Trojan Horse technique to measure nuclear astrophysics rearrangement reactions

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Abstract. The knowledge of nucleosynthesis and of energy production in stars requires an increasingly precise measurement of nuclear fusion reactions at the Gamow energy. Because of the Coulomb barrier reaction cross sections in astrophysics cannot be accessed directly at ultra-low energies, unless very favorable conditions are met. Moreover, the energies characterizing nuclear processes in several astrophysical contexts are so low that the presence of atomic electrons must be taken into account. Theoretical extrapolations of available data are then needed to derive astrophysical S(E)-factors. To overcome these experimental difficulties the Trojan Horse Method (THM) has been introduced. The method provides a valid alternative path to measure unscreened low-energy cross sections of reactions between charged particles, and to retrieve information on the electron screening potential when ultra-low energy direct measurements are available. While the theory has been discussed in detail in some theoretical works, present in the scientific literature, also in relation to different types of excitation functions (e.g. non-resonant and resonant), work on detailed methodology used to extract the events to be considered for the bare nucleus cross section measurements is still on going. In this work we will present some critical points in the application of THM that deserve to be discussed in more detail.

1. Introduction
The new improvements in the field of astrophysics observations and models obtained in the last twenty years have triggered new fusion nuclear reaction measurements at astrophysical relevant energies. Indeed, to understand the energy production in stars, the first phases of the Universe and subsequent stellar evolution, the accurate knowledge of nuclear reaction cross sections close to the Gamow energy $E_G$ is required [1, 2, 3, 4].

In charged-particle induced reactions, occurring during quiescent burning in stars in stars, $E_G$ (in general of order of few keV to 100 keV) is far below the Coulomb barrier $E_C$ for the interacting nuclei, usually of the order of few MeV. This implies that as the energy is lowered the reactions take place via tunneling with an exponential decrease of the cross section for non-resonant reactions. Therefore, their bare nucleus cross sections $\sigma(b)(E)$ drop exponentially with decreasing energy, $\sigma_b(E) \propto exp(-2\pi \eta)$ were $\eta$ is the Sommerfeld parameter $\eta = \frac{Z_1 Z_2 e^2}{\hbar v}$ depending on the atomic numbers $Z_1, Z_2$ of the colliding nuclei and on their relative velocity $v$ in the entrance channel.

Thus, direct $\sigma_b(E)$ measurements of the non resonant cross section often suffer for a low-energy limit, which usually extends to energies that are typically much larger than $E_G$. The
extrapolation procedure of the bare nucleus cross section $\sigma_b(E_{\text{cm}})$, from measurements at higher energies supported by theoretical arguments, would be the standard way to obtain $\sigma_b(E_G)$. But extrapolation is complicated because of the exponential variation of the cross section. To obtain the value at the Gamow energy, $\sigma_b(E)$ is usually parameterized in terms of the bare nucleus astrophysical factor $S_b(E)$, as given by:

$$S_b(E) = E \cdot \sigma_b(E) \cdot \exp(2\pi\eta)$$  \hfill (1)

where $\sigma_b(E)$ is the bare nucleus cross section at the energy $E_{\text{cm}}$ on the center of mass and $\exp(2\pi\eta)$ is the inverse of the Gamow factor, which removes the dominant energy dependence of $\sigma_b(E_{\text{cm}})$ due to barrier penetrability.

The astrophysical $S_b(E)$-factor shows a much weaker energy dependence than the cross section $\sigma_b(E)$, thus it is used to extrapolate the cross section from higher energies ($E > 4E_G$) to the Gamow peak. But an extrapolation by using Eqn.(1) into the unknown can lead to considerable uncertainty and important contributions to the excitation functions, e.g. resonance tails of sub-threshold resonances, can be missed [3].

In the available few accurate direct measurements within, or close to, the Gamow peak (e.g. [5]), an unexpected effect showed up, attributed to the presence of atomic electrons [6]. has been considered as responsible for the increase of the cross section at low energies, the so-called ‘electron screening effect’ [6].

In the extrapolation of the cross section using Eqn.(1), it is assumed that the Coulomb potential of the target nucleus and the projectile is that resulting from bare nuclei (this is why an index $b$ is used for both the cross section and the $S(E)$ factor). However, for nuclear reactions studied in the laboratory, the target nuclei and the projectiles are usually in the form of neutral atoms or molecules and ions, respectively. The electron clouds surrounding the interacting nuclei screen the nuclear charges: the projectile effectively sees a reduced Coulomb barrier, both in height and radial extension. This leads, in turn, to a cross section for the screened nuclei $\sigma_s(E)$, higher than the one we would get in the case of bare nuclei, $\sigma_b(E)$, with an exponential increase of $\sigma_s(E)$ (or equivalently of the astrophysical factor $S_s(E)$) with decreasing energy.

To parameterize the cross section rise due to the screening effect, an enhancement factor $f(E)_{\text{lab}}$ is usually introduced [3]. This $f(E)_{\text{lab}}$ is defined as:

$$[f(E)]_{\text{lab}} = \sigma_s(E)/\sigma_b(E) \sim \exp(\pi\eta(U_e)_{\text{lab}}/E), \hfill (2)$$

where $(U_e)_{\text{lab}}$ is the electron screening potential in laboratory experiments.

