Higgs couplings in the MSSM at large $\tan \beta$

M. Beneke$^{a,b}$, P. Ruiz-Femenía$^a$, and M. Spinrath$^c$

$^a$ Institut für Theoretische Physik E, RWTH Aachen University, D–52056 Aachen, Germany

$^b$ Institut für Theoretische Physik, Universität Zürich, CH – 8057 Zürich, Switzerland

$^c$ Max-Planck Institut für Physik, Föhringer Ring 6, D–80805 München, Germany

Abstract

We consider $\tan \beta$-enhanced quantum effects in the minimal supersymmetric standard model (MSSM) including those from the Higgs sector. To this end, we match the MSSM to an effective two-Higgs doublet model (2HDM), assuming that all SUSY particles are heavy, and calculate the coefficients of the operators that vanish or are suppressed in the MSSM at tree-level. Our result clarifies the dependence of the large-$\tan \beta$ resummation on the renormalization convention for $\tan \beta$, and provides analytic expressions for the Yukawa and trilinear Higgs interactions. The numerical effect is analyzed by means of a parameter scan, and we find that the Higgs-sector effects, where present, are typically larger than those from the “wrong-Higgs” Yukawa couplings in the 2HDM.
1 Introduction

It is well-known that loop effects in the minimal supersymmetric extension of the standard model (MSSM) can become large when the ratio \( \tan \beta = v_u/v_d \) of the two Higgs vacuum expectation values is sizeable. An important example is the Higgs coupling to bottom quarks, \( H b \bar{b} \) \([1,2]\). In the limit of heavy superpartner particle masses, the relevant couplings are \([3]\)

\[
y_D H_d e^C Q D - \delta \tilde{y}_D H^C U e^D + \text{h.c.}
\]

\[
\rightarrow - (y_b v_d + \delta \tilde{y}_b v_u) \bar{b}_L b_L - y_b H^0_d \bar{b}_R b_L - \delta \tilde{y}_b H^0_\ast \bar{b}_R b_L + \text{h.c.}
\]

(1)

This includes a coupling \( \delta \tilde{y}_b \) to the “wrong” Higgs doublet, which is forbidden by supersymmetry at tree level. The quantity \( \delta \tilde{y}_b \) is an ordinary one-loop effect and not enhanced in any way. However, due to the large ratio of vacuum expectation values (vevs) the bottom quark mass

\[
m_b = y_b v_d + \delta \tilde{y}_b v_u = m_b^{(0)} \left( 1 + \frac{\delta \tilde{y}_b}{y_b} \tan \beta \right)
\]

receives large \( \tan \beta \)-enhanced corrections to its tree-level value \( m_b^{(0)} = y_b v_d \) \([4]\). So does the \( H b \bar{b} \) coupling, when \( y_b \) is eliminated in favour of \( m_b \) as is usually done. In general, while the loop effect is not large by itself, it provides large corrections to couplings or observables, which are suppressed by the small vacuum expectation value \( v_d \) or which vanish at tree level. This type of relative enhancement is clearly a one-loop effect, which does not repeat itself in higher loop orders.

Here we investigate other effects of this type from the Higgs sector, where loop effects can lead to a mixing of the \( H_u \) and \( H_d \) fields through two- and four-point interactions, which indirectly modify the Yukawa couplings to the physical Higgs bosons. We also address the issue of renormalization conventions for \( \tan \beta \) and their stability against radiative corrections, as investigated in a numerical approach in \([5,6]\). The indirect effects from the Higgs sector have not been considered explicitly in the literature, although our results are implicit in one-loop calculations of MSSM Higgs decay rates \([1,2]\). However, these calculations are usually available only in numerical form. Having simple analytic expressions at hand is often useful to acquire a better understanding of parameter dependences, especially for the MSSM with its large parameter space. Higgs-sector effects from two-point interactions have been considered recently in \([7]\), while their consequences for \( B \bar{B} \) mixing are discussed in \([8]\).

The theoretical issues can be exposed most clearly in the decoupling limit, assuming that all superpartner particles are heavy, of order \( M_{\text{SUSY}} \), while the standard model (SM) particles together with all five physical Higgs bosons are light, at most \( O(M_{\text{EW}}) \), the electroweak scale. In practice, the decoupling limit may be a good approximation for \( M_{\text{SUSY}} \) as small as a few hundred GeV. The assumed hierarchy implies a certain amount of fine-tuning, since the soft SUSY breaking parameters and the \( \mu \) parameter must be \( O(M_{\text{SUSY}}) \), while \( \hat{m}_{u,d}^2 \equiv |\mu|^2 + m_{u,d}^2 \) are \( O(M_{\text{EW}}) \) to make the Higgs bosons light. The
The Higgs and Yukawa terms of the MSSM Lagrangian are given by

\[ \mathcal{L} = (D_\mu H_d)\dagger(D^\mu H_d) + (D_\mu H_u)\dagger(D^\mu H_u) - \hat{m}_d^2 H_d\dagger H_d - \hat{m}_u^2 H_u\dagger H_u - b(H_d\epsilon H_u - H_u\epsilon H_d) - \frac{g_2^2}{2}(H_d\dagger H_u)(H_u\dagger H_d) - \frac{1}{8}(g_2^2 + g_1^2)(H_d\dagger H_d - H_u\dagger H_u)^2 - y_t H_u\epsilon Q U + y_b H_d\epsilon Q D + y_\tau H_d\epsilon L E + \text{h.c.} \]  

The conventions are as follows: the antisymmetric 2×2 matrix \( \epsilon \) is defined with \( \epsilon_{12} = -1 \). The hypercharge U(1)\( _Y \) and SU(2) couplings are \( g_1 \) and \( g_2 \), respectively. The quark fields are \( Q^\alpha = (t_L^{\alpha}, b_L^{\alpha})^T \), \( U^{\alpha} = (t_R^{\alpha})^\alpha \), \( D^{\alpha} = (b_R^{\alpha})^\alpha \), the leptons \( L^{\alpha} = (\nu_L^{\alpha}, \tau_L^{\alpha})^T \), \( E^{\alpha} = (\tau_R^{\alpha})^\alpha \), where \( \alpha = 1,2 \) denotes a Weyl spinor index. For the Higgs doublets we also use the notation \( \Phi_1 = H_d \), \( \Phi_2 = H_u\dagger = \epsilon H_u^* \), such that \( \Phi_1 \) and \( \Phi_2 \) both have hypercharge \(-1/2\). Our MSSM conventions follow [9] except for \( y_b = -[y_b]_{\text{Rosiek}} \), \( y_\tau = -[y_\tau]_{\text{Rosiek}} \), \( b = -[m_{12}^2]_{\text{Rosiek}} \) and \( A_\tau = -[A_\tau]_{\text{Rosiek}} \). In this way our definitions of the Yukawa couplings and \( A \) parameters are consistent with the SPA convention [10].

### 2.1 Parameters and renormalization of the MSSM

The relevant parameters of the MSSM Higgs sector are \( g_1, g_2, \hat{m}_{u,d}^2, b, y_{b,t} \) and the two vevs \( v_{u,d} \). It appears to be contrary to the spirit of interpreting the MSSM as the high-scale theory and the 2HDM as the low-energy effective theory to introduce the vevs as MSSM parameters, since the vevs should be found \textit{a posteriori} by minimizing the 2HDM effective potential. However, since we wish to use standard MSSM renormalization conventions, and moreover introduce \( \tan \beta \) as one of the MSSM parameters, we substitute

\[ H_d \rightarrow \begin{pmatrix} v_d + H_d^0 \\ H_d^- \end{pmatrix}, \quad H_u \rightarrow \begin{pmatrix} H_u^+ \\ v_u + H_u^0 \end{pmatrix} \]  

in the Lagrangian (3). In this way, after further fixing the renormalization convention, the parameter \( \tan \beta = v_u/v_d \) is well-defined, contrary to the general 2HDM, where \( \tan \beta \)
is unphysical, since it can assume any value by performing a rotation in the \((\Phi_1, \Phi_2)\) fields. In the MSSM such rotations are excluded by holomorphy of the superpotential.

An equivalent set of parameters consists of \(e, M_W, M_Z, M_A, \tan \beta, t_{u,d}, m_{b,t},\) related to the previous set by

\[
e = \frac{g_1 g_2}{\sqrt{g_1^2 + g_2^2}},
\]

\[
M_W = \frac{g_2}{\sqrt{2}} v, \quad M_Z = \frac{1}{\sqrt{2}} \sqrt{g_1^2 + g_2^2} v,
\]

\[
M_A^2 = \hat{m}_u^2 + \hat{m}_d^2, \quad \tan \beta = \frac{v_u}{v_d},
\]

\[
t_d = \hat{m}_d^2 v_d + \frac{1}{4} (g_1^2 + g_2^2) v_d (v_d^2 - v_u^2) - b v_u,
\]

\[
t_u = \hat{m}_u^2 v_u + \frac{1}{4} (g_1^2 + g_2^2) v_u (v_u^2 - v_d^2) - b v_d,
\]

with \(v = \sqrt{v_u^2 + v_d^2}\). The quark masses are given by \(m_{b,t} = y_{b,t} v_{d,u}\) at tree level. For vanishing tadpole couplings \(t_{u,d}\), \(M_A\) is the mass of the CP-odd neutral Higgs boson.

The parameters so far are bare parameters. We introduce renormalized parameters by substituting \(p \rightarrow p_0 \rightarrow p + \Delta p\), where now \(p_0\) \((p)\) is the bare (renormalized) parameter and \(\Delta p\) the corresponding counterterm. The renormalization conventions are specified by imposing conditions on the parameters of the second set: \(e, M_W, M_Z, M_A\) are defined by the conventional on-shell renormalization conditions for the electric charge and the particle pole masses, the tadpole couplings \(t_{u,d}\) by the condition that the tadpoles (one-point functions) vanish. The renormalization condition for \(\tan \beta\) and the Yukawa couplings, and the definition of \(m_{b,t}\) beyond tree level will be discussed later. The counterterms \(\Delta p\) for the parameters of the first set are then defined by requiring that (6) holds for the bare as well as the renormalized parameters. Specifically, for \(\tan \beta\) this implies

\[
\tan \beta + \Delta t_{\beta} = [\tan \beta]_0 - [\tan \beta]_0 = \frac{v_u}{v_d} + \frac{\Delta v_u}{v_d} = \frac{v_u}{v_d} + \left(\frac{\Delta v_u}{v_u} - \frac{\Delta v_d}{v_d}\right) + \ldots
\]

Employing \(\tan \beta = v_u/v_d\), the one-loop relation

\[
\Delta t_{\beta} = \tan \beta \left(\frac{\Delta v_u}{v_u} - \frac{\Delta v_d}{v_d}\right)
\]

follows. Similarly, one obtains a set of equations relating the shifts of all parameters of the second set to those of the first.
We do not need to discuss the renormalization of superpartner masses and the \( A \) and \( \mu \) terms, since these occur only in loops. In a one-loop calculation, it is sufficient to know these parameters at tree-level. Other SM parameters like the strong coupling or CKM matrix play no role here. For the subsequent discussion of tan \( \beta \)-enhanced terms field renormalization in the MSSM can also be ignored. Thus, we shall not introduce renormalization factors \( Z_{u,d} \) for the SU(2)-doublet fields. The divergent parts of the vev counterterms \( \Delta v_{u,d} \) are identical to those in \( Z_{u,d} \), since the theory in the broken phase can be rendered finite by the same counterterms as the unbroken theory before the shift (4). Thus, when we ignore field renormalization, we should formally consider \( \Delta v_{u,d} \) and consequently \( \Delta t_{\beta} \) as finite quantities, which indeed they are in the large-tan \( \beta \) approximation, whose values are nevertheless fixed by the imposed renormalization conventions.

