Quantum Nucleardynamics as an $SU(2)_N \times U(1)_Z$ Gauge Theory

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It is illustrated that quantum nucleardynamics (QND) as an $SU(2)_N \times U(1)_Z$ gauge theory, which is generated from quantum chromodynamics (QCD) as an $SU(3)_C$ gauge theory through dynamical spontaneous symmetry breaking, successfully describes nuclear phenomena at low energies. The proton and neutron assigned as a strong isospin doublet are identified as a colorspin plus weak isospin doublet. Massive gluon mediates strong interactions with the effective coupling constant $G_R/\sqrt{2} = g^2_\text{w}/8M_w^2 \approx 10 \text{ GeV}^{-2}$ just like Fermi weak constant $G_F/\sqrt{2} = g^2_\text{w}/8M_w^2 \approx 10^{-5} \text{ GeV}^{-2}$ in the Glashow-Weinberg-Salam model where $g_w$ and $g_\text{w}$ are the coupling constants and $M_w$ are the gauge boson masses. Several explicit evidences such as cross sections, lifetimes, nucleon-nucleon scattering, magnetic dipole moment, nuclear potential, gamma decay, etc. are shown in support of QND. The baryon number conservation is the consequence of the $U(1)_Z$ gauge theory and the proton number conservation is the consequence of the $U(1)_f$ gauge theory.

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I. INTRODUCTION

One of the longstanding problems in physics is the establishment of nuclear dynamics at low energies as strong interactions based on a fundamental theory. The knowledge of the nuclear force is phenomenologically very good for $r > 2.0 \text{ fm}$ but it is known only partly for $0.8 < r < 2.0 \text{ fm}$ and only poorly for $r < 0.8 \text{ fm}$. This implies that there is no systematic approach based on a fundamental theory applicable in the wide range from the short distance to the long distance for nuclear physics. Quantum chromodynamics (QCD) as an $SU(3)_C$ gauge theory is generally believed as the fundamental theory for strong interactions and is successful in various perturbative tests. Nevertheless, there are, in the one hand, two distinct problems in QCD with quarks and gluons as fundamental constituents. One is the confinement, which is not rigorously explained in the low energy region and the other is the $\Theta$ vacuum $\Theta$, which is a superposition of the various false vacua, violating CP symmetry. At lower energies, on the other hand, many nuclear effective models as the alternatives of QCD were proposed but their applications are not complete and limited to a few aspects. It is thus the motivation of quantum nucleardynamics (QND), which is derived from QCD as the consequence of the confinement and $\Theta$ vacuum, to explain diverse nuclear phenomena at lower energies consistently. For examples, nuclear issues to be clarified are as follows: Lande’s spin $g$-factor for nucleon, constant nucleon density, intrinsic quantum number, nucleon-nucleon scattering data, baryon number conservation, proton number conservation, baryon asymmetry, etc. These phenomena at relatively lower energies may be partly explained by effective models but each effective model is only applicable to limited issues. To resolve these problems all together, a new concept of colorspin is introduced so that QND as an $SU(2)_N \times U(1)_Z$ gauge theory is dynamically spontaneous symmetry broken from QCD as an $SU(3)_C$ gauge theory in terms of the local gauge symmetry of colorspin $\Theta$. This paper attempts to demonstrate that QND is an $SU(2)_N \times U(1)_Z$ gauge theory for nuclear interactions just as the Glashow-Weinberg-Salam (GWS) model $\Theta$ is an $SU(2)_L \times U(1)_Y$ gauge theory for weak interactions.

QND is generated from QCD through the dynamical spontaneous symmetry breaking (DSSB) mechanism: $SU(3)_C \to SU(2)_N \times U(1)_Z \to U(1)_f$. QND as an $SU(2)_N \times U(1)_Z$ gauge theory for strong interactions of nucleons and the GWS model $\Theta$ as an $SU(2)_L \times U(1)_Y$ gauge theory for weak interactions of quarks and leptons have analogous properties $\Theta$. The effective strong coupling constant $G_R/\sqrt{2} = g^2_\text{w}/8M_w^2 \approx 10 \text{ GeV}^{-2}$ like Fermi weak constant $G_F/\sqrt{2} = g^2_\text{w}/8M_w^2 \approx 10^{-5} \text{ GeV}^{-2}$ and the color mixing angle $\sin^2 \theta_R = 1/4$ like the Weinberg mixing angle $\sin^2 \theta_W = 1/4$ thus play important roles in nuclear interactions. The proton and neutron assigned as a strong isospin doublet are identified as a colorspin plus weak isospin doublet. The magnetic dipole moment and nucleon-nucleon scattering give the explicit evidence for the color intrinsic angular momentum of hadrons. Several explicit evidences such as cross sections, lifetimes, nucleon-nucleon scattering, meson-nucleon scattering, magnetic dipole moment, nuclear potential, nuclear binding energy, gamma decay, baryon asymmetry, etc. are shown in support of QND. The nuclear radius $r = r_0 A^{1/3} = r_0 n^2$ with the nucleon number $A$ introduces the extrinsic principal quantum number $n = A^{1/6}$ and the extrinsic angular momentum $l$ originated from color charges. It is suggested that the baryon number conservation is the consequence of the $U(1)_Z$ gauge theory and the proton number conservation is the consequence of the $U(1)_f$ gauge theory.

This paper is organized as follows. In Section II, QND as an $SU(2)_N \times U(1)_Z$ gauge theory with the $\Theta$ term is introduced as the consequence of DSSB of $SU(3)_C$ gauge theory. The conclusive evidences and applications of QND in various nuclear phenomena are discussed in Section III. Section IV describes comparison between QND and effective models. Section V is devoted to conclusions.
II. DYNAMICAL SPONTANEOUS SYMMETRY BREAKING IN STRONG INTERACTIONS

The DSSB mechanism of QCD to QND is briefly described before discussing the evidences of QND in nuclear dynamics. According to the reference [2], SU(3)C → SU(2)N × U(1)Z → U(1)f.

The SU(3)C gauge-invariant Lagrangian density with the Θ term is, in four vector notation, given by

\[ \mathcal{L}_{QCD} = -\frac{1}{2} Tr G_{\mu \nu} G^{\mu \nu} + \sum_{i=1}^{n} \bar{\psi}_i \gamma^\mu D_\mu \psi_i + \bar{\Theta} \frac{g_s^2}{16\pi^2} Tr G^{\mu \nu} \tilde{G}_{\mu \nu}. \]  

(1)

where the subscript \( i \) stands for the classes of pointlike spinors, \( \psi \) for the spinor and \( D_\mu = \partial_\mu - ig_s A_\mu \) for the covariant derivative with the coupling constant \( g_s \). Particles carry the local SU(3)C charges and the gauge fields are denoted by \( A_\mu = \sum_{a=0}^{8} A^a_\mu \lambda^a / 2 \) with the Gell-Mann matrices \( \lambda^a \), \( a = 0, \ldots, 8 \). The field strength tensor is given by \( G_{\mu \nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig_s \langle A_\mu, A_\nu \rangle \) and \( \tilde{G}_{\mu \nu} \) is the dual field strength tensor. In the Lagrangian density, the explicit quark mass term is not contained but the \( \Theta \) vacuum term is added. The fine structure constant \( \alpha_s = g_s^2 / 4\pi \) is the only free parameter.

Gluon interactions in the effective SU(3)C gauge invariant Lagrangian density with the bare \( \Theta \) term are

\[ \Delta \mathcal{L} = -\frac{1}{2} Tr G_{\mu \nu} G^{\mu \nu} + \Theta \frac{g_s^2}{16\pi^2} Tr G^{\mu \nu} \tilde{G}_{\mu \nu}. \]  

(2)

Apart from charge nonsinglet gauge bosons, four singlet gauge boson interactions are parameterized by the SU(3) symmetric scalar potential:

\[ V_\epsilon(\phi) = V_0 + \mu^2 \phi^2 + \lambda \phi^4 \]  

(3)

which is the typical potential with \( \mu^2 < 0 \) and \( \lambda > 0 \) for spontaneous symmetry breaking. The first term of the right hand side corresponds to the vacuum energy density representing the zero-point energy by non-axial singlet bosons. The axial vacuum field \( \phi \) is shifted by an invariant quantity \( \langle \phi \rangle \), which satisfies \( \langle \phi \rangle^2 = \phi_0^2 + \phi_1^2 + \phi_2^2 = \phi_3^2 \) with the condensation of the axial singlet gauge boson \( \langle \phi \rangle = (\phi_3^2)^{1/2} \). DSSB is connected with the surface term \( \Theta \frac{g_s^2}{16\pi^2} Tr G^{\mu \nu} \tilde{G}_{\mu \nu} \), which explicitly breaks down the SU(3)C gauge symmetry through the condensation of axial singlet gauge bosons and breaks down the axial current. The \( \Theta \) can be assigned by an dynamic parameter by

\[ \Theta = 10^{-61} \rho_G / \rho_m \]  

(4)

with the matter energy density \( \rho_m \) and the vacuum energy density \( \rho_G = M_G^2 \).

The Lagrangian density of QND as an SU(2)N × U(1)Z gauge theory has the same form with QCD as an SU(3)C gauge theory without the explicit mass term:

\[ L_{QND} = -\frac{1}{2} Tr G_{\mu \nu} G^{\mu \nu} + \sum_{i=1}^{n} \bar{\psi}_i \gamma^\mu D_\mu \psi_i + \bar{\Theta} \frac{g_s^2}{16\pi^2} Tr G^{\mu \nu} \tilde{G}_{\mu \nu}, \]  

(5)

where the bare \( \Theta \) term is a nonperturbative term added to the perturbative Lagrangian density with an SU(2)N × U(1)Z gauge invariance. The \( \Theta \) term apparently odd under both P, T, C, and CP operation. The nuclear coupling constant \( g_s^2 = c_f g_s^2 = \sin^2 \theta_R g_s^2 = g_s^2 / 4 \) is given in terms of the strong coupling constant \( g_s \) and the color factor \( c_f \). The effective strong coupling constant at low energy is expressed in analogy with the phenomenological, electroweak coupling constant \( G_F \):

\[ \frac{G_R}{\sqrt{2}} = \frac{c_f^2 g_s^2}{8(k^2 - M_G^2)} \approx \frac{c_f^2 g_s^2}{8M_G^2}. \]  

(6)

where \( M_G \) indicates the mass of gluon, \( k \) denotes the four momentum, and \( c_f^2 = \sin^2 \theta_R \) represents the nuclear color factor. The gluon mass is thus generally reduced by the singlet gluon condensation \( \langle \phi \rangle \):

\[ M_G^2 = M_H^2 - c_f g_s^2 \langle \phi \rangle^2 = c_f g_s^2 [A_0^2 - \langle \phi \rangle^2] \]  

(7)

where \( M_H = \sqrt{c_f^2 g_s^2} A_0 \) is the gauge boson mass at the grand unification scale, \( A_0 \) is the singlet gauge boson, and \( \langle \phi \rangle \) represents the condensation of the axial singlet gauge boson. The color factor \( c_f \) becomes the symmetric factor with even parity for singlet gauge boson and is the asymmetric factor with odd parity for axial singlet gauge boson. The vacuum energy due to the zero-point energy, represented by the gauge boson mass, is thus reduced by the decrease of the color factor and the increase of the axial singlet gluon condensation as temperature decreases. This process makes the breaking of discrete symmetries P, C, T, and CP due to the gluon mass.

