SUPERSYMMETRY AND QUBIT FIELD THEORY

Jaroslav HRUBÝ
Institute of Physics AV CR, Czech Republic
e-mail: hruby@gcucmp.cz

Abstract
Supersymmetric extension of the Deutsch’s quantum field theory is presented and a new solution of quantum information paradox via quantum-fields $\bar{\psi}\psi$ condensate is presented.

1 Introduction

In recent time it is believed that quantum mechanics has the potential to bring about a spectacular revolution in quantum computing and quantum information theory [1, 2].

Supersymmetry [3] is subject of great interest among physicists and mathematicians and play the fundamental role in particle physics and in gravity theories. Here we show that it can play crucial role in quantum information theory, when energy becomes sufficiently high and where Bekenstein’s information bound plays role [4].

In the 1970s theoretical physics developed a new fruitful concept in supersymmetry, i.e. the concept of treating states and superpartner states equally, where anticommuting $c$-numbers play the important role.

The glue supersymmetric quantum mechanics and supersymmetry was first given by the superposition Lagrangian in $(1+1)$ space-time dimensions [5] in 1977:

\[
L = \frac{1}{2}[(\partial_\mu \phi)^2 - V^2(\phi) + \bar{\psi}(i + V'(\phi))\psi]
\] (1)

where $\phi$ was a solitonic Bose field and $\psi$ was a Fermi field and $V(\phi)$ was some nonlinear potential.
Substituting into (1) the following restriction to (1 + 1) space-time dimension

\[ \phi \to x(t) \quad \partial_n \to \partial_t \quad (2) \]

\[ \bar{\psi} \to \psi^\top \sigma_2 \quad i \to i \partial_t \sigma_2 \quad (3) \]

where \( \psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} \) with components being interpreted as anticommuting \( c \)-numbers, \( \sigma_h \) denotes the Pauli matrices, then from (1) the Lagrangian of the supersymmetric quantum mechanics is obtained.

\[ L_{SSQM} = \frac{1}{2} [(\partial_t x)^2 - V^2(x) + \psi^\top (i \partial_t + \sigma_2 V'(x)) \psi] \quad (4) \]

and the corresponding Hamiltonian has the known form

\[ H_{SSQM} = \frac{1}{2} p^2 + \frac{1}{2} V^2(x) + \frac{1}{2} i[\psi_1, \psi_2] V'(x) \quad (5) \]

which was proposed as supersymmetric quantum mechanics (SSQM) [6]. If we move from (0+1) dimension to (1+1) dimension we can define superfields.

Superfields provide an elegant and compact description of supersymmetry representations. and are defined on superspace. We show it for the case (1+1) dimension and it can be extended to higher dimensions as is usually.

We shall define the superspace \( E \) with elements \( z^A \in E, \ A = 1, \ldots, 8 \), which are:

\[ (x^\mu, \theta^\alpha) ; \quad \mu = 1, \ldots, 4, \ \alpha = 1, \ldots, 4. \quad (6) \]

It can be viewed as a geometrical fiber space \( E(V, W, P^\dagger_+) \), the basis \( V \) is odd part of the superspace and the fibre \( W \) is the even part. \( P^\dagger_+ \) is the Poincar group, acting on a fibre and \( E \) is the cartesian product between the basis and the fibre.

On the superspace we have the differential one-form:

\[ \omega^\mu = \Omega^\mu + i \tilde{\omega}^\alpha (\gamma^\mu)_{\alpha\beta} \theta^\beta \quad (7) \]

\[ \omega^\alpha = \alpha \theta^\alpha \quad (8) \]

For the form \( \Omega^\mu = dx^\mu \) is valid the invariance \( \Omega^\mu \Omega^\mu \) under \( P^\dagger_+ \).

From the fiber structure follows:

\[ \omega^\mu = \Omega^\mu + \tilde{\omega}^\alpha \Gamma^\mu_{\alpha} \quad (9) \]

where \( \Gamma^\mu_{\alpha} \).

\[ \Gamma^\mu_{\alpha} = i (\gamma^\mu)_{\alpha\beta} \theta^\beta \quad (10) \]

It means \( \varepsilon^\alpha \) in the anticommuting basis

\[ \theta^\alpha \to \theta^\alpha + \varepsilon^\alpha \]
makes change the $x^\mu$ on the

$$x^\mu \to x^\mu + i \bar{\varepsilon} \gamma^\mu \theta.$$  

and it is the supersymmetry transformation, which preserve the invariance of the one-forms $\omega^\mu$ and $\omega^\alpha$.

We can see:

$$\Omega^\mu = \frac{dx^\mu}{d\theta^\alpha} \omega^\alpha = \left( \frac{\partial x^\mu}{\partial \theta^\alpha} \right) \omega^\alpha.$$  

Because the following is valid: $\frac{\partial x^\mu}{\partial \theta^\alpha} = \bar{\theta}^\beta (\gamma^\mu)_{\beta\alpha}$, we obtain: $\Omega^\mu = \bar{\theta} (\gamma^\mu) \omega$.

If we know how to realize the Lorentz transformation in the basis

$$\theta' \to S(\Lambda) \theta', \quad \omega \to S(\Lambda) \omega,$$

we obtain the Lorentz transformation on the fiber $x^\mu$.

We can see that the Lorentz transformation $x^\mu = \Lambda^\mu_\nu x^\nu$ follows from the transformation $\Omega^\mu$:

$$\Omega^\mu \to \Omega^\mu S(\Lambda) \gamma^\mu S(\Lambda) \omega = \theta' S^{-1}(\Lambda) \gamma^\mu S(\Lambda) \omega = \bar{\theta}' \Lambda^\mu_\nu \gamma^\mu \omega = \Lambda^\mu_\nu \omega.$$  

In this way we obtained both the SUSY and the Lorentz transformations on the superspace $E(V,W,P^\dagger)$.

