Budgeted Influence Maximization via Boost Simulated Annealing in Social Networks

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Abstract—Due to much closer to real application scenarios, the budgeted influence maximization (BIM) problem has attracted great attention among researchers. As a variant of the influence maximization (IM) problem, the BIM problem aims at mining several nodes with different costs as seeds with limited budget to maximize the influence as possible. By first activating these seed nodes and spreading influence under the given propagation model, the maximized spread of influence can be reached in the network.

Several approaches have been proposed for BIM. Most of them are modified versions of the greedy algorithm, which work well on the IM but seems inefficient for the BIM because huge time consuming is inevitable. Recently, some intelligence algorithms are proposed in order to reduce the running time, but analysis shows that they cannot fully utilize the relationships between nodes in networks, which will result in influence loss.

Inspired by this, we propose an efficient method based on boosted simulated annealing (SA) algorithm in this paper. Three heuristic strategies are proposed to improve the performance and speed up the proposed algorithm. Experimental results on both real world and synthetic networks demonstrate that the proposed boosted SA performs much better than existed algorithms on performance with almost equal or less running time.

Index Terms—Budgeted Influence Maximization, Social Networks, Simulated Annealing, Ensemble Learning.

I. INTRODUCTION

With the development of information technology, many well-known large-scale network cites, such as Wechat, Facebook, Twitter and Youtube, have been widely used in information dissemination and influence diffusion. The current researches indicated that people can be more willing to accept recommendation from their families and close friends than from other channels such as Handbill and newspaper [1], [2]. This so called word-of-mouth social phenomenon can be utilized in the information spread for people to achieve a wide range of influence. Meanwhile, compared with offline social relationships, the online social relationships have more frequent connections with each others and are easier to be computed, which means companies may get higher profits with less investment.

As an important question originated from viral marketing, the IM problem plays a critical role in the influence spread [3], [4]. The original problem scenario is that one company tends to promote their products to consumers, they decide to pay some money or product discount to some online public figures for making advertisements to their followers [5]. There is no doubt that the company wants to get the most influence spread with a given number of selected seeds. Inspired by this social demand, Domingos and Richardson first investigated the problem. They translate it into a network structure and treat it as an optimization problem [6]. Kempe et.al first pointed out that the IM is an NP-hard problem and can only get approximate result [7]. They put forward a greedy algorithm and achieved $(1 - 1/e)$ optimal spread.

However, when trying to use the IM model into practical scenarios, it is found that the IM model can not describe the characteristics of real social networks in many cases. As a matter of fact, as public figures, they have greater social influence than ordinary people, which means that they deserve to be paid more money to promote products or make advertisements. This condition is different from the setting in the IM problem, which assumed that all seed nodes have uniform cost. Usually the company has a given limited budget instead of a given number of seeds. Nguyen and Zheng investigated the BIM problem firstly and a greedy algorithm was proposed [8].

In fact, variants of the IM problem have been investigated recently, such as the time-constrained IM [9], targeted IM [10], [11], community-aware IM [12], opinion maximization [13], etc. In BIM, each seed node has different costs. Leskovec et.al put forward an improved greedy algorithm to deal with the BIM [14]. Although several strategies have been made to reduce the runtime, it is impossible to overcome the intrinsic flaw of enormous computation in a greedy algorithm. Without using a greedy strategy, Han et.al come up with a combination simulated annealing (Combination SA) for the BIM [15], which reduces the runtime dramatically. In fact, the combination SA provides a mechanism which gets balance between the accuracy and efficiency, but it ignores the relationships among nodes.

In this paper, we propose a seed selection method based on boosted simulated annealing (Boost SA) to deal with the BIM problem by skillfully using the topology of networks. Our main contributions are listed as follows:

1) By using the information of the node’s connections, the accuracy is improved and the candidate space of seed nodes is dramatically reduced, which reduces the runtime.
2) The voting mechanism is introduced to strengthen random substitution process based on the idea of ensemble learning, which improves the accuracy.
3) Adaptive interrupt mechanism is also used to further reduce the runtime. With fixed iteration number, the SA may waste time on useless computation. The adaptive
interrupt mechanism can stop the iteration if there is no further improvement.

Experiments on three real world networks and synthetic networks show that the proposed Boost SA algorithm performs much better than the state-of-art Combination SA algorithm [15] with equal or less runtime.

The rest of this paper is organized as follows: Section 2 is the related works and motivations; Section 3 is the problem formulation and typical diffusion models. Section 4 provides the detailed of the proposed Boost SA. Section 5 is the experimental results and analysis. Section 6 is the conclusion.

II. RELATED WORKS AND MOTIVATIONS

A. Related Works

Influence maximization (IM) problem. Without considering the budget, the IM problem is formulated as how to find $k$ nodes (as seeds) such that the expected number of propagated nodes by the $k$ seeds is the largest as possible. As one of the main problems originated from viral marketing, the IM has attracted great attention among researchers. The IM was first studied by Domingos and Richardson [6]. Kempe et.al proved that it is NP-hard and can only get approximate solution [7]. They put forward a greedy algorithm under the independent cascade (IC) model and linear threshold (LT) model. This algorithm gets $(1-1/e)$ optimal spread with expensive computation. Chen et.al come up with two methods [16], named as NewGreedy and MixedGreedy, to improve the greedy algorithm. Kundu and Pal proposed a deprecation based greedy strategy [17], in this strategy, they estimate the performance of each node and mark the nodes to be depreciated. They also proved that if the influence function is monotonic and sub-modular, this strategy can provide a guaranteed seed set.

In addition to these methods based on a greedy algorithm, some simple heuristic approaches are proposed [7], [18], [19], including selecting the seed nodes by their degree, centrality, and many other indexes to measure the importance of the nodes. These strategies are easy to carry out but inevitably ignore the connections between nodes. In order to deal with the expensive Monte Carlo simulations in greedy methods, Jiang et.al introduce an influence approximation index called the expected diffusion value (EDV) to compute the potential influence spread of each node [20]. Meanwhile, they artfully combine SA with the proposed EDV.