### 2.2 The effective 2HDM Lagrangian

We integrate out the heavy SUSY particles and expand the MSSM effective action in local operators. The result is a general 2HDM, where the Higgs-Yukawa terms read

\[
\mathcal{L}_{2\text{HDM}} = Z_{ab}(D_\mu \Phi_a)^\dagger (D^\mu \Phi_b) - m_{ab}^2 \Phi_a^\dagger \Phi_b \\
- \frac{\lambda_1}{2} (\Phi_a^\dagger \Phi_1)^2 - \frac{\lambda_2}{2} (\Phi_2^\dagger \Phi_2)^2 - \lambda_3 (\Phi_a^\dagger \Phi_1) (\Phi_2^\dagger \Phi_2) - \lambda_4 (\Phi_a^\dagger \Phi_2) (\Phi_2^\dagger \Phi_1) \\
- \left[ \frac{\lambda_5}{2} (\Phi_1^\dagger \Phi_2)^2 + \left( \lambda_6 \Phi_a^\dagger \Phi_1 + \lambda_7 \Phi_2^\dagger \Phi_2 \right) \Phi_a^\dagger + \text{h.c.} \right]
\]

\[ + Y_b \Phi_1^\dagger \epsilon Qb + Y_t \Phi_2^\dagger \epsilon Qt + Y_{\tau} \Phi_1^\dagger \epsilon L\tau - \delta y_b \epsilon Qb + \delta y_t \epsilon Qt - \delta y_\tau \epsilon L\tau + \text{h.c.} \tag{9} \]

neglecting operators with dimension higher than four. The last line equals

\[ Y_b H_d \epsilon Qb - Y_t H_u \epsilon Qt + Y_{\tau} H_d \epsilon L\tau - \delta y_b H_u^C \epsilon Qb + \delta y_t H_d^C \epsilon Qt - \delta y_\tau H_u^C \epsilon L\tau + \text{h.c.} \]

in terms of the \( H_{u,d} \) fields, so the Yukawa couplings take their standard form. Since we assumed CP conservation in the MSSM, the matrices \( Z_{ab}, m_{ab}^2 \) \((a,b = 1,2)\) and the couplings \( \lambda_i \) are real. Note that the kinetic terms of the Higgs fields are not yet canonical, because we match 1PI Green functions. The kinetic terms of the fermions can be disregarded, since their renormalization is an ordinary one-loop correction to an unsuppressed tree term. At tree-level

\[
Z_{ab} = \delta_{ab}, \\
m_{11}^2 = m_d^2, \quad m_{22}^2 = m_u^2, \quad m_{12}^2 = m_{21}^2 = b, \\
\lambda_1 = \lambda_2 = \frac{1}{4}(g_1^2 + g_2^2), \quad \lambda_3 = \frac{1}{4}(g_2^2 - g_1^2), \quad \lambda_4 = -\frac{1}{2}g_2^2, \\
\lambda_5 = \lambda_6 = \lambda_7 = 0, \\
Y_{b,t,\tau} = y_{b,t,\tau}, \quad \delta y_b = \delta y_t = \delta y_\tau = 0. \tag{10} \]
In the following the one-loop corrections to the couplings that vanish or are small at tree-level are of particular interest: $\delta Z_{12} \equiv z_{12}$, $\delta m_{12}^2$, $\delta \lambda_{5,6,7}$, and $\delta \tilde{y}_{b,t,\tau}$. The result of the calculation is summarized in the next section.

### 2.3 One-loop results

All matching calculations are performed in the SU(2)×U(1)$_Y$ symmetric “phase” of the MSSM. The couplings $z_{12}$ and $m_{12}^2$ follow from an expansion of the off-diagonal $H_u H_d$ two-point function in its external momentum. The relevant one-loop diagrams with heavy superpartner particles in the loop are shown in Figure 1. For the kinetic-mixing term we find

$$z_{12} = \mu^* \left\{ 3y_t A_t^* H_2(m_{\tilde{t} R}, m_{\tilde{Q}}) + 3y_b A_b^* H_2(m_{\tilde{b} R}, m_{\tilde{Q}}) + y_\tau A_\tau^* H_2(m_{\tilde{\tau} R}, m_{\tilde{L}}) 
+ 3g_2^2 M_2 H_2(|\mu|, M_2) + g_1^2 M_1 H_2(|\mu|, M_1) \right\}$$

(11)

For the off-diagonal mass term, we write $m_{12}^2 = b + \delta b + [\Delta b]_{\text{heavy}}$ and obtain for $\delta b$ the expression (11) with the replacement $H_2(m_1, m_2) \rightarrow -I_2(m_1, m_2)$, where

$$I_2(m_1, m_2) = \frac{1}{16\pi^2} \left( \frac{1}{\epsilon} - \ln \frac{m_1^2}{\nu^2} + i_2(x) \right),$$

$$i_2(x) = 1 + \frac{x \ln x}{1 - x},$$

(12)

$$H_2(m_1, m_2) = \frac{1}{16\pi^2} \frac{1}{2m_1^2} h_2(x),$$

$$h_2(x) = \frac{1 - x^2 + 2x \ln x}{(1 - x)^3},$$

(13)

and $x \equiv m_2^2/m_1^2$. The kinetic-mixing correction $z_{12}$ is finite, as it should be, while the correction to $m_{12}^2$ is ultraviolet divergent and requires regularization (here dimensional regularization, space-time dimension $d = 4 - 2\epsilon$ and renormalization scale $\nu$). The $1/\epsilon$
pole is cancelled by the counterterm contribution $[\Delta b]_{\text{heavy}}$ to $m_{12}^2$. The counterterm contribution is the difference between the MSSM and the 2HDM counterterm, which survives the matching relation, and cancels the singularity in the heavy-particle loops. In the following we shall drop the label “heavy” in the counterterm contributions. We remark that we have kept complex conjugation on the $\mu$ and $A$ parameters, where required, if they were complex. However, by our assumption of CP-conservation, $\mu$ and $A_{t,b,\tau}$ are real. Here and below the masses that enter the arguments of the loop functions are the gaugino and sfermion mass parameters in the soft supersymmetry-breaking part of the MSSM Lagrangian and not the physical mass parameters. The physical masses receive additional contributions at tree level proportional to the vevs $v_{u,d}$ from electroweak symmetry breaking, which are of higher order in the $M_{\text{EW}}/M_{\text{SUSY}}$ expansion.

For the four-point Higgs couplings $\lambda_{5,6,7}$, which vanish at tree level, we obtain the ultraviolet-finite one-loop expressions

\[
\delta \lambda_5 = -3(\mu A_t)^2 y_t^2 K_2(m_\tilde{Q}, m_{\tilde{i}_R}) - 3(\mu A_b)^2 y_b^2 K_2(m_\tilde{Q}, m_{\tilde{i}_R}) - (\mu A_t)^2 y_t^2 K_2(m_{\tilde{L}}, m_{\tilde{\tau}_R}) + (\mu)^2 M_1^2 g_1^4 K_2(|\mu|, M_1) + 3(\mu)^2 M_2^2 g_2^4 K_2(|\mu|, M_2) + 2(\mu)^2 M_1 M_2 g_1^2 g_2^2 K_3(|\mu|, M_1, M_2), \tag{14}
\]

\[
\delta \lambda_6 = -3\mu A_t y_t^3 \left\{ J_2(m_\tilde{Q}, m_{\tilde{i}_R}) + J_2(m_{\tilde{i}_R}, m_\tilde{Q}) \right\} - \mu A_t y_t^3 \left\{ J_2(m_{\tilde{L}}, m_{\tilde{\tau}_R}) + J_2(m_{\tilde{\tau}_R}, m_{\tilde{L}}) \right\} + 3\mu y_t A_t^* \left( -\frac{1}{4} \right) \left\{ g_2^2 - \frac{g_1^2}{3} \right\} J_2(m_\tilde{Q}, m_{\tilde{i}_R}) + \left[ \frac{4 g_1^2}{3} \right] J_2(m_{\tilde{i}_R}, m_\tilde{Q}) + 3\mu y_b A_b^* \left( -\frac{1}{4} \right) \left\{ g_2^2 + g_1^2 \right\} J_2(m_\tilde{Q}, m_{\tilde{b}_R}) + \left[ -\frac{2 g_1^2}{3} \right] J_2(m_{\tilde{b}_R}, m_\tilde{Q}) + \mu y_t A_t^* \left( -\frac{1}{4} \right) \left\{ g_2^2 + g_1^2 \right\} J_2(m_{\tilde{L}}, m_{\tilde{\tau}_R}) + \left[ -2 g_1^2 \right] J_2(m_{\tilde{\tau}_R}, m_{\tilde{L}}) - 3\mu |A_t|^2 y_t^3 K_2(m_\tilde{Q}, m_{\tilde{i}_R}) - 3\mu |A_b|^2 y_b^3 K_2(m_\tilde{Q}, m_{\tilde{b}_R}) - \mu |A_t|^2 A_t y_t^3 K_2(m_\tilde{L}, m_{\tilde{\tau}_R}) - 3\mu M_2 g_2^4 L_2(|\mu|, M_2) - \mu M_1 g_1^4 L_2(|\mu|, M_1) - \mu M_2 + M_1) g_2^2 g_1^2 L_3(|\mu|, M_2, M_1), \tag{15}
\]

