Long-range spin-qubit interaction mediated by microcavity polaritons

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We study the optically-induced coupling between spins mediated by polaritons in a planar micro-cavity. In the strong coupling regime, the vacuum Rabi splitting introduces anisotropies in the spin coupling. Moreover, due to their photon-like mass, polaritons provide an extremely long spin coupling range. This suggests the realization of two-qubit all-optical quantum operations within tens of picoseconds with spins localized as far as hundreds of nanometers apart.

Planar micro-cavities are semiconductor devices that confine the electromagnetic field by means of two parallel semiconductor mirrors. When a quantum well is placed inside a micro-cavity the optical excitations (excitons) in the well couple to the electromagnetic modes of the dielectric structure. In the so-called strong-coupling regime, excitons and cavity photons give rise to new states, cavity polaritons, which appear in two branches separated by a vacuum Rabi splitting. In the long wavelength limit, and for a cavity exactly at resonance with the exciton energy, polaritons can be seen as hybrid states that are exactly half-matter and half-light. However, even with a half-matter character, polaritons have photon-like dispersive properties, determined by the reduced mass of the cavity-photon/exciton system. A photon confined in a $\lambda/2$ planar cavity has an effective mass of $m_{\text{eff}} = \hbar n \pi/eL$ ($n$ is the refraction index and $L$ is the length of the cavity) which is typically four to five orders of magnitude smaller than the exciton mass. The small polariton mass is known to affect the dynamics of optical excitations with phonons, with interface disorder, and it suggests the possibility of polariton Bose-Einstein condensates at room temperature. In this paper, we show that the small polariton mass has also a strong effect on the mutual interaction between spins localized in a quantum well.

Proposals for quantum computers based on spin degrees of freedom require that individual qubits are placed close enough so to have a significant exchange interaction between them. This exchange interaction can be direct (i.e. induced by a controlled overlap of the wave-functions), or indirect when mediated by spin excitations in a 2D electron gas or by optical excitation across the semiconductor bandgap. In the indirect schemes, the range of the spin coupling is related to the mass of the mediating particles, and the coupling decreases exponentially as a function of the distance between the spins. Here, we show that the small polariton mass gives an extremely long range for the spin coupling and introduces a non-exponential behavior. This implies that spin-qubits can be located several hundreds of nanometers apart while still retaining control on pair interaction through the use of polaritons.

We study two localized spins in a semiconductor quantum well embedded in a planar micro-cavity. Our numerical results consider the case of two shallow neutral donors in GaAs (e.g. Si), but the theory is valid with minor modifications to other impurities and host semiconductors, as well as to charged quantum dots. Under strong coupling conditions, optically active excitons and intra-cavity photons combine into polaritons, while dark excitons remain unaffected. We consider a cavity excited by an extra-cavity continuous wave (cw) laser with frequency below the polariton resonance. In response to the laser, the cavity becomes polarized, without energy absorption, and this dynamic polarization is used to control the localized spins. Microscopically, the polarization is described in terms of a density of virtual polaritons, proportional to the laser intensity, that mediates an interaction between pairs of spins.

Our first step is to use the Hopfield canonical transformation which allows us to treat the coupling between excitons and cavity-photons non-perturbatively. Due to spin-orbit and quantum confinement we can restrict our discussion to heavy-hole excitons only. Heavy-holes have a total spin component in the growth direction $S_z = \pm 3/2$. Optical excitations in the system are then described by lower polaritons with $J_z = \pm 1$, upper polaritons with $J_z = \pm 2$, and dark excitons with $J_z = \pm 2$. In the case of donors considered here the exciton-impurity spin interaction is given by an Heisenberg-like exchange interaction involving the electron in the exciton and the electron in the donor. Therefore, if we restrict the excitation to $\sigma_+$ circularly polarized light, optically excited polaritons with $J_z = +1$ can only couple to states with $J_z = +2$ by scattering with the donors. Moreover, since we are considering microcavities in the strong coupling regime excited near the lower polariton resonance, we will neglect the upper polariton branch. This approximation will be further discussed below. The final Hamiltonian consists of a term $H_0$ describing lower polaritons with $J_z = +1$ and dark excitons with $J_z = +2$, a $H_1$ term representing the coupling of the circularly polarized cw laser with the lower polariton at $k = 0$, and an interaction $H_2$ describing polaritons and dark excitons...
interacting with two localized impurities

\[ H_0 = \sum_{k} \Omega_k^P p_k^\dagger p_k + \Omega_k^X b_k^\dagger b_k \]

\[ H_L = \sqrt{A} \phi_{1s} V r_0 e^{i\omega_0 t} P_0 + h.c. \]

\[ H_I = J \sum_{ij} e^{i(k-k')j} R_{ij} v_{ik} v_{kj} H_{ij}^\lambda, \]

