Two Variable-Weather-Parameter Models and Linear Equivalent Models Expressed by Them for Photovoltaic Cell

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ABSTRACT
For the conventional four-parameter model of photovoltaic (PV) cell, to simplify the number of its mathematical equations, in this paper, a variable-weather-parameter cell model (VWP model) is proposed. Furthermore, to decrease the number of its model parameters, a simplified variable-weather-parameter cell model (SVWP model) is also proposed. Based on them, the model parameters of linear equivalent models can be replaced by the conventional four-parameter model parameters, which finds the direct relationships between linear equivalent cell model and four-parameter cell model. Finally, some simulation experiments are done by comparing the proposed VWP and SVWP models with conventional four-parameter model of PV cell. The results show that, on the one hand, their output characteristics almost correspond with each other. On the other hand, these proposed cell models are very accurate under varying irradiance or temperature conditions. Meanwhile, some simulation experiments also verify that the linear equivalent models expressed by the VWP model and SVWP model are feasible, available and reasonable. By this work, both the conventional four-parameter model and linear equivalent models are greatly evolved and improved. On the one hand, the conventional four-parameter model can be greatly simplified, which makes its modeling and application easier. On the other hand, the bridge between it and linear equivalent models can be built successfully, which makes these linear models more standard and practical.

INDEX TERMS
PV cell, VWP model, SVWP model, MPP, linear equivalent models.

NOMENCLATURE
MPPT maximum power point tracking
VWP variable-weather-parameter
MPP maximum power point
PV photovoltaic
SVWP model simplified VWP model
V output voltage of PV cell
I output current of PV cell
STC standard test conditions
Isc short circuit current at STC
Im MPP current at STC
Voc open circuit voltage at STC
Vm MPP voltage at STC
S solar irradiance
T cell temperature
Sref reference value of S
Tref reference value of T
α temperature coefficient of current variation
β temperature coefficient of voltage variation
Io reverse saturation current of diode of PV cell
q electron charge (1.602 × 10−19C)
A dimensionless junction material factor of PV cell
K Boltzmann constant (1.38×10−23 J/K)
T temperature (K) of PV cell

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I. INTRODUCTION

Hitherto, many research works on the mathematical models of PV cell, especially for its one-diode model and four-parameter model, have been done. On the one hand, lots of MPPT methods are proposed by using the one-diode cell model or four-parameter cell model. The different MPPT methods for PV cell were compared by Danandeh et al. [1]. To design a new MPPT strategy, the single-diode cell model was used by Moshksar and Ghanbari [2]. Based on the one-diode cell model, a tangent error MPPT method was proposed by Peng et al. [3], two perturb and observe methods (P&O method) were analyzed by Yang et al. [4] and Veerapen et al. [5], and a dual-tracking MPPT method under rapidly changing environmental conditions was presented by Jately and Arora [6]. To extract the solar cell parameters of the single-diode model, some approaches were presented and compared by Humada et al. [7], [8]. On the basis of the four-parameter model of PV cell, an improved P&O method was proposed by Li et al. [9]. To gain the fastest tracking speed, some variable-weather-parameter (VWP) methods based on four-parameter cell model were proposed by Li et al. [10]−[13]. In Refs. [10] and [11], three fundamental VWP methods were proposed. In Ref. [12], a VWP MPPT method was proposed by analyzing the input resistance. In Ref. [13], a VWP MPPT method for PV system was proposed by a defined characteristic resistance based on PV cell. However, by these works, the mathematical expressions of the four-parameter cell model is still not simplified, especially for the number of its mathematical equations. To address this issue, on the basis of these VWP methods, a VWP cell model is proposed in this paper. By it, not only the advantages of the VWP methods can be obtained, but also the number of the mathematical equations of the four-parameter cell model can be decreased from five to three.

On the other hand, lots of works have been also done to study the model parameters of PV cell. A method to identify the cell model parameters of the one-diode circuit was proposed by Laudani et al. [14]. To predict the cell behaviour, an improved one-diode model was studied by Salmi et al. [15]. A matrix equation of PV cell was analyzed to describe the parallel-parallel topology by Kadri et al. [16]. A simple and complete cell model based on datasheet with varying environment was studied by Vergura [17]. An optimization method of the parameters and temperature dependence for the two-diode and one-diode models was presented by Barth et al. [18]. After simplifying the five-parameter cell model, the four-parameter model was presented by Nobuyoshi et al. [19]. Two linear cell models at the MPP were presented to linearize the $V-I$ characteristic of the conventional four-parameter model of PV cell by Li [20]. It is obvious that the one-diode model which is regarded as the main model of PV cell has been analyzed and used widely. However, by these works, the number of model parameters of the four-parameter model is still not greatly simplified. To solve this problem, on the basis of the proposed VWP model of PV cell, a simplified variable-weather-parameter (SVWP) cell model is proposed in this paper. By it, not only the number of the model parameters of the four-parameter model can be decreased from four to two, but also the number of the mathematical equations of the four-parameter model can be decreased from five to one.

