D-branes in some near-horizon geometries

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ABSTRACT: I review some properties of D-branes in the SU(2) and SL(2,R) WZW models. I comment on a potential difficulty for the realization of 'warped brane worlds' in string theory. This short note is based on a talk given at the Strings’01 conference in Mumbay.

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1. Introduction

In this talk I will review some properties of D-branes in the SU(2) and SL(2,R) WZW models. These models enter in some of the earliest, exact and stable string-theory backgrounds \[1, 4, 3, 2\], that have returned to center-stage recently in the light of the AdS/CFT correspondence \[3, 3\]. Specifically, the level-\(k\) SU(2) model describes, together with a Feigin-Fuchs field, the near-horizon geometry of \(Q_5 = k + 2\) Neveu-Schwarz fivebranes \[3\]. The background near the horizon of \(Q_5\) NS5-branes and \(Q_1\) fundamental strings involves, on the other hand, both an SU(2) and an SL(2,R) WZW model (see for example \[3\]). In these settings one hopes, in principle, to go beyond the gauge theory/gravity correspondence, and test the conjectured duality between a full string theory and a field theory.

The study of D-branes in these backgrounds is important for a variety of reasons:
- D-branes are an essential ingredient of the corresponding string theories, and have interesting holographically dual interpretations;
- despite substantial progress,* the SL(2,R) WZW model is still only partially understood. The semiclassical analysis of D-branes could help ‘solve’ this important CFT; and
- these backgrounds are controllable playgrounds, in which to see whether ‘warped brane world’ scenarios \[10\] can be realized in string theory.

I will barely touch upon these questions in my talk. For lack of space, I will not even mention many works that have analyzed branes in near-horizon geometries with Ramond-Ramond fluxes. My ‘excuse’ is that these do not have, at present, an exact CFT description. Finally, as this talk was being written, there have appeared several papers discussing related issues \[51, 52, 53, 54, 55, 56\]. I will comment on them succinctly in the appropriate places.

2. D-branes in SU(2)

According to Cardy’s general prescription \[11\], applicable to any rational CFT, the basic conformal boundary states of the SU(2) WZW model are

\[ | n \gg C = \sum_{m=1}^{k+1} \frac{S_n^m}{\sqrt{S_{1,m}^m}} | m \gg I \].

(2.1)

Here \(k\) is the level of the current algebra, and \(S_n^m\) is the modular-transformation matrix for the characters \(\chi_m\). The representations of the chiral algebra are labelled by the dimension of the highest-weight subspace \(m = 1, \cdots, k+1\). The ‘character’ or Ishibashi states \(|m \gg I\) only couple to closed-strings in the representation \(\mathcal{H}_m \otimes \mathcal{H}_m\), and with unit strength. Using Verlinde’s formula one can transform the cylinder

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*In reference \[3\], in particular, a complete proposal was made for the spectrum of closed-string excitations. A recent review and many more references on the SL(2,R) model is \[3\].
diagram, describing the exchange of closed strings between the $n$ and $n'$ Cardy states, to the open-string (annulus) channel with the result
\[ A_{nn'} = \sum_r N_{nn'}^r \chi_r(q) , \] (2.2)
where $N_{nn'}^r$ are the fusion coefficients. These are non-negative integers, as required by the identification of Cardy states with regular D-branes.

Although this algebraic construction was known for several years, its geometric meaning has been only clarified recently. A simple but illuminating remark is that the identification of left and right currents on the worldsheet boundary, which is automatically imposed by all Cardy states, translates into Dirichlet conditions for directions normal to conjugacy classes of the group $[12]$. Explicitly, if we parametrize the strip worldsheet by $(\sigma, \tau)$, then the identification $J_a = \overline{J}_a$ implies
\[ [1 + \text{Ad}(g)] g^{-1} \partial_{\tau} g = [1 - \text{Ad}(g)] g^{-1} \partial_{\sigma} g . \] (2.3)
Here $\text{Ad}(g)L \equiv gLg^{-1}$, so that $[1 - \text{Ad}(g)]$ projects onto the tangent space of the conjugacy class of $g$. It follows that the worldsheet boundaries, parametrized by the coordinate $\tau$, are stuck on conjugacy classes of the group. In the case of SU(2), these are spherical D2-branes.