As an intelligent algorithm, SA was proposed by Metropolis et.al in 1953 [21]. The main idea of SA is to simulate the process of metal annealing. Ref [20] is the first work utilizing SA for the IM problem, which contains two layers of circulation, the outer circulation is controlled by temperature parameter $T(t)$ and the inner circulation is controlled by constant parameter $q$. The method operates as follows:

1) From the initial seed set $A$, initial temperature $T_0$, calculate the influence spread $\sigma(A)$ by EDV.
2) Create a neighbor set $A'$ of $A$ and calculate $\sigma(A')$. If $\Delta E = \sigma(A') - \sigma(A) \geq 0$, replace $A$ with $A'$; else if $\Delta E = \sigma(A') - \sigma(A) < 0$, replace $A$ with $A'$ by probability $exp(\Delta E/T)$. The iteration will go on until the temperature comes to the given threshold.

Swarm intelligence methods are also proposed to solve the IM problem. Gong et.al proposed a discrete particle swarm optimization method (DPSO) which combines with the extended EDV and get improvements in both influence spread and runtime [22].

Budgeted influence maximization (BIM) problem. The BIM is formulated as how to select a number of seeds with given limited budget to maximize the influence spread. Obviously, the IM problem is a special case of the BIM when the costs to activate the nodes are equal, thus the algorithms developed for the BIM can be directly applied to the traditional IM problem.

Leskovec et.al first build up a method called Cost-Effective lazy Forward (CELF) for the BIM, which uses the submodularity property to speed up the algorithm and it is much faster than a simple greedy algorithm [14]. Nguyen and Zheng identify the linkage between the computation of marginal probabilities in Bayesian networks and the influence spread, they propose a method by estimating the influence spread via belief propagation on a underlying directed acyclic graph [8].

Although several strategies have been made to speed up the greedy strategy, it is not a good choice due to its intrinsic flaw of expensive computation. Instead of using a greedy strategy for evaluating the influence spread of a node or node set, Han et.al provide two strategies, named Billboard strategy and Handbill strategy for evaluating and selecting seed node, then by balanced using the two strategies via simulated annealing, a new method (Combination SA) is proposed [15].

As shown in Fig.1, the main processes of Combination SA are summarized as follows.

Step 1. Dividing the nodes of the network into two subsets by their degrees: the set of the nodes with the top 20% largest degree (denoted as $T$) and the set of the nodes with the bottom 80% smallest degree (denoted as $H$).

Step 2. Constructing the candidate seed set (named billboard set and denoted as $S_h$) by the billboard strategy, which iteratively selects the most influential node from $T$ by an influence indicator until the budget $B$ is reached. The influence indicator evaluates a node’s influence with only the node’s 1-hop and 2-hop neighbors information (see Eq.5).

Step 3. Constructing the candidate seed set (named handbill set and denoted as $S_h$) by the handbill strategy, which iteratively selects the most cost-effective node from $H$ by an indicator until the budget $B$ is reached. The cost-effective indicator considers not only the node’s influence but also its cost with it’s 1-hop and 2-hop neighbors information (see Eq.6).

Step 4. Set $S_h$ as the initial seed set $S$, then try to replace each seed node in $S$ with nodes in $S_h$ that has equal costs. Acceptance or rejection of the replacement is decided by SA. If accepted the replacement, go to Step 5.

Step 5. Randomly replace a node in $S$ with nodes in $S_h$ with equal costs. Similarly, acceptance or rejection of the replacement is judged by SA. This operation execute $q$ times.

When all nodes in set $S$ have been processed by Step 4 and Step 5, the algorithm is finished. As a replacement of
traditional greedy algorithm, SA is a good choice to find a solution for the BIM problem. In fact, the Combination SA has achieved good results on the BIM problem.

Yang et al. formulate the BIM as a Multi-Objective optimization problem and utilize Discrete Particle Swarm Optimization algorithm (MODPSO) to deal with [23], which is much faster than a greedy algorithm but is slower than the Combination SA.

B. Motivations

Motivation 1: Make use of the relations between nodes of $T$ and $H$. As introduced above, Combination SA constructs two candidate sets: billboard set ($S_b$) and handbill set ($S_h$), which come from $T$ and $H$ respectively. The separation of $T$ and $H$ is merely based on the nodes’ degrees. Thus, the separation of $S_b$ and $S_h$ ignores the relations (edges) between the nodes of $T$ and $H$, which results in unnecessary budget waste (performance descending). For example, a node is selected into $S_b$, its direct neighbor may be selected into $S_b$ if the edge information is ignored, which is a waste of budget. So, the first motivation is to construct only one candidate seed set.

Motivation 2: Reducing the searching space. As a whole, the two candidate sets including billboard set $S_b$ and handbill set $S_h$ are too large, iteratively search in which will result in long runtime. As introduced above, the billboard set $S_b$ is constructed by selecting the most influential node from $T$ one by one until the budget $B$ is reached. The handbill set $S_h$ is constructed by selecting the most cost-effective node from $H$ one by one until the budget $B$ is reached. The candidate set $S_b + S_h$ has nodes with total cost of $2B$, by constructing only one candidate set $C$ the search space can be reduced to nodes with total cost of about $1.5B$.

Motivation 3: The searching scheme of Combination SA is trying to replace each node in $S_h$ by several nodes in $S_b$, the resulted set is the obtained seed set. In fact, starting from different initial sets, after the replacing process by simulated annealing, the resulted seed sets are different. The third motivation is to start from $k$ different initial sets selected from the candidate set $C$ with beget $B$, obtains $k$ initial seed sets. By designing a voting strategy to obtain the initial seed set from the $k$ seed sets. The voting strategy is also used to obtain the final seed set, which improves the accuracy but increases the runtime.

