An accurate fatigue damage model for welded joints subjected to variable amplitude loading

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Abstract. Researchers in the past have proposed several fatigue damage models to overcome the shortcomings of the commonly used Miner’s rule. However, requirements of material parameters or S-N curve modifications restrict their practical applications. Also, application of most of these models under variable amplitude loading conditions have not been found. To overcome these restrictions, a new fatigue damage model is proposed in this paper. The proposed model can be applied by practicing engineers using only the S-N curve given in the standard codes of practice. The model is verified with experimentally derived damage evolution curves for C 45 and 16 Mn and gives better agreement compared to previous models. The model predicted fatigue lives are also in better correlation with experimental results compared to previous models as shown in earlier published work by the authors. The proposed model is applied to welded joints subjected to variable amplitude loadings in this paper. The model given around 8% shorter fatigue lives compared to Eurocode given Miner’s rule. This shows the importance of applying accurate fatigue damage models for welded joints.

1. Introduction
Fatigue is one of the main causes of failures in both onshore and offshore steel structures subjected to variable amplitude loading (VAL) [1]. The fatigue damage is generally determined using the Miner’s rule due to its simplicity and ease of application [2]. Moreover, the current design codes and standards such as Eurocode [3], Det Norske Veritas [4] etc. also recommend its use in the design practice. However, Miner’s rule may lead to inaccurate life predictions as it does not consider the damage due to loading sequence accurately [5][6]. Several damage theories have been proposed since then to overcome these shortcomings.

Among the first improvements are the nonlinear Palmgren-Miner rule [7] and the Marco-Starkey model in 1954 [8]. The models have a constant parameter C that depends on the physical variables of the material and requires fatigue tests [9]. Several other models were proposed subsequently towards the end of the 1950s, but they required the determination of similar material parameters. Manson proposed a double linear damage rule (DLDR) in 1966 by replacing the linear Miner’s rule with a set of two lines converging at a knee point [10]. The synergetic effect of the application of this rule in conjunction with a probabilistic approach based on P-S-N curves is recently shown by Correia et al.
The DLDR was further improved and the double damage curve approach (DDCA) was proposed by Manson and Halford in 1981 [12]. Many other damage models were proposed in the 1980s and 1990s such as models by Lemaitre and Plumtree [13] and by Chaboche and Lesne [14]. These models were again based on material parameters \( p, \alpha, \beta \) which can be determined only through extensive material testing. Although some of the above-mentioned models have shown good agreement with the experimental data for specific materials, determination of the knee point location and additional material parameters restricted their use in applied engineering problems [15]. A more detailed review of the damage models developed before 1998 can be found in an article by Fatemi and Yang [16]. These damage models are based on crack growth concepts, damage curve modifications and energy based theories, as well as on continuum damage mechanics. However, the application of these models is not found in any of the design standards as they require testing for the determination of material parameters.

Material testing was performed by some researchers to establish fatigue damage behaviour in 1999. These tests were based on the exhaustion of material ductility and estimated the instantaneous damage in material for a given stress amplitude or range [17]. As a result, the damage evolution curves for several materials were established, and these represent the variation in experimental observed damage with number of cycles to failure. Testing was also performed to develop damage models based on hardness increase during the fatigue of material [18]. Subsequently, damage models were proposed to fit such experimental data but were again dependent on material parameters with application to specific materials. In 2005, Mesmacque et al. proposed a sequential law, which does not require any material parameters other than the full-range S-N curve [6]. The application of this model was demonstrated in steel bridges in 2008 [19]. Though the model can capture the loading sequence and predicts the fatigue life accurately, its requirement for a full-range S-N curve restricts industrial application by practicing engineers. Also, the design codes given S-N curves are based on detail categories, and the physical meaning of the intercept used for such details is not clear. Moreover, this law cannot be used with the design codes and standards having bilinear and trilinear S-N curves. Some other proposed models do not require material testing and overcome the shortcomings of Miner’s rule using load interaction factors [20-22]. However, both the damage evolution curves and the fatigue life predictions differ significantly from experimental results, as will be shown later.

