A problem of estimating the velocity of subsidence of a column of charge materials using non-contact methods was considered. This is important because the level of furnace charge materials and the velocity of their subsidence are main indicators of melting intensity determining the furnace productivity.

The design of a blast furnace and its blast path were described and existing methods and means of controlling the velocity of charge materials in the blast furnace were analyzed. A mathematical model was presented for estimating the velocity of subsidence of charge materials in a blast furnace based on the magnitude and fluctuations of gas pressure along the furnace shaft height. The model is based on the fact that the furnace gases rise up in the furnace shaft through elementary channels in the column of charge materials consisting of a combination of capacitances and resistances. Volume of capacities and values of resistance of elementary channels are constantly changing. This changes hydraulic resistance to gas movement in the blast furnace. The system of differential equations describes the dependence of the amplitude of pressure fluctuations on the amplitude of change in coefficients of resistance and frequency of pressure fluctuations on the frequency of change in coefficients of resistance. The experimental data on velocity of the column of charge materials and fluctuations in the pressure differential in the furnace were processed and their significant relationship was shown to confirm the previous theoretical study results. To assess the model adequacy, the simulation method was used. The results of the simulation model work were confirmed by experimental data.

The developed mathematical model can be introduced into production. This will make it more economical and safer through better and more predictable control and improved flexibility in operation under different production conditions.

Keywords: blast furnace, blast path, mathematical model, adequate estimate, experimental data.
the common shop collector of blast furnace gas 15 through the droplet eliminator 14.

Fig. 1. Blast furnace and its blast path: 1 — furnace housing; 2 — charge material loader; 3 — skip; 4 — furnace top; 5 — column of charge materials; 6 — hot blast pipeline; 7 — ring pipeline; 8 — cold blast pipeline; 9 — air heaters; 10 — pipeline of fuel additives; 11 — furnace hearth; 12 — top gas flue; 13 — gas cleaning station; 14 — drop catcher; 15 — shop collector of blast-furnace gas

Combustion of coke and melting of iron ore materials in the lower part of the furnace leads to a decrease in the charge volume and subsiding of the entire column of charge materials. Thus, the process of blast-furnace smelting is a continuous flow of physical and chemical transformations and a continuous subsiding of the column of charge materials occurs with different speeds. Movement of the loose column of charge materials (it is loose on most of the column except its bottom part where softening and melting occurs) leads to a continuous change in the volume of voids and channels through which the gas flow rises. This causes pressure fluctuation in the gas flow both in the column of charge materials and in the entire gas path of the furnace. It is obvious that these pulsations are in a certain way related to the charge subsidence velocity.

The level of charge materials in the furnace and the velocity of their subsidence are main indicators of melting intensity determining the furnace productivity.

Production workers and scientists have always paid great attention to methods and means of controlling the charge subsidence velocity. Various methods and technical means are used to control the level and speed of the column of charge materials in the BF. The charge level (the upper level of the column of charge materials in the BF) and velocity of the column subsidence are controlled at many BFs by electromechanical probes [1]. Two such probes (left and right) are installed in the furnace. The probe is designed in such a way that the sensor signal is proportional to the length of the shoe movement and, accordingly, reflects the position of the charge level and the change in movement after a certain time corresponds to the velocity of subsidence of the column of charge materials. The advantage of this method is sufficient operation reliability. Disadvantages include the complexity of technical implementation, insufficient accuracy (the ability to control only two points on the charge surface), and significant operating costs. Besides, in a case of uneven subsidence of charge materials, the probe shoe are not kept on the surface and are immersed in the charge which leads to an error in level measurement.

Non-contact methods of monitoring the level of charge materials were tested at a number of BFs [2]. Radar devices are installed at several points along the top perimeter to scan the charge surface. The sensor signal is processed by software and the result in the form of a picture of the charge surface with corresponding digital marks is output to the computer of the BF shop foreman. The equipment used in this method requires special dust and heat protection measures and its maintenance on the top takes place in conditions that are dangerous for personnel.

Along with technical means, methods of mathematical modeling are used to determine parameters and indicators of blast-furnace smelting. Mathematical models of blast-furnace processes make production more economical and safer due to better, more predictable control, fewer disturbances, and higher flexibility in various external conditions. Therefore, the development of simpler and, preferably, non-contact methods for controlling the velocity of subsidence of charge materials is still relevant.

2. Literature review and problem statement

Blast-furnace process models are divided into complex models, models of concrete zones, and data-based models [3]. Complex models are further classified as lumped parameter models, 1-D, 2-D, and 3-D stationary and transient models, and CFD-DEM models.

Complex models include the model [4] where Rist diagrams were used for optimizing costs at the operational level and exploring new opportunities, such as injection of hot reducing gas, recirculation of top gas, and use of natural gas [5].

First steady-state models were proposed back in the 1970s and are being developed to this day. Current solutions in this direction are described in the study [6] which presents a one-dimensional dynamic model used to predict the thermal state of the process.

Static models are also developed by many authors. A 2D model of a blast furnace taking into account non-mixed layers of ore and coke with distinct reactions in the ore and coke layers is described in [7]. A fine mesh was used to discretize the furnace. After initial coarse subsidence, all sub-models are re-run to predict the adhesion zone. The algorithm is repeated until the process stabilizes. The model describes thermal and chemical reserve zones in a realistic way. The temperature difference between the gas and solid phases turned out to be maximum at the top as well as in the cohesive zone. Fluctuating isolines of gas composition, gas temperature and charge are also well fixed by the model. Fluctuations in composition and temperature of gas and charge are described by the model [8] which was constructed similarly to [7]. It is integrated with a tuyere zone model and a charge loading model to investigate the effect of carbonaceous layers on furnace operation.

