Large Rapidity Gap Processes in Proton-Nucleus Collisions

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The cross sections for a variety of channels of proton-nucleus interaction associated with large gaps in rapidity are calculated within the Glauber-Gribov theory. We found inelastic shadowing corrections to be dramatically enhanced for such events. We employ the light-cone dipole formalism which allows to calculate the inelastic corrections to all orders of the multiple interaction. Although Gribov corrections are known to make nuclear matter more transparent, we demonstrate that in some instances they lead to an opaqueness. Numerical calculations are performed for the energies of the HERA-B experiment, and the RHIC-LHC colliders.

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I. INTRODUCTION

Large rapidity gap (LRG) events in hadronic collisions at high energies contain important information about the interaction dynamics. The dominant contribution to these events comes from colorless gluonic exchange which is usually associated with diffraction or Pomeron exchange. The probability to have a colored exchange, but radiate no gluons within a LRG, attenuates exponentially as function of the rapidity gap, similar to what is known for secondary Reggeons.

A colorless gluonic exchange may lead to excitation of the colliding proton, or the nucleus, or both. We assume the nuclei to be mainly a composition of colorless clusters. Therefore, a nucleus may be excited either with or without excitation of the clusters. The latter possibility is usually called quasielastic or quasidiffractive scattering.

The proton can also be excited via different mechanisms, with or without excitation of gluonic degrees of freedom. The latter possibility means that the gluons are not resolved by the interaction, but only the valence quark skeleton of the proton is excited. This channel of excitation greatly affects the survival probability for a proton propagating through the nuclear matter. Its dynamics is very much model dependent. In terms of the Regge phenomenology, this excitation channel is related to the $\Gamma \Pi \Pi \Pi$ triple-Regge graph.

The other possibility of proton excitation, due to diffractive gluon radiation, is usually associated with the triple-Pomeron ($3 \Pi \Pi$) mechanism. It has been known since the 70s that the triple-Pomeron coupling is amazingly small, which means that diffractive gluon bremsstrahlung is unusually weak. This signals that the gluons in the proton are located within small spots. Indeed, large mass diffraction provides a unique access to the process of gluon radiation. The existence of the $1/M^2$ tail in the effective mass distribution of the diffractively excited hadron is a clear evidence for radiation of a vector particle. In order to explain the smallness of the radiation cross section one should assume that gluonic clouds in the proton are as small as $0.3\text{fm}$ [1–4].

The observed smallness of of the triple-Pomeron diffraction in pp collisions also leads to weak gluon shadowing in nuclei [1]. This is indeed confirmed by the latest DGLAP analysis of deep-inelastic data in the next-to-leading approximation [5].

Thus one should conclude that diffraction, in particular in nuclei, provides also sensitive tools for the study of the hadronic structure.

Elastic scattering should be also classified as a LRG process. There is, however, an important difference between elastic and inelastic diffraction, in particular, how much they are affected by absorptive corrections. To see that one can just glance at the data. While the total and elastic cross sections rise with energy with a rate corresponding to the Pomeron intercept $\alpha_P(0) \approx 1.1$, the single diffraction cross section is nearly independent of energy [6, 7]. This may be related to the difference in the absorption corrections which look especially simple in the eikonal approximation and impact parameter representation,

\begin{align}
\tilde{f}_{el}(b) &= i \left(1 - e^{if_{el}(b)}\right) , \\
\tilde{f}_{sd}(b) &= f_{sd}(b) e^{if_{el}(b)}. \tag{2}
\end{align}

Here $f_{el}$, $f_{sd}$ and $\tilde{f}_{el}$, $\tilde{f}_{sd}$ are the input and absorption corrected elastic and single diffractive amplitudes respectively. Expanding the exponentials up to the lowest order correction one gets a correction factor $[1 + \xi i f_{el}(b)]$, where $\xi = 1/2$ for the elastic amplitude, Eq. (1), and $\xi = 1$ for the diffractive amplitude, Eq. (2). We conclude that even for the hadronic amplitude the absorption correction for the diffractive amplitude is twice as big a for the elastic one. Correspondingly, the result of the absorptive corrections is more pronounced in diffraction than in elastic scattering. In particular, the energy rise of the exponent in the absorption factor in (2) considerably slows down the energy dependence of the diffraction cross section. In the case of diffraction on nuclear targets absorptive corrections produce dramatic reduction of the cross sections.
Below we demonstrate that the probability of diffractive excitation of the valence quark skeleton in the proton is also quite small, only 6.5% of the elastic scattering. However, on the contrary to gluon radiation such a smallness is due to small overlap of the initial and final wave functions. Therefore, this probability is sensitive to the proton structure as well as to the form of the dipole cross section.

In the case of nuclear targets the LRG processes are especially sensitive to the Gribov inelastic shadowing. Indeed, those corrections affect the nuclear transparency, i.e. the exponential term in (1), (2). For a heavy nucleus this term is tiny, therefore the inelastic corrections to the elastic amplitude Eq. (1) cannot be large even if the exponential term changes considerably. At the same time, single diffraction and other LRG channels are directly affected by inelastic shadowing and may undergo considerable modification. In what follows, we perform calculations for different LRG processes within the Glauber model, as well as within the light-cone dipole approach which incorporates the inelastic corrections in all orders of multiple interaction. We find that for quasielastic scattering inelastic shadowing make nuclear matter more opaque, rather than transparent. This result is at variance with the conventional wisdom.

There is also a practical reason to perform reliable calculations for the LRG cross sections. Many experiments at HERA-B, RHIC and future measurements at LHC have triggering systems which are set up for the central rapidity region and miss LRG events. For the purpose of normalization one has to know the part of the inelastic cross section covered by the trigger, i.e. the total cross section minus the LRG contributions.

The paper is organized as follows. We formulate the light-cone (LC) dipole description of soft hadronic reactions in Sect. II. In particular, Sect. II A presents the phenomenological dipole cross section fitted to data, and Sect. II B describes the models for the valence quark proton wave functions used throughout the paper.

Sect. III is devoted to the process of single diffractive excitation in \( pp \) collisions. The analysis of data performed in Sect. III A demonstrates a surprisingly small cross section of diffraction related to excitation of the valence quark skeleton of the proton. Attempting to explain this smallness within different models listed in Sect. III B we conclude that only a saturated shape of the dipole cross section may be considered realistic. As for the models for the valence quark distribution in the proton, the truth seems to lie between the two extremes, the symmetric 3q configuration and the quark-diquark structure with vanishing diquark size.

The triple-Pomeron part of the diffraction related to diffractive gluon radiation is considered in Sect. III C. This contribution is unambiguously identified in data via its \( 1/M^2 \) tail in the effective excitation mass distribution. This part of diffraction is well described by the model, since the quark-gluon LC wave function has been fitted to this data previously.

Single diffraction, as well as any off-diagonal LRG gap process is subject to unitarity, or absorptive corrections. These corrections evaluated in Sect. III D substantially slow down the energy dependence of diffraction.

Unfortunately, the calculation of the double diffractive cross section goes beyond the employed phenomenology of dipole-proton cross section. It needs more refined information about the dipole-dipole cross section. Therefore, we make a simple estimate of this cross section in Sect. III E, based on Regge factorization.

In Sect. IV we switch to proton-nucleus collisions and present the Glauber model approach to LRG processes. Unfortunately most of LRG channels cannot be accessed in this approximation which neglects all off-diagonal diffractive reactions. Therefore, one should include the Gribov inelastic shadowing corrections which are introduced in Sect. V within the eigenstate formalism. It turns out that at high energies the interaction eigenstates are the color dipoles, thus the Gribov corrections lead to the color transparency effect [8].

In Sect. VI cross sections of coherent processes in which the nucleus remains intact are calculated. These include elastic scattering and diffractive excitation of the projectile proton, as well as the total cross section related via unitarity to the elastic amplitude. We employ different combinations of the two models for the proton wave function and two models for the dipole cross section. The results are presented in Table I for the energy of HERA-B, \( \sqrt{s} = 41.6 \text{ GeV} \), in Table II for RHIC, \( \sqrt{s} = 200 \text{ GeV} \), and in Table III for LHC, \( \sqrt{s} = 5.5 \text{ TeV} \).

The reactions leading to nuclear break-up, quasielastic scattering and diffractive excitation of bound nucleons are considered in Sect. VII. One can calculate the cross section making use of completeness.

Besides the excitation of the proton valence quark skeleton, diffractive interaction can shake gluons off the proton. In terms of Regge phenomenology this process is related to the triple Pomeron part of diffraction. On the other hand, it causes a reduction of gluon density in the proton, an effect usually called gluon shadowing. These phenomena are considered in Sect. IX.

II. LIGHT-CONE DIPOLE REPRESENTATION

A. The dipole cross section

The cross section of interaction of a \( \bar{q}q \) dipole with a nucleon, introduced in [8], is usually assumed to be flavor independent, and to depend only on the \( qq \) transverse separation and energy. This independence of quark flavor has been proven only perturbatively, but good agreement between data and a parameter free calculation [9] of the \( J/\Psi \) photoproduction cross section shows that this is a good approximation.

The dipole cross section calculated perturbatively [8] within the two-gluon model for the Pomeron [10, 11] depends only on the transverse dipole size \( r_T \) and is in-
dependent of energy and the relative fractions of the momentum carried by the quark and antiquark. However, higher order corrections bring forth a dependence on energy (or Bjorken $x$-dependence in DIS) and on the fractions $\alpha$ and $1-\alpha$ of the dipole light-cone momentum carried by the $q$ and $\bar{q}$. This energy dependence is rather mild (especially in soft processes under consideration) and may lead to sizeable effects only for large variations of the collisions energy. Since the variation of $\alpha$ is usually not very large, its dependence can be neglected. This approximation may be questionable in DIS where the end-point region of the $\alpha$ distribution is enhanced by the increasing cross section [12]. However, this is not the case in soft reactions where the $\alpha$-independence should be a good approximation.

In what follows we will test two popular models for the dipole cross section:

(i) Simple extrapolation of the small-$r_T$ behavior,

$$\sigma_{\bar{q}q}(r_T, s) = C(s) r_T^2 \, ;$$

(3)

(ii) The saturated cross section rises as $r_T^2$ at small $r_T^2$, but levels off at large $r_T^2$,

$$\sigma_{\bar{q}q}(r_T, s) = \sigma_0(s) \left[ 1 - \exp \left( -\frac{r_T^2}{R_0^2(s)} \right) \right] \, ,$$

(4)

where $R_0(s) = 0.88 \text{ fm}(s_0/s)^{0.14}$ and $s_0 = 1000 \text{ GeV}^2$ [1]. The energy dependent factor $\sigma_0(s)$ is defined as,

$$\sigma_0(s) = \sigma_{\bar{q}q}^{\text{sat}}(s) \left[ 1 + \frac{3R_0^2(s)}{8\langle r_{ch}^2\rangle_p} \right] \, ,$$

(5)

where $\langle r_{ch}^2\rangle_p = 0.44 \pm 0.01 \text{ fm}^2$ [13] is the mean square of the pion charge radius.

This dipole cross section is normalized to reproduce the pion-proton total cross section, $\sigma_{\bar{q}q}^{\text{sat}} = \sigma_{\bar{q}q}^{\text{tot}}(s)$. The saturated shape of the dipole cross section is inspired by the popular parametrization given in Ref. [14], which is fitted to the low-$x$ and high $Q^2$ data for $F_2^p(x, Q^2)$ from HERA. However, that should not be used for our purpose, since it is unable to provide the correct energy dependence of hadronic cross sections. Namely, the pion-proton cross section cannot exceed 23 mb [50]. Besides, Bjorken $x$ is not a proper variable for soft reactions, since at small $Q^2$ the value of $x$ is large even at low energies. The $s$-dependent dipole cross section Eq. (4) was fitted [1] to data for hadronic cross sections, real photoproduction and low-$Q^2$ HERA data for the proton structure function. The cross section (3) averaged with the pion wave function squared (see below) automatically reproduces the pion-proton cross section.

