Optimized network structure
and routing metric
in wireless multihop ad hoc communication

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Abstract

Inspired by the Statistical Physics of complex networks, wireless multihop ad hoc communication networks are considered in abstracted form. Since such engineered networks are able to modify their structure via topology control, we search for optimized network structures, which maximize the end-to-end throughput performance. A modified version of betweenness centrality is introduced and shown to be very relevant for the respective modeling. The calculated optimized network structures lead to a significant increase of the end-to-end throughput. The discussion of the resulting structural properties reveals that it will be almost impossible to construct these optimized topologies in a technologically efficient distributive manner. However, the modified betweenness centrality also allows to propose a new routing metric for the end-to-end communication traffic. This approach leads to an even larger increase of throughput capacity and is easily implementable in a technologically relevant manner.

Key words: structure of and dynamics on complex networks, information and communication networks, wireless multihop ad hoc communication, packet traffic
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1 Introduction

Nowadays, the complexity of many engineered networks has increased to such a high level, that conceptually new approaches for their operational control have to be looked for. Key aspects like selforganization and artificial intelligence become increasingly important. It is here where engineering and computer science is beginning to exchange ideas and concepts with the natural sciences like physics and biology. In particular, the new cross-disciplinary branch known as the Statistical Physics of complex networks [1,2,3] appears to catalyze such efforts.

We pick up on this latest momentum to continue the discussion of a challenging complex communication network in abstracted form [4,5,6]. In wireless multi-hop ad hoc networks [7,8,9] nodes are connected by wireless links. A central control infrastructure is missing. Each node does not only act as a communication source and sink, but also forwards communication for others. All of this requires a lot of selforganized and decentralized coordination amongst the nodes. The outstanding complexity of this communication network is revealed by mentioning the key mechanisms and the associated problems in some more detail:

With regulation of its transmission power, each node is able to modify its transmission range and its neighborhood, to which it builds up wireless communication links. Here the node faces frustration for the first time. On the one hand it wants to save energy and to keep its transmission power as low as possible, but on the other it might have to choose a larger neighborhood in order to help the network to gain strong connectivity, so that each node will be able to communicate to any other via multihop routes. This brings us to another protocol layer, from link control to routing control. End-to-end routes have to be explored and maintained. During their execution the communication hops from one node to the next. This is where yet another protocol layer, called medium access control, sets in. It blocks all neighbors attached to an ongoing one-hop transmission in order to avoid interference within the same communication channel. This is the origin of another frustration, now across layers. Whereas routing efficiency prefers short end-to-end routes with the consequence of large one-hop neighborhoods, medium access control prefers to block small neighborhoods with the consequence of long end-to-end routes. A delicate balance between these two layers is necessary for the overall network to gain a large end-to-end throughput capacity, which measures the amount of communication traffic the network is able to handle without overloading.

In a previous paper [4] we have already addressed the connectivity issue, with special emphasis on the development of a simple selforganizing topology control. More selforganization has been proposed in Ref. [6], where a reactive
routing & congestion control has been discussed, which adapts to the current congestion state of the wireless multihop ad hoc network. Another investigation [5] has demonstrated that the end-to-end throughput capacity does sensitively depend on the underlying network structure. It is exactly here where we continue and begin to ask for throughput optimization: What is the optimized network structure? What are its properties? Is it possible to construct the optimized network structure with a selforganizing topology control? Are there also other means to enhance the end-to-end throughput and how do these compare to the approach with optimized network structures?

These are a lot of questions. We group them into the following organizational form of the Paper. Section 2 addresses the network structure optimization of wireless multihop ad hoc communication networks. It explains the level of abstraction needed to construct an objective function for the global maximization of end-to-end throughput. A modified version of betweenness centrality is introduced, the cumulative betweenness centrality, which suites well the particular throughput needs of wireless multihop communication. The optimality of the resulting networks is checked with generic packet traffic simulations. Scaling laws for the end-to-end throughput with respect to network size are given. The structure of the globally optimized networks is analyzed and found to be difficult to construct with a decentralized, selforganizing topology control. As a consequence, a new approach is advocated in Section 3 to increase the end-to-end throughput. It uses the cumulative betweenness centrality as key input into a routing metric. This allows to determine throughput-optimizing end-to-end routes iteratively in a selforganizing manner. The increase in throughput turns out to become even slightly larger than for the optimized network structure of the previous Section. Conclusion and outlook are given in Section 4.

2 Optimization of network structure

2.1 Abstraction: geometric minimum-node-degree networks

Some level of abstraction is required to make wireless multihop ad hoc communication amenable to the Statistical Physics of complex networks. The first simplification is to neglect mobility and to distribute $N$ nodes onto a unit square in a random homogeneous way. The transmission power $P_i$ then decides which other nodes $j$ are able to be reached by node $i$ via directed links $i \rightarrow j$. These are the nodes which, according to a simple propagation-receiver model, fulfill the inequality $P_i/R_{ij}^{\alpha} \geq \text{snr}$. $R_{ij}$ denotes the relative Euclidean distance. The path-loss exponent $\alpha$ is assumed to be constant and \text{snr} represents the signal-to-noise ratio. With these simplifications, wireless multihop ad hoc communication networks can be modeled as geometric graphs. The $N$
nodes are networked together via the set \( \{ i \rightarrow j \} \) of all directed links.

Not all of the directed links will be used for wireless multihop ad hoc communication. Since operation requires instant one-hop feedback, only bidirected links \( i \leftrightarrow j \) qualify for the routing of communication traffic. It is the bidirected links attached to node \( i \) which define its communication neighborhood \( \mathcal{N}_i \) and its node degree \( k_i \).

