THE EFFECT OF NONLINEAR LANDAU DAMPING ON ULTRARELATIVISTIC BEAM PLASMA INSTABILITIES

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ABSTRACT

Very high energy gamma-rays from extragalactic sources produce pairs from the extragalactic background light, yielding an electron–positron pair beam. This pair beam is unstable to various plasma instabilities, especially the “oblique” instability, which can be the dominant cooling mechanism for the beam. However, recently, it has been claimed that nonlinear Landau damping renders it physically irrelevant by reducing the effective damping rate to a low level. Here we show with numerical calculations that the effective damping rate is $8 \times 10^{-4}$ the growth rate of the linear instability, which is sufficient for the “oblique” instability to be the dominant cooling mechanism of these pair beams. In particular, we show that previous estimates of this rate ignored the exponential cutoff in the scattering amplitude at large wave numbers and assumed that the damping of scattered waves entirely depends on collisions, ignoring collisionless processes. We find that the total wave energy eventually grows to approximate equipartition with the beam by increasing energy deposition into long-wavelength modes. As we have not included the effect of nonlinear wave–wave interactions on these long-wavelength modes, this scenario represents the “worst case” scenario for the oblique instability. As it continues to drain energy from the beam at a faster rate than other processes, we conclude that the “oblique” instability is sufficiently strong to make it the physically dominant cooling mechanism for high-energy pair beams in the intergalactic medium.

Key words: BL Lacertae objects: general – gamma rays: general – instabilities – magnetic fields – plasmas

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1. INTRODUCTION

The very high energy gamma-ray (VHEGR, $E \geq 100$ GeV) extragalactic sky is dominated by a subset of blazars, which we refer to as TeV blazars. These extragalactic VHEGR emitters produce TeV photons that are greatly attenuated through annihilation upon soft photons in the extragalactic background light (EBL) and produce pairs (see, e.g., Gould & Schréder 1967; Salamon & Stecker 1998; Neronov & Semikoz 2009).

It has been assumed that these ultrarelativistic pairs produced by VHEGR annihilation lose energy exclusively through inverse-Compton (IC) scattering off of the cosmic microwave background (CMB), transferring the energy of the original VHEGR to gamma-rays with energies $\lesssim 100$ GeV. As the gamma-rays are in the Large Area Telescope bands of Fermi, this is a significant target for Fermi observations.

The absence of observed secondary IC emission leads a number of authors to argue that this lack of observed emission places lower bounds upon the intergalactic magnetic field (IGMF; see, e.g., Neronov & Vovk 2010; Tavecchio et al. 2010, 2011; Dermer et al. 2011; Taylor et al. 2011; Takahashi et al. 2012; Dolag et al. 2011) with typical numbers ranging from $10^{-19}$ G to $10^{-15}$ G. In addition, the persistent belief in this IC emission has led other workers to argue that based on the IC contribution to the Fermi extragalactic gamma-ray background (EGRB), the comoving number density of gamma-ray bright blazars must grow slowly with increasing redshift, if at all (Kneiske & Mannheim 2008; Venters 2010; Abazajian et al. 2011; Inoue & Ioka 2012), implying that these TeV blazars cosmologically evolve qualitatively differently compared to other active galactic nuclei (AGNs).

These two conclusions depend on IC cooling dominating the evolution of the ultrarelativistic pairs. However, it was recently found that plasma instabilities driven by the ultrarelativistic pair beams is likely the dominant cooling mechanism (Broderick et al. 2012; Schlickeiser et al. 2012, 2013), depositing this energy as heat in the intergalactic medium (IGM; Chang et al. 2012; Pfommer et al. 2012). Therefore, the lack of observed IC halo emission from TeV blazars does not imply the existence of the IGMF as previous groups have argued (Broderick et al. 2012; Schlickeiser et al. 2012, 2013).

This heating of the IGM may resolve a variety of cosmological puzzles, including naturally explaining anomalies in the statistics of the high-redshift Ly$\alpha$ forest (Puchwein et al. 2012) and potentially explaining a number of the X-ray properties of groups and clusters and anomalies in galaxy formation on the scale of dwarfs (Pfrommer et al. 2012; Lu et al. 2014). We have recently shown that if the IC halos are ignored, it is possible to quantitatively reproduce the redshift and flux distributions of nearby hard gamma-ray blazars and the EGRB spectrum above 3 GeV simultaneously with a unified model of AGN evolution (Broderick et al. 2014a, 2014b). All of these empirical successes provide circumstantial evidence for the presence of virulent plasma beam instabilities.

