Integrated design of structure and anisotropic material based on the BESO method

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Abstract. An Integrated structural and material topology optimization method considering optimal material orientation is presented based on bi-directional evolutionary structural optimization (BESO) method. It is assumed that the macrostructure is composed of uniform cellular material but with different orientation. The homogenization method is used to calculate the effective material properties which builds a connection between material and structure. The continuous material orientation design variables and the discrete topology design variables are treated hierarchically in an iteration. The principal stress method is adopted and embedded to determine the optimal material orientation, meanwhile the topologies of the macrostructure and its material microstructure are concurrently optimized by using the BESO method. Numerical examples are conducted to demonstrate the effectiveness of the proposed optimization algorithm.

Keywords. integrated design; multiscale; material orientation; BESO

1. Introduction

In the context of growing shortage of resource, how to make full use of materials according to different material properties and working conditions has become a significant task for industrial community. Structural topology optimization technique is an effective approach to achieve the best structural performance with limited amount of materials\textsuperscript{[1-3]}. To date, topology optimization has been used to solve one-scale design problems either for macrostructures to improve their structural performance or for materials to develop new microstructures with prescribed or extreme properties\textsuperscript{[4-6]}.

Compared with designing structures or materials solely, the integrated topology optimization design of structure and material can provide more design freedoms to achieve even better design performance. Such as, Yan and Guo et al.\textsuperscript{[7]} studied the multi-scale concurrent topology optimization of structural compliance under mechanical and thermal loads by employing the Porous Anisotropic Material with Penalization (PAMP) model. Yan and Huang et al.\textsuperscript{[8-9]} introduced a two-scale topology optimization algorithm based on the bi-directional evolutionary structural optimization (BESO) method to concurrently design materials and structures for maximizing the structural stiffness and minimizing the material thermal conductivity. Xu et al.\textsuperscript{[10-11]} extended the concurrent design method to the optimization problems under dynamic loadings. Chen et al.\textsuperscript{[12]} presented a new MIST (moving iso-surface threshold) formulation and algorithm for the concurrent design of structures and cellular materials in order to maximize the structural stiffness. Using the two-scale concurrent topology design method. Numerical examples are conducted to demonstrate the effectiveness of the proposed optimization algorithm.
optimization method, Long et al.\cite{13} optimized the frequency of composite macrostructure composed of two isotropic materials with distinct Poisson’s ratios through introducing the interpolation of Poisson’s ratios of different constituent phases. Li et al.\cite{14} presented a novel concurrent design for cellular structures consisting of multiple patches of material microstructures by using the level set-based topology optimization method. However, in these studies, the material orientation is presupposed. Once the material orientation layout is optimized, the performance of the resulted structures can be further improved.

In fact, while concurrently designing structures and materials by topology optimization, the resulting materials are often anisotropic. Reasonably re-arranging the material orientation will significantly improve the performance of the constructed structure, and this is of great importance in the design of composite structures, laminates, etc. However, the research on multiscale topology optimization of structures and materials considering optimal material orientation is still inadequate. In this paper, an integrated topology optimization algorithm which takes the orientation of the material into consideration is proposed based on the BESO method. To improve the computational efficiency, an analytical approach is adopted and embedded to determine the optimal material orientation while concurrently optimizing the material and the structural topologies.

2. Problem Formulation

2.1. Optimization model and sensitivity analysis

As shown in Figure 1(a), the external force $F$ and displacement boundary $\Gamma$ are applied to design domain $\Omega$ of the macrostructure, which is composed of cellular materials with periodic microstructure (Figure 1(b)). Since the cellular material in the macrostructure is anisotropic, the material orientation (as shown in Figure 1(c)) can be designed simultaneously with topologies of macrostructure and material microstructure.

Figure 1. Design domain composed of cellular materials: (a) macro design domain; (b) microstructure of cellular material; (c) a base unit cell with an orientation angle $\theta_i$.

