Nonlinear breathing vibrations of a circular mesh antenna based on an equivalent model of a composite laminated circular cylindrical shell with membranes

T Liu\(^1\), W Zhang\(^2\), Y Zheng\(^3\), and H Liu\(^4\)

\(^1\)College of Mechanical Engineering, Beijing University of Technology, Beijing, 100124, PR China
\(^2\)E-mail: aaron6LT@163.com
\(^3\)E-mail: sandy_zhang0@yahoo.com
\(^4\)E-mail: yan_zh12@163.com
\(^5\)E-mail: 2586608569@qq.com

Abstract. This paper is focused on the nonlinear breathing vibrations of the circular mesh antenna based on a composite laminated circular cylindrical shell with radially pre-stretched membranes at both ends and clamped along a generatrix. The finite element model of the circular mesh antenna are established, we study the effects of different mesh stiffness on the first six orders frequencies. It is found that there is an approximate threefold relationship between the first-order frequency and the forth-order frequency. Based on the particular integer multiple relationship, we can conclude that there is the 1:3 resonance between the first-order and the forth-order vibrations of the circular mesh antenna. The method of multiple scales is employed to obtain the four-dimensional nonlinear averaged equation based on the two degree of freedom non-autonomous nonlinear equations of the equivalent model of circular mesh antenna and the 1:3 internal resonance is considered here. Then, based on the numerical method, the chaotic dynamics of the equivalent model of circular mesh antenna are studied by the bifurcation diagrams, the phase portraits, the waveforms, the power spectrums and the Poincaré map. The temperature parameter excitation shows that the complex chaotic phenomena occur under the certain initial conditions.

1. Introduction

The demand of ring truss antennas is generated due to the rapid development of the modern communication satellites. In order to save room for launching into space, the truss antennas should be designed foldable and deployable. The ring truss antenna is considered to be a reliable and promising scheme to realize its complete deployment [1-3]. After arriving at the predetermined orbit, the ring truss antenna can be fully deployed to form a circular cylindrical shell shape clamped by one beam along its generatrix and with the pre-tightening force meshes covered on both ends, as shown in Figure 1(a).

Fig.1 The geometric model of the circular mesh antenna and the composite laminated circular cylindrical shell system are shown.
The traditional numerical methods for the circular mesh antennas have a low credibility and lead to the huge amount of computation. However, engineers want to understand the nonlinear vibrations of the circular mesh antennas in a macroscopic view through the theoretical analysis based on the equivalent continuous model. The main structure of the circular mesh antennas consists of two parts, namely the ring truss antenna and the three-way mesh. Firstly, the ring truss antenna is usually shaped as the circular shell with the repetitive beam-like lattice grids due to the actual applications in space. Not neglecting that the double meshes are stretched in the ring truss antenna at both ends due to the antennas. It is necessary to consider the coupling effects between the ring truss antenna and the double meshes on the nonlinear oscillations of the ring truss antenna.

Zhang et al. [4,5] established a model of the continuum circular cylindrical shell clamped along one generatrix to substitute the ring truss structure with the repetitive beam-like lattice based on the equivalence principle. It is found that after the circular mesh antennas are deployed in the space environment, there exist four basic nonlinear oscillations including the radial breath nonlinear oscillations, the torsional, the bending and the rolling nonlinear oscillations around support beam. Sun et al. [6] investigated the homoclinic bifurcations and the Shilnikov type multi-pulse chaotic dynamics by using the energy phase method and the existence of the Shilnikov type multi-pulse orbits was studied. Liu et al. [7] studied a three-way mesh structure of the ring truss antenna and obtained a dynamic model which is equivalent to a membrane via the energy equivalence principle. The validity of the equivalent model is verified by the finite element simulation.

Although the continuum circular cylindrical shell model [4,5] and the membrane model [7] have been given respectively, their researches only consider the ring truss antenna or three-way mesh structure. Liu et al. [8] simplified the composite laminated circular cylindrical shell with the membranes at both ends, as shown in Figure 1(b). They studied the nonlinear oscillations of this model in the case of 1:2 internal resonances and also found that the system exhibits the chaotic motions. The new equivalent model is basically suitable to the real structure of the circular mesh antenna by considering the coupling effect of the ring truss and the membranes.

