A Toy Model for Magnetic Extraction of Energy from Black Hole Accretion Disc

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Abstract

A toy model for magnetic extraction of energy from black hole (BH) accretion disk is discussed by considering the restriction of the screw instability to the magnetic field configuration. Three mechanisms of extracting energy magnetically are involved. (1) The Blandford-Znajek (BZ) process is related to the open magnetic field lines connecting the BH with the astrophysical load; (2) the magnetic coupling (MC) process is related to the closed magnetic field lines connecting the BH with its surrounding disk; and (3) a new scenario (henceforth the DL process) for extracting rotational energy from the disk is related to the open field lines connecting the disk with the astrophysical load. The expressions for the electromagnetic powers and torques are derived by using the equivalent circuits corresponding to the above energy mechanisms. It turns out that the DL power is comparable with the BZ and MC powers as the BH spin approaches unity. The radiation from a quasi-steady thin disk is discussed in detail by applying the conservation laws of mass, energy, and angular momentum to the regions corresponding to the MC and DL processes. In addition, the poloidal currents and the current densities in BH magnetosphere are calculated by using the equivalent circuits.

Key words: Black holes, Physics of black holes, Infall, accretion, and accretion disks, Relativity and gravitation
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1 INTRODUCTION

As is well known, the Blandford-Znajek (BZ) mechanism has been regarded as a reasonable process for powering the radio jets in AGNs (Blandford

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Recently much attention has been paid on the magnetic coupling (MC) process, where energy and angular momentum are transferred from a fast-rotating BH to its surrounding disk by virtue of the closed field lines connecting them (Blandford 1999; Li 2002a, hereafter L02; Wang et al. 2002b, 2003a, hereafter W02, W03a, respectively).

In W02 we worked out an equivalent circuit to calculate the BZ and MC powers. Very recently we discussed the condition for the coexistence of the BZ and MC processes (CEBZMC), and found that the state of CEBZMC always accompanies the screw instability of the magnetic field connecting a rotating BH with its surrounding disk (Wang et al. 2003b, 2004, hereafter W03b and W04, respectively). It turns out that the screw instability will occur at some place far away from the inner edge of the disk, if the BH spin and the power-law index for the variation of the magnetic field are greater than some critical values.

In this paper a new scenario for extracting rotational energy of the disk matter is proposed by considering the configuration of the magnetic field restricted by the screw instability. To facilitate description, this mechanism is referred to as the DL process, implying that energy and angular momentum are extracted magnetically from disk to load. By using another equivalent circuit we derived the expression for the DL power and torque under some assumptions on the unknown astrophysical load.

This paper is organized as follows. In x2 the restriction of the screw instability to the configuration of the magnetic field is discussed, in which the three energy mechanisms are contained. In x3 the expressions for the powers and torques of the three mechanisms are derived in the two kinds of equivalent circuits. We compare these powers with the variation of the two parameters, i.e., the BH spin and the power-law index of the magnetic field on the disk. It turns out that the DL power is generally less than the BZ and MC powers, and it is comparable with the latter two when the BH spin approaches unity. In x4 the radiation from a quasi-steady thin disk is discussed in detail by applying the conservation laws of mass, energy and angular momentum to the
regions corresponding to the MC and DL processes. In x5 the poloidal current densities owing from the BH magnetosphere into the horizon and disk are calculated in the regions corresponding to the three energy mechanisms. Finally, in x6, we summarize our main results and argue that this model can be used to extract clean energy from a rotating BH for powering GRBs.

Throughout this paper the geometric units $G = c = 1$ are used, and the assumptions for BH accretion disk and the magnetic field are adopted as given in W 03b and W 04.

2 RESTRICTION OF SCREW INSTABILITY TO CONFIGURATION OF MAGNETIC FIELD

It is well known that the magnetic field configurations with both poloidal and toroidal components can be screw unstable (Kadomtsev 1966; Bateman 1978). According to the Kruskal-Shafranov criterion (Kadomtsev 1966), the screw instability will occur, if the toroidal magnetic field becomes so strong that the magnetic field line turns around itself about once. Recently some authors discussed the screw instability with different conditions in the BH magnetosphere (Gruzinov 1999; Li 2000a; Tominaga et al. 2001). In W 04 we discussed the screw instability of the magnetic field in the MC process, and argued that the instability will occur if the following criterion is satisfied,

\[ (2) \frac{S_b}{L} B_D^p B_D^T < 1; \]

where $L$ is the poloidal length of the closed field line connecting the BH with the disk, $B_D^p$ and $B_D^T$ are the poloidal and toroidal components of the magnetic field on the disk, respectively, and $S_b$ is the cylindrical radius on the disk and reads

\[ S_b = M^{\frac{2}{m_s}} \left( 1 + a^2 \right)^{\frac{q}{2}} \left( 1 + a^2 \right)^{\frac{2}{m_s}} \left( 1 + a^2 \right)^{\frac{4}{m_s}} \left( 1 + a^2 \right)^{\frac{6}{m_s}}; \]

