Analysis of the $\Xi_{cc}\Xi_{cc}$ hexaquark molecular state with the QCD sum rules

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Abstract

In this work, we construct the color-singlet-color-singlet type six-quark pseudoscalar current to investigate the $\Xi_{cc}\Xi_{cc}$ hexaquark molecular state with the QCD sum rules, the predicted mass $M_X \sim 7.2$ GeV supports assigning the $X(7200)$ to be the $\Xi_{cc}\Xi_{cc}$ hexaquark molecular state with the quantum numbers $J^{PC} = 0^{-+}$. The $X(7200)$ can decouple through fusions of the $cc$ and $\bar{q}\bar{q}$ pairs, we can search for the $X(7200)$ and explore its properties in the $J/\psi J/\psi$, $\chi_{c1}\chi_{c1}$, $D^* D^*$ and $D_s D_s$ invariant mass spectrum in the future.

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Key words: Hexaquark molecular state, QCD sum rules

1 Introduction

In 2017, the LHCb collaboration observed the doubly charmed baryon state $\Xi_{cc}^{++}$ in the $\Lambda_c^+ K^- \pi^+ \pi^+$ invariant mass distribution [1]. The observation of the $\Xi_{cc}^{++}$ makes a great progress on the spectroscopy of the doubly charmed baryon states, tetraquark states and pentaquark states.

In 2020, the LHCb collaboration investigated the $J/\psi J/\psi$ invariant mass distributions and observed a narrow structure about 6.9 GeV and a broad structure just above the $J/\psi J/\psi$ threshold at $p_T > 5.2$ GeV with the global significances larger than 5$\sigma$ [2]. The Breit-Wigner mass and width of the $X(6900)$ are $M_X = 6905 \pm 11 \pm 7$ MeV and $\Gamma_X = 80 \pm 19 \pm 33$ MeV, respectively. Furthermore, they also observed some vague structures around 7.2 GeV, which coincides with the $\Xi_{cc}\Xi_{cc}$ threshold 7242.4 MeV [2]. The energy is sufficient to create a baryon-antibaryon pair $\Xi_{cc}\Xi_{cc}$ containing the valence quarks $cc\bar{q}\bar{q}$.

In the dynamical diquark model, the first radial excited states of the D-wave tetraquark states with the valence quarks $cccc$ have the masses about 7.2 GeV, the insignificant enhancement $X(7200)$ may be a combination of some $2P$ and (or) $2D$ diquark-antidiquark type $cccc$ states with threshold effects of the transitions $\Xi_{cc}\Xi_{cc} \rightarrow J/\psi J/\psi$ [3]. The assignment of the $Y(4630)$ serves as a benchmark, in this model, the $Y(4630)$ can be assigned to be a diquark-antidiquark type tetraquark state, fragmentation of the color flux-tube connecting the diquark-antidiquark pair $c\bar{q}\bar{q}\bar{q}$ leads to the lowest-lying baryon-antibaryon pair $\Lambda_c^+\Lambda_c^-$ [4]. Just like the $Y(4630)$, fragmentation of the color flux-tube connecting the diquark-antidiquark pair $cccc$ in the $X(7200)$ leads to the lowest-lying baryon-antibaryon pair $\Xi_{cc}\Xi_{cc}$, then translates to the $J/\psi J/\psi$ pair.

In the V-baryonium tetraquark scenario and the string-junction scenario, which share the same feature, the exotic $X$, $Y$ and $Z$ states are genuine tetraquarks rather than molecular states, they have a baryonic vertex (or string-junction) attaching a $cc$-diquark in color antitriplet, which is connected by a string to an anti-baryonic vertex (or string-junction) attaching a $cc$-antidiquark in color triplet. In the V-baryonium tetraquark scenario, the un-conformed structure $X(7200)$ is assigned to be the first radially excited state of the $X(6900)$, the decays to the baryon-antibaryon pair $\Xi_{cc}\Xi_{cc}$ can occur via breaking the string and creating a quark-antiquark pair [5]. In the string junction scenario, the $X(6900)$ and $X(7200)$ are assigned to be the 2S tetraquark states with the quantum numbers $J^{PC} = 0^{++}$ and $2^{++}$, respectively [6].

