Study on $\Lambda nn$ Bound State and Resonance

Thiri Yadanar Htun$^{1,2}$ and Yupeng Yan$^{1}$

$^1$School of Physics and Center of Excellence in High Energy Physics and Astrophysics, Suranaree University of Technology, Nakhon Ratchasima 30000, Thailand
$^2$Department of Physics, University of Mandalay, 05032 Mandalay, Myanmar

(Dated: November 4, 2022)

We perform the ab initio no-core shell model (NCSM) calculation to investigate the bound state problem of three-body $\Lambda nn$ system in chiral next-to-next-to-leading-order NN and chiral leading-order YN interactions. The calculations show that no $\Lambda nn$ bound state exists, but predict a low-lying $\Lambda nn$ resonant state near the threshold with the energy of $E_r = 0.124$ MeV and the width of about $\Gamma = 1.161$ MeV. In searching for $\Lambda nn$ resonances, we extend the NCSM calculation to the continuum state by employing the J-matrix formalism in the scattering theory with the hyperspherical oscillator basis.

I. INTRODUCTION

Microscopic calculations of few- and many-body systems with strangeness have been a focus in hypernuclear physics to explore the new dynamical features of the structure of hypernuclei and to improve understanding of hyperon-nucleon interactions. Indeed, hyperon-nucleon scattering data is very limited to fully determine the YN interactions. The existing data of few body hypernuclei could provide the important constrain on YN interaction. In hypernuclear physics, hypetriton is used as the first testing ground for YN interaction and a better understanding of the nature of $\Lambda N-\Sigma N$ interaction and a better understanding of the nature of $\Lambda N-\Sigma N$ coupling.

There have been a number of theoretical calculations for the $\Lambda nn$ system as a serious doubtful bound state problem. Nonexistence of $\Lambda nn$ bound state was first revealed by Dalitz and Downs$^3$ using a variational approach. Garcilazo$^4$ investigated the $\Lambda nn$ system by solving Faddeev equations using YN and NN interactions derived from a chiral constituent quark model and revealed that $\Lambda nn$ bound state was not found. Later the various approach such as hyperspherical harmonics (HH)$^5$, Faddeev calculations$^2$ [6–11], variational calculations$^{12}$, pionless effective field theory$^{13–15}$ with various kinds of baryon-baryon interactions have been used to analyze the $\Lambda nn$ system and all reported that it is highly unlikely to form a bound system in the theoretical analysis without a significant altering nuclear and hypernuclear forces.

The $\Lambda nn$ hypernucleus could not be produced in the earlier experiments due to no charge of its bound state. However, the HypHI collaboration at GSI$^{16}$ reported the first evidence of the existence of the $\Lambda nn$ bound state from analysis of the observed two- and three-body decays mode without describing any statement for the value of binding energy. Their observation was inconsistent with the claim of the above theoretical analysis.

In this paper, we analyze the $\Lambda nn$ bound state problem using the ab initio no-core shell model (NCSM)$^{17–19}$ technique. The calculation of the $\Lambda nn$ system ($J^P = 1/2^+, T = 1$) is performed in Jacobi coordinate HO basis using the NN and YN interactions derived from chiral effective field model. In the extension into the continuum state, we apply the SS-HORSE$^{20–25}$ formalism, which is a single state harmonic oscillator representation of scattering equations, to calculate the low-energy phase shifts and scattering amplitudes at the NCSM eigenergies by employing hyperspherical harmonic oscillator basis. The low-lying $\Lambda nn$ resonance energy and width are extracted from the scattering amplitude parametrization. The NCSM-SS-HORSE method$^{24}$ has been successfully applied to study a tetranucleon unbound system considered as true four-body scatterings. Here we first apply this method to study the three-body system with strangeness.

