The Age of the Universe, the Hubble Constant and QSOs in a Locally
Inhomogeneous Universe

J. W. Moffat † & D. C. Tatarski ‡
Department of Physics
University of Toronto
Toronto, Ontario M5S 1A7, Canada

Talk given at the XIV Moriond Workshop on Particle Astrophysics, Atomic Physics and
Gravitaton, Villars sur Ollon, Switzerland, January 22-29, 1994.

Abstract

A local void in the globally Friedmann-Robertson-Walker cosmological model with the critical
density ($\Omega_0 = 1$) is studied. The inhomogeneity is described using a Lemaître-Tolman-Bondi
solution for a spherically symmetric distribution of matter. The scale of the central underdense
region is $\sim 150$ Mpc. We investigate the effects this has on the cosmological time scale, the
measurement of the Hubble constant and the redshift–luminosity distance for moderately and very
distant objects ($z \sim 0.1$ and more). The results indicate that if we happened to live in such a void,
but insisted on interpreting cosmological observations through the FRW model, we could go wrong
in a few instances. For example, the Hubble constant measurement could give results depending
on the separation of the source and the observer, the quasars could be younger than we think and
also less distant (less energetic).

†: moffat@medb.physics.utoronto.ca
‡: tatarski@medb.physics.utoronto.ca

UTPT-94-09, Revised: May 1994.
1 Introduction

It seems, particularly after the introduction of the inflationary paradigm [1], that the isotropic and homogeneous Friedmann-Robertson-Walker (FRW) cosmological models are best suited for the description of the global structure and the evolution of the universe. However, a similar statement is not necessarily true when cosmologically moderate scales are thought of.

There exists direct observational evidence in favour of the isotropy of the observed universe, namely, the COBE data confirming a high degree of isotropy of the cosmic microwave background radiation (CMBR) [2, 3]. However, there is no observationally based reason supporting the assumption of homogeneity. On the contrary, there seems to exist observational evidence in favour of larger and larger structures [4].

Recent work on modelling voids in the expanding universe [5] has shown that it is possible to construct an asymptotically FRW universe containing an expanding spherical void of the Lemaître-Tolman-Bondi type [6]. Moreover, the void can be a region of under-density rather than vacuum.

We think that there exists sufficient observational evidence (briefly discussed later in this paper) to support a conjecture that we may live in a relatively large underdense region embedded in a globally FRW universe. Exploring physical properties of such a model is the aim of the present paper.

In the following section, we briefly discuss the LTB model. Section 3 consists of a brief discussion of the observational background and the simple toy model of a local void presented here. The closing section contains a description of our results of numerical calculations as well as conclusions.

Throughout this paper we use units in which $G = c = 1$, unless stated otherwise.

2 The Model

First, for the sake of notational clarity, let us recall the FRW line element:

$$ds^2 = dt^2 - a^2(t) \left[ \frac{dr^2}{1 - kr^2} + r^2 d\Omega^2 \right],$$

with $d\Omega^2 = d\theta^2 + \sin^2 \theta d\phi^2$.

---

1 A cosmological solution spherically symmetric about one point was first proposed by Lemaître [6]. However, it is usually called the Tolman–Bondi solution [7, 8].
Now, let us consider a Lemaître-Tolman–Bondi model for a spherically symmetric in-
homogeneous universe filled with dust. The line element in comoving coordinates can be written as:
\[
\mathrm{d}s^2 = \mathrm{d}t^2 - R'(t,r) f^{-2} \mathrm{d}r^2 - R^2(t,r) d\Omega^2,
\] (2)
where \( f \) is an arbitrary function of \( r \) only, and the field equations demand that \( R(t,r) \) satisfies:
\[
2R\dot{R}^2 + 2R(1 - f^2) = F(r),
\] (3)
with \( F \) being an arbitrary function of class \( C^2 \), \( \dot{R} = \partial R/\partial t \) and \( R' = \partial R/\partial r \). We have three distinct solutions depending on whether \( f^2 < 1, = 1, > 1 \) and they correspond to elliptic (closed), parabolic (flat) and hyperbolic (open) cases, respectively.

