Analysis of the $X(1835)$ and related baryonium states with Bethe-Salpeter equation

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Abstract

In this article, we study the mass spectrum of the baryon-antibaryon bound states $p\bar{p}$, $\Sigma\Sigma$, $\Xi\Xi$, $\Lambda\bar{\Lambda}$, $p\bar{N}(1440)$, $\Sigma\Sigma(1660)$, $\Xi\Xi$ and $\Lambda\bar{\Lambda}(1600)$ with the Bethe-Salpeter equation. The numerical results indicate that the $p\bar{p}$, $\Sigma\Sigma$, $\Xi\Xi$, $p\bar{N}(1440)$, $\Sigma\Sigma(1660)$, $\Xi\Xi$ bound states may exist, and the new resonances $X(1835)$ and $X(2370)$ can be tentatively identified as the $p\bar{p}$ and $p\bar{N}(1440)$ (or $N(1400)p\bar{p}$) bound states respectively with some gluon constituents, and the new resonance $X(2120)$ may be a pseudoscalar glueball. On the other hand, the Regge trajectory favors identifying the $X(1835)$, $X(2120)$ and $X(2370)$ as the excited $\eta'/(958)$ mesons with the radial quantum numbers $n = 3$, 4 and 5, respectively.

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1 Introduction

In 2003, the BES collaboration observed a significant narrow near-threshold enhancement in the proton-antiproton ($p\bar{p}$) invariant mass spectrum in the radiative decay $J/\psi \rightarrow \gamma p\bar{p}$ [1]. The enhancement can be fitted with either an $S$-wave or a $P$-wave Breit-Wigner resonance function. In the case of the $S$-wave fitted form, the mass and the width are $M = (1859^{+3}_{-10} +^{+5}_{-25})$ MeV and $\Gamma < 30$ MeV, respectively. In 2005, the BES collaboration observed a resonance state $X(1835)$ in the $\eta'/\pi^+\pi^-\eta'$ invariant mass spectrum in the process $J/\psi \rightarrow \gamma \pi^+\pi^-\eta'$ with the Breit-Wigner mass $M = (1833.7\pm 6.2\pm 2.7)$ MeV and the width $\Gamma = (67.7\pm 20.3\pm 7.7)$ MeV, respectively [2]. Recently, the $X(1835)$ was confirmed by the BES collaboration in the radiative decay $J/\psi \rightarrow \gamma \pi^+\pi^-\eta'$ with a statistical significance larger than 20$\sigma$, the fitted mass and width are $M = (1836.5\pm 3.0^{+5.6}_{-2.1})$ MeV and $\Gamma = (190\pm 9^{+38}_{-36})$ MeV, respectively [3, 4]. The mass is consistent with the BESII result [2], while the width is significantly larger. Furthermore, the BES collaboration observed two new resonances $X(2120)$ and $X(2370)$ in the $\pi^+\pi^-\eta'$ invariant mass spectrum with statistical significances larger than 7.2$\sigma$ and 6.4$\sigma$, respectively. The measured masses and widths are $M_{X(2120)} = (2122.4\pm 6.7^{+1.7}_{-2.2})$ MeV, $M_{X(2370)} = (2376.3\pm 8.7^{+3.2}_{-4.3})$ MeV, $\Gamma_{X(2120)} = (83\pm 16^{+31}_{-11})$ MeV and $\Gamma_{X(2370)} = (83\pm 17^{+44}_{-6})$ MeV, respectively [3, 4].

Many theoretical works were stimulated to interpret the nature and the structure of the new resonance $X(1835)$, such as the $p\bar{p}$ bound state [5], the pseudoscalar glueball [6, 7], and the radial excitation of the $\eta'$ [8, 9, 10], the threshold cusp [11], etc.

In Ref. [12], Liu, Ding and Yan study the decay widths of the second and the third radial excitations of the pseudoscalar mesons $\eta$ and $\eta'$ with the $3P_0$ model, and observe that the interpretation of the $\eta(1760)$ and $X(1835)$ as the second radial excitations of the $\eta$ and $\eta'$ crucially depends on the measured mass and width of the $\eta(1760)$, and suggest that

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there may be sizable $p\bar{p}$ content in the $X(1835)$, and the $X(2120)$ and $X(2370)$ cannot be understood as the third radial excitations of the $\eta$ and $\eta'$ respectively, the $X(2370)$ probably is a mixture of the $\eta'$ and glueball.