In the case of a proton beam one needs a cross section for a three-quark dipole, $\sigma_{3q}(\vec{r}_1, \vec{r}_2, \vec{r}_3)$, where $\vec{r}_i$ are the transverse quark separation with a condition $\vec{r}_1 + \vec{r}_2 + \vec{r}_3 = 0$. In order to avoid the introduction of a new unknown phenomenological quantity, we express the three-body dipole cross section via the conventional dipole cross section $\sigma_{\bar{q}q}$ [16],

$$\sigma_{3q}(\vec{r}_1, \vec{r}_2, \vec{r}_3) = \frac{1}{2} \left[ \sigma_{\bar{q}q}(r_1) + \sigma_{\bar{q}q}(r_2) + \sigma_{\bar{q}q}(r_3) \right] \, .$$

(6)

This form satisfies the limiting conditions, namely, turns into $\sigma_{\bar{q}q}(r)$ if one of three separations is zero. Since all these cross sections involve nonperturbative effects, this relation hardly can be proven, but should be treated as a plausible assumption.

B. The light-cone wave function of the proton

Since the dipole cross section is assumed to be independent of the sharing of the light-cone momentum among the quarks, the wave function squared of the valence Fock component of the proton, $|\Psi(\vec{r}_i, \alpha_j)|^2$ should be integrated over fractions $\alpha_i$. The result depends only on transverse separations $\vec{r}_i$. The form of the nonperturbative valence quark distribution is unknown, therefore for the sake of simplicity we assume the Gaussian form,

$$|\Psi_N(\vec{r}_1, \vec{r}_2, \vec{r}_3)|^2 = \frac{1}{\prod_{i=1}^{3} \alpha_i} |\Phi(\vec{r}_i, \alpha_j)|^2 \delta \left( 1 - \sum_{j=1}^{3} \alpha_j \right)$$

$$= \frac{2 + r_p^2/R_p^2}{(\pi r_p R_p)^2} \exp \left( -\frac{r_p^2}{r_p^2 - r_T^2 - \langle r_{ch}^2 \rangle_p} \right) \times \delta(\vec{r}_1 + \vec{r}_2 + \vec{r}_3) \, ,$$

(7)

where $\vec{r}_i$ are the interquark transverse distances. The two scales $r_p$ and $R_p$ characterizing the mean transverse size of a diquark and the mean distances to the third quark. In what follows we consider two extreme possibilities:

(i) The quark-diquark structure of the proton. There are evidences from data that the dominant configuration of valence quarks in the proton contains a small isoscalar ud diquark of a size $r_{ch}^2 \approx 0.2 - 0.4 \text{ fm}^2$ [17–19], i.e. $r_p^2 \ll R_p^2$. Neglecting the diquark radius we arrive at a meson-type color dipole structure,

$$|\Psi_N(\vec{r}_1, \vec{r}_2, \vec{r}_3)|^2 = \frac{2}{\pi R_p^2} \exp \left( -\frac{2r_p^2}{R_p^2} \right) \, ,$$

(8)

where $\vec{r}_1 = \vec{r}_2 = \vec{r}_3$, and $R_p$ is related to the mean charge radius squared of the proton as $R_p^2 = 16\langle r_{ch}^2 \rangle_p$.

(ii) The symmetric proton structure. Another extreme would be to say that the forces binding the valence quarks are of an isoscalar nature, therefore the quark distribution is symmetric, i.e. $r_p = R_p$ in (7). In this case the mean interquark separation squared is $\langle \vec{r}_T^2 \rangle = \frac{4}{3} r_p^2 = 2\langle r_{ch}^2 \rangle_p$.

Thus, we consider two possibilities for the shape of the dipole cross section and two choices for the proton wave function. In what follows we will test these models comparing to data.
III. SINGLE DIFFRACTION pp → Xp

Now we have a choice for the key ingredients of the dipole approach [8], which are the universal dipole cross section and the light-cone wave function of the incoming hadron. First of all, we should make sure that each model correctly reproduces the total pp cross section, which is the main entry for the calculation of nuclear effects. A further important and rigorous condition for a realistic model would be the magnitude of the single diffractive cross section.

A. Experimental data

The cross section of proton excitation, pp → pX, integrated over large Feynman x_F > 0.95 is about 3.5 mb at E_{lab} = 920 GeV and nearly saturates at the value of 4 mb at higher energies [6, 7]. This cross section, however, includes two parts: (i) excitation of the quark skeleton of the hadron without gluon radiation; (ii) Excitations including emission of gluons. The former, in terms of the Regge phenomenology, corresponds to the triple Regge graph P P R and behaves like 1/ M^3 at large excitation masses. The latter corresponds to the triple-Pomeron graph 3P which provides the 1/ M^2 tail at large masses.

In terms of the dipole description the lowest Fock component at which we are concentrating in this section corresponds to the first part, P P R. The higher Fock components containing gluons will be considered in Sect. IX.

At high energies and large x_F → 1 one can write the diffractive cross section in the triple-Regge form [20],

\[ \frac{d \sigma_{sd}}{d x_F dp_T^2} = \sqrt{s} \frac{G_{PPP}(0)}{(1 - x_F)^{9/2}} e^{-B_{PPP} p_T^2} + \frac{G_{3P}(0)}{(1 - x_F)} e^{-B_{PPP} p_T^2} + G_{RRP}(0) e^{-B_{RRR} p_T^2} \cdot (9) \]

Here s_1 = 1 GeV^2,

\[ B_{PPP} = R_{PPP}^2 - 2\alpha_{p} \ln(1 - x_F) \]

\[ B_{RRR} = R_{RRR}^2 - 2\alpha_{r} \ln(1 - x_F) \]

i = P, R; \( \alpha_{p} \approx 0.25 \text{ GeV}^{-2} \) and \( \alpha_{r} \approx 0.9 \text{ GeV}^{-2} \) are the slopes of the Pomeron and secondary Reggeon trajectories. Note that we keep in (10) only the triple-Regge terms which are either singular in (1 - x_F), or do not vanish as powers of energy.

Since the data show no substantial rise of the diffractive cross section with energy [6, 7], which is apparently caused by strong absorptive corrections, we incorporate this fact fixing the effective Pomeron intercept at \( \alpha_P(0) = 1 \). This also allows us to use the results of the comprehensive triple-Regge analysis of data performed in Ref. [20], in which \( \alpha_P(0) = 1 \) was used. They found the following values for the parameters: \( G_{PPP}(0) = G_{PPP,R}(0) = 3.2 \text{ mb/GeV}^2 \); \( G_{RRR}(0) = 13.2 \text{ mb/GeV}^2 \);

\( R_{PPP}^2 = 4.2 \text{ GeV}^{-2} \); \( R_{PPP,R}^2 = 1.7 \text{ GeV}^{-2} \); \( R_{RRR}^2 = 0 \text{ GeV}^{-2} \).

We are now in a position to evaluate the diffractive cross section integrating the distribution Eq. (9) over \( p_T^2 \) and \( x_F \), from \( x_{min} \) up to \( x_{max} \). We fix \( x_{min} = 0.95 \) as is usually done to separate diffractive from nondiffractive contributions, and \( x_{max} = 1 - M_{P}^2/s \) with \( M_{P}^2 = (m_N + m_p)^2 \text{ GeV}^2 \). Of course such small masses do not satisfy the condition of the triple-Regge dynamics, but we appeal to the duality between s-channel resonances and t-channel Reggeons which is known to work well if one averages over the resonances. Then we get at the energy of HERA-B, \( \sigma_{pp}^B = 3.27 \text{ mb} \) which agrees well with the value \( \sigma_{sd}^B = 3.5 \text{ mb} \) suggested by data. We expect \( \sigma_{sd}^B \approx 4 \text{ mb} \) at RHIC and LHC energies.

As was already mentioned, in this section we are only interested in the part of diffraction related to excitation of the valence quark skeleton without gluon radiation. This part corresponds to the first term, \( P P R \), in Eq. (9). Integrating it over \( p_T^2 \) and \( x_F \) we get \( \sigma_{sd}^B(PPR) = 1.13 \text{ mb} \), i.e. about the third of the total single diffractive cross section.

For further model tests we need to know the forward diffraction which turns out to be quite small with respect to the forward elastic cross section,

\[ \sigma_{sd} = \frac{d \sigma_{sd}}{d \sigma_{el} / dp_T^2}_{p_T=0} = 5.5 \text{ mb/GeV}^2 = 84.5 \text{ mb/GeV}^2 = 0.065 \]

at the energy of HERA-B.

It is worth reminding that we rely on the hypothesis of duality and should not expect this result to be very accurate. It just shows the scale of the effect, namely, that diffractive excitation of the projectile valence quark system is of an order of magnitude weaker than the elastic channel.

In hadronic representation, in particular within the one pion exchange model, one can explain rather well such a weak diffractive excitation of the valence Fock component of the proton [20]. It is a challenge, however, to reproduce this result within the QCD based dipole model.

B. Models

Since we have models for the dipole cross section and the proton wave function, it is therefore possible to calculate the total and forward single diffractive cross sections [1, 8],

\[ \sigma_{tot} = \langle \sigma_{q\bar{q}}(r_i) \rangle \quad (12) \]

\[ \int dM_X^2 \frac{d \sigma_{sd}(pp \rightarrow pX)}{dM_X^2 dp_T^2} \bigg|_{p_T=0} = 1 \frac{1}{10\pi} \left\{ \langle \sigma_{q\bar{q}}(r_i)^2 \rangle - \langle \sigma_{q\bar{q}}(r_i) \rangle^2 \right\} \quad (13) \]
Here the averaging weight is the proton wave function squared,

\[
\langle \ldots \rangle = \int \prod_{i=1}^{3} d^2 r_i |\Psi(r_j)|^2 \langle \ldots \rangle .
\]  

(14)

We are going to test the models mentioned before regarding their ability to explain the strong suppression of the diffractive relative to the elastic channels, \( R_{sd} \sim 0.1 \) in Eq. (11).

1. Model I: quark-diquark proton and \( \sigma_{q\bar{q}} \propto r_T^2 \)

The simple quadratic \( r_T \) dependence of the dipole is quite popular in the literature and is frequently used for calculations of nuclear effects (e.g. see [16, 21, 22]). Since the energy dependent factor is unknown, it is usually fitted to reproduce the total cross section. In this case one has to explain the relative values of total cross sections for different hadronic species, in particular, the ratio \( \sigma_{pp}^{\text{tot}}/\sigma_{pp}^{\text{tot}} \approx 1.6 \). Indeed, the ratio of charged radiuses squared agrees with this value within the diversity of the results of different measurements [23].

However, the cross section of single diffraction turns out to be dramatically overestimated. Indeed, the diffraction to elastic ratio defined in (11) reads,

\[
R_{sd} = \frac{\langle r_T^2 \rangle}{\langle r_T^2 \rangle^2} - 1 = 1 .
\]  

(15)

This is more than one order of magnitude larger than the experimental value Eq. 11.

2. Model II: symmetric proton and \( \sigma_{q\bar{q}} \propto r_T^2 \)

In this case we again treat the unknown factor in the dipole cross section as a free parameter and adjust it to the total cross section,

\[
\langle \sigma_{q\bar{q}}(r_T) \rangle = \sigma_{pp}^{\text{tot}} ;
\]  

(16)

Then we can calculate the mean value of the dipole cross section squared,

\[
\langle \sigma_{q\bar{q}}^2(r_T) \rangle = \frac{3}{2} (\sigma_{pp}^{\text{tot}})^2 ,
\]  

(17)

and eventually the diffractive cross section according to (13). Then we arrive at the ratio,

\[
R_{sd} = \frac{1}{2} ,
\]  

(18)

which is closer to the experimental value Eq. (11), but still quite overestimates the data.

