Heat transfer and skin-friction in a nonequilibrium adverse pressure gradient

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Abstract. The paper presents the results of an experimental study of influence of a weak and moderate nonequilibrium adverse pressure gradient (APG) on the parameters of the dynamic and thermal boundary layers. The Reynolds number based on the momentum thickness at the beginning of the APG region was \( \text{Re}^{*} = 5500 \). The section of the channel with APG was a slotted channel with an opening angle of the upper wall of 0-14°. The values of the relative (referred to the parameters in a zero pressure gradient flow at the same \( \text{Re}^{*} \)) friction and heat transfer coefficients, as well as the Reynolds analogy factor depending on the longitudinal pressure gradient, are obtained. The values of the relative friction coefficient decreased to \( c_{f}/c_{f0} = 0.7 \) and those of the heat transfer to \( St/St_0 = 0.9 \). A maximum value of the Reynolds analogy factor \( (St/St_0)/(c_{f}/c_{f0}) = 1.16 \) was reached for the pressure gradient parameter \( \beta = 2.9 \). The ratio of the heat transfer and drag coefficients of the dimpled to smooth surfaces remained approximately constant regardless of the presence or magnitude of an adverse pressure gradient.

1. Introduction

Turbulent Boundary Layers (TBL) with a adverse pressure gradient (APG) can be found almost everywhere. The scientific interest is caused by the presence of additional (in addition to the shear stress on the wall) parameters that control the development of the boundary layer. This causes significant difficulties in modeling, both numerical and experimental.

The experimental study of isothermal, incompressible, equilibrium two-dimensional turbulent boundary layers on flat surfaces was first undertaken by Clauser [1]. In this paper, a dimensionless pressure gradient parameter \( \beta \) is introduced. It was defined as the ratio of the force caused by the longitudinal pressure gradient and the force caused by the shear stress on the wall: \( \beta = (\delta' / \tau_w) (dp/dx) \), where \( \delta' \) is a characteristic boundary layer thickness, \( \tau_w \) is the shear stress on the wall, and \( dp/dx \) is the longitudinal pressure gradient. Also Clauser showed that the displacement thickness \( (\delta') \) can be taken as the characteristic boundary layer thickness. In subsequent works (including this one), the pressure gradient parameter is determined as follows \( \beta = (\delta' / \tau_w) (dp/dx) \). In addition, in Clauser's works, the definition of the equilibrium of the boundary layer was given - this is the constancy of the pressure gradient parameter \( \beta \).

However, in the absence of proper geometry profiling, which presupposes the condition of constant pressure gradient parameter - under the conditions of real currents (diffusers, aerodynamic profiles, etc.), the currents will probably never be in equilibrium. In this regard, modern research in the field of APL is increasingly devoted to non-equilibrium boundary layers.[2–4].
The difficulty in studying such boundary layers lies in the need to take into account the prehistory of the development of the boundary layer, which affects the area of the trace of the boundary layer, together with the local equivalence of its inner area.

In the present work, experimental studies of the influence of a nonequilibrium APG on the heat and momentum transfer on a smooth and dimpled surface are carried out.

2. Description of the experimental setup
Experimental studies were carried out on a subsonic wind tunnel with a working part in the form of a slotted rectangular channel [5–8] (width - $B=300$ mm, height - $H=30$ mm, $(L+L_{APG})=1190$ mm see figure 1). The flow velocity at the channel entry was measured by a Pitot—Prandtl tube and was remained constant ($V_{in}=50$ m/s).

![Figure 1. Schematics of the setup (a) and working channel (b).](image)

The first section of the working channel had a length of 920 mm and was made in the form of a slot. The second section, 270 mm long, was used to create the APG. To create a thermal boundary layer on the lower wall of the channel, it was made heated. To study the friction and drag coefficients of smooth and dimpled models, they were suspended on floating elements in a section with an APG on the bottom wall. The longitudinal pressure gradient was created by changing the geometry of the channel: the slope of the rectilinear upper wall in the range of angles 0–14° with a step of 1°.

