Universal Policies for Software-Defined MDPs

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Abstract

We introduce a new programming paradigm called oracle-guided decision programming in which a program specifies a Markov Decision Process (MDP) and the language provides a universal policy. We prototype a new programming language, Dodona, that manifests this paradigm using a primitive choose representing nondeterministic choice. The Dodona interpreter returns either a value or a choicepoint that includes a lossless encoding of all information necessary in principle to make an optimal decision. Meta-interpreters query Dodona’s (neural) oracle on these choicepoints to get policy and value estimates, which they can use to perform heuristic search on the underlying MDP. We demonstrate Dodona’s potential for zero-shot heuristic guidance by meta-learning over hundreds of synthetic tasks that simulate basic operations over lists, trees, Church datastructures, polynomials, first-order terms and higher-order terms.

1. Introduction

One of the most encouraging discoveries in machine learning over the past several years has been that simple neural network architectures can become meta-learners when trained on sufficiently large datasets. This phenomenon is most apparent in the language model GPT-3 (Brown et al., 2020), which after having been trained in an unsupervised fashion on a large corpora of text was able to perform competently on many diverse language tasks, querying the model on a new task requires an ad-hoc process whereby a human devises a prompt such that the most likely completion of the prompt on a random website may somehow include the desired information. Some tasks do admit standard prompts—for example, “TL;DR” for requesting summarization—but most tasks do not. For example, for a pre-trained language model to provide heuristic guidance deep inside a theorem prover, somehow a lot of context must be provided to the network. What is to be proved? What is known or assumed? What kind of prover is it? What is the current state of the prover? How does the current decision affect what the prover does downstream? It may even be necessary to indicate explicitly in the prompt that one prefers a good decision, i.e. one that will lead to finding a proof quickly. Needless to say, all these descriptions might be intricate and awkward to elicit. The second problem is the verification problem. Since the tokens emitted by language models cannot in general be trusted, they are only useful when it is easier to verify their outputs than to simply solve the problem from scratch. This limitation is particularly salient in program synthesis as it is notoriously difficult to understand code one did not write.

In this work, our aim is to better leverage the tantalizing power of large-scale neural networks for problems that can be specified precisely. Our main insight is that whereas the prompt is necessarily a dark art when natural language is involved, we can construct a principled analogue of the idea for Markov Decision Processes (MDPs) that are defined by software. For a given decision point within a software-defined MDP, the current state of the program along with the code describing the downstream MDP together provide a precise representation of all that is necessary in principle to make an optimal decision at the state in question. Thus a lossless encoding of this information can serve as a complete and unambiguous prompt for a (neural) oracle, that can provide a universal policy for all computable MDPs. We show how such an encoding can be synthesized automatically, providing a solution to the specification problem discussed above. The verification problem is trivially addressed by encoding the MDP precisely in software in the first place.

Manifesting the framework just described requires a new kind of programming language whose programs define MDPs rather than deterministic computations and whose
runtime performs the bookkeeping to synthesize the prompts at each decision point. We refer to this broad paradigm as oracle-guided decision programming, and prototype a new programming language, Dodona, that manifests this paradigm. Dodona—named after the oldest Hellenic oracle—is a minimalist, stand-alone language based on Scheme (Sussman & Steele, 1998), designed to isolate the key ideas of oracle-guided decision programming while avoiding unnecessary complexity. §2 describes the basics of Dodona, both its semantics and how the interpreter produces the datastructures that encode the relevant information at each choicepoint. §3 describes how we embed these datastructures into a graph-aware transformer (Vaswani et al., 2017; Hellendoorn et al., 2019). This embedding represents only one small point in design space; there are many ways of embedding the information but the trade-offs among them are not our present concern. §4 describes an experiment where we generate data for hundreds of synthetic tasks that simulate basic operations over lists, trees, Church datastructures, polynomials, first-order terms and higher-order terms; we see that Dodona performs competently in the zero-shot setting on previously unseen tasks. Finally, §5 includes extensive discussion of related work, missing features, and open problems.

