On the impossibility of non-static quantum bit commitment between two parties

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Recently, Choi et al. proposed an assumption on Mayers-Lo-Chau (MLC) no-go theorem that the state of the entire quantum system is invariable to both participants before the unveiling phase. This means that the theorem is only applicable to static quantum bit commitment (QBC). This paper find that the assumption is unnecessary and the MLC no-go theorem can be applied to not only static QBC, but also non-static one. A non-static QBC protocol proposed by Choi et al. is briefly reviewed and analyzed to work as a supporting example. In addition, a novel way to prove the impossibility of the two kinds of QBC is given.

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I. INTRODUCTION

Bit commitment allows a sender (Alice) to commit a bit $b \in \{0,1\}$ to a receiver (Bob) in the following way: 1) Alice can not change the value of the committed bit after the commitment phase (binding property); 2) Bob can not obtain the value of the committed bit before the unveiling phase (concealing property). Bit commitment is an important cryptographic primitive and can be used as a building block for some other cryptographic protocols, such as coin flipping [1], oblivious transfer [2], zero-knowledge proof [3], and multiparty computation [4].

A secure bit commitment protocol should satisfy the binding property and the concealing property at the same time. However, unconditionally secure classical bit commitment protocols do not exist. There are only some unconditionally binding and computationally concealing bit commitment protocols [5] or unconditionally concealing and computationally binding commitment ones [6].

Since unconditionally secure quantum key distribution protocols were proposed in [7],[8], some quantum bit commitment (QBC) protocols have been proposed with the hope that QBC can provide unconditional security [9–12]. The most famous one is the bit commitment protocol proposed by Brassard et al. in [11], which was claimed to be unconditionally secure. Unfortunately, the protocol was showed to be insecure afterwards [13]. Furthermore, Mayers, Lo and Chau proved that general secure QBC protocols are impossible [14],[15], which is called MLC no-go theorem.

Although discovery of the MLC no-go theorem depressed much study on QBC protocols, researchers try to design secure QBC by adopting certain restrictions or weakening some security requirements. For instance, Kent proposed two bit commitment protocols based on special relativity theory [16,17]; Damgård et al. designed a secure QBC protocol in a bounded quantum-storage model [18]; Hardy and Kent gave a secure cheat sensitive QBC protocol ensuring that if either a committer or a committee cheat, the other can detect it with a nonzero probability [19]. Besides these, secure QBC protocols are implemented in a noisy-storage model under the assumption that the dishonest party can not access large-scale reliable quantum storage [20].

Recently, Choi et al. proposed a secure non-static QBC protocol with help of a trusted third party (TTP) [21] and pointed out that the MLC no-go theorem is based on an assumption that the whole quantum state is static before Alice reveals the committed bit. That is to say, the MLC no-go theorem was thought to be adapted to static QBC only. D’Ariano et al. also hold this opinion and gave another strengthened and explicit proof involving impossibility of some non-static QBC protocols [22]. However, we find that the assumption given by Choi et al. in [21] is unnecessary and non-static QBC is also impossible just due to the MLC no-go theorem. Although Choi et al. proposed a secure non-static QBC protocol by adopting a TTP [21], the protocol is still different from a general two-party QBC protocol and is somewhat similar to a quantum secret sharing protocol. Interestingly, the non-static QBC protocol without the TTP can just serve as an example to show that the MLC no-go theorem can be applied to non-static QBC also. In addition, we prove the impossibility of the two kinds of QBC in a different way: prove any binding QBC protocols is not concealing, while the related proofs proposed in [14,15,21,22] show any concealing protocols are not binding.

The rest of this paper is organized as follows. In the next section, it will be shown that the assumption of the MLC no-go theorem given by Choi et al. is unnecessary and the MLC no-go theorem can be adapted to both static QBC and non-static QBC. The non-static QBC protocol proposed by Choi et al. is reviewed and analyzed in Sec. III. Then the impossibility of QBC is proved in Sec. IV in a different way. The last section concludes the paper.
II. APPlicability of the MLC No-GO theorem to non-static QBC

In [13, 12], the MLC no-go theorem was proved in the following basic idea. Suppose the initial states of Alice and Bob are |b⟩_A (b ∈ {0, 1}) and |ϕ⟩_B, respectively, and let U_{AB} denote all the algorithms that Alice and Bob may implement. Then the final quantum state shared by Alice and Bob is |ϕ_b⟩_{AB} = U_{AB}(|b⟩_A ⊗ |ϕ⟩_B). If a QBC protocol is perfectly concealing, namely

\[ ρ^B = Tr_A(|ϕ_b⟩_{AB}⟨ϕ_b|) = Tr_A(|ϕ_1⟩_{AB}⟨ϕ_1|) = ρ^B, \]

then there exists a local unitary transformation S_A satisfying

\[ (S_A ⊗ I)U_{AB}(|b⟩_A ⊗ |ϕ⟩_B) = U_{AB}(1 - b⟩_A ⊗ |ϕ⟩_B) \]

according to Gisin-Hughston-Jozsa-Wootters theorem given in [23, 24]. So, by postponing measurements and implementing local unitary operations, Alice can change the value of the committed bit arbitrarily without being discovered by Bob. If the QBC protocol is supposed to be unconditionally concealing, similar results can be derived also.

