Intrinsic Spin Hall Effect

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Abstract. A brief review is given on a spin Hall effect, where an external electric field induces a transverse spin current. It has been recognized over 30 years that such effect occurs due to impurities in the presence of spin-orbit coupling. Meanwhile, it was proposed recently that there is also an intrinsic contribution for this effect. We explain the mechanism for this intrinsic spin Hall effect. We also discuss recent experimental observations of the spin Hall effect.

1 Introduction

In the emerging field of spintronics [1], it is important to understand the nature of spins and spin current inside semiconductors. There have been many proposals for semiconductor spintronics devices, whereas their realization remains elusive. One of the largest obstacles is an efficient spin injection into semiconductors. One way is to make semiconductors ferromagnetic, such as (Ga, Mn)As [2]. The Curie temperature is, however, still lower than the room temperature, and there are still rooms for improvement towards practical use.

Spin Hall effect (SHE) can be an alternative way for efficiently injecting spin current into semiconductors. In the first proposal of the spin Hall effect by D’yakonov and Perel [3], followed by several papers [4,5], the SHE has been considered as an extrinsic effect, due to impurities in the presence of spin-orbit (SO) coupling. Nevertheless, quantitative estimate for this extrinsic SHE is difficult, and this extrinsic effect is not easily controllable.

In 2003 two groups independently proposed an intrinsic spin Hall effect in different systems. Murakami, Nagaosa, and Zhang [6] proposed it in p-type semiconductors like p-GaAs. On the other hand, Sinova et al. [7] proposed a spin Hall effect in n-type semiconductors in two-dimensional heterostructures. This induced spin current is dissipationless, and can flow even in nonmagnetic materials. It shares some features in common with the quantum Hall effect.

Because the predicted effect is large enough to be measured, even at room temperature in principle, this intrinsic SHE attracted much attention, and many related works, have been done. Nevertheless there remain several issues, relevant also for experiments. One of the important questions is disorder effect. While there are a lot of works on disorder effect, the most striking one is by Inoue et al. [8]. They considered dilutely distributed impurities with short-ranged potentials, and calculate the SHE, incorporating the vertex
correction in the ladder approximation. Remarkably the resulting spin Hall conductivity is exactly zero in the clean limit. This work made many people to consider that the SHE is “fragile” to impurities; namely, only a small amount of impurities will completely kill the intrinsic SHE. However, this is not in general true. In fact, the spin Hall conductivity is in general nonzero even in the presence of disorder, as we see later.

In such circumstances, two seminal experiments on the SHE have been done. Kato et al. [10] observed spin accumulation in n-type GaAs by means of Kerr rotation. Wunderlich et al. [11] observed a circularly polarized light emitted from a light-emitting diode (LED) structure, confirming the SHE in p-type semiconductors. Separation between the intrinsic and extrinsic SHE for these experimental data is not straightforward, and is still under debate.

The paper is organized as follows. In Sect. 2 we explain basic mechanisms and features for the intrinsic SHE. Section 3 is devoted to a disorder effect on the SHE. In Section 4 we collect a number of recent interesting topics on the SHE. In Section 5 we introduce two recent experimental reports on the SHE. We conclude the paper in Sect. 6.

2 Intrinsic spin Hall effect

In this section we explain the two theoretical proposals for the intrinsic SHE.

2.1 Spin Hall effect in p-type semiconductors

We begin with semiclassical description of the SHE, and apply it to the p-type semiconductors [6]. In this description, we introduce a “Berry phase in momentum space”. The Berry phase [12, 13, 14] is a change of a phase of a quantum state caused by an adiabatic change of some parameters. As we explain later, Berry phase in momentum space gives rise to the Hall effect, as first demonstrated for the quantum Hall effect [15, 16, 17]. Here, the wavevector $k$ is regarded as adiabatically changing due to a small external electric field. In two-dimensional systems, for example, the Hall conductivity $\sigma_{xy}$ in a clean system is calculated from the Kubo formula as

$$\sigma_{xy} = -\frac{e^2}{2\pi\hbar} \sum_n \int_{BZ} d^2k \ n_F(\epsilon_n(k)) B_{nz}(k),$$

