Empirical traffic data and their implications for traffic modeling

Dirk Helbing

II. Institute of Theoretical Physics, University of Stuttgart, Pfaffenwaldring 57/III, 70550

Stuttgart, Germany

Abstract

From single vehicle data a number of new empirical results about the temporal evolution, correlation, and density-dependence of macroscopic traffic quantities have been determined. These have relevant implications for traffic modeling and allow to test existing traffic models.
With the aim of optimizing traffic flow and improving today’s traffic situation, several models for freeway traffic have been proposed, microscopic \[1,3\] and macroscopic ones \[4,14\]. Only some of them have been systematically derived from the underlying laws of individual vehicle dynamics \[11,16\]. Most models are phenomenological in nature \[4,9\]. These base on various assumptions, the correctness of which has not been carefully discussed up to now, mainly due to a lack of empirical data or difficulties to obtain them. Therefore, this paper presents some fundamental empirical observations which allow to test some of the models.

The empirical relations have been evaluated from single vehicle data of both lanes of the Dutch freeway A9 between Haarlem and Amsterdam (cf. Fig. 1) \[17\]. These data were detected by induction loops at discrete places \(x\) below the lanes \(i\) of the roadway and include the passage times \(t_\alpha(x, i)\), velocities \(v_\alpha(x, i)\) and lengths \(l_\alpha(x, i)\) of the single vehicles \(\alpha\). Consequently, it was possible to calculate the number \(N_i(x, t)\) of vehicles on lane \(i\) which passed the cross section at place \(x\) during a time interval \([t, t + T]\), the traffic flow

\[
Q_i(x, t) := N_i(x, t)/T, \tag{1}
\]

and the macroscopic velocity moments

\[
\langle v^k \rangle_i := \frac{1}{N_i(x, t)} \sum_{t_\alpha < t < t + T} [v_\alpha(x, i)]^k. \tag{2}
\]

If nothing else is mentioned, the interval length was chosen \(T = 5\) min, since this allowed to separate the systematic temporal evolution of the macroscopic traffic quantities from their statistical fluctuations \[10\]. The vehicle densities \(\rho_i(x, t)\) were calculated via the flow formula

\[
Q_i(x, t) = \rho_i(x, t)V_i(x, t). \tag{3}
\]

A detailed comparison with other available methods for the determination of the average velocities \(V_i := \langle v \rangle_i\) and the vehicle densities \(\rho_i\) from single vehicle data will be given in \[16\].

Finally, the lane averages of the above quantities were defined according to

\[
Q(x, t) := \frac{1}{T} \sum_i N_i(x, t) = \frac{1}{T} \sum_i Q_i(x, t), \tag{4}
\]
\[ \langle v^k \rangle := \frac{1}{\sum_j N_j(x, t)} \sum_i \sum_{t \leq t_i < t+T} [v_{i}(x, t)]^k, \]  

(5)

and

\[ \rho(x, t) := \frac{Q(x, t)}{V(x, t)}, \]  

(6)

where \( I \) denotes the number of lanes and \( V(x, t) := \langle v \rangle \). Therefore, we have the following relation:

\[ \langle v^k \rangle = \sum_i \frac{N_i(x, t)}{\sum_j N_j(x, t)} \langle v^k \rangle_i. \]  

(7)

For reasons of simplicity, most macroscopic traffic models describe the dynamics of the total cross section of the road in an overall manner by equations for the density \( \rho \) and the average velocity \( V \). However, one would expect that a realistic description requires a model of the traffic dynamics on the single lanes and their mutual coupling due to overtaking and lane-chaning maneuvers \[13,16\]. This could cause a more complex dynamics like density oscillations among the lanes \[18\].

