Transmission through a short interacting wire

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Abstract

We investigate electron transmission through a short wire with electron-electron interactions which is coupled to noninteracting metallic leads by tunneling matrix elements. We identify two temperature regimes (a) $T_{\text{Kondo}} < T \leq T_{\text{wire}} = \hbar v_F/k_B d$ ($d$ is the length of the interacting wire) and (b) $T < T_{\text{Kondo}} \ll T_{\text{wire}}$. In the first regime the effective (renormalized) electron-electron interaction is smaller than the tunneling matrix element. In this situation the single particle spectrum of the wire is characterized by a multilevel “quantum dot” system with magnetic quantum number $S = 0$ which is higher in energy than the $SU(2)$ spin doublet $S = \pm 1/2$. In this regime the single particle energy is controlled by the length of the wire and the backward spin dependent interaction. The value of the conductance is dominated by the transmitting electrons which have an opposite spin polarization to the electrons in the short wire. Since the electrons in the short wire have equal probability for spin up and spin down we find $G = G_\uparrow + G_\downarrow$, $e^2/h \leq G < 2e^2/h$. In the second regime, when $T \to 0$ the effective (renormalized) electron-electron interaction is larger than the tunneling matrix element. This case is equivalent to a Kondo problem. We find for $T < T_{\text{Kondo}}$ the conductance is given by $G = 2e^2/h$. These results are in agreement with recent experiments where
for $T_{\text{Kondo}} < T < T_{\text{wire}}$ the conductance $G$ obeys $e^2/h \leq G < 2e^2/h$, and for $T < T_{\text{Kondo}}$, $G = 2e^2/h$. In both regimes the current is not spin polarized and the $SU(2)$ symmetry is not broken. Our model represents a good description of the experimental situation for an interacting wire with varying confining potential in the transverse direction.

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I. INTRODUCTION

Recent experiments in quantum wires show that the spin degrees of freedom combined with finite size effects give rise to new interesting effects in the ballistic transport regime. As an example we point out the new quasi-plateau with conductance $G \approx 2(0.7 \ e^2/h)$ observed by the Cavendish group [1]. A number of possible explanations have been suggested. Some of these explanations introduce spin polarization [2-4] and others are based on the Kondo effects [5,6]. Recently a number of groups have reported similar results. In particular we mention the results obtained by Reilly et al. [7,8] which have reported that the conductance varies in the range $0.5 - 0.7 \times 2 e^2/h$ as a function of the electron density, length of the wire and temperature.

In order to clarify this problem we investigate electron transmission through a short wire of length $d$ with electron-electron ($e$-$e$) interaction coupled to non-interacting leads. Inspired by the experiments in Ref. [8] we consider a short wire of length $d \sim 0.5-1 \ \mu m$, which corresponds to a temperature $T_{\text{wire}} \approx \hbar v_F/k_B d$, where $v_F$ is the Fermi velocity and $k_B$ is the Boltzmann constant. We expect that the conductance will change drastically when the temperature changes from $T \ll T_{\text{wire}}$ to $T \leq T_{\text{wire}}$ [9].

We argue that the model of a short interacting wire coupled to non-interacting leads represents a good approximation to the experimental situation. In the experimental case the wire widens smoothly from the contact region to the reservoirs. In the contact region the wire is narrow, tuning the gate voltage allows a situation with only one propagating channel. In the reservoirs the wire is wide allowing for several propagating channels. (Due to the continuity condition from the reservoirs region to the contact region only one channel in the reservoirs is a pure propagating channel and the rest have a complex wave vector in the wire.) The presence of the $e$-$e$ interaction can be separated into two parts, intrachannel and interchannel. Projecting out the channels with complex wave number we obtain an effective one dimensional channel.

In the reservoirs the interchannel interaction renormalizes strongly the propagating in-
teracting channel. On the other hand, in the wire region the interchannel renormalization is negligible. As a result the reservoir region is described by a Fermi liquid contrary to the unrenormalized interaction in the wire region which is described by a one dimensional Luttinger liquid. The transmission coefficient between the two regions is described by the tunneling matrix element $\lambda$ which couples the one dimensional Fermi liquid in the leads to the one dimensional Luttinger liquid in the wire. The matrix element $\lambda$ is further reduced by projecting out the non-propagating channels.

To solve this transport problem we construct the effective spectrum of the short wire. We find that the spectrum is built from charge-spin collective excitations and fermionic particle excitations. Using the renormalization group (RG) we find that the spectrum of the short wire is controlled by the zero mode fermionic single particle states. The bosonic degrees of freedom are integrated out giving rise to an effective model for the short wire. As a result we obtain that transmission across the short wire is equivalent to transmission through a “multilevel state” at temperatures $T \leq T_{\text{wire}}$. The effective model describes the physics of length scale longer than $d$ and contains new exchange terms generated by the interaction of the bosonic degrees of freedom of the short wire. We find that, since the conductance is given as the sum of the conductances for the different levels [see eq. (15b)], the $S = 0$ state is higher in energy than the doublet $S = \pm 1/2$, i.e. the $SU(2)$ doublet dominates the conductance and $G \sim e^2/h$.

For temperature $T \to 0$ so that $T < T_{\text{Kondo}} < T_{\text{wire}}$ the problem can be mapped into a Kondo problem which gives rise at $T \to 0$ to perfect transmission with a conductance $G = 2e^2/h$. This result is obtained only if the effective electron-electron interaction is larger than the effective tunneling matrix element. Therefore, this result is sensitive to temperature and length of the wire $d$.

The plan of this paper is as follows. In the next section we present the model relevant to a short interacting wire coupled to two noninteracting leads. In Sec. III we obtain an effective model at (a dimensionless logarithmic) length scale $l > l_d = \log(d/a)$, where $d$ is the length of the interacting region and $a$ is the inter-electron distance. In Sec. IV we present the
non-universal conductance at intermediate temperature and find that $G \simeq e^2/h$. Section V presents the computation of the conductance in the low temperature regime, $T < T_{\text{Kondo}}$, for which we find perfect transmission with the conductance $G = 2e^2/h$. Finally, we summarize our main findings in Sec. VI. Calculational details are relegated to an Appendix.

II. MODEL

We consider a model of two non-interacting metallic leads coupled to a short interacting wire by tunneling matrix element,

$$H = H^{\text{leads}} + H^{\text{wire}} + H_T.$$ (1a)

The method used in this paper is as follows: (a) The two reservoirs are described by two one dimensional non-interacting chiral fermions. (b) The one dimensional interacting chiral fermion describes a short one dimensional interacting region restricted to $d \sim 0.5 - 1 \, \mu\text{m}$. (c) The transmission between the two regions is described by a matrix element $\lambda$ which couples the leads to the wire.

Such a model represents the experimental situation in which an electronic waveguide is confined to the region $-L/2 \leq x \leq L/2$ and $|y| \leq D(x)$, where $D(\pm L/2) = W$ is the width in the reservoir and $D(|x| \leq d/2) = D_0$ is the width in the wire region with the conditions $L \gg d$ and $W > D_0$. In the absence of e-e interaction this problem is solved within the Born-Oppenheimer approximation (see Ref. [10]). One finds that in the $y$ direction we have a square well with transversal energies $E_n(x) = (\hbar^2/2m)[n\pi/2D(x)]^2$, $n = 1, 2, ...$, where $n$ corresponds to the index of the channel. In addition, we have a negligible matrix element $Z_{n,m}(D(x))$ which couples the channels. The value $D_0$ determines the number of conducting channels $n_{\text{max}} = 2k_F D_0/\pi$, where $k_F$ is the Fermi wave vector. For simplicity we consider the case with $n_{\text{max}} = 1$. This is obtained by tuning the gate voltage $\mu^{\text{wire}}$. Due to the fact that the width $D(x)$ varies in the $x$ direction, the transversal energy acts as a one-body potential, $E_n(x)$, and gives rise to backscattering. In the presence of a
1D Luttinger liquid we expect that the scattering potential $E_n(x)$ will give rise to a low transmission coefficient between the two regions. [In the presence of $e-e$ interaction the non-propagating interaction channels and the matrix element $Z_{n,m}(x)$ renormalize strongly the properties of the propagating channels in the reservoir. We know that for the case of coupled Luttinger chains the presence of small matrix elements between the channels is enough to give rise to a Fermi liquid. Therefore we expect that such a renormalization will take place in the reservoirs. Consequently the reservoirs can be described by a one dimensional Fermi liquid. In the wire region this renormalization is absent and we are left with a one dimensional interacting Luttinger liquid. The interaction will reduce the transmission coefficients between the two regions (Fermi liquid-Luttinger liquid).] Therefore we can describe the scattering matrix between the two regions by a matrix element $\lambda < 1$, see Eqs. (6a) and (6b).

The solution of our model, with leads coupled to a short wire, is obtained as a linear combination of the basis chiral operators $c_{R,\sigma}$, $c_{R,\sigma}^\dagger$ (right leads), $c_{L,\sigma}$, $c_{L,\sigma}^\dagger$ (left leads) and $\chi_\sigma$, $\chi_\sigma^\dagger$ (short wire). The fermionic operators $c_{R,\sigma}$, $c_{R,\sigma}^\dagger$; $c_{L,\sigma}$, $c_{L,\sigma}^\dagger$ and $\chi_\sigma$, $\chi_\sigma^\dagger$ are constructed in each region separately for the case $\lambda = 0$. The chiral fermion operators are obtained as a product of the zero mode fermion field and an exponential of the particle-hole bosonic fields. The bosonic field is periodic in each region. The boundary consions are controlled in each region by the chemical potentials $\mu_L^\sigma$ (left leads), $\mu_R^\sigma$ (right leads) and $\mu_{\text{wire}}$ (short wire).

We construct an effective model for (a dimensionless logarithmic) length scale $l > \log(d/a)$, where $d$ is length of the interacting region $d \sim 1 \mu$m and $a$ is the inter-electron distance in the wire. Typical values of $d/a$ are in the range $10^2$–$10^3$. In the leads the inter-electron distance is different than in the wire but, since the leads are noninteracting, the Hamiltonian is scale invariant. Therefore, one can use the same inter-electron distance as in the wire. The only difference between the two inter-electron distances is incorporated by rescaling the coupling constant $\lambda$.

This construction is performed by using the Renormalization Group. At this length scale the bosonic degrees of freedom of the wire have been completely integrated out and
the fermion fields $\chi_\sigma(x), \chi^\dagger_\sigma(x)$ [Eq. (2c)] have been replaced by the fermionic zero mode $V_\sigma, V^\dagger_\sigma$ [Eq. (2d)]. The short wire contains renormalized interactions $\hat{g}_s$, renormalized matrix elements and induced exchange interaction [e.g. Eq. (A6)]. The renormalized ratio $\hat{\lambda}/\hat{g}_s$ and $\mu_{\text{wire}}$ [Eq. (7c)] control the boundary condition at the interface between the reservoirs and the leads. For $\hat{\lambda}/\hat{g}_s < 1$ we reproduce the Kondo boundary conditions. In the opposite limit we obtain the non-universal conductance $G \simeq e^2/h$.

A. The leads

The $H^{\text{leads}} = H^L + H^R$ represents the left and right leads restricted to $-L/2 \leq x < -d/2$ and $d/2 < x \leq L/2$, respectively. We will replace in each lead the fermion operators with chiral right movers, $c_{L,\sigma}(x), c^\dagger_{L,\sigma}(x)$ (left lead) and $c_{R,\sigma}(x), c^\dagger_{R,\sigma}(x)$ (right lead) [11]. This will be achieved by using open boundary conditions (OBC) for each lead. As a result the wave function of the electron vanishes at $x = \pm d/2$ (i.e., the interface between the leads and the short wire). The transmission between the leads and the short wire is described by a matrix element introduced at the interface between the two regions and is controlled by the chemical potential $\mu_{\text{wire}}$. At the interface we will assume that the two wavefunctions in the two regions have an overlapping region of the order of the lattice constants. The transmission coefficient between the leads and the wire can be adjusted by varying the matrix element, the overlapping region and the chemical potential $\mu_{\text{wire}}$. (The concept is similar to the tight-binding model where localized states are used as a basis. By varying the hopping matrix elements we can describe the physics of free delocalized electrons.)

We start with the chiral fermion representation of the electron operators in the leads. The electron field operators take the form:

$$c_{L,\sigma}(x) = e^{i k_{\text{F}}(x+d/2)} L_{\sigma}(x+d/2) - e^{-i k_{\text{F}}(x+d/2)} L_{\sigma}(-(x+d/2)); \quad -L/2 \leq x < -d/2. \quad (1b)$$

Here $c_{L,\sigma}(x)$ is the fermion in the left lead expressed in terms of the chiral right moving
fermion $L_\sigma(x)$. Similarly, for the fermion in the right lead we have:

$$c_{R,\sigma}(x) = e^{ik_F(x-d/2)}R_\sigma(x-d/2) - e^{-ik_F(x-d/2)}R_\sigma(-(x-d/2)); \quad d/2 < x \leq L/2,$$

where $c_{R,\sigma}(x)$ is the fermion in the right lead with the chiral right moving fermion $R_\sigma(x)$.

Next, we perform the continuum limit and construct the Hamiltonian for the leads. We start with the Hamiltonian for the leads on the lattice (for each lead we have $N$ sites):

$$H_{\text{leads}} = -t_0 \sum_{\sigma=\uparrow,\downarrow} [c_{L,\sigma}(Na-d/2)c_{L,\sigma}(-(N-1)a-d/2) + \cdots + c_{L,\sigma}(-a-d/2)c_{L,\sigma}(-d/2-\varepsilon) + H.c]$$

$$-t_0 \sum_{\sigma=\uparrow,\downarrow} [c_{R,\sigma}^\dagger(Na+d/2)c_{R,\sigma}((N-1)a+d/2) + \cdots + c_{R,\sigma}^\dagger(a+d/2)c_{R,\sigma}(d/2+\varepsilon) + H.c].$$

Here $a$ is the lattice spacing ($\sim$ inter-electron distance), $\varepsilon \leq a$ is the overlap region between the leads and the wire, $t_0$ is the hopping matrix element and $Na = L/2$ is the length of the lead. Equation (1d) represents the tight-binding Hamiltonian for the left lead with fermion operators $c_{L,\sigma}$, $c_{L,\sigma}^\dagger$ and the right lead with $c_{R,\sigma}$, $c_{R,\sigma}^\dagger$.

