An absolute scale for measuring the utility of money

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Abstract. Measurement of the utility of money is essential in the insurance industry, for prioritising public spending schemes and for the evaluation of decisions on protection systems in high-hazard industries. Up to this time, however, there has been no universally agreed measure for the utility of money, with many utility functions being in common use. In this paper, we shall derive a single family of utility functions, which have risk-aversion as the only free parameter. The fact that they return a utility of zero at their low, reference datum, either the utility of no money or of one unit of money, irrespective of the value of risk-aversion used, qualifies them to be regarded as absolute scales for the utility of money. Evidence of validation for the concept will be offered based on inferential measurements of risk-aversion, using diverse measurement data.

1. Introduction to utility
The notion of utility was introduced by Daniel Bernoulli in 1738 to solve a gambling problem (the "St. Petersburg Paradox") where the mathematical expectation of money won gave poor guidance to the decision maker [1]. Used by economists such as Jeremy Bentham in the 19th century, it was put on a firm mathematical foundation in a ground-breaking work by von Neumann and Morgenstern [2], who showed that the decision maker could use probabilities to combine the utilities of uncertain outcomes to produce the expected utility, which could then be used as the basis for rational decision making. Economists took up the idea for public planning purposes in the 20th century, and it is now used widely in order to interpret individual and public preferences [3], [4], [5], [6], [7]. Utility theory is also used extensively in the insurance industry [8]: "The very existence of insurers can be explained by way of the expected utility model" – Kaas et al. [9].

Some economists have seen utility as an ordinal concept only, able to indicate a preference between two options, but give no indication of the strength of the preference. However, a severe restriction arises with an ordinal scale, in that if several decision-makers considering the same problem (e.g. the board of a hospital) have utility functions that are different for each, then no best preference ordering for the group can emerge, other than the adoption of the preference ordering of one of the group – he becomes the dictator. This is Arrow's Impossibility Theorem[10], where dictatorship is precluded by assumption (see also [5] and [6]). Driven by pragmatism, other economists have ascribed increasing degrees of measurability to utility. Taking as their point of departure the ordinal scale just discussed, the scales progress to a cardinal scale (where quantitative differences may be measured), then on to a ratio scale (unique to a scaling multiplier), and finally to a fully unique scale, with the assumptions becoming increasingly restrictive along the way.
Von Neumann and Morgenstern were able to justify on an axiomatic basis utility functions, \( v(x) \), of the form

\[
v(x) = ku(x) + h
\]

(1)

where \( h \) is an arbitrary constant, \( k \) is an arbitrary positive constant and \( u(x) \) is the characteristic utility function (CUF) found by putting \( h = 0 \) and \( k = 1 \). In a very important case, \( x \) is money, either wealth (\( £ \)) or income (\( £/\text{year} \)). By equation (1), differences between utilities of different sums of money, \( x_1 \) and \( x_2 \) will yield:

\[
v(x_1) - v(x_2) = k(u(x_1) - u(x_2))
\]

(2)

From equation (2) the sign of \( v(x_1) - v(x_2) \) will be the same as the sign of \( u(x_1) - u(x_2) \), and so a decision based on selection of the greater utility will be the same whether the CUF, \( u(x) \), is used or the positive linear transformation, \( v(x) \). A typical CUF for money is given in Figure 1.

Interestingly, von Neumann and Morgenstern did not rule out the possibility that further research and insight could produce a more closely defined scale for utility, citing the case of temperature: "The latter[temperature] is also numerical, but in a much more subtle way; it is not additive in any immediate sense, but a rigid numerical scale for it emerged from the study of the concordant behavior of ideal gases, and the role of absolute temperature in connection with the entropy theorem. ... the history of the experience in the theory of heat may repeat itself, and nobody can foretell with what ramifications and variations."

Up to this time, however, there has been no universally agreed measure for the utility of money. For example, Kaas et al. cite five families of "suitable utility functions" with "interesting properties" for use in the insurance industry [9]. But in the spirit of the observation just quoted, we shall combine an important result of Pratt [11] with some insights into the behaviour of risk-aversion gained in investigating the general decision on whether or not to invest in a protection system [12], [13], [14], and go on to derive a single family of utility functions, which will have risk-aversion, \( \varepsilon \), as the only free parameter. Evidence of validation for the concept will be offered using inferential measurements based on the theory developed in [13].

