Newly observed exotic doubly charmed meson $T_{cc}^+$

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In this work, we treat the newly observed doubly charmed four-quark state $T_{cc}^+$ as an axial-vector tetraquark with content $ccar{c}ar{d}$, and calculate its spectroscopic parameters and width. The mass and current coupling of the tetraquark $T_{cc}^+$ are found by means of the QCD two-point sum rule method by taking into account quark, gluon and mixed condensates up to dimension 10. The width of the $T_{cc}^+$ is evaluated using partial widths of decay processes $T_{cc}^+ \rightarrow \bar{T}^0\pi^0$ and $T_{cc}^+ \rightarrow T^0_c\pi^+\pi^+$, where $\bar{T} = ccar{c}ar{d}$ and $T^0_c = ccar{c}ar{d}$ are scalar tetraquarks. To compute the partial width of the first process, we apply the QCD three-point sum rule approach and extract numerical value of the strong coupling $g$ that corresponds to the vertex $T_{cc}^+\bar{T}^0$. The width of the second decay is estimated using isospin symmetry and the prediction obtained for the first channel. Our results for the mass $m = (3868 \pm 124)\, \text{MeV}$ and width $\Gamma = (489 \pm 92)\, \text{keV}$ of the tetraquark $T_{cc}^+$ are in a nice agreement with recent measurements of the LHCb collaboration.

I. INTRODUCTION

Exotic mesons $QQ\bar{q}\bar{q}$ containing two heavy quarks $Q$ are already on focus of theoretical studies starting from pioneering works [1–4]. An important problem analyzed in these articles was stability of four-quark states $QQ\bar{q}\bar{q}$ against strong decays, and processes in which they may be discovered. It was shown that tetraquarks $QQ\bar{q}\bar{q}$ might be stable if a ratio $m_Q/m_q$ is sufficiently large. Evidently, tetraquarks composed of a diquark $bb$ and a light antidiquark are first candidates to stable four-quark mesons. Indeed, the isoscalar axial-vector tetraquark $T_{bb\bar{c}\bar{d}}^-$ is expected to be below the two $B$-meson threshold and strong-interaction stable state [4]. In last years, investigations performed using different methods and models confirmed stable nature of the $T_{bb\bar{c}\bar{d}}^-$ [5–8]. Moreover, it became possible to identify other $bb\bar{q}\bar{q}$ tetraquarks as states stable against strong and electromagnetic decays, and calculate their full width via allowed weak transformations [9–13].

The status of particles $bc\bar{q}\bar{q}$ and $cc\bar{q}\bar{q}$ in this sense is not quite clear: they may exist either as bound or resonant states. Nevertheless, these particles are interesting objects for investigations, and deserve detailed analysis. In this paper, we consider the axial-vector tetraquark $ccar{c}ar{d}$, and therefore, in what follows, concentrate on properties of states $cc\bar{q}\bar{q}$. Thus, the axial-vector tetraquark $ccar{c}ar{d}$ was studied using the QCD sum rule method in Ref. [3]. Exotic mesons with a general content $cc\bar{q}\bar{q}$ and quantum numbers $J^P = 0^-, 0^+, 1^-$ and $1^+$ were investigated in Ref. [14] in the framework of the same approach. The axial-vector state $ccar{c}ar{d}$ was modeled as a hadronic molecule composed of the conventional mesons $D^0$ and $D^{*+}$ as well [15,16].

Recent intensive analyses of heavy tetraquarks were triggered by observation of doubly charmed baryon $\Xi_{cc}^{*+} = cc\bar{u}$ [17]. Parameters of this particle were utilized as input information in a phenomenological model to evaluate mass of the axial-vector tetraquark $T_{cc\bar{c}\bar{d}}^+$ [6]. It was demonstrated that $T_{cc\bar{c}\bar{d}}^+$ is unstable particle and can decay to $D^0D^{*+}$ mesons. Similar conclusions about $T_{cc\bar{c}\bar{d}}^+$ were also made in Refs. [7,18–21]. Contrary, existence of stable axial-vector tetraquark was predicted in Ref. [22], in which the authors used a constituent quark model and found that the mass of $T_{cc\bar{c}\bar{d}}^+$ is 23 MeV below the two-meson threshold. In accordance with lattice simulations the mass of the axial-vector state $cc\bar{c}\bar{d}$ is below the two-meson threshold and this gap is equal to $(23 \pm 11)\, \text{MeV}$ [23].

The pseudoscalar and scalar exotic mesons $cc\bar{c}\bar{d}$ were considered in a detailed form in our paper [24]. Our studies proved that these tetraquarks are unstable resonances, and decay strongly to ordinary mesons. To compute widths of these particles, we utilized their kinematically allowed decays to $D^0 D^{*+}$ (2007)$^0$, $D^0 D^{*+}$ (2010)$^+$, and $D^0 D^{*+}$ meson pairs. Another subclass of doubly charmed tetraquarks contains particles and bear two units of electric charge. Spectroscopic parameters and widths of the such pseudoscalar states $cc\bar{s}\bar{s}$ and $cc\bar{d}\bar{d}$ were computed in Ref. [25].

Investigation of doubly charmed exotic mesons is not limited by calculation of their parameters using different methods such as the chiral quark model, dynamical and relativistic quark models: Production of these particles in ion, proton-proton and electron-positron collisions, in $B_c$ and $\Xi_{bc}$ decays was investigated as well (see Ref. [26], and references in [24,25]).

