Angular momentum transport by stochastically excited oscillations in rapidly rotating massive stars

Umin Lee\(^1\), Coralie Neiner\(^2\), and Stéphane Mathis\(^3\)

\(^1\)Astronomical Institute, Tohoku University, Sendai, Miyagi 980-8578, Japan
\(^2\)LESIA, UMR 8109 du CNRS, Observatoire de Paris, UPMC, Univ. Paris Diderot, 5 place Jules Janssen, 92195 Meudon Cedex, France
\(^3\)Laboratoire AIM Paris-Saclay, CEA/DSM-Université Paris Diderot-CNRS, IRFU/SAp Centre de Saclay, 91191 Gif-sur-Yvette, France

ABSTRACT

We estimate the amount of angular momentum transferred by the low-frequency oscillations detected in the rapidly rotating hot Be star HD 51452. Here, we assume that the oscillations detected are stochastically excited by convective motions in the convective core of the star, that is, we treat the oscillations as forced oscillations excited by the periodic convective motions of the core fluids having the frequencies observationally determined. With the observational amplitudes of the photometric variations, we determine the oscillation amplitudes, which makes it possible to estimate the net amount of angular momentum transferred by the oscillations using the wave-meanflow interaction theory. Since we do not have any information concerning the azimuthal wavenumber \(m\) and spherical harmonic degree \(l\) for each of the oscillations, we assume that all the frequencies detected are prograde or retrograde in the observer’s frame and they are all associated with a single value of \(m\) both for even modes (\(l = |m|\)) and for odd modes (\(l = |m| + 1\)). We estimate the amount of angular momentum transferred by the oscillations for \(|m| = 1\) and \(2\), which are typical \(|m|\) values for Be stars, and find that the amount is large enough for a decretion disc to form around the star. Therefore, transport of angular momentum by waves stochastically excited in the core of Be stars might be responsible for the Be phenomenon.

Key words: stars: oscillations – stars: rotation

1 INTRODUCTION

Be stars are rapidly rotating active late O, B, or early A stars hosting a circumstellar decretion disc fed by discrete mass loss events. The mass ejections and disc produce emission lines in the optical spectrum of Be stars (see, e.g., Porter & Rivinius 2003; Rivinius et al. 2013 for recent reviews on Be stars), and it is believed that the circumstellar discs around Be stars are viscous Keplerian discs (Lee, Saio & Osaki 1991). Be stars of late O and early B type are also known as \(p\)-mode pulsators, while those of late B and early A type are \(g\)-mode pulsators, where both the \(p\)-mode and \(g\)-mode pulsations are excited by the opacity mechanism associated with the opacity bump produced by iron-peak elements at the temperature regions of \(T \sim 2 \times 10^5 \text{K}\) in their envelope (e.g., Pamyatnykh 1999). In addition, the convective core of intermediate-mass and massive stars are able to stochastically excite oscillation modes (e.g., Belkacem, Dupret, Noels 2010; Samadi et al. 2010, Shiode et al. 2013). Propagative gravity (gravito-inertial in the case of rapid rotation) waves and modes are excited (e.g., Browning, Brun, Toomre 2004; Rogers et al. 2013; Mathis, Neiner & Tran Minh 2014), because of the convective motions and of their penetration into the surrounding radiative envelope. These waves are able to transport angular momentum because of their dissipation along their propagation and at corotation layers (see e.g., Zahn, Talon, Matias 1997 and Alvan, Mathis, Decressin 2013 for gravity waves, and Pantillon et al. 2007 and Mathis et al. 2008 for gravito-inertial waves).

Mechanisms for the mass ejections and disc formation, however, have not yet been identified for Be stars. For disc formation mechanisms, several models have been proposed such as stellar wind models (e.g., Bjorkman & Cassinelli 1993; Owocki, Cranmer, & Blondin 1994; Cranmer & Owocki 1995), a model making use of magnetic fields to support a disc (Cassinelli et al. 2002; Owocki & ud Doula 2002), an evolutionary model that assumes angular momentum redistribution in the interior of rotating stars (e.g., Ekström et al. 2008; Granada et al. 2013), and a model using \(\kappa\)-driven pulsations as a carrier of angular momentum to...
the surface region of rapidly rotating stars (e.g., Ando 1983, 1986; Lee & Saio 1993; Crammer 2005, 2009; Ishimatsu & Shibahashi 2013). The stellar wind models are not necessarily successful in producing Keplerian discs around the stars. Magnetic fields strong enough to support circumstellar discs are not common among Be stars (e.g., Neiner et al. 2012b). On the other hand, as discussed above, stellar pulsations in Be stars are excited by the opacity bump mechanism or/and probably by stochastic excitation in the convective core. In this sense using the pulsations as the angular momentum carrier can be a promising mechanism for disc formation around Be stars, particularly when both stellar evolution and pulsation play their own parts. In fact, as suggested by Lee (2013), steady viscous decretion disc solutions are possible if a sufficient amount of angular momentum is deposited in the surface layers.

