Experimental Bell inequality violation without the postselection loophole

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We report on an experimental violation of the Bell-Clauser-Horne-Shimony-Holt (Bell-CHSH) inequality using energy-time entangled photons. The experiment is not free of the locality and detection loopholes, but is the first violation of the Bell-CHSH inequality using energy-time entangled photons which is free of the postselection loophole described by Aerts et al. [Phys. Rev. Lett. 83, 2872 (1999)].

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Introduction.—The violation of Bell inequalities [1, 2] has both profound implications for our understanding of the universe, and striking applications like eavesdropping-proof communication [3], reduction of communication complexity [4], and randomness certification [5]. Testing Bell inequalities requires reliable methods for entangling, distributing, and measuring particles in spatially separated regions. So far, no experiment [3, 7, 8, 9, 10] has shown a conclusive violation of a Bell inequality. Local hidden variable (HV) models exist which reproduce the results of any performed experiment. These local HV models exploit the locality and the detection loopholes [11, 12].

In 1989, Franson [13] introduced a setup which allows to create and measure two energy-time entangled photons over large distances. Franson’s setup used the essential uncertainty in the time of emission of a pair of photons to make undistinguishable two alternative paths that the photons can take. As a result, photons detected in coincidence become entangled in path. This type of entanglement is usually called “energy-time” or “time-bin” entanglement, depending on the method used to have uncertainty in the time of emission [14]. Franson’s setup is widely used for testing violations of Bell inequalities [15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26].

However, in the Franson’s scheme a new loophole, the “postselection” loophole, is present. Indeed, Aerts et al. [27] showed that, even in the ideal case of two-particle preparation, enough spatial separation between the local measurements (which closes the locality loophole), and perfect detection efficiency (which closes the detection loophole), there are local HV models that reproduce the quantum predictions for the violation of the Bell-Clauser-Horne-Shimony-Holt (Bell-CHSH) inequality [2] using Franson’s setup. This is because, in Franson’s setup, the fact that photons are detected in coincidence or not could depend on the local measurement settings. This can be exploited to build local HV models which simulate the quantum predictions for this experiment [27, 28].

One way to deal with this “postselection loophole” that affects to all performed Bell tests using energy-time or time-bin entangled photons [14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26] is to add an extra assumption, the fact that a photon is detected at a specific time is independent of the local experiment performed on that photon [27, 28]. Therefore, even assuming ideal devices, Franson’s setup can only rule out local LHV theories when this extra property is assumed. However, without it, the results of all previous tests of the Bell-CHSH inequality using Franson’s setup can be reproduced with a local HV model like those given in Refs. [27, 28].

Three different strategies have been proposed to avoid the postselection loophole in Bell tests with energy-time...
entanglement:

Aerts et al. [27] showed that general local HV models are not possible using Franson’s setup when very fast local switching is used and a specific three-setting Bell inequality (instead of the standard two-setting Bell-CHSH inequality) is violated. This approach has two problems: It requires a very difficult to achieve fast switching, and a violation which is very close to the maximum violation predicted by QM (i.e., it requires nearly perfect visibility).

Brendel et al. [14] proposed replacing the two beam splitters which are closer to the source in Franson’s setup, by switches synchronized with the source. However, these active switchers do not exist for photonic sources, thus in actual experiments they are are replaced by passive beam splitters (see, e.g., [14]), so the resulting setup suffer from the same problem of the original Franson’s setup. Recently, it has been pointed out that active switchers could be feasible if photons are replaced by molecules [30].

More recently, some of us have proposed a modification of Franson’s scheme in which both the short path of the first (second) photon and the long path of the second (first) photon ends in the same observer [28]. In this scheme, the rejection of events is local (i.e., it does not require communication between the distant observers, as in Franson’s scheme), and the selection of events is independent of the local settings. This property is the one that makes that this scheme do not suffer from the postselection loophole that affect all previous Bell-CHSH experiments with energy-time or time-bin entangled photons. This scheme has inspired a new source of electronic entanglement [31] and has been theoretically extended to multiparty Bell experiments with energy-time entangled photons [32].

In this Letter we present the first postselection loophole-free violation of the the Bell-CHSH inequality using energy-time entangled photons. The experiment is based on the scheme proposed in [28].

