Prospects for constraining the dark energy potential

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Abstract. We generalize to non-flat geometries the formalism of Simon et al (2005 Phys. Rev. D 71 123001 [0412269]) to reconstruct the dark energy potential. This formalism makes use of quantities similar to the horizon-flow parameters in inflation, can, in principle, be made non-parametric and is general enough to be applied outside the simple, single-scalar-field quintessence. Since currently available and forthcoming data do not allow a non-parametric and exact reconstruction of the potential, we consider a general parametric description in terms of Chebyshev polynomials. We then consider present and future measurements of $H(z)$, baryon acoustic oscillation (BAO) surveys and supernovae type 1A surveys, and investigate their constraints on the dark energy potential. We find that relaxing the flatness assumption increases the errors in the reconstructed dark energy evolution but does not open up significant degeneracies, provided that a modest prior is imposed on the geometry. Direct measurements of $H(z)$, such as those provided by BAO surveys, are crucially important for constraining the evolution of the dark energy potential and the dark energy equation of state, especially for non-trivial deviations from the standard ΛCDM (CDM: cold dark matter) model.

Keywords: dark energy theory, supernova type Ia, power spectrum

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1. Introduction

Recent observations, e.g., [1]–[7] indicate that the present-day energy density of the Universe is dominated by a ‘dark energy’ component, responsible for the current accelerated expansion.

The leading dark energy candidates are a cosmological constant and a slowly varying rolling scalar field; see, e.g., [8]–[14] and [15] for a review, although explanations in terms of modifications of the Friedmann equation ([16]–[25] and references therein, under the title of ‘modifications of gravity’, and, e.g., [26]–[29], under the title of ‘inhomogeneous models’) are also being widely investigated.

An extended observational effort is being carried out (e.g., SNLS, SDSS, PanStarrs etc) and ambitious plans for the future are being proposed or planned (e.g., DES, PAU, BOSS, WFMOS, SNAP, ADEPT, DUNE, SPACE, SKA, LSST) with the goal of shedding light on the nature of dark energy. With few exceptions, current constraints on the nature of dark energy measure an integrated value over time of the Hubble parameter, \( H(z) \), which in turn is an integral of its equation of state parameter (\( w = p/\rho \), with \( p \) denoting pressure and \( \rho \) density). While these constraints are very tightly centered around the cosmological constant value, with a 15% error, the finding that the average value of \( w \) is consistent with \(-1\) does not exclude the possibility that \( w \) varied in time; see, e.g., [30].

An emerging technique in dark energy studies uses observations of the so-called baryon acoustic oscillations (BAO) [31]–[33]. The BAO yield a measurement of the sound horizon at recombination, a standard ruler visible at different epochs in the lifetime of the Universe: at the last scattering surface through cosmic microwave background observations and at lower redshifts through galaxy clustering. In galaxy surveys, the BAO scale can be
measured both along and perpendicular to the line of sight. In particular the line-of-
sight measurement offers a unique opportunity to measure directly \( H(z) \), rather than its
integral.

To improve our understanding of dark energy, it is important not only to ask whether
this dark energy component is dynamical or constant, but also to constrain possible shapes
of the dark energy potential. As different theoretical models are characterized by different
potentials, a reconstruction of the dark energy potential from cosmological observations
could help discriminating among different, physically motivated, models.

Different approaches to the reconstruction of the dark energy equation of state or
potential have been proposed in the literature; see, e.g., [34]–[39]. In this paper we build
upon and generalize the reconstruction technique proposed in [37] to non-flat universes.
In fact [40,41] showed that there can be a degeneracy between geometry and dark energy
properties; thus analyses for constraining dark energy parameters should be carried out
by jointly varying the geometry of the Universe.

We then apply this reconstruction to existing determinations of \( H(z) \) from ages of
passively evolving galaxies [37] to new supernovae data [5], and we forecast the constraints
on the dark energy potential achievable with the next generation of BAO and supernova
surveys. We find that relaxing the assumption of flatness increases the error in the
reconstructed dark energy evolution but does not open up degeneracies, provided that
a modest prior is imposed on the geometry: a Gaussian prior on \( \Omega_k \) with rms \( \sigma_k = 0.03 \).
Measurements of \( H(z) \) such as those provided by BAO surveys are crucial for constraining
the dark energy evolution and for breaking degeneracies among dark energy parameters,
for non-trivial deviations from the simplest \( \Lambda \)CDM model.

The rest of the paper is organized as follows. In section 2 we present our reconstruction
of the dark energy potential. In section 3 we describe the priors used in addition to
the present and future data sets that we consider. Our results on the dark energy
potential reconstruction are reported in section 4. For completeness, we present a similar
reconstruction applied to the dark energy equation of state parameter as a function of
redshift (in section 5). We conclude in section 6.

