Neutrino Spectra from Low and High Luminosity Populations of Gamma Ray Bursts

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Abstract

The detection of GRB 060218 at z=0.033 by Swift within 1.5 years of operation, together with the detection of GRB 980425 at z = 0.0085 by BeppoSAX, suggest that these low-luminosity (LL) GRBs have a much higher event rate than the canonical high-luminosity (HL) GRBs, and they form a distinct new component in the GRB luminosity function. We explore the contribution of this previously neglected GRB population to the diffuse neutrino background within the internal shock model and compare it with that of the canonical HL population. By considering a wide range of distributions of various parameters (e.g. luminosity, spectral break energy, duration, variability time, Lorentz factor, redshift) for both populations, we find that although it is difficult to detect neutrinos from the individual LL GRBs, the contribution of the LL population to the diffuse neutrino background is more than the HL population above about $10^8$ GeV.

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1 Introduction

The possibility of high energy neutrino emission from GRBs in the burst and the afterglow phases has been studied earlier by different groups [1, 2, 3, 4]. It has been found that only the nearby high luminosity (HL) GRBs (e.g. GRB 030329) are expected to be detected individually by neutrino telescopes like ICECUBE [3, 5]. In general the diffuse neutrino background from the whole GRB population is of great observational interests. Recently a low-luminosity (LL), long-duration event, GRB 060218/SN 2006aj was detected by Swift [6] at a redshift of 0.0331. Earlier, another low luminosity event, GRB 980425/SN 1998bw was detected by BeppoSAX at an even lower redshift of 0.0085 [7, 8]. These two LL events share some common characteristics such as low luminosity, long duration, and low isotropic γ-ray energy. More importantly, their detections at very low redshifts within a relatively short period of time (1.5 years for Swift and 6 years for BeppoSAX) imply that they have a much higher event rate than the canonical HL GRBs [9, 10, 11, 12]. A straightforward estimate of the local rate of LL-GRBs $\rho_{0}^{LL}$ can be derived from

$$\rho_{0}^{LL} V(z < 0.033) \left( \frac{\Omega_{Beppo}}{4\pi} T_{Beppo} + \frac{\Omega_{Swift}}{4\pi} T_{Swift} \right) \sim 2,$$

where $V(z < 0.033) \sim 1.2 \times 10^{-2}$ Gpc$^3$ is the volume enclosed by $z = 0.033$, $\Omega_{Beppo} = 0.123$ and $\Omega_{Swift} = 1.33$ are the solid angles of the GRBM on board BeppoSAX and the BAT on

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board *Swift* respectively. The durations of observation are $T_{\text{Beppo}} \sim 6$ yr and $T_{\text{Swift}} \sim 1.5$ yr (when this paper has been written) for *BeppoSAX* and *Swift*, respectively. The number 2 on the right hand side of Eq. (1) accounts for the detection of GRB 980425 and GRB 060218. This gives a rough estimate of $\rho_{\text{LL}}^0$ as $800 \pm 300 \text{ Gpc}^{-3} \text{yr}^{-1}$. Several groups independently estimated this event rate [9, 11], e.g. $\rho_{\text{HL}} \sim (300 - 1000) \text{ Gpc}^{-3} \text{yr}^{-1}$, which are comparable to the rate of LL GRBs $550^{+700}_{-430} \text{ Gpc}^{-3} \text{yr}^{-1}$ obtained by Liang et al. [12]. These are all much higher than the event rate of HL GRBs, $\rho_{\text{HL}}^0 \sim 1 \text{ Gpc}^{-3} \text{yr}^{-1}$ [13], and its simple extrapolation to the LL regime assuming a same population, e.g. $\rho_0 \sim 10 \text{ Gpc}^{-3} \text{yr}^{-1}$ [14]. If one assumes that the two LL GRBs within $z \leq 0.033$ belong to the population of HL GRBs, then the expected number of detection of GRBs within $z \leq 0.033$ in the *BeppoSAX* and *Swift* era is nearly 0.0026 for $\rho_{\text{HL}}^0 \sim 1 \text{ Gpc}^{-3} \text{yr}^{-1}$, and is 0.026 for $\rho_0 \sim 10 \text{ Gpc}^{-3} \text{yr}^{-1}$. The Poisson probability of detecting two events during the observational period is $3.4 \times 10^{-6}$ for the former and $3.3 \times 10^{-4}$ for the latter. Hence, the detection of GRB 060218 strongly suggests the presence of a distinct population (LL), which is not a subset of the canonical HL population. This implies two distinct components in the GRB luminosity function [12]. One may characterize the luminosity function of each population by a broken power law, i.e.

