Determinations of form factors for semileptonic $D \rightarrow K$ decays and leptoquark constraints

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May 3, 2018

Abstract

By analyzing all existing measurements for $D \rightarrow Kl^{+}\nu_l$ ($l = e, \mu$) decays, we find the determinations of both the vector form factor $f^K_{+}(q^2)$ and scalar form factor $f^K_{0}(q^2)$ for semileptonic $D \rightarrow K$ decays from these measurements are feasible. With the analysis results on these measurements together with magnitude of CKM matrix element $|V_{cs}|$, we obtain these form factors. Both $f^K_{+}(q^2)$ and $f^K_{0}(q^2)$ determined from experiments are consistent within error with those in recent lattice calculations. With our analysis results in conjunction with the average of form factor in $N_f = 2 + 1$ lattice calculations, we extract $|V_{cs}|$, which is in good agreement within error with the value obtained from SM global fit. With $|V_{cs}|$ obtained in this analysis together with the value obtained from leptonic $D_s$ decays, we obtain the average of $|V_{cs}|$, which is in good agreement with the one from PDG’2016. Moreover, using the analysis results in the context of the SM as input parameters, we re-analyze these measurements in the context of new physics. Constraints on scalar leptoquarks are obtained for different final states of semileptonic $D \rightarrow K$.

1 Introduction

Semileptonic $D \rightarrow P(P = K, \pi)$ decays have long been of great interest in the field of flavor physics, because they are important in validating the lattice QCD (LQCD) calculations, extracting the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements, and searching for New Physics (NP) beyond the Standard Model (SM) of particle physics [1].

For $D \rightarrow Kl^{+}\nu_l$ ($l = e, \mu$) decays, strong and weak interaction portions can be well separated and the effects of strong interactions can be parameterized by form factors. In the SM, the differential decay rate for $D \rightarrow Kl^{+}\nu_l$ is given by

$$\frac{d\Gamma_{D \rightarrow Kl^{+}\nu_l}}{dq^2} = \frac{G_F^2 |V_{cs}|^2}{24\pi^3} \left(1 + \frac{m_l^2}{q^2}\right)^2 |p| \cdot \left\{\left(1 + \frac{m_l^2}{2q^2}\right)|p|^2 f^K_{+}(q^2)^2 + \frac{3m_l^2}{8m_D^2q^2}(m_D^2 - m_K^2)^2 f^K_{0}(q^2)^2\right\},$$

where $G_F$ is the Fermi constant, $p$ represents the three momentum of the $K$ meson in the $D$ rest frame. $q \equiv p_D - p_K$ is the four momentum transferred to $l\nu_l$ and $q^2$ ranges from $m_l^2$ where
the daughter meson $K$ has the maximum possible momentum to $(m_D - m_K)^2$ where the $K$ meson at rest. The vector form factor $f_+^K(q^2)$ and the scalar form factor $f_0^K(q^2)$ are defined via

$$
\langle K(p_R)|\bar{s}\gamma^\mu c|D(p_D)\rangle = \left(p_D^\mu + p_R^\mu - \frac{m_D^2 - m_K^2}{q^2}q^\mu\right)f_+^K(q^2) + \frac{m_D^2 - m_K^2}{q^2}q^\mu f_0^K(q^2)
$$

and

$$
\langle K(p_R)|\bar{s}c|D(p_D)\rangle = \frac{m_D^2 - m_K^2}{m_c - m_s}f_0^K(q^2).
$$

At the maximal recoil point, kinematic constraints lead $f_+^K(0) = f_0^K(0)$.

In the last 30 years, various measurements for $D \to Kl^+\nu_l$ decays were performed at more than ten experiments. For $D^0 \to K^-e^+\nu_e$ and/or $D^+ \to K^0e^+\nu_e$ decays, branching fractions and/or decay rates in different $q^2$ bins were measured at experiments such as the E691 [2], Mark-III [3], CLEO [4], CLEO-II [5], BaBar [6], BES-II [7,8], CLEO-c [9] and BES-III [11–12]. The FOCUS Collaboration measured non-parametric relative form factor for $D^0 \to K^{+}\mu^+\nu_{\mu}$ decays at the FOCUS experiment in 2005 [13]. And in 2006, vector form factor for $D^0 \to K^-l^+\nu_l(l = e, \mu)$ decays was measured by the Belle Collaboration at the Belle experiment [14]. By comprehensive analysis of these measurements, one can obtain, the product of the hadronic form factor at $q^2 = 0$ and the magnitude of CKM matrix element $V_{cs}$, $f_+^K(0)|V_{cs}|$. Then with $f_+^K(0)|V_{cs}|$ together with $|V_{cs}|$ from SM global fit one can determine the form factor $f_+^K(0)$, or with $f_+^K(0)|V_{cs}|$ together with $f_0^K(0)$ calculated in lattice QCD one can extract $|V_{cs}|$ [15]. In 2014, Ref. [15] reported their determination of $f_+^K(0)$ and extraction of $|V_{cs}|$ by comprehensively analyzing all the existed measurements for $D \to Kl^+\nu_l$ decays before 2014. In these experimental and theoretical studies, the contributions of $f_0$ term are neglected, since they are suppressed by the mass squared of lepton.

With the increase of measurements for $D \to Kl^+\nu_l$ decays and the improvement of measurement accuracy, we believe that both the vector and scalar form factors can be determined by comprehensive analysis of these measurements. Here, for the first time, we try to determine both the vector form factor $f_+^K(q^2)$ and scalar form factor $f_0^K(q^2)$ by comprehensively analyzing all the existing measurements for $D \to Kl^+\nu_l$ decays. As the result of this analysis, we report the product $f_+^K(0)|V_{cs}|$ and shape parameters, $r_+$ and $r_0$, of vector and scalar form factors, which are the best fitting results to the experimental data in the context of the SM. Then we determine $f_+^K(0)$ by considering the product $f_+^K(0)|V_{cs}|$ in conjunction with $|V_{cs}|$ from SM global fit. With the determined $f_+^K(0)$ and shape parameters $r_+$ and $r_0$, we chick the lattice calculations for the vector form factor $f_+^K(q^2)$ and the scalar form factor $f_0^K(q^2)$. With the product together with $f_+^K(0)$ calculated in lattice QCD, we extract $|V_{cs}|$.

In addition, a comprehensive analysis of these measurements is also important for searching or constraining the contributions of non-Standard interactions beyond the Standard weak to $D \to Kl^+\nu_l$ decays. One candidate of the non-Standard interactions is the exchange a scalar leptoquark [16,18]. Leptoquarks are hypothetical color-triplet bosons that carry both baryon number and lepton number, and can thus couple directly to a quark and a lepton [19,20]. Leptoquark can be of either vector (spin-1) or scalar (spin-0) nature according to their properties under the Lorentz transformations. Some scalar leptoquarks can lead to the effective $s\bar{c}d\bar{l}$ vertex. Searching the signals or obtaining more constraints of scalar leptoquarks from $D \to Kl^+\nu_l$ decays is thus one of the goals of this article. Using the analysis results in the context of the SM and $V_{cs}$ from SM global fit as input parameters, we re-analyze existing measurements for $D \to Kl^+\nu_l$ decays in the context of new physics and obtain constraints on scalar leptoquark from these measurements.
The article is organized as follows: We first review the parametrization of form factors in Section 2. The details of experimental data for $D \rightarrow Kl^+\nu_l$ decays are presented in Section 3. In Section 4, we first discuss the feasibility of the scalar form factor $f^{K^0}(q^2)$ obtained from analyzing these experimental data. Then we describe our analysis procedure of fitting to the experimental data in the context of the SM. At the last of this section, we present the analysis result of these measurements. In Section 5, we re-analyze these measurements in the context of new physics, and give the bounds for leptoquarks. Finally, conclusions for this work are given in Section 6.

