Efficiency Analysis of Concurrently Driven Power Amplifiers

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ABSTRACT In this work, the properties of a two-tone signal driven concurrent power amplifier (CPA) are analyzed extensively. Firstly, the characteristics of two-tone signals are discussed to explain the nonlinearity and efficiency of two-tone signal driven CPA. Secondly, a method of how to solve the efficiency of a two-tone signal driven CPA is proposed in this paper with a detailed calculation procedure and complete theory. Finally, a general empirical expression to calculate the drain efficiency (DE) of CPA versus the frequency ratio and the amplitude ratio of the two carriers is proposed. Simulation results and experimental verifications are given to validate the proposed analytical formulation to predict the efficiency of CPAs when driven with balanced or imbalanced two-tone signals.

INDEX TERMS Concurrent, dual-band, efficiency, elliptic Integral, Fourier Series, imbalance, power amplifiers, two-tone.

I. INTRODUCTION

The rapid evolution of wireless communication systems (WCS), especially the ongoing 5G, along with the Industrial 4.0 [1], [2], leads to a variety of communication standards and additional frequency bands. With the application of technologies like Carrier Aggregation (CA), Software Defined Radio (SDR) and Multiple Input Multiple Output (MIMO), these standards and bands generally result in increasing requirements for radio frequency (RF) transmitters operating in multi-band or multi-standard mode [2]–[5]. As the most challenging and expensive component in the RF front-end of the transmitter, RF power amplifiers (PAs) are expected to handle diverse signals at different frequencies while meeting the requirements of signal quality and power efficiency [6]. The use of a dedicated PA for each frequency is relatively straightforward to meet the above-mentioned requirements, however, it is not cost-effective from the network’s capital and operating expenses perspective [2], [7]. Hence, a transmitter capable of operating at multiple frequencies concurrently is urgently needed [8].

To satisfy the requirements of multi-band or multi-standard mode RF transmitters, PAs should be able to operate concurrently within different frequencies, while maintaining competitive performance. These requirements necessitate an important migration from dedicated single frequency PAs to the concurrent mode PAs.

CPA is defined as to be capable of simultaneous operation at two different frequencies at least. In recent years, the CPA has attracted more and more scholars, and many approaches have been reported regarding the study of related CPA technologies. Examples of these studies are the traditional CPA which operates at two fixed frequencies [3], [7], [9], re-configurable CPA, which could work at several different concurrent combinations [10], [11] or applications for other amplifier’s structures like concurrent...
The remainder of this paper is organized as follows: in Section II, the definition of some signal properties needed for the analytical development, such as the periodicity of the signal, the orthogonality characteristics of the two-tone signal, the nonlinearity, and the power distribution of a two-tone signal of a driven CPA are given. Theory and mathematical derivation methods to analyze the efficiency of CPA are derived in Section III; the impact of the amplitude imbalance and frequency ratio on the power efficiency of CPA are discussed in this section. In Section IV, simulation and experimental verifications are provided to verify the proposed CPA theory to predict accurately their power efficiency. Finally, a concise conclusion is drawn in Section V.

II. THE BASIC CHARACTERISTICS OF TWO-TONE SIGNALS AND TWO-TONE SIGNAL DRIVEN CPA

When a PA is driven concurrently, the PA is considered to be operating concurrently at two different frequencies. It is necessary to briefly report the basic characteristics of the two-tone signals before modeling the properties of two-tone signal driven CPA.

The equivalent schematic circuit of a PA is shown in Fig. 1. For simplicity of the analysis and without loss of generality, it is assumed that all components are ideally lossless and the load is fixed and equal to $Z_L$. In addition, all the harmonics are shorted by the circuit designated as the tuner in Fig. 1. The gate of the PA is biased at $V_{GS}$. The drain of the PA is biased at a drain supply voltage of $V_{DC}$ with a drain supply current $I_{DC}$. When driven by the input signal $v_{gs}$, the transistor can be regarded as an input-voltage-controlled current source, the waveform of drain-to-source voltage $v_{ds}$ and drain-to-source current $i_{ds}$ are the key to evaluate the performance of PA. $Z_{opt}$ is the optimum output impedance that the transistor must be loaded with such that it can deliver its maximum RF power and OMN is the output matching network.

![Equivalent schematic circuit of power amplifier.](image)

The analysis in this manuscript is based on the following assumptions: all harmonics and inter-modulation products are ideally shorted through the tuner, and the power gain of the transistor is ideally absolutely flat with frequency.

A. THE COMMON EXPRESSION OF A TWO-TONE SIGNAL

A two-tone signal with the same initial phase could be expressed as follows [20]:

$$v_{gs}(t) = A\cos\omega_1 t + \cos\omega_2 t$$

where $A$ is the amplitude ratio of the two carriers. If $A \neq 1$, $v_{gs}$ is considered as an imbalanced two-tone signal with two unequal amplitude carriers. When $A = 1$, $v_{gs}$ is considered as a balanced two-tone signal with two equal amplitude carriers, which means the balance is only one of the special cases for the imbalance case. When $A \to 0$ or $A \to \infty$, $v_{gs}(t)$ is actually equivalent to the single-tone signal.

low noise amplifier [8], [12], concurrent Doherty amplifiers [13], uneven concurrent Doherty amplifiers [7], continuous Class-I mode concurrent Doherty [3] dual-band class-E CPA using right/left transmission lines [14], harmonic tune CPAs [5], [15], and Class-F CPA using load coupling networks [16]. These studies are of great values, and stimulated the development of CPA related technologies, like new concurrent structures [10] and digital pre-distortion technologies for CPAs [17], [18], multi-mode PAs [4], [11], [12] and CA technologies [19].

However, most reported works about CPA technologies, are mainly aimed at the new amplifier’s architectures for CPAs, configurable solutions to realize the dual-band matching networks or concurrent applications for different PA structures. The driven concurrent signals in most reported works are usually regarded as “balanced” with equal amplitude for each carrier. The frequency ratio of the two concurrent carriers is also often neglected or assumed small. The complexity of expressions and the tediousness of computation makes it challenging for many scholars to study the basic operating theories of a CPA driven by balanced two-tone signals. The analytical development becomes even more challenging when considering more factors such as the frequency ratio of the two concurrent carriers and the amplitude imbalance between the two-tone driving signal.