However, Choi et al. observed that the local unitary operation S_A performed by Alice is related to Bob’s initial state |ϕ⟩_B [21]. If |ϕ⟩_B is random and unknown to Alice, she can find a suitable local unitary operation to change the committed value. Thus, a necessary assumption of the MLC no-go theorem is that the state of quantum system should be static to both participants. This means the MLC no-go theorem was considered to be applicable to static QBC only.

As shown above, the proof of the MLC no-go theorem is based on the following strategy: a QBC protocol is first supposed unconditionally concealing and it is then proved that unconditionally binding is impossible. So, Theorem 1 can be obtained, which means that the assumption of the MLC no-go theorem suggested by Choi et al. is unnecessary.

**Theorem 1** The MLC no-go theorem is also applied to non-static QBC.

**Proof:** Assume |ϕ⟩_B is random and unknown to Alice. Let U_{AB} = \( ∑_{ijkl} a_{ijkl} |i⟩_A |j⟩_B A_k |l⟩ \) and |ϕ⟩_B = \( ∑_m c_m |m⟩_B \), then we have

\[ |ϕ_b⟩_{AB} = U_{AB}(|b⟩_A ⊗ |ϕ⟩_B) = ∑_{ijkl} a_{ijkl} |i⟩_A |j⟩_B A_k |b⟩_A |l⟩ \left( ∑_m c_m |b⟩_A |m⟩_B \right) = ∑_{ijkl} a_{ijkl} |b⟩_A |l⟩_B, \]

and

\[ ρ^B = Tr_A(|ϕ_b⟩_{AB}⟨ϕ_b|) = Tr_A(|ϕ_1⟩_{AB}⟨ϕ_1|) = ρ^B \]

which is the MLC no-go theorem applicable to non-static QBC.

Suppose a non-static QBC protocol is perfectly concealing, then ρ^B and ρ^B should be identical for any |ϕ⟩_B = \( ∑_m c_m |m⟩_B \), i.e.

\[ ρ^B = ∑_{ijkl} a_{ijkl}^* |c^*⟩_{ijkl} |j⟩_B ⟨q| = ∑_{ijkl} a_{ijkl}^* |c^*⟩_{ijkl} |j⟩_B ⟨q| = ρ^B. \]

Since the above formula always holds for any |ϕ⟩_B, we have

\[ a_{ijkl}^* |c^*⟩_{ijkl} = a_{ijkl}^* |c^*⟩_{ijkl}. \] (1)

Let S_A = \( ∑_{xy} s_{xy} |x⟩_A ⟨y| \), where

\[ \begin{align*}
    s_{xy} &= 0, & \text{if } x \neq y, \\
    s_{xx} &= 0, & \text{if } a_{ijkl}^* = 0 \text{ for any } q \text{ and } r, \\
    s_{xx} &= a_{ijkl}^* & \text{otherwise.}
\end{align*} \]

Equation (11) makes S_A always satisfy

\[ (S_A ⊗ I_B)|ϕ_b⟩_{AB} = \left( ∑_{xy} s_{xy} |x⟩_A ⟨y| ∑_k |k⟩_B⟨k| \right) \left( ∑_{ijkl} a_{ijkl} c_{ijkl} |i⟩_A |j⟩_B \right) = ∑_{ijkl} a_{ijkl} c_{ijkl} |i⟩_A |j⟩_B = ∑_{ijkl} a_{ijkl} |i⟩_A |j⟩_B = |ϕ_1⟩_{AB}. \]

for any |ϕ⟩_B. Thus the non-static QBC protocol is not binding. If assume it is unconditionally concealing, similar results can be obtained also.

