Anisotropic Compact Stars in $f(R)$ Gravity

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Abstract

In this paper we have investigated the possibility of forming of anisotropic compact stars in $f(R)$ gravity, one of the competent candidates of dark energy. To this end, we have applied the analytical solution of Krori and Barua metric to a static spherically symmetric spacetime in $f(R)$ gravity. The unknown constants in Kröri and Barua metric have been determined by using masses and radii of class of compact stars like 4U 1820-30, Her X-1, SAX J 1808-3658. The properties of these stars have been analyzes in detail. Furthermore, we have checked the regularity conditions, energy conditions, anisotropic behavior, stability and surface redshift of the compact stars 4U 1820-30, Her X-1, SAX J 1808-3658.

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1 Introduction

The discovery of cosmic acceleration is one the major advancements in modern cosmology. The observation of type Ia supernovae (SNe Ia) combined with observational probes of numerous mounting astronomical evidences like the cosmic microwave background (CMB), large scale structure surveys (LSS) and Wilkinson Microwave Anisotropy Probe (WMAP) \[1\]-\[4\] reveal that the cosmos at the present is dominated by exotic energy component named as dark energy (DE). The investigation of current cosmic expansion and nature of DE has been widespread among the scientists. For this purpose, numerous efforts have been made based upon different strategies. These efforts can be grouped in two categories: introducing new ingredients of DE to the entire cosmic energy and modification of Einstein-Hilbert action to obtain modified theories of gravity such as $f(R)$ \[5\], $f(T)$ \[6\] where $T$ being the torsion, $f(R, T)$ \[7\] where $R$ and $T$ represent the scalar curvature and trace of the energy-momentum tensor, $f(R, T, R_{\mu\nu}T^{\mu\nu})$ \[8\] and Gauss-Bonnet gravities \[16\].

Recently, it has been shown that such theories of gravity would provide such models which have the capability to reproduce the Hubble diagram derived from SNelave surveys \[9, 10\]. However, this approach require some experimental results in order to be accepted or rejected. Some exotic compact objects which cannot be addressed by standard gravity, could constitute a powerful tool to address this problem. The strong field of relativistic objects could explain the difference between General Relativity and its modified forms. The modeling of relativistic stars in the modified theories of gravity have very interesting consequences \[11, 12\]. According to Psaltis \[13\] the strong gravitational fields could be considered as modified theories of gravity if one assume General Relativity as the weak field limit of some more complicated effective gravitational theory. More specifically, considering the extension of General Relativity, as the $f(R)$ gravity, some models like Chameleon Mechanism \[14\] can explain the stability of stars. Recently, Astashenok et al. \[15\] have discussed the stability of neutron stars in $f(R)$ gravity.

It has been the subject of great interest to study the models of anisotropic stars during the last decades. Egeland \[17\] investigated that the cosmological constant would exist due to density of the vacuum, this is consequence of modeling the mass and radius of the Neutron star. In order to prove this fact Egeland used the relativistic equation of hydrostatic equilibrium with fermion equation of state (EoS). As $f(R)$ model with constant $R$ gives cosmological
constant, therefore motivated by this fact, we study the structure of strange stars and concluded that $f(R)$ gravity with model $f(R) = R + \lambda R^2$ (where $\lambda$ is constant) can describes the anisotropic compact stars candidates X-ray bruster 4U1820-30, X-ray pulsar Her X-1, Millisecond pulsar SAX J 1808-3658. During the recent years, Dey et al. [18], Usov [19], Ruderman [20], Mak and Harko [21, 22] have studied the physical properties of strange stars by using different approaches.

Using spherical symmetry of compact stars, Mak and Harko [23] presented a new class of exact solutions of the field equations, which are the standard models of strange stars. They investigated behavior of energy density as well as tangential and radial pressure, inside the stars these quantities were found as finite and positive. Anisotropic matter distribution has been modeled in terms of a mathematical algorithm by Chaisi and Maharaj [24]. Rahaman et al. [25] provided the extension of Krori-Barua [26] models using the Chaplygin gas EOS. Lobo [27] presented the anisotropic exact models with a barotropic EOS for compact objects. He also generalized the Mazur-Mottola gravastar models by considering the matching of an interior solution governed by the dark energy EOS $\omega = \frac{p}{\rho} < \frac{-1}{3}$ to an exterior Schwarzschild vacuum solution at a junction interface. In this paper, we have formulated the spherically symmetric models of strange stars in $f(R)$ gravity which have been proposed earlier by Alcok et al. [28] and Haensel et al. [29]. Hossein et al. [30] have formulated the anisotropic compact stars in the presence of cosmological constant.