First experimental information on doubly charmed exotc meson was announced recently by the LHCb collaboration [27,28]. The axial-vector exotic meson $T_{cc}^+$ with content $cc\bar{c}\bar{d}$ was discovered in $D^0 D^{*+}$ invariant mass distribution as a narrow peak. Its mass is below, but very close to the $D^0 D^{*+}$ (2010)$^+$ threshold. The two-meson $D^0 D^{*+}$ threshold amounts to 3875.1 MeV, whereas $T_{cc}^+$
has the mass
\[ m_{\text{exp}} = 3875.1 \text{ MeV} + \delta m_{\text{exp}}, \]
where \( \delta m_{\text{exp}} \) was measured to be equal to
\[ \delta m_{\text{exp}} = -273 \pm 61 \pm 5^{+11}_{-14} \text{ KeV}. \]
The tetraquark \( T_{cc}^+ \) has the width
\[ \Gamma = 410 \pm 165 \pm 43^{+18}_{-38} \text{ KeV}, \]
and is longest living exotic meson discovered till now.

Because \( T_{cc}^+ \) is very narrow state, its decay channels attracted close attention of researches \([29,31]\). In these papers relevant problems were addressed using different methods and models.

In the present article, we treat \( T_{cc}^+ \) as a doubly charmed axial-vector diquark-antidiquark state with quark content \( cc\bar{c}\bar{d} \) and calculate its spectroscopic parameters, as well as evaluate width of this resonance. Calculation of the mass \( m \) and current coupling \( f \) of the \( T_{cc}^+ \) are carried out by means of the QCD two-point sum rule method, which is one of powerful nonperturbative approaches to evaluate parameters of conventional hadrons \([32,33]\). But it can be applied to extract masses and couplings of multiquark particles \([34]\), which was successfully demonstrated to explore numerous tetraquarks (see, for example, Ref. \[33\]).

The four-quark exotic meson \( T_{cc}^+ \) was discovered in the \( D^0\bar{D}^0\pi^+ \) mass distribution, and hence decays strongly to these mesons. One of possible and most discussed ways to explain this transformation is the chain of decays \( T_{cc}^+ \to D^0\bar{D}^0 \to D^0\bar{D}^0\pi^+ \). But the process \( T_{cc}^+ \to D^0\bar{D}^0 \) is kinematically forbidden, because the mass of \( T_{cc}^+ \) is smaller than \( D^0\bar{D}^0 \) threshold. Alternatively, production of \( D^0\bar{D}^0\pi \) can proceed through decay of \( T_{cc}^+ \) to a scalar tetraquark \( T_{0,cc}^\pi \) and \( \pi^+ \) followed by the process \( T_{0,cc}^\pi \to D^0\bar{D}^0 \). Another decay channel of \( T_{cc}^+ \) is \( T_{cc}^+ \to T_{cc}^0 \to D^0\bar{D}^0\pi^0 \), where \( T_{cc}^0 \) is again a scalar tetraquark. In other words, we consider allowed decays of \( T_{cc}^+ \) to scalar tetraquarks as a main mechanism for transformation of \( T_{cc}^+ \). Partial widths of these decays to scalar tetraquarks \( T_{0,cc}^\pi \) and \( T_{cc}^0 \) can be employed to evaluate the full width of \( T_{cc}^+ \). Of course, all these arguments are valid only if masses of intermediate scalar tetraquarks meet necessary kinematical restrictions. The mass and coupling of the scalar tetraquark \( T \) and width of its decay to a pair of mesons \( D^0\bar{D}^0 \) were calculated in our work \([24]\). It turns out that parameters of \( \tilde{T} \) satisfy required constraints provided one takes into account theoretical uncertainties of the sum rule computations. In the present article, we concentrate on the decay \( T_{cc}^+ \to T\pi^0 \) and evaluate its partial width. To this end, we calculate the strong coupling \( g \), which corresponds to the vertex \( T_{cc}^+\tilde{T}\pi^0 \), and extract its numerical value applying the QCD three-point sum rule approach. The partial width of the process \( T_{cc}^+ \to T_{cc}^0\pi^+ \) can be estimated using the isospin symmetry and a result obtained for the first decay channel.

This paper is organized in the following way: In Sec. \[II\] we calculate the mass and current coupling of the tetraquark \( T_{cc}^+ \) in the framework of the QCD two-point sum rule method by taking into account various vacuum condensates up to dimension 10. Section \[III\] is devoted to analysis of the decay channel \( T_{cc}^+ \to T\pi^0 \), to calculation of the strong coupling \( g \) and partial width of this process. The width of the second channel \( T_{cc}^+ \to T_{cc}^0\pi^+ \) and full width of the tetraquark \( T_{cc}^+ \) are evaluated in this section as well. We reserve Sec. \[IV\] for discussion and concluding notes.

\[ \text{II. MASS AND CURRENT COUPLING OF } T_{cc}^+ \]

We consider \( T_{cc}^+ \) as an axial-vector diquark-antidiquark state composed of axial-vector diquark \( c^T C\gamma_\mu c \) and light scalar antidiquark \( \pi\gamma_5 C\bar{d}_T^\dagger \). The interpolating current for such state is given by the expression
\[ J_\mu(x) = c^T_0(x)C\gamma_\mu c_0(x)\pi_0(x)\gamma_5 C\bar{d}_T^\dagger(x), \]
where \( a \) and \( b \) are color indices and \( C \) is charge-conjugation matrix. The current \( J_\mu \) belongs to the \([\overline{3}_c]c_c \otimes [\overline{3}_c]\overline{c}_c \) representation of the color group \( SU_c(3) \), and should have lowest mass in its class \([36]\).

