NUCLEAR $\beta$- AND $\gamma$- COLLECTIVE BANDS IN THE SU$_q$(2) ROTATOR MODEL

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ABSTRACT

The SU$_q$(2) rotator model is used for describing the $\beta_1$- and $\gamma_1$-bands of even-even rare earth and actinide collective nuclei. Good results are obtained in nuclei with valence pair number $N > 10$. It is shown that in the excited bands the violation of the exact SU(2) symmetry is generally stronger than in the ground state bands, indicating the presence of a nonadiabatic perturbation caused by the excited vibrational degrees of freedom. The physical content of the parameter $q$ is discussed. Predictions of the SU$_q$(2) model for B(E2) intraband transitions in excited bands are presented and the need for specific experimental data is pointed out.

1. Introduction

The quantum algebra SU$_q$(2)$^{1,2}$ is a nonlinear generalization (having the structure of a Hopf algebra$^3$) of the corresponding Lie algebra SU(2) to which it reduces when the deformation parameter $q$ is set equal to one. It has been found that the SU$_q$(2) algebra can be used for describing the deviations of rotational spectra of collective nuclei$^{4-7}$ and diatomic molecules$^{8-10}$ from the rigid rotator symmetry of SU(2), the deformation parameter $q$ being related$^5$ to the softness parameter of the Variable Moment of Inertia (VMI) model$^{11}$. Furthermore the implications of the SU$_q$(2) symmetry on B(E2) transition probabilities within the ground state bands (gsb) of deformed nuclei have been considered$^{12}$, indicating that the B(E2) values do not saturate but continue to increase with increasing angular momentum $I$, a result also obtained in the framework of other models$^{13,14}$.

So far the SU$_q$(2) symmetry has been tested only in relation to levels and B(E2) transition probabilities of the ground state band of deformed nuclei. It is the purpose of the present work to examine the applicability of the SU$_q$(2) symmetry to excited collective bands, the $\beta_1$- and the $\gamma_1$-band in particular. Such an investigation is naturally motivated by the question: "It is already known that the quantum algebra SU$_q$(2) is appropriate to characterize nuclear rotations built on the ground state, but what is the $q$-rotator in the case of a given excited band where besides
the rotational motion there is a presence of other collective (vibrational) degrees of freedom?” In this respect it is interesting to study whether the $q$-deformation “detects” the presence of the additional nonrotational degrees of freedom. As will be seen below, the study of the energy levels of the excited bands illuminates the above question and leads to interesting conclusions about the physical content of the deformation parameter $q$, while the study of the intraband B(E2) transition probabilities emphasizes the need for specific experimental data for testing the deviations from the pure SU(2) behavior appearing there.

2. $q$-rotator definition

The Hamiltonian of the $q$-rotator model is proportional to the second order Casimir operator $C_2[SU_q(2)]$ of the quantum algebra $SU_q(2)^4$:

$$H = \frac{1}{2\theta} C_2[SU_q(2)] + E_0,$$

(1)

where $\theta$ is the moment of inertia parameter and $E_0$ is the bandhead energy (for the gsb $E_0 = 0$). The corresponding energy eigenvalues are:

$$E_I = \frac{1}{2\theta} [I][I+1] + E_0,$$

(2)

where $I$ is the angular momentum and the square brackets indicate $q$-numbers, according to the following definition:

$$[x] = \frac{q^x - q^{-x}}{q - q^{-1}}.$$

(3)

In the case of $q$ being a phase ($q = e^{i\tau}$ with $\tau$ a real parameter), eq. (2) gives:

$$E_I = \frac{1}{2\theta} \sin(\tau I) \sin(\tau(I+1)) \sin^2(\tau) + E_0.$$

(4)

In the limit $\tau \to 0$, the first term in eq. (4) gives the spectrum of the usual SU(2) rigid rotator\textsuperscript{15}. It has been proved\textsuperscript{5} that the deformation parameter $\tau$ is connected to the softness parameter of the VMI model, thus indicating that $q$-deformation is an alternative way of taking into account nuclear stretching.