2. How to reconstruct the dark energy potential from observations

Reference [37] presented a non-parametric method for reconstructing the redshift evolution
of the potential and kinetic energy densities of the dark energy field, using quantities
similar to the horizon-flow parameters in inflation; see, e.g., [42,43]. As a fully non-
parametric reconstruction would require the knowledge of \( H(z) \) and that of \( H(z) \), [37] also
presented a general parameterization of the dark energy potential as a function of redshift,
\( V(z) \), in terms of Chebyshev polynomials and showed how to reconstruct the potential as
a function of the scalar field \( V(\phi) \) from \( V(z) \). In this section we will follow [37] to directly
relate the dark energy potential \( V(\phi) \) with observable quantities, but we generalize their
approach to non-flat universes.

We restrict ourselves to classical configurations \( \phi = \phi(t) \), that do not break the
homogeneity and isotropy of space–time. The energy–momentum tensor of this scalar
field configuration is that of a perfect fluid, with density \( \rho_\phi \) and pressure \( p_\phi \) given by

\[
\rho_\phi = K(\phi) + V(\phi), \quad p_\phi = K(\phi) - V(\phi) \quad \text{and} \quad K \equiv \frac{1}{2} \dot{\phi}^2, \quad (1)
\]
where $K$ denotes the kinetic energy of the field. The Friedmann equations then read

$$H^2 = \frac{\kappa}{3} (\rho_T + \rho_\phi + \rho_k),$$  \hspace{1cm} (2)$$

$$\frac{\ddot{a}}{a} = \frac{1}{2H} \frac{dH^2}{dt} = -\frac{\kappa}{6} (\rho_T + 3\rho_T + \rho_\phi + 3\rho_\phi),$$  \hspace{1cm} (3)$$

where $\kappa = 8\pi/m_p^2$ (or $\kappa = 8\pi G$) and $\rho_k = -k(3c^2/\kappa a^2)$, where $k$ is the curvature. In equation (3) we introduced the compact notation $\rho_T$ and $\rho_T$ for the total energy density and pressure. Using both of Friedmann’s equations to solve for the kinetic energy of the field and considering a single matter component we obtain

$$3H^2(z) - \frac{1}{2}(1+z) \frac{dH^2(z)}{dz} = \kappa \left( V(z) + \frac{1}{2}\rho_m(z) + \frac{2}{3}\rho_k(z) \right).$$  \hspace{1cm} (4)$$

An exact non-parametric reconstruction of $V(z)$ is possible only if $H(z)$ and $\dot{H}(z)$ are known (see [37] for details). Unfortunately present and near future prospective data do not allow this level of precision. However if $V(z)$ can be parameterized, equation (4) can be integrated analytically:

$$H^2(z) = \left( H_0^2 - \frac{\kappa}{3}(\rho_{m,0} + \rho_{k,0}) \right) (1+z)^6 + \frac{\kappa}{3}(\rho_m(z) + \rho_k(z))$$

$$- 2(1+z)^6 \int_0^z V(x) (1+x)^{-7} \, dx.$$  \hspace{1cm} (5)$$

Hereafter the 0 subscript denotes the quantity evaluated at $z = 0$.

An interesting parameterization of the potential involves the Chebyshev polynomials, which form a complete set of orthonormal functions on the interval $[-1, 1]$. They also have the interesting property of being the minimax approximating polynomial, that is, the approximating polynomial which has the smallest maximum deviation from the true function at any given order. We can thus approximate a generic $V(z)$ as

$$V(z) \simeq \sum_{n=0}^N \lambda_n T_n(x),$$  \hspace{1cm} (6)$$

where $T_n$ denotes the Chebyshev polynomial of order $n$ and we have normalized the redshift interval so that $x = 2z/z_{max} - 1$; $z_{max}$ is the maximum redshift at which observations are available and thus $x \in [-1, 1]$. Since $|T_n(x)| \leq 1$ for all $n$, for most applications, an estimate of the error introduced by this approximation is given by $\lambda_{N+1}$. With this parameterization, the relevant integral in equation (5) becomes

$$\int_0^z V(y)(1+y)^{-7} \, dy = \frac{z_{max}}{2} \sum_{n=0}^N \lambda_n \int_{-1}^{2z/z_{max} - 1} T_n(x)(a+bx)^{-7} \, dx$$

$$= \frac{z_{max}}{2} \sum_{n=0}^N \lambda_n F_n(z),$$  \hspace{1cm} (7)$$

where $a = 1 + z_{max}/2$ and $b = z_{max}/2$. These integrals can be solved analytically for any order $n$ and the $F_n$ are known analytic functions. Substituting in (5) we
finally obtain

\[ H^2(z, \lambda_i) = (1 + z)^6 H_0^2 \left[ 1 - 3 z_{\text{max}} \sum_{n=0}^{N} \frac{\lambda_n F_n(z)}{\rho_c} \right] - \Omega_{m,0} \left( 1 - \frac{1}{(1+z)^3} \right) - \Omega_{k,0} \left( 1 - \frac{1}{(1+z)^4} \right), \tag{8} \]