III. PROBLEM DESCRIPTION AND PRELIMINARIES

In this section, we first formulate the BIM problem in mathematical language. Then typical diffusion models are introduced which describe the diffusion behavior in different scenarios. Finally, the influence indicators for a node or nodes set are analyzed. Some important notations are listed in Table I as follows:

| Notation | Details |
|----------|---------|
| $G(V, E)$ | $G$ is the target network, $V$ and $E$ are the nodes and edges set in $G$ respectively. |
| $S$ | Seed node set. |
| $B$ | The given constant budget. |
| $c(S)$ | The total cost of seed set $S$. |
| $\sigma(S)$ | The influence spread of seed set $S$. |
| $\sigma(v_i)$ | The cost-effective value of node $v_i$. |
| $N(v_i)$ | The set of direct neighbors of node $v_i$. |
| $N_{in}(v_i)$ | The set of direct in-degree neighbors of node $v_i$. |

3.1 Formulation of the BIM Problem: Before formulating of the BIM problem, the ability of the influence spread of a single node and a seed set should be explained.

Influence spread. The ability of influence spread of a single active node $v_i$ is defined as the expected number of nodes which can be activated by $v_i$, which is denoted as $\sigma(v_i)$. Similarly, the ability of influence spread of a seed set $S$ is defined as the expected number of nodes which may be activated by $S$, which is denoted as $\sigma(S)$.

Budgeted influence maximization (BIM). A social network is usually modeled as a graph $G(V, E)$, where $V = \{v_1, v_2, \ldots, v_n\}$ is the set of individuals (nodes) and $E = \{e_{ij}\}$ is the set of edges. For each $v_i \in V$, it has a cost $c(v_i)$ when it is selected to be a seed node. Given the budget $B$, the BIM problem aims at obtaining the most influence spread $\sigma(S)$ by selecting a set of seed nodes $S$ under the budget constraint, which is formulated as follows:

$$S = \arg \max_{S \in V} \sigma(S), \quad S \in V,$$

subject to $\sum_{v_i \in S} c(v_i) \leq B.$

(1)

When $c(v_i) \equiv 1$, $\forall v_i \in V$, the BIM equals to the original IM problem, thus the BIM is a generalization of IM.

3.2 Diffusion Model: The BIM problem can be seen as the combination of the seed selection and influence diffusion. There are several diffusion models to describe the influence diffusion process in previous literatures. Here we introduce three main typical diffusion models and some of their variants.

Independent Cascade (IC) Model. In this model, after a node (e.g. $v_i$) is activated in step $t$, $v_i$ has only one chance to activate each of its inactive neighbors (e.g. $v_j$) with probability in step $t+1$ [15], [20]. Whether $v_j$ is successfully activated by $v_i$ or not is independent of the history of diffusion before $v_i$ is activated. If node $v_j$ has $L$ ($L \geq 2$) neighbors which are activated in step $t$, $v_j$ will have $L$ chances to be activated by its neighbors sequentially in step $t+1$.

Linear Threshold (LT) Model. In this model, each inactive node $v_j$ is influenced by all its active neighbors according to the summation of edges weight: $p_j = \sum p_{ij}$, where $v_i$ is active. $\Theta_j$ is set as the threshold for activating $v_j$. If $p_j \geq \Theta_j$ in step $t$, node $v_j$ is activated in that step. The threshold $\Theta_j$ can be defined node-specific, e.g. uniformly distributed in the
interval $[0, 1]$, or adopt an identical value like $1/2$ [1]. The process will stop until no more node can be activated.

**Triggering Model.** In this model, each node $v_j$ independently select a set of nodes from its neighbors according to some distribution, denoted as $T_j$. If $v_i$ is activated in step $t$ and $v_i \in T_j$, then $v_j$ will be activated in step $t + 1$ [7], [24].

In recent years, there are researches about model independent for the IM problem [25] and model considering heterogeneous nodes [26]. The same as that in Refs [15], [20], in this paper, we study the BIM problem in the IC model.

3.3 Influence Approximation Indicator: Seed selection strategy is an important issue for the BIM problem. It’s necessary to come up with an effective indicator to evaluate the influence of a node or nodes set. Since the computation of $\sigma(S)$ is NP-hard [27], [28], some simple indicators have been proposed.

**Degree centrality** [7]. The main idea is that if a node has more neighbors, it’s more likely to have greater influence on the network. So degree centrality selects nodes with the maximum degree in descending order until the budget runs out. The degree of node $v_i$ is computed by:

$$d_i = \sum_{i \neq j} e_{ij} \quad (2)$$

In Eq.2, if node $v_i$ and node $v_j$ have an edge, $e_{ij} = 1$, otherwise $e_{ij} = 0$. Degree centrality may be the simplest indicator.

**Expected diffusion value (EDV) for seeds set** [20]. EDV considers not only the degree of a node but also its activation probability, which is computed by:

$$EDV(S) = k + \sum_{i \in N(S) \setminus S} 1 - (1 - p)^{\tau(i)}, \quad (3)$$

In Eq.3, $k$ is the number of nodes in set $S$, $N(S)$ is the set of direct neighbors of nodes in $S$, $v_i \in N(S) \setminus S$ indicates that $v_i$ is a direct neighbor of nodes in $S$ but $v_i$ itself doesn’t belong to $S$, $p$ is the activation probability with preset value, and $\tau(v_i)$ is the number of edges between $v_i$ and the nodes in $S$.

In fact, Eq.3 only considers 1-hop nodes, in other words, the seeds and the direct neighbors of the seeds set.

**2-hop influence spread for seeds set** $\sigma_2(S)$ [29]. As well known, the computation of $\sigma(S)$ is expensive, degree centrality ignores more long-distance information. A compromise method is only computing up to the 2-hop nodes, which results in a fast influence indicator $\sigma_2(S)$. Specifically, $\sigma_2(S)$ is the nodes that may be activated by any a node in $S$ along a path equal or less than 2 hops. The indicator is computed by:

$$\sigma_2(S) = \sum_{v_i \in S} \sigma_2(v_i) - \left( \sum_{v_i \in S} \sum_{v_i \in N(v_i) \cap S} p(v_i, v_i) (\sigma_1(v_i) - p(v_i, v_i)) \right) \quad (4)$$

where $\sigma_2(v_i)$ is the nodes that may be activated within 2 hops by $v_i \in S$. Obviously, if a node is in 2-hop of $v_i \in S$ and is also in 2-hop of $v_j \in S (v_i \neq v_j)$, the node maybe counting two times in $\sum_{v_i \in S} \sigma_2(v_i)$, the second the third terms aim at removing those repetitive nodes.

