Mathematical model’s parameters identification in a prototype of solar panel’s motor

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Abstract. Solar panel can optimally convert energy when its position is upright to the sun, thus the innovation of solar tracker is needed. Solar tracker needs a motor so that the solar tracker is able to track the sun. This research studies a motor of solar panel prototype called motor DC. This research identifies parameter value of motor DC mathematical model. Identification is performed on resistance (\(R_a\)), inductance (\(L_a\)), back emf constant (\(K_b\)), torque constant (\(K_m\)), rotor inertia (\(J\)), coefficient of viscous friction (B). Furthermore, simulation is applied to motor DC mathematical model with the identified parameter. Simulation is repeated 4 times with combinations of back emf constant and torque constant. Result of the simulation system is angular velocity. Then, angular velocity from simulation is compared to the result of angular velocity from the experiment. Comparison indicates that the third simulation shows the best value with the confidence level of absolute average relative deviation of 97.317587123%.

1. Introduction

World energy demand is increasing rapidly. According to International Energy Agency, 80\% of world energy is supplied by fossil fuels, and until 2030, the world energy demand will increase about 45\% or 1.6\% per year. World energy demand will keep on increasing concomitant with world population growth. Thus, to determent energy scarcity, there is a need to develop alternative energy resource.

Alternative energy is a term refer to all energy which can be used to substitute nonrenewable energy, some of which are biomass, solar energy, natural gas, and geothermal. As a tropical country, one of the most potential alternative energy sources in Indonesia is solar energy. The intensity of solar energy range of 0.6 – 0.7 kw/m\(^2\), and the energy can be converted to electricity using solar cell.

Solar cells have varied performance which are presented by efficiency. Solar cell efficiency is often used as reference to evaluate the quality of solar cell. Solar cell efficiency is defined by solar cell output energy ratio towards energy input that derives from sun.

Practically, up to now the energy increase achieved by PV in simulation is 59.82\%, whereas experiments achievement is 60.42\% as stated in [1]. According to Mintorogo, there are 6 factors of solar panel operation; those are ambient air temperature 25 \(^\circ\)C, solar radiation, wind velocity, earth atmosphere condition, solar panel orientation and angle position of solar panel [2]. The last factor has got the biggest impact on the amount of energy obtained. The solar panel position that is upright
towards the sun will give 1000 w/m2 energy. These facts show that solar panel position control has high importance and high urgency to be developed.

Some research has been carried out to observe solar panel position [1,2,3,4,6,9], some of which is a research conducted by Yahya Efprianto that discussed the design and simulation of control system on solar panel using type 2 fuzzy sliding mode control method (T2FSMC). This research summed up that T2FSMC in motor system of solar panel worked robust with various interferences.

In the development research of solar panel, now there is a solar panel prototype that is able to track the sun, the motor of solar panel is an important part. In order to design position control, mathematical approach is needed in the motor of solar panel. This research discusses identification parameter of mathematic model, so as the model is able to represent solar panel motor, in hope of that this mathematic model could be designed as position control that suits the prototype condition.

2. Parameter’s Test
Parameter test is done to DC Motor Hosiden 21.6 V 0.6 A. This motor type is used as a motor on solar panel prototype. The parameter test processes in motor mathematic model is conducted experimentally. LCR meter is electronic test tool that is used to quantify Inductance (L), Capacitance (C), and Resistance (R) from component. In this research, inductance and resistance can be obtained through direct measurement by connecting LCR meter to DC motor. In order to get a valid data, measurement is repeated 50 times. From 50 times measurement using LCR meter, it is found that value from each parameter that is an average of 50 data which is resistance 18.2214 and inductance 0.00866.

Further parameter test is torque constant \( (K_m) \) and back emf constant \( (K_b) \). In ideal condition, the values of torque constant and back emf constant are on the same number \([1]\). In the process of testing both parameters, motor function is diverted to generator where the mechanic energy is converted to electric energy. Testing tool that is used in the research is CNC Milling machine and Avometer. In the testing mechanism, CNC Milling machine is used to input angular velocity of DC motor, afterward the generator convert mechanic energy to electric energy. The output of electric energy that is the result of generator is measured by Avometer.

In torque constant and back emf constant parameters test, angular velocity that is given to motor are 200, 400, 600, 800, 1000, 1200, 1400, 1600, 1800, 2000, 2400, 2600, 2800, 300 (in RPM unit). However, according to sensor of CNC Milling shows that actual spin of DC motor that is being tested are 179, 359, 540, 899, 1079, 1259, 1439, 1619, 1799, 1979, 2159, 2339, 2519, 2699 (in RPM unit). Therefore, to obtain back emf constant and torque constant, spin data that is being used is actual spin.