It is not a surprise that both above models based on the dipole cross section Eq. (3) steeply rising with quark separation grossly overestimate the diffractive cross section, since the main contribution comes from large distances. Therefore, both models I and II are not realistic and should not be used for comparison with data.

Actually, Eq. (13) shows that diffraction is given by the dispersion of the dipole cross section dependence on the transverse \( \bar{q}q \) separation. Apparently, \( \sigma_{q\bar{q}}(r_T) \propto r_T^2 \) used so far varies with \( r_T \) too steeply and the dispersion is too large. One needs a flatter dependence in order to suppress diffraction.

3. Model III: quark-diquark proton and saturated cross section

There is no freedom in this case, the dipole cross section Eq. (4) if fixed to reproduce the pion-proton cross section. The calculated value of proton-proton total cross section is rather close to the experimental value, but somewhat smaller. This is not a surprise, since this model employs the approximation of a point-like diquark, which apparently leads to an underestimation of the pp cross section. It is risky, however, to rely on such an approximation for calculating nuclear effects. Even a small deviation of \( \sigma_{pp}^{\text{tot}} \) from the experimental value will cause nuclear effects which may be misinterpreted as inelastic shadowing. To make sure that this cross section is exactly reproduced, we redefine the function \( \sigma_0(s) \) in (5),

\[
\sigma_0^{\text{III}}(s) = \sigma_{pp}^{\text{tot}}(s) \left( 1 + \frac{1}{\delta} \right) ,
\]  

(19)

where the parameter \( \delta = 8\langle r_{ch}^2 \rangle_p/3R_0^2(s) \), and we use the mean proton charge radius squared \( \langle r_{ch}^2 \rangle_p = 0.8 \text{ fm}^2 \) [23].

Using the proton wave function Eq. (8) and the saturated cross section Eq. (4) we get the single diffractive cross section in the form,

\[
\frac{d\sigma_{sd}^{pp}}{dp_T^2} \bigg|_{p_T=0} = \frac{\delta^2}{(1 + 2\delta)^2(1 + 2\delta)} \frac{\sigma_0^2(s)}{16\pi} .
\]  

(20)

Dividing by the elastic cross section we get,

\[
R_{sd} = \frac{1}{1 + 2\delta} .
\]  

(21)

This ratio is rather small at the energy of HERA-B, \( R_{sd} = 0.13 \), which is compatible with the data. The fraction of single diffraction decreases with energy down to \( R_{sd} = 0.06 \) at the energy of RHIC \( (\sqrt{s} = 200 \text{ GeV}) \), and \( R_{sd} = 0.01 \) at the energy of LHC \( (\sqrt{s} = 5.5 \text{ TeV}) \).

One may wonder why the saturated cross section leads to such a weak diffraction and why decreases with energy? This is easy to interpret. Indeed, if the cross section were completely flat, i.e. \( \sigma_{q\bar{q}}(r_T) = \text{const} \), no diffraction would be possible because of orthogonality of the initial and final valence quark wave functions. Only the drop of \( \sigma_{q\bar{q}}(r_T) \) at \( r_T \lesssim R_0(s) \) makes diffraction possible. However, \( R_0(s) \) decreases with energy, therefore
the shape of the dipole cross section is getting flatter and diffraction vanishes. As far as diffraction gets its main contribution from small \( r_T \lesssim R_0 \), note that it is less probable to find three quarks with small separations, than a two-quark system. Therefore, one should expect less diffraction for a symmetric three-quark wave function of the proton as is demonstrated in the next section.

4. Model IV: symmetric proton and saturated cross section

Using the wave function Eq. (7) with \( r_p = R_p \) and the cross section (4) we get the following forward elastic cross section and diffraction to elastic ratio,

\[
\frac{d\sigma_{\text{el}}}{dp_T^2} \bigg|_{pp} = \frac{\gamma^2}{(1 + \frac{2}{3} \gamma)^2} \sigma_0^2(s) \frac{16\pi}{\gamma}, \tag{22}
\]

\[
R_{sd} = \frac{(1 + \frac{2}{3} \gamma)^2 - \frac{1}{2}}{(1 + \frac{2}{3} \gamma)(1 + \frac{1}{3} \gamma)(1 + \gamma)}, \tag{23}
\]

The parameter \( \gamma \) is related to previously introduced \( \delta \),

\[
\gamma = 3 \frac{\langle r_{ch}^2 \rangle_p}{R_0^2(s)} = \frac{9}{8} \delta. \tag{24}
\]

Then, for the energy of HERA-B we arrive at a very small fraction of the single diffractive cross section \( R_{sd} \approx 0.07 \) which is in excellent agreement with the experimental result Eq. (11). We expect a substantial reduction of this fraction at higher energies, \( R_{sd} \approx 0.03 \) at the energy of RHIC, and \( R_{sd} \approx 0.0045 \) at the energy of LHC.

Note that the the parameter \( \gamma \) in Eqs. (22)-(23) can be defined differently,

\[
\sigma_{\text{tot}}^{pp} = (\sigma_{3q}) = \sigma_0(s) \frac{-\gamma}{1 + \frac{2}{3} \gamma}, \tag{25}
\]

where \( \sigma_0(s) \) and \( \sigma_{3q}(r_1) \) are defined in (5) and (6) respectively.

Both definitions (24) and (25) nicely agree at the energies of fixed target experiments. More problematic is to apply Eq. (25) at high energies of colliders, RHIC and LHC. No data for \( \sigma_{\text{tot}}^{pp} \) are available at energies above \( \sqrt{s} = 30 \) GeV. The usual extrapolation with a universal energy dependence as in \( pp \) collisions is just an educated guess not supported by any dynamic theory. Moreover, one should expect a steeper rise of \( \sigma_{\text{tot}}^{pp} \) than \( \sigma_{\text{tot}}^{pp} \). Indeed, the rising part of the cross section related to gluon radiation is nearly independent of hadronic size, while the constant part of the cross section related to dipole-dipole collision followed by no gluon radiation apparently depends on the dipole sizes and is smaller for \( pp \), than for \( pp \). Thus, pion-proton cross section rises steeper with energy, in accordance with the general trend of steeper energy dependence for smaller dipoles discovered at HERA.

One can combine Eqs. (24) and (25) in order to find the unknown energy dependence of \( \sigma_{\text{tot}}^{pp}(s) \). This is a more reliable procedure, since the dipole cross section, in particular the parameter \( R_0(s) \), is fitted to data at energies much higher than those where data for \( \sigma_{\text{tot}}^{pp}(s) \) are available. Then the pion to proton ratio of the cross sections reads,

\[
R_{\pi/p} = \frac{\sigma_{\pi/p}^{pp}(s)}{\sigma_{\text{tot}}^{pp}(s)} = \frac{2}{3} + \frac{1}{3} \frac{R_0^2(s)}{\gamma^2} \frac{R_\pi^2}{(r_{ch})^2}. \tag{26}
\]

This ratio slowly rises with energy. At the energy of HERA-B \( R_{\pi/p} = 0.6 \) in excellent agreement with data, but it becomes 10\% larger, \( R_{\pi/p} = 0.66 \), at the energy of LHC. Eventually, at very high energy when unitarity will be saturated, all the cross sections reach the universal Froissart limit corresponding to an expanded black disk. Correspondingly, \( R_{\pi/p}(s) \to 1 \).

C. Diffractive gluon radiation

Besides excitation of the valence quark skeleton of the proton, a valence quark itself can be excited followed by gluon radiation. In terms of Regge phenomenology this process corresponds to the triple-Pomeron contribution to the diffraction cross section. This is the second term in Eq. (9).

It has been known since the 70s that the triple-Pomeron coupling is quite small. To appreciate this statement one can express diffraction in terms of the Pomeron-proton total cross section which should be expected to be twice as large as a meson-proton one. Indeed, the Pomeron is a gluonic system, therefore one should have an extra Casimir factor \( 9/4 \) compared to a quark dipole. Thus, one may expect \( \sigma_{\text{tot}}^{pp} \approx 50 \text{ mb} \). This expectation is in dramatic contradiction with data [6] which show only \( \sigma_{\text{tot}}^{pp} = 2 \text{ mb} \) at large excitation masses. Apparently, it is much more difficult to shake gluons off valence quarks, compared to pQCD expectations. The way out of this puzzle is to suggest that gluons in the proton are located within small spots which are hardly resolved by soft interactions. The mean transverse size of these spots was fitted to single diffraction data with large effective masses and found to be \( \langle r_T \rangle \approx r_0 = 0.3 \text{ fm} \).

Such gluons have much more intensive Fermi motion than massless perturbative ones, and they are less sensitive to an external kick, i.e. gluon radiation is suppressed.

Such a picture is quite successful in explaining the energy dependence of the total hadronic cross sections and elastic hadronic slopes [2]. It also correctly predicts the reduced value of \( \alpha_s^p \), the slope of the Pomeron trajectory for the process of elastic photoproduction of \( J/\Psi \) [3]. Similar statements about gluonic structure of the proton has been done in [4] recently.

The light-cone wave function of the quark-gluon Fock component of a quark was calculated in [1] within a model describing the nonperturbative interaction of gluons via a phenomenological light-cone potential taken in an os-
The result reads,
\[ \Psi_{qG}(\tilde{r}_T) = \frac{2i}{\pi} \sqrt{\frac{\alpha_s}{3}} \frac{\tilde{r}_T \cdot \tilde{e}^*}{r_T^2} \exp \left( -\frac{r_T^2}{2\alpha_0^2} \right). \] (27)

Here we assume that the gluon, which is a vector particle and possesses a 1/x distribution, is carrying a negligible fraction \( \langle x \rangle \ll 1 \) of the quark momentum. Of course the concrete shape of the light-cone potential is not crucial. What is only important is the smallness of the mean quark-gluon separation. Notice that Eq. (27) can be viewed as a source of small gluonic spots in the proton [3].

Since the light-cone quark-gluon distribution function is known, one can calculate the cross section of diffractive gluon radiation by a high-energy quark interaction with a nucleon [1],
\[ \frac{d\sigma(qN \rightarrow qGN)}{dx_F dp_T^2} \bigg|_{p_T=0} = \frac{1}{16\pi(1-x_F)} \times \int d^2r_T \left| \Psi_{qG}(r_T) \right|^{-2} \tilde{\sigma}^2(r_T, s). \] (28)

Here the cross section \( \tilde{\sigma}(r_T, s) \) is not just a cross section of interaction of a qG dipole. This dipole is not even colorless. As usual, diffractive excitation is possible due to a difference between elastic amplitudes for different Fock states, in this case the bare quark \( |q_0\rangle \) and the \( |qG\rangle \) pair. Since they have the same color, the difference emerges from the color-dipole moment of the \( q-G \) pair. It is shown in [1] (see in particular Appendix A.2) that \( \tilde{\sigma}(r_T, s) = \frac{2}{\pi} \sigma_{qG}(r_T, s) \).

The next step is to integrate over \( p_T \) the cross section of diffractive gluon radiation provided that the forward one, Eq. (28), is known. In terms of the Regge phenomenology diffractive radiation corresponds to the triple-Pomeron term in the cross section of single diffraction. Data agree with a Gaussian \( p_T \)-dependence of the triple-Pomeron term with the slope [20],
\[ B_{3\,p}^{pp}(x_F) = B_{3\,p}^0 + 2\alpha_f' \ln \left( \frac{1}{1-x_F} \right), \] (29)
where \( B_{3\,p}^0 = 4.2 \text{GeV}^{-2} \) and \( \alpha_f' = 0.25 \text{GeV}^{-2} \).

