Efficient multi-objective shape optimization using proper orthogonal decomposition with variable fidelity concept

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Abstract

A variable fidelity concept is introduced in a re-parameterization approach based on the proper orthogonal decomposition (POD) to efficiently solve multi-objective aerodynamic shape optimization problems. The re-parameterization approach enables to extract dominant shape deformation modes from a database of good designs and to reduce the number of design variables. The present variable fidelity approach is proposed by utilizing low-fidelity functional evaluations to select the good designs. The proposed approach is investigated in two multi-objective aerodynamic shape optimization problems of 2D airfoil in which the combinations of viscous/inviscid simulations or fine/coarse grid simulations are treated as the high/low-fidelity evaluation methods. It can be confirmed that dominant POD modes obtained from low-fidelity evaluations are qualitatively equivalent with that obtained from high-fidelity evaluations. Non-dominated solutions obtained from a conventional optimization approach can be reproduced with smaller numbers of design variables using the dominant POD modes. The computational costs to solve the multi-objective aerodynamic shape optimization problems can be dramatically reduced by introducing the variable fidelity concept.

Keywords: Variable fidelity approach, Proper orthogonal decomposition, Multi-objective aerodynamic shape optimization, Kriging model, Computational fluid dynamics

1. Introduction

Recently, aerodynamic shape optimization has attracted much attention with the maturity of computational fluid dynamics (CFD) and progress of computer performance. It has been widely utilized in academia as well as industries for the design of numerous aerospace products, such as turbomachineries, airplanes and so on. Multi-objective aerodynamic shape optimizations have also become common in recent years, that can obtain non-dominated (Pareto) optimal designs that clarify tradeoff relationship between plural objective functions.

Geometry parameterization is one of the most crucial part for efficient shape optimization and usually impacts to the efficiency of the optimization. Therefore, many geometry parameterization methods have been proposed such as the Hicks-Henne function (Hicks and Henne, 1978), Bezier curve (Desideri et al., 2007), parametric section airfoil parameterization (Sobieczky, 1998), class/shape function transformation method (Kulfan, 2008), free form deformation (FFD) (Samareh, 2004; Yamazaki et al., 2010), radial basis function (RBF) approach (Rendall and Allen, 2008; Yamazaki et al., 2010), proper orthogonal decomposition (POD) approach (Toal et al., 2008; Poole, et al., 2015; Li et al., 2018) and so on. In addition, the effect of the geometry parameterization methods on the results of airfoil shape fitting / aerodynamic shape optimization problems has been investigated religiously (Samareh, 2001; Masters et al., 2017a; Masters et al., 2017b). According to the reference of Masters et al., 2017b, it was concluded that the POD approach gave the most efficient coverage of airfoil design space.

POD is also known as the Karhunen–Loève expansion which can extract dominant modes from a large scale database. It has been applied to an inverse design (Bui-thanh, et al., 2004), aerodynamic data reconstruction (Bui-thanh, et al., 2004), reconstruction of flowfield measurement data (Druault et al., 2005; Gunes et al., 2006; Yamazaki et al., 2014), quantitative comparison of two spatial/temporal data (Andrianne et al., 2012), airfoil shape parameterization
(Toal et al., 2008; Poole et al., 2015; Masters et al., 2017a; Masters et al., 2017b; Li et al., 2018) and so on. By using POD, a set of ranked POD modes can be obtained from the large scale input database and its first POD mode corresponds to the most dominant mode to express the large scale input database. The obtained dominant POD modes are useful to extract additional information from the input database. In the reference of Toal et al., 2008, POD was utilized for the re-parameterization of an original optimization problem. In this case, a number of good designs obtained from a (first stage) conventional optimization approach was used as the input database for POD. Then, extracted dominant POD modes, expressing the dominant geometry deformation for the good designs, were used in the subsequent (second stage) optimization in which coefficients of the POD modes were treated as design variables. In the reference of Poole et al., 2015, the input database for POD was constructed from existing airfoil database with the criteria of the airfoil technology factor. In the references of Masters et al., 2017a, Masters et al., 2017b, Li et al., 2018, existing airfoil shapes on the airfoil database of University of Illinois Urbana-Champaign were used as the input database. All approaches enabled the reduction of the number of design variables with maintaining the degree of freedom to express optimal airfoil shapes.