In order to check this, we will investigate the correlation between neighboring lanes. Figure 2 shows that the temporal course of the densities \( \rho_1(x, t) \) and \( \rho_2(x, t) \) is almost parallel. A similar thing holds for the average velocities \( V_1(x, t) \) and \( V_2(x, t) \) \[19\]. The difference between the curves is mainly a function of density (cf. Figure 3): At small densities, the vehicles can move faster on the left lane than on the right one, whereas at high densities the left lane is more crowded than the right one. In addition, Figure 4 shows that the variances

\[ \theta_i(x, t) := \langle (v - V_i)^2 \rangle_i = \langle v^2 \rangle_i - \langle v \rangle_i^2 \]  

(8)

behave almost identically on the neighboring lanes (although the order of magnitude of the average velocities \( V_i(x, t) \) changes considerably). This strong correlation between neighboring lanes probably arises from overtaking and lane-changing maneuvers. It justifies the common practice to describe the dynamics of the total cross section of the road in an overall manner.
Now we face the question, how a realistic traffic model must look like. Due to the conservation of the number of vehicles, the dynamics of the vehicle density is given by the continuity equation

$$\frac{\partial \rho(x, t)}{\partial t} + \frac{\partial Q(x, t)}{\partial x} = \nu^+(x, t) - \nu^-(x, t), \quad (9)$$

where $\nu^+(x, t)$ and $\nu^-(x, t)$ are the rates of vehicles which enter or leave the freeway at on- and off-ramps, respectively. Lighthill, Whitham and Richards have suggested to specify the flow $Q(x, t)$ in accordance with an empirical flow-density relation $Q_e(\rho)$ [4,5]:

$$Q(x, t) = Q_e(\rho(x, t)). \quad (10)$$

This relation has been called into question, since the resulting model cannot describe the emergence of phantom traffic jams or stop-and-go traffic [10,16]. Therefore, some researchers have introduced an additional dynamical equation for the average velocity $V(x, t)$ which allows to describe instabilities of traffic flow [7–10,14]. However, others have interpreted these phenomena as effects of fluctuations or of phantom bottlenecks caused by slow, overtaking vehicles like trucks [20]. Hence we check relation (10) in Figure 5. It is found that (10) becomes invalid above a density of about 12 vehicles per kilometer and lane, where a hysteresis effect occurs [21]. This indicates a transition from stable to unstable traffic flow.

An empirical proof of emerging stop-and-go traffic is presented in Figure 6. During the rush hours between 7:30 am and 9:30 am, average velocity breaks down at place $x = 41.75$ km because of the on-ramp at $x = 41.3$ km. Nevertheless, the traffic situation recovers at the successive cross sections, i.e. average velocity increases again. In spite of this, the initially small velocity oscillations at $x = 41.75$ km grow considerably in the course of the road. This corresponds to emerging stop-and-go traffic (i.e. alternating periods of acceleration and deceleration). At the same time, the wavelength of the oscillation increases. This is in good agreement with computer simulations which show a merging of density clusters leading to larger wave lengths [8].

After we have found that we need a dynamic velocity equation for an adequate description
of the spatio-temporal evolution of traffic flow, we have to clear up the question, whether we also need a dynamic equation for the variance

$$\Theta := \langle (v - V)^2 \rangle = \theta + \langle (V_i - V)^2 \rangle $$

or not. Theoretical considerations on the basis of gas-kinetic approaches have shown that the velocity equation depends on the variance, for which a separate equation can be derived \[11-16\]. Nevertheless we will try out the equilibrium approximation

$$\Theta(x, t) = \Theta_e(\rho(x, t)),$$

where $\Theta_e(\rho)$ is the empirical variance-density relation (cf. Figure 7). Figure 8 shows that this approximation fits the temporal evolution of the variance in a satisfactory way as long as the average velocity $V$ does not rapidly change. However, when the velocity breaks down or increases, the variance shows mysterious peaks. These are a consequence of having built the temporal averages

$$\langle v^k \rangle(x, t) \equiv \langle v^k \rangle_a(x, t) := \frac{1}{T} \int_{t'}^{t+T} dt' \langle v^k \rangle_a(x, t')$$

over finite time intervals $T$, where $\langle v^k \rangle_a(x, t) := \int dv v^k P_a(v; x, t)$ with the actual velocity distribution $P_a(v; x, t)$. In linear Taylor approximation we find

$$V(x, t) \approx \frac{1}{T} \int_{t'}^{t+T} dt' \left[ V_a(x, t) + \frac{\partial V_a(x, t)}{\partial t} (t' - t) \right]$$

$$= V_a(x, t) + \frac{T}{2} \frac{\partial V_a(x, t)}{\partial t} .$$

(14)