Recently, some probabilistic approaches have been proposed by considering a probabilistic S-N field and providing a statistical distribution of the Miner’s damage based on log-normal distribution [23] [24]. The Miner’s damage is related to a normalized variable \( V \), which represents percentile curves in the S-N field unequivocally associated to probability of failure. These models can be applied for fatigue design of structural component based theories, as well as on continuum damage mechanics. More details about these models can be found in several of the recently reviewed articles regarding fatigue damage theories such as by Santecchia et al. [28] and by Silitonga et al. [29].

Although many fatigue damage models have been proposed in the literature, the existing codes and standards still use Miner’s rule because of its simplicity and ease of application. This indicates the need for an equally simple, easy to apply and accurate fatigue damage model for engineering applications/structural engineering problems. This becomes even more important for estimating the
remaining life of ageing steel structures, where loading histories might be known as well. Such a model will not only predict the remaining life but will also help maintain the existing infrastructure more efficiently. The risk of failure and subsequent consequences will also be reduced using a more accurate model.

To overcome above mentioned problems, an accurate and easy-to-apply fatigue damage model is proposed in this paper. The proposed damage model does not require any material testing and depends only on the commonly available S-N curve of the material or its corresponding detail category. Moreover, it does not require a full-range S-N curve and can be easily applied by practicing engineers using the partially known S-N curves given in design standards. The proposed model can be applied to structural details subjected to variable amplitude loading conditions unlike most of the earlier models applied only to block loading cases. As a result, the proposed model can be easily implemented by practicing engineers for fatigue analysis of several practical problems involving design detail categories subjected to variable amplitude loadings. The proposed model considers the loading sequences, along with the interactions between them, and provides better agreement with experimental data compared to previously proposed models. Due to its unique features and better accuracy, the proposed model can provide a platform to design and maintenance communities for an efficient use of existing ageing steel infrastructure by predicting their remaining safe life more accurately. Initially, the paper presents the proposed damage model in detail. The model is verified with experimentally derived damage evolution curves for C 45 and 16 Mn and gives better agreement compared to previous models. The practical application of the proposed model is shown on welded joints used in several engineering applications. The model is applied to butt and fillet welded joints subjected to block loading and predicted lives are compared with experimental results. The application of the model under variable amplitude loading is also shown on these joints and the results are compared with Eurocode predicted lives. Hence, the applicability, validity and significance of the proposed model is confirmed.

2. Proposed fatigue damage model

A new and easy-to-apply fatigue damage model is proposed as shown in equation (1). The damage for a given stress level is determined using the damage evolution curve of the considered material. This damage is then transferred to the next stress level by determining the effective number of cycles using the proposed load interaction factor. The physical meaning of the concept is to assume the same damage state of the material, while transferring the loading state from one stress level to the next and determining the effective number of cycles required to cause this damage.

\[
D_i = 1 - \left[1 - \frac{n_i}{N_i}\right]^{\delta_i} = 1 - \left[1 - \frac{n_{(i+1)\text{eff}}}{N_{i+1}}\right]^{\delta_{i+1}}
\]

where \(D_i\) is the damage at load level \(i\) when member is subjected to a certain stress amplitude (or range) \(\sigma_i\) for \(n_i\) number of cycles and \(n_{(i+1)\text{eff}}\) is the effective number of cycles corresponding to the stress range \(\sigma_{i+1}\) at level \(i+1\). \(N_i\) and \(N_{i+1}\) are the corresponding number of cycles to failure and can be determined from the S-N curve given in design standards and codes. \(\delta_i\) and \(\mu_i\) are the model parameters and depends only on \(N\) and given stress levels as shown in equation (2) and equation (3). The proposed model also represents some of the earlier proposed damage models for specific values of \(\delta_i\) as shown in Table 1.

\[
\delta_i = \frac{-1.25}{\ln N_i}
\]

\[
\mu_i = \left(\frac{\sigma_{i-1}}{\sigma_i}\right)^2
\]

The damage at load level \(i+1\) can be determined using equation (4) and equation (5).
\[ D_{i+1} = 1 - \left[ 1 - \frac{n(\text{total}, i+1)}{N_{i+1}} \right]^{\frac{1}{\delta_i}} \]  

where

\[ n_{(i+1),\text{total}} = n_{(i+1),\text{eff}} + n_{(i+1)} \]  

where \( n_{i+1} \) is the number of cycles for stress state \( \sigma_{i+1} \). Also, \( n_{(i+1),\text{total}} \) is the total number of cycles for stress state \( \sigma_{i+1} \) obtained using the proposed concept.

