Light Confinement by Local Index Tailoring in Inhomogeneous Dielectrics

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The engineering of light confinement is a topic with a long history in optics and with significant implications for the control of light-matter interaction. In inhomogeneous and disordered media, however, multiple scattering prevents the application of conventional approaches for the design of light fields with desired properties. This is because any local change to such a medium typically affects these fields in a non-local and complicated fashion. Here, we present a theoretical methodology for tailoring an inhomogeneous 1D Hermitian dielectric index distribution that allows us to control the intensity profile of an incoming light field purely locally, that is, with little or no influence on the field profile outside of a designated region of interest. Strongly increasing or decreasing the light’s intensity at arbitrary positions inside the medium thereby becomes possible without, in fact, changing the external reflectance or transmittance. These local modifications of the medium can thus be made undetectable to far field measurements. We apply this approach to locally control the confinement of light inside 1D materials with inhomogeneous continuous refractive index profiles and extend it to multilayer films as well as to chains of coupled micro-resonators.

1. Introduction

The propagation of light through dielectric structures can nowadays be tailored to a remarkable degree.[1–4] In this respect, the understanding of the spatial localization of electromagnetic fields inside complex dielectric geometries has proven highly relevant for engineering light–matter interactions.[5] Progress in this direction has already resulted in a broad range of designs for optical devices such as lasers, switches, and memories.[6] The key benefit of using dielectric geometries for trapping and manipulating light is the miniaturization, which in turn enhances light–matter interaction and enables integration of optical elements in a compact structure.[7]

A common approach of confining light is to rely on resonant modes, for which the field intensity is concentrated in a given region of space, and drops sharply outside of it. In photonic crystals, such a localization can, for example, be achieved by placing a defect inside the periodic lattice.[1] Structures that are much easier to fabricate and that also feature strongly localized resonances are disordered media, whose potential for applications has been widely studied in recent years.[3,4,8–18] The multiple scattering of light, however, prevents a straightforward control of the resonances inside these media such that engineering their location and shape both in the spatial and in the spectral domain remains difficult. This is due to the fact that any local change of the refractive index of the disordered medium will have a considerable influence on the entire intensity distribution of a wave propagating through this medium (provided, of course, that the intensity is not negligible in the region where the refractive index is changed). This non-local dependence of the field’s intensity on the entire scattering environment renders any attempt to optimize the field pattern a challenging and potentially impossible task.

In this article we demonstrate how this severe limitation, which constitutes a major hurdle for the design of complex media, can be conveniently overcome. We present an exact approach for tailoring the wave’s intensity profile inside an inhomogeneous 1D Hermitian dielectric, and apply it to engineer the confinement of light in disordered structures. Our methodology is based on a local, but judiciously calibrated modification of the medium’s refractive index distribution by mapping the scattering fields of the given structure (reference medium) onto those of the desired structure (design medium). Specifically, these two scattering fields are proportional to each other with a space-dependent proportionality factor that we can choose to be strongly peaked or reduced in any desired region of space. The flexibility of our method allows us not only to select the exact position where such states are localized, but also to precisely engineer the shape of the corresponding intensity peaks by

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Figure 1. Schematic depiction of our optical design principle. a) A plane wave $e^{i n_0 k_0 x}$ with wavenumber $k_0 = 2\pi/\lambda_0$ enters an inhomogeneous dielectric medium (left) with a refractive index $n_0(x)$ ($n_0$: refractive index of the homogeneous background medium, $\lambda_0$: wavelength of light in vacuum). The complex scattering processes inside the medium give rise to a highly modulated electric field intensity distribution $|\phi(x)|^2$ (right). b) By locally modifying the refractive index $n_E(x)$ according to Equation (4), we create a new medium $n_E(x)$ (left), which has the desired property of light confinement at the center, as depicted in the intensity distribution $|E(x)|^2$ (right), for the same input. Moreover, outside of the confinement region, the electric field $E(x)$ is indistinguishable in amplitude from the field $\phi(x)$ (compare to (a), right, and see Figure 2a,b), and can also be made indistinguishable in phase from the field $\phi(x)$.

tailoring the shape of the local dielectric function. The spatial extent of the engineered intensity peaks is not limited to the scale of the optical wavelength, but can be made arbitrarily small (in principle). Notably, to achieve these effects, we do not need to work with non-Hermitian materials for which finite values of gain or loss would be necessary, \cite{19–22} nor are any other exotic materials required, that would feature properties like non-reciprocal transmission characteristics or a vanishing/negative index of refraction. More specifically, the Hermitian media we consider here are fully described by real valued permittivity profiles.