where \( \{i,j\} \) refer to the type of particle, either polariton (P) or dark exciton (X) with energy \( \Omega_k^P \) and creation operators \( p_k^\dagger \) and \( b_k^\dagger \) respectively. The exciton energy is \( \Omega_k^X = \epsilon_0 + \hbar^2 k^2 / 2M_X \) while the lower polariton energy is

\[ \Omega_k^P = \frac{\Omega_k^X + \Omega_k^C}{2} - \sqrt{\frac{(\Omega_k^X - \Omega_k^C)^2}{4} + g_k^2}, \tag{1} \]

where \( \Omega_k^C = \hbar c / n \sqrt{k^2 + (\pi/L)^2} \) is the energy of cavity photons, and \( g_k \) is the exciton-cavity photon coupling. We will consider microcavities at resonance, i.e. satisfying the condition \( \Omega_0^C = \Omega_0^X \), with a vacuum Rabi splitting of \( 2\omega_0 \). The index \( \lambda \) identifies the two localized impurities that we will call \( \Lambda \) and \( \beta \). \( \mathcal{V} \) and \( \omega_0 \) are the Rabi energy and frequency of the exciting laser, and \( \phi_{1s} \) is the excitonic enhancement in the light-matter interaction we use an exciton-impurity exchange interaction to take into account the finite range of the exchange interaction to this effective spin coupling, which is the one that can be used, for instance, to control spin entanglement and make optically controlled quantum gates.

The relevant terms in the level shift operator that contribute to the spin coupling are proportional to

\[ H_{eff}^{(2)} \propto H_{P_{\lambda \chi}}^{A} e^{i\delta} H_{X \lambda}^{B} R_{\lambda \lambda}^{A} + H_{\lambda \chi}^{A} e^{i\delta} H_{X \lambda}^{B} R_{\lambda \lambda}^{A} + (A \leftarrow B), \]

where \( G^{0}_{P_{\lambda \chi}} \) is the bare Green’s operator for the polariton (exciton), the superscript (2) indicates the second order contribution in \( J \) only, and \( \phi \) is the phase arising from the separation between impurities.

After some algebra, and using units of \( \hbar = 1 \), we obtain the final expression for the effective coupling as

\[ H_{eff}^{(2)} = \frac{C}{\delta^2} \left[ F_{R_{p}} s_x^A s_x^B + F_{R_{x}} (s_x^A s_x^B + s_y^A s_y^B) \right] \tag{3} \]

where \( C = J^2 |\phi_{1s}|^2 |r_0|^2 \), and \( \delta = \Omega_0^P - \omega_0 \) is the laser-polariton detuning. The two functions \( F_{R_{p}} \) and \( F_{R_{x}} \) describe the polariton-mediated and the exciton-mediated contributions to the spin coupling and are defined as

\[ F_{R_{p}} = \int_0^{\infty} \frac{dk}{2\pi} \frac{\nu_{ik}^2 v_{ik}^2 k J_0(kR)}{(\omega_0 - \Omega_k^P)}, \]

\[ F_{R_{x}} = \int_0^{\infty} \frac{dk}{2\pi} \frac{\nu_{ik}^2 v_{ik}^2 k J_0(kR)}{(\omega_0 - \Omega_k^X)}, \]

where \( J_0 \) is the Bessel function of order zero and \( R \) is the inter-qubit separation. \( F_{R_{x}} \) can be explicitly written in terms of modified Bessel functions \( K_0 \) and \( K_1 \). In the large \( R \) limit the function \( F_{R_{x}} \) has the 2D Yukawa form

\[ F_{R_{x}} \sim \frac{e^{-\sqrt{2\pi M_X s R}}}{\sqrt{R}}, \tag{4} \]
Notice that the exponential behavior is characterized by a range $\ell \sim 1/\sqrt{M_X}$. The integral $F_{R\nu}$ has to be calculated by numerical quadrature.

Eq. 4 contains the main features of the polariton mediated spin coupling. The vacuum Rabi splitting resolves dark and optical active excitations, making the indirect interaction mediated by polaritons and dark excitons different in strength and form. The overall interaction is spin-anisotropic. We can define a cut-off wave vector $k_c$ such that for $k > k_c$ both $r_k \sim 1$ and $\Omega_{k}^{\nu} \sim \Omega_X^\nu$. The integrands of $F_{R\nu}$ and $F_{RX}$ coincide in that region since the polaritonic branch is exciton-like.

