LIMITS ON EXTRA DIMENSIONS
IN ORBIFOLD COMPACTIFICATIONS OF SUPERSTRINGS

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ABSTRACT

Perturbative breaking of supersymmetry in four-dimensional string theories predict in general the existence of new large dimensions at the TeV scale. Such dimensions can be consistent with perturbative unification up to the Planck scale in a class of string models and open the exciting possibility of lowering a part of the massive string spectrum at energies accessible to future accelerators. The main signature is the production of Kaluza-Klein excitations which have a very particular structure, strongly correlated with the supersymmetry breaking mechanism. We present a model independent analysis of the physics of these states in the context of orbifold compactifications of the heterotic superstring. In particular, we compute the limits on the size of large dimensions used to break supersymmetry.

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Our observable world is a four dimensional space-time and there is no experimental evidence of the presence of extra dimensions. However, in trying to unify gravity with the other fundamental interactions, we are led to consider Kaluza-Klein (KK) type theories living in bigger spaces [1]. There, spacetime is assumed to be a product $\mathcal{M}_4 \times K$ of the Minkowski space $\mathcal{M}_4$ described by four non-compact coordinates $x^\mu$ where $\mu = 0, ..., 3$, with some internal space $K$ formed by new dimensions $X^i$ where $i = 4, ..., 3 + D$. In general, these extra dimensions can be compactified to very small size, of the order of the Plank length $M_p^{-1}$, so that they have no effect on present experimental observations.

The most promising of such extensions of general relativity is superstring theory which provides the only known example of consistent quantum gravity [2]. One of the main problems of this theory is to break spacetime supersymmetry at energies close to the electroweak scale in order to protect the gauge hierarchy. In contrast to the situation in field theory where one has the freedom to introduce arbitrary soft breaking masses, in string theory the scale of supersymmetry breaking is not a new independent parameter. In perturbation theory, it has to be of the same order with the inverse size of some internal dimension(s) [3]. This leads to a new interest in models with at least one large extra dimension lying in an energy domain where experiments can be performed.

In quantum field theories such models have serious problems: they are not renormalizable, and moreover the coupling constants grow very rapidly to non-perturbative domain above the compactification scale. In the framework of string theory, these problems can be avoided in a class of four-dimensional models which include orbifold compactifications [4]. The crucial property is that all Kaluza-Klein modes which are associated with the large internal coordinate are organized in multiplets of $N = 4$ (spontaneously broken) supersymmetry, leading to cancellations of large radiative corrections among particles of different spins. In particular the evolution of gauge couplings remains logarithmic, as in a four-dimensional theory, up to the Planck scale.

An explicit realization of the minimal supersymmetric standard model in the context of this perturbative breaking mechanism was studied recently and it was found to be extremely restrictive [5]. Quarks and leptons, unlike gauge bosons, should be identified with “twisted” states having no KK-excitations. On the other hand there is a simple choice for the Higgs-sector where the second Higgs doublet can be identified with the first KK-excitation of the first doublet having opposite chirality. In this way, a Higgs-mixing term appears in the effective superpotential
with a coefficient $\mu$ also related to the compactification scale. Another important feature of this mechanism is the absence of quadratic divergences in the vacuum energy. This allows the possibility of generating the large hierarchy between the compactification and Planck lengths radiatively, by minimizing the full one loop effective potential.

It is then important to extract experimental limits on the size of new large dimensions. In this work we obtain upper bounds in the context of orbifold compactifications where exact calculations can be performed. In particular we restrict to the class of models studied in [5] and based on $\mathbb{Z}_N$ ($N > 2$) orbifolds which require at least two large internal coordinates. The main signature of the existence of these dimensions is the appearance of KK-excitations of gauge bosons and higgses (as well as their superpartners). All these massive states are instable and can desintergrate into massless quarks and leptons (twisted states). This is due to the absence of any conserved quantum numbers associated to the massive modes, in contrast to ordinary Kaluza-Klein theories. This property seems to be related to the chiral character of the compactification and it implies that the theory remains four dimensional at all energy scales below $M_p$, as the massive states cannot propagate longer than their short lifetime. However, their exchange gives rise at low energy to effective four-fermion interactions which modify known cross sections or lead to new physical processes. We compute the strength of such interactions and use the present experimental limits to get bounds on the size of extra dimensions.

Similar limits have been derived in the past by a model independent analysis of various experimental data [6]. In the general case there are many parameters related to the couplings of massive to massless states, and thus the simplification of taking into account only the first KK-excitation was made. As we will see, this is not always valid. In fact assuming equal couplings for all massive modes, this approximation is legitimate only in the case of one extra dimension, since the summation over the infinite tower of KK-states leads to a small correction [5]. However, in the case of more than one internal coordinates the sum diverges and the approximation breaks down. To get a finite answer the couplings should decrease with the mass and the result in general depends on their exact form.

