The Research of Hybrid Bond Graph Modeling

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Abstract. Hybrid powertrain is a form of energy coupling. Involve two or more forms of energy. Using bond graph analysis the vehicle and engine, motor, batteries and other components. Using bond graph analysis the vehicle and the components of engine, motor, batteries, etc. Research the bond graph modeling of the return flow stepless variable speed drive system of hybrid electric vehicle drive system convenient for the simulation analysis.

1. Introduction
Bond graph can use graphics show the dynamic performance of the vehicle system, deal with a variety of modeling and analysis the energy system, reflect the internal information system and power flow. The structure and layout of hybrid electric vehicle as figure 1.

2. Assembly Modeling
Transmission system can be divided into powertrain, return flow stepless variable speed drive system and driving system. Powertrain modeling includes the power of the engine and motor modeling. Use Me and Mm respectively represents the engine and motor torque, I represents the energy storage and release of the inertial components. R represents the loss of torque.
Return flow stepless variable speed drive system modeling include the clutch, gear type power coupling device, metal belt continuously variable transmission device, constant speed ratio gear and Single row planetary gear mechanism. Use C represents the energy storage and release of the torsion deformation, Is, Ir and Ic represents the energy storage and release of the sun wheel, ring gear and planet carrier, a represents the structure parameters of the planetary gear train.

Figure 2 Bond Graph Model of the Engine
Figure 3 Bond Graph Model of the Motor

Figure 4 Bond Graph Model of the Clutch
Figure 5 Bond Graph Model of the Gear Type Coupling Device

Figure 6 Bond Graph Model of the Metal Belt Continuously Variable Transmission

Figure 7 Bond Graph Model of the Constant Speed Ratio Gear

Figure 8 Bond Graph Model of the Single Planetary Gear Mechanism
Rear transmission system modeling including the bond graph model of the main reducer assembly, differential assembly, driver shaft, ride wheels and the vehicle body.

![Bond Graph Model of the Main Reducer and Differential Assembly](image9)

Figure 9  Bond Graph Model of the Main Reducer and Differential Assembly

![Bond Graph Model of the Driver Shaft](image10)

Figure 10  Bond Graph Model of the Driver Shaft

![Bond Graph Model of the Ride Wheel](image11)

Figure 11  Bond Graph Model of the Ride Wheel

![Bond Graph Model of the Vehicle Body](image12)

Figure 12  Bond Graph Model of the Vehicle Body

3. Bond Graph Model

Together the bond graph model of the assembly, give each key a number, use the arrow represents the flow of the power. We can establish the vehicle transmission system under low gear conditions of bond graph model. As shown in figure 12.
The parameters and the subscript in the model are as follows:

TF represents convertor, I represents inertial element, C represents Capacitive component, R represents impedance element, Me represents engine torque, Mm represents motor torque, Mf represents rolling resistance moment, Fw represents wind resistance, $i_3$ represents Speed ratio of motor reduction gear, $i$ represents Metal band speed ratio, $i_0$ represents main transmission ratio, $i_f$, $i_{f_2}$ represents the speed ratio of the two-stage speed ratio gear ratio. The number of 1~65 represents bond key.

The subscript of variable parameters in the model: e represents engine; m represents motor/generator; 0 and 3 represent the active plate and driven plate of the Clutch L0; 1 and 2 represent the coupler of the driving gear and driven gear; 4and 17 represent the driving pulley and driven pulley; 5and 6 represent the active plate and driven plate of the Clutch L1; 7, 9 and 10 represent the driving wheel, idler wheel and driven wheel of the constant speed ratio gear; 8 represents brake B; 11 and 13 represent the active plate and driven plate of the Clutch L2; 12 and 14 represent the active plate and driven plate of the Clutch L4; 15, 16, 20 represent the planets, sun wheel, gear ring of the planet gear mechanism; 18 and 21 represent the active plate and driven plate of the Clutch L3; 19 represent intermediate shaft; 22 and 23 represent the driving wheel and driven wheel of the main reducer; 24 and 25 represent semiaxis; 26 and 27 represent tire; 28 represents body.

