ACDC: $\alpha$-Carving Decision Chain for Risk Stratification

Yubin Park  
Accordion Health, Inc., 4200 N. Lamar Blvd., Austin, TX 78756 USA

Joyce Ho  
Emory University, 400 Dowman Dr, Atlanta, GA 30322 USA

Joydeep Ghosh  
The University of Texas at Austin, Austin, TX 78712 USA

Abstract

In many healthcare settings, intuitive decision rules for risk stratification can help effective hospital resource allocation. This paper introduces a novel variant of decision tree algorithms that produces a chain of decisions, not a general tree. Our algorithm, $\alpha$-Carving Decision Chain (ACDC), sequentially carves out “pure” subsets of the majority class examples. The resulting chain of decision rules yields a pure subset of the minority class examples. Our approach is particularly effective in exploring large and class-imbalanced health datasets. Moreover, ACDC provides an interactive interpretation in conjunction with visual performance metrics such as Receiver Operating Characteristics curve and Lift chart.

1. Introduction

Data analytics has emerged as a vehicle for improving healthcare (Meier, 2013) due to the rapidly increasing prevalence of electronic health records (EHRs) and federal incentives for meaningful use of EHRs. Data-driven approaches have provided insights into diagnoses and prognoses, as well as assisting the development of cost-effective treatment and management programs. However, there are two key challenges to the development of health data analytic algorithms: 1) noisy and multiple data sources issues, and 2) interpretability issues.

A decision tree is a popular data analytics and exploratory tool in medicine (Podgorelec et al., 2002; Lucas & Abu-Hanna, 1997; Yoo et al., 2012) because it is readily interpretable. Decision tree algorithms generate tree-structured classification rules, which is written as a series of conjunctions and disjunctions of the features. Decision trees can produce either output scores (a positive class ratio from a tree node) or binary classes (0/1). Not only are the classification rules readily interpretable by humans, but also the algorithms naturally handle categorical and missing data. Therefore, various decision trees have been applied to build effective risk stratification strategies (Fonarow et al., 2005; Chang et al., 2005; Goto et al., 2009).

We believe that decision trees for risk stratification can be improved from two aspects. First, many existing approaches to class-imbalance problems typically rely on either heuristics or domain knowledge (Chawla & Bowyer, 2002; Japkowicz, 2000; Domingos, 1999). Although such treatments may be effective in some applications, many of them are post-hoc; the splitting mechanism of a decision tree usually remains invariant. Second, even the logical rules from a decision tree can be overly complex, especially with class-imbalanced data. Furthermore, the conceptual gap between decision thresholds and decision rules complicates interpretation on visual performance metrics such as the receiving operating character (ROC) curve and the lift chart.

We propose $\alpha$-Carving Decision Chain (ACDC), a novel variant of decision tree algorithms, that produces a chain of decisions rather than a tree. Conceptually, ACDC is a sequential series of rules, applied one after another, where the ratio of positive class increases over the sequence of decision rules (i.e. monotonic risk condition). Figure 1 presents a comparison between a decision tree and a decision chain. Thus, the decision order creates a noticeable difference in the number of distinct rules. The idea of constructing a decision chain has been recently explored using Bayesian models (Wang & Rudin, 2015; Yang et al., 2016). These models have shown
promising results in terms of interpretability and predictive performance. ACDC can be viewed as an alternative to such models. Our greedy chain growing strategy is particularly well-suited when exploring large and class-imbalanced datasets.

ACDC is based on the $\alpha$-Tree framework (Park & Ghosh, 2012) developed for imbalanced classification problems. The key idea of ACDC is to sequentially carving out “pure” subsets of majority examples (the definition of purity is given in Section 2.2). Each subsequent step in ACDC yields a higher minority class ratio than the previous steps. The step-wise approach allows our algorithm to scale readily to large data and handle class-imbalance problems. We demonstrate ACDC on two real health datasets and show that our algorithm produces outputs that are concise and interpretable with respect to visual performance metrics, and achieves predictive performance comparable to traditional decision trees. ACDC can be used for various healthcare applications, including (i) symptom development mining, (ii) step-wise risk-level identification, (iii) early characterization of easily identifiable pure subsets, and (iv) decision thresholds determination with decision rules.