Clearly, a good understanding of $(U_e)_{\text{lab}}$ is essential in order to calculate $\sigma_b(E)$ from the experimental $\sigma_s(E)$ using Eqn.(2). In turn, the understanding of $(U_e)_{\text{lab}}$, which is different from the screening potential in the stellar plasma $(U_e)^{pl}$, may help to better understand $(U_e)^{pl}$ itself, needed to calculate $\sigma(U_e)_{\text{pl}}$ [6, 7]:

$$\sigma_{\text{pl}}(E) = \sigma_b(E) \cdot \exp(\pi\eta(U_e)^{pl}/E)) = \sigma_s(E) \cdot \exp\left(\frac{\pi\eta(U_e)^{pl} - (U_e)_{\text{lab}}}{E}\right), \hfill (3)$$

Since the astrophysical $S_b(E)$-factor at $E_G$ is not available experimentally, the extrapolation approach is needed in order to evaluate the bare one. It means that extrapolation is needed even when measured cross sections are available in the Gamow energy range, because of the presence of the electron screening effects.

In addition, experimental studies of reactions involving light nuclides [5, 7] have shown that the expected enhancement of the cross section at low energies was, in all cases, significantly larger than what could be accounted for by available atomic-physics models. This aspect deserves special attention because one may have a chance to predict the effects of electron
Figure 1. Diagram representing the quasi-free process $A+B \rightarrow C + D + S$. The upper vertex describes the "virtual break up" of the THM-nucleus $A$ into the clusters $x$ (participant) and $S$; the cluster $S$ is then considered to be spectator to the $A+x \rightarrow C+D$ reaction which takes place in the lower vertex.

screening in an astrophysical plasma only if it is preliminarily understood under laboratory conditions (Eqns.(3)). In astrophysical applications we need to estimate the plasma cross section $\sigma_{\text{pl}}(E)$. The $|f(E)|^2$ factor can be calculated if the plasma potential energy $(U_e)^{\text{pl}}$, which depends on detailed properties of the plasma, such as the Debye-Huckel radius. Clearly, a good understanding of $(U_e)^{\text{lab}}$ is needed in order to calculate $\sigma_b(E)$ using Eqn.(2).

To overcome these experimental difficulties of direct measurements, several indirect methods such as Coulomb Dissociation [8, 7], Asymptotic Normalization Coefficients [9, 10, 11], and Trojan Horse Method (THM) ([12, 13, 14, 15, 16] and reference therein) have been developed in the last twenty years.

2. Basic theory

The Trojan Horse Method (THM) has been successfully applied to the bare nucleus cross sections measurements of reactions between charged particles [see Table 1] at sub-Coulomb energies. Among them, the THM is at present the only powerful technique to measure $\sigma_b(E)$ of reactions between charged particles at never reached energies (see references in Table 1). It makes it possible also to retrieve, by using the Eqn.(2), independent information on the electron screening potential $U_e^{\text{lab}}$ when direct ultra-low energy measurements ($\sigma_s(E)$ or $S_s(E)$) are available.

The THM indirect technique selects the quasi-free (QF) contribution of an appropriate three-body reaction $A+B \rightarrow C+D+S$ [12, 13], performed at energies well above the Coulomb barrier, to extract a charged-particle two-body reaction cross section $B+x \rightarrow C+D$ at astrophysical energies free of Coulomb suppression [38].

It has its scientific background in the theory of direct reaction mechanisms (see [40] for instance), and in particular in the studies of the QF reaction mechanisms at low energy ([13] and reference therein). In particular the application to nuclear astrophysics is an extension to the ultra-low energies of the well assessed higher-energy measurements of QF three-body reactions [41, 42, 43].

In the present paper, the THM is presented within the Plane Wave Impulse Approximation (PWIA) framework and applied to non resonant reactions with dominant $l = 0$ partial wave
A reaction. Following the simple PWIA, the three-body reaction cross section can be factorized (triangle graphs) indicating rescattering between the reaction products, are neglected [47, 48].

More sophisticated theoretical formulations in the resonant and non-resonant cases, with different $l$ partial waves contributions, can be found e.g. in references [12, 14, 30, 44, 45]. The QF $B(A,CD)S$ reaction between $B$ and the projectile $A$, whose wave function is assumed to have a large amplitude for the $A = x + S$ cluster configuration, can be described by a Feynman diagram (Fig. 1), where only the first term of the Feynman series is retained. The diagram represents the dominant process (pole approximation), while other graphs (triangle graphs) indicating rescattering between the reaction products, are neglected [47, 48].