One specific consequence of the mapping we employ in our approach is that the reflection coefficients of the original (reference) and the modified (design) medium at the design wavenumber of the incident wave are equal (modulo $2\pi$ in the phase). Moreover, the transmission coefficients of these two media have equal moduli and can be engineered to have also equal phases. This makes the two interrelated media indistinguishable to far field measurements for incoming light at the design wavenumber. This aspect is highly interesting from the standpoint of concealing and mimicking the reflection and transmission signals of scattering objects. By establishing a connection between fields in an inhomogeneous reference medium and a design medium with equal scattering coefficients, our optical design technique generalizes the interesting results of refs. \cite{23, 24} where the amplitude manipulation was limited to homogeneous reference media only, which considerably simplified the problem.

2. Mapping Between Two Inhomogeneous Hermitian Media

The situation we investigate is schematically depicted in Figure 1: we consider the propagation of light through linear, non-magnetic dielectrics that are inhomogeneous, but isotropic in the sense that the permittivity tensor can be replaced by a spatially-dependent scalar permittivity. For the case that this permittivity varies only along one direction (labeled as the $x$-coordinate in our case), but stays constant in the two orthogonal directions ($y, z$)—such as for layered media—the Maxwell equations governing the light fields can be reduced to the much simpler Helmholtz equation in 1D (see below). Specifically, the Helmholtz equation describes the orthogonal component of the time-harmonic electric field ($E_x$) propagating in $x$-direction at a fixed frequency $\omega_0 = ck_0$, where $c$ is the vacuum speed of light and $\lambda_0$ is the vacuum wavelength of light with $k_0 = 2\pi/\lambda_0$ being the corresponding wavenumber. Our methodology provides a systematic way to locally modify the dielectric function $\varepsilon_{ref}(x) = n_0^2(x)$ of the reference medium in order to achieve a desired electric field intensity distribution inside a design medium with a dielectric function $\varepsilon_E(x) = n_E^2(x)$.

When both media satisfy the same general material properties outlined at the beginning of the previous paragraph, the corresponding equations for the propagation of the electric field $\phi(r)$ in the reference medium $\varepsilon_{ref}(x)$, and of the electric field $E(r)$ in the design medium $\varepsilon_E(x)$ read as follows:\cite{1, 25}

$$\begin{align*}
\nabla \times \nabla \times \phi(r) &= k_0^2 \varepsilon_{ref}(x) \phi(r) \\
\nabla \times \nabla \times E(r) &= k_0^2 \varepsilon_E(x) E(r)
\end{align*}
$$

For waves incident in the $x$-direction, their polarized field components $\phi(r) = \phi(x)x$ and $E(r) = E(x)x$ are described by the following 1D Helmholtz equations for the two media:

$$\begin{align*}
\frac{d^2}{dx^2} + k_0^2 \varepsilon_{ref}(x) \phi(x) &= 0 \\
\frac{d^2}{dx^2} + k_0^2 \varepsilon_E(x) E(x) &= 0
\end{align*}
$$
As depicted in Figure 1, our goal is to associate the reference medium, with a real-valued dielectric function $\varepsilon_\phi(x)$, to a design medium with a real-valued dielectric function $\varepsilon_d(x)$, such that the corresponding scattering solutions are related as:

$$E(x) = R(x)\phi(x)$$  \hspace{1cm} (3)

where the space-dependent factor $R(x) = A(x)e^{i\theta(x)}$ is complex-valued and $A(x)$, $\theta(x)$ are the real-valued functions of amplitude and phase, respectively. It can be shown (see Supporting information), that if both media are Hermitian, the modified dielectric function $\varepsilon_d(x)$ is related to the reference dielectric function $\varepsilon_\phi(x)$ by:

$$\varepsilon_d(x) = \varepsilon_\phi(x) + \Delta\varepsilon(x)$$  \hspace{1cm} (4)

where

$$\Delta\varepsilon(x) = \frac{1}{k_0^2} \left[ \left( \frac{d\eta}{dx} \right)^2 + 2\eta I(\eta) \frac{d\eta}{dx} - \frac{d^2 A}{A \frac{dA}{dx}} - \frac{2\eta I(\eta)}{A} \frac{dA}{dx} \right]$$  \hspace{1cm} (5)

with $\frac{d}{dx} \eta = \eta_R + i\eta_I$, and $\theta(x)$ satisfying the equation:

$$\frac{d\theta}{dx} + 2\left( \frac{1}{A} \frac{dA}{dx} + \eta_R \right) \frac{d\theta}{dx} + 2\eta_I \frac{dA}{dx} = 0$$  \hspace{1cm} (6)

