Strapdown electrostatic gyro with and without self-compensation

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Abstract. Various applications of strapdown electrostatic gyroscopes (ESG), depending on the type and purpose of the vehicle on which the gyro is installed, as well as operation conditions, impose substantially different requirements on this gyro, including accuracy of information it provides. One of traditional methods used to ensure the gyro accuracy is self-compensating rotation of the housing. The paper considers some applications of a strapdown ESG with a solid rotor and the self-compensation methods used in these applications. This gyroscope was developed 20 years ago at the CSRI Elektropribor, but the work on its improvement goes on continuously. Theoretical aspects of the ESG operation are discussed, taking into account the developed drift model, as well as their experimental validation based on the analysis of ESG operation in polar orientations.

Introduction
Consider a strapdown ESG, hereinafter referred to as ESG developed at the CSRI Elektropribor [1], the 10-mm beryllium spherical rotor of which is suspended in the electric field generated by several pairs of orthogonally arranged electrodes (Fig. 1). The voltage across the suspension electrodes is 200 V, which provides an overload capacity of about 7g. The rotor spin up with a frequency of 3000 Hz and damping of its nutations is implemented with the help of six symmetrically arranged voltage coils. The drive is used only to spin up the rotor to the operating speed, whereupon it is turned off, and the rotor frequency is stabilized by the electric forces of the suspension. A magnetic discharge pump creates vacuum in the operating gap of about $10^{-6}$–$10^{-7}$ mmHg. The gyro sensing element is surrounded by an array of magnetic shields. The angular orientation of the rotor is readout in an unlimited range of angles by six orthogonally arranged optical sensors using the raster picture on the rotor.

The main errors typical of this gyro are the following:
- errors due to the interaction between the unbalanced rotor and the suspension field;
- errors due to the interaction between the nonspherical rotor and the suspension field;
- errors due to the interaction between the unbalanced rotor and the uneven-stiffness suspension;
- errors due to the interaction between the misaligned rotor and the suspension field.

Here, the rotor drift caused by the residual gas, magnetic fields of different nature is assumed negligible.

With this in view, we developed a basic deterministic drift model, which is represented by analytic functions relating the geometrical parameters of the non-spherical unbalanced and misaligned rotor to the parameters of physical fields – sources of the rotor drift [2].
In particular, drift rate $\omega$, projected on one of the axes of the gyro housing, has the following form:

$$\omega_i = k_0\gamma_1 \left[ -\left( 1 - \gamma_i^2 \right) \gamma_i^2 + \gamma_i^4 + \gamma_i^6 \right] +$$

$$+ k_1 \left[ -\left( 1 - \gamma_i^2 \right) V_1 + \gamma_i \gamma_2 V_2 + \gamma_i \gamma_3 V_3 \right] +$$

$$+ k_2 \gamma_i \left[ -\left( 1 - \gamma_i^2 \right) V_i^2 + \gamma_i^2 \gamma_2 V_2^2 + \gamma_i^2 \gamma_3 V_3^2 \right] +$$

$$+ k_3 \gamma_i \left[ -\left( 1 - \gamma_i^2 \right) \gamma_2 V_1 + \gamma_i^2 \gamma_2 V_2 + \gamma_i^2 \gamma_3 V_3 \right] +$$

$$+ k_4 \gamma_i \left[ -\left( 1 - \gamma_i^2 \right) \gamma_2^2 V_1^2 + \gamma_i^2 \gamma_2^2 V_2^2 + \gamma_i^2 \gamma_3^2 V_3^2 \right] +$$

$$+ \gamma_1 \left( \mu_{12} \gamma_i^2 - \mu_{31} \gamma_i^2 \right) + \gamma_2 \gamma_3 \gamma_{23} +$$

$$+ \left[ \left( 1 - \gamma_i^2 \right) C_{11} + \gamma_i \gamma_2 C_{12} + \gamma_i \gamma_3 C_{13} \right] +$$

$$+ \left[ \left( 1 - \gamma_i^2 \right) \left( 1 - 5 \gamma_i^2 \right) C_{21} + \left( 1 - 5 \gamma_i^2 \right) \gamma_i \gamma_2 C_{22} + \left( 1 - 5 \gamma_i^2 \right) \gamma_i \gamma_3 C_{23} \right]$$