$$S(L) = S_0 \left[ \left( \frac{L}{L_b} \right)^{\alpha_1} + \left( \frac{L}{L_b} \right)^{\alpha_2} \right]^{-1}$$

(2)

The spectral indices and break luminosities of this function for LL and HL populations have been derived/constrained in [12]. The values of $\alpha_1$, $\alpha_2$, $L_b$ are 0.6, 4.5, $10^{47}$ erg/sec and 1.15, 2.5, $10^{51}$ erg/sec for LL and HL GRBs respectively. The values of $S_0$ depend on the upper and lower limits in the values of luminosity for each population.

Although the high energy neutrino flux of these individual LL GRBs is low, the contribution to the diffuse neutrino flux by the whole LL population is non-negligible due to their very high event rate. In this paper we explore the neutrino emission of this hitherto ignored LL-GRB population, and compare the contributions of the LL- and HL-GRB populations to the diffuse neutrino background.

## 2 Diffuse Neutrino Spectra from HL and LL GRBs

We consider the prompt high energy neutrino emission from GRBs in the burst phase due to internal shocks. Protons and photons interact to produce pions which subsequently decay to neutrinos, $p + \gamma \rightarrow \pi^+ \rightarrow \mu^+ + \nu_\mu \rightarrow e^+ + \nu_e + \bar{\nu}_\mu + \nu_\mu$. This is the dominant mechanism of high energy neutrino production inside GRBs. We derive the neutrino spectrum using the power law nature of photon spectrum as observed by different satellite experiments BATSE [16], HETE [15], Swift [17]. GRB 060218 is an X-ray flash (XRF) [6]. Sakamoto et al. [18] have discussed in detail about the characteristics of 45 GRBs observed by HETE-2 (High Energy Transient Explorer) [15], some of them were XRFs. They found no statistically significant deviation in the distribution of photon spectral indices depending on the type of the source. We therefore describe the comoving-frame photon spectrum for both HL and LL GRBs as

$$\frac{dn_\gamma}{d\epsilon_\gamma} = A \left\{ \begin{array}{ll} \epsilon_\gamma^{-\gamma_1} & \epsilon_\gamma \leq \epsilon_{br,c} \\ \epsilon_{br,c}^{-2-\gamma_1} & \epsilon_\gamma > \epsilon_{br,c} \end{array} \right. \quad \epsilon_\gamma \leq \epsilon_{br,c} \quad \epsilon_\gamma > \epsilon_{br,c}$$

(3)
The break energy of the photon spectrum in the comoving frame $\epsilon_{br,c}$ is related to its value in the source rest frame as $\epsilon_{br} = \Gamma \epsilon_{br,c}$, where $\Gamma$ is the bulk Lorentz factor. The normalisation constant $A$ is related to the internal energy density $U$ by,

$$A = \frac{U \epsilon_{br,c}^{\gamma_1^2 - 2}}{\frac{1}{\gamma_2^2 - \gamma_1^2}}$$

