Abstract. When powder forms a weak interaction particle cluster in a gas-solid flow, the rate of fall of the cluster exceeds the terminal velocity of the individual particles (Slack, 1963; Marzocchella et al., 1991). However, the relationship between the unsteady characteristics of the free-fall of the particle cluster and the geometric condition of the experiment is not clear. We performed a simple experiment in which powder of a certain mass falls in a vertical pipe. When the powder falls in the vertical pipe, the distribution length of the powder expands, and the particle volume fraction is dense in the lower part, and is thin in the upper part. The fall velocity of the lower edge of the powder cluster and the flow rate of air generated by the powder fall were measured. We obtained the following results. The relative velocity of free-fall of the particle cluster has no relation to the individual particle diameters. The characteristic of a particle cluster exists unless the cluster has very high void fraction.

1. INTRODUCTION

In industry, we can often see an operation in which powder of a certain mass falls freely in a vertical pipe, for example, a powder packaging machine. A powder packaging machine is one which fills up small bags with a powdery material, such as food, medical supplies or a chemical industry product, individually with a certain constant mass. When the powdery material is supplied to a small bag, a vertical pipe is used for a supply chute in the powder packaging machine, as in Fig. 1. In such a case, powder of a certain mass falls in a vertical pipe.

Concerning a problem in this operation, if the particle diameter or the particle density is small, automatic control of the powder packaging machine is difficult, because particles are scattered and the distribution length of the powder is prolonged. In the real operation of the powder packaging machine, most of the powdery materials are A or B particles of Geldart's classification. And if the physical properties of particles approach the C group of Geldart's classification, the powder packaging machine causes a serious sanitary error in production activity because of the powder scattering. Therefore, a quantitative survey of the operation in which powder of a certain mass falls in a vertical pipe is important in the field of food machine industry for determining the most suitable design, development and use.

Concerning the flow characteristics of sedimentation of solid particles under gravity while still fluid, if the particles behave with uniform dispersion and uniform velocity, a method that corrects the drag force of a single particle based on Stoke's law by function of void fraction is generally used (Steinour, 1944; Richardson and Zaki, 1954; Richardson and Meikle, 1961; Famularo and Happel, 1965). And when the void fraction is comparatively high this method is effective, for example \( \epsilon > 0.6 \) according to the study of Steinour (1944).
When particles fall with a low void fraction and non-uniform dispersion, the powder forms particle clusters (Slack, 1963). When the powder forms a particle cluster, it is thought that the particles and the fluid included in the cluster, stand still relatively, and move together. It is thought that the particle cluster has begun to attract attention from Slack's experiments (1963). In this experiment, the spherical clusters (25-90 [cm] diameter) made up of spherical particles belonging to A group of Geldart's classification were caused to fall through still air. Slack (1963) showed that the fall velocity of the cluster exceeded the terminal velocity of the individual particles in all cases of the experiment, and he expected that if powder forms a particle cluster the cluster can be stable with a bowl shape. On the other hand, Marzocchella et al. (1991) investigated the release of a cylindrical cluster of solid particles in still air inside a two-dimensional column observed by a high-speed video camera. Powders belonging to A group, B group and D group of Geldart's classification were used in this experiment. Marzocchella et al. (1991) expected that clusters consisting of A group particles fall almost like rigid bodies and that clusters can be considered as bodies impermeable to gas. Subsequently, particle clusters attracted attention in the field of fluidized beds. Some investigation was performed using a high-speed camera, and the shape and the scale structure of clusters formed in a fluidized bed are clarified by the following studies.

Rhodes et al. (1992) investigated the flow pattern of a cluster near the wall of a fluidized bed. Hatano et al. (1994) investigated the flow pattern of a cluster near the wall and central area of a fluidized bed by using a microscope visualization. Tsukada et al. (1997) investigated the three-dimensional structure of clusters formed in a circulating fluidized bed by using a scanning laser sheet technique. Lackermier et al. (2001) investigated the internal flow structure of the circulating fluidized bed by using optical endoscopes. Three characteristic shapes of the particle cluster were observed: paraboloid heading downward, parabola in vertical cross-section heading upward and vertical film along the channel.

There are a few studies on the flow characteristics of the free-fall of the particle cluster. Marzocchella et al. (1991) evaluated the fall velocity of the particle cluster quantitatively in an experiment of circular clusters with two-dimensional flow. They showed that the nonsteady curve of the free-fall velocity of a particle cluster could be calculated based on a one-dimensional equation of motion on a single particle. In that calculation, the cluster diameter and the bulk density which were measured before the fall were substituted for the particle diameter and the true density respectively.