III. QUANTUM NUCLEARDYNAMICS AS A GAUGE THEORY

In QND, the most important concepts are that nucleons are treated as a color doublet and gluons are massive as hinted by the analogy of the GWS model [2]. QCD as the color SU(3)C symmetry generates quantum nucleardynamics (QND) as the SU(2)N × U(1)Z symmetry, which governs nuclear strong dynamics, and the U(1)f symmetry, which governs nuclear electromagnetic dynamics, with the condensation of singlet gluons. The SU(2)N × U(1)Z symmetry and U(1)f symmetry using the symmetric color factors, \( c_f = (c_f^1, c_f^2, c_f^3) = (1/3, 1/4, 1/12, 1/16) \), are applied to the typical strong interactions: color asymmetric configuration with odd parity is not observed but color symmetric configuration with even parity is observed. The factors described above are the pure color factors due to
color charges but the effective color factors used in nuclear dynamics must be multiplied by the isospin factor $i_{1f}^w = \sin^2 \theta_W = 1/4$ with the weak Weinberg angle $\theta_W$ since the proton and neutron are an isospin doublet as well as a color doublet: $c_{1f}^f = i_{1f}^w c_f = \sqrt{3}/2(c_f^u, c_f^d, c_f^c, c_f^\gamma)$ for symmetric configurations. Note that the color mixing is the origin of the Cabibbo angle for quark flavor mixing in weak interactions. For example, the electromagnetic color factor for the $U(1)_f$ gauge theory becomes $\alpha_{1f}^\gamma = \alpha_s/64 \simeq 1/137$ when $\alpha_s \simeq 0.48$ at the strong scale $\Lambda_{\text{QCD}}$. Nucleons as spinors possess up and down colorspins as a color doublet just like up and down strong isospins:

$$\left( \begin{array}{c} 1 \\ c \end{array} \right)_c, \uparrow = \left( \begin{array}{c} 1 \\ 0 \end{array} \right)_c, \downarrow = \left( \begin{array}{c} 0 \\ 1 \end{array} \right)_c.$$  

This implies that conventional, global $SU(2)$ strong isospin symmetry in $\text{hadrons}_c$ is postulated as the combination of local $SU(2)$ colorspin and local $SU(2)$ weak isospin symmetries: the static $SU(3)$ quark model is extended as the combination of $SU(2)_N \times U(1)_Z$ gauge theory for nuclear interactions and $SU(2)_L \times U(1)_Y$ gauge theory for weak interactions. Color charge numbers for the proton and neutron, which will be discussed more, are shown in Table 1 where the subscript $d$ denotes the colorspin doublet and the subscript $s$ denotes the colorspin singlet. Nucleons as the color spin doublet are governed by QND as the $SU(2)_N \times U(1)_Z$ gauge theory just as leptons or quarks as the isospin doublet are governed by the GWS model as the $SU(2)_L \times U(1)_Y$ gauge theory in weak interactions. This is also compatible with the confinement of the color electric monopole inside the hadron space but the confinement of the color magnetic monopole inside the vacuum space. The overall wave function for a nucleon may thus be expressed by $\psi_N = \psi(\text{colors}) \psi(\text{isospin}) \psi(\text{spin}) \psi(\text{space})$.

There are several longstanding problems in nuclear physics, which may be resolved if QND as an $SU(2)_N \times U(1)_Z$ gauge theory is applied. The typical evidences discussed are multipole expansion, lifetime and cross section, excitation and decay, conservation and violation of symmetries, strong isospin and colorspin, nuclear mass and charge, nuclear magnetic dipole moment, deuteron, nucleon-nucleon scattering, meson-nucleon scattering, strong isospin and colorspin, nuclear potential, shell model, and nucleus binding energy.

A. Intrinsic and Extrinsic Multipole Expansion

There are two kinds of quantum numbers, intrinsic and extrinsic quantum numbers, described in the reference [2]. Intrinsic quantum numbers mainly play important role inside the hadron size while extrinsic quantum numbers play important role outside the hadron size. There are, on the other hand, two kinds of strong interactions; one produces the Coulomb potential for effectively massless gluons and the other one produces the Yukawa potential for massive gluon. The Coulobm potential is a special case of the Yukawa potential with the zero gauge boson mass. The Yukawa potential derives the confinement of quarks and gluons in the region of low energies. The effective interaction amplitude becomes $M = \frac{G_R J_\mu J_\mu}{\mu}$ where only the color vector current is conserved. The effective coupling constant of the Yukawa potential, $G_R = \frac{2c_f g^2}{8M^2_G}$, is not a fixed value but it changes according to the condensation of singlet gauge bosons; as energy goes down it increases. The gluon mass is the QCD cutoff energy at QCD confinement phase transition. Therefore, the interaction range is roughly the size of the hadron 1 fm; the interaction range by the massive gluon is the QCD cutoff scale. According to the hadron size, the gluon mass becomes a value around 300 MeV for the $SU(2)_N$ gauge theory or 140 MeV for the $U(1)_Z$ gauge theory in hadron formation at the QCD cutoff scale.

Inside the hadron, the Yukawa potential is decomposed with several terms corresponding multipoles: for a few terms of lower orders, the constant potential ($\sim \Lambda_{\text{QCD}}$), the linearly increasing potential ($\sim \alpha \Lambda_{\text{QCD}} r^2/2$), the harmonic oscillator potential ($\sim \beta r^2/2 = \Lambda_{\text{QCD}}^2 r^2/6$), etc. These terms are dependent on intrinsic color angular momenta. The constant energy as the monopole potential due to the gluon mass is relevant for the vacuum energy: the vacuum energy density $M^2_G = \Lambda_{\text{QCD}}^2$ corresponds to the bag constant in the bag model. The linearly increasing potential as the dipole potential is the dominant contribution to quark and gluon confinement: the coefficient is $\alpha \simeq \Lambda_{\text{QCD}}^2/8$ with $\Lambda \simeq 0.3$ GeV. The harmonic oscillator potential as the quadrupole potential, which has the coefficient $\beta = \Lambda_{\text{QCD}}^2/3 \approx 0.01$ GeV$^2$, leads to the dynamics for photons as quanta.

Outside the hadron, the Yukawa potential is also decomposed with several terms corresponding multipoles: for a few terms of low orders, the Coulomb potential ($\propto -1/r$), the dipole potential ($\propto 1/r^2$), the quadrupole potential ($\propto -1/r^3$), etc. These terms are dependent on extrinsic angular momenta. The Coulomb potential as the monopole potential represents the repulsive interaction for photon charges with the energy $E \leq \Lambda_{\text{QCD}}$. This term is the origin of the electromagnetic potential in a nucleus. Dipole and quadrupole potentials deform a nucleus from the spherical shape. Due to the requirement of even parity for nucleons, the monopole moment and quadrupole moment are left as electric moments while the dipole moment and octopole moment are left as magnetic moments.

Nuclear strong interactions are governed by an $SU(2)_N \times U(1)_Z$ gauge theory due to massive gluons while nuclear electromagnetic interactions are governed by a $U(1)_f$ gauge theory due to massless photons. Mas-
sive gluons in the vacuum space is spatially quantized by the maximum wavevector mode \(N_R \approx 10^{20}\), the total gluon number \(N_G = 4\pi N_R^3/3 \approx 10^{91}\), and the gluon number density \(n_G = \frac{3}{\Lambda^2_{QCD}} \approx 10^{-2} \text{GeV}^3 \approx 10^{39} \text{cm}^{-3}\). Baryon matter represented by massive baryons is quantized by the maximum wavevector mode (Fermi mode) \(N_F \approx 10^{26}\) and the total baryon number \(B = N_B = 4\pi N_F^3/3 \approx 10^{78}\). Massless photons are quantized by the maximum wavevector mode \(N_c \approx 10^{20}\) and the total photon number \(N_{\gamma} = 4\pi N_c^3/3 \approx 10^{68}\). Massless phonons in the matter space are spatially quantized by the maximum wavevector mode (Debye mode) \(N_D \approx 10^{25}\) and the total phonon number \(N_{\pi} = 4\pi N_D^3/3 \approx 10^{55}\). The QCD confinement due to massive gluons may be interpreted as the dual Meissner effect; the color electric field is confined and interacts pointlike at the longer range than the hadron size. Nuclear force thus interacts in terms of the interchange of the massive gluon with color charges. This implies that nuclear force interacts via the Yukawa potential due to the massive gluon both inside and outside the hadron size.

**B. Nuclear Lifetime and Cross Section as an \(SU(2)_N \times U(1)_Z\) Gauge Theory**

There are several, explicit examples about lifetimes and cross sections, supporting QND as an \(SU(2)_N \times U(1)_Z\) gauge theory in analogy with the GWS model as an \(SU(2)_L \times U(1)_Y\) gauge theory.

Quark-antiquark pair annihilation \(q + \bar{q} \rightarrow g + g\) gives the decay rate \(\Gamma\)

\[
\Gamma = \frac{8\pi}{3} \left(\frac{G_F^2}{m_q}\right)^2 |\psi(0)|^2
\]

in the spin, color singlet configuration with the quark at rest. Using the values \(\alpha_s = 0.48\), \(m_q = 310 \text{ MeV}\), and \(|\psi(0)|^2 = (148 \text{ MeV})^3\), the decay rate \(\Gamma = 135 \text{ MeV}\) or the lifetime \(\tau = 3.6 \times 10^{-24} \text{ s}\) is obtained. The state \(\Sigma^0\) is formed as a resonance of central mass 1385 MeV in a \(K^-\pi\) interaction:

\[
K^- + p \rightarrow \Sigma^0 \rightarrow \Lambda + \pi^0
\]

where the Q-value in the decay 130 MeV and the lifetime \(\tau = 1/\Gamma \approx 10^{-23} \text{ s}\) are estimated from the measured decay width \(\Gamma = 36 \text{ MeV}\). If the decay rate \(\Gamma \approx G_F^2 m_{\pi}^3\) is used in analogy with the muon decay rate \(\Gamma = G_F^2 m_{\mu}^5/192\pi^3\) of weak interactions, the \(\Sigma^0\)-hyperon lifetime becomes the order of \(10^{-23} \text{ s}\) compared with the muon lifetime in the order of \(10^{-6} \text{ s}\). In the strong decay process, the exchange of the massive gluon is taken into account like the exchange of massive intermediate vector boson in the weak decay process. The strong decay \(\Delta \rightarrow n + p\) and the weak decay \(\Sigma^+ \rightarrow n + \pi^+\) have almost same 0.12 GeV kinetic energy, approximately. Their lifetime ratio becomes

\[
\frac{\tau(\Delta \rightarrow n + p)}{\tau(\Sigma^+ \rightarrow n + \pi^+)} \approx \frac{1/G_F^2 m_{\Delta}^3}{1/G_F^2 m_{\Sigma}^3} \approx \frac{10^{-23}}{10^{-10}} \text{ s}.
\]