\[
\delta \lambda_7 = -3\mu y_t A_t y_t^2 \left\{ J_2(m_\tilde{Q}, m_{\tilde{i}_R}) + J_2(m_{\tilde{i}_R}, m_\tilde{Q}) \right\}
\]
\[-3 \mu^* y_t A_t^* \left( \frac{1}{4} \right) \left\{ \left[ g_2^2 - \frac{g_1^2}{3} \right] J_2(m_{\tilde{Q}}, m_{\tilde{t}_R}) + \left[ \frac{4g_1^2}{3} \right] J_2(m_{\tilde{t}_R}, m_{\tilde{Q}}) \right\} \]
\[-3 \mu^* y_b A_b^* \left( \frac{1}{4} \right) \left\{ \left[ -g_2^2 - \frac{g_1^2}{3} \right] J_2(m_{\tilde{Q}}, m_{\tilde{b}_R}) + \left[ -\frac{2g_1^2}{3} \right] J_2(m_{\tilde{b}_R}, m_{\tilde{Q}}) \right\} \]
\[-\mu^* y_t A_t \left( \frac{1}{4} \right) \left\{ \left[ -g_2^2 + g_1^2 \right] J_2(m_{\tilde{L}}, m_{\tilde{t}_R}) + \left[ -2g_1^2 \right] J_2(m_{\tilde{t}_R}, m_{\tilde{L}}) \right\} \]
\[-3 \mu^* |A_t|^2 A_t^* y_t K_2(m_{\tilde{Q}}, m_{\tilde{t}_R}) - 3 \mu^* |y_b \mu^*|^2 A_b^* y_b K_2(m_{\tilde{Q}}, m_{\tilde{b}_R}) \]
\[-\mu^* |y_t \mu^*|^2 A_t^* y_t K_2(m_{\tilde{L}}, m_{\tilde{t}_R}) \]
\[-3 \mu^* M_2 g_2^4 L_2(|\mu|, M_2) - \mu^* M_1 g_1^4 L_2(|\mu|, M_1) \]
\[-\mu^* (M_2 + M_1) g_2^2 g_1^2 L_3(|\mu|, M_2, M_1) \]  

(16)

The diagrams relevant for the computation of \( \delta \lambda_7 \) are shown in Figure 2. The loop functions read

\[ J_2(m_1, m_2) = \frac{1}{16\pi^2} \left( -\frac{1}{m_1^2} \right) j_2(x), \]
\[ j_2(x) = \frac{1}{1 - x} + \frac{x \ln x}{(1 - x)^2}, \]

(17)

\[ K_2(m_1, m_2) = \frac{1}{16\pi^2} \left( -\frac{1}{m_2^2} \right) k_2(x), \]
\[ k_2(x) = \frac{2}{(1 - x)^2} + \frac{(1 + x) \ln x}{(1 - x)^3}, \]

(18)

\[ K_3(m_1, m_2, m_3) = \frac{1}{16\pi^2} \left( -\frac{1}{m_3^2} \right) k_3(x, y), \]
\[ k_3(x, y) = \frac{1}{(1 - x)(1 - y)} + \frac{x \ln x}{(x - y)(1 - x)^2} + \frac{y \ln y}{(y - x)(1 - y)^2}, \]

\[ L_2(m_1, m_2) = \frac{1}{16\pi^2} \left( -\frac{1}{m_1^2} \right) l_2(x), \]
\[ l_2(x) = \frac{1 + x}{(1 - x)^2} + \frac{2x \ln x}{(1 - x)^3}, \]

(19)

\[ L_3(m_1, m_2, m_3) = \frac{1}{16\pi^2} \left( -\frac{1}{m_2^2} \right) l_3(x, y), \]
\[ l_3(x, y) = \frac{1}{(1 - x)(1 - y)} + \frac{x^2 \ln x}{(x - y)(1 - x)^2} + \frac{y^2 \ln y}{(y - x)(1 - y)^2}, \]

(20)
Figure 2: Diagrams entering the matching of $\delta\lambda_7$.

with $x \equiv m_2^2/m_1^2$, $y \equiv m_3^2/m_1^2$.

The loop-induced Yukawa couplings to the “wrong” Higgs field are given by

$$\begin{align*}
\delta \tilde{y}_b &= -\frac{8}{3} \mu^* y_b M_3 g_3^2 J_3(M_3, m_{\tilde{Q}}, m_{\tilde{b}_R}) - \mu^* y_b y_t A_b^* J_3(|\mu|, m_{\tilde{Q}}, m_{\tilde{t}_R}) \\
&\quad + \frac{3}{2} \mu^* y_b M_2 g_2^2 J_3(|\mu|, M_2, m_{\tilde{Q}}) + \frac{1}{9} \mu^* y_b M_1 g_1^2 J_3(M_1, m_{\tilde{Q}}, m_{\tilde{b}_R}) \\
&\quad + \frac{1}{3} \mu^* y_b M_1 g_1^2 J_3(|\mu|, M_1, m_{\tilde{b}_R}) + \frac{1}{6} \mu^* y_b M_1 g_1^2 J_3(|\mu|, M_1, m_{\tilde{Q}}), \\
\delta \tilde{y}_t &= \frac{8}{3} \mu^* y_t M_3 g_3^2 J_3(M_3, m_{\tilde{Q}}, m_{\tilde{t}_R}) + \mu^* y_b y_t A_b^* J_3(|\mu|, m_{\tilde{Q}}, m_{\tilde{b}_R}) \\
&\quad - \frac{3}{2} \mu^* y_t M_2 g_2^2 J_3(|\mu|, M_2, m_{\tilde{Q}}) + \frac{2}{9} \mu^* y_t M_1 g_1^2 J_3(M_1, m_{\tilde{Q}}, m_{\tilde{t}_R}) \\
&\quad - \frac{2}{3} \mu^* y_t M_1 g_1^2 J_3(|\mu|, M_1, m_{\tilde{t}_R}) + \frac{1}{6} \mu^* y_t M_1 g_1^2 J_3(|\mu|, M_1, m_{\tilde{Q}}), \\
\delta \tilde{y}_r &= \frac{3}{2} \mu^* y_r M_2 g_2^2 J_3(|\mu|, M_2, m_{\tilde{L}}) - \mu^* y_L M_1 g_1^2 J_3(M_1, m_{\tilde{L}}, m_{\tilde{\tau}_R})
\end{align*}$$

(21)
\[ + \mu^* y_T M_1 g_1^2 J_3(\mu, M_1, m_{\tilde{\tau}}) - \frac{1}{2} \mu^* y_T M_1 g_1^2 J_3(\mu, M_1, m_L), \]  

(23)

where

\[ J_3(m_1, m_2, m_3) = \frac{1}{16\pi^2} \left( \frac{1}{m_1^2} \right) j_3(x, y), \]

and

\[ j_3(x, y) = \frac{x \ln x}{(x-y)(1-x)} + \frac{y \ln y}{(y-x)(1-y)}. \]  

(24)

The expression for \( \delta y_b \) agrees with the well-known result for the \( \tan \beta \)-enhanced correction to the bottom quark mass [3,4], when the bino contribution and terms of order \( M_{\text{EW}}/M_{\text{SUSY}} \) are neglected. Our full results for \( \delta y_b/y_b \) and \( \delta y_t/y_t \) agree with the expressions for the correspondent quantities \( \epsilon_0 + \epsilon_y y_b^2 \) and \( -[\epsilon_0' + \epsilon_y' y_b^2] \) given in [7], which were written in terms of the rescaled \( A \)-parameters, \( A_{b,t} \to y_{b,t} A_{b,t} \). Our result for \( \delta \lambda_5 \) agrees with [8], where the bino contributions have been neglected.

It is no surprise that the one-loop effects of interest are all proportional to the \( \mu \)-parameter, since the off-diagonal two-point terms and the \( \lambda_{5,6,7} \) couplings are protected by a U(1)\(_{\text{PQ}}\) symmetry that is broken in the MSSM only by the \( \mu \) and \( b \) term [4]. Note that the quantum correction to \( m_{12}^2 \) is of \( O(M_{\text{SUSY}}^2) \), while the \( b \) (or rather \( m_{12}^2 \)) parameter was assumed to be of order \( M_{\text{EW}}^2/\tan \beta \). Thus, the large-\( \tan \beta \) scenario requires more severe fine-tuning (by a factor of \( \tan \beta \)) than the one to maintain the “little hierarchy” \( M_{\text{EW}} \ll M_{\text{SUSY}} \). It therefore appears less natural in this respect than the generic MSSM.

### 3 Higgs couplings

We are now ready to derive the one-loop corrected Higgs couplings, which are \( \tan \beta \)-enhanced relative to their tree-level expressions. For any quantity \( \mathcal{G} \) we can write its NLO MSSM expression as

\[ \mathcal{G} = \mathcal{G}_{\text{tree}} + \mathcal{G}_{\text{heavy}} + \mathcal{G}_{\text{light}}, \]  

(25)

where \( \mathcal{G}_{\text{heavy}} \) is the contribution from the scale \( M_{\text{SUSY}} \), in practice from loops containing SUSY particles including the corresponding counterterms, and \( \mathcal{G}_{\text{light}} \) the remaining contribution from the electroweak scale (standard model particles and Higgses).

We will be interested only in effective interactions that vanish or are suppressed at tree-level by a factor of \( \tan \beta \) relative to their natural size, and we shall compute them at one loop only at leading order in \( 1/\tan \beta \). In this case \( \mathcal{G}_{\text{light}} \) is suppressed relative to \( \mathcal{G}_{\text{heavy}} \) and can be set to zero. This can be understood from the fact that these interactions are protected by the PQ-symmetry mentioned above. This symmetry is broken strongly by the \( \mu \) parameter that enters the high-scale contributions \( \mathcal{G}_{\text{heavy}} \), but only weakly in the low-energy contributions by the small \( b \) (or rather \( m_{12}^2 \), but in one-loop diagrams there is no distinction) parameter, \( b \sim M_3^2/\tan \beta \). The low-energy contributions \( \mathcal{G}_{\text{light}} \) all involve an insertion of \( b \), or an explicit coupling to the small vev \( v_d \sim M_{\text{EW}}/\tan \beta \), and are therefore negligible. On the other hand, the contribution
\( G_{\text{heavy}} \) from the superpartner particle loops is already encoded in the effective couplings of the 2HDM (9). We therefore conclude that the leading one-loop contributions to the tree-suppressed Higgs couplings in the MSSM are simply obtained by computing the physical Higgs fields with the general tree-level 2HDM Lagrangian (9). Subsequently, the specific expressions for the couplings \( z_{12}, m_{12}^2, \delta \lambda_{5,6,7}, \) and \( \delta y_{b,t,\tau} \) given in the previous section are inserted.

