In crystals, symmetries of continuous rotations and translations are broken due to discrete nature of the molecular lattice structure. Similarly, it is usually expected that isometries of Minkowski space, described by the Poincaré group, are broken or deformed due to granularity of space at the Planck scale. Nevertheless, one can imagine that some types of discontinuity may preserve the large-scale symmetries.

Let us see what it looks like in loop quantum gravity (LQG) [1], which is a background independent approach to quantize gravity. In LQG, “atomic” nature of space is reflected by discrete spectra of geometrical operators, such as area and volume. However, this feature alone does not guarantee violation of Lorentzian symmetry [2]. Similarly, a discrete spectrum of square of the angular momenta operator does not imply breaking of the rotational invariance. In canonical LQG, as pointed out by Rovelli and Speziale [3], kinematical sector can be locally defined in the Lorentz covariant manner. However, LQG is not only kinematics but also constraints that must be imposed in order to extract physical states. Furthermore, the off-shell algebra of these constraints carries information about symmetries of the theory.

At the classical level, this algebra corresponds to hypersurface deformation algebra, encoding general covariance, a cornerstone of general relativity (GR). However, the fate of this algebra at the quantum level is unknown due to the complicated form of the quantum constraints. Nevertheless, significant progress has recently been made in analysis of the effective off-shell algebra, where LQG effects were introduced through systematic modifications of the classical constraints. This analysis was performed for both, spherically symmetric configurations [4,5] and perturbative inhomogeneities on the flat Friedmann-Robertson-Walker (FRW) background [6,7]. Main message coming from these studies is that the algebra of effective constraints is deformed with respect to its classical counterpart. Namely, it takes the following form:

\[
\{D[M^a], D[N^a]\} = D[M^b\partial_b N^a - N^b\partial_b M^a],
\]

\[
\{D[M^a], S^Q[N]\} = S^Q[M^a\partial_a N - N\partial_a M^a],
\]

\[
\{S^Q[M], S^Q[N]\} = \beta D [q^{ab}(M\partial_b N - N\partial_b M)].
\]

The \(q^{ab}\) is an inverse of the spatial metric \(q_{ab}\). \(D\) is the diffeomorphism constraint and \(S\) is the scalar constraint. The superscript \(Q\) indicates that the scalar constraint \(S\) is quantum corrected, whereas the diffeomorphism constraint \(D\), in the spirit of LQG, holds the classical form. Because the algebra is of the first class (the Poisson bracket of two constraints is proportional to the constraints) the constraints are generators of the gauge transformations as well. The scalar constrain generates deformations of the spatial hypersurface \(\Sigma\) in the time direction while the diffeomorphism constraint generates
deformations of $\Sigma$ in the spatial direction. The deformations are parametrized by the laps functions ($N$) and the shift vectors ($N^a$) respectively. The $\beta$ is a quantum deformation factor, equal to one in the classical theory with Lorentzian signature. In this second meaning, “deformation” denotes that the algebra is modified without breaking of the symmetries. The number of generators is preserved.

The functional form of the factor $\beta$ depends on whether the so-called inverse volume or holonomy quantum corrections are applied. For inverse-volume corrections, $\beta$ depends on spatial metric $q_{ab}$, while holonomy corrections introduce dependence on extrinsic curvature. Interestingly, for both types of corrections, $\beta$ falls to zero within the regime of strong quantum gravitational effects. In consequence, the algebra of constraints becomes reduced to the ultralocal form [8] which prohibits any causal contacts between space points. Because information (waves) cannot propagate spatially, this phase of gravity is also known as asymptotic silence [9]. What is extremely interesting, is that such fancy behavior is expected also in the high-curvature limit of GR. This phenomenon is known as Belinsky-Khalatnikov-Lifshitz (BKL) conjecture [10,11].