| \( \delta \) | Damage model | Remarks |
|------------|--------------|---------|
| -1.25/\ln N \_i | Proposed model | only based on commonly available S-N curves |
| 1 | Miner’s rule, 1945 [2] | most commonly used, but is inaccurate under VAL |
| 1/p+1 | Lemaitre and Plumtree, 1979 [13] | \( p \) is a material constant determined by damage evolution curves based on cyclic stress ranges |
| 1/1-\alpha | Shang and Yao, 1999 [17] | \( \alpha \) is a material constant determined by damage evolution curves based on static ductility change |

This damage transfer technique is continued until the fatigue damage \( D \) becomes one, denoting fatigue failure. The proposed damage transfer concept is explained graphically, as shown in Figure 1. More details of the proposed model and associated parameters can be found in recently published work by the authors [30].

**Figure 1.** Graphical representation of the proposed damage transfer concept
2.1. Verification of proposed model with experimental damage evolution
The proposed fatigue damage model is verified by comparing the experimental results for damage evolution curves. C 45 and 16 Mn steels are used for this verification [17].

2.1.1. C 45 steel. This material is of interest to the authors due to its currently increasing use in offshore structures [31]. The yield strength and ultimate tensile strength for this material are 371.7 MPa and 598.2 MPa, respectively [17]. In the past, experiments were conducted on this material for constant amplitude stresses, and the fatigue damage was determined by measuring the static relative ductility change in the material [17]. The experiments were conducted at 330.9 MPa and 405.8 MPa stress amplitudes (with zero mean stress); the results are shown in Figure 2 (a) and Figure 2 (b), respectively. Fatigue damage curves are also plotted for each of the stress amplitudes using the proposed model. The proposed model is also compared with some of the recently developed models which do not require additional material testing, other than their S-N curve.

2.1.2. 16 Mn steel. The damage evolution curves were also determined experimentally for 16 Mn steel by measuring the static relative ductility change in material [17]. For this material, the yield strength is 382.5 MPa, and the ultimate tensile strength is 570.7 MPa. The experiments were conducted at 337.1 MPa and 373.5 MPa stress amplitudes (with zero mean stress); the damage evolution curves are shown in Figure 3 (a) and Figure 3 (b), respectively. Fatigue damage curves are also plotted for each of the stress amplitudes using the proposed model. The proposed model is also compared with some of the recently developed models which do not require additional material testing, other than their S-N curve.

2.1.3. Discussion of verification study. From Figure 2 (a) and Figure 2 (b), it is seen that the proposed model gives good prediction of the damage curves for material C 45 and is in good agreement with the experimental results. The damage evolution curves for 16 Mn steel are shown in Figure 3 (a) and Figure 3 (b) for the two stress amplitudes. Again, it is seen that the proposed model is in good agreement with the experimental results and can predict the damage behaviour more accurately. The proposed model is also compared with some of the other recently developed models which do not require additional material testing, other than their S-N curve. It is seen that the model developed by Kwofie and Rahbar [20] is not in good agreement with the experimental data and is equivalent to Miner’s rule while predicting the damage behaviour. The models proposed by Liakat and Khonsari [21] and Lv et al. [22] seem to give the same damage evolution curves for both considered materials. The sequential law proposed by Mesmacque et al. [6] is also compared with the experimental data. It is seen that the model gives good agreement with the experimental results. However, the model requires the determination of the full-range S-N curve, thereby restricting its use by practicing engineers. After comparing all the curves shown in Figure 2 and Figure 3, it is concluded that the proposed model predicts the real damage evolution behaviour quite accurately for the two materials considered, when compared to previous models. The damage curves can be plotted using the proposed model without determining additional material parameters and using only code-given S-N curves.