For pedagogical reasons, we first recall these computations in the simple case where the large internal dimension is a circle of radius $R$. Then any field $\Phi$ satisfies the periodicity condition

$$\Phi(x, X^4 + 2\pi R) = \Phi(x, X^4),$$  \hspace{1cm} (1)
with $X^4$ the circle coordinate, and it has the Fourier expansion

$$\Phi(x, X^4) = \sum_n \Phi_n(x)e^{i\frac{nX^4}{R}}.$$  \hspace{1cm} (2)

This means that the momentum $P_4$ associated to $X^4$ is quantized in integer multiples of $\frac{1}{R}$. Therefore the particle spectrum forms a tower of KK-states with the same quantum numbers except their masses which are given by:

$$m_n^2 = m_0^2 + \frac{n^2}{R^2},$$ \hspace{1cm} (3)

where $m_0$ is the higher-dimensional mass. For compactifications where $P_4$ is not conserved, the exchange of massive gauge bosons between quarks and leptons gives rise at low energy to an effective interaction described by dimension six four-fermion operators of the form:

$$L_n = -\frac{g_n^2}{2m_n^2}O_{\psi\psi\psi\psi},$$ \hspace{1cm} (4)

$$O_{\psi\psi\psi\psi} = [\eta_{LL}(\bar{\psi}_L\gamma^\mu\psi_L)^2 + \eta_{RR}(\bar{\psi}_R\gamma^\mu\psi_R)^2 + 2\eta_{RL}(\bar{\psi}_R\gamma^\mu\psi_R)(\bar{\psi}_L\gamma^\mu\psi_L)],$$

where $\eta_{IJ}$ parametrize the relative strength of left-left ($LL$), right-right ($RR$) and left-right ($LR$) interactions, while $\eta_{IJ}g_n^2$ are the corresponding coupling constants. In (4) we used for simplicity the same generic symbol $\psi$ for all fermions which in principle can be different species. Note that the exchange of massive scalars with Yukawa couplings lead also to effective four-fermion interactions which can be put in the same form as (4) by appropriate Fierz-transformations.

Assuming $g_n = g$ independent of $n$ and using the mass formula (3), one can perform the sum over all massive modes with the same quantum numbers ($|n| \geq 1$) to get:

$$L_{\text{eff}} = -\frac{g^2}{2}(\frac{\pi}{Rm_0}\coth(\pi m_0 R) - \frac{1}{R^2m_0^2})R^2O_{\psi\psi\psi\psi}.$$ \hspace{1cm} (5)

This result remains valid for large $R$ even if the condition of equal couplings is recovered only asymptotically as $R \to \infty$, provided $g_n/\sqrt{|n|}$ decreases with $|n|$ fast enough that the series converges. In the case of two such large extra dimensions, the same considerations lead to summing on two integer numbers associated to the two quantized momenta but this sum diverges logarithmically. A non-trivial dependence of the couplings on $R$ is then expected to arise from a meaningful theory in order to regularize the divergent expression.
PHYSICS OF KALUZA-KLEIN STATES IN ORBIFOLD MODELS

We turn now to the case of orbifold compactifications of the heterotic string theory [7]. In general, one starts with four-dimensional vacua having $N = 4$ spacetime supersymmetry, like toroidal compactifications, and divide by a discrete symmetry group of the internal space to obtain a chiral spectrum with $N = 1$ supersymmetry. The latter implies a complexification of the internal coordinates, while to realize a $\mathbb{Z}_N$ symmetry with $N > 2$ one needs at least one complex coordinate $X$ compactified on a two-dimensional lattice defined by:

$$X \equiv X + 2\pi R(n_1 + n_2 \theta) ; \quad \theta = e^{\frac{2\pi i}{N}}.$$  \hspace{1cm} (6)

Note that in the limit $R \to \infty$ both dimensions become large. The associated Hilbert space contains two sectors of states:

- The untwisted sector where, going around the string, $X$ is periodic up to a lattice translation. This sector contains the states of the toroidal compactification which are also invariant under the orbifold projection $\mathbb{Z}_N$. Their masses are given in (3) with $n$ replaced by a two-component vector $\vec{n} = (n_1, n_2)$ or, equivalently, by a complex number $n \equiv n_1 + n_2 \theta$. Here, we neglect the string winding modes, whose masses are quantized in units of $R$, and thus they become irrelevant in the limit of large $R$. In the class of models we consider, the KK-excitations are organized in multiplets of $N = 4$ supersymmetry. Moreover, all states of the toroidal compactification are in representations of the $\mathbb{Z}_N$ discrete group. These representations contain states with different quantum numbers, since $\mathbb{Z}_N$ transformations act also on the 2d fermionic superpartner of the internal coordinate $X$. As a result, on the “massless” spectrum ($\vec{n} = 0$), the orbifold acts as a chiral projection which also breaks the gauge group. However the massive KK-states ($\vec{n} \neq 0$) have an additional $\mathbb{Z}_N$-degeneracy associated to the lattice, and all quantum numbers of the $N = 4$ theory remain present even after the orbifold projection.

