THE ELECTROWEAK CHIRAL LAGRANGIAN AS AN EFFECTIVE FIELD THEORY OF THE STANDARD MODEL WITH A HEAVY HIGGS

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ABSTRACT

Using effective field theory methods, we integrate out the standard model Higgs boson to one loop and represent its non-decoupling effects by a set of gauge invariant effective operators of the electroweak chiral Lagrangian. We briefly discuss the relation between the renormalization of both the standard model and the effective theory, which is crucial for a correct understanding and use of the electroweak chiral Lagrangian. Some examples have been chosen to show the applicability of this effective Lagrangian approach in the calculation of low energy observables in electroweak theory.

1. Introduction

The original motivation for studying chiral Lagrangians in the context of electroweak (EW) interactions was to find an effective theory that could parametrize at low energies the physics of the $SU(2)_L \times U(1)_Y$ breaking dynamics. The basic assumption made in this approach is that whatever the interactions that trigger the symmetry breaking may be, the particles involved in this breaking dynamics are heavier than the $W$ and $Z$ bosons. In that case, it is possible to describe electroweak physics at energies
$\approx M_Z$ by a low energy effective Lagrangian formulated just in terms of the relevant degrees of freedom at low energies, namely the gauge and would-be Godstone boson fields. The Higgs field is removed from the physical spectrum of the theory, either because it supposed to be heavy or because one wants to describe some other alternative symmetry breaking scenario.

The lowest order term of this effective lagrangian is a gauged non-linear sigma model that respects the basic symmetries of the standard model (SM), namely $SU(2)_L \times U(1)_Y$ gauge symmetry spontaneously broken to $U(1)_{em}$ and custodial $SU(2)_C$ symmetry of the scalar sector, but does not include a particular dynamics for the symmetry breaking interactions. This non-linear Lagrangian is non-renormalizable in the usual sense, because as one increases the number of loops in a calculation, new divergent structures appear of higher and higher dimension. The non-renormalizability of the theory is directly related to the fact that the SM Higgs boson does not decouple at low energies. In fact, this was the reasoning followed by Appelquist and Bernard and Longhitano when they made the first systematic analysis of the leading logarithmic Higgs mass dependent terms in the SM. Their strategy was to classify the new counterterms needed to absorb all the new divergences generated in a one loop calculation with the non-linear sigma model. Then, taking into account that the Higgs mass acts as a regulator of the linear theory, they identified these new counterterms with the logarithmic Higgs mass dependent terms in the SM.

Another important consequence that can be obtained just from the analysis of the non-linear Lagrangian is that these leading logarithmic terms are not sufficient to discriminate a heavy Higgs possibility from an alternative symmetry breaking scenario to which one requires to respect, at low energies, the same symmetries as the SM. These logarithmic contributions are a consequence of the general gauge and custodial symmetry requirements of the low energy structure of EW interactions and therefore they will be the same irrespective of the particular choice for the breaking dynamics, with the Higgs mass being replaced by some alternative physical mass. Thus, if one wants to reveal the nature of the symmetry breaking from low energy observables, one has to go beyond the leading logarithmic effects.

In order to discriminate between different symmetry breaking scenarios from low energy observables with chiral Lagrangians, one has to use the full machinery of effective field theories. The first step to be taken is to construct a low energy effective theory that can be consistently applied up to one loop, that we call the electroweak chiral Lagrangian (EChL). The EChL consists in the $SU(2)_L \times U(1)_Y$ gauged non-linear sigma model introduced before, plus the whole set of gauge invariant effective operators up to dimension 4, that were classified by Longhitano. This effective theory can be renormalized order by order in the loop expansion, in a similar way as it is done in chiral perturbation theory. In particular, at one loop order, the new divergences generated by a one loop calculation with the non-linear sigma model can be absorbed into redefinitions of the effective operators. These effective operators parametrize the low energy effects of the underlying fundamental dynamics of the symmetry breaking.

* The fermionic interactions, that are ignored in all the discussion, are assumed to be the same as in the SM.
Apart from the logarithmic dependence already mentioned, the effective theory does not give any further theoretical insight on the values of the effective operators for every particular choice of the underlying symmetry breaking dynamics. To get this information, a second step has to be taken and regard the effective operators as the result of integrating out the heavy fields of some underlying fundamental dynamics. In a perturbative approach, the fundamental and effective theories can be related by doing explicit calculations of the relevant loop diagrams and by matching the predictions of the full underlying theory (in which heavy particles are present) and those of the low energy effective theory (with only light degrees of freedom) at some reference scale.

In the rest of this talk, we will describe how to find the values of the effective chiral operators when the underlying theory is the standard model with a heavy Higgs. We will calculate the complete non-decoupling (leading logarithms plus constant contributions) effects of a heavy SM Higgs boson by integrating out the Higgs field to one loop and by matching the SM predictions in the large $M_H$ limit with the predictions from the electroweak chiral Lagrangian to one loop order.

Although the non-decoupling effects of the SM Higgs boson have been extensively analyzed for most of the low energy observables, we believe that the classification of these effects in terms of the effective EChL operators may be interesting for several reasons. First of all, the EChL approach provides a gauge invariant and systematic way of separating the non-decoupling Higgs boson effects from the rest of the EW radiative corrections in low energy processes. On the other hand, the EChL is a general framework in which one can analyze the low energy effects not only of a heavy Higgs in the SM, but of more general symmetry breaking dynamics characterized by the absence of light modes. It is therefore desirable to have the EChL that parametrizes an SM Higgs as a fundamental reference model.

In section 2 we give the precise formulation of the EChL and the one-loop renormalized Green’s functions of the effective theory. In section 3, we discuss the matching procedure used to relate the effective theory and the SM. We will comment briefly in section 4 the renormalization of the SM, that has been taken to be on-shell. In section 5, we set the matching conditions necessary to completely determine the EChL parameters for a heavy Higgs. In that section, we also give and comment the values of the effective operators obtained from the matching. Section 6 is devoted to make a more detailed analysis of the on-shell renormalization of the effective theory, that is necessary to calculate, in section 7, several examples of EW radiative corrections to low energy observables.

