Theoretical and Experimental Investigations of Spatio-Temporally Modulated Metasurfaces with Spatial Discretization

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A dual-polarized, spatio-temporally modulated metasurface is designed and measured at X-band frequencies. Each column of subwavelength unit cells comprising the metasurface can be independently biased, to provide a tunable reflection phase over a range of 330°. In this work, the bias waveform applied to adjacent columns is staggered in time to realize a discretized traveling-wave modulation of the metasurface. An analytic model for the metasurface is presented that accounts for its discretized spatial modulation. The analysis considers a finite unit cell size and thus provides increased accuracy over earlier analysis techniques for space-time metasurfaces that commonly assume continuous spatial modulation. Theoretical and experimental results show that for electrically-large spatial modulation periods the space-time metasurface allows simultaneous frequency translation and deflection. When the spatial modulation period on the metasurface is electrically small, new physical phenomena such as subharmonic frequency translation can be realized. When the spatial modulation period of the metasurface is wavelength-scale, simultaneous subharmonic frequency translation and deflection can be achieved. For certain incident angles, retroreflective subharmonic frequency translation is demonstrated.

I. INTRODUCTION

Metasurfaces are two dimensional structures textured at a subwavelength scale to achieve tailored control of electromagnetic waves. Developments in tunable electronic components have allowed dynamic control over the electromagnetic properties of metasurfaces. Devices such as varactors, transistors and MEMS [1–3] in addition to 2D and phase change materials [4–6] can be integrated into metasurfaces to tune their electric, magnetic and magneto-electric responses. Often, the properties of a metasurface are spatially modulated to shape electromagnetic wavefronts and achieve focusing, beam-steering, and polarization control [7–10]. By incorporating tunable elements into their design, the properties of metasurfaces can also be modulated in time [11–13]. While spatial modulation redistributes the plane-wave spectrum of the scattered field, temporal modulation provides control over the frequency spectrum. Applying both spatial and temporal variation is known as spatio-temporal modulation, and has recently been applied to metasurfaces [14–22]. Space-time modulation can simultaneously allow frequency conversion and beam steering and shaping. It can also be used to break Lorentz reciprocity and enable magnetless nonreciprocal devices such as gyrators, circulators and isolators [23–26].

Spatio-temporally modulated structures are typically analyzed and designed as continuous surfaces. That is, the unit cell size of the physical structure is assumed to be deeply subwavelength. However, accounting for the discretization of the unit cell provides increased accuracy and can yield useful effects which are not predicted in the continuum limit. For example, when the spatial period of the modulation is smaller than a wavelength, subharmonic frequency translation can be achieved in the specular direction. In this case, scattered waves can radiate at a higher order frequency harmonic determined by the number of unit cells in a subwavelength spatial period. Such a behavior does not arise from a continuous analysis of sub-wavelength modulation periodicity, since higher-order spatial harmonics introduced by the spatial discretization are not considered.

In this paper, we demonstrate a spatio-temporally modulated reflective metasurface. The incident wave (dark red) can be reflected (light blue) at an angle and frequency determined by the space-time dependence of the bias voltage.

FIG. 1. A spatio-temporally modulated reflective metasurface. The incident wave (dark red) can be reflected (light blue) at an angle and frequency determined by the space-time dependence of the bias voltage.
sulting in Doppler-like (serrodyne) frequency translation. Structures of a similar design have been subsequently reported in [27].

Here, we consider a discretized, traveling-wave modulation in which the capacitance variation of adjacent columns is staggered in time. This modulation scheme is reminiscent of N-path networks that have received significant attention in the circuits community [28–33] as of late. In the context of electronic circuits, an N-path network consists of a set of linear, periodically time-varying (LPTV) signal paths connected to a common input and output. Each path includes at least one time-varying circuit component. The time-modulation of adjacent LPTV paths is staggered in time by \( T_p/N \), where \( T_p \) is the modulation period, and \( N \) is the number of paths in the network. The N-path configuration suppresses certain harmonic mixing products. Specifically, for a modulation frequency \( f_p = 1/T_p \) and excitation frequency \( f_0 \), the only harmonics present at the input and output are those at \( f = f_0 + rNf_p \), where \( r \in \mathbb{Z} \) [28–33]. N-path networks have attracted widespread attention in the circuits community due to their filtering capabilities [31, 34, 35] and as a method to break time-reversal symmetry and realize non-reciprocal devices such as circulators [36, 37], and isolators [26, 30]. The non-reciprocal behavior of N-path networks can find various applications in full duplex wireless communication and radar. Non-reciprocal devices are also needed in optical fiber communications and the protection of sensitive electronic equipment from high-power microwaves.

Periodic space-time modulation of a metasurface imparts tangential momenta (an impressed wavenumber) onto each frequency harmonic of the scattered field. The tangential wavenumber of the modulation is given by the spatial modulation period. Provided that each unit cell is sub-wavelength, the behavior of the metasurface can be divided into three regimes based on whether the spatial period of the modulation is (1) electrically small (much smaller than a wavelength), (2) electrically large (much greater than a wavelength), or (3) on the order of a wavelength. When the spatial modulation period is electrically small, the columns of the metasurface appear collocated and the behavior approaches that of an N-path network. The staggered modulation scheme in this case results in harmonic cancellation which can be exploited to achieve subharmonic frequency translation. Reflected harmonics at frequencies \( f = f_0 + rNf_p \) (\( r \in \mathbb{Z} \)) correspond to propagating wavenumbers, while the remaining frequencies are evanescent. In contrast, when the spatial modulation period is much larger than a wavelength, the capacitance variation between adjacent sub-wavelength cells is reduced. In this limit, the discretized metasurface approaches a continuous spatiotemporally modulated structure: the incident wave undergoes frequency translation [12, 13] and angular deflection. Finally, when the spatial period is comparable to the wavelength, both subharmonic frequency translation and angular deflection can be simultaneously achieved. At certain incident angles, the metasurface can perform retroreflective subharmonic frequency translation.

In this paper, the proposed metasurface design will be explored both in theory and experiment. In Section II, the semi-analytical procedure for computing the response of the presented metasurface is discussed. This includes the homogenization of each unit cell, followed by a treatment of both time and space-time variation. This procedure is carried out for various modulation schemes in Section III. Specifically, effects such as specular subharmonic frequency translation, deflec-tive/retroreflective serrodyne frequency translation, and deflec-tive/retroreflective subharmonic frequency translation are examined. The results of the theoretical study are then validated experimentally in Section IV.

II. ANALYSIS OF THE TEMPORALLY AND
SPATIO-TEMPORALLY MODULATED
META SURFACE

The spatio-temporally modulated metasurface is depicted in Fig. 1. It is a reflective, electrically-tunable impedance surface [38], consisting of a capacitive sheet above a grounded dielectric substrate. The capacitive sheet is realized as an array of metallic patches interconnected by varactor diodes. It can be modulated in both space and time with a bias signal that is applied through the metallic vias that penetrate the substrate.

A unit cell of the designed metasurface is shown in Fig. 2a. The varactor diodes connecting the metallic patches are biased through the vias located at the edges of the unit cells, while the the via at the central patch is connected to ground. The remainder of the biasing network is shielded behind the ground plane. The biasing network and diode orientations allow the reflection phase of the metasurface to be independently tuned for two orthogonal (TE and TM) polarizations. Bias waveforms \( V_{bias}(t, x) \) and \( V'_{bias}(t, x) \), as shown in Fig. 1, control the sheet capacitance for the two orthogonal polarizations. A detailed description of the fabrication and biasing network are provided in Section IV. A cross section of the metasurface is shown in Fig. 3 under TE and TM excitations. The biasing vias can be seen perforating the

![Fig. 2. (a) Unit cell of the dual-polarized, spatio-temporally modulated metasurface. (b) Circuit model for each polarization.](image-url)
dielectric substrate.

Here, we derive a semi-analytical procedure for computing the response of the metasurface shown in Fig. 1, with a discretized traveling-wave modulation. In Section II A, the metasurface unit cells are homogenized and represented by an equivalent circuit model. The semi-analytical procedure for obtaining the scattered fields in the presence of time-modulation alone is examined in Section II B. Building upon this framework, a procedure for computing the scattered fields produced by the space-time modulated metasurface is then presented in Section II C.

A. Homogenization of the spatio-temporally modulated metasurface

In the analysis that follows, the unit cells of the metasurface are homogenized. Within each unit cell, the metallic patches interconnected by varactor diodes will be treated as a capacitive sheet. This sheet can be modulated in time, independently of adjacent unit cells. The dielectric substrate perforated by vias ($-l < z < 0$), will be treated as a uniaxial anisotropic material, with a relative permittivity tensor [39]:

$$\mathbf{\epsilon}_r = \begin{pmatrix} \epsilon_h & 0 & 0 \\ 0 & \epsilon_h & 0 \\ 0 & 0 & \epsilon_{zz} \end{pmatrix},$$

where $\epsilon_h$ is the relative permittivity of the host medium and $\epsilon_{zz}$ is the effective relative permittivity along the vias. Since the metasurface is electrically thin at the operating frequency of 10 GHz ($l = 0.016\lambda = 0.508$ mm), a local model can be used to describe the wire medium [39]:

$$\epsilon_{zz} = \epsilon_h(1 - \frac{k_p^2}{k_0^2\epsilon_h}),$$

where $k_p = 541.81$ rad·m$^{-1}$ is the plasma wavenumber of the wire medium extracted from a full-wave simulation of the unit cell shown in Fig. 2a, and $k_0$ is the free-space wavenumber of the incident wave. The anisotropic substrate supports TE (ordinary mode) and TM (extraordinary mode) polarizations. The normal wavenumber for each polarization in the substrate can be written as,

$$k_{sz}^{TE} = \sqrt{k_0^2\epsilon_h - k_p^2},$$
$$k_{sz}^{TM} = \sqrt{k_0^2\epsilon_h - k_p^2\epsilon_h/\epsilon_{zz}}, \quad \text{where} \quad k_x \text{ is the tangential wavenumber of the incident wave.}$$

Each unit cell can be modeled with the shunt resonator depicted in Fig. 2b. The circuit model consists of a tunable capacitance (representing the capacitive sheet) backed by shorted transmission-line section (representing the grounded dielectric substrate) that acts as an inductance. As a result, the bias voltage applied to the varactor diodes can be used to tune the reflection phase. The phase range of this topology is $2\pi - \Delta \phi$, where $\Delta \phi$ is the round trip phase delay through the substrate. Details on the phase range of the realized metasurface are provided in Section IV. In this paper, two different reflection phase waveforms are considered. A sawtooth reflection phase with respect to time is studied, which allows serrodyne frequency translation [12, 13], as well as a sinusoidal reflection phase with respect to time.

As mentioned earlier, each column of unit cells can be biased independently, allowing for space-time modulation along a single ($x$) axis. As a result, the homogenized model consists of capacitive strips whose widths are given by the unit cell size $d_0 = \lambda_0/\lambda = 6$ mm, where $\lambda_0$ is the wavelength in free space at 10 GHz. The capacitance seen by each polarization can be controlled independently and is uniform over the strip.

