I. INTRODUCTION

Using the 3D GL EM modeling [1-2] and GL inversion [3], we propose an EM double layered cloak in this paper which is called as GL double layered cloak. The single layer cloak proposed by Pendry et al. [4] is named as PS cloak. The GL double layered cloak is consist of two sphere annular layers, \(R_1 \leq r \leq R_2\) and \(R_2 \leq r \leq R_3\). Two type cloak materials are proposed and installed in the each layer, respectively. The outer layer cloak of the GL double layered cloak has the invisible function, the inner layer cloak has fully absorption function. The GL double layered metamaterials are weak degenerative and weak dispersive. When the source is located outside of the GL double layered cloak, the excited EM wave field propagation outside of the double layered cloak is as same as in free space and never be disturbed by the cloak; also, the exterior EM wave can not penetrate into the inner layer and concealment. When local sources are located inside of the GL double cloaked concealment with the normal EM materials, the excited EM wave is propagating under Maxwell equation governing, it is complete absorbed by the inner layer cloak of GL double cloak and never propagate to outside of the inner layer of the GL cloak, moreover, the EM wavefield in concealment never be disturbed by the cloak.

The GL double layered cloak is a robust cloak and has complete and sufficient invisibility functions. Its concealment is the normal electromagnetic environment. Our EM GL double layered cloak is different from conventional common cloak. The 3D GL EM modeling simulations for the double layered cloak are presented. The GL method is an effective physical simulation method and is fully different from the conventional methods. It has double abilities of the theoretical analysis and numerical simulations to study the cloak metamaterials and wide materials and field scattering problem in physical sciences.

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dispersion from the interaction between the field and materials. The frequency limitation is a difficulty of FEM and FD method.

The GL method is a significant scattering process which reduces the numerical dispersion and is suitable to simulate physical wavefield scattering in the materials, in particular, for dispersive materials. Born Approximation is a conventional method in the quantum mechanics and solid physics, however, it is one iteration only in whole domain which is not accurate in the high frequency and high contrast materials. The GL method divides the domain as a set of small sub domains or sub lattices. The Global field is updated by the local field from the interaction between the global field and local subdomain materials successively. Once all subdomain materials are scattered, the GL field solution is obtained which is much more accurate than the Born approximation.

Moreover, the GL method can be meshless, including arbitrary geometry subdomains, such as rectangle, cylindrical and spherical coordinate mixed coupled together. It is a full parallel algorithm. These advantages of the GL method to overcome historical difficulties have been detailed described in the paper [1]. The theoretical foundation of the GL method is described in the paper [2].

We have used the 3D GL modeling [1-2] and inversion [3] to simulate many cloak metamaterials, nanometer materials, periodic photonic crystals etc. When the point source is located outside or inside of the various geometry cloaks, the 3D GL EM modeling simulations for the EM wavefield propagation through the cloaks have been done. These simulations show that the GL method is fast and accurate. We have submitted a paper titled the 3D GL EM modeling to simulate single layer cloaks to PRE [11].

In this paper, the 3D GL EM modeling simulations of the EM wave field propagation through the new GL double layered cloak is presented. When the local sources are located outside or inside of the outer layer cloak, EM wave propagation through outer layer cloak and never penetrate into the inner layer and the concealment, i.e. $r \leq R_2$. The exterior EM wavefield propagation outside of GL double layered cloak never be disturbed by the cloak. The outer layer cloak has the invisibility function. When the local sources inside of the GL double layer cloak concealed with normal materials, the excited EM wave normally propagating under Maxwell equation governing, the EM field is complete absorbed by the inner layer cloak and can not propagate outside of the inner cloak. The EM environment in the GL double cloak concealment is normal, in which there exist Maxwell EM wave field excited by nonzero local sources, have no reflection from the boundary $r = R_1$, and never propagate outside of boundary $r = R_2$.

By using the 3D GL EM modeling [1-2] simulation and its theoretical analysis, we found and verified a phenomenon that there exists no Maxwell EM wave field can be excited by nonzero local sources inside of the single layer cloak concealed with normal materials. Our GL double layered cloak overcomes the drawback and difficulty in the single cloak. Pendry et al. in paper [4] used a coordinate transformation and ray tracing to propose the annular cloak in which the ray being bending and reflection around central sphere object and can not penetrate into it. The cloak device is empty and does not disturb exterior wave field. There are several other papers to simulate the exterior plane wave propagation through the cloak [12-14]. Cummer et al. in paper [12] proposed numerical simulations by using the COMSOL Multiphysics finite element-based electromagnetic solver for the 2D plane wave propagation through cylindrical cloak. Chen et al. proposed the Mie analytical TEM model to simulate the plane wave through the spherical cloak [13]. Argyropoulos et al. proposed a dispersive finite difference method in time domain (FTFD) in [14] to simulate 2D TEM plane wave field through cylindrical cloak, in which authors considered the difficulty of conventional FDTD scheme for dispersive materials. In papers [12] and [14], authors introduced many papers for cloak research works. Because the plane wave is excited by plane source which can not be located inside of the cloak or concealment. To study the EM wave excited from local sources inside of the cloaked concealment is absent from these papers. In paper [15] and [16], authors studied the effect on invisibility of active devices inside the cloaked region. Author in [16] stated that "when these conditions are over determined, finite energy solutions typically do not exist."

We use 3D GL method to do many simulations for studying the behavior of EM field excited inside of the single layer cloaked concealment. These simulation are divergent or become chaos when the EM wave propagates to the inner boundary of the single layer cloak. Our statement is that "there exists no Maxwell EM wave field can be excited by nonzero local sources inside of the single layer cloaked concealment with normal materials". The detailed proof and 3D GL simulations are presented in this paper. Before the practice production of the single layered cloak, the electromagnetic field environment inside of the concealment can not be studied in physical experiment. Our GL double layered cloak proposed in this paper overcomes the drawback and difficulty of the single layer cloak and avoid the disputing on the EM phenomenon inside of the single layered cloak concealment with normal materials. The GL double layered EM cloak metamaterials inventive and fabrication technology right and 3D GL EM modeling software are patented by GL Geophysical Laboratory. We thanks to GL Geophysical Laboratory to approve us to publish the GL modeling method, GL double layered cloak theory, and simulations.