The OBC causes the boundary terms $c_{L,\sigma}(-d/2)$ and $c_{R,\sigma}(d/2)$ to vanish. This will occur when we substitute $\varepsilon = 0$ in Eq. (1d). As a result we obtain $H_{\text{leads}}^0 = H_{\text{leads}}(\varepsilon = 0)$. For $\varepsilon \neq 0$ we will have a transmission term from the leads to the short wire and in addition a non-zero boundary term. We express $H_{\text{leads}}^0$ in terms of the chiral fermions.

$$H_{\text{leads}}^0 = \hbar v_F \sum_{\sigma=\uparrow,\downarrow} \int_{-L/2}^{L/2} dx [R_{\sigma}^\dagger(x)(-i\partial_x)R_\sigma(x) + L_\sigma^\dagger(x)(-i\partial_x)L_\sigma(x)]; \quad \hbar v_F = 2t_0a \sin(k_Fa).$$

We also express Hamiltonian (1d) in terms of the even and odd chiral fermions

$$\psi_{e,\sigma}(x) = \frac{1}{\sqrt{2}}[R_\sigma(x) + L_\sigma(x)]; \quad \psi_{o,\sigma}(x) = \frac{1}{\sqrt{2}}[R_\sigma(x) - L_\sigma(x)].$$

Following the derivation in Appendix B we replace the leads Hamiltonian (1d) by

$$H_{\text{leads}} = H_{\text{leads}}^0 + H_{\text{BC}},$$

$$H_{\text{leads}}^0 = \hbar v_F \sum_{\sigma=\uparrow,\downarrow} \int_{-L/2}^{L/2} dx [\psi_{e,\sigma}^\dagger(x)(-i\partial_x)\psi_{e,\sigma}(x) + \psi_{o,\sigma}^\dagger(x)(-i\partial_x)\psi_{o,\sigma}(x)]; \quad \hbar v_F = 2t_0a \sin(k_Fa).$$
$H_{BC}$ is the boundary term between the leads and the short interacting wire. This term takes a simplified form in the Kondo regime where the “symmetric state” $\psi_{\sigma}(x), \psi_{\sigma}^\dagger(x)$ is screened out [the designation of the name symmetric or antisymmetric is given according to the original fermions $c_{R,\sigma}(x)$ and $c_{L,\sigma}(x)$]. $H_{BC}$ is replaced by the antisymmetric state $\psi_{e,\sigma}(x), \psi_{e,\sigma}^\dagger(x)$. In this limit we obtain for $H_{BC}$,

$$H_{BC} \simeq -2t_0(2\sin(2k_Fa))^2 \sum_{\sigma=\uparrow,\downarrow} \psi_{e,\sigma}^\dagger(0)\psi_{e,\sigma}(0). \quad (1i)$$

In this paper the zero-mode method [12] is used to solve the transport problem. Since the fermions in the leads are non-interacting, we will use the fermionic representation given by Eqs. (1a)-(1i). For the short interacting wire we will use the zero-mode Bosonization [see Eqs. (2a), (2b)]. For the sake of completeness we will also present the Bosonization for the leads. Because the leads obey $L \to \infty$ the common belief is that the zero mode does not play any role. This belief is incorrect once we add particles to the ground state, in particular when we couple the leads to the external reservoirs [see Eq. (7b)]. For this situation it was shown in Ref. [13] that in order to study transport with Bosonization a nonvanishing DC current implies the presence of gapless modes, in analogy with the Goldstone theorem. The calculation in Ref. [13] is based on anomalous commutators. Similar results are obtained using the zero mode Bosonization [12]:

$$R_\sigma(x) = \frac{1}{\sqrt{2\pi a}} e^{-i\alpha_{R,\sigma}} e^{i\frac{2\pi}{L}(N_{R,\sigma}-1/2)x} e^{i\sqrt{4\pi}\theta_{R,\sigma}(x)} = \frac{1}{\sqrt{2\pi a}} e^{i\sqrt{4\pi}X_{R,\sigma}(x)}. \quad (1k)$$

For $L_\sigma(x)$ we replace $\alpha_{R,\sigma} \to \alpha_{L,\sigma}$, $N_{R,\sigma} \to N_{L,\sigma}$, $\theta_{R,\sigma}(x) \to \theta_{L,\sigma}(x)$ and $X_{R,\sigma}(x) \to X_{L,\sigma}(x)$, where $\alpha_{R,\sigma}, \alpha_{L,\sigma}$ are the zero mode coordinates; $N_{R,\sigma}, N_{L,\sigma}$ are the number operators; and $\theta_{R,\sigma}(x), \theta_{L,\sigma}(x)$ are the particle-hole excitations. We have the following commutation rules, $[\alpha_{R,\sigma}, N_{R,\sigma}] = i\delta_{\sigma,\sigma'}, [\alpha_{L,\sigma}, N_{L,\sigma}] = i\delta_{\sigma,\sigma'}$. The zero mode contribution to the Bosonized Hamiltonian is negligible since in the limit $L \to \infty$, $N_{R,\sigma}/L \to 0$ and $N_{L,\sigma}/L \to 0$. 

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B. The short interacting wire

Next, we consider the short interacting wire of length $d \ll L$. We will use the same Fermi momentum for the short wire as for the leads. According to the experimental situation we expect that the Fermi momentum in the wire region is smaller than the one in the leads (due to the change of the width). This shift of the Fermi momentum in the wire is considered by adding to the wire Hamiltonian the chemical potential $\mu^{\text{wire}}$ [see eq. (7c)] which shifts the wire ground state.

The finite size of the system and the boundary conditions for the short wire allow us to introduce zero mode excitations. The fermion $d_\sigma(x)$, $d^\dagger_\sigma(x)$ on the short wire is represented by anti-periodic right chiral fermions $\chi_\sigma(x)$,

\[
d_\sigma(x) = e^{ik_F x} \chi_\sigma(x) + e^{-ik_F x} \chi_\sigma(-x). \tag{2a}
\]

Here $d_\sigma(x = d/2) = d_\sigma(x = -d/2) = 0$ is obtained by demanding anti-periodic boundary conditions for the chiral fermion in the short wire.

\[
\chi_\sigma(x + d) = \chi_\sigma(x) e^{i\pi}. \tag{2b}
\]

Using the zero mode Bosonization introduced in Ref. [12], we obtain the representation

\[
\chi_\sigma(x) = V_\sigma e^{(i2\pi/d)(p_\sigma - 1/2)x} e^{i\sqrt{4\pi}\phi_\sigma(x)}, \tag{2c}
\]

\[
V_\sigma = \frac{1}{\sqrt{2\pi a}} e^{-iq_\sigma} \equiv \frac{1}{\sqrt{2\pi a}} \hat{V}_\sigma, \quad \sigma = \uparrow, \downarrow. \tag{2d}
\]

Here $q_\sigma$ and $p_\sigma$ are the zero mode coordinate and “momentum”, $[q_\sigma, p_\sigma'] = i\delta_{\sigma\sigma'}$, respectively. The “momentum” $p_\sigma = 0, 1, 2, \ldots$ measures the change of fermion number with respect to the filled Fermi sea. $\phi_\sigma(x)$ is the non-zero mode (the particle-hole) bosonic excitation. The operators $\hat{V}_\sigma^\dagger, \hat{V}_\sigma$ act as creation and annihilation fermion operators, $\hat{V}_\sigma^\dagger |p_\sigma\rangle = |p_\sigma + 1\rangle$. The operators $\hat{V}_\sigma^\dagger, \hat{V}_\sigma$ obey anticommutation relations, $\{\hat{V}_\sigma^\dagger, \hat{V}_\sigma\} = 2\delta_{\sigma\sigma'}, \{\hat{V}_\sigma^\dagger, \hat{V}_\sigma^\dagger\} = \{\hat{V}_\sigma, \hat{V}_\sigma\} = 0$ and commutation relations with the number operator $p_\sigma$; $[p_\sigma, \hat{V}_\sigma^\dagger] = \delta_{\sigma\sigma'} \hat{V}_\sigma'$; $[p_\sigma, \hat{V}_\sigma] = -\delta_{\sigma\sigma'} \hat{V}_\sigma'$.
The form and condition in Eqs. (2a)-(2c) are chosen for the following reasons: (a) We intend to construct the bosonic representation of the chiral fermion field \( \chi_\sigma(x) \) which is given as a product of the zero mode fermion \( V_\sigma \exp \left( i \frac{2\pi}{d} (p_\sigma - \delta) x \right) \) and the exponential of a periodic bosonic field \( \phi_\sigma(x) \). The boundary condition of the fermion field \( d_\sigma(x) \) in Eq. (2b) is determined by the value of \( \delta \). In Eqs. (2b) and (2c) we have \( \delta = 1/2 \). The representation in Eq. (2c) is given for \( \delta = 1/2 \) and has no fermions in the ground state, \( \langle p_\sigma \rangle = 0 \), \( p_\sigma = :p_\sigma: \), where \( :p_\sigma: \) stands for normal order and \( \langle p_\sigma \rangle \) means the expectation value with respect to the filled Fermi sea.

Equation (2c) is obtained in the absence of a chemical potential of the wire. Once a gate voltage \( 2V_G \) is applied, we must add to the Hamiltonian a term \( \mu_{\text{wire}} (p_\uparrow + p_\downarrow) \) [see Eq. (7c)]. The presence of the nonzero chemical potential \( \mu_{\text{wire}} \) shifts the ground state occupation. Consequently \( \langle p_\sigma \rangle \neq 0 \) and \( p_\sigma - 1/2 \) is replaced by \( :p_\sigma: + \langle p_\sigma \rangle - 1/2 \). The shift \( \langle p_\sigma \rangle - 1/2 \) can give rise to a change in the boundary conditions. We observe that the coupling Hamiltonian in Eqs. (9d) and (11c) below is controlled by \( p_\sigma = \langle p_\sigma \rangle + :p_\sigma: \), with \( \langle p_\sigma \rangle \) determined by the renormalized chemical potential \( E_\sigma(d) \) [see Eq. (9h)]. In the intermediate range of temperatures (see Sec. IV) the effect of the chemical potential in the wire is considered by taking the expectation value with respect to \( p_\sigma \) [see Eq. (15b)].

The physical reason for the representation in Eqs. (2a-2c) has to do with the fact that at the (dimensionless) length scale \( l > \log(d/a) \) the short wire is replaced by an impurity atom which in the limit \( T \to 0 \) corresponds to a single impurity. In this regime \( \lambda \ll 1 \) and the e-e interaction \( \hat{g}_s \) is large (see Sec. V). This gives rise to the Kondo physics.

Using the chiral representation given by Eq. (1e), we obtain the Hamiltonian for the short wire Hubbard model

\[
H^{\text{wire}} = H_0^{\text{wire}} + H_u^{\text{wire}} + H_B^{\text{wire}},
\]

where

\[
H_0^{\text{wire}} = \int_{-d/2}^{d/2} dx \left\{ \sum_{\sigma=\uparrow,\downarrow} \hbar v_F \chi_\sigma^\dagger(x) (-i \partial_x) \chi_\sigma(x) + \tilde{U} : \chi_\downarrow^\dagger(x) \chi_\downarrow(x) : \chi_\uparrow^\dagger(x) \chi_\uparrow(x) \right\}
\]
+\tilde{U} : \chi^{\dagger}_{\uparrow}(x)\chi_{\uparrow}(x) : \chi^{\dagger}_{\downarrow}(-x)\chi_{\downarrow}(-x) \right \} \quad (3b)

and \( U \) is the original Hubbard interaction, \( \tilde{U} = 2U \) and \( \tilde{v}_F = 2v_F \). The Umklapp and the backward term Hamiltonians are respectively given by [14]:

\[
H_{u}^{\text{wire}} = \frac{1}{2} \tilde{U} \int_{-d/2}^{d/2} dx \{ \chi^{\dagger}_{\uparrow}(x)\chi^{\dagger}_{\downarrow}(-x)\chi_{\uparrow}(-x)e^{i4k_Fx} + H.c. \}, \quad (3c)
\]

\[
H_{B}^{\text{wire}} = \frac{1}{2} \tilde{U} \int_{-d/2}^{d/2} dx \{ \chi^{\dagger}_{\uparrow}(-x)\chi^{\dagger}_{\downarrow}(x)\chi_{\uparrow}(x) + H.c. \}. \quad (3d)
\]

Next we bosonize the Hamiltonian given in Eqs. (3a)-(3d). \( H_{0}^{\text{wire}} \) can be replaced by \( H_{0}^{(n=0)} \) (the zero mode) and \( H_{0}^{(n \neq 0)} \) (non-zero mode part):

\[
H_{0}^{\text{wire}} = H_{0}^{(n \neq 0)} + H_{0}^{(n=0)}, \quad (4a)
\]

\[
H_{0}^{(n \neq 0)} = H_{0,c}^{(n \neq 0)} + H_{0,s}^{(n \neq 0)}. \quad (4b)
\]

Here \( H_{0,c}^{(n \neq 0)} \) represents the charge density and \( H_{0,s}^{(n \neq 0)} \) is the spin density.

\[
H_{0,c}^{(n \neq 0)} = \int_{-d/2}^{d/2} dx \tilde{V}_c(\partial_x \tilde{\phi}_c(x))^2, \quad (4c)
\]

\[
H_{0,s}^{(n \neq 0)} = \int_{-d/2}^{d/2} dx \tilde{V}_s(\partial_x \tilde{\phi}_s(x))^2. \quad (4d)
\]

The renormalized bosonic fields, \( \tilde{\phi}_c(x) \) and \( \tilde{\phi}_s(x) \) (charge, spin) are related to \( \phi_c(x) \) and \( \phi_s(x) \) by:

\[
\phi_c(s)(x) = \frac{K_{c(s)}^{-1/2}}{2} \left( \tilde{\phi}_c(s)(x) + \tilde{\phi}_c(s)(-x) \right) + \frac{K_{c(s)}^{1/2}}{2} \left( \tilde{\phi}_c(s)(x) - \tilde{\phi}_c(s)(-x) \right), \quad (4e)
\]

with \( K_c \) and \( K_s \) given by:

\[
K_c = \sqrt{\frac{1 - U/\pi v_+}{1 + U/\pi v_+}}, \quad v_+ = \tilde{v}_F \left( 1 + \frac{U}{\pi \tilde{v}_F} \right), \quad (4f)
\]

\[
K_s = \sqrt{\frac{1 + U/\pi v_-}{1 - U/\pi v_-}}, \quad v_- = \tilde{v}_F \left( 1 - \frac{U}{\pi \tilde{v}_F} \right), \quad (4f)
\]

and charge and spin velocities

\[
v_c = \frac{\tilde{v}_F}{K_c}, \quad v_s = \frac{\tilde{v}_F}{K_s}. \quad (4g)
\]
The zero mode part of $H_0$ allows viewing the short interacting wire as a multilevel “quantum dot” with single particle energy given by:

$$H_0^{(n=0)} = \frac{\hbar v_F}{2d} (p_\uparrow^2 + p_\downarrow^2) + \frac{U(4\pi)^2}{d} p_\uparrow p_\downarrow, \quad (5a)$$

where $p_\sigma = 0, 1, 2, \ldots$ represents the additional charges with respect to the filled Fermi sea $|p_\uparrow = 0, p_\downarrow = 0\rangle$. Typical values considered in Ref. [8] are $d_0 \sim 0.5 \mu m$. This corresponds to energies $\epsilon_0 = \hbar v_F/2d = 10^{-19} (v_F/c)(d_0/d)$ Joule, since $v_F/c \sim 10^{-3}$, $\epsilon_0$ is of the order of meV or temperature $T_{wire} = \epsilon_0/k_B = 7.6 \times 10^3 (v_F/c)(d_0/d)$ which corresponds to a few Kelvins.