which relates observable quantities such as the Hubble parameter and \( \Omega_{m,0} \) to the coefficients of the Chebyshev expansion of the potential. Determining the coefficients \( \lambda_i \) in this manner allows one to reconstruct \( V(z) \). To obtain \( V(\phi) \), however, we would also need to reconstruct \( \phi(z) \). This can also be accomplished from the determination of the coefficients \( \lambda_i \), through the kinetic energy of the field. From the first Friedmann equation we have

\[ K(z) = \frac{1}{2} \left( \frac{d\phi}{dz} \right)^2 (1+z)^2 H^2(\lambda_i, z) = 3\kappa^{-1} H^2(\lambda_i, z) - \rho_m(z) - \rho_k(z) - V(\lambda_i, z), \tag{9} \]

which can be integrated to obtain \( \phi(z) \) and thus \( V(\lambda_i, \phi) \) from \( V(\alpha_i, z) \):

\[ \phi(z) - \phi(0) = \pm \int_0^z \sqrt{\frac{6\kappa^{-1} H^2(\lambda_i, z) - 2\rho_m(z) - 2\rho_k(z) - 2V(\lambda_i, z)}{(1+z) H(\lambda_i, z)}} \, dz, \tag{10} \]

where the ambiguity in sign comes from the quadratic expression for the kinetic energy. Typically, if we think of a scalar field rolling slowly along its potential, the plus sign will be the relevant one.

In what follows we will only consider a three-parameter model (i.e. \( N = 2 \)). Even with only three parameters, if the fiducial model is \( \Lambda \)CDM, the forecast constraints on deviations from a flat potential are rather weak, especially at \( z > 0.5 \). However, with the formulation presented here, one can pose the question of how many dark energy parameters are required by the data. Techniques such as cross-validation [44]–[46] could be used to address the issue.

In section 3 we describe the different observables that we consider to probe the dark energy potential.

3. Data sets and priors

Most probes of dark energy measure integrated quantities; for example supernovae measure the luminosity distance \( d_L(z) \). There are two known techniques for reconstructing directly \( H(z) \). One is through the measurement of the BAO scale in the radial direction through \( dr = c/H(z) \, dz \). Current surveys do not yet have sufficient statistical power to do this and thus current measurements are angle averaged. However, forthcoming and future surveys promise to deliver \( H(z) \) determination with % accuracy. BAO surveys, of course, also provide measurement of the angular diameter distance \( d_A(z) \).

The other technique relies on the measurement of ages of passively evolving galaxies. It has been demonstrated from recent observations that massive \((L > 2L_\star)\) luminous red galaxies have formed more than 95% of their stars at redshifts >4. Since then, stars in
These galaxies have been evolving passively. They are, therefore, excellent cosmic clocks, where the age of their stars can be inferred from the integrated stellar spectrum using stellar evolution theory.

Surveys of BAO along with large samples of type 1A supernovae are among the leading techniques for constraining dark energy and an extensive experimental effort is being carried out. For this reason we consider currently available ‘cosmic clock’ data, currently available supernova data and future BAO and type 1A supernova surveys.

3.1. Priors

In what follows we assume a flat ΛCDM with $H_0 = 73.2 \text{ km s}^{-1} \text{ Mpc}^{-1}$ and $\Omega_m = 0.24$ in the fiducial model. In all cases we consider Gaussian priors of $\sigma_H = 8 \text{ km s}^{-1} \text{ Mpc}^{-1}$ for $H_0$, $\sigma_{\Omega_m} = 0.01$ for $\Omega_m h^2$ and $\sigma_k = 0.03$ for $\Omega_k$. This is motivated by the fact that current data already constrain these parameters at this level; see e.g., [47,2,4]. We will assume here that these parameters can be constrained at this level by combination of, e.g., cosmic microwave background experiments (e.g., Planck [48]) and local determinations of the Hubble parameter [49].

3.2. Baryon acoustic oscillations

Dark matter overdensities in the early Universe produce acoustic waves in the photon–baryon plasma that propagate with the speed of sound until the recombination era is reached, when photons decouple from baryons and free stream. The baryon wave then stops propagating, leaving an imprint at a characteristic distance from the original dark matter overdensity: the sound horizon length. This process thus provides a standard ruler at which the correlation function of dark matter (and thus of galaxies) should peak (e.g., [31]). Evidence of this peak has already been reported in galaxy surveys, e.g., [32,33]. Measuring this standard ruler at different redshifts would provide a powerful probe of the expansion history of the Universe and thus of the dark energy potential.

