Distinguishing between $R^2$-inflation and Higgs-inflation

F. L. Bezrukov$^{a,b}$, D. S. Gorbunov$^b$

$^a$Arnold Sommerfeld Center for Theoretical Physics, Department für Physik, Ludwig-Maximilians-Universität, Theresienstr. 37, 80333 München, Germany
$^b$Institute for Nuclear Research of the Russian Academy of Sciences, 60th October Anniversary prospect 7a, Moscow 117312, Russia

Abstract

We present three features which can be used to distinguish the $R^2$-inflation Higgs-inflation from with ongoing, upcoming and planned experiments, assuming no new physics (apart from sterile neutrinos) up to inflationary scale. (i) Slightly different tilt of the scalar perturbation spectrum $n_s$ and ratio $r$ of scalar-to-tensor perturbation amplitudes. (ii) Gravity waves produced within $R^2$-model by collapsing, merging and evaporating scalaron clumps formed in the post-inflationary Universe. (iii) Different ranges of the possible Standard Model Higgs boson masses, where the electroweak vacuum remains stable while the Universe evolves after inflation. Specifically, in the $R^2$-model Higgs boson can be as light as 116 GeV. These effects mainly rely on the lower reheating temperature in the $R^2$-inflation.

Early time inflation is a very attractive idea allowing to solve many serious problems of the Hot Big Bang cosmological model originating in the mystery of the initial conditions of our Universe, see e.g. [1]. The inflation can be arranged with a specific dynamics of only one degree of freedom—a scalar field. Remarkably, many simplest models give different predictions for the tilt of the scalar perturbation spectrum $n_s$ and for the ratio $r$ of squared amplitudes of tensor and scalar perturbations, and thus can be distinguished experimentally by CMB observations. On the contrary, if predictions for ($n_s$, $r$) coincide, it will be generally difficult to determine which model is realized in Nature, if the measured value of ($n_s$, $r$) is close to the prediction.

One could expect this to be the situation for a pair of two minimal inflationary models: $R^2$-inflation and Higgs-inflation [2]. They are minimal, because in order to solve the well-known problems of the Hot Big Bang model they introduce very little of new physics. Indeed, in the first model with nonlinear modification of the Einstein–Hilbert action, one and the same interaction—
gravity—takes care of both inflation and subsequent reheating of the Universe. Only one new degree of freedom—a scalar in the gravity sector—emerges and only one new parameter (in front of the $R^2$-term) is present. In the second model no new degrees of freedom appear: thanks to the non-minimal coupling to gravity (with only one new corresponding parameter) the Standard Model (SM) Higgs boson plays the role of inflaton and its couplings to the SM fields are responsible for subsequent reheating. Remarkably, these minimal inflationary models can be augmented with a small amount of new interaction terms and degrees of freedom to accommodate all currently firmly established experimental evidences of beyond the SM physics (neutrino oscillations, Dark Matter, and baryon asymmetry of the Universe), see e.g. [5, 6] and [7, 8]. These modifications do not interfere with the inflationary and reheating dynamics, and hence are irrelevant for the present study. A particular example of the renormalizable model is $\nu$MSM [9, 10], which is a SM extension with three right handed neutrinos capable of explaining simultaneously the active neutrino masses (by see-saw like mechanism), DM (sterile neutrino at keV scale) and baryon asymmetry of the Universe (by a specific variant of leptogenesis), for a review see [11]. Other examples may also be suggested, e.g. [5, 6, 8]. We stress, however, that to make exact predictions for the inflationary parameters the evolution of the Universe needs to be known both during and after the inflation. This is certainly achieved in any extension, which does not introduce new scales between the electroweak and inflationary scales, and this is true in the $\nu$MSM.

Let us return to the inflationary models proper. The similarity between the two models is clearly seen in the Einstein frame, where the potentials for canonically normalized inflatons $\chi$ (scalaron and the Higgs boson, respectively) are identical for the large fields $\chi \gg U^2/M_P$, relevant for inflationary dynamics,

$$V(\chi) = \frac{U^4}{4} \left( 1 - \exp\left( -\frac{2\chi}{\sqrt{6} M_P} \right) \right)^2.$$  \hspace{1cm} (1)

Here $M_P \equiv 1/\sqrt{8\pi G_N} = 2.44 \times 10^{18}$ GeV is the reduced Planck mass, and $U$ is the only parameter controlling the scale of the potential. For $\chi \gtrsim M_P$ the slow roll conditions are satisfied and inflationary stage takes place. The expressions of the constant $U$ in terms of the fundamental constants of the theory ($\xi$ and $\lambda$ for the Higgs-inflation [2] and $\mu$ for the $R^2$-inflation [3]) are different in the two models, but as far as its value $U = M_P/\sqrt{47000}$ is uniquely defined by the normalization of the amplitude of the primordial density perturbations, both models seem to predict the same set of $(n_s, r)$.