In Ref.\[13\], we take the $X(1835)$ as a baryonium state with the quantum numbers $J^{PC} = 0^{-+}$, and calculate the mass spectrum of the baryon-antibaryon bound states $p\bar{p}$, $\Sigma\bar{\Sigma}$, $\Xi\bar{\Xi}$, and $\Lambda\bar{\Lambda}$ in the framework of the Bethe-Salpeter equation with a phenomenological potential. The numerical results indicate that the $p\bar{p}$, $\Sigma\bar{\Sigma}$ and $\Xi\bar{\Xi}$ bound states maybe exist, and the $X(1835)$ can be tentatively identified as the $p\bar{p}$ bound state. In this article, we extend our previous work to study whether or not there exist the baryon-antibaryon bound states $p\bar{N}(1440)$, $\Sigma\bar{\Sigma}(1660)$, $\Xi\bar{\Xi}$ and $\Lambda\bar{\Lambda}(1600)$ (or $N(1440)\bar{p}$, $\Sigma(1660)\bar{\Sigma}$, $\Xi'\bar{\Xi}$ and $\Lambda(1600)\bar{\Lambda}$), here the $t$ denotes the first radial excited state. In the scenario of the coupled channel effects or the hadronic dressing mechanism [14], the pseudoscalar mesons $X(1835)$, $X(2120)$ and $X(2370)$ can be taken as having small pseudoscalar $q\bar{q}$ kernels of the typical $q\bar{q}$ meson size. The strong couplings to the virtual intermediate hadronic states (for example, the $p\bar{p}$, $p\bar{N}(1440)$, $\Sigma\bar{\Sigma}$, $\Xi\bar{\Xi}$, etc) can change the bare masses which originate from the quark-gluon interactions significantly, and enrich the pure $q\bar{q}$ states with some baryon-antibaryon components. In the present case, the input parameters of the Bethe-Salpeter equation should be readjusted to obtain reasonable predicted masses if the coupled-channel effects are considered. However, it is beyond the scope of the present work. Such a scenario needs detailed studies.

The article is arranged as follows: we solve the Bethe-Salpeter equation for the baryon-antibaryon bound states in Sec.2; in Sec.3, we present the numerical results and discussions; and Sec.4 is reserved for our conclusions.

### 2 Bethe-Salpeter equation

The Bethe-Salpeter equation is a conventional approach in dealing with the two-body relativistic bound state problems, and has given many successful descriptions of the hadron properties [15, 16]. We write down the ladder Bethe-Salpeter equation for the pseudoscalar bound states in the Euclidean spacetime, which can be derived from the Euclidean path-integral formulation of the theory,

\[ S_{1}^{-1}(q + \xi_{1}P)\chi(q,P)S_{2}^{-1}(q - \xi_{2}P) = \int \frac{d^{4}k}{(2\pi)^{4}} \gamma_{5}\chi(k,P)\gamma_{5}G(q - k), \]

\[ S_{1/2}^{-1}(q \pm \xi_{1/2}P) = i (\gamma \cdot q \pm \xi_{1/2}\gamma \cdot P) + M_{1/2}, \]

\[ \xi_{1/2} = \frac{M_{1/2}}{M_{1} + M_{2}}, \]

where the $P_{\mu}$ is the four-momentum of the center of mass of the baryon-antibaryon bound state, the $q_{\mu}$ is the relative four-momentum between the baryon and antibaryon, $\gamma_{5}$ is the bare baryon-meson vertex, the $\chi(q,P)$ is the Bethe-Salpeter amplitude of the baryon-antibaryon bound state, and the $G(q - k)$ is the interaction kernel.

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\[ \text{In this article, we use the metric } \delta_{\mu\nu} = (1,1,1,1), \{\gamma_{\mu}\gamma_{\nu} + \gamma_{\nu}\gamma_{\mu}\} = 2\delta_{\mu\nu}, \text{ the momentums } k_{\mu} = (k_{1}, \vec{k}), q_{\mu} = (q_{4}, \vec{q}) \text{ and } P_{\mu} = (iE, \vec{P}) \text{ with } P^{2} = -M_{X}^{2}. \]
In the flavor $SU(3)$ symmetry limit, the interactions among the ground state octet baryons and the pseudoscalar mesons can be described by the lagrangian $\mathcal{L}$,

$$\mathcal{L} = \sqrt{2} \left( D \text{Tr} \left( \bar{B} \{ P, B \} \right) + F \text{Tr} \left( \bar{B} [ P, B ] \right) \right), \quad (2)$$

where

$$B = \begin{pmatrix}
\frac{1}{\sqrt{2}} \Sigma^0 + \frac{1}{\sqrt{6}} \Lambda & \Sigma^+ \\
\Sigma^- & -\frac{1}{\sqrt{2}} \Sigma^0 + \frac{1}{\sqrt{6}} \Lambda \\
\Xi^- & \Xi^0 \\
\Xi^+ & \Xi^0 \\
\frac{1}{\sqrt{2}} \pi^0 + \frac{1}{\sqrt{6}} \eta & \pi^+ \\
\pi^- & -\frac{1}{\sqrt{2}} \pi^0 + \frac{1}{\sqrt{6}} \eta \\
K^- & K^0 \\
K^+ & K^0 \\
\end{pmatrix}, \quad (3)$$

and the $D$ and $F$ are two parameters for the coupling constants. From the lagrangian, we can obtain

$$g_{\pi^0 pp} = -g_{\pi^0 nm} = D + F, \quad g_{\pi^0 \Sigma^+ \Sigma^-} = -g_{\pi^0 \Sigma^- \Sigma^-} = 2F,$$

$$g_{\pi^0 \Xi^- \Xi^-} = -g_{\pi^0 \Xi^0 \Xi^0} = D - F, \quad g_{\eta ppm} = g_{\eta nm} = -\frac{D - 3F}{\sqrt{3}},$$

$$g_{\eta \Sigma^+ \Sigma^-} = g_{\eta \Xi^- \Xi^-} = g_{\eta \Sigma^0 \Sigma^0} = -g_{\eta \Lambda \Lambda} = \frac{2D}{\sqrt{3}},$$

$$g_{\eta \Xi^- \Xi^-} = g_{\eta \Xi^0 \Xi^0} = -\frac{D + 3F}{\sqrt{3}}, \quad (4)$$

and write down the kernel $G(q - k)$ explicitly,

$$G(q - k) = \frac{g^2(q - k)C_\pi}{(q - k)^2 + m_\pi^2} + \frac{g^2(q - k)C_\eta}{(q - k)^2 + m_\eta^2}, \quad (5)$$

where the coefficients $C_\pi = (1 + \alpha)^2$, $4\alpha^2$, $\alpha^2$, 0 and $C_\eta = (1 - 3\alpha)^2$, $\frac{4}{3}$, $(1 + 3\alpha)^2$, $\frac{1}{3}$ for the $pp$, $\Sigma \Sigma$, $\Xi \Xi$, $\Lambda \Lambda$ bound states respectively; $g^2(k) = D^2$ and $\alpha = \frac{F}{D}$.

