Storage, Splitting, and Routing of Optical Peregrine Solitons in a Coherent Atomic System

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We propose a scheme to realize the storage and retrieval of optical Peregrine solitons in a coherent atomic gas via electromagnetically induced transparency (EIT). We show that optical Peregrine solitons with very small propagation loss, ultraslow motional velocity, and extremely low generation power can be created in the system via EIT. We also show that such solitons can be stored, retrieved, split, and routed with high efficiency and fidelity through the manipulation of control laser fields. The results reported here are useful for the active control of optical Peregrine solitons and promising for applications in optical information processing and transmission.

Keywords: electromagnetically induced transparency, rogue waves, Peregrine solitons, optical memory, optical routing

1 INTRODUCTION

Rogue waves, first observed in ocean surfaces, are highly isolated spatial-temporal wave packets with very large amplitudes when some special conditions are attained [1]. Such waves are ubiquitous in nature and quite intriguing, since they "appear from nowhere and disappear without a trace" and have extremely destructive power [2]. Except for ocean waves, the study on rogue waves has been extended to many other different physical contexts, including atmosphere [3], superfluid helium [4], capillary waves [5], water waves [6], photorefractive ferroelectrics [7], plasmas [8], ferromagnetic materials [9], and so on [10, 11].

Peregrine soliton, firstly suggested by D. H. Peregrine in the early 1980s for nonlinear dynamics of deep waters [12], is commonly taken as a prototype of rogue waves [13, 14]. Such soliton, i.e., localized rational solution of nonlinear Schrödinger equation, can be taken as a limiting case of the one-parameter family of Kuznetsov–Ma breathers [15] or Akhmediev breathers [16]. There have been considerable interests on Peregrine solitons occurring in a variety of physical systems [17–39]. Many efforts have also been devoted to the new understanding of Peregrine solitons through the analysis of other types of nonlinear partial differential equations [40–51].

Among various rogue waves, optical rogue waves have received much attention due to their interesting properties and promising applications [10, 11, 25–39, 52–54]. However, the creation of the optical rogue waves is not an easy task in conventional optical media (such as optical fibers and waveguides). The reason is that the nonlinear optical effect in such media is very weak, and hence a large input optical power is needed to obtain a significant optical nonlinearity required for the formation of rogue waves. Although some resonance mechanisms may be exploited to enhance nonlinear effects, near resonances significant optical absorptions occur, which result in serious attenuation and distortion of optical pulses during propagation.
In recent years, many efforts have been focused on the investigation of electromagnetically induced transparency (EIT), a typical quantum interference effect occurring in three-level atomic systems, by which the light absorption due to resonance may be largely suppressed and giant Kerr nonlinearity may be obtained simultaneously [55]. By means of EIT, it has been shown that weak-light solitons and their storage and retrieval can be realized [56–58]. Recent works [59, 60] have demonstrated that it is possible to generate optical Peregrine solitons with low generation power in EIT-based atomic systems.

In this work, we suggest a scheme to realize the memory of optical Peregrine solitons in a Λ-shaped three-level atomic gas via EIT. We show that such solitons may have very small propagation loss, ultrasonic motional velocity, and extremely low generation power; they can be stored, retrieved, split, and routed with high efficiency and fidelity through the manipulation of control laser fields. The results reported here are helpful for the active control of optical Peregrine solitons and promising for practical applications in optical information processing and transmission.

The article is arranged as follows. In Section 2, the physical model and ultrasonic weak-light Peregrine solitons and their propagation are described. In Section 3, the storage, retrieval, splitting, and routing of such solitons are presented. Finally, Section 4 gives a summary of the main results obtained in this work.

2 MODEL AND ULTRASOUND WEAK-LIGHT PEREGINE SOLITONS

2.1 Model

We start to consider a cold three-state atomic gas with Λ-shaped level configuration, interacting with a weak, pulsed probe laser field (center wavenumber \(k_p\) and center angular frequency \(\omega_p\)) and a strong, continuous-wave (CW) control laser field (wavenumber \(k_c\) and angular frequency \(\omega_c\)). The probe (control) field drives the transition \(|1\rangle \leftrightarrow |3\rangle \) (|2\rangle \leftrightarrow |3\rangle); see Figure 1A.

