Self-consistent Solutions of Canonical Proper Self-gravitating Quantum Systems

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Abstract Generic self-gravitating quantum solutions that are not critically dependent on the specifics of microscopic interactions are presented. The solutions incorporate curvature effects, are consistent with the universality of gravity, and have appropriate correspondence with Newtonian gravitation. The results are consistent with known experimental results that indicate the maintenance of the quantum coherence of gravitating systems, as expected through the equivalence principle.

Keywords Quantum gravitation · Canonical gravitation

1 Introduction

The incorporation of quantum mechanics into gravitational dynamics remains a perplexing issue in modern physics. In contrast to other interactions like electromagnetism, the trajectory of a gravitating system is independent of the mass coupling to the gravitational field. Thus the gravitation of arbitrary test particles can be described in terms of local geometry only. However, the equations of general relativity are quite complex and non-linear in the interrelations between sources and geometry. This makes solutions of even classical systems somewhat complicated.

The behavior of quantum objects in Minkowski space-time is well described by standard quantum theory. The various interpretations (Copenhagen, many worlds, etc.) of the underlying fundamentals of the quantum world must all be consistent with the standard theory, which has yet to be contradicted by experiment. There have been experiments that examined the behaviors of quantum objects in gravitational fields. Gravitating quantum systems do maintain their coherence, demonstrating that...
the structure of the interaction need not break coherence in order to localize the system in the field. Moreover, those systems continue to gravitate after coherence is broken by detection, as well as themselves serve as source energy densities. Even highly dynamic gravitational environments such as the big bang can redshift cosmic microwave background radiation without breaking the coherence of the individual quanta. These behaviors are sensible using fundamental principles of relativity. Due to the principle of equivalence, the motions of detectors and screens should not break the coherence of inertial (freely falling) systems prior to detection. These basics will be briefly discussed in Sect. 2.

Often, a problem considerably simplifies if the parameters are properly chosen. In this treatment, space-like surfaces of simultaneity will be defined by fixed proper time \( \tau \), which are generally not coincident with space-like surfaces of simultaneity defined by coordinate time \( t \). The canonical proper time formulation is particularly useful for describing gravitational dynamics using proper time. This formulation will be discussed in Sect. 3.

Dynamics described using the proper time and convenient spatial coordinates will provide straightforward solutions for generic self-gravitating quantum systems. The description will be particularly robust with regards to arbitrary microscopic interactions that might contribute to generate the inertial masses. In fact, no form of microscopic interaction is mentioned or utilized in the discussion. The self-gravitating solutions will be developed and discussed in Sect. 4.

2 Quantum Mechanics and Gravity

Gravitation is an interaction of considerable mathematical subtlety, despite its familiarity. The geometrodynamics of classical general relativity is most directly expressed using localized geodesics. However, quantum dynamics incorporate measurement constraints that disallow complete localization of physical systems. The subtleties of observed gravitation of quantum systems should offer insight into the fundamentals of quantum self-gravitation, as will here be examined.

2.1 Coherence of Gravitating Systems

There is now considerable experimental evidence that quantum coherence is maintained by the nearly static gravitational field near Earth’s surface. During the early and mid 1970’s, experiments performed by Overhauser et al. [1, 2] examined the gravitation of coherent neutrons diffracting from an apparatus whose orientation could be changed relative to the Earth’s gravitational field. The gravitating neutrons were seen to maintain spatial coherence, exhibiting a pattern consistent with self-interference through two apertures at different gravitational potentials. A more recent experiment measured the small difference between the ticks of two interfering quantum clocks [3]. In that experiment, very cold cesium atoms gravitated vertically across a laser beam that superposed single atoms into states at differing gravitational potentials. The resultant difference in the phase demonstrated interference in the quantum oscillations associated with the relativistic energy of the atoms. Thus, gravitating atoms have also been shown to maintain temporal coherence.
The experimental results discussed imply that the quasi-static near-Earth gravitational field does not break the phase coherence of neutrons or atoms as needed for interference. The experiments also test the principle of equivalence, since motions of the observer do not break the coherence of the inertial (gravitating) particles. The experiments involve both Newton’s gravitational constant $G_N$ and Planck’s constant $\hbar$ in a single equation form.

