Multi-level and two-level models of the decay out of superdeformed bands

A. J. Sargeant*, M. S. Hussein* and A. N. Wilson†**

*Instituto de Física, Universidade de São Paulo, Caixa Postal 66318, 05315-970 São Paulo, SP, Brazil
†Department of Nuclear Physics, Research School of Physical Sciences and Engineering, Australian National University, Canberra, ACT 0200 Australia
**Department of Physics and Theoretical Physics, Faculty of Science, Australian National University, Canberra, ACT 0200 Australia

Abstract. We compare a multi-level statistical model with a two-level model for the decay out of superdeformed rotational bands in atomic nuclei. We conclude that while the models depend on different dimensionless combinations of the input parameters and differ in certain limits, they essentially agree in the cases where experimental data is currently available. The implications of this conclusion are discussed.

INTRODUCTION

Superdeformed (SD) rotational bands occur in the second minimum of the potential energy surface in deformation space. They have been observed in several mass regions around \( A = 20, 40, 80, 130, 150, 165, 190 \) and \( 240 \) [1]. Characteristic of the decay of most superdeformed bands is the sudden disappearance of the total intra-band decay intensity when a certain spin is reached. The intensity reappears as electromagnetic transitions in the normal deformed (ND) minimum. In certain special cases the decay path subsequent to the SD band is known completely as it is for \(^{133}\)Nd [2], or almost completely as for \(^{59}\)Cu [3]. More typical however is the situation in the \( A = 80, 150 \) and 190 regions, where the decay from superdeformed to normal states is spread over many different available paths, making observation of discrete \( \gamma \) rays linking SD and ND states very difficult. Because of this fragmentation of the SD intensity, experimentalists have only been able to identify a small number of the strongest paths in a few cases [4, 5, 6, 7, 8, 9, 10, 11] and these only when very large data sets have been obtained. In these more typical regions it is reasonably clear that there remains barrier between the minima at the decay-out spin and consequently that the decay-out occurs by tunneling [12, 13, 14, 15, 16].

The fact that the decay path is unknown has been thought to indicate that a statistical treatment of the ND states and their coupling to the SD band [17, 18] is appropriate. This approach is expected to be valid when the ND level density is so high as to make a useful description of the coupling to individual ND states unfeasible. However, given that there is uncertainty concerning the ND level density around the SD band there remains the possibility that the decay-out occurs entirely through a single ND level. The ND level density depends strongly on the excitation energy of the SD state; as SD bands have been observed at relatively low energies in the Pb isotopes, the point of view encompassed by a two-level model where the decaying SD state couples to a single ND state (itself decaying) could be more appropriate. Indeed, in Ref. [19] the decay-out in the \( A = 190 \) region is attributed to the crossing of the SD band with the nearest neighboring excited ND band.

In this contribution we propose to compare the two limiting descriptions mentioned in the previous paragraph as embodied by the multi-level statistical model of Refs. [20, 21] which exploit analogies of the SD decay with the theory of compound nucleus reactions [22] and the two-level model of Refs. [23, 24, 25]. Neither approach provides a microscopic description of the decay-out such those provided by the cranked Nilsson-Strutinsky model of Refs. [12, 13, 14, 15], the cluster model of Ref. [13] and the generator coordinate model of Ref. [26]. Rather, both models considered here are phenomenological models which attempt to provide simple formulas for the total intra-band decay intensity in terms of the most relevant parameters. Instead of analyzing the experimental data, in what follows we shall only compare the statistical and two-level models in their formal aspects.
FIGURE 1. Schematic diagram of the multi-level statistical model. The decaying SD state under consideration has energy $\epsilon_S$ and spin $J$ and mixes via the coupling $V_{JS}$ with ND states of the same spin whose energies, $\epsilon_j$, are the eigenvalues of the GOE Hamiltonian $H_{NN'}$. The SD (ND) states have a common electromagnetic width $\Gamma_S (\Gamma_N)$.