The spherically symmetric models of the compact stars proposed here are associated with $f(R)$ theory of gravity and we have study the stability of these models by calculating the speed of sound using the anisotropic property of the model. Finally, the surface redshift has been calculated using the observational data of a class of anisotropic stars. The plan of the paper is the following. In the next section, we present the anisotropic source and Einstein field equations in $f(R)$ gravity. Section 3 deals with the physical analysis of the proposed models. In the last section, we conclude the results of the paper.
2 Anisotropic Matter Configuration in $f(R)$ Gravity

The action of $f(R)$ theory of gravity in the presence of matter is given by \[ I = \int dx^4 \sqrt{-g} [f(R) + \mathcal{L}_{\text{matter}}], \] (1)

where $8\pi G = 1$, $R$ is the scalar curvature, $f(R)$ is an arbitrary function of $R$ as well as its higher powers and $\mathcal{L}_{\text{matter}}$ denotes the Lagrangian density of matter part. Variation of action (1) with respect to $g_{\mu\nu}$ yields the field equations

$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = T_{\mu\nu}^{\text{curv}} + T_{\mu\nu}^{\text{matter}},$ (2)

where $T_{\mu\nu}^{\text{matter}}$ is the stress-energy tensor of the matter scaled by a factor of $\frac{1}{f'(R)}$ and $T_{\mu\nu}^{\text{curv}}$ denotes the contribution that arises from the curvature to the effective stress-energy tensor given by

$T_{\mu\nu}^{\text{curv}} = \frac{1}{F(R)} \left[ \frac{1}{2} g_{\mu\nu} (f(R) - RF(R)) + F(R)^{\alpha\beta}(g_{\mu\alpha} g_{\nu\beta} - g_{\mu\nu} g_{\alpha\beta}) \right],$ (3)

where $F(R) = f'(R)$, prime denotes derivative with respect to the Ricci scalar $R$.

The general spherically symmetric metric which describes the strange star stellar configuration is given by \[ ds^2 = -e^{\mu(r)} dt^2 + e^{\nu(r)} dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2), \] (4)

where $\nu = Ar^2$, $\mu = Br^2 + C$ and $A$, $B$ and $C$ are arbitrary constant to be evaluated by using some physical conditions. To obtain some particular strange star models we assume the anisotropic fluid as interior of compact object and it is defined by

$T_{\alpha\beta}^{\text{matter}} = (\rho + p_t) u_{\alpha} u_{\beta} - p_r g_{\alpha\beta} + (p_r - p_t) v_{\alpha} v_{\beta},$ (5)

where $u_{\alpha} = e^{\mu/2} \delta_{\alpha}^0$, $v_{\alpha} = e^{\nu/2} \delta_{\alpha}^0$, $\rho$, $p_r$ and $p_t$ correspond to energy density, radial and transverse pressures, respectively. The modified field equations
corresponding to spacetime (4) lead to

\[ \rho = -e^{-\nu} F'' + e^{-\nu} \left( \frac{\nu'}{2} - \frac{2}{r} \right) F' + \frac{e^{-\nu}}{r^2} \left( \frac{\mu'' r^2}{2} + \frac{\mu'^2 r^2}{4} - \frac{\mu'\nu' r^2}{4} + \mu' r \right) F' - \frac{1}{2} \hat{f}, \tag{6} \]

\[ p_r = e^{-\nu} \left( \frac{\mu'}{2} + \frac{2}{r} \right) F' - \frac{e^{-\nu}}{r^2} \left( \frac{\mu'' r^2}{2} + \frac{\mu'^2 r^2}{4} - \frac{\mu'\nu' r^2}{4} - \nu' r \right) F + \frac{1}{2} \hat{f}, \tag{7} \]

\[ p_t = -e^{-\nu} F'' + e^{-\nu} \left( \frac{\mu'}{2} - \frac{\nu'}{2} + \frac{1}{r} \right) F' - \frac{e^{-\nu}}{r^2} \left( \frac{\mu' r}{2} - \frac{\nu' r}{2} - e' + 1 \right) F + \frac{1}{2} \hat{f}. \tag{8} \]

