Abstract

Starting from the quantum hydrodynamic model and transforming to the coupled driven pseudoforce system the plasmonic excitations of electron beam with arbitrary degree of degeneracy are studied. Using the conventional normal-mode analysis a dual-length-scale wavefunction theory for a monochromatic electron beam is developed. The beam plasmon excitations are shown to have both wave and particle character, de Broglie wavenumbers of which depend on various parameters such as average beam speed, the chemical potential, the charge screening and the background electrostatic energy. The deterministic single electron dynamics in the beam is investigated using the dual-wavelength electrostatic potential energy. It is shown that electrons can be either trapped or traveling along the beam direction depending on the ambient plasma parameter values. The problem of monochromatic beam transport through binary as well as ternary metallic configurations are investigated and some technological applications of these system are pointed out. The theory is extended to multistream phenomenon and electron beam transport through multimedia junction. We believe that current study can have major impact on understanding of the quantum beam-plasma interactions and instabilities and can further elucidate the mechanisms involving in the collective quantum wave-particle phenomena.
I. INTRODUCTION

Plasmons are elementary quantized excitations of plasma electron oscillations \([1, 2]\). They play fundamental role in many characteristic properties of plasmas and solid state phenomena such as the electric, heat and optical transport \([3, 4]\). These entities make an ideal platform for fast THz device communications \([5]\) and beyond where conventional wired communications fail. They may also have myriad of other foreseeable applications in the emerging nanotechnology \([6]\), plasmonics \([7–9]\), optoelectronics \([10]\), etc. for miniaturized nano-fabricated integrated semiconductor circuitry \([11, 12]\). Plasmon screening effect as the main cause of distinct optical edge in free electron systems makes many of the metallic alloys optically unique among other solids. Plasmons, on the other hand, almost rule every optical and dielectric aspect of plasmas \([13–15]\) and semiconductors. Low dimensional semiconductors \([16]\) such as gapped graphene illustrate surface plasmon effects as recently confirmed by the infrared spectroscopic techniques. The plasmonic property of graphene makes it an ideal object for multilayer composite devices such as compact ultrafast switches, optical modulators, optical lattices, photodetectors, tandem solar cells and biosensors \([17–19]\). The first theoretical development of the idea by Bohm and Pines dates back to mid nineties where they coined plasmon for the excitations due to the long-range collective electron interactions \([20–24]\). The theoretical as well as experimental aspects of collective electron dynamics in quantum gases has been the subject of intense investigations over the past few decades \([25–30]\) due to its fundamental importance in many field of physics and chemistry.

Pioneering developments of quantum statistical theories \([31–35]\) over the years has furnished a pavement for modern theories of quantum plasmas \([36–39]\). Many interesting new aspects of quantum effects in astrophysical and laboratory plasmas has been recently revealed using various quantum plasma theories \([40–53]\). The more effective quantum kinetic theories are however less analytic compared to the hydrodynamic counterparts due to mathematical complexity and require computational efforts. The more simple quantum hydrodynamic or density functional approaches on the other hand have their own limitations in grasping the quantum kinetic phenomena associated with collective electron excitations. Recently, it has been shown \([54]\) that hydrodynamic approaches based on the density functional formalism can reach beyond the previously thought kinetic limitations such as the collisionless damping if accurately formulated. One of the best candidate formulations for studying the quantum
effects in plasmas is the Schrödinger-Poisson model [55, 56] which is based on the Madelung transformation originally attempted for single-electron quantum fluid model. It has been recently shown that the analytic investigation of linearized Schrödinger-Poisson system for arbitrary degenerate electron gas provides routes to many novel quantum feature of collective plasmon excitations [57–61]. In current study we use the model in order investigate the dual-tone plasmon excitations due to monochromatic electron beam transport and instabilities. Here, we extend the theory for the case of multimedia electron beam transport and reveal some interesting transport features regarding the particle- and wave-like beam excitations.

We present the derivation of appropriate pseudoforce system for plasmon excitations with monochromatic electron beam effect starting from the conventional quantum hydrodynamic and Schrödinger-Poisson models in Sec. II. In Sec. III the normal mode analysis and general dual-scalelength solution for spatial electrostatic field distribution is given. The single electron deterministic dynamic in the beam is analysed in Sec. IV. The electron beam transport is studied in Sec. V and conclusions are drawn in VI.

II. HYDRODYNAMICS OF MADELUNG QUANTUM FLUID

A relatively simple way to simulate the one-dimensional dynamics of an arbitrary degenerate electron gas flowing within an inertial neutralizing ion background and in the absence of magnetic force iv via the quantum hydrodynamic method consisting of a closed set of three equations which include the continuity, the momentum balance including the Bohm quantum force and Poisson’s equation, as follows

\[ m \frac{\partial n}{\partial t} + \frac{\partial np}{\partial x} = 0, \]  
\[ \frac{\partial p}{\partial t} + \frac{p}{m} \frac{\partial p}{\partial x} = -eE - \frac{\partial \mu}{\partial x} + \frac{h^2}{2m} \frac{\partial}{\partial x} \left( \frac{1}{\sqrt{n}} \frac{\partial^2 \sqrt{n}}{\partial x^2} \right), \]  
\[ \frac{\partial E}{\partial x} = 4\pi e (n_0 - n), \]  

where \( n, p, \mu \) and \( E \) refer to the electron number density, average fluid momentum, the chemical potential and electrostatic field, respectively. Moreover, \( n_0 \) denotes the equilibrium electron or background ion charge number density. The quantum hydrodynamic set of equations (II) obviously becomes closed once coupled with the appropriate equation of state.
(EoS) relating the charge density to the chemical potential. The most general EoS valid for an electron fluid with arbitrary degree of degeneracy is given in terms of Fermi function. For an isothermal electron fluid the EoS follows as \[62\]

\[
n(\mu, T) = \frac{2^{7/2} \pi^{3/2}}{h^3} F_{1/2}(\mu, T) = -\frac{2^{5/2} (\pi m k_B T)^{3/2}}{h^3} \text{Li}_{3/2}[- \exp(\mu/k_B T)],
\]

(2a)

\[
P(\mu, T) = \frac{2^9 \pi^{3/2}}{3h^3} F_{3/2}(\mu, T) = -\frac{2^{5/2} (\pi m k_B T)^{3/2}}{h^3} (k_B T) \text{Li}_{5/2}[- \exp(\mu/k_B T)],
\]

