Lorentz symmetry from Lorentz violation in the bulk

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We consider the mechanism of spontaneous symmetry breaking of a bulk vector field to study signatures of bulk dimensions invisible to the standard model confined to the brane. By assigning a non-vanishing vacuum expectation value to the vector field, a direction is singled out in the bulk vacuum, thus breaking the bulk Lorentz symmetry. We present the condition for induced Lorentz symmetry on the brane, as phenomenologically required, noting that it is related to the value of the observed cosmological constant.

1. Introduction

Braneworld scenarios have changed our view of the extra dimensions. The various models predict that gravity in our braneworld can exhibit significant deviations from that described by Einstein’s general relativity. In particular, in string theory inspired scenarios which assume that the background bulk spacetime is anti-de Sitter, it is possible to cancel out any 4-dimensional brane contribution to the cosmological constant (see e.g. [1] and references therein). Although not on its own a solution for the cosmological constant problem, it is suggestive that braneworld scenarios might be an important feature of a consistent description of the world.

It is therefore relevant to investigate the implications of the braneworld scenarios to the formulation of fundamental symmetries, another fundamental ingredient of the physical description. Lorentz symmetry, being from the phenomenological point of view one of the most well and stringently tested symmetries of physics, is particularly suitable to test the relation between bulk and brane symmetries as a possible signature for the existence of extra dimensions.

The possibility of violation of Lorentz invariance has been extensively discussed in the recent literature (see e.g. [4]) and in particular its astrophysical implications have been studied.\textsuperscript{5} Furthermore, a connection between the cosmological constant and the violation of Lorentz invariance has been conjectured in the context of the string field theory.\textsuperscript{6}

In this contribution we report on a recent study whose motivation was to un-
understand the way spontaneous Lorentz violation in the bulk is related to Lorentz symmetry on the brane. We consider a bulk vector field coupled non-minimally to the graviton which, upon acquiring a non-vanishing expectation value in the vacuum, introduces spacetime anisotropies in the gravitational field equations through the coupling with the graviton. After deriving the equations of motion in the bulk, we project them parallel and orthogonal to the surface of the brane. The brane is assumed to be a distribution of $\mathbb{Z}_2$-symmetric stress-energy about a shell of thickness $2\delta$ in the limit $\delta \to 0$. Derivatives of quantities discontinuous across the brane will generate singular distributions on the brane which relate to the localization of the stress-energy. This relation is encompassed by the matching conditions across the brane obtained by the integration of the corresponding equation of motion in the direction normal to the brane. The matching conditions provide the boundary conditions on the brane for the bulk fields, thus constraining the parallel projected equations to produce the induced equations on the brane. Spontaneous symmetry breaking is then treated by assuming that the bulk vector field acquires a non-vanishing expectation value which reflects on the brane the breaking of the Lorentz symmetry in the bulk.

2. Bulk Vector Field Coupled to Gravity

Aiming to examine the gravitational effects of the breaking of Lorentz symmetry in a braneworld scenario, we consider a bulk vector field $B$ with a non-minimal coupling to the graviton in a five-dimensional anti-de Sitter space. The Lagrangian density consists of the Hilbert term, the cosmological constant term, the kinetic and potential terms for $B$ and the $B$–graviton interaction term, as follows

$$\mathcal{L} = \frac{1}{2\kappa^2} R - 2\Lambda + \xi B^\mu B^\nu R_{\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} - V(B^\mu B_\mu \pm b^2),$$

where $B_{\mu\nu} = \nabla_\mu B_\nu - \nabla_\nu B_\mu$ is the tensor field associated with $B_\mu$ and $V$ is the potential which induces the spontaneous global symmetry breaking when the $B$ field is driven to the minimum at $B^\mu B_\mu \pm b^2 = 0$, $b^2$ being a real positive constant. Here, $\kappa^2 = 8\pi G_N = M^3_{Pl}$, $M_{Pl}$ is the five-dimensional Planck mass and $\xi$ is a dimensionless coupling constant that we have inserted to track the effect of the interaction. In the cosmological constant term $\Lambda = \Lambda_{(5)} + \Lambda_{(4)}$ we have included both the bulk vacuum value $\Lambda_{(5)}$ and that of the brane $\Lambda_{(4)}$, described by a brane tension $\sigma$ localized on the locus of the brane, $\Lambda_{(4)} = \sigma \delta(N)$.

