Spin entanglement induced by spin-orbit interactions in coupled quantum dots

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We theoretically explore the possibility of creating spin quantum entanglement in a system of two electrons confined respectively in two vertically coupled quantum dots in the presence of Rashba type spin-orbit coupling. We find that the system can be described by a generalized Jaynes-Cummings model of two modes bosons interacting with two spins. The lower excitation states of this model are calculated to reveal the underlying physics of the far infrared absorption spectra. The analytic perturbation approach shows that an effective transverse coupling of spins can be obtained by eliminating the orbital degrees of freedom in the large detuning limit. Here, the orbital degrees of freedom of the two electrons, which are described by two modes of bosons, serve as a quantized data bus to exchange the quantum information between two electrons. Then a nontrivial two-qubit logic gate is realized and spin entanglement between the two electrons is created by virtue of spin-orbit coupling.

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I. INTRODUCTION

Control and manipulation of the spin degree of freedom become one of the most important topics both in spintronics and in quantum information processing. The spin-orbit interactions (SOI) in semiconductor heterostructures provides the ways to couple spin with spatial degree of freedom, and consequently has attracted more and more attention in recent years. The properties of a few electrons confined in semiconductor nanostructures, such as quantum dots, quantum rings and quantum wires, have been studied. The results show that the carrier spin properties are strongly affected in the presence of the SOI, and novel features emerge in these nanostructures compared with the traditional ones without SOI.

On the other hand, spin confined in quantum dot is a natural choice for the physical realization of qubit. This kind of system is considered as an important candidate for solid state based quantum computing. Among various approaches to implement quantum information processing using quantum dot systems, the optical method, which help us to understand the underlying physics of the spectra. To get an effective Hamiltonian of spin-spin interaction, we perform the Fröhlich transformation in the large detuning limit. This effective Hamiltonian can dynamically drive a two-qubit logic gate operation. By using the conventional material parameters, our numerical estimation shows that the effective spin interaction induced by SOI is strong enough, compared to the spin decoherence in low dimensional semiconductor structures. It is feasible experimentally to implement a two-qubit logic gate and thus produce quantum entanglement.
and the Coulomb interaction

$$V_{\text{coul}}(|r_1 - r_2|) = \frac{e^2}{4\pi \varepsilon_0 |r_1 - r_2|}.$$

The total Hamiltonian reads:

$$H = \sum_{i=1,2} \left( \frac{\Pi_i^2}{2m_0} + V(r_i) + H_{SO}^{(i)} + H_Z^{(i)} \right) + V_{\text{coul}}. \quad (4)$$

Here, $-e$, $\mu_B$, and $\varepsilon_0$ are the electron charge, Bohr magneton and dielectric constant in vacuum, $m_0$, $g$ and $\varepsilon$ are the material related parameters of effective mass, Landé g-factor and the relative dielectric constant, respectively. $\Pi_i = p_i + eA(r_i)$ is the canonical momentum and $A(r_i) = B(-y_i/2, x_i/2, 0)$ is the vector potential for magnetic field $B = B \mathbf{z}$.

In order to simplify the Coulomb interaction, we consider a special case in which the interdot separation $z_0$ is much larger than the lateral confinement characteristic length $l_0 = \sqrt{\hbar/\mu_0 \varepsilon_0}$, i.e. $(l_0/z_0)^2 << 1$. Then we expand the Coulomb interaction as a power series of the relative coordinate $r = |r_1 - r_2|$ up to the second order:

$$V_{\text{coul}}(r) \simeq V_0 - \frac{1}{2} m_0 \omega_0^2 r^2. \quad (5)$$

Here, we have defined $V_0 = e^2/(4\pi \varepsilon_0 \varepsilon_0)$, and $\hbar \omega_1 = \sqrt{\hbar^2 V_0/m_0 z_0^2}$. We also assume that electrons are strictly confined in each quantum dots planes, and then neglect the overlap of their wavefunctions.

