TDOA-based adaptive observer trajectory optimisation algorithm for tracking in passive coherent location system

Tong Jing\(^1\), Wei Tian\(^1\), Gaoming Huang\(^1\)\(^\text{✉}\)

\(^1\)School of Electronic Engineering, Naval University of Engineering, Wuhan, People’s Republic of China

E-mail: hgaom@163.com

Abstract: In the passive coherent location system with fixed observers, the target tracking performance is limited by the inappropriate localisation geometry. This study proposes an adaptive observer trajectory optimisation (AOTO) algorithm in terms of minimising the geometric dilution of precision (GDOP) based on the time difference of arrival (TDOA) measurements gained from multiple emitters. The target states are estimated by the extended Kalman filter (EKF) algorithm. Simulation results demonstrate that the AOTO algorithm can effectively improve the tracking accuracy compared with the corresponding tracking approaches with fixed observers.

1 Introduction

The passive coherent location system (PCLS) detects targets with measurements obtained by correlating the signals arriving directly from non-cooperative emitters and the echo signals reflected by targets. These signals are usually potentially commercial signals in the environment, such as digital video broadcast (DVB) signals [1], FM broadcast signals [2] etc. Recently, more and more attention has been paid to PCLS because of its advantages in anti-stealth, anti-low altitude penetration, anti-interference, and anti-radiation missile [3–5].

In the PCLS, the target-tracking accuracy depends on the positions of emitters and observers, measurement errors, and the targets states. Hence, the tracking performance may deteriorate because of the inappropriate localisation geometry with fixed observer. This paper aims at improving the tracking accuracy by optimising the trajectory of mobile observer.

A considerable amount of work has been proposed to solve the problem of optimising the observer’s trajectory in passive tracking utilising the angle of arrival (AOA) of signals emitted by transmitters in the literature [6, 7]. However, these methods are not applicable for PCLS tracking. The problem for PCLS tracking with a mobile observer was studied in [8] using the Doppler and AOA measurements from a single emitter. Yet, this method has several limitations. First, the AOA measurements have to be high-precision to maintain tracking accuracy, which puts stringent requirements on equipment performances. Moreover, the Doppler measurement depends on the signal frequency as part of the measuring function. Hence, the non-ideal signal frequency will decrease the estimation accuracy of the target states. Second, the observability and estimability of this method are limited due to the only one emitter utilised in the system. Third, once the unique emitter is unable to continue as a good signal source, the system will fail to work.

This paper studies the problem of optimal target tracking in the PCLS with multiple emitters and one mobile observer. An adaptive observer trajectory optimisation (AOTO) algorithm with time difference of arrival (TDOA) observations from multiple emitters in PCLS is proposed. Assume that the observer knows its own states and the positions of the multiple emitters. The observer can formulate its trajectory while tracking the target with the extended Kalman filter (EKF) algorithm. Similar to [8], geometric dilution of precision (GDOP) minimisation is applied to perform the criterion to optimally control the observer’s trajectory. This criterion finally converts to a one-dimensional non-linear programming problem, which produces the subsequent manoeuvring angle of the observer. Compared to [8], the proposed method has advantages in many aspects. First, the TDOA measurements with acceptable precision are easier to obtain than the AOA measurements. Second, multiple emitters can extend the observation scope of the system. Third, the robustness of the system can also be enhanced.

The paper proceeds as follows. The system model including the state model, observation model, and observer manoeuvre model are described in Section 2. Section 3 proposes the AOTO algorithm. Section 4 exhibits simulation results illustrating the superiority of the proposed method. Conclusions are shown in Section 5.

2 Model description

2.1 State model

Considering the PCLS with \(N\) fixed emitters and one mobile observer tracking a single target, this paper established a two-dimensional model. The geometrical situation for this model is depicted in Fig. 1. The target is assumed to manoeuvre on a straight line with a constant velocity \( \mathbf{v} \) and a fixed angle \( \beta \) with respect to the \(x\)-axis. Assume that the system does not have state noise. Hence, the target’s discretised states at a sampling interval \(T\) are given by

\[
x_t(k+1) = A x_t(k),
\]

where

\[
A = \begin{bmatrix} I_{2 \times 2} & TI_{2 \times 2} \\ 0_{2 \times 2} & I_{2 \times 2} \end{bmatrix}
\]

and \(x_t^T = [r_1^T, \mathbf{v}^T]\) is the target states vector, where \(r_1^T = [x_1, y_1]\) is the target’s position states and \(\mathbf{v}^T = [v_x, v_y]\) is the target’s velocity states.