The sum rules for the mass \( m \) and current coupling \( f \) of the tetraquark \( T_{cc}^+ \) can be obtained from analysis of the two-point correlation function \( \Pi_{\mu\nu}(p) \) given by the formula
\[ \Pi_{\mu\nu}(p) = i \int d^3x e^{ipx} \langle 0 | \mathcal{T} \{ J_\mu(x) J_\nu^\dagger(x) \} | 0 \rangle. \]

To extract required sum rules and determine their phenomenological side, we express the correlation function \( \Pi_{\mu\nu}(p) \) in terms of the tetraquarks’ physical parameter. Because \( T_{cc}^+ \) has lowest mass in the class of axial-vector states with the same quark content, we treat it as ground-state particle, and keep explicitly only the first term in \( \Pi_{\mu\nu}^{\text{Phys}}(p) \)
\[ \Pi_{\mu\nu}^{\text{Phys}}(p) = \frac{\langle 0 | J_\mu | T_{cc}^+(p,\epsilon) \rangle \langle T_{cc}^+(p,\epsilon) | J_\nu^\dagger | 0 \rangle}{m^2 - p^2} + \cdots. \]

The \( \Pi_{\mu\nu}^{\text{Phys}}(p) \) is derived by saturating the correlation function \([5]\) with a complete set of states with quantum numbers \( J^P = 1^+ \) and performing the integration over \( x \). The dots in Eq. \([6]\) stand for contributions to \( \Pi_{\mu\nu}^{\text{Phys}}(p) \) arising from higher resonances and continuum states.

The function \( \Pi_{\mu\nu}^{\text{Phys}}(p) \) can be rewritten using the matrix element
\[ \langle 0 | J_\mu | T_{cc}^+(p,\epsilon) \rangle = fm_\epsilon \mu, \]
where $\epsilon_\mu$ is the polarization vector of the state $T^+$. It is easy to show that in terms of $m$ and $f$ the function $\Pi_{\mu\nu}^{\text{phys}}(p)$ takes the following form

$$\Pi_{\mu\nu}^{\text{phys}}(p) = \frac{m^2 f^2}{m^2 - p^2} \left(-g_{\mu\nu} + \frac{p_\mu p_\nu}{m^2}\right) + \cdots.$$  

(8)

The QCD side of the sum rules $\Pi_{\mu\nu}^{\text{OPE}}(p)$ should be computed in the operator product expansion (OPE) with certain accuracy. To get $\Pi_{\mu\nu}^{\text{OPE}}(p)$, we insert into Eq. \[5\] the interpolating current $J^\mu(x)$, and contract relevant heavy and light quark fields. After these manipulations, we find

$$\Pi_{\mu\nu}^{\text{OPE}}(p) = i \int d^4xe^{ip\cdot x} \left\{ \text{Tr} \left[ \gamma_\mu \tilde{S}_d^{ab}(-x) \right] \gamma_\nu S_u^{a'b}(x) \right\}.$$  

(9)

In Eq. \[9\] $S_d^{ab}(x)$ and $S_u^{a'b}(x)$ are the $c$ and $q(u,d)$-quark propagators: their explicit expressions can be found in Ref. \[35\]. Here, we also use the shorthand notation

$$\tilde{S}_{c(q)}(x) = CS_{c(q)}^{T}(x)C.$$  

(10)

The QCD sum rules can be extracted by employing the same Lorentz structures both in $\Pi_{\mu\nu}^{\text{phys}}(p)$ and $\Pi_{\mu\nu}^{\text{OPE}}(p)$. The structures proportional to $g_{\mu\nu}$ are appropriate for our purposes, because they receive contributions only from spin-1 particles. We denote corresponding invariant amplitudes by $\Pi_{\mu\nu}^{\text{phys}}(p^2)$ and $\Pi_{\mu\nu}^{\text{OPE}}(p^2)$, respectively.

Another problem to be solved is suppression of contributions coming from the higher resonances and continuum states. To this end, one should apply the Borel transformation to both sides of the sum rule equality. At the next stage using the quark-hadron duality hypothesis, one has to subtract higher resonance and continuum terms from the physical side of the equality. As a result, the sum rule equality depends on the Borel $M^2$ and continuum threshold $s_0$ parameters.

The Borel transformation of $\Pi_{\mu\nu}^{\text{phys}}(p^2)$ is trivial. The Borel transformed and continuum subtracted invariant amplitude $\Pi_{\mu\nu}^{\text{OPE}}(p^2)$ has rather complicated form

$$\Pi(M^2, s_0) = \int_{4m^2}^{s_0} ds \rho^{\text{OPE}}(s)e^{-s/M^2} + \Pi(M^2).$$  

(11)

Here, $\rho^{\text{OPE}}(s)$ is the two-point spectral density, whereas second component of the invariant amplitude $\Pi(M^2)$ includes nonperturbative contributions calculated directly from $\Pi_{\mu\nu}^{\text{OPE}}(p)$. We compute $\Pi(M^2, s_0)$ by taking into account nonperturbative terms up to dimension 10.

