Study on Prediction and Optimization of Furnace Temperature Curve Based on Temperature Change Model

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Abstract. For the furnace temperature curve when heating electronic components in the reflow furnace, with Fourier's law, the Newton's cooling law and the law of conservation of energy as the theoretical basis, combined with the relevant data, and based on heat conduction and thermal convection of the welding area, a temperature change model is established, and single objective optimization model is established as well. By using MATLAB and EXCLE to conduct ergodic search algorithm of solving, the ideal temperature curve and velocity of conveyor belt of the furnace is obtained.

1. Introduction
Reflow soldering is the most widely used soldering process in the production of electronic products such as integrated circuit boards. It can automatically solder electronic components to the circuit board by heating. This process will directly affect the soldering quality and reliability of electronic products [1]. This paper makes an in-depth study on part of the questions in the 2020 National College Students Mathematical Modeling Competition A, in order to ensure the quality of products in the reflow furnace, a design method of temperature in each low temperature zone in the reflow furnace is proposed. The interior of the reflow furnace is divided into four major temperature zones: preheating zone, constant temperature zone, reflow zone and cooling zone. It has 11 low temperature zones, and the length of which is 30.5cm, and there is a gap of 5cm between adjacent low temperature zones. In addition, the length of the furnace front area and the furnace back area are both 25cm, and the temperature sensor works when the central temperature of the welding area reaches 30℃. The key of reflow soldering is to set the temperature curve of reflow furnace: too high the temperature inside will cause damage to the components, and too low the temperature will cause welding defects [2]. Therefore, keeping the temperature of the reflow furnace as required in the production process is very important for the welding quality of components. However, nowadays, the temperature in each temperature zone can only be controlled and adjusted through experimental tests.

Many scholars in China and abroad have conducted research on the regulation, control and optimization of reflow furnace temperature curve. For example, Xia Jianting [3] drew the temperature distribution curves of four reflow areas through research, and analyzed the characteristics of each part, pointed out that the temperature distribution curves may be changed if a related factor is changed. Wang Xuejun[4], through the establishment of temperature zone automatic measurement and control system model, he achieved temperature regulation of each temperature zone, and mastered the multi-temperature zone reflow soldering temperature characteristics, therefore realized the precise temperature
regulation of each temperature zone. Wang Mingquan et al [5] studied the furnace temperature curve regulation model based on the thermodynamic principle, and obtained the furnace temperature variation in each temperature zone by establishing a differential equation model and adopting the finite difference method. Li Nan et al [6] studied the optimal furnace temperature curve and the problem of determining its process parameters, established a new furnace temperature heat transfer model, and thus proposed a method of determining the optimal furnace temperature curve and solving its process parameters. Zhou Hangyan et al [7] studied the analysis and optimization of furnace temperature curve based on genetic algorithm, and analyzed the furnace temperature curve and its optimization using genetic algorithm with cross mutation. Gabriel Takyi et al [8] used the Taguchi method to optimize the reflow furnace temperature curve. In actual production, the key to control the product quality is to ensure the parameters in the reflow soldering process within the process limits, so continuous regulation of the reflow furnace temperature curve is essential.

This paper aims to establish a temperature change model and a single-objective optimization model to give a more ideal furnace temperature curve under the condition of determined temperature in each temperature zone, and through the ergodic search algorithm to find the global optimum, and find out the best conveyor belt passing velocity in the allowable range, so as to provide a reference and basis for the adjustment and control of the furnace temperature and passing-furnace speed in the reflow furnace.

2. Data Sources and Model Assumptions
The data in this paper are from the 2020 National College Students' Mathematical Modeling Competition A. To facilitate the problem, the following assumptions are put forward: (1) the welding components are the same in layer temperature, and the circuit board material is uniform. 2) The gap temperature and the temperature of adjacent temperature zones have a negligible effect on the temperature of a certain temperature zone. (3) The thermal conductivity of the same temperature zone is independent of time and is a constant value. (4) The temperature distribution function of the furnace temperature curve is continuous, that is, the temperature of each layer varies continuously. (5) There is no heat loss, which means all the heat transferred by air thermal convection is transferred to the center of the welded components by heat conduction.