Although it is relatively difficult to give complete closed-form analytic solutions in terms of operating parameters to predict the performance of CPAs like in the case of single tone driven PA, this paper presents an attempt to propose a general analysis method to evaluate the Class-A and Class-B modes CPA when driven by two-tone concurrent signal with arbitrary frequency and amplitude ratios.

The main contributions of this work are the theoretical derivations leading to closed-form expressions of the power efficiency of amplifiers when driven with a two-tone signal reported in Section III. The second contribution is the analysis, along with simulation and experimental validation, of the power efficiency when an imbalanced two-tone signal is driving the power amplifier are reported in Section IV.

The remainder of this paper is organized as follows: in Section II, the definition of some signal properties needed for the analytical development, such as the periodicity of the signal, the orthogonality characteristics of the two-tone signal, the nonlinearity, and the power distribution of a two-tone signal of a driven CPA are given. Theory and mathematical derivation methods to analyze the efficiency of Class-B CPAs are derived in Section III; the impact of the amplitude imbalance and frequency ratio on the power efficiency of CPA are discussed in this section. In Section IV, simulation and experimental verifications are provided to verify the proposed CPA theory to predict accurately their power efficiency. Finally, a concise conclusion is drawn in Section V.
B. VARIABLE ENVELOPE CHARACTERS OF THE TWO-TONE SIGNAL

The two-tone signal $v_{gs}$ can be regarded as a variable envelope signal with the peak envelope power (PEP) of:

$$P_{p_{gs}} = \frac{(1 + A)^2}{2Z_L} \quad (2)$$

While the mean power of $v_{gs}$ can be easily deduced by sum two individual fundamental carrier’s mean powers up, hence, in this case the overall mean power $P_{m_{gs}}$ can be reached as:

$$P_{m_{gs}} = \frac{1 + A^2}{2Z_L} \quad (3)$$

Therefore the Peak-to-Average Power Ratio (PAPR) of $v_{gs}$ can be derived from equation (2) and (3), and is given as:

$$PAPR_{v_{gs}} = \frac{P_{p_{gs}}}{P_{m_{gs}}} = \frac{(1 + A)^2}{1 + A^2} \quad (4)$$

From equation (4), obviously, when $A = 1$, the balanced two-tone signal can be regarded as a variable envelope signal with $3db$ PAPR.

C. THE PERIOD CHARACTERS OF TWO-TONE SIGNALS

Assuming herein the CPA is driven by the two-tone signal with two carriers having two different frequencies: $w_1 (= 2\pi/T_1)$ and $w_2 (= 2\pi/T_2)$, related as follows:

$$k = \frac{w_1}{w_2} = \frac{T_2}{T_1} \quad (5)$$

where $k > 0$ and $k \neq 1$.

Mathematically, the ratio of the periods of two periodic signals needs to be a rational number to make sure that the composite signal of these two periodic signals to be periodical. Therefore, $v_{gs}$ can be regarded as a periodic signal only when $k$ is a rational number [21]. In practice, different sinewave signals generated from the same signal generator generally have a rational ratio to each other, since the signal generator includes a reference crystal oscillator from which signals with different frequencies are usually generated using frequency multipliers and/or phase-locked loop (PLL) architectures. Therefore, multi-carrier signals generated within the same signal generators are inherently periodical. The discussion in this paper is carried out under the condition that $k$ is a rational number.

The period of the composite signal $v_{gs}(t)$, $T$ is indeed the generalized least common multiple (GLCM) of $T_1$ and $T_2$:

$$T = \frac{2\pi}{T_1} = \frac{M \cdot 2\pi}{w_1} = \frac{N \cdot 2\pi}{w_2} = GLCM[T_1, T_2]$$

$$= GLCM[T_1, kT_2] = GLCM[T_2, \frac{T_2}{k}]$$

$$= T_1GLCM[1, k] = T_2GLCM[\frac{1}{k}, 1] \quad (6)$$

where $M$ and $N$ are positive integer numbers. $GLCM[x, y]$ means the smallest positive number that is divisible by both $x$ and $y$.

D. WAVEFORM CHARACTERS OF THE TWO-TONE SIGNAL

The two-tone signal in (1) can be transformed and written as follows:

$$v_{gs} = (1 + A)\cos(w_m t)\cos(w_e t) + (1 - A)\sin(w_m t)\sin(w_e t) \quad (7)$$

where, $w_m = \frac{w_1 + w_2}{2}$; $w_e = \frac{w_1 - w_2}{2}$.

The waveforms of $(A + 1)\cos(w_m t)\cos(w_e t)$, $(1 - A)\sin(w_m t)\sin(w_e t)$ and $v_{gs}$ are drawn in Fig. 2. Apparently, $v_{gs}$ (Fig. 2.c) can be split into the sum of two signals, the even signal in (Fig. 2.a) and odd signal in (Fig. 2.b), respectively.

E. TRIGONOMETRIC SERIES (Fourier Series) EXPRESSIONS

For a CPA, due to the non-linearity of the transistor, intermodulation products, and the harmonic components will be generated at the drain reference plane. For the ideal PA, as is shown in Fig. 1, the drain-to-source current $i_{ds}$ and drain-to-source voltage $v_{ds}$ in the time domain can be described using
Trigonometric Series or Fourier Series as:

\[ i_{ds}(t) = \sum_{m,n=0}^{\infty} I_{m,n} \cos(mw_1 \pm nw_2)t \]

\[ v_{ds}(t) = \sum_{m,n=0}^{\infty} V_{m,n} \cos(mw_1 \pm nw_2)t \]  \hspace{1cm} (10)

\[ I_{m,n} \text{ and } V_{m,n} \text{ are the drain current and voltage coefficients of the } (m, n)\text{th order products (including harmonics and inter-modulation products). Herein, it is assumed that } I_{DC} \text{ and } V_{DC} \text{ are the direct components in the drain current and voltage.} \]

**F. THE POWER DISTRIBUTION AT THE DRAIN OF A CPA**

The corresponding \((m, n)\text{th}\) order impedance seen by two-tone signal driven CPA can be derived from (10):

\[ Z_{Lm,n} = \frac{V_{m,n}}{I_{m,n}} \] \hspace{1cm} (11)

The DC supply power can be expressed as:

\[ P_{DC} = V_{DC}I_{DC} \] \hspace{1cm} (12)

Then the whole power consumption of the PA, dissipated by the PA as heat, can be expressed as:

\[ P_{diss} = \frac{1}{T} \int_{0}^{T} v_{ds}(t)i_{ds}(t)dt \]

\[ = V_{DC}I_{DC} - \frac{1}{2} \sum_{m,n=0}^{\infty} V_{m,n}I_{m,n} \] \hspace{1cm} (13)

where \((m + n) > 0\).