In (1), $k_i$ are coefficients of the drift model (DMC), characterizing the moments due to axial disbalance and rotor nonsphericity harmonics, $\gamma_i$ - direction cosines, $V_i$ - relative control voltages across the suspension electrodes (ratio between the control voltages and reference voltage $V_0$); $\mu_{ij}$ - coefficients characterizing the conservative part of the moment caused by the interaction between the uneven-stiffness suspension and the unbalanced rotor, and $\gamma_{ij}$ are coefficients characterizing the dissipative part of this moment; $C_{ij}$ are coefficients caused by the rotor displacement with respect to the electrodes.

In accordance with this drift model, the equations of the gyro motion in the axes of the body-fixed coordinate frame take the following form:

$$\dot{\gamma}_1 = \omega_{43} \gamma_2 - \omega_{43} \gamma_3 + k_1 (V_3 \gamma_2 - V_2 \gamma_3) + \gamma_2 \gamma_3 [-k_0 (\gamma_2^2 - \gamma_3^2) + k_2 (V_3^2 - V_2^2)] +$$

$$+ k_3 (V_3 \gamma_3 - V_2 \gamma_2) + k_4 (V_2^2 \gamma_3^2 - V_2^2 \gamma_2^2) + \mu_{23}] + (C_{12} \gamma_3 - C_{13} \gamma_2) + (1 - 5 \gamma_2^2) C_{22} \gamma_3 -$$

$$- \left( 1 - 5 \gamma_2^2 \right) C_{23} \gamma_2 + \gamma_1 (V_3 \gamma_2 - V_2 \gamma_3) \gamma_1 (0)$$

$$\dot{\gamma}_2 = \omega_{43} \gamma_3 - \omega_{43} \gamma_1 + k_2 (V_1 \gamma_3 - V_3 \gamma_1) + \gamma_3 \gamma_1 [-k_0 (\gamma_3^2 - \gamma_1^2) + k_2 (V_1^2 - V_3^2)] +$$

$$+ k_3 (V_3 \gamma_1 - V_2 \gamma_3) + k_4 (V_2^2 \gamma_3^2 - V_2^2 \gamma_1^2) + \mu_{23}] + (C_{13} \gamma_1 - C_{11} \gamma_3) + (1 - 5 \gamma_3^2) C_{23} \gamma_3 -$$

$$- \left( 1 - 5 \gamma_3^2 \right) C_{21} \gamma_3 + \gamma_2 (V_2 \gamma_1 - V_3 \gamma_2) \gamma_2 (0)$$

$$\dot{\gamma}_3 = \omega_{43} \gamma_1 - \omega_{43} \gamma_2 + k_1 (V_1 \gamma_1 - V_2 \gamma_2) + \gamma_1 \gamma_2 [-k_0 (\gamma_1^2 - \gamma_2^2) + k_2 (V_1^2 - V_2^2)] +$$

$$+ k_3 (V_2 \gamma_2 - V_3 \gamma_1) + k_4 (V_3^2 \gamma_1^2 - V_3^2 \gamma_2^2) + \mu_{23}] + (C_{11} \gamma_1 - C_{12} \gamma_1) + (1 - 5 \gamma_1^2) C_{21} \gamma_1 -$$

$$- \left( 1 - 5 \gamma_1^2 \right) C_{23} \gamma_1 + \gamma_3 (V_3 \gamma_1 - V_2 \gamma_3) \gamma_3 (0)$$
where $\omega_i (i = 1,2,3)$ are angular rates of the body-axes rotation.

Self-compensating rotation of the gyro housing is a well-known method to increase ESG accuracy [3,4,5]. In this paper we describe how this method affects the ESG rotor motion and its accuracy characteristics.

