Improvement of Edge Brightening by Means of Q Factor Minimization in Circular Antenna Apertures: High Efficient Taylor-Like Patterns

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\section*{ABSTRACT}
Implications and improvements of edge brightening effects led by $Q$ factor minimization restricted to keep the same level of directivity for high efficiency continuous circular aperture distributions are here reported. In this manner, an optimization strategy for a minimum $Q$ value \textemdash keeping the same level of efficiency and restricting the maximum sidelobe level (SLL) \textemdash is envisaged. As application of the method, a design procedure devoted to reduce the $Q$ factor of the antenna aperture distributions while keeping a high level of efficiency is outlined. Then, these optimal Taylor distributions are used as initial point to develop an optimization strategy. This procedure is devoted to search Taylor-like distributions which offer a good compromise between low $Q$ factor and high efficiency values with potentials for the antenna design scenario, based on a decrease in edge brightening effects led by the minimization of the aforementioned $Q$ ratio.

\section*{INDEX TERMS} Circular Taylor distributions, edge brightening, optimization techniques, $Q$ factor, supergain ratio.

\section*{I. INTRODUCTION}
The presence of edge brightening effects in continuous aperture distributions introduces problems in terms of practical realization of different array patterns \cite{1}.

Additionally, parameters as directivity must be taken into account into a synthesis procedure for improving the performance of these design techniques.

Alternatively, the so-called $Q$ factor represents a crucial parameter in the antenna design arena. More concretely, in \cite{2}, Chu established the basis of the theoretical description of the $Q$ factor concept for an omni-directional antenna. It was defined as the ratio between the stored energy and the radiated energy and exploiting in the analysis of the physical limitations of the antenna in terms of bandwidth versus gain.

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From this milestone, the small antennas framework started to attract the attention of the researchers and authors as Rhodes faced this topic trying to adapt these new possibilities to different scenarios. More concretely, he focused on planar antennas \cite{3} and also he analyzed the stored energy term of planar apertures \cite{4}. Another foundational work that deserves attention is the one developed by Hansen in 1981 \cite{5}, where the $Q$ factor concept has been discussed to review the limitations of different types of antennas. In line with these seminal developments of Rhodes and Hansen, Thiele \textit{et al.} \cite{6} devised an alternative formal approach for predicting the realizable radiation $Q$ of an electrically small antenna. Also, another example can be found in the work of Jonsson \textit{et al.} \cite{7}, where the $Q$ factor bounds for super-directive antennas have been analyzed by using an example with electrical size $ka = 0.51$ (where $k$ is the wavenumber and $a$ the radius of the minimum sphere that contains the antenna \textemdash also known as the...
Chu radius [2]–). Then, recent examples as the interesting review made by Schab et al. [8] have been devoted to compare the impact and the determination of this parameter within the radiation properties of the antenna systems. Another interesting and recent approach was discussed by Gustafsson and Capek [9] and it represents a deep and careful study about a trade-off between gain and $Q$ factor for antenna systems.

Other studies regarding infinite planar phased arrays have been conducted. For instance, Kwon and Pozar [10] reported antenna configurations involving arrays of dipoles with lengths from $0.15\lambda$ to $0.45\lambda$. In the same line, but generalizing the study to a wider range of dipole lengths and overcoming the limit $ka = 1$, more recent approaches can be found in the works of Ludvik-Øisipov and Jonsson [11], [12]. Here, they calculated the $Q$ factor and compared their results with MoM simulations in the framework of two-dimensional periodic arrays for both scenarios with and without ground plane.

Considering these descriptions, but focusing on the standard array pattern synthesis scenario and more concretely on its application to cases where superdirectivity effects are moderately low, we can translate the $Q$ factor of the distribution as the ratio between the expression of the radiated far field and the expression of the field within the antenna [13]. Then, the understanding of the $Q$ factor by means of the super gain ratio relation [14], [15] suggests that this parameter can give us an idea about the level of inefficiency of the finite analytical aperture distribution to reproduce the infinite pattern distribution. In addition and diversely to the cases referred in the standard literature of small antennas (i.e., with $ka < 1$), improvements on different radiation characteristics are here reported by reducing the levels of $Q$. Consequently, due to the range of application that we are proposing, the values here analyzed will be less than the unit. Thus, in this framework, by following the definition of this parameter, an improvement in terms of edge brightening effects led by a decrease in this $Q$ factor is here reported, which is highly interesting in terms of practical realization of the distribution.

