Goppa codes over Edwards curves

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Abstract

Given an Edwards curve, we determine a basis for the Riemann-Roch space of any divisor whose support does not contain any of the two singular points. This basis allows us to compute a generating matrix for an algebraic-geometric Goppa code over the Edwards curve.

Keywords: Algebraic Geometric Goppa code; Edwards curve; Riemann-Roch space

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1 Introduction

The literature on elliptic curves and their applications in cryptography is well consolidated. Besides the well-known ECC (Elliptic Curve Cryptography) in which the group law, defined on these curves, is exploited to encrypt messages, and the ECDSA (Elliptic Curve Digital Signature Algorithm), another example can be found in the Lenstra algorithm for the factorization of integers. Moreover, there are as well applications to coding theory based on the Riemann-Roch space \( \mathcal{L}(D) \) associated with a rational divisor \( D \) of these curves. In particular, this space is a fundamental ingredient to construct Goppa codes, first introduced in 1983 [9]. Goppa codes over the Hermitian curve, as well as over maximal curves and hyperelliptic curves, have been extensively studied in [1, 6–8, 11, 12, 15], as they have become an important topic both in coding theory and in cryptography, where they play a central role in McEliece public-key cryptographic systems [14].

To the best of our knowledge, AG Goppa codes for Edwards curves have not been considered until now. In this paper, we compute the generating matrices for AG Goppa codes over Edwards curves. These curves are already the subject of many papers in cryptography [2–5, 10, 13], in particular in their twisted version. Compared to the classic elliptic curves in Weierstrass form, they can be more efficient for cryptographic use and for the (single or multiple) digital signature.

In section 2 we describe Edwards curves and their relationship with elliptic curves in Weierstrass form. In section 3 we compute a basis for \( \mathcal{L}(D) \) over Edwards curves, while in section 4 we construct AG Goppa codes over Edwards curves and their generating matrices. In particular, in subsec. 4.3 we give a small example of a Goppa MDS code where we use the AG Goppa code defined in subsec. 4.1 over Edwards curves.

2 Edwards curves and elliptic curves in Weierstrass form

In this section we introduce Edwards curves \( \mathcal{E} \), that is, algebraic curves, defined over a field \( \mathbb{K} \), which can be represented in a suitable coordinate system by the equation \( \hat{x}^2 + \hat{y}^2 = 1 + d\hat{x}^2 \hat{y}^2 \), with \( d(d - 1) \neq 0 \). We present these curves as a birationally equivalent version of elliptic curves in Weierstrass form.

Recall that, over a field \( \mathbb{K} \) of characteristic different from 2, a (smooth) elliptic curve (possessing at least a \( \mathbb{K} \)-rational point) can be represented in a suitable coordinate system by the Weierstrass equation \( y^2 = x^3 + ax^2 + bx \), having one point at infinity \( \Omega = [Z : X : Y] = [0 : 0 : 1] \) on the y axis.

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Remark 2.1. Note that, unlike those in Weierstrass form, curves in Edwards form $E$ have two points at infinity, that is, $\Omega_1 = \left[ Z : X : Y \right] = [0:1:0]$ on the $x$ axis and $\Omega_2 = \left[ Z : X : Y \right] = [0:0:1]$ on the $y$ axis, which are ordinary singular points for $E$ as this curve is non-smooth.

Remark 2.2. Edwards curves have four remarkable points: $O = (0,1)$, $O' = (0,-1)$, $H = (1,0)$, $H' = (-1,0)$. In particular, $O - O$ is the identity element of the group law defined on them, that is, $(P - O) + (O - O) = P - O$, for any point $P = (a,b) \in E(K)$, and these four points form the cyclic group $C_4$, where $O' - O$ have order 2, while $2(H - O) = 2(H' - O) = O' - O$.

