Electroweak Baryogenesis with Embedded Domain Walls

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We consider electroweak baryogenesis mediated by embedded domain walls. Embedded domain walls originating from a symmetry breaking phase transition are stabilized by thermal plasma effects, so that the electroweak symmetry is unbroken in their cores. For this reason, the cosmological evolution of such domain walls can generate a sufficiently large baryon asymmetry, irrespective of the order of the electroweak phase transition. For embedded domain walls, the condition that the energy of the universe not be dominated by the energy of the domain walls is relaxed significantly, and it is shown to be compatible with our scenario of electroweak baryogenesis.

§1. Introduction

The origin of the baryon asymmetry in the universe is one of the greatest mysteries of modern cosmology and particle physics. The magnitude of this asymmetry has been determined by two methods. One method is Big Bang nucleosynthesis, which yields \( \eta \equiv n_B/n_\gamma = (3.4 - 6.9) \times 10^{-10} \), where \( n_B \) is the net baryon number density and \( n_\gamma \) is the photon number density. The other method involves the study of the angular power spectrum of cosmic microwave background anisotropies (which is effective because changing the baryon number density changes the height of the acoustic peaks in the spectrum), which yields \( \eta = (6.14 \pm 0.25) \times 10^{-10} \). These two estimates are consistent within a 95% confidence level.

Electroweak baryogenesis is one of the most promising scenarios to explain the net baryon asymmetry, because it requires very little physics beyond the Standard Model (SM) of particle physics. In particular, as first emphasized in Ref. 3), by embedding the SM in standard Big Bang cosmology, it is possible to satisfy Sakharov’s conditions for successful baryogenesis: (1) the existence of baryon number violating processes; (2) that these processes violate \( C \) and \( CP \) symmetries; and (3) the existence of deviations from thermal equilibrium.⁴ In particular, the baryon number violating processes in the SM are generated through the chiral anomaly.

Unfortunately, the minimal SM cannot produce a sufficiently large baryon asymmetry, because the degree of \( CP \) violation is too small and, in addition, within this model it is difficult to realize large deviations from thermal equilibrium (see, e.g., Ref. 5) for recent reviews of electroweak baryogenesis). Thus, to realize electroweak baryogenesis an extension of the SM must be considered. Sufficient \( CP \) violation can be obtained, for example, by considering two-Higgs models which explicitly contain \( CP \) violating terms in the Higgs mass matrix in addition to the small amount of \( CP \)
violation in the fermion mass matrix present in the SM.

The nucleation and expansion of critical bubbles — which are produced if the phase transition is strongly first order — are processes out of thermal equilibrium. In the SM, the phase transition is not strongly first order, and it has been pointed out that in fact the electroweak phase transition is absent in the SM with experimentally allowed Higgs masses. However, a sufficiently strong phase transition can be realized by considering supersymmetric extensions of the SM. But in the minimal extension, severe conditions are required: In particular, the Higgs bosons must have masses just above the present experimental bounds and the stop quark must be lighter than the top quark.

Another scenario satisfying the out-of-equilibrium condition is to consider topological defects in the cores of which the electroweak symmetry is unbroken. Specifically, in Ref. 10) the role of electroweak strings (which are non-topological solitons arising in the SM and its extensions) was explored. There, it was shown that the contraction of the loops of such strings generates a departure from thermal equilibrium, just as does the expansion of a bubble wall. In theories which admit defects, defects are formed for any order of the phase transition. Thus, the out-of-equilibrium state is realized even if the electroweak phase transition is of second order. However, electroweak strings are unstable for realistic values of the Weinberg angle.

As an extension of the above idea, one can consider cosmic strings originating from a symmetry breaking that takes place before the electroweak phase transition. If the electroweak symmetry is restored and sphaleron processes are unsuppressed in the defect cores, then electroweak baryogenesis is possible. Initially, this baryogenesis scenario was investigated while taking into account only local processes. In such studies, it was shown that baryogenesis is most efficient when the new symmetry breaking scale is just above the electroweak scale. In this case, the density of strings present at the electroweak symmetry breaking scale is largest. Later, the analysis was improved by including decay and nonlocal effects, which makes the defect-mediated mechanism more efficient. Nonetheless, the amplitude of the baryon asymmetry which could be produced this way still lies below the observed value. The main reason for the ineffectiveness of this mechanism is that cosmic strings are one-dimensional objects, and therefore, even if the strings are moving at relativistic speeds, they sweep out too small a volume of space. An additional necessary constraint is that the defects must be thick enough that the electroweak sphalerons fit into their cores.

