Dilatons in curved backgrounds by the Poisson–Lie transformation

L. Hlavatý
Faculty of Nuclear Sciences and Physical Engineering,
Czech Technical University,
Břehová 7, 115 19 Prague 1, Czech Republic
hlavaty@fjfi.cvut.cz

January 17, 2018

Abstract

Transformations between group coordinates of three–dimensional conformal \( \sigma \)–models in the flat background and their flat, i.e. Riemannian coordinates enable to find general dilaton fields for three–dimensional flat \( \sigma \)–models. By the Poisson–Lie transformation we can get dilatons for the dual \( \sigma \)–models in a curved background. Unfortunately, in some cases the dilatons depend on inadmissible auxiliary variables so the procedure is not universal. The cases where the procedure gives proper and nontrivial dilatons in curved backgrounds are investigated and results given.

1 Introduction

In the paper [1] we have investigated conformally invariant three–dimensional \( \sigma \)–models on solvable Lie groups that were Poisson–Lie T–dual or plural to \( \sigma \)–models in the flat background. Several of them were nontrivial in the sense that they lived in a curved background and had nonvanishing torsion. In some cases we were not able to find the dilaton fields by the plurality procedure given in [2] because necessary conditions for application of Poisson–Lie transformation were not satisfied for the constant dilaton.

Recently we have found explicit forms of transformations between the group coordinates of the flat \( \sigma \)–models and their flat coordinates, i.e. we expressed the Riemannian coordinates of the flat metric in parameters of its solvable isometry subgroups [3]. This enables us to write down the general form of the dilaton field satisfying the vanishing \( \beta \) equations for the flat model in terms of the group coordinates and consequently the dilaton fields on the dual or plural nontrivial models.

To set our notation let us very briefly review the construction of the Poisson–Lie T–plural \( \sigma \)–models by means of Drinfel’d doubles (For more detailed description see [4], [5], [2], [1]). The Lagrangian of dualizable \( \sigma \)–models can be written in terms of right–invariant fields on a Lie group \( G \) that is a subgroup of the Drinfel’d double as

\[
L = F_{ij}(\phi)\partial_\phi^i \partial_\phi^j = E_{ab}(g)(\partial_\gamma gg^{-1})^a(\partial_\gamma gg^{-1})^b, \tag{1}
\]

where

\[
\phi : \mathbb{R}^2 \to \mathbb{R}^n, \quad F_{ij}(y) = e_i^a(g(y))E_{ab}(g(y))e_j^b(g(y)), \tag{2}
\]
\(e_i^a\) are components of right–invariant forms (vielbeins) \(e_i^a(g) = ((dg)_i . g^{-1})^a\) and \(y^i\) are local coordinates of \(g \in G\).

\[
E(g) = (E_0^{-1} + \Pi(g))^{-1}, \quad \Pi(g) = b(g)a(g)^{-1} = -\Pi(g)^t,
\]

(3)

and \(a(g), b(g), d(g)\) are submatrices of the adjoint representation of the group \(G\) on the Lie algebra of the Drinfel’d double \(^1\)

\[
Ad(g)^t = \begin{pmatrix} a(g) & 0 \\ b(g) & d(g) \end{pmatrix}.
\]

(4)

The fact that for a Drinfel’d double several decompositions of its Lie algebra \(\mathcal{D}\) into Manin triples \(\langle G|\tilde{G}\rangle\) may exist leads to the notion of Poisson–Lie T–plurality \(^2\). Namely, let \(\{X_j, \tilde{X}^k\}, j, k \in \{1, \ldots, n\}\) be generators of Lie subalgebras \(G, \tilde{G}\) of the Manin triple associated with the Lagrangian \(^1\)

\[
E(g) = (E_0^{-1} + \Pi(g))^{-1}, \quad \Pi(g) = b(g)a(g)^{-1} = -\Pi(g)^t,
\]

(3)

and \(a(g), b(g), d(g)\) are submatrices of the adjoint representation of the group \(G\) on the Lie algebra of the Drinfel’d double \(^1\)

\[
Ad(g)^t = \begin{pmatrix} a(g) & 0 \\ b(g) & d(g) \end{pmatrix}.
\]

(4)

where

\[
\tilde{X} = (X_1, \ldots, X_n)^t, \quad \tilde{U} = (\tilde{U}^1, \ldots, \tilde{U}^n)^t.
\]