Now, we are in a position to evaluate the effective triple-Pomeron part of the single diffraction cross section for \( pp \) collisions employing the the wave function Eq. (27) and the saturated cross section Eq. (4),
\[ \left[ \frac{d\sigma(pp \rightarrow pX)}{dx_F dp_T^2} \right]_{3\,p} = \frac{81}{16\pi} \alpha_s \sigma_{qG}^2(s) \left( 16\pi f_{pp}^0(x_F) \right)^2 \ln \left[ 1 + \frac{\epsilon^2(s)}{1+2\epsilon(s)} \right] \exp \left[ -\frac{r_T^2}{2f_{pp}^0(x_F)} \right]. \] (30)

where \( \epsilon(s) = r_T^2/R_0^2(s) \). In the energy range under discussion \( \epsilon(s) \) is rather small, then the single diffractive cross section Eq. (30) is proportional to \( r_T^2 \). This is why this process is quite sensitive to the value of \( r_0 \) and provides an efficient way to determine the size of gluonic spots in the proton, \( r_0 \approx 0.3 \text{fm} \) [1].

Notice that the interference between the diffractive amplitudes of gluon radiation by different quarks in the proton should not be appreciable, since \( r_0 \) is small compared to the proton radius. Explicit calculations performed in [1] confirm this.

### D. Unitarity corrections

Any large rapidity gap process is subject to unitarity or absorptive corrections which may be substantial. Indeed, initial/final state interactions tend to fill the rapidity gap by producing particles, and one may treat such corrections as a survival probability of the rapidity gap. Such corrections become especially large and may completely terminate the gap in the vicinity of the unitarity limit, which is also called black disk regime.

Since the phenomenological dipole cross section \( \sigma_{qG}(s) \) is fitted to data, we assume it to include by default all the unitarity corrections. The same is true for the off-diagonal amplitude of diffractive excitation of the valence quark system without gluon radiation, since it is a linear combination of elastic dipole amplitudes. Thus, our calculations for the \( P P P \) term in (9) do not need any unitarity corrections.

The situation is different for the \( P P P \) term, or gluon diffractive radiation. Although the amplitude of diffractive excitation of a quark, \( q \rightarrow qGN \), does include all the absorptive corrections contained in the phenomenological dipole cross section, the presence of other projectile valence quarks, the spectators, should not be ignored. Indeed, any inelastic interactions of the large-size three quark dipole in the proton, will cause particle production which will fill the rapidity gap. Thus, one may expect large absorptive corrections to the cross section of diffractive gluon radiation.

Data for elastic \( pp \) scattering show that the partial amplitude \( f_{el}^{pp} (b, s) \) hardly varies with energy at small impact parameters \( b \rightarrow 0 \), while rises as function of energy at large \( b \) [2, 24, 25]. This is usually interpreted as a manifestation of saturation of the unitarity limit, \( \text{Im} f_{el}^{pp} \leq 1 \). Indeed, this condition imposes a tight restriction at small \( b \), where \( \text{Im} f_{el}^{pp} \approx 1 \), leaving almost no room for further rise. Correspondingly, the amplitude of any off-diagonal process including single triple-Pomeron diffraction acquires a suppression factor
\[ f_{el}^{pp}(b, s) \Rightarrow f_{el}^{pp}(b, s) \left[ 1 - \text{Im} f_{el}^{pp}(b, s) \right]. \] (31)
due to unitarity or absorptive corrections. This factor related to the survival probability of LRG is known to decrease with energy [26]. Interplay of the rising and falling energy dependences of the two factors in (31) may explain the observed flat behavior of the single diffractive cross section [6, 7].

Since \( f_{el}^{pp}(b, s) \) is known directly from data, it would be straightforward just to fit the data with any proper
parametrization and use the result in Eq. (31). Alternatively, one can use any model which provides a good description for $\text{Im} f_{cl}^{dd}(b, s)$. It is demonstrated in [2] that even the simple model treating the Pomeron as a Regge pole with no unitarity corrections describes reasonably well not only the total hadronic cross sections, but even the data for $f_{cl}^{dd}(b, s)$. In this model,

$$\text{Im} f_{cl}^{dd}(b, s) = \frac{\sigma_{tot}^{dd}(s)}{4\pi B_{cl}^{dd}(s)} \exp\left[\frac{-b^2}{2B_{cl}^{dd}(s)}\right].$$

(32)

Here and for further applications we use the parametrization from [27], $\sigma_{tot}^{dd}(s) = 18.76\text{ mb} \times (s/M_0^2)^{0.083} + \sigma_R^{dd}(s)$, where $M_0 = 1\text{ GeV}$, and the Reggeon part of the cross section $\sigma_R^{dd}(s)$ is small at high energies and can be found in [27]. The elastic slope $B_{cl}^{dd}(s) = B_0^{el} + 2\alpha'_p \ln(s/M_0^2)$ with $B_0^{el} = 8.9\text{ GeV}^{-2}$ and $\alpha'_p = 0.25\text{ GeV}^{-2}$. Note that the elastic amplitude at small impact parameters, i.e. the pre-exponential factor in (32), hardly changes with energy imitating saturation of unitarity. This fact has been known back in the 70s as a geometrical scaling. It is demonstrated in [2] (see Fig. 9) that not only at $b = 0$, but in the whole range of impact parameters the model Eq. (32) reasonably well describes the energy dependence of the partial amplitude $f_{cl}^{dd}(b, s)$.

Using Fourier transformed Eq. (32) we arrive at the following cross section for single diffraction integrated over momentum transfer,

$$\frac{d\sigma(pp \to pX)}{dx_F} = \frac{81 \alpha_s \sigma_0^2(s)}{(16\pi)^2} \frac{B_{cl}^{dd}(x_F)}{B_{cl}^{dd}(s)} \ln\left[1 + \frac{e^2(s)}{1 + 2e(s)}\right] \left\{1 - \frac{\alpha_R^{tot}(s)}{\pi B_{cl}^{dd}(x_F) + 2B_{cl}^{dd}(s)} + \frac{[\sigma_{tot}^{dd}(s)]^2}{(4\pi)^2 B_{cl}^{dd}(s)[B_{cl}^{dd}(x_F) + B_{cl}^{dd}(s)]}\right\}.$$  

(33)

At the scale corresponding to the mean transverse momentum of gluons, $\alpha_s(1/r_0^2) \approx 0.4$ [2], (33) agrees with data reasonably well. Nonetheless, we think that one should perform more elaborated calculations when $\text{Im} f_{cl}(s)$ is close to the unitarity limit. Indeed, the absorptive correction factor in (31) is so small in this case that any little variations of $\text{Im} f_{cl}(s)$ may lead to dramatic effects. Such a fine tuning goes beyond the scope of the paper, but we plan to work more on this elsewhere.

**E. Double diffraction $pp \to XY$**

If the Pomeron were a true Regge pole, one would expect Regge factorization relating the forward single and double diffusive cross sections,

$$\left(\frac{d\sigma_{dd}}{dt}\right)_{t=0} = \left(\frac{d\sigma_{pp}}{dt}\right)_{t=0}^2 = \left(\frac{d\sigma_{pp}}{dt}\right)_{t=0}^2.$$  

(34)

However, even within the Regge model this relation is broken due to presence of Regge cuts. The usual accuracy of relations based on Regge factorization is 10–20%.

Besides, neither perturbative QCD calculations [28], nor phenomenological dipole cross sections fitted to DIS data [14] confirm Regge factorization. The very $Q^2$ dependence of the Pomeron intercept observed at HERA is a direct evidence of lack of factorization.

Unfortunately, the phenomenological dipole cross section $\sigma_{qg}$ which is averaged over the size of the target proton, is not sufficient for calculating double diffraction. Therefore, we will employ the approximate relation (34) in what follows, hoping that the corrections are not much larger than in other known cases.

For the integrated double diffusive cross section relation (34) can be rewritten as,

$$\int_0^s \sigma_{tot}^{dd}(s) \, ds = \left(\frac{\alpha_R^{tot}}{\sigma_{tot}^{dd}}\right)^2 \left(\frac{B_{el}^{dd}(s)}{B_{tot}^{dd}(s)}\right)^2$$  

(35)

At the energy of HERA-B the elastic slope $B_{el}^{dd} = 12.6\text{ GeV}^{-2}$; $\sigma_{tot}^{dd} = (\sigma_{tot}^{dd})^2/(16\pi B_{el}^{dd}) = 6.7\text{ mb}$; $\sigma_{tot}^{dd} = 3.5\text{ mb}$. The slope of the first part of single diffraction relevant for Eq. (35), varies with Feynman $x_F$, $B_{el}^{dd} = B_0 - 2\alpha'_p \ln(1 - x_F)$, where $\alpha'_p = 0.25\text{ GeV}^{-2}$, $B_0 = 2\text{ GeV}^{-2}$ [20]. The mass distribution of diffusive excitation of the valence quark skeleton of the proton strongly peaks in the resonance region at $M_0 \approx 1.5\text{ GeV}$, which corresponds to $1 - x_F = M_0^2/s$. Then $B_{el}^{dd} = 5.3\text{ GeV}^{-2}$. As for the double diffusive slope, it contains only the Pomeron contribution, $B_{el}^{dd} = -2\alpha'_p \ln(1 - x_F) = 3.3\text{ GeV}^{-2}$. Eventually, we arrive at an estimate for the double diffusive cross section at the energy of HERA-B, $\sigma_{tot}^{dd} = 1.25\text{ mb}$. The same Eq. (35) leads to the double diffraction cross sections $\sigma_{tot}^{dd} = 1.18\text{ mb}$ and $\sigma_{tot}^{dd} = 0.47\text{ mb}$ at the energies of heavy ions at RHIC, $\sqrt{s} = 0.2\text{ TeV}$, and at LHC, $\sqrt{s} = 5.5\text{ TeV}$, respectively.

**IV. $pA$ COLLISIONS: GLAUBER MODEL**

The $pA$ elastic amplitude at impact parameter $b$ has the eikonal form [29],

$$\Gamma^{pA}(\tilde{b}, \{\tilde{s}_j, z_j\}) = 1 - \prod_{k=1}^A \left[1 - \Gamma^{pN}(\tilde{b} - \tilde{s}_k)\right],$$  

(36)

where $\{\tilde{s}_j, z_j\}$ denote the coordinates of an $i$-th target nucleon; $i\Gamma^{pN}$ is the elastic scattering amplitude on a nucleon normalized as,

$$\sigma_{tot}^{pN} = 2 \int d^2b \text{Re} \Gamma^{pN}(b).$$  

(37)

In the approximation of single particle nuclear density[51],

$$\rho_A(\tilde{b}_1, z_1) = \int \prod_{i=2}^A d^3r_i \left|\Psi_A(\{\tilde{r}_j\})\right|^2,$$  

(38)
the matrix element between the nuclear ground states reads,

$$\langle 0 | \Gamma^{pA} (\vec{b}; \{\vec{s}_j, z_j\}) | 0 \rangle$$

$$= 1 - \left[ 1 - \frac{1}{A} \int d^2 s \Gamma^{pN}(s) \int dz \rho_A (\vec{b} - \vec{s}, z) \right]^A$$  \hspace{1cm} (39)

Correspondingly, the total $pA$ cross section has the form,

$$\sigma^{pA}_{tot} = 2 \text{Re} \int d^2 b \left\{ 1 - \left[ 1 - \frac{1}{A} \int d^2 s \Gamma^{pN}(s) T_A (\vec{b} - \vec{s}) \right]^A \right\} \approx 2 \int d^2 b$$

$$\times \left\{ 1 - \exp \left[ - \frac{1}{2} \sigma^{pN}_{tot} (1 - i \alpha_{pp}) T^h (b) \right] \right\} , \hspace{1cm} (40)$$

where $\alpha_{pp}$ is the ratio of the real to imaginary parts of the forward $pp$ elastic amplitude;

$$T^h_A (b) = \frac{2}{\sigma^{pN}_{tot}} \int d^2 s \text{Re} \Gamma^{pN}(s) T_A (\vec{b} - \vec{s}) \; ; \hspace{1cm} (41)$$

and

$$T_A (b) = \int_{-\infty}^{\infty} dz \rho_A (b, z) \; , \hspace{1cm} (42)$$

is the nuclear thickness function. We use the Gaussian form of $\Gamma^{pN}(s)$,

$$\text{Re} \Gamma^{pN}(s) = \frac{\sigma^{pN}_{tot}}{4 \pi B^N_{el}} \exp \left( \frac{-s^2}{2 B^N_{el}} \right). \hspace{1cm} (43)$$

Note that the accuracy of the optical (exponential) approximation in (40) is quite good, $\sim 10^{-3}$ for heavy nuclei, but for numerical calculations we rely on the exact Glauber expressions throughout the paper. In what follows we neglect the real part of the elastic amplitude which gives quite a small correction, $\sim \rho_{pp}/A^{2/3}$, and can be easily implemented.