In this article, we solve Equation (6) numerically as a differential equation for the relative phase $\theta(x)$ inside the scattering medium for a given amplitude function $A(x)$, which we call the design function. The equation can also be reformulated as an equation for $A(x)$ at a given $\theta(x)$ (see ref. [24]) however we do not follow this approach here. An equivalent formulation of the problem, which does not require the numerical solution of Equation (6), is given in Section 3 of the Supporting Information. However, such a formulation makes the relationship between the solutions in terms of $n_d(x)$ and $n_\phi(x)$ less transparent, so it was not used for the basic results of this article (see Section 4 and the Supporting Information). Equation (6) stems from the fact that the designed medium is Hermitian, and a related equation in ref. [24] was aptly named the “energy conservation condition”. The coefficients $\eta_R(x)$, $\eta_I(x)$ are calculated from the numerical solution of the Helmholtz equation for the reference medium. Here we consider functions $A(x)$, which approach the value one outside the modified region (see below), which means that outside of this region $|\phi(x)| = |E(x)|$.

Since we demand $\varepsilon_d(x \to \pm\infty) = \varepsilon_\phi(x \to \pm\infty) = n_0^2$ (where $n_0$ is the homogeneous background refractive index), we numerically solve Equation (6) with imposed Neumann boundary conditions $\frac{d\eta}{dx} = 0$ at the boundaries of our system. It can be shown that for these boundary conditions, $\theta(x)$ is always constant and $\varepsilon_d(x) = \varepsilon_\phi(x)$ in the region where $A(x) = 1$ (see Supporting Information).

For the wavenumber $k = k_0$ and the reference field $\phi(x)$ incoming from the left, the reflection and transmission coefficients of the two media are intimately connected with each other through Equation (6) (see Supporting Information):

$$r_\phi(k_0) = r_d(k_0) \mod 2\pi \text{ in phase}, t_\phi(k_0) = e^{i\delta(k_0)} t_d(k_0),$$

$$r_\phi'(k_0) = e^{2i\delta(k_0)} r_d'(k_0) \mod 2\pi \text{ in phase}, t_\phi'(k_0) = e^{i\delta(k_0)} t_d'(k_0),$$  \hspace{1cm} (7)

where the non-primed (primed) quantities correspond to a beam incoming from the left (right), and $\delta(k_0)$ is the relative transmission phase shift acquired in the design medium with respect to the reference medium at $k = k_0$. We note that, although in this article we consider solutions $\phi(x)$, which are right-propagating incident waves, the same theory could also be used for left-propagating incident waves. Also in this case the design condition Equation (3), leads to Equation (6) for Hermitian media, and thus determines the relation between the asymptotic properties of the family of dielectric functions related by Equation (4). Such a change of the input beam direction for $\phi(x)$ results in a switching of the primed and non-primed scattering coefficients in Equation (7).

Most importantly, for a judiciously chosen design function, the shift $\delta(k_0)$ can be completely eliminated (see below). In other words, our strategy should also allow us to choose the design medium $n_d(x)$ in such a way that it cannot be distinguished from the reference medium $n_\phi(x)$ by far-field measurements for both a left- and right-propagating input plane wave with $k = k_0$. Note, however, that as relation (3) connects the design field $E(x)$ with a specific reference field $\phi(x)$, this will in general not mean that the design behavior within the medium (such as the light confinement with a desired location and shape) will be achieved for both the right- and left-incident waves. Instead, the desired field profile will only be realized for the input corresponding to $E(x)$ related to $\phi(x)$ via Equation (3). For the examples studied in this article we always work with right-propagating input waves (see the SI for a discussion of the opposite case).