Thus, the POD-based re-parameterization approach is promising for efficient aerodynamic shape optimization while it requires a certain amount of computational cost to construct appropriate input database for POD. In this research, a variable fidelity (VF) concept, which is to utilize multi fidelity functional evaluation methods in an optimization system (Kennedy and O'Hagan, 2000; Alexandrov et al., 2001; Han and Görtz, 2012; Yamazaki and Mavriplis, 2013; Yamazaki, 2017), is introduced in the POD-based approach to further reduce the computational cost of multi-objective aerodynamic shape optimizations. So far, the VF optimization approaches were mostly proposed with advanced VF response surface methods and/or sequential approximate optimization systems. The present research can be another VF optimization approach in which the concept of VF is introduced in the geometry parameterization process. This paper is organized as follows. The computational methodologies utilized in this research are concisely described in section 2. In section 3, the proposed approach for efficient multi-objective shape optimization is explained. In sections 4 and 5, results of two multi-objective aerodynamic shape optimization problems are discussed. Finally, we provide concluding remarks and future prospects in section 6.

2. Computational methods

In this research, compressible Reynolds-averaged Navier-Stokes (RANS) simulations and inviscid Euler simulations are performed using FaSTAR (FAST Aerodynamic Routines) code (Hashimoto et al., 2012) which has been developed by Japan Aerospace Exploration Agency. The Spalart-Allmaras turbulent model (Spalart and Allmaras, 1992) without \(f_{\alpha} \) term and with rotation correction (Dacles-Mariani et al., 1995) is utilized in the RANS simulations. Governing equations are discretized by a finite volume scheme. The numbers of grid points are about 40,000 in both RANS and inviscid simulations, that are visualized in Fig. 1. The computational results of the RAE2822 airfoil at the freestream Mach number (\(M_{\infty} \)) of 0.729, angle of attack (AoA) of 2.31 degrees and Reynolds number (\(Re \)) of 6.5 million are compared with experimental data (NPARC Alliance Validation Archive) in Fig.1 which indicates the RANS computation shows better agreement with the experimental data.

With respect to the optimization approach, a surrogate model-based global design optimization method is utilized in this research which makes use of an ordinary Kriging surrogate model (Jones et al., 1998; Yamazaki and Mavriplis, 2013). For the Kriging model, the Wendland’s C4 RBF (Becket and Wendland, 2001) is adopted as the correlation function, whose hyper-parameters are optimized using a real-coded genetic algorithm (GA) (Fonseca and Fleming, 1993) in this research. Firstly, initial sample points are generated in the design variable space using a Latin hypercube sampling (LHS) approach, and then the performance values on the initial points are evaluated. After the construction of the Kriging model, the search of a promising location in the design variable space is also executed by the real-coded GA on the surrogate model. The promising locations are explored by the criteria of expected improvement (EI) (Jones et al., 1998) as well as expected hyper-volume indicator (EHVI) (Hupkens et al., 2015) in this research. The function of EI expresses a potential for improvement in design variable space with respect to an objective function. The function of EHVI expresses a potential for improvement of hyper-volume indicator in design variable space, which is appropriate for multi-objective optimization problems. In this research, additional performance evaluations are performed for the explored promising locations (configurations) where EI is maximal for each objective function, and where EHVI is maximal. In multi-objective optimization problems with two objective functions, three (two from EI
and one from EHVI) additional performance evaluations are required at each iteration, and then new surrogate model is created by adding the information of the three additional samples. By the iterative process described above, the accuracy of the surrogate model is efficiently increased around the promising locations in the design variable space. The flowchart of this (conventional) optimization approach is shown in Fig.2.

In this research, aerodynamic shape optimization problems are solved not only by the proposed approach but also by the conventional approach for comparison purposes. In the conventional approach, the FFD approach with the Bezier blending function (Yamazaki et al., 2010) is adopted as its parameterization. An initial airfoil shape (RAE2822) is enclosed by the control box of 8×6 points, and then the vertical displacements of inner control points (6×4) are considered as design variables as shown in Fig.2.