(2b)

where \(k_B\) is the Boltzmann constant, \(T\) is the electron fluid temperature and \(F_k\) is the well-known Fermi integral of order \(k\) given as \[3\]

\[
F_k(\mu, T) = \int_0^\infty \frac{x^k}{\exp(x - \mu/k_B T) + 1} dx.
\]

(3)

The \(\text{Li}_k\) function is also known as the polylog function defined below

\[
F_k(\mu, T) = -\Gamma(k + 1)\text{Li}_{k+1}[- \exp(\mu/k_B T)],
\]

(4)

where \(\Gamma\) is the gamma function. Alternatively, the quantum electron fluid hydrodynamic formulation may be cast into the Schrödinger-Poisson system \[55\] using the Madelung transformations with the state function \(\mathcal{N} = \sqrt{n(x,t)} \exp[iS(x,t)/\hbar]\) characterising the electron gas excitations in the fluid. Note that the Madelung transformations appropriately relate the hydrodynamic dependent parameters \(n\) and \(p\) to the state function via \(\mathcal{N} \mathcal{N}^* = n(x,t)\) and \(p(x,t) = \partial S(x,t)/\partial x\). The coupled equations characterizing the dynamic of electron fluid may be written as

\[
i\hbar \frac{\partial \mathcal{N}}{\partial t} = -\frac{\hbar^2}{2m} \frac{\partial^2 \mathcal{N}}{\partial x^2} - e\phi \mathcal{N} + V_0 \mathcal{N} + \mu(n,T)\mathcal{N},
\]

(5a)

\[
\frac{\partial^2 \phi}{\partial x^2} = 4\pi e (|\mathcal{N}|^2 - n_0),
\]

(5b)

where \(\mathcal{E} = -\partial\phi/\partial x\). The system \[5\] can be modeled for a monochromatic beam of electrons with average momentum \(p\) using separation of variables \(\mathcal{N}(x,t) = \psi(x,t) \exp(ipx/\hbar)\) where \(\psi(x,t) = \sqrt{n(x,t)} \exp[i f(t)/\hbar]\) with \(S(x,t) = px + f(t)\). The linearized coupled driven pseudoforce system using new variables \(\mathcal{N}(x,t) = \mathcal{R}(x) T(t)\) becomes

\[
\frac{\hbar^2}{2m} \frac{d^2 \mathcal{R}(x)}{dx^2} + e\phi(x) \mathcal{R}(x) + [\epsilon - \mu(n,T)] \mathcal{R}(x) = V_0 \mathcal{R}(x),
\]

(6a)

\[
\frac{d^2 \phi(x)}{dx^2} = 4\pi e (|\mathcal{R}(x)|^2 - n_0),
\]

(6b)

\[
i\hbar \frac{dT(t)}{dt} = \epsilon T(t),
\]

(6c)
where $\epsilon$ is the energy eigenvalues and $V_0$ is the arbitrary constant potential. In a compact normalized form the pseudodriver system is given as

\[
\frac{d^2\Psi(x)}{dx^2} + \Phi(x) + 2E\Psi(x) = U_0 \exp(ik_dx),
\]

\[
\frac{d^2\Phi(x)}{dx^2} - \Psi(x) = 0,
\]

where $\mu_0$ is the chemical potential assumed to be constant and $E = (\epsilon - \mu_0)/2E_p$ with the plasmon energy $E_p = \hbar\omega_p$ and the electron plasma frequency $\omega_p = \sqrt{4\pi e^2n_0/m}$. The normalization scheme follows as $\Psi(x) = \mathcal{R}(x)/\mathcal{R}(0)$ with $\mathcal{R}(0) = \sqrt{n_0}$ being the value of $\mathcal{R}(x)$ at $x = 0$, $\Phi(x) = e\phi/E_p$ and $U_0 = V_0/E_p$ are fractional electrostatic and pseudodriver energies. Also, $x$ is normalized to the plasmon wavelength $\lambda_p = 2\pi/k_p$ with $k_p = \sqrt{2meE_p}/\hbar$ being the plasmon wavenumber and the parameter $k_d = p/\hbar k_p$ is the normalized characteristic de Broglie’s matter wavenumber. The pseudodriver term $V_0\mathcal{R}(0) \exp(ipx/\hbar)$ plays an essential role in the coupled linear differential system (7) due to the presence of the electron beam. The more general system including the pseudodamping or charge screening effect is as follows

\[
\frac{d^2\Psi(x)}{dx^2} + 2\xi \frac{d\Psi(x)}{dx} + \Phi(x) + 2E\Psi(x) = U_0 \cos(k_dx),
\]

\[
\frac{d^2\Phi(x)}{dx^2} + 2\xi \frac{d\Phi(x)}{dx} - \Psi(x) = 0,
\]

in which $\xi = K/k_p$ with $\xi^2 = (E_p/2)\partial n/\partial \mu = (1/2\theta)Li_{1/2}[ -\exp(\mu/\theta)]/Li_{3/2}[ -\exp(\mu/\theta)]$ in which $\theta = T/T_p$ and $\mu = \mu_0/E_p$ is the normalized temperature dependent screening parameter. Note that $T_p = E_p/k_B = \hbar\omega_p/k_B$ is the plasmon temperature of the system. The system (8) has the following lengthy solution (9)

\[
\Phi(x) = \Phi_h(x) - \frac{U_0e^{-\xi x}}{2\alpha_1\eta_2} \left[ \eta_1 (k_d^2 - k_2^2) \cos(\beta_2 x) - \eta_2 (k_d^2 - k_1^2) \cos(\beta_1 x) \right] + \frac{U_0e^{-\xi x}}{2\alpha_1\eta_2} \left[ \eta_1 (k_2^2 + k_2^2) \frac{\sin(\beta_2 x)}{\beta_2} - \eta_2 (k_2^2 + k_1^2) \frac{\sin(\beta_1 x)}{\beta_1} \right] + \frac{U_0}{\eta_1 \eta_2} \times \]

\[
\{ [(k_d^2 - k_1^2) (k_d^2 - k_2^2) - 4k_d^2\xi^2] \cos(k_dx) + 2k_d\xi (k_1^2 + k_2^2 - 2k_d^2) \sin(k_dx) \},
\]