2 Parameterization of the form factors

The form factors $f^K_+(q^2)$ and $f^{K^0}(q^2)$ can be parameterized according to the constraints of their general properties of analyticity, cross symmetry, and unitarity [21]. Various parameterizations for these form factors exist in literatures such as the single pole model [22], the modified pole model [22], the ISGW2 model [23] and the series expansion [24]. However, experimental data seems do not support previous three models [15]. Thus we use the series expansion in this article. In this parameterization, form factors transformed from $q^2$-space to $z$-space, where

$$z(q^2, t_0) = \frac{\sqrt{t_+ - q^2 - \sqrt{t_+ - t_0}}}{\sqrt{t_+ - q^2 + \sqrt{t_+ - t_0}}},$$

with $t_\pm = (m_D \pm m_K)^2$ and $t_0 = t_+(1 - \sqrt{1 - t_-/t_+})$. The form factors is then expressed as

$$f^K_{+(0)}(q^2) = \frac{1}{\mathcal{P}(q^2)\phi(q^2, t_0)} \sum_{k=0}^{\infty} a_k(t_0)[z(q^2, t_0)]^k,$$

where $a_k(t_0)$ are real coefficients. The function $\mathcal{P}(q^2)$ is $\mathcal{P}_+(q^2) = z(q^2, m^2_D)$ for the vector form factor $f^K_+(q^2)$ or $\mathcal{P}_0(q^2) = z(q^2, m^2_D)$ for the scalar form factor $f^{K^0}(q^2)$. And $\phi(q^2, t_0)$ is

$$\phi(q^2, t_0) = \left(\frac{\pi m^2_e}{3}\right)^{1/2} \left(\frac{z(q^2, 0)}{-q^2}\right)^{5/2} \left(\frac{z(q^2, t_0)}{t_0 - q^2}\right)^{-1/2} \times \left(\frac{z(q^2, t_-)}{t_- - q^2}\right)^{-3/4} \left(\frac{t_+ - q^2}{t_+ - t_0}\right)^{1/4},$$

where $m_e$ is the mass of the charm quark.

To some extent, a two-parameter series expansion of the form factors is fine. Using the relation $1 = f^K(0)\mathcal{P}(0)$

$$\phi(0, t_0)/\left(\sum_{k=0}^{\infty} a_k(t_0)[z(0, t_0)]^k\right)$$

deduced from Eq.5 and setting $k = 1$, one can obtain a vector (scalar) form factor with two free parameters $f^K(0)$ and $r_{+(0)}$

$$f^K_{+(0)}(q^2) = \frac{f^K(0)\mathcal{P}_{+(0)}(0)\phi(0, t_0)(1 + r_{+(0)}z(q^2, t_0))}{\mathcal{P}_{+(0)}(q^2)\phi(q^2, t_0)(1 + r_{+(0)}z(0, t_0))},$$

where $r_{+(0)} = a_1^{+(0)}(t_0)/a_0^{+(0)}(t_0)$.

3 Experiments

The existing measurements for $D^0 \rightarrow K^-e^+\nu_e$ and $D^+ \rightarrow \bar{K}^0e^+\nu_e$ can be divided into three categories:
Table 1: Ratios $R_{q(+)0}$ measured at different experiments.

| Experiment | $q^2$ (GeV$^2$) | $R_{q(+)0}$ |
|------------|-----------------|-------------|
| E691 [2]   | $(m_e^2, q_{max}^2)$ | $R_0 = 0.91 \pm 0.07 \pm 0.11$ |
| CLEO [4]   | $(m_e^2, q_{max}^2)$ | $R_0 = 0.90 \pm 0.06 \pm 0.06$ |
| CLEO-II [5]| $(m_e^2, q_{max}^2)$ | $R_0 = 0.978 \pm 0.027 \pm 0.044$ |
| CLEO-II [5]| $(m_e^2, q_{max}^2)$ | $R_+ = 2.60 \pm 0.35 \pm 0.26$ |
| BaBar [6]  | $(m_e^2, q_{max}^2)$ | $R_0 = 0.927 \pm 0.007 \pm 0.012$ |

Table 2: Branching fractions $B(D \to K\pi\nu)$ measured at different experiments.

| Experiment | $q^2$ (GeV$^2$) | $B_{q(+)0}$ (%) |
|------------|-----------------|-----------------|
| Mark-III [3]| $(m_e^2, q_{max}^2)$ | $B_0 = 3.4 \pm 0.5 \pm 0.4$ |
| BES-II [7] | $(m_e^2, q_{max}^2)$ | $B_0 = 3.82 \pm 0.40 \pm 0.27$ |
| BES-II [8] | $(m_e^2, q_{max}^2)$ | $B_+ = 8.95 \pm 1.59 \pm 0.67$ |
| BES-III [11]| $(m_e^2, q_{max}^2)$ | $B_+ = 8.59 \pm 0.14 \pm 0.21$ |

(i) Ratio of decay rates or of branching fractions $R_{q(+)0}$, where $R_0 = \Gamma(D^0 \to K^-\pi^+)/\Gamma(D^0 \to K^-\pi^+)$ or $R_0 = B(D^0 \to K^-\pi^+)/B(D^0 \to K^-\pi^+)$, and $R_+ = B(D^+ \to K^0\pi^+)/B(D^0 \to K^0\pi^+)$. The ratios measured at different experiments are shown in Table 1.

(ii) Decay branching fraction $B_{q(+)0}$, where $B_0$ is the absolute branching fraction for $D^0 \to K^-\pi^+\nu$ decay and $B_+$ is the absolute branching fraction for $D^+ \to K^0\pi^+\nu$. The absolute branching fractions measured at different experiments are shown in Tab. 2.

(iii) Decay rate $\Delta\Gamma$, where $\Delta\Gamma$ represents the partial decay rate for $D^0 \to K^-\pi^+\nu$ or $D^+ \to K^0\pi^+\nu$ decay in a certain $q^2$ bin.

Measurements of the first two categories could not be used directly to determine $f^K_+(0)|V_{cs}|$ and the shapes of form factors. To use these measurements, we should first transfer them into absolute decay rates in certain $q^2$ ranges [15].

One can obtain the absolute decay rates from the categories (i) and (ii) measurements respectively by

$$\Delta\Gamma = R \times B(D \to K\pi) \times \frac{1}{\tau_D};$$

and

$$\Delta\Gamma = B(D \to K\pi\nu) \times \frac{1}{\tau_D},$$

where $B(D \to K\pi)$ is the branching fraction for $D^0 \to K^-\pi^+$ or $D^+ \to K^0\pi^+$ decays, and $\tau_D$ is the lifetime of $D^0$ or $D^+$ meson. To avoid the possible correlations, we use $B(D^0 \to K^-\pi) = (3.89 \pm 0.04)%$, $B(D^+ \to K^-\pi^+) = (2.93 \pm 0.094)%$ (the sum of $B(D^+ \to K^0\pi^+) = (1.47 \pm 0.08)%$ and $B(D^+ \to K^0\pi^+) = (1.46 \pm 0.05)%$), $\tau_{D^0} = (410.1 \pm 1.5) \times 10^{-15}$ s, $\tau_{D^+} = (1040 \pm 7) \times 10^{-15}$ s from PDG [25].