Recently, Miniati & Elyiv (2013, hereafter ME13) argued that these instabilities are physically irrelevant for the cooling
of these pair beams. First, the “oblique” instability would saturate at a very low level due to nonlinear Landau damping (NLD). ME13 argue that this process occurs when $3 \times 10^{-6}$ of the electron–positron beam energy is contained within the waves, significantly limiting the instability cooling rate. Second, inhomogeneities in the IGM prevent this linear instability from even growing by shifting the unstable waves out of the region of resonance. The second point relates to the properties of the linear instability in the kinetic regime, which has been shown by Schlickeiser et al. (2013) to be physically relevant, contrary to linear instability in the kinetic regime, which has been shown by of resonance. The second point relates to the properties of the inhomogeneities in the IGM prevent this linear instability from saturate at a very low level due to nonlinear Landau damping. The Astrophysical Journal

by discussing the physics in Section 2. Using a numerical calculation, we derive the saturation level of the “oblique” instability in the nonlinear regime in Section 3. We discuss why our results differ from those presented by ME13 in Section 4 and present the implications in Section 5. Finally, we close with a summary of results and pathway to future work in Section 6.

2. THE PHYSICS OF NONLINEAR LANDAU DAMPING

We consider an unstable wave that is driven by a beam of electrons and positrons in a background plasma of electrons and protons (or other ions). As the unstable wave grows in amplitude, it becomes subject to nonlinear particle–wave and wave–wave interactions. In nonlinear particle–wave interactions, the most important interaction is induced scattering by thermal ions (Kaplan & Tsytovich 1968; Smith & Fung 1971; Brežman et al. 1972; Melrose 1986), which is also referred to as NLD (Melrose 1986).

Wave–particle interactions induce the transformation of one plasma wave, characterized by a frequency and wave vector $(\omega, \mathbf{k})$, into another $(\omega', \mathbf{k}')$ via nonlinear scattering on the particles that constitute the plasma. The kinetic equation for these waves in the presence of wave–particle interactions is (Kaplan & Tsytovich 1968; Brežman et al. 1972)

$$\frac{d W_k}{dt} = 2\Gamma_k W_k - \frac{W_k \omega_p}{8(2\pi)^{3/2} n_e m_e v_e^2} \int \frac{(k \cdot k')^2}{k^2 k'^2} \phi(k, k') W_k dk' ,$$

where $W_k$ is the spectral energy density of the waves, normalized such that the total energy density is given by

$$W = \frac{1}{(2\pi)^3} \int W_k dk ,$$

(1)

$$\Gamma_k = \Gamma_{gr}(k) + \Gamma_{LD}(k)$$

is the sum of the unstable wave growth rate, $\Gamma_{gr}(k)$, and the linear Landau damping rate, $\Gamma_{LD}(k)$, $\omega_p \equiv \sqrt{4\pi n_e e^2/m_e}$ is the electron plasma frequency of the background plasma, given in terms of the proper electron density ($n_e$) and rest mass ($m_e$) and $v_e$ and $v_t$ are the electron and ion thermal velocities, respectively. We also note that wave–particle interactions also convert these electrostatic waves into electromagnetic waves as noted in Kaplan & Tsytovich (1968). However, we ignore electromagnetic modes in this work to focus on the electrostatic waves. Including these modes as well other nonlinear processes is the subject of future work.

The growth rate is given by the oblique growth rate, which is

$$\Gamma_{gr}(k) \equiv \frac{1}{\tau_{gr}} \approx 0.4 \frac{n_b}{n_e} \gamma_b \omega_p \Theta(1 - v_{ph}(k)/c) ,$$

where $n_b$ is the beam density and $\gamma_b$ is the Lorentz factor of the ultrarelativistic beam, $\Theta$ is the Heaviside function, and $v_{ph}(k) = \omega/k$ is the phase speed of the Langmuir wave. Equation (3) was first found by Bret et al. (2010) by fitting the maximum growth rate in the kinetic regime. We have confirmed this growth rate in the electrostatic approximation for the kinetic regime (A. E. Broderick et al., in preparation; see also Schlickeiser et al. 2013). More importantly, this result holds true for a large range in $k$. The reason for this is that electrostatic waves oscillate at $\omega_p$, almost independently of $k$ and their phase speed along the $z$-axis (arbitrarily defined) is given by $v_{ph} = \omega/k \cos \theta$, where $\theta$ is the angle between the direction in question and the wave vector. Hence for any $k \geq \omega_p/c$, there exist some $\cos \theta$ such that the $v_{ph} \approx c$ and hence these waves are in resonance with a relativistic beam. In other words, the oblique instability grows for any $k \geq \omega_p/c$, but the angle between the most unstable wave vector and beam varies.