2.2 Observation model

The TDOA observations gathered by the observer on the \(i\)th emitter evolve according to

\[
r_t(k) = \frac{\|r_0(k) - r_1\|_2 + \|r_1 - r_i\|_2 - \|r_i - r_0(k)\|_2}{c} + w_t(k),
\]

\((i = 1, 2, \ldots, N)\)
where $c$ represents the speed of light, $r_i^o = [x_i, y_i]$ and $r_i^e = [x_{o_i}, y_{o_i}]$ represent the observer and the $i$th emitter position states vector, respectively, and $w_i$ represents the observation noise, which is considered as a zero-mean white Gaussian sequence with variance $\sigma_i^2$.

Converting (3) to the matrix form yields the observation model as follows:

$$z(k+1) = h[x_i(k+1)] + w(k+1), \tag{4}$$

where $z_i^k = [t_1, t_2, \cdots, t_N]$ is the observation vector, $h(\cdot)$ is the measuring function, and $w$ is the observation noise vectors, which is assumed to be zero-mean, mutually uncorrelated, white noise sequences with covariance matrix $R$, and

$$R = \begin{bmatrix} \sigma^2_i & 0 & 0 \\ 0 & \sigma^2_i & 0 \\ 0 & 0 & \sigma_N^2 \end{bmatrix}. \tag{5}$$

### 2.3 Observer manoeuvre model

The observer is assumed to manoeuvre with a constant velocity $v_o$ and an alterable angle $\theta_o(k+1)$ with respect to the $x$-axis. In this paper, $\theta_o(k+1)$ represents the observer’s manoeuvring direction from the time step $k$ to $k+1$. It will be updated by the new measurements obtained at the time step $k+1$. Assume that the observer has a precise knowledge about its own states. Hence, the observer’s manoeuvre state model is

$$x_o(k+1) = Ax_o(k), \tag{6}$$

where $x_i^o = [v_{x, o}, v_{y, o}]$ is the observer's position vector, and $v_{i, o}^o = [v_{x, o}, v_{y, o}]$ is the observer's velocity states, where

$$v_{x, o} = v_o \cos \theta_o(k+1),$$

$$v_{y, o} = v_o \sin \theta_o(k+1). \tag{7}$$

### 3 Trajectory optimisation algorithm

This section presents the proposed AOTO algorithm for optimal target tracking in the PCLS. The main idea of the method is that, after estimating target's states at the time step $k$, the observer will find a subsequent manoeuvring angle $\theta_o(k+1)$ to determine its next position. At this position, the system will have the minimal GDOP. The loop of the AOTO algorithm is illustrated in Fig. 2.

Here, the measurements gathered by the observer are locally fused and utilised to formulate the observer's trajectory. Since TDOA-based PCLS tracking model is non-linear, EKF is used in this paper. The EKF estimator will produce a state estimate $\hat{x}_l(k+1)$ and an associated estimated error covariance $P(k+1)$ at each loop. The AOTO algorithm is designed to formulate the observer’s manoeuvring to minimise the $\sqrt{\text{trace}[P(k+1)]}$ for the purpose of making $\hat{x}_l(k+1)$ more precise.

This paper takes the GDOP minimisation as the criterion to achieve optimal control of the observer’s trajectory from a particular time step $k$ to $k+1$. It is well known that the GDOP can reflect whether the system’s geometry is suitable for positioning, namely, the lower the GDOP, the more precise the localisation accuracy [9]. In other words, the mobile observer seeks for the most favourable geometry of the system at each moment for the most precise target tracking accuracy. The GDOP is defined as

$$\text{GDOP}(k+1) = \sqrt{\text{trace}[P(k+1)]}.$$  

According to EKF algorithm, the estimation error covariance $P(k+1)$ depends on the prediction error covariance $P(k+1|k)$ and the Jacobian matrix of the measuring function $H(k+1)$, (see (8)), where $\rho_{ho}$ represents the distance between the target and the observer, and $\rho_{eo}$ is the distance between the target and the $i$th emitter.