Here, the design objective is to minimize the structural mean compliance, that is, to maximize the stiffness of the macrostructure by optimizing the topologies of macrostructures and material microstructures, and material orientation under the prescribed volume constraints for macrostructures and material microstructures. With the framework of finite element analysis (FEA), the two-scale optimization problem can be formulated as

$$\begin{align*}
\text{find:} & \quad x_i, \theta_i, x_j \quad (i = 1, 2, \ldots, N \text{ and } j = 1, 2, \ldots, n) \\
\text{minimize:} & \quad C(x_i, \theta_i, x_j) = \frac{1}{2} \sum_{i=1}^{N} U_i^T K_i(x_i, \theta_i, x_j) U_i \\
\text{subject to:} & \quad K U = F \\
& \quad \sum_{i=1}^{N} x_i V_i = V_1 \\
& \quad \sum_{j=1}^{n} x_j V_j = V_2 \\
& \quad -\frac{\pi}{2} \leq \theta_i \leq \frac{\pi}{2} \\
& \quad x_i, x_j = x_{\text{min}} \text{ or } 1
\end{align*}$$

(1)
where $x_i$ and $x_j$ are binary design variables of the macrostructure and its material microstructure, respectively. $x_i = 1$ denotes the $i$th element of the macrostructure is solid and $x_i = \chi_{\text{min}}$ ($\chi_{\text{min}}$ is a small value, e.g. 0.001) denotes a void element. The same representations are defined for the design variable $x_j$ of the material unit cell. $N$ and $n$ are the total element number of the macrostructure and the unit cell material microstructure, respectively. $C$ is the mean compliance of the macrostructure. $U$ is the nodal displacement vector of the macrostructure. $K$ and $K_i$ are structural and elemental stiffness matrices of the macrostructure, respectively. $V_i$ and $V_j$ are the elemental volumes in the macrostructure and the unit cell, respectively. $V_1$ and $V_2$ are the corresponding material volume fractions at two scales.

When the material base unit cell is rotated by an angle $\theta_i$, as shown in Figure 1(c), the rotated elasticity matrix of $i$th can be expressed as

$$
D_i = T_i^T D^H_i T_i
$$

(2)

where $D^H_i$ is the effective elasticity matrix which can be computed according to classical homogenization method\cite{15} and $T_i$ is standard rotation matrix and is expressed as

$$
T_i = \begin{bmatrix}
\cos^2 \theta_i & \sin^2 \theta_i & \frac{1}{2} \sin 2 \theta_i \\
\sin^2 \theta_i & \cos^2 \theta_i & -\frac{1}{2} \sin 2 \theta_i \\
-\sin 2 \theta_i & \sin 2 \theta_i & \cos 2 \theta_i
\end{bmatrix}
$$

(3)

At the micro scale, the elasticity matrix of the $j$th element can be express by using SIMP interpolation model as

$$
D_j^{\text{m}} = x_j^{p} D_0
$$

(4)

where $p$ is the exponent of penalization and $p=3$ is used in this paper. $D_0$ denotes the elasticity matrix of base material.

Similarly, at the macro scale, the element elasticity matrix of the $i$th element can be express by

$$
D_i^{\text{m}} = x_i^{p} D_i
$$

(5)

According to homogenization method, the effective elasticity matrix of the material microstructure can be calculated on the material base unit cell as\cite{17}

$$
D^H = \frac{1}{|Y|} \int_Y D^{\text{m}} (I - bu) dY
$$

(6)

where $I$ is an identity. $u$ denotes the displacement field of the base unit cell caused by the unit strain fields. $Y$ is total area or volume of the unit cell.

Before implementing the integrated topology optimization, the sensitivity analysis must be conducted. According to equations (1), (2) and (5), the derivate of mean compliance with respect to macro design variable, $x_i$ can be derived as

$$
\frac{\partial C}{\partial x_i} = -\frac{p}{2} x_i^{p-1} U_i^T \left( \int_V B_i^T T_i^T \frac{\partial D^H_i}{\partial x_i} T_i B_i V_i \right) U_i
$$

(7)

Likewise, the derivation of mean compliance with respect to the micro design variable $x_j$ can be derived as

$$
\frac{\partial C}{\partial x_j} = -\frac{1}{2} \sum_{i=1}^N x_i^{p-1} U_i^T \left( \int_V B_i^T T_i^T \frac{\partial D^H_i}{\partial x_j} T_i B_i V_i \right) U_i
$$

