Design of a freeform uniformity corrector lens for extended sources in elliptical reflectors

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Abstract
Illumination design usually requires the collection of a large solid angle of radiation from the light source. However, it is known that elliptical reflectors in combination with extended uniform light sources result in a non-uniform irradiance profile at the secondary focus. Within this paper we propose a design method based on phase space transformations, which includes the source extension from the very beginning. We show that an analysis of the local mapping of the source to the target radiance distribution allows a profound understanding of the effects and in consequence a design concept for an additional freeform lens to correct the uniformity at the secondary focus.

1. Introduction

Many commonly used high power light sources, such as light emitting diodes (LED) or plasma sources, are emitting their radiation into a large solid angle. In consequence a major task in illumination design is usually to collect a large portion of the intensity profile of a light source and to create a specified irradiance profile at a target plane [1]. During the design of a collection system it is quite common to have idealized assumptions about the light source. Very often non-physical light sources, like point sources, are assumed since they allow for the application of mathematical construction methods [2] for the illumination elements. As a simple example point sources quite naturally lead to conic reflector geometries, since they allow for perfect point to point transfer, even for very large angles. Also elements for more complex irradiance patterns are often constructed by mathematical methods, which are based on point sources [3–5]. Common examples are freeform illumination reflectors for various applications [6–8].

However, in contrast to point sources realistic light sources are exhibiting a finite area or volume, which together with the large angular emission characteristics correspond to a noticeable etendue. Especially LED, due to their emission area and Lambertian intensity pattern, exhibit a large angle-area product, which needs to be considered during illumination design [9, 10], because it leads to undesired changes in the attempted irradiance pattern. In the design process the etendue of such light sources is often considered via a feedback step during optimization [11, 12]. Unfortunately the iterative numerical process lacks a deeper understanding of the effect and limitations of the source extensions.

This work presents an approach which includes the etendue of the source from the very beginning. To do so, the method of phase space in optics is employed, which allows for a direct visualization of the radiance distribution [13, 14]. Phase space methods have already proven to be promising for improved raytracing of illumination systems [15]. Within this paper, we will try to employ this approach for the design of illumination systems, in particular for a uniformity-corrector lens [16, 17] used in combination with an elliptical reflector. The analysis of the local transformation in phase space allows us to find an appropriate design approach to optimize a thick freeform correction lens. The method can be employed in a general way for other and more complex illumination design problems.
2. Phase space definition and radiance propagation

A planar light source of a certain emission area $A$, e.g. a LED, which is radiating into some solid angle $\Omega$, can be represented by the corresponding etendue $H$ via the relation:

$$H = n^2 \cdot \int_\Omega \int_A dA \cdot \cos \theta \cdot d\Omega = \int_0^\pi \int_0^{\frac{\pi}{2}} dx \cdot dy \cdot du \cdot dv. \quad (1)$$

Here $n$ is the index of refraction, $dA$ is the surface element and $\cos(\theta) \, d\Omega$ is the differential projected solid angle. If the source is planar in the $xy$-plane then the etendue is conveniently expressed in terms of the differential area $dA = dx \, dy$ and the projected solid angle element as $du \, dv$, where $u = nL$ and $v = nM$, where $(L, M, N)$ represents the direction vector. The amount of optical power $d\phi$ contained inside a differential etendue element defines the local radiance $L$ via:

$$L(x, y, u, v) = \frac{d\varphi(x, y, u, v)}{dx \cdot dy \cdot du \cdot dv} = n_r^2 \cdot \frac{d\varphi(x, y, u, v)}{dA \cdot \cos \theta \cdot d\Omega}. \quad (2)$$

Thus the radiance distribution is a four-dimensional function of the variables $(x, y, u, v)$. In this work however we will only consider rotational-symmetric systems and only concentrate on tangential rays. Any tangential ray $r$ corresponds to a pair of $xu$-values, where $x$ is the radial ray position and $u$ is associated with the ray-angle relative to optical axis $u = n \sin \theta$. For this simplified case we can limit our analysis to two dimensions. An illustration of rays in this two-dimensional space $(xu)$-diagram is called a phase space diagram and defines the concept of phase space in optics. Following from equation (2) the radiance in the two-dimensional case is associated with an area in phase space

$$L(x, u) = \frac{d\varphi(x, u)}{dx \cdot du} \approx \frac{\Delta \varphi(x, u)}{\Delta x \cdot \Delta u}. \quad (3)$$

For paraxial systems the transformation properties of the rays and the corresponding radiance distribution is closely related to the ABCD-matrix formalism, which corresponds to a linear transformation within phase space, since:

$$r' = \begin{pmatrix} x' \\ u' \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \cdot \begin{pmatrix} x \\ u \end{pmatrix} = M \cdot r. \quad (4)$$

Mathematically, any linear matrix operation $M$ in phase space corresponds to a general shear, scaling, or rotation of the radiance distribution. For a simple paraxial imaging situation the phase space transformation is illustrated in figure 1.

Here the radiance transformation mostly corresponds to a pure magnification, corresponding to a transformation matrix of the type

$$M = \begin{pmatrix} \beta & 0 \\ 0 & 1/\beta \end{pmatrix}. \quad (5)$$

where the parameter $\beta$ defines the paraxial magnification.

In consequence the main effect is a re-scaling of the rectangular source radiance (figure 1(c)) to the target (figure 1(d)), where the size of the radiance distribution in $x$ is increased, whereas due to etendue conservation, the angles $u$ are decreased. The other effect of the system is slight shear of the distribution into the angular direction. This corresponds to the not exactly telecentric image and could easily be removed by a field lens at the image. Not that this effect is also present in the later analysis (figures 3 and 5) and not of further relevance.

