Laboratory characterisation of anisotropic and heterogeneous damage of rock sample using acoustic emission and ultrasonic monitoring technologies

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Abstract. Rock damage characterisation of Gosford sandstone samples under triaxial loading was achieved using an integrated approach, combining passive acoustic emission (AE) and active ultrasonic wave monitoring. The location of AE events is firstly obtained using isotropic wave velocity field measured by continuous ultrasonic wave. Then, passive AE tomography is conducted to progressively-update the anisotropy and heterogeneous P-wave velocity field of rock samples. According to the variance in P-wave velocity, damage induced anisotropy and heterogeneity can be identified. The results suggest that the inhomogeneous damage evolution highly dependent on the loading stages and can well reflect the microcrack behaviour. Anisotropic damage variable along the whole loading process is also analysed, by a second order damage tensor with a transversely anisotropy assumption. It is observed that the lateral component of damage is much higher than the axial component. This is mainly due to the dilation behaviour of rock sample. This paper will cast more light on anisotropic and heterogeneous damage characterisation using acoustic/ultrasonic lab experiments.

1. Introduction
Sandstone is one of the most common rock materials in the mining, civil and oil and gas industries. The understanding of rock mass properties and its mechanical response plays an essential role in improving the engineering design efficiency and ensuring workspace safety. Hence, numerous pioneering studies were conducted focus on investigating various rock properties, such as dilation[1], strain-hardening[2], discontinuity[3], anisotropy and heterogeneity[4].

An important rock property is the abovementioned anisotropy and heterogeneity, which can be classified as original (by unevenly distributed grains and discontinuities) or induced (by plasticity and damage evolution). In this paper, we mainly focus on the latter, as it is directly related to the non-linear behaviour of rock samples and highly correlated with the cracking process during rock failure. Normally for an intact sandstone specimen, it can be considered as isotropic and homogenous, whereas the strong anisotropy and heterogeneity (such as AE localisation, irreversible strain, damage propagation) occurs as the non-linear loading stage is reached [5,6].

The theory of damage mechanics was proposed in 1950s and has attracted extensive attention soon after[7]. The core of damage mechanics is to use a damage variable to describe the deterioration of material properties, especially during the non-linear loading stage, where microcracking failure (slippage and opening) predominates and highly affects the material properties. The damage variable can be defined as a scalar, vector, second or fourth order tensor [8–10]. In order to represent the anisotropic damage evolution, the second or fourth order tensor is normally applied in a few constitutive models, which can express the anisotropic damage as well as the damage induced...
anisotropy of rock mass\cite{11}. The second order damage tensor contains nine independent variables, or 81 for the fourth order, which can be challenging to solve explicitly. A pragmatic approach is to use a scalar or second order tensor as the damage variable, which has already been adopted by a few previous studies \cite{12–16}. However, the damage variable is normally treated as an internal variable, since the internal microcracks are invisible and hard to be detected, which hinders the application of damage mechanics in engineering practices.

The non-destructive monitoring technology (NDT), as a fast-developing approach to reflect the inner material behaviour, has innovatively improved the understanding on the material damage. For instance, acoustic emission (AE), computed tomography (CT), and ultrasonic wave velocity measurement are all considered as classic NDT methods. A number of recent studies suggested that these technologies can well capture the change of internal material properties and contributes to the deeper understanding of rock non-linear behaviour \cite{17–19}.

In this paper, we introduced the application of two NDTs (AE and P-wave velocity measurement) to assist the characterisation of rock material internal damage. The damage evolution based on AE and ultrasonic results was presented. Also, the anisotropy (second order tensor) and heterogenous distribution (scalar) of damage variables were both analysed.