In contrast to cosmic strings, a scaling network of domain walls (see, e.g., Refs. 16–18) for reviews of topological defects in cosmology) moving at relativistic speeds will sweep out a large fraction of the universe. Thus, a sufficiently large baryon asymmetry can be easily generated by making use of domain walls, provided that the microscopic Lagrangian contains a sufficiently large CP violation. However, domain walls produced at energy scales comparable to or higher than the electroweak scale quickly dominate the energy density of the universe. This causes severe cosmological problems. Thus, in order to make domain wall-mediated electroweak baryogenesis viable, we must introduce ad hoc processes that remove domain walls at some time after the electroweak symmetry breaking but before the time of nucleosynthesis. For
example, a slight tilt is often introduced into the scalar field potential, which breaks the degeneracy of the vacua such that domain walls decay.

In this paper, we consider another method for wall-mediated electroweak baryogenesis which avoids the domain wall problem. Instead of stable domain walls, we make use of embedded domain walls. These have the nice feature that their energy density decreases with the expansion of the universe. The embedded defects\(^{19}\) of a field theory are solutions of the classical field equations whose energy distribution is similar to that of a topological defect. However, in contrast to topological defects, they are not topologically stable and thus decay in the vacuum. However, if some of the fields are fixed, then the field configuration becomes a topological defect in the constrained space.

It was recently argued\(^{20}\) that many types of embedded defects can be stabilized by interactions with a thermal plasma. To illustrate this effect, let us consider a multi-component real scalar field with a “Mexican hat” type potential. We consider the epoch after spontaneous symmetry breaking when the scalar field is no longer in thermal equilibrium. In the case that all scalar field components except one are charged, the charged fields acquire thermal masses from gauge interactions with the photon bath. (Photons are still in thermal equilibrium.) If the thermal mass is sufficiently large, then the symmetry is broken only in the direction of the neutral scalar field. In this way, embedded domain walls are produced and stabilized. If the square of the induced plasma mass exceeds the absolute value of the negative square mass of the scalar field at the top of its potential, the symmetry is restored in the center of the defect. As the temperature decreases further, the thermal masses also decrease, so that a core phase transition occurs.\(^{21}\) This means that the symmetric state of the charged scalar fields in the core evolves into a slightly asymmetric state, without destroying the overall defect. Thus, the domain wall configurations survive even for small thermal masses. However, the properties and cosmological evolution of these domain walls differ from those of the conventional domain walls, as shown below. The conditions necessary to avoid the cosmological domain wall problem are thereby relaxed significantly.

In the next section, we investigate the properties and cosmological evolution of embedded domain walls. Next, electroweak baryogenesis mediated by these defects is investigated. In the final section, we give discussion and conclusions.

§2. Embedded domain wall formation and dynamics

Let us consider a multi-component real scalar field \(\phi_i \ (i = 1, \ldots, N)\) charged under a gauge group \(G\), which breaks into a gauge group \(H\) that includes the gauge group of the SM as a subgroup. The Lagrangian density \(\mathcal{L}\) is given by

\[
\mathcal{L} = \frac{1}{2} (D_\mu \phi_i)(D^\mu \phi_i) - V(\phi_i) - \frac{1}{4} F^{a}_{\mu \nu} F_{a \mu \nu}, \tag{2.1}
\]

where

\[
F^{a}_{\mu \nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f_{abc} A^b_\mu A^c_\nu \tag{2.2}
\]
is the field strength, $A_\mu^a$ is a gauge field, $g$ is a gauge coupling constant, $D_\mu \phi_i$ is a covariant derivative defined as $D_\mu \phi_i = (\partial_\mu - ig A_\mu^a T^a) \phi_i$, and $T^a$ is a generator of the gauge group G. Furthermore, we assume a zero-temperature effective potential for these scalar fields of the Coleman-Weinberg type,

