Comparing the ensemble mean and the ensemble standard deviation as inputs for probabilistic medium-range temperature forecasts

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Abstract

We ask the following question: what are the relative contributions of the ensemble mean and the ensemble standard deviation to the skill of a site-specific probabilistic temperature forecast? Is it the case that most of the benefit of using an ensemble forecast to predict temperatures comes from the ensemble mean, or from the ensemble spread, or is the benefit derived equally from the two? The answer is that one of the two is much more useful than the other.

1 Introduction

We consider medium range probabilistic site-specific forecasts of temperatures. There are a number of methods that can be used to produce such forecasts. These methods differ in terms of a) the underlying numerical forecast model, b) the predictors that are taken from the forecast model and c) the statistical methods used to transform these predictors into predictions. For instance, the underlying model could be the ECMWF or the NCEP model (inter alia); the underlying predictors could be the ensemble mean and the standard deviation or just the ensemble mean (inter alia) and the statistical transform used could be based on the rank histogram or spread regression (inter alia).

Much work has gone into trying to understand the sources of medium range forecast skill, and to improve medium range forecasts. Some of this has aimed at determining which of the various numerical models produce the best predictors (point "a" above). There has also been some investigation into which of the various statistical transforms are the most appropriate (point "c" above). We will now address a third question but closely related question: what is the relative importance of the different predictors that can be derived from the numerical models (point "b" above). In particular, we will consider the relative importance of the mean and the standard deviation of ensemble forecast temperatures, when used to make a site-specific probabilistic forecast. We consider only the mean and the standard deviation, rather than all the individual ensemble members, since, for the dataset we will use, it has been shown by Jewson (2003b) that there seems to be no useful information in the ensemble beyond the mean and the spread.

The statistical models we use to address this question are all derived from the spread regression model of Jewson et al. (2003). This model takes the mean and the standard deviation from an ensemble forecast and converts them into a probabilistic forecast. This model is particularly useful for addressing the question of relative importance of the different predictors since it allows us to turn each of them on and off rather easily.

We do not think that it is a priori obvious which of the ensemble mean and ensemble spread are more useful for making a probabilistic forecast. For instance, one can imagine dynamical systems for which the expectation varies very little, but for which the uncertainty varies a lot. Similarly one can imagine dynamical systems for which the expectation varies a lot, but for which the uncertainty varies little. In the first of these it is likely that forecasts of the expectation are more important, while in the latter it is likely that forecasts of the uncertainty would be more important. We see, therefore, that our question is, in part, a question about the dynamics of the atmosphere. One can also imagine a particular forecast system that, for whatever reason, does very well in predicting the expectation, but very poorly in predicting the uncertainty, or vice versa. From this we see that our question is also a question about specific forecast systems.

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In section 2 we describe the data we use for this study. In section 3 we describe the methodology and the statistical models we will use to address this question. In section 4 we present the results of our analysis and in section 5 we discuss the implications.

## 2 Data

We will base our analyses on one year of ensemble forecast data for the weather station at London’s Heathrow airport, WMO number 03772. The forecasts are predictions of the daily average temperature, and the target days of the forecasts run from 1st January 2002 to 31st December 2002. The forecast was produced from the ECMWF model \cite{Molteni1996} and downscaled to the airport location using a simple interpolation routine prior to our analysis. There are 51 members in the ensemble. We will compare these forecasts to the quality controlled climate values of daily average temperature for the same location as reported by the UKMO.

There is no guarantee that the forecast system was held constant throughout this period, and as a result there is no guarantee that the forecasts are in any sense stationary, quite apart from issues of seasonality. This is clearly far from ideal with respect to our attempts to build statistical interpretation models on past forecast data but is, however, unavoidable: this is the data we have to work with.

Throughout this paper all equations and all values are in terms of double anomalies (have had both the seasonal mean and the seasonal standard deviation removed). Removing the seasonal standard deviation removes most of the seasonality in the forecast error statistics, and partly justifies the use of non-seasonal parameters in the statistical models for temperature that we propose.

## 3 Models

We will address the question of whether the useful information in ensemble forecasts comes to a greater extent from the ensemble mean or from the ensemble spread by considering the site-specific forecasts for temperature at London Heathrow described above. Our forecasts are produced by four ensemble interpretation models, which is the name we give to the statistical models that take the non-probabilistic predictors from the ensemble forecasts and convert them into probability density functions for the predicted temperatures. Such models are essential if one wants to produce probability forecasts from ensemble output. The models we choose are ideally suited for answering the question at hand because they allow us to turn on and off the various sources of information in the ensemble very easily and see what impact that has on the skill of the resulting forecast.

The simplest model we consider consists of the use of linear regression between a single member of the ensemble and the observed temperature. The regression model serves to correct biases, to adjust the amplitude of the variability, and to predict the uncertainty. We will write this model as:

\[
T_i \sim N(\alpha + \beta x_i, \gamma) \tag{1}
\]

where \(T_i\) is the double anomaly temperature on day \(i\) and \(x_i\) is the ensemble member on day \(i\). The skill of this model gives an indication of the ability of single integrations of the numerical model to predict the distribution of future atmospheric states. It is a baseline that the ensemble forecast should beat if it has any value.