The Einstein equation is given by:

$$\frac{1}{\kappa^2} G_{\mu\nu} + \Lambda g_{\mu\nu} - \xi L_{\mu\nu} - \xi \Sigma_{\mu\nu} = \frac{1}{2} T_{\mu\nu},$$

where

$$L_{\mu\nu} = \frac{1}{2} g_{\mu\nu} B^\rho B^\sigma R_{\rho\sigma} - (B_\mu B^\rho R_{\rho\sigma} + R_{\mu\rho} B^\rho B_\sigma),$$

$$\Sigma_{\mu\nu} = \frac{1}{2} \left[ \nabla_\mu \nabla_\rho (B_\nu B^\rho) + \nabla_\nu \nabla_\rho (B_\mu B^\rho) - \nabla^2 (B_\mu B_\nu) - g_{\mu\nu} \nabla_\rho \nabla_\sigma (B^\rho B^\sigma) \right].$$
are the contributions from the interaction term and
\[ T_{\mu\nu} = B_{\mu\rho}B_{\nu\rho} + 4V'B_{\mu}B_{\nu} + g_{\mu\nu} \left[ -\frac{1}{4}B_{\rho\sigma}B^{\rho\sigma} - V \right] \] (5)
is the contribution from the vector field for the stress-energy tensor. For the equation of motion for the vector field \( B \), we find that
\[ \nabla^\nu (\nabla^\mu B_{\mu} - \nabla^\mu B_{\nu}) - 2V'B_{\mu} + 2\xi B^\nu R_{\mu\nu} = 0. \] (6)
where \( V' = dV/dB \).

When the bulk vector field \( B \) acquires a non-vanishing, covariantly conserved vacuum expectation value by spontaneous symmetry breaking, the bulk vacuum acquires an intrinsic direction determined by \( \langle B_{A} \rangle \), thus inducing the breaking of the Lorentz symmetry in the bulk. In order to obtain a vanishing cosmological constant and ensure that Lorentz invariance holds on the brane, we take the Einstein equation induced on the brane and impose respectively that
\[ \Lambda_{(5)} = \frac{1}{2}(1 - 2(\xi - 1))K\sigma \] (7)
and that
\[ \frac{1}{\kappa_{(5)}^2} \left[ 2K_{AC}K_{BC} - \left( \frac{1}{2} + \xi - 1 \right) K_{AB}K \right. \]
\[ + \frac{1}{2}g_{AB} \left( R^{(ind)} - 2K_{CD}K_{CD} - (1 - 2(\xi - 1))K^2 \right) \]
\[ = \frac{\xi}{2} \left[ \left( \frac{5}{2} - 2 + \frac{2}{\xi} \right) \left( \langle B_{A} \rangle \langle B_{C} \rangle R_{CB}^{(ind)} + \langle B_{B} \rangle \langle B_{C} \rangle R_{AC}^{(ind)} \right) - 4(4 + 2)K_{AC}K_{BD} \langle B_{C} \rangle \langle B_{D} \rangle \right] \]
\[ + \frac{\xi}{2} g_{AB} \left[ \langle B_{C} \rangle \langle B_{D} \rangle R_{CD}^{(ind)} + 2(\xi - 1)K_{CE}K_{ED} \langle B_{C} \rangle \langle B_{D} \rangle \right], \] (8)
which for \( \xi = 1 \) reduce to the results presented in [2].

3. Discussion and Conclusions

In this contribution we examine the spontaneous symmetry breaking of Lorentz invariance in the bulk and its effects on the brane. For this purpose, we considered a bulk vector field subject to a potential which endows the field with a non-vanishing vacuum expectation value, thus allowing for the spontaneous breaking of the Lorentz symmetry in the bulk. This bulk vector field is directly coupled to the Ricci tensor so that, after the breaking of Lorentz invariance, the breaking of this symmetry is transmitted to the gravitational sector. We assign a non-vanishing vacuum expectation value to the component of \( B \) parallel to the brane (the generality of this procedure has been discussed in [8]). We observe that there is a connection between
the vanishing of the cosmological constant and the reproduction of the Lorentz invariance on the brane. The conditions above were enforced so that the higher dimensional signatures encapsulated in the induced geometry of the brane cancel the Lorentz symmetry breaking inevitably induced on the brane, thus reproducing the observed geometry. Naturally, the first condition, Eq. (7), can be modified to account for any non-vanishing value for the cosmological constant induced on the brane. A much more elaborate fine-tuning, however, is required for the Lorentz symmetry to be observed on the brane, as expressed by the condition Eq. (8). We believe that this is a new feature in braneworld models, as in most such models Lorentz invariance is a symmetry shared by both the bulk and the brane. Notice that a connection between the cosmological constant and Lorentz symmetry had been conjectured long ago.\(^6\) We shall examine further implications of this mechanism in a forthcoming publication where we will also discuss the inclusion of a bulk scalar field.\(^9\)

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