$$V_{CW}(\phi_i) = \frac{\lambda}{4} \phi^4 \left( \ln \frac{\phi}{v} - \frac{1}{4} \right) + \frac{\lambda}{16} V^4,$$  \hspace{1cm} (2.3)$$

where $\phi = \sqrt{\sum_i \phi_i^2}$ and $\lambda$ is a coupling constant. This potential can be obtained by considering a massless scalar theory with one-loop corrections. Note that $V_{CW}(\phi_i)$ also possesses global $O(N)$ symmetry. That is, in the case that gauge interactions are negligible, the theory has approximate $O(N)$ symmetry and can accommodate embedded domain wall solutions. Below, we consider gauge interactions that break this global symmetry.

At high temperature, the components $\phi_i$ acquire finite temperature corrections and the symmetry is restored. At the critical temperature, the symmetry is broken. After a series of phase transitions, only $U(1)_{em}$ remains unbroken. Here, we further assume that one component, $\phi_n$, is neutral under the would be $U(1)_{em}$; the other components, $\phi_{c1}, \phi_{c2}, \ldots$, are charged. In such a situation, only the charged scalar fields acquire thermal masses, $V_T(\phi_c)$, originating from gauge interactions with an electromagnetic coupling constant, $e$.\(^\ast\ast\)

$$V_T(\phi_c) = \begin{cases} 
\frac{1}{2} e^2 T^2 \phi_c^2 & \text{for } \phi_c \lesssim T/e, \\
\frac{\pi^2 T^4}{45} & \text{for } \phi_c \gtrsim T/e,
\end{cases}$$

where $\phi_c = \sqrt{\sum_c \phi_c^2}$. Then, the total effective potential becomes $V_{eff}(\phi_i) = V_{CW}(\phi_i) + V_T(\phi_c)$. Note that this asymmetry between the neutral scalar field and the charged scalar fields originates from the gauge interactions, which break the global $O(N)$ symmetry possessed by the potential term. Thus, this theory admits an embedded domain wall solution based on the approximate $O(N)$ symmetry in which the charged scalar fields vanish and the neutral one takes the form of the usual single scalar field domain wall solution. In this domain wall solution, the vacuum expectation value takes the form $(\phi_{c1}, \phi_{c2}, \ldots, \phi_n) = (0, 0, \ldots, \pm v)$, except inside the cores of the walls, even after the core phase transition.\(^\ast\)

The electroweak symmetry is kept unbroken inside the defect cores by introducing interactions with the electroweak Higgs field.

\(^\ast\) We thank Guy D. Moore for informing us that $V_T(\phi_c)$ changes for large $\phi_c$ due to the decoupling of heavy particles.

\(^\ast\ast\) In fact, for charged fields, gauge interactions also generate a zero-temperature effective potential, which breaks the symmetry possessed by the potential (2.3). However, the global minimum of such a potential can remain zero, even though the field values may be slightly changed at that point. Furthermore, when we consider the configuration of domain walls, this term changes the coefficient of the potential (2.3) at $\phi_n = 0$ only slightly, and the resulting effect is smaller than these thermal effects.
\(H\) of the form \(hH^2(\phi^4 - v^4)/v^2\), where \(h\) is a coupling constant.\(^*)\)

We now consider this model in a cosmological context. At very high temperatures, the symmetry is unbroken, and the time averages of all scalar fields vanish. Then, as the temperature decreases, the symmetry is broken in the direction of the neutral component \(\phi_n\) at some critical temperature \(T_c\), and (embedded) domain walls are formed. As long as the photon field remains in thermal equilibrium, i.e. before the time of the last scattering, these embedded walls persist, due to the finite plasma masses for the charged scalar fields.

