The influence of negative-energy states on proton-proton bremsstrahlung

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We investigate the effect of negative-energy states on proton–proton bremsstrahlung using a manifestly covariant amplitude based on a T-matrix constructed in a spectator model. We show that there is a large cancellation among the zeroth-order, single- and double-scattering diagrams involving negative-energy nucleonic currents. We thus conclude that it is essential to include all these diagrams when studying effects of negative-energy states.

I. INTRODUCTION

Recent efforts in the study of the proton-proton bremsstrahlung ($pp\gamma$) reaction have been directed towards investigating the various reaction mechanisms of this process at and beyond the pion production threshold energy $\pi^0$. Among various higher-order (in the photon momentum) processes studied so far, the most important contribution turns out to come from $\Delta$-decay diagrams (containing an $N\Delta\gamma$ vertex). Even at pion-threshold this contribution can enhance the cross-section up to 30% $\pi^0$. Another higher-order process stems from vector-meson-decay vertices. Of these, the $\omega$-decay into a pion and a photon is the most important one. These contributions have an effect opposite to that of

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\(\Delta\)-decay diagrams, i.e., they reduce the \(pp\gamma\) cross section. However, their strength is only about one third of the \(\Delta\)-decay diagrams \[5\]. An important aspect of the \(\Delta\)- and meson-decay diagrams is that at pion-threshold their effect is due to interference with the dominant positive-energy nucleonic current (NN\(\gamma\) vertex) contribution. The absolute value of these diagrams is small. The absolute value of the \(\Delta\)-decay diagrams only becomes comparable to the nucleonic contribution at higher energies. In addition to these higher-order processes, the question of pseudo-scalar/pseudo-vector (ps/pv) mixing of the \(\pi\text{NN}\) coupling has been addressed in connection with the \(pp\gamma\) reaction \[6,7\].

In this work, which is an effort \[1,4,5\] to better understand the role of various reaction mechanisms contributing to the \(pp\gamma\) process, we study the effects of nucleon negative-energy states. The investigation of negative energy states in \(pp\gamma\) reactions has received limited attention \[8,7\] because its contribution is also of higher-order in the photon momentum. In addition, since the most popular realistic NN interactions do not contain negative-energy states dynamically, it is difficult to add this contribution to \(pp\gamma\) calculations based on these interactions in a consistent way, specially, satisfying charge conservation. We note that beyond the soft-photon approximation the negative energy contribution is essential for obtaining a fully gauge invariant \(pp\gamma\) amplitude. An effort to achieve this consistency is reported in the work by M"undel \[8\], who uses the Bonn potential to explore the role of nucleon negative-energy states. This contribution is taken into account through an effective two-body current in a \(p/m\) expansion and considering the \(pp\gamma\) amplitude only in the zeroth-order scattering. M"undel finds that this can interfere effectively with the positive-energy nucleonic contribution, leading to about 10\% effects on the cross section. Eden and Gari \[7\] use the Ruhrpot potential in their \(pp\gamma\) calculations. The authors \[7\] report a relatively large contribution from negative-energy states, which they can control by tuning the ps/pv ratio of the \(\pi\text{NN}\) coupling. However, although they do not use a \(p/m\) expansion of the effective two-body current, they also include this contribution only in the zeroth-order scattering. In doing so, contributions from the single- and double-scattering (rescattering) diagrams are ignored. As we will point out later, when these diagrams are added to the zeroth-order
scattering diagram, the total contribution to the cross section from nucleon negative-energy states practically vanishes due to a large cancellation among these diagrams.