2. ACDC: $\alpha$-Carving Decision Chain

ACDC is motivated by the need to interpret large and class-imbalanced healthcare datasets. While exploring several such datasets, we have frequently observed that negative class examples can be easily carved out with simple rules. We initially attempted to apply various heuristics, such as cost-weighted scoring functions (Buja & Lee, 2001; Buja et al., 2005), to construct such rules, but realized that this approach does not scale with different types of datasets. Every time we encountered a new dataset, we needed new domain knowledge to filter out negative class examples. Thus, we developed ACDC, a novel variant of decision tree algorithms. ACDC produces a chain of decisions by sequentially carving out pure subsets of the majority class ($Y = 0$) examples, and provides a systematic approach to construct such filtering rules.

To achieve this goal, we introduce (i) a new criterion for selecting a splitting feature, (ii) its implication, and then (iii) using the criteria, a simple dynamic strategy to grow a one-sided tree.

2.1. Selecting a Splitting Feature

The splitting criterion for the $\alpha$-Tree (Park & Ghosh, 2012) selects the feature with the maximum $\alpha$-divergence (Amari, 2007; Cichocki & ishi Amari, 2010) between the following two distributions:

$$P(X_i, Y) \leftrightarrow P(X_i)P(Y)$$

The reference distribution is set to the product of the two marginals; if both are independent then the reference distribution is equivalent to the joint distribution. In other words, $\alpha$-Tree selects a splitting feature that maximizes the dissimilarity between the joint and marginal distributions.

Although the $\alpha$-Tree criterion is conceptually simple, it is difficult to control and analyze. Instead, we simplify the reference distribution as follows:

$$U(X_i, Y) = U(Y | X_i)U(X_i)$$

s.t. $U(Y | X_i) = \frac{1}{2}$ and $U(X_i) = P(X_i)$

The reference distribution $U(X_i, Y)$ changes with respect to a feature $X_i$. Integrating $U(X_i, Y)$ over $Y$ yields a distribution that is the same as the marginal of $X_i$. Furthermore, given $X_i$, the reference distribution becomes the uniform distribution as it has no information on $Y$.

We modify the $\alpha$-divergence criterion to select the feature that provides the maximum distance between $P(X_i, Y)$ and $U(X_i, Y)$. Therefore, the ACDC-criterion is the following:

$$\arg\max \frac{1}{\alpha(1-\alpha)}(1 - \frac{1}{2} \sum_{x_i,y} P(x_i)(2P(y | x_i))^\alpha)$$

This particular choice of the reference distribution may appear somewhat contrived. However, the reference distribution automatically captures the splitting criteria of both C4.5 and CART as special cases. From the ACDC-criterion, we can obtain the information gain criterion (C4.5) by setting $\alpha = 1$ and the Gini criterion (CART) by setting $\alpha = 2$. For example, using $\alpha \rightarrow 1$ and L'Hôpital’s rule, we can obtain the information gain criterion.

2.2. Meaning of ACDC-criterion

We define a new quantity $A(p, \alpha)$, the $\alpha$-zooming factor (az.factor), as follows:
\[ A(p, \alpha) = p^\alpha + (1 - p)^\alpha \]

The az.factor can have different interpretations:

- \( \| (p, 1 - p) \|_\alpha^2 \), where \( \| \cdot \|_\alpha \) is \( L^\alpha \) norm
- a generalized entropy index of \( P(Y) = (p, 1 - p) \) in economics literature (Ullah & Ciles, 1998)
- a generalized diversity index of \( P(Y) = (p, 1 - p) \) in ecological literature (Simpson, 1949; Moreno & Rodriguez, 2010; Jost, 2006)

In this paper, we simply use az.factor as a parametrized purity measure and are more interested in its functional role in the ACDC-splitting criterion.