2. The Electroweak Chiral Lagrangian

The EChL is the most simple effective theory of EW interactions that parametrizes, at low energies, the effects of symmetry breaking sectors whose typical mass scale is larger than $M_Z$. It is formulated just in terms of the "light" gauge and would-be Goldstone fields, satisfying the basic requirement of $SU(2)_L \times U(1)_Y$ gauge invariance
spontaneously broken to $U(1)_{\text{em}}$:

$$\mathcal{L}_{\text{ECL}} = \mathcal{L}_{\text{NL}} + \sum_{i=0}^{13} \mathcal{L}_i. \quad (1)$$

Its basic structure is a gauged non-linear sigma model $\mathcal{L}_{\text{NL}}$, where a non-linear parametrization of the would-be Goldstone bosons is coupled to the $SU(2)_L \times U(1)_Y$ gauge fields

$$\mathcal{L}_{\text{NL}} = \frac{v^2}{4} \text{Tr} \left[ D_\mu U D^\mu U \right] + \frac{1}{2} \text{Tr} \left[ W_{\mu\nu} W^{\mu\nu} + B_{\mu\nu} B^{\mu\nu} \right] + \mathcal{L}_{R_\xi} + \mathcal{L}_{\text{FP}}, \quad (2)$$

where the bosonic fields have been parametrized as

$$U \equiv \exp \left( i \frac{\vec{r} \cdot \vec{\pi}}{v} \right), \quad v = 246 \text{ GeV}, \quad \vec{\pi} = (\pi^1, \pi^2, \pi^3),$$

$$W_\mu \equiv -i \frac{2}{v} \vec{W}_\mu \cdot \vec{r},$$

$$B_\mu \equiv -i \frac{2}{v} B_\mu \tau^3, \quad (3)$$

and the covariant derivative and the field strength tensors are defined as

$$D_\mu U \equiv \partial_\mu U - g W_\mu U + g' U B_\mu,$$

$$W_{\mu\nu} \equiv \partial_\mu W_\nu - \partial_\nu W_\mu - g [W_\mu, W_\nu],$$

$$B_{\mu\nu} \equiv \partial_\mu B_\nu - \partial_\nu B_\mu. \quad (4)$$

The physical fields are given by

$$W^\pm_\mu = \frac{W^1_\mu \mp i W^2_\mu}{\sqrt{2}},$$

$$Z_\mu = c \, W^3_\mu - s \, B_\mu,$$

$$A_\mu = s \, W^3_\mu + c \, B_\mu, \quad (5)$$

where $c = \cos \theta_w$, $s = \sin \theta_w$ and the weak angle is defined by $\tan \theta_w = g'/g$.

The second term in Eq.(1) includes the set of $SU(2)_L \times U(1)_Y$ and CP invariant
operators up to dimension four that were classified by Longhitano\textsuperscript{1}: 

\[
\begin{align*}
\mathcal{L}_0 &= a_0 g^2 \frac{\phi^2}{4} [\text{Tr} (TV_\nu)]^2 \\
\mathcal{L}_1 &= a_1 \frac{g}{2} B_{\mu \nu} \text{Tr} (TW^{\mu \nu}) \\
\mathcal{L}_2 &= a_2 \frac{g}{2} B_{\mu \nu} \text{Tr} (T[V_\mu, V_\nu]) \\
\mathcal{L}_3 &= a_3 g \text{Tr} (W_{\mu \nu} [V_\mu, V_\nu]) \\
\mathcal{L}_4 &= a_4 [\text{Tr} (V_\mu V_\nu)]^2 \\
\mathcal{L}_5 &= a_5 [\text{Tr} (V_\mu V_\nu)]^2 \\
\mathcal{L}_6 &= a_6 \text{Tr} (V_\mu V_\nu) \text{Tr} (TV_\mu) \text{Tr} (TV_\nu)
\end{align*}
\]

\[ T \equiv U \tau^3 U^\dagger, \quad V_\mu \equiv (D_\mu U) U^\dagger. \tag{6} \]

We have worked in a generic $R_\xi$ gauge; the gauge fixing term $\mathcal{L}_{R_\xi}$ and the Faddeev-Popov lagrangian $\mathcal{L}_{\text{FP}}^\text{NL}$ in Eq.(\textsuperscript{5}) were given in our previous work.\textsuperscript{2} We refer the reader to this work for the detailed formulas and a discussion on these terms. It is worth just recalling here that $\mathcal{L}_{\text{FP}}^\text{NL}$ does not coincide with the usual Faddeev-Popov lagrangian of the SM due to the non-linearity of the would-be Goldstone bosons under infinitesimal $SU(2)_L \times U(1)_Y$ transformations.

It is also important to mention that because of the non-linear realization of the gauge symmetry some of the couplings in $\mathcal{L}_{\text{NL}}$ have different Feynman rules than in the SM.\textsuperscript{3}

As we have mentioned in the introduction, the non-linear sigma model lagrangian in Eq.(\textsuperscript{2}) is not renormalizable, as increasing the number of loops in a calculation implies the appearance of new divergent structures of higher and higher dimension. However, the EChL is an effective theory that can be renormalized order by order in the loop expansion. In particular, at one loop order, the new divergences generated by a one loop calculation with $\mathcal{L}_{\text{NL}}$ can be absorbed into redefinitions of the effective operators given in Eq.(\textsuperscript{3}). Therefore, one can obtain finite renormalized Green’s functions if one makes a suitable redefinition of the fields and parameters of the EChL, among which the chiral parameters $a_i$ must be included.\textsuperscript{4} Formally, one defines the renormalized quantities in the effective theory by the following relations

\[
\begin{align*}
B_\mu^b &= \tilde{Z}_B^{1/2} B_\mu^a, \\
\tilde{W}_\mu^b &= \tilde{Z}_W^{1/2} \tilde{W}_\mu^a, \\
\tilde{\pi}^b &= \tilde{Z}_\phi^{1/2} \tilde{\pi}^a, \\
\xi_B^b &= \xi_B (1 + \tilde{\delta} \xi_B), \\
\xi_W^b &= \xi_W (1 + \tilde{\delta} \xi_W), \\
a_i^b &= a_i (\mu) + \delta a_i.
\end{align*}
\]

\textsuperscript{†}The relation with Longhitano’s notation can be found in our work.\textsuperscript{1}
where the renormalization constants of the effective theory are $\hat{Z}_i \equiv 1 + \delta Z_i$ and the superscript $b$ denotes bare quantities. We use the hatted notation to distinguish counterterms and Green’s functions in the effective theory from the corresponding quantities in the SM.

