Contact Analysis of Thread Pairs in a Planetary Roller Screw Based on Finite Element Method

Shangjun Ma\textsuperscript{1,2,*}, Zhenhao Duan\textsuperscript{1}, Maodong Niu\textsuperscript{1}, Chuangchuang Li\textsuperscript{1}

\textsuperscript{1}Shaanxi Engineering Laboratory for Transmissions and Controls, Northwestern Polytechnical University, Xi’an, 710072, China
\textsuperscript{2}Hubei Jiangshan Heavy Industries Co., LTD, Xiangyang, 441057, China

mashangjun@nwpu.edu.cn

Abstract. This paper investigates the contact characteristics of the thread pairs in the planetary roller screw mechanism. Firstly, the analytical model of thread contact based on Hertz contact theory is established. Secondly, the contact models of the screw and roller threads and of the roller and nut threads based on FE method are developed. The correctness of the numerical results is verified via the comparison with analytical results, the relative error between the two sets of results is less than or equal to 2\%. Lastly, according to FE model, the contact state, contact positions and contact stresses of thread pairs at the screw-roller interface and at the roller-nut interface are derived.

1. Introduction

A planetary roller screw mechanism (PRSM) is a mechanical transmission device used to convert rotary motion to linear motion. As shown in Fig. 1, the main components of PRSM are nut, screw and rollers, in which the key transmission components are rollers between screw and nut. Compared to the ball screw mechanism (BSM), the PRSM provides more contact points than conventional BSM at a specific length. The advantages of the PRSM are high stiffness, high load capacity, long travel life and compact structure, etc. Therefore, the PRSM is most commonly used as an actuator mechanism in various applications, such as machine tool, medical equipment, port equipment of ship and flight control equipment of more-electric aircraft and so on.

![Figure 1. Structures of the PRSM.](image)

In recent years, many research works have been published that support the engineering applications of the PRSM, such as kinematic modeling and force analysis [1-2], the meshing mechanism and contact position calculation [3], frictional heat [4] and tribological behavior [5], and load distribution [6].
In the study of contact characteristics of threaded surfaces, Jones et al. [7] studied the contact characteristics of threaded surface and the curve equations of the screw, roller and nut were established. Abevi et al. [8] studied the static behavior of the inverted PRSM through 3D finite element method and experiment. Until now, few contact models have been developed for contact analysis of threaded surface, and the contact positions and contact stresses on the threads at the screw-roller interface and at the roller-nut interface are not revealed intuitively.

Therefore, a more intuitive 3D contact model has been established based on FE method in the current paper. After introducing the basic theory of Hertzian contact, the contact pairs between the screw and roller and between the roller and nut are developed respectively. Then, the correctness of the numerical model presented in this paper is verified by the comparison with analytical model. Finally, the contact positions and contact stresses on the threads at the screw-roller interface and at the roller-nut interface are derived.

2. Analytical model based on hertz contact theory
The overall elastic deformation of the contact surfaces is \( \delta \). Then [9],

\[
\delta = \left( \frac{9F_n^2}{16E^2 R_e^2} \right)^{1/3} F_2(e) \quad (1)
\]

\[
F_2(e) = 2\pi (b/a)^{1/2} \{ F_1(e) \}^{-1} K(e) \quad (2)
\]

\[
F_1(e) = \left[ \frac{4}{(πe^2)} \right]^{1/3} \left( \frac{b/a}{1/2} \left( \frac{(a/b)^2 E(e)-K(e)}{K(e)-E(e)} \right) \right)^{1/6} \quad (3)
\]

\[
\frac{1}{E^*} = (1-\mu_1^2)/E_1 + (1-\mu_2^2)/E_2 \quad (4)
\]

\[
R_e = \frac{1}{2}(AB)^{1/2} \quad (5)
\]

where \( F_n \) is the normal force at the contact point, \( E_1 \) and \( E_2 \) are the Young’s moduli of the two elastic bodies, and \( \mu_1 \) and \( \mu_2 \) are the Poisson’s ratios of the bodies. \( a \) and \( b \) are semimajor axis and semiminor axis of the contact ellipse. \( A \) and \( B \) are positive constants. \( e \) is the ellipse eccentricity and \( K(e) \) and \( E(e) \) are complete elliptic integrals of the first and second type, respectively.

The maximum contact stress in the contact surface is as follows:

\[
p_0 = \left( \frac{6F_nE^2}{\pi^3 R_e^2} \right)^{1/3} \{ F_1(e) \}^{-2} \quad (6)
\]

In the contact ellipse, the contact stress at each point follows an elliptical distribution, and the stress distribution is as follows:

\[
p = p_0 \left[ 1 - (x/a)^2 - (y/b)^2 \right]^{1/2} \quad (7)
\]

The semimajor axis of the contact ellipse can be calculated:

\[
a = 3F_n K(e)/(2\pi E^* \delta) \quad (8)
\]

3. FE contact model

3.1. Structural parameters of PRSM
The structural parameters of thread are shown in Table 1. According to the characteristics of the circular symmetrical structure of the PRSM, in this paper, only a pair of contact pairs at the screw-roller interface and at the roller-nut interface is established.
Table 1. PRSM structural parameters.