### 2. The form of the CUF when the risk-aversion, \( \varepsilon \), is constant

As argued in [13], the risk-aversion will vary during the decision process, perhaps by very large amounts, as the decision-maker ponders his choices – this is what is happening during the process of...
mulling the problem over. But this operation will occur in the context of the starting assets of the person or the organisation for which he has responsibility [12], [13], and the risk-aversion will be constant during each application of the utility function. That this must be the case can be shown by a simple *reductio ad absurdum* argument. The intelligent agent, making his decision now, will act as if he is governed by his utility function. This implies that the utility function must be lodged in some form in his brain, let us say "known". So, for the risk-aversion in the utility function to change continuously with wealth, information on how his attitude to risk will change with financial circumstances must be available to the agent now. This will necessitate, for example, a very poor man "knowing" the different feelings and reactions he will experience if he is extremely rich, and vice-versa. Moreover, the poor man (or the rich man) will need to possess internal knowledge of how differently he will feel about risk at each of a representative number of intervals in between the two extremes. But such knowledge can come only with the experience of living with the level of wealth specified. Realistically, even if the poor man has been rich previously, he will almost certainly not have experienced the full set of intermediate wealth levels, especially if his loss of wealth was sudden. In any case, he will be unlikely to retain a reliable memory of how he felt when richer. We may deduce that it is impossible for a rational decision-maker to know now how his attitude to risk will vary at different wealths, some or all of which he will not have experienced, with others remembered imperfectly at best. A prescient being might know the appropriate risk-aversions at all different wealths, but any rational, human decision-maker will be left with his present attitude to risk, conditioned on his current wealth, as his only reliable guide. Thus there is insufficient information to allow the utility function associated with any rational human being to be governed by a risk-aversion that varies with wealth. Since utility functions are taken to characterise the judgements of rational humans, it follows that any practical utility function in wealth must have a constant risk-aversion, although that constant might be related to the starting wealth.

Nor is there a case for using discretely different risk-aversions at the different wealths resulting from the decision. Changing risk-aversion mid-decision in contemplation of a change in wealth implies the same prescience as discussed previously. Thus when any comparison of utilities is made, the decision-maker will apply the same risk-aversion and hence the same utility function to the money resulting from each of the possible outcomes. Such a result is, in fact, implicit in equation (2), which compares \( u(x_1) \) and \( u(x_2) \), not \( u_1(x_1) \) and \( u_2(x_2) \).

Successive differentiation of equation (1) shows that equation (3) will be valid when \( v(x) \) replaces \( u(x) \). Moreover

\[
\frac{v''(x)}{v'(x)} = \frac{u''(x)}{u'(x)} = \frac{d}{dx} \ln u'(x) = \frac{d}{dx} \ln v'(x)
\]

Substituting from equation (3) into equation (4) and integrating gives:

\[
v(x) = \int e^{-\int \frac{\varepsilon}{x} dx} dx
\]

Since we have established that we may legitimately regard \( \varepsilon = \text{constant} \), we may write

\[
-\int \frac{\varepsilon}{x} dx = -\int \frac{\varepsilon}{x} dx = -\varepsilon \ln x + \alpha = \ln x^{-\varepsilon} + \alpha
\]

where \( \alpha = \text{constant} \). If follows further that

\[
v(x) = \int e^{\ln x^{-\varepsilon} + \alpha} dx = \int e^{\ln x^{-\varepsilon}} e^{\alpha} dx = e^{\alpha} \int x^{-\varepsilon} dx = \begin{cases} e^{\alpha} x^{1-\varepsilon} + \beta & \text{for } \varepsilon \neq 1 \\ e^{\alpha} \ln x + \gamma & \text{for } \varepsilon = 1 \end{cases}
\]
where $\alpha$, $\beta$, and $\gamma$ are constants. Under the assumption of constant risk-aversion, $\varepsilon$, $1/(1-\varepsilon)$ will be a positive constant when $\varepsilon < 1$, and $-1/(1-\varepsilon)$ will be a positive constant when $\varepsilon > 1$. Hence, removing the constant terms, $\beta$ and $\gamma$, and dividing by the appropriate positive constant gives the CUF, $u(x)$, as:

$$u(x) = \begin{cases} 
  x^{1-\varepsilon} & \text{for } \varepsilon < 1 \\
  \ln x & \text{for } \varepsilon = 1 \\
  -x^{1-\varepsilon} & \text{for } \varepsilon > 1
\end{cases}$$

(8)

which is the result established by Pratt [11] and provides the theoretical justification for the Power Utility function, as well as the Logarithmic Utility function.

