Microwave assisted optical waveguide in Rydberg atoms

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We theoretically demonstrate an efficient scheme to build a micro-wave (MW) assisted optical waveguide in an inhomogeneously broadened vapor medium that is made of active $^{87}$Rb atoms and inactive buffer gas atoms. We exploit the sensitive behaviour of MW field coupled between highly excited Rydberg states to create distinctly responsive and tunable atomic waveguide. The buffer gas induced collision further manipulates the features of the waveguide by widening the spatial transparency window and enhancing the contrast of the refractive index. We numerically solve Maxwell’s equations to demonstrate diffractionless propagation of $5\,\mu\text{m}$ narrow paraxial light beam of arbitrary mode to several Rayleigh lengths. The presence of buffer gas significantly enhances output intensity of diffraction controlled light beam from 10% to 40%. This efficient diffraction elimination technique has important applications in high-resolution imaging and high-density optical communication.

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I. INTRODUCTION

The ability to guide a narrow width optical beam holds promise for applications in high-density optical communication [1] and high-resolution imaging [2, 3]. The main obstacle for realisation of narrow beam based optical technology comes from diffraction and absorption of the medium [4]. The divergence angle of a narrow beam is significantly larger as compared to broad beam due to its geometrical shape [5]. Consequently, the narrow beam encounters severe spatial distortions along the transverse direction as it propagates a few Rayleigh wavelengths distance through the medium. Ultimately diffraction induced image blurring prevents the important light based applications [1, 2]. Hence, the complete elimination of diffraction for the narrow width beam becomes a long-standing goal.

To achieve this goal, different methods based on Raman self-focusing technique [6, 8], and manipulation of refractive index [9, 12] have been proposed in bulk medium [8, 10] as well as in atomic vapor medium [9, 12]. Suitable tailoring of refractive index along transverse direction leads to the form of waveguide like structure inside cold atomic medium [11, 13, 14] and also in hot vapor cell [9, 12, 16]. Truscott et al. published their seminal paper establishing that atomic vapor can produce a waveguide which controls beam propagation without diffraction [15]. This experiment opened a floodgate for numerous experiments [15, 17, 21] in addition to theoretical investigations [9, 12, 14, 16, 17, 21, 22]. A suitably chosen spatial profile of the control field that creates spatial modulation of refractive index which enables weak optical beams to propagate through the medium without loss of generality. Taking advantage of different spatial profiles of control field such as Gaussian [13, 17, 23], super-Gaussian [13, 16, 24] and different mode of Laguerre-Gaussian ($L_{n}^{l}$) [12, 21, 24] results an undistorted probe beam dynamics. Off-resonance [12, 16] or nearly resonance atomic transitions [21, 27] can also support guiding and steering of optical beams. However, the optically written waveguide is based on normal atom with low principal quantum number and is often associated with considerable amount of absorption which limits lossless beam propagation for several Rayleigh lengths. Another drawback of normal atomic waveguide appear due to lack of high contrast in refractive index that fails to support narrow width beam propagation. These limitations can be overcome by exploiting the exaggerated optical properties of Rydberg atoms with high principal quantum number [26]. An atomic waveguide with narrow core and high contrast refractive index between the core and cladding is a very fundamental criteria for guiding a narrow beam with size of $5\,\mu\text{m}$.

In this article, we use highly excited Rydberg energy states of rubidium atom to create high contrast and narrow core optical waveguide. The inspiration of our work comes from recent experimental demonstrating by Shaffer et al. 27 wherein MW field becomes highly responsive to the Rydberg energy states [27, 30]. Even a very weak MW field ($8\,\mu\text{V/cm}^{-1}$) can be able to modify the probe response drastically [27]. We exploit this sensitive behavior of the Rydberg energy states to create a highly efficient and extremely tunable atomic waveguide. A high contrast refractive index modulation of probe can be produced by application of LG shaped MW beam which couples two highly excited Rydberg states $|30D_2\rangle$, $|31P_2\rangle$ [28]. The desired spatial shape of the beam either in MW or optical domain can be found experimentally [31, 32]. Note that dipole-dipole interaction mediated through the residual occupation in Rydberg states are very small in the considered model system [27, 28] and can be neglected safely.

