Eliminating the ‘flatness problem’
with the use of Type Ia supernova data

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Abstract
Recent measurements of the cosmological constant or cosmic vacuum in Type Ia SN observations (Riess et al.1998, Perlmutter et al. 1999) imply that $\Omega(t)$ is exactly unity or nearly unity at any epoch of cosmic evolution. No fine tuning is needed to describe this phenomenon with the standard cosmological model based on the Type Ia SN data. A time-independent symmetry relation between vacuum and matter is basically behind the observed near-parabolic expansion.

Key words: cosmology: theory, dark matter, miscellaneous.
1 Introduction

It was first mentioned long ago (Dicke 1970), that the universe must be ‘extremely finely tuned’ to yield the present observed balance between the kinetic energy of expansion $K$ and the gravitational potential energy of cosmic matter $U$. The balance is usually quantified in terms of $\Omega$, which is the energy ratio, $|U|/K$, and also the ratio of the total matter density, $\rho$, to the critical density, $\rho_c$. Thirty years ago, observational limits on $\Omega$ were described as $0 < \Omega_0 < 10$, and it was argued that such an apparently wide range implies a very narrow range at earlier epochs. It has been estimated more than once since that $\Omega$ departs from unity by one part in $10^{16}$ at the epoch of light element production, or by one part in $10^{60}$ at the Planck epoch. Why was there such a remarkably fine balance between the kinetic and potential energies in the ‘initial conditions’ for the cosmic evolution?

This argument has become known as the ‘flatness problem’, since the energy ratio is associated with the sign of the spatial curvature in the Friedmann model. It serves as one of the main motivations for the idea of inflation (Guth 1981); this idea suggests an elegant solution to the problem (Guth 1981, Linde 1990). Various aspects of the problem have discussed by Frieman & Waga (1998), Kaloper et al. (1999), Berera et al. (1999), Albrecht & Magueijo (1999), Barrow & Magueijo (1999), Kaloper & Linde (1999), Clayton & Moffat (1999), Avelino & Martins (1999), Arkani-Hamed et al. (2000), Piazza et al. (2000), Kavasaki et al. (2000), Roos & Harun-or-Rashid (2000), Kaganovich (2001), Tegmark et al. (2001), Barrow & Kodama (2001), Starkman et al. (2001), Youm (2001), Deffayet et al. (2001), to mention only some publications of the last 2-3 years.

In this Letter, I show that the recent Type Ia supernova measurements (Riess et al. 1998, Perlmutter et al. 1999) displays the fine-tuning argument in a new light. In Sec.2, a brief account of the concordant data and the new standard model which is based on them is given; new general constraints on $|\Omega(t) - 1|$ for any epoch of the cosmic evolution are presented in Sec.3; the results are discussed in Sec.4.

2 Standard model re-visited

The dynamical equation of the Friedmann cosmology has a form of energy conservation ($K + U = \text{Const}$):

$$a^2 = (a/A_V)^2 + (a/A_D)^{-1} + (a/A_B) + (a/A_R)^{-2} - k,$$

where $a(t)$ is the curvature radius and/or scale factor of the model; $k = +1, 0, -1$ for elliptic ($K + U < 0$) expansion with positive spatial curvature, parabolic expansion ($K + U = 0$) with zero curvature and hyperbolic expansion ($K + U > 0$) with negative curvature, respectively.

The constants $A$ in Eq.(1) represent the four major cosmic energy components which are vacuum (V), dark matter (D), baryons (B) and radiation or relativistic energy (R). The constants are the integrals of the Friedmann ‘thermodynamical’ equation:

$$A = \left[\frac{8\pi G}{3} \rho a^{3(1+w)} \right]^{1/1+w},$$

\[2\]}
where $w$ is the pressure-to-density ratio which is $-1,0,0,1/3$ for vacuum, dark matter, baryons and radiation, respectively. Hereafter, the integrals $A$ are called the Friedmann constants. The constants are related to the ‘initial condition’, under which the cosmic energy forms, vacuum including, were generated in the early universe.

