Probabilistic Estimation of Soil-Water Characteristic Curves of Fine-grained Soils

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Abstract. The soil-water characteristic curve (SWCC), which describes the relationship between water content in soil and its matric suction, is important for studying the mechanical properties of unsaturated soil. Because measuring SWCC is time-consuming and labor-intensive, many empirical methods have been proposed for assessing SWCC. However, the uncertainty associated with empirical methods is rarely quantified. The purpose of this paper is to propose a probabilistic method for assessing the SWCC of fine-grained soils based on the particle size distribution and index properties. First, the van Genuchten model, which is a widely used SWCC model, is briefly presented. A comprehensive soil database was then compiled and the parameters of the van Genuchten model were calibrated for each soil sample. After that, a regression analysis was performed to estimate the mean value of the parameters of the van Genuchten model, and a correlation analysis was performed to estimate the covariance matrix of the parameters of the van Genuchten model. Finally, a joint probability density function of the parameters of the van Genuchten model is constructed. In this paper, Johnson distributions are used to construct the joint probability distribution of the parameters in the van Genuchten model that do not obey the multivariate normal distribution. The proposed method has been rigorously tested to assess its accuracy and applicability. The method proposed in this paper can provide a practical tool for estimating SWCC with limited data and accounting for the associated uncertainty.

1. Introduction
The soil-water characteristic curve (SWCC) is of great importance in the study of unsaturated soil [1]. SWCC simulates various engineering characteristics of unsaturated soil and reflects the relationship between moisture content or soil saturation and its matric suction [2], which plays an important role in
studying the physical and mechanical properties of unsaturated soil. The main parameters of SWCC include air entry value, residual suction, and residual saturation. These parameters are often used to assess the shear strength and seepage parameters of unsaturated soil [3, 4]. For the convenience of engineering applications, many scientists have proposed various fitting equations for describing the SWCC, such as Brooks and Corey model (BC model) [5], van Genuchten model (VG model) [4], Fredlund and Xing model (FX model) [3], etc. Compared to the BC model and FX model, the VG model is more accessible due to its simpler form and less required input.

Since the flow of water in unsaturated soil is a very slow process, it takes a long time to reach a steady-state of water and air, therefore measuring SWCC is a time-consuming and laborious process [6]. Currently, although the soil-water characteristic curve, measured by indoor or on-site tests, has been developed to a certain extent, it is still difficult to accurately obtain the SWCC [7]. To evaluate the SWCC effectively, many scientists have proposed to indirectly predict SWCC by establishing the transfer function of SWCC, that is, to predict the relationship between matric suction and water content using basic soil properties (such as particle size distribution, void ratio, dry density, organic matter, etc.) as the measurement of these soil properties can be more reliable than direct measurement of SWCC. Currently, transfer function methods mainly include the point estimation method [8], the parameter estimation method [9], and the physical experience method [10]. While these methods provide a good idea for quickly predicting SWCC, they are all empirical and inevitably contain prediction errors. However, there is limited literature on assessing the prediction error in SWCC and considering whether it is reasonable to directly assume that the parameters obey a normal distribution.

In this paper, the VG model is used to establish an SWCC probabilistic prediction method based on particle size distribution curve and void ratio for fine-grained soils. Compared to conventional methods, the proposed method can strictly take into account the SWCC estimation error, judge whether the characteristic parameters obey the normal distribution, and propose a probabilistic forecasting method provided that these parameters do not obey the normal distribution and, therefore, characterize the SWCC prediction uncertainty. This paper is organized as follows: first, a database of unsaturated fine-grained soils is created, and the parameters of the VG model are fitted using the maximum likelihood method; then, linear equations of the parameters of the VG model are established using the particle size distribution curve and the void ratio; thereafter, taking into account the uncertainty in the prediction of the parameters, the joint probability distribution of the SWCC parameters is established using the Johnson transformation, and a probability density function is obtained. Finally, a 90% SWCC confidence interval is obtained. The method proposed in this paper can help predict a probabilistically reliable SWCC by including the prediction error.

