Hagino, K.; Rowley, N.
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Brazilian Journal of Physics, vol. 35, núm. 3B, september, 2005, pp. 890-893
Sociedade Brasileira de Física
São Paulo, Brasil

Available in: http://www.redalyc.org/articulo.oa?id=46435547
Quasi-Elastic Barrier Distribution as a Tool for Investigating Unstable Nuclei

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Received on 14 November, 2004

The method of fusion barrier distribution has been widely used to interpret the effect of nuclear structure on heavy-ion fusion reactions around the Coulomb barrier. We discuss a similar, but less well known, barrier distribution extracted from large-angle quasi-elastic scattering. We argue that this method has several advantages over the fusion barrier distribution, and offers an interesting tool for investigating unstable nuclei.

I. INTRODUCTION

It has been well recognized that heavy-ion collisions at energies around the Coulomb barrier are strongly affected by the internal structure of colliding nuclei [1, 2]. The couplings of the relative motion to the intrinsic degrees of freedom (such as collective inelastic excitations of the colliding nuclei and/or transfer processes) results in a single potential barrier being replaced by a number of distributed barriers. It is now well known that a barrier distribution can be extracted experimentally from the fusion excitation function \( \sigma_{\text{fus}}(E) \) by taking the first derivative of the ratio of the quasi-elastic cross section \( \sigma_{\text{qel}}(E) \) with respect to energy, that is, \( d\sigma_{\text{qel}}(E)/dE \) [3]. The extracted fusion barrier distributions have been found to be very sensitive to the structure of the colliding nuclei [1, 4], and thus the barrier distribution method has opened up the possibility of exploiting the heavy-ion fusion reaction as a “quantum tunneling microscope” in order to investigate both the static and dynamical properties of atomic nuclei.

The same barrier distribution interpretation can be applied to the scattering process as well. In particular, it was suggested in Ref. [5] that the same information as the fusion cross section may be obtained from the cross section for quasi-elastic scattering (a sum of elastic, inelastic, and transfer cross sections) at large angles. Timmers et al. proposed to use the first derivative of the ratio of the quasi-elastic cross section \( \sigma_{\text{qel}} \) to the Rutherford cross section \( \sigma_{\text{R}} \) with respect to energy, \(-d(d\sigma_{\text{qel}}/d\sigma_{\text{R}})/dE\), as an alternative representation of the barrier distribution [6]. Their experimental data have revealed that the quasi-elastic barrier distribution is indeed similar to that for fusion, although the former may be somewhat smeared and thus less sensitive to nuclear structure effects (see also Refs. [7–9] for recent measurements). As an example, we show in Fig. 1 a comparison between the fusion and the quasi-elastic barrier distributions for the \(^{16}\text{O} + ^{154}\text{Sm}\) system [10].

In this contribution, we undertake a detailed discussion of the properties of the quasi-elastic barrier distribution [10], which are less known than the fusion counterpart. We shall discuss possible advantages for its exploitation, putting a particular emphasis on future experiments with radioactive beams.

II. QUASI-ELASTIC BARRIER DISTRIBUTIONS

Let us first discuss heavy-ion reactions between inert nuclei. The classical fusion cross section is given by,

\[
\sigma_{\text{fus}}(E) = \pi R_0^2 \left( 1 - \frac{B}{E} \right) \theta(E - B),
\]

where \( R_0 \) and \( B \) are the barrier position and the barrier height, respectively. From this expression, it is clear that the first derivative of \( E\sigma_{\text{fus}}(E) \) is proportional to the classical penetrability for a 1-dimensional barrier of height \( B \) or equivalently the s-wave penetrability,

\[
\frac{d}{dE}(E\sigma_{\text{fus}}(E)) = \pi R_0^2 \theta(E - B) = \pi R_0^2 P_\text{cl}(E),
\]

and the second derivative to a delta function,

\[
\frac{d^2}{dE^2}(E\sigma_{\text{fus}}(E)) = \pi R_0^2 \delta(E - B).
\]

In quantum mechanics, the tunneling effect smears the delta function in Eq. (3). If we define the fusion test function as

\[
G_{\text{fus}}(E) = \frac{1}{\pi R_0^2} \frac{d^2}{dE^2}(E\sigma_{\text{fus}}(E)),
\]

this function has the following properties: i) it is symmetric around \( E = B \), ii) it is centered on \( E = B \), iii) its integral over \( E \) is unity, and iv) it has a relatively narrow width of around \( h\Omega\ln(3 + \sqrt{5})/\pi \sim 0.56 h\Omega \), where \( h\Omega \) is the curvature of the Coulomb barrier.