Now we can ask what is the covariant derivative on the superspace? From the differential geometry we know, that the covariant derivative is

$$dY^J + Y^J_\mu \omega^\mu = \bar{\omega}^\alpha Y^J_\alpha.$$  

(10)

If the object $Y^J$ transforms as the scalar under the supersymmetry transformation, we call it the “superfield” i.e. the local field $x^\mu \in W$ and $\theta^\alpha \in V$. The index $J$ denotes the transformation under the Lorentz group; we suppose the scalar superfield $\Phi(x,\theta)$.

we also get:

$$\nabla \Phi(x,\theta) = d\Phi(x,\theta) + \Phi_\mu(x,\theta) + \Omega^\mu.$$  

(11)

and

$$\Omega^\mu = \omega^\mu - \bar{\omega}^\alpha \Gamma^\mu_\alpha$$

a dosadme-li tento vraz do rovnice (11), dostaneme:

$$\nabla \Phi(x,\theta) = d\Phi(x,\theta) + \Phi_\mu(x,\theta)(\omega^\mu - \bar{\omega}^\alpha \Gamma^\mu_\alpha).$$  

(12)
So we obtain:

\[ \Phi_\mu(x, \theta) \omega^\mu = \bar{\omega}^\alpha \Phi_\alpha(x, \theta) - d \Phi(x, \theta) \]

and

\[ \nabla \Phi(x, \theta) = \bar{\omega}^\alpha \Phi_\alpha(x, \theta) - \Phi_\mu(x, \theta) \bar{\omega}^\alpha \Gamma^\mu_\alpha \]

\[ = \bar{\omega}^\alpha (\Phi_\alpha(x, \theta) - \Phi_\mu(x, \theta) \Gamma^\mu_\alpha) \]

\[ = \bar{\omega}^\alpha \Phi_\rho(x, \theta). \tag{13} \]

So we get:

\[ \Phi_\rho(x, \theta) = \partial_\alpha \Phi(x, \theta) - i(\gamma^\mu \theta)_\alpha \partial_\mu \Phi(x, \theta) = D_\alpha \Phi(x, \theta), \]

where \( D_\alpha = \partial_\alpha - i(\gamma^\mu \theta)_\alpha \partial_\mu = \partial_\alpha - \Gamma^\mu_\alpha \partial_\mu \) is the covariant derivative.

It is known \([3]\) \( \omega^\mu \) and \( \omega^\alpha \) are invariant under the supertransformation:

\[ \delta x^\mu = \bar{\varepsilon}^\alpha \Gamma^\mu_\alpha, \quad \delta \theta^\alpha = \varepsilon^\alpha \]

and:

\[ [\delta, \delta'] x^\mu = 2i \bar{\varepsilon}'^\gamma \gamma^\mu \varepsilon, \quad [\delta, \delta'] \theta^\alpha = 0, \]

where \( \varepsilon, \varepsilon' \in V \) a element \( 2i \bar{\varepsilon}'^\gamma \gamma^\mu \varepsilon \) is the real element from \( W \), so the \([\delta, \delta'] x^\mu \) is the infiniteimal translation. In this way we obtain the superalgebra:

\[ \{Q_\alpha, \bar{Q}_\beta\} = -2(\gamma^\mu)_{\alpha\beta} P_\mu, \]

\[ [P_\mu, Q_\alpha] = 0, \]

\[ [P_\mu, P_\nu] = 0. \]

where:

\[ P_\mu \sim -i \partial_\mu, \quad [P_\mu, x^\nu] = -i \delta^\nu_\mu. \]

and:

\[ Q_\alpha \sim \eta_{\alpha\beta} \partial^\beta, \quad \{\partial_\alpha, \theta^\beta\} = \delta_{\alpha\beta}, \]

The symbol \( \eta_{\alpha\beta} \) denotes the metric, and so \( \eta_{\alpha\beta} = i C_{\alpha\beta} \) where \( C \) is the charge conjugation.

We can expand the superfield in

\[ \theta = \left(\theta_\alpha \right) \]

and

\[ \theta_{\alpha_1} \cdots \theta_{\alpha_N} \] so we get:

\[ Q(x, \theta, \bar{\theta}) = q(x) + \theta \varphi(x) + \bar{\theta} \bar{\chi}(x) \]

\[ + \theta \theta m(x) + \bar{\theta} \bar{\theta} n(x) + \theta \sigma^\mu \bar{\theta} v_\mu(x) \]

\[ + \theta \bar{\theta} \bar{\lambda}(x) + \theta \theta \psi(x) + \theta \bar{\theta} \bar{\theta} d(x) \tag{14} \]
and all higher powers of $\theta$, $\bar{\theta}$ vanish.

The coefficients in expansion are the ordinary Bose and Fermi fields. The scalar chiral superfield is defined

$$\bar{D}_\dot{\alpha} \Phi(x, \theta, \bar{\theta}) = 0.$$  \hspace{1cm} (15)

and it means:

$$\Phi(x, \theta, \bar{\theta}) = e^{\theta_\alpha \bar{\theta}^\dot{\alpha} \partial_{\alpha \dot{\alpha}}} \Phi(y, \theta),$$

where $e^{\theta_\alpha \bar{\theta}^\dot{\alpha} \partial_{\alpha \dot{\alpha}}} = 1 + i \theta \sigma^\mu \bar{\theta} \partial_\mu + \frac{1}{4} \theta^2 \bar{\theta}^2 \Box$, and $y^\mu = x^\mu + i \theta \sigma^\mu \bar{\theta}$.

This means $\Phi(x, \theta, \bar{\theta})$ is the function of $\theta$, but not the $\bar{\theta}$. The Bose variable is the function of $y^\mu = x^\mu + i \theta \sigma^\mu \bar{\theta}$, because the following is valid: $\bar{D}_\dot{\alpha} y^\mu = 0$, $\bar{D}_\dot{\alpha} \theta = 0$.