Then, the sum rules for $m$ and $f$ read

$$m^2 = \frac{\Pi(M^2, s_0)}{\Pi(M^2, s_0)}.$$  

(12)

and

$$f^2 = \frac{\epsilon m^2/M^2}{m^2 - \Pi(M^2, s_0)},$$  

(13)

where $\Pi(M^2, s_0) = d/d(-1/M^2)\Pi(M^2, s_0)$.

The expressions \[12\] and \[13\] contain various quark, gluon and mixed condensates, which are universal parameters:

$$\langle \bar{q}q \rangle = -(0.24 \pm 0.01)^3 \text{ GeV}^3,$$

$$\langle \bar{q}gGq \rangle = m_0^2 \langle \bar{q}q \rangle,$$

$$m_0^2 = (0.8 \pm 0.1) \text{ GeV}^2,$$

$$\langle \alpha_s G^2 \rangle = (0.012 \pm 0.004) \text{ GeV}^4,$$

$$\langle q_i^G q_j^G \rangle = (0.57 \pm 0.29) \text{ GeV}^6,$$

$$m_c = 1.275 \pm 0.025 \text{ GeV}.$$  

(14)

The mass of the $c$ quark is also included into this list. The Borel and continuum threshold parameters $M^2$ and $s_0$ are auxiliary quantities of computations and their choice should satisfy constraints imposed on the pole contribution (PC) and convergence of OPE. A minimum sensitivity of the extracted quantities to the Borel parameter $M^2$ is also among important requirements. The maximum value of $M^2$ can be obtained from the restriction on PC

$$\text{PC} = \frac{\Pi(M^2, s_0)}{\Pi(M^2, \infty)}.$$  

(15)

In the present work, we apply the constraint PC $\geq 0.2$, which is usual for the multiquark hadrons. The low limit of the working region for the Borel parameter is fixed from convergence of the operator product expansion, i.e., from analysis of the ratio

$$R(M^2) = \frac{\Pi_{\text{DimN}}(M^2, s_0)}{\Pi(M^2, s_0)},$$  

(16)

where $\Pi_{\text{DimN}}(M^2, s_0)$ denotes contribution of the last three terms in OPE, in other words DimN = Dim(8 + 9 + 10). The convergence of the operator product expansion at minimum of $M^2$ is ensured by the requirement $R(M^2) \leq 0.01$.

Our analysis demonstrates that working regions for $M^2$ and $s_0$

$$M^2 \in [4, 6] \text{ GeV}^2, \ s_0 \in [19.5, 21.5] \text{ GeV}^2,$$  

(17)

meet all aforementioned constraints. Thus, in these regions PC changes on average within limits

$$0.62 \leq \text{PC} \leq 0.20.$$  

(18)

At the minimum $M^2 = 4 \text{ GeV}^2$ contributions to $\Pi(M^2, s_0)$ coming from last three terms in OPE do not exceed 1% of the full result.

Central values of the mass $m$ and coupling $f$ are evaluated by averaging results for these parameters over working regions \[17\]. Obtained mean values correspond to
predictions of the sum rules approximately at middle point of these regions, i.e., to results at \( M^2 = 5 \text{ GeV}^2 \) and \( s_0 = 20.4 \text{ GeV}^2 \). At this point the pole contribution is \( \text{PC} \approx 0.55 \), which guarantees the ground-state nature of \( T_{cc}^+ \). Our results for \( m \) and \( f \) are

\[
\begin{align*}
m &= (3868 \pm 124) \text{ MeV}, \\
 f &= (5.03 \pm 0.79) \times 10^{-3} \text{ GeV}^4.
\end{align*}
\]

In general, quantities extracted from the sum rules should not depend on the choice of the parameter \( M^2 \). But, in real calculations, one observes a residual dependence of \( m \) and \( f \) on \( M^2 \). In Fig. 1 we depict the mass \( m \) of the tetraquark as a function of \( M^2 \). It is seen, that the region for \( M^2 \) shown in this figure can be considered as a relatively stable plateau, where parameters of \( T_{cc}^+ \) can be evaluated. This dependence on \( M^2 \) allows us also to estimate uncertainties generated by the sum rule calculations. The second source of theoretical errors is a choice of the continuum threshold parameter \( s_0 \). The working window for \( s_0 \) should satisfy limits arising from dominance of PC and convergence of OPE as well. Additionally, \( s_0 \) carries physical information about first excitation of the tetraquark \( T_{cc}^+ \). The self-consistent analysis implies that \( \sqrt{s_0} \) is smaller than mass of such state. In the case under discussion, the mass gap is \( \sqrt{s_0} - m \approx 650 \text{ MeV} \) which can be considered as a reasonable estimate for exotic mesons containing two heavy \( c \) quarks.

Effects connected with a choice of parameters \( M^2 \) and \( s_0 \) are two main sources of theoretical uncertainties in sum rule computations. In the case of the mass \( m \) they equal to \( \pm 3.2\% \), whereas for the coupling \( f \) ambiguities are \( \pm 16\% \) of the full result. Theoretical uncertainties for \( f \) are larger than that for the mass, but they do not exceed accepted limits.

\[
\begin{align*}
1 &= \frac{1}{m^2 - p^2 - i\sqrt{m^2 - p^2} \Gamma(p)}.
\end{align*}
\]

where \( \Gamma(p) \) is the finite width of the tetraquark generated by two-meson states. Investigations demonstrate that these effects rescale the original coupling \( f \) and leave stable the mass \( m \) of the tetraquark \( T_{cc}^+ \). Detailed analyses proved that two-meson contributions are small even for tetraquarks with width around a hundred MeV \( T_{cc}^+ \). In the case under discussion, the width of the \( T_{cc}^+ \) is less than one MeV, therefore two-meson effects can be safely neglected.

