Primordial Baryon Asymmetry and Sphalerons
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I show that a cosmological baryon asymmetry generated at the GUT scale is in general safe against washout due to sphalerons and generic $B$- or $L$-violating effects. This result is mainly due to the (almost) conserved number of right-handed electrons at high temperatures $T \gtrsim 10^{10}$ TeV, but also the mass corrections, in particular the thermal masses of leptons act as the protector of the primordial baryon asymmetry.

1. Introduction

It was realized a long time ago that the evident excess of baryons over antibaryons in the universe could be generated during the early stages of its evolution by generic out of equilibrium $B$- and $CP$-violating interactions. First quantitative implementations of these ideas came in the context of grand unified theories (GUTs), where the out of equilibrium conditions, $B$-violation and a large enough $CP$-violation were rather easily realized. Later other scenarios of baryon asymmetry generation that work in the very early universe have been proposed. It took surprisingly long time before it was realized that all the conditions required for baryon number generation are qualitatively satisfied in the Standard Model during the electroweak phase transition. Whether the electroweak baryogenesis can be made to work quantitatively remains still an open and undoubtedly one of the most challenging questions in the modern physics.

Standard model baryogenesis became a possibility when it was understood that the anomalous baryon number violating sphaleron interactions are unsuppressed at high temperatures. This very same phenomenon however, appeared to imply the death of any primordial (as opposed to the electroweak) baryogenesis mechanisms. Roughly speaking this is because sphaleron interactions destroy any net excess in $B + L$-number, whereas e.g. the simplest GUTs predict $B - L = 0$; combining these one immediately gets $B = L = 0$. A way out would seem to be offered by more complicated models, like SO(10)-GUT which can generate a nonzero $B - L$. Such models, however, necessarily contain a number of nonrenormalizable $B$- and/or $L$-violating interactions. The danger of such interactions to primordial baryon asymmetry was first noticed by Fukugita and Yanagida (FY) who observed that the lepton number must be at least a fairly good approximate symmetry. Their reasoning was that if the lepton violating processes were significant, then effectively $L$ would be driven to zero and since sphalerons cause $B + L \to 0$, one is again reduced to having zero baryon asymmetry.

It should be noted that the presence of $B$ and/or $L$-violating interactions does not automatically imply the vanishing of baryon number; it is possible to adjust all the parameters of the theory in such a way that these interactions are too weak to affect the asymmetries significantly. In essence the GUT baryogenesis could then be made to work if certain set of “consistency constraints” between the parameters of the theory were satisfied. In the light of the subsequent developments these constraints appeared to become rather strong however, so as to make the primordial baryogenesis much more unappealing. Indeed, since the observation made by FY a great deal of effort has been spent to strengthen their result and to generalize it to other baryon and/or lepton number violating operators. The strongest bounds were obtained by requiring that the $B$- and/or $L$-violating interactions

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had to be out of equilibrium ever since the temperature at which the sphalerons first came into equilibrium at \( T_m \sim 10^{12} \) GeV, rather than the much lower electroweak phase transition temperature used by FY.

The situation changed again after a somewhat surprising result was pointed out in ref. [11]: most of the efforts to strengthen the original FY-type constraint are invalidated by a rather mundane feature of the Standard Model, namely the smallness of the Yukawa coupling of the right-handed electron. The key observation is that any \( e_R \)-asymmetry remains untouched until around \( T_* \simeq \mathcal{O}(10) \) TeV, when the small Yukawa interactions with left-handed electrons and Higgs bosons finally become fast enough to convert the \( e_R \)'s into \( e_L \)'s. Because sphalerons interact only with the left-handed particles, they can only directly deplete the latter. Therefore, as long as any additional lepton and/or baryon violating interactions have gone out of thermal equilibrium before the right-handed electrons come into equilibrium, the initial \( e_R \) asymmetry is protected from being washed out. When the temperature eventually falls below \( \sim \mathcal{O}(10) \) TeV, sphalerons will be able to convert a sizeable fraction of the initial \( e_R \) asymmetry into the baryon excess that exists today.

The new conservation law of \( e_R \)-number above \( T_* \) has important consequences for the evolution of the primordial baryon number. Firstly, it tremendously weakens the consistency constraints on the parameters of the theories with initial \( B - L \neq 0 \). This will be demonstrated below for the particular lepton number violating operator considered by FY. Secondly, with theories having initially \( B - L = 0 \), the old vice turns into a new virtue: any \( B \) and/or \( L \)-violating operator in equilibrium at \( T_* \lesssim T \lesssim 10^{12} \) GeV would, with rapid sphaleron transitions and \( e_R \)-conservation, help to generate a baryon asymmetry. This can occur because the rapid \( B \) and/or \( L \)-violation imposes new equilibrium between the chemical potentials of the interacting species which necessarily corresponds to \( (B - L)_{eq} \neq 0 \), because \( e_R \)-species does not partake in these processes and yet carries a charge quantum number. When the fermion number violation ceases the asymmetries evolve in the standard way but with a new initial condition \( (B - L)_i \to (B - L)_{eq} \neq 0 \).