After these transformations, the absolute decay rates obtained from categories (i) and (ii) measurements together with category (iii) measurements, partial decay rates in different $q^2$ bins for $D \to K\pi\nu$, are shown in Tabs. 3 and 4.

We also consider the non-parametric relative form factors $f^K_+(q^2)$ for $D^0 \to K^-\mu^+\nu_\mu$ decays measured by the FOCUS Collaboration at the FOCUS experiment in 2005 [13]. The relative
Table 3: Partial decay rates $\Delta \Gamma$ of $D^0 \rightarrow K^- e^+ \nu_e$ decays in $q^2$ ranges.

| Experiment | $q^2$ (GeV) | $\Delta \Gamma$ (ns$^{-1}$) |
|------------|-------------|-----------------------------|
| E691 [2]   | $(m_e^2, q_{max}^2)$ | 86.32 ± 12.40 |
| CLEO [4]   | $(m_e^2, q_{max}^2)$ | 85.37 ± 8.10 |
| CLEO-II [5] | $(m_e^2, q_{max}^2)$ | 92.77 ± 5.00 |
| BaBar [6]  | $(m_e^2, q_{max}^2)$ | 87.92 ± 1.63 |
| Mark-III [3] | $(m_e^2, q_{max}^2)$ | 82.91 ± 15.62 |
| BES-II [7] | $(m_e^2, q_{max}^2)$ | 93.15 ± 11.77 |
| CLEO-c [9] | $(m_e^2, 0.2)$ | 17.82 ± 0.43 |
|            | (0.2, 0.4)   | 15.83 ± 0.39 |
|            | (0.4, 0.6)   | 13.91 ± 0.36 |
|            | (0.6, 0.8)   | 11.69 ± 0.32 |
|            | (0.8, 1.0)   | 9.36 ± 0.28 |
|            | (1.0, 1.2)   | 7.08 ± 0.24 |
|            | (1.2, 1.4)   | 5.34 ± 0.21 |
|            | (1.4, 1.6)   | 3.09 ± 0.16 |
|            | (1.6, $q_{max}^2$) | 1.28 ± 0.11 |
| BES-III [10] | $(m_e^2, 0.1)$ | 8.812 ± 0.187 |
|            | (0.1, 0.2)   | 8.743 ± 0.162 |
|            | (0.2, 0.3)   | 8.295 ± 0.159 |
|            | (0.3, 0.4)   | 7.567 ± 0.153 |
|            | (0.4, 0.5)   | 7.486 ± 0.152 |
|            | (0.5, 0.6)   | 6.446 ± 0.138 |
|            | (0.6, 0.7)   | 6.200 ± 0.134 |
|            | (0.7, 0.8)   | 5.519 ± 0.126 |
|            | (0.8, 0.9)   | 5.028 ± 0.119 |
|            | (0.9, 1.0)   | 4.525 ± 0.111 |
|            | (1.0, 1.1)   | 3.972 ± 0.103 |
|            | (1.1, 1.2)   | 3.326 ± 0.093 |
|            | (1.2, 1.3)   | 2.828 ± 0.085 |
|            | (1.3, 1.4)   | 2.288 ± 0.077 |
|            | (1.4, 1.5)   | 1.737 ± 0.068 |
|            | (1.5, 1.6)   | 1.314 ± 0.058 |
|            | (1.6, 1.7)   | 0.858 ± 0.050 |
|            | (1.7, $q_{max}^2$) | 0.379 ± 0.039 |
Table 4: Partial decay rates $\Delta \Gamma$ of $D^+ \to \bar{K}^- e^+ \nu_e$ decays in $q^2$ ranges.

| Experiment  | $q^2$ (GeV) | $\Delta \Gamma$ (ns$^{-1}$) |
|-------------|-------------|------------------------------|
| CLEO-II [5] | $(m_e^2, q_{max}^2)$ | $73.25 \pm 12.52$ |
| BES-II [8]  | $(m_e^2, q_{max}^2)$ | $86.06 \pm 16.60$ |
| BES-III [11]| $(m_e^2, q_{max}^2)$ | $82.60 \pm 2.49$ |
| CLEO-c [9]  | $(m_e^2, 0.2)$ | $17.79 \pm 0.65$ |
|             | $(0.2, 0.4)$ | $15.62 \pm 0.59$ |
|             | $(0.4, 0.6)$ | $14.02 \pm 0.54$ |
|             | $(0.6, 0.8)$ | $12.28 \pm 0.49$ |
|             | $(0.8, 1.0)$ | $8.92 \pm 0.41$ |
|             | $(1.0, 1.2)$ | $8.17 \pm 0.37$ |
|             | $(1.2, 1.4)$ | $4.96 \pm 0.27$ |
|             | $(1.4, 1.6)$ | $2.67 \pm 0.19$ |
|             | $(1.6, q_{max}^2)$ | $1.19 \pm 0.13$ |
| BES-III [12]| $(m_e^2, 0.2)$ | $16.97 \pm 0.60$ |
|             | $(0.2, 0.4)$ | $15.29 \pm 0.53$ |
|             | $(0.4, 0.6)$ | $13.57 \pm 0.47$ |
|             | $(0.6, 0.8)$ | $11.65 \pm 0.40$ |
|             | $(0.8, 1.0)$ | $9.33 \pm 0.34$ |
|             | $(1.0, 1.2)$ | $7.06 \pm 0.28$ |
|             | $(1.2, 1.4)$ | $4.96 \pm 0.20$ |
|             | $(1.4, 1.6)$ | $2.97 \pm 0.14$ |
|             | $(1.6, q_{max}^2)$ | $1.01 \pm 0.07$ |
Table 5: Non-parametric relative form factors $f_K^+(q^2)$ measured at the FOCUS experiment.

| $i$ | $q^2$ (GeV) | $f_K^+(q^2)$     |
|-----|-------------|------------------|
| 1   | 0.09        | 1.01 ± 0.03      |
| 2   | 0.27        | 1.11 ± 0.05      |
| 3   | 0.45        | 1.15 ± 0.07      |
| 4   | 0.63        | 1.17 ± 0.08      |
| 5   | 0.81        | 1.24 ± 0.09      |
| 6   | 0.99        | 1.45 ± 0.09      |
| 7   | 1.17        | 1.47 ± 0.11      |
| 8   | 1.35        | 1.48 ± 0.16      |
| 9   | 1.53        | 1.84 ± 0.19      |

form factors $f_K^+(q^2)$ at the central values of nine $q^2$ bins were obtained in assuming $f_K^+(0)$ has been normalized to 1 and the ratio $f_K^+(q^2)/f_K^+(q^2) = -0.7$, where $f_K^+(q^2) = (f^K_0(q^2) - f^K_0(q^2)) (m_D^2 - m_K^2)/q^2$. We list these measurements in Tab. 5.

In 2006, the Belle Collaboration reported their results on the measurements of the vector form factors $f_K^+(q^2)$ for $D^0 \rightarrow K^- l^+ \nu_l$ [14]. Both considering the signal events for $D^0 \rightarrow K^- e^+ \nu_e$ decays and $D^0 \rightarrow K^- \mu^+ \nu_\mu$ decays, they obtained the form factors $f_K^+(q^2)$ in 27 $q^2$ bins with bin size of 0.067 GeV$^2$. It is worthy to note that these measurements were obtained in the case of the masses of leptons neglected in the definition of differential decay rate for $D \rightarrow Kl^+ \nu_l$ decays. Thus the vector form factor is different from the one defined in this article. In the following, to make a distinction between these vector form factors, we use $f_{NL}^+(q^2)$ to represent the vector form factor in the case of neglecting the lepton mass. These $f_{NL}^+(q^2)$ measured at the Belle experiment can’t be used directly in this analysis. To use these measurements, we should translate them into products $f_{NL}^+(q^2)|V_{cs}|$ by using the magnitude of CKM matrix element $|V_{cs}| = 0.97296 \pm 0.00024$, which was used by the Belle Collaboration to obtain these $f_{NL}^+(q^2)$ in their article. The measurements $f_{NL}^+(q^2)|V_{cs}|$, collected in Ref. [15], are listed in Tab. 5.

