Research on calculation model of penetration depth when anchor touches seabed

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Abstract—In order to provide a reference for the safe buried depth of underwater pipeline in the navigable waters, this paper on the basis of the famous penetration formula, studied the calculation model of ship’s anchor dropped to seabed. The model considers the main factors that affect the penetration depth, such as the type of anchor, the weight, the speed of bottoming and the substrate condition. The model makes clear the bottom coefficient values, shows the method of hall anchor's velocity of touching bottom and cross sectional area, and combines with the existing cast anchor experiment results to determine the shape factor of anchor. Using the calculation model, typical scenario was calculated and the calculation result of different methods were compared. The results show that the calculation model is better ease and compliance. It is very important to study the factors affecting the penetration of ship anchor and to determine and evaluate the buried depth of underwater pipeline in navigable waters.

1. Introduction

There is no denying that mooring activities may cause damage to submarine pipelines. According to the Subsea Pipeline and Standpipe Failure Report (PAROC-2001) compiled by Mott MacDonald Company in the UK, 115 third-party damage accidents of submarine pipelines were counted, of which 44 were anchor accidents, the primary source of third-party accidents[1]. In the normal anchoring operation, the anchor usually does not hit bottom at a high speed. In extreme anchoring situations, such as improper anchoring, emergency anchoring and high altitude anchoring loss, the anchor will hit bottom at a high speed. In this case, the drop of the anchor is close to the free fall movement. The higher the anchor height is, the faster the speed will be when the anchor hits the bottom. In serious cases, not only the failure of the anchor chain brake, the damage of the windlass and even the chain breaking and anchor dropping may be caused, but also the deformation or damage of the anchor caused by the violent collision between the anchor and the mud bed may be caused[2]. A more adverse situation is that there are pipelines, tunnels and other underwater facilities in the area below the anchoring point. If the depth of the ship anchor penetration into the mud bed is too large, exceeding or approaching the buried depth, it will cause great damage to the submarine pipeline, which may lead to...
the rupture (fracture) of the underwater pipeline, and then cause serious catastrophic consequences. Therefore, the penetration after the anchor hits the bottom becomes one of the important reference factors to determine the buried depth of the submarine pipeline.

Article 3.3.11 of Code for Design of Oil and Gas Pipeline Crossing Engineering (GB 50423-2007) : "The buried depth of the pipe section crossing the water area of passing ships shall prevent damage to the pipe section caused by ship anchors or dredging devices". However, it does not give how to consider such influence. Therefore, some scholars have carried out research on the penetration of ship anchors touching the bottom. Among them, the literature [3] [4] proposed a method to determine the penetration of anchor contact based on theoretical calculation. The method proposed in [3] makes an ideal assumption of the resistance in the process of the anchor hitting the bottom and penetrating, and considers it to be equal to the component force of the anchor holding force along the vertical direction. There is a large error between the calculated results and the experimental results of anchor dropping. In [4], the soil resistance during the penetration process of anchor touching the bottom was divided into soil bottom support force, lateral support force and inertial resistance. The variation of soil resistance during the penetration process was not considered and the calculation of penetration quantity was complicated. In general, the calculation process of existing research results is complex and the physical significance of relevant parameters is not obvious. To simplify the calculation of quantity of anchor the bottom throughout method, improve the calculation methods of identification, based on the existing cast anchor the result of the experiment, the famous penetration formula (Young formula) is applied to anchor the bottom through calculating model, put forward a kind of experience and theory of the combination of new determine anchor through the calculation formula of calculation results with the experimental results have higher compliance and greatly simplify the calculation process. Since Young's formula is mainly aimed at the penetration of the projectile body, it is necessary to study the values of parameters such as the bottom-touching velocity, the shape coefficient and bottom-touching area of the anchor for the penetration of the anchor on the sea bottom (penetration depth).

