Influence of acoustic type waves on the vortex wake behind a wing in the supersonic flow

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Abstract. In this paper an investigation of an acoustic type waves influence on a wingtip vortex wake parameters was performed in a supersonic flow. The supersonic flow around a straight wing with sharp leading and trailing edges was considered at an attack angle of 10 degrees at Mach number of incoming flow M = 2. Perturbations were introduced into a steady state stream in the form of monochromatic plane waves with small amplitude. Numerical simulations were carried out at the Keldysh Institute of Applied Mathematics RAS using the parallel algorithm for turbulent flow simulation. For the numerical studies an approach based on the URANS method with using the SA turbulence model was applied. Numerical simulations were carried out on the supercomputing system K-60. Numerical data were obtained in the area exceeding 30 wing chords downstream from a wing axis. A numerical data on the pulsation of gasdynamic parameters in the vortex wake influenced by disturbance are obtained, in particular, frequencies and disturbance ranges on the wingtip vortex axis are investigated depending on a distance from the wing - vortex generator.

1. Introduction

The understanding of the influence of disturbance in the incoming flow on the flow parameters at supersonic flow around solid bodies is of great theoretical and practical interest [1], especially with regard to the exploitation of high-speed aircrafts. This interest is caused, firstly, by pervasiveness of acoustic waves or acoustic noise at the atmosphere, and secondly, by existence of an acoustic background in a test section of wind tunnels during the tests [2, 3].

Many authors have studied the influence of disturbances on the boundary layer. In particular, an extensive work has been carried out to study the influence of disturbances on the change of the boundary layer parameters on a flat plate and on a wedge in a supersonic flow [4, 5].

The study of the perturbations influence on the wingtip vortex is no less interesting, but this question is studied quite little. Earlier the authors have investigated a vortex wake behind a wing in a supersonic flow in the absence of disturbances [6]. The aim of this work is to determine the effect of acoustic type disturbances on the wingtip vortex parameters in a supersonic flow. The disturbances were set in the form of a monochromatic plane wave in which all values are simple periodic functions from time. This allows us not to limit the generality, since any sound wave can be represented as a...
superposition of monochromatic plane waves with different wave vectors and frequencies [7]. For this aim a Fourier analysis was used that allows to get a spectral representation of a wave perturbation.

2. Model Statement

2.1. Geometry
The supersonic flow behind a wing – wingtip vortex generator was studied. The wing was straight with sharp leading and trailing edges. The wing had a diamond-shaped base, a chord $b = 30$ mm, a half-span $95$ mm (figure 1 – a). The wing attack angle was 10 degrees, the Mach number of the incoming flow was $M = 2$ and Reynolds number was $1 \times 10^7$.

![Figure 1. Wing – generator model schema: model (a) and numerical interpretation with surface mesh (b).](image)

2.2. Numerical method
For describing a supersonic flow of a perfect viscous compressible fluid we used a system of unsteady Reynolds averaged Navier–Stokes equations (URANS) with the one-parameter Spalart–Allmaras turbulence model for compressible flows [8] with Edwards modification [9]. The finite volume method based on the reconstruction schemas of the second order (TVD) or the third order (WENO) was used for the discretization of equations. Time approximation was performed by means of the implicit schema based on the LU-SGS method and by explicit schema. A more complete description of the numerical algorithms and the mathematical model used in this paper is given in [10].

The numerical simulations have been performed at the Keldysh Institute of Applied Mathematics RAS using the hybrid supercomputer system K-60 [11]. The numerical data were obtained in the wide region exceeding 30 chords downstream from the wing axis.

We used an unstructured grid with hexagonal cells. Number of cells was $15572304$. The grid was thickened in the zone of vortex formation (especially near the tip chord of the wing, figure 1 – b) and in the zone of the wingtip vortex throughout all simulation area, that allowed for more accurate simulations.

3. Wingtip vortex in steady state incoming flow
Earlier the authors have been obtained detailed numerical-experimental data on the position and dimensions of the vortex core with quantitative data on the distribution of gas-dynamic characteristics of the flow [6].

Figure 2 shows pressure distribution $P$ on the wing-vortex generator surface, tip edge streamtraces and helicity contours $H$ in the cross section $x = 0.12$ for $M = 2$. Wingtip vortex develops in the form of such a longitudinal structure with an axis which parameters are one of the principal characteristics. In numerical data the position of wingtip vortex axis was obtained by a minimum of the tangential Mach
number. Helicity have a topological interpretation as a measure of knottiness of vortex lines in the flow \( H = \mathbf{V} \cdot \nabla \times \mathbf{V} \), where \( \mathbf{V} = (u, v, w) \) is a velocity vector.

![Figure 2](image)

**Figure 2.** The wing-generator with a pressure on it, the streamtraces passing through/near the wingtip and helicity in the cross section \( x = 0.12 \) for \( M = 2 \).

With increasing coordinate \( x \) (i.e. downstream from the wing), the intensity of the wingtip vortex decreases, which is accompanied, for example, by an increasing in pressure coefficient \( C_p = (P - P_\infty) \rho_\infty V^2 / (2 \rho_\infty V^2) \) (figure 3 – a). As it shown \( C_p \) tends to reach the values of undisturbed flow at the end of the computational region. The pressure coefficient is a dimensionless number which describes the relative pressures throughout a flow field in fluid dynamics.

It was also obtained that in the case of Mach number 2 there is a zone behind a wing where Mach number on wingtip vortex axis is higher, than a main flow Mach number (figure 3 – b).