The value of the longitudinal pressure gradient $dp/dx$, Pa/m was determined using static pressure taps on the bottom wall. Velocity profiles were measured using a hot-wire anemometer DISA 55M01 CTA equipped with a Dantec Dynamics small-size single-wire 55P81 probe with thermal compensation (the wire length was 1.25 mm and its thickness was 5 μm).

By weighing the models on floating elements [6,7,9], the friction coefficient of the smooth and the drag coefficient of the dimpled models were determined as follows. In the experiment, the total force arising on the floating elements was recorded, including the resultant of friction/drag forces and the pressure drop acting on the ends of the elements. By registering the pressure drop over the length of the floating element, the contribution of this component to the total force on the floating element is
determined. Floating elements occupied the APG section from $x_0=80$ mm to $x_0=205$ mm. The width of the elements was 100 mm. The dimpled surface was set parallel to the smooth model. The staggered array of the spherical dimples was made by milling a dimple on an initially smooth model. The longitudinal and transverse spacing of the dimples was 8 and 9 mm, respectively. The diameter of the sphere of the instrument was 16 mm, and the depth of the dimples was 1 mm. The print diameter of the spherical dimple was 7.75 mm.

To determine the heat transfer coefficient (Stanton number, $St=\frac{\alpha}{\rho \cdot c_p \cdot V}$, $\alpha$ – heat transfer coefficient, W/(m$^2$K), $\rho$ – density of air, kg/m$^3$, $c_p$ - heat capacity at constant pressure, J/(kg·K), $V$ – core velocity above the models, m/s), the transient heat transfer method was used. At the initial moment of experiment the IR camera recorded the temperature field of the model surface. Inhomogeneity of the initial temperature of the dimpled surface did not exceed 5 K, smooth one - 2 K. The process of model cooling was recorded during 60s. The one-dimensional equation of transient cooling of a wall of finite thickness was used. The value thus obtained was averaged over the entire model surfaces.

The uncertainty of measurements of the local friction coefficients by using a hot-wire anemometer was 2.5%. The uncertainty of measurements of the friction and drag coefficients using floating elements was 3.1-9.6% (large values correspond to large $\beta$). The uncertainty of measurements of the heat transfer coefficients did not exceed 5.6%, relative heat transfer coefficients - 7.9%. The uncertainty of measurements of the pressure gradient parameter $\beta$ did not exceed 7.6%.

3. Results of experimental studies
Static pressure profile (expression $c_{p0}=\frac{(p_i-p_{out})}{(\rho \cdot V_{in}^2)/2}$, $p_{out}$ corresponds to the static pressure in the last section considered, $x_0=265$ mm) and the pressure gradient $\frac{dp}{dx}$ are shown in figure 2. The profile

![Image](image_url)

**Figure 2.** Variation in the pressure coefficient $c_{p0}$ (a), the pressure gradient (b) and pressure gradient parameter $\beta$ (c) along the length of the APG region for the channel expansion angles 0…14°.
of the pressure gradient parameter $\beta$ is shown in figure 2, c. For small opening angles the value of $\beta$ remains almost constant in the area of installation of smooth and dimpled models. For large opening angles, a maximum is observed in the range $x_0=80-120$ mm in accordance with the $dp/dx$ distribution along the channel. The parameter of the pressure gradient $\beta$ in the area of installation of the models increases up to the opening angles of $12^\circ$ and then remains almost constant. The influence of the prehistory [4] of the development of the boundary layer in a nonequilibrium pressure gradient can be traced by analyzing the velocity profiles with the same parameters of the pressure gradient (figure 3 for $\beta=1$ and $\beta=2$), but different longitudinal coordinates. As the duration of the action of the longitudinal pressure gradient increases, the velocity profile in the wake region becomes less and less filled and approaches the equilibrium one. The inner part of the boundary layer remains practically unchanged (within the experimental operating parameters).