Under these hypotheses, particle $B$ is considered to interact only with a part ($x$ cluster) of the nucleus $A$, while the other part ($S$ cluster) is considered as spectator to the $B(x,C)D$ virtual reaction. Following the simple PWIA, the three-body reaction cross section can be factorized into two terms corresponding to the vertices of Fig. 1. The three-body reaction cross section $A + B \rightarrow C + D + S$ is proportional to the cross section of the binary reaction $B + x \rightarrow C + D$ and it is given by

$$\frac{d^3\sigma}{d\Omega_C d\Omega_D dE_C} \propto (KF) \cdot |\phi(p_{E_S})|^2 \cdot \left[ \frac{d\sigma(E)}{d\Omega} \right]^{HOES},$$

\[ (4) \]

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**Table 1.** Two-body reactions studied via the THM

| No. | indirect reaction | Direct reaction | $E_{lab}^{inc.}$ (MeV) | $Q_2$, (MeV | THM-Nucleus (x Cluster) | Reference |
|-----|-------------------|----------------|-------------------------|-----------------|------------------------|----------|
| 1   | $^2H(^7Li, o\alpha)n$ | $^1H(^7Li, \alpha)^4He$ | 19-22 | 15.12 | $^2H (p)$ | [17, 18, 19] |
| 2   | $^3He(^7Li, o\alpha)d$ | $^1H(^7Li, \alpha)^4He$ | 33 | 11.85 | $^3H(p)$ | [20, 21] |
| 3   | $^2H(^6Li, o^3He)n$ | $^1H(^6Li, \alpha)^3He$ | 22-34 | 1.79 | $^2H (p)$ | [22] |
| 4   | $^6Li(^6Li, \alpha)^4He$ | $^2H(^6Li, \alpha)^4He$ | 5 | 20.89 | $^6Li (d)$ | [14, 23] |
| 5   | $^2H(^9Be, o^6Li)n$ | $^1H(^9Be, \alpha)^6Li$ | 22 | -0.10 | $^2H (p)$ | [24] |
| 6   | $^2H(^ {10}B, o^7Be)n$ | $^1H(^{10}B, \alpha)^7Be$ | 27 | -1.08 | $^2H (p)$ | [25] |
| 7   | $^2H(^{11}B, o^8Be)n$ | $^1H(^{11}B, \alpha)^8Be$ | 27 | 6.36 | $^2H (p)$ | [26, 27] |
| 8   | $^2H(^{15}N, o^ {12}C)n$ | $^1H(^{15}N, \alpha)^ {12}C$ | 60 | 2.74 | $^2H (p)$ | [28, 29] |
| 9   | $^2H(^{18}O, o^ {15}N)n$ | $^1H(^{18}O, \alpha)^ {15}N$ | 54 | 1.75 | $^2H (p)$ | [30, 31] |
| 10  | $^2H(^ {19}F, o^ {16}O)n$ | $^1H(^{19}F, \alpha)^ {16}O$ | 50 | 5.89 | $^2H (p)$ | [32] |
| 11  | $^2H(^{17}O, o^ {14}N)n$ | $^1H(^{17}O, \alpha)^ {14}N$ | 41 | -1.03 | $^2H (p)$ | [33] |
| 12  | $^6Li(^3He, p^4He)\alpha$ | $^2H(^3He, p)^4He$ | 5-6 | 16.88 | $^6Li (d)$ | [34] |
| 13  | $^2H(^6Li, p^4He)n$ | $^2H(d, p)^3H$ | 14 | 2.56 | $^6Li (d)$ | [35] |
| 14  | $^6Li(^2C, o^ {12}C)^2H$ | $^4He(^{12}C, ^{12}C)^4He$ | 20,16 | -1.47 | $^6Li (\alpha)$ | [36] |
| 15  | $^2H(^6Li, t^4He)n$ | $n(^6Li, t)^4He$ | 14 | 2.56 | $^2H (n)$ | [37] |
| 16  | $^2H(p, pp)n$ | $^1H(p, \alpha)^4H$ | 5-6 | 2.22 | $^2H (p)$ | [38] |
| 17  | $^2H(^3He, p^4He)n$ | $^2H(d, p)^3H$ | 18 | -1.46 | $^3He (d)$ | [39] |
| 18  | $^2H(^3He, n^3He)n$ | $^2H(d, n)^3He$ | 18 | -2.22 | $^3He (d)$ | [39] |
where:

(i) $\left[ \frac{d\sigma(E)}{d\Omega} \right]_{\text{HOES}}$ is the half-off-energy-shell (HOES) differential cross section for the binary $B(x, C)D$ reaction at the center-of-mass energy $E$ given in post-collision prescription (PCP) by the relation

$$E = E_{CD} - Q_{2b}, \quad (5)$$

where $Q_{2b}$ is the $Q$-value for the binary $x + B \rightarrow C + D$ reaction and $E_{CD}$ is the $CD$ relative energy in the exit channel.

(ii) $K_F$ is a kinematical factor containing the final-state phase-space factor and is a function of the masses, momenta, and angles of the outgoing particles [46]

(iii) $\phi(p_{xS})$ is the Fourier transform of the radial wave function for the $\chi(r_{xS})$ intercluster motion usually described in terms of Hankel, Eckart, or Hulthén functions depending on the $xS$ system.

In all the QF reactions already investigated (see Table I) the two-cluster $xS$ system in the nucleus $A$ are most likely in $s$ state, thus the expected momentum distribution has a maximum at $p_{xS} = 0$ MeV/c.