![Figure 3](image)

**Figure 3.** Pressure coefficient \( C_p \) (a), and normalized Mach number (b) on the wingtip vortex axis along the coordinate \( x \).
A comparison of numerical and experimental data was effectuated. In the figure 4 a distribution of mass flow at the wingtip vortex and at its vicinity along the horizontal coordinate $z$ in the cross section $x/b = 2$ is presented for Mach number $M = 2$. This graphs show a satisfactory agreement between results of numerical simulations and experimental data.

Figure 4. Distribution of mass flow at the wingtip vortex and at its vicinity along the horizontal coordinate in the cross section $x/b = 2$ at $M = 2$.

4. Acoustic type disturbance

4.1. Form of disturbance

Disturbances were introduced in steady state incoming flow in the form of a monochromatic plane wave with small amplitude at the inlet boundary:

$$
\begin{pmatrix}
  u' \\
  v' \\
  p' \\
  \rho'
\end{pmatrix} = A \begin{pmatrix}
  \pm \cos \theta \\
  \mp \sin \theta \\
  1 \\
  1
\end{pmatrix} \cos \left( k_x x + k_y y - \omega t \right)
$$

When $u'$, $v'$, $p'$ and $\rho'$ – pulsations of $u$ and $v$ velocity components, of pressure and of density respectively; $\theta$ – angle of wave incidence; $A$ – disturbance amplitude; $t$ – time; $k_x = k \cos \theta$, $k = \omega \left( \frac{M \cos \theta \pm 1}{\rho_\infty} \right)$; $\omega$ is dimensionless frequency; the upper (lower) sign corresponds to a fast (slow) acoustic wave.

4.2. Numerical results under disturbance influence

In this section the numerical results under slow acoustic type disturbances introduced in incoming flow on the inlet boundary are presented. Amplitude of the disturbances was $A = 0.0286 \cdot P_\infty$ and angle of wave incidence was $\theta = 0$.

The general view of the pressure field in the longitudinal section $z = 0.093$ along the calculation domain is shown in figure 5: (a) under steady state incoming flow; (b) in the presence of slow acoustic type disturbances. In the middle of each part of the figure, a zone of the wingtip vortex wake formed behind the wing-generator is discernible in steady state incoming flow (figure 5 – a) and under the influence of the given perturbation (figure 5 – b). Pressure legends are different because otherwise the disturbances are not visible against the background of the free flow pressure.

Fluctuations of pressure in several points on the vortex axis are presented in figure 6. The coordinate $x$ of the points is equal to 0.2, 0.5 and 0.9 (figure 6 – a, 6 – b and 6 – c respectively). It is seen that the amplitude of the pressure fluctuation decreases with distance downstream from the wing.
(i.e. from the inlet boundary). This is natural, because the presence of viscosity leads to the dissipation of the energy of sound waves, therefore the sound is absorbed, i.e. its intensity gradually decreases [7].

Figure 5. The pressure field in the longitudinal section \( z = 0.093 \): (a) steady state incoming flow; (b) incoming flow with acoustic type disturbances.

Figure 6. Pressure fluctuations in few points on the vortex axis at the different coordinate \( x \): (a) \( x = 0.2 \); (b) \( x = 0.5 \); (c) \( x = 0.9 \).
5. Spectral analysis

Fourier analysis was applied to processing the obtained numerical results. The discrete Fourier transform (DFT) was used for this purpose [12, 13]. DFT is one of the most commonly used signal processing procedures. Today DFT is used in almost all areas of engineering. This is a mathematical procedure used to determine the harmonic, or frequency, composition of discrete signals. In our case, a discrete signal is a set of values obtained as a result of periodic sampling of a continuous signal in the time domain [14].

In the figures 7, 8 and 9 the single-sided amplitude spectrum of the pressure is presented in the several points on the wingtip vortex axis at the coordinate \( x = 0.2, 0.5 \) and \( 0.9 \) (figure 7, 8 and 9 respectively).

![Figure 7](image7.png)

**Figure 7.** Single-sided amplitude spectrum of pressure \( P \) in a point on the vortex axis at the coordinate \( x = 0.2 \).

![Figure 8](image8.png)

**Figure 8.** Single-sided amplitude spectrum of pressure \( P \) in a point on the vortex axis at the coordinate \( x = 0.5 \).

Figures 7, 8 and 9 show that the first harmonic frequency is preserved when perturbations move across computational region and coincides with the angular frequency of a given perturbation at the
input boundary of the region. In the first half of the region there are additional harmonics with frequencies that are multiple of the first harmonic frequency (figures 7 and 8). Their amplitudes are small. Amplitudes of all harmonics decrease when moving towards the end of the computational domain. Additional harmonics disappear by the end of the computational domain.

![Figure 9. Single-sided amplitude spectrum of pressure P in a point on the vortex axis at the coordinate x = 0.9.](image)

6. Conclusions
This paper presents the results of the study of the influence of disturbances in the incoming flow on the parameters of the vortex wake behind the wing-generator in the supersonic flow at Mach number \( M = 2 \). Perturbations were introduced in the form of monochromatic plane wave with small amplitude at the input boundary of computational domain. A numerical simulation was performed for the large region exceeding 30 chords within the framework of the URANS approach with the Spalart-Allmaras turbulence model.

Numerical data on the pulsation of gas-dynamic parameters in the vortex wake under the influence of acoustic disturbances were obtained as a result of calculations. The results were processed using Fourier analysis.

The analysis of the results showed that when the perturbation passes through the computational domain, the frequency of the first harmonic is kept equal to the frequency of the perturbation harmonic specified at the input boundary of the domain. In the middle of the region there are additional harmonics which frequencies are multiple of the first with small amplitude of little import. But by the end of the computational domain these harmonics disappear. Downstream from the wing-generator (i.e. from the input boundary) the amplitude of the pressure fluctuations decreases, which is natural in view of sound absorption.

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