However if, on the one hand it is important to reduce the effect as the edge brightening, on the other hand it is also fundamental not to provoke critical reductions of the antenna directivity. Due to this reason, a search devoted to reach a compromise between directivity and $Q$ factor is mandatory.

Examples of this edge brightening alleviation without exploiting the $Q$ factor concept are reported in [16] and [17]. In [16], the problem of minimizing edge brightening effects was faced by filling nulls into the array pattern generated by the distribution. As a drawback, although a multiplicity of solutions has been reached, the generation of a complex aperture distribution and a loss in directivity –due to the null filling– can be highlighted. Alternatively, a direct minimization of the edge brightening phenomenon has been performed in [17]. In this case, although a best level of this parameter could be reached, the $Q$ factor strategy will assure a monotonic behavior of the aperture that also can be highlighted as interesting thinking about practical realization. Until now, no relation between $Q$ factor and this performance in the border of the aperture has been analyzed.

So, the idea here proposed of a reduction in the $Q$ factor of the continuous aperture distribution –by keeping the same efficiency/directivity level– is linked to an energy decreasing into the invisible region. This fact uniquely can be understood as a reduction of the distribution peaks at the edge of the aperture (i.e. the edge brightening effect [18]) in comparison with the reference case, as well as a smoother transition of the excitation at the end of its tail. Thus, the extreme variability of this continuous aperture will be minimized as a result and a smoother transition (mainly near the edge) has to be expected. In this manner, a more stable aperture regarding errors or bandwidth will be produced. So, based on the energy considerations and restrictions abovementioned, the minimization of these parameters of the aperture would make the antenna (without any kind of bound to a concrete size requirement) a more efficient radiation system [19].

Therefore, by focusing on this idea, a procedure keeping the optimal directivities of circular Taylor distributions [20] and improving the $Q$ factor for generating pencil, also called sum, beams is here depicted. Circular Taylor distributions will play a main role in this work, because they represent a direct approach to obtain a pattern of maximum efficiency for a fixed sidelobe level (SLL) requirement. As it is well-known, these analytical distributions can be synthesized to obtain array excitations for reproducing similar patterns from an antenna array [21].

In this work, analytical Taylor distributions are defined by means of suitable optimization strategies in order to minimize the antenna $Q$ factor by also guaranteeing a high efficiency and desired SLL values. The main novelties of this contribution are as follows: 1) a closed-form equation by means of a well-defined finite integral of the $Q$ factor parameter for circular aperture distributions so as to yield a less complex evaluation (with high potentials, if we mainly think about introducing this property in optimization procedures); 2) the definition of a robust optimization method for obtaining high efficient distributions which also present improved edge brightening effects.

Towards this end, the paper is organized as follows. In section II, the theoretical bases of the method are stated. Then, a description of the optimization procedures for $Q$ factor values of the distribution by guaranteeing a specified SLL and a certain directivity is included and discussed in section III. Section IV is devoted to show the results of the optimization methodology here devised by means of circular Taylor patterns and by outlining the advantages in terms of dynamic range ratio ($I_{max}/I_{min}$) and edge brightening (EB) of the distributions. Finally, some conclusions and remarks are drawn in section V.

II. THEORETICAL BASIS

A. Q FACTOR AND DIRECTIVITY

Let us consider a general circular aperture distribution with a radius $a$ (Fig. 1) which generates a radiation pattern $F(\theta, \phi)$. 
elements and considering that the expression of the directivity of a circular aperture of radius $a$ uniformly illuminated is $D_0 = 4\pi^2a^2/\lambda^2$ (p. 261, [21]), the specific directivity can be defined as

$$G = D_i/D_0 = \frac{\lambda^2 |F(\theta_m, \phi_m)|^2}{\pi a^2 \int_0^\infty \int_0^\pi |F(\theta, \phi)|^2 \sin \theta d\theta d\phi}$$

(3)

where $F(\theta_m, \phi_m)$ is the value of the pattern function in its maximum point.