We describe this paper in the following order: The introduction is described in Section 1. In Section 2, we propose a GL double layered cloak materials. The EM integral equations are presented in Section 3. The 3D GL EM modeling are described in Section 4. The theoretical analysis of properties and functions of the GL double
layered cloak are proposed in Section 5. The simulations of the EM wave propagation through the GL double layered cloak by using the GL EM modeling are presented in Section 6. The advantages of the GL double layered cloak is presented in Section 7. In Section 8, we conclude this paper.

II. GL DOUBLE LAYERED CLOAK MATERIALS

A. GL Inner Layered Cloak Anisotropic Material

On the inner sphere annular domain, \( \Omega_{\text{GLI}} = \{ r : R_1 \leq r \leq R_2 \} \), by the GL EM modeling and inversion [1-3], we propose an anisotropic material as follows,

\[
[D]_{\text{GLI}} = \text{diag}[\varepsilon_i, \mu_i], \quad \varepsilon_i = \text{diag}[\varepsilon_{r,i}, \varepsilon_{\theta,i}, \varepsilon_{\phi,i}] \varepsilon_0, \quad \mu_i = \text{diag}[\mu_{r,i}, \mu_{\theta,i}, \mu_{\phi,i}] \mu_0, \quad \varepsilon_{r,i} = \mu_{r,i} = \left( \frac{R_2^2 - R_i^2}{R_2^2 - R_i^2} \right) \frac{R_2^2 - r^2}{R_2^2 - R_i^2},
\]

\[
\varepsilon_{\theta,i} = \varepsilon_{\phi,i} = \mu_{\theta,i} = \mu_{\phi,i} = \frac{R_2^2 - R_i^2}{R_2^2 - r^2} \frac{R_2^2 - r^2}{R_2^2 - R_i^2}. \tag{1}
\]

The \( \Omega_{\text{GLI}} \) is called as GL inner layered cloak, the materials, \( [D]_{\text{GLI}} = \text{diag}[\varepsilon_i, \mu_i] \) in (1), are the anisotropic GL inner layered cloak material tensor.

B. GL Outer Layered Cloak Anisotropic Material

Let the outer sphere annular domain \( \Omega_{\text{GLO}} = \{ r : R_2 \leq r \leq R_3 \} \) be the GL outer layered cloak with the following anisotropic GL outer layered cloak materials,

\[
[D]_{\text{GLO}} = \text{diag}[\varepsilon_o, \mu_o], \quad \varepsilon_o = \text{diag}[\varepsilon_{r,o}, \varepsilon_{\theta,o}, \varepsilon_{\phi,o}] \varepsilon_0, \quad \mu_o = \text{diag}[\mu_{r,o}, \mu_{\theta,o}, \mu_{\phi,o}] \mu_0, \quad \varepsilon_{r,o} = \mu_{r,o} = \frac{R_3^2 - R_2^2}{R_3^2 - r^2} \sqrt{R_3^2 - r^2 - R_2^2},
\]

\[
\varepsilon_{\theta,o} = \varepsilon_{\phi,o} = \mu_{\theta,o} = \mu_{\phi,o} = \frac{R_3^2 - R_2^2}{R_3^2 - R_2^2}, \tag{2}
\]

The GL inner cloak \( \Omega_{\text{GLI}} \) domain and GL outer cloak \( \Omega_{\text{GLO}} \) domain are bordering on the interface surface \( r = R_2 \). We assemble the \( \Omega_{\text{GLI}} \) as the inner layer sphere annular domain and \( \Omega_{\text{GLO}} \) as the outer layer sphere annular domain and make them coupling on their interface boundary annular surface \( r = R_2 \) as follows,

\[
\Omega_{\text{GL}} = \Omega_{\text{GLI}} \cup \Omega_{\text{GLO}} = \{ r : R_1 \leq r \leq R_2 \} \cup \{ r : R_2 \leq r \leq R_3 \} = \{ r : R_1 \leq r \leq R_3 \}, \tag{3}
\]

and offer the coupled anisotropic dielectric and susceptibility tensor \( [D]_{\text{GL}} \) on the \( \Omega_{\text{GL}} \) as follows,

\[
[D]_{\text{GL}} = \begin{cases} [D]_{\text{GLI}}, & r \in \Omega_{\text{GLI}} \\ [D]_{\text{GLO}}, & r \in \Omega_{\text{GLO}}. \end{cases} \tag{4}
\]

The GL inner layer cloak material \( [D]_{\text{GLI}} = \text{diag}[\varepsilon_i, \mu_i] \) in (1) on the \( \Omega_{\text{GLI}} \) and GL outer layer cloak material \( [D]_{\text{GLO}} = \text{diag}[\varepsilon_o, \mu_o] \) in (2) on \( \Omega_{\text{GLO}} \) are assembled into the GL anisotropic double layered cloak material on the domain \( \Omega_{\text{GL}} \). The domain \( \Omega_{\text{GL}} \) with the metamaterial \( [D]_{\text{GL}} \) in (4) is called as the GL double layered cloak.

III. 3D ELECTROMAGNETIC INTEGRAL EQUATION

The 3D EM integral equation in frequency domain has been proposed in authors’ papers [1] and [2]. In this paper, we proposed the EM integral equation in time domain as follows:

\[
\begin{bmatrix} E(r,t) \\ H(r,t) \end{bmatrix} = \begin{bmatrix} E_b(r,t) \\ H_b(r,t) \end{bmatrix} + \int_{\Omega_{\text{E},H}} \begin{bmatrix} G^{J,M}_{E,H}(r',r,t) & \ast_t \delta[D(r')] \\ G^{J,M}_{E,H,b}(r',r,t) & \ast_t \delta[D(r')] \end{bmatrix} \begin{bmatrix} E_b(r',t) \\ H_b(r',t) \end{bmatrix} \, dr', \tag{5}
\]

and

\[
\begin{bmatrix} E(r,t) \\ H(r,t) \end{bmatrix} = \begin{bmatrix} E_b(r,t) \\ H_b(r,t) \end{bmatrix} + \int_{\Omega_{\text{E},H,b}} \begin{bmatrix} G^{J,M}_{E,H,b}(r',r,t) & \ast_t \delta[D(r')] \\ G^{J,M}_{E,M}(r',r,t) & \ast_t \delta[D(r')] \end{bmatrix} \begin{bmatrix} E_b(r',t) \\ H_b(r',t) \end{bmatrix} \, dr'. \tag{6}
\]