B. Time modulation of the metasurface: serrodyne frequency translation

It is instructive to first consider the analysis of the metasurface when it is uniformly biased across all unit cells. In this case, there is no spatial variation in the homogenized model and the reflected power will will spread into discrete frequency harmonics due to the periodic time variation of the reflection phase. The tangential incident and reflected fields above the metasurface ($z = 0^+$) can be expanded into frequency harmonics as

$$E^{\text{inc}}_t = \sum_{m=-\infty}^{\infty} V_m e^{j\omega m t - k_{sx} z},$$
$$E^{\text{ref}}_t = \sum_{m=-\infty}^{\infty} V_m e^{j\omega m t - k_{sx} z},$$
$$H^{\text{inc}}_t = \sum_{m=-\infty}^{\infty} I_m e^{j\omega m t - k_{sx} z},$$
$$H^{\text{ref}}_t = \sum_{m=-\infty}^{\infty} I_m e^{j\omega m t - k_{sx} z},$$

where $\omega_0$ is the radial frequency of the incident wave, $\omega_p$ is the radial frequency of the modulation, and $k_x$ is the tangential wavenumber of the incident wave. The incident and reflected electric field harmonics can be written
FIG. 4. (a) Calculated capacitance modulation of the time-modulated metasurface for TE polarization. (b) Analytical reflection spectrum of the homogenized, lossless, time-modulated metasurface for TE polarization. (c) Calculated capacitance modulation of the time-modulated metasurface for TM polarization. (d) Analytical reflection spectrum of the homogenized, lossless, time-modulated metasurface for TM polarization.

in vector form as \( \mathbf{V}^{\text{inc}} \) and \( \mathbf{V}^{\text{ref}} \) respectively. The vector \( \mathbf{V}^{\text{inc}} \) represents the incident tangential electric field and contains only a single entry \( (\mathbf{V}^{\text{inc}})_m = V^{\text{inc}}_m \delta_m \) since the incident field is monochromatic. The vector \( \mathbf{V}^{\text{ref}} \) contains all the reflected tangential electric field coefficients \( V_m \). Based on the detailed derivation in Supplemental Material I and II, the reflected electric field can be calculated for each polarization,

\[
\mathbf{V}^{\text{ref}} = (\mathbf{Y}^{TX} + \mathbf{Y}^{TX}_0)^{-1}(\mathbf{Y}^{TX}_0 - \mathbf{Y}^{TX})\mathbf{V}^{\text{inc}}, \quad (9)
\]

where the superscript “X” is “E” for TE polarized waves and “M” for TM polarized waves. \( \mathbf{Y}^{TX} \) is the free-space tangential wave admittance matrix. It is a diagonal matrix containing entries of the free-space admittances at the corresponding frequency harmonics. \( \mathbf{Y}^{TX}_0 \) is the input admittance matrix of the time modulated metasurface. It is not a diagonal matrix since the time-modulated capacitive sheet introduces coupling between different harmonics.

This analysis procedure can be used to predict the scattered field for arbitrary time-periodic modulating waveforms. Suppose the reflection phase is modulated by a sawtooth waveform. In this case, serrodyne frequency translation is expected \[12\]. The frequency of the incident wave is \( f_0 = 10 \) GHz and the modulation frequency is \( f_p = 25 \) kHz. The incident wave impinges on the metasurface at an oblique angle of 25°. The capacitance modulation needed to upconvert the wave to \( f_0 + f_p \) is calculated in Supplemental Material I, and shown in Fig. 4 for each polarization. The calculation includes 141 temporal harmonics for the field expansion and 101 temporal harmonics for the capacitance modulation. The reflected spectra for the two orthogonal polarizations reveal a Doppler shift to a frequency of \( f_0 + f_p \). For TE polarization, a 0.107 dB conversion loss and 22.83 dB sideband suppression are achieved. For TM polarization, a 0.125 dB conversion loss and 21.64 dB sideband suppression are achieved. As mentioned earlier, the equivalent circuit model of the unit cell provides a phase range that is slightly less than 2\( \pi \) (1.6\( \pi \) for TE polarization and 1.54\( \pi \) for TM polarization used in the analysis), resulting in undesired sidebands. For TM polarization, the reflection phase range is slightly smaller than TE at the oblique angle of 25°, resulting a slightly higher conversion loss.

C. Space-time modulation of the metasurface

A space-time gradient can be applied to the metasurface by introducing a time delay between the capacitance modulation applied to adjacent columns. This staggered modulation scheme is shown in Fig. 5. It provides a discretized travelling wave (N-path) modulation. The capacitance modulation on each path is chosen to either produce a sawtooth or sinusoidal reflection phase with respect to time. In this staggered modulation scheme, there are \( N \) columns of subwavelength unit cells within one spatial modulation period \( d \). This impresses a modulation wavenumber \( \beta_p = 2\pi/d \) onto the metasurface. \( N \) adjacent columns of the metasurface are modulated with bias signals staggered in time by an interval \( T_p/N \), where \( T_p = 1/f_p \) is the temporal modulation period. One can view each column as a path in a N-path network. The capacitance on each path \( v \) is related to that of the adjacent
path by a time delay,

\[ C^\nu(t, x) = C^{\nu-1}\left(t - \frac{T_p}{N}, x - \frac{d}{N}\right). \]  

(10)

Here \( C^\nu(t, x) \) is a pulse function in space, and periodic function in time (see Supplemental Material III).

In contrast to an N-path circuit, the paths (columns of unit cells) are not connected to a common input and output. Instead, each path is displaced by a subwavelength distance \( d_0 = 6 \text{ mm} \) \((d_0 = \lambda_0/3)\) from its adjacent paths. Examples of 2- and 3-path spatio-temporal modulation schemes are shown in Fig. 6. At any given time, the spatial variation of the reflection phase is a discretized sawtooth (blazed grating) ranging from 0 to approximately \( 2\pi \) over a period \( d = N d_0 \).

As shown in Supplemental Materials III and IV, the capacitance relationship given by Eq. (10) allows the spatio-temporally modulated sheet capacitance to be expanded in the following form,

\[ C(t, x) = \sum_{r = -\infty}^{\infty} \sum_{q = -\infty}^{\infty} C_{rq} e^{i q (\omega_0 t - k_x x)} e^{-i r \beta_d x}, \]  

(11)

where \( \beta_p = 2\pi/d \) is the modulation wavenumber, and \( \beta_d = 2\pi/d_0 = N \beta_p \) is an additional wavenumber which results from the discretization of the spatial modulation into paths (unit cells). The summation over \( r \) accounts for the discontinuity in capacitance at the boundary of each path as well as the microscopic variation of capacitance within the paths (which in this case is constant). The summation over \( q \) accounts for the macroscopic capacitance variation over one spatial modulation period \( d \). The sheet capacitance of each path, is capable of generating a staggered sawtooth reflection phase in time, as shown in Fig. 7.

The N-path symmetry of the system establishes a relation between the fields on adjacent paths [28]. The total electric field must satisfy

\[ E(t, x, y, z) = e^{i \left( \omega_0 t - k_x x \right)} E(t - \frac{T_p}{N}, x - \frac{d}{N}, y, z). \]  

(12)

Eq. (12) is used in Supplemental Materials IV to show that the fields can be expanded in terms of a modified Fourier series. At \( z = 0^+ \), the tangential incident and reflected fields on the metasurface can be expressed as

\[ E_{\text{inc}}^t = V_{00}^\text{inc} e^{i (\omega_0 t - k_x x)}, \]  

(13)

\[ E_{\text{ref}}^t = \sum_{r, q = -M}^{M} V_{rq}^\text{ref} e^{-i r \beta_d x} e^{i q (\omega_t - \beta_p z)} e^{i (\omega_0 t - k_x x)}, \]  

(14)

\[ H_{\text{inc}}^i = I_{00}^\text{inc} e^{i (\omega_0 t - k_x x)}, \]  

(15)

\[ H_{\text{ref}}^i = \sum_{r, q = -M}^{M} I_{rq}^\text{ref} e^{-i r \beta_d x} e^{i q (\omega_t - \beta_p z)} e^{i (\omega_0 t - k_x x)}. \]  

(16)

In Eq. (13-16), spatio-temporal harmonic pair \((r, q)\) of the electromagnetic field on the surface has a tangential wavenumber

\[ k_{x rq} = q \beta_p + r \beta_d + k_x = (q + r N) \beta_p + k_x, \]  

(17)

and a corresponding radial frequency

\[ \omega_{rq} = \omega_0 + q \omega_p, \]  

(18)

where \( N \) is the number of paths within a spatial period of the metasurface. It can be seen that the staggered modulation between paths impresses a tangential wavenumber of \( q \beta_p \) onto the \( q^\text{th} \) harmonic frequency harmonic. The reflected angle of each harmonic pair is equal to

\[ \theta_{rq} = \arcsin \frac{k_{x rq}}{\omega_{rq}/c} = \arcsin \left( \frac{(q + r N) \beta_p + k_0 \sin \theta_i}{\omega_{rq}/c} \right). \]  

(19)
The coefficients of the incident/reflected electric and magnetic fields are related by the free-space tangential wave admittance defined for each spatio-temporal harmonic pair. Meanwhile, the spatio-temporally modulated sheet capacitance provides coupling between different harmonic pairs.

To solve for the scattered field, the incident and reflected tangential electric field harmonics are once more organized into vectors, \( \mathbf{V}^{\text{inc}} \) and \( \mathbf{V}^{\text{ref}} \). Each entry corresponds to a unique spatio-temporal harmonic pair \((r, q)\). The reflected electric field can then be calculated for each polarization using Eq. \((9)\). A detailed derivation of the entries of the metasurface admittance matrix \( \mathbf{Y}_TX \) and the free-space tangential admittance matrix \( \mathbf{Y}_0TX \) is provided in Supplemental Materials IV.

Note that when the unit cell size is infinitesimally small \((d_0 \ll d)\), the variation of field across a unit cell is negligible. Therefore the harmonic pairs that remain are only those with \( r = 0 \). The capacitance modulation of the metasurface can thus be seen as continuous,

\[
C(t, x) = \sum_{q = -\infty}^{\infty} C_q e^{i2\pi \omega_q t - \beta_q x}.
\]

For such a modulation, the metasurface supports harmonics at frequency \( f_0 + qf_p \), with a corresponding wavenumber \( k_x + q\beta_p \). Note that \((20)\) is of the form of a traveling wave, \( C(t, x) = C(t - x/v_p) \), where \( v_p = \omega_p/\beta_p \). This model case the continuum limit, in which the spatial discretization of the traveling wave bias can be neglected. With the modulation waveform (sawtooth reflection phase in time) shown in Fig. 4a and Fig. 4c, the metasurface converts the incident wave at \((f_0, k_x)\) to \((f_0 + f_p, k_x + \beta_p)\), as shown in Fig. 8b. When \( k_x + \beta_p > (\omega_0 + \omega_p)/c \), the metasurface can convert an incident wave to a surface wave (as shown in Fig. 8a), provided that the corresponding surface wave is supported by the metasurface. However, when the corresponding surface wave is not supported, the metasurface reflects all the power back in the specular direction at the same frequency \( f_0 \).

More generally, the transverse resonance condition can be used to identify surface modes supported by the spatially discretized metasurface. For harmonic pairs with tangential wavenumbers larger than free space \((k_{x,q} > \omega_q/c)\), the transverse resonance condition can be used to judge if the corresponding surface waves are supported.

\[
\det (\mathbf{Y}_TX + \mathbf{Y}_0TX) = 0,
\]

Solving Eq. \((21)\) yields the \( \omega_0 - k_x \) dispersion relationship for the supported surface wave. Note that when a surface wave is supported, the reflection coefficient \( V_{\text{ref}}/V_{\text{inc}} \) in Eq. \((9)\) diverges. This is not the case for the incident angle and path number \( N \) combinations considered in this paper. Thus, for all the proceeding examples presented in this paper, a surface wave is not supported by the metasurface.

For our metasurface, the unit cell size is chosen to be \( d_0 = \lambda_0/5 \). Since the unit cell size of the metasurface is fixed, the total spatial period \( d = Nd_0 \) can be controlled by changing \( N \), the number of paths. This enables the same metasurface to achieve different functions depending on the number of paths included in a spatial period. If the modulation period \( d \) is electrically small \((N \text{ is small})\), then the spatial modulation wavenumber \( \beta_p \) is large. As depicted in Fig. 8a, this can lead to a number of higher order harmonics existing outside the light cone. Since the spatial modulation period is electrically small, the paths can be viewed as collocated and a \( N \)-path circuit model can be used to approximate the physical structure. The equivalent circuit model of the spatio-temporally modulated metasurface for \( d \ll \lambda_0 \) is depicted in Fig. 9. If the spatio-temporally modulated metasurface does not support surfaces waves at the operating frequency, power is only coupled to radiating harmonics: those within the light cone. Based on Eq.\((17)\), the radiated harmonics are those with:

\[
q + rN = 0.
\]

Since \( r \in \mathbb{Z} \), Eq. \((22)\) implies that propagating harmonics correspond to \( q = 0, \pm N, \pm 2N, \ldots \). Therefore, the radiated reflected wave only contains frequency harmonics at \( f_0 + rNf_p \), where \( r \in \mathbb{Z} \). This phenomena can only be observed when the spatial discretization of the metasurface is considered. In the continuum limit, sub-wavelength
spatial modulation results in specular reflection at the same frequency as the incident wave. However, the spatial discretization introduces additional spatial harmonics (the summation over \( r \) in (13-16)) that can couple to the incident wave. As a result, the metasurface can achieve subharmonic frequency translation. Note that the tangential wavenumbers of the radiated subharmonic mixing terms are all equal to that of the incident tangential wavenumber (since \( k_{rxq} = k_x \), when \( q + N r = 0 \)). With the capacitance modulation shown in Fig. 4a and 4c, the sawtooth reflection phase on each path enables the metasurface to upconvert the frequency to the first propagating harmonic pair. In this case, the metasurface performs subharmonic frequency translation from \( f_0 \) to \( f_0 + N f_p \).