Similarly, the lifetime ratio between \(\Sigma^0 \rightarrow \Lambda + \pi^0\) and \(\Sigma^- \rightarrow n + \pi^-\) is found to be

\[
\frac{\tau(\Sigma^0 \rightarrow \Lambda + \pi^0)}{\tau(\Sigma^- \rightarrow n + \pi^-)} \approx \frac{1/G_F^2 m_{\Sigma}^3}{1/G_F^2 m_{\Sigma}^3} \approx \frac{10^{-23}}{10^{-10}} \text{ s}.
\]

Since the typical cross section in weak interactions can be estimated by \(\sigma \approx G_F^2 T^2 \approx 10^{-44} \text{ m}^2\), the typical cross section in strong interactions is \(\sigma \approx G_F^2 T^2 \approx 10^{-30} \text{ m}^2\), which gives good agreement with experiment result. In the decay of \(\Delta^{++}\), which hints the color quantum number, \(\Delta^{++} \rightarrow \pi^+ + p\), the lifetime of \(\Delta^{++}\) is \(10^{-23} \text{ s}\). This can be interpreted by the distance of about 1 fm estimated by the gluon mass of about 300 MeV in this scheme. In the strong interaction of \(\pi + p \rightarrow \pi + p\) and the weak interaction of \(\nu + p \rightarrow \nu + p\), the cross section ratio around the kinetic energy \(T = 1 \text{ GeV}\) becomes

\[
\frac{\sigma(\pi + p \rightarrow \pi + p)}{\sigma(\nu + p \rightarrow \nu + p)} \approx \frac{G_F^2 T^2}{G_F^2 T^2} \approx \frac{10 \text{ mb}}{10^{-11} \text{ mb}}.
\]

These examples described above explicitly show the typical lifetimes and cross sections in strong interactions as expected.

**C. Nuclear Excitation and Decay as a \(U(1)_f\) Gauge Theory**

QND as a \(U(1)_f\) gauge theory, where the photon is the massless gauge boson and the proton is positively charged, is proposed just like QED as a \(U(1)_e\) gauge theory, where the photon is the massless gauge boson and the electron is negatively charged, is used. Electromagnetic dynamics in nucleus as a \(U(1)_f\) gauge theory is applied to evaluate nuclear excitation and decay around the order of 1 MeV.

The coupling constant \(\alpha_f = \alpha_s/16\) for a \(U(1)_f\) gauge theory at the strong scale is used to evaluate excitation levels. The pure color coupling constant \(\alpha_f = \alpha_s/16\) has about four times stronger than the coupling constant \(\alpha_e = \alpha_i/16\) for a \(U(1)_e\) gauge theory at the weak scale: the effective coupling constant \(\alpha^{eff}_f = i^0\alpha_f = \alpha_s/64 = \alpha_e\). The emission of energetic photons as gamma radiation is typical for a nucleus deexcitation from some high lying excited state to the ground state. This represents a reordering of the nucleon in the nucleus with a lowering of mass from the excited mass to the lowest mass. Electromagnetic transition due to the proton charge and gamma decay are well established subjects. Alpha decay is a Coulomb repulsion effect, which is important for heavy nuclei since the disruptive Coulomb force increases with size at a faster rate than the nuclear binding force. For another example, nuclei with closed shell plus one valence nucleon is considered in analogy with the hydrogen atom. The Coulomb potential originated from color
D. Conservation and Violation in Nuclear Dynamics

The proton number conservation is the consequence of the \( U(1) \) gauge theory just as the electron number conservation is the consequence of the \( U(1)_e \) gauge theory and the baryon number conservation is the result of the \( U(1)_Z \) gauge theory for strong interactions just as the lepton number conservation is the result of the \( U(1)_Y \) gauge theory for weak interactions. Discrete symmetries are perturbatively conserved but are nonperturbatively violated in strong interactions.

1. Conservation of Proton Number and Baryon Number

The conservation of the proton number is the result of the \( U(1)_f \) local gauge theory just as the conservation of the electron number is the result of the \( U(1)_e \) local gauge theory. The charge quantization is given by \( Q_f = C_3 + \frac{Z_c}{2} \) where \( C_3 \) is the third component of the colorspin operator \( C \) and \( Z_c \) is the hyper-color charge operator. This form has the analogy with the electric charge quantization \( \frac{q_0}{2} + \frac{Y_0}{2} \) in weak interactions and \( Q_e = \frac{I^3_3}{2} + \frac{Y^u_3}{2} \) in the quark model. The hyper-color charge operator may be defined by \( Z_c = B + S + C + \cdots \) with the baryon number operator \( B \), the strangeness number operator \( S \), and the charm number operator \( C \) just as the hypercharge operator is defined by \( Y = B - L \) with the baryon number operator \( B \) and the lepton number operator \( L \). Color charge quantum numbers for the proton and neutron are shown in Table I where nucleons are classified by color doublet and color singlet. Colorspin doublet nucleons are governed by the \( SU(2)_W \times U(1)_Z \) gauge theory just as isospin doublet leptons or quarks are governed by the \( SU(2)_L \times U(1)_Y \) gauge theory in weak interactions. As noted by the observation of the proton decay, the lifetime of the proton is more than \( 10^{32} \) years. The conservation of the baryon number is analogous to the conservation of the proton number. The conservation of the baryon number is the consequence of the \( U(1)_Z \) local gauge theory just as the conservation of the lepton number is the consequence of the \( U(1)_Y \) local gauge theory. Table I shows relations between conservation laws and gauge theories. Baryons are conserved as the colorspin doublet but are not conserved as the color singlet; this is analogous to the conservation of leptons as the isospin doublet but the nonconservation of leptons as the isospin singlet in weak interactions. The immediate result of the proton number conservation or the baryon number conservation is shown in the mass density and charge density of nuclear matter. The effective charge unit of the charge operator \( Q_f \) is \( q^{eff}_f = \sqrt{\pi \alpha_s/64} \) while the charge unit of the charge operator \( Q_e \) is \( e = \sqrt{\pi \alpha_s/4} \): the absolute magnitude of \( q^{eff}_f \) is the same with that of \( e \) since \( \alpha_s \approx 0.48 \) at the strong scale and \( \alpha_i \approx 0.12 \) at the weak scale. The concept of the effective charge \( q^{eff}_f \) is consistent with one of colorspin and isospin intrinsic angular momenta. Note that there is no discrete symmetry breaking due to the \( U(1)_f \) local gauge theory.

Conservation laws of the proton and baryon can be applied in analyzing nuclear interactions including nuclear scattering and reaction as well as conservation laws of the total energy and linear momentum. Angular momentum conservation law enables interacting particles to assign the intrinsic angular momenta.

2. Violation of Discrete Symmetries

Discrete symmetries are perturbatively conserved in strong interactions. The violation of discrete symmetries is thus the nonperturbative indication of DSSB from an SU(3)C gauge theory to an SU(2)N × U(1)Z gauge theory and then to a U(1)f gauge theory. P violation is manifest since there are no parity partners of pseudoscalar mesons, vector mesons, and baryons. This resolves the \( U(1)_A \) problem for the non-observation of the sigma meson as the parity partner of the pion. The fact that the color doublet vector current of the proton and neutron is conserved but the color singlet axial current is not conserved indicates parity violation. The evidence of CP and T violation appears in the magnitude of the \( \Theta \) constant, which is measured by the electric dipole moment of the neutron: \( \Theta < 10^{-9} \). C violation predicts that the number ratio of antibaryons to baryons is extremely small in the matter space: this is connected with the baryon-antibaryon number asymmetry \( \delta_B \approx 10^{-10} \).

E. Strong Isospin and Colorspin

The proton and neutron are considered as two different manifestations of the nucleon in terms of strong isospin symmetry introduced by Heisenberg. Strong isospin has formal analogy with ordinary spin but it has nothing to do with rotation with the coordinate space, contrary...
to spin. However, if approximate strong isospin symmetry is replace with the mixed symmetry of colorspin and weak isospin, a nucleon state possesses local colorspin, weak isospin, and spin degrees of freedom in intrinsic coordinate space since colorspin and weak isospin are intrinsic angular momenta like spin angular momentum. The effects of colorspin will thus appear in the magnetic dipole moment and electric quadrupole moment in nucleons and they may be investigated by QND. Quantum numbers of general nucleon-nucleon systems are summarized in Table III when colorspin degrees of freedom are taken into account in addition to isospin and spin degrees of freedom; there exist six distinctive states because of two colorspins, two weak isospins, and two spins.

Quark model, isospin quantization, colorspin quantization, and colorspin applications are addressed in the following.

1. Quark Model, Isospin, and Colorspin

The quark model introduced by Gell-Mann and Zweig is extremely successful in describing static hadrons. In this part, its relation to the GWS model and QND is considered. As described above, quarks possess three types of intrinsic charges, color, isospin, and spin. Color charge and weak isospin charge are related to strong isospin charge. The quark model with strong isospins as charges is applied to static hadrons composed of quarks, which are governed by the GWS model with weak isospin charges at the weak energy scale and QND with color charges at strong scale. QND and GWS model are the dynamical extensions of the static quark model as a local gauge theories. $SU(2)$ strong isospin is thus regarded as just the combination of $SU(2)$ colorspin and $SU(2)$ weak isospin: for example, the proton with strong isospin $1/2$ consists of three quarks (udd) and the neutron with strong isospin $-1/2$ consists of three quarks (udd) where $u$ has colorspin and weak isospin and $d$ has colorspin and weak isospin simultaneously.

The charge quantization $\hat{Q}_c = \hat{I}_s^c + \hat{Y}_s^c/2$ in the quark model is analogous to the charge quantization $\hat{Q}_c = \hat{I}_s^e + \hat{Y}_s^e/2$ in weak interactions where $\hat{I}^e$ is the strong isospin operator, $\hat{I}^w$ is the weak isospin operator, $\hat{Y}^s$ is the strong hypercharge quantum operator, and $\hat{Y}^w$ is the weak hypercharge quantum operator. $Y^s = B + S + C$ is decomposed of the baryon number $B$, the strangeness number $S$, and the charm number $C$. The longitudinal component of strong isospin is expressed by $\hat{I}_s^c = (Z - A)/2$ in terms of the nuclear number $Z$ and the mass number $A$. The color electric charge is similarly quantized by $\hat{Q}_f = \hat{C}_f + \hat{Z}_f/2$ to describe the proton charge where $\hat{C}_f$ is the longitudinal component of colorspin $C$ and $\hat{Z}_f$ is the hyper-color charge. The hyper-color charge operator may be defined by $\hat{Z}_f = B + S + C + \cdots$ with the baryon number $B$, the strangeness number $S$, and the charm number $C$ just as the weak hypercharge operator is defined by $\hat{Y}^w = B - L$ with the baryon number $B$ and the lepton number $L$.