Circular cylindrical shell structures have been widely used in civil engineering, mechanical engineering and aerospace and aeronautic engineering. Amabili et al. [9] studied the non-linear response of empty and fluid-filled circular cylindrical shells to harmonic excitations considering both modal and point excitations. Then, Amabili et al. [10] investigated the response-frequency relationship in the vicinity of a resonant frequency, the occurrence of travelling wave response and the presence of internal resonances for simply supported, circular cylindrical shells for the first time. Breslavsky et al. [11,12] also investigated the nonlinear dynamics of empty and fluid-filled circular cylindrical shells containing inviscid incompressible fluid flow with simply supported, simply supported, and clamped boundary conditions. Breslavsky et al. [13] studied the nonlinear response of a water-filled, thin circular cylindrical shell, simply supported at the edges subjected to multi-harmonic excitation and explored the nonlinear dynamics by using bifurcation diagrams of Poincaré maps and time responses.

In this paper, we investigate the chaotic dynamics of a composite laminated circular cylindrical shell with radially pre-stretched membranes at both ends and clamped along a generatrix. Based on the two-degree-of-freedom nonlinear equation given in reference [8], the method of multiple scales is applied to obtain the four-dimensional averaged equation in the case of 1:3 internal resonance. The thermal excitation is considered as the control parameter to analyze the nonlinear oscillations and chaotic dynamics of the circular cylindrical shell by using the bifurcation diagrams, the phase portraits, the waveforms and the Poincaré maps.

**Finite Element Analysis and Formulation**

To study the relationship between different resonances of the circular mesh antenna, we establish the finite element model of the circular mesh antenna by using MSC.Nastran, as shown in Figure 1(a).
Based on the finite element model, the stiffness coefficient of the meshes are set as $K$. By using numerical analysis for finite element model, the first six orders frequencies of the circular mesh antenna are obtained in the three cases of 0, $K$ and 1.3$K$, as shown in Figure 2(a), where stiffness equal to 0 denote ring truss structure without meshes. We find that the first, third, fourth and sixth order frequencies increased significantly, while the change ranges of the second and fifth order frequencies are small. In addition, it is found that there is an approximate threefold relationship between the first-order and the forth-order vibrations of the circular mesh antenna. In order to verify this possibility, we have given the rule of the first and fourth order frequency changes with the stiffness, and found that the system does have 1:3 internal resonance under a certain stiffness, as shown in Figure 2(b).

Then, the comparison of the first six vibration modes between the ring truss antenna and the circular mesh antenna is given in Figure 3. It can be found that both the truss antenna and the network antenna have breaching vibration at the first, third, fourth and sixth order and torsional vibration at the second and fifth order. Based on these results, we conclude that the vibration frequency of the second and fifth order does not change much precisely in Figure 2(a) because the stiffness change has little impact on torsional vibration. On the other hand, the modal contrast diagram clearly shows that the presence or absence of the mesh has little effect on the antenna vibration mode. Therefore, it can be explained that in reference [8], it is reasonable to use the modal function of the equivalent cylindrical shell model of ring truss antenna as the modal function of the equivalent circular mesh antenna model.

\[ f = K_0 (l_0 + w_0 - w) + K_1 (l_0 + w_0 - w)^3 + \zeta_0 (\dot{w}_0 - \dot{w}_0) + \zeta_1 (\ddot{w}_0 - \ddot{w}_0), \]  

where $K_0$ and $K_1$ are the stiffness coefficients, $\zeta_0$ and $\zeta_1$ are the damping coefficients, $w_0$ and $w$ are the displacements functions, $l_0$ is the pre-stretched length and $l_0 + w_0 \geq w$ refers to the characteristic of the membranes which can only be stretched.

According to reference [8], the origin of coordinate is chosen at the middle surface of the composite laminated circular cylindrical shell clamped along a generatrix. The displacements of an arbitrary point within the shell along the axial, circumferential and radial directions are represented as $x$, $\theta$ and $z$, respectively, as shown in Figure 1(d).

Based on the Reddy’s third-order shear deformation theory and the von Kármán strain-displacement relation, the nonlinear governing equations of motion are given by Hamilton’s principle. The suitable approximation functions are desired to expand the middle surface displacements in order to reduce the composite laminated circular cylindrical shell to the finite-dimension by the Galerkin discretization. The dimensionless governing differential equations of transverse motion are given as

