Sum Rule for Heavy Meson Decay Widths

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Abstract

A sum rule relating the widths of the decays of mesons belonging to heavy quark multiplets, having the same parity and light quark spin $j$, into the low lying $0^-$ and $1^-$ multiplet is obtained. As this sum rule follows from properties of the axial charges, it is protected from heavy quark spin dependent $1/m_Q$ corrections. The sum rule works well for mesons containing a heavy charmed quark and, surprisingly, for resonances containing a strange quark.

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The heavy quark effective theory, recently reviewed in [1–3], has been applied, with varied success, to the strong decays of meson resonances containing one such quark. For example, the theoretical value for the ratio

$$\frac{\Gamma (D^*_2(2460) \to D + \pi)}{\Gamma (D^*_2(2460) \to D^* + \pi)} = 2.3$$

is in good agreement with the experimental result $2.3 \pm 0.6$ or $1.9 \pm 1.1$ [4] (depending on the charge of the resonance). However, the theoretically obtained ratio

$$\frac{\Gamma (D^*_1(2420) \to D^* + \pi)}{\Gamma (D^*_2(2460) \to D + \pi) + \Gamma (D^*_2(2460) \to D^* + \pi)} = 0.3$$

is in disagreement with the experimentally determined one [5] of $0.82 \pm 0.23$. Eq. (1) gets no correction to order $1/m_Q$, ($m_Q$ is the heavy quark mass) whereas the one of eq. (2) does [3].

In this work we will present a sum rule relating widths of resonances decaying to $D$ and to $D^*$ (and their analogs for other heavy quark mesons); there results are robust to $1/m_Q$ corrections. As we shall see, not only are these sum rules satisfied for resonances containing a charmed quark, but, surprisingly, work well for $K$ resonances.

The results we shall find are valid in the soft pion limit and, in part, are derived in a manner similar to the one used to obtain the Adler-Weisberger relation [6]. With $Q^\pm_5$, the isospin plus/minus component of the axial charge, we consider

$$I_H(p; \Delta)\delta^3(p - p') = \sum_{m^2_\pi \Delta} \left( \langle H, p|Q_5^-|n\rangle\langle n|Q_5^+|H, p'\rangle - \langle H, p|Q_5^+|n\rangle\langle n|Q_5^-|H, p'\rangle \right);$$

in the above, $f_\pi = 93$ MeV is the pion decay constant, $T^{(\pi^-H)}(s)$ is the amplitude, at center of mass energy squared $s$, for $\pi^-H$ scattering and the integration over $s$ is restricted to the interval $\Delta$. (Relaxing this restriction yields the Adler-Weisberger sum rule.)
We shall apply eq. (4) in cases where the state $H$ is the low lying 0$^-$ or 1$^-$ meson containing one heavy quark, for definiteness let us say the $D$ and $D^*$. As in the heavy quark limit interactions of pions with these mesons is independent of the spin of the heavy quark we obtain

$$\int_{\Delta} \frac{ds}{(s - m_D^2)^2} \text{Im} \left( T^{(\pi^- D)}(s) - T^{(\pi^+ D)}(s) \right) = \int_{\Delta} \frac{ds}{(s - m_{D^*}^2)^2} \text{Im} \left( T^{(\pi^- D^*)}(s) - T^{(\pi^+ D^*)}(s) \right);$$

(5)

$T^{(\pi \pm D^*)}$ is the $\pi \pm D^*$ amplitude for any $D^*$ spin or helicity state. Eq. (5) is obviously valid in the $m_Q \to \infty$ limit and we didn’t need the previous discussion to obtain it; we shall show that as as a result of its connection to the axial charge it survives $1/m_Q$ corrections that depend on the heavy quark spin. As there are no isospin 3/2 resonances, in a resonance approximation this relation this becomes

$$\sum_{R \in \Delta} (2j_R + 1) \frac{\Gamma(R \to D \pi)}{p^3} = \frac{1}{3} \sum_{R \in \Delta} (2j_R + 1) \frac{\Gamma(R \to D^* \pi)}{p^3};$$

(6)

the summation extends over resonances whose mass squared is in the interval $\Delta$ and $p$ is the decay momentum. As eq. (5) involves a difference of amplitudes, nonresonant scattering is expected to cancel and eq. (6) should be a good approximation.

