Graph Operations that are Good for Greedoids

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Abstract

$S$ is a local maximum stable set of a graph $G$, and we write $S \in \Psi(G)$, if the set $S$ is a maximum stable set of the subgraph induced by $S \cup N(S)$, where $N(S)$ is the neighborhood of $S$. In [4] we have proved that $\Psi(G)$ is a greedoid for every forest $G$. The cases of bipartite graphs and triangle-free graphs were analyzed in [5] and [6], respectively.

In this paper we give necessary and sufficient conditions for $\Psi(G)$ to form a greedoid, where $G$ is:

(a) the disjoint union of a family of graphs;
(b) the Zykov sum of a family of graphs;
(c) the corona $X \circ \{H_1, H_2, \ldots, H_n\}$ obtained by joining each vertex $x$ of a graph $X$ to all the vertices of a graph $H_x$.

Keywords: Corona, Zykov sum, greedoid, local maximum stable set

1 Introduction

Throughout this paper $G = (V, E)$ is a simple (i.e., a finite, undirected, loopless and without multiple edges) graph with vertex set $V = V(G)$ and edge set $E = E(G)$. If $X \subset V$, then $G[X]$ is the subgraph of $G$ spanned by $X$. By $G - W$ we mean the subgraph $G[V - W]$, if $W \subset V(G)$. We also denote by $G - F$ the partial subgraph of $G$ obtained by deleting the edges of $F$, for $F \subset E(G)$, and we write shortly $G - e$, whenever $F = \{e\}$.

The neighborhood of a vertex $v \in V$ is the set $N_G(v) = \{w : w \in V \text{ and } vw \in E\}$. We denote the neighborhood of $A \subset V$ by $N_G(A) = \{v \in V - A : N(v) \cap A \neq \emptyset\}$ and its closed neighborhood by $N_G[A] = A \cup N(A)$, or shortly, $N(A)$ and $N[A]$, if no ambiguity.

$K_n, P_n, C_n$ denote respectively, the complete graph on $n \geq 1$ vertices, the chordless path on $n \geq 2$ vertices, and the chordless cycle on $n \geq 3$ vertices, respectively.

A stable set in $G$ is a set of pairwise non-adjacent vertices. A stable set of maximum size will be referred to as a maximum stable set of $G$, and the stability number of $G$, denoted by
\(\alpha(G)\), is the cardinality of a maximum stable set in \(G\). In the sequel, by \(\Omega(G)\) we denote the set of all maximum stable sets of the graph \(G\).

Any stable set \(S\) is maximal (with respect to set inclusion) in \(G[N[S]]\), but is not necessarily, a maximum one. A set \(A \subseteq V(G)\) is a local maximum stable set of \(G\) if \(A\) is a maximum stable set in the subgraph induced by \(N[A]\), i.e., \(A \in \Omega(G[N[A]])\), [4]. Let \(\Psi(G)\) stand for the set of all local maximum stable sets of \(G\).

Clearly, every stable set containing only pendant vertices belongs to \(\Psi(G)\). Nevertheless, there exist local maximum stable sets that do not contain pendant vertices. For instance, \(\{e, g\} \in \Psi(G)\), where \(G\) is the graph from Figure 1.

Figure 1: A graph having various local maximum stable sets.

The following theorem concerning maximum stable sets in general graphs, due to Nemhauser and Trotter Jr. [10], shows that some stable sets can be enlarged to maximum stable sets.

**Theorem 1.1** [10] Every local maximum stable set of a graph is a subset of a maximum stable set.

Nemhauser and Trotter Jr. interpret this assertion as a sufficient local optimality condition for a binary integer programming formulation of the weighted maximum stable set problem, and use it to prove an outstanding result claiming that integer parts of solutions of the corresponding linear programming relaxation maintain the same values in the optimal solutions of its binary integer programming counterpart. In other words, it means that a well-known branch-and-bound heuristic for general integer programming problems turns out to be an exact algorithm solving the weighted maximum stable set problem.

The graph \(W\) from Figure 1 has the property that every \(S \in \Omega(W)\) contains some local maximum stable set, but these local maximum stable sets are of different cardinalities: \(\{a, d, f\} \in \Omega(W)\) and \(\{a\}, \{d, f\} \in \Psi(W)\), while for \(\{b, e, g\} \in \Omega(W)\) only \(\{e, g\} \in \Psi(W)\).