MULTI-LEVEL STATISTICAL MODELS

Let us denote the decaying SD state by $|S\rangle$, its energy by $\epsilon_S$ and its electromagnetic width by $\Gamma_S$. It is coupled to the ND states $|N\rangle$, $N = 1, ..., K$ by $V_{NS}$. The ND states are modeled by the GOE Hamiltonian $H_{NN'}$ and are assumed additionally to have a common electromagnetic width $\Gamma_N$. The intra-band decay (see Fig. 1) is described by the Green’s function \[ (g^{-1})_{NN'} = E \delta_{NN'} - H_{NN'} + \frac{i}{2} \Gamma_N \delta_{NN'} \] and the decay-out by the Green’s function \[ G_{NS}(E) = \sum_N (e^{-1})_{NN'} \frac{H_{NS}}{(E - \epsilon_S + \frac{i}{2} \Gamma_S)}; \quad e_{NN'} = E \delta_{NN'} - H_{NN'} + \frac{i}{2} \Gamma_N \delta_{NN'} - \frac{H_{NS}H_{SN'}}{(E - \epsilon_S + \frac{i}{2} \Gamma_S)}. \]

The total intra-band decay intensity is then given by \[ F_S = \frac{\Gamma_S}{2\pi} \int_{-\infty}^{\infty} dE |G_{SS}(E)|^2 \] and the total decay-out intensity by \[ F_N = \frac{\Gamma_N}{2\pi} \int_{-\infty}^{\infty} dE \sum_N |G_{NS}(E)|^2. \]

The ensemble average is defined by \[ \overline{F}_S = \int F_S(H_{NN'}) P(H_{NN'}) d|H_{NN'}|, \] where $P(H_{NN'})$ is the probability distribution of the GOE and $d|H_{NN'}|$ is the product of the differentials of all independent matrix elements of $H_{NN'}$. Gu and Weidenmüller calculated $\overline{F}_S$ by adapting the result for the ensemble average of the compound nucleus cross-section derived by Verbaarschot et al. to obtain, in the limit $K \to \infty$, \[ \overline{F}_S = F_S^\text{av} + F_S^\text{fl}, \]
where \(F_{av}^S = \frac{1}{1 + \Gamma / \Gamma_S}\) \((7)\) and \(F_{av}^T = \frac{1}{16 \Gamma_S^2} \int_{-\infty}^{\infty} dE \int_0^\infty d\lambda_1 \int_0^\infty d\lambda_2 \int_0^\infty d\lambda \frac{(1-\lambda_2)^2}{(1+\lambda_1)(1+\lambda_2)^{1/2}(1+\lambda_1)^{1/2}} \exp[-\pi \Gamma / D (\lambda_1 + \lambda_2 + 2\lambda)]\)

\[
\frac{1}{1+T_1^2/(1+T_1)^{1/2}} \left( |S(E)|^2 T^2 \left( \lambda_2^{1/2} + \frac{\lambda_1}{T_1}\right) + 2 T^2 \left( \frac{\lambda_1}{T_1} + \frac{\lambda_2}{T_1}\right) \right),
\]

with \(S(E) = \frac{E - E_0 - i \Gamma_S/2 + \Gamma/2}{E - E_0 + i \Gamma_S/2 + \Gamma/2}\), \(T = 1 - |S(E)|^2\) \((8)\).

The density of ND levels is denoted by \(D\). From Eq. \(8\) Gu and Weidenmüller deduced the fit formula \(F_{av}^T = \left[1 - 0.9139 (\Gamma_N/D)^{0.2172}\right] \exp\left[-\left[\frac{0.4343 \ln(\Gamma/\Gamma_S) - 0.45 (\Gamma_N/D)^{-0.1303}}{(\Gamma_N/D)^{-0.1477}}\right]^2\right]\) \((9)\).