\[
\Psi(x) = \Psi_h(x) + \frac{U_0e^{-\xi x}}{2\alpha_1\eta_2} \left[ \eta_1 k_2^2 (k_2^2 - k_2^2) \cos(\beta_2 x) - \eta_2 k_1^2 (k_2^2 - k_2^2) \cos(\beta_1 x) \right] - \frac{U_0e^{-\xi x}}{2\alpha_1\eta_2} \left[ \eta_1 k_2^2 (k_2^2 + k_2^2) \frac{\sin(\beta_2 x)}{\beta_2} - \eta_2 k_1^2 (k_2^2 + k_1^2) \frac{\sin(\beta_1 x)}{\beta_1} \right] - \frac{U_0}{\eta_1 \eta_2} \times \]

\[
\{ [1 - (k_1^2 + k_2^2 - k_d^2) (k_1^2 + 4\xi^2)] k_d^2 \cos(k_dx) + 2k_d\xi (1 + k_1^2 - 4\xi^2k_d^2) \sin(k_dx) \},
\]
where $\beta_1 = \sqrt{k_1^2 - \xi^2}$, $\beta_2 = \sqrt{k_2^2 - \xi^2}$, $\eta_1 = (k_d^2 - k_1^2)^2 + 4k_d^2\xi^2$ and $\eta_2 = (k_2^2 - k_d^2)^2 + 4k_2^2\xi^2$.

The functions $\Psi_h$ and $\Phi_h$ are the solutions of homogenous system given as

$$
\Phi_h(x) = e^{-\xi x} \left\{ \left( k_2^2 \Phi_0 + \Psi_0 \right) \left[ \cos(\beta_1 x) + \frac{\xi}{\beta_1} \sin(\beta_1 x) \right] - \right\}, \quad (10a)
$$

$$
\Psi_h(x) = e^{-\xi x} \left\{ \left( \Phi_0 + k_2^2 \Psi_0 \right) \left[ \cos(\beta_2 x) + \frac{\xi}{\beta_2} \sin(\beta_2 x) \right] - \right\}. \quad (10b)
$$

Note that the wavenumbers $k_1$ and $k_2$ are given in terms of the energy eigenvalues $\epsilon$

$$
k_1 = \sqrt{E - \alpha}, \quad k_2 = \sqrt{E + \alpha}, \quad \alpha = \sqrt{E^2 - 1}. \quad (11)
$$

The fundamental wavenumbers follow the complementarity relation $k_1 k_2 = 1$. The matter-wave energy dispersion relation is $E = \left[ 1 + (k^2 + \xi^2)^2 \right] / 2(k^2 + \xi^2)$ where $k = k_1$.

### III. NORMAL MODES ANALYSIS OF BEAM EXCITATIONS

It is seen from the solution (9) that the beam-plasmon excitations has two distinct pseudo-resonance peaks at $k_{1,2} = k_d$. These constitute the normal modes of beam-plasmon excitations. Using the plasmon dispersion relation the wavenumber of normal modes can be written in the following generalized forms

$$
k_{1,2} = \sqrt{\frac{\gamma^2 - \mu + \Phi_0}{2} - \xi^2} \pm \sqrt{\frac{(\gamma^2 - \mu + \Phi_0)^2}{4} - 1}, \quad (12)
$$

where minus/plus sign refers to the wave-like/particle-like excitation branch and $\gamma = v/v_p$ in which $v$ and $v_p = \sqrt{2E_p/m}$ denote the average beam speed and the plasmon speed. Also, $\Phi_0 = e\phi_0/E_p$ is intended to generalized the solution for the case of a constant background electrostatic potential $\phi_0$. Note that in obtaining the generalized de Broglie’s wavenumbers for arbitrary degenerate electron beam we have assumed the plasmon excitation energy to be equal to the average energy per electron in the beam, i.e. $\epsilon = p^2/2m$, using the generalized plasmon excitation dispersion relation. The fundamental wavenumbers may be given in terms of conventional de Broglie’s matter wavenumber as $k_{1,2} = \chi_{1,2} k_d$ with

$$
\chi_{1,2}(\mu, \theta, E_K) = \sqrt{\frac{1}{2} \left( 1 - \frac{\mu}{E_K} \right) - \frac{1}{2} \frac{1 - \exp(\mu/\theta)}{\theta E_K \text{Li}_3[\exp(\mu/\theta)]} \pm \sqrt{\frac{1}{4} \left( 1 - \frac{\mu}{E_K} \right)^2 - \frac{1}{E_K}}. \quad (13)
$$
where $E_K$ is the nonrelativistic kinetic energy per electron in the beam as scaled to the plasmon energy. Having known the normal modes of beam plasmon excitations we may expand the electrostatic potential energy of the system as

$$\Phi(x) = \Phi_1 \exp (ik_1 x) + \Phi_2 \exp (ik_2 x),$$  \hspace{1cm} (14)

in which $\Phi_1$ and $\Phi_2$ can be obtained using the initial values of normalized electrostatic potential energy, $\Phi$, and electric force $F = -\partial\Phi/\partial x$ at the position $x = 0$ ($F_0 = -eE_0$) as

$$\Phi_1 = -\frac{k_2 \Phi_0 - iF_0}{k_1 - k_2}, \quad \Phi_2 = \frac{k_1 \Phi_0 - iF_0}{k_1 - k_2}. \hspace{1cm} (15)$$

As compared to the free electron solution to the ordinary Schrödinger equation (14) is the double lengthscale version one due to the single electron excitation which leads to particle-like branch and the other with wave-like character due to collective electrostatic excitations in the beam. The solution (14) can be readily generalized to multistream case as

$$\Phi(x) = \sum_{\gamma} g(\gamma) \left[ \Phi_{1s} \exp (ik_{1s} x) + \Phi_{2s} \exp (ik_{2s} x) \right], \hspace{1cm} (16)$$

where the index $s$ denoting the stream parameters and

$$\Phi_{1s} = -\frac{k_{2s} \Phi_0 - iF_0}{k_{1s} - k_{2s}}, \quad \Phi_{2s} = \frac{k_{1s} \Phi_0 - iF_0}{k_{1s} - k_{2s}}, \hspace{1cm} (17)$$

with the de Broglies’ wavenumbers given as

$$k_{1s,2s} = \sqrt{\frac{\gamma_{s}^2 - \mu + \Phi_0}{2} - \xi^2 \mp \sqrt{\left(\frac{\gamma_{s}^2 - \mu + \Phi_0}{2}\right)^2 - 4}}, \hspace{1cm} (18)$$

in which $\gamma_s = v_s/v_{sp}$. The function $g(\gamma_s)$ can be obtained using the normalization condition

$$\sum_{\gamma} g(\gamma_s) \left( k_{1s}^2 \Phi_{1s} + k_{2s}^2 \Phi_{2s} \right) = -1. \hspace{1cm} (19)$$