3 Canonical Proper Time Dynamics

Experiments such as those discussed in the previous section provide illustrative examples of the usefulness of the canonical proper time formalism developed in references [4, 5] for describing gravitating systems. The canonical proper time formulation of relativistic dynamics provides a framework from which one can describe the dynamics of classical and quantum systems using the clocks of those very systems. The approach presumes that any gravitating quantum system maintains coherence on surfaces defined by its proper time. The various regions across a coherent state propagate through varying gravitational potentials, with space-like surfaces of simultaneity defined by fixed proper time $\tau$. The formulation utilizes a canonical transformation on the time variable conjugate to the Hamiltonian that is used to describe the dynamics, but does not transform other dynamical variables such as momenta or positions. This gives insight into the fundamentals of an interaction, since the response and back-response of the interacting system is best parameterized using its proper-scaled dynamics.

3.1 Proper Time Heisenberg Equations

For quantum systems, Heisenberg’s equations describe the dynamics of an observable $W(q, p, t)$ in terms of the commutator of that observable with the Hamiltonian:

$$\frac{d\hat{W}(x, p, t)}{dt} = \frac{i}{\hbar}\left[\hat{H}, \hat{W}(x, p, t)\right] + \left(\frac{\partial \hat{W}}{\partial t}\right)_{x, p}. \quad (3.1)$$

In special relativity, the inertial time $t$ is related to the proper time $\tau$ through the standard Lorentz factor $\gamma$ using

$$dt = \gamma d\tau = \frac{H}{Mc^2} d\tau. \quad (3.2)$$

The second form in (3.2) follows from the relationship between the energy of a system compared to its rest energy. The canonical proper energy form $K$ is defined to generate dynamic changes with regards to the proper time of that system:

$$\frac{d\hat{W}}{d\tau} = \frac{d\hat{W}}{dt} \frac{dt}{d\tau} \equiv \frac{i}{\hbar}\left[\hat{K}, \hat{W}\right] + \left(\frac{\partial \hat{W}}{\partial \tau}\right)_{x, p}. \quad (3.3)$$

From this equation, along with Heisenberg’s equation, it then follows that $[K, W] = \frac{H}{Mc^2}[H, W]$. 

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The canonical proper energy form $K$ is expected to correspond to the Hamiltonian when the Hamiltonian itself corresponds to the rest energy, $K|_{H=Mc^2} = H = Mc^2$. Holding the system mass $M$ fixed during the canonical “boost” from $H$ to $K$ results in the form

$$\hat{K}[H] = \frac{\hat{H}^2}{2Mc^2} + \frac{Mc^2}{2}. \quad (3.4)$$

As an example, a direct substitution of the non-interacting relativistic form $H_0 = \sqrt{(pc)^2 + (Mc^2)^2}$ into this equation yields

$$\hat{K}_o = \frac{p^2}{2M} + Mc^2. \quad (3.5)$$

In this case, both temporal parameters are inertial. A few points of interest should be noted:

- The form of the equation for $K_o$ is that of a non-relativistic free particle, despite the system being completely relativistic;
- The momentum in $K_o$ is the same canonical momentum of the particle in the Hamiltonian formulation. This is clearly not a Lorentz transformation of the dynamical parameters of the system;
- The sometimes troublesome square root does not appear in the expression for $K_o$.

Since the positions and momenta of a gravitating particle are typically described relative to fiducial observers, rather than the proper coordinates of the gravitating particle, this formulation is particularly useful for describing gravitational dynamics.

