Using Gray-Markov Model and Time Series Model to Predict Foreign Direct Investment Trend for Supporting China’s Economic Development

Yanyan Zheng, Tong Shu, Shou Chen and Kin Keung Lai

Abstract

Foreign direct investment (FDI) is one of the important factors affecting China’s economic development, the prediction of which is the basis of its development and decision-making. Based on elaborating the significant role in China’s economic growth and the status quo of utilizing foreign investment over the period between 2000 and 2016, this chapter attempts to construct Gray-Markov model (GMM) and time series model (TSM) to forecast the trend of China’s utilization of FDI and then compares the precision of two different prediction models to obtain a better one. Results indicate that although it is qualified, traditional Gray model needs to be optimized; GMM is built to help modify the result, improve Gray-related degrees, and narrow the gap with real value. Comparing the accuracy of GMM with that of TSM, we can conclude that the fitting effect of GMM is better. To increase the credibility of these results, this chapter is based on the data of Beijing and Chongqing from 1990 till 2016, also verifying that the fitting effect of GMM is superior to that of the TSM. Then, we can safely draw a conclusion that the prediction model of GMM is more credible, which has a certain referencing value for the utilization of FDI.

Keywords: foreign direct investment (FDI), Gray-Markov model (GMM), time series model (TSM)

1. Introduction

In the light of the definition of the International Monetary Fund (IMF) and the Organization for Economic Cooperation and Development, foreign direct investment (FDI) is an investment in the form of a controlling ownership in a business in one country by an entity based in another country. The primary purpose of the host country in attracting FDI is to promote the country’s economic development and industrial upgrading. This will facilitate domestic enterprises to improve their technology and quality, gradually supporting the development of foreign enterprises to enter the global value chain [1]. Influencing the supply chain system, FDI has significantly promoted the sound and rapid development of the national economy.
Therefore, it is necessary to focus on the future tendency of FDI in the supply chain system when we investigate the transformation and innovation of Chinese economy.

Since the late 1970s, FDI attracted by China has been steadily increasing, regardless of the changes and fluctuation of the international economic environment and the total flow of FDI globally. Statistically, over the period from 1979 to 2010, China’s actual use of FDI amounted to $1048.31 billion \[2\], and FDI keeps a rapid growth. According to the data of Ministry of Commerce of the People’s Republic of China (PRC) (**Figure 1**), the FDI in China presented a rising trend over the period from 1990 to 2016. The vital roles in the economic development of China are as follows. Firstly, the proportion of basic industries in China declines generally, and the proportion of agricultural output drops by 18% over the period between 1978 and 2011 \[3\]. Secondly, for a long time, FDI mainly concentrates in secondary and tertiary industries, accelerating the restructuring and upgrading of China’s industries \[4\]. Finally, FDI provides investment capital and promotes the rapid development of China’s import and export trade, improving China’s status in international trade.

Due to the remarkable role of FDI, a multitude of scholars began to track and study the FDI in developing countries, build analytical framework, and launch a new field of research of FDI in developing countries. The statistics shows that China has become an emerging market for FDI. Dees indicates that FDI has positive effects on the GDP, technological progress, and the improvement of management system \[5\]. Nourzad considers that FDI promotes economy development through technology transfer \[6\], while Mah argues that the latter one promotes the former one \[7\]. Taking the reform policy (implemented in July 2005) as the boundary, Pan and Song explore the impact of the effective exchange rate of RMB on FDI \[8\]. Research shows that they are in a long-term equilibrium relationship before implementing reform policy. After the policy, the exchange rate of RMB has the Granger causality for FDI, and the appreciation of RMB can promote the flow of FDI. Additionally, De Mello shows that FDI can increase the added value associated with it \[9\]. Based on the data from 1971 to 2012, Dreher et al. conclude that the membership in international organizations is an essential and decisive factor of FDI.

![Figure 1. The horizontal curve of FDI in China.](image-url)
liquidity and has a promoting effect on FDI mobility [10]. Badr and Ayed do a quantitative study of the relationship between FDI and economic development in South American countries, and they find that FDI can be determined by some economic factors, having no important effect on economic development [11]. Kathuria et al. apply panel data to examining the effectiveness of public policy in attracting FDI [12]. Lin et al. divide the FDI company into five strategies [13]; Brülhat and Schmidheiny estimate the rivalness of state-level inward FDI [14].

The trend of FDI in the future is an important reference for China’s economic development. However, much literature focuses on the development of FDI itself and its influencing factors, and there is little research on the future development. This is what we do in this chapter. Currently, the predictive analysis model for economic and trade development can be divided into linear prediction method and nonlinear prediction one. The linear prediction method mainly includes historical average level prediction method, time series prediction method, and Kalman filter prediction method, to name just a few. The nonlinear prediction methods concern Gray theory, Markov chain, support vector machine, and boom prediction method. The historical average prediction algorithm is simple and easy to understand and the parameters can be estimated by using the least squares method. However, it is too simple to accurately reflect the randomness and nonlinearity, and therefore it cannot be applied to unexpected events. The Kalman filter uses the flexible recursive state space model, with the advantages of linear, unbiased, and minimum mean variance. Nevertheless, because the Kalman filter prediction model belongs to the linear model, its performance becomes worse in the nonlinearity and uncertainty [15]. The time series model is simple in modeling, with high prediction accuracy in the case of full historical data. The Gray model can be modeled with less information, handling data easily and having higher accuracy, which can be extensively used in several fields [15–18]. However, Gray model becomes less attractive for time series with large stochastic fluctuation. Markov stochastic process predicts the development and changes of dynamic system according to the transfer probability of different states, and the transfer probability reflects the influence degrees of various stochastic factors and the internal law of the transition states. Therefore, it is more suitable to predict the problems with large stochastic fluctuation. What cannot be ignored is that Markov model requires data to meet the characteristics of no effect. Consequently, when using a simple model, it is very difficult to obtain a better prediction result, and the combination method becomes a popular method.