When the modulation period \( d \) is electrically large (\( N \) is large), the spatial modulation wavenumber \( \beta_p \) is small, as depicted in Fig. 8b. When \( N \) is a very large value, according to Eq. (S.39) in the Supplemental Material, the capacitance coefficient \( C_{rq} \) is zero for \( r \neq 0 \). For this case, the field variation across each unit cell is small, and the capacitance modulation waveform is simplified to the continuum limit given by Eq. (20). In other words, the metasurface shows a similar performance to one with an infinitely small unit cell size. Serradyne frequency translation to a deflected angle can be achieved using the sawtooth waveform given in Fig. 4.

In addition, when the modulation period \( d \) is on the order of a wavelength, the metasurface can simultaneously perform subharmonic frequency translation and angular deflection. The deflected angle of the harmonic pair of interest is given by Eq. (19). Setting \( \theta_{rxq} = -\theta_i \) in Eq. (19) yields an expression for the incidence angles at which retroreflection occurs for a particular spatio-temporal harmonic. The deflective and retroreflective behavior of the metasurface is showcased in Section III for various scenarios.

### III. SCATTERING FROM A SPACE-TIME MODULATED METASURFACE FOR DIFFERENT SPATIAL MODULATION PERIODS

Computed results are given here for various spatio-temporal modulation cases. In Section III A, space-time modulation schemes are designed to achieve subharmonic frequency translation. The spatial modulation period is kept electrically small. In Section III B, the spatial modulation period is electrically large such that beam deflection and frequency translation can be achieved simultaneously. Finally, in Section III C, the spatially modulation period is on the order of a wavelength, allowing simultaneous retroreflection and subharmonic frequency translation. For each of the cases that follow, the conversion loss and sideband suppression for the desired frequency harmonic at a given observation angle are provided in Table I. The table will be referred to throughout this section. In all of the cases studied, the incident signal frequency is \( f_0 = 10 \) GHz. The modulation frequency, \( f_p = 25 \) kHz, which is the maximum frequency which could be experimentally validated using the available equipment (see Section IV). For each angle of incidence, the capacitance modulation is calculated based on Eq. (S.4) to achieve the desired time-varying reflection phase. Unless specifically stated otherwise, the reflection phase of each column (path) is a sawtooth function in time. For both polarizations, the field is expanded into \( 141 \times 141 \) harmonic pairs. The temporal capacitance modulation on each path is truncated to 101 temporal harmonics.

#### A. Small spatial modulation period (\( |k_x + \beta_p| > k_0 \))

In this section, electrically small spatial modulation periods (\( |k_x + \beta_p| > k_0 \)) are considered. In this regime, both the +1 \((k_x + \beta_p)\) and -1 \((k_x - \beta_p)\) spatial harmonics are outside of the light cone. Since the unit cell size of the metasurface is fixed to \( d_0 = \lambda_0/5 \), 2- and 3-path modulation \( (N = 2, 3) \) are chosen to satisfy the small period condition. The incident wave is chosen to impinge on the metasurface with an oblique angle of 25°. The modulation schemes for the 2-path \( (N = 2) \) and 3-path \( (N = 3) \) examples are shown in Fig. 6 and Fig. 7. As explained in section II C, the metasurface performs subharmonic frequency translation at the specular angle.
The spectra for both polarizations, shown in Fig. 10, clearly demonstrate subharmonic frequency translation, where reflected harmonics are only radiated at \( f = f + rNf_p \) with \( r \in \mathbb{Z} \). Doppler-like frequency translations are observed for both polarizations, where the dominant propagating reflected wave is at frequency \( f_0 + Nf_p \).

The conversion loss and sideband suppression for both polarizations using 2-path and 3-path modulation are provided in examples 1 and 2 of Table I. As mentioned earlier, the unit cell provides a phase range that is slightly smaller than \( 2\pi \), resulting in conversion loss and undesired sidebands. It can be seen that, as the converted frequency harmonic (which is equal to the path number \( N \) in this case) is increased, the conversion loss increases and the sideband suppression decreases. This is because the \( N \)-path metasurface upconverts the frequency to the first propagating harmonic pair. The higher the upconverted frequency, the longer this process takes and the larger the conversion loss due to the formation of sidebands that results from the imperfect reflection phase range.

B. Large spatial modulation period \( (|k_x + \beta_p| < k_0) \)

In this section, we consider two cases where the modulation period \( d \) is larger than the free space wavelength \( \lambda_0 \) (\( N \) is large). In this regime, both the +1 \( (k_x + \beta_p) \) and −1 \( (k_x - \beta_p) \) spatial harmonics are inside the light cone. In addition, when \( N \) is a large value, the harmonic pairs \( (r, q) \) with \( r = 0 \) dominate (Eq. (S.38)). In the first case, the metasurface exhibits serrodyne frequency translation to a deflected angle. In the second case, the incident angle is specifically chosen to achieve serrodyne frequency translation in retroreflection.

1. Case 2: Deflective Serrodyne frequency translation

First, let us consider the example shown in Fig. 12a, where a wave is incident at an angle \( \theta_1 = 25^\circ \) and the number of paths is large, \( N = 20 \). From Eq. (17), the tangential wavenumbers of the reflected harmonic pairs

\[
f_0, k_x = f_0 + Nf_p, k_x
\]

The spectra for both polarizations, shown in Fig. 10, clearly demonstrate subharmonic frequency translation, where reflected harmonics are only radiated at \( f = f + rNf_p \) with \( r \in \mathbb{Z} \). Doppler-like frequency translations are observed for both polarizations, where the dominant propagating reflected wave is at frequency \( f_0 + Nf_p \).

The conversion loss and sideband suppression for both polarizations using 2-path and 3-path modulation are provided in examples 1 and 2 of Table I. As mentioned earlier, the unit cell provides a phase range that is slightly smaller than \( 2\pi \), resulting in conversion loss and undesired sidebands. It can be seen that, as the converted frequency harmonic (which is equal to the path number \( N \) in this case) is increased, the conversion loss increases and the sideband suppression decreases. This is because the \( N \)-path metasurface upconverts the frequency to the first propagating harmonic pair. The higher the upconverted frequency, the longer this process takes and the larger the conversion loss due to the formation of sidebands that results from the imperfect reflection phase range.

B. Large spatial modulation period \( (|k_x + \beta_p| < k_0) \)

In this section, we consider two cases where the modulation period \( d \) is larger than the free space wavelength \( \lambda_0 \) (\( N \) is large). In this regime, both the +1 \( (k_x + \beta_p) \) and −1 \( (k_x - \beta_p) \) spatial harmonics are inside the light cone. In addition, when \( N \) is a large value, the harmonic pairs \( (r, q) \) with \( r = 0 \) dominate (Eq. (S.38)). In the first case, the metasurface exhibits serrodyne frequency translation to a deflected angle. In the second case, the incident angle is specifically chosen to achieve serrodyne frequency translation in retroreflection.

1. Case 2: Deflective Serrodyne frequency translation

First, let us consider the example shown in Fig. 12a, where a wave is incident at an angle \( \theta_1 = 25^\circ \) and the number of paths is large, \( N = 20 \). From Eq. (17), the tangential wavenumbers of the reflected harmonic pairs

\[
f_0, k_x = f_0 + Nf_p, k_x
\]
Here, the number of paths is chosen to be \( N = 20 \), and the retroreflection angle is calculated to be \( \theta_i = -7.18^\circ \).

\[
\begin{align*}
\text{FIG. 13. Analytical reflection spectrum of the homogenized, lossless, spatio-temporally modulated metasurface, with an incident angle of 25°. (a) 20-path (\( N = 20 \)) modulation for TE polarization. (b) 20-path (\( N = 20 \)) modulation for TM polarization.}
\end{align*}
\]

\[
\begin{align*}
\text{FIG. 14. Analytical reflection spectrum of the homogenized, lossless, spatio-temporally modulated metasurface, with an incident angle of -42°. (a) 20-path (\( N = 20 \)) modulation for TE polarization. (b) 20-path (\( N = 20 \)) modulation for TM polarization.}
\end{align*}
\]

are given by

\[
k_{xq}|_{r=0} = q\beta_p + k_0 \sin(25^\circ) = \frac{q}{4} k_0 + k_0 \sin(25^\circ),
\]

given that \( d = 20d_0 = 4\lambda_0 \). The harmonics located inside the light cone (propagating harmonics) are those with \( q = 0, \pm 1, \pm 2, -3, -4, -5 \). For the capacitance variation shown in Fig. 4a and Fig. 4c, the metasurface acts as a serrodyne frequency translater. It upconverts the incident wave to the harmonic pair \((r = 0, q = 1)\) with frequency \( f = f_0 + f_p \). Note that in this case, each radiated harmonic has its own tangential wavenumber, and thus reflects at a different angle given by Eq. (19). The harmonic of interest \((f_0 + f_p)\) reflects to \( \theta_2 = 42^\circ \), as shown in Fig. 12a. The reflected spectra for both polarizations are given in Fig. 13. The conversion loss and sideband suppression for both polarizations are provided in example 3 of Table I.

Let’s consider another example (shown in Fig. 12b)

\[
\begin{align*}
\text{where the spatio-temporal modulation of the metasurface and incident frequency are kept the same, but the incident and reflected angles are swapped. The incident angle is } \theta_2 = -42^\circ. \text{ Each radiated (propagating) harmonic pair of the reflected field has tangential wavenumber}
\end{align*}
\]

\[
k_{xrq} = \frac{q}{4} k_0 - k_0 \sin(42^\circ).
\]

\[
\text{2. Case 3: Retroreflective serrodyne frequency translation}
\]

Here, we consider the case where the incident angle is chosen to achieve retroreflection. According to Eq. (19), setting \( \theta_{rq} = -\theta_i \) yields an expression for the incidence angles at which retroreflection occurs for the converted spatio-temporal harmonic. For this case, the modulation wavenumber \( \beta_p = 2k_x \), as shown in Fig. 15. The reflected wave propagates back to the source with an upconverted frequency. The retroreflection angle \( \theta_i \) can be calculated by solving \( \theta_{0,1} = -\theta_i \) in Eq. (19),

\[
\theta_i = -\arcsin \frac{\beta_p}{2k_0} = -\arcsin \frac{\lambda_0}{2Nd_0}.
\]
The calculated reflection spectra for both polarizations are shown in Fig. 16. The spectra clearly show a Doppler shift to frequency \( f_0 + f_p \). The conversion loss and sideband suppression for both polarizations are provided in example 5 of Table I. Note that in Fig. 16, only the harmonic pair \((r = 0, q = 1)\) at frequency \( f_0 + f_p \) is retroreflective. The reflection angle of other harmonics can be calculated based on Eq. (19).