3. $SU_q(2)$ symmetry in exited collective bands

In the case of excited bands one needs an appropriately formulated $q$-rotator definition which should take into account the circumstance that the rotational energy levels are built on a given excited vibrational state\textsuperscript{16}. For this purpose it is convenient to use eq. (4) in the form:

$$\overline{E}_I = E_I - E_0(I_{bh}) = \frac{1}{2\theta} \frac{\sin(\tau I) \sin(\tau(I+1))}{\sin^2(\tau)}$$

(5)
with \( I > I_{bh} \), where it is supposed that the energy scale of collective rotations has its origin in the bandhead energy \( E_0(I_{bh}) \), and \( I_{bh} \) is the bandhead angular momentum, which is 0 for \( \beta \)-bands and 2 for \( \gamma \)-bands. Thus, after subtracting the bandhead energy we determine the rotational parts of the bandlevels. However, it is important to remark that the so obtained energies are still perturbed by the vibrational motion as far as even in the well deformed nuclei the collective rotations are not separated completely from the vibrational degrees of freedom\(^{16}\). Taking into account this nonadiabatic perturbation we suppose that in the excited bands the \( q \)-deformations of the SU(2) symmetry should be generally larger in magnitude than the corresponding ones obtained in the gsb’s. Hence one could expect that the quantum algebraic parameter \( \tau \) will be able to indicate the presence of excited vibrational modes. Below it will be seen that the calculations essentially support this supposition.

So, the theoretical predictions (eq. 5) are compared to the experimental quantities \( E_{I}^{\exp} = E_{I}^{\exp} - E_{0}(I_{bh}) \). For obtaining the fits an autoregularized iterational method of the Gauss-Newton type\(^{17}\) has been used, the quality of the fits being measured by

\[
\sigma = \sqrt{\frac{1}{n} \sum_{I=I_{min}}^{I_{max}} (E_{I}^{\exp} - E_{I}^{th})^2},
\]

where \( n \) is the number of levels used in the fit and \( I_{min} = 2 \) for \( \beta \)-bands, while \( I_{min} = 3 \) for \( \gamma \)-bands. We have included in the fitting procedure rare earth and actinide nuclei in the rotational region (with \( 3 \leq R_{A}^2 = E_{4}/E_{2} \leq 10/3 \)) for which at least 5 levels of the \( \beta_{1} \)- or \( \gamma_{1} \)-band are known\(^{18,19}\). The results for the \( \beta_{1} \)- and/or \( \gamma_{1} \)-bands of 28 rare earths and 3 actinides, along with the results for the corresponding gsb are shown in Table 1.

The following comments can now be made:

i) The parameters \( \tau_{\beta} \) and \( \tau_{\gamma} \) generally obtain values in the region 0.03–0.07, close to the typical \( \tau_{g} \) values of 0.03–0.06 (see also \(^{4–7}\)). Nevertheless it is clearly seen that for almost all considered nuclei, the \( \tau \) values obtained in the excited bands lie above the corresponding gsb values (see also Fig. 1). It turns out that in the excited bands the quantum algebraic parameter \( \tau \), which characterizes the deviation of the spectrum from the pure SU(2) symmetry of the rigid rotator\(^{20}\), indicates the presence of additional nonrotational degrees of freedom. Moreover, some \( \tau_{\gamma} \geq 0.1 \) values occur for nuclei with valence pair number \( N \) relatively small (10–13), indicating that the rotational character of the \( \gamma_{1} \)-band is not yet well developed in this \( N \) region. So, the sensitivity of the SU\(_{q}(2) \) rotator description to the structure of the different types of bands is obvious.
Table 1. Parameters of the fits of $\beta_1$- and $\gamma_1$-bands in the rare earth and actinide regions using eq. (5). The deformation parameters $\tau_\beta$ and $\tau_\gamma$, the quality factors $\sigma_\beta$ and $\sigma_\gamma$ (in keV) (eq. 6) accompanied by the numbers $n_\beta$ and $n_\gamma$ of the experimental levels used in the fit, and the inertial parameters $1/(2\theta_\beta)$ and $1/(2\theta_\gamma)$ (in keV$^{-1}$) for the $\beta_1$- and $\gamma_1$-bands respectively are shown. The corresponding deformation parameters $\tau_g$ of the ground state band and the valence pair numbers $N$ are also given. The experimental data are taken from $^{18,19}$. 

