ELLIPTICAL-WEIGHTED HOLICs FOR WEAK LENSING SHEAR MEASUREMENT. II. POINT-SPREAD FUNCTION CORRECTION AND APPLICATION TO A370

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ABSTRACT

We developed a new method (E-HOLICs) of estimating gravitational shear by adopting an elliptical weight function to measure background galaxy images in our previous paper. Following the previous paper, in which an isotropic point-spread function (PSF) correction is calculated, in this paper we consider an anisotropic PSF correction in order to apply E-HOLICs to real data. As an example, E-HOLICs is applied to Subaru data of the massive and compact galaxy cluster A370 and is able to detect double peaks in the central region of the cluster consistent with the analysis of strong lensing. We also study the systematic error in E-HOLICs using STEP2 simulation. In particular, we consider the dependences of the signal-to-noise ratio (S/N) of background galaxies in the shear estimation. Although E-HOLICs does improve the systematic error due to the ellipticity dependence as shown in Paper I, a systematic error due to the S/N dependence remains, namely, E-HOLICs underestimates shear when background galaxies with low S/N objects are used. We discuss a possible improvement of the S/N dependence.

Key words: galaxies: clusters: individual (A370) – gravitational lensing: weak

Online-only material: color figures

1. INTRODUCTION

Weak gravitational lensing has been widely recognized as a unique and very powerful method for studying not only the mass distribution of the universe but also the cosmological parameters (see, for example, Mellier 1999; Van Waerbeke & Mellier 2003; Schneider 2006; Munshi et al. 2008). In particular, cosmic shear recently has attracted much attention because of its potential to determine the so-called cosmic equation of state, namely, the relation between the effective energy density and pressure of the dark energy. There are some detections of cosmic shear (Bacon et al. 2000, 2003; Kaiser et al. 2000; Maoli et al. 2001; Van Waerbeke et al. 2001, 2005; Refregier et al. 2002; Hamana et al. 2003; Casertano et al. 2003; Massey et al. 2005; Hoekstra et al. 2006). However, the lensing signal of cosmic shear is very small; thus, highly accurate shear measurement is required. The most popular method of shear estimation is given by Kaiser et al. (1995, called the KSB method; see also Luppino & Kaiser 1997; Hoekstra et al. 1998; Viola et al. 2011), where change in the moments of the galactic light distribution by lensing is extracted from the measurement. Other methods of shear estimation have also been developed (Bernstein & Jarvis 2002; Refregier 2003; Kuijken 2006; Miller et al. 2007; Kitching et al. 2008; Melchior 2011). In relation to our E-HOLICs method, we mention the work of Bernstein & Jarvis (2002) and Melchior (2011), who also introduced an elliptical weight for shape measurement. All of these are very challenging and successful for cluster lensing. Although some of them have already been used for cosmic shear, it is argued that none of them have achieved the required level of accuracy to measure the cosmic equation of state in a few percent level.

On the other hand, there are planned cosmic shear observations in the very near future for the purpose of measuring the cosmic equation of state. In this situation, it is urgent to develop a sufficiently accurate method of shear estimation. In our previous paper (Okura & Futamase 2011, which we call here Paper I), we developed a new method of evaluating shear based on the KSB method by adopting an elliptical weight function to measure the shape of background galaxies. Our method is a natural development of our previous studies of weak lensing analysis, which uses higher-order multiple moments of the shape of background galaxies. We called the method HOLICs (Higher Order Lensing Image Characteristics), which are quantities with definite spin properties made of higher-order multipole moments (Okura et al. 2007). We have shown that Oct-HOLICs is an unbiased measure of flexion and is successfully applied to galaxy cluster A1689 to reveal the substructure in the central part of the cluster (Okura et al. 2008). Spin-2 HOLICs, which is a spin-2 combination of higher-order moments, increases the lensing information of the image and is a useful method to apply cosmic shear measurement (Okura & Futamase 2009). We generalize our method as the elliptical-weighted HOLICs (E-HOLICs) method because of the elliptical weight. We have shown that the HOLICs method with an elliptical weight is able to measure the lensing distortion more accurately by heavily weighting a region of the image that is brighter than that in the standard KSB method; thus, it can reduce effects of the systematic error and random noise more effectively than the KSB method. We have also calculated an isotropic point-spread function (PSF) correction in Paper I and tested its accuracy using STEP2 simulation data. Following Paper I, we present a method of correcting PSF containing an anisotropic part in this paper. It is necessary to apply the method to real data. In the case of an isotropic PSF correction, it was shown that the corrected shear can be obtained by solving a polynomial equation. However, since the anisotropic part has an arbitrary direction, it is natural to expect that the shear after PSF correction is obtained by solving two coupled polynomial equations. We succeed in reducing these coupled equations into tractable forms by dividing the anisotropic part of PSF into a parallel part and an orthogonal part against the direction of shear (see Section 2 for details). We test the E-HOLICs method with STEP2 simulation data. In Paper I, we can only test the data with isotropic PSF, but in this paper we test all PSF sets and obtain more detailed results. After establishing PSF
correction, we are able to apply our method to the real data of galaxy cluster A370.

The organization of this paper is as follows: In Section 2, we give a brief review of weak lensing and the E-HOLICs method. In Section 3, we describe the correction of PSF with an anisotropic part in the E-HOLICs method by dividing the anisotropic part of PSF into an anisotropic part, a parallel part, and an orthogonal part. In Section 4 we test the E-HOLICs method by using STEP2 simulation data; in particular, we investigate the dependences of the signal-to-noise ratio (S/N) and size of background galaxies in the shear estimation. Then we apply the E-HOLICs method to the real data of galaxy cluster A370. It will be shown that E-HOLICs is able to detect two peaks in the central part of the cluster consistent with strong lensing analysis. Finally, we summarize our results and give some discussion in Section 6.

