Optimal Squeezing in the Resonance Fluorescence of Single Photon Emitters

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The possibilities and perspectives of squeezed light emission are studied for coherently driven single photon sources, such as atoms and quantum dots. Maximal squeezing is realized, if the electronic subsystem of the emitter is in a pure quantum state. The purification is achieved by using a cavity as a second decay channel, besides the incoherent coupling to the electromagnetic vacuum. For realistic cavities this yields a purity of the electronic state of more than 99%. Aside from numerical calculations, we also derive approximate analytical results. Based on the approximations, effects are studied which originate from the environment of the emitter, including radiationless dephasing and incoherent pumping of the emitter and the cavity mode. The fragility of squeezing against decoherence is substantially reduced, so that squeezing persists even under hostile conditions. The measurement of squeezing from such light sources is also considered.

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I. INTRODUCTION.

Light fields having a cutoff in the photon statistics are known to exhibit nonclassical features. Of particular interest are single photon emitters (SPEs), whose emitted field consists of one photon in a properly defined mode volume. A single, laser-driven two-level atom was the first SPE under study [1,2]. The first experimental demonstration of the quantum nature of light through the photon antibunching effect was realized with a low-density atomic beam [3]. In a related experiment a sub-Poisson photon statistics was observed [4]. Later on, photon antibunching has also been demonstrated with single trapped ions [5,6]. Nowadays, artificial atoms, such as quantum dots in semiconductor microsystems, are established SPEs, showing photon antibunching and sub-Poisson photon statistics [7–10].

Squeezed light was also predicted to occur in the resonance fluorescence of driven SPEs [11]. Squeezing can also be realized and enhanced in the fluorescence of many atoms in different scenarios. They include the regular arrangement of the atoms [12], the detection in the forward direction with respect to the pump-beam [13], and the bistability in a strong driving field [14]. The latter two cases could be experimentally demonstrated [15,16]. Recently it was shown, that the output field of a driven cavity, containing an atom, shows weak squeezing [17].

A direct observation of squeezing from a single driven SPE could neither be demonstrated in atoms nor in quantum dots; hence, this is an issue of fundamental interest. The standard method for detecting squeezing is balanced homodyne detection. In this case, the observable effects in the resonance fluorescence are tiny due to the small collection efficiency of the light field. Based on homodyne correlation measurements, efficient measurement techniques were proposed [18–20], which are not limited by the small collection efficiency. The feasibility of such techniques has been demonstrated recently, in resonance fluorescence of a single trapped ion [21].

Recently we have shown that squeezed light in the fluorescence of an atom can be optimized [22]. The maximal possible squeezing depends on the excitation of the emitter. It is achieved when the electronic subsystem of the emitter is in a pure quantum state. Specifically, we found, that the maximal attainable squeezing is twice as large as for an atom in free space. This indicates the unfavorable situation of an atom which only couples to the vacuum field modes. We presented a purification scheme, solely based on a second coupling of the atom to a cavity field, which yields over 99% purity and 94% of the maximal possible squeezing. The fluorescence light must be observed out the side of the cavity, so that the latter acts as a passive environment. We predicted the squeezing to be substantially less fragile against radiationless dephasing.

In the present paper we study the fluorescence of general SPEs and investigate the possible squeezing and its limitations. We derive an analytical approximation of the purification procedure, which allows us to interpret the underlying physics. Furthermore, based on the approximation we analyze different environmental effects, as they occur in a quantum dot in a semiconductor microcavity. The results indicate, that squeezing persists even under certain perturbations, so that quantum dots in semiconductor microcavities may be a promising source of squeezed light, to be used in integrated optical systems. Finally, we deal with a simple method to detect the emitted squeezed light.

The article is organized as follows. In Sec. II we study the squeezing in the resonance fluorescence of a SPE. The purification method is considered in Sec. III. In Sec. IV we introduce our analytical approximations, which we apply in Sec. V to the environmental effects of dephasing and incoherent pumping. The possibility to detect the squeezing is analyzed in Sec. VI. Finally, in Sec. VII we give some conclusions and an outlook.

II. FLUORESCENCE SQUEEZING OF SPEs

A light field \( \hat{E} \) (dependencies on space and time are suppressed throughout the paper, unless needed) is squeezed, if its variance \( \langle (\Delta \hat{E})^2 \rangle \) is below the variance in the vacuum state,
\((\Delta \hat{E})^2)_{\text{vac}}\). Equivalently, the normally ordered variance,
\[
\langle (\Delta \hat{E})^2 \rangle = \langle (\Delta \hat{E})^2 \rangle - \langle (\Delta \hat{E})^2 \rangle_{\text{vac}},
\]
(1)
attains negative values, \(\langle (\Delta \hat{E})^2 \rangle < 0\). The "\(\ldots\)" prescription denotes normal ordering. The advantage of the normal ordering prescription is, that it separates free fields, \(\hat{E}_i\), from source fields, \(\hat{E}_s\), where \(\hat{E} = \hat{E}_i + \hat{E}_s\). If only the atomic source field hits the detector, the free field does not contribute to normally ordered correlation functions, so that
\[
\langle (\Delta \hat{E})^2 \rangle = \langle (\Delta \hat{E}_s)^2 \rangle.
\]
(2)
A detailed treatment of the source fields, in particular for the case of atomic resonance fluorescence, can be found in [23]. In the following we will omit the source field index 's'.

Based on the dipole- and rotating-wave approximation for the light matter coupling, a quasi-monochromatic source field can be written as an effective single-mode field,
\[
\hat{E} = \chi (\hat{b} e^{-i \varphi} + \hat{b}^\dagger e^{i \varphi}),
\]
(3)
\(\hat{b} (\hat{b}^\dagger)\) being the annihilation (creation) operator of the matter excitation inducing the dipole, while \(\varphi\) describes the phase of the field. The scaling factor \(\chi\) has the dimension of the electric field strength. For this case, the normally ordered field variance becomes
\[
\langle (\Delta \hat{E})^2 \rangle = 2 |\chi|^2 \left[ |\langle \hat{b} \rangle|^2 - |\langle \hat{b}^\dagger \rangle|^2 \right] + \Re \left\{ |\langle \hat{b}^\dagger \rangle|^2 - |\langle \hat{b} \rangle|^2 - e^{-2i \varphi} \right\}.
\]
(4)
Optimizing with respect to \(\varphi\), we obtain the phase \(\varphi_+ (\varphi_-)\) of maximal (minimal) normally ordered variance, \(\langle (\Delta \hat{E})^2 \rangle_\pm\), which read as
\[
e^{-2i \varphi_{\pm}} = \pm \sqrt{\frac{|\langle \hat{b}^\dagger \rangle|^2 - |\langle \hat{b} \rangle|^2}{|\langle \hat{b} \rangle|^2 - |\langle \hat{b}^\dagger \rangle|^2}},
\]
(5)
\[
\langle (\Delta \hat{E})^2 \rangle_\pm = 2 |\chi|^2 \left[ |\langle \hat{b} \rangle|^2 - |\langle \hat{b}^\dagger \rangle|^2 \right] \pm |\langle \hat{b} \rangle|^2 - |\langle \hat{b}^\dagger \rangle|^2 \right].
\]
(6)
In the following, except Sec[V], we only consider the phase optimized, minimal field fluctuation, which is the maximal squeezing. Hence, we omit the index '+'.'