The 1PI renormalized Green’s functions of the effective theory to one loop will be generically denoted by

$$\hat{\Gamma}_R = \hat{\Gamma}_T + \hat{\Gamma}_C + \hat{\Gamma}_L,$$

where the superscript $R$ denote renormalized function and the superscripts $T$, $C$ and $L$ denote the tree level, counterterm and loop contributions respectively. We will discuss in section 6 the on-shell renormalization of the effective theory, giving explicit expressions for the counterterms introduced in Eq.(7). For the moment, in order to discuss the matching procedure, we will treat the counterterm contributions to the renormalized functions of Eq.(8) just at a formal level.

3. The Matching Procedure

We would like to focus now our attention on the procedure to obtain the chiral effective parameters for a heavy Higgs. As we have already said, once a particular renormalization scheme has been chosen to fix the counterterms of the effective theory, the renormalized $a_i(\mu)$ parameters remain as free parameters that can not be determined within the framework of the low energy effective theory. The values of the renormalized chiral parameters can be constrained from the experiment, as they are directly related to different observables in scattering processes and in precision electroweak measurements (see also section 7); but to have any theoretical insight on their values, one has to relate the effective theory with a particular underlying fundamental dynamics of the symmetry breaking.

If the underlying fundamental theory is the standard model with a heavy Higgs, the chiral parameters can be determined by matching the predictions of the SM in the large Higgs mass limit and those of the EChL, at one loop level. By heavy Higgs we mean a Higgs mass much larger than any external momenta and light particle masses ($p^2, M_Z^2 \ll M_H^2$) so that one can make a low energy expansion, but smaller than say 1 TeV, so that a perturbative loop calculation is reliable.

We will impose here the strongest form of matching by requiring that all renormalized one-light-particle irreducible (1LPI) Green’s functions are the same in both theories at scales $\mu \leq M_H$. The 1LPI functions are, by definition, the Green’s functions with only light particles in the external legs and whose contributing graphs cannot be disconnected by cutting a single light particle line. This matching condition is equivalent to the equality of the light particle effective action in the two descriptions. Some other forms of matching have been discussed in the literature, by requiring the equality of the two theories at the level of the physical scattering amplitudes or connected Green’s functions. These requirements, however, complicate the calculation unnecessarily while give at the end the same results for the physical observables.

In order to completely determine the chiral parameters in terms of the parameters of the SM, it is enough to impose matching conditions in the two, three, and four-point
1LPI renormalized Green’s functions with external gauge fields. We have worked in a general \( R_\xi \)-gauge to show that the chiral parameters \( a_i \) are \( \xi \)-independent. We use dimensional regularization and the on-shell substraction scheme.

The SM Green’s functions are non-local; in particular, they depend on \( p/M_H \) through the virtual Higgs propagators. One has to make a large \( M_H \) expansion to represent the virtual Higgs boson effects by the local effective operators \( \mathcal{L}_i \). In this step, care must be taken since clearly the operations of making loop integrals and taking the large \( M_H \) limit do not commute. Thus, one must first regulate the loop integrals by dimensional regularization, then perform the renormalization with some fixed prescription (on-shell in our case) and at the end take the large \( M_H \) limit, with \( M_H \) being the renormalized Higgs mass. From the computational point of view, in the large \( M_H \) limit we have neglected contributions that depend on \( (p/M_H) \) and/or \( (M_V/M_H, M_V = M_W, M_Z) \) and vanish when the formal \( M_H \rightarrow \infty \) limit is taken. An illustrative example of how to take the large \( M_H \) expansion of the loop integrals can be found in our second work.\(^\dagger\)

The matching procedure can be summarized by the following relation among renormalized 1LPI Green’s functions

\[
\Gamma_{SM}^R(\mu) = \hat{\Gamma}_{\text{EChL}}^R(\mu), \quad \mu \leq M_H, \tag{9}
\]

where the large Higgs mass expansion of the SM Green’s functions has to be made as explained above. This matching condition imposes a relation between the renormalization of the SM and the renormalization of the effective theory. We have chosen to renormalize both theories in the on-shell scheme, so that the renormalized parameters are the physical masses and coupling constants. Therefore, the renormalized parameters are taken to be the same in both theories and the matching conditions will provide relations between the SM and the EChL counterterms \(^\dagger\)

The matching condition (9) represents symbolically a system of tensorial coupled equations (as many as 1LPI functions for external gauge fields) with several unknowns, namely the complete set of parameters \( a_i^b \) that we are interested in determining and some constraints relating the counterterms in both theories. In section 5, we will give the solution of matching equations for the two, three and four point Green’s functions but before that, we have to set a renormalization prescription for the standard model.

