Looking for a vector charmonium-like state $Y$ in $e^+e^- \rightarrow \bar{D}D_1(2420) + c.c.$

Jian-Rong Zhang  
Department of Physics, College of Liberal Arts and Sciences,  
National University of Defense Technology, Changsha 410073, Hunan, People’s Republic of China

Inspired by the first observation of a vector charmonium-like state $Y(4626)$ decaying to a meson pair $D_s^+D_s(2536)^-$, which could be viewed as a $P$-wave scalar-scalar $[cq][\bar{c}\bar{q}]$ tetraquark state, we predict a potential vector charmonium-like state $Y$ with $P$-wave scalar-scalar $[cq][\bar{c}\bar{q}]$ configuration. The corresponding mass spectrum of $Y$ state is calculated to be $4.33^{+0.16}_{-0.23}$ GeV in the framework of QCD sum rules. We suggest that the predicted $Y$ state could be looked for in an open-charm $e^+e^- \rightarrow \bar{D}D_1(2420) + c.c.$ process.

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I. INTRODUCTION

In recent years, a series of vector charmonium-like $Y$ states have been observed in the initial-state radiation processes $e^+e^- \rightarrow \gamma_{ISR}\pi^+\pi^- J/\psi(\psi(2S))$ \cite{11,12}, or in the direct processes $e^+e^- \rightarrow \pi^+\pi^- J/\psi(\psi(2S))$ \cite{13}. These experiments show that $Y$ states mainly couple to hidden-charm final states. In contrast, Belle newly reported the first observation of $Y(4626)$ in an open-charm process $e^+e^- \rightarrow D_s^+D_{s1}(2536)^- + c.c.$ with a significance of 5.9σ \cite{13}, which has promptly attracted much attention \cite{14,23,24}. Theoretically, some authors pointed that $Y(4626)$ can be well interpreted as a $P$-wave $[cq][\bar{c}\bar{q}]$ state with a multiquark color flux-tube model \cite{21}. Moreover, we studied $Y(4626)$ from two-point QCD sum rules, and finally arrived at that it could be a $P$-wave scalar-scalar $[cq][\bar{c}\bar{q}]$ state \cite{23}. On analogy of $Y(4626)$’s observation in the open-charm process, we propose that a novel vector charmonium-like state $Y$ could be looked for in an open-charm $e^+e^- \rightarrow \bar{D}D_1(2420) + c.c.$ process. In theory, the predicted $Y$ state could correspondingly be regarded as a $P$-wave scalar-scalar $[cq][\bar{c}\bar{q}]$ tetraquark state.

In this work, we endeavor to explore the charmonium-like state $Y$ with $P$-wave scalar-scalar $[cq][\bar{c}\bar{q}]$ configuration. To deal with the hadronic state, one has to confront the complicated nonperturbative QCD problem. As one trusty method for evaluating nonperturbative effects, the QCD sum rule \cite{24} is firmly founded on the basic QCD theory, and has been successfully applied to plenty of hadronic systems (for reviews see Refs. \cite{25,28} and references therein). Accordingly, we intend to study this $Y$ state by making use of the QCD sum rule approach.

The paper’s organization is as follows. In Sec. \ref{sec:qcdsrule} the QCD sum rule is derived for $Y$ with $P$-wave scalar-scalar $[cq][\bar{c}\bar{q}]$ structure, along with numerical analysis and discussions in Sec. \ref{sec:results}. The last part includes a brief summary.

