Toward a Calculus of Redundancy:
The feedback arrow of expectations in knowledge-based systems

Loet Leydesdorff, a* Mark W. Johnson, b & Inga Ivanova c

Abstract:
Whereas the generation of Shannon-type information is coupled to the second law of thermodynamics, redundancy—that is, the complement of information to the maximum entropy—can be increased by further distinctions: new options can discursively be generated. The dynamics of discursive knowledge production thus infuse the historical dynamics with a cultural evolution based on expectations (as different from observations). We distinguish among (i) the communication of information, (ii) the sharing of meaning, and (iii) discursive knowledge. Meaning is provided from the perspective of hindsight as feedback on the entropy flow and thus generates redundancy. Specific meanings can selectively be codified as discursive knowledge; knowledge-based reconstructions enable us to specify expectations about future states which can be invoked in the present. The cycling among the dynamics of information, meaning, and knowledge in feedback and feedforward loops can be evaluated empirically: When mutual redundancy prevails over mutual information, the sign of the resulting information is negative indicating reduction of uncertainty because of new options available for realization; innovation can then be expected to flourish. When historical realizations prevail, innovation may be locked-in because of insufficient options for further development.

Keywords: redundancy, discursive knowledge, incursion, options, innovation

---

a University of Amsterdam, Amsterdam School of Communication Research (ASCoR), PO Box 15793, 1001 NG Amsterdam, The Netherlands; email: loet@leydesdorff.net; * corresponding author
b …
c Institute for Statistical Studies and Economics of Knowledge, National Research University Higher School of Economics (NRU HSE), 20 Myasnitskaya St., Moscow, 101000, Russia; and School of Economics and Management, Far Eastern Federal University, 8, Sukhanova St., Vladivostok 690990, Russia; inga.iva@mail.ru
1. Introduction

When one assigns a description to a message containing Shannon-type information, one provides meaning to this yet meaningless uncertainty. Since different descriptions are possible, one adds a degree of freedom to the communication. By adding a degree of freedom, however, the maximum information content $H_{\text{max}}$ (or equivalently, the maximum entropy) of the system is changed from $\log(N)$—the logarithm of the number of previously available options $[N = \sum_i p_i]$—to $\log(M \times N)$ where $M$ indicates the number of possible descriptions. In this case, $H_{\text{max}} = \log(\sum_i p_i) + \log(\sum_i q_i)$. The redundancy is thus enlarged with $\log(\sum_i q_i)$ without necessarily changing the Shannon-type information.

The descriptions can be changed without changing the underlying information and information may change without consequences for the meanings attributed. Descriptions can again be communicated and be re-described. However, inter-human communication requires that the messages carry meaning; otherwise, they cannot be received. Meaning—symbolic value—is generated and processed at the level of inter-human communications by exchanging both descriptions and information; but foreseeably to different extents. For example, a text can be expected to contain information and to convey meaning.

Re-descriptions are constrained by the meaning of previous descriptions. Recursion in the communication can be expected to lead to refinements and therefore further enlargement of the number of options. When descriptions, for example, are translated into another language, the other coding scheme may add options. Each coding scheme adds a degree of freedom; but also a
specific selection mechanism, since the codes delimit what can be received. The refinement follows from the decrease of relative information \( \frac{H}{H_{\text{max}}} \) when the phase space of possible realizations is extended by adding dimensions.

The degrees of freedom span a multi-dimensional space of which the codes shape the dimensions. The codes can be considered as selection mechanisms operating upon one another in this multi-dimensional space. Different codes provide different meanings to the events and thus instantiate different semantic resources for further communication and action (Giddens, 1979). For example, a patent on an invention provides legal protection of intellectual property in a specific (juridical) discourse; but the patent can also have economic value and it may be the outcome of a research project and thus add to the reputation and status of the inventor-researcher. The three frames of reference can lead to three descriptions with different semantics. Providing the same (or similar) information with other meanings can be expected to generate overlaps and thus redundancy.

In other words, redundancy is not naturally given or naturalistically observable, but it is the result of the dynamics of (inter-)human communication and understanding. Human understanding can be considered as the receptive capacity for the semantics, and this capacity can further be developed in terms of semantic resources enabling us to specify further redundancy, that is, unrealized options and new dimensions. As a form of action, however, communication unavoidably generates uncertainty—that is, Shannon-type information. The codes operate by leaving footprints in the instantiations.
Both redundancy and uncertainty are generated in the communication; information increases the prevailing uncertainty and redundancy reduces it. The mechanisms, however, are very different: information is generated relationally, whereas redundancy is based on (partially) overlapping positions among systems; positions are defined in a multi-dimensional space of *correlations*. Whereas relations can be analyzed in terms of graphs, redundancy is generated in a vector space of correlations spanned in terms of latent dimensions (“eigenvectors;” von Foerster, 1960). The resulting trade-off between information and redundancy generation can have a positive or negative sign in terms of bits of information. In this study, we explore the question of whether a calculus of redundancy can be developed, relating to—but also different from—the information calculus originating from Shannon’s (1948) information theory (e.g., Bar-Hillel, 1955; Krippendorff, 1986; Theil, 1972; Weaver, 1949; cf. Dretske, 1981)?

In common language, “redundancy” is connoted with repetitive messaging, i.e., as an operation over time. In information theory, however, redundancy is defined as the complement of the information relative to the maximum information content of a system at each moment of time. How can this static definition be translated into the dynamics of redundancy? Whereas information is communicated, the transfer of the message is already complete without communication of redundancy (Weaver, 1949, p. 13). Shannon (1945) considered redundancy as “excess information” (Shannon, 1945) needed for error-correction. In other words, redundancy is generated because meaning is provided from the perspective of hindsight. However, a perspective of hindsight assumes a position (Leydesdorff *et al.*, in press). Positions and therefore meaning can be shared, but not communicated. The sharing generates redundancy.
2. The perspective of information theory

Redundancy $R$ is defined in Shannon’s theory, as follows:

$$R = \frac{H_{\text{max}} - H_s}{H_{\text{max}}} = 1 - \frac{H_s}{H_{\text{max}}}$$  \hspace{1cm} (1)

The maximum information content of a system ($H_{\text{max}}$) is equal to the logarithm of the number of possible states $N$: $H_{\text{max}} = \log(N)$. In Eq. 1, $H_{\text{max}}$ is composed of two parts: the system states hitherto realized [$H_s = -\sum_i p_i \log(p_i)$] and the states which are possible given the definition of the communication system, but hitherto not realized [$H_{\text{max}} - H_s$]. While the information content and redundancy thus are lynch-pinned at each moment of time, the relation with the intuitive concept of redundancy as repetition requires a translation—another description—because one then conceptualizes redundancy as a *dynamic* potentially different from the dynamics of information processing (Leydesdorff, 1991).