3. Prediction of Furnace Temperature Profile Based on Temperature Change Model

3.1. Research ideas
We determine that temperature variation law of the welding area by establishing a mathematical model. Both heat conduction and heat convection exist in the low temperature range and the heat exchange process of the electronic components. Neglecting the influence of irrelevant factors, the heat absorbed by the electronic components is equal to the heat transferred from the outside to the electronic components. According to Fourier's law and Newton’s cooling law, the temperature change model of the welding area was established, and a series of Ki values were solved using the set temperature in the low temperature zone and the existing temperature curve data. Ki is a function of thermal conductivity and convective heat transfer coefficient. The relationship between the temperature Ti and Ki in the low temperature zone is fit by the least squares method, and finally the center temperature of the welding area corresponding to each time point is solved in sequence by the recursive method.

3.2. Research methodology

3.2.1. Introduction to the theory. (1) Heat transfer mode. Heat transfer is the transfer of energy due to the presence of temperature differences. Heat is always spontaneously transferred from the part with a lower-higher temperature to the part with a lower temperature, and the basic ways of heat transfer can be divided into three types: heat conduction, heat convection and heat radiation.
Heat conduction can occur when there is a temperature difference in solid, liquid and gas, and it is related to the medium, thermal conductivity and temperature gradient. Thermal convection occurs within a fluid and is divided into forced convection and natural convection. Heat radiation transmits heat to the outside in the form of electromagnetic waves, which are absorbed by another object to be re-transformed into heat energy[9].

(2) Fourier’s law and Newton’s cooling law

Pure heat conduction only occurs in a closed container, and convection occurs when there is a temperature difference between the gas and the liquid under the influence of the earth’s gravity, so both the conduction phenomenon and the convection phenomenon are considered in this paper. In the process of solving the temperature distribution, heat diffusion with time, and the two laws of heat transfer in heat conduction and heat convection are presented below respectively.

Fourier’s law of heat conduction

Heat and heat conduction flux

Heat is always conducted in the direction of temperature reduction, and the heat $Q$ transferred through the cross-section with $S$ as the cross-sectional area per unit time is proportional to the temperature gradient and the heat conduction area in the direction perpendicular to the cross-section. Fourier’s law expression is:

$$ dQ = -\lambda dS \frac{\partial T}{\partial x} $$

Where $dQ$ indicates the amount of heat transferred per unit time, i.e., the differential heat transfer rate, in the opposite direction to the temperature gradient; $\lambda$ denotes the heat conductivity of indicator substance; $dS$ denotes the differential heat transfer area perpendicular to the direction of heat transfer; $\partial T / \partial x$ denotes the temperature gradient. In the equation, the minus sign indicates that the direction of heat conduction is opposite to the direction of temperature gradient, i.e., heat is transferred toward the direction of temperature decreasing.

Expressed in terms of heat conduction flux as:

$$ q = -\lambda \frac{\partial T}{\partial x} $$

Single-layer flat wall steady heat conduction

![Figure 1. Single-layer flat-wall heat conduction](image)

The temperature inside the flat wall only changes along the direction perpendicular to the flat wall. Compared with the entire flat wall area, the thickness of the flat wall is relatively small, and the effect of heat loss can be neglected [11]. According to Fourier’s law:

$$ Q = -\lambda S \frac{\partial T}{\partial x} $$

The boundary condition is that the $x = 0$, $T = T_1$, $x = b$, $T = T_2$. Integration of the above equation is organized as:
In which, \( b \) denotes the thickness of the flat wall; \( \frac{\lambda S}{b} \) denotes the heat conductivity of the flat wall and \( T_1 - T_2 \) denotes the temperature difference between the two sides of the flat wall.

Expressed in terms of heat conduction flux as:

\[ q = \frac{\lambda}{b} (T_1 - T_2) \]

Newton’s cooling law

The Newton’s cooling law is a law that is followed when an object with a temperature higher than that of its surroundings cools gradually by transferring heat into the surrounding medium. When there is a difference in temperature between the surface of the medium and the surrounding temperature, and this proportionality factor is called the heat transfer coefficient, this law is used to calculate the amount of heat transferred in convective heat transfer [12], namely:

\[ dQ = \alpha dS \Delta T. \]

Where: \( dQ \) denotes the differential convective heat transfer rate; \( \alpha \) denotes convective heat transfer coefficient; \( dS \) denotes the convective area perpendicular to the direction of indication and heat transfer and \( \Delta T \) denotes the temperature difference between the surface of the solid body and the fluid body.

3.2.2. Model establishment. A mathematical model of heat transfer between the hot air layer and the inside of the circuit board under the heating condition is proposed for the system of "reflow furnace heating and soldering circuit board". Figure 3 is a schematic diagram of a certain temperature zone of this system.