The transformed model is then given by the Lagrangian of the form \(^1\) but with \(E(g)\) replaced by

\[
E_U(g_u) = M(N + \Pi_U M)^{-1} = (\tilde{E}_0^{-1} + \Pi_U)^{-1},
\]

(6)

where

\[
M = S^tE_0 - T^t, \quad N = P^t - R^tE_0, \quad \tilde{E}_0 = MN^{-1}
\]

(7)

and \(\Pi_U\) is calculated by \(^4\) from the adjoint representation of the group \(G_U\) generated by \(\{U_j\}\). Note that for \(P = S = 0, T = R = 1\) we get the dual model with \(\tilde{E}_0 = E_0^{-1}\), corresponding to the interchange \(G \leftrightarrow \tilde{G}\) so that the duality transformation is a special case of the plurality transformation \(^5\) – \(^7\).

### 2 Poisson–Lie transformation of dilatons

In quantum theory the duality or plurality transformation must be supplemented by a correction that comes from integrating out the fields on the dual group \(\tilde{G}\) in path integral formulation. In some cases it can be absorbed at the 1-loop level into the transformation of the dilaton field \(\Phi\) satisfying the so called vanishing \(\beta\) equations

\[
0 = R_{ij} - \nabla_i \nabla_j \Phi - \frac{1}{4} H_{imn} H_{j}^{mn},
\]

(8)

\[
0 = H_{kij} \nabla^k \Phi + \nabla^k H_{kij},
\]

(9)

\[
0 = R - 2 \nabla_k \nabla^k \Phi - \nabla_k \Phi \nabla^k \Phi - \frac{1}{12} H_{kmn} H_{k}^{mn}
\]

(10)

where the covariant derivatives \(\nabla_k\), Ricci tensor \(R_{ij}\) and Gauss curvature \(R\) are calculated from the metric

\[
G_{ij} = \frac{1}{2}(F_{ij} + F_{ji})
\]

(11)

\(^1\)t denotes transposition.
that is also used for lowering and raising indices, and the torsion is

\[ H_{ijk} = \partial_i B_{jk} + \partial_j B_{ki} + \partial_k B_{ij} \]  

(12)

where

\[ B_{ij} = \frac{1}{2}(F_{ij} - F_{ji}). \]  

(13)

The Poisson–Lie transformation of the tensor \( F \) that follows from (6) then must be accompanied by the transformation of the dilaton \( \Phi \)

\[ \Phi_U = \Phi + \ln|\text{Det}(N + \Pi_U M)| - \ln|\text{Det}(1 + \Pi E_0)| + \ln|\text{Det} a_U| - \ln|\text{Det} a| \]  

(14)

where \( \Pi_U, a_U \), are calculated by (3) and (4) but from the adjoint representation of the group \( G_U \). The transformed dilaton \( \Phi_U \) then satisfy the vanishing \( \beta \) equations if the dilaton \( \Phi \) does.

Unfortunately, the right-hand side of the formula (14) may depend on the coordinates of the auxiliary group \( \tilde{G} \). That’s why the transformation of the dilaton field cannot be applied in general but only if the following theorem holds

**Theorem 1** The dilaton (14) for the model defined on the group \( G_U \) exists if and only if

\[ \tilde{U} \Phi^{(0)}(g, \tilde{g}) = \frac{d}{dt} \Phi^{(0)} \left( g, \tilde{g}, \exp(t\tilde{U}) \right) |_{t=0} = 0, \quad \forall g \in G_U, \quad \forall \tilde{g} \in \tilde{G}_U, \quad \forall \tilde{U} \in \tilde{G}_U, \]

where \( \tilde{U} \in \tilde{G}_U \) is extended as a left–invariant vector field on \( D \) and

\[ \Phi^{(0)}(g) = \Phi(g) - \ln|\text{Det}(1 + \Pi(g)E_0)| - \ln|\text{Det} a(g)|. \]

(15)

For applications it is much easier to check a weaker necessary condition.

**Theorem 2** A necessary condition for the existence of the dilaton (14) for the model defined on the group \( G_U \) is

\[ \tilde{U} \Phi^{(0)}(e) = \frac{d}{dt} \Phi^{(0)}(\exp(t\tilde{U}))|_{t=0} = 0, \quad \forall \tilde{U} \in \tilde{G}_U, \]

(16)

where \( e \) is the unit of the Drinfel’d double \( D \).