In equation (2) $M$ and $a$ are the BH mass and spin, respectively. The parameter $r/r_{m_s}$ is the radial coordinate on the disk, which is defined in terms of the radius of the marginally stable orbit, and $r_{m_s}$ is related to $M$ and $a$ by the following relation (Novikov & Thorne 1973),

\[ r_{m_s} = M^{\frac{2}{m_s}}; \]

\[ m_s = 3 + A_2 \left( \frac{3}{A_1} + A_1 \right)^{1/2} \left( A_1 - 2A_2 \right)^{-1/2}; \]

\[ A_1 = 1 + (1 + a^2)^{1/3} \left( 1 + a \right)^{1/3} \left( 1 - a \right)^{1/3}; \]

\[ A_2 = (3a^2 + A_1^2)^{1/2}; \]
Fig. 1. Configuration of the magnetic field restricted by the screw instability

It is found, from the criterion (1), that the screw instability will occur, provided that the BH spin $a$ and the power-law index $n$ are great enough. For the given values of $a$ and $n$ we can determine the disk region for the screw instability by using the criterion (1) as follows,

$$s < \frac{a}{n} < 1;$$

where $s = r_s/r_{ms}$ is the minimum radial coordinate for the screw instability. We think that the screw instability prevents the closed field lines to reach the region indicated by inequality (4). It seems reasonable that the configuration of the magnetic field in this region might consist of the open field lines connecting the disk with the astrophysical load, and this region is referred to as the DL region corresponding to the DL process. Considering the restriction of the screw instability, we modify the configuration of the magnetic field as shown in Figure 1, and the three mechanisms of extracting energy magnetically from the BH accretion disk are included: (1) the BZ process involved the open field lines connecting the BH with the astrophysical load, (2) the M C process involved the closed field lines connecting the BH with the surrounding disk, and (3) the DL process involved the open field lines connecting the disk with the astrophysical load.

Now we give a brief description for the configuration in Figure 1. In W 03b we discussed the angular boundary between the open and closed field lines on the BH horizon. It is shown that the boundary angle $\alpha$ exists in CEB2M C, if the parameters $a$ and $n$ are great enough. In W 04 we argued that the state of CEB2M C always accompanies the screw instability, and the minimum radial coordinate $s = r_s/r_{ms}$ for the screw instability can be determined by the
criterion (1). Considering the restriction of the screw instability to the closed eld lines and the action of the magnetic pressure on the horizon, we think that the boundary angle will be extended from $M$ to $S$. The angles $M$ and $S$ are related respectively to inative and the minimum radial coordinate $S$ by the mapping relation (W 04):

$$\cos \cos L = G (a ; ; n) d ;$$

(5)

where

$$G (a ; ; n) = \frac{1 + a^2}{2} \left( 1 + a^2 \right)^{2} + \left( 1 + a^2 \right)^{6} \right) \left( 1 + a^2 \right)^{6} \right)$$

(6)

Therefore the angular region of the open eld lines for the BZ process is given by

$$0 < S ;$$

(7)

and the angular region of the closed eld lines for the MC process is given by

$$S < L ;$$

(8)

where $L$ is the lower boundary angle of the closed eld lines. Throughout this paper $L = 0.45$ is assumed in calculations. Accordingly the value range of the radial coordinate for the MC process is given by

$$1 < S ;$$

(9)

where $= 1$ and $S$ correspond to $L$ and $S$ by the mapping relation (5), respectively.

Following Blanford (1976), we assume that the poloidal magnetic eld varies with the parameter on the disk as a power law (W 03a, W 03b),

$$B^P_D = B^P_H \left[ \frac{\xi}{S_D (r_{ms})} \right]^n ; 1 < S ;$$

(10)

where $B^P_H$ is the poloidal component of the magnetic eld on the horizon, and $n$ is the power-law index of $B^P_D$ varying with . The quantities $\xi$ and $S_D (r_{ms})$ are the radius of the horizon and that of the inner edge of the disk, respectively.

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According to the above discussion, the value range of the radial coordinate for the DL process is given by equation (4), and the corresponding poloidal magnetic field varies with the parameter as follows,

$$B_D^P = B_H^P \left( \frac{r_h}{r_S} \right)^n; \quad s < < 1$$  \hspace{1cm} (11)

Equation (11) is similar to equation (10), where $S_0$ ($r_{m,s}$) and $\in_s = 1$ are replaced by $r_S$ and $s$, respectively.