Let our SPE be a general two-level system in an arbitrary environment. In such a scenario, the source fields are proportional to the atomic operators, see [23]. We can thus identify the annihilation and creation operators with the atomic flip operators \(\hat{A}_{ij} = |i\rangle \langle j| (i, j = 1, 2)\), with |1⟩ and |2⟩ being the ground and excited state, respectively. The source field reads as
\[
\hat{E} = \chi (\hat{A}_{12} e^{-i \varphi} + \hat{A}_{21} e^{i \varphi}),
\]
(7)
and the minimal normally ordered variance can be written as
\[
\langle (\Delta \hat{E})^2 \rangle = 2 |\chi|^2 |\langle \hat{A}_{22} \rangle - 2 |\langle \hat{A}_{12} \rangle|^2|.
\]
(8)
For any atomic excitation, \(\langle \hat{A}_{22} \rangle\), maximal squeezing is obtained for maximal atomic coherence, \(|\langle \hat{A}_{12} \rangle|^2\).

The expectation values in Eq. (8) are readily derived from the density operator of the SPE, \(\sigma = \sum_{ij} \sigma_{ij} \hat{A}_{ij}\). The density matrix \(\sigma\) reads as
\[
\sigma = \begin{pmatrix}
\langle \hat{A}_{11} \rangle & \langle \hat{A}_{12} \rangle \\
\langle \hat{A}_{12} \rangle & \langle \hat{A}_{22} \rangle
\end{pmatrix}.
\]
(9)
Combining the positive semi-definiteness of quantum states, \(\det \sigma \geq 0\), with the completeness relation of the SPE, we get
\[
|\langle \hat{A}_{12} \rangle|^2 \leq |\langle \hat{A}_{11} \rangle| |\langle \hat{A}_{22} \rangle| = |\langle \hat{A}_{22} \rangle| - |\langle \hat{A}_{22} \rangle|^2,\]
(10)
which yields the maximal coherence, \(|\langle \hat{A}_{12} \rangle|^2\), as function of the excitation \(\langle \hat{A}_{22} \rangle\).

Using this result in Eq. (9), the minimal variance follows as
\[
\langle (\Delta \hat{E})^2 \rangle_{\text{min}} = 2 |\langle \hat{A}_{22} \rangle| (2 |\langle \hat{A}_{22} \rangle| - 1),
\]
(11)
with the absolute minimum being given for \(\langle \hat{A}_{22} \rangle = 1/4\).
\[
\langle (\Delta \hat{E})^2 \rangle_{\text{min}} = \frac{1}{4}.
\]
(12)
The purity of the state of the SPE reads as
\[
\text{Tr} (\hat{\sigma}^2) = 1 - 2 (|\langle \hat{A}_{12} \rangle|^2 - |\langle \hat{A}_{22} \rangle|^2 - |\langle \hat{A}_{12} \rangle|^2).
\]
(13)
Compared with Eq. (10), a pure SPE state is equivalent to maximal SPE coherence and hence optimal squeezing.

Knowing the maximal possible squeezing, the question appears: how much squeezing can one achieve in the fluorescence of a SPE in free space? Squeezing of the fluorescence light was predicted [11, 23], but not yet experimentally confirmed. Let \(\omega_{\lambda}\) be the frequency difference between ground and excited state. The pump laser field is in a coherent state of frequency \(\omega_{\lambda} \pm \delta\lambda\). The coupling strength between SPE and laser field is given by the Rabi-frequency \(\Omega_R\), the laser phase is included in the phase of the source field. The SPE couples to the vacuum modes (spontaneous emission) with rate \(\Gamma\). In the frame rotating with \(\omega_{\lambda}\), the Hamiltonian and the master equation read as
\[
\hat{H}_0 = \hbar \delta\lambda \hat{A}_{22} + \hbar \Omega_R (\hat{A}_{12} + \hat{A}_{21}),
\]
(14)
\[
\dot{\hat{\sigma}} = \frac{1}{i \hbar} [\hat{H}_0, \hat{\sigma}] + \frac{\Gamma}{2} \mathcal{L}_{\omega_{\lambda}} [\hat{\sigma}],
\]
(15)
\[
\mathcal{L}_{\omega_{\lambda}} [\hat{\sigma}] = 2 \hat{O} \hat{\sigma} \hat{O}^\dagger - \{ \hat{O}^\dagger \hat{O}, \hat{\sigma} \}.
\]
(16)
This system can be solved analytically. The structure of the solution will be helpful for the discussion of environmental effects in the following sections.

The steady state values of both excitation and coherence can be given by a single variable \(z\),
\[
z = \frac{\Omega_R^2}{(\Gamma)^2 + \delta\lambda^2},
\]
(17)
\[
\langle \hat{A}_{22} \rangle = \frac{z}{1 + 2z}, \quad |\langle \hat{A}_{12} \rangle| = \frac{z}{(1 + 2z)^2}.
\]
(18)
Inserting these results into Eq. (13), we get
\[
\text{Tr}\{\hat{\sigma}^2\} = 1 - 2\langle \hat{A}_{22} \rangle^2.
\] (19)

The purity of the (stationary laser-driven) atom requires \(\langle \hat{A}_{22} \rangle = 0\), i.e. the SPE being in the ground state. In this case it obviously cannot emit fluorescent light. With increasing excitation the purity of the SPE state diminishes. For saturation, \(\langle \hat{A}_{22} \rangle = 1/2\) for \(z \to \infty\), the state of the emitter is fully mixed, without any coherence. Thus, the regime of maximal squeezing cannot be reached in free space fluorescence. Squeezing can only be observed for low atomic excitation. The free space normally ordered variance, Eq. (8), reads as
\[
\frac{\langle (\Delta \hat{E})^2 \rangle_{\text{fs}}}{|\chi|^2} = 2z(2z - 1)\frac{(1 + 2z)^2}{(1 + 2z)^2}.
\] (20)

Squeezed light is obtained for \(z \leq 1/2\), maximal squeezing in free space is realized for the parameters
\[
z = \frac{1}{6}, \text{ or } \langle \hat{A}_{22} \rangle = \frac{1}{8},
\] (21)
\[
\frac{\langle (\Delta \hat{E})^2 \rangle_{\text{fs},\text{min}}}{|\chi|^2} = -\frac{1}{8}.
\] (22)

Altogether, in free-space fluorescence from a SPE the possible squeezing is limited by the impurity of the quantum state of the emitter. This impurity results from the coupling to the vacuum modes described by the master equation (13). For an optimization of squeezing in resonance fluorescence from a SPE, the task is to realize a purification of the atomic state for non-vanishing excitation. This can be achieved by a proper environment of the SPE, such as a cavity \[22\]. In the following we will provide analytical approximations, which will help to better understand such scenarios, with the aim to optimize the resistance of squeezing against various perturbations.