C. Wavelength-scale spatial modulation period
\[ (|k_x| + |\beta_p| > k_0, \ & |k_x| - |\beta_p| < k_0) \]

In this section, we consider three cases where the spatial modulation period is on the order of the wavelength of radiation \((|k_x| + |\beta_p| > k_0, \ & |k_x| - |\beta_p| < k_0)\). In this regime, either the +1 \((k_x + \beta_p)\) or the −1 \((k_x - \beta_p)\) spatial harmonic is inside the light cone. For the fixed unit cell size of \( d_0 = \lambda_0/5 \), 4-path modulation \((N = 4)\) is chosen to satisfy the wavelength-scale period condition. In the first case, the metasurface exhibits simultaneous subharmonic frequency translation and deflection. In the second case, the incident angle is specifically chosen to achieve subharmonic frequency translation in retroreflection. In the last case, we show that the retroreflective frequency can be switched by changing the temporal modulation waveform to sinusoidal.

1. Case 4: Deflective/retroreflective subharmonic frequency translation

First, let us consider the example shown in Fig. 17a, where a wave is incident on the metasurface with a positive \( k_x \) value. According to Eq. (17), the radiated harmonics are those with:

\[ q + rN = 0 \text{ or } -1. \tag{26} \]

Eq. (26) implies that the radiated reflected wave contains frequency harmonics at \( f_0 + rNf_p \) and \( f_0 + (rN - 1)f_p \).

where \( r \in \mathbb{Z} \). Under a capacitance variation that generates sawtooth reflection phase, the reflected wave is upconverted to the first radiated frequency harmonic, which in this case is the harmonic pair \((r = -1, q = 3)\). Therefore, the reflected wave is Doppler shifted to a frequency \( f_0 + 3f_p \). In addition, we choose an incident angle such that the wave is retroreflected. As explained in Section III B 2, the modulation wavenumber is set to \( \beta_p = 2k_x \) (see Fig. 17a). The retroreflection angle can be calculated by setting \( \theta_{-1,3} = -\theta_i \) in Eq. (19), which is \( 39^\circ \) for a path number of \( N = 4 \).

The calculated retroreflection spectra are shown in Fig. 18a and 18c. Doppler-like frequency translation to frequency \( f_0 + 3f_p \) occurs for the incident angle of \( 39^\circ \), for both polarizations. The frequency of interest \( f_0 + 3f_p \) exhibits retroreflective subharmonic frequency translation. The conversion loss and sideband suppression for both polarizations are provided in example 6 of Table I. Note that in Fig. 18a and 18c, only the harmonics represented by a solid line are propagating in the retroreflective direction. The harmonics represented by dashed lines are propagating in the specular direction.

Note that, with a wavelength-scale spatial modulation period, the performance of the metasurface is direction-dependent. When the incident angle is −39°, as shown in Fig. 17b, it is clear that the radiated harmonics are those with:

\[ q + rN = 0 \text{ or } 1. \tag{27} \]

In this case, the harmonic pair \((q = 1, r = 0)\) is inside the light cone. Therefore, the metasurface performs serrodyne frequency translation: upconversion to a frequency \( f_0 + f_p \). Since \( \beta_p = 2k_x \), the frequency of interest \( f_0 + f_p \) is also retroreflected. The calculated reflection spectra are shown in Fig. 18b and 18d. Doppler-like frequency translation to frequency \( f_0 + f_p \) is observed for both polarizations. The conversion loss and sideband suppression for both polarizations are provided in example 7 of Table I. An illustration of the direction-dependent retroreflective behavior of the metasurface, with a wavelength-scale spatial modulation period, is depicted in Fig. 19a.
The harmonics denoted by solid lines retroreflect. The harmonics denoted by the dashed lines reflect in the specular direction. (a) 4-path \((N = 4)\) modulation for TE polarization, for an incident angle of \(39^\circ\). (b) 4-path \((N = 4)\) modulation for TE polarization, for an incident angle of \(-39^\circ\). (c) 4-path \((N = 4)\) modulation for TM polarization, for an incident angle of \(39^\circ\). (d) 4-path \((N = 4)\) modulation for TM polarization, for an incident angle of \(-39^\circ\).

The path number is \(N\). (d) 4-path \((N = 4)\) modulation for TM polarization.

Fig. 17 shows that for a positive \(k_x\) incident wavenumber, the reflected \(+1\) frequency harmonic is outside of the light cone \((|k_x + \beta_p| > k_0)\), and the reflected \(-1\) frequency harmonic is inside the light cone \((|k_x - \beta_p| < k_0)\). Since the \(+1\) frequency harmonic does not radiate and is not supported by the metasurface as a surface wave, the power is reflected from the metasurface with a frequency of \(f_0 - f_p\). In other words, for a wavelength-scale spatial modulation period, the metasurface supports single-sideband frequency translation with the sinusoidal modulation. When the incident angle is \(39^\circ\), the frequency of interest \(f_0 - f_p\) is retroreflective. Again, the retroreflective behavior of the metasurface is directionally dependent. When the incident angle is \(-39^\circ\), the \(+1\) frequency harmonic is inside the light cone, while the \(-1\) frequency harmonic is outside, as shown in Fig. 17b. For the capaci-

2. Case 5: Retroreflective frequency translation with a staggered sinusoidal reflection phase

Here, retroreflection is also achieved using a sheet capacitance that generates a staggered sinusoidal reflection phase with respect to time on adjacent columns. The capacitance modulation waveform is shown in Fig. 20a and Fig. 20c for each polarization. Each column of the metasurface generates a sinusoidal reflection phase in time. When all the columns of the metasurface are biased with the same waveform, the reflection spectra take the form of a Bessel function, as shown in Fig. 20b and Fig. 20d. The peak-to-peak modulation amplitude is chosen to be \(276^\circ\) to suppress the zeroth harmonic in reflection [26]. Unlike the sawtooth modulation, the sinusoidal reflection phase excites both \(+1\) \((r = 0, q = 1)\) and \(-1\) \((r = 0, q = -1)\) frequency harmonics.
IV. METASURFACE DESIGN AND FABRICATION

In this section, a prototype metasurface is described and measurements are reported for several of the space-time modulation cases described earlier. Details of the metasurface realization, as well as the measurement setup used to characterize its performance, are given in Section IV A. The static performance of the metasurface under various DC bias conditions is presented in Section IV B. Based on this static (DC) characterization, the required bias waveform the time-modulated metasurface is determined in Section IV C. This section also includes the measured reflection spectra for time-variation alone. Finally, measured results are given in Section IV D for several of the spatio-temporal modulation cases explored theoretically in Section III.

A. Metasurface design and measurement setup

A unit cell of the dual-polarized, ultra thin (0.06λ) metasurface is shown in Fig. 2a. Varactor diodes (MAVR-000120-1411 from MACOM [40]) are integrated onto the metasurface to act as tunable capacitances for two orthogonal polarizations. The biasing networks for each of these polarizations were printed behind the ground plane of the unit cell, as shown in Fig. 22a and Fig. 22c. Each bias layer consists of 28 metallic lines that can independently modulate all 28 columns of the metasurface. A total of 3136 MAVR-000120-1411 varactor diodes were mounted onto the metasurface. The varactor diodes are biased through vias located on the center of the metallic patches. A photo of the fabricated metasurface is shown in Fig. 22b. The metallic traces of the bias layers are routed to four D-SUB connectors edge mounted to the metasurface. Rogers 4003C ($\epsilon_r = 3.55$

![Image of the metasurface](image_url)

Note that the retroreflection angle for both of the last two cases (with sawtooth and sinusoidal reflection phase) was ±39° for a path number of $N = 4$. By simply changing the temporal modulation waveform, the retroreflection frequency was changed between $f_0 - f_p$ and $f_0 + 3f_p$, for the same incident angle of 39°.

![Image of the metasurface](image_url)
and \( \tan \delta = 0.0027 \) substrate with a thickness of 0.508 mm was chosen for each layer. Rogers 4450F (\( \epsilon_r = 3.52 \) and \( \tan \delta = 0.004 \)) bondply, with a thickness of 0.101 mm was used as an adhesive layer. A cross section of the material layers used to fabricate the metasurface is shown in Fig. 22c. The total thickness of the fabricated metasurface is 1.726 mm (0.06\( \lambda \)).

The metasurface was experimentally characterized using the quasi-optical Gaussian beam system shown in Fig. 23. In the experimental setup, the fabricated metasurface is illuminated by a spot-focusing lens antenna (SAQ-103039-90-S1). The antenna excites a Gaussian beam with a beamwidth of 50 mm at the focal length of 10 cm. The width of the fabricated metasurface is larger than 1.5 times the beamwidth to limit edge diffraction. A continuous wave signal provided by Anritsu MS4644B vector network analyzer at \( f_0 = 10 \) GHz was used as the incident signal. The amplitude of the incident signal impinging on the metasurface was measured to be \(-20 \) dBm. An Agilent E4446A spectrum analyzer was used to capture the reflected spectrum. The path loss of the system was measured and calibrated out of the measurements. The metasurface was modulated by four Keysight M9188A 16-channel D/A converters. Each channel of the D/A converter was synchronized and staggered in time. The D/A converter has an output voltage range from 0 V to 30 V, and a maximum modulation frequency of \( f_p = 25 \) kHz.

The D/A converter has an output voltage range from 0 to 30 V, and a maximum modulation frequency of \( f_p = 25 \) kHz.

B. Measurements of a DC biased metasurface: tunable reflection phase

We will first look at the simulated and measured DC performance of the proposed metasurface. The capacitance provided by the varactors ranges from 0.18 pF to 1.2 pF. Using the commercial electromagnetic solver ANSYS HFSS, a full-wave simulation of the unit cell shown in Fig. 2a is conducted in the absence of time-variation. In the simulations, each varactor diode is modeled as a lumped capacitance in series with a resistance. The capacitance and resistance values of the varactor diode were extracted as a function of bias voltage from its SPICE model [40].

The simulated reflection coefficients of the metasurface for various varactor capacitance values are given in Fig. 24. The incident angle is set to 25\(^\circ\). At the operating frequency of 10 GHz, the reflection phase of the metasurface can be varied from \(-181.1^\circ\) to 155\(^\circ\) for TE polarization, providing a maximum phase range of 336\(^\circ\). For TM polarization, the reflection phase of the metasurface can be varied from \(-181.8^\circ\) to 146.3\(^\circ\), providing a maximum phase range of 328.1\(^\circ\). At the operating frequency of 10 GHz, the simulated reflection amplitude for both polarizations remains greater than \(-3 \) dB across the entire phase range. Note that at the resonant frequency of the unit cell, the input susceptance goes to zero, and the surface admittance becomes purely resistive. The effective resistance seen by the incident wave is determined by the losses within the dielectric, the finite conductivity of the metallic patches and the losses of the varactor. As a result, the reflection coefficient magnitude dips at resonance dips the reflection phase becomes zero (a high impedance condition). The highest return loss at 10 GHz is 3.41 dB for TE polarization, occurring for a varactor capacitance of 0.313 pF. For TM polarization, the highest return loss at 10 GHz is 2.47 dB, occurring at a varactor capacitance of 0.30 pF. The metasurface suffers higher
loss for TE polarization than TM polarization. This is because, at the incident angle of 25°, the value of the free-space tangential wave impedance for TE polarization is closer (impedance matches better) to the purely resistive input impedance of the metasurface at resonance. The simulated cross-polarization behavior of the metasurface is lower than −50 dB for all the orthogonal varactor capacitance combinations.

The static (DC biased) performance of the metasurface was measured under an oblique angle of 25°. The measured TE and TM reflection coefficients under various bias voltages are given in Fig 25. The bias voltage used in measurement ranged from 0 V to 15 V, providing a varactor capacitance range of 0.18 pF to 1.2 pF. At the operating frequency of 10 GHz, the measured reflection phase of the metasurface could be varied from −182.7° to 149.9° for TE polarization, providing a phase range of 332°. For TM polarization, the measured reflection phase could be varied from −176.9° to 147.6°, providing a phase range of 324.5°. At resonance, the measured reflection amplitude was found to be much lower than in simulation, indicating higher losses in the fabricated metasurface. This could be attributed to additional ohmic loss within the diode as well as losses introduced by the tuning and soldering procedures used to mount the diodes. Nevertheless, the simulated and measured static performances of the metasurface are in good agreement. A detailed comparison between simulation and measurement for each bias voltage is given in Supplemental Materials VI.

