Relative distributions of W’s and Z’s at low transverse momenta

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Despite large uncertainties in the $W^\pm$ and $Z^0$ transverse momentum ($q_T$) distributions for $q_T \lesssim 10$ GeV, the ratio of the distributions varies little. The uncertainty in the ratio of $W$ to $Z$ $q_T$ distributions is on the order of a few percent, independent of the details of the nonperturbative parameterization.

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I. INTRODUCTION

With the precise measurements of the $Z^0$ mass from LEP experiments, as well as other electroweak parameter measurements, the standard model of electromagnetic and weak interactions is being tested beyond tree level. Even without knowing the masses for the top quark and the Higgs boson, physical observables when compared to theoretical predictions constrain new physics. An improvement in the measurement of the $W$ mass, currently with a value of $80.22 \pm 0.26$ GeV from CERN and Fermilab Collider experiments, would even further those constraints. With the higher statistics expected in the 1993-1994 Fermilab Tevatron Collider runs, the theoretical and systematic errors must be also be reduced to make a significant improvement in the $W$ mass measurement. Even though the dependence on the $W$ transverse momentum ($q_T$) is not very strong in the $W$ mass sample, the QCD predictions for the low $q_T$ production of $W$’s, including nonperturbative effects, must be better understood.

The low $q_T$ behavior of $W$’s and $Z$’s has been the subject of much theoretical work. The Collins-Soper-Sterman formalism for the low transverse momentum distribution in Drell-Yan has been applied to $W$’s and $Z$’s by Davies, Stirling and Webber, Altarelli et al., and the matching of low and high $q_T$ by Arnold and Kauffman. A study of the parameterization of nonperturbative effects in the low $q_T$ region has recently been done by Ladinsky and Yuan. Theoretical uncertainties in the separate $W$ and $Z$ $q_T$ distributions are discussed in detail in Ref. Here, we discuss the relative uncertainties between the $W$ and $Z$ transverse momentum distributions. By considering the ratios

$$R(q_T, y) \equiv \left( \frac{d\sigma}{dq_T dy} (W^\pm) \right) / \left( \frac{d\sigma}{dq_T dy} (Z^0) \right),$$

and

$$R(q_T) \equiv \left( \frac{d\sigma}{dq_T} (W^\pm) \right) / \left( \frac{d\sigma}{dq_T} (Z^0) \right)$$

the theoretical errors are reduced because the uncertainties tend to cancel. By comparing these quantities with experimental results, one can test the theoretical treatment of low $q_T$
vector bosons. Alternatively, the ratios can be used in Monte Carlo models for the $W$ mass measurement.

The estimation of the theoretical uncertainties in $R(q_T, y)$ and $R(q_T)$ due to the parameterization of nonperturbative effects is the main topic of this paper. In Section 2, we review the theoretical issues associated with the low transverse momentum behavior of $W$’s and $Z$’s. Our numerical results are displayed in Section 3. Section 4 contains our conclusions. As all of the relevant formulae are collected in the Appendix of Ref. 8, we do not include detailed formulae here.

II. THEORETICAL ISSUES

The calculation of the low transverse momentum behavior of $W$’s and $Z$’s is not completely straightforward because of the presence of large logarithms associated with powers of $\alpha_s$. At low $q_T$, the perturbative expansion in $\alpha_s$ is ruined because of the accompanying large logarithms. However, it is possible to resum the leading and next-to-leading $\ln(Q^2/q_T^2)$ powers for each power of $\alpha_s$ to get an exponential factor. Collins, Soper and Sterman have done the resummation, 3 explicitly including transverse momentum conservation for multiple parton emission at each order in $\alpha_s$. The momentum conservation appears through a Fourier transform involving the conjugate variable to $q_T$, namely the impact parameter $b$. The resummation appears as a Sudakov-like exponent $S$, which is incorporated in a form factor, here called $\tilde{W}$. The quantity $\tilde{W}$ can be computed perturbatively over the range of $\Lambda_{QCD} \ll 1/b < Q$. Schematically

$$\tilde{W}_{\text{pert}}(b, Q, x_A, x_B) \sim \sum_{ij} e_{ij}(V)f_{i/A}(x_A, \mu)f_{j/B}(x_B, \mu) \exp[-S_{\text{pert}}(b, Q)].$$  (3)