**Experimental setup.**—The experimental setup used for testing the Bell-CHSH inequality with energy-time entangled photons is schematically shown in Fig. 1. A single-longitudinal-mode laser operating at 266 nm and with an average power of 70 mW is focused into a 5-mm-long β-barium borate (BBO) crystal cut for type-I parametric down-conversion luminescence [33, 34]. The generated energy-time correlated photons are collimated by a 15 cm focal length lens put at focal distance from the crystal, and then sent through two symmetrical interferometers. These interferometers are unbalanced, thus one can refer to their arms as short (S) and long (L). The optical paths of the down-converted photons are such that coincidences counts between detectors $D_A$ and $D_B$ can be seen only when they propagate through the short-short or long-long two-photon paths. These detectors are composed of interference filters with small bandwidths (5 nm FWHM), single-mode optical fibers and pigtailed avalanche photo-counting modules, that are connected to a circuit used to record the singles and the coincidences counts. The phases of local measurements ($\phi_A$ and $\phi_B$) are set by moving piezoelectric driven stages on which the beam splitters are placed as it is shown in Fig. 1. The output $D_{1A}$ ($D_{2A}$) corresponds to the projection onto $|\phi_A\rangle = \sqrt{2}/2 (|L\rangle + e^{i\phi_A}|S\rangle)$ ($|\phi_A\rangle = \sqrt{2}/2 (|L\rangle - e^{i\phi_A}|S\rangle)$). A similar relation holds for the $D_B$ detection.

The four probabilities of coincidence detection after the interferometers can be easily calculated [35]. They depend on the initial two-photon state, $\rho$, and in the case of down-converted photons belonging to the Bell state $\rho = |\Phi^+\rangle \langle \Phi^+|$, where $|\Phi^+\rangle \equiv \sqrt{2}/2 (|SS\rangle + |LL\rangle)$, these probabilities will be given by

$$P_{ij}(\phi_A, \phi_B) = \frac{1}{4} \left[ 1 - (-1)^{ij} V_{ij} \cos (\phi_A + \phi_B) \right], \quad (1)$$

where $P_{ij}(\phi_A, \phi_B)$ is the probability of coincidence detection between $D_{iA}$ and $D_{jB}$, and $V_{ij}$ is the corresponding visibility of the interference.

A Bell inequality can be tested by considering the coincidence counts recorded by two detectors placed behind of the interferometers. Nevertheless, in this case the experiment requires an extra fair sampling assumption (e.g., that detected photons are a statistically representative subsample of the photons detected in the case where four detectors are present), and reduces the credibility of the conclusion. This extra assumption is not necessary in a test of the Bell-CHSH inequality, where the coincidence counts between all the four output ports of these interferometers are considered.

In our experiment, we recorded the coincidence counts between the detector $D_{1A}$ ($D_{2A}$) and the detectors $D_{1B}$ and $D_{2B}$ illustrated in Fig. 1. We used these coincidences to test the Bell-CHSH inequality and to reconstruct the density operator of the energy-time entangled two-photon state used in the test of the Bell-CHSH inequality.

Our experiment is not free of the locality loophole, since the distance between Alice and Bob’s local experiments is around one meter, which is not enough to prevent reciprocal influences between the two local phase settings. Furthermore, our experiment, like any other photonic Bell experiment, is also not free of the detection loophole, since the overall detection efficiency is less than 15%. The main aim of the experiment is to provide a proof of principle that, with enough separation and more efficient detectors, a conclusive violation of the Bell-CHSH inequality can be observed with energy-time entangled photons.