III. REVIEW AND ANALYSIS OF CHOI ET AL’S NON-STATIC QBC PROTOCOL

In [21], Choi et al. proposed an unconditionally secure non-static QBC protocol with aid of a TTP. We briefly reviewed it in the following four phases and then show its simplified version can serve as an example demonstrating the MLC no-go theorem is applicable to non-static QBC.
a. Preparing phase: First, Alice and TTP share $N$ maximally entangled states in the form $|\psi^\pm\rangle_{AT} = \frac{|01\rangle - |10\rangle}{\sqrt{2}}$. This kind of entangled state has a special property, namely equation

$$|\psi^-\rangle_{AT} = (U \otimes U)|\psi^-\rangle_{AT}$$

holds up to the global phase for any unitary transformation $U$. Then, TTP applies random projection measurements represented as

$$M_i = \{|f_i\rangle_T (f_i), |f_i\rangle_T (f_i)\}$$

to its qubit of each entangled state $|\psi^-\rangle_{AT}$ for $i = 1 \sim N$. If the measurement outcome of TTP is $|f_i\rangle_B ((f_i)\rangle_B)$, then Alice’s measurement result should be $|\psi_i\rangle_A = |f_i\rangle_B ((f_i)\rangle_B)$. But TTP does not announce $M_i$ now, so Alice can not know the result $|\psi_i\rangle_A$.

b. Commitment phase: To committee the bit $b$, Alice applies the corresponding operations $P_i \in \{M, N, J, K\}$, where

$$M = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, N = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix},$$

$$J = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & i \\ 1 & -i \end{pmatrix}, K = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & i \\ -1 & i \end{pmatrix}. $$

If Alice chooses to commit $b = 0$, she randomly sends $M|\psi_i\rangle_A$ or $N|\psi_i\rangle_A$ to Bob. Otherwise, she sends $J|\psi_i\rangle_A$ or $K|\psi_i\rangle_A$ instead with the same probability. To guarantee the randomness, Alice introduces an auxiliary system $A'$ whose initial state is $|+\rangle_{A'} = \frac{1}{\sqrt{2}} |0\rangle_{A'} + |1\rangle_{A'}$. Then the state of the whole system $A'A$ is $|+\rangle_{\psi_i}$. If $b = 0$, Alice applies $|0\rangle_{A'} \langle 0 | \otimes M + |1\rangle_{A'} \langle 1 | \otimes N$ to $A'A$ to obtain

$$|\varphi_0\rangle_{A'A} = \frac{|0\rangle_{A'} \otimes M|\psi_i\rangle_A + |1\rangle_{A'} \otimes N|\psi_i\rangle_A}{\sqrt{2}}. $$

Otherwise, she implements $|0\rangle_{A'} \langle 0 | \otimes J + |1\rangle_{A'} \langle 1 | \otimes K$ on $A'A$ and gets

$$|\varphi_1\rangle_{A'A} = \frac{|0\rangle_{A'} \otimes J|\psi_i\rangle_A + |1\rangle_{A'} \otimes K|\psi_i\rangle_A}{\sqrt{2}}. $$

Due to the randomness of $|\psi_i\rangle_A$, the resulting state $|\varphi_0\rangle_{A'A}$ is also different, thus Alice cannot control the relationship between $|\varphi_0\rangle_{A'A}$ and $|\varphi_1\rangle_{A'A}$ without knowing the exact state $|\psi_i\rangle_A$.

c. Sustaining phase: In this phase, both Alice and Bob do nothing.

d. Revealing phase: Alice unveils all $P_i$’s, and then TTP opens all $M_i$’s and the corresponding measurement outcomes. After knowing all the information, Bob measures $P_i^o P_i \langle \psi_i \rangle$ with $M_i$ and compares all the measurement results with those TTP announced. If all the measurement outcomes are opposite, Alice is honest and the committed value has not been changed; otherwise Alice is dishonest.

In [21], Choi et al. claimed that the protocol is unconditionally secure. However, the usage of TTP makes the above non-static QBC protocol do not correspond to the fact that only two parties is involved in a general QBC protocol, although TTP plays a little role in offering quantum sources and is not involved in communication between two parties directly. In a way, the protocol is more like a quantum secret sharing protocol. For instance, the cooperation between Bob and TTP can get Alice’s committed value while one of them cannot. If the actions implemented by TTP are replaced by Bob, the protocol will not be secure. As shown by Choi et al. in [21], if the non-static QBC protocol without a TTP is perfectly concealing, a local unitary operator

$$S_A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

such that $J = aM + bN$ and $K = cM + dN$, can be used to freely change the committed bit. Thus it can be seen that Choi et al.’s non-static QBC protocol without a TTP can serve as an specific example to demonstrate that the MLC no-go theorem is applicable to non-static QBC also.

IV. PROOF OF IMPOSSIBILITY OF QBC BY ANOTHER WAY

Although non-static QBC between two participants is also impossible due to the MLC no-go theorem, it provides us another way to prove the impossibility of both non-static and static QBC.