According to [23], for aperture sizes with $a \geq 5\lambda$, it could be accurately approximated (a detailed discussion can be also found in the appendix of the present paper) by:

$$G = \frac{\lambda^2 |F(\theta_m, \phi_m)|^2}{\pi a^2 \int_0^{2\pi} \int_0^\pi |F(\sin \theta, \phi)|^2 \sin \theta d\theta d\phi}$$

(4)

Otherwise, the effective directivity is the specific directivity when the aperture size tends to infinity ([14], [15]). So, let us define as

$$G_\infty = \frac{\lambda^2 |F(\theta_m, \phi_m)|^2}{\pi a^2 \int_0^{2\pi} \int_0^\pi |F(\sin \theta, \phi)|^2 \sin \theta d\theta d\phi}$$

(5)

Then, by exploiting the properties of the Fourier transformation procedure which leads the relationship between the radiation pattern and the aperture distribution, we can express the infinite integral of the pattern function by means of the finite integral of its circular aperture distribution. This expression is the so-called Parseval’s formula [15]

$$\int_0^{2\pi} \int_0^\pi |F(\sin \theta, \phi)|^2 \sin \theta d\theta d\phi = \left(\frac{2a}{\lambda}\right)^2 \int_0^{2\pi} \int_0^\pi |g(p)|^2 p dp d\phi$$

(6)

where $g(p)$ is the antenna aperture distribution which generates the far field radiation power pattern $F(\theta, \phi)$, and $p = \pi p/\lambda$ (and $p \in [0, a]$). This expression of the aperture is related with the circular aperture shown in Fig. 1 by the formula (p. 214, [21])

$$g(p) = \frac{2a^2}{\pi} K(\alpha p/\pi, \beta)$$

(7)

where $K(\alpha p/\pi, \beta)$ is $K(\alpha p/\pi)$ due to the symmetry of the problem.

Therefore, a compact expression of the $SGR$ –as it was aforementioned, defined as the ratio between the specific directivity and the effective directivity of a given aperture distribution ([14], [15])– is as follows:

$$SGR = G/G_\infty = \frac{\left(\frac{2a}{\lambda}\right)^2 \int_0^{2\pi} \int_0^\pi |g(p)|^2 p dp d\phi}{\frac{\lambda^2 |F(a)|^2}{\int_0^{2\pi} \int_0^\pi |F(a)|^2 u du d\phi}}$$

(8)
number of continuous circular aperture distributions is based on altering a circular Taylor distribution with
expression of the radiation far field pattern in the uniform case
roots of the uniform distribution, it is, when the aperture
aperture distribution and the local minimum previous to this difference between the level of amplitude in the border of the EB
brightening effect, let us mathematically define a quality
distribution. So, in order to evaluate the impact of the edge
The edge brightening [1], [16], [17] can be defined as a
aperture distribution
FIGURE 2. Graphical description of the determination of the parameter devoted to evaluate the effect of edge brightening (EB) in the continuous aperture distribution g(πρ/a). Case in example: relative aperture of a circular Taylor distribution with n = 5. SLLd = –25dB.
and, accordingly, the Q factor is determined through Eq. (1) by means of a formulation based on finite integrals
B. EDGE BRIGHTENING
The edge brightening [1], [16], [17] can be defined as a pronounced rise in amplitude at the boundary of the aperture distribution. So, in order to evaluate the impact of the edge brightening effect, let us mathematically define a quality parameter EB. To determine this quality parameter, the difference between the level of amplitude in the border of the aperture distribution and the local minimum previous to this value (in this same amplitude curve) can be proposed (Fig. 2).
Therefore, the proposed definition is
where ρmin is the local minimum previous to the edge of the continuous aperture distribution (ρ = π)
C. CIRCULAR TAYLOR DISTRIBUTION
As it was above mentioned, the present work is focused on the use of circular Taylor distributions due to their high efficiency.
The basis of this approach is the modification of the roots of the uniform distribution, it is, when the aperture is uniformly excited. This represents the most directive distribution for a given radius a of a circular antenna. The expression of the radiation far field pattern in the uniform case becomes [21]:
where u = (2a/λ) sin θ and J1(πu) is the Bessel function of first order.
More concretely, the method devised by Taylor for continuous circular aperture distributions is based on altering a number of $\tilde{n} - 1$ roots of the pattern and controlling the level of the innermost $\tilde{n} - 1$ ring side lobes [20]. Thus, the sum pattern produced by the circular Taylor distribution would be given by [20], [21]:

where $\gamma_{1,n}$ is the n-th root of the Bessel function of first order $J_1(\gamma_{1,n}π) = 0$ and $u_n$, n = 1,...N the manipulated roots of the pattern by means of this methodology are

where A is related with the desired sidelobe level (SLLd), through the expression $SLLd = -20\log_{10}(\text{cos}(πA))$.
Finally, the expression of the continuous aperture which generates the pattern is [20], [21]

where $p = πρ/a$, $ρ$ is the radial coordinate of the aperture, so it is in the range $ρ ∈ [0, a]$ and $J_0(\gamma_{1,m}π)$ is the Bessel function of the first kind and order zero, evaluated in the roots of the Bessel function of the first order.