Through the vector autoregressive moving average (VARMA), Bhattacharya et al. compare and analyze the consumer price index sequence (CPI) and improve the forecasting accuracy [16]. The Gray model (proposed by a Chinese scholar, Professor Deng) and the Markov model (proposed by a Russian mathematician, Markov) have been combined very early, which is called Gray-Markov model (GMM). Based on the Gray prediction model, GMM is used to solve the inaccurate problems resulting from the large random fluctuation of the data and widely promoted in the fields of financial economy, agricultural economy, and resource and energy [17–20]. On the basis of GM(1,1), Li et al. propose an improved GM(2,1) model [21]. Based on the model of GM(1,1) and Markov stochastic process and combining Taylor formula approximation method, Li et al. construct a model of T-MC-RGM(1,1) and verify its validity by the example of thermal power station in Japan [22].

The level of FDI in China is influenced by many factors such as fixed investment, laws and regulations, corporate culture, innovation ability, and financial market stability, among others. To clearly recognize and describe the role of FDI, the foreign investment system is abstracted as a Gray system with no physical prototype and incomplete information, which can be predicted with GM(1,1)
model. Meanwhile, the FDI level in the previous year has no direct influence on that in the next year, in line with the no-effect characteristic of Markov stochastic process. On the basis of the previous study of Gray-Markov model, it is used to predict the tendency of FDI in China, addressing the shortcomings of the Gray model for the low precision of the data sample with large fluctuation and compensating for the limitation that the Markov model requires the data to have a smooth process. As a comparison, the time series prediction model is introduced to evaluate FDI. Then, the fitting results are compared to decide the optimal prediction model.

2. Gray-Markov model

Gray-Markov model is a forecasting method integrating the Gray theory with the Markov theory \[17–25\]. Firstly, GM(1,1) is constructed to obtain the predicted residual value. Then, the error state can be divided according to the residual values, and the error state can be obtained in light of the Markov prediction model. Then, based on the error state and transition matrix, the predicted sequence from GM(1,1) can be adjusted to obtain more precise predicting internals. The traditional GM(1,1) has its advantage in short-term prediction, while it has a poor fitting effect in forecasting the long-range and fluctuating data series. And the benefit of Markov stochastic process is the prediction of the large data series with random volatility. GMM has been proposed by He to predict the yield of cocoon and oil tea in Zhejiang Province. Subsequently, this model is widely used in the prediction of transportation, air accidents, and rainfall. Accordingly, we use GMM to predict FDI of China \[26–28\].

2.1 Gray model

The Gray system theory, founded and developed by Chinese scholar Deng, extends the viewpoints and methods of general system theory, information theory, and cybernetics to the abstract system of society, economy, and ecology, incorporating the development of mathematical methods to develop the theory and method of Gray system. The modeling process is as follows.

1. Raw series are
   \[ X^{(0)} = \{x^{(0)}(1), x^{(0)}(2), \ldots, x^{(0)}(m)\} \]  

2. To weaken the randomness of the original data, the accumulated generating series is derived:
   \[ X^{(1)}(k) = \sum_{i=1}^{k} x^{(0)}(i). \]  

3. Based on the sequence of \(X^{(1)}(i)\), a new sequence \(Z^{(1)}(i)\) is derived as follows:
   \[ Z^{(1)}(k) = \frac{1}{2} x^{(1)}(k) + \frac{1}{2} x^{(1)}(k - 1) \]  

4. Then, whitened differential equation is obtained:
   \[ x^{(0)}(k) + aZ^{(1)}(k) = b \]  

In Eq. (4) \(a\) is development coefficient, \(b\) is the parameter of Gray action, and \(\Phi\) is identification parameter vector. Then, the least squares estimation of parameters satisfies the following equation:
\[ \Phi = \begin{pmatrix} \hat{a} \\ \hat{b} \end{pmatrix} = (B^T B)^{-1} B^T Y \] (5)

and

\[ B = \begin{pmatrix} -Z^{(1)}(2) & 1 \\ -Z^{(1)}(3) & 1 \\ \vdots & \vdots \\ -Z^{(1)}(m) & 1 \end{pmatrix}, Y = \begin{pmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ \vdots \\ x^{(0)}(m) \end{pmatrix} \] (6)

By differentiating \( x^{(1)}(k) \), a whitened differential equation can be written as

\[ \frac{dx^{(1)}(k)}{dt} + ax^{(1)}(k) = b \]

(5) The whitened time response is as follows:

\[ \hat{x}^{(1)}(k + 1) = \left( x^{(1)}(1) - \frac{\hat{b}}{\hat{a}} \right) e^{-\hat{a}k} + \frac{\hat{b}}{\hat{a}} \] (7)

Reducing the sequence of \( \hat{x}^{(1)}(k + 1) (k = 1, 2, \ldots, m - 1) \), the following sequence is obtained:

\[ \hat{X}^{(0)} = \left\{ \hat{x}^{(0)}(1), \hat{x}^{(0)}(2), \ldots, \hat{x}^{(0)}(m) \right\} \] (8)

