Protecting the entropic uncertainty lower bound in Markovian and non-Markovian environment via additional qubits

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Abstract. The uncertainty principle is an important principle in quantum theory. Based on this principle, it is impossible to predict the measurement outcomes of two incompatible observables, simultaneously. The uncertainty principle basically is expressed in terms of the standard deviation of the measured observables. In quantum information theory, it is shown that the uncertainty principle can be expressed by Shannon’s entropy. The entropic uncertainty lower bound can be altered by considering a particle as the quantum memory which is correlated with the measured particle. We assume that the quantum memory is an open system. We also select the quantum memory from N qubit which interact with common reservoir. In this work we investigate the effects of the number of additional qubits in reservoir on entropic uncertainty lower bound. We conclude that the entropic uncertainty lower bound can be protected from decoherence by increasing the number of additional qubit in reservoir.

1 Introduction

Uncertainty principle is one of the most fundamental and important issues in quantum theory which is determined the distinction between quantum theory and classical theory. It sets a nontrivial limit on our ability to predict the outcomes of two incompatible measurements on a quantum system. The first and well-known uncertainty relation was proposed by Heisenberg [1]. It is associated with incompatibility between the observables. If the state of the outcome of a measurement on the quantum state of the prepared particle \( \hat{Q} \) and \( \hat{R} \) to a universal form as

\[
\Delta \hat{Q} \Delta \hat{R} \geq \frac{\hbar}{2},
\]

Robertson [3] and Schrodinger [4], improved the above uncertainty relation for arbitrary pairs of incompatible observables \( \hat{Q} \) and \( \hat{R} \) to a universal form as

\[
\Delta \hat{Q} \Delta \hat{R} \geq \frac{1}{2} \left| \langle \psi | \hat{Q} \hat{R} | \psi \rangle \right|, 
\]

where \( \Delta \hat{O} = \sqrt{\langle \psi | \hat{O}^2 | \psi \rangle - \langle \psi | \hat{O} | \psi \rangle^2} \) is the standard deviation of measuring outcomes of the incompatible observables position \( \hat{x} \) and momentum \( \hat{p} \), the Heisenberg uncertainty relation is formulated by Kennard as [2]

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\[
\Delta \hat{p} \Delta \hat{x} \geq \frac{\hbar}{2}.
\]
Bob tracks to minimize his uncertainty about the outcome of Alice measurement. The minimum of the uncertainty of Bob about Alice’s measurement is bounded by equation (4). Let us consider the game with two particles. Bob prepares a correlated bipartite state \( \rho_{AB} \) and sends one of them to Alice and the other one is kept as the quantum memory. In this game, the minimum of the uncertainty of Bob about Alice’s measurement is bounded by the Quantum-memory-assisted EUR as [7]

\[
S(Q|B) + S(R|B) \geq \log_2 \frac{1}{c} + S(A|B),
\]

where \( S(O|B) = S(\rho^{OB}) - S(\rho^B) \) denotes the conditional von Neumann entropies of the post measurement states

\[
\rho^{OB} = \sum_i \langle o_i | o_i \rangle \otimes I \rho^{AB} \langle o_i | o_i \rangle \otimes I,
\]

where \( \{ | o_i \rangle \} \)s are the eigenstates of the observable \( O \), and \( I \) is the identity operator. In comparison with equations (3) and (4), one can see that the uncertainty lower bound in equation (5) reduces for the negative conditional entropy \( S(A|B) \). So, Bob can guess Alice’s measurement outcomes with better accuracy. Quantum-memory-assisted EUR has a wide range of applications including entanglement detection [8–10] and quantum cryptography [11,12].

Much efforts have been made to find the tighter uncertainty bounds for Quantum-memory-assisted EUR [13–22]. In reference [19] Adabi et al. proposed a tighter lower bound for Quantum-memory-assisted EUR. They showed that the uncertainty bound of equation (5) can be tightened as

\[
S(Q|B) + S(R|B) \geq \log_2 \frac{1}{c} + S(A|B) + \max\{0, \delta\},
\]

where

\[
\delta = I(A; B) - (I(Q; B) + I(R; B)),
\]

and

\[
I(\hat{O}; B) = S(\rho^B) - \sum_i p_i S(\rho_i^B)
\]

is the Holevo quantity. It shows Bob’s accessible information about Alice’s measurement \( \hat{O} \). When Alice measures observable \( \hat{O} \), the \( i \)th outcome with probability \( p_i = tr_{AB}(\Pi_i^A \rho_{AB} \Pi_i^A) \) is obtained and Bob’s state is left in the corresponding state \( \rho_i^B = \frac{tr_A(\Pi_i^A \rho_{AB} \Pi_i^A)}{p_i} \). Adabi’s uncertainty bound given on the right-hand side (RHS) of equation (7) is tighter than other bound which have been introduced by others.

