A mesh moving technique
with minimum-height-based stiffening
for fluid-structure interaction analysis

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Received: 6 December 2016; Revised: 26 December 2016; Accepted: 15 January 2017

Abstract
The practical use of the fluid-structure interactions (FSI) analysis system requires the robustness for the parametric design of artifacts. In the FSI analysis employing interface tracking methods, mesh moving technique is significant for avoiding the failure of the analysis due to the mesh distortion. Although many conventional techniques have been introduced in the system, there still remain some cases where the analysis fails because of the mesh distortion. For the further improvement of the robustness, we propose a new mesh moving technique, minimum-height-based stiffening technique, where the mesh deformation of the fluid domain is virtually governed by the linear elastic equations and the stiffness of each element is determined according to its minimum height. The proposed technique is applied to two-dimensional benchmark problems with three types of prescribed motions or deformation: translation, rotation, and bending. The results were compared with those with Jacobian-based-stiffening technique, which is one of the most effective approaches, in terms of mesh quality factors. As a result, the proposed technique shows better performance, i.e. the improvement by more than 10% for our mesh quality factor. In addition, low sensitivity of the mesh quality factors to the optimum value of the control parameter was observed. This low sensitivity can contributes to the usability of our proposed technique.

Key words: Mesh moving technique, Pseudo elastic smoothing scheme, Interface tracking method, Finite element method, Fluid-structure interaction analysis

1. Introduction
Fluid-structure interaction (FSI) analysis attracts much attention in the fields of blood flow simulation (Yang et al., 2007), innovative design of bio mimetic robots subjected to fluid flows, such as micro air vehicles (MAV) (Schenato et al., 2004) and so on. The authors have been developing general purpose parallel FSI analysis systems for large scale problems such as flapping motion of MAV (Yamada et al., 2016) and fuel assemblies of the boiling water reactor (Kataoka et al., 2014). In the FSI analysis for flows with moving boundaries and interfaces, the interface tracking method is widely used, i.e. the arbitrary Lagrangian-Eulerian (ALE) formulation with finite element discretization, for well describing the large deformation of the fluid domain enforced by the motion and deformation of the structure. In the interface tracking method, the update scheme of the fluid mesh is required to track the moving interfaces. When the deformation of the mesh becomes too large, the analysis sometimes fails. Robustness of FSI analysis with any condition is essential to use the FSI analysis system practically. Hence, the mesh controlling and updating techniques are significant for FSI analysis.

For mesh controlling and updating, there are two steps: (1) moving (smoothing) the mesh and (2) remeshing fully or partially. In a mesh moving step, the mesh is updated with the motion of the nodes on the interfaces at every incremental or time step. On the other hand, remeshing is conducted when the deformation of the mesh becomes too
large. Remeshing generally requires an automatic mesh generator and the projection procedure of the solution from old mesh to new one. These procedures involve large computation costs and are difficult to be parallelized efficiently. Therefore, mesh controlling and updating by mesh moving techniques are more suitable than those by remeshing as the scale of analysis increases. One of the mesh moving techniques widely used is a pseudo elastic smoothing scheme (Tezduyar et al., 1992) with Jacobian-based stiffening technique (Stein et al., 2003). In the pseudo elastic smoothing scheme, the mesh deformation of the fluid domain is virtually governed by the linear elastic equations and the mesh is updated with the forced displacement on the interfaces. In the Jacobian based stiffening technique, the stiffness of each element for the elastic equation is determined according to its Jacobian to avoid the distortion of tiny elements in addition to the pseudo elastic smoothing scheme. Although this technique is applied to our FSI analysis system, there still remain some cases where the analysis fails because of the mesh distortion.

In this paper, we propose a new mesh moving technique by the extension of the pseudo elastic smoothing scheme for improving the robustness of FSI analysis, where the stiffness of each element is determined according to its minimum height instead of its Jacobian. The minimum height is commonly calculated as the minimum of the three heights corresponding to the three sides of two-dimensional triangle element in the fields of meshing research. For comparing the new technique with the conventional one, we applied both techniques to the two-dimensional benchmark problems and investigated their performance.

2. A mesh moving technique with minimum-height-based stiffening

We developed a new mesh moving technique on the premise of the pseudo elastic smoothing scheme. As linear elastic equations and the mesh is updated with the forced displacement on the interfaces.

The governing equation of linear elastic body is defined as

$$\nabla \cdot \sigma + f = 0 \quad \text{on } \Omega_f$$  \hspace{1cm} (1)

where $\Omega_f$ is spatial fluid domain, $\sigma$ is the Cauchy stress tensor, $f$ is external force vector. In pseudo elastic smoothing scheme, the domain $\Omega_f$ is bounded only by the Dirichlet boundary $\Gamma_D$ corresponding to the interface between fluid and structure, and there is no external force. The Dirichlet boundary conditions are represented as

$$y = d \quad \text{on } \Gamma_D$$  \hspace{1cm} (2)

where $y$ is the displacement vector, and $d$ is the forced displacement vector at the interface.