In order to investigate the dynamics of the charged components \(\phi_c\), we temporarily set \(\phi_n = 0\). Then, the local maximum of the potential in the direction of the charged components is given by

\[
\phi_{c,\text{max}} \sim \frac{eT}{\sqrt{\lambda}},
\]

up to logarithmic corrections. Then, the global minimum is given by

\[
\phi_{c,\text{min}} \sim v,
\]

with the potential energy

\[
V_{\text{eff}}(\phi_{c,\text{min}}) \sim \frac{\pi^2 T^4}{45}.
\]

Here we have assumed

\[
T \ll e v \quad \text{and} \quad e^4 \ll \lambda.
\]

The width of the domain wall is determined by the balance between the potential energy and the surface tension, and is given by

\[
\delta_b \sim \frac{1}{\sqrt{\lambda} v},
\]

which yields the following energy per unit area \(\sigma_b\) of the domain wall:

\[
\sigma_b \sim \sqrt{\lambda} v^3.
\]

The scaling of the domain wall network before the core phase transition can be roughly estimated to be

\[
\rho_{b,\text{DW}} \sim c_b \frac{\sigma_b t^2}{t^3} \sim c_b \frac{\sigma_b}{t},
\]

\(^*)\) The electroweak Higgs field may also be embedded in the \(\phi\) fields, for example, in a more complicated model, in which the electroweak symmetry is automatically unbroken inside the cores, just as in the standard GUT model.

\(^**\) Such scaling properties have been confirmed for local strings,\(^23\) global strings,\(^24\) and global monopoles.\(^25\) In our case, the tension of the domain walls changes with the cosmic time. It has not yet been confirmed that the scaling property holds for such walls. For this reason, we need to investigate this topic further. It has already been shown, however, that a cosmic string network with a time-dependent tension does obey the scaling law.\(^26\)
where \(c_b\) is a constant of order unity. Then note that the total energy density of the universe is given by
\[
\rho_{\text{total}} \sim d \frac{M_G^2}{t^2},
\]
(2.11)
where \(d\) is a number of order unity and \(M_G \approx 2.4 \times 10^{18}\) GeV is the reduced Planck scale. Thus, the time \(t_{eq}\) at which domain walls begin to dominate the energy density of the universe is given by
\[
t_{eq} \sim \frac{M_G^2}{\sigma_b} \sim \frac{M_G^2}{(\sqrt{\lambda} v^3)},
\]
(2.12)
up to numerical coefficients, which corresponds to the temperature
\[
T_{eq} \sim \sqrt{\sqrt{\lambda} v^3/M_G}.
\]
(2.13)
We have assumed that the universe is in the radiation-dominated epoch.

Though the effective mass squared at the origin is always positive, the potential barrier decreases as the universe expands, and thus it can be overcome through thermal fluctuations below some critical temperature \(T_{\text{tran}}\). Then, a core phase transition takes place.\(^\text{21}\) Note that, even after this core phase transition, the domain wall configurations persist, due to the small thermal masses, even though \(\phi_n \neq 0\). However, the properties of the walls are changed significantly. For example, the width of the wall after the core phase transition is given by
\[
\delta_a \sim \frac{v}{T^2},
\]
(2.14)
and the energy per unit area \(\sigma_a\) of the wall becomes
\[
\sigma_a \sim v T^2 \propto T^2.
\]
(2.15)
Note that \(\sigma_a\) decreases in proportion to the temperature squared. Thus, the energy density of such a domain wall network is estimated to be
\[
\rho_{a,\text{DW}} \sim c_a \frac{\sigma_a}{t} \sim c_a v \frac{T^2}{t} \propto \begin{cases} a^{-4} & \text{in the radiation-dominated universe,} \\ a^{-7/2} & \text{in the matter-dominated universe,} \end{cases}
\]
where \(c_a\) is a constant of order unity. Therefore, embedded domain walls after core phase transitions never dominate the energy density of the universe as long as
\[
v \ll M_G.
\]
(2.16)
Thus, \(v\) can be larger than the electroweak scale without encountering an overabundance problem. In previous papers,\(^\text{20, 28, 29}\) it was assumed that embedded defects stabilized by thermal plasmas decay after recombination. However, the scalar fields have effective masses as long as the root mean squared of the photon fields is nonzero, whether or not these fields actually interact. Thus, though a more detailed analysis is needed, we conjecture that the effective masses will not disappear and the embedded defects will remain stable even after recombination. If the embedded domain walls
disappear after recombination, there will be no domain wall problem, but even if they do not, the severity of this problem is significantly lessened, as we have shown.