Edwards curves and elliptic curves in Weierstrass form are closely related. In particular, over a field $K$ of characteristic different from 2, one has that an elliptic curve $W$ defined by the equation $y^2 = x^3 + ax^2 + bx$, and an Edwards curve $E$ defined by the equation $\hat{x}^2 + \hat{y}^2 = 1 + dx^2\hat{y}^2$, where $d$ is not a square, are birationally equivalent (cf. [2]). Furthermore, this equivalence is given by the following two rational maps:
\[
\alpha : E(K) \rightarrow W(K) \quad (2.1a)
\]
\[
(\hat{x}, \hat{y}) \mapsto (x,y) = \left( x_1 + \hat{y}, \frac{1}{1 - \hat{y}}, \frac{1 + \hat{y}}{x(1 - \hat{y})} \right)
\]
\[
\beta : W(K) \rightarrow E(K) \quad (2.1b)
\]
\[
(x,y) \mapsto (\hat{x}, \hat{y}) = \left( \frac{y_1x}{x_1y}, \frac{x - x_1}{x + x_1} \right),
\]
where $P = (x_1, y_1) \in W(K)$ is such that the divisor $2(P - \Omega) = (0,0) - \Omega$.

Remark 2.3. The two rational maps $\alpha$ and $\beta$ defines a birational equivalence between $W$ and $E$. Moreover, one extends the definition of $\alpha$ and $\beta$ by putting $\alpha((0,1)) = \Omega$, $\beta(\Omega) = (0,1)$, $\alpha((0,-1)) = (0,0)$ and $\beta((0,0)) = (0,-1)$; and $\beta((-x_1, s)) = \beta((x_2, 0)) = \Omega_1$, $\beta((-x_1, \pm s)) = \Omega_2$, where $(t_1, 0), (t_2, 0), (-x_1, \pm s) \in W(K)$, with $t_1, t_2 \neq 0$.

The value $\Omega_1$ of the two images $\beta((t_1, 0))$, $\beta((t_2, 0))$ and the value $\Omega_2$ of the two images $\beta((-x_1, \pm s))$ can be directly found by passing to homogeneous coordinates. As for the value of $\beta(\Omega)$, we have that, for $K = \mathbb{C}$, if $\varphi(z)$ and $\varphi'(z)$ are the Weierstrass elliptic functions and $P = \left( \varphi(z) + \frac{a'}{3} \frac{1}{2} \varphi'(z) \right) \in W(K)$, then,
\[
\lim_{z \to 0} \beta(P) = \lim_{z \to 0} \left( \frac{2y_1(3\varphi(z) + a')}{3x_1} \frac{3\varphi(z) + a' - 3x_1}{\varphi'(z)} \right) = (0,1) = O,
\]
where $3\varphi(z) + a' = o(z)$, that is, $\beta$ is continuous in $P = \Omega$.

Remark 2.4. Since there are two points mapped by $\beta$ onto $\Omega_1$ and two points onto $\Omega_2$, one sees that it is not possible to coherently define $\alpha(\Omega_1)$ and $\alpha(\Omega_2)$. For this reason the maps $\alpha$ and $\beta$ define a birational equivalence between the two forms. Note that $W$ is, indeed, a smooth projective resolution of the non-smooth curve $E$.

Remark 2.5. Since the map $\beta$ in (2.1b) transforms a line through $P \in W(K)$ and $Q \in W(K)$ onto the hyperbola through $\beta(P)$, $\beta(Q)$, $O'$, $2\Omega_1$ and $2\Omega_2$, and maps vertical lines onto horizontal lines, then $\beta$ induces a group homomorphism of the corresponding divisor classes group (cf. [2] for further details about the group law of Edwards curves).

3 The Riemann-Roch space $\mathcal{L}(D)$ over Edwards curves

In this section, given a divisor $D \in \text{Div}(E)$, we provide a basis of the Riemann-Roch vector space
\[
\mathcal{L}(D) = \{ f \in \mathbb{K}(E)^* : \text{div}(f) + D \text{ is effective} \} \cup \{0\}
\]
for an Edwards curve $E$, under the assumption that the support of $D$ does not contain the two singular points $\Omega_1$ and $\Omega_2$.

We recall that a divisor is, in this context, an element of the free abelian group $\text{Div}(E)$ on the points of $E$, that is, a formal sum $D = \sum_{P \in E} n_P P$, with $P \in E(K)$, where only finitely many integers $n_P$ are not zero, and that a principal divisor $D = \text{div}(g)$ of a function $g$ is the sum of the zeros of $g$ on $E$ minus the poles of $g$ on $E$. The integer $\delta = \sum n_P$ is the degree of the
divisor $D$ and principal divisors give a subgroup of the subgroup $\text{Div}^0(\mathcal{E})$ of divisors having degree equal to zero, because any function $g$ on $\mathcal{E}$ has by Bezout theorem the same number of zeros and poles on $\mathcal{E}$. The group taken into account is formally the quotient group $\frac{\text{Div}^0(\mathcal{E})}{\text{Princ}(\mathcal{E})}$.