For the superfield in $y$, $\theta$ is valid:

$$\Phi(y, \theta) = A(y) + \sqrt{2} \bar{\theta} \psi(y) + \theta \theta F(y),$$

$$= A(x) + i \theta \sigma^\mu \bar{\theta} \partial_\mu A(x) + \frac{1}{4} \theta \theta \bar{\theta} \bar{\theta} \Box A(x)$$

$$+ \sqrt{2} \bar{\theta} \psi(x) - \frac{i}{\sqrt{2}} \theta \partial_\mu \psi(x) \sigma^\mu \bar{\theta} + \theta \theta F(x),$$

$$\Phi|_{\theta=0} = A(x),$$

$$D_\alpha \Phi|_{\theta=0} = \sqrt{2} \psi_\alpha(x),$$

$$D_\alpha D^\alpha \Phi|_{\theta=0} = F(x).$$

We can also see:

$$D_\alpha = \frac{\partial}{\partial \theta_\alpha} + 2 i \bar{\theta}^\dot{\alpha} \sigma^\mu_{\alpha \dot{\alpha}} \frac{\partial}{\partial y^\mu},$$

$$\bar{D}_{\dot{\alpha}} = -\frac{\partial}{\partial \bar{\theta}^{\dot{\alpha}}}. $$

The same way we can it obtain for the antichiral field:

$$D_\alpha \bar{\Phi}(x, \theta, \bar{\theta}) = 0.$$  \hspace{1cm} (17)

It can be see that chiral superfields are connected with the complex superfield. If we denote $R^{4|4}$ real superspace with the 4 real Bose variable and 4 real Fermi variable (like the Majoran spinor), the chirality conditions represent in $R^{4|4}$ superspace the extension of the real variable $x^\mu$ by the imaginary part $\pm i \theta \sigma^\mu \bar{\theta}$.

We move from the Bose variable $x^\mu$ to the $y \equiv x^\mu_L = x^\mu + i \theta \sigma^\mu \bar{\theta}$, and only one two component spinor $\theta_\alpha \bar{\theta}^{\dot{\alpha}} = (x^\mu_L)^* = (x^\mu_R)^*.$

We can define the complex superspace $C^{4|2}$, which is parametrized by the complex 4-vector $x^\mu_L$ ($x^\mu_R$) and only one two component spinor $\theta_\alpha \bar{\theta}^{\dot{\alpha}}$, as $x^\mu_L, \theta_\alpha,$
then we can see $R^{4|4}$, as real hypersurface $Im \, x^\mu_L = \theta \sigma^\mu \bar{\theta}$ in complex space $C^{4|2}$ (with the 2x4 + 2x2 real variables).

In the complex superspace the supersymmetry transformation has the form

$$x_L^\mu = x^\mu + 2i \theta \sigma^\mu \bar{\varepsilon}, \quad \theta^\alpha = \varepsilon^\alpha.$$

The advantage is that here the role plays only $\theta$, but not $\bar{\theta}$, as in the $R^{4|4}$.

The superanalyticity and antisuperanalyticity will play important role in extended supersymmetries.

The construction of the supersymmetry Lagrangians as is usual [3]:

$$\delta \int d^4x \, L = \int d^4x \left( \bar{\varepsilon} \frac{\partial}{\partial \theta} + \frac{i}{2} (\bar{\varepsilon} \gamma_\mu \theta) \frac{\partial}{\partial x_\mu} L \right)$$

$$= \bar{\varepsilon} \frac{\partial}{\partial \theta} \int d^4x \, L + \text{"surface term"} = 0 .$$

and for ordinary fields we can obtain [3] :

$$L = i \bar{\partial}_\mu \bar{\psi} \sigma^\mu \psi + A^*_i \Box A_i + F^*_i F_i$$

$$+ \left[ m_{ij} (A_i F_j - \frac{1}{2} \bar{\psi}_i \psi_j)$$

$$+ g_{ijk} (A_i A_j F_k - \psi_i \psi_j A_k) + \lambda_i F_i + \text{h.c.} \right] .$$

The auxiliary fields may be eliminated through their Euler equations:

$$\frac{\partial L}{\partial F^*_k} = F_k + \lambda^*_k + m_{ik} A^*_i + g_{ijk} A^*_i A^*_j = 0 ,$$

$$\frac{\partial L}{\partial F_k} = F^*_k + \lambda_k + m_{ik} A_i + g_{ijk} A_i A_j = 0 ,$$

Then $L$ can be express as:

$$L = i \bar{\partial}_\mu \bar{\psi} \sigma^\mu \psi + A^*_i \Box A_i + \frac{1}{2} m_{ik} \bar{\psi}_i \psi_k - \frac{1}{2} m^*_k \bar{\psi}_i \psi_k$$

$$- g_{ijk} \bar{\psi}_i \psi_j A_k - g_{ijk} m_{ik} \bar{\psi}_i \psi_j A^*_i - U (A_i, A^*_i) ,$$

where $U(A_i, A^*_i) = F^*_i F_i$.

In this paper in Sect.2 we repeat some basic ideas from quantum information theory.

In Sect.3 we define a form for the antiquibits on the quantum mechanical level, but the full sense it has in the anticommuting qubit field level.

In Sect.4,5 we show the role of qubits for Feynman diagrams and error correction codes in quantum computing.

Also we present a new interesting idea of the application the superpartner potentials and partner–superpartner energy levels as two level system for quantum computing and we show an analog between supersymmetric square root and square root of not.

In Sect.6 we show the possibility of the existence the information qubit quarks.

In sect.7 we present supersymmetric extension of the qubit field theory which has been presented by D.Deutsch [8].
2 Some ideas of quantum information theory

A quantum bit (qubit) is a quantum system with a two-dimensional Hilbert space, capable of existing in a superposition of Boolean states and of being entangled with the states of other qubits [1].

More precisely a qubit is the amount of the information which is contained in a pure quantum state from the two-dimensional Hilbert space $\mathcal{H}_2$.

A general superposition state of the qubit is

$$|\psi\rangle = \psi_0 |0\rangle + \psi_1 |1\rangle,$$

where $\psi_0$ and $\psi_1$ are complex numbers, $|0\rangle$ and $|1\rangle$ are kets representing two Boolean states. The superposition state has the propensity to be a 0 or a 1 and $|\psi_0|^2 + |\psi_1|^2 = 1$.