A three-dimensional CFD-based blast furnace simulator along with load distribution and a model of charge
movement is described in [9]. The complete Navier-Stokes equation with turbulence was solved for gas flow. The cohesive zone is determined based on the calculated temperature of softening and melting of local charge composition. The authors of [10] found that 2D modeling of liquid fraction distribution near the tuyere zone gives erroneous results and only a 3D model correctly describes the process. Results of the slot model and the sector model of the blast furnace were compared in another study [11] for a given gas flow and charge velocity. It turned out that the sector model gives slightly better results than the slot model.

The study [12] presents an analysis of the 3D CFD-DEM model of a blast furnace in a reduced geometry. The model takes into account the flow of solids and gas but does not take into account thermal effects, i.e. this is an isothermal model. Particular attention is given to the correct choice of simulation conditions (for example, gas supply flow velocity).

A two-dimensional simulation of the process in an experimental blast furnace built in Luleå, Sweden, is described in [13]. It takes into account non-isothermal conditions and basic chemical reactions. The model can describe general conditions in the furnace, predict the level and shape of the cohesive zone. The results of the model operation were verified by comparison with the results of probe measurements in this blast furnace.

The latest solutions in the field of mathematical modeling of different zones of a blast furnace from top to bottom are discussed in [14]: loading systems, furnace housing, and tuyere zone. Zonal models are focused on charge distribution in a blast furnace.

The model from [15] makes it possible to determine the distribution of charge materials over the entire cross-section of the BF taking into account the gas permeability of the charge material layers in dry and viscoplastic zones and heat redistribution between phases and the furnace walls. The simulation results are in good agreement with the readings of the microwave profile meter. A mixed portion (agglomerate, pellets) is loaded as an additional type of material with mechanical and chemical properties of agglomerate and pellets in proportion to their participation in the mixture formation. This model also takes into account the segregation of the material when it is unloaded from the tray.

A detailed review of procedures applied in modeling the drop formation zones is presented in [16]. The models describe the movement of ore and coke and are of decisive importance for controlling and optimizing the process state. Ore to coke ratio affects production and the process of fuel consumption. Radial distribution of the charge materials greatly affects conditions in the upper part of the furnace where the indirect reduction of iron ore occurs.

A large number of data-based models are presented in review [17] which shows the level of development of the issue in this area at that time.

Models based on data analysis represent the blast furnace as a "black box". They use various experimental data and analytics tools such as artificial neural networks (ANN), principal component analysis (PCA), least square method (LSM), machine learning algorithms (MLA) to classify data. Combinations of such models with fuzzy logic were used to develop expert systems for predictive control and advanced non-linear optimization procedures (genetic algorithms, GA) are used to solve multi-parameter problems of optimization, e.g. minimize fuel consumption while maximizing productivity. Some of these procedures are also used to predict anomalies.

Autoregressive models for predicting silicon content in cast iron have been developed since the 1970s and first attempts to predict silicon content using neural networks have been made since the early 1990s. The authors of [18] used the concept of mutual information (MI) when developing a system for predicting the content of silicon. MI measures the overall dependence of random variables without making any assumptions about the nature of their underlying relationships. Initially, a set of 15 input parameters was considered as the input data for the predictive system. Subsequently, the least significant variables were removed using MI. The remaining set of nine parameters was found to be sufficient for the development of a prediction system based on SVM.

Evolutionary and multi-objective genetic algorithms were used in [19] to determine the loading matrix to ensure desired gas distribution in the furnace. The load distribution model [20] in combination with simplified gas flow distribution was used to create a large dataset needed to train neural networks using an evolutionary approach. The loading system was optimized to match the target radial gas temperature distribution at the top taking into account limitations on total pressure drop and ore to coke ratio.

It follows from the above analysis that in relation to existing methods and means of controlling the speed of movement of charge materials in a blast furnace, there are problems associated with the complexity of technical implementation, insufficient accuracy, and substantial operating costs. Non-contact control means use equipment requiring special measures to protect against dust and temperature and its maintenance on the furnace top takes place under conditions dangerous to personnel.

With regard to existing approaches to modeling the processes occurring in the blast furnace shaft, the following shortcomings have been identified: bulkiness of complex models, the difficulty of customizing them for the operation of a particular blast furnace in given conditions, and the need for highly qualified service personnel. Models of concrete zones do not solve the problem of complex analysis of the processes which is necessary for the correct assessment of the velocity of charge descent in real time. Data-driven models require long training and a large training sample size which also makes them difficult to operate under changing conditions and on a different resource base.

Given the above information, there is a need to develop simpler and preferably non-contact methods for monitoring the charge subsidence velocity.

3. The aim and objectives of the study

The study objective consisted in developing a mathematical model for estimating the velocity of subsidence of charge materials in a blast furnace based on the magnitude and fluctuation of gas pressure along the furnace shaft height. This will make it possible to develop a new method for controlling the velocity of charge subsidence and to quickly control it based on this method.

To achieve the objective, the following tasks were set:
– collect and analyze the experimental material (production data) on the relationship between fluctuations in gas pressure in the furnace and the velocity of charge subsidence;
– develop a mathematical model describing fluctuations in gas pressure in the furnace;
– construct a simulation model of the process and evaluate the adequacy of the model according to experimental data.

4. The study materials and methods

Existing methods and means of controlling the speed of movement of charge materials in a blast furnace were analyzed.

To test the basic principle of model construction, we used data on the actual movement of the column of charge materials obtained from mechanical level gauges for 1 day of BF operation in a normal mode and values of pressure fluctuation according to corresponding differential pressure gauges (Fig. 2).

Mechanical probes make it possible to determine the charge level only at two points. Besides, when the charge surface is uneven, the probes are sometimes not kept on the surface and immersed in the material which results in inaccurate readings. When the probes are raised and lowered, no measurement is taken. The data obtained in this way were averaged over the readings of 2 probes and the gaps in the intervals of the rise of the probes were filled using the exponential smoothing method.