![Figure 2. System diagram of "reflow furnace heating and soldering circuit board".](image)

When the circuit board is transported to the low temperature zone along with the conveyor belt, the low temperature zone heats the electronic components so that they are automatically welded onto the circuit board. The energy of external radiation from the temperature in the reflow furnace liner is constant and less. In the study of the furnace temperature curve, the effect of thermal radiation is ignored, and only the effect of heat conduction and heat convection on the furnace temperature curve is considered.

In the soldering process, the heat conduction exists in the surface layer of the circuit board and the central region of the circuit board welding, and the heat flow is set as \( Q_i \); there is convection heat transfer in hot air between a low temperature zone and an electronic component, set this heat flow as \( Q_{i2} \).

The heat balance equation of the surface layer of the circuit board and the welding center area of the circuit board based on Fourier heat conduction law is as the following:
In this equation, the boundary layer temperature of the circuit board is \( T_w \), and the center temperature of the circuit board is \( T_r \).

The relationship between the low temperature zone and the electronic components is based on the following equation:

\[
Q_2 = \alpha_i S_0 (T_j - T_w)
\]

In this equation, \( T_i \) is the set value temperature of the low temperature zone.

Neglecting the loss of the heat from the board, i.e., all the heat transferred to the surface layer of the board from the low temperature zone is absorbed by the electronic components, and according to the conservation of heat, it is obtained that \( Q_1 = Q_1 = Q_2 \)

Thus there are:

\[
Q_1 = \frac{\lambda_i S_0}{b} (T_w - T_r) = \alpha_i S_0 (T_j - T_w)
\]

To wit:

\[
Q_1 = \frac{T_i - T_w}{1 - \frac{b}{\alpha_i S_0}} = \frac{T_w - T_r}{b - \frac{\lambda_i S_0}{\alpha_i S_0}}
\]

The above two heat transfer processes are similar to the series of heat transfer, and using the principle of addictivity, collating the above equation yields:

\[
Q = \frac{T_i - T_r}{1 - \frac{b}{\alpha_i S_0 + \lambda_i S_0}}
\]

Set \( \alpha \) and \( \lambda \) to be constant within each temperature zone of the same temperature environment, so

\[
K_i = \frac{1}{\alpha_i} + \frac{b}{\lambda_i}
\]

Then it follows that:

\[
Q = \frac{(T_i - T_r)S_0}{K_i}
\]

Neglecting heat losses, the heat absorbed by the electronic component at elevated temperature, \( Q \), is:

\[
Q = C_p \rho b S_0 \Delta T
\]

In the equation, \( C_p \) is the heat capacity of the electronic components, \( \rho \) is the density of the electronic components, \( b \) is the distance of the heat transfer on the electronic components, \( S_0 \) is the heat transfer area, and \( \Delta T = T_i - T_{r,0.5} \) is the difference between the temperature at a time when the indicator board is welded in the central area and the temperature of this area 0.5s before.

Based on the law of conservation of energy, establish the conservation of energy relationship, and associate equations (1) and (2) yields:

\[
\frac{(T_i - T_r)S_0}{K_i} = C_p \rho b \Delta T
\]

Rectifying the above equation yields:

\[
\frac{T_i - T_r}{T_r - T_{r,0.5}} = C_p \rho b K_i
\]

In literature [13], \( C_p \) and \( \rho \) values can be obtained. Based on the furnace temperature curve given in the annex at the known low temperature zone, the temperature in the central region of each
temperature zone and the corresponding soldering center, a series of $K_i$ value can be obtained, and the corresponding $K_i$ of each temperature zone can be confirmed through MATLAB fitting $T_i - K_i$ curve. With the help of the least squares method of fitting the low temperature zone, the temperature $K_i$ and $T_i$ and their relationship are obtained. By adopting the given $T_i$ in question 1, the corresponding $K_i$ can be determined. By using the recursive method and substitute $T_i1$ and $T_i2$ in sequence and according to the length of each temperature zone and gap and the conveyoelt over the furnace speed to find the corresponding time, and the temperature change curve of the central welding area can be finally obtained.