However, there exists a graph \(G\) satisfying \(\Psi(G) = \Omega(G)\), e.g., \(G = C_n\), for \(n \geq 4\).

A greedoid is a set system generalizing the notion of a matroid.

**Definition 1.2** [11, 12] A greedoid is a pair \((E, \mathcal{F})\), where \(\mathcal{F} \subseteq 2^E\) is a non-empty set system satisfying the following conditions:

**Accessibility:**

For every non-empty \(X \in \mathcal{F}\) there is an \(x \in X\) such that \(X - \{x\} \in \mathcal{F}\);

**Exchange:**

For \(X, Y \in \mathcal{F}\), \(|X| = |Y| + 1\), there is an \(x \in X - Y\) such that \(Y \cup \{x\} \in \mathcal{F}\).

Let us observe that \(\{d, g\} \in \Psi(W)\), while \(\{d\}, \{g\} \notin \Psi(W)\), where \(W\) is the graph depicted in Figure 1. However, it is worth mentioning that if \(\Psi(G)\) is a greedoid and \(S \in \Psi(G)\), \(|S| = k \geq 2\), then according to the accessibility property, one can build a chain

\(\{x_1\} \subset \{x_1, x_2\} \subset ... \subset \{x_1, ..., x_{k-1}\} \subset \{x_1, ..., x_{k-1}, x_k\} = S\)
such that
\[ \{x_1, x_2, \ldots, x_j\} \in \Psi(G), \text{ for all } j \in \{1, \ldots, k-1\}. \]

For example, \( \{a\} \subset \{a, b\} \subset S \) is an accessibility chain of the set \( S = \{a, b, c\} \in \Psi(G_2) \), where \( G_2 \) is presented in Figure 2.

In [4] it is proved the following result.

**Theorem 1.3** For every tree \( T \), \( \Psi(T) \) is a greedoid on its vertex set.

The case of bipartite graphs owning a unique cycle, whose family of local maximum stable sets forms a greedoid is analyzed in [3] (for an example, see the graph \( G_1 \) from Figure 2). In general, local maximum stable sets of bipartite graphs were treated in [5], while for triangle-free graphs we refer the reader to [6] for details. Nevertheless, there exist non-bipartite and also non-triangle-free graphs whose families of local maximum stable sets form greedoids. For instance, the families \( \Psi(G_2), \Psi(G_3), \Psi(G_4) \) of the graphs in Figure 2 are greedoids.

![Figure 2: Graphs whose family of local maximum stable sets form greedoids.](image)

In this note we present ”if and only if” conditions for \( \Psi(G) \) to be a greedoid, where \( G \) is the disjoint union, or the Zykov sum, or the corona of a family of graphs.

## 2 Disjoint union and Zykov sum of graphs

Let \( G \) be the disjoint union of the family of graphs \( \{G_i : 1 \leq i \leq p\}, p \geq 2 \), i.e.,

\[
V(G) = V(G_1) \cup V(G_2) \cup \ldots \cup V(G_p) \quad \text{and} \quad E(G) = E(G_1) \cup E(G_2) \cup \ldots \cup E(G_p),
\]

under the assumption that \( V(G_i) \cap V(G_j) = \emptyset, 1 \leq i < j \leq p \). Clearly, \( \alpha(G) = \alpha(G_1) + \alpha(G_2) + \ldots + \alpha(G_p) \) and \( S \subseteq V(G) \) is stable if and only if every \( S \cap V(G_i), 1 \leq i \leq p, \) is stable. Moreover, one can easily prove the following result.

**Proposition 2.1** If \( G \) is the disjoint union of the family of graphs \( \{G_i : 1 \leq i \leq p\}, p \geq 2 \), then:

(i) \( S \in \Psi(G) \) if and only if \( S \cap V(G_i) \in \Psi(G), 1 \leq i \leq p; \)

(ii) \( \Psi(G) \) is a greedoid if and only if every \( \Psi(G_i), 1 \leq i \leq p, \) is a greedoid.

Recall that the **Zykov sum** of the graphs \( G_i, 1 \leq i \leq p, p \geq 2 \), is the graph \( Z = Z[G_1, \ldots, G_p] = G_1 + G_2 + \ldots + G_p \) having

\[
V(Z) = V(G_1) \cup \ldots \cup V(G_p),
\]

\[
E(Z) = E(G_1) \cup \ldots \cup E(G_p) \cup \{v_iv_j : v_i \in V_i, v_j \in V_j, 1 \leq i < j \leq p\}.
\]
Clearly, $\alpha(Z) = \max\{\alpha(G_i) : 1 \leq i \leq p\}$. If all $G_1, G_2, ..., G_p, p \geq 2$, are complete graphs, then $Z$ is complete, as well. In this case, we have

$$\Psi(Z) = \Omega(Z) = \left\{ \{v\} : v \in V(Z) \right\}$$

and $\Psi(Z)$ is, evidently, a greedoid.

