Paraconsistency of Interactive Computation *

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Abstract. The goal of computational logic is to allow us to model computation as well as to reason about it. We argue that a computational logic must be able to model interactive computation. We show that first-order logic cannot model interactive computation due to the incompleteness of interaction. We show that interactive computation is necessarily paraconsistent, able to model both a fact and its negation, due to the role of the world (environment) in determining the course of the computation. We conclude that paraconsistency is a necessary property for a logic that can model interactive computation.

1 Introduction

The paradigm shift from algorithms to interactive computation captures the technology shift from mainframes to workstations and now to wireless devices and intelligent appliances, from number-crunching to embedded systems and graphical user interfaces, and from procedure-oriented to object-based and distributed computation. Interactive computation, modeled by interactive systems, is a more powerful paradigm of problem solving than algorithmic computation, modeled by Turing Machines.

The view that TMs completely express the intuitive notion of computing is a common misinterpretation of Church-Turing Thesis. This view has led computer science theorists to assume that questions of expressiveness of finite computing agents had been settled once and for all, so that exploration of alternative models of computability has seemed unnecessary. The claim that interactive computing agents are more expressive than Turing Machines, meaning able to solve a larger set of computational problems, requires a reexamination of fundamental assumptions about models of computation, about computation itself, and about computational logics.

In section 3 we introduce interactive computation and demonstrate that it is more powerful than algorithmic computation of Turing Machines, in that it can solve a wider class of computational problems. In section 3, we show that

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classical first-order logic cannot model interactive computation. In section 4, we examine areas of computing where the principle of consistency plays a major role; in each case, these areas are fundamentally not interactive. Finally, in section 5, we consider embedding logic models in the broader context of interactive models, and conclude that paraconsistency is necessary to model interaction.

2 The power of interactive computation

In this section, we introduce interactive computation and demonstrate that it is more powerful than the algorithmic computation of the Turing Machines, in that it can solve a wider class of computational problems.

2.1 Interactive computation

The notion of algorithms well precedes computer science, having been a concern of mathematicians for centuries. It can be dated back to a 9th century treatise by a Muslim mathematician Al-Koarizmi, after whom algorithms were named.

Algorithm: systematic procedure that produces – in a finite number of steps – the answer to a question or the solution of a problem [2].

Algorithmic computation, modeled by Turing Machines, was adopted as the traditional model of computation when computer science emerged as a new academic discipline in the 1960s.

Algorithmic computation: the computation is performed in a closed-box fashion, transforming a finite input, determined by the start of the computation, to a finite output, available at the end of the computation, in a finite amount of time.

Whereas algorithmic computation is like the data processing mode associated with punch cards of the 1950s, today’s computational systems are interactive. In these systems, which are expected to remain up and running continuously, input is consumed and output is generated throughout the computation. More importantly, there is an interdependence of input and output: output can influence later input, and vice versa.

2.2 The WH problem

Let us consider an automatic car whose task is to drive us across town from point W (work) to point H (home); we shall refer to it as the WH problem. The output for this problem should be a time-series plot of signals to the car’s controls that enable it to perform this task autonomously. At issue is what form the inputs should take.

The car can be equipped with a map of the city. In the algorithmic scenario, where all inputs are provided a priori of computation, this map needs to include
every pothole and even grain of sand along the road. By the principles of chaotic behavior, such elements can greatly affect the car’s eventual course – like the Japanese butterfly that causes a tsunami on the other end of the world.

In a static world, such a map is in principle obtainable; but ours is a dynamic environment. The presence of mutable physical elements such as the weather affect the car’s course both directly (e.g., the wind blowing at the car) and indirectly (e.g. sand shifting location in the path of the car as a result of blowing wind). It is arguable whether the elements of the weather can be precomputed to an accuracy required to produce the proper answer to the WH problem.

We can remain optimistic until we remember that the world also includes humans, as pedestrians or drivers. To avoid collisions, we must precompute the motion of everyone who might be near the car as it drives. An assumption that human actions can be computed ahead of time is tantamount to an assertion of fatalism – a doctrine that events are fixed in advance so that human beings are powerless to change them – clearly beyond the purview of any computer scientist. Therefore, we must assume it cannot be done.

The claim that computational tasks situated in a dynamic world that includes human agents are solvable algorithmically is tantamount to a assertion of fatalism.

2.3 The power of interaction

Nevertheless, the WH problem is solvable – interactively. In this scenario, the inputs, or percepts, consist of a stream of images produced by a video camera mounted on the car, as it is driving from W to H. The signals to the car’s controls are generated on-line in response to these images, to avoid steering off the road or running into obstacles. This change in the scenario is akin to taking the blindfolds off the car’s electronic driver, who was driving from memory and bound to a precomputed sequence of actions. Now, he is aware of his environment and uses it to guide his steering.