The total electric field in the system reads \( \mathbf{E} = \mathbf{E}_p + \mathbf{E}_c = \sum_{\lambda=p,c} e\mathbf{e}\exp\{i(k_\lambda z - \omega_\lambda t)\} + c.c., \) where \( e\) (\( \mathbf{E} \)) is the unit polarization vector (envelope) of the electric field \( \mathbf{E}_c \). To suppress Doppler effect, both the probe and control fields are assumed to propagate along \( z \) direction.

The Hamiltonian of the system in the interaction picture reads

\[
\hat{H}_{\text{int}} = -\hbar \left( \sum_{\lambda=p,c} \Delta_\lambda \right) \hat{\sigma} \hat{j} + \Omega_\lambda |3\rangle\langle 1| + \Omega_\lambda |3\rangle\langle 2| + \text{H.c.},
\]

where \( \Delta_\lambda = \omega_p - (E_3 - E_1)/\hbar \) (\( \Delta_\lambda = \omega_c - \omega_p - (E_2 - E_1)/\hbar \)) is one-(two-) photon detuning; \( \Omega_\lambda = (e_p \cdot \mathbf{p}_{13})/\hbar \) (\( \Omega_\lambda = (e_c \cdot \mathbf{p}_{23})/\hbar \)) is the half Rabi frequency of the probe (control) field; \( \mathbf{p}_\lambda \) is the electric-dipole matrix element associated with levels \( |\lambda\rangle \) and \( \hat{\sigma} \). The atomic dynamics is described by a \( 3 \times 3 \) density matrix \( \sigma \), obeying the optical Bloch equation

\[
\frac{\partial \sigma}{\partial t} = -i\hbar \left[ \hat{H}_{\text{int}}, \sigma \right] - \Gamma [\sigma],
\]

where \( \Gamma \) is a relaxation matrix characterizing the spontaneous emission and dephasing [61]. The explicit form of Eq. 1 is presented in Section 1 of the Supplementary Material.

The evolution of the probe field \( \mathbf{E}_p \) is governed by the Maxwell equation \( \nabla^2 \mathbf{E}_p - (1/c^2)\partial^2 \mathbf{E}_p/\partial t^2 = (1/\varepsilon_0 c^2)\partial^2 \mathbf{P}_p/\partial t^2 \), where \( \mathbf{P}_p = N_a |\mathbf{p}_{13}\rangle \sigma_{31} \exp\{i(k_p z - \omega_p t)\} + c.c. \) is the electric polarization intensity, with \( N_a \) the atomic density. Under slowly varying envelope and paraxial approximations, the Maxwell equation is reduced into the form

\[
\left( \frac{\partial}{\partial z} + \frac{1}{c} \frac{\partial}{\partial t} \right) \Omega_p + \kappa_{13} \sigma_{31} = 0,
\]

with \( \kappa_{13} = N_a \omega_p |\mathbf{p}_{13}|^2 / (2e_0 \varepsilon_0). \) Note that we have assumed that the probe field has a large transverse size so that its diffraction effect is negligible. The model described here may be realized, e.g., by a cold \(^{87}\)Rb atomic gas [62], with the levels selected by \(|1\rangle = |5^2S_{1/2}, F = 1, m_F = 0\rangle\), \(|2\rangle = |5^2S_{1/2}, F = 2, m_F = 0\rangle\), and \(|3\rangle = |5^2P_{3/2}, F = 1, m_F = 0\rangle\). Thus we have \( \omega_p = 2.37 \times 10^{15} \text{ Hz}, \) \( |\mathbf{p}_{13}| = 2.54 \times 10^{-27} \text{ C m}. \) If the atomic density \( N_a = 8.8 \times 10^{11} \text{ cm}^{-3} \), \( \kappa_{13} \) takes the value of \( 2.4 \times 10^{10} \text{ cm}^{-1} \text{s}^{-1}. \) This set of parameters will be used in the following analysis and calculation.