Overall, the combination of the GWS model and QND as quantum flavordynamics is the dynamical extension of the static quark model for hadrons. This might be a clue for the mass difference between constituent quarks and current quarks. Current quarks with the mass around 5 MeV possess the bigger difference number $N_{ud}$, presumably $64 = (2 \times 2)^3$ times bigger from colorspin and isospin degrees of freedom, than constituent quarks since current quarks have combined triplet color, triplet isospin, and doublet spin degrees of freedom and constituent quark has doublet colorspin, doublet isospin, and doublet spin degrees of freedom.

2. Applications of Colorspin

Clear examples of the colorspin assignments can be found in nuclear analog states with the same $A$ but different $N$ and $Z$. For example, the nuclei with the $A = 14$ system, $^{14}_6$C, $^{14}_7$N, and $^{14}_8$O are considered. Since they are conventional strong isospin triplet states ($i^s = 1$), all three nuclei have the almost same energies, apart from the electromagnetic energy: the respective shifts from the strong isospin singlet state of $^{14}_7$N are 2.36 MeV in $^{14}_6$C, 2.31 MeV in $^{14}_7$N, and 2.44 MeV in $^{14}_8$O. The strong isospin triplet energy in $^{14}_7$N is slightly different due to the mixing of two states with quantum numbers $(i = 1, s = 0, c = 1)$ and $(i = 1, s = 1, c = 0)$. The strong isospin singlet state of $^{14}_7$N is also the mixed state of two states with quantum numbers $(i = 0, s = 1, c = 0)$ and $(i = 0, s = 0, c = 1)$. Similar arguments are applicable to the three isobar nuclei $^{10}_4$Be, $^{10}_5$B, and $^{10}_6$C and to the three isobar nuclei $^{6}_2$He, $^{6}_3$Li, and $^{6}_4$Be. Further applications of Table III will be given in the deuteron state and the nucleon-nucleon scattering.

The reason for the mass difference for hadrons may come from the contribution of colorspin or isospin charges. For example, quarks u and d are an isospin doublet but their masses are slightly different. The mass difference, $m_u \neq m_d$, is the isospin violation by strong interactions, in which color degrees of freedom play important role. It might provide the clue for the mass difference between isospin multiplets in hadrons, which needs to be further speculated. The mass difference of the proton (uud) and neutron (udd) (~1.3 MeV) has three possible reasons related to colorspin-colorspin and isospin-isospin interactions: the difference in masses of the u and d quarks ($m_d > m_u$), the energy difference in electric and magnetic interactions. The mass difference from electromagnetic interactions mainly comes from the difference in the u-d quark interaction of the proton and the d-d quark interaction of the neutron since u-d interactions are common in both nucleons. Coulomb energy difference between the u-d and d-d quark interaction becomes $\left| (2/3)^2 - (-1/3)^2 \right| m_u \alpha^2 / 2 \approx 0.5$ MeV and magnetic energy difference becomes $\left| (2/3)^2 - (-1/3)^2 \right| m_d \alpha^2 / 2 \approx 0.5$ MeV.
(−1/3)^2 |\psi(0)|^2 / 3m_\gamma^2 \simeq 0.5 \text{ MeV} \text{ where } 2/3 \text{ represents the electric charge of the u quark and } -1/3 \text{ does the electric charge of the d quark. According to Dashen sum rule, } m^2(\pi^\pm) - m^2(\pi^0) = m^2(K^\pm) - m^2(K^0) \text{ if the electromagnetic interaction is the only source for the mass difference of isospin multiplets. However, the fact that the mass difference sign between two isospin multiplets is wrong indicates another interaction for the difference: } \Delta m(K^0 - K^\pm) \simeq 4 \text{ MeV and } \Delta m(\pi^0 - \pi^\pm) \simeq -4.6 \text{ MeV. Similarly, } \Delta m(\Sigma^0 - \Sigma^\pm) \simeq 3.1 \text{ MeV, } \Delta m(\Sigma^0 - \Sigma^-) \simeq -4.9 \text{ MeV, and } \Delta m(\Xi^0 - \Xi^\pm) \simeq -6.5 \text{ MeV are observed. These examples show the explicit requirement of color interactions due to color degrees of freedom in addition to electromagnetic interactions due to isospin degrees of freedom. Note that the coupling constant for pure color charges is stronger than that for pure isospin charges. For instance, three pion states form a color triplet but the neutral pion is the mixed state of color triplet and color singlet, which makes the lower mass compared with the charge pions. Another example for mixing between color and isospin degrees of freedom is the Cabbibo angle as the mixing angle between d quark and s quark with the same electromagnetic interaction −e/3. The Kobayasi-Maskawa matrix is a more extended version of the flavor mixing due to intrinsic color charges.

F. Nuclear Mass, Charge, and Size: Principal Quantum Number

According to the multipole expansion of the Yukawa potential, there exists the constant potential as the intrinsic monopole term depending on the gauge boson mass M_G, which is the origin of the almost constant mass and there exists the Coulomb potential as the extrinsic monopole term depending on the color factors, which is the origin of the almost constant charge. They are related to the U(1)_Q gauge theory and U(1)_I gauge theory respectively. In this approach, the QCD vacuum and baryon matter energies are spatially quantized as well as photon and phonon energies. These total particle numbers N_G ≃ 10^{91}, N_B ≃ 10^{78}, N_{\gamma} ≃ 10^{81}, and N_{\rho} ≃ 10^{75} are conserved good quantum numbers.

Nuclear matter is quantized by the maximum wavevector mode \( N_F \approx 10^{26} \) and the total baryon number \( B = N_B = 4\pi N_F^3/3 \approx 10^{78} \) as the consequence of the baryon number conservation or the baryon asymmetry \( \delta_B \approx 10^{-10} \). Baryon matter quantization is consistent with the nuclear number density \( n_n = n_B = A/(4\pi r^3/3) \approx 1.95 \times 10^{38} \text{ cm}^{-3} \) with the nuclear mass number \( A \) and the nuclear matter radius \( r \) at the strong scale \( M_G \approx 10^{-3} \text{ GeV} \) and is consistent with Avogadro’s number \( N_A = 6.02 \times 10^{23} \text{ mol}^{-1} \approx 10^{19} \text{ cm}^{-3} \) at the atomic scale. This is also compatible with the nuclear number density \( n_B = 2k_B^2/3\pi^2 \) with the Fermi momentum of a free nucleon \( k_F = 1.33 \text{ fm}^{-1} \) and the Fermi energy \( \epsilon_F = k_F^2/2m_n \approx 37 \text{ MeV} \) and compatible with the nucleus radius

\[
r = r_0 A^{1/3}
\]

with the nuclear radius \( r_0 \approx 1.2 \text{ fm} \). In analogy with the electron orbit radius of the hydrogen atom \( r_e = a_0 h^2/\pi m_e \) with the atomic radius \( a_0 = 1/2m_e \gamma_0 \) or the Bohr radius \( a_B = 1/m_n \gamma_0 = 5.29 \times 10^{-9} \text{ cm} \) and the principal quantum number \( n_e \), the above equation \( (14) \) may be written by

\[
r = r_0 n_e^2
\]

if the nucleon number \( A = B \), the nucleus radius \( r = r_0 A^{1/3} \), and the mass radius \( r = 1/2m_\alpha \alpha_z \) are used.

The QCD vacuum represented by massive gluons is quantized by the maximum wavevector mode \( N_R \approx 10^{30} \), the total gluon number \( N_G = 4\pi N_R^3/3 \approx 10^{91} \), and the gluon number density \( n_G = N_{QCD} \approx 10^{-2} \text{ GeV}^{3/2} \approx 10^{39} \text{ cm}^{-3} \). Massless photons are quantized by the maximum wavevector mode \( N_\gamma \approx 10^{29} \) and the total photon number \( N_{\gamma} = 4\pi N_\gamma^3/3 \approx 10^{88} \). Massless phonons in the matter space are quantized by the maximum wavevector mode (Debye mode) \( N_\rho \approx 10^{25} \) and the total phonon number \( N_{\rho} = 4\pi N_\rho^3/3 \approx 10^{75} \). Vacuum, matter, photon, and phonon energies are also thermodynamically quantized. Quantum states of vacuum, matter, photons, and phonons have average occupation numbers \( f_\gamma = 1/(e^{E/\gamma_0} - 1) \) for gauge bosons, \( f_B = 1/(e^{E/B} - 1) + 1 \) for baryons, \( f_\gamma = 1/(e^{E/\gamma_0} - 1) \) for photons, and \( f_\rho = 1/(e^{E/\rho} - 1) \) for phonons under the assumption of free particles in thermal equilibrium. The quantization unit of energy due to a gauge boson in strong interactions is \( \Lambda_{QCD}/N_R \approx 10^{-31} \text{ GeV} \).

G. Nuclear Magnetic Dipole Moment

The nuclear magnetic dipole moment shows good illustration for colorspin as well as isospin as the intrinsic angular momentum in addition to spin as the intrinsic angular momentum. The electric dipole moment of a nucleon disappears to satisfy the parity of nucleus but it is not zero even though it is extremely small as reflected in the \( \Theta \) parameter.

The magnetic dipole moment of a nucleon comes from two sources, the intrinsic dipole moments of constituent
quarks and the orbital motions of quarks. The three quarks are symmetric in the spatial parts of their wave functions, which have relative motion between them in the $l = 0$ states, so that no contribution to the magnetic dipole moment comes from quark orbital motion.

The magnetic dipole moment of a nucleus is given by $\mu_i = \frac{e}{2m_p}l_i$ where $l_i$ is the orbital momentum of the $i$-th proton and $m_p$ is the proton mass. The magnetic dipole moment for orbital motion may be rewritten by $\mu_i = g_i l_i \mu_N$ where the Lande g-factor $g_i = 1$ for a proton and $g_i = 0$ for a neutron with the definition of nuclear magneton $\mu_N = e/m_p = 3.15 \times 10^{-17}$ GeV/T. Similarly, the contribution from the intrinsic spin of each nucleon may be expressed in the form

$$\mu_i = \frac{e}{2m_p} s_i = g_s s_i \mu_N.$$  \hspace{1cm} (17)

Since $s = 1/2$, the Lande spin g-factor for a free nucleon is $g_s^0 = 2p_p/m_N = 5.59$ for a proton and $g_s^0 = 2\mu_n/\mu_N = -3.83$ for a neutron \cite{14}. The g-factors are different with $g_s = 2$ for a pointlike electron and $g_s = 0$ for a pointlike neutral particle. Formerly, these differences between theoretical and experimental values were ascribed to the cloud of pion mesons that surround nucleons, with positive and neutral pions around protons and with negative and neutral pions around neutrons.