Further, we assimilate inactive buffer gas atoms in addition to active Rydberg atoms inside the vapor cell to
enhance the efficiency of the waveguide [18, 20, 38]. The active atoms frequently collide with the buffer gas atoms in which the velocity of the active atoms alter from one velocity group into another velocity group. This velocity changing collision (VCC) leads to the phenomena of Dicke narrowing [39, 40] in presence of buffer gas. In this article, we exploit buffer gas induced VCC process in order to create a high contrast and efficient atomic waveguide. The perspective of the current scheme is substantially unique from the preceding articles by two ways [18, 20, 21]. First, the key difference is the employing of spatially modulated MW beam between the Rydberg states. The spatial dependent MW LG\_n^\_m generates a sharply varying fiber-like refractive index profile which is tightly confined in the central region of the transverse position (r⊥). Second, the presence of buffer gas further manipulates the features of the waveguide by widening the transparency window and enhancing the contrast of the refractive index profile. Therefore, the transmission of the weak diffraction controlled probe beam at medium output enhances from 10% to 40% in the presence of the refractive index profile. Therefore, the transparency window and enhancing the contrast ther manipulates the features of the waveguide by widening the central region of the transverse position (r⊥). Also the enhanced contrast in refractive index focused the probe beam tightly towards the center of the waveguide. Along with that the waveguide possesses an exclusive and handy feature in which the absorption and refractive index profile of the waveguide are squeezed from both side with the increase of MW intensity which makes this waveguide very efficient in guiding the probe beam of arbitrary width. Narrow beam broadens much faster than the wide beam because the divergence angle is inversely proportional to the beam width. Therefore, this high contrast and tunable atomic waveguide is essential for diffraction elimination from the narrow beam of any arbitrary mode.

The paper is structured as follows. In section III we introduce the model configuration and describe the interaction of Rydberg atoms with the optical and MW fields by a semi-classical density matrix formalism. In section III we point out the advantage of using Rydberg atomic system over its normal counterpart by studying the probe susceptibility. Section IV provides how the spatial structure of the MW beam permits us to build an optical waveguide. In section V we discuss the tunability of the atomic waveguide. Section VI demonstrates the propagation of weak probe field having different beam profiles through the atomic waveguide. Finally, we briefly conclude our work in section VII.

II. THEORETICAL MODEL

A. Model Configuration

In this work, we study the collective behavior of active Rydberg atoms in the presence of inactive buffer gas atoms at room-temperature. The geometry of the model system under consideration is shown in Fig. 1(a) where two counter propagating optical fields and MW field interact with the active \(^{87}\)Rb atoms. Figure 1(b) shows four energy levels of active atoms which include one metastable ground state |1\rangle and three excited states |2\rangle, |3\rangle, |4\rangle. The ground state |1\rangle = |S\_\_ >= |S\_\_ F = 2, m\_F = 2\rangle is coupled to an excited state |2\rangle = |P\_\_ >= |P\_\_ F = 3, m\_F = 3\rangle by a weak probe field. Two highly excited Rydberg states |3\rangle = |D\_\_ >= |D\_\_ m\_J = 3\rangle and |4\rangle = |P\_\_ >= |P\_\_ m\_J = 3\rangle are coupled by a moderately intense MW field of frequency 84.2 GHz. A strong control field connects two states |2\rangle and |3\rangle. The electric fields associated with the electro-magnetic (EM) radiations are described as

\[\hat{E}_j(\vec{r}, t) = \hat{e}_j \hat{E}_j(\vec{r}) e^{(ik_j z - \omega_j t)} + c.c., \]

where \(\hat{E}_j(\vec{r})\), \(k_j\), \(\omega_j\) and \(\hat{e}_j\) are the slowly varying envelope, wave number, frequency and unit polarization vector of the EM fields respectively. The indices \(j \in \{p, c, m\}\) refer to the probe, control and MW field. The EM fields only interact with the active atoms and the interaction can be expressed as a time-dependent Hamiltonian under the electric dipole approximation :

