Asymmetric Weyl semimetals, which possess an inherently chiral structure, have different energies and dispersion relations for left- and right-handed fermions. They exhibit certain effects not found in symmetric Weyl semimetals, such as the quantized circular photogalvanic effect and the helical magnetic effect. In this work, we derive the conditions required for breaking chiral symmetry by applying an external field in symmetric Weyl semimetals. We explicitly demonstrate that in certain materials with the $T_d$ point group, magnetic fields along low symmetry directions break the symmetry between left- and right-handed fermions; the symmetry breaking can be tuned by changing the direction and magnitude of the magnetic field. In some cases, we find an imbalance between the number of type I left- and right-handed Weyl cones (which is compensated by the number of type II cones of each chirality.)

**I. INTRODUCTION**

Dirac [1–10] and Weyl [11–18] semimetals have linear and gapless dispersion relations, forming effective low-energy massless fermions (see Ref. [19] for a review). They exhibit interesting effects such as the chiral magnetic effect [20] and negative longitudinal magnetoresistance [21, 22].

Massless fermions have a well-defined chirality given by the sign of the product of momentum and (pseudo)spin. Dirac semimetals have doubly degenerate bands, with fermions of both chiralities at the same place in the Brillouin zone; their band crossings are not topologically protected, but can be symmetry protected [6]. In contrast, in Weyl semimetals, fermions of different chiralities are separated in momentum space. Consequently, Weyl semimetals exhibit some chirality-dependent effects not found in Dirac semimetals, such as a photocurrent induced by circularly polarized light [23, 24], and elliptically polarized terahertz emission in response to pulsed circularly polarized near infrared light [25].

Certain phenomena require the left- and right-handed fermions to have different energies or velocities. They include the quantized circular photogalvanic effect [22], the helical magnetic effect [23] and the chiral magnetic effect without a source of chirality [24]. These effects are possible only in materials that have chiral crystal lattices which lack non-orientation-preserving symmetries. We refer to such materials as asymmetric Weyl materials, and materials that do have orientation-reversing symmetries as symmetric Weyl materials. Asymmetric Weyl materials include RhSi [25] and CoSi [26]. The quantized circular photogalvanic effect has been observed in both these materials [27, 28].

However, asymmetric Weyl materials are rare in nature compared to symmetric Weyl materials. One way to observe effects that depend on the absence of non-orientation symmetries is to break these symmetries by an external perturbation. It has recently been shown that a symmetric Weyl material can become asymmetric upon ordering magnetically if the magnetic moments break all non-orientation-preserving symmetries of the crystal [39].

In this work, we investigate chiral symmetry breaking more generally. Using the concept of true and false chirality introduced by Barron [40], we derive a criterion to determine whether an external field or perturbation produces an asymmetric material, which depends on the symmetry of the perturbation and the space group of the crystal. We then explicitly show that in the zincblende material InAs, applying a magnetic field in a low symmetry direction breaks all non-orientation-preserving symmetries, and causes left- and right-handed fermions to have different velocities and energies. This induced asymmetry allows for the observation of effects present only in asymmetric Weyl materials. Furthermore, we show examples where the number of type I Weyl fermions [11] of left- and right-chirality are not equal; the imbalance is compensated by the number of type II Weyl fermions of each chirality.

**II. PROPERTIES OF WEYL CONES**

The chirality of a Weyl fermion is $\chi = \text{sgn}(\vec{s} \cdot \vec{p})$ where $\vec{s}$ is the pseudospin and $\vec{p}$ is the momentum. The chirality is positive for right-handed fermions and negative for left-handed fermions. Since Weyl cones are monopoles of Berry curvature and the total Berry charge in the Brillouin zone must be zero, there is always an equal number of left- and right-handed Weyl cones in a microscopic Hamiltonian.