II. THE QCD SUM RULE FOR $Y$ WITH $P$-WAVE SCALAR-SCALAR $[cq][\bar{c}\bar{q}]$ STRUCTURE

Generally speaking, one could have several choices on diquarks to characterize a $P$-wave tetraquark state with $J^P = 1^-$. It is worth noting that there have been broad discussions on the so-called “good” or “bad” diquarks for the tetraquark states \cite{29}, and then the $Y$ state with $P$-wave $[cq][\bar{c}\bar{q}]$ structure could be represented basing on following considerations \cite{30}. A “good” diquark operator in the attractive anti-triplet color channel can be $\bar{q}_c\gamma_5q$ with $0^+$, and a “bad” diquark operator can be $\bar{q}_c\gamma q$ with $1^+$. Similarly, operators with $0^-$ and $1^-$ can be written as $\bar{q}_c q$ and $\bar{q}_c\gamma_5\gamma q$, respectively. Further, it is suggested that diquarks are preferably formed into spin 0 from lattice results \cite{31}. Comparatively, the solid tetraquark candidates tend to be composed of $0^+$ “good” diquarks. For example, the final results from QCD sum rules favor the scalar diquark-scalar antidiquark case after comparing different diquark configurations \cite{32}. Thereby,
the predicted $Y$ state would be dominantly structured as the $P$-wave scalar diquark-scalar antidiquark, which contains the flavor content $[eq][\bar{e}q]$ with momentum numbers $S_{[eq]} = 0$, $S_{[\bar{e}q]} = 0$, $S_{[eq][\bar{e}q]} = 0$, and $L_{[eq][\bar{e}q]} = 1$. Here $q$ can be $u$ or $d$ quark, and $e$ is the charm quark. Considering that both light $u$ and $d$ quark masses are taken as current-quark masses in the paper, they are so small comparing with the heavy running charm mass $m_c$ that they will be neglected in the calculation complying with the usual treatment of heavy hadrons. Thus it is not concretely differentiated whether $q = u$ or $q = d$ for brevity. The corresponding current could be constructed as

$$j_\mu = \epsilon_{def} \epsilon_{d'e'f}(q_{d'}^T C \gamma_5 c_e) D_\mu(q_{d'}^T C \gamma_5 c_{e'}),$$

in which the index $T$ denotes matrix transposition, $C$ means the charge conjugation matrix, $D_\mu$ is the covariant derivative to generate $L = 1$, and $d$, $e$, $f$, $d'$, and $e'$ are color indices.

Generally, the two-point correlator $\Pi_{\mu\nu}(q^2) = i \int d^4xe^{iq.x}(0)|T[j_\mu(x)j_\nu^+(0))|0\rangle$ can be parameterized as

$$\Pi_{\mu\nu}(q^2) = \frac{g_\mu g_\nu}{q^2}(0)\Pi^{(0)}(q^2) + \frac{g_\mu g_\nu}{q^2} - g_\mu g_\nu)\Pi^{(1)}(q^2).$$

To yield the sum rule, the part $\Pi^{(1)}(q^2)$ can be evaluated in two different ways. At the hadronic level, it can be expressed as

$$\Pi^{(1)}(q^2) = \frac{\lambda^2}{M_H^2 - q^2} + \frac{1}{\pi} \int_{s_0}^{\infty} ds \frac{\text{Im}\Pi^{(1)}(s)}{s - q^2},$$

where $\lambda$ is the hadronic coupling constant and $M_H$ is the hadron’s mass. At the quark level, it can be written as

$$\Pi^{(1)}(q^2) = \int_{4m_c^2}^{\infty} ds \frac{\rho(s)}{s - q^2},$$

for which the spectral density $\rho(s) = \frac{1}{4}\text{Im}\Pi^{(1)}(s)$.

In deriving $\rho(s)$, one could work at leading order in $\alpha_s$ and consider condensates up to dimension 8. To keep the heavy-quark mass finite, one uses the heavy-quark propagator in momentum space [33]. The correlator’s light-quark part is calculated in the coordinate space and Fourier-transformed to the $D$ dimension momentum space, which is combined with the heavy-quark part and then dimensionally regularized at $D = 4 - 2\epsilon$. The correlator is given by $\rho(s) = \rho^{\text{pert}} + \rho^{(\bar{q}q)} + \rho^{(s^2G^2)} + \rho^{(g\bar{s}G^2)} + \rho^{(g\bar{s}G^2)} + \rho^{(g\bar{s}G^2)} + \rho^{(g\bar{s}G^2)} + \rho^{(g\bar{s}G^2)}$, detailedly with

$$\rho^{\text{pert}} = \frac{1}{3 - 2^{11/6}} \int_{\alpha_{\text{min}}}^{\alpha_{\text{max}}} \frac{d\alpha}{\alpha^2} \int_{\beta_{\text{min}}}^{1-\alpha} \frac{d\beta}{\beta^2 (1 - \alpha - \beta) \kappa r^5},$$