According to the quark model, the magnetic dipole moment of the proton is given by $\mu_p = 4\mu_u/3 - \mu_d/3$ in terms of the magnetic dipole moments of $u$ and $d$ quarks. The magnetic dipole moment of the neutron is given by $\mu_n = 4\mu_d/3 - \mu_u/3$. Under the assumption that the masses of $u$ and $d$ quarks involved are equal and their ratio of magnetic dipole moments is $\mu_u = -2\mu_d$, the ratio between the magnetic dipole moments of the proton and neutron is $\mu_u/\mu_p = -2/3$, which is in good agreement with the observed value of $-1.913/2.793 = -0.685$. However, the quark model does not give any clue for the absolute values of the magnetic dipole moments.

The intrinsic principal number $n_m = (n_c, n_t, n_s)$ in intrinsic space quantization is, on the other hand, introduced as the analogy to the principal number $n$ in extrinsic space quantization and the intrinsic angular momenta are analogous to the extrinsic angular momentum so that the total angular momentum has the form

$$\vec{J} = \vec{L} + \vec{S} + \vec{I} + \vec{C}.$$  \hspace{1cm} (18)

One of the explicit examples to illustrate the extension of the total angular momentum might be the nuclear magnetic dipole moment: the g-factors of the proton and neutron are respectively $g^0_p = 5.59$ and $g^0_n = -3.83$, which are shifted from 2 and 0 expected for pointlike particles. It is strongly suggested in order to resolve the problem of the nuclear magnetic moment that contributions from color spin and isospin degrees of freedom are included to nucleons. The shifted values for the proton and neutron, 3.59 and -3.83 are almost identical and they mostly come from the combined contribution of color spin, isospin, and spin. The mass ratio of the proton and the constituent quark, $m_p/m_q \sim 2.79$, also represents combined color spin, isospin, spin degrees of freedom. The excess contribution $|g_s| \geq 3.83$ common for $\mu_p$ and $\mu_n$ comes from the magnetic dipole moment due to the $SU(2)_N \times U(1)_Z$ symmetry for color charges and the $SU(2)_L \times U(1)_Y$ symmetry for isospin charges while the contribution $g_s \approx 1.76$ only for $\mu_p$ comes from the magnetic dipole moment due to the $U(1)_f$ gauge symmetry. This interpretation is easily justified if the coupling constant $g_f = \sqrt{e/\alpha_s} = \sqrt{\alpha_s/\lambda_0} \approx 1.7$ for the $U(1)_f$ gauge theory and the coupling constant $g_b = \sqrt{e/\alpha_s} = \sqrt{\alpha_s/\lambda_0} \approx 3.8$ for the $SU(2)_N \times U(1)_Z$ gauge theory. The description above reflects the mixed contribution of color spin, isospin, and spin degrees of freedom and the total angular momentum $\vec{J} = \vec{L} + \vec{C} + \vec{I} + \vec{S}$: the total nuclear magnetic moment $\mu_{SN} = (g_s s + g_w c + g_w + g_l) \mu_N$ with the respective coupling constants $g_s, g_w$, and $g_n$ for spin, isospin, and color spin interactions.

H. Deuteron

A deuteron ($^2$H) is the weak bound state of a proton and a neutron. The ground state of the deuteron becomes the state of strong isospin singlet $i^s = 0$ and spin triplet $s = 1$. The reason why no bound states of proton-proton and neutron-neutron are observed is explained by the violation of discrete symmetries in color pairing mechanism. In strong interactions, the bindings are regarded as color electric dipoles with intrinsic odd parity, which must be suppressed. The deuteron state can also be slightly modified if color spin is taken into account. The ground state of the deuteron may be the mixed state of two states possible by the symmetry argument: one is the dominant state with weak isospin singlet $i = 0$, spin triplet $s = 1$, and color singlet $c = 0$ and the other is the contaminating state with weak isospin singlet $i = 0$, spin singlet $s = 0$, and color triplet $c = 1$.

The binding energy $-2.23$ MeV is relatively less than the average binding energy for the other nucleus. This is interpreted as that the interaction of the deuteron is governed by the nuclear interaction of the $U(1)_Z$ gauge theory. The coupling constant $\alpha^2_2 = -\frac{e^2}{4\pi} = -0.08$ for an asymmetric $U(1)_Z$ gauge symmetry seems to produce the deuteron binding energy by $E_{DB} = -\frac{e^2}{2} \alpha^2_2 \frac{3}{2} \mu = -2.2$ MeV, which is close to experimental value $-2.23$ MeV. The factor $3/4$ might come from the mixing of two states or from the difference in the principal quantum number between the ground state and the first excited state.

In terms of the nuclear magneton $\mu_N$, the magnetic dipole moment of the deuteron may be written as a function of the orbital angular momentum operator and the intrinsic angular momentum operator of each nucleon:
\[ \mu_d = (g_p s_p + g_n s_n + l_p) \mu_N \] where \( l_p \) is the angular momentum of the proton. For \( l_p = 0 \), the expectation value of the magnetic dipole operator reduces to a sum of the intrinsic dipole moment of a proton and a neutron: \( \mu_d = \mu_p + \mu_n = 0.8798 \mu_N \) which is different with the observed value of 0.8574 \( \mu_N \). The small difference \(-0.022339 \mu_N \) is conventionally expressed by the mixing of d wave to s wave; the observed value is consistent with 96 percent s wave and 4 percent d wave contributions. However, the small discrepancy may alternatively be ascribed by the color anomalous magnetic moment, which may be estimated by \( (g^d - 2)/2 \approx \alpha_s/2 \pi \approx 0.02 \) with the coupling constant \( \alpha_s = \alpha_s/3 \approx 0.12 \) if the analogy of the electric anomalous magnetic moment \( (g_s - 2)/2 \approx \alpha_e/2 \pi \) is used. The anomalous magnetic moment might come from the mixing of two intrinsic states.

I. Nucleon-Nucleon Scattering

The nucleon-nucleon (NN) scattering is one of excellent examples whose cross sections can be evaluated by QCD as an \( SU(2)_N \times U(1)_Z \) gauge theory. The ground states of the proton and neutron are considered as a color-spin doublet in QND. Quantum numbers of NN systems shown in Table III are applied to analyze the NN scattering. Furthermore, it is realized that the NN scattering has analogous properties with weak decay process since strong interactions are much stronger than electromagnetic interactions. The NN scattering is basically the combination of two interactions, which are explained by \( SU(2)_N \) gauge theory at relatively high energies and by the \( U(1)_Z \) gauge theory at relatively low energies. This investigation has a complete analogy with the beta decay of the neutron in weak interactions, \( n \rightarrow p^+ + e^- + \nu \); the distinction of Fermi type and Gamow-Teller type scattering is possible.

The invariant amplitude in strong interactions as an \( SU(2)_N \) gauge theory at relatively high energies is given by

\[ \mathcal{M} = -\frac{g^2}{4} \left( \frac{1}{k^2 - M_G^2} J^\mu J^\mu \right) \]

where the gluon mass \( M_G \) is inserted in the gluon propagator. This provides the cross section for the pp, nn, or pn scattering as a colorspin triplet expressed by

\[ \sigma = \frac{4G_R^2 T^2}{\pi} \left( \frac{1}{1 + 4T^2/M_G^2} \right) \]

in the center of mass energy \( T \) since \( \mathcal{M} \sim G_R J^\mu J^\mu \) in terms of the effective strong coupling constant \( G_R = \sqrt{2} g_s^2/8M_G^2 \). At high energy \( T >> M_G \), the cross section converges to \( \sigma = \frac{4G_R^2 M_G^2}{\pi} \), while at low energy \( T < M_G \), the cross section becomes \( \sigma = 4G_R^2 T^2/\pi \). Cross sections for the NN scattering in terms of the massive gluon exchange show excellent agreement with measurement data \([16, 17]\). According to strong isospin invariance, three types of scattering such as the nn, pp, and pn scattering with strong isospin one (spin zero) exhibit almost the same cross sections. The theoretical cross section of the NN scattering as an \( SU(2)_N \) gauge theory at high energies about from 2 GeV to \( 10^3 \) GeV is saturated to the experimental one of about 40 mb \([14]\) and the cross section at low energies from 0.6 GeV to 2 GeV is roughly proportional to \( T^2 \): \( c_f = 1/4, \alpha_s = 0.48, M_G \approx 300 \text{ MeV} \), and \( G_R \approx 10 \text{ GeV}^{-2} \) are used in this evaluation. The symmetric \( SU(2)_N \) colorspin interaction for the isospin triplet and spin singlet contribution is commonly involved in the above three types of nucleon-nucleon interactions with the massive gluon exchange.

In addition to the \( SU(2)_N \) gauge symmetry, there is another process, which explicitly appears at relatively low energies. The cross section in strong interactions as a \( U(1)_Z \) gauge theory at relatively low energies is nonrelativistically obtained using the Yukawa potential \( V(r) = \sqrt{c_f} \alpha_s \pi^{-1} (r - \Lambda_{QCD})/r \) with the strong scale \( \Lambda_{QCD} = 1/\Lambda_{QCD} \approx 10 \text{ GeV}^{-1} \):

\[ \sigma = 4\pi c_f^2 \alpha_s m_n^2 \frac{1}{M_G^2 + 4m_n T/M_G^2} \]

where \( c_f^2 = -1/6 \) (or 1/12) is the asymmetric (symmetric) color factor, \( m_n \) is the nucleon mass \( m_n \approx 940 \text{ MeV} \), \( T \) is the incident particle energy, and the gauge boson mass \( M_G \approx 140 \text{ MeV} \). The cross section at relatively long range \( r > 2 \text{ fm} \) is dependent on angular momentum and is well explained by the massive gauge boson \( M_G \approx m_n \approx 140 \text{ MeV} \); this is confirmed by a pion exchange in effective models. The cross section data in the region below the energy 0.3 GeV or above the range 1.5 fm are obtained by using the above formula, which is definitely dependent on angular momenta. The comparison between theoretical and measurement data shows good agreement. The calculated cross section data in the limit \( T \rightarrow 0 \) give agreement with observed data for cross sections \( (\sigma = 4\pi a^2) \sigma_{pp} \approx \sigma_{nn} \approx 35 \text{ b} \) and \( \sigma_{pn} \approx 66 \text{ b} \) for spin singlet and \( \sigma_{pn} \approx 4 \text{ b} \) for spin triplet as shown in Table IV \([18]\). The effective (running) coupling constant \( \alpha_s = c_f^2 \alpha_s \approx 12 \) becomes stronger at lower energies but is still less than 1 so that the higher order corrections are perturbatively possible: \( \alpha_s \approx 0.04 \) around 100 MeV. A comparison of cross sections at low energies may give aid to clarify such nuclear interactions. The cross section difference in strong isospin triplet is mainly due to the contribution of colorspins. The value of \( a_{pn} = -23.7 \text{ fm} \) is noticeably larger than \( a_{pp} \) and \( a_{nn} \) at \( s = 0 \) and \( i = 1 \) channel: this is explained conventionally by the exchange of the charged pion and the exchange of the neutral pion. This may alternatively be explained by the \( SU(2)_N \) gauge theory: there is only the color triplet \( c = 1 \) contribution for \( a_{pp} \) and \( a_{nn} \) but there are both...
color triplet $c = 1$ and singlet $c = 0$ contributions for $a_{pn}$ if color degrees of freedom are adopted in addition to isospin and spin degrees of freedom. Therefore, the additional contribution for the scattering length ($\sim -6$ fm) is due to one of the color singlet configuration as an isospin triplet and spin triplet state. The mixing of intrinsic and extrinsic waves is imposed as seen in the magnetic dipole moment of the deuteron.