(8)

where according equation (4) and (6), the derivation of $D^H_i$ with respect to $x_j$ can be calculated on the basis of adjoint variable method\cite{16} as

$$
\frac{\partial D^H_i}{\partial x_j} = \frac{p x_j^{p-1}}{|Y|} \int_V (I - bu_j)^T D_0 (I - bu_j) dV_j
$$

(9)
2.2. Optimal orientation determination

In this section, an analytical method is presented to determine the optimal material orientation. If the finite element mesh is dense enough, we can safely assume the strain and stress are constant inside each element, and they can be approximated by the values at the centroid. Thus, the material orientation can be determined according to the strain and stress states element by element. Directly taking the principal stress or strain directions as the optimal material orientation is an efficient way compared with the iterative method. Gea and Luo[17] also proved that, for the shear “weak” material, the optimal material orientation is collinear with one of the principal stress directions. Therefore, the optimal material orientation can be computed by using the principle stress method as

$$\theta_{\text{opt}} = \frac{1}{2} \arctan \left( \frac{2 \sigma_{12}}{\sigma_{11} - \sigma_{22}} \right)$$

where $\sigma_{11}, \sigma_{22}, \sigma_{12}$ are the components of the elemental stress vector.

It should be noted that for the shear “strong” materials[17], the optimal material orientation may not coincide with the principal stress or strain direction any more. In this case, the energy-based method[18] can be used to determine the optimal orientation instead. However, while concurrently designing structures and materials by topology optimization, the resulting materials are often shear “weak”, this will be demonstrated in the following numerical examples. Therefore, the principle stress method is adopted in this paper.

3. Numerical implementation procedure

To implement the two-scale topology optimization with optimal material orientation by using the BESO method, the material optimal orientation determination and the two-scale topology optimization are treated hierarchically. For each iteration, the physical property of microstructural material is first extracted by using the homogenization method. Then, effective material parameters are transmitted to the finite element analysis of macrostructure. Next, the optimal material orientation for each macrostructural element is determined directly according to the macrostructural stress state. In order to improve the convergence, FEA of macrostructure can be conducted again with updated material orientation. Thereafter, the topologies of structure and material are updated concurrently according to the elemental sensitivities at both scales by using the BESO method. More details about the numerical implementation of the BESO method can also refer to the references[8,9]. This process will be repeated again and again until the solution is convergent stably. The whole procedure is summarized as follow

Step 1: Define design domain, set the objective material volume fraction, evolutionary ratio, filter radius and initial material orientation.
Step 2: Carry out FEA on the material base cell and homogenization using equation (6).
Step 3: Carry out FEA on the macrostructure.
Step 4: According to the stress state in the macrostructure, determine the optimal material orientation by using equation (10).
Step 5: Carry out FEA on the macrostructure again.
Step 6: Calculate the macro and micro sensitivities according to equation (7) and (8).
Step 7: Add and delete macro and micro elements respectively by sorting their elemental sensitivity numbers.
Step 8: Construct new macrostructure and material microstructure
Step 9: Repeat step 2 to 8 until both two volume constraints and objective function is convergent.

4. Example and discussion

In this section, a classical example, the MBB beam with dimensions and loading shown in Figure 2, is illustrated to demonstrate the capability of the proposed optimization algorithm. Due to the structural symmetry, only right half of the MBB beam is considered and is discretized into 70x40 4-node quadrilateral elements. The material base unit cell is assumed to be a square with side length of 1 and is discretized into 100x100 4-node quadrilateral elements. The base material is isotropic with
Young’s modulus $E_0 = 1.0$, Poisson’s ratio $\mu = 0.3$. The evolution rates are set $ER_1 = ER_2 = 1\%$ and filter radii $r_{1\text{min}} = r_{2\text{min}} = 3$. The target volume fractions in the macrostructure and the material microstructure are both set to be 50%. The initial material orientation angles for all elements are set to be zero (horizontal).

![Figure 2. Design domain of the MBB beam](image)

To investigate the influence of the material orientation layout, the integrated topological design of structure and material without considering material orientation optimization (MOO) is also carried out (all elemental material orientation angles are kept as the initial value of zero). The evolution histories of the objective mean compliance are plotted in Figure 3. It can be seen the mean compliance without MOO is 108.98, while the mean compliance with MOO is 66.0. The structural compliance is decreased by about 39.4%. Therefore, for the structures composed of anisotropic materials, optimization of the material orientation can significantly improve the structural performance.