### 3.2 Canonical Proper Time Gravitation

The equations of motion generated using the canonical proper time formulation insures that the canonical proper energy is conserved ($\frac{dK}{d\tau} = 0$) if there is no explicit temporal dependence in the functional form of any interactions. For generic proper potential energy forms $U(r)$, the canonical proper energy can often be expressed

$$K = \frac{p \cdot p}{2m} + U(r) + mc^2. \quad (3.6)$$

A potential form consistent with standard gravitation will next be developed.

#### 3.2.1 Geodesic Motion

The potential energy is expected to take the form of Newtonian gravitation to lowest order in the gravitational constant $G_N$. However, space-time curvature effects are expected to modify the classical result. To construct the relativistic energy form, the equations of motion resulting from (3.6) with $U(r) = mV(r)$ will be examined:

$$\frac{dp_j}{d\tau} = -m\partial_j V(r), \quad \frac{dr_j}{d\tau} = \frac{p_j}{m}. \quad (3.7)$$

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For a constant mass, this form is analogous to the geodesic equation incorporating space-time curvature:
\[
\frac{d^2 x^j}{d\tau^2} + \Gamma_{\alpha\beta}^j \frac{dx^\alpha}{d\tau} \frac{dx^\beta}{d\tau} = 0. \tag{3.8}
\]
The present exploration is interested in the behaviors of stationary quantum gravitating systems. As is the case with electronic distributions in stable atoms, the mass distribution should be stationary in a quantum gravitating system. For a stationary gravitating distribution, assume that \( \frac{dx^\alpha}{d\tau} = \frac{dx^0}{d\tau} \delta^\alpha_0 \) (consistent with quantum expectation values). Therefore, substituting the form of the connections \( \Gamma_{\alpha\beta}^j \) for a metric space-time (Riemannian manifold) in the geodesic equation (3.8), the proper interaction form must satisfy
\[
\frac{d^2 x^j}{d\tau^2} = -\partial_j V(r) = \frac{1}{2} g^{j\mu} g_{0\mu} \left( \frac{dx^0}{d\tau} \right)^2. \tag{3.9}
\]
This form will be generated for a straightforward static energy density.

### 3.2.2 Form of the Metric

The space-time metric for a spherically symmetric, static space-time will be chosen to be a generalization of Schwarzschild geometry with non-vanishing local densities. The metric will be given by
\[
ds^2 = -\left(1 - \frac{R_M(r)}{r}\right)(dt)^2 + \frac{dr^2}{1 - \frac{R_M(r)}{r}} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2, \tag{3.10}
\]
where the Jacobian \( \sqrt{-\text{det} g} \equiv \sqrt{-g} = r^2 \sin \theta \). In this equation, a finite radial mass scale \( R_M(r) \equiv 2G N M(r)/c^2 \) is the length scale of the interior mass-energy content of the system, with the mixed Einstein tensor given by \( G^0_0 = \frac{1}{r^2} \frac{\partial}{\partial r} R_M(r) \). For finite mass distributions, the metric takes the form of Minkowski space-time both asymptotically \( (r \gg R_M) \) as well as wherever the radial mass scale vanishes. The Ricci scalar
\[
\mathcal{R} = -\frac{1}{r^3} \frac{d}{dr} \left( r^2 \frac{dR_M(r)}{dr} \right) \tag{3.11}
\]
for such distributions is non-singular as long as the mass density decreases rapidly enough for small \( r \).