Before we interpret what it means let us focus on example of $\frac{1}{3}$-momentum deformation factor, equal to one in the classical theory. Because the loop-deformed Poincaré algebra can be obtained from the foliation deformation algebra by considering linear laps function $N$ and the shift vector $N^a$ [14]:

$$N(x) = \Delta t + v_a x^a, \quad N^a(x) = \Delta x^a + R^a_{\,\,b} x^b,$$

(2)

together with the flat spatial metric $q_{ab} = \delta_{ab}$. Taking this into account, they showed that the resulting deformation of the Poincaré algebra is a special case of a broader class of deformations, with modifications of generators of boosts, studied in ref. [15]. However, as we show here, modification of boosts is not required if $\beta$ depends on generator of time translation only. In that case, the loop-deformed Poincaré algebra can be obtained from the following Heisenberg algebra:

$$[X_\mu, X_\nu] = 0, \quad [P_\mu, P_\nu] = 0, \quad [X_\mu, P_\nu] = ig_{\mu\nu}(P_0),$$

(3)

where $g_{\mu\nu}(P_0) = \text{diag}(\beta(P_0), 1, 1, 1)$. By introducing classical generators of rotations $J_i \equiv \frac{1}{2} \epsilon_{ijk}(X_j P_k - X_k P_j)$ and boosts $K_i \equiv X_i P_0 - X_0 P_i$, we find that the following algebra is fulfilled:

$$[J_i, J_j] = i\epsilon_{ijk} J_k, \quad [J_i, K_j] = i\epsilon_{ijk} K_k,$$
$$[K_i, J_j] = -i\beta(P_0) \epsilon_{ijk} J_k, \quad [J_i, P_j] = i\epsilon_{ijk} P_k,$$
$$[K_i, P_j] = i\delta_{ij} P_0, \quad [K_i, P_0] = i\beta(P_0) P_i,$$
$$[J_i, P_0] = 0, \quad \{P_i, P_j\} = 0, \quad \{P_i, P_0\} = 0.$$  

(4)

This algebra has the same form as the one found in ref. [14]. However, here the boosts are classical and $\beta$ depends on $P_0$ only. The classical Poincaré algebra is recovered for $\beta \to 1$, while for $\beta = 0$ we obtain Carrollian limit [16] describing ultralocal space-time. The Carrollian limit is described by the Carroll Lie algebra, being one of the eight algebras for kinematical groups for which: space is isotropic, parity and time-reversal are automorphisms of the group and the one-dimensional subgroups generated by the boosts $K_i$ are non-compact [17]. Furthermore, for $\beta = -1$, the algebra describes isometries of the 4D Euclidean space. Therefore, we have full compatibility with the conclusions derived at the level of algebra of constraints.

It is worth stressing that, while the loop-deformed Poincaré algebra is recovered from the algebra of constraints in the limit of flat space-time, the metric entering Heisenberg algebra reveals explicit dependence on $P_0$. 

\footnote{The term “deformation”, as mentioned previously, means that the symmetry is modified but not broken— the number of generators of the symmetry is preserved while their physical interpretation may change.}
This can be associated with curvature of momentum space, which suggests relation of our approach with recently introduced concept of *relative locality* [18,19]. An unavoidable consequence of the momentum space curvature is modification of dispersion relation of particles, e.g. photons.

Because the first Casimir invariant for our loop-deformed Poincaré algebra is

\[ C_1 = - \int_0^{P_0} \frac{2y}{\beta(y)} dy + P_i^2, \]

the dispersion relation of photon takes the form \( \int_0^E \frac{2y}{\beta(y)} dy = P_i^2 \), where we defined \( E = P_0 \) and \( P_i^2 = (P_i)^2 \).

For \( \beta = 1 \), the classical relation \( E^2 = P_i^2 \) is correctly recovered. By differentiating the deformed dispersion relation with respect to \( E \) one can show that \( v_{gr}v_{ph} = \beta \), where \( v_{gr} := \frac{dE}{dP_0} \) and \( v_{ph} := \frac{E}{P_0} \) are group and phase velocities respectively. These velocities are no longer constant but vary as a function of energy. Furthermore, under the following assumptions: \( 0 < \beta \leq 1, \beta(0) = 1 \) and \( \frac{d\beta}{dE} \leq 1 \), one can prove that they never exceed the speed of light in vacuum (\( v_{ph} < 1, v_{gr} < 1 \)).