The power of \((m, n)\text{th}\) order products can be written as:

\[ P_{out,m,n} = \frac{1}{2} V_{m,n}I_{m,n} \] \hspace{1cm} (14)

The output power of fundamental carriers:

\[ P_{out,f} = P_{out,1,0} + P_{out,0,1} = P_{w_1} + P_{w_2} \] \hspace{1cm} (15)

According to the law of conservation of energy:

\[ P_{DC} = P_{diss} + P_{out,f} + \sum_{m,n=0}^{\infty} P_{out,m,n} \] \hspace{1cm} (16)

where, \((m + n) > 1\).

**G. CONDITIONS OF HIGH-EFFICIENCY CPA**

Only the power converted to the fundamental carriers are required to calculate the drain efficiency (DE), therefore, the DE of the CPA can be expressed as:

\[ DE = \frac{P_{out,f}}{I_{DC}V_{DC}} = \frac{P_{out,f}}{P_{diss} + P_{out,f} + \sum_{m,n=0}^{\infty} P_{out,m,n}} \] \hspace{1cm} (17)

where \((m + n) > 1\).

According to equation (17), to achieve 100% DE, the following condition must be satisfied:

\[ P_{diss} + \sum_{m,n=0}^{\infty} P_{out,m,n} = 0 \] \hspace{1cm} (18)

where \((m + n) > 1\).

Equation (18) leads to:

\[ \frac{1}{T} \int_{0}^{T} v_{ds}(t)I_{ds}(t)dt = 0 \] \hspace{1cm} (19)

and

\[ \sum_{m,n=0; m+n>1}^{\infty} P_{out,m,n} = 0 \] \hspace{1cm} (20)

Equation (19) means there is no overlap of drain voltage \(v_{ds}\) and drain current \(i_{ds}\). Equation (20) means there is no power consumption of all harmonics and inter-modulation products.

**III. THE EFFICIENCY ANALYSIS OF CPAs**

Complete theories about the efficiency of single-tone driven PA have already be given and discussed by many scholars [20]. However, when it comes to CPA, especially the imbalanced driven CPA, to the authors’ best knowledge, there is almost no publicly reported study about this matter.

In this section, the proposed method with a detailed calculation process to evaluate the performance of a CPA will be discussed completely.

**A. THE ANALYSIS OF CPA BIASED AT CLASS-A**

For a class-A mode CPA, the relationship between drain current \(i_{DS}\), drain voltage \(v_{DS}\) and input concurrent two-tone signal \(v_{gs}\) are shown in Fig. 3. The ideal Class-A PA is regarded as linear PA, there are no harmonics or inter-modulation products at the output.
The drain current $i_{DS}$ and drain voltage $v_{DS}$ can be written as:

$$i_{DS}(t) = \frac{I_{\text{max}}}{2} + \frac{I_{\text{max}}}{2(A+1)}(A\cos w_1t + \cos w_2t)$$

$$v_{DS}(t) = V_{DC}(1 - \frac{1}{A+1}(A\cos w_1t + \cos w_2t))$$ (21)

Due to the linear property of a Class-A PA, any other products except for the DC and fundamental components do not exist. The value of fundamental carriers and DC component can be easily obtained:

$$I_{1,0} = \frac{A I_{\text{max}}}{2(A+1)}; \quad I_{0,1} = \frac{I_{\text{max}}}{2(A+1)}$$

$$V_{1,0} = \frac{A V_{DC}}{(A+1)}; \quad V_{0,1} = \frac{V_{DC}}{(A+1)}$$

$$I_{0,0} = \frac{I_{\text{max}}}{2}; \quad V_{0,0} = V_{DC}$$ (22)

Then the DE of CPA biased at Class-A mode can be easily deduced:

$$DE = \frac{P_f}{P_{DC}} = \frac{1}{2} \frac{I_{\text{max}}}{V_{DC}}((\frac{A}{A+1})^2 + (\frac{1}{A+1})^2)$$

$$= \frac{A^2 + 1}{2(A+1)^2}$$ (23)

Obviously, the DE of Class-A CPA is function only the amplitude ratio $A$ and is independent of frequency ratio $k$. The DE curve of Class-A CPA versus $20\log A$ is drawn in Fig. 4. The minimum value of DE is achieved at $A = 1$. The DE values when $A \to 0$ and $A \to \infty$ are approaching with the performance of single-tone signal driven PA.

![FIGURE 4. The DE of Class-A CPA versus $20\log A$.](image)

### B. THE ANALYSIS OF CPA BIASED AT CLASS-B

According to the definition of DE in equation (17), the fundamental components of the two-carriers and DC components at the drain are needed to calculate the DE of CPA.

Considering the CPA is biased at Class-B, and assuming the relationship between drain current $i_{DS}$, drain voltage $v_{DS}$ and input concurrent two-tone signal $v_{gs}$ are shown in Fig. 5.

The current and voltage at the drain of the Class-B CPA can be written as:

$$v_{DS,B} = V_{DC}(1 - \frac{1}{A+1}(A\cos w_1t + \cos w_2t))$$

$$i_{DS,B} = \begin{cases} \frac{I_{\text{max}}}{A+1}(A\cos w_1t + \cos w_2t), & v_{gs}(t) \geq 0 \\ 0, & v_{gs}(t) < 0 \end{cases}$$ (24)

Obviously, $i_{DS,B}$ and $v_{DS,B}$ have the same period ($T$) as $v_{gs}$. With arbitrary values of $A$, $w_1$ and $w_2$, it is relatively difficult to obtain a close-form expression valid for every harmonic and inter-modulation product. In this paper, a novel method is first proposed to calculate the values of the direct voltage and current components and the fundamental carriers in the drain current of Class-B CPA.

