Synchronization of Spin Torque Oscillators through Spin Hall Magnetoresistance

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Abstract—Spin torque oscillators placed onto a nonmagnetic heavy metal show synchronized auto-oscillations due to the coupling originating from spin Hall magnetoresistance effect. Here, we study a system having two spin torque oscillators under the effect of the spin Hall torque, and show that switching the external current direction enables us to control the phase difference of the synchronization between in-phase and antiphase.

Index Terms—spintronics, spin torque oscillator, synchronization, spin Hall magnetoresistance

I. INTRODUCTION

SUBSTANTIAL efforts, such as material investigation, structural improvement, and theoretical analysis, have been made to develop high-performance spin torque oscillators because of its potential to be applied in microwave generators and magnetic recording head [1-7]. In particular, synchronization of spin torque oscillators is becoming an exciting topic because of the possibility in enhancing emission power and extending the technology to new practical applications such as bio-inspired computing [8-10]. Several methods have been proposed and demonstrated to stabilize the synchronizations, based on spin wave propagation [11,12], electric current injection [13,14], mediation of antivortex [15], microwave field [16], stochastic noise in current [17], or dipole coupling [18].

The spin torque oscillator based on the spin Hall effect [19,20] has been developed recently. It has the advantage of easier fabrication and that it is unnecessary to apply electric current to the ferromagnet directly. A stable auto-oscillation of the magnetization around the in-plane easy axis induced by the spin Hall effect was observed in CoFeB/Ta heterostructure in 2012 [21]. The synchronization of the spin torque oscillators in the spin Hall geometry induced by the spin wave propagation was also demonstrated [22].

The spin Hall effect causes another interesting phenomenon related to magnetoresistance effect. It was recently found that the resistance of ferromagnetic/nonmagnetic bilayers depends on the magnetization direction in the ferromagnet, even when the ferromagnet is an insulator [23-26]. This new type of magnetoresistance effect, named as spin Hall magnetoresistance, originates from additional electric currents generated by the charge-spin conversion due to the direct and inverse spin Hall effects [27]. The spin Hall magnetoresistance has been confirmed by measuring the longitudinal and transverse voltages. Let us imagine that another ferromagnet is placed onto the nonmagnetic heavy metal in the longitudinal or transverse direction. The electric current generated through the spin Hall magnetoresistance effect will be injected into this second ferromagnet as spin current by the spin Hall effect, and excite the spin torque on the magnetization, and vice versa. As a result, the coupled motions of the magnetizations in the ferromagnets are expected. This coupling is unavoidable whenever several ferromagnets are placed onto the same nonmagnet. Recently, we have confirmed a tangible synchronization of magnetizations having several different phase differences, depending on the material parameters, by numerically solving the Landau-Lifshitz-Gilbert (LLG) equation [28].

In this paper, the synchronization of spin torque oscillators through the spin Hall magnetoresistance effect is studied theoretically. In particular, the control of the phase difference between the oscillators is investigated. Considering a system having two oscillators, it is shown that switching the direction of the external current applied in one oscillator enables us to control the phase difference between in-phase and antiphase.

This paper is organized as follows. In Sec. II, we show the LLG equation including the torque related to the spin Hall magnetoresistance. In Sec. III, we compare the synchronized auto-oscillations in two-spin torque oscillators for the external electric currents flowing in the same and opposite directions. The time necessary to synchronize the oscillators is discussed in Sec. IV. The role of the coupling for the oscillators having different anisotropies is discussed in Sec. V. The conclusion is summarized in Sec. VI.