where $\gamma_1 < 2$, and $\gamma_2 > 2$. The threshold energy for pair production ($\gamma + \gamma \rightarrow e^+ + e^-$) in the comoving frame is $\sim 1$ MeV. Since the pair production cross section is much larger than $p\gamma$ interaction cross section, the upper cut-off energy in the photon spectrum which produces high energy neutrinos should be limited by the threshold energy of pair production. Photon energy 1 MeV in the comoving frame corresponds to 300 MeV and 10 MeV in the source rest frame for HL and LL GRBs, with Lorentz factor of the order of 300 and 10, respectively. For HL GRBs, 300 MeV photons can produce neutrinos with energy 3000 GeV in the source rest frame in $p\gamma$ interactions through $\Delta$ resonance. For LL GRBs, 10 MeV photons can produce neutrinos with energy 100 GeV in the source rest frame. The break energies of LL and HL GRBs are assumed to be of the order of keV and MeV, respectively, in the source rest frame. Thus the upper limits in photon energies are much higher than their break energies for both LL and HL GRBs. The average comoving time is longer than the average synchrotron cooling time of protons for HL GRBs, and for LL GRBs they are comparable. As we discuss later in detail the average maximum neutrino energies are generally constrained by synchrotron cooling of protons. The corresponding lower limits in photon energies for $p\gamma$ interactions are much lower than photon spectral break energies for both HL and LL populations. The expression for parameter $A$ has been derived in Eq.(4) using the fact that the minimum photon energies are much lower than the break energies and the maximum photon energies are much higher than the break energies. The threshold energies for synchrotron self absorption [19] of photons are of the order of eV for HL GRBs and are much lower for LL GRBs. The lower limit in photon energies derived by the maximum proton energies (applying the synchrotron cooling constraint) is higher than the typical synchrotron self absorption energies for LL GRBs, and this is also generally the case for HL GRBs. If we take $\gamma_1 = 1$, $\gamma_2 = 2$ and a photon energy range of 30KeV to 3MeV in the source rest frame, then we get $A = \frac{U}{2\epsilon_{br,c}}$ as derived in [1].

In the comoving frame, the energy loss time ($t_\pi$) of a proton of energy $E_{p,c}$ due to pion production can be derived from

$$t_\pi^{-1}(E_{p,c}) = \frac{1}{E_{p,c}} \frac{dE_{p,c}}{dt}$$

The variability time is of the order of milliseconds for HL GRBs. For LL GRBs, observations show that their lightcurves are generally very smooth; for GRB 060218 the width of the broad peak was 431 sec [6]. In $p\gamma$ interactions the maximum contribution to neutrino production comes from the $\Delta$ resonance as discussed in [1]. For $\gamma_1 < 2$ and $\gamma_2 > 2$, we obtain the fractional energy loss of protons to pions as a function of proton energy $E_p \equiv E_{p,obs}(1 + z)$ in the source rest frame.

$$f_\pi(E_p) = f_0 \begin{cases} 1.34^{\gamma_2 - 1}(E_{pb})^{\gamma_2 - 1} & E_p < E_{pb} \\ 1.34^{\gamma_1 - 1}(E_{pb})^{\gamma_1 - 1} & E_p > E_{pb} \end{cases}$$

(6)
where, $f^{HL}_0 = \frac{0.9 L_{\gamma,51}}{\Gamma_{300} t_v/3 \epsilon_{br,MeV} (\frac{1}{\Gamma_{300}^2} - \frac{1}{\Gamma_1^2})}$ for HL GRBs and $f^{LL}_0 = \frac{0.729 L_{\gamma,47}}{\Gamma_{10} t_v/2 \epsilon_{br,keV} (\frac{1}{\Gamma_{10}^2} - \frac{1}{\Gamma_2^2})}$ for LL GRBs. Here $L_{\gamma,47} = L_\gamma / (10^{47} \text{ ergs s}^{-1})$ and $t_{v,2} = t_v / (10^2 \text{ s})$. For, $\gamma_1 = 1$ and $\gamma_2 = 2$ we get an expression of $f_\pi(E_p)$ for photon energy range 30keV to 3MeV similar to that derived in [1],

$$f_\pi = 0.2 \frac{E_p/E_{pb}}{\Gamma^{300} t_v/3 \epsilon_{br,MeV}} \begin{cases} E_p/E_{pb} & E_p < E_{pb} \\ 1. & E_p > E_{pb} \end{cases}$$

