Long-time asymptotic of temporal-spatial coherence function for light propagation through time dependent disorder

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Abstract

Long-time asymptotic of field-field correlator for radiation propagated through a medium composed of random point-like scatterers is studied using Bethe-Salpeter equation. It is shown that for plane source the fluctuation intensity (zero spatial moment of the correlator) obeys a power-logarithmic stretched exponential decay law, the exponent and preexponent being dependent on the scattering angle. Spatial center of gravity and dispersion of the correlator (normalized first and second spatial moments, respectively) prove to weakly diverge as time tends to infinity. A spin analogy of this problem is discussed.

Keywords: time-dependent disorder, light coherence, Bethe-Salpeter equation, series summation technique.
1. Introduction

The fluctuation phenomena in scattering of electromagnetic waves propagating in random media was the subject of extensive studies during the last decade. This activity was inspired by experimental evidence for a number of the phenomena (e.g. enhanced back-scattering [1,2], spatial and frequency correlation of back-scattered and transmitted radiation [3,4]; for a review see [5]) as well as by realizing a close similarity of these phenomena to fluctuation phenomena in electron transport in disordered systems (mesoscopic) [6,7]. The latter brought new physical insight and theoretical methods into the field established in optics many years ago [8]. The study of fluctuation effects was extended also to dynamic disorder in which the scatterers move at random. Golubentsev showed that the contribution of interference back-scattering to static field auto-correlator (in particular, average intensity), which is essential for immobile scatterers, is suppressed when they move [9]. The similar result for temporal-spatial auto-correlator (dynamic coherence function) was obtained by Stephen [10]. Short-time fluctuations were studied using diffusion approximation in [10] for the cases of a point source in an infinite medium and a plane wave incident on semi-infinite medium (Milne problem). However, the validity of this approximation for time-dependent disorder remained questionable and deserves special study, to which the present paper is devoted.

2. Definitions and Statement of the Problem

Let the electric field $E(r,t)$ of light obeys scalar wave equation with a source current density $j(r,t)$. Coherent field (CF) and field coherence function (FCF) are defined respectively by

$$E_c(r,t) = \langle E(r,t) \rangle,$$

$$\Gamma(r_1,t_1; r_2,t_2) = \langle E(r_1,t_1) E^*(r_2,t_2) \rangle - \langle E(r_1,t_1) \rangle \langle E^*(r_2,t_2) \rangle,$$  

where brackets mean average over realizations of underlying random process. In the case under consideration it is the scatterers motion that causes the randomness. Point-
like scatterers are considered, so individual scattering can be described by scattering amplitude $f_s$ (see e.g. [8], vol.1). In weak multiple scattering limit $\lambda \ll l$ (where $\lambda$ is the radiation wavelength, $l = 4\pi/n_s|f_s|^2$ is radiation mean free path, and $n_s$ is the scatterers density) CF is easily calculated. For space-time Fourier transform of CF one obtains

$$E_c(p, \omega) = \frac{4\pi i k_0}{c} j(p, \omega) G(p, \omega), \quad G(p, \omega) = \frac{1}{p^2 - k_0^2 - n_s f_s},$$

(3)

where $k_0 = 2\pi/\lambda$ is the wave number, $\omega = ck_0$ is the radiation frequency and $G(p, \omega)$ is ‘one-particle’ Green function (see e.g. [6,7]). When considering FCF slow coordinate $r = (r_1 + r_2)/2$ and time $t = (t_1 + t_2)/2$ as well as fast ones: $\rho = (r_1 - r_2)/2$ and $\tau = (t_1 - t_2)/2$ are introduced. Then Fourier transform of FCF (Eq.2) with respect to the fast variables

$$\hat{\Gamma}(r, t; p, \omega) = \int \int \exp\left[i\omega \tau - i\rho \cdot \rho\right] \Gamma(r_1, t_1; r_2, t_2) \frac{d^3 p d\tau}{(2\pi)^4},$$

(4)

is commonly used. This quantity may be interpreted as energy density in incoherent portion of the radiation with the frequency $\omega$ and propagation direction $n = p/p$ at the point $r$ and time instant $t$ (see e.g. [8], Vol. 2).

