Superluminal Black Holes

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ABSTRACT

The new solution of the Einstein equations in empty space is presented. The solution is constructed using Schwarzschild solution but essentially differs from it. The basic properties of the solution are: the existence of a horizon which is a hyperboloid of one sheet moving along its axis with superluminal velocity, right signature of the metric outside the horizon and Minkovsky-flatness of it at infinity outside the horizon. There is also a discussion in the last chapter, including comparing with recent astronomical observations.

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1 The metric

Let $g_{\mu\nu}(\mu, \nu = 0, 1, 2, 3)$ be a Schwarzschild metric [2]:

$$dl^2 = g_{\mu\nu}dx^\mu dx^\nu = -(1 - \frac{2M}{r})dt^2 + (1 - \frac{2M}{r})^{-1}dr^2 + r^2(d\theta^2 + \sin^2 \theta d\phi^2)$$

(1)

where $r, \theta, \phi$ are connected with Cartesian coordinates via:

$$x = r \sin \theta \cos \phi$$

$$y = r \sin \theta \sin \phi$$

$$z = r \cos \theta$$

This metric is a well-known solution of the Einstein equations with energy-momentum tensor $T_{\mu\nu}$ equals to zero:

$$R_{\mu\nu} = 0$$

(2)

where $R_{\mu\nu}$ is a Ricci tensor.

This metric describes the geometry of a spherically symmetric black hole and the external gravitational field of a spherically symmetric mass $M$.

Now let us write down the metric which corresponds to a moving black hole (with some velocity, say $v$). In order to do this write down the Schwarzschild metric (1) in Cartesian coordinates $(t, x, y, z)$:

$$dl^2 = -qdt^2 +$$

$$+ \frac{1}{qr^2}(x^2dx^2 + y^2dy^2 + z^2dz^2 + 2xydxdy + 2xzdxz + 2yzdydz) +$$

$$+ \frac{1}{r^2}((y^2 + z^2)dx^2 + (x^2 + z^2)dy^2 + (x^2 + y^2)dz^2 -$$

$$- 2xydxdy - 2xzdxz - 2yzdydz)$$

(3)

where we denote $r \equiv \sqrt{x^2 + y^2 + z^2}$ and $q \equiv 1 - \frac{2M}{r}$.

Now passing to a moving along $x$-axis system of coordinates and performing Lorentz transformation (boost along $x$-axis):

$$t' = \frac{t + vx}{\sqrt{1 - v^2}}$$

$$x' = \frac{x + vt}{\sqrt{1 - v^2}}$$

$$y' = y$$

$$z' = z$$
we obtain a metric which certainly satisfies the Einstein equations (2) (they are generally covariant) and corresponds to a moving black hole or a star:

\[
\begin{align*}
\text{d}t^2 &= -\frac{1}{(1-v^2)r^2}(qr^2 - \frac{q^{-1}v^2}{1-v^2}(x-vt)^2 - v^2(y^2+z^2))\text{d}t^2 - \\
&\quad - \frac{2v}{(1-v^2)r^2}(-qr^2 + \frac{q^{-1}}{1-v^2}(x-vt)^2 + y^2 + z^2)\text{d}tdx - \\
&\quad - \frac{2v(q^{-1} - 1)}{1-v^2}(x-vt)y\text{d}t\text{dy} - \frac{2v}{1-v^2}v(q^{-1} - 1)(x-vt)z\text{d}t\text{dz} + \\
&\quad + \frac{1}{(1-v^2)r^2}(-v^2qr^2 + \frac{q^{-1}}{1-v^2}(x-vt)^2 + y^2 + z^2)\text{d}x^2 + \\
&\quad + \frac{1}{r^2}(-\frac{1}{1-v^2}(x-vt)^2 + q^{-1}y^2 + z^2)\text{d}y^2 + \frac{1}{r^2}\frac{1}{1-v^2}(x-vt)^2 + y^2 + q^{-1}z^2\text{d}z^2 + \\
&\quad + \frac{2q^{-1} - 1}{1-v^2}(x-vt)y\text{d}x\text{dy} + \frac{2q^{-1} - 1}{1-v^2}(x-vt)z\text{d}x\text{dz} + 2(q^{-1} - 1)\frac{y^2}{r^2}\text{d}y\text{dz} + 2(q^{-1} - 1)\frac{z^2}{r^2}\text{d}x\text{dz} + 2(q^{-1} - 1)\frac{y^2}{r^2}\text{d}y\text{dz}
\end{align*}
\]

(4)

where we denote \( r \equiv \sqrt{\frac{1}{1-v^2}(x-vt)^2 + y^2 + z^2} \) and omit the primes.

