Hard and Soft Contributions in Diffraction: 
A Closer Look

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Abstract: Disentangle the hard and soft dynamics in diffractive DIS is 
one of the main open questions of the strong interactions. We propose the 
study of the logarithmic slope in $Q^2$ of the diffractive structure function as a 
potential observable to discriminate between the Regge and the QCD-based 
approaches. Our results indicate that a future experimental analyzes could 
evidentiate the leading dynamics at $ep$ diffractive processes in the HERA 
kinematical regime.

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The study of electroproduction at small $x$ has lead to the improvement of our understanding of QCD dynamics at the interface of perturbative and non-perturbative physics. However, many important problems remain at present unsolved. A longstanding puzzle in the particle physics is the nature of the Pomeron. This object, with the quantum numbers of the vacuum, was introduced phenomenologically in the Regge theory as a simple moving pole in the complex angular momentum plane, to describe the high-energy behavior of the total and elastic cross-sections of the hadronic reactions [1]. Within the framework of the perturbative QCD (pQCD), the Pomeron is associated with the resummation of leading logarithms in $s$ (center of mass energy squared) and at lowest order is described by the two-gluon exchange [2]. Due to its zero color charge the Pomeron is associated with diffractive events, characterized by the presence of large rapidity gaps in the hadronic final state, which are exponentially suppressed [3]. Diffractive processes in deep-inelastic scattering (DIS) are of particular interest, because the hard photon in the initial state gives rise to the hope that, at least in part, the scattering amplitude can be calculated in pQCD. Moreover, DIS exhibits the nice feature of having a colorless particle, the virtual photon, in the initial state. The main theoretical interest in diffraction is centred around the interplay between the soft and hard physics. Hard physics is associated with the well established parton picture and perturbative QCD, and is applicable to processes for which a large scale is present. Soft dynamics on the other hand, linked for example with the total cross section of hadron scattering, is described by nonperturbative aspects of QCD. The ability to separate clearly the regimes dominated by soft and hard processes is essential in exploring QCD at both quantitative and qualitative level.

In DIS the partonic fluctuations of the virtual photon can lead to configurations of different sizes when analysed in the proton rest frame. The size of the configuration will depend on the relative transverse momentum $k_T$ of the $q\bar{q}$ pair. The small size configurations are calculated using perturbative QCD and at small Bjorken scaling variable $x$ (large $s$) the smallness of the cross section (color transparency) is compensated by the large gluon distribution. For large size configurations one expects to be in the regime of soft interactions. In the inclusive measurement of diffractive final states, where the diffractive structure function is derived, one sums over both small-distance and large-distance configurations. So far there is no theoretical framework which allows one to predict the relative magnitudes of the ”soft” and the
"hard" components of the diffractive cross section. One possibility is the analysis of the energy dependence of the cross section, since we expect that the "soft" component rises weakly with the energy for any fixed mass of the diffractive system, whereas the "hard" part should rise faster. As in the diffractive cross section we integrate over both the perturbative and nonperturbative regions of the phase space, there is a competition between these two pieces. At first sight, the large momentum region seems to be rather subdominant. However, the large gluon distribution function provides an enhancement in this region, and in this way weakening the dominance of the soft nonperturbative region. As a result, the effective value of the exponent $n$ of the energy dependence lies between the hard ($n_{\text{hard}} \approx 1.4$) and the soft ($n_{\text{soft}} \approx 1.12$) values [4].

Since the first observation of diffractive DIS at HERA, several attempts have been made to compare the data with the Regge and QCD-based models [5, 6]. In general, these models provide a reasonable description of the present data on the diffractive structure function $F_2^D$, although based on quite distinct frameworks, demonstrating the inclusive character of this observable to delimit the interplay of soft and hard QCD in diffraction. In this letter we propose the analysis of the logarithmic slope of the diffractive structure function as a potential observable to clarify the dynamics in this process. Our analyzes is motivated by the recent discussions in the literature about the behavior of the logarithmic slope of the inclusive structure function $F_2(x, Q^2)$ as a possible signal of one new regime of QCD [7]. At the moment $e^p$ HERA data on the $F_2$ slope cannot clearly demonstrate the presence of a new dynamics in its kinematical regime, but new studies in $eA$ should distinguish the distinct regimes of QCD [8]. We believe that the experimental analyzes of the logarithmic slopes of $F_2^D$ will allow to discriminate the different contributions to the dynamics already in the current HERA kinematical region.

