On Cell Association in Multi-Tier Full-Duplex Cellular Networks

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Abstract

This paper addresses the cell association problem in multi-tier in-band full-duplex (FD) networks. Specifically, we consider the case of decoupled cell association (DCA) in which users are not necessarily served by the same base station (BS) for uplink and downlink transmissions. Instead, users can simultaneously associate to different BSs based on two independent weighted path-loss cell association criteria for the uplink and downlink. We use stochastic geometry to develop a comprehensive modeling framework for the proposed system model where BSs and users are spatially distributed according to independent point processes. We derive closed-form expressions for the mean interference experienced at users and BSs as well as the overall mean rate coverage to evaluate the performance of the network. We also investigate the effect of interference in FD networks on legacy users with only half-duplex (HD) capabilities and how it can be mitigated by controlling the cell association weighting factors. In particular, we formulate and solve two optimization problems that aim at maximizing the mean rate of the legacy users as well as uplink and downlink mean rate by optimizing the cell association criteria. We show the effect of varying different network parameters such as the spatial density of BSs, receiver sensitivity, and weighting factors. In addition, we discuss several special cases and provide guidelines on the extensions of the proposed framework. Finally, we show that DCA outperforms coupled cell association (CCA) in which users associate to the same BS for both uplink and downlink transmissions.

Keywords: Multi-tier cellular networks, in-band full-duplex (FD) communication, decoupled cell association (DCA), coupled cell association (CCA), stochastic geometry.

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In-band full-duplex (FD) communication has recently attracted significant attention as a potential enabler for 5G networks to support higher data rates and meet the ever-increasing user demand for broadband wireless services. In contrast to half-duplex (HD) communications in which a time-frequency resource block is only used for either transmission or reception, in-band FD communications imply simultaneous transmission and reception of information in the same frequency band [1]. This, in turn, introduces extra interference between uplink and downlink networks which affects the network performance gains especially for uplink transmissions that suffer from excessive downlink-to-uplink interference [2]. The performance of FD networks is also limited by the capability of users and base stations (BS) to cancel self-interference (SI), which results from its own transmitter to its own collocated receiver. Fortunately, this technology is becoming feasible thanks to the recent advancements in antenna and digital baseband technologies where SI can be reduced close to the level of noise floor in low-power devices [3].

In this paper, we focus on the cell association problem in multi-tier cellular networks (i.e., networks that consist of different classes of BSs) that support in-band FD communications. We consider the case when a cellular user can be simultaneously served by two different BSs for the uplink and downlink data transmission (i.e., \textit{decoupled cell association}) [4]. In addition, we aim at optimizing cell association criteria in order to maximize the average transmission rate in a generic link in the network or to minimize the impact of the interference resulting from FD transmissions on legacy HD users who do not support FD communication. In order to evaluate and optimize the system performance, we use a statistical approach based on stochastic geometry to capture the network randomness [5], [6]. Specifically, in order to derive closed-form expressions for the mean rate in a generic link in the network, we use independent Poisson Point Processes (PPPs) to model the locations of BSs due to their analytical tractability [7]. The results from the analysis enable us to optimize cell association to meet different objectives (such as maximizing the rate of legacy users) and to understand the impact of network parameters (such as BS spatial density and receiver sensitivity) on the performance to provide insightful guidelines for system design. We also show that DCA is superior to \textit{coupled cell association} (CCA) in which users associate to only one BS for both uplink and downlink transmissions.
A. Related Work and Motivations

In the context of decoupled cell association (DCA) in multi-tier cellular networks, the authors in [8], [9] propose a framework for performance evaluation of multi-tier HD uplink and downlink cellular networks. In this model, the locations of BSs in each tier are modeled by independent PPPs where each network tier differs in the transmit power and spatial density. Using PPP assumption and weighted path-loss cell association, [8] derives expressions for the rate coverage in HD uplink and downlink networks as well as the joint uplink-downlink rate coverage. On the other hand, closed-form expressions for the mean logarithm of the rate and spectrum of both uplink and downlink transmissions are derived in [9]. Furthermore, utility maximization problems are solved to optimize both cell association and spectrum partitioning. The authors in [10] use stochastic geometry to derive the achievable capacity in an HD network with DCA where a real-world simulation tool (Atoll) is used to verify the accuracy of the expressions. In [11], the DCA problem is formulated as a matching game in which users and BSs in a two-tier FD cellular network rank one another based on some preference metric, which is a function of the achievable signal-to-interference-plus-noise ratio (SINR), in order to maximize the total throughput.

The authors in [12]–[16] evaluate the performance of FD networks using statistical modeling. In [12], a hybrid ad-hoc network is considered in which nodes with HD or FD capabilities are randomly deployed. For an ALOHA MAC protocol, it shows that FD networks achieve a 0−30% higher throughput, compared to HD networks, for practical values of path-loss exponent. It also considers the effect of imperfect SI cancellation and how it plays an important role to determine which of FD or HD transmissions is preferable. The authors in [13] presents the effects of transmission duration, SI cancellation, and ratio of HD-to-FD nodes on the performance of ad-hoc networks with asynchronous ALOHA MAC protocol. The paper also highlights different optimal operating regions in which FD networks are more preferable compared to HD networks. In [14], the authors propose a system design that controls the partial overlap between uplink and downlink channels in order to maximize the overall rate of FD cellular networks. The paper compares the performance of two realizations for FD networks: a 2-node topology with FD users and BSs and a 3-node topology with FD BSs and HD users. It shows that a 3-node topology achieves performance results comparable to those for a 2-node topology, which paves the road
to harvest the gains of FD transmissions with HD user terminals. In [15], a hybrid multi-tier FD cellular network is considered in which BSs are operating either in FD mode or HD downlink mode. PPP assumption is used to derive expressions for the successful transmission probability and network throughput. The authors in [16] consider a single-cell scenario to derive the outage probability and achievable sum rate under a 3-node topology considering the effect of SI. It is worth mentioning that nodes (i.e., users or BSs) in [12]–[16] use CCA to solve the association problem in FD networks.

Although statistical modeling can be used for the long-term performance evaluation of FD networks, it does not necessarily provide sufficient insights on the short time-scale. Therefore, tools from optimization theory can be used to evaluate the short-term performance of networks and to find optimal parameters that maximize certain objective functions [17]–[20]. For example, [17] proposes a joint resource management scheme to mitigate the effect of imperfect SI cancellation in an OFDMA-based two-tier FD cellular network. This is achieved by jointly assigning users and transmit power for each resource block in both uplink and downlink transmission based on the level of SI to maximize a total utility sum of the network. The authors in [18] propose an iterative algorithm to jointly perform subcarrier assignment and power allocation in order to maximize the sum-rate performance in a single-cell FD network. In [19], the authors propose a joint uplink/downlink user scheduling and power allocation algorithm to maximize the system throughput by investigating the feasibility conditions of FD operation with 3-node topology. The authors in [20] propose two distributed power control and interference management methods to manage interference in FD networks with 3-node topology. The proposed MAC protocol with transmit power optimization is shown to outperform its HD counterpart in terms of total throughput.

B. Contributions, Organization, and Notations

The contributions of the paper can be summarized as follows:

- Using tools from stochastic geometry, we provide a tractable analytical framework for statistical analysis of multi-tier FD cellular networks. We derive closed-form expressions for the mean interference received at each terminal (i.e., BSs and users) in both uplink and downlink under weighted path-loss cell association. In addition, we derive the mean rate coverage offered by a generic link in the network.
• We investigate the effect of interference resulting from concurrent uplink and downlink in FD networks on legacy HD terminals that do not support FD communication and we provide closed-form expressions for their mean rate coverage. In addition, we solve two optimizations problems to maximize the mean rate of uplink transmissions, downlink transmissions, and both legacy uplink and downlink users.

• We show that decoupled cell association is superior to coupled cell association for wide ranges of cell association weighting factors in FD networks which is achieved by giving an extra degree of freedom for the network designer to jointly optimize both uplink and downlink cell association in order to maximize the overall mean rate of the system.

• We show how the proposed framework can be extended to different special cases and models in the literature such as the traditional HD uplink and HD downlink networks. In addition, we highlight the different tradeoffs in the system and show the effect of varying network parameters such as densities of BSs, sensitivity of the receivers, and cell association weighting factors on the system performance.

• Via numerical results, we show the feasibility of FD communication to increase the mean rate of the network while protecting legacy users with HD transmissions.

The rest of the paper is organized as follows. System model and the assumptions are described in Section II. In Section III, both joint distance distributions and cell association probabilities are derived for a typical user. Section IV presents the analysis for FD interference as well as mean rate of uplink, downlink, and FD transmissions. Two optimization problems are formulated in Section V to maximize the mean rate of both uplink and downlink transmissions in FD networks. Finally, the numerical results and discussion are presented in Section VI before the paper is concluded in Section VII.