The second term is the repetitive nodes in 1-hop. In details, $N(v_i)$ is the direct neighbors of $v_i$, $p(v_i, v_i)$ is the probability that active node $v_i$ may activate inactive node $v_i$, $\sigma_1(v_i) = 1 + \sum_{v_k \in N(v_i)} p(v_i, v_k)$, which is 1-hop influence of node $v_i$. The third term $\chi$ is the repetitive nodes in 2-hop, which is computed by:

$$\chi = \sum_{v_i \in S} \sum_{v_j \in N(v_i) \cap S} \sum_{v_k \in N(v_i) \cap S} p(v_k, v_j) p(v_i, v_d),$$

which represents repeated influence calculation of two seed nodes with a distance of 2-hops.

**Expected diffusion value (EDV) for a node** [15]. Han et.al. extend Eq.3 for evaluating the influence of a node by considering 2-hop nodes, which is computed by:

$$\sigma_2(v_i) = 1 + \sum_{v_j \in N(v_i)} (1 + outd(v_j) * p) * p \quad (5)$$

where, $outd(v_i)$ is the out-degree of $v_i$. In Eq.5, $\sum_{v_j \in N(v_i)} p$ is the number of the potentially influenced 1-hop neighbors, and $\sum_{v_j \in N(v_i)} outd(v_j) p^2 - \sum_{v_k, v_j \in N(v_i), e_{uk} \in E} p^3$ is the number of the potentially influenced 2-hop neighbors.

**Cost-effective diffusion value (CEDV) for a node** [15]. Eq.5 does not considering the cost of selecting a seed node. With considering the cost, the cost-effective diffusion value is computed by:

$$ce\sigma_2(v_i) = \sigma_2(v_i) / c(v_i), \quad (6)$$

where $c(v_i)$ is the cost to activate $v_i$, $ce\sigma_2(v_i)$ is the 2-hop EDV of $v_i$ and is computed by Eq.5.

As introduced above, the CEDV computed by Eq.6 considered up to 2-hop neighbors and the cost for selecting a seed, which is also computation efficient. As shown in Eq.1 for the BIM problem, the sum of the costs of the seed nodes should not larger than the budget $B$, so $ce\sigma_2(v_i)$ is chosen as the influence indicator in this paper.

IV. **Boost SA for the BIM**

The effects of the topology of a network are analyzed and a strategy for candidate seed set selection is introduced in Subsection 4.1. Details of the proposed Boost SA are introduced in Subsection 4.2. Two tricks of the Boost SA are summarized in Subsection 4.3.

4.1 Candidate seed set Selection

As well known, different nodes have different influence usually. Based on statistical results of many real networks, the degree of nodes conforms to the power law distribution. Meanwhile, in many real scenarios, a little part of entities (nodes) usually take control of most of the resources. This interesting phenomenon is often summarized as “2/8 rules”:
The main idea of this combination is to find cost-effective nodes or several nodes in $S$, a good choice to select nodes merely focus on its influence. It is obvious that the degree distribution follows a power law.

When considering the BIM problem by utilizing the 2/8 rules on the structure of a network, there is no doubt that the top 20% nodes are of great value to achieve great influence spread. However, for the BIM, the node cost is about proportional to the number of out-degree, which means nodes with greater influence has more expensive cost. If we try to find out suitable seed set with cost-performance ratio, it is not a good choice to select nodes merely focus on its influence.

From Eq.6, it is obvious that the indicator $ce_2(v_i)$ tends to find nodes with low degrees but have powerful neighbors. In other words, the indicator pays more attention to nodes whose neighbors contain more out-degrees rather than the node itself has more direct neighbors. Since it’s not economy to select nodes with high degree centrality, as a concession, it is better to select their neighbors with low degrees so that the most influential nodes can be activated by the seeds.

For convenience, the set of top 20% nodes with the maximum out-degree in the network is denoted as $T$, the set of bottom 80% nodes with the minimum out-degree in the network is denoted as $H$. In the Combination SA [15], two candidate seed sets are proposed, which are called Billboard set ($S_b$) and Handbill set ($S_h$) respectively (see Section II.A for details). $S_b$ is selected from $T$ and $S_h$ is selected from $H$. Both $S_b$ and $S_h$ are selected with budget $B$. The final seed set $S$ is obtained by iteratively replacing each node in $S_b$ by a or several nodes in $S_h$ with less or equal cost. Although the main idea of this combination is to find cost-effective nodes by the replacement, but the separation of two candidate sets ignores the connections of nodes between the two sets, which will result in unnecessary budget waste.

This paper proposes to use only one candidate set $C$ to reduce the solution space and utilize the connections between nodes, detailed flow chat is shown in Algorithm 1 and the components of $C$ are shown in Fig.4. Algorithm 1 calculates the cost-effective value $ce_2(v_i)$ of $v_i$ in $H$ by Eq.6 and arrange them in descending order (lines: 4-7). Select $v_i$ as candidate seed node if the total budget is less than $\alpha B$ and the out-direction neighbors of $v_i$ are not in candidate set $C$ (lines: 9-16)($C_1$ in Fig.4). When the budget runs out, check node $v_i \in T$, if $v_i$ has no 1-hop and 2-hop in-degree neighbor in $C(C_1)$, check all its neighbors and find out the nodes belongs to top $\beta\%$ of set $H$ then $C \leftarrow C \cup v_i$ (lines: 17-28)($C_2$ in Fig.4). Here, $\alpha$ and $\beta$ are two parameters to control the search space.

Instead of selecting influential nodes in $T$, the main idea of Algorithm 1 is selecting cost-effective nodes in $H$ to activate influential but costly nodes in $T$. As shown in Fig.4, candidate set $C$ composed of two parts $C_1$ and $C_2$. When select nodes from $H$ for $C_1$, the total budget is $\alpha B$ instead of $B$, where $1 < \alpha < 2$, which controls the solution space. In the experiments
of this paper, $\alpha = 1.5$. Too small value of $\alpha$, e.g., $\alpha = 1$ (selecting candidate seed set with budget $B$), will loss too much potential optimal nodes.