### 3.1 Vacuum expectation value \( v_d \)

We first derive a relation between the shift (counterterm) of the vev \( v_d \) and \( \tan \beta \) and the MSSM parameter \( b \), which feeds into all other Higgs couplings. The tadpole counterterm is determined by requiring that the one-loop \( H_d \) tadpole (1PI one-point function) vanishes:

\[
\Gamma_{H_d} = t_d + \Delta t_d + [\Gamma_{H_d}]_{0, \text{heavy}} + [\Gamma_{H_d}]_{0, \text{light}} = 0. \tag{26}
\]

The tadpole coupling \( t_d \) is given by (6) and set to zero by our renormalization convention. The tadpole counterterm in terms of the shifts of the vevs is also obtained from (6) and given by

\[
\Delta t_d = \Delta \hat{m}_d^2 v_d + \hat{m}_d^2 \Delta v_d + \frac{1}{4}(\Delta g_1^2 + \Delta g_2^2) v_d(v_d^2 - v_u^2) - \Delta b v_u - b \Delta v_u \\
+ \frac{3}{4}(g_1^2 + g_2^2) v_d^2 \Delta v_d - \frac{1}{4}(g_1^2 + g_2^2) v_u^2 \Delta v_d - \frac{1}{2}(g_1^2 + g_2^2) v_d v_u \Delta v_u. \tag{27}
\]

The (unrenormalized) SUSY loop contribution to \( \Gamma_{H_d} \) follows from the linear term in the 2HDM Lagrangian (9) after shifting the neutral Higgs fields by the vevs, which leads to

\[
[\Gamma_{H_d}]_{0, \text{heavy}} = \delta m_{11}^2 v_d - \delta b v_u + \delta \lambda_1 v_d^3 - 3\delta \lambda_6 v_d^2 v_u + (\delta \lambda_3 + \delta \lambda_4 + \delta \lambda_5) v_d v_u^2 - \delta \lambda_7 v_u^3, \tag{28}
\]

where \( \delta m_{11}, \delta \lambda_{1-4} \) denote the SUSY loop contributions to the corresponding 2HDM parameters, which, however, will not be needed later on. The low-energy contributions can be neglected, as explained above, so the desired relation for \( \Delta v_d \) follows from \( \Delta t_d + [\Gamma_{H_d}]_{0, \text{heavy}} = 0. \)

This condition simplifies in the large-\( \tan \beta \) limit. Applying \( v_d \ll v_u, b \ll M_{EW}^2, \) (27) reads

\[
\Delta t_d \approx \hat{m}_d^2 \Delta v_d - \Delta b v_u - \frac{1}{4}(g_1^2 + g_2^2) v_u^2 \Delta v_d \approx M_A^2 \Delta v_d - \Delta b v. \tag{29}
\]

The second approximation uses

\[
g_1^2 = \frac{2(M_Z^2 - M_W^2)}{v^2}, \quad g_2^2 = \frac{2M_W^2}{v^2}, \quad b = \frac{1}{2} M_A^2 \sin(2\beta),
\]

\[
\hat{m}_d^2 = \frac{1}{2} M_A^2 \sin(2\beta) \tan \beta - \frac{1}{2} M_Z^2 \cos(2\beta) \approx M_A^2 + \frac{M_Z^2}{2},
\]

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\[ \hat{m}_u^2 = \frac{1}{2} M_A^2 \sin(2\beta) \cot \beta + \frac{1}{2} M_Z^2 \cos(2\beta) \approx -\frac{M_Z^2}{2}, \]

\[ v_d = v \cos \beta, \quad v_u = v \sin \beta \approx v, \quad (30) \]

which follow from inverting (6) for the renormalized parameters together with \( t_u = t_d = 0 \). Similarly,

\[ [\Gamma_{H_d}]_{0, \text{heavy}} \approx -\delta b v - \delta \lambda_7 v^3. \quad (31) \]

Solving for \( \Delta v_d \) results in

\[ \Delta v_d = \frac{(\delta b + \Delta b) v + \delta \lambda_7 v^3}{M_A^2}. \quad (32) \]

We see that unless the counterterm \( \Delta b \) is fine-tuned the shift of the vev is of order \( M_{\text{SUSY}}^2/M_{\text{EW}} \), which is very large relative to its assumed value of order \( M_{\text{EW}}/\tan \beta \). As discussed above, the shift of the vev is a finite quantity at leading order in large \( \tan \beta \), since \( \Delta b \) cancels the divergences in \( \delta b \). Similar statements now apply to the shift of \( \tan \beta \). Inserting (32) into (8), we find

\[ \Delta t_\beta \approx -\tan^2 \beta \frac{\Delta v_d}{v} = -\tan^2 \beta \frac{\delta b + \Delta b + \delta \lambda_7 v^2}{M_A^2} + O(\tan \beta). \quad (33) \]

The Higgs masses themselves receive no \( \tan \beta \)-enhanced one-loop corrections, so \( M_A \) corresponds to the physical mass of the CP-odd Higgs boson even at the one-loop level in our approximations.

### 3.2 Renormalization of \( \tan \beta \)

Contrary to the other parameters of the second set, there is no natural renormalization convention for the parameter \( \tan \beta \). Gauge-independence and numerical stability of various schemes have been investigated in [5,6]. Quite generally, we expect problems with stability of observables against adding perturbative corrections, if the finite part of a counterterm is large, since this leads to a large shift between the tree-level and one-loop corrected values of parameters and large counterterm contributions to observables.

From (33) we conclude that \( \tan \beta \) receives large relative corrections, i.e. \( \Delta t_\beta/\tan \beta \propto \tan \beta \), unless

\[ \delta b + \Delta b + \delta \lambda_7 v^2 = O(1/\tan \beta). \quad (34) \]

Some of the stability features at large \( \tan \beta \) discussed in [5,6] can be understood on the basis of this equation. The \( \overline{\text{DR}} \) scheme for \( \tan \beta \) has \( \Delta t_\beta = 0 \) by definition and fulfills (34) as does the DCPR definition [11,12], for which \( \Delta t_\beta/\tan \beta \propto 1 \). Indeed, these two schemes were found to exhibit good stability. On the other hand, the \( m_3 \)-scheme (\( \overline{\text{DR}} \) for the \( b \) parameter), implies

\[ \frac{\Delta t_\beta}{\tan \beta} \approx -\tan \beta \frac{[\delta b]_{\text{fin}} + \delta \lambda_7 v^2}{M_A^2}, \quad (35) \]
which is very large, of order $M_{\text{SUSY}}^2/M_{\text{EW}}^2 \times \tan \beta$. The Higgs mass and tadpole scheme [5] also violate (34), and are unstable.

Good perturbative renormalization schemes should thus have $\delta b + \Delta b + \delta \lambda_\tau v^2 = 0$ in the large-$\tan \beta$ limit. Since $b \sim M_{\text{EW}}^2/\tan \beta$, this implies fine-tuning of the counterterm for the $b$ parameter as expected.

### 3.3 Physical Higgs fields

To determine the physical Higgs fields, we first bring the kinetic terms of the 2HDM (9) into canonical form by a field redefinition. This is not necessary to determine the Higgs masses, which receive no $\tan \beta$-enhanced corrections, since there are no tree-level suppressions, or observables that involve only highly virtual internal Higgs lines, as for instance in low-energy flavour physics. The latter statement can be illustrated with the help of Figure 3, representing part of a diagram with an internal Higgs propagator. The expression for this diagram using the Lagrangian (9) is $V_b^\dagger \left[ (Z q^2 - m^2)^{-1} \right]_{ba} V_a$, where the propagator is a $2 \times 2$ matrix in Higgs doublet space. In low-energy scattering the momentum transfer satisfies $q^2 \ll m^2$, so the term involving the kinetic-mixing matrix $Z_{ab}$ in the 2HDM Lagrangian drops out at leading order in the low-energy expansion, and cannot affect physical observables. If one performs a field rotation to diagonalize $Z_{ab}$, the rotation affects the vertices $V_a$ and mass matrix $m_{ab}$ in such a way, that once again the effect disappears at leading order in $q^2/m^2$. However, in this paper our aim is to calculate the couplings of on-shell Higgs bosons ($q^2 \sim m^2$) to quarks and the Higgs fields themselves, relevant to Higgs production and decay, so we need the fields corresponding to the physical states.

The field redefinition that diagonalizes the kinetic terms (and includes the shift of the fields by the vevs (4)) is

$$
\begin{align*}
\Phi_1^1 &= H_d^1 \rightarrow [v_d]_0 + \left(1 - \frac{z_{11}}{2}\right) H_d^0 + \frac{1+a}{2} z_{12} H_u^{0*}, \\
\Phi_1^2 &= H_d^2 \rightarrow \left(1 - \frac{z_{11}}{2}\right) H_d^- - \frac{1+a}{2} z_{12} H_u^{+*}, \\
-\Phi_2^1 &= H_u^2 \rightarrow [v_u]_0 + \left(1 - \frac{z_{22}}{2}\right) H_u^0 + \frac{1-a}{2} z_{12} H_d^{0*}, \\
\Phi_2^2 &= H_u^1 \rightarrow \left(1 - \frac{z_{22}}{2}\right) H_u^+ - \frac{1-a}{2} z_{12} H_d^{-*},
\end{align*}
$$

(36)
where \( z_{ab} \) is the one-loop correction to \( Z_{ab} \), and we used that the hermitian matrix \( Z_{ab} \) is also symmetric due to CP-conservation. The diagonal coefficients \( z_{11}, z_{22} \) can be set to zero, since they are ordinary one-loop corrections to a non-vanishing tree term. The interesting terms are those that mix \( H_d \) with the complex conjugate of \( H_u \). The arbitrary quantity \( a \) parameterizes a real field rotation in \((\Phi_1, \Phi_2)\) space, which preserves the diagonal form of the kinetic term. We could set \( a = 0 \), but prefer to keep it to demonstrate explicitly the independence of physical quantities on \( a \) below. Note that we do not rotate the fields and then shift them by the vevs, since the vevs (and \( \tan \beta \)) have been defined as parameters of the MSSM Lagrangian before matching to the 2HDM.