One can notice that the waveform of the drain current $i_{DS}(t)$ in Fig. 5 give by equation (24) is an exact copy of the upper and positive half of the waveforms of the Class-A case shown in Fig. 3. Furthermore, the AC components of the drain current of Class A given in (21) can be decomposed into two parts: the positive part $i_{\text{dp}}(t) = (A\cos w_1t + \cos w_2t) |_{v_{gs}(t) \geq 0}$ above zero (similar to the waveform with solid line of the $i_{DS}$ in Fig. 5) and the negative part $i_{\text{dn}}(t) = (A\cos w_1t + \cos w_2t) |_{v_{gs}(t) \leq 0}$ below zero (similar to the waveform with dotted line of the $i_{DS}$ in Fig. 5). These current’s components can be represented by the following expressions:

$$i_{\text{dp}}(t) = \begin{cases} \frac{I_{\text{max}}}{A+1}(A\cos w_1t + \cos w_2t), & v_{gs}(t) \geq 0 \\ 0, & v_{gs}(t) < 0 \end{cases}$$ (25)
From equation (25), one can deduce that, \( i_{ds,B} \) given by (24) is the same as \( i_{dsp} \), and:

\[
i_{dsp}(t) + i_{dsn}(t) = \frac{I_{\text{max}}}{A + 1}(\cos w_1 t + \cos w_2 t) \tag{26}
\]

The right-hand side of equation (26) does not include the direct, harmonic and inter-modulation products. This means the corresponding direct, harmonic and inter-modulation products in \( i_{dsp}(t) \) and \( i_{dsn}(t) \) have the same in amplitude but opposite phases. The sum of fundamental components is also equal to the value in the right-hand side of equation (26).

Similar to equation (10), \( i_{dsp}(t) \) and \( i_{dsn}(t) \) can also be written in the form of the sum of all harmonics and inter-modulation products (the Trigonometric Series) as:

\[
i_{dsp}(t) = \sum_{m,n=0}^{\infty} I_{P(m,n)} \cos(mw_1 \pm nw_2) t
\]

\[
i_{dsn}(t) = \sum_{m,n=0}^{\infty} I_{N(m,n)} \cos(mw_1 \pm nw_2) t \tag{27}
\]

In fact, the fundamental components in \( i_{dsp}(t) \) are not only \( I_{P(0,1)} \) or \( I_{P(1,0)} \), but also should include the inter-modulation products \( I_{P(m,n)} \cos(mw_1 \pm nw_2) t \) having exactly the same frequencies as the carriers \( mw_1 \pm nw_2 = \pm w_1, \pm w_2 \). Similarly, the DC component in \( i_{dsp}(t) \) is also not only \( I_{P(0,0)} \), but also include the inter-modulation products \( I_{P(m,n)} \cos(mw_1 \pm nw_2) t \) that generated DC components \( mw_1 \pm nw_2 = 0 \).

By combining equations (26) and (27), the DC component \( I_{PDC} \) and the fundamental components \( I_{Pw_1} \) and \( I_{Pw_2} \) of the current waveform \( i_{dsp}(t) \), and the DC component \( I_{NDC} \), the fundamental components \( I_{Nw_1} \) and \( I_{Nw_2} \) of the current waveform \( i_{dsn}(t) \) should satisfy:

\[
I_{Pw_1} + I_{Nw_1} = \frac{A I_{\text{max}}}{A + 1}; \quad I_{Pw_2} + I_{Nw_2} = \frac{I_{\text{max}}}{A + 1}
\]

\[
I_{Pnm} + I_{Nnm} = 0; \quad I_{PDC} + I_{NDC} = 0 \tag{28}
\]

where \( I_{Pnm} \) and \( I_{Nnm} \) are the currents associated with the \( m \)-th and \( n \)-th order of the inter-modulation products and \( mw_1 \pm nw_2 \neq \pm w_1, \pm w_2, 0 \).

From the definition, \( i_{dsp}(t) \) is the positive part of the current waveform and \( i_{dsn}(t) \) is the negative part of the current waveform, therefore one can write:

\[
i_{dsp}(t) - i_{dsn}(t) = \left| I_{\text{max}} \right| \frac{A + 1}{A}(\cos w_1 t + \cos w_2 t) \tag{29}
\]

It is difficult to derive analytical expressions for the DC, fundamental and distortion components in the two-tone driven class B represented by (24). Therefore, equations (26) and (29) will be used concurrently to extract the expression of these components. This will be explained in the following sections.

**C. THE ANALYSIS OF BALANCED CLASS-B CPA**

Considering the case of a balanced CPA case where \( A = 1 \), the amplitudes of the driven two carriers are equal. Then, the drain current and voltage can be written as:

\[
v_{ds,B} = V_{DC}(1 - \frac{1}{2}(\cos w_1 t + \cos w_2 t))
\]

\[
i_{ds,B} = \begin{cases} \frac{I_{\text{max}}}{2}(\cos w_1 t + \cos w_2 t), & v_{gs}(t) \geq 0 \\ 0, & v_{gs}(t) < 0 \end{cases} \tag{30}
\]

According to the definition, the overall DC component \( I_{PDC} \) and fundamental components \( I_{Pw_1} \) and \( I_{Pw_2} \) in \( i_{ds,B} \) can be written as:

\[
I_{Pw_1} = \frac{2}{T} \int_0^T i_{dsp}(t) \cos w_1 t dt
\]

\[
I_{Pw_2} = \frac{2}{T} \int_0^T i_{dsp}(t) \cos w_2 t dt
\]

\[
I_{PDC} = \frac{1}{T} \int_0^T i_{dsp}(t) dt \tag{31}
\]

The overall DC component \( I_{NDC} \) and fundamental components \( I_{Nw_1} \) and \( I_{Nw_2} \) in \( i_{dsn}(t) \) can be written as:

\[
I_{Nw_1} = \frac{2}{T} \int_0^T i_{dsn}(t) \cos w_1 t dt
\]

\[
I_{Nw_2} = \frac{2}{T} \int_0^T i_{dsn}(t) \cos w_2 t dt
\]

\[
I_{NDC} = \frac{1}{T} \int_0^T i_{dsn}(t) dt \tag{32}
\]

According to the analysis above in equation (28):

\[
I_{Pw_1} + I_{Nw_1} = \frac{I_{\text{max}}}{A}
\]

\[
I_{Pw_2} + I_{Nw_2} = \frac{I_{\text{max}}}{A} \tag{33}
\]