Baryon acoustic oscillations (BAO) can be measured both along and perpendicular to the line of sight. An angular measurement of the BAO scale at redshift $z$ would then give

$$\Delta \theta = \frac{r_{\text{BAO}}}{(1 + z) d_A(z)},$$

(11)

where $d_A(z) = 1/(1 + z) \int_0^z c/H(z) \, dz$ and $r_{\text{BAO}}$ is the BAO scale. This measurement of $d_A(z)$ can then be compared to equation (8) or (21), below in section 5, to derive constraints on the coefficients $\lambda_i$ and $w_i$ respectively. Alternatively, if the redshift precision of the survey is good enough, the BAO scale could be measured along the line of sight as

$$\Delta z = H(z) r_{\text{BAO}},$$

(12)

thus providing a direct measurement of $H(z)$.

To forecast the errors with which $H(z)$ and $d_A(z)$ will be recovered we make use of the formulae derived in [50], where a grid of BAO surveys was simulated with different survey parameters and the accuracy found for the observables was fitted to the

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Table 1. Survey parameters of the two BAO surveys considered.

| Survey    | Area \((dg^2)\) | \(z_{\text{min}}\) | \(z_{\text{max}}\) | Bins in \(z\) |
|-----------|-----------------|---------------------|---------------------|----------------|
| Ground    | 10 000          | 0.1                 | 1                   | 9              |
| Space     | 30 000          | 1                   | 2                   | 10             |

following formulae:

\[
\sigma_d(z_i) = x_0^d \frac{4}{3} \sqrt{\frac{V_0}{V_i}} f_{nl}(z_i),
\]

\[
\sigma_H(z_i) = x_0^H \frac{4}{3} \sqrt{\frac{V_0}{V_i}} f_{nl}(z_i),
\]

where

\[
f_{nl}(z_i) = \begin{cases} 
1 & z < z_m \\
\left(\frac{z_m}{z_i}\right)^\gamma & z > z_m.
\end{cases}
\]

\(V_i\) is the volume of the redshift bin \(z_i\). The fitting formula was motivated by the assumption that the accuracy achievable in the observables will be proportional to the fractional error with which the power spectrum can be recovered:

\[
\frac{\Delta P}{P} \simeq \sqrt{\frac{2}{N_m}} \left(1 + \frac{1}{nP}\right),
\]

where \(N_m \propto V_i\) is the number of Fourier modes contributing to the measurement and \(n\) the number density of galaxies surveyed. Non-linearities tend to erase the acoustic peaks via mode coupling: the function in (15) takes into account that, at increasing redshift, increasingly small scales are in the linear regime. The fitting parameters were calibrated for \(N\)-body simulations by [50] and found to be \(x_0^H = 0.0148, x_0^d = 0.0085, V_0 = 2.16 h^{-3}, z_m = 1.4, \gamma = 0.5\).

Here we will consider two setups that roughly encompass ground-based (‘ground’) and space-based (‘space’) perspective BAO surveys. The survey parameters are summarized in table 1. In both cases we assume that shot noise is unimportant at the scale of interest and that the redshift determination is good enough to measure the radial BAO signal. Along the way we will also report the results for the angular-only BAO. This case will be relevant to photometric surveys that can achieve photometric errors better than \(\sim 4\%\) [51,52].

3.3. Galaxy ages

The Hubble parameter depends on the differential age of the Universe as a function of redshift via \(H(z) = \frac{dz}{dt}(1+z)^{-1}\). The feasibility of measuring \(H(z)\) from high resolution, high signal to noise spectra of passively evolving galaxies was demonstrated in [53,37,54]. Here we use the \(H(z)\) determination obtained by [37] and publicly available at [55], from a compilation of data at \(0 < z < 1.8\), and generalize the analysis of [37] to non-flat universes. Recent studies [56,57] have clearly established that massive (>2.2\(L_*\)) luminous red galaxies have formed more than 95% of their stars at redshifts higher than 4. These
galaxies, therefore, form a very uniform population, whose stars are evolving passively after the very first short episode of active star formation [56]–[58]. Because the stars evolve passively, these massive LRG are excellent cosmic clocks, i.e. they provide a direct measurement of $dt/dz$; the observational evidence discards further star formation activity in these galaxies. Dating of the stellar population can be achieved by modeling the integrated light of the stellar population using synthetic stellar population models, in a similar way to what is done for open and globular clusters in the Milky Way. The dating of the stellar population needs to be done on the integrated spectrum because individual stars are not resolved and therefore the requirements on the observed spectrum are stringent as one needs a very wide wavelength coverage, spectral resolution and very high signal to noise. Reference [57] has shown that the spectra of these massive LRG at a redshift $\sim 0.15$ are extremely similar, with differences of only 0.20 mmag, which is further evidence of the uniformity of the stellar populations in these galaxies. There have already been examples of accurate dating of the stellar populations in LRGs ([59, 60, 53, 37]) where it has been shown that galaxy spectra with sufficient wavelength coverage (the UV region is crucial), wavelength resolution (about 3 Å) and enough S/N (at least 20 per resolution element of 3 Å) can provide sensible constraints on cosmological parameters. More details can be found in [37, 61].