3. Application of proposed model to welded joints
The proposed model is applied on welded joints subjected to both block loading as well as variable amplitude loading. These welded joints are used in several engineering applications such as vehicles, electrical mobile units (EMUs), etc., and an accurate prediction of life is necessary for the safety of both passengers and vehicles. There has been a significant advancement in technology over the past couple of decades. As a result, conventional diesel locomotives are increasingly being replaced by faster electrical mobile units (EMUs). These EMUs can reach speeds of 250 km/h and above, as compared to the diesel locomotives which used to have typical speeds of 160 km/hr [32]. While the advancement of the vehicle technology is good for the fast commuting of passengers, it also puts vehicle safety, as well as the safety of people, at huge risk. Such risks arise from the increasingly daily passenger loads, inefficient maintenance of vehicles, as well as ageing of the car bodies, which, in these high-speed trains are generally made from aluminium alloy due to its light weight, security and comfort [33].
Figure 2. Comparison of theoretically predicted fatigue damage evolution with experimental damage of C 45 steel for stress (a) $\sigma_a = 330.9$ MPa (b) $\sigma_a = 405.8$ MPa

Figure 3. Comparison of theoretically predicted fatigue damage evolution with experimental damage of 16 Mn steel for stress (a) $\sigma_a = 337.1$ MPa (b) $\sigma_a = 373.5$ MPa

However, as a result of the high thermal conductivity coefficient and high linear expansion coefficient, aluminum alloys can have complex welding deformations and stresses at the welded joints. The softening and embrittlement of these welded joints can have a serious effect on the structural strength of the car body. It is reported that the failure of the car body usually occurs in the welded joint,
and the major cause is fatigue loading [33] [34], which is most critical for the welded connections; it is therefore important to estimate the fatigue life accurately to minimize the risks.

Much research has been carried out in the past on the fatigue properties of welded aluminum joints. Recently, a group of researchers performed fatigue tests on such aluminum joints to thoroughly understand the fatigue properties and predict the life more accurately under the block loading conditions [33] [34]. Firstly, the experimental setup and considered welded joints are explained in detail in this section. The proposed model is then applied to these joints under the given block loading conditions and predicted lives are compared with experimental results. The applicability and accuracy of proposed model under block conditions is henceforth confirmed. The application of the proposed model is also shown on these welded joints subjected to random variable amplitude loadings (VAL). An in-situ measured VAL is considered for this study. The results are compared with Eurocode recommended Miner’s rule. Hence, the applicability of the proposed model under VAL is also confirmed in this study.

3.1. Considered welded joint details

The fatigue tests were performed for both butt and fillet welded joints by using the PLG-200 fatigue testing machine. The test specimens are shown in Figure 4 (a) and Figure 4 (b) respectively. The base material of the specimen is aluminum alloy ENAW6005 having the main chemical composition in mass percentage as: 0.35Fe, 0.6-0.9Si, 0.1Mn, 0.1Ti, 0.4-0.6Mg, 0.1Cu, 0.1Zn and 0.1 Cr [33] [34]. The tensile strength and yielding strength of the base material are 270 and 225 MPa, respectively. The specimens were tested under a four-point bending load cycle with cyclic stress ratio \( R = -1 \). The load-time history follows the sine function. The loading frequency is 110 Hz, and room temperature is 20~30°C [33] [34].

3.2. Application of proposed model under block loading

The proposed model is applied to welded joints under block loading conditions and the predicted lives are compared with experimental lives. The experiments were conducted by previous researchers under two-stage block loading for each of the considered welded joints [33] [35]. The load sequence considered for the butt-welded joint are 104–74 MPa and 89–74 MPa for high-low loading sequence, and 74–89 MPa and 74–104 MPa for low-high loading sequences. The load sequences considered for the fillet welded joint a re 93–73 MPa and 83–73MPa under high-low loading, and 73–83 MPa and 73–93 MPa under low-high loading, respectively. The experimentally obtained fatigue lives for the butt and fillet welded joint are shown in Table 2 and Table 3, respectively [33] [35].

The proposed damage model is compared with experimentally obtained fatigue lives for the considered welded joints. The loading sequences are given in Table 2 and Table 3. Some of the recently proposed models have also been included in the comparison. The comparison of fatigue life is shown in Figure 5 (a) and Figure 5 (b) for the butt and fillet welded joints, respectively. The maximum and average deviations from the experimental results are 20% and 9% respectively. The solid line
refers to ideal conformity of the results. The dotted lines represent a scatter band with a coefficient of two.