**Notation:** $\Gamma[a] = \int_0^\infty x^{a-1}e^{-x}dx$ is the gamma function, $\Gamma[s,b] = \int_b^\infty x^{s-1}e^{-x}dx$ denotes the upper incomplete gamma function, and $\gamma[s,b] = \int_0^b x^{s-1}e^{-x}dx$ denotes the lower incomplete gamma function. $\mathcal{G}[m, n; x] = {}_2F_1\left[\frac{4+m+n}{2}, \frac{2+n}{2}; 4+n; -x\right]$ is the Gauss Hypergeometric function. $f(\cdot)$ and $F(\cdot)$ denote the probability density function (PDF) and cumulative distribution function (CDF), respectively. Finally, $\mathbb{E}[\cdot]$ denotes the expectation operator and $1_{\{\cdot\}}$ is the indicator function. The key mathematical notations used in this paper are summarized in Table I.
II. SYSTEM MODEL, ASSUMPTIONS, AND METHODOLOGY OF ANALYSIS

A. Multi-Tier Network Model

We consider a cellular network that consists of $K$ tiers of BSs. We use an independent homogeneous PPP $\Phi_k = \{x_{i,k} : i = 1, 2, \ldots\}$ with spatial density $\lambda_k$ to model the locations of BSs belonging to the $k$-th tier where $1 \leq k \leq K$ and $x_{i,k} \in \mathbb{R}^2$ denotes the location of the $i$-th BS in that tier. For all BSs in $k$-th tier, the transmit power is fixed and equal to $P_k$. Locations of users are modeled in $\mathbb{R}^2$ according to an arbitrary independent point process. Saturation condition is assumed where each transmitter (i.e., BS or user) has at least one packet ready for transmission at the beginning of each time slot in a time-slotted transmission scenario.

We assume a co-channel deployment where the wireless channels are subject to both large-scale (i.e., path-loss) and small-scale fading. Let $\|x - y\|$ be the propagation distance between a generic transmitter (i.e., BS or user) located at $x$ and a receiver located at $y$. The path-loss of this link is defined as $L(x, y) = \|x - y\|^\alpha$ where $\alpha > 2$ is the path-loss exponent. In order to avoid the impracticality of this model for $\|x - y\| < 1$, only when necessary, we adopt an alternative distance model by replacing $\|x - y\|$ with $\max\{d_o, \|x - y\|\}$, where $d_o$ is the minimum length of a communication link, hence $d_o = 0$ unless otherwise needed. In this case, the path-loss is $L(x, y) = L_o \max\{d_o, \|x - y\|\}^\alpha$ where $L_o$ is the path-loss at a reference distance of 1 meter. In addition to the distance-dependent path-loss, the small-scale fading component of a channel is modeled by Rayleigh fading with unit average power where different links are assumed to be independent and identically distributed (i.i.d.). Hence, the power gain of a channel from a generic transmitter located at $x$ toward a generic point $y$ is exponentially-distributed and denoted by $h \sim \text{Exp}(1)$. Channel reciprocity is not assumed where, at a given time slot, the channel gain from a transmitter at $x$ to a receiver at $y$ is different than that from a transmitter at $y$ to a.
Cellular users use fractional channel inversion power control where a user at $x$ adjusts her transmit power to $\rho_k \|x - y\|^{\alpha}$ to compensate for the large-scale fading such that the average received signal power at the serving BS at $y$ is equal to $\rho_k \|x - y\|^{-\alpha(1-\epsilon)}$. Note that $0 \leq \epsilon \leq 1$ is the power control factor and $\rho_k$ is the open loop power spectral density (or receiver sensitivity). Here, we use $\gamma_k$ to denote the instantaneous transmit power of a user transmitting to a BS from the $k$-th tier where $\Gamma_k$ is the corresponding random variable. Users have a limited transmit power budget of $P_{\text{max}}$ where users who are unable to perform channel inversion transmit with that maximum power. In addition, we assume that there is no intra-cell interference between uplink (downlink) transmissions where different users in a cell are served in uplink (downlink) using orthogonal time-frequency resources (e.g. OFDMA). Hence, there are only one active user in uplink and one active user in downlink per BS at a certain time slot and channel.

**B. Mode of Operation and Cell Association**

Besides FD links in which a user simultaneously transmits and receives data in the same channel, we also consider users and BSs that do not support FD transmissions. That is, we assume that some transmission links between users and BSs operate either in HD uplink or HD downlink transmission mode. Therefore, a user is refereed to as a *legacy uplink user* when the user-BS link is only carrying data from the user to serving BS. On the other hand, when the user-BS link is only carrying data from the BS to served user, the user is referred to as a *legacy downlink user*.

In this work, for users with FD capabilities, we consider the case when cell association in the uplink is decoupled from that of the downlink. Hence, a user in FD mode is *not necessarily* served by the same BS for both the uplink and downlink. That is, based on the association criteria, the user may simultaneously receive data from one BS and transmit data to another in the same channel. To illustrate, without loss of generality, Fig. 1 shows a realization of a two-tier FD cellular network where a macro-cell network tier is overlaid with a denser and lower power small-cell network tier. It shows that the coverage area of each cell in the downlink is different from the uplink. For example, although user 1 is served by a macro-cell BS in the downlink, it is located in the uplink coverage area of another small-cell BS, hence, served by two different BSs. On the other hand, user 2 is located in the coverage area of one small-cell BS for both
Fig. 1. A two-tier FD cellular network with a macro-cell BSs (red circles) overlaid with lower power and denser small-cell BSs (green triangles). Solid blue lines show the coverage area of each cell for the downlink transmissions, while the dotted black lines show that of the uplink transmissions.

downlink and uplink, hence, served by the same BS.

We assume weighted path-loss cell association criteria similar to [21], [22] where each user independently associates to the BS(s) that minimizes the weighted path-loss of the uplink and/or downlink. That is, for a user at \( y \), with a slight abuse of notation, let \( x_i, x_{UL} \), and \( x_{DL} \) denote the BS with the minimum path-loss from the \( i \)-th tier, the serving BS in the uplink, and the serving BS in the downlink, respectively. Then, the association criteria for the uplink can be described as:

\[
x_{UL} = \arg\min_{x \in \{x_i\}} U_i \|x - y\|\alpha
\]  

(1)

where \( x_i = \arg\min_{x \in \Phi_i} \|x - y\|\alpha \), \( i = \{1, 2, \ldots, K\} \) and \( U_i \) is the weighting factor for the uplink cell association for a BS belonging to the \( i \)-th tier. Similarly, the cell association criteria for the downlink can be described as:

\[
x_{DL} = \arg\min_{x \in \{x_i\}} D_i \|x - y\|\alpha
\]  

(2)

where \( D_i \) is the weighting factor for the downlink cell association for a BS from the \( i \)-th tier.

Note that \( U_i \) and \( D_i \) are design parameters which are not necessarily equal and varying them can result in different association scenarios. For example,

- Case I - coupled cell association (CCA): Setting \( U_i = D_i \), each user associates to the same BS for both downlink and uplink transmissions.
• Case II - *minimum-distance cell association* (MinDCA): Setting $U_i = U$ (or $D_i = D$), each user associates to the nearest BS for uplink (or downlink) transmission.

• Case III - *maximum-received power cell association* (MaxPCA): Setting $D_i = P_i^{-1}$, each user associates to the BS that offers the strongest received power for downlink transmission.

• Case IV - *minimum-transmit power cell association* (MinTrCA): Setting $U_i = \rho_i$, each user associates to the BS that requires lowest transmit power for uplink transmission.

Let $U_{jk} = \frac{U_j}{U_k}$, $D_{jk} = \frac{D_j}{D_k}$, and $\mu_i = \frac{U_i}{D_i}$. Without loss of generality, we assume that network tiers are ordered such that $\mu_1 \leq \mu_2 \leq \cdots \leq \mu_K$.

**C. Methodology of Analysis**

Based on the system model described above, we aim at quantifying the performance of a generic FD link in terms of mean rate in nats/sec/Hz. We first derive the cell association probability and distance distributions based on decoupled and weighted path-loss cell association. Then, we obtain the mean of interference experienced at a generic BS in the uplink and at a generic user in the downlink. Next, we derive the mean rate coverage of the network as well as that of uplink and downlink transmissions. We also present some special cases. Finally, we optimize the cell association weighting factors such that the rates of both uplink and downlink transmissions in an FD network are maximized.

**III. Analyses of Distance and Association Probabilities in FD Networks**

**A. Analysis of Association Probabilities**

Note that, with a certain probability, a user operating in FD mode can associate to the same BS in the $k$-th tier for both uplink and downlink transmissions. It is worth mentioning that this scenario is different from the CCA as it does not necessitate $U_k$ and $D_k$ to be equal (as in the CCA). That is, although the user still uses two different decision criteria for uplink and downlink cell association (i.e., DCA in (1) and (2)), one BS meets both criteria. This event occurs depending on the network realization. Let $\psi_{jk}$ denote the *joint association probability* that a user is served by a BS from the $j$-th tier in the downlink and a BS from the $k$-th tier in the uplink. The following lemma characterizes this probability.
Lemma 1. (Joint association probability) The probability that a user in the FD mode is served by the $j$-th tier for the downlink and $k$-th tier for the uplink transmissions is

$$
\psi_{jk} = \begin{cases} 
\lambda_j \left( \sum_{i=1}^{K} \max \{D_{ji}, U_{ji} \} \right)^{\frac{2}{\alpha}} \lambda_i^{-1}, & j = k \\
\lambda_j \lambda_k \left( \frac{D_{jk}}{U_{jk}} \right)^{\frac{2}{\alpha}} \left( \frac{\mu_k^{\frac{2}{\alpha}}}{1+\mu_k^{\frac{2}{\alpha}} \Omega_i} - \frac{\mu_j^{\frac{2}{\alpha}}}{1+\mu_j^{\frac{2}{\alpha}} \Omega_i} \right), & k < j \\
0, & k > j 
\end{cases}
$$

(3)

where $\Upsilon_l(j) = \sum_{i=l+1}^{K} D_{ji}^{\frac{2}{\alpha}} \lambda_i$ and $\Omega_i = \left( \sum_{i=1}^{l} \frac{\lambda_i}{U_{ji}^{\frac{2}{\alpha}}} \right) \left( \sum_{i=l+1}^{K} \frac{\lambda_i}{D_{ji}^{\frac{2}{\alpha}}} \right)^{-1}$.