The parton distribution functions $f_{i/A}$ are labeled for parton $i$ in hadron $A$. The full perturbative form for $\tilde{W}$ also includes convolutions of the parton distribution functions with other functions which are suppressed by $\alpha_s/(2\pi)$. The coupling constants $e_{ij}(V)$ depend on whether $V = W$ or $V = Z$. 3
Including a normalization parameter $N_V$, $V = W$ or $Z$, the differential cross section has the form

$$\frac{d\sigma}{dq_T^2 dy} = \frac{\pi\alpha}{3s} N_V \int_0^\infty d^2b \exp(i\vec{q}_T \cdot \vec{b}) \cdot \tilde{W}(b, Q, x_A, x_B, \mu)$$

in the low $q_T$ limit. Eqn. 4 applies only to the low $q_T$ limit because not included is the part of the cross section which is regular as $q_T \to 0$. Arnold and Kauffman have shown that the regular part of the differential cross section for $q_T^W$ and $q_T^Z$ has little effect below $q_T = 10$ GeV. As we shall see below, the nonperturbative contributions are sizable only for $q_T \sim 10$ GeV, so we consider this region of low $q_T$ and neglect the regular part throughout. The form factor $\tilde{W}$ depends on the impact parameter, the mass of the weak gauge boson and on $x_A = (Q/\sqrt{S})e^y$ and $x_B = (Q/\sqrt{S})e^{-y}$, where $\sqrt{S}$ denotes the hadron collider center of mass energy, and $y$ is the weak gauge boson rapidity. The form factor also includes the dependence on the parton distribution functions, the resummed terms, a nonperturbative factor and the unphysical factorization scale $\mu$.

For $b \leq 1/Q$, we roughly approximate $\tilde{W}$, since in the integral in eqn. 4, this range of $b$ contributes little for $q_T \sim 10$ GeV. However, for $b \gtrsim 1/\Lambda_{QCD}$, the perturbative expression for $\tilde{W}$ breaks down. A cutoff in $b$ is introduced in the standard way by

$$b \to b_* = \frac{b}{(1 + b^2/b_{\text{max}}^2)^{1/2}}$$

where $b_{\text{max}} \sim 1/Q_0$ is characterized by a scale $Q_0$ at which perturbation theory is still valid. The substitution of $b \to b_*$ in $\tilde{W}_{\text{pert}}$ is accompanied by a nonperturbative factor. Thus, $\tilde{W}$ defined over the full range of $b$ is written this way:

$$\tilde{W}(b, Q, x_A, x_B, \mu) = \tilde{W}_{\text{pert}}(b_*, Q, x_A, x_B, \mu) \cdot e^{-S_{np}(b, Q)}.$$
Not indicated in the formulae above is an additional dependence on three unphysical constants denoted \( C_1, C_2 \) and \( C_3 \) in Ref. [8]. Arnold and Kauffman showed that the dependence of \( d\sigma/dq_T \) for \( W \)'s and \( Z \)'s on these three parameters is weak for a range of parameters. [8]

The nonperturbative factor \( S_{np} \) has the form:

\[
S_{np}(b, Q) = G_1(x_A, x_B, b, b_{max}) + G_2(b, b_{max}) \ln(Qb_{max}) .
\]

(7)

In principle, \( G_1 \) depends on the incoming parton type and is hadron dependent. Following Davies, Stirling and Webber [6], we will assume that \( G_1 \) is independent of parton type and independent of \( x_A \) and \( x_B \). Ladinsky and Yuan include a dependence on \( x_A \) and \( x_B \) in their parameterization, [9] which we comment on below. The standard assumption is that \( G_1 \) and \( G_2 \) are proportional to \( b^2 \), [5,6] and \( S_{np}(b, Q) \) is written in terms of constants \( g_1 \) and \( g_2 \):

\[
S_{np}(b, Q) = (g_1 + g_2 \ln(Qb_{max}/2))b^2 .
\]

(8)

This is, at large \( b \), roughly equivalent to a Gaussian distribution of intrinsic momentum. Ignoring the \( b \) dependence in \( \tilde{W}_{\text{pert}} \), one is left with the Fourier transform of a Gaussian, which is itself a Gaussian. The average transverse momentum squared in the Gaussian is

\[
\langle q_T^2 \rangle = 4[g_1 + g_2 \ln(Qb_{max}/2)].
\]