Under inversion symmetry ($P$), the pseudospin, mo-
momentum, and chirality transform as
\[
\begin{align*}
\vec{s} &\to -\vec{s} \\
\vec{\tilde{p}} &\to -\vec{\tilde{p}} \\
\chi &\to -\chi
\end{align*}
\] (1)

Under time-reversal symmetry (T), the pseudospin, momentum, and chirality transform as
\[
\begin{align*}
\vec{s} &\to -\vec{s} \\
\vec{p} &\to -\vec{p} \\
\chi &\to \chi
\end{align*}
\] (2)

In a material that has both inversion and time-reversal symmetries, the chirality flips under P and remains invariant under T. Crystal momentum \(k_i\) flips sign under both \(P\) and \(T\). Therefore, under \(PT\),
\[
\begin{align*}
k_i &\to k_i \\
\chi &\to -\chi
\end{align*}
\] (3)

In such a material, the left- and right-handed fermions coincide in the Brillouin zone; thus, it has Dirac cones, not Weyl cones. Therefore, all Weyl materials lack time-reversal, inversion or both.

Non-orientation-preserving symmetries transform left-handed fermions into right-handed fermion and vice versa. Therefore, in materials with these symmetries, each left-handed cone has a right-handed partner cone at the same energy and with the same velocities. Consequently, effects such as the quantized photogalvanic effect, which requires an asymmetry between left- and right-handed cones, are possible only in materials that have a chiral crystal lattice without orientation-reversing symmetries.

The cones of Weyl fermions are generally tilted. Fermions with small tilts and elliptical Fermi surfaces are called type I Weyl fermions, while those with large tilts and hyperbolic Fermi surfaces are called type II Weyl fermions [41]. A sketch of the dispersion relations of type I and type II Weyl cones is shown in Figure 1. Many Weyl semimetals have exclusively type I fermions, such as TaAs [15–17, 42]. Some materials have only type II fermions, such as WTe [43]. The Weyl semimetal OsC\(_2\) is unusual in that it has both kinds of Weyl cones—24 of type I and 12 of type II [44].

The linearized Hamiltonian for a general Weyl cone is
\[
H = v_W^i q_i + \frac{1}{2} \sigma_a q_i q_j + E,
\] (4)

where \(v_W^i\) is the “tilt” velocity and \(\sigma_a\) represents the untitled part of the Hamiltonian. \(E\) is the energy of the Weyl point. The matrices \(\sigma_a\) act on spin or pseudospin. The chirality is
\[
\chi = \text{sgn} (\det (v_W^i)),
\] (5)

which is positive for a right-handed cone and negative for a left-handed cone. We define the product of velocities
\[
v_1 v_2 v_3 = |\det (v_W^i)|,
\] (6)

FIG. 1: Dispersion relation (top) and Fermi surface (bottom) of type I (left) and type II (right) Weyl cones with tilt 0.95 and 1.05 respectively. The dashed line shows the Fermi level. Color indicates filling; dark blue indicates both bands are filled, light blue indicates one band is filled, and white indicates both bands are empty.

The velocity tensor \((v_W^2)^{ij} = v_W^i v_W^j\) and its inverse \((v_W^{-2})^{ij}\). We also define a dimensionless measure of the tilt
\[
\text{tilt parameter} = \sqrt{(v_W^{-2})^{ij} v_W^i v_W^j}.
\] (7)

The cone is type I if the tilt parameter is less than 1 and type II if it is greater than 1 [41].

### III. TRUE AND FALSE CHIRALITY

L. D. Barron introduced the idea of true and false chirality of a system [40]. A Weyl material is said to have false chirality if it possesses a symmetry \(MT\), where \(M\) is some non-orientation-preserving symmetry, but does not have the symmetry \(M\) itself. Such a material transforms to its mirror image under time reversal. Systems with true chirality retain their chirality even under time reversal. Examples of systems with true chirality include glucose and DNA molecules and the electroweak part of the Standard Model. Systems with false chirality include magnetic fields.

Asymmetric Weyl materials, such as RhSi and CoSi, have crystal structures with true chirality. Materials with false chirality cannot be asymmetric Weyl materials, because the chirality of each cone is invariant under \(T\) but flips under \(M\) (since, by definition, \(M\) reverses orientation), so left- and right-handed cones would be related by \(MT\).