- The twisted sectors $\{\theta^k\}$ where, going around the string, the internal coordinate $X$ picks up a phase $\theta^k$ (up to a lattice translation). The string center of mass is sitting on a fixed point of the lattice under the transformation $\theta^k$ and the corresponding states do not carry internal momenta. The presence of these states, which do not have KK-excitations, is required from the consistency of the string theory and is related to its chiral character.

The perturbative mechanism of spontaneous supersymmetry breaking in these models, at a scale proportional to $1/R$, leads to a simple pattern of soft breaking
masses at the tree-level [5]. Only the fermions from the untwisted sector acquire a common mass-shift while untwisted bosons and twisted matter remain intact. In particular there is a universal gaugino mass, since gauge bosons arise in the untwisted sector. Moreover, quarks and leptons must be identified with twisted states having no KK-excitations. On the other hand, the higgses can be chosen either in the untwisted or in the twisted sector. The former choice also offers an interesting explanation for the origin of the second Higgs doublet, present in any supersymmetric extension of the standard model, as well as its mixing with the first doublet, which forbids the presence of an unwanted electroweak axion. In fact starting with one massless Higgs doublet at the level of the supersymmetric theory, one can identify the second doublet with its lowest KK-excitation of opposite chirality (and opposite hypercharge) [5]. For the purpose of this work, the effects of supersymmetry breaking are not important and will be neglected in the rest of our analysis.

In the following, we study the main characteristics of the physics of KK-excitations. As already mentioned above, these exhibit a larger symmetry which is present even before the orbifold projection, at the level of the $N = 4$ supersymmetric theory. An $N = 4$ multiplet contains one vector boson, four two-component fermions and six real scalars, or equivalently, one vector and three chiral $N = 1$ multiplets. The interactions between the massive and massless untwisted states are therefore determined entirely by the $N = 4$ symmetry once the gauge group is specified. Considering only the excitations associated to the large complex dimension $X$, and neglecting gravitational interactions, this infinite dimensional gauge group of the $N = 4$ theory, in the large radius limit, is given by [8]:

$$[T^a_n, T^b_m] = f^{ab}{}_c T^c_{n+m}, \quad (7)$$

where $T^a_0$ are the generators of the gauge group $G$ of the massless states and $f^{ab}{}_c$ are the corresponding structure constants. $T^a_n$ with $n \neq 0$ are associated to the massive modes and they extend the gauge algebra to an infinite dimensional Kac-Moody type symmetry.

The discrete orbifold group $Z_N$ corresponds to an automorphism of the algebra (7). In an appropriate basis where $Z_N$ transformations are diagonalized, the generators $T^a_0$ are divided into $N$ sets of eigenvectors with eigenvalues $\theta^k$ ($k = 0, 1, \ldots, N - 1$). The orbifold projection breaks the gauge symmetry, since it keeps in the massless spectrum only the invariant set of generators which form a maximal subgroup $H$ of $G$. At the massive levels, the orbifold group acts also
on the internal momentum taking \( n \) to \( \theta^k n \). This allows to construct \( \mathbb{Z}_N \) invariant excitations for all the generators of \( G \), which are given by the following linear combinations:

\[
\tilde{T}^k_n \equiv \frac{1}{N} \sum_{r=0}^{N-1} \theta^k r \theta^r n ,
\]

where \( T^k_0 \) denote the generators of \( G \) transforming with a phase \( \theta^k \) under \( \mathbb{Z}_N \), and the corresponding gauge indices \( \{ a \} \) were omitted for notational simplicity. \( \tilde{T}^k_n \) satisfy the algebra:

\[
[\tilde{T}^k_n , \tilde{T}^l_m ] = \frac{1}{N} \sum_{r=0}^{N-1} \theta^{l r} \tilde{T}^{k+l}_{n+\theta^r m} ,
\]

where the structure constants of \( G \) were also omitted in the r.h.s.

