A robust correlation between growth rate and amplitude of solar cycles: consequences for prediction methods

R. Cameron
cameron@mps.mpg.de

and

M. Schüssler
schuessler@mps.mpg.de

Max-Planck-Institut für Sonnensystemforschung, 37191 Katlenburg-Lindau, Germany

ABSTRACT

We consider the statistical relationship between the growth rate of activity in the early phase of a solar cycle with its subsequent amplitude on the basis of four datasets of global activity indices (Wolf sunspot number, group sunspot number, sunspot area, and 10.7-cm radio flux). In all cases, a significant correlation is found: stronger cycles tend to rise faster. Owing to the overlapping of sunspot cycles, this correlation leads to an amplitude-dependent shift of the solar minimum epoch. We show that this effect explains the correlations underlying various so-called precursor methods for the prediction of solar cycle amplitudes and also affects the prediction tool of Dikpati et al. (2006) based upon a dynamo model. Inferences as to the nature of the solar dynamo mechanism resulting from predictive schemes which (directly or indirectly) use the timing of solar minima should therefore be treated with caution.

Subject headings: Sun: Activity

1. Introduction

Solar activity is the driver of space weather, which has practical consequences for human activities in space. This is one reason why the search for methods to predict its (short-term and long-term) future levels has found much interest in the literature. Further motivation arises from the potential implications for understanding the origin of solar activity: a reliable
method of predicting the amplitude of future solar cycles could provide a constraint on dynamo models. On the other hand, the converse proposition is not necessarily true, as has been pointed out by Bushby & Tobias (2007): the nonlinear dynamics of the dynamo might be such as to make mid- to long-term prediction impossible even if an almost perfect physical understanding of the dynamo mechanism is achieved.

In many cases, recipes for prediction are inferred from correlations found in historical records of measured quantities, which are (directly or indirectly) related to solar activity. As illustrated by Wilson (1994, cf. his Fig. 14.2) and Lantos & Richard (1998, cf. their Fig. 6), the success of most methods in actually predicting the unknown amplitude of a future cycle is rather disappointing. Nevertheless, the correlations between several ‘precursors’, i.e., quantities measured during the descending or minimum phase of a cycle and the amplitude of the subsequent cycle (e.g., Hathaway et al. 1999; Schatten 2003) might have a non-random origin and thus call for a physical explanation. This could have implications for dynamo models.

In this paper, we consider the effect of the overlapping of solar cycles in combination with their asymmetric shape on the correlations between precursors and following cycle amplitudes. In this connection, the important aspect of the asymmetry is the difference of the amplitude-dependent ascent rate in the early cycle phase (related to the so-called Waldmeier effect) compared to the decay rate near the end of a cycle. Since sunspot cycles overlap for typically 2 to 3 years (Harvey 1992), this asymmetry affects the timing of the activity minima, which are pivotal epochs for most precursor methods. We show that these effects can explain the correlations upon which such methods are based, without necessarily implying a direct physical connection between the precursor quantity and the following cycle. We also show that the essence of the Waldmeier effect, i.e., that stronger cycles tend to show a faster rise of activity levels during their ascending phase than weaker cycles, is a robust property present in all activity indices. On this basis, we explain how cycle asymmetry and cycle overlapping may also affect the dynamo-based prediction method of Dikpati & Gilman (2006), in spite of recent claims to the contrary (Dikpati et al. 2008b).

2. Overlapping asymmetric cycles and precursors

Precursor methods are based upon the existence of a correlation between some physical quantity measured during the descending or minimum phase of a cycle and the amplitude of the following cycle. The correlation is established by considering historical records of data, the longest of which is the record of sunspot numbers. Assuming a non-random origin of such a correlation, there are three possible explanations for its existence:
1. The precursor is a feature of the old cycle that represents or is related to an input quantity for the dynamo process, so that its magnitude directly affects the amplitude of the next cycle. For example, it has been suggested that the strength of the polar field during solar minimum is a measure of the poloidal field from which the toroidal field for the next cycle is generated (e.g., Schatten et al. 1978, Choudhuri et al. 2007).

2. The precursor quantity represents an early manifestation of the new cycle, which already affects the high latitudes of the Sun but does not yet produce sunspots. The notion of such an ‘extended solar cycle’ (Wilson et al. 1988) is supported by observations of coronal activity (Altrock 1997), ephemeral magnetic regions (Harvey 1994), and zonal flows (Howe et al. 2006, Altrock et al. 2008).