We now apply the above approach to increase or reduce the intensity of light inside a 1D Hermitian dielectric by locally modifying its refractive index profile $n_d(x)$. As our first example, we consider the following form for the design function $A(x)$:

$$A(x) = 1 - \left( 1 - \frac{1}{|\phi(x)|} \right) e^{-|x-x_c|^2/\sigma^2} + \frac{a}{|\phi(x)|} e^{-|x-x_c|^2/2\sigma_x^2}$$  \hspace{1cm} (8)

The first two terms of the above expression affect the field profile $|E(x)|$, since they produce a flat (super-Gaussian) region of height equal to one and width $\sigma$ centered at $x = x_c$, while the last term adds to this background a Gaussian function with amplitude $a$ and width $\sigma_x$ at the same center position. To obtain a Gaussian shaped confinement, it is necessary to add the super-Gaussian term, since otherwise the confinement region would have a background varying as $|\phi(x)|$. Depending on the sign of $a$, the solution is increasing or reducing the light’s intensity at $x = x_c$, as shown in the two examples of Figure 2.

In particular, Figure 2 demonstrates the principle of local intensity engineering based on a refractive index modification around $x = x_c$. In the first case, the refractive index in the dark gray region reduces below the background value $n_i = 2$, forming a structure that resembles a well. The interference of the waves reflecting at the edges of the well then creates a region with a peaked electric field intensity. The well is shaped precisely such that the intensity peak is Gaussian, while the reflection and transmission properties of this well structure result in an intensity distribution outside of the well that is identical to the one of the reference medium (see Equation (7)). In the opposite case of intensity suppression at $x_c = x_s$, the refractive index rises above the background value, creating a barrier. The field amplitude inside
Figure 2. Local intensity control inside an inhomogeneous scattering medium. a) Refractive index distribution of the reference medium \( n_\phi(x) \) (blue dashed) and of the designed medium \( n_E(x) \) (orange solid), leading to light confinement of a Gaussian intensity profile centered around \( x = x_c = 0 \). b) Normalized electric field intensity for the reference medium, \( |\phi(x)|^2 \) (blue dashed) and for the designed medium, \( |E(x)|^2 \) (orange solid), corresponding to the refractive index distributions depicted in (a). c) Refractive index distribution of the reference medium \( n_\phi(x) \) (blue dashed) and the designed medium \( n_E(x) \) (red solid), for which the intensity of light is decreased at \( x = x_c = \lambda_0 \), as shown in (d). d) Normalized electric field intensities for the reference medium, \( |\phi(x)|^2 \) (blue dashed) and for the designed medium, \( |E(x)|^2 \) (red solid) with refractive index profiles depicted in (c). In all plots the areas shaded in light gray color mark the scattering region, while the dark shaded parts indicate the region where also \( \Delta\varepsilon(x) \neq 0 \). The green arrows in (b) and (d) denote the incident waves’ propagation direction. The refractive index profile of the reference medium was constructed as a superposition of \( N = 18 \) Gaussians with equal widths and spacing, but with varying amplitudes, such that \( n_\phi(x) = n_0 + f(x) \), where \( f(x) = -f(-x) \). \( A(x) \) is given by Equation (8), while the parameters for the plots (a) and (b) are: \( x_c = 0, \sigma = 0.55\lambda_0, \alpha = 0.4, \sigma_c = 0.07\lambda_0, \) where \( \lambda_0 \) is the vacuum wavelength. The parameters for the plots (c) and (d) are: \( \alpha = -0.45, \sigma_c = 0.0265\lambda_0, x_c = \lambda_0, \sigma = 0.15\lambda_0 \).

With our methodology we can thus, in principle, modify any reference refractive index distribution \( n_\phi(x) \) and its associate scattering wave solution \( \phi(x) \) with finite and continuous \( \eta_R(x) \) and \( \eta_I(x) \) (where \( \frac{1}{2} \ln \phi = \eta_R + i\eta_I \)). However, for a Hermitian \( n_\phi(x) \), the modified medium \( n_E(x) \) will not be dielectric and Hermitian in general, since this depends on the specific parameters of the modified electric field intensity distribution. For instance, if one wants to confine light inside a Hermitian dielectric, then increasing the amplitude of the confined light beyond a certain value will result in the refractive index \( n_E(x) \) falling below unity and/or having complex values at some region of space. In this article we consider only parameters for which both \( n_\phi(x) \) and \( n_E(x) \) correspond to Hermitian dielectric media.