2. Testing Procedures

The equipment used in this study is a Multiphysics high-pressure high-temperature rock testing system developed in the UNSW mining geomechanics laboratory, with displacement measuring, servo-control loading, AE monitoring and P wave velocity testing capacity. All components of the testing system are introduced in Figure 1. The loading frame provides axial load on the rock sample, stiff enough compared with the rock sample, motivated by a pump with the capacity of 1000kN. The loading rate is controlled by a one-way valve, downstreaming the pump, managed by a servo control system (the MOOG smart test controller), so that both force and displacement of loading can be controlled at a constant rate in the test. The loading cell is HTRX-140XL rock triaxial testing cell provided by GCTS (USA), enabling to install AE sensors and ultrasonic nodes on the sample and perform simultaneous testing during the whole stress-strain loading curve. The ultrasonic wave is generated from the loading platen with a high frequency in an attempt to reduce the wave velocity attenuation inside the rock sample. The ULT-200 unit from GCTS is responsible for the data acquisition and pre-processing of ultrasonic waves. The wave arrival time is automatically picked and manually verified. The ultrasonic wave test is continuously performed in a regular time interval (5 seconds) throughout the whole test, in order to probe the continuous damage evolution inside the rock sample. To lubricate and avoid possible wave attenuation caused by void between the platen and rock sample, honey is used as coupling material. Eight AE sensors are attached on the rock sample to monitor the seismic emission during rock failure. All of the AE signals are collected via the data acquisition system and processing software provided by MISTRA, where automatic post-processing of acoustic signals such as wave arrival time pick-up, wave amplitude and energy calculation and events localisation are implemented. The location algorithm applied is the Geiger method, assuming that the wave velocity field is isotropic and the ray path is linear inside the rock sample. The confinement pressure is applied by a portable pump with the maximum capacity of 140MPa. In this research, we conducted four triaxial tests with the confinement at 5MPa, 10Mpa, 15MPa and 20MPa.
3. Fundamental Theory and Assumptions

3.1 Damage definition

According to the continuum damage mechanics, the definition of scalar damage and second order damage variable can be both identified by the reduction of effective cross-sectional area, showing as Figure 2. Where, $\omega$ is the scalar damage variable, which is adopted in the isotropic case; $D_i$ is the damage component along the $x_i$ direction for the anisotropic second order damage tensor. Note that this damage definition is in the principal direction of damage variable, where the shear components of damage variable are ignored. A common assumption is that the stress and damage share the same principal direction so that $x_i$ also represents the principal stress direction [13].

$\omega = \frac{A_{\text{eff}}}{A}$

$D_1 = \frac{A_{11}}{A}$

$D_2 = \frac{A_{22}}{A}$

$D_3 = \frac{A_{33}}{A}$

$D = \begin{bmatrix} D_1 & 0 & 0 \\ 0 & D_2 & 0 \\ 0 & 0 & D_3 \end{bmatrix}$

Figure 2. Identification of the scalar and second damage tensor.
Based on the equivalence of strain assumption, the damage can be referred as the reduction of stiffness matrix (Young’s modulus and Poisson’s ratio). Therefore, the effect of damage can be further rewritten as[20]:

**Isotropic damage:**

\[
\tilde{E} = (1 - \omega)E
\]  

(1)

**Anisotropic damage:**

\[
\tilde{C}_{ij}^{-1} = \begin{bmatrix}
\frac{1}{E_1} & -\tilde{\nu}_{21} & -\tilde{\nu}_{31} & 0 & 0 & 0 \\
-\tilde{\nu}_{12} & \frac{1}{E_2} & -\tilde{\nu}_{32} & 0 & 0 & 0 \\
-\tilde{\nu}_{13} & -\tilde{\nu}_{23} & \frac{1}{E_3} & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{1}{2G_{23}} & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{1}{2G_{31}} & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{1}{2G_{12}} \\
\end{bmatrix}
\]  

(2)

Where,

\[
\tilde{E}_i = E_i(1 - D_i) \quad i = 1,2,3
\]  

(3)

\[
\tilde{\nu}_{ij} = \nu_{ij} \frac{1 - D_i}{1 - D_j} \quad i,j = 1,2,3
\]  

(4)

\[
\tilde{G}_{ij} = G_{ij} \frac{(1 - D_i)^2 (1 - D_j)^2}{(1 - D_i)^2 + (1 - D_j)^2} \quad i,j = 1,2,3
\]  

(5)

\(E\) and \(\tilde{E}\) are the Young’s modulus and equivalent Young’s modulus of the intact rock for isotropic scenario, \(E_i\) and \(\tilde{E}_i\) represent the Young’s modulus and equivalent Young’s modulus at the \(i\) th direction of the intact anisotropic rock . \(\nu_{ij}, \tilde{\nu}_{ij}, \tilde{G}_{ij}\) are the Poisson’s ratio and shear modulus and their equivalent components; \(\tilde{C}_{ij}\) is the stiffness matrix for an anisotropic material.