2.2 ULTRASOUND WEAK-LIGHT PEREGINE SOLITONS AND THEIR PROPAGATION

We first investigate the linear propagation of the probe field. When a very weak probe pulse is applied, the system undergoes a linear evolution. In this case, the Maxwell–Bloch (MB) (Eqs. 1 and 2) admit the solution \( \Omega_p = F \exp\{i(Kz - \omega_2 t)\}, \) where \( F \) is a constant,

\[
K(\omega) = \frac{\omega - \kappa_{13}}{\omega + d_{13}} \left( \omega + d_{13} \right) - |\Omega_p|^2 \quad (3)
\]

is linear dispersion relation, and \( d_{13} = \Delta_3 - \Delta_2 + i\gamma_{13}^\text{R} \) (\( \gamma_{13}^\text{R} \equiv (\Gamma_\alpha + \Gamma_\beta)/2 + \gamma_{13}^\text{L}, \) \( \gamma_{13}^\text{L} \equiv \sum_\omega \gamma_{13}^\text{L}, \) and \( \gamma_{13}^\text{L} \) is the dephasing rate associated with the states \(|\alpha\rangle \) and \(|\beta\rangle\). Shown in Figure 1B is the imaginary part \( \Im(K) \) and the real part \( \Re(K) \) as functions of \( \omega. \) Due to the quantum interference effect induced by the control field, an EIT transparency window is opened in \( \Im(K) \) (dashed line), which implies that the probe field can propagate in this resonant atomic gas with a very small absorption. Parameters used for plotting the figure are \( \Delta_2 = -2\pi \times 0.64 \text{ MHz}, \) \( \Delta_3 = -2\pi \times 9.6 \text{ MHz}, \) \( \gamma_{21} = 2\pi \times 1.09 \text{ kHz}, \) \( \gamma_{31} = 2\pi \times 2.5 \text{ MHz}, \) and \( \Omega_\alpha = 2\pi \times 31.8 \text{ MHz}. \)

From the MB Eqs. 1 and 2 and using the method of multiple-scales [63], we can derive the controlling equation governing the nonlinear evolution of the probe-field envelope \( F \) (see Section 2 of the Supplementary Material), which reads

\[
i \frac{\partial F}{\partial z} - \frac{1}{2} K_2 \frac{\partial^2 F}{\partial t^2} + W|F|^2 F = 0,
\]

where \( \tau = t - z/V_g \) \( \left[ V_g \equiv (\partial K/\partial \omega)^{-1} \right] \) is the group velocity of the envelope; here and in the following, the quantity with a tilde represents the corresponding real part; \( K_2 = \partial K/\partial \omega^2 \) is the
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where \( \Omega_c \) and \( \Omega^c \) are respectively the characteristic half Rabi frequency and time duration of the probe field, and \( L_{\text{Non}} \equiv 1/(U_0^2 W) \) is the characteristic nonlinearity length (which has been assumed to equal the dispersion length defined by \( L_{\text{Dis}} \equiv \tau_0^2/|K(c)| \) for simplicity). One sees that the Peregrine soliton consists of a CW background and a bump in its envelope that first grows and then decay rapidly on the background. The physical reason for the formation of such optical Peregrine soliton can be understood as follows. When a plane-wave probe field with a finite amplitude is applied to and propagates in the atomic gas, the Kerr nonlinearity brings a modulational instability and a phase modulation to the probe field; due to the role played by the group-velocity dispersion, the phase modulation is converted into amplitude modulation and peak amplification. Because of the joint phase and amplitude modulations, the probe field reorganizes its spatial distribution and hence the Peregrine soliton is generated in the system.

As an example, we take \( \tau_0 = 2.36 \times 10^{-7} \) s, \( U_0 = 2\pi \times 8.0 \) MHz, and other system parameters which are the same as those used in Figure 1B. Then we obtain \( K_0 = -1.70 + i 0.02 \) cm\(^{-1} \), \( K_1 = \partial K/\partial \omega \approx (4.5 - i 0.05) \times 10^{-7} \) cm\(^{-1} \) s, \( K_2 \approx (-1.5 - i 0.1) \times 10^{-14} \) cm\(^{-1} \) s\(^2 \), and \( W \approx (1.05 - 0.00) \times 10^{-16} \) cm\(^{-1} \) s\(^2 \) (estimated at \( \omega = 0 \)). We see that the imaginary parts of \( K_j \) \((j = 0, 1, 2)\) and \( W \) are much smaller than their corresponding real parts, which is due to the EIT effect that results in the suppression of the optical absorption in the system. Based on these results, we obtain \( L_{\text{Non}} = L_{\text{Dis}} = 3.8 \) cm and

\[
V_\ell = 7.34 \times 10^{-5} c. \tag{6}
\]