Apart from the Ruhrpot T-matrix, we are aware of two NN T-matrices that include nucleon negative-energy states [9,10]. In the present work we will use the T-matrix of Gross et al [10]. First we will discuss shortly this model for the NN interaction; secondly, we will discuss how we use this T-matrix in our model for the \( pp\gamma \) reaction and, finally, we will present results and compare them with other calculations and with the Triumf data of Ref. [11]. It should be mentioned that, in contrast to our previous work [1,4,5], we do not consider the contribution of the \( \Delta \)-decay diagrams in this work, since there are no T-matrices available that include negative-energy states and \( \Delta \) degrees of freedom. This is beyond the scope of the present work. Moreover, we believe that the role of nucleon negative energy states in \( pp\gamma \) reactions can be explored ignoring the \( \Delta \)-decay diagrams, at least in this initial study. The contribution of vector-meson-decay diagrams is also ignored in the present work.

II. THEORETICAL FRAMEWORK

The formalism we use to describe the hadronic system leading to our NN T-matrix is described in detail in Ref. [10]. Starting from a field-theoretic formulation one employs a three-dimensional reduction of the Bethe-Salpeter equation that restricts one of the interacting particles to be on its mass-shell (the other particle is allowed to be off-shell) and finds a manifestly covariant expression for the T-matrix which is symmetrized with respect to the on- and off-shell particles. The resulting equation is also known as the spectator equation. The propagator of the off-shell particle is retained in full, i.e., both the negative- and positive-energy parts of the propagator are kept. Gross et al. [10] present four different models, all giving a good fit to the NN data below pion threshold with a \( \chi^2 \) that is comparable with those of the Paris and the Bonn potentials. In this work we use the IA model whose underlying interaction has only four mesons (\( \pi, \rho, \sigma, \omega \)) and a very limited number of free parameters. Apart from the coupling constants and the \( \sigma \)-mass, there are two cut-off
parameters and a parameter describing the ps/pv ratio of the $\pi$NN coupling. The specific T-matrix we use is calculated *independently* from Ref. [10]. It is expressed in a plane-wave basis and reproduces the phase-shifts as reported in Ref. [10].

In the model we use for the hadronic interaction, phenomenological formfactors are introduced on the meson-NN vertices. The use of phenomenological formfactors poses immediately a problem of introducing the electromagnetic interaction without violating current conservation. Gross and Riska [12] presented a method describing how this can be done; although it does not yield unique electromagnetic couplings to hadrons (this is impossible with phenomenological formfactors where the underlying structures are unknown), the method is quite general. We, therefore, follow their approach: interpret the formfactors as self-energies in the hadron propagators and demand that the electromagnetic vertices obey the corresponding Ward-Takahashi identities. To achieve this, the hadronic formfactors are chosen to be separable, giving a meson-NN vertex

$$\Gamma_{\text{NNmeson}} = h(p^2)h(p'^2)f(q^2)\Gamma_{\text{NNmeson}}^R, \quad (1)$$

with $p, p'$ the momenta of the incoming and outgoing nucleon, respectively, and $q = p' - p$, the momentum of the meson. $h(p^2)$ is the nucleonic formfactor (nucleonic in the sense that it depends only on the nucleon momentum $p$), $f(q^2)$, the mesonic formfactor and $\Gamma_{\text{NNmeson}}^R$, the reduced meson-NN vertex. To retain the unit-residue at the poles of the nucleon and meson propagators one has the restriction on the formfactors, $h(p^2 = m_N^2) = 1$ and $f(p^2 = m_{\text{meson}}^2) = 1$. In addition, both $h(p^2)$ and $f(q^2)$ should decrease at least like a power of their arguments as they approach infinity and have no zeros. In $pp\gamma$ reactions we do not have meson-exchange currents, and thus, the mesonic formfactor $f(q^2)$ does not enter in the discussion of gauge invariance. Recall that we do not consider vector-meson-decay contributions in this work. The nucleonic formfactor leads to the Ward-Takahashi identity for the $pp\gamma$ vertex

$$k_\mu \Gamma_R^\mu(p', p) = e \left( S^{-1}(p') - S^{-1}(p) \right), \quad (2)$$
where $k = p - p'$ denotes the photon momentum, $\Gamma_{R}^{\mu}$, the reduced $pp\gamma$ vertex and $e$, the proton charge. $S(p)$ is the modified nucleon propagator which includes the formfactor as a self-energy. It is expressed in terms of the Feynman propagator $S_{F}(p) = 1/(\not{p} - m_{N})$ as $S(p) = h^{2}(p^{2})S_{F}(p)$.

