Abstract

Observations of the solar butterfly diagram from sunspot records suggest persistent fluctuation in parity, away from the overall, approximately dipolar structure. We use a simple mean-field dynamo model with a solar-like rotation law, and perturb the $\alpha$-effect. We find that the parity of the magnetic field with respect to the rotational equator can demonstrate what we describe as resonant behaviour, while the magnetic energy behaves in a more or less expected way. We discuss possible applications of the phenomena in the context of various deviations of the solar magnetic field from dipolar symmetry, as reported from analysis of archival sunspot data. We deduce that our model produces fluctuations in field parity, and hence in the butterfly diagram, that are consistent with observed fluctuations in solar behaviour.

1 Introduction

The contemporary symmetry of the solar magnetic field is close to dipolar, i.e. the large-scale magnetic field is antisymmetric with respect to the solar equator. Historical telescopic observations indicate however that sometimes
the solar magnetic field can deviate substantially from this symmetry. This happened at the end of the Maunder minimum (Ribes & Nesme-Ribes 1993), and less clear evidence is available for the epoch just before the Maunder minimum (Nesme-Ribes et al. 1994). In each case an activity wave propagated in just one (Southern) solar hemisphere, in what can be described as simultaneous excitation of dipolar and quadrupolar magnetic configurations with comparable field strengths (Sokoloff & Nesme-Ribes 1994). Indeed, spherical dynamo models are known to be able to excite magnetic configurations of either symmetry (e.g. Brandenburg et al. 1989). Recent investigations of sunspot drawings from the XVIIIth century (Arlt at al. 2013) give a hint that even an almost pure quadrupolar magnetic configuration is sometimes excited (Sokoloff et al., 2010). Some quantities based on sunspot data indicate substantial deviations from dipolar symmetry even in the XXth century (Lisseu et al. 2016).

On one hand, spherical dynamo models allow excitation of dipolar and quadrupolar configurations in addition to what is known as mixed parity configurations (e.g. Moss et al. 2008a). Models which include stochastic fluctuations of dynamo drivers, the $\alpha$-effect primarily, can mimic episodes with deviations from dipolar symmetry such as are known from observations in form of fluctuations of magnetic field parity (e.g. Moss et al. 2008b; Usoskin et al. 2009).

On the other hand however these parity fluctuations appear as rather rare cases in the parametric space of spherical dynamo models, which mainly result in the excitation of magnetic configurations of a particular polarity. A feeling that dynamo driver fluctuations associated with a coincidence of some characteristics of dynamo excited modes underly the fluctuations in parity follows from the investigations cited above. This expectation appears quite specific to the dynamo problem. However it perhaps has a similarity with certain resonant phenomena, and for the sake of definiteness we refer to it here as stochastic resonance.

Several attempts have been undertaken to isolate resonant effects in spherical shell dynamos (Gilman & Dikpati 2011; Moss & Sokoloff 2013), and also in disc dynamos (Kuzanyan & Sokoloff 1993; Moss 1996), that are relevant for the generation and maintenance of galactic magnetic fields. These papers however concentrated attention on enhancement of magnetic energy due to the resonant effects, and in general their results are interesting but are limited in applicability.

Here we readdress the problem of phenomena with substantial fluctua-
tions of parity, that occur in a rather small region of parameter space, and are associated with stochastic variations of the dynamo drivers (although for comparison we do also briefly consider periodic variations). The important point is that we focus on stochastic rather than periodic variations of the dynamo drivers.

As we expect that resonant phenomena are rather general, we consider parity fluctuations in the framework of a simple mean-field dynamo model, with the aim of clarifying the conditions associated with parity fluctuations in our models. This is the aim of this paper.

A dynamo operating in a spherical shell, driven by mirror asymmetric convection and differential rotation, is believed to be the physical process that underlies the magnetic solar activity cycle (e.g. Stix 2004). Spherical shell dynamos seem to be responsible for the activity cycles observed on various late-type stars (Baliunas et al. 1996). As resonance seems to be a general physical phenomenon relevant for various periodic processes it seems natural to search for resonant behaviour in spherical shell dynamos, and possibly to associate various properties of solar and stellar cycles with resonant effects (e.g. Gilman & Dikpati 2011).