\[ \begin{align*}
\ddot{w}_1 + \omega^2_0 w_1 + \beta_1 \left( x \cos(\Omega t) \right) \ddot{w}_1 &+ \beta_2 \left( x \cos(\Omega t) \right) \dot{w}_1 + \beta_3 \dot{w}_1 + \beta_4 \dot{w}_1 + \beta_5 \dot{w}_1 + \beta_6 w_1 + \beta_7 w_1 + \beta_8 w_1 + \beta_9 w_1 + \beta_{10} w_1 + \beta_{11} w_1 + \beta_{12} w_1 + \beta_{13} w_1 + \beta_{14} w_1 + \beta_{15} w_1 + \beta_{16} w_1 + \beta_{17} w_1 + \beta_{18} w_1 + \beta_{19} w_1 + \beta_{20} w_1 + \beta_{21} w_1 + \beta_{22} w_1 + \beta_{23} w_1 + \beta_{24} = 0, \\
\ddot{w}_4 + \beta_{21} \ddot{w}_1 + \beta_{22} \dot{w}_1 + \beta_{23} (x \cos(\Omega t)) \ddot{w}_4 &+ \beta_{24} \dot{w}_4 + \beta_{25} \dot{w}_4 + \beta_{26} \dot{w}_4 + \beta_{27} \dot{w}_4 + \beta_{28} \dot{w}_4 + \beta_{29} \dot{w}_4 + \beta_{30} \dot{w}_4 = 0.
\end{align*} \]  

Fig.3 the comparison of the first six vibration modes between the ring truss antenna and the circular mesh antenna are obtained.

In reference [8], the membranes are made of a viscoelastic material, and an effective way to characterize the dynamic behaviors of the membranes is assuming them as a combination of the elastic and viscous elements in a parallel arrangement [14,15]. One of the combinations is shown as in Figure 1(e), the mechanical properties of the membranes are expressed as

\[ w_1 = w_1^0 + w_1^1, \]  

where $w_1^0$ and $w_1^1$ are the pre-stretched and the characteristic of the membranes which can only be stretched.
where the coefficients are the functions of the geometric, physical and temperature field parameters.

Based on the two-dimensional ordinary differential equation (2) and the 1:3 internal resonance relation between the newly discovered first and fourth order vibrations in this paper, we use the method of multiple scales to solve four dimensional average equation of the system in the state space, which are too long to be listed here.

4. Numerical Simulations

In this section, we will study the periodic motions and the chaotic motions with the increase of the temperature parameter excitation $T$ in this section.

In the following investigation, the fourth-order Runge-Kutta algorithm [16,17] is utilized to numerically analyze the periodic and chaotic motions of the composite laminated circular cylindrical shell based on the averaged equation. We choose the temperature parametric excitation $T$ as the controlling parameter to study the complicated nonlinear dynamics of the composite laminated circular cylindrical shell.

![Fig.4 The bifurcation diagrams and largest Lyapunov exponents of the system are given for the temperature parameter excitation.](image)

![Fig.5 The period-1 motion of the circular cylindrical shell is obtained.](image)

![Fig.6 The quasi-periodic motion of the circular cylindrical shell is obtained.](image)
It is observed from Figures 5-9 that the following three variation intervals can be given for the temperature parameter excitation.

(1) For P1 corresponding to $T=0-3.1$, there exists the period-1 motion $\rightarrow$ the quasi-periodic motion $\rightarrow$ the chaotic motion. As shown in Figure 5, the period-1 motion of the circular cylindrical shell is obtained when the temperature parameter excitations are chosen as $T=2.3$. Figures 6 shows the quasi-periodic motion when the temperature parameter excitation is chosen as $T=2.7$. Figures 7 shows the chaotic motion when the temperature parameter excitation is chosen as $T=3.03$.

(2) For P2 corresponding to $T=3.1-20$, there exists the period motion $\rightarrow$ the chaotic motion. As shown in Figure 8, the period motion of the circular cylindrical shell is obtained when the temperature parameter excitations are chosen as $T=4.5$. Figures 9 shows the chaotic motion when the temperature parameter excitation is chosen as $T=16$.

It can be found that P1 and P2 demonstrate the Pomeau-Manneville type intermittent chaos in the local intervals. The periodic window of P2 is distinctive and complex, which alternately exhibits the periodic and quasi-periodic motion. Moreover, the amplitudes of the periodic motion and the quasi-periodic motion are smaller than the amplitude of the chaotic motion in each local interval of P1 and P2, respectively.

6 Conclusions

The chaotic dynamics of a composite laminated circular cylindrical shell with radially pre-stretched membranes at both ends and clamped along a generatrix on one side are investigated in this paper. Based on the two degree of freedom non-autonomous nonlinear equation, the resonant case considered here is the 1:3 internal resonance.