| Nucleus | $N$ | $\tau_g$ | $\tau_\beta$ | $\tau_\gamma$ | $\sigma_\beta[n_\beta]$ | $\frac{1}{2\theta_\beta}$ | $\sigma_\gamma[n_\gamma]$ | $\frac{1}{2\theta_\gamma}$ |
|---------|-----|----------|--------------|---------------|-------------------------|---------------------------|-------------------------|---------------------------|
| $^{152}$Sm | 10  | 0.0622   | 0.0695       | 0.1030        | 22.57[8]               | 15.39                     | 15.13[8]               | 21.63                     |
| $^{154}$Sm | 11  | 0.0500   | 0.1306       |               |                        | 1.14[5]                  |                         | 18.67                     |
| $^{156}$Gd | 12  | 0.0521   | 0.0641       | 0.0668        | 5.20[6]                | 12.36                     | 11.29[10]              | 14.92                     |
| $^{158}$Gd | 13  | 0.0419   | 0.1345       |               |                        |                          | 4.96[5]                | 14.87                     |
| $^{160}$Gd | 14  | 0.0392   | 0.0507       |               |                        |                          | 0.31[5]                | 11.68                     |
| $^{156}$Dy | 12  | 0.0733   | 0.0727       |               |                        | 17.72[11]                |                         | 18.66                     |
| $^{160}$Dy | 14  | 0.0489   | 0.0715       |               |                        | 0.71[5]                  |                         | 14.24                     |
| $^{162}$Dy | 15  | 0.0368   | 0.0456       | 0.0339        | 7.76[8]                | 8.56                      | 19.01[13]              | 12.07                     |
| $^{164}$Dy | 16  | 0.0391   | 0.0672       |               |                        |                          | 3.40[5]                | 11.70                     |
| $^{160}$Er | 12  | 0.0839   | 0.1158       |               |                        |                          | 4.76[5]                | 22.53                     |
| $^{162}$Er | 13  | 0.0538   | 0.0605       |               |                        |                          | 12.19[11]              | 16.39                     |
| $^{164}$Er | 14  | 0.0463   | 0.0531       |               |                        |                          | 19.94[13]              | 14.49                     |
| $^{166}$Er | 15  | 0.0461   | 0.0932       | 0.0520        | 18.10[7]               | 12.47                     | 5.97[13]                | 12.59                     |
| $^{168}$Er | 16  | 0.0353   | 0.0400       | 0.0321        | 0.50[5]                | 9.79                      | 0.07[7]                 | 12.50                     |
| $^{170}$Er | 17  | 0.0348   | 0.0438       |               |                        |                          | 10.11[6]                | 13.15                     |
| $^{166}$Yb | 13  | 0.0610   | 0.0743       |               |                        |                          | 8.84[6]                | 17.18                     |
| $^{168}$Yb | 14  | 0.0499   | 0.0674       |               |                        |                          | 2.20[6]                | 13.91                     |
| $^{170}$Yb | 15  | 0.0428   | 0.0577       | 0.0342        | 6.49[7]                | 11.08                     | 11.57[8]                | 13.13                     |
| $^{172}$Yb | 16  | 0.0327   | 0.0584       |               | 7.75[8]                | 12.12                     |                         |                           |
| $^{172}$Hf | 14  | 0.0503   | 0.0687       |               |                        | 9.94[10]                 |                         | 17.55                     |
| $^{174}$Hf | 15  | 0.0496   | 0.0439       |               | 5.07[5]                | 11.76                     |                         |                           |
| $^{176}$Hf | 16  | 0.0449   | 0.0632       | 0.0673        | 3.79[6]                | 12.05                     | 17.17[7]                | 16.26                     |
| $^{178}$Hf | 15  | 0.0470   | 0.0867       |               |                        | 1.90[5]                  |                         | 16.22                     |
| $^{180}$Hf | 14  | 0.0357   | 0.0434       |               |                        | 2.24[6]                  |                         | 15.15                     |
| $^{178}$W  | 15  | 0.0537   | 0.0545       |               | 6.23[8]                | 13.61                     |                         |                           |
| $^{180}$W  | 14  | 0.0591   | 0.0883       |               |                        | 8.20[7]                  |                         | 18.89                     |
| $^{182}$W  | 13  | 0.0607   | 0.1140       |               |                        | 9.70[5]                  |                         | 18.93                     |
| $^{184}$W  | 12  | 0.0476   | 0.0681       |               |                        | 1.10[5]                  |                         | 17.22                     |
| $^{232}$Th | 12  | 0.0314   | 0.0378       | 0.0424        | 1.50[8]                | 7.07                      | 4.80[13]                | 7.44                      |
| $^{232}$U  | 12  | 0.0364   | 0.0393       | 0.34[6]       | 7.15                    |                         |                         |                           |
| $^{234}$U  | 13  | 0.0295   | 0.0363       | 0.0514        | 0.45[5]                | 6.92                      | 0.29[6]                 | 7.12                      |
ii) It is known\textsuperscript{20} that for the ground state bands of the rare earths and the actinides the parameter $\tau_g$ decreases with increasing valence pair number $N$ (or, equivalently, with increasing neutron valence pair number $N_n$ in a given group of isotopes) approximately as