The Bosonization of the Umklapp and backward terms involves both the bosonic degrees $\phi_\sigma(x)$ and the zero modes $p_\sigma$, and fermionic operator $\hat{V}_\sigma$.

$$H_{wire}^u = \int_{-d/2}^{d/2} dx \left\{ \frac{U}{2\pi^2 a^2} \hat{V}_\uparrow \hat{V}_\uparrow \hat{V}_\downarrow \hat{V}_\downarrow \cos \left[ (4k_F - G)x + \frac{2\pi}{d} (p_\uparrow + p_\downarrow) + \sqrt{8\pi} (\phi_c(x) - \phi_c(-x)) \right] \right\}. \quad (5b)$$

We find that the Umklapp term is controlled by the charged boson $\phi_c(x)$, the charge of the “dot” $p_c = p_\uparrow + p_\downarrow$ and the reciprocal lattice vector $G = 2\pi/a$. This term is highly sensitive to the electron density, namely $k_F = (\pi/2a)(1 - \delta)$. For density $\delta$ and wires of length $d$ which satisfy $(4k_F - G)d \geq 2\pi$ or $\delta(d/a) \gg 1$, the Umklapp term can be neglected. Since $d \sim 0.5 - 1 \mu m$ the deviation from half filling must be of the order less than 1%. Under this condition the Umklapp term will not give rise to a charge gap.

Next, we consider the backward term, $H_{wire}^B$. In the limit $d \to \infty$ this term renormalizes to zero driving $K_s \to 1$. For finite $d$ this is not the case:

$$H_{wire}^B = \int_{-d/2}^{d/2} dx \left\{ \frac{U}{2\pi^2 a^2} \hat{V}_\uparrow \hat{V}_\uparrow \hat{V}_\downarrow \hat{V}_\downarrow \cos \left[ \frac{2\pi}{d} (p_\uparrow - p_\downarrow)x + \sqrt{8\pi} (\phi_s(x) - \phi_s(-x)) \right] \right\}. \quad (5c)$$

The background term is controlled by the bosonic spin density $\phi_s(x)$ and spin excitations $p_s = p_\uparrow - p_\downarrow$. 

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C. The coupling Hamiltonian

The coupling Hamiltonian is given by

\[ H_T = \sum_{\sigma=\uparrow,\downarrow} \left[ -t_L c_{L,\sigma}^\dagger (-d/2 - \varepsilon) d_{\sigma} (-d/2 + \varepsilon) - t_R c_{R,\sigma}^\dagger (d/2 + \varepsilon) d_{\sigma} (d/2 - \varepsilon) + H.c. \right]. \quad (6a) \]

In Eq. (6a) we have introduced an “overlap” \( \varepsilon \sim a \) between the electrons in the wire and leads in order to allow transmission. (Due to the boundary condition when \( \varepsilon = 0 \) we have \( H_T \equiv 0 \)). Next we assume \( t_L = t_R = t \) and make use of the chiral representation [see Eqs. (1b,1c,1f,2a)]. We find:

\[ H_T = i\lambda \sum_{\sigma=\uparrow,\downarrow} \sum_{R=\pm} \int_{-d/2}^{d/2} dx \delta(x)[\psi_{0,\sigma}^\dagger(x)e^{ik_F R} \chi_{\sigma}(R) - e^{-ik_F R} \chi_{\sigma}^\dagger(R) \psi_{0,\sigma}(x)]. \quad (6b) \]

The tunneling matrix element \( \lambda = \lambda(\Lambda)^0 \), where \( \Lambda = 1/a \) is the cutoff and \( \lambda = 2\sqrt{2}t \sin(k_Fa) \equiv (t/t_0)E_F \sin(k_Fa), \ E_F \equiv \hbar v_F/2a. \)

The matrix element \( \lambda \) obeys \( \lambda < 1 \) (this being a result of the e-e interactions in the leads which have been integrated out). This value of \( \lambda \) is fixed by the transmission strength \( t/t_0 \) and the overlapping region of the wavefunction \( \varepsilon \sim a \). We considered the short wire as a localized state which is coupled to the leads. Controlling the strength of \( \lambda \) we can describe extended solutions in spite of the fact that we have started from a localized picture (this is the philosophy of the tight-binding method).

We argue that for our problem this is a good starting point since renormalization effects further decrease \( \lambda \) and give rise, for the (dimensionless) length scale \( l > \log(d/a) \), to an impurity problem which can be mapped into a Kondo problem for \( T \to 0 \).

D. Computation of the current

To compute the transmission current we add the reservoir Hamiltonian \( Y \):

\[ Y = Y^{\text{leads}} + Y^{\text{wire}}, \quad (7a) \]

\[ Y^{\text{leads}} = \sum_{\sigma=\uparrow,\downarrow} (\mu_L^{(\sigma)} N_{L,\sigma} + \mu_R^{(\sigma)} N_{R,\sigma}), \quad (7b) \]
with \( \frac{1}{2}(\mu^+_L + \mu^+_L) - \frac{1}{2}(\mu^+_R + \mu^+_R) = eV_{DS} \) the voltage difference, and \( N_{L,\sigma} = \int_{-L/2}^{L/2} dx L^\dagger_\sigma(x)L_\sigma(x) \), \( N_{R,\sigma} = \int_{-L/2}^{L/2} dx R^\dagger_\sigma(x)R_\sigma(x) \) are the fermion densities in the leads.

\[
Y^\text{wire} = \mu^\text{wire}(p_t + p_\perp), \tag{7c}
\]

with \( \mu^\text{wire} = eV_G \) being the gate voltage applied to the wire. The term \( Y^\text{wire} \) in Eq. (7c) allows for a nonzero number of fermions with respect to the leads Fermi sea. When \( \mu^\text{wire} \neq 0 \), \( k_F \) in Eqs. (2a)-(2c) is shifted to \( k^\text{wire}_F \equiv k_F + (2\pi/d)\langle p_\sigma \rangle \) and \( p_\sigma \) is replaced by : \( p_\sigma :. \) This procedure accommodates the experimental situation, \( k^\text{wire}_F < k_F \) (\( \mu^\text{wire} \) can be tuned to \( \langle p_\sigma \rangle < 0 \) needed in the experiments). We note that the conductance formula given below in Eq. (15) is sensitive to this tuning. The current operator \( I_\sigma \) is given by

\[
I_\sigma(t) = \frac{e}{2}\frac{d}{dt}(N_{R,\sigma} - N_{L,\sigma}) = \frac{e}{i2\hbar}[N_{R,\sigma} - N_{L,\sigma}, H_T]
= \frac{e\lambda}{2\hbar} \sum_{R=\pm(d/2-\epsilon)} \left[ \psi^\dagger_{e,\sigma}(0)e^{ik_FR}\chi_\sigma(R) + e^{-ik_FR}\chi^\dagger_\sigma(R)\psi_{e,\sigma}(0) \right]. \tag{7d}
\]

The non-equilibrium value of the current is obtained by performing the thermal expectation with respect to the grand canonical Hamiltonian, \( \hat{H} = H - Y \), where \( H \) is given in Eq. (1a) and \( Y \) in Eq. (7a). As a result we obtain

\[
\langle \langle I_\sigma \rangle \rangle = \frac{Tr(e^{-\beta\hat{H}}I_\sigma)}{Tr(e^{-\beta\hat{H}})}. \tag{7e}
\]

Equation (7e) will be used below for the computation of the current.

Equation (7e) with the reservoir Hamiltonian \( Y^{\text{leads}} = eV_{DS}N_L/2 - eV_{DS}N_R/2 \) [see Eq. (7b)], with \( \mu_L = eV_{DS}/2 \), \( \mu_R = -eV_{DS}/2 \) and \( N_L \equiv N_{L\uparrow} + N_{L\downarrow} \), \( N_R \equiv N_{R\uparrow} + N_{R\downarrow} \) allows to compute the current taking into account the boundary imposed by the reservoirs. Consequently, the density in the right and left leads obey the boundary conditions given in Eq. (14c). This method has been used in Ref. [13]. In particular, it has been shown in Ref. [13] that a one-dimensional interacting fermion (with no backward and Umklapp interaction) coupled to non-interacting leads has a universal conductance \( G = 2e^2/h \). This result has been shown to follow from the dynamical requirements, \( [N_R, N_L] = [H, N_R] = [H, N_L] = 0 \). (\( H \) is the interacting Hamiltonian constructed in Refs. [15–17]). The non-renormalization of
the conductance by electron-electron interactions has been shown originally [15–17] to be a consequence of the strong influence of the boundary conditions imposed by reservoirs. The same results have been obtained in Refs. [12,13] using the $Y^{leads}$ Hamiltonian [see Eq. (7b)]. The advantage of the formalism used in Ref. [13] and the zero mode calculation given in Ref. [12] is the simplicity. On the other hand the result derived in Ref. [15–17] are based on the exact integration of the electrostatic potential $V(x)$ along the one dimensional channel $\int_{-L/2}^{L/2} \rho(x)V(x)dx$ where $\rho(x)$ is the electronic density. For problems with backscattering [induced by the one body potential $E_n(x)$] and backward interaction, the method given in Ref. [13] is easy to use. The simplicity of the reservoir Hamiltonian $Y^{leads} = \mu_R N_R + \mu_L N_L$ (see Ref. [13]) causes only the zero mode coordinates (or the Goldstone modes) to be affected by the external boundary conditions. At finite temperature we use Eq. (7e) and at $T = 0$, $Y^{leads}$ is part of the Hamiltonian which will affect the zero mode variables. When $Y^{leads} \neq 0$ the ground state is shifted, consequently the zero mode coordinates become $\alpha_{R,\sigma}(t) \rightarrow \alpha_{R,\sigma}(t) - eV_{DST}/2\hbar$ and $\alpha_{L,\sigma}(t) \rightarrow \alpha_{L,\sigma}(t) + eV_{DST}/2\hbar$. As a result the chiral fermion in the leads will be modified. The current will be computed by replacing the tunneling Hamiltonian given in Eq. (6b) by a time-dependent Hamiltonian. Therefore, $R_\sigma(x)$ and $L_\sigma(x)$ in Eq. (6b) and (7d) will be modified: $R_\sigma(x,t) \rightarrow \exp(ieV_{DST}/2\hbar)R_\sigma(x,t)$ and $L_\sigma(x,t) \rightarrow \exp(-ieV_{DST}/2\hbar)L_\sigma(x,t)$.

The boundary condition affects only the zero modes $\alpha_{R,\sigma}$, $\alpha_{L,\sigma}$ (leads) and $q_\sigma$ (wire). Therefore, when we integrate short distance behavior given by $\phi_\sigma(x)$ in Eq. (2c), this has no effect on the zero modes and thus on the boundary condition. According to Refs. [15–17] this does not appear to be correct since the result depends on the integral of the electrostatic potential $V(x)$ along the one-dimensional channel. The resolution of this problem can be achieved by replacing the electrostatic potential $V(x)$ with an averaged electrostatic potential. Using this approximation will allow us to neglect the effect of the boundary on the short distance renormalization processes. For the rest of the paper we will work at finite temperature. We will use $Y^{leads} = \mu_R N_R + \mu_L N_L$ according to Eq. (7b) and will compute the current using Eq. (7e) in agreement with Ref. [13]. By doing so we ignore the short
length fluctuations of the electrostatic potential considered in Refs. [15–17].

III. EFFECTIVE MODEL

The effective model at the (dimensionless) length scale, \( l \geq l_d \equiv \log(d/a) \), \( T \leq \hbar v_F/k_B d \) can be mapped to an impurity model. To obtain the effective model we integrate out the short wire degrees of freedom. The wire field operators are given by

\[
\chi_{\sigma}(x) = V_{\sigma} e^{i(2\pi/d)(p_{\sigma} - 1/2)x} e^{i\sqrt{4\pi} \phi_{\sigma}(x)}.
\]

This operator contains bosonic degrees of freedom \( \phi_{\sigma}(x) \), which can be integrated out as regular bosonic fields, and a discrete fermion number which measures the added changes to the wire, \( p_{\sigma} = 0, 1, 2, ... \) and \( \hat{V}_{\sigma}, \hat{V}_{\sigma}^\dagger \) are the fermionic operators (see Eq. (2c)).

This integration is performed in two stages: (a) Integration of the bosonic degrees \( \phi_{\sigma}(x) \). This step is performed with the help of the Renormalization Group. As a result of this RG the short wire Hamiltonian and the tunneling matrix element \( \lambda \) are renormalized according to the sine-Gordon scaling equations. In addition, new magnetic exchange terms are induced by the RG. After performing the RG we use experimental considerations, namely that the studies of conductance have been performed below a few Kelvins for wires with length of the order of 1 \( \mu \)m. This means that the relevant physics occurs at a (dimensionless) length scale \( l > l_d = \log(d/a) \). Consequently we compute the relevant coupling constants at the scale \( l = l_d \). We substitute \( \tilde{\Lambda} = 1/d \) as our new cutoff and rewrite the model in terms of the scaled coupling constant and new fields,

\[
\chi_{\sigma}(R = \pm(d/2 - \epsilon)) \to \hat{V}_{\sigma} e^{i(2\pi/d)(p_{\sigma} - 1/2)R}.
\]

Note that in Eq. (8) the bosonic field has completely disappeared.

