Research on Two Types of Estimation Algorithms in Carrier Tracking Loop

Hangtian Qi, Xiaolin Zhang
School of Electronic Information Engineering, Beihang University, Beijing 100191, China
qihangtian925@qq.com

Abstract. This paper analyzes two estimation algorithms in the carrier loop, i.e. maximum likelihood estimation (MLE) and modern filtering based on Bayesian recursion. The limitations of the MLE algorithm are pointed out by establishing the MLE model. In addition, several modern filters based on Bayesian recursion are analyzed. In this process, the article draws two inferences affecting high dynamics and low signal-to-noise ratio (SNR) carrier tracking from two aspects of state equation and measurement equation. Thus, the optimal algorithm using unscented Kalman filter (UKF) is proposed by the inferences mentioned. At the end, simulation results demonstrate the combined strong tracking algorithm has advantages in case of high dynamic and low SNR.

1. Introduction
For a communication system, carrier synchronization is a critical step in signal reception. On the one hand, it has an effect on the quality of the follow-up signal processing such as demodulation, and on the other hand, physical information of the communication target (such as speed and direction) is contained in the carrier. However, the receipt signal of deep space communication is characterized by low signal-to-noise ratio (SNR) and high Doppler dynamic [1]. Carrier synchronization in extreme environments such as Mars EDL process is yet to be resolved. Carrier synchronization is usually divided into acquisition and tracking, wherein, acquisition aims to implement initial traction and identification of the frequency, while tracking tends to extend with a negative feedback loop as the basic architecture to reproduce the received signal carrier frequency continuously. The conventional second-order frequency-locked loop-assisted third-order phase-locked loop [2] (FLL-PLL) has a fixed loop bandwidth, which is subject to restrictions in the deep space communication scenario. Modern scholars have proposed two ways to improve the tracking loop:

- Adaptive control loop filter bandwidth that controls the digital loop filter parameters in real time according to the carrier tracking results [3]. It is easily feasible, although it is essentially a compromise to low SNR and high dynamic, as it fails to meet both of the needs.
- More advanced estimation algorithm instead of the identification structure in the loop, commonly the statistical estimation method (such as the classic maximum likelihood estimation (MLE)) and the modern filtering method, wherein, modern filtering implements optimal estimation of the state variables in noise under certain conditions. Most of the modern filtering algorithms in carrier tracking are based on Bayesian recursion and Kalman iteration. Taking the traditional Kalman filter [4] (CKF) as an example, the paper [5] adopts the extended Kalman filter (EKF), explores the application of unscented...
Kalman filter [6] (UKF) and Cubature Kalman filter in a tracking loop. Yet the particle filter (PF) is inapplicable due to the heavy calculation burden and particle degradation.

In the context that scholars have seldom compared algorithms in principle, this paper explores the MLE method and Kalman iteration-based modern filtering, analyzes the MLE method in detail, summarizes two inferences affecting low SNR and high dynamic in the filtering algorithm, and provides specific design for appropriate algorithm selection according to the discussion results.

2. MLE Parameter Estimation

2.1 Signal Model
It is supposed that the input signal in the carrier tracking loop is a BPSK modulated signal:

\[ s(n) = Ab(n) \cos(o_n T + \theta) \]  \hspace{1cm} (1)

where \( b(n) \) is the information bit, \( A \) is the signal amplitude, \( T \) is the sampling interval, \( \omega \) and \( \theta \) are the IF frequency and the initial phase of the input signal respectively. After the high frequency components are filtered from the multiplied input signal and the NCO signal, two orthogonal signals can be obtained:

\[ i(n) = Ab(n) \cos(\Delta \omega n T + \Delta \theta) \]
\[ q(n) = Ab(n) \sin(\Delta \omega n T + \Delta \theta) \]  \hspace{1cm} (2)