The NN scattering such as the pp, nn, or pn scattering with strong isospin one (spin zero) is the Fermi type scattering in strong interactions while the NN scattering such as the pn scattering or np scattering with strong isospin zero (spin one) is the Gamow-Teller type scattering in strong interactions; the concept of the Fermi type scattering and Gamow-Teller type scattering is just named after the concept of the Fermi decay and Gamow-Teller decay in weak interactions. For the pn scattering, the Gamow-Teller type scattering for an isospin singlet and spin triplet state is expected in addition to the Fermi type scattering as an isospin triplet and spin singlet state as seen in the deuteron as an isospin singlet and spin triplet state. This is explicitly shown in the observation of cross sections for the pn scattering: the scattering length $a_{pn}^s \approx -23.7$ fm for spin singlet as the Fermi type scattering and the scattering length $a_{pn}^t = 5.4$ fm for spin triplet as the Gamow-Teller type scattering.

There exist the color charged current mediated by charged massive gluons $A^\pm$ with the mass $M_A$ and the color neutral current mediated by neutral massive gluon $B^0$ with the mass $M_B$. The NN scattering is good example to illustrate the existence of these massive gluons. The invariant amplitude of the charged current is given by

$$\mathcal{M}^c = -\frac{4G_R}{\sqrt{2}} J^\mu_c J^{\dagger \mu}_c,$$  \hspace{1cm} (22)

and the invariant amplitude of the neutral current is given by

$$\mathcal{M}^n = -\frac{4G_R}{\sqrt{2}} 2 \rho J^\mu_c J^{\dagger \mu}_c.$$  \hspace{1cm} (23)

It is identified that the relative strength of the color neutral and charged current in strong interactions becomes

$$\rho = \frac{M_B^2}{M_A^2 \cos^2 \theta_R}.$$  \hspace{1cm} (24)

The NN scattering data approximately show $\rho \simeq 1$ as confirmed by isospin invariance: $\sigma_{pn} \simeq \sigma_{pp} \simeq \sigma_{nn}$ for isospin singlet and $\cos^2 \theta_R = \frac{M_A^2}{2M_B^2}$. Summarizing, the effective coupling constant $G_R \simeq 10$ GeV$^{-2}$ and the gauge boson mass $M_G \simeq 300$ MeV for the SU(2)$_N$ gauge theory can be evaluated from the saturated cross section $\sigma = 40$ nb at high energy and the effective coupling constant $G_R \simeq 10$ GeV$^{-2}$ and the gauge boson mass $M_G \simeq 140$ MeV for the U(1)$_Z$ gauge theory can also be evaluated from the lower cross section $\sigma = 23$ nb. The cross section $\sigma \approx 35$ b as spin singlet or $\sigma \approx 4$ b as spin singlet at low energy ($T \rightarrow 0$) is dependent on the lowest extrinsic angular momentum ($l = 0$). This implies that gluons with the higher mass play the dominant role depending on the color-spin angular momentum at the short range ($r < 1$ fm) while gluons with the lower mass play the dominant role depending on the orbital angular momentum at the long range ($r > 1$ fm). It is thus emphasized that QND as the SU(2)$_N \times U(1)_Z$ gauge theory produces the cross section data, which do not have divergence problems so that QND is renormalizable, from the zero energy limit to almost $10^3$ GeV.

Figure 1 shows the theoretical cross section of proton-proton scattering as a function of the center of mass energy in terms of equations (22) and (23); for example, the figure is obtained by using input data such as the effective coupling constant $G_R = 71$ GeV$^{-2}$ and gluon mass $M_G = 0.27$ GeV for equation (22) and such as the gluon mass $M_G = 0.15$ GeV and running coupling constant $\alpha_s = 0.02$ at 1 GeV for equation (21). This illustrates good agreement in general trend and in absolute magnitude with the experimental data, which are well shown as a function of the laboratory energy in Perkins [17], in the energy scale from almost zero to $10^3$ GeV. It is also emphasized that only the coupling constant is, in principle, necessary as an input parameter in the calculation of cross section. More fine tuning is required to determine parameters such as the strong coupling constant $\alpha_s$ or $\alpha_z$, gluon mass $M_G$, and effective coupling constant $G_R$ since the cross section data are sensitive to input parameters.

### J. Meson-Nucleon Scattering

QND as the SU(2)$_N \times U(1)_Z$ gauge theory may be applied to the meson-nucleon scattering. At higher energies above 5 GeV, the cross section becomes the same for particle and antiparticle according to the Pomeranchuk theorem and it is moreover isospin independent. Putting all these together, an expected cross section ratio $\sigma(\pi N)/\sigma(NN) \simeq 2/3$ follows from simple quark counting. The ratio 2/3 might stem from the color factor ratio of the meson to the baryon since the color factor of color symmetric octet for the quark-antiquark interaction is $c_f = 1/6$ and the color factor of color symmetric sextet for the quark-quark interaction is $c_f = 1/4$. The cross sections for the $\pi^- p$, $\pi^+ p$, $\pi^- n$, and $\pi^+ n$ scattering become almost equal from charge independence: the measurement data show about $\sigma \simeq 25$ nb. Likewise, the same argument is applied to cross sections for the $K^- p$, $K^+ p$, $K^- n$, and $K^+ n$ scattering: the measurement data show about $\sigma \simeq 20$ nb. The experiment data for the meson-nucleon scattering as well as the NN scattering illustrate massive gluons with about the mass $M_G \simeq 300$ MeV responsible for QND. At lower energies, it is conventionally known that cross sections for
the $\pi^-p$ and $\pi^+p$ scattering are dominated by the $P_{33}$ channel ($l = 1, i = 3/2, s = 3/2$). This is not contradictory to the concept of colorspin since the dominant channel may become the channel with quantum numbers $c = 1, i = 3/2$, and $s = 3/2$ by replacing the extrinsic angular momentum $l = 1$ with the intrinsic colorspin angular momentum $c = 1$ at relatively higher energies. This interpretation may be possible since the lifetime of the resonance $\Delta^{++}$ particle shows the typical strong lifetime $\tau \approx 1/G^2_H m^5_\Delta \approx 10^{-23}$ sec and the cross section data except resonance energies have the typical cross sections for strong interactions $\sigma \approx G^2_H T^2 \approx 30$ mb, originated by massive gluons with about the mass $M_G \approx 300$ MeV.

Other meson-nucleon interactions $\pi^\pm + p \to K^\pm + \Sigma^\pm$, $\pi^- + p \to K^0 + \Sigma^0$, $\pi^- + p \to K^0 + \Lambda$, $K^- + p \to \pi^0 + \Lambda$, etc. similarly indicate strong interactions by the exchange of massive gluons. In these interactions, the reaction $d \bar{d} \to s \bar{s}$ is commonly included in the quark level. They show the typical cross sections for strong interactions and conservation law for the strangeness number as noted by electric charge quantization $Q_f = C_3 + Z_c/2$ with $Z_c = B + S$. The strong decay $\Sigma^{3\pm}(1385) \to \Lambda + \pi^\pm$ holding the typical lifetime and decay width is also explained by the exchange of massive gluons.

If the description above is correct, the meson-nucleon scattering as well as the hadron decay can be studied over much wider energy range in terms of the unified view of QND.

K. Nuclear Potential

The nuclear potential is discussed from QND as a gauge theory point of view; the derivation and extension of each nuclear potential term are in principle possible. The colorspin-colorspin interaction is introduced as the central potential and the isospin-orbit potential and colorspin-orbit potential are introduced as non-local potentials in analogy with the spin-orbit potential. Colorspin and isospin as internal degrees of freedom play roles of intrinsic angular momenta.

The nuclear central potential is generally expressed by

$$V_c(r) = V_0 + V_S \vec{\sigma} \cdot \vec{\sigma} + V_L \vec{\sigma} \cdot \vec{\tau} + V_{2L} \vec{\sigma} \cdot \vec{\sigma} \cdot \vec{\tau}$$

where the first term denotes the pure radial distance dependent potential, the second term the spin-spin interaction, the third the strong isospin-isospin interaction, and the fourth the spin-spin and isospin-isospin interaction. This can be generalized as follows if strong isospin is decomposed with colorspin and weak isospin:

$$V_c(r) = V_0 + V_C \vec{\zeta} \cdot \vec{\zeta} + V_S \vec{\sigma} \cdot \vec{\sigma} + V_L \vec{\sigma} \cdot \vec{\tau} \cdot \vec{\tau} + V_{2L} \vec{\sigma} \cdot \vec{\sigma} \cdot \vec{\tau} \cdot \vec{\tau}$$

where fine interactions are colorspin-colorspin, (weak) isospin-isospin, spin-spin interactions and hyperfine interactions are colorspin-colorspin and isospin-isospin, colorspin-colorspin and spin-spin, spin-spin and isospin-isospin interactions, and the combination interaction of colorspin, isospin, and spin. Fine and hyperfine interactions appear in several places; the nuclear magnetic moment, the nucleon-nucleon scattering, and the nucleus decays of $\Delta$ and $\Sigma^0$ are a few examples as discussed in the previous subsections. This is consistent with the concept of the intrinsic angular momentum $J = \vec{C} + \vec{L} + \vec{S}$ and central potentials between intrinsic angular momenta. The potential $V_0$ is relevant for the $U(1)_Z$ gauge theory, the colorspin-colorspin interaction for the $SU(2)_N$ gauge theory, the isospin-isospin interaction for the $SU(2)_N$ gauge theory, and the spin-spin potential for the $SU(2)$ spin symmetry.

