Identification of abrupt stiffness changes of structures with tuned mass dampers under sudden events

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Summary
This paper presents a recursive system identification method for MDoF structures with tuned mass dampers (TMDs) considering abrupt stiffness changes in case of sudden events, such as earthquakes. Due to supplementary nonclassical damping of the TMDs, the system identification of MDoF + TMD systems poses a challenge, in particular, in case of sudden events. These identification methods may be helpful for structural health monitoring of MDoF structures controlled by TMDs. A new adaptation formulation of the unscented Kalman filter allows the identification method to track abrupt stiffness changes. The paper, firstly, describes the theoretical background of the proposed system identification method and afterwards presents three parametric studies regarding the performance of the method. The first study shows the augmented state identification by the presented system identification method applied on a MDoF + TMD system. In this study, the abrupt stiffness changes of the system are successfully detected and localized under earthquake, impulse, and white noise excitations. The second study investigates the effects of the state covariance and its relevance for the system identification of MDoF + TMD systems. The results of this study show the necessity of an adaptive definition of the state covariance as applied in the proposed method. The third study investigates the effects of modeling on the performance of the identification method. Mathematical models with discretization of different orders of convergence and system noise levels are studied. The results show that, in particular, MDoF + TMD systems require higher order mathematical models for an accurate identification of abrupt changes.

KEYWORDS
abrupt stiffness changes, adaptive unscented Kalman filter, Kalman filter, stiffness identification, system identification, tuned mass dampers

1 | INTRODUCTION
In the past decades, the importance of system identification in civil engineering has grown continuously. Response measurements using accelerations, velocities, etc., are common and can be applied for the identification of important parameters, such as the natural frequencies and damping ratios of linear structures. However, the identification of nonlinear structures, including system damages, got more attention recently. As Devin and Fanning1 mentioned, even partition...
walls or in general nonstructural elements have high influence on natural frequencies and damping ratios, so that, for instance, in case of an earthquake, even small damages can deteriorate the dynamic performance. The damage detection is, therefore, an important research field of nonlinear system identification.

Numerous system identification methods have been proposed so far, which can be split into offline and online, that is, recursive, methods. For offline methods, a whole data set of system responses is required, while recursive methods enable stepwise system identification based on a priori system informations and states. One field of offline identification is the operational modal analysis, which can be further divided in time domain methods, including autoregressive moving average methods or stochastic subspace identification, and the frequency domain methods, such as the frequency domain decomposition, respectively. The aim of the operational modal analysis is the a posteriori system identification under white noise input signals and, therefore, output-only measurements.

On the other hand, online identification methods are necessary to identify nonlinear system changes in real-time. In contrast to offline identification methods, only time domain methods can be applied for the online identification, since basic frequency domain methods are generally nonrecursive. Some examples for the time domain methods, which were successfully applied to civil engineering structures, are, for instance, least square estimation, particle filters, and Kalman filter (KF). Among these methods, Kalman filter methods, in particular, have become one of the common methods for system identification, since a combined parameter and state estimation is possible and the computational cost is moderate.

The KF is a recursive method for estimating states, considering, for example, displacements and velocities, and can be applied for any type of excitation signal. However, only linear system behavior can be covered by the KF. On this account, the extended Kalman filter (EKF) has been proposed. In general, the EKF uses the same concept as the KF, except for the nonlinear state and observation equations, which are linearized in each calculation step by setting up their Jacobians. Although nonlinear systems can be covered by this linearization, EKF has two main disadvantages: first, it is very costly to set up the Jacobians in each time step, which makes a recursive application more difficult, and second, for highly nonlinear systems, the linearization approach is not accurate enough. Therefore, a further development, the unscented Kalman filter (UKF) by Julier et al. has been proposed, avoiding the disadvantages of the EKF. Instead of the linearization, an unscented transformation (UT), based on sampling points, is used, and thus, systems with higher nonlinearities can be covered.

In addition, the main advantage of both EKF and UKF is the possibility of a joint state and parameter estimation, which includes, besides the common displacement and velocity states, also system parameters, such as stiffness coefficients. The joint state and parameter estimation of the UKF was applied to MDoF civil engineering structures both numerically and experimentally. Within these studies, however, only nonlinearities due to initial stiffness deviations were investigated, and apart from that, systems were assumed to behave linearly. Further studies have shown the applicability of the parameter estimation using UKF to the nonlinear Bouc–Wen material behavior, as well as for negative stiffness devices in frame structures.

To cover abrupt system changes of MDoF structures, the UKF or respectively EKF has to be adaptive. On this account, several attempts have been made in the past. For instance, Yang et al. propose a recursively determined forgetting factor introduced in the EKF, which is calculated by an optimization step based on stiffness estimates. Lei et al. instead, propose a three-step algorithm, where, firstly, the initial system parameters are identified by an EKF. For each following time steps, the damages are detected by the innovation error, subsequently, identified and localized by an optimization step and, finally, the new states are identified using a KF. In contrast, Bisht and Singh propose an adaptive UKF. Damages herein are detected by an adaptation criterion based on the innovation error and a posteriori known system response data. Finally, the estimation of abrupt changes is enabled by the adaptation of the state covariance. Although Rahimi et al. use a similar approach like Bisht and Singh, they modified the adaptation criterion, where the adaptation threshold is calculated nonrecursively based on sensitive floating variances.

Tuned mass dampers (TMDs) introduce supplementary damping and restoring forces on structures. Therefore, MDoF structures with TMDs respond to dynamic excitations with lower amplitudes and shorter vibration duration than systems without TMDs. Consequently, in particular, in case of sudden events with abrupt changes, the system identification performance of MDoF structures with TMDs is expected to deteriorate. On this account, the accuracy of stiffness identification is more challenging for systems with TMDs in contrast to those without additional damping devices. However, to the best of authors’ knowledge, no previous study has investigated the performance of the recursive system identification approaches for the estimation of abrupt changes of MDoF + TMD systems. In this context, in particular, the accuracy of the chosen mathematical model is important. The previous studies mostly used linearized mathematical models, which cannot reach the required accuracy level for MDoF + TMD systems. A comparison and a careful choice of existing mathematical models in nonlinear system identification is absolutely necessary. Furthermore, the so far proposed adaptation
algorithms for the UKF either require a completed system response time window in a nonrecursive manner or include highly sensitive nonrobust calculation procedures. Since for real-time measurement scenarios, the response data are available only stepwise, and signals are biased by noise, a robust recursive adaptation criterion is required.