Also, we recall that any divisor $D'$ on $\mathcal{E}$ of degree $k + 1$, such that $\Omega_1$ and $\Omega_2$ do not belong to the support of $D'$, is linearly equivalent to $P + kO$, for a suitable point $P \in \mathcal{E}(\mathbb{K})$ (or $(k + 1)O$, in the case where $P = O$), that is, $D' = P + kO + \text{div}(g)$, for a suitable function $g$. Since the map

$$
\chi : \mathcal{L}(D') \rightarrow \mathcal{L}(P + kO)
$$

$$
F \mapsto gF
$$

is an isomorphism between $\mathcal{L}(D')$ and $\mathcal{L}(P + kO)$, we confine ourselves to the latter space.

**Theorem 3.1.** Let $\mathcal{E}$ be an Edwards curve defined, over a field $\mathbb{K}$ of characteristic different from 2, by the equation $x^2 + y^2 = 1 + dx^2y^2$, where $d$ is not a square. If $P + kO = D \in \text{Div}(\mathcal{E})$ is a divisor of positive degree $k + 1$, where $P = (a, b)$, then $\dim(\mathcal{L}(D)) = k + 1$ and

$$
\mathcal{L}(D) = \begin{cases}
\langle F_0, F_1, \ldots, F_k \rangle & \text{if } P \neq O \\
\langle F_0, F_2, \ldots, F_{k+1} \rangle & \text{if } P = O
\end{cases}
$$

where $F_0, F_1, \ldots, F_{k+1}$ are rational homogeneous functions defined as follows:

$$
F_0 = \frac{Z}{X}
$$

$$
F_1 = \begin{cases}
\frac{Z}{X} & \text{if } P = O' = (0, -1) \\
\frac{Z}{(X + Z)(Y + Z)} & \text{if } P = H = (1, 0) \\
\frac{Z}{(X - Z)(Y + Z)} & \text{if } P = H' = (-1, 0) \\
\frac{Z}{(Y + bZ) \cdot X} & \text{if } P \notin \{O', H, H'\} \\
\frac{Z}{(X - aZ) \cdot (Y - Z)} & \text{if } P \notin \{O', H, H'\}
\end{cases}
$$

$$
F_i = \begin{cases}
\frac{Z^h}{(Y - Z)^h} & \text{if } i = 2h \\
\frac{Z^h}{(Y + Z)Z^h} & \text{if } i = 2h + 1
\end{cases}
$$

for $2 \leq i \leq k + 1$.

**Proof.** Since $P$ is different from $\Omega_1$ and $\Omega_2$, we can take the point $\alpha(P) \in \mathcal{W}(\mathbb{K})$, where $\alpha : \mathcal{E}(\mathbb{K}) \rightarrow \mathcal{W}(\mathbb{K})$ is the map defined in (2.1a). Hence, the (surjective) map

$$
\alpha^{-1} = \beta : \mathcal{W}(\mathbb{K}) \rightarrow \mathcal{E}(\mathbb{K})
$$

in (2.1b), induces an (injective) homomorphism $g \mapsto g \circ \beta$ from $\mathcal{L}(P + kO)$ to $\mathcal{L}(\alpha(P) + k\Omega)$, because $\beta(\text{div}(g \circ \beta)) = \text{div}(g)$ for any function $g \in \mathcal{L}(P + kO)$.

Since $\mathcal{W}$ is smooth, by the formula of Riemann-Roch, the dimension of $\mathcal{L}(\alpha(P) + k\Omega)$ is $k + 1$, and we are left with exhibiting $k + 1$ linearly independent functions in $\mathcal{L}(P + kO)$, as manifestly $\mathcal{L}(P + (i - 1)O)$ is contained in $\mathcal{L}(P + iO)$, for $i = 1, \ldots, k + 1$.