### 3.4. Supernovae

The intrinsic luminosity of type IA supernovae (SN) can be accurately predicted from the decay rate of the supernovae brightness. This provides bright standard candles that can be observed up to redshifts $z > 1$. Measuring SN apparent magnitude, the luminosity distance can thus be inferred:

$$d_L(z) = (1 + z) \int_0^z \frac{c}{H(z)} dz.$$  \hspace{1cm} (17)

If their redshift is also measured, type IA supernova provide information on the integral of $1/H(z)$ and hence on the cosmological parameters. In this way supernovae provided the first direct evidence for the accelerated expansion of the Universe [62, 63].

We consider present and forecast type IA supernovae data in the analysis of section 4. For the present supernovae data we use the sample of [64]. For future supernovae data we assume 1000 supernovae distributed in five redshift bins between 0.8 and 1.3 plus a sample of 500 supernovae at low redshift [65, 25]. Table 2 summarizes the distribution of the supernovae considered.

We consider a statistical error on $\mu = 5 \log d_L + K$ of $\sigma_{\mu,\text{stat}} = 0.1$ due to the uncertainty of the corrected apparent magnitudes. We also consider a systematic error given by [66] $\sigma_{\mu,\text{syst}} = 0.02(1 + z)/2.7$.

For both supernovae samples, present and forecast, we marginalize over the absolute magnitude of the sample.

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**Table 2.** Redshift distribution of the forecast supernovae sample.

| Mean $z$ | 0.1 | 0.85 | 0.95 | 1.05 | 1.15 | 1.25 |
|----------|-----|------|------|------|------|------|
| SN       | 500 | 231  | 219  | 200  | 183  | 167  |
4. Results

For the different data sets we compute (or forecast, for future data) the constraints on the first three coefficients of the Chebyshev expansion, assuming a flat ΛCDM with $H_0 = 73.2 \text{ km s}^{-1} \text{ Mpc}^{-1}$ and $\Omega_m = 0.24$ in the fiducial model with the priors described above and errors on $H(z)$ and $d_A(z)$ and $d_L(z)$ as outlined in section 3. For supernovae and galaxy ages data the analysis is performed exploring the likelihood surface via a Markov chain Monte Carlo. For the BAO surveys we use both a Markov chain Monte Carlo and a Fisher matrix approach, finding good agreement between the results from the two techniques. Here we present the results of the Fisher matrix analysis.

The constraints on the $\lambda_i$ thus derived can be translated into constraints on the potential using equation (6). In figure 1 we show the results of this reconstruction for supernovae (present and future, top panels) galaxy ages (left middle panel), ‘ground’ BAO survey (middle right), ‘space’ BAO survey (lower left) and the combination of the two BAO surveys (lower right).

Notice that the constraints set by all the data sets considered are strongest between $z \approx 0.1–0.3$. This is a consequence of the recent dominance of dark energy in the cosmological history and translates into a strong linear degeneracy between $\lambda_0$ and $\lambda_1$ as discussed below.

4.1. Interpretation of the reconstructed $V(z)$

It can be seen from equations (13) and (14) that the fractional error in $d_A(z)$ is approximately half of the error in $H(z)$. However, there is a direct dependence of $H(z)$ on $\lambda_i$ coefficients through equation (8), while the relation to $d_A(z)$ involves an integral and the bounds derived from the information on $d_A(z)$ are generally weaker than those derived from the measurement of $H(z)$. As an example, in figures 2 and 3, we show the $1\sigma$, $2\sigma$ and $3\sigma$ constraints that a ‘ground’ or ‘space’ BAO experiment could respectively place on $\lambda_0$, $\lambda_1$ and $\lambda_2$ using only the information on $d_A(z)$ (left) and on $H(z)$ (right). Notice that the constraints derived from the information on $H(z)$ are much tighter. Indeed, we found that, with information on $d_A(z)$ or $d_L(z)$ alone, there is a degeneracy between $\lambda_0$ and $\lambda_2$, as shown in the bottom left panel of figures 2 and 3 and the right panel of figure 4 in [37]. This degeneracy is lifted by data constraining $H(z)$ as can be seen in the bottom right panel of figures 2, 3 and the right panels of figures 3 and 6 in [37]. This favors spectroscopic surveys, which can measure $H(z)$, over photometric surveys with large photo-z errors which can only measure $d_A(z)$.

