Numerical analysis of near and far field patterns of second-harmonic generation with tiling nonlinear optical crystals

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Abstract. We report numerical analysis of near- and far-field patterns of the second-harmonic wave in a large-scale high-power laser used in the laser fusion with type I KDP crystals arranged in a tiling style. The thickness and phase-matching angle of the tiling crystals are designed based on the near-field pattern of the second-harmonic wave. The tilting angle error and thickness difference of the tiling crystals are evaluated by the energy distribution included in the Airy spot of the far-field pattern at the focal point. The parallelism and flatness of the tiling crystals can also be estimated with the same method.

1. Introduction

A high-power laser system with a large-scale beam used for the fast ignition is developing based on the 10-kJ PW FIREX program [1]. It will be thought that the short-wavelength generation of the laser with large-scale nonlinear optical crystals is necessary in the near future. However, a single nonlinear optical crystal with a large size more than 1 m is difficult to be obtained. This can be solved by using several pieces of nonlinear optical crystals arranged in a tiling style, which has been demonstrated in the sub-nanosecond and nanosecond regimes [2]. In this paper, we describe the optimum design for the second-harmonic generation of the FIREX program with tiling KDP crystals. We report numerical results on the near- and far-field patterns of the large-scale second-harmonic generation in the tiling KDP crystals with picosecond fundamental laser pulses.

2. Schematic diagram of the tiling crystals

Figure 1 is the schematic diagram that consists of four pieces of type I KDP crystals located in the coordinate system of $(x, y, z)$. When a fundamental laser wave ($\omega$) enters the tiling KDP crystals, a near-field pattern of the second-harmonic wave (2$\omega$) is obtained behind the tiling crystals, and is further focused by a concave mirror or a lens to form a far-field pattern at the focal point located in the coordinate system of $(x_0, y_0, z)$. The crystal 1 is used as a standard crystal, and the crystals 2, 3, and 4 are aligned referred to the standard crystal. Assembling angular parameters include the tilting angles $\alpha$ and $\beta$ around the $x$ and $y$ axes, and the rotation angle $\gamma$ around the $z$ axis [3], respectively, resulting in phase differences between two different tiling crystals. The thicknesses of the tiling crystals are determined so that the conversion efficiency of the second-harmonic generation is highest, while the temporal profile of the second-harmonic laser pulse is almost the same as that of the incident...
fundamental laser pulse. The tiling crystals should be optimized and assembled so that they have the same properties as those of the standard crystal. The assembly of the tiling crystals should be optimized according to the positions and tilting angles of the crystals. In addition, the optimization design for the tiling crystals should also involve the phase or time difference between two different crystals and the parallelism as well as the flatness. The near-field pattern of the second-harmonic laser beam is used to achieve the optimization of the crystal thickness and the phase-matching angle, and the far-field pattern is capable of estimating the phase relationship of the crystals such as the phase difference.

Figure 1. Schematic diagram of the tiling crystals with the phase-matching angle $\theta_m$.

3. Theoretical model for numerical calculation

The near-field pattern of the second-harmonic laser beam behind the tiling crystals can be numerically evaluated with nonlinear wave equation, which is given by

$$\frac{\partial A_{1\omega}}{\partial z} + \frac{1}{v_{1\omega}} \frac{\partial A_{1\omega}}{\partial t} = -i \frac{\omega}{n_{1\omega}c} d_{\text{eff}} A_{1\omega} A_{2e} \exp(-i\Delta k z)$$  \hspace{1cm} (1)

$$\frac{\partial A_{2e}}{\partial z} + \frac{1}{v_{2e}} \frac{\partial A_{2e}}{\partial t} = -i \frac{\omega}{n_{2e}c} d_{\text{eff}} A_{2e} \exp(i\Delta k z),$$  \hspace{1cm} (2)

where $A_{1\omega}$ and $A_{2e}$ are the fundamental and second-harmonic amplitudes polarized at the ordinary $(1\omega)$ and extraordinary $(2e)$ directions, respectively, $v_{1\omega}$ and $v_{2e}$ are the group velocities, $n_{1\omega}$ and $n_{2e}$ are the indexes, $z$ is the propagation distance, $t$ is the time, $c$ is the light velocity, $\omega$ is the angular frequency of the fundamental laser wave, $d_{\text{eff}}$ is the nonlinear constant, $\Delta k$ is the wave-number difference, and $i$ is the imaginary unit.