Main issue of the present paper is the study of $\hat{\Gamma}(r, t; p, \omega)$ at long times neglecting the coherence effects. By 'long times' I mean the regime when $\Delta(t)$, mean square displacement a scatterer moves during time $t$, is larger than the wavelength:

$$\Delta(t) > \lambda.$$  \hspace{1cm} (5)

### 3. Bethe-Salpeter Equation

Bethe-Salpeter equation in weak individual scattering regime was derived in [10]. In general case, except some parametric factors, the structure of this equation remains unchanged. For $\hat{\Gamma}(q, t; p)$, spatial Fourier transform of $\hat{\Gamma}(r, t; p)$, it reads

$$\hat{\Gamma}(q, t; p) = n_s|f_s|^2 G(p - q/2) G^*(p - q/2) \times$$

$$\int \exp\left[-\frac{1}{2}\Delta(t) |p - p_1|^2\right] \left[\hat{\Gamma}(q, t; p_1) - E_c(p_1 - q/2) E^*_c(p_1 + q/2)\right] \frac{d^3 p_1}{(2\pi)^3}. \hspace{1cm} (6)$$
The argument $\omega$ here and further is omitted for brevity. It is worth noting that Eq.6 may be also derived using Foldy-Twersky ansatz (see [8], Vol. 2). At distances much larger than wavelength, that is at $q/k_0 \ll 1$, the approximation

$$G(p - q/2) G^\ast (p - q/2) \simeq \frac{\pi l}{2k_0^2} \cdot \frac{\delta (|p| - k_0)}{1 - ilq \cdot p/k_0}$$ (7)

holds well, and Eq.6 with account of Eq.3 will be satisfied by the substitution

$$\hat{\Gamma}(q, t; p) = \frac{(2\pi)^3 l}{c^2} \delta (|p| - k_0) \gamma(q, t; p/k_0)$$ (8)

that transforms Eq.6 to the transport equation for the on-shell FCF $\gamma(q, t; n)$

$$(1 - ilq \cdot n) \gamma(q, t; n) - \int_{|n_1|=1} \exp \left[ -\frac{\mathcal{J}}{2} |n - n_1|^2 \right] \gamma(q, t; n_1) \frac{d\Omega_1}{4\pi}$$

$$= \int_{|n_0|=1} \exp \left[ -\frac{\mathcal{J}}{2} |n - n_0|^2 \right] j(k_0n_0 - q/2) j^* (k_0n_0 + q/2) \cdot \frac{d\Omega_0}{4\pi} , \quad \mathcal{J} = (k_0\Delta)^2.$$. (9)

For point source Eq.9 coincides with that derived by Golubentsev [9] using a special diagram technique whose rules take explicitly into account Eq.7. The sources of other types were considered [11] within real-space approach (RSA) after Rosenbluh et al. [12]. In the next section a connection between Bethe-Salpeter equation and RSA will be discussed.

4. Real-Space Trajectory Decomposition and Spin Analogy

Solving Eq.9 by iterations of the integral term represents its solution as an average of $\gamma(q, t; n_0, n)$, source-detector directional FCF, over the propagation directions $n_0$ allowed for radiation

$$\gamma(q, t; n) = \int_{|n_0|=1} j(k_0n_0 - q/2) j^* (k_0n_0 + q/2) \gamma(q, t; n_0, n) \frac{d\Omega_0}{4\pi}$$ (10)

with the series expansion in the integrand:

$$\gamma(q, t; n_0, n) =$$
Here \( W_N \) are scattering configuration weights given by

\[
W_N (q; n_0, n_1, \ldots, n_N) = \prod_{j=0}^{N} \frac{1}{1 - i q \cdot n_j}.
\]

At \( N \gg 1 \) and \( |r| \gg l \) the inverse transform Fourier of this product is strongly peaked at \( r = l \sum_{j=0}^{N} n_j \), so in real space FCF appears to be a sum of path integrals over polygon trajectories terminated at the point \( r \). Each \( j \)th (\( 0 \leq j \leq N \)) straight-line part of them has the length \( l \) and directed along the vector \( n_j \). The initial \( n_0 \) and final \( n_N \) directions are determined by a source and a detector, respectively, and each change of ‘free-flight’ direction occurs after a scattering act. Scattered directions \( n_1, \ldots, n_{N-1} \) are distributed with correlation if \( J \neq 0 \).