With a simple replacement

$$B \rightarrow B', \quad D \rightarrow D', \quad F \rightarrow F', \quad \alpha \rightarrow \alpha', \quad g \rightarrow g', \quad (6)$$

where

$$B' = \begin{pmatrix}
\frac{1}{\sqrt{2}} \Sigma(1660) + \frac{1}{\sqrt{6}} \Lambda(1600) & \Sigma(1660) \\
\Sigma(1660) & -\frac{1}{\sqrt{2}} \Sigma(1660) + \frac{1}{\sqrt{6}} \Lambda(1600) \\
\Xi'(?) & \Xi'(?) \\
\end{pmatrix}, \quad (7)$$

we can obtain the corresponding couplings among the first radial excited octet baryons and the pseudoscalar mesons. The corresponding coefficients are $C_\pi = (1 + \alpha)(1 + \alpha')$, $4\alpha\alpha'$, $(1 - \alpha)(1 - \alpha')$, 0 and $C_\eta = (1 - 3\alpha)(1 - 3\alpha')$, $\frac{4}{3}$, $(1 + 3\alpha)(1 + 3\alpha')$, $\frac{1}{3}$ for the bound states $p\bar{N}(1440)$, $\Sigma \Sigma$, $\Xi \Xi$, $\Lambda \Lambda$, respectively. In this article, we can take the approximation $\alpha = \alpha'$ and $g = g'$ (i.e. we assume that the ground
As in our previous work, we take the approximation with \( n = 3 \) for simplicity. We can denote the baryon fields as \( \Psi(x) \). The state can be decomposed as \([16]\). A Gaussian distribution warrants the integral in the Bethe-Salpeter equation is convergent.

The annihilations to the intermediate mesons can be estimated as \( \propto (\frac{g^2(M_X^2)}{M_X^2 + m_{\pi,\eta,\cdots}^2})^n \) with \( n \geq 1 \). If we take the typical momentum \( q = \sqrt{2M_p|E|} \approx 200 \text{ MeV} \), where the \( M_p \) is the reduced mass in the \( p\bar{p} \) system, and the \( E \) is the bound energy of the \( X(1835) \) as the \( p\bar{p} \) bound state, \( M_X \gg q \), and \( (\frac{g^2(M_X^2)}{M_X^2 + m_{\pi,\eta,\cdots}^2})^n \ll \frac{g^2(q^2)}{q^2 + m_{\pi,\eta,\cdots}^2} \), such annihilation effects can be neglected. The annihilation effects in other channels can also be neglected with analogous arguments.

In Ref.\([13]\), we choose the value \( \alpha = 0.6 \) from the analysis of the hyperon semi-leptonic decays \([18]\), and take the coupling constant \( g^2(k) \) as a modified Gaussian distribution \( g^2(k) = A \left( \frac{k^2}{\mu^2} \right)^2 \exp \left( -\frac{k^2}{\mu^2} \right) \) in the Euclidean spacetime, where the strength \( A \) and the distribution width \( \mu \) are two free parameters. The ultraviolet behavior of the modified Gaussian distribution warrants the integral in the Bethe-Salpeter equation is convergent.

The Euclidean Bethe-Salpeter amplitude of the pseudoscalar baryon-antibaryon bound state can be decomposed as \([16]\)

\[
\chi(q, P) = \gamma_5 \{ F(q, P) + i \, P F_1(q, P) + i \, q F_2(q, P) + \{ q, P \} F_3(q, P) \} . \quad (8)
\]

As in our previous work, we take the approximation

\[
\chi(q, P) = \gamma_5 \{ F(q, P) + i \, P F_1(q, P) \} , \quad (9)
\]

for simplicity. We can denote the baryon fields as \( \Psi(x) \), and perform the Fierz re-ordering to study the contributions from different spinor structures\([3]\):

\[
\Psi_\alpha(0) \Psi_\beta(x) = -\frac{1}{4} \delta_{\alpha\beta} \Psi(x) \Psi(0) - \frac{1}{4} (\gamma^\mu)_{\alpha\beta} \Psi(x) \gamma_\mu \Psi(0) - \frac{1}{8} (\sigma^{\mu\nu})_{\alpha\beta} \Psi(x) \sigma_{\mu\nu} \Psi(0)
+ \frac{1}{4} (\gamma^5)_{\alpha\beta} \Psi(x) \gamma_5 \Psi(0) + \frac{1}{4} (i\gamma_5)_{\alpha\beta} \Psi(x) i\gamma_5 \Psi(0) . \quad (10)
\]