It has been assumed so far that the utility function will be twice continuously differentiable, with a positive first derivative and with a constant risk-aversion. However, the concept of mulling the problem over by varying the risk-aversion imposes a further condition on the utility function, namely that it is should be continuous and regular in risk-aversion. Equation (8) conforms to this requirement only when $\varepsilon < 1$, as may be seen from Figure 2. But clues on how to achieve the required function are given in equations (7) and (8). We may first note that $\ln x = \ln x - \ln 1$, implying that the central term in equation (8) may be seen as the difference between the utility of wealth, $x$, and the utility of one unit of money. Transferring the same datum to the other two terms in equation (8) gives:

$$u(x) = \begin{cases} 
  x^{1-\varepsilon} - 1^{1-\varepsilon} = x^{1-\varepsilon} - 1 & \text{for } \varepsilon < 1 \\
  \ln x - \ln 1 = \ln x & \text{for } \varepsilon = 1 \\
  -x^{1-\varepsilon} - (-1)^{1-\varepsilon} = 1 - x^{1-\varepsilon} & \text{for } \varepsilon > 1
\end{cases}$$

(9)

The final step is to add the positive scaling factors evident from equation (7) (which had previously been removed), namely $1/(1-\varepsilon)$ when $\varepsilon < 1$, and $-1/(1-\varepsilon)$ when $\varepsilon > 1$. This allows:

$$u(x) = \begin{cases} 
  \frac{x^{1-\varepsilon} - 1}{1 - \varepsilon} & \text{for } \varepsilon \neq 1 \\
  \ln x & \text{for } \varepsilon = 1
\end{cases}$$

(10)

a utility function, which, following Cowell and Gardner [15], we have attributed to Atkinson [16], although its basic formulation was obviously apparent earlier to Pratt [11]. The limiting behaviour of $u(x)$ when $\varepsilon \neq 1$ but $\varepsilon \to 1$ is given by l'Hôpital's rule:

$$\lim_{\varepsilon \to 1} \frac{d}{d\varepsilon} \left(\frac{x^{1-\varepsilon} - 1}{1 - \varepsilon}\right) = \lim_{\varepsilon \to 1} \frac{\frac{d}{d\varepsilon} (x^{1-\varepsilon} - 1)}{\frac{d}{d\varepsilon} (1 - \varepsilon)} = \lim_{\varepsilon \to 1} \frac{-x^{1-\varepsilon} \ln x}{-1} = \ln x \lim_{\varepsilon \to 1} (x^{1-\varepsilon}) = \ln x$$

(11)

This desirable behaviour is confirmed by Figure 3, which also demonstrates the regularity of the Atkinson Utility function over the wide range of $x$ and for $\varepsilon \geq 1$ as well as $\varepsilon < 1$. It is proposed that, since it assumes a risk-aversion that is constant at the point of decision, and, moreover is smooth and regular in risk-aversion as it increases past unity, the Atkinson Utility function should be adopted as the utility function of choice for application to money. Moreover, harking back to von Neumann and Morgenstern, the fact that the Atkinson Utility function is measured from the datum of the utility of one unit of money means that it provides a ratio scale for the utility of money: it is correct to a positive
multiplier, in the same way as absolute temperature, which may be measured in Kelvin or Rankine. Coincidentally, a key, dimensionless parameter used in decision making is the reluctance to invest, \( R_{120A} \) \[12\]. This has a similar functional form to the dimensionless thermodynamic efficiency, \( \eta = (T_1 - T_2)/T_1 \), where \( T_1 \) and \( T_2 \) are absolute temperatures:

\[
R_{120A}(\varepsilon) = \frac{E(u_1|\varepsilon) - E(u_2|\varepsilon)}{u_0(\varepsilon)}
\]

in which \( u_0(\varepsilon) \) is the starting utility at a given risk-aversion, \( \varepsilon \), while \( E(u_1|\varepsilon) \) and \( E(u_2|\varepsilon) \) are the expected utilities arising from choices 1 and 2, measured using the same risk-aversion, with all utilities of money measured using the Atkinson Utility function.

While the scales of utility based on the Atkinson Utility function will vary with risk-aversion, \( \varepsilon \), the fact that all such scales will return a utility of zero when applied to one unit of money suggests that they may be regarded as absolute scales.