When select nodes from $H$ for $C_2$, Algorithm 1 checks all nodes in $T$ and finds out nodes which can’t be reached in 1-hop or 2-hop paths from a node in $C_1$. As shown in Fig. 4, some nodes in set $T$ (denoted as $(T_1)$) can be directly reached by nodes in $C_1$. Some other nodes in set $T$ (denoted as $T_2$) can be reached by nodes in $T_1$, thus can be reached by nodes in $C_1$ within 2 hops. Apart from $T_1$ and $T_2$, there are still many nodes in $T$ (denoted as $T_3$), which are influential and costly. Experiments on three real world networks are shown in Tables II-IV, which indicate that with the increase of budget, more and more nodes in $T$ can be reached, but there are still some nodes can’t be reached within 2-hops. Find a neighbor for some nodes in set $\alpha$, e.g., within 2 hops. Apart from $T_1$, $T_2$, and $T_3$, which have promising value of $ce_2$ and add it into $C$ (line 20-26), denoted as $C_2$ in Fig.4. In order to activate the nodes in $T_3$ as much as possible and select cost-effective nodes as candidate seeds, we set $\beta\% = 60\%$.

**Time Complexity of Algorithm 1.** To calculate out-degree of each node takes $O(n)$ time (line 1-3), where $n$ represents the number of network nodes. The cost-effective value calculation of nodes needs $O(n^2)$ time (line 4-6); the arrangement of nodes in $H$ spends $O(n)$ time (line 7); nodes selection needs $O(kn)$ time (line 9-16); check of $T$ and construct $C_2$ needs $O(n^2)$ time (line 17-25). Therefore, the time complexity of Algorithm 1 is $O(n^2)$.

When selection of the candidate seed set $C$ is finished, the next step of the proposed Boost SA is selecting nodes from $C$ for initial seed set $S$ (initialization).

**TABLE II:** $T_1$, $T_2$, and $T_3$ in the URV email network

| Budget | $C$ | $T_1$ & $T_2$ | $T_3$ |
|--------|-----|--------------|-------|
| 50     | 62  | 187          | 39    |
| 100    | 75  | 217          | 9     |
| 150    | 114 | 224          | 2     |
| 200    | 156 | 224          | 2     |
| 250    | 207 | 225          | 1     |
| 300    | 262 | 225          | 1     |

**TABLE III:** $T_1$, $T_2$, and $T_3$ in the Wiki-Vote network

| Budget | $C$ | $T_1$ & $T_2$ | $T_3$ |
|--------|-----|--------------|-------|
| 1000   | 901 | 450          | 1209  |
| 1500   | 1237| 541          | 1118  |
| 2000   | 1658| 564          | 1095  |
| 2500   | 2179| 602          | 1057  |
| 3000   | 2787| 653          | 1006  |
| 3500   | 3426| 682          | 977   |
| 4000   | 4118| 687          | 972   |

**TABLE IV:** $T_1$, $T_2$, and $T_3$ in the NetHEPT network

| Budget | $C$ | $T_1$ & $T_2$ | $T_3$ |
|--------|-----|--------------|-------|
| 1000   | 941 | 1259         | 1106  |
| 1500   | 1354| 1337         | 1064  |
| 2000   | 1840| 1350         | 1051  |
| 2500   | 2393| 1359         | 1042  |
| 3000   | 2995| 1359         | 1042  |
| 3500   | 3625| 1359         | 1042  |
| 4000   | 4173| 1359         | 1042  |

**4.2 The proposed Boost SA**

The proposed Boost SA is based on the idea of simulated annealing. The random initialization strategy is used to produce the initial seed set. The pseudocode of initialization is shown in Algorithm 2. At first, $k$ initial seed sets $S_i$, $i = 1, ..., k$, are randomly selected from $C$, each with budget $B$ (line 2). For each $S_i$, a node in $S_i$ is randomly replaced to produce its neighbor set (line 7), and the optimal solution is selected based on the idea of simulated annealing algorithm. That is to choose the optimal solution as much as possible, and select the sub-optimal solution with a certain probability to jump out of the local optimal solution (line 9-16). At the same time, the number of times each node is selected is recorded (line 18). Finally, the initial seed set $S$ is obtained according to the number of times each node in $C$ is selected (line 20-21).

The pseudocode of Boost SA is shown in Algorithm 3. As shown in Algorithm 3, from the candidate seed set $C$, Boost SA has two stages of iteration. The outer iteration is controlled by the initial temperature $t_0$ and the threshold temperature $t_f$. In outer iteration, we create $GP$ groups of $S$ and each group will go through the inner iteration.

The inner iteration is controlled by the number of total groups $GP$ and the number of loops $q$. In inner iteration, randomly select a node in set $S$ and replace it with the equal (or less) costs of nodes in $C$. If the influence difference is bigger than zero, accept it; otherwise, create a random value in $(0, 1)$ and compare it with $exp(-\Delta E/T_i)$ and decide whether to accept or reject the replacement. It is worth pointing out that this comparison with random value allows to accept a worse solution, which can help of jumping out of local optimal and find a better solution. The replacement above will be performed for $q$ time. Calculate the number of times that each nodes are selected in all $GP$ groups, which is a nodes list, denoted as $L(C)$. In $L(C)$, the nodes are sorted in descending order for selecting nodes for $S$ (see Algorithm 4).

We record the influence value of set $S$ per iteration in $max_{influence}$. If the value of $max_{influence}$ has repeated for $num$ times, the iteration is stopped. Then break the iteration and return the final set $S$ (line 22-24 in Algorithm 3).