This paper presents an UKF-based system identification method for MDoF + TMD systems. In Section 2, an adaptive approach with robust, recursive algorithms is proposed, which enables the detection of abrupt stiffness changes of MDoF + TMD systems during sudden events with a high-level accuracy. In particular, the proposed approach needs no special knowledge of a posteriori system responses. Thus, a constant adaptation criterion based on known sensor properties is driven by statistical signal properties. In Section 3, the presented method is investigated on a MDoF + TMD system by three parametric studies. In the first study, using the proposed system identification method a stiffness and state estimation considering abrupt stiffness changes is performed for several load scenarios. The remaining two studies focus on the filter setup of the system identification method and its influence regarding the identification performance. Therefore, the state covariance influence regarding the identification speed, especially in terms of TMD equipped structures, is investigated in the second study. Finally, the accuracy of four Taylor expansion-based mathematical models of different orders of convergences is analyzed in the third study. In particular, the relationship between the system noise level and mathematical model is explored. A conclusion of the work is presented in Section 4.

2 | SYSTEM IDENTIFICATION METHOD FOR MDOF STRUCTURES WITH TMDs

2.1 | System identification method

For the system identification of MDoF + TMD systems, a UKF-based system identification method is proposed. Similar to the linear KF, the UKF consists of a prediction as well as a correction step. To cover the nonlinearities, the UT is applied. In the UT, sampling points $\tilde{X}_i^k$ of size $n$, where $\tilde{\cdot}$ denotes corrected values, are created on the basis of the known mean $\bar{x}_k$ and the current state covariance $\tilde{P}_k$, which is assumed to be Gaussian distributed. For the calculation of the sampling points, weighting factors $W_m^i$ and $W_c^i$ are introduced for mean and respectively covariance values. The scaling parameters $\lambda, \alpha, \beta,$ and $\kappa$ are standard values and are mostly chosen based on a Gaussian distribution:

$$\tilde{X}_i^k = \bar{x}_k \pm \left( \sqrt{(n + \lambda)\tilde{P}_k} \right)_i, \quad i = 1, \ldots, 2n,$$

$$\tilde{X}_i^0 = \bar{x}_k,$$  \hspace{1cm} (1)

$$W_m^0 = \frac{\lambda}{n + \lambda},$$  \hspace{1cm} (2)

$$W_c^0 = \frac{\lambda}{n + \lambda} + 1 - \alpha^2 + \beta,$$  \hspace{1cm} (3)

$$W_m^i = W_c^i = \frac{1}{2(n + \lambda)},$$  \hspace{1cm} (4)

$$\lambda = \alpha^2(n + \kappa) - n.$$  \hspace{1cm} (5)

Using a nonlinear time variant state equation $f(\cdot)$, all sampling points are transformed to the estimates $\hat{X}_{i+1}^k$ at the next time step $k + 1$, where $\hat{\cdot}$ denotes estimation values. Under assumption of a Gaussian distribution for both time steps $k$ and $k + 1$, the state estimate $\hat{x}_{k+1}$, as well as the state covariance estimate $\hat{P}_{k+1}$ at time step $k + 1$, can be predicted by summing up all weighted sampling points. Applying the observation equation $h(\cdot)$ at first, the sampling points of the output vector $\hat{Y}_{i+1}^k$ at time $k + 1$ can be found and finally weighted to the estimated output vector $\hat{y}_{k+1}$ as well:

$$\hat{X}_{i+1}^k = f(\hat{X}_i^k, u_k),$$  \hspace{1cm} (6)

$$\hat{x}_{k+1} = \sum_{i=0}^{2n} W_m^i \hat{X}_i^k,$$  \hspace{1cm} (7)

$$\hat{P}_{k+1} = \sum_{i=0}^{2n} W_c^i(\hat{X}_i^k - \hat{x}_{k+1})(\hat{X}_i^k - \hat{x}_{k+1})^T + Q,$$  \hspace{1cm} (8)

$$\hat{y}_{k+1} = h(\hat{X}_{i+1}^k, u_{k+1}).$$  \hspace{1cm} (9)
Adaptation scheme for the system identification method

For systems with both initial nonlinearities and abrupt changes, an adaptation procedure is presented as follows. As the measurement signal evolves, the estimated state \( \hat{x}_{k+1} \) is corrected using the Kalman gain \( K_{k+1} \), which is calculated by the covariances \( P_{yy,k+1} \) and \( P_{xy,k+1} \). Finally, it yields the predicted and corrected state \( \hat{x}_{k+1} \) and covariance \( P_{k+1} \), respectively:

\[
\hat{y}_{k+1} = \sum_{i=0}^{2n} W^i_m \hat{y}^i_{k+1}. \tag{11}
\]

On basis of the innovation error \( e_{k+1} = y_{k+1} - \hat{y}_{k+1} \), the estimated state \( \hat{x}_{k+1} \) is corrected using the Kalman gain \( K_{k+1} \), which is calculated by the covariances \( P_{yy,k+1} \) and \( P_{xy,k+1} \). Finally, it yields the predicted and corrected state \( \hat{x}_{k+1} \) and covariance \( P_{k+1} \), respectively:

\[
\hat{P}_{yy,k+1} = \sum_{i=0}^{2n} W^i_m (\hat{Y}^i_{k+1} - \hat{y}_{k+1})(\hat{Y}^i_{k+1} - \hat{y}_{k+1})^T + R, \tag{12}
\]

\[
\hat{P}_{xy,k+1} = \sum_{i=0}^{2n} W^i_m (\hat{X}^i_{k+1} - \hat{x}_{k+1})(\hat{Y}^i_{k+1} - \hat{y}_{k+1})^T, \tag{13}
\]

\[
K_{k+1} = P_{xy,k+1} [P_{yy,k+1}]^{-1}, \tag{14}
\]

\[
\hat{x}_{k+1} = \hat{x}_{k+1} + K_{k+1} e_{k+1} = \hat{x}_{k+1} + K_{k+1}(y_{k+1} - \hat{y}_{k+1}). \tag{15}
\]

\[
P_{k+1} = P_{k+1} - K_{k+1} P_{yy,k+1} K_{k+1}^T. \tag{16}
\]

