Damage identification in plate using wavelet transform and artificial neural network

Muyideen Abdulkareem1*, Abideen Ganiyu2 and Muhd Z. Abd Majid1

1Construction Research Centre, Universiti Teknologi Malaysia, Johor, Malaysia
2Department of Civil Engineering and Quantity Surveying, Military Technological College, Muscat, Oman

*Corresponding author: muyikareem@gmail.com

Abstract. Detection of damage at early stage is necessary to prevent catastrophic collapse of civil structure. The quantification of small damage has however, not received the much needed attention from researchers. Amongst various non-destructive methods studied by researchers to identify structural defects are Wavelet Transform (WT) and Artificial Neural Network (ANN). In this paper, a technique is proposed to quantify damage by combining WT and ANN. The study is divided into two phases. The first phase involves detection and location of damage in a plate numerical model by WT decomposition of the mode shape difference. Due to difficulties in obtaining higher mode shapes in practice, the difference of the first mode shapes of the damaged and undamaged plates are applied. After obtaining the damage location, the coordinates of the damage location and the mean values of the obtained WT moduli are applied as input to the designed neural network. The output of the ANN is the severity of damage in the plate model. This method is demonstrated by using a numerical square steel model with all the four sides fixed. The results indicate the ease of the method and that reliable damage identification can be obtained by combining ANN and WT.

1. Introduction
Civil structures are the major assets of any nation. The health status of these structures are paramount to safety of lives and properties, as well as rising cost of maintenance. The health status of structures can be improved by early detection of damage. Such detection has gained attention of researchers over the past two decades. Researchers have applied different vibration-based damage detection methods such as change in frequency [1], mode shape [2], frequency response function (FRF) [3], modal curvature [4] and artificial neural network (ANN) [5] to identify and assess defects in different civil structures.

In applying WT to detecting damage, Garstecki et al. [6] applied dynamic and static numerical models to detect defects in structures by using discrete wavelet. Spanos et al. [7] identified crack damage in beams by establishing a WT modulus map. The WT modulus map had the boundary distortion eliminated and the local maxima due to damage were easily identified. Their technique was verified by employing pseudo-experimental and numerical test of beams with cracks up to three. Wu and Wang [8] applied Gabor wavelet to detect cracks of different depths in an aluminum cantilever beam. Static displacement was imposed on the structure and laser sensors were used to measure the deflection profile that was then decomposed. Rucka [9] examined the effects of higher modes in
damage detection. The WT technique considered Gaussian wavelets Gaus4, Gaus6, and Gaus8 and a cantilever beam by imposing varying depth rectangular notches at different locations. Cao et al. [10] employed a WT method to deal with measurement noise in modal curvature from mode shapes. A Teager energy operator was applied to analyse the modal curvature to produce Teager energy operator-WT modal curvature that has distinct suppressing noise capabilities, thus boosting the main feature caused by damage. The technique was validated using an experimental test consisting mode shapes acquired from a damaged aluminum beam obtained by a scanning laser vibrometer. Montanari et al. [11] proposed a spatial Continuous Wavelet Transform (CWT) damage identification method to determine the minimum number of sampling intervals required for optimal damage detection. In achieving this, the first three free vibration modes of a cantilever and a simple supported beam with an edge open crack were numerically simulated. In addition, the authors carried out a parametric analysis by considering the main parameters governing the problem.

More studies were done applying WT when Douka et al. [12] applied continuous wavelet transform (CWT) to detect crack damage in a plate by analyzing the plate’s mode shapes. Law et al. [13] proposed a WT technique to for assessing noise by analyzing the structure’s acceleration response. Even though the technique was able to detect local damage by taking measurements at few points, the technique showed flaws due to its sensitivity to modelling error and measurement noise. In the works of Sarrafi and Mao [14], a probabilistic WT method was applied to quantify uncertainty by using WT coefficients, and using outlier analysis to obtain the optimal detection threshold. Recently, Abdulkareem et al. [15] proposed a non-probabilistic WT technique to detect damage in plate structures. The WT coefficient values of the decomposed damaged and undamaged mode shape structures are obtained. Thereafter, interval analysis method was utilised to identify the position of damage.

