Possible considerations to teleport fermionic particles via studying on teleportation of two-particle state with a four fermionic-particle pure entangled state

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In this paper we have firstly recapped some evolutionary debates on conceptual quantum information matters, followed by an experiment done by Lamei-Rashti and his collaborator, by which the bell inequality on p-p scattering is violated. We then, by using the goal of his experiment, thought to arrange POVM formalism for a possible teleportation of two particle states, via nuclear magnetic spin of four entangled hydrogen like atoms.

I. INTRODUCTION

The paradox of Einstein, Podolsky and Rosen [1] was advanced as an argument that quantum mechanics could not be a complete theory but should be supplemented by additional variables, by which the causality and locality theories are restored. According to the local realistic theories, objects should have definite properties whether they are measured or not (reality), and there is no action-at-a-distance in nature (locality). Some attempts were made to explain quantum mechanical phenomena from a view of the local realistic theories. Bell, however, showed quantum mechanical correlations between entangled systems, can be stronger than those by the local realistic theories [2]. Since Bell’s proof was given by an inequality which could be tested experimentally [3] and [4]. The mentioned movement to pursue and check experimentally the bell inequality in QCD interactions started with the key paper of Clauser, Horne, Shimony and Holt (CHSH) [3] who generalized Bell’s theorem such that it applies to realizable experimental tests with pair detection of all local hidden variable theories. Quantum mechanically thinking, the mentioned inequality can not be totally true though. To check this non reliability for photons and most important for QCD like scatterings such as Proton-Proton (called P-P scattering) in low and rarely for high energies to actually observe such a violation of Bell’s inequality in a laboratory some experiments was done. Most of the experiments to test Bell’s inequality performed so far have used spin correlations of a two photon system. The only one exception is the experiment by Lameli-Rashti and Mittig (LRM) [6] with protons in entangled systems. They used strong interaction to test the Bell’s inequality. Since the strong interaction is a short range interaction, entangled particles are produced with extremely short coherence length. It is of considerable interest to investigate whether an entanglement between two particles is robustly maintained even if the two particles are spatially separated from each other by a distance extremely beyond their coherence length. Measured spin-correlations between two protons in the spin-singlet state which was produced by the proton-proton S-wave elastic scattering. This is because proton-proton scattering at large angle and low energy, say a few MeV, goes mainly in S wave. But the antisymmetry of the final wave function then requires the antisymmetries singlet spin state. In this state, when one spin is found ‘up’ the other is found ‘down’. This follows formally from the quantum expectation value, \( < \text{singlet} \sigma_z(1) \sigma_z(2) \text{singlet} > = -1 \) in these experiments, protons of 14 MeV lab energy are scattered at a lab angle of 45, and spin correlation of scattered and recoil protons are measured. This experiment anyhow, shows agree with quantum mechanics and disagree with the locality inequality, and are the first serious test of Bell inequality by spin=1/2 fermion system with mass. Violating the Bell inequality for the sake of EPR states, opens up the world of teleportation for us, that has recently been the source of many researches in this domain.

II. QUANTUM TELEPORTATION

Quantum teleportation is a technique for moving quantum states around, even in the absence of a quantum communications channel linking the sender of quantum state to the recipient. Now it seems necessary to explain some crucial concepts, namely, entanglement, qubit and Bell state, which have key roles to perceive teleportation.

A. Entanglement

In order to understand the concept of quantum communication the phenomena called entanglement, described for the first time at 1935 by Einstein, Podolsky and Rosen, the EPR interactions have three main characteristics which make them unique and ex-
extremely interesting:

- Any modification applied to one of the particles, from now on particle 1, would imply a variation on its couple (particle 2). The alteration on particle 2 would depend on the modification done on particle 1. Different modifications would imply different variations.
- The alteration of particle 2, due to the modification of particle 1 does not depend on the distance between them.
- Particle 2 is instantly altered once particle 1 is modified.

B. Measurement-POVM

Quantum measurements are described by a collection \{M_m\} of measurement operators. These operators acting on the state space of the system being measured. The index m refers to the measurement outcomes that may occur in the experiment. Distinguishing quantum states is the important problem in Quantum information and Quantum computation. In such instances there is a mathematical tool known as the POVM (Positive Operator-Value Measure) formalism which is especially well adapted to the analysis of the measurements. The elements of a POVM are not necessarily orthogonal, with the consequence that the number of elements in the POVM, n, can be larger than the dimension, N, of the Hilbert space they act in. Any measurement process can be described in terms of a quantum operation in the following way:

1. To the set of possible outcomes \{m\} from a measurement a set of Quantum operations \{E_m\} is associated.
2. Each \(E_m\) describes the dynamics of system when outcome m is found.
3. The probability \(P_m\) of the outcome m is \(\text{Tr}[E_m(\rho)]\) and the post-measurement state is given by \(\frac{E_m(\rho)}{\text{Tr}[E_m(\rho)]}\).
4. The total Quantum operation \(E = \sum_m E_m\) is trace Preserving, because the probabilities \(p_m\) of the distinct outcomes sum to One.