For the parameter estimation, the previous state vector \( x_k \), including displacement, velocity, or acceleration information, has to be augmented by a parameter vector \( \theta_k \), containing all to be identified parameters. It yields the augmented state vector \( x^a_k \). Afterwards, the state equation has to be changed, and the UKF can be applied using \( x^a_k \).

\[
x^a_k = \begin{bmatrix} x_k \\ \theta_k \end{bmatrix}. \tag{17}
\]

### 2.2 Adaptation scheme for the system identification method

For systems with both initial nonlinearities and abrupt changes, an adaptation procedure is presented as follows. As the corrected state covariance \( P_k \) describes the confidence of the estimated and corrected state \( \hat{x}_k \), it can be used to influence the upcoming parameter estimation step, that is, a high state covariance yield more sensitive system identification and nonlinearities can be identified better.

For this purpose, firstly, similar to Bisht and Singh,\(^22\) the trigger parameter \( \gamma \) based on the innovation error \( e_{k+1} \) is introduced. In contrast to Bisht and Singh, the innovation error is normalized by the measurement noise covariance \( R \) instead of the measurement covariance \( P_{yy} \), allowing the trigger parameter \( \gamma \) to be independent from the measurement noise level:

\[
\gamma = e_{k+1}^T R^{-1} e_{k+1}. \tag{18}
\]

For \( m \) sensors with the identical constant measurement noise covariance \( R_i = R \), \( \gamma \) reads:

\[
\gamma = \begin{bmatrix} e_1 \\ \vdots \\ e_m \end{bmatrix}^T \begin{bmatrix} R_1 & \ldots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \ldots & R_m \end{bmatrix}^{-1} \begin{bmatrix} e_1 \\ \vdots \\ e_m \end{bmatrix} = \frac{e_1^2}{R_1} + \ldots + \frac{e_m^2}{R_m} = \frac{e_1^2 + \ldots + e_m^2}{R}. \tag{19}
\]

To detect system changes, a threshold \( \gamma_0 \) is defined as an adaptation criterion, which has to be exceeded by \( \gamma \) in adaptation cases:

\[
\gamma \geq \gamma_0 \Rightarrow \text{adaptation}. \tag{20}
\]

Bisht and Singh\(^22\) proposed to choose a constant threshold based on the a posteriori known covariance of the measurement signal \( y_k \), whereas Rahimi et al.\(^23\) compensated the unknown a posteriori information by introducing variable thresholds over time. However, different than the previous approaches, in this paper, the proposed threshold avoids both the necessity of a posteriori knowledge as well as the high sensitivity resulting from variable thresholds. On basis of known sensor numbers \( m \) and measurement noise covariance \( R \), the threshold \( \gamma_0 \) is calculated.

Figure 1 shows the individual steps of the derivation of the threshold, including the approximation of the innovation error with noise terms, the corresponding statistics, the realization of the trigger parameter \( \gamma \), and the threshold \( \gamma_0 \).

The innovation error \( e_{k+1} \) contains both the system and measurement errors. System changes correspond to system errors. For the identification of system changes from the innovation error, the threshold \( \gamma_0 \) must cover with a certain probability the measurement error portion of the innovation error. If the trigger parameter \( \gamma \) exceeds this threshold \( \gamma_0 \), the existence of a system error, that is, system change, is ensured.
For the definition of such a threshold, we consider firstly a constant system behavior without any changes, that is, no system errors. In this case, the innovation error can be solely approximated by the measurement error, Figure 1 (Step 1: Approximation of innovation error). For the sake of simplicity, the time steps are not explicitly given for each parameter, since each parameter corresponds to the same time step. The measurement error is computed from the difference between the true measurement signal $y_i$ and the predicted measurement signal $\hat{y}_i$. The predicted measurement signal is calculated by the observation equation by the output matrix $C$ with the predicted state $\hat{x}$ and the transition matrix $D$ with the input $u$ (Section 2.4). The predicted state is independent from the measurement error. Accordingly, output $n_y$ and the input $n_u$ govern the measurement error. If the system motion is observed by displacement and velocity sensors, the transition matrix $D$ becomes zero, so that the innovation error $e_i$ solely depends on the output measurement noise $n_y$. If acceleration sensors are used, $D$ is an identity matrix, and, consequently, $e_i$ is approximated by the difference of both the output $n_y$ and the input $n_u$ measurement noises.

Both measurement noises are assumed to be Gaussian, and each has a covariance of $R$. Accordingly, their superposition can be treated as Gaussian as well. Figure 1 (Step 2: Statistical properties of innovation error). Consequently, $e_i$ has a mean of zero, and its variance $R_i$ can be written as the sum of both variances as $2R$. The innovation error $e_i$ can now be expressed for each sensor by a standard normally distributed variable $z_e$ instead of $e_i$ and $R_i$.

