1 Introduction

Aircraft wake is an inevitable phenomenon generated by a flying aircraft. It is composed of two rotating vortices and can threaten the safety of a following aircraft. To avoid the accidents caused by wakes, International Civil Aviation Organisation (ICAO) released a series of regulations for the minimal temporal – and spatial – intervals between flights in the 1970s. However, the regulations are somewhat conservative and have limited the capacity of airports to a certain degree [1]. The detection and characterisation of aircraft wakes can help to optimise the air traffic management and improve the capacity of airports. After years of research, there have been some algorithms to retrieve the characteristic parameters of wake vortex.

Another type of algorithms are quick methods, and a representative one of them is the algorithm proposed in Ref. [7], namely the tangential velocity method (TV method). This method uses the velocity envelopes for vortex-core locating and retrieves the circulation with the TV (max RV). It is computationally very cheap, but the vortex-core location does not work well for low SNR cases [2] and the maximum RV is vulnerable to the background noise.

In real applications, a feasible parameter-retrieval algorithm of wake vortex is generally supposed to work quickly with an acceptable accuracy. To meet this end, this paper proposes to retrieve the characteristic parameters from the velocity max–min distribution in the RHIs. Simulations and filed detection data both show that the algorithm has the benefits of low computation cost and good accuracy.

2 Methodology

2.1 Vortex-core locating method

Theoretically, measurement cells above and below the vortex cores have opposite RV. This property can be used to locate the vortex cores. At a range gate $R_k$, the max–min of velocity is defined as the difference between the maximum and minimum of the RV $V(R_k, \phi)$:

$$\Delta V(R_k) = \max_{\varphi} \{V(R_k, \varphi)\} - \min_{\varphi} \{V(R_k, \varphi)\}$$

This variable should have two peaks along the ranges, which correspond to the radial positions of the two vortex cores.

Another type of algorithms are accurate methods. They retrieve the characteristic parameters of wake vortex with theoretic model and generally have good robustness and accuracy. For example, the radial velocity (RV) method [2] fits the measured RV with theoretic values, and the methods introduced in Ref. [3–6] fit the Doppler spectrum for better accuracy. Normally, these algorithms adopt least-square minimisation or maximum likelihood (ML) method to execute the optimisation, so the computation cost could be an issue some time.

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2.2 Circulation calculation

From the Burnham–Hallock velocity model [8], the TV of a single vortex whose core located at \((R_i, \Psi_i)\) follows:

\[
V^o = \frac{\Gamma_i}{2\pi} \frac{R^2}{R' + r_c}
\]  

where \(R\) is the distance from the measurement cell to the vortex core \((R_i, \Psi_i)\), \(\Gamma_i\) is the circulation, \(r_c\) is the vortex-core radius.

In Fig. 2, the Lidar scans up and down alternately at angular speeds of \(\omega\) and \(-\omega\), the dashed lines represent the Lidar beams, and the dot-dashed lines indicate the distances from the measurement cells to the vortex cores. The Lidars’ scanning plane is XOY and Z-axis is along the runway direction. To clearly illustrate the velocities, a pair of measurement cells above and below the left vortex core are plotted as examples. Their radial velocities are the projections of wake vortex velocities and background wind velocity on the line-of-sight (LOS):

\[
V_i^r = \frac{\Gamma_i}{2\pi} \frac{R_i}{d_{1i} + r_c} \sin(\phi_i - \phi_c)
\]

\[
V_1^r = \frac{\Gamma_1}{2\pi} \frac{R_1}{d_{11} + r_c} \sin(\phi_1 - \phi_c)
\]

\[
V_2^r = \frac{\Gamma_2}{2\pi} \frac{R_2}{d_{12} + r_c} \sin(\phi_1 - \phi_c) + V_{\text{wind}} \cos \phi_1
\]

where \(d_{1i}, d_{12}, d_{2i}\) (\(i = 1, 2\)), shown as the dot-dashed line in Fig. 3, are the distances from the measurement cells (above and below the vortex core) to the \(i^{th}\) vortex core. \(\phi_i, (\phi_{1i})\) is the elevation angle of the measurement cell above (below) the vortex core. Since the distance from Lidar to wake vortices is generally long, we suppose \(V_{\text{wind}} \cos \phi_i \approx V_{\text{wind}} \cos \phi_{1i}\). In this sense the effect of background wind is mitigated by the subtraction of the two velocities:

\[
\Delta V_i^r = V_i^r - V_{1i}^r = \frac{\Gamma_1}{2\pi} \left[ \frac{R_1}{d_{11} + r_c} - \frac{R_2}{d_{12} + r_c} \right] \sin(\phi_1 - \phi_{1i}) + \frac{V_{\text{wind}}}{2} \left[ \frac{\cos \phi_1}{d_{11} + r_c} - \frac{\cos \phi_{1i}}{d_{12} + r_c} \right]
\]

where

\[
\Psi_1 = \frac{R_1}{2\pi} \left[ \frac{\sin(\phi_1 - \phi_{1i})}{d_{11} + r_c} + \frac{\sin(\phi_{1i} - \phi_i)}{d_{2i} + r_c} \right]
\]

\[
\Psi_2 = -\frac{R_2}{2\pi} \left[ \frac{\sin(\phi_1 - \phi_{1i})}{d_{12} + r_c} + \frac{\sin(\phi_{1i} - \phi_i)}{d_{2i} + r_c} \right]
\]

with \(r_c = 0.052b_0\) being the vortex core size, and \(b_0\) being the separation between the two vortex cores.

To enhance the robustness, more measurement cell pairs \([(R_{ci}, \Psi_{ci}), \cdots (R_{ck}, \Psi_{ck}), K\text{ pairs in total}]\) are used to characterise the circulations:

\[
\begin{align*}
& \Delta V_1 \ldots \Delta V_K \\
& \Psi_1 \ldots \Psi_K
\end{align*}
\]

\[
\begin{bmatrix}
\Delta V_1 \\
\vdots \\
\Delta V_K
\end{bmatrix} =
\begin{bmatrix}
\Psi_1^r & \Psi_2^r & \cdots & \Psi_K^r
\end{bmatrix}
\begin{bmatrix}
\Gamma_1^r \\
\vdots \\
\Gamma_K^r
\end{bmatrix}
\]

from which, \([\Gamma_1^r, \Gamma_K^r]\) can be obtained with the linear least squares method. In this process, the selection of measurement cell pairs is crucial and may deteriorate the accuracy of the estimation. Normally, the distances from the used measurement cells to the vortex cores should be bigger than the vortex-core radius and smaller than half of vortex-core separation \([\{r_c, b_0/2\}]\) [7], but the effect of spatial averaging in Lidar data processing may underestimate the RV. As shown in Fig. 3, if the measurement cell is close to the vortex core (smaller than 15 m), a big difference between the measured velocity and the theoretical velocity is observed; otherwise, the two velocities agree quite well. In this manner, we choose the distances of [15 m, \(b_0/2\)] (shown as the grey rectangle region) to calculate the circulation.

3 Performance verification of the new method

3.1 Simulation experiments

3.1.1 RHI simulation of Doppler velocity: To verify the performance of the proposed method, a RHI simulator is
established to obtain the Lidar scattering signal reflected by the aerosols in the wake. The Lidar's parameters are listed in Table 1.

The background wind is with an eddy dissipation rate of 0.008 m2 s−3 and a mean velocity of −1 m/s (towards the Lidar).

According to Ref. [9], the echo signal of a measurement cell can be simplified as:

\[ z(t_i) = \sum_{i=1}^{N} \text{rect} \left( \frac{t_i - (2\pi c / \lambda)}{T_w} \right) \times \exp(2\pi jTV_i / \lambda) + n_i \]  

where \( t_i \) is the sample time, \( T_w \) is the time window of the transmitted rectangle pulse, \( \lambda \) is the wavelength, \( N \) is the total number of aerosols inside the measurement cell, \( V_i \) is the velocity of the \( i \)th aerosol in the measurement cell (entrailed by the wake vortices and the background wind), and \( n_i \) is an additive random noise. Afterwards, seven consecutive signal samples are used to obtain the spectrum of a measurement cell by fast Fourier transformation, and the overlapping method is used to get a better spatial resolution of 3 m. Finally, for each measurement cell \((R_c, \theta_c)\) the RV is obtained from the spectrum peak.