The exchange of the massive states associated to the generators of \( G/H \), \( \tilde{T}^k_n \) with \( k \neq 0 \), could give rise to new low energy interactions which are in general subject to more stringent experimental constraints. In particular they could lead to phenomenological problems like fast proton decay or flavor changing neutral currents. A part of such potential problems are already avoided since we identified quarks and leptons with twisted states which have no KK-excitations.

In the minimal case where the unbroken low energy group \( H \) is just the Standard model \( SU(3)_c \times SU(2)_w \times U(1)_Y \) with the Higgs doublets in the untwisted sector, there are only three distinct possibilities for the gauge group \( G \) of the \( N = 4 \) supersymmetric theory, before the orbifold projection:

\[
G = SU(3)_c \times SU(3) , \quad G_2 \times SU(3) , \quad F_4 ,
\]

where \( F_4 \) has two different embeddings. The other rank-four groups which have \( SU(3) \times SU(2) \times U(1) \) as maximal subgroup were excluded because they do not contain in their adjoint representations a Higgs like doublet. Moreover \( SU(3)_c \times G_2 \) is not considered because it leads to color singlets KK-modes with fractional electric charges. \( SU(3)_c \times SU(3) \) is the minimal choice which contains only the excitations of one Higgs doublet in addition to those of the Standard model gauge bosons. In fact the other two groups in (10) are simple extensions of this case since they contain \([SU(3)]^2\) as subgroup.

Note that \( G \) is not a real grand-unified group above the decompactification scale because the matter generations are not present at the level of the \( N = 4 \) theory. They appear as twisted states after the orbifold projection and they cannot in general be embedded in representations of \( G \). Consider for instance the simple
$Z_3$ orbifold which breaks $G = E_8$ down to $H = E_6 \times SU(3)$. The matter multiplets in the untwisted sector transform as $(27, 3)$ which is part of the adjoint of $E_8$, while the twisted states transform as $(27, 1)$ or $(1, 3)$ which cannot be embedded in any representation of $E_8$.

The presence of twisted states introduces new interactions which are not constrained by the $N = 4$ symmetry. However, it is easy to show by group theory arguments that ordinary matter fermions couple only to the vector excitations of the Standard model gauge bosons through gauge interactions, and to the scalar excitations of the Higgs doublet through Yukawa interactions. Thus, there are no new effective four-fermion operators and it is sufficient to examine the modifications to known low energy cross sections. Moreover, the Higgs excitations are not expected to provide important model-independent limits as long as the lowest Higgs scalar has not yet been observed. Therefore, our bounds on the size $R$ will be obtained from the exchange of the excitations of gauge bosons.

The resulting four-fermion interaction is of the form (4) where the index $n$ is replaced by a two-dimensional vector $\vec{n}$ associated to the $\vec{n}$-th KK excitation of one of the Standard model gauge bosons. Furthermore, the sum must be done over all $Z_N$ invariant linear combinations $\{\vec{n}\}$ corresponding to the generators $\tilde{T}_n^0$ defined in (8). The coupling constant $g_{\{\vec{n}\}}$ of the $\{\vec{n}\}$-th KK mode (properly normalized) to two massless states from the twisted sectors $\{\theta^k, \theta^{-k}\}$ is [9]:

$$g_{\{\vec{n}\}} = g e^{i \pi \vec{\epsilon} \cdot \vec{n} \delta - \frac{\pi^2}{2 R^2}} ,$$

(11)

where our mass units are defined by setting the string tension $\alpha' = 2$, and $\delta$ is given by $\ln \delta = 2\psi(1) - \psi(k/N) - \psi(1 - k/N)$ with specific values $\delta = 16, 27, 64, 432$ for $k/N = 1/2, 1/3, 1/4$ and $1/6$, respectively. The phase factor in (11) depends on the fixed point associated to the twisted state under consideration; its position is given by $\vec{\epsilon} \pi R$. The strongest bound comes when all four fermions arise at the same fixed point, in which case the phases disappear in the amplitude. Then, the sum over the massive $Z_N$ invariant states, in the region where $m_0 R < 1$, gives:

$$L_{eff} = -\frac{g^2}{2} (c_1 \ln (R^2) + c_2) R^2 O_{\psi \psi \psi \psi} ,$$

(12)

where $c_1 = \frac{2 \pi}{3 \sqrt{3}}, \frac{\pi}{4}, \frac{\pi}{3 \sqrt{3}}$ for $Z_3, Z_4$ and $Z_6$, respectively. The constant $c_2$ has a value of order unity and it is negligible, in the large radius limit, compared to the $\ln (R^2)$ term. The same computation for the case of a $Z_2$ twist with just one large dimension leads to:

$$L_{eff} = -\frac{g^2}{2} \frac{\pi^2}{6} R^2 O_{\psi \psi \psi \psi} .$$

(13)

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Note that in the case where the two pairs of fermions correspond to different fixed points one can take the limit $R \to \infty$ in the expression of the couplings (11), since the sum now converges because of the presence of the phase factor proportional to the relative distance between the two fixed points. The strength of the four-fermion operator behaves now as $R^2$ like in the one-dimensional case.

