Graph kinematics of discrete physical objects:
beyond space - time. I. General

V. E. Asribekov
All - Russian Institute for Scientific and Technical Information, VINITI, Moscow 125315, Russia
(e-mail:peisv@viniti.ru)

Abstract

The necessity of an introduction of discrete physical objects in physics conception is analysed taking into consideration an optimum stage for postulating of some like objects in microworld as well as in macroworld including the new “physical graph” as a discrete microobject and carrying out its analogy with “Kirchhoff’s laws graph” for an electric network as a prototype of discrete macroobject which correspond both to discrete sets of trees — root trees (for microobjects) or skeleton trees (for macro networks). The transitions are found connecting the usual $S$-matrix theory with Feynman integrals and Feynman diagrams and the new physical graph kinematics formalism which uses the natural root trees basis for the treatment of the structure of an arbitrary complicated physical microobject with a specific “graph microgeometry” — beyond space-time consideration. Accordingly to the QCD results the proton (nucleon) mass is determined in terms of the root trees number $T^v_{v=11}=1842$ which corresponds to $v=11$ physical graph vertices. It is supposed that by means of a double- and a triple-splitting of the root trees numbers could be estimated the masses of the various series of another microobjects.

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1 INTRODUCTION

In our days the continuous physical objects fashion especially based on their field-theoretical description is very strong. In this connection it is really hard for any other point of view to gain a hearing.

Of course, it is naturally that in case of the macroobjects it is not necessary obviously to introduce some alternatives for this standard approach within a customary “external” geometry. Nevertheless, beginning from the most elementary viewpoint for describing atomic events we could not picture how the jump from one electronic orbit to another took place and we just had to accept it as a kind of discontinuity.

The following evolution of a picture of the nature towards the nuclear and subnuclear microobjects picture leads us to more open discontinuities and at last to the evident discrete physical objects perhaps with the proper “internal” geometry corresponding to their inner probably discrete structure. It it notable that a well-known field-theoretical problem of the divergence difficulties at the small distances for now structural physical microobjects
eliminates automatically what could be considered as an essential result in the area of discrete objects physics.

In parts I–III of paper we describe a possible transition to the some representation of discrete physical objects using the graph theory formalism.

It is important to have in view that in general a discrete mathematics as well as the physical theories and models with any discrete structural objects are not derived from or reduced to the continuous mathematics and the physical theories and models with corresponding continuous objects. Therefore the real appearance in physics of various discrete physical objects can be made solely by the introduction of an adequate mathematical postulate without some additional justifications but using any typical “discrete-like” results from an initial quasi-continuous physical theory; nevertheless the continuous theories altogether are not excluded from a following consideration. For this purpose however it is necessarily to find a definite stage in continuous physics development setting up the insufficiency of its continuous theory. Inasmuch as the above-mentioned difficulties arise in phenomena involving very small distances (or very high energies) we may choose as a such stage the transition to microworld. In this connection part I includes Feynman diagram technique within $S$-matrix theory for microobjects. Taking into account a non-equivalence of some following singularities–analysed diagram technique (post–Feynman), based on the stable microobjects only, to a perturbation theory we postulate a new derivative physical graph formalism as a first step to discrete microobject. Owing to an analogy of such physical graph with Kirchhoff’s laws graph for an electric network there exist various discrete physical objects which may be presented through the discrete sets of graphs—skeleton or root trees, beyond usual space-time. In part II a proposed graph formalism is applied to the calculation of some qualitative and numerical characteristics for different microobjects without using of the continuous theories formalism (QED, QCD, etc.) itself but only its results. And part III is devoted to a possible realization of the Heisenberg—Dyson’s two-layer physics.

1.1 Heisenberg’s $S$-matrix of 1943 and its state vectors basis in the Hilbert space

It is known that proposed by Heisenberg in 1943 the $S$-matrix contains the only physically measurable quantities and supposes the existence of a corresponding Hilbert space with the positive metric and the possibility to construct the complete Hilbert space basis. The $S$-matrix must be unitary and have so much analyticity that it represents what observed as causality; it also must have an invariance for the Lorentz group and for $TCP$, an approximate invariance for the isospin group, and so on.