2. Penetration formula and anchor bottom contact area

2.1. Introduction to Young Formula

Sandia National Laboratories has been working on soil penetration projects since 1960. Over the decades, some 3,000 tests have been conducted and an important test database has been established[5]. Young's 1997 report "The Penetration Formula"[5] On the basis of the above experiments, a formula of the same form is proposed for soil, rock and concrete, but the coefficients are different. The empirical calculation formula (Young formula) of the depth of the object penetrating the soil is given[5][6]:

When: \( V \leq 61 \text{m/s} \)

\[
D = 0.0008SN(W/A)^{0.7} \ln(1 + 2.15V^2 10^{-4})
\]

When: \( V \geq 61 \text{m/s} \)

\[
D = 0.000018SN(W/A)^{0.7}(V - 30.5)
\]

Where: \( D \) is penetration depth (m), \( N \) is the shape coefficient of the object, \( S \) is the soil coefficient, \( W \) is the mass of the object (kg), \( V \) is the velocity of the object when it touches the soil (m/s), \( A \) is the cross-sectional area of the object (m\(^2\)). The penetration situation of anchor hitting bottom in extreme anchoring is applicable to the formula form \( V < 61 \text{m/s} \), because the anchor hitting bottom velocity is more than 61 m/s when the free falling height is more than 185m and the water depth is relatively shallow, which obviously does not conform to the actual situation of anchor hitting bottom.

2.2. Values of substrate coefficient

The value of substrate coefficient is given in [6]. In order to correspond to the general seabed and combine with the actual situation, 10–20 is recommended for the silt sediment; Sand bottom advisable
6–9;Ooze can be 20~30;The mixed substrate of silt and sand bottom is recommended to be 8~15, with 15 for silt and 8 for sand bottom, which can also be interpolated according to the mixing ratio.

2.3 Calculate the cross-sectional area
Hall anchor is one of the most commonly used anchor types on ships. Therefore, the Hall anchor is taken as an example for calculation. The structure and some dimensions of the Hall anchor are shown in Fig. 1.

![Fig 1. Hall anchor structure and dimensions](image)

The cross-sectional area of the anchor when it touches the bottom is \( A \approx L \cdot B \), where \( L \) and \( B \) respectively take the length and width of the anchor crown. Dimensional parameters of Hall anchor are usually given directly by the outfitting situation of the design representing the ship type, or can be calculated according to the size parameters given in "Hall Anchor" (GB/T546-2016).

In order to facilitate the calculation of the bottom-touching area of the Hall anchor, the relationship between the weight \( m' \) of the Hall anchor and the cross-sectional area \( A' \) of the anchor is fitted quadratic, and the fitting formula is given as follows:

\[
A = -0.0027m'^2 + 0.3067m' + 0.4592
\]  

(1)

In the formula, \( A \) is the bottom-touching area \( (m^2) \), \( m \) is the mass of the anchor \( (t) \). For example, for Hall anchors with weights of 1t and 2t, their cross-sectional areas \( (A) \) are about 0.76 \( m^2 \) and 1.06 \( m^2 \), respectively, according to Equation (1).

3. Calculation model of anchor bottom-striking velocity
According to the different environment and stress state in the process of anchor falling, the process of anchor falling can be divided into two stages, namely, the stage of falling in the air (from the anchor being released to the water surface) and the stage of falling in the water (from the anchor entering the water to the bottom).

3.1 Air-falling stage
Assuming that in the extreme case, the anchor falls directly to the water from a certain height \( h_1 \) away from the water, ignoring the buoyancy and resistance generated by the air, the anchor makes free fall downward from the static state under the action of gravity. According to the kinetic energy theorem, it can be known that:

\[
mgh_1 = \frac{1}{2}mv_1^2
\]  

(2)

Where, \( m \) is the weight of the anchor \( (kg) \), \( h_1 \) is the falling height of the anchor before it touches the water \( (m) \), \( g \) is the acceleration of gravity \( (m/s^2) \), and \( v_1 \) is the velocity of the anchor when it touches the water \( (m/s) \). The velocity of the anchor \( (v_1 = \sqrt{2gh_1}) \) when it touches the water can be obtained.
3.2. Water falling stage

When the anchor enters the water, it is subjected to gravity \( mg \) in the vertical direction and increases buoyancy \( f_B \) and fluid resistance \( f \). According to the buoyancy law:

\[
f_B = \rho_w gV = \rho_w mg / \rho
\]

(3)

Where, \( \rho_w \) is densities of water and \( \rho \) is anchor materials (kg/m³) respectively, \( V \) is the volume of the anchor (m³).