While not an experiment for the particle cluster, Ogata et al. (2001) investigated a powder jet exhaust positioned downward from a hopper. In this study, the velocity distribution of particles at the jet central axis was compared with the curve calculated based on the one-dimensional equation of motion of a single particle as in Marzocchella et al. (1991). The powder jet resembles the free-fall of the particle cluster at the points that the void fraction is low at the hopper exit and that the state of low void fraction is kept downward. Therefore, though Ogata et al. (2001) measured in a steady state, it seems that the curve of a single particle suggests free-fall characteristics of the particle clusters.

On the other hand, approximately realistic simulation of numerical analysis considering interplay between fluid and particles became possible by the contribution of the development of calculation technology. Zhang et al. (2008) calculated a two-dimensional fluidized bed by coupling the Discrete Element Method (DEM) and Direct Numerical Simulation (DNS). Tsuji et al. (2008) calculated a three-dimensional fluidized bed by coupling a hard sphere model and DNS. These studies explained visually the formation of the cluster and the process of the dispersion. The U-shaped cluster structure that was observed in experiments with a high-speed camera appeared in these calculation, too.

If limited to a vertical pipe flow, there are few studies on powder free-fall. Most studies of powder flow in a vertical pipe are analyzed for pneumatic transport or for a standpipe of the circulating fluidized bed. As an example of powder free-fall in a vertical pipe, there is Matsushita's experiment (2002, 2003) on transition of powder slugging in a field of the statistical physics of the pattern formation. In this experiment, it was found that there exists a critical in-flow rate for the formation of density waves, and this result may suggest the condition for generation of the particle cluster.

These conventional studies often pay attention to dynamic mechanisms, for example the
mathematical characteristics of the phenomenon, the shape of the cluster or the process of formation and dispersion. However most of these studies are not useful in the design of a powder packaging machine. Therefore, in this study, we performed a simple experiment in which powder of a certain mass falls through still air in a vertical pipe, and measured the velocity of the lower edge of the powder cluster and the discharged air volume generated by falling powder by video camera, and investigated the characteristic distribution of the powder free-fall. The results in this paper are basic data which are useful for the design of powder packaging machines.

![Fig. 1 Schematic view of a supply chute in the powder packaging machine](image)

2. EXPERIMENTAL APPARATUS AND CONDITIONS

Fig. 2 shows a schematic of the experimental apparatus in this work. It is constituted from transparent acrylic pipe for the powder fall section and glass pipe for the volumetric flowmeter. The diameter and length of the pipe through which powder falls is \( \phi \) 18 [mm] and 1000 [mm] respectively. The lower part of the acrylic pipe is connected to conductive materials which are grounded, so that the experiment was not affected by static electricity. The cross section of the device where powder starts to fall is shown in the upper left in Fig. 2. It is constituted from two acrylic pipes which are connected by a flange. PET resin film is inserted between the flange, and the powder is built up naturally on the film. When the film between the flange is pulled out the powder starts falling. The diameter of the volumetric flowmeter is \( \phi \) 9.2 [mm]. In all experiments, the tube connected to the flowmeter is removed and the soap film is prepared in the upper part of the flowmeter. The flowmeter is set up so that the soap film moves from the upper to the lower, so that movement resistance by the weight of the soap film is reduced. The experiments are recorded on videotape (the frame interval is 1/30 [s]). The coordinate \( z \) is for vertical downward. The \( z \)-axis of the lower edge of the powder cluster, of the upper edge of the powder cluster and of the soap film of the flowmeter were measured.

Table 1 shows powder properties and experimental conditions. The mean particle diameter and the powder mass per experiment are \( d_p \) and \( M \) respectively. The powders are glass beads which were sieved (mesh sizes are 53 [\( \mu \) m], 63 [\( \mu \) m], 75 [\( \mu \) m], 90 [\( \mu \) m], 125 [\( \mu \) m], 150 [\( \mu \) m], 180 [\( \mu \) m], 212 [\( \mu \) m] and 250 [\( \mu \) m]) and separated into a uniform size for each group. The mean particle diameters are the average of ten particles that were picked out from individual groups. An electron
microscope was used for the measurement of the diameter. The conditions of the powder mass per experiment were 5 [g], 10 [g] and 15 [g]. Concerning these mass conditions, when the powder filled up a $\phi$ 18 [mm] pipe, the filling heights became 13 [mm], 26 [mm] and 39 [mm] respectively. The variety of powder had six different diameters in Table 1 all belonging to A or B group of Geldart's classification. The powder particles which have a diameter from 58.3 [ $\mu$ m] to 84.0 [ $\mu$ m] belong to A group of Geldart's classification, and the powder particles which have a diameter from 123.2 [ $\mu$ m] to 220.4 [ $\mu$ m] belong to B. A total of 18 different experiments were performed in all combinations for six kinds of $d_p$ and three kinds of $M$.