The Friedmann constants can be evaluated quantitatively with the use of the current concordant figures for the four energy densities (Riess et al.1998, Perlmutter et al. 1999, see also for a review Primack 2000):

$$\Omega_V = 0.7 \pm 0.1, \Omega_D = 0.3 \pm 0.1, \Omega_B = 0.02 h_{100}^{-2}, \Omega_R = 0.6 \alpha \times 10^{-4}, 1 < \alpha < 10^{-30}. \tag{3}$$

These figures are in general agreement with the Hubble constant $h_{100} = 0.70 \pm 0.15$ and the cosmic age $t_0 = 15 \pm 3$ Gyr. More rigorous recently published upper and lower bounds on the cosmic age are $13.2^{+1.2}_{-0.8}$ Gyr for $h_{100} = 0.72 \pm 0.08$ (Ferreras et al. 2001).

The present-day value of $a(t)$ is determined by these figures; approximately, on the order of magnitude, one has: $a_0 = a(t_0) \sim A_V$, for all the three types of expansion and the signs of spatial curvature.

The four Friedmann constants calculated with the concordant data are proved to be coincident, on the order of magnitude (Chernin 2001):

$$A_V \sim A_D \sim A_B \sim A_R \sim A \sim 10^{60 \pm 1} M_{Pl}^{-1}. \tag{4}$$

Here, units are used in which the speed of light, the Boltzmann constant and the Planck constant are all equal unity: $c = k = \hbar = 1$. The Planck mass is $M_{Pl} = G^{-1/2} \simeq 1.2 \times 10^{19}$ GeV.

The time-independent coincidence of the Friedmann constants described by Eq.(4) looks like a symmetry relation that brings vacuum into association with non-vacuum cosmic energies. (A similar relation for baryons and radiation was recognized soon after the discovery of the CMB – Chernin 1968). As I show below, the symmetry relation between vacuum and matter puts a robust upper bound on $|\Omega(t) - 1|$ at any epoch of cosmic evolution.

### 3 Setting constraints on $|\Omega(t) - 1|$.

The concordant data of Sec.2 narrow essentially the observational bounds on $\Omega(t_0) = \Omega_0$, in comparison with what was discussed three decades ago (see Sec.1). A conservative restriction can be seen from Eq.(3) above: $0.8 < \Omega_0 < 1.2$. More stringent limits (like $\Omega_0 = 0.97 \pm 0.05$) are also advocated in current literature (Roos & Harun-or-Rashid 2001). From the point of view of the fine-tuning argument, the new data make the problem harder than it was in 1970, but not too much – it was hard enough from the very beginning. It is more significant that the Type Ia supernova measurements (Riess et al.1998, Perlmutter et al. 1999) have changed the very sense of the problem.

In the new standard model described in Sec.2, the dynamics of expansion exhibits the parabolic ($K + U = 0$) behaviour as $t$ goes to both zero and infinity for flat, closed and open models, as is seen from Eq.(1). Considerable (maximal) deviations from the regime can be expected for $k \neq 0$ in an ‘intermediate’ cosmic time interval.
As for spatial geometry, the ratio of the horizon radius to the radius of 3-curvature, \( \frac{ct}{a} \), may be a practical measure of non-flatness. This ratio goes to zero when time goes to both zero and infinity, as is also seen from Eq.(1). It means that non-flatness vanishes at early and late epoch of the cosmic evolution. In both closed and open models, the radius of 3-curvature is comparable with the radius of the horizon at the intermediate times. In this sense, non-flatness is finite and, generally, not small in the intermediate time interval, including the present epoch at which \( a \sim A_V \sim ct \) (see. Sec.2).

To put these considerations in a quantitative way, let us follow the evolution of \( \Omega \) as a function of time. It is easy to see from Eq.(1) that

\[
\Omega(t) = \left[ 1 - k \left( \frac{8\pi G}{3} \rho(t) a(t)^2 \right)^{-1} \right]^{-1},
\]

where \( \rho(t) \) is the total density:

\[
\frac{8\pi G}{3} \rho = A_V^{-2} + A_D a^{-3} + A_B a^{-3} + A_R^2 a^{-4}.
\]

In the limits \( t \to 0 \) and \( t \to \infty \), the function \( \Omega(t) \) reaches the unity level, and the maximal deviation from unity may be expected in the intermediate time interval mentioned above. In this time interval (starting, say, with \( a \geq (0.01 - 0.1) a_0 \)), the most significant contributions to the total density \( \rho \) are provided by the vacuum density \( \rho_V \) and the dark matter density \( \rho_D \). Thus, with good accuracy, one can re-write Eq.(5) in the simple form:

\[
\Omega(t) \simeq \left[ 1 - k \left( \frac{a^2}{A_V^2} + \frac{A_D}{a} \right)^{-1} \right]^{-1}.
\]