$$\tau = \sqrt{3}(8N^2 + 22N - 15)^{-\frac{1}{2}},$$  \hfill (7)

indicating that $\tau_g$, as a measure of deviation from the rigid rotator symmetry, indirectly reflects the nuclear shell structure. We remark that the same trend is seen for the $\tau_\gamma$ values, especially in the case of the Er isotopes (shown in Fig. 1) and the Yb isotopes. In the excited bands it is difficult to derive an analytical relation between $\tau$ and $N$, but Fig. 1 clearly shows that such a correlation actually exists. We thus conclude that in the $\gamma$-bands the SU\textsubscript{q}(2) symmetry quite well characterizes the deterioration of the nuclear rotational properties away from the midshells.

\textbf{Fig. 1.} Deformation parameters $\tau_g$ (circles, connected by solid lines) and $\tau_\gamma$ (triangles, connected by dashed lines) for ground state bands and $\gamma_1$-bands respectively of Er isotopes (taken from Table 1) are plotted versus the valence pair number $N$.

iii) we remark that the above behavior of the parameter $\tau_\gamma$ allows one to make some additional conclusions. It has been shown\textsuperscript{21} that in the gsb’s the correlation between $\tau$ and $N$ given approximately by eq. (7) allows one to connect $\tau$ with the axial deformation parameter $\beta$:

$$\beta \sim \left(\frac{B}{[3(2B + 60.25)^{1/2} - 22.5]}\right)^{1/2},$$  \hfill (8)
where $B = 1/(1 - \tau \cot \tau)$. Thus it has been obtained that $\beta$ decreases with the increase of $\tau$ and that $\tau$ could be considered as a relevant measure of decrease in deformation as well as in rotational collectivity of the nuclei in a given rotational region. Though in the excited bands we have not such analytical estimates, Fig. 1 implies that in the $\gamma$-bands the decrease of $\tau_\gamma$ towards the midshells, could be associated similarly with the corresponding increase of nuclear deformation and rotational collectivity. In this case the relevance of the quantum algebraic approach is obvious. The data on $\beta$-bands are not enough for drawing any conclusions about the $\tau_\beta$ values.