2. The Dodona Language

2.1. Quick tour

The Dodona language is based upon a pure subset of the functional language Scheme (Sussman & Steele, 1998), which is itself a minimalist dialect of Lisp (McCarthy, 1959b). Dodona is endowed with a choose primitive that takes a list as argument and that evaluates nondeterministically to one element of the list. Thanks to choose, a Dodona program defines a binary, deterministic Markov Decision Process (MDP), i.e. a search problem, where choosing from an empty list indicates failure. The extension to intermediate, real-valued rewards and probabilistic transitions is relatively straightforward but not necessary for our present aims, so we relegate discussion to §5. The syntax of Dodona is otherwise like Scheme:

\[
\begin{align*}
\text{expr} & ::= \text{c} | \text{x} | (\text{choose} \ e) \\
& | (\text{lambda} \ (\text{x} \ldots) \ e) | (\text{if} \ e1 \ e2 \ e3) \\
& | (\text{define} \ (\text{x} \ldots) \ e) | (\text{quote} \ e) \\
& | (\text{el} e2 \ldots)
\end{align*}
\]

where the prefix c signifies a value (e.g. a bool, int or list), x a symbol and e an expression. We write (quote e) as ‘e, and it indicates that its argument should be returned without being evaluated, so that e.g. the expression ‘(0 1) evaluates to the list containing 0 and 1. The last form (el e2 ...) represents function application. Dodona also includes standard syntactic sugar, e.g. (let ((x v)) b) is sugar for ((lambda (x) b) v) and let* is sugar for nested lets.

We illustrate Dodona with a few examples. We define fail to be a function that chooses from an empty list: (define (fail) (choose '())), and choose-bool to be a function that chooses a bool: (define (choose-bool) (choose '(#f #t))). Here is an expression that chooses a bool and returns 7 if it is true and otherwise fails: (if (choose-bool) 7 (fail)). For our present purposes, all non-failing return values represent the same reward in the induced MDP. Here is a higher-order, recursive program for producing lists:

\[
\begin{align*}
\text{(define (choose-list choose-elem)} \\
& | (\text{if} \ (\text{choose-bool}) '() \\
& | (\text{cons} \ (\text{choose-elem}) \\
& | (\text{choose-list} \ (\text{choose-elem}))))
\end{align*}
\]

Here is a higher-order, recursive program for producing trees:

\[
\begin{align*}
\text{(define (choose-tree choose-leaf choose-node)} \\
& | (\text{if} \ (\text{choose-bool}) \\
& | (\text{choose-leaf}) \\
& | (\text{let} ((\text{fn-nargs} \ (\text{choose-node}))) \\
& | (\text{cons} \ (\text{first} \ \text{fn-nargs}) \\
& | (\text{replicate} \ (\text{second} \ \text{fn-nargs}) \\
& | (\text{lambda} () \\
& | (\text{choose-tree} \\
& | (\text{choose-leaf} \\
& | (\text{choose-node}))))))
\end{align*}
\]

where choose-node is expected to produce a two-element list (node-value, num-args), and where (replicate n thunk) returns a list of length n with each element generated by evaluating the thunk.

We remark that Dodona—like Scheme—may be minimalist but that it inherits the extreme power and flexibility of the Lisp family.

2.2. Evaluation

Whereas a Scheme program evaluates to a value, a Dodona program evaluates to an (binary, deterministic) MDP. We define our restricted notion of MDP as either a (terminal) value or a choicepoint, where a choicepoint includes (a) a list of possible values to choose from and (b) a continuation that takes whichever value may be chosen to another MDP. The continuation in a choicepoint serves two roles in Dodona: the computational role, to enable continued evaluation for a particular choice, and also the prompt role, to represent all relevant information about the MDP in a way that can be embedded. The latter role requires that the continuation be represented as a concrete datastructure, as opposed to remaining implicit in the program state of the interpreter.