The propagation of the second-harmonic laser pulse between the tiling crystals and the concave mirror or lens is then calculated with the following parabolic equation [4],

$$\frac{\partial A_{2e}}{\partial z} + \frac{1}{c} \frac{\partial A_{2e}}{\partial t} - \frac{g_{2e}}{2} \frac{\partial^2 A_{2e}}{\partial t^2} = 0,$$

where $g_{2e}$ is the group-velocity dispersion of the second-harmonic laser pulse, which can be generally ignored in the picosecond regime.

The frequency-space distribution of the second-harmonic laser beam near the focal point can be calculated by the following integration,

$$A_{2f}(2\omega, x_0, y_0) = \frac{i\omega}{\pi f} \exp \left[-i \frac{2\omega}{c} (z_0 + f) \right] \exp \left[- \frac{i\omega}{c f} \left(1 - \frac{z_0}{f} \right) \left(x_0^2 + y_0^2 \right) \right] \times \int \int A_{2e}(t, x, y) \exp \left[ \frac{i\omega}{c f} \left(1 - \frac{f}{z} \right) (x^2 + y^2) \right] \exp \left[ i \left(2\omega t + \frac{2\omega}{c} (x_0 x + y_0 y) \right) \right] dt \, dx \, dy,$$  \hspace{1cm} (3)
where \( f \) is the focal length of the concave mirror or lens, and \( 2\omega \) is the angular frequency of the second-harmonic laser pulse. The temporospatial distribution near the focal point is calculated by taking the inverse Fourier transform of eq. (3). Equation (3) is used to calculate the far-field pattern of the second-harmonic laser beam just at the focal point by \( z = f \). The corresponding energy distribution is therefore obtained by applying the Parseval’s theorem to eq. (3),

\[
E_{2f} = \sum_f \int I_{2f}(t, x_0, y_0) dt dx_0 dy_0 = \frac{1}{2\pi} \sum_f \int I_{2f}(2\omega, x_0, y_0) d(2\omega) dx_0 dy_0,
\]

where \( \Sigma_f \) is the integrating range, and \( I_{2f} \) is the intensity with respect to \( A_{3f} \).

The phase shift in the standard crystal 1 can be expressed as \( \Delta k_1 d_1 / 2 \) where \( \Delta k_1 \) is the wave-number difference due to the phase-mismatching angle \( \Delta\theta_{m1} \) and \( d_1 \) is the crystal thickness. The phase shifts of the other crystals are similarly expressed as \( \Delta k_j d_j / 2 \) (\( j = 2, 3, 4 \)). Therefore the phase difference between the standard crystal and other crystals is \( (\Delta k_j d_j / 2 - \Delta k_1 d_1) / 2 \), generally resulting in an interferometrical far-field pattern at the focal point. In addition, even the phase-matching condition \( [\Delta k_n (\theta_{mn}) = 0, n = 1, 2, 3, 4] \) is satisfied, the thickness difference \( \Delta d_n \) between the standard crystal and other crystals also causes an additional phase difference represented by the expression of \( \Delta \Phi (\Delta d_n) = 2\omega (n_{t0} - 1) \Delta d_n / c \). Therefore, the interference effect at the focal point occurs due to the thickness difference.