Note that a change of approach from algorithmic to interactive such as above involves much more than a restructuring of inputs. The nature of computation itself is at stake. Algorithmic computation is off-line; it takes place before the driving begins. Interactive computation is on-line; it takes place as the car drives. The values of inputs and outputs for interactive computation are interdependent; decoupling them, such as by replacing the videocamera with a prerecorded videotape of the road, will usually lead to an incorrect solution.

This interpretation of the concept of computation is different from the one that Church and Turing had in mind for their famous thesis, where they specifically referred to computation of functions over finite strings that encode natural numbers (i.e., algorithmic computation). Interactive computation falls outside the bounds of the Church Turing thesis. The WH example shows that interaction pulls us out of the Turing tarpit, by providing solutions to problems that are not solvable in the algorithmic setting.
By allowing us to solve computational tasks that cannot be solved algorithmically, interaction is shown to be a more powerful computational paradigm.

3 Logic and interactive computation

In this section, we discuss why classical first-order logic cannot model interactive computation.

3.1 Tradeoffs between reasoning and modeling

Logic comprises both syntactic rules for noninteractively (in a closed-box fashion) proving theorems from axioms by rules of inference (proof theory) and semantic notions of soundness and completeness that relate syntactic processes of proof to semantic properties of a modeled domain in which formulae are interpreted (model theory). A logic is sound if all its theorems are true, and complete if all true assertions of the modeled domain are theorems.

In sound and complete formal systems such as first-order logic, every true assertion about the domain has a finite proof that is constructed by applying the rules to the axioms. With the number of rules and axioms also finite, the resulting set of assertions must be recursively enumerable (RE).

**Necessary condition for completeness**: A sound and complete first-order logic (SCL) can model only domains with a countable set of properties.

Completeness is a desirable mathematical property that certifies formalizability of the modeled domain but by the same token restricts expressiveness. We may wish to model classes of systems (mathematical, computational, or physical) which have a non-RE set of properties. Modeling is observational rather than constructive, based on greatest fixpoints rather than least fixpoints. The gap between least- and greatest-fixpoint semantics is also the gap between operational (algorithm) and denotational (observation) semantics. It is also the same as the gap between deduction and abduction.

Greatest fixpoints allow us to define larger domains. Lacking a constructive foundation, we cannot systematically enumerate the members of such domains. However, it can be observed whenever two members are distinct; distinguishability certificates are always finite. The bisimulation relation for labeled transition systems is an example of a relation with greatest fixpoint semantics and finite distinguishability certificates.

**Sufficient condition for incompleteness**: Any system with an uncountable set of properties cannot be expressed by a sound and complete logic.
As a result, there are tradeoffs between reasoning and modeling: purely rule-based syntactic reasoning is too weak to capture all semantic properties of mathematical, computational, or physical systems. Model builders must choose between complete formalizability and modeling power. Turing Machines focus on formalizability at the expense of modeling power.

(As an aside, this focus on formalizability is intentional; TMs were invented as part of Turing’s proof that Hilbert’s challenge to formalize mathematics cannot be carried out. The halting problem presented a constructive counterexample to this challenge [6]. Turing himself never claimed that TMs formalize all of computation, and in fact made claims that suggested the opposite [11].)

3.2 Incompleteness of interactive systems

Gödel showed that arithmetic over the integers cannot be completely formalized by logic, because not all its properties are syntactically expressible as theorems. We extend Gödel’s reasoning to show that interactive and empirical systems are likewise incomplete because they have too many properties to be expressible as theorems of a sound and complete logic.

As discussed above, sound and complete first-order logics (SCLs) have an RE set of theorems and can formalize only semantic domains with a countable number of distinct properties. Incompleteness occurs when the number of true facts or observable properties of a system cannot be recursively enumerated by theorems. For example, arithmetic over the integers is incomplete because its true properties cannot be recursively enumerated; that is, for any sound formalization we can find a true property of the integers that cannot be proved.

For interactive systems or interaction machines (IMs), properties correspond to observations, and the number of properties corresponds to the number of distinct observations. Algorithms and TMs have only a countable set of properties, while IMs have an open-ended set of properties that is generally not countable and therefore cannot be expressed as theorems of a complete first-order logic. Gödel’s incompleteness result holds for any domain whose true properties cannot be recursively enumerated, including empirical domains modeled by IMs.

Expressiveness for IMs is defined in terms of the ability of observers to make observational distinctions [8]. Observational expressiveness allows behavior of algorithms and IMs to be measured by the same metric. The class of systems or programs which admits finer observational distinctions is the one capable of a greater range of behaviors, and hence able to express solutions to a larger set of problems. From this viewpoint, TM behavior is completely characterized by a single observation, while IM behavior may depend on unbounded sequences or patterns of observations. This gives another proof of greater expressive power of interaction.