3. Indistinguishable Disordered Media

The most striking application of our novel approach is related to the indistinguishability of disordered media. More specifically, we consider a 1D disordered medium that was constructed by a
random superposition of many Gaussian functions, as the one depicted in Figure 3. Whereas the media in the previous section were indistinguishable for far-field measurements of the intensity (at $k = k_0$), we will now also remove any effect on the transmission phase induced by the modification of the medium $\delta a(x) = 0$ in the case of light confinement (see Figure 3). To achieve this, we add to the design function $A(x)$ of Equation (8) a part $\delta a(x)$ of the form:

$$
\delta a(x) = -\frac{\beta}{|\phi(x)|^2} e^{-(x-x_c)^2/\sigma^2}
$$

where $\beta > 0$. Adding this contribution $\delta a(x)$ to the design function $A(x)$ creates a barrier in the refractive index distribution at position $x = x_c$. We have numerically tuned the height $\beta$ of the barrier such that it compensates for the phase shift caused by the wave’s confinement.

Figure 3a clearly shows how the reference medium is modified in this case. As explained above, the confinement of light is achieved by the constructive interference induced by the dip in the refractive index around $x = x_c$. The shape of the dip is precisely tailored to produce the desired profile of the confined field, while maintaining the same amplitude as in the reference medium outside of the confinement region. The additional part $\delta a(x)$ now modifies the medium only slightly, in a region behind the confinement (for $x < x_c$). Figure 3b displays a perfect correlation between $|E(x)|^2$ and $|\phi(x)|^2$, apart from the confinement region centered at $x_c$ and inside the phase shifting region around $x_c$, where a small deviation between these two quantities is observable. Moreover, around $x = x_c$, the refractive index $n_{r}\delta a(x)$ is tailored to reach the value of 1. This part of the modified medium is free space (air), which can be accessed by small emitters such as quantum dots or atoms, enabling them to couple to the engineered electric field intensity $|E(x_c)|$ (see, e.g., Ref. [26]).

The above methodology for tailoring the intensity profile is a delicate wave interference effect and the explicit analytical relation between the reference and the designed medium, Equation (4), holds for a specific design wavenumber $k_0$. To test the spectral robustness of our procedure, we examine how the solutions of the Helmholtz equation in the reference and in the design medium change when the wavenumber $k$ of the incident wave is detuned away from the design wavenumber $k_0$.

In particular, the absolute value of the difference between reflection phases $|\delta, k_0| = |\arg[\phi_0(k)] - \arg[\phi_0(k_0)]|$ and transmission phases $|\delta, k| = |\arg[\phi(k)] - \arg[\phi(k_0)]|$ is plotted in Figure 4a for the case of $A(x) + \delta a(x)$ of Figure 3. The quantities $|\delta, k_0|$ and $|\delta, k|$ reach the minimum values of $2.8 \times 10^{-5}$ and $2.6 \times 10^{-5}$ at $k = k_0$, respectively, and rapidly increase when moving away from $k_0$. The sharpness of the troughs in $|\delta, k_0|$ and $|\delta, k|$ are a signature of the fact that the effects leading to the confinement of light are interferometric in origin. Moving even further away from $k = k_0$, we observe discrete values of $k$ for which $|\delta, k|$ and $|\delta, k|$ drop or rise again sharply, indicating interesting resonance effects.

Moreover, the amplitudes of the transmission coefficients of the reference medium $t_{\delta, 0}$ and of the design medium $t_{\delta, \delta}$, are also shown in Figure 4b. As we can see, they depend less sharply on the variation of $k$, as compared to the changes on the transmission phase. This is a consequence of the particular choice of the refractive index profile, i.e., the fact that the transmission of this structure is still relatively large. Likewise, concerning the sensitivity of the media to design imperfections, we have found that several factors, such as the refractive index contrast, the length scale of variations in the refractive index, the degree of light confinement, and the strength of disorder.
in the system, all play a role in the robustness of our design. We do note, however, that, since the methodology we apply to design \(n_d(x)\) is fairly general (including several free parameters), it is expected that the indistinguishability of \(n_d(x)\) and \(n_e(x)\) can become broadband for an optimized choice of the design parameters.

To complement the above frequency scans, we have also performed scans of the input tilt angle, parametrized by the transverse wavevector component \(k_y\), at the design frequency \(\omega_0 = c k_x\).