3. If the definition of the precursor quantity directly or indirectly depends on the timing of the solar minimum, the amplitude-dependent shift of the minimum due to the overlap of asymmetric cycles can lead to a correlation of the precursor with the amplitude of the new cycle (Cameron & Schüssler 2007, see their Fig. 10). This does not necessarily involve a physical connection between the precursor and the dynamo process, in principle even permitting the prediction of random cycle amplitudes.

The third possibility arises from the observation that information about the height of the maximum of a cycle is already contained in its early rise phase (Waldmeier 1935, 1955). In fact, the shapes of most historical solar cycles can reasonably well be described by simple functions containing a few parameters (e.g., Hathaway et al. 1994, Li 1999), so that an estimate of the further development of a cycle is possible a few years after sunspot minimum (Waldmeier 1936, Elling & Schwentek 1992). Since cycles overlap, i.e., the rise of the new cycle starts already when the decay of the old cycle is still ongoing, the timing of the cycle minimum (the epoch when the sum of the activities of both cycles is minimal) and its height are affected by the amplitude of the new cycle: the minimum occurs earlier and is higher for a subsequent strong cycle than for a weak cycle. In fact, stronger cycles tend to be preceded by shorter cycles (earlier minima, Solanki et al. 2002) with higher minimum activity levels. Both quantities can be considered as precursors with correlation coefficients with the amplitude of the next cycle of about 0.7 (Hathaway et al. 1999).

Cameron & Schüssler (2007) have noted that the amplitude-dependent shift of the minimum epoch is enhanced by the amplitude-dependent asymmetry of solar cycles: strong cycles tend to grow faster in activity than weak cycles, while the decay rate in the late descending phase is largely independent of the cycle amplitude. This is related to, but not identical to, the so-called ‘Waldmeier effect’ (Waldmeier 1935) which is often stated in the form: the time between cycle minimum and maximum is shorter for stronger cycles (e.g., Hathaway et al.
However, the crucial quantity for the minimum shift is not the time between minimum and maximum but the *steepness* of the activity rise (decay) in the initial (late) phase of the cycle, which can be determined independently of the timing of the activity minima and maxima by considering the slope of the activity curve. We show in the next section that the correlation between initial growth rate and cycle amplitude is a very robust feature shown by all available global activity indices.

3. The robust amplitude-dependent cycle asymmetry

Hathaway et al. (2002) found that the ‘classical’ Waldmeier effect, i.e., the correlation between the rise time (time interval between minimum and maximum) and the cycle amplitude, is by a factor of about two weaker for the group sunspot numbers (Hoyt & Schatten 1998) than for the Wolf sunspot numbers. For both datasets, the historically strongest cycles (numbers 18, 19, 21, and 22) do not fit the linear relationship very well, showing somewhat long rise times. This weakening of the correlation can be understood by the shift in the timing of the minima: stronger, rapidly growing cycles lead to early preceding minima, thus increasing the time between minimum and maximum. This effect becomes even more pronounced in the analysis of sunspot area data by Dikpati et al. (2008b) owing to the fact that the sunspot areas for cycles 21, 22 and 23 reach a slightly higher second ‘Gnevyshev maximum’ (Gnevyshev 1967) a few years after the first maximum coinciding with the sunspot number maximum. It is not surprising that determining the rise time in terms of these later maxima destroys the correlation with the cycle amplitude.

As we have pointed out in the previous section, the relevant quantity for the minimum shift of overlapping cycles is not the time interval between minimum and maximum but the *steepness* of the activity growth (decay) in the initial (late) phase of the cycle. An inspection of Fig. 5 of Hathaway et al. (2002) and of Fig. 1 of Dikpati et al. (2008b) already suggests that the different datasets do not differ strongly in this respect. In order to give a quantitative account, we need to measure the steepness without reference to the epochs of maximum and minimum; in particular, the latter is already affected by the correlations that we wish to study. For instance, Lantos (2000) considered the slope of the sunspot number curve at the inflexion point during the ascending part of cycles 9 to 22 and found a correlation coefficient of $r = 0.88$ with the cycle amplitude, while the (anti)correlation with the rise time from minimum to maximum only yields $|r| = 0.61$. In our case, we need to consider specifically the early rise and late decay phases, so that we estimate the rise and decay rates by determining the time required for the activity index under consideration to cover a fixed interval of values. This interval is chosen such that it is not significantly
affected by the cycle overlap. The rise and decay rates are then defined as the ratios of the
value intervals and the corresponding rise and decay times, respectively.