If the near-field distribution of the second-harmonic laser beam is an ideal flat-top profile, its far-field pattern in the focus is near the ideal distribution in proportion to the square of the Bessel function. The radius of the Airy spot with respect to the central bright range, which includes 80% energy of the far-field distribution, is approximately estimated by \( 2.44 \frac{\pi c f}{\omega D} \), where \( D \) is the beam diameter. If the interference effect between the beams from different tiling crystals cannot be suppressed enough, the central bright spot grows big, and the energy within the Airy spot is generally small. The Airy spot size is used as a criterion for estimating the far-field pattern of the second-harmonic laser beam and subsequently the allowable assembling accuracy of the tiling crystals.

### 4. Numerical results

The numerical calculation is based on the parameters of the FIREX laser system. We assume that the beam diameter is 1 m with a super-Gaussian spatial distribution and the pulse duration is 0.5 ps with a Gaussian temporal profile. In addition, the central wavelength is 1053 nm and the initial intensity is 30 GW/cm². The second-harmonic laser beam at 526.5 nm is focused by a concave mirror or lens with a focal length of 10 m.

The numerical results showed that the optimized thicknesses \( d_n \) of the type I tiling KDP crystals are 5.0 ± 0.1 mm to obtain the maximum conversion efficiency. The phase-matching angle \( \theta_{m0} \) of the type I KDP crystal is 41 degree. The phase-mismatching angle \( \Delta \theta_{mu} \) should be limited within 150 µrad to guarantee the conversion efficiency to be larger than 95% of the ideal case of the phase-matching condition. The allowable range for the tilting angle \( \alpha \) around the \( x \) axis is from -0.15 to 0.1 mrad. The allowable ranges for the tilting angle \( \beta \) around the \( y \) axis and the rotation angle \( \gamma \) around the \( z \) axis are equally varying from -10 to 10 mrad, respectively.

Figure 2(a) is the image plot of the near-field pattern of the second-harmonic laser beam behind the tiling crystals. The cross line is the spatial interval between the adjacent crystals, which is chosen as 1.9 cm. Figure 2(b) is the far-field pattern of the second-harmonic laser beam under the conditions of \( \Delta \theta_{mu} = 0 \) and \( \Delta d_n = 0 \). The central portion of the image plot is overexposed by a factor of 50 to increase the visibility of the high-order diffraction portions. If \( \Delta \Phi (\Delta d_n) = 0 \) and \( \Delta \theta_{mn} < 150 \mu \text{rad} \), there are almost no variation in the far-field pattern of the second-harmonic laser beam in comparison with that of the ideal case shown in Fig. 2(b). The breaking of the ideal Airy spot can be observed from Fig. 2(c) if \( \Delta \Phi (\Delta d_n) \neq 0 \). The corresponding energies within the range of the ideal Airy spot deteriorate to 84% of the ideal case shown in Fig. 2(b).

Although the influence of the parallelism and flatness of the tiling crystals on the near-field pattern can be ignored under the generally assembled accuracy, the influence on the far-field pattern should be
considered. To ensure the energy included within the Airy spot to be larger than 90% of the ideal case, the parallelism and flatness of the tiling crystals are required to be better than 0.09 µrad and 1/30 of the second-harmonic wavelength, respectively. In addition, the influence of the time delay due to the thickness difference between the standard crystal and other crystals can be ignored if the thicknesses of the tiling crystals are 5.0 ± 0.1 mm.

5. Conclusions
We reported the numerical analysis of the near- and far-field patterns of the second-harmonic laser beam with type I tiling KDP crystals. For the laser system of the FIREX program with the 1053-nm central wavelength, the 30-GW/cm² peak intensity, and the 0.5-ps pulse duration, the tilting angle and phase error due to thickness difference of the tiling crystals should be respectively less than 150 µrad and π/2.7 to ensure that the energy within the Airy spot is over 90% of the ideal case without any assembling errors. In addition, the parallelism and flatness of the tiling crystals should be better than 0.09 µrad and 1/30 of the second-harmonic wavelength, respectively.

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