### 3.2 P wave velocity

The P wave velocity is related to the stiffness of the rock material. In our study, a conventional triaxial test is conducted with the same lateral stress \((\sigma_2 = \sigma_3)\) and therefore a transverse isotropy is sufficient to describe the damage induced anisotropy here. Therefore, the P wave velocity can be written as:

**Isotropic damage:**

\[
D = 1 - \frac{V_p^2}{V_{p(MAX)}^2} \times \frac{\rho}{\rho_{MAX}}
\]  

(6)

**Anisotropic damage:**
\[ D_1 = 1 - \frac{V_1^2}{V_{1(\text{MAX})}^2} \times \frac{\rho}{\rho_{\text{MAX}}} \]  

(7)

\[ D_2 = D_3 = 1 - \frac{V_3^2}{V_{3(\text{MAX})}^2} \times \frac{\rho}{\rho_{\text{MAX}}} \]  

(8)

Where, \( V_p \) is the P wave velocity by ultrasonic wave measurement, \( V_1 \) is the axial P wave velocity and \( V_{1(\text{MAX})} \) is its maximum value measured over a loading test. \( \rho \) is the density of rock specimen and \( \rho_{\text{MAX}} \) is its maximum value.

3.3 AE tomography

The location of AE events was obtained from the MISTRA AEwin software, by assuming an isotropic wave velocity field. However, in the actual loading process, the localised stress concentration of the unevenly distributed microcrack tips can induce anisotropy easily, which then violates the assumption of an isotropic P wave velocity field over the loading test. In order to obtain the actual P wave field, AE tomography has been proved to be an effective method, which will correct the wave velocity distribution based on AE results. Herein, we used FaATSO, a package enables to perform passive AE tomography using AE arrival time and active ultrasonic wave velocity measurement [18]. The detailed algorithm can be found in a few publications [18,21–23], which will not be repeated here. For the \( i \)th acoustic event, its ray path is \( L_i \) and transmission time is \( T_i \), then:

\[ T_i = \int_{L_i} \frac{dL}{V(x,y,z)} = \int_{L_i} S(x,y,z) dL \]  

(9)

Where, \( V(x,y,z) \) is the P wave velocity for a point \((x,y,z)\) in rock specimen, and \( S(x,y,z) \) is the slowness of that point. If the whole rock sample is discretised into \( m \) grids, then:

\[ T_i = \sum_{j=1}^{m} d_{ij} S_j \quad (i = 1, \ldots, n) \]  

(10)

d_{ij} is the \( i \)th ray path length in the \( j \)th grid. \( n \) is the total number of ray path. Then, the difference of \( T_i \) is assumed to be purely owing to the change of slowness \( S_j \) and then the slowness field is updated to match the actual arrival time. In this paper, two slowness fields are proposed. The first one is an isotropic field, which is measured by the active ultrasonic wave platen while loading. Meanwhile, the isotropic field is used to update the input P-wave velocity to locate the AE events throughout the loading test. The second velocity field is anisotropic. AE events recorded during loading are considered as sources in the other progressively updated velocity field, where the wave velocity is isotropic and continuously changing based on Equation 10. To consider the transversely anisotropy wave velocity, we proposed an anisotropy factor \( \phi \) [18].

\[ V(x,y,z,\theta) = V(x,y,z) \times (1 + \phi \cos^2 \theta) \]  

(11)

Where, \( \theta \) is defined as the angle between the sample axial direction and wave propagation direction, as shown in Figure 3.
4. Results and discussions

4.1 Sample mechanical response

The stress-strain curves of all rock specimens are plotted in Figure 4. Over the whole test process, different loading stages can be identified, which are also highlighted on the stress-strain curves. As a result of the closure of microcracks, all tests begin with an initial non-linear stage and it lasts until the axial stress reaches $\sigma_c$ (crack closure stress). Then the linear elastic stage initiates, where a pure linear stress-strain relationship is observed. The irreversible plastic strain does not occur until the crack initiation stress ($\sigma_{ci}$) is approached and the non-linear stress-strain relationship indicates the presence of yielding and stress hardening. A clear intensification of non-linear behaviour is noticed when the rock sample reaches the crack instable growth stress ($\sigma_{cc}$), where a typical dilation phenomenon (the inverse of volumetric strain) is detected. The lateral strain is much larger than the axial strain, displaying a strong anisotropic non-linear behaviour.
4.2 Heterogeneous damage characterised by AE tomography