\[H^\_i = \hbar\omega_{21} |2\rangle \langle 2| + h(\omega_{21} + \omega_{32}) |3\rangle \langle 3| + \hbar(\omega_{21} + \omega_{32} - \omega_{34}) |4\rangle \langle 4| - \hbar\Omega_p e^{-i\omega_p t} |2\rangle \langle 1| - \hbar\Omega_c e^{-i\omega_c t} |3\rangle \langle 2| - \hbar\Omega_m e^{-i\omega_m t} |3\rangle \langle 4| + h.c., \]

where \(\Omega_p, \Omega_c, \Omega_m\) are the Rabi frequencies of the probe, control and MW fields respectively. The expression of
Rabi frequencies are
\[ \Omega_p = \frac{\hbar}{\Delta k} \hat{e}_p, \quad \Omega_c = \frac{\hbar}{\Delta k} \hat{e}_c \quad \text{and} \quad \Omega_m = \frac{\hbar}{\Delta k} \hat{e}_m. \] (3)

In order to acquire the time-independent Hamiltonian, we execute the following unitary transformation
\[ H = U \hat{H} U^{-1} - i\hbar U \frac{\partial U}{\partial t}, \] (4)

where \( U \) is defined as
\[ U = e^{-i(\omega_p t/2) + (\omega_p + \omega_c)/3 + (\omega_p + \omega_c - \omega_m)/4} \] (4)t.

Now the Hamiltonian transforms into the following time-independent form
\[ H = -\hbar \Delta_p |2\rangle \langle 2| - h(\Delta_p + \Delta_m) |3\rangle \langle 3| - h\Delta_c |3\rangle \langle 2| - h\Delta_m |3\rangle \langle 3| + h c. \] (6)

where \( \Delta_p = \omega_p - \omega_{21}, \quad \Delta_c = \omega_c - \omega_{32}, \quad \text{and} \quad \Delta_m = \omega_m - \omega_{34} \) are the detuning of the probe, control and MW field respectively.

### B. Dynamical Equations

The dynamics of the active atoms inside the vapor cell are governed by the following Liouville’s equation:
\[ \dot{\rho} = -\frac{i}{\hbar} [H, \rho] + \mathcal{L}_\rho \] (7)

where the second term incorporates various radiative and non-radiative decay processes in the presence of buffer gas atoms. The collisions between the active atoms and buffer gas atoms interrupt the velocity distribution of the active atoms and also the phase coherence between the atomic energy levels which modifies the life-time of the active atoms and also the phase coherence between the non-radiative decay processes in the presence of buffer gas. The collision induced \( \Gamma \) can be included in the dynamical Eq. (7) by adding the atomic coherence \([41–45]\).

The effect of such collision can be included in the dynamical Eq. (7) by adding the following term \([41]\) by adding the following term \([41, 42, 44]\)
\[ \left[ \frac{\partial \rho_{jk}(v, t)}{\partial t} \right]_{\text{collision}} = -\gamma_{ph} (1 - \delta_{jk}) \rho_{jk}(v, t) - \Gamma_{jk} \rho_{jk}(v, t) \] (8)

In the above Eq. (8), \( \Gamma_{jk} \) is known as velocity changing collision rate and \( \gamma_{ph} \) is the rate of collisional dephasing of the atomic coherence. The collision kernel, \( K(v' \rightarrow v) \) represents the probability density per unit time that active atoms have their velocity changed from \( v' \) to \( v \) as a result of collisions with buffer gas atoms \([11]\). For simplicity, the collision kernel can be written in terms of \( \Gamma_{jk} \) as shown in the following expression
\[ K(v' \rightarrow v) = \Gamma_{jk} M(v), \]
\[ M(v) = \frac{1}{\sqrt{\pi} v_{th}} e^{-\frac{v^2}{v_{th}^2}}, \quad v_{th} = \sqrt{\frac{2 k_B T}{m_A}}, \] (9)