The values of $P(k+1|k)$ and $H(k+1)$ actually depend on the observer’s states $x_o(k+1)$. Assume that the observer knows its own states at the time step $k$. Hence, $x_o(k+1)$ is the only one unknown

\[
H(k+1) = \begin{bmatrix} \frac{1}{c} \frac{x_i - x_o}{\rho_{e1}} + \frac{y_i - y_o}{\rho_{e1}} & \frac{1}{c} \frac{y_i - y_o}{\rho_{e1}} + \frac{y_i - y_o}{\rho_{o1}} & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots \\ \frac{1}{c} \frac{x_i - x_o}{\rho_{eN}} + \frac{y_i - y_o}{\rho_{eN}} & \frac{1}{c} \frac{y_i - y_o}{\rho_{eN}} + \frac{y_i - y_o}{\rho_{oN}} & 0 & 0 \end{bmatrix}. \tag{8}
\]
in process of calculating GDOP\((k+1)\). For getting the observer's subsequent states \(x_o(k+1)\), the goal is to decide the value of \(\theta_o(k+1)\) calculated by minimising the GDOP\((k+1)\). Hence, the trajectory optimisation problem now converts to solve the optimal value of \(\theta_o(k+1)\)

\[
\text{minimise} \sqrt{\text{trace}[P(k+1)]}.
\]

Looking for a minimum of GDOP\((k+1)\), formula (9) is actually a one-dimensional non-linear programming problem with subject to \(\theta_o(k+1) \leq \pi\). After acquiring \(\theta_o(k+1)\), the observer can subsequently determine where to manoeuvre next. An \(M\)-step loop of the proposed algorithm is demonstrated in Fig. 3.

### 4 Simulations

#### 4.1 Simulation results

This section exhibits simulation results illustrating the superiority of the proposed algorithm. Here, we define the target position estimation error as \(\text{RMS} = \sqrt{(\Delta x)^2 + (\Delta y)^2}\), where \(\Delta x\) and \(\Delta y\) represent the error between the estimated value and the actual value of the target's coordinates, respectively. Assume that three fixed emitters are deployed in the PCLS and the system can acquire correct observations at each time step ignoring the effects of clutter. The simulation parameters are demonstrated in Table 1.

First, we will compare the performances of the AOTO algorithm with the fixed observer tracking and the mobile observer tracking with two additional non-optimal trajectories randomised. The observer's velocity is assumed to be 100 m/s. As illustrated in Fig. 4, the green dash-dotted line and red solid line represent the results of two non-optimal trajectories, respectively. The pink-dashed line represents the results of the optimal trajectory formulated by the AOTO algorithm. Then, the blue dotted line represents the results of the target tracking with a fixed observer located in \(0\ 0\ T\) (km). Moreover, the green square, red circle, and yellow triangle represent the three positions of emitters, respectively.

Second, to verify the superiority of the proposed method, a lot of experiments were tested in different geometries of the system. However, due to the space constraints of this paper, only one situation is listed. Here, three emitters' positions are changed to \(r_{e1} = 0\ 5\ T\) (km), \(r_{e2} = -21\ -25\ T\) (km), and \(r_{e3} = 15\ 0\ T\) (km), respectively. We define the emitters' positions presented in Table 1 as 'geometry 1'. The new positions after the change are defined as 'geometry 2'. As exhibited in Fig. 5, the green dash-dotted line and red solid line represent the results of the trajectory optimisation and the fixed observer tracking in 'geometry 1', respectively. The pink dashed line and blue dotted line represent the results of the target tracking with a fixed observer located in \(0\ 0\ T\) (km). Moreover, the green square, red circle, and yellow triangle represent the three positions of emitters, respectively. Emitters' graphics remain unchanged.