**Lemma 2.2** If $Z = Z[G_1, ..., G_p]$, then

$$\min\{|S| : S \in \Psi(Z)\} \geq \max_2\{\alpha(G_i) : 1 \leq i \leq p\},$$

where $\max_2\{\alpha_i : 1 \leq i \leq p\}$ is a second largest number of the sequence.

**Proof.** Notice that if $S \subseteq V(Z)$ is stable, then there is some $i \in \{1, 2, ..., p\}$ such that $S \subseteq V(G_i)$. Hence, if $S \in \Psi(Z)$, then $S \in \Psi(G_k)$ for some $k \in \{1, 2, ..., p\}$, and, in addition,

$$|S| \geq \max\{\alpha(G_i) : 1 \leq i \leq p, i \neq k\}.$$

Since

$$\max_2\{\alpha(G_i) : 1 \leq i \leq p\} = \min_{1 \leq k \leq p} \left( \max\{\alpha(G_i) : 1 \leq i \leq p, i \neq k\} \right),$$

we get that

$$\min\{|S| : S \in \Psi(Z)\} \geq \max_2\{\alpha(G_i) : 1 \leq i \leq p\},$$

which completes the proof.  

Let us observe that for the graphs $G_1 = Z[K_2, P_3]$ and $G_2 = Z[P_3, P_3]$ (depicted in Figure 3), $\Psi(G_1)$ is a greedoid, while $\Psi(G_2)$ is not a greedoid, because $\{v\} \notin \Psi(G_2)$, for every $v \in V(G_2)$.

**Proposition 2.3** Let $Z = Z[G_1, ..., G_p]$ be such that $\alpha(Z) > 1$. Then $\Psi(Z)$ is a greedoid if and only if the following assertions are true:

(i) all $\Psi(G_i), 1 \leq i \leq p$, are greedoids;

(ii) there is a unique $k \in \{1, 2, ..., p\}$ such that $G_k$ is not complete;

(iii) $\Psi(Z) = \Psi(G_k)$.

**Proof.** Taking into account the definition of $Z$, it follows that at least one of the graphs $G_i$ is not complete, because and $\alpha(Z) > 1$.

Assume that $\Psi(Z)$ is a greedoid and let $\{a\} \in \Psi(Z)$. Hence we infer that

$$\min\{|S| : S \in \Psi(Z)\} = 1.$$
Consequently, by Lemma 2.2, we get $1 \geq \max_2 \{\alpha(G_i) : 1 \leq i \leq p\}$.

Thus all $G_i, 1 \leq i \leq p$ but one must be complete graphs. Suppose $G_k$ is the unique non-complete graph. Then $\alpha \in V(G_k)$ and $\alpha(Z) = \alpha(G_k)$.

Clearly, all $\Psi(G_i), 1 \leq i \leq p, i \neq k$, are greedoids. In addition, $\{v\} \notin \Psi(Z)$, for every $v \in V(Z) - V(G_k)$, because $V(G_k) \subseteq N_Z(v)$ and $\alpha(G_k) > 1$. It follows that $S \subseteq V(G_k)$, for every $S \in \Psi(Z)$. Moreover, one can say that $S \in \Psi(G_k)$, i.e., $\Psi(Z) \subseteq \Psi(G_k)$.

Otherwise, if some $A \in \Psi(Z)$ does not belong to $\Psi(G_k)$, it follows that there is a stable set $B$ in $N_{G_k}[A]$, larger than $A$. Since $B$ is stable in $Z$, as well, and $B \subseteq N_{G_k}[A] \subseteq N_Z[A]$, it implies $A \notin \Psi(Z)$, in contradiction with the choice of $A$. On the other hand, taking into account that no stable set in $Z$ can meet both $V(G_k)$ and $V(Z) - V(G_k)$, it follows that $\Psi(G_k) \subseteq \Psi(Z)$ is true, as well. In other words, we infer that $\Psi(Z) = \Psi(G_k)$, which ensures that $\Psi(G_k)$ is a greedoid.