where \( M(v) \) and \( v_{th} \) are the Maxwell-Boltzmann velocity distribution along the \( z \) direction and most probable velocity of the active atoms of mass \( m_A \) at a temperature \( T \). The spontaneous decay rates from the excited state \( |j\rangle, \ (j \leq 2, 3, 4 \) to the ground state \( |1\rangle \), are denoted by \( \gamma_{j1} \). Note that collision rate of different atomic states \( \Gamma_{j1}, \ j \leq 2, 3, 4 \) are all similar in strength and are indicated by \( \Gamma_{21} \approx \Gamma_{31} \approx \Gamma_{41} = \Gamma_c \) [14]. In the considered model system, \( k_p \) \((\geq 2\pi \times 1.3 \text{ um}^{-1}) \) is nearly equal to \( k_c \)(\( \geq 2\pi \times 2.0 \text{ um}^{-1}) \) such that wave vector difference \( |\delta \hat{k}| = |\hat{k}_p - \hat{k}_c| \) becomes minimal. Further, we restrict our analysis for the moderate collision case in which \( \Gamma_c \) is comparatively smaller than spontaneous decay rate \( \gamma_{21} \) and Doppler width \( \gamma_{d} \) \( i.e. \Gamma_c \ll \gamma_{21}, \gamma_{d} \) [27, 44]. The degree of collisions can be realized experimentally by controlling the density of the buffer gas inside the vapor cell. The collision induced \( \Gamma_c \) significantly influences the absorptive and dispersive features of the atomic medium.

The following three coupled density matrix equations are sufficient for describing the dynamics of the active atoms in the buffer gas environment under weak probe approximation
\[ \dot{\rho}_{21}(v, t) = -A_{21}(v) \rho_{21}(v, t) + i\Omega_p (\rho_{12}(v, t) - \rho_{22}(v, t)) + i\Omega_c^* \rho_{31}(v, t), \]
\[ \dot{\rho}_{31}(v, t) = -A_{31}(v) \rho_{31}(v, t) + i\Omega_c \rho_{21}(v, t) - i\Omega_p \rho_{32}(v, t) + i\Omega_m \rho_{41}(v, t) + \Gamma_{31} M(v) \int \rho_{31}(v', t) dv', \]
\[ \dot{\rho}_{41}(v, t) = -A_{41}(v) \rho_{41}(v, t) + i\Omega_m^* \rho_{31}(v, t) - i\Omega_p \rho_{42}(v, t) + \Gamma_{41} M(v) \int \rho_{41}(v', t) dv', \]

where
\[ \begin{align*}
A_{21}(v) &= i(\hat{k}_p \cdot \vec{v} - \Delta_p) + \gamma_{21} + \Gamma_{21} + \gamma_{ph}, \\
A_{31}(v) &= i\{(\hat{k}_p + \hat{k}_c) \cdot \vec{v} - (\Delta_p + \Delta_c)\} + \gamma_{31} + \Gamma_{31} + \gamma_{ph}, \\
A_{41}(v) &= i\{(\hat{k}_p + \hat{k}_c) \cdot \vec{v} - (\Delta_p + \Delta_c - \Delta_m)\} + \gamma_{41} + \Gamma_{41} + \gamma_{ph}.
\end{align*} \]

The perturbative solution of the atomic coherence and population in the limits of weak probe approximation can be defined as
\[ \rho_{jk} = \rho_{jk}^{(0)} + \rho_{jk}^{(1)} \] (10)

The zeroth order solution in absence of probe field is \( \rho_{11}^{(0)} = M(v) \) [44]. The first order solution in presence of weak probe field can be obtained in the following form
\[ \begin{align*}
\dot{\rho}_{21}^{(1)}(v, t) &= -A_{21}(v) \rho_{21}^{(1)}(v, t) + i\Omega_{p}^{*} \rho_{31}^{(1)}(v, t), \\
\dot{\rho}_{31}^{(1)}(v, t) &= -A_{31}(v) \rho_{31}^{(1)}(v, t) + i\Omega_{c} \rho_{21}^{(1)}(v, t) + i\Omega_{m} \rho_{41}^{(1)}(v, t) + \Gamma_{31} M(v) \int \rho_{31}^{(1)}(v', t) dv', \\
\dot{\rho}_{41}^{(1)}(v, t) &= -A_{41}(v) \rho_{41}^{(1)}(v, t) + i\Omega_{m}^{*} \rho_{31}^{(1)}(v, t) + \Gamma_{41} M(v) \int \rho_{41}^{(1)}(v', t) dv'.
\end{align*} \]
The steady state response of the above coupled equations i.e., $\rho_{jk}^{(1)} \equiv 0$ can be used for finding the analytical expression of first order atomic coherence $\langle \rho_{21} \rangle$, $\langle \rho_{31} \rangle$ and $\langle \rho_{12} \rangle$. We also incorporated the thermal agitation of the atom by performing the velocity averaging of the atomic coherence