Let us show the case on non-static QBC first. Premise of the proof of the MLC no-go theorem is that the QBC protocol is supposed to be perfectly concealing,

$$F(\rho_0^B, \rho_1^B) = 1,$$

or unconditionally concealing,

$$F(\rho_0^B, \rho_1^B) = 1 - \delta,$$

where $\delta > 0$.

For non-static QBC protocols, different initial states $|\varphi\rangle_B$ may lead to different $\rho^B$, so the value of $F(\rho_0^B, \rho_1^B)$ may vary and it is difficult to make the concealing property be satisfied. On the other hand, since $|\varphi\rangle_B$ is totally random and unknown to Alice, it is difficult for Alice to find an appropriate local unitary operator $S_A$ such that

$$F((S_A \otimes I)U_{AB}(|b\rangle_A \otimes |\varphi\rangle_B), U_{AB}(|1 - b\rangle \otimes |\varphi\rangle_B)) = 1$$

or

$$F((S_A \otimes I)U_{AB}(|b\rangle_A \otimes |\varphi\rangle_B), U_{AB}(|1 - b\rangle \otimes |\varphi\rangle_B)) = 1 - \delta$$

is satisfied for all $|\varphi\rangle_B$.

Thus, it is better to suppose a non-static QBC protocol is perfectly or unconditionally binding and then prove it cannot be perfectly or unconditionally concealing.
Assume a non-static QBC protocol is perfectly binding, i.e., there does not exist a local unitary operator $S_A$ such that Eq. (4) holds for any $|\varphi\rangle_B$. Then there must be some $|\varphi\rangle_B$ such that

$$\rho^B_A = Tr_A(|\varphi_0\rangle_{AB} \langle \varphi_0 |) \neq Tr_A(|\varphi_1\rangle_{AB} \langle \varphi_1 |) = \rho^B_1.$$

Otherwise, the assumption violates the MLC no-go theorem. In other words, if Eq. (2) holds for any $|\varphi\rangle_B$, Alice can find a local unitary operation $S_A$ to freely change the committed bit according to the MLC no-go theorem. Thus Bob can choose such $|\varphi\rangle_B$ to get some information of Alice’s committed bit and the non-static QBC is not perfectly concealing. If a non-static QBC protocol is assumed to be unconditionally binding, similar conclusions can be made.

This new approach also can be used to prove impossibility of static QBC. Given a fixed $|\varphi\rangle_B$, if a static QBC protocol is supposed to be perfectly binding, then there is no local unitary operator $S_A$ satisfying Eq. (4). According to the Uhlmann’s theorem in [22], we can find $|\phi\rangle$, a purification of $\rho^B_A$, such that

$$F(\rho^B_A, \rho^B_1) = |\langle \phi | \varphi_1 \rangle|,$$

where $|\phi\rangle = U_{AB}(|1\rangle_A \otimes |\varphi\rangle_B)$ is a purification of $\rho^B_A$. Between two purifications of $\rho^B_A$, $|\phi\rangle$ and $|\phi_0\rangle = U_{AB}(|0\rangle_A \otimes |\varphi\rangle_B)$, there always exist a local unitary operator $S_A$ such that $(S_A \otimes I)|\phi_0\rangle = |\phi\rangle$. Besides, from the assumption, we know that there does not exist a local unitary operator $S_A$ such that $(S_A \otimes I)|\phi_0\rangle = |\phi_1\rangle$. Thus $|\phi\rangle$ cannot be equal to $|\phi_1\rangle$ and

$$F(\rho^B_A, \rho^B_1) = |\langle \phi | \varphi_1 \rangle| \neq 1,$$

which means the static QBC protocol is not perfectly concealing. If assume a static QBC protocol is unconditionally binding, we can prove it is not unconditionally concealing employing the similar method.

V. CONCLUSION

In this paper, we show the assumption given by Choi et al. on the MLC no-go theorem in [21] that the entire quantum state should be static to both participants before the unveiling phase, is unnecessary, and the MLC no-go theorem can be applied to both static QBC and non-static QBC. In addition, a secure non-static QBC protocol proposed by Choi et al. in [21] is found more like to a quantum secret sharing protocol, instead of a general two-party QBC protocol. Just inspired by the non-static QBC, we prove the impossibility of QBC in another way: suppose a QBC protocol is binding first, then show it is not concealing. Now, we can say that the MLC no-go theorem lets any two-party QBC protocol satisfying concealing property is not binding and the novel proof for the impossibility of QBC given by us makes any two-party QBC protocol satisfying binding property is not concealing. In all, any two-party QBC protocol, no matter static or non-static, is not secure.

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