Finally, some works have been done to linearize the $V-I$ curve of the conventional four-parameter model. For example, in Ref [20], two linear equivalent models (Norton equivalent model and Thevenin equivalent model) were proposed by Shaowu Li. The main aim of these proposed linear cell models is to use the nonlinear cell model in linear control theory more easily and conveniently. Meanwhile, some parameters which are the conclusions of three fundamental VWP methods were used as their model parameters. However, there exist some shortcomings for these two linear cell models. On the one hand, a piecewise function is used as their model parameters, which leads to the complex mathematical calculation. On the other hand, their model parameters are not expressed by the conventional cell models, so the fundamental characteristics of PV cell can not be reflected clearly. To make up for these demerits, in this paper, the model parameters of the proposed VWP model and SVWP model are used to express these two linear models. By this work, not only the trouble arising from piecewise function can be prevented, but also the direct relationships between four-parameter model and linear equivalent models can be built.

The main innovations and contributions of this work can be illustrated as follows:

- A VWP cell model, which can decrease the number of the equations of the conventional four-parameter model from five to three, is proposed.
- A SVWP cell model, which can decrease the number of the equations of the conventional four-parameter model from five to one, is proposed.
- The proposed SVWP cell model can decrease the number of the model parameters of the conventional four-parameter model from four to two.
- The model parameters of two linear equivalent models are expressed by the VWP and SVWP cell models.
- The direct relationships between linear equivalent models and four-parameter cell model are firstly built by this work.

This paper is arranged as follows: the VWP cell model and SVWP cell model are studied and proposed by the analysis of the four-parameter mathematical model in Section II. The model parameters of linear equivalent models are analyzed and expressed by the VWP model and SVWP model in Section III. The output characteristics and accuracy of the proposed VWP and SVWP models are analyzed, and the linear models with VWP and SVWP model parameters are

- $R_s$: series resistance of PV cell
- $R_{sh}$: parallel resistance of PV cell
- $P_{o max}$: ideal maximum output power of PV system
- $D_{max}$: duty cycle corresponding to $P_{o max}$
tested in Section IV. There are some discussions and conclusions in Sections V and VI, respectively.

II. PROPOSITION OF THE VWP MODELS

A. THEORETICAL BASIS

Fig. 1 shows the equivalent circuit of the one-diode cell model [14], and Eq. (1) shows its mathematical expression [21], [22].

![One-diode model](image)

**FIGURE 1.** One-diode model.

Eqs. (2)-(4) show the four-parameter model of PV cell [9] [19]. They can be given by simplifying Eq. (1) when some unimportant parameters are ignored [19].

\[
I = I_{sc}[1 - C_1(e^{\frac{V_{oc}}{S_{ref}}} - 1)]
\]

(2)

\[
C_1 = \left[1 - \frac{I_m}{I_{sc}}\right] e^{-\frac{V_{oc}}{C_2 S_{ref}}}
\]

(3)

\[
C_2 = \frac{V_{oc}}{\ln(1 - \frac{I_m}{I_{sc}})}
\]

(4)

According to Ref. [9], when the solar irradiance (S) and cell temperature (T) keep changing, the four-parameter model expressed by Eq. (2) is usually improved as Eqs. (5)-(7). Now, the four-parameter model of PV cell does actually include five equations (Eqs. (3)-(7)).

\[
I = I_{sc}[1 - C_1(e^{\frac{V_{oc}}{S_{ref}}} - 1)] + \Delta I
\]

(5)

\[
\Delta V = -\beta \times (T - T_{ref}) - R_s \times \Delta I
\]

(6)

\[
\Delta I = \alpha \times \frac{S}{S_{ref}} \times (T - T_{ref}) \times (\frac{S}{S_{ref}} - 1) \times I_{sc}
\]

(7)

In this work, Eqs. (3)-(7) will be used to analyze the characteristics of the parameters C_1 and C_2, and to build the relationships between four parameters ($I_{sc}$, $V_{oc}$, $I_m$ and $V_m$) and S, T. Meanwhile, they are also used as the mathematical basis to propose two VWP models of PV cell.

B. PRINCIPLE OF THE VWP MODEL

When S and T are changing, the parameters $I_{sc}$, $V_{oc}$, $I_m$ and $V_m$ will also keep varying. Therefore, they can be all defined as the function of S, T, and represented by $I_{sc}(S, T)$, $V_{oc}(S, T)$, $I_m(S, T)$ and $V_m(S, T)$, respectively. It is obvious that these defined functions illustrate the relationships between four parameters ($I_{sc}$, $V_{oc}$, $I_m$ and $V_m$) and S, T. Meanwhile, according to Eqs. (3) and (4), $C_1$ and $C_2$ are also defined as the function of S, T, and represented by $C_1(S, T)$ and $C_2(S, T)$, respectively. Then Eqs. (2)-(4) can be replaced by Eqs. (8)-(10), respectively.

\[
I = I_{sc}(S, T)\left\{1 - C_1(S, T)\left[e^{\frac{V}{S_{ref}} - 1}\right]\right\}
\]

(8)

\[
C_1(S, T) = \left[1 - \frac{I_m(S, T)}{I_{sc}(S, T)}\right] e^{-\frac{V_{oc}(S, T)}{C_2 S_{ref}}}
\]

(9)

\[
C_2(S, T) = \ln\left[1 - \frac{I_m(S, T)}{I_{sc}(S, T)}\right]
\]

