DEVELOPMENT OF ENERGY ANALYZER OF CHARGED PARTICLES BASED ON THE BASIS NON-UNIFORM ELECTROSTATIC FIELD

Saulebekov A.O., Vénos D., Kambarova Zh.T., Saulebekova D.A., Vassilyeva K.I., Seidualy Kh.B.

1Lomonosov Moscow State University, Kazakhstan branch, Nur-Sultan, Kazakhstan
2Nuclear Physics Institute, Czech Academy of Sciences, 250 68 Řež, Czech Republic
3E.A. Buketov Karaganda State University, Karaganda, Kazakhstan, kambarova@bk.ru

The electron-optical characteristics of a mirror energy analyzer based on a non-uniform, hyperbolic decreasing electrostatic field were investigated. In work an approximate-analytical method for calculating the trajectory of charged particles in an electrostatic field, acting in the mirror reflection regime, was used. The equation of motion of charged particles in the integro-differential form is obtained. The scheme of the most optimal variant of the analyzer with a non-uniform field is found.

Keywords: non-uniform field, energy analyzer, mirror reflection regime, electron-optical characteristics, trajectory of charged particles.

Introduction

The great potential of electron spectroscopy, associated with the broad information that the energy spectrum of these particles carries about various physical processes, about the matter structure, stimulates further improvement of the known analysis methods and the development of new directions of electron spectroscopy.

At the initial stage of the development of electron optics, the main research focused on the study of axially symmetric fields with the stigmatic focusing properties of a charged particles beam and the formation of a correct scale-free undistorted image. Electrostatic mirror energy analyzers based on cylindrical, spherical and hyperbolic fields turned out to be the most advanced and widely used devices in the study of small and medium energy electron beams. Among the known classical type fields an electrostatic mirror with a uniform field is the simplest in construction and widely used energy analyzer of charged particle beams.

Further progress in the development of effective methods for studying the solid surface requires a significant modernization of existing or the creation of qualitatively new analyzing systems based on the further development of the theory. The development of high-resolution electron analyzers based on the synthesis of multipoles and a cylindrical field can be attributed to qualitatively new methods [1-10]. To confirm the universality of the obtained results, it is necessary to continue research of analytical systems based on the synthesis of multipoles with other types of classical electrostatic fields. This class of potential fields includes an non-uniform electrostatic field decreasing by a hyperbolic law, which is a superposition of a dipole with a uniform field.

1. Calculation and analysis of the electron-optical properties of an electrostatic non-uniform field

The object of the study is electrostatic non-uniform field acting in the mirror reflection regime, that decreasing by hyperbolic law. The potential is described by the expression...
\begin{align}
U = \frac{U_0}{d} y(1 - A z),
\end{align}

where \( A \) is dimensionless parameter, at \( A = 0 \) - field (1) is uniform.

The profile of the outer deflecting electrode is determined by calculation the equipotential lines in a non-uniform field. Fig. 1 shows a portrait of equipotential lines in an electrostatic non-uniform field at \( A = 0.01 \).

![Fig.1. Equipotential lines in a non-uniform field at \( A = 0.01 \)](image)

A field (1) is formed in the space between two electrodes, one of which remains flat and is under the ground potential, the deflecting potential \( U_0 \) is supplied to the other electrode, which has a hyperbolic profile (Fig. 2).

![Fig.2. The electrostatic mirror based on non-uniform field. The dashed line is a flat electrode in the limiting case of a flat mirror (\( B \) is source of charged particles; \( C \) is electron-optical image of the source)](image)

In the mirror reflection regime a charged particles beam enters field (1) at an angle \( \theta_0 \) to the \( z \) axis, moves along a “return” trajectory having a vertex \( m \) in the electrostatic field region, and returns to the lower electrode at an angle \( \theta_1 \). The return trajectory of charged particles consists of right and left branches, asymmetrical about the trajectory vertex \( m \), therefore, a separate calculation of each of its branches and their subsequent joining is necessary. The condition for joining the right
and left branches of the trajectory is the equality of the functions describing the particles trajectory and their derivatives at the point \( m \) for both branches. For calculate the branches of the return trajectory, let’s move to a new coordinate system, the beginning of which is located at the trajectory vertex \( m \).

