Search for Lorentz Violation using Short-Range Tests of Gravity

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Experimental tests of the newtonian inverse square law at short range, one at Indiana University and the other at the Huazhong University of Science and Technology, have been used to set limits on Lorentz violation in the pure gravity sector of the nonminimal Standard-Model Extension. In the nonrelativistic limit, the constraints derived for the 14 independent SME coefficients for Lorentz violation acting simultaneously are of order $(\kappa_{\text{eff}}) \sim 10^{-9}$ m$^2$.

1. Introduction

Local Lorentz invariance is at the foundation of both the Standard Model and General Relativity, but is not as well tested for the latter theory. Violation of Lorentz symmetry would break the isotropy of spacetime, permitting the vacuum to fill with "background" fields with a preferred direction. Interaction of the masses in a terrestrial gravity experiment with these fields could result in sidereal modulations of the force between the masses, providing a test of Lorentz invariance in gravity.

A quantitative description of Lorentz violation consistent with local field theory is given by the Standard-Model Extension (SME), which has been expanded to include gravitational effects by introducing a Lagrange density containing the usual Einstein-Hilbert term, plus an infinite series of operators of increasing mass dimension $d$ representing corrections to known physics at attainable scales.\textsuperscript{1} To date, the minimal ($d = 4$) and nonminimal cases have been investigated theoretically up to $d = 6$.\textsuperscript{2} In the nonrelativistic limit, the expression for the $j$th component of the force between two point masses $m_1$ and $m_2$ in the nonminimal SME is given by:

$$F^j = -Gm_1m_2 \left( \frac{\hat{R}^j}{R^2} - \frac{\kappa_{4}(\hat{R}, T)}{R^4} \right) ,$$

where the first term is newtonian gravity and the second is an SME correction. Here, $\hat{R}$ is the vector separating $m_1$ and $m_2$, and $\hat{R}^j$ is the projection of the unit vector along $\hat{R}$ in the $j$th direction. The SME correction term
where \((\kappa_{\text{eff}})_{jklm}\) contains 14 independent coefficients for Lorentz violation with units \(m^2\) in the standard laboratory frame. Motivated by Ref. 2, this report presents a test of Eq. (1) in laboratory gravity experiments.

2. The Indiana short-range experiment

The Indiana experiment is optimized for sensitivity to macroscopic forces beyond gravity at short range, which in turn could arise from exotic elementary particles or even extra spacetime dimensions. It is described in detail elsewhere; here we concentrate on the essential features.

The experiment is illustrated in Fig. 1 of Ref. 5. The test masses consist of 250 \(\mu\)m thick planar tungsten oscillators, separated by a gap of 100 \(\mu\)m, with a stiff conducting shield in between them to suppress electrostatic and acoustic backgrounds. Planar geometry concentrates as much mass as possible at the scale of interest, and is nominally null with respect to \(1/r^2\) forces. This is effective in suppressing the newtonian background relative to exotic short-range effects, and would be expected to be ideal for testing Eq. (1), in which the SME correction term varies as \(1/r^4\). The force-sensitive “detector” mass is driven by the force-generating “source” mass at a resonance near 1 kHz, placing a heavy burden on vibration isolation. The 1 kHz operation is chosen since at this frequency it is possible to construct a simple vibration isolation system. This design has proven effective for suppressing all background forces to the extent that the only effect observed is thermal noise due to dissipation in the detector mass. After a run in 2002, the experiment set the strongest limits on forces beyond gravity between 10 and 100 \(\mu\)m. The experiment has since been optimized to explore gaps below 50 \(\mu\)m, and new force data were acquired in 2012.

Analysis of the 2002 and 2012 data for evidence of Lorentz violation requires a theoretical expression for the Lorentz violating force for the particular geometry. Equation (1) is evaluated by Monte Carlo integration with the geometrical parameters listed in Refs. 3 and 5. The result can be expressed as a Fourier series of the time dependence,

\[
F = C_0 + \sum_{m=1}^{4} S_m \sin(m \omega_{\oplus} T) + C_m \cos(m \omega_{\oplus} T),
\]
where \( C_m, S_m \) are functions of the SME coefficients and test mass geometry (and the laboratory colatitude angle). In Eq. (3), the SME coefficients are expressed in the Sun-centered celestial equatorial frame, which are related to the laboratory-frame coefficients by \( (\mathbf{K}_{\text{eff}})_{jklm} = M^{ij}M^{jk}M^{kl}M^{lm}(\mathbf{K}_{\text{eff}})_{JKLM} \), where the matrix \( M \) is given by Eq. (10) of Ref. 2. The term \( \omega^\oplus \) is the Earth’s sidereal rotation frequency, and the time \( T \) is measured in the Sun-centered frame. The result for the constant term is:

\[
C_0 = \left[ -(1.8 \pm 2.3)(\mathbf{K}_{\text{eff}})_{XX} - (1.8 \pm 2.3)(\mathbf{K}_{\text{eff}})_{YY}
- (3.6 \pm 4.7)(\mathbf{K}_{\text{eff}})_{XY} + (13.5 \pm 7.5)(\mathbf{K}_{\text{eff}})_{XZ}
- (13.5 \pm 7.5)(\mathbf{K}_{\text{eff}})_{YZ} \right] \text{nN/m}^2.
\]  

(4)

Each term is quite sensitive to uncertainties in the test mass geometry (which determine the errors); in fact the means are smaller than might be expected for a simple \( 1/r^4 \) force for planar geometry.

Some insight can be gained from examination of the force in Eq. (1) for the case of a point mass \( m \) suspended a distance \( d \) above the center of a circular plate of radius \( \rho \), which can be solved analytically. The result can be expressed as \( 1/d^2 \) times a linear combination of oscillatory angular functions \( \Gamma^{jkm}(\theta, \phi) \), each function weighted by an SME laboratory-frame coefficient. Here, \( \theta \) and \( \phi \) are the polar and azimuthal angles of the vector between \( m \) and a mass element \( dm \) in the plate. In particular, nine of the \( \Gamma \) vanish upon integration of \( \phi \) over \( 2\pi \) radians. The remaining terms vanish upon integration of \( \theta \) from 0 to \( \pi/2 \) (the case of an infinite plate), and are strongly suppressed for \( \rho/d > 8 \), as the oscillatory structure of the \( \Gamma \) averages out. Thus, the force in Eq. (1) is suppressed in geometries with high symmetry and which subtend large solid angles; both are characteristics of the geometry of the IU experiment.

3. Limits on Lorentz violation coefficients

Analysis of the 2002 and 2012 data sets for signals of Lorentz violation has been completed, following Ref. 5. Time stamps in the data are extracted and offset relative to the effective \( T_0 \) in the Sun-centered frame (taken to be the 2000 vernal equinox). Discrete Fourier transforms of the data at each frequency component of the signal (0, \( \omega^\oplus \), 2\( \omega^\oplus \), 3\( \omega^\oplus \), 4\( \omega^\oplus \)) are computed, with errors, and corrected for discontinuous time data. Results, shown in Table I of Ref. 5, are consistent with zero with errors of order \( \sim 10 \) fN. Gaussian probability distributions at each signal frequency
component are constructed, using the difference between the Fourier transforms and the predicted signals (e.g., Eq. (4)) as the means. A global probability distribution is constructed from the product of the 18 component distributions. Means and errors of particular \( (k_{\text{eff}})_{JKLM} \) (for example, \( (k_{\text{eff}})_{XXXX} \)) are then computed by integration of the distribution over all \( (k_{\text{eff}}) \) except \( (k_{\text{eff}})_{XXXX} \). Results are of order \( (k_{\text{eff}}) \leq 10^{-5} \) m².

Results improve significantly on inclusion of data from the short-range experiment at the Huazhong University of Science and Technology, a torsion balance with planar test masses separated by \( \sim 300 \) µm. Terms in the Lorentz-violating torque for this experiment (the analog of Eq. (4)) are of order 10 nNm/m², while measured torque Fourier components have errors \( \sim 10 \) aNm. Improvement in sensitivity by a factor of \( \sim 10^3 \) would be expected; the resulting constraints on the \( (k_{\text{eff}}) \) are typically \( 10^{-9} \) m². The \( (k_{\text{eff}}) \) are derived from the 336 coefficients \( (k_1^{(6)}) \) and \( (k_2^{(6)}) \) in the fully relativistic SME; the simultaneous constraints on \( (k_{\text{eff}}) \) translate into comparable constraints on 131 fundamental coefficients taken one at a time.

Further study of Eq. (1) reveals that (i) the total force between point masses contains six components (each weighted by a separate \( (k_{\text{eff}}) \)) directed along the vector between the masses with the remaining components directed orthogonally, (ii) variation of the force in a terrestrial experiment is maximized when the sensitive axis is orthogonal to the Earth’s rotation axis, and (iii) the point-plate force tends to a maximum when the separation of the masses is on the order of the plate radius. Future experiments taking advantage of these features are expected to have greater sensitivity.

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