A low complexity detection algorithm for \( m \)-QAM in spatial modulation systems

Qiong Wang, Youzhi Wu, Xiaole Du, Zhijie Yin

School of Communication and Information Engineering, Chongqing University of Posts and Telecommunications, Chongqing, China

Abstract. This paper proposes a low complexity detection method for M-QAM (M-ary quadrature amplitude modulation) in spatial modulation (SM) systems. The new method utilizes the thought of constellation layering to modify constellation points search mode in QAM, which reduces the calculated amount significantly, and the total complexity will not increase with the size of constellation linearly. Simulation results show that the method can work well in high order QAM transmission systems. The novel method provides a significant reduction in computational complexity while keeping the near-optimal bit-error-rate (BER) performance.

1 Introduction

Spatial modulation is a novel transmission technology for multiple-input-multiple-output (MIMO) wireless systems, which was proposed originally in the shape of Space Shift Keying (SSK)\(^1\) in 2001. This technology transmits information by making full use of the channel difference of different transmit antennas. In 2006, to increase the spectral efficiency of SSK, R.Y.Mesleh and H.Haas expanded SSK to a novel multiple antennas transmission technology in \(^2\), and officially named it SM. Different from the conventional two dimension Amplitude Phase Modulation (APM), SM can increase spectral efficiency by employing the additional spatial dimension. This is because both transmit antenna index and constellation symbol carry the bit information.

SM has multiple transmit antennas but only one antenna is activated while other antennas keep silent at each time instance, and the constellation symbol is transmitted through the activated antenna. So the problems of inter-channel interference (ICI) and time synchronization among antennas which exist in MIMO are solved effectively in SM systems. Compared with MIMO, SM has obvious advantages in detection complexity and energy efficiency, which meets the demand of energy efficiency in future mobile communications\(^3\).

It is because SM has many advantages that scholars research its detection algorithm widely. The optimal detection algorithm in SM is maximum-likelihood (ML) method, which provides an excellent performance by exhaustively searching over the transmit antennas and constellation points\(^4\). The complexity of ML increases linearly with the number of antennas and the size of constellation. Theoretically, ML has the optimal BER performance but the highest computational complexity. To reduce the complexity, several decoders called sphere decoding (SD) are proposed. In \(^5\), receiver-centric sphere decoding (RX-SD) decoder decreases the receive search space to reduce the complexity, and transmitter-centric sphere decoding (TX-SD) decoder in \(^6\) reduces the complexity by restricting the constellation points at the transmitter. In \(^7\), a new tree-search-based method named M algorithm to ML (M-ML) is proposed, which applies parallel search structure to reduces the “receive search space”. In 2014, a detection method for M-PSK named low complexity ML optimal detection
algorithm (LC-ML) was proposed in [8], the method reduces the calculated amount significantly by searching the phase of the constellation points. However, this method is just suitable for $M$-PSK systems, which limits its application.

A novel method is proposed for $M$-QAM in this paper, different from traditional constellation points search method, the proposed method layers constellation according to the amplitudes of constellation points firstly, then by searching the phases of constellation points to reduce calculated amount of constellation point search.

2 System model
In SM system, the input bit stream is divided into two part, one part is mapped into constellation point, and the other part is mapped into transmit antenna index. Therefore, the spectral efficiency of SM will increase with constellation size and transmit antenna amount. The diagram of spatial modulation system model is shown in Figure 1.

![Spatial modulation system model](image)

Figure 1: Spatial modulation system model

Considering the SM systems with $N_t \times N_r$ antennas and 'M-ary' constellation set. The Rayleigh fading channel model and additive white Gaussian noise (AGWN) is considered, meanwhile a perfect channel state information (CSI) is known in advance at receiver. The received symbol vector at the receiver is expressed as follows:

$$y = Hx + n = h_i s + n$$

(1)

where $H$ is the $N_t \times N_r$ channel matrix, $x$ is the transmit vector which contains the value of constellation symbol and the index of activated antenna, $n$ is the additive noise vector, $h_i$ is the $i$th column of $H$, $i \in \{1,2,...,N_t\}$, $s$ denotes the transmit constellation symbol, $s \in \{s_1,s_2,...,s_M\}$.

The ML algorithm can be written as:

$$[\hat{i}, \hat{s}] = \arg \min_{i,s} \|y - h_i s\|_F^2.$$  

(2)

where $\| \cdot \|_F$ is the Frobenius norm, $\hat{i}$ and $\hat{s}$ are the estimated value of activated antenna index and constellation symbol, respectively.