**POVM measurement** A set of Operators \{E_m\} satisfying

- \(E_m\) Positivity
- \(\sum_m E_m = 1\) Completeness
- \(p_m = \text{Tr}[E_m\rho] = \langle \Psi | E_m | \Psi \rangle\) Probability rule

Defining \(M_m = \sqrt{E_m}\) we see that \(\sum_m M_m^\dagger M_m = E_m = I\), and therefore the set \{\(M_m\)\} describes a measurement with POVM\{\(E_m\)\}.

C. Qubit

just as a classical bit has a state, either 0 or 1 a qubit also has a state, which can be thought of as a vector in a two-dimensional Hilbert space and will be denoted as \(|0 > and |1 >\) from now on. the main and important difference between bits and qubits is that the latter can also be in a linear combination of states, i.e., a coherent superposition: \(|\Psi > = \alpha|0 > + \beta|1 >\) , where \(\alpha\) and \(\beta\) are complex numbers. Qubit is a fundamental element for quantum computation and quantum information. In the early days of quantum mechanics the qubit structure was not at all obvious, and people struggle with phenomena that we may now understand in terms of qubits, that is, in the terms of two level quantum systems.

One very useful picture when thinking about qubits is the geometrical representation of polarization states on the so-called Bloch-sphere and often serves as an excellent testbed for ideas about quantum computation and quantum information. one can rewrite the state as

\[|\Psi > = \cos(\Theta/2)|0 > + e^{i\Phi} \sin(\Theta/2)|1 >\]  

where the angle \(\Theta\) and \(\Phi\) define a point on the three-dimensional unit sphere shown in

D. Bell state

For two classical bits there are four possible states, 00, 01, 10 and 11, but a pair of qubits can also exist in a superposition of this states, therefore spanning a 4-dimensional Hilbert space. One remarkable feature of such states is that they cannot be built as single and separable qubit states |a > and |b > such that |\Phi > = |a > |b >. Thus, they cannot be written as a product of states of their component systems, which is a very crucial property of entangled states. Einstein, Podolsky and Rosen pointed out these strange properties of such states and they have been named Bell-States in honour of John Bell, who showed that correlations in such entangled states are stronger than could possibly exist between classical systems. For a two-qubit system there are four distinct entangled states, the Bell-States,

\[|\Phi^\pm > = (1/\sqrt{2})(|00 > \pm |11 >)\]

\[|\Psi^\pm > = (1/\sqrt{2})(|01 > \pm |10 >)\]

which form an orthonormal basis for the two-qubit state space, and can therefore be distinguished by appropriate quantum measurements. The basic idea in quantum teleportation is the following: Suppose we have two parties, Alice and Bob. Say Alice wishes to transfer a certain quantum particle to Bob, but cannot do so directly. According to the rules of quantum
mechanics if she measured the qubit this action would
destroy the quantum state of the particle without re-
vealing her all the necessary information which she
could then send to Bob to reconstruct the qubit.
how Alice can provide Bob with her quantum particle.
The solution is once again entanglement. In outline,
the steps of the solution are as follows: Alice interacts
the qubit |Ψ⟩ with half of EPR pair (singlet state), and
then measures the two qubits in her possession, ob-
taining one of four possible classical results, 00, 01, 10
and 11. She sends this information to Bob. Depending
on Alice’s classical message, Bob performs one of four
operations on his half of the EPR pair. Amazingly,
by doing this he recovers the original state |Ψ⟩!
The state to be eleopted is |Ψ⟩ = α|0⟩ + β|1⟩, where α
and β are unknown amplitudes. The state input into
the quantum circuit |Ψ0⟩ is:

|Ψ0⟩ = (1/√2)[α|0⟩ + β|1⟩] = (1/√2)[α|0⟩ + β|1⟩] + β|1⟩

The first two qubits (on the left) belong to Alice,
and the third qubit to Bob. Alice sends her qubits
through a CNOT gate, and then sends the first qubit
through a Hadamard gate, obtaining,