The functional form of \( \beta(P_0) \) can be deduced from the deformation parameter (1). The formal procedure of relating both quantities requires expressing gravitational field variables (in particular \( p \) and \( k \) present in (1)) in terms of asymptotic generators of symmetries. The Brown-York or ADM-type energy and momenta can be used for this purpose [14]. The procedure is, however, ambiguous due to different allowed definitions of the generators of symmetries. Furthermore, asymptotic flatness of space is required in the ADM case. As discussed in the appendix, alternatively one can consider some general form of \( \beta \) and constrain it with use of the Jacobi identities. The method, however, does not allow to recover \( \beta \) in a unique way.

The attempt of deriving explicit form of \( \beta(P_0) \) from (1) requires more extended analysis, which will be addressed in our further studies. Here, instead of this, we consider three exemplary expressions for the \( \beta \) function: \( \beta_1(E) = \cos \left( \frac{\pi}{2} \frac{E}{E_p} \right) \), \( \beta_2(E) = 1 - \frac{E^2}{E_p^2} \) and \( \beta_3(E) = 1 - \frac{E}{E_p} \). The corresponding deformation parameter of the gravitational group velocity, given by the following formulas:

\[
\begin{align*}
v_{gr,1} & = \cos \left( \frac{\pi}{2} \right) \sqrt{\sum_{m=0}^{\infty} \frac{(-1)^m E_{2m}}{(2m)!} \left( \frac{\pi}{2} \right)^{2m} x^{2m}} \\
& = 1 - \frac{3x^2}{32} x^2 + O(x^4), \\
v_{gr,2} & = \left( 1 - \frac{x^2}{x} \right) \sqrt{-\log (1 - x^2)} \\
& = 1 - \frac{3}{4} x^2 + O(x^4), \\
v_{gr,3} & = \frac{1 - x}{x} \sqrt{-2(\log (1 - x) + 1)} \\
& = 1 - \frac{2}{3} x + O(x^2),
\end{align*}
\]

where \( x := E/E_p \), are shown in fig. 1. Furthermore, the \( E_n \) are the Euler numbers. Because functions \( \beta_1 \) and \( \beta_2 \) are even, the models 1 and 2 lead to very similar predictions. In these two cases the leading term in the expression for \( v_{gr} \) is quadratic in \( E/E_p \). The case 3, while qualitatively similar to the 1st and 2nd, leads to linear leading term in the expression for \( v_{gr} \). This difference significantly improves possibility of constraining the model 3 observationally. However, from the theoretical viewpoint the case 3 is less likely. Namely, for the general expression \( \beta = \cos f(E) \) with polynomial function \( f(E) \) one can apply the models \( \beta_1 \) and \( \beta_2 \). A linear term in the development of \( \beta \), as present in the model \( \beta_3 \), would require non-polynomial \( f(E) \), for example \( f(E) \sim \sqrt{E} \).