**III. WIDTH OF THE TETRAQUARK \( T_{cc}^+ \)**

As was noted above, \( T_{cc}^+ \) was discovered in \( D^0 D^0 \pi^+ \) mass distribution, and hence, can decay strongly to these mesons. This process may run through \( T_{cc}^+ \rightarrow D^0 D^+ \) followed by the decay \( D^+ \rightarrow D^0 \pi^+ \). The experimental measurements showed, however, that the mass of \( T_{cc}^+ \) is not enough to trigger this mechanism. In this situation, the final state \( D^0 D^0 \pi^+ \) can be achieved due to production of the intermediate scalar tetraquark \( T_{cc}^+ \). But the \( T_{cc}^+ \) may decay to mesons \( D^0 D^+ \pi^0 \), as well.

In this section, we consider the decay \( T_{cc}^+ \rightarrow \bar{T} \pi^0 \), and calculate its partial width. The reason is that parameters of the tetraquark \( T \) are known and were calculated in our paper \( [24] \)

\[
\begin{align*}
m_{\bar{T}} &= (3845 \pm 175) \text{ MeV}, \\
f_{\bar{T}} &= (1.16 \pm 0.26) \times 10^{-2} \text{ GeV}^4.
\end{align*}
\]

To realize the process \( T_{cc}^+ \rightarrow \bar{T} \pi^0 \rightarrow D^0 D^+ \pi^0 \) the mass of \( \bar{T} \) should satisfy the constraints \( m_{D^0 D^+} < m_{\bar{T}} \) \( < m - m_\pi \). In other words, \( \bar{T} \) has to be heavier than \( m_{D^0 D^+} \approx 3735 \text{ MeV} \). In this section, we use for the
mass of the $T^+_c$ experimental value $m \approx 3875$ MeV, therefore $\bar{T}$ should be lighter than 3740 MeV. It is seen that the narrow region allowed for the mass of the tetraquark $\bar{T}$ between 3735 MeV $< m_{\bar{T}} <$ 3740 MeV is consistent with prediction for $m_{\bar{T}}$ from Eq. (21). For our computations, we fix $m_{\bar{T}} = 3736$ MeV and use it in what follows. Accordingly, for the current coupling, we employ $f_{\bar{T}} = 1.42 \times 10^{-2}$ GeV$^3$.

Here, some comments concerning strong decay channels of $\bar{T}$ are necessary. They were analyzed in Ref. [24], in which it was demonstrated that the exotic state $\bar{T}$ in $S$-wave decays to a pair of the mesons $D^0 D^+$, but its $P$-wave channels require a master particle $\bar{T}$ to be considerably heavier than 3736 MeV, which is not the case. Stated differently, only open strong decay channel of $\bar{T}$ is the process $\bar{T} \rightarrow D^0 D^+$. Hence, width of the tetraquark $\bar{T}$ is determined by this decay and to equal

$$\Gamma(\bar{T} \rightarrow D^0 D^+) = (12.4 \pm 3.1) \text{ MeV}.$$  

(22)

Below we investigate the decay $T^+_c \rightarrow \bar{T} \pi^0$ and calculate the strong coupling corresponding to the vertex $T^+_c \bar{T} \pi^0$. To derive the QCD three-point sum rule for this coupling and extract its numerical value, we start from analysis of the correlation function

$$\Pi_\mu(p, p') = i^2 \int d^4 x d^4 y e^{i(p' \cdot y - px)} \langle 0 \{ \pi \bar{T} \} (y) \rangle = J_\mu(0) J^\dagger_\mu(x) \langle 0 \rangle,$$

where $J_\mu(x), \bar{T}(x)$ and $J_\pi(x)$ are the interpolating currents for the tetraquarks $T^+_c$ and $\bar{T}$ and the pion $\pi^0$, respectively. The $J_\mu(x)$ is given by Eq. (4), for two remaining currents, we employ

$$J_\pi(x) = \epsilon^{abc} \epsilon_{abc} C^{\gamma}_{\pi c} (x) \langle \pi d(x) \gamma^\mu \nabla^T_{\pi c} (x) \rangle,$$

and

$$J_\mu(x) = \frac{1}{\sqrt{2}} \epsilon^{abc} \epsilon_{abc} C^{\gamma}_{\mu c}(x) \langle \bar{T} d(x) i \gamma_\mu d(x) \rangle.$$  

(24)

Here, $\epsilon^{abc} \epsilon_{abc}$ and $a, b, c, d, e$ and $i$ are color indices. The 4-momenta of the tetraquarks $T^+_c$ and $\bar{T}$ are denoted by $p$ and $p'$, as a result, the momentum of the pion $\pi^0$ is $q = p - p'$.