Fig. 1 Inviscid/RANS simulations around RAE2822 airfoil. a) Computational grid for inviscid simulation. b) Computational grid for RANS simulation. c) Comparison of pressure coefficient (Cp) distributions.

![Flowchart of conventional optimization process using Kriging model.](image)

![Free-form deformation of RAE2822 airfoil via 24 design variables.](image)

Fig. 2 Introduction of conventional optimization approach. Left: Flowchart of conventional optimization process using Kriging model. Right: Free-form deformation of RAE2822 airfoil via 24 design variables.
3. Proposed optimization method

3.1 POD-based re-parameterization

The POD-based re-parameterization approach utilized in this research is almost same as that proposed in references of Toal et al., 2008, Poole, et al., 2015, Li et al., 2018. Here, the approach is briefly explained based on an airfoil shape parameterization while it can be applied to the parameterization of arbitrary objects. With respect to an airfoil shape, its vertical coordinates $y_i$ ($i = 1, \cdots, n$) are considered as a vector $y = [y_1 \ y_2 \ \cdots \ y_n]^T$ where $n$ is the number of points on the airfoil (horizontal coordinates of all points are assumed to be fixed). Then the database matrix of good airfoil designs is expressed as follows:

$$Y = [y_1 \ y_2 \ \cdots \ y_m] \quad (1)$$

where $m$ is the number of good airfoil designs. The arithmetic mean of the good airfoil designs is defined as $y_{ave}$ and then the database matrix can be rewritten as

$$Y = [y_{ave} \ y_{ave} \ \cdots \ y_{ave}] + [\Delta y_1 \ \Delta y_2 \ \cdots \ \Delta y_m] = Y_{ave} + \Delta Y \quad (2)$$

In the snapshot POD approach, the POD modes can be obtained by solving the eigenvalue problem of

$$R\phi_i = \lambda_i\phi_i \quad (i = 1, \cdots, m) \quad (3)$$

where $R$ is the covariance matrix defined as $R = \Delta Y^T\Delta Y$. $\lambda_i$ and $\phi_i$ are respectively i-th eigenvalue and eigenvector. The order of the eigenvalues/vectors is defined from:

$$\lambda_1 > \lambda_2 > \cdots > \lambda_m > 0 \quad (4)$$

Here, the eigenvalue expresses the energy of each POD mode. The POD modes $\Phi_i$ and POD coefficients $\xi_i$ can be respectively obtained as:

$$\Phi_i = \Delta Y\phi_i/\sqrt{\lambda_i} \quad (5)$$

$$\xi_i = \Delta Y^T\Phi_i \quad (6)$$

When all POD modes and POD coefficients are obtained, the original database matrix can be reconstructed as:

$$\Delta Y = \sum_{i=1}^{m}\Phi_i\xi_i^T \quad (7)$$

Usually, several dominant POD modes (that have a dominant amount of the total energy) are approximately enough to reconstruct the original database matrix as

$$\Delta Y \cong \sum_{i=1}^{m'}\Phi_i\xi_i^T \quad (8)$$

where $m' (\leq m)$ is the number of considerable POD modes. The concept of this re-parameterization approach is to extract dominant shape deformation modes from the database of good airfoil designs, and then to reduce the number of design variables by utilizing only a small number of POD modes. When one decides to use $m'$ POD modes for the re-parameterization, a new airfoil shape $y_{new}$ can be defined as

$$y_{new} = y_{ave} + \sum_{i=1}^{m'}\Phi_i\xi_i \quad (9)$$

where $\xi_i$ are the POD coefficients to define the new airfoil shape and these are the design variables for this re-parameterization. The reduced number of design variables becomes $m'$ in this approach. With respect to the range of each design variable, it is defined from the range of POD coefficients to express the good airfoil designs. $\xi_i$ of
Eq. (6) corresponds to the i-th POD coefficient values to express the good airfoil designs, so that the range of each design variable is defined from maximum/minimum values of the correspondent POD coefficient in this research.