(6) Model testing

Model test is divided into residual test and Gray-relating test. Residual test is to obtain the difference between predicting value and the actual value. Firstly, the absolute residuals and relative residuals about \( X^{(0)} \) and \( \hat{X}^{(0)} \) are calculated:

\[ \Delta^{(0)}(i) = \hat{x}^{(0)}(i) - x^{(0)}(i) (i = 1, 2, \ldots, n) \] (9)

\[ \phi(i) = \frac{\Delta^{(0)}(i)}{\hat{x}^{(0)}(i)} (i = 1, 2, \ldots, n) \] (10)

Then, below is the average value of relative residuals:

\[ \Phi = \frac{1}{n} \sum_{i=1}^{n} \phi_i \] (11)

Given the value of \( \alpha \), it is called residual qualification model when \( \Phi < \alpha \). The value of \( \alpha \) can be 0.01, 0.05, or 0.10, and the corresponding model is perfect, qualified, and barely qualified.

As shown in Eq. (12), Gray correlation degree measures the correlating coefficient between the original sequence and the reference sequence:

\[ e_i(k) = \min_k \min_i |x(k) - x_i(k)| + \rho \max_k \max_i |x(k) - x_i(k)| \]

\[ \frac{|x(k) - x_i(k)| + \rho \max_k \max_i |x(k) - x_i(k)|}{|x(k) - x_i(k)| + \rho \max_k \max_i |x(k) - x_i(k)|} \] (12)

\( i \) denotes the \( i \)th group of fitting data, and \( k \) denotes the \( k \)th one in a certain group. \( \rho \) denotes the distinguish coefficient varying from 0 to 1, which is always set as 0.5. However, the correlation coefficient varies with moments, which results in disperse information. Combining the correlation coefficient in different moments
together, we can obtain the correlation degree between the original curve and the fitting curve:

\[ r_i = \frac{1}{n} \sum_{k=1}^{n} e_i(k) \] (13)

### 2.2 Markov model

Markov chain is proposed by Andrey Markov (1856–1922), and it is a discrete time stochastic process with Markov property in mathematics. Given the current knowledge and information, historical information has no impact on the future. To improve prediction accuracy, Markov model is used to handle the data obtained by GM(1,1). It is critical to divide state and build transition matrix.

#### 2.2.1 Dividing states

To divide states, four rules are suggested to follow. Firstly, the partition state must have at least one true value in each state. Secondly, elements in a one-step transition matrix cannot be the same. Thirdly, the actual values must fall into one state. Finally, the state must pass Markov test. The numbers vary according to the original data. In this chapter, the overall level of FDI in China is on the rise while fluctuating in detail. Therefore, the level of FDI is a non-stable stochastic process. Taking the curve of \( \hat{Y}(k) = \hat{x}(0)(k + 1) \) as reference, the sequence can be divided into \( n \) states. The intervals can be denoted as \( Q_i = [Q_{1i}, Q_{2i}] \) and \( i = 1, 2, ..., n \), in which \( Q_{1i} = \hat{Y}(k) + E_{1i} \) and \( Q_{2i} = \hat{Y}(k) + E_{2i} \).

#### 2.2.2 Transition matrix

Assuming that there are \( n \) states denoting as \( E_1, E_2, ..., E_n \), the transition probability amounts to frequency approximately in general, namely, \( P_{ij} = \frac{M_{ij}^{(l)}}{M_{jj}} \). Then, we can get the \( l \)th step transition matrix \( P(l) = (P_{ij}^{(l)})_{n \times n} \). \( M_{ij}^{(l)} \) is the data of raw series transferring \( l \) step from the state \( Q_i \) to the state \( Q_j \).

#### 2.2.3 The forecasting value

The eventual forecast is in the center of the Gray zone, which is denoted as \( Y'(k) = \frac{1}{2} (Q_{1i} + Q_{2i}) = \hat{Y}(k) = \frac{1}{2} (E_{1i} + E_{2i}) \). Eventually, the forecasting sequence is obtained as \( Y'(k) = \{Y'(1), Y'(2), ..., Y'(m)\} \).

### 3. Time series model (TSM)

Burg suggests that recursive algorithm estimated by the AR\( (P) \) model is the most practical one [29], while Hannan proposes time series with multidimensional linear stationary RMA\( (p,q) \). The times series model mainly includes the autoregressive model and the moving average model [30–32], and generally the modeling steps are as follows.
3.1 Preliminary analysis of data and modeling identification

Time series prediction is a statistical method processing dynamic data, which is a random sequence arranged in chronological order or a set of ordered random variables defined in probabilistic space \( \{X_t, t = 1, 2, ..., n\} \), in which the parameter \( t \) represents time. In the TSM, if the samples’ autocorrelation function \( \{\hat{\rho}_k\} \) decreases to zero based on the negative exponential function, then it can be preliminarily judged that this sequence is a stationary autoregressive moving average model (ARMA). If the absolute value of the sample autocorrelation function in the \( q \)-step delay \( \hat{\rho}_k(k \leq q) \) is greater than twice of the standard deviation and the value of \( \hat{\rho}_k(k > q) \) is less than twice of the standard deviation, then the sequence is \( q \)-step moving average model (MA(\( q \))). In a similar vein, we can judge \( p \)-step autoregressive moving average model (AR(\( p \))) according to the truncation situation of partial autocorrelation function \( \{\hat{\phi}_{kk}\} \).

3.2 Parameter estimation

In order to fit the TSM, we need to estimate the autoregressive coefficient \( \phi_i \), the moving average coefficient \( \theta_i \), the mean \( \mu \), and the variance \( \sigma^2 \) of the white noise sequence in the ARMA model.