Energy averaging provides an alternative approach to calculating the average values of observables which depend on a random Hamiltonian. The energy average of \(G(E)\) is given by \(\overline{G(E)} = \int G(E') p(E,E') dE'\) \((11)\), where the smoothing function \(p(E,E')\) is normally a Lorentzian or a box function \((22)\). The energy average is carried out for a single realization of the GOE Hamiltonian \(H_{NN'}\) and is expected to be equal to the ensemble average to the extent that the GOE is ergodic. In practice calculations proceed by choosing a representation - the optical background representation of Kawai, Kerman and McVoy \((30)\) - which is defined such that the couplings \(V_{NS}\) have statistical properties which are convenient for analytical calculation. (It was checked numerically in Ref. \(31\) that the properties of the \(V_{NS}\) which are normally assumed do indeed obtain from the underlying random Hamiltonian.) In order to obtain analytical results it is further necessary to assume that \(\Gamma_N/D \gg 1\) which is the principal limitation of the energy averaging technique. The optical background representation was used in Ref. \(21\) to calculate \(T_S\). The result has the same form as Eq. \(6\) with \(F_{av}^S\) still given by Eq. \(7\) but now \(\overline{F_S^T} = 2(\pi \Gamma_N/D)^{-1} F_{av}^T (1 - F_{av}^S)^2\), \(\Gamma_N/D \gg 1\) \((12)\).

An advantage of the energy averaging technique is that it also yields an analytical expression for the variance of the decay intensity \((21)\):

\[
\frac{\Delta F_S^2}{F_S^2} = \frac{(F_S - \overline{F_S})^2}{\overline{F_S}} = \left(\frac{\Gamma_S}{2\pi}\right)^2 \int_{-\infty}^{\infty} dE \int_{-\infty}^{\infty} dE' \left[ G_{SS}^0(E) G_{SS}^0(E')^* + 2ReG_{SS}(E)^* G_{SS}(E') + G_{SS}(E)^* G_{SS}(E')^* \right] + 2 F_{av}^T F_{av}^S \overline{F_S^T} \overline{f_3(\xi)} + 2 F_{av}^S \overline{f_2(\xi)},
\]

where \(\xi = (\Gamma_S + \Gamma) / \Gamma_N\) and

\[
f_1(\xi) = \frac{1}{1+\xi} + \frac{\xi}{1+\xi}^2 + \frac{\xi^2}{2(1+\xi)^3}, \quad f_2(\xi) = \frac{1}{2(1+\xi)}. \]

From Eqs. \(6\), \(7\), \(8\), \(9\) and \(12\) it is seen that \(\overline{F_S}\) depends only on the two dimensionless variables \(\Gamma / \Gamma_S\) and \(\Gamma_N / D\). However it is possible to construct three independent dimensionless variables from the input variables \(\Gamma\), \(\Gamma_S\), \(\Gamma_N\) and
D. In Ref. [20] it is conjectured using supersymmetry arguments [29, 32] that the entire probability distribution of $F_S$ depends only on $\Gamma/\Gamma_S$ and $\Gamma_N/D$. In contradiction to this conjecture the expression for the variance of $F_S$ derived using the energy averaging technique, Eq. (13), depends on the additional dimensionless variable $(\Gamma_S + \Gamma)/\Gamma_N$.

At present the energy dependence of $|G_{SS}(E)|^2$ and $|G_{NS}(E)|^2$ cannot be resolved experimentally and the focus of theory has been on the analysis of $F_S$ or $F_N$. Normally, $\Gamma_S$, $\Gamma_N$ and $D$ are estimated theoretically and a value of $V$ is extracted from the experimental $F_S$ [33]. An intrinsic problem of the statistical model in its application to the decay out of SD bands is that the variance of $F_S$ is large when $\Gamma_N/D \ll 1$ reflecting the fact that in this limit the positions of the individual ND states relative to the SD state are important [20]. Since $\Gamma_N/D \ll 1$ for the cases which have been studied experimentally the preceding argument suggests that the error in the extracted $V$ must also be large. However, from Eqs. (13) and (14) it can be seen that the variance of $F_S$ is suppressed not only for $\Gamma_N/D \gg 1$ but also for $(\Gamma + \Gamma_S)/\Gamma_N \gg 1$. This is physically plausible since in the decay of an ND state to the SD state the larger the value of $\Gamma + \Gamma_S$ the less important the exact position of the SD state.

