2D Self-Similar Profile for Laser Beam Propagation in Medium with Saturating Multi-Photon Absorption

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Abstract. We study a self-similar mode of 2D laser beam propagation in media with multi-photon absorption (MA) taking into account a resonant nonlinearity and nonlinear absorption saturating. An analytical solution of the corresponding equations describing the problems under consideration is derived using an eigenvalue problem method generalization for soliton-like solution finding. The developed solution is used as incident beam profile and phase front for computer simulation of the 2D laser beam propagation. In particular, we demonstrate numerically that the laser beam propagation in a self-similar mode occurs within a certain distance, which depends on medium properties. Under certain relations between the nonlinear absorption and resonant nonlinearity, and cubic nonlinear response, we observe the super long distance of the beam propagation without any beam profile distributions.

1. Introduction
Soliton (or soliton-like) profile of a laser beam (shape of a laser pulse) is very attractive problem among other nonlinear optics problems (see, for example [1-20]). In most researches, the soliton mode of laser pulse or beam propagation is investigated for transparent media or media with linear absorption. This can be explained by wide practical applications. Nevertheless, in some cases for soliton formation understanding, a study of laser pulse soliton appearance due to a nonlinear absorption of a medium is needed. Thus, in [21] it was emphasized that the two-photon absorption (TPA) and free-carrier effects impact on the soliton behaviour in silicon. The paper authors reported the first experimental demonstration of soliton-effect pulse compression in silicon despite TPA and free carrier generation at the picosecond pulse propagation. Analytical and numerical study of multi-photon absorption (MA) influence and free carrier influence, as well as disorder-induced linear scattering influence on laser pulse propagation in slow-light photonic crystal waveguides was discussed in [22]. In [23] a soliton evolution was demonstrated experimentally and numerically for the plasma multi-photon generation regime in highly-nonlinear and highly-dispersive photonic crystal. Numerical simulation of femtosecond pulse propagation in a silicon photonic nano-wire waveguide was conducted using the generalized nonlinear Schrödinger equation taking into account a TPA. The soliton propagation in optical fibers with non-Kerr nonlinearities was investigated analytically and numerically in [24-25].
Among a number of methods, developed for soliton finding, an approach based on the appropriate nonlinear eigenvalue problem solution, should be emphasized. Let us note, that each this problem requires a unique iterative method for soliton finding because the corresponding equation is a nonlinear one. Therefore, it is also true for the medium with nonlinear absorption. However, in this case, the soliton (self-similar profile of intensity distribution) finding is a more difficult task. Such kind of laser pulse propagation modes was discussed in [26-28] for both homogeneous and layered medium. In these papers we investigated also a cubic nonlinear response influence on the self-similar mode realization for the laser pulse propagation. In [29-30] we demonstrated the possibility of laser pulse propagation with the self-similar profile in a medium, possessing a resonant or cubic nonlinearity (self-focusing or defocusing) and MA of optical energy. Below we generalize our approach for 2D problem solution and carry out the corresponding computer simulation.

2. Formulation of the problem

2D laser beam propagation in a medium with MA is described by the dimensionless Schrödinger equation

\[ \frac{\partial A}{\partial z} + iD_x \frac{\partial^2 A}{\partial x^2} + iD_y \frac{\partial^2 A}{\partial y^2} + i\beta A + \beta(\delta(1+i\Delta)) \frac{|A|^4}{1+|A|^2 + I_{sat}^{2+1}} + iz|A|^2 A = 0, \quad 0 < z \leq L_z, \quad 0 < x < L_x, \quad 0 < y < L_y \] (1)

with the following initial and boundary conditions

\[ A(z,0,y) = A(z,L_x,y) = A(z,x,0) = A(z,x,L_y) = 0, \quad 0 \leq z \leq L_z, \quad A(0,x,y) = A_0(x,y), \quad 0 \leq x \leq L_x, \quad 0 \leq y \leq L_y. \] (2)

In (1)-(2) \( A(z,x,y) \) is the dimensionless complex amplitude; \( z \) is the dimensionless coordinate, along which a laser beam propagates, \( x \) and \( y \) are normalized coordinates which are transverse to the propagation direction, \( L_x, L_y \) are their maximal values. Parameters \( D_x, D_y \) characterize the laser beam diffraction. Parameter \( \beta \) describes a constant phase shift at a laser beam propagation, in particular. The integer constant \( k \) determines the MA type. The value \( k=2 \) corresponds to two-photon absorption (TPA), and \( k=4 \) means three-photon absorption (ThPA). The constant \( \delta \) characterizes an amplitude of the medium nonlinear absorption (or amplification). The sign of this constant depends on the medium type: \( \delta > 0 \) means a medium with absorption, and vice versa, if \( \delta < 0 \), we deal with an amplifying medium. The constant \( \alpha \) describes a cubic nonlinear response of the medium. The parameter \( \Delta \) relates to the detuning between the carrier frequency of a wave packet and the energy transition frequency for an atom or a molecule. Parameter \( I_{sat}^{2+1} \) describes absorption saturation.