In addition, the paper presents the results of analytical solutions and experimental data for the ESG operation in polar orientations when the housing does not move relative to the base, and also, for uniaxial and biaxial rotations of the ESG housing. The accuracy characteristics were estimated by determining the coefficients of model (1) with the use of the Kalman filter and uncompensated residual between the real and predicted motion.

1. ESG without self-compensation

Let us solve Equations (2) for the following orientation of the axes of the housing, which does not move with respect to the base: axis Xk is directed to the East, axis Zk – along the celestial axis, axis Yk is in the equator plane. The angular momentum vector is directed along axis Zk.

Then, we have the following relations:

$$V'_1 = 0, \ V'_2 = V \cos \varphi, \ V'_3 = -V \sin \varphi, \ (3)$$

$$\omega_{k1} = 0, \ \omega_{k2} = 0, \ \omega_{k3} = U, \ (4)$$

where $\varphi$ is latitude, $U$ is the Earth angular velocity.

Taking into account (3), (4), Equations (2) are solved as follows:

$$\gamma_{m1}(t) = \frac{U}{\omega_2} \alpha_2 - \frac{C_{11} + C_{21}}{\omega_2} + \frac{\exp(V_3 t - V_2)}{\omega_2} \cos \omega_{pn} t - \frac{U}{\omega_2} \alpha_1 \cos \omega_{pn} t -$$

$$-k_1 V \cos \varphi - U \alpha_1 \sin \omega_{pn} t + \frac{C_{12} + C_{22}}{\omega_{pn}} \sin \omega_{pn} t, \ (5)$$

$$\gamma_{m2}(t) = \frac{k_1 V \cos \varphi - U \alpha_1}{\omega_1} - \frac{C_{11} + C_{21}}{\omega_1} + \frac{\exp(V_3 t - V_2)}{\omega_1} \cos \omega_{pn} t -$$

$$-k_1 V \cos \varphi - U \alpha_1 \cos \omega_{pn} t + \frac{U}{\omega_2} \alpha_2 \sin \omega_{pn} t - \frac{C_{11} + C_{21}}{\omega_{pn}} \sin \omega_{pn} t, \ (6)$$

$$\gamma_{m3}(t) \approx 1,$$

where

$$\omega_{pn} = \sqrt{\omega_1 \omega_2},$$

$$\omega_1 = U + k_0 - (k_1 + k_2) V \sin \varphi + (k_2 + k_3) V^2 \sin^2 \varphi - k_2 V^2 \cos^2 \varphi - \mu_{23} - (C_{13} - 4 C_{23}) \ (7)$$

$$\omega_2 = U + k_0 - (k_1 + k_3) V \sin \varphi + (k_2 + k_3) V^2 \sin^2 \varphi - \mu_{23} - (C_{13} - 4 C_{23}) \ (8)$$

Formulas (5), (6) also make allowance for the angles of gyro measurement axes $x_u, y_u, z_u$ misalignment with the body axes: $\alpha_1$ – angle of rotation about axis $x_k$, $\alpha_2$ – angle of rotation about axis $y_k$, $\alpha_3$ – angle of rotation about axis $z_k$ (in what follows, we call them references).

Based on the analysis of the above expressions, we can make the following conclusions:
1. The trajectory of unit vector \( \vec{y} \) ends in an ellipsoidal spiral line, described with respect to the center \((\gamma_1^0, \gamma_2^0)\) with a period \(1/\omega_{pn}\) defined by the DMC values.

2. Depending on the values of the dissipative part of DMC, the spiral can be either converging or diverging.

3. Coordinates of the hodograph center mainly depend on the values of the references, the rotor displacement in the suspension, and the moment caused by axial disbalance.

4. The hodograph ellipticity depends on the moments of even harmonics of the rotor shape (the second and the following) and with the conservative part of the moment due to interaction between the rotor radial disbalance and uneven stiffness of the suspension channels.