$$\rho^{(\bar{q}q)} = \frac{m_c^2}{3 - 2^{11/6}} \int_{\alpha_{\text{min}}}^{\alpha_{\text{max}}} \frac{d\alpha}{\alpha^2} \int_{\beta_{\text{min}}}^{1-\alpha} \frac{d\beta}{\beta^2 (2 - \alpha - \beta) r^3},$$

$$\rho^{(s^2G^2)} = \frac{m_c^2}{3 - 2^{11/6}} \int_{\alpha_{\text{min}}}^{\alpha_{\text{max}}} \frac{d\alpha}{\alpha^2} \int_{\beta_{\text{min}}}^{1-\alpha} \frac{d\beta}{\beta^2 (1 - \alpha - \beta) (\alpha^3 + \beta^3) r^2},$$

$$\rho^{(g\bar{s}G^2)} = \frac{m_c^2}{2^{8/4}} \left\{ - \int_{\alpha_{\text{min}}}^{\alpha_{\text{max}}} \frac{d\alpha}{\alpha^2} \int_{\beta_{\text{min}}}^{1-\alpha} \frac{d\beta}{\beta^2 (\alpha + \beta - 4\alpha \beta) r^2} + \int_{\alpha_{\text{min}}}^{\alpha_{\text{max}}} \frac{d\alpha}{\alpha^2} \left[ \frac{m_c^2 - \alpha (1 - \alpha) s}{\alpha (1 - \alpha)} \right] \right\},$$

$$\rho^{(\bar{q}q)} = \frac{m_c^2}{3 - 2^{11/6}} \int_{\alpha_{\text{min}}}^{\alpha_{\text{max}}} \frac{d\alpha}{\alpha^2} \int_{\beta_{\text{min}}}^{1-\alpha} \frac{d\beta}{\beta^2 (1 - \alpha - \beta) \kappa [(\alpha^3 + \beta^3) r + 4\alpha^4 + \beta^4] m_c^2 r},$$

$$\rho^{(g\bar{s}G^2)} = \frac{m_c^2}{2^{8/4}} \left\{ - \int_{\alpha_{\text{min}}}^{\alpha_{\text{max}}} \frac{d\alpha}{\alpha^2} \int_{\beta_{\text{min}}}^{1-\alpha} \frac{d\beta}{\beta^2 (2 - \alpha - \beta) (\alpha^3 + \beta^3) m_c^2 - 3\alpha (\beta - 1) + \beta^2 (\alpha - 1) r} \right\}. $$
and
\[ \rho^{(\bar{q}q)}(g\bar{q}q \cdot G_q) = \frac{m_3^2 \langle g\bar{q}q \cdot G_q \rangle}{3 \cdot 2^5 \pi^2} \int_{\alpha_{\text{min}}}^{\alpha_{\text{max}}} d\alpha (6\alpha^2 - 6\alpha + 1), \]
where \( \kappa = 1 + \alpha - 2\alpha^2 + \beta + 2\alpha\beta - 2\beta^2, \)
\( r = (\alpha + \beta)m_1^2 - \alpha\beta s, \)
\( \alpha_{\text{min}} = (1 - \sqrt{1 - 4m_1^2/s})/2, \)
\( \alpha_{\text{max}} = (1 + \sqrt{1 - 4m_1^2/s})/2, \)
and \( \beta_{\text{min}} = \alpha m_1^2/(\alpha - m_1^2). \)
For the four-quark condensate, a general factorization relation
\( \langle g\bar{q}q \rangle = \rho(q\bar{q})^2 \) has been employed, in which \( \rho \) may be equal to 1 or 2.

Equating the two expressions and adopting quark-hadron duality, and making a Borel transform, the sum rule can be turned into
\[ \lambda^2 e^{-M_H^2/M^2} = \int_{4m_c^2}^{s_0} ds \rho e^{-s/M^2}. \]
Taking the derivative of Eq. (5) with respect to \( -\frac{1}{M^2} \) and then dividing the result by Eq. (5) itself, one can obtain the hadron’s mass sum rule
\[ M_H^2 = \int_{4m_c^2}^{s_0} ds \rho e^{-s/M^2} / \int_{4m_c^2}^{s_0} ds \rho e^{-s/M^2}, \]
in which light \( u \) and \( d \) current-quark masses have been safely neglected as they are so small comparing with the heavy \( m_c. \)

