Sensitivity of core+$n$ potential on configuration mixing in ground state of neutron-rich exotic nuclei

Jagjit Singh$^{1,2}$, W. Horiuchi$^3$

$^1$Nuclear Reaction Data Centre, Faculty of Science, Hokkaido University, Sapporo 060-0810, Japan
$^2$Research Center for Nuclear Physics (RCNP), Osaka University, Ibaraki 567-0047, Japan
$^3$Department of Physics, Hokkaido University, Sapporo, 060-0810 Japan

E-mail: jsingh@rcnp.osaka-u.ac.jp

Abstract. We study the configuration mixing in the ground state of the neutron-rich exotic nuclei. The results for weakly-bound two-neutron halo nuclei $^{11}\text{Li}$ and $^{22}\text{C}$, and the two-neutron unbound nucleus $^{26}\text{O}$ are reported. For the present study, we use a three-body (core + $n + n$) structure model developed for describing the two-neutron halo system by explicit coupling of unbound continuum states of the subsystem (core + $n$). We use a density-dependent contact-delta interaction to describe the neutron-neutron interaction and its strength is varied to fix the binding energy. We report the two-neutron correlations in the ground state of these systems.

1. Introduction

Since the discovery of neutron halos [1], astounding advancements have been made in technology of radioactive ion beam (RIB) facilities. These upgrades in RIB facilities has provided the access to the neutron-rich sea of the nuclear chart. Due to this, the neutron-rich side of the nuclear chart has gained extensive attention of the nuclear physics community, particularly the two-neutron (2n) halo nuclei sitting right on the top of neutron driplines and decays of 2n-unbound systems beyond the neutron dripline, consisting of a core and two valence neutrons. The structure of these systems is very much different from well known stable systems lying in the stability valley, due to which these systems demand a genuine three-body (3b) description with proper treatment of continuum. The stability of such 3b-system (core+$n+n$) is linked to the continuum spectrum of the two-body (core+$n$) subsystem. In this context, to explore the sensitivity of choice of a core+$n$ potential with the configuration mixing in the ground state of 3b-systems (core+$n+n$), we will discuss the results of the 2n-halos $^{11}\text{Li}$, $^{22}\text{C}$ and 2n-unbound system $^{26}\text{O}$.

Recently all these systems have been experimentally investigated at TRIUMF and RIKEN, the major experimental facilities of Canada and Japan, respectively. Although $^{11}\text{Li}$ is the first observed two-neutron halo four decades ago [1]. Since then a lot of experimental and theoretical studies have been reported on the structure of the $^{11}\text{Li}$. Recently the role of $^{10}\text{Li}$ resonances is investigated in the halo structure of $^{11}\text{Li}$ via $^{11}\text{Li}(p, d)^{10}\text{Li}$ transfer reaction at TRIUMF [3] and at the same facility the first conclusive evidence of a dipole resonance in $^{11}\text{Li}$ having an isoscalar character has been reported [5, 6]. Also for $^{22}\text{C}$ a high precision measurement of the interaction cross-section was made on a carbon target at 235 MeV/nucleon [7] and $^{26}\text{O}$ has been investigated, using the invariant-mass spectroscopy [8] at RIKEN. The structural spectroscopy
of the two-body subsystem plays a vital role in the understanding of the 3b-system. These high precision measurements and the sensitivity of the structural spectroscopy of subsystem with the structure of 3b-system (core+n+n), are the motivation for selecting these nuclei for the present study.

Very recently, we have studied in detail the pairing collectivity in the ground state of the 2n-halo $^{22}$C [9, 10] and in the the 2n-unbound system $^{26}$O [10]. Here we will discuss the results for these two systems along with our new results on $^{11}$Li. For this study we have used our recently implemented 3b-structure model (core+n+n) for the ground and continuum states of the 2n-halo nuclei [9, 11, 12]. We have reported the configuration mixing in the ground-state of these systems with a particular choice of core+n potential depending upon available experimental information.

2. Model Formulation

In our approach we consider a 3b-system consisting of an inert core nucleus and two valence neutrons, which is specified by the Hamiltonian

$$H = -\frac{\hbar^2}{2\mu} \sum_{i=1}^{2} \nabla_i^2 + \sum_{i=1}^{2} V_{\text{core}+n}(\vec{r}_i) + V_{12}(\vec{r}_1, \vec{r}_2)$$