Proof: See Appendix A-I.

Note that the joint association probability $\psi_{jk}$ is different from the per-tier association probability which is defined as the probability that a user is served by a BS belonging to a certain tier for downlink (uplink) regardless of the serving uplink (downlink) BS. This probability can be obtained directly from the joint association probability $\psi_{jj}$ when $D_i = U_i$. The following lemma provides expressions for this probability for both downlink and uplink transmissions.

Lemma 2. (Per-tier association probability) The probability that a user associates to a BS from the $j$-th tier for either the downlink or uplink is defined, respectively, as follows

$$
A_{DL}^j = \lambda_j \left( \sum_{i=1}^{K} D_{ji}^{\frac{2}{\alpha}} \lambda_i \right)^{-1}, \quad \text{and} \quad A_{UL}^j = \lambda_j \left( \sum_{i=1}^{K} U_{ji}^{\frac{2}{\alpha}} \lambda_i \right)^{-1}.
$$

(4)

B. Analysis of Distance to Serving BS(s)

Based on the system model and cell association criteria described above, for users operating in FD mode, the marginal PDFs of the distances to the serving BSs in downlink and uplink are presented in the following lemma where the proof [22, Appendix A] is omitted.

Lemma 3. (Marginal distance distributions) The CDF of the distance between a generic user associated with the $j$-th tier for the downlink (or uplink) and her serving BS is defined as

$$
P[R_{m}^j \leq r] = 1 - \exp \left[ -\pi \frac{\lambda_j}{A_{m}^j} r^{2} \right], \quad r \geq d_o
$$

(5)

and its $n$-th moment is given by

$$
E[R_{m}^n] = \kappa_j d_o^n + \Gamma \left[ \frac{2+n}{2}, \pi \frac{\lambda_j}{A_{m}^j} d_o^2 \right] \left( \pi \frac{\lambda_j}{A_{m}^j} \right)^{-\frac{n}{2}}
$$

(6)
where \( m \in \{ DL, UL \} \), \( \kappa_j = 1 - \exp\left( -\pi \frac{\lambda_j}{A_j m} d^2_o \right) \), and \( A_j^m \) is given in Lemma 2.

We also need to derive the joint PDF of the distance to the serving BSs for downlink and uplink transmissions for users operating in FD mode. The following lemma provides the joint distance distribution.

**Lemma 4.** *(Joint distance distribution)* The joint PDF of the distance between a generic FD user and her serving BS(s) when associated with the \( j \)-th for downlink and tier \( k \)-th for uplink is

\[
 f_{R_j, R_k}(r_j, r_k) = \begin{cases} 
 2\pi \lambda_j r_j \exp \left[ -\pi \sum_{i=1}^{K} \max\{ D_{ji}, U_{ji} \}^2 \alpha \lambda_i r_j^2 \right], & j = k, r_k = r_j \\
 4\pi^2 \lambda_j \lambda_k r_j r_k \exp \left[ -\pi \sum_{i=1}^{K} \max\{ D_{jir_k^\alpha}, U_{kiri_k^\alpha} \}^2 \alpha \lambda_i \right], & k < j, (r_j, r_k) \in \mathcal{A}
\end{cases}
\]

(7)

where

\[
 \mathcal{A} = \left\{ (r_j, r_k) : r_j \geq 0, D_{jkr_j}^{1/2} < r_k < U_{jkr_j}^{1/2} \right\},
\]

and their joint expectation is given by

\[
 \mathbb{E}[R_j^m R_k^n] = \begin{cases} 
 \frac{\lambda_j}{\pi^2} \Gamma \left[ \frac{2+m}{2} \right] \left( \sum_{i=1}^{K} \max\{ D_{jri}^\alpha, U_{jri}^\alpha \}^2 \alpha \lambda_i \right)^{-\frac{2+m}{2}}, & j = k, \\
 2\lambda_j \lambda_k \frac{\Gamma \left[ \frac{4+m+n}{2} \right]}{(2+n)\pi} \left( \frac{D_{j}^{1/\alpha}}{U_{k}^{1/\alpha}} \right)^{2+n} \sum_{l=k}^{j-1} \frac{\mu_i^{n+1} G \left[ m,n,\mu_i^{n+1} \Omega_l \right] - \mu_j^{n+1} G \left[ m,n,\mu_j^{n+1} \Omega_l \right]}{\Upsilon_l(j)}, & k < j,
\end{cases}
\]

(8)

*Proof:* See Appendix A-II.

**C. Analysis of Uplink Transmission Power**

Based on the described system model, users served in the uplink by a BS from \( k \)-th tier are assumed to perform fractional channel inversion with open loop power spectral density \( \rho_k \). Users are also assumed to have a constraint \( P_{\text{max}} \) on the transmit power. Thus, we define the required amount of transmit power of a user when it associates with a BS from the \( k \)-th tier as:

\[
 \gamma_k = \min\{ \rho_k R_k^{\alpha \alpha}, P_{\text{max}} \}
\]

(9)

where \( R_k \) is the distance to the serving BS from tier \( k \) for the uplink transmission and its CDF is given in Lemma 3.
Therefore, we use (5) and (9) to derive the CDF and the \( n \)-th moment of a typical user’s transmit power when it associates to the \( k \)-th tier as in the following lemma where the proof follows directly from Lemma 3 such that \( \mathbb{P}[\Gamma_k \leq t] = \mathbb{P}\left[R_{UL}^k \leq \left( \frac{t}{\rho_k} \right)^{\frac{1}{\epsilon_\alpha}} \right] \).

**Lemma 5.** *(Transmit power distribution)* The CDF of the transmit power of a user associated to tier \( k \) in the uplink and performing fractional channel inversion power control is

\[
\mathbb{P}[\Gamma_k \leq t] = 1 - \exp\left[-\pi \frac{\lambda_k}{A_{UL}^k} \left( \frac{t}{\rho_k} \right)^{\frac{2}{\epsilon_\alpha}} \right] \cdot 1\{t<P_{\text{max}}\}.
\]

and its \( n \)-th moment is given by

\[
\mathbb{E}[\Gamma_k^n] = \frac{n \epsilon_\alpha \rho_k^n}{2} \gamma \left[ \frac{n \epsilon_\alpha}{2}, \pi \frac{\lambda_k}{A_{UL}^k} \left( \frac{P_{\text{max}}}{\rho_k} \right)^{\frac{2}{\epsilon_\alpha}} \right] \left( \frac{\pi}{\rho_k} \right)^{-\frac{n \epsilon_\alpha}{2}}.
\]

**IV. Analysis of Rate Coverage of FD Transmissions**

In this section, we characterize the rate coverage for a generic FD link based on the described system model. To evaluate the rate coverage, we assume that during a transmission interval, a receiver (i.e., BS or a user) using a particular channel to communicate with its corresponding transmitter experiences interference from all other BSs as well as users reusing the same channel. Now, let us define the rate coverage probability as the probability of uplink (downlink) transmission rate to be higher than a required target rate threshold \( R_{UL}^{\text{th}} \) (\( R_{DL}^{\text{th}} \)) for a given FD link. In other words, for the uplink (downlink) transmission to achieve the target rate, the signal-to-interference ratio (SIR) received at the BS (user) must be greater than a prescribed threshold \( \tau_{UL} \) (\( \tau_{DL} \)) that can be given using Shannon’s formula as follows:

\[
\tau^m = 2^{R_{\text{th}}^m} - 1, \quad m \in \{\text{DL}, \text{UL}\}.
\]

Hence, the rate \( R_{jk} \) of a generic FD link measured in nats/sec/Hz for a user served by a BS from \( j \)-th tier for downlink and a BS from tier \( k \) for uplink can be defined as:

\[
R_{jk} = \mathbb{P}(\text{SIR}_{\text{UL}}^{jk} > \tau_{UL}) \ln \left[ 1 + \tau_{UL} \right] + \mathbb{P}(\text{SIR}_{\text{DL}}^{jk} > \tau_{DL}) \ln \left[ 1 + \tau_{DL} \right]
\]

which corresponds to a fixed data rate transmission (i.e., \( R_{\text{DL}} = \ln[1+\tau_{DL}] \) and \( R_{\text{UL}} = \ln[1+\tau_{UL}] \)) when the SIR of the downlink and uplink transmissions (i.e., \( \text{SIR}_{\text{UL}}^{jk} \) and \( \text{SIR}_{\text{DL}}^{jk} \), respectively) exceed the predefined thresholds, otherwise, the transmission rate is zero.
Furthermore, we characterize the performance of the system using the logarithm of the rate of a generic FD link. This performance metric is proposed in the literature in order to achieve fairness among users [9]. Hence, hereafter, we consider the following performance metric for a generic user operating in the FD mode:

\[
\ln R_{jk}^m = \ln R_0^m + \ln \left[ \mathbb{P}(\text{SIR}_{jk}^m > \tau^m) \right], \quad m \in \{\text{DL, UL}\}.
\] (14)