Our second model is the same as the first, but the predictor is the ensemble mean, rather than a single ensemble member. We write this model as:

\[
T_i \sim N(\alpha + \beta m_i, \gamma) \tag{2}
\]

where \(m_i\) is the ensemble mean on day \(i\). This model should hopefully perform better than the first model.

Our third model is an extension of the first model to include the ensemble standard deviation as a predictor of the uncertainty:

\[
T_i \sim N(\alpha + \beta x_i, \gamma + \delta s_i) \tag{3}
\]

where \(s_i\) is the ensemble standard deviation.

Finally our fourth model combines the second and the third models by using the ensemble mean as a predictor for the temperature, and the ensemble standard deviation as a predictor for the uncertainty around the temperature. This is the spread regression model of \cite{Jewson2003}.
$$T_i \sim N(\alpha + \beta m_i, \gamma + \delta s_i)$$

(4)

These models have been designed to allow us to answer the question at hand. For instance, comparison of the results from model one and model two, or between model three and model four, gives the benefit of using the ensemble mean versus a single integration. Comparison between model one and model three, or between model two and model four, gives the benefit of using the ensemble spread versus only using past forecast error statistics.

We fit all of these models using the standard maximum likelihood method from classical statistics. More details of this method as applied to this problem are given in Jewson (2003c).

3.1 Scoring the models

In order to compare the results from our four models we need a measure for how good the resulting probabilistic forecast is. To our knowledge only two scores have been suggested in the literature for comparing continuous probabilistic forecasts: the "ignorance" by Roulston and Smith (2002), and the likelihood, by Jewson et al. (2003). It turns out that these two are, for this case, more or less the same thing. We will therefore use this score to compare our forecasts, in the form of the log-likelihood.

4 Results

4.1 Parameter values

We first look at the parameter values for our various statistical models. Figure 1 shows the values of alpha for the four models. The two models that are based on the single ensemble member have very similar values of alpha, while the two models that are based on the ensemble mean also have similar values. The values from the models based on the single ensemble member are somewhat smaller.

Figure 2 shows the values of beta for the four models. The values from the four models pair up in the same way as for the values of alpha, with the values for the single model based forecasts being much lower than the values for the ensemble mean based forecasts.

We can understand these values of beta as follows. We can consider a single integration of the numerical model to consist of the ensemble mean plus a noise term. The noise terms are at least somewhat different for the different members of the ensemble. Thus, in forming the ensemble mean the noise terms tend to cancel out and the overall noise is smaller. When we regress a single member onto observed temperatures, the regression coefficient is rather small because of the large noise term. When we regress the ensemble mean onto observed temperatures, the regression coefficient is much larger, because the noise level is lower. The mathematics of this effect have been given in detail by Jewson and Ziehmann (2002).

The value of alpha can be understood in terms of the values of beta. Alpha arises due to a combination of bias, and the beta term. We would expect the bias to be the same for a single ensemble member and the ensemble mean: the differences we see can be explained as being due to the differences in the beta.

Figure 3 shows the values of gamma for the four models. Because gamma plays a slightly different role in the different models, there is no single physical interpretation of what it means. In the two regression models gamma represents the uncertainty: we see that the uncertainty is greater in the model that only uses the single ensemble member, as would be expected. In the spread regression models, gamma represents only a part of the total uncertainty, and can only be interpreted in combination with delta, which is shown in figure 4. Confidence intervals for the value of delta in the spread regression model are given by Jewson et al. (2003), and show that there is significant sampling uncertainty about these estimates. This is not surprising since delta is related to the second moment of the second moment of observed temperatures. This sampling uncertainty is reflected in figure 4 by the jaggedness of the lines. Nevertheless we can see a general trend towards lower values of delta at high leads for both models.

4.2 Total Uncertainty

Comparing the gamma and delta parameter values for the four statistical models is somewhat dissatisfactory since, as we have seen, gamma does not have consistent interpretation across the four models. It makes more sense to compare the total uncertainty predicted by the four models. In figures 5 and figure 6 we compare the mean and the standard deviation of this predicted uncertainty respectively. In figure 5 we see that the two models based on the ensemble mean have much lower mean uncertainty than the two models based on the single integration, as would be expected. The ensemble mean gives significantly better forecasts than individual ensemble members, especially at longer leads. Figure 6 shows the variability
in the predicted uncertainty for the two spread regression models. The two regression models, both of which ignore the ensemble spread, are not shown because they predict constant levels of uncertainty. We see that the two spread regression models predict roughly the same levels of variability in the uncertainty. There are some differences between the two, but we suspect these differences are mostly due to sampling error.