In this setup we consider an observationally reliable \( f(R) \) model namely Starobinsky model \[31\]

\[ f(R) = R + \lambda R^2, \tag{9} \]

where \( \lambda \) is an arbitrary constant. One can set \( \lambda = 0 \) to find the results in
Using the relations of GR. For this model the equation (6)-(8) become

\[
\rho = \frac{e^{-2\nu}}{8r^4} \left\{ r^4 \lambda \mu^4 - 2r^3 \lambda \mu^3 (-4 + r\nu') + r^2 \lambda \mu^2 (16 + 8r\nu' - 11r^2 \nu'^2 + 4r^2 \mu'') + 8r^2 \nu'' + 4r^2 \lambda \mu' (-16r\nu' + 3r^2 \nu'^2 + \nu' (-4 + 9r^2 \mu'' - 7r^2 \nu'') + 2r (-2\mu'') + 6\nu'' - 2r \mu'''' + r \nu''''\right) + 4(-2e^\nu r^2 + 2e^{2\nu} r^2 - 20\lambda + 24e^\nu \lambda - 4e^{2\nu} \lambda
\]

\[+ 12r^3 \lambda \nu'^3 - 3r^4 \lambda \mu'^2 - r^2 \nu'^2 (12 + 11r^2 \mu'') + 8r^2 \lambda \nu'' + 8r^4 \lambda \mu'' \mu'' - 16 \times r^3 \lambda \mu'' + 2r \nu'(e^\nu r^2 - 8\lambda + 16r^2 \lambda \mu'' - 14r^2 \lambda \nu'' + 6r^3 \lambda \mu''') + 8r^3 \lambda \nu''''
\]

\[- 4r^4 \lambda \mu''(iv)\right\},
\]

\[
(10)
\]

\[
\rho_r = \frac{e^{-2\nu}}{8r^4} \left\{ -r^4 \lambda \mu^4 - 2r^4 \lambda \mu^3 \nu' + r^3 \lambda \mu^2 (-24\nu' + 3r\nu'^2 + 4r(\mu' - \nu'')) - 4(-2e^\nu r^2 + 2e^{2\nu} r^2 + 28\lambda - 24e^\nu \lambda - 4e^{2\nu} \lambda - 12r^2 \nu'^2 - 16r^2 \lambda \mu''
\]

\[+ 8r^3 \lambda \nu'^3 + r^4 \lambda \mu'^2 + 16r^2 \lambda \nu'' - 8r^3 \lambda \mu'' + 8r \mu'(e^\nu r^2 - 8\lambda + 3r^2 \lambda \nu'^2
\]

\[+ 6r^2 \lambda \mu'' - r \nu''/(8 + r^2 \mu'') - 4r^2 \lambda \nu'' + r^3 \lambda \mu''')\right\},
\]

\[
(11)
\]

\[
\rho_t = \frac{e^{-\nu}}{8r^4} \left\{ r^4 \lambda \mu^4 + 2r^3 \lambda \mu^3 (2 - 3r\nu') + r^2 \nu'^2 (-32r \lambda \nu' + 17r^2 \lambda \nu'^2 + 2(e^r r^2
\]

\[+ 8\lambda + 6r^2 \lambda \mu'' - 6r^2 \lambda \nu'')\right) - 2r \mu' (-38r^2 \lambda \nu'^2 + 6r^3 \lambda \nu'^3 + r \nu' (e^r r^2 - 24\lambda
\]

\[+ 28r^2 \lambda \mu'' - 14r^2 \lambda \nu'') - 2(e^r r^2 - 4\lambda + 12e^\nu \nu \lambda + 10r^2 \lambda \mu'' - 14r^2 \lambda \nu''
\]

\[+ 6r^3 \lambda \mu'' - 2r^3 \lambda \nu''\right) - 4(12r^3 \lambda \nu'^3 - 11r^4 \lambda \mu'' \nu'^2 - 5r^4 \lambda \mu'^2 + \mu'' (e^r r^4
\]

\[+ 8r^2 \lambda + 8r^4 \lambda \nu'') + r \nu' (e^r r^2 - 28\lambda + 12e^\nu \lambda + 28r^2 \lambda \mu'' - 28r^2 \lambda \nu'' + 12r^3
\]

\[\times \lambda \mu''') + 4\lambda(-7 + 6e^\nu + e^{2\nu} - 3r^3 \mu'' + 2r^3 \nu'' - r^4 \mu''(iv))\right\}.
\]

\[
(12)
\]