In the center of mass (CM) reference of frame defined by $\mathbf{R} = (r_1 + r_2)/2$ and $\mathbf{r} = r_1 - r_2$, we have the CM and relative momentums, and the corresponding angular momentums $\mathbf{P} = \mathbf{p}_1 + \mathbf{p}_2$, $\mathbf{p} = \frac{1}{2} (\mathbf{p}_1 - \mathbf{p}_2)$, $\mathbf{L} = \mathbf{R} \times \mathbf{P}$, and $\mathbf{l} = \mathbf{r} \times \mathbf{p}$, where $M = 2m_0$, $m = m_0/2$, $\mathbf{r}$ is the relative coordinate and $\mathbf{R}$ the CM coordinate. The orbital part of the Hamiltonian is expressed in a quadrature form of these coordinates:

$$H_{\text{orbit}} = \sum_{i=1,2} \left( \frac{\Pi_i^2}{2m_0} + V(r_i) \right) + V_{\text{coul}}$$

$$= \frac{P^2}{2M} + \frac{1}{2} M \Omega^2 R^2 + \frac{1}{2} \omega_c L_z$$

$$+ \frac{p^2}{2m} + \frac{1}{2} m \omega^2 r^2 + \frac{1}{2} \omega_c l_z. \quad (6)$$

Here, the cyclone frequency is $\omega_c = eB/m_0$, and frequency of CM and relative motion $\Omega = \sqrt{\omega_0^2 + \omega_c^2}/4$ and $\omega = \sqrt{\Omega^2 - 2\omega_0^2}$, respectively. Note that the effect of the Coulomb repulsion is reducing the relative motion to a lower frequency compared to the CM motion. In our model the requirement $(l_0/z_0)^2 << 1$ ensures that the $e > 0$ is satisfied even when $B = 0$.

We define the boson operators $a_x, a_x^+, a_X, a_X^+$ of $x$ com-
A, a, B, in terms of four mode bosons straightforward algebra, can be rewritten as:

\[ H_{\text{orbit}} = \hbar \omega_A (A^+ A + 1/2) + \hbar \omega_B (B^+ B + 1/2) \]
\[ + \hbar \omega_a (a^+ a + 1/2) + \hbar \omega_b (b^+ b + 1/2) \]

in terms of four mode bosons \( A, a, B, \) and \( b, \) with their frequencies respectively

\[ \omega_A = \Omega - \frac{1}{2} \omega_c, \quad \omega_B = \Omega + \frac{1}{2} \omega_c, \]
\[ \omega_a = \omega - \frac{1}{2} \omega_c, \quad \omega_b = \omega + \frac{1}{2} \omega_c \]

These four frequencies, together with the Zeeman energy \( \hbar \omega_z = |g| \mu_B B \) are drawn in Fig. 2 with respect to the magnetic field \( B. \) Note that the Zeeman energy \( \hbar \omega_z \) and the boson frequencies \( \omega_A \) and \( \omega_a \) reach the resonant regime at \( B \approx 11T \) with the parameters listed in Table I. We also draw the lower energy spectra of the orbit Hamiltonian \( H_{\text{orbit}} \) in Fig. 2(b). We will focus on how these states are affected in the presence of SOI in the following sections.

In terms of the four boson operators defined by Eqs. 7 and 8, the SOI Hamiltonian \( H_{SO} \), after some straightforward algebra, can be rewritten as:

\[ H_{SO}^{\text{RWA}} = g_A \cdot A (\sigma_{1+} + \sigma_{2+}) \]
\[ + g_a \cdot a (\sigma_{1+} - \sigma_{2+}) + h.c. \]

Note that, due to the negative value of the Landé g-factor, a unitary rotation \( \sigma_z \mapsto -\sigma_z \) and \( \sigma_\pm \mapsto -\sigma_\pm \) has been performed. To obtain the interaction Hamiltonian above, we have used the RWA to neglect the counter-rotating terms like \( \sigma_+ B^+, \sigma_+ b^+, \sigma_+ B, \) and \( \sigma_- b. \) This approximation has been verified numerically for single electron case in Ref. 3. The coupling strengths \( g_A \) and \( g_a \) are defined as follows:

\[ g_A = \alpha \sqrt{\frac{m_0 \Omega}{2 \hbar}} \left( 1 - \frac{\omega_c}{2 \Omega} \right) \]
\[ g_a = \alpha \sqrt{\frac{m_0 \omega}{2 \hbar}} \left( 1 - \frac{\omega_c}{2 \omega} \right) \]

So far, we have obtained a generalized JC model where two-mode bosons interact with two spins. In the following sections we will further demonstrate how the orbit motion induce a spin-spin entanglement in the presence of SOI.