Since 1948–49 the Feynman version of QED became the prototype of what is now called $S$-matrix theory which gave directly the rules for calculating $S$-matrix elements by means of Feynman integrals and corresponding Feynman diagrams. It is important the Feynman theory is a pure physical object theory, and the Feynman diagram describes any elementary process naively as a propagation of physical object from one vertex to another along a connected line.
1.2 “Singularities matrix” for Feynman integrals and networks of the new physical graphs in momentum space

A consistent analysis of all possible singularities of Feynman integral as a key quantity of the $S$-matrix formalism in Hilbert space (see for example, Ref. [1])

\[
\int \prod_{r} d^{4}k_{r} \prod_{s} f^{-n} \delta \left( \sum \alpha - 1 \right) d\alpha_{s},
\]

\[(1) \]

\[
f = \sum_{i} \alpha_{i} (m_{i}^{2} - q_{i}^{2}),
\]

\[(2) \]

determining the contribution of an arbitrary Feynman diagram with $N$ external 4-momenta $p_{j}$ ($j = 1, 2, ..., N$), $n$ internal 4-momenta $q_{i}$ ($i = 1, 2, ..., n$) and $l$ loop momenta $k_{r}$ ($r = 1, 2, ..., l$) can in principle be performed by solving a set of corresponding $v$ laws of conservation of 4-momenta in $N$ external (algebraic sum over $[j]$) and $v - N$ internal (algebraic sum over $(i)$) vertices and $l$ Landau extremal independent loop equations (algebraic sum over $< r >$) in fact already for a new derivative (post–Feynmanian) physical graph with the same characteristics

\[
\sum_{[j]} \epsilon q = p_{j}; \quad j = 1, 2, ..., N,
\]

\[(3) \]

\[
\sum_{(i)} \epsilon q = 0; \quad i = 1, 2, ..., v - N,
\]

\[(4) \]

\[
\sum_{<r>} \alpha q = 0; \quad r = 1, 2, ..., l.
\]

\[(5) \]

Actually the set from (3) and (4) contains only $v - 1$ independent equations, since one of the equations corresponds to the law of conservation for the external 4-momenta $p_{j}$

\[
\sum_{j} N \epsilon p = 0
\]

\[(6) \]

(everywhere $\epsilon = 0, \pm 1$).

Therefore the total number of independent equations (3), (4) and (5) is equal precisely to the number of the unknown internal 4-momenta $q$

\[
[N + (v - N)] - 1 + l = v - 1 + l = n.
\]

The rank of the obtained square matrix of coefficients from equations (3), (4) and (5) — so-called “singularities matrix” for the Feynman integral (1)

\[
M_{(n \times n)} = \left\{ \begin{array}{c}
I_{(v-1 \times n)} \\
A_{(l \times n)}
\end{array} \right\}
\]

\[(7) \]
as can be shown is also equal to $n$.

This composite matrix $M_{(n \times n)}$ contains the incidence matrix $I_{(v-1 \times n)}$ and the independent loop matrix

$$A_{(l \times n)} = C_{(l \times n)} D_{n(\alpha)}$$

(where $\epsilon_{ij} = 0, \pm 1$); here $C_{(l \times n)}$ is an usual cyclic (circuit) matrix and $D_{n(\alpha)}$ — diagonal matrix

$$D_{n(\alpha)} = \begin{pmatrix} 
\alpha_1 \\
\alpha_2 \\
\vdots \\
\alpha_n 
\end{pmatrix}.$$ 

It is known (see Ref. [2]) that

$$C_{(l \times n)} \times I_{(n \times v-1)}^T \equiv 0.$$

The rest of Landau extremal equations for internal 4-momenta

$$q_i^2 = m_i^2; \quad i = 1, 2, ..., n$$

(8)

together with the initial conditions for external 4-momenta

$$p_j^2 = M_j^2; \quad j = 1, 2, ..., N,$$

(9)

where $m_i$ and $M_j$ are the masses of internal and external lines in a new derivative physical graph, set up that all 4-momenta — an internal $q_i$ as well as an external $p_j$ — can be located according to Landau (see Ref. [3]) factually on the mass shell.

Thus the physical graph includes in fact only the real stable physical objects and the matrix equation

$$M_{(n \times n)} (Q_{n(1)}) = (P_{n(1)})$$

(10)

where by means of $(Q_{n(1)})$ and $(P_{n(1)})$ are denoted the column vectors

$$(Q_{n(1)}) = \begin{pmatrix} 
q_1 \\
q_2 \\
q_3 \\
q_4 \\
\vdots \\
q_{n-1} \\
q_n 
\end{pmatrix}, \quad (P_{n(1)}) = \begin{pmatrix} 
p_1 \\
p_2 \\
\vdots \\
p_{N-1} \\
0 \\
\vdots \\
0 
\end{pmatrix}$$

(11)

may serve as a starting point for the following consistent consideration of the networks of these physical graphs in momentum space. Although it is necessarily to note that the direct solution of the full set of algebraic equations (3)–(6), (8), (9) — for a separated as well as for a complicated network of such physical graphs — is enough complex and labor-intensive.
but quite feasible. The fact is that the results may be classified on the base of the forms of construction of the special \( l, v \) — sequences of physical graphs, namely the ladder and the parquet \( l, v \) — sequences.

