HEAVY-LIGHT QUARK SYSTEMS IN THE INSTANTON VACUUM
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Abstract

Assuming the gluon field is well approximated by instanton configurations we derive a partition function and calculate the specific correlators. Namely, the heavy quark propagator and heavy quark-antiquark correlator with the account of the light quark determinant and QCD instanton vacuum properties. With these knowledge we calculate the light quark contribution to the interaction between heavy quarks, which might be essential for the properties of a few heavy quarks systems like oniums and double-heavy baryons.

Introduction.

The physics of the heavy mesons and baryons with open and hidden heavy quarks is very reach and hot topic. Understanding the heavy-meson physics is important for evaluation of the components of the $CKM$-matrix, verification of the Standard Model and probing the physics beyond it, as well as production of different exotic meson states. Currently the experiments with $B$- and $D$-mesons are intensively studied by Belle [1], BaBar [2] and CDF collaborations, where unprecedented integrated luminosities were achieved, as well as neutrino-production of open and hidden charm in neutrino-hadron processes studied by K2K [3], MiniBoone [4], NuTeV [5] and Minerva [6] collaborations.

Theoretically, in pre-QCD era some success was achieved by the quantum-mechanical models which use effective potentials to describe heavy hadrons and their excitations (see e.g. [7] and references therein). However, such description inevitably introduces undefined phenomenological constants. The relation of these constants to QCD parameters is quite obscure: due to interaction with gluons and virtual light quark pairs all the constants contain nonperturbative dynamics. The numerical values of these constants are determined from fits to experimental data, which limits the predictive power of such models.

An advanced version of the potential model is NRQCD [8], however in this model light quarks and their interactions with heavy quarks via gluons is

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done in a phenomenological way. For this reason it is limited to description of systems with two heavy quarks. Alternatively, the heavy mesons are described in the Heavy Quark Effective Theory (HQET) proposed in [9], which treats the heavy mesons using the pQCD methods but does not take into account nonperturbative effects.

We propose to study the heavy quark physics in the framework of the instanton vacuum model. This model was developed in [10] and provided a consistent description of the light mesons physics [14].

One of the most prominent advances of the instanton vacuum model is the correct description of the spontaneous breaking of the chiral symmetry ($S\chi$SB), which is responsible for properties of most hadrons and nuclei [15]. The $S\chi$SB is due to specific properties of QCD vacuum, which is known to be one of the most complicated objects due to perturbative as well as nonperturbative fluctuations and is a very important object of investigations by methods of Nonperturbative Quantum Chromo Dynamics (NQCD). In the instanton picture $S\chi$SB is due to the delocalization of single-instanton quark zero modes in the instanton medium. One of the advantages of the instanton vacuum is that it is characterized by only two parameters: the average instanton size $\rho \sim 0.3$ fm and the average inter-instanton distance $R \sim 1$ fm. These essential numbers were suggested in [16] and were derived from $\Lambda_{\text{MS}}$ in [10]. These values were recently confirmed by lattice measurements [17].

In case of the heavy quarks, the instanton vacuum description was discussed in [12, 13]. For the heavy quarks even the charmed quark mass $m_c \sim 1.5$ GeV is larger than the typical parameters of the instanton media—the inverse instanton size $\rho^{-1} \approx 600$ MeV and the interinstanton distance $R^{-1} \approx 200$ MeV and thus the quark mass determines the dynamics of the heavy quarks.