II. NCSM-SS-HORSE FORMALISM

The hypernuclear Hamiltonian for two nucleon and a hyperon system can be written

\[
H = -\sum_{i=1}^{3} \frac{\hbar^2}{2m_i} \nabla_i^2 + V_{NN}(\vec{r}_1, \vec{r}_2) + \sum_{i=1}^{2} V_{YN}(\vec{r}_i, \vec{r}_3) + \Delta M,
\]

(1)

where the coordinates $\vec{r}_i$ and masses $m_i$ are for the two nucleon with $i = 1, 2$ and the hyperon with $i = 3$. We
work with nonrelativistic two-body NN and YN potentials, employing the leading-order chiral hyperon-nucleon interactions with regulator cutoff $\Lambda_{YN} = 600$ MeV [27] and a family of 42 different nuclear interactions at next-to-next-to-leading order (also called chiral NNLO$_{\text{sim}}$ family of NN interactions) [28]. These nuclear interactions were constructed by varying the chiral regulator cutoff $\Lambda_{NN}$ between 450 and 600 MeV in steps of 25 MeV and the truncation of the input NN scattering $T_{\text{lab}} \leq T_{\text{lab}}^{\text{max}}$ between 125 and 290 MeV in six steps, which were obtained from a simultaneous optimization of all 26 low-energy constants (LECs) to different sets of NN and πN scattering plus bound state observables [28]. In this work, we mainly use the NN interactions with $\Lambda_{NN} = 500$ and $T_{\text{lab}}^{\text{max}} = 290$ MeV. The effect of $\Lambda N$-$\Sigma N$ coupling is taken into account [19].

In NCSM, three active particles are considered in the three-dimensional harmonic oscillator (HO) basis. In the construction of HO basis states for such a few-body $\Lambda NN$ system, it is more effective to use the relative Jacobi coordinates where the center of mass (c.m.) coordinate $\vec{\xi}_0$ is separated, which allows us to perform NCSM calculations in large model space. The relative Jacobi coordinates in terms of the rescaled version of the single-particle coordinates $\vec{x}_i = \sqrt{m_i} \vec{r}_i$ are defined as

$$\tilde{\xi}_1 = \sqrt{\frac{1}{2}} (\vec{x}_1 - \vec{x}_2),$$
$$\tilde{\xi}_2 = \sqrt{\frac{2m_N m_Y}{2m_N + m_Y}} \left[ \frac{1}{2\sqrt{m_N}} (\vec{x}_1 + \vec{x}_2) - \frac{1}{\sqrt{m_Y}} \vec{x}_3 \right],$$

where $m_N$ and $m_Y$ are the masses of nucleon and hyperon. $\tilde{\xi}_1$ is the relative coordinate of the two-nucleon pair and $\tilde{\xi}_2$ is the relative coordinate of the hyperon with respect to the c.m. of the two-nucleon pair. Following the general Jacobi coordinate formulation in Ref. [19], we construct the JT-coupled HO basis states for the system of a two-nucleon pair and a hyperon,

$$|\langle n_N l_{NN} s_{NN}, N_Y, L_Y, J_Y, T_Y \rangle JT \rangle,$$

depending on the coordinates $\tilde{\xi}_1$ and $\tilde{\xi}_2$ respectively. $n_N$, $l_{NN}$, $s_{NN}$, $j_{NN}$, $l_{NN}$, $N_Y$ ($L_Y$, $J_Y$, $T_Y$) are the HO radial quantum number, orbital angular momentum, spin, angular momentum and isospin of the relative two-nucleon (hyperon) state. $J$ and $T$ are the total angular momentum and total isospin of the system. The basis [3] is antisymmetrized with respect to the exchange of two nucleon by restricting the two nucleon relative quantum numbers with the condition $(-1)^{l_{NN} + s_{NN} + t_{NN}} = -1$. The basis [3] is suitable for evaluating two-body YN interaction matrix elements but not for evaluating two-body $YN$ interaction matrix elements.

For a subsystem including $YN$ pair and a nucleon, another set of Jacobi coordinate is correspondingly introduced,

$$\tilde{n}_1 = \sqrt{\frac{(m_N + m_Y)m_Y}{2m_N + m_Y}} \left[ \frac{1}{\sqrt{m_N}} \vec{x}_1 - \frac{1}{\sqrt{m_Y}} \vec{x}_3 \right],$$
$$\tilde{n}_2 = \sqrt{\frac{m_N m_Y}{m_N + m_Y}} \left( \frac{1}{\sqrt{m_N}} \vec{x}_2 - \frac{1}{\sqrt{m_Y}} \vec{x}_4 \right),$$

where $\tilde{n}_1$ is the relative coordinate of a nucleon with respect to the c.m. of the $YN$ pair and $\tilde{n}_2$ is the relative coordinate of the $YN$ pair. By using orthogonal transformation, the antisymmetrized HO basis [3] can be expanded as