The imposition of a longitudinal pressure gradient leads to a decrease in the friction coefficient (from $c_f/c_{f0}=1.03$ at $\beta=-0.25$ to $c_f/c_{f0}=0.75$ at $\beta=2.9$ for average over the $x_0=80-200$ mm values (figure 4)).

![Figure 3](image)

Figure 3. Influence of flow prehistory on the velocity profile in APG TBL: a, $\beta=1$; b, $\beta=2$. Equilibrium boundary layer profiles are plotted using the corresponding $V^*$ according to [10].

The dependence of averaged over the $x_0=80-200$ mm region relative heat transfer coefficient $St/St_0$ on the pressure gradient parameter $\beta$ is presented in figure 4. Data of heat transfer coefficient, calculated in paper [11] for the Reynolds number $Re^*=5000$ and the turbulent Prandtl number $Pr_t=1.0$ are also presented in figure 4.

As can be seen from figure 4, the change in $St/St_0$ is not as significant as $c_f/c_{f0}$. In the encompassed range of operating parameters, the thermal boundary layer is less affected by the positive pressure gradient. Nevertheless, the data obtained allow one to assert that in the APG flows the Reynolds analogy factor $(St/St_0)/(c_f/c_{f0})$ is higher than in zero pressure gradient flow.

In general, the results obtained on the heat transfer and friction coefficients fit into the generally accepted trend of violation of the Reynolds analogy towards heat transfer in flows with a adverse pressure gradient (figure 5). Maximum value $(St/St_0)/(c_f/c_{f0})=1.16$ is achieved at $\beta=2.9$. Also in figure 5, the uncertainties of the experimentally obtained values of $(St/St_0)/(c_f/c_{f0})$ are plotted.

According to the results of the experimental study, the drag coefficient of the dimpled surface, referred to the friction coefficient of a smooth surface in the same APG condition does not depend on the presence or magnitude of a APG (within the measurement accuracy): the value $c_x/c_{x0}=1.05-1.12$ for all values of $\beta$, which corresponds to the value obtained during channel flow (figure 5).
The relative heat transfer coefficient related to the heat transfer coefficient of a parallel-standing smooth model also remains practically constant within the parameters covered in the experiment: \( St/St_0 = 1.08 - 1.15 \). Consequently, a further violation of the Reynolds analogy towards heat transfer on dimpled surfaces in the presence of a APG is observed (in comparison with a smooth surface) (figure 5).

4. Summary and conclusion

An experimental investigation of the influence of weak and moderate non-equilibrium adverse pressure gradient on the boundary layer parameters and the heat transfer and friction/drag coefficients of a smooth and dimpled surface is carried out. On the basis of this investigation the following conclusions concerning the effect of the non-equilibrium streamwise adverse pressure gradient on the heat transfer and friction/drag coefficients and the Reynolds analogy factor can be made.

Both local and average relative friction coefficients \( c_f/c_{f0} \) of smooth surface essentially depend on the dimensionless pressure gradient parameter: they diminish with increase in \( \beta \) in the entire parameter range covered in the experiment.

On the smooth surface the dimensionless relative heat transfer coefficient \( St/St_0 \) diminishes in the presence of APG but this reduction is not so considerable as that of \( c_f/c_{f0} \).

**Figure 4.** Dependence of the relative heat transfer and friction coefficients on the pressure gradient parameter \( \beta \).

**Figure 5.** Dependence of the Reynolds analogy factor of a smooth surface (a) and heat-hydraulic efficiency of dimpled surface (b) on the pressure gradient parameter \( \beta \).
The Reynolds analogy factor increases with increase in $\beta$ in APG flows. The maximum value $(St/St_0)/(c_f/c_{f0})=1.16$ is reached at the maximum value of the pressure gradient parameter $\beta=2.9$. Moreover, further violation of the Reynolds analogy towards heat transfer on dimpled surfaces in the presence of APG is observed.

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