A Computerized Boundary Element Algorithm for Modeling and Optimization of Complex Magneto-Thermoelastic Problems in MFGA Structures

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ABSTRACT

Aims: The aim of this article is to propose a boundary integral equation algorithm for modeling and optimization of magneto-thermoelastic problems in multilayered functionally graded anisotropic (MFGA) structures.

Study design: Original research paper.

Place and Duration of Study: Jamoum laboratory, January 2018

Methodology: A new dual reciprocity boundary element algorithm was implemented for solving the governing equations of magneto-thermoelastic problems in MFGA structures.

Results: A numerical results demonstrate validity, accuracy, and efficiency of the presented technique.

Conclusion: Our results thus confirm the validity, accuracy, and efficiency of the proposed technique. It is noted that the obtained dual reciprocity boundary element method (DRBEM) results are more accurate than the FEM results, the DRBEM is more efficient and easy to use than FEM because it only needs the boundary of the domain needs to be discretized.

Keywords: Predictor-Corrector; Shape optimization; Magneto-thermoelasticity; Functionally graded Anisotropic structures; Dual Reciprocity Boundary Element Method.

2010 mathematics subject classification: 65M38 - 65K05 - 74B05 - 74E05 - 74F05 - 74H05 - 74H15 - 74S20 - 90C31.

1. INTRODUCTION

An understanding of behaviour of functionally graded anisotropic magneto-thermoelastic materials has great practical applications in applied sciences and engineering. In recent years, many researchers discussed the behavior of MFGA structures. With the new advances in computer hardware and software, it is now possible to solve complex magneto-thermoelastic problems by using the DRBEM, which proposed by Nardini and Brebbia [1]. The interested readers can find more details in the following references [2-6].

The aim of this article is to propose a new DRBEM algorithm for solving the governing equations of magneto-thermoelastic problems in MFGA structures. The obtained numerical results demonstrate validity, accuracy, and efficiency of the proposed technique.

2. FORMULATION OF THE PROBLEM

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Consider a MFGA structure occupies the region \( R = \{(x, y, z): 0 < x < h, 0 < y < b, 0 < z < a \} \). At each and every point on the boundary \( C \), the temperature and displacement are suitably specified.

According to Green and Naghdi theory, the governing equations of MFGA structures for the \( i \)th layer, can be expressed as [7-10]:

\[
\sigma_{ab} = (x + 1)^{m}C_{ab}^{i}\left[ \frac{1}{2} u_{a}^{i} - \frac{1}{2} u_{a}^{i} \right] + \tau_{ab} \frac{\partial}{\partial t} (T^{i} - T_{0})
\]

\[
\tau_{ab} = \mu^{i}(x + 1)^{m}\left[ \frac{1}{2} u_{a}^{i} - \frac{1}{2} u_{a}^{i} \right]
\]

where \( \sigma_{ab}, \tau_{ab}, u_{a}^{i} \) and \( T^{i} \) are respectively stress, displacement and temperature, \( T_{0}, C_{ab}^{i}, \mu^{i}, \mu, \eta, k_{ab}^{i}, \rho^{i}, \tau \), are respectively reference temperature, constant elastic moduli, stress-temperature coefficients, magnetic permeability, perturbed magnetic field, thermal conductivity coefficients, new material coefficients associated with the GN theories, density and time. \( c^{i} \) is the specific heat capacity, \( \tau_{1} \) is the relaxation times, \( \chi \) is the heat source, \( i = 1, 2, \ldots, n - 1 \) represents the parameters in multilayered plate, respectively, and \( f(x) \) is a given nondimensional function of space variable \( x \). We take \( f(x) = (x + 1)^{m} \), where \( m \) is a dimensionless constant.

### 3. DRBEM IMPLEMENTATION

Using the same technique of Fahmy [11-13] for the current problem and implementing the DRBEM, we can write the boundary integral representation formula of coupled thermoelasticity as follows:

\[
U_{bd}^{i}(\xi) = \int_{C} (U_{db}^{i}(\xi) - \bar{T}_{db}^{i}(\xi)) dC + \sum_{q=1}^{E} \left( \int_{C} \left( T_{dA}^{i q} U_{dA q}^{i q} - U_{dA q}^{i q} \right) dC \right) a_{e}^{q}
\]

According to Fahmy [14], the DRBEM equation (5) can be written as:

\[
\zeta U - \eta T = (\zeta U - \eta T)_{0}
\]

An implicit-implicit staggered algorithm based on DRBEM was implemented for for solving the governing equations which can be written using (6) as follows:

\[
\tilde{M} U^{i} + \tilde{T} U^{i} + \tilde{K} T^{i} = \tilde{Q}^{i}
\]

(7)

\[
\tilde{X} \tilde{T}^{i} + \tilde{R} \tilde{T}^{i} = \tilde{Z} \tilde{U}^{i} + \tilde{R}
\]