By using the $H$ in Gibb’s formulation of the entropy ($S = k_B \times H$), Shannon (1948) chose to define information as probabilistic entropy.\(^1\) As a consequence, the development of information follows the second law of thermodynamics and can therefore only be positive (Krippendorff, 2009a). The generation of redundancy, however, can be positive or negative depending on the feedback and feedforward loops in the meaning processing as different from information processing (Krippendorff, 2009b). Feedback and feedforward loops can propel information and

\(^1\) $k_B$ is the Boltzmann constant which provides the thermodynamic entropy $S$ with the dimensionality Joule/Kelvin. $H$ is dimensionless.
meaning in clockwise or counter-clockwise cycles; that is, with potentially opposite signs (Ulanowicz, 2009; Ivanova & Leydesdorff, 2014). The relative information content of a message \( (H_s/H_{max}) \) can then be enlarged or reduced by adding or constraining redundancy. In other words, options other than already historically realized are added or removed by mechanisms very different from the second law.

Shannon (1948, at p. 3) deliberately distanced himself from the study of meaning processing in loops by stating that “(t)hese semantic aspects of communication are irrelevant to the engineering problem.” The engineering problem focuses exclusively on the communication of information. Shannon’s co-author Weaver (1949), however, noted that this analysis “has so penetratingly cleared the air that one is now, perhaps for the first time, ready for a theory of meaning” (p. 27). He suggested that “redundancy” would be a prime candidate for developing such a theory; but also warned that “information must not be confused with meaning” (Shannon & Weaver, 1949, p. 8). During the half century since then, however, confusion has prevailed about the definitions of information, meaning, and their relationship(s). Meaning has mainly been defined in relation to information and not in terms of redundancy (Hayles, 1990, pp. 59 ff.).

The number of options in a social system can increase much faster than the realizations because one is able to generate, entertain, and communicate new perspectives reflexively. In a model, for example, the realizations are special cases among possible states. Furthermore, models enable us to envisage and construct new technologies: new technologies can be expected to add degrees of freedom to the development, and thus multiply the number of options. As long as transportation over the Alps, for example, is constrained by passes such as the Brenner and the Gotthard, the
number of these passes determines the maximum capacity. Railways which can be tunneled under the mountains and airplanes which can cross the Alps independently of the conditions on the ground multiply the number of options. These options, however, have to be invented and their realization requires a plan. A plan anticipates on future states; furthermore, a plan can be communicated and improved.

3. Anticipation and the arrow of time

Plans, models, blueprints, etc., are developed reflexively in the linguistic domain. Dubois (2003, at pp. 112f.) distinguished between systems which entertain a predictive model versus anticipatory ones which entertain a model of the modeling system itself. A system which entertains a model of itself is able to adapt its phenotypical appearance or behavior to the expected conditions (Rosen, 1985). An anticipatory system thus can select its historically relevant trajectory from the perspective of hindsight. Since different reconstructions are possible, one gains a degree of freedom. In biology, however, the number of phenotypes is genotypically determined (Hodgson & Knudsen, 2011).

In the context of developing artificial life, Langton (1989, at pp. 22f.) noted that “genotypes” are theoretical constructs in non-biological domains (since there is no DNA). Theories can then be considered as “genotypical:” The various theories specify different dynamics using reflections from specific angles. Each perspective operates with a specific code or set of codes. The codes are linguistically constructed and reconstructed, but in the non-biological domain they can be symbolic.
In other words, the codes are not to be reified (as “DNA”), but remain tendencies in the communication with the status of hypotheses (“eigenvectors”; Lazarsfield & Henry, 1968; von Foerster, 1960). Parsons (1968) proposed to call these genotypical constructs “symbolically generalized media.”² Luhmann ([1997, pp. 202f.], 2012, pp. 120 ff.) added that symbolically generalized media can be performative, more than media which serve the communication itself such as language. The specificity of the codes makes it possible to accelerate the communication and thus to process more complexity. The codes coordinate specific communications selectively. Subsystems with specific functions can be expected to use different codes.

A prime example of symbolically codified communication (used by both Parsons and Luhmann) is money which enables us to replace negotiation over the price in language with a system of symbols (bank notes, coins) without annihilating the option of deconstruction by returning to “elaborate communication” in language (Bernstein, 1971; Coser, 1975). Abstracting from the communicative embedding in language, money can be further developed into banking, credit, etc. The functional need to process increasingly complex financial transactions drives the institutionalization of these contexts. Without the function carried by the code, however, the institutions would lose their rationale and gradually fade away (Boudon, 1979). The functions at the (next-order) regime level renew the institutions developing along historical trajectories from the perspective of hindsight. What began as a feedback mechanism can grow into a reverse arrow of control when the communication is increasingly coded and the codes are symbolically generalized.

² Distin (2012, at p. 95) considered the expression as a misnomer, since power, truth, love, etc., “cannot meaningfully be called either languages or media.”
4. “A difference which makes a difference”

Shannon’s counter-intuitive definition of information as uncertainty has provoked alternative definitions of information as “reduction of uncertainty” or neg-entropy ((Brillouin, 1953; Schrödinger, 1944). The anthropologist Gregory Bateson (1972, at p. 315), for example, proposed to define information as “a difference which makes a difference” (MacKay, 1969). However, a difference may make a difference for one system of reference, but not for another. Varela (1979, p. 266) argued that “information” should be defined as informative in accordance with the semantic root of the Latin word of “in-formare.” In our opinion, a difference which makes a difference for a receiving system can be considered as meaningful information. The meaning is provided by the receiving system for which the difference(s) make a difference.