### III. CAVITY INDUCED PURIFICATION

Pure states of SPEs have attracted significant attention over recent years due to their application as qubits in quantum information theory. Hence, protocols for purification \[24\] \[25\], and the determination of purity \[26\] \[28\] have been established. Recently we have shown that an optical cavity may act as a passive environment to purify the electronic state and to optimize squeezing of an atom undergoing resonance fluorescence \[22\]. A sketch of our setup is given in Fig. 1.

In addition to the system in Eqs. (14) (16), the SPE is coupled to a single-mode cavity of frequency \(\omega_c\), with coupling strength \(g\). The cavity excitation is described by bosonic creation and annihilation operators, \(\hat{a}^\dagger\) and \(\hat{a}\), respectively, and the cavity has an emission rate \(\kappa\). The full Hamiltonian for this system reads as
\[
\hat{H} = \hat{H}_0 + \hbar \delta_c \hat{a}^\dagger \hat{a} + \hbar g (\hat{a}^\dagger \hat{A}_{12} + \hat{A}_{21} \hat{a}),
\] (23)
where \(\delta_c = \omega_c - \omega_x\). The density operator \(\hat{\sigma}\) of the full system obeys the master equation
\[
\frac{d\hat{\sigma}}{dt} = -\frac{i}{\hbar}[\hat{H}, \hat{\sigma}] + \frac{\Gamma}{2} \mathcal{L}_{\hat{A}_{12}}[\hat{\sigma}] + \frac{\kappa}{2} \mathcal{L}_{\hat{a}}[\hat{\sigma}].
\] (24)

A general analytical solution of this system is unknown. For our numerical approach, we refer to appendix A.

![FIG. 1. Sketch of the coherently driven SPE inside a lossy cavity.](image)

The fluorescent light is detected (D) out the side of the cavity. Wavy lines indicate light fields driving the SPE, emitted into the cavity, out of the cavity, or out the side of the cavity. Straight arrows indicate the frequencies of the SPE transition and the cavity mode.

In the scenario discussed in \[22\], the SPE undergoes strong offresonant pumping, such that \(\Gamma \ll \Omega_R, |\delta_c|\) and hence \(z \approx \Omega_R^2/\delta_c^2\). As the cavity detuning is \(|\delta_c| > |\delta_1|\) and large compared to the SPE-cavity coupling \(g\), the intracavity-field is nearly unexcited, \(\langle \hat{a}^\dagger \hat{a} \rangle \ll 1\). However, following the argumentation in \[29\] \[30\], the intracavity field is enhanced if a fluorescence sideband hits the cavity frequency,
\[
\delta_c^2 = (2\Omega_R)^2 + \delta_c^2.
\] (25)

In the following, the cavity frequency is tuned to the lower Rabi sideband of the SPE. For simplicity, we will denote this scenario as cavity resonance.

At such a cavity resonance, excitation and emission of the cavity are enhanced. The excitation of the cavity increases slightly, which is consistent with the argumentation in \[29\]. The cavity emission, however, increases substantially, due to \(\kappa \gg \Gamma\). A similar situation was considered in \[31\], where steady-state inversion of a two-level atom in a cavity was predicted. In our scenario the cavity mode diverts a significant portion of the energy from the SPE, which would otherwise contribute to the fluorescent light. This yields a reduction of \(\langle \hat{A}_{22} \rangle\). It is noteworthy, that for our system, a not too good cavity is preferable, in order to avoid a too strong backaction onto the SPE. On the other hand, the coupling of the cavity and SPE has to be strong enough to preserve the coherence of the SPE. The coherent part of the SPE, \(\langle \hat{A}_{12} \rangle^2\), is increased by the coupling to the cavity. Due to the decrease of \(\langle \hat{A}_{22} \rangle\), from Eqs. (10), (13) the SPE state is expected to be purified.

The strength of the purification will be discussed in the next section. A critical condition in this setup is the requirement of \(\kappa \gg \Gamma\) and \(g \approx \kappa\), so that the SPE-cavity coupling significantly exceeds the spontaneous emission, \(g \gg \Gamma\). In experiments \[32\], a rate of \(g/\Gamma \approx 23\) was realized, which will be used throughout the paper.

In \[22\] we considered the following cavity scenario: \(\Gamma = 1/23g\), \(\Omega_R = 14g\), \(\delta_c = -34g\), and \(\kappa = 1.58g\). With these
parameters, we numerically evaluate the systems parameters and the squeezing of the fluorescence in dependence on $\delta_z$, see Fig. 2. The sought cavity resonance is obtained for $\delta_z = -1.9g$, corresponding to $z \approx 0.54$. Note that, for $z \geq 1/2$ there is no squeezing in free space fluorescence.

![Graph](image)

**FIG. 2.** (color online). Comparison of the behavior of the SPE in free space (solid, black curve) and in the cavity (red, dashed curve). The excitation $\langle \hat{A}_{x2} \rangle$ (left top), the coherence $\langle \langle A_{x12} \rangle \rangle^2$ (right top), the purity (left bottom) and the phase-optimized normally ordered field variance of the fluorescence (right bottom) are shown as a function of $\delta_z$. Two straight lines (right bottom) at $-1/8$ and $0$ indicate maximal and changing free-space squeezing, respectively. The parameters are: $\Omega_R/g = 14$, $\kappa/g = 1.58$, $\Gamma/g = 1/23$, $\delta_z/g = -34$.

At the cavity resonance the minimal normally ordered field variance is $-0.236$. This is more than $94\%$ of the maximum possible squeezing of $-1/4$. The purity $\text{Tr} \{ \hat{\sigma}^2 \}$ of the SPE subsystem even reaches a value of about $99.5\%$. The SPE excitation of $\langle \hat{A}_{x2} \rangle \approx 0.220$ is reduced compared with its free space value, while the coherence drastically increases. Note that, for larger values of $g/\Gamma$ one could achieve even more than $99\%$ of the absolute squeezing limit, $\langle (\Delta E)^2 \rangle_{\text{abs}}$.

**IV. APPROXIMATE ANALYTICAL DESCRIPTION**

Now we will provide an analytical approximation to the system under study. Although its numerical precision is limited, it gives insight in the basic physics of the cavity-assisted purification. Furthermore it will help to predict environmental influences in the next section. All the predictions will be confirmed by numerical calculations.

In the steady state regime, we obtain from Eqs. (23), (24) the following exact relations:

\[
\langle \hat{a} \rangle = \frac{-ig}{\delta_z + \frac{\Gamma}{2}} \langle \hat{A}_{x2} \rangle,
\]

\[
[i\delta_z + \frac{\Gamma}{2}] \langle \hat{A}_{x2} \rangle = ig \{ 2\langle \hat{A}_{x2} \hat{a} \rangle - \langle \hat{a} \rangle \} - i\Omega_R (1 - 2 \langle \hat{A}_{x2} \rangle),
\]

\[
\langle \hat{A}_{x2} \rangle = \frac{2\Omega_R}{\Gamma} \langle \hat{A}_{x1} \rangle - \frac{\kappa}{\Gamma} \langle \hat{a} \rangle \hat{a} \rangle.
\]

The proportionality between the coherent amplitudes of SPE and intracavity field, cf. Eq. (26), is connected to the Purcell-effect. Combining Eq. (26) and (27), we obtain

\[
\langle \hat{A}_{x2} \rangle = \frac{-i\Omega_R}{V} (1 - 2 \langle \hat{A}_{x2} \rangle),
\]

\[
\langle \hat{A}_{x1} \rangle = \frac{-i\Omega_R}{V} (1 - 2 \langle \hat{A}_{x2} \rangle).
\]

It resembles the free-space result, with a change of the scaling factor $V$. For the parameters of the above simulations, the change of $V$ in Fig. 2 relative to free space, $g = 0$, is negligible.