One further step is needed to fully specify wireless multihop ad hoc network graphs: assignment of the transmission power \( P_i \) for all nodes. The simplest procedure is to assign the same value \( P \) to all nodes \([10,11,12,13]\). We prefer to employ a different procedure \([4]\), which contrary to the first one is distributive, self-organizing and adaptive. In a nutshell, each node \( i \) forces its \( k_{\text{min}} \) closest nodes \( j \) to adjust their transmission powers to at least \( P_j = R_{ij}^\alpha \), while adopting the value \( P_i = \sup_j P_j \) for itself. Its own value can be increased further whenever another, still close-by node, which seeks for its minimum communication neighborhood, forces \( i \) in return to have an even larger transmission power. In this respect each node has at least \( k_{\text{min}} \) bidirected neighbors. As a consequence transmission power values differ from node to node. This heterogeneity leads to the occasional emergence of directed links.

We will call wireless multihop ad hoc network graphs generated with this heterogeneous power assignment as geometric minimum-node-degree networks. Already the choice \( k_{\text{min}} = 8 \) is sufficient to guarantee strong network connectivity almost surely for network sizes up to several thousand nodes \([4]\). A realization of a minimum-node-degree network with \( k_{\text{min}} = 8 \) is shown in Fig. 1. For comparison, network realizations generated with the same spatial node pattern, but with \( k_{\text{min}} = 12 \) and 20 are also shown.

### 2.2 Objective function for end-to-end throughput

Maybe the most important performance measure of wireless multihop ad hoc networks is given by the end-to-end throughput capacity \( T_{\text{e2e}} \). It represents the amount of end-to-end communication traffic the network is able to handle without overloading. Ref. \([5]\) has shown that \( T_{\text{e2e}} \) does depend on the underlying network structure. We are now interested to find network structures with increased throughput capacity. The necessary first step for this search is the construction of a suitable objective function in analytical form that can be calculated with reasonable computational effort.

For simplicity, the end-to-end packet traffic is assumed to use only one communication channel and to be completely random, that is on average all possible \( N(N-1) \) end-to-end routes are equally loaded. With rate \( \mu \) each of the \( N \) nodes creates a new packet, for which it selects a random final destination,
Fig. 1. Geometric wireless multihop ad hoc network graphs of minimum-node-degree type: (a) $k_{\text{min}} = 8$, (b) 12 and (c) 20. The same random homogeneous spatial node pattern with $N = 300$ nodes has been used for all three cases. Bidirected links are shown in solid, whereas dashed links exist only in one direction.
and then sends the packet along the respective end-to-end route. As a consequence, the in-packet flux rate \( \mu_i^{\text{in}} \) of end-to-end communications which hop via node \( i \) can be described as

\[
\mu_i^{\text{in}} = \mu N \frac{B_i}{N(N-1)} .
\]  

(1)

Out of all \( N(N - 1) \) end-to-end combinations the betweenness centrality

\[
B_i = \sum_{m \neq n=1}^{N} \frac{b_{mn}(i)}{b_{mn}}
\]

(2)

counts the number of end-to-end routes going through \( i \). To be more precise, \( B_i \) counts the number of end-to-end routes, for which \( i \) has to forward a packet. If \( i \) is the final recipient of such an end-to-end route, then \( i \) is excluded from the count. Whereas if \( i \) is the initial sender, then it is included into the count. \( b_{mn} \) represents the number of used routes from \( m \) to \( n \), of which \( b_{mn}(i) \) pass through \( i \).

Betweenness centrality has already been introduced for networks in general [14] and communication as well as power networks in particular [15,16]. Throughout this section we will base it on shortest multihop routes. Note however, that betweenness centrality is not restricted to the concept of shortest paths [17]. Other generalizations, i.e. from a node-based to a link-based betweenness centrality, have also been discussed [5,16].

Compared to the in-packet flux rate \( \mu_i^{\text{in}} \), the modeling of the out-packet flux rate

\[
\mu_i^{\text{out}} = \frac{1}{\tau_i^{\text{send}}}.
\]

(3)

is more delicate. Given that node \( i \) has buffered at least one packet to forward, \( \tau_i^{\text{send}} \) represents the characteristic time it takes on average to send the next packet. Due to the competition between neighbors to gain medium access for one-hop transmissions, node \( i \) might not be allowed to send a packet right away. If neighbors also have packets to forward or are prospected one-hop receivers, and if one of them succeeds, then this node automatically blocks all its neighbors, including \( i \), by the interference-avoiding medium access control.

Before we propose an empirical modeling of the sending time \( \tau_i^{\text{send}} \), we write down the general expression for the end-to-end throughput. Equating relations (1) and (3) yields an expression for the critical packet creation rate of node \( i \),

\[
\mu_i^{\text{crit}} = \frac{N - 1}{B_i \tau_i^{\text{send}}}.
\]

(4)

For \( \mu < \mu_i^{\text{crit}} \) node \( i \) is able to forward its incoming packet traffic in time and remains subcritical. However, above \( \mu_i^{\text{crit}} \) not all packets can be forwarded,
leading to an increase of its packet queue with time. This defines its super-critical regime. Out of all $N$ nodes, it is the node with the smallest critical packet creation rate, which determines the overall critical network load

$$
\mu^{\text{crit}} = \min_i \mu_i^{\text{crit}}.
$$

The packets accumulate at this bottleneck node and from there congestion spreads over the entire network. Eq. (5) is related to the end-to-end throughput via

$$
T_{e2e} = \mu^{\text{crit}} N = \min_i \left( \frac{N(N-1)}{B_i \tau_{i_{\text{send}}}} \right).
$$

This capacity-like quantity represents the maximum rate of end-to-end communications which can be completed successfully without network overloading.