The eq.(1) can be written as

$$|\psi\rangle = \psi_0 \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \psi_1 \begin{pmatrix} 0 \\ 1 \end{pmatrix},$$

where we labeled $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ two basis states zero and one.

The Clifford algebra relations of the $2 \times 2$ Dirac matrices is

$$\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu},$$

where

$$\eta^{\mu\nu} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}.$$

We choose the representation

$$\gamma^0 = i\sigma^2 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

and

$$\gamma^1 = \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},$$

where $\sigma$ are Pauli matrices and $\gamma^5 = \gamma^0\gamma^1$.

The projectors have the form

$$P_0 = \frac{1 + \gamma^1}{2} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

and

$$P_1 = \frac{1 - \gamma^1}{2} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}.$$
These projectors project qubit on the basis states zero and one:

\[ P_0|\psi\rangle = \psi_0 \begin{pmatrix} 1 \\ 0 \end{pmatrix}, P_1|\psi\rangle = \psi_1 \begin{pmatrix} 0 \\ 1 \end{pmatrix} \]  

(26)

and represent the physical measurements - the transformations of qubits to the classical bits.

In classical information theory the Shannon entropy is well defined:

\[ S_{CL}(\Phi) = -\sum_{\phi} p(\phi) \log_2 p(\phi), \]

(27)

where the variable \( \Phi \) takes on value \( \phi \) with probability \( p(\phi) \) and it is interpreted as the uncertainty about \( \Phi \).

The quantum analog is the von Neumann entropy \( S_Q(\rho_\Psi) \) of a quantum state \( \Psi \) described by the density operator \( \rho_\Psi \):

\[ S_Q(\Psi) = -Tr_{\Psi} [\rho_\Psi \log_2 \rho_\Psi], \]

(28)

where \( Tr_{\Psi} \) denotes the trace over the degrees of freedom associated with \( \Psi \). The von Neumann entropy has the information meaning, characterizing (asymptotically) the minimum amount of quantum resources required to code an ensemble of quantum states.

The density operator \( \rho_\psi \) for the qubit state \( |\psi\rangle \) in (1) is given:

\[ \rho = |\psi\rangle \langle \psi| = |\psi_0|^2 |0\rangle\langle 0| + \psi_0\psi_1^* |0\rangle\langle 1| + \psi_0^*\psi_1 |1\rangle\langle 0| + |\psi_1|^2 |1\rangle\langle 1| \]

(29)

and corresponding density matrix is

\[ \rho_{kl} = \begin{pmatrix} |\psi_0|^2 & \psi_0\psi_1^* \\ \psi_0^*\psi_1 & |\psi_1|^2 \end{pmatrix} \]

(30)

and \( k, l = 0, 1 \).

The von Neumann entropy reduces to a Shannon entropy if \( \rho_\Psi \) is a mixed state composed of orthogonal quantum states.

In the interesting work [2], N.J.Cerf and C.Adami proposed the conditional von Neumann entropy as follows:

for the combined system from two states \( |i\rangle \) and \( |j\rangle \) the von Neumann entropy

\[ S_Q(|ji\rangle) = -Tr \left( \rho_{|ji\rangle} \log_2 \rho_{|ji\rangle} \right), \]

(31)

\[ S_Q(|i\rangle) = -Tr \left( \rho_{|i\rangle} \log_2 \rho_{|i\rangle} \right) \]

(32)

and

\[ \rho_{|j\rangle} = -Tr_{|i\rangle} \rho_{|ji\rangle}. \]

(33)
The von Neumann conditional entropy has the form

$$S_Q(j|i) = -Tr \left( \rho_{ji} \log_2 \rho_{ji} \right).$$

(34)

The appearance of negative values for the von Neumann conditional entropy follows from (17), where the conditional density matrix $\rho_{ji}$ is based on conditional amplitude operator [2]:

$$\rho_{ji} = \exp \left( -\sigma_{ji} \right),$$

(35)

where

$$\sigma_{ji} = I_{ji} \ln \rho_{ji} - \ln \rho_{ji}$$

(36)

and $I_{ji}$ being the unit matrix.

Because the information cannot be negative the question arise what precisely it means.

The quantum entropy of a given quantum state (1) in $H_2$ is the difference between the maximum of the information contained in (1) and information of the information vacuum.

For the qubit

$$S_Q(\psi|v) = -Tr \left( \rho_{\psi v} \log_2 \rho_{\psi v} \right) =$$

$$S_Q(\psi|v) = S_Q(\psi v) - S_Q(v) := S_Q(\psi) = 1,$$

(37)

where $|v\rangle$ is the information vacuum state $S_Q(v) = 0$.

Now we can start to think about anti-quantum bits as the anti-quantum state in the dual Hilbert space $\bar{H}_2$.

3 Anti-quantum bits

We shall define anti-quantum bits (antiqubits) by analogy with antiparticles, as a quantum state from the dual Hilbert space, capable of existing in a superposition of Boolean states and of being entangled with the states of other qubits.

This definition agree formally with these previous ones in [2,3], where antiqubit is called as a quantum of negative information, which is equivalent to a qubit traveling backwards in time.

But here the information of antiqubit is positive only entropy $S_Q(|\bar{\psi}\rangle|0\rangle) = S_Q(\bar{\psi}) := -1$, where $|\bar{\psi}\rangle$ denotes antiqubit state.

For the antiqubit

$$S_Q(|\bar{\psi}\rangle|v\rangle) = -Tr \left( \rho_{\bar{\psi} v} \log_2 \rho_{\bar{\psi} v} \right) =$$

$$S_Q(\bar{\psi}|v\rangle = S_Q(\bar{\psi} v) - S_Q(v) := S_Q(\bar{\psi}) = -1.$$

(39)

To obtain the form of $|\bar{\psi}\rangle$ we use the Dirac adjoint and old ideas about anti-states from Particle Physics.
The antiparticles positrons was defined by Dirac as the energy holes so we can define the antiquubits as the information holes. As the mass in the Dirac eq. has the opposite sign here the entropy of qubit and antiqubit has the opposite sign.