Since $\partial V_a / \partial t$ is varying around zero, the measured value $V$ fluctuates around the actual value $V_a(x, t) := \langle v \rangle_a(x, t)$. For the variance we find

$$\Theta \equiv \langle (v - V)^2 \rangle_a = \Theta_a + [V_a - V]^2$$

$$= \Theta_a + \frac{T^2}{4} \left( \frac{\partial V_a}{\partial t} \right)^2 .$$

(15)

Therefore, time averaging leads to a positive correction term which becomes particularly large, where the average velocity changes rapidly, but vanishes in the limit $T \to 0$. This
correction term describes the variance peaks in Figure 8 quite well. Consequently, the dynamics of the variance can be reconstructed from the dynamics of the vehicle density $\rho(x,t)$ and the average velocity $V(x,t)$.

Summarizing our results, we were able to demonstrate the following by empirical data:

1. The dynamics of neighboring lanes is strongly correlated so that the total freeway cross section can be described in an overall way.
2. There is a transition from stable to unstable traffic flow at a critical density $\rho_{cr}$ of about 12 vehicles per kilometer and lane.
3. Emergent stop-and-go traffic exists, so that a realistic traffic model must contain a dynamic velocity equation.
4. The variance can be well approximated by an equilibrium relation, if corrections due to time averaging are taken into account. These conclusions seem to be also valid for other stretches of freeway systems, at least European ones.

The empirical findings question the fluid-dynamic model by Lighthill, Whitham and Richards [4,5]. They are in favour of the phenomenological models by Payne [6], Phillips [11], Kühne [7], Kerner and Konhäuser [8], Hilliges [9] as well as a recent model by Helbing [14,16] which has been systematically derived from the microscopic vehicle dynamics via a gas-kinetic level of description. The last of these models fits the instability region best, in particular the surprisingly low critical density $\rho_{cr}$ [14,16].

ACKNOWLEDGMENTS

The author is grateful to Henk Taale and the Ministry of Transport, Public Works and Water Management for supplying the freeway data.
REFERENCES

[1] K. Nagel and M. Schreckenberg, J. Physique I France 2, 2221 (1992); K. Nagel and S. Rasmussen, in Artificial Life IV, edited by R. A. Brooks and P. Maes (MIT Press, Cambridge, MA, 1994); M. Schreckenberg, A. Schadschneider, K. Nagel, and N. Ito, Phys. Rev. E 51, 2939 (1995); K. Nagel, Phys. Rev. E 53, 4655 (1996).

[2] M. Bando, K. Hasebe, A. Nakayama, A. Shibata, and Y. Sugiyama, Phys. Rev. E 51, 1035 (1995).

[3] T. Nagatani, J. Phys. A 28, L119 (1995); T. Nagatani, Physica A 223, 137 (1996).

[4] M. J. Lighthill and G. B. Whitham, Proc. R. Soc. A 229, 317 (1955).

[5] P. I. Richards, Oper. Res. 4, 42 (1956).

[6] H. J. Payne, in Mathematical Models of Public Systems, edited by G. A. Bekey (Simulation Council, La Jolla, CA, 1971), Vol. 1.

[7] R. D. Kühne, in Proceedings of the 9th International Symposium on Transportation and Traffic Theory, edited by I. Volmuller and R. Hamerslag (VNU Science, Utrecht, 1984).

[8] B. S. Kerner and P. Konhäuser, Phys. Rev. E 48, R2335 (1993); B. S. Kerner and P. Konhäuser, Phys. Rev. E 50, 54 (1994).

[9] M. Hilliges and W. Weidlich, Transportation Research 29, 407 (1995).

[10] D. Helbing, Phys. Rev. E 51, 3164 (1995).

[11] W. F. Phillips, Transportation Planning and Technology 5, 131 (1979).

[12] D. Helbing, Phys. Rev. E 53, 2366 (1996).