The non-parametric relative form factors $f_K^+(q^2)$ measured at the FOCUS experiment and the vector form factor $f_{NL}^+(q^2)$ measured at the Belle experiment are very important for the determination of scalar form factor $f^K_0(q^2)$ for semileptonic $D \rightarrow K$ decays. We will discuss this issue in the next section.

4 Fits to experimental data in the context of the SM

Our goal is to obtain the product $f_K^+(0)|V_{cs}|$ and the shapes of the vector and scalar form factors for semileptonic $D \rightarrow K$ decays from the existing measurements. The first task we should deal with is to confirm the feasibility of obtaining the scalar form factor $f^K_0(q^2)$ by analyzing these experimental data, which is depending on the relative errors of these measurements and the contribution ratios of $f^K_0$ term to these measurements. If the contributions of the scalar form factor term to these measurements are much smaller than the errors of these measurements, then the fitting result of the scalar form factor will not be credible. Thus the confirmation of the feasibility is very important. The second task is to construct a Chi-squared function $\chi^2$, by minimizing this function we can arrive to the results of this fit.
Table 6: $f^{NL}_{+}(q^2_i)|V_{cs}|$ measured at the Belle experiment, collected in Ref. [15].

| $i$ | $q^2$ (GeV) | $f^{NL}_{+}(q^2_i)|V_{cs}|$ |
|-----|-------------|-----------------------------|
| 1   | 0.100       | $0.688 \pm 0.029$          |
| 2   | 0.167       | $0.762 \pm 0.029$          |
| 3   | 0.233       | $0.743 \pm 0.029$          |
| 4   | 0.300       | $0.811 \pm 0.032$          |
| 5   | 0.367       | $0.762 \pm 0.032$          |
| 6   | 0.433       | $0.817 \pm 0.036$          |
| 7   | 0.500       | $0.856 \pm 0.039$          |
| 8   | 0.567       | $0.915 \pm 0.039$          |
| 9   | 0.633       | $0.882 \pm 0.039$          |
| 10  | 0.700       | $0.798 \pm 0.039$          |
| 11  | 0.767       | $0.996 \pm 0.042$          |
| 12  | 0.833       | $0.970 \pm 0.045$          |
| 13  | 0.900       | $0.921 \pm 0.045$          |
| 14  | 0.967       | $1.015 \pm 0.052$          |
| 15  | 1.033       | $1.070 \pm 0.052$          |
| 16  | 1.100       | $0.911 \pm 0.055$          |
| 17  | 1.167       | $1.083 \pm 0.065$          |
| 18  | 1.233       | $1.067 \pm 0.068$          |
| 19  | 1.300       | $1.219 \pm 0.078$          |
| 20  | 1.367       | $1.343 \pm 0.084$          |
| 21  | 1.433       | $1.278 \pm 0.101$          |
| 22  | 1.500       | $1.158 \pm 0.107$          |
| 23  | 1.567       | $1.378 \pm 0.120$          |
| 24  | 1.633       | $1.433 \pm 0.169$          |
| 25  | 1.700       | $1.375 \pm 0.214$          |
| 26  | 1.767       | $1.116 \pm 0.331$          |
| 27  | 1.833       | $1.411 \pm 0.892$          |
4.1 The contribution ratios of scalar form factor

The contribution ratio of $f_0$ term to the decay rate for $D \to Ke^+\nu_e$ decay at a certain $q^2$ can be described by the ratio

$$r^\Gamma(q^2) = \frac{3m^2_{D} - m^2_{K}}{8m^2_{D}q^2} \bigg| \frac{3m^2_{D} - m^2_{K}}{2q^2} \bigg| p^2 + \rho \bigg( \frac{3m^2_{D} - m^2_{K}}{8m^2_{D}q^2} \bigg| p^2 \bigg| + \frac{3m^2_{D} - m^2_{K}}{8m^2_{D}q^2} \bigg| p^2 \bigg| \bigg)$$

(10)

where $\rho$ represents $|f^K_+(q^2)/f^K_0(q^2)|^2$ and the other portion of the first item of the denominator is coefficient of the $|f^K_+(q^2)|^2$, the numerator is the coefficient of $|f^K_0(q^2)|^2$. These coefficients can be easily obtained from Eq. 1.

The contributions of $f_0$ term to the relative form factor $f^K_+(q^2)$ defined in the FOCUS Collaboration’s article [13] and the vector form factor $f^{NL}_+(q^2)$ defined in the Belle Collaboration’s article [14] can be described by

$$r^J(q^2) = \sqrt{\frac{\kappa S_{\gamma}(q^2) + S_{\mu}(q^2)}{\rho(\kappa V_{\gamma}(q^2) + V_{\mu}(q^2)) + \kappa S_{\gamma}(q^2) + S_{\mu}(q^2)}}$$

(11)

where $\rho = |f^K_+(q^2)/f^K_0(q^2)|^2$, $S_{\gamma,\mu}(q^2) = 3m^2_{e,\mu}(m^2_{D} - m^2_{K})^2/(8m^2_{D}q^2)$, $V_{\gamma,\mu}(q^2) = (1 + m^2_{e,\mu}/(2q^2))|p|^2$, and $\kappa$ is a constant, where $\kappa = 0$ is for the relative form factor $f^K_+(q^2)$ and $\kappa = 1$ is for the vector form factor $f^{NL}_+(q^2)$.

Figure 1: The contribution ratio of $f_0$ term to the decay rate for $D \to Ke^+\nu_e$ decay (a), the contributions of $f_0$ term to the relative form factor $f^K_+(q^2)$ ($\kappa = 0$) and the vector form factor $f^{NL}_+(q^2)$ ($\kappa = 1$) (b).

$r^\Gamma(q^2)$ and $r^J(q^2)$ varying with $q^2$ were shown in Figs. 1 (a) and (b), respectively. The $\rho$ in Eqs. (10) and (11) is set to $\rho = 1.0, 1.5$ or $2.0$, which is according to previous lattice calculations. $\rho = 1.0$ is a good approximation at low $q^2$ range, $\rho = 2.0$ can be used for high $q^2$ range, and $\rho = 1.5$ is suitable for the middle $q^2$ range. Fig. 1 (a) shows that the contributions of $f^K_0$ term to the partial decay rates for $D \to Ke^+\nu_e$ decay are only $10^{-7} \sim 10^{-6}$ in the most $q^2$ range except $q^2 \to 0$ and $q^2 \to (m_D - m_K)^2$. The $r^\Gamma(q^2)$ are much smaller than the relative errors of the partial decay rates for $D \to Ke^+\nu_e$ measured at experiments, these relative errors ($>0.02$) can be calculated from Tabs. 3 and 4. Fig. 1 (b) shows the contribution rates of $f^K_0$ term to
the relative form factor $f^K_0(q^2)$ of $D^0 \rightarrow K^- \mu^+ \nu_\mu$ ($\kappa = 0$) and the contribution rates of $f^K_0$ term to the vector form factor $f^N_+(q^2)$ of $D^0 \rightarrow K^- l^+ \nu_l$ ($\kappa = 1$). As comparison, relative errors of $f^K_0(q^2)$ measured by the FOCUS Collaboration and relative errors of $f^N_+(q^2)$ measured by the Belle Collaboration, $\Delta_{FOCUS}$ and $\Delta_{Belle}$, are also presented in Fig.1(b). We can find that all these relative errors $\Delta_{FOCUS}$ and $\Delta_{Belle}$ are smaller than the contribution rates of $f^K_0$ term to the relative form factor for $D^0 \rightarrow K^- \mu^+ \nu_\mu$ ($\kappa = 0$) and the contribution rates of $f^K_0$ term to the vector form factor for $D^0 \rightarrow K^- l^+ \nu_l$ ($\kappa = 1$), except $\Delta_{Belle}$ at the range of $q^2 > 1.7$ GeV$^2$ measured at the Belle experiment.

