Tilt-Induced Phase Transitions in Even-Denominator Fractional Quantum Hall States at the ZnO Interface

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(Dated: December 28, 2017)

Even denominator fractional quantum Hall states in a ZnO quantum well reveal interesting phase transitions in a tilted magnetic field. We have analyzed the planar electron gas in ZnO, confined in a parabolic potential in the third dimension, perpendicular to the plane of the electron gas. Since the Landau level gap is very small in this system we have included the screened Coulomb potential in order to include the effects of all the Landau levels. We observe an incompressible state - compressible state phase transition induced by the tilted field. Additionally, the 5/2 state has been experimentally found to be missing in this system. We however propose that a wider quantum well may help to stabilize the incompressible phase at the 5/2 filling factor.

The ‘enigmatic’ even-denominator fractional quantum Hall effect (FQHE) at the filling factor \( \nu = \frac{5}{2} \) was first discovered in GaAs heterojunctions \([1, 2]\). It is a special member in the FQHE family since its ground state and excitations can not be described by the Laughlin wave function \([3, 4]\). Although there are some aspects of this state that still remains unclear as yet, it is generally believed that a Pfaffian state with non-Abelian excitations is the most likely candidate to describe this extrordinary FQHE \([3, 4]\). Numerical studies of the even denominator FQHE were also helpful to understand the nature of this state \([5, 6]\). The ground state at this filling factor is incompressible (just as for the odd-denominator states), so that the attributes of the transport experiments are the same as those of the odd-denominator FQHE. Recently, the FQHE has been discovered in an oxide material, the ZnO interface with high mobility \([11, 13]\). There is heightened expectations that the stronger Coulomb interactions in this system will perhaps display unusual effects related to strong electron correlations \([14]\). Interestingly, in the MgZnO/ZnO interface the \( \nu = \frac{5}{2} \) state was found to be missing. The ZnO quantum well is quite different from the conventional GaAs system because here the Landau level (LL) gap is comparable to the Zeeman gap and the ratio \( k \) of the Coulomb interaction to the LL gap is very large. In order to explain the missing FQHE, we earlier introduced the screened Coulomb interaction which includes the effect of all the LLs in our model \([15]\). The system was indeed found to be compressible, thereby explaining the absence of the \( \nu = \frac{5}{2} \) state in ZnO.

A tilted magnetic field is a very useful means to study the nature of the fractional quantum Hall systems \([10–18]\). It modifies the transport properties to provide additional information about the systems, especially the spin polarization and the excitations associated with electron spins \([19, 20]\). In the experiment by Falson et al. involving the ZnO \([13]\), the tilted magnetic field reflects very unusual behavior in the transport properties. At the \( \nu = \frac{3}{2} \) filling factor the FQHE was found to appear only when the tilt angle is large. On the other hand, for the \( \nu = \frac{5}{2} \) filling factor the FQHE disappeared when the tilt angle was increased. We believe that there are phase transitions associated with these experiential observations; since the LL energy gap which is only related to the perpendicular component of the magnetic field can be easily exceeded by the Zeeman coupling which is proportional to the total magnetic field. In particular, the spin transition is also involved in these phase transitions. The spin polarization measurements could therefore be used to observe these phase transitions. With the increase of the tilt angle the screening potential is changed. So the transport properties of the electron gas in a variable tilted field must become very different from those in a perpendicular magnetic field alone. Here we focus only on the even-denominator filling factors.

In our work, the electron gas is confined in a parabolic potential in the \( z \) direction, thereby making the system quasi-two-dimensional. Motivated by the experimental work of Falson et al. \([13]\), we consider this system in a tilted magnetic field. In the ZnO quantum well, unlike in the GaAs system, the LL crossing is easily achieved in a tilted field. Since the Zeeman coupling is about 0.94 of the LL gap in a perpendicular magnetic field, a spin polarized state may not satisfactorily describe the system. We therefore use a spin-mixed Hamiltonian to study the ground states and the phase transitions involving spin flipping.