The proper density can be expressed as:
\[
\rho = \frac{F'}{16\pi R'R^2},
\] (4)
Whatever the curvature, the total mass within comoving radius \( r \) is:
\[
M(r) = \frac{1}{4} \int_0^r d rf^{-1} F' = 4\pi \int_0^r d r \rho f^{-1} R'R^2,
\] (5)
so that
\[
M'(r) = \frac{dM}{dr} = 4\pi \rho f^{-1} R'R^2.
\]
Also for \( \rho > 0 \) everywhere we have \( F' > 0 \) and \( R' > 0 \) so that in the non-singular part of the model \( R > 0 \) except for \( r = 0 \) and \( F(r) \) is non-negative and monotonically increasing for \( r \geq 0 \). This could be used to define the new radial coordinate \( \bar{r}^3 = M(r) \) and find the parametric solutions for the rate of expansion.

In the flat (parabolic) case \( f^2 = 1 \), we have
\[
R = \frac{1}{2}(9F)^{1/3}(t + \beta)^{2/3},
\] (6)
with \( \beta(r) \) being an arbitrary function of class \( C^2 \) for all \( r \). After the change of coordinates \( R(t,\bar{r}) = \bar{r}(t + \beta(\bar{r}))^{2/3}, \) the metric becomes:
\[
\mathrm{d}s^2 = \mathrm{d}t^2 - (t + \beta)^{4/3} \left( Y^2 \mathrm{d}r^2 + r^2 d\Omega^2 \right),
\] (7)
where
\[
Y = 1 + \frac{2r\beta'}{3(t + \beta)},
\] (8)
and from (4) the density is given by

$$\rho = \frac{1}{6\pi(t + \beta)^2 Y}. \quad (9)$$

Clearly, we have that ($t \to \infty$) the model tends to the flat Einstein–de Sitter case.

For the closed and open cases the parametric solutions for the rate of expansion can be written as [9]:

$$R = \frac{1}{4} F \left(1 - f^2\right)^{-1} [1 - \cos(v)], \quad f^2 < 1, \quad (10a)$$

$$t + \beta = \frac{1}{4} F \left(1 - f^2\right)^{-3/2} [v - \sin(v)], \quad f^2 < 1, \quad (10b)$$

and

$$R = \frac{1}{4} F \left(f^2 - 1\right)^{-1} [\cosh(v) - 1], \quad f^2 > 1, \quad (11a)$$

$$t + \beta = \frac{1}{4} F \left(f^2 - 1\right)^{-3/2} [\sinh(v) - v], \quad f^2 > 1, \quad (11b)$$

with $\beta(r)$ being again a function of integration of class $C^2$ and $v$ the parameter.

The flat case ($f^2 = 1$) has been rather extensively studied elsewhere [10]. The model depends on one arbitrary function $\beta(r)$ and could be specified by assuming the density on some space-like hypersurface, say $t = t_0$.

The cases of interest to us, (10) and (11), correspond to closed and open models, respectively.

Before we proceed (in the next section) to discuss the observational grounds for modelling a local void, we need to amplify the discussion of the LTB model by introducing basic features of the propagation of light in our model. The high degree of isotropy of the microwave background forces us to the conclusion that we must be located very close to the spatial centre of the void. In our discussion, for the sake of simplicity, we place an observer at the centre ($t_{ob} = t_0, r_{ob} = 0$).

The luminosity distance between an observer at the origin of our coordinate system ($t_0, 0$) and the source at ($t_e, r_e, \theta_e, \phi_e$) is [8]:

$$d_L = \left(\frac{L}{4\pi F}\right)^{1/2} = R(t_e, r_e)[1 + z(t_e, r_e)]^2, \quad (12)$$

where $L$ is the absolute luminosity of the source (the energy emitted per unit time in the source’s rest frame), $F$ is the measured flux (the energy per unit time per unit area as measured by the observer) and $z(t_e, r_e)$ is the redshift (blueshift) for a light ray emitted at ($t_e, r_e$) and observed at ($t_0, 0$).
The light ray travelling inwards to the centre satisfies:

\[ ds^2 = dt^2 - R'(t, r)^2 f^{-2} dr^2 = 0, \quad d\theta = d\phi = 0, \]

and thus

\[ \frac{dt}{dr} = -\frac{R'(t, r)}{f(r)}. \] \hspace{1cm} (13)

Without getting into a detailed discussion, which can be found in [8, 10], let us state that if the equation of the light ray travelling along the light cone is:

\[ t = T(r), \] \hspace{1cm} (14)

using (13) we get the equation of a ray along the path:

\[ \frac{dT(r)}{dr} = -\frac{R'(T(r), r)}{f[T(r), r]}, \] \hspace{1cm} (15)

where

\[ \dot{R}'[T(r), r] = \frac{\partial^2 R}{\partial t \partial r} \bigg|_{r, T(r)} = \frac{\partial R'}{\partial t} \bigg|_{r, T(r)}. \]