Any local HV model should satisfy the Bell-CHSH-inequality

$$S \leq 2,$$

(2)
FIG. 2: The four coincidence curves obtained while $\phi_B = 0$.

where

$$S \equiv E(\phi_A, \phi_B) + E(\phi_A', \phi_B')$$

$$+ E(\phi_A', \phi_B) - E(\phi_A, \phi_B'),$$  \hspace{1cm} (3)

and

$$E(\phi_A, \phi_B) = P_{11}(\phi_A, \phi_B) + P_{22}(\phi_A, \phi_B)$$

$$- P_{12}(\phi_A, \phi_B) - P_{21}(\phi_A, \phi_B).$$  \hspace{1cm} (4)

If all the four possible interference curves have the same average visibility $V$, the initial two-photon state is $\rho = |\Phi^+\rangle\langle\Phi^+|$, and the phase settings are

$$\phi_A = \frac{\pi}{4}, \ \phi_B = 0, \ \phi_A' = -\frac{\pi}{4}, \ \phi_B' = \frac{\pi}{2},$$  \hspace{1cm} (5)

then, the expected value for $S$ is $S = 2\sqrt{2}V$. Therefore, we will have an experimental violation of $S$ whenever the average visibility of the two-photon interferences is larger than $1/\sqrt{2} \approx 0.71$.

The correlation functions $E$ that appear in the Bell-CHSH inequality can be determined directly from the coincidence counts recorded by the four detectors. The experimental coincidence counts recorded between the four detectors, when $\phi_B = 0$ and $\phi_A = \frac{\pi}{2}$, are shown in Fig. 2 and Fig. 3 respectively. From these curves we calculated the values of the probability correlation functions $E$ and, therefore, $S$. We calculated $S$ in two different ways: directly from the experimental points obtained, and from the values of the fit of the experimental data. Both are shown in Table I.

The mean visibility observed in the curves of Figs. 2 and 3 is $V = 0.90 \pm 0.015$, and therefore the expected value of $S$ is $S_{\text{exp}} = 2\sqrt{2}V = 2.54 \pm 0.04$. The experimental results obtained for $S$ (see Table I) are slightly lower than $S_{\text{exp}}$ due to small phase-shifts that exists between the eight curves recorded. However, they correspond to a violation of the Bell-CHSH inequality by 20 standard deviations.

The energy-time two photon state was tested by quantum tomography. The reconstructed density operator was obtained considering an analogy between the projective phase-measurements and the measurements used for doing the standard quantum tomography of polarization two-photon states [36]. The data acquired and the analogy between these measurements are shown in Table II.

The density operator $\rho_{\text{exp}}$ obtained after numerical op-
timization is shown in Fig. 4. It has a fidelity \( S = 0.93 \pm 0.02 \) with the Bell state \( |\Phi^+\rangle = |\Psi^+\rangle = |\Phi^+\rangle = |\Psi^+\rangle \). For this reconstruction, we did not consider accidental substraction. In this case, the predicted maximum value of \( S \) is \( \text{tr}(S\rho_{\text{exp}}) = 2.488 \). The actual measured values of \( S \) (see Table I) are very close to this value. By considering accidental subtraction, the fidelity of the reconstructed state was \( 0.97 \pm 0.02 \).

Conclusions.—We have presented the first violation of the Bell-CHSH inequality using energy-time entangled photons which is free of the postselection loophole reported by Aerts et al. [27] and which affects all previous Bell experiments using energy-time or time-bin entangled photons [13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26].

The experiment is not free of the locality and detection loopholes. Increasing the length of the interferometers and adding a fast switch between the local setting is required to escape from the locality loophole. The required values and the stability problems were discussed in [28]. Increasing the efficiency of the photodetectors or replacing low-energy photons by easily detectable particles, like massive particles, is required to escape from the detection loophole. However, the experiment provides a proof of principle that the scheme proposed in [28] is suitable for a testing the Bell-CHSH inequality with energy-time entangled particles. Further developments might include nonphotic versions of the setup and larger-scale implementations.