Let us consider more closely the strong degeneracy between $\lambda_0$ and $\lambda_1$ in the top panels of figures 2 and 3 and in figures 3, 4 and 6 of [37]. This degeneracy is present in all the data sets that we considered but it is more pronounced when no information on $H(z)$ is available and the sensitivity to the $\lambda_i$ coefficients lies in integrals like $d_A(z)$ or $d_L(z)$ as for supernovae data. This degeneracy is described by a linear relation between $\lambda_0$ and $\lambda_1$ of the form

$$\lambda_0 = \alpha \lambda_1 + \beta.$$  \hspace{1cm} (18)

This implies, to first order in the Chebyshev expansion of equation (6),

$$V(z) = (\alpha - 1)\lambda_1 + \beta + \lambda_1 \frac{2z}{z_{\text{max}}},$$  \hspace{1cm} (19)
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Figure 1. $1\sigma$ and $2\sigma$ constraints on the reconstructed potential as a function of redshift $V(z)$ from present SN data (top left), SN data from a future space-based experiment (top right), galaxy ages (middle left), ‘ground’ BAO survey (middle right), ‘space’ BAO survey (bottom left) and the combination of the two BAO surveys (bottom right).

Then, for any value of $\lambda_1$ along this degeneracy, there is a redshift $z = z_{\text{max}}(1 - \alpha)/2$ for which the value of the potential is fixed to $V = \beta$. For all the data sets we found linear degeneracies between $\lambda_0$ and $\lambda_1$ with $\alpha \lesssim 1$ and $\beta \sim \Omega_\Lambda$; this means that for all data sets the potential is better constrained at low redshift, in order to have $V \sim \Omega_\Lambda$. This reflects the fact that cosmological data are more sensitive to the dark energy properties for small $z$, since at larger redshifts the matter component dominates and the dependence on $V(z)$ is subdominant. However the exact transition redshift (where $\Omega_m = \Omega_{\text{DE}}$) depends on the shape of the dark energy potential.
Figure 2. $1\sigma$, $2\sigma$, and $3\sigma$ contours for $\lambda_0$, $\lambda_1$ (upper panels) and $\lambda_0$, $\lambda_2$ (lower panels) from measurements of $d_A(z)$ (left) and $H(z)$ (right) alone, for a ‘ground’ BAO survey. While the fractional error in $d_A(z)$ is approximately half of the error in $H(z)$, the direct dependence of $H(z)$ on $\lambda_i$ coefficients through equation (8) yields stronger constraints on these parameters.

As for the effect of considering non-flat geometries we find that, with the prior of $\sigma_k = 0.03$ in $\Omega_k$ that we consider, the effect on the extraction of the DE properties is very small, only slightly increasing the error in the reconstructed parameters. However, loosening the prior on $\Omega_k$ can severely spoil the constraints shown here, significantly worsening the degeneracies among the $\lambda_i$ coefficients. As an example we show in figure 4 the effect on the constraints of the first three $\lambda_i$ from the ‘space’ BAO survey for different values of the prior on $\Omega_k$, 0, 0.03, 0.1 and 1.

Notice that, in spite of the lack of data for $z < 1$, the ‘space’-type survey is placing strong constraints also in that redshift region. This is a consequence of (a) the priors imposed at $z = 0$, (b) the very accurate data for $z > 1$ and (c) the information on $w(z)$ enclosed in the $d_A(z)$ constraints, making it possible to interpolate the potential given by our smooth parameterization to the low redshift regime. Therefore, the combination of the two surveys does not provide a significant improvement of the constraints on $V(z)$ over the ‘space’ survey alone; however these constraints are much more robust since both experiments now cover the whole redshift range shown in figure 1.
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Figure 3. 1σ, 2σ and 3σ contours for \(\lambda_0\), \(\lambda_1\) and \(\lambda_2\) from measurements of \(d_A(z)\) (left) and \(H(z)\) (right) alone, for a ‘space’ BAO survey. While the fractional error in \(d_A(z)\) is approximately half of the error in \(H(z)\), the direct dependence of \(H(z)\) on \(\lambda_i\) coefficients through equation (8) yields stronger constraints on these parameters.

4.2. Reconstructed \(V(\phi)\)

Equation (10) also enables us to reconstruct \(\Delta \phi(z)\) from \(V(z)\), thus constraining \(V(\phi)\). Figure 5 shows the results of such a reconstruction for the 68% best models for the different data sets. Note that, upon integration of equation (10) up to \(z_{\text{max}}\), a range of \(\Delta \phi(z)\) can be obtained up to a maximum value when \(z = z_{\text{max}}\). This maximum value will strongly depend on the actual model that is integrated and on how strongly the field evolves in that model. Thus, not all values for \(\Delta \phi\) are allowed and showing the 1σ and 2σ contours would not be fully correct. Indeed, for ΛCDM, equation (10) will always yield \(\Delta \phi(z) = 0\) regardless of \(z_{\text{max}}\). If the constraints placed by a given data set on the model are tightly centered around ΛCDM, very small values of \(\Delta \phi(z)\) will be recovered from such models.