The equation for the redshift considered as a function of \( r \) along the light cone is:

\[ \frac{dz}{dr} = (1 + z) \dot{R}'[T(r), r], \] \hspace{1cm} (16)

and the shift \( z_1 \) for a light ray travelling from \((t_1, r_1)\) to \((t_0, 0)\) is:

\[ \log(1 + z_1) = -\log(1 - a_1) - \int_{0}^{r_1} dr \frac{M'(r)}{r(1 - a_1)}, \] \hspace{1cm} (17)

where

\[ a_1(r) = \dot{R}[T(r), r], \]

and, in obtaining the second equation, we used \((9)\) and \((10)\). Thus we have two contributions to the redshift. The cosmological redshift due to expansion, described by the first term with \( a_1 = \dot{R} \), and the gravitational shift due to the difference between the potential energy per unit mass at the source and at the observer. Obviously, in the homogeneous case \((M'(r) = 0)\) there is no gravitational shift.

3 The modelling of the local void

If we restrict ourselves to spatial scales that have been well probed observationally, i.e. up to a few hundred Mpc, the most striking feature of the luminous matter distribution is the existence of large
voids surrounded by sheet-like structures containing galaxies (e.g. [11]). The surveys [11], [12] give a
typical size of the voids of the order 50–60 $h^{-1}$ Mpc. There has also been some evidence [13] –with
less certainty– for the existence of larger underdense regions with characteristic sizes of about 130
$h^{-1}$ Mpc. Also, dynamical estimates of the FRW density parameter $\Omega_0$ give very different results on
different scales. The observations of galactic halos on scales less than about 10 to 30 Mpc typically
give (see e.g. [14]) $\Omega_{10-30} \simeq 0.2 \pm 0.1$. On the other hand, smoothing the observations over larger
scales ($> 20$ Mpc, say $\sim 100$ Mpc ) indicates (e.g. [12]) the existence of a less clustered component
with a contribution exceeding 0.2, and perhaps as high as $\Omega_{\sim 100} \simeq 0.8 \pm 0.2$.

At the same time, the large scale galaxy surveys (some of the recent literature is given in
[15]) firmly indicate a considerable excess in the number–magnitude counts for faint galaxies rel-
tive to predictions of homogeneous, “no-evolution” models. This excess could be the result of
a non-standard galactic evolution or could be caused by rather exotic FRW cosmology (i.e. the
deceleration parameter $q_0 \ll 0.5$ or a non-zero cosmological constant $\Lambda$). However, it can also be
treated as an observational indication of a very large (on the scale of the redshift $z \sim 0.5$) void.
A model of a local void with a density distribution based on the faint galaxies number counts is
presently being studied [16].

In the model presented here, we confine ourselves to the simple density distributions. We
study two cases: a void with the central density equal to that of an FRW model with the density
parameter $\Omega_0 = 0.2$, asymptotically approaching the FRW model with $\Omega_0 = 1$, and a very similar
void “distorted” on an intermediate scale by a peak ($\Omega \lesssim 1$) in the density distribution. The two
distributions are:

$$\Omega_v(r) = \Omega_{\min} + (\Omega_{\max} - \Omega_{\min}) \left[1 - \left(\frac{r}{L}\right)^2 \frac{exp(r/L)}{[exp(r/L) - 1]^2}\right], \quad (18a)$$

and

$$\Omega_{vp}(r) = \Omega_{\min} + (\Omega_{\max} - \Omega_{\min}) \left[1 - \left(\frac{r}{L}\right)^2 \frac{exp(r/L)}{[exp(r/L) - 1]^2}\right] + \left(\frac{r}{l}\right)^2 e^{-(r/l)^2}. \quad (18b)$$

In the numerical calculations presented in the next section we used the values $\Omega_{\min} = 0.2$, $\Omega_{\max} = 1$
and $L = l = 30$Mpc. This assures that the void converges satisfactorily fast to the outside critical
FRW universe ($\Omega \simeq 0.86$ for $r \simeq 150$Mpc and $\Omega \simeq 0.95$ for $r \simeq 200$Mpc) and that the intermediate
peak is observationally acceptable ($\Omega \simeq 0.64$ for $r \simeq 33$Mpc ).
4 The results and discussion