In Ref. [5], the blockings induced by the medium access control of one- and two-hop neighbors have been directly modeled into the sending time $\tau_{i_{\text{send}}}$. Upon employment of a simple queuing behavior for the nodes’ buffers, a set of coupled linear equations for the sending times of all nodes has been derived. Although this approach has led to a good qualitative understanding of the end-to-end throughput for various multihop ad hoc network structures, its analytical form is too complicated and numerically too expensive to be suited as an objective function. Another, much simpler description has to be found.

Guidance for such a description will be given by two particular network examples. In a fully connected network each node has direct communication links to all other nodes. For all nodes the degree is $k_i = N - 1$. Initial sender and final recipient of each end-to-end communication are only one hop away from each other. Then, due to medium access control, each single one-hop end-to-end communication blocks the overall network, leading to a maximum throughput $T_{e2e} = 1$ of one completed end-to-end communication per time step. Any detailed proposition of (6) has to fulfill this $T_{e2e} = 1$ for fully connected networks.

The second particular network example comes with a central hub, which has direct communication links to all other nodes. All end-to-end communications first go from the initial sender to the central hub, and from there to the final recipient. For simplicity, we discard the central hub as initial sender as well as final recipient. In each of the two one-hop transmissions involved in an end-to-end communication again the overall network is blocked by medium access control. First the receiving central hub blocks all its neighbors, and then all its neighbors are again blocked during the subsequent forwarding to the final recipient. This limits the maximum throughput to $T_{e2e} = 0.5$, i.e. one completed end-to-end communication per two time steps. Again, any further proposition based on (6) also has to fulfill this $T_{e2e} = 0.5$ for a central-hub network.
If all neighbors of node $i$ are currently not involved in prospected one-hop transmissions, then $i$ is able to send its packet right away. In this case the sending time of (3) would be $\tau_{i}^{\text{send}} = 1$ in units of a typical one-hop packet-transmission time interval. For the other extreme, when all neighbors either also attempt to forward a packet or are expecting packet receipt from some other node, the average sending time takes on its upper limit

$$\tau_{i}^{\text{send}} = 1 + k_{i}^{\text{in}}, \quad (7)$$

where $k_{i}^{\text{in}}$ represents the ingoing node degree. It is tempting to insert this estimate into (6), leading to

$$T_{e2e} = \min_{i} \left( \frac{N(N-1)}{B_{i}(1 + k_{i}^{\text{in}})} \right) . \quad (8)$$

In fact, this expression is consistent with $T_{e2e} = 1$ of a fully connected network, where $k_{i}^{\text{in}} = B_{i} = N - 1$. However, it is not consistent with $T_{e2e} = 0.5$ of the central-hub network. With $k_{c.h.} = N$ and $B_{c.h.} = N(N - 1)$ it is the central hub itself which produces the maximum of $B_{i}(1 + k_{i}^{\text{in}})$, leading to the wrong estimate $T_{e2e} = 1/(N + 1)$. Consequently, the ansatz (7) for the sending time is ruled out.

We propose another empirical ansatz for the sending time. Node $i$ will be blocked, if one node in its vicinity is involved in a packet transmission. We have to consider all nodes that have a directed link towards node $i$, because these block $i$ via medium access control during their active participation in a one-hop transmission. This set of $i$’s ingoing neighbors is abbreviated as $\mathcal{N}_{i}^{\text{in}}$. As a proposition, we set the activity of an ingoing neighbor $j$ equal to the relative betweenness centrality $B_{j}/B_{i}$. Then, the sending time is estimated as

$$\tau_{i}^{\text{send}} = \frac{1}{B_{i}} \left( B_{i} + \sum_{j \in \mathcal{N}_{i}^{\text{in}}} B_{j} \right) . \quad (9)$$

It introduces the cumulative betweenness centrality

$$B_{i}^{\text{cum}} = B_{i} + \sum_{j \in \mathcal{N}_{i}^{\text{in}}} B_{j} . \quad (10)$$

Insertion of (9) into (6) produces

$$T_{e2e} = \min_{i} \left( \frac{N(N-1)}{B_{i}^{\text{cum}}} \right) \quad (11)$$

for the end-to-end throughput.

This expression is consistent with $T_{e2e} = 1$ for fully connected networks. In this case $B_{i} = N - 1$ and, consequently, $B_{i}^{\text{cum}} = N(N - 1)$. Moreover, expression
Fig. 2. Network-size dependent end-to-end throughput for minimum-node-degree networks with $k_{\text{min}} = 8$ (squares), 12 (circles), 20 (up-triangles), 40 (down-triangles). Curves with full symbols represent the estimate (11). Respective curves with open symbols result from a generic packet traffic simulation. An average over 100 independent network realizations has been performed for each symbol $N$.

(11) also agrees with the $T_{e2e} = 0.5$ of a central-hub network. There, it is again the central hub which yields the largest $B_{i}^{\text{cum}}$. Its betweenness centrality amounts to $B_{c.h.} = N(N-1)$. Each of its $N$ neighbors counts $B_{\text{ngb}(c.h.)} = N-1$. All this totals to $B_{c.h.}^{\text{cum}} = B_{c.h.} + NB_{\text{ngb}(c.h.)} = 2N(N-1)$.

