Effect of Oriented Nuclei on the Competing Modes of $\alpha$ and One-Proton Radioactivities in the Vicinity of $Z = 82$ Shell Closure

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ABSTRACT

The purpose of the present work is to investigate the alpha ($\alpha$) emission as competing mode of one proton emission using the preformed cluster decay model (PCM). PCM is based on the quantum-mechanical tunneling mechanism of penetration of the preformed fragments through a potential barrier, calculated within WKB approximation. To explore the competing aspects of $\alpha$ and one proton radioactivity, we have chosen emitters present immediately above and below the $Z = 82$ shell closure i.e. $^{177}$Tl and $^{185}$Bi by taking into account the effects of deformations ($\beta$) and orientations of outgoing nuclei. The minimized values of fragmentation potential and maximized values of preformation probability ($P_0$) for proton and alpha fragment demonstrated the crucial role played by even $Z$- even $N$ daughter and shell closure effect of $Z = 82$ daughter, in $^{177}$Tl and $^{185}$Bi, respectively. The higher values of $P_0$ of the one proton further reveal significance of nuclear structure in the proton radioactivity. From the comparison of proton and $\alpha$ decay, we see that the former is heavily dominating with larger values of $P_0$ in comparison to the later. Theoretically calculated half-lives of one proton and $\alpha$ emission for spherical and deformed considerations have also been compared with available experimental data.

1. Introduction

One of the most exciting topics in current nuclear physics research is the search for the nuclear limits of stability within nuclear landscape. The knowledge of many new radioactive decay modes from ground state of spherical as well as deformed nuclei provides information of nuclei far from limits of stability. The novel explorations led to the prediction of existence and properties of a large number of proton rich isotopes and established the limits of stability of the isotopes with respect to decay with proton emission i.e. the existence of new type of nuclear decay mode [1]. For nuclei with an odd number of protons ($Z$) one-proton (1p) radioactivity i.e. the emission of a proton from a nuclear ground state with a certain half-life was predicted, whereas even-$Z$ nuclei were predicted to decay through two-proton (2p) radioactivity, at proton drip line.

The experimental observation of 1p radioactivity was first reported in September 1970 by Jackson et al. from the isomeric state of $^{169}$Co [2] and promptly confirmed by Cerny et al. [3]. The emission of proton depends strongly upon the Q-value and its positive value, portrays spontaneous proton emission. One proton emission has been studied in two regions: $51 \leq Z \leq 67$ and $69 \leq Z \leq 83$, theoretically, first region is considered to be deformed and other one to be spherical in their ground state [4]. Different theoretical approaches have been used to obtain the half-lives of spherical and deformed proton emitters. In the earliest attempts, the Coulomb potential plus the Woods-Saxon potential have been put forwarded to explore the same [5]. Furthermore, several theoretical models such as the unified fission model (UFM) [6], the generalized liquid drop model [7] and the Gamow-like model [8] provide excellent estimates for the lifetimes of proton radioactivity. Therefore, all the above approaches are considered as standard tools to describe proton radioactivity. These studies help to obtain structural information and the related aspects of the proton-rich nuclei in the vicinity of the limits of nuclear landscape.

For the $Z > 72$ odd-$Z$ nuclei lying beyond the proton drip line ($Qp > 0$), proton radioactivity becomes energetically possible in close analogy to the familiar $\alpha$ decay, where the proton tunnels through the potential barrier comprising the superposition of the Coulomb and centrifugal potentials. Protons experience a relatively low Coulomb potential and a relatively high centrifugal potential when comparing with the alpha decay case, due to their lower charge and mass compared to alphas [9]. In the present work we will study the one proton radioactivity in $^{177}$Tl ($Z = 81$) and $^{185}$Bi ($Z = 83$) nuclei since they reside immediately above and
below the $Z = 82$ shell closure, respectively. The decay of p-rich $^{177}$Tl and $^{185}$Bi nucleus has also been explored by UFM with spherical consideration and assuming preformation probabilities $P_0 = 1$ [6] for 1p radioactivity. However, in the present approach of preformed cluster decay model (PCM) we intend to investigate the dynamics of competing $\alpha$ and 1p emissions within spherical as well as deformations ($\beta$) and orientation effects included of outgoing nuclei, within the collective clusterization approach of Quantum Mechanical Fragmentation Theory (QMFT).