### Table 2. Experimental fatigue lives for fatigue testing on butt welded joint [33] [35].

| Load Set | σ₁ (MPa) | σ₂ (MPa) | n₁ (*1000) | n₂ (*1000) | N₁    | N₂    |
|----------|----------|----------|-------------|-------------|-------|-------|
| Set 1    | 104      | 74       | 109.9       | 797.6       | 549300| 1540100 |
| Set 2    | 89       | 74       | 176.1       | 1029.2      | 880500| 1540100 |
| Set 3    | 74       | 89       | 770.1       | 545.6       | 1540100 | 880500 |
| Set 4    | 74       | 104      | 770.1       | 418.9       | 1540100 | 549300 |

### Table 3. Experimental fatigue lives for fatigue testing on fillet welded joint [33] [35].

| Load Set | σ₁ (MPa) | σ₂ (MPa) | n₁ (*1000) | n₂ (*1000) | N₁    | N₂    |
|----------|----------|----------|-------------|-------------|-------|-------|
| Set 1    | 93       | 73       | 309.9       | 587.5       | 619800| 1546100 |
| Set 2    | 83       | 73       | 476.1       | 681.1       | 952300| 1546100 |
| Set 3    | 73       | 83       | 509.2       | 708.2       | 1546100| 952300 |
| Set 4    | 73       | 93       | 773.0       | 426.4       | 1546100| 619800 |

### Figure 5. Comparison of calculated fatigue life with experimental life for (a) butt welded joint (b) fillet welded joint

3.3. Application of proposed model under variable amplitude loading (VAL)

The proposed model is also applied to considered welded joints under variable amplitude loading conditions. The fatigue life of both butt and fillet welded joints is predicted using the proposed model. The lives are also predicted using the Miner’s rule recommended by standard codes and standards such as Eurocode. The detail category S-N curves for the joints are taken from the Eurocode [36]. The cycle counting is performed using the rainflow counting method [37]. Finally, the damage accumulation curves are obtained and fatigue lives from proposed model are compared with those obtained using Miner’s rule.
3.3.1. Considered variable amplitude loading (VAL). The variable amplitude loading considered for this study is shown in Figure 6. The stress values are shown for every 0.13 seconds and for a period of one hour. The same stress history is assumed to be repeated until the end of the fatigue life (i.e. the same loading block is assumed to be repeated until the failure point).

3.3.2. Rainflow counting and mean stress correction. The cycle counting is performed on the considered VAL using the rainflow counting method [37]. This is followed by the mean stress correction as recommended by Goodman rule [5]. The resulting stress amplitude history is shown in Figure 7.

3.3.3. Considered detail category S-N curves for welded joints. The detail category S-N curves of the welded joint details are taken from Eurocode [36]. The parameters of these curves including the fatigue endurance stress limit are shown in Table 4. The stress history after the removal of endurance limit are shown in Figure 8 and Figure 9 for the butt welded and fillet welded joint respectively.

| Joint   | S-N curve parameters |
|---------|----------------------|
|         | $\Delta \sigma_L$  | $\Delta \sigma_D$ | $\Delta \sigma_C$ | $m_1$ | $m_2$ |
| Butt    | 20.1                 | 32.3              | 40                | 4.3   | 3.4   |
| Fillet  | 12.3                 | 21.4              | 28                | 6.3   | 5.4   |

Table 4. S-N curve parameters for considered joints given in Eurocode [36]

Figure 6. Considered variable amplitude loading for welded joints
Figure 7. Stress history after mean stress correction

Figure 8. Stress history above endurance for butt welded joint
3.3.4. Comparison of damage curve and fatigue life. The proposed model is applied to calculate fatigue life of the considered welded joints. The stress amplitude histories shown in Figure 8 and Figure 9 are used for these calculations. The interaction factor in the proposed model is taken as one for this study and the accumulated fatigue damages until failure is shown in Figure 10 and Figure 11 for butt and fillet welds respectively. The damage accumulations using the Eurocode recommended Miner’s rule are also shown in Figure 10 and Figure 11.

The fatigue life for the considered joints can be determined from Figure 10 and Figure 11. The fatigue lives are calculated in terms of number of blocks of the VAL considered (i.e. number of repeated blocks) and are shown in Table 5.

