Pico pulse response analysis of Josephson weak-link using time-dependent Ginzburg-Landau model

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Abstract The pulse wave whose width is picosecond is called the pico pulse. In this paper, the pico pulse response analysis of high Tc superconducting Josephson weak-link made of Bi$_2$Sr$_2$Ca$_2$Cu$_3$O$_{10}$ in the Terahertz (THz) region is reported. Since the photon energy of the THz radiation is greater than the energy gap of high Tc superconductor, the one dimensional time-dependent Ginzburg–Landau (TDGL) equation is used and the TDGL model as the equivalent circuit of Josephson weak-link instead of the Resistively-Shunted-Junction model is derived by use of the TDGL equation. It is found that the peak value of pulse response decreases when the pulse width decreases.

1. Introduction
The Josephson junctions are applied to ultrahigh sensitive detectors in the millimeter and Terahertz (THz) wave regions since they have the properties of high sensitivity and low noise. The THz wave is the electro-magnetic wave whose frequency is ranging from 0.1 to 10 [THz] and is going to be useful for high speed and high capacity telecommunicating, imaging, biological and chemical analysing.

Since discovery of high Tc superconductor (HTS), thin film deposition and device fabrication technology of HTS have been developed [1], [2], [3], [4]. Hua T et al. recently reported on the bicrystal HTS Josephson mixer’s conversion efficiency at THz region [5].

In this paper, we call the pulse wave whose width is picosecond the pico pulse. Since the photon energy of the THz radiation is greater than the energy gap of HTS, we use the one dimensional time-dependent Ginzburg–Landau (TDGL) equation proposed by Gor’kov and Eliashberg and derive the TDGL model instead of the Resistively-Shunted-Junction(RSJ) model as the equivalent circuit of Josephson weak-link [6], [7], [8].

We assume that the current voltage characteristics of the bicrystal Josephson junction made of Bi-based cuprate, Bi$_2$Sr$_2$Ca$_2$Cu$_3$O$_{10}$ can be described as the Josephson weak-link whose length is L.

The pico pulse response analysis of HTS Josephson weak-link in the THz region is reported. It is found that the peak values of pulse response decreased when the pulse width decreased.
2. TDGL model of Josephson weak-link

Derivation of the TDGL model of Josephson weak-link was described in detail in Ref. [8].

Figure 1 is the equivalent circuit of the TDGL model, where \( I_d \) is the DC driving current, \( I_p \) the current component of the irradiated pico pulse, \( R(V) \) the nonlinear resistance of the Josephson junction, \( \phi \) the phase difference between both electrodes, \( I_{J1} \) the current amplitude of \( \sin \phi \) term well known as the Josephson current and \( I_{J2} \) the current amplitude of \( \cos \phi \) term corresponding to the interference of the superconducting wave function with the quasi particle wave function.

From figure 1, we obtain the following equation.

\[
I_d + I_p(t) = I_{J1} \sin \phi + I_{J2} \cos \phi + \frac{V}{R(V)}
\]  

(1)

Here \( R(V) \) is given by

\[
R(V) = R_n \left[ 1 + \frac{3}{4\nu} \tanh \left( \frac{\nu}{\lambda} \right) \right]^{-1}
\]

(2)

where \( R_n \) is the normal resistance in the limit \( V \to 0 \) and \( \nu = \frac{V}{I_c R_n} \).

The Greek letter \( \lambda \) is defined as follows where \( \tau \) is the relaxation time of the Cooper pair [6].

\[
\lambda = \left\{ \frac{4\tau}{45} \left( \frac{2e}{\hbar} I_c R_n \right) \left( \frac{L}{\xi} \right)^2 \right\}^{-1}
\]

(3)

As can be easily understood from equation (2) in the TDGL model of the Josephson weak-link, the normal resistance of the junction is not constant but dependent on the voltage difference \( V \). However, \( R(V) \) is constant in good approximation when the weak-link length \( L \) is short enough in comparison with the coherence length \( \xi \). Thus, \( R(V) \) is assumed to be equal to \( R_n \) in this analysis. Moreover, \( I_{J1} \) and \( I_{J2} \) can be expressed in good approximation in the THz region as follows [8].

**Figure 1.** TDGL model for picosecond pulse response analysis of Josephson weak-link.
\[ I_{J1} = I_c \]  
\[ I_{J2} = -\frac{17d_c}{735} \frac{2eV}{h} \left( \frac{L}{\xi} \right)^2 \]  
where \( I_c \) is the critical current of the Josephson weak-link.

The irradiated rectangle pulse in figure 1, \( I_p(t) \) is expressed as follows.

\[ I_p(t) = I_{p0} \left\{ \frac{1}{2} + \frac{2}{\pi} \sum_{n=0}^{\infty} \sin \left( \frac{(2n+1)\pi}{2n+1} t \right) \right\} \]  

Here \( I_{p0} \) and \( l_p \) are the irradiated pulse height and pulse width, respectively.

From figure 1 with equation (4) and equation (5), the equation of motion of the phase difference \( \phi \) is given by

\[ \frac{d\phi}{dt} = \frac{i_d + i_p(t) - \sin \phi}{\alpha - \beta \cos \phi} \]  

where \( i_d \) and \( i_p(t) \) are equal to \( i_d = \frac{I_d}{I_c} \) and \( i_p(t) = \frac{I_p(t)}{I_c} \), respectively.

Parameters \( \alpha \) and \( \beta \) are defined as follows.

\[ \alpha = \frac{\hbar}{2eI_c R_n} \]  
\[ \beta = \frac{17\pi}{735} \left( \frac{L}{\xi} \right)^2 \]

3. Results and Discussions

The assumptions in order to analyze the pico pulse response of HTS Josephson weak-link are as follows.

(1) The critical temperature of \( \text{Bi}_2\text{Sr}_2\text{Ca}_2\text{Cu}_3\text{O}_{10} \) is 110[K].
(2) The energy gaps of \( \text{Bi}_2\text{Sr}_2\text{Ca}_2\text{Cu}_3\text{O}_{10} \) at 4.2[K] and 50[K] are 30[meV] and 28[meV], respectively.
(3) The normal resistance is 1.0[\Omega].
(4) Values of the \( LdR_n \) product at 4.2[K] and 50[K] are 18.5[mV] and 17.1[mV], respectively.
(5) The weak-link length is equal to the coherence length of HTS. The coherence lengths of \( \text{Bi}_2\text{Sr}_2\text{Ca}_2\text{Cu}_3\text{O}_{10} \) at 4.2[K] and 50[K] are 1.02[nm] and 1.35[nm], respectively.
(6) The DC driving current normalized by the critical current, \( i_d \) is 1.1.
(7) The pulse height normalized by the critical current, \( i_p = \frac{I_p}{I_c} \) is 3.0.
We analyze the pulse width dependence of the peak value of pico pulse response and show results in figure 2 at the liquid nitrogen temperature, 4.2[K] and figure 3 at 50[K] achieved by use of compact cryocooler. We found from figure 2 and figure 3 that the pulse width dependence of the peak value is not dependent on the temperature and that the peak values of the pico pulse response at both temperatures decrease with decrease of pulse width. We think that the decrease of the peak value of pulse response is due to the harmonic component of pico pulse. According to equation (6), for example, pico pulse of 0.5[ps] wide consists of the fifth harmonic component greater than 10[THz]. Influence of the fifth harmonic component is non-negligible.

**Figure 2.** Peak value of pulse response as a function of pulse width as a parameter at 4.2[K].

**Figure 3.** Peak value of pulse response as a function of pulse width as a parameter at 50[K].
since the amplitude of the fifth harmonic component is greater than a tenth of the fundamental component.

The TDGL model of HTS Josephson weak-link is promising when we analyze the properties of HTS Josephson weak-link applied to developing research such as astronomical observation in the THz region.

4. Conclusions
We analysed the pico pulse response of the Josephson weak-link made of Bi$_2$Sr$_2$Ca$_2$Cu$_3$O$_{10}$ at 4.2[K] and 50[K] in the THz region using the TDGL model.

At both temperatures, we found that the peak values of pulse response decreased when the pulse width decreased.

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