What stops these branes from shrinking to a point is a quantized worldvolume flux $\int F$, whose interaction with the background Neveu-Schwarz $B$-field is described by the Dirac-Born-Infeld (DBI) action (see for instance [13]). The semiclassical analysis based on this action [14] (see also [13]) reproduces many detailed properties of the SU(2) D-branes, and offers a nice geometric interpretation of the algebraic data of this CFT. Furthermore, most of these semiclassical results turn out to be exact, to all orders in the $\alpha'$ expansion, thanks presumably to the existence of supersymmetric embeddings. I will not review these calculations here – the reader can consult the original articles.

Let me instead focus on the subtle question of how to define the Ramond-Ramond charge of these D-branes. The issue has been elucidated from various angles in references [16, 17, 18, 19, 20, 21, 22]. The two natural candidates [14] for the charge are: (i) the integral of the two-form $F = B + 2\pi \alpha' F$, which is gauge-invariant but not quantized; and (ii) the integral of $F \simeq dA$ which is quantized, but changes under large gauge transformations $\delta B \simeq d\Lambda$. As it turns out, the former gives the local (‘source’) coupling to RR fields, while the latter, after periodic identification modulo $k + 2$, is the invariant charge of a twisted version [23, 22] of K theory [24, 25, 26]. From the viewpoint of the effective supergravity, the existence of different notions of 'charge' can be attributed to the Chern-Simons terms in the action [13, 19].

The fact that the flux of $F \simeq dA$ should be quantized was an early source of confusion. One can argue for this by requiring that the $\sigma$-model on a worldsheet
Let $\Sigma$ have the topology of a disk, and denote by the same symbol its spacetime embedding. The boundary $\partial \Sigma$ is a loop inside the worldvolume of some D-brane. Let $\mathcal{D}$ be a disk inside this worldvolume such that $\partial \Sigma = \partial \mathcal{D}$, and let $\mathcal{M}_3$ be a 3-manifold bounded by the two disks, $\partial \mathcal{M}_3 = \Sigma - \mathcal{D}$ (the minus sign refers to inverse orientation). This is illustrated in figure 1. The $\sigma$-model action can now be written as follows:

$$ S = \int_{\Sigma} \text{tr} \hat{G} + \int_{\mathcal{D}} F + \int_{\mathcal{M}_3} H , \quad (2.4) $$

where $\hat{G}$ is the pull-back metric, $H \simeq dB$ is the NS 3-form, and $F$ is the gauge-invariant field on the D-brane. The measure is independent of the choice of $\mathcal{M}_3$ provided the $H$-flux through the three-sphere is quantized, $\int H = k + 2$. Likewise, quantization of the $F$-flux threading any closed two-manifold ensures that the measure is independent of the choice of $\mathcal{D}$. Explicitly, given two disks $\mathcal{D}$ and $\mathcal{D}'$, and a ‘ball’ $\mathcal{B}_3$ such that $\partial \mathcal{B}_3 = \mathcal{D} - \mathcal{D}'$, we must have

$$ \int_{\mathcal{D} - \mathcal{D}'} F - \int_{\mathcal{B}_3} H = n \in \mathbb{Z} . \quad (2.5) $$

The ambiguity in the choice of $\mathcal{B}_3$ implies that we can only associate a flux in $\mathbb{Z}_{k+2}$ to any closed two-manifold $\mathcal{D} - \mathcal{D}'$ on the D-brane.

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**Figure 1:** The spacetime image of the worldsheet $\Sigma$, and a disk $\mathcal{D}$ on the D-brane worldvolume, which together form the boundary of a 3-manifold $\mathcal{M}_3$. These enter in the $\sigma$-model action for open strings, equation (2.4).