The term proportional to $\Omega_R$ in Eq. (28) also resembles the free-space term. The second term,

\[
R = \frac{\kappa}{\Gamma} \langle \hat{a} \hat{a} \rangle > 0,
\]

describes the sharing of the excitation between SPE and cavity mode discussed in the previous section. While $\langle \hat{a} \hat{a} \rangle \ll 1$, it is scaled up by a factor of $\kappa/\Gamma \approx 36$, making it a significant contribution. This quantity $R$ causes the purification. From now on, we will call $R$ the purification rate.

Considering $R$ as a parameter of the calculations, we end up with the following two equations

\[
\langle \hat{A}_{x2} \rangle = \frac{2\Omega_R}{\Gamma} \langle \hat{A}_{x1} \rangle - R,
\]

\[
\langle \hat{A}_{x1} \rangle = \frac{-i\Omega_R}{V} (1 - 2 \langle \hat{A}_{x2} \rangle).
\]

Inserting these results into each other, we can conclude

\[
\langle \hat{A}_{x1} \rangle = \frac{\Omega_R}{\Gamma |V|^2} \langle 1 - 2 \langle \hat{A}_{x2} \rangle \rangle,
\]

\[
\langle \hat{A}_{x2} \rangle = \frac{2\Omega_R}{\Gamma |V|^2} \langle 1 - 2 \langle \hat{A}_{x2} \rangle \rangle - R,
\]

\[
\Rightarrow \langle \hat{A}_{x2} \rangle = \frac{\bar{z} - R}{1 + 2\bar{z}} \bar{z} = \frac{2\Omega_R^2}{\Gamma |V|^2} |V|, \quad (37)
\]

\[
|\langle \hat{A}_{x2} \rangle|^2 = \frac{\Gamma}{2|V|^2} \frac{\bar{z} + (1 + 2\bar{z})^2}{(1 + 2\bar{z})^2}.
\]

For $g \to 0$, we get the free space case of $|V| = \Gamma/2$ and $\bar{z} = z = \Omega_R^2/|V|^2$. As stated above, $V$ does change marginally for our parameters. We can set $|V| \approx \Gamma/2$ and $\bar{z} \approx z$ to obtain

\[
\langle \hat{A}_{x2} \rangle = \frac{\bar{z} - R}{1 + 2\bar{z}} < \frac{z}{1 + 2z},
\]

\[
|\langle \hat{A}_{x2} \rangle|^2 = \frac{z(1 + 2R)^2}{(1 + 2z)^2} > \frac{z}{(1 + 2z)^2}.
\]
The positivity of $\Gamma$ diminishes the excitation of the SPE at the expense of increasing its coherence. Even a small cavity excitation $\langle a \dagger a \rangle$, scaled up by the prefactor in $\Gamma$, yields a substantial purification of the quantum state of the SPE.

Inserting these approximations into Eq. (8), we obtain in the cavity-assisted setup the result for the minimal field fluctuation, that is for maximal squeezing:

$$\frac{\langle (\Delta \hat{E})^2 \rangle_{\text{cw}}}{|x|^2} = \frac{1}{2} (\langle \hat{A}_{22} \rangle - 2 |\langle \hat{A}_{12} \rangle|^2) = \frac{1}{1 + 2z} \left( 1 + \frac{8(1 + R)}{1 + 2z} \right).$$

(41)

Here we have used the expression $\langle (\Delta \hat{E})^2 \rangle_{\text{cw}}$ according to Eq. (29). As expected, we have a clear decrease of the normally ordered variance, or an increase of squeezing, as the second term, proportional to $R$, is always positive.

V. ENVIRONMENTAL DISTURBANCES

Based on the above approximations, let us study the important problem of environmental disturbances. We will consider three types of disturbances. Nonradiative or pure dephasing is caused by laser fluctuations or by atomic motion. In semiconductor microcavities containing quantum dots, two types of incoherent gains exist, either for the quantum dot or the cavity field. They are caused by the interaction of the quantum dot with phonons, which will be modeled by Lindblad terms. Alternative descriptions of phonon-induced dephasing are given, e.g., in [35–37]. Semiconductor microcavities are currently intensely studied. They can be useful as nonclassical light sources in integrated optical systems. Our results will indicate that squeezing persists even under strong environmental disturbances.

A. Nonradiative dephasing

Nonradiative dephasing or pure dephasing is the radiationless decay of coherence of a system. In case of a SPE this obviously destroys the squeezing. Let us first reconsider the impact of pure dephasing in free space [23], and then compare it with the cavity-assisted squeezing scenario.

In addition to dephasing due to radiative damping, let there be radiationless dephasing described by the rate $\Gamma_D$. We supplement the equations of motion (15) for the atom in free space by another Lindblad-term,

$$\frac{d\hat{\rho}}{dt} = \frac{i}{\hbar} [\hat{H}_0, \hat{\rho}] + \frac{\Gamma}{2} \mathcal{L}_{\hat{A}_{12}}[\hat{\rho}] + \frac{\Gamma_D}{2} \mathcal{L}_{\hat{A}_{22}}[\hat{\rho}].$$

(43)

The additional dephasing only enhances the decay of the off-diagonal matrix elements of the density operator, that is, of the coherence of the SPE. We can again solve this system analytically and obtain

$$z_D = \left( 1 + \frac{\Gamma_D}{\Gamma} \right) \frac{\Omega_R^2}{(1 + 1/2)^2 + \delta_x^2}.$$  

(44)

$$\langle \hat{A}_{22} \rangle = \frac{z_D}{1 + 2z_D},$$

(45)

$$|\langle \hat{A}_{12} \rangle|^2 = \frac{1}{1 + 1/2} \left( 1 + 2z_D \right)^2.$$  

(46)

Structurally, the solution for $\langle \hat{A}_{22} \rangle$ resembles the case without pure dephasing, with a scaled value $z_D$. In our scenario of large detuning, the variation of the denominator in $z_D$ is negligible,

$$z_D \approx (1 + \frac{\Gamma_D}{\Gamma}) \frac{\Omega_R^2}{\delta_x^2} = (1 + \frac{\Gamma_D}{\Gamma})z.$$  

(47)

With increasing dephasing rate $\Gamma_D$ the atomic excitation increases, while the coherence decreases as

$$|\langle \hat{A}_{12} \rangle|^2 \approx \frac{z}{(1 + 2z_D)^2}.$$  

(48)

As the excitation increases, the pumping $\Omega_R$ has to be reduced to preserve squeezing. For $\Gamma_D = \Gamma$ squeezing vanishes as

$$\langle (\Delta \hat{E})^2 \rangle_{\text{cw}} = \left( \frac{2z_D}{1 + 2z_D} \right)^2 > 0.$$  

(49)

This limit for squeezing can be physically understood, as the time needed to emit a photon is as long as the coherence time of the emitted light.