A harmonic balance simulation with the Keysight ADS circuit solver was used to verify the theoretical analysis and compute reflection spectrum. However, to use the harmonic balance circuit solver, a circuit equivalent of the fabricated metasurface needed to be extracted for each polarization. The equivalent circuits are extracted from full-wave scattering simulations. A voltage-dependent resistance is added to it to account for the added losses observed in measurement. The equivalent circuits for the two polarizations under an oblique incident angle of 25° are given in Supplemental Materials VI. From the equivalent circuits, the capacitance modulation required to obtain a given reflection phase versus time dependence can be obtained.

C. Measurements of a time-modulated metasurface: serrodyne frequency translation

As discussed in Section II B, if all columns of the metasurface are biased with the same modulation waveform, providing a sawtooth reflection phase versus time, the metasurface can perform serrodyne frequency transla-
tion. As shown in Fig. 24 and Fig. 25, the reflection amplitude is not unity due to the loss in the metasurface. Therefore, the capacitance modulation waveform had to be numerically optimized. The detailed optimization process is detailed in Supplemental Materials VII. In the experiment, the optimized waveform was sampled at 20 data points per period $T_p = 40 \mu s$, and the sampled waveform was entered into the D/A converter. All channels of the D/A converter were synchronized with the same bias waveform. The bias waveform across several diodes was measured using a differential probe (Tektronix TMDP0200) and Tektronix oscilloscope MDO3024. The optimized and measured bias voltage waveforms are shown in Fig. 26a for TE polarization and Fig. 26c for TM polarization. The measured reflection spectrum for an oblique angle of $25^\circ$ is shown in Fig. 26b for TE polarization and Fig. 26d for TM polarization. Both polarizations show serrodyne frequency translation to $f = f_0 + f_p$. For TE polarization, a 4.604 dB conversion loss and 9.196 dB of sideband suppression are achieved. For TM polarization, a 3.67 dB conversion loss and 9.86 dB sideband suppression are achieved. For each polarization, the measured reflection spectrum in Fig. 26b and 26d generally agrees with harmonic balance simulations of its extracted circuit models, as shown in Fig. S6 of Supplemental Materials VIII.

D. Measurements of a space-time modulated metasurface

As shown in Section III, various functions can be realized by spatio-temporally modulating the metasurface, including specular subharmonic frequency translation, deflective/retroreflective serrodyne frequency translation, and deflective/retroreflective subharmonic frequency translation. Measured results are given here as a validation of our analysis. Again, the incident frequency $f_0$ and modulation frequency $f_p$ are set to 10 GHz and 25 kHz, respectively. Unless stated otherwise, the reflection phase of each column (path) is a sawtooth function in time. It is optimized as described in Supplemental Material VII. In Section IV D 1, the spatial modulation period of the metasurface is set to be electrically small ($N = 2, 3$). Subharmonic frequency translation is demonstrated for specular reflection. In Section IV D 2, the spatial modulation period is electrically large. The number of paths is chosen to be $N = 20$. Simultaneous beam steering and frequency translation is demonstrated. In Section IV D 3, the spatial modulation period is on the order of a wavelength. The number of paths is set to $N = 4$, and retroreflective subharmonic frequency translation is demonstrated. In Section IV D 4, the spatial modulation period is still on the order of a wavelength ($N = 4$); however, a staggered sinusoidal reflection phase is used to demonstrate retroreflective frequency translation. The measured conversion loss and sideband suppression for each of the examples are provided in Table II, and will be referred to throughout this section.

1. Reflective (specular) subharmonic frequency translation

In this section, electrically-small spatial modulation periods are considered ($|k_x \pm \beta_p| > k_0$). The incident wave is chosen to impinge on the metasurface with an oblique angle of $25^\circ$. The measured reflection spectra for 2-path ($N = 2$) and 3-path ($N = 3$) modulation schemes are given in Fig. 27. The reflection spectra are measured at a reflection angle of $\theta = 25^\circ$ (see Fig. 11). The measured spectra for both polarizations clearly demonstrate subharmonic frequency translation, where the only radiated harmonics are those reflected at frequencies $f = f + rN f_p$ and $r \in Z$. Doppler-like frequency translation is observed for both polarizations. The measured conversion loss and sideband suppression for both polarizations are shown as examples 1 and 2 in Table II. Compared to the homogenized, lossless metasurface presented in Section III A, the conversion loss and sideband suppression degrade more as the number of paths is increased. This is attributed to the evanescent harmonic pairs on the surface of the structure. This is discussed further in Supplemental Material VIII.

2. Measured deflective serrodyne frequency translation

In this section, the spatial modulation period is chosen to be electrically large ($|k_x \pm \beta_p| < k_0$). The path number is set to $N = 20$. For the capacitance variation shown in Fig. 26a and Fig. 26c, the metasurface acts...
as a serrodyne frequency translator, and simultaneously deflects the wave to a different angle. When the incident angle is $\theta_1 = 25^\circ$, the measured reflection spectra at the reflection angle $\theta_2 = 42^\circ$ is shown in Fig. 28. The spectra for both polarizations clearly show a Doppler shift to frequency $f_0 + f_p$. The measured conversion loss and sideband suppression for both polarizations are provided in example 3 in Table II. Note that the reflection angles for harmonics with frequency $f_0$ and $f_0 + 2f_p$ are $25^\circ$ and $67^\circ$ respectively. We can see from Fig. 28 that a fraction of the reflected power from these two harmonics was still captured by the finite aperture of the receive antenna due to its relatively close placement.

By simply interchanging the transmitting and receiving antennas, we can measure the reflection spectra for the case where the incident angle is $\theta_2 = -42^\circ$. The measured reflected spectra at the reflection angle of $\theta_1 = -25^\circ$ are given in Fig. 29. Again, the spectra for both polarizations clearly show a Doppler shift to a frequency $f_0 + f_p$. The measured conversion loss and sideband suppression for both polarizations are shown in example 4 in Table II. The harmonics with frequency $f_0$ and $f_0 + 2f_p$ are reflected at $-42^\circ$ and $9.74^\circ$ respectively, which are also captured by the receiving antenna. Note that, the reflected spectra in Fig. 28 and Fig. 29 are almost identical. This is due to the fact that the modulation frequency is far lower than the signal frequency. Otherwise, the reflection angle would differ from $\theta_1 = -25^\circ$, as discussed in Supplemental Material V.

### 3. Measured retroreflective subharmonic frequency translation

In this section, the spatial modulation period is on the order of the wavelength of radiation ($|k_x| + |\beta_p| > k_0$, & $||k_x| - |\beta_p|| < k_0$). The path number is chosen to be $N = 4$. As in Section III C, the retroreflective angle is chosen to be $\pm 39^\circ$. In experiment, a 3 dB directional coupler (Omni-spectra 2030-6377-00) was attached to the antenna in order to measure the retroreflected spectra. Note that the modulation waveform on each column is optimized with the same procedure given in Supplemental material VII, for an incident angle of $39^\circ$. The measured retroreflection spectra at an oblique angle of $39^\circ$ are given in Fig. 30a and 30c for TE and TM polarization, respectively. As expected, frequency translation to
$f_0 + 3f_p$ is observed for both polarizations. The measured conversion loss and sideband suppression for both polarizations are shown in example 5 in Table II. Note that, comparing Fig. 30a and 30c to Fig. 18a and 18c, only the harmonics in solid lines are captured by the antenna.

For an incident angle of $-39^\circ$, the measured retroreflection spectra are shown in Fig. 30b and 30d for TE and TM polarization, respectively. As expected, the spectra for both polarizations show a Doppler shift to frequency $f_0 + f_p$. The measured conversion loss and sideband suppression for both polarizations are shown in example 6 in Table II. Again, comparing Fig. 30b and 30d to Fig. 18b and 18d, only the harmonics in solid lines are captured by the antenna.

4. Measured retroreflective frequency translation with a staggered sinusoidal reflection phase

In this section, the bias waveform on adjacent columns generates staggered sinusoidal reflection phases. The modulation waveform on each column is optimized with the same procedure given in Supplemental material VII, for an incident angle of $39^\circ$. Again, a wavelength-scale spatial modulation period ($N = 4$) is used. As in Section III C, the retroreflection angle is $\pm 39^\circ$. The measured retroreflection spectra at an oblique angle of $39^\circ$ are given in Fig. 31a and 31c. As expected, frequency translation to $f_0 - f_p$ is observed for both polarizations. The measured conversion loss and sideband suppression for both polarizations are provided in example 7 in Table II. The measured retroreflection spectra at an oblique angle of $-39^\circ$ are given in Fig. 31b and 31d. Frequency translation to $f_0 + f_p$ is observed for both polarizations. The measured conversion loss and sideband suppression for both polarizations are provided in example 8 in Table II.

Both of the two previous retroreflection cases used 4 paths per spatial modulation period for a retroreflection angle of $39^\circ$ (examples 5 and 7 in Table II). The only difference between the two cases was the time-dependence of the reflection phase (sawtooth versus sinusoidal). Thus, we have shown that simply changing the temporal modulation waveform, the retroreflection frequency can be changed. In this case, it changed from $f_0 - f_p$ to $f_0 + 3f_p$. 
V. CONCLUSION

We reported a spatio-temporally modulated metasurface that can simultaneously control the reflected frequency and angular spectrum. A proof-of-principle metasurface was designed and fabricated at X-band frequencies. Additionally, a theoretical treatment of the spatio-temporally modulated metasurface was presented which accounts for the spatial discretization of the structure. The theoretical treatment provides an accurate model of the metasurface as well as insight into the subharmonic frequency translation possible with subwavelength spatial modulation periods.

Specifically, when the spatial modulation is electrically large, the metasurface exhibits serrodyne frequency translation, where the metasurface can upconvert or downconvert the incident frequency $f_0$ by the modulation frequency $f_p$. Meanwhile, tuning the spatial modulation period allows the metasurface to steer the reflected beam, and even exhibit retroreflection. When the spatial modulation is electrically small, the metasurface exhibits subharmonic frequency translation. In this case, all the radiated harmonics are reflected in the specular direction. When the spatio modulation period is on the order of a wavelength, retroreflective subharmonic frequency translation can be achieved. The retroreflected wave carries a frequency that can be switched by changing the temporal modulation waveform.

The designed metasurface provides a new level of reconfigurability. Multiple functions including beamsteering, retroreflection, serrodyne frequency translation, and subharmonic frequency translation can all be achieved with one ultra-thin (0.06X) metasurface by appropriately tailoring the space-time modulation waveform. The designed metasurface can find various applications in next-generation communication, imaging and radar systems.