(10)

According to Eqs. (8)-(10), it is obvious that the functions $I_{sc}(S, T)$, $V_{oc}(S, T)$, $I_m(S, T)$ and $V_m(S, T)$ are the key. On the one hand, according to Ref[12], $I_m(S, T)$ and $V_m(S, T)$ can be expressed by Eq. (11) and Eq. (12), respectively.

\[
V_m(S, T) = 4.4 \times 10^{-6} \times (S - 638.25)^2 + 16.918
\]

\[
+ 0.0504 \times (25 - T)
\]

(11)

\[
I_m(S, T) = 2.145 \times 10^{-5} \times (T - 25) \times S
\]

\[
+ 8.58 \times 10^{-3} \times S
\]

(12)

On the other hand, in order to obtain the expressions of $I_{sc}(S, T)$ and $V_{oc}(S, T)$, some simulation results shown by Figs. 2-3 are used. Here, the four cell parameters $I_{sc}$, $V_{oc}$, $I_m$ and $V_m$ are selected as 9.19A, 22V, 8.58A and 17.5V at STC, respectively. The $V_{oc}$ and $I_{sc}$ keep changing, the four-parameter model can be shown by Fig. 2(a) and Fig. 2(b), respectively. The $I_{sc}$ and $I_{sc} - T$ curves are shown by Fig. 3(a) and Fig. 3(b), respectively.

According to Fig. 2(a), by fitting $V_{oc}$ with $S$ curve, the approximation function shown in Eq. (13) can be given when $T$ keeps at 25°C.

\[
V_{oc} = 5.5095 \times 10^{-6}S^2 - 7.0531 \times 10^{-3}S + 23.522
\]

(13)

Meanwhile, according to Fig. 2(b), the approximation function shown in Eq. (14) can be given by fitting $V_{oc} - T$ curve when $S$ keeps at 1000W/m².

\[
V_{oc} = -0.06336T + 23.584
\]

(14)

Take Eqs.(13) and (14) into account, the expression between $V_{oc}$ and S, T, is shown by Eq. (15).

\[
V_{oc} = V_{oc}(S, T) = 5.51 \times 10^{-6} \times (S - 638.25)^2
\]

\[
+ 21.2776 + 0.06336 \times (25 - T)
\]

(15)

According to Fig. 3(a), by fitting $I_{sc} - S$ curve, the approximation function shown in Eq. (16) can be given when $T$ keeps at 25°C.

\[
I_{sc} = 9.19 \times 10^{-3}S - 1.3991 \times 10^{-17}
\]

(16)

Meanwhile, according to Fig. 3(b), the approximation function shown in Eq. (17) can be given by fitting $I_{sc} - T$ curve when $S$ keeps at 1000W/m².

\[
I_{sc} = 0.022975T + 8.6156
\]

(17)
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Take Eqs. (16) and (17) into account, the expression between $I_{sc}$ and $S$, $T$, is shown by Eq. (18).

$$I_{sc} = I_{sc}(S, T) = 2.2975 \times 10^{-5} \times (T - 25) \times S + 9.19 \times 10^{-3} \times S$$  \hspace{1cm} (18)

Finally, according to Eqs. (8)-(12), (15) and (18), the four-parameter cell model expressed by Eqs. (3)-(7) can be replaced by Eqs. (8)-(10). In this cell model, all model parameters (including $I_{sc}(S, T)$, $V_{oc}(S, T)$, $I_{m}(S, T)$ and $V_{m}(S, T)$) are the function of the variable weather parameters (including $S$ and $T$), so it can be named as “variable-weather-parameter model” (VWP model).

C. SIMPLIFIED VWP MODEL

It is clear that the VWP model of PV cell expressed by Eqs. (8)-(10) is still complex, so it is inconvenient to use in practical application. Therefore some simulation experiments must be done to simplify it and the results can be shown by Figs. 4 and 5. Here, Fig.4(a) and Fig.4(b) show the $C_1 - S$ and $C_1 - T$ curves, respectively. Fig.5(a) and Fig.5(b) show the $C_2 - S$ and $C_2 - T$ curves, respectively.

It can be seen from Figs. 4(a) and 4(b) that $C_1$ approximately keeps constant all the time regardless of the changing irradiance or temperature, and its value is about $1.7416 \times 10^{-6}$. Meanwhile, it can be also seen from Figs. 5(a) and 5(b), $C_2$ approximately keeps constant all the time regardless of the changing irradiance or temperature, and its value is about $7.5411 \times 10^{-2}$.
According to Eq. (8) and characteristics of $C_1$ and $C_2$, the VWP model of PV cell can be simplified and expressed by Eq. (19). This cell model can be named as “simplified variable-weather-parameter model” (SVWP model). It is obvious that, according to Eq. (19), the SVWP model of PV cell is only determined by $I_{sc}(S, T)$, $V_{oc}(S, T)$, $C_1$ and $C_2$, which makes the VWP model expressed by Eqs. (8)-(10) greatly simplified when $S$ and $T$ keep varying.

$$I = I_{sc}(S, T) \left\{ 1 - C_1 \left[ e^{\frac{V_{oc}(S, T)}{T}} - 1 \right] \right\}$$

(19)

Compare Eqs. (3)-(7) with Eq. (19), it is clear that, by the SVWP model, the conventional four-parameter cell model can be simplified more greatly than VWP model.