III. OPTIMIZATION METHOD
In this section, the formulation highlighted in the previous section is exploited within an optimization strategy devoted to afford low Q factor levels by guaranteeing high efficiencies for a desired SLL. The proposed optimization process can be understood as a single objective problem (SOP) [24] and it can be summarized as follows:
1. As starting point, a circular Taylor pattern is established by selecting the $\tilde{n}$ value which brings the maximum efficiency for a certain $SLL$. $u_n^0$ denotes the roots of this pattern.
2. Small perturbations $\delta u$ to these initial roots are introduced. The main objective of these perturbations is to synthesize a pattern in which the efficiency is maximum, the Q factor is low and the SLL is controlled.
First of all, it can be concluded that there are three objectives to be considered in the design concerning the efficiency, Q factor, and SLL. Moreover, the optimization variables are continuous and real.
In this process, the following scalar cost function is minimized:

where $\eta$ denotes the aperture illumination efficiency (3), $\eta_{\text{Taylor}}$ is the aperture illumination efficiency of the reference aperture: the initial Taylor aperture distribution, $Q$ is the quality factor (expressed in natural units), whereas $c_1$ are the weights to control the importance given to each term
of (15). The function \( f_{SLL} \), which penalizes the increasing in the sidelobe level, is defined by

\[
f_{SLL} = \begin{cases} 
(SLL_0 - SLL_d)^2, & \text{if } SLL_0 \geq SLL_d \\
0, & \text{otherwise}
\end{cases}
\]

(16)

where \( SLL_0 \) and \( SLL_d \) are, respectively, the obtained and the desired sidelobe level of the pattern. The squared difference is here adopted for amplifying the error of this term. In such a way, the biggest error gets amplified more and, in consequence, it will obtain more priority for improvement. Moreover, \( \delta \bar{u} = [\delta u_1, \delta u_2, \ldots, \delta u_{n-1}]^T \) is the vector of perturbations applied to the roots of the initial circular Taylor pattern.

To develop this procedure, a hybrid approach described on section 3 of [25], which is based on the procedure developed in [26] and involves the Simulated Annealing (SA) algorithm [27] with a modification based on the Downhill Simplex (DS) method [28] has been implemented. More concretely, as it is referred in [25], the procedure devoted to found the minimum is equal to the one developed by DS with the following main difference: A \( T \) parameter analogous to the temperature for the SA algorithm [27] is introduced. In such a way, a probabilistic component (modeled exactly in the same manner as in the SA) is added in the DS strategy for comparing with the other values of the cost function represented in the Simplex [28]. The aforementioned \( T \) is set in order to explore a wider space of solutions. In this manner, the algorithm improves the searching and prevents to always fall in the same local minimum. In our case, we set \( T \) as 100.

IV. RESULTS

Let us first consider the case of a circular Taylor-like pattern with a \( SLL_d \) of \(-25\)dB. So, in order to proceed with the optimization of the \( Q \) factor and improve the solution in terms of edge brightening for distributions with a high efficiency, a circular Taylor pattern distribution with \( \bar{n} = 5 \) has been chosen (see Figs. 3 and 4). The initial roots, as well as the directivity, the \( SLL \), the \( Q \) factor, the dynamic range ratio (\( |l_{\max}/l_{\min}| \)) and the edge brightening quality value (\( EB \)) of this pattern are shown in Table 1.