3 Proposed detection algorithm

3.1 LC-ML
LC-ML is a low complexity detection algorithm for $M$-PSK proposed in [8]. This decoder reduces plenty of calculated amount by searching the phases of constellation points, the total computational complexity of LC-ML is independent of constellation size and it will not increase with $M$. For LC-ML, the estimated constellation symbol $\hat{s}$ is detected as follows:

$$[\hat{i}, \hat{s}] = \arg \min_{i,s} \|y - h_i s\|_F^2 \Rightarrow$$

$$\hat{s} = \arg \min_{s} \|y - h_i s\|_F^2 \Rightarrow$$

$$\hat{s} = \arg \min_{s} \|h_i^H s\|^2 - 2 \Re(s^H y).$$

(3)

In $M$-PSK constellation, all the amplitudes of constellation points are 1, so equation (3) can be written as:
\hat{s} = \arg\min_{s} (\|h_{s}\|^{2} - 2\text{Re}(s^{H}h^{H}y))
\hat{s} = \arg\min_{\varphi_{s}} (\|h_{s}\|^{2} - 2\text{Re}(e^{-j\varphi_{s}} h^{H}y|e^{\imath\theta_{s}}|))
\hat{s} = \arg\min_{\varphi_{s}} (\|h_{s}\|^{2} - 2|h_{s}^{H}y|\cos(\theta_{s} - \varphi_{s})).

where $\varphi_{s}$ denotes the phase of constellation points, and $\theta_{s}$ denotes the included angle between $h_{s}$ and $y$. when the value of $i$ is given, equation (4) has the equivalent form as follows:

$$\varphi_{i} = \arg\max_{\varphi_{i}} (\cos(\theta_{i} - \varphi_{i})).$$

The transmit symbol is the constellation point whose phase is closest to $\theta_{i}$, the phases of constellation points with 8PSK and included angle between $\theta$ and $\varphi$ is shown in Figure 2.

$$\varphi_{i} = \text{mod}(\text{rnd}(Q_{i}),M) * 2\pi / M.$$  

$$(i, \hat{s}) = \arg\min_{i,s} (\|h_{s}\|^{2} - 2\text{Re}(s^{H}h^{H}y)).$$

where $\text{rnd}(\cdot)$ is the integer which is nearest to a variable, $\text{mod}(\cdot, \cdot)$ is the modulo operation, $Q_{i} = \theta_{i} / (2\pi / M)$, $i$ is the activated antenna index, the $N_{t}$ corresponding $\varphi_{i}$ which meet equation (5) can be obtained from equation (6), then it is easy to get the transmit signal $s$ according to $\varphi_{i}$. Finally, traversing the $N_{t}$ transmit antennas indexes $i$ and corresponding constellation points $s$, the combination $i$ and $s$ which meets equation (7) is the optimal solution. The detection process of LC-ML method is described as follows:

### Algorithm 1: LC-ML detection algorithm for $M$-PSK

1: for $i$ = 1 to $N_{t}$
   2: compute $h^{H}y, \|h_{s}\|^{2}$
   3: compute $\varphi_{i}$ by equation (6), then get $s$ according to $\varphi_{i}$
   4: $B(i,s) = \arg\min_{i,s} (\|h_{s}\|^{2} - 2\text{Re}(s^{H}h^{H}y))$
   5: endfor
   6: $[i, \hat{s}] = \arg\min_{i,s} (B(i,s))$

### 3.2 Proposed method

It is obvious that the complexity of LC-ML reduces significantly. However, the decoder is just applicable for $M$-PSK, it can not work in $M$-QAM. Actually, the performance of $M$-PSK is much worse than $M$-QAM when $M$ is large. To resolve this problem, a novel algorithm for $M$-QAM is proposed. For $M$-QAM, the amplitudes of constellation points is not 1, the proposed method layers constellation with $M$-QAM into $L$ layers according to the amplitudes of constellation points. For example, the layered constellation diagram of 16QAM is shown in Figure 3, and more detail is shown in Table 1.

