Monopole clusters and critical dynamics in four-dimensional U(1)†

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We investigate monopoles in four-dimensional compact U(1) with Wilson action. We focus our attention on monopole clusters as they can be identified unambiguously contrary to monopole loops. We locate the clusters and determine their properties near the U(1) phase transition. The Coulomb phase is characterized by several small clusters, whereas in the confined phase the small clusters coalesce to one large cluster filling up the whole system. We find that clusters winding around the periodic lattice are absent within both phases and during the transition. However, within the confined phase, we observe periodically closed monopole loops if cooling is applied.

1. INTRODUCTION

Ever since the work of DeGrand and Tous-saint[1], who gave a prescription for constructing monopoles in compact QED, there is no doubt that monopoles can yield a clear signal as to the location of the U(1) phase transition in four dimensions and that one can characterize the phase structure from the dynamical behavior of monopoles. Their results have been confirmed by Barber who found a strong correlation between the average plaquette action and the total monopole length[2]. Further evidence for the influence of monopoles on the phase structure has been given in a recent work of Bornyakov et al. [3]. These authors suppressed configurations containing monopoles, on the level of the partition function. This led to a deconfined phase throughout a β-range from 0 to 2.

The latter result suggests that monopoles play an active role in controlling the phase structure of U(1) rather than being just another signal for the phase change. Such a scenario is supported by the observation of very strong hysteresis effects[4], which lead to supercritical slowing down (SCSD) for the U(1) updating. As to a possible mechanism how monopoles affect the phase structure and handicap the efficiency of the updating, the authors of Ref. [4] argued that periodically closed monopole current loops, i. e. loops closed due to the finiteness of the lattice and the periodical boundary conditions imposed, are responsible for long lived metastable states[4]. It remains an open question, how in the infinite volume limit—where periodically closed loops should be absent—SCSD can be explained by such a mechanism. Or is SCSD just a finite-size phenomenon in U(1)?

Our present approach is to proceed from previous global analyses to an investigation of detailed spatial structures by locating closed monopole loops on the lattice and determining their properties.

We have no well-founded criterion, however, which would allow to identify single closed loops unambiguously, in view of the observed proliferation of monopole line crossings. Therefore, we restrict our considerations to clusters of loops, i. e. objects composed out of connected monopole lines. We compute observables like the winding number and the length of single clusters. We will also try to uncover topological structures, which might be hidden by the local fluctuations.

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4Additionally, the authors traced back small action gaps—deep in the weak coupling phase—to closed Dirac-sheets.
2. MONOPOLE LINES AND CLUSTERS

We adopt the usual definition for monopole world lines in four dimensions [1]:

\[ m_\mu(x) = \frac{1}{2} \epsilon_{\mu\nu\rho\sigma} \left[ n_{\nu\sigma}(x + \nu) - n_{\nu\sigma}(x) \right], \]

with

\[ n_{\mu\nu}(x) = \frac{1}{2\pi} \left( \Phi_{\mu\nu}(x) - \Phi_{\mu\nu}(x) \right). \]

The plaquette flux for the Wilson action varies between \(-4\pi\) and \(4\pi\), the physical flux lies in the range between \(-\pi\) and \(\pi\), and \(n_{\mu\nu}\) can take on integer values between \(-2\) and \(2\). The divergence of the monopole links vanishes, \(\partial_\mu m_\mu(x) = 0\), giving rise to closed loops of monopole lines. The monopole lines can intersect, and the identification of single monopole loops is ambiguous as depicted in Fig. 1. In the first example, either two plaquette-loops or one large loop with one crossing can be identified. The second example can either be viewed as two periodically closed loops and one square loop or as one large loop homotopic to a point.

We emphasize again that the actual monopole clusters turn out to be utterly entangled. Numerous loop crossings occur. Therefore, we have concentrated our investigation on monopole clusters, i.e. objects composed out of connected closed loops.

3. RESULTS

Our numerical work has been done on a connection machine CM-5. We designed data-parallel and message-passing algorithms in order to locate the clusters [2,3]. We worked on lattices of size \(8^4\) and \(16^4\). For each size, we performed about 100000 sweeps using standard Metropolis updating and analyzed more than 10000 configurations. In each case, we worked at the respective coexistence point \(\beta_c\).

3.1. Length distribution and loop crossings

In Fig. 2, we present the distribution of the length of the clusters for both phases on an \(8^4\)- and a \(16^4\)-lattice. The confined phase is characterized by the occurrence of large clusters. In particular, on the \(16^4\)-lattice the contribution of smaller loops nearly vanishes and one cluster is filling up the whole lattice. In the Coulomb phase, the large cluster is split into several smaller pieces. This goes along with a decreasing total length of monopole lines. We interpret a tun-
nelling event from the Coulomb to the confined phase as a condensation from a dilute monopole current gas to a condensed cluster. The simulation algorithm should be capable to drive tunnelling, i. e. to create and to destroy large clusters.

A four-dimensional visualization of the tunnelling reveals an entangled jumble of monopole lines throughout the system. In an attempt to look into those clusters, we analyzed the number of crossings, \( N_c \). We found nearly linear correlation between \( N_c \) and the length of the clusters, \( L_c \), cf. Fig. 3.

![Figure 3. Loop crossings on the 16\(^4\)-lattice.](image)

This provides evidence that small cluster condense into larger structures.

### 3.2. Periodically closed clusters

We define the winding number for the direction \( \mu \) as

\[
W(\mu) = \frac{1}{L_\mu} \sum_{x \in \text{cluster}} m_\mu(x),
\]

where \( L_\mu \) is the lattice extension in \( \mu \)-direction. If this number is non-zero the considered cluster is wrapping around the lattice along the specified direction.

Among 10000 analyzed configurations of the 8\(^4\)-lattice, we found only 8 events with non-zero winding numbers. These clusters came up within the flip region from the confined to the Coulomb phase. The lifetimes of the non-zero winding numbers ranged from one to two sweeps. On the 16\(^4\)-lattice, we did not encounter any periodically closed cluster within the confined phase and in particular within the flip region between the two phases.

These findings cast doubt on periodically closed monopole clusters being responsible for long lived metastabilities at \( \beta_c \).

### 3.3. Cooling

One might argue, however, that topologically important structures within the monopole clusters might be hidden under the local fluctuations. In an attempt to remove the fluctuations and to reveal those structures, we imposed two different types of cooling on the configurations, the first by minimizing the action and the second by cooling adiabatically.

In the Coulomb phase, where the clusters are small, no non-zero winding numbers appear through the cooling procedure. Within the confined phase, however, we observe that the cooling process gives rise to the creation of such winding numbers. The time history under cooling depends strongly on the type of cooling, generating winding numbers in different directions. Furthermore, for a given cooling procedure, the directions \( \mu \), Eq. 3, fluctuate frequently from sweep to sweep. This is another aspect of the fact that it is impossible to identify single closed loops unambiguously.

Clearly, further investigations are needed in order to answer the question whether or not periodically closed loops are an artifact of cooling.

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