### Table 1 Simulation settings

| Parameter  | Value                                   |
|------------|-----------------------------------------|
| \(r_{e1}\) | \([0\ 5]\ T\) (km)                     |
| \(r_{e2}\) | \([15\ 0]\ T\) (km)                     |
| \(r_{e3}\) | \([0\ -33]\ T\) (km)                   |
| \(v_t\)   | 100 m/s                                 |
| \(\beta_t\) | \(\frac{3}{\pi}\) (rad)               |
| \(\tau\)  | 1 s                                    |
| \(\{m_0\}_{i=1}\) | \(1\) (μs)       |
| \(\hat{x}(0)\) | \([120.2\ 100.3\ -70.7\ -70.7]\ T\) |
Third, the effects of signal-to-noise ratio (SNR) on the proposed algorithm will be analysed. Since the AOTO algorithm processes at the data level, the probability of getting correct observations in the clutter is used to represent the value of SNR. We define this probability as $P_c$. Furthermore, in order to intuitively exhibit the performance of the AOTO algorithm under different SNR, the degree of improvements produced by the proposed method is defined as $D_{fo} = \text{RMS}_f - \text{RMS}_o$, where $\text{RMS}_f$ and $\text{RMS}_o$ represent the RMS created by the fixed observer tracking and the trajectory optimisation, respectively. As shown in Fig. 6a, the lines with three colours represent the results of tracking with trajectory optimisation and fixed observer under the values of $P_c$ being 70, 80 and 95%, respectively. Among them, the lines with circle, asterisk, and triangle represent the results of the fixed observer tracking. In Fig. 6b, the lines with red (circle), blue (asterisk) and black (triangle) represent the results of $D_{fo}$ under the values of $P_c$ being 70, 80 and 95%, respectively.

4.2 Results discussion

This sub-section comes to the conclusions according to the presented simulation results. First, as shown in Fig. 4, the observer

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*Fig. 4* Performances of the optimal trajectory versus the fixed observer tracking and the tracking with non-optimal trajectories
(a) Trajectories of the target and the observer, (b) Comparison of RMS, (c) Comparison of GDOP

*Fig. 5* Performances of the optimal trajectory and the fixed observer tracking versus geometries
(a) Trajectories of the target and the observer in geometry 2, (b) Comparison of RMS, (c) Comparison of GDOP
manoeuvres in the direction of making the system’s GDOP better. According to RMS, obviously, the AOTO algorithm presents superior performances to both the tracking with non-optimal trajectories and the tracking with the fixed observer. The proposed algorithm could keep the estimation errors to a minimum at each moment. Second, the results indicate that the GDOP first increases and then decreases with the time increases. This is reasonable, because it is exactly the trend of estimation error covariance. Furthermore, the results of the GDOP produced by the optimal trajectory could keep the lowest all the time. This indicates that the proposed method could achieve the most favourable geometry of the system at each sampling interval. Third, to our knowledge, changing the geometry can not only reformulate the observer trajectory but also affect the target tracking accuracy. This conclusion can be proved by Fig. 5. Moreover, when the geometry is appropriate for target tracking, such as ‘geometry 1’, the AOTO algorithm can obviously improve the estimation accuracy. Later, when the geometry is non-ideal caused by changing the positions of emitters, such as ‘geometry 2’, the improvement in tracking accuracy is even greater. Hence, it is shown that the proposed algorithm has robustness. Fourth, the proposed method can still present better performances when $P_c$ gets lower. However, the degree of the improvement provided by the AOTO algorithm will decrease with the SNR decreases.

5 Conclusion

An observer trajectory optimisation algorithm was proposed to improve the performance of target tracking in the PCLS. This algorithm was derived based on the TDOA measurements from multiple emitters under the criterion of GDOP minimisation. The trajectory optimised is used for formulating relatively advantageous geometry of the whole system for minimising the estimate errors of target states at every time step. The proposed algorithm can achieve more precise tracking accuracy than the fixed observer tracking as well as the mobile observer tracking with non-optimal trajectories. Moreover, the robustness of the proposed algorithm was exhibited. The AOTO algorithm can still perform well in the non-ideal geometry and under lower SNR.

6 Acknowledgments

The work was supported by the General Program Supporting Fund of China Postdoctoral Science Foundation (No. 2017M613370).

7 References

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