Numerical simulation of electrical network left ventricular circulatory system with biological valve motion model

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Abstract. This paper presents a new concentrated parameter model represented by an electrical network model for cardiovascular dynamics that includes an innovative model of biological heart valve dynamics. We obtained the hemodynamic waveform of the heart by solving the equations of state through programming. The results show that using the biological valve motion model can reflect the phenomenon of blood return flow that is not found in the ideal diode model. This model can simulate the biological characteristics of heart valve reasonably and accurately, and it is simple and easy to control.

1. Introduction
The calculation model of cardiovascular system can reflect the relationship between physiological characteristic parameters and hemodynamic variables of human body. As the method and tool in vitro, the numerical model and electric network model can provide technical support for clinical noninvasive diagnosis.[1] Through the study of the simulation model of cardiovascular system, it can provide a feasible theoretical basis for the generation mechanism of normal or pathological hemodynamic waveform [2]. As important accessories to the heart, heart valves function to keep the one way flow direction in the cardiovascular system. Normal function of the valves has a great impact on the pathophysiological processes in the cardiovascular system. In many previous studies, the valve model is assumed that valve can be opened and closed immediately, and it ignores the process of valve closure. Thus, the ideal valve model cannot simulate the valve flow rate during valve closure. This apparently simple cyclic process presents unexpected fluid dynamic complexities in the details. The exact mechanism of heart valve motion has puzzled researchers for decades. To clarify this simple yet complex valve motion mechanism, extensive numerical and experimental studies have been carried out.

In this study, we improved the modeling of heart valves on the basis of the cardiovascular circuit model proposed in reference [3]. Based on the related basic theories of hydrodynamics and electrical network, this paper establishes a cardiovascular system simulation model based on lumped parameters. The simulation model of cardiovascular system is divided into two sub-models: systemic circulation sub-model and cardiac sub-model. We use the state variable analysis method to establish the mathematical expression of the model. The system model was simulated on the Matlab platform, and the simulation results of the pressure and volume of atrium and ventricle and arterial blood flow were obtained [4]. The results were consistent with the physiology of a healthy heart. According to the results, the valve motion model we proposed can reflect the actual blood regurgitation in human blood circulation.
2. The cardiovascular lumped parameter model

2.1. Cardiovascular circulatory system

A fifteen-order lumped parameter circuit model which can reproduce the left ventricle hemodynamics of the heart is shown in Figure 1. In this model, preload and pulmonary circulations are represented by the compliance \( C_{la} \); the mitral valve is represented by resistor \( R_{MVR} (\theta) \) and inductor \( L_{MVR} \); the aortic valve is represented by resistor \( R_{AVR} (\theta) \) and inductor \( L_{AV} \); the aortic compliance is represented by \( C_{ao} \) and afterload is represented by the four-element Windkessel model comprising \( R_s, R_c, L_s, \) and \( C_s \). Left ventricle chamber is composed of Windkessel elements representing hemodynamic parameters. This method can not only reduce the number of parameters to improve computational efficiency, but also obtain global hemodynamics. In this paper, the cardiovascular circulation system was divided into two parts: left heart, systemic circulation. Table 1 lists the various system parameters and their physiological meaning.

![Cardiovascular circuit model](image)

### Table 1. System parameters

| Parameters     | Value   | Physiological meaning          |
|---------------|---------|---------------------------------|
| Resistance    |         |                                 |
| \( R_s \)     | 1.0000  | Systemic Vascular Resistance    |
| \( R_c \)     | 0.0398  | Characteristic Resistance       |
| \( R_{MVR} (\theta) \) | time-varying | Mitral Valve Resistance |
| \( R_{AVR} (\theta) \) | time-varying | Aortic Valve Resistance |
| Inductance    |         |                                 |
| \( L_{MVR} \) | 0.0002  | Mitral Valve Inductance         |
| \( L_{AV} \)  | 0.0004  | Aortic Valve Inductance         |
| \( L_s \)     | 0.0001  | Inertance of blood in Aorta     |
| Compliance    |         |                                 |
| \( C_{la} \)  | 4.4000  | Left Atrial Compliance          |
| \( C_{lv}(t) \) | time-varying | Left Ventricular Compliance |
| \( C_{ao} \)  | 0.0800  | Aortic Compliance               |
| \( C_s \)     | 1.3300  | Systemic Compliance             |