2. BASES OF WEAK LENSING AND ELLIPtical-WEIGHTED HOLICs

In this section, we present bases and definitions of the E-HOLICs method. More details can be found in Paper I.

2.1. Notations and Definitions

Here, we describe briefly our notation and the concept of zero image. The relation between the zero image, source, and observed image will be shown in Figure 1. The bases of weak lensing can be found, for example, in Bartelmann & Schneider (2001).

We use a complex notation for angular positions (e.g., \( \theta = \theta_1 + i \theta_2 \) in the image plane and \( \beta = \beta_1 + i \beta_2 \) in the source plane).

For notational simplicity, we set the centroid of the image as the origin in our coordinates (therefore the centroid position is \( \bar{\theta} = \theta_1, +i \theta_2 = 0 \) and \( \bar{\beta} = \beta_1 + i \beta_2 = 0 \)). We introduce an imaginary plane called zero plane where shapes of all sources are perfect circles, and we regard the intrinsic shear as the result of an imaginary lensing distortion. The sources in the zero plane are called zero images. This plane is introduced in order to define naturally an elliptical window in measuring shapes of background galaxy images. Then the lens equation gives the following relation between the displacement in the zero plane (\( \bar{\beta} \)) and image plane (\( \theta \)) as shown in Paper I:

\[
\bar{\beta} = (1 - \kappa)(\theta - g^C \theta^*),
\]

where \( \kappa \) is the dimensionless surface mass density and \( g^C = g^L_1 + ig^L_2 \) is the “combined shear” defined as

\[
g^C = \frac{g' + g^L}{1 + g^L g^L*},
\]

where \( g^L \) is the shear induced by lensing and \( g' \) is the intrinsic reduced shear. Using the combined shear, we can divide lensing shear analysis into two steps: measuring \( g^C \) from each background galaxy and then determining lensing shear by statistical averaging such that

\[
\langle g' \rangle = \left\{ \frac{g^C - g^L}{1 - g^L g^L*} \right\}.
\]

Henceforth, because the purpose of E-HOLICs is measuring \( g^C \), we note \( g = g^C \) for simplicity. The complex distortion is defined as

\[
\delta = \frac{2g}{1 + g^2},
\]

and their absolute values are notated as

\[
g = |g|,
\]

\[
\delta = |\delta|.
\]

We define complex moments of brightness distribution \( I(\theta) \) measured with weight function which has ellipticity \( \delta \) as

\[
Z_N^N(I, \delta) = \int d^2 \bar{\theta} I(\theta)\delta_N^N W \left( \frac{\theta_N^2 - \text{Re}[\delta^* \theta_N^2]}{\sigma^2} \right),
\]

where

\[
\theta_N^N = \theta N \theta^N - \frac{\text{Re}[\delta^* \theta_N^2]}{\sigma^2},
\]

and \( \sigma \) is the arbitrary scale (usually the typical scale of each object). Then HOLICs is defined as

\[
\mathcal{H}_N^N(I, Z_L^L, \delta) = \frac{Z_N^N(I, \delta)}{Z_L^L(I, \delta)}.
\]

3. PSF CORRECTION IN THE E-HOLICs METHOD

In this section, we give a formulation for the correction of PSF with anisotropy in the E-HOLICs method. Generally, PSF will be an arbitrary function, but we assume PSF has only an elliptical part.

First, we define \( I^L \) as the brightness distribution of a lensed image which has ellipticity \( \delta \) and \( I^Z \) is the brightness distribution of a zero image, so

\[
I^Z(\bar{\beta}) \equiv I^L(\theta).
\]

Because a zero image is defined as not having ellipticity with a circular weight function, we can obtain

\[
\mathcal{H}_Z^Z(I^L, Z_0^0, \delta) = \delta
\]

\[
\mathcal{H}_Z^Z(I^Z, Z_0^0, 0) = 0,
\]

where \( \delta \) is the scale of weight function in the zero plane and \( \tilde{\delta} \) is slightly different from that in the image plane \( \sigma \), because lensing changes the scale of metric. Next, we consider \( I^{iso} \).
which is smeared $I^L$ by circular PSF $\tilde{P}^{\text{iso}}(\tilde{\theta})$; therefore, we have the following relation:

$$\tilde{I}^{\text{iso}}(\tilde{\theta}) \equiv \int d^2\tilde{\psi} I^L(\psi) \tilde{P}^{\text{iso}}(\tilde{\theta} - \tilde{\psi}), \quad (14)$$

where $\tilde{P}(\theta)$ is a PSF function of $\tilde{P}^{\text{iso}}(\tilde{\theta})$ in the image plane, so

$$\tilde{P}(\theta) \equiv \tilde{P}^{\text{iso}}(\tilde{\theta}). \quad (15)$$

Because $I^L$ and $\tilde{P}^{\text{iso}}$ do not have ellipticity and this relation is the same as Equations (12) and (13), we can obtain

$$\mathcal{H}_2^2(\tilde{I}^L, Z^2_0, \delta) = \delta \quad (16)$$

$$\mathcal{H}_2^2(\tilde{P}^{\text{iso}}, Z^2_0, 0) = 0. \quad (17)$$

We define $\tilde{I}^L$ as a lensed image from $\tilde{I}^{\text{iso}}(\tilde{\theta})$, so

$$\tilde{I}^L(\theta) \equiv \tilde{I}^{\text{iso}}(\tilde{\theta}). \quad (18)$$