The pseudoscalar current \( \Psi(x) i\gamma_5 \Psi(0) \), the axialvector current \( \Psi(x) \gamma_\mu \gamma_5 \Psi(0) \) and the tensor current \( \Psi(x) \sigma_{\mu\nu} \Psi(0) \) have nonvanishing couplings with the pseudoscalar meson.
the coefficients $A_0$, $A_1$, $B_0$, $B_1$, $C_0$ are functions of the $q_\mu$ and $P_\mu$. We can take the estimations $P_\mu \sim M_X \approx 1.8$ GeV, $q_\mu \sim \sqrt{2M_F|E|} \approx 0.2$ GeV, $q,P \sim \sqrt{2M_F|E|M_X} \approx 0.36$ GeV$^2$, translate the $P_\mu$ and $q_\mu$ into dimensionless quantities, $P_\mu \rightarrow \tilde{P}_\mu \Lambda$, $q_\mu \rightarrow \tilde{q}_\mu \Lambda$ with $\Lambda = 1$ GeV, and absorb the $\Lambda$ into the coefficients $A_0$, $A_1$, $B_0$, $B_1$, $C_0$, then $P_\mu \sim 1.8$, $\tilde{q}_\mu \sim 0.2$, $[\tilde{q},\tilde{P}] \sim 0.36$. Compared with the term $\gamma_5 P$, the term $\gamma_5 \tilde{q}$ is greatly suppressed and can be neglected. Furthermore, we expect that couplings of the tensor currents to the pseudoscalar mesons are weaker than that of the pseudoscalar and axialvector currents, and neglect the term $\gamma_5 [\tilde{q},P]$, which is also suppressed as the $\gamma_5 \tilde{q}$.

The Bethe-Salpeter amplitudes $F(q,P)$ and $F_1(q,P)$ can be expanded in terms of Tchebychev polynomials $T_n^\frac{1}{P} (\cos \theta)$ \cite{19},

\[
F(q,P) = \sum_{n=0}^{\infty} i^n F^n(q^2, P^2) q^n P^n T_n^\frac{1}{P} (\cos \theta),
\]

\[
F_1(q,P) = \sum_{n=0}^{\infty} i^n F_1^n(q^2, P^2) q^n P^n T_n^\frac{1}{P} (\cos \theta),
\]

where $\theta$ is the included angle between $q_\mu$ and $P_\mu$. If we translate the momenta $q_\mu$ and $P_\mu$ into the dimensionless quantities $\tilde{q}_\mu$ and $\tilde{P}_\mu$ respectively, and absorb the $\Lambda$ into the $F^n(q^2, P^2)$ and $F_1^n(q^2, P^2)$, then $\tilde{q}\tilde{P} T_\frac{1}{P}^\frac{1}{P} (\cos \theta) \sim 0.36 \cos \theta \sim 0$, $\tilde{q}^2 \tilde{P}^2 T_\frac{1}{P}^\frac{1}{P} (\cos \theta) \sim 0.13 \cos 2\theta \sim -0.03$, here we have taken the average $\cos n\theta \approx \frac{1}{\pi} \int_0^{\pi} \cos n\theta \sin^2 \theta d\theta$. It is impossible to solve an infinite series of coupled equations of the $F^n(q^2, P^2)$ and $F_1^n(q^2, P^2)$, we have to make truncation in one or the other ways. In this article, we neglect the small terms with $n \geq 1$. Numerical calculations indicate that taking only the terms with $n = 0$ can give satisfactory results. If we take into account the small terms with $n \geq 1$, the predictions may be improved mildly. In the following, we will smear the index 0 for simplicity.

Multiplying both sides of the Bethe-Salpeter equation by $\gamma_5 [\tilde{q},P]$ and carrying out the trace in the Dirac spinor space, we can obtain an simple relation $F = (M_1 + M_2)F_1$, the amplitudes $F(q^2, P^2)$ and $F_1(q^2, P^2)$ are not independent. The Bethe-Salpeter amplitude can be written as

\[
\chi(q,P) = \gamma_5 \left( 1 + \frac{iP}{M_1 + M_2} \right) F(q^2, P^2),
\]

and the Bethe-Salpeter equation can be projected into the following form,

\[
\left( q^2 + M_1 M_2 + \frac{M_1 M_2}{(M_1 + M_2)^2} P^2 \right) F(q^2, P^2) = \int \frac{dk}{(2\pi)^4} F(k^2, P^2) G(q-k). \]

X(1835),

\[
i\gamma_5(0)\bar{\Psi}(x)i\gamma_5 \Psi(0)|X(P)\rangle \propto \gamma_5 A_0 + \gamma_5 q \cdot PA_1 + \cdots,
\]

\[
\gamma^\mu\gamma_5(0)\bar{\Psi}(x)\gamma_\mu\gamma_5 \Psi(0)|X(P)\rangle \propto \gamma_5 PB_0 + \gamma_5 \tilde{q}B_1 + \cdots, \tag{11}
\]

\[
s_{\mu\nu}(0)\bar{\Psi}(x)s_{\mu\nu} \Psi(0)|X(P)\rangle \propto \gamma_5 [\tilde{q},P] C_0 + \cdots, \tag{12}
\]
We can introduce a parameter $\lambda(P^2)$ and solve above equation as an eigenvalue problem. If there really exists a bound state in the pseudoscalar channel, the mass of the bound state $X$ can be determined by the condition $\lambda(P^2 = -M_X^2) = 1$,

$$\left(q^2 + M_1 M_2 + \frac{M_1 M_2}{(M_1 + M_2)^2} P^2\right) F(q^2, P^2) = \lambda(P^2) \int \frac{d^4 k}{(2\pi)^4} F(k^2, P^2) G(q - k).$$