### 3 Power of Magnetic Extraction from BH Accretion Disk

It is tempting for us to discuss the power extracted from BH accretion disk based on the above magnetic field configuration. In W02 we derived the expressions for the BZ and MC powers by using an equivalent circuit as shown in Figure 2 (henceforth circuit I), where segments $P_1 S_1$ and $Q_1 R_1$ represent the two adjacent magnetic surfaces, and segments $P_1 Q_1$ and $R_1 S_1$ represent the BH horizon and the loads (either the astrophysical load or the disk load) sandwiched by the two magnetic surfaces, respectively.

By using circuit I, we derive the expressions for the BZ and MC powers and those for the BZ and MC torques corresponding to the modified configuration of the magnetic field as follows,

$$P_{BZ}/P_0 = 2a^2 \int_0^Z \frac{k}{2} \left( \frac{1}{q} \right) \sin^3 \frac{d}{\sin^2} d; \quad 0 < < s$$  \hspace{1cm} (12)

$$T_{BZ}/T_0 = 4a (1 + q) \int_0^Z \frac{k}{2} \left( \frac{1}{q} \right) \sin^3 \frac{d}{\sin^2} d; \quad 0 < < s$$  \hspace{1cm} (13)
\[
P_{MC}/P_0 = 2a^2 \frac{Z}{s} \frac{(1/q) \sin^3 d}{2} \quad s < < L \quad (14)
\]

\[
T_{MC}/T_0 = 4a (1+q) \frac{Z}{s} \frac{(1/q) \sin^3 d}{2} \quad s < < L \quad (15)
\]

where \( P_{BZ}, T_{BZ}, P_{MC} \) and \( T_{MC} \) are related by

\[
P_{BZ} = F T_{BZ}; \quad P_{MC} = D T_{MC}; \quad (16)
\]

In equations (12) | (15) the parameters \( k \) and \( q \) are respectively the ratios of the angular velocities of the open and closed field lines to the angular velocity of the horizon and read

\[
k = \frac{F}{H}; \quad D = \frac{2(1+q)}{a} \frac{q - 3}{m_s + 3}=: (17)
\]

where \( F \) is the angular velocity of the magnetic field line, \( H \) and \( D \) are respectively the angular velocities of the horizon and the disk and read

\[
H = \frac{a}{2r_H}; \quad D = \frac{1}{M \left( \frac{3/2}{m_s + 3} + a \right)}; \quad (18)
\]

Usually \( k = 0.5 \) is taken for the optimal BZ power \( (M T 82) \), while depends on the BH spin and the place where the field line penetrates on the disk. In equations (12) | (15) we use the parameter \( q \) and \( a \), and the parameters \( P_0 \) and \( T_0 \) are defined as

\[
P_0 \geq (B_H^4)^2 M^2 B_4^2 M /M \geq 6.59 \times 10^{28} \text{erg s}^{-1};
\]

\[
T_0 \geq (B_H^4)^3 M^2 B_4^2 M /M \geq 3.26 \times 10^{23} \text{g cm}^2 \text{s}^{-2}; \quad (19)
\]

where \( B_4 \) is the strength of the poloidal magnetic field on the horizon in units of \( 10^4 \) gauss.

Considering that the poloidal magnetic field will exert a braking torque on the current in the DL region, we can calculate the DL power and torque by using the equivalent circuit as shown in Figure 3. Henceforth this equivalent circuit is referred to as circuit II, where segments \( P_2 S_2 \) and \( Q_2 R_2 \) represent the two adjacent magnetic surfaces consisting of the open field lines in the DL region, and segments \( P_2 Q_2 \) and \( R_2 S_2 \) represent the disk surface in the
Fig. 3. Equivalent circuit for the DL process (circuit II)

DL region and the load sandwiched by the two adjacent magnetic surfaces, respectively. The quantities $Z_A$ and $\Delta \epsilon_i$ (the subscript \(i\) is omitted) are the resistance of the load and EMF due to the rotation of the disk, respectively. The disk load is neglected in calculations by considering the perfect conductivity of plasma.

The following equations are used to derive the DL power in circuit II, which are similar to those given in deriving the BZ power and MC power in W02.

$$P_{DL} = I_{DL}^2 Z_A; \quad I_{DL} = \frac{\epsilon}{Z_A}; \quad \epsilon = \left( \frac{D}{2} \right)_D; \quad (20)$$

where $I_{DL}$ is the current in each loop of circuit II. The minus sign in $\epsilon_D$ arises from the direction of the magnetic flux between the two adjacent magnetic surfaces, which is given by

$$D = B_D^P 2 \int \frac{-P}{r^2 + a^2} \cos \frac{1}{2} \, dr:\quad (21)$$

The concerned Kerr metric coefficients are given by (Thorne, Price & MacDonald 1986)