On the basis of the above discussion one can identify two possible loopholes that could lead to the destruction of primordial baryon asymmetry. The first is that the \( B \)- and/or \( L \)-violation persists below \( T_* \), but this would require adjusting the parameters of the theory to get into the trouble, not vice versa! The second is more important: if one has a theory with \( (B - L)_i = 0 \) and no additional fermion number violation, then the anomalous situation would persist only until \( T_* \), after which the equilibrium with \( B = L = 0 \) would be re-established [13,11]. However, even then a fraction of the primordial baryon number is restored due to the finite mass effects. Depending on the order of the phase transition this would be due to vacuum mass effects [13] or due to the thermal mass corrections [15,16].

2. Examples of \( L \)- and \( B \)-violation

2.1. Neutrino see-saw mass

The lepton number violating \( \Delta L = 2 \), \( D = 5 \) operator first considered by FY can be written as:

\[
\mathcal{O}_5 = \frac{1}{v^2} \sum_{ij} m_{ij} (\bar{L}_i H)(H^T L^c_j). \tag{1}
\]

Here \( v = 246 \) GeV is the usual higgs vev and \( m \) is the see-saw mass matrix of light neutrinos. The rate of lepton number violation induced by \( \mathcal{O}_5 \) scales with the temperature cubed, \( \Gamma_{\Delta L} \sim T^3 \). Hence it is more effective at higher temperatures and drops out of equilibrium at low enough \( T \). An accurate expression for the rate \( \Gamma_{\Delta L} \) by which \( e_L \)-type lepton number asymmetry is destroyed was computed in [11]:

\[
\Gamma_{\Delta L} = \frac{9}{\pi^2} \frac{T^3}{v^4} \mu^2, \tag{2}
\]

In addition to the obvious possibility of having yet new exotic interactions that would bring \( e_R \)-species into chemical equilibrium with the left-chiral world.

\[\text{In the supersymmetric case it was realized earlier} \]\n
that above a certain scale associated with supersymmetry breaking, \( T_m \sim 10^8 \) GeV, the presence of new anomalies would cause the baryon number to be encoded in supersymmetric particles, saving it from erasure until temperatures below \( T_* \).
where $\mu^2 \equiv \frac{2}{3}|m_{ee}|^2 + |m_{\mu\mu}|^2 + |m_{\tau\tau}|^2$. Comparing this rate to that of the Hubble expansion ($\approx 17T^2/M_P$) yields a freezeout temperature of

$$T_f = 174(\text{keV}/\mu^2)\text{TeV}. \quad (3)$$

FY's original constraint $\mu \lesssim 50$ KeV essentially results from equating $T_f$ with the weak scale although their computation of the rate (3) was less accurate. The improved constraints found in [11] were obtained by equating $T_f$ with the sphaleron equilibration temperature $T_m \sim 10^{12}$ GeV.

The rate of $e_R - e_L$ interactions on the other hand is determined by the Higgs decays and inverse decays as well as scattering processes such as $tRtL \to e_R\bar{e}_L$ and $eLH \to e_RW$ etc. The rate of these interactions scale as $\Gamma_{LR} \sim T$ so that they are more effective at low $T$ and are out of equilibrium at high $T$. The total rate of all processes was computed in [11] and it was found to correspond to an equilibration temperature

$$T_s \approx 1.3f(x)\text{TeV}, \quad (4)$$

where the function $f(x) \approx 1.0 + (-1.1 + 3.0x) + h_0(0.6 - 0.1x)$ depends on the thermal higgs mass $x \equiv m_h(T)/T$ and the top quark Yukawa coupling $h_0$; assuming that $m_h = 60$ GeV and $m_t = 174$ GeV gives $x \approx 0.6$ and $T_s \approx 3$ TeV [11]. In any case $T_s$ is significantly below the sphaleron equilibration scale $\sim 10^{12}$ GeV and relatively close to the weak scale. Using $T_s = 3$ TeV one finds

$$\mu \approx (\theta_{ee}^2m_{ee}^2 + \theta_{\tau\tau}^2m_{\tau\tau})^{1/2} \approx 8 \text{ keV}, \quad (5)$$

where the matrix element $m_{ee}$ constrained by the double beta decay experiments to $m_{ee} < 1$ eV was neglected and the remaining elements $m_{\mu\mu}$ and $m_{\tau\tau}$ were related to the observable mixing angles and mass eigenstates. One quickly realizes that the bound (5) must already be satisfied due to other laboratory and cosmological constraints [11] and thus presents no danger to the primordial baryon asymmetry.