II. LLG EQUATION

Figure 1 schematically shows the system in this study, which consists of two ferromagnets $F_k$ ($k = 1, 2$) placed onto a nonmagnet, $N$. The ferromagnets are aligned along the $y$ direction. External voltages are applied along the $x$ direction, generating the external electric current densities $J_{0y}^{(k)}$ passing under the $F_k$ layer. The directions of the external currents are independently controllable. We assume that the ferromagnets
are placed onto the nonmagnet at the center along the $x$ direction, the electric field in the nonmagnet along the $x$ direction is uniform, and the current magnitudes, $|J_0^{(1)}|$ and $|J_0^{(2)}|$, are the same. Therefore, if the spin Hall effect is absent, the electric potentials near the $F_1/N$ and $F_2/N$ interfaces are the same, and electric current does not flow between these interfaces. In the presence of the spin-orbit interaction, on the other hand, the charge-spin conversion by the direct and inverse spin Hall effects near the $F_b/N$ interface gives an additional electric current density flowing in the $y$ direction,

$$J_{c}^{F_b/N} = - \left( \chi m_{kz} m_{ky} + \chi' m_{kz} \right) J_0^{(k)}, \quad (1)$$

where $m_k = (m_{kx}, m_{ky}, m_{kz})$ is the unit vector pointing in the magnetization direction in the $F_b$ layer. The dimensionless coefficients, $\chi$ and $\chi'$, in metallic ferromagnetic/nonmagnetic bilayers are given by [27,29,30]

$$\chi = \frac{d^2 \ell_N}{dN} \left[ \frac{\Re \left\{ g^+ \right\}}{g_N + g^+ \coth(d_N/\ell_N)} - \frac{g^+}{g_N} \right] \tanh^2 \left( \frac{d_N}{2\ell_N} \right), \quad (2)$$

$$\chi' = -\frac{d^2 \ell_N}{dN} \Im \left\{ \frac{g^+}{g_N + g^+ \coth(d_N/\ell_N)} \right\} \tanh^2 \left( \frac{d_N}{2\ell_N} \right), \quad (3)$$

where $\vartheta$, $\ell_N$, and $d_N$ are the spin Hall angle, spin diffusion length, and thickness of the nonmagnet, respectively. The dimensionless mixing conductance is denoted as $g^+/\ell_N$ [31,32], whereas $g_N/S = h\sigma_N/(2e^2\ell_N)$ with the cross section area of the ferromagnetic/nonmagnetic interface, $S$, and the conductivity of the nonmagnet, $\sigma_N$. We introduce $g^+$ as $1/g^+ = \{2/[1 - (\ell_N)^{-1}]\} + \{1/|g\Re \left( d_N/\ell_N \right) \} + \{1/|g\Im \left( d_N/\ell_N \right) \}$, where $g$ and $p_g$ are the dimensionless interface conductance and its spin polarization, $\ell_N$, the conductivity, $\sigma_F$, and its spin polarization, $p_{\sigma_F}$, of the ferromagnet. The thickness of the ferromagnet is $d_F$. The interface resistance is related to $g$ via $S = (h/e^2)/r$. Using the values of the parameters estimated from the experiments in CoFeB/W heterostructure [29] and the first principles calculation [32], $\rho_F = 1/\sigma_F = 1.6$ kΩnm $\rho_N = 0.72$, $\ell_F = 1.0$ nm, $\rho_N = 1.25$ kΩnm, $\ell_N = 1.2$ nm, $r = 0.25$ kΩnm$^2$, $\rho_N = 0.50$, $\Re[g^+/S] = 25.0$ nm$^{-2}$, $\Im[g^+/S] = 1.0$ nm$^{-2}$, $d_F = 2$ nm, $d_N = 3$ nm, and $\vartheta = 0.27$, we set $\chi \simeq 0.01$ and $\chi' \simeq -0.0002$.

The external electric current $J_0^{(k)}$ is converted to spin current near the $F_b/N$ interface by the direct spin Hall effect, and excites the spin torque on the magnetization, $m_k$. In addition, in the present system having two ferromagnets, the electric current density $J_{c}^{F_{b}'/N}$ originated near the $F_{b}'/N$ interface through the spin Hall magnetoresistance effect and moved to the other ferromagnetic/nonmagnetic interface, $F_b/N$ ($k \neq k'$), is converted to spin current again by the direct spin Hall effect, and excites the spin torque on the magnetization, $m_k$. Therefore, the LLG equation of the magnetization is [28]