3.3 Diagnostic test

The purpose of diagnostic test is to check and test the rationality of the model, including residual test, autocorrelation function of residual error and partial autocorrelation function test, and the significance test of parameters in the model.

3.4 Optimal model selection

Model recognition is only a preliminary selection of TSM. Considering the actual observed errors and statistical errors, several models are taken as candidate models. And the most common methods of selecting optimal models include F-test method, criterion function method (AIC criterion, BIC criterion, SBC criterion).

4. Comparison of GMM and TSM

4.1 GMM predicting FDI of China

Take the FDI value of China over the period from 1990 to 2016 as the original data (unit, $100 million; data source, Ministry of Commerce of the PRC):

\[
X^{(0)} = \{34.87, 43.66, 110.08, ..., 1260\}
\]

Based on Eq. (5) and using the software MATLAB, the least squares estimation (LSE) of FDI is as follows:

\[
\Phi = \begin{pmatrix} \hat{a} \\ \hat{b} \end{pmatrix} = \begin{pmatrix} -0.0697 \\ 243.795 \end{pmatrix}
\]

Based on Eq. (7), time-response function can be written as \( \hat{x}(k + 1) = 3530.59e^{0.0697k} - 3495.72 \). Residual values can be obtained according to relative...
error based on the prediction value of GM(1,1) model. To improve the predicting accuracy, the relative error can be divided into five states (E1, E2, E3, E4, E5) between 1990 and 2010. The relative error status can be seen in Tables 2.

According to the original FDI value over a period from 1990 to 2010 and the relative error of prediction value in GM(1,1), the transition matrixes of different steps $P_i^{(1)} (i = 1, 2, 3, 4, 5)$ are shown as follows:

\[
P_1^{(1)} = \begin{pmatrix}
3 & 1 & 0 & 1 & 0 \\
5 & 5 & 0 & 5 & 0 \\
1 & 0 & 0 & 0 & 0 \\
11 & 1 & 1 & 1 & 0 \\
4 & 4 & 0 & 2 & 0 \\
0 & 0 & 1 & 1 & 0 \\
0 & 0 & 1 & 2 & 0 \\
0 & 2 & 0 & 1 & 2
\end{pmatrix},
P_1^{(2)} = \begin{pmatrix}
3 & 0 & 0 & 2 & 0 \\
5 & 0 & 0 & 5 & 0 \\
1 & 0 & 0 & 0 & 0 \\
1 & 1 & 1 & 1 & 0 \\
4 & 4 & 4 & 4 & 0 \\
1 & 1 & 1 & 0 & 0 \\
6 & 0 & 3 & 2 & 0 \\
0 & 1 & 1 & 2 & 0
\end{pmatrix},
P_1^{(3)} = \begin{pmatrix}
1 & 0 & 0 & 3 & 0 \\
4 & 0 & 0 & 4 & 0 \\
1 & 0 & 0 & 0 & 0 \\
1 & 1 & 1 & 1 & 0 \\
2 & 0 & 1 & 1 & 0 \\
1 & 1 & 1 & 1 & 0 \\
6 & 6 & 3 & 3 & 0 \\
1 & 1 & 2 & 0 & 0
\end{pmatrix},
P_1^{(4)} = \begin{pmatrix}
0 & 0 & 1 & 3 & 0 \\
0 & 4 & 4 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 \\
2 & 3 & 0 & 0 & 1 \\
3 & 3 & 0 & 0 & 1 \\
1 & 1 & 1 & 1 & 0 \\
6 & 6 & 3 & 3 & 0 \\
1 & 1 & 2 & 0 & 0
\end{pmatrix},
P_1^{(5)} = \begin{pmatrix}
0 & 0 & 1 & 3 & 0 \\
0 & 4 & 4 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 \\
1 & 1 & 1 & 1 & 0 \\
3 & 3 & 3 & 0 & 0 \\
2 & 0 & 1 & 2 & 0 \\
5 & 0 & 5 & 5 & 0 \\
1 & 0 & 0 & 0 & 0
\end{pmatrix},
\]

| Residual State | E1 | E2 | E3 | E4 | E5 |
|----------------|----|----|----|----|----|
| Meaning        | Extremely underestimated | Underestimated | Reasonable | Overestimated | Extremely overestimated |
| Range          | $[-0.17, -0.10]$ | $[-0.10, 0.02]$ | $[0.02, 0.07]$ | $[0.07, 0.12]$ | $[0.12, 0.83]$ |

Table 1.
Relative error status of FDI level in China.

| Year | Original | Relative error of GM | State | Year | Original | Relative error of GM | State |
|------|----------|----------------------|-------|------|----------|----------------------|-------|
| 1990 | 34.87    | 0                    | E3    | 2001 | 468.78   | 0.0848               | E4    |
| 1991 | 43.66    | 0.8288               | E5    | 2002 | 527.43   | 0.0397               | E3    |
| 1992 | 110.08   | 0.5974               | E5    | 2003 | 535.05   | 0.0914               | E4    |
| 1993 | 275.15   | 0.0615               | E3    | 2004 | 606.30   | 0.0398               | E3    |
| 1994 | 337.67   | –0.0741              | E2    | 2005 | 603.25   | 0.109                | E4    |
| 1995 | 375.21   | –0.1131              | E1    | 2006 | 630.21   | 0.1318               | E4    |
| 1996 | 417.26   | –0.1545              | E1    | 2007 | 747.68   | 0.0394               | E3    |
| 1997 | 452.57   | –0.1679              | E1    | 2008 | 923.95   | –0.1071              | E1    |
| 1998 | 454.63   | –0.0941              | E1    | 2009 | 900.33   | –0.0061              | E2    |
| 1999 | 403.19   | 0.095                | E4    | 2010 | 1057.35  | –0.102               | E1    |
| 2000 | 407.15   | 0.1477               | E4    |      |          | –                    | –     |

Data source: China Statistical Yearbook over the period from 2000 to 2006, Ministry of Commerce of the PRC.