We study in detail the predictions to this observable considering two distinct approaches: i) a Regge inspired model [9, 10], where the diffractive production is dominated by a nonperturbative Pomeron, and the diffractive structure function is obtained using the Ingelman-Schlein ansatz [11]. ii) a pQCD approach [12] where the diffractive process is modeled as the scattering of the photon Fock states with the proton through a gluon ladder exchange (in the proton rest frame). Before the proper analyzes of the models we need to define the diffractive processes and the usual kinematical variables.
The most important observable at diffractive DIS (DDIS) is the associated structure function $F_2^D$. In this work we are concerned to the $t$-integrated structure function, denoted $F_2^{D(3)}$. The main variables used for the description of DDIS are the total hadronic energy $W$ of the $\gamma^*\text{proton}$ system and the diffractive produced mass $M$. In the analyses of $F_2^D$, it is convenient to use also the variables $\beta$ and $x_{IP}$. In terms of $W$ and $M$, one has $\beta = Q^2/(Q^2 + M^2)$ and $x_{IP} = (M^2 + Q^2)/(W^2 + Q^2)$, where we have neglected the proton mass and the momentum transfer $t$. To connect these variables with the Bjorken scaling variable $x$, we remind that $x = Q^2/(W^2 + Q^2)$, which immediately leads to $x = \beta x_{IP}$. In the kinematic domain of the present experimental measurements, $x_{IP}$ may be interpreted as the fraction of the four-momentum of the proton carried by the diffractive exchange, the Pomeron, if such a picture is invoked. The $\beta$ is the fraction of the four-momentum of the diffractive exchange carried by the parton interacting with the virtual boson.

Diffraction dissociation of virtual photons, observed at HERA $ep$ collider, furnishes the details on the nature of the Pomeron and on its partonic structure. Capella-Kaidalov-Merino-Tran Thanh Van (CKMT) proposed a few years ago a model to diffractive DIS based on Regge theory [9, 10] and the Ingelman-Schlein ansatz, which is based on the intuitive picture of a Pomeron flux associated with the proton beam and on the conventional partonic description of the Pomeron-photon collision. In this case, deep inelastic diffractive scattering proceeds in two steps (the Regge factorization): first a Pomeron is emitted from the proton and then the virtual photon is absorbed by a constituent of the Pomeron, in the same way as the partonic structure of the hadrons. In the CKMT model the structure function of the Pomeron, $F_{IP}(\beta, Q^2)$, is associated to the deuteron structure function through the arguments given above. The Pomeron is considered as a Regge pole with a trajectory $\alpha_{IP}(t) = \alpha_{IP}(0) + \alpha' t$ determined from soft processes, in which absorptive corrections (Regge cuts) are taken into account. Explicitly, $\alpha_{IP} = 1.13$ and $\alpha'_{IP} = 0.25$ $GeV^{-2}$. The diffractive contribution to DIS is written in the factorized form:

$$F_2^D(x, Q^2, x_{IP}, t) = \frac{[g_{pp}^D(t)]^2}{16\pi} x_{IP}^{-1-2\alpha_{IP}(t)} F_{IP}(\beta, Q^2, t),$$

where $g_{pp}^D(t) = g_{pp}^D(0) \exp(C t)$ is the Pomeron-proton coupling, with $[g_{pp}^D(0)]^2 = 23$ mb and $C = 2.2$ $GeV^{-2}$. In this approach, $F_{IP}$ is determined using Regge
factorization and the values of the triple Regge couplings determined from
soft diffraction data. Namely, the Pomeron structure function is obtained
from $F^p_2$, or more precisely from the combination $F^d_2 = \frac{1}{2}(F^p_2 + F^n_2)$, by re-
placing the Reggeon-proton couplings by the corresponding triple reggeon
couplings (see Ref. [9] for details). The following parametrization of the
deuteron structure function $F^d_2$ at moderate values of $Q^2$, based on Regge
theory, was introduced:

$$F^d_2(x, Q^2) = A x^{-\Delta(Q^2)}(1 - x)^{n(Q^2)} \left( \frac{Q^2}{Q^2 + a} \right)^{1+\Delta(Q^2)}$$

$$+ B x^{1-\alpha_R}(1 - x)^{n(Q^2)} \left( \frac{Q^2}{Q^2 + b} \right)^{\alpha_R}$$

(2)

where $1 + \Delta(Q^2)$ is the Pomeron intercept, which depends on the photon
virtuality, and $\alpha_R$ is the intercept of the secondary reggeon (the $f$ trajectory).
The Pomeron structure function $F^p_2$ is identical to $F^d_2$, given above, except
for the following changes in its parameters:

$$F^p_2(\beta, Q^2, t) = F^d_2 \left( x \rightarrow \beta; A \rightarrow eA, B \rightarrow fB, n(Q^2) \rightarrow n(Q^2) - 2 \right).$$

(3)

The values of $e$ and $f$ in $F^p_2$ are obtained from conventional triple reggeon
fits to high mass single diffraction dissociation for soft hadronic processes.
The remaining parameters are given in Refs. [9, 10].

The comparison of the CKMT model with data is quite satisfactory [9, 10].
A remark is that here we use the pure CKMT model [9] rather than to include
QCD evolution of the initial conditions [11], which has been used to improve
the model at higher $Q^2$. Such procedure ensures that we take just a pure
Regge model, without contamination from QCD inspired phenomenology.

On the other hand, the pQCD framework has been recently used by
some authors to describe the diffractive structure function [13], and their
main properties are very similar. We consider for our analyzes the Bartels-
Wüsthoff model and its further parameterization to experimental measure-
ments [12]. The physical picture is that, in the proton rest frame, diffractive
DIS is described by the interaction of the photon Fock states ($q\bar{q}$ and $q\bar{q}g$
configurations) with the proton through a Pomeron exchange, modeled as a
two hard gluon exchange. The corresponding structure function contains the
contribution of $q\bar{q}$ production to both the longitudinal and the transverse
polarization of the incoming photon and of the production of $q\bar{q}g$ final states from transverse photons.

The basic elements of this approach are the photon light-cone wave function and the nonintegrated gluon distribution (or dipole cross section in the dipole formalism). For elementary quark-antiquark final state, the wave functions depend on the helicities of the photon and of the (anti)quark. For the $q\bar{q}g$ system one considers a gluon dipole, where the pair forms an effective gluon state associated in color to the emitted gluon and only the transverse photon polarization is important. The interaction with the proton target is modeled by two gluon exchange, where they couple in all possible combinations to the dipole. Then the diffractive structure function can be written as

$$F_D^2(x_{IP}, \beta, Q^2) \sim \beta \int dt \int \frac{k_t^2 d^2k_t}{(1-\beta)^2} \left| \int \frac{d^2l_t}{l_t^2} D\Psi(\alpha, k_t) F(l_t^2, k_0^2; x_{IP}) \right|^2, \quad (4)$$

where $D\Psi$ is a combination of the concerned wave functions, $l_t$ is the transverse momentum of the exchanged gluons and $F(l_t^2, k_0^2; x_{IP})$ defines the Pomeron amplitude (nonintegrated gluon distribution). The $k_0^2$ sets the hadronic scale which splits the regions of soft and hard QCD. With a suitable anzatz for the $l_t$ dependence of the two-gluon Pomeron, or more precisely the non-integrated gluon distribution, it is possible to interpolate between the hard region where the parton model applies and the soft region where the aligned jet configuration dominates, as emphasized in Ref. [14]. Regarding the $x_{IP}$ behavior, the hypothesis is that for small transverse momentum of the quarks (soft) the energy dependence should be the same as in hadron-hadron scattering. At higher $k_t$ values one expects the Pomeron to be described by the two-gluon model, i.e., the energy dependence will be provided by the square of the gluon structure function of the proton, and consequently a steeper growth. In this model the diffractive structure function is given by:

$$F_D^{D(3)} = F_D^{D(3),I} + F_D^{D(3),II} + F_D^{D(3),III}, \quad (5)$$

where the (I) and (II) contributions correspond to the production of a quark-antiquark pair and the production of a quark-antiquark-gluon system with transversely polarized photon. The third component (III) corresponds to the production of a quark-antiquark pair from a longitudinally polarized photon, which is a contribution at higher twist (twist-4). Here we desconsider the
secondary reggeon contribution dominant at low $\beta$. In the leading twist transverse contribution to $q\bar{q}$ production there is no $\ln(Q^2/Q_0^2)$ enhancement from the phase-space integral, whereas $q\bar{q}g$ production is of higher order in $\alpha_s$ and presents an $\alpha_s \ln(Q^2/Q_0^2)$ dependence. The longitudinal contribution belongs to higher twist and the phase-space integral gives a $\ln(Q^2/Q_0^2)$ enhancement. In a comparison with data, the transverse $q\bar{q}$, $q\bar{q}g$ production and the longitudinal $q\bar{q}$ production dominate in distinct regions in $\beta$, namely medium, small and large $\beta$ respectively [12]. The $\beta$ spectrum and the $Q^2$-scaling behavior follow from the evolution of the final state partons, and are derived from the light-cone wave functions of the incoming photon, decoupling from the dynamics inside the Pomeron, while the energy dependence and the normalizations are free parameters.

Before to perform the analyzes of the presented models, some comments are in order. In the first extraction of the $F_2$ slope data at HERA, the so-called Caldwell plot [15], the variables $x$ and $Q^2$ were strongly correlated due to the poor statistics. Since a similar situation should be present in the first studies of the diffractive slope, in this work we consider a kinematical constraint which relates the variables $x$ and $Q^2$, taken from the most recent global analyzes of the MRST group [16], where the behavior of the proton structure function slope was considered. We will address the behavior of the $F_2^{D(3)}$ slope without such constraint in a forthcoming paper. Below, we present our results for the logarithmic slopes of the diffractive structure function considering the kinematical constraint.

Starting by the pure CKMT model, we show the dependence on $x_{IP}$ of the logarithmic slope at three distinct fixed $\beta$ values in Fig. 1 (a). The slope is ever positive for small $\beta = 0.04$. For medium and high $\beta$ (0.4 and 0.9 values) the slope is negative for $x_{IP} < 10^{-3}$. Moreover, the CKMT provides a transition between positive and negative slope values at $\beta = 0.4$. This behavior is consistent since the Pomeron structure function in this model is related to the nucleon structure function $F_2$ [Eq. (3)], which presents that feature due to the scaling violation.

The pQCD model provides a quite different result, as presented in the Fig. 1 (b). The slope is predominantly positive in almost all $\beta$ range, taking negative values only at $\beta = 0.9$ for the interval $x_{IP} < 0.0004$. A remarkable feature is the existence of a $\beta$ dependent turn over, which is shifted to greater $x_{IP}$ values as $\beta$ increases. The positive behavior of the slope at low values of $\beta$ is associated to the $q\bar{q}g$ contribution, while for intermediate $\beta$ the $q\bar{q}T$
state dominates, producing an almost constant function on $Q^2$. The high $\beta$ behavior is consistent with the H1 measurements, in the region $x_{IP} > 10^{-3}$, which prefer a positive slope in $Q^2$, corresponding to a large $q\bar{q}G$ contribution in this region [12].

