Decrease of the effective cosmological constant in the Poincare gauge theory of gravity with a scalar field

O V Babourova\textsuperscript{1} and B N Frolov\textsuperscript{2}

\textsuperscript{1}Moscow Automobile and Road Construction State Technical University (MADI), Department ‘Physics’, Leningradsky pr., 64, Moscow, 125319, Russia
\textsuperscript{2}Moscow Pedagogical State University (MPGU), Institute of Physics, Technology and Information Systems, M. Pirogovskaya ul. 29, Moscow 119992, Russia

E-mail: ovbaburova@madi.ru, bn.frolov@mpgu.su

Abstract. Cosmological consequences of the Poincare gauge theory of gravity are considered. An effective cosmological constant depending from the Dirac scalar field is introduced. It is proved that at the super-early Universe, the effective cosmological constant decreases exponentially from a huge value at the Big Bang to its extremely small value in the modern era, while the scale factor sharply increases and demonstrates inflationary behavior. This fact solves the well-known “cosmological constant problem” also in the Poincare gauge theory of gravity.

1. Introduction
Earlier in [1-3] it has been shown that in the framework of the Poincare–Weyl gauge theory of gravity there is a solution in which one has the sharp exponential decrease of the effective cosmological constant at small time and the scale-invariant inflation-like exponential increase of the scale factor.

These results represent a development of the authors’ ideas since 2012, based on the assumption made by Harrison and Zeldovich about the approximate scale invariance of the early stage of the evolution of the Universe, which underlies the calculation of the initial part of the spectrum of primary fluctuations of matter density in the early Universe (Harrison–Zeldovich Plateau, [4,5]) and have been confirmed by results of the COBE experiment for the study of the anisotropy of the brightness of the background radiation.

We shall show that a similar result can be obtained in the Poincare gauge theory of gravity in the Riemann–Cartan space without nonmetricity. But only if at the same time we admit the existence of a cosmological scalar field, which simulates dark energy.

2. Lagrangian density and field equations
A truncated version of Poincare gauge theory of gravity is considered, the Lagrangian of which contains the squares of the torsion tensor (which thus plays a dynamic role in the theory), but excludes the squares of the curvature tensor:

\begin{equation}
L = 2f_{\alpha} \left[ (l/2) R^\alpha_b \wedge \eta_a^b + \beta^4 \lambda_\eta + \rho_1 \beta^2 T^a \wedge *T_a + \rho_2 \beta^2 (T^a \wedge \theta_b) \wedge *(T^b \wedge \theta_a) + \rho_3 \beta^2 (T^a \wedge \theta_b) \wedge *(T^b \wedge \theta_a) \right] + l \, d\beta \wedge *d\beta \right],
\end{equation}
where $\theta^a (a = 0,1,2,3)$ are basis 1-forms, $\ast$ is the Hodge dualization operation, $\eta$ is a volume 4-form with components $\eta_{abcd}$, and $\eta_{ab} = (1/2)! \theta^c \wedge \theta^d \eta_{abcd}$.

According to E. Gliner [6], the cosmological constant $\Lambda$ in the Einstein equations is interpreted as the energy of the hypothetical vacuum-like medium, which is currently called vacuum energy and also dark energy. The term $\beta^4 \Lambda_0$ in (1) describes the effective cosmological constant (interpreted here as dark energy), depending on the Weyl–Dirac scalar field $\beta$ [7] ($\Lambda_0$ is the theory parameter providing the correct rate of inflation).

The derivation of the variational field equations is based on the use of the lemma on commutation of variation and dualization operations of external forms in the Riemann–Cartan space, proved in [8] and then modified taking into account the presence of the scalar Weyl–Dirac field.

As a result of the variation procedure, we have obtained three field equations: $\Gamma$-equation, $\theta$-equation, and $\beta$-equation:

**$\Gamma$-equation:**

\[
\frac{1}{2} \mathcal{R}^a_{\ b} \wedge \eta^b_{\ a} + \beta^2 \Lambda_0 \eta_a + \rho_1 [2D \ast T_a + T_b \wedge \ast (T^b \wedge \theta_a) + \ast (T^a \wedge \theta_b) \wedge T_b + 4 \text{d} \ln \beta \wedge \ast T_a] + \rho_2 [2D(\theta_b \wedge \ast (T^b \wedge \theta_a)) + 2T^b \wedge \ast (\theta_b \wedge T_a) - \ast (T^b \wedge \theta_c \wedge \theta_a)(T^c \wedge \theta_b) - \ast (T^a \wedge \theta_c \wedge \theta_b)(T^c \wedge \theta_a) + 4 \text{d} \ln \beta \wedge \theta_b \wedge \ast (T^b \wedge \theta_a)] + \rho_3 [2D(\theta_a \wedge \ast (T^b \wedge \theta_b)) + 2T_a \wedge \ast (T^b \wedge \theta_b) - \ast (T^b \wedge \theta_c \wedge \theta_b)(T^c \wedge \theta_a) - \ast (T^b \wedge \theta_c \wedge \theta_a)(T^c \wedge \theta_b) + 4 \text{d} \ln \beta \wedge \theta_a \wedge \ast (T^b \wedge \theta_b)] + l [\ast \ast (\theta_a \wedge \ast \ast \text{d} \ln \beta) - \ast (\theta_a \wedge \ast \ast \ast \text{d} \ln \beta)] = 0.
\]

