ABSTRACT

The effect of electric field on the electron resonant tunnelling into a double barrier structure is studied. We show for particular field strengths an increase of the tunnelling time which leads us to explain the Stark-ladder localization and to discuss Bloch oscillations and the quenching of luminescence in multiple quantum wells.

Keywords: Resonant Tunnelling, Double Barrier, Stark-Ladder Localization, Bloch Oscillation, Quantum Wells
1 INTRODUCTION

Since its prediction \[1, 2\] the Wannier-Stark ladder localization has been difficult to observe experimentally \[3\]. This effect takes place when the electron wave-vector becomes of the order of the reciprocal lattice parameter leading to Bragg reflections. During the last decade several papers examined this effect theoretically \[4\] and experimental evidence has been provided by photoluminescence measurements \[5\]. The effect has now a wide application to the optical properties of quantum wells \[6\] and photodetectors based on the effect have been developed recently \[7\]. Moreover, useful resonant tunnelling current-voltage peaks can be made by engineering the well width and the barrier height \[8\]. Resonant tunnelling is a quantum phenomenon in which the electron wave-function interference becomes constructive. In the case of a single quantum well it corresponds to a quasi-bound state while in a symmetric double barrier structure it shows a transmission coefficient peak characterized by the energy position \(E\) and the width \(\Delta E\). The analytical resonant tunnelling condition determines these parameters for a given double barrier structure \[9\] and shows a strong dependence on the width of the well. The applied electric field provides a way to increase the Fermi energy of the conduction band electrons which shifts the resonant peaks and minibands. The resonant tunnelling width can be related to the corresponding lifetime by \[10\]:

\[
\tau = \hbar/\Delta E
\]  

We can interpret the motion of the carriers semi-classically by using the group velocity of an electron wave-packet. We also expect that, in an electric field, the electron wave-packet will be accelerated leading to a decrease of its lifetime. The resonant tunnelling width will therefore broaden in an electric field. This has been shown by the recent work of Gonzalez et al. \[11\]. However their calculations were made for high fields (from 200 to 400 kV/cm) for which the electric field can lead to the localization of the electron states which confines them in a small region of the sample. In particular, it has been shown recently for one-dimensional \(\delta\)-peak potentials, that localization appears at lower fields and should disappear when the field strength is increased \[12\]. Therefore, we expect different behaviours of the tunnelling time in such a case in the quantum well and double barrier systems. In this paper we discuss the behaviour of the electron tunnelling time by calculating the transmission coefficient in the case of the Stark-ladder localization for a double barrier system. An extensive study of electron transmission in an electric field leads us to give a new interpretation of the origin of the field induced localization in periodic systems and the transmission oscillations observed recently \[12\]. We expect a decrease of the tunnelling time to yield a deceleration of the electron wave-packet. Similar considerations are involved in the quenching of the luminescence in the \(GaAs - Ga_{1-x}Al_xAs\) multiple quantum wells \[13\].
2 THE MODEL

We consider a single electron in a rectangular double barrier potential in a constant electric field. The corresponding Schrödinger equation can be read:

\[
\frac{\hbar^2}{2m^*} \frac{\partial^2}{\partial x^2} \Psi + V(x)\Psi - |e| F x = E \Psi
\]

where the potential \( V(x) \) is defined for a well and a barrier of width \( L_w \) and \( L_b \) respectively by:

\[
V(x) = \begin{cases} 
V_0 & : 0 < x < L_b \quad \text{or} \quad L_b + L_w < x < L_w + 2L_b \\
0 & : \text{otherwise}
\end{cases}
\]

The solutions of Eq. (2) are a combination of independent Airy functions. We use the transfer matrix method to determine the transmission coefficient through the structure (the detailed method is given by Cota et al. [4]). The resonant energy width is identified with the half maximum width by assuming that the transmission spectrum is Lorentzian near resonance.

3 DISCUSSION OF THE RESULTS

We are interested in the electron confinement in the growth direction for a double barrier structure in the presence of an electric field. We therefore calculate the transmission spectrum for field strengths between 0 and 200 kV/cm since above 200 kV/cm the tunnelling time decreases [11] leading to the acceleration of the electron wave-packet. Furthermore, Zekri et al. [12] show that Stark localization can be observed clearly for weak fields and tends to disappear for higher ones. These authors also show (using a chain of \( \delta \)-peak potentials) that a breakdown of the minibands takes place and a set of localized states appears. This localization does not depend on the potential model and can also be observed in a chain of rectangular potentials as shown in Fig.1 where the transmission corresponding to only a part of the minibands disappears because of the electric field.

In the following calculations the barrier and the well effective mass are taken to be 0.067 and 0.108 respectively with widths 20 and 50 Å respectively while the potential height of both barriers is 500 meV. With the chosen parameters we have two resonant transmission peaks and can compare the effect of the field on each peak.