The metric (3.10) is not the most general static spherically symmetric form, since in general the components \( g_{00} \) and \( g_{rr} \) need only have an inverse relationship in the exterior. However, since the state of the system will remain quantum coherent, considerations of macroscopic properties such as pressure that must be statistically derived will be minimal. The metric form has been chosen as the simplest generalization of the Schwarzschild metric giving a radial dependence to the mass. The formulation remains valid for more complicated metrics.
3.2.3 Proper Potential Form

Substituting the metric into (3.9), one should note that $g^{rr} = -g_{00}$ and $(\frac{dx_0}{d\tau})^2 = -\frac{c^2}{g_{00}}$, giving the equation

$$\partial_r V(r) = -\frac{c^2}{2} \partial_r (g_{00}). \quad (3.12)$$

Using the standard condition $V(\infty) = 0$, the form of the interaction for the proper canonical energy form is therefore given by

$$V(r) = -\frac{G_N M(r)}{r}. \quad (3.13)$$

This relativistic form is the same as the usual Newtonian interaction. This form for the potential will be more complicated for more general metrics, since it is incorporated in a non-linear manner in the geodesic equation.

4 Static Field Quantum Self-gravitation

The gravitational potential energy from (3.13) will next be incorporated in the quantum form of the canonical proper energy equation (3.6). The equation developed will include both the local dynamics of special relativity as well as the curvature effects of general relativity. Self-consistent solutions of this equation will be developed in this section.

4.1 Proper Time Quantum Gravitating Particles

Consider the stationary gravitation of a mass $m$ due to an interior source mass distribution $M(r)$. An invariant probability form measuring the likelihood that the particle will be measured by an observer in the space-time interval $\Delta ct \Delta \mathcal{V}$ is expected to take the form

$$P_{\Delta ct \Delta \mathcal{V}} = \int_{\Delta ct \Delta \mathcal{V}} d\tau d^3r \sqrt{-g} |\psi(\tau, r)|^2. \quad (4.1)$$

General quantum systems will be temporally dynamic. However, stationary state probability densities are not expected to have time dependencies. The wave function that satisfies the stationary state canonical proper energy equation for this mass, and represents the likelihood for measurement within the time interval $\Delta ct$, is given by

$$\left[\frac{\hat{p} \cdot \hat{p}}{2m} - \frac{G_N m M_{\ell}(r)}{r} + mc^2\right] \psi_{n\ell\zeta}^{\Delta ct}(r, \theta, \phi) = K_{n\ell}\psi_{n\ell\zeta}^{\Delta ct}(r, \theta, \phi),$$

$$\psi_{n\ell\zeta}^{\Delta ct}(r, \theta, \phi) = \frac{1}{\sqrt{\Delta ct}} R_{n\ell}(r) Y_{\ell\zeta}^{\ell}(\theta, \phi). \quad (4.2)$$
The proper energy eigenvalues \( K_{n\ell} \) are expected to include relativistic velocities and temporal curvature effects.

The form of the canonically conjugate momentum components in (4.2) must be consistent with the Heisenberg equations of motion

\[
\left\{ \frac{d \hat{p}_r}{d\tau} \right\} = \frac{i}{\hbar} [\hat{K}, \hat{p}_r] = -m \partial_r V(r),
\]

(4.3)

where \( V(r) \) was obtained from the geodesic equation (3.12). This implies that the scale factor of the momentum conjugate to \( r \) in the proper energy form should be unity, \( \hat{p}_r = \frac{\hbar}{T} \partial_r \). Therefore, the spatial curvature effects are evidently already incorporated in the functional form of the potential \( V(r) \) and the given conjugate momentum operator.

The square of the momentum for the metric form (3.10) is thus given by

\[
\hat{p} \cdot \hat{p} = -\hbar^2 \left\{ \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial}{\partial r} \right) - \frac{\hat{L}_r^2}{\hbar^2 r^2} \right\},
\]

(4.4)

while in contrast the spatial Laplacian for this metric satisfies

\[
\nabla \cdot \nabla = \frac{1}{r^2} \sqrt{1 - \frac{R_m(r)}{r}} \frac{\partial}{\partial r} \left( r^2 \sqrt{1 - \frac{R_m(r)}{r}} \frac{\partial}{\partial r} \right) - \frac{\hat{L}_r^2}{\hbar^2 r^2} \}
\]

(4.5)