Note that if the embedded domain walls do not disappear after recombination, the $U(1)_{em}$ symmetry is broken inside the walls, and this induces photon masses. Then, a photon wave which hits the embedded wall can no longer propagate as a free wave. The mass profile in the equation of motion of photons will induce nontrivial transmission and reflection, so that CMB photons may be affected slightly. However, today there will only be one wall per Hubble radius. We will not detect the mass in our local experiments. None the less, effects from the reflected photons might be detected. We will analyze this topic in the future.

In order to find the critical temperature $T_{\text{tran}}$ at which the core phase transition takes place, we estimate the transition rate. This transition occurs through bubble nucleation due to thermal fluctuations. The typical radius $r_b$ of a bubble (strictly speaking, the thickness of the bubble wall) is given by the curvature of the potential at the top of the potential barrier, which is estimated as

$$r_b \sim \frac{1}{\sqrt{\left| \frac{\partial^2 V_{\text{eff}}}{\partial \phi^2} (\phi_{c, \text{max}}) \right|}} \sim \frac{1}{eT}.$$  \hspace{1cm} (2.17)

In the high temperature approximation, the transition rate $\Gamma$ is found by considering the three-dimensional Euclidean action $S_3$,

$$\Gamma \propto \exp \left( -S_E \right),$$  \hspace{1cm} (2.18)

where the Euclidean action $S_E$ is approximated as

$$S_E = \frac{S_3}{T},$$

$$= \frac{1}{T} \int d^3 x \left[ \frac{1}{2} (\nabla \phi)^2 + V_{\text{eff}}(\phi_c) - V_{\text{eff}}(0) \right].$$  \hspace{1cm} (2.19)

Minimizing $S_3$ gives the transition rate.

However, in our case, the transition takes place only inside the cores of domain walls, and the typical radius $r_b$ is larger than $\delta_b$. Thus, we should consider a two-dimensional action $S_2$ inside the cores rather than the three-dimensional action $S_3$. Then, the Euclidean action is given by

$$S_E = \frac{1}{T} S_3 = \frac{\delta_b}{T} S_2,$$  \hspace{1cm} (2.20)

with

$$S_2 = \int d^2 x \left[ \frac{1}{2} (\nabla \phi_c)^2 + V_{\text{eff}}(\phi_c) - V_{\text{eff}}(0) \right].$$  \hspace{1cm} (2.21)

In order to minimize $S_2$, we consider a circularly symmetric Gaussian profile given by

$$\phi_c \equiv \phi_0 \exp \left( -\frac{r^2}{R^2} \right),$$  \hspace{1cm} (2.22)
which, as we will see below, is justified. Here \( r \) is a radial coordinate, \( R \) is a typical radius, and \( \phi_0 > 0 \) is assumed for simplicity. Then, the two-dimensional action \( S_2 \) becomes

\[
S_2 \simeq 2\pi \left[ \frac{\phi_0^2}{4} + \frac{\lambda}{32} \phi_0^4 R^2 \left( \ln \frac{\phi_0}{v} - \frac{1}{2} \right) + \frac{1}{8} e^2 T^2 \phi_0^2 R^2 \right],
\]

(2.23)

where we have also assumed that \( \phi_0 \ll T/e \), which we also show is justified below. Varying \( S_2 \) with respect to \( \phi_0 \) and \( R \) yields

\[
\frac{\partial S_2}{\partial \phi_0} = 2\pi \left[ \frac{\phi_0}{2} + \frac{\lambda}{8} \phi_0^3 R^2 \left( \ln \frac{\phi_0}{v} - \frac{1}{4} \right) + \frac{1}{4} e^2 T^2 \phi_0^2 R \right],
\]

\[
\frac{\partial S_2}{\partial R} = 2\pi \left[ \frac{\lambda}{16} \phi_0^4 R \left( \ln \frac{\phi_0}{v} - \frac{1}{2} \right) + \frac{1}{4} e^2 T^2 \phi_0^2 R \right].
\]