2 The dynamo model

We consider a standard mean-field dynamo based on the joint action of differential rotation and the $\alpha$-effect, operating in a spherical shell. For convenience, we use a synthetic rotation curve, as presented in Jouve et al. (2008). The isorotation contours are illustrated in Fig. 1. Our model extends through a tachocline to solid body rotation at fractional radius $r=0.67$. The diffusivity is taken to be uniform. The boundary condition at the surface is that the interior field fits smoothly onto a vacuum exterior field, and at the lower
boundary we use perfect conductor conditions. A simple algebraic alpha-quenching is implemented, with further details such as the unquenched form of alpha given by case A of Jouve et al. (2008). Salient results are given (NDYND code) in Jouve et al. (2008). We take a standard dynamo number for our investigations here that is about twice the supercritical value. The excitation conditions for dipolar and quadrupolar modes are well separated, with the dipolar solution strongly preferred. (To verify this we made several experiments starting with initial fields of very close to purely even parity: in each case the system evolved rapidly to the same pure odd parity state.)

We then add fluctuations of the dynamo driver $\alpha$ as follows. The unper-
turbed $\alpha$ is multiplied by a factor $1 + f_r r_i$ where $r_i$ is a sequence of Gaussian random quantities (with zero mean and r.m.s. value $s$) that are independent at time intervals of fixed duration $t_c$; $f_r$ is an adjustable scaling factor. In one set of models $r_i$ is regenerated periodically at fixed intervals $t_c$. Then our reference case has standard deviation $s = 0.2$, and rerandomizations occur.

Figure 2: Extract from a mature solution showing variations in parity (top), ratio of poloidal to toroidal energies (middle) and total energy for the model with equatorially symmetric alpha perturbations (case b).
Figure 3: Extracts from time series for models with $\alpha$-fluctuations (s=0.2), all case a). a: $t_c = 0.019$, $f_r = 0.075$, b: $t_c = 0.05$, $f_r = 0.075$, c: $t_c = 0.005$, $f_r = 0.005$, d: $t_c = 0.019$, $f_r = 0.150$. The rerandomization time interval $t_c$ is constant. In each subfigure the upper panel shows the parity, the middle panel the ratio of global poloidal to toroidal energies, and the lower panel gives the global magnetic energy.
at fixed intervals $t_c = 0.0075$. In another set of computations the interval $t_c$ is replaced by $t_{c0}(1 + s)$ where $s$ is also taken from a Gaussian distribution. For comparison we consider also periodic $\alpha$-variations with amplitude $r$ and period $t_\alpha$.

3 Resonant effects

3.1 Stochastic $\alpha$-fluctuations

For simplicity and clarity we consider a model in which fluctuations in the alpha coefficient are applied to each hemisphere as a whole. We consider two possibilities: a) $r_i(t, N) = -r_i(t, S)$ and b) $r_i(t, N) = r_i(t, S)$, where N and S refer to the corresponding hemispheres. (More general perturbations could, of course, be considered, cf Moss et al. 1992.) Our choice is in the spirit of Moss et al. (2007), and in some ways can be considered as an extension of that 1D model to two dimensions.

We find that that with the antisymmetric perturbations (case a)) the destabilization of the initial odd parity state is almost immediate, and that the solution subsequently shows deviations from even parity.

Case b) with equatorially symmetric perturbations maintains pure parity with fluctuations in energy similar to those shown in Fig. 3a. In this case, the pure parity state is preserved, and the magnetic energy shows irregular variations. Thus we consider below case a) only.

Fluctuations in parity and energy for a selection of mature solutions are shown in Fig. 3. The form of the parity fluctuations as well as their amplitude depend significantly on the fluctuation parameters. In particular, the parity fluctuations shown in Fig. 3c: are noticeably stronger and longer than in the other cases. The magnetic energy and the ratio between the energy in toroidal and poloidal magnetic fields do not demonstrate any unusual behaviour that could be attributed to resonant effects. Here the results are similar to those obtained previously by Moss & Sokoloff (2013) earlier. The novelty is in the unusual behaviour of the parity, which appears chaotic if not quasiperiodic.

We further investigated models in which the injection rerandomization interval $t_c$ also varies randomly. In Fig. 3h we show an extract from the mature solution obtained with the same parameters as Fig. 3a but with $t_c$ replaced by $t_{c0}(1 + s)$ where $s$ is taken from a Gaussian distribution with mean 0.0 and standard deviation 0.2, and with $t_{c0}$ the reference time interval
Reducing the standard deviation of the variable \( s \) from 0.2 to 0.1 we do not see substantial reorganization of fluctuations observed.

