A Simple Circuit to Visualize Space Vectors by an Oscilloscope

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Abstract—The paper presents a simple circuit for analog computation of space vector components on the basis of phase voltages. The space vector trajectory could be directly visualized by an oscilloscope. Proposed circuit consists of nine resistors only. An algorithm to compute the resistor values and programs that support the design are provided. The method is experimentally verified using a three-phase voltage source inverter model. Three methods to visualize recorded data are proposed, each capturing different aspect of the space vector trajectory.

Index Terms—DC-AC power converters, Measurement techniques, Oscilloscopes, Pulse width modulation converters, Space vector pulse width.

I. INTRODUCTION

Space vectors are a useful tool in visualizing three-phase voltage systems, and since the space vector modulation has been introduced [1] is a dominant method to control inverters both in motor drives [2]–[4] and in power factor correction [5]. Numerous extensions of space vector modulation to cover complex inverter structures exist, like [6], [7].

Regardless the space vector approach being widely spread, visualizing of space vectors by an oscilloscope is not a straightforward task. Aim of this paper is to provide a simple circuit for analog computation of space vector components that could be directly visualized on an oscilloscope screen.

II. SPACE VECTORS

Consider a three-phase voltage system with line voltages that satisfy

\[ v_{12} + v_{23} + v_{31} = 0 \]  \hspace{1cm} (1)

which is required by Kirchhoff’s voltage law. Constraint (1) reduces the voltage system to two degrees of freedom, making it possible to characterize the voltages with two independent variables. Having in mind an additional constraint that that applies for balanced voltage systems

\[ v_{12}(t) = v_{23}(t + \frac{1}{3} T) = v_{31}(t + \frac{2}{3} T) \]  \hspace{1cm} (2)

it is convenient to visualize the three-phase voltage system by a space vector \( \vec{v} \) consisting of components \( v_X \) and \( v_Y \)

\[ \vec{v} = (v_X, v_Y) \]  \hspace{1cm} (3)

such that line voltages are obtained as dot products of the space vector and unit vectors that correspond to line voltage axes, as depicted in Fig. 1, such that

\[ v_{12} = \vec{v} \cdot (1, 0) = v_X \]  \hspace{1cm} (4)

\[ v_{23} = \vec{v} \cdot \left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right) = -\frac{1}{2} v_X + \frac{\sqrt{3}}{2} v_Y \]  \hspace{1cm} (5)

and

\[ v_{31} = \vec{v} \cdot \left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right) = -\frac{1}{2} v_X - \frac{\sqrt{3}}{2} v_Y. \]  \hspace{1cm} (6)

In this manner, phase delay between line voltages is visualized as the space vector rotates.

At this point it is worth to mention that the only requirement for the system of three voltages to be represented by a space vector is (1), while (2) is not a requirement. In the case of an unbalanced system, where (2) is not met, space vector representation would lead to the space vector trajectory which is not circular, but elliptic. One of the applications of space vector representation is to visualize and observe distortions and unbalances in three-phase systems.

III. THREE-PHASE VOLTAGE SOURCE INVERTER

The hardware used to verify concepts presented in this paper is based on a three phase voltage source inverter [3] model.
TABLE I
THREE-PHASE VOLTAGE SOURCE INVERTER, SWITCH STATE VARIATIONS AND LINE VOLTAGES

| state | S1 | S3 | S5 | $v_{12}$ | $v_{23}$ | $v_{31}$ |
|-------|----|----|----|---------|---------|---------|
| 0     | 0  | 0  | 0  | 0       | 0       | 0       |
| 1     | 0  | 0  | 1  | $-V_{IN}$ | 0       | $V_{IN}$ |
| 2     | 0  | 1  | 0  | $V_{IN}$  | $-V_{IN}$ | 0       |
| 3     | 0  | 1  | 1  | $V_{IN}$  | $-V_{IN}$ | 0       |
| 4     | 1  | 0  | 0  | $V_{IN}$  | 0       | $-V_{IN}$ |
| 5     | 1  | 0  | 1  | $V_{IN}$  | 0       | $V_{IN}$  |
| 6     | 1  | 1  | 0  | $V_{IN}$  | $-V_{IN}$ | 0       |
| 7     | 1  | 1  | 1  | 0       | 0       | 0       |

built as a teaching aid using Arduino Uno board [8], [9]. All the design files are provided at [10] under an open license. The inverter model is built with an intention to illustrate six step operation of a voltage source inverter.