Damping in the linear regime is given by linear Landau damping, whose rate is

$$\Gamma_{LD}(k) = - \left( \frac{\pi}{8} \right)^{1/2} \frac{\omega_p}{v_e} \left( \frac{v_{ph}}{v_e} \right)^3 \exp \left( - \frac{v_{ph}^2}{2v_e^2} \right) .$$

(4)

The overlap integral $\phi$ is given by (Kaplan & Tsytovich 1968)

$$\phi(k, k') = \frac{3v_e^2 (k^2 - k'^2)}{4\omega_p |k - k'| v_i} \exp \left[ -2 \frac{3v_e^2 (k^2 - k'^2)}{4\omega_p (k - k') |v_i|} \right] ^2 .$$

(5)

An important feature of Equation (5) is the dependence on $k^2 - k'^2$, which sets the sign of $\phi(k, k')$. Scattering of a wave with wave vector $k$ into another wave with wave vector $k'$ can only proceed if $\phi(k, k') > 0$, i.e., the wave energy in the $k$ wave is damped, while the $k'$ wave energy grows. This requires that $k' < k$, i.e., the scattered wave has a longer wavelength than the incident wave. The demand that induced scattering drives waves to longer wavelength arises from the transfer of some momentum from the incident wave into the polarization clouds surrounding the ions (Smith & Fung 1971).

3. NUMERICAL STUDIES

We now solve Equation (1) numerically assuming that linear growth and nonlinear damping via NLD are the two mechanisms that control the initial evolution of Langmuir waves. However, because $k$ is three-dimensional, we adopt the simplifying assumption that the Langmuir waves are isotropic, i.e., $W_k = W$. This simplifying assumption is reasonable as long as the induced scattering processes are sufficiently rapid that it isotropizes the waves (Kaplan & Tsytovich 1968). Equation (1) reduces then to

$$\frac{d W_k}{dt} = 2\Gamma_k W_k - \frac{W_k \omega_p}{8(2\pi)^{3/2} n_e m_e v_e^2} \times \int k' \cos^2 \theta \phi(k, k') W_k dk' d \cos \theta ,$$

(6)

where $\theta$ is the angle between $\mathbf{k}$ and $\mathbf{k}'$. Without loss of generality, we have fixed $\mathbf{k}$ along the $z$-axis. Here $\phi(k, k')$ can be simplified

7 Further support for this approximation emerges if the number of TeV blazars that contribute to the heating of a given patch of the intergalactic medium exceeds 100.

8 Equation (5) can also be simplified if we set $(k_1 \cdot k_2)^2/k_1^2 k_2^2$ to the angle-averaged value of 1/3 as done by Kaplan & Tsytovich (1968). We computed this integral both ways and found little difference in the saturation amplitude.
to

\[ \phi(k, k') = \frac{3\nu_e^2 (k^2 - k'^2)}{4\omega_p v_i \sqrt{k^2 + k'^2 - 2kk' \cos \theta}} \times \exp \left( 2 \left( \frac{3\nu_e^2 (k^2 - k'^2)}{4\omega_p v_i \sqrt{k^2 + k'^2 - 2kk' \cos \theta}} \right)^2 \right). \] (7)

We calculate Equation (6) numerically for \( N_{\text{modes}} = 300 \) logarithmically spaced modes from \( k = 10^{-6} \omega_p/c \) to \( 10^3 \omega_p/c \) and have verified this calculation using \( N_{\text{modes}} = 1000 \). The lower limit of \( k_{\text{min}} \) was chosen to fulfill the requirement \( k_{\text{min}} \ll \omega_p/c \). The upper limit of \( k_{\text{max}} = 10^3 \omega_p/c \) was set because it is significantly larger than the estimated \( k \) where linear Landau damping would suppress the instability. We have found that our calculations are not affected by extending the upper and lower limits on \( k \). We also set the initial \( k^3 W_k \) for a small value of the initial beam energy, i.e., \( 10^{-15} \) and confirmed that our results are independent of this initial value.