$$
\langle \rho_{21} \rangle = \int \rho_{21}^{(1)}(v) dv,
$$

$$
=\Omega_{p} f_{10}(r) + i \Omega_{m} \Gamma_{31} f_{3}(r) \langle \rho_{31}^{(1)} \rangle
- \Omega_{m}^{2} \Omega_{m} \Gamma_{41} f_{1}(r) \langle \rho_{41}^{(1)} \rangle,
$$

$$
\langle \rho_{31}^{(1)} \rangle = \int \rho_{31}^{(1)}(v) dv,
$$

$$
=\Omega_{p} \Omega_{c} \left[ \frac{|\Omega_{m}|^2 f_{1}(r) L_{2} - f_{3}(r) L_{3}}{L_{1} L_{3} + |\Omega_{m}|^2 L_{2} L_{4}} \right],
$$

$$
\langle \rho_{41}^{(1)} \rangle = \int \rho_{41}^{(1)}(v) dv,
$$

$$
=-i \Omega_{p} \Omega_{c} \Omega_{m}^{*} \left[ f_{1}(r) L_{1} + f_{3}(r) L_{4} \right],
$$

where

$$
L_{1} = 1 + |\Omega_{c}|^2 \Gamma_{31} f_{5}(r) - \Gamma_{31} f_{5}(r),
L_{2} = \Gamma_{41} f_{6}(r) - |\Omega_{m}|^{2} \Gamma_{41} f_{6}(r),
L_{3} = 1 - |\Omega_{c}|^2 |\Omega_{m}|^{2} \Gamma_{41} f_{9}(r)
- \Gamma_{41} f_{2}(r) + |\Omega_{m}|^{2} \Gamma_{41} f_{7}(r),
L_{4} = \Gamma_{31} f_{4}(r) - |\Omega_{c}|^{2} \Gamma_{31} f_{6}(r).
$$

The expression of the spatial functions $f_{j}(r)$, $(j \in 1, 2, 3, \ldots, 10)$ are shown in the appendix section 15. Finally, the velocity averaged linear probe susceptibility, $\langle \chi_{21} \rangle$ at frequency $\omega_{p}$ can be written as

$$
\langle \chi_{21} \rangle = \frac{N |d_{21}|^{2}}{h \Omega_{p}^{*}} \langle \rho_{21} \rangle
$$

where $N$ is the atomic density.

### III. MICROWAVE FIELD SENSITIVITY

In this section, we distinguish the advantage of using Rydberg atomic system over its normal counterpart by studying the probe susceptibility in the absence and presence of the MW field. In Rydberg atomic system, MW field couples two Rydberg states with high principal quantum number as shown in Fig. 1(b). Whereas MW field connects two states with very low principal quantum number for normal atomic system. In Fig. 2(a) and 2(b), we compare the probe absorption lineshape in case of normal as well as Rydberg atomic system. In Fig. 2(a) and 2(b), we compare the probe absorption lineshape in case of normal as well as Rydberg atomic system for three different values of MW field intensity $\Omega_{m}=0$, 0.01$\gamma_{d}$, 0.05$\gamma_{d}$. In absence of MW field ($\Omega_{m}=0$), both the normal and Rydberg atomic system displays electromagnetically induced transparency (EIT) under two-photon resonance condition $i.e.$, $\Delta_{p} = \Delta_{c}$ as shown by solid red curve in Fig. 2. We have observed that Rydberg system offers complete flat transparency window (no absorption) due to the low decay rate ($\sim$ KHz) of the Rydberg states as compared to a shallow window exists in the normal atomic states (decay rate $\sim$ MHz). We next study how a weak MW field ($\Omega_{m}=0.01\gamma_{d}$) drastically modify the probe response in Rydberg system which is distinct from normal system. It is clear from Fig. 2(b) that a complete sharp electromagnetically induced absorption (EIA) peak presence in Rydberg system. On the other hand, the EIA peak in normal atomic system just build up as shown with dash green curve in Fig. 2(a). With the increase of MW field power ($\Omega_{m}=0.05\gamma_{d}$), Rydberg EIA peak experiences power broadening, while the normal EIA peak is still growing towards its maximum peak value. These observations clearly confirms that Rydberg energy states are strongly responsive to the MW field unlike the normal atomic states. We exploit this responsive behavior of MW field in Rydberg atomic system to create a highly tuneable atomic waveguide. The effective modulation of spatial susceptibility due to spatial structure of the MW beam holds the main essence behind the formation of waveguide inside the Rydberg...
IV. FORMATION OF ATOMIC WAVEGUIDE

