Impact Parameter Dependence of Inelasticity in $pp$ / $p\bar{p}$ Collisions

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Abstract

We study the impact parameter dependence of inelasticity in the framework of an updated geometrical model for multiplicity distribution. A formula in which the inelasticity is related to the eikonal is obtained. This framework permits a calculation of the multiplicity distributions as well as the inelasticity once the eikonal function is given. Adopting a QCD inspired parametrization for the eikonal, in which the gluon-gluon contribution dominates at high energy and determines the asymptotic behavior of the cross sections, we find that the inelasticity decreases as collision energy is increased. Our results predict the KNO scaling violation observed at LHC energies by CMS Collaboration.

Key words: Eikonal approximation, $pp$ / $p\bar{p}$ Inelasticity, Multiplicity distribution

PACS: 13.85.Hd, 12.40.Ee, 13.85.Ni

1 Introduction

Inelasticity is defined as the fraction of the available energy released for multiple particle production in inelastic hadronic interactions. The remaining part of the incident energy is carried away by the participant’s remnants, so called leading particles. The energy dependence of inelasticity is a problem of great interest from both theoretical and experimental standpoints [1]. However, the experimental data are scarce and, on the theoretical side, the existing models are largely in conflict with each other even in explaining a simple aspect as the center-of-mass energy dependence of the inelasticity [2], [3]. For example, the decrease in inelasticity with energy is advocated by some authors, while others believe that the inelasticity is an increasing function of energy [1]. Hence the problem remains unsolved. Naturally, multiplicity distributions are connected to inelasticity ones, so one can study multiplicity distribution features in order to derive information on the inelasticity behavior. Following this way, we have updated an existing phenomenological procedure [4], referred as Simple One String Model, which allows simultaneous description of several experimental data from elastic and inelastic channels through the Unitarity Equation [4]. Thus, based on the Simple One String Model formalism, able to describe the charged multiplicity distributions from ISR to Collider energies (30.4 - 900 GeV), we have computed the impact parameter dependence of inelasticity at fixed center-of-mass energies, $\sqrt{s}$. We have also inferred information on energy dependence of inelasticity. The plan of the paper is as follows. In the section 2 we present the basic formalism of the Simple One String Model and the predictions for $pp/p\bar{p}$ overall multiplicity distributions compared with the experimental data. In Section 3 we apply the theoretical framework computing the inelasticity of hadronic reactions. The final remarks are the content of Section 4.

2 Simple One String Model for Multiplicity Distributions

The Simple One String Model has been discussed in references [4], [5] and in order to define the notation and also update the model, we shall review the main points here. We work in impact parameter space, $b$, and to guarantee unitarity, the inelastic cross sections, $\sigma_{in}$, is calculated via the relation:

$$\sigma_{in}(s) = \int d^2b \, Gin(s, b)$$  \hspace{1cm} (1)

where

$$Gin(s, b) = 1 - e^{-2\chi_I(s, b)}$$ \hspace{1cm} (2)
is the Inelastic Overlap Function. In this work we have update the model adopting the complex eikonal function \( \chi_{pp}(s, b) = \chi_I(s, b) + i \chi_I(s, b) \), from Ref. [6], as will be discussed in subsection 2.1. In Ref. [4] has been adopted the Henzi Valin parametrization for \( G_{in}(s, b) \). The probabilities of \( n \) particle production, namely multiplicity distribution, \( P_n \), is the most general feature of the multiparticle production processes [7] and measurements of charged particle multiplicity distributions have revealed intrinsic features in \( pp / p\overline{p} \) interactions [8]. The multiplicity distribution is defined by the formula [9]

\[
P_n(s) = \frac{\sigma_n(s)}{\sum_{n=0}^{\infty} \sigma_n(s)} = \frac{\sigma_n(s)}{\sigma_{in}(s)},
\]

where \( \sigma_n \) is the cross section of an \( n \)-particle process (the so-called topological cross section). The charged multiplicity distribution, in the impact parameter formalism, may be constructed by summing contributions coming from hadron-hadron collisions taking place at fixed impact parameter. In this way, the idea of a normalized multiplicity distribution at each impact parameter \( b \) is introduced [10]. Thus the multiplicity distribution is written as

\[
P_n(s) = \frac{\sigma_n(s)}{\sigma_{in}(s)} = \frac{\int d^2b G_{in}(s, b) \left[ \frac{\sigma_n(s, b)}{\sigma_{in}(s, b)} \right]}{\int d^2b G_{in}(s, b)},
\]