We performed numerical calculations at $E_{lab} = 920 \text{GeV}$, having in mind the needs of the experiment HERA-B at DESY. At this energy we used $\sigma^{pp}_{tot} = 41.2 \text{mb}, B_{pp} = 12.6 \text{GeV}^{-2}$ and $\sigma^{el}_{pp} = (\sigma^{pp}_{tot})^2/16\pi B_{pp} = 6.7 \text{mb}$. The Wood-Saxon form was used for the nuclear density with parameters fixed by data on electron-nucleus elastic scattering [30], except carbon whose density was taken in an oscillatory form [31],

$$\rho_C (r) = \left( \frac{2a}{\pi} \right)^\frac{3}{2} \left( 1 + \frac{4}{3} \frac{a}{r^2} \right) e^{-a r^2}. \hspace{1cm} (44)$$

The parameter $a$ was fitted to data for electron-carbon scattering, $a = 0.0143 \text{GeV}^{-2}$, and we assumed identical distributions for protons and neutrons. The results are depicted in Table I.

We also performed calculations at the energies of RHIC and LHC, $\sqrt{s} = 5.5 \text{TeV}$ using the input cross sections listed at the end of Sect. III E. The results are presented in Tables II and III respectively.

A. Elastic cross section

The simplest process with a large rapidity gap (LRG) is elastic scattering. It worth noting, however, that this channel is enhanced by absorptive corrections, while other LRG processes considered below are suppressed by these corrections.

The elastic cross section according to (39) reads,

$$\sigma^{pA}_{el} = \int d^2 b \left[ 1 - \exp \left[ -\frac{1}{2} \sigma^{pN}_{tot} T^h_A (b) \right] \right]^2. \hspace{1cm} (45)$$

Numerical results are shown in Table I.

B. Diffractive excitation of the beam and target

Diffractive excitation of the beam and/or target is another example of a LRG process. Unfortunately, the Glauber model is a single channel approximation and cannot treat properly diffractive excitation of the beam. Nevertheless, the cross section of diffractive excitation of the nucleus can be calculated, provided that the elastic and single diffractive cross sections for $NN$ collisions are known.

If the nucleus is excited without new particle production, i.e. it breaks up to nucleons and nuclear fragments, such a process, $pA \rightarrow pA^*$, is called quasielastic scattering. Summing over final states of the nucleus $|F\rangle$ and applying the condition of completeness, one gets,

$$\sigma^{pA}_{qel} (pA \rightarrow pA^*) = \sum_F \int d^2 b \left[ \langle 0 | \Gamma^{pA} (b) | F \rangle \right]^4$$

$$\times \left( F | \Gamma^{pA} (b) | 0 \rangle - | \langle 0 | \Gamma^{pA} (b) | 0 \rangle |^2 \right)$$

$$= \int d^2 b \left[ \langle 0 | \Gamma^{pA} (b) | 0 \rangle \left( | \Gamma^{pA} (b) | 0 \rangle \right)^2 - | \langle 0 | \Gamma^{pA} (b) | 0 \rangle |^2 \right]. \hspace{1cm} (46)$$

Here the cross section of elastic $pA \rightarrow pA$ scattering, Eq. (45), is subtracted.

The fist term in this expression contains the quadratic term $\int d^2 s T_A (\vec{b} - \vec{s}) \left[ \Gamma^{pN}(s) \right]^2 = T_A^2 (b) \sigma^{pN}_{el}$. Then the quasielastic cross section gets the form,

$$\sigma^{pA}_{qel} (pA \rightarrow pA^*)$$

$$= \int d^2 b \left\{ \exp \left[ -\sigma^{pN}_{el} T^h_A (b) \right] \right\} \left\{ \exp \left[ -\sigma^{pN}_{el} T^h_A (b) \right] \right\}$$

$$\approx \sigma^{pN}_{el} \int d^2 b T_A (b) \exp \left[ -\sigma^{pN}_{el} T^h_A (b) \right]. \hspace{1cm} (47)$$

Another possibility to excite the nucleus is to excite a bound nucleon, $pA \rightarrow pY$. To specify the terminology, following [32] we call this channel target single diffraction (tsd). Since the nucleus breaks up anyway, and the debris of the bound nucleon stay in the target fragmentation region, they cannot fill the rapidity gap, therefore their final state interaction do not affect the LRG cross section.
Then it must have the same form as the quasiparticle cross section Eq. (47), except the normalization factor,

\[
\sigma^{pA}_{tot}(pA \to pY) = \frac{\sigma^{pp}}{\sigma^{el}_{pp}} \sigma^{pA}_{qel}(pA \to pA^+) .\tag{48}
\]

We fix the single diffractive \( pp \) cross section at \( \sigma^{pp}_{sd} = 3.5 \text{ mb} \) for \( E_{lab} = 920 \text{ GeV} \) in accordance with [6]. For the energies of RHIC and LHC we extrapolate the Tevatron data [6] assuming no energy independence, \( \sigma^{pp}_{sd} = 4 \text{ mb} \).

The numerical results for the target dissociation cross sections are shown in Tables II and III.

V. COLOR TRANSPARENCY

The light-cone dipole representation was proposed in [8] as an effective tool for calculation of hadronic cross sections and nuclear shadowing, relying on the observations that color dipoles are the eigenstates of hadronic interactions at high energies, and the eigenstate method [33] is a powerful tool for summing up all Gribov inelastic corrections.

The key ingredient of this approach, the cross section of the dipole-nucleon interaction, \( \sigma_{qq}(r_T) \), is an universal and flavor independent function which depends only on transverse separation \( r_T \) and energy. Applications of the dipole formalism to nuclei are especially simple, if the energy is sufficiently high to freeze the fluctuations of the dipole size during its propagation through the nucleus. Otherwise one should rely on the light-cone Green function technique [34–36], which takes care of these fluctuations.

Due to color screening colorless point-like dipoles cannot interact with an external color field. Since the underlying theory is non-abelian, the interaction cross section for such dipoles vanishes at \( r_T \to 0 \) as \( \sigma_{qq}(r_T) \propto r_T^{-2} \). [8], the phenomenon called color transparency [52]. At high energies nuclei are transparent for small-size fluctuations of the incoming hadron, therefore the exponential attenuation suggested by the eikonal Glauber formula cannot be correct and should underestimate the nuclear transparency, i.e. overestimate the total hadron-nucleus cross section.

Thus the Glauber approach is subject to modifications called Gribov’s corrections. Originally those corrections were proposed in hadronic representation where they look like intermediate diffractive excitations [37]. The lowest order correction is expressed via the single diffraction cross section \( \sigma^{hN}_{sd} (hN \to XN) \) measured experimentally [37, 38],

\[
\Delta \sigma^{hA}_{tot} = -4\pi \int d^2b \exp \left[ -\frac{1}{2} \frac{\sigma^{hN}_{tot}}{\sigma^{tot}_{tot}} T_A(b) \right] \times \int \frac{dM^2}{M^2_-} \frac{d\sigma^{hN}}{dM^2_- dp_T^2} \bigg|_{p_T=0} \int d\xi_1 \rho_A(b, \xi_1) \times \int d\xi_2 \rho_A(b, \xi_1) e^{i\xi_L(z_2-z_1)} , \tag{49}
\]

where

\[
q_L = \frac{M^2 - m_h^2}{2E_h} . \tag{50}
\]

Therefore, one might think that this is a model-independent calculation. However, the cross section of interaction of the intermediate inelastic state \( X \) is unknown and is assumed to be equal to \( \sigma^{hN}_{tot} \).

Formally, all the observables calculated either in color-dipole, or hadronic representations must be identical. Nevertheless, although the correction Eq. (49) is negative, i.e. it makes the nuclear matter more transparent, it cannot reproduce the effect of color transparency [39] which results from compensation of many diagonal (positive) and off-diagonal (negative) amplitudes. Indeed, for heavy nuclei this correction vanishes exponentially with nuclear thickness \( T_A(b) \).

The dipole representation allows to sum up the inelastic corrections to all orders [8]. For a dipole of a fixed size \( r_T \) the eikonal form is exact, since the dipole is the eigenstate of interaction. Therefore nuclear transparency, which is the non-interaction probability of propagation of a \( qq \) dipole fixed separation \( r_T \) through nuclear medium, reads,

\[
T_r(r_T) = e^{-\sigma_{qq}(r_T)T_A} . \tag{51}
\]

In averaging over dipole sizes important contribution comes only from small \( \sigma_{qq}(r_T) \lesssim 1/T_A \). Therefore, for a sufficiently long path in the nuclear medium only very small values of \( r_T \) contribute, and any model for the dipole cross section must have the same behavior \( \sigma_{qq}(r_T) \propto r_T^{-2} \). Then [8],

\[
T_r = \left( e^{-\sigma_{qq}(r_T)T_A} \right) \propto \frac{1}{T_A} . \tag{52}
\]

This result should be compared with the exponential attenuation in the Glauber model, \( T_r = \exp(-\sigma^{pN}_{tot} T_A) \). Such a difference cannot result from the lowest order inelastic correction Eq. (49) which has the same exponential dependence on \( T_A \). This is a full color transparency effect which must include all the higher order inelastic corrections. It is characterized by the nuclear saturation scale \( Q^2_s \propto T_A \) and can be treated perturbatively for sufficiently heavy nuclei or at very high energies. In reality, such large nuclear thicknesses are beyond the reach
of existing nuclei. For this reason, one should not rely on the limiting behavior Eq. (52), but employ realistic models for the dipole cross section. In what follows we demonstrate how important is this fact.

VI. \( pA \) COLLISIONS: THE TOTAL, ELASTIC AND SINGLE DIFFRACTIVE CROSS SECTIONS

A. Excitation of the valence quark skeleton of the proton

Once models for the proton wave function and the dipole cross section are chosen one is in a position to perform calculations for Gribov’s corrections to different nuclear reactions, in particular LRG processes. The total and elastic cross sections read,

\[
\sigma_{\text{tot}}^{pA} = 2 \int d^2b \left[ 1 - \left< e^{-\frac{1}{2} \sigma_{qq}(r_i)} T_A^b(b) \right> \right] \\
\sigma_{\text{el}}^{pA} = \int d^2b \left[ 1 - \left< e^{-\frac{1}{2} \sigma_{qq}(r_i)} T_A^b(b) \right> \right]^2
\]

Single diffractive excitation of the projectile proton, \( pA \rightarrowXA \) cannot be treated properly within the single-channel Glauber model approximation which assumes that the projectile hadron is an eigenstate of the interaction. To generalize the model one should introduce off-diagonal diffractive amplitudes, but then one faces the same problem as in the case of higher order Gribov’s corrections: lack of experimental information for those amplitudes. And the remedy is the same, to switch to the dipole representation.