In Figure 1, we plot the wave energy \( W \) in units of the initial beam energy density \( n_b \gamma_b m_c c^2 \) as a function of growth e-folding times \( \Gamma_{gt} \) for a 1 TeV beam and a 10 TeV beam, where the beam density is what is expected at \( z = 0 \) for a TeV blazar with equivalent isotropic luminosity \( E L_E = 10^{45} \text{erg s}^{-1} \). The wave energy grows exponentially up to a time \( \Gamma_{gt} \approx 15 \), where exponential growth ends and transitions to a slow linear growth in \( W \). The wave energy, \( W \), is equal to the energy of the initial beam when \( \Gamma_{gt} \approx 2000 \). Therefore, in the absence of a significant back-reaction, the beam experiences one e-folding reduction in energy at \( \Gamma_{gt} \approx 1300 \), giving a damping rate of

\[ \Gamma_{\text{NLDD}} \approx 8 \times 10^{-4} \Gamma_{gt}. \] (8)

In Figure 2 we plot the wave energy as a function of wave vector \( k \), for three different times, \( \Gamma_{gt} = 12 \) (solid line), 15 (dotted line), and 30 (dashed line). These times are also marked in Figure 1 by vertical lines of the same type as in Figure 2. For \( \Gamma_{gt} = 12 \) (solid line), NLD is not important and the instability grows for all \( k \gtrsim \omega_p/c \). As the unstable waves grow, the effect of NLD begins to become important and long-wavelength modes

\( k \ll \omega_p/c \) begin to grow at the expense of short-wavelength modes. This is clearly seen for \( \Gamma_{gt} = 15 \) (dotted line) and 30 (dashed line). However, the largest wave vector modes are not suppressed by the effect of NLD and remain at a level of \( \approx 10^{-3} \) of the beam energy density. These large wave vector modes survive for the duration of the calculation and continually pump energy into long-wavelength modes.

To understand the origin of the survival of these large wave vector modes and the development of the empty region between \( k \approx 10^{-2} \omega_p/c \) and \( \sim 20 \omega_p/c \), it is helpful to return to the coupling term (Equation (5)). Because the argument in the exponent is dominated by \( k' \approx k \), modes that are spaced closely to each other are strongly scattered. When the difference is large, i.e., \( k^2 - k'^2 \) is large, the scattering is exponentially suppressed. Therefore, NLD is strongest on modes near each other. As a result, as the long-wavelength modes grow, they quickly sap energy from the nearest modes, leaving the short-wavelength modes untouched due to the exponential suppression. This suppression becomes important where the exponent is order unity or

\[ \frac{\nu_e^2 (k^2 - k'^2)}{\omega_p |k - k'| v_i} \gtrsim \frac{\nu_e^2 k}{\omega_p v_i} \gtrsim 1. \] (9)

This gives the condition

\[ k \gtrsim \sqrt{\frac{m_e \omega_p \, c}{m_i \, c v_e}} \approx 20 \left( \frac{T}{10^3 \text{K}} \right)^{-1/2} \omega_p/c, \] (10)

which is of the order of where the suppression occurs (see Figure 2).

4. RELATIONSHIP TO PREVIOUS WORK

4.1. Comparison with Miniati & Elyiv (2013)

Our result differs from ME13, who found that the effect of NLD on the “oblique” instability is to drive the saturation of the excited Langmuir waves to a physically irrelevant amplitude. ME13 estimates the excited wave energy to be \( W/(n_b \gamma_b m_c c^2) \approx 3 \times 10^{-6} \), whereas our numerical calculation finds a value that is over two orders of magnitude larger. There are two crucial differences. First, ME13 based their estimate on the order-of-magnitude calculation which ignores the effect of the
suppression of NLD at large wave numbers. Second, ME13 assumes that the damping of these scattered waves is completely due to collisions, which are extraordinarily slow. In this case, the wave energy of the excited waves is reduced by \( W_r / \gamma_n m_e c^2 = \Gamma_{\text{NLD}} / \Gamma_{\text{gr}} \), where \( W_r \) is the wave energy of resonant waves, \( \Gamma_{\text{gr}} \) is the electron collision rate, typically given by electron–ion collisions, and \( \Gamma_{\text{NLD}} \) is the maximum growth rate of the resonant waves that are unstable to growth due to NLD of the linearly unstable mode.

