Topological defects and conditions for baryogenesis in the Left-Right symmetric model

U. A. Yajnik\textsuperscript{a*}, Hatem Widyan\textsuperscript{b†}, Shobhit Mahajan\textsuperscript{b‡}, Amitabha Mukherjee\textsuperscript{b§} and Debajyoti Choudhury\textsuperscript{c¶}

\textsuperscript{a}Physics Department, Indian Institute of Technology Bombay, Mumbai 400 076, India
\textsuperscript{b}Department of Physics and Astrophysics, University of Delhi, Delhi 110 007, India
\textsuperscript{c}Mehta Research Institute of Mathematics and Mathematical Physics, Chhatnag Road, Jhusi, Allahabad 221 506, India

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Abstract

It is shown that the minimal Left-Right symmetric model admits cosmic string, domain wall and conditionally, monopole solutions. The strings arise when the $SU(2)_R$ is broken and can either be destabilized at the electroweak scale or remain stable through the subsequent breakdown to $U(1)_{EM}$. The monopoles and domain wall configurations exist in the $SU(2)_L \otimes U(1)_Y$ symmetric phase and disappear after subsequent symmetry breaking. Their destabilization provides new sources of non-equilibrium effects below the electroweak scale. Several defect-mediated mechanisms for low energy baryogenesis are shown to be realisable in this model.

1 Introduction

Spontaneously broken gauge theories typically possess topological solutions\cite{1}. The presence of such objects in the early universe has been shown to be natural\cite{2, 3}. Some of these objects, e.g. cosmic strings, have been investigated for their role in structure formation and baryogenesis \cite{4, 5, 6, 7, 8}. On the other hand, monopole and domain wall solutions \cite{9, 10, 11, 12} have undesirable cosmological consequences, barring a few exceptional circumstances. The requirement of their absence puts stringent limits on the theory.

Currently, several unification schemes are being investigated in detail, specially for their signatures in the planned particle accelerators. Some of the unification schemes have interesting consequences for cosmology. A rich variety of cosmic string solutions was demonstrated\cite{11, 12}.
in the context of $SO(10)$ unification and has received fresh attention\[13\]. Furthermore, as the non-viability of several models for electroweak baryogenesis is becoming apparent\[14, 15\], it is interesting to search for new mechanisms for low energy baryogenesis in other unified models\[8, 16\].

In this paper we investigate the minimal Left-Right symmetric model for the presence of topological solutions. In section II we discuss the topology of the vacuum sector of the theory. Two possibilities for the same are distinguished depending upon the nature of the Higgs potential. In section III we show that one possibility leads to stable strings and no monopoles. The other possibility leads to monopoles and metastable cosmic strings. The monopoles disappear after electroweak symmetry breaking. The fate of the string depends on several factors, but at least some are shown to survive to the present epoch. We also show the existence of zero modes for neutrinos. In section IV we discuss a domain wall solution, also unstable below the electroweak scale. All these objects can play an important role in cosmology, which is discussed in the concluding section.

2 Topological considerations

The Left-Right symmetric unification group $SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$ possesses a $U(1)$ whose gauge charge turns out, in a natural and compelling way, to be the $B-L$ number of the observed fermions. We use the conventions of Mohapatra\[17\], except that the $U(1)$ charge assignments are the value of $X = (1/2)(B-L)$. We begin with the phase in which only the first stage of symmetry breaking $SU(2)_R \otimes U(1)_{B-L} \rightarrow U(1)_Y$ has occurred. The field signalling this breakdown is the $(1,0,1)$ field $\Delta_R$ which acquires the vacuum expectation value (vev) with the $(2,1)$ entry of the matrix being the only non-trivial component, $\langle \Delta_R \rangle_{21} = v_R$. This choice leads to the $U(1)$ generated by $Y = T^3_R + X$ left unbroken. The hypercharge $Y$ is unbroken also by all other matrices of this form, i.e., $\langle \Delta_R \rangle_{21} = v_R \exp{i\theta}$ with $0 \leq \theta < 2\pi$ for a given $v_R$. These therefore constitute the vacuum manifold.