To completely determine the kinematical properties of the $S$ particle (“spectator” – cluster), in particular its momentum distribution $|\phi(p_{xS})|^2$, energies $E_C$ and $E_D$ and their emission angles $\theta_C$ and $\theta_D$ must be measured. The resulting two-dimensional energy spectrum $(E_C - E_D)$ obtained from such a measurement is usually reduced to a one-dimensional spectrum by projecting the coincidence yield onto one of the energy axes ($E_C$ or $E_D$) (see [17, 26]).

3. Energy and Momentum Prescriptions

3.1. Incident energy prescriptions.

Beam energies must been carefully chosen to optimize the kinematical conditions for the presence of quasi-free mechanism, under the assumptions of the IA. The chosen beam energy overcome the Coulomb barrier $(E_{AB})_{\text{CB}}$ in the entry channel. Thus, particle $x$ is brought inside the nuclear interaction zone to induce the relevant reaction $x + B \rightarrow C + D$. In addition the QF kinematical conditions must be chosen in such a way that the $E_{xB}$ relative energy can span the astrophysical region of interest below the Coulomb barrier $(E_{xB})_{\text{CB}}$ [8]

$$E_{xB} < (E_{xB})_{\text{CB}}. \quad (6)$$

Eqn (5) gives the $E_{cm}$ as a function of $E_{CD}$ and $Q_{2b}$. From this equation it is clear that the relative energy of the fragments in the initial channel $x + B$ of the binary reaction can be very low and even negative. This is possible because the initial projectile velocity is compensated for by the binding energy of particle $x$ inside $A$ ([15, 17, 26] and references therein). Thus, the $xB$ relative energy of the fragments can be very low (Eqn. (8)).

The idea [17] is that, in the quasi-free kinematic condition ($E_{xS}=0$) the initial projectile velocity is compensated for by the binding energy of particle $x$ inside $A$.

$$E_{cm} = E_{x-B} - B_{xS} \quad (7)$$

with $E_{xB}$ projectile energy in the two-body center of mass system and $B_{x-S}$ the binding energy of the nucleus $A$. We also conclude from Eqn. (8) that the energy $E_{cm}$ is uniquely determined by the incident beam energy in QF kinematics. Hence the determination of the energy dependence of the binary reaction cross section from the $TH$ reaction requires a continuous change of the beam energy. From the experimental point of view it is more convenient to fix the beam energy and vary the relative momentum $p_{x-S}$ within few tens of $MeV/c$. In this case the kinematics
of the experiment slightly deviates from the QF condition, but one can easily fix the accessible astrophysical energy region of interest.

To experimentally validate the correctness of the pole approximation in each specific case [48] some validity tests are very important. Note that the applicability of the pole approximation [48] is limited to small $p_{xS}$ with a prescription given by relation:

$$0 \leq |\vec{p}_{xS}| \leq k_{xS}$$

with $p_{xS}$ being the half-off energy shell (HOES) momentum of the cluster $x$ when it interacts with the particle $B$, and $k_{xS}$ defined by the relation

$$k_{xS} = \sqrt{2\mu_{xS}B_{xS}}$$

where $\mu_{xS}$ is the reduced mass.

Note that the quasi-free QF processes were previously investigated in high $Q$-value three-body reactions induced on light nuclei ([49] and references therein) at low energy. QF effects are usually expected in reactions where high transferred momenta ($\geq 300$ MeV/c) are involved, even with low incident energy if the $Q$-value are high enough [49, 51, 52]. These studies have been demonstrated that the condition of applicability of the IA is connected to the transferred momentum $q_t$ hence in the case of the QR processes we must consider the $Q$-value of the three body reactions $A + B \rightarrow C + D + S$. In particular the "ideal" condition is that the magnitude of $|\vec{q}_t|$ is

$$|\vec{q}_t| >> |\vec{k}_{xS}|$$

where $|\vec{k}_{xS}|$ is the inter-cluster on-shell energy momentum.

4. DATA ANALYSIS

4.1. Experimental Momentum Distribution in Impulse Approximation

A sensitive way to check the presence of the QF mechanism in a three body $A + B \rightarrow C + D + S$ reaction is by means of a shape analysis of the measured momentum distribution $|\phi(\vec{p}_{xS})_{\text{exp.}}|^2$ [15, 17, 26, 36]. In the first study of the QF processes [50] the energy-sharing or angular methods were used. With the improvements in energy and angular resolution of new detectors and of analysis programs, a new method to extract the momentum distribution $|\phi(p_{xS})_{\text{exp.}}|^2$ has introduced [26]. If the factorization of Eqn.(3) is applicable, dividing the quasi-free coincidence yield ($Y$) by the kinematic factor, a quantity which is proportional to the product of the momentum distribution by the two-body cross section is obtained.

$$|\phi(\vec{p}_{xS})_{\text{exp.}}|^2 \propto \frac{Y}{KF \cdot \left[\frac{d\sigma(E)}{d\Omega}\right]_{\text{HOES}}.}$$