(2.24)

Then \( S_2 \) is minimized for \( \phi_0 \sim e T/\sqrt{\lambda} \ll T/e \) and \( R \sim 1/(e T) \) and has a value of \( S_2 \sim e^2 T^2/\lambda \), which yields the following Euclidean action \( S_E \):

\[
S_E \sim \frac{\delta b}{T} S_2 \sim \frac{e^2 T}{\lambda^{\frac{3}{2}} v}.
\]

(2.25)

Thus, the transition temperature \( T_{\text{tran}} \) becomes

\[
T_{\text{tran}} \sim \frac{\lambda^{\frac{3}{2}} v}{e^2},
\]

(2.26)

with a logarithmic correction coming from the prefactor of the transition rate. Because \( r_b \sim R \), the thick wall (Gaussian) ansatz is justified. Note, again, that after the core phase transition, the electroweak symmetry is broken even inside the cores of domain walls, so that electroweak baryogenesis stops.

For successful baryogenesis, \( T_{\text{tran}} \) should be lower than the electroweak scale \( T_{\text{EW}} \sim 100 \text{ GeV} \), to prevent the electroweak symmetry from breaking even inside the cores at that temperature. This is the temperature at which electroweak baryogenesis is most effective. Furthermore, \( T_{\text{tran}} \) should be higher than \( T_{\text{eq}} \), the temperature at which the walls with symmetric cores would start to dominate the energy density; otherwise the walls would dominate the universe, and their subsequent decay would produce too large an entropy, significantly diluting the baryon asymmetry and distort the black-body nature of the CMB.\(^{29}\) These conditions yield the following constraints on the breaking scale \( v \):

\[
v \ll \frac{\lambda^{\frac{3}{2}}}{e^4} M_G, \quad v \ll \frac{e^2}{\lambda^{\frac{3}{2}}} T_{\text{EW}}.
\]

(2.27)

Note that condition (2.7) should be satisfied at least at the temperature \( T_{\text{tran}} \). Together (2.7) and (2.26) give the constraint

\[
e^4 \ll \lambda \ll e^2.
\]

(2.28)

Under this constraint, the two conditions in (2.27) are easily satisfied. Thus, we have a large parameter region in which the conditions (2.16), (2.27), and (2.28) are satisfied.
§3. Electroweak baryogenesis

Assuming that the criteria derived at the end of the previous section are satisfied, we now investigate electroweak baryogenesis. In our scenario, the embedded domain walls play the role of nucleated bubble walls in standard electroweak baryogenesis. Other ingredients, such as CP asymmetry, are assumed to be the same as in the standard case.

First, we consider local baryogenesis, in which the baryon asymmetry is produced only at the edges of domain walls, because they are the locus where CP violation in the scalar field sector is localized. Defining the core passage time $\tau \equiv \delta b / v_D$, where $v_D$ is the typical wall velocity, the net baryon asymmetry after decay is given by (see e.g. 14)

$$ n_{lB}^f = n_{lB}^{l0}(1 - e^{-T_s \tau}), \quad (3.1) $$

where $n_{lB}^{l0}$ is the baryon (anti-baryon) number density produced at either edge of the defect. Here, $T_s$ is defined as $T_s \equiv 6N_f \Gamma_s / T^3$, using the weak sphaleron rate $\Gamma_s = \kappa(\alpha_w T)^4$, with $\kappa = 0.1 \sim 1.31^1$. The quantity $N_f$ is the number of families, and $\alpha_w \sim 1/29$ is the weak coupling constant. Considering a chemical potential originating from the one-loop effect, the baryon to entropy ratio is given by $14$)

$$ \frac{n_{lB}^l}{s} \sim 3.9 \kappa \alpha_w^4 g_*^{-1} \left( \frac{m}{T} \right) \Delta \theta_{CP}(1 - e^{-T_s \tau}) \times (SF), \quad (3.2) $$

where $m$ is the mass of the particles that make the dominant contribution to the chiral anomaly, $\Delta \theta_{CP}$ is the magnitude of the CP violation (assumed to be of order unity), and (SF) is the geometrical suppression factor, which represents the ratio of the volume in which baryogenesis actually occurs to the total volume. The entropy density is given by

$$ s = \frac{2\pi^2}{45} g_* T^3, \quad (3.3) $$

where $g_*$ is the effective number of degrees of freedom.