Table 2.
Comparison of GM(1,1) prediction value and original value of FDI of China.
Based on the transition matrix, we can obtain the error state over a period from 2011 to 2016 (see Table 3). Taking the middle value of the error state to modify the prediction value of GM(1,1) model, then the modified value can be seen in Table 3. And $x^{(0)}(k), \hat{x}^{(0)}(k)$, and $\phi(i)$ represent the original value, predicting value and relative error of GM(1,1). $\hat{x}^{(0)}(k)$ and $\phi'(i)$ represent the modified value and relative error of GMM.

In the light of Eqs. (9)–(11), the relative residual error of GM(1,1) and GMM is $0.0584 < \alpha = 0.1$ and $0.0458 < \alpha = 0.05$, respectively. Therefore, the GM(1,1) model is barely qualified, and the modified GMM model is qualified. Gray correlation degrees of the two models are 67 and 79.9%, respectively. In summary, the

| Year | $x^{(0)}(k)$ | $\hat{x}^{(0)}(k)$ | $\phi(i)$ | $\hat{x}^{(0)}(k)$ | $\phi'(i)$ |
|------|--------------|------------------|-----------|------------------|-----------|
| 1990 | 34.87        | 0.0349           | 0         | 33.4752          | -0.0471   |
| 1991 | 43.66        | 0.2550           | 0.8288    | 127.5082         | 0.6739    |
| 1992 | 110.08       | 0.2734           | 0.5974    | 136.7182         | 0.2332    |
| 1993 | 275.15       | 0.2932           | 0.0615    | 281.4594         | 0.0173    |
| 1994 | 337.67       | 0.3144           | -0.0741   | 326.9385         | -0.0328   |
| 1995 | 375.21       | 0.3371           | -0.1131   | 379.2043         | 0.0193    |
| 1996 | 417.26       | 0.3614           | -0.1545   | 406.5945         | -0.0172   |
| 1997 | 452.57       | 0.3875           | -0.1679   | 435.9630         | -0.0289   |
| 1998 | 454.63       | 0.4155           | -0.0941   | 467.4528         | 0.0360    |
| 1999 | 403.19       | 0.4455           | 0.0950    | 392.0632         | 0.0000    |
| 2000 | 407.15       | 0.4777           | 0.1477    | 420.3821         | 0.0582    |
| 2001 | 468.78       | 0.5122           | 0.0848    | 450.7465         | -0.0113   |
| 2002 | 527.43       | 0.5492           | 0.0397    | 527.2409         | -0.0056   |
| 2003 | 535.05       | 0.5889           | 0.0914    | 518.2134         | -0.0040   |
| 2004 | 606.30       | 0.6314           | 0.0398    | 606.1574         | -0.0055   |
| 2005 | 603.25       | 0.6770           | 0.1090    | 595.7787         | 0.0154    |
| 2006 | 630.21       | 0.7259           | 0.1318    | 638.8121         | 0.0407    |
| 2007 | 747.68       | 0.7784           | 0.0394    | 747.2223         | -0.0059   |
| 2008 | 923.95       | 0.8346           | -0.1071   | 938.8998         | 0.0246    |
| 2009 | 900.33       | 0.8949           | -0.0061   | 930.6540         | 0.0326    |
| 2010 | 1057.35      | 0.9595           | -0.1020   | 1079.4327        | 0.0291    |
| 2011 | 1160.11      | 1.0288           | -0.1276   | 1157.4006        | 0.0065    |
| 2012 | 1117.20      | 1.1031           | -0.0128   | 1241.0002        | 0.1077    |
| 2013 | 1175.90      | 1.1828           | 0.0058    | 1330.6383        | 0.1241    |
| 2014 | 1195.60      | 1.2682           | 0.0573    | 1116.0363        | -0.0417   |
| 2015 | 1262.70      | 1.3598           | 0.0714    | 1196.648         | -0.0260   |
| 2016 | 1260.00      | 1.4580           | 0.1358    | 1283.0826        | 0.0451    |

Annotations: The unit of $x^{(0)}(k)$ and $\hat{x}^{(0)}(k)$ is 1 billion dollars. The unit of $\hat{x}^{(0)}(k)$ is $10^3$ billion dollars. Data source: China Statistical Yearbook.

Table 3. Residual checklist of Markov model and GM(1,1).
prediction accuracy of GMM has been improved, and its fitting effect exceeds the model of GM(1,1).

### 4.2 TSM predicting FDI of China

Now we will build a TSM based on the FDI value of China over the period from 1990 to 2016, obtain the predicting data, compare the difference between the predicted data and the original date, and evaluate the accuracy of this model.

**Figure 1** shows the changing tendency of FDI in China over the period between 1990 and 2016. The raw data series show the seasonal change and overall growth, but the data series are not stable. Through the seasonal difference method to process the data, the seasonal difference order of three was selected. After the differential processing, the data sequence has been stabilized, eliminating the growing trend (**Figure 2**).