The fluid resistance of the anchor in the water is proportional to the square of the moving velocity, which can be estimated by the following equation:

\[
f = \rho_w v^2 A c
\]

(4)

Where, \( v \) is the falling velocity of the anchor in water (m/s), and \( A \) is the cross-sectional area of the anchor touching the bottom (m²), \( c \) is the resistance coefficient of the anchor in water, about 0.6 in [7].

From the Angle of partial safety, the critical falling velocity \( v_0 \) of the anchor in the water can be obtained when the gravity \( mg \) is equal to the velocity resistance \( f \), ignoring the buoyancy force of the anchor in the water:

\[
v_0 = \sqrt{\frac{mg}{0.6 \rho_w A}}
\]

(5)

At that time \( v_1 < v_0 \), after the anchor enters the water, because the gravity is greater than the resistance, the anchor will do variable acceleration motion, until the vertical force of the anchor in the water reaches the equilibrium state \( (v = v_0) \). At that time \( v_1 > v_0 \), after the anchor enters the water surface, because the gravity is less than the resistance, the anchor will start to do deceleration motion, also at that time to reach the equilibrium state \( (v = v_0) \).

When the anchor falls in the water, according to the kinetic energy theorem, it can be known:

\[
(mg - f)h = \frac{1}{2}mv_1^2 - \frac{1}{2}mv_2^2
\]

(6)

In Equation (6), \( h \) is the falling height (m) of the anchor in the water, and the velocity (m/s) when the anchor touches the bottom can be obtained \( (h = h_2) \):

\[
v_2 = \sqrt{\frac{(mgh_2 + 0.5mv_1^2)}{(0.5m + 0.6\rho_w Ah_2)}}
\]

(7)

When \( v_1 = v_0 \), the critical height of airborne fall can be obtained \( (h_0 = 0.5m/(0.6\rho_w A)) \).

Taking Hall anchor of 10t as an example, it can be obtained that \( A = 3.2m^2 \), \( \nu = 7.15m/s \), \( h_0 = 2.56m \). Analysis shows that the 10t hall anchor in distance from the surface height less than 2.56 m drop, contact with the surface will be lower than the critical velocity of 7.15 m/s, after entering the water will be accelerated motion, the bottom of the largest speed 7.15 m/s, the bottom of the speed is not as depth increases further, if the speed of the inadequacy of the depth of the water bottom may not reach the critical speed 7.15 m/s. If the anchor in distance from the surface height more than 2.56 m in the fall, when in contact with the surface will be more than the critical speed 7.15 m/s, after entering the water will do become slow motion, the bottom of the largest speed is close to zero under the condition of the anchor in depth contact when the speed of the water, with the increase of the depth of the water, the bottom of the speed will be reduced, but not less than the critical speed 7.15 m/s.

The above analysis shows that the anchor to a position above the water dropping, the bottom of the speed there is a critical value, not necessarily as the height increases with the increase of the depth of the water or air, on the contrary the air height more than the critical height, with the increase of the depth of the water bottom velocity is reduced, the result has proved that the higher the anchor from the height of the base bottom velocity, the greater the understanding is wrong. In addition, research shows that the anchor will be affected by the strong reaction force of water at the moment of entering the water, and its speed will decrease to a certain extent. If the speed decrease in this process is not taken into account, the calculation result of the anchor's bottom-touching speed will be too large to some extent, which ensures that the calculation result is more inclined to safety.
4. Calculation model of penetration to the bottom

Japan conducted an anchor dropping experiment in 1975, aiming to determine the optimal depth of the submarine pipeline through the experimental data. These experiments have very important guiding significance and reference value for future generations to carry out related researches[4]. The results of foreign anchor dropping experiments are shown in Table 1:

| Serial number | Anchor weight (t) | Water depth (m) | Bottom | Permeability (m) |
|---------------|-------------------|-----------------|--------|------------------|
| 1             | 2.0               | 18              | Silt and sand | 1.5              |
| 2             | 2.5               | 12              | Under the soft mud is the hard mud | 1.8              |
| 3             | 3.4               | 12              | sand     | 0.3              |
| 4             | 6.0               | 16              | silt     | 1.9              |
| 5             | 9.7               | 18              | silt     | 3.5              |
| 6             | 18.0              | 45              | Sand, silt | 2.6              |