In addition, compare Eqs. (8)-(10) with Eq. (19), it is also clear that, by the SVWP model, not only the number of the model parameters of the VWP model can decreased from four to two, but also the number of the mathematical equations of the VWP model can be decreased from three to one.

In a word, both the VWP model and the SVWP model can greatly simplify the conventional four-parameter model of PV cell so that this conventional model can be used more easily and conveniently.

### III. LINEAR EQUIVALENT MODELS BASED ON VWP MODEL PARAMETERS

After the VWP cell model and SVWP cell model have been proposed, their model parameters can be used to express the linear equivalent models. According to Ref [20], at the MPP, PV cell can be linearized as the Thevenin equivalent model (Fig.6(a)) and Norton equivalent model (Fig.6(b)). Their model parameters including $R_sM$, $V_sM$ and $I_sM$ can be expressed by Eqs. (20)-(22), respectively. Where $C$ can be
According to Eqs. (20)-(23), for two linear cell models, their model parameters can be expressed by some state parameters of the DC/DC circuit (including \( C \) and \( P_{o, \max} \)). However, there exist some shortcomings arising from these parameters. On the one hand, the state parameters of the DC/DC circuit are changing with time, so the measured data are usually depended on to obtain the values of these parameters, which makes the use of these linear cell models inconvenient. On the other hand, for PV cell, these state parameters can not clearly illustrate the relationship between its linear models and its four-parameter model, so the cell characteristics of the linear models can not be shown. Therefore, in order to overcome these shortcomings, the question how the model parameters of the VWP model and SVWP model are used to express these linear equivalent models will be analyzed.

**A. LINEAR MODELS EXPRESSED BY VWP MODEL PARAMETERS**

According to Section 2.2, the model parameters of the VWP cell model include \( I_{oc}(S, T) \), \( V_{oc}(S, T) \), \( I_m(S, T) \), and \( V_m(S, T) \), so the linear models expressed by \( I_m(S, T) \) and \( V_m(S, T) \) will be firstly analyzed.

On the one hand, for the model parameter \( R_{sM} \), it is well known that, at the MPP, Eqs. (24) and (25) must be satisfied according to Fig. 6.

\[
R_{sM} = R_{MPP} = \frac{V_{MPP}}{I_{MPP}} \quad (24)
\]

\[
V_{MPP} = C \quad (25)
\]

According to Ref[12], at the MPP, Eqs. (26) and (27) are satisfied when the MPPT unit is the buck circuit.

\[
R_{MPP} = \frac{R_L}{D_{max}} \quad (26)
\]

\[
D_{max} = \sqrt{\frac{I_m(S, T)R_L}{V_m(S, T) + 0.79}} \quad (27)
\]

Eq. (28) can be obtained by submitting Eq. (27) into Eq. (26).

\[
R_{MPP} = \frac{V_m(S, T) + 0.79}{I_m(S, T)} \quad (28)
\]

On the other hand, for the model parameter \( V_{sM} \), according to Ref[12], Eq. (31) can be given.

\[
V_{MPP} = V_m(S, T) + 0.411 \quad (31)
\]

Eq. (32) can be obtained by submitting Eq. (31) into Eq. (21).

\[
V_{sM} = 2V_m(S, T) + 0.822 \quad (32)
\]

Take the range of \( V_m \) into account, Eq. (32) can be simplified as Eq. (33). Where \( C_\beta \approx 2.045 \).

\[
V_{sM} = C_\beta \cdot V_m(S, T) \quad (33)
\]

Finally, for the model parameter \( I_{sM} \), submit Eqs. (32) and (29) into Eq. (22), Eq. (34) can be given.

\[
I_{sM} = \frac{V_{sM}}{R_m} = \frac{[2V_m(S, T) + 0.822 \cdot I_m(S, T)]}{V_m(S, T) + 0.79} \quad (34)
\]

In order to simplify Eq. (34), submit Eqs. (33) and (30) into Eq. (22), Eq. (35) can be given. Where \( C_\beta/C_\alpha \approx 1.966 \).

\[
I_{sM} = \frac{V_{sM}}{R_{sM}} = \frac{C_\beta}{C_\alpha} \cdot I_m(S, T) \quad (35)
\]

In a word, according to Eqs. (29), (32) and (34) or Eqs. (30), (33) and (35), the linear equivalent models shown by Fig. 6 can be expressed successfully by the VWP model parameters \( I_m(S, T) \) and \( V_m(S, T) \).
B. LINEAR MODELS EXPRESSED BY SVWP MODEL PARAMETERS

According to Section 2.3, the model parameters of the SVWP cell model include $I_{sc}(S, T)$ and $V_{oc}(S, T)$, so the linear models expressed by $I_{sc}(S, T)$ and $V_{oc}(S, T)$ will be analyzed.