Table 5. Fatigue life comparison using proposed model and Eurocode (in blocks)

| Joint | Number of blocks of considered VAL | % difference |
|-------|-----------------------------------|--------------|
| Butt  | Eurocode 899                      | Proposed 829 | 7.80         |
| Fillet| Eurocode 544                      | Proposed 514 | 5.3          |

3.4. Discussion and comparison of the results

The proposed model is applied to predict the fatigue lives of welded joints used in several engineering applications. Butt and fillet welded joints were considered in the case study. The application of the fatigue damage model to these joints is shown under both block loadings and variable amplitude loading conditions. The comparisons of the fatigue lives under block loading conditions are shown in Figure 5 (a) and Figure 5 (b) for butt welded and fillet welded joint respectively.
**Figure 10.** Fatigue damage accumulation using proposed model and Eurocode for butt welded joint

**Figure 11.** Fatigue damage accumulation using proposed model and Eurocode for fillet welded joint
The predicted lives using the proposed model are in good agreement with the experimental lives compared to Eurocode given Miner’s rule. The proposed model predictions are also better compared to earlier models. The sequential law model could not be applied to these joints since full range S-N curve is not available for these detail categories. Using the proposed model, the maximum and average deviations from the experimental results are 20% and 9% respectively for these joints. The scatter bands with a coefficient two is also plotted in all the figures where experimental fatigue life is compared with predicted life. The predicted fatigue lives are all lying within the scatter band.

The application of proposed model is also shown under variable amplitude loading conditions. The variable amplitude loading considered is taken from in-situ measurements for a period of one hour and is assumed to be repeated until the failure. The mean stress zero equivalent stress amplitude history is then derived using the rainflow counting algorithm followed by the mean stress correction. The proposed model requires only the S-N curve parameters of the detailed categories and these were taken as given in the Eurocode. The fatigue damage accumulation curves are shown in Figure 10 and Figure 11 for the butt welded and fillet welded joint respectively. These are compared for both proposed model and the Eurocode (i.e. using the Miner’s rule). The predicted fatigue lives are shown in Table 5.

From Figure 10 and Figure 11, it is seen that the damage accumulation curves using the proposed model are nonlinear and are very similar to the damage evolution curves discussed earlier in this paper. This further confirms the real physical damage accumulation behaviour in the material instead of the linear accumulation as assumed by the commonly used Miner’s rule given in Eurocode and other design standards. From Table 5, it is seen that the predicted lives using the proposed model are around 8% shorter compared to the Eurocode. This difference may not seem too significant for the design of new structures. However, the proposed model can be used for a more accurate prediction of remaining lives of existing infrastructures such as old bridges, ageing oil/gas platforms and industrial buildings. The remaining life calculations based on Eurocode might result in longer lives than the structures can stand. This shows the importance of applying an accurate fatigue damage model.

4. Conclusions
The paper presents a new fatigue damage model. The proposed model does not require any material parameters, other than the commonly available S-N curve parameters, which are generally used with Miner’s rule. The major advantage of proposed model is that it does not require detail material testing or modifications to the S-N curve. Also, unlike earlier models, the proposed model can be applied to design detail categories using the corresponding partially known S-N curve in the design standards. Therefore, the proposed model can be easily implemented by practicing engineers for fatigue analysis of several engineering problems. The model is verified with both the damage evolution curves and fatigue life estimations. It is concluded that the damage curves plotted using the proposed model are in good agreement with the available experimental data for two considered materials. It is also concluded that the fatigue life predictions are more accurate using the proposed model compared to Eurocode given Miner’s rule. The application of the proposed model is shown to welded joints used in several engineering applications. The both butt and fillet welded joints are commonly subjected to both block loading and variable amplitude loading conditions. The obtained damage accumulation curves using proposed model confirms the real physical damage accumulation behaviour in the material instead of the linear accumulation as assumed by the commonly used Miner’s rule given in Eurocode. It is therefore concluded that the proposed model can be used for better fatigue life predictions and can be easily applied by practicing engineers using only the code-given S-N curves. The applicability, significance and validity of the proposed fatigue damage model should be further verified in the future by applying the model to more case studies.
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