We have analyzed four datasets: monthly Wolf sunspot numbers\(^1\) (SN) for cycles 7 to 23, monthly group sunspot numbers\(^2\) (GSN, Hoyt & Schatten 1998) for cycles 7 to 22, sunspot areas (SAR, Balmaceda et al. 2005) for cycles 12 to 23, and 10.7-cm solar radio flux\(^3\) (SRF) for cycles 19 to 23. While we have used the full available data sets for SRF and SAR, the SN and GSN data are only considered from cycle 7 on because a) we wish to compare the results for the different datasets during roughly the same period of time, and b) for very low-amplitude cycles like those during the Dalton minimum and the first cycles after the Maunder minimum we cannot use the same intervals for the determination of the rise and decay times. Fig. 1 shows the datasets, also indicating the intervals chosen for the determination of growth and decay rates: 30–50 (SN, GSN), 300–600 microhemispheres (SAR), and 900–1100 sfu (SRF). All data have been smoothed by a Gaussian with a FWHM of 1 year.

\(^1\)http://sidc.oma.be
\(^2\)http://www.ngdc.noaa.gov/stp/SOLAR/ftpsumspotnumber.htm
\(^3\)http://www.drao.nrc.ca/icarus/www/solhome.shtml
Fig. 1.— Data sets used in this study: Wolf sunspot number (SN, top left), group sunspot number (GSN, top right), sunspot area (SAR, bottom left), and 10.7-cm solar radio flux (SRF, bottom right). SN and GSN are considered from cycle no. 7 onward (after the Dalton minimum). The dashed horizontal lines indicate the intervals used for the determination of the rise and decay rates.
Fig. 2.— Scatter diagrams of rise rates (left panels) and decay rates (right panels) versus cycle amplitude for four sets of solar activity indices: Wolf sunspot number (SN, top row), group sunspot number (GSN, second row), sunspot area (SAR, third row, cycles 21–23 indicated by triangles, two of which nearly coincide on the left-hand panel), and 10.7-cm solar radio flux (bottom row). The corresponding linear correlation coefficients are given.
Scatter diagrams of the resulting rise and decay rates with the cycle amplitudes are shown in Fig. 2 together with the corresponding correlation coefficients. The figure demonstrates that high correlation coefficients \( r = 0.83\ldots0.89 \) between rise rate and cycle amplitude exist for all activity indices. Assuming a Gaussian distribution, the corresponding significance levels are between 95\% (SRF) and over 99\% (SN, GSN, SAR). In particular, the group sunspot numbers and the sunspot areas exhibit an even higher correlation than the Wolf sunspot numbers. Note that cycles 21–23 (indicated by triangles in the third row of Fig. 2), which, owing to their higher Gnevyshev maxima in the SAR data, largely destroyed the correlation of the ‘classical’ Waldmeier effect in the analysis of Dikpati et al. (2008b), nicely fit the correlation for the growth rate. On the other hand, the decay rates show no significant correlation with the cycle amplitude, so that we find an \emph{amplitude-dependent asymmetry} between the rise rates early in a cycle and the decay rates in the late phase. This asymmetry acts to make the behavior near sunspot minimum rather insensitive to the amplitude of the old cycle but sensitive to that of the new.

We have checked how strongly the correlation coefficients depend on the chosen intervals for the determination of the growth and decay rates, and found them to be rather insensitive. For instance, in the case of the growth (decay) rates for the SN data, we have \( r = 0.79 (0.05) \) for the interval [25, 45], \( r = 0.82 (0.22) \) for [30, 50] (the interval chosen for Fig. 2), \( r = 0.85 (0.42) \) for [35, 55], and \( r = 0.88 (0.54) \) for [40, 60]. Similar results are found for the other datasets.