### 3.2 How to select good designs for POD

In this POD-based approach, the selection of the good designs to construct the database matrix is very important and requires a certain amount of computational cost. By using existing airfoil shapes as references of Masters et al., 2017a, Masters et al., 2017b, Li et al., 2018, the cost can be reduced while it will be difficult to apply to arbitrary design conditions since it is questionable whether the existing airfoils can be the good designs or not in the design conditions. In addition, there is no such many existing reference shapes for arbitrary configurations other than the airfoil. Although airfoil shape optimization problems are only discussed in this paper, the present optimization approach is proposed to be utilisable for shape optimization problems of arbitrary configurations.

In this research, we consider to select the set of the good designs utilizing functional predictions via the Kriging model. In a multi-objective optimization problem, the good designs in a precise sense should be Pareto optimal designs that can be obtained by exactly solving the multi-objective optimization problem. There is a conflict that one has to solve the optimization problem for the application of the POD-based approach. Therefore, we consider to utilize approximate non-dominated optimal designs obtained using the Kriging model prediction as the alternate of the good designs. In this case, initial sample points are generated by LHS using a conventional parameterization approach (in this research via FFD) and then the performance of the initial sample points is evaluated. Next, the Kriging models of performance functions are constructed from the information of the initial sample points. The multi-objective optimization problem is solved just once using the functional prediction of the Kriging models, which yields the set of approximate non-dominated optimal designs. This problem is also solved using the real-coded GA (population size of 200, number of generations of 200) since the computational cost of the Kriging model functional prediction is not expensive. The major computational cost for this pre-process is to evaluate the initial sample points (that can be executed in parallel), and then the approximate non-dominated optimal designs can be utilized to construct the database matrix for the POD-based approach. The flowchart of this approach is shown in Fig. 3. Its procedure at the latter half (main optimization part) is almost the same as that of the conventional approach (Fig. 2) while the POD-based re-parameterization approach enables to reduce the number of design variables.

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![Flowchart of proposed optimization process with POD-based re-parameterization. Left: Single fidelity case, Right: Variable fidelity case. Gray boxes indicate important points of the proposed optimization process.](image-url)
3.3 Introduction of variable fidelity concept

The variable fidelity concept is introduced for further reduction of the computational cost of the pre-process. In the proposed approach, the absolute accuracy is not required in the pre-process to obtain POD modes. It is enough to obtain only the trends of non-dominated optimal designs qualitatively. Therefore, the variable fidelity concept is introduced by replacing the performance functional evaluations required in the pre-process with its low-fidelity functional evaluations. The flowchart of the proposed variable fidelity approach is also shown in Fig.3. The proposed approach introduces the VF concept in its geometry parameterization process.

4. Validity of proposed approach

In this section, the proposed optimization approach is applied and discussed in a multi-objective aerodynamic shape optimization problem for 2D airfoil.

4.1 Definition of optimization problem

In this optimization problem, the maximization of lift coefficient ($C_l$) as well as the minimization of drag coefficient ($C_d$) are considered for 2D airfoil at $M_a$ of 0.729, AoA of 2.31 degrees and $Re$ of 6.5 million. A constraint is given on the sectional area, which has to be larger than that of the RAE2822 airfoil. To treat the constraint, the Kriging model to predict the value of the sectional area is also iteratively constructed in the optimization process. In the search process using the GA, the predicted value of the Kriging model is used to judge whether a new individual is feasible or not.

In this problem, RANS and inviscid CFD simulations are respectively utilized as the high and low-fidelity evaluations. In the low-fidelity case, the skin friction drag coefficient ($C_{df}$) is estimated from the following simple algebraic model (Raymer, 2006):

$$C_{df} = \frac{0.455}{(\log_{10} Re)^{2.58}} (1 + 0.144 M_a^2)^{-0.65} \frac{S_{wet}}{S_{ref}}$$

(10)

where $S_{ref}$ and $S_{wet}$ are respectively the reference area and wetted area of airfoil. One of the objective functions $C_d$ is defined as the sum of the estimated $C_{df}$ and the pressure drag coefficient obtained from the inviscid CFD simulation in the low-fidelity case.