In Equation (2), \( \Delta \omega \) and \( \Delta \theta \) are frequency and phase differences of the input signal and NCO. Thus, the receipt signal can be expressed as follows:

\[ r(n) = Ab(n) \exp(j \Delta \omega k T + \Delta \theta) + w(n) \]  \hspace{1cm} (3)

where \( w(n) \) is the Gaussian white noise at a power spectral density of \( N_0 \). Suppose that the integral time is \( T_s, T = NT_s \). After integral clearance, Equation (4) can be obtained:

\[ I(k) = Ab(k) \sin c\left(\frac{1}{2} \Delta \omega T \cos[\Delta \omega (t_{k-1} + \frac{T}{2}) + \Delta \theta]\right) \]
\[ Q(k) = Ab(k) \sin c\left(\frac{1}{2} \Delta \omega T \sin[\Delta \omega (t_{k-1} + \frac{T}{2}) + \Delta \theta]\right) \]  \hspace{1cm} (4)

where \( t_{k-1} = (k - 1)NT_s \) is the previous integral time.

2.2 MLE algorithm
The MLE algorithm, independent of the prior signal distribution, figures out the minimum error variance estimation of a parameter based on certain observation data in white noise. Rigorous probability and statistics methodologies make it advantageous at low SNR [7]. To ensure concise expression in subsequent mathematical derivation, suppose \( \phi = \Delta \omega k T + \Delta \theta \), i.e.:

\[ i(n) = Ab(n) \cos(\phi) \]
\[ q(n) = Ab(n) \sin(\phi) \]  \hspace{1cm} (5)

The joint probability density function for consecutive N receipt signals is as follows:

\[ p(A, \Delta \omega, \Delta \theta | r_N) = \frac{1}{(\pi N_0)^N} \exp\left(-\frac{1}{N_0}(r_N - \hat{r}_N)^T W (r_N - \hat{r}_N)\right) \]  \hspace{1cm} (6)

where \( r_N = [r(0), r(1), \cdots, r(N-1)]^T \) is the N-dimensional observation vector, \( \hat{r}_N = [\hat{r}(0), \hat{r}(1), \cdots, \hat{r}(N-1)]^T \) is the estimation of \( r_N \), and \( W \) is the weight matrix of \( r_N \), representing the weight of each element in \( r_N \). The unit matrix is used herein. By taking the logarithm of Equation (6), the log-likelihood cost function is obtained:

\[ L(A, \Delta \omega, \Delta \theta | r_N) = -\frac{1}{N_0} |r_N - \hat{r}_N|^2 - N \ln(\pi N_0) \]  \hspace{1cm} (7)

Remove the extraneous items of \(-N \ln(\pi N_0)\) and expand the equation.
Take the partial derivative of $A$ and $\Delta \theta$ of Equation (8).

$$\frac{\partial L}{\partial A} = -\frac{2}{N_0} \sum_{n=0}^{N-1} A - [i(n) \cdot \cos(\phi) + q(n) \sin(\phi)]$$

(9)

$$\frac{\partial L}{\partial \Delta \theta} = \sum_{n=0}^{N-1} [-i(n) \cdot \sin(\phi) + q(n) \cos(\phi)]$$

(10)

Suppose that Equations (9) and (10) are 0. The maximum likelihood estimations of $A$ and $\Delta \theta$ are obtained:

$$\hat{A} = \frac{2}{N} \sqrt{\left(\sum_{n=0}^{N-1} i(n)\right)^2 + \left(\sum_{n=0}^{N-1} q(k)\right)^2}$$

(11)

$$\Delta \hat{\theta} = \arg\left(\sum_{n=0}^{N-1} i(n) - \sum_{n=0}^{N-1} q(n)\right)$$

(12)

Equation (15) shows the maximum likelihood estimation of $\Delta \omega$. As first and second-order partial derivative expressions contain the accumulation of $n$, a direct solution is unavailable.

$$\frac{\partial^2 L}{\partial \Delta \omega^2} = -T_s \sum_{n=0}^{N-1} n^2 [-i(n) \cdot \sin(\phi) + q(n) \cos(\phi)]$$

As the optimal solution is needed in only one dimension, the Hessian matrix is not complicated. Thus, the classical Newton method is more convenient and accurate. Equation (14) shows the iterative process.

$$\Delta \hat{\omega}_{m+1} = \Delta \hat{\omega}_m - H^{-1}_m g_m$$

(14)

$H_m$ is the Hessian matrix $\frac{\partial^2 L}{\partial \Delta \omega^2}$, and $g_m$ is the gradient $\frac{\partial^2 L}{\partial \Delta \omega^2}$. Although the MLE algorithm is suitable for low SNR scenarios, it requires the optimal solution of nonlinear equations. In addition to the computational complexity, the dependence of the solution process on initial value selection may probably lead to unsuccessful convergence. Moreover, since the MLE requires data observation as demonstrated by Equation (8), it may fail to meet some real-time requirements.