The choice of collisional damping likely underestimates the true damping rate. In particular, collisionless processes like wave–wave scattering and wave–particle interaction are likely to produce a damping rate for these scattered waves that far exceeds collisional damping. Indeed, collisionless simulations (Davidson 1972) have shown that nonlinear wave–wave interactions can lead to particle heating and wave damping in the absence of collisions.

Here we presume that the damping rate of the scattered waves are sufficiently rapid to give

\[
\frac{W_r}{\gamma_n n_b m_e c^2} = \frac{\Gamma_{\text{NLD}}}{\Gamma_{\text{gr}}} \approx 8 \times 10^{-4}.
\]

(11)

We are motivated to use this estimate by comparison with the work of Ziebell et al. (2008a, 2008b) who found while NLD effects dominate the scattering of Langmuir waves, the rate of three-wave interactions is competitive with NLD and results in the quasi-isotropic heating of the electrons. The results of Ziebell et al. (2008a, 2008b) suggest that collisional processes are irrelevant to the damping of scattered waves. Moreover, nonlinear wave–wave coupling will likely lead to a more equitable distribution of mode energies among the different wave vectors whereas the effect of NLD moves energy toward smaller wave vectors. Thus, inclusion of nonlinear wave–wave coupling will lead to a stronger damping rate for the oblique instability by counteracting the effect of NLD. As a result, the calculation that we present here likely represents the worst case scenario for the “oblique” instability.

4.2. Comparison with Sironi & Giannios (2014)

Recent numerical simulations have been brought to bear upon these dilute beam–plasma processes to study the physics of nonlinear saturation by Sironi & Giannios (2014), hereafter SG14. However, due both to their spectral resolution and constraints on the parameters simulated, these appear to be unable to capture the physics of NLD.

The range of \( k \) over which NLD redistributes energy is limited by the particular form of the overlap integral, Equation (5). To estimate this range, we note that the overlap integral has a maximum width for \( k' \) antiparallel to \( k \), i.e., the backward scattering case; in this case, we take \( k' = -(k - \Delta k) \hat{k} \) and apply this to Equation (5) to give

\[
\phi(\Delta k) = \frac{3v_i^2 \Delta k}{4\omega_p v_i} \exp \left[ -2 \left( \frac{3v_i^2 \Delta k}{4\omega_p v_i} \right)^2 \right],
\]

(12)

for \( \Delta k / k \ll 1 \). This peaks at \( 3v_i^2 \Delta k / 4\omega_p v_i = 1 / 2 \), falling shortly thereafter. Thus, to marginally resolve NLD requires spectral resolutions in excess of

\[
\frac{\Delta k}{\omega_p / c} = \frac{2v_i c}{3v_i^2} = \frac{2}{3} \left( \frac{m_e c^2}{m_e T_e} \frac{m_p}{m_e} \frac{T_e}{T_i} \right).
\]

(13)

SG14 found that the temperature of the background electrons approached relativistic temperatures (see Figure 3 of SG14) in their simulations and hence \( \Delta k \approx 0.02 \omega_p / c \), assuming \( T_i = T_e \).

The spectral resolution of a numerical computation is set by the simulated domain’s physical size. In SG14, the reported size is \( 128 c / \omega_p \), implying a spectral resolution of \( \Delta k_{\text{min}} = (2\pi / 128) \omega_p / c \approx 0.05 \omega_p / c \), larger than the minimum required to resolve the NLD. Therefore, the calculations described in SG14 are unable to capture the impact of NLD even for the most optimistic case of backward scattering. Moreover, it is likely that \( T_i \ll T_e \) as the collisions needed to maintain this equilibrium are absent and electromagnetic interactions are inefficient because of the large mass ratio; a significantly lowered \( T_i \) would make this disparity even more substantial.

In addition, the high temperatures reached in SG14’s simulations suppresses the effectiveness of NLD. To see this, we estimate the NLD term in Equation (6) for the backward scattering case discussed above. Again taking \( k' = -(k - \Delta k) \hat{k} \) and integrating over \( \Delta k \), we find Equation (6) becomes

\[
\frac{dW_k}{dt} \approx 2\Gamma_k W_k - \frac{W_k \omega_p}{8(2\pi)^{5/2} n_e m_e v_i^2} \frac{4\pi}{3} k^2 \Delta k W_k,
\]