Fluctuations in the lower pressure drop of gases were analyzed in terms of frequency and compared with the average velocity of charge subsidence by calculating the correlation coefficient. The value of the correlation coefficient obtained in this case equal to 0.83 shows a possible relationship between fluctuations in the gas flow and the velocity of subsidence of the column of charge materials in the furnace.

The channel through which gases move in a blast furnace starts from the hot blast pipeline. Then it goes through a hearth, a column of charge materials, a furnace top, gas cleaning, and enters the shop blast-furnace gas collector. For a theoretical description of the process, each part of this channel can be represented as a kind of container with corresponding hydraulic resistances at its inlet and outlet.

In the blast furnace shaft itself, the hearth gases rise up through elementary channels of the charge material column during the smelting process. Each such elementary channel consists of a combination of capacitances and resistances. Since the charge materials are constantly moving (subside), the number of such channels, their lengths and volumes are constantly changing which means that hydraulic resistance to the movement of gases in the blast furnace changes as well. This should cause a pulsating change in pressure in the channels and in the furnace shaft as a whole which is confirmed by the practice of blast furnace operation (Fig. 2).

Significant pressure peaks in Fig. 2 correspond to the transition from a cooled air heater to a heated one. The main characteristics of the channel through which the gas moves include its capacity $C$ and hydraulic resistance $R$. Capacity of the channel is determined by the change in a gas amount which corresponds to a single pressure change in it [3]:

$$ C = \frac{\Delta V}{\Delta p} $$

Hydraulic resistance $R$ of the channel is a reciprocal of the tangent of the slope of the flow curve, that is [3]:

$$ R = \frac{dP}{dF} $$

An elementary channel was presented as a set of $n$ vessels with certain capacities $C_i$ and hydraulic resistances $R_i$ at the inlet and hydraulic resistances $R_{i+1}$ at the outlet. The mathematical model of fluctuations in the pressure of gases during their movement in the furnace is presented as a system of $n$ differential inhomogeneous equations of the first order (1).
The resulting mathematical model was simplified by passing to the steady-state regime of gas flow. For each change in $C_i$ and $R_i$, the gas pressure in these vessels has already been established at a new level and remained constant until a new change in $C_i$ and $R_i$ as a result of the charge motion. The model in steady-state is a system of algebraic equations (6). Each solution of this system of equations must be sought for certain values of resistance coefficients $K_j(i)$ at each given $j$-th moment.

This has made it possible to construct a simulation model of the process of change in the gas pressure in a blast furnace with fluctuations in velocity of the charge subsidence. Values of the resistance coefficients for each moment are randomized using a sensor of normally distributed pseudo-random numbers with optimal parameters obtained by the least-square method based on values of technological parameters. As a result of each randomization, values of eight coefficients were obtained and the system of algebraic equations (6) was solved at these values.

The solution of this system has resulted in the next point of gas pressure values. Each such randomization of coefficients and finding pressures corresponds to a certain time intervals of subsidence of the column of charge materials which is also random. For these moments, the values of the velocity of the charge subsidence were found as a realization of a normally distributed quantity with parameters corresponding to the real process. The time intervals for the charge column to pass through a section of the furnace of constant length were found from the calculated velocity values.

The value of the coefficient of correlation between the obtained model sequences of pressure values and those read from instruments indicates the identity of the processes and correctness of the approach to considering this phenomenon.

5. The results obtained in the study of the dependence of the frequency of pressure fluctuations and the velocity of charge subsidence in the blast furnace shaft

5.1. Collection and processing of experimental material on the relationship between gas pressure fluctuation in the furnace and the charge subsidence velocity

To test the basic principle of model construction, we used data on the actual motion of the column of charge materials obtained by monitoring the technological process with the help of sensors. As a result of their processing, numerical values of parameters of gas pressure fluctuations and velocity of the charge motion were obtained. This information was used to construct the process models and assess their adequacy.

Mechanical probes make it possible to determine the charge level only at two points. In addition, when the charge subsidence is uneven, the probes are sometimes not kept on the surface and are immersed in the material which leads to inaccurate readings.

A fragment of an averaged piecewise linear function of the charge subsidence velocity according to the readings of two probes is shown in Fig. 3.

Velocities of the charge subsidence were averaged over those periods when values of probe readings were considered correct and exponential smoothing was used in those intervals when correct readings were not observed.

A fragment of the averaged function of the charge subsidence velocity is shown in Fig. 4.

A fragment of the graph of lower pressure differential in 1 hour of furnace operation is shown in Fig. 5.
Frequencies of lower gas pressure differential were analyzed by frequency over the time intervals from 5 to 8 min. The graph of sliding control (Fig. 6) shows the dependence of the value of the coefficient of correlation between frequency of pressure differentials and the estimate of the average velocity of the charge subsidence on the value of the time interval of frequency calculation.

Fig. 7 shows normalized curves of estimation of the charge subsidence velocity and frequency of fluctuations of the lower pressure differential. The maximum coefficient of correlation between them was 0.83. In this case, frequencies of fluctuation the lower pressure differential were calculated with an averaging window equal to 5.63 min.
The obtained dependences give reason to consider that value of the charge subsidence velocity can be estimated on the basis of analysis of the frequency of gas pressure fluctuation. To do this, it is necessary to establish real-time control of the frequency of pressure fluctuations and find for each furnace mathematical dependence of the velocity of movement of the charge material column on the frequency of pressure fluctuations over a certain period.

