The numerical study of the rake angle of impeller blade in centrifugal compressor

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Abstract: Investigated impellers have blade surfaces formed by straight generatrix. Blade profiles on shroud and disc surfaces are optimized by velocity diagram control (inviscid, quasi-three dimensional calculations). The blade profiles at hub and shroud blade-to-blade surfaces must be coordinated. A designer can choose the generatrix position at a trailing edge for it. The position is defined by the rake angle that is the angle between a trailing edge generatrix and a meridional plane. Two stages with 3D impellers, vaneless diffusers and return channels were investigated. Seven candidates of impellers of these stages with rake angles in range plus-minus 30 degrees were designed and investigated by quasi-three-dimensional inviscid calculation. CFD-calculations were made for the stages with these impellers. The optimal rake angle is minus 20 degrees for the high flow rate impeller due to lesser blade surface area and favorable meridian velocity field. Zero rake angle is optimal for the medium flow rate impeller where blade surface area is not so important. The combination of inviscid and viscid calculations is the informative instrument for further studies.

Nomenclature

- $b$: blade height
- $c$: absolute velocity
- $c_m$: meridional component of absolute velocity
- $c'$: velocity in view of a blade blockage factor
- $C_p$: isobaric heat capacity
- $D$: diameter
- $k$: isentropic coefficient
- $T_{mc}$: relative length in meridional plane
- $L_r$: relative axial length of impeller
- $\bar{m}$: mass flow rate
- $M_o$: blade Mach number
- $n$: RPM
- $p$: pressure
- $R$: gas constant
- $R_h$: hub radius of curvature
- $R_s$: shroud radius of curvature
1. Research aim

Impellers with three-dimensional blade cascades (3D impellers) have complicated and not completely studied configuration. Figure 1 illustrates geometry parameters of a 3D impeller meridional plane. The meridian configuration of the impeller in Figure 1 is a combination of arcs and straight lines. Figure 2 illustrates an impeller geometry as it is presented by the PC program for quasi-three inviscid calculations [1, 2]. A meridian configuration and quasi-orthogonal lines (red) to calculate axis-
symmetric flow are shown in the Figure 2 on the left, a blade cascade view and blade angles at three blade-to-blade surfaces are presented amidst. Inviscid velocity diagrams on three blade-to-blade surfaces (hub, mean, shroud) are shown on the right.

Figure 1. Meridional dimensions of 3D impeller (left), Blade meridional cross section (amidst) and blade generatrix position on a conical surface “a-a” (right).

3D impellers under investigation have blade surfaces formed by straight generatrix. Blade profile on shroud and disc surfaces are optimized by velocity diagram control. In accordance with [1, 2, 3] a maximum velocity on a suction side must be minimized. A flow deceleration and non-dimensional blade load must be chosen in accordance with an impeller parameters $\Phi_{des}$, $\psi_{Rdes}$. The blade profiles at hub and shroud must be coordinated in 3D space. An angle $\gamma_z$ of the generatrix at a trailing edge can be chosen by a designer. Rake angle $\chi$ (figure 1) is the angle between a generatrix and a meridional surface.

Figure 2. Graphic information on a 3D impeller #1 in course of design (PC program 3DM.023 [1])

Rake angle is positive - $\chi > 0$ - if a generatrix is tilted in the direction of an impeller rotation. The geometric equation presented in [4] connects a meridian configuration, blade angles and rake angle:

$$\frac{1}{\cos^2 \chi} \frac{\partial \chi}{\partial \beta} = -\frac{\partial \beta_h}{\partial b} + \sin \beta_m \cos \beta_h \left( \frac{1}{R_m} + \frac{\cos \gamma}{r} \right).$$

Here $R_m$ - curvature radius of a blade-to-blade surface, $\gamma$ - angle between a radial direction and a normal to blade-to-blade surface.

The rake angle values along the meridional coordinate are function of the meridional shape and the blade shape that are chosen in a course of design. The chosen value of the rake angle $\chi_z$ defines rake
angles along the entire blades surface in accordance with the Eq. (1). The equilibrium equation for the axis-symmetric meridional flow is presented in [5]:

\[
\frac{\partial c_m}{\partial b} = \frac{c_m}{r} + \frac{\operatorname{ctg} \beta \partial (c_r)}{\partial b} - \frac{P_u \operatorname{tg} \chi}{c_m}.
\]  

(2)

Here the force \( P_u = \frac{2\pi b}{2\pi r} (p_s - p_p) \) that represents influence of a blade load \( p_s - p_p \) on meridian flow in a real three-dimensional flow. The meridional non-uniformity of the flow is caused by change of its direction in a meridional plane. In accordance with Eq. (2) the negative rake angle reduces the velocity gradient. This can be considered as a positive factor. Uniform flow has less kinetic energy, which should be considered as an advantage. But in case \( \chi \neq 0 \) the blade surface is bigger:

\[
S_{bl} = \int_{\chi_m}^{\chi_b} b \cos \chi \, d\chi \, (m).
\]  

(3)

It increases the friction losses. Blade blockade factor is smaller. It is a negative effect too:

\[
\tau = 1 - \frac{Z_{imp} \operatorname{tg} \beta_{bl}}{\pi D \cos \chi}.
\]  

(4)

There are other, less obvious factors probably. Design experience shows that if zero rake angle \( \chi_2 \) is chosen by a designer, a rake angle of a leading edge is positive and can reach up to 40-50° and more. Blade height and velocity are the highest at the impeller entrance. There's also the greatest flow blockade by blades. It is possible that better flow uniformity if \( \chi_1 > 0 \) gives greater positive effect than listed above negative ones. Aim of the research is to study rake angle \( \chi_2 \) choice as design recommendation.

In the paper [6] the research of a high-pressure aviation centrifugal compressor with a different rake angle \( \chi_2 \) is carried out. The values of the angle \( \chi_2 \) within the limits of +45 - -45 degrees were investigated. However, the outlet blade angle varied on its height, depending on the rake angle. For an angle \( \chi_2 \) of 45 degrees, the angle \( \beta_{bl2} \) at the shroud is less than the angle on the hub by about 10 degrees. For an angle \( \chi_2 \) of -45 degrees, the angle \( \beta_{bl2} \) is bigger than the angle on the hub by 5 degrees. Calculations were made by the ANSYS CFX program. The results had shown a high efficiency of the compressor with rake angle \( \chi_2 = +45 \) degrees and a larger stall margin for a compressor with a \( \chi_2 = -45 \) degrees.

2. Research objects

Two stages with different design parameters were chosen as research objects. Both stages have 3D impellers, vaneless diffusers and return channels. The stage #1 has a high flow rate coefficient \( \Phi_{des} = 0.105 \). The stage #2 has a medium flow rate coefficient \( \Phi_{des} = 0.097 \). The similarity criteria are \( M_{u} = 0.7 \), \( \operatorname{Re}_{u} = 560000 \), \( k = 1.4 \). The values are typical for industrial compressor model stage tests. Stages’ parameters are presented in table 1.

| Table 1. Gas dynamic and geometry parameters of impellers ## 1, 2 |
|------------------|---------|---------|
| Impeller         | #1      | #2      |

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Despite the development of computational fluid dynamics, the world's turbomachinery manufacturers and scientists [8, 9] insist that mean line calculations are obligatory as a first step of design. The stages were designed by means of the so-called “Universal modelling method” [1-2, 7]. Mean line programs based on the mathematical model are applied to optimize stage main dimensions. Program 3DM.023 mentioned above (Figure 2) was applied for blade cascade design. The samples of this design method successful application are presented in [10].

A large number of CFD-calculations for candidate stages with 3D impellers were made by the authors of [3, 11]. The following are design recommendations:

- inlet diameter must be somewhat smaller than calculated by formula \( \bar{D}_{\text{in} \text{w.min}} = \sqrt{\bar{D}^2 + \frac{2}{3} \left( \frac{\Phi_{\text{des}}}{v'_\text{ch} \nu} \right)^2} \).

The relative diameter \( \bar{D}_{\text{w.min}} \) corresponds to minimal inlet velocity at a shroud blade-to-blade surface,

- leading edge of the blades is located at the distance 0.25% of meridian length from the section "0",

- exit blades angle must be increased from hub to shroud: \( \beta_{32s} > \beta_{32h} \).

The information on the impeller #1 with \( \Phi_{\text{des}} = 0.105 \) is shown in Figure 2 above. The candidate impellers ## 1 and 2 have different rake angles \( \chi_2 \): 30, 20, 10, 0, -10, -20, -30 degrees. All candidates have identical meridional configuration and blade cascade shape. Only the rake angles \( \chi_2 \) are different.

3. Q3D flow analysis

Results of flow parameters measurement in rotating impellers are presented in [1, 5]. Good agreement between calculated and measured velocity diagrams determines the successful application of Q3D calculations in design practice [10]. The analysis of inviscid flow structure provides useful information. The information on the candidates – inviscid and viscid calculations - is presented in Table 2.

| Imp. | \( \chi_2 \) | \( \chi_1 \) | \( \bar{c}_{\text{ms}} - \bar{c}_{\text{mh}} \) | \( S_{\text{h2L}} / S_o \) | \( \Psi_{\text{f.des}} \) | \( \zeta_{\text{3D.imp}} \) | \( \eta_{\text{des}} \) |
|------|--------|--------|----------------|----------------|----------------|----------------|----------------|
| 1    | -30    | 32.54  | 0.1863         | 1.23545        | 0.630          | 0.148          | 86.88          |
| 1    | -20    | 35.03  | 0.1310         | 1.24616        | 0.633          | 0.148          | 87.03          |
| 1    | -10    | 37.37  | 0.1230         | 1.26098        | 0.634          | 0.151          | 87.01          |
| 1    | 0      | 39.08  | 0.1150         | 1.27888        | 0.632          | 0.163          | 86.78          |
The rake angles $\chi_i$ for the candidate impeller #1 are 32.5-41 degrees and are 46.6-53 degrees for the candidate impeller #2. The larger value of rake angles $\chi_i$ in case of the impeller #2 takes place due to the greater curvature in the meridional plane – the term $1/R_m$ in equation (1).

The design parameter $\chi_2$ is varied in the range 60 degrees. The variation of the rake angle at the entrance $\chi_1$ is 8.5 degree for the candidate impeller #1, -7. The variation of the rake angle at the entrance $\chi_1$ is 6.4 degree for the candidate impeller #2.

Blades’ surface of the candidate # 1 with $\chi_2 = 0$ is by 28% more than the mean blade-to-blade surface. Blades’ surface of the candidate # 2 with $\chi_2 = 0$ is by 32% less than the mean blade-to-blade surface. Blades’ surface varies in the range 1.24 – 1.36 of disk surface for the candidate impeller #1. Blades’ surface variation is 12%. Blades’ surface varies in the range 0.68-0.70 of the average blade-to-blade surface for the candidate impeller #2. There is practically no influence of $\chi_2$ on blade surface in case of the medium flow rate impeller #2.

Graphic information of inviscid calculations is presented in Figure 3. Positive rake angles suppress meridional velocities non-uniformity. The difference of non-dimensional meridian velocities along a blade height in a middle of a blade channel is shown in the column 5 Table 1. This parameter is reduced from 18 to 6% for the impeller #1 candidates in the range of $\chi_2$ -30 ...+30 degrees. It is important that the velocity $\tau_{ms}$ is visibly lower at negative $\chi_2$. Non-uniformity is reduced from 6 to 1% for the candidate impeller #2 in the same range of $\chi_2$.
Figure 3. Impeller #1 (above) and impeller #2 (below). The meridional velocity of eight quasi orthogonals. $\chi_2$ of the candidates are: -30$^\circ$ - left, 0$^\circ$ - amidst, 30$^\circ$ - right

Velocity diagrams on three blade-to-blade surfaces of impellers #1 and 2 with rake angles $\chi_2$ = -30, 0, +30 degrees are shown in Figure 4. The rake angle variation does not lead to significant changes of velocity diagrams. There is slight increase of a load on the leading edge of impeller #1 with $\chi_2 = 30^\circ$. It can lead to slight shift of performances to higher flow rates.

Figure 4. Impeller #1 (above) and impeller #2 (below). Velocity diagram on the hub on the mean surface and the shroud. $\chi_2 = -30^\circ$ left, 0$^\circ$ amidst, 30$^\circ$ right

4. Numerical research method

Authors’ experience and known sources [3, 12-14] show that CFD-calculation satisfactorily predicts stage maximum efficiency. Loading factor is usually overestimated by 6-12% at a design flow rate. Performances at flow rates below design flow rate are not predicted satisfactorily.

Performances of two stages with vaneless diffusers and return channels were calculated. Each stage included seven candidates of the impellers analyzed above. Computational fluid dynamics program package NUMECA Fine/Turbo was applied:

- computational grid consists of 1.8 million cells;
- the turbulence model Spalart-Allmaras was applied;
- one blade channel was calculated;
- boundary conditions at a stage inlet: the total pressure and total temperature;
- boundary conditions at a stage exit: mass flow rate.

Disc friction losses and leakage through the labyrinth seals were not modeled. The boundary condition for the rotating and stationary coordinate systems – “mixing plane”.

Non-dimensional gas dynamic performances are presented as functions the flow rate coefficient:

$$\Phi = \frac{4m}{\rho_{10} \pi D_2^2 u_2}.$$  \hspace{1cm} (5)

Loading factor is:
\[\psi_T = \frac{C_p (T_{z2} - T_{r0})}{u_z^2}. \quad (6)\]

Total polytrophic efficiency is calculated as:

\[\eta_t = \frac{\ln \left( \frac{p_{\text{out}}}{p_{\text{ind}}} \right)}{k-1} \cdot \frac{\ln \left( \frac{T_{\text{out}}}{T_{\text{ind}}} \right)}{k}. \quad (7)\]

An impeller loss coefficient is:

\[\zeta_{\text{imp}} = \frac{2 \cdot \psi_T \cdot (1 - \eta_{\text{imp}})}{(w_t / u_z)^2}. \quad (8)\]

5. Results of CFD-calculations

Efficiency performances of stages are presented in Figure 5. Efficiency of the candidate stage #1 increases monotonously within 1.2% in the range of \(\chi_z\) from plus 30 to minus 10 degrees. The maximum efficiency is the same for the candidates with rake angles minus 10 and minus 20 degrees. Evidently, blades surface area influences mainly efficiency level of the candidates.

Efficiency of the candidate stage #2 changes within 1.1% in the range of \(\chi_z\) from plus 30 to minus 30 degrees. The maximum efficiency has the candidate with \(\chi_z = 0\) degree. The candidates with \(\chi_z > 0\) are less effective. Obviously, the above recommendation \(\chi_{\text{opt}} < 0\) is not valid for impellers with medium specific speed. Loss coefficient performances of the candidate impeller are presented in Figure 6.

Figure 5. Candidates of the stage #1 (left) and impeller #2 (right) efficiency performances
Loss coefficients (Eq. (8)) of the candidate impeller #1 vary within 0.145-0.175 at the design flow rate $\Phi_{des} = 0.105$. Minimal loss coefficients have the candidate impellers of the most effective stages. It means that a rake angle $\chi$ does not influence stator part efficiency but only an efficiency of an impeller. Loss coefficients of the candidate impeller #2 are significantly larger in absolute value $\zeta_{imp,des} = 0.34 - 0.38$. Velocity diagrams of all candidates indicate the favorable flow character. The blade profiling is OK. Perhaps large loss coefficients are the result of large friction losses on discs. The channels of the impeller #2 are narrower in comparison with the impeller #1. The lowest loss coefficient has the candidate with the rake angle minus 20 degrees. Rake angle has little effect on the surface area of the blades. The most effective stage has the impeller with zero rake angle. Its meridian non-uniformity is twice less (column 4 in Table 2). It may be important for stators.

The information on flow structure is presented in figures below. Exit flow angle along blade height for candidate impeller with three rake angles is shown in Figure 7. Flow angle is averaged on mass flow in the section "2".

Uneven exit angle diagram with low angles near a shroud is typical for high flow rate impellers. Low angles lead to flow separation in a vaneless diffuser. The candidate impeller #1 with the negative rake angle 30 degree has the better diagram. This candidate’s inviscid meridian velocities in accordance with Figure 3 are bigger near a shroud. It helps maybe to rise flow angles of a viscid flow. The candidate impeller #2 with lower $\Phi_{des} = 0.067$ has practically the same angle diagrams that are rather uniform. The results confirm the conclusions drawn from the analysis of the Eq. (2).

Velocity fields in the blade cascades of the impellers ## 1 and 2 are presented in Figure 8. Flow seems to be well organized in all candidates. There are no low-energy zones near blades’ suction side ("wake"). This is typical for well-designed impellers with low loading factor less than 0.55. Low-energy zone is on the blade pressure side in all candidates of the impeller #1. This can be due to
increased shear stress, as the large deviation of flow from its inertial direction. The intensity of this zone increases with algebraic increasing of the $\chi_2$.

![Figure 8. Relative velocity fields in the blade cascades of the impeller #1 (above) and impeller #2 (below). Blade-to-blade surface near the shroud. Design flow rate. $\chi_2 = -30^\circ$ left, $\chi_2 = 0^\circ$ amidst, $\chi_2 = 30^\circ$ right](image)

Loading factor performances calculated by Eq. (6) are shown in Figure 9. The design flow rate loading factor of the candidate impeller #1 is practically constant, with the exception of the candidate with $\chi_2 = -30^\circ$ degrees. The loading factor is bigger for the candidate impeller #2 with negative $\chi_2$. The loading factor at the optimal zero rake angle is by 2% higher than at rake angle $+30^\circ$. This trend is not confirmed by the analysis of non-viscous flow velocity diagrams.

![Figure 9. Loading factor performances of the candidate impeller #1 (left) and #2 (right)](image)

**Conclusion**

In case of the stage with $\phi_{des} = 0.105$ the efficiency of candidates with $\chi_2 = -10 \ldots -20$ degrees is by 0.7% higher against the candidate with $\chi_2 = 0$ degrees. The optimal candidates have smaller blade surface area and higher inviscid meridian velocities on a shroud surface. The last probably is a reason of more uniform viscous exit velocity angles near a shroud.
The optimal rake angle is zero for the medium flow rate stage with $\phi_{drv} = 0.067$. The surface of blades is less than the surface of the discs and its role in the overall friction losses is less. Rake angle $\chi_2$ has little effect on the surface of the blades in this medium flow rate impeller. The rake angle $\chi_2$ influences little the loading factor performances of the candidate impeller #1. The loading factor of the optimal candidate impeller #2 is by 2% higher than that of the candidate with $\chi_2=30^\circ$. Inviscid velocity diagrams do not explain this fact. But the CFD-calculated efficiency of the candidate with $\chi_2=30^\circ$ is by 1% lower. An impeller's lower efficiency leads to lower loading factor. However, these considerations are not supported by the candidate impeller #1 performances.

As a whole the combination of inviscid and viscid calculations is a proper instrument for choice of better candidates of three-dimensional blade cascades. The authors recommend applying it in design optimization and will continue the study of the rake angles in impellers of different design parameters.

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