By inserting the fields \(\tilde{\phi}'(x) = \phi'(x)e^{i k_y y} z\) and \(\tilde{E}'(x) = E'(x)e^{i k_y y} z\) into the Equations (1), we get the equations for propagation of a tilted beam through media with dielectric functions \(\varepsilon_\phi(x)\) and \(\varepsilon_E(x)\), respectively:

\[
\begin{align*}
\frac{d\tilde{\phi}'(x)}{dx} + k_y^2 \varepsilon_\phi(x) - n_\phi^2 k_y^2 \tilde{\phi}'(x) &= 0, \\
\frac{d\tilde{E}'(x)}{dx} + k_y^2 \varepsilon_E(x) - n_E^2 k_y^2 \tilde{E}'(x) &= 0.
\end{align*}
\]

The incoming field is now of the form \(e^{i \alpha_0(k_x x + k_y y)}\), with \(k_x, k_y\) satisfying the dispersion relation \(k_0^2 = k_x^2 + k_y^2\). The refractive index distributions of the two media are plotted in Figure 3a.

The scans are presented in Figure 5. Both the reflected \((\tilde{\phi})\) and transmitted \((\tilde{\phi}'\) and \(\tilde{E}'\)) phase difference moduli stay below the value of 0.1 rad for \(k_y\) values up to 0.099\(k_0\), corresponding to input angles of \(|\alpha| = |\arcsin(k_y/k_0)|\) is 5.2°. Moreover, as the transmission coefficients \(t_\phi\) and \(t_E\) stay similar in both phase and amplitude (see Figure 5b) for \(k_y\) values up to 0.15\(k_0\), the initial and design medium are approximately indistinguishable to transmission measurements at input angles in the relatively broad range of \([-8.6°, 8.6°]\) with respect to the normal. As the \(k_y\) component of the input wavevector changes by \(\approx 0.04k_0\) in the whole scan range of Figure 5, the robustness of the transmission and refraction properties to changes of the incident beam’s tilt is thus similar to the robustness for corresponding changes of the incident beam’s frequency.

4. Potential Experimental Implementations

In this section we study how our theoretical and numerical results could in principle be realized in an experiment. Certainly a challenging aspect is the fact that the assumed continuous refractive index profiles might be difficult to implement in practical photonic setups, due to their highly oscillatory \(x\)-dependence. We will thus investigate here, how to implement our theory in more realistic experimental settings.

The first system we consider is a multilayer medium consisting of a 1D stack of dielectric slabs, which is a physical system that is frequently encountered in photonics.\(^{[1,25]}\) We have found that Equation (4) can be applied to discontinuous \(n_d(x)\) distributions. In particular, for certain forms of the amplitude design function \(A(x)\), the resulting continuous refractive index distribution \(n_d(x)\) can be made to vary without fast oscillations on a scale of the typical manufacturing resolution of photonic crystals. Implementing such \(n_d(x)\)-distributions in an experimental setting would be a fascinating prospect for photonic design. To illustrate such a potential implementation, we provide in the following one specific example of such an amplitude function \(A(x)\):

\[
A(x) = \begin{cases} 
|\phi(x)|^{-1} \times |\phi(x_\ell)|, & 0 \leq x < 0, \\
A(x) = \begin{cases} 
\alpha |\phi(x)|^{-1} \times |\phi(x_\ell)|, & 0 \leq x \leq x_\ell, \\
\alpha (|\phi(x)| - |\phi(x_\ell)|) \times |\phi(x_\ell)|^{-1}, & x_\ell \leq x < 0. 
\end{cases}
\end{cases}
\]

where \(x_\ell\) and \(x_r\) are the left and right boundaries of the region where \(\Delta x(x) \neq 0\), and \(A(x < x_\ell) = A(x > x_r) = 1\). We have found that by choosing the \(x_{\ell, r}\) boundaries to be at the local minima of \(|\phi(x)|^2\), the distribution \(n_d(x)\) can be made to vary smoothly inside this region (see the Supporting Information for a discussion on designing smooth \(n_d(x)\) functions). The results shown in Figure 6 demonstrate an example of using Equation (11) to create light confinement by modifying a discontinuous distribution \(n_d(x)\), describing a stack of randomly alternating dielectric slabs. In this example, the resulting intensity distribution \(|E(x)|^2\) has a shape described by Equation (11), giving light confinement at \(x = x_\ell = 0\) (see also Equation (S17), Supporting Information). As in Figure 3, the refractive index of the region around the origin reaches the value of free space (air), making the design appealing for interfacing light and matter.\(^{[26]}\)