For the system model described above, the SIR can be expressed as follows:

\[
\text{SIR}_{jk}^{\text{DL}} = \frac{P_j h \|x_{jk}^{\text{DL}}\|^\alpha}{I_{\text{DL}} + I_{\text{SE}}^{\text{DL}}} \quad \text{and} \quad \text{SIR}_{jk}^{\text{UL}} = \frac{\rho_k g \|x_{jk}^{\text{UL}}\|^{-\alpha(1-\epsilon)}}{I_{\text{UL}} + I_{\text{SE}}^{\text{UL}}}
\] (15)

where \(x_{jk}^{\text{DL}} \in \Phi_j\) and \(x_{jk}^{\text{UL}} \in \Phi_k\) denote the serving BS for downlink and uplink transmissions, respectively, as defined in (1)-(2). \(h\) and \(g\) are the channel power gains where subscripts are dropped for simplicity, and \(I_{\text{DL}}\) and \(I_{\text{UL}}\) are the interference received, respectively, at the tagged user (i.e., downlink) and BS (i.e., uplink) from all other BSs and users sharing the same channel. Note that, unlike HD networks, the interference signals received from the downlink and uplink transmissions are correlated due to the relative distance between each user-BS pair and the observation point. In order to characterize the interference, using the fact that each BS performs exactly one downlink and one uplink transmissions, we pair each user and her serving BS in the uplink together as an interfering pair. Hence, the aggregate interference at a typical receiver (i.e., a user or a BS) located at the origin \((0, 0)\) can be defined as follows:\footnote{Due to the PPP assumption and since the network is stationary, interference statistics are independent of the observation point and interference measured at any point in the network has the same PDF [5].}

\[
I_{\text{DL}} = \sum_{i=1}^{K} \sum_{x \in \Phi_i \setminus \{x_{\text{DL}}\}} \frac{P_i h}{L(x,0)} + \sum_{y \in \Psi_i \setminus \{u[y]\}} \frac{\gamma_i g}{L(y,0)}
\] (16)

\[
I_{\text{UL}} = \sum_{i=1}^{K} \sum_{x \in \Phi_i \setminus \{x_{\text{UL}}\}} \frac{P_i h}{L(x,0)} + \sum_{y \in \Psi_i \setminus \{u[y]\}} \frac{\gamma_i g}{L(y,0)}
\] (17)

where \(u[x]\) returns the user served by BS \(x\) in the uplink, \(\gamma_i = \min\{\rho_i \|y - u[y]\|^\alpha, P_{\text{max}}\}\) is the transmit power of the user, and \(\Psi_i\) is the point process that represents active users. Note that this point process is not a PPP in general and is assumed to be independent of \(\Phi_i\).

\(I_{\text{SE}}^{\text{DL}}\) is the self-interference resulting from the uplink transmission at the user, and \(I_{\text{SE}}^{\text{UL}}\) is the self-interference resulting from the downlink transmission at the BS. Therefore, since the self-interference incurred at a given receiver depends on its own transmit power, we define the residual
self-interference power after cancellation as follows:

\[ I_{DL}^{SE} = \sigma_U \min \{ \rho_k \| x_{UL} \|^{\alpha}, P_{\max} \} \quad \text{and} \quad I_{UL}^{SE} = \sigma_k P_k \]

(18)

where \( \sigma_k \) and \( \sigma_U \) represent the self-interference cancellation capability of BSs from tier \( k \) and users, respectively.

A. Mean Rate Coverage

Now, we derive the mean rate coverage \( \mathbb{E}[\ln R_{jk}^{m}] \) expressed in (14) where the expectation is with respect to \( R, \Phi_i, \) and \( \Psi_i \). First, we focus on the uplink rate coverage which can be obtained as follows:

\[ \ln [\mathbb{P} (\text{SIR}_{jk}^{UL} > \tau_{UL})] = \mathbb{E} \left[ \ln \left( g > \frac{\tau_{UL} (I_{UL}^{UL} + I_{UL}^{SE})}{\rho_k R_k^{\alpha(1-\epsilon)}} \right) \right] \]

\[ = -\tau_{UL} \mathbb{E} \left[ I_{UL}^{UL} + I_{UL}^{SE} \right] \rho_k R_k^{\alpha(1-\epsilon)} \]

(19)

where \( a \) follows from (15) and \( b \) follows from the Rayleigh fading assumption. The mean of \( I_{UL}^{UL} \) and \( I_{UL}^{SE} \) are presented in the following lemma.

**Lemma 6. (Mean of UL interference)** The mean of the interference received at a typical BS belonging to the \( k \)-th tier is given by

\[ \mathbb{E} [I_{UL}] = \sum_{i=1}^{K} \frac{2\pi \lambda_i}{\alpha - 2} \left( \frac{\alpha d_o^{2-\alpha}}{2 L_o} P_i + U_{ik}(i)^{2-\alpha} \right) \]

(20)

and the mean of the self-interference is

\[ \mathbb{E} [I_{SE}] = \sigma_k P_k \]

(21)

where

\[ \mathcal{K}(i) = \kappa_i \rho_i d_o^{2-\alpha(1-\epsilon)} + P_{\max} \Gamma \left[ \frac{4 - \alpha}{2}, \frac{\lambda_i}{A_{UL}^{i}} \right] \left( \frac{P_{\max}}{\rho_i} \right)^{\frac{2}{\alpha}} \left( \frac{\lambda_i}{A_{UL}^{i}} \right)^{\frac{\alpha - 2}{2}} \]

(22)

\[ + \rho_i \left( \Gamma \left[ \frac{4 - \alpha(1-\epsilon)}{2}, \frac{\lambda_i}{A_{UL}^{i}} d_o^{2} \right] - \Gamma \left[ \frac{4 - \alpha(1-\epsilon)}{2}, \frac{\lambda_i}{A_{UL}^{i}} \right] \left( \frac{P_{\max}}{\rho_i} \right)^{\frac{2}{\alpha}} \right) \left( \frac{\lambda_i}{A_{UL}^{i}} \right)^{\frac{\alpha(1-\epsilon)-2}{2}} \]

**Proof:** See Appendix B-I.

We can also derive the mean of uplink interference when there is no limit on the transmit power of the user (i.e., \( P_{\max} \rightarrow \infty \)). In this case, we have

\[ \lim_{P_{\max} \rightarrow \infty} \mathcal{K}(i) = \kappa_i \rho_i d_o^{2-\alpha(1-\epsilon)} + \rho_i \Gamma \left[ \frac{4 - \alpha(1-\epsilon)}{2}, \frac{\lambda_i}{A_{UL}^{i}} d_o^{2} \right] \left( \frac{\lambda_i}{A_{UL}^{i}} \right)^{\frac{\alpha(1-\epsilon)-2}{2}} \]

(23)
Next, we consider the downlink rate coverage which can be obtained similar to that of the uplink as follows:

$$\ln \left[ \mathbb{P}(\text{SIR}_{j,k}^\text{DL} > \tau_{\text{DL}}) \right] = -\frac{\tau_{\text{DL}}^{\text{DL}}}{P_j R_j^{-\alpha}} \mathbb{E} [I_{\text{DL}} + I_{\text{DL,SE}}]$$

(24)

where the mean of $I_{\text{DL}}$ and $I_{\text{DL,SE}}$ are presented in the following lemma.

**Lemma 7. (Mean of DL interference)** The mean of the interference received at a typical user served in the downlink by the $j$-th tier and in the uplink by the $k$-th tier is given by

$$\mathbb{E} [I_{\text{DL}}] = \sum_{i=1}^{K} \frac{2 \pi \lambda_i}{\alpha - 2} \left( \frac{\alpha}{2} \frac{D_{ji}^2 R_j^{2-\alpha} P_i + \alpha d_o^{2-\alpha}}{2 L_o} \mathbb{E} [\Gamma_i] \right)$$

(25)

and the mean of the self-interference is

$$\mathbb{E} [I_{\text{DL,SE}}] = \frac{\sigma_U \epsilon \alpha \rho_k}{2} \gamma \left[ \frac{\epsilon \alpha}{2}, \pi \frac{\lambda_k}{A_k^\text{UL}} \frac{(P_{\text{max}}/\rho_k)^{2-\alpha}}{\lambda_k/ \rho_k} \right] \left( \frac{\pi \lambda_k}{A_k^\text{UL}} \right)^{-\frac{\epsilon \alpha}{2}}$$

(26)

where $\mathbb{E} [\Gamma_i]$ is given in Lemma 5.