In the EM integral equation (5),

\[
G^{J,M}_{E,H}(r',r,t) = \begin{bmatrix} E^J(r',r,t) \\ H^J(r',r,t) \end{bmatrix}, \quad G^{J,M}_{E,H,b}(r',r,t) = \begin{bmatrix} E^M(r',r,t) \\ H^M(r',r,t) \end{bmatrix}, \tag{7}
\]

and \( G^{J,M}_{E,H,b} \) is the EM Green’s tensor in the background medium, where, \( E(r,t) \) is the electric field, \( H(r,t) \) is the magnetic field, \( E_b(r,t) \) and \( H_b(r,t) \) is the incident electric and magnetic field in the background medium, \( E^J(r',r,t) \) is electric Green’s tensor, \( H^J(r',r,t) \) is magnetic Green’s tensor, they are excited by the point impulse current source, \( E^M(r',r,t) \) and \( H^M(r',r,t) \) are electric and magnetic Green’s tensor, respectively, they are excited by the point impulse magnetic moment source, \( \ast_t \) is convolution with respect to t, \( \delta[D] \) is the electromagnetic material parameter variation matrix,

\[
\delta[D] = \begin{bmatrix} \delta D_{11} & 0 \\ 0 & \delta D_{22} \end{bmatrix}, \quad \delta D_{11} = (\tilde{\sigma}(r) - \sigma_0 I) + (\tilde{\varepsilon}(r) - \varepsilon_0 I) \frac{\partial \tilde{\sigma}}{\partial \varepsilon}, \quad \delta D_{22} = (\tilde{\mu}(r) - \mu_0 I) \frac{\partial \tilde{\mu}}{\partial \mu}, \tag{8}
\]

\( \delta D_{11} \) and \( \delta D_{22} \) are a 3 \times 3 symmetry, inhomogeneous diagonal matrix for the isotropic material, for anisotropic material, they are an inhomogeneous diagonal or full matrix, \( I \) is a 3 \times 3 unit matrix, \( \tilde{\sigma}(r) \) is the conductivity tensor, \( \tilde{\varepsilon}(r) \) is the dielectric tensor, \( \tilde{\mu}(r) \) is susceptibility tensor which can be dispersive parameters depend on the angular frequency \( \omega \), \( \sigma_0 \) is the conductivity, \( \varepsilon_0 \) is the permittivity, \( \mu_0 \) is the permeability in the background free space, \( \Omega \) is the finite domain in which the parameter variation matrix \( \delta[D] \neq 0 \), the \( (\varepsilon(r) - \varepsilon_0 I)E \) is the electric polarization, and \( (\tilde{\mu}(r) - \mu_0 I)H \) is the magnetization.
IV. 3D GL EM MODELING

We propose the GL modeling based on the EM integral equations (1) and (2) in the space-time domain.

(3.1) The domain $\Omega$ is divided into a set of $N$ sub domains, $\{\Omega_k\}$, such that $\Omega = \bigcup_{k=1}^{N} \Omega_k$. The division can be mesh or meshless.

(3.2) When $k = 0$, let $E_0(r, t)$ and $H_0(r, t)$ are the analytical global field, $E_0^J(r', r, t)$, $H_0^M(r', r, t)$, and $H_0^M(r', r, t)$ are the analytical global Green’s tensors in the background medium. By induction, suppose that $E_k-1(r, t)$, $H_k-1(r, t)$, $E_k^J(r', r, t)$, $H_k^M(r', r, t)$, and $H_k^M(r', r, t)$ are calculated in the $(k-1)^{th}$ step in the subdomain $\Omega_k-1$.

(3.3) In $\{\Omega_k\}$, upon substituting $E_k-1(r, t)$, $H_k-1(r, t)$, $E_k^J(r', r, t)$, $H_k^M(r', r, t)$, and $H_k^M(r', r, t)$ into the integral equation (1), the GL Green’s tensor integral equation (1) in $\Omega_k$ is reduced into $6 \times 6$ matrix equations. By solving the $6 \times 6$ matrix equations, we obtain the Green’s tensor field $E_k^J(r', r, t)$ and $H_k^M(r', r, t)$ are updated by the interaction scattering field between the Green’s tensor and local polarization and magnetization in the subdomain $\Omega_k$ as follows,

$$
\begin{bmatrix}
E_k(r, t) \\
H_k(r, t)
\end{bmatrix} = \begin{bmatrix}
E_k-1(r, t) \\
H_k-1(r, t)
\end{bmatrix} + \int_{\Omega_k} \begin{bmatrix}
E_k^J(r', r, t) \\
H_k^M(r', r, t)
\end{bmatrix} \cdot \delta[D(r')] 
\begin{bmatrix}
E_k-1(r', t) \\
H_k-1(r', t)
\end{bmatrix} \, dr'.
\tag{9}
$$

(3.5) The steps (3.2) and (3.4) form a finite iteration, $k = 1, 2, \cdots, N$, the $E_N(r, t)$ and $H_N(r, t)$ are the electromagnetic field of the GL modeling method. The GL electromagnetic field modeling in the space time domain is short named as GLT method.

The GL EM modeling in the space frequency domain is proposed in the paper [2], we call the GL modeling in frequency domain as GLF method.

V. THEORETICAL ANALYSIS OF INTERACTION OF THE EM WAVE FIELD THROUGH THE CLOAKS

A. Theoretical Analysis Of Interaction Of The EM Wave Field Through The GL Double Layered Cloaks

We propose the theoretical analysis of the interaction between the EM wave and GL cloaks in this section.

Statement 1: Let domain $\Omega_{GL}$ in (3) and the metamaterial $D_{GL}$ in (4) be GL double layered cloak, and $\varepsilon = \varepsilon_b, \mu = \mu_b$ be basic permittivity and permeability, respectively, inside of the central sphere concealment $|\vec{r}| < R_1$ and outside of the GL cloak $|\vec{r}| > R_3$, we have the following statements: (1) provide the source is located inside of the concealment of GL double layered cloak, $|\vec{r_s}| < R_1$, the excited EM wave field propagates inside of the concealment and never be disturbed by the cloak; (2) provide the source is located inside of concealment or inside of the inner layer of the GL double layered cloak, $|\vec{r_s}| < R_2$, the EM wave field is vanished outside of the inner layer of GL cloak and is always propagating to the boundary $r = R_2$ and is absorbed by the boundary $r = R_2$. (3) provide the source is located outside of the GL double layered cloak, $|\vec{r_s}| > R_3$, the excited EM wave field outside of the double layered cloak is as same as in free space and never be disturbed by the double layered cloak; (4) provide the source is located outside of double layered cloak or located inside of the outer layer of GL cloak, $|\vec{r_s}| > R_2$, the excited EM wave field never propagate into the inner layer of GL cloak and the concealment.