According to equations (7) and (29), when \( A = 1 \):

\[
I_{Pw_1} - I_{Nw_1} = \frac{2}{T} \int_0^T (i_{dsp}(t) - i_{dsn}(t)) \cos w_1 t dt
\]

\[
= \frac{2}{T} \int_0^T \left| I_{\text{max}} \right| (\cos w_1 t + \cos w_2 t) \cos w_1 t dt
\]

\[
= \frac{2I_{\text{max}}}{T} \int_0^T \cos w_1 t |\cos w_2 t| \cos w_1 t dt \tag{34}
\]

It is well known that the Fourier Series of \( \cos x \) can be written as:

\[
|\cos x| = \frac{2}{\pi} \sum_{m \in \mathbb{Z}} (-1)^m \frac{1}{4m^2 - 1} \cos(2mx) \tag{35}
\]

By defining:

\[
S(x) = \frac{(-1)^m}{1 - 4m^2} \cos(2mx) \tag{36}
\]

It is also known that, the value of the integral of \( |\cos x|^2 \) over a period is:

\[
\frac{1}{\pi} \int_0^\pi (\cos x)^2 dt = \frac{1}{2} \tag{37}
\]
Then $I_{p_{w_1}} - I_{N_{w_1}}$ can be further derived (See Appendix A, equation (56)):

$$I_{p_{w_1}} - I_{N_{w_1}} = \frac{2I_{\text{max}}}{\pi^2} \sum_{p,q \in U_{w_1}} \frac{(-1)^{p+q}}{(1 - 4p^2)(1 - 4q^2)}$$

(38)

where, $U_{w_1}$ is sub-space of integer numbers $p$ and $q$, that satisfy the following constraint:

$$p, q \in Z, \quad 2p_{m} \pm 2q_{w} = \pm w_{1}$$

$$\Leftrightarrow q \in Z, \quad p = \frac{\pm q(k - 1) \pm k}{k + 1} \in Z$$

(39)

The expression of $I_{p_{w_2}} - I_{N_{w_2}}$ can also be deduced in the same way:

$$I_{p_{w_2}} - I_{N_{w_2}} = \frac{2I_{\text{max}}}{T} \int_{0}^{T} |\cosw_{mt}| |\cosw_{2t}| \cosw_{2t} dt$$

$$= \frac{2I_{\text{max}}}{\pi^2} \sum_{p,q \in U_{w_2}} \frac{(-1)^{p+q}}{(1 - 4p^2)(1 - 4q^2)}$$

(40)

where, $U_{w_2}$ is sub-space of integer numbers $p$ and $q$, that satisfy the following constraint:

$$q \in Z, \quad p = \frac{\pm q(k - 1) \pm 1}{k + 1} \in Z$$

(41)

Combining equations (33), (38) and (40), $I_{p_{w_1}}$ and $I_{p_{w_2}}$ can be obtained as:

$$I_{p_{w_1}} = I_{\text{max}} \frac{\pi}{4} + I_{\text{max}} \sum_{p,q \in U_{w_1}} \frac{(-1)^{p+q}}{(1 - 4p^2)(1 - 4q^2)}$$

$$I_{p_{w_2}} = I_{\text{max}} \frac{\pi}{4} + I_{\text{max}} \sum_{p,q \in U_{w_2}} \frac{(-1)^{p+q}}{(1 - 4p^2)(1 - 4q^2)}$$

(42)

Then DC components in $i_{dp}(t)$ can be expressed as (See Appendix B, equation (57)):

$$I_{p_{DC}} = \frac{I_{\text{max}}}{\pi^2} \sum_{p,q \in U_{DC}} \frac{(-1)^{p+q}}{(1 - 4p^2)(1 - 4q^2)}$$

(43)

where, $U_{DC}$ is sub-space of integer numbers $p$ and $q$, that satisfy the following constraint:

$$q \in Z, \quad p = \frac{k + 1}{k - 1} \in Z$$

(44)

Finally, the DE of balanced CPA can be obtained as:

$$DE = \frac{\frac{1}{2}(I_{p_{w_1}} + I_{p_{w_2}}) V_{DC}}{I_{p_{DC}} V_{DC}} = \frac{I_{p_{w_1}} + I_{p_{w_2}}}{4I_{p_{DC}}}$$

(45)

Variation of DC, fundamental components and the DE of the class-B balanced CPA versus the frequency ratio of the two carriers $k$ are shown in Fig. 6. The values in Fig. 6 are obtained using the analytical formulation developed above, using the step of $k = 0.001$ and the maximum iteration order of $p, q = 10^6$.

Apparently, the calculated results of each curve in Fig. 6 has an asymptotic limit value, except for some special $k$ values. Both the fundamental components of $w_1$ and $w_2$ in $i_{dp}(t)$ are tending to be 0.25, the DC component in $i_{dp}(t)$ is approaching an ideal value: $\frac{2}{2\pi} \approx 0.2026$, which the final ideal DE (where $GLCM[1, k]$ is large enough to ensure there is ideally no effect from the inter-modulation products) is approaching $\frac{2}{10} \approx 61.69\%$. The calculated values fluctuate for some $k$ values like $k = 1.5, 2, 2.5, 3 \cdots$.

To make a better comparison, some of the calculated results at some selected $k$ values are listed in Table 1. The reason behind the numerical fluctuation at some $k$ values is due to the fact that the $(m + n)$-th orders’ inter-modulation products may generate DC or fundamental components (as is discussed above when $m \cdot w_1 \pm n \cdot w_2 = 0, \pm w_1$ or $\pm w_2$). Also, the generated DC or the fundamental components will directly affect the efficiency of the PA. Generally, the first generated DC or the fundamental components occur at a higher order inter-modulation products (a larger value of $GLCM[1, k]$), the less is the impact on the PA’s efficiency will be. For example, when $k = 2(GLCM[1, k] = 2)$, the first generated DC or fundamental components happened at the 2-nd order inter-modulation products $w_1 - w_2 = 2w_2 - w_2 = w_2$, the DE is 77.8%, which is 16.11% higher than the ideal DE value; When $k = 1.5(GLCM[1, k] = 3)$, the first generated DC or fundamental components happened at the 4-th order inter-modulation products $3w_2 - w_1 = 3w_2 - \frac{3}{2}w_1 = w_1$, the DE is 59.59%, which is 2.28% lower than the ideal DE value; When $k = 10(GLCM[1, k] = 10)$, the first generated DC or fundamental components happened at the 10-th order inter-modulation products $w_1 - 9w_2 = 10w_2 - 9w_2 = w_2$, the DE is 61.68%, which is only 0.01% lower than the ideal DE value.