Now, starting from the most general NN$\gamma$ vertex [13], we can uniquely construct the longitudinal part of the $pp\gamma$ vertex for a real photon which satisfies Eq. (2),

$$\Gamma_{R}^{\mu}(p', p) =$$
$$\frac{-ie}{p'^{2} - p^{2}} \left[ \gamma^{\mu} \left( \frac{p'^{2}}{h^{2}(p'^{2})} - \frac{p^{2}}{h^{2}(p^{2})} \right) + (\not{p}'\gamma^{\mu}\not{p}' + m_{N}\not{p}'\gamma^{\mu} + m_{N}\gamma^{\mu}\not{p}') \left( \frac{1}{h^{2}(p'^{2})} - \frac{1}{h^{2}(p^{2})} \right) \right].$$

(3)

This vertex reduces to the conventional one, $-ie\gamma^{\mu}$, when $h(p^{2}) = 1$. Also, note that the vertex is finite when $p' = p$. The Ward-Takahashi identity does not provide a constraint on the transverse part of the vertex, and in particular, the effect of the meson-NN formfactors on the magnetic part of the $pp\gamma$ vertex. To remove this ambiguity one needs to calculate the formfactors in a microscopic model, as is done e.g. in Ref. [14]. Therefore, we retained the conventional magnetic vertex,

$$\Gamma_{mag}^{\mu}(p', p) = -e\kappa \frac{\sigma^{\mu\nu}k_{\nu}}{2m_{N}},$$

(4)

with $\kappa = 1.79$, the anomalous magnetic moment of the proton. Note in the above equation that the electromagnetic formfactor as discussed, e.g., in Ref. [15] is also set to its on-shell value. Our full $pp\gamma$ vertex is, therefore, the sum of the vertices given by Eqs. (3) and (4),

$$\Gamma^{\mu} = \Gamma_{R}^{\mu} + \Gamma_{mag}^{\mu},$$

(5)

and satisfies, by construction, the charge conservation requirement imposed by the particular NN interaction used.

The pp bremsstrahlung amplitude is obtained by sandwiching the vertex given by Eq. (5) with the two-nucleon wave functions calculated within the spectator model. These wave functions are expressed in terms of the T-matrix described in the beginning of this section. Note that, in this way, the electromagnetic interaction is taken into account to first-order
in the coupling, while the strong interaction is taken to all orders. The resulting amplitude is shown diagrammatically in Fig. 1 and is the sum of the single-scattering diagrams, (a) and (b), and the rescattering diagram, (c). The cross in Fig. (1) indicates where the particle is restricted to be on its mass-shell; the ± denotes intermediate states which are off-shell and have both positive- and negative-energy components. The T-matrix is *symmetrized* with respect to the on- and off-shell particles, leading to an amplitude that fulfills the Pauli-principle. Note that our single-scattering diagrams, Figs. 1a,b, incorporate the zeroth-order scattering diagrams mentioned in the introduction (Figs. 1a,b with the T-matrix replaced by the potential).

Now, in order to obtain the complete \( pp\gamma \) amplitude within the spectator model, one has to add the rescattering diagrams Figs. 1d,e to diagrams Figs. 1a-c. This is peculiar to this type of model in which one of the interacting particles is restricted to its mass-shell. Since photons cannot couple to an on-shell particle, these diagrams are necessary in order to account for the complete \( pp\gamma \) amplitude \(^{12}\). They are also needed to ensure gauge invariance. It should be stressed that the complete amplitude obtained in this way is manifestly covariant.