### III. NUMERICAL ANALYSIS AND DISCUSSIONS

In the numerical analysis, the running charm mass \( m_c \) is 1.27 \pm 0.02 GeV \[ \text{[34]}, \]
and other input parameters are \[ \text{[24, 28]}. \]
\( \langle \bar{q}q \rangle = -(0.24 \pm 0.01)^3 \) GeV \(^3, \)
\( m_0^2 = 0.8 \pm 0.1 \) GeV \(^2, \)
\( \langle g\bar{q}q \cdot G_q \rangle = m_0^2 \langle \bar{q}q \rangle, \)
\( \langle g^2G^2 \rangle = \) \( 0.88 \pm 0.25 \) GeV \(^4, \)
as well as \( \langle g^3G^3 \rangle = 0.58 \pm 0.18 \) GeV \(^6. \)
According to the standard criterion of sum rule analysis, one could find proper work windows for the threshold parameter \( \sqrt{s_0} \) and the Borel parameter \( M^2. \)
The lower bound of \( M^2 \) is obtained from the OPE convergence, and the upper one is found in view of that the pole contribution should be larger than QCD continuum one. Meanwhile, the threshold \( \sqrt{s_0} \) describes the begining of continuum state, which is about \( 400 \sim 600 \) MeV bigger than the extracted \( M_H \) empirically.

At the very start, all the input parameters are kept at their central values and the four-quark condensate factor is taken as \( \rho = 1. \) To get the lower bound of \( M^2, \) the OPE convergence is shown in FIG. 1 by comparing the relative contributions of different condensates from sum rule \[ \text{[4]} \] for \( \sqrt{s_0} = 4.9 \) GeV. Numerically, some main condensates could cancel each other out to some extent and the relative contribution of perturbative could play a predominant role in OPE at \( M^2 = 2.5 \) GeV \(^2, \) which is increasing with the enlarging of Borel parameter \( M^2. \) In this way, it is taken as \( M^2 \geq 2.5 \) GeV \(^2 \) with an eye to the OPE convergence analysis. Besides, the upper bound of \( M^2 \) is attained with a view to the pole contribution dominance in phenomenological side. In FIG. 2, it is compared between pole contribution and continuum from sum rule \[ \text{[5]} \] for \( \sqrt{s_0} = 4.9 \) GeV. The relative pole contribution is close to 50% at \( M^2 = 3.0 \) GeV \(^2 \) and descending with the Borel parameter \( M^2. \) Thus the pole contribution dominance could be fulfilled while \( M^2 \leq 3.0 \) GeV \(^2. \) Accordingly, the Borel window of \( M^2 \) is restricted to be \( 2.5 \sim 3.0 \) GeV \(^2 \) for \( \sqrt{s_0} = 4.9 \) GeV. Analogously, the reasonable window of \( M^2 \) is acquired as \( 2.5 \sim 2.9 \) GeV \(^2 \) for \( \sqrt{s_0} = 4.8 \) GeV, and \( 2.5 \sim 3.2 \) GeV \(^2 \) for \( \sqrt{s_0} = 5.0 \) GeV. In the work windows, one can expect that the two sides of QCD sum rules have a good overlap and it is reliable to extract information on the resonance. The dependence on \( M^2 \) for the mass \( M_H \) of \( \Lambda \) state is shown in FIG. 3, and its value is computed to be \( 4.33 \pm 0.11 \) GeV in work windows.

Next varying the input parameters, the mass \( M_H \) is obtained as \( 4.33 \pm 0.11^{+0.05}_{-0.08} \) GeV (the first error due to variation of \( s_0 \) and \( M^2 \), and the second one resulted from the uncertainty of QCD parameters) or shortly \( 4.33^{+0.16}_{-0.19} \) GeV. In the end, paying attention to the variation of four-quark condensate factor \( \rho, \) the
FIG. 1: The OPE convergence for the $Y$ state with $P$-wave scalar-scalar $[cq][\bar{c}\bar{q}]$ configuration is shown by comparing the relative contributions of perturbative, two-quark condensate $\langle \bar{q}q \rangle$, two-gluon condensate $\langle g^2G^2 \rangle$, mixed condensate $\langle g\bar{q}\sigma \cdot Gq \rangle$, four-quark condensate $\langle \bar{q}q \rangle^2$, three-gluon condensate $\langle g^3G^3 \rangle$, $\langle \bar{q}q \rangle\langle g^2G^2 \rangle$, and $\langle \bar{q}q \rangle\langle g\bar{q}\sigma \cdot Gq \rangle$ from sum rule (5) for $\sqrt{s_0} = 4.9$ GeV.