**Light quark determinant with the quark sources term.**

Instanton vacuum field is assumed as a superposition of $N_+$ instantons and $N_-$ antiinstantons:

$$A_{\mu}(x) = \sum_{I}^{N_+} A_{\mu}^I(\xi_I, x) + \sum_{A}^{N_-} A_{\mu}^A(\xi_A, x).$$  \hspace{1cm} (1)

Here $\xi = (\rho, z, U)$ are (anti)instanton collective coordinates—size, position and color orientation (see reviews [10, 18]. The main parameters of the model are the average inter-instanton distance $R$ and the average instanton
size $\rho$. The estimates of these quantities are

$$
\rho \simeq 0.33 \text{ fm}, \ R \simeq 1 \text{ fm}, \ (\text{phenomenological}) \ [10, 18],
$$

$$
\rho \simeq 0.35 \text{ fm}, \ R \simeq 0.95 \text{ fm}, \ (\text{variational}) \ [10],
$$

$$
\rho \simeq 0.36 \text{ fm}, \ R \simeq 0.89 \text{ fm}, \ (\text{lattice}) \ [17]
$$

and have $\sim 10 - 15\%$ uncertainty.

Our main assumption is the interpolation formula:

$$
S_i = S_0 + S_0 \hat{\rho} \frac{\Phi_{0i} < \Phi_{0i}}{c_i} \hat{\rho} S_0, \quad S_0 = \frac{1}{\hat{\rho} + i m}, \quad c_i = i m < \Phi_{0i} | \hat{\rho} S_0 | \Phi_{0i} >
$$

The advantage of this interpolation is shown by the projection of $S_i$ to the zero-modes:

$$
S_i | \Phi_{0i} > = \frac{1}{i m} | \Phi_{0i} > , \quad < \Phi_{0i} | S_i = < \Phi_{0i} | \frac{1}{i m}
$$
as it must be, while the similar projection of $S_i$ given by [10] has a wrong component, negligible only in the $m \to 0$ limit.

Summation of the re-scattering series leads to the total quark propagator and making few further steps we get the fermionized representation of low-frequencies light quark determinant in the presence of the quark sources, which is relevant for our problems, in the form [14]:

$$
\text{Det}_{\text{low}} \exp(-\xi^+ S \xi) =
$$

$$
= \int \prod_f D\psi_f D\psi_f^\dagger \exp \left[ \sum_f \left( \psi_f^\dagger (\hat{\rho} + i m_f) \psi_f + \psi_f^\dagger \xi_f + \xi_f^+ \psi_f^\dagger + \psi_f^\dagger \xi_f^+ \psi_f^\dagger \right) \prod_f \prod_{\pm} V_{\pm,f}[\psi^\dagger, \psi] \right],
$$

where

$$
V_{\pm,f}[\psi^\dagger, \psi] = i \int d^4 x \left( \psi_f^\dagger(x) \hat{\rho} \Phi_{\pm,0}(x; \xi_\pm) \right) \int d^4 y \left( \Phi_{\pm,0}(y; \xi_\pm)(\hat{\rho} \psi_f(y)) \right),
$$

The averaging over collective coordinates $\xi_\pm$ is a rather simple procedure, since the low density of the instanton medium ($\pi^2 \left( \frac{\rho}{F} \right)^4 \sim 0.1$) allows us to average over positions and orientations of the instantons independently.

Light quark partition function from (5) at $N_f = 1$ and $N_\pm = N/2$ is exactly given by

$$
Z[\xi, \xi^+] = e^{-\xi^+(\hat{\rho} + i(m + M(p))^{-1} \xi} \exp \left[ \text{Tr} \ln (\hat{\rho} + i(m + M(p))) + N \ln \frac{N/2}{\lambda} - N \right]
$$

$$
N = \text{Tr} \frac{i M(p)}{\hat{\rho} + i(m + M(p))}, \quad M(p) = \frac{\lambda}{N_c} (2\pi \rho F(p))^2.
$$
Here the form-factor $F'(p)$ is given by Fourier-transform of the zero-mode. The coupling $\lambda$ and the dynamical quark mass $M(p)$ are defined by the Eq. (10).

At $N_f > 1$, and in the saddle-point approximation (no meson loops contribution) $Z[\xi_f, \xi_f^+]$ has a similar form as the Eq. (7).

Heavy quark propagator.