In finite element method, week formulation of Eq. (1) is discretely solved by assembling the element stiffness matrices to form the global stiffness matrix as

$$\int_{\Omega_f} \left[ \cdots \right] d\Omega_f = \sum_e \int_{\Xi} \left[ \cdots \right] f^e d\Xi$$  \hspace{1cm} (3)

where $\left[ \cdots \right]$ symbolically represents the terms being integrated, $\Xi$ is the whole finite element domain, superscription $e$ represents the element number, and $f^e$ is the Jacobian for element $e$, which is defined as

$$f^e = \det \left( \frac{\partial x}{\partial \xi} \right)$$  \hspace{1cm} (4)

where $x$ and $\xi$ represent the physical coordinate and the elemental one, respectively. The Jacobian corresponds to the area of the element in two-dimensional analysis.

Here, the terms of minimum-height based stiffening and Jacobian based one are introduced to Eq. (3) as

Minimum-height based stiffening:

$$\sum_e \int_{\Xi} \left[ \cdots \right] f^e d\Xi \to \sum_e \int_{\Xi} \left[ \cdots \right] f^e \left( \frac{h^0}{h^e} \right)^x d\Xi$$  \hspace{1cm} (5)

Jacobian based stiffening:

$$\sum_e \int_{\Xi} \left[ \cdots \right] f^e d\Xi \to \sum_e \int_{\Xi} \left[ \cdots \right] f^e \left( \frac{f^0}{f^e} \right)^x d\Xi$$
dimension of the equations, and $\chi$, a non-negative number, determines the degree of the stiffening power. With $\chi = 0.0$, the technique corresponds to the pseudo elastic smoothing scheme. When $\chi \neq 0.0$, the technique stiffens each element by a factor of $(h^e)^{-\chi}$ or $(J^e)^{-\chi}$.

3. Benchmark problems
3.1 Schematics of benchmark problems

We set the benchmark problems referring to the previous study (Stein et al., 2003) for investigating the performance of mesh moving techniques. The benchmarks are constructed in a two-dimensional problem. Figure 1 shows the mesh and its close-up view near the structure. An unstructured mesh composed of linear triangular elements is used, and a virtual structure with zero thickness is embedded at the center of the whole domain. The mesh spans a region of $|x| \leq 1.0$ and $|y| \leq 1.0$, while the structure spans $y = 0.0$ and $|x| \leq 0.5$. A thin layer of the elements is placed along the structure. The height of this thin layer is $l_x = 0.02$ and $l_y = 0.01$, so that 50 element edges along the structure. This thin layer corresponds to boundary layer elements in FSI analysis.

The three different types of motions or deformation are prescribed for the structure as shown in Fig. 2: rigid-body translation in the $y$-direction, rigid-body rotation around the origin, and prescribed bending.

In the translation cases, the prescribed translation is in the $y$-direction, with the displacement magnitudes ranging from $\Delta y = 0.0$ to 0.5. In the rotation cases, the rotation magnitudes range from $\Delta \theta = 0.0\pi$ to $0.25\pi$. In the bending cases, the structure bends from a straight line to a half-circle with no net vertical and horizontal displacement without stretching of the structure. Let $\theta$ denote the arc length (in radians) for the deformed structure, and the bending magnitudes range from $\Delta \theta = 0.0$ to $\pi$.

These benchmarks are performed with two techniques for comparison: the conventional technique defined by Eq. (5), Jacobian-based stiffening, and the proposed technique, minimum-height-based stiffening, where $\chi$ takes every 0.1 from 0.0 to 4.0.
3.2 Mesh update condition and mesh quality factors

In all benchmark cases, the maximum displacement or deformation is reached at the 50th step. The mesh is updated at each incremental step by solving the linear elastic equations. The new mesh is updated based on the displacements calculated over the current mesh, which has been stiffened by Eq. (5). Therefore, the Jacobian or the minimum height of each element used in stiffening is also updated at each increment. In other words, the values obtained from the most current mesh are used for determining the stiffness of each element.

For the evaluation of mesh moving techniques, three factors of mesh quality are employed referring to the previous studies (Stein et al., 2003 and Johnson et al., 1996). One is aspect ratio, and the others are element shape change and element area change, respectively. The aspect ratio is defined as

\[ AR^e = \frac{\left[ (l_{e_{\max}})^2 \right]}{A^e} \tag{6} \]

where \( l_{e_{\max}} \) is the maximum edge length for element \( e \).

The element shape change and element area change are defined as

\[ f_{AR}^e = \left| \log \frac{AR^e}{AR_o^e} \right| / \log(2.0) \tag{7} \]

\[ f_A^e = \left| \log \frac{A^e}{A_o^e} \right| / \log(2.0) \tag{8} \]

where subscript \( o \) refers to the undeformed (original) mesh, \( A^e \) is area for element \( e \).

For a given mesh, global aspect ratio, and shape and area changes are defined to be the maximum values among all the elements of the mesh, respectively.

3.3 Results of benchmarking

In this section, the minimum-height-based stiffening technique is denoted as “Height” and the Jacobian-based stiffening technique is denoted as “Jacobian”, respectively, for convenience.