Beyond these two limiting network examples, the quality of (11) is decided with the minimum-node-degree network structures. Generic packet traffic simulations with shortest-path routing, as described in Ref. [5], have been used to determine their end-to-end throughput. For various $k_{\text{min}}$ and in dependence of the network size $N$, these results are shown in Fig. 2. The same network realizations, as used for the simulations, have then been taken to determine (11). The betweenness centrality based on shortest multihop paths has been calculated with an algorithm similar to that described in Ref. [14]. The overall agreement between the estimate (11) and the throughput curves obtained from the generic packet traffic simulations is remarkable for all the various $k_{\text{min}}$ values. This proves the high quality of expression (11), which will now serve as objective function for the network-structure optimization of the end-to-end throughput.

Note, that for large network sizes the results of Fig. 2 suggest a scaling law
\[ T_{e2e} \sim (N - N_0)^\gamma \]

| \( k_{\text{min}} \) | \( N_0 \) | \( \gamma \) | \( N_0 \) | \( \gamma \) |
|-------------------|--------|--------|--------|--------|
| 8                 | 24     | 0.23   | 65     | 0.22   |
| 12                | 69     | 0.25   | 127    | 0.22   |
| 20                | 179    | 0.24   | 219    | 0.24   |
| 40                | (400)  | (0.22) | (389)  | (0.29) |
| \text{opt}(k_{\text{min}} = 8) | 36 | 0.41 | 37 | 0.43 |
| \text{opt}(k_{\text{min}} = 12) | 84 | 0.40 | (52) | (0.49) |
| \text{opt}(k_{\text{min}} = 20) | (94) | (0.47) | (65) | (0.54) |
| \( k_{\text{min}} = 8, B_{i\text{cum}}^\text{-metric} \) | 0 | 0.42 | 23 | 0.41 |
| \( k_{\text{min}} = 12, B_{i\text{cum}}^\text{-metric} \) | 22 | 0.43 | 63 | 0.41 |
| \( k_{\text{min}} = 20, B_{i\text{cum}}^\text{-metric} \) | 93 | 0.40 | 127 | 0.41 |

Table 1
Scaling exponent \( \gamma \) and parameter \( N_0 \) of the end-to-end throughput \( T_{e2e} \sim (N - N_0)^\gamma \). Rows 1–4 are for minimum-node-degree networks and rows 5–7 are for the respective structure-optimized networks, all with shortest-multihop-path routing. Rows 8–10 are again for minimum-node-degree networks, but now with use of the \( B_{i\text{cum}}^\text{-routing metric} \). Column blocks 2 and 3 represent the estimate (11) and the generic packet traffic simulations, respectively. In some cases, extracted parameters depend to some extend on the interval size used for the fit; brackets indicate these less reliable values.

of the form \( T_{e2e} \sim (N - N_0)^\gamma \). Fitted values for \( N_0 \) and \( \gamma \) are given in Tab. 1. Except for \( k_{\text{min}} > 20 \), the scaling exponent \( \gamma = \gamma(k_{\text{min}}) \) is found to depend only weakly on the minimum-node degree. For all cases it falls well inside the range \( 0 < \gamma < 0.5 \). The upper estimate \( \gamma < 0.5 \) has been first given by [11]. Despite a homogeneous end-to-end traffic pattern this overestimation neglects the heterogeneities in the one-hop traffic as a consequence of the spatial network geometry. In general, nodes in the spatial center of the network have to carry a higher load than nodes in the periphery. Taken alone, \( \gamma > 0 \) proves that for sufficiently large enough network sizes multihop networks produce a larger throughput than central-hub networks. However, this statement is much too modest, since for all curves of Fig. 2 and for all network sizes, the absolute value of the end-to-end throughput is always larger than \( T_{e2e} = 0.5 \). For network sizes slightly above \( N_0 \), the end-to-end throughput of the various minimum-node-degree networks becomes larger than \( T_{e2e} = 1 \) of a fully connected network.
Based on (11), the search for optimized network structures is challenging. First of all, the expression for the end-to-end throughput depends on the network structure in a non-linear and non-local manner. Local addition or removal of links might change the end-to-end routes and thus the traffic distribution on a global scale. Moreover, the search space of all testable network configurations is very large. It is of the order \((N - 1)^N\). Each of the N nodes has its own transmission power ladder with \(N - 1\) rungs. Being on rung \(k\) means that the picked node is able to reach its \(k\) closest neighbors. Of course not all of these configurations are meaningful for wireless multihop ad hoc communication. Hence, it is important on the one hand to confine the search operations only to the meaningful ones and on the other hand to start with good initial network configurations.

As initial configurations the geometric minimum-node-degree networks are chosen. For the moment we stick to \(k\text{_{min}} = 8\). This sets a minimum node degree \(k_i\text{_{min}} \geq k\text{_{min}}\) for each node \(i\). During subsequent optimization operations, the respective transmission power values of all nodes are not decreased below their initial value, thus ensuring strong connectivity for all times [4]. Search operations are performed in rounds. Per round, each node is randomly picked once. A picked node explores in two directions. In the first move it increases its transmission power by one rung and, if the newly reached node does not already have a large enough transmission power, forces the latter to climb up its ladder until its rung suffices to successfully build a new mutual bidirectional communication link. In the other move the picked node steps down its transmission power ladder by one rung, implying that the lost neighbor might also move down its ladder until it reaches the rung just before another communication link is broken. Both moves modify the local network structure, require a global update of the shortest end-to-end routes and the betweenness centralities for all nodes, and lead to two modified estimates of the end-to-end throughput (11), which are then compared to the old estimate before the two explorative moves. The network structure yielding the largest estimate is accepted. This gradient update procedure guarantees meaningful wireless multihop ad hoc network structures and keeps the occurrence of interfering one-directed links to a minimum.