For parametrization of \( g \in G \) in the form

\[ g(y) = \exp(y_1X_1) \exp(y_2X_2) \exp(y_3X_3), \]

(17)

where \( y_j \) are coordinates on the group manifold and \( X_j \) are the group generators the condition (16) can be rewritten (see [1]) as

\[ R_{ijk} \frac{\partial \Phi^{(0)}(y_j)}{\partial y_j} |_{y=0} = 0, \]

(18)

where \( R \) is the submatrix in [3].

The condition (18) could not be satisfied for some of the \( \sigma \)–models with constant dilaton field so that we were not able to find the transformed dilaton \( \Phi_U \) that satisfy the vanishing \( \beta \) equations. The possibility to find the general dilaton fields for the flat models offers a possibility to overcome this obstacle and obtain more general dilatons in curved backgrounds.

### 3 Dilatons of \( \sigma \)–models on solvable three-dimensional groups

All models investigated in the following admit nonsymmetric tensor \( F_{ij} \) but their torsions vanish so without loss of generality we shall deal with models having \( F_{ij} = F_{ji} = G_{ij} \).
3.1 General dilatons in flat backgrounds

It follows from the construction of classical dualizable models that they are given by decompositions (Manin triples) \((G|\tilde{G})\) and matrices \(E_0\). Most of the flat and torsionless models found in \([1]\) can be formulated on the Drinfeld doubles with semiabelian decompositions \((X|1)\) where \(1\) is the three–dimensional abelian algebra. From the form of the vanishing \(\beta\) equations \((8–10)\) it is easy to see that the general form of their dilaton fields is

\[
\Phi(y) = c_1 \xi_1(y) + c_2 \xi_2(y) + c_3 \xi_3(y) + c_0,
\]

where \(\xi_j(y)\) are coordinates that bring the flat metric \(G_{ij}(y)\) to a constant form \(G'_{ij}\) (see \([3]\)) and \(c_j\) are real constants satisfying

\[
\sum_{j=1}^{3} G'^{ij} c_i c_j = 0.
\]

By the Poisson–Lie transformation of \((19)\) we can get dilatons for the dual \(\sigma\–models\) but, as mentioned before, only if the necessary conditions are satisfied. Due to \((15)\) and \((19)\) the condition \((18)\) reads

\[
R^{jk}(c_j - \partial \partial y_j \ln |\text{Det} a(g)|) |_{y=0} = 0.
\]

Moreover, the matrix \(\Pi(g)\) vanishes for \((X|1)\) and the flat coordinates can be chosen to satisfy \(\partial \partial y_j(0) = \delta_{mj}\). The condition \((21)\) then simplifies to

\[
R^{jk}(c_j - \partial \partial y_j \ln |\text{Det} a(g)|) |_{y=0} = 0.
\]

3.2 Dilatons for \(\sigma\–models dual to \(5|1)\)

The first \(\sigma\–model\) in the curved background we are going to investigate is given by the metric

\[
\tilde{G}_{ij}(u) = \begin{pmatrix}
    e^{-2u_3}Q & e^{-2u_3}Q & V \cosh u_3 - H \sinh u_3 \\
    e^{-2u_3}Q & e^{-2u_3}Q & H \cosh u_3 - V \sinh u_3 \\
    V \cosh u_3 - H \sinh u_3 & H \cosh u_3 - V \sinh u_3 & J
\end{pmatrix},
\]

(23)

where \(\epsilon = \pm 1\) and \(Q, V, H, J\) are constants. This metric has nonvanishing Ricci tensor but its Gauss curvature is zero. It belongs to the \(\sigma\–model\) corresponding to the \((60|1)\) decomposition of the \(DD11\) (for notation see \([6]\)) and \(\tilde{E}_0 = \tilde{G}(0)\). On the other hand, it can be obtained by the Poisson–Lie transformation \((6), (7)\) from the metric

\[
G_{ij}(y) = \begin{pmatrix}
    0 & 0 & v e^{-y_1} \\
    0 & q e^{-2y_1} & 0 \\
    v e^{-y_1} & 0 & 0
\end{pmatrix},
\]

(24)

where \(q, v\) are constants. The latter metric is flat and corresponds to the \((5|1)\) decomposition of the \(DD11\) and \(E_0 = G(0)\).