![Figure 2: Constellation diagrams of 8PSK](image-url)
Figure 3: Layered constellation diagrams of 16QAM

| Layer | Amplitude | Phase amount | Set of phase |
|-------|-----------|--------------|--------------|
| 1     | $r_1 = \sqrt{5}/5$ | 4             | $\theta_1 = (4/3, 4/5, 4/7, 4/4)$ |
| 2     | $r_2 = 1$      | 8             | $\theta_2$                                           |
| 3     | $r_3 = \sqrt{5}/5$ | 4             | $\theta_3 = (4/3, 4/5, 4/7, 4/4)$ |

Table 1: Amplitudes and phases of layered constellation with 16QAM

Then equation (4) can be written as:

$$\hat{s} = \arg \min_s \left( \| h^T_s s \|^2 - 2 | h_i^H y | | s |^2 \cos(\theta_i - \phi_{i,s}) \right)$$

$$= \arg \min_{i,s} \left( \| h^T_s s \|^2 - 2 | h_i^H y | | s |^2 \cos(\theta_i - \phi_{i,s}) \right). \tag{8}$$

where $r_i$ denotes the amplitude of constellation points in $l_{th}$ layer, when $i$ and $l$ is given, equation (8) has the equivalent form as follows:

$$\phi_{i,s} = \arg \max_{\phi_{i,s}} \cos(\theta_i - \phi_{i,s}). \tag{9}$$

$$(\hat{i}, \hat{s}) = \arg \min_{i,s} \left( \| h^T_s s \|^2 - 2 \Re(s^H h_i^H y) \right). \tag{10}$$

where $\phi_{i,s}$ denotes the corresponding phase of $i_{th}$ transmit antenna and $l_{th}$ layer constellation points.

After computing $\phi_{i,s}$ by equation (6), it is easy to get the corresponding constellation point $s$ according to $\phi_{i,s}$ and $r_i$, by traversing transmit antenna indexes $i$, number of layers $l$ and constellation points $s$, the combination of $i$ and $s$ which meets equation (10) is the optimal solution. The specific detection procedure of proposed method is described as follows:

Algorithm 2: proposed detection method for $M$-QAM

1: compute amplitudes and phases of all the constellation points, then layer the constellation into $L$ layers according to amplitudes
2: for $i = 1 : N_t$
3: compute $h_i^H y, \| h_i \|^2$
4: for $l = 1 : L$
5: compute $\phi_{i,l}$ by equation (6), then get $s$ according to $\phi_{i,l}$ and $r_i$
6: $B(i,s) = \arg \min_{i,s} \left( \| h^T_s s \|^2 - 2 \Re(s^H h_i^H y) \right)$
7: endfor
8: endfor
9: $[f_{\text{Proposed}}, x_{\text{Proposed}}] = \arg \min_{i,s} (B(i,s))$
Table 2 shows the specific layer amount of constellation with different $M$-QAM.

| Modulation order | 4QAM | 16QAM | 64QAM | 256QAM |
|------------------|------|-------|-------|--------|
| Layer amount     | 1    | 3     | 9     | 32     |

Table 2: Layer amount of different modulation order

4 Complexity analysis

In this subsection, the computational complexity of proposed method and the methods introduced in section 1 are provided, the complexity is defined as the total times of real multiplications or real division.

1) ML decoder

For ML decoder, $6N_r$ real multiplications are required for $\|y - h_i s\|_2^0$, so the computational complexity of ML is $6N_r N_M$.

2) RX-SD decoder

The computational complexity of RX-SD decoder is $6\sum_{i=1}^{N_r} \sum_{s=1}^{M} N_r(i,s)$, where $N_r(i,s) \in \{1,2,\ldots,N_r\}$.

3) TX-SD decoder

For TX-SD decoder in [6], the upper-bound of computational complexity is $C_{\text{Pre-Comp}} + C_{\text{internal}} + 6N_r \text{card}\{\theta_R\}$, where $C_{\text{Pre-Comp}} = 4N_r^2 / 3 + N_t(4N_t N_r + 6N_r + 6N_t + 3)$, $C_{\text{internal}} = 2N_t + (2N_t + 3)N_i(19)$, $N_i(19)$ denotes the calculation times of equation (19) in [6], card\{\cdot\} denotes the cardinality of a set, $\theta_R$ denotes the set of selected constellation points.

4) M-ML decoder

The computational complexity of M-ML decoder is $6\sum_{i=1}^{N_r} M_i + 6N_t M$, where $M_i$ denotes the retained search node number of $i$th layer in the tree search structure.