Numerical simulations of the composite laminated circular cylindrical shell are carried out. The numerical results are presented to investigate the complex nonlinear behaviors of this system by the bifurcation diagrams, the phase portraits, the waveforms and the Poincaré map. It is shown that the Pomeau-Manneville type intermittent chaos is found under the certain initial conditions. In addition, the amplitudes of the chaotic motion are larger than the amplitudes of the periodic and quasi-periodic motion in each local interval. From the aforementioned analysis, it is easily found that the temperature parameter can control the nonlinear oscillations of the composite laminated circular cylindrical shell.

Acknowledgments

The authors gratefully acknowledge the support of National Natural Science Foundation of China (NNSFC) through Grant Nos. 11832002, 11290152 and 11427801, the Funding Project for Academic Human Resources Development in Institutions of Higher Learning under the Jurisdiction of Beijing Municipality (PHRILB).
References

1. BM. Mobrem, S. Kuehn, C. Spier and E. Slinko, 2012, Design and performance of astromesh reflector onboard soil moisture active passive spacecraft, IEEE Aerospace Conference, pp.1-10.
2. J. Santiago-Prowald and H. Baier, 2013, Advances in deployable structures and surfaces for large apertures in space, CEAS Space Journal, 5, pp89-115.
3. H. Y. Hu, Q. Tian, W. Zhang, D. P. Jin, G. K. Hu and Y. P. Song, 2013, Nonlinear dynamics and control of large deployable space structures composed of trusses and meshes, Advances in Mechanics, 43, pp.390-414.
4. W. Zhang, J. Chen and Y. Sun, 2016, Nonlinear breathing vibrations and chaos of a circular truss antenna with 1:2 internal resonance, International Journal of Bifurcation and Chaos, 26, p1650077-1-33.
5. W. Zhang, J. Chen, Y. F. Zhang and X. D. Yang, 2017, Continuous model and nonlinear dynamic responses of circular mesh antenna clamped at one side, Engineering Structures, 151, pp.115-135.
6. Y. Sun, W. Zhang and M. H. Yao, 2017, Multi-pulse chaotic dynamics of circular mesh antenna with 1:2 internal resonance, International Journal of Applied Mechanics, 9(4), 1750060.
7. F. S. Liu, X. M. Gao and D. P. Jin, 2015, Equivalent membrane model for the dynamic analysis of a prestressed paraboloidal cable nets, Journal of Vibration Engineering and Technologies, 3, pp.589-600.
8. T. Liu, W. Zhang and J. F. Wang, 2017, Nonlinear dynamics of composite laminated circular cylindrical shell clamped along a generatrix and with membranes at both ends, Nonlinear Dynamics, 90, pp.1393-1417.
9. M. Amabili, 2008, Nonlinear Vibrations and Stability of Shells and Plates, Cambridge University Press.
10. M. Amabili, F. Pellicano and M. P. Paidoussis, 1999, Non-linear dynamics and stability of circular cylindrical shells containing flowing fluid, Part II: large-amplitude vibrations without flow, Journal of Sound and Vibration, 228(5), pp.1103-1124.
11. M. Amabili, F. Pellicano and A. F. Vakakis, 2000, Nonlinear vibrations and multiple resonances of fluid-filled, circular shells, Part I: Equations of motion and numerical results, ASME Journal of Vibration and Acoustics, 122(4), pp.346-354.
12. F. Pellicano, M. Amabili and A. F. Vakakis, 2000, Nonlinear vibrations and multiple resonances of fluid-filled, circular shells, Part II: Perturbation analysis, ASME Journal of Vibration and Acoustics, 122(4), pp.355-364.
13. I. Breslavsky, M. Amabili, 2018, Nonlinear vibrations of a circular cylindrical shell with multiple internal resonances under multi-harmonic excitation, Nonlinear Dynamics (available online).
14. M. Salama and C. Jenkins, 2001, Intelligent gossamer structures - A review of recent developments and future trends, AIAA Applied Aerodynamics Conference.
15. S. Nakasuka and T. Aoki, et al, 2001, “Furoshiki Satellite” - A large membrane structure as a novel space system, Acta Astronautica, 48, pp.461-468.
16. T. S. Parker and L. O. Chua, 1989, Practical numerical algorithms for chaotic systems, Springer Verlag.
17. Y. Q. Wang, Y. H. Wan and Y. F. Zhang, 2017, Vibrations of longitudinally traveling functionally graded material plates with porosities, European Journal of Mechanics-A/ Solids, 66.