As shown in Figure 1 (Step 3: Trigger parameter and threshold), substituting $z_e$ instead of $e_i$ and $R_i$ in Equation (19) yields for the case of accelerometers $\gamma \approx 2mz_e^2$, which solely depends on the number of sensors $m$, the variable $z_e$, and the Scalar 2, which results from the choice of accelerometers. The scalar changes to $\gamma \approx mz_e^2$ for displacement and velocity sensors. Consequently, a parameter $\delta$ is introduced in the calculation of the trigger parameter as

$$\gamma \approx \delta mz_e^2$$

with $\delta = 1$ for displacement and velocity sensors and $\delta = 2$ for accelerometers, respectively. Accordingly, the corresponding threshold is given by

$$\gamma_0 = \delta mz_0^2$$

Now, since $\delta$ and $m$ are system dependent preset parameters, $z_0$ governs the threshold based on the exceeding probability of the Gaussian distribution. For the variable $z_0 = 3\sqrt{2}$, which corresponds to an exceeding probability of 99.998%, the threshold yields $\gamma_0 = 72$ for two accelerometers. This threshold value will be used in the performance studies in Section 3.1. The presented threshold, Equation (22), is valid for monitoring systems consisting of either only displacement and velocity sensors or only accelerometers. Considering mixed sensor types in the monitoring system, instead, the threshold has to be derived individually as shown above.

After detecting abrupt system changes, a localization algorithm has to follow. For this purpose, the localization scheme of Bisht and Singh is extended as shown in Figure 2. The flowchart of the proposed adaptive UKF presents besides the detection of system changes the adaptation step, in particular, consisting of localization and covariance adaptation. In the following paragraph, the subscript $\theta$ denotes covariances $P$, which are dependent on the system parameters $\theta$ only, and subscript $x$ analogously denotes the state dependent covariances only. To localize system changes, an additional UKF estimation step is shown in Figure 2 for the next time step $k+1$. The state covariance component $\hat{P}_{\theta,k+1}[i,i]$ is set to $P_{\text{adapt}}$ for each $i = 1, \ldots , n$ individually, where $P_{\text{adapt}}$ is a high constant covariance value, which is introduced to
increase the sensitivity of the parameter identification. Since only stiffness degradations are expected, each parameter $\theta_i$ with corresponding index $i$ is additionally decreased by 5\%, different than previous studies, in order to facilitate the localization. Accordingly, for each index $i$, now a different set of $\tilde{P}_{k+1}$ and $\tilde{\theta}_{k+1}$ exists. For each of these sets and otherwise unchanged conditions a single calculation step of the UKF is executed and finally the trigger parameter $\gamma_i$ of Equation (18) is recalculated. Now assuming that the lowest value of $\gamma_i$ describes the lowest system error and, thus, yields the best estimate for the system properties, the related index $i$ belongs to the degrading parameter $\theta_i$. For the next simulation step $k+1$, solely the state covariance component $\tilde{P}_{k+1}[i, i]$ of the localized index $i$ is substituted by the new state covariance value $P_{\text{adapt}}$, which has to be chosen in advance and is highly dependent on the chosen system noise covariance $Q$ and the present measurement noise covariance $R$. The parameter has to be chosen as high as possible to enable a system identification of abrupt changes. Section 3.3 will give a detailed simulation example of how to choose $P_{\text{adapt}}$.

2.3 Application of the system identification method on MDoF + TMD systems

The theory is introduced using the example case, at which a TMD is attached at the top DoF of a MDoF frame structure, Figure 3. The structure is instrumented with a monitoring system and the proposed system identification method will be implemented on this system to obtain its abrupt stiffness changes. Thus, the equation of motion with stiffness, damping, and mass matrices $(K, C, M)$ can be set up for a seismic ground excitation distributed equally over the height of the system as

$$M \ddot{x}(t) + C \dot{x}(t) + K x(t) = M \Gamma \ddot{y}_g(t) \quad \text{with} \quad \Gamma = [1 \ldots 1]^T.$$  \hspace{1cm} (23)

The stiffness matrix $K$ is assumed to be time variant with abrupt changes assembling a nonlinear structural behavior. Whereas the to be identified stiffness parameters $k_1(t), \ldots, k_n(t)$ are time variant, the damping constants $c_1, \ldots, c_n$ and masses $m_1, \ldots, m_n$ remain constant during the simulation as well as the initially adjusted TMD parameters $k_d$, $c_d$, $m_d$.

A monitoring system is set up to observe the actual system responses. As it is shown in Figure 3, sensors are assumed to be placed on each DoF $i = 1, \ldots, n$. However, for other systems, e.g. high-rise structures with a large number of DoFs, a different sensor layout with a reduced amount of sensors is possible with a reduced accuracy. For the monitoring system all motion sensors (e.g., displacement, velocity, acceleration, forces) are possible. In the scope of this paper, accelerometers are used only, since they are the most commonly used sensor types for vibration measurements. Sensor properties, such as offset and RMS value of the measurement noise, are required for later system identification steps and the adaptation step, in particular. Using the response $y$ and input signals $u$, the adaptive system identification method, introduced in Sections 2.1 and 2.2, is applied for the joint state and parameter estimation computing the corrected and estimated state vectors, consisting of displacements, velocities, and system stiffnesses.

The challenge for identification of highly damped systems (e.g., TMD) is to deal with rapidly decreasing vibration amplitudes compared to lightly damped systems. For such a system, a system identification is, therefore, only possible during a significantly smaller time period. In particular, for strongly (nonclassically) damped systems, special attention has to be
paid on the sensitivity or filter settings, respectively, of the system identification as well as the used mathematical models. This aspect will be elaborated in Section 3 by three parametric studies on a MDoF structure with and without TMD.

2.4 Modeling of the MDoF + TMD systems

As described in Section 2.1, the UKF requires a state equation $f(\cdot)$ and an observation equation $h(\cdot)$, which are herein assumed as stepwise linear state-space representations. Starting with the state equation, the equation of motion of the previously in Section 2.3 described system can be rewritten to a differential equation of 1st order as follows:

$$\dot{x}(t) = A(t)x(t) + Bu(t) + w(t).$$  \hspace{1cm} (24)

The system matrix $A(t)$ and input matrix $B$ contain the nonlinear system properties and information of input signals, respectively. An additive noise $w(t)$ is added to the state equation describing the system noise, including errors of the mathematical model. The input signal $u(t)$ is given as ground acceleration

$$u(t) = \Gamma x_g(t)$$  \hspace{1cm} (25)

with the incidence vector $\Gamma$ from Equation (23).