After substituting (36) into (9), we perform a unitary (in fact orthogonal, on account of CP-conservation) field rotation to diagonalize the Higgs mass matrix. The transformation to the physical Higgs fields \( h^0, H^0, A^0, H^\pm \), including the pseudo-Goldstone fields \( G^0, G^\pm \), is

\[
\begin{align*}
\left( \text{Im} H_u^0 \right) &= \frac{1}{\sqrt{2}} \begin{pmatrix} s_\beta + \delta s_\beta & c_\beta + \delta c_\beta \\ -[c_\beta + \delta c_\beta] & s_\beta + \delta s_\beta \end{pmatrix} \begin{pmatrix} G^0 \\ A^0 \end{pmatrix}, \\
\left( \text{Im} H_d^0 \right) &= \frac{1}{\sqrt{2}} \begin{pmatrix} c_\alpha + \delta c_\alpha & s_\alpha + \delta s_\alpha \\ -[s_\alpha + \delta s_\alpha] & c_\alpha + \delta c_\alpha \end{pmatrix} \begin{pmatrix} h^0 \\ H^0 \end{pmatrix},
\end{align*}
\]

where \( \delta s_\beta, \delta c_\beta, \delta s_\alpha, \delta c_\alpha \) parameterize the correction to the corresponding MSSM tree-level rotation, and we use the conventional notation \( s_\phi = \sin \phi, c_\phi = \cos \phi \). We already incorporated here that the correction \( \delta c_\beta \) to the tree-level mixing matrix turns out to be the same for the CP-odd and the charged Higgs fields. The mixing angle \( \alpha \) is given by

\[
\tan 2\alpha = \frac{M_A^2 + M_Z^2}{M_A^2 - M_Z^2} \tan 2\beta.
\]

The correction terms \( \delta s_\beta, \delta c_\beta \) are of the size of an ordinary loop correction, and hence relevant only if the corresponding tree contribution is suppressed. This is the case for the off-diagonal elements, since \( c_\beta \propto 1/\tan \beta \). We therefore neglect the \( \delta s_\beta \) terms relative to \( s_\beta \approx 1 \). For the off-diagonal correction we obtain

\[
\delta c_\beta = -\frac{1}{2} z_{12} + \frac{\delta b + \Delta b + \delta \lambda_{7} v^2}{M_A^2}.
\]

The second term vanishes in “good” renormalization schemes.

In determining the correction to \( \alpha \), the cases \( M_A > M_Z \) and \( M_Z > M_A \) should be distinguished. In the following we discuss explicitly only the case \( M_A > M_Z \). The other case follows roughly (that is, up to some signs) from interchanging \( h^0 \) and \( H^0 \). For large
\[ \tan \beta \text{ we have } \tan 2\beta \approx -\frac{2}{\tan \beta}, \] so (38) implies that either \( c_\alpha \) or \( s_\alpha \) is small, unless \( M_A \) is very close to \( M_Z \) to compensate the \( \tan \beta \) suppression. For \( M_A > M_Z \) the relevant case is \( s_\alpha \) small and negative and \( c_\alpha \) near 1. The correction to \( \tan 2\alpha \) is (valid for both cases \( M_A > M_Z \), and \( M_A < M_Z \))

\[
\delta t_{2\alpha} = az_{12} + \frac{M_A^2 + M_Z^2}{M_A^2 - M_Z^2} z_{12} \\
- \frac{2}{M_A^2} \left( \frac{M_A^2 + M_Z^2}{M_A^2 - M_Z^2} \left[ \delta b + \Delta b + \delta \lambda_7 v^2 \right] + \frac{2M_A^2}{M_A^2 - M_Z^2} \delta \lambda_7 v^2 \right).
\]

For \( M_A > M_Z \) we have \( \delta s_\alpha = \frac{1}{2} \delta t_{2\alpha} \), \( \delta c_\alpha \approx 0 \) (relative to \( c_\alpha \approx 1 \)). The field redefinitions that lead to the mass eigenstates therefore read

\[
H_0^u = \frac{1}{\sqrt{2}} \left( c_\alpha h^0 + [s_\alpha + \delta s_\alpha] H^0 + i[c_\beta + \delta c_\beta] A^0 + is_\beta G^0 \right), \\
H_0^d = \frac{1}{\sqrt{2}} \left( -[s_\alpha + \delta s_\alpha] h^0 + c_\alpha H^0 + is_\beta A^0 - i[c_\beta + \delta c_\beta] G^0 \right), \\
H^- = s_\beta H^- - [c_\beta + \delta c_\beta] G^-, \\
H^+ = [c_\beta + \delta c_\beta] H^+ + s_\beta G^+.
\]

We briefly comment on the case \( M_A = M_Z \), since (40) appears to be singular for this parameter input. The masses used here are tree-level masses, since we keep loop corrections only when they are enhanced by \( \tan \beta \) relative to the tree term. When \( M_A = M_Z \) the CP-even Higgs bosons \( h^0, H^0 \) are mass-degenerate and in fact physically indistinguishable. Eq. (40) is derived under the assumption that the difference between the diagonal elements of the neutral Higgs boson mass matrix is not small. Otherwise one must keep the one-loop correction to the diagonal elements in the calculation of \( \delta t_{2\alpha} \), and the approximation discussed here cannot be used. Thus the resonant enhancement in (40) is a real effect as long as \( M_A^2 - M_Z^2 \) is large compared to \( M_{EW}^2 \times \) loop factor. However, if the widths of the Higgs bosons are so large that the \( h^0 \) and \( H^0 \) intermediate states are not separated by some measurement, nothing distinguishes the large tree-unsuppressed \( c_\alpha H^0 \) component from the smaller \( [s_\alpha + \delta s_\alpha] h^0 \) component in (41) and there is no observable \( \tan \beta \)-enhancement in this measurement. In the following, we do not consider this degenerate case further.

### 3.4 Yukawa couplings

The Yukawa couplings and quark mass terms follow from inserting (41) into (9). The bottom mass term is of special interest. The relevant bilinear terms are

\[
\mathcal{L}_{b, \text{mass}} = -\left[ (y_b + \delta y_b + \Delta y_b)(v_d + \Delta v_d) + \delta \bar{y}_b(v_u + \Delta v_u) \right] \bar{b}_R b_L + \text{h.c.}
\]

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The physical (pole) quark mass \( m_b \) is obtained from this mass term plus low-energy self-energy contributions calculated in the 2HDM. As before we keep one-loop corrections only when they are enhanced relative to the tree expressions. This allows us to neglect the low-energy contributions (see [13,14] for the standard model electroweak loops) as well as \( \Delta v_u \), and \( \delta y_b \) in (42). \( \Delta y_b \) is the counterterm that cancels the divergences from superpartner loops in \( \delta y_b \). The finite part of \( \Delta y_b \) defines the relation between the physical bottom quark mass and the renormalized Yukawa coupling \( y_b \). There is no reason to artificially define this finite part to be parametrically larger than the one of \( \delta y_b \), so we may drop \( \Delta y_b \) as well. Introducing the definitions

\[
\epsilon_t = \frac{\delta \tilde{y}_t}{y_t}, \quad \epsilon_b = \frac{\delta \tilde{y}_b}{y_b}, \quad \epsilon_\tau = \frac{\delta \tilde{y}_\tau}{y_\tau}
\]  

we obtain the mass terms

\[
\mathcal{L}_{\text{mass}} = -m_t \bar{t}_R t_L - m_b \bar{b}_R b_L - m_\tau \bar{\tau}_R \tau_L + \text{h.c.}
\]  

with

\[
m_t = y_t v_u = y_t v s_\beta,
\]

\[
m_b = y_b (v_d + \Delta v_d) + \delta \tilde{y}_b v_u = y_b v s_\beta \left( \frac{1}{\tan \beta + \Delta t_\beta} + \epsilon_b \right).
\]  

Note that we do not expand in \( \Delta t_\beta \), since according to (33) it counts as a \( \tan \beta \)-enhanced loop correction relative to \( \tan \beta \), just as \( \epsilon_b \tan \beta \) does relative to 1. No such enhancements are present for the top quark mass, which retains its tree expression. The expression for \( m_\tau \) is obtained from the one for \( m_b \) by replacing \( (y_b, \epsilon_b) \rightarrow (y_\tau, \epsilon_\tau) \).

Solving (45) for \( y_b \) and eliminating \( y_{b,t} \) by \( m_{b,t} \), as it is conventionally done, we obtain the Higgs-Yukawa interactions

\[
\mathcal{L}_{\text{Yuk}} = -\frac{m_t}{\sqrt{2} v} \left\{ \left( \frac{1}{\tan \beta} + \epsilon_t + \delta c_\beta - \frac{1 - a}{2} z_{12} \right) \left( i \bar{t}_R t_L A^0 - \sqrt{2} \bar{t}_R b_L H^+ \right) + \frac{c_\alpha}{s_\beta} \bar{t}_R t_L \bar{h}_0 + \frac{s_\alpha + \delta s_\alpha - \epsilon_t + \frac{1 - a}{2} z_{12}}{s_\beta} \bar{t}_R t_L \bar{H}^0 \right. \\
\left. + i \bar{t}_R t_L \bar{G}^0 - \sqrt{2} \bar{t}_R b_L \bar{G}^+ \right\} \\
- \frac{m_b [\tan \beta + \Delta t_\beta]}{\sqrt{2} v (1 + \epsilon_b [\tan \beta + \Delta t_\beta])} \left\{ i \bar{b}_R b_L A^0 - \sqrt{2} b_R t_L H^- \\
- \frac{s_\alpha + \delta s_\alpha - \epsilon_b - \frac{1 + a}{2} z_{12}}{s_\beta} \bar{b}_R b_L \bar{h}_0 + \frac{c_\alpha}{s_\beta} \bar{b}_R b_L \bar{H}^0 \\
- \left( \frac{1}{\tan \beta} + \epsilon_b + \delta c_\beta + \frac{1 + a}{2} z_{12} \right) \left( i \bar{b}_R b_L \bar{G}^0 - \sqrt{2} b_R t_L \bar{G}^- \right) \right\} \\
+ \text{h.c.},
\]  

(46)
where as before we neglect ordinary one-loop corrections relative to unsuppressed tree terms, and the $\tau$ Yukawa couplings are given by the bottom ones after replacing all bottom parameters and fields by the corresponding expressions for tau. Several comments can be made. First, the explicit dependence on the arbitrary parameter $a$ cancels with the implicit dependence in (39), (40). Second, our result is independent on the renormalization convention for $\tan \beta$. A change of renormalization prescription is effected by a change in the finite part of $\Delta t_\beta$, or (33) $\Delta v_d$, or (32) $\Delta b$. From (39), (40), we therefore find that

$$[\tan \beta + \Delta t_\beta, \left[\frac{1}{\tan \beta} + \delta c_\beta\right], [s_\alpha + \delta s_\alpha], s_\beta, c_\alpha]$$

are invariant under a change of renormalization prescription that contains only $\tan \beta$-enhanced one-loop terms.