The model which is obtained for the short wire corresponds to a multilevel “quantum dot” system characterized by the quantum number \( p_{\sigma} \). Equation (1a) is replaced by:

\[
\tilde{H}(d) = \tilde{H}_{\text{leads}}(d) + \tilde{H}_{\text{wire}}(d) + \tilde{H}_T(d) + \tilde{H}_{\parallel}(d) + \tilde{H}_{\perp}(d) + \tilde{H}_{p-p}(d) + \tilde{H}_\mu(d).
\]

(9a)
Here, $\tilde{H}^{\text{leads}}(d)$ is the same as $H^{\text{leads}}$ except for the fact that $\psi_{e,\sigma} \to \hat{\psi}_{e,\sigma}$ and $\hat{\psi}_{o,\sigma} \to \hat{\psi}_{o,\sigma}$, where $\hat{\psi}_{e,\sigma}^\dagger$, $\hat{\psi}_{e,\sigma}$, $\hat{\psi}_{o,\sigma}^\dagger$ and $\hat{\psi}_{o,\sigma}$ represent the fermions in the leads with the cutoff $\tilde{\Lambda} = 1/d$ instead of $\Lambda = 1/a$.

$\tilde{H}^{\text{wire}}(d)$ represents the renormalized short wire Hamiltonian and is given by:

$$\tilde{H}^{\text{wire}}(d) = \frac{\epsilon_0}{2} p_1^2 + \frac{\epsilon_0}{2} p_2^2 + \eta p_1 p_2 + \hat{V}_1^l \hat{V}_1^\dagger \hat{V}_2^l \hat{V}_2^\dagger [\hat{g}_c(l_d) \delta_{p_1 + p_2, \text{even}} + \hat{g}_s(l_d) \delta_{p_1 - p_2, 0}].$$

In Eq. (9b), $\eta = 2U(4\pi)^2/d\epsilon_0 \sim 1$ is the dimensionless interaction parameter; $\hat{g}_c(l_d)$ and $\hat{g}_s(l_d)$ are the renormalized Umklapp and backward interactions, respectively, and $p_\sigma$ represents the quantum number of the multilevel system. When $(4k_F - G)d \geq 2\pi$ we can neglect the Umklapp term. On the other hand the backward interaction term vanishes in the limit $d \to \infty$.

The value of $\hat{g}_s(l_d)$ is determined by the sine-Gordon scaling equations [14]. For a finite $d$ and $K_s > 1$ we find,

$$\hat{g}_s(l_d) \simeq \frac{U}{2\pi^2} \left( \frac{d}{a} \right)^{2(1-K_s)} \frac{d}{a^2} = \frac{U}{2\pi^2} \left( \frac{d}{a} \right)^{3-2K_s} \frac{1}{d}. \quad (9c)$$

Since $K_s(0) > 1$ and $d$ is finite, $K_s(l_d) > 1$ and therefore the critical scaling $d \to \infty$, $K_s \to 1$ with $\hat{g}_s(l \to \infty) \sim \hat{g}_s(0)/(1 + \hat{g}_s(0)d)$ is not applicable.

We also approximate the Hubbard interaction $U$ by a screened Coulomb interaction with a dielectric constant $\kappa = \varepsilon/\varepsilon_0 \simeq 1$. We approximate $U/a \simeq e^2/(4\pi \varepsilon_0) \kappa a$. As a result we find that $U$ is related to the energy of the short wire $\epsilon_0 = h v_F/d$, $U \simeq \frac{\epsilon_0}{\kappa \sqrt{137}} (c/v_F)d$, where $c \simeq 3 \times 10^8$ m/sec is the speed of light, $c/v_F \simeq 10^3$ and $\kappa \simeq 10$. Using these values we estimate $K_s(0) = \sqrt{1 + U/\pi t_0 v_-}/(1 - U/\pi t_0 v_-) \simeq \sqrt{1 + (1/137)(c/v_F)(1/\pi \kappa))/(1 - (1/137)(c/v_F)(1/\pi \kappa))} \simeq 1.5$. This estimate allows us to replace Eq. (9c) by $\hat{g}_s(l_d) \simeq (\epsilon_0/2\pi^2 \kappa)(1/137)(c/v_F)(d/a)^{3-2K_s}$. For $d/a \simeq 10^3$ we find that $\hat{g}_s(l_d) \sim \epsilon_0$, which is a few meV.

The next term is the coupling Hamiltonian $\tilde{H}_T(d)$, which replaces Eq. (6b) by

$$\tilde{H}_T(d) = 2i\hat{\lambda} \sum_{\sigma = \uparrow, \downarrow} \int_{-L/2}^{L/2} dx \delta(x)[\hat{\psi}_{o,\sigma}^\dagger(x) \hat{V}_\sigma \cos \left( k_F(d/2 - \varepsilon) + \frac{2\pi}{d}(p_\sigma - 1/2)(d/2 - \varepsilon) \right)$$
\[-\cos \left( k_F(d/2 - \varepsilon) + \frac{2\pi}{d} (p_\sigma - 1/2)(d/2 - \varepsilon) \right) V_\sigma^\dagger \hat{\psi}_{o,\sigma}(x), \right) \tag{9d}
\]

where \( \hat{\lambda} = (2/\sqrt{\pi}d) t \sin(k_Fa) = \tilde{\lambda} \lambda^{1/2}, \tilde{\lambda} = 1/d \) and \( \lambda = (t/\sqrt{\pi}) \sin(k_Fa) \). This result was obtained within an RG calculation; see Appendix A.

The next three terms represent the induced magnetic interaction and effective impurity energy:

\[ \tilde{H}_||(d) = 2 \sum_{\sigma=\uparrow,\downarrow} \left[ \hat{J}_||(l_d) + \hat{I}_||(l_d) \cos \left( 2k_F(d/2 - \varepsilon) + \frac{2\pi}{d} (p_\sigma - 1/2)(d/2 - \varepsilon) \right) \right] \hat{\psi}_{o,\sigma}^\dagger(0) \hat{\psi}_{o,\sigma}(0) \hat{V}_\sigma^\dagger \hat{V}_\sigma. \tag{9e} \]

The coupling constants \( \hat{J}_\perp \) and \( \hat{I}_\perp \) are given in Appendix A (see Eqs. (A7,A8)). The transversal exchange term is given by

\[ \tilde{H}_\perp(d) = 2 \hat{J}_\perp(l_d) \left[ \cos \left( \frac{2\pi}{d} p_s(d/2 - \varepsilon) \right) + \cos \left( 2k_F + \frac{2\pi}{d} p_c(d/2 - \varepsilon) \right) \right] \times \left[ \hat{\psi}_{o,\uparrow}^\dagger(0) \hat{\psi}_{o,\downarrow}^\dagger \hat{V}_\downarrow^\dagger \hat{V}_\uparrow + \hat{\psi}_{o,\downarrow}^\dagger(0) \hat{\psi}_{o,\uparrow}^\dagger(0) \hat{V}_\uparrow^\dagger \hat{V}_\downarrow \right], \tag{9f} \]

where \( p_s = p_\uparrow - p_\downarrow \) and \( p_c = p_\uparrow + p_\downarrow \). The coupling constant \( \hat{J}_\perp(l_d) \) is given in Appendix A (see Eq. (A9)). We observe that the coupling Hamiltonian in Eq. (9d) and the induced interactions in Eqs. (9e)-(9g) are sensitive to the value of the overlapping region \( \varepsilon \) and chemical potential \( \mu_{\text{wire}} \). The use of \( \varepsilon \neq 0 \) and \( \mu_{\text{wire}} \neq 0 \) changes the boundary condition to \( \delta \neq 1/2 \) [see Eqs. (2a)-(2c)]: \( (p_\sigma - 1/2)(d/2 - \varepsilon) = (p_\sigma : -\delta)(d/2 - \varepsilon) \), therefore \( \delta = 1/2 - \langle p_\sigma \rangle \).

The induced two particle term is given by \( \tilde{H}_{p-p}(d) \),

\[ \tilde{H}_{p-p}(d) = \hat{J}_\perp(l_d) \left[ \cos \left( \frac{2\pi}{d} p_s(d/2 - \varepsilon) \right) + \cos \left( 2k_F + \frac{2\pi}{d} p_c(d/2 - \varepsilon) \right) \right] \times \left[ \hat{\psi}_{o,\uparrow}^\dagger(0) \hat{\psi}_{o,\downarrow}^\dagger(0) \hat{V}_\downarrow^\dagger \hat{V}_\uparrow + \hat{V}_\uparrow^\dagger \hat{V}_\downarrow^\dagger \hat{\psi}_{o,\downarrow}^\dagger(0) \hat{\psi}_{o,\uparrow}(0) \right]. \tag{9g} \]

\( \tilde{H}_{p-p}(d) \) gives rise to two particle transmission at low temperature. The two particle term dominates when the single particle transmission controlled by \( \lambda \) vanishes.

Equations (9e) and (9f) represent the induced magnetic interaction. These equations are obtained from the first two terms given in Eq. (A6). The particle-particle term given in Eq. (9g) is obtained from the last two terms in Eq. (A6). The explicit form in Eq. (9e) is
obtained once we integrate out completely the bosonic degrees of freedom \( \phi, \sigma \) of the short wire. At scale \( l > \log(d/a) \) we replace in Eq. (A6) fermion fields \( \chi, \chi^\dagger \) by the zero mode fermions \( \hat{V}, \hat{V}^\dagger \).

The last term in Eq. (9a) is the induced single particle energy

\[
\tilde{H}_\mu(d) = - \sum_{\sigma = \uparrow, \downarrow} \tilde{E}_\sigma \hat{\psi}^{\dagger}_{o,\sigma}(0) \hat{\psi}_{o,\sigma}(0) - \sum_{\sigma = \uparrow, \downarrow} E_\sigma(d) \hat{V}^{\dagger}_\sigma \hat{V}_\sigma,
\]

where

\[
E_\sigma(d) = \left( \frac{1}{\pi d} \right)^2 \left[ \hat{J}_\parallel(l_d) + \hat{I}_\parallel(l_d) \cos \left( (2k_F + \frac{2\pi}{d} p_\sigma)(d/2 - \varepsilon) \right) \right] > 0,
\]

and \( \tilde{E}_\sigma \approx E_\sigma(d) \) are obtained after performing the normal ordering in Eq. (A6). When a gate voltage is applied, \( E_\sigma(d) \) is shifted by the chemical potential of the wire \( \mu_{wire} \) [this will be the case in Eq. (15f) below where the single particle energy \( \epsilon_1 \) is shifted, \( \epsilon_1 \rightarrow \epsilon_1 - eV_G \)].

The set of Eqs. (9a)-(9h) complete step one of the renormalization group.

In the second step we map the problem to an impurity model. This step is performed using the impurity Hamiltonian (Eq. 9b). We have to project out high energy states: we project out the states with \( p_\sigma = 2, -1, -2, ... \). As a result of this projection we keep the same structure as given in Eqs. (9a)-(9h) with the difference that \( p_\sigma \) is restricted to \( p_\sigma = 0, 1, \sigma = \uparrow, \downarrow \) and the coupling constants \( \hat{J}_\parallel(l_d), \hat{I}_\parallel(l_d) \) and \( \hat{J}_\perp(l_d) \) are replaced by \( \langle \hat{J}_\parallel(l_d) \rangle \equiv \overline{J}_\parallel(l_d), \langle \hat{I}_\parallel(l_d) \rangle \equiv \overline{I}_\parallel(l_d), \) and \( \langle \hat{J}_\perp(l_d) \rangle \equiv \overline{J}_\perp(l_d) \) with “\( \langle ... \rangle \)” standing for the averages. From Appendix A we find that in the limit \( d \rightarrow a \) the induced exchange coupling constants \( \overline{J}_\parallel(l_d), \overline{I}_\parallel(l_d) \) and \( \overline{J}_\perp(l_d) \) vanish. The projected Hamiltonian \( h \) replaces \( \tilde{H}(d), \)

\[
\tilde{H}(d) \rightarrow h
\]

\[
h = \tilde{H}^{leads}(d) + h_{imp} + h_T + h_\parallel + h_\perp + h_p,
\]

where \( \tilde{H}^{wire}(d) + \tilde{H}_\mu(d) \rightarrow h_{imp}, \tilde{H}_T(d) \rightarrow h_T, \tilde{H}_\parallel(d) \rightarrow h_\parallel, \tilde{H}_\perp(d) \rightarrow h_\perp, \) and \( \tilde{H}_{p-p} \rightarrow h_p.\]
IV. CONDUCTANCE IN THE INTERMEDIATE TEMPERATURE REGIME:

For $T_{Kondo} \ll T \leq \hbar v_F/k_B d \simeq T_{wire}$ we use the impurity model obtained in Eq. (9b). In particular, the impurity Hamiltonian with $p_\sigma = 0, 1$ and $\sigma = \uparrow, \downarrow$ is given by,

$$h_{imp} = \frac{\epsilon_0}{2} [p_\uparrow^2 + p_\downarrow^2 + \eta p_\uparrow p_\downarrow] + \hat{g}_s(l_d) \hat{V}_\uparrow \hat{V}_\downarrow \hat{V}_\downarrow \delta_{p_\uparrow - p_\downarrow, 0} - E_d (\hat{V}_\uparrow \hat{V}_\downarrow + \hat{V}_\downarrow \hat{V}_\uparrow),$$  \hspace{1cm} (11a)

where $\hat{g}_s(l_d)$ is the renormalized backscattering term given in Eq. (9c). [We have neglected the Umklapp term $(4k_F - G)d > 2\pi$. In Eq. (9h) we replace $E_\sigma(d)$ by $E_d \sim \langle \hat{J}_\parallel \rangle / \pi d$.

In this temperature regime we ignore the two particle transmission and replace $h_{\parallel} + h_{\perp} + h_{p-p}$ by $h_{\parallel}$:

$$h_{\parallel} \simeq \langle \hat{J}_\parallel \rangle \sum_{\sigma = \uparrow, \downarrow} \hat{\psi}_{\sigma,0}^\dagger(0) \hat{\psi}_{\sigma,0}(0) \hat{V}_\sigma \hat{V}_\sigma,$$  \hspace{1cm} (11b)

with $\langle \hat{J}_\parallel \rangle \simeq (2t^2/\pi v_F) \log(d/a)$.

The transmission term in Eq. (9d) is replaced by $h_T$.

$$h_T \simeq i2\hat{\lambda} \sum_{\sigma = \uparrow, \downarrow} \cos(\pi p_\sigma) [\hat{\psi}_{\sigma,0}^\dagger(0) \hat{V}_\sigma - \hat{V}_\sigma^\dagger \hat{\psi}_{\sigma,0}(0)].$$  \hspace{1cm} (11c)

We have approximated the cosine term in Eq. (9d) by $\cos(\pi p_\sigma)$. This approximation has a negligible effect on the renormalized tunneling matrix element $\hat{\lambda}$. The matrix element $\hat{\lambda}$ is fixed by using $\varepsilon \neq 0$, $\delta = 1/2$ or equivalently $\varepsilon = 0$, $\delta \neq 1/2$, see Eq. (2c).