Figures 3 and 4 show the relation between parameter \( \chi \) and mesh quality factors at the 50th step, when the displacement or deformation reaches maximum. As can be seen from the minimum value of global aspect ratio in Fig. 3, the aspect ratio is improved with the Height for every cases. Here the quantitative improvements are 11.2% in the translation cases, 13.8% in the rotation cases, and 13.8% in the bending cases. The optimum values of aspect ratio are obtained at \( \chi = 1.6 \) for the Height and \( \chi = 0.9 \) for the Jacobian in the translation cases, \( \chi = 1.7 \) for the Height and \( \chi = 0.8 \) for the Jacobian in the rotation cases, and \( \chi = 2.3 \) for the Height and \( \chi = 1.2 \) for the Jacobian in the bending cases. As for the optimum values of \( \chi \), it is found that the optimum values in the Height are larger than those of the Jacobian. In addition, the aspect ratio in the Jacobian deteriorates more rapidly as the distance from the optimum value of \( \chi \) increases than that in the Height.

![Figure 3 Aspect ratio vs. \( \chi \) at the 50th step](image-url)
On the other hand, as shown in Fig. 4, the shape change and the area change are slightly improved with the Height. The most improvement was shown in the translation cases. The optimum values of $\chi$ in the Height are larger than those of the Jacobian, similar to the aspect ratio. The behavior of the mesh qualities around the optimum value of $\chi$ also resembles that of the results in the aspect ratio.

Figure 4 Shape and area changes vs. $\chi$ at the 50th step

Figure 5 shows the slope (discrete derivative) of the aspect ratio at the 50th step, that is, the sensitivity with regards to $\chi$. The dashed line shows that the slope is equal to the zero. When $\chi$ is set to be less than the optimum value of $\chi$, the slope is relatively high in both the techniques. On the other hand, when $\chi$ increases from the optimum value, the slope in Height gets constant near zero, and the lower sensitivity is clearly shown in the Height than that in the Jacobian. This lower sensitivity can contributes to the usability of our proposed technique for the practical use.

Figure 5 Slope of aspect ratio vs. $\chi$ at the 50th step
Figures 6 and 7 show the incremental step histories of mesh quality factors with the optimum values of $\chi$ in the 50th step. The aspect ratio is improved by the Height at each step as is shown in Fig.6. As for the aspect ratio, it can be clearly seen that the Height is superior to the Jacobian.

For the shape and area changes, the improvement is shown at each step in every case of the benchmarks except for the shape change in the rotation and bending cases. The Height gets superior to the Jacobian as the magnitude reaches the 50th step although the shape change in Jacobian is slightly better than that in the Height in the middle of the magnitude.

Figure 8 shows the deformed mesh colored by the aspect ratio at the 50th step. The close-up view of each mesh is shown at the right tip of the structure. In the translation cases, not only the elements near the structure tips but also those in the upper part of the domain are well controlled by the Height. In the rotation cases, the tangling mesh at structure tips shows the improvement in the Height. In the bending cases, the element at structure tips shows less tangling, similar to the rotation cases. Moreover, the deformation of the elements crushed by the structure is well distributed. The superiority of the proposed technique, the Height, is confirmed by the comparison in the benchmarks.
3.4 Discussion of benchmark results of minimum-height-based stiffening technique

Finally, the tendency of the optimum values of control parameter, $\chi$, for the three types of prescribed motions or deformation on their magnitude is investigated. Figure 9 shows the contour of the aspect ratio in the Height as the
functions of $\chi$ for different magnitudes of motions or deformation i.e. different incremental steps. The bold curve with circle marker denotes the optimum value of $\chi$ in the Height. In the range of low magnitude, when $\chi$ decreases from the optimum value of $\chi$, the aspect ratio deteriorates rapidly. On the other hand when $\chi$ is set to be larger than the optimum value, the aspect ratio tends not to deteriorate so much. In the range of high magnitude, although the tendency is the same as that of low magnitude, the mesh deteriorates as $\chi$ increases.

4. Conclusions

The present paper proposed a new mesh moving technique, minimum-height-based stiffening technique, for further improvement of the robustness of FSI analysis. The proposed technique is developed by the extension of the pseudo elastic smoothing scheme. In the proposed technique, the mesh deformation of the fluid domain is virtually governed by the linear elastic equations, and the stiffness of each element is determined according to its minimum height.

For comparing the new technique with the conventional one, we applied both techniques to the two-dimensional benchmark problems with three types of prescribed motions or deformation: translation, rotation, and bending. The results were compared with those with conventional technique in terms of mesh quality factors. As a result, the proposed technique shows the improvement with any benchmark cases. The aspect ratio was improved by 11.2% in the translation cases, 13.8% in the rotation cases, and 13.8% in the bending cases. The shape change and the area change also showed the improvement in Height. In addition, low sensitivity of the mesh quality factors to the optimum value of $\chi$ was observed. This low sensitivity can contributes to the usability of our proposed technique.

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