**$\theta$-equation:**

\[
\theta^a \wedge \chi^b_{\ a} + 4 \beta^2 \Lambda_0 \eta_a + \rho_1 [2T^a \wedge \ast T_a + 2T^a \wedge \ast (T^b \wedge \theta_a) + \ast (T^a \wedge \theta_b) \wedge T_b + 2 \text{d} \ln \beta \wedge \ast T_a] + \rho_2 [2T^a \wedge \ast (T^b \wedge \theta_a) + \ast (T^a \wedge \theta_b) \wedge T_b + 2 \text{d} \ln \beta \wedge \ast T_a] + \rho_3 [2T^a \wedge \ast (T^b \wedge \theta_a) + \ast (T^a \wedge \theta_b) \wedge T_b + 2 \text{d} \ln \beta \wedge \ast T_a] + l [\ast \ast \ast (\theta_a \wedge \ast \ast \ast \text{d} \ln \beta) - \ast (\theta_a \wedge \ast \ast \ast \ast \text{d} \ln \beta)] = 0.
\]

**$\beta$-equation:**

\[
\mathcal{R}^a_{\ b} \wedge \eta^b_{\ a} + 4 \beta^2 \Lambda_0 \eta_a + \rho_1 [2T^a \wedge \ast T_a + 2T^a \wedge \ast (T^b \wedge \theta_a) + \ast (T^a \wedge \theta_b) \wedge T_b + 2 \text{d} \ln \beta \wedge \ast T_a] + \rho_2 [2T^a \wedge \ast (T^b \wedge \theta_a) + \ast (T^a \wedge \theta_b) \wedge T_b + 2 \text{d} \ln \beta \wedge \ast T_a] + \rho_3 [2T^a \wedge \ast (T^b \wedge \theta_a) + \ast (T^a \wedge \theta_b) \wedge T_b + 2 \text{d} \ln \beta \wedge \ast T_a] + l [\ast \ast \ast \ast (\theta_a \wedge \ast \ast \ast \ast \text{d} \ln \beta) - \ast (\theta_a \wedge \ast \ast \ast \ast \ast \text{d} \ln \beta)] = 0.
\]

3. Cosmological model

The scale-invariant super-early stage of the Universe evolution is considered. Using the spatially flat FRW metric with the scale factor $a(t)$,

\[
ds^2 = dt^2 - a^2(t)(dx^2 + dy^2 + dz^2),
\]

let us solve the field equations (2)-(4). In this case the torsion 2-form is completely determined by the 1-form of its trace $T$, $T_a = (1/3) T \wedge \theta_a$. 


Then the $\Gamma$-equation (2) becomes equivalent to the equations,
\[
\mathcal{T} = s (d\beta / \beta), \quad s = -3, \quad \rho_1 - 2 \rho_2 = 0. \tag{6}
\]

For the $\theta$-equation (3) we obtain the only non-null expressions
\[
3H^2 + 6HU + U^2 (l + 3) = \Lambda_0 \beta^2, \tag{7}
\]
\[
2H_u + 2U_u + 4HU + 3H^2 - U^2 (l - 1) = \Lambda_0 \beta^2, \tag{8}
\]
where we use notations,
\[
H = a_i / a, \quad U = \beta_i / \beta, \quad a_i = da_i / dt, \quad \beta_i = d \beta_i / dt. \tag{9}
\]

For the $\beta$-equation (4) we have the only non-null expression
\[
3H_u + 6H^2 + (U_u + U^2 + 3HU) (l + 3) = 2\Lambda_0 \beta^2. \tag{10}
\]

After subtracting the equation (7) from the equation (8) we have,
\[
H_u + U_u - HU - U^2 (l + 1) = 0. \tag{11}
\]

Then let us subtract from the equation (10) the double equation (8) taking into account the equation (11). As a result we obtain the equation
\[
l (U_u + 3HU + 2U^2) = 0, \quad l \neq 0. \tag{12}
\]

When $l \neq 0$, we have three equations (7), (11) and (12) for two unknown functions $a(t)$ and $\beta(t)$, and the system of equations is over-determined.