3. Bayesian Recursion Criteria-based Algorithm

Equation (15) shows a basic nonlinear model of the carrier tracking loop.

$$\begin{align*}
x_k &= f(x_{k-1}) + W_{k-1} \\
z_k &= h(x_k) + V_k
\end{align*}$$

(15)

$x_k$ and $z_k$ are the state vector and the observation vector at the moment $k$, $h$ is the measurement equation, $f$ is the state transition equation, $W$ is the Gaussian state transition noise vector with a covariance matrix of $Q$, and $V$ is the Gaussian observation noise vector with a covariance of $R$. Bayesian recursion criteria are as follows:

$$p(x_{k+1} | Z^t) = \int p(x_k | Z^t) p(x_{k+1} | x_k) dx_k$$

(16)
\[ p(x_{k+1} \mid Z^{k+1}) = \frac{p(x_k \mid Z^k) p(z_{k+1} \mid x_{k+1})}{\int p(x_k \mid Z^k) p(z_{k+1} \mid x_{k+1}) dx_{k+1}} \] (17)

It is known that the posterior probability density function at the moment \( k \) is \( p(x_k \mid Z^k) \), and a new measured value \( Z^{k+1} \) is obtained at the moment \( k+1 \). The Bayesian recursion aims to obtain \( p(x_{k+1} \mid Z^{k+1}) \) through the steps of prediction update and measurement update. \( p(x_{k+1} \mid x_k) \) is the single-step state transition probability density function. \( p(z_{k+1} \mid x_{k+1}) \) is the measurement probability density function.

The calculation of Equations (16) and (17) is challenging, for which the accurate analytical solution is available by being supplemented with estimation criteria (such as the minimum variance) in the linear system, i.e., the classical Kalman filter. On this basis, many scholars have worked out different Kalman iterative structure-based nonlinear filtering algorithms like EKF, UKF and CKF [8].

The author analyzes the influence of the filtering algorithm on the carrier tracking from state equation and measurement equation models below.

3.1 Approximate linearization of the state equation model
It is obvious that the complete nonlinear motion equation of the target is unknown, and so is its probability distribution density. Therefore, the nonlinear filtering methods are not available for the unknown nonlinear state transition equations. For this reason, the only way is to approximate state equations [9]. Suppose that the n-dimensional carrier state quantity in carrier tracking is as follows:

\[ x = [\Delta \theta, \Delta \theta^{(1)}, \Delta \theta^{(2)}, \ldots, \Delta \theta^{(n-1)}]^T \] (18)

\( \Delta \theta \) is the phase difference. \( \Delta \theta^{(1)} \) is the first-order derivative of the phase position, i.e., frequency difference \( \Delta \omega \). \( \Delta \theta^{(n-1)} \) is the n-order derivative. Perform Taylor expansion in time \( T \) and abandon the higher order items after \( n-1 \) to obtain the following:

\[ \Delta \theta^{(n)} = \Delta \theta^{(n)}_{k+1} + \Delta \theta^{(n)}_{k-1} + \sum_{i=1}^{n-1} \frac{T^{n-i}}{(n-i)!} \Delta \theta^{(i)}_{k-1} \] (19)

The time before and after is unchanged by default, namely:

\[ \Delta \theta^{(n)} = \Delta \theta^{(n)}_{k-1} \quad n = m+1 \] (20)

It means that \( \Delta \theta^{(n-1)} \) can be estimated, but it does not change with true values in real time. The following is obtained from Equations (19) and (20):

\[ x_k = \begin{bmatrix} 1 & \cdots & \frac{T^{n-2}}{(n-2)!} & \frac{T^{n-1}}{(n-1)!} \\ 0 & 1 & \cdots & \frac{T^{n-2}}{(n-2)!} \\ 0 & 0 & \cdots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{bmatrix} x_{k-1} \] (21)