(1)

where $\mu = \frac{A_c m_N}{A_c + 1}$ is the reduced mass, and $m_N$ and $A_c$ are the nucleon mass and mass number of the core nucleus, respectively. The recoil term is neglected in the present study. $V_{\text{core}+n}$ is the core-n potential and $V_{12}$ is n-n potential. The neutron single-particle unbound s-, p-, d- and f-wave continuum states of the subsystem $^{10}$Li, $^{21}$C and $^{25}$O are calculated in a simple shell model picture for the converged model parameter, bin width ($\Delta E = 0.1$), by using the Dirac delta normalization and are checked with a more refined phase-shift analysis. These core+n continuum wave functions are used to construct the two-particle states of the core+n+n system by proper angular momentum couplings. We use a density-dependent (DD) contact-delta pairing interaction [19], given by

$$V_{12} = \delta(\vec{r}_1 - \vec{r}_2) \left( v_0 + \frac{v_\rho}{1 + \exp[(r_1 - R_\rho)/a_\rho]} \right).$$

(2)

The first term in Eq. (2) with $v_0$ simulates the free n-n interaction, which is characterized by its strength and the second term in Eq. (2) represents density-dependent part of the interaction. The strengths $v_0$ and $v_\rho$ are scaled with the $\Delta E$ by following relation from Ref. [9]. The $v_\rho$ is the parameter which will be fixed to reproduce the ground-state energy. For a detailed formulation one can refer to [9, 11, 12].

3. Two-body unbound subsystems (core+n)

The investigation of the two-body (core+n) subsystem is crucial in understanding the three-body system (core+n+n). The interaction of the core with the valence neutron (n) plays a vital role in the binding mechanism of the core+n+n system. The elementary concern over the choice of a core+n potential is the scarce experimental information about the core-neutron systems. We employ the following core+n potential

$$V_{\text{core}+n} = \left( V_0^l + V_{ls}^l \cdot \frac{1}{r} \frac{d}{dr} \right) \left( \frac{1}{1 + \exp \left( \frac{r-R_c}{a} \right)} \right),$$

(3)

where $R_c = r_0 A_c^{\frac{1}{3}}$ with $r_0$ and $a$ are the radius and diffuseness parameter of the Woods-Saxon potential.
Table 1. Parameter sets of the core-\(n\) potential for different \(j\) states of \(^{10}\text{Li}+n\), \(^{20}\text{C}+n\) and \(^{24}\text{O}+n\). The possible resonances with resonance energy \(E_R\) and decay width \(\Gamma\) in MeV are also tabulated.

| System | \(l_j\) | \(r_0\) (fm) | \(a_0\) (fm) | \(V_0\) (MeV) | \(V_{ls}\) (MeV) | \(E_R\) (MeV) | \(\Gamma\) (MeV) |
|--------|--------|--------------|-------------|--------------|--------------|-------------|-------------|
| \(^{10}\text{Li}\) | \(s_{1/2}\) | 1.27 | 0.67 | -47.50 | 40.00 | 21.02 | 0.46 | 0.36 |
| \(^{10}\text{Li}\) | \(p_{1/2}\) | 1.27 | 0.67 | -47.50 | 40.00 | 21.02 | 2.98 | 1.39 |
| \(^{21}\text{C}\) | \(s_{1/2}\) | 1.25 | 0.65 | -47.50 | 35.00 | 0.92 | 0.09 |
| \(^{21}\text{C}\) | \(d_{3/2}\) | 1.25 | 0.65 | -42.00 | 35.00 | 6.69 | 2.93 |
| \(^{25}\text{O}\) | \(d_{3/2}\) | 1.25 | 0.72 | -44.10 | 22.84 | 0.74 | 0.09 |
| \(^{25}\text{O}\) | \(p_{3/2}\) | 1.25 | 0.72 | -48.67 | 22.84 | 0.57 | 1.38 |
| \(^{25}\text{O}\) | \(f_{7/2}\) | 1.25 | 0.72 | -44.10 | 22.84 | 2.44 | 0.21 |

3.1. \(^{10}\text{Li}\)
For \(^{10}\text{Li}\) (\(N = 7\) and \(Z = 3\)), we use a potential set consistent with the \(\text{P}4\) model of Ref. [13] and this potential reproduces the observed \(p_{1/2}\) resonance at 0.45 MeV [3, 4] and \(d_{5/2}\) resonance, which lies at higher energy around 2.98 MeV, this position is consistent with high-lying structure of \(^{10}\text{Li}\) reported in [4]. For present calculations we have ignored the spin of the core \(^{3}\text{Li}\). The neutron number 6 is assumed for the core configuration given by \((0s_{1/2})^2(0p_{3/2})^2\). The four valence neutron continuum orbits, i.e., \(p_{1/2}, d_{3/2}, s_{1/2}\) and \(d_{3/2}\) are considered in the present calculations for \(^{10}\text{Li}\). The parameter set used in the present study is tabulated in Table 1.