One can think of the integer $n$ as the number of D-particles, which expand out in the background $B$-field by a generalized ‘dielectric’ effect [28]. The boundary RG flow, describing the formation of the $n$-particle bound state, is the same that describes the screening of a magnetic-impurity spin $s = (n - 1)/2$ by $k$ species of conduction electrons (the Kondo problem) [29, 30]. The fact that $n \simeq k + 2 - n$

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$I$ thank Cumrun Vafa for discussions of this point.
can be understood differently in various string theory contexts. Consider for instance $Q_5 \equiv k + 2$ NS5-branes, and a D3-brane extending in the transverse dimensions. The D-strings stretching between the D3- and NS5-branes have a chiral, fermionic ground state, so that pulling the D3-brane through the fivebranes creates (destroys) $k + 2$ oriented D-strings \[31, 32, 33, 34\]. In the near-horizon geometry of the fivebranes, this process changes precisely $n$ to $k + 2 - n$ \[35\].

A related argument, explained to me by Juan Maldacena, starts with a D3-brane wrapping the SU(2) manifold. The DBI action includes a term

$$\int B \wedge *F = - \int H \wedge \tilde{A},$$

with $\tilde{A}$ the (magnetic) gauge field dual to $A$. Equation (2.6) shows that the $H$-flux induces $k+2$ units of magnetic charge, that can be cancelled by $k+2$ D-strings ending on the D-brane. This is analogous to the baryon vertex in AdS5 \[36\]. Now think of the D-strings as Euclidean D-particle trajectories, which terminate on a spherical D-brane at fixed (Euclidean) time. This is an instanton configuration, describing a process in which $k + 2$ units of D-particle charge disappear. D-particle number is thus only conserved modulo $k + 2$, as advertised.

### 3. D-branes in SL(2,R)

In the SL(2,R) WZW model we lack, at present, an algebraic construction of D-branes as conformal boundary states à la Cardy (see, however, the recent work \[55\] for some steps in this direction). What has been worked out are the possible ‘gluing conditions’ for worldsheet currents, and their semiclassical interpretation based on the DBI action \[37, 38\]. Let me summarize very briefly the results (the reader should consult the references for details):

(a) The SL(2,R) group elements can be written

$$g = \frac{1}{L} \begin{pmatrix} X^0 + X^1 & X^2 + X^3 \\ X^2 - X^3 & X^0 - X^1 \end{pmatrix},$$

with the $X^M$ parametrizing a pseudosphere in flat space of signature ($-++++$). The group manifold is Lorentzian AdS3 of radius $L$. Identifying the currents by an inner automorphism, as in (2.3), leads to two generic types of D-branes corresponding to the elliptic or hyperbolic conjugacy classes of the group \[37\]. Their geometry is two-dimensional de Sitter (dS2) or the two-dimensional hyperbolic plane (H2). There are in addition two special cases: pointlike D-instantons (the conjugacy class of the identity) and the half lightcones in AdS3.

\[\text{Lifted to M theory, such a process can describe the elastic scattering of a Kaluza-Klein graviton transferring } Q_5 \text{ units of momentum to a bound state of fivebranes. I thank Ed Witten for a discussion of this point.}\]
(b) A third generic class of D-branes is obtained for the gluing conditions [38]:

\[ J^a T_a = J^a \omega_0 T_a \omega_0^{-1} \quad \text{with} \quad \omega_0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}. \] (3.2)

Here \( T_a \) are the generators of the group, and \( \omega_0 \) is an outer automorphism. By an extension of the argument of the previous section, one can show that these D-brane worldvolumes are ‘twined conjugacy classes’ [39] for which \( \text{tr}(\omega_0 g) \) is fixed. Their geometry is two-dimensional anti-de-Sitter (AdS2). The various D-branes preserving a SL(2,R) symmetry are collected in the table below. There are, in addition, stable non-symmetric branes that I won’t discuss.

| Conjugacy class | D-brane          |
|-----------------|------------------|
| \(-\infty < \text{tr} (\omega_0 g) < \infty\) | AdS\(_2\)         |
| \(|\text{tr} g| < 2\) | \(H_2\)         |
| \(|\text{tr} g| > 2\) | \(dS\(_2\)\) |
| \(|\text{tr} g| = 2\) | light cone       |
| \(g = 1\) | point            |

**Table 1:** The different regular and twined conjugacy classes of \(SL(2,\mathbb{R})\), and the geometry of the corresponding D-brane worldvolumes.

