Reliability of Certain Class of Transportation Networks

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Abstract. The reliability of a special class of technical objects - tree-like transportation networks is considered. It is argued that the indicators traditionally used to assess reliability in relation to such objects are not very informative and are ambiguously interpreted from a physical point of view. As a quantitative measure, an indicator of operational reliability is proposed and an engineering method for its calculation has been developed. It uses the features of the object, is based on an analogue of the Y-shaped structure-forming fragment of the network and is reduced to a repeating step-by-step computational procedure, when at each step the results of the calculation are obtained at the previous stage and used as initial data.

Relationships are derived that allow extending this approach to the case when at the junction point of network elements not two, but, in general, n elements are combined. The advantages of the introduced indicator of the operational reliability of tree-like transportation networks are discussed and the ways of its further generalization and use are outlined.

1. Introduction

In many branches of industrial production, transport, housing and communal services, etc. Transportation networks are widely used. In general, a transportation network is understood as functionally interconnected motor roads designed for the purposeful movement of a certain "product" in space. Regardless of the physical content of the concept of "product", transportation networks can be classified according to many criteria, one of which is their configuration. Here, consideration is limited to tree-like networks, a characteristic feature of which is the "collective" function, i.e., the movement of the "product" from several inputs, where it enters, to the only output - the place of delivery [1]. It is customary to represent such networks in the form of a simply connected acyclic directed graph; from the point of view of graph theory - "tree".

As with any technical facility, one of the most important qualities of the transportation network is its reliability. During operation, network elements can fail, be repaired and re-commissioned. The consequence of this is that for some time T the part of the product arriving at the inputs of the network does not arrive at its output. This circumstance should be taken into account when forming the discipline of object maintenance: determining the frequency of preventive and overhaul repairs, the range and volumes of required spare parts, the timing of replacement or modernization of equipment, staffing in the person of maintenance personnel, and other similar measures. Thus, the study of the reliability of transportation networks is not only of theoretical but also of direct practical interest.
The effectiveness of measures to improve the reliability of the transportation network is largely determined by what is meant by this term, i.e. which is considered an indicator of the reliability of objects of this class. In the "classical" theory, the reliability of a repaired (restored) object is usually assessed by a number of formal indicators (probability of no-failure operation, mean time between failures, availability factor, etc. [2, 3], which, as a rule, are easily interpreted physically and do not contradict). When it comes to the reliability of a tree-like transportation network, such indicators, while remaining "correct" from the point of view of the classical theory, turn out to be of little informative and, as a rule, require additional explanations and clarifications. Constructive and functional reliability [4], and for the development of methods for its engineering calculation.

2. Calculation method

The indicator of operational reliability is proposed as such an indicator \( \gamma \). It was originally introduced in [5] to assess the reliability of the drainage system of a large city, which is a typical example of a tree-like transportation network, and methods for its generalization and engineering calculations were developed in a number of subsequent publications [6-9].

The indicator of operational reliability \( \gamma \) is understood as the ratio of the "product volume" \( \Delta Q \) not delivered to the network output due to failures of its elements to the "product volume" \( Q \) received at the network inputs for some time, that is:

\[
\gamma = \Delta Q / Q.
\]  

(1)

The calculation of \( \Delta Q \) is carried out using a specially developed method of decomposition and equivalence (MDE) [10]. MDE uses a characteristic feature of transportation networks of the class under consideration - their tree structure. The network is presented as a kind of composition of Y-shaped fragments that form its structure. Each such structure-forming fragment is virtually replaced by one fictitious element, the reliability parameters of which are determined from additional considerations. This equivalence procedure is illustrated in figure 1.