4. Implications for precursor methods

The overlapping of cycles with an amplitude-correlated asymmetry leads to a dependence of the minimum epoch on the amplitude of the following cycle: since the initial activity of a large-amplitude cycle increases faster while the decline in the decay phase is not or less amplitude-dependent, the minimum level of activity between overlapping cycles occurs the earlier the higher the amplitude of the following cycle. Such a minimum shift affects the value of any precursor quantity which is defined (directly or indirectly) with regard to the minimum epoch; in the extreme, it may completely explain the predictive power of the precursor. Cameron & Schüssler (2007) have demonstrated this for the case of a very simply defined precursor, namely, the activity level (such as sunspot number or area), three years before minimum. The earlier the minimum occurs (i.e., the stronger the new cycles is), the higher is the precursor value, simply because it is measured at an earlier time when the activity of the old cycle had not declined as much. This explains why the method can, in principle, even predict cycles with random amplitudes. It is important to note, however, that
such methods always require that the minimum epoch is already known. For a real prediction (as opposed to a postdiction of past cycles) such a method can be applied only some time into the new cycle, when proper averaging can be performed to define the minimum time in view of the strong fluctuations of activity around minimum (cf. Harvey & White 1999).

Precursors that are defined (directly or indirectly) in relation to the timing of the minimum are affected by the amplitude-dependent minimum shift, which thus may (partly or fully) explain their correlation with the amplitude of the next cycle (i.e., their predictive power). Among the various precursor methods affected we mention just a few examples: geomagnetic activity before and around solar minimum (e.g., Ohl 1966; Legrand & Simon 1981; Lantos & Richard 1998; Kane 2007), sunspot activity around minimum (Hathaway et al. 1999), length of the preceding sunspot cycle (Kane 2008), skewness of the preceding cycle (Ramaswamy 1977; Lantos 2006), as well as properties of the decay phase of the preceding cycle (Podladchikova et al. 2008).

5. Implications for the model of Dikpati et al.

Cameron & Schüssler (2007) suggested that at least part of the predictive skill of the model of Dikpati et al. (2006) and Dikpati & Gilman (2006), henceforth referred to as the DGT model, may result from the overlap of asymmetric cycles in the sunspot area data from which their source term is derived. We have shown above that the underlying effect, i.e., the amplitude-dependent asymmetry between the growth and decay phases, is present in all global activity datasets, including the sunspot area data used in the DGT model.

The effect of this crosstalk between cycles on the DGT model results from the assumption of a fixed speed and range of latitude drift of the source term in the course of a cycle. The shortening of the cycle length (defined as the time between minima) by the shift of the minimum preceding a strong cycle then leads to larger source amplitudes (sunspot areas) assigned to the low-latitude part of the artificial butterfly diagram used in the DGT model. As a consequence, the magnetic flux crossing the equator as the potentially relevant quantity for the dynamo amplitude of the next cycle is higher, while the opposite is true for a weak following cycle. In this way, the correlation in the data survives the artificial stretching/compressing of cycles carried out in the DGT model in order to obtain a constant cycle period. In fact, Dikpati et al. (2008a) found that, in their model, the flux crossing the equator during a cycle is significantly correlated ($r = 0.76$) with the amplitude of the next cycle. This is in accordance with Fig. 8a in Dikpati & Gilman (2006), which demonstrates that the memory in their model does not extend much longer than one cycle, but it is in contrast to the claim of Dikpati et al. (2008a) that it takes 17-21 years for the poloidal sur-
face field to reach the tachocline and generate the new toroidal field, so that 2-3 preceding cycles contribute to the toroidal flux of a cycle. **A stronger diffusive coupling between surface and tachocline than indicated by the explicit value of their magnetic diffusivity (for example, due to numerical diffusion) could possibly explain this discrepancy (cf. Yeates et al. 2008).**

By disregarding the observed sunspot latitudes in favor of an imposed fixed latitude drift, the DGT model becomes affected by the amplitude-dependent minimum shift. The extent to which this contributes to the predictive skill of the model could be easily tested by replacing the actual sunspot area data as input by a synthetic dataset with random cycle amplitudes (without any memory), but keeping the correlation resulting from the minimum shift of overlapping cycles. Cameron & Schüssler (2007) used this approach to demonstrate the effect on precursor methods. The amount of predictive skill shown by the DGT model for such an input will directly indicate how strongly this correlation affects the prediction.