$$|\langle n_{NN} l_{NN} s_{NN}, N_Y, L_Y, J_Y, T_Y \rangle JT \rangle,$$

in terms of HO basis states

$$|\langle n_{NN} l_{NN} s_{NN}, N_Y, L_Y, J_Y, T_Y \rangle JT \rangle,$$

depending on the coordinates $\tilde{n}_2$ and $\tilde{n}_1$ respectively. The general HO bracket $\langle n_{NN} l_{NN} s_{NN}, N_Y, L_Y, J_Y, T_Y \rangle_d$ follows the agreement of Ref. [29]. YN interaction matrix elements involving $\Lambda$ and $\Sigma$ hyperons are evaluated in the antisymmetrized basis [3] through its expansion in the basis [6] as

$$\langle \frac{2}{V_{NN}(\vec{r}_i, \vec{r}_3)} \rangle = 2 \langle V_{YN}(\tilde{n}_2) \rangle,$$

where the matrix elements on the right-hand side are diagonal in all quantum numbers of the basis states [6], except for $n_{NN}$ and $l_{NN}$. The lowest eigenstates of the $\Lambda nn$ system are calculated by the diagonalization of the truncated Hamiltonian matrix.

To look for resonances, we extend our study to the continuum state by employing J-Matrix formalism, also known as Harmonic oscillator representation of scattering equation (HORSE) formalism, which arms one to study continuum spectrum using only positive energies obtained from bound state approach like NCSM applying HO basis. The HORSE method can be used to describe the open channels in the external subspace while the internal subspace is associated with the NCSM approach.
For details of the HORSE formalism, we may refer to Refs. [22 30].

In the extension into continuum, the three-body extension of the J-matrix formalism for all three-body decay channels is very complicated. We apply the democratic decay approximation (also known as true three-body scattering or 3 → 3 scattering) [31] which employs the hyperspherical harmonic (HH) basis to describe the Ann system decaying through only three-body break-up channel and it does not allow for other possible two-body channels associated with two-body sub-bound states.

The hyperspherical oscillator basis can be labeled as |κKγ⟩, where κ is the principal quantum number and K is the hypermomentum, γ = {l1, l2, L, s1, s2, S, t1, t2, T} collects all possible quantum numbers corresponding to the Jacobi coordinates for a three-body system. The external subspace is spanned by hyperspherical oscillator functions with N = 2κ + K > N_{\text{max}} where the Hamiltonian H = T is used. Here N_{\text{max}} is the maximum number of excitation quanta defining the many-body NCSM basis space. Because of high centrifugal barrier L(L + 1)/ρ^2, the HH states with larger K can be neglected in the case of no sub-bound Ann system [ρ is hyper radius with the mass scaled Jacobi coordinates and L = K + 3/2 is the effective momentum]. It is adequate to consider a single hyperspherical channel with minimum hypermomentum K_{\text{min}} = 0 to describe democratic three-body decays.

We follow the SS-HORSE approach [21 22 26] to compute the 3 → 3 scattering phase shifts at the eigenenergies E_{\nu} > 0 obtained directly from NCSM calculation,

\[ \tan \delta(E_{\nu}) = -\frac{S_{N_{\text{max}} + 2, L}(E_{\nu})}{C_{N_{\text{max}} + 2, L}(E_{\nu})}, \] (8)

where \(S_{N_{\ell}}\) and \(C_{N_{\ell}}\) are regular and irregular solutions of free Schrödinger equation in the hyperspherical oscillator representation, which can be applied in the case of arbitrary L (both integer and half integer), taking simple analytical expressions [21 23 31]

\[ S_{N_{\ell}}(E) = \frac{(N - L + \frac{9}{2})!}{\lambda \Gamma(\frac{N}{2} + \frac{L}{2} + \frac{9}{4})} q^{L+1} e^{-\frac{2}{\rho^2}} L_{(N-L+\frac{1}{2})/2}^{-\frac{1}{2}}(q^2), \] (9)

\[ C_{N_{\ell}}^{(\pm)}(E) = \frac{1}{\pi \sqrt{\lambda}} \frac{(N - L + \frac{3}{2})!}{\Gamma(\frac{N}{2} + \frac{L}{2} + \frac{9}{4})} \Psi\left(\frac{N}{2} + \frac{L}{2} + \frac{9}{4}, \frac{3}{2}, e^{\mp i \pi} q^2\right) q^{L+1} e^{-\frac{2}{\rho^2}} e^{\mp i \pi (L+\frac{1}{2})}, \] (10)