According to Fig.2,

\[ z = z_m \pm \xi, \quad x = y_m - y. \]

(2)

Here, the upper sign in front \( \xi \) corresponds to the right branch \( (\xi > 0) \), the lower sign to the left branch of the trajectory \( (\xi < 0) \). The distribution of the field (1) in the coordinate system \( x, \xi \) has the following form:

\[ U(x, \xi) = \omega (1 \pm A' \xi)(y_m - x), \]

(3)

where

\[ \omega = \frac{U_0 (1 - Az_m)}{d}, \quad A' = \frac{A}{1 - Az_m}. \]

(4)

According to the law of conservation of energy when moving in an electrostatic potential field, the kinetic energy of a charged particle is determined by the passed potential difference. For a particle moving in the field (3) from point \( m \) to an arbitrary point \( A \), we can write:

\[ \frac{m}{2} (x^2 + \xi^2) - \frac{mv^2_m}{2} = -q(U_m - U) = q\omega G_{1,2} (x, \xi), \]

(5)

where

\[ G_{1,2} (x, \xi) = x \pm A' \xi (y_m - x). \]

(6)

Here and below, the number 1 in the subscript corresponds to the functions for the right branch, the number 2 to the functions of the left branch of the trajectory.

The law of conservation of energy for the longitudinal component of the motion of a charged particle in the field (3), taking into account the condition \( v_m^2 = \hat{\xi}_m^2 \), because \( \dot{x}_m = 0 \), as well as the ratio \( \frac{d\xi}{dt} = \frac{dx}{dt} = \frac{dx}{d\xi} \), will be written as follows

\[ \frac{m\hat{\xi}^2}{2} - \frac{mv^2_m}{2} = -q \int_0^x \frac{\partial U(x, \xi)}{\partial \xi} \xi' d\xi = \mp q\omega A' \int_0^x (y_m - x) \xi' d\xi, \]

(7)

where the derivative \( \xi' \) is greater than zero for both branches, as \( x \) and \( \xi \) are taken in absolute value.

At \( x = y_m \), \( m\hat{\xi}^2 \) = \( \frac{mv^2_m}{2} \cos^2 \theta_{0,1} = W \cos^2 \theta_{0,1} \), therefore, expression (7) can be rewritten with respect to \( \frac{mv^2_m}{2} \) the following form:

\[ \frac{mv^2_m}{2} = q\omega \left( S' \cos^2 \theta_{0,1} \pm A' \right), \]

(8)
The value of \( S \) has the dimension of length and is some characteristic size of the electron mirror. Solving equations (5), (7) and (8) relatively to \( \xi' \), we come to the integro-differential equation of the trajectory of a charged particle in a non-uniform field (3)

\[
\left( \xi'_{1,2} \right)^2 \left[ g_{1,2} (x, \xi) \pm A' \int_0^x (y_m - x) \xi'^{'}_1 dx \right] = S' \cos^2 \theta_{0,1} \pm A' \int_0^x (y_m - x) \xi'^{'}_1 dx.
\]

The integral-differential equation (10) has a singular point at \( x=0 \), since the denominator in this case vanishes, so the solution of the equation is sought as a generalized power series [11]:

\[
\xi = \sqrt{x} \sum_{n=0}^{\infty} c_n x^n + \sum_{n=1}^{\infty} a_n x^n.
\]

The second-order angular focusing regime is determined from the condition

\[
\left[ \frac{d^2L}{d\theta^2} \right]^{\ast} = 0
\]
\[
\cos^2 \theta_o - 3 \sin^2 \theta_o + (SA) \sin 4\theta_o + \\
+ \frac{4}{3} (SA)^2 \left( \cos^2 \theta_o - 3 \sin^2 \theta_o \right) + \sin^2 \theta_o \left( 16 \cos^2 \theta_o - 10 \sin^2 \theta_o \right) \right] = 0.
\]

From the analysis of equation (16) it has been established that second-order angular focusing regime can be realized only for the analyzer scheme with parameters \( A=0, \ h_1 + h_2 = 0.5 \), i.e. for a flat mirror. To search for the parameters of the most optimal variant of analyzer with a non-uniform field decreasing by hyperbolic law, it is necessary to determine the functions \( f(\Delta \theta, SA, h_1/h_2) = l(\theta) - l(\theta_o) \) characterizing the longitudinal aberration smearing of the image near the Gaussian focus by the formula (12) for different values of \( SA \) and the selected divergence angle \( \Delta \theta \) of the analyzed beam.

The aberration smearing functions of electron-optical mirrors with a non-uniform field (Table 1), whose schemes correspond to different values of \( SA \), were calculated and tuned to the second-order angle focusing regime of the flat mirror \( h_1 + h_2 = 0.5, \ \theta_o = 30^0 \).

| \( SA \)  | \( \theta_0 \) (deg) | \( l_o \)  | \( f(\Delta \theta, SA, h_1/h_2) \) |
|---------|----------------------|----------|-------------------------------|
| 0.000   | 30                   | 2.5981   | 0.0268                        |
| 0.005   | 29.9043              | 2.6056   | 0.0154                        |
| 0.010   | 29.8082              | 2.6133   | 0.0117                        |
| 0.015   | 29.7118              | 2.6210   | 0.0079                        |
| 0.020   | 29.6149              | 2.6289   | 0.0063                        |
| 0.025   | 29.5178              | 2.6368   | 0.0074                        |
| 0.030   | 29.4202              | 2.6449   | 0.0097                        |
| 0.035   | 29.3224              | 2.6530   | 0.0125                        |
| 0.040   | 29.2242              | 2.6613   | 0.0157                        |
| 0.045   | 29.1257              | 2.6697   | 0.0188                        |
| 0.050   | 29.0269              | 2.6781   | 0.0220                        |