### 3.2 Periodic \( \alpha \)-perturbations

Just for orientation, we also performed similar experiments with models in which the variations in \( \alpha \)-fluctuations were periodic. We agree immediately that there is no obvious physical application, but felt that such experiments might provide some general insight. Now there is no random input into the models, but \( \alpha \) varies with period \( t_\alpha \). With the same parameters otherwise as in the models of Sect. 3.1 we increased \( t_\alpha \) from about half of the unperturbed period of the model to about 4 times this value, again starting from an odd parity seed field.

Solutions maintain pure parity \( P = -1 \), but expectedly show fluctuations in total energy. We found no evidence for resonant behaviour in these examples. (It is possible that in a model in which excitation conditions for dipolar and quadrupolar fields are close, the behaviour in this case could be more complex.)

### 4 Discussion and conclusions

Summarizing, we found the cases studied show strong fluctuations of parity, which appear as chaotic or even quasi-periodic, in contrast to the fluctuations in energy which do not display any such remarkable characteristics. Fluctuations of parity becomes especially pronounced, i.e. large amplitude and time scale, for particular tunings of the dynamo governing parameters (Fig. 3c) and in this sense we are speaking about a resonant effect. The parity fluctuations found are a consequence of the stochastic fluctuations of the dynamo drivers. In this sense we refer the case as stochastic resonance.

As emphasized, we note that substantial deviations from dipolar symmetry have been reported for the solar magnetic field at the end of the Maunder minimum (Ribes & Nesme-Ribes 1993; Sokoloff & Nesme-Ribes 1994), at the times just before the Maunder minimum (Nesme-Ribes et al. 1994), and in the epoch following the Maunder minimum (Sokoloff et al. 2010). In these cases, a tracer of parity demonstrates a significant deviation from dipolar parity. Substantial deviations from dipolar parity are claimed even for a cycle at the end of the XXth century (Leussu et al. 2016). Of course, the
reliability of archival observations is necessarily lower than for contemporary data (Zolotova & Ponyavin 2015), and even the detailed reliability of sunspot data in the XXth century is currently a matter for discussion (e.g. Clette et al. 2014). However unusual behaviour of solar activity as recorded in parity data is accepted for the time being at least as lying within the mainstream of solar physics (Usoskin et al. 2015). Stochastic resonance similar to that investigated here should be considered as a possible physical effect underlying the above phenomenology.

Further, fluctuations of dynamo drivers are considered as the most probable factor that underlies Grand minima and other related phenomena in solar activity (e.g. Moss et al. 2008; Olemskoy et al. 2013). It seems plausible to suggest that resonant excitation of parity variations by random fluctuations of dynamo drivers could be a viable addition to the understanding of the Maunder minimum. We note here that the resonant nature of such phenomena was recently discussed by Stefani et al. (2016).

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References

[1] Arlt, R., Leussu, R., Giese, N., Mursula, K., and Usoskin, I.G., Sunspot positions and sizes for 1825-1867 from the observations by Samuel Heinrich Schwabe. Mon. Not. Roy. Astron. Soc. 2013, 433, 3165-317.

[2] Baliunas, S.L., Donahue, R.A., Soon, W.H., Horne, J.H., Frazer, J., Woodard-Eklund, L., Bradford, M., Rao, L.M., Wilson, O.C., Zhang, Q., Bennett, W., Briggs, J., Carroll, S.M., Duncan, D.K., Figueroa, D., Lanning, H.H., Misch, T., Mueller, J., Noyes, R.W., Poppe, D., Porter, A.C., Robinson, C.R., Russell, J., Shelton, J.C., Soyumer, T., Vaughan, A.H., and Whitney, J.H., Chromospheric variations in main-sequence stars. ApJ 1995, 438, 269–287.

[3] Brandenburg, A., Krause, F., Meinel, R., Moss, D., and Tuominen, I., The stability of nonlinear dynamos and the limited role of kinematic growth rates. A&A 1989, 213, 411-422.
4 Clette, F., Svalgaard, L., Vaquero, J.M., and Cliver, E.W., Revisiting the sunspot number. A 400-year perspective on the solar cycle, *Sp. Sci. Rev.* 2014, *186*, 35-103.