In symmetric Weyl materials, it is possible to break mirror symmetries by applying external perturbations.
Such symmetry breaking may produce either true or false chirality, as we will demonstrate. As a first example, consider the transition metal monopnictide class of materials, which includes TaAs, TaP, NbAs, and NbP [13-17]. These compounds have tetragonal symmetry and are in the space group $I4_1md$ (No. 109) with fourfold rotation about the [001] axis, reflection symmetry about the [100], [010], [110], [110] planes, and time-reversal symmetry, but no inversion symmetry or reflection symmetry about the [001] plane. If we apply a magnetic field along the [001] direction, $B_z$ flips sign under reflections and therefore breaks all mirror symmetries. However, this perturbation does not introduce true chirality as $B_z$ also flips sign under time-reversal; therefore the perturbed system remains invariant under symmetries $MT$, where $M$ is a mirror reflection symmetry of the unperturbed system. Thus, TaAs with a magnetic field along the $c$ axis has false chirality and would therefore still be a symmetric Weyl material.

Only systems with true chirality will display effects that depend on a difference in energy or velocity between left- and right-handed fermions. Such effects will correspond to the expectation value of an operator, $Q$, such that $Q$ is invariant under all orientation-preserving symmetries of the unperturbed system, but flips sign under all non-orientation-preserving symmetries. For a perturbation $\lambda$ to produce true chirality in the system, there must exist a function $Q(\lambda)$ with this property.

For systems that possess time-reversal symmetry, a quantity that is of odd order in magnetic field does not qualify as $Q(\lambda)$ because it is odd under time-reversal, which preserves chirality. For example, in the transition metal monopnictides mentioned above, $B_z$ is invariant under rotation, and flips under reflections, but since it also flips under time-reversal, which preserves the chiralities of fermions, operators that are odd in $B_z$ will yield a vanishing expectation value.

In TaAs, the lowest order $Q$ as a function of magnetic field is $Q(\vec{B}) = B_x B_y (B_x^2 - B_z^2)$. Similarly, if we consider chirality induced by strain along a low symmetry axis, the lowest order $Q(S)$ is $S_{xy} (S_{xx} - S_{yy})$. Therefore, in TaAs, we expect physical phenomena such as the helical magnetic effect to be of fourth order in $\vec{B}$ or second order in $S_{ij}$. The effects of a low symmetry perturbation are illustrated in Figure 2. The rare earth carbides studied in [29] have the space group $Amnm$2 (No. 38). In systems with this symmetry, the lowest order function of magnetization $\vec{m}$ that transforms in the same way as the chirality of fermions under all symmetries of the unperturbed system is $Q(\vec{m}) = \mu_x e_y + \mu_y e_x$.

In materials that possess inversion symmetry, external magnetic fields or uniform strain cannot induce chirality, because both magnetic field and strain are even under parity, and any functions of these quantities will also be even under parity. Since most Dirac materials such as ZrTe$_5$ and Na$_3$Bi have inversion symmetry, we cannot transform them into asymmetric Weyl materials through uniform strain or uniform external magnetic field. However, it is possible to create symmetric Weyl cones and other topological phases in materials such as ZrTe$_5$ with Zeeman splitting [48].

We now specialize to the case of chirality breaking by a magnetic field. Out of the 32 point groups, 11 are enantiomorphic and already asymmetric. Another 11 are centrosymmetric; as discussed in the previous paragraph, magnetic field cannot induce chirality in these groups. In the other 10 groups, we can use a magnetic field to induce chirality breaking. We list the function $Q(\vec{B})$ in Table I for these 10 groups. The symmetry analysis in Table I holds only for perturbations that can be described by uniform time-odd pseudovectors, such as a uniform magnetic field and ferromagnetism.

But a magnetic field is not the only perturbation that can create asymmetry in Weyl materials. Weyl points can be created or manipulated by strain [49], antiferromagnetism [51], incident light [52], ferroelectricity [53, 54], and a superconducting condensate [55], for example. The symmetry analysis in Table I is not applicable to these perturbations. For example, an electric polarization along a low symmetry direction, which is described by a time-even vector, can break chiral symmetry even in materials with the $Oh$ point group. The relevant chirality breaking parameter in that case is $Q(\vec{P}) = P_x P_y P_z (P_z^2 - P_y^2)(P_y^2 - P_x^2)(P_x^2 - P_z^2)$.