Table 1. Results of anchor dropping experiment (anchor without rod)

| Serial number | Bottom-touching speed V (m/s) | The substrate coefficient S | The cross-sectional area is A (m²) | Shape factor N | Calculated value of penetration D (m) |
|---------------|-------------------------------|-----------------------------|-----------------------------------|----------------|-------------------------------------|
| 1             | 5.37                          | 15                          | 1.06                              | 102            | 1.4                                 |
| 2             | 5.49                          | 30                          | 1.21                              | 55             | 3.1                                 |
| 3             | 5.76                          | 6                           | 1.47                              | 39             | 0.7                                 |
| 4             | 6.31                          | 20                          | 2.20                              | 55             | 3.3                                 |
| 5             | 6.67                          | 20                          | 3.18                              | 83             | 4.0                                 |
| 6             | 7.42                          | 8                           | 5.11                              | 113            | 2.2                                 |

Table 2. The coefficient of Young's formula and the calculation results of Equation (7) in anchor dropping experiment

Assuming that the height above the water surface is h₁=0m and the water depth is h₂=18, 12, 16, 18 and 45m, the velocity when the anchor hits the bottom can be obtained from Equation (7) as 5.37, 5.49, 5.76, 6.31, 6.67 and 7.42 m/s respectively.

According to the condition of the base material, the base material coefficient S is 15, 30, 6, 20, 20 and 8 respectively.

The cross-sectional area A of the anchor can be obtained from Equation (1), which are respectively 1.06, 1.21, 1.47, 2.20, 3.18 and 5.11 m².

By substituting the above correlation coefficient and penetration results of anchor dropping experiments into Young's formula, the shape coefficients N determined by the six anchor dropping experiments are 102, 55, 39, 55, 83, 113 respectively. In order to ensure that the calculation results are more inclined to safety, the average value of the first three numbers (113, 102, 83) from the largest to the smallest is =100 (N). Therefore, based on Young's formula and existing anchor dropping experiment results, the calculation model of penetration of rodless anchor on the seabed is given as follows:

\[
D = 0.077 \cdot S(m/A)^{0.7} \ln\left(1 + 2.15V^2 \cdot 10^{-4}\right) \tag{8}
\]

Equation (8) is used to calculate the penetration of anchor hit bottom under the anchor dropping experiment condition, which are about 1.4, 3.1, 0.7, 3.3, 4.0 and 2.2 m respectively, as shown in Table 2. Comparison amount through the results of table 1 and table 2 shows the results of the calculation results and cast anchor experiment of certain error, from the size and distribution of error, equation (8) can be used for the calculation of penetration depth of rod anchor, but in the practical application of
related values should be taken into account to safety and possible error, the error source is mainly anchor bottom velocity and sediment coefficient.

5. Model checking and calculation examples

5.1. Comparison and analysis of model calculation results and finite element results

In addition to the empirical formula proposed in this paper, the pure theoretical finite element method can also be used to calculate the penetration of the anchor when it hits the bottom. In order to compare the differences between the two methods in calculating the penetration of the anchor hitting the bottom, the two methods are respectively applied in Table 3 for calculation:

Table 3. Comparison of results between empirical method (proposed in this paper) and finite element method

| Type of ship (10,000 tons) | Anchor weight (t) | Bottom sediment types | Anchor distance from water (m) | Water depth (m) | The bottom of the speed | Model calculation results | Finite element calculation results (influence depth) |
|----------------------------|-------------------|-----------------------|-------------------------------|----------------|------------------------|--------------------------|---------------------------------------------|
| 3                          | 6.0               | Silty clay            | 12                            | 11.3           | 29.18                  | 7.64                     | 2.93                         | 3.01                                     |
|                            |                   | silt                  |                               |                |                        |                          |                               |                                           |
| 5                          | 8.7               | Silty clay            | 12                            | 11.5           | 29.18                  | 7.98                     | 3.40                         | 3.85                                     |
|                            |                   | silt                  |                               |                |                        |                          |                               |                                           |
| 10                         | 14.7              | Silty clay            | 12                            | 13.0           | 29.18                  | 8.59                     | 4.27                         | 5.30                                     |
|                            |                   | silt                  |                               |                |                        |                          |                               |                                           |
| 20                         | 35.5              | Silty clay            | 12                            | 20.5           | 29.18                  | 10.60                    | 7.93                         | 8.17                                     |
|                            |                   | silt                  |                               |                |                        |                          |                               |                                           |

As can be seen from the calculation results of the two methods in Table 3, the difference between the two methods is small, especially in the case of silt clay (coefficient 8). Under the condition of soft substrate, the calculation results given by empirical method are larger than those by finite element method.