4. $B_q(E2)$ transitions in the exited bands

We now turn to the study of the $B(E2)$ transition probabilities within $\beta$- or $\gamma$-bands. In the usual case the $B(E2)$ values are given by

$$B(E2; I_i \rightarrow I_f) = \frac{5}{16\pi} Q_0^2 |C_{K,0,K}^{I_i,2,2I_f}|^2,$$  \hspace{1cm} (9)

where $Q_0$ is the intrinsic quadrupole moment and $C_{m_1,m_2,m}^{j_1,j_2,j}$ are the Clebsch-Gordan coefficients of the Lie algebra SU(2). In the case of SU$_q(2)$ one should use the $q$-generalized angular momentum theory, in which the irreducible tensor operators for the quantum algebra SU$_q(2)$ as well as the $q$-generalized version of the Wigner-Eckart theorem are available. The $q$-deformed versions of the Clebsch-Gordan coefficients needed for the $q$-generalization of eq. (9),

$$B_q(E2; I_i \rightarrow I_f) = \frac{5}{16\pi} Q_0^2 |qC_{K,0,K}^{I_i,2,2I_f}|^2,$$  \hspace{1cm} (10)

are also known. In the case of intraband transitions with $\Delta I = I_i - I_f = 2$ one needs

$$qC_{K,0,K}^{I+2,2,I} = q^{-2K} \left( \frac{[3][4][I + K + 1][I + K + 2][I - K + 2][I - K + 1]}{[2][2I + 2][2I + 3][2I + 4][2I + 5]} \right)^{\frac{1}{2}},$$  \hspace{1cm} (11)

while in cases with $\Delta I = 1$

$$qC_{K,0,K}^{I+1,2,I} = -q^{-2K+2}([I + K] - q^{2I} [I - K]) \left( \frac{[2][3][I + K + 1][I - K + 1]}{[2I][2I + 2][2I + 3][2I + 4]} \right)^{\frac{1}{2}},$$  \hspace{1cm} (12)

is needed, where the square brackets again indicate $q$-numbers as defined in eq. (3) with $q = e^{i\tau}$.

Therefore in the case of $\beta$-bands ($K = 0$) one finds

$$B_q(E2; I + 2 \rightarrow I) = \frac{5}{16\pi} Q_0^2 \frac{[3][4][I + 1]^2[2I]}{[2][2I + 2][2I + 3][2I + 4][2I + 5]}. $$  \hspace{1cm} (13)
In the case of $\gamma$-bands ($K = 2$) for $\Delta I = 2$ transitions one has

$$B_q(E2; I + 2 \rightarrow I) = \frac{5}{16\pi} Q_0^2 \frac{[3][4][I - 1][I + 3][I + 4]}{[2][2I + 2][2I + 3][2I + 4][2I + 5]} ,$$

while for $\Delta I = 1$ transitions one finds

$$B_q(E2; I + 1 \rightarrow I) = \frac{5}{16\pi} Q_0^2 \left( \frac{[2][3][I + 3][I - 1]}{[2I][2I + 2][2I + 3][2I + 4]} \right).$$

On these results the following comments apply:

i) eq. (13), concerning the $\beta$-bands, is exactly the same as the one obtained in the case of gsb$^{4,12}$. It has been shown that this equation gives $B(E2)$ values increasing with increasing $I$, while the corresponding usual SU(2) expression (obtained here for $\tau \rightarrow 0$) exhibits saturation with increasing $I$. This is illustrated in Fig. 2. In the case of gsb’s some experimental examples supporting this prediction have been given in $^{12}$. Similar predictions also occur in the framework of other models$^{13,14}$. The existing data for $\beta_1$-bands do not suffice for testing this prediction.