2.2 Left ventricular model

The elastance describes the relationship between the ventricle’s pressure and volume according to the expression

\[
E_v(t) = \frac{P_v(t)}{V_v(t) - V_0}
\]

where \( P_v(t) \) is the left ventricular pressure, \( V_v(t) \) is the left ventricular volume, and \( V_0 \) is a reference volume, which corresponds to the theoretical volume in the ventricle at zero pressure. In our study, we use the mathematical expression

\[
E_v(t) = \frac{P_v(t)}{V_v(t) - V_0}
\]
to approximate the elastance function $E_n(t)$. $E_n(t)$ is described by the following expression

$$E_n(t) = 1.55 \times \left[ \frac{(t_n / 0.7)^{0.9}}{1 + (t_n / 0.7)^{0.9}} \right] \times \frac{1}{1 + (t_n / 1.17)^{2.1}}$$

(3)

where $t_n = t / T_m$, $T_m = 0.2 + 0.15 t_c$, $t_c = 60 / HR$ is the cardiac cycle and $HR$ is heart rate.

Figure 2 shows the waveform of the left ventricular elasticity function $E_n(t)$ during a cardiac cycle.

2.3 Valve model

We assume that the base of the valve is a ring. Figure 3 shows a simplified model of the heart valve. $P_{in}$ and $P_{out}$ are input side pressure and output side pressure respectively. $Q_{value}$ is the blood through the valve. The length of the leaflet is $r$. $\theta$ is the angle at which the leaflet open.

According to the analysis of factors affecting the movement of valve in reference [5], the main factor affecting the movement of valve leaflets is the pressure on both sides.

$$\frac{d^2\theta}{dt^2} = K_1 (P_{in} - P_{out}) \cos(\theta) - K_2 \frac{d\theta}{dt} - K_3 \theta$$

(4)

The opening area changes during valve movement. The resistance of the valve is inversely proportional to the opening area. In our work, we use

$$R(\theta) = k \frac{(1 - \cos \theta_{\text{max}})^2}{(1 - \cos \theta)^2}$$

(5)

where $k = 0.004$, and $\theta_{\text{max}} = 85^\circ$ is the maximum valve opening angle[6].
The constants in Formula (4) and the maximum and minimum valve opening angles are shown in Table 2.

### Table 2. Parameter values used in the heart valve.

| Part       | Parameter | Value | Unit       |
|------------|-----------|-------|------------|
| Mitral     | $K_{mv,1}$ | 1e5   | deg·s⁻²·mmHg⁻¹ |
|            | $K_{mv,2}$ | 5e3   | deg·s⁻²·mmHg⁻¹ |
|            | $K_{mv,3}$ | 5e4   | deg·s⁻²·mmHg⁻¹ |
|            | $K_{mv,mv}$ | 250  | mL·mmHg⁻¹·s⁻¹ |
|            | $\theta_{mv,max}$ | 85  | degrees    |
|            | $\theta_{mv,min}$ | 9.905 | degrees   |
| Aortic     | $K_{av,1}$ | 4e5   | deg·s⁻²·mmHg⁻¹ |
|            | $K_{av,2}$ | 20e3  | deg·s⁻²·mmHg⁻¹ |
|            | $K_{av,3}$ | 16e4  | deg·s⁻²·mmHg⁻¹ |
|            | $K_{av,mv}$ | 250  | mL·mmHg⁻¹·s⁻¹ |
|            | $\theta_{av,max}$ | 85  | degrees    |
|            | $\theta_{av,min}$ | 9.905 | degrees   |
|            | $L_{av}$ | 0.0003 | mmHg·s⁻²·mL⁻¹ |

### 2.4 Equation of state

According to Kirchhoff's law, we write the differential equation of voltage and current and convert them to equations of state [7]:

\[
\begin{align*}
\dot{x}_1 &= -\frac{C_{ls}'(t)}{C_{ls}(t)} x_1 + \frac{x_6 - x_7}{C_{ls}(t)} \\
\dot{x}_2 &= \frac{x_3 - x_2}{R_s * C_{ls}} - \frac{x_8}{C_{ls}} \\
\dot{x}_3 &= \frac{x_2 - x_1}{R_s * C_s} + \frac{x_1}{C_s} \\
\dot{x}_4 &= \frac{x_3 - x_2}{C_{av}} \\
\dot{x}_5 &= \frac{1}{L_a} (x_2 - x_3 - x_5 * R_a) \\
\dot{x}_6 &= \frac{1}{L_{av}'(t)} (x_2 - x_1 - x_6 * R_{av}'(\theta)) \\
\dot{x}_7 &= \frac{1}{L_{av}'(t)} (x_1 - x_4 - x_7 * R_{av}'(\theta)) \\
\dot{x}_8 &= x_9 \\
\dot{x}_9 &= k_{av,1} * (x_2 - x_1) * \cos(x_8) - k_{av,2} * x_9 - k_{av,3} * x_8 \\
\dot{x}_{10} &= x_{11} \\
\dot{x}_{11} &= k_{av,1} * (x_1 - x_4) * \cos(x_{10}) - k_{av,2} * x_{11} - k_{av,3} * x_{10} \\
\end{align*}
\]