4. Renormalization of the Standard Model

We start by writing down the SM lagrangian

\[
\mathcal{L}_{SM} = (D_\mu \Phi)^\dagger(D^\mu \Phi) + \mu^2 \Phi^\dagger \Phi - \lambda(\Phi^\dagger \Phi)^2 + \frac{1}{2} Tr(\mathcal{W}_{\mu \nu} \mathcal{W}^{\mu \nu} + \mathcal{B}_{\mu \nu} \mathcal{B}^{\mu \nu}) + \mathcal{L}_{R_\xi} + \mathcal{L}_{\text{FP}}, \tag{10}
\]

\(^\dagger\) In some related literature on effective field theories\(^\dagger\) the choice of a mass-independent subtraction prescription (\( \overline{\text{MS}} \)) in both theories has also been discussed. In that case, the matching procedure relates the running \( \overline{\text{MS}} \)-renormalized parameters, that are different in the fundamental and the effective theories.
where
\[ \Phi = \frac{1}{\sqrt{2}} \left( \phi_1 - i\phi_2 \right) \sigma + i\chi \], \quad (\pi_1, \pi_2, \pi_3) \equiv (-\phi_2, \phi_1, -\chi), \]
\[ D_\mu \Phi = (\partial_\mu + \frac{1}{2} i g \vec{W}_\mu \cdot \vec{\tau} + \frac{1}{2} i g' B_\mu) \Phi. \] (11)

\( W_\mu, B_\mu \) are defined in Eq.(3,4), \( L_{R_\xi} \) and \( L_{FP} \) are the usual \( R_\xi \) gauge fixing and Faddeev–Popov terms of the standard model.

We rescale the fields and parameters as follows
\[ B^b_\mu = Z^{1/2} B_\mu, \quad \tilde{W}^b_\mu = Z^{1/2} \tilde{W}_\mu, \]
\[ \Phi^b = Z^{1/2} \Phi, \quad v^b = Z^{1/2} (v - \delta v), \]
\[ g^b = Z^{-1/2} (g - \delta g), \quad g'^b = Z^{-1/2} (g' - \delta g'), \]
\[ \mu^b = Z^{-1/2} (\mu - \delta \mu), \quad \lambda^b = \lambda (1 - \delta \lambda / \lambda), \]
\[ \xi^b_B = \xi_B (1 + \delta \xi_B), \quad \xi^b_W = \xi_W (1 + \delta \xi_W). \] (12)

where the renormalization constants of the SM are \( Z_i \equiv 1 + \delta Z_i \) and the superscript \( b \) denotes bare quantities.

We have chosen to renormalize the SM in the on-shell scheme. We choose the physical masses, \( M_H, M_W, M_Z \) and \( g \) as our renormalized parameters. The weak mixing angle is defined in terms of physical quantities, as it is usual in the on-shell scheme
\[ \cos^2 \theta_W \equiv \frac{M_W^2}{M_Z^2}. \] (13)

The 1LPI renormalized Green’s functions in the standard model to one loop will be generically denoted by
\[ \Gamma^R = \Gamma^T + \Gamma^C + \Gamma^L, \] (14)
where one has to consider the tree, counterterm and loop contributions of all the one light particle irreducible diagrams in the SM; that is, all the diagrams that cannot be disconnected by cutting a light (non-Higgs) particle line. The details on the on-shell SM counterterms and the renormalization that we have chosen for the tadpole and the scalar self-coupling \( \lambda^{20} \) can be found in our second work.

5. Solution to the Matching Equations

In this section we present the results of our calculation of the two, three and four Green’s functions for external gauge fields, giving the set of matching equations that we have imposed and their solution. The master equations that summarize the complete set of matching conditions are the following:
\[ \Pi^{T\mu\nu}_{ab} + \Pi^{C\mu\nu}_{ab} + \Pi^{L\mu\nu}_{ab} = \hat{\Pi}^{T\mu\nu}_{ab} + \hat{\Pi}^{C\mu\nu}_{ab} + \hat{\Pi}^{L\mu\nu}_{ab} \]
\[
V_{abc}^T + V_{abc}^C + V_{abc}^L = \hat{V}_{abc}^T + \hat{V}_{abc}^C + \hat{V}_{abc}^L
\]
\[
M_{abcd}^T + M_{abcd}^C + M_{abcd}^L = \hat{M}_{abcd}^T + \hat{M}_{abcd}^C + \hat{M}_{abcd}^L,
\]
(15)

where \(ab\) stand for \(WW, ZZ, \gamma\gamma\) and \(\gamma Z\); \(abc\) represent \(\gamma WW\) and \(ZWW\) and \(abcd = \gamma\gamma WW, ZZZW, WWWW, ZZZZ\).

The calculation of the one loop contributions is the most involved part. One must include all the 1PI diagrams in the EChL and all the 1LPI diagrams in the SM. 1LPI diagrams are those that cannot be disconnected by cutting a single light particle line, that is, a non-Higgs particle line. One must, in principle, account for all kind of diagrams with gauge, scalar and ghost fields flowing in the loops. However, some simplifications occur. Firstly, a subset of the diagrams that have only light particles in it is exactly the same in both models, and their contribution can be simply dropped out from both sides of the matching equation (15). This is the case, for instance, of the subset of diagrams with gauge, scalar and ghost fields flowing in the loops. However, some simplifications occur. Firstly, a subset of the diagrams that have only light particles in it is exactly the same in both models, and their contribution can be simply dropped out from both sides of the matching equation (15). This is the case, for instance, of the subset of diagrams with only gauge particles in them. Secondly, we have checked explicitly by analyzing the large \(M_H\) expansion of every diagram that in the case of the three and four point Green’s functions, only those diagrams with just scalar (Goldstone bosons or Higgs) particles in the loops do contribute with non-vanishing corrections in the large \(M_H\) limit to the matching equations. In the case of the 2-point functions, however, both pure scalar and mixed gauge-scalar loops do contribute in the large \(M_H\) limit. Finally, among the diagrams with pure scalar loops, there are some with only Goldstone boson particles. One would expect that these diagrams give the same contributions in the SM and the EChL, however they do not. The reason is the already mentioned differences between the SM and the EChL Feynman rules of some vertices. Therefore, care must be taken to include these diagrams in both sides of the matching equations.

In our works, the explicit expressions for the tree, counterterms and loop contributions to the matching equations for the different Green’s functions can be found.