In order to show that there are no $1/m_Q$ corrections depending on the heavy quark spin and for other further discussions it is useful to review some of the heavy quark spectroscopy and effective field theory. The heavy quark multiplets, both resonant and non-resonant, are labeled by the total angular momentum, $j = l + s$, of the light quark; this in turn is coupled to the spin-1/2 heavy quark. For $l = 0$, $j = 1/2$ resonant states are $D$ and $D^*$ with spin-parity 0$^-$ and 1$^-$ respectively; for $l = 1$, $j = 1/2$ we have a $D_{0+}$ and a $D_{1+}'$, while for $l = 1$, $j = 3/2$ we find a second $D_{1+}$ (the reason for the prime in the notation for the previous state) and a $D_{2+}$. The latter may be identified with the $D_1(2420)$ and $D_2(2460)$ respectively, while the former has not yet been seen. It is convenient to introduce fixed velocity fields $\hat{\Pi} \hat{\mathcal{B}}$, which for $v = 0$ are $2 \times 2$ matrices.

$$H = \frac{1}{\sqrt{2}} (D + \sigma_a D^{*a});$$
The axial charges $Q^i_5$ connect ground state the $l = 0, j = 1/2$ multiplet with all $l = 1, j = 1/2$ multiplets.

$$Q^i_5 = \sum_r g_r \text{Tr} H^{\tau^i_2} S^{(r)\dagger} + \text{h.c.};$$

the summation is over all $l = 0, j = 1/2$ multiplets and the $g_r$ are constants determining the strengths of the matrix element of the axial charge between the states in question. Using the fixed velocity field equal time commutation relations

$$\begin{align*}
[H_{ab}, H_{cd}^\dagger] & = i\delta_{ad}\delta_{bc}, \\
[S^{(r)}_{ab}, S^{(s)\dagger}_{cd}] & = i\delta^{rs}\delta_{ad}\delta_{bc}. \quad (9)
\end{align*}$$

the current algebra relations

$$\begin{align*}
[Q^i_5, Q^j_5] & = if^{ijk}Q^k, \quad (10)
\end{align*}$$

with $Q^k$ the vector isospin charge, imply $\sum_r g_r^2 = 1$, independent of the heavy quark mass. Corrections to the axial charge matrix elements that are not invariant under heavy spin rotations, such as

$$\delta Q^i_5 = \sum_r h_r \text{Tr} \sigma^a H^{\tau^i_2} S^{(r)\dagger} + \text{h.c.}, \quad (11)$$

are not allowed as these could not be accommodated by current algebra; due to this eq. (6) has no heavy quark spin dependent $1/m_Q$ corrections. For an interval $\Delta$ dominated by one heavy quark multiplet with light quark angular momentum $j$ we obtain

$$\frac{1}{3} \sum_{S_R = j \pm 1/2} (2S_R + 1) \frac{\Gamma(R \rightarrow D^*\pi)}{p^3} = \sum_{S_R = j \pm 1/2} (2S_R + 1) \frac{\Gamma(R \rightarrow D\pi)}{p^3}; \quad (12)$$

this is the main result of this paper.
For the charmed heavy quark the $D_{1^+}(2420)$ and the $D_{2^+}(2460)$ form such a multiplet with $j = 3/2$. Table II contains the experimental data and the last column gives the numerical values for the left and right hand sides of eq. (12); this sum rule is very well satisfied. We note that this is an $D$ wave decay and the widths are proportional to $p^5$; yet with a broad range of decay momenta it is the $\Gamma/p^3$’s that are related.

There are no data on other $D$ multiplets to compare with eq. (12), however it is tempting to look at the $K$ meson system, even though the mass of the strange quark is generally taken to be to light for the heavy quark formalism to apply. Results for this system analogous to the one shown in Table II are presented in Table III; again the agreement is very good. In the $K$ system data exist for the $l = 1, j = 1/2$ resonances, namely $K_{1^+}(1400)$ and $K_{0^+}(1430)$. The result shown in Table III, although not as spectacular as the previous two, is reasonable.