Because $\tilde{I}^{\text{iso}}(\tilde{\theta})$ is made by convolution of circular functions, $\tilde{I}^{\text{iso}}(\tilde{\theta})$ also does not have ellipticity, so we obtain

$$\mathcal{H}_2^2(\tilde{I}^L, Z^2_0, \delta) = \delta \quad (19)$$

$$\mathcal{H}_2^2(\tilde{I}^{\text{iso}}, Z^2_0, 0) = 0. \quad (20)$$

Finally by writing Equation (14) in the image plane with Equations (11), (15), and (18), we obtain

$$\tilde{I}^L(\theta) = \int d^2\psi I^L(\psi) \tilde{P}(\theta - \psi). \quad (21)$$

These equations mean that if the image (e.g., $I^L$) and the smearing function (e.g., $\tilde{P}$) have the same ellipticity, the smeared image (e.g., $\tilde{I}^L$) also has the same ellipticity with them. Then $\delta$ can be determined by observing E-HOLICs, Equation (21), using $\tilde{I}^L$, Equation (19). On the other hand, the observed image is the smeared $I^L$ by real PSF $P(\theta)$:

$$I^{\text{obs}}(\theta) = \int d^2\psi I^L(\psi) P(\theta - \psi). \quad (22)$$

Thus, by finding a transformation from the observed real PSF $P(\theta)$ to $\tilde{P}(\theta)$, $\delta$ can be determined. This is what we do below.

In the KSB method, PSF is divided into an anisotropic part and an isotropic part. However, the E-HOLICs method divides PSF into an orthogonal part and a parallel part against the direction of lensing distortion. Since the direction of PSF ellipticity is not always the same as the direction of lensing distortion, it is natural to expect that the equations of correcting PSF are a two-dimensional form. By dividing PSF into orthogonal and parallel parts, PSF correction can be written in two simple one-dimensional forms as shown below.

3.1. General Form of PSF Correction

Here we demonstrate the general form of anisotropic PSF correction.

Let $q(\theta)$ be a part of the anisotropic part (it is not necessary to be the whole of the anisotropic part). $P(\theta)$ can be divided into

$$P(\theta) \equiv \int d^2\psi P^o(\theta - \psi)q(\psi). \quad (23)$$

This decomposition is one of the basic assumptions of the KSB method, but there has been some discussion of the validity of this decomposition (Kuijken 1999). Kuijken (1999) shows an example with double Gaussian PSF. Therefore, $I^{\text{obs}}$ is

$$I^{\text{obs}}(\theta) = \int d^2\psi I^L(\psi)P(\theta - \psi)$$

$$= \int d^2\psi I^L(\psi) \int d^2\phi P^o(\theta - \psi - \phi)q(\phi)$$

$$= \int d^2\phi I_q(\theta - \phi)q(\phi), \quad (24)$$

where

$$I_q(\theta) \equiv \int d^2\psi I^L(\psi)P^o(\theta - \psi). \quad (25)$$

Let us define $f(\theta) = \theta^2 MW\left(\left(\theta^2 - \text{Re}\left[\delta^* \delta^2\right]\right)/\sigma^2\right)$; by calculating moments, we obtain

$$\int d^2\theta f(\theta)I^{\text{obs}}(\theta) = \int d^2\theta f(\theta) \int d^2\psi I^L(\psi)P(\theta - \psi)$$

$$= \int d^2\theta f(\theta) \int d^2\phi I_q(\theta - \phi)q(\phi)$$

$$= \int d^2\varphi \int d^2\phi I_q(\varphi)f(\varphi + \phi)q(\phi). \quad (26)$$

By expanding $f(\varphi + \phi)$, moments of $I^{\text{obs}}$ can be expressed by combination of moments of $I_q$ and $q$. Especially, $Z^2_2$ and $Z^2_0$ are obtained as

$$Z^2_2(I^{\text{obs}}, \delta) \approx QZ^2_2(I_q, \delta)$$

$$+ Qq^2_2 \left[ Z^0_0 - \frac{1}{2\sigma^2} \left(4Z^0_0 - 5\delta^* Z^2_2\right) \right]$$

$$+ \frac{1}{2\sigma^4} \left(4Z^4_0 - 2\delta^* Z^2_2 + \delta^2 Z^4_2\right) \right] (I_q, \delta)$$

$$+ Qq^2_2 \left[ - \frac{1}{2\sigma^2} (-\delta Z^2_0) + \frac{1}{2\sigma^4} \left(4Z^4_0 - 2\delta Z^2_2 + \delta^2 Z^4_0\right) \right] (I_q, \delta) \quad (27)$$

$$Z^2_0(I^{\text{obs}}, \delta) \approx QZ^2_0(I_q, \delta)$$

$$+ Qq^2_2 \left[ - \frac{1}{2\sigma^2} (2Z^2_2 - 3\delta^* Z^0_0) + \frac{1}{2\sigma^4} \right.$$

$$\times \left(2Z^4_2 - 2\delta^* Z^2_0 + \delta^2 Z^4_2\right) \right] (I_q, \delta)$$

$$+ Qq^2_2 \left[ - \frac{1}{2\sigma^2} (2Z^2_2 - 3\delta Z^0_0) + \frac{1}{2\sigma^4} \right.$$

$$\times \left(2Z^4_2 - 2\delta Z^2_0 + \delta^2 Z^4_2\right) \right] (I_q, \delta), \quad (28)$$

where

$$q^2_M(\psi) = \frac{\int d^2\theta q^2_M(\theta)W\left(\frac{\theta^2}{\sigma^2}\right)}{Q}. \quad (29)$$
Instead of the above decomposition, we decompose PSF into parallel and orthogonal parts as

\[ P(\theta) \equiv \int d^2 \psi P_{\text{iso}}(\psi) q_{\text{iso}}(\theta - \psi). \]  