(1)

where $n$ is the band index, and the integral is over the entire Brillouin zone. $B_{nz}(k)$ is defined as the $z$ component of $B_n(k) = \nabla_k \times A_n(k)$, where

$$A_n(k) = -i \left\langle nk \left| \frac{\partial}{\partial k_l} \right| nk \right\rangle \equiv -i \int_{\text{unit cell}} u_{nk}^\dagger \frac{\partial u_{nk}}{\partial k_l} d^2x,$$

(2)

and $u_{nk}(x)$ is the periodic part of the Bloch wavefunction $\phi_{nk}(x) = e^{ik \cdot x} u_{nk}(x)$. This $B_{nz}(k)$ represents the effect of Berry phase in momentum space. $n_F(\epsilon_n(k))$
is the Fermi distribution function for the \( n \)-th band. This intrinsic Hall conductivity was first recognized in the paper by Karplus and Luttinger. This Berry phase in momentum space has been studied in the recent works on anomalous Hall effect (AHE) and those on the SHE.

By incorporating the effect of \( B(k) \), the Boltzmann-type semiclassical equation of motion (SEOM) acquires an additional term:

\[
\dot{x} = \frac{1}{\hbar} \frac{\partial E_n(k)}{\partial k} + k \times B_n(k), \quad \hbar \dot{k} = -e(E + \dot{x} \times B(x)).
\]

The term \( k \times B_n(x) \) represents the effect of Berry phase, and it is called an anomalous velocity. Under the external electric field, the anomalous velocity becomes perpendicular to the field, and gives rise to the Hall effect. This Hall current is distinct from the usual Ohmic current, which comes from the shift of the Fermi surface from its equilibrium. This Hall effect comes from all the occupied states, not only from the states on the Fermi level. By summing up the anomalous velocity over the filled states, one can reproduce the Kubo formula result. Given the Hamiltonian, the vector field \( B(k) \) is calculable, and we can get the intrinsic Hall conductivity, as in the ab initio calculation of the AHE in. Due to remarkable similarity of the two equations in Eq. (3), \( B_n(k) \) can be regarded as a “magnetic field in \( k \)-space. \( B_n(k) \) can have monopoles, and such monopoles can give nontrivial topological structure for magnetic superconductors.

**Fig. 1.** Schematic band structure for GaAs. CB, HH, LH, SO represent the conduction, heavy-hole, light-hole and split-off bands, respectively.
This anomalous velocity leads to the SHE in semiconductors with diamond structure (e.g. Si) or zincblende structure (e.g. GaAs). The valence bands consist of two doubly degenerate bands: heavy-hole (HH) and light-hole (LH) bands. They are degenerate at \( k = 0 \) as shown in Fig. 1. Near \( k = 0 \), the valence bands are described by the Luttinger Hamiltonian \[ H = \frac{\hbar^2}{2m} \left[ \left( \gamma_1 + \frac{5}{2} \gamma_2 \right) k^2 - 2 \gamma_2 (k \cdot S)^2 \right], \]

where \( S \) is the spin-3/2 matrices representing the total angular momentum. For simplicity, we employed the spherical approximation for the Luttinger Hamiltonian, while a calculation without it is also possible \[27\]. In this Hamiltonian, a helicity \( \lambda = \frac{k \cdot S}{k} \) is a good quantum number, and can be used as a label for eigenstates. The HH and LH bands have \( \lambda_H = \pm \frac{3}{2} \) and \( \lambda_L = \pm \frac{1}{2} \), respectively. The SEOM reads as

\[ \dot{x} = \frac{1}{\hbar} \frac{\partial E_{\lambda}(k)}{\partial k} + k \times B_{\lambda}(k), \quad \hbar \dot{k} = e E. \]

Because we are considering holes, the sign of the charge has been changed.

By straightforward calculation, we get \( B_{\lambda}(k) = \lambda (2 \lambda^2 - \frac{7}{2}) k/k^3 \). Hence, the anomalous velocity due to Berry phase is along the direction \( E \times k \). By integration in terms of the time \( t \), we get a trajectory of the holes as shown in Fig. 2. This shows the motion projected on a plane perpendicular to \( E \).