A complementary way to test the presence of QF and/or SD mechanism is to investigate the correlation between the $E_{CD}$ relative energy and the momentum $p_{xS}$ for all coincidence events, in the whole angular range covered by the detectors. In the case of the $^2H(^{11}B,\alpha^8Be)n$ reaction [26], $E_{\alpha^8Be}$ vs. $p_n$ two dimensional plot is reported in Fig.2. The sharp line at 8.7 MeV in the region of low $p_{mn}$ is associated with 16.11 MeV excited state of $^{11}C$. In these restricted relative energy $\Delta E$ and center-of-mass angular range $\Delta\theta_{cm}$, the differential binary cross section can be considered almost constant $\left[\frac{d\sigma(E)}{d\Omega}\right]_{\text{HOES}} = \text{const}$, and

$$|\phi(\vec{p}_{xS})_{\text{exp.}}|^2 \propto \frac{Y}{KF}.\quad (12)$$
Figure 2. $E_{\alpha-8Be}$ relative energy as a function of the spectator momentum $p_s (= p_{pn})$ [26].

Figure 3. Deuteron experimental momentum distribution (full dots) extracted from $^2H(^{11}B, \alpha^{8}Be)n$ reaction compared with the theoretical one (dashed line) [26].

In Fig. 3 the experimental momentum distribution for the proton inside the deuteron for the $^2H(^{11}B,\alpha^{8}Be)n$ reaction at $E_{11B} = 27$ MeV is obtained by projection, of the events selected in Fig.2 in the two horizontal lines, on the $p_s = p_{pn}$ axis [26]. The squared Hulthén function (dashed line) in momentum space is superimposed onto the data. The theoretical momentum distribution extracted from Eqn.(10) reproduces quite well the shape of experimental data. The theoretical FWHM = 58 MeV/c is in good agreement with the experimental one [26].

The momentum distributions calculated in the PWIA might differ both in their absolute value and their shape from those calculated in DWIA. However, at high incident energies (more than about 100 MeV for ($p,p\alpha$) (see Fig. 5 and Fig.6), ($p,pd$) and ($\alpha, 2\alpha$) reactions) the two normalized calculations have essentially the same shape at least for values of the spectator cluster momenta smaller that about 60 MeV/c. In the DWIA [53, 54], the radial wave functions are deduced from optical-model potentials and the following considerations apply:

(i) at low relative momenta, the shapes of the $|\phi(p_{xS})_{theor.}|^2_{PWIA}$ and $|\phi(p_{xS})_{theor.}|^2_{DWIA}$ distributions coincide. However, while the PWIA introduces unphysical zeros in the momentum distribution, the DWIA represents a more realistic approach;

(ii) in DWIA treatment the absolute value of the momentum distribution undergoes a dramatic decrease due to the wave absorption effects, which are not taken into account in PWIA.

At lower energies, absorption and distortion effects become more important, so that the validity of the PWIA is less and less justified [49]. The usual treatment in the PWIA yields, in fact, narrower momentum distributions with a FWHM between 40 and 60 MeV/c, so that in the standard procedure to fit data, a cut-off distance in the radial part $R_B(r)$ of the inter-cluster wave function $\chi(r)$ is introduced. Many experiments [41, 49, 50] were performed to study quasi-free processes in high $Q$ value three-body reactions induced on light nuclei at low incident energy.

An important point to be stressed is that in the THM only events with a spectator momentum close to the QF condition (cut-off of few tens of MeV/c) are taken into account. In the case:

(i) the essential features of the $|\phi(p_{S})|^2$ are the same in both procedures $|\phi(p_{xS})_{theor.}|^2_{PWIA}$ and $|\phi(p_{xS})_{theor.}|^2_{DWIA}$ within the experimental uncertainties;
In the case of non-resonant reactions at low incident energies but with high $Q$-value, the standard procedure is totally reliable. Conversely, in reactions with both low incident energies and low $Q$-values, distortions might intervene introducing more marked differences between the $|\phi(p_{S})_{\text{theor.}}|_{\text{PWIA}}^2$ and $|\phi(p_{S})_{\text{theor.}}|_{\text{DWIA}}^2$ momentum distributions. These distortions are responsible for a drastic change of the width (FWHM) in the spectator momentum distribution $|\phi(p_{\text{s}})_{\text{exp.}}|_{\text{PWIA}}^2$. The line in Fig. 4 shows the trend of the momentum distribution widths from experimental data available in the literature as a function of transferred momentum $q_t$ from $A$ to the system $B = C + D$; it is determined as the Galilean invariant transferred momentum [52].