Completeness restricts the expressiveness of domains modeled by an SCL: domains whose properties are not countable are necessarily incomplete. IMs, based on infinite streams of inputs and outputs created in partnership with an (uncomputable) environment, have an uncountable set of computations: the distinct streams of an IM are not enumerable.
By viewing Godel’s incompleteness result as a corollary of the more general result that SCLs cannot model systems with an uncountable set of properties we gain insights into the nature of incompleteness. Incompleteness is seen to be a ubiquitous phenomenon that applies not only to mathematical systems like the integers but also to IMs.

\[
\text{The set of behaviors of an interactive system cannot in general be formalized by any sound and complete logic.}
\]

Incompleteness shows the limitations of syntactic formalisms in expressing semantic behavior. The completeness/incompleteness dichotomy distinguishes algorithms from IMs, closed from open systems, and rationalist from empiricist models [9]. Formal incompleteness of computing systems is related to descriptive incompleteness of physical models of the real world.

Interactive problem solving (section 2.2) is a first-class form of computation that should be included in any intuitive notion of computability. Hence, logics need to go beyond soundness and completeness to capture computation.

### 3.3 Logic for physical theories

The relatively recent development of \textit{paraconsistent logic} challenges the logical principle that contradictory premises lead to an explosion of meaningless conclusions.

\textbf{Paraconsistency:} Let $\Rightarrow$ be a relation of logical consequence. Let us say that $\Rightarrow$ is explosive iff for every formula $A$ and $B$, $\{A, \neg A\} \Rightarrow B$. Classical logic and most other standard logics are explosive. A logic is said to be \textit{paraconsistent} iff its relation of logical consequence is not explosive [4].

The most telling reason for paraconsistent logic is the fact that there are non-trivial theories which are inconsistent. Some of the best examples of such theories come from the history of physics.

The interactive view of computing allows us to bridge the gap between closed-box algorithmic computing and real physical systems. Interactive computation spans the range from closed-box algorithmic computation at one end to situated environment-aware computing agents (e.g. the car in the \textit{WH} problem) in the middle, to the behavior of real world objects at the other extremum.

The notion of \textit{observation} as a metric of expressiveness and equivalence for IMs is a natural extension of the role of observation in the formulation of physical theories [9]. The idea that physical objects are not completely describable or knowable but that they may have describable parts or views is a basic tenet of the scientific method. An analogous idea for IMs is that \textit{there is no silver bullet} – when testing a software system, one can never be sure of having found all the bugs.

\textit{Just as physical theories provide an important motivation for paraconsistent logics, so does the theory of interactive computation.}
4 Consistency in computation

In this section, we examine some of the areas of computing which have been founded on the principle of consistency, i.e., that it is impossible for $A$ and $\neg A$ to coexist. We show that in each case, these areas are fundamentally not interactive.

4.1 Logic programming

Logic programming corresponds to closed-system computing, whereas interactive systems are inherently open. The Japanese 5th generation computing project that aimed to reduce computation to logic programming proved a failure. It was shown in [7] that this was due not to lack of cleverness on the part of logic programming researchers, but to the theoretical impossibility of such a reduction.

The key argument is the inherent incompatibility between reactivity of interactive computation, realized by committed choice at each interaction step, and logical completeness, realized by backtracking. Committed choice is inherently incomplete because commitment cuts off branches of the proof tree that might contain the solution. However, commitment is essential to interactive computation, since execution of an observable action by a system cannot be reversed. The power of interaction can be achieved only by sacrificing logical completeness. Pure logic programming is inherently too weak to serve as a model for interactive computation.

4.2 Data Management

Despite significant overlaps, the research areas of databases (DB) and information systems (IS) are based on fundamentally different assumptions, due to the different jobs they perform. The “job of a database” is to store data and answer queries. This entails addressing issues like data models, schema design, handling distributed data, maintaining data consistency, query evaluation, etc.

Given a query and a database instance, the behavior of a database system consists of answering queries, which is algorithmic in nature. In particular, a database management system implements an algorithmic mapping of the form:

$$(DB \text{ contents, user query}) \rightarrow (DB \text{ contents’, query answer}).$$

On the other hand, the job of an IS is to provide information-based services, which entails considerations that span the life cycle of the larger system. Services over time are interactive in nature, involving user sessions with one or more users.

It is not surprising then that consistency is taken for granted in database research, given its algorithmic nature. However, most large contemporary database systems are to some degree IS’s, providing additional services beyond simply storing data and running queries and updates. When lines between the two are blurred in this fashion, the tension between consistency and interaction can result.
4.3 Artificial Intelligence

Classical logic has always played a prominent role in Artificial Intelligence (AI), where a typical problem would be to form a plan or to solve a problem, given a complete description of the “world” in question. Closed-box computation was the modus operandi, even in problems like rearranging blocks in Shakey’s world \[5\] that were meant to offer a simplified version of situated real-world behavior.