The nuclear non-local spin-orbit potential may be expressed by

$$V_{LS} = V_{LS}(r) \vec{L} \cdot \vec{\sigma}$$

which might have the form of $V_{LS} \propto \alpha_f |\psi(0)|^2/m_i m_j$. The spin-orbit interaction plays an important role in the formation of nucleus as shown in the shell model. The nuclear spin-orbit coupling splits a nuclear energy into two levels $j = l \pm s$ except for $l = 0$, but $j = l - 1/2$ lies higher in energy and $j = l + 1/2$ lies lower in energy. The order of the levels is inverted from that in an atom because of the change of sign of the spin-orbit coupling in nuclei compared to atoms. This is the explicit evidence for the difference between angular momenta of atom and nucleus. The nuclear non-local isospin-orbit potential may be analogously added if isospin is the intrinsic angular momentum like spin:

$$V_{LI} = V_{LI}(r) \vec{L} \cdot \vec{\tau}$$

which might have the form of $V_{LI} \propto \alpha_f |\psi(0)|^2/m_i m_j$. The isospin-orbit interaction plays a major role in the level splitting between protons and neutrons and the level splitting among isobaric nuclei as shown in the shell model. The nuclear isospin-orbit coupling splits a nuclear energy into two levels $j = l \pm i$, but the neutron with $j = l - 1/2$ lies lower in energy and the proton with $j = l + 1/2$ lies higher in energy as expected in the shell model. The isospin-orbit coupling is also relevant to the splitting of the energy levels for isobaric nuclei. The nuclear non-local colorspin-orbit potential may be analogously added if colorspin is also regarded as the intrinsic angular momentum like spin:

$$V_{LC} = V_{LC}(r) \vec{L} \cdot \vec{\zeta}$$

which might have the form of $V_{LC} \propto \alpha_f |\psi(0)|^2/m_i m_j$. The colorspin-orbit interaction in nucleus is significant at the surface of the nucleus. For example, it prevents the harmonic oscillator potential from diverging at long range in the shell model.
L. Shell Model

The nuclear shell model \([19]\) is to describe nucleon states in terms of nucleon degrees of freedom and the existence of magic numbers may be explained by independent particle model. The \(U(1)_Z\) gauge theory, apart from electromagnetic interactions due to the \(U(1)_f\) gauge theory, may be very useful in studying the scattering, decay, and excitation of nuclei. It provides quantum numbers such as the radial principal number \(n\), the angular momentum number \(l\), and the third component number of the angular momentum \(m_l\) but they have different origin compared to quantum numbers, \((n_c, l_c, m_{lc})\), associated with quantum electromagnetism (QED) as a \(U(1)_e\) gauge theory. The average potential of nuclear matter is the sum of Yukawa potentials for individual particles and it creates quantum numbers \((n, l, m_l)\). This is supported by the fact that the orbital angular momentum \(l\) of the nucleon has the different origin from the color charge with the orbital angular momentum \(l_c\) of the electron from the isospin charge since two angular momenta have opposite directions from the information of spin-orbit couplings in nucleus and atoms.

The independent particle shell model is roughly successful in the magnetic dipole moment. Including both \(l\) and \(s\) terms, the magnetic dipole moment becomes

\[
\mu = (g_1 l + g_s s_z)\mu_N \quad \text{with } j = j_z.
\]

Taking the expectation value, the result is

\[
\langle \mu \rangle = [g_l j + (g_s - g_l)\langle s_z \rangle] \mu_N
\]

since the expectation value of \(s_z\) is given by \(\langle s_z \rangle = \frac{2j(j+1)}{(2j+1)}[j(j+1) - l(l+1) + s(s+1)]\): for \(j = l + 1/2\), \(\langle s_z \rangle = 1/2\), while for \(j = l - 1/2\), \(\langle s_z \rangle = -j/2(j+1)\). The corresponding magnetic moments are expressed by

\[
\langle \mu \rangle = [g_l (j - 1/2) + g_s/2] \mu_N \quad \text{for } j = l + 1/2 \quad \text{and} \quad \langle \mu \rangle = [g_l j(j+3/2) + g_s/2(j+1)] \mu_N \quad \text{for } j = l - 1/2.
\]

Theoretical values as shown in Schmidt lines \((\langle \mu \rangle/\mu_N)\) for odd-\(A\) nuclei show the success in the general trend of observed values but experimental values are overall agreement for \(g_s = 0.6g_1^2\) with the g-factor of the free nucleon \(g_1^2\). The factor 0.6 might indicate the contribution of color degrees of freedom to the magnetic dipole moment since the actual spin g-factor \(g_s\) in \((30)\) may be reduced due to the contribution from colorspin angular momentum to the total angular momentum just as shown in its reduction in the deuteron.

In prospect, it may be possible to establish a periodic table for nuclei just like the periodic table for atoms in terms of nuclear extrinsic quantum numbers \((n, l, m_l)\) and magic numbers if colorspin degrees of freedom in nucleons are taken into account.

M. Nucleus Binding Energy

QND may be applied to interpret the semi-empirical mass formula and eventually extended to evaluate nucleus mass spectra as well as excitation energies. In this part, only the binding energy for nucleus is briefly discussed how to apply QND to nucleus.

Nucleus mass formula is expressed by

\[
m_N = Z^n m_p + N^n m_n + E_B
\]

where \(Z\) is the proton number, \(N\) is the neutron number, and \(E_B\) is the total binding energy. The form of this mass formula is very analogous to the mass formula for the hadron by the constituent quark model as discussed in the previous section. The binding energy \(E_B\) semi-empirically constitutes of the volume, surface, Coulomb, symmetric, and pairing terms \([23]\):

\[
E_B(Z, N) = a_v A + a_s A^{2/3} + a_C Z^2 / A^{1/3} + a_i(N - Z)^2 / A - \delta(A)
\]

where \(a_v \approx -16\text{ MeV}, a_s \approx 18\text{ MeV}, a_i \approx 0.7\text{ MeV}, a_s \approx 28\text{ MeV},\) and the pairing term \(\delta\). The binding energy \(E_B\) may be rewritten in terms of the extrinsic principal number \(n\) by

\[
E_B(n) = a_v n^6 / 3 + a_s n^4 + a_C Z^2 / n^2 + a_i(N - Z)^2 / n^6 - \delta(n)
\]

since \(A = n^6\).

The color \(SU(3)_c\) symmetry generates the \(SU(2)_N \times U(1)_Z\) symmetry or the \(U(1)_f\) symmetry, which governs nuclear dynamics. In terms of \(\alpha_s \approx 0.48\), the nuclear coupling constant of the nucleon-nucleon interaction becomes \(-f_n^2 = -G_R m_n^2 = -\sqrt{2}G_f g_s^2 m_n^2 / 8M_G^2 \approx -15.5\) with the nucleon mass \(m_n = 0.94\text{ GeV}\) and \(G_R = \sqrt{2}G_f g_s^2 / 8M_G^2\). The asymmetric colorspin-colorspin interaction between nucleons might be the major contribution of the volume term representing the saturation: \(V_c = \sum_{i>j} V_c(r) \vec{z}_i \cdot \vec{z}_j\), which is proportional to the limited number of nucleons \(A\) and the energy per particle approximately becomes constant. This reflects the fundamental symmetry of nuclear force, which might be related to the saturation of the cross section between nucleons at higher energy than the gluon mass. The color neutral coupling constant \(\alpha_b = \alpha_s / \cos \theta_R^2 = -\frac{4}{3}\alpha_s = -0.32\) produces the depth of nuclear effective potential in terms of the hydrogen atom analogy, \(V_0 = -\frac{\alpha_s^2 m_n}{2} = -48\text{ MeV}\), which might be relevant for the depth of the Woods-Saxon potential \(V_0 \approx -50\text{ MeV}\): this can be also produced by the constituent quark mass and g-factor, that is, \(V_0 = -g\frac{\alpha_s^2 m_n}{2} \approx -48\text{ MeV}\) with the mass ratio \(g = m_n / m_u \approx 2.79\). The binding energy of the nucleon in infinite nuclear matter is obtained by \(E_B = -\frac{\alpha_s^2 m_n}{2} = -18.3\text{ MeV}\) in terms of the constituent quark mass \(m_n\): it corresponds to the binding energy due to the volume contribution in the semi-classical mass formula. The surface energy might originate from the colorspin-orbit coupling, in which the cross section between nucleons increases as the energy increases. The symmetry energy is roughly
understood when Fermi-gas model, where nucleons are
treated like non-interacting particles, are used but it can
be studied as color interactions as a color doublet of the
proton and neutron.

The description above may be easily estimated as fol-
lows. The empirical binding energy ~−2.23 MeV for the
deuteron is governed by the nuclear interaction as the
U(1)Z gauge theory. The average binding energy for the
nuclei greater than the nuclear mass number A ≈ 12 may
be obtained by ~−2.23×4 = −8.9 MeV since there are four
gauge bosons if their binding energy is governed by the
nuclear interaction as the SU(2)N × U(1)Z gauge theory.
The coefficient of binding energy due to infinite nuclear
volume is also estimated by ~−2.23×8 = −17.8 MeV since
there are eight gauge bosons if their binding energy is
governed by the nuclear interaction as the SU(3)C gauge
theory; the volume binding energy is proportional to the
nuclear mass A. The coefficient of binding energy due to
nuclear surface is also estimated by 8.9 × 2 = 17.8 MeV
since there are two degenerated fermions if their surface
binding energy is governed by the intrinsic and extrinsic
orbit interactions; the surface energy is proportional to
the average of the potential $V_{LC} \vec{L} \cdot \vec{\zeta}$. The depth of
the average potential is obtained by −17.8 × 2.79 = −49.6
MeV with the mass ratio $g = 2.79$. The estimated values
are reasonably closed to observed values.

IV. COMPARISON BETWEEN QUANTUM
NUCLEAR DYNAMICS AND EFFECTIVE
MODELS

In the previous section, nuclear phenomena supporting
QND are addressed and scientific merits of QND com-
pared with nuclear effective models are presented in this
section. This section is thus devoted to summarize and to
convince QND applicable to whole nuclear phenomena.

Comparison between QND and effective models is
given in Table 1. Effective models, regardless of non-
relativistic or relativistic, stand for Bonn, Paris, Reid
phenomenological potentials, meson exchange models,
Walecka model [21], linear sigma model [22], nonlinear
sigma model [23], Nambu-Jona-Lasinio model [24], chiral
effective models [25], bag model [26], quark model,
QCD sum rule [27], Skyrme model [28], constituent quark
model [29], other QCD inspired models, etc.

QND as an SU(2)N gauge theory has the similarity
to the effective models with the omega meson (ω) ex-
change and QND as a U(1)Z gauge theory has the simi-
larity to the effective models with the pion exchange.
Short range repulsion in the nucleon-nucleon interaction
(r < 0.5 fm) is due to the gluon A8 in the symmetric
configuration of quark-quark interactions and long range
attraction (r ~ 1 fm) is due to the gluon A4 in the asym-
metric configuration of quark-quark interactions; this is
related to the relativistic effective models since the roles of
A8 are similar to the attractive roles of the pion me-
sion ($\pi$) and sigma meson (σ), respectively, and the role
of A4 is similar to the repulsive role of the omega meson
(ω) [21]. The coupling constant for repulsion is predicted
by $\alpha_s = \alpha_s/12 \approx 0.04$ at the QCD cutoﬀ. This scheme
provides the derivation of nuclear parameters, which can
be used to study the nucleon-nucleon interaction, and
the periodic table of nuclear properties, which are char-
acterized by the shell model, from the viewpoint of the
microscopic theory for strong interactions, QCD.