The monitoring system is transferred to the observation equation, where the output matrix $C$ and transition matrix $D$ describe the sensor layout of number, type, and position. The result is finally enhanced by the noise component $v(t)$, representing measurement noise:

$$y(t) = Cx(t) + Du(t) + v(t).$$  \hspace{1cm} (26)

Both state and measurement equations are given in continuous time so far. However, the system identification method requires a discrete time formulation, since the measurement data has a discrete form. A discretization can be realized by many methods, e.g. Euler or 4th-order Runge Kutta method. Although every method has different characteristics and calculation rules, all of them can be compared by the order of convergence $p$, defined by discretization errors. Higher orders of convergences generally yield more accurate results but also have higher computational costs. In case of the explicit Euler method, the order of convergence is $p = 1$ and for 4th order Runge Kutta $p = 4$, respectively. In this paper, however, the discretization is done by a Taylor expansion developed from the analytical solution with orders of convergence $p = 1 - 4$. This approach is preferred here, since all $p = 1 - 4$ easily can be implemented based on one model only allowing a parametric study of the influence of model accuracy, Section 3.4. The discretization yields the matrices $A_d$ and $B_d$:

$$A_d = e^{AT_s} = \sum_{i=0}^{\infty} \frac{1}{i!} A^i T_s^i \approx I + AT_s + \ldots + \frac{1}{p!} A^p T_s^p,$$  \hspace{1cm} (27)
Using the above discretization, the state and observation equation can be easily transformed into the discrete domain assuming real sampling, with the sampling time $T_s$:

$$x_{k+1} = A_{d,k}x_k + B_{d}u_k + w_k, \quad y_k = Cx_k + Du_k + v_k.$$  \hfill (29) 

$$\begin{align*}
B_d &= \int_0^{T_s} e^{\mathbf{A}t} \mathbf{B} dt = \sum_{i=0}^{\infty} \frac{1}{(i+1)!} A^i BT_s^{i+1} 
&\approx 0 + BT_s + \ldots + \frac{1}{(p+1)!} A^p BT_s^{p+1}. \quad (28)
\end{align*}$$

$3$ | PERFORMANCE STUDIES

In this section, investigations on a 2-DoF structure with and without TMD will be presented under seismic, white noise and impulse excitations. Detailed parameter studies are done regarding the system accuracy and convergence behavior of the system identification method considering TMD influence and abrupt stiffness changes of the structure. Recommendations to the filter and model setup are given for the investigated systems.

3.1 | Description of the investigated MDoF + TMD systems

Two different systems are investigated: solely a 2-DoF structure as well as the same 2-DoF structure with a TMD attached at the top DoF, Figure 4. Stiffness, damping, and mass matrices ($\mathbf{K}, \mathbf{C}, \mathbf{M}$) can be set up corresponding to the system parameters,\textsuperscript{26} listed in Table 1, as:

$$\begin{align*}
\mathbf{K} &= \begin{bmatrix} k_1 + k_2 & -k_2 & 0 \\
-k_2 & k_2 + \kappa_d & -k_d \\
0 & -k_d & k_d \end{bmatrix}; \\
\mathbf{C} &= \begin{bmatrix} c_1 + c_2 & -c_2 & 0 \\
-c_2 & c_2 + \kappa_d & -c_d \\
0 & -c_d & \kappa_d \end{bmatrix}; \\
\mathbf{M} &= \begin{bmatrix} m_1 & 0 & 0 \\
0 & m_2 & 0 \\
0 & 0 & m_d \end{bmatrix}. \quad (31)
\end{align*}$$

The masses ($m_1, m_2$) and the damping constants ($c_1, c_2$) are time invariant values. The stiffness values ($k_1, k_2$) consider abrupt system changes of $10\%$ at each DoF corresponding to a $5\%$ decrease of the $1$st natural frequency, which are realistic values to observe during load cases, such as earthquakes. The initial stiffness values are chosen as $k_1 = 12 \text{ kN/m}$ and $k_2 = 10 \text{kN/m}$. For each load case, abrupt stiffness changes appear at a defined time step $t_f$ according to high interstory drifts between the individual DoFs.

The attached TMD is defined by the time invariant parameters $\kappa_d, \kappa_d, m_d$, which are chosen in the initial time step. The undamaged 2-DoF structure has a natural frequency of $f_1 = 0.33 \text{ Hz}$ and a damping ratio of $D_1 = 0.92\%$ for the $1$st eigenmode and analogously for the $2$nd eigenmode $f_2 = 0.84 \text{ Hz}$ and $D_2 = 2.49\%$. Accordingly, the structure (2-DoF) is assumed to be classically damped and with supplementary damping of the TMD the damping matrix of the system (2-DoF + TMD) becomes nonclassically damped. Table 1 provides the remaining natural frequencies of the 2-DoF + TMD system. To adjust the damper parameters, several possible approaches are proposed in the literature. In this paper, we
focus on the system identification and use the classical approach of Warburton.\textsuperscript{27} Assuming the structure to be lightly damped ($D_1 = 0.92\%$), an application of Warburton is reasonable. The TMD is tuned to the 1st natural frequency of the 2-DoF structure. Therefore, the mass ratio $\mu$, describing the relation of damper mass $m_d$ and generalized mass of the 1st mode $m_1$; the optimal damper frequency $f_{opt}$, dependent on $f_1$ and $\mu$; and finally the optimal damping ratio $D_{opt}$, dependent on $\mu$, are calculated:

$$\mu = \frac{m_d}{m_1} = 0.076; \quad f_{opt} = f_1 \sqrt{1 - \frac{2}{1 + \mu}} = 0.30 \text{ Hz}; \quad D_{opt} = \sqrt{\frac{\mu(1 - \frac{\mu}{4})}{4(1 + \mu)(1 - \frac{\mu}{2})}} = 13.42 \%. \quad (32)$$