where the topological cross section \( \sigma_n \) is decomposed into contributions from each impact parameter \( b \) with weight \( G_{in}(s, b) \). In the original formulation [4] the quantity \( m \) in brackets scales in KNO sense, and the Eq. (4) can be written as

\[
P_n(s) = \frac{\int d^2b G_{in}(s, b) \left[ < n(s, b) > \sigma_n(s, b) \right]}{\int d^2b G_{in}(s, b)},
\]

where \( < n(s, b) > \) is the average number of particles produced at \( b \) and \( \sqrt{s} \) due to the interactions among hadronic constituents involved in the collision and, in this model, \( < n(s, b) > \) factorizes as [4]

\[
< n(s, b) > \propto < N(s) > f(s, b),
\]

where \( < N(s) > \) is the average multiplicity at \( \sqrt{s} \) and \( f(s, b) \) is the so called multiplicity function. Similarly to KNO, is introduced, for each \( b \), the elementary multiplicity distribution

\[
\psi^{(1)} \left( \frac{n}{< n(s, b) >} \right) = < n(s, b) > \sigma_n(s, b) \sigma_{in}(s, b).
\]

Thus, with Eqs. (6) and (7), Eq. (5) becomes

\[
\Phi(s, z) = \frac{\int d^2b G_{in}(s, b) \psi^{(1)} \left( \frac{z}{f(s, b)} \right)}{\int d^2b G_{in}(s, b)},
\]

where \( \Phi(s, z) =< N(s) > P_n(s) \) and \( z = n/ < N(s) > \).

Here \( z \) represents the usual KNO scaling variable. Now, to obtain the multiplicity function \( f(s, b) \) in terms of the imaginary eikonal \( \chi_I \), it has been assumed that

1. the fractional energy \( \sqrt{s} \) that is deposited for particle production in a collision at \( b \) is proportional to \( \chi_I \):

\[
\sqrt{s} = \beta(s) \chi_I(s, b).
\]

The physical motivation of this equation is that eikonal may be interpreted as an overlap, on the impact parameter plane, of two colliding matter distributions [10];

2. the average number of produced particles depends on the energy \( \sqrt{s} \) in the same way as in \( e^- e^+ \) annihilations, which is approximately represented by a power law in \( \sqrt{s} \) [4]

\[
< n(s, b) > = \gamma \left( \frac{s}{s_0} \right)^A,
\]

where \( s_0 = 1 \text{ GeV}^2 \). In Ref. [11] a power law energy dependence of multiplicity in both \( pp \) and \( Pb + Pb \) collisions has been analyzed based on the gluon saturation scenario. Now, combining Eqs. (9), (10) and (6), we have

\[
f(s, b) = \frac{\gamma}{< N(s) >} \left[ \frac{\beta(s)}{\sqrt{s_0}} \right]^{2A} \chi_I(s, b)^{2A} = \xi(s) \chi_I(s, b)^{2A},
\]

with \( \xi(s) \) determined by the usual normalization conditions on \( \Phi [4] \) and that serves to determine \( \xi(s) \) as an energy dependent quantity, explicitly [4]

\[
\xi(s) = \frac{\int d^2b G_{in}(s, b) [\chi_I(s, b)]^{2A}}{\int d^2b G_{in}(s, b) [\chi_I(s, b)]^{2A}}.
\]

Thus, adopting an appropriate parametrization for \( G_{in} \) and \( \psi^{(1)} \), as well as an adequate value for \( A \), we can test the formalism embodied in Eq. (8) and (12), making direct comparisons with multiplicity distribution data. In the following, we will discuss the results obtained in the context of our updated model.

### 2.1 Inputs and Results on Multiplicity Distributions

The Simple One String Model is based on the idea of multiparticle creation due to the interactions between hadronic constituents in collisions taking place at \( b \). It is assumed that in parton-parton collision there is formation of a string, in which probably one of the 

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\Phi(s, z) =< N(s) > P_n(s) \text{ and } z = n/ < N(s) >.
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< n(s, b) > = \gamma \left( \frac{s}{s_0} \right)^A,
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f(s, b) = \frac{\gamma}{< N(s) >} \left[ \frac{\beta(s)}{\sqrt{s_0}} \right]^{2A} \chi_I(s, b)^{2A} = \xi(s) \chi_I(s, b)^{2A},
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with \( \xi(s) \) determined by the usual normalization conditions on \( \Phi [4] \) and that serves to determine \( \xi(s) \) as an energy dependent quantity, explicitly [4]