The converse is clear. ■

3 Corona of graphs

Let $X$ be a graph with $V(X) = \{v_1 : 1 \leq i \leq n\}$, and $\{H_i : 1 \leq i \leq n\}$ be a family of graphs. Joining each $v_i \in V(X)$ to all the vertices of $H_i$, we obtain a new graph, which we denote by $G = X \circ \{H_1, H_2, ..., H_n\}$ (see Figure 4 for an example, where $X = K_3 + v_3v_4$). If $H_1 = H_2 = ... = H_n = H$, we write $G = X \circ H$, and in this case, $G$ is called the corona of $X$ and $H$.

Figure 4: $G = (K_3 + v_3v_4) \circ \{K_3, K_2, P_3, K_1\}$ is a well-covered graph.

Let us notice that $G = X \circ \{H_1, H_2, ..., H_n\}$ has $\alpha(G) = \alpha(H_1) + \alpha(H_2) + ... + \alpha(H_n)$.

Let us consider the graph $G$ depicted in Figure 4. Notice that:

- $S_1 = \{x, z, v_4\} \in \Psi(G)$ and also $S_1 \cap V(H) \in \Psi(H)$, for every $H \in \{K_3, K_2, P_3, K_1\}$;
- the set $S_2 = \{y, v_2\}$ is stable, but $S_2 \notin \Psi(G)$, because $\{y, u, v_3\} \subseteq N_G[S_2]$ and it is stable and larger than $S_2$;
- $\{v_4\}, \{v_2, v_4\} \in \Psi(K_3 + v_3v_4)$, but $\{v_4\}, \{v_2, v_4\} \notin \Psi(G)$;
- $\{y, v_4\} \notin \Psi(G)$, since $\{y, t, v_3\} \subseteq N_G[\{y, v_4\}]$ and it is stable and larger than $\{y, v_4\}$;
- $\{y\} \in \Psi(K_3), \{x, z\} \in \Psi(P_3)$ and also $\{x, y, z\} \in \Psi(G)$;
- the set $S_3 = \{y, v_3\}$ is stable and $S_2 \cap V(H) \in \Psi(H)$, for each $H \in \{K_3, K_2, P_3, K_1\}$, but $S_3 \notin \Psi(G)$.  


Lemma 3.1 Let $G = X \circ \{H_1, H_2, ..., H_n\}$, where $V(X) = \{v_i : 1 \leq i \leq n\}, n \geq 2$. Then the following assertions are true:

(i) $\Psi(H_i) \subseteq \Psi(G), 1 \leq i \leq n$;
(ii) if $v_i \in S \in \Psi(G)$, then $H_i$ is complete, and $S \cap V(H_k) \neq \emptyset$, for each $v_k \in N_X(v_i)$;
(iii) if $S \in \Psi(G)$, then $S \cap V(H_k) \in \Psi(H_i), 1 \leq i \leq n$;
(iv) if $S$ is a stable set in $G$ such that: $S \cap V(H_i) \in \Psi(H_i), 1 \leq i \leq n$, and for every $v_i \in S \cap V(X), H_i$ is a complete graph, while $S \cap V(H_k) \neq \emptyset$, for all $v_k \in N_X(v_i)$, then $S \in \Psi(G)$.

Proof. (i) Let $A \in \Psi(H_i)$. Then, $N_G[A] = N_{H_i}[A] \cup \{v_i\}$ and, thus, $A$ is a maximum stable set in $N_G[A]$, as well, i.e., $A \in \Psi(G)$. Consequently, $\Psi(H_i) \subseteq \Psi(G)$ for each $i \in \{1, 2, ..., n\}$.

(ii) If $v_i \in S$ and there are non-adjacent vertices $x, y \in V(H_i)$, then the set $S \cup \{x, y\} - \{v_i\}$ is stable in $N_G[S]$, larger than $S$, in contradiction with $S \in \Psi(G)$. Therefore, $H_i$ must be a complete graph.

Assume that $S \cap V(H_k) = \emptyset$ for some $v_k \in N_X(v_i)$.

If $N(v_k) \cap S = \{v_i\}$, then for every $x \in V(H_i)$, the set $S \cup \{v_k, x\} - \{v_i\}$ is stable in $N_G[S]$ and larger than $S$, in contradiction with $S \in \Psi(G)$.