Thus, the propagation velocity of the optical Peregrine soliton is much slower than the light speed \( c \) in vacuum. If the transverse cross-section area of the probe pulse takes the value \( S = 8.0 \times 10^{-3} \) cm\(^2 \), the generation power of the soliton (which can be estimated by using the Poynting vector \([56]\)) reads

\[
P_{\text{max}} \approx 1.8 \mu W, \tag{7}
\]

i.e., very small power needed for creating such soliton. Consequently, the Peregrine solitons given here are different from those obtained in conventional optical systems \([25, 27, 28, 31]\).

We now investigate the propagation of the ultrasonic Peregrine soliton by exploiting Runge–Kutta method based on solving the MB Eqs. 1 and 2 numerically. Since solution (5) has an infinite
energy due to the existence of the CW background, it cannot be generated in a real experiment. To avoid this, we assume the probe field at $z = 0$ has the form

$$\Omega_p(0, t) = \Omega_p(0) \left[ \frac{1}{2} \tanh \left( \frac{t - T_{\text{on}}}{T_p} \right) - \frac{1}{2} \tanh \left( \frac{t - T_{\text{off}}}{T_p} \right) \right].$$

(8)

Here $\Omega_p(0) = 6.67 [1 - 3.2/(1 + 4t^2/\tau_0^2)]$ is chosen to match the analytical solution (5); the hyperbolic tangent function is used to impose temporal boundaries on both sides of CW background (far from the pump part), which can make the soliton have finite energy and also have a clear illustration on its waveshape (similar to the case for generating dark solitons [64, 65]); $T_p^\text{on} = 3.0 \tau_0$ is the switching time when turning on and off the probe field; $T_p^\text{on} = -80 \tau_0$ and $T_p^\text{off} = 4 \tau_0$ are parameters characterizing the two temporal boundaries, respectively. The waveshape of the input probe field at $z = 0$ is shown by a solid blue line in the upper part of Figure 2A, where the dashed vertical lines represent temporal boundaries.

The lower part of Figure 2A illustrates the result of a numerical simulation on the propagation of the Peregrine soliton (with $\Delta_3 = -2\pi \times 95.5$ MHz, $\tau_0 = 1.5 \times 10^{-7}$ s, and other parameters the same as those used in Figure 1B), by taking $[\Omega_p(t_0)]$ as a function of $t/\tau_0$ and $z$. The orange line is the input Peregrine soliton at $z = 0$; the red line denotes the Peregrine soliton propagating to $z = 4.3$ cm; the maximum value $[I(\Omega_p(t_0))]$ of the soliton along the trajectory appears sharply around $z = 4.3$ cm at $t = 16.9 \tau_0$. Figure 2B shows the contour map for the propagation of the Peregrine soliton, which can be taken as a projection of Figure 2A onto the $t$-$z$ plane. One sees that the Peregrine soliton (indicated by the red dashed circle in Figure 2B) appears sharply and disappears suddenly; a secondary peak (soliton) emerges at longer distance, as a result of phase modulation when the first soliton is excited.

3 STORAGE, RETRIEVAL, SPLITTING, AND ROUTING OF THE OPTICAL PEREGRINE SOLITONS

We now turn to consider the memory of the optical Peregrine solitons and related applications in optical splitting and routing through the manipulation of the control fields.

3.1 Storage and Retrieval of the Optical Peregrine Solitons

We first consider the storage and retrieval of optical Peregrine solitons obtained above, which can be implemented by switching off and on the control field described by the following switching function:

$$\Omega_s = \Omega_0 \left[ 1 - \frac{1}{2} \tanh \left( \frac{t - T_{\text{off}}}{T_p^s} \right) + \frac{1}{2} \tanh \left( \frac{t - T_{\text{on}}}{T_p^s} \right) \right],$$

(9)

where $\Omega_0$, is a constant, $T_p^s$ is the time interval for switching off and on the control field (switching time), and $T_p^\text{on} (T_p^\text{off})$ is the time when the control field is switched on (off).