In our previous work \([15]\) we proposed that the missing fractional quantum Hall state at \( \nu = \frac{3}{2} \) is due to the screened Coulomb potential. We also use that screened Coulomb potential to study the FQHE in a tilted magnetic field. With a non-zero tilt angle \( \alpha_0 \), it is more realistic to suppose that the electron gas is confined in a parabolic potential with the frequency \( \omega_z \) in the \( z \) direction which is perpendicular to the plane of the electron gas. The advantage of this approximate potential is that the wave function can be analytically obtained for any value of the tilted field. This approximation to a certain extent should be similar to other finite-thickness approaches such as the infinite square well \([8]\) or a trian-
regular well. We choose the Landau gauge with the vector potential $A = (0, B_x x - B_z z, 0)$. The wave function is then given by \[ 10, 21, 23 \]

$$
\psi_{k,n_1,n_2}(u) = \frac{e^{iky}}{\sqrt{T_y}} \phi_{n_1}^{(1)} \left[ (x + k \ell^2) \cos \theta - z \sin \theta \right] \\
\times \phi_{n_2}^{(2)} \left[ (x + k \ell^2) \sin \theta + z \cos \theta \right],
$$

where $k$ is the guiding center index, $n_{1,2}$ is the sub LL index, and

$$
\theta = \arctan \frac{\omega_b^2 - \omega_1^2 + \sqrt{(\omega_b^2 - \omega_1^2)^2 + 4 \omega_2^2 \omega_b^2}}{2 \omega_1 \omega_b},
$$

with $\omega_1 = eB_z/m^*$, $\omega_\parallel = eB_x/m^*$ and $\omega_2 = \omega_1^2 + \omega_2^2$. The wave function of a harmonic oscillator is $\phi_n(x) = \frac{1}{\sqrt{\pi^{2n} n!}} \exp \left( -\frac{x^2}{2 \theta_n^2} \right) H_n \left( \frac{x}{\theta_n} \right)$, with a Hermite polynomial $H_n$. The magnetic length is $\ell = \sqrt{\epsilon_e/eB_z}$ and the magnetic length for each subband is $\ell_{1,2} = 1/\sqrt{m^* \omega_{1,2}}$, where $m^*$ is the effective mass and $\omega_{1,2} = 1/\sqrt{\omega_b^2 + \omega_1^2}$. The energies of the LLs are $E_{n_1,n_2} = (n_1 + \frac{1}{2}) \hbar \omega_1 + (n_2 + \frac{1}{2}) \hbar \omega_2$, where $n_{1,2}$ are sub LL indices. In a parabolic potential the LL indices must be described by two branches. One is related to the original LL in a pure two-dimensional (2D) case, while the other one is essentially the energy levels of the parabolic potential.

The interaction Hamiltonian including spin is given by

$$
H = \frac{1}{2} \sum_{\sigma,\beta} \sum_{m^n, n'^n} \sum_{j_1,j_2} \int du_1 du_2 \psi_{j_1,m,n}(u_1) \psi_{j_2,m',n'}(u_2) V(\mathbf{u}_1 - \mathbf{u}_2) \psi_{j_3,m',n'}(u_2) \psi_{j_4,m,n}(u_1) \\
\psi_{j_2,m',n'}(u_2) V(\mathbf{u}_1 - \mathbf{u}_2) \psi_{j_3,m',n'}(u_2) \psi_{j_4,m,n}(u_1) \\
c_{j_1,m,n} \psi_{j_1,m,n} \psi_{j_2,m',n'} \psi_{j_4,m,n} \psi_{j_3,m',n'} c_{j_3,m',n'} c_{j_4,m,n} c_{j_2,m',n'} c_{j_1,m,n},
$$

where we only consider one LL or two LLs with different spins. In the rectangular space, the discrete momenta is $q_x = \frac{2\pi}{L_x} s$, $q_y = \frac{2\pi}{L_y} t$. So the Coulomb interaction is then given by \[ 21, 24 \]

$$
V_C = \frac{2e^2}{\ell \epsilon} \sum_{q_x,q_y} \delta_{j_1,j_4} + \delta_{j_2,j_3} - q_y e^{i(j_3-j_1)q_x} \ell^2 - q_y^2 - q_x^2 \\
\int \frac{dq_y}{\epsilon_s(q)} \exp \left( -q_y^2 - \frac{q_x^2}{2} \right) \Xi(m,n;q) \Xi(m',n';q),
$$