$$8 \overset{(\it a)}{\overset{\it b}{\overset{\it c}{\overset{\it d}{\overset{\it e}{\overset{\it f}{\overset{\it g}{\overset{\it h}{\overset{\it i}{\overset{\it j}{\overset{\it k}{\overset{\it l}{\overset{\it m}{\overset{\it n}{\overset{\it o}{\overset{\it p}{\overset{\it q}{\overset{\it r}{\overset{\it s}{\overset{\it t}{\overset{\it u}{\overset{\it v}{\overset{\it w}{\overset{\it x}{\overset{\it y}{\overset{\it z}{\overset{\it A}{\overset{\it B}{\overset{\it C}{\overset{\it D}{\overset{\it E}{\overset{\it F}{\overset{\it G}{\overset{\it H}{\overset{\it I}{\overset{\it J}{\overset{\it K}{\overset{\it L}{\overset{\it M}{\overset{\it N}{\overset{\it O}{\overset{\it P}{\overset{\it Q}{\overset{\it R}{\overset{\it S}{\overset{\it T}{\overset{\it U}{\overset{\it V}{\overset{\it W}{\overset{\it X}{\overset{\it Y}{\overset{\it Z}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}{}{}}}
Each open field line intersects with the disk and the plane \( P \) at the cylindrical radii \( r \) and \( r^0 \), respectively. The both radii are related by

\[
r^0 = r;
\]

where \( r \) is assumed to be a constant.

The surface resistivity \( L \) of the unknown load is assumed to obey the following relation,

\[
L = 2 \bar{z} = 4 \bar{z};
\]

where \( \bar{z} \) is a parameter, and \( \bar{z} = 4 \) = 377 ohm is the surface resistivity of the BH horizon. Throughout this paper \( \bar{z} = 1 \) is assumed in calculations. The load resistance \( Z_A \) between the two adjacent magnetic surfaces can be written as

\[
Z_A = \frac{dr^0}{r^0} = 2 \frac{dr}{r};
\]

where equations (23) and (24) are used in the last step. Incorporating equation (11) with equations (20) \( \mid \) (25), we have

\[
P_{DL}/P_0 = f (a; ; n) \text{d} ;
\]

where the function \( f (a; ; n) \) is expressed by

\[
f (a; ; n) = \frac{(1 + g)^2 (1 + a^2 \frac{4}{m_s})^2 + 2a^2 \frac{6}{m_s} (\frac{4}{m_s})^{2n + 3}}{2 (3 - 2 \frac{3}{m_s} + a)^2 (1 + a^2 \frac{4}{m_s} + 2 \frac{2}{m_s} + a^2 \frac{4}{m_s} + a^2 \frac{4}{m_s})};
\]

Similarly, the DL power is related to the DL torque by

\[
P_{DL} = \int T_{DL};
\]

and we have

\[
T_{DL}/T_0 = g (a; ; n) \text{d} ;
\]

where the function \( g (a; ; n) \) is expressed by

\[
g (a; ; n) = \frac{(1 + g)^2 (1 + a^2 \frac{4}{m_s})^2 + 2a^2 \frac{6}{m_s} (\frac{4}{m_s})^{2n + 3}}{2 (3 - 2 \frac{3}{m_s} + a)^2 (1 + a^2 \frac{4}{m_s} + 2 \frac{2}{m_s} + a^2 \frac{4}{m_s} + a^2 \frac{4}{m_s})};
\]
Integrating equations (26) and (29) over the DL region, we have the DL power and torque expressed by

\[
P_{DL}(a; ;n)/P_0 = \int_a^Z f(a; ;0; n) d^0; \tag{31}
\]
\[
T_{DL}(a; ;n)/T_0 = \int_a^Z g(a; ;0; n) d^0. \tag{32}
\]

For the given values of \(a\) and \(n\) the strength of \(P_{BZ}/P_0, P_{MC}/P_0\) and \(P_{DL}/P_0\) are compared by using equations (12), (14) and (31) as shown in Figures 4.
From Figure 4 we obtain the following results,

(1) For the increasing $a$ with the given $n$, both the BZ and the MC powers vary non-monotonically, attaining their maxima as the BH spin approaches unity, while the DL power increases monotonically.

(2) For the increasing $n$ with the given $a$, the BZ power increases monotonically, the MC power decreases monotonically, while the DL power varies non-monotonically, attaining its maximum value $P_{DL} = 0.034$ with $n = 6.39$.

(3) The MC power is generally greater than the BZ power, if the power-law index $n$ is not very big. The DL power is generally less than the BZ and MC powers, and it is comparable with the latter two powers as the BH spin approaches unity.