For comparison purposes, the present optimization problem is solved by the conventional optimization approach with the FFD parameterization in which 24 design variables (DVs) are considered. 100 initial sample points are generated and then 99 sample points are added to obtain non-dominated optimal designs. The performance of the obtained optimal designs is shown in Fig.4 which indicates the tradeoff relationship between $C_l$ and $C_d$. With respect to the plot of “Optimals (Inviscid, FFD, 24DVs, Evaluated by RANS)”, its optimization was performed with inviscid evaluations, and then RANS evaluations were performed for the obtained optimal designs. It can be confirmed that the performance of the optimal designs using inviscid evaluations is inferior to that using RANS simulations.

4.2 Comparison of dominant POD modes

For the POD-based approach, 100 initial sample points are generated in the 24 dimensional design variable space and then those performance is evaluated. Kriging models are constructed from the performance of the 100 samples to obtain approximate Pareto optimal designs. The obtained approximate Pareto optimal designs in both fidelity cases are also shown in Fig.4. The locations of Pareto front are different from that obtained by the conventional optimization approach due to the inaccuracies of the Kriging models constructed from the 100 initial samples. The correlations between the high and low-fidelity evaluations of the 100 initial samples are shown in Fig.5 in which the correlation coefficients of $C_l$ and $C_d$ are 0.97 and 0.93, respectively.
Fig. 4 Pareto optimal designs obtained by conventional FFD parameterization which indicates the tradeoff relationship between objective functions. POD modes are extracted from approximate Pareto optimal designs in proposed approach.

The POD-based re-parameterization is applied to the sets of the approximate Pareto optimal designs shown in Fig.4 that obtained with high or low-fidelity evaluations. Obtained averaged shapes as well as dominant POD modes are compared in Fig.6, in which the effects on the upper surface are indicated by dashed lines. With respect to the averaged shapes and first/second POD modes, qualitative agreements can be observed between the high and low-fidelity results. Since the upper/lower distributions of the first POD mode are almost same, this means that the major effect of the first POD mode is to deform the camber line of the airfoil especially at the rear side of the airfoil. The major effect of the second mode is, on the other hand, to deform the camber line at the front side of the airfoil. For more quantitative comparison, the modal assurance criterion (MAC) (Andrianne et al., 2012) is evaluated for the dominant POD modes. MAC is defined as follows:

\[
MAC_{ij} = \frac{\langle \Phi_i^T \Phi_j \rangle^2}{\langle \Phi_i^T \Phi_i \rangle \langle \Phi_j^T \Phi_j \rangle}
\]  

(11)

MAC becomes one when the two mode vectors are similar while zero when the two mode vectors are orthogonal. The results of the MAC evaluation are summarized in Table 1. We can observe higher degree of agreement in the first and second modes between the high and low-fidelity cases. Although the degree of agreement becomes lower in the higher POD modes due to the fidelity difference, the third (fourth) mode of the high-fidelity case shows a certain level of agreement with the fourth (third) mode of the low-fidelity case. Since the upper/lower distributions of the third/fourth POD modes are largely different, this means that the major effect of these POD modes is to deform the thickness distribution of the airfoil. In Fig.6, the cumulated energy distributions of the dominant POD modes are also indicated. This shows that 99% of total energy is included in the four dominant POD modes. Although the order of the third/fourth POD modes is changed, this MAC analysis indicates that the dominant POD modes obtained from the low-fidelity evaluation are approximately equivalent to that obtained from the high-fidelity evaluation.

| High, RANS | Low, Inviscid |
|------------|---------------|
| \(\Phi_1\)  | 0.83          | 0.09       | 0.04       | 0.02       | 0.01       |
| \(\Phi_2\)  | 0.06          | 0.79       | 0.01       | 0.04       | 0.00       |
| \(\Phi_3\)  | 0.06          | 0.08       | 0.00       | 0.64       | 0.06       |
| \(\Phi_4\)  | 0.02          | 0.00       | 0.50       | 0.04       | 0.23       |
| \(\Phi_5\)  | 0.00          | 0.03       | 0.02       | 0.16       | 0.08       |
Fig. 5 Correlations between objective function values evaluated by high and low-fidelity models. Left: lift coefficient, Right: drag coefficient. High correlations can be observed between them.