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I Finding the time-modulated sheet capacitance

In this section, the relation between the reflection phase and the sheet capacitance will be derived. A homogenized model of the spatio-temporally modulated metasurface is shown in Fig. 5. It consists of a discretized, space-time modulated capacitive sheet over a grounded dielectric substrate. The spatial modulation period of the capacitive sheet is \( d \), and its temporal modulation period is \( T_p = \frac{2\pi}{\omega_p} \). Each spatial modulation period is discretized into \( N \) paths (unit cells of width \( d_0 \)) over which the sheet capacitance is uniform. At the operating frequency of \( f_0 = \frac{\omega_0}{2\pi} = 10 \) GHz, the wave admittance in the substrate for TE and TM polarizations are,

\[
Y_{s0}^{TE} = \frac{E_y}{H_x} = \frac{I_{s0}^{TE}}{\mu_0\omega_0},
\]

(\( S.1 \))

\[
Y_{s0}^{TM} = \frac{E_x}{H_y} = \frac{\epsilon_0\epsilon_h\omega_0}{k_{s0z}^{TM}},
\]

(\( S.2 \))

where subscripts \( s, 0 \) and \( z \) denote the substrate, the operating frequency \( \omega_0 \), and the \( z \)-component of the wavenumber, respectively. The relative permittivity of the substrate is \( \epsilon_h \). Note that the substrate admittance looking down from \( z = 0^- \) is simply a ground plane translated by a distance \( l \), which provides an inductive input reactance (see Fig. 2b)

\[
Y_{sub}^{TX} = -jY_{s0}^{TX} \cot(k_{s0z}^{TX}l),
\]

(\( S.3 \))

where “\( X \)” is either “\( E \)” for TE polarized waves or “\( M \)” for TM polarized waves. If there is no spatial variation, the entire capacitive sheet has the same time variation. The sheet capacitance \( C^{TX}(t) \) is assumed to be a periodic function of time,

\[
C^{TX}(t) = C_0^{TX} + \Delta C^{TX}(t).
\]

(\( S.4 \))

where \( C_0^{TX} \) is static capacitance designed to resonate with the inductive reactance given by (\( S.3 \)) at frequency \( f_0 \) for each polarization. It is given by

\[
C_0^{TX} = \frac{Y_{s0}^{TX} \cot(k_{s0z}^{TX}l)}{\omega_0}.
\]

(\( S.5 \))

Since \( Y_{s0}^{TX} \) resonates with \( C_0^{TX} \) at frequency \( f_0 \), the reflection phase \( \phi(t) \) at the incident frequency is fully controlled by \( \Delta C^{TX}(t) \),

\[
\phi(t) = -2 \arctan \left( \frac{\omega_0 \Delta C^{TX}(t)}{Y_{s0}^{TX}} \right),
\]

(\( S.6 \))
where \( Y_{00}^{TX} \) is the tangential wave admittance in free space at radial frequency \( \omega_0 \) for each polarization:

\[
Y_{00}^{TE} = \frac{\sqrt{k_0^2 - k_x^2}}{\mu_0 \omega_0},
\]
\[ Y_{00}^{TM} = \frac{\epsilon_0 \omega_0}{\sqrt{k_0^2 - k_x^2}}. \]

where \( k_0 \) is the free space wavenumber at radial frequency \( \omega_0 \), and \( k_x \) is the tangential wavenumber of the incident wave. In this paper, the reflection phase of each column of the metasurface is either a sawtooth (\( \phi(t) = \omega_p t \)) or a sinusoidal function (\( \phi(t) = A_0 \sin(\omega_p t) \)) in time; where \( \omega_p \) is the radial frequency of the modulation. From Eq. S.6, the capacitance \( \Delta C^{TX}(t) \) can be found for a desired time-varying phase \( \phi(t) \)

\[
\Delta C^{TX}(t) = -\frac{Y_{00}^{TX}}{\omega_0} \tan \left( \frac{\phi(t)}{2} \right). \quad \text{(S.9)}
\]

The capacitance \( C^{TX}(t) \) is a periodic function in time. Therefore, it can be expressed as a Fourier series:

\[
C^{TX}(t) = \sum_{q=-\infty}^{\infty} C_q^{TX} e^{j\omega_q t}, \quad \text{(S.10)}
\]

where the coefficient \( C_q^{TX} \) is equal to

\[
C_q^{TX} = \frac{1}{T_p} \int_0^{T_p} C^{TX}(t) e^{-j\omega_q t} dt. \quad \text{(S.11)}
\]

This Fourier representation of the capacitance is used to find the fields scattered from the metasurface: a time-modulated capacitive sheet over a grounded uniaxial dielectric substrate.

### II Calculation of the reflected spectrum from the time-modulated metasurface

The total tangential fields above (\( z = 0^+ \)) and below (\( z = 0^- \)) the time-modulated capacitive sheet take the form,

\[
E_t|z=0^+ = E_t|z=0^- = \sum_{q=-\infty}^{\infty} V_q e^{j\omega_q t} e^{j(\omega_0 t - k_x z)}, \quad \text{(S.12)}
\]

\[
H_t|z=0^+ = \sum_{q=-\infty}^{\infty} I_q e^{j\omega_q t} e^{j(\omega_0 t - k_x z)}, \quad \text{(S.13)}
\]

\[
H_t|z=0^- = \sum_{q=-\infty}^{\infty} -jV_q Y_{sq}^{TX} \cot(k_{sq} z) e^{j\omega_q t} e^{j(\omega_0 t - k_x z)}. \quad \text{(S.14)}
\]

\( Y_{sq}^{TX} \) and \( k_{sq}^{TX} \) are the tangential wave admittance and normal (\( z \)-directed) wavenumber in the substrate at the frequency \( \omega_q = \omega_0 + q\omega_p \), and are given by

\[
k_{sq}^{TE} = \sqrt{\frac{\omega_q^2}{c^2} \epsilon_h - k_x^2}, \quad k_{sq}^{TM} = \sqrt{\frac{\omega_q^2}{c^2} \epsilon_h - k_x^2 \epsilon_{zz}}. \quad \text{(S.15)}
\]
\[
Y_{sq}^{TE} = \frac{k_{sq}^{TE}}{\mu_0 \omega_q}, \quad Y_{sq}^{TM} = \frac{\epsilon_0 \epsilon_h \omega_q}{k_{sq}^{TM}}. \tag{S.16}
\]

If the magnitude of the voltage signal that modulates the varactors comprising the time-modulated capacitive sheet is much larger than the incident signal at \( f_0 \), the sheet can be treated as a linear, time-varying capacitance. Therefore, the boundary condition [1] at \( z = 0 \) is given by

\[
H_t|_{z=0^+} - H_t|_{z=0^-} = \frac{d}{dt}(C_{TX}^T(t)E_t). \tag{S.17}
\]

Inserting the Fourier series expansion for capacitance from Eq. (S.10) and the field expressions from Eqs. (S.12-S.14) into (S.17) yields

\[
I_q = -jV_q Y_{sq}^{TX} \cot(k_{sq}^{TX}l) + j(q \omega_p + \omega_0) \sum_{q'=-\infty}^{\infty} C_{q-q'}^{TX} V_{q'}. \tag{S.18}
\]

In matrix form, this can be written as

\[
I^{TX} = Y^{TX} V^{TX}, \tag{S.19}
\]

where \( I^{TX} \) is a vector with the complex coefficients \( I_q \) of the total tangential magnetic field above the metasurface, \( V^{TX} \) is a vector with the coefficients \( V_q \) of the total tangential electric field, and \( Y^{TX} \) is the input admittance matrix of the metasurface (looking into the metasurface at \( z = 0^+ \)) with entries

\[
Y_{qq'}^{TX} = j \omega_q C_{q-q'}^{TX} - j \delta(q - q') Y_{sq}^{TX} \cot(k_{sq}^{TX}l). \tag{S.20}
\]

The incident and reflected tangential fields above the metasurface are given by Eq. (5-8). Further, the coefficients of the incident electric and magnetic fields, as well as the reflected electric and magnetic fields, are related by the free-space tangential admittance:

\[
I^{inc} = Y_0^{TX} V^{inc}, \quad I^{ref} = -Y_0^{TX} V^{ref}. \tag{S.21}
\]

For each polarization, the diagonal admittance matrix \( Y_0^{TX} \) contains entries

\[
Y_{TE}^{0qq'} = \frac{\delta(q - q') \sqrt{\omega_q^2 / c^2 - k_x^2}}{\mu_0 \omega_q}, \tag{S.22}
\]

\[
Y_{TM}^{0qq'} = \frac{\delta(q - q') \epsilon_n \omega_q}{\sqrt{\omega_q^2 / c^2 - k_x^2}}. \tag{S.23}
\]

From Eq. (S.19) and (S.21), the reflected electric field can be calculated for each polarization using Eq. (9) in the main text.

### III Finding the discretized, space-time modulated sheet capacitance

The space-time capacitance, \( C(t, x) \), of the capacitive sheet can be expanded as a 2-D Fourier series:

\[
C(t, x) = \sum_{m=-\infty}^{\infty} \sum_{q=-\infty}^{\infty} C_{mq} e^{-jm \beta_p x} e^{jq \omega_p t}, \tag{S.24}
\]
where $\beta_p = 2\pi/d$ is the spatial modulation wavenumber and $\omega_p = 2\pi/T_p$ is the radial frequency (temporal modulation wavenumber) of the modulation. The coefficients $C_{mq}$ of the 2-D Fourier series can be calculated as:

$$C_{mq} = \frac{1}{dT_p} \int_0^d \int_0^{T_p} C(t, x)e^{jm\beta_p x}e^{-jq\omega_p t}dt dx. \tag{S.25}$$

As noted in the main text, the capacitive sheet is assumed to be spatially invariant across a given path. According to Eq. (10), the capacitance modulation of a path is staggered in time by $T_p/N$ with respect to its adjacent path. Therefore, if there are $N$ unit cells in one spatial modulation period $d$ (N-path configuration), the sheet capacitance, of a path can be expressed as

$$C^n(t, x) = \begin{cases} C^{TX}(t - (v - 1)\frac{T_p}{N}), & \frac{v-1}{N}d < x < \frac{v}{N}d \smallskip \\
0, & \text{otherwise} \end{cases} \tag{S.26}$$

which is a pulse function in space, and periodic function in time (see Eq. (S.10)). The spatio-temporally varying sheet capacitance can then be expressed as

$$C(t, x) = \sum_{v=1}^{N} C^n(t, x). \tag{S.27}$$

The capacitance of path $v$, $C^n(t, x)$, can also be expanded as a 2-D Fourier series,

$$C^n(t, x) = \sum_{m=\infty}^{\infty} \sum_{q=\infty}^{\infty} C_{mq}^v e^{-jm\beta_p x}e^{jq\omega_p t}, \tag{S.28}$$

where

$$C_{mq}^v = \frac{1}{dT_p} \int_0^d \int_0^{T_p} C^n(t, x)e^{jm\beta_p x}e^{-jq\omega_p t}dt dx \smallskip$$

$$= \frac{1}{dT_p} \int_{\frac{v-1}{N}d}^{\frac{v}{N}d} \int_0^{T_p} C^{TX}(t - (v - 1)\frac{T_p}{N})e^{jm\beta_p x}e^{-jq\omega_p t}dt dx. \tag{S.29}$$

The equation above can be used to derive the following relationship between the Fourier coefficients of the sheet capacitance on adjacent paths,

$$C_{mq}^v = C_{mq}^{v-1} e^{jm\frac{2\pi}{N}} e^{-jq\frac{2\pi}{N}}. \tag{S.30}$$

The Fourier coefficients of the overall capacitive sheet, $C_{mq}$, given by Eq. (S.25), can be found by
summing the capacitance over all the paths and employing Eq. (S.30),

\[ C_{mq} = \frac{1}{dT_p} \left( \int_0^d \int_0^{T_p} C(t, x) e^{jm\beta_p x} e^{-jq\omega_p t} dtdx + \cdots \right) \]

\[ = \frac{1}{dT_p} \left( \int_0^d \int_0^{T_p} C^1(t, x) e^{jm\beta_p x} e^{-jq\omega_p t} dtdx + \cdots \right) \]

\[ = \frac{1}{dT_p} \left( \int_0^d \int_0^{T_p} C^N(t, x) e^{jm\beta_p x} e^{-jq\omega_p t} dtdx \right) \]

\[ = \sum_{v=1}^N C_v^{mq} = \sum_{v=1}^N C^1_{mq} e^{j\frac{2\pi}{N}(m-q)(v-1)}. \] (S.31)

It is clear from Eq. (S.31) that the coefficient \( C_{mq} \) is zero except when

\[ m - q = rN, \text{ where } r \in \mathbb{Z}. \] (S.32)

Therefore,

\[ C_{mq} = \begin{cases} NC^1_{mq}, & m - q = rN, \text{ where } r \in \mathbb{Z} \\ 0, & \text{otherwise} \end{cases}. \] (S.33)

Given the staggered modulation of the paths (unit cells), the metasurface functions as an N-path system, and the indices \( m \) and \( q \) are related by Eq. (S.32). Inserting Eq. (S.32) into Eq. (S.24), the sheet capacitance can be rewritten as,

\[ C(t, x) = \sum_{r=-\infty}^{\infty} \sum_{q=-\infty}^{\infty} C_{q+rN, q} e^{-j(q+rN)\beta_p x} e^{jq\omega_p t} \]

\[ = \sum_{r=-\infty}^{\infty} \sum_{q=-\infty}^{\infty} C_{rq} e^{j(q\omega_p t-\beta_p x)} e^{-jr\beta_p x}, \] (S.34)

where the wavenumber \( \beta_d = N\beta_p = 2\pi/d_0 \) is an additional wavenumber resulting from the discretization of the spatial modulation. The summation over \( r \) accounts for the discontinuity in capacitance at the boundary of each path as well as the microscopic variation of capacitance within the paths (which in this case is constant). The summation over \( q \) accounts for the macroscopic capacitance variation over one spatial modulation period \( d \).