A local maximum of (11) is reached, if during a complete search round no improvement of the throughput estimate is found. Fig. 3 shows a typical evolution of the end-to-end throughput in dependence of the number of search rounds until the first local maximum is reached. It only takes a modest number of rounds. The increase of the throughput performance is remarkable. Once a local maximum is reached, the respective network realization is perturbed by forcing a small, randomly chosen fraction of the nodes to step up or down by
Fig. 3. Evolution of end-to-end throughput with progressing meta rounds. Each meta-round is kicked off with a random perturbation of \( n_p = 1 \) (squares), 2 (circles), 4 (triangles) randomly selected nodes from a local-maximum network configuration. Within the first meta round, the evolution in terms of optimization rounds is shown until the first local maximum of expression (11) is reached. A minimum-node degree network based on \( k_{\text{min}} = 8 \) and \( N = 300 \) has been chosen as initial network realization. The upper family of curves (with filled symbols) is for (11), whereas the lower family of curves (with open symbols) represents its counterpart from packet traffic simulations.

one rung on their transmission power ladder, including respective new or lost neighbor operations as explained before. We denote the period until the next local maximum is found as meta-round. Fig. 3 also illustrates the evolution of the end-to-end throughput in terms of meta-rounds. The striking feature is that if more than one node is perturbed out of its local-maximum state, the throughput performance decreases with the number of meta-rounds. If only one node is perturbed, the throughput performance remains more or less the same as found for the first local-maximum network realization.

If we use other minimum-node-degree networks as initial network configurations instead of \( k_{\text{min}} = 8 \), like \( k_{\text{min}} = 12 \) and 20, we arrive at the same qualitative findings. Independent of the network size, the first local throughput maximum is reached after only a few update rounds. Subsequent meta rounds do not lead to a significant performance increase; on the contrary, those initiated by larger perturbations again result in a decrease of end-to-end throughput.
It is important to check all of these results with packet traffic simulations. As also demonstrated in Fig. 3, a strong correlation between the simulation results and the throughput estimate \((11)\) is found. This is a non-trivial and important statement. It has been clear from the beginning that the empirical expression \((11)\) is not fully exact. A small discrepancy to the unknown true expression remains. A subsequent optimization with respect to \((11)\) further broadens the gap. The important statement is that the end-to-end throughput of the packet traffic simulation also increases significantly and in correlation to \((11)\).

Taken together, the results obtained from \((11)\) and from packet traffic simulations show that the first found local maximum yields the largest throughput. All further maxima show a lower performance. This allows to terminate the optimization after the first meta-round. We are well aware that such an early termination most likely will not provide the global maximum, maybe even not a close-by network realization. However in view of an engineer’s pragmatism, our maximization policy produces a well defined and fast search into a strong local maximum for this hard and very costly optimization problem.

2.4 Scalability of optimized end-to-end throughput

For various network sizes ranging from \(N = 100\) to 2000 and in dependence of the initial minimum-node degree \(k_{\min}\), ensembles consisting of 5 to 25 throughput-optimized network realizations have been generated. Besides the optimized estimate \((11)\) the end-to-end throughput has also been calculated from packet traffic simulations. It is the numerical cost of the network structure optimization, which forbids larger ensemble sizes. Results are shown in Fig. 4.

The optimized network topologies have an end-to-end throughput significantly larger than their initial counterparts. The end-to-end throughput of the optimized topologies again reveals the scaling behavior \(T_{e2e} \sim (N - N_0)^\gamma\) in the limit of large network sizes. Fitted parameter values \(N_0, \gamma\) are given in Tab. 1. The values found for the scaling exponent are very close to the upper bound \(\gamma = 0.5\) given in Ref. [11]. The increase of \(\gamma\) to almost 0.5 demonstrates that within the optimized network topologies the heterogeneities of the one-hop traffic have been considerably reduced. With other words, the network structure has been modified in such a way that the new shortest-path end-to-end routes distribute the overall network traffic more evenly and reduce the peak traffic loads of the bottleneck nodes.
2.5 Structural properties of optimized networks

The optimized network structures resulting from the initial $k_{\text{min}} = 8$ minimum-node-degree networks produce the largest end-to-end throughput; consult again Fig. 4. Optimized counterparts resulting from initial $k_{\text{min}} = 12$ have almost the same end-to-end throughput. However, for the larger initial $k_{\text{min}} = 20$ the respective optimized networks already come with a noticeably smaller end-to-end throughput. These findings are in accordance with the intuitive philosophy expressed in Ref. [11]: the largest throughput is obtained once the network is just barely strongly connected and blockings due to medium access control are smallest. The minimum-node degree $k_{\text{min}} = 8$ just barely guarantees strong network connectivity and due to the small neighborhoods the blockings from medium access control are also small.