In second-order systems theory, this receiving system has been characterized as an “observer” (e.g., Maturana, 1978; Von Foerster, 1969). In a next step, the observer is abstracted from contexts; Luhmann (e.g., 1989) even envisaged a general theory of observation (Kauffman, 2001; Spencer Brown, 1969; cf. Leydesdorff, 2006). If one wishes to avoid the further abstraction of an observer to a transcendental subject, however, the systems of reference have to be specified in which the incoming information can be provided with potentially different meanings.

In sum, a “difference which makes a difference” denotes “meaningful information” as different from Shannon-type information or uncertainty. The system of reference for the analysis is in this
case the receiving system or “the observer;” but no longer the message itself and its information content. Warren Weaver’s dictum that “information must not be confused with meaning” is thus violated, and the information-theoretical perspective—including the development of a perspective on redundancy—tends to be lost, since the definitions of information, meaning, and knowledge can easily be entangled and confused. Hidalgo (2015, at p. 165), for example, defines “information” with reference “to the order embodied in codified sequences, such as those found in music or DNA, while knowledge and knowhow refer to the ability of a system to process information.” However, knowledge does not have to be “embodied;” codified knowledge can be distinguished from embodied knowledge and knowhow (e.g., Cowan, David, & Foray, 2000).

Meaning, information, and knowledge can be analytically distinguished. Without these distinctions, one loses degrees of freedom in the model and theorizing. Shannon-type information is defined mathematically and in this respect yet meaningless. Meaning is provided to information by (the specification of) a system of reference. Specific meanings can be selected for codification as knowledge. Codified knowledge can circulate using a global dynamics different from the local dynamics of knowledge carriers who historically embed the knowledge.

5. The biological perspective

Living systems need room for adaptation—autonomous “error correction”—and therefore surplus options that have not yet been realized (or that were perhaps abandoned in a previous adaptation). As noted, Shannon (1945) called this “excess information.” In their book entitled
“Evolution as Entropy,” Brooks & Wiley (1986, p. 43) visualized the biological perspective on a system that continuously proliferates realizations (e.g., diversity), but in which order can increase “so long as the realized entropy increases at a slower rate than the maximum possible entropy” (at p. 40). Note that this implies that the redundancy ([H_{max} - H_{obs}] in Figure 1) has to increase at a rate higher than the realized entropy (H_{obs}).

![Diagram of information capacity, content, and redundancy over time.](image)

**Figure 1:** the development of information capacity, information content, and the redundancy over time. Source: Brooks & Wiley, 1986, p. 43.

In the legend on the right side of this figure, “historically realized” are distinguished from “excluded” options. System states above $H_{max}$ are labeled as “impossible;” but the authors added that “what is impossible at one time period may become possible at a later time period.” However, they did not draw the conclusion that the dynamics of redundancy may be different from the dynamics of the capacity or the observable information.

Kauffman (2000, at pp. 142f.) discussed the border between the historically realized and excluded as a dynamic frontier along which “adjacent” possibilities can be realized step-by-step;
in other words, along a trajectory. Focusing on the chemical synthesis of biological molecules, he added that “the adjacent possible is indefinitely expendable. Once members have been realized in the current adjacent possible, a new adjacent possible, accessible from the enlarged actual that includes the novel molecules from the former adjacent possible, becomes available.” In a further reflection (at p. 258) he formulated that by “mere constructive interference” the various trajectories may resonate into a phase transition about which “one can hope” that it provides evolutionary advantages. However, such an interference would remain historically coincidental. The evolutionary mechanism of this next-order change was not further specified.

![Figure 2: Hitherto impossible options are made possible because of cultural and technological evolution.](image)

We argue (Figure 2) that cultural evolution may lead to changes in the number of options by orders of magnitude different from biological evolution which is historically constrained in terms of genotypes. What was previously impossible can be made possible by constructing
technological artifacts (e.g., airplanes) induced to variable extents by advances in the relevant sciences (Rosenberg, 1982). Using rationalized expectations, one is able to explore options beyond the historically realized or excluded ones. In other words, the discursive exploration can lead to new techno-scientific realizations, and thus to an additional loop in an emerging dimension. However, this reconstructive communication cannot be modeled using the Shannon-model.

From a biological perspective, redundancy can only be considered as a residual because this non-information cannot be naturalistically specified or observed. Ulanowicz (2014), for example, compared this refusal to specify redundancy with “negative” or “apophatic” theology, which holds the position that one can only specify God in terms of what He is not. In the author’s opinion, the apophasis (A)—of the redundancy—cannot teach us anything about historical events, unlike the observable information which the author labels D as an abbreviation of didactic. The difference Φ between D and A (Φ = D – A) can be considered as a measure of the degree to which the interactions (mutual informations) “remain independent of each other, i.e., the lack of constraints among the flows” (p. 24). However, Φ is defined as positive, and thus the apophasis is limited by this constraint (Ulanowicz, 1986, p. 92). From this perspective, a biological system with excess options more than realized (A > D) would be vulnerable to perturbations to the extent that a catastrophe would be unavoidable (Ulanowicz, 2014, p. 26).

In our opinion, psychological and social systems operate with this type of volatility on top of biological systems such as bodies and populations carrying the communication. Their internal reflexivity provides these systems with next-order buffering capacity so that they can maintain
identity beyond historically observable stability. In psychological (action) systems stability is also provided by the underlying body as a biological substrate. Inter-human communication, however, can proliferate in more than a single direction; but distributions of carriers would not provide sufficient stability without codes operating on the basis of correlations among them. The latent codes shape an additional layer of reflexivity providing meta-stability at a next-order (and therefore relatively globalized) level.