Through the above analysis we can come to the conclusion that relative form factors measured at the FOCUS experiment and vector form factors measured at the Belle experiment can be used for the determination of the scalar form factor for semileptonic $D \rightarrow K$ decays, while the measurements of partial decay rates for $D \rightarrow Ke^+\nu_e$ can not. But note that all these measurements are important for the determination of the vector form factor for semileptonic $D \rightarrow K$ decays.

### 4.2 Construct Chi-squared function

To obtain $f^K_0(0)|V_{cs}|$ and shapes of the vector and scalar form factors, we perform our fit to these experimental measurements by minimizing the Chi-squared function

$$
\chi^2 = \chi^2_\Delta + \chi^2_{FOC} + \chi^2_{Belle},
$$

(12)

where $\chi^2_\Delta$ is constructed for measurements of partial decay rates in different $q^2$ ranges for $D \rightarrow Ke^+\nu_e$ as shown in Tabs. 3 and 4, $\chi^2_{FOC}$ is for the non-parametric form factors $f^K_0(q^2)$ measured at the FOCUS experiment, and $\chi^2_{Belle}$ corresponds to the Belle Collaboration measured products $f^N_+(q^2)|V_{cs}|$.

Since there are correlations between the measurements of partial decay rates for $D^0 \rightarrow K^- e^+\nu_e$ decays and/or $D^+ \rightarrow \bar{K}^0 e^+\nu_e$ decays, the $\chi^2_\Delta$ is given by

$$
\chi^2_\Delta = \sum_{i=1}^{54} \sum_{j=1}^{54} (\Delta \Gamma_i - \Delta \Gamma_i^{th})(C_{\Delta \Gamma}^{-1})_{ij}(\Delta \Gamma_j - \Delta \Gamma_j^{th}),
$$

(13)

where $\Delta \Gamma$ is the partial decay rate measured in experiment, $\Delta \Gamma^{th}$ denotes its theoretical expectation, and $C_{\Delta \Gamma}^{-1}$ is the inverse of the covariance matrix $C_{\Delta \Gamma}$, which is a $54 \times 54$ matrix. To compute the covariances of these 54 partial decay rates measured in different $q^2$ ranges and at different experiments, we adopt the concept proposed in Ref. [13]: (a) at the same experiment, the statistical and systematic errors of these partial decay rates, and corresponding correlations between these partial decay rates are used to compute their covariances; (b) the systematic uncertainties caused by the lifetime of $D^{0(+)}$ meson are fully correlated among all of the partial decay rates for $D^{0(+)} \rightarrow K^-(\bar{K}^0)e^+\nu_e$ decays measured at different experiments; (c) the systematic uncertainties related to $D^0 \rightarrow K^-e^+\nu_e$ are full correlated among all of the measurements of category (i) in Section 3 for $D^0 \rightarrow K^-e^+\nu_e$ decays.

Due to the correlations between measurements of the non-parametric form factors at the FOCUS experiment, the $\chi^2_{FOC}$ in Eq. [12] is defined as

$$
\chi^2_{FOC} = \sum_{i=1}^{9} \sum_{j=1}^{9} (f_i - f_i^{th})(C_{FOC}^{-1})_{ij}(f_j - f_j^{th}),
$$

(14)

where $f_i$ is the $i$th non-parametric form factor measured at the FOCUS experiment, $f_i^{th}$ is the $i$th theoretical value, and $C_{FOC}^{-1}$ is the inverse of the covariance matrix $C_{FOC}$.
where $f_i$ and $f_i^{th}$ are the measured value from the FOCUS experiment and the theoretical expectation of $f^K_+(q^2)$ at the center of $i$-th $q^2$ bin, respectively. It is worth noting that vector form factor $f^K(q^2)$ in Eq. (7) can not be used as the theoretical form factor $f_i^{th}$ directly, even $f^K_+(0)$ has been normalized to 1, because some assumptions about the expression of differential decay rate for $D^0 \to K^-\mu^+\nu_\mu$ process are different between this article and Ref. [13]. By comparing Eq. (11) in this article and Eq. (2) in Ref. [13], we can obtain

$$f_i^{th} = \sqrt{\frac{\int_{q_{i,min}^2}^{q_{i,max}^2} \xi \left\{ V_\mu(q^2)|f^K_+(q^2)|^2 + S_\mu(q^2)\right\} dq^2}{\int_{q_{i,min}^2}^{q_{i,max}^2} \xi \left\{ V_\mu(q^2) + S_\mu(q^2) \left( 1 + \frac{q^2\beta}{\alpha} \right)^2 \right\} dq^2}},$$

(15)

where $\xi = (1 - m_\mu^2/q^2)^2|p|$, $S_\mu(q^2) = 3m_\mu^2(m_D - m_K^2)^2/(8m_D^2q^2)$, $V_\mu(q^2) = (1 + m_\mu^2/(2q^2))|p|^2$, $\alpha = (m_D^2 - m_K^2)^2$ and $\beta = f^K(q^2)/f^K_+(q^2) = -0.7$. To obtain Eq. (15) we make an approximation

$$\int_{q_{i,min}^2}^{q_{i,max}^2} q^2f^{th}(q^2)\xi \left\{ V_\mu + S_\mu \left( 1 + \frac{q^2\beta}{\alpha} \right)^2 \right\} dq^2 \approx \int_{q_{i,min}^2}^{q_{i,max}^2} q^2f^{th}(q^2)\xi \left\{ V_\mu + S_\mu \left( 1 + q^2\beta/\alpha \right)^2 \right\} dq^2.$$

(16)

The $C^{-1}_{FOC}$ in Eq. (14) is the inverse of the covariance matrix $C_{FOC}$, which is a $9 \times 9$ matrix. We can construct the covariance matrix $C_{FOC}$ by the relation $(C_{FOC})_{ij} = \sigma_i\sigma_j\rho_{ij}$, where $\sigma_i$ ($\sigma_j$) is the standard error of $f^K_+(q^2)$ at the central value of the $i$-th ($j$-th) $q^2$ bin measured at the FOCUS experiment, and $\rho_{ij}$ is the correlation coefficient of measurements of $f^K_+(q^2)$ at $i$-th $q^2$ bin and $j$-th $q^2$ bin. The $\chi^2_{Bel}$ in Eq. (12) is built for the products $f^{NL}_{i}(q_i^2)|V_{cs}|$ measured at the Belle experiment. The $\chi^2_{Bel}$ is defined as

$$\chi^2_{Bel} = \sum_{i=1}^{27} \left( \frac{F_i - F_i^{th}}{\sigma_i} \right)^2,$$