In order to overcome this result, we hypothesize that the interaction constant $l$ is extremely small. In this case in equations (7) and (11), we may neglect the terms with the coefficient $l$. Then the equation (7) can be reduced to the equation
\[
H + U = (\lambda / 3) \beta, \quad \lambda = \sqrt{3\Lambda_0}. \tag{13}
\]

The sign “+” has been chosen here.

In this case we make sure that the equation (11) with regard to (12) becomes a consequence of the equation (13), and now the system of equations includes the two equations (12) and (13) for two unknown functions $a(t)$ and $\beta(t)$. Therefore the system of equations is well defined.

The equations (12) and (13) yield the equation,
\[
U_u + \lambda \beta U - U^2 = 0, \tag{14}
\]
which is equivalent to the equation
\[
\beta \beta_u + \lambda \beta \beta^2 - 2 \beta^2 = 0. \tag{15}
\]

Using the substitution $\beta_i = \beta^2 z (\beta)$, this equation is integrated with the first integral,
\[
U = \beta_i / \beta = - \lambda \beta \ln(C \beta), \tag{16}
\]
($C$ is an integration constant).

The second equation of the system is obtained by substituting (16) into the equation (13),
\[ H = \frac{a_t}{a} = \lambda \beta \left( \ln(C\beta) + 1/3 \right). \]

(17)

We shall use a boundary condition,

\[ C\beta \rightarrow 1, \text{ when } t \rightarrow \infty. \]

(18)

The limit \( \beta \rightarrow 1/C \) occurs exponentially, \( \sim e^{-\frac{t}{C}} \), for large \( t \).

To choose the value of the constant \( C \) we require that the value of the effective cosmological constant \( \beta^2(t)\Lambda_0 \) currently coincides with the modern cosmological constant value \( \Lambda \),

\[ \beta^2(t_U)\Lambda_0 = \Lambda. \]

(19)

Here \( t_U \) is the Universe lifetime \( (t_U = 13.8 \text{ billion years}) \), moreover the equality \( \beta_0^2\Lambda_0 = 10^{120} \Lambda \) should be fulfilled (see [9-11]), where \( \beta_0 \) is the Weyl–Dirac scalar field value at \( t = 0 \).

The result of numerical integration of the system of equations (16) and (17) (see [2,3]) with the boundary condition (18) is shown in Fig. 1 and Fig. 2, on which one can see for small and large time values the inflationary behavior of the scale factor \( a(t) \), as well as a sharp exponential decrease in the effective cosmological constant \( \beta^2(t)\Lambda_0 \) from a huge value at the beginning of the Big Bang to an extremely small (but not zero) value in the modern epoch, which coincides with its observed value.

**Figure 1.** The functions \( a(t), \beta(t) \) for small values of time \( t \).

**Figure 2.** The functions \( a(t), \beta(t) \) for large values of time \( t \).
4. Conclusion

We have shown that the sharp exponential decrease of the effective cosmological constant and the inflationary increase of the scale factor at the early stage of the Universe evolution can be realized not only in the Cartan–Weyl space with curvature, torsion and nonmetricity, but also on the basis of the Poincare gauge theory of gravity in the presence of only torsion along with curvature under the following conditions:

1) A cosmological scalar field (the Weyl–Dirac scalar field), which simulates dark energy, exists in Nature.
2) Dark energy has extremely weak self-action.

We follow the idea of E. Gliner about a homogeneous vacuum-like medium generating the gravitational field subsequently named as dark energy. In our theory this medium determines the non-constant effective cosmological “constant”, which depends on the Weyl–Dirac scalar field.

We also follow the idea of Harrison and Zeldovich that the Universe at the early stage has the property of approximate scale invariance. As a consequence, we get a scale invariant inflation.

Under the condition that the constant of the self-action of the field $\beta$ is substantially small (but not equal to zero), a cosmological solution is found, which is similar to that obtained in [1,2]. The peculiarity of this solution consists in an exponentially sharp decrease in the effective cosmological constant (within the time of inflation) with a transition to its present value at a large value of time. At the same time, the scale factor $a(t)$ demonstrates inflationary behavior.

The hypothesis on the sharply decrease of the effective cosmological constant as a consequence of the fields dynamics in the super-early Universe was expressed in the monograph [12] and the earlier papers [13–15]. The main thing of the solution obtained and that found earlier by the authors is that these solutions demonstrate for the effective cosmological constant sharply decrease at small $t$ and approach $\Lambda$ (but not zero) at large values of $t$.

It can be assumed that the found solution can serve as a basis for solving the well-known cosmological constant problem also in the Poincare gauge theory of gravity, and not only in the Poincare–Weyl gauge theory of gravity [1,2].

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