|Ψ1⟩ = (1/√2)[α|0⟩ + β|1⟩] = (1/√2)[α|0⟩ + β|1⟩] + β|1⟩

The latter state may be re-written in the following
way, simply by regrouping terms:

|Ψ2⟩ = (1/√2)[|0⟩ + β|1⟩] = (1/√2)[|0⟩ + β|1⟩] + β|1⟩

|0⟩ = α|0⟩ + β|1⟩

Obviously Alice obtains one of the four possible two-
bit results among 00, 01, 10 or 11 as measurement
outcome. Each of them is in close connection with
the state of Bob’s qubit hence Alice sends these two
classical bits to Bob. After a short hesitation Bob
compares |Ψ⟩ to the potential states of his half Bell
pair. It is easy to realize that 00 → a(0) + b(1) = I|Ψ⟩
01 → a(1) + b(0) = X|Ψ⟩ 10 → a(0) − b(1) = Z|Ψ⟩
11 → a(1) − b(0) = Z.X|Ψ⟩. Therefore Bob has only to
apply the inverse of the appropriate transform(s) in
compliance with the received classical bits.
So as it can be seen, quantum teleportation is a pro-
cess of transmission of an unknown quantum state
via a previously shared EPR pair with the help of
only two classical bits transmitted through a classi-

cal channel. It was regarded as one of the most
striking progress of quantum information theory.
Suppose that the sender Alice has two particles 1,2 in
an unknown state |00⟩

|Φ12⟩ = a(00) + b(01) + c(10) + d(11)⟩,

where a, b, c, d are arbitrary complex numbers, and
satisfy |a|2 + |b|2 + |c|2 + |d|2 = 1. We also suppose that
Alice and Bob share quantum entanglement in the
form of following partly pure entangled four-particle
state, which will be used as the quantum channel,

|Φ⟩ = (a|0000⟩ + b|1011⟩ + c|0110⟩ + d|1111⟩⟩.

(6)

The particles 3 and 4, and particle pair (1, 2) are in
Alice’s possession, and particles 5 and 6 are in Bob’s
possession. The overall state of six particles is

|Φ⟩ = |Φ12⟩ ⊗ |Φ⟩.

In order to realize the teleportation, firstly Alice per-
forms two Bell state measurements on particles 2,3
and 1,4. If the outcomes of the Alice’s two Bell state
measurements are |Φ⟩⟩⟩⟩2 and |Φ⟩⟩⟩⟩14 then the particle 5
and 6 are in the state,

|Ψ⟩⟩⟩⟩56 = 14(Φ+23)(Φ+25)(Φ+15)⟩⟩⟩⟩56

= 1

√ |a|2 + |b|2 + |c|2 + |d|2

= (a|00⟩ + b|01⟩ + c|10⟩ + d|11⟩⟩⟩⟩56,

(8)

Now, we will not write out the states of the particles
5 and 6 corresponding to the other outcomes of Alice’s
two Bell state measurements. Then Alice informs
Bob her two Bell state measurements on particles 2, 3
and 1, 4 Without loss of generality, we give the case
for |Ψ⟩⟩⟩⟩56 all other cases can be deduced similarly. In
order to realize the teleportation, Bob introduces two
auxiliary qubits a and b in the state |00⟩a b. Thus the
state of particles 5,6,a, and b becomes, |Ψ⟩⟩⟩⟩56|00⟩a b Then
Bob performs two controlled-not operations
(CNOT gate) with particles 5 and 6 as the control
qubits and the auxiliary particles a and b as the target
qubits respectively. After completing this operation
the particles 5,6, a, and b are in the following state,

|Ψ⟩⟩⟩⟩56ab = 1

√ |a|2 + |b|2 + |c|2 + |d|2

= (aa|00⟩ + bβ|01⟩ + cγ|10⟩ + dδ|11⟩⟩⟩⟩56ab.

(9)

After some rearrangement one obtains,

|Ψ⟩⟩⟩⟩56ab = 1

4√ |a|2 + |b|2 + |c|2 + |d|2

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\( (a|00\rangle + b|01\rangle + c|10\rangle + d|11\rangle)_{ab} \)
\( (\alpha|00\rangle + \beta|01\rangle + \gamma|10\rangle + \delta|11\rangle)_{ab} \)
\( +(a|00\rangle + b|01\rangle - c|10\rangle - d|11\rangle)_{ab} \)
\( (\alpha|00\rangle + \beta|01\rangle - \gamma|10\rangle - \delta|11\rangle)_{ab} \)
\( +(a|00\rangle - b|01\rangle + c|10\rangle - d|11\rangle)_{ab} \)
\( +(a|00\rangle - b|01\rangle - c|10\rangle + d|11\rangle)_{ab} \)