Fig. 5 (top right) shows a typical realization of an optimized network. It still looks similar to the initial $k_{\text{min}} = 8$ network, which is illustrated in Fig. 5 (top
Fig. 5. Analysis of network structure optimization: (upper left, see also top of Fig. 1) typical initial $k_{\text{min}} = 8$ network configuration and (upper right) respective optimized network configuration. Only bidirectional communication links are shown; for reasons of readability, one-directed links have been suppressed. The gray scale of the nodes encodes $B_i^{\text{cum}}$, with zero (white) to maximum (black). From left to right, the subfigures in the lower row illustrate the most important 5, 15, 50 and 391 (all) new links. Here, light colored nodes come with strongly reduced $B_i^{\text{cum}}$ values, whereas dark nodes have experienced a strong increase.

left). For this example, only 391 new communication links have been added within the $N = 300$ nodes during the optimization procedure. Fig. 5(bottom right) shows the bidirected links added during the optimization. Almost none of them attaches to the spatially centered nodes, which are the bottleneck nodes with the largest overall cumulative betweenness centralities. Instead, nearly all of the new links are located in the greater surrounding of the most loaded nodes, including the outer parts of the network. The introduction of these new communication links modifies the shortest-path end-to-end routes in such a careful way that on the one hand the largest $B_i^{\text{cum}}$ values of the most-loaded nodes are decreased and on the other the end-to-end throughput performance is increased significantly.

Not all of the 391 new links are important. A ranking of the new links reveals that the five most important new links are in charge of already 49% of the throughput increase, found with the generic packet traffic simulations. The ranking has been performed in the following way: at first, each of the 391
new links has been added singly to the minimum-node-degree network and
the respective throughput (11) has been determined. The single link addition
which yields the largest throughput defines the most important link. This
link is then included into the network. Next, this procedure is repeated for
the remaining 390 new links, leading to the second most important new link,
and so on. For the 15 and 50 most important links 73 % and 104 % of the final
throughput increase are reached, respectively. For the remaining 341 new links
the throughput increase fluctuates closely around 100%. This demonstrates
that a fraction of only about 50 out of the 391 new links is necessary to reach
the optimal performance.

Similar findings hold for all other examined network realizations and for all
network sizes. From initial $k_{\text{min}} = 8$ minimum-node-degree network to opti-
mized network the average node degree has increased from more-or-less $N$-
independent $\langle k \rangle = 9.9$ to 12.8 for $N = 100$ nodes and 13.8 for $N = 2000$.

From left to right, the lower row of Fig. 5 shows the 5, 15, 50 and 391 most
important out of the 391 new links for this typical optimized network struc-
ture. The change of the nodes’ $B_{\text{cum}}^i$ values is shown with a gray scale, where
black/light gray means increase/decrease. A close investigation of the five most
important links reveals that two of them are formed between one- and two-hop
neighbors of the bottleneck nodes. The other three are further away. All five
form shortcuts in the periphery of the network. By reducing the hop distance
between certain nodes they create new shortest path routes. This leads to a
redistribution of traffic away from the highly loaded core of the network to
the low-loaded periphery. For the 15, respectively 50 most important links this
behavior even becomes more visible. All of them are also only added in the
periphery and not in the center of the network.

2.6 Attempts for greedy optimization

So far, the throughput optimization of network structure has been from a
global perspective. To become technologically more relevant, a distributive
optimization analogue has to be found. From the previous Subsection we have
learned that the additional, throughput-enhancing new links are not directly
attached to the spatially centered, most-loaded nodes, but further away within
the periphery of the network. The distributive control of this strong non-local
dependence between network structure and end-to-end throughput will be
facilitated for sure, once the nodes would have access to geographical infor-
mation. However, this is not necessarily the case. Moreover, to some extend
this also contradicts the advocated spirit of selforganization, since positioning
information has to be provided to the nodes from an outside infrastructure.
Without geographical information it will be very difficult for any distributive
implementation of the throughput optimization to find the few, really important new communication links. We will now demonstrate our point with three different, non-geographical greedy-like algorithms. Of course, given the previous experience, all of them are based on cumulative betweenness centrality and all of them start with an initial \( k_{\text{min}} = 8 \) minimum-node-degree network.

In greedy attempt one, each single node \( i \) compares its \( B_i^{\text{cum}} \) to the respective values \( B_j^{\text{cum}} \) of its bidirected neighbors \( j \in \mathcal{N}_i \). Then, out of the set \( \{i\} \cup \mathcal{N}_i \) it tags the node with the smallest value for the cumulative betweenness centrality. After network-wide tagging is completed, each tagged node increases its transmission power by one rung on the transmission power ladder and builds up a new bidirectional communication link, the latter requiring that the new neighbor might be forced to step up on its own transmission power ladder, too. This completes the first round of network structure change. For a couple of rounds this procedure is repeated. This attempt assumes that the new links, which are attached to the least loaded nodes, take away some of the end-to-end routes passing through the most loaded nodes. In this respect, on the one side the cumulative betweenness centrality for the least loaded nodes would increase and on the other side it would decrease for the most loaded nodes, thus increasing the end-to-end throughput. Fig. 6 depicts the end-to-end throughput (11) as a function of the number of rounds. Right from the first round on the end-to-end throughput decreases.

Greedy attempt two is related to the previous one. The difference is, that a single node only tags itself whenever its cumulative betweenness is smallest compared to each of its communication neighbors. Then, again after one round of tagging, each tagged node increases its transmission power to build up a new communication link, which eventually requires also some cooperation from the new neighbor. Fig. 6 also shows the respective outcome for the end-to-end throughput as a function of the number of rounds. This time the throughput does not decrease. However, the increase is only very modest and peaks after rounds 3–4.