Dodona achieves this by representing a continuation as a stack of segments, where a segment is a triple consisting
of: a syntax function, \( i.e. \) a function in the host language that maps a value to a Dodona expression, an environment to evaluate that expression in (a mapping from symbols to values), and the number of arguments in the expression that will have already been evaluated. Consider the following Dodona program:

\[
\text{(let ((x 0))} \\
\text{ (+ x} \\
\text{ (let ((x 2)) (if (choose-bool) x 1))} \\
\text{ x))}
\]

The Dodona interpreter will evaluate this expression in applicative order until reaching the first (and only) choice, and will return a choicepoint with choices \((\#f \ #t)\) and continuation given by the following stack of segments: \([ (z \mapsto (\text{if }z \ x \ 1), \{x:2\}, 1), (z \mapsto (<\text{prim:}+> 0 z x), \{x:0\}, 3) ] \). In the second segment, the <prim:+> represents the primitive procedure that \( + \) evaluates to. Note that in this segment, the first summand \( x \) of the original expression has already evaluated to 0, whereas the last summand \( x \) has yet to be evaluated. This is reflected in the number 3 in the last field of the segment, which indicates that three of the four arguments in the list \(<\text{prim:+}> 0 z x)\) have been evaluated already.

To summarize, the Dodona interpreter takes as input an expression, an environment, an integer indicating the number of arguments that have already been evaluated, and a stack of segments, and returns as output either a value or a choicepoint as described above. We now present pseudocode for a few snippets of the interpreter. The main function is step, which dispatches to helper functions step-atom, step-if, step-quote, step-lambda, step-app based on the syntax of the expression it is evaluating. Our actual implementation is tail-recursive, but we present a naively-recursive version here for simplicity. To evaluate an atom, we first check if the continuation stack is empty; if it is, we simply return the atom, and if it is not, we pop the next segment from the continuation stack and continue evaluating:

\[
\text{(define (step-atom x env i cstack)} \\
\text{ (if (null? cstack))} \\
\text{ x} \\
\text{ (let ((seg (first cstack)))} \\
\text{ (step ((seg->fn seg) x)} \\
\text{ (seg->env seg)} \\
\text{ (seg->idx seg)} \\
\text{ (rest cstack))})
\]

To evaluate an if-statement, we first check if we have already evaluated the condition. If not, we evaluate the condition while adding a segment to the continuation stack indicating that we should resume evaluating the expression at argument index 1; if we have already evaluated the choices, we simply return a choicepoint:

\[
\text{(define (step-if x env i cstack)} \\
\text{ (if (= i 0)} \\
\text{ (let ((f (lambda (y) \\
\text{ (list 'if y \\
\text{ (if->then x)} \\
\text{ (if->else x)))))} \\
\text{ (step (if->cond x)} \\
\text{ env 0 \\
\text{ (cons (seg->mk f env 1) cstack))}) \\
\text{ (let ((y (if (if->cond x) \\
\text{ (if->then x)} \\
\text{ (if->else x)))) \\
\text{ (step y env 0 cstack))}))})
\]

To evaluate a choicepoint, we first check if we have already evaluated the choices. If not, we evaluate the choices while adding a segment to the continuation stack indicating that we should resume evaluating the expression at argument index 1; if we have already evaluated the choices, we simply return a choicepoint:

\[
\text{(define (step-choicepoint x env i cstack)} \\
\text{ (if (= i 0)} \\
\text{ (let ((f (lambda (y) \\
\text{ (list 'choose y)))))} \\
\text{ (step (second x)} \\
\text{ env 0 \\
\text{ (cons (seg->mk f env 1) cstack))}) \\
\text{ (cp->mk (second x) cstack))})
\]

The other cases in the interpreter follow the same pattern and so we omit them.

Our representation of a choicepoint as a list of choices and a stack of segments is clearly sufficient for its computational role. On the other hand, the host-language syntax functions in the segments may seem problematic at first glance for the choicepoint’s prompt role, since we cannot easily inspect or embed a host-language function. Our representation turns out to be very convenient for the prompt role as well: when we build the graph of the choicepoint for the oracle, we apply these host-language functions to the nodes that represent the values these functions will be applied to during evaluation. We revisit this in §3.

2.3. Meta-evaluation

The Dodona interpreter will usually return a choicepoint as opposed to a value; the power of Dodona comes from the meta-evaluators that we can write on top of the base interpreter. The simplest is rollout, which executes one path through the Dodona program, querying a callback decide for each decision that gets to observe the choicepoint:

\[
\text{(define (rollout decide x env i cstack)} \\
\text{ (let ((y (step x env i cstack))} \\
\text{ (if (value? y) y \\
\text{ (let* ((idx (decide y)) \\
\text{ (step-if x env i cstack))})})})
\]
We now explain how we embed the choicepoints returned by rollout. Thus, some kind of analysis is required to find the subset of values in the environments that are actually used. A compiled oracle-guided decision language may simply embed a lower-level representation of the prompt that already has its symbols resolved; in Dodona, we simply perform this analysis online as we build the embedding.