The cross section of single diffraction on a nucleus related to excitation of the valence quark skeleton, \( \sigma_{sd}^{pA} \), \( pA \rightarrow p + R + l \), is given by Eq. (13), but with the replacement, \( \sigma_{qq} \Rightarrow \sigma_{qq} = \int d^2b \left\{ 1 - \exp\left[ -\frac{1}{2} \sigma_{qq} T_A(b) \right] \right\} \).

\[
\frac{d \sigma_{sd}(pA 
X A)}{dM_X^2} = \int d^2b \left[ e^{-\sigma_{qq}(r_i)} T_A^b(b) \right] \\
- \left< e^{-\frac{1}{2} \sigma_{qq}(r_i)} T_A^b(b) \right>^2
\]

Both terms in this expression are vanishingly small for heavy nuclei, except at the very periphery. Therefore, the cross section of single diffraction is expected to rise as \( A^{1/3} \), but the coefficient should be sensitive to the inelastic shadowing corrections.

Although comparison with single diffraction performed in previous sections for four different variants of the dipole model clearly demonstrated that only the last two, which employ the saturated dipole cross section, may be realistic, we will try all four cases for nuclear reactions to get an idea of the theoretical uncertainties.

The inelastic corrections to \( \sigma_{\text{tot}}^{pA} \) have been well detected experimentally [40, 41] and found to be rather small, about 10\%. However, one should not think that inelastic shadowing is always a weak effect. In fact, for heavy nuclei it affects dramatically the exponential term in (40), but the term itself is very small compared to one, since the unitarity limit is almost saturated. However, LRG channels (except elastic scattering), e.g. the single diffraction Eq. (55), may be very sensitive to these corrections, since their cross section is proportional to nuclear transparency.

It was demonstrated in Sect. V that for hadrons heavy nuclei are much more transparent than the Glauber model predicts, due to color transparency and the presence of small-size dipoles in the hadronic wave function. As a result, the exponential attenuation switches to an inverse linear dependence on \( T_A \), Eq. (52). This is, however, an asymptotic behavior valid in the limit of \( T_A^b(b) \rightarrow \infty \), since for real nuclei the result depends on the actual modeling of \( \Psi_N(r_i) \) and \( \sigma_{qq}(r_T) \).

Model I

Here we employ the quark-diquark model Eq. (8) for the proton wave function and the dipole cross section \( \sigma_{qq}(r_T) \propto r_T^2 \). Then the averaged exponential terms in (53)-(55) read,

\[
\left< \exp\left[ \frac{1}{2} \sigma_{qq}(r_T) T_A^b(b) \right] \right> = \frac{1}{1 + \frac{1}{2} \sigma_{\text{tot}}^{NN} T_A^b(b)}
\]

The results of computing the total and elastic \( pA \) cross sections are compared with the Glauber model in Table I. The effect of inelastic shadowing is rather large, in fact the total cross section is reduced by about 20\%. Although no data are available at this energy, extrapolation from lower energies [40] is hardly compatible with such a correction. Apparently this is closely related to the found overestimation of diffraction by this model of the dipole cross section.

Model II

Since the probability to be found in a point like configuration is less for three- than two-body system, one should expect more opaque nuclei in this variant. Indeed,

\[
\left< \exp\left[ \frac{1}{2} \sigma_{qq}(r_i) T_A^h(b) \right] \right> = \int d^2r_i |\Psi_N(r_1, r_2, r_3)|^2 e^{-\frac{1}{2} \sigma_{qq}(r_1, r_2, r_3) T_A^h(b)} \\
= \frac{1}{1 + \frac{1}{2} \sigma_{\text{tot}}^{NN} T_A^h(b)}
\]

In this case the nuclear transparency is quadratic, rather than linear function of the inverse nuclear thickness [16].
The results of the calculation, depicted in Table I, show that the inelastic correction is about 10% of the total cross section for heavy nuclei. This looks much better than for the previous model, and is compatible with what may be expected as an extrapolation of data at lower energies. However, it is too early to jump to conclusions: the triple-Pomeron part of shadowing has not been included yet.

Naturally, the elastic cross section follows the total cross section and is reduced by the inelastic corrections as well. As we mentioned, it may be at variance with other LRG channels which one may expect to be enhanced, if nuclei are more transparent. However, further calculations show that the situation is more complicated.

**Model III**

The steep rise with $r_T$ of the dipole cross section, $\sigma_{dip}(r_T) \propto r_T^2$, used above is justified only for small, but not for large $r_T$. This is why it overestimates diffraction. More reliable calculations can be done using a more realistic phenomenological cross section which levels off at large $r_T$, as described in Sect. II A.

We can perform analytic calculations with the saturated cross section. Then the mean value of the exponential terms in Eqs. (53)-(55) for the total cross section gets the form,

$$
\left\langle \exp \left[ -\frac{1}{2} \sigma_{dip}(r_T) T_A^h(b) \right] \right\rangle_{III} = e^{-\frac{1}{2} \sigma_0(s) T_A^h(b) \sum \frac{n!}{2^n (1 + n \delta) n!} \sigma_0(s) T_A^h(b)^n},
$$

(58)

where $\sigma_0(s)$ and $\delta$ are defined in (19).

As could be expected, the numerical results shown in Table I demonstrate a weaker effect of inelastic shadowing compared to the previous variants.

**Model IV**

This case involves the symmetric 3q-wave function of the proton and the dipole cross section. Expanding again the Glauber exponential and performing part of the integrations analytically, we arrive at the following result,

$$
\left\langle \exp \left[ -\frac{1}{2} \sigma_{dip}(r_T) T_A^h(b) \right] \right\rangle_{IV} = \frac{3}{4} e^{-\frac{3}{2} \sigma_0(s) T_A^h(b)} \times \int_0^\infty d\xi \sum_{n=0}^\infty e^{-\frac{3}{4} \frac{\sigma_0(s) T_A^h(b)^n}{(1 + n \gamma) n!}} \gamma^3,
$$

(59)

where $\gamma$ is defined in (25).

As far as the Glauber exponentials are averaged over the eigenstates of interaction, for four different models, Eqs. (56)-(59), we are in a position to calculate the total, elastic and inelastic $pA$ cross sections given by Eqs. (53), (54) and (55) respectively.

The numerical results for four nuclei and energy $\sqrt{s} = 41.6$ GeV of the experiment HERA-B are displayed in Table I in parentheses for four different combinations of the models for the valence quark wave function of the proton and the dipole cross section.

| Obs. | Nucl. | Glauber model | Model I | Model II | Model III | Model IV |
|------|-------|---------------|---------|----------|-----------|----------|
|      |       |               |         |          |           |          |
| W    | 3073  | (2462)        | (2727)  | (2908)   | (3000)    |          |
| Ti   | 1159  | (938)         | (1028)  | (1103)   | (1130)    |          |
| Al   | 726   | (594)         | (647)   | (692)    | (707)     |          |
| C    | 380   | (349)         | (372)   | (363)    | (369)     |          |
|      |       |               |         |          |           |          |
| W    | 1196  | (748)         | (931)   | (1061)   | (1137)    |          |
| Ti   | 352   | (217)         | (268)   | (313)    | (331)     |          |
| Al   | 196   | (122)         | (149)   | (174)    | (183)     |          |
| C    | 79.1  | (51.6)        | (61.5)  | (70.5)   | (73.3)    |          |

Our predictions for $p - Au$ collisions at RHIC and for
$p-Pb$ collisions at LHC are depicted in Tables II and III respectively.

As one could have expected, the nonrealistic models I and II, which grossly overestimate single diffraction, also overestimate the inelastic corrections. On the other hand, the more realistic models III and IV lead to quite moderate deviation from the Glauber model expectations. However, this is not the end of the story, since the inelastic shadowing related gluonic excitations is still to be included.

Note that the result for single diffraction in Tables I, II, III demonstrate much higher sensitivity to inelastic corrections than for elastic scattering. This is easy to understand, in the black disc limit the elastic cross section reaches its maximum, $\pi R_A^2$, and cannot be varied by any corrections. At the same time, diffraction vanishes, therefore is extremely sensitive to the transparency of the nucleus, it is maximal for model I, but minimal for IV.

**B. Diffractive excitation of the gluonic degrees of freedom**

The higher Fock components of the proton wave function contain more partons, gluons, sea quarks. Apparently, the more degrees of freedom, the larger are the Gribov corrections. In particular, diffractive gluon radiation, corresponding to the triple-Pomeron term, makes nuclear matter more transparent, i.e. leads to a further reduction of the total $pA$ cross section. These corrections will be taken into account in Sect. IX.

Diffractive gluon radiation also contributes to the single diffractive process $pA \rightarrow XA$ and creates the triple-Pomeron tail $1/M^2$ in the effective mass distribution. Correspondingly, the single-diffraction cross section Eq. (55) must be corrected for this excitation channel. The cross section of coherent gluon radiation on a nucleus is given by a straightforward generalization of Eq. (28),

$$
\left[ \frac{d\sigma(pA \rightarrow XA)}{dx_F} \right]_{pA} = \frac{3}{4\pi(1-x_F)} \int d^2b \left( e^{-\frac{1}{2} \sigma_{sd}(r_1,s) T_A(b)} \right)^2 \times \int d^2r_T \left| \Psi_G(r_T) \right|^2 \left[ 1 - e^{-\frac{1}{2} \tilde{\sigma}(r_T,s) T_A(b)} \right]$, \quad (60)
$$

with the same notations. Here the first factor implies the absorptive corrections, analogous to those in (31). They are, however, much stronger than in $pp$ collisions and almost terminate diffraction on heavy nuclei, except at the very periphery. This factor is averaged over the proton size weighted with its wave function squared, as in (14). Therefore it depends on the models for the proton wave function considered above. Since we employ the phenomenological cross section fitted to data, the possibility of gluon radiation during propagation of the proton though the nucleus is included.

The integral over $r_T$ in (60) takes into account the diffractive channels containing gluons coherently radiated by a valence quark propagating through a nucleus of thickness $T_A(b)$. We make use of the smallness of the mean quark-gluon separation and add up incoherently the gluons radiated by different projectile valence quarks. Besides, the two models for the dipole cross section under discussion are almost identical at such small separations, $r_0 \sim 0.3$ fm.

