Boundary Conditions and Predictions of Quantum Cosmology

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Abstract

A complete model of the universe needs at least three parts: (1) a complete set of physical variables and dynamical laws for them, (2) the correct solution of the dynamical laws, and (3) the connection with conscious experience. In quantum cosmology, item (1) is often called a ‘theory of everything,’ and item (2) is the quantum state of the cosmos. Hartle and Hawking have made the ‘no-boundary’ proposal, that the wavefunction of the universe is given by a path integral over all compact Euclidean 4-dimensional geometries and matter fields that have the 3-dimensional argument of the wavefunction on their one and only boundary. This proposal has had several partial successes, mainly when one takes the zero-loop approximation of summing over a small number of complex extrema of the action. However, it has also been severely challenged by an argument by Susskind.
1 Introduction

A complete model of the universe needs at least three parts:

1. A complete set of physical variables (e.g., the arguments of the wavefunction) and dynamical laws (e.g., the Schrödinger equation for the wavefunction, the algebra of operators in the Hilbert space, or the action for a path integral.) Roughly speaking, these dynamical laws tell how things change with time. Typically they have the form of differential equations.

2. The correct solution of the dynamical laws (e.g., the wavefunction of the universe). This picks out the actual quantum state of the cosmos from the set of states that would obey the dynamical laws. Typically a specification of the actual state would involve initial and/or other boundary conditions for the dynamical laws.

3. The connection with conscious experience (e.g., the laws of psycho-physical experience) These might be of the form that tells what conscious experience occurs for a possible quantum state for the universe, and to what degree each such experience occurs (i.e., the measure for each set of conscious experiences [1]).

Item 1 alone is called by physicists a TOE or ‘theory of everything,’ but it is not complete by itself. Even Items 1 and 2 alone are not complete, since by themselves they do not logically determine what, if any, conscious experiences occur in a universe.

2 The Hartle-Hawking Proposal for the Quantum State

Here I shall focus on Item 2, the quantum state of the cosmos, and in particular focus on a proposal by Hawking [2] and by Hartle and Hawking [3] for this quantum state. They have proposed that the quantum state of the universe, described in canonical quantum gravity by what we now call the Hartle-Hawking wavefunction, is given by a path integral over compact four-dimensional Euclidean geometries and matter fields that each have no boundary other than the three-dimensional geometry and matter field configuration that is the argument of the wavefunction.

In particular, the wavefunction for a three-geometry given by a three-metric $g_{ij}(x^k)$, and for a matter field configuration schematically denoted by $\phi^A(x^k)$, where the three-metric and the matter field configuration are functions of the three spatial coordinates $x^k$ (with lower-case Latin letters ranging over the three values \{1, 2, 3\}), is given by the wavefunction

$$\psi[g_{ij}(x^k), \phi^A(x^k)] = \int D[g_{\mu\nu}(x^\alpha)]D[\phi^\Omega(x^\alpha)]e^{-I[g_{\mu\nu},\phi^\Omega]},$$

(1)
where the path integral is over all compact Euclidean four-dimensional geometries that have the three-dimensional configuration \([g_{ij}(x^k), \phi^A(x^k)]\) on their one and only boundary. Here a four-geometry are given by a four-metric \(g_{\mu\nu}(x^\alpha)\), and four-dimensional matter field histories are schematically denoted by \(\phi^\Omega(x^\alpha)\), both functions of the four Euclidean spacetime coordinates \(x^\alpha\) (with lower-case Greek letters ranging over the four values \(\{0, 1, 2, 3\}\)).

3 Incompleteness of the Hartle-Hawking Proposal

The Hartle-Hawking ‘one-boundary’ proposal is incomplete in various ways. For example, in quantum general relativity, using the Einstein-Hilbert-matter action, the path integral is ultraviolet divergent and nonrenormalizable \([4]\). This nonrenormalizability also occurs for quantum supergravity \([5]\). String/M theory gives the hope of being a finite theory of quantum gravity (at least for each term of a perturbation series, though the series itself is apparently only an asymptotic series that is not convergent.) However, in string/M theory it is not clear what the class of paths should be in the path integral that would be analogous to the path integral over compact four-dimensional Euclidean geometries without extra boundaries that the Hartle-Hawking proposal gives when general relativity is quantized.

Another way in which the Hartle-Hawking ‘one-boundary’ proposal is incomplete is that conformal modes make the Einstein-Hilbert action unbounded below, so the path integral seems infinite even without the ultraviolet divergence \([6]\). If the analogue of histories in string/M theory that can be well approximated by low-curvature geometries have actions that are similar to their general-relativistic approximations, then the string/M theory action would also be unbounded below and apparently exhibit the same infrared divergences as the Einstein-Hilbert action for general relativity. There might be a uniquely preferred way to get a finite answer by a suitable restriction of the path integral, but it is not yet clear what that might be.