Afterwards, the voltage from each angular velocity input is measured by avometer. Voltage data that is obtained in this research are minimum and maximum voltage which was resulted from the conversion of angular velocity from each input. In order to get torque constant and back emf constant, referring to the formula that is used to get back emf constant. The voltage of conversion result is divided by angular velocity input \([1]\). Formula is shown below in Equation 1.

\[
K_b = \frac{Voltage (volt)}{Angular velocity (\text{rad/second})} \tag{1}
\]

In the given input, unit of angular velocity is RPM, thus need to be converted to radian/second that is shown in equation 2.

\[
1 \text{ RPM} = \frac{2\pi \text{ rad}}{60 \text{ second}} = 0.104719755 \text{ radian/second} \tag{2}
\]

Conversion is done by multiplying angular velocity with 0.104719755. In this research, testing to identify torque constant and back emf constant parameter is repeated twice. Because too many spin at motor causing brush on the motor becomes exhausted and affect to angular velocity. So as the brush wear out, the angular velocity becomes smaller. According to the test, the data of angular velocity by CNC Milling and Voltage conversion is shown in Figure 1 and 2.
In Figure 1 and 2, blue and red indicates number of voltage obtained in each angular velocity input. Blue is minimum voltage and red is maximum voltage. Next step is to calculate value of back emf constant refer to equation 1. In the parameter calculation, angular velocity in RPM unit must be converted to radian/second. The result of back emf constant calculation is shown in Figure 3 and 4.

Figure 3 and 4 shows the constant value of back emf constant in every input of angular velocity during the test. With any constant value which we had previously, we can obtain the average value as a valid parameter. In ideal condition, both of torque constant and back emf constant will have the same value. The value of both torque constant and back emf constant are 0.031034441, 0.031092118, 0.030941093, 0.031071821.

![Figure 1. The First Data of Spin and Voltage](image1)

![Figure 2. Second Data of Spin and Voltage](image2)

![Figure 3. Kb From First Data](image3)
Afterward, we have to determine the parameter value of rotor inertia and coefficient of viscous friction. Rotor inertia and coefficient of viscous friction obtained by referring to a specification approached within the motor that being tested [8]. Based on that reference values, the obtained value for rotor inertia is 0.000090 and 0.000025 for coefficient of viscous friction.

3. Mathematical Model Analysis

Mathematical model analysis can be done by substituting the equation on DC motor mathematical model functioning as a prototype of a solar panel [4]. Mathematical model analysis done out of the simulation necessity on Simulink MATLAB. Mathematical model analysis is divided into two observations, mechanical observation and electrical observation. Mathematical model analysis on solar panel will be described as follows

\[ e_a(t) = R_a i_a(t) + L_a \frac{di_a(t)}{dt} + e_b(t) \]
\[ e_b(t) = K_b \omega(t) \]
\[ T_m(t) = K_m i_a(t) \]
\[ T_m(t) = J \frac{d\omega(t)}{dt} + B \omega(t) \]

Where:
- \( e_a(t) \): Value of voltage input to the motor (volt)
- \( e_b(t) \): Back emf (volt)
- \( i_a(t) \): Current (Ampere)
- \( R_a(t) \): Resistance (Ohm)
- \( L_a(t) \): Inductance (Henry)
- \( K_b \): Back emf constant (Volt-sec/rad)
- \( K_m \): Torque Constant (N-m/Ampere)
- \( J \): Rotor inertia (Kg – m²)
- \( B \): Coefficient of viscous friction (N-m/rad/sec)
- \( T_m(t) \): Motor torque (N-m)
- \( \omega(t) \): Angular velocity of motor (rad/sec)
From the mathematical model above, further calculations are \[5,10\],
\[
e_a(t) = R_a i_a(t) + L_a \frac{d i_a(t)}{d t} + e_b(t)
\]
with \(\omega(t) = \frac{d \theta}{d t}\) dan \(e_b(t) = K_b \omega(t)\), then
\[
e_a(t) = R_a i_a(t) + L_a \frac{d i_a(t)}{d t} + K_b \frac{d \theta}{d t}
\]
\[
L_a \frac{d i_a(t)}{d t} = e_a(t) - R_a i_a(t) - K_b \frac{d \theta}{d t}
\]
So
\[
\frac{d i_a(t)}{d t} = \frac{1}{L_a} \left( e_a(t) - R_a i_a(t) - K_b \frac{d \theta}{d t} \right)
\]
and
\[
\int \frac{d i_a(t)}{d t} = i_a
\]
The next equation is obtained as follows
\[
T_m(t) = J \frac{d \omega(t)}{d t} + B \omega(t)
\]
With \(\omega(t) = \frac{d \theta}{d t}\) dan \(T_m(t) = K_m i_a(t)\), so
\[
K_m i_a(t) = \int \frac{d^2 \theta}{d t^2} + B \frac{d \theta}{d t}
\]
\[
J \frac{d^2 \theta}{d t^2} = K_m i_a(t) + B \frac{d \theta}{d t}
\]
\[
\frac{d^2 \theta}{d t^2} = \frac{1}{J} \left( K_m i_a(t) + B \frac{d \theta}{d t} \right)
\]
\[
\int \frac{d^2 \theta}{d t^2} = \frac{d \theta}{d t} = \omega
\]

According to the process of mathematical model analysis shown above, we obtain two equations that state a system with electrical observation in Equation 3 and mechanical observation in Equation 4.