For $i = 0$ the assertion follows, because $\text{div} \left( \frac{Z}{Z} \right) = 0$ and for every $P \in \mathcal{E}(\mathbb{K})$ we have that $\text{div}(F_0) + P$ is effective.
Recalling that $O' = (0, -1)$, $H = (1, 0)$, $H' = (-1, 0)$, and putting $R = (a, -b)$ and $R' = (-a, -b)$ for $P = (a, b)$ as in figure 3.1, for $i = 1$ we have that:

$$
\operatorname{div} \left( \frac{Z}{X} \right) = 2H' + 2\Omega_2 + 2O' + 2\Omega_1 - (H + H' + 2\Omega_1 + O + O' + 2\Omega_2) = H' + O' - H - O,
$$

$$
\operatorname{div} \left( \frac{(X + Z)(Y + Z)}{XY} \right) = (2H' + 2\Omega_2 + 2O' + 2\Omega_1) - (H + H' + 2\Omega_1 + O + O' + 2\Omega_2) = H' + O' - H - O,
$$

$$
\operatorname{div} \left( \frac{(X - Z)(Y + Z)}{XY} \right) = (2H + 2\Omega_2 + 2O' + 2\Omega_1) - (H + H' + 2\Omega_1 + O + O' + 2\Omega_2) = H + O' - H' - O,
$$

$$
\operatorname{div} \left( \frac{Y + bZ \cdot X}{(X - aZ) \cdot (Y - Z)} \right) = (R + R' + 2\Omega_1 + O + O' + 2\Omega_2) - (P + R + 2\Omega_2 + 2O + 2\Omega_1) = R' + O' - P - O.
$$

Hence, we have that $\operatorname{div}(F_1) + P + O$ is effective for any suitable $P$.

Additionally, for $i \geq 2$, we have that:

$$
\operatorname{div} \left( \frac{Z^h}{(Y - Z)^h} \right) = (2h\Omega_1 + 2h\Omega_2) - (2hO + 2h\Omega_1) = 2h\Omega_2 - 2hO,
$$

$$
\operatorname{div} \left( \frac{(Y + Z)Z^h}{X(Y - Z)^h} \right) = (2O' + 2\Omega_1 + 2h\Omega_1 + 2h\Omega_2) - (O + O' + 2\Omega_2 + 2hO + 2h\Omega_1) = O' + 2\Omega_1 + (2h - 2)\Omega_2 - (2h + 1)O,
$$

hence, $\operatorname{div}(F_i) + P + iO$ is effective in both the cases $i = 2h$ and $i = 2h + 1$.

So, every function $F_i$ is such that $\operatorname{div}(F_i) + D$ is effective if $D = P + kO$ or $D = (k + 1)O$. In order to complete the proof, it is necessary to show that all these functions are linearly independent, but this follows from standard, elementary, arguments of linear algebra.

We note that in the case $D = (k + 1)O$ we simply remove $F_1$ and we add $F_{k+1}$, thus, also in this case, we have $k + 1$ linearly independent functions. $
$ ■

**Remark 3.1.** We note that it is not possible to extend the proof about $L(D)$ in theorem 3.1 when $P$ is equal to $\Omega_1$ or $\Omega_2$, because the map $\beta$ is not invertible on these points (see remark 2.4). Riemann-Roch spaces on curves having singular points are the subject of § IV.2 in [16].

### 3.1 Computational cost

Recalling that the costs of modular addition, multiplication, and inversion over $GF(q)$ are $O(\ln(q))$, $O(\ln^2(q))$, $O(\ln^3(q))$, respectively, we now compute the cost of evaluating at a point $P \in E(GF(q))$ each element of the basis of $L(D)$. 

\[\]
We firstly note that we can compute \( F_{2h} \) from \( F_{2h-2} \), and \( F_{2h+1} \) from \( F_{2h} \), as

\[
\begin{align*}
F_{2h} &= F_2F_{2h-2} & \text{if } h \geq 2, \\
F_{2h+1} &= \frac{Y + Z}{X} F_{2h} & \text{if } h \geq 1,
\end{align*}
\]

that is, at each step we have to perform a single multiplication times the last (or the second-last) value. Moreover, we can pre-calculate the value of the function \( \frac{Y + Z}{X} \) at \( P \) with a cost \( \mathcal{O}(\ln(q)) + \mathcal{O}(\ln^2(q)) + \mathcal{O}(\ln^3(q)) \approx \mathcal{O}(\ln^3(q)) \) further speed up the computation.