3.3. Solution of the problem

3.3.1. Coefficient determination. In equation (3), $b$ denotes the distance of heat transfer on the electronic components, which is $7.5 \times 10^{-5}$m. Because the upper and lower temperature in the reflow furnace blow hot air to heat the circuit board at the same time, the upper and lower surface of the circuit board absorbs the equal heat, and the distance of heat transfer to the central welding area of the circuit board through the electronic components is the half of the thickness of the welding area. $C_p$ and $\rho$, by consulting literature, are $0.39 \times 10^3$ J/Kg·℃ and $8.9 \times 10^3$ Kg/m3 respectively.

3.3.2. Determination of relationship between temperature $T_i$ and $K_i$ in the low temperature zone. The temperature of each low temperature zone and its corresponding temperature of the center of the welding area are substituted into equation (3) respectively, to obtain a series of $K_i$ values corresponding to each low temperature zone. Take low temperature zones 10~11 as an example, the relationship between $K_i$ and center board temperature $T_i$ by MATLAB fitting is shown in Figure 4:

![Figure 3. Correlation diagram of $K_i$ and $T_i$.](image)

By analyzing the calculated $K_i$ and the temperature data of each low temperature zone, it is found that $K_i$ is linearly related to the temperature $T_i$ of the low temperature zone, assuming that the relationship formula between $K_i$ and the temperature of the low temperature zone $T_i$ is: $K_i = aT_i + b$

The linear correlation curves of $K_i$ and $T_i$ were fitted by MATLAB using the least squares method, as shown in Figure 5.
3.3.3. Determine the furnace temperature curve. Substitute the temperature $T_i$ of the low temperature zones into the equation (4), and the corresponding $K_i$ values of each low temperature zone are obtained. Substitute $C_p$, $B$, $\rho$, $K_i$, and $T_i$ into equation (3), and substitute $Tt-0.5$ value step by step by recursive method to obtain the corresponding $Tt$ value.

1) Determination of the initial $Tt-0.5$

The initial $Tt-0.5$ is to be determined here, the temperature in the low temperature zones 10–11 is 25ºC, because the temperature sensor starts to work when the center temperature of the welding area reaches 30ºC. Through the recursive calculation by MATLAB, the temperature reaches 30.22ºC from 25ºC through four 0.5 seconds after the circuit board enters the low temperature zone 1. Therefore, the initial displayed temperature of the temperature sensor is determined to be 30.22ºC. The time for the circuit board to pass through the furnace front area plus the four 0.5 seconds is the initial $Tt-0.5$, and the initial $Tt-0.5$ is calculated to be 21.23 seconds.

Substituting the obtained initial $Tt-0.5$ value into equation (3), and recursively solving $Tt$ every 0.5 second by using MATLAB in sequence, we can get the temperature change in the center of the welding area as follows: the temperature in the preheating zone increases rapidly, the temperature in the constant temperature zone remains basically unchanged, the temperature in the reflow zone increases slowly, and the temperature in the cooling zone decreases rapidly.

2) Solution for the temperature at the center of the welding area

The temperature at the midpoint of the welding area in the low temperature zones 3, 6, and 7 and the temperature at the center of the welding area at the end of the low temperature zone 8 are solved, as shown in Table 2.

| Location | At the midpoint of low temperature zone 3 | At the midpoint of low temperature zone 6 | At the midpoint of low temperature zone 7 | At the end of low temperature zone 8 |
|----------|------------------------------------------|------------------------------------------|------------------------------------------|-------------------------------------|
| Temperature (ºC) | 127.9 | 167.69 | 190.86 | 238.64 |

(3) Plotting of furnace temperature curves

The corresponding furnace temperature curves made by EXCEL is shown in Figure 5.
3.3.4. Calibration of temperature change model. Considering the actual soldering situation, temperature measurement errors will inevitably occur in the measurement when determining the center temperature of electronic components, and it is necessary to analyze the sensitivity of the model we have developed to temperature. Taking temperature zones 1-5 as an example, we make an adjustment of $K_i = \frac{a_i}{\lambda_i} + \frac{b_i}{\lambda_i}$ with an error of $\pm 10\%$, and made the temperature curves then compared it with the predicted value.

![Figure 5. Furnace temperature curve](image)

From Figure 5, it can be seen that when $Ki$ fluctuates up and down by 10%, the trend of $T$ is unchanged or in small change. Therefore, the established temperature change model is reasonable, indicating that our model is insensitive to the influence of $Ki$, and the solution error of $Ki$ does not have a significant impact on the final result.