According to the method mentioned above, 10 Lidar detection scenarios in terms of different aerosols' initial random positions and turbulence velocity are simulated. Each scenario contains 13 consecutive RHIs. One RHI is shown in Fig. 4 as an example.

### 3.1.2 Retrieval results of simulated RHIs:

To verify the performance of the newly proposed method, the TV method is adopted for comparison. The comparison is shown in Fig. 5.

For locating the vortex-core positions, the new method can give more robust results. As shown in Fig. 5a, the new method always gives positions very close to the theoretical values, but the results of the TV method gradually deviate from the true values. For the circulation estimation, overestimation is observed in TV method, while for the new method, only a slight underestimation occurs. The overestimation of TV method is mainly caused by the error of background wind estimation, while the underestimation of the new method is related to the spatial averaging error which is inevitable in Lidar data processing.

Quantitative comparisons of relative error \( E_r \) and relative root mean square errors \( \text{RMSE}_r \) between the TV method and the new method are further carried out. They are defined as:

\[ E_r = \frac{1}{N} \sum_{i=1}^{N} \left( P_i^r - P_i^T \right) \times 100\% \]  

\[ \text{RMSE}_r = \frac{1}{N} \sum_{i=1}^{N} \left[ \left( P_i^r - P_i^T \right) \right]^2 \times 100\% \]  

where \( P_i^r \) is the estimated value of a parameter for the \( i \)th scan, \( P_i^T \) is the corresponding true value of the parameter, and \( N \) is the total number of considered scans. The errors are listed in Table 2, from which it is observed that the new method can provide estimations with higher accuracy both for the vortex-core positions and circulations.

### 3.2 Field detection experiments

The new proposed algorithm has been applied on the field data measured in Hong Kong international airport (HKIA) in 2014. The campaign adopted a WindCube 200 s Lidar, whose pulse duration is 200 ns and the wavelength is 1.54 μm.

One of the obtained RHIs of RV in the campaign is shown in Fig. 6. More detailed descriptions of the measurement campaigns can be found in [10, 11]. In the field measurement campaigns, we did not record the aircraft parameters, so the reference parameters of circulation and vortex cores are available. To make the
Table 2: Relative errors and relative RMSEs in the estimation of wake parameters

| Method         | Left vortex core height, % | Right vortex core height, % | \( \frac{\Gamma_1 + \Gamma_2}{2} \), % |
|----------------|-----------------------------|-----------------------------|--------------------------------------|
|                | \( E_{r} \)                | \( \text{RMSE}_{r} \)       | \( E_{r} \)                | \( \text{RMSE}_{r} \)       |
| new method     | 0.78                        | 1.32                        | 1.26                                | 1.68                        | 6.53                        | 6.17                        |
| TV method      | 4.39                        | 4.19                        | 4.63                                | 3.95                        | 15.4                        | 11.5                        |

Fig. 6: Radial velocity distribution of the field measurement campaigns in HKIA in 2014

Fig. 7: Vortex-core trajectories (a) And circulations (b) Retrieved by three methods (RV method, the new method, TV method) for comparison

4 Conclusion

This paper proposes a parameter-retrieval method based on the velocity max–min in the RHIs. The method locates the vortex cores according to the distribution of max–min along the range and velocity peaks along the elevation. Then, the velocity differences of measurement cells above and below the vortex cores are used to calculate the circulations of the two vortices. Good performance has been observed in the simulations and field detection data.

Compared to the existing fast retrieval methods, the new method has a higher accuracy, and the effects of two vortices are both considered for a measurement cell. While compared to the fitting methods, the new method has a quicker speed to accomplish the parameter-retrieval. Furthermore, the impact of background wind is mitigated in the method, which means that the number of unknown variables (vortex-core parameters and background wind velocity) is declined. In all, the new method works quickly and has an acceptable accuracy and can be helpful to real-time wake detection in the air traffic management. However, the ground effect has not been considered in the new method, which will be studied in the near future.

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