We follow the usual recipes of the sum rule method and, first calculate the correlation function $\Pi_\mu(p, p')$ using phenomenological parameters of the involved particles. Separating the ground-state contribution to the correlation function [25] from effects of higher resonances and continuum states, for physical state of the sum rule $\Pi^\text{phys}_\mu(p, p')$, we obtain

$$\Pi^\text{phys}_\mu(p, p') = \frac{\langle 0 \{ \pi \bar{T} \} (p') \rangle \langle \pi 0 \rangle}{(p^2 - m_\pi^2)(q^2 - m_\pi^2)} \langle \bar{T} (p, \epsilon) \rangle J_\mu(0) J^\dagger_\mu(x) \langle 0 \rangle \langle \pi 0 \rangle (p^2 - m^2) + \cdots,$$

(25)

with $m_\pi$ being the mass of the pion.

To simplify further the function $\Pi^\text{phys}_\mu(p, p')$, it is convenient to use matrix elements of the tetraquarks and pion. To this end, we introduce the matrix elements

$$\langle 0 \{ \pi \bar{T} \} (p') \rangle = m_\pi f_{\pi \bar{T}},$$

$$\langle \pi 0 \rangle (q) = \frac{1}{\sqrt{2}} f_{\pi \mu_\pi}, \quad \mu_\pi = \frac{2(q\bar{q})}{f_\pi^2},$$

where $f_\pi$ and $(\bar{q}q)$ are the pion decay constant and the quark vacuum condensate, respectively. The matrix element of the $d$ quark field $d_\pi \gamma_5 d$ is given by the similar expression.

We model $\langle \pi 0 \rangle (p') \langle T^+_c(p, \epsilon) \rangle$ in the form

$$\langle \pi 0 \rangle (p') \langle T^+_c(p, \epsilon) \rangle = g(q^2) p'_\mu \epsilon^\mu,$$

and denote by $g(q^2)$ the strong coupling at the vertex $T^+_c \bar{T} \pi^0$. Then, it is easy to find that

$$\Pi^\text{phys}_\mu(p, p') = g(q^2) f_{\pi \mu_\pi} f_{\pi \bar{T}} f_m \frac{m}{(p^2 - m^2)(q^2 - m_\pi^2)}$$

$$\times \left( \frac{m^2 + m_\pi^2 - q^2}{2m^2} p_\mu - p'_\mu \right) + \cdots.$$  

(28)

The double Borel transformation of the correlation function over variables $p^2$ and $q^2$ is given by the following formula

$$\Pi^\text{phys}_\mu(p, p') = g(q^2) f_{\pi \mu_\pi} f_{\pi \bar{T}} f_m \frac{m}{(p^2 - m^2)(q^2 - m_\pi^2)}$$

$$\times \left( \frac{m^2 + m_\pi^2 - q^2}{2m^2} p_\mu - p'_\mu \right) + \cdots.$$  

(29)

The function $\Pi^\text{phys}_\mu(p, p')$ contains Lorentz structures proportional to $p_\mu$ and $p'_\mu$. We work with the invariant amplitude $\Pi^\text{phys}_\mu(p^2, p'^2, q^2)$ corresponding to the structure proportional to $p_\mu$. The Borel transform of this amplitude forms the phenomenological side of the sum rule.

To find the QCD side of the three-point sum rule, we compute $\Pi_\mu(p, p')$ in terms of the quark propagators and get

$$\Pi^\text{OPE}_\mu(p, p') = i^3 \sqrt{2} \int d^4 x d^4 y e^{i\epsilon^\dagger \epsilon <(p' - px)}$$

$$\times \text{Tr} \left[ \gamma_5 S_u^{x}(-y) \gamma_\mu S_u^{y} \right] \langle \bar{y} \rangle \langle \bar{y} \rangle$$

$$\times \left\{ \text{Tr} \left[ \gamma_\mu S_{bb}^{x}(-y) \gamma_\nu S_{bb}^{y} \right] \langle \bar{y} \rangle \langle \bar{y} \rangle \right\} - \text{Tr} \left[ \gamma_\mu S_{cc}^{x}(-y) \gamma_\nu S_{cc}^{y} \right] \langle \bar{y} \rangle \langle \bar{y} \rangle \right\}.$$  

(30)

where $\epsilon^\dagger = \epsilon_{ab'} \epsilon_{ac} \epsilon_{nd} \epsilon_{de'}$. In deriving of Eq. (30), we have taken into account that in the chiral limit $m_u = m_d$ adopted in the present article, both components of the pion interpolating current give the same results: In Eq. (30), $\Pi^\text{OPE}_\mu(p, p')$ is twice of the $\pi \gamma_5 u$ contribution.
The correlation function $\Pi_{\mu}^{\text{OPE}}(p, p')$ is calculated with dimension-6 accuracy, and has the same Lorentz structures as $\Pi_{\mu}^{\text{phys}}(p, p')$. The double Borel transformation $B\Pi_{\mu}^{\text{OPE}}(p^2, p'^2, q^2)$, where $\Pi_{\mu}^{\text{OPE}}(p^2, p'^2, q^2)$ is the invariant amplitude that corresponds to the term proportional to $p_{\mu}$, constitutes the second part of the sum rule. By equating $B\Pi_{\mu}^{\text{OPE}}(p^2, p'^2, q^2)$ and Borel transformation of $\Pi_{\mu}^{\text{phys}}(p^2, p'^2, q^2)$, and performing continuum subtraction we find the sum rule for the coupling $g(q^2)$.