5. Reconstruction of the equation of state

Parameterizing dark energy not by the scalar field potential but by its equation of state is a widespread practice. As long as the equation of state \(w\) is \(> - 1\), parameterizing
Figure 4. 1σ, 2σ and 3σ contours for $\lambda_0$, $\lambda_1$ and $\lambda_2$ for a ‘space’ BAO survey for different values of the prior on $\Omega_k$, 0, 0.03, 0.1 and 1.
Figure 5. Instead of reporting the 1σ and 2σ contours, we plot the reconstructed $V(\phi)$ for the 68% best models for the different data sets (see the text for more details).

The dynamics of dark energy through the evolution of its effective equation of state is equivalent to considering the redshift evolution of the dark energy potential. However if we want to allow $w < -1$, then the scalar field description as presented above fails.

Thus, considering the evolution of an effective equation of state is more general than considering the potential of a scalar field. In this case it is easier to relate $w(z)$ to the observables. Expanding the redshift dependence of the equation of state in Chebyshev polynomials analogously to the expansion of $V(z)$:

$$w(z) \simeq \sum_{i=0}^{N} \omega_i T(x(z)),$$

(20)
and substituting this into the first Friedmann equation we would have

\[ H^2(\omega_i, z) \simeq H_0^2 \left[ \Omega_{m,0}(1 + z)^3 + \Omega_{k,0}(1 + z)^2 \right. 
\]

\[ + \left. (1 - \Omega_{m,0} - \Omega_{k,0})(1 + z)^3 \exp \left( \frac{3}{2} z_{\text{max}} \sum_{n=0}^{N} \omega_n G_n(z) \right) \right] \] 

(21)

where now

\[ G_i(z) = \int_{-1}^{2z/z_{\text{max}}-1} T_i(x)(a + bx)^{-1} \, dx. \] 

(22)

Note that in this parameterization the present-day value of \( w \) is given by

\[ w_0 = \sum_{i=0}^{N} (-1)^i \omega_i. \] 

(23)

Equations (20)–(22) are a generalization of section 3 of [37] to non-flat geometries.

We study the constraints that the data sets above can put on the DE equation of state through (21) expanding the dark energy equation state up to second order in Chebyshev polynomials. We perform forecasts using MCMCs. As in section 4, \( H_0, \Omega_m \) and \( \Omega_k \) are parameters which we marginalize over. The same priors as were quoted before are also assumed here. We show the results in figure 6. As for the reconstruction of \( V(z) \), dark energy properties are best constrained at \( z \lesssim 0.3 \) and the \( H(z) \) determination is crucial in constraining the dark energy evolution especially for non-trivial deviations from a constant equation of state parameter.

Analogously to the case of the reconstruction of \( V(z) \), since the dependence of \( H(z) \) on \( w(z) \) is through an integral, for quantities that depend on integrals of \( 1/H(z) \) such as \( d_A(z) \) or \( d_L(z) \), the information on \( w(z) \) is even more diluted. This explains the weak constraints found in figure 6 for all the data sets except the two BAO surveys. The deterioration of the bounds comes mainly through a degeneracy between \( w_0 \) and \( w_1 \) that can span down to \( w_0 \sim -100 \) and \( w_1 \sim -100 \) if only information on \( d_A \) or \( d_L \) is available. This degeneracy is solved with information on \( H(z) \). As an example let us consider a model lying within this degeneracy. The parameters for this model are \( H_0 = 70.38 \text{ km s}^{-1} \text{ Mpc}^{-1}, \Omega_m = 0.28, \Omega_k = 0.025, w_0 = -57.51, w_1 = -82.63, \) \( w_2 = -28.31 \) and its equation of state parameter is shown in figure 7 along with the \( \Lambda \)CDM values \( w = -1 \) for comparison. From equation (23), we can see that these parameters will still give \( w = -3.19 \) today, however for \( z = 2, w = -168 \).

In figure 8 we show the comparison between \( H(z) \) and \( d_A(z) \) for this model and for the \( \Lambda \)CDM model. From the figure it is clear that information on \( d_A(z) \) alone does not suffice for discriminating between the two, while the differences in \( H(z) \) between the two models are large. Even with these extreme values of the parameters, this model mimics the \( d_A(z) \) behavior of the \( \Lambda \)CDM, but has a significantly different \( H(z) \) which oscillates around the \( \Lambda \)CDM. Thus, upon integrating \( H(z) \) to obtain \( d_A(z) \), the regions where the model is above the \( \Lambda \)CDM compensate for the ones where it is below. A measurement of \( H(z) \), however, can easily distinguish the two models. We were not able to reproduce this behavior to the same degree with a two-parameter description of the dark energy equation of state dynamics. This example highlights an important open issue in dark energy studies:
Figure 6. $1\sigma$ and $2\sigma$ constraints on the DE equation of state from present SN data (top left), SN data from a future space-based experiment (top right), galaxy ages (middle left), ‘ground’ BAO survey (middle right), ‘space’ BAO survey (bottom left) and the combination of the two BAO surveys (bottom right).