### 2 KIRCHHOFF’S LAWS MATRIX FOR AN ELECTRIC NETWORK ANALOGY OF 1847

In agreement with the classical Kirchhoff’s work of 1847 (see Ref. [4]) initiated a development of the graph theory (especially in part of the particular graphs — the trees) the investigation of an arbitrary electric network with \( n \) wires is carried out by means of two laws:

\[
\begin{align*}
\sum_{[j]} \epsilon_j J_j &= J_j^*; \quad j = 1, 2, \ldots, N, \quad (12a) \\
\sum_{(i)} \epsilon_i J_i &= 0; \quad i = 1, 2, \ldots, v - N, \quad (12b) \\
\sum_{<r>} J_\omega &= \sum_{<r>} \mathcal{E}; \quad r = 1, 2, \ldots, l. \quad (13)
\end{align*}
\]

The first, current law for \( N \) external (algebraic sum over \([j]\) in (12a)) and for \( v - N \) internal (algebraic sum over \((i)\) in (12b)) vertices of the electric network states that the algebraic sum of the currents \( J \) flowing through all the network wires that meet at a vertex is the external current \( J^* \) or zero. The second, voltage law for \( l \) circuits (loops) states that the algebraic sum of the electromotive forces \( \mathcal{E} \) within any closed circuit is equal to the algebraic sum of the products of the currents \( J \) and the resistances \( \omega \) in the various portions of the circuit.

The Kirchhoff’s laws (12a), (12b), (13) for an electric network including only the real measurable quantities \( J, \mathcal{E} \) (and also \( \omega \)) are formally the same as the linear equations (3)–(5) for an identical topologically physical graph with real objects quantities \( q, p \). The electric circuit analogy still with the suitable Feynman diagram has been carried by Bjorken, T. T. Wu and Boiling earlier in 1959–64 (see Ref. [1]). It is easy to show (see for example, Ref. [5]) however that the number of independent equations among (12a)–(12b) as in the case of set (3) and (4) is equal to \( v - 1 \) and therefore the rank of the resulting square matrix of coefficients

\[
M_{(n\times n)} = \begin{pmatrix}
I(\epsilon)_{(v-1\times n)} \\
C(\epsilon)D_n(\omega)_{(l\times n)}
\end{pmatrix}
\]

is exactly corresponded to the number of unknown components of the column vector \( (J_n) \), i.e., to \( n \).

In this way the soluble matrix equation for a typical Kirchhoff’s matrix (14) may be written in the obvious form.
\[
\left\{ \begin{array}{c}
I(\epsilon) \\
C(\epsilon) D_n(\omega)
\end{array} \right\}_{(v-1 \times n)} (J_n)_{(n \times 1)} = \left( \begin{array}{c}
(J^*_n)_{(v-1 \times 1)} \\
C(\epsilon) (\mathcal{E}_n)
\end{array} \right)_{(l \times n)}
\]

where by means \((J_n)_{(n \times 1)}\), \((J^*_n)_{(v-1 \times 1)}\) and \((\mathcal{E}_n)_{(n \times 1)}\) are denoted the column vectors

\[
(J_n)_{(n \times 1)} = \begin{pmatrix}
J_1 \\
J_2 \\
J_3 \\
\vdots \\
J_{n-1} \\
J_n
\end{pmatrix}, \quad (J^*_n)_{(v-1 \times 1)} = \begin{pmatrix}
J^*_1 \\
\vdots \\
J^*_{N-1} \\
0
\end{pmatrix}
\]

\[
(\mathcal{E}_n)_{(n \times 1)} = \begin{pmatrix}
\mathcal{E}_1 \\
\mathcal{E}_2 \\
\mathcal{E}_3 \\
\vdots \\
\mathcal{E}_{n-1} \\
\mathcal{E}_n
\end{pmatrix}
\]

### 2.1 Skeleton trees basis for electric network

In accordance to Kirchhoff’s theorem (see Refs. [2], [5]) every electric network can be substituted by a corresponding graph with the same number of vertices \(v\). The solution of the equations (12) and (13) for this adequate graph may be presented through the set of skeleton \(v\)-trees. The maximum set of independent skeleton \(v\)-trees for the corresponding full \(K_v\)-graph, with \(\binom{v}{2}\) lines, forms the skeleton trees basis for electric network. By using of this basis can be constructed the all possible solutions for various concrete networks (lots of such electric network examples described in Ref. [5]).