4 RADIATION FROM A MAGNETIZED ACCRETION DISK

4.1 Radiation from the MC region

Radiation from a relativistic steady thin disk around a Kerr BH has been discussed based on the three conservation laws of mass, energy and angular momentum with the "no-torque boundary condition" by some authors (Novikov & Thorne 1973, Page & Thorne 1974). By combining the MC effects with the above conservation laws and the "no-torque boundary condition", the following equation of radiation from a relativistic quasi-steady thin disk was derived in L02,

$$F_{MC}^{\text{total}} = F_{IA}^{DA} + F_{MC};$$

where $F_{IA}^{DA}$ and $F_{MC}$ are the radiation fluxes due to disk accretion and the MC effects, respectively, and they are expressed by

$$F_{IA}^{DA} = \frac{M_D}{4} \frac{\left(\theta_D / \theta_r\right)}{r (E^+_{D} \cdot L^+_{D})^2} \int_{i_{in}} r \left(\frac{\partial}{\partial r}\right) E^+_{D} \cdot L^+_{D} \cdot \theta_r \ dr;$$

$$F_{MC} = \frac{(\theta_D / \theta_r)}{r (E^+_{D} \cdot L^+_{D})^2} \int_{i_{in}} r H_{MC} \ dr;$$

In equation (35) $H_{MC}$ is the flux of angular momentum transferred between the BH and the disc by the magnetic field, and $E^+$ and $L^+$ are the specific
energy and angular momentum of a particle in the disc, respectively, and they read (Novikov & Thorne 1973)
\[ E^+ = 1 \frac{a^3}{2^2} + a \frac{1}{3} 3^2 + 2a \frac{3}{3} \] ; \ (36)
\[ L^+ = M 1 \frac{2a}{3} + a^2 \frac{4}{2} 1 \frac{3}{3} 2 + 2a \frac{3}{3} \] ; \ (37)

From equations (36) and (37) we have \( E^+ = E_{ms} \) and \( L^+ = L_{ms} \) for \( = m_s \). The expression for \( H_{MC} \) can be worked out by using the following relation,
\[ \frac{\partial T_{MC}}{\partial r} = \frac{\left( \frac{\partial T_{MC}}{\partial \rho} \right) \left( \frac{d}{d\rho} \right)}{M^2 m_s} = 4 \rho H_{MC} ; \] \ (38)

where \( \frac{\partial T_{MC}}{\partial \rho} \) and \( \frac{d}{d\rho} \) can be calculated using equations (15) and (5). In W 03a we expressed \( H_{MC} \) as follows,
\[ H_{MC} / H_0 = A (a ; ) \; ; \; \; 1 < < s ; \] \ (39)

where
\[ H_0 = (B_H^p)^2 M = 1 \times 10^{13} \; \; B_H^2 \; M \; /M \; g \; s^2 ; \] \ (40)

\[ A (a ; ) = \frac{a}{\left( \frac{1}{2} \frac{a}{h} \frac{1}{a} \frac{1}{a} \frac{1}{a} \frac{1}{a} \right)} \; F_H (a ; ) ; \] \ (41)

\[ F_H (a ; ) = \frac{1 + a^2 + a^2}{2 a \left( 1 + a^2 \right) \left( 1 + a^2 \right)} \; \frac{1}{1 + a^2 + a^2} \; \frac{1}{1 + a^2 + a^2} \; \frac{1}{1 + a^2 + a^2} \; \frac{1}{1 + a^2 + a^2} \; \frac{1}{1 + a^2 + a^2} \ ; \] \ (41)

Since the magnetic field on the horizon is brought and held by its surrounding magnetized disk, there must exist some relations between the magnetic field and the accretion rate. As a matter of fact these relations might be rather complicated, and would be very different in different situations. One of them is given by considering the balance between the pressure of the magnetic field on the horizon and the ram pressure of the innermost parts of an accretion flow (Moderski, Sikora & Lasota 1977), i.e.,
\[ (B_H^p)^2 (8 ) = P_{\text{ram}} c^2 M_D 4 \; r_H^2 ; \] \ (42)

From equation (42) we define \( F_0 \) as
\[ F_0 (B_H^p)^2 = \frac{2M_D}{M^2 \left( 1 + q \right)^2} ; \] \ (43)
By using equations (34) and (35) we have the radiation fluxes, $F_{DA}^I F_0$ and $F_{MC}/F_0$, versus the radial parameter for the given values of the power-law index and the BH spin as shown in Figure 5.

Expecting Figure 5 we find that the peak of $F_{MC}/F_0$ is not only closer to the inner edge of the disk, but also is greater than that of $F_{DA}^I F_0$. Furthermore, $F_{MC}/F_0$ varies more steeply with than $F_{DA}^I F_0$ does. Thus the radiation flux due to the MC mechanism can result in a very steep emissivity in the inner region of the disk, which is consistent with the recent XMM-Newton observation of the nearby bright Seyfert 1 galaxy MCG-6-30-15 (W im s et al. 2001; Li 2002b; W 03a).