3. Beam propagation in a medium with nonlinear absorption only

Firstly, we consider a beam propagating in a medium without Kerr and resonant nonlinear responses \((\alpha, \Delta = 0)\) without an absorption saturation \((I_{sat}^{2+1} \to \infty)\). Let us represent the complex amplitude in the form

\[ A(z,x,y) = B(x,y) e^{-izc}. \] (3)

Substituting (3) in (1) and solving the resulting eigenvalue problem, we get the following soliton-like solution

\[ B(x,y) = ch^{-n}(c_1 x + c_2 y) e^{if(x,y)}, \quad n = \frac{2}{k}, \quad D_x c_1^2 + D_y c_2^2 = \frac{\beta \delta k^2}{3 \sqrt{(k+2)(k+4)}}, \]

\[ f(x,y) = \frac{k}{\sqrt{2}} \left( \frac{1}{2} + \ln(ch(c_1 x + c_2 y)) \right) + const. \quad \lambda = \frac{\beta - \frac{2k \beta \delta}{\sqrt{(k+2)(k+4)}}}{\frac{4 \beta \delta}{(k+4)}}. \] (4)
In (4) \( c_1 \) and \( c_2 \) are constants, \( f(x, y) \) is the phase distribution, and \( \lambda \) is the eigenvalue, determined by the MA type. Function \( B(x, y) \) is the corresponding eigenfunction. It is important that the self-similar solution (4) is in a perfect accordance with 1D solution derived in [27]. Let us note that the equalities (4) determine a variety of the eigenfunctions since for each value of the parameter \( k \) we obtain different combinations of \( c_1 \) and \( c_2 \) values, satisfying the third expression in (4).

Obviously, beam radii as well as its maximal intensity change at the beam propagation in a medium with nonlinear absorption. To take into account both these changes and eigenfunction (4) one can search a self-similar beam profile evolution in the following form

\[
A(z, x, y) = \rho(z, x, y)e^{i\phi(z, x, y)}, \quad \rho = \rho_0(z)ch^{-n}(c_1(z)x + c_2(z)y), \quad s = \sqrt{n^2 + n\ln(ch(c_1(z)x + c_2(z)y))}. \quad (5)
\]

Substituting the functions (5) into equation (1), and neglecting the derivatives of \( c_1 \) and \( c_2 \), with respect to the longitudinal coordinate, after some transformations we get

\[
n = \frac{2}{k} \phi(z) = \frac{1}{2\sqrt{2(2 + k)}} \ln(z + z_0) - \beta z, \quad z_0 = \frac{k + 4}{4k \beta \delta}. \quad (6)
\]

From (6) it is easy to see that the expression for \( D_x c_1^2 + D_y c_2^2 \) turns into (4) in section \( z = 0 \).

Obviously, disregarding of derivatives of \( c_1 \) and \( c_2 \) is in a contradiction with the assumption (5). Nevertheless, it is necessary to keep in mind two circumstances. First, the maximal intensity of the beam connects with its radii via the power integral and we can see this dependence from (6). Second, these derivatives are less with respect to the derivative from other functions on \( z \)-coordinate. Third, if we consider the laser beam propagation in a medium with TPA (\( k = 2 \)), for simplicity, and neglect the diffraction influence, the amplitude \( \rho_0(z) \) evolution is similar to a solution of the simplified equation (see (1))

\[
\frac{dA}{dz} + \beta(i + \delta |A|^2)A = 0, \quad 0 < z \leq L_x, \quad 0 < x < L_x, \quad 0 < y < L_y. \quad (7)
\]

Multiplying both parts of (7) by \( A^* \) and supposing a equality \( |A(z = 0)| = 1 \), we come to the following formula which will be referenced below

\[
|A(z)|^2 = (2\beta\delta z + 1)^{-1}. \quad (8)
\]

This intensity evolution law is similar to the corresponding expression for the function \( \rho_0(z) \) from (6). Therefore, it is to be expected that at least for weak diffraction case, our analytical consideration will be valid. As will be shown below, the application field of the expressions (6) is much wider.