We now investigate how the spatial structure of the MW beam permits us to build an optical waveguide inside the atomic medium. A waveguide like refractive index can be formed by considering the transverse profile of the MW beam to be Laguerre-Gaussian \((\text{LG}_m^0)\) together with a Gaussian \((\text{LG}_0^0)\) shaped control beam. The spatial shape of the \(\text{LG}_m^0\) beam in cylindrical coordinate can be expressed as 

\[
\Omega_j(r, \phi, z) = \Omega_j^0 \frac{w_j}{w_j(z)} \left( \frac{r\sqrt{z}}{w_j(z)} \right) \left| \frac{1}{e^{\frac{-r^2}{w_j^2(z)}}} e^{i\phi} \right| \times L_n^l \left( \frac{2r^2}{w_j^2(z)} \right) e^{-i\frac{r^2}{w_j^2(z)}} e^{-i(2n+l+1) \tan^{-1}\left( \frac{r}{z} \right)},
\]

\[ r = \sqrt{x^2 + y^2}, \quad \phi = \tan^{-1}\left( \frac{y}{x} \right). \tag{12} \]

The input amplitude is denoted by \(\Omega_j^0\), \(R_j(z) = z + \left( z_j^2 / z \right)\) is the radius of curvature and \(z_j = \lambda_j w_j / \lambda_j\) is the Rayleigh length of the beam. The spot size of the beam is defined as \(w_j(z) = w_j \sqrt{1 + \left( z / z_j \right)^2}\), where \(w_j\) is the minimum beam waist at \(z=0\). The indices \(j \in \{ m, c, p \}\) denotes the MW, control and probe beams, respectively.

The spatial inhomogeneous intensity distribution of \(\text{LG}_m^0\) beam as shown in Fig. 3(a). It is clear from Fig. 3(a) that the doughnut shaped \(\text{LG}_m^0\) MW beam has zero intensity at the central region whereas maximum intensity occurs in the ring-shaped region. Therefore, \(\text{LG}_m^0\) MW beam together with Gaussian control beam can be used to obtain the desired spatial refractive profile of the probe beam as follows. Fig. 3(b) shows the transverse variation of the probe absorption. A complete transparency window exists at the core because of dominant characteristics of control beam over the MW field. The diminishing intensity of the control beam toward the wing region yields absorption at the cladding. Simultaneously MW field gain maximum intensity in the bright ring which causes high probe absorption due to EIA as shown in Fig. 3(b). Hence, considering suitable spatial structure of the two fields allows us to achieve probe transparency at the core and opaqueness at cladding both at resonance condition and near resonance detuned situation. Fig. 3(d) display that the refractive index attains a maximum value at EIT dominant region and forms the core of the atomic waveguide. The cladding section of the waveguide can be cast by EIA, since EIT is ineffective in the ring-shaped region. The induced waveguide structure consists of refractive variation between core and cladding accompanied with a small width of core. Hence the spatial response of the medium for the probe field exhibits waveguide like structure at blue detuned condition whereas at red detuned condition, it changes to the anti-waveguide like structure as shown in Fig. 3(c). In both the cases, the core region of the atomic waveguide display minimum absorption as shown in Fig. 3(c). Finally, we have chosen the blue detuned probe field condition, i.e., \(\Delta_p = 0.001 \gamma_{21}\) for efficient guiding of various narrow Gaussian and Hermite Gaussian with arbitrary modes.