The matrix \((5)\) that transform the Manin triple \((5|1)\) to \((60|1)\) and the metric \((24)\) to \((23)\) is

\[
\begin{pmatrix}
P & T \\
R & S
\end{pmatrix} = \begin{pmatrix}
-\beta + \frac{1}{2} \alpha & \epsilon(\beta + \frac{1}{2} \alpha) & -\epsilon & 0 & 0 & 0 \\
0 & 0 & 0 & \epsilon & 1 & \alpha \\
0 & -\epsilon & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -\epsilon \\
\frac{1}{2} \epsilon & -\frac{1}{2} \epsilon & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -\frac{1}{2} \epsilon & \frac{1}{2} & \beta
\end{pmatrix},
\]

(25)

\[4\]
where relations between the constants are

\[ q = Q^{-1}, \quad v = V - \epsilon H, \quad \alpha = \frac{\epsilon V + H}{2 Q}, \quad \beta = \frac{\alpha^2 Q - J}{2 v}. \quad (26) \]

In fact, the metric \( 23 \) is the most general that can be obtained by the Poisson–Lie transformation from a flat metric corresponding to the \( (5|1) \) decomposition of the \( DD11 \).

General form of the dilaton field for the metric \( 24 \) is given by \( 19 \) where

\[ \xi_i(y_1, y_2, y_3) = -e^{-y_1}, \quad \xi_2(y_1, y_2, y_3) = e^{-y_1}y_2, \quad \xi_3(y_1, y_2, y_3) = \frac{q}{2v}e^{-y_1}y_2^2 + y_3. \quad (27) \]

These are the coordinates that bring the flat metric to its constant form \( \sigma_{ij} \).

The formula \( 14 \) for the general dilaton of the \( \sigma \)-model given by \( 23 \) yields

\[ \Phi_U(y) = -2y_1 + c_1 e^{-y_1} + c_2 e^{-y_1}y_2 + c_3 \left( \frac{q}{2v}e^{-y_1}y_2^2 + y_3 \right) + c_0, \quad (28) \]

where the coefficients satisfy the equation \( 20 \) that in this case reads

\[ v c_2^2 + 2q c_1 c_3 = 0. \quad (29) \]

However, this is not yet the final form of the dilaton field because it is expressed in terms of the coordinates \( y \) of the \( \sigma \)-model given by \( 24 \) and it must be transformed to the coordinates \( u \) of the \( \sigma \)-model given by \( 23 \). The transformation formulas between these coordinates follow from two different decompositions of elements of the Drinfel’d double \( DD11 \), namely from the relation

\[ e^{-y_1}X_1 e^{-y_2}X_2 e^{-y_3}X_3 e^{-\tilde{y}_1}\tilde{X}_1 e^{-\tilde{y}_2}\tilde{X}_2 e^{-\tilde{y}_3}\tilde{X}_3 = e^{-u_3}U_3 e^{-u_2}U_2 e^{-u_1}U_1 e^{-\tilde{u}_3}\tilde{U}_3 e^{-\tilde{u}_2}\tilde{U}_2 e^{-\tilde{u}_1}\tilde{U}_1, \quad (30) \]

where \( X_j, \tilde{X}_j \) are generators corresponding to the decomposition \( (5|1) \) of the Drinfel’d double \( DD11 \) and \( U_j, \tilde{U}_j \) are generators of the decomposition \( (6_0|1) \). They can be related by \( 25 \).

Coordinates \( y \) in terms of \( u \) are then expressed as

\[
\begin{align*}
y_1 &= -\epsilon u_3, \\
y_2 &= \frac{\epsilon \, \bar{u}_1 + \bar{u}_2}{2}, \\
y_3 &= \frac{-\epsilon u_1 + u_2}{2} + \beta u_3, \\
\tilde{y}_1 &= \beta(-\bar{u}_1 + \epsilon \, \bar{u}_2) - \epsilon \, \bar{u}_3 + \frac{1}{2}(\bar{u}_1 + \epsilon \, \bar{u}_2)(\alpha + \epsilon u_1 + \epsilon u_2 + \epsilon \alpha u_3), \\
\tilde{y}_2 &= \epsilon u_1 + u_2 + \alpha u_3, \\
\tilde{y}_3 &= -\epsilon \, \bar{u}_1 + \bar{u}_2.
\end{align*}
\]