As already mentioned, in the analysis of THM applications the PWIA is usually adopted since it fairly describes the experimental data. Anyway, to check the validity of the PWIA approach and to reproduce those data where distortions cannot be neglected, the more realistic Distorted Wave Born Approximation (DWBA) should be employed. The experimental data [27, 30, 33, 37] show that even in the DWBA if we limit our event selection to the region close to the maximum of the momentum distribution, $|\phi(p_{\text{pn}})_{\text{exp.}}|_{\text{PWIA}}^2$ and $|\phi(p_{\text{pn}})_{\text{theor.}}|_{\text{DWBA}}^2$ in PWBA and DWIA approaches give the same results. For example, we will consider the case of data of the $^{18}\text{O}(p,\alpha)^{15}\text{N}$ reaction studied using deuteron as the TH nucleus [30]. The $pn$ PWIA momentum distribution $|\phi(p_{\text{pn}})_{\text{exp.}}|_{\text{PWIA}}^2$ from $^{18}\text{O}(p,\alpha)^{15}\text{N}$ QF reaction is shown in Fig. 8 as a dashed curve. The dashed curve reproduces the experimental behavior for $p_{\text{pn}} \leq 50 \text{ MeV}/c$ while for larger momenta it departs from the experimental data. The normalization factor is obtained by scaling the calculated distribution to the experimental one (see [30]). The DWBA calculations is instead given by a dotted line in the same figure. It has been performed by means of the FRESCO computer code, taking the optical-model potential parameters from Perey and Perey [55]. In the range 0 to 50 MeV/c, the difference between and $|\phi(p_{\text{pn}})_{\text{exp.}}|_{\text{PWIA}}^2$ and $|\phi(p_{\text{pn}})_{\text{theor.}}|_{\text{DWBA}}^2$ momentum distributions is negligibly small, about 4%, in comparison with other uncertainties. Same considerations apply to the $^{17}\text{O}(p,\alpha)^{14}\text{N}$ reaction [33]. These results strengthen the importance and reliability of the PWIA approach.

In general relatively simple experimental set-ups in the THM experiments are needed.
Figure 5. $^6$Li momentum distribution obtained through the $^6$Li(p, p$^\alpha$)$^2$H reaction (full dots). The normalization is done equalizing the maximum value of the theoretical distributions to the maximum value of the experimental data. The full (dashed) curve is the DWIA (PWIA) distribution [53, 54, 15].

Figure 6. $^9$Be(p, p$^\alpha$)$^5$He cross section. The curve are DWIA calculations for $L=0$ and $L=2$ knockout with equal spectroscopic strengths [53, 54, 15].

The main requirements are high energy and angular resolutions. Usually the detection set-up consists of two or more couples of coincidence telescopes arranged at opposite side of the beam direction at quite forward angles, the experiments being usually performed in inverse kinematics. These calculations, in general, show the strong dependence of the $E_{xB}$ uncertainty on the angular resolution of both ejectiles. That is why in our experiments an angular resolution of the order of $0.1^\circ$–$0.5^\circ$ is required.

A number of steps are involved in the data analysis before the two-body cross section of astrophysical relevance can be extracted. These steps include:

(i) selection of the "Trojan Horse nucleus";
(ii) identification of events due to the QF mechanism contribution in the three-body reaction of interest $A + B \rightarrow C + D + S$;
(iii) measurement of the HOES two-body cross section in arbitrary and absolute units;
(iv) validity tests on the selected events;
(v) extraction of the astrophysical S(E)-factor and electron screening potential, when ultra-low energy measurements are available.

4.2. Selection of the "Trojan Horse" Nucleus

In Table 2 a list of possible TH nuclei is reported with the corresponding virtual $x$ participant. It is clear that different choices of TH nuclei are available to get the same virtual $x$-cluster. A number of indirect investigations with the THM has been focused on the study of $(p, \alpha)$ reactions that play a key role in many stellar nucleosynthesis paths. These measurements are usually performed in inverse kinematics with a ‘deuteron target’ as a ‘virtual-proton’ target. We note from Table 2 that, for example, a virtual participant proton can be hidden either inside a deuteron $^2$H=($p + n$) with $n =$ spectator ($^2$H binding energy $B_{pn} = 2.225$ MeV) or inside...
Figure 7. $^2\text{H}(^{10}\text{B}, \alpha^{7}\text{Be})n$ reaction: deuteron experimental momentum distribution (full dots), best fit of the experimental distribution obtained by leaving as a free parameter the FWHM of the square of the Hulthén function in momentum space (full black line) and theoretical momentum distribution given by the square of the Hulthén function with the asymptotic FWHM (full blue line) [51, 52].

$^3\text{He} = (p + d)$ with $d =$ spectator ($^3\text{He} \ B_{pn} = 5.49 \text{MeV}$). The choice of particle $^2\text{H}$ as TH nucleus to study $(p, \alpha)$ two-body reactions is suggested by a number of reasons:

- its binding energy $B_{pn}$ is low;
- its wave function is well known;
- it has a unique simple cluster structure (proton plus neutron);
- the cluster spectator is not charged;
- the inter-cluster $xS$ motion takes place at $l=0$, thus the momentum distribution has a maximum for $|\vec{p}_{xS}| = 0 \text{MeV}/c$.

Another very important point to emphasize is that all TH nuclei ($^2\text{H}$, $^6\text{Li}$, $^3\text{He}$, $^{16}\text{O}$) used so far in the performed experiments (Table 1) are always characterized by an $l = 0$ orbital angular momentum for the inter-cluster $xS$ motion. In this condition, the momentum distribution shows a peak at $p_{xS} = 0 \text{MeV}/c$. This choice is linked not only to the reduction of experimental difficulties when selecting the QF mechanism but also to theoretical considerations for the applicability of the pole approximation [48]. Indeed, the extension to measurements with TH nuclei having $l = 1$ is desirable as it would allow for the investigation of nuclear reactions induced by virtual $^3\text{H}$ and $^3\text{He}$, obtained from the cluster systems $^7\text{Li} = t - \alpha$ and $^7\text{Be} = \alpha - ^3\text{He}$ respectively (see Table 2).