(17)

where $F_i$ and $F_i^{th}$ are experimental and theoretical values of $f^{NL}_{i}(q_i^2)|V_{cs}|$ in the $i$-th $q^2$ bin respectively, and $\sigma_i$ represents the standard deviation of $F_i$. In Eq. (17) we neglect some possible correlations among the measurements of $f^{NL}_{i}(q_i^2)|V_{cs}|$. Similar to the analysis of the measurements at the FOCUS experiment above, by comparing Eq. (11) in this article and Eq. (1) in Ref. [13], the expression of $f^{NL}_{i}(q_i^2)|V_{cs}|$ is

$$F_i^{th} = \left( \frac{\int_{q_{i,min}^2}^{q_{i,max}^2} d\Gamma_e dq^2 + \int_{q_{i,min}^2}^{q_{i,max}^2} d\Gamma_\mu dq^2}{2 \int_{q_{i,min}^2}^{q_{i,max}^2} \frac{G_F^2}{24\pi^3}|p|^3 dq^2} \right)^{1/2},$$

(18)

where $d\Gamma_e/dq^2$ and $d\Gamma_\mu/dq^2$ are respectively the Eq. (11) for $D \to Ke^+\nu_e$ and $D \to K\mu^+\nu_\mu$ decays. To obtain Eq.(15), we make an approximation

$$\int_{q_{i,min}^2}^{q_{i,max}^2} \frac{G_F^2}{24\pi^3}|f^{NL}_{i}(q_i^2)|V_{cs}|^2|p|^3 dq^2 \approx |f^{NL}_{i}(q_i^2)|V_{cs}|^2 \int_{q_{i,min}^2}^{q_{i,max}^2} \frac{G_F^2}{24\pi^3}|p|^3 dq^2,$$

(19)
Table 7: Returns of the fit to experimental measurements.

| $f^K(0)|V_{cs}|$ | $r_+ |$ | $r_0 |$ | $\chi^2/d.o.f.$ |
|-----------------|-----------|-----------|----------------|
| 0.718 ± 0.003   | -2.17 ± 0.07 | 1.4 ± 3.5 | 94.5/87       |

4.3 Fit to experimental data

We fit the experimental data with the vector and scalar form factors parameterized as Eq. (7). All of the SM input parameters such as the Fermi constant $G_F$, the masses of mesons and charged leptons are taken from PDG [25]. The optimized $f^K(0)|V_{cs}|$, the shape parameter of the vector form factor $r_+$, the shape parameter of the scalar form factor $r_0$ and the minimized $\chi^2$ value are shown in Tab. 7.

Figs. 2, 3 and 4 present the fitting results of the experimental data for semileptonic $D \rightarrow K$ decays. In Fig. 2 (a) and (b), we list the branching fractions for the vector form factor $r_+$, the shape parameter of the scalar form factor $r_0$ and the minimized $\chi^2$ value are shown in Tab. 7.

To check the quality of the fit to differential decay rates of $D \rightarrow Ke^+\nu_e$ decays, these measurements are usually used to estimate the product $f^K(q^2)|V_{cs}|$ via

$$f^K(q^2)|V_{cs}| = \sqrt{\left( \frac{d\Gamma}{dq^2} \right)_i \frac{24\pi^3}{G_F^2|p_i|^2}},$$

where the differential decay rate $(d\Gamma/dq^2)_i = \Delta\Gamma_i/\Delta q^2$ is obtained by the size of $i$-th $q^2$ bin dividing the partial decay rate measured in the $q^2$ bin, and $p_i$ is the momentum of $K$ meson for $i$-th $q^2$ bin. From Eq. (11) one can obtain

$$|f^K(q^2)|V_{cs}|^2 = \left( 1 + \frac{m_e^2}{q^2} \right)^2 V_{cs} \cdot \left\{ \left( 1 + \frac{m_e^2}{2q^2} \right)|p|^2|f^K(q^2)|^2 + \frac{3m_e^2}{8m_D^2q^2}(m^2_D - m^2_K)|f^K(q^2)|^2 \right\}.$$  (21)

$f^K(q^2)|V_{cs}|$ as a function of $q^2$ together with $f^K(q^2)|V_{cs}|$ with errors measured at CLEO-c [9] and BES-III [10,12] experiments are shown in Fig. 5.

4.4 Determinations of $f^K(0)$ and $|V_{cs}|$

Note that the fit to experimental data returns just the product of the hadronic form factor $f^K(0)$ and the magnitude of CKM matrix element $V_{cs}$. To determine $f^K(0)$ or extract $|V_{cs}|$, we need more inputs. Since the CKM matrix element $|V_{cs}|$ can be obtained precisely from the SM global fit, by considering the product $f^K(0)|V_{cs}| = 0.718 ± 0.003$ shown in Tab. 7 together with $|V_{cs}| = 0.97351 ± 0.00013$ obtained from global fit in the SM, one can obtain

$$f^K(0) = 0.738 ± 0.003 ± 0.000,$$  (22)

where the first error is from the uncertainties in the partial decay rate measurements, and the second is the contribution of the uncertainty of $|V_{cs}|$. The form factor $f^K(0)$ from the
Figure 2: Measurements of branching fractions for (a) $D^0 \rightarrow K^- e^+ \nu_e$ and (b) $D^+ \rightarrow \bar{K}^0 e^+ \nu_e$ at different experiments. Differential decay rates measured at the CLEO-c and BES-III experiments for (c) $D^0 \rightarrow K^- e^+ \nu_e$ and (d) $D^+ \rightarrow \bar{K}^0 e^+ \nu_e$.

Figure 3: The relative form factor $f_{K}^{K}(q^2)$ measured at the FOCUS experiment for $D^0 \rightarrow K^- \mu^+ \nu_{\mu}$ decays.
Figure 4: The product $f^{NL}_+(q^2)|V_{cs}|$ measured at the Belle experiment for $D^0 \to K^- l^+ \nu_l (l = e, \mu)$ decays.

Figure 5: The product $f^{NL}_+(q^2)|V_{cs}|$ measured at the CLEO-c and the BES-III experiments for $D \to K e^+ \nu_e$ decays.
recent lattice QCD calculations \cite{26,27}, the $N_f = 2 + 1$ lattice average before 2017 \cite{28} and experimental fit in 2014 \cite{15} are compared in Fig. 6. Our fitting result is consistent with these theoretical calculations and presents a good consistency with the previous fitting result, but is with higher precision.

When $f_+^K(0)$ has been determined, vector form factor $f_+^K(q^2)$ and scalar form factor $f_0^K(q^2)$ for semileptonic $D \to K$ decays are determined, since the shape parameters of vector and scalar form factors, $r_1$ and $r_0$, have been obtained as returns in the fit. We show the comparisons of $f_+^K(q^2)$ and $f_0^K(q^2)$ obtained in this analysis with those obtained in the recent $N_f = 2 + 1$ lattice calculations by JLQCD collaboration \cite{20} and recent $N_f = 2 + 1 + 1$ lattice calculations by V. Lubicz et al \cite{27} in Fig. 7 (a) and (b), respectively. As shown in Fig. 7 (a), vector form factor $f_+^K(q^2)$ obtained from the fit to experimental measurements presents more consistent within error with the prediction by JLQCD collaboration calculations, but with higher precision. Fig. 7 (b) shows the shapes and the error bands of scalar form factors $f_0^K(q^2)$ obtained in this analysis and these values calculated by $N_f = 2 + 1$ and $N_f = 2 + 1 + 1$ lattice QCDs. The scalar form factors $f_0^K(q^2)$ obtained in this analysis shows consistent within error with the predictions of both $N_f = 2 + 1$ and $N_f = 2 + 1 + 1$ lattice QCDs, but with less precision at high $q^2$ range. The comparisons of scalar form factors $f_0^K(q^2)$ obtained from the fit to measurements for $D \to K\nu_l$ decays and these values calculated by lattice QCDs indicate that although the contributions of scalar form factor term to the decay of $D \to K\mu\nu_\mu$ are small, the determination of scalar form factor from $D \to K\mu\nu_\mu$ decays is feasible. And the precision of $f_0^K(q^2)$ determined from the fit to experimental measurements will improved with the increase of measurements for $D \to K\mu\nu_\mu$ decays and the improvement of measurement accuracy.