In the cavity assisted fluorescence scenario, we repeat the calculations from Eqs. (26)-(40), and obtain the following differences. The parameters $V$ and $z$ are changed to

$$V_D = V + \frac{\Gamma_D}{2}, \quad z_D = \frac{2\Omega_R^2}{\Gamma|V_D|^2} |\Re[V_D]|.$$  

(50)

Again the real and imaginary part of $V_D$ are only marginally different from the free space values with pure dephasing. The purification rate $R$ on the other hand remains unchanged, as neither the excitation of the SPE nor the cavity are directly coupled to $\Gamma_D$. The SPE averages then read as

$$\langle \hat{A}_{22} \rangle = \frac{z_D - R}{1 + 2z_D},$$

(51)

$$|\langle \hat{A}_{12} \rangle|^2 = \frac{1}{1 + 1/2} \left( z_D(1 + 2R)^2 \right)^2.$$  

(52)

Similar to the case without radiationless dephasing, Eqs. (39) and (40), the excitation $\langle \hat{A}_{22} \rangle$ is diminished while $|\langle \hat{A}_{12} \rangle|^2$ is enhanced by the positivity of $R$. Combining them to obtain the squeezing, Eq. (8), we may compare with the result (42) for $\Gamma_D = 0$,

$$\frac{\langle (\Delta \hat{E})^2 \rangle_{\text{cw}}}{|x|^2} = \frac{1}{1 + 2z} \left( 1 + \frac{1}{1 + \frac{\Gamma_D}{\Gamma}} \frac{8(1 + R)}{1 + 2z_D} \right).$$

(53)
The second term in the brackets is now diminished by the dephasing prefactor.

Our results reveal that cavity-assisted purification increases the stability of squeezing against dephasing. The enhancement of the coherence is given by the ratio \((1 + 2R)^2\) to \(1 + \Gamma_D/\Gamma\) in Eq. \((52\)). As \(R \ll 1\) (see above), one might not expect a significant effect. However, the dephasing also affects the intracavity excitation. The radiationless dephasing suppresses the coherence of the SPE, while it does not modify the coupling strength \(g\) to the cavity. As the SPE is strongly excited for increasing \(\Gamma_D\), \(\langle \hat{a} \hat{a}^\dagger \rangle\) and hence \(R \propto \langle \hat{a} \hat{a}^\dagger \rangle\) substantially increases at the cavity resonance.

The purification rate is depicted in Fig. \[3\] We see, that the increase of \(R\) at the cavity resonance becomes more pronounced with increasing \(\Gamma_D\).

<Figure 3>

FIG. 3. (color online). Purification rate \(R\) over \(\delta_s\) for different dephasing rates \(\Gamma_D\). From bottom to top: \(\Gamma_D/\Gamma = 0, 2, 4, 6, 8\). All other parameters are as in Fig. \[2\]

<Figure 4>

FIG. 4. (color online). SPE excitation \(\langle \hat{A}_{22} \rangle\) over \(\delta_s\) for different dephasing rates \(\Gamma_D\). The solid lines represent the free space case, the dashed ones the corresponding cavity assisted scenario. From bottom to top (for each solid and dashed lines separately): \(\Gamma_D/\Gamma = 0, 2, 4, 6, 8\). All other parameters are as in Fig. \[2\]

In Fig. \[3\] we compare the SPE excitation for different dephasing rates with and without cavity-assisted purification. In the latter case, the excitation of the SPE is suppressed at the cavity resonance even below the free space value. The coherence near the resonance remains almost constant, as the terms \((1 + 2R)^2\) and \(1 + \Gamma_D/\Gamma\) in Eq. \((52\)) are nearly equal.

<Figure 5>

FIG. 5. (color online). Squeezing of the SPE fluorescence over \(\delta_s\) for different dephasing rates \(\Gamma_D\). From bottom to top: \(\Gamma_D/\Gamma = 0, 2, 4, 6, 8\). All other parameters are as in Fig. \[2\] The horizontal lines indicate maximal free space squeezing (-1/8) and vanishing squeezing (0).

These effects imply, that the resistance of squeezing against dephasing is significantly enhanced. The phase-optimized normally ordered variance Eq.(8) for different dephasing in the cavity setup is shown in Fig \[5\]. Due to the behavior of \(\langle \hat{A}_{22} \rangle\) and \(\langle \hat{A}_{12} \rangle\), the suppression of the field noise sensitivity depends on \(\delta_s\). For \(\Gamma_D < 3.24\Gamma\), the minimal variance is still below \(-1/8\), being the maximal squeezing in free space. The squeezing in the cavity setup under study vanishes for \(\Gamma_D \approx 7.47\Gamma\). This value is, however, not the actual limit. From Eqs. \((48\), \((51)\) and the increase of \(R\), it seems reasonable to look for squeezing at lower pump rates \(\Omega_R\). For lower pumping, the emitter frequency \(\omega_s\) shifts towards the cavity frequency \(\omega_c\), cf. Eq. \((25)\). While the squeezing in this region is not as strong as in Fig. \[5\] it is even more persistent. As an example, for \(\Omega_R = g\), squeezing still persists for \(\Gamma_D = 19\Gamma\).

These findings are of great interest for condensed matter systems, where dephasing plays a significant role \[35,37,39\]. The observation of coherence effects, such as squeezing, under these hostile conditions is a demanding task. Note that, the needed variation of \(\Omega_R\) and \(\delta_s\) can be easily realized for a semiconductor quantum dot inside a cavity.

B. Incoherent Pumping of SPE

The light emitted by a quantum dot in a semiconductor first passes the medium, where it excites phonons. The phonons can incoherently drive the quantum dot. The incoherent pumping of the SPE will be included by a rate \(P_c\). We will again start to consider the corresponding effects in free space, before analyzing the SPE inside the cavity.