We note that a semiclassical trajectory can be calculated directly from the Heisenberg equation of motion; the resulting trajectory agree well with the one from \[19\], but with small rapid oscillations \[25\]. This justifies validity of the SEOM \[5\] for adiabatic transport, which was questioned in \[29\].

**Fig. 2.** Trajectory of holes from the semiclassical equation of motion with Berry-phase terms. This is a projection on the plane perpendicular to the electric field \( E \). The transverse shift of the trajectory is to the opposite direction, depending on the sign of the helicity \( \lambda = k \cdot S \). The bold arrows represent the direction of spin \( S \).

Due to the anomalous velocity, the motion of the holes is deflected from an otherwise straight motion along \( k \) (dashed line). The shift of the motion is opposite for the signs of the helicity \( \lambda \), referring to whether the spin \( S \) and the wavevector \( k \) are parallel or antiparallel. This shift amounts to the SHE. By summing up this shift over the occupied states, we can calculate a spin.
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If the electric field is along \( l \)-axis, the spin current with \( S^l \) spin flowing toward the \( j \)-direction is

\[
j^j_l = \frac{e}{12\pi^2} (3k^H_F - k^L_F) \epsilon_{ijl} E_l,
\]

where \( k^H_F \) and \( k^L_F \) are the Fermi wavenumber for the HH and LH bands, respectively. This is schematically shown in Fig. 3. Nominal values obtained for p-GaAs are of the similar order of magnitude as the conductivity at room temperature. In GaAs, the nominal energy difference of the two bands is larger than the room temperature, and the effect can in principle survive even at room temperature.

![Fig. 3. Schematic of the spin current induced by an electric field](image)

### 2.2 Spin Hall effect in n-type semiconductors in heterostructure

In n-type semiconductors with diamond or zincblende structure, the SO coupling is small. On the other hand, however, if they are incorporated into two-dimensional heterostructure, the inversion symmetry is broken, and the SO coupling becomes relevant. The Hamiltonian is approximated as

\[
H = \frac{k^2}{2m} + \lambda (\sigma \times k) z,
\]

where \( \sigma_i \) is the Pauli matrix. The second term is called the Rashba term \[30,31\], representing the SO coupling. The coupling constant \( \lambda \) can be experimentally determined, and can be controlled by the gate voltage \[32\].

Sinova et al. applied the Kubo formula to this Rashba Hamiltonian \[7\]. For this procedure, they defined the spin current \( J^S_y \) to be a symmetrized product of the spin \( S^z \) and the velocity \( v_y = \frac{\partial H}{\partial k_y} \). By assuming no disorder, the resulting spin Hall conductivity is \( e/(8\pi) \), which is independent of the Rashba coupling \( \lambda \). The Rashba term in \( \lambda \) can be regarded as a \( k \)-dependent effective Zeeman field \( B_{\text{eff}} = \lambda (\hat{z} \times \mathbf{k}) \). In an equilibrium the spins are pointing either parallel or antiparallel to \( B_{\text{eff}} \) for the lower and upper
bands, respectively. An external electric field $E \parallel \hat{x}$ changes the wavevectors $k$ of Bloch wavefunctions, and $B_{\text{eff}}$ also changes accordingly. The spins will then precess around $B_{\text{eff}}$, and tilt to the $\pm z$-direction, depending on the sign of $k_y$. This appears as the SHE, and the spin Hall conductivity is calculated to be $\frac{e}{8\pi}$, in agreement with the Kubo formula. We can incorporate also the Dresselhaus term representing the bulk inversion-symmetry breaking of the zincblende structure. The Hamiltonian becomes

$$H = \frac{k^2}{2m} + \lambda(k_x\sigma_y - k_y\sigma_x) + \beta(k_x\sigma_x - k_y\sigma_y). \quad (8)$$

The spin Hall conductivity $\sigma_s$ is given as follows; $\sigma_s = \frac{e}{(8\pi)}$ for $\lambda^2 > \beta^2$, and $\sigma_s = -\frac{e}{(8\pi)}$ for $\lambda^2 < \beta^2$ [33, 34].