B. There Exists No Maxwell EM Wavefield Can Be Excited By Nonzero Local Sources Inside Of The Single Layered Cloaked Concealment With Normal Materials

Statement 2: Suppose that a 3D anisotropic inhomogeneous single layered cloak domain separates the whole 3D space into three sub domains, one is the single layered cloak domain $\Omega_{clk}$ with the cloak material; the second one is the cloaked concealment domain $\Omega_{cont}$ with normal EM materials; other one is the free space outside of the cloak. If the Maxwell EM wavefield excited by a point source or local sources outside of the concealment $\Omega_{cont}$ is vanished inside of the cloak domain $\Omega_{cont}$, then there is no Maxwell EM wave field excited by the local sources inside of the cloaked concealment $\Omega_{cont}$.

The Maxwell EM wavefield is the EM wave field which satisfies the Maxwell equation and tangential continuous interface boundary conditions. We call the Maxwell EM wavefield as the EM wavefield and use inverse process to prove the statement 2 as follows: Suppose that there exists Maxwell EM wavefield excited by the local sources inside the concealment with the normal materials, the wavefield satisfies the Maxwell equation in the 3D whole space $\mathcal{R}^3$ which includes the anisotropic inhomogeneous cloak domain $\Omega_{clk}$ and concealment $\Omega_{cont}$, and satisfies the tangential continuous interface conditions on the interface boundary surface $S_1$ and $S_2$. The $S_1$ is the interface boundary surface between the cloak domain $\Omega_{clk}$ and the concealment $\Omega_{cont}$, it also is the inner boundary surface of the cloak domain $\Omega_{clk}$. The $S_2$ is the interface boundary surface between the cloak domain $\Omega_{clk}$ and the free space, it also is the outer boundary surface of the cloak domain $\Omega_{clk}$.

Let $R_c = R_3 - \Omega_{clk} \cup \Omega_{cont}, R_d = R_3 - \Omega_{cont}$, and by
the EM integral equation (1), the EM wave field satisfies
\[
\begin{bmatrix}
E(r, t) \\
H(r, t)
\end{bmatrix}
= \begin{bmatrix}
E_b(r, t) \\
H_b(r, t)
\end{bmatrix} +
\int_{\Omega_{clk}} G_{E,H}^{J,M}(r', r, t) \ast_t \delta[D] \begin{bmatrix}
E_b(r', t) \\
H_b(r', t)
\end{bmatrix} \, dr',
\tag{10}
\]
where \(G_{E,H}^{J,M}(r', r, t)\) is the EM Green’s tensor, its components \(E^J, H^J, E^M,\) and \(H^M(r', r, t)\) are the EM Green’s function on \(\Omega_{clk} \cup \Omega_{conl} \cup R_c\), excited by the point impulse sources outside of the concealment, \(r \in R_d\). By the assumptions, \(G_{E,H}^{J,M}(r', r, t)\) exists on \(\Omega_{clk} \cup \Omega_{conl} \cup R_c\) and when \(r' \in \Omega_{conl}\), \(G_{E,H}^{J,M}(r', r, t) = 0\). The integral equation (10) becomes to
\[
\begin{bmatrix}
E(r, t) \\
H(r, t)
\end{bmatrix}
= \begin{bmatrix}
E_b(r, t) \\
H_b(r, t)
\end{bmatrix} +
\int_{\Omega_{clk}} G_{E,H}^{J,M}(r', r, t) \ast_t \delta[D] \begin{bmatrix}
E_b(r', t) \\
H_b(r', t)
\end{bmatrix} \, dr'.
\tag{11}
\]

We consider the Maxwell equation in \(R_d\), the virtual source is located \(r, r_0 \in R_d\) and the point source is located \(r_s, r_s \in \Omega_{conl}\) and \(r_s \notin R_d\), we have
\[
\begin{align*}
\left[-\nabla \times \nabla \times \right] G_{E,H}^{J,M}(r', r, t)
&= [D] G_{E,H}^{J,M}(r', r, t) + I\delta(r', r)\delta(t),
\tag{12}
\end{align*}
\]
and
\[
\begin{align*}
\left[-\nabla \times \nabla \times \right] \begin{bmatrix}
E_b \\
H_b
\end{bmatrix}(r', r_s, t)
&= [D_b] \begin{bmatrix}
E_b \\
H_b
\end{bmatrix}(r', r_s, t),
\tag{13}
\end{align*}
\]

By using \([E_b(r, t), H_b(r, t)]\) to convolute (12), and \(G_{E,H}^{J,M}(r', r, t)\) to convolute (13), to subtract the second result equation from the first result equation and make their integral in \(R_d\), and use integral by part and make some manipulations, we can prove
\[
\begin{bmatrix}
E_b(r, t) \\
H_b(r, t)
\end{bmatrix} +
\int_{\Omega_{clk}} G_{E,H}^{J,M}(r', r, t) \ast_t \delta[D] \begin{bmatrix}
E_b(r', t) \\
H_b(r', t)
\end{bmatrix} \, dr' =
\int_{\Omega_{conl}} G_{E,H}^{J,M}(r', r, t) \ast_t [E(r, r_s, t), H(r, r_s, t)] \, dr',
\tag{14}
\]
where \(\bigotimes_t\) is cross convolution. From the assumption of the statement 2 that “the Maxwell EM wavefield excited by a point source or local sources outside of the concealment \(\Omega_{conl}\) is vanished in side of the concealment \(\Omega_{conl}\)”, and virtual source \(r\) is located outside of the concealment, \(r \in R_d\), if \(r' \in \Omega_{conl}\), \(G_{E,H}^{J,M}(r', r, t) = 0\). By continuity, when \(r' \in S_1\), we have \(G_{E,H}^{J,M}(r', r, t) = 0\). By tangential continuous interface conditions of \(G_{E,H}^{J,M}(r', r, t)\), the term in right hand side of (14) is vanished, we have
\[
\begin{bmatrix}
E_b(r, t) \\
H_b(r, t)
\end{bmatrix} +
\int_{\Omega_{clk}} G_{E,H}^{J,M}(r', r, t) \ast_t \delta[D] \begin{bmatrix}
E(r', t) \\
H(r', t)
\end{bmatrix} \, dr' = 0.
\tag{15}
\]