$$\frac{dm_k}{dt} = -\gamma m_k \times H_k + \alpha m_k \times \frac{dm_k}{dt} - \frac{\gamma h}{2eMD_F} m_k \times (e_y \times m_k) - \frac{\gamma h}{2eMD_F} m_k \times (e_x \times m_k) - \frac{\gamma h}{2eMD_F} (\chi m_{kz} m_{ky} + \chi' m_{kz}) J_0^{(k)} m_k \times e_x, \quad (4)$$

where $M$, $\alpha$, and $\gamma$ are the saturation magnetization, the Gilbert damping constant, and gyromagnetic ratio of the ferromagnet, respectively, and are assumed as 1500 emu/cc, 0.005, and $1.764 \times 10^7$ rad/(Oe s) from the experiments [28,32]. We also introduce

$$\vartheta_{R(1)} = \vartheta \tanh \left( \frac{d_F}{2eF} \right) \Re \left( \frac{g^+}{g_N + g^+ \coth(d_N/\ell_N)} \right), \quad (5)$$

and $\beta = -\vartheta_1/\vartheta_R$, which are also estimated from the parameters written above as $\vartheta_R \simeq 0.167$ and $\beta \simeq -0.01$.

The first and second terms on the right hand side of Eq. (4) represent the torque due to the magnetic field $H_k$ and the Gilbert damping torque. The third and fourth terms are the conventional spin Hall torques corresponding to the anti-damping (or Slonczewski-like [34]) torque and the field-like torque, respectively. Since the auto-oscillation of the magnetization by this spin Hall torque was observed in the in-plane magnetized ferromagnet [21], we assume that the magnetic field consists of an in-plane anisotropy field $H_k$ along the $y$ direction and a shape anisotropy field along the $z$ direction as

$$H_k = H_K m_{ky} e_y - 4\pi M m_{kz} e_z, \quad (6)$$

where $H_K$ is set as 200 Oe. Note that there are two stable states of the magnetization, $m_k = \pm e_y$. It is known that the auto-oscillation around the $y$ axis is excited when the external current density is larger than a critical value [35],

$$J_c = \pm \frac{2\alpha e M d_F}{h\vartheta_R} (H_K + 2\pi M), \quad (7)$$

where the double sign means that the upper (lower) when the initial state is near $m_k = (+(-)e_y$. On the other hand, the last two terms in Eq. (4) originate from the electric current given by Eq. (1), contributing to the spin Hall magnetoresistance effect. These coupling torques are on the order of $\vartheta^3$, whereas the conventional spin Hall torque is proportional to $\vartheta$. Thus, the coupling torque is at least two orders of magnitude smaller than the conventional torque. Nevertheless, these torques result in coupled motions of the magnetizations in the $F_1$ and $F_2$ layers.

The right hand side of Eq. (4) becomes zero when the magnetization points to the $y$ direction. In the following, small deviations of $\mathbf{m}_1$ and $\mathbf{m}_2$ from the $y$ axis are assumed as $\mathbf{m}_1 = (\cos 80^\circ, \sin 80^\circ, 0)$ and $\mathbf{m}_2 = (\cos 95^\circ, \sin 95^\circ, 0)$ at the initial state, except Fig. 4 where the dependences of the time necessary to synchronize the oscillations on the initial conditions is investigated.
III. SYNChronization of auto-oscillations

A key quantity in the synchronization of oscillators is the phase difference. Its precise control is of interest for both nonlinear science and practical applications [9,10]. For example, in the theoretical study of the self-synchronization by the delayed feedback [36], the phase difference between a vortex oscillator and the feedback current is controlled by the delay time. In the present system based on the spin Hall effect, it was found that the phase difference can be varied when the value of the field-like torque strength, $\beta$, is altered [28]. However, $\beta$ is determined by the material parameters, and once a sample is fabricated experimentally, its value, as well as sign, cannot be altered. Therefore, for practical studies, alternative proposal to control the phase difference will be necessary. We note here that the direction and magnitude of the external currents in Ref. [28] were assumed to be identical among the oscillators.