The QMFT based fragmentation potential comprises of binding energy (B.E) (which is the sum of liquid drop energy ($V_{\text{LDM}}$) and shell corrections ($\delta U$)), Coulomb potential ($V_C$) and nuclear proximity potential ($V_P$) of the decaying fragment and daughter nuclei in the ground state decay of the proton rich nuclei. The structure information of the CN enters the model via the $P_0$ (also known as the spectroscopic factors) of the fragments. The preformation profile of all competing fragments shows that 1p-emission is more probable than $\alpha$-emission for spherical and $P_0$ for $\alpha$ further reduced for deformed cases [10]. This approach enabled us to calculate the proton decay and alpha-decay half-lives via fragmentation potential, preformation probability, scattering potential for both the spherical and deformed considerations, of chosen nuclei. Theoretically calculated half-lives of spherical and deformed proton emitters are compared with experimental data. The brief methodology is given in Section 2 and results and discussion of the presented calculations (graphically) are described in section 3. The work is summarized in section 4.

2. Methodology: Dynamical Cluster Decay Model (DCM)

The Quantum mechanical fragmentation theory (QMFT) based Preformed cluster model (PCM) [11-14] uses the collective coordinates of mass and charge asymmetries $\eta_A$ and $\eta_V$, relative separations coordinate $R_{\alpha}$ and the multiple deformation coordinates $\beta_{\alpha\nu}$; the orientation degrees of freedom $\theta_i$ (i = 1, 2). In terms of these collective coordinates, the decay constant in PCM is defined as

$$\lambda = \frac{\ln 2}{T} = \nu_0 P_0 P$$  \hspace{1cm} (1)

Here, $\nu_0$ is the assault frequency with which the cluster hits the barrier. $P_0$ corresponds to cluster preformation probability and $P$ the barrier penetrability. $P_0$ is obtained by solving the stationary Schrödinger equation in $\eta$, at a fixed $R = R_a$

$$P_0 = \sqrt{\frac{B_{\eta\eta}}{2A}} |\psi(\eta(A))| (2/A)$$  \hspace{1cm} (2)

using QMFT based fragmentation potential. Fragmentation potential consists of Binding energies (B.E), Shell Corrections ($\delta U$), Coulomb potential ($V_C$), Proximity Potential ($V_P$) and angular momentum Potential ($V_\ell$) as:

$$V(\eta) = \sum_{i=1}^n B(A_i, Z_i) + V_C + V_P + V_\ell$$  \hspace{1cm} (3)

The penetrability, $P$ calculated as the WKB tunneling probability,

$$P = \exp \left[ -\frac{2}{h} \int_{R_a}^{R_b} \left\{ 2\mu [V(R) - Q_{\text{eff}}] \right\}^{1/2} dR \right]$$  \hspace{1cm} (4)

Here, $R_a$ and $R_b$ are the first and second turning points of the penetration path used to calculate the $P$. Here, $V(R)$ is given by the $\ell$-dependent scattering potential, consists of $V_c$, $V_P$, and $V_\ell$.

3. Results and Discussions

The Fig. 1(a-d) presents fragmentation profile of favored fragments for $^{177}$Tl and $^{185}$Bi for spherical and $\beta$ deformed and oriented consideration of nuclei. It shows that in both the cases one proton (1p) decay is more minimized in comparison to the $\alpha$ decay. Minimized value of the fragmentation potential means 1p decay is more probable or favorable than $\alpha$-decay. Secondly for 1p decay in $^{177}$Tl and $^{185}$Bi, the presence of even $Z$ ($Z = 80$) even $N$ ($N = 96$) in the daughter nuclei $^{176}$Hg and shell closure effect of $Z = 82$ in daughter nuclei $^{184}$Ph, respectively, results into the minimum fragmentation potential than $\alpha$-decay as shown in Fig. 1(a) and 1(c). A similar type of observation is seen for quadrupole deformations as in Fig. 1(b) and 1(d), there is no significant change in the potential energy surface for the cluster mass upto $A \sim 35$. For $A > 35$ many new minimas are observed as we go from spherical to $\beta$ consideration. But we are not interested in the structure of the other fragments except 1p and $\alpha$ decay. We observe that for 1p decay fragmentation potential reduced minutely and for $\alpha$-decay enhanced minutely with the inclusion of $\beta$ deformation as in Fig. 1(b) and 1(d). Following the fragmentation potential, the preformation probability ($P_0$) is also calculated for the decay of proton rich $^{177}$Tl and $^{185}$Bi for spherical and deformed consideration respectively.