In (12)-(13), $g$ is the tree level gauge coupling constant at the string scale. However in order to compare with experiments, one has to compute the renormalized four-fermion interaction at low energies. The massive vector bosons exchanged in these processes are associated to spontaneously broken gauge symmetries via a stringy Higgs mechanism [8]. Because of the corresponding Ward identities, the radiative corrections to the propagators of the external fermions cancel against the corrections to the interaction vertices (for an appropriate choice of gauge). Thus, the only contribution to the renormalization of $L_{\text{eff}}$ comes from the radiative corrections to the propagators of the exchanged gauge bosons. These propagators are the sum of two terms, one proportional to the four dimensional flat metric $g_{\mu\nu}$ and another proportional to the corresponding momenta $p_\mu p_\nu$. The latter does not contribute to the amplitude because of the conservation of the external currents.

Let us then denote by $g_{\mu\nu}S(p, R, \delta)$ the sum of all tree level vector boson propagators weighted by a factor $\delta^{-\frac{a^2}{\pi^2}}$ from the interaction vertices (11). Neglecting $m_0$, $S(p, R, \delta)$ is the sum of the massless gauge boson propagator $\frac{1}{p^2}$ with a correction $\delta S$ appearing in the effective four-fermion interaction $L_{\text{eff}}$:

$$S(p, R, \delta) = \frac{1}{p^2} + \delta S \ , \quad (14)$$

where

$$\delta S = -\left(\frac{\pi^2}{6} + \cdots\right)R^2 \quad (15)$$

in the one dimensional example, while in the two dimensional case described above,

$$\delta S = -(c_1 \ln (R^2) + c_2 + \cdots)R^2 \ , \quad (16)$$

where the dots stand for terms which are vanishing in the limit $pR \to 0$.

The one loop self-energy correction has the form $g^{\mu\nu}I_b(p^2) = g^{\mu\nu}p^2g^2bL(p^2)$, where $L(p^2) \equiv \ln \frac{p^2}{M_p^2}$ and $b$ is the $\beta$-function coefficient dependent on the massless modes that propagate in the loop. The massive states do not contribute because of the $N = 4$ supersymmetry. When supersymmetry is broken, one expects small modifications due to threshold corrections of massive particles. The renormalized
one loop effective propagator is obtained by summing over all one loop bubbles. In
the simplest case where all twisted states come from the same fixed point, the result
is:

\[ S_{\text{ren}}(p, R, \delta) = \frac{S(p, R, \delta)}{1 - \frac{1}{p^2} I_b(p^2) - \delta S(p, R, \delta) I_{bT}(p^2)} , \quad (17) \]

where \( bT \) denotes the contribution to the one loop \( \beta \)-function of the twisted states
only. Using (14), the above expression takes the form:

\[ g^2 S_{\text{ren}}(p, R, \delta) = \frac{g^2(p^2)}{p^2} + \frac{g^4(p^2)}{g_U(p^2)} \frac{\delta S}{1 - p^2 \delta S \left( \frac{g^2(p^2)}{g_U(p^2)} - 1 \right)} , \quad (18) \]

where \( g^2(p^2) = \frac{g^2}{1 - g^2 bL(p^2)} \) is the running coupling constant associated to the mass-
less gauge boson, and \( g_U(p^2) \) is the effective coupling which takes into account only
the contribution of the massless untwisted states\(^1\). The first term in (18) can be
identified as the contribution to the amplitude of the massless gauge boson with the
usual running coupling constant, while the second term contains the contribution
of the tower of KK-modes.

Before obtaining the numerical bounds, in order to justify the effective field
theory approach used above, we present a direct derivation of the effective four-
fermion interactions in string theory.