(16)

If we take $q^2 = 0$ and assume that there exists a physical solution, then

$$\left(M_1 M_2 - \frac{M_1 M_2}{(M_1 + M_2)^2} M_X^2\right) F(0, -M_X^2) = \int \frac{d^4 k}{(2\pi)^4} F(k^2, -M_X^2) G(0 - k).$$

(17)

In numerical calculations, we observe that the Bethe-Salpeter amplitude $F(k^2, -M_X^2)$ has the same sign in the region $k^2 \geq 0$,

$$M_1 M_2 - \frac{M_1 M_2}{(M_1 + M_2)^2} M_X^2 = \int \frac{d^4 k}{(2\pi)^4} F(k^2, -M_X^2) G(0 - k) > 0,$$

(18)

and obtain an simple relation (or constraint),

$$M_X^2 < (M_1 + M_2)^2,$$

(19)

which survives for $q^2 > 0$ (although the relation is not explicit for $q^2 > 0$), i.e. the bound energy $E_X$ originates from the interacting kernel $G(k)$ and should be negative, $E_X = M_X - M_1 - M_2 < 0$. On the other hand, if the Bethe-Salpeter amplitude $F(k^2, -M_X^2)$ changes sign in the region $k^2 \geq 0$, which does not warrant the positive value $\int \frac{d^4 k}{(2\pi)^4} F(k^2, -M_X^2) G(0 - k) > 0$, and the relation $M_X^2 < (M_1 + M_2)^2$ fails to survive.

3 Numerical results and discussions

The input parameters are taken as $m_\pi = 135$ MeV, $m_\eta = 548$ MeV, $M_p = 938.3$ MeV, $M_{\Sigma^+} = 1189.4$ MeV, $M_{\Sigma^-} = 1321.7$ MeV, $M_\Lambda = 1115.7$ MeV, $M_{\Lambda(N(1440))} = (1420 - 1470)$ MeV $\approx 1440$ MeV, $M_{\Sigma(1660)} = (1630 - 1690)$ MeV $\approx 1660$ MeV, $M_{\Lambda(1600)} = (1560 - 1700)$ MeV $\approx 1600$ MeV, and $M_{\Lambda(1835)} = 1833.7$ MeV from the Review of Particle Physics [20]. The first radial excited state of the $\Xi$ has not been observed yet, we take the approximation $M_{\Xi^*} = M_\Xi + \frac{M_{\Xi(1660)} - M_\Xi + M_{N(1440)} - M_p + M_{\Lambda(1600)} - M_\Lambda}{3} \approx M_\Xi + 486$ MeV. The strength $A$ and the distribution width $\mu$ are free parameters, we take the values $A = 215$ and $\mu = 200$ MeV for the $p\bar{p}$ bound state as in Ref. [13], and take the simple replacements $\mu \rightarrow \mu \frac{M^2}{M^2_p}$, $\mu \frac{M^2}{M^2_p}$ and $\mu \frac{M^2}{M^2_p}$ to take into account the flavor SU(3) breaking effects for the $\Sigma\Xi$ ($\Sigma\Sigma(1660)$), $\Xi\Xi$ ($\Xi\Xi^*$) and $\Lambda\Lambda$ ($\Lambda\Lambda(1600)$) bound states, respectively.

We solve the Bethe-Salpeter equations as an eigen-problem numerically by direct iterations, and observe that the convergent behaviors are very good. For the $p\bar{p}$, $\Sigma\Xi$, $\Xi\Xi$, $p\bar{N}(1440)$, $\Sigma\Sigma(1660)$ and $\Xi\Xi^*$ bound states, there exists a solution with $\lambda(P^2 = -M_X^2) = 1$ and $E_X < 0$. On the other hand, we cannot obtain a solution to satisfy the condition $\lambda(P^2 = -M_X^2) = 1$ for the $\Lambda\Lambda$ and $\Lambda\Lambda(1600)$ bound states. Experimentally, there are $\Lambda\Lambda$ near threshold enhancements in the decays $B^+ \rightarrow \Lambda\Lambda K^+$, $B^0 \rightarrow \Lambda\Lambda K^0$, $\Lambda\Lambda K^{*0}$ [21, 22, 23].
Table 1: The masses $M_X$ and bound energies $E_X$ of the baryon-antibaryon bound states, the $*$ denotes that the upper bounds of the masses of the $N(1440)$ and $\Sigma(1660)$ baryons are taken.

| $M_X$ [MeV] | $p\bar{p}$ | $\Sigma\Sigma$ | $\Xi\Xi$ | $p\bar{N}(1440)$ | $\Sigma\Sigma(1660)$ | $\Xi\Xi'$ |
|-------------|-------------|---------------|---------|-----------------|-----------------|--------|
| $M_X$ [MeV] | (μ = 200 MeV, τ = 0.55) | 2378.2/2408.2* | 2375.3/2405.3* | 2849.3/2879.3* | 2832.9/2862.9* | 3192.3 |
| $M_X$ [MeV] | (μ = 200 MeV, τ = 0.78) | 2363.2/2393.2* | 2813.9/2843.9* | 3118.1 |
| $M_X$ [MeV] | (μ = 200 MeV, τ = 1.00) | 2344.0/2374.0* | 2798.4/2828.4* | 3103.4 |
| $M_X$ [MeV] | (μ = 400 MeV, τ = 0.44) | 2378.2/2408.2* | 2849.3/2879.3* |
| $M_X$ [MeV] | (μ = 400 MeV, τ = 0.52) | 2361.8/2391.8* | 2832.3/2862.3* | 3129.3 |
| $M_X$ [MeV] | (μ = 400 MeV, τ = 0.60) | 2336.8/2366.8* | 2784.4/2814.4* | 3084.4 |
| expt [MeV] | ?2376.3 | ?2376.3 | ?2376.3 | ?2376.3 | ?2376.3 | ?2376.3 |