On the other hand, artificial systems where the medium’s index varies in a discrete fashion (unlike bulk materials), such as micro-resonators,\(^{[27,28]}\) waveguides\(^{[29,30]}\) and loudspeaker arrays,\(^{[31]}\) or time-multiplexed photonic crystals,\(^{[32]}\) have in recent years been employed for proof-of-principle demonstrations of a plethora of wave physical phenomena. We will thus consider, as our second example, the case of such a discrete system consisting of a chain of microresonators.

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Image 48x736 to 137x757

Figure 5. Sensitivity of the scattering characteristics to variations in the incident beam's angle (quantified by the \(y\)-component of the input wavevector, \(k_y\)) for the reference and the designed refractive index distributions shown in Figure 3a. a) Modulus of the difference of reflection phases \(|\phi(k_y,0) - \phi(k_y,\pm 0.1\text{ rad})|\) (black dashed) and transmission phases \(|\phi(k_y,\pm 0.1\text{ rad})|\) (red solid) for the two media (in radians). b) Modulus squared of the frequency difference for the reference (blue dashed) and the design medium (orange solid). The parameters for the design medium are given in the caption of Figure 3.
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Figure 6. Demonstrating light confinement by modifying a 1D stack of dielectric slabs. We directly apply our method, by choosing an amplitude design function \( A(x) \) as given in Equation (11). a) Refractive index distributions of the reference medium, \( n_R(x) \) (blue dashed) and of the design medium, \( n_E(x) \) (orange solid). The refractive index \( n_E(x) \) describes a collection of 50 slabs of width \( 0.5 \lambda_0 \), alternating randomly between the values of 1.82, 2, and 2.18. The area shaded in light gray denotes the scattering region, while the dark shaded part is the region where the medium differ, that is, \( \Delta n(x) \neq 0 \). Electric field intensity (normalized to input) for the solution (b) in the reference medium, \( |\phi|_R^2 \) (blue) and (c) in the design medium, \( |E(x)|^2 \) (orange). The (false colored) density plots in the backgrounds of (b) and (c) are schematic depictions of \( n_E(x) \) and \( n_R(x) \), respectively. The green arrows in (b) and (c) indicate the propagation direction of the incident wave. The related parameters of the designed medium are: \( x_{\text{L}} = -2.583 \lambda_0, x_R = 2.997 \lambda_0, x_0 = 0.533, x_c = 0, \gamma_f = 0.7 \lambda_0 \), and \( \sigma_c = 0.05 \lambda_0 \).

More specifically, in the stationary regime, the system is conveniently described under the coupled-mode approximation \(^{27,28}\) by the following equations:

\[
(\gamma_n - \omega) \phi_n + \kappa(\phi_{n-1} + \phi_{n+1}) = 0
\]

\[
(\Gamma_n - \omega) E_n + \kappa(E_{n-1} + E_{n+1}) = 0
\]

where \( \omega \) is the input frequency, \( \kappa \) is the nearest resonator coupling strength, \( \phi_n \) is the mode amplitude in the \( n \)th resonator for the distribution of resonance frequencies \( \{\gamma_n\} \), while \( E_n \) is the corresponding quantity for the distribution \( \{\Gamma_n\} \). As before, our goal here is to create a modified medium, described by a frequency distribution \( \{\Gamma_n\} \subset \mathbb{R} \), supporting a solution \( E_n \) which is related to the reference distribution \( \{\phi_n\} \) as \( E_n = A_n \exp(i \sum w_i) \phi_n \). Moreover, \( \{A_n\} \subset \mathbb{R} \) is the amplitude distribution (analogous to the design function), and \( \{w_n\} \subset \mathbb{R} \) is a distribution of coefficients, where the relative phase at the \( n \)th resonator is given by \( \theta_n = \sum w_i \).