5.2. The mathematical model describing the gas pressure fluctuations in the furnace

An elementary channel of movement of gases in a column of charge materials in the furnace was represented as a set of \( n \) vessels with certain capacities \( C_i \), hydraulic resistances \( R_i \) at the inlet, and hydraulic resistances \( R_{i-1} \) at the outlet (Fig. 8).

\[
P_0 \quad R_1 C_1 P_1 \quad R_2 C_2 P_2 \quad R_3 C_3 P_3 \quad \ldots \quad R_n C_n P_n \quad R_{n+1} P_{n+1}
\]

Fig. 8. Schematic representation of an elementary gas movement channel in a column of charge materials in a blast furnace

The rate of pressure change \( \frac{dP_i}{dt} \) in the first (in the gas flow direction) vessel is determined by capacity \( C_1 \), pressure \( P_0 \) and resistance \( R_1 \) at the inlet as well as pressure \( P_2 \) and resistance \( R_2 \) at the inlet of the next channel according to the expression [5]:

\[
C_1 \frac{dP_i}{dt} = \frac{P_0 - P_1}{R_1} - \frac{P_1 - P_2}{R_2}.
\] (1)

For the second and further (in the gas flow direction) channels, there are similar expressions:

\[
C_2 \frac{dP_2}{dt} = \frac{P_1 - P_2}{R_2} - \frac{P_2 - P_3}{R_3};
\]

\[
C_3 \frac{dP_3}{dt} = \frac{P_2 - P_3}{R_3} - \frac{P_3 - P_4}{R_4};
\]

\[
C_{i-1} \frac{dP_i}{dt} = \frac{P_{i-1} - P_i}{R_{i-1}} - \frac{P_i - P_{i+1}}{R_i};
\]

\[
\ldots
\]

\[
C_{n-1} \frac{dP_n}{dt} = \frac{P_{n-1} - P_n}{R_{n-1}} - \frac{P_n - P_{n+1}}{R_n}.
\]

The resulting system is a system of \( n \) differential inhomogeneous equations of the first order. Each equation of the system contains the rate of pressure change in the \( i \)-th cavity (vessel) of the column of charge materials. It depends on pressure and resistances in previous \((i-1)\)-th and next \((i+1)\)-th vessels, therefore, it does not have the property of detecting pressure changes in each channel vessel. This complicates the solution of such systems and makes it impossible to apply conventional methods (e.g., Laplace transform, etc.) for solving.

If we consider the gas flow in the BF as a whole, then pressure \( P_0 \) in the hot blast pipeline and the gas pressure at those points along the shaft height where it is controlled are known. \( P_M \) is pressure in the middle of the furnace shaft, \( P_T \) is pressure on the top and \( P_C \) is pressure in the blast furnace gas collector downstream the throttle group and the drop catcher. All other downstream pressures are unknown. When the charge materials move in the gas flow channels of the charge column, capacitances (volumes) \( C_i \) and hydraulic resistances \( R_i \) randomly change. Thus, the coefficients \( C_i \) and \( R_i \) are random variables in the system of inhomogeneous first-order differential equations (1), that is, we have a system of inhomogeneous differential equations of first order with random variable coefficients. Obviously, the frequency and magnitude of change in these coefficients depend on the velocity of the column of charge materials and this, in turn, leads to a change in pressure \( P_i \) in the channels with a certain amplitude and frequency. The pressure differentials will change similarly: \( \Delta P_i = P_0 - P_M \) for lower and \( \Delta P_{i+1} = P_M - P_T \) for the upper differentials. Therefore, an idea arose to control the velocity of movement of the column of charge materials in the blast furnace according to the frequency (and possibly taking into account the amplitude) of pressure fluctuations and gas pressure differentials in the blast furnace.

Finding pressures and their fluctuations by solving the complete system of equations (1) is practically impossible, so the problem was simplified. To describe the entire gas flow from the hot blast pipeline to the shop blast-furnace gas collector by a similar system of equations, the entire gas flow path was presented in four parts (volumes):

- the furnace hearth;
- the lower part of the column of charge materials (up to the middle of the column);
- the upper part of the column (up to the top);
- the tops with a chimney, a throttle group, and a drop catcher (up to the blast-furnace gas collector).

To simplify the analysis, a totality of all elementary channels along the section of the lower and upper parts of the column was presented as one common channel with corresponding capacitances and resistances. According to this assumption, we have the following diagram of the common channel for the gas flow in the BF (Fig. 9).

\[
P_0 \quad R_1 C_1 P_1 R_2 C_2 P_2 R_3 C_3 P_3 \quad \ldots \quad R_n C_n P_n \quad R_{n+1} P_{n+1}
\]

Fig. 9. A common channel for movement of gas flow in a blast furnace

The first part of the common channel (furnace hearth):

- inlet: the hot blast pipeline (pressure \( P_1 \) and resistance \( R_0 \) at the inlet);
- outlet: the furnace hearth (pressure \( P_1 \) and resistance \( R_1 \) at the outlet).

The second part of the common channel (lower part of the column of charge materials):

- inlet: the furnace hearth (pressure \( P_1 \) and resistance \( R_1 \) at the inlet);
Industry control systems

Outlet: the lower part to the middle of the column of charge materials (pressure $P_2$ and resistance $R_2$ at the outlet).

The third part of the common channel (upper part of the column of charge materials (top):

- inlet: middle of the column (pressure $P_2$ and resistance $R_2$ at the inlet);
- outlet, upper part: the top (pressure $P_3$ at the top and resistance $R_3$ at the outlet).

The fourth part of the common channel is the gas duct, gas cleaning station, throttle group, and drop eliminator (up to the blast-furnace gas collector):

- inlet: the upper part, i.e. the top (pressure $P_3$ at the top and resistance $R_3$ at the inlet);
- outlet: the gas duct, gas cleaning station, throttle group, and drop eliminator to the blast-furnace gas collector (pressure $P_4$ after the drop eliminator and resistance $R_4$ at the outlet, pressure $P_5$ in the shop blast-furnace gas collector).