By analogy with the Dirac adjoint for the two component spinor, we define antiqubit $\bar{\psi}$ as follows:

$$\bar{\psi} = \psi^+ \gamma^0 = (\psi_0^*, \psi_1^*) \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} = (-\psi_1^*, \psi_0^*) = \psi_0^* |1\rangle - \psi_1^* |0\rangle.$$  \hspace{1cm} (41)

The corresponding density matrix for the antiqubit $\bar{\psi}$ is

$$\rho_{kl} = \begin{pmatrix} |\psi_0|^2 & -\psi_0^* \psi_1^* \\ -\psi_0^* \psi_1 & |\psi_1|^2 \end{pmatrix}$$  \hspace{1cm} (42)

and $k,l = 0,1$.

The information vacuum is $|v\rangle \in \mathcal{H}_2 \otimes \bar{\mathcal{H}}_2$.

The two members of a spatially separated Einstein-Podolsky-Rosen (EPR) pair are maximally entangled qubits so called ebits [2,4]. EPR pair $e\bar{e}$ is created from the information vacuum, which is the pure quantum state without information $S_Q(\psi_e \psi_{\bar{e}}) = S_Q(\psi_e \bar{\psi}_{\bar{e}}) = S_Q(e\bar{e}) = S_Q(|v\rangle) = 0$.

As the EPR pairs $e\bar{e}$ after creation from the information vacuum contain no readable information they represents the virtual information elementary objects. The conditional quantum entropy between $e$ and $\bar{e}$ is $S_Q(e|\bar{e}) = S_Q(e\bar{e}) - S_Q(\bar{e}) = 0 - (-1) = 1$,

$$S_Q(e|e) = S_Q(e\bar{e}) - S_Q(e) = 0 - 1 = -1.$$  

Quantum information processes can be described by quantum information diagrams similar to Feymann diagrams in Particle Physics, where elementary objects interact. The information flow is conserved in each vertex of these diagrams. The law of the conservation of quantum information has no analog in classical information.

The elementary objects of the quantum information dynamics (QID) can be classical bits $c$, qubits $q$, antiqubits $\bar{q}$, ebits $e$ and antiebits $\bar{e}$. But there could be some more elementary virtual quantum information objects "the information quarks".

Moreover these virtual elementary objects of the QID can be combined into the more complicated information objects in analogy with quarks in the bag models. There is shown that quarks play no role for the energy of the bag [5] and so virtual quantum information entangled quarks (equarks) play no role for the information of the information bag.

### 4 Feymann quantum information diagrams

We show QID for the information processes teleportation and superdense coding, which are connected via time reversal operation.
In quantum teleportation [see Fig.1] a qubit $\psi_q$ is transported with perfect fidelity between two vertices $M$ and $U$ through the transmission of two classical $2c$ bits and shared EPR pair $ee$:

$$\psi_q \rightarrow M \rightarrow U \rightarrow \psi_q \quad 2c$$

Fig.1 Feynmann quantum information diagram for teleportation.

Here $M$ (the measurement) and $U$ (the unitary transformation) mean the two processes in QID and their description is written elsewhere [6]. In vertex $M$ qubit interacts with ebit and it gives four orthogonal maximally entangled $|eq\rangle$ states, which are transported via 2 cbits.

In vertex $U$ is the reconstruction of $\psi_q$ the qubit from 2 c bits via applying to antiebit one of four possible unitary transforms in the one state Hilbert space $\mathcal{H}_2$.

The information flow is conserved in each vertex of the diagram on Fig.1. For the vertex $M$ the entropy conservation rule is

$$S_Q(q) + S_Q(e) = 1 + 1 = 2 \quad (43)$$

since qubit and ebit are initially independent. At vertex $U$ we have

$$S_Q(q) = S_Q(2c) + S_Q(\bar{e}) = 2 - 1 = 1 = S_Q(q\bar{e}) = S_Q(qe) + S_Q(\bar{e}|qe). \quad (44)$$

From eq.(2) we can see that outgoing $e$ from vertex $U$ is equivalent to the incoming $\bar{e}$ to $U$.

In superdense coding [see Fig.2] we transport 2 cbits via 1 qubit. The two cbits can be packed into one qubit via applying the unitary transform what is equivalent to the interaction $2c$ with $\bar{e}$. At vertex $M$ the interaction between qubit and ebit gives recovering of the 2 cbits.

$$\bar{\psi}_e \leftarrow 2c \rightarrow U \rightarrow M \rightarrow 2c \quad \psi_q$$

Fig.2 Feynmann quantum information diagram for superdense coding.
The diagram in Fig.2 is equivalent to the Feynman quantum information diagram where ebit $\psi_e$ is sent backwards in time (it is equivalent to antiebit $\bar{\psi}_e$):

![Diagram](image)

Fig.3 Feynman quantum information diagram for superdense coding with $\psi_e$. In this case at vertex U we have

$$S_Q(2c) + S_Q(e) = 2 - 1 = 1 = S_Q(2ce|e) = S_Q(2c\bar{e}|e) = S_Q(2c) + S_Q(e).$$

since $2c$ and $\bar{e}$ are initially independent. At M is

$$S_Q(2c) = S_Q(qe) = S_Q(q|e) + S_Q(e) = 1 + 1 = 2.$$ (46)

The quantum information conservation law can have important consequences in cosmology.

5 Quantum error correction using antiqubits and SSQM in quantum computing

The main problem in quantum computing are error processes, such as decoherence and spontaneous emission, on the state of a quantum memory register. For example if a quantum memory register is described by an isolated qubit (1) with the density matrix (13) at initial moment, the density matrix changes over time:

$$\rho_{kl} = \begin{pmatrix} |\psi_0|^2 & e^{-\frac{i}{\tau}\psi_0\psi_1^*} \\ e^{-\frac{i}{\tau}\psi_0^*\psi_1} & |\psi_1|^2 \end{pmatrix}$$ (47)

and $k, l = 0, 1$.