When a perpendicular magnetic field is applied, the spin Hall conductance will have a resonant behavior as a function of the magnetic field [35, 36]. The Zeeman splitting induces degeneracies between different Landau levels, and if this degeneracy occurs at the Fermi level, the spin Hall conductance is divergent. An in-plane magnetic field also affects the SHE, as studied in [37].

Close relationship between the SHE and the Pauli spin susceptibility [38, 39, 40] or the dielectric function [41] has been argued for these models. An effect of electron-electron interaction has been investigated by use of this relationship [40]. In the Luttinger model, on the other hand, these three have different frequency dependence [42]. Thus it is not clear whether this relationship remains for disordered case or for generic SO-coupled models.

### 2.3 General properties of the intrinsic spin Hall effect

We consider the intrinsic SHE to be robust against spin relaxation. Momentum relaxation induces rapid spin relaxation via the SO coupling. Namely, if the Fermi surface deviates from its equilibrium position, the momentum and spin distributions will rapidly relax. Nevertheless, because the spin Hall current comes from the anomalous velocity, it will survive even when the momentum and spin relaxation is in equilibrium. On the other hand, near sample boundaries, the spin Hall current will induce spin accumulation. Hence, the spin distribution is deviated largely from equilibrium, and the spin relaxation becomes effective. The amount of accumulated spins is roughly estimated as a product of spin current and spin relaxation time $\tau_s$ [6]. The spin accumulation affects also the spin current itself near the boundaries [43, 44].

Because spin current is even under time-reversal, it can be induced even when the time-reversal symmetry is preserved. It also implies that the spin Hall current is dissipationless. In doped semiconductors, however, the longitudinal conductivity is finite and the system undergoes Joule heating. Nevertheless, there exist spin-Hall insulators, which are band insulators with nonzero SHE; in such systems the longitudinal conductivity is zero, and the SHE accompanies no dissipation.
3 Disorder effect and extrinsic spin Hall effect

One can include an effect of the self-energy broadening $1/\tau$ by disorder. The intrinsic SHE is reduced as expected. In the clean limit the spin Hall conductivity reproduces its intrinsic value. Inoue et al. found an important result. They assumed dilutely distributed impurities with a $\delta$-function potential. They calculated the self-energy within the self-consistent Born approximation, and the vertex correction within the ladder approximation. In a clean limit, they obtained a vertex-correction contribution $-\frac{e}{8\pi}$ to the spin Hall conductivity, exactly cancelling the intrinsic value $\frac{e}{8\pi}$. This result was rediscovered and generalized by several people. In particular, by the Keldysh formalism, it is found that the spin Hall current appears only near the electrodes whereas in the bulk of the sample the spin Hall current vanishes irrespective of the lifetime $\tau$. The Keldysh formalism is used for more generalized cases in the Rashba model and related models.

One may wonder whether the SHE vanishes in other systems, and there remain some controversies in this respect. It is well-established that the SHE vanishes in the Rashba model with $\delta$-function impurities in the clean limit. Meanwhile, even for the Rashba model, it is still under debate whether it vanishes for finite $\tau$ or for finite-ranged impurities. Here we note that the Rashba model is exceptional, in that the spin current operator $J^z$ is proportional to $S_y = i[H, S_y]$. In fact one can check that the SHE does not vanish in general models; for example, when the Rashba model is generalized to include a higher-order term in $k$, the spin Hall conductivity no longer vanishes. In addition, there are some models where the vertex correction does not cancel the intrinsic value, or even vanishes by symmetry.

In retrospect, extrinsic SHE has been considered since more than thirty years ago, as mentioned in the Introduction. The relationship with the disorder effect on the intrinsic SHE is, however, unclear at present. We note that there have been a similar debate in the disorder effect on the AHE over decades. To summarize, the studies on the disorder effect are so far restricted to special models; general and exhaustive understanding for the disorder effect on the SHE is still lacking.