In a real sense, quantum systems interact with their surrounding subjected to information loss in the form of dissipation and decoherence. In this work we consider the case in which \( N \) qubits interact with common environment. We select the quantum memory from these \( N \) qubits. We investigate how these additional qubits effect on entropic uncertainty lower bound EULB. The work is organized as follows: in Section 2, we will review the dynamical model which is used in this work. We will examine some examples in Section 3. The manuscript will close with a conclusion in Section 4.

2 Dynamical model

Let us consider \( N \) single-qubit which are located in a common dissipative reservoir. We suppose that each qubit is independently coupled to common zero temperature thermal reservoir that consists of harmonic oscillators. The Hamiltonian \( H \) of the total system contains two parts, \( H_0 \) the free Hamiltonian and \( H_I \) describes the interaction Hamiltonian. So, the Hamiltonian of the total system \((N \) single qubit + reservoir) is given by [25,26]

\[
H = \hat{H}_0 + \hat{H}_I
\]

\[
= \omega_0 \sum_{i=1}^N \sigma_i^+ \sigma_i^- + \sum_k \omega_k b_k^\dagger b_k + \sum_{i=1}^N (g_k b_k^\dagger \sigma_i^- + g_k b_k \sigma_i^+),
\]

where \( g_k \)s are the coupling strength between the \( i \)th qubit with transition frequency \( \omega_0 \) and \( k \)th field mode with frequency \( \omega_k \), \( b_k^\dagger \) and \( b_k \) are creation and annihilation operators of the \( k \)th field mode, respectively. \( \sigma_i^- \) and \( \sigma_i^+ \) are the \( i \)th qubit raising and lowering operators, respectively. We consider the case in which there exists one excitation in the total system and reservoir in the vacuum state \( |0\rangle_E \), initially. We also suppose that the initial state of the whole system is given by

\[
|\psi(0)\rangle = C_0(0)|0\rangle_s \otimes |0\rangle_E + \sum_{i=1}^N C_i(0)|i\rangle_s \otimes |0\rangle_E,
\]

where \( |0\rangle_s \) means that all qubits are in ground state \( |0\rangle \), and \( |i\rangle_s \) shows that \( i \)th qubit in the excited state \( |1\rangle \) and the others are in ground state \( |0\rangle \). The dynamics of the whole system can be written as

\[
|\psi(t)\rangle = (C_0(t)|0\rangle_s + \sum_{i=1}^N C_i(t)|i\rangle_s) \otimes |0\rangle_E
\]

\[
+ \sum_{j=1}^N C_j(t)|0\rangle_s \otimes |1_j\rangle_E,
\]

where \( |1_j\rangle_E \) is the state of the reservoir with single excitation in the \( j \)th field mode. The Schrodinger equation in the interaction picture has the form

\[
i \frac{d}{dt} |\psi(t)\rangle = \hat{H}_I(t)|\psi(t)\rangle,
\]

where \( \hat{H}_I(t) = e^{i\hat{H}_I t} \hat{H}_I e^{-i\hat{H}_I t} \). By solving equation (13) and following the method which has been outlined in reference [26], the dynamics of the \( i \)th qubit can be obtained as

\[
\rho_i(t) = \begin{pmatrix} \rho_{00} & C_i(t) \\ C_i(t)^\dagger & \rho_{11} \end{pmatrix}
\]

\[
= \begin{pmatrix} \rho_{00} & C_i(t) \\ C_i(t)^\dagger & 1 - \rho_{00} \end{pmatrix}
\]

From equation (13), we have \( \dot{C}_0(t) = 0 \) and so \( C_0(t) = C_0(0) = C_0 \). The other probability coefficients satisfies
the following integro-differential equations
\[
d\frac{d}{dt} C_i(t) = -\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} J(\omega)e^{i(\omega_0-\omega)t}\sum_{j=1}^{N} C_j(\tau)d\omega d\tau,
\]
where \( J(\omega) \) is the spectral density of the reservoir. Let us consider a Lorentzian spectral density
\[
J(\omega) = \frac{1}{2\pi} \frac{\gamma_0 \lambda^2}{(\omega_0 - \omega)^2 + \lambda^2},
\]
where the spectral width of the coupling \( \lambda \) is related to the correlation time of the environment \( \tau_E \) via \( \tau_E \approx 1/\lambda \). The parameter \( \gamma_0 \) is connected to the relaxation time \( \tau_E \), over which the state of the system changes, by \( \tau_E \approx 1/\gamma_0 \).