4. Self-similar mode of beam propagation in a medium with TPA

In addition to a soliton-like profile developing it is necessary to demonstrate that this profile is rather stable at beam propagation. In this section, we present the computer simulation results of the problem (1)-(2) for \( k = 2, \alpha = 0, \Delta = 0, I_{iak}^{k/2+1} \to \infty, c_1 = c_2 \). The last equality corresponds to the symmetrical beam propagation.

As initial complex amplitude distribution \( A_0(x, y) \), the derived solution for the eigenvalue problem in the form (4) is used. For definitely, in our computations the parameter \( \beta \) is equal to \( \beta = 1 \), if other its value is not specified. For practice, the preservation of the incident profile of a beam, propagating
at significant diffraction influence \(D_x = D_y = 1\), is of interest. It is a crucial key for developed solution. For example, computer simulation results, obtained at \(\delta = 5\), is depicted in Fig. 1. We compare the analytical dependences (6) with the numerically computed maximum intensity \(I_{\text{max}} = \max |A(x, y, z)|^2\) in Fig. 1a. It is evident that two curves are in a concordance, although we see a certain discrepancy between them due to the laser beam diffraction influence. Almost the same maximal intensity changing occurs if we change only the diffraction coefficients down to \(D_x = D_y = 0.1\). This fact can be explained by the prevailed influence of TPA. Absorption coefficient decreasing to \(\delta = 2.5\) results in almost perfect coincidence between \(I_{\text{max}}\) and \(\rho_0^2(z)\) in the first stage of the laser beam propagation up to \(z=0.2\) (Fig. 1b). It is also important a correlation between \(\rho_0^2(z)\) and a solution of (8), written in the frame-work of negligible diffraction assumption. In Fig. 1c we can see a good agreement between these two curves.

![Fig. 1. Maximum intensity computed using (1) (a, b) or (8) (c) (solid line) in comparison with \(\rho_0^2(z)\) (dashed line) for \(D_x = D_y = 1\), \(\delta = 5\) (a), 50 (c); \(D_x = D_y = 0.1\), \(\delta = 2.5\) (b).](image)

5. **Self-similar profile of a beam propagating in a medium with TPA and cubic nonlinearity**

In this section we consider simultaneous action of nonlinear cubic response and TPA i.e. \(k = 2, \alpha \neq 0, \Delta = 0\), \(I_{\text{sat}}^{(2)} \rightarrow \infty\). Using the representation (3), substituting it into (1), and analyzing the arising eigenvalue problem, we write the following expressions

\[
B(x, y) = c h^{\alpha} (c_1 x + c_2 y) e^{f(x, y)} f (x, y) = a \ln [h (c_1 x + c_2 y)] + \text{const},
\]

\[
(D_x c_1^2 + D_y c_2^2) (a^2 - 2) = -\alpha \beta, \quad 3a(D_x c_1^2 + D_y c_2^2) = \beta \delta, \quad \lambda = (\beta - \beta (\frac{\delta}{3a} - \alpha) - \frac{2\beta \delta}{3}).
\]  

(9)

Let us note that the parameter \(a\) in (9) is positive and can be defined as follows

\[
a = -\frac{3\alpha}{2\delta} + \sqrt{\frac{9\alpha^2}{4\delta^2} + 2}.
\]

It is possible to take into account the dependence of the derived solution on \(z\) if we include in (9) the factor \(e^{2\text{Im} z}\) in the following way:

\[
(D_x c_1^2 + D_y c_2^2) (a^2 - 2) = -\alpha \beta e^{2\text{Im} z}, \quad 3a(D_x c_1^2 + D_y c_2^2) = \beta \delta e^{2\text{Im} z}.
\]
Then, for simplicity, assuming an equality validity \( D_y = D_x = D \) and \( c_1 = c_2 = c \), we finally get

\[
\alpha = -\frac{3\alpha}{2\delta} + \frac{9\alpha^2}{4\delta^2} + 2, \quad c^2 = \frac{\beta\delta}{6D(-\frac{3\alpha}{2\delta} + \frac{9\alpha^2}{4\delta^2} + 2)}(1 + \frac{4\beta\delta}{3} z)^{\frac{1}{3}}, \quad \text{Im} \lambda = -\frac{1}{2} \ln(1 + \frac{4\beta\delta}{3} z),
\]