QND is applicable to various aspects in the wide en-
ergy range but effective models are only effective to
few aspects of nuclear phenomena in the rather small
energy range. The effective strong coupling constant
$G_R/\sqrt{2} = c_f g_\pi^2/8M_\pi^2 \approx 10 \text{ GeV}^{-2}$ like Fermi weak con-
stant $G_F/\sqrt{2} = g_\omega^2/8M_\omega^2 \approx 10^{-5} \text{ GeV}^{-2}$ are used to
study nuclear interactions. The proton number conser-
vation is the result of the U(1)F gauge theory and the
baryon number conservation ($B = N_B \approx 10^{38}$) is the
consequence of the U(1)Z gauge theory for strong inter-
actions; the charge quantization $\hat{Q}_I = \hat{C}_3 + \hat{Z}_c/2$ with the hyper-color operator $\hat{Z}_c = \hat{B} + \hat{\Sigma}$ for the U(1)F
gauge theory. The mass density and charge density of
nuclear matter are the consequence of the proton num-
ber conservation or the baryon number conservation.
In analogy with the electron orbit radius of the hydrogen
atom $r_e = a_0 n_e^2$ with the atomic radius $a_0 = 1/2m_\alpha y$$\approx 0$ and the principal quantum number $n_e$, the nuclear ra-
dius may be written by $r = r_0 n^2$ with a nucleon orbit
radius $r_0 = 1/2m_\alpha y \approx 1.2 \text{ fm}$ and the principal quan-
tum number $n = A^{1/3} = B^{1/6}$. The proton and neutron
are assigned as a colorspin plus weak isospin doublet in-
stead of as a strong isospin doublet. The magnetic dipole
moment and electric quadrupole moment further suggest
that colorspin and isospin may be introduced as intrinsic
angular momenta in addition to the spin angular mo-
mentum. The extension of the total angular momentum
$\vec{J} = \vec{L} + \vec{\mathcal{S}} + \vec{\mathcal{C}} + \vec{T}$ from $\hat{J} = \hat{L} + \hat{\mathcal{S}}$ may explain the
Landau spin g-factors of magnetic dipole moments for the
proton and neutron. $g_L^p = 5.59$ and $g_L^n = −3.83$, respec-
tively. The nucleon-nucleon scattering can be investi-
gated in terms of QND as an SU(2)N × U(1)Z gauge
theory in analogy with the beta decay of the neutron in
weak interactions. The nucleon-nucleon scattering (pp,
nn, or np) as a spin singlet is known as the Fermi type
scattering and the nucleon-nucleon scattering (np or pn)
as a spin triplet is known as the Gamow-Teller type scat-
tering. Further testable predictions or already conﬁrmed
predictions from QND are given as follows: parity viola-
tion in meson and baryon spectra, charge conjugation vi-
olation in baryon spectra, time reversal and CP violation
in the electric dipole moment for the neutron ($\Theta \leq 10^{-9}$),
the nonconservation of color singlet proton and neutron,
the nonconservation of the axial vector current, coupling
constant hierarchy, the color mixing angle $\sin^2 \theta_R \approx 1/4$, the
assignment of strong isospin as colorspin plus weak
isospin. Moreover, QND possessing colorspin degrees of
freedom may apply to explain the various nuclear phe-
nominal such as lifetimes and cross sections of nuclear scattering and reaction, shell model, meson-nucleon scattering, nuclear potential, nuclear binding energy, gamma ray, etc. over the wide energy range.

V. CONCLUSIONS

This paper claims that QCD as an $SU(3)_C$ gauge theory produces quantum nuclear dynamics (QND) as an $SU(2)_N \times U(1)_Z$ gauge theory in terms of dynamical spontaneous symmetry breaking (DSSB) through the condensation of singlet gluons. Important concepts are colorspin, massive gauge boson, and discrete symmetry breaking. The proton and neutron assigned as a strong isospin doublet are identified as a colorspin plus weak isospin doublet; nucleons as color doublets are governed by QND. Quantized gauge bosons, i.e., gluons, are massive and yield the Yukawa potential; this is understood as the confinement mechanism of massive gluons limited to relatively short range propagation. Phase transition occurs when singlet gluons, rather than other scalar bosons, acquire vacuum expectation values and reduce the masses of other bosons: the gauge boson mass square is $M^2_\alpha = M^2_H - c_f g^2 \langle \phi \rangle^2 = c_f g^2 (\alpha^2 - \langle \phi \rangle^2)$ with the grand unification mass $M_H$, the strong coupling constant $g_s$, and the singlet gluon condensation $\langle \phi \rangle$.

QND as an $SU(2)_N \times U(1)_Z$ gauge theory or as a $U(1)_f$ gauge theory is applied to evaluate lifetimes and cross sections in strong interactions or to study alpha and gamma decays. QND is applicable to study the nucleon-nucleon scattering in analogy with the beta decay of the neutron in weak interactions: the Fermi type scattering and Gamow-Teller type scattering in strong interactions are suggested just like the Fermi decay and Gamow-Teller decay in weak interactions. It is also useful to the meson-nucleon scattering so that cross sections over wider energy regions are analyzed by the exchange of massive gluons. The strong isospin symmetry and magnetic dipole moment for the proton and neutron suggest the introduction of the colorspin intrinsic angular momentum. The color factors described above are the color factors purely due to color charges but the effective color factors used in nuclear dynamics must be multiplied by the isospin factor $i^w_f = \sin^2 \theta_W = 1/4$ since the proton and neutron are a colorspin and isospin doublet: $c^w_f = i^w_f c_f = i^w_f (c_f^1, c_f^2, c_f^3, c_f^4) = (1/12, 1/16, 1/48, 1/64)$ for symmetric configurations. The conservation of the proton number is the result of the $U(1)_f$ local gauge theory just as the conservation of the electron number is the result of the $U(1)_e$ local gauge theory. The baryon number conservation is the result of the $U(1)_Z$ gauge theory for strong interactions just as the lepton number conservation is the result of the $U(1)_Y$ gauge theory for weak interactions. The constant mass density and charge density of nuclear matter are the consequence of the proton number conservation or the baryon number conservation. The colorspin-colorspin interaction potential is introduced in the nuclear local central potential as the major contribution and the colorspin-orbit coupling potential and isospin-orbit coupling potential as non-local potentials are suggested in addition to the spin-orbit coupling potential. Several contributions of nuclear binding energies are also interpreted from the gauge theory point of view. This proposal thus provides the connection between QCD and QND, which are both renormalizable just like the GWS model. This implies that nuclear dynamics can be studied from the gauge theory without free parameters except the strong coupling constant. Lots of applications for QND are expected toward the complete understanding of nuclear matter.

Notable consequences of this work are summarized in the following. QCD as an $SU(3)_C$ gauge symmetry generates QND as an $SU(2)_N \times U(1)_Z$ gauge theory or a $U(1)_f$ gauge theory through DSSB induced by the condensation of singlet gluons. The nonperturbative solution of QCD in the low energy limit is explicitly obtained from DSSB. The concepts of colorspin, massive gluon, and discrete symmetry breaking play important roles in nuclear interactions. The analogy property emphasizes that QND is the analogous partner of the GWS model and that massless gauge bosons mediate nuclear electromagnetic interactions. QND as an $SU(2)_N \times U(1)_Z$ gauge theory emerges as the dynamics for nuclear interactions so that cross sections, lifetimes, nucleon-nucleon scattering, meson-nucleon scattering, magnetic dipole moment, nuclear potential, nuclear binding energy, gamma decay, etc are studied. The baryon number conservation is the consequence of the $U(1)_Z$ gauge theory and the proton number conservation is the consequence of the $U(1)_f$ gauge theory. This proposal may also provide a turning point toward the understanding of nuclear forces at relatively low energies since QND is more or less applicable to all the nuclear phenomena.

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FIG. 1. The cross section of proton-proton scattering as a function of the center of mass energy.
### TABLE I. Color Quantum Numbers of Nucleons

| Baryons | $C_1$ | $C_2$ | $Z_8$ | $Q_4$ |
|---------|-------|-------|-------|-------|
| $p_d$   | $1/2$ | $1/2$ | 1     | 1     |
| $n_d$   | $1/2$ | $-1/2$| 1     | 0     |
| $p_s$   | 0     | 0     | 2     | 1     |
| $n_s$   | 0     | 0     | 0     | 0     |

### TABLE II. Relations between Conservation Laws and Gauge Theories

| Force          | Conservation Law | Gauge Theory                |
|----------------|------------------|-----------------------------|
| Electromagnetic| Proton           | $U(1)_f$                    |
| Strong         | Baryon           | $U(1)_Z$                    |
| Strong         | Color Vector     | $SU(2)_N \times U(1)_Z$    |
| Strong         | Color            | $SU(3)_C$                   |
| Electromagnetic| Electron         | $U(1)_e$                    |
| Weak           | Lepton           | $U(1)_Y$                    |
| Weak           | V-A              | $SU(2)_L \times U(1)_Y$    |

### TABLE III. Quantum Numbers of Nucleon-Nucleon Systems ($i^s$: strong isospin, $i^w$: weak isospin)

| State | $i^s = 1$ | $i^w = 0$ |
|-------|-----------|-----------|
| pp    | $i = 1, s = 0, c = 1$ |           |
| nn    | $i = 1, s = 0, c = 1$ |           |
| pn    | $i = 1, s = 0, c = 1$ | $i = 0, s = 1, c = 0$ |
|       | $i = 1, s = 1, c = 0$ | $i = 0, s = 0, c = 1$ |

### TABLE IV. Nucleon-Nucleon Scattering Length $a$ (fm) and Effective Range $r_e$ (fm)

| Scattering | $i^s = 1, s = 0$ | $i^w = 0, s = 1$ |
|------------|-----------------|-----------------|
| pp         | $a = -17.1, r_e = 2.79$ |           |
| nn         | $a = -16.6, r_e = 2.84$ |           |
| pn         | $a = -23.7, r_e = 2.73$ | $a = 5.4, r_e = 1.73$ |
TABLE V. Comparison between Quantum Nucleardynamics and Effective Models

| Classification                              | QND                               | Effective Models               |
|---------------------------------------------|-----------------------------------|--------------------------------|
| Exchange Particles                          | massive gluons                   | massive mesons (model dependent) |
| DSSB                                        | yes                               | no                             |
| Discrete symmetries (P, C, T, CP)           | breaking                          | no                             |
| Confinement                                 | yes                               | no                             |
| Θ vacuum                                    | yes                               | no                             |
| Baryon number conservation                  | $U(1)_Z$ gauge theory ($N_B \approx 10^{78}$) | unknown                       |
| Proton number conservation                  | $U(1)_f$ gauge theory             | unknown                       |
| Nuclear electromagnetic interaction         | $U(1)_f$ gauge theory             | no                             |
| Intrinsic angular momenta                   | colorspin, weak isospin, spin     | spin                          |
| Hadron mass generation                      | yes                               | unknown                       |
| NN scattering cross section                 | $G^2_f T^2$ (from high to low energy) | only at low energy            |
| Hadron decay time                           | $1/G^2_f m^2_f$                   | no                            |
| Neutron electric dipole moment              | $\Theta \approx 10^{-12}$         | no                            |
| Free parameters                             | coupling constant                 | many                          |
| Renormalization                             | yes                               | model dependent               |