Codes of communication and their embodiments in technological artifacts (e.g., the internet) can stabilize the communication so that footprints can be left in the historical substrate that serves the embeddedness. This “natural” environment is continuously reconstructed by the technological imprint as a retention mechanism for the further development of the reconstructing communication. The coordination in the communication is not only historically achieved in instantiations, but also supported and reconstructed evolutionarily by (interactions among) codes emerging in the communication.

From this perspective, the biological metaphor can be left behind: the observables develop historically along trajectories, but the next-order systems develop as expectations. Whereas biological systems may be able to develop semantic domains (Maturana, 1978), reflexive human agency cannot only provide the information with meaning, but also specify expectations in language. This sharing of meaning is error-prone and thus coupled again to the communication of uncertainty: the generation of uncertainty remains basic since grounded in the second law. The generation of redundancy, however, can be expected to follow other rules such as control by codes in the communication.
In summary: both the engineering discourse of information theory and the evolutionary discourse among biologists have failed to specify the dynamics of redundancy otherwise than as a residual term. However, this specification is central to understanding the dynamics of reflexivity in inter-human communications and the role of communicative agency both reflecting on and contributing to these exchanges.

6. The perspective of meaning processing

Meaning is provided to the historical events from the perspective of hindsight; that is, against the arrow of time. Whereas entropy increases with the arrow of time, meaning processing works as feedback on the information processing by generating redundancies in both senses that were specified above: (i) different meanings can be provided to the same information and thus rewrite the information more than once (for example, from different perspectives); (ii) by considering the historically observable state of the art as “not necessarily so:” other options than the instantiated ones can reflexively be specified. Options that were hitherto not realized, can be entertained reflexively or hypothesized on the basis of theorizing.

The mathematical biologist Robert Rosen (1985) theorized that systems which entertain a model of themselves can be considered anticipatory: these systems are able to explore states other than the realized ones in the model that they entertain, and thus optimize the states that are instantiated. For example, a plant can vary its phenotype given the climate prevailing. Dubois (1998) added that a strongly anticipatory system can shape its present state on the basis of the
model entertained by it; in other words, self-referentially and relatively distanced from the environment. The following three dynamics along the time axis were distinguished by the author:

1. *recursive* dynamics in which the next state is a function of the previous one ($x_t = f(x_{t-1})$); the system develops with the arrow of time;
2. *incursive* dynamics, in which the present state of the system is a co-determinant of the further development: ($x_t = f(x_{t-1}; x_t)$;
3. and a *hyper-incursive* system in which the present state of the system is (re-)constructed on the basis of future states: $x_t = f(x_{t+1})$.

Dubois’ prime example has been the logistic equation which can be written, as follows:

\[
\begin{align*}
\text{recursively:} & \quad x_t &= ax_{t-1}(1 - x_{t-1}) \\
\text{incursively:} & \quad x_t &= ax_{t-1}(1 - x_t) \\
& \quad \text{or} & \quad x_t &= ax_t(1 - x_{t-1}) \\
\text{hyper-incursively:} & \quad x_t &= ax_{t+1}(1 - x_{t+1})
\end{align*}
\]

Whereas Eq. 2 is the standard formulation of the logistic equation—well-known for its bifurcation points at $a \geq 3.0$ and increasingly chaotic development for $a \geq 3.57$—Eq. 4 models a system that is driven exclusively by expectations of future states. Such a system cannot be found in “nature”; it is by definition a cultural construct. For example, the rule of law on the basis of the *trias politica* is such a counterfactual, attributed as an invention to Montesquieu (1748).

Since these structures of expectations operate against the arrow of time, hyper-incursions (like in
Eq. 4) generate redundancy or reduction of uncertainty as opposite to the generation of uncertainty in accordance with the second law of thermodynamics which prevails in natural systems (Eq. 2).

Eqs. 3a models selection processes in the present, such as the ones made by embodied understanding and decision making. For example, we build on our previous understanding, but select in the present. Analogously, a new technology builds on previous technology, but the selection term in the market \((1 - x_t)\) operates in the current situation or perhaps even on the basis of expectations \((1 - x_{t+1})\). Advanced markets such as financial ones, for example, can be expected to operate on the basis of model-based expectations more than on the basis of previous states of the industry. The instantiations such as the new technologies are historical and thus entrained in the entropy flow. Incursion provides the mechanism for inscribing the dynamics of cultural evolution into the historical dynamics.

In other words, we distinguish between historical dynamics (following the arrow of time and generating entropy) and evolutionary dynamics providing meaning to the historical developments and generating other options, i.e., redundancy, against the arrow of time. Whereas the rules for redundancy generation are genotypically given in biological systems, the genotypes generating new options have to be hypothesized in non-biological systems. The hypotheses can discursively be anchored in theoretical expectations, conventions, or opportunities. However, the historical origins of the expectations are relatively irrelevant, since an anticipatory dynamics reconstructs previous configurations. A reflexive system is able to rewrite its history incursively, a hyper-reflexive one can be expected to operate in terms of a reflexive reconstructions.
Figure 3: Bifurcation diagram for recursive, incursive, and hyper-incursive formulations of the logistic map; $a$ is the bifurcation parameter. Source: Leydesdorff, 2008: 35; Leydesdorff & Franse, 2009: 113.

Figure 3 shows the bifurcation diagram of the logistic map using the set of Eqs. 2, 3a, and 4 for modeling (i) the biological domain ($a < 4$), (ii) the psychological one—or equivalently, another system following a trajectory in history—and (iii) the sociological one of structures of expectations operating for $a > 4$, respectively. On the left side one recognizes the well-known bifurcation diagram for $a < 4$; for values of $a$ larger than four, this natural system vanishes in complete chaos. However, the opposite is true for the map of the cultural system on the right side ($a > 4$) based on Eq. 4.