If the $\Xi_{cc}^{(*)}\Xi_{cc}^{(*)}$ system and $D^{(*)}D^{(*)}$ system are related to each other via heavy antibaryon-diquark symmetry, several molecular states can be predicted based on the contact-range effective field theory, the $\Xi_{cc}\Xi_{cc}$ molecular states which have the quantum numbers $J^{PC} = 0^{-+}$ and (or) $1^{--}$ maybe contribute to the $X(7200)$ [7].

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The exotic $X$, $Y$, $Z$ and $P$ states always lie near two-particle thresholds, such as

\[
egin{align*}
DD^*/DD^* &: X(3872), \ Z_c(3885/3900), \\
D^* D^* &: Z_c(4020/4025), \\
D^*_s/D^*_s &: Z_{cs}(3985/4000), \\
D^*_c/D^*_c &: X(4140), \\
D\bar{D}_1/\bar{D}D_1 &: Y(4260/4220), \ Z_c(4250), \\
D^*D_0/D^*D_0 &: Y(4360/4320), \ Z_b(10610), \\
\bar{D}\Sigma_c &: P_c(4312), \\
\bar{D}\Xi_c'/: P_{cs}(4459), \\
D^*\Sigma_c &: P_c(4380), \\
\bar{D}^*\Sigma_c &: P_c(4440/4457), \\
\Lambda^+_c\Lambda^-_c'/f_0(980)\psi': Y(4630/4660), \\
BB^*/BB^* &: Z_b(10650), \\
B^*\bar{B}^*: Z_b(10650), \ (1)
\end{align*}
\]

it is natural to assume that they are molecular states composed of two color-singlet constituents and investigate their properties with the QCD sum rules \cite{8, 9, 10, 12, 13, 14}. The QCD sum rules approach is a powerful theoretical tool in exploring the hadron properties, and has given many successful descriptions of the masses and decay widths of the tetraquark and pentaquark (molecular) states \cite{8, 9, 10, 12, 13, 14}, and has been successfully applied to investigate the dibaryon states and baryonium states \cite{15, 16}.

In the present work, we explore the color-singlet-color-singlet type hexaquark molecular state $\Xi_{cc}\Xi_{cc}$ with the quantum numbers $J^{PC}=0^{-+}$, and make possible assignment of the $X(7200)$.

The article is arranged as follows: we derive the QCD sum rules for the mass and pole residue of the $\Xi_{cc}\Xi_{cc}$ molecular state in section 2; in section 3, we present the numerical results and discussions; section 4 is reserved for our conclusion.

## 2 QCD sum rules for the $\Xi_{cc}\Xi_{cc}$ hexaquark molecular state

Let us write down the two-point correlation function $\Pi(p^2)$ firstly,

\[
\Pi(p^2) = i \int d^4x e^{ip\cdot x} \langle 0 | \{ J(x) J^\dagger(0) \} | 0 \rangle , \quad (2)
\]

where

\[
\begin{align*}
J(x) &= \bar{J}_{cc}(x) i\gamma_5 J_{cc}(x) , \\
J_{cc}(x) &= e^{ikT_i} c_i(x) C_\gamma c_j(x) \gamma^\alpha \gamma^5 q(x) ,
\end{align*}
\]

$q = u, d$, the $i$, $j$, $k$ are color indexes, the current $J_{cc}(x)$ has the same quantum numbers as the doubly charmed baryon state $\Xi_{cc}$, and couples potentially to the color-singlet clusters with the same quantum numbers as the $\Xi_{cc}$ \cite{17}.

At the hadron side, we isolate the ground state contribution of the pseudoscalar hexaquark molecular state, we obtain the result,

\[
\Pi(p^2) = \frac{\lambda_X^2}{M_X^2 - p^2} + \cdots , \quad (4)
\]

where the pole residue $\lambda_X$ is defined by

\[
\langle 0 | J(0) | X(p) \rangle = \lambda_X . \quad (5)
\]
In Eq. (4), we have neglected the two-particle scattering state contributions, just like in the QCD sum rules for the tetraquark (molecular) states. In Ref. [18], Lucha, Melikhov and Sazdjian obtain the conclusion that "A possible exotic tetraquark state may appear only in $N_c$-subleading contributions to the QCD Green functions" based on the naive large-$N_c$ analysis by S. Narison et al at the real word $N_c = 3$ shows the opposite due to the induced loop factor missed in a naive $N_c$ counting rule [19]. In fact, as pointed in Ref. [20], without excluding the contributions of factorizable Feynman diagrams in the color space to the QCD sum rules by hand, we cannot obtain the conclusion that the factorizable parts of the operator product expansion series cannot have any relationship to the possible tetraquark bound states. For the baryon, tetraquark, pentaquark states with string junctions [21], the standard large-$N_c$ counting rules for the ordinary mesons cannot be trivially extrapolated to the exotic hadrons but should be modified for being properly applied at the real word $N_c = 3$. All in all, we can examine the predictions of the multiquark states based on the QCD sum rules in different channels in the future.

In the QCD side, we carry out the operator product expansion up to the vacuum condensates of dimension 10 in a consistent way. There are four heavy quark propagators and two light quark propagators in the correlation function $\Pi(p^2)$ after accomplishing the Wick’s contractions. If each heavy quark line emits a gluon, and each light quark line contributes a quark-antiquark pair, we obtain a quark-gluon operator $g_s G_{\mu\nu} g_s G_{\alpha\beta} g_s G_{\rho\sigma} g_s G_{\lambda\gamma} q q q q q q q q q q q q q q q q q q$, which is of dimension 14, we should take account of the vacuum condensates of dimensions up to dimension 14. In the QCD sum rules for the hidden-charm or hidden-bottom tetraquark (molecular) states, pentaquark (molecular) states, we usually take the truncation $O(\alpha_s^k)$ with $k \leq 1$ [10, 13, 22, 23]. If we also take the truncation $k \leq 1$ to discard the quark-gluon operators of the orders $O(\alpha_s^{k+1})$ in the present work, the operator product expansion is terminated at the vacuum condensates of dimension 10.

In the QCD sum rules for the triply-charmed diquark-diquark-diquark type hexaquark states and triply-charmed dibaryon states, there are three light quark propagators and three heavy quark propagators in the correlation functions after accomplishing the Wick’s contractions [15, 24]. Again, if each heavy quark line emits a gluon and each light quark line contributes quark-antiquark pair, we obtain a quark-gluon operator $g_s G_{\mu\nu} g_s G_{\alpha\beta} g_s G_{\rho\sigma} g_s G_{\lambda\gamma} q q q q q q q q q q q q q q q q q q$, which is of dimension 15. If we take the truncations $k \leq 1$, the operator product expansion is terminated at the vacuum condensates of dimension 13. The operator $g_s G_{\mu\nu} g_s G_{\alpha\beta} g_s G_{\rho\sigma} g_s G_{\lambda\gamma} q q q q q q q q q q q q q q q q q q$ leads to the vacuum condensates $(\frac{\sigma G q}{s}) (\langle q \bar{q} \rangle)$, $(\langle \sigma G q \rangle (\langle q \bar{q} \rangle)^2$, $(\langle \sigma G q \rangle (\langle q \bar{q} \rangle)^3$, and $(\langle \sigma G q \rangle (\langle q \bar{q} \rangle)^4$, we calculate the vacuum condensate $(\langle \sigma G q \rangle (\langle q \bar{q} \rangle)^k$, and observe that its small contribution can be neglected safely [15, 24]. All the vacuum condensates $(\langle \sigma G q \rangle (\langle q \bar{q} \rangle)^k$, which are the vacuum expectation values of the quark-gluon operators of the orders $O(\alpha_s^k)$ with $k \leq 1$. The vacuum condensates $(\langle \sigma G q \rangle (\langle q \bar{q} \rangle)^3$, $(\langle \sigma G q \rangle (\langle q \bar{q} \rangle)^4$ and $(\langle \sigma G q \rangle (\langle q \bar{q} \rangle)^5$ have the dimensions 6, 8, 9 and 9, respectively. However, they are the vacuum expectation values of the quark-gluon operators of the orders $O(\alpha_s^2)$, $O(\alpha_s^3)$, $O(\alpha_s^2)$ and $O(\alpha_s^3)$, respectively, and are neglected, direct calculations indicate that those contributions are tiny indeed [24].

We obtain the spectral density at the quark level through dispersion relation, take the quark-hadron duality below the continuum threshold $s_0$, and perform Borel transform in regard to the variable $P^2 = -p^2$ to obtain the QCD sum rules:

$$\chi^2 \exp \left( -\frac{M_X^2}{T^2} \right) = \int_{16m_t^2}^{s_0} ds \rho_{QCD}(s) \exp \left( -\frac{s}{T^2} \right),$$  \tag{6}
Table 1: The energy gaps between the ground states (1S) and first radial excited states (2S) of the hidden-charm tetraquark candidates with the possible assignments.

| $J^{PC}$ | 1S    | 2S    | energy gaps | References |
|----------|-------|-------|-------------|------------|
| 1$^+$    | $Z_c(3900)$ | $Z_c(4430)$ | 591 MeV | [30] [36] [37] |
| 0$^+$    | $X(3915)$   | $X(4500)$   | 588 MeV | [28] [39] [40] |
| 1$^+$    | $Z_c(4020)$ | $Z_c(4600)$ | 576 MeV | [11] [12] |
| 1$^+$    | $X(4140)$   | $X(4685)$   | 566 MeV | [43] |

where the $\rho_{QCD}(s)$ is the spectral density at the quark level.

We derive Eq. (6) with respect to $\tau = \frac{t}{\Lambda^2}$, then eliminate the pole residue $\lambda_X$, and obtain the QCD sum rules for the mass of the pseudoscalar hexaquark molecular state $\Xi_{cc}\Xi_{cc}$,

$$M_X^2 = -\frac{d}{ds} \frac{r_{s0}}{f_{16m_c^2}} ds \rho_{QCD}(s) \exp\left(-\frac{s}{\Lambda^2}\right). \tag{7}$$

3 Numerical results and discussions

We adopt the standard values of the vacuum condensates $\langle \bar{q}q \rangle = -(0.24 \pm 0.01)$ GeV$^3$, $\langle \bar{q}q, \sigma Gq \rangle = m_0^2 \langle \bar{q}q \rangle$, $m_0^2 = (0.8 \pm 0.1)$ GeV$^2$, $\langle \alpha_s G^2 \rangle = 0.012 \pm 0.004$ GeV$^4$ at the energy scale $\mu = 1$ GeV [27] [28] [29] [30] [31], and take the $\overline{MS}$ mass of the charm quark, $m_c(m_c) = (1.275 \pm 0.025)$ GeV, from the Particle Data Group [32]. In addition, we take account of the energy-scale dependence of all the input parameters [33].

$$\langle \bar{q}q \rangle (\mu) = \langle \bar{q}q \rangle (1 \text{GeV}) \left[ \frac{\alpha_s(1 \text{GeV})}{\alpha_s(\mu)} \right]^{\frac{12}{\pi^2} t},$$

$$\langle \bar{q}q, \sigma Gq \rangle (\mu) = \langle \bar{q}q, \sigma Gq \rangle (1 \text{GeV}) \left[ \frac{\alpha_s(1 \text{GeV})}{\alpha_s(\mu)} \right]^{\frac{2}{\pi^2} t},$$

$$m_c(\mu) = m_c(m_c) \left[ \frac{\alpha_s(\mu)}{\alpha_s(m_c)} \right]^{\frac{12}{\pi^2} t},$$

$$\alpha_s(\mu) = \frac{1}{b_0 t} \left[ 1 - \frac{b_1}{b_0^2} \log \frac{t}{b_0^2} + \frac{b_2^2}{b_0^4} \left( \log^2 t - \log t - 1 \right) + \frac{b_3}{b_0^4 t^2} \right], \tag{8}$$

where $t = \log \frac{\mu^2}{\Lambda^2}$, $b_0 = \frac{33-2n_f}{12\pi}$, $b_1 = \frac{153-19n_f}{24\pi^2}$, $b_2 = \frac{2857-303n_f+325n_f^2}{128\pi^4}$, $\Lambda = 213$ MeV, 296 MeV and 339 MeV for the quark flavor numbers $n_f = 5, 4$ and 3, respectively [32]. In the present work, we explore the hidden-charm hexaquark molecular state $\Xi_{cc}\Xi_{cc}$ and choose $n_f = 4$, then evolve all the input parameters to a typical energy scale $\mu$ to extract the hexaquark molecular mass. Furthermore, we present the predictions based on the updated parameters obtained by S. Narison, $m_s(m_c) = (1.266 \pm 0.006)$ GeV and $\langle \alpha_s G^2 \rangle / \pi = 0.021 \pm 0.001$ GeV$^4$ [34], and in this case the energy scales of other vacuum condensates are taken at $\mu = 1$ GeV.

We should choose suitable continuum threshold $s_0$ to avoid contamination from the first radial excited state. In the scenario of the tetraquark states, the possible assignments of the exotic states $Z_c(3900)$, $Z_c(4430)$, $X(3915)$, $X(4500)$, $Z_c(4020)$, $Z_c(4600)$, $X(4140)$ and $X(4685)$ are presented plainly in Table 1 according to the (possible) quantum numbers, decay modes and energy gaps. From the table, we can obtain the conclusion tentatively that the energy gaps between the ground states and first radial excited states of the hidden-charm tetraquark states are about 0.58 GeV. We can choose the continuum threshold parameter as $\sqrt{s_0} = M_X + 0.6 \pm 0.1$ GeV. If the mass of the $\Xi_{cc}\Xi_{cc}$ hexaquark molecular state and the energy scale of the spectral density at the quark
Table 2: The energy scale of the QCD spectral density, Borel parameter, pole contribution, mass and pole residue of the hexaquark molecular state Ξ_{cc}Ξ_{cc}, where the superscript * denotes the c-quark mass and gluon condensate are taken from Ref. [34].

| μ (GeV) | T^2 (GeV^2) | pole | M_{X/Y} (GeV) | λ (10^{-3}GeV^6) |
|---------|-------------|------|--------------|------------------|
| 1       | 4.7 – 5.3   | (18 – 37)% | 7.21 ± 0.13 | 6.35 ± 2.19     |
| 1.275   | 5.3 – 5.9   | (18 – 35)% | 7.21 ± 0.11 | 13.1 ± 3.5      |
| 1.5     | 5.7 – 6.3   | (17 – 32)% | 7.21 ± 0.12 | 18.4 ± 4.6      |
| 2       | 6.1 – 6.7   | (18 – 32)% | 7.18 ± 0.11 | 27.9 ± 6.2      |
| 1*      | 5.0 – 5.6   | (22 – 41)% | 7.21 ± 0.12 | 13.3 ± 2.8      |

The energy scale formula \( M_X = \sqrt{\mu^2 + (4M_c)^2} \), the lower bound of the mass \( M_X \geq \sqrt{(1 \text{GeV})^2 + (4 \times 1.85 \text{GeV})^2} = 7.47 \text{GeV} > 2M_{\Xi_{cc}} \).

Now let us suppose that the multiquark states \( X, Y, Z \) and \( P \) have \( N_Q + N_q \) valence quarks, where the \( N_Q \) and \( N_q \) are the numbers of the heavy quarks and light quarks, respectively. Generally speaking, if \( N_Q \leq N_q \), we can apply the energy scale formula \( \mu = \sqrt{M_{X/Y/Z/P}^2 - (N_QM_Q)^2} \) to enhance the pole contributions and improve the convergent behavior of the operator product expansion [10, 13, 15, 22, 23, 24]. In the present case, \( N_Q = 4 \times N_q = 2 \), the energy scale formula is not applicable.

After trial and error, we obtain the continuum threshold parameter \( \sqrt{s_0} = 7.36 \pm 0.1 \text{GeV} \), and Borel parameters which are shown in Table 2 for four typical energy scales \( \mu = 1.0 \text{GeV}, \mu_c(m_c), 1.5 \text{GeV}, 2.0 \text{GeV} \) and \( \mu^* \), where the superscript * denotes the c-quark mass \( m_c(m_c) = (1.266 \pm 0.006) \text{GeV} \) and gluon condensate \( \langle \bar{q}q \rangle = 0.021 \pm 0.001 \text{GeV}^4 \) are taken for Ref. [34]. In the Borel windows, the ground state contributions are about (18 – 37)% or (22 – 41)% and the pole contributions cannot reach 50%, the contributions of the vacuum condensates of dimension 10 are about (2 – 6)% and (1 – 2)% respectively. The operator product expansion converges very well.

Then we take account of all uncertainties of the parameters, and obtain the values of the mass and pole residue of the pseudoscalar hexaquark molecular state \( \Xi_{cc} \Xi_{cc} \), which are shown explicitly in Table 2 and Fig. 1. From Fig. 1, we can see that the predicted mass is rather stable with variation of the Borel parameter, the uncertainty comes from the Borel parameter is rather small. From Table 2 we can see that the predicted mass \( M_X \) is almost independent on the energy scale of the QCD spectral density, while the pole residue depends heavily on the energy scale of the QCD spectral density, which is qualitatively consistent with the evolution behavior of the current operator from the re-normalization group equation, \( J(x, \mu) = L^{-\gamma J} J(x, \mu_0) \) and \( \lambda_X(\mu) = L^{-1} \lambda_X(\mu_0) \), where \( L = \frac{\alpha_s(\mu_0)}{\alpha_s(\mu)} \), and the \( \gamma_J \) is the anomalous dimension of the current operator \( J(x) \). The predicted mass \( M_X \sim 7.2 \text{GeV} \) is compatible with the vague structure around \( 2 \times 10^{-4} \text{GeV} \) in the \( J/\psi J/\psi \) invariant mass spectrum observed by the LHCb collaboration [2], and supports assigning the \( X(7200) \) as the \( \Xi_{cc} \Xi_{cc} \) hexaquark molecular state with the quantum numbers \( J^{PC} = 0^{-+} \). On the other hand, direct calculations based on the QCD sum rules [44] do not support assigning the \( X(7200) \) as the first radial excited state of the diquark-antidiquark-type \( (3, 3_s^- - \text{type}) \) \( c\bar{c}c\bar{c} \) tetraquark state claimed in Refs. [3, 5, 6].

The decays of the \( \Xi_{cc} \Xi_{cc} \) hexaquark molecule candidate \( X(7200) \) can take place through fusions of the \( \bar{c}c \) and \( \bar{q}q \) pairs,

\[ X(7200) \rightarrow \Xi_{cc} \Xi_{cc} \rightarrow J/\psi J/\psi, \chi_{c1} \chi_{c1}, D^* \bar{D}^*, D_1 \bar{D}_1, \]

we can search for the \( \Xi_{cc} \Xi_{cc} \) hexaquark molecular state in the \( J/\psi J/\psi, \chi_{c1} \chi_{c1}, D^* \bar{D}^* \) and \( D_1 \bar{D}_1 \) invariant mass spectrum at the BESIII, LHCb, Belle II, CEPC, FCC and ILC in the future.
Figure 1: The mass of the $\Xi_{cc}\Xi_{cc}$ hexaquark molecular state with variation of the Borel parameter $T^2$, where the $m_c(m_c) = 1.266$ GeV denotes the $c$-quark mass and gluon condensate are taken from Ref. [34].
4 Conclusion

In the present work, we construct the color-singlet-color-singlet type six-quark pseudoscalar current to interpolate the $\Xi_{cc}\Xi_{cc}$ hexaquark molecular state, then accomplish the operator product expansion by calculating the vacuum condensates up to dimension 10, which are vacuum expectation values of the quark-gluon operators of the orders $\mathcal{O}(a_s^k)$ with $k \leq 1$, and obtain the QCD sum rules for the mass and pole residue. The predicted mass $M_X \sim 7.2$ GeV supports assigning the $X(7200)$ to be the $\Xi_{cc}\Xi_{cc}$ hexaquark molecular state with the quantum numbers $J^{PC} = 0^{-+}$. Moreover, direct calculations based on the QCD sum rules do not support assigning the $X(7200)$ as the first radial excited state of the diquark-antidiquark-type ($\bar{3},\bar{3}$- type) $cc\bar{c}\bar{c}$ tetraquark state. The decays of the $X(7200)$ can take place through fusions of the $\bar{c}c$ and $\bar{q}q$ pairs, we can search for the $\Xi_{cc}\Xi_{cc}$ hexaquark molecular state and explore its properties in the $J/\psi J/\psi$, $\chi_{c1}\chi_{c1}$, $D^* \bar{D}^*$ and $D_1 \bar{D}_1$ invariant mass spectrum at the BESIII, LHCb, Belle II, CEPC, FCC and ILC in the future.

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