Suppose now that $L$ violation is occurring above $T_s$ and below $10^{12}$ GeV. Under these conditions, the $L$-violating and sphaleron reactions will establish equilibrium between all the interacting species, with the boundary condition that the $e_R$ asymmetry is conserved. Because $e_R$ carries charge, this constraint carries over to the interacting species because the universe is charge-neutral. One can easily show that above $T_s$, $(B - L)_{eq} = -\frac{3}{10}L_{e_R,p}$, where the sub $p$ refers to the primordial value of the quantity, regardless of the initial values of $B$ and $L$. Assuming there was no lepton number violation below $T_s$, one obtains the final baryon asymmetry according to the standard analysis [11]

$$B_f = \frac{28}{79}(B_s - L_s) \approx -0.11L_{e_R,p}. \quad (6)$$

Eq. (6) holds independent of the initial $B - L$ and in particular for $(B - L)_i = 0$. The strength of $L$-violation needed is quite modest: all we require is that either an $L$-violating decay or scattering remain in equilibrium to temperatures below $T_m \sim 10^{12}$ GeV. This could be accomplished by a tau neutrino mass of at least $10^{-2}$ eV.

### 2.2. $n - \bar{n}$-oscillation

As the second example consider the $\Delta B = 2$, $D = 9$ operator that would induce the neutron antineutron oscillations:

$$C_9 = \frac{1}{M^3}(\bar{u}_Rd_R)(\bar{d}_Rd_R^c)(\bar{u}_Rd_R^c). \quad (7)$$

This operator obviously conserves the differences between the leptonic asymmetries, in particular $C_p = 2L_e - L_\mu - L_\tau$ and imposes a constraint on the chemical potentials $2\mu_{u_R} + 4\mu_{d_R} = 0$. One then readily finds the equilibrium value $(B - L)_{eq} = \frac{3}{\sqrt{5}}(C_p - 9L_{e_R,p})$, valid for $T \gtrsim T_s$, and the final baryon asymmetry

$$B_f = \frac{21}{134}(C_p - 9L_{e_R,p}). \quad (8)$$

The important thing to notice here is that in order to generate a nonzero $B_f$ in this example it was not necessary even to have a primordial asymmetry in the $e_R$-species, all that was needed was the conservation of $e_R$-number.

Finally, using $T_s \sim 3$ TeV from (3) one finds that the limit on the heavy mass scale $M$ suppressing the operator becomes $M \gtrsim 1 \times 10^9$ GeV, which is comparable to the current experimental bound of $M > 10^9 - 10^8$ GeV.
3. Finite mass effects

The zero result obtained for the final baryon asymmetry in the \((B-L)_i = 0\) case is an artifact of the massless approximation. Mass effects enter through the conserved global charge densities that appear as boundary conditions for the network of equilibrium equations for chemical potentials. For example, above the electroweak phase transition one might have

\[
Q_{em} = 0, \quad Q_3 = 0, \quad B - L = 0, \quad (9)
\]

where \(Q_{em}\) is the electric charge density and \(Q_3\) is the isospin density. In the massless approximation these charges are directly related to the chemical potentials. These relations, however, get corrections due to finite vacuum masses and due to thermal interactions. For example

\[
Q_{em} = \sum_f \frac{\mu_f T^2}{6} (1 - \frac{3x_f^2}{\pi^2}) + ..., \quad (10)
\]

where the ellipses refer to the bosonic contribution and e.g. above the phase transition \(x_f = m_f(T)/T\) \([3]\). It appears natural to expect that these perturbations should bring about nonzero final asymmetries roughly of the order of perturbations. This indeed turns out to be the case with three important refinements: (i) corrections in the quark sector alone are not sufficient because all the quarks have equal chemical potentials, (ii) one must have unequal corrections between lepton families, so that only Yukawa couplings contribute and (iii) one must have nonzero differences between the leptonic asymmetries (these differences are conserved). Then, if the phase transition is 1st order, the contribution from \(T > T_e\) equilibrium values get frozen to the universe implying \([4]\)

\[
B_f \simeq 3 \times 10^{-7}(\Delta L_e\tau + \Delta L_{\mu\tau}), \quad (11)
\]

where \(\Delta L_{ij} \equiv L_i - L_j\). If the electroweak phase transition is of second order, then the relevant equilibrium is the one at the broken phase right after the phase transition. Nevertheless, even then a final baryon asymmetry of the same order as in \([4]\) results due to the vacuum mass effects \([4, 5]\).

In conclusion, due to the approximate \(e_R\)-conservation and the finite (thermal or vacuum) mass effects the mechanisms of primordial baryon asymmetry generation remain completely viable in explaining the origin of the baryon asymmetry in the universe.

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