On the one hand, for the model parameter $R_{sM}$, Eq. (36) can be obtained by submitting Eq.(25) into Eq. (19).

$$I_{MPP} = I_{sc}(S, T) \left\{ 1 - C_1 \left[ e^{\frac{C_2}{V_{oc}(S, T)}} - 1 \right] \right\} \quad (36)$$

Then submit Eqs. (23), (25) and (36) into Eq. (24), Eq. (37) can be satisfied.

$$R_{sM} = \frac{V_{MPP}}{I_{MPP}} = \frac{V_{oc}(S, T)}{I_{sc}(S, T)} \cdot \frac{C_2 [\text{lambertw}(e^{1+C_1} + C_1 - 1)]}{1 - C_1 (e^{\text{lambertw}(e^{1+C_1}) - 1} - 1)} \quad (37)$$

Eq. (37) can be simplified as Eq. (38).

$$R_{sM} = C_Y \cdot \frac{V_{oc}(S, T)}{I_{sc}(S, T)} \quad (38)$$

where

$$C_Y = \frac{C_2 [\text{lambertw}(e^{1+C_1} + C_1) - 1]}{1 - C_1 (e^{\text{lambertw}(e^{1+C_1}) - 1} - 1)} \quad (39)$$

In the SVWP model, $C_1$ and $C_2$ are two constants, so $C_Y$ is also a constant according to Eq. (39).

On the other hand, for the model parameter $V_{sM}$, according to Eqs. (21) and (23), Eq. (40) can be given.

$$V_{sM} = C_3 \cdot V_{oc}(S, T) \quad (40)$$

where

$$C_3 = 2C_2 [\text{lambertw}(e^{1+C_1} + C_1) - 1] \quad (41)$$

It is clear that $C_3$ is also a constant because $C_1$ and $C_2$ are two constants in the SVWP model.

Finally, for the model parameter $I_{sM}$, submit Eqs. (38) and (40) into Eq. (22), Eq. (42) can be given.

$$I_{sM} = \frac{V_{sM}}{R_{sM}} = \frac{C_3}{C_Y} \cdot I_{sc}(S, T) \quad (42)$$
In order to obtain the values of $C_\gamma$, $C_\delta$ and $C_\delta/C_\gamma$, some simulation results shown by Figs. 7 and 8 can be given.

Figs. 7 and 8 show that $C_\gamma$ and $C_\delta$ can be regarded as the constants when $S$ and $T$ keep varying, and their values are 0.8893 and 1.6278, respectively. Meanwhile, the calculated value of $C_\delta/C_\gamma$ is 1.8304.

In a word, according to Eqs. (38), (40) and (42), the linear equivalent models shown by Fig. 6 can be expressed successfully by the SVWP model parameters $I_{sc}(S,T)$ and $V_{oc}(S,T)$. Here, $I_{sc}(S,T)$ and $V_{oc}(S,T)$ are the SVWP model parameters rather than VWP model parameters, although there are the same expressions, because Eqs. (38), (40) and (42) can be satisfied only by using these assumed constants $C_1$, $C_2$, $C_\gamma$ and $C_\delta$.

IV. SIMULATION EXPERIMENTS
A. OUTPUT CHARACTERISTICS OF THE VWP MODEL AND SVWP MODEL
1) OUTPUT CHARACTERISTICS OF THE VWP MODEL
To test the output characteristics of the proposed VWP model, some simulations are done. Figs. 9-12 show the simulation results. Where "VWP-1000", "VWP-800" and "VWP-500" are the results of the VWP model under 1000W/m$^2$, 800W/m$^2$ and 500W/m$^2$ conditions, respectively, when $T$ keeps 25°C; "FP-1000", "FP-800" and "FP-500" are the results of the four-parameter model under 1000W/m$^2$, 800W/m$^2$ and 500W/m$^2$ conditions, respectively, when $T$ keeps 25°C; "VWP-0", "VWP-20" and "VWP-40" are the results of the VWP model under 0°C, 20°C and 40°C conditions, respectively, when $T$ keeps 1000W/m$^2$; "FP-0", "FP-20" and "FP-40" are the results of the four-parameter model under 0°C, 20°C and 40°C conditions, respectively, when $T$ keeps 1000W/m$^2$.

Figs. 9 and 10 show that, when $T$ keeps at 25°C and $S$ keeps varying, both the $V-I$ and $P-V$ curves of the VWP model almost correspond with those of the conventional four-parameter model of PV cell, and there exist a little error between them. Meanwhile, it can be seen from Figs. 11 and 12 that, when $S$ keeps at 1000W/m$^2$ and $T$ keeps varying, both the $V-I$ and $P-V$ curves of the VWP model almost correspond with those of the conventional four-parameter
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model of PV cell, and there also exist a little error between them.

Therefore, a conclusion can be drawn that, when the solar irradiance or cell temperature keeps varying, the output characteristics of the VWP cell model can correspond with the conventional four-parameter model of PV cell to a great extent.

2) OUTPUT CHARACTERISTICS OF THE SVWP MODEL

To test the output characteristics of the proposed SVWP model of PV cell, some simulations are also done. Figs. 13-16 show the simulation results. Where “SVWP-1000”, “SVWP-800” and “SVWP-500” are the results of the SVWP model under 1000W/m², 800W/m² and 500W/m² conditions, respectively, when $T$ keeps 25°C; “SVWP-0”, “SVWP-20” and “SVWP-40” are the results of the SVWP model under 0°C, 20°C and 40°C conditions, respectively, when $T$ keeps 1000W/m²; “FP-1000”, “FP-800”, “FP-500”, “FP-0”, “FP-20” and “FP-40” are the same as Section 4.1.1.