In this work, we focus on a class of noncentrosymmetric materials that have the space group $F\bar{4}3m$ (No. 216) and the point group 43m($T_d$). This class includes the half-Heusler compound GdPdBi [50] and the zincblende compounds HgTe and InSb. These materials have several axes of rotation and planes of reflection, as well as time-reversal symmetry, but they lack inversion symmetry. Their electronic structure is characterized by a fourfold degeneracy at the $\Gamma$ point and no Weyl cones. However, if we apply an external magnetic field [57] or strain [49], or if there is magnetic or-
pressed as field is along a low-symmetry axis. all the emergent Weyl fermions come in pairs of opposite field along high symmetry axes, such as [111] and [100], appear. When the degeneracy is broken by a magnetic

| Group                | $Q(\vec{B})$                                      |
|----------------------|---------------------------------------------------|
| $m(C_3)$             | $B_x B_z \text{ OR } B_y B_z$                    |
| $mm2(C_{2v})$        | $B_x B_y$                                        |
| $4(S_4)$             | $B_x B_y \text{ OR } B_x^2 - B_y^2$              |
| $4mm(C_{4v})$        | $B_x B_y B_z \text{ OR } B_x^2 - B_y^2$          |
| $42m(D_{2d})$        | $B_z^2 - B_y^2$                                  |
| $3m(C_3v)$           | $(B_x - B_z)(B_x - B_y)(B_x - B_z)(B_1 + B_2 + B_3)$ |
| $6(C_{3h})$          | $B_x B_y B_z \text{ OR } (B_x - B_y)(B_x - B_z)(B_1 - B_2 - B_3)$ |
| $6mm(C_{6v})$        | $B_x B_y B_z (B_x - B_y)(B_x - B_z)(B_1 - B_2 - B_3)$ |
| $6m2(D_{3h})$        | $B_y B_z B_x (B_x - B_y)(B_x - B_z)(B_1 - B_2 - B_3)$ |
| $43m(T_d)$           | $(B_x^2 - B_y^2)(B_x^2 - B_z^2)(B_y^2 - B_z^2)$  |

TABLE I: $Q(\vec{B})$ for the 10 point groups where $\vec{B}$ can break chirality. In the first five rows and the last row, $x, y, z$ indicate the orthorhombic crystal axes; in the sixth row, 1, 2, 3 indicate the rhombohedral crystal axes; in the remaining rows, $z, a, b, c$ indicate hexagonal lattice vectors, with $\hat{a} + \hat{b} + \hat{c} = \hat{0}$. When multiple functions $Q$ are listed, it is enough for one of these quantities to be non-zero to break chiral symmetry.

dering [51] [50], the degeneracy splits, and Weyl points appear. When the degeneracy is broken by a magnetic field along high symmetry axes, such as [111] and [100], all the emergent Weyl fermions come in pairs of opposite chirality related by symmetry [57]. In the next section, we will show that this is not the case when the magnetic field is along a low-symmetry axis.

**IV. MODEL**

We consider the Hamiltonian given in Ref. [57]:

$$H = \frac{1}{2} \hat{k}^2 I_4 + C \left[ (k_x^2 - k_y^2) \Gamma_1 + \frac{1}{\sqrt{3}} (2k_x^2 - k_y^2 - k_z^2) \Gamma_2 \right]$$

$$+ E(k_x k_y \Gamma_3 + k_x k_z \Gamma_4 + k_y k_z \Gamma_5)$$

$$+ D(k_x U_x + k_y U_y + k_z U_z)$$

$$+ \mu_B g (B_x J_x + B_y J_y + J_z B_z),$$

where $J_{x,y,z}$ are the spin-3/2 matrices, which can be expressed as

$$J_x = \begin{pmatrix}
0 & 0 & 0 & 0 \\
0 & k_x^2 & 0 & 1 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & k_x^2
\end{pmatrix},$$

$$J_y = \begin{pmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & -\frac{\sqrt{3}}{2} & 0 \\
0 & 0 & 0 & -\frac{\sqrt{3}}{2} \\
0 & 0 & -\frac{\sqrt{3}}{2} & 0
\end{pmatrix},$$

$$J_z = \begin{pmatrix}
\frac{1}{2} & 0 & 0 & 0 \\
0 & \frac{1}{2} & 0 & 0 \\
0 & 0 & \frac{1}{2} & 0 \\
0 & 0 & 0 & \frac{1}{2}
\end{pmatrix}.$$  