Let us note four essential points about the obtained metric:

1. It is a solution of the Einstein equation at any value of \(|v| < 1\);
2. It is asymptotically flat, i.e. becomes Minkovsky at infinity \((r \to \infty)\);
3. It has Schwarzschild horizon (singularity) at \( r = 2M \);
4. It has the right signature \((-+++)\) outside the horizon, i.e. when

\[
\frac{1}{1-v^2}(x-vt)^2 + y^2 + z^2 > (2M)^2
\]

(5)

The crucial observation about the metric is that (4) does not contain the expressions \( \sqrt{1-v^2} \), although the Lorentz transformation does (this is due to the fact that (3) contains only even powers of \( x \) together with \( dx, y \) together with \( dy \) etc.). This means the expression (4) is symmetric bilinear and real-valued form also at the values of \(|v| > 1\) in the domain of definition (see below) and we call it the metric. Now we forget about the Lorentz transformations and the way we obtained (4). We know that for \(-1 < v < 1\) the metric (4) is a solution of the Einstein equations which themselves do not depend on parameter \( v \). Hence, for \(|v| > 1\) the expression (4) gives us the solution of the Einstein equations as well (let us note that the components of the metric (4) are holomorphical functions on \( v \) except points \( v = \pm 1 \)).

Now we should examine the basic properties of the obtained solution. Let’s fix some value of \( v \) \((|v| > 1)\). Then the region of the space-time where components of the metric take real values (i.e. the domain of definition of the metric) is defined by:

\[
\frac{1}{1-v^2}(x-vt)^2 + y^2 + z^2 \geq 0
\]

(6)
At any fixed value of $t$ this is 3-dimensional space except elliptic cone, axis of which coincides with $x$-axis. And as $t$ grows the picture moves along the $x$-axis with velocity $v$ ($|v| > 1$). This cone is the analogue of the point singularity in the Schwarzschild metric.

We want to stress that the obtained metric at $|v| > 1$ cannot be transformed via coordinate transformations to usual Schwarzschild metric (because the domain of definition of this metric is not topologically equivalent to the domain of definition of the Schwarzschild metric) and, hence, gives us another solution of the Einstein equations.

The obtained solution has a singularity, corresponding to spherical Schwarzschild horizon in a black hole. Now the singularity is a hypersurface defined by:

$$\frac{1}{1 - v^2}(x - vt)^2 + y^2 + z^2 = (2M)^2 \tag{7}$$

When $|v| > 1$ this is a hyperboloid of one sheet coaxial to $x$-axis and moving along it with a velocity $v$.

Next important feature of the metric is that it has the right signature $(-+++)$ in the region (5). This region now is an exterior of the hyperboloid at any fixed value of $t$.

It is easy to see that the obtained solution becomes Minkovsky at infinity at arbitrary direction in 3-dimensional space outside the cones (6)(at all these directions $q \to 1$ at infinity). That is, the solution is asymptotically flat at any direction in the domain of definition (6).

Also it is easy to show that, despite of $|v| > 1$, the horizon (7) is a time-like surface (i.e. normal vector is always space-like), as in case of usual Schwarzschild metric. Due to it there is possibility for a particle to avoid a fall on the horizon with its velocity less than that of light. In fact, we believe the particle in the given geometry will move in the following way: it moves faster and faster, approaching to the horizon and simultaneously sliding along it and its velocity approaches to that of light. The work of finding exact trajectories of the particles is now in progress.

The solution (4) can be rewritten in a simple form, using the new coordinates-$(t', \rho, \chi, \psi)$ defined as follows:

$$t' = \frac{t - vx}{\sqrt{v^2 - 1}}$$
$$\rho \sinh \chi = \frac{x - vt}{\sqrt{v^2 - 1}}$$
$$\rho \cosh \chi \sin \psi = y$$
$$\rho \cosh \chi \cos \psi = z \tag{8}$$

The metric (4) becomes:

$$dl^2 = (1 - \frac{2M}{\rho})dt'^2 + (1 - \frac{2M}{\rho})^{-1}d\rho^2 + \rho^2(-d\chi^2 + \sinh^2 \chi d\psi^2) \tag{9}$$
Direct calculations show that this metric satisfies the Einstein equations (2).

But these coordinates are unphysical because of the signs in the expression above. For example, \( t' \) cannot be considered as a time variable since the term containing \( dt'^2 \) has the wrong sign. But let us stress that the metric itself is "good" in the former coordinates \((t, x, y, z)\), which can be considered as physical(Cartesian) coordinates.

In fact, the solution (4) is not the parametrized family of solutions (with parameter \( v \)) of the Einstein equations but only one solution and all others can be obtained via Lorentz transformation of the given one. We may chose this standard solution in order to factor \( \frac{1}{1-v^2} \) equals to \(-1\), i.e. \( v = \sqrt{2} \). In that case, for obtaining the solution (4) at some value \( v_0 \) (including \( v_0 = \infty \)) one performs Lorentz transformations(boost along \( x \)-axis) with

\[
v_L = \frac{\sqrt{2} - v_0}{\sqrt{2}v_0 - 1}
\]

It is possible to chose as the standard solution the solution (4) with \( v = \infty \). This case will be examined in the next chapter.