Here $\tau$, called the "decoherence time", sets the characteristic time-scale of the decoherence process.

For the antiqubit (21) we obtain

$$\rho_{kl} = \begin{pmatrix} |\psi_0|^2 & -e^{-\frac{i}{\tau}\psi_0\psi_1^*} \\ -e^{-\frac{i}{\tau}\psi_0^*\psi_1} & |\psi_1|^2 \end{pmatrix}$$ (48)

and $k, l = 0, 1$. 

12
Quantum error-correction strategy can be based on the idea of mirror computers, which are working in parallel regime with qubits and antiqubits.

It is known that quantum error-correcting code [7] works on the idea that several physical qubits are used to encode one logical qubit. The actual value of the logical qubit is stored in the correlations between the several classical physical qubits so that even if there is disruption to a few qubits in the codewords, there is still sufficient information in the correlation when error occurs and then correct the qubit.

In mirror computation we can use the correlation between ebits and antiebits to anihilate the error information.

We know that any error in a single bit can be described by the action of linear combination of Pauli matrices:

\[ \sigma_1 \left( \begin{array}{c} \psi_0 \\ \psi_1 \end{array} \right) = \left( \begin{array}{cc} 0 & 1 \\ 1 & 0 \end{array} \right) \left( \begin{array}{c} \psi_0 \\ \psi_1 \end{array} \right) = \left( \begin{array}{c} \psi_1 \\ \psi_0 \end{array} \right), \]

i.e. bit flip error,

\[ \sigma_2 \left( \begin{array}{c} \psi_0 \\ \psi_1 \end{array} \right) = \left( \begin{array}{cc} 0 & -i \\ i & 0 \end{array} \right) \left( \begin{array}{c} \psi_0 \\ \psi_1 \end{array} \right) = i \left( \begin{array}{c} \psi_1 \\ \psi_0 \end{array} \right), \]

i.e. phase shift plus bit flip error,

\[ \sigma_3 \left( \begin{array}{c} \psi_0 \\ \psi_1 \end{array} \right) = \left( \begin{array}{cc} 1 & 0 \\ 0 & -1 \end{array} \right) \left( \begin{array}{c} \psi_0 \\ \psi_1 \end{array} \right) = \left( \begin{array}{c} \psi_0 \\ -\psi_1 \end{array} \right), \]

i.e. phase shift error.

For antiqubits we obtain the same in the dual space.

In general, an \( N \)-qubit can be in an arbitrary superposition of all \( 2^N \) classical states:

\[ |\psi_N\rangle = \sum x |\alpha_x x\rangle, \quad x \in \{ |0\rangle, |1\rangle \}^N \quad \text{and} \quad \sum_x |\alpha_x|^2 = 1. \]

It is known that two bit gates are universal for quantum computation, which is likely to greatly simplify the technology required to build quantum computers.

In our application the quantum interference can be given by the superposition of \( N \) two-level solitonic system.

To understand this we start with a simple idea of quantum algorithm square root of not in the form from [1] via partner–superpartner case.

We begin with supersymmetric square root in SSQM.

SSQM is generated by supercharge operators \( Q^+ = (Q^-)^+ \) which together with the Hamiltonian \( H = 2H_{SSQM} \) of the system, where

\[ H_{SSQM} = \frac{1}{L} \begin{pmatrix} \frac{d^2}{dx^2} + v^2 + v' & 0 \\ 0 & -\frac{d^2}{dx^2} + v^2 - v' \end{pmatrix} \]

\[ H = \begin{pmatrix} H_0 & 0 \\ 0 & H_1 \end{pmatrix} = \begin{pmatrix} A^+A^- & 0 \\ 0 & A^-A^+ \end{pmatrix} = -\left( \frac{d^2}{dx^2} \right) + \sigma_3 \psi' \]
fulfil the superalgebra
\[(Q^\pm)^2 = 0, \quad [H, Q^-] = [H, Q^+], \quad H = \{Q^+, Q^-, Q^+ + Q^-, Q^2\}\]
where
\[Q^- = \begin{pmatrix} 0 & 0 \\ A^- & 0 \end{pmatrix}, \quad Q^+ = \begin{pmatrix} 0 & A^+ \\ 0 & 0 \end{pmatrix}, \quad Q = Q^+ + Q^-\]
and
\[A^\pm = \pm \frac{d}{dx} + v(x), \quad v' = \frac{dv}{dx} .\]
Such Hamiltonians \(H_0, H_1\) fulfil
\[H_0 A^+ = A^+ H_1, \quad A^- H_1 = H_0 A^-\]
The coefficients \(\alpha_0\) and \(\alpha_1\) are called the amplitudes of the \(|0\rangle\) and \(|1\rangle\), respectively.

The previous relations lead to the double degeneracy of all positive energy levels, of belonging to the “0” or “1” sectors specified by the grading state operator \(S = \sigma_3\), where
\[[S, H] = 0 \text{ and } \{S, Q\} = 0 .\]

The \(Q\) operator transforms eigenstates with \(S = +1\), i.e. the null-state \(|0\rangle\) into eigenstates with \(S = -1\), i.e. the one-state \(|1\rangle\) and vice versa.

With this notation the square root of not is represented by the unitary matrix \(U[1]\):
\[U = \frac{1}{2} \begin{bmatrix} 1 - i & 1 + i \\ 1 + i & 1 - i \end{bmatrix},\]
that solves:
\[U^2|0\rangle = |1\rangle\]
\[U^2|1\rangle = |0\rangle\]

In such way this supersymmetric double degeneracy represents two level quantum system and we can see the following:
\[Q = Q^+ + Q^- = \begin{pmatrix} 0 & a^+ \\ a^- & 0 \end{pmatrix}, \]
\[\tau = \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \]
\[\{Q, \tau\} = 0 .\]

It implies that the operator supercharge \(Q\) really transforms the state \(|0\rangle, |1\rangle\) as the operator square root of not quantum algorithm operator \(U^2\). In such a way supersymmetric “square root” corresponds the “square root of not”
6 The information quarks

The quantum information-theoretical formalism defined above can be generalized to multipartite system. Consider for example a tripartite entangled quantum system $u$, $d$, $s$. The chain rule for quantum entropies is

$$S_Q(uds) = S_Q(u) + S_Q(d) + S_Q(s|ud).$$

(52)

The von Neumann conditional mutual entropy

$$S_Q(u : d | s) = S_Q(u | s) + S_Q(d | s) - S_Q(us | d) = S_Q(us) + S_Q(ds) - S_Q(s) - S_Q(uds).$$

(53)

which reflects the quantum mutual entropy between $u$ and $d$ when $s$ is known.