![Figure 3. Iteration histories of the objective mean compliance with and without MOO](image)

The corresponding optimized macrostructural and material microstructural topologies and material orientation layout are given in Figure 4 and 5. Comparing Figure 4 with Figure 5, it is shown that in the integrated optimization, the optimized topologies of macrostructures and material microstructures with and without MOO are totally different. The topology of the material microstructure without MOO is absolutely asymmetric, and its material elasticity matrix is anisotropic. By contrast, the resulted material elasticity matrix with MOO is orthotropic, and the shear modulus is much smaller than the horizontal and vertical moduli. This can be confirmed from the macrostructure that the material principal axes of each element are all placed along the tensile or compressive direction. While in the situation without MOO, the resulted material has relatively bigger shear modulus which is due to bearing multiaxial stress state in the macrostructure.
Figure 4. Optimized results with MOO: (a) Macrostructural topology (black short lines represent the material principal axis); (b) Material base unit cell topology; (c) Effective material elasticity matrixes $D^H (\times 10^{-2})$

Figure 5. Optimized results without MOO: (a) Macrostructural topology; (b) Material base unit cell topology; (c) Effective material elasticity matrixes $D^H (\times 10^{-2})$

Table 1. Resulting topologies of macrostructure and its material microstructure with different volume fractions under a prescribed total volume fraction $\bar{V}_1 \times \bar{V}_2 = 25\%$

| Mean compliance $C$ and material volume fraction | Macrostructure | Material microstructure |
|-------------------------------------------------|----------------|------------------------|
| $C = 74.39$                                      | ![Macrostructure](image1) | ![Material microstructure](image2) |
| $\bar{V}_1 = 62.5\%$                            |                |                        |
| $\bar{V}_2 = 40\%$                             |                |                        |
| $C = 62.90$                                      | ![Macrostructure](image3) | ![Material microstructure](image4) |
| $\bar{V}_1 = 41.67\%$                          |                |                        |
| $\bar{V}_2 = 60\%$                             |                |                        |
| $C = 57.51$                                      | ![Macrostructure](image5) | ![Material microstructure](image6) |
| $\bar{V}_1 = 31.25\%$                          |                |                        |
| $\bar{V}_2 = 80\%$                             |                |                        |
| $C = 52.65$                                      | ![Macrostructure](image7) | ![Material microstructure](image8) |
| $\bar{V}_1 = 25\%$                             |                |                        |
| $\bar{V}_2 = 100\%$                            |                |                        |
To further investigate the influence of material allocations at different scales. In this case, the total volume fraction of the base material, \( V_f = V_1 \times V_2 \), is restricted to 25%. The resulted topologies of macrostructure and its material microstructure, and the mean compliance are listed in Table 1 when \( V_2 \) increases from 40% to 100%. Table 1 indicates that, at the micro scale, the topologies of the material microstructures are generally similar and the materials always maintain the characteristics of weak shear and orthogonality. Additionally, with the base material shifting gradually from the macro scale to the micro scale, the mean compliance of the macrostructures decreases monotonously. When the microstructure is fully filled with the base material and the material becomes uniform and isotropic, the mean compliance of the macrostructure reaches to the minimum. In this case, using solid material may be the best choice for minimizing the structural mean compliance.

5. Conclusions
Based on the BESO method, an integrated multiscale topology design method with material orientation optimization is proposed to optimally design macrostructure, material microstructure and material orientation distribution simultaneously. The principle stress method is adopted and integrated to determine the optimal material orientation while concurrently optimizing the material and the structural topologies. Numerical examples demonstrate that the structural and material designs interact strongly with each other. In the integrated two scale topology optimization, the resulting material is usually anisotropic. The resulting material with material orientation optimization (MOO) has obvious orthogonality and weak shear characteristics, which is totally different from that without MOO. Optimally designing the topologies of macrostructures and material microstructures, together with the material orientation in the macrostructure can significantly improve the structural performance.

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