2 Limit \( v \to \infty \)

Now let’s see what solution do we obtain when \( v \) approaches to infinity. Substituting \( v \to \infty \) into (4) we find:

\[
dl^2 = -\frac{1}{r^2}(-q^{-1}t^2 + y^2 + z^2)dt^2 - 2(q^{-1} - 1)\frac{ty}{r^2}dtdy - 2(q^{-1} - 1)\frac{tz}{r^2}dtdz + qdx^2 + \frac{1}{r^2}(-t^2 + q^{-1}y^2 + z^2)dy^2 + \frac{1}{r^2}(-t^2 + y^2 + q^{-1}z^2)dz^2 + 2(q^{-1} - 1)\frac{yz}{r^2}dydz
\]

where \( r \equiv \sqrt{y^2 + z^2 - t^2} \) and \( q \equiv 1 - \frac{2M}{r} \).

There is a singularity in this metric, corresponding to the Schwarzschild horizon, when \( q = 0 \), i.e. when

\[
y^2 + z^2 - t^2 = (2M)^2
\]

The sections of this hypersurface by the hyperplanes \( t = \text{const} \) are cylinders coaxial with \( x \)-axis and their radius depend on \( t \) as follows:

\[
R(t) = \sqrt{(2M)^2 + t^2}
\]

The arbitrary solution (4) at some particular value \( v \) can be obtained from this solution via boost along \( x \)-axis with \( v_L = 1/v \).

Calculation shows that in this case\((v = \infty)\) the light, emitted at the horizon (7) will not leave the horizon (as in usual black hole). Since any other solution at any value of \(|v| > 1\) can be obtained from this one via Lorentz transformation, the situation is the same for any \( v \) (at arbitrary \( v \) the horizon is a one-sheet hyperboloid moving along its
axis with a velocity \( v \). Thus, we may say that the horizon (moving with a velocity greater than that of light) has a definite physical properties and it is not an imaginable surface.

3 Discussion, Speculations and Conclusion

The obtained solution was called ”new”, because it was not found in the book [1].

The basic question arising about the obtained solution is its physical meaning. It is well-known that the Einstein equations have nonphysical solutions even if we consider solutions, which become Minkovsky-flat at infinity. The main problem arising is the causality principle violation. The presented solution has the right signature in the exterior of the horizon (hyperboloid) and that’s why the particle in the gravitational field, corresponding to the solution, will move slower then light. Also it seems that the particle cannot fall under the horizon. These are arguments that the causality principle is not violated in this case.

It is possible to make an essential refinement of the solution if we map the domain of definition of the metric (i.e. exterior of elliptic cone at fixed \( t \)) to the whole 4-dimensional space except 2-dimensional plane (which sections by hyperplanes \( t = const \) are the lines, coinciding with hyperboloid axis). These are isotopic, so this mapping(isotopy) is possible. This mapping is ambiguous and it can be chosen in order to transform the horizon from the hyperboloid to a ”spindle” continuing to infinity in the both directions of its edges. Such a solution seems to be more physically valuable. Yet it is not clear, is it possible to arrange the mapping in order to make the metric Minkovsky-flat at infinity at any direction in the resulting space (at least in some range of directions this can be done). This point is very important because if it is so, one can consider the ”spindle”- solution as possibly existing in the Universe.

Thus the General Theory of Relativity(GTR) together with the causality principle do not forbid a movement with a superluminal velocity. Probably, the important condition of such a movement is the presence of a horizon which itself moves faster then light, through which it is impossible for a particle to pass in a finite time and, hence, it is impossible to transfer an information (even just increasing a mass of a superluminal object) with a superluminal velocity. Since the existence of horizons is one of predictions of the GTR, finding out in the Universe superluminal objects would be else one argument in favor of it.

Let us suppose that there are superluminal objects (relatively local) in the Universe and we can see their traces (or maybe radiation of the object). How we would see their movement? The answer is: we would see first appearing of two objects at some place in the Universe and second scattering of them into opposite directions. There is an illusion of a backward movement when an object approaches an observer. And when an object
moves off an observer there is no such illusion. Hence, an observer see a pair of resembling objects scattering into opposite directions.

The presented solution of the Einstein equation gives some theoretical possibility of the existence of such superluminal object. Is there an astronomical evidence of such objects (since the pairs of scattering objects should be quite noticeable)?

The answer is yes. One of the first observations of that kind was made by Arp in 1967 [3]. He observed, when studying peculiar galaxies, that very often a peculiar galaxy lies on the line, joining two radio-sources between them. Such examples are statistically significant, so one may say that these are physically connected and the radio sources was ejected from the galaxy. But, in this case, there arise some problems: first, usually one thinks that radio sources are farther then associated peculiar galaxy and, second, one should explain the locality of radio sources. In addition these objects often have close values of their parameters (such as redshifts and magnitude) The assumption that these pairs are superluminal objects clearly explains these observations. Let me note that except these pairs there are also many other examples of pairing and lining quite local objects in Universe (for example, pairing of quasars with large separation on the sky[7] and lining of superclusters of galaxies).

Probably the assumption of existence in the Universe superluminal objects also can explain some strange phenomena recently discovered by astronomers[3,4,5,6].

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