5. The model coefficients are included in the formulas for the precessional motion frequency as a linear combination, i.e. on a stationary base, it is impossible to separate coefficient \( k_{1,y} \) from the references and coefficients caused by the interaction between the misaligned rotor with the field of the suspension; neither it possible to separate DMC from each other.

Figure 2 shows a typical experimental hodograph of the ESG rotor motion in the fixed housing. The hodograph curls, decreasing in diameter; the coordinate of the hodograph center in the eastern direction is about 25 arc min.

![Hodograph in the absence of self-rotation of the housing](image)

The ESG uncompensated drift estimated with the use of the Kalman filter is in the range from 0.01 to 0.03 \(^\circ\)/h.

It should be noted that without self-compensation, ESGs operate in spacecraft inertial attitude control systems developed at the CSRI Elektropribor [6].

2. **ESG with uniaxial self-compensating rotation**

For uniaxial self-compensating rotation of the housing about axis \( z_k \) the following relations are valid:

\[
\begin{align*}
V_1 &= V \cos \varphi \sin(\omega_{sr} t), \\
V_2 &= V \cos \varphi \cos(\omega_{sr} t), \\
V_3 &= -V \sin \varphi \\
\omega_{k1} &= 0, \\
\omega_{k2} &= 0, \\
\omega_{k3} &= U + \omega_{sr},
\end{align*}
\]

where \( \omega_{sr} \) is the angular rate of self-rotation.
Let us give the analytical solution to motion Equations (2) of the gyro angular momentum vector in the axes of the equatorial coordinate frame in the case of the housing self-rotation, taking into account the drift model (1), formulas (9), as well as “references” of the gyro measurement axes to the housing axes (for simplicity, the initial values of direction cosines \( \gamma(0) \) and \( \gamma(0) \) in the expressions are assumed equal to zero):

\[
\gamma_{m1}(t) = \frac{U + \omega_p}{\omega_p + \omega_{sr}} \left[ (\alpha_1 \sin \omega_p t + \alpha_2 \cos \omega_p t) + (\alpha_3 \sin \omega_p t - \alpha_4 \cos \omega_p t) \right] -
\frac{k_V \cos \varphi}{\omega_p} \sin \omega_p t + \left( \frac{C_{11} + C_{21}}{\omega_p + \omega_{sr}} \cos \omega_p t + \frac{C_{12} + C_{22}}{\omega_p + \omega_{sr}} \sin \omega_p t \right) -
\frac{C_{11} + C_{21}}{\omega_p + \omega_{sr}} \cos \omega_{sr} t - \frac{C_{12} + C_{22}}{\omega_p + \omega_{sr}} \sin \omega_{sr} t,
\]

\[
\gamma_{m2}(t) = \frac{U + \omega_p}{\omega_p + \omega_{sr}} \left[ (\alpha_1 \sin \omega_p t - \alpha_4 \cos \omega_p t) + (\alpha_3 \cos \omega_p t + \alpha_2 \sin \omega_p t) \right] +
\frac{k_V \cos \varphi}{\omega_p} (1 - \cos \omega_p t) + \left( \frac{C_{12} + C_{22}}{\omega_p + \omega_{sr}} \cos \omega_p t + \frac{C_{11} + C_{21}}{\omega_p + \omega_{sr}} \sin \omega_p t \right) -
\frac{C_{12} + C_{22}}{\omega_p + \omega_{sr}} \cos \omega_{sr} t + \frac{C_{11} + C_{21}}{\omega_p + \omega_{sr}} \sin \omega_{sr} t,
\]

\[
\gamma_{m3}(t) \approx 1,
\]

\( \omega_p \) - frequency of the gyro precessional motion,

\[
\omega_p = U + k_0 - (k_1 + k_3) V \sin \varphi + (k_2 + k_4) V^2 \sin^2 \varphi - 0.5 k_2 V^2 \cos^2 \varphi + 0.5(\mu_2 - \mu_3) - (C_{13} - 4C_{23})
\]

\( \alpha_1, \alpha_2 \) are misalignment angles between the gyro measurement axes and the body axes.