**D. THE ANALYSIS OF IMBALANCED CLASS-B CPA**

By considering $A$ as a variable, one can simulate the imbalanced CPA’s CPA case. The current and voltage wavforms given by equation (24) are used to calculate the overall DC component and the fundamental components of $i_{ds,b}$. The calculation of the following integrals are needed:

$$\int_{0}^{T} |\cosw_{1t} + \cosw_{2t}| dt$$

$$\int_{0}^{T} |\cosw_{1t} + \cosw_{2t}| \cosw_{2t} dt$$

$$\int_{0}^{T} |\cosw_{1t} + \cosw_{2t}| \cosw_{2t} dt$$

(46)

However, it becomes very difficult, maybe even impossible to be calculated with analytical solutions or expressed using analytical methods.

Computer-assisted numerical analysis is adopted herein to obtain numerically the results. By up sampling $10^{6}$ times sampling of the lower fundamental frequency in the length of a period, the final drain efficiency of the imbalanced CPA versus the frequency ratio $k$ (from 1 to 10, with a step of 0.001) and the logarithmic function $20\log A$ (from $-40$ to 40 with a step of 0.001) are drawn in Fig. 7. The surface in Fig. 7 is consist of DE curves versus $A \in (0.01, 100)$ with different $k$ values. Comparing with Fig. 6 and Table 1, the predict
As discussed in Section III.C, the DE of CPA will be influenced by the generated DC or fundamental components in the inter-modulation products. Thus resulting in the DE fluctuation around a general convergence approaching the ideal DE curve. Generally, the results of numerical analysis

special points at the list \( k \) values are vividly 3D-depicted in Fig. 7.

For a better comparison, the calculated DEs of Class-B CPA versus 20log\( A \) at selected values of \( k \) are drawn in Fig. 8.

| \( k \) | 1.25 | 1.5 | 1.6 | 1.75 | 2 | 2.25 | 2.5 | 3 | 3.5 |
|--------|------|-----|-----|------|---|------|-----|---|-----|
| \( GLCM[1,k] \) | 5 | 3 | 8 | 7 | 2 | 9 | 5 | 3 | 7 |
| \( I_{Pw1} \) | 0.25 | 0.2616 | 0.2509 | 0.25 | 0.27 | 0.25 | 0.2471 | 0.25 | 0.25 |
| \( I_{Pw2} \) | 0.2452 | 0.25 | 0.25 | 0.2516 | 0.3734 | 0.2503 | 0.2542 | 0.25 | 0.2506 |
| \( I_{PDc} \) | 0.2031 | 0.2277 | 0.2026 | 0.2027 | 0.2027 | 0.2027 | 0.2027 | 0.2027 | 0.2027 |
| \( DE(\%) \) | 60.96 | 59.94 | 61.79 | 61.87 | 77.8 | 61.71 | 61.83 | 64.43 | 61.75 |
| \( 1/k \) | 4 | 4.5 | 5 | 6 | 7 | 8 | 9 | 10 | \( \infty \) |
| \( GLCM[1,k] \) | 4 | 9 | 5 | 6 | 7 | 8 | 9 | 10 | \( \infty \) |
| \( I_{Pw1} \) | 0.2557 | 0.2499 | 0.25 | 0.2508 | 0.25 | 0.2502 | 0.25 | 0.2501 | 0.25 |
| \( I_{Pw2} \) | 0.2402 | 0.2502 | 0.25 | 0.249 | 0.25 | 0.2497 | 0.25 | 0.2499 | 0.25 |
| \( I_{PDc} \) | 0.2028 | 0.2026 | 0.2019 | 0.2027 | 0.2025 | 0.2026 | 0.2026 | 0.2026 | 0.2026 |
| \( DE(\%) \) | 61.14 | 61.69 | 61.91 | 61.65 | 61.74 | 61.68 | 61.7 | 61.68 | 61.69 |

Values of \( I_{Pw1}, I_{Pw2} \) and \( I_{PDc} \) are normalized by \( I_{max} \).
are a very good approximation of the real case. This numerical analysis helped to draw $7.2 \times 10^8$ points in the given 3D format in Fig. 7.

As discussed before, the larger the value of $GLCM[1, k]$ the less impact on the PA’s efficiency from the intermodulation products will be. When the least common multiple $GLCM[1, k]$ is large enough, the contribution of the DC and fundamental components generated from the intermodulation products could be neglected and their impact on the DE becomes non-significant.

As is shown in Fig. 7, only a fraction of DE curves with small $GLCM[1, k]$ values are values are considerably different in shape compared to most of the curves. These differences decrease for large values of $GLCM[1, k]$, where the curves tend to converge to the general ideal case where there is ideally no effect from the inter-modulation products. The ideal DE curve as the function of the amplitude imbalance tends to the DE curves as shown in Fig. 8 ($k = 10$) and is ideally plotted in Fig. 9.

To address the fundamental components for the ideal case (when $GLCM[1, k] \to \infty$), only the DC component in $|\cos(w_1 t) + \cos(w_2 t)|$ is considered in equation (31), according to the orthogonal properties of trigonometric function (see Appendix C), the component of $w_1$ can be extracted as shown below:

$$I_{Pw_1} - I_{Nw_1} = \frac{2I_{\max} \int_0^T |\cos(w_1 t) + \cos(w_2 t)| \cos(w_1 t) dt}{T(A + 1)} \approx \lim_{L \to \infty} \frac{2I_{\max} \int_0^{T_1 M} g(w_1 t) \cos(w_1 t) dt}{(A + 1)T_1 M L}$$

(47)

where $L = GLCM[1, k]$, $g(w_1 t) = G_L \frac{1}{k} \cos(w_1 t)$ is the lowest order $((L + \frac{1}{k} - 1)$-th order, see Appendix D) intermodulation product that has the same frequency of the fundamental component in $|\cos(w_1 t) + \cos(w_2 t)|$. $G_L \frac{1}{k} \cos(w_1 t)$ is the coefficient of $(L + \frac{1}{k} - 1)$-th order intermodulation product, when $GLCM[1, k] \to \infty$, $G_L \frac{1}{k} \cos(w_1 t)$ will be infinitely close to 0.