Constraints on dark energy parameters coming from measurement of integrated quantities depend crucially on the choice of the dark energy parameterization [67]; in the absence of a theoretical motivation for a parameterization of dark energy properties, forecasts and constraints become crucially model dependent.

We have checked that fixing $\omega_2 = 0$ and $\Omega_k = 0$, with just a two-parameter description of the dynamics of the dark energy equation of state via $\omega_0$ and $\omega_1$, we are
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Figure 7. Equation of state parameter for a model lying along the degeneracy given by data set that can only constrain $d_A$ or $d_L$. For comparison, also the ΛCDM case is shown ($w = -1$).

Figure 8. Comparison of $H(z)$ and $d_A(z)$ for the example model described in the text and in figure 7 and for the ΛCDM model.

The analysis shown here shows that increasing the number of parameters seriously spoils our ability to constrain $w(z)$ except at small redshifts. As for the constraints on the potential, the stronger constraints at low redshifts are related to very pronounced degeneracies between $w_0$ and $w_1$. This degeneracies are much less important for the two BAO surveys, but the rest of the data sets considered can only effectively constrain the dark energy equation of state at small values of $z$. For a constant equation of state ($\omega_i = 0$ for $i > 0$) the bound derived will roughly correspond to the narrowest allowed region at small redshifts.
6. Conclusions

We have generalized to non-flat geometries the formalism of [37] for reconstructing the dark energy potential. This approach makes use of quantities similar to the horizon-flow parameters used to reconstruct the inflation potential [42,43]. The method can, in principle, be made non-parametric, but present and forthcoming data do not allow a fully non-parametric reconstruction. We have therefore considered a parametric description in terms of Chebyshev polynomials which, for all our applications, we have truncated to second order. For completeness we have also considered a reconstruction of the dark energy equation of state redshift dependence in terms of Chebyshev polynomials, also generalizing to non-flat geometries the results of [37].

We have considered present measurements of $H(z)$ from ages of passively evolving galaxies [53,37], future baryon acoustic oscillation (BAO) surveys and present and future type IA supernova surveys, and investigated their constraints on dark energy properties.

We present present and forecast constraints both on $V(z)$ (figure 1) and, more interestingly, on $V(\phi)$ (figure 5), in section 4. Model building for dark energy which relies on simple single-field models and provides physically motivated potentials should satisfy the constraints shown in the left top and middle panels of figure 5. In the future, the expected constraints can be as tight as those shown in the two bottom panels of figure 5. More complicated models (multifields etc) should produce a redshift evolution of the effective dark energy potential which satisfies the constraints in the left top and middle panel of figure 1. The expected future constraints can be as tight as those shown in the bottom panels of figure 1.

We find that relaxing the flatness assumption slightly increases the errors on the reconstructed dark energy evolution, but does not generate significant degeneracies, provided that a modest prior on geometry is imposed $\sigma_k = 0.03$ (e.g., figure 4).

Dark energy properties are best constrained at $z \lesssim 0.3$: this is the result of the dominance at late time of dark energy. Under the assumptions made here, the most crucial being the assumption of a smooth $V(z)$ or $w(z)$, we find that high redshift ($z < 2$) measurements of both $H(z)$ and $d_A$ are more powerful than low $z$ measurements.

When constraining the redshift evolution of both the dark energy potential $V(z)$ and the dark energy equation of state parameter $w(z)$ with measurements of integrated quantities such as $d_A$ or $d_L$, there are large degeneracies among the parameters. These degeneracies are greatly reduced or removed with measurements of $H(z)$, such as those to be provided, e.g., by future BAO surveys. This is illustrated in figures 2 and 3 for the potential reconstruction. While the $H(z)$ constraint is generally weaker than the $d_A(z)$ constraints, $H(z)$ is more directly related to the dark energy properties and thus offers more powerful dark energy constraints. We have illustrated this with an example of a model which lies in the ‘$d_A$ degeneracy’ for the reconstruction of $w(z)$. This model produces a $d_A(z)$ and a $d_L(z)$ virtually indistinguishable from those of $\Lambda$CDM; however the $H(z)$ are different and easily distinguishable from BAO measurements with $H(z)$ information.

This highlights an important open issue in dark energy studies: constraints on dark energy parameters coming from measurement of integrated quantities such as $d_A$ and $d_L$ depend crucially on the choice of the dark energy parameterization (see, e.g., [67]): in the absence of a theoretical motivation for a parameterization of dark energy properties,
forecasts and constraints become crucially model dependent. The dependence of the constraints on the assumed dark energy parameterization becomes evident only when considering non-trivial deviations from a ΛCDM model (e.g., deviations from a constant $w$ or generic shape of the potential). This issue is greatly alleviated by measurements that carry information on $H(z)$.

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