### 6. Discussion

The shift of the solar minimum epoch due to the amplitude-dependent asymmetry between the growth and decay phases of solar cycles can explain the rather high correlation coefficients between various precursor quantities and the strength of the subsequent cycle (e.g., Hathaway et al. 1999). In this connection, we should note that even a statistically highly significant correlation does not automatically imply a high skill of the precursor for the actual prediction of a future cycle:

1. A typical value of, say, \( r = 0.8 \) for a ‘good’ precursor leaves about one third of the variance in amplitude unexplained by the correlation, so that the prediction is prone to considerable statistical uncertainty. Even the value of \( r = 0.97 \) shown by the method of Thompson (1993, see Hathaway et al. 1999) did not prevent an utterly inaccurate prediction for cycle 23 (30% too high).

2. The data used to determine the correlations in most cases cover only part of the known range of variability in the solar cycle. Periods such as the Maunder and Dalton minima indicate that nonlinear and stochastic effects may severely limit the predictability since it is unclear how strongly these effects are represented in the observed surface flux (Bushby & Tobias 2007). ‘Tuning’ of free parameters in prediction schemes can lead to seemingly high correlations for past cycles while the predictive skill for future cycles in fact is much lower.
The dependence of many precursor methods on knowing the minimum epoch further limits their practical usefulness: the large relative fluctuations of the activity indices around sunspot minimum make it necessary to average the data (over 1 year, for instance) in order to obtain a proper definition of the minimum epoch. Consequently, a sensible prediction can only be made one or two years after the minimum has passed, so that there is no big advantage compared to methods that use the information provided by the early phases of the new cycle itself, like the steepness of the activity rise (Waldmeier 1936; Elling & Schwentek 1992).

The relation between the shift of the minimum time and the amplitude of the next cycle due to cycle overlapping is strongest for a dynamo where the time interval between the onsets of activity of subsequent cycles is constant. Fluctuations in the timing of the onset of activity introduces noise into the relation between minimum shift and activity and thus reduces the predictive skill of precursor methods. Quantifying such fluctuations in the solar cycle data is difficult because the length of the available datasets is not sufficient to reliably determine the phase fluctuations and drifts (Gough 1978; Hoyng 1996) and to decide on the existence or otherwise of a solar ‘clock’ (cf. Dicke 1978, 1988). However the expected shifts due to the overlapping of the cycles and the amplitude dependent asymmetry produce variations in the cycle length (as measured from minimum to minimum) which are consistent with those observed since the Dalton minimum. This means the effect we have outlined is relevant. In passing, we also note that in the case of advection-dominated dynamo models the cycle period is determined by the meridional flow, which supports phase stability (Charbonneau & Dikpati 2000; Charbonneau 2005).

For our understanding of the solar dynamo, it would certainly be useful to clarify to which extent the skill of some prediction methods results 1) from correlations in the input data (such as discussed here), 2) from capturing early high-latitude manifestations of an extended new cycle, and 3) from capturing mechanisms that connect properties of the old cycle to the strength of the new cycle (such as a dynamo model with a memory of at least one cycle). It is reasonable to suppose that there is some memory in the solar cycle i.e., that the solar cycle amplitudes do not constitute a purely random sequence. Potentially important information could be gleaned from identifying a quantity that unequivocally represents this memory and is not ‘contaminated’ by early information leaking in from the new cycle. Clarifying whether the memory extends over more than one cycle would help to decide between diffusion-dominated and advection-dominated dynamo models (Yeates et al. 2008).
7. Conclusions

We have confirmed a highly significant correlation between the growth rate of activity during the early phase of a solar cycle and its maximum amplitude for all global activity indices, i.e., Wolf and group sunspot numbers, total sunspot area, and 10.7-cm radio flux. On the other hand, there is no significant correlation between the decay rate in the late cycle phase and the cycle amplitude.

Owing to the overlapping of individual cycles, this asymmetry leads to an amplitude-dependent shift of the minimum epoch, thus explaining (fully or partly) the predictive power of precursor methods which (directly or indirectly) use the timing of the activity minimum as a pivotal point. The resulting correlation in the sunspot area data probably also affects the predictions with the dynamo-based model of Dikpati & Gilman (2006).

For our understanding of the origin of the solar magnetic field, it is important to disentangle the effects of ‘real’ physical precursors, i.e., properties of the old cycle directly affecting the flux generation for the next cycle or early high-latitude manifestations of the new cycle, from apparent precursors, which derive their predictive power from the the amplitude-dependent shift of the minimum epoch.