The later is obtained through the Poisson’s relation at $x = 0$. Note that in the absence of charge screening effect $\xi = 0$ the complementarity relation $k_{12} = 1$ leads to

$$\sum_{\gamma} g(\gamma_s) = 1, \hspace{1cm} (20)$$

which is analogous to $\int f(v)dv = 1$ in which $f(v)$ denotes the distribution function. For $\gamma < \sqrt{\mu - \Phi_0 + 2}$ the wavenumbers $k_{1,2}$ are imaginary and complex conjugate of each other.
In the complex plane both the wave-like and particle-like wavevectors as originate from the origin of coordinate will end up to an ellipse of equation

\[ k_r^2 + k_i^2 = \sqrt{\left(\frac{\gamma^2 - \mu + \Phi_0}{2} - \xi^2\right)^2 - \left(\frac{\gamma^2 - \mu + \Phi_0}{2}\right)^2} + 1, \quad (21) \]

in which \( k_r \) and \( k_i \) are the real and imaginary parts of the wavenumbers, respectively.

Note that for \( \xi = 0 \) the complex plane one obtains \( k_r = k_i = 1 \). It is found that for \( \gamma < \sqrt{\mu - \Phi_0 + 2} \) the beam propagation is unstable in which the wave-like branch always grows and the particle-like one damps. Moreover, in the fractional momentum range \( \sqrt{\mu - \Phi_0 + 2} < \gamma < \sqrt{\xi^2 + \xi^{-2} + \mu - \Phi_0} \) and \( \xi < 1 \) both of \( k_1 \) and \( k_2 \) are purely real. However, for \( \gamma > \sqrt{\xi^2 + \xi^{-2} + \mu - \Phi_0} \) and \( \xi < 1 \), \( k_1 \) is purely imaginary and \( k_2 \) is purely real. Beyond the critical charge screening limit \( \xi > 1 \) the wave-like branch becomes imaginary in the whole range of the fractional beam speed, \( \gamma \). In the later case the collective plasmon excitations are critically damped but particle branch continues to propagate although it is still spatially damped in the range \( \gamma < \sqrt{\mu - \Phi_0 + 2} \).

Figure 1 shows the spatial variation of electrostatic energy in the beam for various parameter. In Fig. 1(a) quantum beating takes place at the critical point \( k_1 \approx k_2 \approx k_p \) or \( \gamma \gg \sqrt{\mu - \Phi_0 + 2} \). In this limit wave-like and particle-like energy oscillations have almost the same wavelength and largest amplitude as seen from Eqs. (14) and (15). Note that the quantum beating condition is independent of the charge screening parameter \( \xi \). As the beam speed increases in Fig. 1(b) the wave-like branch wavelength and amplitude increases while those of particle-like decreases and their (spatial) oscillations becomes out of phase. For instance, the parameters of the electron beam and plasma in Fig. 1(c) are chosen so that the wavelength of the wave-like branch is nearly twice that of the particle-like. Finally, in Fig. 1(d) for largest value of \( \gamma \) the particle-like oscillation wavelength and amplitude decrease further while that of wave-like has increased significantly. However, comparing all plots in Fig. 1 reveals that the overall oscillation amplitude decreases as the fractional beam increases away from the critical value \( \gamma_{cr} = \sqrt{\mu - \Phi_0 + 2} \). Note that for all plots in Fig. 1 the generalized de Broglie’s electron-beam wavenumbers are purely real.

Figure 2 shows the variation in spatial electrostatic energy distribution in the beam for separate wave-like and particle-like oscillation branches. In Figs. 2(a) and 2(b) the beam speed is in the regime \( \sqrt{\mu - \Phi_0 + 2} < \gamma < \sqrt{\xi^2 + \xi^{-2} + \mu - \Phi_0} \) and both wave-like and particle-like oscillations are stable. On the other hand, in Fig. 2(b) and 2(c) the increase
FIG. 1: The spatially stable electrostatic energy oscillations $\Phi(x)$ for given parameters such as the over critical $\gamma > \sqrt{\mu - \Phi_0 + 2}$ fractional beam speed $\gamma$, the normalized chemical potential, $\mu$, the charge screening parameters $\xi$ and the initial electrostatic energy value $\Phi_0$. The initial value of normalized electrostatic force is set to zero for all plots, i.e., $d\Phi/dx|_{x=0} = F_0 = 0$. The space parameter $x$ is normalized to the plasmon wavelength $\lambda_p = 2\pi/k_p$.

In the charge screening parameter leads to the beam-speed regime $\gamma > \sqrt{\xi^2 + \xi^{-2} + \mu - \Phi_0}$ in which the wave-like oscillation is strongly damped. Note that the increase in screening parameter has also the slight effect on amplitude and wavelength of the particle-like branch. Due to this increase, the amplitude of the particle-like branch decreases while its wavelength decreases. It is evident that in the beam-speed regime, $\gamma > \sqrt{\xi^2 + \xi^{-2} + \mu - \Phi_0}$, over the
FIG. 2: The wave-like $\Phi_w(x)$ and particle-like $\Phi_p(x)$ electrostatic energy oscillations for given parameters such as the fractional beam speed $\gamma$, the normalized chemical potential, $\mu$, the charge screening parameters $\xi$ and the initial electrostatic energy value $\Phi_0$. The initial value of normalized electrostatic force is set to zero for all plots, i.e., $d\Phi/dx|_{x=0} = F_0 = 0$. The space parameter $x$ is normalized to the plasmon wavelength $\lambda_p = 2\pi/k_p$.