Taking into account these assumptions, we have a system of four equations:

$$
\begin{align*}
C_1 \frac{dP_1}{d\tau} &= \frac{P_1 - P_2}{R_1} + \frac{P_2 - P_3}{R_2}; \\
C_2 \frac{dP_2}{d\tau} &= \frac{P_2 - P_3}{R_2} + \frac{P_3 - P_4}{R_3}; \\
C_3 \frac{dP_3}{d\tau} &= \frac{P_3 - P_4}{R_3} + \frac{P_4 - P_5}{R_4}; \\
C_4 \frac{dP_4}{d\tau} &= \frac{P_4 - P_5}{R_4} + \frac{P_5 - P_6}{R_5};
\end{align*}
$$

(2)

To find the change in pressure of gases during their movement for each of the BF parts, it is necessary to solve the system of differential equations (2). To this end, equation (2) was transformed in such a way as to obtain a system of differential equations of the first order (3) describing the change in pressure in one full channel.

$$
\begin{align*}
R_2 R_3 C_1 \frac{dP_1}{d\tau} + P_1 &= \frac{R_3 R_4}{R_3 + R_4} + \frac{R_1 R_2}{R_1 + R_2}; \\
R_3 R_4 C_2 \frac{dP_2}{d\tau} + P_2 &= \frac{R_2 R_3}{R_2 + R_3} + \frac{R_1 R_2}{R_1 + R_2}; \\
R_4 R_5 C_3 \frac{dP_3}{d\tau} + P_3 &= \frac{R_2 R_3}{R_2 + R_3} + \frac{R_3 R_4}{R_3 + R_4}; \\
R_5 R_6 C_4 \frac{dP_4}{d\tau} + P_4 &= \frac{R_3 R_4}{R_3 + R_4} + \frac{R_4 R_5}{R_4 + R_5};
\end{align*}
$$

(3)

After entering notations in these equations: $T_i = C_i \frac{R_i R_{i+1}}{R_i + R_{i+1}}$ for a time constant and $K_i = \frac{R_i R_{i+1}}{R_i + R_{i+1}}$ for the dimensionless coefficient of resistance, the new system of equations (4) takes the form:

$$
\begin{align*}
T_1 \frac{dP_1}{d\tau} + P_1 &= K_1 P_2 + K_2 P_3; \\
T_2 \frac{dP_2}{d\tau} + P_2 &= K_2 P_3 + K_3 P_4; \\
T_3 \frac{dP_3}{d\tau} + P_3 &= K_3 P_4 + K_4 P_5; \\
T_4 \frac{dP_4}{d\tau} + P_4 &= K_4 P_5 + K_5 P_6;
\end{align*}
$$

(4)

where $K_1 = \frac{R_1}{R_1 + R_2}$; $K_2 = \frac{R_2}{R_2 + R_3}$; $K_3 = \frac{R_3}{R_3 + R_4}$; $K_4 = \frac{R_4}{R_4 + R_5}$; $K_5 = \frac{R_5}{R_5 + R_6}$.

To find average values of resistances $R_i$ the system of equations (2) was considered in the steady state:

$$
\begin{align*}
0 &= \frac{P_1 - P_2}{R_1} + \frac{P_2 - P_3}{R_2}; \\
0 &= \frac{P_2 - P_3}{R_2} + \frac{P_3 - P_4}{R_3}; \\
0 &= \frac{P_3 - P_4}{R_3} + \frac{P_4 - P_5}{R_4}; \\
0 &= \frac{P_4 - P_5}{R_4} + \frac{P_5 - P_6}{R_5};
\end{align*}
$$

(5)

$$
\tau_{EF} = \frac{L_{EF}_V}{V_{EF}} = \frac{28}{2.82} = 10 \text{ s}.
$$

Average pressure values in respective parts of the furnace were taken as follows:

- $P_1 = 300 \text{ kPa}$, $P_2 = 290 \text{ kPa}$, $P_3 = 190 \text{ kPa}$,
- $P_4 = 150 \text{ kPa}$, $P_5 = 16 \text{ kPa}$, $P_6 = 11 \text{ kPa}$.

To find the value of resistance $R_i$, we have set the value of $R_5$ since pressure $P_4$ is constant and known.

$$
0 = \frac{150 - 16}{R_4} - \frac{P_4 - P_5}{R_5} = \frac{134}{R_4} - \frac{5}{R_5}; \quad 134 R_5 = 5 R_4.
$$

Let’s take the blast flow rate under normal conditions $\Delta F_{BFN}=67 \text{ m}^3/\text{s}$, then the average resistance value is

$$
R_4 = \frac{\Delta P}{\Delta F_{BFN}} = \frac{P_4 - P_5}{5000 \text{ m}^3/\text{s}} = 75 \text{ Pa}/\text{m}^3/\text{s}.
$$

Then $R_4=(134/5) 74.6=2010 \text{ Pa}/\text{m}$.s.

After substituting values of corresponding average pressures and average resistances into equations of system (5), we have obtained values given in Table 1. When taking into account average values of pressures $P_i$ and resistances $R_i$, we have obtained average values of the coefficients presented in Table 2.

Since the rate of pressure change $\frac{dP}{d\tau}$ and value of coefficients of resistance $K_i$ in the system of equations (4) depend on the velocity of subsidence $V_{\text{sub}}$ of charge materials and velocity of gas movement $V_{\text{g}}$ in elementary channels of the furnace filled with charge, let us estimate and compare these velocities.