(14)

where we have approximated the \( \phi(k, k') \approx \phi(\Delta k) \approx 1 / 3 \) over the interval \( \Delta k \) given by Equation (13). Substituting \( \Delta k \) by Equation (13) and \( \Gamma_k \) by Equation (3), the ratio between the second and first terms, indicating the importance of NLD, is

\[
\Gamma_k^{-1} \frac{4\pi \omega_p k^2 \Delta k W_k}{48(2\pi)^{5/2} n_e m_e v_i^2} \approx 10^{-4} \frac{\omega_p}{\omega_p} \frac{k^3 W_k}{k c \gamma_n n_b m_e c^2} \frac{(m_e c^2)}{(k_B T_e)}^{3/2}.
\]

(15)

For conditions relevant to SG14’s simulations, \( k^3 W_k / \gamma_n n_b m_e c^2 \approx 0.1, k_B T_e / m_e c^2 \approx 1 \), and \( \omega_p / k_B T_e \approx 1 \), implying NLD is suppressed by five orders of magnitude compared to linear growth.

It remains unclear if quasi-linear relaxation plays an important role. In the simulations performed by SG14 no more than 10% of the energy of the original beam is drained by the oblique instability, which they attribute to quasi-linear relaxation processes. Assuming this to be the case, extrapolating to the parameters of intergalactic TeV-driven beams results in even more stringent limits on the efficiency of plasma instabilities. However, the accuracy and applicability of these extrapolations, from \( \gamma \approx 10^5 \) and \( n_b / n_e \approx 10^{-2} \) to \( \gamma \approx 10^6 - 10^8 \) and \( n_b / n_e \approx 3 \times 10^{-18} \), is far from clear and depends critically on the identification of the physical and potentially numerical causes of the instability saturation. An exhaustive study of these effects, quasi-linear relaxation, and nonlinear wave–wave coupling in conjunction with NLD is left for future work.

5. IMPLICATIONS

We now compare the damping rate given by Equation (8) to the current (i.e., at \( \Delta z = 0 \)) damping rate due to IC scattering off CMB photons, which is given by Broderick et al. (2012):

\[
\Gamma_{\text{IC}} = \frac{4\sigma_T u_{\text{CMB}}}{3m_e c} \gamma_b \approx 2.2 \times 10^{-13} \left( \frac{E}{\text{TeV}} \right) \left( \frac{1 + z}{2} \right)^{4} \text{s}^{-1}.
\]

(16)

This sets a maximum beam density (Broderick et al. 2012), which we can use to get the effective maximum damping rate
of the beam. For $\Gamma_{\text{NLD}} = 8 \times 10^{-4} \Gamma_{\text{gr}}$, we find

$$\Gamma_{\text{NLD, max}} \approx 6.9 \times 10^{-12} \left(\frac{1 + z}{2}\right)^{3.5 - 5.5} \times \left(\frac{E_L}{10^{45} \text{ erg s}^{-1}}\right) \left(\frac{E}{\text{TeV}}\right)^2 \text{s}^{-1},$$ (17)

where $\zeta$ is our parameterization of the EBL and is given by $\zeta = 4.5$ for $z < 1$ and 0 otherwise.

However, the above calculation is not self-consistent as we have presumed that the beam density is limited only by IC scattering. If instead we assume that the damping of the beam is driven by these plasma instabilities (Broderick et al. 2012) with the damping rate given by Equation (8), we find a self-consistent effective damping rate of

$$\Gamma_{\text{NLD}} \approx 10^{-12} \left(\frac{1 + z}{2}\right)^{6(-3)/4} \left(\frac{E_L}{10^{45} \text{ erg s}^{-1}}\right)^{1/2} \times \left(\frac{E}{\text{TeV}}\right)^{3/2} \text{s}^{-1}.$$ (18)

Hence, we find that NLD, while important, does not appear to be sufficiently strong to prevent the oblique instability from dominating the cooling of the pair beam at energies $\gtrsim 0.8$ TeV. This domination is not complete at $z = 0$: a substantial fraction of the beam energy can now be lost to IC scattering, of order 46% at 1 TeV and 20% at 10 TeV. We caution that this value may be significantly reduced if nonlinear wave–wave interactions reduce the effectiveness of NLD.

Figure 3 shows contours of the ratio between the damping rates due to plasma effects in Equation (18) and due to IC scattering in Equation (16). Lines denoting ratios of 0.1, 0.5, 1.0, 2.0, and 10.0 are shown and the shaded green region denotes where IC scattering dominates plasma effects. For larger photon energies, the dominance of plasma effects becomes more pronounced. In addition, the dominance of plasma effects is also more pronounced for increasing redshifts up to $z = 1$. Beyond $z > 1$, the constant physical density of the EBL implies that the stronger $(1 + z)^4$ scaling of IC scattering will become ever more important for the energy budget of these TeV beams.