**4.3 Two Tricks for Optimization**

**Trick 1. Voting for new seed set $S$.** The computation of traditional Greedy Algorithm is expensive. In order to cut down the running time, SA Algorithm is utilized to replace it. As described above, the main structure of Combination SA Algorithm [15] is two stages of iteration. The outer iteration is controlled by the number of nodes in Billboard set and the inner iteration is controlled by constant value $q$. In essence, Combination SA performs millions of iterations just in order to fully utilize the replacement operation and obtain reliable results. Random replacement is introduced to strengthen the ability to jump out of local optimum. However, this trick will also bring in uncertainty which has a negative influence on final solution due to the fact that set $S$ is a whole group.

As a popular theory in machine learning, Ensemble Learning has been widely used in many famous algorithms (e.g.
Algorithm 2 Initialization.

Input: Graph $G(V, E)$, budget $B$, candidate seed set $C$, initial temperature $t_0$, the number of loop $q$, the number of $k$.

Output: The initial seed set $S$;
1: $temp\_q \leftarrow q$;
2: Randomly generate $k$ initial seed sets $S_i$ from $C$, $i = 1, \ldots, k$, $S_i$ has budget $B$;
3: for $i = 1, \ldots, k$ of $S_i$ do
4: $q \leftarrow temp\_q$, $flag \leftarrow false$;
5: while $q > 0$ do
6: $q \leftarrow q - 1$;
7: Randomly replacing a node in $S_i$ by nodes in $C$ with less than or equal cost, formulate a neighbor set $S'_i$;
8: Compute the influence difference $\Delta E \leftarrow \sigma(S'_i) - \sigma(S_i)$;
9: if $\Delta E > 0$ then
10: $S_i \leftarrow S'_i$, $flag \leftarrow true$;
11: else
12: create a random number $\varepsilon \in U(0, 1)$;
13: if $exp(\Delta E/t_0) > \varepsilon$ then
14: $S_i \leftarrow S'_i$;
15: end if
16: end if
17: end while
18: Record the number of times that each node in set $C$ is selected, denoted as $L(C)$;
19: end for
20: Let nodes in set $C$ are sorted in descending order of the selected number;
21: Select nodes in turn from set $C$ into set $S$ with budget $B$;
22: return initial seed set $S$;

Algorithm 3 Seed set selection based on Simulated Annealing.

Input: Graph $G(V, E)$, budget $B$, candidate seed set $C$, initial temperature $t_0$, temperature drop $\Delta T$, Threshold temperature $t_f$, Objective function $\sigma(C)$, the number of loop $q$, the number of group $GP$ and set number $num$, initial seed set $S$.

Output: The final seed set $S$;
1: $T_i \leftarrow t_0$, $temp\_q \leftarrow q$;
2: $S \leftarrow$ initial seed set $S$;
3: while $T_i > t_f$ do
4: for $i = 1, \ldots, GP$ do
5: $q \leftarrow temp\_q$, $flag \leftarrow false$;
6: while $q > 0$ do
7: $q \leftarrow q - 1$;
8: Randomly replacing a node in $S$ by nodes in $C$ with less than or equal cost, formulate a neighbor set $S'$;
9: Compute the influence difference $\Delta E \leftarrow \sigma(S') - \sigma(S)$;
10: if $\Delta E > 0$ then
11: $S \leftarrow S'$, $flag \leftarrow true$;
12: else
13: create a random number $\varepsilon \in U(0, 1)$;
14: if $exp(\Delta E/T_i) > \varepsilon$ then
15: $S_i \leftarrow S'$;
16: end if
17: end if
18: end while
19: Record the number of times that each node in set $C$ is selected, denoted as $L(C)$;
20: end for
21: Update $S$ and $max\_influence$ by Algorithm 4;
22: if the value of $max\_influence$ is repeated for $num$ times then
23: Break;
24: end if
25: $T_i \leftarrow T_i - \Delta T$;
26: end while
27: return set $S$;

Algorithm 4 Voting for new set $S$.

Input: $L(C)$.

Output: $S$ and $max\_influence$.
1: Let nodes in set $C$ are sorted in descending order of the selected number;
2: $temp\_S \leftarrow S$, $S \leftarrow \emptyset$;
3: Select nodes in turn from set $C$ to set $S$ with the budget $B$;
4: if $\sigma(S) - \sigma(temp\_S) > 0$ then
5: $max\_influence \leftarrow \sigma(S)$;
6: else
7: $S \leftarrow temp\_S$;
8: $max\_influence \leftarrow \sigma(temp\_S)$;
9: end if
10: return $S$ and $max\_influence$;
nodes $K$ in Billboard set, and the inner iteration is controlled by constant value $q$, which means the total number of iterations is $K*q$. As the budget $B$ grows, the number of outer iteration keeps on growing and the runtime is proportional to the budget increase. This property has a negative effect on practical application.

Experiments are done to show the influence spread computed by Eq.5 with different budgets, the results on Wiki-Vote network are shown in Fig.5, which shows that the influence spread remains unchanged after several iterations, more number of outer iteration is only a waste of running time. Similar results are found in the URV email and NetHEPT datasets, which are not shown here. Inspired by this observation, we record influential value $max_{influence}$ of set $S$ in each iteration and compare it with previous $num$ influential values $max_{influence}$. If the past $num$ $max_{influence}$ equal to the latest $max_{influence}$, the algorithm believes further iteration has out of action and break the iteration. Runtime comparison in next section will verify the advantage of this adaptive interrupt strategy.

**Time Complexity.** The initialization by Algorithm 2 takes $O(k*q*n)$ time. In Algorithm 3, the outer iteration is controlled by the value of $GP$ and $(t_0 - t_f)/\Delta T$. The inner iteration is controlled by value $q$. In inner iteration, the calculation of influence indicator in line 9 takes $O(n)$ time. Voting for new set $S$ by Algorithm 4 (line 21 in Algorithm 3) takes $O(n)$ time. Therefore, the running time of Algorithm 3 is $O((t_0 - t_f)*GP + q*n/\Delta T)\approx O(n^2)$. So the time complexity of the Boost SA is $O(n^2)$.