Therefore, the maximal global cost \( \mathcal{O}(\mathcal{L}(D)) \) of evaluating the first \( k \) functions \( F_i \) is \( \mathcal{O}(k \cdot \ln^2(q) + \ln^3(q)) \).

## 4 AG Goppa codes on Edwards curves

In this section, we construct the generating matrix and the parity-check matrix for a \([n,k,d]_q\) AG Goppa code for an Edwards curve \( \mathcal{E} \) over \( \text{GF}(q) \), compute the computational cost, and give a small example.

### 4.1 Goppa code for an Edwards curve

In the following, we adapt the definition of a Goppa code to our case.

**Definition 4.1.** Let \( D = P + (k-1)O \) be a divisor of positive degree \( \delta D = k \) of the Edwards curve \( \mathcal{E} \) over \( \text{GF}(q) \), where \( q = p^r \) and \( p \) is an odd prime number. Let \( \mathcal{L}(D) \) be the Riemann-Roch space, let \( T = \{ P_1, \ldots, P_n \} \) be a set of \( n > k-1 \) points such that, for \( i = 1, \ldots, k \) and \( j = 1, \ldots, n \), \( G_{ij} = F_{i-1}(P_j) \), where \( \{ F_i \} \) is a basis of \( \mathcal{L}(D) \), \( P_j \in \mathcal{E} \), and \( P_j \notin \text{supp}(D) \). Let \( \{ G_{ij} \} = G \in \text{GF}(q)^{k \times n} \) be the \( k \times n \) matrix, we define the \([n,k,d]_q\) AG Goppa code \( C_G = \{ c \in \text{GF}(q)^n : c = a \cdot G, a \in \text{GF}(q)^k \} \).

**Remark 4.1.** We note that \( G \) is well defined because all points \( P_j \in T \) do not belong to the support of \( D \) which contains the poles of each \( F_i \).

**Theorem 4.1.** If \( C_G \) is the AG Goppa code of definition 4.1, then the minimum distance \( d \) of this code is such that \( d \geq n - \delta D = n - k \).

**Proof.** It follows from the same, classic, proof of AG Goppa codes over curves.

If we order the points in \( T \) so that the first \( k \) columns of the generating matrix

\[
G = \begin{pmatrix}
F_0(P_1) & F_0(P_2) & \cdots & F_0(P_n) \\
F_1(P_1) & F_1(P_2) & \cdots & F_1(P_n) \\
\vdots & \vdots & \ddots & \vdots \\
F_{k-1}(P_1) & F_{k-1}(P_2) & \cdots & F_{k-1}(P_n)
\end{pmatrix} \in \text{GF}(q)^{k \times n}
\]

of the Goppa code \( C_G \) are linearly independent, e.g. by applying the Gauss-Jordan method, then \( G \) can be reduced in its standard form \([I_k|M]\), where \( I_k \) is the identity matrix of order \( k \) and \( M \in \text{GF}(q)^{k \times (n-k)} \). Once \( G \) is in standard form, the parity-check matrix \( H \in \text{GF}(q)^{(n-k) \times n} \) of this Goppa code, that is, the matrix such that \( G \cdot H^T = 0 \) and \( H \cdot y^T = 0 \) for every code word \( y \in C_G \), is simply \( H = [-M^T|I_{n-k}] \). Thus, the code \( C_G \) is also defined as \( \{ y \in \text{GF}(q)^n : H \cdot y^T = 0 \} \).

### 4.2 Computational cost of constructing a Goppa code

In order to compute the generating matrix \( G \) we need to evaluate each of the \( n \) points in the set \( T \) for each element of the basis of \( \mathcal{L}(D) \), that is, we have a computational cost of \( \mathcal{O}(n \cdot \mathcal{O}(\mathcal{L}(D))) \) because \( G \) is a matrix of size \( k \times n \). Moreover, the cost of computing the parity-check matrix depends on the method used to solve the linear system \( G \cdot x = 0 \). For instance, if we used the
We now compute the parity-check matrix \( G \) to its standard form, then the cost would be \( \mathcal{O} \left( \max(n,k)^3 \right) = \mathcal{O} (n^3) \).