(c) The semiclassical analysis [38] shows that only the AdS2 branes are ‘physical’. The dS2 geometries, in particular, correspond to motions of closed D-strings carrying a supercritical electric field. This is in line with other statements [40, 41] about the impossibility to realize de Sitter spaces in string theory. The AdS2 branes, on the other hand, are worldvolumes of static \((p,q)\) strings stretching between antipodal points on the AdS boundary. Two of those are drawn schematically in figure 2. The larger the value of \(q/p\), the more the string bends towards the AdS boundary.
Figure 2: The AdS2 D-branes in cylindrical (left) and Poincaré coordinates (right). The straight brane is the worldvolume of a pure D-string. Binding $q$ fundamental strings to it, makes it bend towards the boundary of the ambient AdS3 spacetime.

(d) All of these branes have a supersymmetric embedding [37, 38] in the AdS3×S3 ×(T4 or K3) background arising in the near horizon region of a NS5/F1 black string [7]. For the S3 component of the space, one must use the SU(2) branes of the previous section. The AdS2×S2 branes, in particular, describe the junction of a (p,q) string with the NS5/F1 string of the background [38]. Adding momentum along this latter string amounts to modding out AdS3 by a discrete isometry, exactly as for the BTZ black hole. This breaks all the supersymmetries of the D-brane.

The worldvolume theory of the AdS2×S2 branes is an interesting deformation of N=4 super Yang-Mills. The theory has N=2 supersymmetries, and it approaches in appropriate limits the non-commutative theory on the ‘fuzzy’ sphere [30], and a curved version of the NCOS theory [12, 13]. It would be interesting to study S-duality in this context. Steps towards deriving the full open-string spectrum on the AdS2 branes have been taken recently in references [54, 56]. In the case $q = 0$, in particular, the spectrum is the ‘holomorphic half’ of the closed-string spectrum proposed for the SL(2,R) WZW model in reference [8].

Another very interesting question concerns the interpretation of the AdS D-branes in the dual spacetime CFT. Their holographic ‘images’ turn out to be conformal defects separating different CFTs on either side [52, 44]. One can also extend the geometric considerations of this talk so as to incorporate orientifolds [44], recovering in particular the algebraic results of [46, 47]. I will not discuss these issues here any further – I will zoom instead, in the concluding section, on the intriguing interplay of effective versus induced geometry.
4. Brane worlds and ‘radius locking’

The AdS2\times S2 branes of the previous section have higher-dimensional analogs in other near-horizon geometries. A particularly interesting example are the AdS4\times S2 branes in the AdS5\times S5 geometry with five-form Ramond Ramond background. These have been analyzed recently by Karch and Randall \[52, 53\], as a step towards the realization of warped ‘brane world’ compactifications in string theory. The question of gravity localization requires to go beyond the probe approximation, and to study the back reaction of the branes on the ambient geometry.

Another potential obstruction to the Randall-Sundrum scenario, of purely string-theoretic origin, has been pointed out in reference \[38\]. It has to do with the fact that, in the presence of non-trivial fluxes, the induced and effective metrics on the brane can be drastically different. Let me now explain this in some more detail.