There is just one compatible solution to the complete set of matching conditions given by the following values of the bare electroweak chiral parameters

\[
\begin{align*}
\alpha_0^b &= \frac{1}{16\pi^2} \frac{3}{8} \left( \Delta_\epsilon - \log \frac{M_H^2}{\mu^2} + \frac{5}{6} \right), \\
\alpha_1^b &= \frac{1}{16\pi^2} \frac{1}{12} \left( \Delta_\epsilon - \log \frac{M_H^2}{\mu^2} + \frac{5}{6} \right), \\
\alpha_2^b &= \frac{1}{16\pi^2} \frac{1}{24} \left( \Delta_\epsilon - \log \frac{M_H^2}{\mu^2} + \frac{17}{6} \right), \\
\alpha_3^b &= \frac{-1}{16\pi^2} \frac{1}{24} \left( \Delta_\epsilon - \log \frac{M_H^2}{\mu^2} + \frac{17}{6} \right), \\
\alpha_4^b &= \frac{-1}{16\pi^2} \frac{1}{24} \left( \Delta_\epsilon - \log \frac{M_H^2}{\mu^2} + \frac{17}{6} \right),
\end{align*}
\]
\[ a_5^b = \frac{M_H^2}{2g^2M_H^2} - \frac{1}{16\pi^2} \frac{1}{24} \left( \Delta_\epsilon - \log \frac{M_H^2}{\mu^2} + \frac{79}{3} - \frac{27\pi}{2\sqrt{3}} \right), \]

\[ a_{11}^b = -\frac{1}{16\pi^2} \frac{1}{24}, \]

\[ a_6^b = a_7^b = a_8^b = a_9^b = a_{10}^b = a_{12}^b = a_{13}^b = 0. \quad (16) \]

where

\[ \Delta_\epsilon = \frac{2}{\epsilon} - \gamma_E + \log 4\pi, \quad \epsilon = 4 - D \quad (17) \]

We would like to make some remarks on this result for the chiral parameters:

1. First of all, we agree with the $1/\epsilon$ dependence of the $a_i^b$ parameters that was first calculated by Longhitano\cite{Longhitano} looking at the divergences of the non-linear sigma model. We see therefore that the divergences generated with the $\mathcal{L}_{\text{NL}}$ to one loop are exactly canceled by the $1/\epsilon$ terms in the $a_i^b$'s.

2. The values of $a_5^b$ and $a_5^b$ agree with previous results\cite{other_works} where the equivalence theorem\cite{another_works} was used in comparing the scattering amplitudes for Goldstone bosons in the SM\cite{SM_work} and the EChL. These values, however, do not coincide with the values\cite{previous_results} obtained when only the pure Higgs loops are taken into account.

3. It is important to realize that the matching procedure fixes completely the values of the bare parameters $a_i^b$ in terms of the renormalized parameters of the SM, ensuring the equality of the two theories at low energies.

4. Eqs.(16) give the complete non-decoupling effects of a heavy Higgs, that is, the leading logarithmic dependence on $M_H$ and the next to leading constant contribution to the electroweak chiral parameters. The $a_i$'s are accurate up to corrections of the order $(p/M_H)$ where $p \approx M_Z$ and higher order corrections in the perturbative expansion.

5. We demonstrate that the $a_i$'s are gauge independent, as expected.

6. Only one custodial breaking operator, the one corresponding to $a_0$ which has dimension 2, is generated when integrating out the Higgs at one loop. No custodial breaking operator of dimension four is generated.

7. There is only one effective operator, the one corresponding to $a_5$, that gets a tree level contribution. Its expression in terms of renormalized SM parameters depends on the renormalization prescription that one has chosen in the standard model, on-shell in our case. In our second work\cite{second_work}, we discussed how this effective parameter changes if a different renormalization is chosen for the SM. For instance, in the $\overline{\text{MS}}$ scheme, one would obtain

\[ a_5^b = \frac{1}{16\lambda_{\overline{\text{MS}}}} - \frac{1}{16\pi^2} \frac{1}{24} \left( \Delta_\epsilon - \frac{67}{6} \right). \]
where $\lambda_{\overline{MS}}$ is the renormalized scalar self-coupling in the $\overline{MS}$. We would like to emphasize with this example that the bare chiral parameters for a given underlying theory, the SM in our case, must be computed once a renormalization prescription of the underlying theory is chosen. The explicit expression of the chiral parameters will vary from one prescription to another, but the numerical value remains the same, and the connection between different prescriptions can be clearly and easily established.

From the matching conditions, one also obtains the following relations among the counterterms of the two theories

$$\Delta Z_W = -\frac{g^2}{16\pi^2} \frac{1}{12} \left( \Delta_\epsilon - \log \frac{M^2_H}{\mu^2} + \frac{5}{6} \right),$$

$$\Delta Z_B = -\frac{g'^2}{16\pi^2} \frac{1}{12} \left( \Delta_\epsilon - \log \frac{M^2_H}{\mu^2} + \frac{5}{6} \right),$$

$$\Delta \xi_W = \Delta Z_W, \quad \Delta \xi_B = \Delta Z_B,$$

$$\frac{\Delta g^2}{g^2} = \frac{\Delta g'^2}{g'^2} = 0,$$

$$\Delta Z_\phi - \frac{2}{v} \frac{\Delta v}{v} = \frac{g^2}{16\pi^2} \left[ -\frac{M^2_H}{8M^2_W} + \frac{3}{4} \left( \Delta_\epsilon - \log \frac{M^2_H}{\mu^2} + \frac{5}{6} \right) \right]$$

$$+ \frac{1}{4} \xi_Z \frac{\Delta_\epsilon - \log \xi_Z M^2_W}{\mu^2} + \frac{1}{2} \xi_W \left( \Delta_\epsilon - \log \xi_W M^2_W \right),$$

where

$$\Delta Q \equiv \delta Q - \delta\overline{Q} \quad \text{with} \quad Q = Z_B, Z_W, g^2, \text{etc...} \quad (19)$$

These equations give the differences among the renormalization constants of the SM in the large $M_H$ limit and those in the EChL, when the on-shell renormalization scheme is chosen in both theories. They are obtained here as a constraint imposed by the matching; one can also calculate them from the explicit expressions of the on-shell counterterms of the two theories and verify that these relations are indeed satisfied.