4. Simulation and Validation
At this stage, simulation will be done on the equations that has been obtained in mathematical model analysis stage before (Equation 3 and 4) and by using Simulink MATLAB.

Simulation will be carried out by giving the several variations of the input voltage that are 4V, 5V, 6V, 7V, 8V, 9V, 10V, 11V, 12V, 13V, and 4 possible torque constant and back emf constant. The first simulation is given with the parameter value \(R_a(t) = 18.2214\) ohm, \(L_a(t) = 0.00866\) Henry, \(K_b = 0.031034441\) volt-sec/rad, \(K_m = 0.031034441\) N-m/Ampere, \(J = 0.000090\) Kg − m², \(B = 0.000025\) N-m/rad/sec.
Figure 5. Output From First Simulation

The second simulation is given with the parameter value $R_a(t) = 18.2214$ ohm, $L_a(t) = 0.00866$ Henry, $K_b = 0.031092118$ volt-sec/rad, $K_m = 0.031092118$ N-m/Ampere, $J = 0.000090$ Kg – m$^2$, $B = 0.000025$ N-m/rad/sec.

Figure 6. Output From Second Simulation

Next, The third simulation is given with the parameter value $R_a(t) = 18.2214$ ohm $L_a(t) = 0.00866$ Henry, $K_b = 0.030941093$ volt-sec/rad, $K_m = 0.030941093$ N-m/Ampere, $J = 0.000090$ Kg – m$^2$, $B = 0.000025$ N-m/rad/sec.
The fourth simulation is given with the parameter value $R_a(t) = 18.2214$ ohm, $L_a(t) = 0.00866$ Henry, $K_b = 0.031071821$ volt-sec/rad, $K_m = 0.031071821$ N-m/Ampere, $J = 0.000090$ Kg – m$^2$, $B = 0.000025$ N-m/rad/sec

The result of 4 simulation shows that each parameter presents different responses. Angular velocity output shows insignificant differences. The result of angular velocity from 4 simulations are shown in the Table 1
Table 1. Angular Velocity From Simulation

| V | Angular Velocity (rad/sec) |
|---|----------------------------|
|   | Sim 1 | Sim 2 | Sim 3 | Sim 4 |
| 4 | 87,5028 | 87,4446 | 87,5969 | 87,4651 |
| 5 | 109,3786 | 109,3058 | 109,4961 | 109,3314 |
| 6 | 131,2542 | 131,1669 | 131,3953 | 131,1976 |
| 7 | 153,13 | 153,028 | 153,2945 | 153,0639 |
| 8 | 175,0057 | 174,8892 | 175,1937 | 174,9302 |
| 9 | 196,8814 | 196,7503 | 197,0929 | 196,7965 |
| 10 | 218,7571 | 218,6115 | 218,9921 | 218,6627 |
| 11 | 240,6328 | 240,4727 | 240,8914 | 240,5291 |
| 12 | 262,5085 | 262,3338 | 262,7906 | 262,3954 |
| 13 | 284,3842 | 284,195 | 284,6898 | 284,2616 |

Furthermore, direct experiment with the measurement of angular velocity will be carried by using tachometer after the voltage from power supply is input to DC motor. The result of angular velocity based on experiment are shown in Table 2.

Table 2. Angular Velocity From Experiment

| V | Angular Velocity (rad/sec) |
|---|----------------------------|
|   |                             |
| 4 | 87,19804559 |
| 5 | 107,0864215 |
| 6 | 131,6746199 |
| 7 | 160,0955614 |
| 8 | 185,3958543 |
| 9 | 204,5595694 |
| 10 | 222,6341991 |
| 11 | 233,7763811 |
| 12 | 256,2911284 |
| 13 | 276,6695927 |

After obtaining angular velocity based on simulation and experiment, next step is to measure mathematical model accuracy and its parameter based on angular velocity system output. The method that used is Absolute Average Relative Deviation[7]. The result is shown below.
### Table 3. AARD First Simulation

| V  | Omega experiment | Omega Simulation | ARD (%) |
|----|------------------|------------------|---------|
| 4  | 87,19804559      | 87,5028          | 0,348279605 |
| 5  | 107,0864215      | 109,3786         | 2,095637115  |
| 6  | 131,6746199      | 131,2542         | 0,320309702  |
| 7  | 160,0955614      | 153,13           | 4,548789554  |
| 8  | 185,3958543      | 175,0057         | 5,937037623  |
| 9  | 204,5595694      | 196,8814         | 3,899895783  |
| 10 | 222,6341991      | 218,7571         | 1,772330649  |
| 11 | 233,7763811      | 240,6328         | 2,849328495  |
| 12 | 256,2911284      | 262,5085         | 2,368445827  |
| 13 | 276,6695927      | 284,3842         | 2,712741175  |
|    | AARD             |                  | 2,685279553  |