where $\epsilon$ is the dielectric constant, $S = L_x L_y$ is the area of the sample, $\delta'$ includes the periodic boundary condition and the bar over the summation excludes the term $q_x = q_y = 0$. We also write

$$
q_1 = \frac{1}{\sqrt{2}} \cos \theta q_y \ell^2, \quad q_2 = \frac{1}{\sqrt{2}} \sin \theta q_y \ell^2 \\
q_- = \frac{1}{\sqrt{2}} (q_x \cos \theta - q_y \sin \theta) \ell_1, \\
q_+ = \frac{1}{\sqrt{2}} (q_x \cos \theta + q_y \sin \theta) \ell_2
$$

$$
\lambda_{1,2}(m,m',q) = [\text{sign}(m-m') q_{1,2} \mp i q_{3,4}]^{m-m'} \\
L_{\text{min}(m,m')}^{m-m'} (q_{3,4}^2 + q_{3,4}^2)
$$

with a Laguerre polynomial $L_n$ and

$$
\Xi(m,n;q) = \lambda_1 (m,m,q) \lambda_2 (n,n,q).
$$

Following Ref. \[ 13 \], we introduce the dielectric function of the screening in a general three-dimensional case,

$$
\epsilon_s(q) = 1 - V(q) \chi_0^0 (q) = 1 - \frac{2 \pi e^2}{q^2 \epsilon} \chi_0^0 (q).
$$

The non-interacting density-density response function in the Matsubara frequency $\Omega_n$ is

$$
\chi_{nn}^0 (q,\Omega_n) = \frac{N_n}{S L_z} \sum_{\sigma,m,m',n,n'} \left| G_{mn,m'n'} (-q) \right|^2 \\
\times \frac{\nu_{\sigma,mn} - \nu_{\sigma,m'n'}}{i \hbar \Omega_n + (E_{mn} - E_{m'n'})},
$$

where $N_n$ is the degeneracy of a LL, $\sigma$ is the spin index, $m^{(i)}, n^{(i)}$ are the sub LL indices, $\nu$ is the filling factor, $E_{mn}$ is the kinetic energy of the sub LL $(m,n)$, and the form factor is defined as

$$
G_{mn,m'n'} (q) = \exp \left[ -\frac{1}{2} (q_x^2 + q_y^2 + q_z^2 + q_z^2) \right] \\
\sqrt{\frac{\min(m,m')! \min(n,n')!}{\max(m,m')! \max(n,n')!}} \lambda_1 (m,m',q) \lambda_2 (n,n',q).
$$

For the third dimension, the length in the $z$ direction, $L_z$ is difficult to determine in a parabolic potential. In principle it should be infinity, but for our present purpose we choose a finite value for $L_z$ since the wave functions vanish rapidly in the $z$ direction. The electrons are confined well in the $z$ direction, so it is reasonable to limit $L_z$ in a proper range. Here, we approximate the $L_z$ in terms of the parabolic potential frequency $\omega_z$, $L_z = 2 \sqrt{\ln(2) \frac{q_z}{q_z}} \ell$, which is the width of the wave function of the lowest LL in the $z$ direction. We set $L_z = 1.8$ nm which corresponds to a relatively narrow quantum well, since the electron gas will be split into a "double" layer system in a wide quantum well \[ 23 \] and the consequent transport properties would become very different. The density-density response function is calculated in the noninteracting case, so the filling factors $\nu$ are also the noninteracting filling factors.
We utilize the exact diagonalization scheme in the standard case of finite-size systems in a periodic rectangular geometry \((L_x = L_y)\) with the screening potential included \([15, 28]\). In order to analyze the phase transition near the LL crossing in a tilted field, we take one LL or two LLs with different spin orientations. With an increase of the tilt angle, the Landau level crossings occur. The first crossing happens at about 18°, while the second one at about 62°. We study only the small tilt angles, since the parabolic potential may not be a very good approximation at very large angle. When the quantum well is narrow, the energies of the series of sub LLs \((n < 3)\) are much higher than those of sub LLs \((n > 0, 0)\). When the tilt angle is \(\theta < 60°\), the LL crossing only happens between the LLs \((0, n; \downarrow)\) and \((0, n + 1; \uparrow)\). The magnetic fields of the \(z\)-component are set the same as in Ref. \([13]\), i.e., \(B_z = 6.2, 3.75, 2.75\) T for \(3/2, 5/2, 7/2\), respectively.