It is certainly useful to compare the character and motion parameters of the ESG rotor with the fixed housing and uniaxial self-compensating rotation (Fig. 2, 3). Table 1 presents the values of motion parameters for the same gyro in the polar orientation with and without self-rotation. Here, coefficient \( k_V \) is calculated from the coordinate of the hodograph center in the meridian plane, and in the absence of self-rotation, it contains not only coefficient \( k_V \), as it was shown above, but also the references and coefficients caused by misaligned rotor.

| Date of start | Test conditions | Period Tp, h | \( X_0 \), arc min | \( Y_0 \), arc min | \( \omega_{Kp} \), \( \varphi \)/h | Hodograph diameter, arc min |
|---------------|----------------|-------------|-------------------|------------------|-------------------------------|---------------------------|
| 23.06.18      | without self-rotation | 15.95       | 2.58              | 5.11             | 0.0592                        | 44                        |
| 22.06.18      | with self-rotation    | 15.82       | -0.2              | 2.658            | 0.0395                        | 43.3                      |

Based on the analysis of formulas (5), (6), (10)–(12) as well as the experimental data, we can conclude that in the case of uniaxial self-rotation:

1. the angular momentum vector is in the meridian plane because the coordinates of the hodograph center in the equatorial axes can be written as \( \gamma_{m1}^0 = 0 \), \( \gamma_{m2}^0 = \frac{k_V \cos \varphi}{\omega_p} \). The coordinate of the center in the meridian plane is due to the moment caused by the axial disbalance.
2. The moments due to the interaction of the unbalanced and nonspherical rotor with the field of suspension, moments due to the interaction of the unbalanced rotor with the uneven-stiffness suspension, and dissipative moments are not “zeroed”. The only exception is the moments due to the rotor misalignment.

3. “References” and coefficient $k_1 V$ are separated because they show themselves at different frequencies: “references” – at precession and self-rotation frequencies, and coefficient $k_1 V$ – only at precession frequency. This circumstance makes it possible to improve the accuracy of coefficient $k_1 V$, which determines the gyro drift in the polar orientation.

4. DMC due to the rotor misalignment show themselves in the same way as “references”: they do not affect the hodograph center, but are superimposed on the precessional motion as high-frequency oscillations (Fig. 3); amplitudes of oscillations due to “references” and the rotor misalignment depend on the self-rotation rate and reduce as the latter increases.

5. The hodograph curls decreasing in diameter because during rotation, axes $x_k$ and $y_k$ of the housing are alternately subjected to the same load (gravity projection) and the moments due to the second harmonic of the rotor shape (coefficient $k_2$) along axes $x_k$ and $y_k$ are equal, thus providing uniform motion of the angular momentum vector along the trajectory. On a fixed base, only axis $y_k$ is subjected to the load.

6. The periods of the precessional motion differ insignificantly, which means small variance of the rotor drifts as compared to the fixed housing.

In the case of uniaxial rotation, uncompensated drift is in the range between 0.005 and 0.007 °/h. Note that the CSRI Elektropribor employs an ESG with uniaxial self-compensation in a ground direction keeper.

3. ESG with biaxial self-compensating rotation

Consider biaxial rotation of the gyro housing about axes $z_k$ and $x_k$.