We encode the choicepoint information by building a graph. The encoding is relatively straightforward, so rather than belabor the low-level details we highlight a few decision decisions and then summarize the process.

- Every node stores a single token rather than arbitrary data, so that the initial embedding step can be performed efficiently on GPUs.
- There are many different types of edges for different kinds of relationships between the nodes.
- Every value is summarized by a single node no matter whether it is atomic and embeds to a single node (like a bool) or if it has internal structure and embeds to multiple nodes (like a list or a function). In particular, a function has a distinguished FUNCTION-SUMMARY node that has (typed) incoming edges from its parameters and its body.
- The procedure embed takes an expression (where some terms may be nodes), an environment, and the number of arguments of the expression that have already been evaluated; it adds nodes and edges to the graph as appropriate, and returns the node summarizing its argument.
- We unfold symbol-lookups recursively, but with memoization so that a single value in an environment is embedded at most once no matter how many times the symbol is referenced.
- Lists are represented in terms of their cons representation and so have linear depth, whereas unordered containers (e.g. sets and maps) are embedded in a permutation-invariant way with constant depth.

To embed a choicepoint, we first embed all the choices and connect them to a distinguished CHOICE node. We then feed the CHOICE node into the embedding of the continuation stack. Specifically, for each segment in the stack, we apply the syntax function to the node representing the value that will be passed into it, and embed the result in the designated environment. Note that a subset of the nodes correspond to the possible choices available at the choicepoint.

3.2. Dynamic graph transformer (DGT)

The graph neural network (GNN) architecture that we find most intuitive does not seem to have been tried before, so
we introduce it here and call it the dynamic graph transformer (DGT). Like the GREAT architecture (Hellendoorn et al., 2019), the DGT uses the edge information to bias the attention weights within a transformer encoder (Vaswani et al., 2017). In GREAT, each attention layer has one scalar parameter for each edge type that indicates the bias for the edge type across all nodes and heads; in particular, the edge bias is not conditioned on the nodes at all. In contrast, the DGT learns a linear transformation for each attention layer that maps each node to a $\text{numHeads} \times \text{numEdgeTypes}$ matrix that represents the node’s bias weight for each (attention head, incoming edge type) pair. We implemented the DGT using PyTorch (Paszke et al., 2019), with the help of the PyTorch Lightning library (Falcon, 2019).

4. Experiments

No neural network of fixed size can evaluate arbitrary Dodona programs—not even deterministic Dodona programs—since the language is Turing-complete and hence evaluation may require arbitrarily large amounts of both memory and time. Even Dodona programs that can be rolled out very efficiently (as in §2.3) may still require exponential time to determine the optimal choice at a given choicepoint, e.g. if the program nondeterministically chooses a sequence of bits and then observes that those bits invert a cryptographic hash of a known input sequence. Of course, these limitations do not rule out the possibility of a useful oracle, since many programs of interest require only moderate resources to evaluate, and even exponentially-branching search spaces may permit extremely effective heuristics.

We also note that by nature there can be no single “killer application” for a universal policy. For any particular application with sufficiently large amounts of training data—and with sufficiently motivated ML engineers—there will likely be advantages to designing a custom representation and neural network that need not be isomorphic to the continuation of the choicepoints. The promise of oracle-guided decision programming is in the potential gains in statistical efficiency, i.e. the amount of training data required for a new task. The holy grail for our approach would be zero-shot learning for a wide range of important domains. Based on the empirical findings from large-scale language models, such meta-learning is likely to only be apparent when training at extremely large scales, in terms of both the model complexity and the diversity of the training data (Kaplan et al., 2020; Henighan et al., 2020). While experiments at such large scales are not practically feasible for our first proof-of-concept, we nevertheless demonstrate zero-shot learning over hundreds of synthetic tasks.

**Tasks.** We generate data for hundreds of synthetic tasks that simulate basic operations over lists, trees, Church data-structures, polynomials, first-order terms and higher-order terms; we see that Dodona performs competently in the zero-shot setting on previously unseen tasks.