Using the relations of \mu and \nu in metric (4), we find the expressions for \rho,
\[ P_r \quad \text{and} \quad P_t \quad \text{in the following form} \]

\[
\rho = \frac{1}{r^4} e^{-2Ar^2} \left\{ e^{2Ar^2}(r^2 - 2\lambda) + 2(-5 - 3B^2r^4 + 6B^3r^6 + B^4r^8 + 12A^3r^6) \times (2 + Br^2) - A^2r^4(40 + 68Br^2 + 11B^2r^4) + A(-4r^2 + 48Br^4 + 26B^2r^6 - 2B^3r^8)\right\} + e^{Ar^2}(r^2 - 2 + 2Ar^4 + 12\lambda) \},
\]

\[ (13) \]

\[
p_r = \frac{1}{r^4} e^{-2Ar^2} \left\{ -e^{2Ar^2}(r^2 - 2\lambda) + 2(-7 + 11B^2r^4 + 2B^3r^6 - B^4r^8 - 2Ar^2(4 + 16Br^2 + 9B^2r^4 + B^3r^6)) + e^{Ar^2}(r^2 - 2\lambda) + 2Br^2 + 26B^2r^6 + 2B^3r^8)\right\} + e^{Ar^2}(r^2 - 2 + 2Ar^4 + 12\lambda) \},
\]

\[ (14) \]

\[
p_t = \frac{1}{r^4} e^{-2Ar^2} \left\{ -2(-7 + 6e^{Ar^2} + e^{2Ar^2})\lambda + 16B^3r^6 + 2B^4r^8 - 24A^3r^6(2 + Br^2)\right\} + 2Ar^2(4 + 16Br^2 + 9B^2r^4 + B^3r^6)) + e^{Ar^2}(r^2 - 2\lambda) + 2Ar^4 + 12\lambda) \},
\]

\[ (15) \]

The equation of state (EoS) parameters corresponding to radial and transverse directions can be obtained as

\[
\omega_r = \left\{ \{e^{2Ar^2}(r^2 - 2\lambda) + 2(-7 + 11B^2r^4 + 2B^3r^6 - B^4r^8 + 3A^2r^4(2 + Br^2)\}ight\} + e^{Ar^2}(r^2 - 2\lambda) + 2Br^2(2 + Br^2)\}ight\} + e^{Ar^2}(r^2 - 2\lambda) + 2Ar^4 + 12\lambda) \},
\]

\[ (16) \]

\[
\omega_t = \left\{ \{e^{2Ar^2}(r^2 - 2\lambda) + 2(-7 + 6e^{Ar^2} + e^{2Ar^2})\lambda + 16B^3r^6 + 2B^4r^8 - 24A^3r^6(2 + Br^2)\lambda + 2Ar^2(4 + 16Br^2 + 9B^2r^4 + B^3r^6)) + e^{Ar^2}(r^2 - 2\lambda) + 2Br^2(-6\lambda) + e^{2Ar^2}(r^2 + 6\lambda)\} - Ar^2(4(-7 + 19Br^2 + 25B^2r^4 + 3B^3r^6)) + e^{Ar^2}(r^2 + 6\lambda)\}ight\} + e^{Ar^2}(r^2 - 2\lambda) + 2Br^2(2 + Br^2)\}ight\} + e^{Ar^2}(r^2 - 2\lambda) + 2Ar^4 + 12\lambda) \},
\]

\[ (17) \]

3 Physical Analysis

Here, we discuss some physical conditions which are necessary for the interior solution. In the following, we present the anisotropic behavior and stability
3.1 Anisotropic Constraints

In the first place, we present the evolution of energy density $\rho$, radial pressure $p_r$ and tangential pressure $p_t$ as shown in Figures 1-3 for different strange stars (see Table 1).

Taking derivatives of equations (13) and (14) with respect to radial coordi-
The evolution of \( \frac{dp}{dr} \) and \( \frac{d\rho}{dr} \) is shown in Figures 4 and 5. It can be seen that \( \frac{dp}{dr} < 0 \) and \( \frac{d\rho}{dr} < 0 \).