If \( \gamma_0/\lambda \leq 1/2 \) we have the weak system-reservoir coupling regime and the dynamic is Markovian, while for \( \gamma_0/\lambda > 1/2 \), we have the strong coupling regime and the dynamic is non-Markovian. Using Laplace transformation and its inverse, the probability coefficient \( C_i(t) \), which is independent of subscript \( i \), can be obtained as
\[
C_i(t) = \frac{N - 1}{N} e^{-\lambda t/2} \sum_{j=1}^{N} C_j(\tau),
\]
where \( D = \sqrt{\lambda^2 - 2N\gamma_0\lambda} \).

3 EULB and additional qubits

In this section we use the above mentioned dynamical model to reduce the EULB in the presence of decoherence. Let us suppose that Bob prepare the initial correlated state \( \rho_{AB_i} \) such that he chooses ith single-qubit from \( N \) qubits in common reservoir as the part \( B \). Then, he sends particle \( A \) to Alice and keeps \( B \) as the quantum memory. Then, they reach an agreement on measuring observables by Alice on her particle. Alice does her measurement on the quantum state of the prepared particle and declares her choice of the measurement to Bob. Bob tracks to minimize his uncertainty about the outcome of Alice measurement.

In this model the quantum memory \( B_i \) interacts with reservoir as an open quantum system (The model is sketched in Figure 1). It is expected that, as a result of the interaction between quantum memory and reservoir, the correlation between quantum memory \( B \) and measured particle \( A \) decreases. As mentioned before, the presence of correlation between Alice and Bob reduces the uncertainty of Bob about the outcomes of Alice’s measurement. So, it is logical to expect that the EULB increases due to interaction between quantum memory and reservoir.

3.1 Examples

3.1.0.1 Maximally entangled state:

Let us consider the case in which Bob prepares a maximally entangled pure state \( |\phi\rangle_{AB_i} = 1/\sqrt{2}(|00\rangle_{AB_i} + |11\rangle_{AB_i}) \). If the quantum memory \( B_i \) interacts with environment, the dynamic of the bipartite quantum state \( \rho_{AB_i} = |\phi\rangle_{AB_i}\langle\phi| \) can be obtained as
\[
\rho_{AB_i}(t) = \frac{1}{2} \left[ \begin{array}{cccc} |C_i(t)|^2 & 0 & 0 & C_i(t) \\ 0 & 1 - |C_i(t)|^2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ C_i^*(t) & 0 & 0 & 1 \end{array} \right].
\]

Alice and Bob reach an agreement on measuring two observables \( \sigma_x \) and \( \sigma_z \). The von Neumann entropy of the post measurement states are given by
\[
S(\rho_{\sigma_x, B_i}) = -\frac{1 - \eta}{2} \log_2 \left( \frac{1 - \eta}{4} \right) - \frac{1 + \eta}{2} \log_2 \left( \frac{1 + \eta}{4} \right),
\]
where \( \eta = \sqrt{1 - |C_i(t)|^2} + |C_i(t)|^2 \). The right-hand side (LHS) of equation (7) is obtained as
\[
U_L = \frac{1}{2} - \frac{1 - \eta}{2} \log_2 \left( \frac{1 - \eta}{4} \right) - \frac{1 + \eta}{2} \log_2 \left( \frac{1 + \eta}{4} \right) - \frac{|C_i(t)|^2}{2} \log_2 \left( \frac{|C_i(t)|^2}{2} \right) - \frac{1 - |C_i(t)|^2}{2} \log_2 \left( \frac{1 - |C_i(t)|^2}{2} \right) - S_{\bin} \left( \frac{|C_i(t)|^2}{2} \right),
\]
where \( S_{\bin}(x) = -x \log_2 x - (1 - x) \log_2 (1 - x) \). The right-hand side (LHS) of equation (7) is given by
\[
U_R = 1 + S_{\bin} \left( \frac{1 - |C_i(t)|^2}{2} \right) - S_{\bin} \left( \frac{|C_i(t)|^2}{2} \right) - \max\{0, \delta\},
\]
where
\[
\delta = \frac{1}{2} - \frac{1 - \eta}{2} \log_2 \left( \frac{1 - \eta}{4} \right) - \frac{1 + \eta}{2} \log_2 \left( \frac{1 + \eta}{4} \right) - \frac{|C_i(t)|^2}{2} \log_2 \left( \frac{|C_i(t)|^2}{2} \right) - \frac{1 - |C_i(t)|^2}{2} \log_2 \left( \frac{1 - |C_i(t)|^2}{2} \right) - S_{\bin} \left( \frac{1 - |C_i(t)|^2}{2} \right) - S_{\bin} \left( \frac{|C_i(t)|^2}{2} \right).
\]
of additional qubit. In Figure 3, see, the lower bound decreases by increasing the number of additional qubits. As the second example, let us consider the set of two-qubit states with the maximally mixed marginal states.