### III. LOWER EXCITATION STATES AND FIR SPECTRA

In this section, we calculate the eigenenergies and eigenstates of the low excitation states. Notice that the

| Quantity | Value | Unit |
|----------|-------|------|
| \( m_0 \) | 0.042 | - |
| \( \varepsilon \) | 14.6 | - |
| \( g \) | -14 | - |
| \( \alpha \) | 10 | meV nm |

| Quantity | Value | Unit |
|----------|-------|------|
| \( \omega_0 \) | 20 | meV |
| \( \varepsilon \) | 9.5 | nm |
| \( z_0 \) | 20 | nm |

TABLE I: Parameters used in the calculations

FIG. 2: (a) Energy dispersions with respect to the magnetic field \( B \) of the four modes of the boson frequencies \( \omega_A \) (red solid line), \( \omega_a \) (blue solid line), \( \omega_B \) (red dashed line), and \( \omega_b \) (blue dashed line), and the Zeeman energy \( \omega_z \) (black line). (b) Energy spectrum of \( H_0. \) Different lines correspond to different spin and orbit states, which are indicated explicitly in the figure. The parameters used in calculation are listed in Table I.
total excitation number operator

\[ \hat{N} = a^+a + A^+A + \frac{1}{2}(\sigma_{1z} + \sigma_{2z}) \]  

commutes with \( H = H_{\text{orbit}} + H^{(1)}_z + H^{(2)}_z + H^{RW}_{SO} \). For a given integer \( N \), which is the eigenvalue of \( \hat{N} \), the dimension of the invariant subspace \( V(N) \) is \( 4N + 4 \) for \( N \geq 0 \).

The lowest subspace \( V(-1) \), corresponding to \( N = -1 \), is of one dimension. The the ground state can be directly written as \( |GS\rangle = |0_A,0_a,0_B,0_b,\downarrow,\downarrow\rangle \), which means the excitation numbers of boson modes

\[ A^+A = a^+a = B^+B = b^+b = 0 \]  

and both spins are in the down state. The corresponding eigenenergy is

\[ E_{GS} = \frac{\hbar}{2}(\omega_A + \omega_B + \omega_a + \omega_b - 2\omega_z). \]  

The second lowest subspace \( V(0) \), which is of four dimension. The Hamiltonian can be exactly diagonalized in this subspace. We define

\[ |S\rangle = \frac{1}{\sqrt{2}}(|0_A,0_a,\uparrow,\downarrow\rangle - |0_A,0_a,\downarrow,\uparrow\rangle), \]
\[ |T_0\rangle = \frac{1}{\sqrt{2}}(|0_A,0_a,\uparrow,\downarrow\rangle + |0_A,0_a,\downarrow,\uparrow\rangle), \]  

(18)
\[ |T_{-1}^A\rangle = |1_A,0_a,\downarrow,\downarrow\rangle, |T_{-1}^a\rangle = |0_A,1_a,\downarrow,\downarrow\rangle. \]

The eigenstates of the Hamiltonian in this subspace are

\[ |\Phi_A^\pm\rangle = \sin \frac{\theta_A}{2} |T_{-1}^A\rangle + \cos \frac{\theta_A}{2} |T_0\rangle, \]  

(19)
\[ |\Phi_a^\pm\rangle = \cos \frac{\theta_a}{2} |T_{-1}^a\rangle - \sin \frac{\theta_a}{2} |S\rangle, \]  

(20)
\[ |\Phi^A\rangle = \sin \frac{\theta_A}{2} |T_{-1}^A\rangle + \cos \frac{\theta_A}{2} |S\rangle, \]  

(21)
\[ |\Phi^a\rangle = \cos \frac{\theta_a}{2} |T_{-1}^a\rangle - \sin \frac{\theta_a}{2} |S\rangle. \]  