Many researchers came to realize that algorithmic approaches cannot scale up to solutions for tasks involving the real world \[1\], like the WH problem; a paradigm shift was needed. The new agent-based approach to AI \[5\] is interactive. The change in approach has reinvigorated this field, breaking the inertia that gripped it in the ’80s, and restoring our optimism in the promises that AI was making when originally founded.

Not surprisingly, classical logic does not play the same prominent role in the new AI. The emphasis has shifted to probabilistic approaches, with such models as neural networks, hidden Markov models and belief networks \[5\]. We expect that paraconsistent logics can also play a significant role in the new AI.

5 Extending logic to interaction

By embedding logic models in the broader context of interactive models, we gain insight into the limits of logic in modeling autonomous external worlds. We also understand why paraconsistency is necessary to model interaction.

The number of interpretations of function and predicate symbols of first-order logic (\(\text{fol}\)) is nonenumerable. However, the set of theorems for a fixed interpretation is recursively enumerable (section 3.1). By assuming a static interpretation of the world that is fixed prior to the start of the computation, algorithmic models have a recursively enumerable set of behaviors. By contrast, the nonenumerability of interpretations of \(\text{fol}\)s reflects the nonenumerable external worlds (environments) of interactive computation, which are neither static nor known \textit{a priori}. Interactive models replace complete semantics of \(\text{fol}\)s by weaker abstractions that sacrifice completeness for expressiveness.

Conjecture: Late (dynamic) binding of interpretations in logic is a form of interactive modeling.

Interaction extends models so that the interpretation of nonlogical symbols evolves during inference to take account of new data. However, interactive discovery of facts negates the fundamental property that true facts always remain true. \textit{Nonmonotonic logics} permit reasoning about incompletely described modeled worlds. As the system gains information that contradicts earlier inferences (beliefs), it may retract these inferences.

The contrast between the constructive paradigm of algorithmic models and the observational paradigm of interaction is the contrast between deduction and abduction (section 3.1). Abduction is open-ended and subject to revision; abductive reasoning is \textit{paraconsistent} in that it is possible for incompatible inferences to be drawn from the same premises.
Abduction and nonmonotonicity are not only desirable, but required for interactive computation, where input, inference, and action are interleaved. Paraconsistency, the embodying of contradictory information, is necessary for abductive and nonmonotonic reasoning, and hence for interaction.

6 Conclusion

The goal of computational logic is to allow us to model computation as well as to reason about it. A useful computational logic must be able to model interactive computation. We showed that first-order logic cannot model interactive computation due to its incompleteness. Interactive computation is necessarily paraconsistent, able to model both a fact and its negation, due to the role of the world (environment) in determining the course of the computation. We noted that interactive computation shares much in common with scientific theories, providing a common motivation for the development of paraconsistent logic. We concluded that paraconsistency is a necessary property for a logic that can model interactive computation.

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References

1. Rodney A. Brooks. Intelligence Without Reason. MIT A.I. Memo, 1991. [http://www.ai.mit.edu/people/brooks](http://www.ai.mit.edu/people/brooks)
2. Encyclopaedia Britannica.
3. Dina Goldin, Srinath Srinivasa, Bernhard Thalheim. Information Systems = Databases + Interaction: Towards Principles of Information System Design. ER2000, Salt Lake City, October 2000.
4. Graham Priest, Koji Tanaka. Paraconsistent Logic. Stanford Encyclopedia of Philosophy. 2000. [http://plato.stanford.edu](http://plato.stanford.edu)
5. Stuart Russell, Peter Norvig. Artificial Intelligence: A Modern Approach. Addison-Wesley, 1995.
6. Alan Turing. On Computable Numbers with an Application to the Entscheidungsproblem. Proceedings of the London Mathematical Society, 2(42), pp. 173-198, 1936.
7. Peter Wegner. Tradeoffs Between Reasoning and Modeling. In Research Directions in Concurrent Object-Oriented Computing, Agha, Wegner, Yonezawa (Eds.). MIT Press, 1993.
8. Peter Wegner. Interactive Foundations of Computing. Theoretical Computer Science, February 1998.
9. Peter Wegner. Towards Empirical Computer Science. The Monist, Issue on the Philosophy of Computation, January 1999.
10. Peter Wegner and Dina Goldin. Coinductive Models of Finite Computing Agents. Proc. Coalgebra Workshop (CMCS ’99). *Electronic Notes in Theoretical Computer Science*, Volume 19, March 1999.

11. Peter Wegner and Dina Goldin. *Computation Beyond Turing Machines*. 2002. Accepted for publication.