The impurity spectrum is characterized by the $U(1) \times SU(2)$ Kac-Moody primary states of the form $|p_\uparrow, p_\downarrow\rangle$. When $p_c = p_\uparrow + p_\downarrow$ is even the state $|p_\uparrow, p_\downarrow\rangle = |p_c = even, p_s = 0\rangle$ is an $SU(2)$ singlet with $p_s = p_\uparrow - p_\downarrow = 0$. On the other hand, when $p_c = p_\uparrow + p_\downarrow$ is odd the state $|p_\uparrow, p_\downarrow\rangle = |p_c = odd, p_s \pm 1\rangle$ is an $SU(2)$ doublet $p_s = \pm 1$ and spin $p_s/2 = \pm 1/2$.

The Heisenberg equations of motion are obtained from the Hamiltonian in Eq. (11a) with the zero mode anticommutation relations $\{\hat{V}_\sigma^\dagger, \hat{V}_{\sigma'}\} = 2\delta_{\sigma,\sigma'}$, $\{\hat{V}_\sigma^\dagger, \hat{V}_{\sigma'}^\dagger\} = \{\hat{V}_\sigma, \hat{V}_{\sigma'}\} = 0$ and commutation relations $[p_\sigma, \hat{V}_\sigma^\dagger] = \delta_{\sigma,\sigma'} \hat{V}_{\sigma'}$, $[p_\sigma, \hat{V}_\sigma] = -\delta_{\sigma,\sigma'} \hat{V}_{\sigma'}$.

We replace the number operator $p_\sigma$ by $\hat{V}_\sigma^\dagger \hat{V}_\sigma = 2\pi(d/a)p_\sigma$. The factor $2\pi(a/d)$ is related to the different renormalization of $p_\sigma$ and $\hat{V}_\sigma^\dagger \hat{V}_\sigma$. Next, we determine the eigenvalues for the impurity model [Eq. (11a)],

$$h_{imp} |p_\uparrow, p_\downarrow\rangle = \varepsilon(p_\uparrow, p_\downarrow) |p_\uparrow, p_\downarrow\rangle.$$
We find that due to the fact that \( \hat{g}_s(l_d) \geq \epsilon_0 \) and \( E_d > 0 \), the \( SU(2) \) doublet states \( |p_c = odd, p_s = \pm 1) \) have a lower energy than the singlet state \( |p_c = even, p_s = 0) \). Therefore we expect that the current will be controlled by the \( SU(2) \) doublet.

In the remaining part of this section we compute the current \( I_\sigma \), which is obtained from Eq. (7d) with \( H_T \) replaced by \( h_T \) [see Eq. (11c)],

\[
I_\sigma = \frac{e\lambda}{\hbar} \cos(\pi p_\sigma)[\hat{\psi}_{e,\sigma}^\dagger(0)\hat{V}_\sigma + \hat{V}_\sigma^\dagger\hat{\psi}_{e,\sigma}(0)].
\]

(12a)

To find the current from Eq. (11) we have to solve for the scattering states [18,19]. The scattering states will be obtained within the Heisenberg equations of motion for \( \hat{V}_\sigma^\dagger \), \( \hat{\psi}_{e,\sigma} \) and \( \hat{\psi}_{o,\sigma} \):

\[
i\hbar\hat{V}_\sigma^\dagger = [\hat{V}_\sigma^\dagger, h_{imp} + h_T + h_\parallel] = -\epsilon_0 \left[ p_\uparrow - \frac{\eta}{2} p_\downarrow + \frac{\eta}{2} \mathcal{F} s p_\downarrow \delta p_\uparrow - p_\downarrow, 0 \right] \hat{V}_\sigma^\dagger
\]

\[+ E_d \hat{V}_\sigma^\dagger - i(2\lambda) \cos(\pi p_\uparrow)\hat{\psi}_{o,\sigma}^\dagger(0) + \langle \hat{J}_|| \rangle \hat{\psi}_{o,\sigma}^\dagger(0) \hat{\psi}_{o,\sigma}(0) \hat{V}_\sigma^\dagger.\]

(12b)

\[
i\hbar\hat{\psi}_{o,\sigma}(x) = [\hat{\psi}_{o,\sigma}^\dagger(0), h_{imp} + h_\parallel + \hat{H}^{lead}(d)] = \hbar v_F (-i\partial_x) \hat{\psi}_{o,\sigma}^\dagger(x)
\]

\[+ \mathcal{J}_|| p_\uparrow \hat{\psi}_{o,\sigma}^\dagger(x) \delta(x) + i(2\lambda) \cos(\pi p_\uparrow) \hat{V}_\sigma^\dagger \delta(x).\]

(12c)

\[
i\hbar\hat{\psi}_{e,\sigma}(x) = \hbar v_F (-i\partial_x) \hat{\psi}_{e,\sigma}^\dagger(x).
\]

(12d)

The zero mode \( p_\sigma \) which measures the charge fluctuations with respect to the Fermi energy obeys the Heisenberg equation of motion

\[
i\hbar \hat{p}_\sigma = [p_\sigma, h_{imp} + h_T + h_\parallel] = [p_\sigma, h_T] = -2\lambda \cos(\pi p_\sigma)[\hat{\psi}_{o,\sigma}^\dagger(0)\hat{V}_\sigma + \hat{V}_\sigma^\dagger\hat{\psi}_{o,\sigma}(0)].
\]

(12e)

In Eq. (12b) we have used the notation: \( (1/2)\mathcal{F}_s \equiv \hat{g}_s(l_d)2\pi (d/a)(1/\epsilon_0) \) and \( \mathcal{J}_|| \equiv \langle \hat{J}_|| \rangle 2\pi (a/d) \). From Eq. (12c) we find that \( \hat{\psi}_{o,\sigma}(0) \) scales with \( 2\lambda \) and the spectrum of \( \hat{V}_\sigma \) is controlled by the single particle energy \( \epsilon_0 \). Therefore, we conclude that we can neglect the time dependence of \( p_\sigma \) when the single particle energy is larger than the transmission energy, \( (2\lambda)^2/\epsilon_0 < 1 \). In this limit we can neglect the fluctuations \( \delta p_\sigma(t) \), \( p_\sigma(t) = p_\sigma + \delta p_\sigma(t) \) and we make the approximation \( \langle p_\sigma(t) \rangle \approx p_\sigma \).

Next we use the bosonic representation of \( \hat{\psi}_{o,\sigma} \) and replace, \( \hat{\psi}_{o,\sigma}^\dagger(0)\hat{\psi}_{o,\sigma}(0) = (1/2\pi a) + (1/2\pi)\partial_x \theta_{o,\sigma} \). Here \( \theta_{o,\sigma} \) is the bosonic field in the leads and \( \tilde{E}_d \) is the renormalized impurity
We substitute Eq. (13c) into Eq. (13d) and obtain the scattering state solution

\[ \psi_{o,\sigma}(x, t) = \sum_E A_{E,\sigma}(t)U_{E,\sigma}(x), \]

(13a)

\[ \tilde{V}_{o}(t) = \sum_E A_{E,\sigma}(t)\tilde{V}_{\sigma,E}, \]

(13b)

with \( A_{E,\sigma}(t) = A_{E,\sigma}e^{-i(E/\hbar)t} \). We find from Eqs. (12b), (12d) and the approximation \( \langle \delta p_{o}(t) \rangle = 0 \) that \( \tilde{V}_{o,E}^{*} \) is proportional to \( U_{E,\sigma}(0) \),

\[ \tilde{V}_{o,E}^{*} = -\frac{i2\lambda \cos(\pi p_{\uparrow})U_{E,\uparrow}(0)}{\epsilon_0[p_{\uparrow} - \frac{\eta}{2}p_{\downarrow} + \frac{\eta}{2}p_{\uparrow}\delta_{p_{\uparrow}-p_{\downarrow},0}] - E_d - E}, \]

(13c)

and

\[ (E + \hbar v_{F}(i\partial_{x}))U_{E,\uparrow}(x) = \mathcal{J}_{p_{\uparrow}}\delta(x)U_{E,\uparrow}(x) + i2\lambda \cos(\pi p_{\uparrow})\tilde{V}_{o,E}^{*}\delta(x). \]

(13d)

We substitute Eq. (13c) into Eq. (13d) and obtain the scattering state solution \( U_{E,\uparrow}(x) \):

\[ U_{E,\uparrow}(x) = \frac{1}{\sqrt{L}}e^{i(E/\hbar v_{F})x}e^{i\mathcal{J}_{p_{\uparrow}}\mu(x)} \left[ \mu(-x) + \mu(x) \left( \frac{1 - i(2\lambda)^{2}/S}{1 + i(2\lambda)^{2}/S} \right) \right], \]

(14a)

where \( \mu(x) \) is the step function \( \mu(x \geq 0) = 1, \mu(x < 0) = 0 \) and \( S \) is the resonance energy function

\[ S \equiv S(E; p_{\uparrow}, p_{\downarrow}) = \epsilon_0(p_{\uparrow} - 1/2 + (\eta/2)p_{\downarrow} + \overline{\gamma}_{o}\delta_{p_{\uparrow}-p_{\downarrow},0}) - E_d - E, \]

(14b)

where \( \epsilon_0 = \hbar v_{F}/2d, \eta = U(4\pi)^{2}/\epsilon_0 d, \) and \( \overline{\gamma}_{o} \) is given by Eq. (9c). According to Eq. (12d), \( \hat{\psi}_{E,\sigma} \) is given in terms of plane waves, \( \hat{\psi}_{E,\sigma}(x) = \sum_{E} B_{E,\sigma}(1/\sqrt{L})e^{i(E/(\hbar v_{F}))x} \), and \( \hat{\psi}_{o,\sigma}(x) = \sum_{E} A_{E,\sigma}U_{E,\sigma}(x) \), where \( U_{E,\sigma}(x) \) is the scattering state (Eq. (14a)).

In the leads the presence of the voltage difference \( V_{DS} \) gives

\[ \langle (R_{\sigma}^{\dagger}R_{\sigma})_{E} \rangle = \langle a_{E,\sigma}^{\dagger}a_{E,\sigma} \rangle \equiv f_{FD} \left( E + \frac{eV_{DS}}{2} \right), \]

\[ \langle (L_{\sigma}^{\dagger}L_{\sigma})_{E} \rangle = \langle b_{E,\sigma}^{\dagger}b_{E,\sigma} \rangle \equiv f_{FD} \left( E - \frac{eV_{DS}}{2} \right), \]

(14c)
where $f_{FD}(E)$ is the thermal Fermi-Dirac function. The right and left moving operators $a_{E,\sigma}$ and $b_{E,\sigma}$ are related to $\hat{\psi}_{e,\sigma}$ and $\hat{\psi}_{o,\sigma}$ by the relation:

$$A_{E,\sigma} = \frac{1}{\sqrt{2}}(a_{E,\sigma} - b_{E,\sigma}); \quad B_{E,\sigma} = \frac{1}{\sqrt{2}}(a_{E,\sigma} + b_{E,\sigma}).$$

(14d)

We substitute into Eq. (12a) the Eqs. (13a) and (13b) using the eigenfunction $U_{E,\sigma}(x)$. We take the thermal expectation with respect to the leads reservoir [see Eq. (7b)] and find:

$$\langle I_{\uparrow} \rangle = e\lambda \bar{\hbar} \cos(\pi p_{\uparrow}) \sum_{E} \frac{1}{L} \left[ (B_{E,\uparrow}^\dagger A_{E,\uparrow}) \bar{V}_{\uparrow,E} + (A_{E,\uparrow}^\dagger B_{E,\uparrow}) \bar{V}_{\uparrow,E}^* \right]$$

$$= -\frac{ie}{4\hbar^2} \sum_{E} \left( \frac{2\lambda}{2\lambda + S(E)} \left( U_{E,\uparrow}(0) - U_{E,\uparrow}(0) \right) \left( \langle a_{E,\uparrow}^\dagger a_{E,\uparrow} \rangle - \langle b_{E,\uparrow}^\dagger b_{E,\uparrow} \rangle \right) \right)$$

$$= e\hbar \int dE \left[ \frac{(2\lambda)^4}{(2\lambda)^4 + S^2} \cos(\bar{J}_{\parallel} p_{\uparrow}) + \frac{(2\lambda)^2}{S} \left( \frac{S^2 - (2\lambda)^4}{S^2 + (2\lambda)^4} \right) \sin(\bar{J}_{\parallel} p_{\uparrow}) \right]$$

$$\times \left[ f_{FD} \left( E - \frac{eV_{DS}}{2} \right) - f_{FD} \left( E + \frac{eV_{DS}}{2} \right) \right].$$

(14e)

In Eq. (14e) $S$ is given by Eq. (14b). When we compute the current $\langle I_{\downarrow} \rangle$ we have to replace $p_{\uparrow} \rightarrow p_{\downarrow}$ and $p_{\downarrow} \rightarrow p_{\uparrow}$ in Eq. (14e). Next, we perform the expectation value over the impurity states at temperature $1/\beta$ using the result in Eq. (14b):

$$\langle \langle I_{\sigma} \rangle \rangle = \frac{Tr[e^{-\beta h_{\text{imp}}(p_{\uparrow},p_{\downarrow})} \langle I_{\sigma} \rangle]}{Te^{-\beta h_{\text{imp}}(p_{\uparrow},p_{\downarrow})}}, \quad \sigma = \uparrow, \downarrow.$$  

(15a)

Expanding the Fermi-Dirac function in Eq. (14e) around $E = 0$ gives for the $I_{\uparrow}$ current the conductance:

$$G_{\uparrow} = \frac{\langle \langle I_{\uparrow} \rangle \rangle}{V_{DS}} = \frac{e^2}{\hbar} \left\langle \frac{(2\lambda)^4 \cos(\bar{J}_{\parallel} p_{\uparrow})}{(2\lambda)^4 + (S(p_{\uparrow},p_{\downarrow}))^2} \right\rangle,$$

(15b)

where $S(p_{\uparrow},p_{\downarrow}) \equiv S(E = 0; p_{\uparrow},p_{\downarrow})$. Here $\langle \langle \ldots \rangle \rangle$ stands for the thermodynamic sum over the impurity states: $|p_{\uparrow} = 1, p_{\downarrow} = 0\rangle$, $|p_{\uparrow} = 0, p_{\downarrow} = 1\rangle$, $|p_{\uparrow} = 0, p_{\downarrow} = 0\rangle$, $|p_{\uparrow} = 1, p_{\downarrow} = 1\rangle$. Equation (15b) is dominated by the resonance term $S(p_{\uparrow},p_{\downarrow})$ which obeys $(2\lambda) > |S(p_{\uparrow},p_{\downarrow})|$. In obtaining Eq. (15b) we have neglected the term proportional to $\sin(\bar{J}_{\parallel} p_{\uparrow})$ in Eq. (14e). This approximation is justified given the fact that $\bar{J}_{\parallel} \ll 1$, see Eq. (A14). In order to perform the expectation value in Eq. (15b) we consider the even and odd state eigenvalues $\varepsilon(p_{\uparrow},p_{\downarrow})$ and the resonance term $S(p_{\uparrow},p_{\downarrow})$. 