The matrices $\Gamma_{\mu}$ are defined as

$$\Gamma_1 = \frac{1}{\sqrt{3}} (J_x^2 - J_y^2)$$

$$\Gamma_2 = \frac{1}{3} (2J_x^2 - J_y^2 - J_z^2)$$

$$\Gamma_3 = \frac{1}{\sqrt{3}} \{J_x, J_y\}$$

$$\Gamma_4 = \frac{1}{\sqrt{3}} \{J_y, J_z\}$$

$$\Gamma_5 = \frac{1}{\sqrt{3}} \{J_x, J_z\}$$

These matrices satisfy the anticommutation relations $\{\Gamma_{\mu}, \Gamma_{\nu}\} = 2\delta_{\mu\nu}$. The matrices $U_i$ are defined as

$$U_x = \frac{1}{2i} (\sqrt{3} [\Gamma_1, \Gamma_3] - [\Gamma_2, \Gamma_5])$$

$$U_y = \frac{1}{2i} (\sqrt{3} [\Gamma_1, \Gamma_4] + [\Gamma_2, \Gamma_4])$$

$$U_z = \frac{1}{i} [\Gamma_2, \Gamma_3]$$

The unperturbed Hamiltonian (at zero magnetic field) has time-reversal and $T_d$ symmetry. The coefficient $D$ represents an inversion breaking term.

We seek an expression $Q(\vec{B})$ that indicates true chirality. As discussed in the previous section, $Q(\vec{B})$ must be even(odd) under symmetries that maintain(reverse) the chirality of a Weyl cone. The expression $B_x B_y B_z$ represents false chirality because it is odd under time-reversal and even under $S_4 T$, where $S_4$ is the rotoinversion symmetry about the $z$ axis that maps $(x, y, z) \mapsto (y, -x, z)$. The lowest order polynomial of $B$ that represents a true chirality breaking term is $Q(\vec{B}) = (B_x^2 - B_y^2)(B_y^2 - B_z^2)(B_z^2 - B_x^2)$, as listed in Table I. It is non-zero only when the magnetic field is away from high symmetry axes such as [100], [110], and [111]. In what follows, we will use the dimensionless quantity

$$Q'(\vec{B}) = (B_x^2 - B_y^2)(B_y^2 - B_z^2)(B_z^2 - B_x^2)/B^6$$

as a measure of chirality breaking. $Q'(\vec{B})$ takes a maximum value of 0.0962 when $\vec{B} = (0, 0.460, 0.888)$ or points in a symmetry-related direction.

The Hamiltonian defined by equation (8) is a lowest order expansion in $\hat{k}$ that is only valid for a finite range of crystal momentum and energy; at higher energies, mixing with other bands becomes important. Therefore, when we search for Weyl points in this Hamiltonian with an applied magnetic field, we choose a cut-off in crystal momentum and energy and only focus on Weyl points that are within this cutoff. We ensure that the cutoff always contains the same number of left- and right-handed Weyl points.

We will now focus on the zincblende material InSb. It has six bands closest to the Fermi level, consisting of a set of four valence bands that meet at $\Gamma$ very close to
When a magnetic field is applied along a general low-symmetry direction, the induced Weyl points are not related to each other by any symmetry. There is no analytical expression for their positions and each one must be located numerically by diagonalizing Eq. (8). We will search for Weyl points within the cutoffs $k < 0.032 \, \text{Å}^{-1}$ and $E < 50 \, \text{meV}$. For all magnitudes and directions of magnetic field considered in our work, there is an equal number of left- and right-handed Weyl cones within these cutoffs.

In addition to $Q'(\vec{B})$, we characterize the chirality breaking by a dimensionless parameter

$$\delta_v = \sum \frac{\chi v_1 v_2 v_3}{\sum v_1 v_2 v_3}, \quad (15)$$

where the product of velocities $v_1 v_2 v_3$ is defined in [6] and the sum is over all the cones. We also define a dimensionful chirality breaking parameter

$$\delta_E = \sum \frac{\chi E}{n} \quad (16)$$

where $n$ is the total number of right-handed Weyl cones (equal to the total number of left-handed Weyl cones). Physically, $\delta_v$ represents the average difference in velocities of the left- and right-handed Weyl cones normalized by the average velocity, while $\delta_E$ represents the average difference in energy between left- and right-handed Weyl cones. While $Q'(\vec{B})$ serves as a quick check to determine which low-symmetry directions are likely to have the largest chirality breaking, $\delta_v$ and $\delta_E$ are directly related to known physical observables [32, 33]. Note that $Q'(\vec{B}) \neq 0$ is a necessary condition for the left- and right-handed cones to have different energies and velocities, and thus for $\delta_v, \delta_E$, and relevant physical observables to be non-zero.