A review of previous studies reveals that existing WT-based damage detection techniques are unable to detect and quantify of damage simultaneously. Although a few WT-based techniques tried to quantify damage [16], they have only been successful in estimating the damage to the overall structure [17]. On the other hand, ANN has the ability to recognize linear and non-linear relationships between two variables. It is the most explored artificial intelligent approach in vibration-based damage detection [18]. Another advantage is that it possesses noise filtering ability and can be trained with incomplete data.

One of the pioneers ANN applications in structural damage detection was done by Wu et al. [19]. The authors applied the FRF data of a numerical three-storey structure to identify damage that was initiated by reduction of stiffness of the columns. Although damage was accurately identified when the ANN input data were from the third floor, the results were flawed when the ANN was fed with measurements taken from the second floor and third floor. Sahin and Shenoi [20] applied different structural parameters as input to an ANN. The first three frequencies changes, maximum absolute difference in mode shape curvature, absolute difference in mode shape curvature and their corresponding damage locations as input vector of the neural network. The results indicated that the absolute difference in mode shape curvature provided the best result for damage identification. However, change in frequencies neither predicted damage location nor damage severity. Yam et al. [21] proposed a neuro-wavelet method to detect damage in composite structures. The energy variations of the structural responses were decomposed and damage features extracted were applied to train the neural network. The proposed method accurately located all the damage cases, however, few incorrect predictions were observed in the experimental study.

In addition, Lee et al. [22] utilised a simple beam and a multi-span bridge to compare results obtained by training an ANN with inputs of mode shapes, their ratios and their differences. They concluded that mode shape differences and mode shape ratios provided good results. Rucka and Wilde [23] applied ANN to identify damage in plate and shell structures by using the decomposed mode shape signals. Bagchi et al. [24] successfully obtained damage data of a bridge structure by employing ANN and modal parameters. The location of damage was identified first by using the natural frequencies and mode shapes, and subsequently using ANN to obtain the damage severity. Li et al.
[25] presented an ANN technique by using details of the residual FRF as input. The FRF was reduced by using principal component analysis (PCA) to obtain damage features that were applied to train different neural networks. They further applied an ensemble neural network to fuse the outcome of each neural network. It was shown that the network ensemble provided better results than the individual neural networks. Hakim et al. [26] applied ANN to detect damage in an I-beam by using natural frequencies and mode shapes as inputs. Recently, Khoshnoudian et al. [27] fed damage features from FRF extracted by using 2D PCA into a neural network to identify damage in truss bridge. In addition to using these methods, researchers have combined them with other methods to identify damage in structures. The combination of methods allows complementing of drawbacks of methods. Dackermann et al. [28] utilized ANN and FRF methods to identify defects on a frame structure. The efficiency of the technique showed that the bearing limit of uncertainty was 10%. This work was furthered by Fallahian et al. [29] when they added PCA and replaced the ANN with deep neural network (DNN). The effects of the uncertainties caused by temperature change on the undamaged structures were reduced by DNN. The PCA reduced the FRF to a defined pattern, while using temperature as the input to the DNN. These studies have shown that identification of structural damage is more feasible by combining other methods with ANN.

This study aims at proposing a damage detection method to detect, locate and quantify damage by combining WT and ANN. The proposed neuro-wavelet technique is to identify small damage in a steel plate structure. The plate structure is inflicted with damage at different locations. The proposed method involves applying WT to decompose the mode shape difference to detect and locate damage in the plate structure. Lee et al. [22] showed that the sensitivity of mode shape differences to modelling error is less compared to mode shapes due to the baseline finite element model and the accuracy of the finite element model greatly influences the effectiveness of damage detection algorithms. In addition, the training data applied in ANNs are mostly obtained from finite element models, this property makes mode shape differences a perfect input vector for training neural networks. Upon damage localization, the mean value of the WT moduli of the decomposed signal and the coordinates of the located damage are applied to train the neural network to predict damage severity.

2. Procedure of the neuro-wavelet technique
In the proposed method, data (mode shape) from the modal analysis of plate model in the damaged and undamaged states are used. This is facilitated by numerical data generated from a finite element model that enables collection of sufficient data needed to train the neural network. The plate structure is square and made of steel, and all the side boundaries are fixed. The proposed method is designed to be able to provide information on the three levels of damage identification, namely, detect presence of damage, detect location of damage and detect severity of damage. The first two levels are accomplished by using WT, while the third was achieved by using features extracted from the first WT and damage located coordinates.

