A Comprehensive Study of Past, Present, and Future of Spectrum Sharing and Information Embedding Techniques in Joint Wireless Communication and Radar Systems

Muhammad Fahad Munir, Abdul Basit, Wasim Khan, Ahmad Saleem, and AbdulRahman Al-salehi

Faculty of Engineering & Technology, International Islamic University, Islamabad, Pakistan

Correspondence should be addressed to Abdul Basit; abdulbasit@iiu.edu.pk

Received 22 December 2021; Accepted 1 March 2022; Published 28 March 2022

Wireless spectrum is a limited resource, and the rapid increase in demand for wireless communication-based services is increasing day by day. Hence, maintaining a good quality of service, high data rate, and reliability is the need of the day. Thus, we need to apportion the available spectrum in an efficient manner. Dual-Function Radar and Communication (DFRC) is an emerging field and bears vital importance for both civil and military applications for the last few years. Since hybridization of wireless communication and radar designs provoke diverse challenges, e.g., interference mitigation, secure mobile communication, improved bit error rate (BER), and data rate enhancement without compromising the radar performance, this paper reviews the state-of-the-art developments in the spectrum shared between mobile communication and radars in terms of coexistence, collaboration, cognition, and cooperation. Compared to the existing surveys, we explore an open research issue on radar and mobile communication operating with mutual benefits based on collaboration in terms of spectrum sharing. Additionally, this paper provides important perspectives for future research of DFRC technology.

1. Introduction

To guarantee a high data rate and improved quality of services, increase in the bandwidth is mandatory for wireless communication. Therefore, increased wireless communication applications with aforementioned properties have caused the auction price of available frequency spectrum to a tremendous rise [1–10]. Since the increased data rate communication requirements have enforced the network providers to ponder upon the reuse of available spectrum, which has been currently allotted to other technologies [11], the radar spectrums are the best candidates to be imparted for different communication systems because huge slabs of the spectrum can become easily accessible at radar frequencies [12]. The conventional radar applications around the world include air traffic control (ATC), geophysical checking, climate perception, and exploration for safeguarding and security [13]. However, the radars used for monitoring purposes are, generally, utilized for the communication by sharing the spectrum [14–16].

Currently, the allotted frequency band can be divided into two broad categories, i.e., radar system and a communication system. However, a significant percentage of frequency bands from 1 to 10 GHz has been mainly distributed among radar operations, yet new cohabitation options of the radar with communication systems, e.g., 5G NR, LTE, and Wi-Fi, lead to new directions [17]. On one hand, sharing the high frequency, e.g., millimeter wave band, benefits both communication and radar platforms for high data rates and improved tracking the targets, respectively, but on the other hand, interference issues have raised the concerns from both military and civilian applications for critical radar operations.

In the recent decades, the radars have evolved with good precision and increased list of capabilities that include multifunctioning such as surveillance, tackling of clutter, and back
scanning as well as dealing with false alarm, simultaneously [18]. Therefore, it requires higher frequency bands as compared to traditional radars. Moreover, the growth of civil activities and the emergence of new technologies in social media have raised dramatically, which also put strong pressure on bandwidth allocation board.

Since higher bandwidth for radar as well as communication stand-alone designs is the need of the day, it also warrants the hybridization of both designs (i.e., radar and communication) for getting joint benefits at higher bandwidths. However, all the aforementioned challenges including the identification of frequency bands that could be made available for wireless broadband are to be taken care of for improved overall performance of joint radar and communication designs. International Telecommunication Union (ITU) and World Radio Communication Conference (WRC) review the allocation of the frequency spectrum annually [19], and in United States of America, the National Telecommunications and Information Administration (NTIA) [20] had dedicated its energies to identify frequency bands that could be made available for wireless broadband service provision alongside with radar operations.

In [21–26], it has been reported that in L band, GSM system (GPRS and EDGE) can overlap UHF radars that operate between 1 to 2 GHz, whereas in S band, Long-Term Evolution (LTE) and WiMax overlap with Airport Surveillance or Air Traffic Control (ATC) radar with frequency range between 2 and 4 GHz. Other examples of WiMax and radar overlap are mentioned by [27]. Finally, for millimeter waves, which are used for orthogonal frequency division multiplexing (OFDM), single carrier, WLAN, ranges 11 ft to 33 ft, used for indoor communication overlaps with high-resolution imaging radar. Similarly, the same OFDM-based Wireless LAN (WLAN) used for outdoor activities ranges from 100 m to 5 km using OFDM overlap with weather radar that operates between 2 and 4 GHz in C band. Thus, to fulfill the need of extra bandwidth for wireless communication, one must work on idea of spectrum sharing. This will help us economically as well as politically and socially, while understanding of this concept is unavoidable in near future [28]. Therefore, innovative way out of the effective and reasonable spectrum sharing is needed [19, 29].

In this paper, we review the recent trends in DFRC design with focus on data embedding techniques. We start with reviewing the different spectrum-sharing techniques and then explain the DFRC data model. Then, we discuss the different methods of embedding information in radar waveforms like amplitude and phase shift keying, phase rotation invariance method, and time modulation arrays. After that, we explore the various methods of modifying the communication waveform to facilitate radar functions. Then, we briefly discuss beampattern modulation techniques like subbeam sharing. Additionally, we compare the performance of the diverse DFRC information embedding techniques in terms of interference mitigation, secure communication, improved bit error rate (BER), and data rate enhancement. Finally, we list some of the research challenges in the field and provide directions for future research.

Rest of the paper is organized as follows. Section 2 provides details about the basic spectrum sharing concepts that have evolved with time. In Section 3, data model is presented and methods commonly used for DFRC designs are discussed in detail. Section 4 focuses on information embedding methods via radar waveforms. In Section 5, information embedding by using communication waveform is discussed. Beampattern-based methods are explained in Section 6. Challenges and future work are discussed in Section 7. Conclusion is presented in Section 8.

2. Spectrum Sharing Approaches for DFRC

It is obvious that crowding of the spectrum cannot be addressed by the traditional communication techniques and beamforming approaches [30–33]. Thus, coexistence of both the radar and communication design is highly required for the radar emission and communication on the available spectrum [34–46]. In broader sense, this coexistence can work either by time-based sharing or embedding information into radar emission for a same spectrum. In the following subsections, we provide the details about integrating radar–communication designs and diverse spectrum sharing methods.

2.1. Time Sharing Approach. One of the simple methods to integrate radar with communication is to use time-based sharing of resources [47–59]. A strobe switch is used for switching between radar and communication users [60] as shown in Figure 1. Note that, when a communication process is required, the strobe switch will turn off the radar operation and allows communication modem to transmit and receive and vice versa for the radar operation. For this purpose, some protocol need to be devised so that enabling and disabling communication link do not affect the radar performance. This type of system is also known as radar system joined with communication [60]. Design of such system is simple but it degrades the radar efficiency. The radar operation utilizes the frequency modulated continuous wave (FMCW), while communication symbol is encoded by using either ASK, FSK, or PSK, while higher order schemes can also be used, e.g., QAM. The limitation of this technique is the reduced efficiency and less time slot allocation for radar and communication users, respectively. Thus, this technique is only feasible when small burst of data is used.

The authors in [61] used different waveforms for radar and communication to mitigate the interference issues. $M_T$ transmitted signal can be written as

$$s(t) = s_r(t) + s_c(t),$$

where $s_r(t)$ is radar-based waveform while $s_c(t)$ is communication-based waveform, respectively. The system resources are distributed between radar and communication in time domain, which results in performance degradation [62]. To allocate the resources in different time domains by using nonoverlapping frequency is studied in [63], while random antenna switching in [64] and media access control in [65]. Similarly, subcarrier approach of OFDM in time.
sharing is also implemented in [66], and [67] optimized the subcarrier by using a-priory knowledge target and statistical model is studied.

2.2. Spectrum Sharing Approach. Contrary to the previous approach, both radar and communication operations work simultaneously in spectrum sharing all the time [68–86] as shown in Figure 2. This sharing leads us to the new paradigm of research, which is called as communication and radar spectrum sharing (CRSS) or integrated sensing and communication (ISAC). This CRSS is further divided into two broad categories, one is temporal coexistence or joint radar and communication coexistence, while the other is DFRC.

2.2.1. Joint Radar and Communication Coexistence. This is the first type of the CRSS which is also called as radar-communication coexistence (RCC), where the same frequency band is used to perform radar operations as well as communication operation. Furthermore, it does not require common transmitting hardware as shown in Figure 3. Therefore, this type of a system is also called an opportunistic system or joint radar and communication (JRC). In this technique, the spectrum sensing (i.e., radar operation) acts as a primary user, while the communication operations are performed on secondary basis. Authors of [87] divided the concept of JRC into coexistence, cooperation, and codesign.

In coexistence, interference is mitigated without exchanging information [87], while in cooperation, information is explicitly shared with beneficiaries, i.e., radar and communication system [68]. In later case, both systems improve the system performance by using shared knowledge. Interference is mitigated jointly, and this level of collaboration is the first step towards joint architecture and attempting RF convergence. This branch leads towards cognitive behaviour between radar and communication system, which is explained in the next section.

Eventually, in codesign, both systems need to be designed jointly. There are two possibilities, either by using the same hardware at transmitter side or by using the same waveform [89]. To avoid interference, the communication devices need to keep their power levels under certain threshold levels. Authors in [90–92] studied that communication is only allowed if it will not jeopardize the radar operation. The performance merit for communication is set under following three constraints, which are (i) average data rate for small distance between radar and communication receiver, (ii) half duplex communication only, and (iii) percentage of time that communication user can transmit. Similarly, in [93], another scenario is studied where the radar senses the entire spectrum periodically, while cellular base station (BS) is only allowed to communicate, if it lies in sidelobes of the main radar beam. In this case, the performance will be evaluated by keeping the minimum tolerable distance in terms of interference to noise ratio (INR). The main drawback of this technique is that the communication receiver cannot fully utilize the spectrum, and it must keep its power below certain levels. Misdetection due to spectrum sharing between American integrated naval weapon system and with a cellular system including 100 base stations operating at 3.5 GHz in S Band, which raised the interference management and power allocation, respectively, is investigated in [94]. Thus, to improve the abovementioned drawbacks, authors in [95] studied the regulatory policies for 10 GHz band where sharing occurs between radar and communication users in terms of sensing and relocation techniques. In the same line of action, [96] discussed sharing the spectrum with rotating radar in detail and authors in [97, 98] studied the spectrum sharing when the distance between radar and communication is fairly large given that performance is not affected. Another study is carried out by [99] to investigate the performance of shared spectrum in L band for rotating radar and fixed communication user. In their study, communication user is supposed to limit its transmit power when it senses the radar main beam.

Precoder-based design is another solution to this problem, which uses interference channel state information (ICSI). In this case, radar transmitter first estimates the information of pilot signal being transmitted by the communication receiver and maintains a given ratio of INR [100]. This technique mitigates the interference efficiently. However, it can only be applied in scenarios where radar has primary privileges. Another drawback to this scheme is its computational complexity cost at the radar transmitter. Likewise, the communication receiver first identifies the mode of the radar (i.e., either probing or scanning) and then starts its communication. The other solution is to build an efficient receiver for improved interference mitigation. The primary task of such receiver is to evaluate the target parameters in existence of the BS interference. This receiver can either be at the radar side or at BS side. In [41], null-space precoder-based approach is used on radar.
waveform by using singular value decomposition (SVD); this results in zero-forced interference to communication user. The system is further improved by using optimization-based techniques in precoder design in [38] and [101] in the presence of clutter. The overall drawback of aforementioned precoder-based techniques is they rely on the knowledge of the interfering channel between the radar and the communication user. One way is to have such information is by sending training signal by DFRC transmitter to all communication receivers or by getting through coordination office connected to both radar and communication system [38]; this burdens the system in terms of spectrum and computational complexities. It is concluded by [102] that those communication users who are trying to obtain the spectrum initially allocated to radar must guarantee the target detection rather than obtaining ICSI.

2.2.2. Cognitive Radar and Communication. In contemplation to provide the dual nature to devices, price may be reduced, and spectrum can be managed efficiently. To balance the requirements of both systems in terms of interference, adaptability shall be provided to communication receiver in order to sense the radar environment and modify radars to sense the communication as well. Cognitive behaviour needs to be implemented for both the transmitter side and the receiver [103–107]. Thus, by upgrading the radars and communication users to do cognition, i.e., ability to take their own decisions according to needs, has been warranted. However, without any central administration, all the networks will face the chaos and congestion.

Note that modified version of JRC having cognition between radars and multiple communication users is discussed in [108] where they used centralised control to assign threshold to power levels to communication users as shown in Figure 4.

In [107], similar ideas have been used for the DFRC network as shown in Figure 5. In this approach, a single hardware is used to transmit dual nature waveform which benefits both radar operations and communication users. Similarly, efficient utilization of the bandwidth by using dynamic frequency allocation has been studied in [36–109]. In [110, 111], applied Lagrangian optimization techniques have been used to obtain solutions for power allocation to the communication user. Moreover, [112] studied the dynamic allocation of power to communication users keeping the satisfactory threshold level for transmission. In [113], authors investigated that the communication user can adaptively adjust its transmit power to maximize the data rate. Optimal power control and adaptive data rate were
proposed in [114, 115] to maximize the communication users’ capacity keeping average interference power and peak transmission power constraints. Additionally, fast power allocation to communication user has been studied in [116] with low computational complexity constraint to achieve the optimal solution.

2.2.3. Dual-Function Radar and Communication (DFRC).

The other major branch of CRSS is DFRC [117–120]. In this category, the radar and communication systems work on a single hardware at the transmitter side for both functions as shown in Figure 6. This technique is also called as intentional modulation on a pulse [121] or a coradar [122].

This joint approach provides efficient utilization of power [70], less weight, and reduced system size at one hand and provides compatibility to avoid spectrum congestions on other hand as studied by [63–124]. The DFRC system performs radar operations and communication task simultaneously by using dual-nature waveform. The overarching objective of the DFRC is to utilize radar spectrum to capitalize on the resources by using existing infrastructure. These resources may include multisensor beamforming, high-power and high-gain antennas, and large bandwidth.

Keeping in mind the smart nature of the DFRC to sense nearby environment, these are employed in synthetic aperture radar system (SAR) designs, vehicle to vehicle (V2V), vehicle to network (V2N), vehicle to pedestrian (V2P), and vehicle to cloud (V2C) communication designs as mentioned by [125]. These features enable the DFRC to be the best suitable candidate for the vehicular network applications. The successful communication is mainly aimed at maximizing the data rate by embedding information in the transmitted waveform [126], while radar waveforms focus to maximize detection performance [127–133]. Thus, a dual-function system that performs radar and communication operations simultaneously involves a performance trade-off between these functions. Three different methods are devised to embed information bits in waveform of a DFRC transmitter side. 1st is embedding communication bits into radar waveform [134–136] while 2nd is using communication waveform for radar operations as well [137, 138], and 3rd is by using beamforming-based approaches [139, 140] as shown in hierarchical structure Figure 7.

The DFRC design has been under study for almost a decade, yet which scheme suits the situation is still a challenging task. Therefore, the DFRC researchers aim to benefit from the bond of knowledge hoarding in the communication literature and trailblazing radar techniques.

To flourish the DFRC systems, researchers needed to devise signaling strategies vigorously and materialize the modulation schemes of the radar that would lead to integrate and improve the use of the existing radio spectrum.

The following section presents an overview of DFRC systems from the information-embedding perspective in terms of data model, data rate, computational complexities, and radar capabilities. Note that it discusses the various techniques and implementation strategies that define the state of the art DFRC designs.

3. DFRC Data Model

Consider a DFRC system with uniform linear array (ULA), having an interelement spacing to half of the wavelength, which is used for both transmission of radar pulse and communication symbols, simultaneously. The number of antenna elements at DFRC transmitter is $M_T$. We have one radar receiver array which has the same configuration as a transmitter with $M_R$ antenna elements. Also, we consider one communication receiver having antenna element $N_g$, which is located at some arbitrary location in far field. The DFRC transmitter and radar receiver are placed close to each other, such that both DFRC transmitter and radar receiver observe the same spatial angle, and size of DFRC transmitter array and radar receiver array needs to be same, while the size of communication receiver antenna array may be different.

The primary task of the dual-function transmitter array is to generate pulses for target tracking in an efficient way, while the secondary objective is to embed the information bits without effecting the radar operation. The vector form of the baseband signals at the input of the transmit antennas is given as

$$s(t, \tau) = \sum_{g=1}^{G} \lambda_g(\tau) w_{g}^* \psi_g(t),$$

where $t$ represents the time within each pulse while $\tau$ represents pulse number $\psi_g(t)$, $t = 1, 2, 3, \cdots$ are $G$ orthogonal waveforms, and $w_{g}$, $g = 1, 2, 3, \cdots$ weight vector $\langle . \rangle^*$ denotes the complex conjugate.

It is assumed that waveforms must fulfill the conditions of orthogonality with no time delay and can be written mathematically as $\int_T \psi_g(t) \psi_g^*(t) dt = 0, g \neq g'$, where $T$ is the pulse width. Let us assume $I$ far-field targets within the radar main beam, and the vector of baseband signals received by the radar is expressed as

$$x(t, \tau) = \sum_{m=1}^{M} \beta_m(\tau) \left( a^T(\theta_m) s(t, \tau) \right) b(\theta_m) + e(t, \tau) + n(t, \tau),$$
where $\beta_m(\tau)$ is the reflection coefficient of the $m$th target, $b(\theta_m)$ is the steering vectors in direction of $\theta_m$, $a(\theta_m)$ is the steering vectors in direction of $\theta_m$, $e(t, \tau)$ is the interference vector, and $n(t, \tau)$ is the AWG noise with variance $\sigma^2 I$.

Let us consider $L$ communication receivers which are located somewhere within the sidelobe region. It is assumed that the dictionary of orthogonal waveform used at the transmitter is known to each of the communication receivers. Assume that the $j$th communication receiver with arbitrary linear shape antennas receives the following baseband signal.

$$y_j(t, \tau) = \alpha_j c_j (\psi_j) (a^T (\theta_j) s(t, \tau)) b(\theta_m) + n_j(t, \tau), \quad (4)$$

where $\alpha_j$ is the channel coefficient constant from transmitter array towards $j$th communication receiver, $c_j (\psi_j)$ is the steering vector from receive array in direction of $\psi_j$ communication receiver, $n_j(t, \tau)$ is the AWG noise with variance $\sigma^2 I$, and $(\psi_j)$ is the direction of $j$th communication receiver.

4. Radar Waveform for Information Embedding

Information bits are embedded into radar waveforms. Radar operation is performed in the main lobe, while the communication receiver operation is performed only in the sidelobe regions [141]. In order to embed information into radar waveform-based DFRC, we review the following techniques.

4.1. ASK-Based Method. Most popular among all methods for information embedding in radar waveform is ASK-based waveform design [123] in which the communication bits are mapped to the sidelobe levels of the received signal at communication receiver. Two sets of weight vectors are used for this purpose. If the received signal has higher power than predefined threshold $\varepsilon$, it will be considered as binary one, and if the received power is below a certain threshold, it is considered as a binary zero as shown in Figure 8.

$$\text{sidelobe level} = \begin{cases} \Delta_H = |w^H a(\theta_j)| \geq \varepsilon, \\ \Delta_L = |w^H a(\theta_j)| < \varepsilon. \end{cases} \quad (5)$$

The overall form of a radar waveform with information embedded to it at the transmitter of the DFRC is

$$s(t, \tau) = \sqrt{MT \sum_{g=1}^{L_s} (B_L(\tau)w_1^* + (1 - B_L(\tau))w_H^*) \psi_g(t)} + n_j(t, \tau), \quad (6)$$

Similarly, for the communication receiver, we have the following signals (see Equation (7), next page top)

$$y_j(t, \tau) = \begin{cases} \sqrt{MT \sum_{g=1}^{L_s} [B_L(\tau)w_1^* a(\theta_j) + (1 - B_L(\tau))w_H^* a(\theta_j)] \psi_g(t) + n_j(t, \tau)}, & B_L(\tau) = 1, \\ \sqrt{MT \sum_{g=1}^{L_s} [B_L(\tau)w_1^* a(\theta_j) + (1 - B_L(\tau))w_H^* a(\theta_j)] \psi_g(t) + n_j(t, \tau)}, & B_L(\tau) = 0. \end{cases} \quad (7)$$

By performing a simple ratio test, we obtain

$$B_L(\tau) = \begin{cases} 0, & \text{if } |y_j(t, \tau)| \geq T, \\ 1, & \text{if } |y_j(t, \tau)| \leq T, \end{cases} \quad (8)$$

where $T$ is the threshold.

ASK is used to modulate the data in sidelobes [142]. Using the same analogy in [143] for multiwaveform, multi-waveform designed multiwaveform for multiwaveform-based DFRC, we review the following techniques.

![Figure 8: Amplitude shift keying-based information embedding [123.](image)](image)
information embedding is achieved by two methods; first is by use of time modulated array (which is explained in subsequent section later), while the second approach is based on the convex optimization. That is, \( K \) distinct SLL are achieved by solving a convex beamforming problem to obtain weight vectors. During each radar pulse, one of the \( k \)th weight vector is utilized to transmit signal, where each weight vector represents unique binary symbol. Moreover, a generalized side-lobe canceller method is implemented in [144], by using both the active and the listening modes of the radar. In active mode, mainlobe is used for radar operations while sidelobes were dedicated for communication. In listening mode, there is no radar operation and entire duration is dedicated for the communication. Eight SLL are achieved by the author as shown in Figure 9.

Two beampatterns with the same power in the mainlobe are used for radar operations while variable sidelobes are used to accommodate four communication receivers. The communication receivers were located at \( \theta_{c1} = -60^\circ \), \( \theta_{c2} = -40^\circ \), \( \theta_{c3} = 40^\circ \), and \( \theta_{c4} = 60^\circ \). Red beampattern represents binary zero, while blue beampattern represents binary one, respectively. The first beampattern having red color has four sidelobe levels starting from SLL\(_1\) = −6dB, SLL\(_2\) = −7dB, SLL\(_3\) = −10dB, and SLL\(_4\) = −5dB, respectively. Similarly, for the 2nd beampattern, blue colors line have the following SLL which are SLL\(_{1}\) = −11dB, SLL\(_{2}\) = −9dB, SLL\(_{3}\) = −8dB, and SLL\(_{4}\) = −12dB. Thus, to increase the number of communication receivers, the SLL must be increased.

4.2 PSK-Based Method. Another technique is devised in [145], which uses PSK modulation scheme to embed information. The phase shift will let us know whether the embedded bit is 1 or 0. Similarly, authors in [141] proposed phase modulation (PM) for embedding information into radar waveforms. Binary data is mapped with phase of signal, which is decoded by using phase detector at the receiver side. PM-based information embedding provides more accurate results as compared to AM and multiform ASK-based methods. Another advantage of the said scheme is that we can use it for both directional and broadcast mode and for coherent and noncoherent detection. In [146], authors claim that PSK-based method is more secure as compared to ASK method because interference can disintegrate the SLL as compared to the phases of the waveform. If the communication is coherent, only one waveform is used with 1 beamforming weight vector, and if the communication is incoherent, pair of waveforms and beamforming weight vectors are required, as shown in Figure 10. Communication symbols that are embedded into phase of signal equal the total number of waveforms minus one.

To ensure the radar operation, unity power weight vector has to be used.

\( \psi_p(t) \) and \( \psi_q(t) \) are two orthogonal radar waveforms with unity power, and in each radar pulse, only one bit of information is embedded in the form of phase symbol.
The model of the DFRC radar waveform-based signal is as follows:

\[ s(t, \tau) = \sqrt{\frac{P_T}{2}} \left[ w_g^* \psi_p + w_g^* \psi_q \right]. \tag{9} \]

Similarly, the model of the signal received at the communication receiver is given as

\[ y_p(\tau) = \sqrt{\frac{P_T}{2}} \left[ \alpha_j (w_p^H a(\theta_j)) + n_p(\tau) \right], \tag{10} \]

\[ y_q(\tau) = \sqrt{\frac{P_T}{2}} \left[ \alpha_j (w_q^H a(\theta_j)) + n_q(\tau) \right]. \tag{11} \]

The embedded phase symbol can be extracted by using

\[ \hat{\phi}(\tau) = \angle \frac{y_p(\tau)}{y_q(\tau)}. \tag{12} \]

It is important to note that both waveforms \( \psi_p(t) \) and \( \psi_q(t) \) will be transmitted simultaneously. Hence, at receiver side, difference between both waveform phase will determine the phase symbol. The common terms between both phases will be cancelled out, and this extracted phase value will be compared with original dictionary. Therefore, phase synchronization is not required. Moreover, it is also worth mentioning that if the entire process is noncoherent, and channel coefficient \( \alpha \) is correctly estimated, then two symbols can also be transmitted, and this leads to double the data rate. This technique can hold the benefits of MIMO radar but lacks dual functionality of MIMO radar and MIMO communication. As with the increase in the constellation size, the exact correlation on phase symbol becomes difficult at the receiver end and this affects the communication process.
4.3. Phase Rotation Invariance Method. In [143], phase rotation invariance-based scheme is used. This technique uses two waveforms to embed one bit of information. This technique is easy to implement and gives better data rate, but it needs minimum two matched filters at the communication end. The phase rotation is direction $\theta$ dependent; hence, only the intended communication receiver will receive embedded information that is located at $\theta$. In this case, communication process is directional. In those situations, where the communication receiver location is not known in advance or communication receiver is moving rapidly, either to iteratively calculate the communication receiver location or broadcast mode will be used. Authors in [143] achieved broadcast mode by using $w_p$ as rotated version of $w_q$ beamforming weight vectors:

$$w_p = w_q e^{j\chi},$$  \hspace{1cm} (13)

where

$$\chi = \angle \left\{ \frac{w_p^H a(\theta_r)}{w_q^H a(\theta_i)} \right\}. $$ \hspace{1cm} (14)

From Equation (14), we conclude that phase difference between two signals is constant.

Figure 11 shows comparison of different schemes studied so far in terms of SNR and BER. For $\theta_c = 50^\circ$ and $\theta_r = 0^\circ$, the performance of sidelobe AM-based approach shows worst results as compared to beampattern AM and multiwaveform ASK. Similarly, the beampattern PSK outperforms beampattern ASK, phase modulation, and aforementioned techniques.

4.4. Index Modulation-Based Method. Previous techniques discussed so far were either using amplitudes of waveform or by using phase of transmitter waveform to embed information. Now, we turn to another domain of information embedding which is called index modulation (IM). Index modulation methods use the index or number of antenna elements to convey additional information bits [147]. Multicarrier agile phased array radar (MAPAR) is used to embed information bits for remote user by using the same technique in [147]. Thus, integrating index modulation into a DFRC transmitter side by using radar waveform leads to high spectral and energy efficient system, without degrading radar performance [148]. Sparse array is used by [149] to embed information into orthogonal waveform and permutation of antenna element. However, this reduces transmit power and antenna gain, thus degrading the target detection and overall performance. In [150], the authors propose carrier agile phased array radar (CAESAR), which has the capability to achieve the wideband performance by using narrow band signals. The abovementioned performance is achieved by applying the concept of frequency agile radar (FAR) in which carrier frequency changes from pulse to pulse; thus, combination of unique frequency with different antenna elements provides more degrees of freedom as shown in Figure 12.
Index modulation can be achieved by pairing antenna elements with unique waveform. This pairing is known to both of the transmitter and receiver. In case of MIMO radar, there is no fix binding between waveforms and antenna elements. The system model swaps the antenna elements and waveforms randomly. This swapping does not affect the performance of system. Let us consider MIMO with $M_T$ antenna elements and $G$ orthogonal waveforms; this pairing provides constellation effect in terms of factorial, i.e., $G!$ and the total bit rate becomes

$$\text{bitrate}(R) = \left| \log_2(G!) \right| (f_{\text{PRF}}),$$

signal at the transmitter of DFRC having form in MIMO

$$s(t, \tau) = \sqrt{\frac{P}{M_T}} W \psi(t),$$

$$\psi(t) = P \phi(t),$$

$$\psi(t) = P \phi(t),$$

where $P$ is the permutation matrix of $M_T \times M_T$, $W$ is the beamforming weight matrix, $\psi(t)$ is shuffled waveform matrix, and $\phi(t)$ is waveform matrix. The received signal at communication receiver with index modulated waveform is

$$y_j(t) = a_j P^T W^H a(\theta_j) + n(t),$$

where $a_j$ is channel coefficient and $a(\theta_j)$ is steering vector in the direction of $j$th communication receiver ($\theta_j$).

Comparison is provided for symbol decoding in [151] by using maximum likelihood- (ML-) based decoder, noniterative suboptimal decoder, and iterative low complexity decoder as shown in Figure 13; among all, the computationally complex optimal ML decoder achieves the lowest BER values.

As we increase the number of messages, $N_b$ grows; however, the overall BER performance degrades as shown in Figure 14.

### 4.5. Code Shift Keying-Based Method

Changing radar waveform on pulse-to-pulse basis introduced new horizon for researchers [152]. This technique enables us to assign number to waveform and at receiver by decoding the waveform will give extra information. This is itself an information embedding technique because each waveform represents unique symbol. In this technique, binary data is mapped to Gold codes or Kasami codes, initially, and then embedded to the waveform from the dictionary as shown in Figure 15.

By using this technique, the interference between radar and communication user and due to other targets is minimized to remarkable level because they provide low probability of intercept (LPI) [153]. Monte Carlo simulations were used to check the performance of communication receiver by using various code lengths for both Gold and Kasami codes. PSK modulation can be used to increase bit...

![Figure 14: BER vs. SNR of ML decoder.](image-url)
rate. At DFRC, CSK-based waveform is transmitted via omnidirectional antenna, and at radar receiver, narrow beam width is required to achieve scanning by a phased array radar (PAR) antenna. Suppose we have a dictionary of $G$ waveforms, with $G$ assumed to be power of 2, then by assuming each waveform as communication symbol, the bit rate $R_{bt}$ of transmitted waveform can be written as

$$R_{bt} = \lceil \log_2 G \rceil .$$

If the code length is $N_c$ chips, and duration of each chip is $t_c$, the maximum bit rate to be achieved can be obtained by $R = R_{bt}f_{PRF}$. Similarly, for binary PSK modulated waveform, the bit rate can be achieved by

$$R_{bt} = \lceil \log_2 (G + 1) \rceil f_{PRF} .$$

Furthermore, now the symbol error rate is determined by using Gold codes and Kasami codes in [153], $1 \times 10^7$ trials of Monte Carlo simulations were conducted by using code length of $M = 32$ bits, $M = 64$, $M = 128$, and $M = 512$ bits by using QPSK modulation, respectively. For $M = 32$ and $M = 64$, small Gold codes and Kasami codes were used while for $M = 128$, and $M = 512$, large codes are used. It is concluded that Gold codes for $M = 64$ perform well as compared to Kasami codes for $M = 64$ bits shown in Figure 16.

4.6. Frequency Hopping-Based Method. Authors of [154–156] used frequency hopping waveform for radar purpose only. Keeping the success rate for radar only, they now utilize the same concept in DFRC. In this approach, authors of [157] used PSK symbol into radar waveform to embed information. Subpulse-based architecture is used to cipher the information. The waveform is divided into multiple segments called as hops as shown in Figure 17.

To decode the PSK symbol, communication receiver needs accurate information about the channel and frequency hopping sequence (FHS). This technique improves the data rate because pulse repetition frequency (PRF) is improved, but on the other hand, the requirements of multiple hops, accurate channel estimation, and multiplicative clutter effect due to timing offsets increase the computational complexity. The MIMO-based DFRC by using FH waveforms is proposed in [159]. This architecture uses radar as primary function, while the communication on secondary basis. During each FH interval, only one bit of information is embedded by using PM. However, due to time variant nature of channel, waveform optimization needs to be done successively to obtain target information and other features as well. These features and target information are later on used to increase the MI between the target response and the target returns. Author of [158] used PSK and DPSK-based symbols for FH waveforms. In this proposed technique, $M_f$ antenna elements and $K$ frequencies to generate frequency hopped waveform were used. Greater the number of frequencies, greater will be number of hops ($Q$), and hence higher the number of symbols ($L$) per pulse. The number of symbols can be achieved by using

$$\text{symbol}(L) = M_f C_K = \left( \frac{M_f^L}{K!(M_f-K)!} \right) ,$$

$$\text{bitrate} = \left| \log_2 L \right| (Q)(f_{PRF}) .$$

This is the simple most method to implement, and it gives higher bit rate ($R$) as compared to previous approaches.

Numerical results are based on Monte Carlo simulations to validate the effectiveness of this method, i.e., frequency hopping-based waveform design in [107]. A high PRF is used in X band, which in return gives data rate of megabits, respectively. Figure 18 shows the performance in terms of BER vs. SNR for frequency hopping in phase modulated waveform (uncoded) and convolutional encoder of rate 2/3 in waveform (encoded) compared with method 1, i.e., controlling side lobe levels (SLL) for communication users in [160], respectively.

4.7. Chirp Slope Keying-(CSK-) Based Method. In this technique, information is embedded into radar emission by using chirp subcarriers [161]. These chirps are generated by using fractional Fourier transform (FrFFT) [162]. Linear frequency modulated (LFM) pulse is used to preserve the radar performance. This type of information embedding is used to mitigate the interchannel interference (ICI) caused by quasi chirp subcarriers [163]. Authors of [142–164] used the slope of chirps to represent the digital modulating data, i.e., 1 and 0. Rising slope or up-chirp means bit equals to 1, while falling slope or down-chirp means bit is zero. Higher constellation can be achieved by using large number of up/down chirp levels [165]. Additionally, Direct Sequence Code Division Multiple Access (DS-CDMA) proposed by [166] avoids mutual interference between communication user and radar. In [167], the authors achieved orthogonality between radar and communication signals by implementing up-chirps and down-chirps. [168] implemented stepped frequency continuous waveforms (SFCW), and [83] used BPSK signal modulated by LFM. Similarly, [169] used saw chirp for communication purpose.

4.8. Time Modulated Array-Based Method. Time modulated array or 4-dimension antenna array use predefined time sequence programmed at the transmitter to radiate beampattern [170]. Initially, TMA was limited to the field of radio astronomy only [171] due to slow FR switches, nonavailability of ad hoc design methodologies for on-off sequence of antenna elements, and inefficient implementation of time
From the beginning of new century, demand of TMA increased, when low priced array structure, irregular shaped geometry, and low SLL became the demand of industry with unconventional radiation characteristics [173]. Recent developments of TMA in DFRC with each beampattern represent unique binary information. One of the main advantages of using TMA is the use of wide band instead of narrow band signals. Authors in [174] used ULA at DFRC transmitter side; similarly, another study is carried out by [175–178] to implement TMA for harmonic beamforming, multiprogramming in [179], angle diversity in [180], and [181] conducts a quantitative study on the energy efficiency of the radar and communications integration. The overall efforts were made to reduce the power loses in terms of sideband radiation.

The basic idea of TMA is to use the radar integration time (IT) by dividing it into time slots according to modulation. Specific number of antenna elements was turned off for number of time instant to achieve higher data rate. This switching of antenna element represents 1 bit on or off as shown in Figure 19.
To keep radar operations uninterrupted, Genetic Algorithm- (GA-) based optimization technique is used [47] and 4 different beampatterns were designed to transmit binary information. In the above methods, the data rate is highly dependent on PRF of the radar and can be achieved only when line-of-sight (LoS) channel is used. Similarly, time modulated linear array (TMLA) is utilized for information embedding by [182]; this obtains low SLL by using single and multiple frequencies in different designs. It is concluded that only by controlling time-based sequence, diverse power levels and different beampatterns can be achieved. [183] used TMA for information embedding by proposing two different architectures which are Sparse TMA (STMA) in which phase angle is set to zero while power is set to unity, and the second is Phase Only Synthesis TMA (POSTMA), in which phase is optimized by using GA.

Heretofore, we have presented an overview of different strategies for radar-embedded communication signals. Such strategies are key to establishing dual-function systems that permit simultaneous execution of both radar and communication functions from a shared platform. We have provided a balanced and complete account of existing methods and discussed their respective advantages and disadvantages. In the following section, we will overview the methods that use communication signal for radar operations.

5. Communication Waveform for Radar Operations

As we know, the DFRC shares its resources like spectrum, power, and antenna elements to transmit such a signal which suits both the radar and communication receivers [184–191]. Now we will put a glance over the methods, which utilize the communication-based waveforms that scan the target without any degradation in system efficiency by simply doing small alteration in actual waveform. This approach utilizes digital multiplexing techniques to encode digital data into multiple orthogonal frequency carriers called subcarriers. By using OFDM-based waveform for DFRC, we achieve better characteristics of low side lobes, high Doppler tolerance, and information transmission capacity reported in [65–197]. Decoding at receiver side is done by using fast Fourier transform (FFT). Because of the diverse nature and wide range of application, the OFDM-based...
waveform became feasible option to attract the researchers as alternate solution to fulfill the requirements of the industry [198].

5.1. Mutual Information-Based Design. Mutual information between radar and communication plays vital role in terms of channel capacity and radar performance [199–202]. In this technique, mutual information (MI) between the communication user and radar target is used as optimization objective for radar at transmitter side [203]. Radar MI is used to evaluate the radar performance, while channel capacity calculation is used as performance measure of the communication system. Impact of SNR and number of antenna elements on MI and channel capacity is calculated in [204]. Adaptive OFDM (AOFDM) design-based approach is proposed in [201] in which the conditional MI between the radar and the received signal is used to calculate data information rate (DIR) in frequency selective channel. Similarly, inner bounds on performance of DFRC in terms of DIR and estimation information rate at receiver are also investigated in [205]. Afterwards, the MI maximization is further explored in [206] to minimize the minimum mean square error (MMSE) in terms of target impulse response. Moreover, for communication-based waveform design, a linear test is to maximize the data rate by adaptively assigning the transmit power according to the CSI [207]. To summarize the discussion, by using OFDM-based waveforms, MI maximization can be solved as convex optimization problem. Therefore, it becomes an attractive measure as compared to other optimization criteria, like probability of detection and Cramer-Rao bound, which are generally non-convex problems [67].

In OFDM-based systems, the entire bandwidth is divided into $K$ subcarriers and it is important to note that each subcarrier uses unique frequency. Similarly, each communication receiver utilizes one subchannel only, while radar utilizes all subcarriers for estimation purpose as shown in Figure 20.

DFRC transmitter and communication receiver need to be synchronized in terms of frequency [208]. The signal at the output of transmitter of dual-function antenna array is

$$x = Fs,$$

(21)

where $x$ contains $L$ symbols and $K$ subcarriers and $K \leq L$. $F$ is Inverse Discrete Fourier Transformed matrix, and each row represents OFDM subcarrier.

$s = [s_1, s_2, s_K]^T$, having length $K \times 1$ vector, represents the amplitudes and phases of each subcarrier.

The signal at the radar receiver is given as

$$y_{\text{Radar}} = Hs + n,$$

(22)

where $H$ contains all the diagonal values of channel impulse responses and $n$ is AWG noise vector. Similarly, the signal received at communication receiver is given as

$$y_{\text{comm},j} = G_j s + m_j,$$

(23)

where $G_j = \text{diag} (g_j)$ and $g_j = [g(j, 1), g(j, 2), \ldots, g(j, K)]^T$ denote channel coefficients for the $K$ subcarrier, which are associated with $j^{th}$ communication receiver.

Information is embedded into this OFDM waveform by using QPSK phase as explained in [208]. Each communication receiver is allotted unique subcarrier, which is using unique frequency. Hence, at communication end, the interference is minimum. The main task in OFDM-based waveform design is to manage transmit power of each subcarrier such that radar target identification is improved. The power of each phase can be calculated by $p_k = |s_k|^2$. Hence, the overall transmit power of transmitted signal is

$$P_{\text{total}} = x^H x \Rightarrow s^H P^H Fs = s^H s = \sum_{k=1}^{K} P_k = \text{tr}(P),$$

(24)

where $\text{tr}(.)$ represents the trace of matrix. The maximum power allocated to $k^{th}$ subcarrier is represented as $P_{(k, \text{max})}$; hence, the $P_{\text{max}} = [P_{(1, \text{max})}, P_{(2, \text{max})}]^T$.

The following optimization gives us acceptable radar objectives.

$$\sum_{j=1}^{J} \sum_{k=1}^{K} w_{j,k} \log \left(1 + \frac{p_k \sigma_{g_{j,k}}^2}{\sigma_{m_{j,k}}^2}\right),$$

(25)

such that $\sum_{k=1}^{K} w_{j,k} \log \left(1 + \frac{p_k \sigma_{g_{j,k}}^2}{\sigma_{m_{j,k}}^2}\right) \leq -\alpha_{\text{opt}}$, where $\sigma_{g_{j,k}}^2$ is normalized channel gain for communication receiver, $\sigma_{h_{j,k}}^2$ is normalized channel gain for radar target, $\sigma_{n_{j,k}}^2$ is noise components in the $K$ subcarriers at radar receiver, and $\sigma_{m_{j,k}}^2$ is noise components in the $K$ subcarriers at $j^{th}$ communication receiver. $\alpha_{\text{opt}}$ represents the MI level of the radar and communication users, and $\gamma$ is the flexibility of radar towards communication user. The value of $\gamma$ ranges between 0 and 1. For better radar operations, the value of $\gamma$ is more inclined towards 1.

$$1^T \rho \leq P_{\text{total, max}}, \quad 0 \leq \rho \leq P_{\text{max}}.$$
Radar function allows the dual-purpose transmitter to vary the power allocation such that the radar mutual information does not fall below $\gamma_{\text{opt}}$.

Figure 21 shows power allocation for communication user by assigning 29 subchannels to user 1 shown by red color while 3 subchannels to user 2 shown by blue color, respectively. Only one radar target is present utilizing all 32 subchannels. The maximum power normalization for each subcarrier is set to 10 units. Maximizing the overall communication mutual information is done by using eq (14) in [208]. It is observed that three subchannels which are assigned to user 2 are low powered; this results in better MI. Similarly, for worst-case scenario, 8 subcarriers are assigned to user 1, while 16 subcarriers are assigned to user 2. The mutual information is degraded with poor channel conditions than the communication receiver 1 in radar-favored subcarriers.

5.2. Index Modulation-Based Design:. Authors in [209] used index modulation for increasing data rate by using OFDM waveform. Information is encoded into waveform by using quadrature amplitude modulation (QAM) as well as by using the indices of antenna elements in DFRC transmitter. These two parameters help us to improve the efficiency as compared to the traditional OFDM [210–213], and advantages observed are mentioned in [214]. This presented technique gives good results for radar scanning in terms of energy efficiency, reduced PAPR, robustness to ICI, and improved BER, respectively. Similarly, [151] used different antenna elements and frequencies of subcarrier, which act as constellation space. In this technique, authors split the information bits by using bit splitter module, where each bit is mapped to index selector and Golay code sequence inserter module. Afterwards, subblocks or subcarriers are created in the OFDM block creator. All the subcarriers undergo the IFFT and cyclic prefix (CP); then, data is further converted from parallel to serial and transmitted through DFRC transmitter as shown in Figure 22.

The $m^{th}$ OFDM symbol, which is generated by applying IFFT is given below:

$$x_m = \text{IFFT}(X_m) = [X_{m,1}, X_{m,2}, \ldots, X_{m,N}]^T,$$

where matrix $X$ contains frequency domain transmitted symbols and $X_{m,n}$ represents $m^{th}$ symbol transmitted over $n^{th}$ subcarrier. Cyclic prefix of $C$ samples is added to the beginning of the OFDM symbol after applying IFFT as below.

$$x = [X_{m,N-(C+1)}, X_{m,N-1}, \ldots, X_{m,N}]^T.$$

This baseband discrete signal is processed with digital to analog converter and upconverted to desired carrier frequency. The signal to be transmitted through DFRC Tx becomes...
The received signal is sampled and BER. The radar is calculated in terms of the MSE, while the performance of the well-known maximum likelihood (MLL) approximation is used to decode the QAM modulation. The performance of communication receiver all the time while the remaining portion is used for different radar operations as shown in Figure 21. Recently, authors in [220] used the same concept for 5G communication systems and [221] for two channel selectable down converter for interference mitigation in radar operations, respectively. Recent developments for unified hardware-based radar and communication multibeam are studied in [220–228]. Entire spectrum is divided into portions called subbeam, and one portion is utilized by communication receiver all the time while the remaining portion is used for different radar operations as shown in Figure 23.

Phased array radars can scan in 2 dimensions to fully utilize the available spectrum. This system is capable to transmit and receive simultaneously, and multiple beams are generated using the same transmitter. The general drawback of multiple beams is that total power is divided in multiple beams, and the scan range is reduced. [216] proposed idea to split transmitter antenna array in two parts for joint radar and communication for interference mitigation. The proposed framework implements time division duplex (TDD) for radar operations while OFDM for communication system, respectively. The parameter estimation is done by on-grid compressed sensing-based algorithm.

Finally, for convenience of the reader, Table 1 shows the brief summary of all the algorithms discussed so far in this paper.

6. Beampattern Modulation

Since existing literature mostly focuses the scheme of using a single beam for communication and sensing [79–215], we review beamforming-based approaches for radar and communication in this section. Our aim is to study the performance parameters of separate beams for radar operations as well as communication system generated by single aperture by using signal processing algorithms [216]. It is important to note that both radar and communication system have different requirements for beamforming [71–218]. Working on high frequencies, radio system encounters propagation loss; therefore, communication system at one hand requires stable and LOS beams for large gain, while radar on other hand requires time-varying and directional scanning beams.
at the radar receiver keeping the signal to interference plus noise (SINR) ratio at a threshold level and minimum utilization of power. Those signals that employ channel coding techniques needs extra bits, which in return reduce the data rate. Thus, there is a trade-off between radar parameter estimation, and high data rate occurs. Estimation of carrier frequency for radar-based waveform design and method for cyclic prefix spectrum density and symbol rate for OFDM-based waveform design need to be revised for DFRC in terms of PARP and SINR for both stationary and moving user as well as communication receiver mounted on ships.

7.4. Waveform Diversity and Cognitive Behaviour. Waveform diversity is required at the DFRC transmitter side in case of nonapproved/malicious user and enemy targets. Cognition is highly recommended between dual-function transmitter and communication receiver. In this case, when waveform is trapped or enemy deciphers the waveform, it should be updated simultaneously by using cognitive techniques.

7.5. Effect of Clutter and Scatters. Dealing with clutter in bistatic DFRC is one of the major challenges due to varying nature of beampattern-based modulation, similarly the same for frequency hopping and index modulation-based approaches. Thus, bit detection performance at communication receiver may be degraded due to interference. Therefore, benefit from the wealth of knowledge developed in communication literature may be utilized to overcome this problem.

7.6. Subbeam Parameter Estimation. There are many challenging problems and possible improvements yet to be done for multibeam beamforming, which includes weight vector generation with quantized magnitude and phase values, communication and sensing subbeam combination methods optimized with respect to certain criterion, and sensing algorithms that work for high-dimension and off-grid models. Moreover, diverse efficient methods are required to resolve angle of arrivals beyond the conventional concept of scanning.

Most of the DFRC literature is restricted to OFDM-based waveform design only. However, there are potential waveforms which need to be explored for DFRC designs. Therefore, we present some of the latest waveform that can also be used in DFRC to accommodate the requirements of 5G and beyond.

7.7. Orthogonal Time Frequency Space (OTFS) Modulation. To overcome the shortcomings of the OFDM for dual-function transmitter in fading and time-varying environments, another promising modulation technique can be a
potential candidate known as orthogonal time frequency space modulation (OTFS). This is a two-dimensional technique, which exploits the full diversity in time and frequency. This generalized form inherits the qualities of both OFDM and CDMA. OTFS utilizes the advantage of equalizer and converts time-varying, high Doppler signals and fading channel in time-independent channel with constant gain for almost all subcarriers. In OTFS, each transmitted symbol is modulated over two-dimensional basis function that spans both in time and frequency. OTFS symbol can be reduced to one dimension, i.e., spreading codes, which is CDMA and to subchannels which is OFDM.

7.8. Space-Division Multiple Access (SDMA). When multiple communication users need to communicate with dual-function transmitter simultaneously, modulation techniques are needed which split the channel into parallel spatial pipes. Thus, space-division multiple access is needed. This can be achieved by using phased array antenna keeping healthy and safety standards.

8. Conclusion

This paper presented a review of the recent trends in DFRC design with focus on data embedding techniques. Different spectrum-sharing techniques were discussed, and the DFRC data model was explained. Then, the different methods of embedding information in radar waveforms were discussed. After that, various methods of modifying the communication waveform to facilitate radar functions were reviewed. Then, beampattern modulation techniques like subbeam sharing were explained. Additionally, the paper discussed few major research challenges in the field and provided directions for future research. Finally, for the ease of the reader, a summary of this review paper is provided in a tabular form.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

References

[1] A. Hassanien, M. G. Amin, E. Aboutanios, and B. Himed, “Dualfunction radar communication systems: a solution to the spectrum congestion problem,” IEEE Signal Processing Magazine, vol. 36, no. 5, pp. 115–126, 2019.
[2] B. Li, A. P. Petropulu, and W. Trappe, “Optimum co-design for spectrum sharing between matrix completion based MIMO radars and a MIMO communication system,” IEEE Transactions on Signal Processing, vol. 64, no. 17, pp. 4562–4575, 2016.
[3] DARPA, “Shared Spectrum Access for Radar and Communications (SSPARC) (Archived) spectrum description,” https://www.darpa.mil/program/shared-spectrum-access-for-radar-and-communications.
[4] I. F. Akyildiz, W.-Y. Lee, M. C. Vuran, and S. Mohanty, "Next generation/dynamic spectrum access/cognitive radio wireless networks: a survey," Computer Networks, vol. 50, no. 13, pp. 2127–2159, 2006.
[5] C. Hernandez and D. Giral, "Spectral handoff in cooperative cognitive radio networks.
[6] A. De Domenico, E. C. Strinati, and M.-G. Di Benedetto, "A survey on MAC strategies for cognitive radio networks," IEEE Communications Surveys & Tutorials, vol. 14, no. 1, pp. 21–44, 2012.
[7] H. Nakajo and T. Fujii, “Local 5G mmWave signal measurement and analysis for spectrum database,” in 2021 Twelfth International Conference on Ubiquitous and Future Networks (ICUFN), pp. 350–355, Jeju Island, Republic of Korea, 2021.
[8] N. Mittal, H. Singh, V. Mittal et al., “Optimization of cognitive radio system using self-learning salp swarm algorithm,” CMC-Computers Materials & Continua, vol. 70, no. 2, pp. 3821–3835, 2022.
[9] S. F. Zamanian, M. H. Kahaei, S. M. Razaviadze, and T. Svensson, “Attacking massive MIMO cognitive radio networks by optimized jamming,” IEEE Open Journal of the Communications Society, vol. 2, pp. 2219–2321, 2021.
[10] R. Ghosh, S. Mohanty, P. K. Patnaik, and S. Pramanik, "Performance analysis based on probability of false alarm and miss detection in cognitive radio network," International Journal of Wireless and Mobile Computing, vol. 20, no. 4, pp. 390–400, 2021.
[11] F. Liu, C. Masoursou, A. P. Petropulu, H. Griffiths, and L. Hanzo, "Joint radar and communication design: applications, state-of-the-art, and the road ahead," IEEE Transactions on Communications, vol. 68, no. 6, pp. 3834–3862, 2020.
[12] M. B. Alabd, L. G. de Oliveira, B. Nuss, W. Wiesbeck, and T. Zwick, "Time-Frequency Shift Modulation for Chirp Sequence based Radar Communications," in 2020 IEEE MIT-S International Conference on Microwaves for Intelligent Mobility (ICMIM), pp. 1–4, Linz, Austria, 2020.
[13] A. Hassanien, M. G. Amin, V. D. Zhang, and F. Ahmad, "Signaling strategies for dual-function radar communications: an overview," IEEE Aerospace and Electronic Systems Magazine, vol. 31, no. 10, pp. 36–45, 2016.
[14] A. Alselwi, A. U. Khan, I. M. Qureshi, W. Khan, and A. Basit, "Throughput enhancement for the joint radar-communication systems based on cognitive closed-loop design," IEEE Access, vol. 9, pp. 64785–64807, 2021.
[15] B. Bahram, S. Bhattarai, A. Ullah, J.-M. J. Park, J. Reed, and D. Gurney, "Protecting the primary users’ operational privacy in spectrum sharing," in 2014 IEEE International Symposium on Dynamic Spectrum Access Networks (DYSPAN), pp. 236–247, McLean, VA, USA, 2014.
[16] F. Hessar and S. Roy, "Spectrum sharing between a surveillance radar and secondary Wi-Fi networks," IEEE Transactions on Aerospace and Electronic Systems, vol. 52, no. 3, pp. 1434–1448, 2016.
[17] D. Mcqueen, "The momentum behind LTE adoption [SGPP LTE]," IEEE Communications Magazine, vol. 47, no. 2, pp. 44–45, 2009.
[18] M. Wicks, "Spectrum crowding and cognitive radar," in 2010 2nd International Workshop on Cognitive Information Processing, pp. 452–457, Elba, Italy, 2010.
[19] H. Griffiths, L. Cohen, S. Watts et al., "Radar spectrum engineering and management: technical and regulatory issues," Proceedings of the IEEE, vol. 103, no. 1, pp. 85–102, 2015.
[20] Z. Zhongming, L. Linong, Z. Wangqiang, L. Wei et al., “USDA, FCC, and NTIA Announce Interagency Agreement to Coordinate Broadband Funding Deployment,” 2021.

[21] E C Committee, “The European table of frequency allocations and applications in the frequency range 8.3 kHz to 3000 GHz (ecatable),” in Proceedings of European Conference of Postal and Telecommunications Administrations; Electronic Communications Committee, Copenhagen, Denmark, 2013.

[22] T. Wang, G. Li, B. Huang et al., “Spectrum Analysis and Regulations for 5G,” in 5G Mobile Communications, pp. 27–50, Springer, 2017.

[23] W. Balani, M. Sarvagya, T. Ali et al., “Design techniques of super-wideband antenna–existing and future prospective,” IEEE Access, vol. 7, pp. 141241–141257, 2019.

[24] A. Di Serio, J. Buckley, J. Barton et al., “Potential of sub-GHz wireless for future iot wearables and design of compact 915 MHz antenna,” Sensors, vol. 18, no. 2, p. 22, 2018.

[25] I. Selinis, K. Katsaros, M. Allayioti, S. Vahid, and R. Tafazolli, “The race to 5G era; LTE and Wi-Fi,” IEEE Access, vol. 6, pp. 56598–56636, 2018.

[26] F. Qamar, M. Hindia, K. Dimgi et al., “Investigation of future 5G-IOT millimeter-wave network performance at 38 GHz for urban microcell outdoor environment,” Electronics, vol. 8, no. 5, p. 495, 2019.

[27] L. Cohen, E. Daly, J. DeGraaf, and K. Scheff, “Mitigation of radar interference with WiMAX systems,” in 2010 International Waveform Diversity and Design Conference, pp. 159–164, Niagra Falls, ON, Canada, 2010.

[28] H. T. Hayvaci and B. Tavi, “Spectrum sharing in radar and wireless communication systems: a review,” in 2014 International Conference on Electromagnetics in Advanced Applications (ICEAA), pp. 810–813, Palm Beach, Aruba, 2014.

[29] Z. Geng, H. Deng, and B. Hiamed, “Adaptive radar beamforming for interference mitigation in radar-wireless spectrum sharing,” IEEE Signal Processing Letters, vol. 22, no. 4, pp. 484–488, 2015.

[30] H. Griffiths, S. Blunt, L. Cohen, and L. Savy, “Challenge problems in spectrum engineering and waveform diversity,” in 2013 IEEE Radar Conference (RadarConf), pp. 1–5, Ottawa, ON, Canada, 2013.

[31] L. Zheng, M. Lops, and X. Wang, “Adaptive Interference Removal for Uncoordinated Radar/Communication Coexistence,” IEEE Journal of Selected Topics in Signal Processing, vol. 12, no. 1, pp. 45–60, 2018.

[32] C. Baylis, M. Fellows, L. Cohen, and R. J. Marks II, “Solving the spectrum crisis: intelligent, reconfigurable microwave transmitter amplifiers for cognitive radar,” IEEE Microwave Magazine, vol. 15, no. 5, pp. 94–107, 2014.

[33] E. BouDaher, A. Hassamien, E. Aboutanios, and M. G. Amin, “Towards a dual-function MIMO radar-communication system,” in 2016 IEEE Radar Conference (RadarConf), pp. 1–6, Philadelphia, PA, USA, 2016.

[34] S. Kumar, G. Costa, S. Kant, B. F. Flemming, N. Marchetti, and P. Mogensen, “Spectrum sharing for next generation wireless communication networks,” in 2008 First International Workshop on Cognitive Radio and Advanced Spectrum Management, pp. 1–5, Aalborg, Denmark, 2008.

[35] E. Biglieri, A. J. Goldsmith, L. J. Greenstein, H. V. Poor, and N. B. Mandayam, Principles of Cognitive Radio, Cambridge University Press, 2013.

[36] A. F. Martone, K. I. Ranney, K. Sherbony, K. A. Gallagher, and S. D. Blunt, “Spectrum Allocation for Noncooperative Radar Coexistence,” IEEE Transactions on Aerospace and Electronic Systems, vol. 54, no. 1, pp. 90–105, 2018.

[37] K.-W. Huang, M. Bica, U. Mitra, and V. Koivunen, “Radar waveform design in spectrum sharing environment: coexistence and cognition,” in 2015 IEEE Radar Conference (RadarConf), pp. 1698–1703, Arlington, VA, USA, 2015.

[38] B. Li and A. P. Petropulu, “Joint transmit designs for coexistence of MIMO wireless communications and sparse sensing radars in clutter,” IEEE Transactions on Aerospace and Electronic Systems, vol. 53, no. 6, pp. 2846–2864, 2017.

[39] L. Zheng, M. Lops, X. Wang, and E. Grossi, “Joint Design of Overlaid Communication Systems and Pulsed Radars,” IEEE Transactions on Signal Processing, vol. 66, no. 1, pp. 139–154, 2018.

[40] A. Khawar, A. Abdelhadi, and T. C. Clancy, “Coexistence analysis between radar and cellular system in LoS channel,” IEEE Antennas and Wireless Propagation Letters, vol. 15, pp. 972–975, 2016.

[41] A. Mahal, A. Khawar, A. Abdelhadi, and T. C. Clancy, “Spectral coexistence of MIMO radar and MIMO cellular system,” IEEE Transactions on Aerospace and Electronic Systems, vol. 53, no. 2, pp. 655–668, 2017.

[42] D. W. Bliss, “Cooperative radar and communications signaling: the estimation and information theory odd couple,” in 2014 IEEE Radar Conference, pp. 0050–0055, Cincinnati, OH, USA, 2014.

[43] F. Paisana, N. Marchetti, and L. A. DaSilva, “Radar, TV and cellular bands: which spectrum access techniques for which bands?,” IEEE Communications Surveys & Tutorials, vol. 16, no. 3, pp. 1193–1220, 2014.

[44] F. Liu, C. Masouros, A. Li, and T. Ratnarajah, “Robust MIMO beamforming for cellular and radar coexistence,” IEEE Wireless Communications Letters, vol. 6, no. 3, pp. 374–377, 2017.

[45] A. Basit, W.-Q. Wang, S. Y. Nusenu, and S. Zhang, “Range-angle-dependent beampattern synthesis with null depth control for joint radar communication,” IEEE Antennas and Wireless Propagation Letters, vol. 18, no. 9, pp. 1741–1745, 2019.

[46] A. Basit, W.-Q. Wang, and S. Y. Nusenu, “Adaptive transmit array sideloobe control using FDA-MIMO for tracking in joint radar-communications,” Digital Signal Processing, vol. 97, p. 102619, 2020.

[47] L. Han and K. Wu, “Joint wireless communication and radar sensing systems-state of the art and future prospects,” IET Microwaves, Antennas & Propagation, vol. 7, no. 11, pp. 876–885, 2013.

[48] S. Quan, W. Qian, J. Guq, and V. Zhang, “Radar-communication integration: an overview,” in The 7th IEEE/International Conference on Advanced Infocomm Technology, pp. 98–103, Fuzhou, China, 2014.

[49] P. Ren, A. Munari, and M. Petrova, “Performance analysis of a timesharing joint radar–communications network,” in 2020 International Conference on Computing, Networking and Communications (ICNC), pp. 908–913, Big Island, HI, USA, 2020.

[50] H. Shajiaia, A. Abdelhadi, and C. Clancy, “Spectrum sharing approach between radar and communication systems and its impact on radar’s detectable target parameters,” in 2015 IEEE 81st Vehicular Technology Conference (VTC Spring), pp. 1–6, Glasgow, UK, 2015.
A. Khawar, A. Abdelhadi, and T. C. Clancy, "On the impact of time-varying interference-channel on the spatial approach of spectrum sharing between s-band radar and communication system," in 2014 IEEE Military Communications Conference, pp. 807–812, Baltimore, MD, USA, 2014.

B. Li, H. Kumar, and A. P. Petropulu, "A joint design approach for spectrum sharing between radar and communication systems," in 2016 IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP), pp. 3306–3310, Shanghai, China, 2016.

A. Babaei, W. H. Tranter, and T. Bose, "A practical precoding approach for radar/communications spectrum sharing," in 8th International Conference on Cognitive Radio Oriented Wireless Networks, pp. 13–18, Washington, DC, USA, 2013.

A. Khawar, A. Abdel-Hadi, and T. C. Clancy, "Spectrum sharing between s-band radar and LTE cellular system: a spatial approach," in 2014 IEEE International Symposium on Dynamic Spectrum Access Networks (DYSSPAN), pp. 7–14, McLean, VA, USA, 2014.

H. Deng and B. Himed, "Interference mitigation processing for spectrum-sharing between radar and wireless communications systems," IEEE Transactions on Aerospace and Electronic Systems, vol. 49, no. 3, pp. 1911–1919, 2013.

N. Q. Hieu, D. T. Hoang, N. C. Luong, and D. Niyato, "irdc: an intelligent real-time dual-functional radar-communication system for automotive vehicles," IEEE Wireless Communications Letters, vol. 9, no. 12, pp. 2140–2143, 2020.

A. Khawar, Spectrum Sharing between Radar and Communication Systems, [Ph.D. thesis], Virginia Tech, 2015.

B. Li and A. Petropulu, "MIMO radar and communication spectrum sharing with clutter mitigation," in 2016 IEEE Radar Conference (RadarConf), pp. 1–6, Philadelphia, PA, USA, 2016.

A. Dimas, B. Li, M. Clark, K. Psounis, and A. Petropulu, "Spectrum sharing between radar and communication systems: can the privacy of the radar be preserved?," in 2017 51st Asilomar Conference on Signals, Systems, and Computers, pp. 1285–1289, Pacific Grove, CA, USA, 2017.

L. Li, G. Li, and C. Li, "A communication system based on active phased-array radar," Journal of China Academy of Electronics and Information Technology, vol. 2, 2008.

D. Ma, N. Shlezinger, T. Huang, Y. Liu, and Y. C. Eldar, "Automotive dual-function radar communications systems: an overview," in 2020 IEEE 11th Sensor Array and MultiChannel Signal Processing Workshop (SAM), pp. 1–5, Hangzhou, China, 2020.

A. R. Chiriyath, B. Paul, and D. W. Bliss, "Radar-communications convergence: coexistence, cooperation, and co-design," IEEE Transactions on Cognitive Communications and Networking, vol. 3, no. 1, pp. 1–12, 2017.

G. C. Tavik, C. L. Hilterbric, J. B. Evins et al., "The advanced multifunction RF concept," IEEE Transactions on Microwave Theory and Techniques, vol. 53, no. 3, pp. 1009–1020, 2005.

D. Ma, T. Huang, Y. Liu, and X. Wang, "A novel joint radar and communication system based on randomized partition of antenna array," in 2018 IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP), pp. 3335–3339, Calgary, AB, Canada, 2018.

C. Aydogdu, M. F. Keskin, N. Garcia, H. Wymeersch, and D. W. Bliss, "RadChat: spectrum sharing for automotive radar interference mitigation," IEEE Transactions on Intelligent Transportation Systems, vol. 22, no. 1, pp. 416–429, 2021.

C. Sturm, Y. L. Sit, M. Braun, and T. Zwick, "Spectrally interleaved multi-carrier signals for radar network applications and multi-input multi-output radar," IET Radar, Sonar & Navigation, vol. 7, no. 3, pp. 261–269, 2013.

M. Bică and V. Koivunen, "Multicarrier radar-communications waveform design for RF convergence and coexistence," in ICASSP 2019-2019 IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP), pp. 7780–7784, Brighton, UK, 2019.

B. Paul, A. R. Chiriyath, and D. W. Bliss, "Survey of RF communications and sensing convergence research," IEEE Access, vol. 5, pp. 252–270, 2017.

A. Gameiro, D. Castanheira, J. Sanson, and P. P. Monteiro, "Research challenges, trends and applications for future joint radar communications systems," Wireless Personal Communications, vol. 100, no. 1, pp. 81–96, 2018.

D. Ma, N. Shlezinger, T. Huang, Y. Liu, and Y. C. Eldar, "Joint Radar–Communication Strategies for Autonomous Vehicles: Combining Two Key Automotive Technologies," IEEE Signal Processing Magazine, vol. 37, no. 4, pp. 85–97, 2020.

C. Sturm and W. Wiesbeck, "Waveform design and signal processing aspects for fusion of wireless communications and radar sensing," Proceedings of the IEEE, vol. 99, no. 7, pp. 1236–1259, 2011.

R. Bera, "Generation, Detection and Analysis of Subterahertz over the Air (OTA) Test Bed for 6G Mobile Communication Use Cases," in Generation, Detection and Processing of Terahertz Signals, pp. 97–122, Springer, 2022.

F. Liu, C. Masouros, A. Li, H. Sun, and L. Hanzo, "MU-MIMO communications with MIMO radar: coexistence to joint transmission," IEEE Transactions on Wireless Communications, vol. 17, no. 4, pp. 2755–2770, 2018.

F. Liu, L. Zhou, C. Masouros, A. Li, W. Luo, and A. Petropulu, "Toward dual-functional radar-communication systems: optimal waveform design," IEEE Transactions on Signal Processing, vol. 66, no. 16, pp. 4264–4279, 2018.

L. Zheng, M. Lops, Y. C. Eldar, and X. Wang, "Radar and communication coexistence: an overview, a review of recent methods," IEEE Signal Processing Magazine, vol. 36, no. 5, pp. 85–99, 2019.

D. Ma, N. Shlezinger, T. Huang, Y. Liu, and Y. C. Eldar, "Bit constrained communication receivers in joint radar communications systems," in ICASSP 2021-2021 IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP), pp. 8243–8247, Toronto, ON, Canada, 2021.

X. Mu, Y. Liu, L. Guo, J. Lin, and L. Hanzo, "Noma-aide joint radar and multicast-unicast communication systems," 2021, https://arxiv.org/abs/2110.02372.

L. Reichardt, C. Sturm, F. Grünhaupt, and T. Zwick, "Demonstrating the use of the IEEE 802.11 p car-to-car communication standard for automotive radar," in 2016 6th European Conference on Antennas and Propagation (EUCAP), pp. 1576–1580, Prague, Czech Republic, 2012.

P. Kumari, N. Gonzalez-Prlecic, and R. W. Heath, "Investigating the IEEE 802.11 ad standard for millimeter wave automotive radar," in 2015 IEEE 82nd Vehicular Technology Conference (VTC2015-Fall), pp. 1–5, Boston, MA, USA, 2015.

P. Kumari, J. Choi, N. Gonzalez-Prlecic, and R. W. Heath, "IEEE 802.11 ad-based radar: an approach to joint vehicular
communication-radar system,” *IEEE Transactions on Vehicular Technology*, vol. 67, no. 4, pp. 3012–3027, 2017.

[81] G. Kwon, A. Conti, H. Park, and M. Win, “Joint communication and localization in millimeter wave networks,” *IEEE Journal of Selected Topics in Signal Processing*, vol. 15, no. 6, pp. 1439–1454, 2021.

[82] A. Alselwi, A. U. Khan, I. M. Qureshi, W. Khan, and A. Basit, “Multiuser transmission for the joint radar communication systems based on amplitude phase shift keying modulation and waveform diversity,” *International Journal of Microwave and Wireless Technologies*, pp. 1–15, 2021.

[83] Q. Zhang, X. Wang, Z. Li, and Z. Wei, “Design and performance evaluation of joint sensing and communication integrated system for 5G MmWave enabled CAVs,” *IEEE Journal of Selected Topics in Signal Processing*, vol. 15, no. 6, pp. 1500–1514, 2021.

[84] Q. Gu and J. Sandell, Secure Joint Radar Communications and Risk Assessment for Autonomous Driving, UNIVERSITY OF GOTHENBURG, 2021.

[85] M. Kafafy, A. S. Ibrahim, and M. H. Ismail, “Maximum-service channel assignment in vehicular radar-communication,” *IEEE Access*, vol. 9, pp. 138359–138370, 2021.

[86] H. Ruan, Y. Liu, T. Huang, and X. Wang, “Designing the waveform bandwidth and time duration of radar oriented to collision warning performance for better resource efficiency,” *Digital Signal Processing*, vol. 118, article 103204, 2021.

[87] M. Mert Şahin and H. Arslan, “Multi-functional Coexistence of Radar-Sensing and Communication Waveforms,” in 2020 IEEE 92nd Vehicular Technology Conference (VTC2020-Fall), pp. 1–5, Victoria, BC, Canada, 2020.

[88] N. C. Luong, X. Lu, D. T. Hoang, D. Niyato, and D. I. Kim, “Radio resource management in joint radar and communication: a comprehensive survey,” *IEEE Communications Surveys & Tutorials*, vol. 23, no. 2, pp. 780–814, 2021.

[89] A. Herschelet and D. W. Bliss, “Spectrum management and advanced receiver techniques (SMART): Joint radar communications network performance,” in *2018 IEEE Radar Conference (RadarConf’18)*, pp. 1078–1083, Oklahoma City, OK, USA, 2018.

[90] R. Saruthirathanaworakun, J. M. Peha, and L. M. Correia, “Opportunistic sharing between rotating radar and cellular,” *IEEE Journal on Selected Areas in Communications*, vol. 30, no. 10, pp. 1900–1910, 2012.

[91] R. Saruthirathanaworakun, J. M. Peha, and L. M. Correia, “Performance of data services in cellular networks sharing spectrum with a single rotating radar,” in *2012 IEEE International Symposium on a World of Wireless, Mobile and Multimedia Networks (WoWMoM)*, pp. 1–6, San Francisco, CA, USA, 2012.

[92] S.-S. Raymond, A. Abubakari, and H.-S. Jo, “Coexistence of powercontrolled cellular networks with rotating radar,” *IEEE Journal on Selected Areas in Communications*, vol. 34, no. 10, pp. 2605–2616, 2016.

[93] D. Ciuonzo, A. De Maio, G. Foglia, and M. Piezzo, “Intra-pulse radarembodied communications via multiojective optimization,” *IEEE Transactions on Aerospace and Electronic Systems*, vol. 51, no. 4, pp. 2960–2974, 2015.

[94] A. Khawar, A. Abdelhadi, and T. C. Clancy, “A mathematical analysis of cellular interference on the performance of S-band military radar systems,” in *2014 Wireless Telecommunications Symposium*, pp. 1–8, Washington, DC, USA, 2014.

[95] E. Obregon, K. W. Sung, and J. Zander, “On the sharing opportunities for ultra-dense networks in the radar bands,” in *2014 IEEE International Symposium on Dynamic Spectrum Access Networks (DYSAN)*, pp. 215–223, McLean, VA, USA, 2014.

[96] M. J. Marcus, “Sharing government spectrum with private users: opportunities and challenges,” *IEEE Wireless Communications*, vol. 16, no. 3, pp. 4–5, 2009.

[97] L. Wang, J. McGeehan, C. Williams, and A. Doufexi, “Radar spectrum opportunities for cognitive communications transmission,” in *2008 3rd International Conference on Cognitive Radio Oriented Wireless Networks and Communications (CrownCom)*, pp. 1–6, Singapore, 2008.

[98] M. Tercero, K. W. Sung, and J. Zander, “Temporal secondary access opportunities for WLAN in radar bands,” in *2011 The 14th International Symposium on Wireless Personal Multimedia Communications (WPMC)*, pp. 1–5, Brest, France, 2011.

[99] J. T. Johnson, C. J. Baker, H. Wang, L. Ye, and C. Zhang, “Assessing the potential for spectrum sharing between communications and radar systems in the L-band portion of the RF spectrum allocated to radar,” in *2014 International Conference on Electromagnetics in Advanced Applications (ICEAA)*, pp. 331–334, Palm Beach, Aruba, 2014.

[100] F. Liu, A. Garcia-Rodriguez, C. Masouros, and G. Geraci, “Interfering channel estimation for radar and communication coexistence,” in *2019 IEEE 20th International Workshop on Signal Processing Advances in Wireless Communications (SPAWC)*, pp. 1–5, Cannes, France, 2019.

[101] F. Liu, C. Masouros, A. Li, T. Ratnarajah, and J. Zhou, “MIMO radar and cellular coexistence: a power-efficient approach enabled by interference exploitation,” *IEEE Transactions on Signal Processing*, vol. 66, no. 14, pp. 3681–3695, 2018.

[102] R. M. Rao, V. Marojevic, and J. H. Reed, “Probability of Pilot Interference in Pulsed Radar-Cellular Coexistence: Fundamental Insights on Demodulation and Limited CSI Feedback,” *IEEE Communications Letters*, vol. 24, no. 8, pp. 1678–1682, 2020.

[103] Y.-C. Liang, Y. Zeng, E. C. Peh, and A. T. Hoang, “Sensing-throughput tradeoff for cognitive radio networks,” *IEEE Transactions on Wireless Communications*, vol. 7, no. 4, pp. 1326–1337, 2008.

[104] X. Zhao, Z. Guo, and Q. Guo, “A cognitive based spectrum sharing scheme for LTE advanced systems,” in *International Congress on Ultra Modern Telecommunications and Control Systems*, pp. 965–969, Moscow, 2010.

[105] D. A. Guimaraes, L. dos Santos Costa, and R. A. A. de Souza, “Comparison between eigenvalue fusion and decision fusion for spectrum sensing of OFDMA signals under errors in the control channel,” in *2014 International Telecommunications Symposium (ITS)*, pp. 1–5, Sao Paulo, Brazil, 2014.

[106] P. Si, E. Sun, R. Yang, and Y. Zhang, “Cooperative and distributed spectrum sharing in dynamic spectrum pooling networks,” in *The 19th Annual Wireless and Optical Communications Conference (WOCC)*, pp. 1–5, Shanghai, China, 2010.

[107] B. Tang, H. Wang, L. Qin, and L. Li, “Waveform Design for Dual-function MIMO Radar-communication Systems,” in *2020 IEEE 11th Sensor Array and Multichannel Signal Processing Workshop (SAM)*, pp. 1–5, Hangzhou, China, 2020.

[108] R. Saruthirathanaworakun, J. M. Peha, and L. M. Correia, “Gray-space spectrum sharing between multiple rotating
radars and cellular network hotspots,” in *2013 IEEE 77th Vehicular Technology Conference (VTCSpring)*, pp. 1–5, Dresden, Germany, 2013.

[109] Z. Junhui, Y. Tao, G. Yi, W. Jiao, and F. Lei, “Power control algorithm of cognitive radio based on non-cooperative game theory,” *China Communications*, vol. 10, no. 11, pp. 143–154, 2013.

[110] N. Gatsis and G. B. Giannakis, “Power control with imperfect exchanges and applications to spectrum sharing,” *IEEE transactions on signal processing*, vol. 59, no. 7, pp. 3410–3423, 2011.

[111] I. Kim and D. Kim, “Minimizing source-sum-power consumption in multi-sensor single-relay networks,” *IEEE Transactions on Communications*, vol. 59, no. 9, pp. 2362–2366, 2011.

[112] A. Goldsmith, S. A. Jafar, I. Maric, and S. Srinivasa, “Breaking spectrum gridlock with cognitive radios: an information theoretic perspective,” *Proceedings of the IEEE*, vol. 97, no. 5, pp. 894–914, 2009.

[113] Y. Chen, G. Yu, Z. Zhang, H.-H. Chen, and P. Qiu, “On cognitive radio networks with opportunistic power control strategies in fading channels,” *IEEE Transactions on Wireless Communications*, vol. 7, no. 7, pp. 2752–2761, 2008.

[114] S. Srinivasa and S. A. Jafar, “Soft sensing and optimal power control for cognitive radio,” *IEEE Transactions on Wireless Communications*, vol. 9, no. 12, pp. 3638–3649, 2010.

[115] V. Asghari and S. Aissa, “Adaptive rate and power transmission in spectrum-sharing systems,” *IEEE Transactions on Wireless Communications*, vol. 9, no. 10, pp. 3272–3280, 2010.

[116] S. Wang, F. Huang, and Z.-H. Zhou, “Fast power allocation algorithm for cognitive radio networks,” *IEEE Communications Letters*, vol. 15, no. 8, pp. 845–847, 2011.

[117] A. Hassanien, M. G. Amin, E. Aboutanios, and B. Himed, “Dual-function radar communication systems: a solution to the spectrum congestion problem,” *IEEE Signal Processing Magazine*, vol. 36, no. 5, pp. 115–126, 2019.

[118] H. Wymeersch, G. Seco-Granados, G. Destino, D. Dardari, and F. Tufvesson, “5G mmWave positioning for vehicular networks,” *IEEE Wireless Communications*, vol. 24, no. 6, pp. 80–86, 2017.

[119] C. Yang and H.-R. Shao, “WiFi-based indoor positioning,” *IEEE Communications Magazine*, vol. 53, no. 3, pp. 150–157, 2015.

[120] S. D. Blunt, P. Yatham, and J. Stiles, “Intr pulse radar-embedded communications,” *IEEE Transactions on Aerospace and Electronic Systems*, vol. 46, no. 3, pp. 1185–1200, 2010.

[121] M. Nowak, M. Wicks, Z. Zhang, and Z. Wu, “Co-designed radarcollection using linear frequency modulation waveform,” *IEEE Aerospace and Electronic Systems Magazine*, vol. 31, no. 10, pp. 28–35, 2016.

[122] D. Gaglione, C. Clemente, A. R. Persico, C. V. Ilioudis, I. K. Proudluder, and J. J. Soraghan, “Fractional Fourier Transform Based Co-Radar Waveform: Experimental Validation,” in *2016 Sensor Signal Processing for Defence (SSPD)*, pp. 1–5, Edinburgh, UK, 2016.

[123] A. Ahmed, Y. D. Zhang, and B. Himed, “Multi-user dual-function radar-communications exploiting sidelobe control and waveform diversity,” in *2018 IEEE Radar Conference (RadarConf18)*, pp. 0698–0702, Oklahoma City, OK, USA, 2018.

[124] C. Avedogdu, N. Garcia, and H. Wyneersch, “Improved pedestrian detection under mutual interference by FMCW radar communications,” in *2018 IEEE 29th Annual International Symposium on Personal, Indoor and Mobile Radio Communications (PIMRC)*, pp. 101–105, Bologna, Italy, 2018.

[125] X. Wang, S. Mao, and M. X. Gong, “An overview of 3GPP cellular vehicle-to-everything standards,” *GetMobile: Mobile Computing and Communications*, vol. 21, no. 3, pp. 19–25, 2017.

[126] C. Sahin, J. Jakabosky, P. M. McCormick, J. G. Metcalf, and S. D. Blunt, “A novel approach for embedding communication symbols into physical radar waveforms,” in *2017 IEEE Radar Conference (RadarConf)*, pp. 1498–1503, Seattle, WA, USA, 2017.

[127] D. Ma, T. Huang, N. Shlezinger, Y. Liu, X. Wang, and Y. C. Eldar, “A DFRC system based on multi-carrier agile FMCW MIMO radar for vehicular applications,” in *2020 IEEE International Conference on Communications Workshops (ICC Workshops)*, pp. 1–7, Dublin, Ireland, 2020.

[128] D. Ma, N. Shlezinger, T. Huang et al., “Spatial modulation for joint radar-communications systems: design, analysis, and hardware prototype,” *IEEE Transactions on Vehicular Technology*, vol. 70, no. 3, pp. 2283–2289, 2021.

[129] I. P. Eedara, M. G. Amin, and A. Hassanien, “Analysis of communication symbol embedding in FH MIMO radar platforms,” in *2019 IEEE Radar Conference (RadarConf)*, pp. 1–6, Boston, MA, USA, 2019.

[130] S. Zhou, X. Liang, Y. Yu, and H. Liu, “Joint radar-communications co-use waveform design using optimized phase perturbation,” *IEEE Transactions on Aerospace and Electronic Systems*, vol. 55, no. 3, pp. 1227–1240, 2019.

[131] Y. Dong, G. A. Fabrizio, and M. G. Amin, “Dual-functional radar waveforms without remodulation,” in *2019 IEEE Radar Conference (RadarConf)*, pp. 1–6, Boston, MA, USA, 2019.

[132] A. Ahmed, Y. Gu, D. Silage, and Y. D. Zhang, “Power-efficient multi-user dual-function radar-communications,” in *2018 IEEE 19th International Workshop on Signal Processing Advances in Wireless Communications (SPAWC)*, pp. 1–5, Kalamata, Greece, 2018.

[133] K. Wu, J. A. Zhang, X. Huang, Y. J. Guo, and R. W. Heath, “Waveform design and accurate channel estimation for frequency-hopping MIMO radar-based communications,” *IEEE Transactions on Communications*, vol. 69, no. 2, pp. 1244–1258, 2021.

[134] D. Ma, N. Shlezinger, T. Huang, Y. Liu, and Y. C. Eldar, “Joint radarcommunication strategies for autonomous vehicles: combining two key automotive technologies,” *IEEE Signal Processing Magazine*, vol. 37, no. 4, pp. 85–97, 2020.

[135] C. S. Pappu and T. L. Carroll, “Quasi-FM waveform using chaotic oscillator for joint radar and communication systems,” *Chaos, Solitons & Fractals*, vol. 152, article 111449, 2021.

[136] S. H. Dokhanchi, B. S. Mysore, K. V. Mishra, and B. Ottersten, “A mmWave automotive joint radarcommunications system,” *IEEE Transactions on Aerospace and Electronic Systems*, vol. 55, no. 3, pp. 1241–1260, 2019.

[137] C. B. Barneto, T. Riihonen, M. Turunen et al., “Full-duplex OFDM radar with LTE and 5G NR waveforms: challenges, solutions, and measurements,” *IEEE Transactions on Microwave Theory and Techniques*, vol. 67, no. 10, pp. 4042–4054, 2019.
[138] L. Gaudio, M. Kobayashi, G. Caire, and G. Colavolpe, “On the effectiveness of OTFS for joint radar parameter estimation and communication,” IEEE Transactions on Wireless Communications, vol. 19, no. 9, pp. 5951–5965, 2020.

[139] M. A. Richards, J. A. Scheer, W. A. Holm, B. Beckley, P. Mark, and A. Richards, Principles of Modern Radar Volume I- Basic Principles, Citeseer, 2010.

[140] P. M. McCormick, S. D. Blunt, and J. G. Metcalf, “Simul- taneous radar and communications emissions from a common aperture, part Theory,” in 2017 IEEE Radar Conference (RadarConf), pp. 1685–1690, Seattle, WA, USA, 2017.

[141] A. Hassanien, M. G. Amin, Y. D. Zhang, and F. Ahmad, “Phase modulation based dual-function radar-communications,” IET Radar, Sonar & Navigation, vol. 10, no. 8, pp. 1411–1421, 2016.

[142] G. N. Saddik, R. S. Singh, and E. R. Brown, “Ultra-wideband multifunctional communications/radar system,” IEEE Transactions on Microwave Theory and Techniques, vol. 55, no. 7, pp. 1431–1437, 2007.

[143] A. Hassanien, M. G. Amin, Y. D. Zhang, and F. Ahmad, “Dual-function radar-communications using phase-rotational invariance,” in 2015 23rd European Signal Processing Conference (EUSIPCO), pp. 1346–1350, Nice, France, 2015.

[144] A. R. Al-Salehi, I. M. Qureshi, A. N. Malik, Z. Khan, and W. Khan, “Throughput enhancement for dual-function radar-embedded communications using two generalized sidelobe cancellers,” IEEE Access, vol. 7, pp. 91 390–91 398, 2019.

[145] A. Hassanien, M. G. Amin, Y. D. Zhang, and B. Himed, “A dualfunction MIMO radar-communications system using PSK modulation,” in 2016 24th European Signal Processing Conference (EUSIPCO), pp. 1613–1617, Budapest, Hungary, 2016.

[146] A. Hassanien, M. G. Amin, Y. D. Zhang, F. Ahmad, and B. Himed, “Non-coherent PSK-based dual-function radar-communication systems,” in 2016 IEEE Radar Conference (RadarConf), pp. 1–6, Philadelphia, PA, USA, 2016.

[147] E. Basar, “Index modulation techniques for 5G wireless networks,” IEEE Communications Magazine, vol. 54, no. 7, pp. 168–175, 2016.

[148] T. Huang, X. Xu, Y. Liu, N. Shlezinger, and Y. C. Eldar, “A dualfunction radar communication system using index modulation,” in 2019 IEEE 20th International Workshop on Signal Processing Advances in Wireless Communications (SPAWC), pp. 1–5, Cannes, France, 2019.

[149] X. Wang, A. Hassanien, and M. G. Amin, “Dual-function mimo radar communications system design via sparse array optimization,” IEEE Transactions on Aerospace and Electronic Systems, vol. 55, no. 3, pp. 1213–1226, 2019.

[150] T. Huang, N. Shlezinger, X. Xu, D. Ma, Y. Liu, and Y. C. Eldar, “Multi-carrier agile phased array radar,” IEEE Transactions on Signal Processing, vol. 68, pp. 5706–5721, 2020.

[151] T. Huang, N. Shlezinger, X. Xu, Y. Liu, and Y. C. Eldar, “MajorCom: a dual-function radar communication system using index modulation,” IEEE Transactions on Signal Processing, vol. 68, pp. 3423–3438, 2020.

[152] X. Liu, T. Huang, N. Shlezinger, Y. Liu, J. Zhou, and Y. C. Eldar, “Joint transmit beamforming for multiuser MIMO communications and MIMO radar,” IEEE Transactions on Signal Processing, vol. 68, pp. 3929–3944, 2020.

[153] T. W. Tedesco and R. Romero, "Code shift keying based joint radar and communications for EMCON applications," Digital Signal Processing, vol. 80, pp. 48–56, 2018.

[154] C.-Y. Chen and P. Vaidyanathan, "MIMO radar ambiguity properties and optimization using frequency-hopping waveforms," IEEE Transactions on Signal Processing, vol. 56, no. 12, pp. 5926–5936, 2008.

[155] G. Sharma, P. Srilhari, and K. R. Rajeswari, "MIMO radar ambiguity analysis of frequency hopping pulse waveforms," in 2014 IEEE Radar Conference, pp. 1241–1246, Cincinnati, OH, USA, 2014.

[156] K. Han and A. Nehorai, "Jointly optimal design for MIMO radar frequency-hopping waveforms using game theory," IEEE Transactions on Aerospace and Electronic Systems, vol. 52, no. 2, pp. 809–820, 2016.

[157] A. Hassanien, B. Himed, and B. D. Rigling, "A dual-function MIMO radar-communications system using frequency-hopping waveforms," in 2015 IEEE Radar Conference (RadarConf), pp. 1721–1725, Seattle, WA, USA, 2017.

[158] I. P. Fedara, Novel Signal Embedding Strategies for Mimo Dual Function Radar Communication Systems [PhD dissertation], VILLANOVA UNIVERSITY, 2021.

[159] Y. Yao, X. Li, and L. Wu, "Cognitive frequency-hopping waveform design for dual-function MIMO radar-communications system," Sensors, vol. 20, no. 2, 2020.

[160] A. Hassanien, M. G. Amin, Y. D. Zhang, and F. Ahmad, "Dual-function radar-communications: information embedding using sidelobe control and waveform diversity," IEEE Transactions on Signal Processing, vol. 64, no. 8, pp. 2168–2181, 2016.

[161] M. Hanif and H. H. Nguyen, "Frequency-Shift Chirp Spread Spectrum Communications With Index Modulation," IEEE Internet of Things Journal, vol. 8, no. 24, pp. 17611–17621, 2021.

[162] P. Sriani, C. V. Ilioudis, C. Clemente, and J. J. Soraghan, "Fractional fourier transform based joint radar communication system for multi-user automotive applications," in 2019 IEEE Radar Conference (RadarConf), pp. 1–6, Boston, MA, USA, 2019.

[163] D. Gaglione, C. Clemente, C. V. Ilioudis et al., "Waveform design for communicating radar systems using fractional fourier transform," Digital Signal Processing, vol. 80, pp. 57–69, 2018.

[164] M. Jamil, H.-J. Zepernick, and M. I. Pettersson, "On integrated radar and communication systems using Oppermann sequences," in MILCOM 2008-2008 IEEE Military Communications Conference, pp. 1–6, San Diego, CA, USA, 2008.

[165] E. J. Kaminsky and L. Simanjuntak, "Chirp Slope Keying for Underwater Communications," in Sensors, and Command, Control, Communications, and Intelligence (C3I) Technologies for Homeland Security and Homeland Defense IV, E. M. Carapezza, Ed., vol. 5778, pp. 894–899, SPIE, 2005.

[166] S. Xu, Y. Chen, and P. Zhang, "Integrated radar and communication based on DS-UWB," in 2006 3rd International Conference on Ultrawideband and Ultrashort Impulse Signals, pp. 142–144, Sevastopol, Ukraine, 2006.

[167] M. Robertson and E. Brown, "Integrated radar and communications based on chirped spread-spectrum techniques," in IEEE MTT-S International Microwave Symposium Digest, 2003, vol. 1, pp. 611–614, Philadelphia, PA, USA, 2003.
[199] J. Qian, M. Lu, and N. Huang, "Radar and Communication Co-Existence Design Based on Mutual Information Optimization," *IEEE Transactions on Circuits and Systems II: Express Briefs*, vol. 67, no. 12, pp. 3577–3581, 2020.

[200] Y. Gu and Y. D. Zhang, "Information-theoretic pilot design for downlink channel estimation in FDD massive MIMO systems," *IEEE Transactions on Signal Processing*, vol. 67, no. 9, pp. 2334–2346, 2019.

[201] Y. Liu, G. Liao, J. Xu, Z. Yang, and Y. Zhang, "Adaptive OFDM integrated radar and communications waveform design based on information theory," *IEEE Communications Letters*, vol. 21, no. 10, pp. 2174–2177, 2017.

[202] B. Donnet and I. Longstaff, "Combining MIMO radar with OFDM communications," in *2006 European radar conference*, pp. 37–40, Manchester, UK, 2006.

[203] A. De Maio and M. Lops, "Design principles of MIMO radar detectors," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 43, no. 3, pp. 886–898, 2007.

[204] R. Xu, L. Peng, W. Zhao, and Z. Mi, "Radar mutual information and communication channel capacity of integrated radar-communication system using MIMO," *ICT Express*, vol. 1, no. 3, pp. 102–105, 2015.

[205] A. R. Chiriyath, B. Paul, G. M. Jacyna, and D. W. Bliss, "Inner bounds on performance of radar and communications coexistence," *IEEE Transactions on Signal Processing*, vol. 64, no. 2, pp. 464–474, 2015.

[206] Y. Yang and R. S. Blum, "MIMO radar waveform design based on mutual information and minimum mean-square error estimation," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 43, no. 1, pp. 330–343, 2007.

[207] B. Luo, Q. Cui, H. Wang, and X. Tao, "Optimal joint water-filling for coordinated transmission over frequency-selective fading channels," *IEEE Communications Letters*, vol. 15, no. 2, pp. 190–192, 2011.

[208] A. Ahmed, Y. D. Zhang, A. Hassanien, and B. Himed, "OFDM-based joint radar-communication system: optimal sub-carrier allocation and power distribution by exploiting mutual information," in *2019 53rd Asilomar Conference on Signals, Systems, and Computers*, pp. 559–563, Pacific Grove, CA, USA, 2019.

[209] E. Bașar, U. Aygölü, E. Panayırıcı, and H. V. Poor, "Orthogonal frequency division multiplexing with index modulation," *IEEE Transactions on Signal Processing*, vol. 61, no. 22, pp. 5536–5549, 2013.

[210] E. Basar, M. Wen, R. Mesleh, M. Di Renzo, Y. Xiao, and H. Haas, "Index modulation techniques for next-generation wireless networks," *IEEE access*, vol. 5, pp. 16 693–16 746, 2017.

[211] E. Arslan, A. T. Dogukan, and E. Basar, "Index modulation-based flexible non-orthogonal multiple access," *IEEE Wireless Communications Letters*, vol. 9, no. 11, pp. 1942–1946, 2020.

[212] A. T. Dogukan and E. Basar, "Super-mode ofdm with index modulation," *IEEE Transactions on Wireless Communications*, vol. 19, no. 11, pp. 7535–7562, 2020.

[213] E. Arslan, A. T. Dogukan, and E. Basar, "Sparse-encoded codebook index modulation," *IEEE Transactions on Vehicular Technology*, vol. 69, no. 8, pp. 9126–9130, 2020.

[214] E. Başar, "OFDM with index modulation using coordinate interleaving," *IEEE Wireless Communications Letters*, vol. 4, no. 4, pp. 381–384, 2015.

[215] L. Yan, X. Fang, H. Li, and C. Li, "An mmWave wireless communication and radar detection integrated network for rail- ways," in *2016 IEEE 83rd Vehicular Technology Conference (VTC Spring)*, pp. 1–5, Nanjing, China, 2016.

[216] J. A. Zhang, X. Huang, Y. J. Guo, J. Yuan, and R. W. Heath, "Multibeam for joint communication and radar sensing using steerable analog antenna arrays," *IEEE Transactions on Vehicular Technology*, vol. 68, no. 1, pp. 671–685, 2019.

[217] V. Va and R. W. Heath, "Performance analysis of beam sweeping in millimeter wave assuming noise and imperfect antenna patterns," in *2016 IEEE 84th Vehicular Technology Conference (VTC-Fall)*, pp. 1–5, Montreal, QC, Canada, 2016.

[218] B. D. Van Veen and K. M. Buckley, "Beamforming: a versatile approach to spatial filtering," *IEEE ASSP Magazine*, vol. 5, no. 2, pp. 4–24, 1988.

[219] A. K. Singh, S. E. Avirah, and P. Kumar, "Multi beam antenna system for portable 3-D surveillance radar," in *The Second European Conference on Antennas and Propagation, EuCAP 2007*, pp. 1–4, Edinburgh, 2007.

[220] W. Hong, Z. H. Jiang, C. Yu et al., "Multibeam antenna technologies for 5G wireless communications," *IEEE Transactions on Antennas and Propagation*, vol. 65, no. 12, pp. 6231–6249, 2017.

[221] Y. A. Atesal, B. Cetinoneri, K. M. Ho, and G. M. Rebeiz, "A twochannel 8–20-GHz SiGe BICMOS receiver with selectable ifs for multibeam phased-array digital beamforming applications," *IEEE Transactions on Microwave Theory and Techniques*, vol. 59, no. 3, pp. 716–726, 2011.

[222] H. Ma, Z. Wei, J. Zhang, F. Ning, and Z. Feng, "Three-dimensional multiple access method for joint radar and communication enabled V2X network," in *IEEE International Conference on Signal, Information and Data Processing (ICSIDP)*, pp. 1–5, Chongqing, China, 2019.

[223] H. GV Trunk II and L. PK, *Advanced Multifunction RF System (AMRFs) Preliminary Design Considerations*, Formal Rep, NRL, Washington, DC, 2001.

[224] G. Tavik, "Advanced multifunction radio frequency (AMRF) concept testbed overview," in *Government Microcircuit Application Conf. Dig.*, Mar. 2001, pp. 100–102, 2001.

[225] D. C. Wu and J. Lawrence, *Advanced ECM Transmitter Advanced Technology Demonstration*, Rep NRL/FR/5740-02-10033, NRL, Washington, DC, 2002.

[226] D. H. Schaubert, "Wide-band phased arrays of ViValdi notch antennas," in *Tenth International Conference on Antennas and Propagation (Conf. Publ. No. 436)*, vol. 1, pp. 6–12, Edinburgh, UK, 1997.

[227] S. Lindenmeier, J.-F. Luy, and P. Russer, "A multifunctional antenna for terrestrial and satellite radio applications," in *2001 IEEE MTT-S International Microwave Symposium Digest (Cat. No. 01CH37157)*, vol. 1, pp. 393–396, Phoenix, AZ, USA, 2001.

[228] P. W. Moo and D. J. DiFiilippo, "Multifunction RF systems for naval platforms," *Sensors*, vol. 18, no. 7, p. 2076, 2018.