Parameters analogous to those of Bohr for hydrogenic systems can be developed. The radial scale of the solutions is given by

\[
a \equiv \frac{\hbar^2}{G_N m^3} = \left( \frac{\lambda_m}{L_P} \right)^2, \quad \lambda_m = \left( \frac{M_P}{m} \right)^2 \lambda_m,
\]

(4.6)

where the reduced Compton wavelength is given by \( \lambda_m \equiv \frac{\hbar}{m c} \), the Planck length is labeled \( L_P \), and the Planck mass is labeled \( M_P \). For the present treatment, only \( s \)-wave \( \ell = 0 \) states will be examined. Since probability densities using the metric coordinates will take the form \( r^2 R_{C,\ell=0}(r) \), it is convenient to introduce a central reduced radial wavefunction \( u_C(r/a) \propto r R_{C,0}(r) \) parameterized by dimensionless variable \( \xi \equiv r/a \). The dynamic parameters can also be scaled using the parameter \( a \):

\[
\mathcal{P}(r/a) \equiv \int_0^{r/a} u_C^2(\xi') d\xi', \quad \mathcal{P}(\infty) = 1,
\]

\[
\frac{R_M(r)}{r} = 2 \left( \frac{m}{M_P} \right) ^4 a \frac{r}{r} \mathcal{P}(r/a),
\]

\[
-\frac{2ma^2}{\hbar^2} V(r) = 2 a \frac{M(r/a)}{m},
\]

\[
-\frac{2ma^2}{\hbar^2} (K - mc^2) = -2 \left( \frac{M_P}{m} \right) ^4 \frac{K - mc^2}{mc^2}.
\]

(4.7)
Using these identifications, (4.2) can then be re-written
\[
\epsilon_C u_C(\zeta) = \frac{d^2 u_C(\zeta)}{d\zeta^2} + \left(\frac{2}{\zeta}\right) \left(\frac{M(\zeta)}{m}\right) u_C(\zeta), \quad (4.8)
\]
where the dimensionless parameter \(\epsilon_C \equiv -2 \left(\frac{M_P}{m}\right)^4 \frac{K - mc^2}{mc^2}\) quantifies the gravitational binding energy of the mass. In order to examine the scale of deviations from spatial flatness, an equation for which the spatial Laplacian (which incorporates proper radial distances) replaces \(-\hat{p} \cdot \hat{p} \hbar^2\), will also be examined:
\[
\epsilon^\ast u^\ast(\zeta) = \frac{d^2 u^\ast(\zeta)}{d\zeta^2} + \left(\frac{2}{\zeta}\right) \left(\frac{M^\ast(\zeta)}{m}\right) u^\ast(\zeta) + \left[\frac{P^\ast(\zeta)}{\zeta^2} - \frac{P'_\ast(\zeta)}{\zeta^3}\right] \frac{d u^\ast(\zeta)}{d\zeta} - \frac{2}{\zeta} P^\ast(\zeta) \frac{d^2 u^\ast(\zeta)}{d\zeta^2}. \quad (4.9)
\]
The terms containing \(P^\ast(r)\) demonstrate the modifications to the previous form (with \(\frac{m}{M_P} \to 0\) due to factors \(\frac{M_P}{r}\).

For all solutions, the source mass \(M(\zeta)\) will be presumed to be generated by the interior self-sourcing of probability density:
\[
M(\zeta) = \int_0^\zeta \rho_{\text{mass}}(\zeta') d\zeta' = \int_0^\zeta m u_C^2(\zeta') d\zeta' = m P(\zeta). \quad (4.10)
\]
The distribution indicates that the differential equations (4.8) and (4.9) will be non-linear. For this distribution, it should be noted that the particle mass scale \(m\) appears nowhere in (4.8), while it only appears in the spatial scale terms in (4.9).