One way proposed here is to control the phase difference between the oscillators by reversing the direction of the external electric current. Namely, for example, the electric current under the F1 layer always flows in the positive $x$ direction ($J_0^{(1)} > 0$), whereas that under the F2 layer flows in either the positive ($J_0^{(2)} > 0$) or negative ($J_0^{(2)} < 0$) $x$ direction. We note that the current $J_0^{(2)}$ is a direct current, and our proposal does not mean that an alternative current is applied. When $J_0^{(2)}$ is also positive, the phase difference is zero, i.e., the in-phase synchronization is realized [28]. On the other hand, when $J_0^{(2)}$ is negative, the phase difference becomes antiphase, as shown below.

Before discussing the phase difference, we note that the magnetization direction should be changed to the appropriate direction of $m_k \simeq \pm y$ before exciting the auto-oscillation. In our definition, positive (negative) external current $J_0^{(k)}$ excites the auto-oscillation of the magnetization $m_k$ around the positive (negative) $y$ direction, according to Eq. (7). Since $J_0^{(1)}$ is always positive for the convention, as mentioned above, the initial state of $m_1$ should be close to the positive $y$ direction. On the other hand, the initial state of $m_2$ should be close to the positive (negative) $y$ direction when $J_0^{(2)}$ is positive (negative). Note that the magnetization direction in the F2 layer can be reversed between the positive and negative $y$ directions when $J_0^{(2)}$ exceeds another critical value [37],

$$J^* = \frac{4AeM dF}{h|\nu|} \sqrt{4\pi M (H_K + 4\pi M)}. \quad (8)$$

Then, let us explain the reason why we consider that the phase difference can be altered by changing the direction of the external electric current under the F2 layer. The magnetization in the F1 layer oscillates around the $y$ axis with the counterclockwise direction. When the magnetization in the F2 layer also points to the positive $y$ direction and oscillates by the positive current, its precession direction is also counterclockwise. On the other hand, when $m_2$ oscillates around the negative $y$ direction by the negative current, its precession direction is clockwise. Therefore, the complete in-phase synchronization between $m_1$ and $m_2$ is no longer possible. Note that the coupling torques include the term $(\chi m_{k1}m_{k'y} + \chi'm_{k'z})J_0^{(k')}$, as can be seen from Eq. (4). This term determines the phase difference between the spin torque oscillators. For example, the dynamics of $m_1$ is affected by the term $(\chi m_{2x}m_{2y} + \chi'm_{2z})J_0^{(2)}$, which will be an even function by the reversal of the current direction. We notice that the term $m_{2x}J_0^{(2)}$ in the coupling torque does not change the sign by reversing the direction of $J_0^{(2)}$. Then, it is expected from the term $m_{2x}m_{2y}J_0^{(2)}$ in the coupling torque that the phase difference between $m_{1z}$ and $m_{2x}$ does not change by reversing the direction of $J_0^{(2)}$. On the other hand, the phase difference between $m_{1z}$ and $m_{2z}$ will be altered by changing the current direction because the coupling torque includes the term $m_{2z}J_0^{(2)}$.

To confirm the validity of these considerations, now let us investigate the oscillation behaviors of the magnetizations for positive and negative $J_0^{(2)}$ by numerically solving Eq. (4). Figures 2(a) and 2(b) show the orbits of the auto-oscillations of $m_1(t)$ (red) and $m_2(t)$ (blue) for the external current densities (a) in the same directions, $(J_0^{(1)}, J_0^{(2)}) = (4.0, 4.0)$ A/cm², and (b) in the opposite directions, $(J_0^{(1)}, J_0^{(2)}) = (-4.0, -4.0)$ A/cm².
such structures, where (a) the in-phase oscillating signals are obtained when both $J_0^{(1)}$ and $J_0^{(2)}$ flow in the same directions, whereas (b) the antiphase signals are obtained when $J_0^{(1)}$ and $J_0^{(2)}$ flow in the opposite directions.

IV. TIME NECESSARY TO SYNCHRONIZE OSCILLATORS

As mentioned above, the results shown in Fig. 2 are obtained from the LLG equation with the initial conditions of $m_1 = (\cos 80^\circ, \sin 80^\circ, 0)$ and $m_2 = (\cos 95^\circ, \sin 95^\circ, 0)$. One might be interested in how the initial conditions of the magnetizations affect this conclusion. In this section, let us discuss the role of the initial state on the synchronization.