The radius of the AdS2 branes of the previous section, measured in the induced (closed-string) metric is

\[
\hat{l}_{\text{AdS2}} = \frac{LT(p,q)}{T(p,0)} \geq L, \tag{4.1}
\]

where \(T(p,q)\) is the tension of a \((p,q)\) string, which is always greater or equal than the tension of \(p\) pure D-strings. By taking \(q \to \infty\) one can make \(\hat{l}_{\text{AdS2}}\) arbitrarily large, so that the brane is much more flat than the ambient geometry. The S2 part of the brane, on the other hand, cannot be bigger than the equator two-sphere. A straightforward calculation gives

\[
\hat{l}_{S2} = L \sin \left( \frac{\pi q}{k + 2} \right) \leq L, \tag{4.2}
\]

so that the spherical brane can be arbitrarily more curved than the background geometry. This situation seems, at first sight, paradoxical because unbroken supersymmetry requires the AdS2 and S2 radii to be equal. The point, however, is that the Yang-Mills multiplet couples to an effective open-string metric, which is related to the closed-string metric through the well-known formula \[48, 49\]

\[
G_{\alpha\beta} = \hat{g}_{\alpha\beta} - \mathcal{F}_{\alpha\gamma} \hat{g}^{\gamma\delta} \mathcal{F}_{\delta\beta}. \tag{4.3}
\]

Measured in the open-string metric the two radii are, indeed, equal to each other and to the background radius (which is the same for AdS3 and S3),

\[
L_{S2} = L_{\text{AdS2}} = L, \tag{4.4}
\]

for all values of \(p\) and \(q\). This ‘locking’ of the effective radii to the ambient values has been observed also in a related context in \[51\]. One could have argued for it from the fact that the open- and closed-string spectra are related. Note that
the causal structure is determined by the lightcones of the closed-string metric, in accordance with the general arguments of [50]. In other words nothing travels faster than gravitons.

Relation (4.4) shows that it is impossible to fine-tune the effective geometry of the D-brane to be ‘flatter’ than the bulk geometry. It is unclear whether this phenomenon persists in higher dimensions and for Ramond-Ramond fluxes. The back-reaction of the branes on the ambient metric could also play an important role. If the above conclusion were, however, to persist, it would be an obstruction to the construction of realistic ‘warped brane worlds’ in string theory.

Acknowledgements: I am grateful to the organizers for the invitation to speak. I thank Mike Douglas, Marios Petropoulos and Christoph Schweigert for very pleasant collaborations on the contents of this talk. Finally, I acknowledge the support of the European Networks “Superstring theory” (HPRN-CT-2000-00122) and “the quantum structure of spacetime” (HPRN-CT-2000-00131).

References

[1] J. Balog, L. O’Raifeartaigh, P. Forgacs and A. Wipf, “Consistency Of String Propagation On Curved Space-Times: An SU(1,1) Based Counterexample,” Nucl. Phys. B 325, 225 (1989).

[2] I. Antoniadis, C. Bachas and A. Sagnotti, “Gauged Supergravity Vacua In String Theory,” Phys. Lett. B 235, 255 (1990).

[3] C. G. Callan, J. A. Harvey and A. Strominger, “World sheet approach to heterotic instantons and solitons,” Nucl. Phys. B 359, 611 (1991).

[4] G. T. Horowitz and D. L. Welch, “Exact three-dimensional black holes in string theory,” Phys. Rev. Lett. 71, 328 (1993) [hep-th/9302126].

[5] J. Maldacena, “The large N limit of superconformal field theories and supergravity,” Adv. Theor. Math. Phys. 2, 231 (1998) [Int. J. Theor. Phys. 38, 1113 (1998)] [hep-th/9711200].

[6] O. Aharony, S. S. Gubser, J. Maldacena, H. Ooguri and Y. Oz, “Large N field theories, string theory and gravity,” Phys. Rept. 323, 183 (2000) [hep-th/9905111].

[7] J. Maldacena and A. Strominger, “AdS(3) black holes and a stringy exclusion principle,” JHEP 9812, 005 (1998) [hep-th/9804087].

[8] J. Maldacena and H. Ooguri, “Strings in AdS(3) and SL(2,R) WZW model. I,” [hep-th/0001053].