Fig. 6 Comparison of averaged shapes and dominant POD modes. Dominant POD modes obtained from low-fidelity (inviscid) evaluation are approximately equivalent to that obtained from high-fidelity (RANS) evaluation.

4.3 Optimization results

Firstly, only the first POD modes are considered and examined. Since there is only one design variable in these cases, parametric studies for the first POD coefficient ($d_1$ of Eq.(9)) are performed and the results are shown in Fig.7-a. When the POD mode obtained from the high-fidelity (RANS) evaluation is used, it can be considered as a single fidelity (SF) case, so that it is referred to as POD(SF) hereafter. The performance of the averaged shapes (shown in Fig.6) is also indicated in Fig.7-a. It is interesting that the result of POD(SF) only using the first POD mode is qualitatively equivalent with that obtained by the conventional optimization using FFD. This means that only one design variable with the first POD mode has enough degree of freedom to express the Pareto front obtained using 24
design variables of FFD. Although the result obtained by POD(VF) only using the first POD mode is inferior than the others, the trend of the Pareto front is almost reproduced by one design variable. The inferiority of the POD(VF) result is due to the worse performance of the VF averaged shape as understood from Fig.7-a. In this case, $C_l$ and $C_d$ were decreased with larger $d_1$ as indicated in Fig.7-a. Representative airfoil shapes expressed using the first POD modes (A, B, A’, and B’ of Fig.7-a) are compared in Fig.8. Since the major effect of the first POD modes is to deform the camber line downward at the rear side of the airfoil, the positive camber around the trailing edge is strengthened with smaller (negative values of) $d_1$ to prevent from the generation of shock waves. This can be considered as the most influential geometry deformation factor in this optimization problem.

Next, results obtained using the first and second modes are shown in Fig.7-b. In cases with plural design variables, the optimization process shown in Fig.3 is utilized in which 20 initial and 15 additional sample points are evaluated in its main optimization part. The obtained Pareto front by the POD(VF) approach becomes close to the others. Results using more POD modes are also shown in Fig.7-c/d. The geometries / flow fields of representative optimal designs specified in Fig.7-d are compared in Fig.9. In the optimal designs with larger $C_l$, larger shock waves are generated on the upper surfaces. In the optimal designs with smaller $C_d$, the strength of the shock waves becomes weaker to reduce the wave drag. These trends of aerodynamics as well as trends of airfoil shape are equivalent between the conventional optimization (FFD) and POD(VF) approach. Finally, the computational costs of the representative cases are summarized in Fig.10 in which the computational cost of a high-fidelity (RANS) evaluation is set to one unit while that of a low-fidelity (inviscid) evaluation is set to 0.1 considering actual computational times. Computational costs other than the CFD evaluations are negligible and omitted. The computational cost of the proposed approach is about 23% of the conventional FFD approach while the obtained non-dominated solutions are qualitatively equivalent. This massive reduction of computational cost is due to the reduction of number of design variables as well as the reduction of cost to construct the input database for the POD-based re-parameterization.

![Fig. 7 Comparison of Pareto fronts between conventional (FFD) approach and proposed (POD) approach. a), b), c) and d) indicate results using 1, 2, 3 and 5 dominant POD modes, respectively. Obtained results with the proposed approach show qualitative agreement with that of the conventional approach.](image-url)
Fig. 8 Comparison of representative airfoil shapes expressed using first POD modes. Left: SF case. Right: VF case. The major effect of first POD modes is to deform the camber line at the rear side of the airfoil.

Fig. 9 Comparison of Pareto optimal designs. Left: Airfoil geometries. Right: Mach number distributions. Trends are comparable between the conventional and proposed approaches.

Fig. 10 Comparison of computational costs of representative cases. Computational cost was reduced by 77% in the proposed POD(VF) approach.