Recently, Neiner et al. (2012a) observed the hot Be star HD 51452 with the CoRoT satellite and identified numerous low-frequency oscillations. Since the star is rapidly rotating at the rate 1.22 cycle per day and the oscillation frequencies in the observer’s frame are comparable to or lower than the rotation frequency, the oscillations are low-frequency ones in the corotating frame of the star, suggesting that the effects of rapid rotation of the star can be significant to determine the wave properties propagating in the star. It is also important to note that low-frequency oscillations in early B type main sequence stars such as HD 51452 cannot be excited by the opacity bump mechanism. Neiner et al. (2012a) therefore suggested that the low-frequency oscillations detected are gravito-inertial waves excited stochastically by the convective motions in the core of the star.

In this paper, we estimate a possible amount of angular momentum transferred by the low-frequency oscillations identified by Neiner et al. (2012a) for the Be star HD 51452. We calculate non-adiabatic low-frequency oscillations by taking the effects of rotation of the star into account assuming that the rotation is uniform (Lee & Baraffe 1996), and we employ a wave-meanflow interaction theory (e.g., Andrews & McIntyre 1978ab; Grimshaw 1984; Lee 2013) to estimate the amount of angular momentum transfer. The oscillations are treated as forced oscillations whose frequencies are equal to those identified with CoRoT for the star. Since the CoRoT observations do not provide any information concerning the azimuthal wavenumber $m$ and spherical harmonic degree $l$ for the oscillations, we employ in this paper a working hypothesis that all the oscillations identified have the same $m$ and $l$ when estimating the possible amount of angular momentum. We describe the analytical method of calculation in §2 and numerical results in §3. Conclusions are given in §4.

2 THEORY FOR THE TRANSPORT OF ANGULAR MOMENTUM

2.1 Wave-meanflow equation

We discuss the angular momentum transfer by low-frequency waves excited by the convective motions in the core of a massive star. To estimate the amount of angular momentum transferred by the waves, we employ a theory of wave-meanflow interaction, in which a wave-meanflow equation may be given in the Cowling approximation by

$$\frac{d\ell}{dt} = \sum_{m} \frac{1}{2} \Im \left[ \nabla \cdot \left( m \xi_m p_m^* \right) \right],$$

where $\ell$ is the specific angular momentum in the $z$-direction along the rotation axis, $m$ is the azimuthal wave number around the axis, $\xi$ is the displacement vector, $p^*$ is the pressure Eulerian perturbation associated with the waves, and the asterisk ($^*$) indicates complex conjugation. Here, as a first step, our goal is to estimate the amount of angular momentum possibly carried by low-frequency oscillations to the surface region when the oscillations are excited by periodic convective motions of the fluids in the core. The completely coupled time evolution of both the meanflow and waves as a result of their interactions will be studied in a forthcoming work. We may represent the waves associated with the azimuthal wavenumber $m$ as

$$\xi_m = \sum_{\alpha} \xi_{ma} e^{i\omega_{ma} t + i\delta_{ma}},$$

$$p_m' = \sum_{\alpha} p_{ma}' e^{i\omega_{ma} t + i\delta_{ma}},$$

where $\omega_{ma} = \sigma_{ma} + m\Omega$, with $\Omega$ being the rotation frequency, is the oscillation frequency observed in the corotating frame of the star, $\sigma_{ma}$ is the frequency in an inertial (observer’s) frame, $\delta_{ma}$ is the phase shift, where $\alpha$ indicates the mode indices, for example, $\alpha = (l, n)$ with the degree $l$ and radial order $n$ for a mode in a non-rotating star. Note also that since we assume the time and azimuthal angle dependence of the perturbations are given by exp $i(xt + m\phi)$ or by exp $(i\sigma t + im\phi)$, prograde and retrograde modes are respectively associated with negative and positive values of $m$. For a given value of $m$, the term $\xi_m p_m'$ contains cross terms $\xi_{ma} p_{ma}' e^{i(\Delta \omega_{\alpha} + \Delta \delta_{\alpha})}$ for $\alpha \neq \beta$, where $\Delta \omega_{\alpha} = \omega_{\alpha} - \omega_{\alpha'}$ and $\Delta \delta_{\beta} = \delta_{\beta} - \delta_{\alpha}$. Since $e^{i(\Delta \omega_{\alpha} + \Delta \delta_{\beta})}$ is rapidly oscillating except when $\Delta \omega_{\alpha} \approx 0$, we ignore the terms with $\alpha \neq \beta$ in this paper. In this approximation, we obtain

$$\frac{d\ell}{dt} = \sum_{m, \alpha} \frac{m}{2} \Im \left[ \nabla \cdot \left( \xi_{ma} p_{ma}' \right) \right].$$