At \( a = a_{ex} = \left( \frac{1}{2} A_V A_D \right)^{1/3} \) the function \( \Omega(t) \) has an extremum,

\[
\Omega_{ex} \simeq \left[ 1 - \frac{1}{2} k (\frac{A_D}{A_V})^{2/3} \right]^{-1}.
\]

Putting in accordance with Eq.(4) \( A_V \simeq A_D \), one finds that there are a minimum \( \Omega_{ex} \simeq 2/3 \) for \( k = -1 \) and a maximum \( \Omega_{ex} \simeq 2 \) for \( k = 1 \).

This leads to upper bounds on deviation from the unity level of \( \Omega(t) \) valid for all the times of the cosmic evolution:

\[
\Omega(t) - 1 \leq 1, \quad k = 1; \quad |\Omega(t) - 1| \leq 1/3, \quad k = -1.
\]

Defined and estimated in this way, the upper bounds of Eq.(8) prove to be rather severe. They rule out completely any possibility of large deviations from the near-parabolic dynamics at any epoch of cosmic expansion. Numerically, the bounds are not too different from the present-day observed figures; the latter could be expected, because the epoch of extremum, \( a_{ex} \simeq 2^{-1/3} A_V \), is not far from us in look-back time.

### 4 Discussion

The new standard model inspired by the Type Ia supernova measurements (Riess et al.1998, Perlmutter et al. 1999) describes not only the present transition epoch of the
cosmic expansion, but as well the early epoch of matter (radiation) domination and the later epoch of vacuum domination (Sec.2). The model leads to the strong conclusion that the cosmic expansion is exactly parabolic or nearly parabolic during all the cosmic evolution. Eqs.(5-8) show this in an explicit quantitative way and clarify also the basic cause of the nearly-parabolic dynamics in the standard model.

The physics which is behind the phenomenon is directly related to the Friedmann constants of the standard model. According to Eq.(7), the constraints on possible deviations from the parabolic dynamics are expressed in the terms of the Friedmann constants. The (practically exact) relation of Eq.(7) includes the vacuum constant $A_V$ and the dark matter constant $A_D$. The Type Ia SN data put the constants in a simple relation: $A_V \sim A_D$. This approximate relation is enough to provide the upper bound on possible deviations from the parabolic regime of the cosmic expansion for all the cosmic time.

The quantitative estimate of Eqs.(7,8) and the qualitative conclusion that follows from it are obviously stable and robust: they are insensitive to small variations of the constants involved. Moreover, the estimate does not depend on the absolute values of the individual constants; only the constant ratio $A_D/A_V$, which is unity or near unity, affects the result. In this sense, the presently observed near-parabolic expansion is completely controlled by only one order-of-unity empirical parameter of the model.

Thus, the long-standing problem of the near-parabolic dynamics finds a clear solution: the cosmic expansion is basically as it is because the Friedmann constants are nearly coincident.

There is no need in addressing a hypothetical pre-Friedmann inflation to solve the problem. As for the fine-tuning argument (see Sec.1), it appears now in a quite different light. As is seen from Eqs.(5-7), the seemingly fine-tuned inter-relation between the time-dependent kinetic $K$ and potential $U$ energy at an early epoch is robustly determined by the time-independent symmetry relation of Eq.(4).

Instead of the ‘fine tuning’ argument, one faces now a new question: Why are the Friedmann constants nearly coincident?

The physical nature of the Friedmann constants seems to be associated with the origin of the cosmic energy forms in the very early universe. One may expect that the physical processes that developed at that time might bring the four cosmic energies into a correspondence with each other. A possible approach to the problem (Chernin 2001) assumes that the freeze-out process at the epoch of the electroweak ($\sim$ 1 TeV) temperatures might be responsible for establishing the symmetry relation between vacuum and matter. If so, the ‘initial conditions’ for the observed cosmic dynamics were generated in the terms of the Friedmann constants at cosmic age $t \sim 10^{-12}$ sec.

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