As an example, we take $\Omega_0 = 2\pi \times 31.8$ MHz, $T_p^\text{off} = 10.0 \tau_0$, $T_p^\text{on} = 20.0 \tau_0$, $T_p^s = 3.0 \tau_0$ ($\tau_0 = 1.5 \times 10^{-7}$ s), and other system parameters are the same as those used in Figure 2. The upper part of Figure 3A shows the time sequences of the control field (black line) and the probe field (blue line); the red dashed vertical line (black dashed vertical line) represents the time $T_p^\text{on} (T_p^\text{off})$.

Symbols I, II, and III denote the CW background, the Peregrine soliton, and the low-intensity component of the probe field, respectively. The lower part of the figure shows the result of a numerical simulation on the storage and retrieval of the Peregrine soliton by taking $[\Omega_p(t_0)]$ as a function of $t/\tau_0$ and $z$. Here the orange line is the input Peregrine soliton at $z = 0$; the purple line represents the Peregrine soliton at the storage period; the red line denotes the retrieved Peregrine soliton propagating to $z = 4.3$ cm; the maximum value $[I(\Omega_p(t_0))]$ of the soliton along the trajectory appears sharply around $z = 4.3$ cm at $t = 16.9 \tau_0$. Figure 2B shows the contour map for the propagation of the Peregrine soliton, which can be taken as a projection of Figure 2A onto the $t$-$z$ plane. One sees that the Peregrine soliton (indicated by the red dashed circle in Figure 2B) appears sharply and disappears suddenly; a secondary peak (soliton) emerges at longer distance, as a result of phase modulation when the first soliton is excited.
The efficiency of the Peregrine soliton memory can be characterized by the parameter $\eta = \frac{\int_{t_1}^{t_2} F_{\text{Pere}}^p (t) \text{d}t}{\int_{t_1}^{t_2} F_{\text{Pere}}^\text{in} (t) \text{d}t} \times 100\%$ at $t_1 = T_{\text{off}}^\text{ps}$ and the peak of the retrieved soliton pulse $F_{\text{Pere}}^\text{Pere}$. Here we take $\Delta T = 30.5 \tau_0$ and $T_1 = 10 \tau_0$. We obtain $\eta F^2 = 84.3\%$. We see that the efficiency and fidelity of the storage and retrieval of the Peregrine soliton are quite high.

The numerical result shown in Figure 3C is similar to that of Figure 3A but for $T_{\text{off}}^\text{ps} > T_{\text{off}}^\text{ps}$. In this case, the storage and retrieval of the Peregrine soliton can also be implemented; however, compared with Figure 3A (which is for $T_{\text{off}}^\text{ps} < T_{\text{off}}^\text{ps}$), the retrieved waveshape is much more distorted. Figure 3D illustrates the contour map of the Peregrine soliton in the $t-z$ plane with $T_{\text{off}}^\text{ps}$ for $T_{\text{off}}^\text{ps}$. One sees that the probe field has a nonzero value in the region indicated by the dashed white circle, which means that some parts of the probe field are not stored when the control field is switched off. We obtain the efficiency and fidelity of the Peregrine soliton memory for $T_{\text{off}}^\text{ps} > T_{\text{off}}^\text{ps}$ are $\eta = 77\%$ and $\eta F^2 = 65\%$, respectively. Based on these results, we conclude that...
in order to get a high memory quality, the choice of $T_{\text{off}}^2 < T_{\text{off}}^1$ is better than that of $T_{\text{off}}^2 > T_{\text{off}}^1$.

### 3.2 Splitting of the Optical Peregrine Solitons

To realize an optical splitting [67] of the Peregrine soliton, we generalize the system into a four-level one with a tripod-type level configuration. Here a probe field $\Omega_p$ drives the transition $|1\rangle \leftrightarrow |3\rangle$; two CW control fields $\Omega_{c1}$ and $\Omega_{c2}$ drive respectively the transitions $|2\rangle \leftrightarrow |3\rangle$ and $|4\rangle \leftrightarrow |3\rangle$; $\Gamma_3$ is the decay rate from $|3\rangle$ to $|j\rangle$ ($j = 1,2,4$), $\Delta_3$ and $\Delta_4$ ($l = 2,4$) are respectively one-photon and two-photon detunings (see Figure 4A). The Hamiltonian of the system and the optical Bloch equations controlling the dynamics of the atoms have been presented in Section 5 of the Supplementary Material.