The free space master equation \((15)\) is supplemented with another Lindblad term for the incoherent pumping,

\[
\frac{d\hat{\rho}}{dt} = \frac{1}{i\hbar} [\hat{H}_0, \hat{\rho}] + \frac{\Gamma}{2} \mathcal{L}_{\hat{A}_{12}} [\hat{\rho}] + \frac{P_c}{2} \mathcal{L}_{\hat{A}_{21}} [\hat{\rho}].
\]  

\((54)\)
The solutions in the steady state now read as

$$z_x = \frac{\Omega_R^2}{\Gamma_P^2 + \Delta_x^2}, \quad (55)$$

$$\langle A_{22} \rangle = \frac{z_x + \frac{P}{\Gamma + P_x}}{1 + 2z_x}, \quad (56)$$

$$|\langle A_{12} \rangle|^2 = \frac{z_x(1 - 2P)^2}{(1 + 2z_x)^2}. \quad (57)$$

Similarly to dephasing, the excitation is increased while the coherence is decreased by the incoherent pumping. Restricting $P_x \leq \Gamma$, the saturation case is $P_x = \Gamma$, for which $\langle A_{22} \rangle = 1/2$ and $|\langle A_{12} \rangle|^2 = 0$, independent of the coherent pumping from the laser. We emphasize, that, when defining the quantity $P = P_x/(\Gamma + P_x) > 0$, we can write the solutions as

$$\langle A_{22} \rangle = \frac{z_x + P}{1 + 2z_x}, \quad (58)$$

$$|\langle A_{12} \rangle|^2 = \frac{z_x(1 - 2\Gamma)^2}{(1 + 2z_x)^2}. \quad (59)$$

The term $P$ appears in place of the purification rate $R$, compare Eqs. (59), (40), but with opposite sign, so that the purity of the quantum state of the SPE decreases.

In the cavity-assisted scenario one may expect $R$ and $P$ to directly counteract each other. Complementing the calculations from Eqs. (26)-(40) by incoherent pumping yields

$$V_x = i\delta_x + \Gamma P_x + \frac{\delta^2}{2i\delta_x + \Gamma}, \quad (60)$$

$$z_x = \frac{2\Omega_R^2}{\Gamma + P_x} \frac{\Re[|V_x|^2]}{|V_x|^2} \approx \frac{\Omega_R^2}{|V_x|^2} \approx z, \quad (61)$$

$$R_x = \frac{\kappa}{\Gamma + P_x} \langle \dot{\hat{a}}^\dagger \hat{a} \rangle, \quad (62)$$

$$\langle \hat{A}_{22} \rangle = \frac{z_x + P - R_x}{1 + 2z_x}, \quad (63)$$

$$|\langle \hat{A}_{12} \rangle|^2 = \frac{z_x(1 - 2\Gamma + 2R_x)^2}{(1 + 2z_x)^2}. \quad (64)$$

The excitation parameter $z_x \approx z$ does not change significantly. Likewise the structure of the expectation values itself is identical to the case of no incoherent pumping, if we define

$$\tilde{R}_x = R_x - P = \frac{\kappa \langle \dot{\hat{a}}^\dagger \hat{a} \rangle - P_x}{\Gamma + P_x} \quad (65)$$

as the new purification rate of the cavity setup. For $\tilde{R}_x > 0$ we have purification, $\tilde{R}_x = 0$ corresponds to the free-space scenario, and for $\tilde{R}_x < 0$ we impurify the state. Consequently, squeezing behaves as in the case of no incoherent pumping, Eq. (42), but with $R_x$ replacing $R$.

Two consequences of the solutions (60)-(61) should be noted. First, when the SPE is tuned through a cavity resonance the sign of $\tilde{R}_x$ changes from negative to positive and back, changing the behavior decreasing to increasing purity of the atomic state and back. Compared with the simple cavity-assisted case, where we have purification, as $R > 0$, here we can control the purity of the quantum state of the SPE by tuning its resonance frequency. Second, the effect of incoherent pumping is limited by $P_x \leq \Gamma$ or equivalently $P \leq 1/2$. The saturation case $P = 1/2$ is, however, only a theoretical value. For example in case of phonon-induced pumping, this is equal to infinite temperature. For our scenario with $\kappa \gg \Gamma > P_x$, the purification and thus optimized squeezing is nearly unaffected by the incoherent pumping, at least at the cavity resonance.

![FIG. 6. (color online). As Fig. 4 for different incoherent pumpings $P_x$. The solid lines represent the free space case, the dashed ones the corresponding cavity assisted scenario. From bottom to top (for each solid and dashed ones separately): $P_x/\Gamma = 0, 0.2, 0.4, 0.6, 0.8, 1$. All other parameters are as in Fig. 2.](image1)

![FIG. 7. (color online). Squeezing of the SPE fluorescence over $\delta_x$ for different incoherent pumpings $P_x$. From bottom to top: $P_x/\Gamma = 0, 0.2, 0.4, 0.6, 0.8, 1$. All other parameters are as in Fig. 2. The horizontal lines indicate maximal free space squeezing (-1/8) and vanishing squeezing (0).](image2)

In Fig. 6 we compare $\langle A_{22} \rangle$ for different values of $P_x$ inside and outside the cavity. Similar to the case of dephasing, the pronounced cavity resonance effect indicates an increase of $\langle \dot{\hat{a}}^\dagger \hat{a} \rangle$, cf. Eqs. (62), (63). Remarkably, even for the saturated scenario, in the cavity resonance, the excitation of the SPE remains significantly below the free space value for no
C. Incoherent Pumping of the Cavity

The cavity mode may also be incoherently pumped, either directly from the phonons or from the interaction with the SPE. Nevertheless, this effect is expected to be smaller than the interaction of the cavity mode with phonons. The latter was observed to be negligibly small [40]. The cavity is supposed to be pumped incoherently with a rate \( P_c \leq \kappa \), where equality again represents saturation. Since the cavity mode is bosonic, this implies \( \langle \hat{a}^\dagger \hat{a} \rangle \rightarrow \infty \). As we are interested in the case \( \langle \hat{a}^\dagger \hat{a} \rangle \ll 1 \), we are limited to \( P_c \ll \kappa \).

After applying the formalism of Eqs. (26)-(40) for this scenario, we obtain terms, which resemble the previous case of incoherent SPE pumping:

\[
V_c = i\delta_c + \frac{\Gamma}{2} + \frac{g^2}{i\delta_c + \kappa - P_c}, \tag{66}
\]

\[
z_c = \frac{2\Omega_R^2}{\Gamma} \Re |V_c| \approx \frac{\Omega_R^2}{|V_c|^2} \approx z, \tag{67}
\]

\[
R_c = \frac{\kappa - P_c}{\Gamma} \langle \hat{a}^\dagger \hat{a} \rangle, \tag{68}
\]

\[
\langle \hat{A}_{12} \rangle = \frac{z_c + P_c \Gamma - R_c}{1 + 2z_c}, \tag{69}
\]

\[
|\langle \hat{A}_{12} \rangle|^2 = \frac{z_c(1 - 2P_c\Gamma + 2R_c)^2}{(1 + 2z_c)^2}. \tag{70}
\]

Again, we can define an effective purification rate

\[
\check{R}_c = R_c - \frac{P_c}{\Gamma} = \frac{(\kappa - P_c) \langle \hat{a}^\dagger \hat{a} \rangle - P_c}{\Gamma}, \tag{71}
\]

which is formally similar to \( \check{R}_x \). However, contrary to \( P_x \), \( P_c \) is not limited by \( \Gamma \) but only by \( \kappa \) which is very large compared to \( \Gamma \). Hence, we may have \( P_c \gg \Gamma \), without violating \( \kappa \gg P_c \).