Davies, Stirling and Webber [6] have performed a fit to ISR (R209) and Fermilab (E288) fixed target data. Their numerical values for \( g_1 \) and \( g_2 \) are:

\[
\begin{align*}
g_1 &= 0.30 \text{ GeV}^2 & g_2 &= 0.16 \text{ GeV}^2 \quad (np1) \\
g_1 &= 0.15 \text{ GeV}^2 & g_2 &= 0.41 \text{ GeV}^2 \quad (np2) \\
g_1 &= 0.0 \text{ GeV}^2 & g_2 &= 0.60 \text{ GeV}^2 \quad (np3)
\end{align*}
\]

with \( b_{max} = 0.5 \text{ GeV}^{-1} \). The nonperturbative parameterization \( np2 \) is their preferred fit. These parameters were used by Arnold and Kauffman in their analysis of theoretical errors in \( d\sigma/dq_T \).

Evident from the discussion so far is the fact that the the distributions for \( Q = M_W \) and \( Q = M_Z \) will not be very much different apart from the overall normalization factors and
the combination of parton distribution functions, evaluated at slightly different values of $x_A$ and $x_B$. We now proceed to consider the numerical results.

## III. NUMERICAL RESULTS

In obtaining the numerical results, we have used $M_W = 80$ GeV and $M_Z = 91$ GeV. The weak mixing angle was taken such that $\sin^2 \theta_W = 0.22$. To compare nonperturbative parameterizations, we have used the Harriman et al. \cite{Harriman:2003db} parton distribution functions HMRSB with the four flavor \overline{MS} scale $\Lambda_{QCD} = 0.19$ GeV, based on a fit to the BCDMS data. To compare the effect of a change in parton distribution functions on $R(q_T, y = 0)$, we also use the HMRSE distribution functions fit to the EMC data with $\Lambda_{QCD} = 0.10$ GeV. As in Ref. \cite{Gottfried:1980mu}, the canonical choices of $C_1 = C_3 = 2e^{-\gamma_E} \equiv b_0$ and $C_2 = 1$ are used. Here, $\gamma_E$ is the Euler constant. As indicated above, we neglect the regular part of the differential cross sections, which contribute on the order of less than 1% in the range of $q_T < 10$ GeV. \cite{Gottfried:1980mu} All of the figures show results for the Tevatron Collider at $\sqrt{S} = 1.8$ TeV. Similar results obtain for the CERN Collider.

### A. Nonperturbative parameters $g_1$ and $g_2$

The largest uncertainty for $q_T \lesssim 6$ GeV comes from the choice of parameters $g_1$ and $g_2$ in eqn. (8). Figure 1 shows the distribution $d\sigma/(dq_Tdy)$ at $y = 0$ as a function of $q_T$ of the $W$, including both charges of the $W$ in the rate. The three predictions vary from the average of the three curves by as much as 40% for very low $q_T$. For $q_T \gtrsim 6$ GeV, the the sensitivity to the nonperturbative parameterization is significantly less. Unfortunately, it is precisely below $\sim 6$ GeV where the differential cross section is the largest. A similar uncertainty is obtained for the $Z^0$ transverse momentum distribution as a function of the nonperturbative parameterization.

Because the $W$ and $Z$ masses are close and there is only log dependence on the masses, the nonperturbative parameterization $S_{np}(b, Q)$ and perturbative exponent $S_{pert}(b_*, Q)$ are
nearly identical for \( Q = M_W \) and \( Q = M_Z \). One expects that the ratio of the \( W \) to \( Z \) transverse momentum distributions will be approximately constant for small \( q_T \). Indeed, this is the case. Figure 2 shows the relative size of the \( W \) to \( Z \) distributions at \( y = 0 \) for \( np1 \) \(-\) \( np3 \). Qualitatively, the size of \( R(q_T, y = 0) \) can be understood from a combination of coupling constants and parton distribution functions. Consider the quantity

\[
    D_V(x_A, x_B, \mu) = \sum_{ij} e_{ij}^2(V)f_{i/p}(x_A, \mu)f_{j/\bar{p}}(x_B, \mu)
\]

By taking

\[
    D_W(M_W/\sqrt{S}, M_W/\sqrt{S}, M_W)/D_Z(M_Z/\sqrt{S}, M_Z/\sqrt{S}, M_Z),
\]

one obtains the value of 3.04. If one were to evaluate the \( W \) and \( Z \) parton luminosities at the same values of \( x_A \) and \( x_B \), the ratio would equal 2.34, as a consequence of their different coupling constants. Deviations of the \( np1 \) \(-\) \( np3 \) curves from the average value \( R_{\text{avg}}(q_T, y = 0) \) are less than a percent.