**TWO-LEVEL MIXING MODEL**

In the two-level mixing model the decaying SD state of energy $\epsilon_S$ and electromagnetic width $\Gamma_S$ is coupled to single ND state of energy $\epsilon_N$ and electromagnetic width $\Gamma_N$ by the coupling matrix element $V$ (see Fig. 2). In analogy to Eqs. (1) and (2), the intra-band decay is described by the Green’s function

$$G_{SS}(E) = \frac{1}{E - \epsilon_S + i\Gamma_S/2 - \frac{V^2}{E - \epsilon_N + i\Gamma_N}}$$

(15)

and the decay-out by the Green’s function

$$G_{NS}(E) = \frac{V}{(E - \epsilon_N + i\Gamma_N)(E - \epsilon_S + i\Gamma_S) - V^2}.$$  

(16)

The decay-out intensity, $F_N$, is given by Eq. (3) (instead of a sum over $N$ there is just one term) and an exact calculation yields [23]

$$F_N = \frac{(1 + \Gamma_N/\Gamma_S)V^2}{\Delta^2 + \Gamma^2(1 + 4V^2/\Gamma_N\Gamma_S)}.$$  

(17)
Alternatively, one could calculate the average of to zero to 1. In Ref. [25] it was argued that the average value Eq. (5) is an average over the statistical model. Thus, while Eq. (23) is related to the ensemble average, Eq. (5), it is not identical with it since

\[
\langle F_N \rangle = \frac{\Gamma_N \Gamma_s}{\Gamma_s + \Gamma_N \Gamma_s}. 
\]

Since \( F_S = 1 - F_N \) this implies that

\[
F_S = \left( 1 + \frac{\Gamma_s}{\Gamma_N} \frac{\Gamma_N}{\Gamma_N + \Gamma_s} \right)^{-1}. 
\]

From Eq. (17) it may be seen that \( F_N \) depends on three independent dimensionless variables which may be taken to be \( V/\Gamma, \Gamma/\Delta \) and \( \Gamma_S/\Gamma_N \). The calculations of Ref. [19] suggest that the value of \( \Delta \) is rather important and in fact determines the decay-out spin in the sense that the decay-out occurs very near to where the SD band crosses the nearest excited ND band. The behavior of \( F_S \) as a function of \( \Delta \) for a model related to the two-level model is discussed in Ref. [34]. In the event that \( \Delta \) could be calculated it would be favorable to use Eq. (17) to extract \( V \) from the experimental \( F_N \).

An alternative to calculating \( \Delta \) is to assume that it obeys some probability distribution. In Ref. [25] it was assumed that \( \Delta \) has the distribution

\[
P(\Delta) = \int_0^\infty ds \mathcal{P}_s(\Delta) P(s),
\]

where

\[
P(s) = \frac{\pi}{2} s^2 e^{-\pi s^2/4} \quad \text{and} \quad \mathcal{P}_s(\Delta) = \frac{1}{D} \Theta(\Delta^{-\frac{1}{2}}). 
\]

In Ref. [25] it was argued that the average value \( \langle |\Delta| \rangle = D/4 \) was the appropriate value to use in the calculation of \( F_N \). Alternatively, one could calculate the average of \( F_N \) with respect to the distribution \( P(\Delta) \):

\[
\langle F_N \rangle = \int_{-\infty}^\infty F_N(\Delta) P(\Delta). 
\]

Since \( \Delta = \epsilon_N - \epsilon_S \), Eq. (23) corresponds to an average over the eigenvalues, \( \epsilon_j \), of the GOE Hamiltonian of the statistical model. Thus, while Eq. (23) is related to the ensemble average, Eq. (5), it is not identical with it since Eq. (5) is an average over the \( H_{NN'} \), i.e. an average over both eigenvalues and eigenvectors. It is clear from Eq. (23) that \( \langle F_N \rangle \) depends on the dimensionless variables \( V/D, \Gamma/D \) and \( \Gamma_S/\Gamma_N \). Both \( F_N(\Delta = D/4) \) and \( \langle F_N \rangle \) are compared with \( F_N \) of the statistical model in the next section.