The oscillators in the present model have two energetically stable state at $m_k = \pm e_y$, and these two states are separated by the energy barrier. The phase difference in the synchronized state is zero when two magnetizations initially stay near the same stable direction, whereas it becomes antiphase when they stay near the different directions at the initial state. In other words, the phase difference is zero when $m_{1y}(t = 0)/m_{2y}(t = 0) > 0$, whereas it is antiphase when $m_{1y}(t = 0)/m_{2y}(t = 0) < 0$. This conclusion is not affected by the specific values of $m_{1y}(t = 0) \text{ and } m_{2y}(t = 0)$.

On the other hand, the time necessary to synchronize the oscillators depends on the initial states of the magnetizations. We investigate the relation between the time and phase difference for several initial states from the numerical simulations. The initial states of the magnetizations are distributed by the thermal fluctuations. Due to the large demagnetization field along the $z$ direction, the magnetizations lie almost in the $xy$ plane with the averaged angle from the $y$ axis to the $x$ axis as

$$\langle \theta \rangle = \frac{\int_0^{\pi/2} \theta \exp(\Delta_0 \cos^2 \theta) \sin \theta d\theta}{Z},$$

where $\Delta_0 = M H K V/(2 k_B T)$ with the volume $V$, the Boltzmann constant $k_B$, and the temperature $T$ is the thermal stability, whereas $Z$ is the partition function defined as

$$Z = \int_0^{\pi/2} \exp(\Delta_0 \cos^2 \theta) \sin \theta d\theta.$$

Since the exponentials in Eqs. (9) and (10) are dominated near $\theta \simeq 0$, $\langle \theta \rangle$ becomes

$$\langle \theta \rangle \simeq \frac{\int_0^\infty \theta^2 \exp[\Delta_0(1 - \theta^2)]d\theta}{\int_0^\infty \theta \exp[\Delta_0(1 - \theta^2)]d\theta} = \frac{1}{2} \sqrt{\frac{\pi}{\Delta_0}}.$$

Assuming $\Delta_0 = 60$, which is a required value for memory applications [38], $\langle \theta \rangle \simeq 6.6^\circ$. We consider that the relative angle between the magnetizations affects the time necessary to synchronize the oscillators. Therefore, we change the values of $\varphi_1$ in $m_1(0) = (\cos \varphi_1, \sin \varphi_1, 0)$ around $\varphi_1 = \pi/2 \pm \langle \theta \rangle$, while the initial condition of the other magnetization is fixed as $m_2(0) = (\cos 95^\circ, \sin 95^\circ, 0)$.

In Fig. 3(a), we show the time evolutions of the phase difference between the oscillators for several initial states, $\varphi_1 = 80^\circ, 81^\circ, 85^\circ, 91^\circ, \text{ and } 100^\circ$. We note that we did not calculate it for $\varphi = 95^\circ$ because, in this case, two
magnetizations have the same initial conditions, and therefore, the in-phase synchronization is excited from the initial state. We consider the case that two currents flow in the same direction. For all cases, the phase difference finally becomes zero, i.e., in-phase, as in the results in the previous section. The phase difference in the vertical axis is calculated by \( \phi_0 = \phi_1 - \phi_2 \), where the zero corresponds to the in-phase synchronization whereas 0.5 corresponds to the antiphase synchronization. The results shown in Fig. 4(a) indicate that the time necessary to excite the synchronization is typically on the order of hundred nanoseconds.

An exception is found for \( \phi_1 = 85^\circ \) in Fig. 4(a), where the time almost 1 \( \mu s \) is necessary to fix the phase difference. This result is explained as follows. We note that the initial phase difference of the magnetizations for \( \phi_1 = 85^\circ \) is exactly the antiphase, i.e., \( \mathbf{m}_1(0) = (\cos 85^\circ, \sin 85^\circ, 0) \), whereas \( \mathbf{m}_2(0) = (\cos 95^\circ, \sin 95^\circ, 0) \). As clarified in our previous work \cite{Kimura2020}, the present model has fixed points at the in-phase and antiphase synchronized state, where the former corresponds to a stable fixed point (an attractor) whereas the latter is the unstable one. Since the initial states for \( \phi_1 = 85^\circ \) corresponds to the fixed point of the synchronization, even though it is unstable, a relatively long time is necessary to reach the stable in-phase synchronized state.