3.2. \(^{21}\text{C}\)
For \(^{21}\text{C}\) (\(N = 15\) and \(Z = 6\)), not much information is known beyond that it is unbound. The only available experimental study using the single-proton removal reaction reported the limit to the scattering length \(|a_0| < 2.8\) fm and due to the low statistics of this experimental data at low energies, the possibility of low-lying resonance states can not be ruled out [14]. In the view of exploring the sensitivity of the core-\(n\) potential to the possible resonances and configuration mixing in the ground state of \(^{22}\text{C}\), very recently we examined in detail the four different potential sets (for details see text and Table 1 of Ref. [9]). In the present manuscript the results corresponding to potential set tabulated in Table 1 will be discussed. The subshell closure of the neutron number 14 is assumed for the core configuration given by \((0s_{1/2})^2(0p_{3/2})^2(0p_{1/2})^2(0d_{5/2})^2\). The seven valence neutron continuum orbits, i.e., \(s_{1/2}, d_{3/2}, f_{7/2}, p_{3/2}, f_{5/2}, p_{1/2}\) and \(d_{5/2}\) are considered in the present calculations for \(^{21}\text{C}\).

3.3. \(^{25}\text{O}\)
In the recent measurement conducted at RIKEN [8], along with high accuracy measurement of the ground state of \(^{26}\text{O}\), they have also reported the \(d_{3/2}\) resonance state at 749(10) keV with width of 88(6) keV for \(^{25}\text{O}\). This information will serve as input for fixing the core-\(n\) potential parameters. For \(^{25}\text{O}\) (\(N = 17\) and \(Z = 8\)), we adopt the potential set from our recent study reported in Ref. [10]. For the Wood-Saxon depth parameter \((V_0)\) and the strength of spin-orbit potential \((V_{ls})\) parameter tabulated in Table 1, we use the information for the energy of unbound \(d_{3/2}\) state. Our parameter set is consistent with the one reported in Ref. [19]. The neutron number 16 is assumed for the core configuration given by \((0s_{1/2})^2(0p_{3/2})^2(0p_{1/2})^2(0d_{5/2})^2(1s_{1/2})^2\). The three valence neutron continuum orbits, i.e., \(d_{3/2}, p_{3/2}\) and \(f_{7/2}\) are considered in the present calculations for \(^{25}\text{O}\).
4. Results and Discussions

The 3b-model with two non-interacting particles in the above single-particle levels of $^{10}$Li, $^{21}$C and $^{25}$O produces different parity states, when two neutrons are placed in different unbound orbits. The four configurations $(s_1/2)^2$, $(p_1/2)^2, (d_3/2)^2, (d_5/2)^2$ couple to $J^p = 0^+$ for $^{11}$Li, seven configurations $(s_1/2)^2, (p_1/2)^2, (p_3/2)^2, (d_3/2)^2, (d_5/2)^2, (f_5/2)^2$ and $(f_7/2)^2$ couple to $J^p = 0^+$ for $^{22}$C and three configurations $(d_1/2)^2, (p_3/2)^2$ and $(f_7/2)^2$ couple to $J^p = 0^+$ for $^{26}$O.

In the 3b-calculations, along with the core+n potential the other important ingredient is the $n$-$n$ interaction. We use the DD contact-delta pairing interaction, with the only adjustable parameter being $v_\rho$. In the DD contact-delta pairing interaction (defined by Eq. (2)), the strength of the DI part is given as $v_\rho = 2\pi^2 \hbar^2 \frac{2a_{nn}}{m_N \pi - 2k_c a_{nn}}$, where $a_{nn}$ is the scattering length for the free neutron-neutron scattering and $k_c$ is related to the cutoff energy, $e_c$, as $k_c = \sqrt{\frac{m_N e_c}{\hbar^2}}$. We use $a_{nn} = 15$ fm and $e_c = 30$ MeV [19], which leads to $v_\rho = 857.2$ MeV fm$^3$. For the parameters of the DD part, we determine them so as to fix the ground-state energy of $^{11}$Li, $^{22}$C and $^{26}$O, $E = -0.370$ MeV [15], $E = -0.140$ MeV [16] and 0.018 MeV [8] respectively. The values of the parameters that we employ are $R_\rho = 1.25 \times A_c^{1/6}$ ($A_c = 9, 20, 24$) and $v_\rho = 861.75, 591.55$ and 1058.70 MeV fm$^3$ for $^{11}$Li, $^{22}$C and $^{26}$O respectively.