### V. Experiments

#### A. Network Datasets

Three real world networks and four synthetic networks are used in experiments. The parameters of the three real world datasets are summarized in Table V:

| Network     | Nodes | Edges | Average Degree |
|-------------|-------|-------|----------------|
| URV email   | 1133  | 10903 | 9.62           |
| Wiki-Vote   | 7115  | 103689| 14.57          |
| NetHEPT     | 12008 | 237010| 19.74          |

Several parameters listed in Table VI, the number of nodes are 2000, 5000, 10000 and 30000 respectively.

| Definition | Symbol | Value |
|------------|--------|-------|
| Number of vertices | N | 2000/5000/10000/30000 |
| Average Degree | avg-D | 5/10/10/10 |
| Maximum Degree | max-D | 50 |
| Mixing parameter | $\mu$ | 0.15 |
| Minimum for the community sizes | min-c | 20 |
| Maximum for the community sizes | max-c | 50 |
| Exponent for the degree distribution | Exp-D | 2 |
| Exponent for the community size distribution | Exp-C | 1 |

#### B. Comparing Algorithms

**MODPSO.** The MODPSO algorithm [23] is a modified particle swarm optimization method. It considers the BIM problem as a multi-objective optimization problem and try to maximize the number of activated nodes with least budget. As a novel method, it’s persuasive to make a comparison with it.

**CELF.** The CELF algorithm [14] is an improvement on the greedy algorithm. It can spread up the latter for 700 times. CELF has been the baseline method for IM problem. Since the algorithm is designed for IM problem, we make some changes so that it can satisfy the BIM problem.

**MaxDegree.** The MaxDegree algorithm [7] is a heuristic method that can be easy to come up with. In this algorithm, degree is the only indicator to calculate the ability of influence spread. It sorts nodes in descending order and selects nodes with budget $B$.

**Combination SA.** As introduced in Section II, the Combination SA [15] is the first algorithm that utilizes simulated annealing method to deal with the BIM problem.

**Boost SA.** The proposed algorithm as described in Algorithm 3, which is an improvement over the Combination SA algorithm.

The IC model is adopted for all experiments. In order to keep the accuracy of different algorithms, we adopt simulated method [7] to compute the final influence spread of selected seed set of each algorithm. 10000 times of Monte-Carlo simulation are performed and we take the average influence spread as the expected result. In order to keep the reliability, each algorithm is independently run 30 times on each dataset. All experiments are performed on a computer with 2.10GHz AMD Ryzen5 and 16G memory. All algorithms are written in MATLAB R2017a.
C. Parameter Setting

For Combination SA algorithm, we directly utilize the settings of parameters in Ref [15]: \( t_0 = 1000000, q = 1000, \Delta T = 1000 \) and \( t_f = 1000000 \).

For convenience in comparison between Combination SA and the proposed Boost SA, the same parameter settings are used in the Boost SA. Besides, we set \( k = 10, GP = 3 \), and \( num = 10 \). The cost of a node \( v_i \) is defined as

\[
c(v_i) =\text{outD}(v_i) \times p + 1,
\]

where \(\text{outD}(v_i)\) is the out-degree of \( v_i \) and \( p = 0.1 \) is the propagation probability.

D. Experiments on real networks

The experimental results are shown in Fig.6 with \( q = 1000 \) for both Boost SA and Combination SA. It can be seen that in most cases the Boost SA obtains the best seed set \( S \) with different budgets, which has the greatest influence.

More specifically, for the URV email network, when \( 150 < B < 300 \), Boost SA is better than both Combination SA and MODPSO. When \( B > 300 \), the advantage of Boost SA over other algorithms becomes more and more obvious. When \( B = 500 \), Boost SA is 9.90% better than Combination SA. In general, Boost SA is 4% better than Combination SA algorithm from \( B = 100 \) to 600 in average.

For Wiki-Vote network, the Boost SA always obtains the best seed set \( S \). As the increasing of Budget from 1000 to 6000, the advantage of the Boost SA is more obvious than the Combination SA and other algorithms. Boost SA outperforms Combination SA about 7.5% in average.

For NetHEPT network, Boost SA obviously performs better than the Combination SA and other algorithms when \( B < 7000 \). Boost SA is 3.41% better than Combination SA in average. When \( B > 7000 \), Boost SA almost equivalent to Combination SA. Due to expensive time consuming, we only run MODPSO at two budget points. The results reflect that Boost SA performs better than MODPSO with very little running time.

Fig.6(d) shows the running time of the algorithms on three real networks when \( B = 100 \). As we know, the greedy algorithm CELF consumes the most time. The running time of Boost SA and Combination SA are close together.

For a comprehensive comparison between the Boost SA and Combination SA for different values of \( B \) and \( q \), we set parameter \( q \) as 1000, 500, 100 in Boost SA and Combination SA for different budgets \( B \) on different datasets. The experimental results are shown in Tables VII, VIII and IX, which indicate the influence value increases gradually with the increase of \( q \).

With equal number of \( q \), the influence spread of the Boost SA outperforms Combination SA obviously, the running time of Boost SA is approximate to that of Combination SA. Comparing \( q = 100 \) of Boost SA with \( q = 1000 \) of Combination SA, the influence spread of the Boost SA outperforms Combination SA obviously also, the corresponding running time of Boost SA are all less than the Combination SA. In summary, the Boost SA can outperform Combination SA in influence spread with less running time.
TABLE VII: Influence/running time on URV email network

| Budget | Boost SA (influence/running time) | Combination SA (influence/running time) |
|--------|----------------------------------|----------------------------------------|
| q=1000 | 425/1.237                       | 424/0.955                              |
| 100    | 425/1.237                       | 424/0.955                              |
| 200    | 490/2.908                       | 488/1.574                              |
| 300    | 562/2.471                       | 560/1.824                              |
| 400    | 650/3.121                       | 652/2.289                              |
| 500    | 740/3.399                       | 737/2.930                              |
| 600    | 782/2.204                       | 781/1.688                              |

For network-2000, Boost SA always performs the best for different budgets. Boost SA performs 5.68% better than Combination SA in average. In addition, Boost SA and Combination SA are always better than CELF and MaxDegree. When $B < 300$, MaxDegree is slightly better than CELF. When $B > 300$, CELF is always better than MaxDegree.