![Figure 1. Y-shaped fragment of the network a) and its fictitious equivalent b).](image-url)

In figure 1 the following designations are accepted: 1, 2, 3 - numbers of network elements; \( \lambda_i \) and \( \mu_i \) - the intensity of the flows of failures and restorations of the i-th element, respectively; \( \rho_i = \lambda_i / \mu_i \) - dimensionless parameter characterizing the process of "recovery after failure" of the i-th network element; \( q_1 \) and \( q_2 \) - the cost of production at the inputs of the Y-shaped network fragment.
Applying the MDE to the network shown in Fig. 1, gives for the dummy element 123 [10]:

$$\rho_{123} = \frac{(\rho_1 + \rho_3)q_1 + (\rho_2 + \rho_3)q_2}{q_1 + q_2}.$$  (2)

The calculation of the operational reliability $\gamma$ of the transportation network as a whole is reduced to a recurrent sequential procedure, at each step of which all structural elements of the network are extracted (decomposed) with the replacement (equivalence) of each of them with one virtual element. As a result, the entire network turns out to be "folded" into one element, the parameter $\rho$ of which, as shown in [6], is numerically equal to the value $\gamma$ calculated by formula (1). In this case, the indicator $\gamma$ turns out to be normalized (0 ≤ $\gamma$ ≤ 1) and takes into account both the reliability parameters of the network elements and technological variables - the cost of the product at all its inputs. Calculated examples of using this approach can be found in [1, 6].

3. Generalization of methodology

When using MDE, a Y-shaped, i.e. three-component, fragment of the network is considered as a structure-forming element. Meanwhile, there are cases when not two elements are connected at the point of union, as shown in figure 1, but more. The calculation of the dimensionless parameter $\rho$ of the fictitious element replacing such a fragment of the network can no longer be performed according to (2); you must have an expression that takes into account such cases.

First, consider a fragment of the network shown in figure 2a), when three elements (1, 2 and 3) are connected at point A with parameters $\rho_1$, $\rho_2$ and $\rho_3$ and the costs of the product through them $q_1$, $q_2$ and $q_3$, respectively.

![Figure 2. Four-element network fragment a) and its sequential virtual transformations b), c), d).](image)

Suppose that element 3 is connected to the network not at point A, but at point B and is separated from it by a virtual element characterized by a parameter $\rho_{\phi}$ (Figure 2b)). In accordance with the MDE, we now select the Y-shaped fragment I, circled in figure 2b) with a dashed line, and find the value of the parameter of the element $\rho_I$ that replaces it. Using (2) we have:

$$\rho_I = \frac{(\rho_1 + \rho_\phi)q_1 + (\rho_2 + \rho_\phi)q_2}{q_1 + q_2}.$$  (3)

which makes it possible to go to the subsystem shown in figure 2 c), which, in turn, is a Y-shaped fragment of the network. Repeating the equivalent procedure with respect to it again, for $(\rho_\phi)_I$ we get:
Now, to go to the original subsystem, it is enough to enter (4) \( \rho_0 = 0 \), which corresponds to the zero length of the virtual element connecting points A and B (see figure 2b)). After that, for the dimensionless parameter \( \rho_{\text{exc}} \), we have an equivalent element replacing the network fragment shown in Figure 2a) (Figure 2d)):

\[
\rho_{\text{exc}} = \frac{(\rho_1 + \rho_2)q_1 + (\rho_2 + \rho_3)q_2 + (\rho_3 + \rho_4)q_3 + (\rho_1 + \rho_2)q_4}{q_1 + q_2 + q_3}.
\]

(5)

Applying this procedure of sequential equivalence, after elementary transformations of the resulting expressions, for the general case when \( n \) network elements are connected at one point, we obtain:

\[
\rho_{\text{exc}} = \rho_{n+1} + \sum_{i=1}^{n} \rho_i q_i.
\]

(6)

Expression (6) makes it possible to use the decomposition and equivalence method for almost any tree-like transportation network.

4. Conclusion

Further development of MDE is associated with its application for non-stationary cases, including: for “aging” elements [11, 12] and for elements with seasonally changing failure rates [13-15].

5. References

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