5 Gilman, P.A., and Dikpati, M., Resonance in forced flux-transport dynamos, *ApJ* 2011, *738*, id 108.

6 Jouve, L., Brun, A.S., Arlt, R., Brandenburg, A., Dikpati, M., Bonanno, A., K?pyl?, P.J., Moss, D., Rempel, M., Gilman, P., Korpi, M.J., and Kosovichev, A. G., A solar mean field dynamo benchmark. *A&A* 2008, *483*, 949-960.

7 Kitchatinov, L.L., Do dynamo-waves propagate along isorotation surfaces? *A&A* 2002, *394*, 1135-1139.

8 Kuzanyan, K.M., and Sokoloff, D.D., Parametric resonance in a thin disc dynamo. *Astroph. Sp. Sci.* 1993, *208*, 245-252.

9 Leussu, R., Usoskin, I.G., Arlt, R., and Mursula, K., Properties of sunspot cycles and hemispheric wings since the 19th century. *A&A* 2016, *592*, A160.

10 Moss, D., Parametric resonance and bisymmetric dynamo solutions in spiral galaxies. *A&A* 1996, *308*, 381-386.

11 Moss, D., and Sokoloff, D., Resonances for activity waves in spherical mean field dynamos. *A&A* 2013, *553*, A37.

12 Moss, D., Saar, S.H., and Sokoloff, D. 2008a, What can we hope to know about the symmetry properties of stellar magnetic fields?, *Mon. Not. Roy. Astron. Soc.* 2008, *388*, 416-420.

13 Moss, D., Sokoloff, D., Usoskin, I., and Tutubalin, V., Solar Grand minima and random fluctuations in dynamo parameters. *Solar. Phys.* 2008, *250*, 221-234.

14 Nesme-Ribes, E., Sokoloff, D., Ribes, J.C., and Kremliovsky, The Maunder minimum and the solar dynamo. In *The Solar Engine and its Influence on Terrestrial Atmosphere and Climate* 1994, B., Heidelberg, 71-80.
[15] Olemskoy, S.V., Choudhuri, A.R., and Kitchatinov, L.L., Fluctuations in the alpha-effect and grand solar minima Astron. Rep. 2013, 57, 458-468.

[16] Ribes, J.C., & Nesme-Ribes, E., The solar sunspot cycle in the Maunder minimum AD1645 to AD1715. A&A 1993, 276, 549-563.

[17] Sokoloff, D., Arlt, R., Moss, D., Saar, S.H., and Usoskin, I., Sunspot cycles and Grand Minima. Solar and Stellar Variability: Impact on Earth and Planet, 2010, IAU Symp. 264, 111-119.

[18] Sokoloff, D., and Nesme-Ribes, E., The Maunder minimum: A mixed-parity dynamo mode? A&A 1994, 288, 293–298.

[19] Stefani, F., Giesecke, A., Weber, N., and Weier, T., Synchronized Helicity Oscillations: A Link Between Planetary Tides and the Solar Cycle? Solar Phys. 2016, 291, 2197-2212.

[20] Usoskin, I.G., Arlt, R., Asvestari, E., Hawkins, E., Käpylä, M., Kovaltsov, G.A., Krivova, N., Lockwood, M., Mursula, K., O’Reilly, J., Owens, M., Scott, C.J., Sokoloff, D.D., Solanki, S.K., Soon, W., & Vaquero, J.M., The Maunder minimum (1645-1715) was indeed a grand minimum: A reassessment of multiple datasets. A&A 2015, 581, A95.

[21] Usoskin, I.G., Sokoloff, D., and Moss, D., Grand Minima of Solar Activity and the Mean-Field Dynamo. Solar. Phys. 2009, 254, 345-355.

[22] Zolotova, N.V., and Ponyavin, D.I., The Maunder minimum is not as grand as it seemed to be. ApJ 2015, 800, id42.
Figure 4: Models with fluctuations in $t_c = t_{c0}(1+s)$, where $t_{c0} = 0.0075$ and $s$ has mean 0.0. In panel a), the standard deviation of the interval fluctuations, determined by $s$, is 0.2, and the other parameters are as in panel a) of Fig. [5]. In panel b) the standard deviation $s$ of the random interval is reduced from 0.2 to 0.1.