In Fig.2a the transmission coefficient for a double barrier structure is shown for different values of the field \( F=0, 50 \) and 100 kV/cm. The main effect of the field is to shift the resonant peaks which can be understood in terms of the increase of the kinetic energy of the conduction electrons since, at the resonant energy, the electrons move freely through the barriers. Indeed this is clearly shown in Fig.2b where the energy shift increases linearly with the field for both peaks and its slope has the magnitude of \(-eL\) (\( L \) being
the width the well). Therefore the new shifted resonant energy for this system is:

\[ E_f = E_0 + |e| FL \]  

(4)

where \( E \) is the zero field resonant energy. However, this behaviour is not in agreement with the results of Gonzalez et al. [11] and Bastard et al. [14] where the shift behaves quadratically with the field. In fact they considered a single quantum well while the structure studied here is a double barrier. This difference between the two structures is due to a change of the multiple reflection effect which is strong in the single quantum well case at the bound state because the electron wave function becomes evanescent in the infinite width of the potential barrier while for our structure this reflection disappears at the resonant tunnelling energy and the electron does not "see" the finite width of the potential barrier. However, an infinite barrier width is unrealisable and in the practical realization of the single quantum well, the potential barrier width is large enough to ensure that the outgoing electron wavefunction becomes very small.

The other main effect shown in Fig. 2a is that the first resonant peak becomes narrower with increasing field while the second one becomes broader. This is clearly shown in Fig. 3 where the full width at half maximum is plotted as a function of the field strength for both resonant peaks and a fast drop is observed for the first peak while the increase for the second one is to be linear. From Eq. (1) the decrease of the width of the first peak means an increase of the tunnelling time and leads to the confinement of the electron wave-packet in the potential well. Also shown in Fig. 2a are non-unity resonant peaks when the field is applied. This decrease of the resonant transmission peaks may be responsible for the breakdown [12] of the minibands in a superlattice built from a double barrier since near this resonance the reflection vanishes and the transmission coefficient for the superlattice becomes a high power of the corresponding double barrier value.

The decrease of the width of the first resonance leads to Bloch oscillations in the energy spectrum. We recall that the minibands of a superlattice are produced by the coupled resonant peaks of the multiple potential wells [15]. If the number of potential wells increases, these resonant energies become closer and the corresponding transmission spectrum overlap to produce a continuum. When the electric field is applied it decreases the overlap which leads to an oscillation of the transmission coefficient as is clearly shown in the energy spectrum given in [12].

The field-induced localization disappears for higher fields [12]. Such a delocalization arises from the existence of a critical field at which the width of the first peak reaches a minimum and subsequently increases. This minimum is not observed in Fig. 3 because the first peak is also shifted by the field and disappears before reaching the critical field. By using a different structure, Agullo-Rueda et al. [16] have interpreted this Stark-localization effect in terms of the interaction of the localized and the extended states.
However, this is not the case in our superlattice structure which has no localized states in the absence of the field. The Stark localization observed at the first resonance is not seen at the second one because the electron wavelength at this energy becomes different from the distance between the two barriers and then there is no Bragg reflection.

The increase of the tunnelling time observed in Fig.3 may also be responsible for the quenching of the luminescence observed by Mendez et al. [13]. This phenomena has been interpreted by Bastard et al. [14] as the effect of the increase of the distance between the conduction electron and the hole which leads to a decrease of the recombination between them but from their results this is not sufficient to explain this experimental observation. However, Buttiker and Landauer [17] argued that the electron interacts with a radiation of frequency $\omega$ if its tunnelling time $\tau$ satisfies $\omega \tau \sim 1$. Therefore the field-induced high tunnelling time observed at the first energy resonance can lead to the emission of a low frequency radiation which may not be observed in the luminescence experiments.

4 CONCLUSIONS

In this paper we have studied numerically the effect of the electric field on a double barrier structure on the transmission coefficient. It is found that, for particular field strengths, confinement appears due to the decrease of the resonant tunnelling width and the transmission peak value. These effects allow us to explain the Stark ladder localization and Bloch oscillations recently calculated for a one-dimensional chain in an electric field [12]. The localization can also explain (via the tunnelling time) the quenching of the luminescence observed by Mendez et al. [13]. However, a quantitative study of this effect and an analytical determination of the field values localizing the system are needed to fully interpret the Stark ladder effects. Such a study will allow us to choose the parameters of the structure to adjust the critical field at which the resonant energy width is a minimum. The search for this minimum width is important since, if it is sufficiently small, this can lead to an apparent violation of the Heisenberg uncertainty principle. Moreover, the study of this effect in more realistic structures, e.g. non-abrupt heterojunctions, is required to allow us to make quantitative comparisons with experiments.

5 Acknowledgement

One of the authors (N.Z.) would like to thank the International Center for Theoretical Physics for hospitality during his visit to the centre where part of this work has been done to Professor P.N. Butcher for reading the manuscript. He is very grateful to the Arab Fund for financial support during his visit to the centre.
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Figure Captions

Figure 1 The transmission spectrum for a superlattice of 50 barriers in the absence of electric field (solid curve) and for a field strength of 10 kV/cm (dotted curve).

Figure 2.a The transmission spectrum for a double barrier for field strengths of 0 (solid curve), 10 kV/cm (dotted curve) and 50 kV/cm (long dashed curve).

Figure 2.b The energy shift as a function of the field for both the first and the second peak. The dotted line is used as a guide for the eyes.

Figure 3 The full width at half maximum as a function of the field for the first peak (open losenges) and the second peak (+). The scale of the second resonance widths should be multiplied by 10.