(7)

where $\Gamma_{300} = \Gamma / 300$, $\Gamma_{10} = \Gamma / 10$ denote typical Lorentz factors of HL and LL GRBs, respectively [20, 21]; $\epsilon_{br,MeV}$ and $\epsilon_{br,keV}$ denote the break energy $\epsilon_{br}$ in units of MeV and keV, respectively. The choice of a low $\epsilon_{br}$ for LL GRBs is based on the fact that GRB 060218 has $\epsilon_{br} \sim 5 \text{ keV}$ [6], which well satisfy the so-called Amati-relation [22, 23]

$$\epsilon_{br} = (3.64 \pm 0.04) \left( \frac{E_{iso}^{\gamma}}{7.9 \times 10^{52} \text{ ergs}} \right)^{0.51 \pm 0.01}$$

(8)

Another LL GRB 980425 apparently does not satisfy this correlation. However, Ghisellini et al. [24] argue that GRB 980425 might not be an outlier. In particular, they suspect that *BeppoSAX* only recorded a short duration of hard emission, and it might be similar to GRB 060218 should it be detected by an instrument similar to *Swift* that could record soft emission simultaneously. We therefore assume the Amati-relation for all bursts in this paper [25], but caution that the diffuse neutrino flux of the LL-component would be overestimated (say, a factor of 2) if a fraction (say, 50%) of LL-GRBs do not satisfy the Amati relation. It is worth commenting that if most LL-GRBs satisfy the Amati-relation, they are not detectable by BATSE, as is the case of GRB 060218 [26], so that they do not contribute to the BATSE population. The proton break energy corresponding to the break energy in the photon spectrum can be expressed as

$$E_{pb}^{HL} = 1.3 \times 10^7 \Gamma_{300}^2 (\epsilon_{br,MeV})^{-1} \text{GeV}.$$ 

(9)

for HL GRBs or $E_{pb}^{LL} = 1.45 \times 10^7 \Gamma_{10}^2 (\epsilon_{br,keV})^{-1} \text{GeV}$ for LL GRBs. In $p\gamma$ interactions both $\pi^0$ and $\pi^+$ can be produced with equal probabilities. $\pi^+$ gets on the average 20% of the proton energy and if the final state leptons share the pion energy equally then each neutrino carries 5% of the initial proton energy. The first break energy in the neutrino spectrum, $E_{\nu b}$ is due to the break in the photon spectrum.

$$E_{\nu b}^{HL} = 6.5 \times 10^5 \frac{\Gamma_{300}^2}{\epsilon_{br,MeV}} \text{GeV}$$

(10)

for HL GRBs or $E_{\nu b}^{LL} = 7.23 \times 10^5 \frac{\Gamma_{10}^2}{\epsilon_{br,keV}} \text{GeV}$ for LL GRBs. The total energy to be emitted by neutrinos of energy $E_{\nu}$ can be expressed as,

$$E_{\nu}^2 dN_{\nu}(E_{\nu}) \approx \frac{3 f_\pi}{8 \kappa} \frac{1}{\epsilon_e} (1 - \epsilon_e) E_{\gamma}^{\text{iso}}$$