(31)

Instead of the above decomposition, we decompose PSF into parallel and orthogonal parts as

\[ P(\theta) \equiv \int d^2 \psi P_\parallel(\psi) q_\parallel(\theta - \psi). \]  

(32)

where the ellipticity of \( P_\parallel(\theta) \) is parallel to the ellipticity of the lensed image, and the ellipticity of \( q_\parallel(\theta) \) is orthogonal to the ellipticity of the lensed image. The moments of these parts are defined as

\[ q_{\times M}^N = \frac{\int d^2 \theta \theta^N q_{\times}(\theta) W \left( \frac{\theta^2}{\sigma^2} \right)}{Q_{\times}}. \]  

(33)

\[ Q_{\times} = \int d^2 \theta q_{\times}(\theta) W \left( \frac{\theta^2}{\sigma^2} \right). \]  

(34)

This decomposition might have the same problem as the KSB decomposition in Equation (23), namely, that not all PSF shapes can be precisely decomposed in this way. We note that the accuracy of the PSF correction could be improved by considering higher-order moments, but for the remainder of this work we assess the validity of this PSF correction scheme.

For the moment, we assume the above decomposition and define “parallel image” \( I_{\parallel}(\theta) \), which is seared by \( P_\parallel(\theta) \) as

\[ I_{\parallel}(\theta) = \int d^2 \phi I(\phi) P_\parallel(\theta - \phi). \]  

(35)

We define \( q_{\delta}(\theta) \), which relates \( P_\parallel(\theta) \) with \( \tilde{P} \), as

\[ \tilde{P}(\theta) = \int d^2 \phi P_\parallel(\phi) q_{\delta}(\theta - \phi). \]  

(36)

Moments are defined as

\[ q_{\delta, m}^N = \frac{\int d^2 \theta \theta^N q_{\delta}(\theta) W \left( \frac{\theta^2}{\sigma^2} \right)}{Q_{\delta}}. \]  

(37)

\[ Q_{\delta} = \int d^2 \theta q_{\delta}(\theta) W \left( \frac{\theta^2}{\sigma^2} \right). \]  

(38)

where \( q_{\delta, m}^2 \) have the same direction of \( \delta \). The anisotropic part \( q_{\text{aniso}} \) in KSB decomposition has an ellipticity that has the same direction of \( \delta \).

A catalog of these definitions can be seen in Appendix A. For notational simplicity we set the direction of distortion as real, namely,

\[ \delta = \delta. \]  

(39)

Then, the following expressions are obtained by definitions:

\[ q_{\times 2}^2 = i q_{\times 2}^2 \]  

(40)

\[ q_{\delta 2}^2 = q_{\delta 2}^2. \]  

(41)

3.3. Orthogonal PSF Correction (Determining a Direction of Distortion)

Here, we present a method of orthogonal PSF correction. After correcting the orthogonal part, \( I_{\parallel} \) has the direction of distortion; therefore, this is the same as determining a direction of distortion.

From Equations (27), (28), (33), and (40), the correction of the orthogonal part of PSF is written as

\[ Z_{\parallel, \delta}^{I_{\text{obs}}} \approx Q_{\times} Z_{\parallel, \delta}^{I_{\parallel}} \]

\[ + i Q_{\times} q_{\times 2}^2 \left[ Z_{\parallel, \delta}^{I_{\parallel}} \left( Z_{\parallel, \delta}^{I_{\parallel}} - 2H_{\parallel, \delta}^2 + \delta Z_{\parallel, \delta} \right) \right] \]

\[ + \left( Z_{\parallel, \delta}^{I_{\parallel}} - 2\delta Z_{\parallel, \delta} + \delta^2 Z_{\parallel, \delta} \right) \left( I_{\parallel, \delta} \right) \]

\[ = Q_{\times} Z_{\parallel, \delta}^{I_{\parallel}} \]

\[ + i Q_{\parallel, \delta} q_{\parallel 2}^2 \left[ H_{\parallel, \delta}^2 \left( H_{\parallel, \delta}^2 - \delta H_{\parallel, \delta} \right) \right] \]

\[ + \frac{1}{2\sigma^2} \left( 1 + \delta^2 \right) \left( H_{\parallel, \delta}^2 - H_{\parallel, \delta} \right) \left( I_{\parallel, \delta} \right) \]

\[ = Q_{\times} Z_{\parallel, \delta}^{I_{\parallel}} \]

\[ + i P_{\times}^E (I_{\parallel}, \delta) q_{\times 2}^2 \]  