**Phase transition at \(\nu = \frac{1}{2}\)**: We consider the two LLs \((0, 0; \downarrow)\) and \((0, 1; \uparrow)\) into our exact diagonalization scheme. We did not find any spin coherence irrespective of the tilt angle. There is a phase transition associated with spin polarization at about \(\alpha_1 = 29°\). When \(\alpha_0 < \alpha_1\), all electrons are in LL \((0, 0; \downarrow)\), even though the kinetic energy of \((0, 1; \uparrow)\) is lower when \(18° < \alpha_0 < \alpha_1\). The incompressibility of the ground state may not be stable for different sizes of the system. When \(\alpha_0 > \alpha_1\), all electrons flip to \((0, 1; \uparrow)\). In this LL, the even-denominator FQHE can be found as an incompressible liquid without any LL mixing or screening. For \(\nu = \frac{3}{2}\), the screening is weaker than that of filling factor \(\nu = \frac{5}{2}\). The collective modes for \(\nu = \frac{3}{2}\) when \(\alpha_0 > \alpha_1\) show that the ground state is incompressible. However, in the experiment, the FQHE is only observed when \(\alpha_0 > 38°\) \([13]\).

**Phase transitions at \(\nu = \frac{5}{2}\)**: When the magnetic field is perpendicular to the electron plane, we have already shown that the FQHE survives the screening potential \([15]\). In a tilted field, with the parabolic potential we consider the two LLs \((0, 1; \downarrow)\) and \((0, 2; \uparrow)\) into the exact diagonalization scheme. The phase transition which is found to be at \(\alpha_2 = 18°\) is more or less at the same angle where the non-interacting LLs cross. The incompressible ground state is still found when \(\alpha_0 < \alpha_2\). All electrons are in LL \((0, 1; \downarrow)\), and the collective mode for eleven electrons is shown in Fig. 1.

The collective modes do not change much when \(\alpha_0 < \alpha_2\). The two minima are at about \(ql = 2.2\) and \(3.8\), which is close to the collective mode of the pure 2D case. When \(\alpha_0 > \alpha_2\), the screening is changed due to the fact that the non-interacting filling factors are changed, and the electrons are flipped to LL \((0, 2; \downarrow)\), and the system is no longer incompressible. In the experiment, the phase transition occurs in the range \((21°, 27°)\) \([13]\). The difference between our theoretical work and the measurement is likely due to the LL broadening which is induced by the disorder. The LL broadening is able to shift the LL crossing to a higher tilt angle. The second LL crossing occurs for \(\alpha_0 = 62°\), between \((0, 2; \uparrow)\) and \((0, 0; \downarrow)\). We determine the collective modes of LL \((0, 0; \downarrow)\) and the ground state is still compressible. In the phase transitions at this filling factor, the mixed-spin state is also absent. No spin coherence state has lower energy than the spin polarized state. All phase transitions are first-order.

We note here that all phase transitions involve spin flip. The spin polarization before and after all the phase transitions is changed significantly. If we define the spin polarization as \(\langle S_z \rangle = \frac{1}{2} (\nu_{\uparrow} - \nu_{\downarrow})\), then the spin phase transition is shown in Fig. 2. Therefore, the spin polarization measurement could be a very powerful means to determine the phase transitions. We expect that future experimental work may confirm our present findings.