We take the effects of rotational deformation of the equilibrium structure on the oscillations into account by introducing the mean radius $a$ of the equipotential surface defined as

$$r = a \left[ 1 + \epsilon(a, \theta) \right],$$

where $(r, \theta, \phi)$ are spherical polar coordinates, and $\epsilon(a, \theta) = \alpha(a) + \beta(a)P_2(\cos \theta)$ with $P_2 = (3\cos^2 \theta - 1)/2$ being a Legendre polynomial is a quantity proportional to $\Omega^2$ and represents the deviation of the equilibrium structure from spherical symmetry (see Lee & Baraffe (1995) for the detail of the formulation). The terms $\alpha$ and $\beta$ respectively represent spherical expansion and rotational deformation of the equilibrium structure. In this paper, we ignore the term $\alpha$ in $\epsilon$ since we use evolutionary models calculated by taking account of spherical expansion of the structure assuming uniform rotation (e.g., Walker et al 2005).

As discussed in Lee & Baraffe (1995), in the coordinate system $(a, \theta, \phi)$, the displacement vector $\xi$ is given by

$$\xi = \xi^e_{e_r} + \xi^e_{\theta} + \xi^e_{\phi},$$

where $\xi^e_{e_r} = (1 + \epsilon + a\partial\epsilon/\partial a)e_r$, $\xi^e_{\theta} = (\partial \epsilon/\partial \theta)e_r + (1 + \epsilon)e_\theta$, and $\xi^e_{\phi} = (1 + \epsilon)e_\phi$ and $e_r$, $e_\theta$, and $e_\phi$ are orthonormal vectors in the $r$, $\theta$, and $\phi$ directions in spherical polar coordinates. The determinant of...
Angular momentum transport by stochastically excited oscillations in rapidly rotating massive stars

the metric tensor \( g_{ij} \) in this coordinate system is given by \( g = \text{det}(g_{ij}) = a^2(1 + \epsilon + a \delta t / \partial a)^2(1 + \epsilon) \sin^2 \theta \) and the volume element is given by \( dV = \sqrt{g} \sin \theta d\theta d\phi \). To the first order of \( \epsilon \), we obtain \( \sqrt{g} \approx a^2[1 + \vartheta(\epsilon)] \sin \theta \), where \( \vartheta(\epsilon) = 3\epsilon + a \delta t / \partial a \).

Since separation of variables is not possible to represent oscillation modes in a rotating star, we use a finite series expansion in terms of spherical harmonic functions for a given value of \( m \), assuming axisymmetry of the equilibrium structure. For example, the radial component of the displacement vector, \( \xi_{m\alpha} \), and the pressure Eulerian perturbation, \( p'_{m\alpha} \), are given by

\[
\xi_{m\alpha} = \sum_{j=1}^{J_{\max}} S_{m\alpha,j}(a) Y^m_j(\theta, \phi),
\]

and

\[
p'_{m\alpha} = \sum_{j=1}^{J_{\max}} p'_{m\alpha,j}(a) Y^m_j(\theta, \phi),
\]

where \( l_j = |m| + 2j - 2 \) for even modes, and \( l_j = |m| + 2j - 1 \) for odd modes, and \( j = 1, 2, 3, \ldots, J_{\max} \). Substituting perturbations represented by series expansions of finite length into the linearized basic equations, we obtain a finite system of coupled first order linear ordinary differential equations for the expansion coefficients, which we may call the oscillation equation. The detail of the oscillation equation as well as the boundary conditions imposed at the surface and center of the star is given by Lee & Baraffe (1995).

If we integrate Eq. (4) over an equipotential surface of radius \( a \), we obtain

\[
\int \frac{d\ell}{d\ell} \sqrt{g} d\theta d\phi = \frac{\partial}{\partial a} F(a),
\]

where

\[
F(a) = \frac{1}{2} a^2 \sum_{m, \alpha} \left[ m S^0_{m\alpha} \left[ I + \vartheta(\beta) A_0 \right] P_{m\alpha} \right],
\]

and \( S \) and \( P \) are column vectors whose \( j \)-th components are given by \( S_{ij} \) and \( p'_{ij} \), respectively, \( I \) is the unit matrix, and the definition of the matrix \( A_0 \) is given in Lee (1993). Note that \( S' = S^T \), and \( S'^T \) is the transpose of \( S \). Since the quantity \( F(a) \) is closely related to the work function for oscillations in uniformly rotating stars, acceleration (deceleration) of the rotation velocity occurs in the damping (excitation) regions of prograde modes, while deceleration (acceleration) occurs in the damping (excitation) regions of retrograde modes (e.g., Lee 2013).