Continuous wavelet transform (CWT) is the WT applied to decompose the mode shape difference. Unlike Fourier transform that is only localized in frequency, WF is localized in both frequency and time [30]. WT examines local data (e.g., displacement), thus it is able to detect and locate local damage; it also performs excellently in the presence of noise. The major problem of WT is selection of the most appropriate mother wavelet. The best option usually requires trial and error simulations. Decomposition of the mode shape difference provides information on damage detection and location. The coordinates of detected damage and mean value of the WT moduli serve as input to the neural network to predict damage severity. The procedure of damage severity prediction in the plate structure consists of the following steps:

1. Find the mode shapes of the monitored structures.
2. Evaluate the mode shape difference of the first mode.
3. Calculate the WT moduli of the first mode shape difference to locate damage.
4. Feed the mean value of WT moduli and detected damage coordinates to the ANN system.
5. Evaluate the output regarding damage severity (severity ranges from 0 to 1).
3. Continuous wavelet transform

A mother wavelet function $\psi(x)$ creates a wavelet family $\psi_{u,s}(x)$.

$$\psi_{u,s}(x) = \frac{1}{\sqrt{s}} \psi\left(\frac{x-u}{s}\right)$$  \hspace{1cm} (1)

where $s$ and $u$ are scale and position, respectively.

The deflection of the plate as a one-dimensional signal $f(x)$, the CWT defined by Mallat [31] is:

$$Wf(u,s) = \frac{1}{\sqrt{s}} \int_{-\infty}^{+\infty} f(x) \psi\left(\frac{x-u}{s}\right) dx$$  \hspace{1cm} (2)

where $Wf(u,s)$ is wavelet coefficient of the wavelet $\psi_{u,s}(x)$. The $n$ vanishing moment of a wavelet is vital in detecting singularity of signals, which is based on the equation:

$$\int_{-\infty}^{+\infty} f^k(x) \psi(x) dx = 0, \hspace{0.5cm} k = 0,1,2,\ldots, n-1$$  \hspace{1cm} (3)

Rewriting the above equation gives:

$$Wf(u,s) = \frac{1}{\sqrt{s}} \int_{-\infty}^{+\infty} f(x) \psi\left(-\frac{(u-x)}{s}\right) dx = \frac{1}{\sqrt{s}} f \ast \psi\left(-\frac{u}{s}\right) = f \ast \overline{\psi}_s(u)$$  \hspace{1cm} (4)

and

$$\overline{\psi}_s(u) = \frac{1}{\sqrt{s}} \psi\left(-\frac{x}{s}\right)$$  \hspace{1cm} (5)

Re-writing as the $n$th order derivation of a smooth function $\theta(x)$ [25]:

$$Wf(u,s) = \frac{s^n}{\sqrt{s}} \int_{-\infty}^{+\infty} f(x) \frac{d^n}{dx^n} \theta\left(\frac{x-u}{s}\right) dx = \frac{s^n}{\sqrt{s}} \frac{d^n}{dx^n} \int_{-\infty}^{+\infty} f(x) \theta\left(-\frac{(u-x)}{s}\right) dx$$  \hspace{1cm} (6)

where $f \ast \overline{\theta}_s$ is convolution. In two-dimensional CWT, the function $f(x, y)$ is given by Rucka and Wilde [32] as:

$$W^i f(u,v,s) = \frac{1}{s} f \ast \psi^i\left(-\frac{u}{s}, -\frac{v}{s}\right) = f \ast \overline{\psi}_s(u,v), \hspace{0.5cm} i = 1,2$$  \hspace{1cm} (7)

The horizontal $\psi^1(x,y)$ and vertical $\psi^2(x,y)$ wavelets are constructed with separable products of scaling $\varnothing$ and wavelet function $\psi$

$$\psi^1(x,y) = \varnothing(x)\psi(y) \hspace{0.5cm} \psi^2(x,y) = \psi(x)\varnothing(y)$$  \hspace{1cm} (8)