According to Equation (21), the state equation model is linear at this moment, with errors introduced in two aspects. On one hand, the equation of \( \Delta \theta \) abandons the higher-order items after the n-1 order; on the other hand, at the n-1 order, the items at time \( k \) are equal to those before and after time \( k-1 \). One of the major constraints against the modern filtering is the fixed measurement equation, as shown in the framework of Equations (18) and (19). The following inferences are made on the basis of the above analysis:

Inference 1: In the context of high SNR and accurate system and noise models, the state quantity dimension \( n \) affects the carrier tracking capacity under high dynamic.
3.2 Measurement equation model

In carrier tracking, the measurement equation is connected with the loop structure. Where there is a phase discriminator, only one dimension in the observed variables is the phase discrimination result \( \theta_{\text{err}}(T) \). The average phase discrimination result in \( T \) is as follows:

\[
\theta_{\text{err}}(T) = \frac{1}{T} \int_0^T (\Delta \theta + \Delta \theta^{(1)} \tau + \cdots + \frac{1}{(n-1)!} \Delta \theta^{(n-1)} \tau^{n-1}) d\tau
\]

\[
= \Delta \theta + \frac{1}{2} \Delta \theta^{(1)} T + \cdots + \frac{1}{n!} \Delta \theta^{(n-1)} T^{n-1}
\]

(22)

As shown in Equation (22), the measurement equation is in linear state. Where there is no phase discriminator in the loop, \( I(k) \) and \( Q(k) \) can be observed directly. To eliminate the effect of \( \pm 1 \) on the data bit \( b(k) \), the square method or bit flip method is usually used as follows:

\[
h(x) = \begin{bmatrix} K_1 \cos L_1(\Delta \theta) \\ K_2 \sin L_2(\Delta \theta) \end{bmatrix}
\]

(23)

where \( K_1 \) and \( K_2 \) are constant coefficients, which are generally equal; \( L_1 \) and \( L_2 \) are linear functions, which are generally equal as well. Direct observation gets rid of the nonlinear noise error introduced by phase discrimination, while describing the noise-concerned nonlinear measurement equation in a complete and direct way. Nonlinear filtering is used specifically for nonlinearity of the measurement equation, which is independent of nonlinearity of the state equation. On the basis, Inference 2 is made as follows:

Inference 2: For the same state model and noise model, the selection of measurement equations affects the carrier tracking capability in low SNR.

3.3 Algorithm selection and design

In view of Inference 1 and the calculation complexity of nonlinear filtering, and the fact that the KF method has been widely applied in engineering, KF is used for high SNR herein. For low SNR, UKF is superior to EKF in terms of calculation complexity and accuracy. CKF are not dominant in lower dimension. PF suffers heavy calculation burden and particle degradation. UKF is used for low SNR herein.

In the scenario of KF, through substitution into Equations (21) and (22), the state transition equation and the observation equation in \( T \) can be obtained. \( A \) is the state transition matrix, \( B \) is the input control matrix, and \( H \) is the observation matrix. \( \hat{x}_{k-1} \) is state prediction, \( P_{k-1} \) is mean square error of state prediction, \( K_k \) is filtering gain, \( \hat{X}_k \) is state estimation, \( P_k \) is mean square error of state prediction, and the entire iterative computation process is as follows [10]:

\[
\hat{x}_{k|k-1} = A\hat{x}_{k-1} + Bu_{k-1}
\]

\[
P_{k|k-1} = AP_{k-1} A^T + Q_{k-1}
\]

\[
K_k = P_{k|k-1} H^T (HP_{k|k-1} H^T + R_k)^{-1}
\]

\[
\hat{x}_k = \hat{x}_{k|k-1} + K_k (z_k - H\hat{X}_{k|k-1})
\]

\[
P_k = (I - K_k H)P_{k|k-1}
\]

(24)

Similarly, the transition equation and the observation equation of UKF are available through substitution into Equations (21) and (23). The UKF process is as follows:

1 UT sampling and weight calculation:
\[
\begin{aligned}
\chi'_{i,1} &= \hat{x}_{i,1}, i = 0 \\
\chi'_{i,1} &= \hat{x}_{i,1} + \sqrt{(n+\lambda)}P_{i,1}^{1/2}, i = 1, \cdots, n \\
\chi'_{i,1} &= \hat{x}_{i,1} - \sqrt{(n+\lambda)}P_{i,1}^{1/2}, i = n + 1, \cdots 2n \\
\lambda &= \alpha^2(n+k) - n \\
W^{(m)}_0 &= \frac{\lambda}{n+\lambda} \\
W^{(m)}_0 &= W^{(m)}_0 + (1-\alpha^2 + \beta) \\
W^{(m)}_i &= \frac{1}{2(n+\lambda)}, i = 1, 2, \cdots 2n
\end{aligned}
\]

where \(\chi'\) is sampling point set; \(n\) is the matrix dimension; \(\alpha\) is the scaling factor to determine the distance between point sigma and the mean; \(k\) is the second proportional coefficient to affect the deviation caused by the higher-order moments after the second order; \(\beta\) is related to the prior distribution, affecting the accuracy of the covariance; \(W^{(m)}_i\) and \(W^{(m)}_i\) are the mean value and the variance weight of the sampling point set respectively.

2 State prediction:
\[
\begin{aligned}
\dot{x}_{i,k-1} &= f(x_{i,k-1}, u_{k-1}) \\
\hat{x}_{i,k-1} &= \sum_{i=0}^{2n} W^m_i \chi'_{i,k-1} \\
\end{aligned}
\]

\[
P_{i,k-1} = \sum_{i=0}^{2n} W^c_i \left( \chi'_{i,k-1} - \hat{x}_{i,k-1} \right) \left( \chi'_{i,k-1} - \hat{x}_{i,k-1} \right)^T + Q_{k-1}
\]

3 Measurement update:
\[
\begin{aligned}
z_{i,k-1} &= h(x'_{i,k-1}) \\
\hat{z}_{i,k-1} &= \sum_{i=0}^{2n} W^m_i z_{i,k-1} \\
\end{aligned}
\]

4 Filtering update:
\[
\begin{aligned}
P_{i,k} &= \sum_{i=0}^{2n} W^c_i \left( z_{i,k-1} - \hat{z}_{i,k-1} \right) \left( z_{i,k-1} - \hat{z}_{i,k-1} \right)^T + R_k \\
P_{i,k} &= \sum_{i=0}^{2n} W^c_i \left( z_{i,k-1} - \hat{z}_{i,k-1} \right) \left( z_{i,k-1} - \hat{z}_{i,k-1} \right)^T \\
K_i &= P_{i,k}^{-1} P_{i,k}^{1/2} \\
\hat{x}_{i,k} &= \hat{x}_{i,k-1} + K_i (z_k - \hat{z}_{i,k-1}) \\
P_i &= P_{i,k-1} - K_i P_{i,k} K_i \\
\end{aligned}
\]

To enhance the robustness of the algorithm and eliminate errors caused by the linear state equation and inaccurate noise model, it is necessary to introduce strong tracking filtering (STF). The essence of STF is to use the orthogonality principle of Kalman filtering as follows:
\[
\begin{aligned}
\eta_k &= z_k - h(\hat{x}_{i,k-1}) \\
E(\eta_k \eta_{k-1}^T) &= 0
\end{aligned}
\]
η is known as the information sequence, and the Kalman filtering information is orthogonal in ideal conditions. A fading factor λ is introduced and embedded into the one-step prediction error covariance matrix \( P_{k+1|k} \) to adjust the gain matrix \( P_{k+1|k} \) and facilitate new information orthogonality [12].

In KF:
\[
\lambda_k = \max(1, \lambda_0)
\]
\[
\lambda_0 = \frac{tr(\eta_k \eta_k^T - HQ_k H^T - R_k)}{tr[H P_{k+1|k} H^T]}
\]  
(29)

\[
P_{k+1|k} = \lambda_k A P_k A^T + Q_{k+1}
\]

In UKF:
\[
\hat{\lambda}_k = \max(1, \hat{\lambda}_k)
\]
\[
\hat{\lambda}_0 = \frac{tr[\sum_{i=0}^{2n} W_i^r \left( z_{k|k-1} - \hat{z}_{k|k-1} \right) \left( z_{k|k-1} - \hat{z}_{k|k-1} \right)^T + R_k]}{tr[\sum_{i=0}^{2n} \left( z_{k|k-1} - \hat{z}_{k|k-1} \right) \left( z_{k|k-1} - \hat{z}_{k|k-1} \right)^T]}
\]
(30)

Thus far, the Kalman filtering-assisted phase-locked loop (KAPLL) algorithm has been constructed.