Table 2: The masses $M_X$ of the baryon-antibaryon bound states with variations of the input parameters, the * denotes that the upper bounds of the masses of the $N(1440)$ and $\Sigma(1660)$ baryons are taken.

and above threshold enhancements in the decays $B^+ \rightarrow \Lambda\bar{\Lambda}K^+$ from the Belle collaboration, the above threshold enhancements can be identified as the $J/\psi$ and $\eta_c$ mesons respectively, the decays $J/\psi \rightarrow \Lambda\bar{\Lambda}$ and $\eta_c \rightarrow \Lambda\bar{\Lambda}$ are observed. The $\Lambda\bar{\Lambda}$ near threshold enhancement may be a $\Lambda\bar{\Lambda}$ baryonium state or just a final-state re-scattering effect, more experimental data are still needed to identify it. We can study the baryon-antibaryon scattering amplitudes in unitary Chiral perturbation theory by taking into account the intermediate multichannel baryon-loops (for example, the $p\bar{p}$, $\Lambda\bar{\Lambda}$, $\Sigma\Sigma$, etc.), and adjust the parameters in the phenomenological lagrangian to reproduce the $p\bar{p}$ baryonium state $X(1835)$, and explore whether or not there exists a pole related with the $\Lambda\bar{\Lambda}$ baryonium state.

The numerical results for the Bethe-Salpeter amplitudes are shown in Fig.1 and the values of the bound states are presented in Table 1. From the Table, we can see that the new resonances $X(1835)$ and $X(2370)$ can be tentatively identified as the $p\bar{p}$ and $p\bar{N}(1440)$ (or $N(1440)\bar{p}$) bound states, respectively, while the mass of the $\Sigma\Sigma$ bound state disfavors identifying it as the $X(2120)$ or $X(2370)$, because the energy gaps $M_{\Sigma\Sigma} - M_X(2120) = 195.4$ MeV and $M_{\Sigma\Sigma} - M_X(2370) = -58.5$ MeV.

In Ref.[7], Hao, Qiao and Zhang study the $0^{-+}$ three-gluon glueball with the QCD sum rules, and observe that its mass lies in the region of $(1.9 - 2.7)$ GeV, while the
Figure 1: The Bethe-Salpeter amplitudes of the baryon-antibaryon bound states where $P^2 = -M_X^2$, $A$, $B$, $C$, $D$, $E$ and $G$ denote the $pp$, $\Sigma \Sigma$, $\Xi \Xi$, $p\bar{N}(1440)$, $\Sigma \bar{\Sigma}(1660)$ and $\Xi \bar{\Xi}'$, respectively. The line-shapes of the $D$, $E$ and $G$ are plotted with the replacement $q^2/\mu^2 \rightarrow (q^2 - 10\mu^2)/\mu^2$ to avoid overlapping with the $A$, $B$ and $C$.

Figure 2: The Regge trajectory for the $\eta'$ mesons.
quenched lattice QCD calculations indicate that the pseudoscalar glueballs have masses about 2590(40)(130) MeV 24 or 2560(35)(120) MeV 25. We should bear in mind that it is the quenched lattice QCD not the full lattice QCD, as the fermion determinant is neglected. In fact, the predictions vary with the theoretical approaches, for example, the mass of the low-lying pseudoscalar glueball \( G \) is about 2.22 GeV from the quenched QCD Hamiltonian in the Coulomb gauge 26, 2.62 GeV from the constituent gluons model with the Cornell potential within the helicity formalism 27, 2.28 GeV from the string Hamiltonian derived from the vacuum correlator method 28, 2.19 GeV from the refined Gribov-Zwanziger version of the Landau gauge 29, etc. In Ref. 30, H. Y. Cheng deduces the mass of the pseudoscalar glueball \( G \) from an \( \eta - \eta' - G \) mixing formalism based on the anomalous Ward identity for transition matrix elements, and find a solution \( M_G = (1.4 \pm 0.1) \text{ GeV} \) with the inputs from the KLOE experiment. If the \( \eta(1405) \) is confirmed as a pseudoscalar glueball one day, it is not necessary that the low-lying pseudoscalar glueballs should have masses about 2.5 GeV. The new resonance \( X(2120) \) may be a pseudoscalar glueball, and there may be mixings among the baryonium states and the glueballs, as the baryon-antibaryon bound states can annihilate into three gluons.