The inverter circuit diagram is presented in Fig. 2, and consists of six controlled switches, operated such that states of even indexed switches are complements of the states of switches having the preceding index

$$S_{2k} = S_{2k-1}$$

for $k \in \{1, 2, 3\}$. Since there are three independent switch states, there is a total of $2^3 = 8$ switch state variations, each defining an inverter state. The inverter states are given in Table I alongside the resulting line voltages. Corresponding space vectors achievable by the inverter are given in Fig. 3. Out of the eight inverter states, two provide zero space vectors, not being used in six step operation, but used extensively in space vector modulation [1].

A. The Neutral Point

The inverter of Fig. 2 provides three line voltages. Transformation of these three line voltages into three phase voltages is not unique, since an infinite number of phase voltage sets can provide specified line voltages. This is a consequence of (1) which reduced the three line voltage set to two degrees of freedom. Unless there is constraint that specifies a reference voltage, a phase voltage set is not limited to two degrees of freedom like the line voltage set is by Kirchhoff’s voltage law.

However, there is a specific reference voltage point, the neutral point, defined as

$$v_N = \frac{1}{3} (v_1 + v_2 + v_3)$$

which if used as a reference point for voltage measurements removes zero sequence components from the phase voltages. In the case phase voltages are referenced to the neutral point, transformation from the line voltages to the phase voltages is unique. Potential of the neutral point is obtained as the common point voltage of a symmetric three-phase load.

To illustrate concept of the neutral point in presenting phase voltage waveforms, waveforms of the inverter phase voltages referenced to the negative rail voltage are presented in Fig. 4, while the waveforms referenced to the neutral point obtained using a symmetric three resistor load are presented in Fig. 5. The voltages of Fig. 4 are never negative, and they do not add up to zero. On the other hand, voltages of Fig. 5 during one half of the period take negative values and always add up to zero.

IV. VISUALIZING SPACE VECTORS BY AN OSCILLOSCOPE

Our aim in this section is to design a system to visualize space vectors by an oscilloscope that should meet two design
requirements:

1) common two channel oscilloscopes should be supported, that measure the two voltages with respect to the same reference voltage, and

2) the network that computes the space vector components should use only resistors.

The requirements are imposed in order to create a system as simple as possible.

A. Relating Space Vectors and Phase Voltages

According to (4), \( v_X \) component of the space vector is

\[
v_X = v_{12} = v_1 - v_2.
\]

(9)

This is inconvenient, since a network containing only resistors cannot provide negative weighting coefficients. Having in mind that (8) results in

\[
v_1 + v_2 + v_3 = 0
\]

(10)

for \( v_N \) as a reference, the \( v_X \) component of the space vector could be expressed as

\[
v_X = 2v_1 + v_3
\]

(11)

and a scaled version or \( v_X \) may be obtained by a network consisted of resistors.

On the other axis, \( v_Y \) component of the space vector is obtained solving (5) and (6) over \( v_Y \), resulting in

\[
\sqrt{3} v_Y = v_{23} - v_{31}
\]

(12)

which in terms of the phase voltages reduces to

\[
v_Y = \frac{1}{\sqrt{3}} (v_1 + v_2 - 2v_3).
\]

(13)

Again, to get rid of the negative sign (10) is used, providing

\[
v_Y = \sqrt{3} (v_1 + v_2)
\]

(14)

and its scaled version may be obtained only using resistors.

Conclusion is that (11) and (14) provide components of the space vector as linear combinations of two phase voltages referenced to the neutral point. The linear combinations have positive weighting coefficients.

B. Obtaining Linear Combination of Two Voltages

A circuit that provides linear combination \( v_c \) of voltages \( v_a \) and \( v_b \) is shown in Fig. 6. The circuit is linear and resistive, and it can be solved using Thévenin and Norton theorems, resulting in

\[
v_c = \frac{R_3}{R_1 + R_2 + R_3 + R_2 R_3} (R_2 v_a + R_3 v_b).
\]

(15)

In the case of \( v_X \), let us take \( v_a = v_1 \), \( v_b = v_3 \), which requires \( R_1 = R_a \), \( R_2 = 2R_a \), and let \( R_3 = R_c \). This reduces (15) to

\[
v_x = k v_X = \frac{R_c}{2 + 3 R_c/R_a} (2v_1 + v_3)
\]

(16)

which is a voltage proportional to \( v_X \). The constant which relates \( v_x \) and \( v_X \) is

\[
k = \frac{v_x}{v_X} = \frac{R_c}{2 + 3 R_c/R_a}
\]

(17)

and for \( R_c > 0 \) limits \( k \) to the range \( 0 < k < \frac{1}{3} \). For a given value of \( k \), \( R_c \) is computed as

\[
R_c = \frac{2k}{1 - 3k} R_a.
\]

(18)