It is convenient to introduce a mean \( \langle f \rangle \) of \( f \), defined by

\[
\langle f \rangle \equiv \frac{1}{4\pi} \int f[1 + \vartheta(\beta) P_2(\cos \theta)] d\Omega,
\]

where \( d\Omega = \sin \theta d\theta d\phi \). Here, \( \ell = r^2 \sin^2 \theta \Omega \) for uniformly rotating stars, so we have \( \langle \ell \rangle \approx (2/3)r^2 \Omega^3 \). We define the local timescale of acceleration, \( \tau_a \), as \( \tau_a^{-1} = (d\ell / dt)^{-1} \), and we have

\[
\frac{1}{\tau_a} = \frac{1}{4\pi a^2} \left[ \frac{\partial}{\partial a} F(a) \right],
\]

where positive and negative \( \tau_a \) respectively means local acceleration and deceleration of rotational flows around the axis. If we assume steady mass loss, we may rewrite the left hand side of Eq. (8) as

\[
\int \rho a^2 [1 + \vartheta(\beta) P_2] v_a \frac{\partial}{\partial a} d\Omega = \langle M \partial \ell / \partial a \rangle
\]

to obtain

\[
\langle M \partial \ell / \partial a \rangle = \frac{\partial}{\partial a} F(a),
\]

where \( M = 4\pi a^2 \rho v_a \) can be regarded as a mass loss rate. Integrating Eq. (13), we obtain

\[
\int_{a_0}^{a} \left[ M \partial \ell / \partial a \right] da = F(a) - F(a_0).
\]

If we approximate \( \langle M \partial \ell / \partial a \rangle \approx M \partial \ell / \partial a \) and assume \( M \) is constant in steady state, we obtain

\[
F(a) - F(a_0) = \int_{a_0}^{a} \left[ M \partial \ell / \partial a \right] da \approx J(a) - J(a_0),
\]

where \( J(a) = M \langle \ell \rangle \). For the rotational acceleration to take place near the stellar surface, we need \( F(R_\ast) - F(a_0) > 0 \) for \( a_0 < R_\ast \), where \( R_\ast \) is the radius of the star. For a given \( M \), by calculating the quantity \( \Delta F \equiv F(R_\ast) - F(a_0) \), we may estimate the amount of angular momentum, deposited in the surface region between \( a_0 \) and \( R_\ast \), necessary to accelerate the flow from \( J(a_0) \) to \( J(R_\ast) \).

It is convenient to normalize the quantities such as \( F \), \( M \), and \( \ell \) as

\[
f(a) = \frac{F(a)}{(GM^2/\ell R_\ast)}, \quad \ell = \frac{\langle \ell \rangle}{(R_\ast^2 \Omega_{\text{crit}})}, \quad m = \frac{M}{(M \Omega_{\text{crit}})},
\]

where

\[
\Omega_{\text{crit}} = \sqrt{\frac{GM}{R_\ast^3}},
\]

and \( G \) is the gravitational constant. If the stellar surface at \( a = R_\ast \) reaches the critical rotation rate \( \Omega \approx \Omega_{\text{crit}} \), we have \( \langle \ell \rangle \sim m \langle \ell \rangle \sim m \). If acceleration from \( \Omega / \Omega_{\text{crit}} \approx 0.5 \) to \( \Omega / \Omega_{\text{crit}} \approx 1 \) is to take place between \( a_0/R_\ast \approx 0.95 \) and the surface \( a_0/R_\ast \approx 1 \), we have \( J(R_\ast) - J(a_0) \sim 0.5 m \), which suggests that we need \( \Delta f \gtrsim 0.5 m \) for the matter in the surface layers to reach escape velocities above \( \Omega_{\text{crit}} \) (e.g., Meynet et al. 2010) and, thus, for a decretion disc to form around the star as a result of angular momentum transfer by the oscillations.