For ternary mutual quantum entropy $S_Q(u : d : s)$, i.e., that piece of the mutual entropy between $u$ and $d$ that is also shared by $s$. It has no classical counterpart - for any entangled tripartite system $uds$ in a pure state, the ternary mutual entropy vanish, i.e.

$$S_Q(u : d : s) = S_Q(u) + S_Q(d) + S_Q(s) - S_Q(uds) = 0.$$  

(54)

This results from the fact that $S_Q(uds) = 0$ implies $S_Q(ud) = S_Q(s)$, $S_Q(us) = S_Q(d)$, $S_Q(ds) = S_Q(u)$, as a consequence of the Schmidt decomposition of the state $uds$.

7 Supersymmetric qubit fields

Consider commuting or anticommuting fields which can be coupled via supersymmetry. Let an field of identically-constituted quantum physical systems on spacetime. Following [8] each event is associated with one such system and each of systems has the same algebra of observables and obeys the same dynamical law, the field is continuous in the sense that the fields corresponding physical quantities of each system are continuous and also differentiable.

We shall define the superspace $(x, \theta, \bar{\theta})$ and let $\Phi(x, \theta, \bar{\theta})$ be the superqubit field at event $(x, \theta, \bar{\theta})$, which fullfils:

$$\Phi_j \Phi_k = \delta_j k I + i \epsilon_{jkl} \Phi_l$$

and so for the first component in the expansion we obtain exactly the scalar qubit field as D.Deutsch in [8].

The chiral superqubit field on the superspace $(x, \theta)$:

$$\Phi(x, \theta) = \varphi(x) + i \theta^\alpha \psi_\alpha(x) + \frac{i}{2} \theta^\alpha \theta_\alpha F(x)$$

the supermultiplet $\{\varphi, \psi, F\}$ of ordinary commuting and anticommuting qubit fields (at this moment we do not write the qubit field indeces $j,k,l...$ without lost of generallity)
We can generalize the super $O(3) \sigma$ model:

$$\frac{1}{2} \int d^2 x \, d^2 \theta \, \epsilon^{\alpha \beta} (D_\alpha \Phi^a \cdot D_\beta \Phi^a) ,$$  \hspace{1cm} (56)

with the condition: $\Phi^a(x, \theta) \cdot \Phi^a(x, \theta) = 1, \quad a = 1, 2, 3.$ \hspace{1cm} (57)

For qubit fields we obtain the following Lagrangian:

$$L = \frac{1}{2} [ \partial_\mu \varphi^a \cdot \partial_\mu \varphi^a + i \psi^a \cdot \gamma^n \partial_\mu \psi^a - F^a \cdot F^a] \hspace{1cm} (58)$$

and the condition gives:

$$\varphi^a \cdot \varphi^a = 1, \quad \varphi^a \cdot \psi^a = 0, \quad (59)$$

$$\varphi^a \cdot F^a = \frac{i}{2} \psi^a \cdot \psi^a .$$

As usual:

$$L = \frac{1}{2} \partial_\mu \varphi^a \cdot \partial_\mu \varphi^a + \frac{\alpha}{2} (\varphi^a \cdot \varphi^a - 1) + \frac{i}{2} \psi^a \cdot \gamma^n \partial_\mu \psi^a + \frac{1}{8} (\psi^a \cdot \psi^a)^2 . \hspace{1cm} (60)$$

The equation of motion follows:

$$\Box \varphi^b + (\partial_\mu \varphi^a \cdot \partial_\mu \varphi^a) \varphi^b = 0 , \hspace{1cm} (61)$$

$$i \gamma^b \cdot \partial \psi^b + \frac{1}{2} (\psi^a \cdot \psi^a) \psi^b = 0 . \hspace{1cm} (62)$$

We can see from (61) that anticommuting qubits fields which are given via supersymmetric extension do not change the equation for commuting qubit.

The supercurrent has the form:

$$J_\mu = \gamma^n \partial_n \varphi^a \cdot \gamma^\mu \psi^a \hspace{1cm} (63)$$

and:

$$\theta_{\mu \nu} = \partial_\mu \varphi^a \cdot \partial_\nu \varphi^a - \frac{1}{2} g_{\mu \nu} \partial_\lambda \varphi^a \cdot \partial_\lambda \varphi^a +$$

$$+ \frac{i}{4} [\psi^a \cdot (\gamma_\mu \partial_\nu - \gamma_\nu \partial_\mu) \psi^a - g_{\mu \nu} \psi^a \cdot \gamma^\rho \partial_\rho \psi^a] .$$

The super action (56) and (57) are invariant under the supertransformation:

$$\delta \varphi^a = i \varepsilon \psi^a , \quad \delta \psi^a = (\gamma \cdot \partial \varphi^a + F) \varepsilon , \quad (64)$$

$$\delta F^a = i \varepsilon \gamma^\mu \partial_\mu \psi^a .$$

It is known that (56) is invariant under the extended supersymmetry with the internal symmetry $O(2)$ in the Grassmann variable.
From this complex supersymmetry the current follows:

\[
\bar{J}_\mu = \varepsilon^{abc} \varphi^a \partial \varphi^b \gamma_\mu \psi^c , \quad a, b, c = 1, \ldots, 3, \tag{65}
\]
\[
V_\mu = \varepsilon^{abc} \varphi^a \psi^b \gamma_\mu \psi^c \tag{66}
\]

and an axial current \( A_\nu = \varepsilon_{\mu
u} V^\mu \).