\[ C_{N_{\ell}}(E) = \frac{1}{2} \left(C_{N_{\ell}}^{(+)}(E) + C_{N_{\ell}}^{(-)}(E)\right), \] (11)

where \(q = \sqrt{\frac{2E}{\hbar \omega}}\) is dimensionless momentum, \(L_{\nu}^{L+\frac{1}{2}}(x)\) is the associated Laguerre polynomial, \(\lambda = \sqrt{\frac{2\hbar \omega}{m}}\) is the oscillator radius at \(\Psi(a; c; x)\) which is the Tricomi function.

The SS-HORSE scattering amplitude for neutral particles may be calculated in the standard way,

\[ f(E_{\nu})q = \frac{1}{(\cot \delta_{L}(E_{\nu}) - i)}. \] (12)

We parameterize the scattering amplitude in the method proposed in [22] for the case that a resonance is not sharp, but both the potential scattering (non-resonant background) and resonance contribution are not negli-
The scattering amplitude may be parametrized as

\[ F(E)q = e^{i\delta_0(E)} \sin \delta_0(E) + \frac{-\Gamma/2}{E - E_r + i\Gamma/2} e^{i2\delta_0(E)}, \]

where \( \delta_0(E) \) is the potential scattering phase shift, depending on the energy \( E \). We will fit the SS-HORSE scattering amplitude by the complex-valued function \( F(E)q \) in the next section to determine the form of the \( \delta_0(E) \) and derive the resonance energy \( E_r \) and width \( \Gamma \).

### III. RESULTS AND DISCUSSION

The \( \Lambda nn \) system is analyzed using the NCSM approach with chiral NNLO\(_{\text{sim}}\) NN and LO YN interactions. The NCSM computational model space is characterized by a chosen maximal total HO quanta \( N_{\text{max}} \), that is,

\[ 2n_{NN} + l_{NN} + 2N_Y + L_Y \leq N_{\text{max}} \equiv N_{\text{max}} + N_0, \]  

where the minimal possible number of HO quanta is \( N_0 = 0 \). In \( \Lambda nn \) case, \( N_{\text{max}}^{\text{tot}} = N_{\text{max}} \). We have computed the total energy of \( \Lambda nn \) system in the oscillator basis with model space truncations \( N_{\text{max}} \leq 36 \), and in the range of the HO frequencies 1 MeV \( \leq \hbar \omega \leq 40 \) MeV. It is found that there is no \( \Lambda nn \) bound system. The \( \Lambda nn \) ground-state energy as a function of the model truncation \( N_{\text{max}} \) and HO frequency \( \hbar \omega \) is presented in Figure 1. The NCSM energies decrease with increasing \( N_{\text{max}} \) and with decreasing \( \hbar \omega \). Our model used here can reproduce well the binding energy of hypertriton 33 and also for s-shell hypernuclei, \( \frac{3}{2}H \) and \( \frac{1}{2}He \), which will be a future publication.

The SS-HORSE phase shifts covering all computed NCSM energies calculated by using Eq. \[ 8 \] are shown in Figure 2. The phase shifts obtained with smaller \( N_{\text{max}} \) lie in a wide energy region as the obtained \( \Lambda nn \) ground-state energies spread widely. With \( N_{\text{max}} \) increasing, however, the obtained \( \Lambda nn \) ground-state energies converge to lower values, as shown in Figure 1, and hence the corresponding phase shifts shift to the lower energy region. The first convergence of phase shifts is achieved at smaller energies with larger \( N_{\text{max}} \), almost the same results at \( N_{\text{max}} = 34 \) and 36 MeV. We follow the selection procedure of Ref. \[ 21 \] and select a set of eigenvalues \( E_r \), from the \( N_{\text{max}} = 10-36 \) model spaces, which is illustrated by the shaded area in Figure 1 to produce a single smooth curve of phase shifts for parametrization. The SS-HORSE phase shifts corresponding to these selected smaller eigenvalues are plotted in Figure 3.