At filling factor \(\nu = \frac{5}{2}\), in a pure 2D case the FQHE is absent due to the screened Coulomb potential from other LLs \([13]\). The FQHE is still absent for the parabolic potential. Interestingly, the \(\nu = \frac{3}{2}\) FQHE could occur if we tune the width of the quantum well. When \(L_z \geq 9\) nm, the ground state of \(\nu = \frac{3}{2}\) becomes incompressible. Figure 3 (a) and Fig. 3 (b) display the collective modes of seven electrons for \(L_z = 1.8\) nm and 11 nm in a zero

**FIG. 1**: The collective mode of \(\nu = 7/2\) for 11 electrons in a tilted magnetic field \((\theta = 10°)\).

**FIG. 2**: The phase transitions at the \(\nu = \frac{5}{2}\) filling factor.
to stabilize the FQHE at
imental works indicated that wider quantum well helps
the FQHE state. In GaAs, both theoretical and exper-
higher magnetic field a narrower width can also stabilize

But the collective modes show that the incompressibil-
row the electron gas behaves similar to a pure 2D case.

In order to explore the nature of the ground state, we
have calculated the wave functions of the even-
denominator FQHE states. For simplicity, we consider
the pure 2D case. The many-body wave function of the
ground state at \( \nu = \frac{5}{2} \) in a perpendicular magnetic field
is \( \left| \phi_{\frac{5}{2}} \right\rangle \) with our screening potential. The wave function
of \( \nu = \frac{3}{2} \) or \( \nu = \frac{7}{2} \) without screening is \( \left| \phi_{\frac{3}{2}} \right\rangle \approx \left| \phi_{\frac{7}{2}} \right\rangle \). We
find that the overlap of these two wave functions is close
to unity, \( \left\langle \phi_{\frac{3}{2}} \left| \phi_{\frac{7}{2}} \right\rangle \approx 0.99 \). So the screened Coulomb
potential at \( \nu = \frac{3}{2} \) does not change the wave function. The pair distribution function is also similar to that of
a liquid \[7\]. Note that the screened Coulomb potential at
\( \nu = \frac{3}{2} \) completely destroy the liquid ground state, the
lowest energy state is a density wave.

In summary, we have analyzed the phase transitions
at \( \nu = \frac{3}{2} \) and \( \nu = \frac{7}{2} \) in the exact diagonalization scheme
of the two LLs with different spin polarization when the
magnetic field is tilted from the direction perpendicular
to the electron plane. We find phase transitions be-
tween different spins. The nature of the ground states
is changed significantly in the phase transition. How-
ever, we could not find any spin coherent state in the
phase transition. It seems that the spin polarization is
first-order-like flipped to the other spin orientation when
the tilt angle is increased. The spin flipping could be
observed in a spin-sensitive experiment \[29\]. Experiment-
tally, the \( \nu = \frac{3}{2} \) FQHE was found to be absent in ZnO.
However, we propose that if the width of the quantum
well could be artificially widened then this FQHE state
might be observable. Incidentally, the measured activa-
tion gaps in ZnO are usually one order of magnitude
smaller than those in GaAs \[12\]. This can be qualita-
tively understood as follows: the LL gap of GaAs is about
seven times larger than that of ZnO, and therefore the
screening of ZnO is seven times stronger than that of
the GaAs. Consequently, the excitation energy in ZnO is
about one order smaller than that in GaAs.

The work has been supported by the Canada Research
Chairs Program of the Government of Canada. The com-
putation time was provied by Calcul Qu´ebec and Com-
pute Canada.

\[\text{FIG. 3: The collective mode of } \nu = \nu = \frac{3}{2} \text{ for seven electrons in a perpendicular magnetic field. (a) The characterised width of the parabolic potential } L_x = 1.8 \text{ nm and (b) } L_x = 11 \text{ nm.}\]

\[\text{Energy gap (e}^2/\varepsilon_l)\text{ of quantum well is increased. If the width of the quantum well in ZnO could be artifically fabricated then we hope that this width effect predicted here can indeed be observed.}\]

\[\text{In order to explore the nature of the ground state, we have calculated the wave functions of the even-}
\text{denominator FQHE states. For simplicity, we consider}
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ground state at } \nu = \frac{5}{2} \text{ in a perpendicular magnetic field}
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of } \nu = \frac{3}{2} \text{ or } \nu = \frac{7}{2} \text{ without screening is } \left| \phi_{\frac{3}{2}} \right\rangle \approx \left| \phi_{\frac{7}{2}} \right\rangle. \text{ We}
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