Therefore, from the analysis above and equation (47), one can deduce that:

$$I_{Pw_1} - I_{Nw_1} \approx 0 \approx I_{Pw_2} - I_{Nw_2}$$

(48)

Then the values of $I_{Pw_1}$ and $I_{Pw_2}$ can be obtained from equations (28) and (48):

$$I_{Pw_1} = \frac{AI_{\max}}{2(A + 1)}; \quad I_{Pw_2} = \frac{I_{\max}}{2(A + 1)}$$

(49)

And the DC component in the drain current, without any influence of inter-modulation products taken into consideration, should be:

$$I_{PDC} = \frac{1}{2T} \int_0^T (i_{dsp}(t) - i_{dsn}(t))dt$$

$$= \frac{I_{\max}}{2T(A + 1)} \int_0^T |\cos(w_1 t) + \cos(w_2 t)| dt$$

(50)

For the last integral in equation (50), $|\cos(w_1 t) + \cos(w_2 t)|$ can be regarded as the magnitude of the sum of vectors $\cos(w_1 t)$ and $\cos(w_2 t)$. According to the Law of Cosines, $I_{PDC}$ can be written as:

$$I_{PDC} = \frac{I_{\max} \int_0^T \sqrt{A^2 + 1 + 2A \cos(w_1 t - w_2 t)|cos\psi| dt}{2T(A + 1)}$$

(51)
FIGURE 8. The calculated DEs of Class-B CPA versus \(20 \log A\) at selected values of \(k\). (a) \(k = 1.25\); (b) \(k = 1.5\); (c) \(k = 1.75\); (d) \(k = 2\); (e) \(k = 2.5\); (f) \(k = 3\); (g) \(k = 3.5\); (h) \(k = 4\); (i) \(k = 4.5\); (j) \(k = 5\); (k) \(k = 6\); (l) \(k = 7\); (m) \(k = 8\); (n) \(k = 9\); (o) \(k = 10\).
κ in terms of elementary functions except in degenerate cases. To calculate using elementary math, but when

where λ = arc tan


to help derive the full analytical expression of the two-tone signal driven CPA’s DE. Despite that the derivation process of the DE’s expression is tedious and complex, we believe that it can be used by amplifier designers to quickly and accurately estimate the DE in the case of two-tone signal driven CPA without a need to perform any lengthy simulation that requires behavior models. And the final analytical expression is found to be valuable to accurately predict the energy consumption of a two-tone signal driven CPA.

IV. SIMULATION AND MEASUREMENT RESULTS

For verification, a dual-band CPA operated at 4G band 2.4 – 2.6 GHz and 5G band 3.6 – 3.8 GHz is designed and measured. The photograph and topology with detailed parameters of the fabricated PA are shown in Fig. 10 (a) and (b), respectively. The dual-band matching network should be pre-designed at the center frequencies of the two bands. When designing the dual-band output matching network, the parasitic parameters of the package lead and bonding wires in the packaged transistor should be taken into consideration for meeting the dual-band impedance matching. Computer-aided optimization is also needed to obtain the final optimized parameters.

Eventually the DE of any imbalanced signal driven class-B CPA can be expressed as:

$$DE = \frac{1}{2}\left(\frac{2}{A^2 + 1}\right)^2 + \frac{A^2}{2\pi^2} \left(E\left(\frac{\pi}{2}\right)^2 I_{max} V_{DC}\right)$$

$$= \frac{\pi^2 (A^2 + 1)}{8(A + 1)^2 E\left(\frac{\pi}{2}\right)^2 |\kappa^2|}$$

(55)

Herein we give a simple verification of the validity of equation (55). When A = 0 or A → ∞, in fact, the input signal vgs will no longer a two-tone signal, but the same as single-tone signals cosω1t or Acosω1t respectively. Therefore, the DE is the same as the conventional single-tone signal driven PA, the specific process can refer to [20], as η = π/4 ≈ 78.54%.

When A = 1, the approaching DE is already given in equation (45) and Fig. 6. The DE calculates from Table of Elliptic Integrals versus the logarithmic relatively amplitude 20lgA are shown in Fig. 9. The DE of CPA reaches its only maximal value π/4 ≈ 61.69% in the balanced case (A = 1). When A = 0 or A → ∞, the DE is going to be infinitely close to 78.5% (the typical value of single-tone signal driven PA). It seems that the curve should be monotone increasing at the right of the DE-axis and be monotone decreasing at the left of the DE-axis, however due to the monotonicity properties of the DE function, the ratio of π/4(A^2 + 1) and E(π/2 |κ^2|), there does exists two minimum points near A = 1.52 and 0.66(20lgA = ±3.6 dB), and the minimum value ≈ 61.1%. Finally, the results do consistent with the results of Numerical Analysis.

In this section, Complete Elliptic Integral is introduced to help derive the full analytical expression of the two-tone signal driven CPA’s DE. Despite that the derivation process of the DE’s expression is tedious and complex, we believe that it can be used by amplifier designers to quickly and accurately estimate the DE in the case of two-tone signal driven CPA without a need to perform any lengthy simulation that requires behavior models. And the final analytical expression is found to be valuable to accurately predict the energy consumption of a two-tone signal driven CPA.

\[ I_{PDC} = \frac{I_{max}}{\pi T} E(T|\kappa^2) \]

(52)

where \[ \kappa^2 = \frac{4A}{(A + 1)^2} \in [0, 1]. \]

The final expression in equation (52) is the form of so-called Incomplete Elliptic Integral, which cannot be evaluated in terms of elementary functions except in degenerate cases \( \kappa^2 = 0 \) (⇒ A = 0) or A → ∞) or \( \kappa^2 = 1 \) (⇒ A = 1). In fact, the Incomplete Elliptic Integral is unable to calculate using elementary math, but when \( \cos(w_t) \) have the period of T, which exactly equals the integral length, the Incomplete Elliptic Integral in equation (52) can be converted into the Complete Elliptic Integral (See Appendix E, equation (61)):

\[ E(T|\kappa^2) = \frac{2(M - N)}{w_e} E(\frac{\pi}{2}|\kappa^2) \]

(53)

where \[ E(\frac{\pi}{2}|\kappa^2) \] is a Complete Elliptic Integral, although the general form of \[ E(\frac{\pi}{2}|\kappa^2) \] is impossible to be evaluated in terms of elementary functions, but its value with a specific A can be obtained using table look-up method from published Table of Elliptic Integrals [22] or computer aid calculation software (like the commander function \[ K, E = \text{ellipke}(\kappa^2) \] in Matlab).