relatively long propagation path ($l > \lambda_p$) the electrostatic beam oscillations become purely particle-like. One should note that for real value of $\gamma$ the condition $\xi^2 + \xi^{-2} + \mu \geq \Phi_0$ must be fulfilled. Such effect can have practical applications when a monochromatic beam production is in order by allowing the quantum electron beam to pass through a very thin
FIG. 3: The growing wave-like \( \Phi_w(x) \) and damping particle-like \( \Phi_p(x) \) electrostatic energy oscillations for given parameters such as the under critical fractional beam speed \( \gamma < \sqrt{\mu - \Phi_0 + 2} \), the normalized chemical potential, \( \mu \), the charge screening parameters \( \xi \) and the initial electrostatic energy value \( \Phi_0 \). The initial value of normalized electrostatic force is set to zero for all plots, i.e., \( \frac{d\Phi}{dx}|_{x=0} = F_0 = 0 \). The space parameter \( x \) is normalized to the plasmon wavelength \( \lambda_p = 2\pi/k_p \).

column of strongly screening solid.

Figure 3 depicts the spatial beam oscillation for the speed range \( \gamma < \sqrt{\mu - \Phi_0 + 2} \) separately for wave-like and particle-like components. It is seen from Figs 3(a) and 3(b) that in this regime the wave-like component grows while the particle-like oscillation damps, spatially. Both the wave-like and particle-like branches are unstable in this regime. However,
such an instability is an energy conserving one like the two stream instability in which the
electrostatic energy exchanged between beam and the plasma. Here, while the overall energy
of plasmon is conserved we have an energy flow from particles to the wave which is quite
opposite to the case of collisionless Landau damping effect. There is significant difference
between the amplitudes of wave-like and particle-like branches in Figs. 3(a) and 3(b). The
plasmon oscillation components for beam speed in beating condition are shown in Figs. 3(c)
and 3(d). It is remarked that in such condition the amplitudes of the two excitation branches
are comparable and the damping and growing rates are significantly lowered.

IV. SINGLE ELECTRON CLASSICAL DYNAMICS

Electrons in the beam with average speed \( v = p/m \) are under the influence of electrostatic
forces caused by single electron motion as well as collective interactions. The normalized
electrostatic force acting on each electron is given as \( F = d\Phi(x)/dx \) in which the electrostatic
energy is given by Eq. (14). The normalized energy equation for single electron motion is

\[
\frac{dx}{dt} = \sqrt{\gamma_0^2 - \Phi_0 + \Phi(x)}, \quad \Phi(x) = \Phi_1 \cos(k_1x) + \Phi_2 \cos(k_2x),
\]

(22)

where \( \gamma_0 = v_0/v_p \) in which \( v_0 \) is the initial speed of the given electron in the beam. The
integral form of the solution to equation of motion is

\[
t = \int \frac{dx}{\sqrt{\gamma_0^2 - \Phi_0 + \Phi_1 \cos(k_1x) + \Phi_2 \cos(k_2x)}},
\]

(23)
in which the coordinates \( x \) and \( t \) are scaled to the plasmon wavelength \( \lambda_p \) and inverse plas-
mon frequency \( 1/\omega_p \), respectively. Obviously, the integral form (23) has no straightforward
analytical solution. However, for \( v \gg v_p \) the amplitude of the particle-like branch becomes
vanishingly small and an approximate solution will be

\[
t = \frac{2}{k_1 \sqrt{\gamma_0^2 - \Phi_0 + \Phi_1}} \mathcal{E}_\ell \left( \frac{k_1x}{2}, \frac{2\Phi_1}{\gamma_0^2 - \Phi_0 + \Phi_1} \right),
\]

(24)
in which \( \mathcal{E}_\ell(\varphi, m) \) is the elliptic function of the first-kind of variable \( \varphi \) and modulus \( m \). The
single electron equation of motion in this regime is of cinodal waveform given by

\[
x(t) = \frac{2}{k_1} \mathcal{J}_\ell \left( \frac{k_1 \sqrt{\gamma_0^2 - \Phi_0 + \Phi_1}}{2}, \frac{2\Phi_1}{\gamma_0^2 - \Phi_0 + \Phi_1} \right),
\]

(25)
FIG. 4: The single electron dynamics in a propagating beam with average normalized beam speed $\gamma$ for different parametric valued used in Fig. 1. The parameter $\gamma_0$ denotes the initial electron normalized speed at the origin $x = 0$. The initial value of normalized electrostatic force is set to zero for all plots, i.e., $d\Phi/dx|_{x=0} = F_0 = 0$. The space and time parameters are normalized to the plasmon wavelength $\lambda_p = 2\pi/k_p$ and inverse plasmon frequency $\omega_p = E_p/\hbar$.

where $\mathcal{J}_\varphi(\varphi, m)$ is the Jacobi elliptic function. The exact solution to the electron dynamics can still be evaluated numerically via the force equation

$$\frac{d^2x}{dt^2} = \mathcal{F}(x), \quad \mathcal{F}(x) = \frac{k_1k_2\Phi_0}{k_1 - k_2} \left[ \sin(k_1x) - \sin(k_2x) \right].$$ (26)
With the initial conditions $x(0) = 0$ and $x'(0) = \gamma_0$ the numerical results are shown in Fig. 4 for same values of parameters used in Figure 1. The values of $\gamma_0$ are chosen to be the average beam speed in all cases, for simplicity. However, there are electrons with speeds above and below this average value. In Fig. 4(a) which corresponds to the electrostatic beam-energy variation shown in Fig. 1(a) the specific electron with $\gamma_0 = 1.1$ is trapped inside the beam going back and forth in a nonlinear fashion. this is due to the fact that the initial kinetic energy of the electron is not enough to overcome the maximum electrostatic energy of the beam. Note that in the quantum beating condition due to resonantly large amplitude of electrostatic field oscillations in the beam most of electrons become trapped and only a small fraction with large initial speed $\gamma_0$ penetrate the electrostatic barrier. Same situation is also true for Fig. 4(b) with slightly lower electrostatic energy amplitude shown in Fig. 1(b). However, the trapped electron in this case has an approximately sinusoidal oscillation about its equilibrium position as compared to the case in Fig. 4(a). The motion of electrons in Figs. 4(c) and 4(d) corresponding respectively to Figs. 1(c) and 1(d) is of traveling type together with an oscillation about the equilibrium position. However, the oscillations in Fig. 4(c) is double-tone while that in Fig. 4(d) is almost single-tone. The later is due to the very small amplitude of particle-like branch oscillation compared to that of wave-like which eliminates one of the components of electrostatic energy in (22), as we assumed for obtaining the approximate analytic solution (25).