(22)

and the corresponding eigenvalues are:

\[ E_A^\pm = -\frac{\Delta_A}{2} \pm \sqrt{\left(\frac{\Delta_A}{2}\right)^2 + 2g_A^2}, \]  

\[ E_a^\pm = -\frac{\Delta_a}{2} \pm \sqrt{\left(\frac{\Delta_a}{2}\right)^2 + 2g_a^2}. \]  

Here, the zero point energies of the four boson modes are omitted. \( \theta_{A,a} \) is defined as

\[ \tan \theta_{A,a} = \frac{2\sqrt{2}g_{A,a}}{\Delta_{A,a}} \]  

(25)

where \( \theta_{A,a} \in [0, \pi] \), and \( \Delta_{A,a} = \omega_z - \omega_{A,a} \).

With the help of these exact eigenenergies and the eigenstates obtained above, we can calculate the FIR absorption spectra analytically. To this end, we consider the time-dependent Hamiltonian of the system in an optical field is

\[ \hat{H}(t) = H + H' e^{-i\omega t} \]  

(26)

where \( H' \) is the time dependent term induced by the classical optical field \( \vec{E} \)

\[ H' \propto - \sum_{i=1,2} \frac{e}{m} \vec{e} \cdot (\vec{p}_i + e\vec{A}_i) + \frac{\omega a}{\hbar} (\sigma_i \times \vec{e})_z \]  

(27)

where \( a \) is the radiation field amplitude, and \( \vec{e} = (\hat{x} - i\hat{y})/\sqrt{2} \) is the polarization vectors for circular polarized light.

The absorption coefficient is calculated according to the Fermi golden rule\[19 \ 20]:

\[ \alpha(\omega) \propto \omega \sum_f |\langle f |H'|i\rangle|^2 \delta(\omega - \omega_f), \]  

(28)

Here, \( \hbar \omega_f = E_f - E_i \), \( |i\rangle = |GS\rangle \) is the ground state, and \( |f\rangle \) stands for the excited states. We will focus on
the four lowest excited states $|\Phi^+_A\rangle$ and $|\Phi^+_a\rangle$. According to Eqs. (7) and (8), the perturbation $H'$ is obtained as

$$H' = \hbar \Omega \sqrt{\frac{\omega_0}{\Omega}} \left(1 - \frac{\omega_c}{\Omega}\right) A^+ + \alpha \sqrt{\frac{2m\omega_0}{\hbar}} \left(\sigma_{1+} + \sigma_{2+}\right)$$  (29)

Note that we have omitted the terms related to the $B$ modes, which is not involved in the initial and final states we are considering and thus does not contribute to the absorption coefficient. Our analytical results give the FIR spectra of obvious physical meanings. From Eq. (29) above, we find that the first term is spin independent, and it provides an CM angular excitation. This CM angular excitation, which is the consequence of the Coulomb interaction, exists even in the absence of SOI. The second term, which is spin dependent, is due to the presence of SOI. This term contributes an excitation of the second term, which is spin dependent, and it provides an CM angular excitation. The matrix elements of $\langle H'|\Phi^+_A\rangle$, $\langle H'|\Phi^+_a\rangle$ are inactive in this case. The exact solution of the eigenvalue problem will be given in this section. In the perturbation available regime, we derive an effective transverse spin-spin interaction Hamiltonian. This Hamiltonian can induce a two-qubit logic gate, and can be used to produce a controllable quantum entanglement.

We summarize the Hamiltonian obtained

$$H_0 = \frac{1}{2} \hbar \omega_z (\sigma_{1z} + \sigma_{2z})$$
$$H_1 = g_A \cdot A (\sigma_{1+} + \sigma_{2+}) + g_a \cdot a (\sigma_{1+} - \sigma_{2+}) + h.c.$$  (31)

and then consider its reduction in the large detuning limit, i.e. $\Delta_{A,a} >> g_{A,a}$. In this limit, we perform the Fröhlich transform with the operator

$$S = \left(\frac{g_A}{\Delta_A} A^+ (\sigma_{1-} + \sigma_{2-}) + \frac{g_a}{\Delta_a} a^+ (\sigma_{1-} - \sigma_{2-})\right) - h.c.$$  (32)

and the effective Hamiltonian $\exp(-S)H\exp(S)$ is calculated up to the second order as:

$$H_S \simeq H_0 + \frac{1}{2} [H_1, S].$$  (33)