Fig. 2. Lower bounds of the entropic uncertainty relation of the two complementary observables \(\hat{\sigma}_x\) and \(\hat{\sigma}_z\) as a function of \(\gamma_0 t\), when Bob prepare a maximally entangled state \(|\phi\rangle = 1/\sqrt{2}(|00\rangle + |11\rangle)\) in non-Markovian regime \(\lambda = 0.1\gamma_0\).

Fig. 3. Lower bounds of the entropic uncertainty relation of the two complementary observables \(\hat{\sigma}_x\) and \(\hat{\sigma}_z\) as a function of \(\gamma_0 t\), when Bob prepare a maximally entangled state \(|\phi\rangle = 1/\sqrt{2}(|00\rangle + |11\rangle)\) in Markovian regime \(\lambda = 40\gamma_0\).

In Figure 2, the EULB, \(U_R\) is plotted as a function of time in non-Markovian regime \(\lambda = 0.1\gamma_0\) for various number of additional qubit \(N\). As can be seen from Figure 2, due to the interaction between quantum memory and reservoir, EULB is increased through time. One can see, the lower bound decreases by increasing the number of additional qubit. In Figure 3, \(U_R\) is plotted as a function of time in Markovian regime \(\lambda = 40\gamma_0\) for various number of additional qubits \(N\). As can be seen from Figure 4, \(U_R\) is increased over time, while it is decreased by increasing the number of additional qubits.

3.1.0.2 Bell diagonal state:

As the second example, let us consider the set of two-qubit states with the maximally mixed marginal states.

\[
\rho_{AB} = \frac{1}{4}(I \otimes I) + \sum_{i=1}^{3} r_i \sigma_i \otimes \sigma_i, \quad (24)
\]

where \(\sigma_i\)s are Pauli matrices and \(\vec{r} = (r_1, r_2, r_3)\) belongs to a tetrahedron defined by the set of \((-1, -1, -1)\), \((-1, 1, 1), (1, -1, 1)\) and \((1, 1, -1)\). Bob prepares the state with \(r_1 = 1 - 2p, r_2 = r_3 = -p\), such that the state in equation (24) becomes

\[
\rho_{AB}^{\vec{r}} = p|\psi^-\rangle\langle\psi^-| + \frac{1-p}{2}(|\psi^+\rangle\langle\psi^+| + |\phi^+\rangle\langle\phi^+|), \quad (25)
\]

where \(|\phi^\pm\rangle = \frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle)\) and \(|\psi^\pm\rangle = \frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle)\) are the Bell diagonal states. In the following the Bell-diagonal state with \(p = 1/2\) is considered. The dynamics of density matrix when quantum memory \(B_i\) interacts with reservoir is given by

\[
\rho_{AB_i}(t) = \begin{pmatrix}
\rho_{11}^t & 0 & \rho_{14}^t \\
0 & \rho_{22}^t & \rho_{23}^t \\
\rho_{32}^t & \rho_{33}^t & \rho_{44}^t
\end{pmatrix}, \quad (26)
\]

where

\[
\rho_{11}^t = \frac{1+p}{4}|C_i(t)|^2, \quad \rho_{14}^t = \frac{1-p}{4}|C_i(t)| \\
\rho_{22}^t = \frac{1-p}{4} + \frac{1+p}{4} |C_i(t)|^2, \quad \rho_{23}^t = \frac{1-3p}{2}|C_i(t)| \\
\rho_{32}^t = \frac{1}{4}(1-p)|C_i(t)|^2, \quad \rho_{44}^t = \frac{1+p}{4} + \frac{1}{4}(1-p)(1-|C_i(t)|^2).
\]