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The even states:

\[ \varepsilon(p_\uparrow = 0, p_\downarrow = 0) = 0, \quad S(p_\uparrow = 0, p_\downarrow = 0) = -\left( \frac{\epsilon_0}{2} + \tilde{E}_d \right); \]
\[ \varepsilon(p_\uparrow = 1, p_\downarrow = 1) = 2\epsilon_1 + U_{e-e}, \quad S(p_\uparrow = 1, p_\downarrow = 1) = S(p_\uparrow = 0, p_\downarrow = 0) + 2\epsilon_1 + U_{e-e}. \quad (15c) \]

The odd states:

\[ \varepsilon(p_\uparrow = 1, p_\downarrow = 0) = \epsilon_1, \quad S(p_\uparrow = 1, p_\downarrow = 0) = \epsilon_1; \]
\[ \varepsilon(p_\uparrow = 0, p_\downarrow = 1) = \epsilon_1, \quad S(p_\uparrow = 0, p_\downarrow = 1) = \epsilon_1 + \left( \frac{\eta}{2} - 1 \right) \epsilon_0. \quad (15d) \]

where \( \epsilon_1 \equiv (\epsilon_0/2 - \tilde{E}_d) \) is the “impurity” single particle energy which is below the Fermi energy. \( U_{e-e} \equiv \epsilon_0(\eta/2 + \mathcal{G}_s) \) is the effective “Hubbard” interaction and \( S(p_\uparrow = 0, p_\downarrow = 1) - S(p_\uparrow = 1, p_\downarrow = 0) \equiv (\eta/2 - 1)\epsilon_0 \) is the effective spin polarization energy. The energies \( \varepsilon(p_\uparrow, p_\downarrow) \) control the thermal expectation values. We observe that due to the electron-electron interaction \( \mathcal{G}_s \) and the single particle energy \( \tilde{E}_d \) the odd states, \( \varepsilon(p_c = odd, p_s = \mp 1) \), can have lower energy than the even states.

The denominator function \( S(p_\uparrow, p_\downarrow) \) controls the quantum weights for the different states \( |p_\uparrow, p_\downarrow \rangle \) and it is asymmetric under the transformation \( p_\uparrow \rightarrow p_\downarrow, p_\downarrow \rightarrow p_\uparrow \). For the odd states it has the property for \( I_\uparrow, S(p_\uparrow = 0, p_\downarrow = 1) \neq S(p_\uparrow = 1, p_\downarrow = 0) \). [Similarly, for \( I_\downarrow, S(p_\uparrow = 0, p_\downarrow = 1) \neq S(p_\uparrow = 1, p_\downarrow = 0) \)]. Due to these properties we note that the current is dominated by one of the two states. We use Eqs. (15b)-(15d) to find:

\[ G_\uparrow = \frac{e^2}{h} \left[ \frac{(2\lambda)^4}{(2\lambda)^4 + (S(0, 0))^2} + \frac{(2\lambda)^4 \cos(\mathcal{J}_\parallel)}{(2\lambda)^4 + (S(1, 1))^2} e^{-\beta\varepsilon(1,1)} + \frac{(2\lambda)^4 \cos(\mathcal{J}_\parallel)}{(2\lambda)^4 + (S(1, 0))^2} e^{-\beta\varepsilon(0,1)} \right. \]
\[ + \left. \frac{(2\lambda)^4}{(2\lambda)^4 + (S(0, 1))^2} e^{-\beta\varepsilon(0,1)} \right] \left[ 1 + e^{-\beta\varepsilon(1,1)} + e^{-\beta\varepsilon(1,0)} + e^{-\beta\varepsilon(0,1)} \right]^{-1}. \quad (15e) \]

The conductance in Eq. (15a) depends on the effective single “particle” energy, \( \epsilon_1 \equiv \epsilon_0/2 - \tilde{E}_d < 0 \), the vacuum energy, \( \epsilon_e \equiv \epsilon_0/2 + \tilde{E}_d \equiv -S(p_\uparrow = 0, p_\downarrow = 0) \), the effective two particle energy \( U_{ee} = \epsilon_0(\eta/2 + \mathcal{G}_s) > 0 \) and polarization energy \( \epsilon_s \equiv \epsilon_0(\eta/2 - 1) \). In the presence of the gate voltage \( V_G \) the single particle energy is shifted \( \epsilon_1 \rightarrow \epsilon_1 - eV_G \). (The
single particle energy $\epsilon_1$ is controlled by the renormalized wire energy and by the chemical potential shift $\mu_{\text{wire}} = eV_{G}$.)

Next we introduce dimensionless parameters: $(2\lambda)^2/\epsilon_0 \equiv \Gamma$, $\epsilon_1/\epsilon_0 \equiv \hat{\epsilon}_1$, $eV_{G}/\epsilon_0 \equiv \hat{U}_G$, $U_{ee}/\epsilon_0 \equiv \hat{U}_{ee}$, $\epsilon_c/\epsilon_0 \equiv \hat{\epsilon}_c$, $\epsilon_s/\epsilon_0 \equiv \hat{\epsilon}_s$, $\epsilon_0/k_BT \equiv T_{\text{wire}}/T$, and $\cos(J_{\parallel}) \simeq 1$. As a consequence we find that the conductance takes the form:

$$G_{\uparrow} = \frac{e^2}{\hbar} \left( \frac{1}{Z} \right) \frac{\Gamma^2}{\Gamma^2 + (\hat{\epsilon}_c + \hat{U}_G)^2} + e^{-\hat{\beta}(2(\hat{\epsilon}_1 - \hat{U}_G) + (\eta/2 + \Gamma_s))} \frac{\Gamma^2}{\Gamma^2 + [2(\hat{\epsilon}_1 - \hat{U}_G) + (\eta/2 + \Gamma_s)]^2}$$

$$+ e^{-\hat{\beta}(\hat{\epsilon}_1 - \hat{U}_G)} \left( \frac{1}{\Gamma^2 + (\hat{\epsilon}_1 - \hat{U}_G)^2} + \frac{\Gamma^2}{\Gamma^2 + [(\hat{\epsilon}_1 - \hat{U}_G) + (\eta/2 - 1)]^2} \right), \tag{15f}$$

where $Z = 1 + e^{-\hat{\beta}(2(\hat{\epsilon}_1 - \hat{U}_G) + (\eta/2 + \Gamma_s))} + 2e^{-\hat{\beta}(\hat{\epsilon}_1 - \hat{U}_G)}$. Due to the repulsive interactions $2\hat{\epsilon}_1 + \hat{U}_{ee} > 0$ and $\hat{\epsilon}_1 < 0$, we find that for $T_{\text{wire}} > T$ the even part $p_e = 0, 2$ in Eq. (15f) can be neglected. Consequently for $\hat{U}_G = 0$ Eq. (15f) is replaced by:

$$G_{\uparrow} \simeq \frac{e^2}{\hbar} \left( \frac{1}{2 + e^{(T_{\text{wire}}/T)\hat{\epsilon}_1}} \right) \left[ \frac{\Gamma^2}{\Gamma^2 + (\hat{\epsilon}_1)^2} + \frac{\Gamma^2}{\Gamma^2 + (\hat{\epsilon}_1 + \eta/2 - 1)^2} \right]. \tag{15g}$$

We investigate Eq. (15g) for $\hat{\epsilon}_1 < 0$. As a result the term $e^{(T_{\text{wire}}/T)\hat{\epsilon}_1} \to 0$ when $T_{\text{wire}}/T \gg 1$. Due to the repulsive interaction $\eta/2$ we have a situation where one of the two terms satisfies the condition $(\hat{\epsilon}_1/\Gamma)^2 < 1$ or $(\hat{\epsilon}_1 + \eta/2 - 1)^2/\Gamma^2 < 1$. Choosing $\hat{\epsilon}_1 + \eta/2 - 1 = 0$ we find:

$$G_{\uparrow} \simeq \frac{e^2}{2\hbar} \left[ 1 + \frac{\Gamma^2}{\Gamma^2 + (\eta/2 - 1)^2} \right] \simeq \frac{e^2}{2\hbar}. \tag{15h}$$

A similar calculation for $G_{\downarrow}$ gives $G_{\downarrow} \simeq \frac{e^2}{2\hbar}$. Therefore $G = G_{\uparrow} + G_{\downarrow} \simeq \frac{e^2}{\hbar}$. This result is not based on a spin polarized state but makes use of the asymmetry between odd and even states such that only one state gives a resonant contribution $(2\lambda)^4/((2\lambda)^4 + S^2) \sim 1$ for $S(0,1)$ or $S(1,0)$ but not both.

The result obtained above depends on the number of electrons in the wire. We consider the case $p_{\uparrow} + p_{\downarrow} = p_e = 1$. (The even case $p_e = 0, 2, \ldots$ can be ignored since these correspond to high energy states and therefore give negligible contribution to the current.) When $p_e = 1$ we have one electron in the highest state in the wire. Suppose that an additional electron transmits into the wire. Here we can have the following two situations: (a) The spin of the
electron in the wire is the *same* as the spin of the transmitting electron; (b) The spin of the electron in the wire is opposite to the one which is transmitting into the wire. For case (a) the transmitting electron having the same spin as the one in the wire must occupy the next level and therefore the energy is increased by $\epsilon_0$ (by the level separation). For the case (b) when the spins are opposite, the transmitting electron can occupy the same level with the electron in the wire. As a consequence, the energy of the system will increase only by $\epsilon_0\eta$ (due to the repulsive interaction in the wire). This is the reason for the asymmetric resonant condition $S(p_{\uparrow} = 1, p_{\downarrow} = 0) \neq S(p_{\uparrow} = 0, p_{\downarrow} = 1)$ given in Eq. (15f) and the single particle asymmetry in energy, $\epsilon(p_{\uparrow} - 1/2 + (\eta/2)p_{\downarrow})$ [see Eq. (12b)]. This formula is asymmetric under the transformation $p_{\uparrow} \rightarrow p_{\downarrow}$ and $p_{\downarrow} \rightarrow p_{\uparrow}$ for $p_c = 1$. As a result the current is dominated by electrons which have opposite spins to the one in the wire. It might appear from this result that the transmission current is polarized. Since the electron in the short wire is in contact with a “thermal bath” the electron has equal probability to be in a state $|p_{\uparrow} = 0, p_{\downarrow} = 1\rangle$ or $|p_{\uparrow} = 1, p_{\downarrow} = 0\rangle$. Due to this fact we conclude that the current is *not* polarized since for each polarized incoming electron only one of the two wire states $|p_{\uparrow} = 0, p_{\downarrow} = 1\rangle$ or $|p_{\uparrow} = 1, p_{\downarrow} = 0\rangle$ gives rise to a transmission current. We conclude that the conductance for each polarization is half the value of the noninteracting conductance $G_{\uparrow} \sim G_{\downarrow} \sim e^2/2h$. Consequently we find that $G = G_{\uparrow} + G_{\downarrow} \sim e^2/h$.

We remark that Eq. (15g) gives perfect conductance when both $\hat{\epsilon}_1$ and $(\hat{\epsilon}_1 + (\eta/2 - 1))^2$ obey the condition $\hat{\epsilon}_1^2/\Gamma^2 < 1$, $(\hat{\epsilon}_1 + (\eta/2 - 1))^2/\Gamma^2 < 1$. Under this condition we obtain from Eq. (15g) $G_{\uparrow} \simeq e^2/h$, and consequently $G = G_{\uparrow} + G_{\downarrow} \simeq 2e^2/h$. The result obtained for this case is in a region where our calculation in Eqs. (12b)-(14b) might not be valid since $\Gamma^2$ is large. For a short wire $\overline{\sigma}_a$ is large and Eq. (12e) in conjunction with the subsequent discussion can invalidate the condition $\hat{p}_\sigma \simeq 0$. This case is investigated explicitly in the next section.
V. CONDUCTANCE IN THE LOW TEMPERATURE REGIME

Equation (10) represents our effective impurity model at length scale \( l > l_d \). At this length scale the Hamiltonian impurity model \( h_{imp} \) [see Eq. (11a)] contains an effective Hubbard interaction, \( \hat{g}_s(l_d) \hat{V}_\uparrow \hat{V}_\uparrow \hat{V}_\downarrow \hat{V}_\downarrow \). For finite \( l_d \) we can have a situation where the tunneling matrix element \( \hat{\lambda} \) in the coupling Hamiltonian \( h_T \) [see Eq. (11c)] obeys \( \hat{\lambda} < \hat{g}_s(l_d) \). \( \hat{\lambda} \propto d^{-1/2}, \hat{g}_s \sim (d/a)^3 K_s (1/d) \). Here we investigate this case, namely \( \hat{\lambda} / \hat{g}_s < 1 \). This situation is easily achieved in a short wire where \( \hat{g}_s(l_d) \) is close to the bare Hubbard interaction \( U \), which obeys \( t/U \ll 1 \). (For a long wire \( \hat{g}_s \) decreases, first exponentially and then logarithmically, contrary to \( \hat{\lambda} \) which decreases like \( d^{-1/2} \). Therefore, for a long wire the condition \( \hat{\lambda} / \hat{g}_s < 1 \) is not achieved). In the limit \( \hat{\lambda} / \hat{g}_s < 1 \) we have a strong coupling problem.