Upon substituting integral equation (15) into the integral equation (11), we have
\[
\begin{bmatrix}
E(r, t) \\
H(r, t)
\end{bmatrix} = 0.
\tag{16}
\]

From the continuous property of the EM wave field, we obtain the following over vanish boundary condition on the boundary \(S_1\) of the concealment \(\Omega_{conl}\), we have
\[
\begin{bmatrix}
E(r, r_s, t) \\
H(r, r_s, t)
\end{bmatrix} = 0,
\tag{17}
\]
where \(r_s\) denotes point source location inside of the concealment, \(r\) is EM field receiver point, \(r \in S_1\). Because the EM wave field is excited by local sources inside of the concealment domain \(\Omega_{conl}\), it satisfies the following Maxwell equation,
\[
\begin{bmatrix}
0 \\
0
\end{bmatrix}
\bigotimes_t
\begin{bmatrix}
E_{r,s} \\
H_{r,s}
\end{bmatrix}(r', r_s, t)
= [D_{conl}] \begin{bmatrix}
E_{r,s} \\
H_{r,s}
\end{bmatrix}(r', r_s, t) + \Omega(r', r_s, t) = Q(r', r_s, t),
\tag{18}
\]
where \([D_{conl}] = \text{diag} \{\varepsilon_r \varepsilon_0 \mu_r \mu_0\} \, (\partial / \partial t)\) with the normal EM material parameters, \(\varepsilon_r \geq 1\) and \(\mu_r \geq 1\) are relative EM parameters, \(\varepsilon_0\) is basic permittivity and \(\mu_0\) is basic permeability, \(r_s \in \Omega_{conl}\) is the local source location, \(Q(r', r_s, t)\) is the nonzero local source inside of \(\Omega_{conl}\).

Let \(G_{E,H,conl}^{J,M}(r', r, t)\) be Greens tensor which satisfies
\[
\begin{bmatrix}
0 \\
0
\end{bmatrix}
\bigotimes_t
\begin{bmatrix}
E_{r,s} \\
H_{r,s}
\end{bmatrix}(r', r, t)
= [D_{conl}] G_{E,H,conl}^{J,M}(r', r, t) + \Omega(r', r_s, t)\delta(t)
\tag{19}
\]

By using \([E(r, t), H(r, t)]\) to convolute (19), and \(G_{E,H,conl}^{J,M}(r', r, t)\) to convolute (18), to subtract the second result equation from the first result equation and make their integral in \(\Omega_{conl}\), and use integral by part and make some manipulations, we have
\[
\begin{align*}
\begin{bmatrix}
E(r, r_s, t) \\
H(r, r_s, t)
\end{bmatrix}
= \int_{\Omega_{conl}} G_{E,H,conl}^{J,M}(r', r, t) \ast_t Q(r', r_s, t) \, dr' +
\int_{\partial \Omega_{conl}} G_{E,H,conl}^{J,M}(r', r, t) \bigotimes_t \begin{bmatrix}
E(r', r_s, t) \\
H(r', r_s, t)
\end{bmatrix} \, dr',
\tag{20}
\end{align*}
\]
where \(\bigotimes_t\) denotes the cross convolution, and \(\partial \Omega_{conl} = S_1\). Because the over vanished condition (17),
\[
\begin{bmatrix}
E(r, r_s, t) \\
H(r, r_s, t)
\end{bmatrix} = 0.
\]
we have
\[
\begin{bmatrix}
E(r, r_s, t) \\
H(r, r_s, t)
\end{bmatrix} = \int_{\Omega_c} G^{l,M}_{E,H,conl}(r', r_s, t) * Q(r', r_s, t) dr'.
\] (21)

Because \(G^{l,M}_{E,H,conl}(r', r_s, t) \neq 0\) and \(Q(r', r_s, t) \neq 0\), so,
\[
\begin{bmatrix}
E(r, r_s, t) \\
H(r, r_s, t)
\end{bmatrix} \neq 0.
\] (22)

From the continuity of the EM wavefield, the nonzero EM wave field (22) results that
\[
\begin{bmatrix}
E(r, r_s, t) \\
H(r, r_s, t)
\end{bmatrix}\bigg|_{S_1} \neq 0.
\] (23)

The EM wavefield is nonzero on the boundary \(S_1\) in (23) is an obvious contradiction with the same EM wavefield is zero on the boundary \(S_1\) in (17). Therefore, we proved that there exists no Maxwell EM wave field can be excited by the nonzero local sources inside of the single layered cloak concealment. For more simplicity to derive the nonzero EM wavefield (22) from the integral expression of the EM wave field (21), let the source is point impulse current source with polarization direction in \(\vec{z}\), i.e.,
\[
Q(r, r_s, t) = \delta(r-r_s)\delta(t)\vec{z}.
\] (24)

Upon substituting (24) and \(\varepsilon_r = 1.0\) and \(\mu_r = 1.0\) into the (21), we have
\[
\begin{bmatrix}
E_x(r, r_s, t) \\
H_x(r, r_s, t)
\end{bmatrix} = \begin{bmatrix}
E_x^l(r, r_s, t) \\
H_x^l(r, r_s, t)
\end{bmatrix}
\] (25)

\[
E_x^l(r, r_s, t) = \begin{bmatrix}
E_{xx}(r, r_s, t) \\
E_{xy}(r, r_s, t) \\
E_{xz}(r, r_s, t)
\end{bmatrix}
\] (26)

\[
E_{xx}(r, r_s, t) = \frac{1}{4\pi\varepsilon_0} \frac{Q}{|r-r_s|} \delta(t-\sqrt{r^2-|r-r_s|^2})
\]
\[
+ \frac{1}{4\pi\mu_0} \frac{Q}{|r-r_s|} \delta(t-\sqrt{r^2-|r-r_s|^2})
\] (27)

It is obvious that when \(r \in S_1\)
\[
E_{xx}(r, r_s, t) |_{r \in S_1} \neq 0.
\] (28)

The electric intensity \(E_{xx}(r, r_s, t) |_{r \in S_1} \neq 0\) in (28) and \(E_{xx}(r, r_s, t) |_{r \in S_1} = 0\) in (17) are an obvious contradiction. Therefore, we proved the Statement 2 that there exists no Maxwell EM wavefield can be excited by the nonzero local sources inside of the single layered cloaked concealment with normal materials.