Subsequently, all damper parameters can be calculated using fundamental SDoF relations, Table 1. The detuning effect of the TMD due to abrupt stiffness changes can be observed in Figure 5, where for each presented damage pattern, the corresponding deformation response factor is shown. Here, the structure is excited with the harmonic force $F = F_0 \sin(\Omega t)$ applied on the top floor. Accordingly, the TMD is tuned to Den Hartog's criteria.\textsuperscript{28}

In a final step, $K$, $C$, and $M$ are transformed into the state-space representation. The time variant system matrix $A(t)$ and the input matrix $B$ can be set up according to the nonlinear system properties. We formulate the representation for a ground acceleration $\ddot{x}_g$ as input. Furthermore, the output matrix $C$ and transition matrix $D$ can be calculated as follows, describing a monitoring system of two accelerometers on both DoFs $x_1$ and $x_2$ and one accelerometer for the ground motion $\ddot{x}_g$:

$$A(t) = \begin{bmatrix} 0_{3 \times 3} & I_{3 \times 3} & 0_{3 \times 2} \\ -M^{-1}K(t) & -M^{-1}C & 0_{3 \times 2} \\ 0_{2 \times 3} & I_{2 \times 3} & 0_{2 \times 2} \end{bmatrix}; \quad B = \begin{bmatrix} 0_{3 \times 3} \\ I_{2 \times 3} \\ 0_{2 \times 3} \end{bmatrix}; \quad C = \begin{bmatrix} -M^{-1}K & -M^{-1}C \end{bmatrix}; \quad D = \begin{bmatrix} I_{2 \times 2} \end{bmatrix} \quad (33)$$

with the input vector $u$ and the output vector $y$:

$$u = \begin{bmatrix} \ddot{x}_g \\ \ddot{x}_g \\ \ddot{x}_g \end{bmatrix}; \quad y = \begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_1 \\ \ddot{x}_1 \end{bmatrix}. \quad (34)$$

### Table 1

| System parameter | (a) 2-DoF | (b) 2-DoF + TMD |
|------------------|-----------|-----------------|
| $m_1$            | $t \leq t_f$ | $t > t_f$ |
| $m_2$            | $t \leq t_f$ | $t > t_f$ |
| $m_d$            | - | 0.1$t$ |
| $k_1(t)$         | 12 kN/m | 10.8 kN/m |
| $k_2(t)$         | 10 kN/m | 9 kN/m |
| $k_d$            | - | 0.36 kN/m |
| $c_1$            | 0.1 kN\(s\)/m | 0.1 kN\(s\)/m |
| $c_2$            | 0.1 kN\(s\)/m | 0.1 kN\(s\)/m |
| $c_d$            | - | 0.051 kN\(s\)/m |
| $f_1$            | 0.33 Hz | 0.31 Hz |
| $f_2$            | 0.84 Hz | 0.79 Hz |
| $f_3$            | - | 0.84 Hz |

**FIGURE 5** Detuning effect: Comparison of deformation response factors for undamaged and damaged 2-DoF structures with TMDs tuned according to Den Hartog’s criteria.
Both state and observation equations are calculated for the joint state and parameter estimation, that is, the state vector $\mathbf{x}$ extends to an augmented state vector $\mathbf{x}^a$, including displacements, velocities, and stiffnesses of the system:

$$
\mathbf{x}^a = \begin{bmatrix} x_1 & x_2 & x_d & \dot{x}_1 & \dot{x}_2 & \dot{x}_d & k_1 & k_2 \end{bmatrix}^T.
$$  \tag{35}

For the investigations, one far-field and one near-field earthquake acceleration history is considered according to those input signals in Ohtori et al. 29:

- **El Centro**: N–S component recorded at the Imperial Valley Irrigation District substation in El Centro, California, during the Imperial Valley, California earthquake of May 18, 1940, Figure 6 (left).
- **Northridge**: N–S component recorded at Sylmar County Hospital parking lot in Sylmar, California, during the Northridge, California earthquake of January 17, 1994, Figure 6 (right).

Moreover, a white noise input with an RMS value of $0.57 \text{ m/s}^2$ and an impulse load of $80 \text{ m/s}^2$ at $t = 2 \text{ s}$ are investigated. In all studies, an additive white Gaussian noise of RMS $= 0.01 \text{ m/s}^2$, approximately 2% RMS noise of the El Centro earthquake, is added to input as well as output measurement signals. The simulations are performed with a sampling time of $T_s = 0.02 \text{ s}$.

### 3.2 Study 1: Identification of the augmented state including abrupt stiffness changes

The chosen filter setup of the identification algorithm applied herein is presented in Table 2. The state covariance $\mathbf{P}$ is chosen corresponding to the results of the Parametric Study 2, which will be presented in Section 3.3. According to the results of the parametric study, the new state covariance value after adaptation is chosen as high as possible as $\mathbf{P}_{\text{adapt}} = 10^0$.

The initial covariance value, however, is chosen as low as possible $\mathbf{P}_0 = 10^{-6} \mathbf{I}_{8 \times 8}$, so that the identification algorithm does not show oversensitive reactions regarding the stiffness estimation before the abrupt stiffness change. The system noise covariance $\mathbf{Q}$ is chosen corresponding to the results of the Parametric Study 3, which will be presented in Section 3.4. Both 2-DoF and 2-DoF + TMD systems are modeled based on 3rd-order Taylor expansion. Effects of the used order for the Taylor expansion, in particular, will be also shown in Section 3.4. The measurement noise covariance $\mathbf{R}$ is calculated from the square of the RMS value for the present noise, according to 2% RMS noise of El Centro earthquake. The initial stiffness estimations $\hat{k}_1$ and $\hat{k}_2$ correspond to the real stiffness values $k_1$ and $k_2$, as shown in Table 1.