Hence, the global computational cost of constructing a Goppa code over \( \mathcal{E} (\text{GF}(q)) \) is:

\[
\mathcal{O} (n \cdot C(\mathcal{L}(D))) + \mathcal{O} \left( \max(n,k)^3 \right) = \mathcal{O} \left( n \cdot (k \cdot \ln^2(q) + \ln^3(q)) \right) + \mathcal{O} (n^3).
\]

In particular, for \( k \cdot \ln^2(q) + \ln^3(q) < n^2 \), the computational cost is \( \mathcal{O} (n^3) \). However, if \( k \cdot \ln^2(q) + \ln^3(q) > n^2 \), for instance if we were working with very large finite fields \( (q \gg 2) \), the overall computational cost would be \( \mathcal{O} \left( n \cdot (k \cdot \ln^2(q) + \ln^3(q)) \right) \).

### 4.3 A small example

Let \( \mathbb{K} = \text{GF}(17) \) and let \( \mathcal{E} \) be the Edwards curve defined by the equation \( x^2 + y^2 = 1 + 10x^2y^2 \).

There are 24 affine points on this curve:

\[
\{(0, 1), (0, 16), (1, 0), (2, 2), (2, 15), (3, 6), (3, 11), (5, 8), (5, 9), (6, 3), (6, 14), (8, 5), (8, 12), (9, 5), (9, 12), (11, 3), (11, 14), (12, 8), (12, 9), (14, 6), (14, 11), (15, 2), (15, 15), (16, 0)\},
\]

and the two points at infinity.

Let \( D = (2, 15) + 4O \) be the divisor defining \( \mathcal{L}(D) \), so the degree \( k \) is 5. Let \( T = \{ P_1 = (5, 8), P_2 = (5, 9), P_3 = (6, 3), P_4 = (6, 14), P_5 = (8, 5), P_6 = (8, 12), P_7 = (9, 5) \} \) be the set of points such that \( P_j \notin \text{supp}(D) \) defining the generating matrix \( G \) of \( C_G \), thus \( n = 7 \). Applying theorem 3.1, the vector space \( \mathcal{L}(D) \) has the following basis:

\[
\mathcal{L}(D) = \langle F_0, F_1, F_2, F_3, F_4 \rangle = \left\{1, \frac{x(y + 15)}{(x - 2)(y - 1)}, \frac{1}{y - 1}, \frac{1}{x(y - 1)}, \frac{1}{(y - 1)^2} \right\},
\]

whereas the generating matrix \( G = (G_{ij}) \) of \( C_G \) is defined by putting \( G_{ij} = F_{i-1}(P_j) \), that is,

\[
G = \begin{pmatrix}
1 & 1 & 1 & 1 & 1 & 1 & 1 \\
16 & 5 & 5 & 4 & 1 & 11 & 4 \\
5 & 15 & 9 & 4 & 13 & 14 & 13 \\
9 & 13 & 6 & 10 & 14 & 10 & 3 \\
8 & 4 & 13 & 16 & 16 & 9 & 16
\end{pmatrix}.
\]

We now compute the parity-check matrix \( H \) by solving the linear system \( G \cdot \mathbf{x} = \mathbf{0}, \) which reduces to:

\[
\begin{align*}
x_1 &= 7x_6 + 2x_7 \\
x_2 &= 3x_6 + 12x_7 \\
x_3 &= x_6 + 9x_7 \\
x_4 &= 13x_6 + 12x_7 \\
x_5 &= 9x_6 + 15x_7
\end{align*}
\]

that is,

\[
H = \begin{pmatrix}
7 & 3 & 1 & 13 & 9 & 1 & 0 \\
2 & 12 & 9 & 12 & 15 & 0 & 1
\end{pmatrix}.
\]

Finally, the minimum distance for this code is \( d \geq n - \delta D = 7 - 5 = 2 \). Moreover, we know from the Singleton theorem that \( d \leq n - k + 1 \) for a \([n,k,d]_q \) code, that is, \( d \leq 7 - 5 + 1 = 3 \) as \( k = \delta D = 5 \). Hence, \( d = 2 \) or \( d = 3 \), but one can easily check that 2 columns of \( H \) are always linearly independent, so \( d = 3 \). Therefore, we have a \([7,5,3]_{17}\)-Goppa MDS code.

### 5 Conclusions

Edwards curves have been recently introduced for their applications in cryptography. In this paper, we provided a basis for the Riemann-Roch space of a divisor on these curves, and we used this basis for the construction of the generating matrices of the AG Goppa codes, thus providing a possible application of Edwards curves to Coding theory, as well.
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