### Table 4. AARD Second Simulation

| V  | Omega experiment | Omega Simulation | ARD (%) |
|----|------------------|------------------|---------|
| 4  | 87,19804559      | 87,4446          | 0,281954982  |
| 5  | 107,0864215      | 109,3058         | 2,030430715  |
| 6  | 131,6746199      | 131,1669         | 0,387079314  |
| 7  | 160,0955614      | 153,028          | 4,618475994  |
| 8  | 185,3958543      | 174,8892         | 6,007606103  |
| 9  | 204,5595694      | 196,7503         | 3,96912707   |
| 10 | 222,6341991      | 218,6115         | 1,840113228  |
| 11 | 233,7763811      | 240,4727         | 2,784648294  |
| 12 | 256,2911284      | 262,3338         | 2,303428538  |
| 13 | 276,6695927      | 284,195          | 2,647973149  |
|    | AARD             |                  | 2,687083739  |

### Table 5. AARD Third Simulation

| V  | Omega experiment | Omega simulation | ARD (%) |
|----|------------------|------------------|---------|
| 4  | 87,19804559      | 87,5969          | 0,455329363  |
| 5  | 107,0864215      | 109,3058         | 2,20069805  |
| 6  | 131,6746199      | 131,3953         | 0,212579854  |
| 7  | 160,0955614      | 153,2945         | 4,436598472  |
| 8  | 185,3958543      | 175,1937         | 5,823356806  |
| 9  | 204,5595694      | 197,0929         | 3,788401011  |
| 10 | 222,6341991      | 218,9921         | 1,663118957  |
| 11 | 233,7763811      | 240,8931         | 2,953620984  |
| 12 | 256,2911284      | 262,7906         | 2,473251179  |
| 13 | 276,6695927      | 284,6898         | 2,817174093  |
|    | AARD             |                  | 2,682412877  |
Table 6. AARD Fourth Simulation

| V | Omega experiment | Omega simulatio | ARD (%) |
|---|------------------|-----------------|--------|
| 4 | 87,1980455       | 87,4651         | 0,30532681 |
| 5 | 107,086421       | 109,3314        | 2,05337033 |
| 6 | 131,674619       | 131,1976        | 0,36358892 |
| 7 | 160,095561       | 153,0639        | 4,59393850 |
| 8 | 185,395854       | 174,9302        | 5,98276012 |
| 9 | 204,559569       | 196,7965        | 3,94471924 |
| 10| 222,634199       | 218,6627        | 1,81626730 |
| 11| 233,776381       | 240,5291        | 2,80744364 |
| 12| 256,291128       | 262,3954        | 2,32636380 |
| 13| 276,669592       | 284,2616        | 2,67078187 |
|   | AARD             |                 | 2,68645605 |

Based on Absolute Average Relative Deviation method, the result of AARD in first simulation is 2.685279553%, 2.687083739% in second simulation, 2.682412877% in third simulation, and 2.686456059% in the fourth simulation. Result shows that the confidence level of mathematic model and parameter towards the prototype panel motor truly shows a value above 95% and the highest credibility is shown on the third simulation with 97.317587123%, which indicates that in the third simulation of mathematic models and parameter is the most representative toward the solar panel prototype motor.

5. Conclusion
Simulation results confidence level value is above 95%. The best parameter value obtained in the third simulation with confidence level is 97.317587123% and the parameter value are Resistance ($R_a$) 18.2214 ohm, Inductance ($L_a$) 0.00866 Henry, back emf constant ($K_b$) 0.030941093 Volt-sec/rad, torque Constant ($K_m$) 0.030941093 N-m/Ampere, rotor inertia (J) 0.000090 Kg-m$^2$, coefficient of viscous friction (B) 0.000025. It can be concluded that DC motor mathematical model with that parameter have the most representative output toward solar panel prototype motor.

This research discusses mathematic model as well as its parameters that represent the condition of solar panel prototype motor. In order to get a better accuracy, we suggest developing measurement tool to obtain rotor inertia and coefficient of viscous friction. This research can also be developed by giving control design that suitable with the solar panel system, in hope that the design control would increase the knowledge in comparing the best control design in solar panel prototype. Moreover, this research is also can be developed by correlating between solar panel motor and photovoltaic’s efficiency.

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