5.2. Model calculation examples

Assume that the initial height \( h_1 \) of the anchor on the water surface is 1.5m and 5m respectively. Equation (8) is used to calculate the penetration of 2t and 5t Hall anchors directly anchored under different water depths and different substrates (silt, sand and the mixed substrates of both), and the calculated results are shown in Figure 2 and Figure 3.

From the analysis of Figure 2 and Figure 3, it can be seen that both the bottom material and the anchor weight have a great influence on the penetration of the bottom touch. The penetration of the anchor in the silt bottom is the largest, while the penetration in the sand bottom is smaller, and the influence on the penetration depends on the bottom material coefficient. Under the same conditions, the greater the anchor weight, the greater the penetration. Different from the previous understanding that the water depth has a great influence on the penetration of the anchor touching the bottom, the research shows that the influence of the water depth on the penetration of the anchor touching the bottom is not very obvious, especially when the water depth is large, the penetration will approach to a stable value with the increase of the water depth. Anchor the height of the fall in the air to through the
influence of the amount of water is very clear, this is because the anchor in the air can in a relatively short distance high speed, the value directly affect the speed of anchor when contact surface, thus affect the bottom velocity, especially the water depth is shallow, the bottom of the velocity is almost entirely determined by the air drop height.

Fig 2. Calculation Results of Bottom-contact Penetration (Air Altitude 1.5m, lower than the critical height)

The influence of air height on the penetration is bounded by the critical height. If the anchor is dropped below the critical height, for example, the anchor is dropped at the air height of 1.5m, the penetration will reach a stable value at about 10m water depth. After that, the increase of water depth will have very limited influence on the penetration. If the anchor to throw the anchor from the critical height above, such as air height 5m, when in contact with the surface of the water at a speed of more than the critical speed, into the water after the gravity of the fluid resistance is greater than the anchor will do become slow movement, at the moment of contact with the water velocity, water depth, the greater the volume instead of smaller, in the depth of about 30 m throughout the amount reaches a stable value, after the increase of the depth of influence on throughout the amount can be ignored.

6. conclusion
In order to accurately calculate the penetration after the anchor hits the bottom, firstly, it is necessary to reasonably determine the base mass coefficient. The base mass coefficient has a great influence on the penetration. In the case of uncertainty, it is suggested to choose the large value to be safe. Second, it is necessary to reasonably select the design representative ship type, and then determine the weight of the anchor and the falling height of the anchor in the air, so as to reasonably determine the speed of the anchor when it hits the bottom.

Moreover, in particular, it is unreasonable to overemphasize the impact of anchoring. The influence of mooring activities on submarine pipelines can be subdivided into anchor dropping and anchor towing damage in [1]. In terms of anchoring and towing, towing is more likely to cause submarine pipeline accidents. From the depth of the buried pipeline to the seabed, the depth of about 3m can basically resist the damage caused by anchor dragging activities. And even if the buried depth is increased by 3m, it is difficult to completely avoid the impact of the anchor hitting the bottom of the pipeline after anchoring. Therefore, it is a good choice to adopt cover protection measures such as backfill gravel on the basis of reasonable determination of buried depth for the submarine pipelines that need to be protected.

The calculation model proposed in this paper takes into account the influence of five factors, namely, the mass of the anchor, the cross-sectional area of the anchor hitting the bottom, the speed of the anchor hitting the bottom, the condition of the bottom material and the shape of the anchor. The correlation coefficient can be verified and continuously improved through further anchoring
experiments. In addition, anchoring experiments with different anchor types can be carried out to further determine the shape coefficients corresponding to other anchor types, so as to further improve the accuracy and applicability of the calculation model for anchoring penetration in extreme cases.

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