5. Application in multi-points optimization problem

5.1 Definition of optimization problem

In this optimization problem, the maximization of lift to drag ratio ($L/D$) values at two freestream Mach number conditions is considered. The first condition is same as that of the previous section, $M_\infty$ of 0.729, AoA of 2.31 degrees and $Re$ of 6.5 million. In the second condition, $M_\infty$ is only changed from 0.729 to 0.80. The given constraint for the sectional airfoil area is same as that of the previous section and its treatment in the optimization process is also same as that of the previous section. For comparison purposes, the present optimization problem is also solved by the conventional approach with RANS simulation and FFD parameterization in which 24 design variables are considered. 100 initial sample points are generated and then 99 sample points are added to obtain non-dominated optimal designs in the conventional optimization approach.
In this problem, RANS simulations on fine / coarse computational grids are considered as the high / low-fidelity evaluations for the VF approach. The number of points of the coarse grid is about 5000 as shown in Fig.11, whose computational cost is about 15% of the high-fidelity (fine grid) computation. The correlations between the high and low-fidelity evaluations of 100 initial sample points are shown in Fig.12 in which the correlation coefficients of $L/D$ at $M_{\infty}$ of 0.729 and of 0.80 are 0.97 and 0.99, respectively.

![Correlations between high and low-fidelity evaluations.](image)

**Fig. 11** Fine (left) and coarse (right) computational grids used for high and low-fidelity evaluations.

**Fig. 12** Correlations between high and low-fidelity evaluations. Left: $L/D$ at $M_{\infty}$ of 0.729. Right: $L/D$ at $M_{\infty}$ of 0.80. High correlations can be observed between high and low-fidelity evaluations.

### 5.2 Optimization results

In this optimization problem, the number of initial sample points to obtain approximate Pareto optimal designs (defined as $N$) is changed and investigated. In the SF (fine grid) approach, $N=100$ and 200 are investigated. In the VF (coarse grid) approach, $N=100$, 200 and 300 are investigated. The cumulated energy distributions of the dominant POD modes are shown in Fig.13-a. In addition, the POD analysis results of SF($N=200$) and VF($N=300$) are compared in Fig.13-b/c/d in which the effects on the upper surface are indicated by dashed lines as the same manner as Fig.6.

Firstly, only the first POD modes are considered and examined. As the previous section, parametric studies for the first POD coefficient ($d_1$ of Eq.(9)) are performed and the results are shown in Fig.14-a. It can be confirmed that the results obtained from the first POD mode become closer to the Pareto front of the conventional FFD case by increasing $N$. This is due to the increase in the accuracy of the Kriging models to obtain approximate Pareto optimal designs. Objective functions of this problem seem to be more complicated than the previous problem, so that the results with $N=100$ are much worse than the others because it is difficult to construct accurate Kriging models in 24 dimensional design variable space with $N=100$. This results in the worse performance of the averaged shapes of $N=100$ than that of $N=200$ and 300 as shown in Fig.14-a. In this case, L/D value at $M_{\infty}$ of 0.80 was increased with larger $d_1$ as indicated in Fig.14-a. Representative airfoil shapes expressed using the first POD mode of VF($N=300$) case (E and F of Fig.14-a) are compared in Fig.15. The major effect of the first POD mode is to deform the camber line downward at the front side of the airfoil. The upper surface becomes flattened with larger $d_1$ which can reduce the strength of shock waves on the upper surface at $M_{\infty}$ of 0.80. This can be considered as the most influential geometry deformation factor in this optimization problem.

Next, results obtained using several dominant modes are shown in Fig.14-b/c/d. In these cases, the optimization process shown in Fig.3 is utilized in which 20 initial and 15 additional points are evaluated in its main optimization
part. It can be confirmed that the obtained Pareto front by the POD(VF) approach becomes better with the increase in the number of considering POD modes. In the VF case of $N=300$ considering 5 dominant POD modes, its Pareto front is the closest to the conventional FFD case and can reproduce a part of the Pareto front obtained using the FFD approach. The geometries of representative optimal designs indicated in Fig.14-d are compared in Fig.15 and then those flow field visualizations are shown in Fig.16. The trends of aerodynamics as well as trends of the optimal airfoil shapes are equivalent between the conventional optimization (FFD) and POD (VF) approach, which indicates the effectiveness of the proposed POD (VF) approach in the present multi-points optimization problem. The computational costs of the representative cases are summarized in Fig.17 as in the same manner as Fig.10. The computational cost of the POD (VF) case with $N=300$ is about 40% of the conventional FFD approach while the obtained Pareto front locations are qualitatively equivalent.