### 2.2 Kirchhoff–Maxwell topological analysis of electric networks

Two-layer structure of the typical Kirchhoff’s matrix (14) (or analogous matrix (7)) in a general matrix equation (15) (or in corresponding matrix equation (10)) shows a separation of the functions of the matrix operator (14) (or (7)) acting, firstly, upon the “internal geometry” of electric network by means of the incidence matrix \(I(\epsilon)\) (“upper layer”) and, secondly, upon the topologically other equilibrium conditions along the circuits (loops) of electric network by means of the loop matrix \(C(\epsilon) D_n(\omega)\) (“under layer”) within the framework of the same skeleton trees basis. In this connection it is important to note that
into the structure of the incidence matrix $I(\epsilon)$ is embedded the “internal geometry” of
real electric network including the natural skeleton trees basis factually as a creation of the
specific “graph geometry” beyond the space-time consideration.

By introducing of “circuit currents” of Helmholtz–Maxwell instead of “branch (wire)
currents” of Kirchhoff can be performed formally the analogous concrete calculations on the
base of corresponding topologically equivalent Maxwell rules (see Ref. [5]) within the same
“graph geometry”.

Obviously, the last formalism of the Helmholtz–Maxwell “circuit currents” is like to the
formalism of “$(p, k)$-diagrams” (see Ref. [6]) which may be used as an adequate tool for the
analysis of the physical graphs from section 1. 2. However this problem is beyond the task
of a given paper.

3 “MICROGEOMETRY” INSIDE
OF THE PHYSICAL OBJECTS
CONSIDERING WITHIN THE FRAMEWORK
OF THE INPUT–OUTPUT SCHEME

Heisenberg’s theory of the $S$-matrix connects the input and output of a scattering ex-
periment without seeking to give a localized description of the intervening events including
the inner structure of a propagated physical object.

Introducing a full set of the external 4-momenta $p_j$ ($j=1, 2, \ldots, N$) simultaneously for
$N_1$ ingoing as well as for $N_2$ outgoing physical objects ($N_1 + N_2 = N$) we can obtain the
solution of the equations (3)–(6), together with the mass shell conditions (8)–(9), in the
terms of independent kinematic invariants $s_i = p_i^2$, $s_{ik} = (p_i + p_k)^2$, $s_{ikl} = (p_i + p_k + p_l)^2$, $s_{iklm} = (p_i + p_k + p_l + p_m)^2$, etc. connecting with each other by kinematic and geometrical
conditions in the 4-dimensional momentum space (see Refs. [7–10]). Thus we should be
presented the whole physical picture for the real scattering process where an inner structure
of the participating physical objects is factually omitted and the latter should be introduced
only by insertion of the separate full graph with $v$ vertices $K_v$ as a some complicated vertex-
fragment for concrete physical object.

3.1 Root trees basis for
physical graph of microobject

Within the framework of the input–output scheme for the full physical graph with in-
serted complicated $K_v$-vertex, as a physical microobject vertex-fragment in the general
scheme, it is to be determined the special separate vertex — the root of tree which coinc-
dides with input vertex, and the other part of physical graph is to be arranged in hierarchical
order, creating a path to output vertex (vertices).

Thus the solution of the corresponding linear equations (3)–(5) for this new hierarchical
physical graph may be presented already through the set of root $v$-trees as against of skeleton
$v$-trees for electric network in section 2.1.

If we consider the full physical graph as an unique $K_v$-vertex in the input–output scheme
then may be formulated the problem of the discrete physical object structure. The maximum
set of independent root $v$-trees for corresponding full $K_v$-graph, with the separate root-vertex and $\binom{v}{2}$ lines, forms the natural root trees basis for the hierarchical physical graph, i.e. a set of paths between the root-vertex, as an initial point of input, and the final points of output.

Returning to the two-layer structure of the matrix (7) in a general matrix equation (10) already for the discrete physical microobjects in the $S$-matrix input–output scheme we note again that the incidence matrix $I(e)_{(v-1\times n)}$ ("upper layer") acts upon the "internal microgeometry" of the physical microobjects within the framework of the natural fixed root trees basis. Therefore the incidence matrix $I(e)_{(v-1\times n)}$ contains a specific "graph microgeometry" beyond space-time approach.

3.2 On the proton mass

The fact is that the above-described initial $S$-matrix theory overgrew practically in the specific physical graph kinematics formalism allowing to realize the discrete physical microobject calculations without use of the traditional space-time treatment.