2. Probabilistic prediction model of the SWCC

2.1. SWCC model
The SWCC model used in this paper is the VG model, which is represented as follows:

\[
S = \left[ \frac{1}{1 + (\psi r)^m} \right]^n 
\]

(1)

where \( S \) is the volume saturation of soil, \( a, n, \) and \( m \) are the parameters of the VG model, \( \psi \) is the value of the matric suction of soil. The parameter \( m \) can be expressed as a function of \( n \) as follows [7]:

\[
m = 1 - \frac{1}{2n}
\]

(2)

Based on Eq. (2), the required input parameters of the VG model can be reduced to two: \( a \) and \( n \). Given that SWCC measurement is time-consuming, an indirect method is adopted to predict SWCC in this paper.

Previous studies have shown that SWCC parameters strongly correlate with the parameters of the particle size distribution curve [11, 12]. Li [13] pointed out that SWCC is closely related to the parameters of the particle size distribution curve, such as \( d_{10}, d_{30}, d_{60}, \) non-uniformity coefficient \( C_u \), curvature coefficient \( C_r \), and so on. At the same time, since the water retention capacity of the soil
depends on the pore structure of the soil, the void ratio $e$ has a large influence on the SWCC.

2.2. The Marginal probability distribution prediction model

To build a regression model between particle size distribution, void ratio, and characteristic parameters of the VG model ($a, n$), 244 fine-grained soil samples were taken from the SoilVision database with $P200$ (percentage of soil passing standard sieve No.200) $> 30\%$, and the database of fine-grained soils was established. The measured data were obtained for SWCC (the volume saturation $S$ and the corresponding matric suction $\psi$), $d_{10}, d_{30}, d_{60}$ of particle size distribution, non-uniformity coefficient $C_u$, curvature coefficient $C_c$, and void ratio $e$ of soil. Table 1 summarizes the basic physical properties of 244 fine-grained soil samples.

According to SWCC laboratory data, the parameters $a$, $n$, and $m$ of the VG model were obtained using Solver in a Microsoft Excel spreadsheet. Figure 1 shows the measured SWCC data points and the fitted SWCC from 244 fine-grained soil datasets. Figure 1 shows that the VG model is well suited for the soil matric suction range from $10^{-2}$ kPa to $10^{3}$ kPa, and this range can meet the needs of most projects.

In the subsequent calculation, it was found that the prediction accuracy is higher after taking the logarithm of the parameters. Therefore, the empirical equations of $\ln a, \ln n, \ln m$ with $\ln d_{10}, \ln d_{30}, \ln d_{60}, \ln C_u, \ln C_c$, and $\ln e$ will be established based on the maximum likelihood method. To identify inefficient soil properties in this model, the F-test was performed to evaluate the fitting effect between $\ln a, \ln n$ and $\ln d_{10}, \ln d_{30}, \ln d_{60}, \ln C_u, \ln C_c, \ln e$. This is a test of the fact that the statistical value obeys the F-distribution under the null hypothesis (H0). Moreover, it is commonly used to analyze a statistical model with more than one parameter to judge whether all or part of the parameters in the model are suitable for estimating the population. The results are shown in Table 2. The P-values of the F-test between $\ln a$ and $\ln d_{60}$, $\ln n$ and $\ln d_{60}$, and $\ln n$ and $\ln e$ are 0.42, 0.20, 0.80, respectively, which are greater than 0.05, therefore, cannot pass F-test. Hence, $\ln d_{60}$ will be eliminated from the fitting parameters $\ln a$ and $\ln n$; $\ln e$ will be eliminated from the fitting parameters $\ln n$. In the subsequent regression analysis, the corresponding term coefficient is taken as 0.