Our workhorse is a simple program `predict-app` in Dodona that generates synthetic data for predicting the result of evaluating a deterministic Dodona program. Specifically, `predict-app` takes the following arguments:

- **choose-func-arg.** A non-deterministic program returning a `(fn, arg)` pair.
- **max-results.** The number of `(fn, arg)` pairs to enumerate.
- **choose-output.** A non-deterministic program that can produce the results of evaluating the Dodona expression `(fn arg)` for every `(fn, arg)` in the codomain of `choose-func-arg`.
- **invert-output.** A program that inverts `choose-output`, i.e. that takes a value and produces the sequence of choices such that rolling out `choose-output` using that sequence will produce the desired value.

The program `predict-app` enumerates the first `max-results` outputs of `choose-func-arg`; for each `(fn, arg)` pair, it evaluates `(fn arg)` to yield `target`, and uses the result of `(invert-output target)` as the correct sequence of choices for the Dodona program `(if (= (choose-output) (fn arg)) #t (fail))`. Every call to `predict-app` constitutes one task, and we set `max-results` to be 500 for each task in the suite. Note that this may induce more than 500 datapoints per task since each `(fn, arg)` pair may itself require many choices made in sequence, e.g. to produce a tree of digits.

Here is a sample of the tasks in our suite:

- **Identity.** Predicting the result of applying the identity function to various values, including various types of lists and trees. These tasks are not quite as trivial as they may seem, since the network must be able to make the connection between the embedding of the values downstream, its partially constructed guess, and its current choices.

- **Arithmetic.** Predicting the result of evaluating different kinds of (integer) arithmetic expression trees involving adding, subtracting, multiplying, maximums, minimums, dividing, and computing remainders. The tasks are parameterized by tree-generators, e.g. one task might have the form `(+)` while another may have the form `((+ _ _))` where the underscores indicate nondeterministic integers.
• **Lists.** Predicting the result of many basic functions on lists of digits, such as computing the length, taking and dropping a specified number of elements, finding the \( n \)th element, determining if there exists an element that satisfies a certain property, counting, erasing and filtering the elements that satisfy certain properties, mapping simple unary functions, concatenating two lists, and others. The list tasks also include folding binary Boolean operations over lists of Boolean’s.

• **Trees.** Predicting the result of many basic functions on trees (with all values at the leaves), such as computing the number of inner nodes, the number of leaves, the total number of nodes, the depth, the subtree at a specified path in the tree and the number of leaves satisfying certain properties, mapping simple unary functions over the tree, and folding simple binary Boolean operations over the leaves of Boolean trees.

• **Polynomials.** Predict the result of applying various components of the sparse Horner normal form computation (Grégoire & Mahboubi, 2005) to different classes of polynomials, e.g. with different numbers of differently named variables.

• **First-order simplification.** Predict the result of simplifying (i.e. exhaustively rewriting) first-order terms for a range of different simplification rules, e.g. one task may involve simplifying \( (f \ ?x) \) to \( (g \ ?x) \) whereas another may involve simplifying \( (f \ (f \ ?x)) \) to \( (g \ (g \ ?x)) \) and \( (g \ (g \ ?x)) \) to \( (h \ ?x) \).

• **Higher-order reduction.** Predict the result of performing \( \beta \) and \( \eta \) reduction on higher-order terms with different sets of differently typed metavariables, as well as a few other higher-order tasks such as type inference, and term substitution followed by \( \beta \eta \) reduction (Dowek, 2001).

• **Planted path.** Chart a path through some encoding of a binary tree to find the single non-failing leaf, where the tasks range over different encodings of trees and paths. For example, one task encodes binary trees using nested Church pairs, where a path indicates a sequence of Church pair projections.

The supplementary material includes code for all tasks in the suite.

**Results.** We randomly divided the tasks into a 70-10-20 train-valid-test split, and trained a 12-layer DGT (see §3.2) on the training set using the validation set for early stopping. The results on the held-out test set are shown in Figure 1. Each row corresponds to a held-out task. The horizontal bar for a task indicates the metric \( \log(\text{uniformLoss}/\text{actualLoss}) \) where uniformLoss is the loss achieved by the uniform distribution over choices. This metric is 0 when the trained network is as good as uniform guessing, positive (and green) when it is better than uniform guessing, and negative (and red) when it is worse than uniform guessing. The majority of tasks are in the green, often significantly so, indicating some degree of zero-shot learning.