2.2 Forced oscillation calculation

We assume that the oscillations observed in the star are stochastically excited by the convective motions of the fluids in the convective core and at its boundary. This mechanism is highly complex and non-linear (e.g., Browning et al. 2004, Samadi et al. 2010, Rogers et al. 2013). Indeed, to explore the strength of the angular momentum transport by g- and r- modes in HD 51452, we choose to model their stochastic excitation by turbulent convection at the core-envelope boundary (at \( r = a_i \)) as resulting from periodic convective pressure fluctuations (see also Mathis 2009) with frequencies and amplitudes that are constrained thanks to the observations. This periodic pressure perturbation is incorporated as
a boundary condition and expanded as
\[
p'(a, \theta, \phi, t) = p'_c \Theta_{km}(\cos \theta)e^{im(\theta - \omega t)} = p'_c \sum c_l Y^m_l(\theta, \phi)e^{im(\theta - \omega t)}
\]  
(18)

where \( \omega \) may be regarded as the frequency of the convective motions in the core, which is related to the characteristic turn-over time \( \tau_c = 2\pi/\omega \) in the corotating frame (we thus consider a perfectly resonant excitation), and \( \Theta_{km}(\cos \theta) \) is the Hough function, the eigen-solution to the Laplace's tidal equation (e.g., Lee & Saio 1997), \( c_l \) is the expansion coefficient for the function in terms of spherical harmonics, and we assume \( k = 0 \) for even modes and \( k = 1 \) for odd modes. In this paper, we assume that many periodic oscillations detected in the star are excited by the convective motion of fluids in the core, and that the frequencies detected are those of the convective motions, which means that by detecting many periodicities at the surface we are seeing periodic motions of the fluids in the convective core. In this sense \( p'_c \) indicates the strength (or amplitude) of the perturbations induced by the convective motions. For a given value of \( p'_c \) and the frequency \( \omega \), we can integrate the oscillation equation for non-adiabatic modes to obtain the amplitude of \( \delta L'/L \) at the surface, which is proportional to the value of \( p'_c \). Comparing \( \delta L'/L \) to the observed value, we can determine \( p'_c \) and hence the amplitudes of the perturbations \( S \) and \( P \), which makes it possible to estimate the amount of angular momentum transfer taking place in the surface region of the star, using the meanflow equation.

The oscillation equation we solve for uniformly rotating stars is the same as that given in Lee & Baraffe (1995), in which global oscillation modes are calculated as eigen-modes that satisfy the appropriate boundary conditions at the surface and center of the stars. To calculate oscillations forced by the convective motions of the fluids in the core, we replace the inner mechanical boundary conditions, which are usually imposed at the stellar center, by the condition given by Eq. (18) imposed at the core-envelope boundary. The thermal boundary condition at the boundary is that the oscillations are adiabatic in the deep interior. The surface boundary conditions are the same as those given in Lee & Baraffe (1995). The replacement of the inner mechanical boundary conditions makes the system of linear differential equations inhomogeneous, which makes it possible to integrate the system of differential equations for arbitrary forcing frequencies \( \omega \). We solve the oscillation equation between the core-envelope boundary and the stellar surface, and we employ the Cowling approximation for simplicity.

3 NUMERICAL RESULTS

We calculate forced oscillations for the observed oscillation frequencies and luminosity amplitudes tabulated in Neiner et al. (2012a) for main sequence stars. We compute the main sequence models using a standard stellar evolution code with an OPAL opacity table for \( X = 0.7 \) and \( Z = 0.02 \), and we incorporate the spherical expansion due to uniform rotation by replacing the usual hydrostatic balance equation \( dp/dr = -\rho G M_r/r^2 \) by \( dp/dr = -\rho G M_r/r^2 + (2/3)r \Omega^2 \), where the average on latitude of the centrifugal acceleration is taken into account. We use the observed rotation rate 1.22 cycle per day for the model calculation, and since we fix this rate as a constant, the ratio \( \Omega/\Omega_{\text{crit}} \) increases with the evolution of the star from the ZAMS due to the radial expansion and exceeds unity during the main sequence stage. We stop the evolution calculation for \( \Omega/\Omega_{\text{crit}} \gtrsim 1 \). Figure 1 shows evolutionary tracks from the ZAMS of uniformly rotating stars of different masses. As a fiducial model for the analysis in this paper, we use a rotating main sequence star model of 18M\(_\odot\), whose physical parameters are \( \log (L/L_\odot) = 4.6049, \log T_{\text{eff}} = 4.4691, R_*/R_\odot = 7.1624, \log g = 3.9829 \), and \( \Omega \equiv \Omega/\Omega_{\text{crit}} = 0.6425 \) for the rotation frequency 1.22 cycle per day, where \( \Omega_{\text{crit}} = 1.381 \times 10^{-2} \) s\(^{-1}\). These physical parameters for the model are consistent with those cited in Neiner et al. (2012a) for the Be star HD 51452. For this model, we have \( \dot{M} = 1.26 \times 10^{-5} \) for \( \dot{M} = 10^{-10} M_\odot/\text{yr} \).