We next ask ourselves the question of how best to define a similar test function for a scattering problem. In the pure classical approach, in the limit of a strong Coulomb field, the differential cross sections for elastic scattering at \( \theta = \pi \) is given by,

\[
\sigma_{\text{cl}}(E, \pi) = \sigma_{\text{R}}(E, \pi) \theta(B - E),
\]

where \( \sigma_{\text{R}}(E, \pi) \) is the Rutherford cross section. Thus, the ratio \( \sigma_{\text{cl}}(E, \pi)/\sigma_{\text{R}}(E, \pi) \) is the classical reflection probability \( R(E) = 1 - P(E) \), and the appropriate test function for scattering is [6].

\[
G_{\text{qel}}(E) = -\frac{dR(E)}{dE} = -\frac{d}{dE} \left( \frac{\sigma_{\text{cl}}(E, \pi)}{\sigma_{\text{R}}(E, \pi)} \right).
\]
in the parabolic approximation. \(\alpha(E, \lambda_c)\) in Eq. (7) is given by

\[
\alpha(E, \lambda_c) = 1 + \frac{V_N(r_c)}{ka} \frac{\sqrt{2\mu k \eta}}{E} \times \left[ 1 - \frac{r_c}{Z_p Z_T e^\mu} \cdot 2V_N(r_c) \left( \frac{r_c}{a} - 1 \right) \right],
\]  

where \(k = \sqrt{2\mu E/\hbar^2}\), with \(\mu\) being the reduced mass for the colliding system. The nuclear potential \(V_N(r_c)\) is evaluated at the Coulomb turning point \(r_c = (\eta + \sqrt{\eta^2 + \lambda_c^2})/k\), and \(a\) is the diffuseness parameter in the nuclear potential.

Figure 2 shows an example of the excitation function of the cross sections and the corresponding quasi-elastic test function, \(G_{qel}(E)\), at \(\theta = \pi\) for the \(^{16}\text{O} + ^{144}\text{Sm}\) reaction. Because of the nuclear distortion factor \(\alpha(E, \lambda_c)\), the quasi-elastic test function behaves rather similarly to the fusion test function \(G_{fus}(E)\). In particular, both functions have a similar, relatively narrow, width, and their integral over \(E\) is unity. We may thus consider that the quasi-elastic test function is an excellent analogue of the one for fusion, and we exploit this fact in studying barrier structures in heavy-ion scattering.

In the presence of the channel couplings, the fusion and the quasi-elastic cross sections may be given as a weighted sum of the cross sections for uncoupled eigencanals,

\[
\sigma_{fus}(E) = \sum_\alpha w_\alpha \sigma_{fus}^{(\alpha)}(E),
\]

\[
\sigma_{qel}(E, \theta) = \sum_\alpha w_\alpha \sigma_{el}^{(\alpha)}(E, \theta),
\]
where $\sigma_{\text{fus}}^{(\alpha)}(E)$ and $\sigma_{\text{qel}}^{(\alpha)}(E,\theta)$ are the fusion and the elastic cross sections for a potential in the eigenchannel $\alpha$. These equations immediately lead to the expressions for the barrier distribution in terms of the test functions,

$$D_{\text{fus}}(E) = \frac{d^2}{dE^2}[E\sigma_{\text{fus}}(E)] = \sum_{\alpha} w_\alpha \pi R_\alpha^2 G_{\text{fus}}^{(\alpha)}(E), \quad (13)$$

$$D_{\text{qel}}(E) = -\frac{d}{dE} \left( \frac{\sigma_{\text{qel}}(E,\pi)}{\sigma_R(E,\pi)} \right) = \sum_{\alpha} w_\alpha G_{\text{qel}}^{(\alpha)}(E). \quad (14)$$