The optimization process was initialized by considering these roots as starting solution. Several cases have been run in which the weight of the efficiency term \(-c_2 \) coefficient in (15)– has been varied in order to find a good compromise between high efficiency and low \( Q \) factor values. These coefficients have been set in all the examples here analyzed as \( c_1 = 1200, c_2 = 1500, c_3 = 1000 \). Results keeping the same level of efficiency (i.e., having an efficiency loss of \( \Delta \eta = \eta_{Taylor} - \eta = 0\% \)) and consequently the directivity of the circular Taylor pattern distribution are shown in Table 1. It can be observed that, by keeping the same efficiency, this method led a decrease of the \( Q \) factor of 1.91 dB (a 35.61% in natural units) and both the dynamic range ratio and the edge brightening were reduced a 4.44% and a 18.68%, respectively. The relative amplitude of both apertures (the initial and the optimized) are shown (Fig. 4) as well as a comparison between the generated radiation far field pattern and the regular Taylor case (Fig. 3). Finally, in order to complete the discussion, the roots which generate each one of the patterns can be found in Table 1 with a report of the resulting quality parameters.

For illustrating the behavior of the method and to understand how it works in case of different side lobe topography, \( SLL_d \) of \(-30\)dB and \(-35\)dB are also analyzed.
In the case of a $SLL_d$ of $-30\text{dB}$, the most efficient choice is the circular Taylor pattern distribution with $\bar{n} = 8$ (see Figs. 5 and 6). In the same manner that in the previous case, the initial roots, as well as the $SLL$, the directivity, the $Q$ factor, the dynamic range ratio ($|I_{\text{max}}/I_{\text{min}}|$) and the edge brightening quality value ($EB$) are shown in Table 2.

To outline the performance of the method in terms of aperture distribution, the relative amplitude of both cases (the initial and the optimized continuous aperture distributions) are shown in Fig. 6. A comparison between the two radiation far-field patterns is also shown in Fig. 5, as well as the roots of each one of them (see Table 2).

Additionally, for the case of a circular Taylor pattern distribution of $SLL_d = -35\text{dB}$, a $\bar{n} = 13$ has been chosen (see Figs. 7 and 8) for guaranteeing a high efficiency.

The initial roots, the actual $SLL$ as well as the directivity, the $Q$ factor, the dynamic range ratio ($|I_{\text{max}}/I_{\text{min}}|$) and the edge brightening quality value ($EB$) of the compared patterns are shown in Table 3.

In the same spirit of the previous analysis, the performance of the method is illustrated in Figs. 7 and 8, where both apertures (the initial and the optimized ones) as well as the radiation far field patterns are shown. The roots related with the generated patterns in this case are reported with the results in Table 3.
TABLE 3. Roots, SLL, Directivity, Q Factor, Dynamic Range Ratio and Edge Brightening of Taylor Pattern with $n = 13$ and $SLL_d = -35dB$ and the pattern with optimized $Q$ value.

| Radius of the aperture distribution | Taylor pattern | Optimized pattern |
|-------------------------------------|----------------|------------------|
| $a = \lambda$ | $\pi = 13$ | $SLL_d = -35dB$ |
| $u_1$ | 1.6669 | 1.6775 |
| $u_2$ | 2.2334 | 2.2495 |
| $u_3$ | 3.0694 | 3.0926 |
| $u_4$ | 4.0080 | 4.0439 |
| $u_5$ | 4.9921 | 5.0471 |
| $u_6$ | 5.9993 | 6.0896 |
| $u_7$ | 7.0198 | 7.1962 |
| $u_8$ | 8.0484 | 8.2358 |
| $u_9$ | 9.0824 | 9.2995 |
| $u_{10}$ | 10.1202 | 10.2580 |
| $u_{11}$ | 11.1606 | 11.3664 |
| $u_{12}$ | 12.2031 | 12.4207 |

| SLL (dB) | 35.11 | 35.00 |
| Directivity (dB) | 33.15 | 33.15 |
| $Q$ (dB) | -19.81 | -23.28 |
| $|I_{\text{max}}/I_{\text{min}}|$ | 5.82 | 4.90 |
| $EB$ | 0.1952 | 0.1384 |

of $3.47$ dB (a $55.02\%$ in natural units). In case of dynamic range ratio, the optimized solutions have reached a reduction of a $14.55\%$ and a $15.81\%$ (a $1.26\%$ and $11.37\%$ less than in the $-25dB$ case, respectively). Regarding edge brightening effects, they have been reduced a $23.17\%$ and a $29.10\%$ on each case. They represent improvements of a $4.49\%$ and a $10.42\%$ respectively. So, we confirm how the method tends to alleviate the tendency of the aperture of impinging a much higher level of excitation on its edges than in the medium part of its tail. These results only can be understood due to the fact that a lower $Q$ is related to a decreasing in the level of far field power radiation pattern present in the visible region limit. Therefore, from the inspection of Figs. 4, 6 and 8 it can be stated that generally improvements in terms of $Q$ imply a smooth transition on the tail of the aperture distribution in comparison with the initial case.