Here, the second term in the r.h.s can be written explicitly

$$\frac{1}{2} [H_1, S] = \hbar \xi (\sigma_{1+} \sigma_{2-} + \sigma_{1-} \sigma_{2+}) + \frac{g_A g_a}{2} \left(\frac{1}{\Delta_A} + \frac{1}{\Delta_a}\right) (A^+ a + a^+ A) (\sigma_{1z} - \sigma_{2z})$$
$$+ \left[\frac{g_A^2}{\Delta_A} (A^+ A + 1/2) + \frac{g_a^2}{\Delta_a} (a^+ a + 1/2)\right] (\sigma_{1+} + \sigma_{2+}) + \left(\frac{g_A^2}{\Delta_A} + \frac{g_a^2}{\Delta_a}\right) (\sigma_{1z} + \sigma_{2z})$$  (34)

Note that the first term of Eq. (34) is the effective transverse spin-spin coupling induced by SOI. The effective coupling strength

$$\hbar \xi = g_A^2 / \Delta_A - g_a^2 / \Delta_a$$  (35)

depends on (i) the SOI strength $\alpha$ (see Eqs. (14) and (15)), and (ii) the frequency difference between $\omega_A$ and $\omega_a$, which is the consequence of the Coulomb interaction. We note that, in the subspace $V^{(0)}$, the second term of Eq. (35) vanishes, and the remaining terms commute with the total spin $\sigma^2 = (\sigma_1 + \sigma_2)^2$. Thus we can denote the eigenstates by spin singlet $|S\rangle = (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) / \sqrt{2}$ and triplet $|T_0\rangle = (|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle) / \sqrt{2}$, $|T_1\rangle = |\uparrow\uparrow\rangle$, and $|T_{-1}\rangle = |\downarrow\downarrow\rangle$ in the large detuning limit in subspace $V^{(0)}$. Diagonalizing the Hamiltonian (34), we obtain the eigenenergies

$$E_{|T_{-1}\rangle} = -\Delta_A - \frac{2g_A^2}{\Delta_A},$$
$$E_{|T_{+1}\rangle} = -\Delta_a - \frac{2g_a^2}{\Delta_a},$$
$$E_{|T_0\rangle} = \frac{2g_A^2}{\Delta_A},$$
$$E_{|S\rangle} = \frac{2g_a^2}{\Delta_a}$$  (36-39)

for the different spin states corresponding to the exact solutions in Eq. (29). Here, we also omit the zero point energies.

The above energies are drawn as the function of magnetic field in Fig. 3(c) in comparison with the exact solution. The exact consistency of the two solutions in the
The perturbation treatment is valid when both $g_A/\Delta_A$ and $g_a/\Delta_a$ less than unity. (b) The effective transverse spin coupling strength induced by SOC. (c) A comparison of eigenenergies between exact (Eqs. (23) and (24), lines) and perturbative (Eqs. (20) and (21), lines) solutions in the large detuning regime. Our perturbation treatment is valid in the unshaded region.

FIG. 4: (a) The ratio $g_A/\Delta_A$ (black line) and $g_a/\Delta_a$ (red line). The perturbation treatment is valid when both $g_A/\Delta_A$ and $g_a/\Delta_a$ less than unity. (b) The effective transverse spin coupling strength induced by SOC. (c) A comparison of eigenenergies between exact (Eqs. (23) and (24), lines) and perturbative (Eqs. (20) and (21), lines) solutions in the large detuning regime. Our perturbation treatment is valid in the unshaded region.

large detuning regime confirms the validity of our perturbation treatment. Fig. 4(a) shows effective spin coupling strength induced by SOI in our model. In the perturbation valid regime, for example at $B = 10 T$ (the point denoted by X), we have the spin coupling strength $\hbar \xi \approx 20 \mu eV$. This value is comparable with that in the proposal of the spin-spin coupling induced by the electromagnetic field in cavity\cite{15}. Thus, it is feasible to realize the two-qubit gate operation during the long coherence time of conduction band electrons.