(42)
\[ Z_0^2(I_{\text{obs}}, \delta) \approx Q_x Z_0^2(I_{//}, \delta) \]
\[ + i Q_x q_{x^2} \left[ -\frac{1}{2\sigma^2} (2Z_{x^2}^2 - 3\delta^* Z_0^0) + \frac{1}{2\sigma^4} \right] \times \left( \frac{1}{2\sigma^2} (Z_{x^2}^2 - 2\delta^* Z_0^0 + \delta^2 Z_{x^2}^0) \right) \]
\[ + (i Q_x q_{x^2})^* \left[ -\frac{1}{2\sigma^2} (2Z_{x^2}^2 - 3\delta Z_0^0) + \frac{1}{2\sigma^4} \right] \times \left( Z_{x^2}^2 - 2\delta^* Z_0^0 + \delta^2 Z_{x^2}^0 \right) \quad (I_{//}, \delta) \]
\[ \times (Z_{x^2}^2 - 2\delta^* Z_0^0 + \delta^2 Z_{x^2}^0) \quad (I_{//}, \delta) \]
\[ = Q_x Z_0^2(I_{//}, \delta), \quad (43) \]

where we used the condition that all moments on the right hand have only real parts. Therefore, we obtain
\[ H_{x^2}^2(I_{\text{obs}}, \delta) \approx H_{x^2}^2(I_{//}, Z_0^0, \delta) + i P_x^E(I_{//}, \delta) q_{x^2}. \quad (44) \]

Thus, we can divide the observed ellipticity into real and imaginary parts as follows:
\[ \text{Re} \left[ H_{x^2}^2(I_{\text{obs}}, \delta) \right] = H_{x^2}^2(I_{//}, Z_0^0, \delta) \quad (45) \]
\[ \text{Im} \left[ H_{x^2}^2(I_{\text{obs}}, \delta) \right] = P_x^E(I_{//}, \delta) q_{x^2}. \quad (46) \]

Similarly, the moments of star \( I_{\text{obs}}^* \) can be divided as follows:
\[ \text{Re} \left[ H_{x^2}^2(I_{\text{obs}}^*, \delta) \right] = H_{x^2}^2(I_{//}^*, Z_0^0, \delta) \quad (47) \]
\[ \text{Im} \left[ H_{x^2}^2(I_{\text{obs}}^*, \delta) \right] = P_x^E(I_{//}^*, \delta) q_{x^2}. \quad (48) \]

Thus, we have
\[ q_{x^2} = \frac{\text{Im} \left[ H_{x^2}^2(I_{\text{obs}}, \delta) \right]}{P_x^E(I_{//}, \delta)} = \frac{\text{Im} \left[ H_{x^2}^2(I_{\text{obs}}^*, \delta) \right]}{P_x^E(I_{//}^*, \delta)}. \quad (49) \]

In real analysis, first we must find a direction that satisfies the second equality of Equation (49), and the removal of the imaginary part in Equation (44) must correspond to the correction of orthogonal PSF.

Let us consider that the observed ellipticity of the image is \( A + iB \) and the star is \( a + ib \) in an arbitrary basis. The direction of distortion \( \phi_\delta \) is determined by the following equation:
\[ \phi_\delta = \tan^{-1} \left( -\frac{P_x^E(I_{//}, \delta)}{P_x^E(I_{//}^*, \delta)} B - \frac{P_x^E(I_{//}, \delta)}{P_x^E(I_{//}^*, \delta)} A \right). \quad (50) \]

### 3.4 Parallel PSF Correction (Determining the Absolute Value of Distortion)

Next, we show a method of correction of parallel PSF; it is the same as correcting the absolute value of the real part.

From Equations (27), (28), (36), and (41), we obtain the correction of parallel PSF as follows:
\[ Z_{x^2}^2(I_{//}^L, \delta) \approx Q_\delta Z_{x^2}^2(I_{//}, \delta) \]
\[ + Q_\delta q_{\delta^2} \left[ Z_{0^0}^0 - \frac{1}{2\sigma^2} (4Z_{0^0}^2 - 5\delta^* Z_{x^2}^2) + \frac{1}{2\sigma^4} \right] \times \left( \frac{1}{2\sigma^2} (Z_{x^2}^2 - 2\delta^* Z_0^0 + \delta^2 Z_{x^2}^0) \right) \]
\[ + Q_\delta q_{\delta^2} \left[ -\frac{1}{2\sigma^2} (2Z_{x^2}^2 - 3\delta Z_0^0) + \frac{1}{2\sigma^4} \right] \times \left( Z_{x^2}^2 - 2\delta^* Z_0^0 + \delta^2 Z_{x^2}^0 \right) \quad (I_{//}, \delta) \]
\[ + Q_\delta q_{\delta^2} \left[ -\frac{1}{2\sigma^2} (2Z_{x^2}^2 - 3\delta Z_0^0) + \frac{1}{2\sigma^4} \right] \times \left( Z_{x^2}^2 - 2\delta^* Z_0^0 + \delta^2 Z_{x^2}^0 \right) \quad (I_{//}, \delta) \]
\[ = Q_\delta Z_{x^2}^2(I_{//}, \delta). \quad (51) \]

Because these equations have only real parts, we obtain
\[ H_{x^2}^2(I_{//}^L, Z_0^0, \delta) = \delta \approx H_{x^2}^2(I_{//}, Z_0^0, \delta) \]
\[ + q_{\delta^2} \left[ H_{0^0}^0 - \frac{1}{\sigma^2} ((2 + 3\delta^2)H_0^0 - 5\delta H_{x^2}^2) \right] \frac{1}{2\sigma^4} \]
\[ + (1 + 5\delta^2)H_0^0 - 2(3 + \delta^2)H_{x^2}^2 \]
\[ = H_{x^2}^2(I_{//}, Z_0^0, \delta) + P_x^E(I_{//}, \delta) q_{\delta^2}. \quad (52) \]

When we apply the above equation to a star, we define a distorted image of star \( I^* \). This image is obtained using a delta function for Equation (14). Therefore, \( I^*(\theta) = \tilde{P}^*(\theta) \) and \( H_{x^2}^2(I^*, Z_0^0, \delta) = \delta \).