To investigate the heterogeneity of damage distribution, AE tomography is conducted with the continuous P wave velocity measurement and AE location. To better represent the heterogeneity of damage variable, a simple isotropic damage variable is considered in this section based on Equation 6. The damage evolution and its spatial distribution are presented in Figure 5. A clear correlation between the crack initiation stress ($\sigma_{ci}$), crack unstable growth stress ($\sigma_{cd}$) and the AE counts and damage evolution can be observed. Few AE counts are observed before reaching the crack initiation stress. When the crack unstable growth stress is achieved, AE counts suddenly surge, along with a reduction on the P wave velocity, indicating microcracks have created voids inside rock mass via dilation. The isotropic and heterogenous damage distribution is also obtained from the AE tomography results. In the first stage (before $\sigma_{ci}$ is reached), the overall P wave velocity of rock sample is increasing (referring to Figure 6). Since only few AE events are detected in this period, the P wave velocity field is not fully updated using limited events, so that some zones show a lower P wave velocity and a damage variable at around 0.6-0.7. This could be an unavoidable system deviation for AE tomography defined damage. As the rock specimen enters the second stage ($\sigma_{ci}$ to $\sigma_{cd}$), more AE events are recorded and some zones with extremely high damage can be observed inside the sample, especially zones close to centre, where the ray path density is the highest. An interesting observation is that the system deviation of AE tomography is no longer significant. At the final stage of loading ($\sigma_{cd}$ to peak stress), where massive AE events are collected and the P wave velocity field is updated significantly, the damage shows strong localisation which is an omen of crack coalescence. The propagation of damage zone captured in the 3rd stage is presented.
4.3 Anisotropic damage characterised by AE tomography

The anisotropic damage can be calculated based on Equations 7 and 8 introduced before. For simplicity, we assume that the anisotropic damage is homogenous. Note that we also assume that the principal direction of stress and damage coincide with each other. Thus, the damage tensor can be simplified as a diagonal matrix, and the two lateral damage components in the matrix are identical ($D_2 = D_3$), leaving only two independent variables ($D_1$) and ($D_3$) to be solved.

The experiment results are presented in Figure 6 with the variation of $D_1$, $D_3$ and P wave velocity over the loading cycle. Initially, the axial and lateral components of damage variable are similar. However, after the drop of P wave velocity, the lateral damage is more serious and deviates from the axial damage as the strain-hardening proceeds. This can be explained by the effect of dilation, where the axial damage is induced by compression and most of them are actually closed under high axial stress. In contrast, the lateral damage is due to shear and its induced tensile extension. As a typical brittle material, rock has more compressive tolerance than tension. Therefore, it is acceptable that the tensile induced lateral damage is higher. Also, the microcracks are generally aligned with the axial loading direction and therefore the lateral projection of microcrack induced void is higher, leading to a higher damage in the lateral direction[24]. For the 5MPa confinement scenario, since the rock specimen showed notable brittle behaviour and the AE events are few for the pre-peak stage and therefore the damage evolution is not well captured.
Figure 6. The variance of anisotropic damage and P wave velocity during the whole loading

5. Conclusions
The anisotropic and heterogenous damage is identified with the assistance of acoustic emission and ultrasonic wave velocity measurement. From the concept of continuum mechanics, the P wave velocity is proved be a reasonable measurement of rock sample internal damage. In order to achieve a heterogeneous and anisotropic damage field, AE tomography is conducted based on the AE events and active ultrasonic measurement. For heterogeneous damage analysis, the distribution of damage inside rock sample is presented, mainly generated from the crack initiation stress ($\sigma_{ci}$) to the crack unstable growth stress ($\sigma_{cd}$). The propagation and coalescence of microcracks are observed as the damaged zone propagation along the low wave velocity zone. As for the anisotropic damage, a transverse anisotropic assumption is accepted since only conventional triaxial test is performed in this study. The lateral damage evolution is faster than the axial damage, since the dilation and the orientation of microcracks have a larger projection on the laterally. This study shows that AE and ultrasonic wave velocity have a great potential and proved the availability of those technologies on damage identification.

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