[13] D. Helbing, in Traffic and Granular Flow, edited by D. E. Wolf, M. Schreckenberg, and A. Bachem (World Scientific, Singapore, 1996); D. Helbing and A. Greiner, Modeling and simulation of multi-lane traffic flow, Phys. Rev. E, submitted (1996).
[14] D. Helbing, Derivation and empirical validation of a refined traffic flow model, Physica A, in print (1996).

[15] C. Wagner et al., Second order continuum traffic flow model, Phys. Rev. E, submitted (1996).

[16] D. Helbing, Verkehrsdynamik. Neue physikalische Modellierungskonzepte (Springer, Berlin, in preparation).

[17] Note that the situation on European freeways is somewhat different from American ones due to other legal regulations: Because of the higher speed limit (if there is any), overtaking is only allowed on the left-hand lane. Therefore, trucks mainly use the right lane, on which the average velocity is lower (cf. Fig. 3).

[18] D. C. Gazis, R. Herman, and G. H. Weiss, Oper. Res. 10, 658.

[19] Note that a synchronized state of traffic in the sense of B. S. Kerner and H. Rehborn [Phys. Rev. E 53, R4275] (i.e. with the same velocities $V_i(x, t)$ on all lanes $i$) only occurs above an average density $\rho$ of about 35 vehicles per kilometer and lane (cf. Fig. 3).

[20] D. C. Gazis and R. Herman, Transpn. Sci. 26, 223 (1992).

[21] Similar results have been obtained by J. Treiterer et al. for American freeways [in Proceedings of the 6th International Symposium on Transportation and Traffic Theory, edited by D. Buckley (Sydney, 1974)].
FIGURES

Rottepolderplein  S 17  Badhoevedorp

$\frac{1}{4} \times 2$

43.31 km  42.25  41.75  40.80  39.60  37.60  36.90  36.60  35.89 km

FIG. 1. The investigated stretch of the Dutch two-lane freeway A9 from Haarlem to Amsterdam including on- and off-ramps. Detectors are indicated by vertical lines. The detector at $x = 41.3$ km (---) only evaluates on-ramp traffic and a bus lane. Between $x = 40.8$ km and $x = 37.6$ km traffic flow is not disturbed over more than three kilometers. The speed limit is 120 km/h.

FIG. 2. The temporal course of the average velocities $V_i$ (---: right lane; - - -: left lane) and the vehicle densities $\rho_i$ (---: right lane; · · ·: left lane) on October 14, 1994 at $x = 41.75$ km.
FIG. 3. Average differences between the vehicle densities (▼) and the average velocities (+) on both lanes of the Dutch freeway A9 on November 2, 1994 (\(T = 1\) min).

FIG. 4. The temporal course of the average velocities \(V_i\) (—: right lane; - - -: left lane) and the standard deviations \(\sqrt{\theta_i}\) of vehicle velocities (— —: right lane; · · · : left lane) on November 2, 1994 at \(x = 41.75\) km.
FIG. 5. Comparison of the fundamental diagram $Q_e(\rho)$ (i.e. the average flow-density relation, $\cdots$) with the temporal evolution of traffic flow $Q(x, t) = \rho(x, t)V(x, t)$ (---) at $x = 41.75$ km.

FIG. 6. Temporal evolution of the average velocity $V(x, t)$ at successive cross sections of the freeway on October 14, 1994 ($\cdots$: $x = 41.75$ km; $\cdots$: $x = 40.8$ km; $\cdots$: $x = 39.6$ km; $\cdots$: $x = 37.6$ km).
FIG. 7. Average density-dependence of the standard deviation $\sqrt{\Theta_e}$ of vehicle velocities (for $T = 1\text{ min}, \diamondsuit$) and corresponding fit function (—).

FIG. 8. Temporal evolution of the standard deviation $\sqrt{\Theta(x,t)}$ of vehicle velocities (—) in comparison with the equilibrium approximation $\sqrt{\Theta_e(\rho(x,t))}$ (· · ·). Obviously, the empirical variance $\Theta(x,t)$ has peaks where the average velocity $V(x,t)$ (---) changes considerably. In order to describe these, a correction term due to time averaging must be added (—). (Data from November 2, 1994 at $x = 41.75\text{ km}$)