Thus, \( q_{\delta^2} \) is obtained as
\[ q_{\delta^2} = \frac{\delta - H_{x^2}^2(I_{//}^*, Z_0^0, \delta)}{P_x^E(I_{//}^*, \delta)}. \quad (53) \]

Therefore, the absolute value of the complex distortion is obtained as
\[ |\delta| \approx H_{x^2}^2(I_{//}, Z_0^0, \delta) + \frac{P_x^E(I_{//}, \delta)}{P_x^E(I_{//}^*, \delta)} \left( \delta - H_{x^2}^2(I_{//}^*, Z_0^0, \delta) \right) \]
\[ \approx H_{x^2}^2(I_{//}, Z_0^0, \delta) + \frac{P_x^E(I_{\text{obs}}, \delta)}{P_x^E(I_{\text{obs}}^*, \delta)} \left( \delta - H_{x^2}^2(I_{//}^*, Z_0^0, \delta) \right) \]
\[ \approx \frac{P_x^E(I_{\text{obs}}^*, \delta)}{P_x^E(I_{\text{obs}}^*, \delta)} \left( \delta - H_{x^2}^2(I_{//}^*, Z_0^0, \delta) \right) \]
\[ \approx 1 - \frac{P_x^E(I_{\text{obs}}^*, \delta)}{P_x^E(I_{\text{obs}}^*, \delta)} \frac{H_{x^2}^2(I_{//}, Z_0^0, \delta)}{1 - \frac{P_x^E(I_{\text{obs}}^*, \delta)}{P_x^E(I_{\text{obs}}^*, \delta)}}. \quad (54) \]

This equation is the same as the isotropic PSF correction discussed in Paper I, because the isotropic PSF does not have the orthogonal part.
Finally, we obtain the complex distortion in the two-dimensional form as follows:

\[
\delta = \mathcal{H}_2^i(I_{\|}, Z_0^2, \delta) + \frac{P_{E}(I_{\text{obs}}, \delta)}{P_x(I_{\text{obs}}, \delta)} \left( \delta - \mathcal{H}_2^i(I_{\|}, Z_0^2, \delta) \right)
\]

\[
\approx \Re \left[ \mathcal{H}_2^i(I_{\text{obs}}, Z_0^2, \delta)e^{-i\phi_i} \right] e^{i\phi_i}
+ \frac{P_{E}(I_{\text{obs}}, \delta)}{P_x(I_{\text{obs}}, \delta)} \left( \delta - \Re \left[ \mathcal{H}_2^i(I_{\text{obs}}, Z_0^2, \delta)e^{-i\phi_i} \right] e^{i\phi_i} \right),
\]

(57)

where \(\phi_i\) satisfies as follows:

\[
\Im \left[ \mathcal{H}_2^i(I_{\text{obs}}, Z_0^2, \delta)e^{-i\phi_i} \right] = \frac{P_{E}(I_{\text{obs}}, \delta)}{P_x(I_{\text{obs}}, \delta)} 
\times \Im \left[ \mathcal{H}_2^i(I_{\text{obs}}, Z_0^2, \delta)e^{-i\phi_i} \right].
\]

(58)

3.5. Star Selection

In real analysis, because the ellipticity used in the weight function is obtained by PSF correction, we must use some technique to determine it, for example, by iteration. In each step of iteration, we must determine the centroid and measure the moments of galaxies and stars until the result converges; thus, the E-HOLICs method needs a much longer time than the KSB method to measure the shear. Indeed, in real analysis, which is shown in the following section, we use three stars for PSF correction of each background galaxy; the E-HOLICs method needs a much longer time than the KSB correction of each background galaxy; the E-HOLICs method can avoid systematic error depending on intrinsic complex distortion \(|\delta|^{\text{intrinsic}}\). Thus, we restrict ourselves to the dependencies of ‘S/N’ and size (half-light radius “\(\text{rhs}\)”) in the shear measurement in this paper.

4.1. Star Selection

In real analysis, PSF varies across the field of view; therefore, we must use many stars for determining the PSF distribution. However, because STEP2 simulation data have the same PSF in each field, we can correct PSF from only one star. To avoid the error from PSF measurement, we use only one star, which has a maximum S/N in each field.

4.2. Tests of S/N and Size Dependence

A result of the STEP2 test in Paper I shows that the E-HOLICs method can avoid systematic error depending on intrinsic complex distortion \(|\delta|^{\text{intrinsic}}\). Thus, we restrict ourselves to the dependencies of ‘S/N’ and size (half-light radius “\(\text{rhs}\)”) in the shear measurement in this paper.

Figures 3, 4, 5, and 6 show the results of the STEP2 test by using objects which are S/N>30 or 5.0, \(\text{rhs}>\text{rhs}\), or 3.0 pixel in PSF-A, PSF-B, PSF-C, PSF-D, PSF-E, and PSF-F, where “\(\text{rhs}\)” is the maximum size of the half-light radius of the star in each PSF set. Here the bars with a square sign show the result of error for the direction 1 component of the shear, and the bars with a cross sign show the direction 2 component of the shear. The red, green, purple, pink, blue, and black colors correspond to PSF-A, PSF-B, PSF-C, PSF-D, PSF-E, and PSF-F, respectively. We can see PSF dependence from these figures; however, there is not only PSF dependence, but also other dependences, and the results are affected by their combination.