We determine the order of TSM based on sample autocorrelation function and partial autocorrelation function. After the one-step delay, the sample autocorrelation function falls to a standard error of twice times and has the property of truncation. After the two-step delay, the sample partial autocorrelation function falls to a standard error of twice times and has the property of truncation.

In the light of the calculation of SAS software, now we compare the model of ARMA(2,1), AR(2), and MA(1) (see **Tables 4** and **5**).

Comparing the AIC and SBC values for ARMA(2,1), AR(2), and MA(1) models (see **Table 4**), we find the model MA(1) to be the most inferior. Considering the AIC and SBC criterion values of ARMA(2,1) and AR(2) and the significance of parameters, it is found that fitting effect of the AR(2) model is the best.

As shown in **Table 5**, the P-value (Pr > ChiSq) for self-correlation test of the residual sequence with the 6-step delay, the 12-step delay and the 18-step delay are greater than that of the significant level $\alpha = 0.1$. Therefore, we cannot reject the hypothesis that residuals are non-autocorrelated. That is to say, the residual is regarded as a white noise sequence. This illustrates that the AR(2) model has extracted sufficient information from the raw series and it is a rational model:
Using Gray-Markov Model and Time Series Model to Predict Foreign Direct Investment Trend...
DOI: http://dx.doi.org/10.5772/intechopen.83801

\[
X_t = \varepsilon_t
\]

where \( X = \text{num} - 2.0711 \), \( t = \text{year} \), and \( \text{num} \) represents the FDI value of the corresponding year.

### 4.3 Comparison of prediction results of two models

#### 4.3.1 Accuracy assessment

Regarding how to select the appropriate accuracy evaluation criteria, Yokuma and Armstrong [33] have done a survey of expert opinions. They think that accuracy, clear physical meaning, and being easy to implement can be the critical evaluation criteria [33]. Accordingly, three criteria are used to evaluate the accuracy of the prediction model.

#### Table 4.
Prediction results of TSM.

| Model    | Parameter | Estimate | P-value  | AIC      | SBC     |
|----------|-----------|----------|----------|----------|---------|
| AR(2)    | MU        | 2.0711   | <0.0001*** | 1.0603   | 4.5944  |
|          | AR1,1     | 1.5357   | <0.0001*** |          |         |
|          | AR1,2     | -0.5392  | 0.0102**  |          |         |
| MA(1)    | MU        | 0.4676   | 0.0038***  | 28.7297  | 31.08558|
|          | MA1,1     | -0.7099  | 0.0001***  |          |         |
| ARMA(2,1)| MU        | 1.9380   | <0.0001***  | 0.7062   | 5.4184  |
|          | MA1,1     | -0.4940  | 0.1192     |          |         |
|          | AR1,1     | 1.2352   | 0.0015***  |          |         |
|          | AR1,2     | -0.2358  | 0.5028     |          |         |

Annotations: ***, **, and * indicate a significant level of 0.01, 0.05, and 0.1, respectively.

#### Table 5.
Self-correlation test of AR(2) model.

| To lag | 6    | 12   | 18   |
|--------|------|------|------|
| Chi-square | 3.91 | 5.44 | 8.02 |
| Pr > ChiSq  | 0.4187 | 0.8599 | 0.9481 |

#### Table 6.
Three criteria to evaluate the accuracy of models.

\[
\begin{align*}
\text{MSE} &= \frac{1}{n} \sum_{i=1}^{n} (x_i - \hat{x}_i)^2 \\
\text{MAE} &= \frac{1}{n} \sum_{i=1}^{n} |x_i - \hat{x}_i| \\
\text{MAPE} &= \frac{1}{n} \sum_{i=1}^{n} \left| \frac{x_i - \hat{x}_i}{x_i} \right|
\end{align*}
\]

Annotations: \( \hat{x}_i \) is the predicting value, \( x_i \) is the original value, and \( n \) is the predicting number.
4.3.2 Comparing predicted values with actual values

As shown in Table 6, the prediction accuracy of GMM has been improved manifestly compared with that in GM(1,1) model. Therefore, the forecasting value in GMM is closer to the actual level of China’s FDI. Then, from Figures 3 and 4, we can clearly see that GMM model has a better fitting effect than that in TSM.

5. Empirical analysis of FDI in Chongqing and Beijing

From discussions above, it is found that GMM has higher prediction accuracy and better fitting effects than those of TSM of Chinese FDI level. To further verify the credibility of this result, we construct GMM and TSM based on the FDI level of
Beijing (1990–2016) and Chongqing (1990–2015). The divided states involved in the GMM are shown in Table 7, and the transition matrixes of GMM associated with Beijing and Chongqing are denoted as $P_2$ and $P_3$. For simplification, we only list the form of transition matrix $P_2$. The comparison of GM(1,1) and GMM can be seen in Tables 8 and 9. The average relative errors of GM(1,1) and GMM of Beijing (Chongqing) are 0.0312 (0.5285) and $0.0029 (0.1051)$, respectively. The Gray Area Error state E1 E2 E3 E4 E5

| Area    | Error state | E1     | E2     | E3     | E4     | E5     |
|---------|-------------|--------|--------|--------|--------|--------|
| Beijing | Range       | [−0.47, −0.2] | [−0.2, −0.1] | [−0.1, 0.1] | [0.1, 0.28] | [0.28, 0.65] |
| Chongqing | Range     | [0, 0.28] | [0.28, 0.55] | [0.55, 0.75] | [0.75, 0.81] | [0.81, 0.97] |

Table 7. Residual states of FDI in Beijing and Chongqing.