However, we stress that the primary motivation for this experiment was to demonstrate the Dodona features discussed earlier in the paper rather than to test a rigorous hypothesis. There is no rigorous motivation for the choice of tasks, and although we tried to design the experiment so that we could not predict the outcome, countless design decisions in the task suite could have made the resulting numbers higher or lower in myriad inscrutable ways. Many tree-to-tree tasks such as first-order simplification and higher-order reduction may leave most subtrees unchanged most of the time, potentially allowing the hypothesis that approximates the simplification function with the identity function to beat random significantly. Even worse, many of the higher-order reduction tasks are parameterized by the names and types of metavariables, but some of the terms generated may not include any variables at all; this could cause some datapoints to be included in multiple nominally-different tasks. On the other hand, the unseen task that the oracle performs worst on is list-length, and this red would likely be green if list-length were split into two tasks depending on the type of elements in the list, e.g. into a list-digit-length and list-bool-length. With these considerations in mind, we still consider the predominance of green to be encouraging at the very least.

5. Discussion

Oracle-guided decision programming extends nondeterministic programming, which goes back at least to McCarthy (1959a) in which John McCarthy proposed the \texttt{amb} (for “ambiguous”) operator, which is effectively the same as the \texttt{choose} operator we use in Dodona. Zabih et al. (1987) extended Scheme with \texttt{amb} to produce the nondeterministic Schemer language, with a built-in notion of dependency-directed backtracking. This feature could be productively adopted by Dodona and its descendants as well. Siskind & McAllester (1993) presented a nondeterministic extension of Common Lisp called Screamer while Andre & Russell (2002) made explicit the connection between nondeterminism and Markov decision problems. However, nondeterministic programming is of limited use in the absence of heuristics since most nondeterministic programs of interest will be \textit{a priori} intractable even with dependency-directed
backtracking. Our present work can be viewed in part as revisiting this classic work, where we use machine learning to provide heuristic guidance.

Our work has many parallels with probabilistic programming languages as well and is particularly inspired by the Scheme-based language, Church (Goodman et al., 2012). Nondeterministic programming can in principle be simulated in probabilistic programming languages by sampling all choices from a uniform distribution, implementing fail as observing an event of probability zero, and querying for the maximum a posteriori (MAP) estimate over all the choices. However, the inference algorithms adopted by probabilistic programming languages are not designed for such “hard” observations and most—specifically the ones based on local transitions—do not work at all on problems unless a random starting point is likely to be connected to a solution by a path of nonzero probability. Within probabilistic programming, many projects have used neural networks to amortize the cost of inference within certain families of models (Ritchie et al., 2016; Le et al., 2017), but none that we know of have attempted domain-agnostic variants. An analogue of our universal oracle could be devised for probabilistic programming languages where instead of learning which decisions are preferable the oracle learns which distributions to propose local transitions from.

The idea of improving sample efficiency by training on many tasks in parallel using a shared representation was articulated in the early 1990’s. Baxter (1995) suggested it as a new approach to machine learning: “Instead of only learning the task required, learn as many related tasks as possible.” Caruana (1995; 1997) showed the efficacy of the regime empirically and coined the phrase multitask learning, though Caruana credits Hinton et al. (1986) with the key insight that generalization may be improved by learning underlying (i.e. task-independent) regularities. Thrun & Pratt (2012) generalize the classic definition of learning given in Mitchell (1997) to this setting and call it learning to learn. Co-training on related tasks has become increasingly common over time and there have been many reports of improved generalization as a result; Collobert & Weston (2008) is a notable example while Zhang & Yang (2017) provides a recent survey. Our present work can be seen as taking the approach Baxter (1995) suggests to an extreme, by pooling over tasks spanning a computationally-universal family of decision problems.