| Name  | Nominal radius | Half of thread thickness | Addendum of thread | Dedendum of thread | Flank angle | Pitch | Profile radius of roller thread |
|-------|----------------|-------------------------|--------------------|--------------------|-------------|-------|-------------------------------|
| Screw | 9.75 mm        | 0.54 mm                 | 0.27 mm            | 0.36 mm            | 45°         | 2 mm  | /                             |
| Roller| 3.25 mm        | 0.45 mm                 | 0.27 mm            | 0.45 mm            | 45°         | 2 mm  | 3.536 mm                      |
| Nut   | 16.25 mm       | 0.46 mm                 | 0.27 mm            | 0.44 mm            | 45°         | 2 mm  | /                             |

3.2. FE model of thread pairs

According to the structural parameters of the PRSM listed in Table 1, the contact models of the screw and roller threads and of the roller and nut threads based on FE method are shown in Fig. 2 and Fig. 3, respectively. The contact pairs are created and assembled through the 3D software package SolidWorks, and the 3D FE model is developed using the commercial finite element package ABAQUS.

Meshing is based on the 8-node hexahedron noncoordinated solid element C3D8I, and the local contact area on the thread is densified. The contact model of the screw thread and roller thread has 682,038 elements and 701,012 nodes. The contact model of the roller thread and nut thread has 651,411 elements and 667,240 nodes. In these numerical models, the material is GCr15, whose density is $\rho = 7,810$ kg/m$^3$, elastic modulus is $E = 212$ GPa and Poisson’s ratio is $\mu = 0.29$.

3.3. Boundary conditions

Based on the relative movement relationships of the components in PRSM, at the screw-roller interface, the symmetry constraint is used to constrain the thread sides of the screw and roller. The purpose is to take into account the supporting effect of cyclically symmetric structures. Only the axial displacement degree of freedom of the screw and the roller are released, and the radial translational and rotational degrees of freedoms of the roller and the screw are constrained. Similarly, at the roller-nut interface, only the axial displacement degree of freedom of the roller and the nut are released, and the radial translational and rotational degrees of freedoms of the roller and the nut are constrained.

For the contact pair of screw and roller threads, to facilitate the load applied on the roller, a reference point is created on the roller end surface, that is, the end surface nodes of the roller are coupled to the reference point. As shown in Fig. 2, The corresponding axial load is applied to the reference point. The similar load constrains are applied to the contact pair of roller and nut threads. In addition, contact interactions are applied to the interfaces between the screw and roller and between the roller and nut to achieve load transfer. The contact type is surface-to-surface which is applied to both the screw-roller interface and roller-nut interface.

4. Calculation results

An axial load of 200 N is applied to a single pair of threads. The results from the numerical solution for the maximum contact stress and contact ellipse area are obtained. In order to verify the correctness of the FE model, the analytical model based on Hertz contact theory is also performed in this section. The Comparison of numerical and analytical results are shown in Table 2.
Table 2. Comparison of numerical and analytical results.

| Contact pair           | Parameters        | Numerical solution | Analytical solution | Relative error |
|------------------------|-------------------|--------------------|---------------------|----------------|
| Screw-roller interface | Contact area      | 0.1167 mm$^2$     | 0.1188 mm$^2$       | 1.8            |
|                        | Maximum contact stress | 3655 MPa      | 3583 MPa            | 2.0            |
| Roller-nut interface   | Contact area      | 0.1344 mm$^2$     | 0.1321 mm$^2$       | 1.7            |
|                        | Maximum contact stress | 3144 MPa      | 3205 MPa            | 1.9%           |

As shown in Table 2, the relative error between the two sets of results is less than or equal to 2%, and the range of error is acceptable. Therefore, the numerical model proposed in this paper can be used to evaluate the contact characteristics of the PRSM.

According to the FE model, the von Mises stresses on the screw-roller contact pair and the roller-nut contact pair are shown in Figs. 4 and 5.

Figure 4. Von mises stresses at the screw-roller interface.

Figure 5. Von Mises stresses at the roller-nut interface.

As shown in Figs. 4 and 5, the contact stress distribution indicates that both the screw-roller interface and the roller-nut interface are ellipses, and the contact area at the roller-nut interface is larger than that of the screw-roller interface. The maximum stress value of the roller at the screw-roller interface is 2343 MPa and at the roller-nut interface is 1923 MPa, which indicates that the roller is subject to alternating stress.

Besides, the von Mises stress distribution at the roller-nut interface and at the screw-roller interface are shown in Fig. 6.

Figure 6. Contact stress distribution in the shaft section.
Fig. 6 shows that the contact stress at the roller-nut interface is less than that of the screw-roller interface, this is because the radius of curvature at the screw-roller interface is smaller than that of the roller-nut interface. The contact position at the screw-roller interface is not on the central axis of the cross-sectional plane, while the contact position at the roller-nut interface is lie on the central axis of the cross-sectional plane. This is because the helix angle of the roller is the same as that of the nut.

5. Conclusions
This study develops an 3D numerical model to investigate the contact characteristics of a PRSM based on the Hertz contact theory. The numerical model proposed in this paper is verified by the comparison with analytical model. The results shown that the contact region on both the screw-roller interface and the roller-nut interface are ellipses. The von Mises stress at the screw-roller interface is greater than that of the roller-nut interface, and the contact area at the roller-nut interface is larger than that of the screw-roller interface. The contact point at the screw-roller interface deviates from the central axis of the cross-sectional plane, and the contact point at the roller-nut interface is located at the central axis of the cross-sectional plane. The results can also be used to guide the parameter design and help improve the transmission quality.

6. References
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