Eq. 4 can be rewritten as follows:

$$x_t = ax_{t+1}(1-x_{t+1})$$  \hspace{1cm} (5)
\[ x_t = ax_{t+1} - ax_{t+1}^2 \]
\[ ax_{t+1}^2 - ax_{t+1} + x_t = 0 \]
\[ x_{t+1}^2 - x_{t+1} + x_t / a = 0 \]

This equation has two solutions for \( a > 4 \) which coincide for \( a = 4 \):

\[ x_{t+1} = \frac{1}{2} \pm \frac{1}{2} \sqrt{1 - (4/a) x_t} \]  \hspace{1cm} (6)

With more than a single solution, an instantiation of this system requires a decision (for all values of \( a > 4 \)). The decision makes the system historical, but requires an incursive agent who is able to perceive the options (Eq. 3a). Whereas the hyper-incursive system develops as a regime at the next-order level generating options, the incursive system develops historically along its steady state \( x = (a - 1)/a \) for all values of \( a \).\(^3\) The brown line in Fig. 3 represents this function: the line crosses the nature/culture divide at \( a = 4 \) while it contains both a historically embedded component and a reflexive one. Its individual identity develops along a trajectory.

---

\(^3\) One can rewrite Eq. 3a as follows:

\[ x_{t+1} = ax_t(1 - x_{t+1}) \]  \hspace{1cm} (n1)
\[ x_{t+1} = ax_t - ax_t x_{t+1} \]  \hspace{1cm} (n2)
\[ x_{t+1}(1 + ax_t) = ax_t \]  \hspace{1cm} (n3)
\[ x_{t+1} = ax_t / (1 + ax_t) \]  \hspace{1cm} (n4)

The steady state for \( x_{t+1} = x_t \) follows:

\[ 1 = a / (1 + ax) \]  \hspace{1cm} (n5)
\[ 1 + ax = a \]  \hspace{1cm} (n6)
\[ x = (a - 1)/a \]  \hspace{1cm} (n7)

The steady state of Eq. 3b is the same as that of Eq. 3a (Leydesdorff & Franse, 2009).
The social system is not an individual but a distribution. Luhmann (1984: 625) proposed to use the word “dividuum” for this system (Nietzsche ([1878] 1967: 76). The expectations are instantiated by carriers who entertain a network of relations as a retention mechanism. The carriers of the communication can entertain different perspectives on the social system evolving among them in terms of expectations. As Langton (1989, p. 31) formulated: “local behavior supports global dynamics, which shapes local context, which affects local behavior, which supports global dynamics, and so forth.” The two processes operate as feedback mechanisms upon each other. If the one feedback is dominant, the resulting process has a positive sign; but in the other case the sign is negative.

In summary, three processes can be specified: (i) the historical one, (ii) the evolutionary one, and (iii) the coupling between the two. The historical one can be captured in terms of Shannon’s information theory; the evolutionary one has first to be hypothesized. The two dynamics are coupled in the incursive equation (Eq. 3a) providing a reference to prior and expected states. The first reference generates entropy, the second reduces uncertainty. The resulting trade-off can be modeled using mutual information and mutual redundancy in interactions among systems.

7. Redundancy versus information generation

Figure 3 visualizes two sets of messages #1 and #2 with an overlap.
In this case, the sum of the expected information contents of #1 and #2 is:

\[ H_{12} = H_1 + H_2 - T_{12} \]  \hspace{1cm} (5)

One subtracts the overlap \((T_{12})\) because otherwise one would count this mutual information twice. However, if one sums the two sets as whole sets, one obtains \(Y_{12} = H_1 + H_2 + T_{12} \). In other words, mutual redundancy \(R_{12}\) has the opposite sign: \(R_{12} = -T_{12}\). \(T_{12}\) is Shannon-type information and therefore positive; \(R_{12}\) consequently is expressed in terms of negative bits of information.

Eq. 5 can be written in a more general format:

\[ T_{12} = \sum_{i=1}^{n=2} H(x_i) - H(x_1, x_2) \geq 0 \]  \hspace{1cm} (6)
In the case of more than two dimensions, one has to correct also for the overlaps among the overlaps as follows:

\[
\sum_{i=1}^{n=3} H(x_i) - H(x_1, x_2, x_3) = \sum_{ij} T_{ij} - T_{123}
\]

(7)

\[
\sum_{i=1}^{n=4} H(x_i) - H(x_1, x_2, x_3, x_4) = \sum_{ij} T_{ij} - \sum_{ijk} T_{ijk} + T_{1234}
\]

(8)

\[
(\ldots)
\]

\[
\sum_{i=1}^{n} H(x_i) - H(x_1, \ldots, x_n) =
\]

\[
\sum_{ij}^{(n)} T_{ij} - \sum_{ijk}^{(n)} T_{ijk} + \sum_{ijkl}^{(n)} T_{ijkl} - \cdots + (-1)^{n} \sum_{ijkl\ldots(n-1)}^{(n-1)} T_{ijkl\ldots(n-1)}
\]

(9)

It follows from Eq. 9 that mutual information in \( n \) dimensions is:

\[
\sum_{ijkl\ldots(n)}^{(n)} T_{ijkl\ldots(n)} = [\sum_{i=1}^{n} H(x_i) - H(x_1, \ldots, x_n)]
\]

\[
- [\sum_{ij}^{(n)} T_{ij} - \sum_{ijk}^{(n)} T_{ijk} + \sum_{ijkl}^{(n)} T_{ijkl} - \cdots + (-1)^{1+n} \sum_{ijkl\ldots(n-1)}^{(n-1)} T_{ijkl\ldots(n-1)}]
\]

(10)

And mutual redundancy with the opposite sign is:

\[
R_n = (-1)^{1+n} T_{1234\ldots n} = -[\sum_{i=1}^{n} H(x_i) - H(x_1, \ldots, x_n)]
\]

\[
+ [\sum_{ij}^{(n)} T_{ij} - \sum_{ijk}^{(n)} T_{ijk} + \sum_{ijkl}^{(n)} T_{ijkl} - \cdots + (-1)^{1+n} \sum_{ijkl\ldots(n-1)}^{(n-1)} T_{ijkl\ldots(n-1)}]
\]

(11)

Since \([\sum_{i=1}^{n} H(x_i) - H(x_1, \ldots, x_n)] \geq 0\)—see Eq. 6 above—the first bracketed term of Eq. 11 is negative entropy (that is, redundancy). The sum of the mutual information relations in the right bracketed term contributes a second term (which is positive since Shannon-type information).
In other words, we model here the generation of redundancy on the one side versus the historical process of uncertainty generation in relating on the other. The result is an empirical balance between positive and negative terms: When the resulting $R$ is negative, the evolutionary dynamics prevails over historical organization in an incursion under study, whereas a positive $R$ indicates conversely a predominance of historical organization over evolutionary self-organization. In empirical cases, one is thus able to test which combination of dimensions generates redundancy.