Values of hydraulic resistance of sections of the BF gas path are as follows: $R_1=150 \text{ Pa}/\text{m}^3/\text{s}$, $R_2=1500 \text{ Pa}/\text{m}^3/\text{s}$, $R_3=600 \text{ Pa}/\text{m}^3/\text{s}$, $R_4=2010 \text{ Pa}/\text{m}^3/\text{s}$, $R_5=75 \text{ Pa}/\text{m}^3/\text{s}$

Values of coefficients of resistance of sections of the BF gas path are as follows: $K_1=0.909$, $K_2=0.901$, $K_3=0.286$, $K_4=0.714$, $K_5=0.770$, $K_6=0.230$, $K_7=0.036$, $K_8=0.964$.

The velocity of movement of gases $V_{\text{g}}$ in an empty furnace

$$
V_{\text{g}}=\frac{F_{\text{BF}}}{S_{\text{BF}}},
$$
where \( F_{\text{BFR}} \) is the blast flow rate under operating conditions (without taking into account the increase in the amount of gases in the furnace shaft), \( S_{\text{EF}} \) is the cross-sectional (average) area of the empty furnace.

The blast flow rates under operating conditions can be found from the flow rate under normal conditions:

\[
F_{\text{BFR}} = F_{\text{BNN}} \frac{T_{\text{g}}}{T_{\text{kn}}} \frac{P_{\text{k}}}{P_{\text{BNN}}}
\]

where \( F_{\text{BFR}} \) is flow rate under normal conditions, \( F_{\text{BNN}} = 66.7 \text{ m}^3/\text{s}; T_{\text{g}} = 2273 \text{ K} \) is blast pressure under operating conditions, \( P_{\text{k}} = 395 \text{ kPa}, \)

\[
F_{\text{BFR}} = 66.7 \cdot 2.273 \cdot 101.2 = 66.7 \cdot 2.13 = 142 \text{ m}^3/\text{s}.
\]

The cross-sectional area of the empty furnace:

\[
S_{\text{EF}} = 0.785(D_{\text{ef}})^2 = 50.24 \text{ m}^2.
\]

Then

\[
V_{\text{g,ef}} = \frac{F_{\text{BFR}}}{S_{\text{EF}}} = \frac{142}{S_{\text{EF}}} = 2.82 \text{ m/s}.
\]

The time of passage of gases in an empty furnace:

\[
\tau_{\text{EF}} = \frac{L_{\text{EF}}}{V_{\text{g,EF}}} = \frac{28.9}{2.82} = 10 \text{ s}.
\]

The time of passage of gases in a filled shaft (through the charge column) reaches 4–10 s [2]. If the height of the column of charge materials is 28 m, the velocity of their straight-line movement is:

\[
V_{\text{g,f}} = \frac{28}{(4..7)} = (4..7) \text{ m}^3/\text{s}.
\]

In fact, gases move through channels that can be much longer than a column of the charge materials, so the actual speed of their movement is much greater. Time of passage of gases through a filled oven:

\[
\tau_{\text{EF}} = \frac{L_{\text{EF}}}{V_{\text{g,EF}}} = \frac{L_{\text{EF}}}{V_{\text{g,EF}}}.
\]

Hence: \( \tau_{\text{EF}} = L_{\text{EF}} S_{\text{EF}} F_{\text{BFR}} \). Substitution of values of \( \tau_{\text{EF}} \) and \( F_{\text{BFR}} \) gives \( L_{\text{EF}} S_{\text{EF}} = 7.142 \cdot 994 = 1.142 \cdot 142 \text{ m}^3/\text{s} \) for an empty furnace.

For gases, the flow area of the column of charge materials is less than the cross-sectional area of an empty furnace. Then, taking into account the average porosity of the furnace charge column \( \epsilon = 0.5 \), it was assumed that:

\[
S_{\text{EF}} = 0.5S_{\text{EF}} = 0.5 \cdot 50.24 = 25.12 \text{ m}^2.
\]

Hence, the average length of the gas channels in the column of charge materials:

\[
L_{\text{EF}} = 994/22.12 = 40 \text{ m}.
\]

If the average time of gas passage through the charge column \( \tau_{\text{EF}} = 7 \pm 2 \text{ [2]} \), then the average velocity of gases in the column of charge materials will be

\[
V_{\text{g,f}} = 392/7 = 56 \text{ m/s}.
\]

Average charge column velocity \( V_{\text{cm}} \) in the furnace is 6–8 m/h, or 0.0017–0.0022 m/s (10–14 cm/min) [2].

As can be seen, the velocity of charge materials is much less than the velocity of gases in the channel. This makes it possible to assume that the time of the transient process of the gas pressure change in the channel vessels is much less than the change in the containers themselves and the resistance coefficients because of movement of the charge column. Thus, we can consider the equations of system (4) in the steady state. For each change in \( C_i \) and \( R_i \), the gas pressure in these containers has already been established at a new level and remains constant until a new change in \( C_i \) and \( R_i \) as a result of the charge movement. Then the system of equations (4) takes the following form:

\[
\begin{align*}
\tau_i &= \frac{L_{\text{cm}}}{{V_{\text{cm}}}} = \frac{28.9}{392/7} = 0.56 \text{ s} \quad \text{(for} \ i = 1, 2, 3, 4)\end{align*}
\]

The resulting system has four equations and four unknown pressures \( P_i \), \( P_2 \), \( P_3 \) and \( P_4 \), that is, it has a unique solution at known values of the coefficients of resistance \( (K_i - K_0) \).

It is obvious that the magnitude (amplitude) of pressure fluctuations \( (\Delta P_i) \) in system (6) depends on the magnitude (amplitude) of change in the resistance coefficients and frequency depends on the frequency of change in coefficients.