### III. ADVANTAGES OVER FUSION BARRIER DISTRIBUTIONS

There are certain attractive experimental advantages to measuring the quasi-elastic cross section $\sigma_{\text{qel}}$ rather than the fusion cross section $\sigma_{\text{fus}}$ to extract a representation of the barrier distribution. These are: i) less accuracy is required in the data for taking the first derivative rather than the second derivative, ii) whereas measuring the fusion cross section requires specialized recoil separators (electrostatic deflector/velocity filter) usually of low acceptance and efficiency, the measurement of $\sigma_{\text{qel}}$ needs only very simple charged-particle detectors, not necessarily possessing good resolution either in energy or in charge, and iii) several effective energies can be measured at a single-beam energy, since, in the semi-classical approximation, each scattering angle corresponds to scattering at a certain angular momentum, and the cross section can be scaled in energy by taking into account the centrifugal correction. Estimating the centrifugal potential at the Coulomb turning point $r_c$, the effective energy may be expressed as [6]

$$E_{\text{eff}} \sim E - \frac{\lambda^2 \hbar^2}{2\mu r_c^2} = 2E \frac{\sin(\theta/2)}{1 + \sin(\theta/2)} \cdot (15)$$

Therefore, one expects that the function $-d/dE(\sigma_{\text{qel}}/\sigma_R)$ evaluated at an angle $\theta$ will correspond to the quasi-elastic test function (6) at the effective energy given by eq. (15).

This last point not only improves the efficiency of the experiment, but also allows the use of a cyclotron accelerator where the relatively small energy steps required for barrier distribution experiments cannot be obtained from the machine itself [7]. Moreover, these advantages all point to greater ease of measurement with low-intensity exotic beams, which will be discussed in the next section.

In order to check the scaling property of the quasi-elastic test function with respect to the angular momentum, Fig. 3 compares the functions $\sigma_{\text{qel}}/\sigma_R$ (upper panel) and $-d/dE(\sigma_{\text{qel}}/\sigma_R)$ (lower panel) obtained at two different scattering angles. The solid line is evaluated at $\theta = \pi$, while the dotted line at $\theta = 160^\circ$. The dashed line is the same as the dotted line, but shifted in energy by $E_{\text{eff}} - E$. Evidently, the scaling does work well, both at energies below and above the Coulomb barrier, although it becomes less good as the scattering angle decreases [10].

![Figure 3: Comparison of the ratio $\sigma_{\text{qel}}/\sigma_R$ (upper panel) and its energy derivative $-d/dE(\sigma_{\text{qel}}/\sigma_R)$ (lower panel) evaluated at two different scattering angles.](image-url)
FIG. 4: The excitation function for quasi-elastic scattering (upper panel) and the quasi-elastic barrier distribution (lower panel) for the $^{32}\text{Mg} + ^{208}\text{Pb}$ reaction around the Coulomb barrier. The solid and the dashed lines are the results of coupled-channels calculations which assume that $^{32}\text{Mg}$ is a rotational and a vibrational nucleus, respectively. The single octupole-phonon excitation in $^{208}\text{Pb}$ is also included in the calculations.

We mention that the distorted-wave Born approximation (DWBA) yields identical results for both rotational and vibrational couplings (to first order). In order to discriminate whether the transitions are vibration-like or rotation-like, at least second-step processes (reorientation and/or couplings to higher members) are necessary. The coupling effect plays a more important role in low-energy reactions than at high and intermediate energies. Therefore, we expect that quasi-elastic scattering around the Coulomb barrier will provide a useful means to allow the detailed study of the structure of neutron-rich nuclei in the near future.

Acknowledgments

This work was supported by the Grant-in-Aid for Scientific Research, Contract No. 16740139, from the Japan Society for the Promotions of Science.

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