We also examine the behavior of derivatives of \( \rho \) and \( p_r \) at center \( r = 0 \).
of compact star and it is found that

\[
\frac{d\rho}{dr} = 0, \quad \frac{dp_r}{dr} = 0, \quad \frac{d^2\rho}{dr^2} < 0, \quad \frac{d^2p_r}{dr^2} < 0. \quad (20)
\]

Equation (20) shows the maximality of central \( \rho \) and \( p_r \). Hence \( \rho \) and \( p_r \) attain maximum values at \( r = 0 \) and functional values decreases with the increase in \( r \) as shown in Figures 1-3. We present the evolution of EoS parameters \( \omega_r \) and \( \omega_t \) in Figures 6 and 7 for different strange stars. We call these parameters as effective since these involve the contribution from the additional terms in \( f(R) \) gravity. Here, it is clear that, like normal matter distribution, the bound on the effective EOS in this case is given by 

\[ 0 < \omega_i(r) < 1, \quad (i = r, t). \]
The anisotropy measurement $\Delta = \frac{2}{r}(p_t - p_r)$ for this model is given by

$$
\Delta = \frac{1}{r^5}2e^{-2Ar^2}\{e^{2Ar^2}(r^2 - 4\lambda) - 4(-7 + 3Br^2 - 3B^3r^6 - B^4r^8 + 6A^3r^6(2 + Br^2) - A^2r^4(8 + 31Br^2 + 7B^2r^4) + Ar^2(-11 + 3Br^2 + 16B^2r^4 + 2B^3 \times r^6))\lambda - e^{Ar^2}(Ar^4 + (A - B)Br^6 + 24\lambda + r^2(1 + 12A\lambda - 12B\lambda))}\}. \quad (21)
$$

The measure of anisotropy is directed outward when $p_t > p_r$ which implies $\Delta > 0$ whereas it is directed inward if $p_t < p_r$ resulting in $\Delta < 0$. $\Delta$ represents a force that is due to the local anisotropy of the fluid. In our model, we find that $\Delta > 0$ for the different strange stars as shown in Figure 8. Hence for this case repulsive force exists which allows the construction of more massive configuration.

It is interesting to see that anisotropy vanishes at the center $r = 0$ and the corresponding pressures take the form $p_t(0) = p_r(0) = p_0 = 34A^2\lambda + 2B(1 + 11B\lambda) - A(1 + 64B\lambda)$.
3.2 Matching Conditions

Here, we match the interior metric (3) to the vacuum exterior spherically symmetric metric given by

\[ ds^2 = -\left(1 - \frac{2M}{r}\right)dt^2 + \left(1 - \frac{2M}{r}\right)^{-1}dr^2 + r^2d\theta^2 + r^2d\phi^2, \]  

(22)

At the boundary surface \( r = R \) continuity of the metric functions \( g_{tt}, g_{rr} \) and \( \frac{\partial g_{tt}}{\partial r} \) yield,

\[ g_u = g^+_u, \quad g^+_r = g^+_{rr}, \quad \frac{\partial g^+_u}{\partial r} = \frac{\partial g^+_{tt}}{\partial r}, \]  

(23)

where \(-\) and \(+\), correspond to interior and exterior solutions. From the interior and exterior metrics, we get

\[ A = -\frac{1}{R^2} \ln \left(1 - \frac{2M}{R}\right), \]  

(24)

\[ B = \frac{M}{R^3} \left(1 - \frac{2M}{R}\right)^{-1}, \]  

(25)

\[ C = \ln \left(1 - \frac{2M}{R}\right) - \frac{M}{R} \left(1 - \frac{2M}{R}\right)^{-1}. \]  

(26)

For the given values of \( M \) and \( R \) for given star, the constants \( A \) and \( B \) are given in the table 1.

Table 1: Values of constants for given Masses and Radii of Stars

| Strange Quark Star   | \( M \)  | \( R(km) \) | \( \frac{M}{R} \) | \( A(km^{-2}) \) | \( B(km^{-2}) \) |
|----------------------|---------|-------------|-------------------|----------------|----------------|
| Her X-1 0.88\( M_\odot \) | 7.7     | 0.168       | 0.006906276428    | 0.004267364618 |
| SAX J 1808.4-3658 1.435\( M_\odot \) | 7.07    | 0.299       | 0.01823156974     | 0.01488011569  |
| 4U 1820-30 2.25\( M_\odot \) | 10.0    | 0.332       | 0.01090644119     | 0.009880952381 |
3.3 Energy Conditions

In order to comprehend various cosmological geometries and some general results associated with the strong gravitational fields, energy bounds are of great interest. These conditions include weak energy condition (WEC), null energy condition (NEC), strong energy condition (SEC) and dominant energy condition (DEC). The validity of these energy conditions is necessary for a physically reasonable energy-momentum tensor. For an anisotropic fluid, these are defined as