4 Discussions

In this section we discuss several topics on the SHE, which are still currently under intensive research.
4.1 Definition of the spin current

In the presence of the SO coupling, the total spin is not conserved. Hence there is no unique way to define a spin current. Naively we expect that the “spin current” $J_s$ should satisfy the equation of continuity $\frac{\partial S_i}{\partial t} + \nabla \cdot J_s^i = 0$; this relationship requires the conservation of total spin, namely,

$$0 = \frac{\partial}{\partial t} \int S_i d^d x = -i \left[ \int S_i d^d x , H \right].$$

(9)

In the cases relevant for the SHE, the SO coupling violates this conservation of total spin. In other words, due to the nonconservation of spin, Noether’s theorem is not applicable for a definition of spin current.

One can adopt the symmetrized product $\frac{1}{2}(v_i S_j + S_j v_i)$ between the velocity $v$ and the spin $S$ as a definition of the spin current as in [7]. The result calculated by the Kubo formula with this definition is in general different from that by semiclassical theory described above [62]. This difference comes from noncommutativity between the spin $S$ and the velocity $v$. In other words, this comes from the non-uniqueness of the definition of spin. One can modify the semiclassical theory to give the same result as the Kubo formula, by adding three contributions: spin dipole, torque moment, and change of wavepacket spins due to electric field [63].

An alternative way is to separate the spin $S$ into conserved (intraband) part $S_{(c)}$ and nonconserved (interband) part $S_{(n)}$ [62]. As $[S_{(c)}, H] = 0$, spin current can be uniquely defined for $S_{(c)}$. The resulting spin current is different from (6), and this difference is considered as a quantum correction to (6). The reason why we only take $S_{(c)}$ is because relaxation by impurities will rapidly smear out the non-conserved part $S_{(n)}$. Another attempt for defining conserved spin current is done by introducing torque dipole moment [64]; this definition ensures the Onsager relation. Because there is no unique definition for the spin current, we have to choose one definition which matches the considered experimental setup to measure the spin current.

4.2 Landauer-Büttiker formalism

The four-terminal Landauer-Büttiker formalism can be used to study the SHE. In [65,66], the authors used a tight-binding Hamiltonian with SO coupling on a square lattice, and used the four-terminal Landauer-Büttiker formalism to study mesoscopic SHE for system size up to $\sim 100 \times 100$. In the bottom of the band the tight-binding model reduces to the Rashba model. They first studied the SHE without any disorder, by changing the system size and the SO coupling. The resulting spin Hall conductivity is not equal to the universal value of $\frac{e}{2\pi}$, and is critically dependent on the strength of the SO coupling. They also studied the dependence on on-site disorder. Even in the presence of disorder, the SHE remains nonzero, and depends on the disorder.
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strength. The Luttinger model was studied in a similar fashion \cite{67}. We remark that because of the nonuniqueness of the definition of spin current, the comparison between the results by Landauer-Büttiker formalism and those by Kubo formula is not straightforward.

Non-equilibrium spin accumulation has been studied in a two-terminal geometry \cite{68}. The Keldysh non-equilibrium Green’s function is combined with the Landauer formalism to study numerically the spin accumulation. The spins accumulate at the both edges, with their direction along z-axis opposite for the two edges. Spin accumulation at the edges of ballistic systems is also studied \cite{69}. This accumulation at the edges is qualitatively similar to the experimental result \cite{10}.

In \cite{70}, an H-shaped structure is proposed for a measurement of the SHE via dc-transport properties. With use of the Landauer-Büttiker formalism, the dc voltage response is calculated with realistic parameters. In \cite{71}, the Landauer-Büttiker formalism is applied to a mesoscopic ring with Rashba SO coupling; by tuning the Rashba coupling, the spin Hall current oscillates due to the Aharanov-Casher phase around the ring.

4.3 Criterion for nonzero SHE, spin Hall insulator

The SHE is induced by the SO coupling, which is inherent in every material. However, if the bands within the same multiplet are all filled or all empty, the bands do not contribute to the SHE. Thus, the criterion for nonzero SHE is that the bands within the same multiplet have different fillings. For example, in GaAs, the valence bands \((J = 3/2)\) consist of the LH and the HH; therefore, p-doping brings about the difference of fillings between the LH and the HH, giving nonzero SHE. On the other hand, the conduction band \((J = 1/2)\) is doubly degenerate (if we ignore the bulk inversion symmetry breaking from the zincblende structure), and n-doping does not give rise to nonzero SHE. If we incorporate it into heterostructure, the degeneracy of the conduction band is lifted, and n-doping induces nonzero SHE.