Eq. (46) shows that a resummation of $\tan \beta$-enhanced terms is necessary only in the overall factor multiplying the bottom (and tau) Yukawa couplings [3], where the need for resummation arises from using the quark (lepton) mass as a parameter rather than the MSSM Yukawa coupling. All of the Yukawa couplings which are $\tan \beta$-suppressed at tree-level receive unsuppressed loop corrections, which are incorporated in (46) through the “wrong-Higgs” Yukawa couplings $\epsilon_{t,b,\tau}$, and the quantities $\delta c_\beta$, $\delta s_\alpha$, $z_{12}$ from the one-loop effects in the Higgs sector. The loop-induced couplings may well exceed the tree terms, if $\tan \beta$ is large.

To compare our result with previous results in the literature, see Eq. (21) in [3], we consider the bottom quark coupling to the lightest Higgs boson $h^0$, which is the product of two factors

$$m_b [\tan \beta + \Delta t_\beta] \sqrt{2v (1 + \epsilon_b [\tan \beta + \Delta t_\beta])} \times \frac{s_\alpha + \delta s_\alpha - \epsilon_b - \frac{1}{2} z_{12}}{s_\beta}. \quad (48)$$

With respect to the first factor, note that we do not discuss logarithmic one-loop effects, and hence do not distinguish the physical quark mass $m_b$ from the running $\overline{\text{MS}}$ quark mass $\overline{m}_b(Q)$ used in [3]. In reality, renormalization group evolution from $m_b$ to the electroweak scale is an important effect that should be taken into account as done in [3]. The main difference in the first factor constitutes the presence of the (finite) counterterm $\Delta t_\beta$, which must not be neglected in general, and which renders the resummed result manifestly scheme-independent. This clarifies that the previously known result where $\Delta t_\beta = 0$, is only valid in “good” renormalization schemes (34) where the one-loop shift of $\tan \beta$ is not itself $\tan \beta$-enhanced, as also observed recently in the related context of the muon Yukawa coupling [15]. The resummation formula from [3] without the counterterm contribution should not be used in other schemes, for which $\Delta t_\beta$ is an important correction in the first factor.

Using $s_\beta \approx 1$ in the one-loop corrections, and (33), (40), the second factor can be rewritten as

$$\frac{s_\alpha - \epsilon_b + \frac{M_Z^2}{M_A^2 - M_Z^2} z_{12} - \frac{2\delta \lambda_7 v^2}{M_A^2 - M_Z^2} - \frac{M_A^2 + M_Z^2}{M_A^2 - M_Z^2} \delta b + \Delta b + \frac{\delta \lambda_7 v^2}{M_A^2}}{M_A^2} \cdot (49)$$
The first two terms reproduce the corresponding result in [3], but the Higgs-sector effects embodied in the last three terms have not been included there. Higgs-sector effects related to kinetic mixing have recently been computed in [7]. The DR scheme for \( \beta \) is assumed there, hence the last term involving \( \delta b + \Delta b + \delta \lambda t_v^2 \propto \Delta \beta = 0 \) vanishes. We find several discrepancies in the comparison with [7]. The third term proportional to \( z_{12} \) would be consistent with the result of [7] if \( z_{12} = \epsilon_{GP} \), but our result for \( z_{12} \) in (11) gives \( z_{12} = -\epsilon_{GP} \). We also find that the corrections to the \( t_L t_R H^0 \) coupling are not included in [7]. The most important difference is that the fourth term involving \( \delta \lambda_7 \) is missed, since it is assumed that Higgs-sector effects arise only from kinetic mixing, and not (also) from the Higgs self-couplings. The large quantum correction to the tree-level suppressed charged Higgs \( \bar{t}_R b_L H^0 \) coupling is present in the discussion of the \( b \to s \gamma \) transition in [16,17]. These papers account for the gluino contribution to \( \epsilon_t \). The Higgs-sector terms involving \( \delta \epsilon_\beta \) and \( z_{12} \) are again not considered.

More precisely, in the expression for the third term proportional to \( z_{12} \) Ref. [7] has \( M_{h_0}^2/(M_{H^0}^2 - M_{h_0}^2) \), where we have \( M_{Z}^2/(M_{A}^2 - M_{Z}^2) \). The two expressions are consistent with each other, since in the one-loop correction we may use the large-tan \( \beta \) limits of the tree-level mass relations, which give \( M_{h_0} = M_{Z} \), and \( M_{H^0} = M_{A} \). In reality, the Higgs masses must receive large quantum corrections to satisfy experimental limits, which are not included in our equations, since they are not tan \( \beta \)-enhanced. Since the apparent singularity for \( M_A = M_Z \) arises from the degeneracy of the two CP-even Higgs bosons, as discussed above, it is plausible that \( M_{h_0}^2/(M_{H^0}^2 - M_{h_0}^2) \) provides a better approximation, even though formally the difference to \( M_{Z}^2/(M_{A}^2 - M_{Z}^2) \) is a higher-order effect.

In Section 4 we perform some representative numerical estimates of the new Higgs-sector corrections to the Yukawa couplings and compare them to the previously known term \( \epsilon_b \).

### 3.5 Higgs self-couplings

The Higgs self-couplings are obtained in the same straightforward fashion. Here we focus on the trilinear couplings, since the quartic self-interactions with coefficients induced by quantum effects will be too small to be measured in any foreseeable experiment. The detection of tan \( \beta \)- or loop-suppressed trilinear couplings provides a challenge even to the ILC (see, e.g., [18]).

The Lagrangian for the trilinear couplings

\[
\mathcal{L}_{HHH} = \mathcal{L}_{HHH, \text{large}} + \mathcal{L}_{HHH, \text{small}}
\]

consists of a piece \( \mathcal{L}_{HHH, \text{large}} \) involving interactions that do not vanish at tree level when \( \beta \to \infty \), and another piece \( \mathcal{L}_{HHH, \text{small}} \) with interactions that do. The tree-level couplings in the MSSM are standard and will not be repeated here. We find a compact expression for the tan \( \beta \)-enhanced one-loop corrections relative to the small tree-level interactions in \( \mathcal{L}_{HHH, \text{small}} \), applicable to the case \( M_A > M_Z \):

\[
\mathcal{L}_{HHH, \text{small}}^{\text{one-loop}} = -\frac{v}{\sqrt{2}} \left\{ (-1)(3M_A^2 + 2M_Z^2) C_H h^0 H^0 \right\}
\]
\[ + (M_\Delta^2 C_H - \delta \lambda_6 - \delta \lambda_7) (H^{02} + A^{02}) H^0 \]
\[ + 2 \left( [M_\Delta^2 - 2M_Z^2] C_H - (\delta \lambda_6 + \delta \lambda_7) \right) H^0 H^+ H^- \]
\[ + 2 (M_\Delta^2 - M_Z^2) C_H h^0 A^0 G^0 - 2\delta \lambda_5 H^0 A^0 G^0 - M_\Delta^2 C_H H^0 (G^{02} + 2 G^+ G^-) \]
\[ + 2 (M_\Delta^2 - M_Z^2 + M_W^2) C_H h^0 (H^- G^+ + H^+ G^-) \}, \] (51)

where
\[ C_H = \frac{1}{M_\Delta^2 - M_Z^2} \left( \frac{M_Z^2}{v^2} \left[ \frac{\delta b + \Delta b + \delta \lambda_7 v^2}{M_\Delta^2} - \frac{z_{12}}{2} \right] + \delta \lambda_7 \right). \] (52)

## 4 Numerical estimates

In this section we present numerical estimates of the Higgs-sector corrections by evaluating them for degenerate SUSY masses, and by performing a random parameter space scan. We are especially interested in comparing the new Higgs-sector effects to the known one-loop induced “wrong-Higgs” Yukawa couplings.

### 4.1 Degenerate SUSY masses

We begin the analysis of the size of the new terms in the Yukawa couplings relative to \( \epsilon_{b,t} \) by assuming that all SUSY parameters are equal to a common mass scale \( M_{\text{SUSY}} \) and by allowing for arbitrary signs in the \( \mu \) and \( A \) parameters, but not the gaugino masses. In the following we give analytic results for \( \epsilon_{b,t,\tau} \) and the quantities \( z_{12} \) and \( \delta \lambda_{5-7} \), which determine the Higgs-sector corrections to the Yukawa couplings and the Higgs self-couplings, and numerical results to judge the importance of individual terms.

To make contact with the literature, we introduce the rescaled \( A \) parameters \( \tilde{A} \) by \( A_i = y_i \tilde{A}_i \). In units of \( \text{sgn}(\mu)/(96\pi^2) \), we find

\[ \epsilon_b = 3y_t^2 \text{sgn}(\tilde{A}_t) + 32\pi \alpha_s - 18\pi \alpha_{\text{em}} \left( \frac{1}{s_w^2} + \frac{11}{27c_w^2} \right) \]
\[ = 9.65 + \text{sgn}(\tilde{A}_t) \frac{3.02}{s_b^2}, \]

\[ \epsilon_t = -3y_b^2 \text{sgn}(\tilde{A}_b) - 32\pi \alpha_s + 18\pi \alpha_{\text{em}} \left( \frac{1}{s_w^2} + \frac{5}{27c_w^2} \right) \]
\[ = -9.77 - \text{sgn}(\tilde{A}_b) \frac{2.14}{[50c_b]^2}, \]

\[ \epsilon_{\tau} = -18\pi \alpha_{\text{em}} \left( \frac{1}{s_w^2} - \frac{1}{3c_w^2} \right) = -1.80 \] (53)
for the “wrong-Higgs” Yukawa couplings, and

\[
z_{12} = 3y_t^2 \text{sgn}(\tilde{A}_t) + 3y_b^2 \text{sgn}(\tilde{A}_b) + y_{\tau}^2 \text{sgn}(\tilde{A}_\tau) + 4\pi\alpha_{\text{em}} \left( \frac{3}{s_w^2} + \frac{1}{c_w^2} \right)
\]

\[
= 1.45 + \text{sgn}(\tilde{A}_t) \frac{3.02}{s_\beta^2} + \text{sgn}(\tilde{A}_b) \frac{2.14}{[50\ c_\beta]^2} + \text{sgn}(\tilde{A}_\tau) \frac{0.26}{[50\ c_\beta]^2},
\]

\[
\delta\lambda_5 = \text{sgn}(\mu) \left\{ -3y_t^4 - 3y_b^4 - y_{\tau}^4 + 16\pi^2\alpha_{\text{em}}^2 \left( \frac{3}{s_w^4} + \frac{1}{c_w^4} + \frac{2}{c_w^2 s_w^2} \right) \right\}
\]

\[
= \text{sgn}(\mu) \left\{ 0.712 - \frac{3.04}{s_\beta^4} - \frac{1.59}{[50\ c_\beta]^4} \right\},
\]