[9] P. M. Petropoulos, “String theory on AdS(3): Some open questions,” hep-th/9908189.
[10] L. Randall and R. Sundrum, “A large mass hierarchy from a small extra dimension,” Phys. Rev. Lett. 83, 3370 (1999) [hep-ph/9905221]; “An alternative to compactification,” Phys. Rev. Lett. 83, 4690 (1999) [hep-th/9906064].

[11] J. L. Cardy, “Boundary Conditions, Fusion Rules And The Verlinde Formula,” Nucl. Phys. B 324, 581 (1989).

[12] A. Y. Alekseev and V. Schomerus, “D-branes in the WZW model,” Phys. Rev. D 60, 061901 (1999) [hep-th/9812193].

[13] C. P. Bachas, “Lectures on D-branes,” [hep-th/9806199]; A. A. Tseytlin, “Born-Infeld action, supersymmetry and string theory,” [hep-th/9908105]; J. H. Schwarz, “Comments on Born-Infeld theory,” [hep-th/0103165] in these proceedings.

[14] C. Bachas, M. Douglas and C. Schweigert, “Flux stabilization of D-branes,” JHEP 0005, 048 (2000) [hep-th/0003037].

[15] J. Pawelczyk, “SU(2) WZW D-branes and their noncommutative geometry from DBI action,” JHEP 0008, 006 (2000) [hep-th/0003057].

[16] W. Taylor, “D2-branes in B fields,” JHEP 0007, 039 (2000) [hep-th/0004141].

[17] S. Elitzur, A. Giveon, D. Kutasov, E. Rabinovici and G. Sarkissian, “D-branes in the background of NS fivebranes,” JHEP 0008, 046 (2000) [hep-th/0005052].

[18] A. Alekseev, A. Mironov and A. Morozov, “On B-independence of RR charges,” [hep-th/0005244].

[19] D. Marolf, “Chern-Simons terms and the three notions of charge,” [hep-th/0006117].

[20] S. Stanciu, “A note on D-branes in group manifolds: Flux quantization and D0-charge,” JHEP 0010, 015 (2000) [hep-th/0006145].

[21] J. M. Figueroa-O’Farrill and S. Stanciu, “D-brane charge, flux quantization and relative (co)homology,” JHEP 0101, 006 (2001) [hep-th/0008038].

[22] S. Fredenhagen and V. Schomerus, “Branes on group manifolds, gluon condensates, and twisted K-theory,” JHEP 0104, 007 (2001) [hep-th/0012164].

[23] P. Bouwknegt and V. Mathai, “D-branes, B-fields and twisted K-theory,” JHEP 0003, 007 (2000) [hep-th/0002023].

[24] R. Minasian and G. Moore, “K-theory and Ramond-Ramond charge,” JHEP 9711, 002 (1997) [hep-th/9710230].

[25] E. Witten, “D-branes and K-theory,” JHEP 9812, 019 (1998) [hep-th/9810188].

[26] E. Witten, “Overview of K-theory applied to strings,” Int. J. Mod. Phys. A 16, 693 (2001) [hep-th/0007173].
[27] C. Klimcik and P. Severa, “Open strings and D-branes in WZNW models,” Nucl. Phys. B 488, 653 (1997) [hep-th/9609112].

[28] R. C. Myers, “Dielectric-branes,” JHEP 9912, 022 (1999) [hep-th/9910053].

[29] I. Affleck, “Conformal Field Theory Approach to the Kondo Effect,” Acta Phys. Polon. B 26, 1869 (1995) [cond-mat/9512099].

[30] A. Y. Alekseev, A. Recknagel and V. Schomerus, “Non-commutative world-volume geometries: Branes on SU(2) and fuzzy spheres,” JHEP 9909, 023 (1999) [hep-th/9908040].

[31] A. Hanany and E. Witten, “Type IIB superstrings, BPS monopoles, and three-dimensional gauge dynamics,” Nucl. Phys. B 492, 152 (1997) [hep-th/9611230].

[32] C. P. Bachas, M. R. Douglas and M. B. Green, “Anomalous creation of branes,” JHEP 9707, 002 (1997) [hep-th/9705074].