The timing sequences of the switching-off and -on of $\Omega_3(t)$ for obtaining a Peregrine soliton splitter are shown in Figure 4B, with $T_{\text{off}}^1 < T_{\text{off}}^2 < T_{\text{off}}^3 < T_{\text{off}}^4$. For $j$th control field $\Omega_{cj}$ ($j = 1,2$), $T_{\text{off}}^j (T_{\text{on}}^j)$ is its switching-off (switching-on) time. The corresponding switching functions have been given in Section 5 of the Supplementary Material. When plotting the figure, we have set $\Omega_{c1}(t) = \Omega_{c2}(t) = 2\pi \times 31.8$ MHz, $T_{\text{off}}^3 = T_{\text{off}}^4 = 6.0 \tau_0$, $T_{\text{on}}^3 = 15.0 \tau_0$, $T_{\text{off}}^1 = 35.0 \tau_0$, $T_{\text{on}}^1 = 45.0 \tau_0$, and $T_{\text{off}}^2 = T_{\text{on}}^2 = 3.0 \tau_0$.

Shown in Figure 4C is the numerical result for the simulation of obtaining the Peregrine soliton splitter by taking $\Omega_p \tau_0$ as a function of $t/\tau_0$ and $z$ (with $\tau_0 = 1.5 \times 10^{-7}$ s). The operation steps can be described as follows: 1) Firstly, the two control fields $\Omega_{c1}$ and $\Omega_{c2}$ are applied and a probe field with the waveform $\Omega_0(t) = 6.67 \{1 - 3.2/(1 + 4(t/\tau_0)^2)\} [0.5\tan (t/\tau_0 + 3.0/3.0) - 0.5\tan (t/\tau_0 - 3.0/3.0)]$ is incident to the system. 2) Then, both control fields are simultaneously switched off at time $t = T_{\text{off}}^1 = T_{\text{off}}^2 = 6.0 \tau_0$. Thus the probe field is stored in the two atomic coherences $\sigma_{21}$ and $\sigma_{41}$ simultaneously. 3) Later on, switching off the control $\Omega_{c1}$ at $t = T_{\text{on}}^1 = 15.0 \tau_0$ (but $\Omega_{c2}$ is remained to be switched off), the atomic coherence $\sigma_{21}$ is converted back into the probe field, and hence a new probe pulse is retrieved. At time $t = 23.0 \tau_0$, this retrieved probe pulse turns into a Peregrine soliton (i.e., “1st retrieved PS”, indicated by a red circle in Figure 4B) with the maximum intensity $|\Omega_p \tau_0|_{\text{max}} = 15.4$ at the position $z = 3.7$ cm. 4) By switching off $\Omega_{c2}$ at $t = T_{\text{off}}^3 = 35.0 \tau_0$ and switching on $\Omega_{c2}$ at $t = T_{\text{on}}^2 = 45.0 \tau_0$, the atomic coherence $\sigma_{41}$ converts back into the probe field; this retrieved probe field turns into another Peregrine soliton (i.e., “2nd retrieved PS”, indicated by another red circle in Figure 4B) with the maximum intensity $|\Omega_p \tau_0|_{\text{max}} = 14.0$ at the position $z = 5.5$ cm at $t = 61.5 \tau_0$.

In the simulation, we have taken $\Delta_2 = \Delta_4 = -2\pi \times 0.64$ MHz, $\gamma_{21} = \gamma_{41} = 2\pi \times 1.09$ kHz, with the other parameters the same as those used in Figure 3A. The reason for taking $\Delta_2 = \Delta_4$ and $\gamma_{21} = \gamma_{41}$ is to keep the symmetry of the tripod level configuration, which gives two nearly degenerated EITs in the system; for details, see [67]. The splitting efficiency and fidelity of the first (second) Peregrine soliton are $\eta_1 = 89.8\%$ and $\eta_2/\sqrt{2} = 85.4\%$ ($\eta_2 = 89.3\%$, $\eta_2/\sqrt{2} = 84.9\%$, respectively).