According to this criterion, some band insulators have nonzero SHE, even though the charge conductivity is zero \cite{72}. Two classes of materials have been proposed for such “spin Hall insulators” in \cite{72}. One is zero-gap semiconductors such as HgTe and a-Sn. By introducing uniaxial anisotropy the gap becomes finite. The other is narrow-gap semiconductors such as PbTe. In these semiconductors the gaps come from the SO coupling, and the SHE is nonzero even though they are band insulators. In \cite{73}, on the other hand, a graphene sheet is proposed to be a quantum spin Hall insulator.

One can consider a Hall effect for the orbital angular momentum (OAM) instead of spin \cite{74}. In the Rashba model, the resulting intrinsic Hall conductivity for the OAM is \(-e/(8\pi)\), exactly cancelling the intrinsic SHE. Thus the intrinsic Hall effect for the total angular momentum vanishes. It follows from the conservation of the total angular momentum \(s_z + L_z\) \cite{73}, i.e. from
a continuous rotation symmetry around the $z$-axis. Nevertheless, for general systems it is not true, and the cancellation does not take place in general.

Several first-principle calculations have been done [75,76]. In [76], the intrinsic SHE is calculated for Si, GaAs, W and Au for various values of the Fermi level. It is found that even without doping, Si and GaAs show a small but finite SHE. It is due to a small hybridization. In this sense, these undoped semiconductors are spin Hall insulators. In [75], on the other hand, the intrinsic SHE is calculated for n-type Ge, GaAs and AlAs as a function of the hole concentration and strain. They also calculated the Hall effect for the OAM, and showed that they do not cancel with the spin Hall effect.

5 Experiments

Kato et al. [10] observed the SHE in n-type semiconductors by measuring spin accumulation at the edges of the sample by Kerr rotation. The spin accumulation is uniformly distributed along the both edges. They evaluated from experimental data the amount of spin accumulation and spin lifetime as a function of an external magnetic field. The measured spin Hall resistivity is $2 \text{ M} \Omega \mu \text{m}$. They concluded the observed SHE to be extrinsic for the following reasons; (i) spin splitting is negligibly small in the sample, and (ii) the effect has no dependence on crystal orientation.

Nevertheless, Bernevig and Zhang argued that the observed SHE can be intrinsic, coming from the Dresselhaus term representing bulk inversion-symmetry breaking [61]. They showed that even if the spin splitting due to the Dresselhaus term is negligibly small, the SHE can be as large as the experimental data. It can also account for the absence of dependence on crystal orientation. Thus, the source of the observed SHE is still to be resolved.

The observed distribution of spin accumulation is clearly different from the Keldysh formalism calculation on the Rashba model [47]. As discussed in the previous section, the Rashba model may not be general enough to be useful for comparison with experiments.

On the other hand, Wunderlich et al. [11] observed the SHE in a 2D p-type system, using a p-n junction light-emitting diode. They applied an electric field across the hole channel, and observed a circular polarization of the emitted light, whose sign is opposite for the two edges of the channel. The circular polarization is $\sim 1\%$ at maximum. They argued that it is near to the clean limit, and the obtained SHE is mostly intrinsic. More refined argument, by showing vanishing vertex correction, also supports this conclusion [77].

As we have seen in Sect. 3, disorder effect on the SHE is still under intensive studies. It will take some time to determine whether the above experimental results for the SHE is mostly intrinsic or extrinsic (or both). To pursue this issue experimentally, it will be ideal to change systematically the disorder, and measure the SHE in the same line as in [78].
6 Summary

In this review we summarize recent results for the SHE. In these two years there has been much progress in this field, both in theories and in experiments. Nevertheless, as many results have accumulated, we come up with new questions to be solved. In the present stage there are a lot of ways to approach the problem theoretically and experimentally, and the results from different methods have not yet satisfactorily converged into a unified picture of the SHE. In particular, disorder effect is the key issue to enable comparison between theories and experiments in a systematic fashion.

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