Define the heavy quark propagator as:

$$S_H = \frac{1}{Z} \int D\psi D\bar{\psi} \left\{ \prod_{\pm}^N \bar{V}_{\pm}[\psi^\dagger, \psi] \right\} \exp \int \left( \psi^\dagger (\hat{p} + i m) \psi \right) w[\psi, \psi^\dagger]$$

(9)

$$w[\psi, \psi^\dagger] = \left\{ \prod_{\pm}^N \bar{V}_{\pm}[\psi^\dagger, \psi] \right\}^{-1} \int d\zeta \left\{ \prod_{\pm}^N \bar{V}_{\pm}[\psi^\dagger, \psi] \right\} \frac{1}{\theta - \sum_i a_i}, \quad w_\pm = \frac{1}{\theta - a_\pm},$$

$$< t|\theta|t' > = \theta(t - t'), \quad < t|\theta^{-1}|t' > = -\frac{d}{dt} \delta(t - t'), \quad a_i(t) = i A_{i\mu}(x(t)) \frac{d}{dt} x_\mu(t)$$

Accordingly [12]

$$w^{-1}[\psi, \psi^\dagger] = \theta^{-1} + \frac{N}{2} \sum_{\pm} \frac{1}{\bar{V}_{\pm}[\psi^\dagger, \psi]} \int d\zeta \bar{V}_{\pm}[\psi^\dagger, \psi] \left( \theta - a_{\pm}^{-1} \right)^{-1} + O(N^2/V^2)$$

$$= \theta^{-1} - \frac{N}{2} \sum_{\pm} \bar{V}_{\pm}^{-1}[\psi^\dagger, \psi] \int d\zeta \bar{V}_{\pm}[\psi^\dagger, \psi] \theta^{-1}(w_\pm - \theta)\theta^{-1} + O(N^2/V^2)$$

$$= \theta^{-1} - \frac{N}{2} \sum_{\pm} \frac{1}{\bar{V}_{\pm}[\psi^\dagger, \psi]} \Delta_H[\psi^\dagger, \psi] + O(N^2/V^2).$$

(10)

and finally we get

$$S_H = \frac{1}{\theta^{-1} - \lambda \sum_{\pm} \Delta_H[\psi^\dagger, \psi]} \exp \left[ -\frac{\xi^+ \left( \hat{p} + i (m + M(p)) \right)^{-1} \xi}{\Delta_H[\psi^\dagger, \psi]} \right]_{\xi = \xi^+ = 0}$$

(11)

$$\approx \frac{1}{\theta^{-1} - \lambda \sum_{\pm} \Delta_H[\psi^\dagger, \psi]} \exp \left[ -\frac{\xi^+ \left( \hat{p} + i (m + M(p)) \right)^{-1} \xi}{\Delta_H[\psi^\dagger, \psi]} \right]_{\xi = \xi^+ = 0}$$

(12)

$$S_H^{-1} \approx \theta^{-1} - i tr \int \frac{d^4 k_1}{(2\pi)^4} N_c k_1^2 \frac{\lambda (2\pi)^2 F^2(k_1)}{N_c k_1^2 + i (m + M(k_1))} \frac{1}{2N_c} \sum_{\pm} \int d^4 z \bar{c} \left( \theta^{-1}(w_\pm - \theta)\theta^{-1} \right)$$

$$= \theta^{-1} - \frac{N}{2V N_c} \sum_{\pm} \int d^4 z \bar{c} \left( \theta^{-1}(w_\pm - \theta)\theta^{-1} \right)$$

(13)