Next, we consider non-local baryogenesis, particularly focusing on the case in which baryon number violation is driven by chemical potentials for left-handed leptons. For other cases, the argument is essentially the same. Defining the diffusion root $\lambda_D \equiv v_D / D_L$, with the diffusion constant $D_L \sim 1/(8\alpha_w^2 T)$ for leptons, the net baryon asymmetry is given by

$$ n_{lB}^{nl} = n_{lB}^{nl0}(1 - e^{-\delta b \lambda_D}), \quad (3.4) $$

where $n_{lB}^{nl0}$ is the baryon number density produced in front of the trailing edge. Solving the diffusion equation for the flux of left-handed leptons, we obtain the following.

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$^1$ Bödeker found a different dependence on the weak coupling constant, $\Gamma_s \propto \alpha_w^5 \ln(1/\alpha_w) T^4$. However, we assume the above form for simplicity, because the additional dependence can be absorbed into the numerical uncertainty $\kappa$. 

baryon to entropy ratio,

$$\frac{n_{nl}^{pl}}{s} \sim 0.2\kappa \alpha_w g_s^{-1} \Delta \theta_{CP} \frac{1}{v_D} \left(\frac{m_l}{T}\right)^2 \left(\frac{m_H}{T}\right) \left(\frac{\xi L}{D_L}\right) \times (\text{SF}) \sim 10^{-6} \kappa \Delta \theta_{CP} \frac{y_{\tau}^2}{v_D} \times (\text{SF}),$$

(3.5)

where $m_l$ is the mass of the leptons, $m_H$ is the Higgs mass, $\xi_L$ is the persistence length, and $y_{\tau}$ is the Yukawa coupling constant for tau leptons. In the second step, we have used the fact that contributions coming from tau leptons are dominant. In the case that domain walls evolve for a long time according to the scaling solution, the equilibrium baryon number is reached as long as the sphaleron is active inside the cores, which enhances the baryon number $n_{nl}^{pl}$ by a factor of $v_D^2/(T_s D_L)$.

In our case, the suppression factor (SF) is given by the wall velocity $v_D$, because domain walls are two-dimensional objects. Therefore, as long as wall motion is relativistic, there is no significant suppression, and hence [as shown by (3.5)] a sufficiently large baryon asymmetry can easily be produced in our scenario.

§4. Discussion and summary

In this paper, we have studied electroweak baryogenesis in the context of embedded domain walls. The walls are stabilized by thermal plasma effects, so that the electroweak symmetry is restored and sphaleron processes are active inside their cores. Then, as we have shown, a sufficiently large baryon asymmetry can easily be produced during the cosmological evolution of the wall network. In contrast to the case of stable cosmic strings, the volume factor that suppresses defect-mediated baryogenesis from a baryogenesis process which is effectively uniform in space is not significant, because domain walls are two-dimensional objects, which, when moving relativistically, sweep out a fraction of order unity of space. This makes it easy to produce a sufficient baryon asymmetry.

In general, a network of stable domain walls will easily dominate the energy density of the universe, leading to a severe cosmological problem. But, in our case, the domain walls are stabilized by thermal plasma and undergo a core phase transition at some critical temperature. This transition reduces their energy per unit area as the tension of the domain walls decreases in proportion to the cosmic temperature squared, and this leads to a significant relaxation of the condition needed for the domain walls not to dominate the energy of the universe. The walls may even decay completely after recombination.

Note that our scenario requires the use of a Coleman-Weinberg potential, as opposed to a potential in which the scalar field has a positive mass squared at the origin. In spite of this caveat, the positive results of this investigation warrant further investigation of the fate of embedded defects stabilized by thermal plasmas. This is a topic of future work.
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