A third technical problem with the Hartle-Hawking path integral is that one is supposed to sum over all four-dimensional geometries, but the sum over topologies is not computable, since there is no algorithm for deciding whether two four-dimensional manifolds have the same topology. This might conceivably be a problem that it more amenable in string/M theory, since it seems to allow generalizations of manifolds, such as orbifolds, and the generalizations may be easier to sum over than the topologies of manifolds.

A fourth problem that is likely to plague any proposal for the quantum state of the cosmos is that even if the path integral could be uniquely defined in a computable way, it would in practice be very difficult to compute. Thus one might be able to deduce only certain approximate features of the universe from such a path integral.

One can avoid many of the problems of the Hartle-Hawking path-integral, and achieve some partial successes, by taking a ‘zero-loop’ approximation\([7]\).
4 Partial Successes of the Hartle-Hawking Proposal

Despite the difficulties of precisely defining and evaluating the Hartle-Hawking ‘one-boundary’ proposal for the quantum state of the universe, it has had a certain amount of partial successes in calculating certain approximate predictions for highly simplified toy models:

1. Lorentzian-signature spacetime can emerge in a WKB limit of an analytic continuation [2, 3].
2. The universe can inflate to large size [2].
3. Models can predict near-critical energy density [2, 8].
4. Models can predict low anisotropies [9].
5. Inhomogeneities start in ground states and so can fit cosmic microwave background data [10].
6. Entropy starts low and grows with time [11, 12, 13].

5 Susskind’s Objection to the Hartle-Hawking Proposal

Leonard Susskind [14, 15, 16, 17] has argued that the cosmological constant or quintessence or dark energy that is the source of the present observations of the cosmic acceleration [18, 19] would give a large Euclidean 4-hemisphere as an extremum of the Hartle-Hawking path integral that would apparently swamp the extremum from rapid early inflation. Therefore, to very high probability, the present universe should be very nearly empty de Sitter spacetime, which is certainly not what we observe.

This argument is a variant of Vilenkin’s old objection [20] that the no-boundary proposal favors a small amount of inflation, whereas the tunneling wavefunction favors a large amount. Other papers have also attacked the Hartle-Hawking wavefunction [21, 22, 23]. However, Susskind was the first to impress upon me the challenge to the Hartle-Hawking no-boundary proposal from the recent cosmic acceleration.

Of course, it may be pointed out that most of de Sitter spacetime would not have observers and so would not be observed at all, so just the fact that such an unobserved universe dominates the path integral is not necessarily contrary to what we do observe. To make observations, we are restricted to the parts of the universe which have observers. One should not just take the bare probabilities for various configurations (such as empty de Sitter spacetime in comparison with a spacetime that might arise from a period of rapid early inflation). Rather, one
should consider conditional probabilities of what observers would see, conditional upon their existence [24, 1, 25].

However, the bare probability of an empty de Sitter spacetime forming by a large 4-hemisphere extremum of the Hartle-Hawking path integral dominates so strongly over that of a spacetime with an early period of rapid inflation that even when one includes the factor of the tiny conditional probability for an observer to appear by a vacuum fluctuation in empty de Sitter, the joint probability for that fluctuation in de Sitter dominates over the probability to form an inflationary universe and thereafter observers by the usual evolutionary means. Therefore, the argument goes, almost all observers will be formed by fluctuations in nearly empty de Sitter, rather than by the processes that we think occurred in our apparently inflationary universe.

The problem then is that almost all of these fluctuation observers will not see any significant ordered structures around them, such as the ordered large-scale universe we observe. Thus our actual observations would be highly atypical in this no-boundary wavefunction, counting as strong observational evidence against this theory (if the calculation of these probabilities has indeed been done correctly). As Dyson, Kleban, and Susskind put it in a more general challenge to the theories with a cosmological constant [15], “The danger is that there are too many possibilities which are anthropically acceptable, but not like our universe.” See [26, 27] for further descriptions of this general problem.

The general nature of this objection was forcefully expressed by Eddington 75 years ago [28]: “The crude assertion would be that (unless we admit something which is not chance in the architecture of the universe) it is practically certain that at any assigned date the universe will be almost in the state of maximum disorganization. The amended assertion is that (unless we admit something which is not chance in the architecture of the universe) it is practically certain that a universe containing mathematical physicists will at any assigned date be in the state of maximum disorganization which is not inconsistent with the existence of such creatures. I think it is quite clear that neither the original nor the amended version applies. We are thus driven to admit anti-chance; and apparently the best thing we can do with it is to sweep it up into a heap at the beginning of time.”

In Eddington’s language, Susskind’s challenge is that the Hartle-Hawking no-boundary proposal seems to lead to pure chance (the high-entropy nearly-empty de Sitter spacetime), whereas to meet the challenge, we need to show instead that somehow in the very early universe (near, if not at, the “beginning of time”) it actually leads to anti-chance, something far from a maximal entropy state.

For further details of Susskind’s challenge, see my recent account [29].

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