Figure 8. Deuteron experimental momentum distribution (full dots) from the $^2\text{H}(^{10}\text{B}, \alpha^{7}\text{Be})n$ reaction compared with the theoretical momentum distribution in PWIA (black line, same as Fig.7) and with the DWBA momentum distribution evaluated using the FRESCO code (dotted red line).
Table 2. Main feature of candidates Trojan Horse Nuclei.

| TH nucleus | x-S cluster | Momentum l-relative | Binding Energy (MeV) |
|------------|-------------|---------------------|---------------------|
| [1]        | $^2$H       | p-n                | 0                   | 2.22               |
| [2]        | $^3$H       | d-n                | 0                   | 6.26               |
| [3]        | $^3$He      | d-p                | 0                   | 5.49               |
| [4]        | $^6$Li      | d-$\alpha$         | 0                   | 1.47               |
| [5]        | $^7$Li      | d-$\alpha$         | 1                   | 2.47               |
| [6]        | $^7$Be      | $^3$He-$\alpha$    | 1                   | 1.58               |

4.3. Extraction of Two-Body Cross Section from the Measured Three-Body Reaction

If the experimental momentum distribution $|\phi(p_{xS})_{\text{exp}}|^2$ is measured, one can extract the half-off-energy-shell cross section from the three-body coincidence yield:

$$\frac{d\sigma}{d\Omega}^{\text{HOES}} \propto \frac{Yd}{(KF)|\phi(p_{xS})_{\text{exp}}|^2}.$$  \hspace{1cm} (13)

In the analysis, the product $(KF)|\phi(p_{xS})_{\text{exp}}|^2$ is usually calculated by means of a Monte-Carlo simulation. We stress that this relation is valid as far as the pole approximation is applicable. To get the absolute value, the cross section has to be normalized to the binary reaction in an energy range $\Delta E$ above the Coulomb barrier.

In the THM applications, the reaction $x + B \rightarrow C + D$ is induced inside the short-range $B$ nuclear field. Thus, the HOES cross section represents the nuclear part only. For this reason, the penetration probability of the Coulomb barrier has to be introduced to compare the THM cross section with the direct data from literature in the energy region below the Coulomb barrier.

5. Validity test of pole approximation

In the PWIA as well as in the DWIA the three-body reaction cross section is proportional to the cross section of the virtual two-body reaction (Eq. (4)). Thus, the factorization test can be used. Different tests of the pole approximation mechanism have been proposed at low energies. As already mentioned, the analysis of the experimental results for three-body reactions is complicated by the presence of other reaction mechanisms, constituting a background for QF data. Because of the uncertainties that can be introduced by the presence of this background in the applicability of the IA, a series of experiments were carried out to provide some critical tests of pole approximation or of the IA. We present hereafter some of them.

They apply to the behavior of the indirect excitation function: both excitation function $\sigma(E)$ and angular distributions $\sigma(\theta_{cm})$ of the binary reaction $x+B \rightarrow C+D$ must be compared with the corresponding two-body $\sigma(E)^{\text{HOES}}$ excitation function and angular distributions $\sigma(\theta_{cm})^{\text{HOES}}$, respectively (see[26] and references therein).

5.1. Comparison of excitation functions

5.1.1. $E_{xB} \geq E_{CB}$. If the energy $E_{xB}$ of the $x$ and $B$ relative motion is higher than the Coulomb barrier $E_{CB}$ between the particles in the entrance channel of the $x+B \rightarrow C+D$ reaction, a necessary condition for the pole approximation is:

$$\sigma(E_{cm})^{\text{HOES}} \propto \sigma(E_{cm})^{\text{OES}}.$$  \hspace{1cm} (14)
5.1.2. \(E_{xB} \leq E^{CB}\) If \(E_{xB}\) is smaller than the Coulomb barrier \(E^{CB}\), to compare the results with those from direct measurements it is necessary to correct the HOES cross section for the penetration function \(P_l\) through the Coulomb barrier (see Fig. 8); a necessary condition for the validity of the pole approximation is that the cross section of the two-body reaction \(\sigma(E_{cm})^{TH}\) is proportional to the binary direct reaction \(\sigma(E_{cm})^{OES}\):

\[
\sigma(E_{cm})^{TH} \propto \sigma(E_{cm})^{HOES} P_l(k_{xB}r_{xB}) \propto \sigma(E_{cm})^{OES},
\]

(15)

where the penetration probability of the Coulomb barrier is defined by the equation:

\[
P_l(k_{xB}r_{xB}) = \frac{k_{xB}r_{xB}}{F_l^2(k_{xB}r_{xB}) + G_l^2(k_{xB}r_{xB})}
\]

(16)

with \(F_l\) and \(G_l\) the regular and irregular Coulomb wave functions, \(k_{xB}\) and \(r_{xB}\) the \(xB\) relative wave number and interaction radius, respectively.