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\xi(s) = \frac{\int d^2b G_{in}(s, b) [\chi_I(s, b)]^{2A}}{\int d^2b G_{in}(s, b) [\chi_I(s, b)]^{2A}}.
\]

Thus, adopting an appropriate parametrization for \( G_{in} \) and \( \psi^{(1)} \), as well as an adequate value for \( A \), we can test the formalism embodied in Eq. (8) and (12), making direct comparisons with multiplicity distribution data. In the following, we will discuss the results obtained in the context of our updated model.
this work, we borrow their results. Specifically, assuming a gamma distribution normalized to 2,
\[ \psi^{(1)}(z) = 2 \frac{k^k}{\Gamma(k)} z^{k-1} e^{-kz}, \] (13)
experimental data on \( e^- e^+ \) multiplicity distributions were fitted, obtaining \( k=10.775 \pm 0.064 \) \((\chi^2/N_{DF} = 2.61)\). Also, the average multiplicity data in \( e^- e^+ \) annihilations were fitted by Eq. (10), giving \( A=0.258 \pm 0.001 \) and \( \gamma=2.09 \) \((\chi^2/N_{DF} = 8.89)\) in the interval \( 5.1 \) GeV \( \leq \sqrt{s} \leq 183 \) GeV, and \( A=0.198 \pm 0.004 \) \((\chi^2/N_{DF} = 1.7)\) for the set in the interval \( 10 \) GeV \( < \sqrt{s} \leq 183 \) GeV, respectively [4]. In the analysis done in Ref. [11] the value of \( A=0.11 \) was obtained within the gluon saturation picture. The difference between the values of \( A \) is, probably, associated with the different sets of experimental data used in each analysis. We recall that the value of \( A=0.11 \) was obtained by using experimental data for average multiplicity of hadrons in the gluon and quark jets in \( e^- e^+ \) annihilation, in the interval of the jet energy between \( 0.6 \sim 32 \) GeV [11]. At the end of the section the One String Model formalism will be tested using the three \( A \) values, above mentioned. Now is needed a parametrization for the eikonal function, and we have adopted the QCD-inspired complex eikonal from the work of Block et al. [6] in which the eikonal function is written as a combination of an even and odd eikonal terms related by crossing symmetry \( \chi_{pp}(s,b) = \chi^+(s,b) \pm \chi^-(s,b) \). The even eikonal is written as the sum of gluon-gluon, quark-gluon and quark-quark contributions:
\[ \chi^+(s,b) = \chi_{gg}(s,b) + \chi_{qg}(s,b) + \chi_{qq}(s,b), \] (14)
while the odd eikonal, that accounts for the difference between \( pp \) and \( p\bar{p} \), is parametrized as
\[ \chi^-(s,b) = C^- \sum \frac{m_g}{\sqrt{s}} e^{i\pi/4} W(b; \mu^-). \] (15)

The various parameters and functions involved in last two expressions are discussed in Ref.[6]. By fixing the value of \( A=0.258 \), \( \psi^{(1)} \) given by Eq. (13) with \( k=10.775 \), adopting \( G_{in} \) from analysis by Block et al. and observing that \( \xi(s) \) is obtained by Eq. (12), we have computed the overall multiplicity distributions arising from \( pp/\bar{p}p \) collisions at energies 52.6, 200, 546 and 900 GeV. The theoretical curves are shown in Figs. 1, 2, 3 and 4 together with the experimental data. The curves at \( pp \)-ISR 52.6 GeV and CERN \( p\bar{p} \) Collider 546 GeV shows excellent agreement with the data, Figs. 1 and 3 respectively. At energies \( \sqrt{s}=200 \) and 900 GeV, also in CERN \( p\bar{p} \) Collider, the agreement with data seems reasonable since the curves agree with experimental points for \( z' \geq 1 \), (high multiplicities), Figs. 2 and 4 respectively. In view of the recent results on multiplicity distributions, in \( pp \) collisions at \( \sqrt{s}=0.9, 2.36 \) and 7 TeV at the Large Hadron Collider (LHC), reported by CMS Collaboration [15],

