Calculation of Value of $\pi$ by Splitting Technique (2013)

Fazal Rehman Abasyeen Khan* and Qadeem Khan
Kohat University of Science and Technology (K.P.K) Pakistan

Abstract
This paper proposes a geometrical technique to obtain the value of $\pi$ from splitting a circle in Equilateral triangles, a Square and Sectors. It is first ever study to calculate the value of $\pi$ by splitting a circle in these shapes and then by combining areas of these shapes value of $\pi$ can be calculated.

Introduction
The quest for $\pi$ has a long history [1]. The fact that the ratio of circle’s circumference to its diameter is a constant has long been known. The ratio first enters human consciousness in Egypt. The Egyptian Rhind Papyrus, dating back to 1650 B.C, provides the good evidence that $\pi$ equals $4(8/9)^2 = 3.16$. In the Bible, there are also records of the value of $\pi$. The biblical value of $\pi$ is 3. In ancient Greece, the first approximation of $\pi$ was obtained by Archimedes (287-212 B.C.), who showed that $223/7 < \pi < 22/7$. In ancient China, the calculation of $\pi$ was documented in the 1000 B.C. In Zhoung Bi Suan Jing, one of the oldest Chinese astronomical and mathematical treatises. It states that the circumference of a circle is about 3 times its diameter. Heng Zhang (78-139) approximated $\pi$ by $92/299 (\approx 3.174)$ and $\sqrt{10} (=3.1622)$. Was shown to be an irrational in 1767 by Lambert [6]. It was up to 4 billion decimal places and could be calculated. In1949, the US army obtained the value of $\pi$ up to 806 decimal places [8]. Since Zu, the calculation of $\pi$ was the first attempt made to obtain the value of $\pi$. The notation $\pi$ for the circumference-to-diameter ratio was first introduced in 1706 by William Jones (1675-1749) in his book ”A new introduction to Mathematics; and was made popular by Leonard Euler (1707-1783). The number has remained as $\pi$ ever since.

Steps of Construction
(I) Draw a circle of radius $R$ with centre $O$.
(ii) Draw another circle having radius $r = R/2$ with centre $O$.

Figure 1: Splitting of Circle in square, Equilateral Triangles and Sectors.
(iii) Make angles of 0º, 45º, 90º, 135º, 180º, 225º, 270º, and 315º inside given circle.

(iv) Joining end points of angles 0º and 180º.

(v) Joining end points of angles 45º and 225º.

(vi) Joining end points of angles 90º and 270º.

(vii) Joining end points of angles 135º and 315º.

(vii) It is required split circle.

Obtaining from Splitting A Circle

In this technique a given circle having radius R and Centre is O split into a square, four equilateral triangles and then by adding areas of these geometrical shapes inside given circle value of \( \pi \) can be calculated easily and efficiently.

Let

Radius of given circle=R

Measurement of each side of square=x units

Measurement of each side of equilateral triangle=x units

Measurement of radius of each sector=x units

Area of circle = 4(Area of Equilateral Triangle) + (Area of square) + 4(Area of sector)…………… (1)

\[ \pi R^2 = 4\left(\frac{\sqrt{3}}{4}\right)x^2 + 2x^2 + 4\left(\frac{\pi x^2}{4}\right) \]

\[ \pi R^2 = \left(\sqrt{3} + 1\right)x^2 \]

\[ R^2 = \frac{\left(\sqrt{3} + 1\right)x^2}{\pi} \]  

(2)

\[ R=IO+AI \]  

(3)

\[ IF=IO=\frac{x}{2} \]  

(4)

\[ AF=GH=EG=EF=x \]  

(5)

AIF is right angled triangle

AI=?  

(6)

By Applying Pythagoras theorem value of AI can be calculated as.

\[ (AF)^2 = (AI)^2 + (IF)^2 \]

\[ AI = \sqrt{(AF)^2 - (IF)^2} \]  

(7)

(8)

By putting value of AF and IF

\[ AI = \sqrt{x^2 - \left(\frac{x}{2}\right)^2} \]  

(9)

By putting value of IO and AI in (iii) we get

\[ R = \frac{x}{2} + \sqrt{x^2 - \left(\frac{x}{2}\right)^2} \]  

(10)

Now putting value of R in (ii)

\[ d = \frac{\left(\sqrt{3} + 1\right)x^2}{\sqrt{x^2 - \left(\frac{x}{2}\right)^2} - x^2} \]  

(11)

It is general formula of \( \pi \)

4. Solving general form new formula

Now By solving (XI) we get value of

\[ \pi = \frac{(1 + \sqrt{3})x^2}{x^2 - \frac{x^2}{4}} \]

\[ \pi = \frac{(1 + \sqrt{3})x^2}{x^2 - \frac{x^2}{4}} \]

\[ \pi = \frac{(1 + \sqrt{3})x^2}{\frac{\sqrt{3}}{2} x^2} \]

\[ \pi = \frac{(1 + \sqrt{3})x^2}{\frac{\sqrt{3}}{2} x^2} \]

\[ \pi = \frac{2(1 + \sqrt{3})}{\sqrt{3}} \]

\[ \pi = (1.154700538....) + 2 \]

= 3.154700538....

It is value of \( \pi \)

6. Calculating value of \( \pi \) while using 9 as arbitrarily number

Put x = 9 cm in (xi) we have

\[ \pi = \frac{(1 + \sqrt{3})x^2}{x^2 - \frac{x^2}{4}} \]

\[ \pi = \frac{(1 + \sqrt{3})x^2}{x^2 - \frac{x^2}{4}} \]

\[ \pi = (1.154700538....) + 2 \]

= 3.154700538....
\[\pi = \frac{(1 + \sqrt{3}) \times 81}{4.5 + \sqrt{81 - (4.5)^2} - 81}\]

\[\pi = \frac{(1 + \sqrt{3}) \times 81}{4.5 + \sqrt{81 - 20.25} - 81}\]

\[\pi = \frac{(1 + \sqrt{3}) \times 81}{4.5 + \sqrt{60.75} - 81}\]

\[\pi = \frac{(1 + \sqrt{3}) \times 81}{4.5 + 7.794228634 - 81}\]

\[\pi = \frac{(1 + \sqrt{3}) \times 81}{12.294228634 - 81}\]

\[\pi = \frac{(1 + \sqrt{3}) \times 81}{151.1480577 - 81}\]

\[\pi = \frac{221.2961154}{70.14805771}\]

\[\pi = 3.154700538,\ldots\]

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