Thus, basic assumptions of RSA [12] result from the Bethe-Salpeter equation. In RSA, however, \( W_N \) are calculated using \textit{apriori} random-walk considerations, and the exponential weights due to scatterers motion are approximately averaged by averaging of their exponents [12]. An analysis shows that \( W_N \) of RSA is an average of right-hand side of Eq.12 at \( ql \ll 1 \) and \( N \gg 1 \) over fully random \( n_j \). With the only this less stringent approximation \( \gamma(q, t; n_0, n) \) will coincide with conditional partition function in an ensemble of one-dimensional classical Heisenberg spin systems, each with the same exchange integral \( J \), but different numbers \( N \geq 2 \) of species and weights \( W_N \). This analogy facilitated an estimation of FCF within RSA making use the methods of magnetism [13].

5. Long-Time Asymptotic of Coherence Function

Scatterers motion removes diffusion singularity: \( \gamma(q, t; n_0, n) \propto q^{-2} \) at \( q \to 0 \) specific for immobile scatterers. At \( ql \ll 1 \) it makes sense to consider hydrodynamic expansion

\[
\gamma(q, t; n_0, n) = g_0(n_0, n) + \sum_{k=1}^{\infty} i^k \sum_{\alpha} \frac{g_{\alpha_1} \cdots g_{\alpha_k}}{k!} g_{\alpha_1 \cdots \alpha_k}(n_0, n),
\]

(13)
where $\alpha$’s are the Cartesian indices, and the functions $g_{\alpha_1 ... \alpha_k}(t; n_0, n)$

$$g_{\alpha_1 ... \alpha_k}(n_0, n) = i^{-k} \left. \frac{\partial^k \gamma(q, t; n_0, n)}{\partial q_{\alpha_1} ... \partial q_{\alpha_k}} \right|_{q=0} = \int r_{\alpha_1 ... \alpha_k} \gamma(r, t; n_0, n) \, d^3r$$  \hspace{1cm} (14)

are spatial moments. The first term $g_0(n_0, n)$ of the expansion in Eq.13 (the zero moment) is the intensity of the fluctuations in a macroscopic volume. Next two moments normalized to the zero one have clear physical meanings. The vector and matrix

$$\mathbf{r}_\alpha = \frac{g_{\alpha}(n_0, n)}{g_0(n_0, n)}$$

$$\mathbf{r}_\alpha \mathbf{r}_\beta = \frac{g_{\alpha\beta}(n_0, n)}{g_0(n_0, n)}$$  \hspace{1cm} (15)

are the spatial center of gravity and dispersion matrix, respectively. Eq.13 can be directly obtained by iterations of the term $\propto q \cdot n$ in the transport equation for $\gamma(q, t; n_0, n)$, which differ from Eq.9 only in the right-hand side. Such a procedure creates recurrence equations for the moments, which enable us to calculate them recursively, but analyzing the full set of the moments (same as solving the original equation) is hardly possible. Nevertheless, the asymptotics of the above mentioned three moments at $\mathcal{J} \to \infty$ can be calculated rigorously. This limit is quite reasonable since Eq.5 implies the strong inequality $\mathcal{J} \gg 1$.

5.1. Fluctuations Intensity

Using Eq.10 and Eq.9 one obtains the equation for the zero moment

$$(1 - \mathcal{L}) g_0(n_0, n) = \exp \left[ -\frac{\mathcal{J}}{2} |n - n_0|^2 \right]$$  \hspace{1cm} (16)

where $\mathcal{L}$ is the same integral operator as appears in the left-hand side of Eq.9. The eigenfunctions of the operator $\mathcal{L}$ is well known spherical harmonic $Y_L^{(m)}(n)$ with the eigenvalue $\lambda_L$ degenerated with respect to the azimuthal number $m = 0, \pm 1, ..., L$:

$$\mathcal{L} Y_L^{(m)}(n) = \int_{|n_1|=1} \exp \left[ -\frac{\mathcal{J}}{2} |n - n_1|^2 \right] Y_L^{(m)}(n_1) \frac{d\Omega_1}{4\pi} = \lambda_L Y_L^{(m)}(n).$$  \hspace{1cm} (17)