VI. THE GL EM MODELING SIMULATIONS OF THE EM WAVE FIELD THROUGH THE GL DOUBLE CLOAKS

A. The Simulation Model of The GL Double Layered Cloak

The simulation model: the 3D domain is \([-0.5m, 0.5m] \times [-0.5m, 0.5m] \times [-0.5m, 0.5m]\), the mesh number is 201 × 201 × 201, the mesh size is 0.005m. The electric current point source is defined as
\[
\delta(r-r_s)\delta(t)\vec{e},
\] (29)
where the \(r_s\) denotes the location of the point source, the unit vector \(\vec{e}\) is the polarization direction, the time step \(dt = 0.3333 \times 10^{-10}\) second, the frequency band is from 0.05 GHz to 15 GHz, the largest frequency \(f = 15GHz\), the shortest wave length is 0.02m. The EM GL double layered cloak \(\Omega_{GL} = \Omega_{GL1} \cup \Omega_{GL0}\) consist of the double spherical annular \(\Omega_{GL1}\) and \(\Omega_{GL0}\) with the center in the origin and interior radius \(R_1 = 0.22m\), meddle radius \(R_2 = 0.3m\), and exterior radius \(R_3 = 0.35m\). The cloak is divided into 90 × 180 × 90 cells. The spherical coordinate is used in the sphere \(r \leq R_3\), the Cartesian rectangular coordinate is used in outside \(\Omega_{GL}\) to mesh the domain.

![Electric Wave Exx Propagation in Double Layered EM GL-Cloak](#)

FIG. 1: (color online) At this moment of the time step 39dt, the most part of the front of the First electric wave, \(E_{xx,1}\), propagates enter to the inner GL cloak layer, \(R_1 \leq r \leq R_2\), a few part of the front of the \(E_{xx,1}\) is located in right and top of the concealment; the front of Second electric wave, \(E_{xx,2}\), reaches the outer boundary \(r = R_3\) of the GL double layered cloak.
double layered cloak. In the Figure 2, at time step 75
ered cloak are presented in the Figures 1-3. The two
ing simulations of the EM wave excited by above two
and around the sphere annular \( r = R_2 \) and never pene-
trate into inner domain, \( r < R_3 \). It does disperse and split
into the two phases around the sphere annular \( r = R_2 \).
The first EM wave, \( E_{xx,1} \), is propagating inside of the
inner sphere annular layer of the GL double layered cloak,
\( R_1 \leq r \leq R_2 \).

B. The EM Wave Excited By Point Source In The
Concealment And Other Point Source In The Free
Space Propagates Through The GL Double Layered
Cloak

The configuration of the GL double layered cloak material
is described in the subsection A of this section. Two
point sources are used to excite the EM wave propagation
through the GL double layered cloak. The first point cur-
cent source is located inside of the center sphere conceal-
ment at \((-0.12m, -0.12m, 0.0)\), by which the excited EM
wave is named as First EM wave, its component \( E_{xx,1} \)
is named First electric wave. The second current point
source is located in free space at \((0.518m, 0.518m, 0.0)\)
where is the right and top corner outside of the whole
GL double layered cloak. The EM wave by the second
source is named as Second EM wave. Its component
\( E_{xx,2} \), is named Second electric wave. The GL model-
ling simulations of the EM wave excited by above two
point sources propagation through the GL double lay-
ered cloak are presented in the Figures 1-3. The two
waves are propagating at time step 38\( dt \) that is shown
in the Figure 1, at this moment, the most part of the
front of the First electric wave, \( E_{xx,1} \), propagates en-
ter to the inner GL cloak layer, \( R_1 \leq r \leq R_2 \), a few
part of the front of the \( E_{xx,1} \) is located in right and top
of the concealment. The front of Second electric wave,
\( E_{xx,2} \), reaches the outer boundary \( r = R_3 \) of the GL
double layered cloak. In the Figure 2, at time step 75\( dt \),
the Second electric wave, \( E_{xx,2} \), is propagating inside of
the outer layer cloak of the GL double layered cloak,
\( R_2 \leq r \leq R_3 \), and around the sphere annular \( r = R_2 \)
and never penetrate into the inner layer of GL cloak and the
concealment, i.e. \( r.le.R_2 \). It does disperse and split into
the two phases around the sphere annular \( r = R_2 \). The First electric wave, \( E_{xx,1} \), is still propagat-
ing inside of the inner sphere annular layer of the GL double
layered cloak, \( R_1 \leq r \leq R_2 \).

FIG. 2: (color online) At time step 75\( dt \), the Second electric wave, \( E_{xx,2} \), is propagating inside of the outer
layer cloak of the GL double layered cloak, \( R_2 \leq r \leq R_3 \), and around the sphere annular \( r = R_2 \) and never pene-
trate into inner domain, \( r < R_3 \). It does disperse and split into
the two phases around the sphere annular \( r = R_2 \). The First electric wave, \( E_{xx,1} \), is propagating inside of the
inner sphere annular layer of the GL double layered cloak,
\( R_1 \leq r \leq R_2 \).

FIG. 3: (color online) At time step 98\( dt \), one part of front the
Second electric wave, \( E_{xx,2} \), is propagating inside of the outer
layer cloak, \( R_2 \leq r \leq R_3 \). It has around the sphere annular
\( r = R_2 \) and forward bending in the left of the sphere annular
\( r = R_2 \). The First electric wave, \( E_{xx,1} \), is still propagating
inside of the inner sphere annular layer of the GL double
layered cloak, \( R_1 \leq r \leq R_2 \).
double cloak, $R_1 \leq r \leq R_2$. It can be very closed to the interface boundary $r = R_2$. However, it can not be reached to the interface boundary $r = R_2$ for any long time.