Are there any differences between these two models, which can be resolved at the present level of experimental techniques? The answer is positive and we discuss the details in this Letter.

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1Let us note, that introduction of additional light scalar degree of freedom (or several) also leads to a plethora of good models, see e.g. [3, 4]. However they are naturally leading to obviously very different inflationary parameters, and are less minimal, so we do not consider them in this Letter.
Certainly, the two models are different. $R^2$-inflation modifies the gravitational sector and exploits as the inflaton one new degree of freedom—scalaron—in addition to the SM and hence to the Higgs-inflation. The Higgs-inflation modifies the interaction of gravity with only one of the existing particles—the Higgs boson—making the Higgs field itself the inflaton.

The key observation is that right after inflation, starting from the onset of inflaton homogeneous oscillations, interactions of the two inflatons with the SM fields are absolutely different. Being one of the gravitational excitations the scalaron, similarly to the graviton, interacts with gravitational strength, i.e. all scalaron coupling constants are suppressed by an appropriate power of the Planck mass. The Higgs boson interacts with the weak bosons and top quarks with couplings of order one. This circumstance drastically changes the post-inflationary evolution of the Universe, which takes place between inflation and the hot stage (preheating). Indeed, though in both models at this stage the inflaton exhibits free-field oscillations and the Universe is in a matter dominated stage, the inflaton couplings to the other fields play a crucial role.

In the case of Higgs-inflation one can see, that while at high scales the Higgs boson is nearly decoupled from all other SM fields, at the energy scale of the order $U^2/M_P \sim 10^{13}$ GeV its interaction regain the SM form \cite{2}. So, when the oscillation amplitude of the field after inflation drops below this value, the energy is effectively transferred into all SM degrees of freedom. The detailed analysis \cite{7,8} of the field decay during the matter dominated stage shows that the Higgs-inflaton field rapidly produces weak bosons, which subsequently decay into all other SM particles and reheat the Universe, leading to a slightly higher than $U^2/M_P$ temperature

$$T_{\text{reh}}^H \simeq 6 \times 10^{13} \text{GeV},$$

with uncertainty factor about two.

In the $R^2$-inflation the scalaron coupling to all fields is Planck scale suppressed, and the reheating mainly occurs via its decays into the SM Higgs bosons \cite{12,13}, which immediately recast into SM particles (scalaron couplings to all other fields are additionally suppressed due to conformal symmetry). The reheating temperature in the model is significantly lower \cite{3},

$$T_{\text{reh}}^{R^2} = 3.1 \times 10^9 \text{GeV}.$$  \hfill (3)

The difference in reheating temperatures yields three important consequences each providing with features (potentially) experimentally observable and allowing for discriminating between the models. These consequences are slightly different numbers of e-foldings before the end of inflation for the moment of the horizon exit of the same pivot scale of WMAP, collapses of small scale scalaron perturbations in $R^2$-model in the post-inflationary Universe and different regions of the SM parameter space, where the electroweak vacuum remains sufficiently stable today as well as in the very early Universe. Let us discuss these issues in turn.
Different e-folding numbers. The post-inflationary history of the Universe differs in the two models: the pre-Big-Bang matter dominated stage lasts much longer in $R^2$-inflation. As a result, the matter perturbations of a given wavelength at present (including the scale used for normalization of WMAP) were of different sizes at the time of horizon crossing at the inflationary stage in these two models. Indeed, the wavelength scales as the scale factor, $\propto a(t)$, when the Universe expands, and at matter-domination and radiation-domination the scale factor grows differently (as $a(t) \propto t^{2/3}$ and $a(t) \propto t^{1/2}$, respectively). Hence, at the beginning of the post-inflationary stage the wavelength of a perturbation of a given present scale were different in these two cosmological models. At the end of inflation their wavelength exceeded the horizon size. Their amplitudes have been frozen earlier at the inflationary stage when the wavelength of the stretched perturbations crossed the horizon (see details in e.g. [1]). Thus, the e-folding numbers (and, respectively, the field values in [1], corresponding to this moment of time) of the horizon crossing for the modes of the presently observed pivot scale are different in the two inflationary models.