In Figure 7 (left), the time history of the trigger parameter $\gamma$ is shown. The curve is calculated by the previously, in Section 2.2, introduced Equation (18). In addition, the right diagram shows the time history for a time window around the abrupt stiffness change. The trigger parameter shows a peak value corresponding to the time step of the abrupt stiffness change $t = 9 \text{ s}$. After comparing the threshold $\gamma_0$, which is calculated from Equation (22), the state covariance is adapted. In Figure 7, three different thresholds $\gamma_0 = 10.8$, $\gamma_0 = 26.5$, and $\gamma_0 = 72$ are shown according to the exceeding probability of $90\%$, $99\%$ and $99.998\%$ respectively. For both probabilities $90\%$ and $99\%$ the threshold is exceeded several times with significantly high values (e.g., $\gamma = 53$ at $t = 52 \text{ s}$). Best result is achieved with the threshold value of $\gamma_0 = 72$, which is exceeded only during the abrupt stiffness change.

In the first part of the study, we consider the El Centro earthquake excitation, including a stiffness degradation of $10\%$ at the 1st DoF after $t = 9 \text{ s}$ due to large story drift. Figure 8 compares the true values of the motion (displacement,
velocity, and acceleration) of both DoF of the structure and the stiffness time histories with those time histories, which are estimated by the proposed system identification method. Both cases with and without TMD are presented in the graphics. The abrupt stiffness change can be directly seen from the time histories of $k_1$ and $\hat{k}_1$. Both estimated and true values match with each other. The abrupt stiffness change is identified for both systems.

In the second part of the study, we enhance our investigation by considering besides the both El Centro and Northridge earthquakes also impulse and white noise excitations. Furthermore, we allow an abrupt stiffness change on the 2nd DoF as well. The occurrence times also in this second part of the study correspond to the interstory drift between the 1st and 2nd DoF. In Figures 9 and 10, the corresponding time histories of the estimated and true values of the displacements and stiffness values are shown. A high accuracy of the estimated results is also observed here.
FIGURE 9  Study 1: Time histories of estimated (⋯) and true values of displacement and stiffness of both DoFs during El Centro (a) and Northridge (b) earthquake. Abrupt stiffness changes at both 1st and 2nd DoFs

TABLE 3  Study 2: Filter setup of the system identification method

| Filter parameter | Value                  | Scaling factor | Value |
|------------------|------------------------|----------------|-------|
| $P_0$            | $10^{-8}I_{3x3}$       | $\alpha$       | 0.001 |
| $Q$              | $10^{-9}I_{3x3}$       | $\beta$        | 2     |
| $R$              | $10^{-4}I_{3x3}$       | $\kappa$       | 0     |

3.3  Study 2: Effects of the state covariance

The supplementary damping introduced by the TMD as well as abrupt stiffness changes of the structure shorten the time window, in which the proposed system identification method must complete its estimation. In this regard, the most powerful parameter is the state covariance $P$. By increasing the state covariance, the reaction time of the identification method can be reduced. On the other hand, too high $P$ values can decrease the estimation accuracy. To clarify this effect, this study performs calculations with different constant $P$ values between $10^{-8}$ and $10^0$. Further filter parameters are shown in Table 3. Calculations are performed using 3rd-order Taylor expansion-based models of 2-DoF and 2-DoF + TMD systems under the Northridge earthquake. The initial stiffness estimates of the structure are assigned as $\hat{k}_1 = 14.4 \text{ kN/m}$ and $k_2 = 12 \text{ kN/m}$, which are 20% higher than the true stiffness values of $k_1 = 12 \text{ kN/m}$ and $k_1 = 10 \text{ kN/m}$.

Figure 11 (left) shows the true and estimated values of the 1st DoF stiffness $k_1$ and $\hat{k}_1$. On the right side, in Figure 11, we see the true and estimated values of the displacement of the 1st DoF $x_1$ and $\hat{x}_1$. The displacement time histories are shown for the selected state covariance values of $10^{-8}$ and $10^0$. From the comparison of the displacement time histories, the effect of the TMD can be clearly observed from the short vibration duration. Already after 35 s, the vibration of the 2-DoF + TMD...
The system is reduced below 0.01 m. At the same time step, the vibration of the 2-DoF structure without TMD still continues with an amplitude of 0.30 m. This difference governs the required accuracy level of the identification method.

In the time histories of the stiffness, we observe, in particular for lower $P$ values, that as soon as the vibrations vanish the estimated stiffness of the 2-DoF + TMD system converges to a constant value, which is far away from the real stiffness value. For instance, the estimated stiffness value of 2-DoF + TMD system is for $P = 10^{-8}$ approximately 14 kN/m, which does not match the true stiffness value of 12 kN/m. For the same $P$ value of $10^{-8}$, the estimated stiffness of the 2-DoF structure without TMD converges slowly to the true stiffness value as the structure is still continuing to oscillate.

By increasing the $P$ value, we observe from the results that both systems can be identified with high accuracy. For the 2-DoF + TMD system, the correct stiffness value is estimated with $P = 10^{-4}$. On the other hand, as stated before, the 2-DoF structure is estimated already with $P = 10^{-8}$. A further increase of the $P$ value causes the system identification method to behave oversensitive and the estimated stiffness course begins for both systems to fluctuate. With high $P$ values, we observe at the beginning of the both time histories initially underestimated stiffness values.

Accordingly, the $P$ value must be chosen depending on the expected abrupt changes and the type of the system, which is a challenge for all UKF-based system identification methods. To overcome this effect, as introduced in Section 2.2, the proposed parameter identification algorithm tunes the state covariance in an adaptive manner.
Figure 11: Study 2: Time histories of estimated stiffness ($\hat{k}_1$) of 1st DoF (left). Time histories of estimated ($\hat{x}_1$) and true ($x_1$) displacement of 1st DoF (right). State covariance values $P_0$ are varying. Calculations are performed for (a) 2-DoF and (b) 2-DoF + TMD systems under Northridge earthquake.

