Calculations of \( \beta \)-decay half-lives for neutron rich tin isotopes

M Khiter and F Benrachi
Mathematical and Subatomic Physics Laboratory, Physics Department, Frères Mentouri Constantine 1 University, Algeria
E-mail:khiter.meriem@yahoo.fr

Abstract. Many of the nuclei made temporarily during r-process nucleosynthesis are currently inaccessible to experiment. Their properties, however, help determine the abundances of stable elements which we observe in the solar system. Beta decay half-lives and rates are among the most important of these properties. Nuclei with \( 50 \leq Z \leq 56 \) and \( 82 \leq N \leq 88 \) in the \( \pi(gdsh) \oplus \nu(hfpi) \) valence space above the \( ^{132}\text{Sn} \) core lie on or close to the path of astrophysical r-process flow. The even-N neutron-rich Sn isotopes are the classical “waiting point” nuclei in the \( A=130 \) solar system abundance peak under typical r-process conditions. With respect to the r-process, \( N=86 \) isotope is an important waiting-point nucleus for moderate neutron densities to drive the r-process flow beyond \( A \approx 130 \) peak. For such systems, the consideration of the three-body monopole effects is presently a relevant issue in nuclear structure calculations in order to reproduce their experimental studies. In the present work, the total \( \beta \)-decay half-lives and rates of the exotic Sn isotopes above the \( ^{132}\text{Sn} \) core are calculated using monopole interaction at different temperatures. The calculation has been realized using Oxbash code in the frame work of the nuclear shell model.

1. Introduction

One of the most important open questions in nuclear astrophysics concerns the synthesis of elements heavier than iron in explosive astrophysical scenarios. The creation of approximately half of all elements heavier than iron is via the rapid neutron capture process (r-process) that is characterized by fast neutron captures in a neutron-rich environment. The path of the r-process runs very close to the neutron drip-line, from where the nuclei decay towards the valley of stability after the neutron flux diminishes[1].

The actual site of the r-process is under discussion, with the candidates being core-collapse supernovae and neutron star mergers. Nevertheless, it is generally accepted that it occurs in explosive environments involving relatively high temperatures (up to 1 GK) and neutron densities (\( n_n > 10^{20} \text{ g cm}^{-3} \)). Apart from the complexities involved in simulating the astrophysical environment, the r-process presents a particularly difficult challenge due to the large amount of required input. To properly account for all the relevant reactions and processes that take place during nucleosynthesis, knowledge of a variety of nuclear properties is necessary[2].

Of those, \( \beta \)-decay properties of unstable neutron-rich nuclei critically affect the distribution of abundances of elements as they determine the speed of the flow of matter towards higher atomic numbers, thus setting the time scale of the r-process [3,4]. Nuclides around the closed neutron shells at \( N = 50, 82 \) and \( 126 \) are particular importance. These nuclei display relatively low neutron capture cross sections, thus accumulating matter which results with the observed peaks in the solar system r-process distribution. The even-\( N \) neutron-rich Sn isotopes are the classical “waiting point” nuclei in...
the A=130 solar system abundance peak under typical r-process conditions. With respect to the r-
process, N=86 isotope is an important waiting-point nucleus for moderate neutron densities to drive
the r-process flow beyond A≈130 peak. Lifetimes of these nuclei are very small and production rate
very low presenting challenges to spectroscopic studies. Reliable theoretical results are therefore
necessary and useful.
In this study, the total β-decay half-lives and rates of $^{134,136,138}$Sn nuclei at different temperatures are
calculated using monopole interaction [5]. Basing on three body effects, we have modified the two
body matrix elements (TBMEs) of the original interaction kh5082 [6] and new interaction named
mkh3 is constructed.