**TABLE II: Predictions for RHIC for different LRG channels.** All the cross sections are for proton-gold collisions and are in mb. The cross sections shown in parentheses are corrected for inelastic shadowing related only to valence quark fluctuations, while the bottom numbers are also corrected for gluon shadowing. The cross sections which turned out to be smaller than the numerical accuracy of the calculations are put equal to zero.

| Model | $\sigma_{tot}$ | $\sigma_{el}$ | $\sigma_{sd}$ | $\sigma_{ qq}$ | $\sigma_{qG}$ | $\sigma_{qsd}$ | $\sigma_{sd}$ | $\sigma_{dG}$ |
|-------|----------------|----------------|-------------|-------------|-------------|-------------|-------------|-------------|
| Glauber | 3616.8 | 1446.8 | - | 5.1 | 98.6 | - | 42.3 | - |
| III | (3524.4) | (1367.6) | (31.8) | (7.3) | (95.8) | (3.1) | (41.2) | (3.1) |
| | 3457.5 | 1313.8 | 33.3 | 7.6 | 96.2 | 3.1 | 41.4 | 3.1 |
| IV | (3582.0) | (1417.7) | (8.1) | (5.8) | (98.4) | (0.6) | (42.3) | (0.6) |
| | 3514.1 | 1362.3 | 8.9 | 6.3 | 98.9 | 0.6 | 42.53 | 0.6 |

Therefore for the sake of simplicity we use the simple quadratic form, $\sigma_{qq} \propto r_T^2$. The accuracy of this approximation is greatly enhanced by the cut off imposed on large separations by the first exponential factor in
Table III: The same as in Table II, but for proton-lead collisions at LHC.

| Model | $\sigma_{tot}^{Pb}$ | $\sigma_{el}^{Pb}$ | $\left[ \sigma_{sd}^{Pb} \right]_{G^{Pb}_{F}}$ | $\left[ \sigma_{sd}^{Pb} \right]_{G^{Pb}_{el}}$ | $\sigma_{qsd}^{Pb}$ | $\sigma_{sd}^{Pb}$ | $\sigma_{sd}^{Pb}$ |
|-------|---------------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| Glauber | 4241.5 | 1794.9 | - | 28.8 | 141.43 | - | 22.9 |
| III | (4222.9) | (1778.7) | (5.5) | 31.8 | (142.1) | (0.0) | (23.0) | (0.0) |
| | 4194.2 | 1755.6 | 5.8 | 33.4 | 141.8 | 0.0 | 23.0 | 0.0 |
| IV | (4235.2) | (1789.9) | (0.8) | 29.5 | (142.8) | (0.0) | (23.1) | (0.0) |
| | 4207.1 | 1767.3 | 0.9 | 31.2 | 142.5 | 0.0 | 23.1 | 0.0 |

Eq. (60), which gets the form,

$$\left[ \frac{d\sigma(pA \rightarrow XA)}{dx_{F}} \right]_{G^{Pb}_{el}} = \frac{\alpha_{s}}{2\pi^{2}(1-x_{F})} \int q^{2}b$$

$$\times \left[ e^{-\frac{b}{2}\sigma_{sd}(r_{i})}T_{A}^{1}(b) \right]^{2} \ln \left[ 1 + \frac{e_{A}^{2}(b,s)}{1 + 2\epsilon_{A}(b,s)} \right],$$ (61)

where

$$\epsilon_{A}(b,s) = \frac{9q_{s}^{2}}{16R_{0}^{2}(s)}K(s)\sigma_{0}(s)T_{A}(b).$$ (62)

The absorptive corrections, given in (61) by the exponential factor, are factorized from the coherent diffractive gluon radiation, given by the logarithmic factor. For heavy nuclei these absorptive corrections are much stronger than for $pp$ collisions (see Sect. III D). They practically eliminate diffraction on nuclear targets except the very periphery. Therefore, the cross section Eq. (62) varies as $A^{1/3}$.

Since the main contribution comes from peripheral collisions where the projectile proton finds very few nucleons, the absorptive corrections for $NN$ scattering considered earlier in Sect. III D may be important and have to be included into (62). This is done in the same way as in Sect. III D, namely using the relation Eq. (31) which results in the suppression factor $K(s)$ in Eq. (62),

$$K(s) = 1 - \frac{1}{4\pi} \frac{\sigma_{tot}^{Pb}(s)}{B_{cl}^{Pb}(s) + B_{3g}^{Pb}}.$$ (63)

Note that although this factor decreases with energy, it always remains positive. Indeed, $K > 0$ even in the Froissart limit, where [42] $\sigma_{tot} = 4\pi\alpha_{F}'\epsilon_{n} \ln^{2}(s/M_{0}^{2})$ and $B_{cl} = \sigma_{F}'\epsilon_{n} \ln^{2}(s/M_{0}^{2})$, where $\epsilon = \epsilon_{F}'(0) - 1 \approx 0.08$. The triple Pomeron slope $B_{3g}^{Pb}(s)$ depends very slowly with energy, as a double logarithm. It rises only by 10% between the energies of HERA-B and LHC.

Only the absorptive correction factor in (62) depends on the model for the proton wave function and on the shape of the dipole cross section. It can be evaluated either within the Glauber approximation, or including the inelastic shadowing corrections calculated in accordance with either of the four models considered above. We fix $\alpha_{s} = 0.4$ as was explained in Sect. III D and integrate over $x_{F}$ from 0.9 up to 1 $- M_{0}^{2}/s$. The results are depicted in Table I for the energy of HERA-B, and in Tables II and III for the energies of RHIC and LHC respectively.

The coherent triple-Pomeron diffraction on nuclei at the energy of HERA-B turns out to be amazingly small. This can be understood as follows. At this energy the value of $\epsilon(s)$ in (30) is sufficiently small to expand the logarithm up to the first order of $\epsilon^{2}$. The same is true for Eq. (62) provided that the absorptive corrections eliminate the contribution of large $T_{A}(b)$. Then, comparing the cross section Eq. (30) integrated over $p_{F}$, and the one presented in Eq. (62), we see that they are related via the replacement,

$$\frac{1}{B_{3g}^{Pb}} \Rightarrow A\langle T_{A} \rangle,$$ (64)

where the mean nuclear thickness for this process is given by

$$\langle T_{A} \rangle = \frac{1}{A} \int d^{2}b \left[ e^{-\frac{b}{2}\sigma_{sd}(r_{i})}T_{A}^{1}(b) \right]^{2} T_{A}^{2}(b).$$ (65)

This can be estimated as,

$$\langle T_{A} \rangle \approx \frac{2\pi w R_{A}}{A(\sigma_{sd}^{Pb})^{2}},$$ (66)

where $R_{A} \approx 1.12 \text{ fm} \times A^{1/3}$ and $w \sim 0.5 \text{ fm}$ are the nuclear radius and edge thickness for the Woods-Saxon nuclear density [30]. Thus, the replacement Eq. (64) leads to a reduction of the cross section by about a factor of five for heavy nuclei, $A \sim 200$, and much more for light nuclei. This explains the smallness of the coherent diffractive gluon radiation by extremely strong absorptive corrections. In the black disk limit with vanishing edge thickness ($w \rightarrow 0$) no diffractive gluon radiation is possible.
VII. QUASIELASTIC SCATTERING/EXCITATION OF THE PROJECTILE, pA → p(X)A

The simplest channel of nuclear excitation is a quasielastic breakup of the nucleus, caused by elastic scattering, pN → pN, of the projectile proton on one of the bound nucleons. In this case the nucleus breaks up into fragments without particle production. It is difficult to control this condition experimentally, but is easy to calculate it.

One should modify Eq. (46), first sum the nuclear excitations A* and integrate over impact parameter s, then average over the quark positions \( \vec{r}_i \) and \( \vec{r}_j \) in each of the two amplitudes,

\[
\sigma_{qel}(pA \rightarrow pA^*) = \int d^2b \left\{ \sum_F \left[ \langle 0 | \Gamma^A(b, \vec{r}_i) | F \rangle^\dagger \langle F | \Gamma^A(b, \vec{r}_j) | 0 \rangle \right]_{s,r_i,r_j} \right\}
\]

\[
= \int d^2b \left\{ \langle 0 | \Gamma^A(b, \vec{r}_i) \rangle^\dagger \Gamma^A(b, \vec{r}_j) \langle 0 \rangle_{s,r_i,r_j} \right\}
\]

\[
= \int d^2b \left\{ \langle 0 | \Gamma^A(b, \vec{r}_i) \rangle^\dagger \Gamma^A(b, \vec{r}_j) \langle 0 \rangle_{s,r_i,r_j} \right\}. \tag{67}
\]

Here s is the impact parameter of the projectile proton relative to the bound nucleons, and \( \vec{r}_i \) are the relative transverse positions of the valence quarks in the projectile proton. After integration over impact parameter s [see in (46)] we get,

\[
\sigma_{qel} = \int d^2b \left\{ \langle 0 | \Gamma^A(b, \vec{r}_i) \rangle^\dagger \Gamma^A(b, \vec{r}_j) \langle 0 \rangle_{s,r_i,r_j} \right\}
\]

\[
= \int d^2b \left\{ \langle 0 | \Gamma^A(b, \vec{r}_i) \rangle^\dagger \Gamma^A(b, \vec{r}_j) \langle 0 \rangle_{s,r_i,r_j} \right\}
\]

\[
= \int d^2b \left\{ \langle 0 | \Gamma^A(b, \vec{r}_i) \rangle^\dagger \Gamma^A(b, \vec{r}_j) \langle 0 \rangle_{s,r_i,r_j} \right\}. \tag{68}
\]

This series quickly converges due to smallness of the elastic cross section. Even the first term provides a reasonable accuracy. We control the accuracy to be within 1%.

If we sum over all excitations of the proton containing no radiated gluons and apply the condition of completeness, a delta-function \( \delta(\vec{r}_i - \vec{r}_j) \) emerges, leading to the following expression for the sum of quasielastic and quasi-single-diffractive channels,

\[
\sigma_{qel}(pA \rightarrow pA^*) + \sigma_{qsd}(pA \rightarrow XA^*)
\]

\[
= \int d^2b \left\{ \langle 0 | \Gamma^A(b, \vec{r}_i) \rangle^\dagger \Gamma^A(b, \vec{r}_j) \langle 0 \rangle_{s,r_i,r_j} \right\}
\]

\[
= \int d^2b \left\{ \langle 0 | \Gamma^A(b, \vec{r}_i) \rangle^\dagger \Gamma^A(b, \vec{r}_j) \langle 0 \rangle_{s,r_i,r_j} \right\}
\]

\[
= \int d^2b \left\{ \langle 0 | \Gamma^A(b, \vec{r}_i) \rangle^\dagger \Gamma^A(b, \vec{r}_j) \langle 0 \rangle_{s,r_i,r_j} \right\}. \tag{69}
\]

With these expressions, Eqs. (68)-(69), we can calculate the quasielastic and quasi-diffractive cross sections employing different models for the dipole cross section and the proton wave function.

Model I

In this simplest case of the meson type wave function of the proton and \( \sigma(\vec{r}_i) \equiv \sigma_{qg}(\vec{r}_p) \propto r_i^2 \), the quasielastic and quasi-diffractive cross sections read respectively,

\[
\sigma_{qel} = \sum_{k=1} k! \int d^2b \frac{\sigma_{el}^{pN} T_A^k(b)}{[1 + \frac{1}{2} \sigma_{tot}^A T_A^k(b)]^{2k+2}}. \tag{70}
\]

\[
\sigma_{qel} + \sigma_{qsd} = \sum_{k=1} (2k)! \int d^2b \frac{\sigma_{cl}^{pN} T_A^k(b)}{[1 + \frac{1}{2} \sigma_{tot}^A T_A^k(b)]^{2k+1}}. \tag{71}
\]

Model II

In the case of a symmetric valence quark wave function and dipole cross section \( \sigma(\vec{r}_i) \equiv \sigma_{3q}(\vec{r}_i) \) the quasielastic cross section takes the form,

\[
\sigma_{qel} = \sum_{k=1} \frac{(k+1)(k+1)!}{2^{2k}} \int d^2b \frac{\sigma_{el}^{pN} T_A^k(b)}{[1 + \frac{1}{2} \sigma_{tot}^A T_A^k(b)]^{2k+4}}. \tag{72}
\]

This result may look surprising. Indeed, the quasielastic cross section Eq. (47) is proportional to nuclear transparency, i.e. the survival probability of a proton propagating through the nucleus. That is given by the mean value Eq. (57) squared, i.e. the fourth power of \( T_A \) in the denominator. That would mean more nuclear transparency and larger quasielastic cross section compared to the Glauber model. Eq. (72), however, has the leading term which behaves as the inverse sixth power of \( T_A \). Although at large \( T_A \) the exponential Glauber transparency is less than any power of \( T_A \), it turns out that for real nuclei inelastic shadowing makes nuclei less transparent, at variance with the simplified expectation. The reason why the nucleus is less transparent than is suggested by Eq. (57) is an extra weight factor \( \sigma_{qg}(r) \) in the quasielastic amplitude. This factor suppresses small-size projectile components for which the nucleus is transparent.
Using completeness we can also calculate the sum of the cross sections of quasielastic and quasidiffractive scattering, the result reads,

\[
\sigma_{qel}^A + \sigma_{qsd}^A = \sum_{k=1}^{\infty} \frac{(2k+1)!}{2^{2k} k!} \int d^2r \frac{[\sigma_{el}^N T_A^N(b)]^k}{[1 + \frac{1}{2} \sigma_{tsd}^N T_A^N(b)]^{2k+2}}.
\]

(73)

In this case the leading term behaves like \(T_A^{-3}\) at large \(T_A\), i.e. very heavy nuclei are much more transparent for quasidiffractive, than quasielastic processes.