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{fig1.png}
\caption{Overall scaled multiplicity distribution data for \( pp \) at ISR energy [12], compared to theoretical prediction using the Simple One String Model, Eqs. (8) and (12).}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{fig2.png}
\caption{Overall scaled multiplicity distribution data for \( p\bar{p} \) at Collider energy [13], compared to theoretical prediction using the Simple One String Model, Eqs. (8) and (12).}
\end{figure}

we have also computed the overall multiplicity distributions that the One String Model predicts at LHC energies. The results are shown in Fig. 5 and we can see vio-
lation of KNO scaling, in qualitative agreement with the result obtained by CMS Collaboration in pseudorapidity interval of $|\eta|<2.4$, as discussed in Ref. [15]. As mentioned, two fit values of $A=0.258$ and $A=0.198$ have been obtained in the previous study [4], the first one giving a better account of lower energy data whereas the second one higher energy data. As before, we have computed the corresponding hadronic multiplicity distribution by fixing both the gamma parametrization for $\psi^{(1)}$, Eq. (13), and the complex eikonal, Eqs. (14) and (15), and considered the two parametrizations for the average multiplicity, $\sim s^{0.258} (\xi(s)=1.424)$ and $\sim s^{0.198} (\xi(s)=1.348)$. The results at 546 GeV are shown in Fig. 6 and, for $A=0.198$, we can see the disagreement of the theoretical curve when compared to the data. As pointed out in [4] the parametrization $\sim s^{0.198}$ brings information from data at high energies, while the parametrization $\sim s^{0.258}$ is in agreement with data at smaller energies. However, the information from the $e^-e^+$ average multiplicities at high energies does not reproduce the overall multiplicity distribution. Hence, by using $A=0.256$ the output seems to be more consistent with data. In addition, there is no evidence of gluon saturation at CERN $p\bar{p}$ Collider 546 GeV, however and as a pedagogical exercise, we have also computed $\Phi$ considering the value of $A=0.11 (\xi(s)=1.211)$, Fig. 6. We would like emphasize that when the formalism is applied, considering the three values of $A$ (0.258, 0.198 and 0.11) at energies 52.6, 200 and 900 GeV, the results are essentially the same obtained in Fig. 6. We have expressed $\Phi$ in terms of modified the scaling variable $\tilde{z} = n - N_o/ \langle n - N_o \rangle$ with $N_o=0.9$ representing the average number of leading particles [4].

3 Inelasticity

The concept of inelasticity is essential since it defines the energy available for particle production in high energy hadronic and nuclear collisions. However, the impact parameter dependence of the inelasticity is a problem unsolved. In theoretical works, it is quite natural to assume that the multiplicity distribution and inelasticity are connected. Indeed, some authors has defined multiplicity distributions in terms of inelasticity as [1], [16]

$$P_n(s) = \int_0^1 P(n|K)P(K(s))dK,$$

(16)

where $P[K(s)]$ is the inelasticity distribution and $P(n|K)$ is the probability of the production of $n$ particles at the given inelasticity $K$. Thus, based on the connection between multiplicity and inelasticity, we have explored the parametrization of the One String Model formalism and computed the impact parameter dependence of inelasticity, as will discuss in next two subsections.

Fig. 3. Overall scaled multiplicity distribution data for $p\bar{p}$ at Collider energy [14], compared to theoretical prediction using the Simple One String Model, Eqs. (8) and (12).

Fig. 4. Overall scaled multiplicity distribution data for $p\bar{p}$ at Collider energy [13], compared to theoretical prediction using the Simple One String Model, Eqs. (8) and (12).
The idea of a universal hadronization mechanism is not new and similarities between both processes were indeed observed [16], [17], [18]. For example, the average multiplicities in $pp/p\bar{p}$ and $e^+e^-$ collisions become similar when comparisons are made at the same effective energy for hadron production. In $pp/p\bar{p}$ collisions the effective energy for particle production, $E_{eff}$, is the energy left behind by the two leading protons

$$E_{eff} = (\sqrt{s})_{pp} - (E_{leading,1} + E_{leading,2}),$$  

(17)

or

$$E_{eff} = (\sqrt{s})_{pp} - 2E_{leading},$$

(18)

in the case of symmetric events. (We recall that $(\sqrt{s})_{pp}$ and $\sqrt{s}$ represents both the center-of-mass energy, however, in this subsection the notation $(\sqrt{s})_{pp}$ is helpful to differentiate that from $(\sqrt{s})_{e^+e^-}$). In $e^+e^-$ collisions the effective energy for hadron production coincides with total center-of-mass energy of the beam

$$E_{eff} = (\sqrt{s})_{e^+e^-} = 2E_{beam}.$$  

(19)

Thus, the same equivalent energy for both $pp/p\bar{p}$ and $e^+e^-$ collisions can be written as

$$(\sqrt{s})_{pp} - 2E_{leading} = E_{eff} = (\sqrt{s})_{e^+e^-}.  