6. Computer simulation of a self-similar profile beam propagation in a medium with TPA and Kerr nonlinearity

Below we discuss computer simulations results, obtained for a medium with TPA and Kerr nonlinearity: \( k = 2, \alpha \neq 0, \Delta = 0, c_1 = c_2, R_{sat} \to \infty \). Incident beam with the self-similar profile (9) decays at its propagation in a medium with both focusing and defocusing nonlinearity. However, defocusing nonlinearity (strongly speaking, this nonlinear response can be realized due to resonant nonlinearity) leads to slower intensity attenuation. Moreover, with nonlinearity increasing a plateau on the curve of the maximum intensity evolution can be well seen (Fig. 2a).

![Graphs showing maximum intensity evolution](image)

**Fig. 2.** Maximum intensity evolution for \( \alpha = 7.5, 1 \) (solid lines 1, 2) and -7.5, -1 (dashed lines 1,2) (a); Intensity profile, obtained from computer simulation (solid line 1 \( z=0.18 \), 2 \( z=0.28 \)), and from (9) with the computed radius (dashed lines) for \( \alpha = 7.5 \) (b), -7.5 (c). Other parameters are: \( D_y = D_x = 1, \delta = 5 \).

For rather weak focusing/defocusing nonlinearities \( (\alpha = \pm 1) \), a beam with the initial profile (9) attenuates at a certain distance \( (z=0.3) \) without visible profile distortions. Then, the beam profile becomes distorted near the beam edges. With increasing the nonlinearity strength, a beam profile disturbs at smaller propagation. This regularity is the most pronounced for defocusing nonlinearity. In Fig. 2c, we see the considerably deformed profile of a beam \( (\alpha = -7.5) \) while, for the focusing nonlinearity, the profile is still self-similar one excepting its edges (Fig. 7b). Let us note, for this focusing nonlinearity value, the beam profile keeps its central part even after its intensity has been decreased twenty times from the initial value.

To clarify the beam profile perturbation scenario, we follow the beam phase distribution dynamics (Fig. 3 a - c). It is seen that the beam profile edge changes are accompanied by the phase distribution changes. At the initial stage of laser beam propagation the beam phase curvature corresponds to the beam focusing (Fig. 3 a). Then, phase distribution disturbs near the beam edges and, as consequence of this, the beam profile violates from its initial distribution (Fig. 3 b). Further laser beam propagation
leads to a wave front curvature changing out the central part of laser beam. Thus, two sub-beams appear (Fig. 3 c) and they move from the central sub-beam which has a self-similar profile. It should be emphasized that the phase distribution of a beam, propagating in a focusing medium, preserves its initial form.

Fig. 3. Intensity profile (solid lines) and phase distribution (dashed lines) depicted in consecutive sections z and computed for the parameters: $D_x = D_y = 1$, $\alpha = -7.5$.

Our study of 2D beam propagation in a medium with nonlinearity absorption would not be properly completed without considering self-similar beams with different initial radii on transverse coordinates: parameters $c_1$ and $c_2$. We analyzed the propagation of the beams with various relations between these parameters i.e. $c_1 > c_2$, $c_1 < c_2$, with the aim of their influence showing up. Typical regularities are shown in Fig. 4. It is evident that the beam propagation scenario remains the same for all relations between these parameters. Each picture of Fig. 4 demonstrates the beam profile, obtained using computer simulation, and the beam profile given by (9). If $c_1 > c_2$ (Fig. 4a), we show the beam profile with respect to y coordinate at the centre of the domain along x coordinate. If $c_1 < c_2$ (Fig. 4c)
we change coordinates. Comparing these pictures with the middle one (Fig. 4b), which represents the case of equal radii, we see no essential difference between them. In all cases, the beam central part coincides with the theoretical prediction. It is obvious that the beam loses its profile starting from the edges.

The last but not the least of our results, which we discuss in this section, concerns an important finding described briefly in [29]. Computer simulation of a 1D beam propagation in a medium with a resonant nonlinear response revealed that the beam propagates without any profile distortions of its profile up to the distance of 8 diffraction lengths for certain values of parameters. This scenario is especially apparent for the following set of nonlinear absorption, resonant nonlinearity, and cubic nonlinear response with parameters $\beta = 0.1, \alpha = 1, k = 4, \delta = 5, \Delta = 0.7$ (Fig. 5a). Let us remind that the following equality takes place $k=4$. It means three-photon absorption occurring.