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[1] J. S. Bell, Physics (Long Island City, N.Y.) 1, 195 (1964).
[2] J. F. Clauser, M. A. Horne, A. Shimony, and R. A. Holt, Phys. Rev. Lett. 23, 880 (1969).
[3] J. Barrett, L. Hardy, and A. Kent Phys. Rev. Lett. 95, 010503 (2005).
[4] H. Buhrman, R. Cleve, S. Massar, and R. de Wolf, Rev. Mod. Phys.; arXiv:0907.3584.
[5] S. Pironio et al., arXiv:0911.3427.
[6] S. J. Freedman and J. F. Clauser, Phys. Rev. Lett. 28, 938 (1972).
[7] A. Aspect, J. Dalibard, and G. Roger, Phys. Rev. Lett. 49, 1804 (1982).
[8] G. Weihs, T. Jennewein, C. Simon, H. Weinfurter, and A. Zeilinger, Phys. Rev. Lett. 81, 5039 (1998).
[9] M. A. Rowe, D. Kielpinski, V. Meyer, C. A. Sackett, W. M. Itano, C. Monroe, and D. J. Wineland, Nature (London) 409, 791 (2001).
[10] D. N. Matsukevich, P. Maunz, D. L. Moehring, S. Olmschenk, and C. Monroe, Phys. Rev. Lett. 100, 150404 (2008).
[11] A. Aspect, Nature (London) 398, 189 (1999).
[12] E. Santos, Phys. Rev. Lett. 66, 1388 (1991).
[13] J. D. Franson, Phys. Rev. Lett. 62, 2205 (1989).
[14] J. Brendel, N. Gisin, W. Tittel, and H. Zbinden, Phys. Rev. Lett. 82, 2594 (1999).
[15] P. G. Kwiat, W. A. Vareka, C. K. Hong, H. Nathel, and R. Y. Chiao, Phys. Rev. A 41, 2910 (1990).
[16] Z. Y. Ou, X. Y. Zou, L. J. Wang, and L. Mandel, Phys. Rev. Lett. 65, 321 (1990).
[17] J. Brendel, E. Mohler, and W. Martienssen, Phys. Rev. Lett. 66, 1142 (1991).
[18] P. G. Kwiat, A. M. Steinberg, and R. Y. Chiao, Phys. Rev. A 47, R2472 (1993).
[19] P. R. Tapster, J. G. Rarity, and P. C. M. Owens, Phys. Rev. Lett. 73, 1923 (1994).
[20] W. Tittel, J. Brendel, H. Zbinden, and N. Gisin, Phys. Rev. Lett. 81, 3563 (1998).
[21] W. Tittel, J. Brendel, H. Zbinden, and N. Gisin, Phys. Rev. Lett. 84, 4737 (2000).
[22] G. Ribordy, J. Brendel, J.-D. Gautier, N. Gisin, and H. Zbinden, Phys. Rev. A 63, 012309 (2000).
[23] R. T. Thew, A. Acín, H. Zbinden, and N. Gisin, Phys. Rev. Lett. 93, 010503 (2004).
[24] I. Marcikic, H. de Riedmatten, W. Tittel, H. Zbinden, M. Legré, and N. Gisin, Phys. Rev. Lett. 93, 180502 (2004).
[25] D. Salart, A. Baas, J. A. W. van Houwelingen, N. Gisin, and H. Zbinden, Phys. Rev. Lett. 100, 220404 (2008).
[26] D. Salart, A. Baas, C. Branciard, N. Gisin, and H. Zbinden, Nature (London) 454, 861 (2008).
[27] S. Aerts, P. G. Kwiat, J.-A. Larsson, and M. Žukowski, Phys. Rev. Lett. 83, 2872 (1999); 86, 1909 (2001).
[28] A. Cabello, A. Rossi, G. Vallone, F. De Martini, and P. Mataloni, Phys. Rev. Lett. 102, 040401 (2009).
[29] J. D. Franson, Phys. Rev. A 80, 032119 (2009).
[30] C. Gueingk and K. Hornberger, Phys. Rev. Lett. 101, 260503 (2008).
[31] D. Frustaglia and A. Cabello, Phys. Rev. B 80, 201312(R) (2009).
[32] G. Vallone, P. Mataloni, and A. Cabello, arXiv:0912.4419.
[33] C. K. Hong and L. Mandel, Phys. Rev. A 31, 2409 (1984).
[34] D. C. Burnham and D. L. Weinberg, Phys. Rev. Lett. 25, 1805 (1972).
[35] M. A. Horne, A. Shimony, and A. Zeilinger, Phys. Rev. Lett. 62, 2209 (1989).
[36] D. F. V. James, P. G. Kwiat, W. J. Munro, and A. G. White, Phys. Rev. A 64, 052312 (2001).
[37] R. Jozsa, J. Mod. Opt. 41, 2315 (1994).