On the other hand, a comprehensive consideration of $f_+^K(0)|V_{cs}| = 0.718 \pm 0.003$ and $f_0^K(0) = 0.740 \pm 0.018$ (the average of $N_f = 2 + 1$ lattice calculations $f_0^K(0) = 0.698 \pm 0.046$ \cite{20} and $f_0^K(0) = 0.747 \pm 0.019$ \cite{28}), the magnitude of the CKM matrix element $V_{cs}$ is determined to be

$$|V_{cs}| = 0.970 \pm 0.004 \pm 0.024,$$

where the first error is experimental and the second is theoretical. With comprehensive consideration of the $|V_{cs}| = 0.970 \pm 0.004 \pm 0.024$ extracted in this analysis together with the $|V_{cs}| = 1.008 \pm 0.021$ determined from leptonic $D_s$ decay \cite{25}, the magnitude of the CKM

Figure 6: Comparison of $f_+^K(0)$ determined from experimental data and calculated by lattice QCD.
Figure 7: Comparisons of the result of fit to experimental measurements and lattice calculations for $f_+^K(q^2)$ (a) and $f_0^K(q^2)$ (b).
Figure 8: Comparisons of $|V_{cs}|$ extracted from different analysis.

matrix element $V_{cs}$ is determined to be

$$|V_{cs}| = 0.992 \pm 0.016,$$  \hspace{1cm} (24)

which is consistent with the average value $|V_{cs}| = 0.995 \pm 0.016$ from PDG’2016 \[25\]. Comparisons of $|V_{cs}|$ extracted from different analysis are shown in Fig. 8.

5 Leptoquark constraints from $D \rightarrow K l^+ \nu_l$ decays

If there exist new interactions beyond the Standard $W^+$ mediating $c\bar{s} \rightarrow \bar{c}l\nu_l$, then they alter the decay rate and $q^2$-distribution of $D \rightarrow K l\nu_l$ decay processes. Since experimental data for $D \rightarrow K l\nu_l$ decays, at some level, are in good agreement with the SM, the contributions of the new particles should be small and thus these new particles must be too heavy to directly detect. So an effective Lagrangian extended the SM is considered here. Omitting right-handed neutrinos, the effective Lagrangian is given by \[16\]

$$L_{\text{eff}} = - \left( \sqrt{2} G_F V_{cs}^* + G_V \right) (\bar{s} \gamma^\mu c)(\bar{\nu}_L \gamma_\mu l_L) + \left( \sqrt{2} G_F V_{cs}^* + G_A \right) (\bar{s} \gamma^\mu \gamma_5 c)(\bar{\nu}_L \gamma_\mu l_L) - G_S (\bar{s}c)(\bar{\nu}_L l_R) - G_P (\bar{s}\gamma_5 c)(\bar{\nu}_L l_R) - G_T (\bar{s}\sigma^{\mu\nu} c)(\bar{\nu}_L \sigma_{\mu\nu} l_R) + \text{h.c.},$$  \hspace{1cm} (25)

where the terms parameterized by $G_F V_{cs}^*$ represent the effective SM Lagrangian, the other five arose by new type interactions.

A few possibilities can arise the non-Standard contributions to $D_s \rightarrow l\nu$ decays which have been analyzed in Refs. \[16,17\]. Among the candidates they discussed, the most possible mechanism which can lead to an effective $\bar{s}c\bar{v}d\bar{l}$ vertex is the $u$-channel exchange of a charge $-2/3$ scalar leptoquark $S_0$, which transforms as color-triplet and weak-singlet with the $U(1)$ hyper-charge $-1/3$ under $SU(3)_c \times SU(2)_L \times U(1)_Y$ transformations of the SM \[16,20\]. The interactions between the leptoquark $S_0$ and the SM fermions can be described as \[16\]

$$L_{S_0} = \lambda_{LL}^{2i} (\bar{c}_i f^C_{iL} - \bar{s}_L \nu^C_{iL}) S_0 + \lambda_{RR}^{2i} (\bar{c}_i f^C_{iR} \nu^C_{iR}) S_0 + \text{h.c.},$$  \hspace{1cm} (26)

where the superscript $C$ stands for charge conjugation, the subscript $i$ denotes the generation of lepton, and $\lambda_{LL}^{2i}$ and $\lambda_{RR}^{2i}$ are complex Yukawa couplings. When the leptoquark mass satisfy
\[ m_{S_0} \gg m_D, \quad \text{one can obtain} \]

\[
G_V = G_A = \frac{|\lambda_{21}^{LL}|^2}{4m_{S_0}^2}, \tag{27}
\]

\[
G_P = G_S = \frac{\lambda_{21}^{LL} \lambda_{21}^{RR^*}}{4m_{S_0}^2} = -2G_T. \tag{28}
\]

Then, there are only two unrelated coefficients left for two type fermion-leptoquark-fermion interactions in Eq. 25. We can name the one parameterized by \( |\lambda_{21}^{LL}|^2/m_{S_0}^2 \) the \( LL \) type, which is caused by only the scalar leptoquark \( S_0^L \), and the one parameterized by \( \lambda_{21}^{LL} \lambda_{21}^{RR^*}/m_{S_0}^2 \) the \( LR \) type.

Because of these new interactions, the expression of differential decay rate for \( D \to Kl\nu \) decays, Eq. 11, should be rewritten as

\[
\frac{d\Gamma_{D \to Kl\nu}}{dq^2} = \frac{|p|}{24\pi^3} \left( 1 + \frac{m_l^2}{q^2} \right)^2 \cdot \left\{ \left( 1 - \frac{m_l^2}{2q^2} \right) |p|^2 \left( G_F V_{cs}^* + \frac{G_V}{\sqrt{2}} \right) f_+(q^2)^2 \right. \\
+ \left. \frac{3(m_D^2 - m_K^2)}{8m_D^2} \frac{m_l}{\sqrt{q^2}} \left( G_F V_{cs}^* + \frac{G_V}{\sqrt{2}} \right) + \sqrt{q^2} G_S \right] \frac{q^2}{\sqrt{2}(m_c - m_s)} |f_0^K(q^2)|^2 \\
+ \frac{q^2 + 2m_l^2}{m_D^2} |p|^2 \left| \frac{G_T}{\sqrt{2}} \right|^2 |f_{10}^K(q^2)|^2 \\
- \left. 3 \frac{m_l}{m_D^2} |p|^2 \text{Re} \left[ \left( G_F V_{cs}^* + \frac{G_V}{\sqrt{2}} \right) \frac{G_T^*}{\sqrt{2}} f_0^K(q^2) f_{10}^K(q^2) \right] \right\}, \tag{29}
\]

where the new form factor, \( f_2(q^2) \), is defined for describing the contribution of the tensorial operators via

\[
\langle K | \bar{q} \sigma^{\mu\nu} q | D(p_D) \rangle = i \frac{p_D^\mu p_K^\nu - p_D^\nu p_K^\mu}{m_D} f_2^K(q^2). \tag{30}
\]

In both SCET and HQET limits, one can find \( f_2(q^2) \approx f_+(q^2) \) [22].