In this case, the following expressions are valid:

- for suspension voltage:

$$V_1 = V \cos(\varphi) \sin(\omega_{sr1} t),$$
$$V_2 = V \cos(\varphi) \cos(\omega_{sr1} t) \cos(\omega_{sr2} t) - V \sin(\varphi) \sin(\omega_{sr2} t),$$
$$V_3 = -V \cos(\varphi) \cos(\omega_{sr1} t) \sin(\omega_{sr2} t) - V \sin(\varphi) \cos(\omega_{sr2} t),$$

(13)
for angular rates of the housing rotation:

\[ \omega_1 = \alpha \omega_{v1} , \quad \omega_2 = (\alpha \omega_{v1} + U) \sin(\omega_{v1} t) , \quad \omega_3 = (\alpha \omega_{v1} + U) \cos(\omega_{v1} t) , \tag{14} \]

The relation between the direction cosines for the rotating axes (\( \gamma_i \)) and direction cosines for the fixed axes (\( \gamma_{in} \)) can be written as:

\[
\begin{align*}
\gamma_1 &= \gamma_{1n} \cos(\omega_{v1} t) + \gamma_{2n} \sin(\omega_{v1} t) \\
\gamma_2 &= -\gamma_{1n} \sin(\omega_{v1} t) \cos(\omega_{v2} t) + \gamma_{2n} \cos(\omega_{v1} t) \cos(\omega_{v2} t) + \gamma_{3n} \sin(\omega_{v2} t) \\
\gamma_3 &= -\gamma_{1n} \sin(\omega_{v1} t) \sin(\omega_{v2} t) - \gamma_{2n} \cos(\omega_{v1} t) \sin(\omega_{v2} t) + \gamma_{3n} \cos(\omega_{v2} t) 
\end{align*} \tag{15} \]

Taking (13)–(15) into account, we can rewrite Equations (2) for the fixed axes:

\[
\begin{align*}
\dot{\gamma}_{in} - U \gamma_{2H} &= k_1 V \sin \varphi \gamma_{2H} - V \cos \varphi \gamma_{3H} + \sum_{i=1}^{3} \alpha_i f_{i1} + \sum_{i=2}^{4} k_i r_{i1} + \\
\end{align*} \tag{16} \]

\[
\begin{align*}
\gamma_{2n} &= -k_1 V \sin \varphi \gamma_{1H} + \sum_{i=1}^{3} \alpha_i f_{i2} + \sum_{i=2}^{4} k_i r_{i2} + \sum_{i=2}^{3} \mu_i \dot{g}_{i2} + \sum_{i=1}^{3} c_{i2} z_{i2} \gamma_{2H} (0) \\
\gamma_{3n} &= k_1 V \cos \varphi \gamma_{1H} + f_3 + \sum_{i=3}^{4} k_i r_{i3} + \sum_{i=2}^{4} \mu_i \dot{g}_{i3} + \sum_{i=1}^{3} c_{i3} z_{i3} \gamma_{3H} (0) \\
\end{align*} \tag{16} \]

where

\[
\begin{align*}
f_{i1} &= 0.5(\alpha \omega_{v1} + U) \left[ - \gamma_{1n} \sin 2\omega_{v1} t + \gamma_{2n} \cos 2\omega_{v1} t \sin \omega_{v2} t + 2\gamma_{3n} \cos \omega_{v1} t \cos 2\omega_{v2} t \right] + \\
&+ \omega_{v2} \left[ - \gamma_{1n} \sin \omega_{v1} t + \gamma_{2n} \cos \omega_{v1} t \cos 2\omega_{v2} t + 3\gamma_{3n} \sin 2\omega_{v2} t \right] \sin \omega_{v1} t \\
\end{align*} \tag{16} \]

\[
\begin{align*}
f_{i2} &= (\alpha \omega_{v1} + U) \left[ \gamma_{1n} \cos 2\omega_{v1} t + \gamma_{2n} \sin 2\omega_{v1} t \sin \omega_{v2} t + \gamma_{3n} \cos 2\omega_{v1} t \sin \omega_{v2} t \right] + \\
&+ 0.5 \omega_{v2} \cos \omega_{v1} t \left[ \gamma_{1n} \sin 2\omega_{v1} t - \gamma_{2n} \cos 2\omega_{v1} t + 3\gamma_{3n} \right] \\
\end{align*} \tag{16} \]