Figs. 13 and 14 show that, when $T$ keeps at 25°C and $S$ keeps varying, both the $V-I$ and $P-V$ curves of the SVWP model almost correspond with those of the conventional four-parameter model of PV cell, and there exist a little error between them. Meanwhile, it can be seen from Figs. 15 and 16 that, when $S$ keeps at 1000W/m² and $T$ keeps varying, both the $V-I$ and $P-V$ curves of the SVWP model almost correspond with those of the conventional four-parameter model of PV cell, and there also exist a little error between them.

B. ACCURACY OF THE VWP MODEL AND SVWP MODEL

To analyze the accuracy of the VWP model and SVWP model of PV cell, especially at the MPP (the main operating point of PV system), $E_{I-VWP}$, $E_{I-SVWP}$, $E_{P-VWP}$ and $E_{P-SVWP}$ are defined by Eqs. (43), (44), (45) and (46), respectively. Where $I_{FP}$, $I_{VWP}$ and $I_{SVWP}$ are the output currents of the four-parameter model, VWP model and SVWP model,
TABLE 1. Some primary data corresponding to Figs. 17-22.

| Conditions | 1000 W/m², 25°C | 800 W/m², 25°C | 500 W/m², 25°C |
|------------|-----------------|-----------------|-----------------|
| Parameters | Values at the MPP | Values at the MPP | Values at the MPP |
| $E_{I-VWP}$ (mA) | 7.66 | 2.69 | -130 | -6.55 | -90.2 | -4.70 |
| $E_{I-SVWP}$ (mA) | 7.37 | 0.508 | -130 | -8.75 | -90.2 | -6.38 |
| $E_{P-VWP}$ (W) | 0.166 | 0.0483 | -2.78 | -0.115 | -1.928 | -0.0840 |
| $E_{P-SVWP}$ (W) | 0.162 | 0.00907 | -2.78 | -0.157 | -1.928 | -0.109 |

respectively. $P_{FP}$, $P_{VWP}$ and $P_{SVWP}$ are the output powers of the four-parameter model, VWP model and SVWP model, respectively.

$$E_{I-VWP} = I_{FP} - I_{VWP}$$ (43)

$$E_{I-SVWP} = I_{FP} - I_{SVWP}$$ (44)

$$E_{P-VWP} = P_{FP} - P_{VWP}$$ (45)

$$E_{P-SVWP} = P_{FP} - P_{SVWP}$$ (46)

1) SIMULATIONS UNDER VARYING IRRADIANCE CONDITIONS

Some simulations are done under varying $S$ conditions when $T$ keeps 25°C. Figs. 17-22 show the results. Meanwhile, some primary data including the maximum values and values at the MPP are shown in Table 1.

Figs. 17-22 show that, under varying irradiance conditions, there exist errors for both the VWP model and SVWP model, and their maximum values are usually around the maximum output voltage ($V_{oc}$). According to Table 1, the maximum value of the output current error is 130 mA and the maximum value of the output power error is 2.78 W under 800W/m², 25°C conditions. By contrast, their values at the MPP are 6.55 mA and 0.115 W for the VWP model, respectively, while they are 8.75 mA and 0.157 W for the SVWP model, respectively. Because PV system usually operates around the MPP, the error between VWP model or SVWP model and four-parameter model is very small, which means the good accuracy of the VWP model and SVWP model.

Therefore, a conclusion can be drawn that both the VWP model and SVWP model are very accurate with conventional four-parameter model of PV cell under different irradiance conditions.

2) SIMULATIONS UNDER VARYING TEMPERATURE CONDITIONS

Some simulations are done under varying temperature conditions when $S$ keeps 1000W/m². Figs. 23-28 show the
**TABLE 2.** Some primary data corresponding to Figs. 23-28.

| Conditions                    | 1000 W/m², 0°C | 1000 W/m², 20°C | 1000 W/m², 40°C |
|------------------------------|----------------|-----------------|-----------------|
| Parameters                   | Maximum values | Values at the MPP | Maximum values | Values at the MPP |
| \( E_{I-VWP} \) (mA)         | 6.70           | 2.32            | 7.46           | 2.63            | 8.32           | 2.95            |
| \( E_{I-SVWP} \) (mA)        | 6.42           | 0.434           | 7.18           | 0.495           | 7.98           | 0.56            |
| \( E_{P-VWP} \) (W)          | 0.156          | 0.0372          | 0.164          | 0.0475          | 0.172          | 0.0510          |
| \( E_{P-SVWP} \) (W)         | 0.152          | 0.00633         | 0.161          | 0.00891         | 0.168          | 0.00950         |

Simulation results. Meanwhile, some primary data including the maximum values and values at the MPP are shown in Table 2. Figs. 23-28 show that, under varying temperature conditions, there exist errors for both the VWP model and SVWP model and their maximum values are usually around the maximum output voltage (\( V_{oc} \)). Table 2 show that the maximum value of the output current error is 8.32 mA and the maximum value of the output power error is 0.172 W under 1000W/m², 40°C conditions. By contrast, their values at the MPP are 2.95 mA and 0.05 W for the VWP model, respectively, while they are 0.56 mA and 0.0095 W for the SVWP model, respectively. Because PV system usually operates around the MPP, the error between VWP model or SVWP model and four-parameter model is very small, which also means the good accuracy of the VWP model and SVWP model.