(11)

where $E_{\gamma}^{\text{iso}}$ is the total isotropic energy of the emitted gamma-ray photons in the energy range of 1keV to 10MeV, which is available from the observations [29]; $\epsilon_e \sim 0.3$ is the energy fraction carried by electrons, and $\kappa$ is a normalization factor considering the differential
neutrino spectrum. The relativistic electrons produce the photons by synchrotron radiation and inverse Compton scattering of low energy photons, so four orders of magnitude in photon energy corresponds to two orders of magnitude in the energy of the radiating charged leptons. Photon spectral index $\gamma_2 = 2.25$ corresponds to a spectral index of $-2.5$ of the relativistic electron spectrum. This correspond to $\kappa = 1.8$ assuming photon fluence is proportional to neutrino luminosity. If the electron spectral index is $-2$, one would have $\kappa = \ln(100) \sim 4.6$. The internal shocks of GRBs are typically collisionless and Coulomb interaction is not important between electrons and protons. It is still an open question regarding how some energies of Fermi accelerated protons are transferred to electrons and whether electrons and protons would have a same spectral index. In principle, the spectral indices of electrons and protons in the shocked region could be different. In our calculations, the electron spectral index is taken as $-0.5$ (based on the observed photon spectrum), while the proton spectral index is still assumed to be $\sim (-2)$ as has been adopted by all the previous neutrino calculations.

The break (Eq.(10)) due to the break energy in the photon spectrum is caused by the break in $f_\pi$ (Eq.(6)). If one takes into account muon and pion cooling, a second break appears in the neutrino spectrum

$$E_\nu^2 \frac{dN_\nu(E_\nu)}{dE_\nu} \approx \frac{3 f_\pi}{14.4} \frac{(1 - \epsilon_e)}{\epsilon_e} E_\nu^{iso} \begin{cases} \frac{1}{(E_\nu/E_0)^2} & E_\nu < E_\nu^s \\ E_\nu > E_\nu^s \end{cases}$$

Pion cooling energy is ten times higher than the muon cooling energy. We would be overestimating the total neutrino flux if we use the pion cooling energy to derive the second break energy in the neutrino spectrum. The muon cooling break $E_\nu^s$ can be expressed as a function of GRB parameters.

$$E_\nu^{s, HL} = 2.56 \times 10^6 \epsilon_e^{1/2} \epsilon_B^{-1/2} L_\gamma^{1/2} \Gamma_4^{1/2} t_{vl}^{1/2} m_p GeV$$

for HL GRBs or $E_\nu^{s, LL} = 3.16 \times 10^7 \epsilon_e^{1/2} \epsilon_B^{-1/2} L_\gamma^{1/2} \Gamma_4^{1/2} t_{vl}^{2} m_p GeV$ for LL GRBs. Here the parameter $\epsilon_B$ denotes the fraction of internal energy carried by the magnetic fields. We can see that if $\epsilon_e$ and $\epsilon_B$ are comparable their contributions are cancelled out. They are assumed to be equal to 0.3 in our calculations. Neutrino oscillation only redistributes the number of different flavors of neutrinos. The total number of neutrinos remains unchanged.

The maximum proton energy achievable in HL and LL GRBs can be derived by equating the acceleration time scale with the minimum of the dynamic time scale and synchrotron cooling time scale of protons [27]. In the comoving frame, the synchrotron cooling time can be expressed in terms of proton energy in the source rest frame as,

$$t_{sync} = \frac{\Gamma_4 m_p^3 c^3}{\sigma_t m_e^2 E_p B_c^2},$$

where $m_p$ and $m_e$ are proton and electron masses respectively, $\sigma_t$ is the Thompson scattering cross section and $B_c$ is the comoving magnetic field. The comoving magnetic field strength can be expressed as a function of radial distance, luminosity, Lorentz factor the unknown shock strength (which depends on the relative Lorentz factor of the two colliding shells), and the unknown shock equaiartition parameters [28]. For $\epsilon_B = 0.3$ and reasonable shock
strength, one could estimate $B_{c}^{HL} \simeq (2.5 \times 10^{5} G)L_{51}^{1/2}r_{13}^{-1}$ with $r_{13}^{HL} = 0.27\Gamma_{300}^{2}t_{v,-3}$ for HL and $B_{c}^{LL} \simeq (4.42 \times 10^{4} G)L_{17}^{1/2}r_{13}^{-1}$, $r_{13}^{LL} = 30\Gamma_{10}^{2}t_{v,2}$ for LL GRBs, $r_{13} = r/10^{13}$ is the dissipation radius where internal shocks are formed. The acceleration time scale in the comoving frame in terms of proton energy in the source rest frame is,

$$t_{a} = \frac{E_{p}}{\Gamma B_{c} ec}$$

The dynamical comoving time scale is

$$t_{d y n} = \frac{r}{\Gamma c}.$$  \hspace{1cm} (16)