The Eq. (13) exactly coincide with the similar one from [12].
Now re-write the Eq. (11) introducing heavy quark fields $Q, Q^\dagger$:

$$S_H = e^{-\text{Tr} \ln[\hat{\rho} + i(m + M(p))]} \int D\psi D\psi^\dagger DQDQ^\dagger \exp \left\{ (\psi^\dagger(\hat{\rho} + i(m + M(p)))\psi) \right\}$$

$$+ Q^\dagger \left( \theta^{-1} - \lambda \sum_{\pm} \Delta_{H,\pm}[\psi^\dagger, \psi] \right) Q - \text{Tr} \ln \left( \theta^{-1} - \lambda \sum_{\pm} \Delta_{H,\pm}[\psi^\dagger, \psi] \right),$$

where third term represent the (negligible) contribution of the heavy quark loops, while the second one is the heavy and light quarks interaction action, explicitly represented by

$$-\lambda \sum_{\pm} Q^\dagger \Delta_{H,\pm}[\psi^\dagger, \psi] Q = -i\lambda \sum_{\pm} \int d^4 z_{\pm} \frac{d^4 k_1}{(2\pi)^4} \frac{d^4 k_2}{(2\pi)^4} \exp(i(k_2 - k_1)z_{\pm})$$

$$\times (2\pi\rho)^2 F(k_1) F(k_2) \left[ \frac{1}{N_c} \psi^+(k_1) \frac{1}{2} \psi(k_2) Q^+ \text{tr}_c \left( \theta^{-1}(w_{\pm} - \theta)\theta^{-1} \right) Q \right.$$

$$\left. + \frac{1}{32(N_c^2 - 1)} \psi^+(k_1) \left( \gamma_{\mu} \gamma_{\nu} \frac{1}{2} \psi(k_2) \right) \text{tr}(\tau^+ \tau^+ \lambda^\dagger) Q^+ \text{tr}_c \left( \theta^{-1}(w_{\pm} - \theta)\theta^{-1}\lambda^\dagger \right) \lambda^i Q \right]$$

**Heavy quark anti-quark system.**

Define the correlator for this system as:

$$< T| C(L_1, L_2) | 0 > = \frac{1}{Z} \int D\psi D\psi^\dagger \left\{ \prod_{\pm} V_{\pm}[\psi^\dagger, \psi] \right\} \exp \left\{ (\psi^\dagger(\hat{\rho})\psi) \right\} W[\psi(16)]$$

$$< T| W[\psi, \psi^\dagger] | 0 > = \left\{ \prod_{\pm} V_{\pm}[\psi^\dagger, \psi] \right\}^{-1} \int D\zeta \left\{ \prod_{\pm} V_{\pm}[\psi^\dagger, \psi] \right\}$$

$$< T| \left( \theta^{-1} - \sum_i a_i^{(1)} \right)^{-1} | 0 > < 0 | \left( \theta^{-1} - \sum_i a_i^{(2)} \right)^{-1} | T > ,$$

here the correlator is a Wilson loop along the rectangular contour $L \times r$, where the sides $L_1, L_2$ are parallel to $x_4$ axes and separated by the distance $r$. The $a_i^{(1)}, a_i^{(2)}$ are the projections of the instantons onto the lines $L_1, L_2$.

Accordingly \(11\)

$$W^{-1}[\psi, \psi^\dagger] = w_1^{-1}[\psi, \psi^\dagger] \times w_2^{-1,T}[\psi, \psi^\dagger]$$

$$-\frac{N}{2} \sum_{\pm} V_{\pm}^{-1}[\psi^\dagger, \psi] \int d\zeta_{\pm} V_{\pm}[\psi^\dagger, \psi] \left( w_1[\psi, \psi^\dagger] - a^{(1)-1}_i \right) \times \left( w_2[\psi, \psi^\dagger] - a^{(2)-1}_i \right)^{-1,T}$$

$$= w_1^{-1}[\psi, \psi^\dagger] \times w_2^{-1,T}[\psi, \psi^\dagger]$$

$$-\frac{N}{2} \sum_{\pm} V_{\pm}^{-1}[\psi^\dagger, \psi] \int d\zeta_{\pm} V_{\pm}[\psi^\dagger, \psi] \left( \theta^{-1} \left( w_{\pm}^{(1)} - \theta \right) \theta^{-1} \right) \times \left( \theta^{-1} \left( w_{\pm}^{(2)} - \theta \right) \theta^{-1} \right)^T$$
where, superscript $T$ means the transposition, $\times$ – tensor product and