**Proof:** See Appendix B-II.

The following theorem presents closed-form expressions for the mean rate coverage for both cases when a user is associated to the same tier $j$ (i.e., with probability $\psi_{jj}$) and when a user is associated to tier $j$ for downlink transmission and tier $k$ for uplink transmission (i.e., with probability $\psi_{jk}$). Hence, the mean rate coverage of a generic FD link in the network can be obtained as follows:

$$\tilde{R} = \sum_{j=1}^{K} \psi_{jj} (R_{j,j}^\text{DL} + R_{j,j}^\text{UL}) + \sum_{j=1}^{K} \sum_{k=1}^{j-1} \psi_{jk} (R_{j,k}^\text{DL} + R_{j,k}^\text{UL})$$

(27)

**Theorem 1. (Mean of logarithm of rate coverage)** The mean of logarithms of rate coverage of the considered FD network model described above, when $j = k$, are given by

$$\ln R_{j,j}^\text{DL} = \ln R_0^\text{DL} - \pi \lambda_j \frac{\tau_{\text{DL}} P_j}{A_1(j)} + \frac{2 + \alpha}{2} \frac{A_2(j, j)}{(\frac{\pi \lambda_j}{\psi_{jj}})^{\frac{2+\alpha}{2}}}$$

(28)

$$\ln R_{j,j}^\text{UL} = \ln R_0^\text{UL} - \pi \lambda_j \frac{\tau_{\text{UL}} \rho_j}{A_3(k)} \Gamma \left[ \frac{2 + \alpha (1 - \epsilon)}{2} \right] \frac{A_3(k)}{(\frac{\pi \lambda_j}{\psi_{jj}})^{\frac{2+\alpha(1-\epsilon)}{2}}}$$

(29)

$^2$Using Jensen’s inequality, we can obtain a lower bound on $\mathbb{E}[R]$ such that $\mathbb{E}[R] \geq \exp(\mathbb{E}[\ln R])$. 


and, when \( j > k \), are given by
\[
\ln R_{DL}^{jk} = \ln R_{DL}^{o} - \lambda_j \lambda_k \frac{\tau_{DL}^{jk}}{\rho_k} \left( \frac{D_j}{U_k} \right)^{2} \sum_{l=k}^{j-1} \frac{1}{\Omega_l} \left( \mathcal{H}_l [1, 1; 2] \frac{A_1(j)}{\Upsilon_l^{3}(j)} + \mathcal{H}_l [1, 1; \alpha] \frac{A_2(j, k)}{\Upsilon_l^{4}(j)} \right)
\] (30)
\[
\ln R_{UL}^{jk} = \ln R_{UL}^{o} - \lambda_j \lambda_k \frac{\tau_{UL}^{jk}}{\rho_k} \left( \frac{D_j}{U_k} \right)^{2} \sum_{l=k}^{j-1} - \mathcal{H}_l [\mu_l, \mu_{l+1}; \alpha (1 - \epsilon)] \frac{A_3(k)}{\Upsilon_l^{4}(j)}
\] (31)
where
\[
A_1(j) = \sum_{i=1}^{K} \frac{2 \pi \lambda_i P_i D_{ji}^{2-\alpha}}{\alpha - 2}, \quad A_2(j, k) = \sigma_U \mathbb{E}[\Gamma_k] + \sum_{i=1}^{K} \frac{\pi \alpha \lambda_i d_{i}^{2-\alpha}}{L_o (\alpha - 2)} \mathbb{E}[\Gamma_i],
\]
\[
A_3(k) = \sigma_k P_k + \sum_{i=1}^{K} \frac{2 \pi \lambda_i}{\alpha - 2} \left( \frac{\alpha d_{i}^{2-\alpha} P_i}{2 L_o} + \mathcal{U}_{ik}^{2-\alpha} \mathcal{K}(i) \right),
\]
\[
\mathcal{H}_l [a, b; c] = \frac{\Gamma \left( \frac{a + c}{2} \right)}{\pi \frac{1}{2}} \left( \frac{a^{2+c}}{1 + \mu_l \frac{2}{\alpha} \Omega_l} - \frac{b^{2+c}}{1 + \mu_{l+1} \frac{2}{\alpha} \Omega_l} \right).
\]

**Proof:** See Appendix C. \[\blacksquare\]

### B. Special Cases

Using the lemmas derived above and Theorem 1, the performance of both uplink and downlink transmissions in FD cellular networks is presented in the following two corollaries. It is worth mentioning that the expressions for the mean rate presented in these corollaries are equivalent to the mean rates of legacy uplink and downlink users, respectively.

**Corollary 1. (Mean rate of uplink transmissions)** The mean rate coverage offered by an FD cellular network to a legacy HD uplink user is given by
\[
\ln R_{UL}^{o} = \ln R_{UL}^{o} - \frac{\tau_{UL}^{o}}{\rho_k} \mathbb{E} \left[ R_k^{o (1 - \epsilon)} \right] \sum_{i=1}^{K} \left( \frac{\alpha d_{i}^{2-\alpha}}{2 L_o} P_i + \mathcal{U}_{ik}^{2-\alpha} \mathcal{K}(i) \right)
\] (32)
and \( \mathbb{E} [R_k^{o}] \) is given in Lemma 3.

**Corollary 2. (Mean rate of downlink transmissions)** The mean rate coverage offered by an FD network to a legacy HD downlink user is given by
\[
\ln R_{DL}^{o} = \ln R_{DL}^{o} - \frac{\tau_{DL}^{o}}{P_j} \sum_{i=1}^{K} \pi \lambda_i \left( 2 P_i d_{ji}^{2-\alpha} \mathbb{E} \left[ R_i^{2} \right] + \frac{\alpha d_{i}^{2-\alpha}}{L_o} \mathbb{E} \left[ \Gamma_i \right] \mathbb{E} \left[ R_i^{2} \right] \right),
\] (33)
and $E[R^o_j]$ is given in Lemma 3.

In addition, the performance of FD networks with CCA, HD uplink networks, and HD downlink networks can be obtained as presented in the following corollaries.

**Corollary 3.** *(Coupled cell association)* The rate coverage of FD networks with CCA is given by

$$\bar{R} = \sum_{j=1}^{K} A^{DL}_j (R^{DL}_{jj} + R^{UL}_{jj})$$

where

$$\ln R^{DL}_{jj} = \ln R^o_{jj} - \frac{\tau^{DL}}{P_j} (A_1(j)\mathbb{E}[R^2_j] + A_2(j,j)\mathbb{E}[R^o_j])$$

and

$$\ln R^{UL}_{jj} = \ln R^o_{jj} - \frac{\tau^{UL}}{\rho_j} A_3(j)\mathbb{E}[R^o_{j(1-\epsilon)}]$$

and $E[R^o_j]$ is given in Lemma 3 and $D_i = U_i$.

**Corollary 4.** *(Half-duplex uplink networks)* The mean rate coverage of HD uplink networks with weighted path-loss cell association is given by

$$\bar{R} = \sum_{k=1}^{K} A^{UL}_k R^{UL}_k$$

where

$$\ln R^{UL}_k = \ln R^o_k - \frac{\tau^{UL}}{\rho_k} \mathbb{E}[R^o_{k(1-\epsilon)}] \sum_{i=1}^{K} \frac{2\pi\lambda_i U_i^{2-\alpha}}{2} \frac{2}{\alpha - 2} \mathcal{K}(i).$$

The rate is also given when the is no constraint on the maximum transmit power (i.e., $P_{max} \to \infty$) using (23) as

$$\ln R^{UL}_k \overset{(P_{max} \to \infty)}{\overset{(d_o=0)}{\approx}} \ln R^o_k - \frac{\tau^{UL}}{\rho_k} \Gamma \left[ \frac{2 + \alpha(1 - \epsilon)}{2} \right] \Gamma \left[ \frac{4 - \alpha(1 - \epsilon)}{2} \right] \sum_{i=1}^{K} \frac{2\rho_i U_i^{2-\alpha}}{\alpha - 2} A^U_i. \quad (37)$$

**Corollary 5.** *(Half-duplex downlink networks)* The mean rate coverage of HD downlink networks with weighted path-loss cell association is given by

$$\bar{R} = \sum_{j=1}^{K} A^{DL}_j R^{DL}_j$$

where

$$\ln R^{DL}_j = \ln R^o_j - \frac{\tau^{DL}}{P_j} \mathbb{E}[R^2_j] \sum_{i=1}^{K} \frac{2\pi\lambda_i P_i D_{ji}^{2-\alpha}}{\alpha - 2} \overset{(d_o=0)}{\approx} \ln R^o_j - \frac{\tau^{DL}}{P_j} \sum_{i=1}^{K} \frac{2P_i D_{ij}}{\alpha - 2} A^D_i. \quad (38)$$

Note that the results presented in (37) and (38) are consistent with the previous results in [9] on cell association in multi-tier HD cellular networks.
**Corollary 6.** *(Half-duplex uplink networks with MinTrCA)* The mean rate coverage of HD uplink networks with MinTrCA such that $U_i = \rho$ is given by

$$\ln R_{UL}^{(P_{\text{max}}\rightarrow \infty, d_0=0)} = \ln R_o^{UL} - \frac{2\tau_{UL}}{\alpha - 2} \left[ \frac{2 + \alpha (1 - \epsilon)}{2} \right] \Gamma \left[ \frac{4 - \alpha (1 - \epsilon)}{2} \right].$$  \hspace{1cm} (39)

**Corollary 7.** *(Half-duplex downlink networks with MaxPCA)* The mean rate coverage of HD downlink networks with MaxPCA is given by

$$\ln R_{DL}^{(d_0=0)} = \ln R_o^{DL} - \frac{2\tau_{DL}}{\alpha - 2}.$$  \hspace{1cm} (40)

From (39) and (40), it can be seen that the performance of an interference-limited network is independent of the spatial density, receiver sensitivity, and transmit power of BSs which is consistent with the results in existing literature [8], [9], [21]–[23].