The Borel transformed and subtracted amplitude $\Pi_{\mu}^{\text{OPE}}(p^2, p'^2, q^2)$ can be expressed using the spectral density $\tilde{\rho}(s, s', q^2)$ which is proportional to a relevant imaginary part of $\Pi_{\mu}(s, s')$

$$\Pi(M^2, s_0, q^2) = \int_{4m^2_\pi}^{s_0} ds \int_{4m^2_\pi}^{s'_0} ds' \tilde{\rho}(s, s', q^2) \times e^{-s/M^2_\pi} e^{-s'/M^2_\pi}, \quad \text{(31)}$$

where $M^2 = (M_1^2, M_2^2)$ and $s_0 = (s_0, s'_0)$ are the Borel and continuum threshold parameters, respectively. Then, the sum rule for $g(q^2)$ reads

$$g(q^2) = \frac{2m^2}{f_\pi f_{\pi T} f_m m^2 + m^2_\pi - q^2} \times e^{-m^2/2M^2_\pi^2} QCD$$

The coupling $g(q^2)$ is a function of $q^2$ and depends on the Borel and continuum threshold parameters, which are not shown in Eq. (32) as arguments of $g$. We also introduce a new variable $Q^2 = -q^2$ and denote the obtained function as $g(Q^2)$.

The sum rule Eq. (32) contains masses and couplings of the tetraquarks $T_{cc}^+$ and $\bar{T}$, as well as the mass and decay constant of the pion $\pi^0$. The spectroscopic parameters of the $T_{cc}^+$ have been calculated in the present work and presented in Eq. (19). As the mass and coupling of the tetraquark $T$, we use $m_{T} = 3736$ MeV and $f_{T} = 1.42 \times 10^{-2}$ GeV$^2$, respectively. For the mass and decay constant of the pion, we employ the values: $m_\pi = 134.98$ MeV and $f_\pi = 131$ MeV.

Apart from these spectroscopic parameters, for numerical analysis of $g(Q^2)$ one also needs to fix $M^2$ and $s_0$. The constraints imposed on these auxiliary parameters are usual for sum rule computations and have been discussed above. The regions for $M^2$ and $s_0$ correspond to the $T_{cc}^+$ channel, and coincide with the working windows for these parameters determined in the mass calculations [see, Eq. (17)]. The pair of parameters $(M^2_1, s'_0)$ for the $T_{cc}^+$ channel are chosen within the limits

$$M^2_1 \in [4, 5] \text{ GeV}^2, \quad s'_0 \in [19, 20] \text{ GeV}^2. \quad \text{(33)}$$

The extracted strong coupling $g(Q^2)$ depends on $M^2$ and $s_0$: the working intervals for these parameters are chosen in such a way that to minimize these uncertainties.

The width of the decay under analysis should be computed using the strong coupling at the pion’s mass shell $Q^2 = m^2_\pi$, which is not accessible to the sum rule calculations. We solve this problem by introducing a fit function $G(Q^2)$ that for the momenta $Q^2 > 0$ coincides with QCD sum rule’s predictions, but can be extrapolated to the region of $Q^2 < 0$ to find $g(-m^2_\pi)$. To construct the function $G(Q^2)$, we employ the analytic form

$$G(Q^2) = G_0 \exp \left[ c_1 \frac{Q^2}{m^2} + c_2 \left( \frac{Q^2}{m^2} \right)^2 \right], \quad \text{(34)}$$

where $G_0$, $c_1$ and $c_2$ are fitting parameters. Numerical analysis allows us to fix $G_0 = 72.75$, $c_1 = -1.84$, and $c_2 = -0.03$. In Fig. 2 we depict the sum rule predictions for $g(Q^2)$ and also provide $G(Q^2)$: an agreement between them is evident.

At the pion mass shell $Q^2 = -m^2_\pi$ this function leads to prediction

$$g = G(-m^2_\pi) = 73 \pm 11. \quad \text{(35)}$$

The width of decay $T_{cc}^+ \to \bar{T}\pi^0$ is given by the simple expression

$$\Gamma \left[ T_{cc}^+ \to \bar{T}\pi^0 \right] = \frac{g^2 \lambda^3 (m, m_{\bar{T}}, m_\pi)}{24\pi m^2}, \quad \text{(36)}$$

where

$$\lambda(a, b, c) = \frac{1}{2a} \sqrt{a^4 + b^4 + c^4 - 2(a^2b^2 + a^2c^2 + b^2c^2)}. \quad \text{(37)}$$

Using the strong coupling from Eq. (35), it is not difficult to evaluate width of the decay $T_{cc}^+ \to \bar{T}\pi^0$

$$\Gamma \left[ T_{cc}^+ \to \bar{T}\pi^0 \right] = (163 \pm 41) \text{ keV}. \quad \text{(38)}$$

FIG. 2: The sum rule predictions and fit function for the strong coupling $g(Q^2)$. The red diamond shows the point $Q^2 = -m^2_\pi$. The second process $T_{cc}^+ \to T_{cc}^{0\pi} \pi^+$ can be considered via the same manner. Due to isospin symmetry, the strong coupling $g$ corresponding to the vertex
\( T_{cc}^+T_{cc}^0\pi^+ \) is connected with \( g \) through the simple relation \([41]\)

\[
|g| = \sqrt{2}g. \tag{39}
\]

As a result, the width of the second decay channel of the state \( T_{cc}^+ \) is given approximately by the expression

\[
\Gamma \left[ T_{cc}^+ \to T_{cc}^0\overline{\pi}\pi^+ \right] \approx (326 \pm 82) \text{ keV}. \tag{40}
\]

Then for the full width of the exotic meson \( T_{cc}^+ \), we get

\[
\Gamma = (489 \pm 92) \text{ keV}, \tag{41}
\]

in a nice agreement with the result of the LHCb collaboration.