This property is shared by the Power Utility, provided the risk-aversion is strictly less than 1.0. The Power Utility returns a utility of zero when there is no money, for all risk-aversions strictly less than 1.0. In fact, inspection of Figures 2 and 3 shows that the Power Utility function is geometrically similar to the Atkinson Utility function when \( 1 < \varepsilon \), indicating that the two functions contain similar information. The Power Utility also possesses the desirable property of being equivalent to a utility difference, since \( u(x) = x^{1-\varepsilon} - 0^{1-\varepsilon} = u(x) - u(0) \). The Power Utility may therefore be regarded as an equally good choice of utility function when it is known that \( \varepsilon < 1 \).

3. Zero utility and the quantum of wealth

Let us assume that money is expressed in a general currency, which we will denote by a capital "Q" crossed by a bar, \( \overline{Q} \), following the convention of actual currencies such as £, $, € and ¥. Using the Atkinson absolute utility scale, the utility of any quantity of assets lying in the range \( 0 < x < 1\overline{Q} \) will be negative, while the utility will be undefined for \( x \leq 0 \overline{Q} \). But there is a case for regarding the Atkinson Utility function as being undefined for any monetary value below \( 1\overline{Q} \). Such a stance would be justified if money were regarded as consisting of discrete packages of \( 1\overline{Q} \) each. This would, in turn, imply the existence of a quantum of wealth, the addition of less than which would be regarded by the average adult as giving him or her no discernable increase in wealth.

In support of such a premise is the fact that money has always been discrete. For example, the penny (£0.01) is the lowest denomination coin in the UK, and a difference in wealth less than £0.01 will not register in a UK bank account. Similarly in Europe, €0.01 is the smallest change in wealth that will register, and in the USA, $0.01. So while the dividend of a commercial company might be
declared as, for example, 5.444 pence, this is only to ease calculation of the dividend on large numbers of shares. The cheque sent out to the holder of ten shares would be for £0.54, not for £0.5444. The result is that utility of money and hence the utility of any asset must be of a similarly discrete nature. There will be discrete steps in utility in consequence, even if the steps are small enough to allow the discrete variable to be approximated by a continuous variable.

It would be possible to adopt one penny as the quantum of wealth, and it would, of course, be very easy to translate all monetary sums expressed in pounds into corresponding sums in pennies before carrying out utility calculations. But it is arguable that the smallest quantum of money giving rise to a change in utility will be rather larger than one penny. Thomas et al. reviewed evidence for a decision-maker with assets, \( A \), being unable to discriminate a change in assets of \( 5 \times 10^{-6} A \) or less \([12]\). This translated into an equivalent figure for the average UK adult of about £0.90, based on average assets of about £180,000. Hence about £1 might be regarded as the quantum of wealth in the UK. This would coincide with the minimum amount below which UK taxation authorities declare no interest. By allowing tax payers to round down their personal income to the nearest whole pound, H. M. Revenues and Customs choose to disregard for individual taxation purposes any addition of income less than or equal to £0.99 \( \approx \) £1.00.

A more international perspective is offered by the lowest denomination below which Governments or Central Banks cease issuing bank notes. Because coins are bulky and their storage is inconvenient, this may be interpreted as a demarcation point between wealth, as measured by "folding money", and "loose change", as measured by coins. In the UK, the lowest denomination note is £5, while the lowest denomination Euro bank note is €5, equivalent to about £4.50. Moreover, while the Japanese Yen is a very small unit, similar to a penny and worth only \( \sim 1/150 \)th of a pound, the smallest-denomination, Japanese bank note is ¥1,000, equivalent to about £6.66. Repeated attempts to introduce a coin to replace the dollar bill in the USA have not met with success, and $1 persists as the lowest denomination bank note in the USA, equivalent to about £0.61. Denoting as \( 1Q \) the quantum of money in developed nations that can have an effect on utility, we may deduce from a consideration of the transition from coins to bank notes for the USA, Europe, UK and Japan, the proposition that

\[
0.61 \leq 1Q \leq 6.66
\]  

It is clear from condition (13) that there is a strong similarity in major developed nations for the quantum of money thought to produce a recognisable change in personal wealth and hence utility, based on the transition from coinage to paper notes. Since £1 is an intermediate point in condition (13), it is a candidate for the quantum of wealth: \( 1Q = £1.00 \), would promote ease of use. However, further research might establish a better and possibly larger approximation to \( 1Q \). In all cases, it will be desirable to normalise all assets into \( Q \) sums before using in the utility function. This will avoid the small differences in evaluating the dimensionless reluctance to invest that are possible if the denomination of the assets is changed from pounds to dollars, for example. See \([17]\) for a discussion of this effect.