Table 8. Comparison of predicted errors of GMM and GM(1,1) of Beijing FDI level.

Annotations: In Table 8, the unit of original value is 1 million dollars. GM, GMM, and TSM represent the predicted value of Gray model, Gray-Markov model, and time series model, and their unit is 10^6 million dollars. GR and MR represent the residuals of Gray model and Gray-Markov model. Data source: Beijing Statistical Yearbook (1990–2017), Beijing Municipal Bureau of Statistics.
relational degrees of GM(1,1) and GMM of Beijing (Chongqing) are 64.62% (75.26%) and 79.39% (86.82%), respectively. Therefore, the errors of GM(1,1) and GMM are barely qualified or qualified, and hence GMM is superior to GM(1,1):
Similar to Section 4.2, TSM of Beijing FDI can be modeled as MA (1):

\[ (1 + 0.82673B)(1 - B^2)X_t = \varepsilon_t \]

where \( X = \text{num} - 0.27264 \) and \( t = \text{year} \).

TSM of Chongqing FDI can be modeled as ARMA(1,2,1):

\[ (1 - 0.82442B)(1 + B)(1 - B^2)X_t = \varepsilon_t \]

where \( X = \text{num} - 2.178452 \) and \( t = \text{year} \).

Figure 5 (Figure 6) shows the difference between the original value and the predicting value in Gray-Markov model (time series model) of foreign direct investment in Beijing. It is apparent that the fitting effect of GMM is better than...
that of TSM. The similar conclusion can be drawn from Figures 7 and 8. Tables 9 and 10 show the predicting effect of GMM is better than that of TSM from the point of predicting errors and accuracy. There is no doubt that it is a good thing to predict accurately the foreign direct investment of the forthcoming 5 or 10 years for the domain specialists. Because if the predicting results is lower or higher than they expected, they could pay attention to seeking the critical factors and policy which have impacts on FDI and adjust them in advance.

Figure 6.
Original value and that in TSM of BJ. Annotations: BJ denotes the city of Beijing.

Figure 7.
Original value and that in GMM of CQ. Annotations: CQ denotes the city of Chongqing.
6. Conclusions and future work

Our contributions are threefold. Firstly, comparing the predicting results of the Gray-Markov model and the time series model and the original value, respectively, we can find that the fitting effect of the former (GMM) is better than the latter (TSM) and so does its scientific and practical importance. Secondly, the predicting results of GMM show that the level of foreign investment in China has been increasing by years. Thirdly, in order to further enhance Chinese international status and attract more foreign investment, the government should play a role at a macro level to reduce excessive market administrative intervention, establish a service-oriented government, and reduce the relevant approval procedures for international investment.

Figure 8. Original value and that in TSM of CQ. Annotations: CQ denotes the city of Chongqing.

Table 10. Comparison of predicting accuracy of two models.

| Area     | Index   | $MSE = \frac{1}{n} \sum_{i=1}^{n} (x_i - \tilde{x}_i)^2$ | $MAE = \frac{1}{n} \sum_{i=1}^{n} |x_i - \tilde{x}_i|$ | $MAPE = \frac{1}{n} \sum_{i=1}^{n} \frac{|x_i - \tilde{x}_i|}{x_i}$ |
|----------|---------|----------------------------------------------------------|----------------------------------------------------------|----------------------------------------------------------|
| Beijing  | GM(1,1) | $4.3599e+09$                                             | $3.9369e+04$                                             | $0.1032$                                                |
|          | GMM     | $5.4378e+09$                                             | $4.6731e+04$                                             | $0.1335$                                                |
|          | TSM     | $3.9173e+10$                                             | $1.3478e+05$                                             | $2.9584$                                                |
| Chongqing| GM(1,1) | $5.8230e+09$                                             | $3.4080e+04$                                             | $0.2663$                                                |
|          | GMM     | $3.9173e+10$                                             | $1.3478e+05$                                             | $2.9584$                                                |
|          | TSM     | $4.4524e+10$                                             | $1.0126e+05$                                             | $0.6018$                                                |

Annotations: $MSE$, $MAE$, and $MAPE$ denote mean squared error, mean absolute error, and mean absolute percentage error.
In the future work, the Gray-Markov model and time series model can be combined with other predicting model (e.g., support vector machine and dynamic Bayesian) to improve the accuracy. Also these models have the potential to be applied in the other areas such as finance (e.g., stocks, funds, and security), risk (e.g., financial risk and operational risk), and business (e.g., consumer price index and incomes).

Acknowledgements

This chapter is financially supported by the National Natural Science Foundation of China under grant nos. 71771080, 71172194, 71521061, 71790593, 71642006, 71473155, 71390335, and 71571065.

Author details

Yanyan Zheng¹*, Tong Shu², Shou Chen² and Kin Keung Lai³:⁴

1 School of Management, Xi’an Polytechnic University, Xi’an, China
2 Business School, Hunan University, Changsha, China
3 International Business School, Shaanxi Normal University, Xi’an, China
4 Department of Management Sciences, City University of Hong Kong, Kowloon, Hong Kong, China

*Address all correspondence to: yanzheng@whu.edu.cn; shutong@hnu.edu.cn

IntechOpen

© 2019 The Author(s). Licensee IntechOpen. This chapter is distributed under the terms of the Creative Commons Attribution License (http://creativecommons.org/licenses/by/3.0), which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited. 