Figure 5 shows the electron motion in a beam with corresponding parameters used for Figs. 2 and 3. In Fig. 5(a) which corresponds to electrostatic profiles shown Figs. 2(a) and 2(b) electron motion is traveling type. However, for the case corresponding to Figs. 2(c) and 2(d), in which the wave-like branch is strongly damped, the electron is trapped and its motion although still quasiperiodic is very complex. Moreover, Figs. 5(c) and 5(d) correspond to values used for Fig. 3 in which both wave and particle-like branches are unstable. It is seen that our typical the electron is trapped in both cases due to a very large amplitude of electrostatic energy variations in these cases.

V. ELECTRON BEAM COLLECTIVE TRANSPORT

In order to study the electron beam transport effect we consider the beam with initial conditions that is initial electrostatic energy $\Phi_0$ and force $F_0$ propagating in the first media.
FIG. 5: The single electron dynamics in a propagating beam with average normalized beam speed $\gamma$ for different parametric valued used in Figs. 2 and 3. The parameter $\gamma_0$ denotes the initial electron normalized speed at the origin $x = 0$. The initial value of normalized electrostatic force is set to zero for all plots, i.e, $d\Phi/dx|_{x=0} = \mathcal{F}_0 = 0$. The space and time parameters are normalized to the plasmon wavelength $\lambda_p = 2\pi/k_p$ and inverse plasmon frequency $\omega_p = E_p/\hbar$. 
with characteristic wavenumbers shown below

\[
\begin{align*}
k_{11} &= \sqrt{\frac{\gamma^2 - \mu_1 + \Phi_{01}}{2} - \xi_1^2} - \sqrt{\frac{(\gamma^2 - \mu_1 + \Phi_{01})^2}{4} - 1}, \\
k_{21} &= \sqrt{\frac{\gamma^2 - \mu_1 + \Phi_{01}}{2} - \xi_1^2} + \sqrt{\frac{(\gamma^2 - \mu_1 + \Phi_{01})^2}{4} - 1},
\end{align*}
\]

where \(\Phi_{01}\) is an arbitrary constant electric potential energy characterising the first medium. The electrostatic wavefunction the follows

\[\Phi_1(x) = \Phi_{11} \exp (ik_{11}x) + \Phi_{21} \exp (ik_{21}x),\]

in which \(\Phi_{11}\) and \(\Phi_{12}\) are defined in terms of initial condition as

\[\Phi_{11} = \frac{k_{21}\Phi_0 - i\mathcal{F}_0}{k_{21} - k_{11}}, \quad \Phi_{21} = -\frac{k_{11}\Phi_0 - i\mathcal{F}_0}{k_{21} - k_{11}}.\]

Note that at critical points \(k_{11} = i\mathcal{F}_0/\Phi_0\) and \(k_{11} = i\mathcal{F}_0/\Phi_0\) the spatial variation of electrostatic energy distribution becomes monotonic plane-wave. At \(x = a\) the second media joins continuously so that the electrostatic energy and force acting on it match exactly at the interface between the two media, i.e. \(\Phi_{1a} = \Phi_{2a} = \Phi_{a}\) and \(\mathcal{F}_{1a} = \mathcal{F}_{2a} = \mathcal{F}_a\) from which the initial values of electron beam at the junction \(x = a\) are obtained as

\[\Phi_a = -\frac{e^{ik_{11}a} (i\mathcal{F}_0 - k_{21}\Phi_0) - e^{ik_{21}a} (i\mathcal{F}_0 - k_{11}\Phi_0)}{k_{21} - k_{11}}, \]

\[\mathcal{F}_a = -\frac{k_{11}e^{ik_{11}a} (\mathcal{F}_0 + ik_{21}\Phi_0) - k_{21}e^{ik_{21}a} (\mathcal{F}_0 - ik_{11}\Phi_0)}{k_{21} - k_{11}}.\]

Assuming the wavefunction for \(\Phi_2(x) = \Phi_{21} \exp (ik_{21}x) + \Phi_{22} \exp (ik_{22}x)\) for the second media, the coefficients \(\Phi_{21}\) and \(\Phi_{22}\) are obtained in terms of \(\Phi_0\) and force \(\mathcal{F}_0\) as follows

\[\Phi_{12} = \frac{e^{-ik_{21}a} (e^{ik_{11}a} (k_{11} - k_{22}) (i\mathcal{F}_0 - k_{21}\Phi_0) - e^{ik_{21}a} (k_{21} - k_{22}) (i\mathcal{F}_0 - k_{11}\Phi_0))}{(k_{21} - k_{11}) (k_{22} - k_{12})}, \]

\[\Phi_{22} = \frac{e^{-ik_{21}a} (e^{ik_{21}a} (k_{21} - k_{12}) (i\mathcal{F}_0 - k_{11}\Phi_0) - e^{ik_{11}a} (k_{11} - k_{12}) (i\mathcal{F}_0 - k_{21}\Phi_0))}{(k_{21} - k_{11}) (k_{22} - k_{12})}.\]

Note that for the second media the generalized de Broglie’s wavenumbers are

\[
\begin{align*}
k_{12} &= \sqrt{\frac{\gamma^2 - \mu_2 + \Phi_{02}}{2} - \xi_2^2} - \sqrt{\frac{(\gamma^2 - \mu_2 + \Phi_{02})^2}{4} - 1}, \\
k_{22} &= \sqrt{\frac{\gamma^2 - \mu_2 + \Phi_{02}}{2} - \xi_2^2} + \sqrt{\frac{(\gamma^2 - \mu_2 + \Phi_{02})^2}{4} - 1},
\end{align*}
\]
FIG. 6: The electrostatic energy wavefunction profiles in electron beam transport through binary Aluminium-Silver metallic junction for different parameter values. The parameter $a$ indicates the junction position in each case. Different regions are shown in different colors in color-online version for better resolution. The space parameter $x$ is normalized to the plasmon wavelength $\lambda_p = 2\pi/k_p$.

where $\Phi_a$ is given by (30) and $\Phi_{02}$ is the arbitrary constant electric potential energy in the second medium.