The phrase zero-shot learning has been used in different contexts in the literature. For example, the recent survey Wang et al. (2019) defines it narrowly in terms of the multi-class classification problem where some classes lack labels during training. We use it more broadly to mean success on new
tasks not seen during training. Besides (Brown et al., 2020), the closest to our zero-shot results is perhaps Selsam et al. (2019), which showed that training on a single synthetic distribution of satisfiability problems permits zero-shot transfer to a suite of SAT problems encoding many diverse domains. Central to the approach of Selsam et al. (2019) was to use propositional logic as the shared representation for all the tasks, and as a result the applicability of their approach is severely limited by the inherent limitations of propositional logic. Our present work uses a higher-order, computation-ally universal language for the shared representation and thus has no such limitations.

Our universal policy differs significantly from the AI theory of universal artificial intelligence (Hutter, 2000). AIXI is a learning agent that operates in an environment that is unknown and not software-defined; it is universal in the sense that it maximizes expected future reward with respect to a universal prior probability distribution over environments, i.e. one that assigns nonzero probability to all computable hypotheses (Solomonoff, 1964a,b). In contrast, our oracle is given the source code of the environment, and is universal in the sense that it can provide heuristic guidance for whatever computable environment that it may be given.

Here we survey additional features that a more mature oracle-guided decision language may support. We already mentioned dependency-directed backtracking as developed by (Zabih et al., 1987). A second feature is partial evaluation. The idea is that after some choices have been made, it may be possible to simplify the continuation dramatically before making the next choice. While dependency-directed backtracking can be seen as a generalization of conflict-driven back-jumping in SAT solvers, partial evaluation can be seen as a generalization of unit propagation. A third feature is the ability to decode entire sequences in a single query. Currently Dodona may only query the oracle to decide among a finite set of possibilities; to produce a list, it must query the oracle many times in sequence, each time building the (only slightly evolving) continuation graph. A fourth, as mentioned in §2.1, is to support the full MDP family. We could support arbitrary real-valued rewards at arbitrary times in Dodona by introducing a new effect \( \text{reward <float>} \), though since this effect does not behave functionally, we would also need to introduce blocks into the language so that statements can be sequenced, i.e. using the \( \text{begin ...} \) form from Scheme. Stochastic transitions can be added by simply adding random primitives to the language. However, stochasticity would require additional meta-evaluators to handle effectively, as best-first search would no longer be as sensible an operation.

We have presented a simple, stand-alone oracle-guided decision language, but this paradigm can be realized in various ways within existing languages as well. Ignoring for a moment the challenge of producing embeddable representations of the continuations, nondeterminism can be simulated in a traditional language by writing programs that explicitly return choicepoints where the continuations are simply functions in the language. The catch is that this encoding requires an inordinate number of functions and is syntactically cumbersome in traditional languages. However, nondeterminism constitutes a monad, and this proliferation of functions is merely a special case of the proliferation of functions when using any monad, which the do notation pioneered by Haskell is designed to hide (Jones, 1995; Peterson et al., 1996). Embedding the continuation poses a different challenge though, since if the continuation is represented as a function in the base language, some significant meta-programming capabilities will be required to inspect it. In §3 we also made use of runtime type information to improve the embedding, e.g. by making the embedding for sets invariant to permutations of their elements.

The elephant in the room for oracle-guided decision programming is where will the training data come from? Language models have a clear advantage here: since the data is not required to have any particular structure or meaning, almost any data will do. The benefits of our approach that we discussed in §1 come at the cost of requiring that we be more selective about the data we train on. Ultimately we envision an OGDP framework embedded in a general-purpose programming language with large-scale formal mathematics, software verification, and program synthesis efforts all sharing an oracle. The oracle could also be seeded by training on synthetic data of all sorts. It would even be possible to co-train it as a language model as well, by postulating an opaque next-token function that takes some data representing provenance (e.g. the website the text was scraped from) along with the usual sequence of tokens indicating the context.

Lastly, although the present work has focused on the universality of the oracle, the broader paradigm of oracle-guided decision programming may also be useful when used with domain-specific oracles. The primitive choice can be extended with a second argument that represents some data that is to serve as input to a neural network of the user’s choice. Such a decision language could still provide value in the form of compositional ways of building MDPs and generic (oracle-parameterized) search procedures. Indeed, when using a domain-specific oracle, the MDP need not even be software-defined in the usual sense; the program could be specialized for each individual input, e.g. by including an empirically-observed label for that input.

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