![Experimental apparatus](image)

**Fig. 2**: Experimental apparatus.

**Table 1.** Experimental materials properties and experimental conditions.

| Particle characteristics |  |
|--------------------------|--|
| Material                 | Glass beads |  |
| $\rho_p$ [g/cm$^3$]      | 2.35        |  |
| $d_p$ [$\mu$ m]          | 58.3, 69.4, 84.0, 123.2, 160.4, 220.4 |  |
| $M$ [g]                  | 5, 10, 15   |  |

**3. RESULTS AND DISCUSSION**

**3.1 Measurement of the height (movement distance)**

Fig. 3 (a) shows the heights of the lower edge of powder cluster, of the upper edge of powder cluster and of the soap-film of the volumetric flowmeter as a function of elapsed time. The values plotted in Fig. 3 (a) are the averages of ten experiments. The experiment conditions of Fig. 3 (a) are $d_p = 58.3$ [$\mu$ m] and $M = 15$ [g]. Concerning experimental conditions, the influence that particles had on air was the largest. Consequently the discharged air volume from the vertical pipe was the largest influence in all experiments. The origin of the height of powder is the position of the lower edge of the powder cluster before fall start (the position of PET film), and the origin of the height of soap film is the position of the soap film before starting a fall. The explanatory notes of plotted signs are shown on the upper left in Fig. 3 (a). The height of the soap film in Fig. 3 (a) is converted by multiplying the measured value and the area ratio, because it is easy to compare the discharged air volume with the volume that the powder moved. The area ratio is calculated by dividing the cross section of the flowmeter by the cross section of the pipe. In Fig. 3 (a), it is found that the lower edge of powder cluster $\bullet$ is increasing depending on the elapsed time, and that the increase rate of $z$ is increasing at
ranges from $t = 0.0$ [s] to $t = 0.45$ [s]. It appears that this tendency conforms to the general scheme of the free-fall of a single particle. At about $t = 0.45$ [s] the lower edge of the powder cluster is drained from the pipe. The upper edge of powder cluster $\bigcirc$ is increasing depending on elapsed time, and the increase rate of $z$ is increasing at ranges from $t = 0.0$ [s] to $t = 0.5$ [s], as well. However it is found that the increase rate of $z$ is decreasing after $t = 0.5$ [s]. When we see Fig. 3 (a) as $z = \text{constant}$, at the same $z$, the time difference between the lower edge $\bullet$ and the upper edge $\bigcirc$ indicates the passage time of the powder at the position of $z$. In Fig.3 (a), it is found that the passage time of the powder gets longer depending on the increase of $z$ in excess of about $z = 800$ [mm]. It appears that the particle velocity approaches the terminal velocity of individual particles, because the particle volume fraction is low (void fraction is high) near the upper edge. Therefore, the atmosphere that is just forming sedimentation of the single particles may exist near the upper edge of the powder cluster. On the other hand, it is found that the discharged air volume from the vertical pipe is very low in comparison with the volume of powder moved. It appears that there was less volume of discharged air so that most of the air passes outside or inside of the powder fall.

Fig. 3 (b) shows the heights of the lower edge of the powder cluster, of the upper edge of the powder cluster and of the soap film of the volumetric flowmeter as a function of elapsed time. The experiment conditions are $d_p = 220.4$ [ $\mu$ m] and $M = 15$ [g]. The method of the data reduction is totally the same as in the case of Fig. 3 (a). In Fig. 3 (b), it is found that each distribution ($\bullet$, $\bigcirc$, $\triangle$) has the same tendency as in Fig.3 (a) at the ranges from $t = 0.0$ [s] to $t = 0.6$ [s]. However it is found that the slowdown of the upper edge of the powder cluster is small after $t = 0.6$ [s]. It appears that there are few scattered particles near the upper edge of the powder cluster in the condition of $d_p = 220.4$ [ $\mu$ m].

![Graph of Fig. 3](image1)

Fig. 3 The heights of the lower edge of powder cluster, of the upper edge of powder cluster and of the soap film of the volumetric flowmeter as a function of elapsed time. The height of the soap film $\triangle$ is converted by multiplying the measured value and the area ratio. The area ratio is calculated by dividing the cross section of the flowmeter by the cross section of the pipe.