(20)$$

For the quantitative estimation of the inelasticity, $K$, we can use the definition [1], [2]

$$E_{eff} = K(\sqrt{s})_{pp} \Rightarrow K = \frac{E_{eff}}{(\sqrt{s})_{pp}}.  

(21)$$

In the following, we will explore the One String Model formalism, able to describe the multiplicity distributions in wide interval of energy (30.4 - 900 GeV), to obtain information about inelasticity.

### 3.2 Computation of Inelasticity

The Eq. (9) is a key point of the formalism. Physically, it corresponds to the energy for hadron production deposited at $b$, due to the interactions among hadronic constituents involved in the collision. Thus, and as discussed in last section, the fractional energy, $\sqrt{s'}$ (Eq. (9)), and the effective energy for hadron production, $E_{eff}$ (Eq. (21)), represents both the same physical quantity ($\sqrt{s'} = E_{eff}$). Now, by using the Eq. (9), let us write the inelasticity, Eq. (21), as a function of $\sqrt{s'}$ and $b$ as

$$2K(s, b) = \frac{\sqrt{s'}}{(\sqrt{s})_{pp}} = \frac{\beta(s)\chi_f(s, b)}{\sqrt{s}}.$$  

(22)

The factor 2, in the above equation, is due to the fact that the multiplicity distributions data are normalized to 2. However, we can not calculate the $K(s, b)$ until the value of $\beta(s)$ is known. To estimate $\beta(s)$ we note that
the parameter $\xi(s)$, which is introduced in Eq. (11), is related with $\beta(s)$ by

$$\xi(s) = \frac{\gamma}{<N(s)> \left[ \frac{\beta(s)}{\sqrt{s}} \right]^{2A}}. \quad (23)$$

We recall that $<N(s)>$ is the average multiplicity at $\sqrt{s}$. By using the values of $A = 0.258$, $\gamma = 2.09$, discussed in subsection 2.1, and the values of $<N(s)>$ imputed from experiments [12], [13] and [14], and also observing that $\xi(s)$ is obtained from Eq. (12), we have estimated the values of $\beta$ at various energies. The results are displayed in Table 1. We can see clearly that $\beta$ increases as the collision energy also increases. $\beta$ can be parameterized as $\beta(s) = 77.48\sqrt{s} + 0.4168$ with $\chi^2/N_{DF}=1$.

| $\sqrt{s}$ (GeV) | $\xi(s)$ | $<N(s)>$ | $\beta(s)$ (GeV) |
|------------------|----------|----------|------------------|
| 52.6             | 1.612    | 11.55    | 69.295           |
| 200              | 1.517    | 21.4     | 203.531          |
| 546              | 1.424    | 27.5     | 292.729          |
| 900              | 1.377    | 35.6     | 452.376          |
| 2360             | 1.286    | -        | 1061.13          |
| 7000             | 1.188    | -        | 2995.08          |
| 14000            | 1.130    | -        | 5912.68          |

Table 1 $\beta(s)$ estimated values at various energies. The values of $<N(s)>$ was imputed from Refs. [12], [13] and [14].

Now we proceed to compute the impact parameter dependence of inelasticity and infer some information on its energy dependency.