For network-5000, when $B < 1100$, CELF performs slightly better than Boost SA and Combination SA. When $1100 < B < 1500$, Boost SA is 5.31% better than CELF and CELF is slightly better than Combination SA. Since $B > 1500$, Boost SA is 0.65% better than Combination SA and Combination SA is 34.30% better than CELF. MaxDegree performs worse than other three algorithms with different budgets.

For network-10000, Boost SA always performs the best for different budgets. When $B < 1500$, CELF performs slightly better than Boost SA and Combination SA, but when $B > 1500$, Combination SA is better than CELF and the advantage gradually expands to about 10.10% when $B = 4000$. Boost SA performs about 3.84% better than Combination SA in average with different budgets.

For network-30000, Boost SA always performs the best for different budgets. In particular, Boost SA performs 5.77% better than Combination SA. Combination SA performs almost as well as CELF and Combination SA.

The running time is shown in Fig.8. With the increase of network size, the running time increases gradually. Specifically, CELF consumes the most the running time. Boost SA and Combination SA consume roughly the same running time. It shows that the proposed Boost SA can achieve better result with approximate running time.

TABLE VIII: Influence/running time on Wiki-Vote network

| Budget | Boost SA (influence/running time) | Combination SA (influence/running time) |
|--------|----------------------------------|----------------------------------------|
| q=1000 | 1897/19.312                      | 1891/18.911                            |
| 100    | 1897/19.312                      | 1891/18.911                            |
| 200    | 2169/21.737                      | 2168/21.733                            |
| 300    | 2510/24.581                      | 2509/24.581                            |
| 400    | 3004/30.684                      | 3001/27.468                            |
| 500    | 3351/32.940                      | 3350/30.329                            |

E. Experiments on synthetic networks

The experiment results of synthetic networks are shown in Fig.7. In this experiment, $q = 1000$ for both Boost SA and Combination SA. From results of real networks and expensive runtime consuming, it is unnecessary to run MODPSO algorithm on synthetic networks. We just compare the rest four algorithms on synthetic networks.

TABLE IX: Influence/running time on NetHEPT network

| Budget | Boost SA (influence/running time) | Combination SA (influence/running time) |
|--------|----------------------------------|----------------------------------------|
| q=1000 | 443/197.661                      | 443/188.314                            |
| 100    | 443/197.661                      | 443/188.314                            |
| 200    | 511/208.960                      | 511/206.970                            |
| 300    | 600/219.880                      | 600/215.020                            |
| 400    | 693/231.910                      | 693/227.980                            |
| 500    | 763/241.170                      | 763/233.570                            |

F. Comparison between the random initialization and unified initialization

In Algorithm 2, we randomly generate $k$ initial seed sets $S_i$ for generate an initial seed set $S$ before sending it into the...
Fig. 8: The runtime on synthetic networks

Boost SA. The random initialization has a larger search space than that of the uniform initialization.

To show the effects of the proposed random initialization in Algorithm 2, we keep the parameter settings described in subsection C, and compare the results of the Boost SA with the random initialization and unified initialization. The experimental results are shown in Tables X-XII for three real world networks respectively. It can be seen that the initialization strategy in Algorithm 2 can promote the influence spread of the obtained seed set.

TABLE X: Influence spread on URV email network

| Budget | 100  | 200  | 300  | 400  | 500  | 600  | 700  |
|--------|------|------|------|------|------|------|------|
| Random ini. | 425  | 490  | 563  | 656  | 740  | 782  | 842  |
| Unified ini. | 424  | 487  | 561  | 650  | 732  | 782  | 835  |

TABLE XI: Influence spread on Wiki-Vote network

| Budget | 1000 | 2000 | 3000 | 4000 | 5000 | 6000 |
|--------|------|------|------|------|------|------|
| Random ini. | 2169 | 2817 | 3651 | 4461 | 5506 | 6308 |
| Unified ini. | 2167 | 2801 | 3641 | 4458 | 5485 | 6305 |

TABLE XII: Influence spread on NetHEPT network

| Budget | 1000 | 2000 | 3000 | 4000 | 5000 | 6000 |
|--------|------|------|------|------|------|------|
| Random ini. | 4435 | 5121 | 6005 | 6933 | 7636 | 10035 |
| Unified ini. | 4429 | 5119 | 6001 | 6924 | 7618 | 9965 |

G. Nodes cost sensitivity analysis

The parameter $p$ is fixed as $p = 0.1$ in Eq.7, which means that the activation cost of a node is positively correlated with its outdegree. In practical scenarios, this assumption is too absolute. In this subsection, we randomly select 2% nodes and set $p = 0.05$, then compare Boost SA with Combination SA. The experimental results are shown in Fig.9, which also shows that Boost SA always performs better than Combination SA. The results reflect that Boost SA can deal with more general network application problems.

Fig. 9: Nodes cost sensitivity (98% with $p = 0.1$ and 2% with $p = 0.05$)

H. Parameter sensitivity analysis

To investigate the effect of parameter $\beta$, we show the influence spread and running time results by varying $\beta$ on Wiki-Vote network in Fig.10 and Fig.11. Generally speaking, nodes of $T_3$ in Fig.4 have more chances to be activated with the increasing of $\beta$. As shown in Fig.10, in a wide range of $\beta$ from 20 to 80, Boost SA works better than Combination SA, however, with larger value of $\beta$ the cost performance of the selected seed nodes may be falling. Fig.11 shows that the running time is not sensitive with $\beta$. Taking into account of different datasets and different budgets, we set $\beta = 60$.

Fig. 10: Influence spread on Wiki-Vote network with different $\beta$

VI. Conclusion

Compared with the influence maximization (IM) problem, the budget influence maximization (BIM) is more approaching
to the practical scenarios. In this paper, an efficient algorithm named Boost SA for the BIM problem is proposed, which is based on the simulated annealing algorithm. Simulation results on real networks and synthetic networks show that the proposed algorithm can obtain a seed set with more influence spread with similar running time than state of the art algorithms. Without using a greedy strategy, the proposed Boost SA is scalable to large networks.

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