![Electric Wave Exx Propagation in Double Layered EM GL-Cloak in XY Plane Using 3D GL Modeling in GLGEO](image)

**FIG. 4:** (color online) At moment $21 dt$, one part of the front of the **First electric wave**, $E_{xx,1}$, propagates enter to the concealment; other part of the front is still propagating inside of inner GL cloak layer, $R_1 \leq r \leq R_2$. One part of front of **Second electric wave**, $E_{xx,2}$, reaches the middle interface boundary $r = R_2$ of the GL cloak; other part of the front has propagated outside of the whole GL double layered cloak and in free space with disturbance. At the time step $48 dt$, the **First electric wave**, $E_{xx,1}$, has propagated through the concealment and whole front is inside of the inner layer of the GL cloak $R_1 \leq r \leq R_2$, its speed is smaller than the light speed. The **Second electric wave** never propagates into the inner layer of GL cloak, The part of the front of **Second electric wave**, $E_{xx,2}$, is inside of outer layer of the GL double layered cloak, $R_2 \leq r \leq R_3$, and being forward bending with speed larger than the light speed. Other part of the front has been propagating in free space with disturbance. The EM wave propagation image is presented in an omitted Figure. In the other omitted Figure, at time step $68 dt$, the **Second electric wave**, $E_{xx,2}$, is propagating in free space and outside of the whole GL double layered cloak, and never penetrate into inner domain, $r < R_2$, i.e. never penetrate into the inner layer of GL cloak and the concealment. The **Second electric wave**, $E_{xx,1}$, is still propagating inside of the inner sphere annular layer of the GL double layered cloak, $R_1 \leq r \leq R_2$. At time step $98 dt$, the **Second electric wave**, $E_{xx,2}$, has been propagating out of whole GL cloak and out of the plot frame. Very small part of its front is located in left and low corner of the plot frame, the figure is omitted. The **First electric wave**, $E_{xx,1}$, is still propagating inside of the inner sphere annular layer of the GL double layered cloak, $R_1 \leq r \leq R_2$. It can be very closed to the interface boundary $r = R_2$. However, it can not be reached to the interface boundary $r = R_2$ for any long time.

### C. The EM Wave Excited By The Point Source In The Inner Layer $\Omega_{GLI}$ And Other Point Source In The Outer Layer $\Omega_{GLO}$ Propagates Through The GL Double Layered Cloak

The 3D EM full wave excited by the point source in the inner layer and other point source in the outer layer of GL double Layered cloak are simulated by using GL EM modeling. The simulations of the EM wave excited by the above sources through GL double layered cloak are presented in the Figures 4. The configuration of the GL double layered cloak material is described in the subsection A of this section. Two point sources are used to excite the EM wave propagation through the GL double layered cloak. The first point current source is located inside of the inner layer of the GL cloak at $(-0.165 m, -0.165 m, 0.0)$, by which the excited EM wave is named as **First EM wave**. Its component $E_{xx,1}$ is named **First electric wave**. The second current point source is located in outer layer GL cloak at $(0.23 m, 0.23 m, 0.0)$. The EM wave by the second source is named as **Second EM wave**. Its component $E_{xx,2}$, is named **Second electric wave**. In the Figure 4, at moment $21 dt$, one part of the front of the **First electric wave**, $E_{xx,1}$, propagates enter to the concealment; other part of the front is still propagating inside of inner GL cloak layer, $R_1 \leq r \leq R_2$. One part of front of **Second electric wave**, $E_{xx,2}$, reaches the middle interface boundary $r = R_2$ of the GL cloak; other part of the front has propagated outside of the whole GL double layered cloak and in free space with disturbance. At the time step $48 dt$, the **First electric wave**, $E_{xx,1}$, has propagated through the concealment and whole front is inside of the inner layer of the GL cloak $R_1 \leq r \leq R_2$, its speed is smaller than the light speed. The **Second electric wave** never propagates into the inner layer of GL cloak, The part of the front of **Second electric wave**, $E_{xx,2}$, is inside of outer layer of the GL double layered cloak, $R_2 \leq r \leq R_3$, and being forward bending with speed larger than the light speed. Other part of the front has been propagating in free space with disturbance. The EM wave propagation image is presented in an omitted Figure. In the other omitted Figure, at time step $68 dt$, the **Second electric wave**, $E_{xx,2}$, is propagating in free space and outside of the whole GL double layered cloak, and never penetrate into inner domain, $r < R_2$, i.e. never penetrate into the inner layer of GL cloak and the concealment. The **Second electric wave**, $E_{xx,1}$, is still propagating inside of the inner sphere annular layer of the GL double layered cloak, $R_1 \leq r \leq R_2$. At time step $98 dt$, the **Second electric wave**, $E_{xx,2}$, has been propagating out of whole GL cloak and out of the plot frame. Very small part of its front is located in left and low corner of the plot frame, the figure is omitted. The **First electric wave**, $E_{xx,1}$, is still propagating inside of the inner sphere annular layer of the GL double layered cloak, $R_1 \leq r \leq R_2$. It can be very closed to the interface boundary $r = R_2$. However, it can not be reached to the interface boundary $r = R_2$ for any long time.

### VII. ADVANTAGES

#### A. The EM GL Double Layered Cloak Is Robust For Invisibility

The figure 4 clearly show that the wave front of the **Second electric wave** has propagated outside of the cloak and go to free space with disturbance. The results reminds us that if only single outer layer cloak $\Omega_{GLO}$ is adopted, and there is a little crack loss on the inner boundary surface $\partial \Omega_{GLO}$, some EM or current source inside of the $\Omega_{GLO}$ will excite the EM wave propagation go out to free space and expose the cloak immediately. The GL double layered cloak overcomes the weakness that is also shown in the figure 4. The wave front of the First electric wave, which is excited by a point source inside of the inner layer cloak $\Omega_{GLI}$, is always propagating inside of inner layer cloak $\Omega_{GLI}$ or concealment $\Omega_{conl}$ and never propagate outside of the interface annular $S_1$. Therefore, the EM GL double layered cloak is robust for invisibility.
B. The EM GL Double Layered Cloak Is Complete Invisible

The figure 1-3 clearly show that the outer layer cloak of the GL double layered cloak has the invisibility function, while the inner layer cloak has fully absorption function to absorb the EM wave excited from local sources inside of the concealment. When the source is located outside of the double cloaked concealment, the excited EM wave field propagation outside of the double layer cloak is as same as in free space and never be disturbed by the cloak; also, the exterior EM wave can not penetrate into the inner layer and concealment of the GL double layered cloak. When local sources are located inside of the GL double cloaked concealment with the normal EM materials, the excited EM wave is propagating under Maxwell equation governing, it is complete and successively absorbed by the inner layer cloak of GL double cloak and never propagate to outside of the inner layer of the GL cloak, moreover, the EM wave field in concealment never be disturbed by the cloak.