5) LC-ML decoder

The computational complexity of LC-ML decoder in [8] is $6N_t(N_r + 1)$, where $\|h_i\|_p^2$ and $h_i^H y$ in step 2 of algorithm 1 need $2N_r$ and $4N_r$ real multiplications, respectively. step 3 needs 3 real multiplications to compute $\varphi$, step 4 needs 3 real multiplications, because 2 real multiplications are required for $\text{Re}(s^H h_i^H y) = \text{Re}(h_i^H y) + \text{Im}(h_i^H y) + \text{Im}(s^H y)$.

6) Proposed decoder

The computational complexity of proposed decoder is $(6N_r + 7L)N_i + 3M$. As is shown in algorithm 2, step 1 needs $3M$ real multiplications, $6N_t$ real multiplications are required in step 3, and step 5 and step 6 need 3 and 4 real multiplications, respectively.

5 Simulation results

The BER versus signal-to-noise ratio (SNR) performance and complexity reduction percentage of various methods on different condition are analyzed and compared in this section.

The BER performance of ML, LC-ML and proposed method with different modulation order are shown in Figure 4. We can observe that the curve of proposed decoder overlaps with ML on different conditions, that means the proposed method has negligible BER performance loss compared with ML algorithm. On the contrary, LC-ML decoder lose a lot in BER performance relative to the other two decoders, especially the modulation order is high. This is because the Euclidean distance between constellation points is very close when high order PSK is adopted, which will lead to higher error decision probability at the receiver.
In Figure 5, the computational complexity reduction percentage of five methods relative to ML are shown, where $C_{rel} (%) = (C_{ML} - C_{other})/C_{ML} \times 100\%$. To obtain a near-optimal BER performance, the values of $M$ for M-ML are $[12,6,4]_{M=4}$, $[32,10,4]_{M=16}$, $[64,32,16]_{M=64}$. The simulation results show that all the five methods have significantly complexity reduction compared with ML algorithm. Besides, it is obvious that the complexity reductions percentage of LC-ML and proposed decoders are both much larger than RX-SD, TX-SD and M-ML decoders with different modulation order and SNR, because both the two methods convert traditional constellation points search mode into phase search, which decreases the amount of calculation dramatically. As is shown in Figure 5, the complexity of proposed method is higher than LC-ML, because the procedure of constellation layering needs extra calculated amount. The larger the constellation order are, the higher complexity reduction percentage the proposed method provides, and almost 90% reduction achieved for 64QAM. Obviously, the proposed method is suitable for high order modulation systems.

6 Conclusion
This paper proposed an efficient detection algorithm for $M$-QAM in spatial modulation systems. The proposed method has the near BER performance and enormous reduction in complexity compared with ML. On the basis of LC-ML, the new method makes the thought of phase search applicable for $M$-QAM by layering constellation according to the amplitudes of constellation point, which reduces the complexity dramatically while keeping a near optimal BER performance.
Acknowledgements
This work is partially supported by the Major Project of National Science and Technology of China under Grants No. 2016ZX03002010-003&2015ZX03001033-002.

References
[1] Chau, Yawgeng A, Shi-Hong Yu. “Space modulation on wireless fading channels”, Vehicular Technology Conference, 3, pp. 1668-1671, (2001).
[2] Mesleh, Raed Y, et al. “Spatial modulation”, IEEE Transactions on Vehicular Technology, 57, pp. 2228-2241, (2008).
[3] Di Renzo, Marco, et al. “Spatial modulation for generalized MIMO: Challenges, opportunities, and implementation”, Proceedings of the IEEE, 102, pp. 56-103, (2014).
[4] Jeganathan, Jeyadeepan, Ali Ghrayeb, and Leszek Szczecinski. “Spatial modulation: optimal detection and performance analysis”, IEEE Communications Letters, 12, (2008).
[5] Younis, Abdelhamid, et al. “Reduced complexity sphere decoder for spatial modulation detection receivers”, Global Telecommunications Conference (GLOBECOM 2010), 2010 IEEE. IEEE, (2010).
[6] Younis, Abdelhamid, et al. “Generalised sphere decoding for spatial modulation”, IEEE Transactions on Communications, 61, pp. 2805-2815, (2013).
[7] Zheng, Jianhong, Xiaobo Yang, and Zhe Li. “Low-complexity detection method for spatial modulation based on M-algorithm”, Electronics Letters, 50, pp. 1552-1554, (2014).
[8] Men, Hongzhi, and Minglu Jin. “A low-complexity ML detection algorithm for spatial modulation systems with M-PSK constellation”, IEEE Communications Letters, 18, pp. 1375-1378, (2014).