To obtain constraints on \( LL- \) and \( LR- \) type fermion-leptoquark-fermion interactions from \( D \to Kl\nu \) decays, using the central values of the product \( f_+^K(q^2)|V_{cs}| \) and the shape parameters \( r_+ \) and \( r_0 \) obtained from the fit to experimental measurements in the context of the SM (cf. Tab. 4) together with \( V_{cs}^* = |V_{cs}| = 0.97351 \) (conventionally) obtained from the SM global fit as input parameters, we re-analyze the experimental data via Eq. 29 by one type interaction at a time. \( \chi^2 \) varying with \( \Lambda/m_{S_0}^2 \) are shown in Figs. 9 (a) and (b) for the lepton-neutrino pairs \( e-\nu_e \) and \( \mu-\nu_\mu \), respectively. From Fig. 9 (a) one can find the bounds, at 95% C.L., for the case of the final states with \( e-\nu_e \) pair are

\[
|\lambda_{21}^{LL}|^2/m_{S_0}^2 < 8.8 \times 10^{-7} \text{ (GeV}^{-2}), \tag{31}
\]

\[
|\text{Re}(\lambda_{21}^{LL} \lambda_{21}^{RR^*})/m_{S_0}^2| < 5.7 \times 10^{-6} \text{ (GeV}^{-2}). \tag{32}
\]

And as Fig. 9 (b) shows, the bounds, at 95% C.L., for the case of the final states with \( \mu-\nu_\mu \) pair are

\[
|\lambda_{21}^{LL}|^2/m_{S_0}^2 < 1.5 \times 10^{-6} \text{ (GeV}^{-2}), \tag{33}
\]

\[
|\text{Re}(\lambda_{21}^{LL} \lambda_{21}^{RR^*})/m_{S_0}^2| < 1.2 \times 10^{-5} \text{ (GeV}^{-2}). \tag{34}
\]
Figure 9: $\chi^2$ varies with $\Lambda/m^2_{S_0}$, where $\Lambda = |\lambda_{21}^{LL}|^2$ and $\Lambda = \text{Re}(\lambda_{21}^{LL} \lambda_{21}^{RR*})$ for the case of the final states with $e$-$\nu_e$ pair (a), and $\Lambda = |\lambda_{22}^{LL}|^2$ and $\Lambda = \text{Re}(\lambda_{22}^{LL} \lambda_{22}^{RR*})$ for the case of the final states with $\mu$-$\nu_\mu$ pair (b).

Figure 10: Combined limits on second-generation leptoquark $S^L_0$.

Recently, a search for pair production of second-generation leptoquarks is performed by the CMS Collaboration by using 35.9 $fb^{-1}$ of data collected at $\sqrt{s}=13$ TeV in 2016 with the CMS detector at the LHC [30]. By analyzing the final states with $\mu\mu jj$ and $\mu\nu jj$, they exclude second-generation leptoquarks with masses less than 1530 GeV (1285 GeV) for $\beta = 1(0.5)$ at 95% C.L., where $\beta$ is the branching fraction of a leptoquark decaying to a charged lepton and a quark. Assuming lepton number conservation, with limits Eqs. (33) obtained from semileptonic $D \to K$ decays in conjunction with masses limits for second-generation scalar leptoquarks obtained by the CMS Collaboration, we show the combined limits on second-generation leptoquark $S^L_0$ in Fig. 10.
6 Conclusions

By globally analyzing all existing measurements for $D \to Kl^+\nu_l(l = e, \mu)$ decays in the last 30 years, we find both the vector and scalar form factors of $D \to Kl^+\nu_l$ decays from these experimental measurements. With two-parameter series expansion form factors, we obtain the product of form factor $f^K_+(0)$ and the magnitude $|V_{cs}|$ and the shape parameters of both vector and scalar form factors

$$f^K_+(0)|V_{cs}| = 0.718 \pm 0.003 \pm 0.000,$$

$$r_+ = -2.17 \pm 0.007, \quad 1.4 \pm 3.5.$$

With the product $f^K_+(0)|V_{cs}|$ together with $|V_{cs}|$ obtained from SM global fit, we determine

$$f^K_+(0) = 0.738 \pm 0.003 \pm 0.000,$$

which is consistent within error with the lattice calculations, and presents a good consistency with the previous fitting result, but with higher precision.

Then we check the vector form factor $f^K_+(q^2)$ and the scalar form factor $f^0_0(q^2)$ calculated by lattice QCDs. Both $f^K_+(q^2)$ and $f^0_0(q^2)$ determined from the fit are consistent within error with recent $N_f = 2 + 1$ lattice calculations. But the vector form factor $f^K_+(q^2)$ determined from the fit is more precise than the value calculated by lattice QCD, and so is the scalar form factor $f^0_0(q^2)$ at low $q^2$ range, but at high $q^2$ range $f^K_+(q^2)$ determined from the fit is much less precise than that calculated by lattice QCD.

With the product $f^K_+(0)|V_{cs}|$ in conjunction with the recent average of form factor $f^K_+(0)$ in $N_f = 2 + 1$ lattice calculations, the magnitude of CKM matrix element $V_{cs}$ can be extracted

$$|V_{cs}|^{D \to Kl\nu_l} = 0.970 \pm 0.004 \pm 0.024,$$

where the second error is from the lattice calculated form factor which is 6 times larger than the first error which is from experiments. The determined magnitude $|V_{cs}|$ presents a good consistency within error with the one from SM global fit. Then factoring in $|V_{cs}|^{D_s \to Kl\nu_l} = 1.008 \pm 0.021$ determined from leptonic $D_s$ decay [25], the magnitude of the CKM matrix element $V_{cs}$ is determined to be

$$|V_{cs}| = 0.992 \pm 0.016,$$

which is in good agreement within error with the average value $|V_{cs}| = 0.995 \pm 0.016$ from PDG’2016 [25].

We re-analyze these experimental measurements in the context of new physics. Taking the returns of the fit in the context of the SM and $|V_{cs}|$ from SM global fit as input parameters, we constrain leptoquark $S_0^{-1/3}$ from $D \to Kl\nu_l$, at 95% C.L., for the case of final states with $e-\nu_e$ are

$$\left| \frac{\lambda_{21}^{LL}}{m_{S_0}^2} \right|^2 < 8.8 \times 10^{-7} \text{ (GeV}^{-2}),$$

$$\left| \frac{\text{Re} (\lambda_{21}^{LL} \lambda_{21}^{RR})}{m_{S_0}^2} \right| < 5.7 \times 10^{-6} \text{ (GeV}^{-2});$$

and for the case of final states with $\mu-\nu_\mu$ are

$$\left| \frac{\lambda_{22}^{LL}}{m_{S_0}^2} \right|^2 < 1.5 \times 10^{-6} \text{ (GeV}^{-2}),$$

$$\left| \frac{\text{Re} (\lambda_{22}^{LL} \lambda_{22}^{RR})}{m_{S_0}^2} \right| < 1.2 \times 10^{-5} \text{ (GeV}^{-2}).$$
Considering recent mass constraints for second-generation leptoquarks obtained by the CMS Collaboration, we give a combined limits on second-generation leptoquark $S_L^0$.

Acknowledgement

This work was supported in part by the National Natural Science Foundation of China under Grants No.11275088 and 11747318.

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