V. TUNABILITY OF THE WAVEGUIDE

Next, we discuss how the buffer gas induced collision significantly manoeuvres the features of the atomic waveguide along the transverse direction in the presence of MW field \((\Omega_m^0)\). In order to comprehend the reasons behind these manipulations of atomic waveguide, we plot the probe absorption lineshape as a function of \(\Delta_p\) in the absence and presence of VCC as shown in Fig. 4(a). The MW induced EIA peak in Fig. 4(a) can be enhanced by a notable amount in the presence of VCC. Along with that the EIA lineshape manifests Dicke like narrowing due to the collision process as displayed in the inset of Fig. 4(a). Fig. 4(b) illustrates that the contrast of refractive index significantly enhances in the presence of VCC. The slope of VCC induced refractive index profile is remarkably sharp which makes this waveguide more efficient in guiding the narrow probe beam in comparison to that of a Kerr field induced waveguide [21]. Further, the transparency window of the waveguide becomes much wider and steeper in the presence of buffer gas. As
are \( \Omega \) and do not hallucinate.

a result, a narrow probe beam propagates through the waveguide without significant loss of intensity. These VCC induced Dicke narrowing and enhancing of the EIA peak distinctly facilitate the waveguide characteristics.

Next, we study the propagation of weak probe field having different beam profiles through the atomic waveguide. The beam propagation dynamics is governed by the Maxwell’s wave equations \[12, 21\]. Under slowly varying envelope and paraxial wave approximation, Maxwell’s wave equations for the probe beam transform into the following form

\[
\frac{\partial \Omega_p}{\partial z} = i \frac{\mu}{2k_p} \nabla_+^2 \Omega_p + 2i\pi k_p \langle \chi_21 \rangle \Omega_p. \tag{13}
\]

In Eq. \(13\), second order partial derivative in the \( xy \) plane i.e. \( \nabla_+^2 = (\partial^2/\partial x^2 + \partial^2/\partial y^2) \) incorporates inherent optical diffraction of the probe beam. The last term of Eq. \(13\) is the contribution of linear and nonlinear optical effects which includes MW and buffer gas induced absorption and refractive index profile of the medium in order to suppress the diffraction. We adopt split-step Fourier method (SSFM) to obtain the numerical solution of Eq. \(13\) and demonstrate the effect of spatially varying absorption and refractive index profile on probe beam dynamics. First, we study the propagation dynamics of a Gaussian (LG\(_0^0\)) probe beam. The width of the probe beam is 5 \( \mu \text{m} \) which remains within the limit of paraxial wave approximation, \( \lambda_p/2\pi w_p < 0.1 \) \[21, 51\]. The propagation dynamics of the narrow Gaussian shaped beam is inversely proportional to the beam width \( w_p \). Hence, this high-contrast and squeeze waveguide is highly desired in order to remove diffraction from the narrow beam.

VI. BEAM PROPAGATION THROUGH THE WAVEGUIDE
probe beam through the high contrast atomic waveguide is shown in Fig. 6. The input and output intensity profile of the probe beam in the presence and absence of the MW beam and buffer gas are illustrated clearly. In absence of MW LG$_0^2$ beam, the probe beam suffers diffraction induced broadening along with large absorption as shown with double-dashed-dot magenta curve in Fig. 6. The diffraction of the probe beam is drastically reduced in the presence of MW LG$_0^2$ beam. We notice that the output intensity of the probe beam decreases below 10% in absence of buffer gas after the propagation of $z = 5z_p$. The situation changes in the presence of buffer gas, and the transmissivity of the diffraction controlled probe beam enhances over 40% for the same propagation distance ($z = 5z_p$). This efficient beam propagation without any diffraction is possible due to the presence of high-contrast tunable optical waveguide in buffer gas medium.

