Recent paper \cite{1} presents calculations of the EDM enhancement factor for Tl, which disagrees with previous most accurate calculations \cite{2,3}. The calculations of this kind are used for a search for new physics beyond the standard model in atomic experiments. Therefore, it is important to use the most accurate results for the interpretation of the experiments.

The authors of \cite{1} make many misleading comments about our calculations of thallium EDM \cite{3}. Contrary to the statement in \cite{1} the atomic electric field interacting with electron EDM is calculated in \cite{3} as a derivative of the total potential which includes both nuclear and electron parts. This is done in a same way as in our early work on EDM of Fr and Au \cite{4}. The formula $E = Ze/r^3$ for the leading contribution to the atomic electric field presented on first page of \cite{4} may indeed make an impression that only nuclear field is included. However, the formula (2) few lines below clearly includes screening functions $Q(r)$ and $P(r)$ for both the nuclear Coulomb and the external electric field. By the way, the inclusion of the electron electric field change the matrix elements of the electron EDM for thallium by 0.4% only. This is because the main contribution comes from short distances where the electron electric field is small since the electron potential rapidly tends to a constant inside the 1s orbital.

The authors of \cite{1} claim that atomic core is strongly contracted in the $V^{N-3}$ zero approximation used in our calculations \cite{3}. In fact, it is not. Fig.1 shows the outermost $5d_{5/2}$ core function of Tl calculated in $V^{N-3}$ and $V^{N-1}$ approximations. The difference between the functions is very small. This is due to the fact that the valence 6s and 6p electrons are located outside of atomic core. Their charge distribution creates almost constant potential and no electric field inside the sphere where all inner electrons are located. Therefore, the valence electrons have practically no effect on the core wave functions (see \cite{3} for a detailed discussion). The change is even smaller for other core functions. The core functions enter the configuration interaction (CI) Hamiltonian via core potential $V_{core}$ to which all core electrons contribute (from 1s to 5d). The difference for $V_{core}$ in $V^{N-3}$ and $V^{N-1}$ approximations is very small \cite{3}. Moreover, the corresponding corrections to the configuration interaction (CI) Hamiltonian have been included in \cite{3} using the many-body perturbation theory approach.

As it is well known, the eigenstates of a Hamiltonian do not depend on the basis one uses. The valence states are indeed different in the $V^{N-3}$ and $V^{N-1}$ approximations. However, this should have no effect on the final results as long as the saturation of the basis for valence states is achieved. There are only two conditions the basis states must satisfy: (a) they must be orthogonal to the core, and (b) they must constitute a complete set of states. Therefore, it does not matter whether valence states are calculated in the $V^{N-3}$ or $V^{N-1}$ potential or by any other method (e.g., a popular basis $\psi_n(r) = r^n\psi_0(r)$ \cite{3}), the final results should be the same. If there is any differ-
ence in the results, the most likely reason for this is the incompleteness of the basis set.

In spite of no difference in final results there is a good reason for the use of the $V^{N-3}$ approximation – the simplicity and good convergence of the many-body perturbation theory (MBPT) for the core-valence correlations. When an approximation different from the $V^{N-3}$ is used one has to include the so called subtraction diagrams [5], while there are no such diagrams in the $V^{N-3}$ approximation. Large energy denominators suppress the value of the correlation terms in the $V^{N-3}$ approximation ensuring good convergence of the MBPT [3]. There must be large cancellation between subtraction and other diagrams to ensure the same final results if any other initial approximation is used. This is very similar to the well known fact that the Hartree-Fock basis is the best choice for any MBPT calculations. Initial approximation might be better in some other approximation, however, strong cancellations between the subtraction and other diagrams would lead to poor convergence of the MBPT.

The authors of [1] claim that the major drawback of our work [2] is the inclusion of the core-valence correlations in the second order only. However, the correlations between the valence electrons and core electrons below the 6s state are small which is evident from the fact that their inclusion change the EDM of Tl by 3% only [3]. Therefore, only the correlations between three valence electrons should be treated to all orders. This is done in [3] to a very high precision using the CI technique.

Early calculations of thallium EDM by Liu and Kelly [2] were performed by the same relativistic coupled-cluster (RCC) method as those used in [1]. Table I of [1] presents term by term comparison between the contributions to the enhancement factor calculated in both works. For some terms the agreement is perfect, for others there is strong disagreement. The authors of [1] claim that the disagreement is due to more accurate treatment of higher-order correlations in their work. It might be possible to prove this claim by switching off the higher-order terms and reproducing the results of [2]. The benefit of having the test is enormous. Without it, no other reasons for disagreement can be excluded (e.g., incompleteness of the basis). There is no indication that such test has been performed in [1].

In conclusion, we would like to note that the calculations of the Tl EDM due to electron EDM [1] and SPS interaction [8] do not satisfy a simple consistency test: the ratio of the EDMs due to two operators must be approximately equal to the ratio of the $s-p$ single-electron matrix elements of these operators. This ratio is the same for all important single-electron matrix elements. This is because only short distances, where single-electron energies can be neglected, contribute to the single-electron matrix elements of the $T,P$-odd operators. The ratio approximately equals to $89d_e/C_{SP}10^{-18}$ $e$ cm according to analytical estimates, or $83d_e/C_{SP}10^{-18}$ $e$ cm in more accurate numerical calculations [9]. This ratio must hold in any order of the MBPT if the $s_{1/2} - p_{1/2}$ $T,P$-odd matrix elements dominate at the Hartree-Fock level (see [9] for details). The ratio of the results of [1] and [8] is $115d_e/C_{SP}10^{-18}$ $e$ cm. This may indicate that important many-body effects are missed in one (or both) of the works [1, 8].

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