The limits derived above assume that NLD alone limits the growth rate of the linear “oblique” instability. We have noted that this neglects nonlinear wave–wave coupling and quasi-linear effects. However, the modulation instability may also play an important role in limiting the growth of the long-wavelength modes that are fed by NLD. The modulation instability is a result of the ponderomotive force that results from a spatially inhomogeneous distribution of Langmuir waves. In particular, it can be shown that the change on the time-averaged electron density is $\Delta n_e \propto |E|^2 \propto -W$, i.e., regions of high wave density correspond to regions with lower electron density (Boyd & Sanderson 2003). Such a shift in the electron density leads to a shift in the plasma frequency that is of order

$$\frac{\Delta \omega_p}{\omega_p} = \frac{\Delta n_e}{2n_e}.$$ (19)

As a result, sufficiently long-wavelength Langmuir waves that propagate freely in regions of high wave density (low electron density) can lie below the plasma resonance outside and therefore are trapped. Which modes become trapped depends upon the shift in the plasma frequency and therefore the wave density. Assuming the Langmuir wave dispersion relation, $\omega(k) = \omega_p(1 + 3k^2\lambda_D^2/2)$, where $\lambda_D = \nu_e/\omega_p$ is the Debye length, the condition for trapped modes is given by

$$\frac{W}{n_e\lambda_B T} \gg 3(k\lambda_D)^2.$$ (20)

The late-time development of the modulation instability in multiple dimensions is currently believed to result in strong turbulence, which then rapidly damps the participating Langmuir waves and results in direct heating of the background plasma (ME13; Schlickeiser et al. 2012).

For the long-wavelength modes generated by NLD, $k \approx 10^{-2} \omega_p/c$, Equation (20) implies

$$\frac{f_w n_b \gamma_b m_e c^2}{n_e k_B T} \gg \left(\frac{3 \times 10^{-2} \nu_e}{\omega_p}\right)^2 \approx 6 \times 10^{-10} \left(\frac{T}{10^4 \text{ K}}\right)^4,$$ (21)

where $f_w$ is the saturation amplitude of these waves relative to the beam energy $n_b \gamma_b m_e c^2$. For $f_w = 10^{-3}$ given by the saturation amplitude due to NLD, this gives

$$\frac{f_w n_b \gamma_b m_e c^2}{n_e k_B T} \approx 2 \times 10^{-9} \left(\frac{f_w}{10^{-3}}\right) \left(\frac{n_b}{n_e}\right) \left(\frac{\gamma_b}{10^6}\right) \left(\frac{T}{10^4 \text{ K}}\right)^{-4},$$ (22)

which is above the criterion given in Equation (21) and can allow these waves to rapidly heat the background electrons (Schlickeiser et al. 2012). In doing so, the modulation instability may limit the effectiveness with which NLD can drive long-wavelength modes, and hence the linear growth of the instability, as well as provide a natural mechanism for converting the wave energy into heat. However, more work is required to answer this question.
6. CONCLUSIONS

We have calculated the effects of NLD on the saturation amplitude of the “oblique” instability to which high-energy pair beams in the IGM are linearly unstable. Using a numerical calculation, we find that the “oblique” instability remains the most powerful cooling mechanism for these pair beams contrary to the earlier claims of ME13. In particular, we find that the beam saturates at a rate that is \( \approx 0.1\% \) of the kinetic growth rate of the “oblique” instability. The damping of the beam leads to the transfer of beam energy into long-wavelength non-resonant waves. When comparing our results to the estimate of ME13, we find that our damping rate exceeds their estimate by two orders of magnitude. Using this damping rate, we conclude that the oblique instability is effective in quenching the beam at \( z \approx 1 \), but is less effective at different redshifts.

We caution that the results that we present here are limited to the effects of NLD. As we argue above, the inclusion of nonlinear wave–wave coupling will likely lead to a more equitable distribution of energy among wave vectors. The nonlinear damping rate of the “oblique” instability is likely to increase under these conditions. Thus, the calculation that we present here represents the “worst case” scenario for the “oblique” instability where the magnitude of plasma effects is comparable to the effects of IC scattering.

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