The spin degrees of freedom of the impurity interact with the conduction electrons in the leads. A virtual charge fluctuation in which an electron migrates off or onto the impurity gives rise to a spin-exchange between the impurity and the electrons in the leads. As a result an antiferromagnetic interaction between the impurity and the electron in the leads is induced [9,20]. Following Eq. (10), we consider only the impurity model plus transmission plus the chemical potential part given in Eqs. (9d) and (9h):

\[
\mathcal{H} = h_{imp} + h_T + h_\mu = \frac{\epsilon_0}{2} [\hat{p}_\uparrow + \hat{p}_\downarrow + \eta p_\uparrow p_\downarrow] - \tilde{E}_d [\hat{\psi}^\dagger_{a,\uparrow}(0) \hat{\psi}_{a,\uparrow}(0) + \hat{\psi}^\dagger_{a,\downarrow}(0) \hat{\psi}_{a,\downarrow}(0)] \\
+ \hat{g}_s(l_d) \hat{V}_\uparrow \hat{V}_\uparrow \hat{V}_\downarrow \hat{V}_\downarrow + i2\hat{\lambda} \sum_{\sigma=\uparrow,\downarrow} \cos(\pi p_\sigma) [\hat{\psi}^\dagger_{a,\sigma}(0) \hat{V}_\sigma - \hat{V}^\dagger_{\sigma} \hat{\psi}_{a,\sigma}(0)],
\]

where \( p_\sigma = 0, 1 \), \( \sigma = \uparrow, \downarrow \) and \( \tilde{E}_d \ll E_d \) with \( E_d > 0 \).

Using the projection method we project out the double occupancy for \( \hat{V}_{\sigma,\uparrow} \), \( \hat{V}_{\sigma} \) and \( \hat{\psi}^\dagger_{a,\sigma}(0) \), \( \hat{\psi}_{a,\sigma}(0) \):

\[
\hat{V}_{\sigma,\uparrow} = |0\rangle \langle \sigma | + e_{\sigma,\uparrow} - \sigma \rangle \uparrow, \downarrow |,
\]
\[
\hat{V}_{\sigma} = |\sigma\rangle \langle 0 | + e_{\sigma,\uparrow} | \uparrow, \downarrow \rangle \langle -\sigma |,
\]

and

\[
\hat{\psi}^\dagger_{a,\sigma} = |0\rangle (\sigma | + e_{\sigma,\uparrow} - \sigma \rangle \uparrow, \downarrow |.
\]
\[ \hat{\psi}_{o,\sigma} = |\sigma\rangle(0) + e_{\sigma,-\sigma} |\uparrow,\downarrow\rangle(-\sigma), \quad (17b) \]

where \(e_{\uparrow,\downarrow} = 1 = -e_{\downarrow,\uparrow}\).

The full Hilbert space consists of \(1_V \otimes 1_{\hat{\psi}(0)} \equiv 1\)

\[ 1_V = |0\rangle\langle 0| + |\uparrow\rangle\langle \uparrow| + |\downarrow\rangle\langle \downarrow| + |\uparrow,\downarrow\rangle\langle \downarrow,\uparrow|, \]

\[ 1_{\hat{\psi}(0)} = |0\rangle(0) + |\uparrow\rangle(\uparrow) + |\downarrow\rangle(\downarrow) + |\uparrow,\downarrow\rangle(\downarrow,\uparrow). \quad (17c) \]

We project out the double occupancy

\[ Q = Q_V \otimes Q_{\hat{\psi}(0)} = (|\uparrow,\downarrow\rangle\langle \downarrow,\uparrow|) \otimes (|\uparrow,\downarrow\rangle\langle \downarrow,\uparrow|). \quad (17d) \]

As a result we obtain, in agreement with Refs. [9,20],

\[ h_{\text{eff}} = -P\hbar Q(Q\hbar Q - E)^{-1}Q\hbar P = J_{\text{eff}} \vec{s}(0) \cdot \vec{S}, \quad (18) \]

where \(P = 1 - Q\) and

\[ \vec{s}(0) = \hat{\psi}_{o,\alpha}^\dagger(0) \left( \frac{\vec{\sigma}}{2} \right)_{\alpha,\beta} \hat{\psi}_{o,\beta}(0), \quad \vec{S} = \hat{V}_\alpha^\dagger \left( \frac{\vec{\sigma}}{2} \right)_{\alpha,\beta} \hat{V}_\beta; \]

\[ J_{\text{eff}} = 2(2\lambda)^2 \left[ \frac{1}{\hat{g}_s(\lambda)} - E_d + \frac{1}{E_d} \right]. \quad (19) \]

For a short wire such that \(\lambda/\hat{g}_s(\lambda) < 1\) we can neglect the induced terms \(h_{\parallel}\) and \(h_{\perp}\). (For a long wire this condition is not possible to satisfy.) Therefore, for this case we rely on the result given in Eqs. (14e), (15b) and (15e) for single particle transmission. For the remaining discussion we consider the strong coupling limit.

In the strong coupling limit we consider the Hamiltonian obtained in Eq. (18),

\[ H_K = H^{\text{leads}} + J_{\text{eff}} \vec{S} \cdot \vec{s}(0). \quad (20a) \]

The RG equation for the Hamiltonian in Eq. (20a) are [9,20]

\[ \frac{dJ_{\parallel,\text{eff}}}{dl} = \frac{1}{\pi v_F} J_{\perp,\text{eff}}^2, \quad \frac{dJ_{\perp,\text{eff}}}{dl} = \frac{1}{\pi v_F} J_{\parallel,\text{eff}} J_{\perp,\text{eff}}. \quad (20b) \]
For $J_{\parallel,\text{eff}} = J_{\perp,\text{eff}} = J_{\text{eff}}$ we find [9,20] that $J_{\text{eff}}$ flows to the strong coupling value $J_{\text{eff}}(l) = \frac{J_{\text{eff}}}{1-2J_{\text{eff}}l}$. This allows us to introduce the Kondo temperature $T_K$:

$$T_K = \frac{\hbar v_F}{k_B} \exp \left[ \frac{1}{2J_{\text{eff}}} \right].$$

(20c)

At temperature $T < T_K$ the symmetric electron “orbit” is screened out by the impurity. The symmetric screened electron state is given by $c_{s,\sigma}$:

$$c_{s,\sigma} = \frac{1}{\sqrt{2}} [c_{L,\sigma}(-d/2 - \varepsilon) + c_{R,\sigma}(d/2 + \varepsilon)] = 2i (\sin k_F \varepsilon) \hat{\psi}_{o,\sigma}(0).$$

(20d)

Here $c_{L,\sigma}$ and $c_{R,\sigma}$ are the electrons in the leads. Since the symmetric orbital $\hat{\psi}_{o,\sigma}(0)$ is screened away we project it out into the asymmetric orbital

$$c_{a,\sigma} = \frac{1}{\sqrt{2}} [c_{L,\sigma}(-d/2 - \varepsilon) - c_{R,\sigma}(d/2 + \varepsilon)] = 2i (\sin k_F \varepsilon) \hat{\psi}_{e,\sigma}(0).$$

(20e)

Using the projection operator $P = |\hat{\psi}_{e,\sigma}(\cdot)\rangle \langle \hat{\psi}_{e,\sigma}(\cdot)|$ we replace $R_\sigma$ and $L_\sigma$ by $PR_\sigma P = \frac{1}{\sqrt{2}} \hat{\psi}_{e,\sigma}$ and $PL_\sigma P = \frac{1}{\sqrt{2}} \hat{\psi}_{e,\sigma}$. We substitute these projections into the boundary term $H_{BC}$ given in Eq. (1g) and $H_{\text{leads}}$ given by Eq. (1h). Thus we obtain

$$H_e = \sum_{\sigma = \uparrow, \downarrow} \hbar v_F \int_{-L/2}^{L/2} dx [\hat{\psi}_{e,\sigma}^\dagger(x)(-i\partial_x)\hat{\psi}_{e,\sigma}(x) - \delta(x)(2 \sin k_F a)^2 \hat{\psi}_{e,\sigma}^\dagger(x)\hat{\psi}_{e,\sigma}(x)].$$

(21)

Here $H_e$ represents the effective Kondo screened Hamiltonian expressed in terms of the asymmetric orbitals $\hat{\psi}_{e,\sigma}$. For $k_F a = \pi/2$ Eq. (21) represents the single impurity resonant level model. This model has perfect transmission, namely $G_\uparrow = G_\downarrow = e^2/h$, and $G = 2e^2/h$.

In summary, we have obtained perfect transmission, $G = 2e^2/h$, in the region $T < T_K$ and for a short wire such that $\lambda / g_s(l_d) < 1$. This is in contrast to a long wire and intermediate temperatures, where $G \simeq e^2/h$. At low temperatures and a long wire the conductance is affected by the two particle transmission, see Eq. (9g).

VI. DISCUSSION

To compare our theory with the existing experiments [1,7,8] we used Eq. (15f) to compute the conductance. This equation was obtained under the assumption that $\hat{p}_\sigma \simeq 0$. This
assumption is consistent with the assumption of weak transmission; therefore in the Kondo regime our formula might not be valid. However, we find that Eq. (15f) works rather well and explains both of the regimes of a long wire with weak transmission and a short wire with large transmission. Using Eq. (15f) we observe that the shape of the conductance curve is mainly dependent on the tunneling matrix element and on the gate voltage $\hat{U}_G \equiv eV_G/\epsilon_0$. We plot the conductance $G/(2e^2/h)$, $G = G_\uparrow + G_\downarrow$ as a function of $\hat{U}_G$, for different $\Gamma^2 \equiv (2\hat{\lambda})^2/\epsilon_0$ tunneling matrix elements and different $\hat{\beta} = \epsilon_0/k_BT = T_{\text{wire}}/T$ temperatures.

The conductance is less sensitive to the rest of the parameters $\eta/2$, $\bar{g}_s$, electron-electron interaction and single particle energy. We use $\bar{g}_s \simeq \eta/2 \simeq 0.75$, $\hat{\epsilon}_1 = -0.2$ and $\hat{\epsilon}_c = 1.2$ [see Eq. (15f)]. In Fig. 1 we plot the calculated conductance as a function of gate voltage $\hat{U}_G$ for fixed transmission $\Gamma^2 = 0.1$ (which corresponds to weak transmission) and temperatures $\hat{\beta} = 1, 10$ and 50. At $\beta = 1$ we observe that the conductance takes the value of $G/(2e^2/h) \simeq 0.8$. Decreasing the temperature to $\hat{\beta} = 10$ we observe that the conductance has become 1 and in addition we observe a shoulder in the conductance around 0.7. At $\beta = 50$, the shoulder develops into a separate conductance peak. Note that this figure very much resembles the experimentally measured conductance [1,7,8].

To understand the situation for large tunneling matrix elements (i.e. a short wire and in the Kondo regime) we chose $\Gamma^2 = 5$ and $\hat{\beta} = 100$. In Fig. 2 we plot the calculated conductance as a function of gate voltage for these parameters. We observe that the conductance is $G/2e^2/h = 1$ as is expected from the Kondo solution given in Sec. V. However, we note that in this regime the approximation used in Eq. (15f) is not valid and the Kondo solution given in Sec. V should be used.

**VII. CONCLUSION**

We have investigated the case in which a two dimensional electronic waveguide with a varying width $D(x)$ in the $y$ direction and $e$-$e$ interactions can be projected to a one channel problem. In this case the reservoirs are replaced by a one dimensional Fermi liquid (leads)
coupled to a short wire Luttinger liquid controlled by the gate voltage $\mu_{\text{wire}}$. Using a combined method of Renormalization Group and zero mode bosonization, we have constructed the effective spectrum of a short interacting wire. This spectrum consists of combined charge-spin density waves and is dominated by the zero mode fermionic spectrum. At temperature $T \leq T_{\text{wire}}$ the fermionic spectrum is equivalent to a multilevel spectrum for which we find the conductance formula given in Eq. (15b). Transmission is dominated by the spin $S = \pm 1/2$ doublet giving rise to a conductance $G \sim e^2/h$. This result is relevant for experiments in which the wire is short, $T \leq T_{\text{wire}} \simeq \hbar v_F/k_B d$. On the other hand, at lower temperatures when the effective tunneling matrix $\hat{\lambda}$ is smaller than the effective electron-electron interaction $g_s(l_d)$, $\hat{\lambda}/g_s(l_d) < 1$, we find that the short interacting wire is mapped into a Kondo problem which gives rise, for $T < T_{\text{Kondo}} < T_{\text{wire}}$, to a conductance $G = 2e^2/h$. By tuning the length of the wire and the temperature we obtain $e^2/h \leq G \leq 2e^2/h$ in agreement with experiments [7].

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IX. APPENDIX A

Here, using the RG equation we derive the coupling constants for the model in Eq. (9a). (We use the scaling results for the short wire.) We treat, within a perturbative RG, the transmission term $H_T$. We separate $\phi_\sigma(x) = \phi_\sigma^<(x) + \delta\phi_\sigma(x)$, $\psi_{o,\sigma}(x) = \psi_{o,\sigma}^<(x) + \delta\psi_{o,\sigma}(x)$, $\psi_{e,\sigma}(x) = \psi_{e,\sigma}^<(x) + \delta\psi_{e,\sigma}(x)$, and integrate $\delta\phi_\sigma(x)$, $\delta\psi_{o,\sigma}(x)$, and $\delta\psi_{e,\sigma}(x)$ reducing the cutoff from $\Lambda$ to $\Lambda' = \Lambda - d\Lambda \equiv \Lambda e^{-l}$. As a result of the integration we replace $H_T$ with $H_{T,\text{eff}} = H_T^< + dH_T^{(1)} + dH_T^{(2)}$, where $H_T^<$ represents the transmission term at the reduced cutoff and
fields \( \phi_{\sigma}^{<}(x) \), \( \psi_{\alpha,\sigma}^{<}(x) \), \( \psi_{\epsilon,\sigma}^{<}(x) \), and \( dH_{T}^{(1)} \) represents the first order correction in \( \lambda \):

\[
dH_{T}^{(1)} = -i\lambda \sqrt{4\pi} \frac{2}{dH} \left[ \langle \delta\theta_{\sigma,\tau}^{2} \rangle + \langle \delta\phi_{\tau}^{2} \rangle + \langle \delta\phi_{\tau}^{2} \rangle \right] \sum_{R=\pm(d/2-\epsilon)} \int_{L/2}^{L/2} dx d\delta(x) \left[ \psi_{\alpha,\sigma}^{(1)}(x) e^{ikFR} \chi_{\sigma}^{<}(R) \right. \\
\left. -e^{-ikFR} \chi_{\sigma}^{<}(x) \psi_{\alpha,\sigma}^{<}(x) \right]. \tag{A1}
\]