Fig. 2. $B_q(E2; I + 2 \rightarrow I)$ transition probabilities are plotted as a function of angular momentum $I$ in the cases of $\beta$-bands (eq. 13, solid lines) and $\gamma$-bands (eq. 14, dashed lines) for some typical values of the deformation parameter $\tau$. The numerical values of $B_q(E2)$ correspond to $\frac{5}{16\pi} Q_0^2 = 1$. The limiting case $\tau = 0$ gives the usual rigid rotator predictions.
ii) eq. (14), concerning $\Delta I = 2$ transitions in $\gamma$-bands, gives almost the same behavior as eq. (13), as seen in Fig. 2. It follows that for $\Delta I = 2$ transitions the introduction of $q$-generalized Clebsch-Gordan coefficients leads to a typical modification of the reduced transition probabilities in all considered bands.

iii) eq. (15), concerning $\Delta I = 1$ transitions in $\gamma$-bands, illustrated in Fig. 3, gives an interesting prediction. For typical $\tau$-values (0.03–0.07) one initially observes a decrease of $B_q(E2; I + 1 \rightarrow I)$ with increasing $I$, but further, after reaching some minimum (for example at $I = 5$ when $\tau = 0.05$), a significant increase of $B_q(E2)$ is observed, while in the rigid rotator limit ($\tau \rightarrow 0$) a continuous decrease down to zero at sufficiently large $I > 12$ is predicted. The available data for $E2$ intraband transitions in the excited bands do not suffice for detailed tests of these predictions, due to the short life times and strong $M1$ mixing observed in these transitions. The need for further experimental data is clear. In particular the observation of any $E2$ transitions with $\Delta I = 1$ at $I > 10 − 12$ in the $\gamma$-bands will be useful in testing the predictions of eq. (15).

We now remark that the present investigation outlines the principal limits of the SU$_q(2)$-symmetry approach to the nuclear rotational spectra. It should be emphasized that in the framework of the quantum algebra SU$_q(2)$ as well as in the case of the standard Lie algebra SU(2)$^{25}$, one is able to provide a consistent description of the physical characteristics of only one given rotational band. This is clearly indicated by the distinctions in the magnitudes of the $q$-deformation parameter obtained for the different types of bands (see Table 1). It follows that one
should understand the SU\(_{q}(2)\)-rotator as a one-band model based on the particular intrinsic state or vibrational mode. Hence the unified description of the different rotational bands including the calculation of the interband transition probabilities is beyond the limits of the quantum algebra SU\(_{q}(2)\). Such an extension could be referred to a model based on the \(q\)-deformed algebra SU\(_{q}(3)\) in which the introduction of a bandmixing interaction would be possible. However the realization of such a model is still complicated due to some difficulties in the obtaining of the reduction SU\(_{q}(3)\supseteq SO_{q}(3)\) (for example see \(^{26,27}\)). In this respect the use of the simplest quantum algebra SU\(_{q}(2)\) could be considered as a first approximation in the construction of a more complicated quantum algebraic theory of nuclear collective motion.

5. Conclusion

In conclusion, we have demonstrated the relevance of the SU\(_{q}(2)\) approach beyond the ground state bands, namely in the excited bands of even-even rare earth and actinide nuclei. Good results have been obtained for \(\beta_{1}\) and \(\gamma_{1}\) bands in nuclei with valence pair number \(N > 10\). The quantum algebraic parameter \(\tau\) fitted in these bands obtains values generally shifted above the corresponding ones in the gsb’s. In such a way the \(q\)-deformation specifically indicates the presence of a nonadiabatic perturbation caused by the excited vibrational degrees of freedom. The decrease of \(\tau_{\gamma}\) and \(\tau_{g}\) with increasing \(N\) is in accordance with the interpretation of \(\tau\) as a measure of deviation from the rigid rotator limit equivalent to the nuclear softness\(^{5,20}\). In addition, these correlations (illustrated in Fig. 1) allow one to extend the SU\(_{q}(2)\) symmetry to a wider range of nuclear rotational properties\(^{21}\). The predictions of the SU\(_{q}(2)\) rotator model for the \(B(E2)\) intraband transition probabilities in the excited bands show modifications in comparison to the SU(2) rigid rotator limit, the experimental data needed for testing these predictions having been identified. It is pointed out that SU\(_{q}(2)\) is a simple one-band model, but it can be considered as the first step in the development of more complicated models based on the \(q\)-deformed algebras.

6. References

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