The observation suggests that the star is rapidly rotating, as rapidly as \( \Omega/\Omega_{\text{crit}} \gtrsim 0.5 \), but most of the detected frequencies in the observer’s frame are small compared to the critical frequency \( \Omega_{\text{crit}} \) of the star. This indicates that most of the frequencies may be attributable to low-frequency gravito-inertial waves in the corotating frame, and that the effects of rotation on the oscillations become crucial, particularly for the oscillations whose frequencies in the corotating frame are comparable to or lower than the rotation frequency \( \Omega \).

As is well known, the opacity bump due to iron group elements in the temperature regions of \( T \sim 2 \times 10^7 \)K in the stellar interior can excite high radial order \( g \) modes in slowly pulsating B (SPB) stars and low radial order \( p \) modes in \( \beta \) Cephei stars. Although the opacity bump at \( T \sim 2 \times 10^7 \)K cannot be effective enough to excite low-frequency modes in main sequence stars of \( M \sim 18 M_\odot \) such as HD 51452, it is expected that this high-\( \kappa \) region located close to the stellar surface may contribute to angular momentum transfer in the surface layers of the star by global oscillations excited by convective motions in the core because of their radiative damping (e.g., Zahn, Talon, Matias 1997) as well as
Angular momentum transport by stochastically excited oscillations in rapidly rotating massive stars

Figure 2. $f(a)$ and $1/\tau$ versus $a/R_*$ for forced oscillations of even parity for $m = 1$, where the solid, dashed, and dotted curves are for the forced frequencies $|\bar{\omega}| = 0.5$, 1, and 1.5, respectively, and the black and red curves are for retrograde and prograde waves, respectively. Here, $\bar{\omega} = \omega/\Omega_{\text{crit}}$, and the time-scale $\tau$ is normalized by $\Omega_{\text{crit}}$.

Figure 3. Same as Figure 2 but for forced oscillations of odd parity.

$\kappa$-driving due to the opacity bump (e.g., Lee 2013). Since angular momentum deposition to (extraction from) the meanflow depends on the sign of $m$, it is important to know whether the observationally identified oscillations are prograde or retrograde in the corotating frame of the star when we sum up all contributions from the modes considered.

Assuming forced oscillations, we can calculate the quantities $f(a)$ and $1/\tau$ for arbitrary frequencies. For the case of $m = 1$, we plot $f(a)$ and $1/\tau$ in Figure 2 (even parity waves) and Figure 3 (odd parity waves) for the forced frequencies $|\bar{\omega}| = 0.5$, 1, and 1.5 for prograde (red curves) and retrograde (black curves) waves, where $\bar{\omega} \equiv \omega/\Omega_{\text{crit}}$. Note that the amplitudes of the forced oscillations are determined so that the calculated $\langle \delta L' \rangle_{t_i}/L$ at the surface is equal to the largest observed amplitude mode having the frequency 0.6293 cycle per day. The rapid changes in the function $f(a)$ occurs in the surface regions, and hence the quantity $1/\tau$, which is quite small in the deep interior, has finite values in the surface layers. The functions $f(a)$ and $1/\tau$ for prograde and retrograde waves are roughly symmetric about the $a-$axis. We note that the strong high-$\kappa$ region, due to the opacity bump produced by iron group elements, is located at $a/R_* \simeq 0.95$, and that there appears a strong damping region below this region. As indicated by the right panels of Figs. 2 and 3 for $1/\tau$, a strong acceleration (deceleration) by retrograde (prograde) oscillations takes place in the driving region due to the opacity bump, while a strong deceleration (acceleration) by retrograde (prograde) oscillations takes place in
the damping region. It is also interesting to note that the amount of the difference $\Delta f$ between $a/R_\ast = 0.95$ and 1 can be of order of $10^{-14}$ for the frequency $\omega \approx 1.5$ for both prograde and retrograde oscillations and that it is larger than $\dot{m} \approx 10^{-15}$ for $M = 10^{-10} M_\odot$ yr$^{-1}$. This may suggest that even a single mode can transfer an amount of angular momentum large enough for a decretion disc to form around the star. Of course, if many modes are excited at the same time by the external force, we have to sum up all contributions from the excited modes to both acceleration and deceleration in order to obtain the net amount of angular momentum transferred to the surface layers of the star.

Figure 4 also plots the functions $f(a)$ and $1/\tau$ of $m = 1$ waves for the forcing frequencies $\bar{\omega} = 2m(\Omega - \delta)/[l'_1 (l'_1 + 1)]$ with $\delta = 10^{-3}$, which are close to an asymptotic frequency of $r$-modes, where $l'_j = l_j + 1$ for even modes and $l'_j = l_j - 1$ for odd modes. $R$-modes are retrograde modes and angular momentum deposition occurs in the layers much closer to the surface than $g$-modes. The amount of angular momentum $\Delta f$ transferred by the $r$-modes to the surface layers can be comparable to $\dot{m} \approx 10^{-15}$.