The spin transverse coupling described by the first term in the r.h.s. of Eq. (14) generates an ideal $\sqrt{\text{SWAP}}$ gate\cite{22,10} at a specific time $t_0 = \pi/4\xi$. On the other hand, it is worthy to notice that the unitary transformation $\exp(-S)H \exp(S)$ may induce an excitation of the boson modes, and then causes gate error during the time evolution of the spin states. To explicitly show the unwanted boson modes excitation and to examine the reliability of our model, we consider the time evolution of the initial state $|\psi(0)\rangle = |↓↑, 0_A, 0_a\rangle$:

$$|\psi(t)\rangle = U(t) |\psi(0)\rangle,$$

where

$$U(t) = \exp \left[ \frac{it}{\hbar} \left( e^{S}H_{S}e^{-S} \right) \right] = e^{S} \exp \left( \frac{itH_{S}}{\hbar} \right) e^{-S}.$$  

Notice that $|\psi(0)\rangle$ belongs to the subspace $V(0)$, and the time evolution will be restricted in this subspace. Fig. 4(a) gives the probabilities $P_{i}(t) = |\langle i|\psi(t)\rangle|^{2}$ of finding state $|i\rangle$ at time $t$, where $|i\rangle$ stands for the four bases of the subspace $V(0)$: $|↓↑, 0_A, 0_a\rangle$, $|↓↑, 0_A, 0_a\rangle$, $|↓↓, 1_A, 0_a\rangle$, and $|↓↓, 0_A, 1_a\rangle$ respectively. Besides the states $|↓↑, 0_A, 0_a\rangle$ and $|↓↓, 0_A, 0_a\rangle$, the unwanted boson modes excitation states $|↓↓, 1_A, 0_a\rangle$, and $|↓↓, 0_A, 1_a\rangle$ also have nonzero populations. This populations will induce the gate error. Furthermore, we calculate the fidelity function defined by

$$F(t) = \left| \langle \psi(0) | U_{\sqrt{\text{SWAP}}}^{-1} U(t) | \psi(0) \rangle \right|,$$

where $U_{\sqrt{\text{SWAP}}}$ is the ideal $\sqrt{\text{SWAP}}$ gate operator. We notice that the fidelity function $F(t)$ reaches its maximum slightly less than unity at $t = t_0$, and the high-frequency oscillations appear due to the boson modes excitation mentioned above. We also examine the fidelity function generated by the original Hamiltonian\cite{31}, i.e., $U(t) = \exp \left[ -it \left( H_0 + H_1 \right) \right] / \hbar$. In this case the original Hamiltonian\cite{31} gives a high fidelity $F(t') \approx 1$ at a different time $t' \approx 1.2t_0$, which can be regarded as the higher order correction in comparison to the approximate
Hamiltonian \[33\]. Finally, Fig.5 shows that spin entanglement can be created in this system by adjusting the operation time.

V. CONCLUSION

In this paper, we have considered a system of two vertically CQDs each containing an electron in the presence of Rashba type SOI. We theoretically demonstrate that it is possible to create spin entanglement in this kind of system by using the SO coupling. With the large inter-dot separation case, the Coulomb interaction between the two electrons is approximately expressed in a quadratic form. And then two-boson-two-spin interacting model is derived in the RWA from the original Hamiltonian. We give the exact solution of the low excitation states analytically. This solution helps us reveal the physics under the FIR spectra near the resonant point. Perturbation treatment in the large detuning case shows that, similar to the quantum dot embedded in an optical cavity, the orbital freedoms play a role of quantized data bus via the Coulomb interaction and SOI in this system. An effective Hamiltonian of spin-spin interaction is obtained in the perturbation regime by eliminating the orbital freedoms. This Hamiltonian provides a two-qubit operation, which is essential in quantum information processing.

Finally, we would like to point out that using the effective inter-spin coupling to create spin entanglement is feasible to be controlled and measured. From the discussion above, we know that the effective spin coupling strength can be controlled by external magnetic field. On the other hand, the tuneable strength of SOI \(\alpha\) [11], in principle, also enable us to switch on and off the effective inter-spin coupling by external gates conveniently. To probe the quantum entanglement of spin system, similar method in the protocol proposed in ref. [23] can be used, where the information stored in the spin degrees of freedom is converted to the charge states, and then the charge states can be detected.

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