3.2 Discharged air volume generated by the powder fall

Fig.4 shows the relationship between the air volume that was discharged from the pipe by the fall of powder and elapsed time. All six kinds of mean particle diameters (58.3 [ $\mu$ m], 69.4 [ $\mu$ m], 84.0 [ $\mu$ m], 123.2 [ $\mu$ m], 160.4 [ $\mu$ m], 220.4 [ $\mu$ m]) were plotted. The result of the experiment of $M = 15$
[g], and the influence that particles give to air is the largest. In Fig. 4, it is found that the increase rate of $V$ (i.e. flow rate) is increasing in the ranges from $t = 0.4 \ [s]$ to $t = 0.6 \ [s]$ temporarily, and that the increase rate of $V$ is decreasing after $t = 0.6 \ [s]$, at all $d_p$. Further details of this will be described later. Almost all powder is discharged upon exit of the pipe ($z = 1000 \ [\text{mm}]$) at $t = 0.8 \ [s]$, and the discharge of air by the fall stops. When the maximum value of $V$ at $t = 0.8 \ [s]$ is compared with the volume of the pipe ($= 254.5 \ [\text{cm}^3]$), it was 4.7 \ [%] at $d_p = 58.3 \ [\mu \text{m}]$ it appears that there was less volume of discharged air because most of the air (95.3 \ [%]) passes outside or inside of the powder cluster.

**Fig. 4 Relationship between the discharged air volume generated by the powder fall and elapsed time.**

### 3.3 Relative velocity of the particle cluster to the air flow

Fig. 5 (a)-(f) show the relationship between the relative velocity of the particle cluster to the air flow and the height of the $z$-axis. The powder velocity $v_p$ was measured at the lower edge of the powder cluster. The air velocity $v_f$ was calculated from the discharged air volume in Fig. 2. The bold line represents the calculated local free-fall velocity of a single particle as a sphere by Eq. (1) where the gravitational, buoyant, and drag forces are considered:

$$m_p \frac{dv_p}{dt} = \frac{m_p}{\rho_p} \left( \rho_p - \rho_f \right) g - \frac{1}{2} C_D \rho_f v_f^2 A$$

$A$, $C_D$, $g$, $m_p$, $v_p$, $\rho_p$ and $\rho_f$ are the projected area of a particle, the drag coefficient, gravitational acceleration, the mass of one particle, local free-fall velocity of a single particle in unbounded fluid, the particle true density and the air density respectively. Morsi and Alexander's (1972) approximate equations were used for the $C_D$. The fine line represents the calculated local free-fall velocity of a single particle as a sphere by Eq. (1) with no drag force. It is plotted as the factual basis for the decision to judge measurement. It is evident that the measurement result is approximately correct, because of the measurement results in Fig. 5 (a)-(f) are distributed between the bold line and the fine line. It is also found that the results in Fig. 5 (a)-(f) have the same tendency. The distributions of $v_p-v_f$ exceed the terminal velocity of the individual particles in all cases as well as those of Slack (1963) and Marzocchella et al. (1991). It is inferred that the powder is forming a particle cluster at the lower edge of the powder cluster. Moreover, the $v_p-v_f$ increased from $z = 0 \ [\text{mm}]$ to $z = 600-800 \ [\text{mm}]$ according to the law of free-fall, and decreased after that. It is inferred that the decay of the particle cluster causes the slowdown.
Fig. 5 Relationship between the relative velocity of the particle cluster to the air flow and $z$. These are plotted for all experiment conditions.
Fig. 6 is the relationship between the relative velocity of the particle cluster to the air flow and the height \( z \)-axis, and is a graph that showed only results for \( M = 15\) [g] in Fig.5 (a)-(f). The solid lines are the case which neglects aerodynamic drag, the case of \( d_p = 1.0 \) [mm] and the case of \( d_p = 0.5 \) [mm] respectively, which are the fall velocity of a single particle calculated with Eq. (1). The distributions for \( v_p, v_f \) are almost the same, however the particle diameters are different. And it closely resembles the distribution of the fall velocity for a single particle of \( d_p = 0.5 \) [mm]. When the powder forms the particle cluster, the fall velocity of the particle cluster may depend on the void fraction as well as sedimentation under uniform dispersion.