To estimate the possible amount of angular momentum transferred by oscillations excited by convective motions of the core fluid, we have to sum up contributions from all the
modes. As shown in the previous paragraphs, however, for a correct estimation we need to know, besides the frequencies for forcing, the value of $m$ for each of the modes. This sort of information, however, could not be determined from the observations in Neiner et al. (2012a). In this paper, therefore, to calculate forced oscillations having the detected frequencies, we assume, as a working hypothesis, that the observed oscillations are all prograde waves or all retrograde waves in the observer’s frame, having a single value of the wave numbers, we assume, as a working hypothesis, that the observed oscillations for forcing, the value of $m > 0$, we have $\omega > 0$ for all the oscillations, that is, the oscillations are all retrograde waves in the corotating frame. On the other hand, if the oscillations are all prograde waves in the observer’s frame such that $m \leq 0$ and $\sigma > 0$, we have $\omega > 0$ for all the oscillations, that is, the oscillations are all retrograde waves in the corotating frame. Note that almost all the oscillations could be retrograde for large $|m|$ when $\Omega \sim \Omega_{\text{crit}}$ and $\sigma \lesssim \Omega_{\text{crit}}$.

Since the observed oscillations should have amplitudes large enough to be detected at the stellar surface by the CoRoT satellite, it is reasonable to rule out the oscillations that have large amplitudes only in the deep interior, and such confinement usually takes place for low-frequency oscillations in the corotating frame for given values of $m$ and $l$. To exclude such low-frequency waves confined in the deep interior, defining the kinetic energy of the oscillation as $E_k(a) = \omega^2 \int_0^{R_*} \rho \xi^2 \cdot \xi dV$, we pick up only the oscillations for which $E_k(0.5 R_*/E_k(R_*)) < 0.9$. The oscillations ruled out for a given set of $m$ and $l$ are not necessarily ruled out for other combinations of $m$ and $l$.

We chose to use $|m| = 1$ and 2, since these $|m|$ values are those typically identified in observations of Be stars (e.g., Rivinius et al. 2003).

In Figure 5, we plot the functions $f(a)$ and $1/\tau$ for the observed frequencies for $m = 1$ and 2 for both even and odd parities. Since the detected oscillations are all assumed to be retrograde in the corotating frame, the contributions coming from the waves’ driving zone due to the opacity bump close to the stellar surface all work for increasing the rotation velocity. We also note that the amount of angular momentum deposition there, $\Delta f$, is much larger than $\dot{m} \sim 10^{-13}$ for $\dot{M} = 10^{-10} M_\odot/\text{yr}$, which suggests that the amount of angular momentum deposition is large enough for a decretion disc to form around the star. We have to keep in mind that the estimation of the angular momentum deposition here could be simply an over estimation.

Figure 6 is for $m = -1$ and $-2$ and in this case the oscillations in the corotating frame can be both prograde and retrograde. Since the high opacity zone works for accelerating (decelerating) the rotation velocity for retrograde (prograde) waves in the corotating frame, the net effects of angular momentum deposition at a given radius $a$ are determined by the sum of the contributions from all the retrograde and prograde waves, and this summation leads to a rather complicated behavior of the quantities $f(a)$ and $1/\tau$ as a function of $a/R_*$ in the surface regions. The amount of angular momentum deposition $\Delta f$ in the interval of $0.95 \lesssim a/R_* \lesssim 1$ is of the order of $10^{-13}$ for odd parity oscillations and is larger than the value of $\dot{m} \sim 10^{-15}$ necessary for a disc formation with $\dot{M} \sim 10^{-10} M_\odot/\text{yr}$.

The minimum timescale $\tau$ of acceleration in the surface regions is attained at $a/R_* \approx 0.96$ for positive $m$ (Fig. 5) and at $\approx 0.99$ for negative $m$ (Fig. 6). The minimum timescale $\tau$ is of order of $10^4$ for both cases and since $\Omega_{\text{crit}} \sim 10^{-4} \text{ s}^{-1}$, the acceleration timescale is of order of a few years. This timescale is typical of the recurrence timescale of outbursts in Be stars.