**V. RATE MAXIMIZATION**

There are no closed-form expressions for the optimal cell association weighting factors that maximize the overall mean rate of FD cellular networks presented in Theorem 1 which is not convex with respect to $U_k$ and $D_j$. However, using the results in Section IV, we optimize the cell association criteria in order to independently maximize the mean rate of uplink and downlink transmissions. This, in turn, maximizes the rate of legacy HD transmissions in FD networks. Following a similar procedure as in [9], we firstly derive the optimal cell association probabilities $A_{UL}^k$ and $A_{DL}^j$, then we obtain the optimal weighting factors $U_k$ and $D_j$ using Lemma 2.

Using results in Corollary 2, we derive the optimal cell association to maximize the rate of downlink transmissions in an FD cellular network. That is, after some mathematical manipulations, we formulate the following optimization problem:

$$\min_{A_{DL}^j, A_{UL}^k} \sum_{j=1}^{K} \frac{\tau_{DL} (A_{DL}^j)^{\frac{2+\alpha}{2}}}{P_j \lambda_j^{\frac{\alpha}{2}}} + \sum_{i=1}^{K} \frac{2P_i (A_{UL}^i)^{\frac{2+\alpha}{2}}}{\lambda_i^{\frac{\alpha}{2}}} + \sum_{j=1}^{K} \frac{\tau_{DL} B[A_{UL}^j] (A_{DL}^j)^{\frac{2+\alpha}{2}}}{P_j \lambda_j^{\frac{\alpha}{2}}}$$  \hspace{1cm} (41)

subject to $\sum_{j=1}^{K} A_{DL}^j = 1$, $\sum_{j=1}^{K} A_{UL}^j = 1$, and $A_{DL}^j, A_{UL}^j \geq 0$, $\forall j$.

We assume all network tiers have the same receiver sensitivity (i.e., $\rho_k = \rho$), no constraint on the maximum transmit power (i.e., $P_{\text{max}} \rightarrow \infty$), and perfect interference cancellation.
where $B[A_{UL}] = \frac{\pi^{1-\alpha} \epsilon d^{2-\alpha}}{2 \lambda_o (\alpha-2)} \Gamma \left[ \frac{\alpha}{2} \right] \Gamma \left[ \frac{2+\alpha}{2} \right] \sum_{i=1}^{K} \rho_i \lambda_i^{2-\alpha} \left( A_{ULi} \right)^{\frac{\alpha}{2}}.$

Then, we transform the problem into the following three concave optimization sub-problems subject to the same set of constraints

$$
\min_{A_{DL}^j} \sum_{j=1}^{K} \frac{P_j (A_{DLj})^{2+\alpha}}{\lambda_j^{2+\alpha}}, \quad \min_{A_{UL}^j} \sum_{j=1}^{K} \frac{P_j (A_{ULj})^{2+\alpha}}{\lambda_j^{2+\alpha}} \gamma \sum_{j=1}^{K} B[A_{UL}^j] (A_{DLj})^{2+\alpha}. \quad (42)
$$

Using Lagrangian relaxation and taking the first order partial derivatives with respect to $A_{DLj}$ and $A_{ULj}$, the optimal cell association probability can be obtained as follows:

$$
A_{DLj}^* = \frac{P_j \lambda_j}{\sum_{i=1}^{K} \frac{P_i \lambda_i^{2+\alpha}}{\lambda_i^{2+\alpha}}}, \quad A_{ULj}^* = \frac{\lambda_j}{\sum_{i=1}^{K} \lambda_i}. \quad (43)
$$

which are equivalent to $D_j^* = \frac{D}{P_j}$ and $U_j^* = U$ for arbitrary constants $D$ and $U$. This result is presented in the following theorem without a proof.

**Theorem 2. (Optimal cell association for downlink transmissions)** For an FD network, the downlink transmissions rate is maximized when cell association is based on the maximum received SINR (MaxPCA) in the downlink and on the minimum distance in the uplink (MinDCA).

Next, we consider maximizing the rate of uplink transmissions. As can be seen from Corollary 1, the cell association criteria of the downlink does not impact the performance of uplink transmissions. Therefore, we only optimize uplink cell association criteria $A_{ULk}$. Now, using Corollary 1, maximizing the mean rate is equivalent to solving the following optimization problem:

$$
\min_{A_{UL}^k} \sum_{k=1}^{K} \frac{\tau_{UL} A_{ULk}}{\rho_k} + \sum_{i=1}^{K} \frac{2 \pi \lambda_i^{\alpha-2} \alpha d^{2-\alpha}}{2 L_o} P_i + \sum_{k=1}^{K} \frac{\tau_{UL} (A_{ULk})^{4-\alpha}}{\rho_k \lambda_k^{2+\alpha}} \lambda_k^{2+\alpha} \sum_{i=1}^{K} \alpha - 2 \frac{\alpha \lambda_i^{2+\alpha}}{2}. \quad (44)
$$

subject to $\sum_{k=1}^{K} A_{ULk} = 1, \quad A_{ULk} \geq 0 \quad \forall k.$

Similar to the downlink case, we transform the optimization problem in (44) into the following two concave sub-problems subject to the same set of constraints:

$$
\min_{A_{UL}^k} \sum_{k=1}^{K} \frac{(A_{ULk})^{4-\alpha}}{\rho_k \lambda_k^{2+\alpha}}, \quad \min_{A_{UL}^k} \sum_{k=1}^{K} \frac{\rho_k (A_{ULk})^{\alpha}}{\lambda_k^{2+\alpha}}. \quad (45)
$$

\text{We assume full channel inversion (i.e., $\epsilon = 1$), all network tiers have the same receiver sensitivity (i.e., $\rho_k = \rho$), no constraint on the maximum transmit power (i.e., $P_{\max} \rightarrow \infty$), and perfect interference cancellation.}
Using Lagrangian relaxation and taking the first order derivative with respect to $A_{UL}^k$, the optimal solution is presented in the following theorem.

**Theorem 3.** (Optimal cell association for uplink transmissions) For an FD network, the uplink transmissions rate is maximized when cell association is based on the minimum distance (MinDCA).

It is worth mentioning that the results in Theorem 2 and Theorem 3 show the importance of DCA in FD cellular networks in order to maximize the overall mean rate. In addition, both the results imply that the cell association criteria should be based maximum received SINR for downlink and minimum distance for uplink in order maximize the mean rate of both downlink and uplink transmissions. These insights will be discussed in details in the following section.

VI. NUMERICAL RESULTS AND DISCUSSIONS

A. System Parameters

We use the results obtained above in closed-form to evaluate system performance in different scenarios. We consider the following cases: (1) FD cellular networks with both CCA and DCA, (2) both HD downlink networks (i.e., only downlink transmissions) and HD uplink networks (i.e., only uplink transmissions), and (3) both legacy downlink users (i.e., HD downlink users in an FD network) and legacy uplink users (i.e., HD uplink users in an FD network). For numerical evaluation, unless otherwise stated, the path-loss exponent for all links is $\alpha = 4$ and the reference distance $d_o$ is set to 5 m. For uplink transmissions, the power control factor is $\epsilon = 0.9$ and all BSs have the same receiver sensitivity (i.e., $\rho_i = \rho$). For CCA we assume that $D_i = U_i = P_i^{-1}$, while for DCA $D_i = P_i^{-1}$ and $U_i = 1$. For the evaluation of the mean rate, the rate coverage thresholds $\tau_{DL}$ and $\tau_{UL}$ are set to 0 dB which is equivalent to $\ln(2)$ nats/sec/Hz. We show results when the number of tiers varies between 2 and 3 network tiers.

Fig. 2 shows the effect of varying the density of BSs on the mean rate for three different networks: (1) HD downlink network, (2) HD uplink network, and (3) FD network (both legacy downlink and uplink users). For HD downlink networks, from Lemma 5, it can be seen that the mean rate of downlink transmissions is almost independent of the spatial density of BSs. This is expected because increasing $\lambda$ results in increasing both the useful signal power and interference power and the SIR remains unchanged. For HD uplink networks, from Lemma 4, a
similar conclusion can be made. It is worth mentioning that for HD uplink networks with low density of BSs (e.g., $\lambda_1 < 2 \times 0.5^2 \text{ BS/km}^2$), the power of both the useful signal and interference is limited by the maximum power constraint $P_{\text{max}}$. On the other hand, for both HD downlink and uplink networks, when the spatial density of BSs is very high (e.g., $\lambda_1 > 70 \times 0.5^2 \text{ BS/km}^2$), users become very close to the serving BS and the power of the useful signal received at the corresponding receiver is constrained by $d_o$. Both effects are not evident when $P_{\text{max}}$ is very large as in Lemma 6 and $d_o = 0$ as in Lemma 7.