NEC : \( \rho + p_r \geq 0, \quad \rho + p_t \geq 0, \)
WEC : \( \rho \geq 0, \quad \rho + p_r \geq 0, \quad \rho + p_t \geq 0, \)
SEC : \( \rho + p_r \geq 0, \quad \rho + p_t \geq 0, \quad \rho + p_r + 2p_t \geq 0, \)
DEC : \( \rho > |p_r|, \quad \rho > |p_t|. \)

We find that our model satisfies these conditions for specific values of mass and radius which helps to find the unknown parameters for different strange stars. Here, we present the evolution of these conditions for strange star Her X-1 as shown in Figure 9. It can be seen that energy conditions are satisfied for our model.

3.4 TOV Equation

The generalized Tolman-Oppenheimer-Volkoff (TOV) equation for an anisotropic fluid is of the form

\[
\frac{dp_r}{dr} + \frac{\nu'(\rho + p_r)}{2} + \frac{2(p_r - p_t)}{r} = 0
\]

Following [30], one can express the above equation in terms of gravitational mass and henceforth it results in the equilibrium condition for the strange star, involving the gravitational, hydrostatic and anisotropic forces of the stellar object as

\[
F_g + F_h + F_a = 0,
\]

\[
F_g = -Br(\rho + p_r), \quad F_h = -\frac{dp_r}{dr}, \quad F_a = \frac{2(p_t - p_r)}{r}
\]

Using the effective \( \rho, p_r \) and \( p_t \) [10]-[12], we can find the equilibrium condition for \( f(R) \) theory. In Figure 10, we show the evolution of above forces at
the interior of strange star Her X-1. It can be seen that static equilibrium conditions is achievable for strange star candidates in $f(R)$ gravity as shown in figure 10.

3.5 Stability Analysis

People [32]-[34] have discussed the appearance of cracking in spherical compact objects by using different approaches. Herrera [32] introduced the concept of cracking to identify potentially unstable anisotropic matter configuration. It was considered to explain the behavior of fluid distribution, once the equilibrium configuration has been perturbed and total non-vanishing radial forces of different signs appear within the system. Now, by considering the sound speeds one can assess the potentially stable and unstable regions established through the difference of sound propagation within the matter configuration. The region for which radial sound of sound $v_{sr}^2$ is greater than the transverse speed of sound $v_{st}^2$ is potentially stable.

To analyze the stability of our model we calculate the radial and trans-
verse speeds as

\[
v^2_{sr} = \{ -e^{2Ar^2}(r^2 - 4\lambda) + 4(-7 - B^3r^6 + B^4r^8 + 3A^3r^6(2 + Br^2)^2 - A^2r^4 \\
\times (8 + 38Br^2 + 21B^2r^4 + 2B^3r^6) + Ar^2(-11 + 20B^2r^4 + 4B^3r^6 \\
- B^4r^8))\lambda + e^{Ar^2}(Ar^4 + 2ABr^6 + 24\lambda + Br^2(1 + 12\lambda))\} / \{e^{2Ar^2}(r^2 \\
- 4\lambda) + 4(-5 - 3B^3r^6 - B^4r^8 + 12A^4r^8(2 + Br^2) - A^3r^6(52 + 80Br^2 \\
+ 11B^2r^4) + A^2r^4(-4 + 82Br^2 + 37B^2r^4 - 2B^3r^6) + Ar^2(-7 - 16B^2r^4 \\
+ 8B^3r^6 + B^4r^8))\lambda + e^{Ar^2}(-Ar^4 + 2A^2r^6 + 24\lambda + r^2(-1 + 12\lambda))\}, (29)
\]

\[
v^2_{st} = \{ B^2e^{Ar^2}r^6 + 4(-7 + 6e^{Ar^2} + e^{2Ar^2})\lambda - 12B(-1 + e^{Ar^2})r^2\lambda + 16B^3r^6\lambda \\
+ 4B^4r^8 + 48A^4r^8(2 + Br^2)\lambda - 4A^3r^6(40 + 86Br^2 + 17B^2r^4)\lambda + A^2r^4 \\
\times (4(-14 + 75Br^2 + 67B^2r^4 + 6B^3r^6)\lambda + e^{Ar^2}(r^2 + Br^4 + 12\lambda)) - Ar^2 \\
\times (-8(-7 + 3e^{Ar^2})\lambda + 56B^3r^6\lambda + 4B^4r^8\lambda + B^2r^4(e^{Ar^2}r^2 + 144\lambda) + 3Br^2 \\
\times (-8\lambda + e^{Ar^2}(r^2 + 4\lambda))\} / \{-e^{2Ar^2}(r^2 - 4\lambda) - 4(-5 - 3B^3r^6 - B^4r^8 \\
+ 12A^4r^8(2 + Br^2) - A^3r^6(52 + 80Br^2 + 11B^2r^4) + A^2r^4(-4 + 82Br^2 \\
+ 37B^2r^4 - 2B^3r^6) + Ar^2(-7 - 16B^2r^4 + 8B^3r^6 + B^4r^8)\}\lambda + e^{Ar^2}(Ar^4 \\
- 2A^2r^6 - 24\lambda + r^2(1 - 12\lambda))\}. (30)
\]