We can see there is an error in \(m\) in PSF-B in contrast to the case of PSF-A. The data sets of PSF-A and PSF-B have the corresponding distortion fields obtained from the original shapes in each PSF set by rotating by 90°. For the evaluation of the shear, a first-order polynomial is used to estimate the difference between the estimated shear and the input shear such as

\[
y_{\text{estimated}} - y_{\text{input}} = m y_{\text{input}} + c,
\]

(59)

where \(m\) means over/underestimation and \(c\) means in/oversufficiency of the anisotropic PSF correction. More detailed information of STEP2 can be seen in Massey et al. (2007).
the same PSF, but background objects have different profiles in these sets. The profiles of background objects in PSF-B are purely exponential. We can also see there is much larger $\epsilon$ error for the direction 2 in the PSF-D case and for the direction 1 in the PSF-E case than for other cases, but large ellipticities of input PSF are in the direction 1 in PSF-D and in the direction 2 in PSF-E, respectively. Therefore, these results look rather strange, and a similar tendency is obtained by other studies (Figure 5 of Massey et al. 2007), but only “MJ” in the figure obtained good results. We do not understand these results very well at the moment, but it might be due to the insufficiency of PSF decomposition as in Equation (34).

Figures 7 and 8 show systematic error of $m$ depending on $S/N$ in each component of PSF-A and PSF-F, which have a high precise result of $\epsilon$. These results show clearly the underestimation depending on $S/N$.

4.3. Summary of the STEP2 Test

The above results of the STEP2 test show that the E-HOLICs method has not only $S/N$ dependency, but also the PSF shape and the morphology of the background galaxies like the KSB method. However, the E-HOLICs method can estimate the lensing shear more precisely if we use only high $S/N$ objects or use
an appropriate statistical weight that is a function for S/N. Systematic error depending on size is smaller than that of S/N. This means that the effect of PSF is smaller than the random count noise. We do not know a fundamental reason for the S/N dependence found above. We need more studies for avoiding and/or correcting this systematic error for the precise determination of weak lensing shear (for example, cosmic shear study).

5. A370 GALAXY CLUSTER ANALYSIS

In this section, we apply the E-HOLICs method to the real data. Our data are from the massive compact galaxy cluster A370 at $z = 0.375$, where many distorted images and arcs are found. Recently, A370 has been studied (for example, Richard et al. 2010), and two peaks in the central region and the elliptical mass distribution in the outer region were found by strong lensing analysis. Umetsu et al. (2011) and Medezinski et al. (2011) analyzed this cluster by weak lensing using the Subaru telescope, where the color information is used to make a clear separation of member galaxies from background galaxies. They are interested in the accurate determination of the radial mass profile of A370. The data they used were observed with the wide-field camera Suprime-Cam (Miyazaki et al. 2002) at the prime focus of the 8.3 m Subaru telescope, and are publicly available from the Subaru archive, SMOKA5. Subaru reduction software (SDFRED) developed by Yagi et al. (2002) was used for flat fielding, instrumental distortion correction, differential re-fraction, sky subtraction, and stacking. More detailed information of the data can be seen in Medezinski et al. (2011). We also use the same catalog of background galaxies as Umetsu et al. in this analysis, but we extend their analysis from one dimension to two dimensions. Medezinski et al. (2010, 2011) also analyzed A370 with a weak lensing method.

We used IMCAT (http://www.ifa.hawaii.edu/~kaiser/imcat) and perl language scripts (K. Umetsu 2006, private communication) for detecting objects. Stars for PSF correction are selected with parameters of $1.60 < rh < 1.95$ (pixel), $19.0 < MAG < 21.0$, and $4 < SN$, and Figure 9 shows their ellipticities measured by KSB quadrupole moments. We have not used PSF fitting in the data space to find irregular stars because it will take much more time for analysis. The range of parameters we used rejected many stars darker than $MAG = 21.0$. The number density of stars after the above selection is 0.4 arcmin$^{-2}$ in this analysis. Although the number is slightly small, it is not bad judging from the result of mass reconstruction. However, we have to improve the star selection for the application of cosmic shear analysis. Typically, they have ellipticities around 0.02 along the y-axis. We have used the same sample of background objects selected by using color information provided by Umetsu et al. (2011), and the parameters of $rh > 1.95$ (arcsec) = maximum $rh$ of stars, $25.5 > MAG > 20.0$, and $SN > 5$. We used Fourier transformation (see Paper I) to transform the shear distribution into the mass distribution, where the shear distribution is smeared by Gaussian weight with a 0.1 arcmin scale. Figure 10 shows the results of two-dimensional mass reconstruction of A370 by the E-HOLICs method. For comparison, we show also the two-dimensional mass distribution obtained by the KSB method in Figure 11. The E-HOLICs method uses 803 objects, corresponding to 7.872 (arcmin$^{-2}$). In Figure 10, the minimum contour is $\kappa = 0.1$, the width between nearby contours is $\Delta \kappa = 0.1$, the peak value of $\kappa$ is 1.147, and S/N is 9.034, where we use rms of B-mode as noise. The KSB method uses 776 objects, corresponding to 7.067 (arcmin$^{-2}$). In Figure 11, contours are similarly withdrawn as in Figure 10, but the peak value of $\kappa$ is 1.156 and S/N is 9.788.