### STRING COMPUTATION OF FOUR-FERMION OPERATORS

In string theory the emission or absorption of a fermionic state is described by
the appropriate Ramond vertex operator \( V^R(z, \bar{z}) \) where \( z \) and \( \bar{z} \) are the world-sheet
coordinates which must be integrated on. In the heterotic string, this operator takes
the form (in the -1/2 ghost picture) [9]:

\[ V^R_{\pm}(z, \bar{z}) = e^{-\varphi/2}(z) S_{\pm}(z) \Psi_{\pm}(z, \bar{z}) e^{i p_{\mu} x^\mu} , \quad (19) \]

where \( \varphi \) is the bosonized super-reparametrization ghost, \( S_{\pm} \) is a four-dimensional
spin field with \( \pm \) the chirality index, and \( \Psi_{\pm}(z, \bar{z}) \) is a Ramond field of the internal
(super)-conformal field theory. After bosonizing the four fermionic coordinates in
terms of two free 2d scalars \( \phi_{1,2} \), one has:

\[ S_+(z) = e^{\pm \frac{i}{2}(\phi_1+\phi_2)} , \quad S_-(z) = e^{\pm \frac{i}{2}(\phi_1-\phi_2)} . \quad (20) \]

\(^1\) Note that \( g^2_U(p^2) \) should not be considered as some physical effective coupling;
it can even take negative values for non abelian groups.
Moreover, in the case of orbifolds when the fermions are in the twisted sector, the internal part of the vertex, $\Psi_\pm$, becomes:

$$\Psi_\pm(z, \bar{z}) = \prod_{j=1}^{3} \sigma^{(j)}_{k_j, \epsilon_j} \Psi^{0 \pm}(z, \bar{z}) e^{\pm i \left(\frac{k_j}{N} - \frac{1}{2}\right) H_j(z) \bar{s}(\bar{z})}$$  \hspace{1cm} (21)

where $j$ labels the three internal planes twisted by $e^{\pm 2i\pi k_j/N}$ ($k_j = 1, ..., N$), and $\sigma^{(j)}_{k_j, \epsilon_j}$ are the corresponding twist operators associated with the fixed points $\epsilon_j$. $H_j$ are three free scalars which bosonize the three complex internal fermionic coordinates, while $\bar{s}$ is an additional right-moving part of the vertex operator containing the gauge group dependence. Finally, space-time supersymmetry requires $\sum_j k_j = N$.

To retrieve for instance an effective four-fermion interaction of the form $(\bar{\psi}_0 \gamma^\mu \psi_1)(\bar{\psi}_2 \gamma^\mu \psi_3)$, we need to compute the following amplitude:

$$A = \int d^2z \langle V^R_{0+}(0)V^R_{1-}(z, \bar{z})V^R_{2+}(1)V^R_{3-}(\infty) \rangle,$$  \hspace{1cm} (22)

where $V^R_i$ are the vertices corresponding to $\psi_i$. In (22), as usual, we have used the global conformal invariance $SL(2, \mathbb{C})$ of the world-sheet sphere to fix the positions of three vertices at 0, 1, and $\infty$. We will derive the effective four-fermion operator generated by the exchange of massive string states in the $s$-channel. Consequently, we can restrict the $z$-integration inside a disk of radius one ($|z| \leq 1$), since the integration outside the disk corresponds by duality to the $u$-channel exchanges.

The non-trivial part of the amplitude (22), involves the correlator of four twist operators [9]:

$$< \sigma^{(j)}_{k_j, \epsilon_j}(0) \sigma^{(j)}_{l_j, \epsilon_j}(z, \bar{z}) \sigma^{(j)}_{k_j, \epsilon_j+\epsilon_1}(1) \sigma^{(j)}_{l_j, \epsilon_j+\epsilon_3}(\infty) >,$$  \hspace{1cm} (23)

where we have chosen two pairs of opposite twists, since we need to describe the exchange of massive gauge bosons which come in the untwisted sector. In (23) we have also chosen the first fixed point to sit at the origin. To illustrate the ideas we present the computation in the simplest case of a $\mathbb{Z}_2$ twist, while the generalization to $\mathbb{Z}_N$ is straightforward. Then, there are two fixed points for each dimension parametrized by a number $\epsilon = 0, 1$ corresponding to the position $\epsilon \pi R$. Furthermore all twists are identical, $k_j/N = l_j/N = 1/2$. To compute the correlation function (23), one goes from the sphere where the twisted quantities are multivalued to a
torus where the functions are well defined \[9\]. This torus is described by a complex parameter
\[
\tau = \tau_1 + i\tau_2 \ (\tau_2 \geq 0)
\]
related to the sphere coordinate \(z\) by:
\[
z = \left(\frac{\theta_2(\tau)}{\theta_3(\tau)}\right)^4,
\]
(24)
where \(\theta_i\) are the usual Jacobi-Theta functions.