Within the PCM, the preformed fragments have to penetrate the respective scattering potential barrier, as shown in Fig. 2(a-d) for $^{177}$Tl and $^{185}$Bi nuclei, respectively. The barrier lowering parameter delta $\Delta V_\eta$ is also shown for both 1p in Fig. 2(a), 2(c) and $\alpha$ decay in Fig. 2(b), 2(d). We observe that the barrier modification or lowering in barrier height increases as we go from the 1p decay to $\alpha$ decay.
Lower the value of $\Delta V_b$ in case of $1p$ decay also suggests that $1p$ has more penetration than alpha hence $1p$ decay is more favored than alpha decay. We notice there is small but not that significant change in scattering potential by including the effects of quadrupole deformations and orientations of outgoing nuclei.

The turning point of the scattering potential corresponds to $\Delta R$ i.e. the neck-length parameter that assimilates the neck formation effects. The PCM calculated half-lives ($T_{1/2}$) is obtained by adjusting this only parameter $\Delta R$ given in Table 1. It is important to note that the value of $\Delta R$ is more for $1p$ emission compared to $\alpha$ emission, which indicates that the $\alpha$ decay follows the $1p$ emission. These results are nearly same for both the spherical and deformed considerations. The calculated $T_{1/2}$ for $\alpha$ decay is in good agreement with the experimental data [15] as compared to $1p$ decay. May be the inclusion of higher multiple order deformations will further improve the comparisons.

Figure 1: Fragmentation Potential $V$(MeV) as a function of Fragment Mass $A$ for the decay of $^{177}$Tl and $^{185}$Bi with both the spherical and deformed configuration.
Figure 2: Scattering potential for the one proton and alpha decay of $^{177}$Tl and $^{185}$Bi with spherical and $\beta_2$ deformation consideration of nuclei.

Table 1: The PCM calculated decay half-lives $T_{1/2}$ for 1p and $\alpha$ radioactivity together with their comparison with the available experimental data [15] and other characteristic quantities.

| Parent | Decay | Q-value (MeV) | $\log_{10} T_{1/2}$ DR (fm) | $\log_{10} T_{1/2}$ PCM (fm) | $\log_{10} T_{1/2}$ Deformation $\beta_2$ |
|--------|-------|---------------|-----------------------------|-------------------------------|----------------------------------|
| $^{177}$Tl | p + $^{184}$Pb | 1.180 | 0.936 | 0.941 | -2.429 | -2.537 |
| $^{177}$Tl | $\alpha$ + $^{197}$Tl | 6.907 | 0.750 | 0.655 | -1.739 | -1.745 |
| $^{185}$Bi | p + $^{179}$Hg | 1.624 | 0.958 | 0.959 | -6.604 | -6.620 |
| $^{185}$Bi | $\alpha$ + $^{179}$Au | 7.639 | 0.774 | 0.700 | -3.466 | -3.487 |

Moreover, a systematic study should be developed for calculated $T_{1/2}$ of 1p-emission in other proton emitters which undertakes exploration of significant structural information of decaying nucleus, within the collective clusterization approach of PCM.
Summary

The role of competing $\alpha$ and 1p emission in $^{177}$Tl and $^{185}$Bi proton emitters present immediately above and below the $Z = 82$ shell closure by taking the effects of deformations ($\beta_2$) and orientations of outgoing nuclei have been explored within the collective clusterization approach of Quantum Mechanical Fragmentation Theory (QMFT). Fragmentation and preformation profiles show that the one proton is preborn with much larger $P_0$ values in comparison to the heavier alpha cluster in both the cases. The minimized values of fragmentation potential for proton than alpha fragment demonstrated the crucial role played by shell closure effect of $Z = 82$ daughter and even $Z$ - even $N$ daughter, in $^{185}$Bi and $^{177}$Tl, respectively. The good experimental correlation of the theoretically calculated half-lives of one proton and $\alpha$ emission encouraged us to extend this work to the other proton emitters in this region of $69 \leq Z \leq 83$ and further to the $51 \leq Z \leq 67$ region.

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