Under the condition \( \alpha > 1 \), we can rearrange the terms to obtain:

\[
\max_{X_i} D_{\alpha}(P(X_i, Y) \parallel U(X_i, Y)) = \max_{X_i} A(PPV_i, \alpha) P(Y_i = 1) + A(NPV_i, \alpha) P(Y_i = 0)
\]

where PPV and NPV represent positive and negative predictive values, respectively. Notice that \( \alpha \) emphasizes or zooms on these values: PPV and NPV. As \( \alpha \) increases, the splitting criterion prefers higher \( P(Y | X) \):

- \( \alpha \uparrow \): more focus on the PPV and NPV terms
- \( \alpha \downarrow \): more focus on the balance terms

Therefore, lower values of \( \alpha \) result in more balanced splits, and higher values of \( \alpha \) provide very sharp PPV and NPV values (i.e., a pure subset of either the majority or the minority classes).

### 2.3. Growing a Decision Chain

Our strategy to build a monotonic decision chain is to gradually decrease the value of \( \alpha \). This is motivated by the following two observations. First, at each subsequent stage, we have a smaller number of samples. To prevent biased splits, \( \alpha \) should be appropriately adjusted to the current sample size. Second, as a chain grows, we have a more balanced class ratio at each stage. If \( \alpha \) remains too high, then both PPV and NPV terms numerically have little effect.

We introduce an \( \alpha \)-carving strategy to adjust \( \alpha \) accordingly. At each stage of a decision chain, we set \( \alpha \) as follows:

\[
\text{Find } \alpha \text{ s.t. } \nu = \left. \frac{\partial A(\omega, \alpha)}{\partial \omega} \right|_{\omega = \omega_y}
\]

where \( \nu \) is a predefined velocity parameter, and \( \omega_y \) is defined as \( \max(P(Y_i), 1 - P(Y)) \). As the decision chain builds up, the value of \( \alpha \) decreases.

The overall steps of the ACDC algorithm is as follows:

1. Set the value of \( \nu \)
2. Find an appropriate \( \alpha \)
3. Find a feasible set of splitting variables that satisfy the monotonic risk condition
4. Find a splitting variable from the feasible splitting variable set
5. Discard the majority class examples
6. Repeat from Step 2

Note that ACDC grows only one branch unlike decision trees. The parameter \( \nu \) controls the size of the chain. A low \( \nu \) typically results in a large \( \alpha \) and obtains chains that tend to be longer with small-sized partitions. On the other hand, a large \( \nu \) produces a shorter chain with big partitions.

### 3. Experimental Results

We provide the experimental results of ACDC on MIMIC-II database (Saeed et al., 2011) focusing on two different conditions (septic shock and asystole). For each condition, we will compare the performance with C4.5 and CART and other kinds of alpha trees, show how the cutting plane changes with different values of \( \alpha \), and display rule-annotated ROC and Lift charts resulting from ACDC.

The MIMIC-II database is one of the largest publicly available clinical databases. The database contains more than 30K patients and 40K ICU admission records. For this paper, we concentrate on two subsets of the database, specifically 1) patients with systemic inflammatory response syndrome (SIRS) for septic shock prediction, and 2) patients with or without cardiac arrests for asystole prediction. The features are derived primarily from non-invasive clinical measurements and include blood pressure (systolic and diastolic measurements), body temperature, heart rate, respiratory rate, and pulse oximetry. For each measurement, we use the last observed measurement and three additional sets of derived features: max, min, average values within the last 12 hours.

**Septic Shock.** We first illustrate the results from the septic shock dataset. Septic shock is defined as “sepsis-induced hypotension, persisting despite adequate fluid resuscitation, along with the presence of hypo perfusion abnormalities or organ dysfunction” (Bone et al., 1992). The time of septic shock onset was defined using the criteria outlined in a recent work on septic shock prediction (Ho et al., 2012). For this subset, there is a total of 1359 patients with 213 transitioning to septic shock. We use ACDC and decision trees to predict if a patient will enter septic shock 1 hour prior to shock onset.
ACDC: \(\alpha\)-Carving Decision Chain for Risk Stratification

Figure 2. (a) Different decision cuts with different \(\alpha\) values. The cut with \(\alpha = 64\) provides an extremely pure subset of non-septic shock patients. (b) The performance of ACDC is comparable to that of \(\alpha\)-Trees with \(\alpha = 1\) (C4.5) and \(\alpha = 2\) (CART).