Therefore, a conclusion can be also drawn that, under different temperature conditions, both the VWP model and
SVWP model can keep accurate when they are compared with conventional four-parameter model of PV cell.

In a word, the proposed VWP and SVWP models of PV cell are very accurate, especially at the MPP, regardless of the varying irradiance or temperature, when the conventional four-parameter model is selected as the compared object.

C. SIMULATIONS OF LINEAR MODELS WITH VWP AND SVWP MODEL PARAMETERS

1) CHARACTERISTIC CURVES OF THE MODEL PARAMETERS

In order to analyze the characteristics of the linear equivalent models with VWP model parameters (represented by PVLM-VWP) and linear equivalent models with SVWP model parameters (represented by PVLM-SVWP), and compare them with the linear equivalent models of PV cell (represented by PVLM), some simulations are done. Figs. 29-34 show the results. Where the $R_s M - S$, $V_{oc M} - S$ and $I_{sc M} - S$ curves are shown by Figs. 29-31, respectively, when $T$ keeps 25 °C; the $R_s M - T$, $V_{oc M} - T$ and $I_{sc M} - T$ curves are shown by Figs. 32-34, respectively, when $S$ keeps 1000 W/m$^2$.

Figs. 29-31 show that, when $T$ remains unchanged, the characteristic curves of the PVLM-VWP and PVLM-SVWP are approximately same as the PVLM regardless of the
$R_{SM} - S$, $V_{SM} - S$ or $I_{SM} - S$ curve. Clearly, these results reveal the feasibility, availability and rationality of the PVLM-VWP and PVLM-SVWP under varying irradiance conditions.

Figs. 32-34 show that, when $S$ keeps invariable, the characteristic curves of the PVLM-VWP and PVLM-SVWP are approximately same as the PVLM regardless of the $R_{SM} - T$, $V_{SM} - T$ or $I_{SM} - T$ curve. Clearly, these results reveal the feasibility, availability and rationality of the PVLM-VWP and PVLM-SVWP under varying temperature conditions.

Therefore, a conclusion can be drawn that the characteristics of the PVLM-VWP and PVLM-SVWP can always...
TABLE 3. Simulation results under various weather conditions.

| \((S, T)\) \((\text{W/m}^2, ^\circ\text{C})\) | \(R_{SM}^*\) | \(R_{SM}'\) | \(R_{SM}^pk\) | \(V_{SM}^*\) | \(V_{SM}'\) | \(V_{SM}^pk\) | \(I_{SM}^*\) | \(I_{SM}'\) | \(I_{SM}^pk\) |
|--------------------------------|-----------|-----------|-------------|-----------|-----------|-------------|-----------|-----------|-----------|
| (300, 0)                      | 8.051     | 8.083     | 8.050       | 7.992     | 38.20     | 38.24       | 38.32     | 38.21     | 4.75      |
| (300, 10)                     | 7.631     | 7.660     | 7.675       | 7.641     | 37.17     | 37.21       | 37.20     | 37.18     | 4.87      |
| (300, 20)                     | 7.231     | 7.260     | 7.206       | 7.295     | 36.14     | 36.18       | 35.88     | 36.15     | 5.00      |
| (500, 0)                      | 4.722     | 4.741     | 4.730       | 4.726     | 37.35     | 37.39       | 37.26     | 37.33     | 7.91      |
| (500, 10)                     | 4.473     | 4.491     | 4.524       | 4.496     | 36.32     | 36.35       | 36.40     | 36.30     | 8.12      |
| (500, 20)                     | 4.236     | 4.253     | 4.299       | 4.270     | 35.28     | 35.32       | 35.28     | 35.27     | 8.33      |
| (800, 0)                      | 2.800     | 2.812     | 2.782       | 2.804     | 36.38     | 36.42       | 36.18     | 36.36     | 12.99     |
| (800, 20)                     | 2.652     | 2.663     | 2.653       | 2.657     | 35.35     | 35.39       | 35.24     | 35.33     | 13.33     |
| (800, 30)                     | 2.511     | 2.521     | 2.540       | 2.514     | 34.32     | 34.35       | 34.40     | 34.30     | 13.67     |
| (1000, 20)                    | 2.178     | 2.187     | 2.198       | 2.182     | 36.29     | 36.33       | 36.36     | 36.30     | 16.66     |
| (1000, 30)                    | 2.064     | 2.072     | 2.112       | 2.066     | 35.26     | 35.29       | 35.60     | 35.27     | 17.08     |
| (1000, 40)                    | 1.955     | 1.963     | 1.967       | 1.952     | 34.23     | 34.26       | 34.32     | 34.24     | 17.50     |
| (1200, 20)                    | 1.898     | 1.905     | 1.942       | 1.909     | 37.95     | 37.98       | 38.12     | 38.01     | 19.99     |
| (1200, 30)                    | 1.801     | 1.808     | 1.818       | 1.811     | 36.92     | 36.95       | 36.92     | 36.97     | 20.49     |
| (1200, 40)                    | 1.709     | 1.715     | 1.724       | 1.715     | 35.89     | 35.92       | 36.00     | 35.94     | 21.00     |