We plot the radial and transverse sound speeds of different strange stars in Figures 11 and 12. It is shown that \(v^2_{sr}\) and \(v^2_{st}\) satisfy the inequalities \(0 \leq v^2_{sr} \leq 1\) and \(0 \leq v^2_{st} \leq 1\) within the anisotropic matter configuration.
The difference of \( v_{sr}^2 \) and \( v_{st}^2 \) can be obtained as

\[
\begin{align*}
v_{st}^2 - v_{sr}^2 &= \left\{ e^{2Ar^2}(r^2 - 8\lambda) - 4(-14 + 3Br^2 + 3B^3r^6 + 2B^4r^8 + 12A^4r^8) \right. \\
&+ B^2r^4 - 2A^3r^6(14 + 37Br^2 + 7B^2r^4) + A^2r^4(-22 + 37Br^2 + 46) \\
&\times B^2r^4 + 4B^3r^6) - Ar^2(25 - 6Br^2 + 16B^2r^4 + 10B^3r^6 + 2B^4r^8)) \\
&\times \left. \lambda - e^{Ar^2}((A^2 - AB + B^2)r^6 + A(A - B)Br^8 + 48\lambda + Ar^4(1 \\
&+ 12A\lambda - 12BA) + r^2(1 + 36A\lambda - 12BA)) \right\} / \left\{ e^{2Ar^2}(r^2 - 4\lambda) + 4 \\
&\times (-5 - 3B^3r^6 - B^4r^8 + 12A^4r^8(2 + Br^2) - A^3r^6(52 + 80Br^2) \\
&+ 11B^2r^4) + A^2r^4(-4 + 82Br^2 + 37B^2r^4 - 2B^3r^6) + Ar^2(-7 \\
&- 16B^2r^4 + 8B^3r^6 + B^4r^8))\lambda + e^{Ar^2}(-Ar^4 + 2A^2r^6 + 24\lambda \\
&\left. + r^2(-1 + 12A\lambda)) \right\}. \tag{31}
\end{align*}
\]

The variation of \( v_{st}^2 - v_{sr}^2 \) of different strange stars is shown in Figure 13. Figure 13 shows that difference of the two sound speeds, i.e., \( v_{st}^2 - v_{sr}^2 \) retain similar sign within the specific configuration and it satisfies the inequality \( |v_{st}^2 - v_{sr}^2| \leq 1 \). Thus, our proposed strange star model is stable.
The compactness of the star is given by

\[
u = \frac{M(R)}{R} = \frac{\pi e^{-2AR^2}}{32A^2(A R^2)^{3/2}} \left\{ -4\sqrt{AR^2} (15R^2 B^4 \lambda + 192 A^5 R^4 (2 + R^2 B) \lambda + 2AR^2} \times B^3 (21 + 10 R^2 B) \lambda - 16 A^4 R^2 (22 + 53 R^2 B + 11 R^4 B^2) \lambda + A^2 R^2 B^2 (99 + 56 R^2 B + 16 R^4 B^2) \lambda - 4 A^3 (-49 R^4 B^2 \lambda + 8 R^6 B^3 \lambda + 16 (5 - 6 e^{AR^2})} \times e^{2AR^2}) \lambda + R^2 (-8 e^{AR^2} + 8 e^{2AR^2} - 33 B \lambda)) - 1536 A^4 R^2 e^{2AR^2} E r f (\sqrt{AR^2}) \times \sqrt{\pi \lambda + 3 R^2 (224 A^4 + 44 A^3 B + 33 A^2 B^2 + 14 A B^3 + 5 B^4) e^{2AR^2} \sqrt{2 \pi \lambda} \times E r f (\sqrt{2AR^2}) \right\}.
\]