On the other hand, it is observed that the range of the Pareto front is smaller in the proposed POD-based approach than that of the conventional FFD approach. As the reasons, the effect of poor diversity/inaccuracy of the approximate Pareto optimal designs as well as setting narrow ranges of design variables can be considered. In addition, the design variables extracted by the present POD-based approach are considered to be globally most influential at the center region of the Pareto front, while they may not be locally most influential at both ends of the Pareto front. To extract design variables that are locally most influential may be possible using a part of the database of good designs in the POD analysis. These points may improve the issues at both ends of the Pareto front, which can be considered as our future work.

In the present study, the results of POD-based approaches were compared with the results obtained from the conventional FFD approach, so that the effect of the number of considering POD modes could be examined in detail. At the practical application of the proposed POD-based approach, however, such comparative information is considered to be not available. The decision of the appropriate number of considering POD modes depends on optimization problems to be solved and may be difficult to be generalized. Extracting some knowledge from the present study, the consideration of 5 dominant POD modes was almost enough to provide adequate optimization results in the POD-based approach. Considering 5 dominant POD modes corresponded to include 99% of the total energy in all the cases. Therefore, the appropriate number of considering POD modes can be potentially determined from the value of the cumulated energy.

Fig. 13 Comparison of POD analysis results. a) Cumulated energy distributions. b) Averaged shapes. C) First POD modes. d) Second POD modes.
Fig. 14 Comparison of Pareto fronts between conventional (FFD) approach and proposed (POD) approach using several dominant POD modes. Better results are obtained with larger numbers of considering POD modes and/or with larger numbers of initial sample points.

Fig. 15 Comparison of representative airfoil shapes. Left: Airfoil geometries defined using first POD mode. Right: Comparison of representative Pareto optimal designs. Trends are comparable between the conventional and proposed approaches.
Fig. 16 Comparison of Mach number distributions around representative Pareto optimal designs. Left: at $M_{\infty}$ of 0.729. Right: at $M_{\infty}$ of 0.80. Trends are comparable between the conventional and proposed approaches.

Fig. 17 Comparison of computational costs of representative cases. Computational cost was dramatically reduced in the proposed POD(VF) approach.

6. Conclusions

In this research, a variable fidelity concept was introduced in the POD-based re-parameterization approach to efficiently solve multi-objective aerodynamic shape optimization problems. The low-fidelity functional evaluation was utilized to select good designs that were used as the input database for the POD-based re-parameterization. The proposed approach was applied to two multi-objective aerodynamic shape optimization problems of 2D airfoil, in which the RANS/inviscid simulations or fine/coarse grid simulations were respectively considered as the high/low-fidelity evaluation methods. In the first problem of the 2D airfoil shape optimization, it was confirmed that dominant POD modes obtained from the high and low-fidelity evaluations were qualitatively equivalent. Geometry deformation only using the first POD mode (one design variable) was approximately enough to express the Pareto front solutions obtained using 24 design variables of the FFD approach, which meant the efficiency of the POD based re-parameterization. In the second problem of the 2D airfoil shape multi-points optimization, more number of initial sample points to obtain approximate Pareto optimal designs ($N=300$) and five dominant POD modes were required to reproduce a part of the Pareto front obtained using 24 design variables of the FFD approach. The proposed approach could reduce the computational cost to solve the multi-objective optimization problem dramatically, thanks to the lower burden of the low-fidelity evaluation as well as the decrease in the number of design variables via the POD-based re-parameterization. Furthermore, since the obtained dominant POD modes can be considered as the most influential geometry deformation factors of the optimization problems, the extraction of design knowledge was also possible by...
examining the effect of the dominant POD modes.

These findings obtained in this research are effective only in the present optimization problems at present. Further investigations are essential in practical optimization problems of 3D configurations to demonstrate the general availability of the proposed approach. In addition, design variables extracted by the present POD-based approach are considered to be globally most influential, while they may not be locally most influential for a part of the Pareto front. Additional investigation is needed to extract the design variables locally most influential and to examine the effect of the design variables. Another future work is to utilize a different low-fidelity model in the proposed approach since the different low-fidelity model can be defined from the viewpoints of detailed geometric modeling, convergence criterion and so on.

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