We can see that unless \( c_2 = 0, c_3 = 0 \) the dilaton \( 28 \) depends on the coordinate \( \epsilon \, \bar{u}_1 + \bar{u}_2 \). It is not admissible and thus the general form of dilaton obtained by the Poisson–Lie transformation for the metric \( 23 \) is

\[ \Phi(u) = \Phi_U(y(u)) = 2\epsilon u_3 + c_1 e^{\epsilon u_3} + c_0. \quad (32) \]

We have checked that the vanishing \( \beta \) equations for \( \Phi(u) \) and \( G_{ij}(u) \) given by \( 24 \) are satisfied.
A bit more complicated \( \sigma \) corresponding to the decompositions (1) Poisson–Lie transformation of the dilaton.

By other plurality transformations of (24) we can get \( \sigma \)-models with curved background corresponding to the decompositions (1|60) and (5,ii|60) of the DD11. For the dilaton fields the formula (14) could be again used but we were not able to express the coordinates \( y, \tilde{y} \) in terms of \( u, \tilde{u} \) from the relation (30) in these cases.

**3.3 Sigma models dual to (4|1)**

A bit more complicated \( \sigma \)-model is given by the metric \( \tilde{G}_{ij}(u) \), where

\[
\begin{align*}
\tilde{G}_{11}(u) &= \tilde{G}_{22}(u) = e^{-2u_3}Q \\
\tilde{G}_{12}(u) &= \tilde{G}_{21}(u) = e^{-2u_3}Q \\
\tilde{G}_{13}(u) &= \tilde{G}_{31}(u) = V \cosh u_3 - H \sinh u_3 + Q \frac{\epsilon V - H}{2} u_3 e^{-u_3} \\
\tilde{G}_{23}(u) &= \tilde{G}_{32}(u) = H \cosh u_3 - V \sinh u_3 + Q \frac{\epsilon V - H}{2} u_3 e^{-u_3} \\
\tilde{G}_{33}(u) &= J - Q (V - \epsilon H)^2 u_3^2
\end{align*}
\]

where \( \epsilon = \pm 1 \) and \( Q, V, H, J \) are constants. Again, this metric has nonvanishing Ricci tensor and its Gauss curvature is zero. It belongs to the \( \sigma \)-model corresponding to the (60|2) decomposition of the DD12 and \( \tilde{E}_0 = \tilde{G}(0) \). Besides that it can be obtained by the Poisson–Lie transformation (6), (7) from the metric

\[
\begin{pmatrix}
0 & v e^{-y_1} y_1 & v e^{-y_1} \\
v e^{-y_1} y_1 & q e^{-2y_1} & 0 \\
v e^{-y_1} & 0 & 0
\end{pmatrix}
\]

where \( q, v \) are constants. This metric is flat and corresponds to the (4|1) decomposition of the DD12.

The matrix (5) that transform the metric (34) to (33) is

\[
\begin{pmatrix}
P & T \\
R & S
\end{pmatrix} = \begin{pmatrix}
-\beta - \frac{1}{2} \alpha & \epsilon (\beta - \frac{1}{2} \alpha) & -\epsilon & 0 & 0 & 0 \\
0 & 0 & 0 & -\epsilon & -1 & \alpha \\
-\epsilon & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -\epsilon & 0 \\
-\frac{1}{2} \epsilon & -\frac{1}{2} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -\epsilon & \frac{1}{2} & \beta
\end{pmatrix}
\]

where the relations between the constants are

\[
q = Q^{-1}, \quad v = V - \epsilon H, \quad \alpha = -\frac{\epsilon V + H}{2Q}, \quad \beta = \frac{\alpha^2 Q - J}{2v}.
\]

The dilaton field for the \( \sigma \)-model given by (33), obtained by insertion of the flat coordinates of the metric (34) into the formula (14) is

\[
\Phi(y) = -2y_1 + c_1 e^{-y_1} + c_2 \left( \frac{v}{q} (y_1 + e^{-y_1}) + e^{-y_1} y_2 \right) + c_3 \left( \frac{v}{q} (y_1 - \sinh y_1) + \frac{q}{2v} e^{-y_1} y_2^2 + (e^{-y_1} + y_1 - 1) y_2 + y_3 \right) + c_0,
\]
where the coefficients satisfy the equation (29). To get the final form of the dilaton field we must transform it to the coordinates \( u \). The transformation formulas follow from decompositions of elements of the Drinfel’d double \( DD_{12} \), namely from the relation (30) where \( X_j, \tilde{X}_j \) are generators corresponding to the decomposition (4|1) and \( U_j, \tilde{U}_j \), related by (5) and (35), correspond to the decomposition (6|0|2). Coordinates \( y \) in terms of \( u \) are then expressed as