The two resonance frequency distributions are for \( \{\gamma_n\}, \{\Gamma_n\} \subset \mathbb{R} \) related as:

\[
\Gamma_n = \gamma_n + \kappa \left[ \eta_{n+1}^R + \frac{\eta_{n+1}^R}{\Gamma_n} - \kappa \frac{\eta_{n+1}^R}{\Gamma_n} \sin w_{n+1} \right] + \frac{\Delta \sigma}{\Gamma_n} \eta_{n+1}^R \sin w_{n+1}
\]

\[
\eta_{n+1}^R \cos w_{n+1} - \frac{\Delta \sigma}{\Gamma_n} \left( \frac{\eta_{n+1}^R}{\Gamma_n} \sin w_n + \frac{\eta_{n+1}^R}{\Gamma_n} \cos w_n \right)
\]

Figure 7. Light confinement at the center of a chain of \( N = 130 \) micro-resonators with a disordered resonance frequency distribution. a) Distribution of the microresonator resonance frequencies for the reference medium, \( \gamma_n \) (blue dots), and for two design media with a width \( \sigma_c = 1.5 \) (\( \Gamma_n^R \), green dots) and \( \sigma_c = 4 \) (\( \Gamma_n^G \), orange dots), respectively. b) Field intensity (normalized to input) inside the resonator chain for the reference medium, \( |\phi|^2 \) (blue dots) and the designed medium for the two cases above, \( |E_n|^2 \) (green dots) and \( |E_n|^2 \) (orange dots). The amplitude distribution \( A_n \) is given by \( A_n = 1 - (1 - \frac{1}{2 \gamma_f}) e^{-\frac{(n-1)^2}{4 \sigma_c^2}} + \frac{1}{2 \gamma_f} e^{-\frac{(n-1)^2}{2 \sigma_c^2}} \).

The gray arrow marks the propagation direction of the incident wave. The other relevant parameters are: \( \omega = 2 \omega_c, \alpha = 0.4, n_c = 65, \sigma = 7.1, \lambda_0 = 4 \pi, \Delta \lambda = 0.025, \gamma_N = 0.197 \pi \).

We solve the Equation (12) under boundary conditions \( \phi_N = e^{i k_0 N \Delta_x}, \phi_{N+1} = e^{i k_0 (N+1) \Delta_x}, \) where \( n_0 = \frac{1}{k_0 \Delta x} \sqrt{\frac{\sigma}{\kappa}} \), that is, the situation is analogous to an incident propagating wave traveling to the right. The calculated parameters for the \( \phi_n \) solution are substituted into Equation (14), which is then solved under the boundary conditions of \( w_1 = w_N = 0 \), in order to get the distribution \( \{\Gamma_n\} \).

Figure 7 shows an implementation of this idea in a chain of \( N = 130 \) micro-resonators with a disordered distribution of resonance frequencies. Even for such an irregular reference system, Equation (13) produces an intensity confinement with a desired amplitude and width at the middle of the system, and the same intensity distribution as in the reference medium away from the center. These results provide strong indications that our theoretical study can be realized with current experimental setups.

5. Conclusion

We have proposed a theoretical methodology for the local tailoring of light confinement in 1D inhomogeneous Hermitian
dielectric media. Our approach allows us to locally design the intensity distribution of electromagnetic fields even inside strongly disordered media, where the dependence of the modal structure on the local changes of the refractive index is highly complex. In particular, we have demonstrated that a purely local modification of a given 1D dielectric structure allows us to confine light of a desired frequency at a desired location within the medium. Additionally, the modified structure can be tailored to have scattering coefficients that are equal both in phase and amplitude to those of the initial structure, which makes the two media indistinguishable to far field measurements. Moreover, the theory can be extended to discrete scattering systems, like for example a 1D stack of dielectric slabs, and a discrete chain of coupled microresonators. This makes our approach relevant to possible experimental realizations in the context of photonics.

In future work, we expect to extend our methodology to 2D media, in order to engineer long-lived localized states. A promising direction here would be to connect our results to commonly employed nanofabrication processes in dielectric materials, where numerical optimization techniques were typically used for optical engineering and design. Applying our results to topological photonic media and media supporting bound states in the continuum presents another exciting prospect for future research. Moreover, as our approach is not limited to Hermitian dielectrics, it will be interesting to study its effectiveness in metamaterials and in non-Hermitian media.

Supporting Information
Supporting Information is available from the Wiley Online Library or from the author.

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Conflict of Interest
The authors declare no conflict of interest.

Data Availability Statement
Data available on request from the authors.

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