Applying the addition theorem for spherical harmonics and the eigenfunction expansion of the solution to Eq.16 one gets closed expressions for the eigenvalues:

$$\lambda_L = \Lambda \left( L + \frac{1}{2}, \mathcal{J} \right), \quad \Lambda (\nu, x) = \sqrt{\frac{\pi}{2x}} \exp (-x) I_\nu (x),$$  \hspace{1cm} (18)
and the zero moment:

\[ g_0(n_0, n) = \sum_{L=0}^{\infty} (2L + 1) \frac{\lambda_L}{1 - \lambda_L} P_L(\cos \theta), \quad (19) \]

where \(I_\nu(x)\) is modified Bessel function [14], \(P_L(x)\) are the Legendre polynomials and \(\theta\) is the scattering angle: \(\cos \theta = n \cdot n_0\).

Apparent fluctuation intensity depend on the detection direction \(n\) if the source is directional (i.e. a narrow range of the radiation directions \(n_0\) is allowable). In the opposite case of point source all terms except isotropic one with \(L = 0\) are averaged out when substituting Eq.19 into Eq.10 at \(q = 0\). It is the value of \(J\) that determines how strong the anisotropy of Eq.19 is. Power expansion of \(I_\nu(x)\) with respect to \(x\) leads to \(\lambda_L = J^L/((2L + 1)!! + O(J^{L+2})\) at \(J \ll 1\). Thus, at short times Eq.19 is dominated by isotropic term \(\propto J^{-2}\), and the anisotropic terms are relatively small as \(J^{L+2}\). By an analogy with the case \(J = 0, q \neq 0\) this regime may be called diffusional in the sense that multiple scattering retains the isotropy of individual scattering on macroscopic scale. Quite opposite behavior takes place at long times. Using the asymptotic of \(I_\nu(x)\) at \(x \gg 1\) one obtains \(\lambda_L = (2J)^{-1} + O(J^{-2})\) at \(J \gg 1\). Independence of the leading term in \(\lambda_L\) on \(L\) makes all the terms in the series (19) important. Moreover, since next-to-leading asymptote of \(\lambda_L\) contains the factor \(L(L + 1)\) the terms with large orbital numbers seem to be even more important. This means that the calculation of \(g_0(n_0, n)\) requires summation of the whole infinite series. As a result, scattering angle dependence will be strong.

To exploit classical summation technique the series of interest is represented as a residue sum over points \(Z_L = i(L + \frac{1}{2})\), the zeros of \(\cosh \pi z\)

\[ g_0(n_0, n) = 2\pi i \sum_{L=0}^{\infty} \text{Res} \left[ \frac{-iz\Lambda(-iz, J)}{1 - \Lambda(-iz, J)} \cdot \frac{P_{\frac{1}{2}-iz}(\cos \theta)}{\cosh \pi z} \right]_{z = Z_L} \quad (20) \]

where \(\vartheta = \pi - \theta\), and \(P_\nu(x)\) is the spherical function. Adding the sum of residues at \(z_l\), the zeros of the function \(1 - \Lambda(-iz, J)\) in upper half-plane, to Eq.20 gives the integral
of the function in square brackets over the line \((-\infty + i \cdot 0, +\infty + i \cdot 0)\), and therefore

\[ g_0(n_0, n) = \int_{-\infty}^{+\infty} \frac{-i\zeta \Lambda (-i\zeta, J)}{1 - \Lambda (-i\zeta, J)} \cdot \frac{P_{-\frac{1}{2} - i\zeta} (\cos \vartheta)}{\cosh \pi \zeta} \, d\zeta + 2\pi \sum_i \frac{z_i P_{-\frac{1}{2} - iz_i} (\cos \vartheta)}{\cosh (\pi z_i) \frac{d}{dz} \Lambda (-iz_i, J)}. \]