Figure 6 shows the electron beam transport from metallic Aluminium with $\mu_{Al} = 0.78$ and $\xi_{Al} = 0.25$ to the Silver with parameters $\mu_{Ag} = 1.57$ and $\xi_{Ag} = 0.37$. The temperature parameter for calculation of screening parameter $\xi(\mu, \theta)$ is take $\theta = T/T_p = 0.1$ for all plots in the paper. The background electrostatic energy is taken to be equal for two medium
FIG. 7: The electrostatic energy wavefunction profiles in electron beam transport through binary Aluminium-Cesium metallic junction for different parameter values. The parameter $a$ indicates the junction position in each case. Different regions are shown in different colors in color-online version for better resolution. The space parameter $x$ is normalized to the plasmon wavelength $\lambda_p = 2\pi/k_p$.

$\Phi_{01} = \Phi_{02} = \Phi_0$. Figure 6(a) reveals that the wave-like oscillation wavelength in Silver is comparable larger while that of the particle-like oscillation is nearly the same. Also, the total amplitude of electrostatic energy is the same for both regions. Figure 6(b) shows a plot with same parameters as (6) a but a higher background energy $\Phi_0 = 2$. The amplitude of oscillations in this case becomes slightly higher than that of Fig. 6(a). Figures 6(c) and 6(d)
show similar plot as 6(a) and 6(b) but lowered fractional beam speed parameter $\gamma = 1.5$. The pattern of oscillations greatly changes due to this decrease. The oscillation amplitude of transmitted beam has significantly amplified in both plots 6(c) and 6(d) with different background energies. It is however remarked that the increase in background energy for smaller $\gamma$ has led to decrease in output beam amplitude as is contrary to the case for Fig. 6(b).

Figure 7 shows the same situation for Al-Cs junction beam transport. Cesium has the corresponding parameters $\mu_{Cs} = 0.54$ and $\xi_{Cs} = 1.14$ ($\theta = 0.1$) It is seen that Cesium has over-critical screening value ($\xi > 1$). This means that the wave-like branch is strongly damped for $\gamma > \sqrt{\mu - \Phi_0 + \frac{2}{\xi}}$. Figure 7(a) shows that the beam excitation in Aluminium is dual-tone while that in the Cesium side the long wavelength wave-like branch has been filtered out. Such a metallic junction may have technological application in monotonic beam creation. One of the applications is in the quantum interference effect or crystallography where monotonic beam is needed. Figure 7(b) shows that the amplitude oscillations increase with increase of the background energy similar to the case of Fig. 6. Decrease of the fractional beam speed has similar effect as in Fig. 6(c) and 6(d) which are revealed in Figs. 7(c) and 7(d). It is evident that decrease of the fractional beam speed has also significant effect on the output beam wavelength in this case. For smaller fractional speed $\gamma$ increase in $\Phi_0$ again lowers the overall amplitude of oscillations significantly in Cs regions.

We are now in a position to formulate multijunction beam transport phenomenon. Adding another junction via the third medium with its own parameters we have

$$\Phi_3(x) = \Phi_{13} \exp(ik_{13}x) + \Phi_{23} \exp(ik_{23}x),$$

(33)

with fundamental de Broglie’s wavenumbers in the third region given as

$$k_{13} = \sqrt{\frac{\gamma^2 - \mu_3 + \Phi_{03}}{2} - \xi_3^2 - \sqrt{\frac{(\gamma^2 - \mu_3 + \Phi_{03})^2}{4} - 1}},$$

(34a)

$$k_{23} = \sqrt{\frac{\gamma^2 - \mu_3 + \Phi_{03}}{2} - \xi_3^2 + \sqrt{\frac{(\gamma^2 - \mu_3 + \Phi_{03})^2}{4} - 1}}.$$  

(34b)

The coefficients $\Phi_{13}$ and $\Phi_{23}$ can be obtained through the continuity conditions $\Phi_{1b} = \Phi_{2b} = \Phi_b$ and $\mathcal{F}_{1b} = \mathcal{F}_{2b} = \mathcal{F}_b$ where $x = b$ is the second junction place. If there are $n$ medium with $n - 1$ junctions placed at $x_j$ assuming that $x_0 = 0$ is the start position one may write
the generalized beam wavefunction as

$$\Phi_j(x) = \Phi_{1j} \exp (ik_{1j}x) + \Phi_{2j} \exp (ik_{2j}x),$$

(35)
FIG. 9: The electrostatic energy wavefunction profiles in electron beam transport through ternary Aluminium-Cesium-Silver metallic junction with different configurations and for different parameter values. The parameter \(a\) indicates the junction position in each case. Different regions are shown in different colors in color-online version for better resolution. The space parameter \(x\) is normalized to the plasmon wavelength \(\lambda_p = 2\pi/k_p\).
where

\[
k_{1j} = \sqrt{\frac{\gamma^2 - \mu_j + \Phi_{0j}}{2} - \xi_j^2 - \sqrt{\frac{(\gamma^2 - \mu_j + \Phi_{0j})^2}{4} - 1}},
\]

\[
k_{2j} = \sqrt{\frac{\gamma^2 - \mu_j + \Phi_{0j}}{2} - \xi_j^2 + \sqrt{\frac{(\gamma^2 - \mu_j + \Phi_{0j})^2}{4} - 1}}.
\]

A matrix method can be developed to obtain the wavefunction in the consecutive media based on repetitive application of continuity conditions at junctions. For instance the potential coefficients \(\Phi_{1j}\) and \(\Phi_{2j}\), in which the index \(j\) characterizes the medium number, can be found in terms of potential values at \(j - 1\) junction position as follows

\[
\begin{pmatrix}
\Phi_{1j} \\
\Phi_{2j}
\end{pmatrix} = \frac{1}{k_{2j} - k_{1j}} \begin{pmatrix}
 k_{2j}e^{-i k_{1j} x_{j-1}} & -i e^{-i k_{1j} x_{j-1}} \\
 -k_{1j}e^{-i k_{2j} x_{j-1}} & i e^{-i k_{2j} x_{j-1}}
\end{pmatrix}
\begin{pmatrix}
\Phi_{x_{j-1}} \\
F_{x_{j-1}}
\end{pmatrix}.
\]