In general, an LTB model depends on three arbitrary functions, see section 2, \( F(r), \beta(r) \) and \( f(r) \). Since \( F(r) \) can be interpreted as twice the effective gravitational mass within comoving radius \( r \) \( (5) \), then, in accordance with the discussion following \( (5) \), assuming its form is equivalent to a coordinate choice. In our calculations we used \( F(r) = 4r^3 \). The second function, \( \beta(r) \), sets the initial singularity (“big bang”) hypersurface of the model. Since we want the outside region in our toy model to be fully equivalent to the critical FRW universe, we set \( \beta(r) = 0 \), thereby assuming a universally simultaneous big bang. We also set the time coordinate of constant time hypersurface “now” so that it is equal to the age of the universe \( t_0 \) in the FRW model with \( \Omega_0 = 1 \). In doing so, we give up a very important feature of an LTB model: an extra (with respect to FRW) degree of freedom that would allow the age of the universe to be different from FRW or even position dependent. The third (“curvature”) function, \( f(r) \), is an unknown to be solved for in our calculations. From the work done in \( (5) \), we conclude that the LTB case to be used in modelling an underdense comoving void in an FRW universe is the hyperbolic \( (f^2 > 1) \) one.

In a manner similar to that employed in \( (10) \), we assume that since all cosmological observations are necessarily done by detecting some form of electromagnetic radiation, the solution should progress along the light cone. The final set of equations we solve consists of the equations \( (15) \), \( (16) \) and the equation describing the density distribution \( (4) \) through either of the relations \( (18) \) taken along the light cone, e.g.:

\[
\rho[T(r), r] = \frac{F'(r)}{16\pi R'[T(r), r]R^2[T(r), r]} = \Omega_v(r). \tag{19}
\]

Since \( T(r) \) (the time of emission \( t_e \) of a light ray observed at \( r = 0 \) at \( t_0 \)) is now given by \( (11b) \), the functions to be solved for are \( f(r), z(r) \) and \( v(r) \), where the parameter \( v \) becomes the function of position. The initial conditions for the integration have to be set at \( r \neq 0 \), since the analytic expressions \( (11) \) are singular at \( r = 0 \), where \( f^2 = 1 \) (we have a flat \( \Omega_0 = 0.2 \) FRW universe there). We assume that for the initial radius \( r_i \ll 1 \) (we use dimensionless radius and time in the calculations) corresponding \( z_i \) and \( t_i \) are given by their standard FRW values. Then \( z(r_i), v(r_i) \) and \( f(r_i) \) can be obtained from \( (11b), (15) \) and \( (16) \).

Once the equations have been numerically integrated we use \( (11b) \) to obtain \( t_e \) for a given \( r_e \). The luminosity distance \( d_L \) corresponding to this event is obtained with the use of \( (12) \) and \( z(t) \) (useful in studying the cosmological time scale) is given by the parametric relation \( [T(r), z(r)] \).
The redshift $z$ as a function of comoving radius $r$. The FRW results for $\Omega = 1$ and $\Omega = 0.2$ are denoted by “FRW 1” and “FRW 0.2”, respectively. The results for the LTB modelled voids are denoted “voids”.

The results of our numerical calculations are as follows. Figure 1 depicts $z(r)$, where $r$ is the dimensionless comoving radius used in the calculations. The coordinate distance has no direct physical relevance, but our units here are such that $r = 1$ corresponds to $2997.95h^{-1}$ Mpc, where $h$ is the usual coefficient in the observationally determined value of the Hubble constant: $H_0 = 100h$ km s$^{-1}$Mpc$^{-1}$.

The departure of $z(r)$ from its FRW behaviour does not seem to be dramatic. In fact, on the distance scale used in Figure 1 it is hardly noticeable. More details, on a smaller spatial scale, can be seen in Figure 2. The redshift $z$, after being influenced noticeably by rapid changes in the density $\rho$ on smaller scales, asymptotically tends to a limit that could, in accordance with FRW interpretation, correspond to the universe with the density parameter in the range $\Omega \in (0, 2; 1)$. The increase in $z$ on intermediate scales is clearly induced by the additional gravitational shift caused by the mass distribution of the void. The large scale behaviour is controlled by our assumption of the equality $t_0 = t_{0, FRW}$.