Here \(\Lambda (-i\zeta, J)\) for real \(\zeta\) means the value on the upper branch of real-axis cut. Because \(\Lambda (-iz) = \Lambda (iz)\) both \(z_i\) and \(-z_i\) are the roots. Using this and the properties of the cone function \(P_{-\frac{1}{2} - iz} (x)\) [15] one obtains after a transformation

\[ g_0(n_0, n) = 2 \int_{0}^{+\infty} \frac{\zeta \text{Im} \Lambda (-i\zeta, J)}{[1 - \text{Re} \Lambda (-i\zeta, J)]^2 + [\text{Im} \Lambda (-i\zeta, J)]^2} \cdot \frac{P_{-\frac{1}{2} - i\zeta} (\cos \vartheta)}{\cosh \pi \zeta} \, d\zeta 
+ 4\pi \text{Re} \sum_{\text{Re} z_i > 0} \frac{z_i P_{-\frac{1}{2} - iz_i} (\cos \vartheta)}{\cosh (\pi z_i) \frac{d}{dz} \Lambda (-iz_i, J)}. \tag{21} \]

With this exact representation the desired asymptotic can be now handled. Eq.18 and Bessel functions theory [14] give at \(\text{Im} z > 0\)

\[
\Lambda (-iz, J) = \frac{1}{\sqrt{2\pi J}} \int_{0}^{\pi} \exp \left[ -J (1 - \cos u) \right] \cosh (uz) \, du 
+ i \frac{\sin (\pi z)}{\sqrt{2\pi J}} \int_{0}^{\infty} \exp \left[ -J (1 + \cosh w) + iwz \right] \, dw. \tag{22}
\]

Both integrals in Eq.22 for real \(\zeta\) can be estimated at \(J \to \infty\) using the Laplace method. At \(|\zeta| \gg \sqrt{2J}\) one obtains

\[
\Lambda (-i\zeta, J) \simeq \frac{1}{2J} \left[ \exp \left( \frac{\zeta^2}{2J} \right) + i \sinh (\pi \zeta) \exp \left( -2J - \frac{\zeta^2}{2J} \right) \right]. \tag{23}
\]

Let us assume, in addition, that \(|\zeta| \ll 2J/\pi\). The integral in Eq.21 is now easily calculated because under this condition lorentzian-like factor in the integrand has exponentially small broadening \(\text{Im} \Lambda (-i\zeta, J)\), and hence is well approximated by the delta-function

\[
\frac{\pi \delta (\zeta - \zeta_0)}{\left| \frac{d}{dz} \text{Re} \Lambda (-i\zeta_0, J) \right|},
\]

where \(\zeta_0\) is the positive root of real equation \(\text{Re} \Lambda (-i\zeta, J) = 1\). Substituting here the real part of Eq.23 gives the asymptotic of the root

\[
\zeta_0 \simeq \sqrt{2J \log (2J)}. \tag{24}
\]
This equation is consistent with the assumed inequalities provided that
\[ \frac{\sqrt{2J}}{\pi} \gg \sqrt{\log(2J)} \gg 1. \]  
(25)

An analysis based Eq.22 shows that in this limit the equation \( \Lambda(-iz, J) = 1 \) has in the upper right quadrant only one root \( z_0 \) given asymptotically by
\[ z_0 \simeq \zeta_0 + i \frac{\exp(\pi\zeta_0 - 2J)}{8\zeta_0 J}. \]  
(26)

Due to the inequalities (25) the imaginary part of this root is exponentially small, and can be discarded when calculating the residue term in Eq.21. Then the integral and residue contributions give together
\[ g_0(n_0, n) \simeq 6\pi J \frac{\frac{1}{2} - i\zeta_0 (\cos \vartheta)}{\cosh \pi \zeta_0}. \]  
(27)

At the end, using in Eq.27 the asymptotic of cone function at large argument [15], one obtains for hemisphere \( 0 < \theta \leq \pi \)
\[ g_0(n_0, n) \simeq 12\pi J f(\theta) \exp \left[ -\theta \sqrt{2J \log(2J)} \right], \]  
(28)

where \( f(\theta) = 1 \) at backward scattering \( (\theta = \pi) \), otherwise at \( 2\pi \sin \theta \gg [2J \log(2J)]^{-1/2} \)
\[ f(\theta) \simeq (2\pi \sin \theta)^{-1/2} [2J \log(2J)]^{-1/4}. \]  
(29)

At forward scattering \( (\theta = 0) \) the whole derivation is inapplicable because all terms of the series in Eq.19 are positive. In this case the fluctuation intensity proves to be greater than unity at any time.