Figure 3. ROC curve and Lift chart. Every edge point on the curves is associated with a decision rule.

Figure 2 (a) shows the first cuts of decision trees with the data collected 1 hour before septic shock. The information gain criterion (\(\alpha = 1\)) selects the cut with systolic=96 mmHg, which almost coincides with the definition of the septic shock. However, this cut results in a large portion of false negatives. On the other hand, high values of \(\alpha\) reduce such false negatives. The resultant cuts are rather conservative (smaller number of patients are classified as negatives) but produce a pure subset of non-septic shock patients.

Figure 2 (b) shows the AUC for both 1 hour and 2 hours before shock. The decision trees are grown until they reach tree-depth 3 and ACDCs were grown until they reach chain-depth 4. As can be seen, the performance of ACDC is statistically comparable with that of decision trees. Figure 3 shows the rule-annotated ROC curves and Lift charts.

Cardiac Arrest. For the second MIMIC-II subset, we use decision trees to predict a cardiac arrest event, specifically asystole. Cardiac arrest is a deadly condition caused by a sudden failure of heart and is often synonymous with clinical death. Early response to cardiac arrest can reduce the mortality rate from 80% to 60%. For this subset, there is a total of 3590 patients with 361 diagnosed with asystole.

Figure 4 (a) illustrates the predictive performance measured by AUC. ACDC results in better predictive performance compared to the other tree-based methods. Furthermore, the decision chains are visually interpretable. Figure 4 (b) shows the rule-annotated Lift charts. From these charts, we can observe that asystole patients are characterized by low heart beat rates and low pulse oximetry values. Figure 5 illustrates the extracted decision chain visualized using a pyramid diagram.

4. Discussions

This paper introduces a novel variant of decision tree algorithms, \(\alpha\)-Carving Decision Chain (ACDC). Our algorithm produces a chain of decisions to sequentially carve out “pure” subsets of the majority class samples, leveraging two newly developed techniques: (i) ACDC splitting criterion and (ii) \(\alpha\)-carving strategy. As a result, the chain of decision rules reatively leads to a pure subset of the minority class examples.

ACDC is a greedy alternative to a general framework known as Rule Lists (Wang & Rudin, 2015). Moreover, our approach is particularly well-suited when exploring large and class-imbalanced datasets. While a decision chain may seem too restrictive, our empirical results suggest that a chain structure achieves almost similar predictive performance as normal trees in many cases. Moreover, the resulting chain of decisions provide an intuitive interpretation in conjunction with visual performance metrics such as ROC curve and Lift chart.
References

Amari, Shun-ichi. Integration of stochastic models by minimizing $\alpha$-divergence. *Neural Comput.*, 19(10): 2780–2796, 2007.

Bone, R. C., Balk, R. A., Cerra, F. B., Dellinger, R. P., Fein, A. M., Knaus, W. A., Schein, R. M., and Sibbald, W. J. Definitions for sepsis and organ failure and guidelines for th e use of innovative therapies in sepsis. *The ACCP/SCCM Consensus Conference Committee*, 1992.

Buja, Adreas, Stuetzle, Werner, and Shen, Yi. Loss functions for binary class probability estimation and classification: Structure and applications. *University of Pennsylvania: Technical report*, 2005.

Buja, Andreas and Lee, Yung-Seop. Data mining criteria for tree-based regression and classification. *Proceedings of the seventh ACM SIGKDD International conference on Knowledge discovery and data mining*, 2001.