6. Renormalization of the Effective Theory

In this section we briefly describe the renormalization procedure in the effective theory. Given the effective Lagrangian of Eq.(1), the first step is to redefine the fields and parameters of the Lagrangian according to Eq.(7). It introduces, at a formal level, the set of counterterms of the effective theory $\delta \overline{Z}, \delta g$, etc, that need to be computed once a particular renormalization prescription scheme is chosen. We fix here the on-shell renormalization scheme as we did in the case of the SM. For practical reasons we prefer to choose the renormalization conditions as in reference[23] which are the most commonly used for LEP physics. In terms of the renormalized selfenergies these
renormalization conditions read as follows

\[ \hat{\Sigma}_W^R(M_W^2) = 0, \quad \hat{\Sigma}_Z^R(M_Z^2) = 0, \quad \hat{\Sigma}_1^R(0) = 0, \quad \hat{\Sigma}_2^R(0) = 0. \quad (20) \]

The renormalized self energies are computed in the effective theory as usual, namely, by adding all the contributions from the one loop diagrams and from the counterterms. We get the following expressions:\n
\[ \hat{\Sigma}_{\gamma}^R(q^2) = \hat{\Sigma}_{\gamma}^L(q^2) + \left( s^2 \delta Z_W + c^2 \delta Z_B \right) q^2 + s^2 g^2 (a_8^b - 2a_1^b) q^2. \]
\[ \hat{\Sigma}_W^R(q^2) = \hat{\Sigma}_W^L(q^2) + \delta Z_W \left( q^2 - M_W^2 \right) - \delta M_W^2. \]
\[ \hat{\Sigma}_Z^R(q^2) = \hat{\Sigma}_Z^L(q^2) + \left( c^2 \delta Z_W + s^2 \delta Z_B \right) \left( q^2 - M_Z^2 \right) - \delta M_Z^2 \]
\[ + 2g^2 a_0^b M_Z^2 + \left( 2s^2 g^2 a_1^b + c^2 g^2 a_8^b + (g^2 + g'^2) a_{13}^b \right) q^2. \]
\[ \hat{\Sigma}_{\gamma Z}^R(q^2) = \hat{\Sigma}_{\gamma Z}^L(q^2) + s c \left( \delta Z_W - \delta Z_B \right) q^2 - s c M_Z^2 \left( \frac{\delta g'}{g'} - \frac{\delta g}{g} \right) \]
\[ + \left( s c g^2 a_8^b - (c^2 - s^2) g g' a_1^b \right) q^2. \quad (21) \]

where

\[ \delta M_W^2 = M_W^2 \left( \delta Z_\Phi - 2 \frac{\delta g}{g} - 2 \frac{\delta v}{v} - \delta Z_W \right), \]
\[ \delta M_Z^2 = M_Z^2 \left( \delta Z_\Phi - 2c^2 \frac{\delta g}{g} - 2s^2 \frac{\delta g'}{g'} - 2 \frac{\delta v}{v} - c^2 \delta Z_W - s^2 \delta Z_B \right), \]
\[ M_W^2 = g^2 v^2 / 4, \]
\[ M_Z^2 = (g^2 + g'^2) v^2 / 4. \quad (22) \]

and the superscripts R and L denote renormalized and EChL loops respectively.

From Eq. (22) the following relation among the W and Z mass counterterms is obtained

\[ \frac{\delta M_Z^2}{M_Z^2} - \frac{\delta M_W^2}{M_W^2} = 2s^2 \frac{\delta g}{g} + 2c^2 \frac{\delta g'}{g'} + s^2 \left( \delta Z_W - \delta Z_B \right) \quad (23) \]

Finally, by requiring these renormalized self energies to fulfill Eq. (20) and taking into account that the $U(1)_Y$ Ward identity implies $\delta g' = 0$ one gets the following results for the values of the counterterms in terms of the unrenormalized selfenergies of the effective theory and the bare $a_i$'s:

\[ \delta M_W^2 = \hat{\Sigma}_W^L(M_W^2), \]

\footnote{Notice that in our rotation defining the physical gauge fields, the terms in $s$ have different sign than in reference.}
\[
\delta M_Z^2 = \tilde{\Sigma}^L_Z(M_Z^2) + M_Z^2 \left(2g^2 a_0^b + 2s^2 g^2 a_1^b + c^2 g^2 a_8^b + (g^2 + g'^2)a_{13}^b\right),
\]
\[
\frac{\delta g}{g} = -\frac{1}{s c} \frac{\tilde{\Sigma}^L_Z(0)}{M_Z^2},
\]
\[
\frac{\delta g'}{g'} = 0,
\]
\[
\delta Z_W = \frac{c^2}{s^2} \left(\frac{\tilde{\Sigma}^L_Z(M_Z^2)}{M_Z^2} - \frac{\tilde{\Sigma}^L_W(M_W^2)}{M_W^2}\right) + 2 \frac{c}{s} \frac{\tilde{\Sigma}^L_Z(0)}{M_Z^2} - \tilde{\Sigma}^L_W(0)
+ 2g^2 a_0^b + 2g^2 a_1^b + \frac{c^2 - s^2}{s^2} g^2 a_8^b + \frac{c^2}{s^2} (g^2 + g'^2)a_{13}^b,
\]
\[
\delta Z_B = \frac{\tilde{\Sigma}^L_W(M_W^2)}{M_W^2} - \frac{\tilde{\Sigma}^L_Z(M_Z^2)}{M_Z^2} - 2 \frac{s}{c} \frac{\tilde{\Sigma}^L_Z(0)}{M_Z^2} - \tilde{\Sigma}^L_W(0)
- \left(2g^2 a_0^b + g^2 a_8^b + (g^2 + g'^2)a_{13}^b\right).
\] (24)

Now that we have at hand Eq.(24) the only parameters of the theory that still need to be renormalized are the electroweak chiral parameters \(a_i\). The following formal redefinition of the chiral parameters has already been introduced

\[
a_i^b = a_i(\mu) + \delta a_i.
\] (25)