4.2 Radiation from the DL region

We can show that the radiation flux from the DL region also consists of two terms:

$$F_{DL}^{Total} = F_{DA}^{II} + F_{DL}; \tag{44}$$

where $F_{DL}$ is the electromagnetic flux in the DL process, and $F_{DA}^{II}$ is the radiation flux due to disk accretion in the DL region. The flux $F_{DL}$ can be worked out by

$$F_{DL} = \frac{P_{DL}}{2 \pi r^2} \tag{45}$$

Substituting equation (26) into equation (45), we have

$$F_{DL}/F_0 = \frac{f(a; ;n)}{2 \sqrt{m_s}} \tag{46}$$

The difference between the radiation from the DL region and that from the MC region lies in two aspects.

1. The outgoing flux $F_{MC}$ can be obtained after resolving the equations of the conservation laws with $P_{MC}$ and $T_{MC}$ incoming the disk, while the outgoing flux $F_{DL}$ is given before resolving the equations from the conservation laws.

2. "No-torque boundary condition" can be used at $r_{in}$ for the solution in the MC region, while this boundary condition is not valid at $r_s$ for the solution in the DL region.
Fig. 5. The radiation flux from the MC region, $F_{DA} = F_0$ (solid line) and $F_{MC} = F_0$ (dashed line) versus for $1 < \xi < \infty$ with $n = 5.5$ and different values of the BH spin: (a) $a = 0.5$, (b) $a = 0.7$ and (c) $a = 0.998$. 
Thus we find the flux $F_{DA}^{II}$ by applying the conservation laws of energy and angular momentum to the DL region, i.e.,

$$\frac{\partial}{\partial r} M_D E^+ - 2 r W_r^{D} = 4 r F_{DA}^{II} E^+ + H_{DL}^D ;$$

(47)

$$\frac{\partial}{\partial r} M_D L^+ - 2 r W_r^{D} = 4 r F_{DA}^{II} L^+ + H_{DL}^D ;$$

(48)

where accretion rate $M_D$ is regarded as a constant for a quasi-steady disk, and $W_r^D$ is the internal viscous torque per unit circumference. Incorporating equations (47) and (48), we have

$$W_r^D = \frac{2 (E^+ - D L^+)}{(d D / dr)} F_{DA}^{II} ;$$

(49)

The quantity $H_{DL}$ is the flux of angular momentum, which is related to the electromagnetic torque by

$$\frac{\partial T_{DL}}{\partial r} = \left(\frac{\partial T_{DL}}{\partial \theta}\right) \frac{\partial \theta}{\partial r} = 4 r H_{DL} ;$$

(50)

Substituting equation (32) into equation (50), we have

$$H_{DL} / H_0 = g(a; ; n) \cdot 4 \cdot 4 \cdot m_s ;$$

(51)

Substituting equation (49) into equation (48) and integrating equation (48) over the DL region, we have the expression for $F_{DA}^{II}$ as follows,

$$F_{DA}^{II} = F_A + F_B + F_C ;$$

(52)

where

$$F_A = \frac{M_D}{4} \frac{(\theta D / \theta r) ^{2r}}{r (E^+ - D L^+ ) ^{2r}} \left( E^+ - D L^+ \right) ^{2r} \left( L^+ \right) ^{2r} \left( \theta L^+ \theta r \right) \left( r D \right) \left( r D \right) ;$$

(53)

$$F_B = \frac{(\theta D / \theta r) ^{2r}}{r (E^+ - D L^+ ) ^{2r}} \left( E^+ - D L^+ \right) ^{2r} \left( L^+ \right) ^{2r} \left( \theta L^+ \theta r \right) \left( r D \right) \left( r D \right) ;$$

(54)

$$F_C = \frac{(\theta D \theta r)}{r (E^+ - D L^+ ) ^{2r}} \left( E^+ - D L^+ \right) ^{2r} \left( \theta L^+ \theta r \right) \left( r D \right) \left( r D \right) ;$$

(55)

Expecting equations (52) | (55), we find the following characteristics of $F_{DA}^{II}$:
Fig. 6. The radiation flux from the DL region, $F_{DA}^{II}/F_0$ (solid line) and $F_{DL}/F_0$ (dashed line) versus $s < 4$ with $n = 5.5$ and different values of the BH spin: (a) $a = 0.5$, (b) $a = 0.7$ and (c) $a = 0.998$. 
(1) The ux $F_A$ has the same form as $F_{DA}^{II}$ except the integral region;

(2) The ux $F_B$ has a similar form to $F_{MC}$ except the minus sign in equation (54), which implies the contribution of the DL torque on $F_{DA}^{II}$ is negative;

(3) The ux $F_C$ represents the effect of the boundary condition at $r = r_5$, where "no-torque boundary condition" is not valid.

By using equations (52) and (46) we have the radiation uxes, $F_{DA}^{II} = F_0$ and $F_{DL} = F_0$, versus the radial parameter for the given values of the power-law index and the BH spin as shown in Figures 6.