In such a case however, the incoherent pumping contributes strongly to \( \check{R}_c \) and the squeezing in the cavity resonance is suppressed. On the other hand, out of the cavity resonance the effective coupling between SPE and cavity mode is very weak, which yields \( \check{R}_c \approx 0 \). As the environmental effect is only caused by the cavity, we obtain the free space scenario again with the remaining squeezing of weak effective pumping \( \kappa \gg \Omega_R^2 \). Of course, in this case the atomic-state purification does not occur.

In Fig. 8 we show the squeezing (note the larger region of \( \delta_c \)) for different incoherent pumping rates. For comparison, the free-space squeezing is also given. For large \( |\delta_c| \)-values we approach the free space value, while in the cavity resonance the squeezing is suppressed significantly for increasing \( P_c \). However, for an incoherent pumping equal to the spontaneous emission of the SPE, we still obtain squeezing of about the maximal free-space value.

VI. DETECTION OF SQUEEZING

The prediction of squeezing in the resonance fluorescence of a SPE could not be confirmed in experiments yet. Usually the normally ordered variance of a light field is measured by balanced homodyne detection, for details see e.g. [23]. In the case of single-atom fluorescence the following problems must occur. First, the atomic motion yields phase shifts, which can be eliminated by using trapped ions or well localized excitations in semiconductor systems. Second, the small collection efficiency of the field substantially reduces the observable squeezing. This problem can be resolved by homodyne correlation measurements [18-20].

Let us reconsider the homodyne cross-correlation measurement [19], the setup is shown in Fig. 6. The signal field \( E_{SI} \) is superimposed by a beam splitter with the coherent local oscillator field of amplitude \( E_{LO} \). The cross-correlation between the two outgoing light fields \( E_1 \) and \( E_2 \) is recorded. The measured signal,

\[
G^{(2,2)}(t_1, t_2) = \eta^2 \langle \hat{E}_{1}^{(-)}(t_1) \hat{E}_{2}^{(-)}(t_2) \hat{E}_{2}^{(+)}(t_2) \hat{E}_{1}^{(+)}(t_1) \rangle, \tag{72}
\]

is the given by the intensity cross-correlation function and the (equal) quantum efficiencies \( \eta \) of the two detectors. Note that \( \eta \) includes the (small) collection efficiency of the fluorescence signal. Following [19], for equal times \( (t_1 = t_2 = t) \) we get

\[
G^{(2,2)}(t, t) = \frac{\eta^2}{4} [I_{LO}^2 + I_{SI}^2 - 2I_{LO} \Re \langle E_{SI}^{(+)}(t) \rangle], \tag{73}
\]
with $\hat{I}_{\text{SI}} = \hat{E}_{\text{SI}}^{(-)} \hat{E}_{\text{SI}}^{(+)}$ and $I_{\text{LO}} = E_{\text{LO}}^2$. For a sufficiently large time delay ($t_2 - t_1 \to \infty$) we approach the uncorrelated events,

$$\mathcal{G}_{\text{unc}}^{(2,2)}(t) = \frac{\eta^2}{4} [ (\hat{I}_{\text{SI}})^2 + I_{\text{LO}}^2 ] - 2I_{\text{LO}} \left( \Re(\langle \hat{E}_{\text{SI}}^{(+)} \rangle^2) + |\langle \hat{E}_{\text{SI}}^{(+)} \rangle|^2 - \langle \hat{I}_{\text{SI}} \rangle \right).$$  \hfill (74)

Let us consider the difference $\Delta \mathcal{G}^{(2,2)}$ of $\mathcal{G}^{(2,2)}(t, t)$ and $\mathcal{G}_{\text{unc}}^{(2,2)}(t)$,

$$\Delta \mathcal{G}^{(2,2)} = \frac{\eta^2}{4} \left( \langle (\Delta \hat{I}_{\text{SI}})^2 \rangle - I_{\text{LO}} \langle (\Delta \hat{E}_{\text{SI}})^2 \rangle \right).$$  \hfill (75)

This difference includes the normally ordered variances of the intensity and the field strength of the signal. For a SPE we have $\langle \hat{I}_{\text{SI}}^2 \rangle = 0$, so that we obtain

$$\Delta \mathcal{G}^{(2,2)} = -\frac{\eta^2}{4} \left( I_{\text{SI}}^2 + I_{\text{LO}} \langle (\Delta \hat{E}_{\text{SI}})^2 \rangle \right).$$  \hfill (76)

Only if the field is squeezed, $\langle (\Delta \hat{E}_{\text{SI}})^2 \rangle < 0$, $\Delta \mathcal{G}^{(2,2)}$ can become positive. For squeezed fields and sufficiently strong local oscillator, $\Delta \mathcal{G}^{(2,2)}$ switches the sign for some phase of the signal, and squeezing is detected by $\Delta \mathcal{G}^{(2,2)} > 0$.

From Eq. (76) it may seem that a strong local oscillator is preferential for detection the squeezing. However, since the present simple detection scheme is not balanced, the classical fluctuations of the local oscillator must be considered [41]. In fact, this problem can be avoided by using a more complex balanced homodyne correlation setup [20]. For the present scheme, the dominant classical noise term is

$$\Delta \mathcal{G}_{\text{cl}}^{(2,2)} = \eta^2 I_{\text{LO}} \langle 3 \hat{E}_{\text{LO}}^2 \rangle^2;$$  \hfill (77)

where $\langle 3 \hat{E}_{\text{LO}}^2 \rangle^2$ is the classical amplitude variance of the local oscillator. This classical noise is easily measured by blocking the signal channel. The squeezing condition finally becomes

$$\Delta \mathcal{G}^{(2,2)} > \Delta \mathcal{G}_{\text{cl}}^{(2,2)}.$$  \hfill (78)

Following the discussion in [19], the optimal experiment is performed with a weak local oscillator, whose intensity slightly exceeds the intensity of the fluorescence signal of the SPE.

\section{VII. CONCLUSIONS AND OUTLOOK}

We have studied the optimization of squeezing in resonance fluorescence of a single-photon emitter through cavity-assisted purification of the atomic quantum state. This can be achieved by tuning the cavity on resonance with the lower Rabi-sideband of the emitter. The squeezed light is recorded out the side of the cavity. The maximal squeezing is significantly larger and more robust against disturbance than in free space.

Analytical approximations are given, which yield an interpretation of the basic effects of our purification scenario. A detailed study is given of the resistance of the optimized squeezing against environmental disturbances. In particular, it is shown that squeezing is much more robust against dephasing and incoherent pumping compared with an atom in free space. Consequently, even strong incoherent channels do not fully suppress the squeezing in our optimized setting. All the considered incoherent effects are present for quantum dots in semiconductor microcavities. Our results indicate that such complex devices may be promising integrated squeezed-light sources. A simple homodyne correlation measurement technique is considered, which renders it possible to detect the squeezing of a laser-driven single-photon emitter.

It is of some interest to compare the relation of the squeezed light sources under study with standard sources based on optical parametric oscillators. Both types of sources are very different from two perspectives. The squeezing in resonance fluorescence is diminished by the small collection efficiency, which is not the case for standard sources. On the other hand, our observation scheme is not sensitive to the efficiency, whereas the balanced homodyne detection in the standard case is. Hence, it is a challenging open problem to compare the advantages of both scenarios for practical applications. However, this problem is beyond the scope of our paper as it requires further research.