The divergent part of the \(a_i^b\) parameters, or equivalently the divergent part of the counterterms \(\delta a_i\), are fixed by the symmetries of the effective theory and since the work of Longhitano they are known to be

\[
\delta a_0|_{\text{div}} = \frac{1}{16\pi^2} \frac{3}{8} \Delta \epsilon,
\delta a_1|_{\text{div}} = \frac{1}{16\pi^2} \frac{1}{12} \Delta \epsilon,
\delta a_2|_{\text{div}} = \frac{1}{16\pi^2} \frac{1}{24} \Delta \epsilon,
\delta a_3|_{\text{div}} = \frac{1}{16\pi^2} \frac{1}{24} \Delta \epsilon,
\delta a_4|_{\text{div}} = 0, \quad i = 6, ..., 13.
\] (26)

These universal divergent contributions to the chiral bare parameters imply in turn the already mentioned universal scale dependence of the renormalized parameters

\[
a_0(\mu) = a_0(\mu') + \frac{1}{16\pi^2} \frac{3}{8} \log \frac{\mu^2}{\mu'^2},
a_1(\mu) = a_1(\mu') + \frac{1}{16\pi^2} \frac{1}{12} \log \frac{\mu^2}{\mu'^2},
a_2(\mu) = a_2(\mu') + \frac{1}{16\pi^2} \frac{1}{24} \log \frac{\mu^2}{\mu'^2},
a_3(\mu) = a_3(\mu') - \frac{1}{16\pi^2} \frac{1}{24} \log \frac{\mu^2}{\mu'^2},
\]
$$a_4(\mu) = a_4(\mu') - \frac{1}{16\pi^2} \left( \frac{1}{12} \log \frac{\mu^2}{\mu'^2} \right), \quad a_5(\mu) = a_5(\mu') - \frac{1}{16\pi^2} \left( \frac{1}{24} \log \frac{\mu^2}{\mu'^2} \right),$$

$$a_i(\mu) = a_i(\mu'); \quad i = 6, \ldots, 13. \quad (27)$$

The value of the bare chiral parameters $a_i^b$, on the other hand, is completely determined by the matching procedure in terms of the renormalized parameters of the underlying physics that has been integrated out, as we have seen for the particular case of a heavy Higgs. However, for a given $a_i^b$, we still have to choose how to separate the finite part into the renormalized $a_i(\mu)$ and the counterterm $\delta a_i$ in Eq.(25) such that their sum gives $a_i^b$. This second renormalization scheme concerns only to the effective theory. Therefore, in using a set of renormalized parameters $a_i(\mu)$ for a particular underlying theory, one must specify, in addition, how the finite parts of the counterterms in Eq.(25) have been fixed.

In the case of the SM, where a heavy Higgs has been integrated out to one loop, the bare chiral parameters are given in Eq.(16). They correspond to the on-shell renormalization of the underlying SM. Now, in order to present the corresponding renormalized parameters we have to fix the finite parts of the counterterms. For instance, if we fix the counterterms to include just the $\Delta \epsilon$ terms as in Eq.(26), the renormalized chiral parameters for the SM with a heavy Higgs are

$$a_0(\mu) = \frac{1}{16\pi^2} \left( \frac{3}{8} \left( \frac{5}{6} - \log \frac{M_H^2}{\mu^2} \right) \right), \quad a_3(\mu) = \frac{-1}{16\pi^2} \left( \frac{17}{6} - \log \frac{M_H^2}{\mu^2} \right),$$

$$a_1(\mu) = \frac{1}{16\pi^2} \left( \frac{1}{12} \left( \frac{5}{6} - \log \frac{M_H^2}{\mu^2} \right) \right), \quad a_4(\mu) = \frac{-1}{16\pi^2} \left( \frac{17}{6} - \log \frac{M_H^2}{\mu^2} \right),$$

$$a_2(\mu) = \frac{1}{16\pi^2} \left( \frac{1}{24} \left( \frac{17}{6} - \log \frac{M_H^2}{\mu^2} \right) \right), \quad a_{11}(\mu) = \frac{-1}{16\pi^2} \left( \frac{1}{24} \right),$$

$$a_5(\mu) = \frac{v^2}{8M_H^2} - \frac{1}{16\pi^2} \left( \frac{1}{24} \left( \frac{79}{3} - \frac{27\pi}{2\sqrt{3}} - \log \frac{M_H^2}{\mu^2} \right) \right),$$

$$a_i(\mu) = 0, \quad i = 6, 7, 8, 9, 10, 12, 13. \quad (28)$$

We have discussed also other choices of the renormalization scheme of the effective theory, for instance the one chosen by Gasser and Leutwyler for the linear $O(N)$ sigma model.

7. Calculating Observables with the EChL

In this section we will show, as an example, the explicit calculation of the radiative corrections to $\Delta \rho$ and $\Delta \tau$ within the electroweak chiral Lagrangian approach. These

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This particular renormalization of the chiral parameters was chosen in our first work where we called it MS.
observables are defined in the effective theory in terms of the renormalized self-energies in the same way as in the fundamental SM, namely:
\[ \Delta \rho \equiv \frac{\hat{\Sigma}^R_L(0)}{M_Z^2} - \frac{\hat{\Sigma}^R_W(0)}{M_W^2}, \]
\[ \Delta r \equiv \frac{\hat{\Sigma}^R_W(0)}{M_W^2} + \text{(vertex + box)}, \quad (29) \]
where
\[ (\text{vertex + box}) \equiv \frac{g^2}{16\pi^2} (6 + \frac{7 - 4s^2}{2s^2} \log c^2) \]
and the renormalized self-energies can be computed as we have explained in section 6.