If the potential of the $\Delta_R$ field is such as to allow more general matrices to be possible vevs, $Y$ should turn out to be broken, replaced perhaps by some other generator of the parent group. This can be tolerated before electroweak symmetry breaking, but conflicts with phenomenology at lower energies since electric charge $Q = T^3_L + Y$. The complete potential involving all the Higgs fields of the theory will therefore be assumed to tolerate $Q$ preserving vevs only, but the part involving only the $\Delta_R$, and including temperature dependent corrections may either break or preserve $Y$. We show in the next section that these two possibilities lead to separate interesting results.

Next, we note that in the parent group, each factor $SU(2)$ and $U(1)$ is multiply connected for the purpose of present considerations. The factor $U(1)_{B-L}$ is a compact $U(1)$. This clear because the $X$ charge of the $\Delta_R$ is integer. But more fundamentally, this is because all known and proposed particles carry integer multiples of the basic unit $1/6$ of this charge, carried by the quarks. This factor is therefore not simply connected. Secondly, acting on the $\Delta_R$ field, the $SU(2)$ is effectively an $SO(3)$. This is because the $\Delta_R$ is a 3-dimensional vector, albeit with complex components. $SO(3)$ too is not simply connected. The existence or otherwise of topological objects therefore depends on new nontrivial closed curves or discrete symmetries appearing due to the form of the vev.
3 Cosmic strings and monopoles

Consider first the case where $Y$ is preserved throughout. This makes the manifold of inequivalent vacua isomorphic to $S^1$, a circle. A cosmic string ansatz can be constructed by selecting a map $U^\infty$ from the circle $S^\infty$ at infinity into some broken $U(1)$ subgroup of the original group, such that action of this $U(1)$ makes the vev traverse the complete manifold of inequivalent vacua. Such a $U(1)$ is generated by $\tilde{Y} = T^3_R - X$. Furthermore, we select the internal parameter to be one-half times the spatial cylindrical angle $\theta$. Thus,

$$U^\infty(\theta) = \exp\{\frac{\theta}{2}(T^3_R - X)\}$$

(1)

The $SU(2)$ acts on $\Delta_R$ by similarity transformation, so $\langle\Delta_R(\infty, \theta)\rangle_{21} = e^{-i\theta}v_R$. The vev therefore traces the whole $S^1$; however,

$$U^\infty(2\pi) = e^{-i\pi} \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix} \neq U^\infty(0).$$

(2)

Thus we have identified a $Z_2$ which leaves the vev invariant but not the general matrix $\Delta_R$. The assumption regarding the form of the potential has reduced the problem to that of $U(1) \otimes U(1)_B \rightarrow U(1)_Y$ breaking. This example is easily analysed to show the existence of topological strings. The ansatze for the gauge fields are derived from the $U^\infty(\theta)$ and the required asymptotic forms for $r \rightarrow \infty$ are found to be

$$W_R = \frac{T^3_R}{4rg_R} \quad \text{and} \quad B_\theta = \frac{X}{2rg'},$$

(3)

where $g_R$ and $g'$ are the gauge couplings.

Consider next the possibility that the vacuum manifold is 2-dimensional. The $Z_2$ identified above makes the stability group to be $\exp\{i\theta(T^3_R + X)\}$, with $0 \leq \theta < \pi$. The restriction of the $U(1)$ parameter to half its natural range means a new unshrinkable curve and the kernel of the natural homomorphism $[1]$ of $\Pi^1(H)$ to $\Pi^1(G)$ becoming non-trivial. Thus monopoles become possible. Monopole-antimonopole pairs can also form, connected by string configurations previously identified. The string configurations on the other hand lose their topological stability because the vacuum manifold becomes isomorphic to $S^2$.

The configurations identified above do not survive the subsequent phase transition. For analysing low energy vacuum structure, we must assume only $Q$ preserving vacua. Whereas monopoles cannot exist, cosmic strings can be shown to exist by generalising the analysis of the first case above. The low energy vevs of the $(1, 0, 1)$ field $\Delta_L$ and the $(\frac{1}{2}, \frac{1}{2}, 0)$ field $\phi$ are, respectively, $\langle\Delta_L\rangle_{21} = v_L$ and $\text{diag}(\kappa, -\kappa)$ which are not invariant under the action of $U^\infty(2\pi)$. However, one may think of the above curve $U^\infty(\theta)$ as a projection to the subspace $SU(2)_R \otimes U(1)_{B-L}$ of the more general curve $\bar{U}^\infty(\theta) = \exp\{i(T^3_R + T^3_L - X)\theta/2\}$. This leaves $\Delta_R(\infty, \theta)$ to be as above and leaves the $\phi$ vev invariant, but makes $\langle\Delta_L(\infty, \theta)\rangle_{21} = e^{i\theta}v_L$. The argument for stability remains the same as before.