\section{ACKNOWLEDGMENTS}

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[1] H. J. Carmichael and D. F. Walls, J. Phys. B 9, L43 (1976).
[2] H. J. Kimble and L. Mandel, Phys. Rev. A 13, 2123 (1976).
[3] H. J. Kimble, M. Dagenais, and L. Mandel, Phys. Rev. Lett. 39, 691 (1977).
[4] R. Short and L. Mandel, Phys. Rev. Lett. 51, 384 (1983).
[5] F. Diedrich and H. Walther, Phys. Rev. Lett. 58, 203 (1987).
[6] M. Schubert, I. Siemers, R. Blatt, W. Neuhauser, and P. E. Toschek, Phys. Rev. Lett. 68, 3016 (1992).
[7] A. Muller, E. B. Flagg, P. Bianucci, X. Y. Wang, D. G. Deppe, W. Ma, J. Zhang, G. J. Salamo, and C. K. Shih, Phys., Rev. Lett. 99, 187402 (2007).
[8] S. Gerber, D. Rotter, L. Slodička, J. Eschner, H. J. Carmichael, E. Shchukin and W. Vogel, Phys. Rev. Lett. 96, 200403 (2006).
[9] E. B. Flagg, A. Muller, J. W. Robertson, S. Founta D. G. Deppe, M. Xiao, W. Ma, G. J. Salamo, and C. K. Shih, Nature Physics 5, 203 (2009).
[10] D. F. Walls and P. Zoller, Phys. Rev. Lett. 47, 709 (1981).
[11] W. Vogel and D.-G. Welsch, Phys. Rev. Lett. 54, 1802 (1985).
[12] A. Heidmann and S. Reynaud, J. Physique 46, 1937 (1985).
[13] M. D. Reid and D. F. Walls, Phys. Rev. A 32, 396 (1985).
[14] Z. H. Lu, S. Bali, and J. E. Thomas, Phys. Rev. Lett. 81, 3635 (1998).
[15] M. G. Raizen, L. A. Orozco, Min Xiao, T. L. Boyd, and H. J. Kimble, Phys. Rev. Lett. 59, 198 (1987).
[16] A. Ourjoumtsev, A. Kubanek, M. Koch, C. Sames, P.W.H. Pinske, G. Rempe and K. Murr, Nature 474 623 (2011).
[17] W. Vogel, Phys. Rev. Lett. 67, 2450 (1991).
[18] W. Vogel, Phys. Rev. A 51, 4160 (1995).
[19] E. Shchukin and W. Vogel, Phys. Rev. Lett. 96, 200403 (2006).
[20] S. Gerber, D. Rotter, L. Slodička, J. Eschner, H. J. Carmichael, and R. Blatt, Phys. Rev. Lett. 102, 183601 (2009).
[21] P. Grünwald and W. Vogel, Phys. Rev. Lett. 109, 013601 (2012).
[22] W. Vogel and D.-G. Welsch, Quantum Optics (Wiley-VCH, Berlin, 2006).
[23] G. Kieblisch, G. Schaller, C. Emary, and T. Brandes, Phys. Rev. Lett. 107 050501 (2011).
[24] P. Bowles, M. Guţă, and G. Adesso, Phys. Rev. A 84, 022320 (2011).
[25] R. Filip, Phys. Rev. A 65, 062320 (2002).
[26] A. K. Ekert, C. M. Alves, D. K. L. Oi, M. Horodecki, P. Horodecki, and L. C. Kwek, Phys. Rev. Lett. 88, 217901 (2002).
[27] H. Nakazato, T. Tanaka, K. Yuasa, G. Florio, and S. Pascazio, Phys. Rev. A 85, 042316 (2012).
[28] H. Freedhoff and T. Quang, J. Opt. Soc. Am. B 10, 1337 (1993).
[29] T. Quang and H. Freedhoff, Opt. Comm. 107, 480 (1994).
[30] A. Muller, E. B. Flagg, P. Bianucci, X. Y. Wang, D. G. Deppe, W. Ma, J. Zhang, G. J. Salamo, M. Xiao, and C. K. Shih, Phys., Rev. Lett. 99, 187402 (2007).

Appendix A: Numerical calculations.

The master equation for a SPE in a single mode cavity yield an infinite hierarchy of coupled equations for the density matrix elements

\[ \rho_{n, i} = \langle n, i | \hat{\rho} | m, j \rangle. \]  

Here, the first index is the cavity photon number and the second is the SPE excitation number \((i, j = 1, 2)\). The explicit equations for the cavity assisted system, without further environmental effects, can be written as

\[ \hat{\rho}_{n, 1; m, 1} = -\frac{i}{\hbar} \left[ (\delta_{n, 1} - n - m) \rho_{n, 1; m, 1} - i \hbar \sqrt{n} \rho_{n-1, 2; m, 1} - i \hbar \sqrt{n} \rho_{n, 1; m-1, 2} \right] - i \Omega_R \left[ \rho_{n, 2; 1, m} - \rho_{n, 1; 1, m} \right] + \frac{\Gamma + \gamma(n + m)}{2} \rho_{n, 2; 1, m} - i \hbar \left[ n \rho_{n-1, 2; m, 1} - \sqrt{n} \rho_{n-1, 2; m, 1} \right] \]  

(A2)

\[ \hat{\rho}_{n, 1; m, 2} = \frac{i}{\hbar} \left[ (\delta_{n, 1} - n - m) \rho_{n, 1; m, 2} - i \hbar \sqrt{n} \rho_{n-1, 2; m, 2} - i \hbar \sqrt{n} \rho_{n-1, 2; m, 1} \right] \]  

(A3)

\[ \hat{\rho}_{n, 2; m, 1} = -\frac{i}{\hbar} \left[ (\delta_{n, 1} + n - m) \rho_{n, 2; m, 1} + \sqrt{n} \rho_{n+1, 1; m, 1} - \sqrt{n} \rho_{n+1, 1; m, 1} \right] \]  

(A4)

\[ \hat{\rho}_{n, 2; m, 2} = \frac{i}{\hbar} \left[ (\delta_{n, 1} - n + m) \rho_{n, 2; m, 2} - i \hbar \sqrt{n} \rho_{n+1, 1; m, 2} - i \hbar \sqrt{n} \rho_{n+1, 1; m, 1} \right] \]  

(A5)

Here, the first index is the cavity photon number and the second is the SPE excitation number \((i, j = 1, 2)\). The explicit equations for the cavity assisted system, without further environmental effects, can be written as
We truncate the set of equations at a sufficiently large photon number $N$. By varying $N$, the validity of the calculations can be checked. Using $\text{Tr}\{\hat{\rho}\} = 1$, we can eliminate one element of the main diagonal, in our case, we choose $\rho_{0,1,0,1}$. This introduces an inhomogeneity into the equations, allowing us to calculate the steady state density matrix simply by inverting the matrix of coefficients and multiplying with the inhomogeneity. Finally, the expectation values of interest can be directly obtained.