$$
w_{1}^{-1}[\psi, \psi^\dagger] = \theta^{-1} - \frac{N}{2} \sum_{\pm} \frac{1}{V_{\pm}[\psi^\dagger, \psi]} \int d\zeta \bar{V}_{\pm}[\psi^\dagger, \psi] \theta^{-1}(w^{(1)}_{\pm} - \theta)\theta^{-1} + O(N^2/V^2)
$$

$$
= \theta^{-1} - \frac{N}{2} \sum_{\pm} \frac{1}{V_{\pm}[\psi^\dagger, \psi]} \Delta^{(1)}_{\pm}[\psi^\dagger, \psi] + O(N^2/V^2). \quad (18)
$$

and similar for the $w_{2}^{-1}[\psi, \psi^\dagger]$.

From previous calculations we see that the lowest orders on $\frac{N_{c}}{V}$ in $C(L_{1}, L_{2})$ are given by the integration over $\psi, \psi^\dagger$ of the $W^{-1}[\psi, \psi^\dagger]$. Then, we have the new interaction term between heavy quarks located on the lines $L_{1}$ and $L_{2}$ due to exchange of the light quarks between them.

Explicitly the integration of the first term in $W^{-1}[\psi, \psi^\dagger]$ over $\psi, \psi^\dagger$ leads to:

$$
\frac{1}{Z} \int D\psi D\psi^\dagger \left\{ \prod_{\pm} \bar{V}_{\pm}[\psi^\dagger, \psi] \right\} \exp \left[ \left( \psi^\dagger(\hat{p})\psi \right) w_{1}^{-1}[\psi, \psi^\dagger] \times w_{2}^{-1,T}[\psi, \psi^\dagger] \right]
$$

$$
= \left( \theta^{-1} - \lambda \sum_{\pm} \Delta^{(1)}_{\pm}[\frac{\delta}{\delta \xi^{+}}, \frac{\delta}{\delta \xi^{-}}] \right) \times \left( \theta^{-1} - \lambda \sum_{\pm} \Delta^{(2)}_{\pm}[\frac{\delta}{\delta \xi^{+}}, \frac{\delta}{\delta \xi^{-}}] \right)^{T} e^{-\xi^{+}(\hat{p} + i(m + M(p))^{-1}\xi^{-}|_{\xi^{-}=0}}
$$

Light quarks generated potential is given by

$$
V_{lq} = \left( \lambda \sum_{\pm} \Delta^{(1)}_{\pm}[\frac{\delta}{\delta \xi_{1}}, \frac{\delta}{\delta \xi_{2}}] \right) \times \left( \lambda \sum_{\pm} \Delta^{(2)}_{\pm}[\frac{\delta}{\delta \xi_{1}}, \frac{\delta}{\delta \xi_{2}}] \right)^{T}
$$

$$
\times \exp \left[ -\xi_{2}^{+} (\hat{p} + i(m + M(p))^{-1}\xi_{1} - \xi_{1}^{+} (\hat{p} + i(m + M(p))^{-1}\xi_{2} \right]|_{\xi^{-}=0}
$$

Conclusion.

Approximating the gluon field by the instanton configurations it was derived the low-frequency part of the light quark determinant in the presence of quark sources. This one provided the way to calculate the heavy quark propagator with account of the light quark determinant and QCD instanton vacuum properties and to derive instanon generated light-heavy quarks interaction terms. With these knowledge it was calculated the light quark contribution to the interaction between heavy quarks providing more detailed investigation of the few heavy quarks systems like oniums and double-heavy baryons and of the role of the spontaneous breaking of the chiral symmetry (SBCS) in light quark sector for the heavy-light quarks systems.
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