---

2 This situation does not change considerably if we model the central void in a similar manner but with respect to the observable $z$ as a variable 16.
Figure 2: The redshift $z$ as a function of comoving radius $r$. Smaller scales. Notation as in Fig. 1.

However, one should not forget that the comoving distance is not an observable, whereas the luminosity distance $d_L$ is. In principle, provided we know its absolute luminosity $\mathcal{L}$, we can establish the luminosity distance, defined by $4\pi d_L^2 = \mathcal{L}/\mathcal{F}$, by measuring the energy flux $\mathcal{F}$ of an observed object (for a discussion of usual caveats associated with so-called “standard candles” see e.g. [17]).

Due to the lack of space we do not present the $z(t)$ relation here. There is, again, some small scale $(t_0 - t \ll 1)$ divergence from the FRW behaviour, but for early cosmological times $(t \ll 1)$ the relation tends to the critical FRW ($\Omega = 1$) one. This is in accordance with our assumption of a simultaneous big bang, $\beta(r) = 0$, and with our setting the age of the universe to be equal to that of the critical FRW case. Objects with redshifts of order a few are younger than their FRW counterparts, but not significantly.

Figures 3 and 4 present the redshift–luminosity distance relation on cosmologically large (Figure 3) and intermediate (Figure 4) scales. Now the departure from the FRW behaviour becomes apparent. This comes as no surprise if one recalls the formulae for $d_L$ in both LTB ([12]) and FRW:

$$d_L^2 = a^2(t_0)r_e^2(1 + z_e)^2.$$  \hspace{1cm} (20)

The important question is whether our results contradict the linearity of the Hubble relation $z = H d$, well established on small scales.
Figure 3: The redshift $z$ as a function of the luminosity distance $d_L$ (in Mpc).

Figure 4: The redshift $z$ as a function of the luminosity distance $d_L$ (in Mpc). Smaller scales.
Due to our choice of the cosmological time scale and FRW embedding, the asymptotic \((z \to \infty)\) behaviour of \(z(d_L)\) is that of the critical \((\Omega = 1)\) FRW case. On intermediate scales, however, objects of comparable redshifts are located at smaller luminosity distances. The ratio \(d_{LTB}/d_{FRW}\) is \(\approx 0.7\) \((\approx 0.5)\) for \(z \approx 1\), \(\approx 0.8\) \((\approx 0.4)\) for \(z \approx 2.5\) and \(\approx 0.9\) \((\approx 0.36)\) for \(z \approx 4.5\), where the most distant quasars are observed. (Values in parentheses correspond to the FRW \(\Omega = 0.2\) case.) Since the absolute luminosity \(L\) of the source scales as the square of \(d_L\) this reduces the energy output of QSOs (up to an order of magnitude). Also, the angular diameter distance \(d_A = D/\delta = (1 + z)^{-2}d_L\) now gives a smaller proper distance \(D\) across the source for the same observed angular diameter \(\delta\). This helps resolve problems with seemingly acausal signals (correlations in luminosity bursts) observed across some quasars.

At the same time, inspection of Figure 4 shows that on small scales a very nearly linear (in fact, observationally indistinguishable from linear) “Hubble diagram” is obtained. However, a different value for the Hubble parameter (constant) is inferred (position, or rather \(d_L\), dependent on larger scales), if we insist on interpreting the results of cosmological observations through an FRW model.

To explore this possibility let us recall that in FRW cosmology the exact result for the Hubble relation \((z\) versus \(d_L)\) in the matter dominated universe is \([17]\):

\[
H_0 d_L = q_0^{-2} \left[ zq_0 + (q_0 - 1) \left( \sqrt{2zq_0 + 1} - 1 \right) \right], \tag{21}
\]

where \(q_0 \equiv -\ddot{a}(t_0)/a(t_0)H_0^2\) is the deceleration parameter.

Let us assume that we live in a local LTB void and the \(z\) vs. \(d_L\) relation differs from the FRW one as described in this paper, but we are biased by our theoretical prejudice and interpret cosmological observations through the FRW model.

On cosmologically small distances we measure the same value of \(H_0\) independently of the model (we call this value “the local measurement”). This stems from the fact that, due to our assumptions, very close to the centre \((r \ll 1)\) the model is well approximated by the FRW universe with \(\Omega = 0.2\). Obviously, if the universe were LTB rather than FRW, then the Hubble parameter based on the observed (LTB) values of \(z\) and \(d_L\), but inferred through an FRW relation \([21]\), would be position (redshift) dependent as shown in Figure 3.