Using the GL method theoretical analysis, the statement 2 in section 4 is rigorous proved. It states that “there exists no Maxwell electromagnetic wavefield can be excited by nonzero local sources inside of the single layer cloaked concealment with the normal EM materials”. The invisibility of the single layered cloak and existence of Maxwell EM wave field excited by the local sources inside its concealment is inconsistent. Provide only single outer layered cloak is adopted. The EM field excited by local sources inside of its concealment with normal materials does not satisfy the Maxwell equation. The EM chaos phenomena, which is divorced from the Maxwell equation governing, may damage devices and human inside the concealment, or may degrade the invisibility of the cloak. Invisibility function of the single layered cloak is not complete. The EM GL double layered cloak overcomes the drawback of the single layered cloak. The GL double layered cloak has the complete sufficient invisibility function.

C. The EM GL Double Layered Cloak Can Be Double Ellipsoid Annular

The EM GL double layered cloak can be extended to have double ellipsoid annular and other geometrical double layered closed strips.

D. Frequency Band

Many simulations and theoretical analysis by the GL method show that the idea EM GL double layered cloak is of the invisibility function for all frequencies. However, the practical material has some loss. The frequency band will be depended on the rate of the material loss. Because the EM GL double layered cloak is robust and complete cloak, it has three radius $R_1$, $R_2$, and $R_3$ can be chosen, and GL method reduced numerical frequency limitation in FEM and FD scheme, a reasonable wide frequency band of the GL double layered cloak for low loss rate will be optimization and reported in next paper.

E. Advantages Of The GL Method

The GL EM modeling is fully different from FEM and FD and Born approximation methods and overcome their difficulties. There is no big matrix equation to solve in GL method. Moreover, it does not need artificial boundary and absorption condition to truncate the infinite domain. Born Approximation is a conventional method in the quantum mechanics and solid physics, however, it is one iteration only in whole domain which is not accurate for high frequency and for high contrast materials. The GL method divides the domain as a set of small sub domains or sub lattices. The Global field is updated by the local field from the interaction between the global field and local subdomain materials successively. Once all subdomain materials are scattered, the GL field solution is obtained which is much more accurate than the Born approximation. GL method is suitable for all frequency and high contrast materials.

Moreover, the GL method can be meshless, including arbitrary geometry subdomains, such as rectangle, cylindrical and spherical coordinate mixed coupled together. It is full parallel algorithm. These advantages of the GL method to overcome historical difficulties have been detailed described in the paper [1]. The theoretical foundation of the GL method is described in the paper [2] The GL EM method consistent combines the analytical and numerical approaches together and reduced the numerical dispersion and numerical frequency limitation. The GL method has double abilities of the theoretical analysis and numerical simulations that has been shown in this paper.

The 3D GL simulations of the EM wave field through the single and multiple sphere, cylinder, ellipsoid, and arbitrary geometry cloaks in single layered and double layered profile show that the GLT and GLF EM modeling are accurate, stable and fast. It saves much more storages than the conventional methods. In general, only 10 to 50 minute are needed to run the 3D EM wave field through the cloaks with 64 to 128 frequencies in the PC. The high performance GL parallel algorithm in PC cluster and super parallel computer is very fast and powerful to simulate complex and large scale physical and chemical process.

A double layer cloth phenomenon to prevent the GILD inversion [5-7] detection has been observed in paper [8] in 2001 which is published in SEG online [http://www.segdl.org/journals/doc/SEGLIB-home/dci/searchDCI.jsp]. After the event, we effort improve GILD [5-7] and developed a novel and effective Global and Local field (GL) modeling and inversion to study the meta materials,
periodic photonic crystals and condense physics etc. wide physical sciences. 3D GL EM modeling and inversion [9] and computational mirage [10] have been presented in PIERS 2005 and published in proceeding of PIERS 2005 in Hangzhou, which can be downloaded from http://piers.mit.edu/piersproceedings/piers2k5Proc.php.

We developed 3D FEM for the elastic mechanics first in China in 1972 which has been cited and recorded in [17]. Our 3D FEM paper has been published in [18] in Chinese. We are deeply and clearly to know the merits and drawbacks of FEM and its serious limitation and difficulties to simulate high frequency wave propagation through dispersive materials. The GL method overcome the drawbacks of FEM and FD methods. The history of development of our 3D FEM [18], novel inversion [7], GILD [5] and GL method [1-3] has been described in [2]. The 3D and 2D GL parallel software is made and patented by GLGEO. The GL modeling and its inversion [1-3] and GL EM quantum field modeling are suitable to solve quantization scattering problem of the electromagnetic field in the dispersive and loss metamaterials, cloaks and more wide anisotropic condense materials.

VIII. CONCLUSIONS

Many simulations of the EM wave propagation through the GL double layered cloak by the GL modeling and theoretical analysis verify that the EM GL double cloaked is robust cloak and has complete and sufficient invisibility functions. Its concealment is the normal electromagnetic environment. The outer layer cloak of the GL double layered cloak has the invisible function, the inner layer cloak has fully absorption function. When the source is located outside of the GL double layered cloak, the excited EM wave field propagation outside of the double layered cloak is as same as in free space and never be disturbed by the cloak; also, the exterior EM wave can not penetrate into the inner layer and concealment. When sources are located inside of the GL double cloaked concealment with the normal EM materials, the excited EM wave is propagating under Maxwell equation governing, it is complete absorbed by the inner layer cloak of GL double cloak and never propagate to outside of the inner layer of the GL cloak, moreover, the EM wavefield in concealment never be disturbed by the cloak. The EM GL double layered cloak has advantages to overcome the drawback and difficulty of the single layered cloak.

The GL method is an effective physical simulation method. It has double abilities of the theoretical analysis and numerical simulations to study the cloak metamaterials and wide material and Field scattering in physical sciences.

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