If $N(v_k) \cap S = \{v_i, v_{j_1}, v_{j_2}, ..., v_{j_q}\}$, then

$$S \cup \{x, v_k\} \cup \{x_{j_1}, x_{j_2}, ..., x_{j_q}\} - \{v_i, v_{j_1}, v_{j_2}, ..., v_{j_q}\}$$

is a stable set in $N_G[S]$ for every $x \in V(H_i)$ and each $x_{j_t} \in V(H_{j_t}), 1 \leq t \leq q$, larger than $S$, in contradiction with $S \in \Psi(G)$.

Consequently, $S \cap V(H_k) \neq \emptyset$, for every $v_k \in N_X(v_i)$.

(iii) Assume that $S \in \Psi(G)$.

If $S_j = S \cap V(H_j) \notin \Psi(H_j)$, then $v_j \notin S$ (because $S_j \neq \emptyset$) and there is some stable set $A_j \subseteq N_{H_j}[S_j]$ larger than $S_j$. Since

$$N_{H_j}[S_j] \cup \{v_j\} = N_G[S_j]$$

and $v_j \notin A$, we get that $(S - S_j) \cup A$ is a stable set included in $N_G[S]$ and $|S| < |(S - S_j) \cup A|$, in contradiction with $S \in \Psi(G)$. Therefore, $S \cap V(H_i) \in \Psi(H_i)$ for every $i \in \{1, 2, ..., n\}$.

(iv) We have to prove that $|A| \leq |S|$ for every stable set $A \subseteq N_G[S]$.

Let us define the following partitions of the sets $A$ and $S$:

$$A = A_1 \cup A_2 \cup A_3, \quad S = S_1 \cup S_2 \cup S_3$$

where

$$A_1 = \bigcup_{v_j \in (A - S)} [A \cap (V(H_j) \cup \{v_j\})], \quad S_1 = \bigcup_{v_j \in (A - S)} [S \cap (V(H_j) \cup \{v_j\})]$$

$$A_2 = \bigcup_{v_j \in V(X) - (A \cup S)} [A \cap (V(H_j) \cup \{v_j\})], \quad S_2 = \bigcup_{v_j \in V(X) - (A \cup S)} [S \cap (V(H_j) \cup \{v_j\})]$$

$$A_3 = \bigcup_{v_j \in S} [A \cap (V(H_j) \cup \{v_j\})], \quad S_3 = \bigcup_{v_j \in S} [S \cap (V(H_j) \cup \{v_j\})].$$

Our intent is to show that

$$|A_k| \leq |S_k|, 1 \leq k \leq 3,$$
which will lead us to the conclusion that $|A| \leq |S|$.

Case 1. $v_j \in A - S$.

Since $S$ is maximal in $N_G[S]$, we infer that $v_j \in E(G)$, for some $y \in S$. If $y \in V(H)$, then $S \cap V(H) \neq \emptyset$. Otherwise, $y \in V(X)$ and according with the hypothesis on $S$, again $S \cap V(H) \neq \emptyset$. Therefore, we get that

$$|A \cap (V(H) \cup \{v_j\})| = 1 \leq |S \cap (V(H) \cup \{v_j\})|,$$

which implies $|A_1| \leq |S_1|$.

Case 2. $v_j \in V(X) - (A \cup S)$.

Since $S \cap V(H) \in \Psi(H)$, we have that $|A \cap V(H)| \leq |S \cap V(H)|$. Together with the condition $v_j \in V(X) - (A \cup S)$ it gives

$$|A \cap (V(H) \cup \{v_j\})| \leq |S \cap (V(H) \cup \{v_j\})|.$$

Therefore, it follows that $|A_2| \leq |S_2|$.

Case 3. $v_j \in S$.

According with the hypothesis on $S$, $H_j$ is a clique. Consequently, we obtain

$$|A \cap (V(H) \cup \{v_j\})| \leq 1 \leq |S \cap (V(H) \cup \{v_j\})|,$$

which ensures that $|A_3| \leq |S_3|$. 

The following theorem generalizes some partial findings from [7], [8], [9].

**Theorem 3.2** If $G = X \circ \{H_1, H_2, ..., H_n\}$ and $H_1, H_2, ..., H_n$ are non-empty graphs, then $\Psi(G)$ is a greedoid if and only if every $\Psi(H_i), i = 1, 2, ..., n$, is a greedoid.

**Proof.** Assume that $\Psi(G)$ is a greedoid.

According to Lemma 3.1(i), (iii), we get that

$$\Psi(H_i) = \{S \cap V(H_i) : S \in \Psi(G)\}, 1 \leq i \leq n.$$

Hence, every $\Psi(H_i)$ satisfies both accessibility property and exchange property, i.e., $\Psi(H_i)$ is a greedoid.