IV. DISCUSSION AND CONCLUDING NOTES

In the present work, we have calculated spectroscopic parameters and width of the doubly charmed state \( T_{cc}^+ \) observed recently by the LHCb collaboration. Properties of this state were studied in the context of various methods and presented in numerous publications. Relevant information is collected in Table I. Our result for the mass gap (for all predictions, we consider only their central values) \( \delta m = -7 \text{ MeV} \) is qualitatively comparable with predictions of Refs. \([22, 23]\). The table contains also prediction for \( m \) found in the hadronic molecule model \([15]\), which is very close to experimental datum. All other results are above the two-meson \( D^0D^+ \) threshold, which are excluded by experimental data.

We have investigated strong decay channels \( T_{cc}^+ \to \overline{T}\pi^0 \) and \( T_{cc}^+ \to T_{cc}^0\overline{\pi}\pi^+ \) and by this way calculated full width of the state \( T_{cc}^+ \). Calculation of the partial width of the process \( T_{cc}^+ \to \overline{T}\pi^0 \) has been carried out in the framework of the QCD three-point sum rule method. The partial width of the second channel has been evaluated using isospin symmetry and our result for the first process.

![Table I: Theoretical predictions for the mass (or for a mass gap from the two-meson threshold) of the axial-vector state \( T_{cc}^+ \) obtained using different models and methods.](image)

| Works | \( m \) or \( \delta m \) (in units of MeV) |
|-------|----------------------------------|
| This work | 3868 ± 124 |
| F. S. Navarra et al. \([5]\) | 4000 ± 200 |
| J. M. Dias et al. \([15]\) | 3872.2 ± 39.5 |
| M. Karliner, and J. L. Rosner \([6]\) | 3882 ± 12 |
| E. J. Eichten, and C. Quigg \([7]\) | 3978 |
| Z. G. Wang, and Z. H. Yan \([19]\) | 3900 ± 90 |
| E. Braaten et al. \([20]\) | 3947 ± 11 |
| J. B Cheng et al. \([21]\) | 3929.3 |
| Q. Meng et al. \([22]\) | \( \delta m = -23 \) |
| P. Junnarkar et al. \([23]\) | \( \delta m = -23 \pm 11 \) |

very nice agreement of the \( T_{cc}^+ \) tetraquark’s full width obtained in the present article with the LHCb data considerably strengthen arguments in favor of the diquark-antidiquark nature of the \( T_{cc}^+ \).

But there are still open problems that should be clarified to make firm conclusions about inner structure of the \( T_{cc}^+ \). Main question to be addressed is a molecule model, which was employed in Ref. \([15]\) to estimate the mass of the molecule \( DD^* \). Studies there were performed in the context of the QCD spectral sum rule method by including into analysis vacuum condensates up to dimension-6. The mass of the molecule \( DD^* \) was found below the two-meson threshold \( DD^* \), which means that it is bound state and cannot fall apart to a pair of conventional mesons \( D + D^* \). Due to closeness of the \( DD^* \) molecule’s mass to LHCb data and the same quark content, it may be interpreted as doubly charmed state \( T_{cc}^+ \). Problem is that, in Ref. \([15]\) width of the molecule \( DD^* \) was not computed.

The masses of the tetraquark \( T_{cc}^+ \) and molecule \( DD^* \) have been extracted with theoretical uncertainties, which are unavoidable feature of all sum rule computations. These uncertainties can be reduced only up to some limits by including into analysis higher dimensional condensates. Therefore based only on this information it is impossible to distinguish molecule and tetraquark states from each another. Only way to make strong statements about internal organization of the \( T_{cc}^+ \) is to analyze its decay channels and calculate width of this particle. In the present work, we have carried out such investigation and computed both the mass and full width of the \( T_{cc}^+ \). Our results support a hypothesis about diquark-antidiquark structure of the \( T_{cc}^+ \).

Nevertheless, new investigations are necessary to extract additional information about the \( T_{cc}^+ \). For example, it will be very interesting to study strong decays of the molecule \( DD^* \) using the sum rule method and confront results with the LHCb data and theoretical predictions. Other parameters of the \( T_{cc}^+ \) like the magnetic dipole moment \( \mu \) can also provide useful information on this particle \([42]\).

The molecule picture for the \( T_{cc}^+ \) and its decays were considered in Refs. \([43, 44]\) in the context of alternative approaches. Predictions for the full width of the \( T_{cc}^+ \) obtained in articles \([43, 44]\) are rather small compared with the LHCb data, whereas in Ref. \([45]\) a nice agreement with recent measurements was obtained. Moreover the authors of this work predicted other doubly charmed resonance with parameters \( m = 3876 \text{ MeV} \) and \( \Gamma = 412 \text{ keV} \), which should be object of investigations. As is seen, even in the limits of the same model results for parameters of the \( T_{cc}^+ \) differ considerably.

Another problem in analyses of the doubly charmed state \( T_{cc}^+ \) is connected with essential experimental errors in measurements of \( \Gamma \). More accurate data are required to compare different theoretical models. In other words, physics of the exotic meson \( T_{cc}^+ \) is far from being finished and is waiting for future experimental and theoretical studies.
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