Given Eq. (S.33), the spatio-temporal coefficients of the capacitance variation are given by

\[
C_{r,q} = \frac{1}{dT_p} \int_0^d \int_0^{T_p} C^N(t, x) e^{j(q+rN)\beta_p x} e^{-jq\omega_p t} dtdx
\]

\[ = NC_{r,q} \]

\[ = \frac{N}{dT_p} \int_0^{d/N} \int_0^{T_p} C^{TX}(t) e^{j(q+rN)\beta_p x} e^{-jq\omega_p t} dtdx \]

\[ = \frac{N}{dT_p} \int_0^{d/N} \int_0^{T_p} C^{TX}(t) e^{j(q\omega_p t-\beta_p x)} e^{-jr\beta_p x} dtdx. \] (S.35)
From Eq. (S.36), we see that the capacitance $C^v(t,x)$ along a path is a separable function,
\[ C^v(t,x) = f^v(t)g^v(x), \]  
(S.36)
where $f^v(t)$ is the temporal modulation of capacitance $C^{TX}(t)$ along path 1. In this paper, $f^v(t)$ generates either a sawtooth reflection phase in time (see Fig. 5a and 5c) or a sinusoidal reflection phase in time (see Fig. 20a and 20c). In addition, $g^v(x)$ is a function describing the spatial dependence of capacitance along path 1, which is assumed to be a pulse function,
\[ g^v(x) = \begin{cases} 
1, & \frac{(v-1)d}{N} < x < \frac{vd}{N} \\
0, & \text{otherwise} 
\end{cases} \]  
(S.37)
Inserting Eq. (S.36-S.37) into Eq. (S.35), we obtain
\[ C_{rq} = N \frac{dT_s}{dN} \int_0^d e^{j\omega_0 t} \int_0^{T_s} f^1(t) e^{j\omega_p t} \, dt = e^{j\pi \frac{q+rN}{N}} C^{TX}_q, \]  
(S.38)
where $C^{TX}_q$ are the temporal coefficients of the capacitance modulation for a single path, given by Eq. (S.11).

IV Calculation of the reflected spectrum from a discretized, space-time modulated metasurface

As a result of the symmetry introduced by the staggered modulation scheme, the tangential field above the metasurface satisfies the following N-path field relation [2].
\[ E(t,x) = e^{j\left(\frac{\pi \omega_0 T_p}{N} - \frac{kx}{d}\right)} E(t - \frac{T_p}{N}, x - \frac{d}{N}) \]  
(S.39)
This space-time field distribution on the surface can also be expressed in terms of a modified 2-D Floquet expansion:
\[ E(t,x) = E_t|_{z=0^+} = E_t|_{z=0^=} = \sum_{m=-\infty}^{\infty} \sum_{q=-\infty}^{\infty} V_{mq} e^{-j\beta_p x} e^{j(q+rN)\omega_p t} e^{j(\omega_0 t - k x)} \]  
(S.40)
Substituting (S.40) into (S.39), one again finds that (S.32) must hold. As a result,
\[ E(t,x) = \sum_{r=-\infty}^{\infty} \sum_{q=-\infty}^{\infty} V_{rq} e^{-j\beta_p x} e^{j(q+rN)\omega_p t} e^{j(\omega_0 t - k x)} \]  
(S.41)
\[ E(t,x) = \sum_{r=-\infty}^{\infty} \sum_{q=-\infty}^{\infty} V_{rq} e^{-j\beta_d x} e^{j(q+rN)\omega_p t} e^{j(\omega_0 t - k x)} \]  
(S.42)
where the wavenumber $\beta_d = N\beta_p = 2\pi/d_0$ results from the discretization of the spatial modulation. The summation over $r$ accounts for the microscopic field variation along each path (unit cell size of length $d_0 = d/N$), while the summation over $q$ accounts for the macroscopic field variation over
one spatial modulation period \(d\). Observing the field expression in Eq. (S.42), it can be concluded that the staggered modulation between paths impresses a tangential wavenumber of \(q\beta_p\) onto the \(q^{th}\) harmonic. The total tangential magnetic field on the spatio-temporally modulated metasurface (see Fig. 3) can be expressed as,

\[
H_{1}\rvert_{z=0^+} = \sum_{r,q=-M}^{M} I_{rq}e^{-j\beta_p x}e^{j(q\omega_p t - \beta_p x)}e^{j(\omega t - k_z z)} \tag{S.43}
\]

\[
H_{1}\rvert_{z=0^-} = \sum_{r,q=-\infty}^{\infty} -jV_{rq}Y_{srxq}\cot(k_{srxq} l)e^{-j\beta_p x}e^{j(q\omega_p t - \beta_p x)}e^{j(\omega t - k_z z)} \tag{S.44}
\]

Inserting Eqs. (S.42), (S.43) and (S.44) into the boundary condition given by Eq. (S.17) yields

\[
I_{rq} = j(q\omega_p + \omega_0) \sum_{r',q'=-M}^{M} C_{r' - r, q' - q} V_{r'q'} - jV_{rq}Y_{srxq}\cot(k_{srxq} l), \tag{S.45}
\]

where \(Y_{srxq}^T\) and \(k_{srxq}^T\) are the tangential wave admittance and normal wavenumber in the substrate for each spatio-temporal harmonic pair \((r, q)\),

\[
Y_{srxq}^T = \frac{k_{srxq}^T}{\mu_0\omega_{rq}}, \quad Y_{srxq}^M = \frac{\epsilon_0\epsilon_h\omega_{rq}}{k_{srxq}^M}, \tag{S.46}
\]

\[
k_{srxq}^T = \sqrt{\frac{\epsilon_h^2}{\epsilon_0^2} - k_{srxq}^2}, \quad k_{srxq}^M = \sqrt{\frac{\omega_{rq}^2}{\epsilon_0^2} - \frac{\epsilon_h}{\epsilon_z}}, \tag{S.47}
\]

where \(s, r, q,\) and \(z\) denote the substrate, harmonic pair \((r, q)\), and \(z\)-component of the wavenumber.

We can separate the total tangential field into incident and reflected tangential fields, as given in Eqs. (12-15). The coefficients of the incident electric and magnetic field, as well as reflected electric and magnetic field are related by the free-space wave admittance

\[
Y_{0rq}^T = \frac{k_{0rq}^T}{\mu_0\omega_{rq}}, \quad Y_{0rq}^M = \frac{\epsilon_0\omega_{rq}}{k_{0rq}^M}, \tag{S.48}
\]

\[
k_{0rq}^T = \sqrt{\frac{\omega_{rq}^2}{\epsilon_0^2} - k_{0rq}^2}, \quad k_{0rq}^M = \sqrt{\frac{\omega_{rq}^2}{\epsilon_0^2} - \frac{\epsilon_h}{\epsilon_z}}. \tag{S.49}
\]

In order to simplify the calculation, each harmonic pair \((r, q)\) is mapped to one harmonic index \(\alpha = (r + M)(2M + 1) + q + 1\) [3], as shown in Table S1. The harmonic mapping allows the tangential fields, given by Eqs. (S.42) and (S.43), to be represented as vectors \(V_{s}^{TX}\) and \(I_{s}^{TX}\), where each contains \((2M + 1)^2\) entries: \(V_{s}\) and \(I_{s}\) respectively. The boundary condition and free-space admittance given by Eqs. (S.45) and (S.48) can then be written in matrix form. The size of metasurface admittance matrix \(Y_{s}^{TX}\) and free-space tangential admittance \(Y_{0}^{TX}\) is \((2M + 1)^2\times (2M + 1)^2\). The reflected electric field can be calculated for each polarization using Eq. (9). The metasurface admittance matrix \(Y_{s}^{TX}\) contains entries:

\[
Y_{\alpha\alpha'}^{TX} = j\omega_0 C_{\alpha - \alpha'}^{TX} - j\delta(\alpha - \alpha')Y_{s\alpha}^{TX}\cot(k_{s\alpha}^M l). \tag{S.50}
\]
Table S1: Harmonic mapping relationship used in the analysis

| r   | q   | α   |
|-----|-----|-----|
| −M  | −M  | 1   |
| −M  | −M + 1 | 2   |
| ... | ... | ... |
| −M  | M   | 2M + 1 |
| −M + 1 | −M   | 2M + 2 |
| −M + 1 | −M + 1 | 2M + 3 |
| ... | ... | ... |
| M   | M   | (2M + 1)^2 |

V  Theoretical study of the spatio-temporally modulated metasurface for a high modulation frequency

In this section, we study the spatio-temporally modulated metasurface under a modulation frequency $f_p$ that is comparable to the incident frequency $f_0$. The modulation frequency is chosen to be $f_p = 0.6$ GHz and the incident frequency is $f_0 = 10$ GHz. The incident plane wave is obliquely incident at $\theta_1 = 25^\circ$, and 20 paths per modulation period are chosen. The reflection phase of each column is a sawtooth function in time, at the incident frequency of $f_0$. For both polarizations, the field is expanded into 141 frequency harmonics as well as spatial harmonics. The temporal capacitance modulation on each path is truncated to 101 frequency harmonics.

Based on Eq. (23), the harmonics inside the light cone are those with $q = 0, \pm 1, \pm 2, \pm 3, -4$. Based on Eq. (19), the harmonic of interest ($f_0 + f_p$) reflects at an angle of $\theta_2 = 39^\circ$. The reflected TE and TM spectra are given in Fig. S1a and Fig. S1b, respectively. For TE polarization, a 1.72 dB conversion loss and 7.84 dB sideband suppression are observed. For TM polarization, a 2.1 dB conversion loss and 7.25 dB sideband suppression are observed.

![Figure S1: Analytical reflection spectrum of the homogenized, lossless spatio-temporally modulated metasurface. (a) 20-path ($N = 20$) modulation for TE polarization, for an incident angle of 25°. (b) 20-path ($N = 20$) modulation for TM polarization, for an incident angle of 25°. (c) 20-path ($N = 20$) modulation for TE polarization, for an incident angle of −39°. (d) 20-path ($N = 20$) modulation for TM polarization, for an incident angle of −39°.](image-url)
When the incident signal impinges on the metasurface at $\theta_2 = -39^\circ$, the propagating reflected harmonics are those with $q = 0, \pm 1, 2, 3, 4, 5, 6, 7, 8$. The harmonic of interest $(f_0 + f_p)$ travels with a reflection angle of $\theta_3 = -21^\circ$. Note that the reflected angle $\theta_3 \neq \theta_1$ [4]. The reflected spectrum for TE and TM polarization is given in Fig. S1c and Fig. S1d, respectively. For TE polarization, a 0.89 dB conversion loss and 7.51 dB sideband suppression are observed. For TM polarization, a 0.77 dB conversion loss and 8.23 dB sideband suppression are observed.

VI Reflection from a DC biased unit cell for an incident angle of $25^\circ$

As described in the paper, the tunability of the metasurface is provided by surface-mounted varactor diodes MAVR-000120-1411. In the full-wave simulations, the varactor diode is modeled as a lumped capacitance in series with a resistance. The capacitance and resistance values are extracted as a function of bias voltage from the varactor’s SPICE model [5]. The simulated reflection coefficient of the unit cell for various capacitance values is shown in Fig. 24 for an incident angle of $25^\circ$. In addition, the reflection coefficient of the fabricated metasurface is measured for various bias voltages for the same angle of incidence, and is shown in Fig. 25. Comparing the simulated and measured reflection coefficients, we noticed that the varactor capacitance versus bias voltage characteristic given by the SPICE model did not accurately match the experimental results. Therefore, the varactor capacitance versus experimental bias voltage characteristic was obtained by aligning the measured reflection phase to simulation. In addition, the measured reflection amplitude indicated that there was higher loss in measurement than in simulation. The additional loss can be introduced by a higher measured varactor resistance or by the tinning and soldering processes used to mount the varactor diodes.