To understand the increase in \( R \) as \( q_T \) decreases, it is useful to look at \( \tilde{W} \) for \( W \)’s and \( Z \)’s, which is independent of \( q_T \). In Figure 3a, we show the form factors \( \tilde{W}(b, Q, x_A, x_B, b_0/b_\ast) \) with \( x_A = x_B = Q/\sqrt{S} \) for \( Q = M_W \) and \( Q = M_Z \), and with the nonperturbative parameters \( np2 \), as a function of \( \ln(b \cdot \text{GeV}) \). The low \( b \) behavior is an extrapolation of the value of \( \tilde{W} \) at \( b = 1/Q \), to lower values of \( b \). The integral over \( b \) of \( \tilde{W} \) also includes a factor of \( bJ_0(bq_T) \), so the differential distribution is fairly insensitive to such small values of \( b \) where the extrapolation must be performed. For large values of \( b \), the nonperturbative parameterization cuts off the integral. Figure 3b shows the ratio of the form factors \( \tilde{W} \), for the three sets of nonperturbative parameters. We cut off the range of \( \ln(b \cdot \text{GeV}) \) displayed because at low \( b \), the shape of the ratio is an artifact of the low \( b \) extrapolation. The curves for \( np1 \) \(-\) \( np3 \) separate at \( b \sim 10^{-1}\text{GeV}^{-1} \), because of the mass dependence in \( S_{np} \), with the largest ratio belonging to \( np3 \).

At low \( q_T \), the average value of \( b \) is for “large” \( b \). For example, for \( W \)’s at \( q_T = 0.5 \) \text{GeV}, \( \langle b \rangle = 0.8 \text{GeV}^{-1} \). As \( q_T \) increases, the Bessel function in the integral oscillates more
rapidly as a function of $b$, and $\langle b \rangle$ decreases, moving into the perturbative range. Thus, as $q_T$ increases, the sensitivity to nonperturbative physics decreases and the calculation is more reliable. Figure 3b also shows the trend that $R(q_T)$ and $R(q_T, y)$ decline with increasing $q_T$. If one trusts the form of the nonperturbative parameterization, then the uncertainty in $R(q_T, y = 0)$ is less than a few percent. Even if one estimates the uncertainty in $R$ by the deviation from the ratio of coupling constants and parton distribution functions, one is still left with a small uncertainty due to nonperturbative parameterizations of order 5%.

Figure 4 shows the ratio $R(q_T)$ (where now the rapidity of the gauge boson has been integrated) for $np_1 - np_3$, as a function of $q_T$. The shapes of the distributions are nearly identical to those of Figure 2, however, with an increase in the normalization. This is due to the fact that the ratio of the $W$ to $Z$ parton luminosities increases with increasing $|y|$. We have the same qualitative conclusions about the uncertainty in the ratio $R(q_T)$ as in the $y = 0$ case.

**B. The Ladinsky-Yuan parameterization**

The Ladinsky-Yuan parameterization [9] is characterized by three parameters and depends on $S$ as well as $Q$. In a recent reanalysis of the R209 and E288 low $q_T$ Drell-Yan data, Ladinsky and Yuan (LY) have postulated a modified form for $S_{np}$:

$$S_{np}(b, Q, x_A, x_B) = g_1 b^2 + g_1 g_3 b \ln(100 x_A x_B) + g_2 b^2 \ln(Q b_{\text{max}}/1.6)$$  \hspace{1cm} (11)

The inclusion of the $x_A$ and $x_B$ dependence is to account for changing $\langle q_T^2 \rangle$ as a function of $Q^2/S$. The $W$ and $Z$ $q_T$ distributions have peaks at slightly higher $q_T$ values than in the two parameter cases. We use their central values of $g_1$, $g_2$ and $g_3$ to characterize the effects of this parameterization on $R(q_T, y)$ and $R(q_T)$. Their central values are $g_1 = 0.11 \text{ GeV}^2$, $g_2 = 0.58 \text{ GeV}^2$ and $g_3 = -1.5 \text{ GeV}^{-1}$, and their fit was performed using the CTEQ parton distribution functions. [11] As we are interested primarily in the ratios, we continue to use the HMRSB parton distribution functions. The LY curve for $R(q_T)$ at $\sqrt{S} = 1.8 \text{ TeV}$ is
shown as the dotted line in Figure 4. The deviation from the two parameter result is less than a few percent.

C. Parton distribution dependence

Figure 5 shows the ratio of the $W$ and $Z$ $q_T$ distributions for two different parton distribution functions. We show $R(q_T, y = 0)$ for the HMRSB and HMRSE parton distribution functions with $np2$. The HMRSE distributions yield the slightly lower value of $R$, but the two predictions for $R_{\text{avg}}$ are within 2% of each other at fixed $q_T$.

IV. CONCLUSIONS

The values of $g_1$ and $g_2$ determined by Davies et al. are clearly not the best current values because of the advances in our knowledge of the parton distribution functions. Ladinsky and Yuan, by refitting the data with current parton distributions, have made a significantly better choice. Nevertheless, uncertainties still exist in the form of the parameterization, affecting the extrapolation from $Q \sim 5$ GeV to $Q \sim 80 - 91$ GeV. Parton distribution uncertainties at large $Q$ compound the uncertainties.

We have shown that by taking the ratio of $W$ to $Z$ $q_T$ distributions, the theoretical uncertainties due to the parameterization of nonperturbative effects tend to cancel. For the range of $q_T < 10$ GeV, the ratio $R(q_T)$ is $\sim 3.3$, slightly higher or lower depending on $q_T$. When one takes the ratio of the total cross sections obtained by integrating the resummed, matched differential cross sections for $np2$, one obtains $R = 3.27$. $R$ is less than 3.3 because with increasing $q_T$, $R(q_T)$ declines roughly linearly, with $R(q_T = 100 \text{ GeV}) = 2.66$.

In our focus on the nonperturbative effects, we have neglected the regular part of the differential cross section which comes in at the percent level. A full study would also include an analysis of the other parameter dependence described in the text, e.g., $C_1, C_2, C_3, b_{\text{max}}$.
etc. We expect, however, that in the ratios $R(q_T, y)$ and $R(q_T)$ the dependence should largely cancel.

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[12] See Table 1 in Ref. \[.\]
FIGURES

FIG. 1. The distribution $d\sigma/(dq_Tdy)$ at $y = 0$ as a function of $q_T$ of the $W$ for the sum of $W^+$ and $W^-$ production at $\sqrt{S} = 1.8$ TeV, with nonperturbative parameterizations $np1$ (solid line), $np2$ (dashed line) and $np3$ (dot-dashed line).

FIG. 2. $R(q_T, y = 0)$ defined in eqn. 1 for $np1 - np3$, as a function of $q_T$ for $\sqrt{S} = 1.8$ TeV.

FIG. 3. a) $\tilde{W}_{\text{pert}}(b_*, Q) \exp(-S_{np}(b, Q))$ with $y = 0$, $\sqrt{S} = 1.8$ TeV and $\mu = b_0/b_*$, as a function of $\ln(b \, \text{GeV})$. The solid line is for $Q = M_W$ and the dashed line is for $Q = M_Z$. b) Ratio of $\tilde{W}$ for $W^\pm$ to $\tilde{W}$ for $Z^0$ production, as a function of $\ln(b \, \text{GeV})$, for $np1$ (solid line), $np2$ (dashed line) and $np3$ (dot-dashed line).

FIG. 4. $R(q_T)$ defined in eqn. 2 for $np1 - np3$, (solid, dashed and dot-dashed lines) as a function of $q_T$ for $\sqrt{S} = 1.8$ TeV. The dotted line indicates $R(q_T)$ for the Ladinsky-Yuan parameterization.

FIG. 5. $R(q_T, y = 0)$ for the HMRSB (solid line) and HMRSE (dashed line) parton distribution functions for $\sqrt{S} = 1.8$ TeV and $np2$. 
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