### 4.3 Likelihood scores

Finally we come to the results that answer the question that we set out to address: what are the relative contributions of the ensemble mean and the ensemble standard deviation to the skill of the final forecast? We present the comparisons in terms of the negative of the log-likelihood, for which small values represent a good forecast. We compare the models in pairs to isolate individual effects, as described in section 4.

Figure 7, top left panel, shows a comparison between models one and two. This comparison is designed to illustrate the benefit of using the ensemble mean over using a single ensemble member. We see a very large benefit at all lead times. For instance, the ensemble mean based forecast at lead 6 is roughly as good as the single member based forecast at lead 4. We are left in no doubt that use of the ensemble mean is very beneficial for our probabilistic forecast. The lower left panel shows a comparison between models three and four. This comparison also shows the benefit of using the ensemble mean over using a single ensemble member, but now for the models that also use the ensemble spread. The size of the benefit is roughly the same as before.

The right hand panels show the effects of using the ensemble spread. The top right panel shows the benefit of using the ensemble spread in the models that do not use the ensemble mean, and the lower right panel shows the benefit of using the ensemble spread in the models that do use the ensemble mean. The results are roughly the same: in both cases we see only a very marginal benefit from using the ensemble spread.

Figure 8 now shows the differences that we see in the benefit due to the ensemble mean and the ensemble spread. There are a number of ways one could try and quantify this effect: we choose to look at the numerical differences in the log-likelihood. The top left hand panel shows the differences in the log-likelihood caused by the use of the ensemble mean as a predictor (from comparing models 1 and 2). The values lie between 10 and 70. The top right hand panel shows the differences in the log-likelihood caused by the use of the ensemble spread as a predictor (from comparing models 2 and 4). The values lie between zero and 10. The lower panels show the ratios of these log-likelihood differences, which quantify how much more useful the ensemble mean is as a predictor than the ensemble spread. We see the values for this ratio lie between four and 100, and increase at longer lead times. The ensemble spread is relatively most useful at the shortest lead times, while the ensemble mean is relatively most useful at the longest lead times.

### 5 Discussion

We have investigated the relative contributions of the ensemble mean and the ensemble standard deviation to the skill of probabilistic temperature forecasts. This was done by using four different statistical models to produce the probabilistic forecasts. These models differ in terms of whether they use the ensemble mean or a single ensemble member, and whether they use the ensemble spread or predict uncertainty purely using past forecast error statistics.

The results are very clear: we see that the ensemble mean is much more important than the ensemble spread. The difference is least at short leads, where the ensemble mean is only about four times as useful as the ensemble spread (using the particular measure that we have chosen). At medium leads the ensemble mean is between 10 and 20 times as useful as the ensemble spread, and at the longest lead the ensemble mean is roughly 100 times more useful than the ensemble spread.

There are a number of caveats to this study. In particular, we have only used data from one station. Different results would probably be obtained at other stations. We have also only used one year of forecast data, and we have seen that this leads to rather noisy estimates for some of our parameters. Our estimates would be more accurate if we had more data, as long as that data were stationary. Having said that, our main result, which is that the ensemble mean is much more useful than the ensemble standard deviation, seems to be so clear that we doubt whether it would change even if we did repeat this analysis for a number of other stations and with more data.

Finally, we note that have performed all of our analysis in-sample. This is not ideal: if possible, out of sample results are to be preferred. However, Jewson (2003) has shown that out of sample one cannot detect any beneficial effects of ensemble spread whatsoever using this data-set, and so it would not be
possible to perform this study if we were to do it out of sample. By using an in-sample analysis, and using statistical models with small numbers of parameters to minimize overfitting and artificial skill, we have been able to shed light on a question that could not be otherwise addressed.
Figure 1: The values of alpha for the four models described in the text. Model 1 (solid line), model 2 (dashed line), model 3 (dotted line) and model 4 (dot-dashed line).
Figure 2: The values of beta for the four models described in the text. Model 1 (solid line), model 2 (dashed line), model 3 (dotted line) and model 4 (dot-dashed line).
Figure 3: The values of gamma for the four models described in the text. Model 1 (solid line), model 2 (dashed line), model 3 (dotted line) and model 4 (dot-dashed line).
Figure 4: The values of delta for two of the four models described in the text. Model 2 (solid line) and model 4 (dashed line).
Figure 5: The mean level of uncertainty predicted by the four models described in the text. Model 1 (solid line), model 2 (dashed line), model 3 (dotted line) and model 4 (dot-dashed line).
Figure 6: The standard deviation of the level of uncertainty predicted by two of the four models described in the text. Model 2 (solid line) and model 4 (dashed line).
Figure 7: Comparisons between the minus log-likelihood scores from the four models described in the text.
Figure 8: Comparisons between the minus log-likelihood scores from the four models described in the text. The top left panel shows the differences in the log-likelihood between models 1 and 2, while the top right panel shows the differences between models 2 and 4. The lower panels show the ratio of these differences (with different vertical scales).

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