When we see Fig.5 (a)-(f) and Fig. 6 it is thought that the particle cluster decays near \( z = 600-800 \) [mm] because the relative velocity has maximum value. For calculating void fraction at the maximum point, if it is assumed that the particle cluster forms a sphere shape, the balance of force is given as Eq. (2):

\[
\frac{\pi D^3}{6} \left( 1 - \varepsilon \right) \left( \rho_p - \rho_f \right) g - \frac{\pi D^2}{4} C_D \rho_f \left( v_p - v_f \right) = 0 \tag{2}
\]

Reynolds number based on the particle cluster is given as Eq. (3):

\[
\text{Re}_c = \frac{\rho_p D c (v_p - v_f)}{\mu} \tag{3}
\]

The void fraction was about 99.4 [%], in our calculation (refer to APPENDIX 1) based on Eq. (2), Eq. (3) and Morsi and Alexander’s (1972) approximate equation. It is expected that the particle cluster maintains its characteristic as a particle cluster until high void fraction comparatively, however this is a slightly rough calculation.

4. CONCLUSIONS

The experiment concerning how powder of a certain mass falls in a vertical pipe was performed, using glass beads whose mean diameters were 58.3 [\( \mu \)m], 69.4 [\( \mu \)m], 84.0 [\( \mu \)m], 123.2 [\( \mu \)m], 160.4 [\( \mu \)m], 220.4 [\( \mu \)m] respectively. The velocity of the lower edge of the powder cluster and the
The velocity of air flow generated by the powder were measured by video camera, and obtained the following results.

1. The fall velocity of the particle cluster has no relation to the particle diameter in the range of $58.3 \, [\mu m] < d_p < 220.4 \, [\mu m]$, and approximates to the fall velocity of single particle of 0.5 [mm] in diameter.

2. The free-fall velocity of the particle cluster has the characteristic of exceeding the terminal velocity of the individual particles (Slack, 1963; Marzocchella et al., 1991). It is expected that this characteristic exists unless the cluster has high void fraction.

3. The discharged air volume generated by a powder fall from the pipe exit is low, and most of the air (95.3 [%]) passes to the outside or inside of the particle cluster.

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NOMENCLATURE

$A$ particle projected area

$C_D$ drag coefficient of a particle in unbounded fluid

$d_p$ particle diameter

$D_c$ cluster diameter

$g$ gravity acceleration

$M$ mass of powder falling at one time

$m_p$ mass of a particle

$Re_c$ Reynolds number based on a particle cluster

$Re_p$ Reynolds number based on a particle

$t$ elapsed time from powder fall start

$V$ discharged air volume from vertical pipe generated by powder fall

$v_f$ actual velocity of air

$v_p$ actual velocity of powder (measured at lower edge of powder cluster)

$v_s$ local free-fall velocity of a single particle in unbounded fluid

$z$ fall height from fall start point

Greek Letters

$\varepsilon$ void fraction

$\mu$ viscosity of air

$\rho_f$ density of air

$\rho_p$ true density of particle

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APPENDIX 1

The void fraction calculation when the particle cluster decays

It is assumed that the particle cluster decays immediately before the particle cluster slows down (at a maximum point in Fig. 5 (a)-(f) or Fig.6).
Then, from Fig. 5 (a)-(f) or Fig. 6 (at $z = 700$ [mm])

$$v_p - v_f = 2.6 [m/s] \ (at \ z = 700 \ [mm])$$

In addition, it is assumed that $D_c$ is nearly equal to the pipe diameter.

$$D_c = 18 [mm]$$

The other conditions that are necessary for the calculation are as follows.

$$g = 9.80665 [m/s^2]$$
$$\rho_f = 1.2465 [kg/m^3]$$
$$\rho_p = 2346.518 [kg/m^3]$$
$$\mu = 17.74 \times 10^{-6} [Pa \cdot s]$$

Therefore, Reynolds number based on the particle cluster is calculated by Eq. (3)

$$Re_c = \frac{\rho_f D_c (v_p - v_f)}{\mu} = 3288$$

$C_D$ is given by the following from Morsi and Alexander's (1972) approximate equations.

$$C_D = \frac{148.62}{Re_p} - \frac{4.75 \times 10^4}{Re_p^2} + 0.357 \ (1000 < Re_c < 5000)$$

Where, we substitute Reynolds number based on particle cluster for Reynolds number based on a single particle.

$$C_D = \frac{148.62}{Re_c} - \frac{4.75 \times 10^4}{Re_c^2} + 0.357 = 0.3978$$

When Eq. (2) is arranged

$$\varepsilon = 1 - \frac{3 C_D \rho_f (v_p - v_f)}{4 D_c \rho_p (\rho_p - \rho_f)}$$

When all the conditions are substituted, we obtain $\varepsilon$.

$$\varepsilon = 0.994$$