4. Numerical plate model

The plate model applied in this study is a square steel plate. The dimensions of the steel plate model are: 560mm length, 560mm width and 2mm thickness, and the material properties are: Young Modulus $E = 200$Gpa, Poisson’s ratio $\nu = 0.3$ and steel density $\rho = 7850$kg/m$^3$. The four sides of the plate are fixed. The plate is modelled by using Finite Element Analysis (FEA) program SAP2000 into 672 plate elements of size 20mm x 20mm elements, and the modal parameters (mode shape and frequencies) are obtained. To demonstrate damage detection of WT, three locations (middle, side and top of the plate) of damage are applied in this section. These locations are shown in Figure 1. The three damage locations are simulated by reducing the plate’s thickness at the specified area. The damages are in rectangular shape with the dimension of 40mm x 40mm which represents approximately 0.005% of the plate’s area. Figure 2 shows a cross-section of the plate with a middle damage.

The mode shape difference of the damaged and undamaged states is generated by subtracting the displacement value at each node in the first mode shape. Figure 3 shows the first mode shape of the undamaged plate model. The obtained mode shape differences are analyzed with CWT to obtain the damage locations. To accurately determine the damage location, appropriate mother wavelet needs to be chosen in decomposing the mode shape differences. Paul mother wavelet is chosen after several trials and errors [33]. Figure 4 shows the locations of the detected damage for the three damage locations. The coordinates of the damage are included in Figure 4, and these coordinates and the mean wavelet coefficients serve as input into the neural network.
5. Artificial neural network

5.1. Architecture of neural network

The architecture of the ANN consists of two hidden and one output layers. Each of the hidden layers consists of 20 neurons. Figure 5 shows a typical architecture of a neural network model. The optimum number of hidden layers and the required neurons were estimated by applying trial and error. The input layer has 3 nodes that correspond to the mean value of WT moduli and the x- and y-axes of the detected damage. Levenberg–Marquardt (LM) backpropagation algorithm was applied for training the ANN. The neurons in the ANN are taken to be fully connected and sigmoid function was selected as
the activation function for all hidden layers. The output layer was assigned the linear activation function. To prevent the overcrowding of the ANN, the input and output values were trimmed to between -1 and +1.

![Figure 5. Typical architecture of an ANN model.](image)

5.2. Training of neural network
The training of the neural network is done by using noise-free numerical data. The damage locations and damage orientations are shown in Figure 1 and Figure 2. The damage severity is defined by the thickness loss in the plate and taken as b, as shown in Figure 2. The damage locations are characterized by their coordinates (x- and y-axes), and 25 different damage locations and 50 different damage severities were randomly selected on the plate model. The numerical deflection lines were computed by SAP2000. The Paul mother wavelet was selected for the CWT after several trials and errors.

The total number of input pattern was 672, and the inputs were the x- and y-axes coordinates of the detected damage and the WT moduli mean value. Training of the neural network stopped at 700 epochs or when the msereg reaches $10^{-3}$. The performance of the neural network for best prediction is shown in Figure 6(a). The predictions of the neural network are shown on the y-axis, while the actual values are on the x-axis. From Figure 6(a), all points lie very near the line called the best linear fit. The correlation coefficient is 0.9904 and is very close to 1, this indicates very good compatibility between the outputs and targets.

![Figure 6. Damage severity results.](image)

5.3. Testing of neural network
Noise-free numerical data of the steel plate was applied for testing of the trained neural network. A total of 10 different damage locations and 15 different damage severities were randomly used to test the neural network. The damage set comprised of 100 patterns. These damage patterns were totally different from the damage patterns applied in training the neural network. However, the neural network was able to provide good predictions that were in good agreement with the actual values. The correlation coefficient reaches 0.9882. The results of the testing mode are shown in Figure 6(b). It can
be concluded that neural network can accurately predict damage severity when the appropriate damage WT parameters are applied as input.

6. Conclusion
This paper presented a neuro-wavelet damage detection method for damage identification in plate structures. A numerical example of a square steel model was utilised. Damage identification was done in two phases. The first phase involved WT decomposition of mode shape differences to detect and locate damage. The second phase utilised neural network to predict the damage severity. A backpropagation neural network was trained using damage parameters obtained from the WT moduli and detected damage coordinates. This study showed that WT can successfully detect and locate small damage in plate structure by utilizing the mode shape difference. In addition, neural network can predict damage severity in plate structures with a high degree of precision. The architecture of neural network to damage detection plays a role in obtaining accurate result and reducing computational time.

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