (b) The solid line represents the \( \sigma_{xx}, \sigma_{xy} \) data after averaging. The dashed line shows the “ideal” semicircle relation.
symmetric parts, as made here for the most simple case. So far, the analysis has been applied successfully to the plateaul-insulator (PI) phase transition measured for an InGaAs/InP heterostructure. At the PI transition, \( R_{xy} \sim h/e^2 \) remains quantized which facilitates the analysis. At the PP transition the situation is much more complex, as both \( \rho_{xx} \) and \( \rho_{xy} \) are a function of \( T \) and \( B \). In this case, higher order contributions to \( n_e(x) \) have to be considered as well.

VII. CONCLUSIONS

The effect of macroscopic sample inhomogeneities on magnetotransport data of the two-dimensional electron gas in the Hall bar geometry has been investigated. We find a remarkable antisymmetry in the longitudinal resistances \( R_{tx} \) and \( R_{bx} \) measured on both sides of the Hall bar upon reversing the magnetic field: \( R_{tx}(B) = R_{bx}(-B) \). The antisymmetry in \( R_{xx} \) is explained by a small carrier gradient along the channel direction of the Hall bar. The presence of a carrier density gradient complicates the study of quantum criticality of the plateau-plateau transitions in the quantum Hall effect. We evaluate expressions for the resistances at the plateau-plateau transitions and demonstrate complications that arise in extracting \( \sigma_{xx}, \sigma_{xy} \) flow diagrams.

Finally, we mention that in order to obtain an experimentally accessible large temperature range for scaling, the dominant electron scattering mechanism should be provided by short-range potential fluctuations, as for instance is the case for alloy scattering. From the point of view of sample preparation, this is virtually inherent to macroscopic sample inhomogeneities. Any proper study of quantum criticality of the quantum Hall effect therefore requires an unremitting research effort in understanding and modelling the effect of macroscopic sample inhomogeneities.

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