4 CONCLUSIONS

Using a theory of wave-meanflow interaction, we have estimated the possible amount of angular momentum transferred by gravito-inertial waves having the set of frequencies observationally detected for the Be star HD 51452. Since the Be star is rapidly rotating and the detected frequencies are
low in the observer’s frame in the sense that $\sigma \lesssim \Omega_{\text{crit}}$, the frequencies $\omega$ in the corotating frame of the star are low, that is, the detected frequencies are in the frequency domain of $g$-modes and $r$-modes. Since the opacity $\kappa$ mechanism does not work for excitation of low frequency modes in massive stars of $M_\star \gtrsim 10 M_\odot$ for a solar metallicity (Pamyatnykh 1999, Miglio et al. 2007), we need to look for an alternative mechanism for the excitation of the observationally detected low frequency oscillations. In this paper, we assume that the oscillations are excited by periodic convective motions of the core fluid in the massive star as suggested by Neiner et al. (2012a). We treat therefore the detected oscillation frequencies as forcing frequencies for waves propagating in the radiative envelope. We calculate the forced oscillations having the observed frequencies by imposing pressure perturbations produced by the convective fluid motions at the core-envelope interface as the boundary condition. This procedure makes inhomogeneous the system of linear differential equations for oscillations, which can be integrated for arbitrary frequencies.

In the theory of wave-meanflow interaction, the high-$\kappa$ regions works for accelerating (decelerating) the rotation (mean)flows for retrograde (prograde) waves observed in the corotating frame of the star. Since we have no information on the azimuthal wave number $m$ for the detected oscillations, we calculated the oscillations of the frequencies just assuming that they are all retrograde modes or prograde modes in the observer’s frame. In the former case, all the modes are retrograde in the corotating frame and hence the angular momentum deposition can take place efficiently in the surface layers of the star. In the latter case, however, the modes can be separated into prograde and retrograde modes, which means that even in the high-$\kappa$ regions the net amount of angular momentum deposition depends on the net sum of accelerating and decelerating contributions of retrograde and prograde waves.

Imposing the observed frequencies, we have shown that the amount of angular momentum transferred to the surface regions is large enough for decretion disc formation with the mass loss rate $\sim 10^{-10} M_\odot/yr$, although a definite amount of angular momentum transferred can be estimated only after the details of mode identification are determined. This could possibly be done with a ground-based high-resolution spectroscopic campaign.

Since we assume forced oscillations propagating in the radiative envelope of the star, there exist the possibility that some of the forcing frequencies, that are determined by properties of motions in the convective core, are in resonance with eigenfrequencies of the free oscillation modes such as $g$-modes or $r$-modes in the envelope. Both the forcing frequencies and the eigenfrequencies can change with the star’s evolution. If such a frequency resonance happens between an envelope free mode and a forcing frequency, its amplitude can be increased so that the effects of angular momentum deposition or extraction in the surface regions is enhanced.

In this paper, we have demonstrated, using the case of HD 51452, the ability of gravito-inertial waves to transport angular momentum efficiently in rapidly rotating massive stars and to deposit this momentum just below the surface. This mechanism increases the velocity of the surface layers. These layers can then reach the escape velocity at which material gets ejected from the star. Therefore, this mechanism could be at the origin of matter ejections in Be stars and the reason of the presence of a decretion disc around Be stars.

In a near future, we will improve the physical modelling of this wave-meanflow. First, in this work, we have assumed a uniform rotation to compute the oscillations. However, because angular momentum deposition occurs in a rather short time-scale in the surface layers, it may be important to take differential rotation into account when computing modes and the transport of angular momentum they induce (e.g., Lee & Saio 1993; Mathis 2009). In this way, we will get a complete picture of wave-meanflow interactions. Next, the modelling of the amplitude of waves, which are stochastically excited by the convective core in massive stars, must be improved using theoretical models (e.g., Belkacem et al. 2009; Lecoanet & Quataert 2013; Mathis, Neiner & Tran Minh 2014) and more and more realistic direct numerical simulations (e.g. Browning et al. 2004, Rogers et al. 2013). Besides, differential rotation and related meridional flows and turbulence play an important role in the evolution of massive stars (e.g., Meynet & Maeder 2000). In this context, it would be important to take these mechanisms into account as they also transport angular momentum simultaneously with gravito-inertial waves. This has already been done for the case of solar-type stars (Talon & Charbonnel 2005, Charbonnel et al. 2013, Mathis et al. 2013), in which waves strongly modify the star’s rotational evolution, but the case of massive stars still has to be studied. Such study should be undertaken first in spherical models in which the centrifugal acceleration is treated as a perturbation to unravel the role of each physical processes. Then, complete 2D models must be developed for rapidly rotating stars (e.g., Ballot et al. 2010; Espinosa Lara & Rieutord 2013).

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Angular momentum transport by stochastically excited oscillations in rapidly rotating massive stars  

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