Unfortunately, the diagrams Figs. 1d,e are very difficult to calculate. A close examination, however, reveals that the sum of them can be well approximated by the diagram Fig. 1c. We, therefore, calculate the diagram Fig. 1c and multiply it by a factor of 2 in order to account for the total rescattering diagrams in the present work, i.e., \((c)+(e)+(d) \approx 2 \times (c)\). Of course, this approximation destroys 'exact' gauge invariance of the complete amplitude. However, the violation is minor for we find the contraction of our approximated complete amplitude with the photon momentum, \( k_\mu M \mu \), reasonably close to zero. This means that we have a reasonable numerical fulfillment of gauge invariance. A positive point in the use of a spectator-type three-dimensional reduction is that it allows us to treat the energy variable in all diagrams consistently. The point is that it is kinematically impossible to treat the interacting two nucleons symmetrically when a photon couples to one of them. Therefore, it is difficult to use consistently in NN bremsstrahlung calculations those T-matrices obtained
using symmetric three-dimensional reductions, such as those based on the Blankenbecler-Sugar reduction. We feel that the correct treatment of this aspect is a nice feature of the spectator model.

III. RESULTS

With the model described above we performed calculations for a selected set of kinematics from the coplanar TRIUMF experiment [11]. The results are shown in Fig. 2. The dotted lines denote the full results, including all diagrams with both positive- and negative-energy state contributions; the long-dashed lines are calculated by restricting the $pp\gamma$ vertex to positive-energy states only (positive-energy nucleonic current). Of course, in this case, negative-energy states are still present in the intermediate states of the T-matrix. For comparison, we also show the results based on the Bonn OBEPQ potential [16] with only the positive-energy nucleonic contributions (solid lines). Comparing these results (long-dashed and solid) we observe that they are similar to each other and fall into the trend observed in earlier works, i.e., T-matrices that fit the phase-shifts give similar results for the $pp\gamma$ observables. We now find the same even when the T-matrix includes negative-energy particles in its intermediate states. Also, recall that due to different meson-NN formfactors we have to use different $pp\gamma$ vertices for the spectator model and Bonn T-matrices. Our results indicate that these formfactors have little influence on the observables when the photon couples to a positive-energy nucleon, although this is not the case when the photon couples to a negative energy nucleon as will be discussed later. The most remarkable finding, however, appears when we compare the results with and without the negative-energy nucleonic current (dotted and long-dashed curves, respectively) for the cross section. As can be seen, the negative-energy nucleonic current shows practically no influence on this observable. This is not due to the individual diagrams in Fig. [1] being small; on the contrary, they are large. But the sum of all the negative-energy diagrams has no effect on the cross section.

To detail the last point further, we show in Fig. 3 the results when only the single-
scattering diagrams are considered with (dashed lines) and without (solid lines) the negative-energy nucleonic current. As can be seen, the effect of the negative-energy state on the cross-section from the single-scattering diagrams is large. A closer examination of the matrix elements shows that this large contribution stems from very large $M_{\mu=1}$ and $M_{\mu=2}$ matrix elements of the $pp\gamma$ amplitude in the $++ \rightarrow +-$ spin transition channel. This contribution stems from the part of the $pp\gamma$ vertex given by Eq. (3). However, in this channel the sum of all the negative-energy single-scattering contributions is almost exactly cancelled by the sum of the negative-energy rescattering diagrams, leading to the observed nearly null-result for the cross-section in Fig. 2. Unfortunately, up to now we have not been able to find a specific explanation as to why this cancellation should occur, nor why these particular spin transition matrix elements should be large. Gauge invariance appears to be important here, for the cancellation is much less perfect when we use the conventional vertex ($-ie\gamma^\mu$) in conjunction with the Gross T-matrix instead of the modified one given by Eq. (3) as required by gauge-invariance. Note that we also constructed a Bonn T-matrix with an outgoing negative energy state. Using this T-matrix we observed exactly the same feature as mentioned above. This indicates that the observed cancellation may be a general feature of calculations based on potential models.