It should be reminded that the above calculations are performed for the coupling constant of \( \chi \simeq 0.01 \). We note that the time necessary to synchronize the oscillators depends on the coupling strength. For comparison, we also show the time evolution of the phase difference for \( \chi = 0.001 \) in Fig. 4(b). It is found that the time longer than that shown in Fig. 4(a) is necessary to reach the synchronized state. The time to fix the phase difference is typically on the order of 1 \( \mu s \), except for the initial condition corresponds to the unstable fixed point (\( \phi_1 = 85^\circ \)), where the phase difference does not reach to the in-phase state due to the same reason mentioned above.

V. Oscillators Having Different Parameters

A difficulty using the spin Hall effect as a coupling mechanism of the spin torque oscillators is its small strength. We note that the above calculations have been performed for identical oscillators. When the oscillators have different parameters, a strong strength of the coupling is required to lock the frequencies of the oscillators; if the coupling is weak, the synchronization is easily unlocked. Figure 5(a) shows an example of a desynchronized state, where the in-plane anisotropy field \( H_K \) in the F\(_2\) layer is changed from 200 Oe to 220 Oe, whereas the other parameters are kept to those used in Sec. III. The Fourier transformations of \( m_{1z}(t) \) and \( m_{2z}(t) \) are shown in Fig. 5(b). It is clearly shown that the main peaks of \(|m_{1z}(f)|\) and \(|m_{2z}(f)|\) appear different frequencies \( f \), i.e., the peak frequencies are 3.824 and 4.026 GHz for \( m_{1z} \) and \( m_{2z} \), respectively. We change the value of \( H_K \) in the F\(_2\) layer further down to 205 Oe, but do not observe clear frequency lockings.

The above result on desynchronization is also understood from the following consideration. The coupling force in the unit of the angular frequency is roughly estimated from Eqs. (11) and (12) as

\[
\Omega \sim \frac{\gamma_b \beta R J_0}{2e M R} \sim \alpha \gamma \chi (H_K + 2\pi M),
\]

which is 8.5 MHz for the present parameters. According to the Adler equation \cite{Adler1961}, the frequency difference between the oscillators should be smaller than \( \Omega / (2\pi) \).

Using the ferromagnetic resonance (FMR) frequency
Therefore, the voltage control of magnetic anisotropy will be preferable to excite the self-oscillation by low currents, but it might be still insufficient to excite the synchronization between the oscillators having different frequency.

A possible way to overcome these difficulties is utilizing voltage control of magnetic anisotropy [40,41]. Adding an MgO barrier on the oscillating layers and applying a direct voltage, the magnetic anisotropy, as well as the oscillation frequency, of each ferromagnet can be tuned to a precise value independently. A typical value of the perpendicular anisotropy energy modified by the voltage application is about 200 μJ/m² in FeCo for $d_F = 0.68$ nm [41], which corresponds to the perpendicular anisotropy field of 400 Oe, assuming $M = 1500$ emu/c.c. We note that the three-terminal structure of the spin torque oscillators based on the spin Hall effect is suitable to control the in-plane current exciting the self-oscillation and the perpendicular voltage modifying the anisotropy independently. Therefore, the voltage control of magnetic anisotropy will provide an interesting tool for the synchronization of the spin torque oscillator based on the three terminal devices.

VI. CONCLUSION

In conclusion, the synchronization of the auto-oscillations in spin torque oscillators spontaneously excited by the spin Hall effect was investigated by considering the coupling torques generated from the electric currents contributing to the spin Hall magnetoresistance effect. It is shown that the phase difference between the magnetizations become either in-phase or antiphase, depending on the direction of the external electric current. The result indicates that the precise control of the phase difference between spin torque oscillators is possible by using the spin Hall effect as a coupling mechanism.

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