In our calculations, we use $t_{a} = \min(t_{s y n c}, t_{d y n})$ to calculate the maximum proton energy in the shock. For typical parameters, one usually has $t_{s y n c} < t_{d y n}$, one can then derive the expression of the maximum proton energy using the condition $t_{s y n c} = t_{a}$, which gives

$$E_{p, m a x}^{2} = \frac{\Gamma^{2}6\pi m_{p}^{4}A_{e}}{\sigma_{t}m_{e}^{2}B_{c}}$$  \hspace{1cm} (17)

which gives $E_{p, m a x}^{HL} \approx 6 \times 10^{10} \frac{\Gamma_{300}}{\sqrt{B_{c}^{HL}}}$ GeV for HL GRBs and $E_{p, m a x}^{LL} \approx 2 \times 10^{10} \frac{\Gamma_{10}}{\sqrt{B_{c}^{LL}}}$ GeV for LL GRBs. After averaging over the parameters, including log normal distributions in Lorentz factor and variability time, the broken power-law distribution of the luminosity distribution, and the redshift distribution of GRBs (see below for detail), we obtain the average values of maximum proton energies for LL and HL GRBs.

The neutrino spectrum emitted from the source is to be corrected for the redshift of the GRB to derive the total spectrum to be observed on earth. The total isotropic energy can be expressed as a product of the isotropic luminosity ($L_{\gamma}$) and the duration of the burst ($T_{d}$). The observed neutrino spectrum on earth

$$dN_{\nu}^{ob}(E_{\nu}) dE_{\nu} = dN_{\nu}^{ob}(E_{\nu}) \frac{1}{4\pi d^{2}(z)}(1+z)$$

where $r(z)$ is the comoving radial coordinate distance of the GRB. For a spatially flat universe with $\Omega_{\Lambda} + \Omega_{m} = 1$, the comoving distance can be expressed as

$$d(z) = \int_{0}^{z} \frac{c}{H_{0}} \frac{dz'}{\sqrt{\Omega_{\Lambda} + \Omega_{m}(1+z')}}$$

In our calculations we use $\Omega_{\Lambda} = 0.73$, $\Omega_{m} = 0.27$ and the Hubble constant $H_{0} = 71$ km sec$^{-1}$ Mpc$^{-1}$ from [30]. When calculating the diffuse neutrino background from GRBs we assume that GRBs follow the star formation rate$^{3}$. The diffuse neutrino flux on earth from GRBs distributed up to a redshift of $z_{m a x}$ can be expressed as,

$$M_{\nu}^{ob}(L_{\gamma}, \Gamma, T_{d}, t_{v}, E_{\nu}^{ob}) = \int_{0}^{z_{m a x}} \frac{dN_{\nu}^{ob}(E_{\nu}^{ob})}{dE_{\nu}^{ob}} \frac{R_{GRB}(z)}{1+z} dV(z)$$

$^{3}$If however GRBs do not trace the global star-forming rate, our normalization factor needs to be modified.
where, the star formation rate is given by [31]

$$R_{GRB}(z) = 23\rho_0 \frac{e^{3.4z}}{e^{3.4z} + 22}.$$  (21)