The quantum of money, \( 1Q \), as derived above, could be applicable throughout the developed world, giving a common basis for the valuation of assets in utility calculations.

### 4. Inferential measurement of the average risk-aversion of the average UK adult

An inferential measurement of risk-aversion for the UK in the context of valuing life expectancy may be deduced using actuarial statistics, Gross Domestic Product (GDP) per head, wages as a fraction of GDP and the expected free-time fraction from now on. The figure inferred is \( \varepsilon = 0.82 \) \([18], [19]\), which is close to a number of previous estimates. For example, Cowell and Gardiner \([15]\) suggested that the value of \( \varepsilon \) should be just above or just below 1.0, a conclusion endorsed by H. M. Treasury \([7]\). Blundell et al. recommended \([20]\) \( \varepsilon = 0.83 \). Pearce and Ulph \([21]\) suggested \( 0.8 \leq \varepsilon \leq 0.9 \), while Layard et al. \([22]\) calculated \( \varepsilon = 1.24 \), based on happiness surveys.
In an entirely diverse approach, the theory of personal protection and insurance developed in [13], using Atkinson Utility functions shows that risk-aversion will vary depending on: the individual's assets, the size of the potential loss, the cost of insurance or protection against it and the loss probabilities with and without protection or insurance. It may be shown [13] that an average figure for risk-aversion at the point of investment in a protection system or insurance policy, $\varepsilon_{ppave}$, for the average UK adult can be inferred simply from a measurement of the assets owned by the average UK adult, about £180,000. Applying absolute Atkinson Utility scales and performing the necessary calculations produces a figure of $\varepsilon_{ppave} = 0.85$, with an estimated range of $0.74 \leq \varepsilon_{ppave} \leq 1.13$. The central value of risk-aversion is an average over all potential losses with a probability of occurrence, $\pi_1$, of 0.05 and less, and with the lower, limiting value of the reluctance to invest, $R_{120.4}$, set at $5 \times 10^{-6}$. Reducing the top probability considered, $\pi_1$, has little effect above about $2 \times 10^{-5}$, as shown in Figure 4. This diverse estimate, $\varepsilon_{ppave} = 0.85$, derived using the Atkinson scale of utility, is within 4% of the previous estimate of $\varepsilon = 0.82$, providing useful validation for the approach.

![Figure 4. Average value of the permission point, \varepsilon_{ppave}, for UK adult, averaging over all accidents with probability less than \pi_1; discrimination limit on reluctance to invest set at 5 x 10^{-6} : R_{120.4} \leq 5 \times 10^{-6}](image)

5. Conclusions
Measuring the utility of money is a topic of great importance to the insurance industry and to economists concerned with prioritising public spending schemes. It is also important in deciding how much to spend on safety and protection systems across all industries, particularly those with high hazards. People have worked with several types of utility function up to now, but it is argued here that a single family of utility functions is the most suitable, namely those based on the assumption that, at the point of comparison, the utility function used to evaluate wealth outcomes will be based on a constant risk-aversion, which is the only free parameter. Members of this family, the Power Utility function and the Atkinson Utility function, may be interpreted as utility differences, referred, respectively, to the datum of the utility of no money or the datum of the utility of one monetary unit. Both measurement scales will return a utility of zero at the datum, irrespective of the value of risk-aversion, which justifies denoting them as absolute scales. The Power Utility function is smooth and regular in risk-aversion only below a risk-aversion of unity, while the Atkinson Utility function continues this regularity through risk-aversions of unity and much higher. Thus the Atkinson Utility function has a claim to greater generality.

The Atkinson Utility function prompts the thought that there might a minimum quantum of wealth. This could be set at one penny, but it is suggested that a higher value might be more suitable, in the region of £1 to £5, the exact value being open to further research. It is suggested that consistency will be enhanced if all assets are converted to multiples of this quantum before applying the utility function.
The average value for risk-aversion for the average UK adult was measured inferentially using two, diverse methods, operating on diverse data. The answers agree to within 4%, providing a useful degree of validation for the ideas advanced in the paper.

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