The simulation results show that the parameter values of the PVLM calculated by Eqs. (20), (21) and (22) respectively, when PV system is operating at the MPP by using P&O method, their values are approximately equal to their corresponding parameter values of the PVLM-VWP calculated by Eqs. (30), (33) and (35); the differences among them are always less than 0.09Ω while the average value is only 0.021A.}

2) ACCURACY OF THE MODEL PARAMETERS

Some simulation experiments are done to analyze the accuracy of the PVLM-VWP and PVLM-SVWP, and the results are shown in Table 3. Where \(R_{SM}, V_{SM} \) and \(I_{SM} \) represent the parameter values of the PVLM calculated by Eqs. (20), (21) and (22) respectively; \(R_{SM}^*, V_{SM}^* \) and \(I_{SM}^* \) represent the parameter values of the PVLM-VWP calculated by Eqs. (30), (33) and (35); \(R_{SM}' \), \(V_{SM}' \) and \(I_{SM}' \) represent the parameter values of the PVLM-SVWP calculated by Eqs. (38), (40) and (42). Table 3 shows that, firstly, \(R_{SM}^* \), \(R_{SM}' \) and \(R_{SM}^pk \) are approximately equal to their corresponding \(R_{SM} \), and the differences among them are always less than 0.09Ω. Secondly, \(V_{SM}^* \), \(V_{SM}' \) and \(V_{SM}^pk \) are approximately equal to their corresponding \(V_{SM} \), the differences among them are always less than 0.33V while the average value is only 0.019V. Finally, \(I_{SM}^* \), \(I_{SM}' \) and \(I_{SM}^pk \) are approximately equal to their corresponding \(I_{SM} \), the differences among them are always less than 0.15A while the average value is only 0.021A.

In a word, when the model parameters of PVLM are expressed by the VWP model or SVWP model, their characteristics can be maintained very well and their good accuracy can be ensured, which illustrates the feasibility, availability and rationality of using the VWP and SVWP model to express linear equivalent model parameters.

V. DISCUSSIONS

There are some errors arising from the used curve fitting method to obtain Eqs. (11), (12), (15) and (18). Meanwhile, there are also some errors arising from these assumed constants including \(C_1, C_2, C_4, C_5 \) and \(C_7 \). Therefore, one of the limitations on the VWP model, SVWP model, PVLM-VWP and PVLM-SVWP is the existence of these errors. However, there are some arguments for these errors as follows: firstly, Fig. 4 and Fig. 5 show that \(C_1 \) and \(C_2 \) approximately keep constant all the time regardless of the changing irradiance or temperature, respectively. Meanwhile, Fig. 7 and Fig. 8 also show that \(C_4 \) and \(C_5 \) approximately keep constant all the time regardless of the varying irradiance or temperature, respectively. Therefore, it is reasonable that these parameters are assumed as the constants. Secondly, according to Section IV, these errors arising from not only curve fitting method but also those assumed constants are very small. Therefore, in practical application, these errors can be ignored to make the use of these cell models more convenient. Finally, it is well known that almost all theoretical researches to simplify the mathematical models inevitably lead to some errors. Therefore, these small errors in the theoretical analysis are acceptable.

In addition, all the proposed cell models (including PVLM-VWP and PVLM-SVWP) are studied on the basis of the conventional four-parameter model in this text. Therefore, their accuracy, rationality and applicability are constrained by the four-parameter cell model, which is the other limitation of these proposed models.

Although some drawbacks (or limitations) exist, the excellence and effectiveness of these cell models can still be guaranteed by the combination of the curve-fitting technique, MATLAB simulation analysis and VWP methods. The ways to overcome them include the error calibration, average value and so on.

VI. CONCLUSION AND FUTURE DIRECTIONS

In this paper, a VWP model of PV cell, which can simplify the number of the mathematical equations of the conventional four-parameter model from five to three, has been correspond well with PVLM regardless of varying irradiance or temperature.
proposed. Meanwhile, to simplify this VWP model, a SVWP model, which can make the number of the model parameters decreased to two, has been also proposed. By the VWP model and SVWP model, the model parameters of the linear equivalent models have been expressed, which can build the direct relationship between four-parameter model and linear equivalent models. Finally, by some simulation experiments, the correspondence of the output characteristics between VWP model, SVWP model and conventional four-parameter model has been shown, the accuracy of the proposed VWP model and SVWP model compared with conventional four-parameter model has been illustrated, and the feasibility, availability and rationality of the linear equivalent models expressed by the VWP model and SVWP model has been verified.

Future work on the subject will be focused on the application of the proposed PVLVM-VWP and PVLVM-SVWP in the linear control theory. Here, their accuracy should be one of the research emphases when PV system cannot duly operate at the MPP under fast varying weather conditions.

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