Fig. 5. Intensity profile evolution, obtained using computer simulation for the parameters $\alpha=1$, $\Delta=0.7$, $\Delta=0.7$ in 1D case (a) or in 2D case (b).

Note that we have not yet derived an analytical solution for a 1D beam, nor have done it in 2D case. For the numerical simulation in [29] we used a self-similar profile, computed by numerical solving of the eigenfunction problem, corresponding to (1). This technique was developed in [26-30] for 1D case. 2D case is considerably more complicated and has not yet been realized for the nonlinearities mentioned above. In order to simulate a 2D beam propagation in self-similar mode we constructed the following approximation for the initial beam profile. The 1D beam profile, say discrete complex function $F(x_j)e^{i\sigma(x_j)}$, where index $j$ relates to a mesh nodes, computed from the eigenvalue problem, is used as a base for the intensity profile approximation along both axes x and y. Complex amplitude coincides with this function at the beam centre along y-coordinate. Then, to construct the dependence of the initial distribution of complex amplitude on y-coordinate, we multiply the function $F(x_j)e^{i\sigma(x_j)}$ by $F(y_l)e^{i\sigma(y_l)}$, which coincides with value of $F(x_j)e^{i\sigma(x_j)}$ if values of the discrete coordinates are equal: $y_l = x_j$. In fact, our approximation is given by the formula $F(x_j, y_l) = F(x_j)F(x_l)e^{i\sigma(x_j)+i\sigma(x_l)}$, where indices $j$ and $l$ relate to the grid nodes in a 2D domain.

Fig. 5 sums up our computer simulation results. It is well seen that the 2D beam attenuates at a greater rate than the 1D beam. In addition, the distance of beam travelling at the self-similar profile is shorter than in 1D case. After the distance, which is equal to 5 diffraction lengths, the 2D beam starts to disturb. Its profile is completely broken down and the amplitude decreases more than ten times.
Probably, the achieved length of the 2D beam propagation in a self-similar mode could be explained by a rather rough approximation of the initial profile.

7. Propagation of laser beam in a medium with resonant nonlinearity and saturation

In this section we report the computer simulation results, obtained for the parameters: $\kappa_{sl} = 1.7$, $k = 4$, $\alpha \neq 0$, $\beta = 0.1$, $\Delta \neq 0$, $\delta \neq 0$. It means the joint action of MA, and resonant nonlinearity, and Kerr nonlinearity, as well as absorption saturation. In our knowledge, there are no papers, in which the analytical solution for this problem is developed. However, in [30] we reported about self-similar profile of a 1D beam, propagating the medium with resonant nonlinear response and its saturation. This result was obtained using a computer simulation. Below, we use this beam profile as initial beam profile for the problem (1)-(2). Particularly, remarkable result, got for 1D case, is shown in Fig. 6 for both signs of laser beam frequency detuning.

The beam attenuates greatly for positive sign of $\Delta$, but the central part of its self-similar profile preserves at a longer distance than at the laser beam propagation in the medium with negative resonant response (Fig. 6a,b). The beam phase differs essentially for both signs of $\Delta$. The latter circumstance is illustrated by Fig. 6 c, d.

Similar regularities are revealed for 2D beam propagation. Note that in the process of computations we used a beam with the self-similar profile, described in the last part of the previous section, as initial condition. In Fig. 7 the significant self-similar profile deformation is clearly seen for negative resonant medium response.

![Fig. 6. Intensity profile (a, b) and phase distribution (c, d) at various distances (shown in the pictures), computed for 1D laser beam propagation with parameters: $D_s = 0.4$, $\delta = 5$, $\alpha = 2$, $\Delta = 0.7$ (a, c), - 0.7 (b, d).]
Fig. 7. Intensity profile at different distances (shown in the pictures), computed for 2D laser beam propagation with parameters: $D_x = D_y = 0.4$, $\delta = 5$, $\alpha = 2$, $\Delta = 0$ (a), 0.7 (b), -0.7 (c).

8. Conclusion

We derived analytically the 2D self-similar profile of a laser beam propagating in a media with both various types of MA and various nonlinear phase grating. Our computer simulation confirmed the results of analytical consideration. Under certain nonlinear absorption, and resonant nonlinearity, and cubic nonlinear response, the self-similar beam propagates along super long distance without any disturbance of its profile.

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