4. Mathematical Model

According to the bond graph model in figure 13, the transmission system state variables are:

$$ X = \left[ P_2, P_7, Q_{15}, P_{17}, Q_{25}, P_{27}, Q_{30}, P_{33}, Q_{36}, P_{42}, Q_{48}, P_{50}, Q_{53}, Q_{56}, P_{58}, Q_{61}, P_{64} \right] $$

- P represent generalized momentum; q represent generalized displacement; According to the causation and the power flow calculate the bond graph simulation equation:
\[
\begin{align*}
\dot{p}_2 &= M_e - \frac{R_s}{I_e + I_0} p_2 - \frac{1}{C_{o,3}} q_3, \\
\dot{p}_3 &= \beta_1 \left[ i_f \cdot M_m + \frac{1}{C_{o,3}} q_3 + \frac{i_i^2 \cdot (I_{17} + I_{19}) - I_i^2 \cdot R_s}{I_2 + I_3 + I_4 + I_5} p_3 - \frac{1}{C_{o,6}} q_5 + \frac{i_i^2 \cdot q_6}{C_{o,9}} \right], \\
\dot{p}_{15} &= \frac{1}{I_3 + I_7 + I_9} p_{17} - \frac{1}{I_6 + I_7 + I_8} p_{17}, \\
\dot{q}_{25} &= \frac{(i_{f_1} \cdot i_{f_2})^{-1}}{I_6 + I_7 + I_8} p_{17} - \frac{1}{I_{13} + I_{14} + I_{15}} p_{27}, \\
\dot{q}_{30} &= \frac{1}{I_{13} + I_{14} + I_{15}} p_{27} - a \cdot \frac{1}{1 + a} \cdot I_{20} + I_{21} + I_{22} - \frac{1}{1 + a} \cdot I_{16} + I_{18} - \frac{1}{a} \cdot I_{32} - \frac{1}{a} \cdot I_{33}, \\
\dot{p}_{33} &= \frac{1}{1 + a} \cdot \frac{1}{C_{o,5}} q_{30} - \frac{1}{C_{o,9}} q_{36} - \frac{R_s}{I_{6} + I_{8}} p_{33}, \\
\dot{p}_{42} &= \beta_2 \left[ \frac{a}{1 + a} \cdot \frac{1}{C_{o,5}} q_{30} - \frac{i_0^{-1}}{C_{o,24}} q_{48} - \frac{i_0^{-1}}{C_{o,25}} q_{56} \right], \\
\dot{p}_{50} &= \frac{1}{C_{o,24}} q_{48} - \frac{R_{26}}{I_{24} + I_{26}} p_{50} - \frac{1}{C_{o,26}} q_{53}, \\
\dot{q}_{56} &= \frac{i_0^{-1}}{I_{20} + I_{21} + I_{22}} p_{52} - \frac{1}{I_{25} + I_{27}} p_{58}, \\
\dot{q}_{61} &= \frac{1}{I_{25} + I_{27}} p_{58} - \frac{r^{-1}}{I_{28}} p_{64}, \\
\dot{p}_{36} &= \frac{1}{I_{16} + I_{18}} p_{33} - \frac{1}{I_2 + I_3 + I_4 + I_5} p_{7}, \\
\dot{q}_{48} &= \frac{i_0^{-1}}{I_{20} + I_{21} + I_{22}} p_{42} - \frac{1}{I_{26} + I_{24}} p_{50}, \\
\dot{q}_{53} &= \frac{1}{I_{24} + I_{26}} p_{50} - \frac{r^{-1}}{I_{28}} p_{64}, \\
\dot{p}_{58} &= \frac{1}{C_{o,25}} q_{56} - \frac{1}{C_{o,27}} q_{61} - \frac{R_{27}}{I_{25} + I_{27}} p_{58}, \\
\dot{p}_{64} &= \frac{r^{-1}}{C_{o,26}} q_{53} + \frac{r^{-1}}{C_{o,27}} q_{61} - \frac{R_{26}}{I_{28}} p_{64},
\end{align*}
\]

Among:

\[
\begin{align*}
\beta_1 &= \frac{1}{1 + \frac{i_i^2 \cdot (I_{17} + I_{19})}{I_2 + I_3 + I_4 + I_5} + \frac{i_i^2 \cdot (I_2 + I_4)}{I_2 + I_3 + I_4 + I_5}}, \\
\beta_2 &= \frac{1}{1 + \frac{i_{f_1} \cdot I_4}{I_6 + I_7 + I_8} \cdot \frac{(i_{f_1} \cdot i_{f_2})^{-2} \cdot (I_{10} + I_{11} + I_{12})}{I_6 + I_7 + I_8}}, \\
\beta_3 &= \frac{1}{1 + \frac{i_0^{-2} \cdot I_{23}}{I_{20} + I_{21} + I_{22}}},
\end{align*}
\]

Standard form of the equation of state:

\[
\dot{X} = AX + BU
\]

5. Conclusion

Based on the bond graph theory to establish the transmission system bond graph model. Then according to the mathematical models of the transmission system column write state equation. The establishment of the transmission model lays the foundation for the vehicle simulation.
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