Using the form of the fermion vertices (19)-(21), a straightforward computation of the amplitude (22) gives:
\[
\mathcal{A} = g^2 \int d^2 z |z|^{-2+s} |1 - z|^{-2+t} \{(1 - z)(\bar{\psi}_0 \gamma^\mu \psi_1)(\bar{\psi}_2 \gamma_\mu \psi_3) - z(\bar{\psi}_0 \gamma^\mu \psi_3)(\bar{\psi}_2 \gamma_\mu \psi_1)\}
\times \frac{1}{|\theta_3(\tau)|^{2d}} Z_R(\tau, R) Z_{int}(\tau),
\]
(25)
where \(Z_R\) contains the classical contribution of the lattice with the large dimension(s) and \(Z_{int}\) stands for the remaining internal part (contribution of the other twisted coordinates).

\[
Z_R = \frac{R^d}{\tau_2^{d/2}} \sum_{u,v} e^{-|u+v|^2/4\pi\tau_2} \]
(26)
where \(u = \pi R(2\vec{m} + \vec{e}_1), \ v = \pi R(2\vec{n} + \vec{e}_3)\), and the sum is over the integer components of the \(d\)-dimensional vectors \(\vec{m}, \vec{n}; \ d = 1, 2\) in the case of one or two large dimensions, respectively.

In the limit \(R \to \infty\) there are three possible contributions to the sum (26) which are not exponentially suppressed:

\[
\{u = 0 \ , \ \tau_2 \to \infty\} \ , \ \{v = 0 \ , \ \tau \to 0\} \ \text{and} \ \{u = v = 0\}.
\]
(27)
The condition \(u = 0\) implies \(\vec{e}_1 = 0\), while the condition \(v = 0\) implies \(\vec{e}_3 = 0\). This corresponds to the case where the four fermion vertices contain two pairs of twist-antitwist operators situated at two fixed points, 0 and \(\epsilon_3 \pi R \ (u = 0)\) or \(\epsilon_1 \pi R \ (v = 0)\).

The limit \(\tau_2 \to \infty\) is equivalent to \(z \to 0\) where the amplitude is factorized into the exchange of KK-states in the \(s\)-channel. In fact, in this limit \(\theta_3 \to 1\), \(Z_{int}\) goes to a constant, while \(\pi \tau_2 \sim -\ln |z|\) with \(\delta = 16\). After a Poisson resummation in \(\vec{n}\), one finds:
\[
Z_R \sim \sum_{\vec{n}} (-)^{\vec{n} \cdot \vec{e}_3} e^{-\pi \tau_2 \frac{\vec{e}_3^2}{\delta^2}} \sim \sum_{\vec{n}} (-)^{\vec{n} \cdot \vec{e}_3} \left(\frac{|z|}{\delta}\right)^{\frac{\vec{e}_3^2}{\delta^2}}.
\]
(28)
The limit \( \tau_2 \to 0 \) implies \( z \to 1 \) which corresponds to the \( t \)-channel factorization. It can be obtained from the previous case by the transformation \( \tau \to -1/\tau \) sending \( z \to (1 - z) \) and \( s \to t \). The last case of (27) can contribute only when \( \epsilon_1 = \epsilon_3 = 0 \) implying that all four twist-operators come from the same fixed point.

Putting together all three contributions, one finds that in the large radius limit the amplitude (22) becomes:

\[
\mathcal{A} \sim g^2(\bar{\psi}_0\gamma^\mu\psi_1)(\bar{\psi}_2\gamma_\mu\psi_3)\delta_{\epsilon_1,0} \int d^2z|z|^{-2+s}\left\{ \sum_{\vec{n}}(-)^{\vec{n}\cdot\vec{\epsilon}_3}\left(\frac{|z|}{\delta}\right)^{\frac{\vec{n}^2}{R^2}}
\right.
\]

\[
+ \delta_{\epsilon_3,0} \frac{R^d}{\tau_2^{d/2}}(1 - z)^{-2+t}(1 - z)^{-1}\left(\frac{1}{|\theta_3(\tau)|^{2d}}Z_{\text{int}}(\tau) - 1\right)\}
\]

\[- (s \leftrightarrow t, 1 \leftrightarrow 3), \tag{29}\]

where we normalized \( Z_{\text{int}}(i\infty) = 1 \). After integration, the first term in (29) gives at low energy:

\[
g^2(\bar{\psi}_0\gamma^\mu\psi_1)(\bar{\psi}_2\gamma_\mu\psi_3)\left(\frac{1}{s} + R^2\sum_{\{\vec{n}\}}(-)^{\vec{n}\cdot\vec{\epsilon}_3}\frac{\delta^{\vec{n}^2}}{R^2}\right), \tag{30}\]

which reproduces the field theory result. The same expression can also be obtained for general \( Z_N \) twists. The \( 1/s \) pole in (30) corresponds to the exchange of massless gauge bosons, while the sum represents the contribution of the massive KK-modes which was computed in (12) and (13) (for \( \vec{\epsilon}_3 = 0 \) and the corresponding values for \( \delta \)).