Once a renormalization scheme has been chosen, one can always express \( \Delta \rho \) and \( \Delta r \) in terms of unrenormalized self-energies and the \( a^b_i \)'s. For instance, in the on-shell scheme given by the conditions of Eq.(20), one gets the particular values of the counterterms given in Eq.(24). Next, by plugging these counterterms into Eq.(21), one obtains the renormalized self-energies in terms of the unrenormalized ones and the \( a^b_i \)'s. Finally, by substituting these formulas into Eq.(29) the following expressions for \( \Delta \rho \) and \( \Delta r \) in the on-shell scheme are found
\[ \Delta \rho = \frac{\hat{\Sigma}^L_L(0) - \hat{\Sigma}^L_W(0)}{M_Z^2} + \frac{2s}{c} \frac{\hat{\Sigma}^L_\gamma(0)}{M_Z^2} + 2g^2 a^b_0, \]
\[ \Delta r = \frac{\hat{\Sigma}^L_W(0) - \hat{\Sigma}^L_W(M_W^2)}{M_W^2} + \frac{s^2}{c} \frac{\hat{\Sigma}^L_\gamma(0)}{M_Z^2} + \frac{c^2}{s^2} \left[ \frac{\hat{\Sigma}^L_W(M_W^2)}{M_W^2} - \frac{\hat{\Sigma}^L_L(M_Z^2)}{M_Z^2} - \frac{2s}{c} \hat{\Sigma}^L_\gamma(0) \right] \]
\[ -2g^2 a^b_0 + \frac{s^2 - c^2}{c^2} g^2 (a^b_8 + a^b_{13}) - 2g^2 (a^b_1 + a^b_{13}) + (\text{vertex + box}). \quad (30) \]

The explicit computation of the bosonic loop contributions in the effective theory, as well as the contributions from just \( a^b_0 \) and \( a^b_1 \) were found in.\[ \text{We present here the complete result} \]
\[ \Delta \rho \ = \ \frac{g^2}{16\pi^2} \left[ \frac{3s^2}{4c^2} \left( -\Delta_\epsilon + \log \frac{M_W^2}{\mu^2} \right) + h(M_W^2, M_Z^2) \right] + 2g^2 a^b_0, \]
\[ \Delta r \ = \ \frac{g^2}{16\pi^2} \left[ \frac{11}{12} \left( \Delta_\epsilon - \log \frac{M_W^2}{\mu^2} \right) + f(M_W^2, M_Z^2) \right] \]
\[ -2g^2 a^b_0 + \frac{s^2 - c^2}{c^2} g^2 (a^b_8 + a^b_{13}) - 2g^2 (a^b_1 + a^b_{13}). \quad (31) \]
where
\[ h(M_W^2, M_Z^2) = \frac{1}{c^2} \log c^2 \left( \frac{17}{4s^2} - 7 + 2s^2 \right) + \frac{17}{4} - \frac{5}{8} \frac{s^2}{c^2}. \]
\[ f(M_W^2, M_Z^2) = \log c^2 \left( \frac{5}{c^2} - 1 + \frac{3c^2}{s^2} - \frac{17}{4s^2c^2} \right) - s^2(3 + 4c^2) F(M_Z^2, M_W, M_W) \]
\[ + I_2(c^2)(1 - \frac{c^2}{s^2}) + \frac{c^2}{s^2} I_1(c^2) + \frac{1}{8c^2}(43s^2 - 38) \]
\[ + \frac{1}{18} (154s^2 - 166c^2) + \frac{1}{4c^2} + \frac{1}{6} + \Delta \alpha + \left( 6 + \frac{7 - 4s^2}{2s^2} \log c^2 \right) \]
\[ (32) \]

and \( F, I_1, I_2 \) and \( \Delta \alpha \) can be found in. In Eq.\((31)\) there are apparently a divergent term and a \( \mu \)-scale dependent term. However, when one redefines the bare effective chiral parameters as usual, \( a_i = a_i(\mu) + \delta a_i \), it can be easily seen that the divergent terms are cancelled by the divergent parts of the \( \delta a_i \) and the \( \mu \)-scale dependence is cancelled by the scale dependence of the \( a_i(\mu) \). The observables \( \Delta \rho \) and \( \Delta r \) turn out to be finite and scale and renormalization prescription independent, as it must be. In particular, if we set the substraction scheme for the chiral counterterms to include just the \( \Delta \epsilon \) terms as in Eq.\((26)\), the following expressions for the bosonic contributions to \( \Delta \rho \) and \( \Delta r \) in terms of renormalized chiral parameters are obtained:

\[ \Delta \rho = \frac{g^2}{16\pi^2} \left[ \frac{3}{4} \frac{s^2}{c^2} \log \frac{M_W^2}{\mu^2} + h(M_W^2, M_Z^2) \right] + 2g^2a_0(\mu), \]
\[ \Delta r = \frac{g^2}{16\pi^2} \left[ -\frac{11}{12} \log \frac{M_W^2}{\mu^2} + f(M_W^2, M_Z^2) \right] \]
\[ - 2g^2a_0(\mu) + \frac{s^2 - c^2}{s^2} g^2(a_8 + a_{13}) - 2g^2(a_1(\mu) + a_{13}). \quad (33) \]

Equations \((33)\) are general and can be applied to any underlying physics for the symmetry breaking sector. If we want to recover the values of \( \Delta \rho \) and \( \Delta r \) in the particular case of the SM with a heavy Higgs, one just has to substitute the values of the chiral parameters in Eq.\((28)\) into Eq.\((33)\) to obtain:

\[ \Delta \rho = \frac{g^2}{16\pi^2} \left[ -\frac{3}{4} \frac{s^2}{c^2} \left( \log \frac{M_H^2}{M_W^2} - \frac{5}{6} \right) + h(M_W^2, M_Z^2) \right], \]
\[ \Delta r = \frac{g^2}{16\pi^2} \left[ \frac{11}{12} \left( \log \frac{M_H^2}{M_W^2} - \frac{5}{6} \right) + f(M_W^2, M_Z^2) \right]. \quad (34) \]

which agrees with the result given in. One can similarly obtain the heavy Higgs contributions to other relevant observables in electroweak phenomenology.

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