Let's assume that the velocity of movement of materials in the furnace \( V_{\text{cm}} \) for some minimum time \( \tau_i \), e.g., 1 s, changes randomly and discretely (stepwise). Then, when taking a certain given thickness of the column of these materials \( l_{\text{cm}} \), it is easy to find the time for passage (subsidence) of this section by the column. The time required for the charge to pass the distance \( l_{\text{cm}} \) will be \( \tau_{\text{cm}}(\tau) = \frac{l_{\text{cm}}}{V_{\text{cm}}}(\tau) \) for each value.

Then the frequency \( f(\tau) = \frac{1}{\tau_{\text{cm}}(\tau)} \) of change in the time of passage of this section of the column, that is, frequency of fluctuations in the change in subsidence velocity is:

\[
 f(\tau) = \frac{1}{\tau_{\text{cm}}(\tau)} = \frac{1}{V_{\text{cm}}(\tau)} = kV_{\text{cm}}(\tau).
\]

Capacitances, channel resistance coefficients, and corresponding gas pressures will change with the same frequency.

5.3. A simulation model of the process of gas pressure change in a blast furnace with fluctuations in the charge subsidence velocity

Construction of a simulation model of the process of change in the gas pressure \( (P_1 - P_2) \) in a blast furnace with fluctuation in velocity of the charge subsidence was considered. The system of equations (6) was taken as a basis. Each solution of this system of equations must be sought for certain values of resistance coefficients \( K_0(j) \) at each given \( j \)-th moment. Values of the coefficients of resistance for each moment were found by generating normally distributed random numbers with mathematical expectation equal to the average value of the coefficients \( M(K_0) \) and variance \( \sigma = 0.05M(K_0) \). Moreover, as follows from the sys-
tem of equations (4), expressions for the coefficients in these equations are mutually connected in pairs by resistances $R_i$. Therefore, only the coefficients $K_1$, $K_3$, $K_5$, $K_7$ were randomized and the coefficients $K_2$, $K_4$, $K_6$, and $K_8$ were found by multiplying these values by the corresponding coefficient of proportionality equal to the ratio $M(K_{32})/M(K_{11M})$. As a result of each randomization, values of eight coefficients ($K_1$–$K_8$) were obtained and the system of algebraic equations (4) was solved with these values. The next point of gas pressure values was found with a solution of this system.

Each such randomizing of coefficients and finding of pressures correspond to a certain time interval of subsidence of the column of charge materials which is also random. Therefore, the charge subsidence velocity $V_{n}(k)$ was determined for each moment $j$ as a normally distributed value with an average value $M[V_{n}(k)]=0.00195$ m/s and a root-mean-square deviation $σ=0.0001$ m/s. Based on randomized values of $V_{n}(k)$, time intervals $τ_{n}(k)=\frac{0.1}{V_{n}(k)}$ were found for the charge column to pass through the furnace section of constant length $l=0.1$ m. Fig. 10 shows corresponding graphs of pressure fluctuations in the gas path of the blast furnace.

![Graph of pressure fluctuations](image)

Fig. 10. Graphs of pressure fluctuations in the gas path of the blast furnace: $P_1$ — the gas pressure in the furnace hearth; $P_3$ — the gas pressure in the middle of the furnace shaft; $P_3$ — gas pressure at the top; $P_4$ — pressure downstream the droplet eliminator

In this case, the amplitude of pressure changes at each time interval characterizes the velocity of subsidence of the charge in this period and the frequency of pressure changes characterizes the frequency of change in velocity of the charge subsidence.

The resulting graphs (Fig. 10) reflect the nature of real pressure fluctuations in the gas path of the furnace and the coefficient of correlation between generated sequences and those taken from the instruments is 0.96. This indicates the identity of the processes and correctness of the approach when considering this phenomenon.

### 6. Discussion of the results obtained in studying the estimation of charge subsidence velocity based on gas pressure fluctuations in the furnace

The resulting mathematical model of pressure fluctuations in the furnace gas flow (6) and the simulation model built on its basis correspond to the experimental data. The coefficient of correlation between generated sequences and those taken from the instruments is 0.96. This is explained by the fact that the methods of gas dynamics and data taken from sensors characterizing actual processes in the furnace were used in the model construction.

A peculiarity of the proposed method of controlling the velocity of charge subsidence in the furnace consists in its simplicity and the fact that data from two mechanical probes and pressure sensors are used to adjust the parameters. Only operational data from pressure sensors are required for the control, i.e., conventional data for controlling a blast furnace.

Limitations of the study are related to the use of experimental data obtained from a microcontroller, that is, quantized in time.

A disadvantage of the method consists in the necessity of adapting the obtained models to each individual furnace. This method can be improved in the future by taking into account the effect of charge parameters (granulometric composition, etc.) on gas pressure fluctuations in the furnace.

### 7. Conclusions

1. The experimental material for the study was obtained from existing furnace control systems. When processing this material, quantitative characteristics of pressure (amplitude and frequency of oscillations) and velocity of subsidence of the charge were obtained. These characteristics are used in the theoretical description of the process.

2. Using the methods of gas dynamics and physics of the process of gas flow in the furnace, a mathematical model of change in gas flow pressure was obtained in a form of a system of $n$ differential inhomogeneous equations of first order with random coefficients. Differential equations connect capacitances $C_i$ and hydraulic resistances $R_i$ of $n$ elementary vessels at the inlet and hydraulic resistances $R_{i+1}$ at the outlet.

3. When considering the steady-state mode of the gas flow motion, the system of equations was simplified to a system of four differential inhomogeneous equations of first order with random coefficients. This has made it possible to construct a process simulation model and estimate its adequacy. The coefficient of correlation between the generated sequences and those taken from instruments is 0.96.