From the results of calculation the trajectories performed for the angular spread \( \Delta \theta = \pm 6^\circ \) of particles at the analyzer entrance, a scheme was found for the most optimal variant of analyzer with a non-uniform field: \( h_1 + h_2 = 0.5, \ \theta_o = 30^0 \) and \( SA = 0.02 \). In this case the value of the aberration smearing \( f(\Delta \theta, SA, h_1/h_2) \) is 3 times less than the flat mirror \( (SA = 0) \). This means that in a mirror analyzer built on the basis of a non-uniform electrostatic field that decreasing by a hyperbolic law, which is a superposition of a dipole with a uniform field, the resolution can be improved several times as compared with the case of a flat mirror.
Conclusion

A theoretical study of the electron-optical properties of an analyzing system based on a mirror with a modified electrostatic field has been carried out. Equation of total projection of the particle trajectory on the axis from the source to the image was obtained. The aberration smear functions of electron mirrors with a non-uniform field were calculated. The optimal variant of the analyzer scheme with a non-uniform field has a higher resolution than a flat mirror.

REFERENCES

1. Ashimbaeva B.U., Chokin K.Sh., Saulebekov A.O. Focusing properties of a mirror analyzer with hexapole cylindrical field. J. of E.Spect. and Rel. Phen. 2005, 143 (1). pp. 29-32.
2. Saulebekov A.O., Assylbekova S.N., Kambarova Zh.T., Tussupbekova A.K. Calculation of the charged particles trajectories in the hexapole cylindrical field. Eurasian Physical Technical Journal. 2011, V.8, No.2(16). - pp. 41-49.
3. Ashimbaeva B.U., Chokin K.Sh., Saulebekov A.O., Kambarova Zh.T. Modeling of electron-optical scheme of hexapole-cylindrical analyzer. Applied Physics. 2012, Issue 2, pp.45-48.
4. Assylbekova S.N., Saulebekov A.O., Kambarova Zh.T. Numerical modeling of the electrostatic energy analyzer based on hexapole-cylindrical field with a concave outer electrode. Eurasian Physical Technical Journal. 2015, V.12, No 1 (23). pp.25-29.
5. Saulebekov A.O., Kambarova Zh.T., Saulebekova D.A. Modeling of electrostatic decapole-cylindrical mirror analyzer. IOP Conf. Series: Materials Science and Engineering. 2017, 168, 012078 doi:10.1088/1757-899X/168/1/012078
6. Saulebekov A.O., Kambarova Zh.T. Development of mirror energy analyzer based on electrostatic quadrupole-cylindrical field. Eurasian Physical Technical Journal. 2017, V.14, No 2 (28), pp.42-47.
7. Kambarova Zh.T., Trubitsyn A.A., Saulebekov A.O. Axially symmetric energy analyzer based on the electrostatic decapole-cylindrical field. Technical Physics. 2018, Vol. 63, No. 11, pp. 1667–1671.
8. Saulebekov A.O., Kambarova Zh.T. Calculation and modeling of the motion of charged particles in the quadrupole-cylindrical field. Eurasian Physical Technical Journal. 2018, V.15, No 1 (29), pp.55-60.
9. Saulebekov A.O., Trubitsyn A.A., Kambarova Zh.T., Saulebekova D.A. Electrostatic energy analyzer of charged particles on the basis of a quadrupole-cylindrical field in the “ring-axis” focusing regime. Eurasian Physical Technical Journal. 2018, V.15, No 2 (30), pp.35-39.
10. Wannberg B. An electrostatic mirror spectrometer with coaxial electrodes for multi-detector operation. Nuclear Instruments and Methods. 1973, Vol. 107, pp. 549 – 556.
11. Zashkvar A.V., Ashimbayeva B.U., Chokin K.SH., Masyagin V.Ye. Raschet trayektoriy zaryazhennykh chastits v elektrostaticheskom neodnorodnom pole [Calculation of the trajectories of charged particles in an electrostatic non-uniform field]. News of the National Academy of Sciences of the Republic of Kazakhstan. Series Physico-Mathematical. 2000, №2, pp.72-78.[in Russian]
12. Gurov V.S., Saulebekov A.O., Trubitsyn A.A. Approximate-Analytical Method of Calculating the Charged Particle Trajectories in Electrostatic Fields (Book Chapter). Advances in Imaging and Electron Physics. 2015, V.192, pp. 87-115.