In calculations, we have used a universal coupling constant \( g^2 (g = g') \). In fact, no experimental data warrants that the coupling constants \( g_{NN\pi}, g_{NN(1440)\pi} \) and \( g_{N(1440)N(1440)\pi} \) have equal values. The experimental value \( \frac{g_{NN\pi}}{g_{NN(1440)\pi}} \approx 0.39 - 0.55 \) 31 and the theoretical value from a special non-relativistic quark model \( \frac{g_{NN\pi}}{g_{NN(1440)\pi}} \approx 0.33 \) 32 deviate from 1 obviously, we expect that the values \( |g_{N(1440)N(1440)\pi}| \leq |g_{NN\pi}|. \) In phenomenological applications, we often introduce the monopole (or dipole) form-factors 33 and the exponential form-factors 34 to parameterize the off-shell effects, and there are some form-factors associate with the coupling constants. In this article, we use the modified Gaussian distribution \( g^2(k) = A \left( \frac{k^2}{\mu^2} \right)^2 \exp \left( - \frac{k^2}{\mu^2} \right) \), which is assumed to take into account the form-factors effectively, and introduce a parameter \( \tau \) (with the value \( 0 < \tau \leq 1 \)) to parameterize the difference between the \( g_{NN\pi} \) and the \( g_{N(1440)N(1440)\pi} \), i.e. \( g^2(k) \rightarrow g(k)g'(k) = \tau A \left( \frac{k^2}{\mu^2} \right)^2 \exp \left( - \frac{k^2}{\mu^2} \right) \). We solve the Bethe-Salpeter equations with variations of the parameters \( \tau \) and \( \mu \), where the flavor SU(3) breaking effects for the \( \Sigma\Sigma(1660) \) and \( \Xi\Xi' \) bound states are taken into account by the simple replacements \( \mu \rightarrow \mu \frac{M_0^2}{M_F^2} \) and \( \mu \frac{M_0^2}{M_F^2} \) respectively, the eigenvalues are presented in Table 2. In calculations, we observe that in some regions, there indeed exist solutions in the channels \( p\bar{N}(1440) \), \( \Sigma\Sigma(1660) \) and \( \Xi\Xi' \), and some eigenvalues of the \( p\bar{N}(1440) \) (or \( p\bar{N}(1440) \)) bound state with \( \tau = 0.44 - 1.00 \) and \( \mu = (200 - 400) \text{ MeV} \) are consistent with the mass of the \( X(2370) \).

The radiative decays of the \( J/\psi \) are generally believed to be glue-rich, which can explain the branching ratio of the decay \( J/\psi \rightarrow \gamma \eta' \) is large (about \( 5.28 \pm 0.15 ) \times 10^{-3} \)), while the branching ratio of the \( J/\psi \rightarrow \gamma \eta \) is small (about \( 1.104 \pm 0.034 ) \times 10^{-3} \) 20. The observation of the \( X(1835) \), \( X(2120) \) and \( X(2370) \) in the \( \eta' \) channel not in the \( \eta \) channel maybe due to the intermediate virtual gluons are flavor-neutral and the \( \eta' \) meson is mainly an SU(3) flavor singlet and has considerable gluon constituent via the axial anomaly. It is natural to assume the \( X(1835) \) and \( X(2370) \) have some gluon constituents, which play an important role in the decays to \( \pi^+ \pi^- \eta' \).

The hadronic molecular state \( A \) which consists of a meson pair or a baryon pair \( B + C \) can decay through two typical routines, the first one is \( A \rightarrow B + C \rightarrow B + E + F + \cdots \),
and the second one is $A \rightarrow B + C \rightarrow E + F + G + \cdots$, then the decay widths are

determined by the intermediate process $C \rightarrow E + F$ or the annihilation of the $B + C$, we
can estimate the widths of the molecular states via the decay mechanisms. For example, in
Ref. [35], Guo, Hanhart and Meissner take the $Y(4660)$ as a $\psi' f_0(980)$ molecular state
considering the nominal threshold of the $\psi' - f_0(980)$ system is about $4666 \pm 10$ MeV
[20]. The $Y(4660)$ decays dominantly via the decay of the scalar meson $f_0(980)$, i.e.

$Y(4660) \rightarrow \psi' f_0(980) \rightarrow \psi' \pi \pi$, $\psi' K \bar{K}$, and the width of the $Y(4660)$ originates from the
decay of the $f_0(980)$ mainly. On the other hand, if we take the $X(3872)$ as the $D^* \bar{D} \pm D^* \bar{D}$
molecular state, the decay $X(3872) \rightarrow \bar{D}^0 D^0 \pi^0$ can occur through the decays $D^* \rightarrow D \pi^0$
and $\bar{D}^* \rightarrow \bar{D} \pi^0$ [39], the narrow widths of the $D^*$ and $\bar{D}^*$ mesons warrant that the
width of the $X(3872)$ is not broad, furthermore, the decay is suppressed kinematically in the
phase-space. In the present case, the thresholds $2M_p = 1876$ MeV $> 1835$ MeV and
$M_p + M_{N(1440)} = 2408$ MeV $> 2376$ MeV, the decays $X(1835) \rightarrow p\bar{p} \rightarrow \eta' \pi^+ \pi^-$
and $X(2370) \rightarrow pN(1440), N(1440)p \rightarrow \eta' \pi^+ \pi^-$ can take place via the Okubo-Zweig-
Iizuka super-allowed fall apart mechanism with re-arrangement in the color space. The
decays $X(1835) \rightarrow p\bar{p}$ and $X(2370) \rightarrow pN(1440), N(1440)p$ occur through the higher
tails of the mass distributions, and the widths may be large, although the decays are
suppressed kinematically at the lower tails of the mass distributions. We can search for the
$pN(1440), N(1440)p$ enhancements in the radiative decays $J/\psi \rightarrow \gamma pN(1440), \gamma N(1440)p$.