5.2. Comparison of the angular distribution
As a second test a comparison between the indirectly extracted angular distributions and the direct measurements of the angular distribution behavior is performed. A necessary condition for the pole approximation is that the total cross section of the two-body \(\sigma(\theta_{cm})^{HOES}\) reaction is proportional to the binary direct reaction \(\sigma(\theta_{cm})^{OES}\):

\[
\sigma(\theta_{cm})^{HOES} \propto \sigma(\theta_{cm})^{OES}.
\]

(17)

This applies both above and below \(E^{CB}\). Indeed, the effect of \(P_l(k_{xB}r_{xB})\) on the angular distributions, which is calculated for a given \(E_{cm}\), is only the introduction of an overall scaling factor. Typical results referring to the indirect study of the \(^{11}\)B\((p,\alpha)^{8}\)Be reaction is described in ref [26].

6. Results
The THM provides an independent bare nucleus cross section measurement, \(\sigma_b(E)\) or equivalently the bare nucleus astrophysical factor \(S_b(E)\) (Eqn.(1)). Thus it is possible to extract information on the electron screening potential

\[
[S_b(E)]^{THM} = E \cdot [\sigma_b(E)]^{THM} \cdot \exp(-2\pi \eta).
\]

(18)

Thus, the energy dependence of \([S_b(E)]^{THM}\) should show the same trend of that derived by the direct measurements \([S_b(E)]\), except in the ultra-low energy range \(\Delta E_{ES}\) where the two data sets should differ due to the effects of electron screening:

\[
[S_b(E)]^{THM} \propto S_b(E),
\]

(19)

where \(E > \Delta E_{ES}\). The value for the electron screening potential \(U_e\) can then be obtained by comparing the two data sets.

As already mentioned, the THM does not allow us to extract the absolute value of the astrophysical \(S\) factor. However, the absolute scale for \(S_b(E)]^{THM}\) can be obtained by normalization of the THM data to the direct ones at energies \(E^*\), where the electron screening effects are negligible

\[
N_{\text{abs.value}} = \frac{[S_b(E^*)]^{OES}}{[S_b(E^*)]^{THM}}.
\]

(20)

In case of resonances with only one partial wave (e.g. \(l = 0\)) , the normalization is performed equalizing the areas of the same resonance in both THM and direct data in the case of resonance
reaction. While if the are more than one resonance with different partial waves we need one normalization coefficient for each partial wave. Recent an intense research activity is devoted to the developments of THM for application to resonance reactions. Interesting results were obtained for a number of reactions: \(^2\text{H} (^{15}\text{N}, \alpha^{12}\text{C}) \text{n} \ [28, 29], \ ^2\text{H} (^{18}\text{O}, \alpha^{15}\text{N}) \text{n} \ [30, 31, 56, 57, 58], \ ^2\text{H} (^{19}\text{F}, \alpha^{16}\text{O}) \text{n} \ [32, 60].

The abundances of light elements, such as lithium, beryllium and boron, play a key role in a number of astrophysical problems yet to be completely solved, such as Big Bang nucleosynthesis and the light nuclei depletion. Both production and destruction mechanisms must be studied and their cross sections should be measured at the energies of astrophysical interest. The \((p, \alpha)\) reactions are crucial for the light element destruction in stellar environment \((E_{\text{cm}}=1–50\ \text{keV})\)and the cross sections (or reaction rates) of these reactions are necessary inputs for the astrophysical models of light element abundances in the universe. In this context, great efforts have been devoted to the study, both directly and via THM, of relevant reactions, such as \(^7\text{Li}(p, \alpha)^4\text{He} \ [17, 18, 19], \ ^6\text{Li}(p, \alpha)^3\text{He} \ [22], \ ^6\text{Li}(d, \alpha)^4\text{He} \ [36], \ ^9\text{Be}(p, \alpha)^6\text{Li} \ [24], \ ^{10}\text{B}(p, \alpha)^7\text{Be} \ [25], \ ^{11}\text{B}(p, \alpha)^8\text{Be} \ [25].

**Figure 9.** \(^7\text{Li}(p, \alpha)^4\text{He} \) reaction through \(^2\text{H}(^7\text{Li}, \alpha\alpha)\text{n} \) reaction: (a) Example of nuclear two-body differential cross section from a TH experiment; (b) TH two-body cross section already multiplied by the \(P_l\) as explained in the text [17].
at astrophysical energies.

For some of them, for instance the lithium isotopes, it has also been possible to extract the electron screening potentials \([U_e]^{\text{THM}}\) which were found to be in agreement with the direct \([U_e]^{\text{OES}}\) estimates, but with much smaller errors. More details we can be found in each reference.

7. Conclusions

The THM has proven to be a powerful method to investigate astrophysical relevant reactions without Coulomb suppression or electron screening effects. This method provides the only existing way to measure the relevant bare nucleus \(S(0)\) parameter avoiding extrapolation.

Among future developments, applications of the THM to neutron-induced reactions are planned. As mentioned, a deuteron beam will be used as a virtual source of a neutron beam [37]. Another novel field of application of the THM is that of astrophysical reactions involving radioactive nuclei. The first test on the \(^{18}\text{F}(p,\alpha)^{15}\text{O}\) reaction using the \(^2\text{H}\) as Trojan Horse has been already performed [61].

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