FIG. 2: The phenomenological contribution in sum rule (5) for $\sqrt{s_0} = 4.9$ GeV for the $Y$ state with $P$-wave scalar-scalar $[cq][\bar{c}\bar{q}]$ configuration. The solid line is the relative pole contribution (the pole contribution divided by the total, pole plus continuum contribution) as a function of $M^2$ and the dashed line is the relative continuum contribution.

corresponding Borel curves are presented in FIG. 4 with $\rho = 2$. In comparison with Fig. 3 for $\rho = 1$, one could notice the mass uncertainty when varying $\rho$ from 1 to 2, and could get the final mass $4.33^{+0.16}_{-0.23}$ GeV for the $Y$ state with $P$-wave scalar-scalar $[cq][\bar{c}\bar{q}]$ configuration.

In experiment, one may note that in the hidden-charm $e^+e^- \rightarrow \gamma_{ISR}\pi^+\pi^-\psi(2S)$ process, BABAR observed a broad structure near $4.32$ GeV [2], and Belle subsequently found the charmonium-like state $Y(4360)$ [3]. Afterward, a combined fit to these cross sections measured by BABAR and Belle experiments was performed [38], and the property of $Y(4360)$ was further studied in $e^+e^- \rightarrow \pi^+\pi^-\psi(2S)$ via initial-state radiation at BABAR [7] and at Belle [8]. Taking notice of the close masses of $Y(4360)$ and $Y$ state concerned here, one could conjecture that they may be the same structure attributing to different decay
FIG. 3: The dependence on $M^2$ for the mass $M_H$ of the $Y$ state with $P$-wave scalar-scalar $[car{q}][ar{c}q]$ configuration from sum rule (6) is shown while the four-quark condensate factor $\varrho = 1$. The ranges of $M^2$ are $2.5 \sim 2.9$ GeV$^2$ for $\sqrt{s_0} = 4.8$ GeV, $2.5 \sim 3.0$ GeV$^2$ for $\sqrt{s_0} = 4.9$ GeV, and $2.5 \sim 3.2$ GeV$^2$ for $\sqrt{s_0} = 5.0$ GeV, respectively.

FIG. 4: The dependence on $M^2$ for the mass $M_H$ of the $Y$ state with $P$-wave scalar-scalar $[car{q}][ar{c}q]$ configuration from sum rule (6) is shown while the four-quark condensate factor $\varrho = 2$.

modes. If that true, it would be very important for understanding $Y(4360)$ to search for the predicted $Y$ state, because complementary measurements by other decay modes such as the open-charm process will provide further insights into $Y(4360)$’s internal structure. Whether or not, it is undoubtedly exciting and significative if one could find a vector charmonium-like $Y$ state particularly in an open-charm decay.

Invigoratingly, there has appeared some measurement of Born cross section for $e^+e^- \rightarrow D^- D_1(2420)^+ + c.c.$ [39], in which the cross section line shape is consistent with the previous BESIII’s result based on full reconstruction method [40], and there is some indication of enhanced cross section at the location of $Y(4360)$. Thereby, it seems promising that the predicted $Y$ state could be observed in the open-charm process $e^+e^- \rightarrow \bar{D}D_1(2420) + c.c.$ via either the initial-state radiation or the direct production for the future experiments.
IV. SUMMARY

Activated by the first observation of a vector charmonium-like state $Y(4626)$ in the open-charm $D_s^+ Ds(2536)^-$ decay mode, for which could be a $P$-wave scalar-scalar $[cs][¯c¯s]$ tetraquark state, we predict a novel vector charmonium-like $Y$ state with $P$-wave scalar-scalar $[cq][¯c¯q]$ configuration. Finally, the mass of $Y$ is presented to be $4.33^{+0.16}_{-0.23}$ GeV from QCD sum rules. We suggest that the predicted $Y$ state could be searched for in an open-charm $e^+e^→D_s^+ Ds(2536)$ + $c.c.$ process through the initial-state radiation or the direct production in experiments, for which virtually there has been some indication of enhanced cross section in BESIII’s existing measurements.

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