An example is a method of calibrating the regression equation of $\ln a$. For convenience, let $X$ denote $\{\ln d_{10}, \ln d_{30}, \ln d_{60}, \ln C_u, \ln C_c, \ln e\}$, and $Y$ denote $\ln a$. Assume that the relationship between $Y$ and $X_1, X_2, X_3, X_4, X_5, X_6$ can be expressed by the following equation:

$$Y = b_0 + b_1X_1 + b_2X_2 + b_3X_3 + b_4X_4 + b_5X_5 + b_6X_6 + \varepsilon$$

(3)

where $\varepsilon$ is a random variable with a normal distribution, the mean value of which is 0, and the standard deviation is $\sigma$. In the above equation, the unknown parameters include $B = \{b_0, b_1, b_2, b_3, b_4, b_5, b_6, \sigma\}$. When $B$ is known, based on Eq. (3), the mean value and standard deviation of $Y$ can be obtained from the following equations:

$$\mu_y = b_0 + b_1X_1 + b_2X_2 + b_3X_3 + b_4X_4 + b_5X_5 + b_6X_6$$

(4)

$$\sigma_y = \sigma$$

(5)

Let $X_{i1}, X_{i2}, X_{i3}, X_{i4}, X_{i5}, X_{i6}$ represent the values of $\ln d_{10}, \ln d_{30}, \ln d_{60}, \ln C_u, \ln C_c, \ln e$ of soil samples of group $i$, respectively. Let $\delta_i$ be the $Y$ value of soil samples of group $i$. Given $B$, the probability of observation $\delta_i$ can be obtained from the following equation:

$$f(\delta_i | B) = \phi \left( \frac{\delta_i - b_0 - b_1X_{i1} - b_2X_{i2} - b_3X_{i3} - b_4X_{i4} - b_5X_{i5} - b_6X_{i6}}{\sigma} \right)$$

(6)

where $\phi$ is the probability density function of the standard normal distribution. Suppose there are $s$ groups of soil samples, let $\mathbf{\delta} = \{\delta_1, \delta_2, \ldots, \delta_s\}$. When $B$ is known, the probability of observing all data is:

$$l(\mathbf{\delta} | B) = \prod_{i=1}^{s} \phi \left( \frac{\delta_i - b_0 - b_1X_{i1} - b_2X_{i2} - b_3X_{i3} - b_4X_{i4} - b_5X_{i5} - b_6X_{i6}}{\sigma} \right)$$

(7)

The above expression is the likelihood function of parameter $B$ of the regression equation. According to the principle of maximum likelihood [14], the optimal value of the model parameters is
\( \mathbf{B}^* \), at which \( f(\mathbf{B}) \) in Eq. (7) takes the maximum value. In the actual optimization process, the absolute value of the likelihood function in Eq. (7) is very small due to the existence of multiplication, so the algorithm can be easily influenced by the computation error during the optimization process, which makes it difficult for the algorithm to converge. In this case, the logarithm of the likelihood function can be performed, and the optimal value of the model parameters can be obtained by taking the maximum logarithm of the likelihood function.

Next, we will use the maximum likelihood estimation method to predict \( \ln n \) and \( \ln a \). Let \( \mu_i \) and \( \mu_2 \) be the mean values of \( \ln a \) and \( \ln n \), and let \( \sigma_1 \) and \( \sigma_2 \) represent the standard deviations of \( \ln a \) and \( \ln n \), respectively. First, the maximum likelihood method is used to calibrate the regression equation of \( \ln a \) based on Eq. (7), resulting in \( \mathbf{B} = \{ -2.16, -0.0833, 0.2050, -0.264, -0.178, 0.674 \} \). Taking the above parameters into Eq. (4), we can get the prediction equation of \( \mu_i \) as follows:

\[
\mu_i = -2.16 - 0.0833 \ln d_{10} + 0.2050 \ln d_{30} - 0.264 \ln C_u - 0.178 \ln C_c + 0.674 \ln e
\]

Based on Eq. (5), we obtain \( \sigma_2 = 1.33 \), \( \mu_1 \) is the most likely value of \( \ln a \) or best-predicted value, and \( \sigma \) measures the variability in the \( \ln a \) prediction.

Similarly, the maximum likelihood method is used to calibrate \( \ln n \), and the mean prediction equation is as follows:

\[
\mu_n = 0.167 + 0.0456 \ln d_{10} + 0.0122 \ln d_{30} + 0.00197 \ln C_u + 1.07 \times 10^{-4} \ln C_c
\]

The standard deviation of \( \ln n \) is \( \sigma_1 = 0.233 \).