**Models III and IV**

We skip the cumbersome expressions for the quasielastic and quasi-diffractive cross sections in this case. We use instead equations (68) and (69) respectively and perform calculations numerically. For the averaged Glauber exponential \(\exp \left[ -\frac{1}{2} \sigma_r(\vec{r}) T_A(b) \right] \), we employ Eqs. (58) and (59) for models III and IV respectively.

So far we calculated the quasi-diffractive cross section related to proton excitations without gluon radiation. Now we should include diffractive gluon bremsstrahlung, as it is done for coherent diffraction in Sect. VI B. This can be done via a simple replacement in the above equations (68)-(73),

\[
\sigma_{qsd}^A \Rightarrow \frac{\sigma_{gpb}^P}{\sigma_{qsdp}^P} \sigma_{qsd}^A.
\]

(74)

The results for \(\sigma_{qel}^A\) and \(\sigma_{qsd}^A\) are depicted in parentheses in Table IV for different nuclei at the energy of HERA-B, and in Tables II and III for gold at RHIC and lead at LHC respectively. In both models III and IV the quasi-diffractive and double-diffractive cross sections are very small due to very low nuclear transparency close to the black disc limit, so small at the energy of LHC that in model IV we could not reach a sufficient numerical accuracy. Therefore these cross sections are set to zero in Table III.

One can notice a much higher sensitivity to the models for quasidiffractive scattering compared to quasielastic. In the former case this simply reflects the tremendous model dependence of single diffraction, pointed out in Sect. III. In the latter case there is a partial compensation between the steep rise of the dipole cross section with the dipole size \((x r_A^4)\) at small \(r_T\) and the exponential fall of nuclear transparency.

**VIII. DIFFRACTIVE EXCITATION OF BOUND NUCLEONS**

Another LRG process, not explored so far, is diffractive excitations of bound nucleons in the target. It may happen along with or without excitation of the beam. We call these cases target single diffraction, or double diffraction respectively.

Since the phenomenological dipole cross is averaged over the target proton wave function, it is insufficient for these reactions which need knowledge of a double-dipole cross section. If, however, one assumes that in \(pp\) collisions the dependence on the beam dipole size is the same for reactions with or without target excitation, then

**TABLE IV:** Same as in table I, but involving diffractive break up of the nucleus.

| Nucl. | Glauber model | Model I | Model II | Model III | Model IV |
|-------|---------------|---------|----------|-----------|----------|
| W     | 88.2 (59.1)   | (45.9)  | (73.0)   | (77.3)    |          |
| Ti    | 63.9 (42.5)   | (35.3)  | (53.2)   | (56.0)    |          |
| Al    | 48.8 (32.8)   | (30.0)  | (40.7)   | (42.7)    |          |
| C     | 34.5 (23.2)   | (20.5)  | (28.6)   | (29.7)    |          |
|       |               |         |          |           |          |
| W     | 42.1 (28.2)   | (21.9)  | (34.8)   | (36.9)    |          |
| Ti    | 30.5 (20.3)   | (16.9)  | (25.4)   | (26.7)    |          |
| Al    | 23.3 (15.7)   | (13.3)  | (19.4)   | (20.4)    |          |
| C     | 16.5 (11.0)   | (9.8)   | (13.6)   | (14.2)    |          |
|       |               |         |          |           |          |
| W     | 15.0 (8.1)    | (2.2)   | (2.2)    | (0.9)     |          |
| Ti    | 11.2 (6.7)    | (1.6)   | (1.6)    | (0.7)     |          |
| Al    | 8.9 (5.5)     | (1.3)   | (1.3)    | (0.5)     |          |
| C     | 6.5 (4.3)     | (1.0)   | (1.0)    | (0.4)     |          |
one can relate the cross sections in question to known quasielastic and quasidiffractive ones,

\[ \sigma_{fsd}^{AA}(pA \rightarrow pY) = \sum_{\sigma_{el}} \sigma_{qel}^{AA}(pA \rightarrow pA^*) ; \quad (75) \]

\[ \sigma_{sd}^{AA}(pA \rightarrow XY) = \sum_{\sigma_{el}} \sigma_{qel}^{AA}(pA \rightarrow XA^*) ; \quad (76) \]

Numerical results are depicted in Table IV for HERA-B and in Tables II and III for RHIC and LHC respectively.

**IX. GLUON SHADOWING AND THE TRIPLE-POMERON DIFFRACTION**

Eikonalization of the lowest Fock state \(|3q\rangle\) of the proton done in (53) corresponds to the Bethe-Heitler regime of gluon radiation. Indeed, gluon bremsstrahlung is responsible for the rising energy dependence of the phenomenological cross section (3), and in the eikonal form (36) one assumes that the whole spectrum of gluons is multiply radiated. However, the Landau-Pomeranchuk-Migdal (LPM) effect [43, 44] is known to suppress radiation in multiple interactions. Since the main part of the inelastic cross section at high energies is related to gluon radiation, the LPM effect becomes a suppression of the cross section. This is a quantum-mechanical interference phenomenon and it is a part of the suppression called Gribov’s inelastic shadowing. The way how it is taken into account in the QCD dipole picture is by the inclusion of higher Fock states, \(|3qG\rangle\), etc. Each of these states represents a colorless dipole and its elastic amplitude on a nucleon is subject to eikonalization.

As we already mentioned, the eikonalization procedure requires the fluctuation lifetime to be much longer than the nuclear size. Otherwise, one has to take into account the “breathing” of the fluctuation during propagation through a nucleus, which can be done by applying the light-cone Green function technique [1, 34, 45]. In hadronic representation this is equivalent to saying that all the longitudinal momenta transfers must be much smaller than the mean free path of the hadron in the nucleus. Otherwise, the phase shifts and interferences are to be included [46].

The c.m. energies of HERA-B, RHIC and LHC are sufficiently high to treat the lowest Fock state containing only valence quarks as “frozen” by the Lorentz time dilation during propagation through the nucleus. Indeed, for the excitations with the typical nucleon resonance masses, the coherence length is sufficiently long compared to the nuclear size. This is why we applied eikonalization without hesitation so far. Such an approximation, however, never works for the higher Fock states containing gluons. Indeed, since the gluon is a vector, the integration over effective mass of the fluctuation is divergent, \(dM^2/M^2\), which is the standard triple-Pomeron behavior. Therefore, the colliding energy never can be sufficiently high to neglect the large-mass tail. For this reason the inelastic shadowing corrections related to excitation gluonic degrees of freedom and calculated in the tree-approximation never saturate, and keep rising logarithmically with energy.

There are, however non-linear effects which are expected to stop the rise of inelastic corrections at high energies. This is related to the phenomenon of gluon saturation [47, 48] or color glass condensate [49]. The strength of these nonlinear effect is model dependent. It is a rather mild effect within the model of small gluonic spots explained in Sect. III.C. The reason is simple, in spite of a sufficient longitudinal overlap of gluon clouds originated from different nucleons, there is insufficient overlap in the transverse plane. This fact leads to a delay of the onset of saturation up to very high energies, since the transverse radius squared of the gluonic clouds rise with energy very slowly, logarithmically, with a small coefficient of the order of 0.1 GeV\(^{-2}\).

The details of the calculation of inelastic corrections related to excitation of gluonic degrees of freedom can be found in [1, 16]. The results for nuclear cross sections corrected for gluon shadowing are shown in Tables I, II, III, and IV.

**X. CONCLUSIONS**

In this paper we study the dynamics of LRG processes in \(pp\) and \(pA\) collisions basing on the light-cone dipole approach which allows to develop phenomenology not only for hard, but also for soft reactions.

One of our objectives is an improvement of accuracy of calculations for nuclear effects in LRG processes. First of all, we tested some popular models for the dipole-proton cross section and the proton wave function on their ability to reproduce the cross section of diffractive excitation of the valence quark skeleton of the proton. Data show that the forward diffractive cross section is amazingly small, only about 10% of the elastic one. We conclude that the models based on the dipole cross section which quadratically depends on quark separation fail badly to explain even the order of magnitude of the single diffractive \(pp\) cross section. This model, however, is quite popular in the literature devoted to nuclear effects, in particular color transparency, since it helps to simplify the calculations. Apparently, those results cannot be realistic. On the other hand, we found the saturated shape of the dipole cross section which levels off at large separations to be quite successful in explaining the data on \(pp\) diffraction.

As for nuclear effects, the popular Glauber model cannot treat properly most of the off-diagonal diffractive processes, since this is a single-channel approximation. Based on the color-dipole representation we develop techniques for calculating the cross sections of LRG processes, both diagonal and off-diagonal. This method allows to sum the Gribov inelastic shadowing corrections to all orders. These corrections make nuclear matter more trans-
parent and reduce the total hadron-nucleus cross section. At the same time, their influence on other diffractive reactions depends on a complicated color transparency interplay making nuclei more transparent for small size hadronic fluctuations, but simultaneously suppressing the strength of the interaction with bound nucleons, due to the same effect. This is confirmed by our numerical results for the cross sections of a variety of channels presented in Tables I–IV. We found that models I and II based on the quadratic dependence of the cross section on the dipole size grossly overestimate the Gribov corrections compared to more realistic variants III and IV, based on the saturated form of the dipole cross section.

Available data for the Gribov corrections [40, 41] show that they rise with energy, what results from the falling energy dependence of the longitudinal momentum transfer vanishes, while the triple-Pomeron part keeps rising logarithmically. Comparing, however, the results depicted in Tables I and III we see that the inelastic corrections, i.e. the deviation from the Glauber model, vary from $5 - 7\%$ at the energy of HERA-B down to about $1 - 2\%$ at LHC. Such a dramatic reduction signals the closeness of the unitarity limit. Indeed, in this regime, also called black-disk limit, different Fock components of the proton interact with the same cross section. Therefore the incoming and outgoing waves consist of the same superposition of Fock states, then only elastic scattering is possible. Similar suppression of diffraction is expected for $pp$ scattering when it reaches the Froissart regime, $\sigma_{\text{tot}}^{pp}/\sigma_{\text{tot}}^{\text{sat}} \propto 1/\ln(s)$. However, the onset of this effect on nuclear targets can be observed at much lower energies.

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[50] According to [15] this dipole cross section reproduces well the energy dependence of the photoabsorption cross section $\sigma_{\gamma p}^{\text{tot}}(s)$. This happens only due to the singularity in the light-cone wave function of the photon at small $r_T$, which is a specific property of this wave function and is not applicable to hadrons.
[51] We ignore the effect of motion of the center of gravity assuming the nucleus to be sufficiently heavy.
[52] Actually, the cross section behaves as $r_T^2 \ln(r_T^2) \ [8]$, but with a good accuracy one can fix the logarithm at an effective separation typical for the process under consideration.