Chang, Howard Y., Nuyten, Dimitry S. A., Sneddon, Julie B., Hastir, Trevor, Tibshirani, Robert, Sorile, Theresse, Dai, Hongyue, He, Yudong D, van’t Veer, Laura J., Barterlink, Harry, van de Rijn, Matt, Brown, Patrick O., and van de Vijver, Marc J. Robustness, scalability, and integration of a wound-response gene expression signature in predicting breast cancer survival. *Proceedings of the National Academy of Science of the United States of America*, 102(10), 2005.

Chawla, Nitesh V. and Bowyer, Kevin W. Smote: Synthetic minority over-sampling technique. In *Journal of Artificial Intelligence Research*, volume 16, pp. 321–357, 2002.

Cichocki, Andrzej and ishi Amari, Shun. Families of alpha-beta- and gamma- divergences: Flexible and robust measures of similarities. *Entropy*, 2010.

Domingos, Pedro. Metacost: A general method for making classifiers cost-sensitive. In *Proceedings of the Fifth International Conference on Knowledge Discovery and Data Mining*, pp. 155–164, 1999.

Fonarow, Gregg C, Adams, Kirkwood F., Abraham, William T., Yancy, Clyde W., and Boscardin, John. Risk stratification for in-hospital mortality in acutely decompensated heart failure. *The Journal of the American Medical Association*, 293(5), 2005.

Goto, Masahi, Kawamura, Takashi, Wakai, Kenji, Ando, Masahiko, Endoh, Masayuki, and Tomino, Yasuhiko. Risk stratification for progression of IgA nephropathy using a decision tree induction algorithm. *Nephrology Dialysis Transplantation*, 24(4):1242–1247, 2009.

Ho, Joyce C, Lee, Cheng H, and Ghosh, Joydeep. Imputation-Enhanced Prediction of Septic Shock in ICU Patients. In *ACM SIGKDD Workshop on Health Informatics (HI-KDD 2012)*, 2012.

Japkowicz, Nathalie. The class imbalance problem: Significance and strategies. In *Proceedings of the 2000 International Conference on Artificial Intelligence*, pp. 111–117, 2000.

Jost, Lou. Entropy and diversity. *Oikos*, 2006.

Lucas, Peter and Abu-Hanna, Ameen. Prognostic methods in medicine. *UU-CS*, 1997.

Meier, Carlos. A role for data: An observation on empowering stakeholders. *American Journal of Preventive Medicine*, 2013.

Moreno, Claudia E. and Rodriguez, Pilar. A consistent terminology for quantifying species diversity. *Oecologia*, 2010.

Park, Yubin and Ghosh, Joydeep. Ensembles of $\alpha$-Trees for Imbalanced Classification Problems. *IEEE Transactions on Knowledge and Data Engineering*, 2012.

Podgorelec, Vili, Kokol, Peter, Stiglic, Bruno, and Rozman, Ivan. Decision trees: an overview and their use in medicine. *Journal of Medial Sysstems*, 2002.

Saeed, Mohammed, Villarroel, Mauricio, Reisner, Andrew T., Clifford, Gari, Lehman, Li-Wei, Moody, George, Heldt, Thomas, Kyaw, Tin H., benjamin Moody, and Mark, Roger G. Multiparameter intelligent monitoring in intensive care ii (MIMIC-II): A public access intensive care unit database. *Critical Care Medicine*, 2011.

Simpson, Edward H. Measurement of diversity. *Nature*, 1949.

Ullah, Aman and Ciles, David E. A. *Handbook of Applied Economic Statistics*. CRC Press, 1998.

Wang, Fulton and Rudin, Cynthia. Falling rule lists. In *Proceedings of Artificial Intelligence and Statistics (AISTATS)*, 2015.

Yang, Hongyu, Rudin, Cynthia, and Seltzer, Margo. Scalable bayesian rule lists. In *arXiv:1602.08610*, 2016.

Yoo, Ilhoi, Alafaireet, Patricia, Marinov, Miroslav, Penahernandez, Keila, Gopidi, Rajitha, Chang, Jia-Pu, and Hua, Lei. Data mining in healthcare and biomedicine: A survey of the literature. *Journal of Medical Systems*, 2012.