We show that the action (56) is invariant under the extended supersymmetry \( N = 2 \).

We shall write the superaction (56) for super \( \text{CP}^1 \) model.

We shall define the complex qubit superfield superpole \( \Phi_i(x, \theta), i = 1, 2 \), where \( \{ \varphi_i, \psi_i, F_i \} \) are complex qubit fields and \( \theta \) is the real two component spinor.

Transformation \( \Phi^a = \Phi_i \sigma^a_{ik} \Phi_k \) gives:

\[
S = \frac{1}{4} \int d^2 x d^2 \theta \nabla \Phi_i \cdot \nabla \Phi_i , \tag{67}
\]

where \( \nabla = D - A \) is \( U(1) \) gauge invariant covariant derivative.

Where \( A \) denotes a Fermi qubit superfield, which transforms as Abelian gauge field under \( U(1) \) gauge transformation.

The condition \( \Phi^a \cdot \Phi^a = 1 \) has the form \( \bar{\Phi}^i \cdot \Phi^i = 1 \) and \( A \) can be eliminated as usual:

\[
A = \bar{\Phi}_i \cdot D \Phi_i . \tag{68}
\]

So the super \( \text{CP}^1 \) action has the form of the super \( O(3) \) \( \sigma \) model action. We can see it more directly:

\[
C(x, \theta, \bar{\theta}) \equiv C(x, \theta_1 + i\theta_2, \theta_1 - i\theta_2) .
\]

Super transformation in \( (x, \theta, \bar{\theta}) \) has the form:

\[
\delta x = -\frac{i}{2} [\varepsilon \gamma \bar{\theta} + \bar{\varepsilon} \gamma \theta] , \quad \delta \theta = \varepsilon , \quad \delta \bar{\theta} = \bar{\varepsilon},
\]

and acting on \( C(x, \theta, \bar{\theta}) \):

\[
\delta C = [\varepsilon Q + \bar{\varepsilon} \bar{Q}] C , \tag{69}
\]

where \( Q, \bar{Q} \) are supercharges and \( D, \bar{D} \) supercovariant derivatives. Here the “self-duality” is equivalent the superanalyticity :

\[
\bar{D} C(x, \theta, \bar{\theta}) = 0 . \tag{70}
\]

The condition (70) plays here the role of the following constraint:

\[
C(x, \theta, \bar{\theta}) = C(x - \frac{i}{2} \bar{\theta} \gamma \theta, \theta) . \tag{71}
\]
Complex supersymmetry gives:

\[
\begin{align*}
\delta \varphi_c &= i \varepsilon \bar{\psi}_c , \\
\delta \psi_c &= \varepsilon F_c + \partial \varphi_c \bar{\varepsilon} , \\
\delta F_c &= i \bar{\varepsilon} \partial \psi_c 
\end{align*}
\] (72)

We can see the following identification \( \varphi_i, \psi_i, F_i, i = 1, 2 \) with (67):

\[
\varphi_C = \varphi_1 + i \varphi_2 , \quad \psi_C = \psi_1 + i \psi_2 , \quad F_C = F_1 - i F_2 .
\] (73)

For complex conjugate superfield \( \bar{C} \) supraanalyticity is valid:

\[
D \bar{C} = 0 .
\] (74)

As usual in super CP\(^1\) model we have the vector superfield \( V(x, \theta, \bar{\theta}) \), which has in Wess-Zumino gauge the form:

\[
V = \frac{1}{2} \bar{\theta} \gamma^\mu \theta v_\mu + \text{“koeficienty se dvma a vce } \theta \text{”} ,
\] (75)

and \( C, V \) transforms under \( U(1) \) gauge as follows transformations:

\[
\begin{align*}
\delta C &= i \Lambda C , \\
\delta \bar{C} &= i \bar{\Lambda} \bar{C} , \\
\delta V &= i (\Lambda - \bar{\Lambda}) ,
\end{align*}
\]

where \( \Lambda(x_\mu - \frac{1}{2} \bar{\theta} \gamma_\mu \theta, \theta) \) is chiral gauge superfield.

The complex supersymmetry and gauge invariant action has the form:

\[
S = \frac{1}{8} \int dx^2 dx^2 \theta dx^2 \bar{\theta} (V + C \bar{C}) e^V
\] (76)

and equations of motion follow:

\[
C \bar{C} = e^{-V} = 1 - V ,
\] (77)

and for the first term in the \( \theta \) expansion we get:

\[
\bar{\varphi} \varphi = 1 , \quad \bar{\psi} \cdot \varphi = \psi \cdot \bar{\varphi} = 0 .
\]

So \( V \) acts as an unification force.
8 Conclusions

We obtained in scalar qubit field theory proposed by D.Deutsch an interaction with anticommuting qubit field via natural supersymmetric way. If qubit fields are realised in nature then also antiqubit fields must exist and and then all existing quantum field theories that have empirical corroboration are presumably long range approximations to an exact theory of interacting qubit fields.

The interesting application of this is that the existence of the condensate $\bar{\psi}\psi$ from the anticommuting qubits. It can be interpreted as the information vacuum and it can be broken near the black hole on qubit and antiqubit fermionic field and so the black hole can radiate also an information. It is the quantum information analog of the Hawking radiation. Also the holographic bound given in the work [4] for the entropy $S$ of any region enclosed by surface of area $A$:

$$S = \frac{Ak}{4l_p^2}$$

(78)

where $l_p$ is the Planck length and $k$ the Boltzmann constant. The equality holds if and only if the system is static black hole so it means that that a black hole has at least as much entropy as any other object that could be enclosed in the same surface. The Hilbert space of the quantum fields inside such region therefore cannot have dimension higher then $\exp^\frac{S}{4l_p^2}$ and so the region cannot hold more than $\frac{A^d}{4l_p^6\ln^2}$ bits. This information paradox can be explained via antiqubit radiation of information from the black hole. Entropy inside black hole plus outside must increase. At least here we show an example of theory of interacting qubit fields which can be extended.

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