### NUMERICAL COMPARISON OF THE MODELS

The first thing which may be noted on comparing the statistical model and the two-level model is that they depend on different dimensionless combinations of the input parameters \( V, D, \Gamma_S \) and \( \Gamma_N \). The statistical model depends on \( \Gamma/\Gamma_S = 2\pi(1/2)^2(1/\Gamma_S) \) and \( \Gamma_N/\Gamma_S \) while the two-level model depends on \( V/D, \Gamma/\Gamma_S \) and \( \Gamma_N/\Gamma_S \). In addition, it may be deduced from Eq. (17) that \( F_N \) in the two-level model increases monotonically from zero to \( \Gamma_N/\Gamma_S \) as \( V \) is varied from zero to infinity whereas in the statistical model \( F_N \) increases monotonically from zero to 1.

A second difference occurs in the (physically not realized) limit \( \Gamma_N/D \to \infty \). In this limit \( F_N \) in the statistical model becomes independent of \( \Gamma_N \), being given by \( 1 + \Gamma_S/\Gamma \)^\(-1 \), cf. Eq. (17). The two-level model in the limit that \( \Gamma \to \infty \) yields \( F_N = \Gamma_N/(\Gamma_N + \Gamma_S)[1 + \Gamma_N/\Gamma_S/4V^2]^{-1} \) which clearly does not become independent of \( \Gamma_N \) as \( \Gamma_N/D \to \infty \).

It is useful to note that when a uniform distribution for \( \Delta \) is assumed then

\[
\langle F_N \rangle_U = \frac{1}{D} \int_{-\infty}^\infty F_N(\Delta) d\Delta = \frac{2\pi V^2/\Gamma S}{\sqrt{1 + 4V^2/\Gamma_N \Gamma_S}}, 
\]

where

\[
\Gamma_i = \frac{2V^2 r_i}{\Delta^2 + r_i^2}, \quad \Gamma = (\Gamma_S + \Gamma_N)/2 \quad \text{and} \quad \Delta = \epsilon_N - \epsilon_S. 
\]
FIGURE 3. The figure shows $1 - \sigma_S$ (thick solid), $\langle F_N \rangle$ (thin solid), $\langle F_N \rangle_U$ (dotted) and $F_N(\Delta = D/4)$ (dashed) versus $\log_{10}(V/D)$, calculated using Eqs. (6), (23), (24) and (17) respectively. The left column has $\Gamma_N/\Gamma_S = 1000$ and the right column has $\Gamma_N/\Gamma_S = 1$. The first, second and third rows have $\bar{\Gamma}/D = 10^{-4}$, 0.1 and 10 respectively. The dependence on $\Gamma_N/\Gamma_S$ is clearly seen in the two-level model as is its absence in the statistical model. Eq. (24) is seen to agree with the statistical model for small $V$ but to be unphysical for large $V$. It can also be seen that $\langle F_N \rangle$ and $F_N(\Delta = D/4)$ agree best with the statistical model when $\bar{\Gamma}/D$ is small. Indeed, when $\Gamma_N \gg \Gamma_S$ so that the coherence effects present in the two-level model are suppressed and $\bar{\Gamma} \ll D$ then $\langle F_N \rangle$ and $F_N(\Delta = D/4)$ are close to the statistical model for all $V/D$. It is interesting that $\langle F_N \rangle_U$ agrees better with statistical model than $\langle F_N \rangle$ when $\bar{\Gamma} \gg 1$ suggesting that in this region the average over the uniform distribution, Eq. (24), corresponds more closely to the ensemble average, Eq. (5), than Eq. (23) does.
FIGURE 4. Decay-out intensity for the three-level model. The dotted and long dashed curves are the branching ratios for decay to the nearest ND state and the next nearest ND state respectively while the thin solid curve is the their sum (see Eq. (25) and the discussion which follows it). The thick solid and dashed curves are the same as in Fig. 3 (they are the decay-out intensities for the multi-level statistical and two-level models respectively) as are values of $\bar{\Gamma}/D$ and $\Gamma_N/\Gamma_S$ used to plot the curves.