Table 1. Range of the basic physical properties of 244 fine-grained soil samples.

| \( d_{10} \)(mm) | \( d_{30} \)(mm) | \( d_{60} \)(mm) | \( C_u \) | \( C_c \) | \( e \) |
|-----------------|-----------------|-----------------|-----------|-----------|-------|
| Minimum         | 1.17×10^{-5}    | 1.89×10^{-5}    | 1.41×10^{-3} | 6.26     | 5.73×10^{-4} | 0.425 |
| Maximum         | 7.99×10^{-3}    | 2.97×10^{-2}    | 1.81×10^{-1} | 1.38×10^{4} | 3.19×10^{2} | 10.7  |

Table 2. P values of F-test between SWCC characteristic parameters and soil physical properties.

| \( \ln d_{10} \) | \( \ln d_{30} \) | \( \ln d_{60} \) | \( \ln C_u \) | \( \ln C_c \) | \( \ln e \) |
|-----------------|-----------------|-----------------|-----------|-----------|-------|
| \( \ln a \)     | 1.95×10^{-14}   | 1.06×10^{-7}    | 0.42      | 2.84×10^{-15} | 0.001 | 2.17×10^{-4} |
| \( \ln n \)     | 3.68×10^{-27}   | 1.49×10^{-14}   | 0.20      | 5.29×10^{-22} | 3.48×10^{-5} | 0.80  |

Figure 1. SWCC laboratory data points and fitted curves.

2.3. The joint probabilisty distribution prediction model

The boundary probability distributions of \( \ln a \) and \( \ln n \) are obtained. To obtain the joint probability distribution between \( \ln a \) and \( \ln n \), it is necessary to obtain the correlation between \( \ln a \) and \( \ln n \). To obtain the above correlation, it is necessary to analyze the residual between the predicted and measured values of \( \ln a \) and \( \ln n \) of 244 soil samples. Let \( \delta_{il} \) and \( \delta_{2l} \) represent the measured values of \( \ln a \) and \( \ln n \) of the corresponding \( i \)th soil sample, and let \( e_{1l} \) and \( e_{2l} \) represent the prediction errors of
In \( a \) and \( \ln n \) of the corresponding \( i \)th soil sample. The residual error of the \( i \)th soil sample with respect to \( \ln a \) and \( \ln n \) can be calculated by the following equation:

\[
e_k = \delta_k - \mu_k \quad k = 1, 2
\]  

(10)

Let \( \beta = \{\text{elna}, \text{eln}\} \) be the residual error of 244 groups of soil samples. Kolmogorov-Smirnov (KS) test [15] makes it possible to judge whether the data obey the normal distribution. If the probability value of the test is greater than 0.05, it is considered to have passed the normal distribution test. The residual probabilities of the KS test are 0.0029 and \( 3.8 \times 10^{-29} \) for \( \ln a \) and \( \ln n \), respectively, so they do not obey the normal distribution.

Phoon and Ching [16] suggested that this is a very efficient method for converting the non-normal variable \( Q_i \) into its corresponding standard normal random variable \( P_i \) using the Johnson distribution system since it has a closed-form of the transformation relationship, and its form remains unchanged in the subsequent process of updating the parameters. To facilitate subsequent calculations, the residual is converted to a normal distribution using the Johnson transform.