The local rates \(\rho_0\), of the LL and HL populations have been assumed to be about 550Gpc\(^{-3}\)yr\(^{-1}\) and 1.1Gpc\(^{-3}\)yr\(^{-1}\), respectively [12]. We obtain the neutrino fluxes from LL and HL populations distributed up to a distance of \(z_{\text{max}} = 5\). The neutrino fluxes have been averaged with luminosity, Lorentz factor, variability time and burst duration distribution functions to take into account their variations in the population (the variations in luminosity and duration also reflect variation in isotropic energy)

$$\phi_\nu(E^{ob}_\nu) = \int_{L_{\gamma,1}}^{L_{\gamma,2}} \int_{t_{v,1}}^{t_{v,2}} M^{ob}_{\nu}(L_\gamma, \Gamma, t_d, t_v, E^{ob}_\nu) S(L_\gamma) G(\Gamma) P(t_d) Q(t_v) dL_\gamma d\Gamma dt_d dt_v.$$  (22)

Here \(S(L_\gamma)\) is the broken power law luminosity function (eqn(2)), with \(L^{\text{HL}}_b = 10^{51}\) erg s\(^{-1}\), \(\alpha^{\text{HL}}_1 \sim 1.15, \alpha^{\text{HL}}_2 \sim 2.5\) for HL and \(L^{\text{LL}}_b = 10^{47}\) erg s\(^{-1}\), \(\alpha^{\text{LL}}_1 \sim 0.6\) and \(\alpha^{\text{LL}}_2 \sim 4.5\) for LL GRBs (following [12]). For the HL population the lower and upper limits in luminosities are \(L^{\text{HL}}_{\gamma,1} = 10^{50}\) erg s\(^{-1}\) and \(L^{\text{HL}}_{\gamma,2} = 10^{52}\) erg s\(^{-1}\), respectively, and the corresponding values for the LL population are \(10^{46}\) erg s\(^{-1}\) and \(5 \times 10^{49}\) erg s\(^{-1}\), respectively. The Lorentz factor distribution function \(G(\Gamma)\) is assumed to be log normal. For HL GRBs, we assume the mean of the distribution at \(\log(\Gamma_m) = 2.6\), which correspond to \(\Gamma_m \approx 398\), with a standard deviation of 0.3. For LL GRBs, we assume the log normal distribution of Lorentz factors is peaked at \(\log(\Gamma_m) = 1.1\) [21] with a standard deviation 0.1. We allow the values of Lorentz factors to vary from 100 to 1000 for the HL GRBs, and from 5 to 50 for the LL population. The durations of the bursts are also assumed to follow log normal distributions \(P(t_d)\), with the mean at \(\log(T_{d,m}) = 1.47, 3.5\); which corresponds to about 30sec and 3000sec, for the HL and LL populations, respectively, with standard deviations 0.5 and 0.1. The scattering in Amati’s relation [23] does not affect our results qualitatively. Since the photon index \(\gamma_2\) does not differ significantly among bursts [18], we take \(\gamma_2 = 2.25\) universally for both the HL and LL populations. We also consider variation in variability time of GRBs. Their distributions are assumed to be log normal, represented by \(Q(t_v)\) in eqn.(22) with the mean at \(\log(t_{v,m}) = -1.52, 2.6\) and standard deviations 0.3, 0.1 for HL, LL GRBs respectively. The maximum energies of neutrinos have also been averaged with the Lorentz factor, luminosity and variability time distribution functions as discussed above.

$$E_{\nu,\text{max}} = \int_{L_{\gamma,1}}^{L_{\gamma,2}} \int_{t_{v,1}}^{t_{v,2}} M_{\nu,\text{max}}(L_\gamma, \Gamma, t_v) S(L_\gamma) G(\Gamma) Q(t_v) dL_\gamma d\Gamma dt_v.$$  (23)

Also, the redshift correction on the observed neutrino energy has been averaged over the entire volume between \(z = 0\) to \(z = 5\).