\[
\begin{align*}
f_{i3} &= (\alpha \omega_{v1} + U) \left[ - \gamma_{1n} \sin \omega_{v1} t \cos 2\omega_{v2} t + 3\gamma_{3n} \sin \omega_{v2} t \sin \omega_{v1} t \right] + \\
&+ 0.5 \omega_{v2} \sin \omega_{v1} t \left[ \gamma_{1n} \sin 2\omega_{v1} t - \gamma_{2n} \cos 2\omega_{v1} t + 3\gamma_{3n} \right] \\
\end{align*} \tag{16} \]

\[
\begin{align*}
f_{i4} &= (\alpha \omega_{v1} + U) \left[ - \gamma_{1n} \sin \omega_{v1} t \cos 2\omega_{v2} t - \gamma_{2n} \cos 2\omega_{v2} t \sin \omega_{v1} t \right] + \\
&- \omega_{v2} \left[ - \gamma_{1n} \sin \omega_{v1} t + \gamma_{2n} \cos \omega_{v1} t \cos 2\omega_{v2} t + 2\gamma_{3n} \cos \omega_{v2} t \right] \sin \omega_{v1} t \\
\end{align*} \tag{16} \]

In (16), functions \( r_{ij}, \dot{g}_{ij}, z_{ij} \) contain nonlinear dependencies on direction cosines and harmonic components with periods multiple of self-rotation rates. For example,
\[ r_1 = -\left[ y_2 y_3 (\gamma_2^2 - \gamma_3^2) \cos \omega_{sr1} t - y_1 (\gamma_2^2 - \gamma_3^2) \cos \omega_{sr2} t - y_2 (\gamma_2^2 - \gamma_3^2) \sin \omega_{sr2} t \sin \omega_{sr1} t \right]. \] (17)

Figure 4 presents the experimentally obtained motion of the rotor for the case of biaxial self-rotation, from which we can make the following conclusions:

1) The trajectory of the angular momentum vector is a circle with a period \(1/\omega_p\) and superposition of oscillations with frequencies depending on \(\omega_{sr1,2}\). The amplitude of oscillations depends on the values of “references”.

2) Coordinates of the hodograph center are mainly defined by the axial disbalance (“references” do not define these coordinates). The angular momentum vector is set in the meridian plane.

3) The functionals at \(r_i, c_i, \mu_{ij}\) contain oscillating components with self-rotation frequencies. In other words, double self-rotation “resets to zero” the moments from the 2\(^{nd}\), 3\(^{rd}\), 4\(^{th}\) harmonics of the rotor shape, the moments due to uneven stiffness of the suspension and due to the rotor misalignment in the suspension.

4) The formulas with coefficient \(k_i\) do not depend on the presence or absence of self-rotation. In other words, double self-rotation due to modulation by self-rotation angles does not zero the moment due to by the axial disbalance.

5) Precession frequency of the polar gyro in this mode is written as \(\omega_p \approx U + k V \sin(\varphi)\), i.e. the period is close to 24 h.

![Hodograph in the case of biaxial self-compensation.](image)

The value the ESG uncompensated drift estimated with the use of the Kalman filter in the case of biaxial self-compensating rotation is about 0.001°/h.

The CSRI Elekropribor studied the feasibility of using this variant of the ESG for application in a system of in-tube navigation for oil-and-gas industry where the housing rotation about one axis was provided by the gyro unit, and rotation about the second axis, by rotation of the whole system about the longitudinal axis of the pipeline.

**Conclusion**

We have analyzed how the motion parameters and accuracy of a strapdown ESG are affected by the operation modes with the fixed housing, uniaxial and biaxial self-compensating rotations of the housing.
It has been shown that as compared to the case of the fixed base (housing), the accuracy increases:

- in the case of uniaxial self-rotation due to separation of “references” and drift model coefficients (owing to improved observability owing to the use of the Kalman filter);
- in the case of uniaxial self-rotation, additionally, due to averaging of the moments by self-compensation angles.
- In our experiments we used the real data obtained from an ESG developed by the CSRI Elektropribor. The results have shown good agreement with the results of the theoretical analysis.

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