Eq. (11) shows the conversion formula between \( Q_i \) obeying Johnson distribution and \( P_i \) obeying the lognormal distribution. In the Johnson distribution system, there are three types of distribution [17]: SU (unbounded system distribution), SB (bounded system distribution), and SL (lognormal system distribution). The known lognormal distribution belongs to the SL distribution. Slifker and Shapiro [18] proposed a method for determining the type of distribution and the corresponding parameters \( (a_P, b_P, a_Q, b_Q) \) based on four quantiles of \( Q_i \):

\[
P = \begin{cases} 
  b_P + a_P \sinh^{-1} \left( \frac{Q - b_Q}{a_Q} \right) & \text{SU} \\
  b_P + a_P \ln \left( \frac{Q - b_Q}{a_Q + b_Q - Q} \right) & \text{SB} \\
  b_P + a_P \ln \left( \frac{Q - b_Q}{a_Q} \right) & \text{SL}
\end{cases}
\]

(11)

Using the method proposed by Slifker and Shapiro (1980), the type of distribution and the corresponding parameters \( (a_P, b_P, a_Q, b_Q) \) of \( Q_i \) and \( Q_j \) are obtained as SB, \((1.23, -0.561, -4.42, 7.47)\) and SU, \((0.796, 0.297, 0.0690, 0.0648)\). Then, the transformed data \( P_1 \) and \( P_2 \) corresponding to the prediction residuals of \( \ln a \) and \( \ln n \) can be computed. The KS test probability values for \( P_1 \) and \( P_2 \) are 0.76 and 0.81, respectively. Thus, it is proved that the conversion results meet the criterion of normality.

Figures 2 (a) and (b) show the histogram of \( \text{elna} \) and \( \text{eln} \), respectively, and Figures 2 (c) and (d) show the histograms of \( P_1 \) and \( P_2 \), respectively. The KS test is also shown in Figure 2. According to the cumulative normal probability function curve (CDF), the empirical probability function curve (EDF), and the KS bounds, it assumes that \( P_1 \) and \( P_2 \) obey the normal test after the Johnson transform of \( \text{elna} \) and \( \text{eln} \), and the Johnson transform is very effective for \( \text{eln} \). Let \( \theta \) denote \( \{P_1, P_2\} \), then it obeys multivariate normal distribution.

According to the correlation analysis, there is a positive correlation between the residuals of \( P_1 \) and \( P_2 \), and the correlation coefficient between them is 0.944, i.e., \( \rho_{12} = 0.944 \).

Accordingly, let \( \mu_\theta \) represent the mean value of \( \theta \), and \( C_\theta \) represent the covariance of \( \theta \). \( C_\theta \) can be calculated as follows:

\[
C_\theta = \begin{bmatrix}
\sigma^2_1 & \rho_{12}\sigma_1\sigma_2 \\
\rho_{12}\sigma_1\sigma_2 & \sigma^2_2
\end{bmatrix} = \begin{bmatrix}
1.33 & 1.04 \\
1.04 & 0.916
\end{bmatrix}
\]

(12)

Based on the characteristics of multivariate normal distribution, the joint probability density function of \( \theta \) has the following form:
In equation (13), \( \mu_\theta = \{\mu_{ln a}, \mu_{ln n}\} \) represents the most likely value of \( \theta \), and the obtained SWCC is the most probable; \( C_0 \) gives the estimation error of \( \theta \) and determines the possible range of values \( \theta \). Therefore, the method proposed in this paper can not only provide the best possible SWCC position but also provide the possible range of SWCC values.

In practical application according to Eq. (13), samples of \( \theta \) denoting \( \{P_1, P_2\} \) can first be obtained. Then samples of the predicted residuals \( \{eln a, elnn\} \) can be obtained by the inverse Johnson transform as shown by Eq. (14). Finally, the predicted \( a, n, m \) can be obtained, and the predicted SWCC is obtained by inputting it into Eq. (1).

\[
f(\theta) = (2\pi)^{-1}\left|C_0\right|^{-1/2}\exp\left[-\frac{1}{2}(\theta - \mu_\theta)^T C_0^{-1}(\theta - \mu_\theta)\right]
\]  

(13)

3. Validation of the probability prediction model

To further verify the robustness of the method, four soil samples were randomly selected to predict SWCC. The characteristic parameters of these four soil samples are shown in Table 3.