In Eq. (A1) we have used the bosonic representation of the fermions in the leads, \( \psi_{\alpha,\sigma}(x) = \frac{1}{\sqrt{2\pi a}} e^{i\sqrt{4\pi} \theta_{\alpha,\sigma}(x)} \) and the bosonic part \( (1/\sqrt{2\pi a}) \exp(i\sqrt{4\pi} \phi_{\sigma}(x)) \) of the short wire, see Eq. (2c). We obtain

\[
\sum_{\sigma=\uparrow,\downarrow} \langle \delta\theta_{\sigma,\tau}^{2} \rangle = \frac{2\pi}{8} dl, \tag{A2}
\]

\[
\sum_{\sigma=\uparrow,\downarrow} \langle \delta\phi_{\tau}^{2} \rangle = \frac{K_{c} + K_{s}}{8\pi} dl, \tag{A3}
\]

where \( K_{c} \) and \( K_{s} \) are scale dependent (see [8]). Using Eqs. (A2) and (A3) we find

\[
\frac{d\lambda}{dl} = -\left( \frac{2 + K_{c} + K_{s}}{4} \right) \lambda. \tag{A4}
\]

To a good approximation \( K_{c} + K_{s} \approx 2 \). This gives \( d\lambda/dl \approx \lambda \) with the scaling law \( \lambda(l) = \lambda \frac{e^{l}}{l} \). Therefore \( \lambda(l_{d}) = \lambda(d/a)^{-1} \). Next, we rescale the fields such that \( \psi_{\alpha,\sigma}(x) \) is replaced by \( \hat{\psi}_{\alpha,\sigma}(x) \), \( \psi_{\alpha,\sigma}(x) = \sqrt{d/a} \hat{\psi}_{\alpha,\sigma}(x) \), where \( \hat{\psi}_{\alpha,\sigma}(x) \) depends on the new cutoff \( \tilde{\Lambda} = 1/d \). Since at the scale \( l > l_{d} \) the bosonic field has disappeared, we replace \( \tilde{V}_{\sigma} = 1/\sqrt{2\pi a} \tilde{V}_{\sigma} \) by \( \tilde{V}_{\sigma} \). As a result the new coupling constant becomes

\[
\hat{\lambda} \equiv \lambda(l_{d}) \left( \frac{d}{a} \right)^{1/2} \frac{1}{\sqrt{2\pi a}} \equiv \tilde{\Lambda} \tilde{\lambda}^{1/2}, \quad \tilde{\lambda} = \frac{t}{\sqrt{\pi}} \sin k_{F}a, \quad \tilde{\Lambda} \equiv 1/d. \tag{A5}
\]

The induced RG terms are obtained from the second order term in \( \lambda \), \( dH_{T}^{(2)} = dH_{T,p-h}^{(2)} + dH_{T,p-p}^{(2)} \), where \( dH_{T,p-h}^{(2)} \) represents the particle-hole and \( dH_{T,p-p}^{(2)} \) the two-particle induced interactions:

\[
dH_{T}^{(2)} = \frac{i}{2}(i\lambda)^{2} \sum_{\sigma,\sigma'=\uparrow,\downarrow, R,R'=\pm(d/2-\epsilon)} \left[ \int dt \int d\tau T \left( \right) \right] \chi_{\sigma}^{<}(R, t+\tau) e^{ikFR} \chi_{\sigma'}^{<}(R', t) \\
\psi_{\alpha,\sigma}^{<}(0, t) e^{-ikFR} \chi_{\sigma}^{<}(R, t+\tau) \psi_{\alpha,\sigma'}^{<}(0, t) e^{ikFR'} \chi_{\sigma'}^{<}(R', t) \right] \left[ 4\pi \langle \delta\theta_{\alpha,\sigma}(t+\tau) \delta\phi_{\alpha,\sigma'}(t) \rangle 
\right] \\
+4\pi \psi_{\alpha,\sigma}^{<}(0, t) e^{-ikFR} \chi_{\sigma}^{<}(R, t+\tau) \psi_{\alpha,\sigma}^{<}(0, t) e^{ikFR'} \chi_{\sigma'}^{<}(R', t) \right] \left[ 4\pi \langle \delta\phi_{\alpha,\sigma}(t+\tau) \delta\phi_{\alpha',\sigma'}(t) \rangle \right]
\]
The values \( \hat{I} \) of \( \hat{\psi}_{o,\sigma} \) in Eqs. (A7), (A8) and (A9) obey the scaling equation (A4), \( \Lambda(l) = \Lambda e^{-l} \) and \( K_c(l), K_s(l), V_c(l) \) and \( V_s(l) \) are determined by the sine-Gordon scaling (see [14]).

In Eqs. (A7), (A8) and (A9) \( \lambda \) obeys the scaling equation (A4), \( \Lambda(l) = \Lambda e^{-l} \) and \( K_c(l), K_s(l), V_c(l) \) and \( V_s(l) \) are determined by the sine-Gordon scaling (see [14]).

The values of \( \hat{J}_\parallel(l_d), \hat{J}_\perp(l_d) \) and \( \hat{I}_\parallel(l_d) \) are obtained by solving Eqs. (A7)-(A9) with the initial conditions \( \hat{J}_\parallel(0) = J_\parallel(0) = J_\perp(0) = 0 \). The induced coupling constants are given by \( \hat{I}_\parallel(l_d), \hat{J}_\parallel(l_d) \) and \( \hat{J}_\perp(l_d) \). These couplings are obtained from \( I_\parallel(l_d), J_\parallel(l_d) \) and \( J_\perp(l_d) \).

The values \( \hat{I}_\parallel(l_d), \hat{J}_\parallel(l_d) \) and \( \hat{J}_\perp(l_d) \) depend on the sine-Gordon scaling of the short wire.

Next we consider the two cases:

(a) \( (G - 4k_F)d < 2\pi \). As a result the backward term controls the scaling, \( K_c/v_c \approx 0, K_s/v_s \sim (1/2v_F)(1 - U/\pi v_F) \).

\[
g_c(l_d) = \frac{U}{2\pi^2} \left( \frac{d}{a} \right)^{3-2K_c} \frac{1}{d} \quad K_c < 1.
\]
Using
\[ \frac{U/a}{\epsilon_0} \approx \frac{1}{137} \frac{c}{v_F} \frac{d}{a} \]
we find
\[ \hat{g}_s(l_d) = \frac{\epsilon_0}{k2\pi^2137} \left( \frac{c}{v_F} \right) \left( \frac{d}{a} \right)^{(3-2K_c)} \]
with
\[ K_c = \sqrt{\frac{1 - U/\pi v_+}{1 + U/\pi v_+}} \approx \sqrt{\frac{1 - (1/137)(c/v_F)(1/\pi k)}{1 + (1/137)(c/v_F)(1/\pi k)}}. \]

Using \( g_c(l_d) \) and \( K_c \) we obtain the exchange coupling constants.

\[ \hat{J}_\parallel(l_d) = \frac{\lambda^2(0)}{4\pi v_F} \log \left( \frac{d}{a} \right) \left[ \left( 1 - \frac{U}{2\pi v_F} \right) (1 - \langle \cos \Lambda d \rangle) \right], \quad (A10) \]

\[ \hat{I}_\parallel(l_d) = \frac{\lambda^2(0)}{4\pi v_F} \left[ \left( 1 - \frac{U}{\pi v_F} \right) (1 - \langle \cos \Lambda d \rangle) \right], \quad (A11) \]

and \( \hat{J}_\perp(l_d) \) is given by

\[ \hat{J}_\perp(l_d) = \frac{\lambda^2(0)}{2\pi v_F} \log \left( \frac{d}{a} \right) \left[ \left( 1 - \frac{U}{\pi v_F} \right) (1 - \langle \cos \Lambda d \rangle) \right]. \quad (A12) \]

In Eqs. (A10)-(A12) \( \lambda(0) \) is given by \( \lambda(0) = 2\sqrt{2}t\sin(k_Fa) \). \( U \) is the original Hubbard interaction and \( \langle \cos \Lambda R \rangle \) is given by

\[ \langle \cos \Lambda R \rangle = \frac{1}{l_d} \int_0^{l_d} \cos(\Lambda e^{-l}) dl = \frac{1}{l_d} \int_{R/a}^{R/a} \cos x dx = \frac{1}{\log(d/a)} \int_1^{d/a} dx \frac{\cos x}{x}. \quad (A13) \]

(b) For the generic case \((G - 4k_F)d > \pi\) we can neglect the Umklapp term. We use the relations \( K_c v_c = K_s v_s = \tilde{v}_F = 2v_F \);

\[ \frac{K_c}{v_c} = \frac{K^2_c}{v_F} = \frac{1}{2v_F} \left( \frac{1 - U/\pi v_+}{1 + U/\pi v_+} \right) \]

and

\[ \frac{K_s(l)}{v_s(l)} = \frac{K^2_s}{2v_F} \left[ 1 + \frac{K^2_s \pi}{2v_F} \int_0^l dl' \hat{g}^2_s(l') \right]^{-1} \approx \frac{K^2_s}{2v_F} - O \left( \frac{K^2_s}{2v_F} \right). \]

As a result we obtain

\[ \frac{K_c}{v_c} + \frac{K_s}{v_s} \approx \frac{1}{2v_F} \left( \frac{1 - 2U}{\pi v_+} + 1 + \frac{2U}{\pi v_-} \right) \approx \frac{1}{v_F}. \]
and
\[ \frac{K_e}{v_c} - \frac{K_s}{v_s} \sim \left( \frac{U}{\pi v_F} \right) \frac{1}{v_F}. \]
Substituting these values into the scaling equations (A7)-(A9) we find:
\[ \hat{j}_\parallel(l_d) = \frac{\lambda^2(0)}{4\pi v_F} \log \left( \frac{d}{a} \right), \quad (A14) \]
\[ \hat{j}_\perp(l_d) = \frac{\lambda^2(0)}{4\pi v_F} \log \left( \frac{d}{a} \right) \langle \cos \Lambda d \rangle, \quad (A15) \]
\[ \hat{j}_\perp(l_d) = \frac{\lambda^2(0)}{2\pi v_F} \left( \frac{U}{4\pi v_F} \right) \log \left( \frac{d}{a} \right) [1 - \langle \cos \Lambda d \rangle]. \quad (A16) \]

**X. APPENDIX B**

Here, we consider the boundary term in the leads with the overlapping region \( \varepsilon \sim a \).

\[ H_{BC} = -t_0 \sum_{\sigma=\uparrow,\downarrow} \left[ (e^{ik_F a} L_\sigma^\dagger(-a) - e^{-ik_F a} L_\sigma^\dagger(a)) (e^{-ik_F \varepsilon} L_\sigma(-\varepsilon) - e^{ik_F \varepsilon} L_\sigma(\varepsilon)) \right] \]
\[ - \left[ (e^{-ik_F a} R_\sigma^\dagger(a) - e^{ik_F a} R_\sigma^\dagger(-a)) (e^{ik_F \varepsilon} R_\sigma(\varepsilon) - e^{-ik_F \varepsilon} R_\sigma(-\varepsilon)) \right] \]
\[ = t_0 \sum_{\sigma=\uparrow,\downarrow} \sin(k_F a) \sin(k_F \varepsilon) \left[ (L_\sigma^\dagger(-a) - L_\sigma^\dagger(a)) (L_\sigma(-\varepsilon) - L_\sigma(\varepsilon)) \right] \]
\[ + (R_\sigma^\dagger(a) - R_\sigma^\dagger(-a)) (R_\sigma(\varepsilon) - R_\sigma(-\varepsilon)) \] + H.c. \quad (B1)

The leads Hamiltonian in Eq. (1e) can be written in terms of even and odd chiral fermions:
\[ \psi_{e,\sigma}(x) = \frac{1}{\sqrt{2}} (R_\sigma(x) + L_\sigma(x)); \quad \psi_{o,\sigma}(x) = \frac{1}{\sqrt{2}} (R_\sigma(x) - L_\sigma(x)). \quad (B2) \]
\[ H_{0}^{leads} = \hbar v_F \sum_{\sigma=\uparrow,\downarrow} \int_{-L/2}^{L/2} dx [\psi_{e,\sigma}^\dagger(x)(-i\partial_x)\psi_{e,\sigma}(x) + \psi_{o,\sigma}^\dagger(x)(-i\partial_x)\psi_{o,\sigma}(x)]. \quad (B3) \]

The boundary term \( H_{BC} \) becomes,
\[ H_{BC} = \frac{\hbar}{2} (\sin(k_F a)) \sin(k_F \varepsilon) \sum_{\sigma=\uparrow,\downarrow} \left\{ [(\psi_{e,\sigma}^\dagger(-a) - \psi_{e,\sigma}^\dagger(a)) - (\psi_{o,\sigma}^\dagger(a) - \psi_{o,\sigma}^\dagger(-a))] \right. \]
\[ + (\psi_{e,\sigma}(\varepsilon) - \psi_{e,\sigma}(-\varepsilon)) - (\psi_{o,\sigma}(\varepsilon) - \psi_{o,\sigma}(-\varepsilon)) \left. \right\} + |(\psi_{e,\sigma}(a) - \psi_{e,\sigma}(-a)) + (\psi_{o,\sigma}(a) - \psi_{o,\sigma}(-a)) | \] + H.c. \quad (B4)
The representation (1i) is used in the Kondo regime where the symmetric state \( \psi_{\alpha,\sigma}(x) \) is screened out and \( H_{BC} \) is replaced by the antisymmetric state \( \psi_{e,\sigma}(x) \). In the Kondo limit \( H_{BC} \) is replaced by a “mass” term [the designation symmetric or antisymmetric is given according to the original fermions \( c_{R,\sigma}(x) \) and \( c_{L,\sigma}(x) \)],

\[
H_{BC} \simeq -2t_0 (2 \sin(k_F a))^2 \sum_{\sigma=\uparrow,\downarrow} \psi_{e,\sigma}^\dagger(0) \psi_{e,\sigma}(0).
\] (B5)
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FIGURES

FIG. 1. Conductance (G) as a function of gate voltage (\(\tilde{U}_G\)) for fixed weak transmission (\(\Gamma^2 = 0.1\)) for three different temperatures: \(\beta = 1\) (dotted line), \(\beta = 10\) (dashed line) and \(\beta = 50\) (solid line). Note that decreasing the temperature to \(\beta = 10\) leads to the conductance reaching the value of 1.0 and in addition there is a shoulder around 0.7.

FIG. 2. Conductance as a function of gate voltage for large transmission (\(\Gamma^2 = 5.0\)), i.e., for a short wire in the Kondo regime at very low temperature (\(\beta = 100\)). The conductance is about \(1.0 \times 2e^2/h\), as expected from the Kondo solution.