Alice and Bob reach an agreement on measuring two observables \(\hat{\sigma}_x\) and \(\hat{\sigma}_z\). The von Neumann entropy of the
post measurement states are given by
\[
S(\rho_{s_{B_i}}) = -\frac{|C_i(t)|^2}{2} \log_2 \frac{|C_i(t)|^2}{4} - 2 - \frac{|C_i(t)|^2}{2} \log_2 \frac{2 - |C_i(t)|^2}{4},
\]
\[
S(\rho_{s_{s_{B_i}}}) = -\frac{|C_i(t)|^2}{8} \log_2 \left(\frac{|C_i(t)|^2}{8} - \frac{3|C_i(t)|^2}{8} \log_2 \frac{3|C_i(t)|^2}{8}ight) - 4 - \frac{3|C_i(t)|^2}{8} \log_2 \left(\frac{4 - |C_i(t)|^2}{8}\right) - 4 - \frac{|C_i(t)|^2}{8} \log_2 \left(\frac{4 - |C_i(t)|^2}{8}\right).
\]

So, the left-hand side (LHS) of equation (7) is obtained as
\[
U_L = -\frac{|C_i(t)|^2}{2} \log_2 \frac{|C_i(t)|^2}{4} - 2 - \frac{|C_i(t)|^2}{2} \log_2 \frac{2 - |C_i(t)|^2}{4} - \frac{3|C_i(t)|^2}{8} \log_2 \left(\frac{|C_i(t)|^2}{8} - \frac{3|C_i(t)|^2}{8} \log_2 \frac{3|C_i(t)|^2}{8}\right) - 4 - \frac{3|C_i(t)|^2}{8} \log_2 \left(\frac{4 - |C_i(t)|^2}{8}\right) - 4 - \frac{|C_i(t)|^2}{8} \log_2 \left(\frac{4 - |C_i(t)|^2}{8}\right).
\]

The right-hand side (RHS) of equation (7) is given by
\[
U_R = 1 - (\alpha_+ - \theta) \log_2 (\alpha_+ - \theta) - (\alpha_- - \theta) \log_2 (\alpha_- - \theta)
\]
\[
- (\alpha_+ + \theta) \log_2 (\alpha_+ + \theta) - (\alpha_- + \theta) \log_2 (\alpha_- + \theta) + \max\{0, \delta\} - S_{\text{bin}} \left(\frac{|C_i(t)|^2}{2}\right),
\]

where
\[
\alpha_\pm = \frac{(2 \pm |C_i(t)|^2)/2}{2},
\]
\[
\theta = \left(\sqrt{1 - |C_i(t)|^2 + |C_i(t)|^2}\right)/4,
\]

and
\[
\delta = (\alpha_- - \theta) \log_2 (\alpha_- - \theta) + (\alpha_+ - \theta) \log_2 (\alpha_+ - \theta)
\]
\[
+ (\alpha_- + \theta) \log_2 (\alpha_- + \theta) + (\alpha_+ + \theta) \log_2 (\alpha_+ + \theta) - S_{\text{bin}} \left(\frac{|C_i(t)|^2}{2}\right) + S(\rho_{s_{s_{B_i}}}) + S(\rho_{s_{s_{B_i}}}).
\]

In Figure 4, the EULB, $U_R$ is plotted as a function of time in non-Markovian regime $\lambda = 0.1 \gamma_0$ for various number of additional qubit $N$. As can be seen from Figure 4, due to the interaction between quantum memory and reservoir, EULB is increased through time. One can see, the lower bound decreases by increasing the number of additional qubit. In Figure 5, $U_R$ is plotted as a function of time in Markovian regime $\lambda = 40 \gamma_0$ for various number of additional qubits $N$. As can be seen from Figure 5, $U_R$ is increased over time, while it is decreased by increasing the number of additional qubits.

4 Conclusion

In this work we have studied the quantum-memory assisted EULB when quantum memory interacts with reservoir. The model we have considered here is such that the quantum memory along with $N - 1$ non-interacting qubits located in common reservoir. We assume that these $N$ qubit independently are coupled to a common reservoir. It is logical to expect that the EULB increases over time. However, we have shown that the EULB can be protected from decoherence by controlling the additional qubits in reservoir. It has been shown that for both Markovian and non-Markovian regime the EULB is decreased by increasing the number of additional qubits.

Author contribution statement

All authors have contributed equally.

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