We can then rewrite

$$F_{R\nu} \simeq F_{RX} + \int_{0}^{k_c} (I_P - I_X) \, dk = F_{RX} + D_{PX}$$

where $I_i$ is the integrand of either $F_{R\nu}$ or $F_{RX}$. The term $D_{PX}$ represents then the pure polariton contribution, while all the excitonic effects (dark-excitons plus polaritons at large $k$) are included in $F_{RX}$. The cut-off $k_c$ depends on the exciton cavity detuning and on the strength of the exciton-cavity coupling. Eq. 4 is rewritten as,

$$H_{eff} = \frac{C}{\delta^2} \left( F_{RX} s^A \cdot s^B + D_{PX} s_z^A s_z^B \right).$$

Fig. 1 shows the relative strength of the Ising-like polariton-mediated and the isotropic exciton-mediated contributions for different values of the exciton-cavity coupling $g_0$ as a function of the spin distance. Excitonic atomic units are used, where energy is given in excitonic $Ry^*$ and lengths are in Bohr radius $a_B^*$; for GaAs, they correspond to $1Ry^* = 4.4meV$ and $a_B^* = 125\text{\AA}$. The analytical and numerical values for $F_{RX}$, $F_{R\nu}$ and $D_{px}$ are presented in Fig. 2. Notice the existence of two distinct regions separated by a crossover distance $R_c$. For $R < R_c$, the dominant interaction has an isotropic Heisenberg form, while it changes to Ising-like for $R > R_c$. The first regime corresponds to an exciton-mediated coupling, while the second is a pure polariton-induced effect. Polaritons, due to their light mass, mediate a long range spin coupling interaction. However, since they have a fixed $J_z = +1$ they cannot flip the impurity spins, and can only induce an interaction diagonal in the impurity spin space. The upper polariton has also $J_z = +1$ and can only contribute to the Ising-like term. However, its contribution is reduced by $(\delta + 2g_0)^2$ due to the vacuum Rabi splitting $2g_0$. Fig. 3 shows that the strength of the exciton and polariton induced coupling in logarithmic scale. For the exciton, the interaction decays exponentially, while the polariton mediated term can survive up to extremely long spin-separations and shows a non-exponential behaviour. The long range nature of the polariton-mediated interaction presents important technological advantages for quantum information implementations. Using our parameters (see Fig. 3), we predict that the strength of the interaction is $J_{eff} \sim 10^{-2}Ry^*$ for impurities separated by distances of the order of $R_c = 12\,a_B^* \simeq 150\text{\AA}$. An estimate for the time needed for an operation can be given as $T = \pi/J_{eff} \simeq 40\text{ps}$, which is much smaller than the typical dephasing time for impurity spin qubits. Recent measurements have reported a spin relaxation time of the order of $\mu s$ for donors in GaAs [19]. To our knowledge, the spin decoherence time ($T_2$) of a single donor in GaAs has not been measured, but is also expected to be in the $\mu$s range. Moreover, notice that the time needed for a quantum operation does not change considerably when
we further increase the qubit separation. Even with an inter-qubit separation of 1 micron the time needed for one operation increases only by one order of magnitude to about 400 ps, and is still reasonably smaller than the decoherence time. With such a long range interaction, the realization of electric gates to control one-qubit operations and the use of localized magnetic field becomes feasible. The Ising-like interaction at long separation is not a limitation for quantum gate implementations. The polariton mediated coupling could be also used to control the nuclear spin of the donor in a scheme similar to the one in Ref. 8. In contrast to other cavity QED-based quantum computing implementations 20, the scheme discussed here does not require 0D confined electromagnetic modes, which is much harder to achieve experimentally. In a planar cavity the lateral dimension is not limited by the optical wavelength, which provides a fully scalable geometry for the qubit.

A spin coupling can also be obtained by a real polariton population in a scheme analogue to the RKKY 21 spin coupling mechanism. The spin interaction induced by a 0D cavity and excitons in quantum dots has been recently investigated 22, and also in this 0D case the presence of a strong coupling generates anisotropies in the spin interaction. This approach is not appealing for quantum computing implementation since a real population of photocarriers will add decoherence to the spin-qubit. However, it would be interesting to explore the dynamics of spin in the presence of a dense polaritons population that condense in a phase coherent state, as observed recently in II-VI microcavities 23. High-quality microcavities embedding Mn-doped magnetic quantum wells in the strong coupling regime have recently been realized 24. The polariton mediated spin coupling could be explored in these systems as a method for the ultrafast control of the quantum well magnetization.

In conclusion, we have shown that the optical excitation of microcavity polaritons can couple spins localized in a quantum well. Due to the small polariton mass, the spin coupling has an extremely long range, and at large distances is Ising-like. The interaction is strong enough for the realization of quantum operations with spins located as far as several hundred of nanometers apart and within a time scale much shorter than the spin decoherence time. This interplay of polaritons and localized spins represents a peculiar feature of solid-state cavity QED, which has no equivalent in the atomic case.

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