In order to prove the robustness of the atomic waveguide, we also demonstrate diffraction-less propagation of arbitrary Hermite-Gaussian (HG$_n^m$) modes. A further reason to choose HG$_n^m$ modes of narrow width is its direct application in super-resolution imaging [3, 52]. The spatial profile of different HG$_n^m$ modes at medium entrance ($z = 0$) is given by

$$\Omega_p(x, y, 0) = \Omega_p^0 H_n \left( \frac{\sqrt{2}x}{w_p} \right) H_m \left( \frac{\sqrt{2}y}{w_p} \right) e^{-\frac{x^2+y^2}{w_p^2}},$$ (14)

where $H_n$ and $H_m$ are the Hermite polynomials of order $n$ and $m$ respectively. For demonstration, we propel the Hermite-Gaussian probe beam of mode $n = 1$, $m = 1$ through the waveguide. The intensity profile of the HG$_1^1$ mode at $z = 0$ is shown in Fig. 7(a). The diffraction of the beam in absence of MW LG$_0^2$ beam and buffer gas is displayed in Fig. 7(b). The presence of LG$_0^2$ beam eliminates the diffraction completely as shown in Fig. 7(c) and Fig. 7(d). However, Fig. 7(d) clearly shows that the diffraction-less HG$_1^1$ beam becomes tightly focused towards the centre of the waveguide due to the sharply varying refractive index in the presence of buffer gas medium. The transverse structure of the MW beam and buffer gas play the important role in guiding the weak probe beam of narrow width and arbitrary modes.

**VII. CONCLUSION**

In conclusion, we have demonstrated an efficient scheme to generate MW assisted optical waveguide in an inhomogeneously broadened vapor medium that is made of active $^{87}$Rb atoms and inactive buffer gas atoms. The sensitive behaviour of MW field coupled between two highly excited Rydberg states of Rb atoms allow us to create a responsive atomic susceptibility. The structured MW LG$_0^2$ beam and Gaussian control beam together build an optical waveguide with amenable fiber like refractive index profile. The presence of buffer gas induced collision further manipulates the features of the waveguide by widening the spatial transparency win-
dow and enhancing the contrast of the refractive index. The increasing intensity of the MW field squeezes the high contrast waveguide from both sides which duly guides the probe beam of narrow width. We numerically solve Maxwell’s equations to demonstrate diffractionless propagation of narrow paraxial light beam of arbitrary modes such as Gaussian, Hermite-Gaussian HGₙ to several Rayleigh lengths. The output intensity of diffractionless light significantly enhances in the presence of buffer gas. This efficient technique to eliminate diffraction from narrow light beams have important applications in high-density optical communication [1] and high-resolution imaging [2, 3, 52].

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APPENDIX

\[ f_1(r) = \int \frac{M(v)}{f_D(v)} dv, \quad f_2(r) = \int \frac{M(v)}{A_{41}(v)} dv, \]

where

\[ f_D(v) = A_{21}(v) (A_{31}(v) A_{41}(v) + |\Omega_m|^2) + A_{41}(v)|\Omega_c|^2. \]

\[ f_3(r) = \int \frac{A_{41}(v) M(v)}{f_D(v)} dv, \]

\[ f_4(r) = \int \frac{M(v)}{A_{31}(v) A_{41}(v) + |\Omega_m|^2} dv, \]

\[ f_5(r) = \int \frac{A_{41}(v) M(v)}{A_{31}(v) A_{41}(v) + |\Omega_m|^2} dv, \]

\[ f_6(r) = \int \frac{(A_{31}(v) A_{41}(v) + |\Omega_m|^2) f_D(v)}{M(v)} dv, \]

\[ f_7(r) = \int \frac{M(v)}{(A_{31}(v) A_{41}(v) + |\Omega_m|^2) A_{41}(v)} dv, \]

\[ f_8(r) = \int \frac{A_{31}(v) M(v)}{(A_{31}(v) A_{41}(v) + |\Omega_m|^2) f_D(v)} dv, \]

\[ f_9(r) = \int \frac{M(v)}{(A_{31}(v) A_{41}(v) + |\Omega_m|^2) f_D(v)} dv, \]

\[ f_{10}(r) = \int \frac{M(v) (A_{31}(v) A_{41}(v) + |\Omega_m|^2)}{f_D(v)} dv, \]

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