A mass whose interior probability density provides its local source gravitational field will be referred to as a self-gravitating mass. If, in additional, the overall gravitational bound state energy is that of the mass itself, the self-gravitating mass will be referred to as a self-generating mass. A self-gravitating central mass is expected to have non-vanishing probability density at the center. Such a self-gravitating single mass satisfies (4.8) with mass distribution given in (4.10). This form is clearly non-linear, so that initial conditions and eigenvalues are non-trivially related to the solution.

A solution to (4.8) for a system that is self-gravitating, but has non-vanishing binding energy eigenvalue is demonstrated in Fig. 1. Expressed in the dimensionless form demonstrated, the solution is completely independent of the mass of the system. The binding energy eigenvalue for the normalized probability density was obtained by examining the small \(r\) behavior of (4.8) in a self-consistent manner, yielding a value \(\epsilon_C \simeq 1.212\). The diagram on the left demonstrates the probability density \(|u_C(r/a)|^2\), while the diagram on the right is a density plot of the self-gravitating mass density. For the system, the gravity at a given radial coordinate is a field generated by the integrated mass density within that radial coordinate.
To obtain a solution to (4.9), a mass value must be chosen, since the radial scale factors explicitly appear in the equation. The mass was chosen \((m_{\text{MP}})^4 = 0.01\), or \(m \approx 0.316M_P\) such that some spatial curvature effects would be apparent in the calculations. For this mass, the binding energy eigenvalue \(\epsilon^*\) was found to be larger than \(\epsilon_C\) by 0.38 %, while the central density remained essentially unchanged.

A solution to (4.8) for which the gravitational potential results in vanishing net binding energy can also be found. Figure 2 demonstrates a self-generating solution to this equation with \(\epsilon_C = 0\). The diagram on the left again demonstrates the probability density \(|u_C(r/a)|^2\), while the diagram on the right is a density plot of the self-gravitating mass density consistent with vanishing overall gravitational binding energy. The scales of the diagrams have been chosen to be consistent with the prior self-gravitating mass. The self-generating mass density is seen to be more concentrated at the center relative to the self-gravitating mass density, developing a greater integrated gravitational potential energy, with a commensurate change in integrated kinetic energy. The central density \(|u^*_0|^2\) solving (4.9) (with \(m \approx 0.316M_P\)) is modified from that solving (4.8) by an increase of about 3.8 %.

The solutions demonstrated in Figs. 1 and 2 are independent of mass. However, as previously mentioned, the radial mass scale used to generate the Einstein tensor...
Fig. 3  Metric factor $-g_{00}$ and $-g_{rr}$

is given by $R_M(r) = 2\left(\frac{m}{M_p}\right)^4 a \mathcal{P}(r/a)$. The crucial factor $1 - \frac{R_M(r)}{r}$ in the metric is demonstrated in Fig. 3. If this factor changes sign, space-like behaviors become time-like (and vice versa), and a trapped region for which outgoing photons must propagate towards decreasing radial parameter $r$ will be present. As long as the mass is not chosen to be too large, there is no trapped region. A larger mass lowers the $y$-intercept of this curve. For the self-generating mass, the maximum value the mass can take without introducing a trapped region is given by about $0.63 M_p$. Masses smaller than this will not generate a black hole.

A conformal density plot for this energy distribution is presented in Fig. 4. The center $r = 0$ is the left boundary of the diagram, with both fixed radial coordinate curves and fixed Schwarzschild time surfaces originally graded in units of the surface scale. As usual, each point on the diagram represents the surface of a sphere at a given time. Unlike the interior of Schwarzschild geometry, the center is everywhere a time-like curve. Since there is no horizon in this case, any fixed Schwarzschild time coordinate is also seen to parameterize a space-like surface everywhere.
For completeness, the non-vanishing components of the Einstein tensor $G^{ct}_{ct} = G^r_r$ and $G^{θ_θ} = G^{φ_φ}$ are demonstrated in Fig. 5. The tensor satisfies the vacuum solution $G^{μ_β} = 0$ in the exterior region $\frac{r}{a} > 0.64$. All solutions presented have non-vanishing densities of finite extent.