### 3.3 Routing of the Optical Peregrine Solitons

To realize all-optical routing [67, 68] of optical Peregrine solitons, we consider a four-level atomic system with a double-$\Lambda$-type level configuration. Here, two probe laser fields $\Omega_{p1}$ and $\Omega_{p2}$ drive the transitions $|1\rangle \leftrightarrow |3\rangle$ and $|1\rangle \leftrightarrow |4\rangle$, respectively; two CW control laser fields $\Omega_{c1}$ and $\Omega_{c2}$ drive the transitions $|2\rangle \leftrightarrow |3\rangle$ and $|2\rangle \leftrightarrow |4\rangle$, respectively; $\Delta_1$ and $\Delta_4$ are one-photon detunings, and $\Delta_2$ is two-photon detuning (see Figure 5A).

The Hamiltonian of the system and the MB equations governing the dynamics of the atoms and light fields have been given in Section 6 of the Supplementary Material. For simplicity, here we consider a frequency routing process, i.e., the probe field $\Omega_{p1}$ is converted into the $\Omega_{p2}$ (which has different frequency from $\Omega_{p1}$). The time sequence of the switching off and on of $\Omega_p$ for obtaining route of Peregrine soliton is shown in Figure 5B, with $T_{\text{off}} < T_{\text{off}} < T_{\text{on}}$, where $T_{\text{off}}$ is the switching-off time of $\Omega_{c1}$ and $T_{\text{on}}$ is the switching-on time of $\Omega_{c2}$. The corresponding switching functions have been given in Section 5 of the Supplementary Material. Without loss of generality, the system parameters are set to be $\Omega_{c1} = \Omega_{c2} = 2\pi \times 31.8$ MHz, $T_{\text{off}} = 10.0 \tau_0$, $T_{\text{on}} = 25.0 \tau_0$, and $T_{\text{on}} = T_{\text{off}} = 3.0 \tau_0$ (switching time).

The implementing procedure of the Peregrine soliton routing is as follows. First, by switching on the control field $\Omega_{c1}$ with the initial condition $\Omega_{c0}(t/\tau_0) = 6.67 \{1 - 3.2/(1 + 4(t/\tau_0)^2)\} [0.5\tan (t/\tau_0 + 3.0)/3.0) - 0.5\tan (t/\tau_0 - 3.0/3.0)]$ propagates in the system, as shown in the upper panel of Figure 5C as a function of propagation distance $t/\tau_0$ and $z$. One sees that a trajectory of the soliton shows up before its storage. Second, by switching off $\Omega_{c1}$ at time $t = 10\tau_0$, the probe field $\Omega_{p1}$ is stored in the atomic coherence $\sigma_{21}$. Third, by switching on the control field $\Omega_{c2}$ at $t = 25\tau_0$, another probe pulse $\Omega_{p2}$ appears from the atomic coherence $\sigma_{21}$, i.e., “retrieved PS” in the lower panel of Figure 5C. We stress that during this routing process, the Peregrine soliton in the probe field $\Omega_{p1}$ is annihilated and a new Peregrine soliton in the probe field $\Omega_{p2}$ (which has no input) is created. Since the frequency of $\Omega_{c2}$ is different from that of $\Omega_{c1}$, the system performs as a frequency router of the Peregrine soliton.

## 4 CONCLUSION

We have proposed a scheme for realizing the storage and retrieval of optical Peregrine solitons in a coherent atomic gas via EIT. We have shown that the optical Peregrine solitons with very small propagation loss, ultraslow motional velocity, and extremely low generation power can be generated in the system via EIT. We have demonstrated that such solitons can be stored, retrieved, split, and routed with high efficiency and fidelity through the manipulation of control laser fields.

The scheme can also be generalized to cases with more optical output channels through the use of more control fields, and hence the two-channel splitting and routing processes can be generalized to multiple channel ones. Furthermore, the storage and retrieval of the optical Peregrine solitons can be extended to solid materials, like on-chip optical resonator systems [69]. The research results reported here may be useful for the active control of optical Peregrine solitons and promising for potential applications in optical information processing and transmission.
DATA AVAILABILITY STATEMENT

The raw data supporting the conclusion of this article will be made available by the authors, without undue reservation.

AUTHOR CONTRIBUTIONS

GH proposed the idea and supervised the whole work. CS carried out the analytical and numerical calculation. Both authors contributed to the writing of the manuscript.

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SUPPLEMENTARY MATERIAL

The Supplementary Material for this article can be found online at: https://www.frontiersin.org/articles/10.3389/fphy.2021.594680/full#supplementary-material.
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Conflict of Interest: The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

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