Inspecting Figure 6, we have the following results.

(1) The radiation ux $F_{DL}/F_0$ decreases monotonously and very steeply with the increasing. This result arises from two aspects, i.e., both the angular velocity decreases and the area of the ring from $r$ to $r + dr$ increases with the increasing.

(2) The radiation ux $F_{DA}^{II}/F_0$ generally decreases monotonously with the increasing, while it varies non-monotonously with as the BH spin approaches unity as shown in Figure 6c. This result arises from the conservation laws of energy and angular momentum and the behavior of $F_{DL}/F_0$ near the boundary at $r_5$.

5 CURRENT DENSITIES FLOWING FROM MAGNETOSPHERE INTO HORIZON AND DISK

In W 03b we calculated the current density flowing from the BH magnetosphere into the MC region on the horizon in the state of CEBZMC by using the circuit I. In this paper the three energy mechanisms of the magnetic extraction are described by using the circuits I and II, based on which the current densities flowing from the BH magnetosphere into the horizon and the disk can be calculated and compared. As argued in W 03b the poloidal current flowing in each loop of circuit I can be written as

$$I_{BZ}^H (a ; ) = I_0 \frac{a (1 - k)}{(1 - q)} ; 0 < < s ;$$

$$I_{MC}^H (a ; n) = I_0 \frac{a}{2 \csc^2 (1 - q)} ; s < < L ;$$

where $I_0 = B_0^P M = 148 \times 10^{10} B_4 (M/M_\odot) A$. The currents $I_{BZ}^H$ and $I_{MC}^H$ are respectively the poloidal currents flowing on the BZ and MC regions of the
Fig. 7. Current densities versus with $a = 0.9$, $L = 0.45$, (a) $j_{BZ}^* = j_0$ for $0 < s < S$, where $A$, $B$, $C$ are the end points of the curves corresponding to $n = 4, 5.5, 7$, respectively. (b) $j_{MC}^* = j_0$ for $s < L$ for $n = 4, 5.5, 7$ and $9$ in dashed, solid, dotted and dot-dashed lines, respectively.

horizon, which depend on the angular coordinate for the given values of $a$ and $n$. Inspecting equations (56) and (57), we nd that $j_{BZ}^*$ are generally not equal to $j_{MC}^*$ for $6 k$. The conservation of the current at $s$ is guaranteed by the current owing between the magnetosphere and the horizon. The continuity of the current at the boundary between the BZ and MC regions of the horizon is discussed in W 03b.

Based on the conservation of current, the current densities owing from the BH magnetosphere into the above regions of the horizon are expressed by

$$j_{BZ}^* (a ; s) = \frac{1}{2} \left[ \frac{1}{\sin \theta} \right] \frac{d I_{BZ}^*}{d \theta}$$

$$= \frac{j_0}{4} \left( \frac{k}{\sin^2 \left( \frac{\theta}{a} \right)^2} \right)^* \cos \left( \frac{\theta}{a} \right), \quad 0 < s < S; \quad (58)$$
Fig. 8. Current densities versus with $n = 5S$, $L = 0.45$; (a) $J_{BZ}^{\parallel} = \frac{1}{10}$, (b) $J_{MC}^{\parallel} = \frac{1}{10}$ for $a = 0.36, 0.62, 0.9$ and $0.998$ in dashed, dotted, solid, and dot-dashed lines, respectively.

$$J_{MC}^{\parallel}(a; n) = - \frac{j_0}{2} \left( \frac{M}{\sin^2 (a - q)} \right) \frac{d^2 j_0}{d^2} \sin \frac{d}{d}; \quad s < < L;$$

where $j_0 B_R^P / (2 M) = 0.108 \ B_4 (M / M) \ A \ cm^2$. For the given value of $a$ and $n$, both $J_{BZ}^{\parallel}$ and $J_{MC}^{\parallel}$ vary with angle in the range $0 < < s$ and $s < < L$, respectively.

According to circuit I, the poloidal current owing on the MC region of the disk is equal to that owing on the MC region of the horizon, i.e.,

$$I_{MC}^{\parallel}(a; n) = J_{MC}^{\parallel} = I_0 \frac{a}{2 \csc^2 (a - q)}; \quad 1 < < s;$$

$$I_0 = \frac{B_4}{2}.$$
Fig. 9. Current densities versus with $a = 0.9$, $l = 0.45$, (a) $j^D_{MC} = j_0$, (b) $j_{DL} = j_0$ for $n = 4, 5, 7$ and 9 in dashed, solid, dotted and dot-dashed lines, respectively.