We start with the consideration of the proton mass problem within the framework of the results of QCD-theory [11]. It is shown earlier (in 3.1) that "internal microgeometry" of the discrete physical microobjects reflects their inner structure and therefore is to correspond to the vertex many-point Green function (without free tails). These points are responsible for determination of the number of vertices of the root trees filling in graph microgeometry of the discrete physical microobjects (in a Riemann [15] sense).

If, as usually supposed, the full proton mass is concentrated into an internal gluon field (Refs. [12–13]) — a case of the pure gluodynamics — then from the Gell-Mann–Low function (Ref. [14])

$$\beta(g^2) = -b \frac{g^4}{16\pi^2} + O\left(\frac{g^2}{4\pi}\right)$$

where $g$ — coupling constant, we have a dimensionless constant $b=11$ characterized the number of vertices of the corresponding root trees. For setting up the graph equivalent to the electron mass $m_e$ we choice the corresponding simplest case of the root tree graph with $v=2$ vertices inasmuch as the number of a such root tree $T_{v=2} = 1$ (the case $v=1$ is the trivial graph — an isolate vertex). Therefore the number of the root trees with $v=11$ vertices $T_{v=11} = 1842$ (see Ref. [2]) determines for the proton mass

$$M_p = 1842 m_e.$$

In the standard QCD-model with the number of quark flavours $n_f$ we have a dimensionless constant $b = 11 - 2/3 n_f$ what is responsible for the pion mass at $n_f=3$: $T_{v=9}=286$ (see Ref. [2]) or

$$m_\pi = 286 m_e.$$
3.3 On the “double - and triple - splitting” mechanism in the physics and the biology hierarchy

The investigation of the inner structure of the discrete physical microobjects by using of the analysis of a specific “graph microgeometry” may be continued on the base of the all kinds of root $v$-trees from $v$-sequences (see Table 1) what could permit to classify the various discrete

Table 1. The number of root trees $T_v$ and their relations $T_{v+1}/T_v$ where $v$ — the number of vertices (see Ref. [2])

| $v$ | $T_v$ | $T_{v+1}/T_v$ |
|-----|-------|---------------|
| 1   | 1     | 1             |
| 2   | 1     | 2             |
| 3   | 2     | 2             |
| 4   | 4     | 2,250         |
| 5   | 9     | 2.22(2)       |
| 6   | 20    | 2,400         |
| 7   | 48    | 2,396         |
| 8   | 115   | 2,487         |
| 9   | 286   | 2,514         |
| 10  | 719   | 2,562         |
| 11  | 1842  | 2,587         |
| 12  | 4766  | 2,620         |
| 13  | 12486 | 2,641         |
| 14  | 32973 | 2,663         |
| 15  | 87811 | 2,681         |
| 16  | 235381| 2,697         |
| 17  | 634847| 2,711         |
| $v$  | $T_v$   | $T_{v+1}/T_v$ |
|------|---------|--------------|
| 18   | 1 721 159 | 2,724        |
| 19   | 4 688 676 | 2,736        |
| 20   | 12 826 228 | 2,746        |
| 21   | 35 221 832 | 2,756        |
| 22   | 97 055 181 | 2,764        |
| 23   | 2 688 282 855 | 2,772    |
| 24   | 7 437 249 844 | 2,779    |
| 25   | 2 067 174 645 | 2,786    |
| 26   | 5 759 636 510 |            |

Microobjects and to estimate their masses. It is easy to see from the Table 1 that $T_{v+1}/T_v$ relation is into the interval

$$2 \leq T_{v+1}/T_v < 3$$

and therefore we could produce the double- and triple-splitting operation on the “mass-values” of the various $T_v$ before the experimental fitting. In Table 1 we may have conventionally three zones:

- inequality $1 \leq T_v \leq 115$ for the stable atomic and nuclear discrete objects quantities,
- inequality $286 \leq T_v \leq 1842$ for the pion-nucleon discrete objects quantities,
- inequality $4766 \leq T_v$ for the heavy physical and the complicated biological hierarchical objects quantities.

**4 CONCLUSIONS**

In conclusion it is important to note that the development of the graph kinematics formalism for the discrete physical objects (in a Riemann [15] sense) which has two-layer matrix description (incidence+loop matrices) with natural “graph microgeometry” beyond the traditional space-time consideration leads obviously to the more general results in the area of the creation of the Heisenberg–Dyson’s two-layer physics (see for example, [16]) with the proper “internal geometry”. In the upper layer of this scheme we have the formalism for real physical objects, their momenta, energy, forces, etc. Whereas in the under layer we have the symbolical fields quantities, such as field strength, induction, intensity, etc. which may be discovered only through their energy and forces in the upper layer.
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