If such cosmic strings form, they should exist as relics at the present epoch. At the electroweak phase transition, if the vevs of $\Delta_L$ and $\phi$ in the domains around an existing vortex are not too different from each other, they will destabilize the vortex. If the new vevs wind nontrivially in the internal space while traversing a closed physical path around the existing vortex, then a stable string forms.

It may be noted that the stable strings necessarily contain $SU(2)_R$ charged condensates. Therefore two possible scenarios need to be considered while estimating relic string density at
any epoch. If the vev structure is $Q$ and $Y$ preserving throughout, no new strings arise at the electroweak phase transition since the latter generically does not release latent heat sufficient to excite $SU(2)_R$ charged field condensates. On the other hand if the potential is such as to allow monopoles above the electroweak scale, the corresponding strings are at best metastable. The strings then form only as connecting monopole-antimonopole pairs and continue to contract and disappear. However, those monopoles not attached to strings must disintegrate at the electroweak phase transition. This may release large amounts of latent heat, creating new string-like defects, which can become stable according to the scenario of the previous paragraph.

The cosmic strings also carry fermion zero modes. The $Y$ charge of all the fermions is $\leq 1$. Hence the gauge field of the minimal vortex will not bind any fermions in a zero-energy mode. The Yukawa coupling of the $\nu_R$ and $\nu_L$ for each flavour contains terms $\psi_R^0 C^{-1} \tau_2 \Delta_R \nu_R$ and $\psi_L^0 C^{-1} \tau_2 \Delta_L \psi_L$. Substituting the ansatz for the Higgs fields, we see that above the electroweak scale, the $\Delta_R$ term undergoes a $2\pi$ phase change around the minimal vortex, giving rise to solitary [19] zero mode. For the strings surviving the electroweak breaking transition, additionally, $\nu_L$ possesses a zero mode for the same reason. More zero modes due to both gauge and Higgs coupling are indeed possible for vortices with higher winding number.

### 4 Domain wall

At tree level the Lagrangian is symmetric under the exchange $\Delta_L \leftrightarrow \Delta_R$, reflecting the hypothesis of $L - R$ symmetry. If the vacuum values for these two Higgs fields are assumed to be as in the previous section, it can be shown [17] that their potential assumes the form

$$V(v_L, v_R) = -\mu^2 (v_L^2 + v_R^2) + (\rho_1 + \rho_2)(v_L^4 + v_R^4) + \rho_3 v_L^2 v_R^2,$$

where the parameters are inherited from the original form of the potential [17]. Upon parametrizing $v_R = v \cos \alpha$ and $v_L = v \sin \alpha$, the points $(v, \alpha) = (v_0, 0)$ and $(v_0, \pi/2)$ with $v_0 = \sqrt{\mu^2 / 2(\rho_1 + \rho_2)}$ are the minima, and $(\sqrt{2\mu^2 / (\rho_3 + 2(\rho_1 + \rho_2))}, \pi/4)$ a saddle point, provided $\rho_3 > 2(\rho_1 + \rho_2) > 0$. Electroweak phenomenology dictates that the latter condition be valid.

It is reasonable to assume that the effective potential continues to enjoy the above discrete symmetry, since the same loop corrections enter for both the fields. This means the symmetry is broken spontaneously at the $L - R$ breaking scale, providing requisite topological condition for the existence of domain walls. As the universe cools from the $L - R$ symmetric phase, there should be causally disconnected regions that select either $\alpha = 0$ or $\alpha = \pi/2$. Thus the vev’s are functions of position and the two kinds of regions are separated by domain walls.

The equations governing the wall configuration can be obtained from minimization of the energy. The existence of the required solution can be shown by an extension to two variables of the standard arguments for one-variable solitonic solutions [1]. There exists a first integral of the motion, viz., $h \equiv (1/2)(v^2 + v^2 \alpha^2) + (-V)$ and an analysis of equivalent problem of a point particle in an inverted potential can be carried out. Imposing the requirements of finiteness of total energy ensures that a solution exists. In particular, for $\rho_3 = 6(\rho_1 + \rho_2)$, one finds an exact solution $v_L(x) = (v_0 / \sqrt{2})[1 - \tanh(\mu x)]; v_R(x) = (v_0 / \sqrt{2})[1 + \tanh(\mu x)]$. This establishes the solution at least in a neighbourhood of this set of values of the parameters.