Therefore the current density owing from the BH magnetosphere into the disk is

$$j^D_{MC} (a ; ; n) = \frac{1}{2} \int_{-1/2}^{1/2} d^D_{MC} ; 1 < s$$  \hspace{1cm} (61)$$

Substituting equation (60) into equation (61), we have the current density owing from the magnetosphere into the MC region of the disk as follows,

$$j^D_{MC} (a ; ; n) = \frac{\lambda a \cdot c (a ; )}{2 \csc^2 (1/4 q)} \int_{-1/2}^{1/2} d^D_{MC} ; 1 < s$$  \hspace{1cm} (62)$$
Fig. 10. Current densities versus $\xi$ with $n = 5.5$, $\lambda = 0.45$, (a) $j_{MC}^0 = j_0$, (b) $j_{DL}^0 = j_0$ for $a = 0.36, 0.62, 0.9$ and $0.998$ in dashed, dotted, solid and dot-dashed lines, respectively.

where

$$C (a ; n) = \frac{q}{4} \frac{1 + 2^4 m_s a^2 + 2^1 m_s^2 a^2 + 2^3 m_s^3 a^2}{m_s^2}$$

(63)

Applying the same procedure to circuit II, we can derive the current owing on the DL region of the disk and the current density owing from the BH magnetosphere into the disk as follows,

$$I_{DL} = \frac{I_0 (1 + q) D (a ; n)}{2^n}; \quad s < < 1$$

(64)
\[ j_L (a; n) = \frac{1}{2} \left( \frac{1}{\sqrt{s}} \right) \frac{dL}{dr} \]

\[ = \frac{j_0 (a; n) D (a; n)}{2} ; \quad s < < 1 ; \]

where \( C (a; ) \) is expressed by equation (63) and \( D (a; ) \) is expressed by

\[ D (a; n) = \frac{p}{n+1} \frac{\left( \frac{1}{2} \frac{a^2}{m_s} \right)^{n+2}}{\left( \frac{3}{2} \frac{a^2}{m_s} \right)^{n+2}} \]

By using equations (58) and (59) we have the current densities \( j_{\text{BZ}} \) and \( j_{\text{MC}} \) varying with the angle on the horizon for \( a = 0.9 \) and the different values of \( n \), and those for \( n = 5.5 \) and the different values of \( a \) as shown in Figures 7 and 8, respectively.

By using equations (62) and (65) we have the current densities \( j_{\text{MC}} \) and \( j_{\text{DL}} \) varying with the radial parameter on the disk for \( a = 0.9 \) and the different values of \( n \), and those for \( n = 5.5 \) and the different values of \( a \) as shown in Figures 9 and 10, respectively.

From Figures 7|10, we find that the distribution features of the above current densities depend on both the power-law index \( n \) and the BH spin \( a \). The current densities \( j_{\text{BZ}} \) and \( j_L \) are always positive, while \( j_{\text{MC}} \) and \( j_{\text{DL}} \) might change their signs in some cases, which correspond to the corotation magnetic surface (CRM S) as defined in W 03b.

6 DISCUSSION

In this paper we discuss and compare the three powers of extracting energy magnetically by considering the restriction of the screw instability to the configuration of the magnetic field in BH magnetosphere. These powers are derived by using circuit I for the BZ and MC processes and circuit II for the DL process. It turns out that the DL power is generally less than the BZ and MC powers, and it is comparable with the two powers for the BH spin close to unity. By using the conservation laws of energy and angular momentum we discuss the radiation flux from a quasi-steady thin disk around a Kerr BH. Although the situation at the boundary between the MC and DL regions is not very transparent, we obtain the solution for the radiation by resolving the equation from the conservation laws. By using circuits I and II we discuss the distribution of the currents and the current densities on the BH horizon and on the disk for the three energy mechanisms.
Recently Li (2000b) presented a model for extracting clean energy from a Kerr BH surrounded by a dense plasma torus, and argued that this model may be relevant to GRBs, provided that the magnetic field is strong enough. Li argued that the baryonic contamination from the plasma in the torus is greatly suppressed by the magnetic confinement. In fact, the problem of the baryonic contamination can also be suppressed by the closed field lines in the MC region in our model, provided that the magnetic field is strong enough. Inspecting equations (12) and (19), and Figure 4, we find that the maximum of the BZ power could be in the following range,

\[ 0.05 P_0 < P_{BZ}^{\text{max}} < 0.1 P_0; \]  \hspace{1cm} (67)

which implies that \( P_{BZ}^{\text{max}} \) could attain \( 10^{51} \text{erg s}^{-1} \) for \( M = 7M_\odot \) and \( B_4 = 10^{11} \). Although \( P_{BZ}^{\text{max}} \) seems much more than needed for powering the beam ed GRBs, we can adjust the value of \( P_{BZ} \) to the beam ed GRBs by taking the adequate values of \( M \) and \( B_4 \) in our model. In addition, the DL process discussed in this paper could be another energy mechanism for extracting clean energy to GRBs. We shall apply this model to GRBs and present a detailed calculation in the future work.

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