V. CONCLUSION

In this paper, several numerical optimizations were performed and results of improving the $Q$ factor by preventing efficiency losses have been shown. A relation with the edge brightening (and also the dynamic range ratio) of the distribution has been established and, in this manner, the practical applicability of this process is devised. The minimization of the $Q$ ratio –by keeping the same directivity level– within the far field power generated by an aperture distribution is translated in a aperture distribution with less edge brightening and –generally– on a less dynamic range ratio of the aperture. More concretely, for a $SLL_d = -25dB$, an improvement of a $35.61\%$ of $Q$ factor leads improvements in terms of dynamic range ratio ($4.44\%$) and edge brightening ($18.68\%$). At the same time, for a $SLL_d = -30dB$, through an improvement of a $44.01\%$ of $Q$ factor, the improvement in terms of dynamic range ratio and edge brightening were $14.55\%$ and $23.17\%$ respectively. Finally, for a level of $SLL_d = -35dB$, an improvement of $Q$ about $55.02\%$ leads improvements of $15.81\%$ and $29.10\%$ in terms of edge brightening of the continuous aperture distribution. Based on these results, a method to design good distributions devoted to afford high efficiency and improving the aperture variability (by reducing the $Q$ factor for a required directivity) was developed. This technique shows interesting potentials for the antenna design discipline. At the same time, this strategy overcomes the results of standard techniques –based on a direct optimization of the edge brightening parameter– by alleviating the tendency of the aperture of impinging a much higher level of excitation on its edges than in the medium part of its tail. This statement can be understood from the fact that –diversely from [17], where a $Q$ factor of $-18.64dB$ as well as a flat ending of the aperture (fig. 4 in [17]) can be obtained from the direct optimization of the edge brightening of the aperture distribution analyzed– the result here presented for the same $SLL (-25dB)$ reports a $Q$ of $-21.18dB$ and a tail of the aperture which is descending up to its edge.

Although, this proposed technique could be extended to other distributions, the spirit of the present work is based on the idea of using circular Taylor distributions because of they are optimal in terms of efficiency for a desired $SLL$. Additionally, it can be also highly remarkable that an analogous procedure could be easily implemented for linear Taylor distributions.

APPENDIX

Accuracy of the approximation: Regarding the precision of the $Q$ ratio expressions based on aperture integrals by means the Parseval identity it is necessary to outline the performance of this approximated expression and its level of accuracy versus the formal infinity integral of the far field pattern. This can be done by simply comparing two different performances: the calculation of the method by integrating the exact expressions of the far field pattern generated by the aperture –on the denominator of (3)– and the method here developed and based on the use of the integral of the aperture –on the denominator of (4). The results regarding this comparison in the worst scenario ($SLL_d = -25dB$) are shown in Fig. 9.A.

As an example and due to the fact that it represents the worst scenario in terms of agreement between the results of both methods, the performance of the case for a $SLL_d = -25dB$ has been taken into account. By means of the far field integral in the invisible region, the differences between the computation times of both strategies are reported in Fig. 9.B. It can be appreciated how, as it could be expected, the greater the $n$ parameter is, the more computation time will need. The codes to calculate the integrals have been developed in MATLAB [29], more concretely, they have been run in MatlabR2016b. All the computation times are referred to a
an 11.50% of the nominal value in natural units. For cases of SLL IV is of 0.002 in natural units (more concretely in the case of 7.78% and 3.67% respectively.

machine with a processor Intel i7-2600 CPU @3.40GHz and a RAM memory of 8GB.

Regarding the level of accuracy we can confirm that the maximum level of misalignment between the 2 methods (the exact and the approximated) for the cases studied in Sect. IV is of 0.002 in natural units (more concretely in the case of SLLd = −25dB, $\tilde{n} = 5$). In this concrete case it represents an 11.50% of the nominal value in natural units. For cases of SLLd = −30dB, $\tilde{n} = 8$ and SLLd = −35dB, $\tilde{n} = 13$ are 7.78% and 3.67% respectively.

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