The values of \(H_0\) reported to date span the range 40 to 100 km s\(^{-1}\)Mpc\(^{-1}\) (with standard errors quoted frequently as 10 km s\(^{-1}\)Mpc\(^{-1}\) or less!). Inhomogeneities similar to the LTB void presented here might provide an explanation for this.
Figure 5: The “observed” Hubble constant $H_0$ (in units of the local measurement) as a function of the redshift $z$. The results interpreted through FRW $\Omega = 0.2$ and $\Omega = 1$ models, respectively, are denoted “int. as FRW 0.2” and “int. as FRW 1”.

The LTB void, such as the one presented here, decreases its density contrast (the depth of the void with respect to the FRW background) when evolved back in time [9, 10]. At early times it is almost homogenized (at $t/t_0 \approx 10^{-5}$ we have $|\rho_{LTB}(r)/\rho_{FRW} - 1| < 10^{-6}$). This corresponds to a universe which at the beginning is very similar to the FRW one, but different at late stages.

In this manner, while retaining all accomplishments of the FRW cosmology in dealing with epochs preceding the matter dominated era, we can gain new freedom in modelling the more recent universe. We can solve the age of the universe problem (by assuming $\beta(r) = \text{const} \neq 0$), provide the excess power observed on scales of $5–10,000 \text{ km s}^{-1}$ in modelling structure formation (see [10]), alleviate a few old problems associated with quasars (their age, luminosity and size) and provide an explanation for the wide range of reported values of the Hubble constant.

Acknowledgement
The authors thank Ken-Ichi Nakao for helpful discussions and suggestions.

References
[1] Guth A. H., *Phys. Rev.* D, 23, 347, 1981.

[2] Mather J. C. et al., *Astrophys. J.*, 354, L37, 1990; Hogan C. J., *Nature*, 344, 107, 1990.

[3] Smoot G. F. et al., *Astrophys. J.*, 396, L1, 1993; Wright E. L. et al., *Astrophys. J.*, 396, L13, 1993.

[4] Lynden–Bell D. et al., *Astrophys. J.*, 326, 19, 1988; Geller M. J. & Huchra J. P., *Science*, 246, 897, 1989; Clowes R. G. & Campusano L. E., *M.N.R.A.S.*, 249, 218, 1991.

[5] Bonnor W. B. & Chamorro A., *Astrophys. J.*, 361, 21, 1990; *Astrophys. J.*, 378, 461, 1991.

[6] Lemaître G., *M.N.R.A.S.*, 91, 490, 1931; *Ann. Soc. Sci. Bruxelles* A53, 51, 1933.

[7] Tolman R. C., *Proc. Nat. Acad. Sci.*, 20, 169, 1934.

[8] Bondi H., *M.N.R.A.S.*, 107, 410, 1947.

[9] Bonnor W. B., *M.N.R.A.S.*, 159, 261, 1972; *M.N.R.A.S.*, 167, 55, 1974.

[10] Moffat J. W. & Tatarski D. C., *Phys. Rev.* D 45, 3512, 1992.

[11] Geller M. J. & Huchra J. P., *Science*, 246, 897, 1989.

[12] Efstathiou G. et al., *M.N.R.A.S.*, 247, 10, 1990; Saunders W. et al., *Nature*, 349, 32, 1991.

[13] Broadhurst T. J. et al., *Nature*, 343, 726, 1990.

[14] Sancisi R. & van Albada T. S., in ‘Dark Matter in the Universe’, eds. J. Kormendy & G. Knapp, Reidel, Dordrecht, 1987.

[15] Maddox S. J. et al., *M.N.R.A.S.*, 240, 43, 1990; Tyson J. A., *Astronom. J.*, 96, 1, 1988; Heydon-Dumbleton N. H. et al., *M.N.R.A.S.*, 238, 379, 1989; Lilly S. J., *Astrophys. J.*, 411, 501, 1993.

[16] Moffat J. W. & Tatarski D. C., work in progress.

[17] Weinberg S., ‘*Gravitation and Cosmology*’, John Wiley & Sons Inc., New York, 1972; Kolb E. W. & Turner M. S., ‘*The Early Universe*’, Addison–Wesley Publ. Co., Redwood City, 1990; Peebles P. J. E., *Principles of Physical Cosmology*, Princeton University Press, Princeton, 1993.