\[
\begin{align*}
y_1 &= -\epsilon u_3, \\
y_2 &= \epsilon \frac{u_1 + \tilde{u}_2}{2}, \\
y_3 &= -\epsilon u_1 + u_2 + \beta u_3, \\
\tilde{y}_1 &= \beta(-\tilde{u}_1 + \epsilon \tilde{u}_2) + \frac{1}{2}(\tilde{u}_1 + \epsilon \tilde{u}_2)(-\alpha + u_1 + \epsilon u_2 - \epsilon \alpha u_3) - \frac{1}{4} \tilde{u}_1^2 + \frac{1}{4} \epsilon \tilde{u}_1 \tilde{u}_2 + \frac{1}{4} \epsilon^2 \tilde{u}_2^2 - \epsilon \tilde{u}_3, \\
\tilde{y}_2 &= -\epsilon u_1 - u_2 + \alpha u_3, \\
\tilde{y}_3 &= -\epsilon \tilde{u}_1 + \tilde{u}_2.
\end{align*}
\]

In order that the dilaton does not depend on the coordinate \( \epsilon \tilde{u}_1 + \tilde{u}_2 \) we must set \( c_2 = 0 \) and \( c_3 = 0 \) and the general form of the dilaton obtained by the Poisson–Lie transformation for the metric (33) is again (32). The vanishing \( \beta \) equations are satisfied.

Let us mention in the end that there are still other models with curved backgrounds dual to the flat ones, namely those corresponding to the Manin triples (1|2), (1|7|0) of the Drinfel’d doubles \( DD_{15} \) and \( DD_{19} \). Unfortunately, in these cases all \( y \) coordinates depend on the \( \tilde{u}_1, \tilde{u}_2, \tilde{u}_3 \) so that only \( \Phi = \text{const} \) may be inserted into (14) giving results published in [1].

### 4 Conclusions

We have investigated the possibilities to apply the Poisson–Lie transformation to the general solution of the vanishing \( \beta \) equations for the flat metric. We have obtained dilaton fields for the metrics (28) and (33) having a nontrivial Ricci tensor. They are the most general dilatons that can be obtained by the Poisson–Lie transformation from the general dilatons (28), (37) of the dual flat metrics (24) and (34). An interesting but yet unsolved question is whether the dilaton (32) is the general solution of the vanishing \( \beta \) equations for the curved backgrounds (25) and (36).

On the other hand, we have found that the procedure does not work universally because the transformed dilatons often depend on inadmissible auxiliary variables and the above mentioned cases show that the necessary condition (16) for the applicability of the formula (14) is not sufficient.

### 5 Acknowledgements

This work was supported by the project of the Grant Agency of the Czech Republic No. 202/06/1480 and by the research plan LC527 15397/2005-31 of the Ministry of Education of the Czech Republic. Useful comments of Libor Šnobl are gratefully acknowledged.

### References

[1] L. Hlavatý and L. Šnobl, Poisson–Lie T–plurality of three–dimensional conformally invariant sigma models II : Nondiagonal metrics and dilaton puzzle, J. High En. Phys. 04:10 (2004) 045, [hep-th/0408126].
[2] R. von Unge, *Poisson–Lie T-plurality*, J. High En. Phys. 02:07 (2002) 014, [hep-th/0020524].

[3] L. Hlavatý and M. Turek, *Flat coordinates and dilaton fields for three-dimensional conformal sigma models* [hep-th/0512082].

[4] C. Klimčík and P. Ševera, *Dual non–Abelian duality and the Drinfeld double*, Phys. Lett. B 351 (1995) 455, [hep-th/9502122].

[5] C. Klimčík, *Poisson-Lie T-duality*, Nucl. Phys. B (Proc. Suppl.) 46 (1996) 116, [hep-th/9509095].

[6] L. Snobl and L. Hlavatý, *Classification of 6-dimensional real Drinfel’d doubles*, Int.J.Mod.Phys. A17 (2002) 4043 [math.QA/0202210].