(32)

The surface redshift \((Z_s)\) corresponding to compactness \((u)\) can be obtained as

\[
1 + Z_s = (1 - 2u)^{-1/2} = \left\{ 1 - \frac{\pi e^{-2AR^2}}{16A^2(AR^2)^{3/2}} \left\{ -4\sqrt{AR^2} (15R^2 B^4 \lambda + 192 A^5 R^4 (2 + R^2 B) \lambda + 2AR^2} \times (2 + R^2 B) \lambda + 2AR^2 B^3 (21 + 10 R^2 B) \lambda - 16 A^4 R^2 (22 + 53 R^2 B + 11 R^4 B^2) \lambda - 4 A^3 (-49 R^4 B^2 \lambda + 8 R^6 B^3 \lambda + 16 (5 - 6 e^{AR^2})} \times \lambda + 16 (5 - 6 e^{AR^2}) e^{2AR^2}) \lambda + R^2 (-8 e^{AR^2} + 8 e^{2AR^2} - 33 B \lambda)) - 1536 A^4} \times R^2 e^{2AR^2} E r f (\sqrt{AR^2}) \sqrt{\pi \lambda + 3 R^2 (224 A^4 + 44 A^3 B + 33 A^2 B^2 + 14 A B^3} \times + \quad 5 B^4) e^{2AR^2} \sqrt{2 \pi \lambda} E r f (\sqrt{2AR^2}) \right\}^{-1/2}.
\]

(33)

Figure 13 shows the plot of redshift of compact star Her X-1 of radius 7 km and the maximum redshift turns out to be \(Z_s = 0.845\).
4 Conclusion

$f(R)$ theory of gravity being a scientific and logical explanation of dark energy problem have attracted the much attention of modern cosmologist. This theory has attained a particular interest since the $f(R)$ modifications to general theory of relativity appeared in a very natural way in the low-energy effective actions of the quantum theory of gravity and the quantization of underlying fields in curved spacetime. This theory is also conformally related to general theory of relativity with a self-interacting scalar field [35]. Also, this theory explains simultaneously both the early time inflation and the late-time acceleration of the universe.

In this paper, we have explored the analytical solutions for the compact stars with general static interior source in the framework of $f(R)$ theory of gravity. The investigation has been completed by considering that compact stars are anisotropic in their internal configuration in $f(R)$ gravity. The interior geometry of the compact stars has been handled by metric assumption proposed by Krori and Barua [26]. These assumptions involves some unknown constants that can be expressed in term of masses and radii of the compact stars by matching smoothly the interior solution with Schwarzschild exterior solution. The application of the masses and radii of the compact stars yield the values of constants that determine the nature of the stars. For these values of the constants, we found that the energy conditions hold for the given class of compact strange stars. By the physical interpretation of the results, we conclude that the EOS parameters behave like normal matter distribution, the bound on the effective EOS in this case is given by

![Graph showing evolution of redshift of strange star Her X-1.](image-url)
$0 < \omega_i(r) < 1, (i = r, t)$. This indicates that fact that compact stars are constituted by the combination of ordinary matter and effect of $f(R)$ term. The density and pressure are regular everywhere and attain the maximum value at the center. Therefore, the present model is free from central singularity. The radial pressure $p_r$ and matter density $\rho$ are monotonically decreasing function of $r$, i.e., these have maximum values at the center and it decreases from the center to the surface of the star.

It has been found that the anisotropy will be directed outward when $P_t > P_r$ this implies that $\Delta > 0$. We have found that $\Delta > 0$ for the different strange stars as shown in Figure 8. Hence, in this case repulsive force exists which allows the construction of more massive stellar configuration in $f(R)$ gravity. The subliminal velocity of sound is less than 1, i.e, $0 < v_{sr}^2$, $v_{st}^2 < 1$ and $v_{sr}^2 > v_{st}^2$. The variation of $v_{st}^2 - v_{sr}^2$ for different strange stars is shown in Figure 13, which satisfies the inequality $|v_{st}^2 - v_{sr}^2| \leq 1$. Thus, our proposed strange stars model are stable in the presence of $f(R)$ term. The range of surface redshift $Z_s$ for the compact objects in this case lies on $0 < Z_s \leq 0.845$. In case of isotropic interior configuration without cosmological constant this range has been noted as $Z_s \leq 2$. Hence in present configuration redshift has been decreased to a certain range.

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