One of us developed this indicator when studying innovation systems in terms of “the Triple Helix of university-industry-government relations” (Etzkowitz & Leydesdorff, 2000; Leydesdorff, 2003). The relations among technologies, territories, and organization were considered, for example by Storper (1997, pp. 26f.), as a “holy trinity”: “Territorial economies are not only created, in a globalizing world economy, by proximity in input-output relations, but more so by proximity in the untraded or relational dimensions of organizations and technologies.” The triple helix indicator allows one to analyse at which level this “holy trinity” generates new options: nationally, regionally, in terms of some sectors of the economy more than others, etc.?

In a number of studies of national systems of innovation, we found, for example that the national level adds to the redundancy in some countries (e.g., in Sweden, the Netherlands, and China), but not always. In the German case, we found redundancy generation at the level of states (Länder), but not above this level; in Hungary, the national level did not add redundancy to the sum of
three regional innovation systems in this country: (1) the western part which is oriented towards Europe, (2) the metropolitan system around Budapest, and (3) the eastern part in which the older state-oriented system prevails. In the case of the Russian Federation, historical organization is so prevalent that the synergy at regional levels is disturbed by the national level.

A more extensive discussion of these studies of national systems of innovation is provided by Leydesdorff & Ivanova (2016). However, the model is not limited to studying geographical/territorial constraints on innovation, but can be used whenever three or more independent dimensions can be specified. The three (or more) dimensions can be considered as the degrees of freedom in the communication; the distributions over the units of analysis provide information in the respective dimensions; and the correlations among the sets of relations span a vector space in which the system “lives” by recognizing and acknowledging the information as differences that can make a difference.

8. Discussion and conclusions

We have argued that while defined as each other’s complement to the maximum entropy in Shannon’s information theory, “information” and “redundancy” have a different status. The dynamics of information and redundancy are also very different. Whereas a probability distribution always contains uncertainty which can be expressed in units of information (e.g., bits), redundancy as the difference between the maximum information and the information is not observable; it is absent information. The specification of redundancy is dependent on our specification of the possibility space and thus on our theoretical imagination. Unlike individual
imagination, theoretical imagination is discursive. The specification of redundancy and hence maximum entropy is grounded in intersubjectivity.

The intersubjective dimension as was introduced by the philosopher Edmund Husserl in his ([1936/54], 1970) *The Crisis of European Sciences and Transcendental Philosophy*. According to Husserl, this crisis is generated by an objectivistic self-understanding of the empirical sciences as positive sciences about “natural” phenomena. In his *Cartesian Mediations*, Husserl (1929) returned to Descartes’ (1637) distinction between *res extensa* and *res cogitans*; that is, between an observable reality “out there” and a mental reality which guides us and in which we interpret our observations. Whereas the cliff between these two realities had to be bridged in Descartes’ philosophy transcendentally (by God), Husserl argues that this transcendental dimension can nowadays be recognized as intersubjective intentionality: the sciences are grounded in discourse. Husserl ([1929: 181] 1960:155) concludes that a theory of science should be grounded in this ontology of intentionality and expectations.

Building on Husserl’s philosophy, Luhmann (1986) proposed to proceed to the specification of intersubjective intentionality. Basic is the double contingency in inter-human communication: we are not only contingent to each other in time and space, but also in terms of mutual expectations (Parsons & Shils, 1951): one expects the other to entertain expectations as one entertains them oneself. The expectations can be communicated in language or shared in terms of generalized symbols (Mead, 1934). The second contingency communicates in terms different from the material one.
The specification of the second contingency in terms of redundancy confronts us with the need to explore the theoretical imaginations. Redundancy is not a given, but a consequence of our specification of a knowledge base. When redundancy increases faster than information, the number of not yet realized possibilities becomes more important for the further development of the communication than the past achievements of realized possibilities, and the system becomes increasingly knowledge-based: such a system tends to develop in terms of reconstructions instead of historical continuity and trajectories A technological evolution of expanding possibilities emerges on top of the biological one of knowledge carriers such as reflexive agents and their networks of relations. The latter serve the retention as a condition for further development.

Mutual redundancy enables us to assess the trade-off between the instantiating and the instantiated in terms of bits of information and, even more importantly, in terms of a resulting sign (Yeung, 2009). Does the system generate new options or does it tend towards a lock-in into existing options? For the further development of an innovation system the generation of new options may be more important than the realization of existing ones. One can make this assessment empirical and distinguish among regions and sectors of a knowledge-based economy in terms of bits of information with positive or negative signs.

In summary: against monist programs and philosophies nowadays prevalent (e.g., Damasio’s (2006), The Error of Descartes), we argue in favor of a dualism between facts and values. Although both facts and values can be considered as constructed, their status is different. The systematic elaboration of the analytical distinctions enabled us to distinguish between the communication of information, meaning, and knowledge. However, one cannot measure
information without assuming a coding scheme; the coding or, in other words, the specification of the system(s) of reference, provides meaning to the information. Some meanings can further be specified and be validated as knowledge. The operationalization of “knowledge-based” in terms of generating options (i.e., redundancy) enables us to measure the innovativeness of innovation systems (Carter, 1996; Godin, 2006). Innovation can be considered as the result of the imprint of the evolving system of theoretical expectations on the networks that serve the retention.