\[
\delta\lambda_6 = 15\ y_t^4 \text{sgn}(\tilde{A}_t) - 3y_t^4 \text{sgn}(\tilde{A}_t) + 5y_{\tau}^4 \text{sgn}(\tilde{A}_\tau)
\]

\[
- \frac{3\pi\alpha_{\text{em}}}{c_w^2 s_w^2} \left( -3y_t^2 \text{sgn}(\tilde{A}_t) + 3y_b^2 \text{sgn}(\tilde{A}_b) + y_{\tau}^2 \text{sgn}(\tilde{A}_\tau) \right)
\]

\[
+ 32\pi^2\alpha_{\text{em}}^2 \left( \frac{3}{s_w^4} + \frac{1}{c_w^4} + \frac{2}{s_w^2 c_w^2} \right)
\]

\[
= 1.42 - \text{sgn}(\tilde{A}_t) \left( \frac{0.11}{[50\ c_\beta]^2} - \frac{0.34}{[50\ c_\beta]^4} \right) + \text{sgn}(\tilde{A}_b) \left( \frac{0.91}{[50\ c_\beta]^2} - \frac{7.62}{[50\ c_\beta]^4} \right)
\]

\[
- \text{sgn}(\tilde{A}_\tau) \left( \frac{0.11}{[50\ c_\beta]^2} - \frac{0.34}{[50\ c_\beta]^4} \right),
\]

\[
\delta\lambda_7 = 15\ y_t^4 \text{sgn}(\tilde{A}_t) - 3y_b^4 \text{sgn}(\tilde{A}_b) - y_{\tau}^4 \text{sgn}(\tilde{A}_\tau)
\]

\[
+ \frac{3\pi\alpha_{\text{em}}}{c_w^2 s_w^2} \left( -3y_t^2 \text{sgn}(\tilde{A}_t) + 3y_b^2 \text{sgn}(\tilde{A}_b) + y_{\tau}^2 \text{sgn}(\tilde{A}_\tau) \right)
\]

\[
+ 32\pi^2\alpha_{\text{em}}^2 \left( \frac{3}{s_w^4} + \frac{1}{c_w^4} + \frac{2}{s_w^2 c_w^2} \right)
\]

\[
= 1.42 + \text{sgn}(\tilde{A}_t) \left( \frac{15.2}{s_\beta^2} - \frac{1.29}{s_\beta^2} \right) + \text{sgn}(\tilde{A}_b) \left( \frac{0.91}{[50\ c_\beta]^2} - \frac{1.52}{[50\ c_\beta]^4} \right)
\]

\[
+ \text{sgn}(\tilde{A}_\tau) \left( \frac{0.11}{[50\ c_\beta]^2} - \frac{0.07}{[50\ c_\beta]^4} \right) \tag{54}
\]

for the Higgs-sector couplings. Our results for \(\delta\lambda_{5-7}\) agree with the special case given in [20], which assumed all squark masses equal to \(M_{\text{SUSY}}\) and neglected the gaugino and slepton contributions. The numerical values in (54) have been evaluated with SM parameter input as follows: for the gauge boson masses we choose \(M_W = 80.398\ \text{GeV}\) and \(M_Z = 91.188\ \text{GeV}\), the gauge couplings are fixed to \(\alpha_{\text{em}} = \alpha_{\text{em}}^{\text{MS}}(M_Z) = 1/127.9\) and
Figure 4: Dependence of Higgs-sector couplings, normalized to $\epsilon_b$, on $\tan \beta$. For the ratios $z_{12}/\epsilon_b$, $\delta \lambda_{6,7}/\epsilon_b$ the plots correspond to the sign choices \{sgn($\tilde{A}_t$), sgn($\tilde{A}_b$)\} = \{+, +\}, \{-, +\}, \{+, -\}, \{-, -\}. In the plots for $\delta \lambda_5/\epsilon_b$ we adopt \{sgn($\mu$), sgn($\tilde{A}_t$)\} = \{+, +\}, \{-, +\}, \{+, -\}, \{-, -\}.

$\alpha_s = \alpha_s^{\overline{\text{MS}}}(M_Z) = g_2^{2}/(4\pi) = 0.118$. The cosine of the weak mixing angle is given by $c_w = M_W/M_Z$. For the top Yukawa coupling we use $y_t = m_t/(v s_\beta) \simeq 1.003/s_\beta$, where $m_t = 171.7$ GeV and $v = s_w M_W/\sqrt{2 \pi \alpha_{\text{em}}} \simeq 171.2$ GeV. For the down-type and lepton Yukawa couplings we here use the tree-level relations $y_b = m_b/(v c_\beta) \simeq 0.0169/c_\beta$ and $y_\tau = m_\tau/(v c_\beta) \simeq 0.0102/c_\beta$, where $m_b = 2.89$ GeV and $m_\tau = 1746.24$ MeV. All fermion masses correspond to the renormalized masses in the $\overline{\text{MS}}$ scheme at $\mu = M_Z$ [19] rather than the pole masses.

Since $\epsilon_b$ is nearly independent of $\tan \beta$ for $\tan \beta \gg 1$, we show the $\tan \beta$ dependence of the ratios $z_{12}/\epsilon_b$ and $\delta \lambda_{5,7}/\epsilon_b$ from (53), (54) in Figure 4 for different choices of the signs of the $\tilde{A}$-parameters. The slepton terms give a negligible contribution except for $z_{12}$ at very large values of $\tan \beta$, so we always set the sign of $\tilde{A}_t$ to +1. For $\tan \beta$ smaller than 20 – 30 the value of $z_{12}$, $\delta \lambda_{5,7}$ is determined by the $\tilde{A}_t$ terms and the gaugino contributions. For $\delta \lambda_7$, in particular, which feeds into the Yukawa couplings, the terms proportional to $\tilde{A}_t$ give by far the largest contributions, while the effect of $\tilde{A}_b$ becomes relevant for very large $\tan \beta$ values, reaching up to a 20% contribution. On the other hand we observe from the plots that the $\tilde{A}_b$ terms and a change in the sign
of $\tilde{A}_b$ has a large effect on $\delta \lambda_6$, which is driven by the $\text{sgn}(\tilde{A}_b)/c_3^4$ term in (54) with its sizeable coefficient. In general, under the assumption of a common mass scale for all SUSY parameters, we thus conclude that the quantities $z_{12}$, $\delta \lambda_{5,6}$ and particularly $\delta \lambda_7$ are comparable in size with the “wrong-Higgs” couplings $\epsilon_{b,t}$ even though they do not receive gluino contributions. Note that $|\delta \lambda_7|$ is larger than $|\epsilon_{b,t}|$ for all choices of signs of the $\tilde{A}_{t,b}$.

4.2 Estimates using random parameter-space sampling

We now investigate in more detail the size of the Higgs one-loop corrections to the Yukawa interactions relative to $\epsilon_{b,t}$ for different SUSY parameter input. We consider good renormalization schemes for $\tan \beta$, i.e. schemes satisfying $\delta b + \Delta b + \delta \lambda_7 v^2 = 0$ in the large-$\tan \beta$ limit, in which case the two relevant corrections from (46) are

$$h_1 \equiv z_{12}, \quad h_2 \equiv \frac{M_A^2 z_{12} - 2v^2 \delta \lambda_7}{M_A^2 - M_Z^2}.$$  \hspace{1cm} (55)

In terms of the quantities $h_{1,2}$, the one-loop improved couplings of the Higgs fields to the bottom and top quark are rewritten as, omitting the global factors outside the curly brackets in Eq. (46),

$$\bar{b}_R b_L h^0 : \frac{s_\alpha}{s_\beta} + \delta s_\alpha - \epsilon_b - \frac{1 + a}{2} z_{12} = \frac{s_\alpha}{s_\beta} - \epsilon_b + h_2,$$

$$i \bar{b}_R b_L G^0 : \frac{1}{\tan \beta} + \epsilon_b + \delta c_\beta + \frac{1 + a}{2} z_{12} = \frac{1}{\tan \beta} + \epsilon_b,$$

$$i \bar{t}_R t_L A^0 : \frac{1}{\tan \beta} + \epsilon_t - \delta c_\beta - \frac{1 - a}{2} z_{12} = \frac{1}{\tan \beta} + \epsilon_t - h_1,$$

$$i \bar{t}_R t_L H^0 : \frac{s_\alpha}{s_\beta} + \delta s_\alpha - \epsilon_t + \frac{1 - a}{2} z_{12} = \frac{s_\alpha}{s_\beta} - \epsilon_t + h_1 + h_2.$$ \hspace{1cm} (56)

The function $C_H$, which appears in the one-loop corrections to the small tree-level trilinear Higgs couplings (51), also has a simple expression in terms of $h_2$, given by

$$C_H = -\frac{h_2}{2v^2}.$$ \hspace{1cm} (57)

We perform a random parameter sampling for the SUSY parameters in the following ranges: $500 \text{ GeV} \leq \tilde{m}_i \leq 5 \text{ TeV}$, with $\tilde{m}_i$ being the squark, slepton or gaugino masses, and $500 \text{ GeV} \leq |\mu|, |\tilde{A}_t|, |\tilde{A}_b|, |\tilde{A}_\tau| \leq 5 \text{ TeV}$. The different sign assignments for the parameters $\mu$, $\tilde{A}_t$ and $\tilde{A}_b$ are explored separately, but we fix $\text{sgn}(\tilde{A}_\tau) = +1$. Since the $M_A$ dependence of $h_2$ comes in only through the prefactor $1/(M_A^2 - M_Z^2)$, we shall fix it to the reference value $M_A = 200 \text{ GeV}$. Here contrary to the analytic expressions in the previous subsection, we correctly include the $\tan \beta$-enhanced loop correction to the down-type quark and lepton Yukawa couplings, i.e. $y_{b,\tau} = \frac{\overline{m}_{b,\tau}}{(v c_\beta)} \times 1/(1 + \epsilon_{b,\tau} \tan \beta)$.
Figure 5: The corrections $h_1$ and $h_2$ versus $\epsilon_t$ and $\epsilon_b$ respectively, for $M_A = 200$ GeV and positive $\mu$. The plots in the first row have $\text{sgn}(\tilde{A}_t) = +1$ and those in the second row have $\text{sgn}(\tilde{A}_t) = -1$ ($\text{sgn}(\tilde{A}_b) = +1$ in all plots).

is used. We remark that not all points allowed in the parameter scan are physical, since vacuum stability requirements do not allow large $A$ terms relative to the scale of the SUSY particle masses.