© 2019 The Author(s). Licensee IntechOpen. This chapter is distributed under the terms of the Creative Commons Attribution License (http://creativecommons.org/licenses/by/3.0), which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.
References

[1] Choe JI. Do foreign direct investment and gross domestic investment promote economic growth. Review of Development Economics. 2003;7(1): 44-57

[2] Shu T, Liu CX, Chen S, Wang SY, Li JQ. The influence and demonstration of FDI on China’s economic growth in supply chain system. Systems Engineering - Theory & Practice. 2014; 34(2):282-290

[3] Wei KL. Foreign direct investment and economic growth in China. Beijing: Economic Science Press; 2013

[4] Pan FH, Wang JC. From “passive embedding” to supply Chain Park investment: A new model of FDI. China Soft Science. 2010;25(3):95-102

[5] Dees S. Foreign direct investment in China: Determinants and effects. Economics of Planning. 1998;31(2): 175-194

[6] Nourzad F. Openness and Efficiency of FDI: A panel stochastic production frontier study. International Advances in Economic Research. 2008;14(1):25-35

[7] Mah JS. Foreign direct investment inflows and economic growth of China. Economic Research Journal. 2002;48(3): 69-75

[8] Pan WR, Song Y. Empirical analysis for the impact of RMB real effective exchange rate on foreign direct investment in China. Journal of Chemical and Pharmaceutical Research. 2014;6(5):1830-1836

[9] De Mello LR Jr. Foreign direct in developing countries and growth: A selective survey. The Journal of Development Studies. 1997;34(1):1-34

[10] Dreher A, Mikosch H, Voigt S. Membership has its privileges-The effect of membership in international organizations on FDI. World Development. 2015;66:346-358

[11] Badr OM, Ayed TL. The mediator role of FDI in North Africa: Case of Egypt. 2015;3(1):1-7

[12] Kathuria V, Ray P, Bhangaonkar R. FDI (foreign direct investment) in wind energy sector in India: Testing the effectiveness of state policies using panel data. Energy. 2015;80:190-202

[13] Lin HL, Hsiao YC, Lin ES. The choice between standard and non-standard FDI production strategies for Taiwanese multinationals. Research Policy. 2015;44(1):283-293

[14] Brühlhart M, Schmidheiny K. Estimate the rivalness of state-level inward FDI. Journal of Regional Science. 2015;55(1):139-148

[15] Wang YB, Papageorgiou M. Real-time freeway traffic state estimation based on extended Kalman filter: A general approach. Transportation Research Part B. 2005;39(2):141-167

[16] Bhattacharya PS, Thomakos DD. Forecasting industry-level CPI and PPI inflation: Does exchange rate pass-through matter? International Journal of Forecasting. 2008;24(1):134-150

[17] Li GD, Yamaguchi D, Nagai M. Application GM(1,1)-Markov chain combined model to China automobile industry. International Journal of Industrial and Systems Engineering. 2007;2(3):327-347

[18] Wang CB, Li LH. The forecast of gold price based on the GM(1,1) and Markov chain. Gray Systems and Intelligent Services. 2007;18(1):739-743

[19] Lin CT, Lee IF. Artificial intelligence diagnosis algorithm for expanding a
[20] Aries MB, Veitch JA, Newsham GR. Windows, view, and office characteristics predict physical and psychological discomfort. Journal of Environmental Psychology. 2010;30(4):533-541

[21] Li GD, Masuda S, Yamaguchi D, Nagai M, Wang CH. An improved grey dynamic GM(2,1) model. International Journal of Computer Mathematics. 2010;87(7):1617-1629

[22] Li GD, Masuda S, Nagai M. The prediction model for electrical power system using an improved hybrid optimization model. Electrical Power and Energy Systems. 2013;44(1):981-987

[23] Lin LC, Wu SY. Analyzing Taiwan IC assembly industry by Gray-Markov forecasting model. Mathematical Problems in Engineering. 2013;70(2):717-718

[24] Chen YY, Liu CX. An improved GM (1,1)-Markov model in supply chain disruption and its application in demand prediction. Information Technology Journal. 2014;13(13):2204-2210

[25] Dong S, Chi K, Zhang QY, Zhang XD. The application of a Gray Markov model to forecasting annual maximum water levels at hydrological stations. Journal of Ocean University of China. 2012;11(1):13-17

[26] Syamala P, Vishnu B. Gray model for stream flow prediction. Aceh International Journal of Science and Technology. 2012;1(1):14-19

[27] Sun W, Wang JM, Chang H. Forecasting carbon dioxide emissions in china using optimization grey model. Journal of Computers. 2013;8(1):97

[28] He Y, Bao YD. Gray Markov model and its application. Systems Engineering - Theory & Practice. 1992;12(4):59-63

[29] Burg JP. Maximum Entropy Spectral Analysis. In: Proceedings of the 37th Meeting of the Society of Exploration Geophysicists, Oklahoma City, Oklahoma; 1967

[30] Wang ZX, Chan LW. Learning causal relations in multivariate time series data. ACM Transactions on Intelligent Systems and Technology. 2012;3(4):76-104

[31] Kadri F, Harrou F, Chaabane S, Tahon C. Time series modelling and forecasting of emergency department overcrowding. Journal of Medical Systems. 2014;38(9):1-20

[32] Rivero CR, Pucheta J, Laboret S. Time series forecasting using bayesian method: Application to cumulative rainfall. IEEE Latin America Transactions. 2013;11(1):359-364

[33] Yokum JT, Armstrong JS. Beyond accuracy: Comparison of criteria used to select forecasting methods. International Journal of Forecasting. 1995;11(4):591-597