We can see double peaks and the elliptical mass distribution in the E-HOLICs method, which is consistent with the result of strong lensing (Richard et al. 2010). On the other hand, the KSB method cannot detect these peaks. We guess that the reason for this is the difference of the PSF correction between KSB and E-HOLICs. KSB includes the correction of weight function, which produces galaxies with an ellipticity larger than 1. These galaxies are eliminated from the available background galaxies. This is the reason why the KSB method has used fewer background galaxies. On the other hand, the E-HOLICs method does not produce many galaxies with an ellipticity larger than 1 after the PSF correction, compared with KSB.
6. SUMMARY AND DISCUSSION

It is now widely recognized that there are many sources of systematic errors in the evaluation of weak lensing shear, and their improvement is urgently required for the precise measurement of cosmic shear. We have developed a new method of shear estimation based on the KSB method, called the “E-HOLICs method,” by introducing the elliptical weight function to define multipole moments of the galaxy light distribution in our previous paper (Paper I). The use of the elliptical weight function is expected to avoid the systematic error coming from an expansion of the weight function, which was usually done in some of the previous approaches, including the KSB method, to shape measurement. Bernstein & Jarvis (2002) and Melchior (2011) are also developing schemes that use an elliptical weight for the shape measurement.

Following Paper I, we developed the correction scheme of PSF with anisotropy in the E-HOLICs method. Generally, the direction of the elliptical PSF is not the same as the direction of distortion; thus, we must treat these two directions in PSF correction. By dividing the anisotropic part of PSF into parallel and orthogonal parts against the direction of the distortion, we succeeded in reducing PSF correction to two steps of a simple one-dimensional form. Then we tested E-HOLICs using STEP2 data simulation. In particular, we tested the dependence on the S/N and size of background galaxies. We found that E-HOLICs gives underestimation for low S/N sources. Although the precision is of the same level of other methods, this causes a serious problem in the accurate shear measurement. We have found that at least part of the reason for the underestimation is the errors in the centroid. Although the error in the position of the centroid distributes randomly, the area in the image where the measured ellipticity from the centroid within it is smaller than the correct value is larger than the other area in the image. Thus, the random error in the centroid has a tendency to cause the underestimation of the measured shear. We are investigating this effect, and the result will be shown in a forthcoming publication.

In this paper, we have ignored higher-order moments in PSF that might be important in some cases. We aim to study the higher-order PSF correction in the same future publication.

As we pointed out, the E-HOLICs method requires a longer period of time for measuring moments if we use more stars for PSF correction than used in other methods such as the KSB method. This might cause a practical difficulty in any wide-field surveys. We hope that there will be a way to avoid this problem by using a powerful computer especially programmed for this purpose.

As an application of the E-HOLICs method, we analyzed a massive and compact galaxy cluster A370 using Subaru/S-Cam data. We found that E-HOLICs can detect two mass peaks, which is consistent with the strong lensing analysis. The KSB method cannot detect two peaks, possibly because the number of background objects available is reduced in comparison with E-HOLICs.

Although the E-HOLICs method developed in this paper has the potential to accurately measure the shear, it has still some shortcomings as described above. More studies are necessary to apply E-HOLICs to the planned large-scale cosmic shear observations.

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APPENDIX

LIST OF NEWLY DEFINED FUNCTIONS

| Function | Definition From | Ellipticity | Name |
|----------|----------------|-------------|------|
| \( I^2 (\hat{\beta}) \) | \( = \int d^2 \phi I^2 (\phi) \tilde{P}^{\text{iso}} (\hat{\beta} - \hat{\phi}) \) | Circular | Zero image |
| \( \tilde{I}^{\text{iso}} (\hat{\beta}) \) | | Circular | Smeared zero image |
| \( I^2 (\theta) \) | \( = I^2 (\hat{\theta}) \) | Circular | Lensed image |
| \( \tilde{I}^{\text{obs}} (\theta) \) | \( = \int d^2 \phi I^2 (\phi) P (\theta - \phi) \) | Having a direction of distortion | Observed image |
| \( I_\parallel (\theta) \) | \( = \int d^2 \phi I^2 (\phi) P_\parallel (\theta - \phi) \) | Having an ellipticity of \( \delta \) | Parallel image |
| \( I_\perp (\theta) \) | \( = \int d^2 \phi I^2 (\phi) P_\perp (\theta - \phi) \) | Having an ellipticity of \( \delta \) | Distortional image |
| \( P (\theta) \) | \( = \int d^2 \phi P_\parallel (\phi) q_1 (\theta - \phi) \) | Circular | Real PSF |
| \( \tilde{P} (\theta) \) | | Circular | Isotropic zero PSF |
| \( P_\parallel (\theta) \) | \( = \int d^2 \phi P_\parallel (\phi) q_1 (\theta - \phi) \) | Having an ellipticity of \( \delta \) | Distortional PSF |
| \( q_1 (\theta) \) | \( = \int d^2 \phi P_\parallel (\phi) q_1 (\theta - \phi) \) | Having a direction of distortion | Parallel PSF |
| \( q_2 (\theta) \) | \( = \int d^2 \phi P_\parallel (\phi) q_2 (\theta - \phi) \) | Orthogonal to distortion | Orthogonal part of \( P (\theta) \) |
| \( q_3 (\theta) \) | \( = \int d^2 \phi P_\parallel (\phi) q_3 (\theta - \phi) \) | Having a direction of distortion | Distortional part of \( \tilde{P} (\theta) \) |
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