References

Ashby, W. R. (1958). Requisite variety and its implications for the control of complex systems. Cybernetica, 1(2), 1-17.
Bar-Hillel, Y. (1955). An Examination of Information Theory. Philosophy of Science, 22, 86-105.
Bateson, G. (1972). Steps to an Ecology of Mind. New York: Ballantine.
Bernstein, B. (1971). Class, Codes and Control, Vol. 1: Theoretical studies in the sociology of language. London: Routledge & Kegan Paul.
Boudon, R. (1979). La logique du social. Paris: Hachette.
Brillouin, L. (1953). The negentropy principle of information. Journal of Applied Physics, 24(9), 1152-1163.
Brooks, D. R., & Wiley, E. O. (1986). Evolution as Entropy. Chicago/London: University of Chicago Press.
Carter, A. P. (1996). Measuring the performance of a knowledge-based economy. In D. Foray & B.-Å. Lundvall (Eds.), Employment and growth in the knowledge-based economy (pp. 61-68). Paris: OECD.
Coser, R. L. (1975). The complexity of roles as a seedbed of individual autonomy. In L. A. Coser (Ed.), The idea of social structure. Papers in honor of Robert K. Merton (pp. 237-264). New York/Chicago: Harcourt Brace Jovanovich.
Cowan, R., David, P., & Foray, D. (2000). The Explicit Economics of Knowledge Codification and Tacitness. Industrial and Corporate Change, 9(2), 211-253.
Damasio, A. R. (1994). Descartes' error: emotion, reason, and the human brain. New York: Grosset/Putnam.
Descartes, R. (1637). Discours de la méthode pour bien conduire sa raison, et chercher la vérité dans les sciences. Leiden: Jan Maire.
Distin, K. (2012). Symbolically Generalized Communication Media: A Category Mistake? Constructivist Foundations, 8(1), 93-95.
Dretske, F. I. (1981). Knowledge and the flow of information. Cambridge, MA: MIT Press Mass.
Dubois, D. M. (1998). Computing Anticipatory Systems with Incursion and Hyperincursion. In D. M. Dubois (Ed.), *Computing Anticipatory Systems, CASYS-First International Conference* (Vol. 437, pp. 3-29). Woodbury, NY: American Institute of Physics.

Dubois, D. M. (2003). Mathematical Foundations of Discrete and Functional Systems with Strong and Weak Anticipations. In M. V. Butz, O. Sigaud & P. Gérard (Eds.), *Anticipatory Behavior in Adaptive Learning Systems* (Vol. Lecture Notes in Artificial Intelligence Vol. 2684, pp. 110-132). Berlin: Springer-Verlag.

Etzkowitz, H., & Leydesdorff, L. (2000). The Dynamics of Innovation: From National Systems and 'Mode 2' to a Triple Helix of University-Industry-Government Relations. *Research Policy*, 29(2), 109-123.

Giddens, A. (1979). *Central Problems in Social Theory*. London, etc.: Macmillan.

Godin, B. (2006). The Knowledge-Based Economy: Conceptual Framework or Buzzword? *Journal of Technology Transfer*, 31(1), 17-30.

Hayles, N. K. (1990). *Chaos Bound: Orderly Disorder in Contemporary Literature and Science* Ithaca, etc.: Cornell University.

Hidalgo, C. (2015). *Why Information Grows: The Evolution of Order, from Atoms to Economies*. New York: Basic Books.

Hodgson, G., & Knudsen, T. (2011). *Darwin's conjecture: The search for general principles of social and economic evolution*. Chicago / London: University of Chicago Press.

Husserl, E. ([1935/36] 1962). *Die Krisis der Europäischen Wissenschaften und die Transzendentale Phänomenologie*. Den Haag: Martinus Nijhoff.

Husserl, E. (1929). *Cartesianische Meditationen und Pariser Vorträge [Cartesian Meditations and the Paris Lectures, translated by Dorion Cairns]* (Vol. at http://ia600601.us.archive.org/10/items/CartesiamMeditations/12813080-husserl-cartesian-meditations.pdf ). The Hague: Martinus Nijhoff, 1960.

Ivanova, I. A., & Leydesdorff, L. (2014a). Redundancy Generation in University-Industry-Government Relations: The Triple Helix Modeled, Measured, and Simulated. *Scientometrics*, 99(3), 927-948. doi: 10.1007/s11192-014-1241-7

Ivanova, I. A., & Leydesdorff, L. (2014b). Rotational Symmetry and the Transformation of Innovation Systems in a Triple Helix of University-Industry-Government Relations. *Technological Forecasting and Social Change*, 86, 143-156.

Kauffman, L. H. (2001). The Mathematics of Charles Sanders Pierce. *Cybernetics & Human Knowing*, 8(1-2), 79-110.

Kauffman, S. A. (2000). *Investigations*. Oxford, etc.: Oxford University Press.

Krippendorff, K. (1986). *Information Theory. Structural Models for Qualitative Data*. Beverly Hills, etc.: Sage.

Krippendorff, K. (2009a). W. Ross Ashby’s information theory: a bit of history, some solutions to problems, and what we face today. *International Journal of General Systems*, 38(2), 189-212.

Krippendorff, K. (2009b). Information of Interactions in Complex Systems. *International Journal of General Systems*, 38(6), 669-680.

Künzler, J. (1987). Grundlagenprobleme der Theorie symbolisch generalisierter Kommunikationsmedien bei Niklas Luhmann. *Zeitschrift für Soziologie*, 16(5), 317-333.

Langton, C. G. (1989). *Artificial Life*. In C. G. Langton (Ed.), *Artificial Life* (Vol. VI, pp. 1-47). Redwood, etc.: Addison-Wesley.
Lazarsfeld, P. F., & Henry, N. W. (1968). *Latent structure analysis.* New York: Houghton Mifflin.

Leydesdorff, L. (1991). The Static and Dynamic Analysis of Network Data Using Information Theory. *Social Networks, 13*(4), 301-345.