We report the percentage configuration mixing in the ground state of these three systems in Table 2. We found for this particular choice of core+n potential, $^{11}$Li is p-neutron halo, $^{22}$C is s-neutron halo and where as for $^{26}$O case our results report mixed configuration mixing without any particular angular momentum dominance. These results of configuration mixing are consistent with the results of Ref. [13] for $^{11}$Li, of Ref. [17, 18] for $^{22}$C and of Ref. [19] for $^{26}$O. The detailed results of configuration mixing with different choices of pairing interaction for $^{22}$C and $^{26}$O are reported in Ref. [9, 10]. The detailed investigation of configuration mixing for different choices of pairing interactions and core+n potential for $^{11}$Li are in progress, very soon will be reported elsewhere.

The two particle density of $^{11}$Li, $^{22}$C and $^{26}$O as a function of two radial coordinates, $r_1$ and $r_2$, for valence neutrons, and the angle between them, $\theta_{12}$ in the LS-coupling scheme is calculated by following Refs. [12, 19]. The distribution at smaller and larger $\theta_{12}$ are referred to as “di-neutron” and “cigar-like” configurations, respectively. One can see in Fig. 1 that the two-particle density is well concentrated around $\theta_{12} \leq 90^0$, which is the clear indication of the di-neutron correlation and the di-neutron component has a relatively higher density in comparison to the small cigar-like component for all three cases i.e. $^{11}$Li, $^{22}$C and $^{26}$O. The reflection of dominance of $p$-component and $s$-component in ground state of $^{11}$Li and $^{22}$C can be seen in of Fig. 1(a) and Fig. 1(b) showing extended di-neutron component in comparison to $^{26}$O (in panel (c) of Fig. 1), which has shrunk di-neutron component due to the mixing of $l > 0$ components in its ground-state.

5. Summary

In the present study we report the emergence of bound 2n-halo ground state of $^{11}$Li and $^{22}$C from the coupling of four unbound spd-waves and seven unbound spdf-waves in the continuum of $^{10}$Li and $^{21}$C respectively due to the presence of pairing interaction. Also the emergence of 2n-unbound ground state of $^{26}$O from the coupling of three unbound pdf-waves in the continuum of $^{25}$O due to the presence of the pairing interaction is reported. The configuration mixing in the ground state of these neutron-rich nuclei has been reported for a particular choice of core+n potential fixed in the view of the available recent experimental data. Also two-neutron correlation for these systems showing dominance of di-neutron component is discussed. Investigation with different choices of pairing interactions and core+n potential for $^{11}$Li are in progress and will be reported elsewhere.
Table 2. Components of the ground state (0\(^+\)) of \(^{11}\)Li, \(^{22}\)C and \(^{26}\)O, with the model parameter energy cut \(E_{\text{cut}}\). For \(^{11}\)Li, \(^{22}\)C and \(^{26}\)O the core+n potential tabulated in Table 1 are used. In the last column of the table, the comparison has been made with Ref. [13] for \(^{11}\)Li, Ref. [17] for \(^{22}\)C and Ref. [19] for \(^{26}\)O.

| System | \(E_{\text{cut}}\) (MeV) | \(l_j\) | Present work | Reference |
|--------|------------------------|--------|--------------|-----------|
| \(^{11}\)Li | 5                      | (s\(_{1/2}\))^2 | 0.245 | 0.270 |   |
|         |                        | (p\(_{1/2}\))^2 | 0.596 | 0.670 |   |
|         |                        | (d\(_{5/2}\))^2 | 0.091 | 0.030 |   |
|         |                        | (d\(_{3/2}\))^2 | 0.012 | –     |   |
| \(^{22}\)C | 5                      | (s\(_{1/2}\))^2 | 0.819 | 0.823 |   |
|         |                        | (p\(_{1/2}\))^2 | 0.006 | –     |   |
|         |                        | (p\(_{3/2}\))^2 | 0.035 | 0.010 |   |
|         |                        | (d\(_{3/2}\))^2 | 0.084 | 0.158 |   |
|         |                        | (d\(_{5/2}\))^2 | 0.003 | –     |   |
|         |                        | (f\(_{5/2}\))^2 | 0.0001 | – |   |
|         |                        | (f\(_{7/2}\))^2 | 0.0049 | 0.007 |   |
| \(^{26}\)O | 10                     | (d\(_{3/2}\))^2 | 0.643 | 0.661 |   |
|         |                        | (p\(_{3/2}\))^2 | 0.088 | 0.105 |   |
|         |                        | (f\(_{7/2}\))^2 | 0.268 | 0.183 |   |

Figure 1. Two-particle densities for the ground state of \(^{11}\)Li, \(^{22}\)C and \(^{26}\)O as a function \(r_1 = r_2 = r\) and the opening angle between the valence neutrons \(\theta_{12}\) for settings mentioned in caption of Table 2.

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