Let us calculate the number of e-foldings for the fluctuations of conformal momentum $k$. We are interested in fluctuations which physical momentum today corresponds to the WMAP pivot scale $k/a_0 = 0.002$ Mpc, where $a_0$ is the present scale factor (below we use subscript ‘0’ to denote the present values of parameters). At inflationary stage the fluctuations exit the horizon when the physical momentum $k/a$ drops below the value of the Hubble parameter $H = \dot{a}/a$ determining the expansion rate. Marking the values of all parameters at the horizon crossing with asterisk, we write

\[ H_* = \frac{k}{a_*} = \frac{k}{a_0 a_r a_e a_s} \equiv \frac{k}{a_0 a_r a_e} \equiv \frac{k}{a_0 a_r a_e} e^N, \]

hereafter subscripts ‘e’ and ‘r’ refer to the values of parameters at the end of inflation and at the end of reheating, respectively. The change of the scale factor after reheating is

\[ \frac{a_r}{a_e} = \left( \frac{g_0}{g_r} \right)^{1/3} \frac{T_0}{T_r}, \]

where $g_r$ is the number of degrees of freedom (d.o.f.) in the primordial plasma at reheating and $g_0 = 2 + \frac{3}{2} \cdot 2 \cdot 3 \cdot \frac{1}{6}$ is the present effective number of relativistic d.o.f. taking into account different neutrino temperature. Since both models exhibit matter dominated expansion between inflation and reheating, we get for the change of the scale factor during preheating

\[ \frac{a_r}{a_e} = \left( \frac{V_e}{g_r^{2/3} T_r^4} \right)^{1/3}. \]

Collecting everything and using the Friedman equation for the Hubble param-
eter $H = (V_e/(3M_P^2))^{1/2}$, we get

$$N = \frac{1}{3} \log \left( \frac{\pi^2}{30\sqrt{27}} \right) - \log \left( \frac{k/a_0}{T_{0g_0}} \right)^{1/3} + \log \frac{V_e^{1/2}}{V_e^{1/4}M_P} - \frac{1}{3} \log \frac{10^{13} \text{GeV}}{V_e^{1/4}} - \frac{1}{3} \log \frac{10^{13} \text{GeV}}{T_r},$$

(4)

The first term in (4) contains model-independent numbers, the rest terms in the same line vary with the change of the moments of horizon crossing and end of inflation very mildly (sub-logarithmically),

$$V_* \approx \frac{U^4}{4}, \quad V_e \approx \frac{U^4}{4(1 + \sqrt{3}/4)^2}.$$

The main difference in $N_*$ comes from the last term in (4), so that approximately

$$N_* \approx 57 - \frac{1}{3} \log \frac{10^{13} \text{GeV}}{T_r}.$$

For the models under discussion with the reheating temperatures (2) and (3) one obtains numerically from (4)

$$N_H = 57.66, \quad N_{R^2} = 54.37.$$

This discussion is applicable to both scalar (inflaton) and tensor (graviton) perturbations. The different sizes of horizon imply different values of the inflaton potential (1) and hence different values of the inflaton field $\chi_*$. In this way we finally arrive at different values of parameters of scalar and tensor perturbations (see details in e.g. [1]), which for the present models sharing the same potential (1) at inflationary stage are mainly (up to corrections $O(U/M_P)$) determined by the number of e-foldings, i.e. $n_s \approx 1 - 8/(4N + 9)/(4N + 3)^2$ and $r \approx 192/(4N + 3)^2$ [7]. Using exact formulas (see e.g. [1, 7]) gives numerically

Higgs-inflation: $n_s = 0.967, \quad r = 0.0032,$

$R^2$-inflation: $n_s = 0.965, \quad r = 0.0036.$

The difference is small, at the level of $10^{-3}$, but such an accuracy is close to achievable at Planck experiment (expected precision for $n_s = 0.0045$, see [14]) and CMBPol experiment (precision for $r$ is $10^{-3}$ or even $0.5 \times 10^{-3}$, and for $n_s$ up to 0.0016 [15]). Note that an additional test for all large-field inflationary models could become possible if the tilt of the tensor fluctuations spectrum $n_T \approx -r/8$ is measured.

The primordial tensor perturbations can be also directly detected as gravity waves by the advanced stages of DECIGO project [16]. Note also, that the matter dominated stage causes reduction of the gravity waves amplitude for high frequency modes. For the reheating temperature of $T_r \sim 10^9 \text{GeV}$ the gravity waves are suppressed at frequencies above approximately $10 \text{Hz}$ [17] (and above lower frequencies for lower reheating temperatures). While being not testable by the currently proposed detectors, the value is not far from explorable region.
**Gravitational waves at matter dominated stage.** When the matter perturbations discussed above enter the horizon, they grow proportionally to the scale factor at the matter-dominated stage. The observed CMB anisotropy and large scale structure can be explained with primordial (inflaton) energy density perturbations at the level $\delta \rho/\rho \sim 10^{-5}$. With long post-inflationary matter dominated stage in $R^2$-inflation certain small scale perturbations have enough time to enter horizon and grow up to $\delta \rho/\rho \sim 1$ evolving towards the nonlinear stage. Then one expects production of small scale self-gravitating structures from the scalaron condensate, which does not change the reheating \[5\], but can give rise to gravity waves emission by inflaton clumps.