Leydesdorff, L. (2003). The Mutual Information of University-Industry-Government Relations: An Indicator of the Triple Helix Dynamics. *Scientometrics, 58*(2), 445-467.

Leydesdorff, L. (2006). The Biological Metaphor of a (Second-order) Observer and the Sociological Discourse. *Kybernetes, 35*(3/4), 531-546.

Leydesdorff, L. (2008). The Communication of Meaning in Anticipatory Systems: A Simulation Study of the Dynamics of Intentionality in Social Interactions. In D. M. Dubois (Ed.), *Proceedings of the 8th Intern. Conf. on Computing Anticipatory Systems CASYS'07* (Vol. 1051 pp. 33-49). Melville, NY: American Institute of Physics Conference Proceedings.

Leydesdorff, L. (2012). Radical Constructivism and Radical Constructedness: Luhmann's Sociology of Semantics, Organizations, and Self-Organization. *Constructivist Foundations, 8*(1), 85-92.

Leydesdorff, L., & Franse, S. (2009). The Communication of Meaning in Social Systems. *Systems Research and Behavioral Science, 26*(1), 109-117.

Leydesdorff, L., & Ivanova, I. A. (2016). 'Open Innovation' and 'Triple Helix' Models of Innovation: Can Synergy in Innovation Systems Be Measured? *Journal of Open Innovations: Technology, Market and Complexity, 2*(1), 1-12. doi: 10.1186/s40852-016-0039-7

Leydesdorff, L., Petersen, A., & Ivanova, I. (in press). The self-organization of meaning and the reflexive communication of information. *Social Science Information* (arXiv preprint arXiv:1507.05251).

Luhmann, N. (1984). *Soziale Systeme. Grundriß einer allgemeinen Theorie.* Frankfurt a. M.: Suhrkamp.

Luhmann, N. (1986). Intersubjektivität oder Kommunikation: Unterschiedliche Ausgangspunkte soziologischer Theoriebildung. *Archivio di Filosofia, 54*(1-3), 41-60.

Luhmann, N. (1997). *Die Gesellschaft der Gesellschaft.* Frankfurt a.M.: Suhrkamp.

Luhmann, N. (2012). *Theory of Society, Vol. 1.* Stanford, CA: Stanford University Press.

MacKay, D. M. (1969). *Information, Mechanism and Meaning.* Cambridge and London: MIT Press.

Maturana, H. R. (1978). Biology of language: the epistemology of reality. In G. A. Miller & E. Lenneberg (Eds.), *Psychology and Biology of Language and Thought. Essays in Honor of Eric Lenneberg* (pp. 27-63). New York: Academic Press.

Mead, G. H. (1934). *Mind, Self, & Society from the Standpoint of a Social Behaviourist. Works of G. H. Mead.* Chicago and London: University of Chicago Press.

Montesquieu, C. de Sécondat, Baron de (1748). *De l'esprit des lois:* Paris.

Nelson, R. R., Buterbaugh, K., Perl, M., & Gelpi, A. (2011). How medical know-how progresses. *Research Policy, 40*(10), 1339-1344.

Nietzsche, F. ([1878] 1967). *Menschliches, allzumenschliches: ein Buch für freie Geister.* Berlin: Walter de Gruyter & Co.

Parsons, T. (1968). Interaction: I. Social Interaction. In D. L. Sills (Ed.), *The International Encyclopedia of the Social Sciences* (Vol. 7, pp. 429-441). New York: McGraw-Hill.

Parsons, T., & Shils, E. A. (1951). *Toward a General Theory of Action.* New York: Harper and Row.
Petersen, A., Rotolo, D., & Leydesdorff, L. (2016). A Triple Helix Model of Medical Innovations: Supply, Demand, and Technological Capabilities in Terms of Medical Subject Headings. *Research Policy, 45*(3), 666-681. doi: 10.1016/j.respol.2015.12.004

Rosen, R. (1985). *Anticipatory Systems: Philosophical, mathematical and methodological foundations*. Oxford, etc.: Pergamon Press.

Rosenberg, N. (1982). How exogenous is science? *Inside the Black Box: Technology and Economics*. Cambridge: Cambridge University Press.

Schrödinger, E. (1944). *What is Life?* Cambridge: Cambridge University Press.

Shannon, C. E. (1945). A mathematical theory of cryptography. *Memorandum MM, 45*, 110-102.

Shannon, C. E. (1948). A Mathematical Theory of Communication. *Bell System Technical Journal, 27*, 379-423 and 623-656.

Shannon, C. E., & Weaver, W. (1949). *The Mathematical Theory of Communication*. Urbana: University of Illinois Press.

Spencer Brown, G. (1969). *Laws of Form*. London: George Allen and Unwin.

Storper, M. (1997). *The Regional World - Territorial Development in a Global Economy*. New York: Guilford Press.

Theil, H. (1972). *Statistical Decomposition Analysis*. Amsterdam/ London: North-Holland.

Ulanowicz, R. E. (1986). *Growth and Development: Ecosystems Phenomenology*. San Jose, etc.: toExcel.

Ulanowicz, R. E. (2009). The dual nature of ecosystem dynamics. *Ecological modelling, 220*(16), 1886-1892.

Ulanowicz, R. E. (2014). Reckoning the nonexistent: Putting the science right. *Ecological modelling, 293*, 22-30.

Varela, F. J. (1979). *Principles of biological autonomy*. Amsterdam: North Holland.

von Foerster, H. (1960). On Self-organizing Systems and Their Environments. In M. C. Yovits & S. Cameron (Eds.), *Self-Organizing Systems* (pp. 31-50). London: Pergamon Press.

Von Foerster, H. (1982). *Observing Systems* (with an introduction of Francisco Varela ed.). Seaside, CA: Intersystems Publications.

Weaver, W. (1949). Some Recent Contributions to the Mathematical Theory of Communication. In C. E. Shannon & W. Weaver (Eds.), *The Mathematical Theory of Communication* (pp. 93-117.). Urbana: University of Illinois Press.

Yeung, R. W. (2008). *Information Theory and Network Coding*. New York, NY: Springer.