### References

1. Kaplun, L. I., Malygin, A. V., Onorin, O. P., Parhachev, A. V. (2016). Ustroystvo i proektirovanie domennyh pechey. Ekateriburg: UrFU, 219. Available at: https://elar.urfu.ru/bitstream/10995/44483/1/978-5-321-02486-7_2016.pdf
2. Bol’shakov, V. I., Murav’eva, I. G., Semenov, Yu. S. (2013). Primenenie radiolokatsionnyh sistem izmereniya poverhnosti zasypi shihty dlya kontrolya i upravleniya domennoy plavkoj. Dnepropetrovsk: Porogi, 364. Available at: https://www.researchgate.net/profile/Semenov-Yus/publication/325880653_Primenenie_radiolokacionnyh_sistem_izmereniya_poverhnosti_zasypi_shihty_dlya_kontrolya_i_upravleniya_domennoy_plavkoj/links/5b2a0594aca272993784984/Primenenie-radiolokacionnyh-sistem-izmereniya-poverhnosti-zasypi-shihty-dlya-kontrolya-i-upravleniya-domennoy-plavkoj.pdf

3. Abhale, P. B., Viswanathan, N. N., Saxén, H. (2020). Numerical modelling of blast furnace – Evolution and recent trends. Mineral Processing and Extractive Metallurgy, 129 (2), 166–183. doi: 10.1080/25726641.2020.1733357

4. Pettersson, F., Saxén, H. (2006). Model for Economic Optimization of Iron Production in the Blast Furnace. ISIJ International, 46 (9), 1297–1305. doi: 10.2355/isijinternational.46.1297

5. Jampani, M., Gibson, J., Pistorius, P. C. (2019). Increased Use of Natural Gas in Blast Furnace Ironmaking: Mass and Energy Balance Calculations. Metallurgical and Materials Transactions B, 50 (3), 1290–1299. doi: https://doi.org/10.1007/s11663-019-01538-8

6. Hashimoto, Y., Kitamura, Y., Ohashi, T., Sawa, Y., Kano, M. (2019). Transient model-based operation guidance on blast furnace. Control Engineering Practice, 82, 130–141. doi: https://doi.org/10.1016/j.conengprac.2018.10.009

7. Yu, X., Shen, Y. (2018). Modelling of Blast Furnace with Respective Chemical Reactions in Coke and Ore Burden Layers. Metallurgical and Materials Transactions B, 49 (5), 2370–2388. doi: https://doi.org/10.1007/s11663-018-1332-6

8. Kuang, S. B., Li, Z. Y., Yan, D. L., Qi, Y. H., Yu, A. B. (2014). Numerical study of hot charge operation in ironmaking blast furnace. Minerals Engineering, 63, 45–56. doi: https://doi.org/10.1016/j.mineng.2013.11.002

9. Zhou, C. (2012). Minimization of Blast furnace Fuel Rate by Optimizing Burden and Gas Distribution. United States: N.p. doi: https://doi.org/10.2172/1053052

10. Shen, Y., Guo, B., Chew, S., Austin, P., Yu, A. (2015). Modeling of Internal State and Performance of an Ironmaking Blast Furnace: Slot vs Sector Geometries. Metallurgical and Materials Transactions B, 47 (2), 1052–1062. doi: https://doi.org/10.1007/s11663-015-0557-x

11. Shen, Y., Guo, B., Chew, S., Austin, P., Yu, A. (2014). Three-Dimensional Modeling of Flow and Thermochemical Behavior in a Blast Furnace. Metallurgical and Materials Transactions B, 46 (1), 432–448. doi: https://doi.org/10.1007/s11663-014-0204-y

12. Bambauer, F., Wirtz, S., Scherer, V., Bartusch, H. (2018). Transient DEM-CFD simulation of solid and fluid flow in a three dimensional blast furnace model. Powder Technology, 334, 53–64. doi: https://doi.org/10.1016/j.powtec.2018.04.062

13. Hou, Q., E, D., Kuang, S., Li, Z., Yu, A. B. (2017). DEM-based virtual experimental blast furnace: A quasi-steady state model. Powder Technology, 314, 557–566. doi: https://doi.org/10.1016/j.powtec.2016.12.017

14. Kuang, S., Li, Z., Yu, A. (2017). Review on Modeling and Simulation of Blast Furnace. Steel Research International, 89 (1), 1700071. doi: https://doi.org/10.1002/srin.201700071

15. Danloy, G. (2009). Modelling of the blast furnace internal state with MOGADOR. Revue de Métallurgie, 106 (9), 382–386. doi: https://doi.org/10.1051/metal/2009066

16. Ghosh, S., Viswanathan, N. N., Ballal, N. B. (2017). Flow phenomena in the dripping zone of blast furnace – A review. Steel Research International, 88 (9), 1600440. doi: https://doi.org/10.1002/srin.201600440

17. Saxen, H., Gao, C., Gao, Z. (2013). Data-Driven Time Discrete Models for Dynamic Prediction of the Hot Metal Silicon Content in the Blast Furnace – A Review. IEEE Transactions on Industrial Informatics, 9 (4), 2213–2225. doi: https://doi.org/10.1109/tii.2012.2226897

18. Wang, Y., Gao, C., Liu, X. (2011). Using LSSVM model to predict the silicon content in hot metal based on KPCA feature extraction. 2011 Chinese Control and Decision Conference (CCDC). doi: https://doi.org/10.1007/s10004-011-0985-2

19. Mitra, T., Saxén, H. (2015). Simulation of Burden Distribution and Charging in an Ironmaking Blast Furnace. IFAC-PapersOnLine, 48 (17), 183–188. doi: https://doi.org/10.1016/j.ifacol.2015.10.100

20. Li, H., Saxén, H., Liu, W., Zou, Z., Shao, L. (2019). Model-Based Analysis of Factors Affecting the Burden Layer Structure in the Blast Furnace Shaft. Metals, 9 (9), 1003. doi: https://doi.org/10.3390/met9091003