The second term in (29) which behaves like \( R^d \) takes into account the contribution of KK-excitations of the massive string states. However its contribution is subleading in all cases, and thus, it can be neglected.

In our previous field-theoretical analysis we studied the low energy effects of the “light” KK-modes which are associated to the massless states. In the class of models we examined above (see discussion after eq. (10)), these states do not lead to any new important interactions, but their main effect consists in modifying known cross sections which we analyzed by computing the strength of the effective four fermion operators. However in any string model there are also superheavy states which may generate new interactions whose couplings are normally suppressed by powers of the Planck mass. These states have also a tower of KK-excitations which may introduce large corrections invalidating the power suppression, when a compactification radius becomes large. For instance, if \( Z_{\text{int}} \) vanishes at all boundaries of \( z \) (0, 1 and \( \infty \)), there is only one possible contribution to the amplitude (25), arising from the third
case in (27) which leads to the second term of “stringy” origin in the integrand (29). Note however that in contrast to supersymmetric D-terms we examine here, non-renormalizable F-terms are exponentially suppressed in the large radius limit [10].

**NUMERICAL RESULTS**

We now derive the numerical bounds. As explained above, these bounds can be mainly obtained from modifications of low energy cross sections due to the effective four-fermion interactions of the form (13), (12) induced by the exchange of the KK-excitations of gauge bosons. The presence of similar effective operators have been investigated in searches for quark and lepton compositeness [11]. The best limits come from $e^+e^- \rightarrow l^+l^-$ experiments with $l = e, \mu, \tau$. In our case, the strength of these effective operators depends on the relative positions of the fixed points associated to the external fermions. In fact, the strongest bound is obtained when the four fermions are identified with twisted states sitting at the same fixed point, giving rise to the additional factor $\ln(R^2)$ in the case of two large dimensions. It turns out that in the class of models we discussed previously, the assumption that the Higgs field comes from the untwisted sector implies that the Yukawa couplings are exponentially suppressed in $R^2$ when the two twisted fermions are in two different fixed points. Thus, to obtain sensible quark and lepton masses, the left $SU(2)$-doublet and the corresponding right singlets must always come from the same fixed point. As a result, in our case, the strongest model independent bound is obtained from $e^+e^- \rightarrow e^+e^-$ processes.

Here, we use the data of TASSO collaboration at PETRA which provides actually the strongest limits on compositeness scale [11]$^2$. Their numerical values of differential cross-sections, given in ref.[13], allowed us to perform a $\chi^2$ fit to the renormalized amplitude (18). In our fit, we used the values $\alpha_{em}(M_Z) = 1/127$, $\sin^2 \theta_W(M_Z) = 0.233$, and putted back our mass units from the relation $\alpha' = \frac{4}{\alpha} M_p^{-2}$ where $\alpha = g^2/4\pi$ is the string coupling constant. The determination of the value of $\alpha$, which is also necessary for the computation of the effective coupling $g^2_U$, is related to the problem of the unification scale. For the numerical values given below, we used $\alpha \sim 0.4$ with a unification scale $\sim 10^{18}$GeV. Furthermore, decreasing the

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$^2$ LEP data, recently analyzed, provide bounds for compositeness scale of the same order of magnitude [12].
unification scale by two orders of magnitude our bounds are increased by around 10%.

For the one dimensional $Z_2$ orbifold the bound for $R$ is low and the approximation made in (13) of neglecting $m_0$ is not good for the case of $Z$-boson. Instead, using (5), we obtain:

$$R^{-1} \gtrsim 150 \text{ GeV}.$$  \hfill (31)

On the other hand, for the two dimensional case of $Z_3$, $Z_4$ and $Z_6$ orbifolds we obtain:

$$R^{-1} \gtrsim 0.84 \text{ TeV}, \quad 0.68 \text{ TeV}, \quad 0.60 \text{ TeV},$$

respectively. It is interesting that these bounds leave open the exciting possibility of producing the lightest KK-excitations in future supercolliders (LHC, SSC).

Finally, one of the important features of these models is that all massive KK-modes are unstable. Here we present the computation of lifetime of the lightest one which can decay only to two massless twisted states. It turns out that the result remains of the same order independently of which gauge boson of the electroweak theory one considers to have the lightest excitation. For instance in the case of the first excitation of the photon, $\gamma^*$, we obtain a width:

$$\Gamma_{\gamma^*} \sim \frac{8}{3R} \alpha_{em}(1/R) \sim \frac{0.02}{R},$$

which leads to a lifetime of the order of $10^{-27}$ seconds for $R^{-1} \sim 1\text{TeV}$.

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