### 3.3 Results and Discussion

Based on the connection between multiplicity distribution and inelasticity, we have update and applied the One String Model formalism deriving an expression, Eq. (22), which allows us to study the impact parameter and energy dependence of inelasticity. Adopting the Block et al. QCD-inspired parametrization for $\chi^2_{pp}(s,b)$ [6] and by using the estimated values of $\beta(s)$, Table 1, we have applied the Eq. (22) by fixing the collision energy and computed the inelasticity as a function of $b$. We show, in Fig. 7, the results from our analysis. Naturally, the inelasticity decreases as a function of impact parameter, $b$. The inelasticity behavior is essentially the same at energies 546 and 900 GeV. At energies 52.6 and 200 GeV, we can see appreciable difference just in the region of $b \approx 0$ (central collisions). It is interesting to note that, from the range of collision energy 50 ~ 200 GeV to that one 500 ~ 900 GeV, the inelasticity shows a difference about 60 percent in its values for $b \lesssim 0.5$ fm. We can also see that, at fixed $b$, the inelasticity $K$ decreases as $\sqrt{s}$ increases in the interval 52.6 - 900 GeV. The Eq. (22) depends on the eikonal and $\beta$ parameter. With the eikonal as determined phenomenologically in [6] as input, where high energy cross sections grow with energy as a consequence of the increasing number of soft partons populating the colliding particles ($pp/\bar{p}\bar{p}$), it seems quite natural to expect that multipartion interactions leads to larger multiplicities/inelasticities as consequence to the full development of the gluonic structure. However, looking the same impact parameter dependence of inelasticity functions at 546 and 900 GeV, Fig. 7, we would be tempted to conclude that we are observing saturation effects due gluon recombination in the inelasticity, but there is no evidence of saturation in this range of energy (52.6 - 900 GeV). We have also applied our approach at energies $\sqrt{s}=2.36$ and 7 TeV (LHC), as shown in Fig. 7, and the results suggest that the inelasticity is an increasing function of energy for the interval 2.36 - 7 TeV. As mentioned before, our main purpose is study features of multiplicity distributions deriving information on inelasticity and our analysis is based on the model in which is assumed that in parton-parton collision there is formation of a string. Thus, despite some simplifications made in the One String Model, the results seems to be consistent with the multiplicity distributions data in a wide interval of energy (52.6 - 900 GeV). Hence, the computed inelasticities, in this range of energy, are reliable results. In counterpart, the results at $\sqrt{s}=2.36$ and 7 TeV are inconclusive in the context our analysis, because the One String Model probably underestimates the high multiplicities events due to the lack of the multicomponent structure in its formulation. In fact, recent results reported by CMS Collaboration pointed out the importance of a multicomponent structure in hadron-
hadron inelastic interactions, in agreement with previous experimental results (for details see [15]). Inelasticity also has been studied recently in Ref. [19], where, in the context of both Wdowczyk and Wolfendale model and UHECR data analysis, it was found that the inelasticity decreases in very high energy interactions, and, in the same work and by using the modified Feynman scaling formula, the inelasticity is an increasing function of the energy. It reflects the subtlety of the theme. We note that at ISR Energies (30-60 GeV), where the leading particle spectrum could be measured, the inelasticity is defined to be about 0.5. This value can be identified with $pp$ collision taking place at $b \sim 0.6$ fm, Fig. 7. The One String Model has been used to study the influence on $\Phi$ considering possible values of $A$ parameter at $\sqrt{s}=546$ GeV, Fig. 6. In addition, we have also computed $\Phi$ at energies 52.6, 200 and 900 GeV using the different values of $A$ (0.258, 0.198 and 0.11) and the results, in each energy, are similar with that obtained in Fig. 6. Finally, we emphasize that the curves in Figs. 1-4 has not been fit to data, except for the values of $A$ and $k$ (fixed) no experimental information about multiplicity distribution has gone into the calculation. Hence, the energy evolution of the multiplicity distributions, from ISR to Collider (30 $\sim$ 900 GeV), is correctly reproduced by changing only the Overlap Function, Eq.(2), without changing the underlying elementary interaction, in agreement with what could be expected from QCD.

4 Concluding Remarks

Being the impact parameter $b$ an essential variable in a geometrical description of hadronic collisions, we have investigated the $b$ dependence of inelasticity and also inferred some information on its energetic behavior. By using a geometrical model we have derived an expression for $K$ based on the hypothesis of connection between multiplicity distribution and inelasticity. We have adopted the Block et al. model in our analysis, where the eikonal functions $\chi_{gg}(s, b)$ and $\chi_{qg}(s, b)$ are needed to describe the lower energy forward data, while $\chi_{gg}(s, b)$ contribution dominates at high energy and determines the asymptotic behavior of cross sections. We believe that the same impact parameter dependence of the inelasticity at energies 546 and 900 GeV can have important implications for the underlying gluon-gluon dynamics. In fact, we are testing our formalism by using the QCD Eikonal Model in which the gluon may develop a dynamical mass [20], [21]. At energies $\sqrt{s}=2.36$ and 7 TeV the One String Model parametrization can not be tested, hence the results do not allow for any conclusion. Finally, the results suggest that there are relationships between the inelasticity and the eikonal function.

Acknowledgements

I am grateful to E.G.S. Luna for several instructive discussions.

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