In contrast to the cross section, the analyzing power is much more affected by the negative-energy contribution as is shown in Fig. 2. Already the result with only the positive-energy nucleonic current (long-dashed line) shows some deviation from the corresponding Bonn result (solid line). This deviation takes the result away from the data. However, this is not too discomforting since other effects, like including the $\Delta$-isobar, tend to push the analyzing power towards the data [1,2,3,4]. Including the negative-energy nucleonic current (dotted curve) the minimum in the analyzing power at intermediate photon angles fills up almost completely. With this result it is difficult to imagine an effect that would push the curve back onto the data. The effect of the negative-energy nucleonic current on the analyzing power when only the single-scattering diagrams are considered is also illustrated in Fig. 3. Again, its effect is large and comparing with the result in Fig. 2 we see that there
is a large cancellation among the diagrams involving negative energy states. In view of this, it is possible that a more refined calculation will give better results for the analyzing power, since in the present calculation we 'lost' exact gauge invariance due to the approximation made for evaluating the \( pp\gamma \) amplitude, and that, we might miss higher-order cancellations among the negative-energy contributions. This has to be investigated in the future.

At this point, we note that the above finding forces us to revise the claim made in Refs. [6,7] that one can determine the ps/pv content of the \( \pi NN \) vertex by means of the negative-energy contributions to \( pp\gamma \) observables. They take into account the negative-energy nucleonic current contributions only in the zeroth-order scattering and, thus, miss the cancellation observed in the present calculation. Of course, this does not necessarily imply that the \( pp\gamma \) reaction is not sensitive to the ps/pv mixing. It simply means that one should be careful in drawing conclusions based on calculations that include only the zeroth-order scattering diagrams.

**IV. SUMMARY AND CONCLUSIONS**

We investigated the role of nucleon negative-energy states on the \( pp\gamma \) process. We used a model in which the negative energy states are treated dynamically in the hadronic interaction and the electromagnetic interaction is introduced consistently with the hadronic interaction so that the current conservation is satisfied. The photon is allowed to couple to both positive and negative energy nucleons. For cross sections we found almost no effect of the negative energy states. We showed that this is not due to the separate diagrams being small, but caused by an almost perfect cancellation among all diagrams involving negative-energy nucleonic currents. Therefore, we concluded that it is crucial to include all negative-energy diagrams in the calculation. The results for the analyzing power are less encouraging when compared to the TRIUMF data [1]. However, we believe that there are possible refinements in the model which might improve upon this. If the effect of the negative energy states on the analyzing power found in this work is genuine, i.e., not caused by our approximate
treatment of gauge invariance due to numerical difficulties, reproducing the analyzing power as measured in the TRIUMF data will be a severely restrictive test on potential models of pp-bremsstrahlung reactions which incorporate nucleon negative energy states. Certainly, there is much to be done before this reaction mechanism is thoroughly understood.

Finally, we mention that investigation of the $p_s/p_v$ ratio of the $\pi NN$ coupling using the $pp\gamma$ reaction as suggested in Refs. [6,7] should be revised. The sensitivity of the $pp\gamma$ reaction to this ratio arises basically from the negative-energy state contribution. Therefore, as we have shown in this work, one has to be cautious with the conclusions based on calculations where the negative-energy nucleonic current is accounted for only in the zeroth-order scattering [6,7].

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FIGURES

FIG. 1. Diagramatic representation of the \( pp\gamma \) amplitude in the present model. (a) and (b) are the single-scattering diagrams; (c)-(e), the rescattering diagrams. The cross indicates an on-shell intermediate state, the ± is an off-shell intermediate state which has both positive and negative energy contributions. Diagrams (d) and (e) are peculiar to the present model.

FIG. 2. Results for the cross-section and analyzing power. The dotted line stands for the full calculation including all diagrams; the dashed line with only positive-energy diagrams. The solid line represents the Bonn OBEPQ results. The data are from Ref. [11]; the cross-sections were not multiplied by a factor 2/3; the analyzing power is multiplied by -1.

FIG. 3. Results for the cross-section and analyzing power. The solid line is the result with only positive-energy diagrams. The dashed line is the result when only the single-scattering negative energy diagrams are added. The dotted line is the full result which, for the cross-section, coincides with the solid line.
