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Some torsion classes in the Chow ring and cohomology of $\text{B}\mathbb{PGL}_n$. (English) [Zbl 1475.14009]

J. Lond. Math. Soc., II. Ser. 103, No. 1, 127-160 (2021).

Summary: In the integral cohomology ring of the classifying space of the projective linear group $\mathbb{PGL}_n$ (over $\mathbb{C}$), we find a collection of $p$-torsion classes $y_{p,k}$ of degree $2(p^k+1)+1$ for any odd prime divisor $p$ of $n$, and $k \geq 0$. If, in addition, $p^2 \nmid n$, there are $p$-torsion classes $\rho_{p,k}$ of degree $p^k+1+1$ in the Chow ring of the classifying stack of $\mathbb{PGL}_n$, such that the cycle class map takes $\rho_{p,k}$ to $y_{p,k}$. We present an application of the above classes regarding Chern subrings.

MSC:

14C15 (Equivariant) Chow groups and rings; motives
14B23 Stacks and moduli problems
55R35 Classifying spaces of groups and $H$-spaces in algebraic topology
14L30 Group actions on varieties or schemes (quotients)
55R40 Homology of classifying spaces and characteristic classes in algebraic topology
55T10 Serre spectral sequences

Full Text: DOI arXiv

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