Last in line, greedy attempt three represents yet another modification. In addition to attempt two, nodes are also tagged if they possess a local maximum in cumulative betweenness centrality, i.e. their \( B_i^{\text{cum}} \) is larger than that of their communication neighbors. In such cases these nodes then decrease their transmission power to break one communication link; however this is only allowed if their node degree does not decrease below their initial degree. Lost neighbors are also allowed to decrease their transmission power to the level just before they are about to loose another communication link. The motivation for this additional tagging is straightforward: if the most-loaded nodes loose a neighbor, then also some by-passing end-to-end routes might break away, eventually leading to a relief in load. Another quick look on Fig. 6 reveals that again outcome does not match expectations. Compared to the previous
The outcomes of all three attempts produce a clear message. A distribu-
tive, non-geographical greedy-like network structure optimization of end-to-
end throughput is a highly non-trivial and tough problem. Simple-minded
straightforward attempts do not lead to a significant increase in throughput
performance. Time to think about conceptually other ways!

3 Performance increase with a routing metric based on cumulative
betweenness centrality

A different approach will now be taken to increase the end-to-end throughput.
In the previous Section, the optimized modification of network structure has
been relying on a simple, fixed routing policy. The end-to-end communications
have always been routed along the shortest multihop paths. What about the
other way around? Keeping the network structure fixed and modifying the end-
to-end routes in such a way that the most-loaded nodes get a substantial relief,
thus increasing end-to-end throughput. For this endeavor we will introduce
a new routing metric. Given the experience of the last Section, the latter will be based on cumulative betweenness centrality. This alternative approach will also allow for a distributed implementation with only moderate costs for coordination overhead and computation.

3.1 Routing metric based on cumulative betweenness centrality

In general, a routing metric is needed to determine the length of an end-to-end route. The simplest example is the hop-count metric. In this case the length of an end-to-end route is equal to the number of hops or links traversed along this route. For any pair of initial sender and final recipient, shortest-path routing only picks those routes which have the smallest multihop length. This has the disadvantage that a small number of nodes, especially those in the spatial center of the network, have to carry a large fraction of the overall network traffic, thus reducing the overall end-to-end throughput performance.

As a measure of a node’s load we could choose its betweenness centrality. However, this describes the load only in terms of the in-packet flux; consult again Eq. (1). We prefer to take the product between the node’s betweenness centrality and its sending time (9), i.e. the number of end-to-end routes going through this node and the time it takes to send a packet. This product is equal to the cumulative betweenness centrality $B_{i}^{\text{cum}}$. It is this cumulative betweenness centrality which we will now introduce as routing metric. The length of an end-to-end route between end-points $i, f$ then becomes

$$d_{i \rightarrow f} = \sum_{k \neq f} \sigma_{i \rightarrow f}(k) B_{k}^{\text{cum}}.$$  \hspace{2cm} (12)

All nodes $k$ belonging to the end-to-end route have $\sigma_{i \rightarrow f}(k) = 1$, otherwise $\sigma_{i \rightarrow f}(k) = 0$. The distance (12) sums up the betweenness centralities of all one-hop transmitting nodes along the end-to-end route, including the initial sender $i$, but excluding the final recipient $f$. The shortest end-to-end route between $i$ and $f$ is the one with minimum $d_{i \rightarrow f}$.

Note, that the shortest end-to-end routes are based on the actual routing metric, which itself is determined by the end-to-end routes; consult again Eqs. (2) and (10). Consequently, shortest end-to-end routes and routing metric have to be calculated in alternating order, until some form of convergence is reached.

The self-consistent, iterative determination of all end-to-end routes subject to the $B_{k}^{\text{cum}}$ metric consists of two parts: initialization and iteration. Initially, we set $B_{k}^{\text{cum}} = 1$ for all nodes. In one round of iterations, all nodes are picked one after the other. A picked node, say $i$, explores and updates all shortest
end-to-end routes originating from itself. In doing so, it uses a Dijkstra-like procedure [18] with the presently assigned routing metric. Directly after $i$’s end-to-end routing updates, the routing metric is also updated. All nodes in the network determine their new cumulative betweenness centrality. Formally, due to the decomposition $B_k = \sum_{i,f} B_{i\rightarrow f}(k)$ of the betweenness centrality, only the contribution $B_{i\rightarrow f}(k)$ resulting from all routes with initial sender $i$ needs to be updated. Then the next node in this round is picked. It already uses the freshly updated $B_{k\text{cum}}$ values to proceed further. This procedure can also be performed in a distributive manner. Only information about the link state of all nodes has to be exchanged between the nodes.

In order to fix the number of iteration rounds, generic packet traffic simulations have been performed, as described in Ref. [5]. Minimum-node-degree networks with $k_{\text{min}} = 8$ and $N = 30–300$ have been chosen to investigate the influence of the number of iteration rounds. The simulation results reveal that already two iteration rounds are sufficient. Beyond two rounds the end-to-end throughput does not increase further, although end-to-end routes are still subject to modifications. This defines a weak convergence. It is contrary to strong convergence, for which also the end-to-end routes would become stable. For the following we will only concentrate on the algorithm using two iteration rounds.

### 3.2 Results on end-to-end throughput

For various minimum-node-degree networks with sizes up to $N = 2000$ we have calculated the end-to-end throughput from a generic packet traffic simulation, which uses the end-to-end routes obtained from the routing metric based on cumulative betweenness centrality. Averages over 100 independent network realizations have been taken for $k_{\text{min}} = 8$; for $k_{\text{min}} = 12, 20$ it have only been 20. Fig. 7(top) illustrates $T_{e2e}$ as a function of $N$. Again the scaling expression $T_{e2e} \sim (N - N_0)^\gamma$ produces a good description for $N > 200$. The resulting parameter values are listed in Tab. 1. The scaling exponent $\gamma = 0.41$ is found to be independent of $k_{\text{min}}$. It is much larger than the respective $\gamma = 0.22-0.24$ resulting from the hop-count metric. Moreover, this scaling exponent is of the same order as those obtained from the optimized network structures based on shortest-multihop-path routing. In fact, on absolute scales the end-to-end throughput has become even slightly larger.