At the electroweak scale, the effective potential does not respect $L-R$ symmetry due to the nature of the $\phi$ self coupling. One finds that $v_L v_R \sim \kappa^2$ where the symbols have been introduced in sec. III. Upon choosing $\kappa \sim v_{EW}$ with $v_{EW}$ denoting the electroweak scale, $v_L$ is driven to
be tiny. The $Z_2$ guaranteeing the topological stability of the walls now disappears. Energy minimization requires that the walls disintegrate.

There is a possibility that the L-R symmetry is not exact due to effects of a higher unification scale, in which case, the R breaking minimum should be energetically preferred by small amounts before the electroweak phase transition. This will cause the domain walls to move around till the regions with L breaking minimum have been converted to the true vacuum. Some fractin of the walls would then disappear before the electroweak scale is reached. The fate of the surviving walls is the same as that discussed in the previous paragraph. Further consequences are discussed in the next section.

5 Consequences for cosmology

From the point of view of a predictable baryogenesis, L-R model enjoys the advantage that the primordial value of the $B - L$ number is naturally zero, being the value of an abelian gauge charge. The topological defects studied here can play a significant role in baryogenesis through leptogenesis. It has been shown in [20] and [21], using mechanisms for electroweak baryogenesis that do not rely on topological defects, that the parameters in the potential require unnatural fine tuning to provide sufficient $CP$ violation to explain the observed asymmetry within the context of the minimal model considered here. The cure suggested is a singlet pseudoscalar $\sigma$ of mass either $v_R$ [21] or $\sim v_{EW}$ [20].

Defect mediated leptogenesis mechanisms also need this enhanced $CP$ violation, with one exception to be noted below. For the present purpose we note that the $\sigma$ field does not alter the topological considerations presented above since it is a gauge singlet and its main function is to bias the potential of the $\phi$ which does not enter the topological defects in a significant way. The coupling of $\sigma$ to both $\Delta_R$ and $\Delta_L$ may be assumed to be identical due to Left-Right symmetry.

Then the domain walls present very interesting prospects. Their interaction with other particles in the pre-electroweak scale plasma can result in leptogenesis. A model independent possibility of this kind was considered in [8]. More specific considerations also appear in [22] and [23]. It is likely that the model is descended from a grand unified theory. For this or for some other reason there may be a small asymmetry between the L-prefering and R-prefering minima even above the electroweak scale. If the energy density difference is suppressed by powers of the GUT mass, the walls are still expected to be present long enough to bring about requisite leptogenesis.

The case of exact Left-Right symmetry leads to domain walls that are stable before the electroweak symmetry breaking. In this case the regions trapped in $\langle \Delta \rangle_L \sim v_R$ vacuum will become suddenly destabilised as the $\phi$ acquires a vev. The destabilization can generate large amounts of entropy and the domains should reheat to some temperature $T_H$ greater than $v_{EW}$ but less than $v_R$. The possibility for baryogenesis from situations with large departure from thermal equilibrium was considered by Weinberg[24]. It was argued that in such situations the asymmetry generated should be determined by the ratio of time constants governing baryon number violation and entropy generation respectively. In the present case we expect leptogenesis from the degeneration of $\langle \Delta_L \rangle$ due to the Majorana-like Yukawa coupling mentioned in sec. III. Since the mechanism operates far from equilibrium, $CP$ violation parameter may not enter in a significant way. The generated lepton asymmetry can then convert to baryon asymmetry through the electroweak anomaly. This possibility will be studied separately.

The cosmic strings demonstrated above can play several nontrivial roles in the early universe. They can provide sites for electroweak baryogenesis as proposed in [3]. It has also been proposed
that he fermion zero modes they possess can result in leptogenesis \[16\]. Equally interesting is the process of disintegration of the unstable strings below the electroweak scale. The decay should proceed by appearance of gaps in the string length with formation of monopoles at the ends of the resulting segments. The free segments then shrink, realising the scenario of \[6\]. Monopoles may also catalyse lepton number creation but their contribution should be much smaller than that of strings or domain walls.

The Left-Right symmetric model considered here provides a concrete setting for all of the above scenarios. Several new features that have been demonstrated can alter the scenarios qualitatively and merit further study.

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