4.2 Energy Conditions

Classical gravitating systems are expected to satisfy various energy conditions everywhere. These conditions assert that any observer should locally measure gravitational fields generated by time-like or light-like sources, regardless of their motion. This is consistent with an expectation that no energy source can propagate at a speed greater than that of light. However, quantum systems do exhibit space-like coherent behaviors. Space-like coherence allows the evaporation of black holes, thereby locally violating energy conditions. Also, systems with significant binding might violate these conditions. It is therefore of interest to examine the energy conditions of these self-gravitating systems.

The null and weak energy conditions assert that the form

$$I_{null/weak} \equiv -μ^{μ}_{null/weak} T^{μ_β} u^{β}_{null/weak} \leq 0$$

should be non-positive for light-like (null) and time-like (weak) observer four velocities, where $T^{μ_β}$ represents components of the energy-momentum tensor sourcing the gravitational field in Einstein’s equation. The dominant energy condition directly develops the form of the 4-momentum of the gravitational source as seen by the observer with four velocity $\vec{u}_{observer}$, given by $p^{μ}_{source} \equiv -T^{μ_β} u^{β}_{observer}$. This four-momentum is expected to be time-like or light-like, i.e.,

$$I^{DE}_{observer} \equiv \vec{p}_{source} \cdot \vec{p}_{source} \leq 0,$$

where the dot product is defined by the metric of the geometry. For all of the self-gravitating and the self-gravitating, self-generating solutions given, the null and weak energy conditions are satisfied everywhere for all types of motions. Likewise, the dominant energy condition for fiducial (static) observers and arbitrary radial motions is also satisfied everywhere for all solutions. However, the dominant energy condition for rapid pure azimuthal motions of the observer was found to be violated only in
the region just inside of the surface in each solution, likely due to coherence and gravitational binding from the interior mass distribution. Rapid motions were found to be those motions exceeding the condition

\[ ru_\theta > \frac{2R'_M(r)}{\sqrt{(rR''_M(r))^2 - (2R'_M(r))^2}}. \quad (4.11) \]

Exterior to the region of coherence, as well as proximal to the center, all energy conditions were found to be satisfied. A further exposition of energy conditions, as well as a more detailed development of the general formulation based on the equivalence principle, including co-gravitating masses and cluster decomposability, will be found in reference [6].

5 Conclusions

Self-gravitating quantum solutions, consistent with the equivalence principle, have been found using coherence parameterized by the local proper time of the gravitating system. The solutions required no specific form for micro-physical interactions, consistent with the universal nature of gravitation. The approach considers space-time as an emergent construct of quantum measurement, with curvatures generated by Einstein’s equation in the form \( G_{\mu\nu} = -\frac{8\pi G_N}{c^4} \langle \hat{T}_{\mu\nu} \rangle \). The dynamics developed is consistent with the measurement constraints of standard quantum theory.

The quantum stationary solutions developed incorporate curvature effects. For weak curvatures and slow motions, the solutions exhibit both quantum and classical Newtonian correspondence through proper time Heisenberg equations of motion. The formulation, being generally representation independent, demonstrates that the exhibition of quantum coherent behavior for gravitating systems need not require second quantization of the gravitation field itself. The solutions satisfy sensible conditions of physicality on the energy densities sourcing the gravitational fields, including non-singular behavior everywhere and non-negative mass densities \( R'_M(r) \geq 0 \) everywhere. The natural size scales are consistent with bound quantum systems.

Interaction forms for more general metric spaces have also been developed. The approach is consistent with experimental results demonstrating the coherence of gravitating systems. Because of the non-linearity of the equations, (independent) co-gravitating masses modify the solutions as expected. These results, as well as a more detailed analysis of its general foundations, will be presented elsewhere [6].

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