4. Simulation
The carrier frequency is L1 of GPS, \( f = 1575.42 \text{MHz} \), and the intermediate frequency has no effect to simulation. Suppose that the intermediate frequency in a signal is 1 MHz, the sampling frequency is 8 MHz, and the integral time \( T \) is 1ms. The traditional third-order phase-locked loop has a band-width of 47 Hz, which varies with the signal CNR. To be specific, 25 dB-HZ is used at low CNR, 35 dB-Hz is used at moderate CNR and 50 dB-Hz is used at high CNR. According to the high dynamic model designed by the American Jet Propulsion Laboratory (JPL), when the initial velocity is \( v = 100 \text{m/s} \), the acceleration is \( 25g \) and the jerk is \( 100g/\text{s} \), the Doppler frequency, Doppler acceleration and Doppler jerk are obtained:

\[
\pm 100, \pm 25g, \pm 100g \times f/c
\]  
(31)

where \( g = 9.8 \text{m/s}^2 \), \( c = 299792458 \text{m/s} \).

The conventional third-order phase-locked loop is unable to track a high dynamic model with a CNR of 25dB-HZ. If a jerk of 100g/s is suddenly added at 500ms, the jerk cannot be accurately tracked in real time in the four-dimensional state model. In this case, it is obvious that the CKF has already diverged, and the strong tracking UKF can quickly fulfill regression and convergence.
To explore the impact of the state equation on high dynamic carrier tracking, suppose that the CNR is 35dB-Hz and the acceleration ($\Delta \theta^{(2)}$) is 25g to analyze loss of lock probability for the jerk in different state equation dimensions. Table 1 lists the results.

Table 1 Loss of Lock Probability for Different Dimensions

|     | CKF | EKF | UKF | CKF | EKF | UKF |
|-----|-----|-----|-----|-----|-----|-----|
|     | 0g/s| 0g/s| 0g/s| 100g/s| 100g/s| 100g/s|
| n=3 | 0.35| 0.18| 0.17| 1    | 1    | 1    |
| n=4 | 0    | 0    | 0    | 0.37 | 0.17 | 0.15 |

For a tracking object with acceleration of 25g and without jerk, Fig.2 shows the tracking performance of different algorithms at different CNRs. The results demonstrate that UKF is slightly superior to EKF, and CKF performance approaches nonlinear filtering at 37dB-Hz.

To compare the performance of different algorithms at low SNR, suppose that the input signal CNR is 25dB-HZ to eliminate the high dynamic effect and maintain the tracking target at a constant speed. Fig.3 shows the tracking frequency errors. The following conclusion can be drawn: At low CNR, CKF is obviously the worst with the highest error, KAPLL and EKF are much better, and KAPLL is slightly better than EKF.
To compare the tracking performance of different algorithms in high dynamic and eliminate the influence of low SNR, suppose that the input signal CNR is 50dB-HZ, and the tracking target has acceleration and a constant jerk. Fig.4 shows the tracking frequency errors. The following conclusion can be drawn: The three algorithms have almost equal errors at high CNR, which demonstrates Inference 1 indirectly.

5. Conclusion
This paper explores the role of the filtering algorithm in the tracking loop, proposes two inferences and designs a new combined strong tracking algorithm.

a) The mathematical derivation process of the carrier tracking MLE shows that the algorithm is computationally intensive and dependent on the initial value for the solution, which may result in divergence and low real-time performance.

b) The state model has an effect on the carrier tracking capability at high dynamic, while the measurement model compromises the carrier tracking capability at low CNR.

c) No matter for low CNR or high dynamic signals, the new combined strong tracking algorithm is superior to other algorithms.

d) Based on the principle of filtering modeling, the two inferences are proposed qualitatively according to simulation results, without any strict mathematical derivation. In addition, rather than all the existing algorithms, only the mainstream algorithms are analyzed. The author will further study towards these two directions in the near future.

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