In Ref. [25] the validity of the two-level model was investigated by calculating the decay-out intensity for a three-level model where the decaying SD state is allowed to decay to the nearest neighbor and next-nearest-neighbor ND states. The decay-out intensity may be written

$$ F_N^{(3)} = \sum_{j=1}^{2} F_j, \quad F_j = \frac{\Gamma_N}{2\pi} \int_{-\infty}^{\infty} dE |G_{jS}(E)|^2, $$

where $G_{jS}$ denotes the Green's function of Eq. (2) in the basis of eigenvectors, $|j\rangle$, of $H_{NN'}$. Clearly, $F_N^{(3)}$ depends on $\Delta_1$ and $\Delta_2$, the relative energies of the two ND states and on $V_1$ and $V_2$, their interactions with the SD state. In Ref. [25] a probability distribution for $\Delta_2$ similar to Eq. (21) was assumed resulting in the average value $\langle \Delta_2 \rangle = 3D/4$. 

The case $\bar{\Gamma}/D = 10^{-4}$, $\Gamma_N/\Gamma_S = 1000$ shown in Fig. 3 is representative of the mass 190 region. The considerations in the present contribution thus go some way to explaining why the values of $V$ extracted [33] from experimental data in the mass 190 region using the statistical model and those extracted using the two-level model were very similar.
In Fig. 6 we compare $\overline{F}_N$ of the multi-level statistical model and $F_N(\Delta = D/4)$ of the two-level model with $F_N^{(3)}$, $F_1$ and $F_2$ for $\Delta_1 = D/4$, $\Delta_2 = 3D/4$ and $V_1 = V_2 = V$. In similarity to two-level case, the effects of coherence are pronounced in $F_N^{(3)}$ when $\Gamma_N = \Gamma_S$. It can be seen that $F_N^{(3)}$ moves in the direction of $\overline{F}_N$ relative to $F_N(\Delta = D/4)$. (As mentioned in the caption to Fig. 5, $\overline{F}_N$ appears to be underestimated in the middle row due to use of the approximate Eq. (10).) We expect that $F_N^{(n)}$ calculated analogously to $F_N^{(3)}$ would rapidly approach $\overline{F}_N$ with increasing $n$. The fact that $F_N^{(3)}$ and $F_N(\Delta = D/4)$ are barely distinguishable for the case $\Gamma / D = 10^{-4}$, $\Gamma_N / \Gamma_S = 1000$ again suggests that the two-level model is adequate to describe the SD decay in the mass 190 region. It would be interesting to calculate the average of $F_N^{(3)}$ over $\Delta_1$ and $\Delta_2$ in analogy to Eqs. (23) and (24).

CONCLUSIONS

We have compared a multi-level statistical model and a two-level model of the decay out of a superdeformed band with each other. We conclude that while the models depend on different dimensionless combinations of the input parameters and yield different results for certain limiting cases, they essentially agree in the cases where experimental data are available. However, this conclusion does not mean that we have reached a fully satisfactory description of the decay-out of superdeformed bands. The two-level model and multi-level statistical model level quite naturally only agree in the parameter regime where there are just two well defined levels interacting. However, this agreement, as it is described in the preceding section, surely has much to do with the use of the GOE probability distribution to calculate the ensemble average in Eq. (5) and the use of the Wigner nearest neighbor spacing distribution in obtaining Eq. (21) for the probability distribution of $\Delta$. The use of these two intimately related distributions to account for the unknown parameters of the respective formulations means that level fluctuations must be equally important in the two-level and multilevel statistical models. Consequently, the respective values of $V$ which can be extracted from the experimental $F_N$ using the two models appear to be subject to similar errors. An analytical calculation of the variance of $F_N$, based on either model, which is valid in the physically realized region would be useful in assessing the errors.

ACKNOWLEDGMENTS

We thank D. M. Cardamone for providing us with the calculations for the three-level model shown in Figure 6 and for many comments on a draft of this contribution. We benefited from discussions with DMC and C. A. Stafford during the Nuclei and Mesoscopic Physics Workshop held at the NSCL in October, 2004. AJS is also grateful to DMC for answering questions about Ref. [25] by email. MSH is partly supported by FAPESP and the CNPq and both MSH and AJS are supported by the Instituto de Milênio de Informação Quântica - MCT, all of Brazil.