The recent BESIII data indicates that the $X(1835)$ has the width $\Gamma = (190 \pm 9^{+36}_{-36})$ MeV
[33, 4] it is too large for a pure molecular state. A larger mass glueball constituent $G$ besides the
$p\bar{p}$ component in the $X(1835)$ is needed to take into account the experimental data, the
decays $G \rightarrow ggg \rightarrow q\bar{q}gq\bar{q}$ can take place easily if kinematically allowed, furthermore,
such glue-rich processes prefer the final-state $\eta' \pi^+ \pi^-$. The conventional $q\bar{q}$ components in the
$X(1835)$ and $X(2370)$ can also lead to the decays $X(1835) \rightarrow p\bar{p}$ and $X(2370) \rightarrow pN(1440), N(1440)p$ with the creation of additional two $q\bar{q}$ pairs from the QCD vacuum,
however, the final-state $\eta \pi^+ \pi^-$ (rather than $\eta' \pi^+ \pi^-$) is preferred as such processes are
not glue-rich. It is difficult to calculate the decay widths of the $X(1835)$ and $X(2370)$
quantitatively in the framework of the Bethe-Salpeter equation.

If those bound states presented in Table 1 exist indeed, they can be produced in the
radiative $J/\psi$ decays, i.e. $J/\psi \rightarrow \gamma gg, gg + q\bar{q} \rightarrow p\bar{p}, \Sigma \Sigma, \Sigma \Xi, pN(1440), \Sigma \Sigma(1660)$, those
bound states can decay to the $\eta \pi \pi$, $\eta K \bar{K}$, $\eta' \pi \pi$, $\eta' K \bar{K}$, $\eta' \eta \eta$, $\eta' \eta' \eta$, $\eta \eta \eta$ final states.

We can search for those bound states in the $\eta \pi \pi$, $\eta K \bar{K}$, $\eta' \pi \pi$, $\eta' K \bar{K}$, $\eta' \eta \eta$, $\eta' \eta' \eta$, $\eta \eta \eta$ invariant
mass distributions in the radiative decays of the $J/\psi$ at the BESIII [37] or the charmless
$B$-decays at the KEK-B.

We do not exclude the canonical explanations, the $X(1835)$, $X(2120)$ and $X(2370)$
may be the conventional mesons which originate from the confining QCD forces and consist of the constituent quark-antiquark pairs with (or without) some gluon components. In Ref. [8], Huang and Zhu take the $X(1835)$ as the second radial excited state of the $\eta'$, the ground state nonet pseudoscalar mesons are $\{\pi, K, \eta, \eta'\}$, the first radial excited states are $\{\pi(1300), K(1460), \eta(1295), \eta'(1475)\}$, and the second radial excited states are $\{\pi(1800), K(1830), \eta(1760), X(1835)\}$. In Ref. [9], Klempt and Zaitsev perform detailed analysis of the properties of the $\eta(1295)$, $\eta(1405)$ and $\eta(1475)$, and draw the conclusion that there maybe only one $\eta$ state, the $\eta(1440)$, which has mass
about $1200 - 1500$ MeV, and identify the $X(1835)$ as the first radial excited state of the $\eta'$.
In Fig.2, we plot the $(n, M^2)$ for the $\eta'$ mesons, where the $n$ denotes the ra-
dial quantum numbers, the Regge trajectory favors identifying the $\eta(1475)$, $X(1835)$, $X(2120)$ and $X(2370)$ as the radial excited $\eta'$ mesons with $n = 2$, 3, 4 and 5, respectively. The Regge trajectory alone cannot result in definite identification. The decays $X(1835), X(2120), X(2370) \to \eta'\pi^+\pi^-$ take place through the emission of a pair of $S$-wave $\pi$ mesons, while the decays $X(1835), X(2120), X(2370) \to \eta\pi\pi$ have not been observed experimentally yet. Whether or not there exist those decay modes is of great importance, further experiments are needed to prove or exclude the possibility.

4 Conclusion

In this article, we study the mass spectrum of the baryon-antibaryon bound states $p\bar{p}$, $\Sigma\bar{\Sigma}$, $\Xi\bar{\Xi}$, $\Lambda\bar{\Lambda}$, $p\bar{N}(1440)$, $\Sigma\bar{\Sigma}(1660)$, $\Xi\bar{\Xi}'$ and $\Lambda\bar{\Lambda}(1600)$ in the framework of the Bethe-Salpeter equation with a phenomenological potential. The numerical results indicate that the $p\bar{p}$, $\Sigma\bar{\Sigma}$, $\Xi\bar{\Xi}$, $p\bar{N}(1440)$, $\Sigma\bar{\Sigma}(1660)$, $\Xi\bar{\Xi}'$ bound states maybe exist, and the new resonances $X(1835)$ and $X(2370)$ can be tentatively identified as the $p\bar{p}$ and $p\bar{N}(1440)$ bound states respectively with some gluon constituents, while the new resonance $X(2120)$ may be a pseudoscalar glueball. The other bound states predicted in this work may be observed experimentally in the future in the radiative decays of the $J/\psi$ at the BESIII or the charmless $B$-decays at the KEK-B. On the other hand, the Regge trajectory favors identifying the $\eta(1475)$, $X(1835)$, $X(2120)$ and $X(2370)$ as the excited $\eta'$ mesons with $n = 2$, 3, 4 and 5, respectively.

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