We compute the SS-HORSE low-energy scattering amplitude for the purpose of extracting the resonance parameters from scattering amplitude parametrization. The function \( |f(E_r)q|^2 \) of the scattering amplitude given in Eq. \[ 12 \] is shown by symbols in Figure 4. The fitting to the SS-HORSE result \( |f(E_r)q|^2 \) by the function \( |F(E)q|^2 \) leads the \( \delta_0(E) \) to the form

\[ \delta_0(E) = a_0 + a_2(\sqrt{E})^2 + a_4(\sqrt{E})^4, \]

with the adjustable parameters \( a_0 = 1.856, a_2 = -0.014 \) MeV\(^{-1}\), \( a_4 = 2.959 \times 10^{-4} \) MeV\(^{-2}\). The resonance energy and width are derived, \( E_r = 0.124 \) MeV and \( \Gamma = 1.161 \) MeV. The result is in good agreement with those in Ref. \[ 7 \] and lies within the estimated range of the location and width of a \( \Lambda nn \) pole 35. We look forward to the results of \( \Lambda nn \) bound and resonance states from the ongoing experiment (E12-17-003) at Jefferson Lab (JLab) 36. Such \( \Lambda nn \) bound and resonance states, if any, are expected to provide new perspective on \( \Lambda n \) interactions.

### SUMMARY

We have performed ab initio no-core shell model calculations for the \( \Lambda nn \) system \( (J^\pi = 1/2^+, T = 1) \) without tuning the strength of realistic NN and YN potentials at
various $N_{\text{max}}$ and $\hbar \omega$ values with full inclusion of $\Lambda N$-$\Sigma N$ coupling, and found that no bound state exists. To look for resonance states of the $\Lambda nn$, we have applied the NCSM-SS-HORSE technique to calculate the $\Lambda nn$ scattering phase shifts which suggest a $\Lambda nn$ resonant state at energy $E_r = 0.124$ MeV and $\Gamma = 1.161$ MeV. Further theoretical studies and experimental searches for $\Lambda nn$ resonances would be of great benefit of constraining $\Lambda n$ interactions.

**ACKNOWLEDGMENTS**

We are grateful to Daniel Gazda for providing us with the NCSM codes used to compute the $\Lambda nn$ eigenenergies. This work is supported by the Royal Golden Jubilee Ph.D. Program jointly sponsored by Thailand International Development Cooperation Agency, Thailand Research Fund, Swedish International Development Cooperation Agency and International Science Programme (ISP) at Uppsala University under Contract No. PHD/0068/2558.

[1] D. H. Davis, *Proceedings on 8th International Conference on Hyperrnuclear and strange particle physics (HYP 2003)* Newport News, USA, October 14-18, 2003, Nucl. Phys. A754, 3 (2005).

[2] K. Miyagawa, H. Kamada, W. Gloeckle, and V. Stoks, Phys. Rev. C 51, 2905 (1995).

[3] B. W. Downs and R. H. Dalitz, Phys. Rev. 92, 291 (1959).

[4] H. Garcilazo, Journal of Physics G: Nuclear Physics 13, L63 (1987).

[5] V. Belyaev, S. Rakityansky, and W. Sandhas, Nuclear Physics A 803, 210 (2008).

[6] I. R. Afian and B. F. Gibson, Phys. Rev. C 92, 054608 (2015).

[7] I. Filikhin, V. Suslov, and B. Vlahovic, EPJ Web Conf. 113, 08006 (2016).

[8] H. Garcilazo, I. Fernández-Caramés, and A. Valcarce, Phys. Rev. C 75, 034002 (2007), arXiv:hep-ph/0701275.

[9] H. Garcilazo and A. Valcarce, Phys. Rev. C 89, 057001 (2014), arXiv:1307.08061 [nucl-th].

[10] H. Kamada, K. Miyagawa, and M. Yamaguchi, EPJ Web of Conferences 113, 07004 (2016).

[11] B. Gibson and I. Afian, AIP Conf. Proc. 2130, 020005 (2019).

[12] E. Hiyama, S. Ohnishi, B. Gibson, and T. A. Rijken, Phys. Rev. C 89, 061302 (2014), arXiv:1405.2365 [nucl-th].

[13] S. I. Ando, U. Raha, and Y. Oh, Phys. Rev. C 92, 024325 (2015), arXiv:1507.01260 [nucl-th].