The potential coefficients in \(j\)-th media is also obtained in terms of the potential coefficients in \((j - 1)\)-th media as

\[
\begin{pmatrix}
\Phi_{1j} \\
\Phi_{2j}
\end{pmatrix} = \frac{1}{k_{2j} - k_{1j}} \begin{pmatrix}
 k_{2j}e^{i k_{1j} x_{j-1} - i k_{1j} x_{j-1}} & -i e^{i k_{1j} x_{j-1} + i k_{2j} x_{j-1}} \\
 k_{1j}e^{i k_{1j} x_{j-1} - i k_{2j} x_{j-1}} & k_{2j}e^{i k_{2j} x_{j-1} - i k_{2j} x_{j-1}}
\end{pmatrix}
\begin{pmatrix}
\Phi_{x_{j-1}} \\
\Phi_{x_{j-1}}
\end{pmatrix}.
\]

The potential component in terms of the initial values \(\Phi_0\) and \(F_0\) is given as

\[
\begin{pmatrix}
\Phi_{1j} \\
\Phi_{2j}
\end{pmatrix} = \prod_{j=n} \Theta_j
\begin{pmatrix}
\Phi_0 \\
F_0
\end{pmatrix},
\]

in which \(\Theta_j\) is the transport matrix given by

\[
\Theta_j = \frac{1}{k_{2j} - k_{1j}} \begin{pmatrix}
 k_{2j}e^{i k_{1j} x_{j-1} - i k_{1j} x_{j-1}} & -i e^{-i k_{1j} x_{j-1} + i k_{2j} x_{j-1}} \\
 k_{1j}e^{i k_{1j} x_{j-1} - i k_{2j} x_{j-1}} & k_{2j}e^{i k_{2j} x_{j-1} - i k_{2j} x_{j-1}}
\end{pmatrix}.
\]

In Figure 8 we have depicted the plasmonic beam potential profile in a triple junction system. A metallic sandwich consisting of two configurations Al-Ag-Al and Ag-Al-Al are shown in Fig. 8. Figure 8(a) shows that electrostatic energy oscillations in Aluminium regions are quite similar. Compared to these regions in the middle part (Ag) the wave-like oscillations have larger wavelength while particle-like oscillations have relatively smaller wavelength. However, the overall amplitude of oscillations are nearly the same in all regions. On the other hand, Fig. 8(b) shows the effect of region interchange in metallic junction order on
the oscillations in all regions. It is remarked that this system the oscillation amplitudes and wavelengths transparently transport through the moddile region irrespective of the metal order.

A ternary composition with Aluminium, Cesium and Silver is analyzed in Fig. 9. Figure 9(a) shown a rather interesting effect. It is remarked that the Cesium column indeed acts as a wave-like oscillation filter even if a third column metallic appears at the end. It is seen that the particle-like oscillations proceed to the third media without changes to the particle-like wavenumber. Seemingly the monotonic particle-like excitations are amplified in Cesium contact. This is rather clearly confirmed by Fig. 9(b) where the Cesium column has been moved to the end. For the lower fractional beam speed shown in Figs. 9(c) and 9(d) we see a double-tone oscillation in the Silver region of Fig. 9(c). However, for configuration shown in Fig. 9(d) oscillations in Cesium are purely particle-like.

The electron density distribution in beam may be assumed to be of the similar form as the electrostatic energy in every plasmon region \( j \),

\[ \Psi_j(x) = \Psi_{1j} \exp(ik_{1j}x) + \Psi_{2j} \exp(ik_{2j}x) \]

in which the coefficients \( \Psi_{1j} \) and \( \Psi_{2j} \) can be obtained from the following conditions

\[ \Psi_j(x)\Psi_j^*(x) = n_j, \quad \frac{\hbar}{2m} \left[ \Psi_j^*(x)\frac{d\Psi_j(x)}{dx} - \Psi_j(x)\frac{d\Psi_j^*(x)}{dx} \right] = J_j(x). \]  

(41)

If we assume that at \( x = 0 \) we have \( n = n_0 \) for \( j = 1 \) and use the normalization scheme we find

\[ \Psi_{11} = \frac{2k_{21} - e\gamma k_p}{k_{21} - k_{11}}, \quad \Psi_{21} = -\frac{2k_{11} - e\gamma k_p}{k_{21} - k_{11}}, \]  

(42)

where we have used \( mv_p = \hbar k_p \) and \( \gamma = v/v_p \) in which \( v \) is the average electron beam speed. It is remarked that the two-tone character of spatial distribution in electron density makes the statistical probability of finding an electron at position \( x \) depend on both particle-like and wave-like collective oscillations. Note that the critical values of beam speed \( \gamma = 2k_1/e k_p \) and \( \gamma = 2k_2/e k_p \) to the single-tone plane-wave propagation of the beam. The generalized coefficients for beam density in the \( j \)th media is obtained in a similar manner as Eq. (39). Continuity must hold for \( \Psi_j(x) \) and \( d\Psi_j(x)/dx \) at all junctions.

In current model while using the quantum charge screening effect we have ignored the energy band-gap effect. However, it is possible to incorporate such effect in current model via the effective mass formulation which is beyond the scope of current analysis.
VI. CONCLUSION

We developed a dual-scale-length plasmon theory of monochromatic quantum electron beam propagation using the normal-mode analysis of the linearized pseudoforce method within the framework of quantum hydrodynamic theory. The use of generalized de Broglie wavenumbers allowed us for a detailed analysis of beam-plasmon electrostatic wavefunction and instabilities with variations in large parameter space such as the beam-speed, chemical potential, screening parameter and background electrostatic energy. We applied the theory to realistic binary and ternary metallic junction systems to show specific applications of current investigation. Moreover, current development provides a useful method for separate delicate manipulation of the wave and particle aspects of plasmon excitation in quantum systems specifically the monochromatic (dual-tone and single frequency) electron beam by passing through different. We believed that current study can help in better physical understanding of quantum effects in beam plasma interactions and further illuminates the particle-wave energy exchange in quantum environments.

VII. DATA AVAILABILITY

Data of current analysis is available upon reasonable request from the author.

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