For FD networks, Fig. 2 shows that increasing $\lambda$ has two effects on the mean rate of legacy users. The first effect is captured in Lemma 2 for legacy downlink users in which it can be seen that increasing the number of BSs improves the mean rate. That is because deploying more BSs reduces the distance to the served user which in turn (i) increases the received power at this user and (ii) decreases the transmit power of users and the interference resulting from the uplink transmissions. This happens up to some point of $\lambda$ after which the performance of the network becomes interference-limited and the interference from uplink transmissions becomes very small compared to that from downlink transmissions. The second effect, as shown in Lemma 1, is the degradation in the mean rate of legacy uplink users by increasing the spatial density of BSs. As can be seen in Fig. 2, increasing $\lambda$ in FD networks decreases the distance to the serving BS and thus each user transmits with less power (due to channel inversion power control). However, the
the interference power from the downlink transmissions increases at the same time which highly impacts the performance of uplink transmissions (cf. (15)).

Fig. 3 elaborates more on this effect in FD networks by showing the mean rate of a generic FD link in the network for different cell association criteria. Overall, it can be seen that $\lambda$ should be adjusted to balance the trade-off between the rates of downlink and uplink transmissions in FD networks. As explained earlier, although deploying BSs with low spatial density helps uplink transmissions by reducing interference from downlink transmissions, it degrades the performance of downlink transmissions. While the rate of downlink transmissions is improved in dense networks, uplink transmissions become susceptible to high interference power from the downlink transmissions. Therefore, as shown in Fig. 2, as $\lambda$ increases, the overall mean rate of the FD network increases up to a maximum value, then it starts to decrease. Note that a similar behavior is evident for FD networks with CCA.

Fig. 4 shows the effect of varying $\rho$ on the mean rate of the network under different network configurations. In general, decreasing the sensitivity of the receiver (i.e., increasing $\rho$) increases the amount of transmission power required by each user to perform channel inversion towards the serving BS (cf. (9)). This, in turn, increases both the levels of useful signal and interference power received at other users and BSs. Hence, as shown in Fig. 4, although the mean rate of uplink transmissions improves with increasing $\rho$, the mean rate of downlink transmissions
Fig. 4. Mean rate (in nats/sec/Hz) vs. the sensitivity of BS receivers $\rho$ (in dBm). The network parameters are $K = 3$, $\lambda_1 = 5(500^2\pi)^{-1}$ BS/km$^2$, $\lambda_2 = 5\lambda_1$, $\lambda_3 = 10\lambda_1$, $\{P_1, P_2, P_3\} = \{37, 33, 23\}$ dBm, $P_{\text{max}} = 23$ dBm, and $\rho_1 = \rho_2 = \rho_3$. The results are shown for both CCA and DCA in FD networks, and legacy downlink and uplink users.

deteriorates due to the extra amount of interference. Overall, it can be seen that there exists an optimal value of $\rho$ that maximizes the overall all mean rate and splits the performance of FD networks into two regimes. That is, as $\rho$ increases, the overall mean rate of the network increases up to a maximum value, then it starts to decrease. This behavior can be explained as follows: when $\rho$ is very low (the left side of the optimal point), while downlink transmissions experience low interference power from uplink transmissions, the interference at BSs is very high compared to the useful signal power and the mean rate of uplink transmissions is almost 0. As $\rho$ increases, the mean rate of downlink transmissions stays almost constant while the mean rate of uplink transmissions improves and this increase dominates the overall mean rate performance. This happens until achieving the maximum overall mean rate. After this point, as $\rho$ increases (the right side of the optimal point), the mean rate of downlink transmissions starts to fall and the mean rate of uplink transmissions becomes almost constant. Hence, the overall mean rate of the FD network starts to decrease. A similar behavior is observed for FD networks with CCA. It is worth mentioning that Figs. 3 and 4 show that DCA is always superior to CCA for all ranges of $\lambda$ and $\rho$ where the overall network performance can be optimized by separately optimized the downlink and uplink cell association criteria.

Fig. 5 shows the mean rate of downlink transmissions as a function of the downlink weighting factors in a two-tier network for different transmit power settings. It can be clearly seen that the
maximum mean rate is achieved when the ratio of the weighting factors is equal to the inverse of the transmit power ratio. For example, when $\frac{P_2}{P_1} = 0.2$ (shown by the solid black curve with diamond-shaped markers), the rate of downlink transmissions is maximized when $\frac{D_2}{D_1} = \frac{P_1}{P_2} = 5$. Similar remark can be made for different cases when the ratio $\frac{P_2}{P_1}$ is 1, 0.5, and 0.1, the rate is maximized at $\frac{D_2}{D_1}$ is set to 1, 2, and 10, respectively. This behavior can be explained using (14) and (15) where the SIR of the downlink transmissions can be expresses as:

$$\text{SIR}^{DL} = \left( \frac{\text{sum of rec. power from all BSs and users}}{P_j h \|x^{DL}\|^{-\alpha}} - 1 \right)^{-1}.$$

It can be seen the numerator is not a function of $D_j$ since the transmit power of BSs is constant and that of users is independent of $D_j$ (as shown in Lemma 5). Therefore, the mean transmission rate, as well as SIR, is maximized when the denominator (which is equivalent to the power of the useful signal received from the serving BS) is maximized. Hence, MaxPCA is the optimal cell association criterion to maximize the mean rate of downlink transmissions as well as legacy downlink users. This result agrees with Theorem 2 on the maximization of the mean rate of downlink transmissions where $D_j^* = P_j^{-1}$.

On the other hand, Fig. 6 shows the mean rate of uplink transmissions as a function of the uplink weighting factors in a two-tier network for different receiver sensitivity settings. In contrast to Fig. 5, the maximum rate of uplink transmissions is achieved when the ratio of the weighting
Fig. 6. Mean rate of uplink transmissions (in nats/sec/Hz) vs. the ratio of uplink weighting factors $\frac{U_2}{U_1}$. The network parameters are $K = 2$, $\lambda_2 = 2\lambda_1$, $\{P_1, P_2\} = \{33, 23\}$ dBm, $\{D_1, D_2\} = \{\frac{1}{P_1}, \frac{1}{P_2}\}$, $\rho_1 = \rho_2$, and no constraint on $P_{max}$. The results are shown for different BSs receiver sensitivity.

factors is equal 1. This behavior is evident for different values of $\rho$ and can be explained when $\epsilon = 1$ using (14) and (15) where the SIR of the uplink transmissions can be expresses as:

$$\text{SIR}_{UL}^{\text{UL}} = \left(\frac{\text{sum of rec. power from all BSs and users}}{\rho g} - 1\right)^{-1}.$$ 

Although both the transmit power of BSs and the denominator are independent of $U_k$, the sum of received power from all users in the numerator is a function of $U_k$. In this case, SIR is maximized when the numerator is minimized which is equivalent to minimizing the transmit power of all users. This is achieved when each user is served by the BS that requires the lowest transmit power $\rho\|x_{\text{UL}}^\text{UL}\|^{\alpha}$ (i.e., MinTrCA) which is equivalent to MinDCA when $\rho_i = \rho$. That is, since users perform channel inversion power control, the transmit power of users is minimized when each user associates to the nearest BS. Hence, MinTrCA is the optimal cell association criterion to maximize the mean rate of uplink transmissions as well as legacy uplink users. This result agrees with Theorem 3 on the maximization of the mean rate of uplink transmissions where $U_k^* = \rho$.

Finally, in order to show the effect of cell association criterion on the performance of legacy users, Fig. 7 shows the overall mean rate of the legacy users (i.e., sum of mean rate of downlink and uplink transmissions) as a function of the ratios of downlink and uplink weighting factors $\frac{D_2}{D_1}$ and $\frac{U_2}{U_1}$, respectively. The following remarks can be made:
The maximum overall transmission rate in a generic link is achieved at $\frac{D_2}{D_1} = \frac{P_1}{P_2}$ and $\frac{U_2}{U_1} = 1$. This result is presented by solid black triangle. It is worth mentioning that although this cell association criterion is optimal for both legacy downlink and uplink users, they do not necessarily maximize the overall mean rate of a generic FD link in the network.

- For any given $\frac{U_2}{U_1}$ ratio, the overall mean rate of the legacy users is maximized at $\frac{D_2}{D_1} = \frac{P_1}{P_2}$. This result is presented by the vertical dashed black line.
- For any given $\frac{D_2}{D_1}$ ratio, the overall mean rate of the legacy users is maximized at $\frac{U_2}{U_1} = 1$. This result is presented by the horizontal dashed black line.
- CCA (i.e., presented by the solid black line with slope of 1 such that $\frac{D_2}{D_1} = \frac{U_2}{U_1}$) is not optimal to maximize the overall mean rate compared to DCA which achieves higher rates for wide range of weighting factors.

Note that all aforementioned remarks agree with the results presented in Section V and Figs. 5 and 6.