Let us first study the case of positive $\mu$, where $\epsilon_b$ is preferentially positive, so that $y_b$ is reduced by the one-loop correction. In Figure 5 we show $h_1$ (left) and $h_2$ (right) versus $\epsilon_t$ and $\epsilon_b$, respectively, for both signs of $\tilde{A}_t$ (positive in the upper plots, negative in the lower ones). For positive $\mu$, the value of $\tan\beta$ does not significantly change the shape of the scatter plot, and hence has been fixed to $\tan\beta = 35$. The typical size of the effective couplings is a few percent. Values of $|h_2|$ range up to 0.1, which implies that the one-loop induced coupling may exceed the $\tan\beta$-suppressed tree coupling for values of $\tan\beta$ as small as 10. In comparison, $h_1$, which originates only from kinetic mixing, tends to be significantly smaller. Thus, the Higgs sector correction $h_2$ competes in size with the “wrong-Higgs” Yukawa coupling $\epsilon_b$ even for large values of $M_A$, and is in fact the dominant one-loop correction to the $\bar{b}bh^0$ Yukawa interaction. Some general features that can be observed in Figure 5, or extracted from the analytic expressions are:

1/ In the assumed range of SUSY parameters, the sign of $\epsilon_b$ ($\epsilon_t$) is correlated (anti-correlated) with the sign of $\mu$, since the gluino contribution (first term in (21))
dominates and the function \( J_3 \) is always negative. The term proportional to \( \tilde{A}_t \) can overcome the gluino contribution and change the sign of \( \epsilon_b \) only for negative \( \tilde{A}_t \) in the parameter-space region where \(|\tilde{A}_t|/\mu^2\) is much larger than \(1/M_3\). Similarly, large negative \( \tilde{A}_b \) can make \( \epsilon_t \) positive, when \( y_t^2|\tilde{A}_b|/\mu^2 \gg 1/M_3 \).

ii/ Since \( \epsilon_b \) cannot reach large negative values, there is no significant enhancement of the bottom Yukawa coupling from the large-tan\( \beta \) resummation. The numerical effect of the terms proportional to \( y_t^2 \tilde{A}_b \) is therefore small compared to those proportional to \( y_t^2 \tilde{A}_t \). We verified that the scatter plots for negative \( \tilde{A}_b \) are similar to the ones with positive \( \tilde{A}_b \) shown in Figure 5.

iii/ Larger values of \( |\epsilon_t| (|\epsilon_b|) \) tend to be correlated with larger values of \( |h_1| (|h_2|) \). However, there is no strict relation, since the dominant contribution to the “wrong-Higgs” Yukawa couplings is proportional to the gluino mass, while those to the Higgs-sector couplings involve the \( A \) terms.

iv/ The largest contribution to \( h_1 \) is given by the term proportional to \( \tilde{A}_t \) in (11). However, the sign of \( h_1 \) is not determined uniquely by the sign of \( \tilde{A}_t \) due to the gaugino term proportional to \( g_2^2 \), which is always positive (the function \( H_2 \) is positive) and can become comparable to the \( \tilde{A}_t \) term in some regions of the parameter space.

v/ In most of the explored parameter space the sign of \( h_2 \) is opposite to the sign of \( \tilde{A}_t \). Since the \( \delta \lambda_7 \) term in the definition (55) of \( h_2 \) is the larger of the two, we have \( \text{sgn}(h_2) = -\text{sgn}(\delta \lambda_7) \). The relation \( \text{sgn}(\delta \lambda_7) = \text{sgn}(\tilde{A}_t) \) can be understood as follows: neglecting the smaller terms involving gauge couplings, Eq. (16) for \( \delta \lambda_7 \) contains the terms

\[
-3\mu y_t^4 \tilde{A}_t \left\{ J_2(m_{\tilde{Q}}, m_{\tilde{t}_R}) + J_2(m_{\tilde{t}_R}, m_{\tilde{Q}}) \right\} - 3\mu \tilde{A}_t^3 y_t^4 K_2(m_{\tilde{Q}}, m_{\tilde{t}_R})
\]

proportional to powers of \( \tilde{A}_t \). The first term has the same sign as \( \tilde{A}_t \), since \( J_2 \) is always negative, while the second term has opposite sign, since \( K_2 \) is positive. Defining the ratio

\[
r(x) = -\frac{J_2(m_{\tilde{Q}}, m_{\tilde{t}_R}) + J_2(m_{\tilde{t}_R}, m_{\tilde{Q}})}{m_{\tilde{Q}}^2 K_2(m_{\tilde{Q}}, m_{\tilde{t}_R})}
\]

with \( x = m_{\tilde{t}_R}^2/m_{\tilde{Q}}^2 \), we find that \( r(x) \) is monotonically increasing, satisfying \( 1.6 < r(x) < 169 \) when \( 1/100 < x < 100 \) as allowed by our parameter-space sampling. The third term in (58) can compete with the first only if \( (\tilde{A}_t/m_{\tilde{Q}})^2 > r(x) \). For \( x = 1 \) this relation requires large \( A \) terms, \( (\tilde{A}_t/m_{\tilde{Q}})^2 > 6 \), which are disfavoured by vacuum stability arguments. We also note that the largest gaugino term in \( \delta \lambda_7 \) (the one proportional to \( g_2^4 \)) gives a positive contribution because the function \( L_2 \) is negative.
Figure 6: The correction $h_2$ versus $\epsilon_b$, for $M_A = 200$ GeV and negative $\mu$. The plots in the first row correspond to $\{\text{sgn}(\tilde{A}_t), \text{sgn}(\tilde{A}_b)\} = \{+1, +1\}$ and $\{-1, +1\}$, while those in the second row have $\{\text{sgn}(\tilde{A}_t), \text{sgn}(\tilde{A}_b)\} = \{+1, -1\}, \{-1, -1\}$. Black, lightgray and darkgray points (blue, yellow and red in colored plots) correspond to $\tan \beta = 10, 35$ and $60$, respectively.

Let us now turn to the case $\text{sgn}(\mu) = -1$. The most important difference is that the negative sign of the $\mu$ parameter flips the sign of $\epsilon_b$ to negative values (and the one of $\epsilon_t$ to positive values), which leads to a strong increase of the bottom and $\tau$ Yukawa coupling, when $\epsilon_b$ cancels the $1/\tan \beta$ term in (45). The validity of a perturbative expansion is in doubt when the Yukawa couplings become too large. In the parameter space sampling for negative $\mu$, we therefore only keep points that satisfy $0 < y_{b,\tau} < 2$.

The dependence of the Higgs sector correction $h_2$ versus $\epsilon_b$, for three representative values of $\tan \beta = 10, 35, 60$, $M_A = 200$ GeV, and all other SUSY parameters scanned randomly in the above intervals is shown Figure 6. The perturbativity cut on $y_b$ has a strong effect, since it eliminates points with large negative values $\epsilon_b$ given a value of $\tan \beta$. The value of $\epsilon_b$ at which this happens can be estimated by $\epsilon_b \tan \beta \approx -0.5$, which gives $-\epsilon_b \approx 0.05, 0.014, 0.008$ for $\tan \beta = 10, 35, 60$, respectively, in agreement with the figures. At these points we observe a rapid increase of $|h_2|$ driven by the term

$$-3\mu^3 y_b^2 \tilde{A}_b K_2(m_{\tilde{Q}}, m_{\tilde{R}})$$

in (16), which gives a contribution to $h_2 \sim -\delta \lambda_7$ with sign opposite to $\tilde{A}_b$. Point
Figure 7: The correction $h_1$ versus $\epsilon_t$, for $M_A = 200$ GeV and negative $\mu$. Plots in the first row have $\{\text{sgn}(\tilde{A}_t), \text{sgn}(\tilde{A}_b)\} = \{+1, +1\}$ and $\{+1, -1\}$, and $\{\text{sgn}(\tilde{A}_t), \text{sgn}(\tilde{A}_b)\} = \{-1, +1\}, \{-1, -1\}$ in the second row. Black, lightgray and darkgray points (blue, yellow and red in colored plots) correspond to $\tan \beta = 10, 35$ and 60, respectively.

i/ discussed above also applies to the case of negative $\mu$ taking into the account the reversed signs of $\epsilon_{b,t}$. Finally, Figure 7 shows the the Higgs correction $h_1$ versus $\epsilon_t$ for $\tan \beta = 10, 35, 60$. Compared to the case of positive $\mu$, the sign of $\tilde{A}_b$ has now a relevant effect, especially for larger values of $\tan \beta$, due to the contribution of the $\tilde{A}_b$ term in $z_{12}$. However, the growth with $y_b$ is less pronounced than in case of $h_2$, since $h_1$ does not contain terms proportional to $y_b^4$.

5 Summary

This paper has been motivated by previous work [3] that systematically investigated the resummation of SUSY QCD large-$\tan \beta$ effects in the decoupling limit, where the standard model particles and Higgs scalars remain light, but which did not consider electroweak effects. We performed a complete one-loop matching of the MSSM to a two-Higgs doublet model for all couplings that are absent or suppressed in the MSSM with large $\tan \beta$, keeping all contributions that are $\tan \beta$-enhanced relative to the suppressed tree terms. This includes complete expressions for the kinetic mixing and Higgs self-interactions.
couplings $\delta \lambda_{5-7}$, which have not been given previously.

Our result confirms, as expected, that a resummation of large-$\tan \beta$ effects to arbitrary loop order is necessary only for the bottom and tau Yukawa couplings, if the bottom and tau mass are used as parameters of the MSSM. It clarifies a point left open in [3], namely how the renormalization of $\tan \beta$ affects this resummation. We find that the standard expression for the resummed Yukawa coupling is valid in renormalization schemes where the counterterms (shifts) of $\tan \beta$ and the vacuum expectation value $v_d$ do not receive large-$\tan \beta$ contributions. In other schemes, a finite counterterm must be explicitly included in the resummation formula.

Besides the known “wrong-Higgs” Yukawa couplings, all other Higgs-sector effects that feed into the Yukawa couplings by modifying the definition of the physical Higgs fields at one loop can be parameterized in terms of two couplings $h_1, h_2$ dominated by kinetic mixing and the Higgs self-coupling $\delta \lambda_7$, respectively. The same $h_2$ also enters the effective trilinear Higgs couplings. Our numerical study suggests that, where present, the Higgs-sector effect $h_2$ is more important than the “wrong-Higgs” Yukawa couplings, and we identified the dominant SUSY parameter dependences. Our result extends and corrects a previous result [7] that included the kinetic-mixing effect, but not the one from the one-loop induced Higgs couplings.

Although obtained in the decoupling limit, the effective one-loop couplings derived in this paper should be useful to obtain simple analytic estimates of the leading quantum corrections to those Yukawa and Higgs trilinear interactions, that are suppressed at tree-level, such as the $b \bar{b} h_0$ coupling.

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