Indeed, at the late post-inflationary stage the scalaron behaves like homogeneously oscillating free scalar field (inflaton), which was argued \[18,19\] to be capable of producing short-length gravity waves due to collapses of inflaton perturbations, merging of inflaton clumps (halos), final evaporation of inflaton clumps (halos) during reheating (scalaron decays in the case of $R^2$-inflation). The signal from the latter processes falls \[18\] right in the region to be probed at the advanced stage of DECIGO project \[16\] on gravity waves measurements. This allows for an independent test of the $R^2$-inflationary model.

**Allowed Higgs boson masses.** As far as in both models there are no new physics between electroweak and inflationary scales, one can expect bounds on the Higgs mass from the absence of strong coupling and stability of the electroweak vacuum.

The models have to be in the weak coupling regime for all the SM couplings up to the inflationary scale, which is determined by value of the Hubble parameter at the end of inflation, $H_e \sim 10^{13}\text{GeV}$. In particular, large self-coupling of the Higgs boson is forbidden, if the corresponding Landau pole is at a lower energy scale. This places an upper limit on the Higgs boson mass (proportional to the square root of the self-coupling) in both models at the level \[20–24\]

$$m_h \lesssim 194\text{GeV}.$$  

At present this bound is superseded by the direct searches at LHC, which give a stronger upper limit $m_h \lesssim 146\text{GeV} \[25\].

The lower limit on the Higgs self-coupling comes from the requirement of a sufficient stability of the electroweak vacuum in the SM. At a small self-coupling (corresponding to $m_h \lesssim 129\text{GeV}$, see \[20–24\]) the vacuum becomes unstable at large energy scales, mostly due to the top-quark corrections. The true vacuum in this case appears at a very large value of the Higgs field. The absolute stability of the electroweak vacuum is not required, because the decay rate of our vacuum (at zero temperature) is very small, and it can survive for the time equal to the age of the Universe for any $m_h \gtrsim 111\text{GeV} \[26\]$, which is below the LEP bound.

However, in the early Universe (right after inflation, at the early matter dominated stage, or later at the hot stage) the Higgs field may be caught in this vacuum. As far as this is not the case for the observed Universe, this implies a lower limit on the Higgs self-coupling and hence a lower limit on the Higgs boson mass.
For the case of the $R^2$-inflation the Higgs field does not evolve to the large field minimum during inflation, if the inflation started with a reasonable small value of the Higgs field. At the same time, the metastable vacuum can decay at high temperature right after preheating. The corresponding bound for the reheating temperature (3) can be obtained from Ref. [26],

$$m_h^{R^2} > \left[ 116.5 + \frac{m_t - 172.9 \text{GeV}}{1.1} \times 2.6 - \frac{\alpha_s(M_Z) - 0.1181}{0.0007} \times 0.5 \right] \text{GeV}. $$

The limit depends on the top-quark mass $m_t$, strong coupling constant $\alpha_s$, and also has systematic uncertainty of about 3 GeV from higher loop corrections. Note, that for variations of the $R^2$-inflation with lower reheating temperature the viable interval of the Higgs boson mass may be even wider and almost overlap with the direct lower bound from LEP2, $m_h > 114.4$ GeV [27].

For the Higgs-inflation the lower limit from the thermal decay with reheating temperature (2) is about $m_h > 120$ GeV. But the bound in this case is even stronger, because at the end of inflation the model directly ends up with the large Higgs field value (of the order at least $10^{13}$ GeV). So, if the SM potential has a minimum at that scale the evolution stops there even before reheating, leading to the bound [20] (see also [28–31])

$$m_h^{H} > \left[ 129.0 + \frac{m_t - 172.9 \text{GeV}}{1.1} \times 2.1 - \frac{\alpha_s(M_Z) - 0.1181}{0.0007} \times 0.5 \right] \text{GeV}, $$

also with a systematic error of about 2 GeV, related to higher loop corrections. Note, that this analysis depends on the assumptions about the UV completion of the Higgs inflationary model [32], specifically on the value of the non-minimal coupling $\xi$ after inflation and the shape of the potential during reheating.

**Conclusions.** The models of Higgs-inflation and $R^2$-inflation, though providing identical inflationary potential and thus exhibiting identical dynamics at the inflationary stage, can still be distinguished experimentally by cosmological observations. The key point is that both models, having no additional beyond the SM dynamics at high energies except for the inflationary one, provide the full description of the Universe evolution. Specifically, they both allow to study the reheating process, and have different reheating temperatures. This allows to determine exactly the predictions for the CMB parameters in the models and gives rise to possible distinguishing signals in future gravitational wave experiments. Different reheating mechanisms also lead to a much more constrained region of allowed Higgs masses for the Higgs-inflation, which can be very soon fully explored at LHC.

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