In order to conduct harmonic balance simulation (see Section VIII) and predict the reflection spectrum of the metasurface when modulated, a circuit representation of the fabricated metasurface was extracted for each polarization. The circuit models are extracted from the full-wave scattering

![Figure S2: Extracted circuit models for for a unit cell (see Fig. 2a) of the metasurface under an oblique incident angle of $25^\circ$. (a) TE polarization. (b) TM polarization.](image-url)
Table S2: Detailed information and values of the extracted circuit shown in Fig. S2

| Parameter | Value                  |
|-----------|------------------------|
| $Z_{TE}^0$ | Free space tangential wave impedance (TE) under the oblique angle of 25°, 415.97 Ω |
| $Z_{h}^T$ | Substrate tangential wave impedance (TE) under the oblique angle of 25°, 220.77 Ω |
| $Z_{TM}^0$ | Free space tangential wave impedance (TM) under the oblique angle of 25°, 341.68 Ω |
| $Z_{h}^T$ | Substrate tangential wave impedance (TM) under the oblique angle of 25°, 181.34 Ω |
| $C_{TE}^g$ | Extracted pattern capacitance, 0.0325 pF |
| $C_{TM}^g$ | Extracted pattern capacitance, 0.046 pF |
| $L_{TE}^p$ | Extracted pattern inductance, 0.67 nH |
| $L_{TM}^p$ | Extracted pattern inductance, 0.61 nH |
| $R_{TE}$ | Extracted voltage-dependent resistance |
| $R_{TM}$ | Extracted voltage-dependent resistance |
| $k_{hx}^{TX}$ | Substrate normal wavenumber for TE or TM, electric thickness $k_{hx}^{TX}l = 10.411°$ |
| $l$ | Substrate thickness, 0.508 mm |
| $C_d$ | Varactor diode SPICE model |

Table S3: Varactor (MAVR-000120-1411) capacitance and resistance verses bias voltage characteristic for TE polarization.

| Varactor capacitance (pF) | Additional loss in circuit simulation (Ω) | Bias voltage used in circuit simulation (V) | Bias voltage used in measurement (V) |
|---------------------------|-------------------------------------------|---------------------------------------------|-------------------------------------|
| 0.4                       | 2.08                                      | 4.03                                        | 4                                   |
| 0.329                     | 1.82                                      | 5.2                                         | 5                                   |
| 0.284                     | 1.758                                     | 6.5                                         | 6                                   |
| 0.25                      | 1.74                                      | 7.98                                        | 7                                   |
| 0.22                      | 1.60                                      | 9.58                                        | 8                                   |
| 0.202                     | 1.20                                      | 11.28                                       | 9                                   |

Table S4: Varactor (MAVR-000120-1411) capacitance and resistance verses bias voltage characteristic for TM polarization.

| Varactor capacitance (pF) | Additional loss in circuit simulation (Ω) | Bias voltage used in circuit simulation (V) | Bias voltage used in measurement (V) |
|---------------------------|-------------------------------------------|---------------------------------------------|-------------------------------------|
| 0.385                     | 2.066                                     | 4.16                                        | 4                                   |
| 0.319                     | 1.95                                      | 5.44                                        | 5                                   |
| 0.271                     | 2.00                                      | 6.92                                        | 6                                   |
| 0.239                     | 2.17                                      | 8.4                                         | 7                                   |
| 0.213                     | 2.39                                      | 10.2                                        | 8                                   |
| 0.19                      | 2.12                                      | 12.49                                       | 9                                   |
Figure S3: Reflection phase and magnitude versus varactor capacitance for the realized metasurface from full-wave simulation, circuit model simulation and measurement under an oblique incident angle of 25°. Results are shown for a TE polarization.

Figure S4: Reflection phase and magnitude versus varactor capacitance for the realized metasurface from full-wave simulation, circuit model simulation and measurement under an oblique incident angle of 25°. Results are shown for a TM polarization.

simulations, with an added voltage-dependent resistance $R^{TX}$ to account for the additional loss observed in measurement. The dependence of $R^{TX}$ is obtained by aligning the measured and simulated reflection amplitudes. The extracted circuit models are shown in Fig. S2. The values of the extracted circuit parameters are shown in Table S2. The varactor diode $C_d$ is modeled using the SPICE model for MAVR-000120-1411 varactors. For each varactor capacitance, the corresponding
bias voltages used in circuit simulation (given by the SPICE model) and measurement, as well as
the additional resistances \( R_{TX} \) are given in Table S3 and S4 for the TE and TM polarizations. The
varactor characteristics and additional losses \( R_{TX} \) are slightly different for the two polarizations.
This is likely due to tolerances in the varactor capacitance and resistance values.

The extracted circuits shown in Fig. S2 are simulated with the commercial circuit solver Keysight
Advanced Design System (ADS). Comparisons between full-wave simulation, measurement, and
circuit simulation are shown in S3 and S4 for various capacitance values. The reverse bias voltage
values used in circuit simulation are given in Table S3 and S4. The circuit simulations agree closely
with full-wave simulations and measurements of the metasurface, confirming the accuracy of the
circuit model shown in Fig. S2.

VII Calculating the optimized bias waveform

In order to achieve serrodyne frequency translation, a bias waveform is needed that generates a
sawtooth reflection phase, which varies \( 2\pi \) radians over each modulation period. To obtain the bias
waveform, the following procedure was followed. Using the extracted circuit models shown in Fig.
S2, the reflection amplitude and phase were plotted versus bias voltage at 10 GHz, as shown in Fig.
S5. The plots show that the bias voltage versus reflection phase curves follow a tangent function.
Since the targeted sawtooth reflection phase is linear with respect to time over each modulation
period, the bias waveform was assumed to be of the following form,

\[
V_{bias}^{TX}(t) = \begin{cases} 
A \tan(B(\omega pt + C)) + D, & \text{for } 0 < V_{bias}^{TX} < 14 \\
0, & \text{for } V_{bias}^{TX} < 0 \\
14, & \text{for } V_{bias}^{TX} > 14 
\end{cases}
\]  

(S.51)

over each 40 usec modulation period \((-T_p/2 < t < T_p/2)\). The parameters \( A, B, C, \) and \( D \) were then
numerically optimized to provide optimal serrodyne frequency translation: the highest frequency
conversion and lowest sidebands. The optimized values for TE polarization are \( A = 0.51 \) V, \( B =
0.522, C = 0.266 \) rad, \( D = 5.597 \) V. The optimized values for TM polarization are \( A = 0.8 \) V,
\( B = 0.535, C = 0.339 \) rad, \( D = 5.9 \) V.

Similarly, in order to generate a sinusoidal reflection phase with respect to time, the bias waveform

![Figure S5: Reflection coefficient magnitude and phase of the extracted circuit model for the meta-
surface as a function of reverse bias voltage. (a)TE polarization. (b) TM polarization.](image-url)
was assumed to be of the following form,
\[
V_{bias}^{TX}(t) = \begin{cases} 
A \tan(B(A_0 \sin(\omega_p t) + C)) + D, & \text{for } 0 < V_{bias}^{TX} < 14 \\
0, & \text{for } V_{bias}^{TX} < 0 \\
14, & \text{for } V_{bias}^{TX} > 14
\end{cases}
\]  
(S.52)

where \( A_0 = 138^\circ \), as explained in the paper. The fitting parameters \( A, B, C, \) and \( D \) were again numerically optimized to suppress the zeroth harmonic in reflection. The optimized waveform used in simulation is shown in Figs. S7a and S7c. The optimized values for TE polarization are \( A = 0.45 \) V, \( B = 0.455 \), \( C = 0.286 \) rad, \( D = 5.53 \) V. The optimized values for TM polarization are \( A = 0.65 \) V, \( B = 0.458 \), \( C = 0.389 \) rad, \( D = 5.8 \) V.

A harmonic representation of the optimized bias waveform (referred to as simulated bias waveform) was then used in the harmonic balance simulation, which is detailed in the next section. In addition, a mapping between the bias voltage used in circuit simulation and in measurement for each varactor capacitance value, was obtained from Table S3 and S4. The experimental bias waveform was determined by applying this mapping to the optimized bias waveform. A sampled version (20 points per period) of the experimental bias waveform (referred to as measured bias waveform) was applied to the metasurface through the D/A converter in measurement. The optimized and measured bias waveform used in the measurement are shown in Fig. 26a and 26c for serrodyne frequency translation.

**VIII Harmonic balance simulation of the extracted circuit model**

If all the columns of the metasurface are biased with the same waveform, the metasurface’s response can be predicted by performing a harmonic balance simulation of a single unit cell’s extracted circuit model. Harmonic balance simulations of the circuit model shown in Fig. S2 were performed using Keysight ADS. The incident signal was set to an amplitude of \(-20 \) dBm at frequency \( f_0 = 10 \) GHz. The optimized waveforms \( V_{bias}^y \) and \( V_{bias}^x \) were calculated as described in the previous section.

When the reflection phase is a sawtooth function in time, the simulated reflection spectra are given in Fig. S6b and S6d. The simulation results agree with the measurement results shown in Fig. 26. However, the measured results shows higher conversion loss and lower sideband suppression. This can be attributed to the fact that the measured bias waveform is a coarsely sampled version of the optimized waveform. The sampling rate of the D/A converter used in experiment is 0.5 MHz. Therefore, only 20 samples per period could be taken of the 25 kHz modulation waveform. For a reflection phase that is a sinusoidal function of time, the simulated reflection spectra are given in Fig. S7b and S7d.

As mentioned in the paper, when the spatial modulation period is deeply subwavelength, the metasurface can be viewed as an \( N \)-path system. Subharmonic frequency translation is supported in this case, and the metasurface exhibits Doppler-like frequency translation to a high-order frequency harmonic. The metasurface can be represented using an \( N \)-path circuit model shown in Fig. 9, where there are \( N \) branches of time-varying circuits connected to a common port. Each path (column of metasurface) is represented by a circuit model shown in Fig. S2. The bias waveform of each path is given in Fig. S6a and S6c; and is staggered in time by \( T_p/N \) with respect to that of its adjacent path. The simulated reflection spectra for 2-path and 3-path configurations are shown in Fig. S8. Note that, as the path number \( N \) increases, the conversion loss increases as well. This is because the \( N \)-path metasurface upconverts the frequency to the first propagating harmonic pair. The higher
the upconverted frequency, the more loss there is in the frequency conversion process. The simulated results agree with the measurement results shown in Fig. 27. However, the conversion loss of the measured results degrade more severely as the path number increases. This is due to the fact that when the metasurface is lossy, the evanescent harmonic pairs on the metasurface consume energy as well. Those harmonic pairs are not represented in the N-path circuit network, where the N branches of time-varying circuits are considered perfectly co-located.

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Figure S6: (a) Optimized bias waveform for a TE polarization. (b) Reflection spectrum from harmonic balance simulation of the extracted circuit shown in Fig. S5a. The bias waveform is given in Fig. S6a. (c) Optimized bias waveform for TM polarization. (d) Reflection spectrum from harmonic balance simulation of the extracted circuit shown in Fig. S5b. The bias waveform is given in Fig. S6c.
Figure S7: (a) Optimized bias waveform for a TE polarization. (b) Reflection spectrum from harmonic balance simulation of the extracted circuit shown in Fig. S5a. The bias waveform is given in Fig. S7a. (c) Optimized bias waveform for TM polarization. (d) Reflection spectrum from harmonic balance simulation of the extracted circuit shown in Fig. S5b. The bias waveform is given in Fig. S7c.

Figure S8: Reflection spectrum from harmonic balance simulation of the N-path circuit model depicted in Fig. 9. Each path is represented by the extracted circuit models shown in Fig. S2. (a) 2-path ($N = 2$) modulation for TE polarization. (b) 3-path ($N = 3$) modulation for TE polarization. (c) 2-path ($N = 2$) modulation for TM polarization. (d) 3-path ($N = 3$) modulation for TM polarization.