In the case of \( v_Y \), let us take \( v_a = v_1 \), \( v_b = v_2 \), requiring \( R_1 = R_b \) and \( R_2 = R_a \), and let \( R_3 = R_d \). This results in

\[
v_y = k v_Y = \frac{R_d}{1 + 2 R_d/R_b} (v_1 + v_2)
\]

(19)

where the scaling factor is

\[
k = \frac{v_y}{v_Y} = \frac{1}{\sqrt{3}} \frac{R_d}{1 + 2 R_d/R_b}
\]

(20)

Condition \( R_d > 0 \) limits \( k \) to the range \( 0 < k < \frac{1}{2 \sqrt{3}} \). For a given value of \( k \), corresponding value of \( R_d \) is obtained as

\[
R_d = \frac{\sqrt{3} k}{1 - 2 \sqrt{3} k} R_b.
\]

(21)

Isotropic representation of space vectors requires the same value of \( k \) for \( v_x \) and \( v_y \), and the upper limit for the value of \( k \) is set by the \( v_y \) constraint, being more stringent, to the value

\[
k < \frac{1}{2 \sqrt{3}} \approx 0.28868.
\]

(22)
C. Setting the Neutral Point Voltage

The use of (10) requires \(v_N\) as a reference point for measuring voltages. The circuit used to provide that voltage is the simplest three-phase symmetric load depicted in Fig. 7. Application of Kirchhoff’s current law for the neutral point results in

\[
\frac{v_1 - v_N}{R_a} + \frac{v_2 - v_N}{R_b} + \frac{v_3 - v_N}{R_c} = 0
\]  

(23)

which provides the neutral point voltage

\[
v_N = \frac{v_1 + v_2 + v_3}{3}
\]  

(24)

for any reference point used to measure the phase voltages, and the equivalent Thévenin resistance

\[
R_N = \frac{1}{3} R_n.
\]  

(25)

This resistance should be much lower than the equivalent Thévenin resistance observed at the outputs of circuits of Fig. 6 used to provide \(v_x\) and \(v_y\). In the case of \(v_x\), after algebraic transformations this reduces to

\[
R_a \gg \frac{1}{6 k} R_n
\]  

(26)

while for \(v_y\) reduces to

\[
R_b \gg \frac{1}{3 \sqrt{3 k}} R_n.
\]  

(27)

Since a common choice for independent resistors is \(R_a = R_b\), the conditions (26) and (27) are joined to

\[
R_a, R_b \gg \frac{1}{3 \sqrt{3 k}} R_n.
\]  

(28)

D. The Circuit for Analog Computation of Space Vector Components and Its Numerical Verification

Combining circuits of Figs. 6 and 7, complete circuit to compute scaled space vector components is obtained as shown in Fig. 8. In actual application, for the Arduino based inverter model, the supply voltage is 5 V, thus the highest scaling factor of \(k = \frac{1}{2 \sqrt{3}} \approx 0.28868\) is selected. The resistors are chosen such that \(R_a = R_b = 47 \, \text{k}\Omega\), which for specified value of \(k\) requires \(R_c = 200 \, \text{k}\Omega\), while \(R_d \to \infty\) is omitted. To provide the neutral point, three resistors of \(R_n = 470 \, \text{Ω}\) are used, resulting in (28) being satisfied by a huge margin, \(47 \, \text{k}\Omega \gg 313.33 \, \text{Ω}\).

Due to the finite Thévenin resistance of the neutral point, \(v_n \neq v_N\), and the circuit computes the space vector components only approximately, requiring (28) to be satisfied. Referring \(v_1\), \(v_2\), and \(v_3\) to any reference point, after transformation of independent sources using Norton’s theorem to their equivalents based on current sources, a system of nodal equations over \(v_x\), \(v_y\), and \(v_n\) is obtained as

\[
\begin{align*}
\left(\frac{3}{2 R_a} + \frac{1}{R_c}\right) v_x - \frac{1}{R_c} v_n = \frac{1}{R_a} v_1 + \frac{1}{2 R_a} v_3 \\
\left(\frac{2}{R_b} + \frac{1}{R_d}\right) v_y - \frac{1}{R_d} v_n = \frac{1}{R_b} v_1 + \frac{1}{R_b} v_2
\end{align*}
\]  

(29)

and

\[
\begin{align*}
-\frac{1}{R_c} v_x - \frac{1}{R_d} v_y + \left(\frac{1}{R_c} + \frac{1}{R_d} + \frac{3}{R_n}\right) v_n = \\
= \frac{1}{R_n} v_1 + \frac{1}{R_n} v_2 + \frac{1}{R_n} v_3.
\end{align*}
\]  

(30)