To express Eqs.28, 29 via \( t \) one may use the models of the scatterer motion described in the literature. In two simplest, but practically important, cases of ballistic and diffusive motion: \( 2J = (t/\tau_a)^a \), where \( a = 2 \) and \( a = 1 \), respectively. The formulas for time constants \( \tau_a \) are found, for example, in [9,10]. Thus, at \( t \to \infty \) the fluctuations decay according to power-logarithmic stretched exponential law
\[ g_0(n_0, n) \sim \exp \left[ -\frac{t}{\tau_a} a \left( \frac{t}{\tau_a} \right)^a \log \left( \frac{t}{\tau_a} \right) \right]. \]  
(30)
5.2. The First and Second Spatial Moments

Calculating the asymptotics for $g_\alpha (n_0, n)$ and $g_{\alpha\beta} (n_0, n)$ is a very involved task. To make things easier one might introduce a local coordinate system with $z$-axis parallel to vector $n_0$ and consider local longitudinal components $g_z (n_0, n)$ and $g_{zz} (n_0, n)$. Other components are zero if spatial variation occurs only in the radiation direction. This is the case for an infinite source concentrated at the plane $z = 0$. CF calculated using Eq.3 is one half of the sum of the two attenuated forward and backward plane waves

$$E_\pm (r) \propto \exp \left( \pm i k_0 z - \frac{|z|}{2l} \right),$$

that is there are two allowed radiation directions $n_0 = n_0^\pm = (0, 0, \pm 1)$. Fortunately, the superposition principle holds also for FCF (Eq.10). This turns out to be equal to the sum of half-weighted $\gamma (r, t; n_0^\pm, n)$, both being independent on $x, y$. Therefore, considering CF and FCF for one direction, say $n_0^+$, one could hope to emulate a transmission problem where plane wave impinges on the boundary $z = 0$. Both CF and FCF vanish in the counterpart half-space, and coherent portion of radiation propagates only forward, whereas the fluctuations develop in all directions with the intensity at $t \to \infty$ given by Eqs.28, 29.

The first longitudinal moment, calculated recursively as outlined above, is given by

$$g_z (n_0, n) = l g_0 (n_0, n) + \sum_{L=0}^{\infty} \left[ \frac{(L+1) \lambda_{L+1}}{1 - \lambda_{L+1}} + \frac{L \lambda_{L-1}}{1 - \lambda_{L-1}} \right] \frac{P_L (\cos \theta)}{1 - \lambda_L}. \quad (32)$$

At $\theta > 0$ and $J \to \infty$ the summation technique developed above can be applied to Eq.32 as well. After division by Eq.27 one obtains the asymptotic for normalized first moment

$$\bar{z} \simeq \frac{\xi \sin \theta}{6}, \quad \xi = \sqrt{\frac{2J}{\log (2J)}} = \frac{(t/\tau_a)^{n/2}}{[a \log (t/\tau_a)]^{1/2}} \quad (33)$$

tending to infinity if $0 < \theta < \pi$, and $\bar{z} \to 0$ if $\theta = \pi$. Closed formula for the second longitudinal moment $g_{zz} (n_0, n)$ is derived along the same lines as Eq.32, and long-time asymptotic of normalized second moment is obtained making use the summation method
developed above. Expression for $g_{zz}(n_0, n)$ is rather lengthy, and obtaining the asymptotic involves processing many cumbersome terms without bringing new ideas. This analysis is omitted, and the final result is as follows

$$
\overline{z^2} \simeq \frac{\xi^2}{3} = \frac{l^2}{3} \cdot \frac{(t/\tau_a)^a}{a \log (t/\tau_a)}. \tag{34}
$$

In conclusion, whereas the fluctuations intensity decays at $t \to \infty$ according to Eq.30, spatial center of the FCF 'runs away' according to Eq.33, and due to Eqs.33, 34 spatial dispersion of the fluctuations $\delta = \sqrt{\overline{z^2} - (\overline{z})^2} \simeq \xi \left(1 - \sin^2 \theta/12\right)^{1/2} / \sqrt{3}$ also tends to infinity but with much weaker anisotropy. In other words, at long times random motion of the scatterers weakens the fluctuations, thus tending the radiation to a coherent state, but enhances drastically the spatial range of the field correlations $\xi$ over its value for static disorder $\sim l$.

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