[33] U. Danielsson, G. Ferretti and I. R. Klebanov, “Creation of fundamental strings by crossing D-branes,” Phys. Rev. Lett. 79, 1984 (1997) [hep-th/9705084].

[34] O. Bergman, M. R. Gaberdiel and G. Lifschytz, “Branes, orientifolds and the creation of elementary strings,” Nucl. Phys. B 509, 194 (1998) [hep-th/9705130].

[35] O. Pelc, “On the quantization constraints for a D3 brane in the geometry of NS5 branes,” JHEP 0008, 030 (2000) [hep-th/0007100].

[36] E. Witten, “Baryons and branes in anti de Sitter space,” JHEP 9807, 006 (1998) [hep-th/9805112].

[37] S. Stanciu, “D-branes in an AdS(3) background,” JHEP 9909, 028 (1999) [hep-th/9901122].

[38] C. Bachas and M. Petropoulos, “Anti-de-Sitter D-branes,” JHEP 0102, 025 (2001) [hep-th/0012234].

[39] G. Felder, J. Frohlich, J. Fuchs and C. Schweigert, “The geometry of WZW branes,” J. Geom. Phys. 34, 162 (2000) [hep-th/9909030].

[40] B. de Wit, D. J. Smit and N. D. Hari Dass, “Residual Supersymmetry Of Compactified D = 10 Supergravity,” Nucl. Phys. B 283, 165 (1987).

[41] J. Maldacena and C. Nunez, “Supergravity description of field theories on curved manifolds and a no go theorem,” Int. J. Mod. Phys. A 16, 822 (2001) [hep-th/0007018].

[42] N. Seiberg, L. Susskind and N. Toumbas, “Strings in background electric field, space/time noncommutativity and a new noncritical string theory,” JHEP 0006, 021 (2000) [hep-th/0005040].
[43] R. Gopakumar, J. Maldacena, S. Minwalla and A. Strominger, “S-duality and non-commutative gauge theory,” JHEP 0006, 036 (2000) [hep-th/0005048].

[44] C. Bachas, J. de Boer, R. Dijkgraaf and H. Ooguri, in preparation.

[45] C. Bachas, N. Couchoud and P. Windey, in preparation.

[46] G. Pradisi, A. Sagnotti and Y. S. Stanev, “Planar duality in SU(2) WZW models,” Phys. Lett. B 354, 279 (1995) [hep-th/9503207].

[47] G. Pradisi, A. Sagnotti and Y. S. Stanev, “The Open descendants of nondiagonal SU(2) WZW models,” Phys. Lett. B 356, 230 (1995) [hep-th/9506014].

[48] A. Abouelsaood, C. G. Callan, C. R. Nappi and S. A. Yost, “Open Strings In Background Gauge Fields,” Nucl. Phys. B 280, 599 (1987).

[49] N. Seiberg and E. Witten, “String theory and noncommutative geometry,” JHEP 9909, 032 (1999) [hep-th/9908142].

[50] G. W. Gibbons and C. A. Herdeiro, “Born-Infeld theory and stringy causality,” Phys. Rev. D 63, 064006 (2001) [hep-th/0008052].

[51] J. Maldacena, G. Moore and N. Seiberg, “Geometrical interpretation of D-branes in gauged WZW models,” [hep-th/0105038].

[52] A. Karch and L. Randall, “Localized gravity in string theory,” [hep-th/0105108].

[53] A. Karch and L. Randall, “Open and closed string interpretation of SUSY CFT’s on branes with boundaries,” [hep-th/0105132].

[54] P. M. Petropoulos and S. Ribault, “Some remarks on anti-de Sitter D-branes,” [hep-th/0105252].

[55] A. Giveon, D. Kutasov and A. Schwimmer, “Comments on D-branes in AdS$_3$,” [hep-th/0106003].

[56] P. Lee, H. Ooguri, J. Park and J. Tannenhauser, “Open Strings on AdS$_2$ Branes,” [hep-th/0106129].