REFERENCES

Altrock, R., Howe, R., & Ulrich, R. 2008, in ASP Conf. Ser., Vol. 383, Subsurface and Atmospheric Influences on Solar Activity, ed. R. Howe, R. W. Komm, K. S. Balasubramaniam, & G. J. D. Petrie, 335

Altrock, R. C. 1997, Sol. Phys., 170, 411

Balmaceda, L., Solanki, S. K., & Krivova, N. 2005, Memorie della Societa Astronomica Italiana, 76, 929

Bushby, P. J. & Tobias, S. M. 2007, ApJ, 661, 1289

Cameron, R. & Schüssler, M. 2007, ApJ, 659, 801

Charbonneau, P. 2005, Living Reviews in Solar Physics, LRSP-2005-2, [http://solarphysics.livingreviews.org](http://solarphysics.livingreviews.org)

Charbonneau, P. & Dikpati, M. 2000, ApJ, 543, 1027

Choudhuri, A. R., Chatterjee, P., & Jiang, J. 2007, Phys. Rev. Lett., 98, 131103
Dicke, R. H. 1978, Nature, 276, 676
—. 1988, Sol. Phys., 115, 171
Dikpati, M., de Toma, G., & Gilman, P. A. 2006, Geophys. Res. Lett., 33, 5102
—. 2008a, ApJ, 675, 920
Dikpati, M. & Gilman, P. A. 2006, Astrophys. J, 649, 498
Dikpati, M., Gilman, P. A., & de Toma, G. 2008b, ApJ, 673, L99
Elling, W. & Schwentek, H. 1992, Sol. Phys., 137, 155
Gnevyshev, M. N. 1967, Sol. Phys., 1, 107
Gough, D. 1978, in Pleins Feux sur la Physique Solaire, ed. S. Dumont & J. Roesch (CNRS, Paris), 81
Harvey, K. L. 1992, in The Solar Cycle, ed. K. L. Harvey (San Francisco: Astronomical Society of the Pacific, ASP Conf. Series Vol. 27), 335
Harvey, K. L. 1994, in Solar Surface Magnetism, ed. R. J. Rutten & C. J. Schrijver (Dordrecht: Kluwer), 347
Harvey, K. L. & White, O. R. 1999, J. Geophys. Res., 104, 19759
Hathaway, D. H., Wilson, R. M., & Reichmann, E. J. 1994, Sol. Phys., 151, 177
—. 1999, J. Geophys. Res., 104, 22375
—. 2002, Sol. Phys., 211, 357
Howe, R., Komm, R., Hill, F., Ulrich, R., Haber, D. A., Hindman, B. W., Schou, J., & Thompson, M. J. 2006, Sol. Phys., 235, 1
Hoyng, P. 1996, Sol. Phys., 169, 253
Hoyt, D. V. & Schatten, K. H. 1998, Sol. Phys., 179, 189
Kane, R. P. 2007, Sol. Phys., 243, 205
—. 2008, Sol. Phys., 248, 203
Lantos, P. 2000, Sol. Phys., 196, 221
Lantos, P. & Richard, O. 1998, Sol. Phys., 182, 231
Legrand, J. P. & Simon, P. A. 1981, Sol. Phys., 70, 173
Li, K. 1999, A&A, 345, 1006
Ohl, A. I. 1966, Soln. Dann., 12, 84
Podladchikova, T., Lefebvre, B., & van der Linden, R. 2008, Journal of Atmospheric and Terrestrial Physics, 70, 277
Ramaswamy, G. 1977, Nature, 265, 713
Schatten, K. H. 2003, Adv. Space Res., 32, 451
Schatten, K. H., Scherrer, P. H., Svalgaard, L., & Wilcox, J. M. 1978, Geophys. Res. Lett., 5, 411
Solanki, S. K., Krivova, N. A., Schüssler, M., & Fligge, M. 2002, A&A, 396, 1029
Thompson, R. J. 1993, Sol. Phys., 148, 383
Waldmeier, M. 1935, Mitt. Eidgen. Sternw. Zürich, 14, 105
—. 1936, Astron. Nachr., 259, 267
—. 1955, Ergebnisse und Probleme der Sonnenforschung. (Leipzig, Geest & Portig)
Wilson, P. R. 1994, Solar and stellar activity cycles (Cambridge Astrophysics Series, Cambridge University Press)
Wilson, P. R., Altrock, R. C., Harvey, K. L., Martin, S. F., & Snodgrass, H. B. 1988, Nature, 333, 748
Yeates, A. R., Nandy, D., & Mackay, D. H. 2008, ApJ, 673, 544