For both models the slope converges to a flat behavior at large values of $x_{IP}$, with different behaviors at small $x_{IP}$ corresponding to low virtualities ($Q^2 \leq 5$ GeV$^2$). Indeed, the kinematical constraint implies that at $x_{IP} = 10^{-4}$ we are probing $Q^2 \sim 10^{-3}$, which is far from the current HERA kinematical region. Confronting the approaches, we conclude that both models predict a positive slope up to $\beta \sim 0.4$, with a steeper decreasing in CKMT. The high $\beta$ region discriminates the behaviors. The pQCD results hold a positive slope, while CKMT produces negative values. These come from the fact that the CKMT approach does not include the $q\bar{q}g$ contribution, which is dominant in this region for the pQCD model. Since the $Q^2$-behavior in the CKMT is determined by the $F_2$ scaling violations, then it only includes at most the $q\bar{q}_{T,L}$ contributions. Therefore, the experimental analyzes in this specific region of the slope should clarify the dynamics in diffractive DIS.

We also present the $Q^2$-slope as a function of the variable $\beta$ in Fig. 2 for typical values of $x_{IP}$. Some of the remarkable features are: (i) the CKMT and the pQCD model provide a similar shape (flat behavior) for the whole interval of $\beta$ at $x_{IP} \geq 10^{-3}$; (ii) a noticeable difference between the Regge and the pQCD-inspired model in the region of small values of $x_{IP}$ ($10^{-4}$), with the prediction of a turn-over at $\beta = 0.1$ from CKMT while for the pQCD one expects the turn-over at $\beta = 0.5$. Again, the scaling violations of $F_2$ drive the behavior of the $Q^2$-slope in CKMT, which implies positive values of the slope at $\beta < 0.5$ and negative values at larger values. Moreover, this connection implies the large value of the slope at small $\beta$ and a similar turnover that one found in the first measurements of inclusive structure function slope [15].

In Fig. 3 we show the results for $d\ln F_2^D/d\ln(1/x_{IP})$ (or shortly, $x_{IP}$-slope) as a function of the photon virtuality $Q^2$. Indeed, this quantity gives the Pomeron intercept and its behavior describes the energy dependence of the diffractive structure function. While the CKMT model predicts a constant value, without dependence on $\beta$, the pQCD model presents a dependence on the $\beta$ value considered. This feature is associated to the distinct energy dependence of each term in Eq. (5), which dominates at specific regions of the phase space. A feature in the result is the characteristic shape of this slope at $\beta = 0.9$, providing a clearly hard intercept. In fact a dependence on
\(\beta\) for the Pomeron intercept is expected as shown in Ref. [12]. In principle, the model is only valid above of the starting point \(Q_0^2 = 1\,\text{GeV}^2\), however one extrapolated it for lower virtualities for comparison. For completeness we include the soft Pomeron intercept (Donnachie-Landshoff) [17] in the plot. We verify, therefore, the evident distinction between the prediction from the CKMT and pQCD based approaches.

New quantities to distinguish the regimes of QCD have been argued for future measurements [18]. The available experimental results seem already allow to extract information about the slope of the diffractive structure function, which we propose to study as a potential source to discriminate between the hard and soft contribution in diffraction. Considering two sound models in the literature, we verify that the results are quite distinct, allowing to characterize the dynamics through that quantity.

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**Figure Captions**

Fig. 1: The $x_P$ dependence of the $Q^2$-slope at some typical values of $\beta$ for: (a) the pure CKMT model [9]; (b) the pQCD-inspired model [11]. Kinematical constraint $Q^2 = Q^2(x)$ from the MRST group [13] was used.

Fig. 2: The $\beta$ dependence of the $Q^2$-slope at some typical values of $x_P$ for: (a) the pure CKMT model [9]; (b) the pQCD-inspired model [11]. The kinematical constraint $Q^2 = Q^2(x)$ from the MRST group [13] was used.

Fig. 3: The $x_P$-slope versus $Q^2$ for the pQCD approach (BW) [11] and the CKMT model [9]. The Donnachie-Landshoff intercept [14] is also depicted.
Figure 1:
Figure 2:
Figure 3: