Baxter-Bazhanov Model, Frenkel-Moore Equation and the Braid Group

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Abstract

In this paper the three-dimensional vertex model is given, which is the duality of the three-dimensional Baxter–Bazhanov (BB) model. The braid group corresponding to Frenkel–Moore equation is constructed and the transformations R, I are found. These maps act on the group and denote the rotations of the braids through the angles $\pi$ about some special axes. The weight function of another three-dimensional vertex model related the 3D lattice integrable model proposed by Boos, Mangazeev, Sergeev and Stroganov is presented also, which can be interpreted as the deformation of the vertex model corresponding to the BB model.

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I. Introduction

The braid group has a deep connection with the Yang–Baxter equation which plays an important role in the exactly solvable model in two dimensions in statistical mechanics. The tetrahedron equation is an integrable condition of statistical model in three dimensions due to which gives the commutativity of the layer to layer transfer matrices. The Frenkel–Moore version of the tetrahedral equation is formulated as

$$S_{123}S_{124}S_{134}S_{234} = S_{234}S_{134}S_{124}S_{123},$$

where $S \in \text{End}(V^\otimes 3)$ and each side of the equation acts on $V^\otimes 4 = V_1V_2V_3V_4$, for example, acts on $V_4$ identically. Then we can ask a question that what is the braid group corresponding to Frenkel–Moore equation. An answer will be given in this paper.

Just as the Yang–Baxter equation can describe the scattering in two dimensions, with a factorizable matrix, the tetrahedron equation gives the relations among the scattering amplitudes of three strings. And the three labeling schemes exist. With the cell (or vacuum) labeling Bazhanov and Baxter generalized the two-state Zamolodchikov model to an arbitrary number of state. Then the three-dimensional star-star relation is proved and the star-square relation is discussed. Recently Mangazeev and Boos obtained the solution of modified tetrahedron equation in terms of elliptic functions which generalized the result of Ref. [13]. Frenkel–Moore equation can also be gotten by using string labeling. So we discuss the braid relation of Frenkel–Moore equation in the following section by regarding the three-string cross as the “elementary” braid.

The vertex-type tetrahedron equations were discussed in Refs [5] and [15] and the discrete symmetry groups of vertex models were studied by Boukraa et al. Mangazeev et al. proposed a three-dimensional (3D) vertex model and the weight function of this model can be obtained from Baxter–Bazhanov model when taking some limits. The three-dimensional vertex model corresponding to Baxter–Bazhanov model is constructed in this paper. And the six spectrums with a constrained condition relate to the six spaces in which the vertex-type tetrahedron equation is defined.

This paper is organized as follows. In Sec. II, the three-dimensional vertex model is given, which is a duality of the three-dimensional Baxter–Bazhanov model. The braid group corresponding to the Frenkel–Moore equation is constructed in Sec. III. In Sec. IV, the transformations $R, I$ are discussed. Finally some conclusions are given and Boltzmann weight of another

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three-dimensional vertex model is presented. It corresponds to the 3D lattice integrable models proposed by Boos et al. and can be regarded as the deformation of the vertex model corresponding to BB model.

II. The Three-Dimensional Vertex Model

The Baxter-Bazhanov model is an interaction-round-a-cube (IRC) model. The weight function of it has the form

$$W_P(a|efg|bcd|h) = \frac{\omega^{\frac{1}{2}}}{\omega^2} \left[ \frac{w(x_{14}, x_{23}, x_{12}, x_{24}|a + d, e + f)}{w(x_{14}, x_{23}, x_{12}, x_{24}|g + h, c + b)} \right]^{1/2} \times$$

$$\left[ \frac{w(x_4, x_3, x_2|e + h, d + c)}{w(x_4, x_3, x_2|a + b, f + g)} \right]^{1/2} \left[ \frac{w(x_2, x_1, x_3|c + g, a + c)}{w(x_2, x_1, x_3|d + b, f + h)} \right]^{1/2} \times$$

$$\{ \sum_{x \in Z_N} \frac{w(x_3, x_{13}, x_1|d, h + \sigma)w(x_4, x_{24}, x_2|a, g + \sigma)}{w(x_4, x_{14}, x_1|c, e + \sigma)w(x_3, x_{23}, x_2|f, b + \sigma)} \} \right)_a,$$

where the subscript "0" after the curly brackets indicates that the expression in the braces is divided by itself with the zero exterior spins and we have used the notations

$$w(x, y, z|k, l) = w(x, y, z|k - l)\Phi(l), \quad w(x, y, z|l) = \prod_{j=1}^{l} \frac{y}{z - z^{j}}, \quad k, l \in Z_N,$$

$$x^N + y^N = z^N, \quad \Phi(l) = \omega^{l(l+N)/2}, \quad \omega^{1/2} = \exp\left(\frac{\pi i}{N}\right), \quad x_i^N - x_j^N = x_{ij}$$

for $i < j$ and $i, j = 1, 2, 3, 4$. It satisfies the cube-type tetrahedron equation

$$\sum_d W(a_4|c_2c_3|b_1b_2b_3d)W'(c_1|b_2a_3b_1|c_4d_c|b_4) \times$$

$$W''(b_1|d|c_3a_2b_3|c_5)W'''(d|b_2b_4|c_5c_6a_1) = \sum_d W''(b_1|c_1c_3|a_2a_3|d)W''(c_1|b_2a_3d|c_2c_6a_1) \times$$

$$W'(a_4|c_2d_3|a_2b_3a_1|c_5)W(d|a_1a_3a_2|c_4c_5c_6|b_4),$$

where $W, W', W''$ and $W'''$ are some four sets of Boltzmann weights. By using the symmetry properties of the Boltzmann weights and setting

$$u = \frac{x_1}{x_2}, \quad v = \frac{x_4}{x_3}, \quad z = \frac{x_1}{x_2}, \quad z_1 = \frac{x_3}{x_4}, \quad z_2 = \frac{x_2}{x_3}, \quad z_3 = \frac{x_4}{x_3},$$

the Boltzmann weight of the Baxter-Bazhanov model can be written into the vertex form

$$R(u, v, w)^{i_1j_1j_3} = (-)^j(\omega^{1/2})^{j_1j_2j_3} \left[ \frac{w(u, j_1)w(z_2|\omega z_1), -i_2)w(v, i_3)}{w(u, i_1)w(z_2|\omega z_1), -j_2)w(v, j_3)} \right]^{1/2} \times$$

$$\left\{ \sum_{x \in Z_N} \frac{w(wz_1, \sigma + j_2 + j_3)w(z_2; \sigma) s(\sigma, j_1)}{w(z_1, \sigma + j_2)w(vz_2, \sigma + i_3)} \right\}_a,$$

with

$$\frac{w(v, a)}{w(v, 0)} = [\Delta(v)]^a \prod_{j=1}^{a} (1 - w^j v)^{-1}, \quad \Delta(v) = (1 - v^N)^{1/N},$$

where $s(a, b) = \omega^{ab}$ and the spin variables $i_1, i_2, i_3, j_1, j_2, j_3$ satisfy the conditions $i_1 + i_2 = j_1 + j_2, i_2 + i_3 = j_2 + j_3$. The Boltzmann weights satisfy the vertex-type tetrahedron equation

$$\sum_{i=1}^{6} R(u_1, u_2, u_3)^{k_1, k_2, k_3} R(u_1, u_4, u_5)^{k_1, k_3, k_4} R(u_2, u_4, u_6)^{k_2, k_4, k_6} R(u_3, u_5, u_6)^{k_1, k_2, k_5} = \sum_{i=1}^{6} R(u_3, u_5, u_6)^{k_3, k_5, k_6} R(u_2, u_4, u_6)^{k_2, k_4, k_6} R(u_1, u_4, u_5)^{k_1, k_3, k_4} R(u_1, u_2, u_3)^{k_1, k_2, k_5},$$

where $s(a, b) = \omega^{ab}$ and the spin variables $i_1, i_2, i_3, j_1, j_2, j_3$ satisfy the conditions $i_1 + i_2 = j_1 + j_2, i_2 + i_3 = j_2 + j_3$. The Boltzmann weights satisfy the vertex-type tetrahedron equation

$$\sum_{i=1}^{6} R(u_1, u_2, u_3)^{k_1, k_2, k_3} R(u_1, u_4, u_5)^{k_1, k_3, k_4} R(u_2, u_4, u_6)^{k_2, k_4, k_6} R(u_3, u_5, u_6)^{k_1, k_2, k_5} = \sum_{i=1}^{6} R(u_3, u_5, u_6)^{k_3, k_5, k_6} R(u_2, u_4, u_6)^{k_2, k_4, k_6} R(u_1, u_4, u_5)^{k_1, k_3, k_4} R(u_1, u_2, u_3)^{k_1, k_2, k_5}.$$
where

\[
\begin{align*}
  u_1 &= \frac{x_1'}{\omega z_2} = \frac{x_1'}{\omega z_2}, \\
  u_2 &= \frac{x_1' x_2'}{\omega x_4 x_3} = \frac{x_1' x_2'}{\omega x_4 x_3}, \\
  u_3 &= \frac{x_4'}{x_3} = \frac{x_4'}{x_3}, \\
  u_4 &= \frac{x_1' x_2'}{\omega x_4 x_3} = \frac{x_1' x_2'}{\omega x_4 x_3}, \\
  u_5 &= \frac{x_4'}{x_3} = \frac{x_4'}{x_3}, \\
  u_6 &= \frac{x_4'}{x_3} = \frac{x_4'}{x_3}. 
\end{align*}
\]

(10)

In this way, we get a three-dimensional vertex model[19] which corresponds to the Baxter-Bazhanov model. The spectra \( u_i \) \((i = 1, 2, \ldots, 6)\) appeared in the above tetrahedron equation satisfy the condition

\[
\left[ \sin \left( \frac{\theta_1 + \theta_2 + \theta_3}{2} \right) \sin \left( \frac{-\theta_1 + \theta_2 + \theta_3}{2} \right) \sin \left( \frac{-\theta_3 + \theta_5 + \theta_6}{2} \right) \sin \left( \frac{\theta_3 + \theta_5 - \theta_6}{2} \right) \right]^{1/2} = \\
\left[ \sin \left( \frac{\theta_1 - \theta_2 + \theta_3}{2} \right) \sin \left( \frac{\theta_1 + \theta_2 - \theta_3}{2} \right) \sin \left( \frac{\theta_3 - \theta_5 + \theta_6}{2} \right) \sin \left( \frac{\theta_3 + \theta_5 - \theta_6}{2} \right) \right]^{1/2} = \\
\sin \theta_3 \left[ \sin \left( \frac{\theta_3 + \theta_4 - \theta_6}{2} \right) \sin \left( \frac{-\theta_3 + \theta_4 + \theta_6}{2} \right) \right]^{1/2},
\]

(11)

where we have parametrized the spectra of the Boltzmann weights as

\[
u_i = \omega^{-1/2} \left[ \cot \left( \frac{\theta_i}{2} \right) \right]^{2/N}, \quad i = 1, 2, \ldots, 6.
\]

(12)

III. Braid Group \( \hat{B}_N \)

The shorthand notation of the vertex-type tetrahedron equation (9) is

\[
R_{12} R_{13} R_{14} R_{23} = R_{23} R_{12} R_{14} R_{13},
\]

which can be reformulated as[15]

\[
R_{12131423} R_{13143423} = R_{232434} R_{13143423},
\]

by using the following translation[6]

\[
1 \rightarrow 12, \quad 2 \rightarrow 13, \quad 3 \rightarrow 23, \quad 4 \rightarrow 14, \quad 5 \rightarrow 24, \quad 6 \rightarrow 34.
\]

(15)

Then we can write down the Frenkel-Moore equation (1) (see Ref. [5]). The relations of the braid group corresponding to the Frenkel-Moore equation are discussed as follows. For \( N + 1 \) strings, let us express the “elementary” braid \( \alpha_i \) and \( \alpha_i^{-1} \) \((i = 1, 2, \ldots, N - 1)\) by Fig. 1 and Fig. 2 respectively. In Fig. 1, \((i + 1)\)-string is on the bottom, \((i - 1)\)-string is on the top and \(i\)-string is between the \((i - 1)\)-string and \((i + 1)\)-string. In Fig. 2, \((i - 1)\)-string is on the bottom, \((i + 1)\)-string is on the top and \(i\)-string is also between the \((i - 1)\)-string and \((i + 1)\)-string. When we define the product of \( \alpha_i \) and \( \alpha_j \) \((1 \leq i, j \leq N - 1)\) as that \( \alpha_j \) acts on \( N + 1 \) strings on which \( \alpha_i \) has acted, \( \alpha_i, \alpha_i^{-1}, \alpha_j, \alpha_j^{-1}, \ldots \) and all of the arbitrary products of them form a group \( \hat{B}_N \) in which the identity element is the no crossed \( N + 1 \) strings. The “elementary” braid \( \alpha_i \) satisfies the following relations

\[
\begin{align*}
  \alpha_i \alpha_j &= \alpha_j \alpha_i, \quad |i - j| \geq 3, \\
  \alpha_i \alpha_{i \pm 1} \alpha_i^\pm 1 &= \alpha_i \alpha_{i \pm 1} \alpha_i^\pm 1, \\
  \alpha_i \alpha_{i \mp 1} \alpha_i^\mp 1 \alpha_i \mp 1 &= \alpha_i \alpha_{i \mp 1} \alpha_i^\mp 1 \alpha_i \mp 1, \\
  \alpha_i \alpha_{i \pm 1} \alpha_i^\pm 1 \alpha_i \mp 1 &= \alpha_i \alpha_{i \pm 1} \alpha_i^\pm 1 \alpha_i \mp 1, \\
  \alpha_i \alpha_i^{-1} &= E.
\end{align*}
\]

(16 - 20)

Notice that there are two relations in Eqs (17), (18) and (19), respectively. The first ones of them can be proved graphically by Fig. 3, Fig. 4 and Fig. 5. The second relations of them can be proved similarly. From these relations it can be gotten easily that

\[
\begin{align*}
  \alpha_{i \pm 1} \alpha_i \alpha_{i \mp 1} \alpha_i \mp 1 &= (\alpha_i \alpha_{i \mp 1})^{2n+1} \alpha_{i \pm 1} \alpha_i \mp 1, \\
  \alpha_{i \pm 1} \alpha_i \alpha_{i \pm 1} \alpha_i \pm 1 &= (\alpha_i \alpha_{i \pm 1})^{2m+1} \alpha_{i \pm 1} \alpha_i \pm 1, \\
  \alpha_i \alpha_{i \pm 1} \alpha_{i \mp 1} \alpha_i \pm 1 &= (\alpha_i \alpha_{i \mp 1})^{2l} \alpha_{i \pm 1} \alpha_i \mp 1.
\end{align*}
\]

(21 - 23)
where \( m, n, l \), are all the arbitrary integers.

Fig. 1. \( \alpha_i \)

Fig. 2. \( \alpha_i^{-1} \)

Fig. 3. The graphic proof of \( \alpha_i \alpha_{i+1} \alpha_{i+1} \alpha_i = \alpha_{i+1} \alpha_i \alpha_{i+1} \alpha_i \).

Fig. 4. The graphic proof of \( \alpha_{i+1} \alpha_{i-1} \alpha_{i+1} \alpha_i = \alpha_i \alpha_{i-1} \alpha_{i+1} \alpha_i \).

Fig. 5. The graphic proof of \( \alpha_{i+1} \alpha_{i-1} \alpha_{i+1} = \alpha_i \alpha_{i-1} \alpha_{i+1} \alpha_{i-1} \).
Now we discuss the generators of the braid group $\hat{B}_N (N \geq 2)$ as follows. Firstly, $\hat{B}_2$ and $\hat{B}_3$ have two and four generators respectively. And $\hat{B}_4$ has also four generators owing to

$$\alpha_2 = \alpha_1^{-1} \beta^2 \alpha_1 \beta^{-1} \text{ and } \alpha_3 = \beta \alpha_1 \alpha_3 \beta^{-1} \text{ where } \beta = \alpha_1 \alpha_2 \alpha_3.$$  

For $N \geq 5$, setting

$$\beta = \alpha_1 \alpha_2 \alpha_3 \cdots \alpha_{N-1},$$

we have, from Eq. (19),

$$\alpha_i \beta = \beta \alpha_{i-2}, \quad N - 1 \geq i \geq 3.$$  

Then $\alpha_i$ can be expressed as

$$\alpha_i = \beta^{[\frac{i-1}{2}]} \alpha_{i-2}^{[\frac{i-1}{2}]} \beta^{-[\frac{i-1}{2}]},$$

where $[\frac{i-1}{2}]$ is an integer but $i/2 - 1 \leq [\frac{i-1}{2}] \leq i/2$. So braid group $\hat{B}_N (N \geq 5)$ has six generators: $\alpha_1, \alpha_2, \beta$ and the inverses of them: $\alpha_1^{-1} \alpha_2^{-1}$ and $\beta^{-1}$ due to

$$\alpha_{i-2}^{[\frac{i-1}{2}]} = \begin{cases} 
\alpha_1 & \text{for odd } i, \\
\alpha_2 & \text{for even } i.
\end{cases}$$

Equation (17), that is, Fig. 3, is the braid relation for the Frenkel-Moore equation which is written as [14] $S_{123} S_{214} S_{143} S_{234} = S_{234} S_{143} S_{214} S_{123}$. When we denote the indices of $S$ by the place where the strings cross, we have $S_{123} S_{234} S_{123} S_{234} = S_{234} S_{123} S_{234} S_{123}$. This is just the planar tetrahedral equation.[3]

### IV. Transformations $R$ and $I$

From relations (18) and (19) we get

$$S_{123} S_{145} S_{325} S_{345} = S_{123} S_{234} S_{325} S_{145} S_{123},$$

$$S_{123} S_{145} S_{234} S_{345} = S_{234} S_{123} S_{234} S_{145} S_{123}.$$  

The other two equations corresponding to relations (18) and (19) can be obtained from the above relations by using the index maps: $1 \leftrightarrow 5$ and $2 \leftrightarrow 4$. This means that some relations exist between the two equations in Eq. (17) or Eq. (18), respectively. So we set

$$R(\alpha_i) = \alpha_{\sigma(i)}, \quad R(\alpha_i \alpha_j) = \alpha_{\sigma(i)} \alpha_{\sigma(j)}, \quad R(\alpha_i \alpha_j \alpha_k) = \alpha_{\sigma(i)} \alpha_{\sigma(j)} \alpha_{\sigma(k)}, \quad \cdots;$$  

$$I(\alpha_i) = \alpha_i, \quad I(\alpha_i \alpha_j) = \alpha_j \alpha_i, \quad I(\alpha_i \alpha_j \alpha_k) = \alpha_k \alpha_j \alpha_i, \quad \cdots,$$

where $\sigma(i) = N - i$ and $N$ is the total numbers of strings minus one. It can be proved easily that $[R, I] = 0, R^2 = I^2 = 1$. The transformation $R$ acting on the braids denotes the rotation of the braids through the angle $\pi$ around the axis along which the strings cross. The transformation $I$ acting on the braids denotes that the braids are rotated through the angle $\pi$ about the axis which is in the plane which the braids are in and is perpendicular to the above axis. Then transformation $RI$ describes the rotation of the braids through the angle $\pi$ for the axis which is perpendicular to the plane in which the braids are. So the two relations in Eq. (18) or Eq. (19) can be changed each other by using the transformation $I$ and (then) we can choose only one of them respectively. Acting on the generators: $\Delta = b_1 b_2 \cdots b_{N-1}$ and $\Omega = b_1 \Delta$, of the ordinary braid group $B_N$ by the transformations $R$ and $I$, we have that

$$R(\Delta) = I(\Delta) = \Delta^{N-1} (\Delta^{-1} \Omega \Delta^{-1})^{N-1}, \quad R(\Omega) = \Delta^{N-2} \Omega (\Delta^{-1} \Omega \Delta^{-1})^{N-1},$$

$$I(\Omega) = \Delta^{N-1} (\Delta^{-1} \Omega \Delta^{-1})^{N-1} \Omega \Delta^{-1}.$$  

It is noted that the braid relations of the planar permutohedron equation can be gotten easily from elements of $B_N$ and $\hat{B}_N$.

### V. Conclusions and Remarks

Similar to the above discussions, we can write down the weight function of the three-dimensional vertex model related the 3D lattice integrable model proposed in Refs [11] and
as the following form
\[
R(u_1, u_2, u_3)_{i_1j_1k_1}^{i_2j_2k_2} = (-)^j (\omega^{1/2})^{j_1j_2 + j_2j_3 + j_3j_1} \times \\
\left[ \frac{w(uq_1, j_1)w(q^{-1}(\omega u_2)^{-1}, \sigma j_2)w(q^{-1}u_3, j_3)}{w(q^{-1}u_1, j_1)w(q\omega u_2, \sigma j_2)w(q\omega u_3, j_3)} \right]^{1/2} \times \\
\left\{ \sum_{\sigma \in \mathbb{Z}_N} \frac{w(u\sigma u_3, \sigma + j_2 + j_3)w(\sigma j_1, \omega^{-1}u\sigma j_1)}{w(q^{-1}u_2, \sigma + j_2)w(u'^2\sigma u_3, \sigma + j_3)} \right\}^{1/2} \quad (33)
\]

It satisfies the modified tetrahedron equation \( R_{123} \tilde{R}_{145} \tilde{R}_{246} \tilde{R}_{356} = R_{356} \tilde{R}_{246} \tilde{R}_{145} \tilde{R}_{123} \), where \( \tilde{R}(u_1, u_2, u_3)_{i_1j_1k_1}^{i_2j_2k_2} \) can be obtained from expression (33) by the substitutions \( q \rightarrow q^{-1}, \quad u_2' = u_2, \quad u_3' = u_3' \) with the condition \( u_2' u_3' = u_3' u_2' \). The details will be given elsewhere. When \( q = 1 \), this vertex model reduces to the one in Sec. II. Then it can be regarded as a deformation of the three-dimensional vertex model corresponding to BB model.

As the conclusions we get the three-dimensional vertex model which is a duality of the three-dimensional Baxter–Bazhanov model. The Boltzmann weights of the model satisfy the vertex-type tetrahedron equation. The braid group \( B_N \) corresponding to the Frenkel–Moore equation is constructed and the transformations \( R \) and \( I \) acting on the braids and denoting the rotations of the braids around some special axes through the angle \( \pi \) are given. Using the method presented in this paper, we can construct a new three-dimensional vertex model for which the weight function has the form (33). It is a duality of the 3D lattice integrable model proposed in Refs [11] and [12] and a deformation of the vertex model related BB model. This means that we can interpret the 3D lattice integrable model in Refs [11] and [12] as a deformation of the three-dimensional Baxter–Bazhanov model. The symmetry properties of the weight function (33) can be given similarly as in Ref. [19].

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