[14] F. Hildenbrand and H.-W. Hammer, Phys. Rev. C 100, 034002 (2019), arXiv:1904.05818 [nucl-th].

[15] M. Schäfer, B. Bazak, N. Barnea, and J. Mares, Phys. Rev. C 88, 061302 (2014), arXiv:2007.10264 [nucl-th].

[16] C. Rappold et al., Phys. Rev. C 88, 041001 (2013).

[17] B. R. Barrett, P. Navratil, and J. P. Vary, Prog. Part. Nucl. Phys. 69, 131 (2013).

[18] R. Wirth, D. Gazda, P. Navrátil, A. Calci, J. Langhammer, and R. Roth, Phys. Rev. Lett. 113, 192502 (2014), arXiv:1403.3067 [nucl-th].

[19] R. Wirth, D. Gazda, P. Navrátil, and R. Roth, Phys. Rev. C 97, 064315 (2018), arXiv:1712.05694 [nucl-th].

[20] I. Mazur, A. Shirokov, A. Mazur, and J. Vary, Phys. Part. Nucl. 48, 84 (2017), arXiv:1512.03983 [nucl-th].

[21] A. Shirokov, A. Mazur, I. Mazur, and J. Vary, Phys. Rev. C 94, 063200 (2016), arXiv:1608.05885 [nucl-th].

[22] L. Blokhintsev, A. Mazur, I. Mazur, D. Savin, and A. Shirokov, Phys. Atom. Nucl. 80, 226 (2017).

[23] Y. A. Lurie and A. M. Shirokov, Annals Phys. 312, 284 (2004), arXiv:nucl-th/0312028.

[24] A. M. Shirokov, A. I. Mazur, I. A. Mazur, E. A. Mazur, I. J. Shin, Y. Kim, L. D. Blokhintsev, and J. P. Vary, Phys. Rev. C 98, 044624 (2018), arXiv:1808.03394 [nucl-th].

[25] A. I. Mazur, A. M. Shirokov, I. A. Mazur, L. D. Blokhintsev, Y. Kim, I. J. Shin, and J. P. Vary, Phys. Atom. Nucl. 82, 537 (2019).

[26] A. Shirokov, G. Papadimitriou, A. Mazur, I. Mazur, R. Roth, and J. Vary, Phys. Rev. Lett. 117, 182502 (2016), arXiv:1607.05631 [nucl-th].

[27] H. Polinder, J. Haidenbauer, and U.-G. Meissner, Nucl. Phys. A779, 244 (2006), arXiv:nucl-th/0605050 [nucl-th].

[28] B. D. Carlsson, A. Ekström, C. Forssén, D. F. Strömgren, G. R. Jansen, O. Lilja, M. Lindby, B. A. Mattsson, and K. A. Wendt, Phys. Rev. X6, 011019 (2016), arXiv:1506.02466 [nucl-th].

[29] G. P. Kamuntavicius, R. K. Kalinauskas, B. R. Barrett, S. Mieczkiewicz, and D. Germainas, Nucl. Phys. A 695, 191 (2001), arXiv:nucl-th/0105009.

[30] J. Bang, A. Mazur, A. Shirokov, Y. Smirnov, and S. Zaytsev, Annals of Physics 280, 299 (2000).

[31] S. Zaitsev, Y. Smirnov, and A. Shirokov, Theoretical and Mathematical Physics 117, 1291 (1998).

[32] G.-C. Cho, H. Kasari, and Y. Yamaguchi, Prog. Theor. Phys. 90, 783 (1993).

[33] T. Y. Hsu, D. Gazda, C. Forssén, and Y. Yan, Few Body Syst. 62, 94 (2021), arXiv:2109.09479 [nucl-th].

[34] I. A. Mazur, A. M. Shirokov, A. I. Mazur, I. J. Shin, Y. Kim, and J. P. Vary, in 5th International Conference Nuclear Theory in the Supercomputing Era (2016) pp. 280–292.

[35] M. Schäfer, B. Bazak, N. Barnea, and J. Mares, Phys. Rev. C 103, 2 (2021), arXiv:2007.10264 [nucl-th].

[36] L. Tang, F. Garibaldi, P. Markowitz, S. Nakamura, J. Reinhold, and U. G.M., Proposal to Jefferson Lab PAC 48.