REFERENCES

1. B. Singh, R. Zywna, and R. B. Firestone, Nucl. Data Sheets, 97, 241–592 (2002).
2. D. Bazzacco, F. Brandolini, R. Burch, S. Lunardi, E. Maglione, N. H. Medina, P. Pavan, C. Rossi-Alvarez, G. de Angelis, D. de Acuna, M. de Poli, J. Rico, D. Bucurescu, and C. Ur, Phys. Rev. C, 49, 2281 (1994).
3. C. Andreoiu, T. Døssing, C. Fahlander, I. Ragnarsson, D. Rudolph, S. Aberg, R. A. Austin, M. P. Carpenter, R. M. Clark, R. V. Janssens, T. L. Khoo, F. G. Kondev, T. Lauritsen, T. Rodinger, D. G. Sarantites, D. Seweryniak, T. Steinhardt, C. E. Svensson, O. Thelen, and J. C. Waddington, Phys. Rev. Lett., 91, 232502 (2003).
4. C. J. Chiara, et al., AIP Conf. Proc. (2005), proceedings of the Conference on Nuclei at the Limits, Argonne National Laboratory, July 2004 (to be published).
5. T. Lauritsen, M. P. Carpenter, T. Døssing, P. Fallon, B. Herskind, R. V. Janssens, D. G. Jenkins, T. L. Khoo, F. G. Kondev, A. Lopez-Martens, A. O. Macchiavelli, D. Ward, K. S. Abu Saleem, I. Ahmad, R. Clark, M. Cromaz, J. P. Greene, F. Hannachi, A. M. Heinz, A. Korichi, G. Lane, C. J. Lister, P. Reiter, T. Lauritsen, T. Rodinger, D. G. Sarantites, D. Seweryniak, S. Siem, R. C. Vondrasek, and I. Wiedenhöver, Phys. Rev. Lett., 88, 042501 (2002).
6. A. Lopez-Martens, F. Hannachi, A. Korichi, C. Schück, E. Guerguieva, C. Vieu, B. Haas, R. Lucas, A. Astier, G. Balsdiefen, M. Carpenter, G. de France, R. Duffait, L. Ducroux, Y. Le Coz, C. Finck, A. Gorgen, H. Hübel, T. L. Khoo, T. Lauritsen, M. Meyer, D. Prévost, N. Redon, C. Rigollet, H. Savajols, J. F. Sharpey-Schafer, O. Stezowski, C. Theisen, U. V. Severen, J. P. Vivien, and A. N. Wilson, Phys. Lett. B, 380, 18–23 (1996).
7. K. Hauschild, L. A. Bernstein, J. A. Becker, D. E. Archer, R. W. Bauer, D. P. McNabb, J. A. Cizewski, K.-Y. Ding, W. Younes, R. Krücken, R. M. Diamond, R. M. Clark, P. Fallon, I.-Y. Lee, A. O. Macchiavelli, R. MacLeod, G. J. Schmid, M. A. Deleplanque, F. S. Stephens, and W. H. Kelly, Phys. Rev. C, 55, 2819–2825 (1997).
8. T. L. Khoo, M. P. Carpenter, T. Lauritsen, D. Ackermann, I. Ahmad, D. J. Blumenthal, S. M. Fischer, R. V. Janssens, D. Nisius, E. F. Moore, A. Lopez-Martens, T. Dössing, R. Krücken, S. J. Asztalos, J. A. Becker, L. Bernstein, R. M. Clark, M. A. Deleplanque, R. M. Diamond, P. Fallon, L. P. Farris, F. Hannachi, E. A. Henry, A. Korichi, I. Y. Lee, A. O. Macchiavelli, and F. S. Stephens, Phys. Rev. Lett., 76, 1583–1586 (1996).
9. G. Hackman, T. L. Khoo, M. P. Carpenter, T. Lauritsen, A. Lopez-Martens, I. J. Calderin, R. V. Janssens, D. Ackermann, I. Ahmad, S. Agarwala, D. J. Blumenthal, S. M. Fischer, D. Nisius, P. Reiter, J. Young, H. Amro, E. F. Moore, F. Hannachi, A. Korichi, I. Y. Lee, A. O. Macchiavelli, T. Dössing, and T. Nakatsukasa, Phys. Rev. Lett., 79, 4100–4103 (1997).