It is worthwhile to look at possible sources of corrections to eq. (12) when applied to the $l = 1, j = 1/2$ $K$ multiplet. In this analysis we have paired one of the $1^+$ resonances, the $K_{1^+}(1270)$, with the $K_{2^+}(1430)$ and the other one, the $K_{1^+}(1400)$ with the $K_{0^+}(1430)$. It is not clear whether it should not have been the other way around, or more likely some linear combination. A partial wave analysis [8] indicates that the $K_{1^+}(1400)$ decays to $K^*\pi$ almost exclusively in an S-wave and is thus correctly paired. The $K_{1^+}(1270)$ has, in addition to a D-wave component, an S-wave one. It is unclear how to include mixing of the two $1^+$ states. The $K$ system points out a major source of corrections to eq. (12). The resonance approximation, eq. (6) should hold as long as the interval $\Delta$ is dominated by the resonances in question. We have applied it, eq. (12), to the case where there is only one multiplet in $\Delta$; this should be valid for heavy meson systems. For the $K$ system the spin dependent splitting of the multiplets is large enough that states belonging to other multiplets lie close by or even intervene, e.g. the $K_{1^-}(1410)$ and the $K_{0^-}(1460)$. Testing eq. (6) for the $K$ system by summing over several heavy quark multiplets would be desirable but the necessary data on the partial widths do not exist.

Eq. (12) should work well for other $D$ multiplets and even better for the $B$ states.
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TABLES

TABLE I. \( l = 2 \) D-Meson Decays. Column 2 gives the decay momentum for the reaction in column 1 and the corresponding partial width is in column 3. This partial width, multiplied by the initial spin multiplicity and divided by the final state one, as in eq. (12) are in column 4 and the sum pertaining to a definite final state is shown in the last column; the sum rule in eq. (12), requires an equality of the numbers in the last column.

| \( p \) (GeV) | \( \Gamma \) (MeV) | \( \frac{(2S_f+1)\Gamma}{(2S_i+1)p^3} \) (GeV\(^{-2}\)) | Sum   |
|--------------|-----------------|---------------------------------|-------|
| \( D_{1+} \) \( (2420) \) \( \rightarrow D^* \pi \) | 0.355 | 18.9 ± 4.0\(^a\) | 0.42 ± 0.11 | 0.62 ± 0.12 |
| \( D_{2+} \) \( (2460) \) \( \rightarrow D^* \pi \) | 0.387 | 7.0 ± 1.8\(^b\) | 0.20 ± 0.05 |
| \( D_{2+} \) \( (2460) \) \( \rightarrow D \pi \) | 0.503 | 16.0 ± 1.8\(^b\) | 0.67 ± 0.17 | 0.67 ± 0.17 |

\(^a\)Ref. [4], p.469.
\(^b\)Ref. [4], p.470.

TABLE II. \( l = 2 \) K-Meson Decays. Presentation is as in Table I.

| \( p \) (GeV) | \( \Gamma \) (MeV) | \( \frac{(2S_f+1)\Gamma}{(2S_i+1)p^3} \) (GeV\(^{-2}\)) | Sum   |
|--------------|-----------------|---------------------------------|-------|
| \( K_{1+} \) \( (1270) \) \( \rightarrow K^* \pi \) | 0.301 | 13.9 ± 4.2\(^a\) | 0.51 ± 0.13 | 1.06 ± 0.14 |
| \( K_{2+} \) \( (1430) \) \( \rightarrow K^* \pi \) | 0.423 | 24.8 ± 1.7\(^b\) | 0.55 ± 0.06 |
| \( K_{2+} \) \( (1430) \) \( \rightarrow K \pi \) | 0.622 | 48.9 ± 1.2\(^b\) | 1.02 ± 0.11 | 1.02 ± 0.11 |

\(^a\)Ref. [4], p.432.
\(^b\)Ref. [4], p.434.
TABLE III. $l = 0$ K-Meson Decays. Presentation is as in Table I except that, as each resonance has only one final state no summation is needed; the sum rule in eq. (12) requires an equality of the numbers in the last column.

|                               | $p$ (GeV) | $\Gamma$ (MeV) | $(\frac{(2S_i+1)\Gamma}{(2S_f+1)p^3}$ (GeV$^{-2}$) |
|-------------------------------|-----------|----------------|--------------------------------------------------|
| $K_{1+}^{*}(1400) \to K^*\pi$ | 0.401     | $117 \pm 10^a$ | $1.81 \pm 0.17$                                  |
| $K_{0+}^{*}(1430) \to K\pi$   | 0.621     | $267 \pm 29^b$ | $1.15 \pm 0.13$                                  |

$^a$Ref. [4], p.433.

$^b$Ref. [4], p.434.