In principle, the expression (11) is not restricted to shortest-multihop-path routing. So far, in Sect. 2 it has only been tested for this case and found to produce a good estimate for the end-to-end throughput. A similar quality statement can now also be given for the routing based on the metric with the cumulative betweenness centrality. Fig. 7(bottom) and the bottom
Fig. 7. (Top) $N$-dependent end-to-end throughput obtained with the routing metric based on cumulative betweenness centrality for $k_{\text{min}} = 8$ (open squares), 12 (open circles), 20 (open triangles) minimum-node-degree networks. All curves have been determined from a generic packet traffic simulation. An average over 100 (for $k_{\text{min}} = 8$) and 10 (for $k_{\text{min}} = 12, 20$) independent network realizations has been performed for each symboled $N$. For comparison, respective curves (crosses, rotated crosses, stars; see also Fig. 2) based on the hop-count routing metric are also shown, as well as the analogue (open diamonds; see also Fig. 4) resulting from the $k_{\text{min}} = 8$ network structure optimization. (Bottom) Comparison of the first three curves (open symbols) from (top) with their counterparts (closed symbols) determined from Eq. (11).
Fig. 8. Selected end-to-end routes based on the hop-count metric (left) and on the metric with cumulative betweenness centrality (right). The underlying network is of minimum-node-degree type with $k_{\text{min}} = 8$ and $N = 300$.

of Tab. 1 compare the respective end-to-end throughput obtained from (11) with its counterpart obtained from packet traffic simulations. For the various minimum-node-degree networks with $k_{\text{min}} = 8, 12, 20$ the agreement is remarkable, although not perfect, and of the same order as in the previous discussions illustrated in Figs. 2 and 4. This proves again the high relevance of the cumulative betweenness centrality for the generic modeling of the end-to-end communication traffic in wireless multihop ad hoc networks.

A consequence of the large end-to-end throughputs obtained with the routing metric based on cumulative betweenness centrality must be that the end-to-end communication traffic is then more evenly distributed over the network. Fig. 8 illustrates the point. It shows the routes of three selected end-to-end communication partners. In case of the hop-count metric, two of the routes are very similar. They have the same initial node and neighboring final nodes. Both routes have a long common part, using the same highly congested nodes in the spatial center of the network. In case of the $B^\text{cum}_k$ metric, these two routes are pushed apart. In this way they decrease the load of the highly congested centered nodes. Another property of the new routing metric is illustrated with the third end-to-end route. Whereas the hop-count metric chooses a geometrically rather direct path, the metric based on cumulative betweenness centrality pushes the same end-to-end communication to the periphery of the network. This increases the load of those nodes situated at the periphery to some extent. However, such nodes are not critical to the network, and in return, the other close-by nodes experience a certain amount of traffic relief.

We recapitulate: cumulative betweenness centrality describes well the average traffic load of nodes networked together via wireless multihop ad hoc commu-
A routing metric based on this quantity has the strong tendency to distribute the end-to-end communication traffic evenly over the network. It pushes end-to-end routes towards the periphery. This is achieved without any geographical information and suffices to increase the end-to-end throughput to realms, which even slightly exceed those obtained with the optimization of network structure based on the hop count metric.

4 Conclusion and outlook

We have discussed wireless multihop ad hoc communication networks from the perspective of the Statistical Physics of complex networks. A modification of betweenness centrality, which we have denoted as cumulative betweenness centrality, has been shown to be very relevant for the intricate modeling of the end-to-end throughput performance. The maximization of a respective objective function has led to optimized geometric network structures. Roughly, those are such that the networks are just barely strongly connected and that the size of the blockings resulting from the nodes’ competition to gain wireless medium access are kept to a minimum. However, the significant increase in end-to-end throughput depends on only a few important links. These are not attached to the highly loaded nodes in the spatial center of the geometric network, but are found in the network’s periphery. This pronounced nonlocal relationship makes it almost impossible to construct the optimized network structures in a technologically relevant distributive manner.

A second approach has been presented to increase the end-to-end throughput. It also relies on the cumulative betweenness centrality. The latter is used as routing metric and leads to an iterative determination of end-to-end routes, which decrease the traffic load of the most utilized nodes. With this routing metric the end-to-end throughput performance of non-optimized network structures becomes even larger than for the structure-optimized networks with the hop-count routing metric.

This alternative approach with the new routing metric also allows for a technologically relevant distributed implementation. Only moderate costs for link-state coordination overhead between the nodes and subsequent computation within the nodes are needed. A performance comparison with other distributive implementations for routing and congestion control, like the promising ant algorithms [19], still needs to be done.

Potential further applications of the routing metric based on cumulative betweenness centrality are not restricted to static multihop ad hoc and sensor networks. There is spinoff potential for more cyber physics related to the Internet, Ethernet and computer traffic engineering.
The last remark is again on network structure optimization. A very intriguing distributive approach could be network game theory. In Ref. [20] a coupling of playing games with neighboring nodes and network structure evolution has been introduced. Consequently, the optimization strategy would be: give a game to the nodes, let them play, and by doing so they will automatically end up in a game-dependent network structure, serving the optimization objective. Of course, for the moment this is only an idea and a lot of tough conceptual work is still necessary to prove it right or wrong.

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