In Table IV, we have listed the number of type I and type II Weyl cones of each chirality, and the chirality breaking parameters $\delta_v$ and $\delta_E$, for different magnitudes of magnetic field along the [147] direction. Table IV shows that $\delta_v$, the dimensionless average velocity difference between left- and right-handed Weyl cones, increases with increasing magnitude of the magnetic field, even while $Q'(\vec{B})$ remains constant. The average energy difference, $\delta_E$, also increases with increasing field. Thus, we expect physical observables that depend on a difference in energy or velocity between Weyl cones will increase as the magnetic field is increased along this direction.

The same quantities are recorded in Table V for different directions of magnetic field and fixed magnitude 0.75 T. Here the three measures of chirality-breaking, $Q'(\vec{B})$, $\delta_v$ and $\delta_E$ can be compared: $\delta_v$ and $\delta_E$ show similar trends as $Q'(\vec{B})$, but are not functions of $Q'(\vec{B})$. Because of the complexities of the band structure and topology, they do not even necessarily vary monotonically with $Q(\vec{B})$ or $Q'(\vec{B})$: they can change sign or even be zero for some low-symmetry directions of $\vec{B}$. Both Tables IV and V show that it is not unusual to find different numbers of left- and right-handed Weyl cones of the same type in this model.

In Table V we see that while the number of left and right handed cones of each chirality remains the same as we change the magnitude of the magnetic field, the number type I (and type II) right handed cones changes. Type I and type II cones have very different Fermi surfaces as shown in Figure 1. When the tilt parameter is close to 1, even a small change in the parameters of the Hamiltonian results in a drastic change in the Fermi surface.

As an example, the positions of cones of different types and chiralities for a magnetic field of 0.75 T along the directions [001], [111], and [147] are shown in Figure 2.
\[
\begin{array}{cccccccc}
\hline
k_x & k_y & k_z & \chi & \text{Energy Bands} & v_1v_2v_3 & \text{Tilt} & \text{Type} \\
(10^8 \text{ m}^{-1}) & & & & (eV^3) & (pm^3) & & \\
\hline
-0.229 & -0.569 & 0.782 & L & -1.36 & 2-3 & 5830 & 0.706 & I \\
-0.755 & -0.572 & -0.802 & L & 1.82 & 3-4 & 13200 & 1.19 & II \\
0.812 & -0.432 & -0.782 & L & 1.32 & 2-3 & 9350 & 1.20 & II \\
-0.083 & -0.302 & 1.093 & L & 0.20 & 2-3 & 1780 & 1.87 & II \\
-2.114 & -0.120 & -0.304 & L & 11.72 & 3-4 & 12.6 & 333 & II \\
-0.016 & -0.579 & 1.058 & R & 0.10 & 2-3 & 2950 & 0.585 & I \\
0.310 & 0.462 & -0.733 & R & -1.59 & 1-2 & 4390 & 0.710 & I \\
0.735 & 0.533 & 0.837 & R & 1.81 & 2-3 & 11400 & 1.32 & II \\
-0.762 & 0.344 & 0.865 & R & 1.30 & 2-3 & 6430 & 1.57 & II \\
-1.648 & -0.355 & -0.119 & R & 7.45 & 3-4 & 44.7 & 40.1 & II \\
\hline
\end{array}
\]

**TABLE III:** Weyl cones for a magnetic field of 0.75T along the low symmetry direction [147]. The first three columns specify the crystal momentum of the Weyl point; the next columns indicate its chirality; its energy; the two bands that comprise it, where band 1 has the lowest energy; its product of velocities; its tilt; and whether it is type I or type II. The chirality \( \chi \), product of velocities \( v_1v_2v_3 \) and tilt are defined in Eqs. (5), (6), and (7), respectively.
under Awards DE-SC-0017662 (S. K.) and DE-FG02-88ER40388 (E. J. P.) and by the National Science Foundation under award DMR-1942447 (J. C.). J. C. acknowledges the support of the Flatiron Institute, a division of the Simons Foundation. J. C. and S. K. also acknowledge the support of an OVPR Seed Grant from Stony Brook University.

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