(6)

The meanings of each state variable are shown in Table 3.
Table 3. State variable.

| State variable | Unit | Physiological meaning                      |
|----------------|------|--------------------------------------------|
| $x_1$          | mmHg | Left ventricular pressure                  |
| $x_2$          | mmHg | Left atrial pressure                       |
| $x_3$          | mmHg | Arteriole pressure                         |
| $x_4$          | mmHg | Aortic pressure                            |
| $x_5$          | ml / s | Aortic blood flow                         |
| $x_6$          | ml / s | Mitral valve blood flow                    |
| $x_7$          | ml / s | Aortic valve blood flow                    |
| $x_8$          | rad  | Mitral valve leaf angle                    |
| $x_9$          | rad / s | The first derivative of $x_8$             |
| $x_{10}$       | rad  | Aortic valve leaf angle                    |
| $x_{11}$       | rad / s | The first derivative of $x_{10}$          |

2.5 The simulation protocol

We use Matlab R2016a programming to solve the state equations. The ode45 solver was used to solve time-varying differential equations. The time step was set to 0.001 second. All simulation times were set to 15 seconds. The system reaches a stable periodic solution after about 10 seconds. We set the initial values in the iteration process are shown in the Table 4.

Table 4. The initial value of the state variable.

| State variable | Value | Unit |
|----------------|-------|------|
| $x_1$          | 8.2   | mmHg |
| $x_2$          | 7.7   | mmHg |
| $x_3$          | 65    | mmHg |
| $x_4$          | 80    | mmHg |
| $x_5$          | 52    | ml / s |
| $x_6$          | 50    | ml / s |
| $x_7$          | 30    | ml / s |
| $x_8$          | 1     | rad  |
| $x_9$          | 0.5   | rad / s |
| $x_{10}$       | 1     | rad  |
| $x_{11}$       | 0.5   | rad / s |

3. The simulation results

3.1 Hemodynamic waveform

Through MATLAB programming simulation, we get the hemodynamic curve under the healthy physiological condition.
Fig. 4. Simulated hemodynamic waveforms for a normal heart.

From Figure 4 we derive the following hemodynamic parameters. The left ventricular systolic pressure was 111 mmHg and diastolic pressure was 5 mmHg. The aortic systolic pressure is 111 mmHg and the diastolic pressure is 70 mmHg. The dicrotic notch can be shown in the aortic pressure waveform. The range of left atrial pressure is 5-16 mmHg. The positive peak of aortic flow velocity was 646 ml/s, and the peak reverse flow velocity is 136 ml/s. The aortic flow curve can be integrated to obtain the cardiac output over a cardiac cycle. The dicrotic notch is the result of the combined action of valve closure, inertial effect of blood flow, and pulse wave reflection in the arteries. In this simulation, we obtained the cardiac output of 4.9 L/min. A normal person's cardiac output at rest ranges from 4.5 to 5.5 L/min. The volume of the left ventricle varies from 63 to 134 ml per cardiac cycle. These hemodynamic data are consistent with medical textbooks.

Fig. 5. Blood flow waveform of mitral and aortic valves.

In early diastole accompanying the closure of the aortic valve, there are reverse flows with amplitude of 188 ml/s in the aortic valve. Similarly in early systole, accompanying the closure of the mitral valve, there are reverse flows with amplitude of 47 ml/s in the mitral valve.
4. Conclusion

Human cardiovascular system is a closed circulatory system and its independent parts can interact with each other through hemodynamic parameters. Based on the existing model, this paper improved the mitral valve and aortic model in the cardiovascular lumped parameter model. We improved the valve model in the systemic circulation system by replacing the ideal diode with a valve motion model based on differential pressure drive. This is more helpful to reflect the relationship between valve aperture and blood flow and closer to the physiological mechanism of the valve.

In order to verify the feasibility of the model, we use the model to simulate the cardiac hemodynamic waveform in healthy human condition. Compared with previous studies using ideal diode valves, the results reflect the actual aortic regurgitation. The other simulation results are basically consistent with the medical textbooks, which indicates that the model proposed in this paper is feasible to a certain extent. This concentrated parameter model of the blood–leaflet interaction effect is less detailed than three-dimensional distributed parameter studies, but is an advancement over the diode models, and it satisfies the need of the present article to model the overall cardiovascular system. The study of the characteristics of cardiovascular system is of great significance to the understanding of the pathogenesis of cardiovascular diseases and the treatment of diseases.

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