Table 4: Study 3: Discretizations of the system matrix $A_d$ and the output matrix $B_d$ with up to 4th-order Taylor expansion

| Matrix | Taylor expansions |
|--------|-------------------|
| $A_d$  | $I + AT_1 + \frac{1}{2}AT_2^2 + \frac{1}{6}AT_3^3 + \frac{1}{24}AT_4^4$ |
| $B_d$  | $BT_1 + \frac{1}{2}ABT_2^2 + \frac{1}{6}ABT_3^3 + \frac{1}{24}ABT_4^4$ |

Table 5: Study 3: Filter setup of the system identification method

| Filter parameter | Value | Scaling factor | Value |
|------------------|-------|----------------|-------|
| $P_0$            | $I_{8x8}$ | $\alpha$ | 0.001 |
| $Q$              | $10^{-3}I_{8x8}$ | $\beta$ | 2 |
| $R$              | $10^{-4}I_{3x3}$ | $\kappa$ | 0 |

3.4 Study 3: Modeling effects

The accuracy of recursive system identification methods is directly related with the accuracy of the chosen mathematical model describing the system properties. The error inherent in the chosen mathematical model is considered in the proposed UKF-based identification method by the system noise covariance $Q$. However, due to additional damping of TMDs, the accuracy sensitivity of the identification process increases. Therefore, $Q$ struggles to realize the desired identification efficiency. Accordingly, the necessity of an accurate mathematical model increases for MDoF + TMD systems.

In this section, to show the modeling effect, four mathematical models are investigated using $A_d$ and $B_d$ discretization, introduced in Section 2.4, by Taylor expansions of 1st to 4th order of convergence, Table 4. During the study, different $Q$ matrices, which are constant over simulation time, are introduced varying from $10^{-8}I_{8x8}$ to $10^{-15}I_{8x8}$. Two load scenarios are investigated: the El Centro and the Northridge earthquakes. To determine the accuracy of the final stiffness estimation, the deviation parameter $\Delta k_i$ is introduced, which defines the percentage deviation of the final estimated stiffness $\hat{k}_i$ to the true value $k_i$:

$$\Delta k_i = \frac{|k_i - \hat{k}_i|}{k_i}[\%].$$

In this study, the initial stiffness estimates are chosen to be $k_1 = 14.4 \text{ kN/m}$ and $k_2 = 12 \text{ kN/m}$, which are 20% higher than the true stiffness values of $k_1 = 12 \text{ kN/m}$ and $k_1 = 10 \text{ kN/m}$. Accordingly, a nonlinear parameter identification is required. Besides this fact, in this study, the structure is assumed to behave linearly during the earthquake excitation without any abrupt stiffness changes. All remaining filter setup parameters are shown in Table 5.

Figure 12 compares for the El Centro earthquake the estimated time histories of the 1st DoF stiffness $\hat{k}_1$ with the true values $k_1$. Two different system noise covariance levels $Q[i,i]$ are shown. At $Q[i,i] = 10^{-14}$ (left), the 1st-order Taylor expansion-based model of the 2-DoF + TMD system causes larger deviations than the model of the 2-DoF without TMD. These results show the increased sensitivity of the system identification due to supplementary TMD. By increasing the covariance level to $Q[i,i] = 10^{-9}$ (right) the deviation reduces. In Figure 12, the other investigated higher order models do not show any dependency with the covariance level.
The study is expanded in Figure 13 for further $Q_{[i,i]}$ values. Here, we observe that the Taylor 1st-order expansion-based model of the 2-DoF structure allows for system noise covariance level values higher than $Q_{[i,i]} = 10^{-9}$ a high accuracy system identification with $\Delta k_1 < 0.001\%$. With the same order of the model, the system identification accuracy of the 2-DoF + TMD system also increases by increasing the system noise covariance. However, after reaching its minimum
deviation at $Q[i, i] = 10^{-9}$ with increasing system noise covariance, the deviation of the stiffness estimation increases again. This effect exists invisible small also for the 2-DoF structure without TMD.

Corresponding to the results of Figure 12, also in Figure 13, we see again for higher order models that the accuracy is independent from the system noise covariance level. Accordingly, as introduced before, we emphasize also with these results the necessity of higher order mathematical models for the identification MDoF + TMD systems.

In Figure 14, the study is repeated for the near-field Northridge earthquake. The performance results of the investigated models conform with the conclusions of the in Figures 12 and 13 shown El Centro results. Also, here, the course of the deviation parameter $\Delta k_1$ shows for the 1st-order Taylor expansion model of the 2-DoF structure a stable accuracy after a certain system noise level. On the other hand, for the same order 2-DoF + TMD model the deviation $\Delta k_1$ fluctuates depending on the system noise level. For the estimated 2nd DoF stiffness $\hat{k}_2$, we get similar results, which we do not include here for the sake of brevity.

4 | CONCLUSIONS

In this paper, for MDoF structures with TMDs a recursive system identification method is presented, which is able to detect and localize abrupt stiffness changes during sudden events, such as earthquakes. The method enhances the UKF by a new adaptation formulation, which is modifying the state covariance initiated by a trigger parameter. The proposed adaptation algorithm operates in a recursive manner and calculates the trigger parameter depending on the innovation error, which is normalized by the measurement noise covariance. A constant threshold is formulated based on the sensors. Three parametric studies are conducted on a 2-DoF + TMD system to investigate the performance of the system identification method. In the first study, earthquake, impulse, and white noise excitations are applied. Single and combined abrupt stiffness changes of the DoFs of the structure are simulated. Time histories of estimated and true values of structural motion and stiffness changes are compared. Results show that the proposed identification method is able to detect and localize the abrupt stiffness changes. The estimated state conforms with the true values. The second study investigates the effects of the state covariance. On the 2-DoF + TMD system, an earthquake excitation is applied. An increase of the state covariance improves the parameter estimation performance. However, after a certain value, a further increase causes the identification method to behave oversensitive and loose its accuracy. The results conclude the necessity of an adaptive formulation of the state covariance as applied in the proposed approach. In the third study, the effects of the modeling accuracy are investigated on the 2-DoF + TMD system under earthquake excitation. Besides the effects of the system noise covariance, the study considers also the effects of convergence orders for discretization using Taylor expansion. The results confirm that the identification of abrupt stiffness changes requires a high-level accuracy of the method, in particular, for the identification of MDoF structures with supplementary TMDs.

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