Therefore, the direct component in \[ \tilde{i}_{dsp}(t) \] as is expressed in equation (50) can be finally written as:

\[ I_{PDC} = \frac{I_{max}}{\pi T} E(T|\kappa^2) = \frac{2I_{max}}{\pi^2} E(\frac{\pi}{2}|\kappa^2) \]

(54)
Firstly, to give an overview of the dual-band CPA’s performance, as is shown in Fig. 11, the CPA is measured when $A \rightarrow \infty$, $A = 0$, and $A = 1$, which are equivalent to be tested using single-tone signal (non-concurrent mode) at 2.5GHz, 3.7GHz, and balanced two-tone signal (concurrent mode) at the concurrent combination of 2.5GHz and 3.7GHz, respectively. The measured peak DE of 77.5% achieved at $A \rightarrow \infty$ with a saturated output power of 41.8dBm.

The measured peak DE of 71.9% achieved at $A = 0$ with the saturated output power of 41.2dBm. The measured peak DE of 62.6% achieved at $A = 1$ with a saturated output power of 40.1dBm.

To investigate the DE and output relationship when the CPA is driven by an imbalanced two-tone signal, the measured results of overall output power and DE versus the imbalanced input power are shown in Fig. 12. The DE curve and output power curve are in good agreement with the trend of that in Fig. 9. However, it is obvious that there exists about a 2dB difference between the balanced input line and the minimum value line, which should converge to the same value in theory. The reason causing this discrepancy is the non-constant gain behavior over the operating frequency bandwidth as illustrated in Fig. 11. A drift of the minimum value line towards the frequency with a lower power gain is usually noticed.

The measurement results of the CPA versus an imbalanced output power is also given in Fig. 13. Apparently, the balanced input line and the minimum value line is closer to each other, and the agreement with Fig. 9 is also better than the one in Fig. 12. The gap still exists due to the transistor’s different intrinsic efficiency and power handling capacity for
different frequencies, and generally, for a given transistor, the performance becomes worse with the increasing frequency.

Many measurement results in reported works are obtained with the condition of balanced input power. However, when considering the power gain fluctuates and transistor performance differs at different operating frequencies, the measurement results of CPA will have a large difference when expressed using balanced output power or balanced input power.

V. CONCLUSION

The method to analyze CPA driven by imbalanced two-tone signals, along with analytical formulation is proposed in this paper to predict the DE of CPAs. The detailed method to solve complicated imbalanced two-tone signals is discussed. An analytical expression of general DE of imbalanced two-tone signal driven CPA biased at Class-A and Class-B is proposed.

A general empirical expression to calculate the drain efficiency (DE) of CPA versus the frequency ratio and the amplitude ratio of the two carriers is proposed. Simulation results and experimental verifications are given to validate the proposed analytical formulation to predict the power efficiency of CPAs when driven with balanced and imbalanced two-tone signals. Extension of the proposed multi-concurrent mode, to performance analysis for arbitrary conduction angles, and to concurrent Doherty amplifiers, will be valuable to the CPA designers.

APPENDIX A

The detailed derivation of equation (38) is shown in equation (56), as shown at the bottom of this page.

APPENDIX B

The detailed derivation of equation (43) is shown as below:

\[ I_{PDC} = \frac{I_{PDC} - I_{NDC}}{2T} \]

\[ = \frac{1}{2T} \int_0^T (i_{dsp}(t) - i_{das}(t))dt \]

\[ = \frac{I_{max}}{2T} \int_0^T |\cos\omega_1t||\cos\omega_2t|dt \]

\[ = \frac{2I_{max}}{\pi^2T} \sum_{p,q \in Z} (-1)^{p+q} \int_0^T \cos(2pw_m)\cos(2qw_e)dt \]

\[ = \frac{I_{max}}{\pi^2} \sum_{p,q \in U_{BC}} (-1)^{p+q} \frac{(1 - 4p^2)(1 - 4q^2)}{(1 - 4p^2)(1 - 4q^2)} \]

\[ = I_{max} \int_0^T \cos(2pw_m)\cos(2qw_e)dt \]

\[ = \int_0^T \cos(m_1w_1 + n_1w_2)\cos(m_2w_1 + n_2w_2)\]

\[ \Rightarrow w_1 = Lw_2 - \left( \frac{L}{k} - 1 \right)w_1 \]
The detailed derivation of equation (52) is shown as below:

\[
I_{\text{PDC}} = \int_{0}^{T} \sqrt{A^2 + 1 + 2A \cos(2\omega e T)} dt
\]

\[
= \int_{0}^{T} \frac{\sqrt{A^2 + 1 + 2A - 4A \sin^2 \omega e T}}{2T(A + 1)} dt
\]

\[
= \frac{I_{\text{max}}}{\pi T(A + 1)} \int_{0}^{T} \left(1 - \frac{4A}{(A + 1)^2} \sin^2 \omega e T\right) dt
\]

\[
= \frac{I_{\text{max}}}{\pi T} E(T|k^2)
\]

(61)

The detailed derivation of equation (53) is shown as below:

\[
E(T|k^2) = \int_{0}^{(M-N)\pi} \frac{2(M-N)\pi}{w_0} \sqrt{1 - k^2 \sin^2 \omega e T dt}
\]

\[
= \frac{1}{w_0} \int_{0}^{(M-N)\pi} \frac{2}{\sqrt{1 - k^2 \sin^2 \omega e T dt}} \sin x dx
\]

\[
= \frac{2(M-N)}{w_0} \int_{0}^{\pi/2} \frac{2}{\sqrt{1 - k^2 \sin^2 x}} dx
\]

\[
= \frac{2(M-N)}{w_0} E\left(\frac{\pi}{2} | k^2\right)
\]

(62)

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