Now Bob makes an optimal POVM \[7\] on the ancillary [17, pp.282] particles \( a\ and b\) to conclusively distinguish the above states. We choose the optimal POVM in this subspace as follows:

\[
P_1 = \frac{1}{x}|\Psi_1\rangle\langle \Psi_1|, P_2 = \frac{1}{x}|\Psi_2\rangle\langle \Psi_2|,
\]
\[
P_3 = \frac{1}{x}|\Psi_3\rangle\langle \Psi_3|, P_4 = \frac{1}{x}|\Psi_4\rangle\langle \Psi_4|,
\]
\[
P_5 = I - \frac{1}{x}\sum_{i=1}^{4}|\Psi_i\rangle\langle \Psi_i|,
\]

where

\[
|\Psi_1\rangle = \frac{1}{\sqrt{\frac{1}{\alpha^2} + \frac{1}{\beta^2} + \frac{1}{\gamma^2} + \frac{1}{\delta^2}}}
\]
\[
\left(\frac{1}{\alpha}|00\rangle + \frac{1}{\beta}|01\rangle + \frac{1}{\gamma}|10\rangle + \frac{1}{\delta}|11\rangle\right)_{ab}
\]
\[
|\Psi_2\rangle = \frac{1}{\sqrt{\frac{1}{\alpha^2} + \frac{1}{\beta^2} + \frac{1}{\gamma^2} + \frac{1}{\delta^2}}}
\]
\[
\left(\frac{1}{\alpha}|00\rangle + \frac{1}{\beta}|01\rangle - \frac{1}{\gamma}|10\rangle - \frac{1}{\delta}|11\rangle\right)_{ab}
\]
\[
|\Psi_3\rangle = \frac{1}{\sqrt{\frac{1}{\alpha^2} + \frac{1}{\beta^2} + \frac{1}{\gamma^2} + \frac{1}{\delta^2}}}
\]
\[
\left(\frac{1}{\alpha}|00\rangle - \frac{1}{\beta}|01\rangle + \frac{1}{\gamma}|10\rangle - \frac{1}{\delta}|11\rangle\right)_{ab}
\]
\[
|\Psi_4\rangle = \frac{1}{\sqrt{\frac{1}{\alpha^2} + \frac{1}{\beta^2} + \frac{1}{\gamma^2} + \frac{1}{\delta^2}}}
\]
\[
\left(\frac{1}{\alpha}|00\rangle - \frac{1}{\beta}|01\rangle - \frac{1}{\gamma}|10\rangle + \frac{1}{\delta}|11\rangle\right)_{ab}
\]

\[I\ is an identity operator; \ x\ is a coefficient relating to \alpha, \beta, \gamma, \delta, \ 1 \leq x \leq 4, \ and \ makes \ P_5 \ to \ be \ a \ positive \ operator. \ Obviously, \ we \ should \ carefully \ choose \ x \ such \ that \ all \ the \ diagonal \ elements \ of \ P_5 \ are \ nonnegative. \ If \ the \ result \ of \ Bob’s \ POVM \ is \ P_1, \ then \ Bob \ can \ safely \ conclude \ that \ the \ state \ of \ the \ particles \ 5,6 \ is \]
\[
|\Phi_{56}\rangle = (a|00\rangle + b|01\rangle + c|10\rangle + d|11\rangle)_{ab}.
\]

for other result we can recover the state of particles 5,6. By the similar method we can make the teleportation successful in the other outcomes of Alice’s Bell state measurement. For the sake of saving the space we will not write them out. Evidently, when the Bell states \( |\Phi^+\rangle_{23} \) and \( |\Phi^+\rangle_{14} \) are acquired in Alice’s two Bell state measurements, the probability of successful teleporation is \( \frac{2}{x^2} \). Synthesizing all Alice’s Bell state measurement cases (sixteen kinds in all), the probability of successful teleportation in this scheme is \( \frac{16 + F^2}{x^2} \).

Hence the smallest \( x \) corresponds to the highest probability of successful teleportation.

## III. CONCLUSION

It was shown that the possibility of teleportation of protons as fermions (in low energy scales) can be focused using some available experimental techniques. However lastly, we have used a two-fermion particle state in one hand, with a four-particle pure entangled state in other hand to teleport a fermonic characteristic (spin), but it should be noted that we have no idea on how to prepare a successful setup to examine our suggestion. It seems that the mathematics behind it works well. But it is honestly a long way between a mythic and gedanken idea and a viewable-experimental idea to have reasonable data in laboratory.

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