2. Theoretical framework
The β-decay rates are appropriate for the study of astrophysical scenario depending on the density of
the nuclear material ($\rho$), electron fraction ($Y_e$), and the predominant temperature (T) in the
environment [7].
The excitation energies and the transition densities of $^{134-136}$Sn and $^{134-136}$Sb nuclei above the
inert core $^{132}$Sn are calculated using mkh3 interaction, in order to calculate B(GT) values required in
the calculation of beta decay rates. B(GT) values have been calculated using OXBASH code[8].
It is commonly known that beta decays involving the low lying excited states of parent nuclei can
play a significant role in stellar conditions, where the excited levels are in thermal equilibrium with the
ground-state populations. The temperature dependent effective transition rate [λ(T)] of the parent
nucleus may be written as[9]

$$\lambda(T) = \frac{1}{Z_T} \sum \lambda_i,$$

where the decay rate $\lambda_i$ of the nuclear state i is given by

$$\lambda_i = \sum \lambda_{ij},$$

and

$$\lambda_{ij} = \ln(2) (f_{ij} \epsilon_j) / f_{ij},$$

$Z_T$ is the partition function of the mother nucleus at $\beta_T = 1/\kappa T$

$$Z_T = \sum (2J_i + 1)e^{\beta E_i},$$

where $J_i, \epsilon_i$ is the spin and energy of the state of the mother nucleus.

$f_{ij}$ is the phase-space factor[10] for beta decay. The $(f_{ij})_y$ value of an allowed beta decay is given by

$$f_{ij} = \frac{6250s}{B_y(F) + B_y(GT)},$$

where $g_A, g_V$ are vector and axial-vector coupling constants such as [11]:

$$\left(\frac{g_A}{g_V}\right) = 1.25$$
Here, $B_{ij}(F)$ and $B_{ij}(GT)$ are the Fermi and Gamow-Teller transition probabilities from $i$th mother state to $j$th daughter state.

3. Results and discussions

The low lying states in these isotopes of the parent (Sn) and daughter nuclides are illustrated in ‘figure 1’. For the isotopes of Sn, ground state to ground state beta decays are forbidden transitions. But they are quite fast. The selection rules for GT transitions only allow transitions from single particle $\nu_1 h_{9/2}$ orbital to $\pi_1 h_{11/2}$ orbital in this model space. But the wave function compositions of the relevant low lying states in these isotopes of Sn and Sb have very small contribution from the shell model configurations involving these orbitals. So the calculated allowed GT strengths are generally very small.

![Figure 1](image1)

Figure 1. Schematic representation of $\beta$ decay of the nucleus Sn to Sb.

We have also calculated the Gamow-Teller strengths and the half-lives in the temperature range from $T = 0.01$ to 100 MeV (‘figure 2’, ‘figure 3’ and ‘figure 4’).

While varying the temperature in the nuclear field going from 10 keV to 100 MeV, one can observe that for the interaction, the effective half-life increases with increasing temperature for $^{134,136,138}$Sn (‘figure 2’, ‘figure 3’ and ‘figure 4’). However, the total rate decreases with increasing temperature. In the range 0.01MeV to 0.02 MeV, $T_{1/2}$ is constant at $\sim 1.05s$ ($^{134}$Sn), $0.25s$ ($^{136}$Sn) or $\sim 480ns$ ($^{138}$Sn) corresponding with that obtained in laboratory measurements. Beyond 0.2 MeV ($\sim 10^9K$), there is a deviation of this value for interaction use. The deviation starts at the same temperature and the saturation is reached above 10 MeV.

![Figure 2](image2)

Figure 2. $\beta$-decay rates and half-lives as a function of temperature for $^{134}$Sn.
4. Conclusion

In this paper, we calculated beta decay half-lives and transition rates as a function of temperature $T$ for $A=134-136-138$ isotopes with two, four and six valence particles in addition to the $^{132}$Sn. The calculations are carried out in the framework of the shell model by means of Oxbash nuclear structure code, using monopole interaction.

While varying the temperature in the nuclear field going from 10 keV to 100 MeV, the effective half-life increases with increasing temperature. However, the total rate decreases with increasing temperature for $^{134-136-138}$Sn. By this interaction, the deviation and saturation of half-lives start respectively at around 0.2 MeV and above 10 MeV.

The abundance at each $Z$ value is inversely proportional to the decay constant of the waiting point nucleus for that particular $Z$. So the increase in half-life of isotopes of Sn, like $^{136}$Sn will definitely have substantial impact on the r-process nucleosynthesis.

Acknowledgments

Authors of this article thanks the organizers of the Fourth Algerian Conference on Astronomy and Astrophysics ACAA 2017, March 27th-29th 2017 Khenchela-Algeria for the organization and the support provided during the conference. Special thanks are owed to B. A. Brown for his help in providing us the OXBASH code (Windows Version)

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