The free parameter \( E_p \) can be constrained by studying time lags of high-energy photons arriving from the distant astrophysical sources, such as gamma ray bursts (GRB) [20]. For the considered models, the time lags are expressed as \( \tau_1 \simeq \frac{2L}{c} \left( \frac{E}{E_p} \right)^2, \tau_2 \simeq \frac{2L}{c} \left( \frac{E}{E_p} \right)^2 \) and \( \tau_3 \simeq \frac{L}{2} \left( \frac{E}{E_p} \right) \), where \( L \) is a distance to source. Using constraint from the GRB 090510 detected by the Fermi satellite [21] we find that: 1) \( E_p > 4.7 \cdot 10^{10} \text{GeV} \) 2) \( E_p > 5.2 \cdot 10^{10} \text{GeV} \) 3) \( E_p > 5.1 \cdot E_{Pl} \). The constraints 1 and 2 are weak relatively to the Planck energy \( E_{Pl} = 1.22 \cdot 10^{19} \text{GeV} \) because quantum corrections are quadratic in energy, as suggested by the cosine form of \( \beta \). In turn, the constraint 3 obtained for \( \beta_3 \) (being linear in \( E \)) is strong enough to disfavor the model for \( E_p \sim E_{Pl} \) (which is generally expected). However, this case gains less theoretical support than the other two ones. Nevertheless, a new possibility of testing loop quantum gravity emerges. What is especially worth stressing is that both, dynamics of very early universe and propagation of astrophysical particles are affected by the same deformation \( \beta \). This provides a unique possibility of constraining the same physics by using completely
different observations. This might open a new stage in cosmic search for the quantum gravity effects.

APPENDIX

In this appendix we check whether the loop-deformed Poincaré algebra (4) is a Lie algebra of generators of the symmetries. This will be verified by inspection of the Jacobi identities between all elementary operators forming the algebra. We show that, in the case considered in this letter, the Jacobi identities are fulfilled if the explicit form $J_i \equiv \frac{1}{2} \epsilon_{ijk} (X_j P_k - X_k P_j)$ of the generators of rotations is used. However, by introducing certain dependence of the deformation factor $\beta$ on $P_i^2$, the Jacobi identities can be fulfilled identically.

Inspection of the Jacobi identities reveals that the only non-trivial expression is

$$[K_k, [K_i, K_j]] + [K_j, [K_k, K_i]] + [K_i, [K_j, K_k]] = \beta(P_0) \beta'(P_0) (P_i J_i + P_j J_j + P_k J_k),$$  \hspace{1cm} (A.1)

where relation $[K_i, J(P_0)] = i \beta(P_0) f'(P_0) P_i$, valid for any differentiable function $f$, has been used. The right-hand side of (A.1) is vanishing in the classical limit ($\beta = 1$). However, in general situation, the Jacobi identity is satisfied only if $\vec{P} \cdot \vec{J} = 0$. This condition cannot be fulfilled without referring to explicit form of the generators of symmetries. One possibility is employing the standard spineless generators $\vec{J} = \vec{X} \times \vec{P}$, together with the commutation rules between $X_i$ and $P_j$ given by (3), for which the condition $\vec{P} \cdot \vec{J} = 0$ is satisfied by virtue of the vector identity $\vec{P} \cdot (\vec{X} \times \vec{P}) = 0$. However, in general $\vec{J} = \vec{L} + \vec{S}$, where $\vec{L} := \vec{X} \times \vec{P}$ is the angular momentum part and $\vec{S}$ is the spin part. While $\vec{P} \cdot \vec{L} = 0$ vanishes identically, the contribution $\vec{P} \cdot \vec{S}$ (the helicity operator) is non-vanishing for particles with non-zero spin. Therefore, for $\beta = \beta(P_0)$, the loop-deformed Poincaré algebra accordingly describes space-time symmetries but is not well suited for the finite-dimensional representations (describing transformations of non-scalar fields).

The above drawback can be overcome by promoting $\beta$ to a multivariable function. It fact, it was assumed for the sake of simplicity that $\beta$ is a function of $P_0$ only. In general, one could consider deformation function in the form $\beta = \cos(G)$, where $G$ is some function of the generators of the symmetries. In what follows we show that it is possible to fulfill the Jacobi identities by introducing dependence of the $\beta$ function on $P_i^2$. In order to distinguish this case with the previous one let us rename $\beta(P_0)$ in the loop-deformed Poincaré algebra (4) by $F(P_0, P_i P_i')$. While the functional form of the deformation function changes, the deformed brackets remain the same:

$$[K_i, K_j] = -i F \epsilon_{ijk} J_k,$$  \hspace{1cm} (A.2)

$$[K_i, P_0] = i F P_i.$$  \hspace{1cm} (A.3)

The classical form of the remaining brackets is preserved.