$$E^{\text{av,ob}}_{\nu,\text{max}} = E^{\text{av}}_{\nu,\text{max}} \frac{\int_0^5 R_{GRB}(z) dV(z) / (1 + z)}{\int_0^5 R_{GRB}(z) dV(z)}.$$  (24)

### 3 Results and Discussions

The neutrino spectra from HL and LL populations are plotted in units of GeV cm\(^{-2}\) s\(^{-1}\) sr\(^{-1}\) in Fig.1. for \(\gamma_1 = 1, 1.1, 1.3\) and 1.5 and \(\gamma_2 = 2.25\). We have assumed \(\epsilon_e = 0.3\) and \(\epsilon_B = 0.3\)
in our calculations. The long dashed (green) lines represent neutrino fluxes from the LL population and the solid (red) lines are for the HL population. The Waxman-Bahcall [32] limit with $z$ evolution (including all three flavors) has been shown by short dashed (blue) lines, and the dotted (pink) lines represent the sensitivity of the IceCube detector [33] after three years of operation. The current limit from AMANDA-II data taken between 2000 and 2003 [34] is also shown by triple dotted line above the WB upper bound. The HL component has an average maximum energy of $6.3 \times 10^9$ GeV and for LL population it is $1.5 \times 10^9$ GeV. Above about $10^8$ GeV the contribution of LL population to the diffuse neutrino background is more than the HL population. The LL component continues to below 100 GeV but the HL component has a lower cut-off energy of about 1000 GeV. Between 1000 GeV and $4 \times 10^4$ GeV their fluxes are equal. We have also plotted the GZK neutrino flux from [35] in thin dashed lines for a comparison. Comparing with the Icecube sensitivity, we can see that the GZK neutrinos are generally not detectable by Icecube, while the contribution of the LL-component would be detectable if $\gamma_1 \geq 1.1$. In this case we expect $30 km^{-2} sr^{-1}$ neutrino events per year or more in IceCube at an energy of $10^7$GeV. If we compare our derived neutrino spectra from HL GRBs with some of the previous results [4] the width of the intermediate region between the two break energies is narrower. This is because we have used the muon cooling energy instead of the pion cooling energy in determining the second break energy of the spectra to avoid overestimation of neutrino fluxes. However, in this way we slightly underestimate the $\nu_\mu$ fluxes as they are produced by $\pi^+$ decay unlike $\bar{\nu}_\mu$ and $\nu_e$. We also find that our results are very sensitive to variations in the mean value of Lorentz factor. As the break energies in the neutrino spectra depend on $\Gamma^2$ and $\Gamma^4$ a slight deviation in the value of $\Gamma$ changes the neutrino fluxes significantly. In the future it might be possible to set limits on the values of GRB Lorentz factors if the detection or upper limit of high energy neutrino background from GRBs is set up. Overall, our results show that the previously neglected LL population of GRBs significantly contribute to the diffuse neutrino background as their event rate is very high. This conclusion would be strengthened as Swift detects more LL GRBs in the years to come.

Finally we notice that after we submitted the first version of this paper to astro-ph, similar results were independently reported by Murase et al.[36].

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Figure 1: The diffuse neutrino fluxes expected on earth from low and high luminosity populations of GRBs distributed up to a redshift of 5, being plotted in units of GeV cm$^{-2}$ s$^{-1}$sr$^{-1}$ against the observed neutrino energy. The four panels are for photon spectral indices $\gamma_1 = 1, 1.1, 1.3, 1.5$ (clockwise from left bottom), and $\gamma_2 = 2.25$ are adopted for all of them. The Waxman-Bahcall limit with $z$ evolution [32] has been shown with short dashed (blue) line. The double dotted (pink) line represents IceCube experiment’s sensitivity [33] after 3 years of operation and the current limit from AMANDA-II data (2000-2003) [34] triple dotted (orange) line is just above the Waxman-Bahcall limit. These limits include all the three flavors of neutrinos. GZK neutrino flux has been plotted in thin dashed line from [35] for comparison.