10. A. N. Wilson, G. D. Dracoulis, A. P. Byrne, P. M. Davidson, G. J. Lane, R. M. Clark, P. Fallon, A. Görgen, A. O. Macchiavelli, and D. Ward, Phys. Rev. Lett., 90, 142501 (2003).
11. S. Siem, P. Reiter, T. L. Khoo, T. Lauritsen, P.-H. Heenen, M. P. Carpenter, I. Ahmad, H. Amro, I. J. Calderin, T. Døssing, T. Duguet, S. M. Fischer, U. Garg, D. Gassmann, G. Hackman, F. Hannachi, K. Hauschild, R. V. Janssens, B. Kharraja, A. Korichi, I.-Y. Lee, A. Lopez-Martens, A. O. Macchiavelli, E. F. Moore, D. Nisius, and C. Schück, Phys. Rev. C, 70, 014303 (2004).
12. Y. R. Shimizu, F. Barranco, R. A. Broglia, T. Dössing, and E. Vigezzi, Phys. Lett. B, 274, 253 (1992).
13. Y. R. Shimizu, E. Vigezzi, T. Dössing, and R. A. Broglia, Nucl. Phys., A557, 99c (1993).
14. Y. R. Shimizu, M. Matsuo, and K. Yoshida, Nucl. Phys., A682, 464–469 (2001).
15. K. Yoshida, M. Matsuo, and Y. R. Shimizu, Nucl. Phys., A696, 85–122 (2001).
16. K. Lagergren, and B. Cederwall, Eur. Phys. J. A, 21, 175–177 (2004).
17. E. Vigezzi, R. A. Broglia, and T. Dössing, Nucl. Phys., A520, 179c (1990).
18. E. Vigezzi, R. A. Broglia, and T. Dössing, Phys. Lett. B, 249, 163 (1990).
19. G. G. Adamian, N. V. Antonenko, R. V. Jolos, Y. V. Palchikov, W. Scheid, and T. M. Shneidman, Phys. Rev. C, 69, 054310 (2004).
20. J.-z. Gu, and H. A. Weidenmüller, Nucl. Phys., A660, 197–215 (1999).
21. A. J. Sargeant, M. S. Hussein, M. P. Pato, and M. Ueda, Phys. Rev. C, 66, 064301 (2002).
22. H. Feshbach, Theoretical Nuclear Physics: Nuclear Reactions, Wiley, N.Y., 1992.
23. C. A. Stafford, and B. R. Barrett, Phys. Rev., C60, 051305 (1999).
24. D. M. Cardamone, C. A. Stafford, and B. R. Barrett, Phys. Stat. Sol., 230, 419–423 (2002).
25. D. M. Cardamone, C. A. Stafford, and B. R. Barrett, Phys. Rev. Lett., 91, 102502 (2003).
26. P. Bonche, J. Dobaczewski, H. Flocard, P. H. Heenen, S. J. Krieger, J. Meyer, and M. S. Weiss, Nucl. Phys., A519, 509 (1990).
27. H. A. Weidenmuller, P. von Brentano, and B. R. Barrett, Phys. Rev. Lett., 81, 3603–3606 (1998).
28. T. Guhr, A. Muller-Groeling, and H. A. Weidenmuller, Phys. Rep., 299, 189–425 (1998).
29. J. J. M. Verbaarschot, H. A. Weidenmuller, and M. R. Zimbauer, Phys. Rep., 129, 367–438 (1985).
30. M. Kawai, A. K. Kerman, and K. W. McVoy, Ann. Phys. (N.Y.), 75, 156–170 (1973).
31. N. R. Dagdeviren, and A. K. Kerman, Ann. Phys. (N.Y.), 163, 199–211 (1985).
32. E. D. Davis, and D. Boone, Z. Phys. A, 332, 427–441 (1989).
33. R. Krucken, A. Dewald, P. von Brentano, and H. A. Weidenmuller, Phys. Rev. C, 64, 064316 (2001).
34. A. J. Sargeant, M. S. Hussein, M. P. Pato, N. Takigawa, and M. Ueda, Phys. Rev. C, 69, 067301 (2004).