On the example of soil sample 1, the characteristic parameters of particle size distribution, such as \( d_{10}, d_{50}, C_a, C_v \), and void ratio \( e \), are 2.18 \times 10^{-3} \text{ mm}, 7.15 \times 10^{-3} \text{ mm}, 11.1, 0.97, and 0.98, respectively. When the above parameters are introduced into Eqs. (8) and (9), we can get \( \mu_\theta = \{-3.03, -0.167\} \). The covariance of the distribution can be obtained from Eq. (12), that is, \( C_0 = \{1.33, 1.04; 1.04, 0.916\} \). According to the logarithm of the mean value of SWCC, the most possible SWCC position of
soil sample 1 is given in Figure 3 (a). It can be seen that the shape of the most likely SWCC is similar to the actual SWCC, but there are some errors in the data.

Then, substituting $\mathbf{\mu}_\theta$ and $\mathbf{C}_\theta$ into Eq. (13), the model of the probability distribution of the transformed parameters $\theta$ is obtained, which denotes $\{P_1, P_2\}$. According to the distribution, by generating 1000 samples of $\theta$, and using the method described above, 1000 samples of $\{\ln a, \ln n\}$ and 1000 SWCC are obtained. It is seen that due to certain prediction errors of $\ln a$ and $\ln n$ the obtained SWCC also has a certain variability, i.e., SWCC cannot be unambiguously determined by the soil particle size distribution and void ratio.

Each suction value in the 1000 prediction curves corresponds to 100 saturations. According to the 1000 saturation values, the confidence interval of the significance level of 0.1 below the suction value can be obtained, i.e., the confidence level of 90%. By varying the suction value, the 90% confidence interval of saturation can be obtained at different suction levels. The measured SWCC and 90% confidence intervals of saturation at different suction levels are compared in Figure 3 (a). It can be seen that the measured SWCC is within the 90% confidence interval of the predicted SWCC, and the fitted result is reasonable.

The same method is used to analyze soil samples 2, 3, and 4, and their measured SWCC and predicted SWCC are compared as shown in Figures 3 (b) (c) (d). It can be seen that the most probable SWCC of other soil samples also deviates somewhat from the measured SWCC, but it is still within the 90% confidence interval of the predicted SWCC, so it can provide more accurate information for engineering practice.

Table 3. The maximum width of the 95% confidence interval of the fragility curves.

| Property | Number | $N$ | 11226 | 11036 | 11229 | 11103 |
|----------|--------|-----|-------|-------|-------|-------|
| $d_{10}$ (mm) | 2.18×10^{-3} | 1.48×10^{-3} | 1.66×10^{-3} | 1.47×10^{-3} |
| $d_{30}$ (mm) | 7.15×10^{-3} | 4.53×10^{-4} | 4.07×10^{-3} | 1.41×10^{-2} |
| $d_{60}$ (mm) | 2.42×10^{-2} | 9.63×10^{-3} | 1.13×10^{-2} | 2.98×10^{-2} |
| $C_u$ | 11.1 | 651 | 6.80 | 20.2 |
| $C_c$ | 0.97 | 1.44 | 0.883 | 4.53 |
| $e$ | 0.98 | 2.51 | 1 | 0.98 |
Figure 3. Soil-water characteristic curve and prediction curve of tested soil samples: (a) soil sample 1; (b) soil sample 2; (c) soil sample 3; (d) soil sample 4.

4. Conclusion

Based on the VG model, the relationship between the fit parameters of the SWCC for 244 groups of fine-grained soil samples and their basic physical characteristics ($d_{10}$, $d_{30}$, $d_{60}$, $C_u$, $C_c$, and void ratio $e$) was established. The model for probabilistic prediction of the SWCC was created using residual regression analysis. Not only the most possible position of the SWCC is given, but also the range of model variation is considered and the 90% confidence interval of the predicted curve is determined. It has been established that the SWCC cannot be unambiguously determined based on the characteristic parameters of the particle size distribution of the soil and void ratio. When using particle size distribution curve to predict the SWCC, model errors inevitably occur. In comparison with existing studies, the proposed method can not only provide optimal values of the parameters of the SWCC but also give the range of variation of the SWCC, which lays a foundation for determining the impact of the prediction error of the SWCC in the analysis and evaluation of the mechanical properties of unsaturated soils.

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