For more general non-paraxial systems, especially systems with large angles and tilted complex surfaces, the corresponding radiance transformation is usually much more complex, nonlinear and cannot be expressed by one linear matrix. But still for every small radiance element the local ABCD matrix can be calculated, for example by differential ray-tracing and MATLAB-analysis around the reference ray. It is helpful that the commercial optical design software CodeV [18] provides a function to display this local ABCD-matrix for every ray, even for tilted and decentered systems. So with the help of a ray-tracer we can analyze the transformation of the input radiance from object (light source) to image in a phase space diagram. From the final radiance distribution all radiometric quantities, like irradiance $E(x)$ can be calculated via a simple projection of the radiance distribution [13, 14].

3. Radiance transformation of an elliptical reflector

Let us now consider an ideal conic reflector. Figure 2 shows a situation where a point source is placed at the first focus of an elliptical mirror. An appropriate conic constant $k$ of the mirror will result in a perfect point image in
the secondary focus (target) even for large angles. In this study we consider a maximum $\text{NA} = 0.9$ (corresponding to $\theta = 64^\circ$).

If we now replace the idealized point source by an extended circular source of diameter $d = 1$ mm (figure 3(c)) the properties of the ellipse [19] will result in a nonhomogeneous irradiance profile at the target, as shown in figure 3(b). We here note, that the extended source is also idealized assumed to be self-transparent, so the rays are passing the source without intensity change. For a realistic source the transmission characteristics would need to be included, however for the sake of this study we concentrate on the effect of the source extension only. In this sense the non-uniformity can be understood from the corresponding phase space mapping from source to target, which is illustrated in figure 3(d): as the source is located in the first focal plane of the system, and the image plane is in the second focus, the basic phase space transformation again corresponds to a magnification, resulting in a re-scaling in phase space, as expressed by equation (5). However due to the properties of the ellipse the magnification is not constant, but depending on the angle. In the paraxial region the

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**Figure 1.** Illustration of a simple paraxial imaging system in phase space: (a) paraxial system of one ideal single lens with magnification $\beta = 2 \times$; (b) same paraxial lens, but modified in magnification to $\beta = 1.25 \times$ by a telescope; (c) radiance pattern the light source of $0.4 \text{ mm} \times 0.4 \text{ rad}$; (d) illustrates the corresponding transformed radiance patterns of both systems at the image.

**Figure 2.** Illustration of the collection geometry of the elliptical mirror used in this design study.
magnification is given by \( \beta = s'/s = 5 \times \), so for small angles \( \theta \) the source is imaged onto a \( d' = 5 \) mm diameter disc, whereas for higher angles the magnification is reduced, as shown in figure 3(e).

This effect of angular varying magnification can also be interpreted as third order coma of the elliptical mirror, corresponding to an offense against the sin-condition, meaning that the angular magnification is changing across the pupil. We can determine the local value of the magnification for every angle (as given in figure 3(e)) via analytical equations [19], or just by an analysis of the local ABCD-transformation matrix in a ray-tracer (e.g. CodeV). As a consequence of the magnification variation the uniform irradiance profile of the source is mapped onto differently sized images, resulting in a non-uniform distribution at the target. The situation is also illustrated by the ray-tracing image of figure 3(a). The rays clearly reveal the larger magnification for small angels, versus the reduced magnification at high angles.

4. Design of a uniformity-correction lens

In order to correct the disturbing effect of angular variation of magnification we need an optical element, which can change the magnification without changing the location of the secondary focus. If we fix the shape of the ellipse, a single optical power (or surface) is not able to achieve this. However, a telescopic arrangement of two optical powers very similar to the arrangement in figure 1(b) can perform the required transformation of magnification, while keeping the ellipse and the location of the second focus fixed. Such a telescopic arrangement can be realized by a thick lens, as shown in figure 4. The front and back surface of the thick lens must be highly aspheric, or in the general case a freeform lens, since the telescopic factor is required to drastically change in order to compensate the effect of the varying magnification. We can deterministically design the shape of the freeform lens by optimizing the full system within any design program to a constant magnification. For an optimization to a magnification of \( \beta = 3, 8 \times \) for all angles we find the aspheric deviation form a base sphere as given in the table inside figure 4, for a lens of thickness \( 7 \) mm and made from BK7.

The full system is illustrated in figure 5. The analysis of the system reveals that the nonlinear phase space mapping of the standard ellipse is now almost perfectly counter-compensated by the freeform lens, leading to an almost linear phase space mapping. As a result the image of the planar extended source is uniformly imaged onto the target.

Clearly this example is a very simple one, and the result could have been achieved by other types of analysis and optimization. However the presented analysis of the underlying phase space transformation allows an alternate identification of the basic problems and limitations, especially for extended sources, in this case the varying magnification across the source. In consequence an optical designer is able to find deterministic optical elements to solve this problem, respectively to design the merit function. Here a thick lens with a non-uniform
telescope functionality. In that sense we are confident, that the described method can be employed in the deterministic design of more complex illumination systems for extended sources.

5. Conclusion

We have presented an alternative method to analyze and correct illumination systems, which includes the etendue of a realistic extended source from the very beginning. An illustration of the radiance transformation properties in phase space allows insight into the system and limitations. Based on this analysis a specific correction element can be designed to improve the system. We have demonstrated this procedure by correcting a standard elliptical mirror via a highly aspheric thick lens, such that an extended source can be transferred to the secondary focus without introducing non-uniformity. Moving forward, this can lead to novel design methods for extended sources, which are based on controlling the local transformation properties in phase space.

Acknowledgments

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