Conversely, suppose that every $\Psi(H_i), 1 \leq i \leq n$, is a greedoid.

Firstly, we show that $\Psi(G)$ satisfies the accessibility property.

Let $S \in \Psi(G)$ and $S \neq \emptyset$.

If $v_i \in S \cap V(X)$, then $N_X(v_i) \cap S = \emptyset, V(H_i) \cap S = \emptyset$, while, by Lemma 3.1(ii), $S \cap V(H_i) \neq \emptyset$ holds for every $v_k \in N(v_i)$. Hence, we may infer that $S - \{v_i\} \in \Psi(G)$.

If $S \cap V(X) = \emptyset$, then there is some $i \in \{1, 2, ..., n\}$, such that $S_i = S \cap V(H_i) \neq \emptyset$ and $S_i \in \Psi(H_i)$, according to Lemma 3.1(iii). Since $\Psi(H_i)$ is a greedoid, there is some $x \in S_i$ such that $S_i - \{x\} \in \Psi(H_i)$. Since

$$N_G[S - \{x\}] \cap V(H_i) = N_{H_i}[S_i - \{x\}],$$

while

$$N_G[S - \{x\}] \cap V(H_j) = N_G[S] \cap V(H_j)$$

for every $j \neq i$, we may conclude that $S - \{x\} \in \Psi(G)$.

We check now the exchange property. Let $S_1, S_2 \in \Psi(G)$ be with $|S_1| = |S_2| + 1$.  

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Case 1. $S_1 \cap V(H_j) = S_2 \cap V(H_j)$ for all $j \in \{1, 2, \ldots, n\}$. Then there is some $v_i \in S_1 - S_2$, because $|S_1| > |S_2|$. Hence, it follows $S_1 \cap V(H_i) = \emptyset$, which ensures that also $S_2 \cap V(H_i) = \emptyset$. By Lemma 3.1(ii), we have that, for every $v_k \in N_2(v_i)$, $S_1 \cap V(H_k) \neq \emptyset$ which implies that also $S_2 \cap V(H_k) \neq \emptyset$. Consequently, using Lemma 3.1(iv), we may infer that $S_2 \cup \{v_k\} \in \Psi(G)$.

Case 2. There is some $i \in \{1, 2, \ldots, n\}$, such that $A_1 = S_1 \cap V(H_i)$ is larger than $A_2 = S_2 \cap V(H_i)$. Since $A_1, A_2 \in \Psi(H_i)$ and $\Psi(H_i)$ is a greedoid, there must exist some $x \in A_1 - A_2$, such that $A_2 \cup \{x\} \in \Psi(H_i)$. Hence, we get also that $S_2 \cup \{x\} \in \Psi(G)$.

Consequently, $\Psi(G)$ satisfies the exchange property.

In conclusion, $\Psi(G)$ forms a greedoid on the vertex set of $G$. ■

**Corollary 3.3** $\Psi(X \circ H)$ is a greedoid if and only if $\Psi(H)$ is a greedoid.

### 4 Conclusions and future work

Let $\{H_1, \ldots, H_n\}$ be a family of graphs indexed by the vertex set $\{1, 2, \ldots, n\}$ of a graph $H_0$.

The graph denoted by $H_0[H_1, H_2, \ldots, H_n]$ is defined as:

$$V(H_0[H_1, H_2, \ldots, H_n]) = \{1\} \times V(H_1) \cup \ldots \cup \{n\} \times V(H_n),$$

and $(i, x), (j, y) \in V(H_0[H_1, H_2, \ldots, H_n])$ are adjacent if and only if either (i) $ij \in E(H_0)$ or (ii) $i = j$ and $xy \in E(H_i)$. For instance, $K_n[H_1, H_2, \ldots, H_n]$ is the Zykov sum of $H_1, \ldots, H_n$; while if $H_1 = H_2 = \ldots = H_n$, then $H_0[H_1, H_2, \ldots, H_n]$ is known as lexicographic product $H_0 \bullet H_1$. It seems to be interesting to establish necessary and sufficient conditions ensuring that $\Psi(H_0[H_1, H_2, \ldots, H_n])$ forms a greedoid. When $H_0 \in \{K_n, K_1\}$, Propositions 2.3, 2.7 give the conditions needed.

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