VII. Conclusion

We have presented a comprehensive framework for cell association in multi-tier full-duplex cellular networks. For both uplink and downlink transmissions, we have considered different
cell association criteria including both coupled and decoupled cell association. We have used stochastic geometry to provide a complete framework to model, analyze, and evaluate the performance of the proposed system in terms of mean rate of full-duplex, uplink, and downlink transmissions. Using weighted path-loss cell association, we have derived the optimal weighting factors that maximize the mean rate of both uplink and downlink transmissions in the presence of interference resulting from full-duplex transmissions. In addition, we have shown that in order to maximize the mean rate of legacy half-duplex users in full-duplex networks, a legacy uplink user should associate to the nearest BS whereas a legacy downlink user should associate to the BS that results in the maximum received power. We also conclude that decoupled cell association outperforms coupled cell association for wide range of weighting factors where these parameters should be carefully tuned to optimize the network performance.

APPENDIX A

PROOF OF ASSOCIATION PROBABILITY AND DISTANCE DISTRIBUTION

I. Proof of Lemma 1

We first consider the case when a user associates to different BSs for downlink and uplink. A user associates to different BSs ($x_{DL} \in \Phi_j$ for downlink and $x_{UL} \in \Phi_k$ for uplink where $j \neq k$) under the following four conditions:

1) $x_{UL}$ meets the criteria in (1) for the uplink, i.e., $U_k\|x_{UL}\|^\alpha < \min_{x \in \Phi_i} U_i\|x\|^\alpha \ \forall i \neq j,k$.
2) $x_{DL}$ meets the criteria in (2) for the downlink, i.e., $D_j\|x_{DL}\|^\alpha < \min_{x \in \Phi_i} D_i\|x\|^\alpha \ \forall i \neq j,k$.
3) $x_{DL}$ does not meet the criteria defined for the uplink in (1), i.e., $U_j\|x_{DL}\|^\alpha > U_k\|x_{UL}\|^\alpha$.
4) $x_{UL}$ does not meet the criteria defined for the downlink in (2), i.e., $D_k\|x_{UL}\|^\alpha > D_j\|x_{DL}\|^\alpha$.

Using these four conditions and letting $R_j = \|x_{DL}\|$ and $R_k = \|x_{UL}\|$, the event that $j \neq k$ can be expressed as:

$$\bigcap_{i=1}^{K} \min_{x \in \Phi_i} \|x\| > \max\{D_j R_j^\alpha, U_k R_k^\alpha\} \frac{1}{\alpha} D_j^{\frac{2}{\alpha}} R_j < R_k < U_k^{\frac{2}{\alpha}} R_k \bigg\}.$$  \hspace{1cm} (46)

Hence,

$$\psi_{jk} \stackrel{(a)}{=} \mathbb{E} \left[ \exp \left[ -\pi \sum_{i=1}^{K} \max_{i \neq j,k} \left\{ D_j R_j^\alpha, U_k R_k^\alpha \right\} \frac{2}{\alpha} \lambda_i \right] \bigg| D_j^{\frac{2}{\alpha}} R_j < R_k < U_k^{\frac{2}{\alpha}} R_k \right]$$
\[ (b) = 4\pi^2 \lambda_j \lambda_k \int_0^\infty \int_{D_{jk}^{1/2}} r u \exp \left[ -\pi \sum_{i=1}^K \max \{ D_{ji}^\alpha, U_{ki}^\alpha \} \frac{\lambda_i}{\alpha} \right] dr du \]

\[ = 2\lambda_j \lambda_k \int_{D_{jk}^{1/2}} \left( \sum_{i=1}^K \lambda_i \max \left\{ D_{ji}^\alpha, U_{ki}^\alpha \right\} \right)^{-2} d\alpha \]  

(47)

where (a) follows from: (i) the fact that the minimum distance to a BS from a PPP \( \Phi_i \) is Rayleigh-distributed with CDF \( \mathbb{P} \left[ \min_{x \in \Phi_i} \| x \| \leq t \right] = 1 - \exp\left[ -\pi \lambda_i t^2 \right] \) and (ii) independence assumption for network tiers, (b) follows since the expectation in (a) is with respect to \( R_j \) and \( R_k \) which denote the distances to the serving BSs. Note also that \( \psi_{jk} = 0 \) when \( k > j \) due to ordering the network tiers such that \( \mu_i < \mu_{i+1} \).

For the case when \( j = k \), using (1) and (2) and following a similar procedure, the probability of the event where \( x_{DL} = x_{UL} = x_o \) given that \( x_o \) belongs to the \( j \)-th tier can be expressed as:

\[ \psi_{jj} = \mathbb{E} \left[ \exp \left[ -\pi \sum_{i=1}^K \max \{ D_{ji}^\alpha, U_{ki}^\alpha \} \frac{\lambda_i}{\alpha} R_j^2 \right] \right]. \]  

(48)

Hence, the result in (3) can be then easily verified.

II. Proof of Lemma 4

Following the procedure in Lemma 1, the joint CDF of the distance can be obtained by adding two more conditions such that \( R_j > r_j \) and \( R_k > r_k \). Then, the joint PDF can be obtained by differentiation. \( \mathbb{E}[R_j^\alpha R_k^\alpha] \) follows the definition of the expected value.

**APPENDIX B**

**PROOF OF MEAN INTERFERENCE**

I. Proof of Lemma 6

Following the definition in (17), we have

\[ \mathbb{E} [t_{UL}^{(a)}] = \sum_{i=1}^K 2\pi \lambda_i \left( \frac{P_i}{L_o} \int_0^\infty r \max \{ d_o, r \}^{-\alpha} dr + \mathbb{E}_{R_i} \left[ \min \{ \rho_i R_i^\alpha, P_{\max} \} \int_{U_{ik}^{1/2}}^\infty r^{1-\alpha} dr \right] \right) \]

\[ = \sum_{i=1}^K 2\pi \lambda_i \left( \frac{\alpha d_o^{2-\alpha}}{2L_o} P_i + \frac{\alpha U_{ik}^{2-\alpha}}{2L_o} \mathbb{E}_{R_i} \left[ \min \{ \rho_i R_i^\alpha, P_{\max} \} R_i^{2-\alpha} \right] \right) \]  

(49)

where (a) follows from Campbell’s Theorem [5] knowing that the distance \( R \) of the closest interfering user from tier \( i \) is greater than \( U_{ik}^{1/2} R_i \) (i.e., \( U_i R_i^\alpha < U_k R^\alpha \)). Following Lemma 3, we obtain \( \mathbb{E}_{R_i} \left[ \min \{ \rho_i R_i^\alpha, P_{\max} \} R_i^{2-\alpha} \right] = K(i) \).
II. Proof of Lemma 7

Following the definition in (16), we have

\[ E[I_{DL}]^{(a)} = \sum_{i=1}^{K} 2\pi \lambda_i \left( P_i \int_{D_{ji}^R} r^{1-\alpha} dr + \frac{E[R_i]}{L_o} \int_0^\infty r \max\{d_o, r\}^{-\alpha} dr \right) \]  

(50)

where \((a)\) follows from the fact that \(D_iR_j^\alpha > D_jR_j^\alpha\). Moreover, (26) is obtained from (18) such that \(E[I_{SE}] = \sigma_UE[\Gamma_k]\).

APPENDIX C

PROOF OF THEOREM 1

By combining (14), (19), (24), Lemma 6, and Lemma 7 and rearranging all terms, we have

\[ E[R_j^{DL}] = \ln R_o^{DL} - \frac{\tau^{DL}}{P_j} \sum_{i=1}^{K} \frac{2\pi \lambda_i P_i D_{ji}^D 2^{\alpha \beta}}{\alpha - 2} \mathbb{E}[R_j^2] \] 

\[ - \frac{\tau^{DL}}{P_j} \left( \mathbb{E}[I_{SE}] + \sum_{i=1}^{K} \frac{\pi \alpha \lambda_i d_i^{2-\alpha}}{L_o(\alpha - 2)} \mathbb{E}[\Gamma_i] \right) \mathbb{E}[R_j^\alpha] \] 

(51)

\[ E[R_k^{UL}] = \ln R_o^{UL} - \frac{\tau^{UL}}{\rho_k} \left( \mathbb{E}[I_{SE}] + \mathbb{E}[I_{UL}] \right) \mathbb{E}[R_k^{\alpha(1-\epsilon)}] \] 

(52)

where \(\mathbb{E}_R[\cdot]\) is with respect to \(f_R(r_j, r_k)\) given in Lemma 4. First, we consider the case when \(j \neq k\), the following terms can be simplified as:

\[ E[R_j^{\alpha}] = \lambda_j \lambda_k \left( \frac{D_j}{U_k} \right)^{\frac{2}{\alpha}} \sum_{l=k}^{j-1} \frac{\mathcal{H}_l[1, 1; m]}{\Omega \Gamma_{l+1}^{\frac{4+m}{2}}(j)} \] 

(53)

\[ E[R_k^{n}] = \lambda_j \lambda_k \left( \frac{D_j}{U_k} \right)^{\frac{2+n}{\alpha}} \sum_{l=k}^{j-1} \frac{-\mathcal{H}_l[m; m+1; n]}{\Gamma_{l+1}^{\frac{4+n}{2}}(j)} \] 

(54)

Hence, the above expressions can be easily verified after some mathematical manipulations. Similarly, the expression for \(j = k\) can be obtained.

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