In what follows we will derive exemplary form of the deformation function $F$, for which the Jacobi identities are satisfied. The easiest way to make the deformed algebra a Lie algebra is to choose the function $F$ such that it commutes with all generators. Furthermore, we assume that the dependence on $P_0$ and $P_i^2$ is separable, i.e. one can write $F(P_0, P_i^2) = A(P_0) B(P_i^2)$. Vanishing of the commutators $[F, P_0]$ and $[F, P_i^2]$ is a direct consequence of the brackets $[P_0, P_0] = 0$ and $[P_i^2, P_0] = 0$. In turn, the commutation $[F, J_i] = 0$ is a consequence of $[J_i, P_0] = 0$ and

$$[J_i, P_i^2] = P_k [J_i, P_k] + [J_i, P_k] P_k = 2 i \epsilon_{ijk} P_k P_l = 0.$$  \hspace{1cm} (A.4)

So far, the brackets are vanishing independently on the form of $F$. This is no more the case while considering the $[K_i, F]$ bracket, which can be expressed as

$$[K_i, F] = i P_i \left( 2 A P_0 \frac{dB}{dP_i^2} + AB^2 \frac{dA}{dP_0} \right).$$  \hspace{1cm} (A.5)

Vanishing of the right-hand side of (A.5) leads to a differential equation on $F$, which by requiring $F(0,0) = 1$, has solution

$$F = \frac{P_0^2 - \alpha}{P_i^2 - \alpha}.$$  \hspace{1cm} (A.6)

where $\alpha$ is a constant of integration, which can be interpreted as a deformation parameter. The classical expression is recovered by taking $\alpha \rightarrow \infty$. The Carrollian limit ($F \rightarrow 0$) is approached either for $P_0 \rightarrow \sqrt{\beta}$ if $\alpha > 0$ or for $P_i^2 \rightarrow \infty$ if $\alpha < 0$.

Because $F$ given by (A.6) is commuting with all generators of the Poincaré algebra $[F, P_0] = 0, [F, P_i^2] = 0, [F, J_i] = 0, [F, K_i] = 0$ it is a Casimir invariant. By an appropriate rescaling, the Casimir operator being in agreement with the classical mass operator in the $\alpha \rightarrow \infty$ limit can be defined:

$$C_1 = \alpha (F - 1) = \frac{-P_0^2 + P_i^2}{1 - \frac{\alpha}{\beta}}.$$  \hspace{1cm} (A.7)

For the massive representation ($C_1 = -m^2$) it leads to the following dispersion relation:

$$P_0^2 = m^2 + P_i^2 \left( 1 - \frac{m^2}{\alpha} \right).$$  \hspace{1cm} (A.8)

It is worth noticing that, for the massless particles, the dispersion relation seems to take a classical form while for the massive particles momenta it is rescaled by a constant factor. This is, however, not in contradiction with the conclusion of the letter. The example presented here was based on the assumption of separability of $P_0$ and $P_i^2$, which is not supported by the original loop deformations of the hypersurface deformation algebra (1). Moreover, while considering the $F$-deformations we assumed that $[K_i, F] = 0$. This condition can be satisfied only if $F$ depends non-trivially on both $P_0$ and $P_i^2$. For e.g. $A(P_0) = \beta(P_0)$ and $B(P_i^2) = 1$ the condition $[K_i, F] = 0$ cannot be satisfied for non-trivial $\beta(P_0)$ and one gets (A.1). Furthermore,
the dispersion of photon (as well as massive particles) is dependent on the definition of the d’Alembert operator in the momentum space, which in general can be written as

\[ M^2(p) = C_1 + \sum_{n=2} c_n \alpha \left( \frac{C_1}{\alpha} \right)^n, \tag{A.9} \]

where \( c_n \) are some dimensionless constants.

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