Symbolic solution of the system is too bulky to follow, but the system could easily be transformed into a matrix form and solved numerically, providing

\[
\begin{bmatrix}
v_x - v_n \\
v_y - v_n
\end{bmatrix} = \begin{bmatrix}
c_{1,1} & c_{1,2} & c_{1,3} \\
c_{2,1} & c_{2,2} & c_{2,3}
\end{bmatrix} \begin{bmatrix}
v_1 \\
v_2 \\
v_3
\end{bmatrix}.
\]  

(32)

This expresses displayed components of space vectors \(v_x - v_n\) and \(v_y - v_n\) in terms of phase voltages \(v_1\), \(v_2\), and \(v_3\). In the case that \(R_n \to 0\) and that \(R_a\), \(R_b\), \(R_c\), and \(R_d\) take their computed values exactly, the matrix of (32) takes its ideal value

\[
\begin{bmatrix}
v_x - v_n \\
v_y - v_n
\end{bmatrix} = k \begin{bmatrix}
\frac{1}{\sqrt{3}} & -1 & 0 \\
\frac{1}{\sqrt{3}} & 1 & -\frac{2}{\sqrt{3}}
\end{bmatrix} \begin{bmatrix}
v_1 \\
v_2 \\
v_3
\end{bmatrix}.
\]  

(33)

A program to compute the coefficients numerically is given in [10], and besides the actual coefficient values it provides their comparison to ideal values (33) and the relative difference. For chosen resistors, the differences in coefficients are by far less than 1% of their corresponding ideal values.
Described circuit for analog computation of space vectors is built, connected to the Arduino based inverter model, and resulting components of space vectors are shown in Fig. 9 in the six step inverter operation. The inverter operated at the frequency about 33 Hz, to make each of six generated space vectors last about 5 ms. Switching to $x-y$ presentation of data by the oscilloscope, the diagram of Fig. 10 is obtained setting the trace persistence to 1 s. Since the space vectors change over time, to capture the trajectory an adequate setting for the persistence is essential. For infinite persistence, complete trajectory of the space vector is captured, as depicted in Fig. 11. The results indicate that visualization depends on the oscilloscope persistence settings, where the persistence simulating algorithm applied in the digital oscilloscope is not under our complete control.

To take complete control over visualization of the space vector trajectory, the oscilloscope is connected to a computer using software written in Python [11]. Taking the data corresponding to one screen like the one presented in Fig. 9 results in the plot of Fig. 12 where red plus signs correspond to theoretically expected positions of space vectors. The data illustrate that captured values primarily correspond to “steady state” space vectors that are intended to be generated, while only a few of the points are captured during the space vector transitions.

To capture more data and to capture transitions of the space vectors, let us note that the oscilloscope being used applies 8-bit analog to digital converters, resulting in 256 discrete values that can be obtained, out of which 200 fit the oscilloscope screen. In $x-y$ data acquisition mode, this results in a $200 \times 200$ matrix being addressed by $v_x$ and $v_y$ values, where the addressed matrix element corresponds to the space vector location. In this manner, data acquisition can be iterated over many oscilloscope screens, as long as needed to provide an adequate data set. Measurement results are stored in a moderately sized matrix, where each data point increments corresponding matrix entry. After 25 million data points, the resulting diagram in which the intensity of black corresponds to the number of occurrences of the space vector is presented in Fig. 13. As expected, the space vectors reside primarily near expected positions, and transitions between them cannot be observed.

To observe transitions and to keep some information about the number of space vector occurrences, the same data used to produce Fig. 13 are presented in Fig. 14 such that intensity of
black color is proportional to the logarithm of the number of space vector occurrences. Now, with the nonlinear scale, the transitions became visible.

Finally, the infinite persistence could be implemented, which reduces the data set to a bitmap visualized in Fig. 15, where black dot represents presence of the space vector in at least one data point. The diagram corresponds to the diagram of Fig. 11, but without unexplained variations in the color intensity.

VII. CONCLUSIONS

In this paper, a simple circuit to facilitate direct observation of space vectors by an oscilloscope is presented. The circuit is designed to consist only of resistors and to provide space vector visualization using oscilloscopes with channels referred to a common ground. The final design of the circuit consists of nine resistors divided into three groups, two of the groups providing linear combinations of phase voltages, while the third group provides the neutral point used as a voltage reference for the oscilloscope. To verify the circuit, a three-phase voltage source inverter model in six step operation is used. The inverter model is primarily used as a teaching aid. Programs used to compute the resistor values and to verify the design approximations are provided. Experimental results are in complete agreement with the theoretical predictions. Three visualization methods are proposed, two of them using nonlinear scale to expose space vector transitions.

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