4. Determination of the maximum furnace speed based on the single-objective optimization model

4.1. Research ideas

We establish a single-objective optimization model with the objective function being the maximum value of velocity. The constraints are determined according to the process boundaries, and the step size is set as 1 by using the ergodic search algorithm. The possible values of the velocity are enumerated and the known conditions are substituted into the heat conduction function to obtain the furnace temperature curve corresponding to each velocity. If the furnace temperature curve meets the constraints, the velocity corresponding to this curve is the obtained solution. If not, the next step would be carried out until the allowable maximum conveyor belt boiler velocity is found [14].

![Figure 6. Calibration diagram of furnace temperature curve](image)
4.2. Research methods

We establish a single objective optimization model to determine the maximum allowable conveyor passing speed. Within the allowable range, the greater the speed of the conveyor belt passing through the furnace, the shorter the time from 150 °C to 190 °C and the time when the temperature rises above 217 °C. At the same time, based on the established temperature distribution model, the constraints of the optimization problem are determined, so that a univariate optimization model of the optimal speed is established. Finally, the model is solved by using the ergodic search algorithm for the adjustable range of velocity from 65 to 100 cm/min, with the enumeration of stepsize of 1 from high to low velocity performed to find the maximum velocity that meets the constraints.

In combination with the conditions given in the problem, the final optimization model of the allowable maximum furnace speed of the conveyor belt is given as follows:

\[
Z = \max v \\
0 < \frac{dT}{dt} < 3, 0 < t < \frac{20670}{v} \\
-3 < \frac{dT}{dt} < 0, t > \frac{20670}{v} \\
240 < T(\frac{20670}{v}) < 250 \\
71 < v < 100
\]

4.3. Problem solving

The ergodic search algorithm is adopted to optimally solve the furnace temperature curve model and solve the allowable maximum conveyor passing speed, as shown in Figure 8.

MATLAB is used to enumerate the adjustable speed range [71, 100] from high to low with a step size of 1, and the set temperature of each temperature zone is known, each enumerated speed value will be a known condition substituted into the temperature change model, each speed will get a corresponding furnace temperature curve, and observe whether the obtained furnace temperature curve meets the requirements of constraints. If the requirements are met, an allowable maximum conveying belt passing speed is obtained; if not, the next enumeration step is performed. Where ΔV is the step size of the search, and p = 10–4, which means the precision of the final output maximum furnace velocity, V, is 10–4.

We give the analysis by taking v=90cm/min as an example. According to the established heat conduction model, and substituting the known data, we can get the center temperature of the welding area in different temperature zones every 0.5 seconds from room temperature 25 degrees, and then we use the cftool toolbox in MATLAB to fit and get the curves of the center temperature of the welding area in five temperature zones changing with time respectively. The following only lists the furnace temperature curves in the 1–5 temperature zones, as shown in Figure 9. And then whether the obtained furnace temperature curve meets the requirements of the constraints is observed through derivation.

Through the above search algorithm using Matlab software programming solution, the maximum allowed conveyor belt furnace speed is of 81cm/min.

5. Conclusion

In this paper, the temperature curve of a reflow furnace is studied based on Fourier's law, Newton's cooling law and law of conservation of energy, and the temperature change model of the welding area based on heat conduction and heat convection is established. The temperature change in the center of the welding area is calculated by recursive method, which is in good agreement with the given data. In addition, we have checked the temperature change model and found that when K_i fluctuates up or down by 10%, the temperature change with time is small and the trend is constant. It shows that the established temperature change model is not sensitive to the change of K_i, and the solution error of K_i will not have
a significant impact on the final result. Therefore, the model in this paper is relatively accurate. On the premise of accurate model, the single-objective optimization model is established. By seeking the global optimum with the aid of ergodic search algorithm, the ideal furnace temperature curve is determined and the allowable maximum conveyor passing velocity under given conditions is found.

If the optimal furnace temperature system of a certain type of reflow furnace is to be designed, the temperature variation of each temperature zone of the reflow furnace can be determined according to the model in this paper, and the furnace temperature curve can be drawn, then the parameters in this model can be adjusted according to the actual data. If the specified process boundary values are different, the constraint conditions of the temperature change model in this paper can be adjusted, and then the optimal furnace temperature system can be determined. This paper provides reference and basis for the adjustment and control of each low temperature zone and the passing speed of the conveyor belt in the reflow furnace, and provides a method to design the optimal furnace temperature system, which has high practical engineering significance and strong practicability.

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