(8)

where the matrices in (7) and (8) are as follows:

\[
V = (\eta \tilde{T} - \zeta U)^{-1}, \quad \tilde{M} = V \left( \tilde{I} + \tilde{\alpha}_{db} \right), \quad \tilde{T} = V \tilde{B},
\]

\[
\tilde{R} = -\zeta + V \left( B^{i} \tilde{T}^{i} + \tilde{\alpha}_{db} \right), \quad \tilde{X} = -\eta \tilde{T} + V \tilde{S}, \quad \tilde{Z} = -\rho^{i} c^{i}(x + 1)^{m},
\]

\[
\tilde{K} = k_{ab}^{i} \frac{\partial}{\partial x_{a}} \frac{\partial}{\partial x_{b}} \tilde{X}, \quad \tilde{B} = k_{ab}^{i} \frac{\partial}{\partial x_{a}} \frac{\partial}{\partial x_{b}} \tilde{X} = \tilde{B} \tilde{T}^{i}, \quad \tilde{R} = -\rho^{i} \tilde{T}^{i}.
\]

Equations (7) and (8) yield the following system [15]:

\[
\tilde{M} \tilde{U}_{n+1}^{i} + \tilde{T} \tilde{U}_{n+1}^{i} + \tilde{K} \tilde{T}_{n+1}^{i} = \tilde{Q}_{n+1}^{ip}
\]

(9)

\[
\tilde{X} \tilde{T}_{n+1}^{i} + \tilde{R} \tilde{T}_{n+1}^{i} = \tilde{Z} \tilde{U}_{n+1}^{i} + \tilde{R}
\]

(10)

where \( \tilde{Q}_{n+1}^{ip} = \eta T_{n+1}^{ip} + V S^{i} \) and \( \tilde{T}_{n+1}^{ip} \) is the predicted temperature.

Integrating Eq. (7) and using Eq. (9), we get

\[
\tilde{U}_{n+1}^{i} = \tilde{U}_{n+1}^{i} + \frac{\Delta t}{2} \left( \tilde{U}_{n+1}^{i} + \tilde{U}_{n}^{i} \right)
\]

\[
= \tilde{U}_{n+1}^{i} + \frac{\Delta t}{2} \left[ \tilde{U}_{n+1}^{i} + \tilde{M}^{-1} \left( \tilde{Q}_{n+1}^{ip} - \tilde{T} \tilde{U}_{n+1}^{i} - \tilde{R} \tilde{U}_{n+1}^{i} \right) \right]
\]

(11)

\[
\tilde{U}_{n+1}^{i} = \tilde{U}_{n+1}^{i} + \frac{\Delta t^{2}}{4} \left[ \tilde{U}_{n+1}^{i} + \tilde{M}^{-1} \left( \tilde{Q}_{n+1}^{ip} - \tilde{T} \tilde{U}_{n+1}^{i} - \tilde{R} \tilde{U}_{n+1}^{i} \right) \right]
\]

(12)

From Eq. (11) we have...
Substituting from Eq. (13) into Eq. (12), we have
\[ \dot{U}_{i+1}^n = \ddot{U}^{-1}_i \left[ \frac{\Delta t}{2} \left( \ddot{U}^i + \dddot{U}^i \left( \tilde{Q}^i_n + \tilde{R} U_{i+1}^n \right) \right) \right] \]
(13)
where \( \dddot{Y} = \left( I + \frac{\Delta t}{2} \dddot{M}^{-1} \right) \).

Substituting from Eq. (13) into Eq. (10) we derive
\[ U_{i+1}^n = U_n^i + \Delta t \dot{U}^i_n + \frac{\Delta t^2}{4} \left( \dddot{U}^i_n + \dddot{R} U_{i+1}^n \right) \]
(14)
Substituting \( \dddot{U}_{i+1}^n \) from Eq. (13) into Eq. (9) we obtain
\[ \dddot{U}_{i+1}^n = \dddot{M}^{-1} \left( \dddot{Q}^i_n + \dddot{R} U_{i+1}^n \right) \]
(15)
Integrating Eq. (8) and using Eq. (10) we have
\[ T_{i+1}^n = T_n^i + \frac{\Delta t}{2} \left( \dddot{T}^i_n + \dddot{R} \dddot{T}^i_{i+1} \right) \]
(16)
\[ T_{i+1}^n = T_n^i + \Delta t \dddot{T}^i_n + \frac{\Delta t^2}{4} \left( \dddot{T}^i_n + \dddot{R} \dddot{T}^i_{i+1} \right) \]
(17)
From Eq. (16) we get
\[ T_{i+1}^n = \dddot{Y}^{-1} \left( T_n^i + \Delta t \left( \dddot{Y}^{-1} \left( T_n^i + \dddot{R} \dddot{T}^i_{i+1} \right) + \dddot{T}^i_n \right) \right) \]
(18)
where \( \dddot{Y} = \left( I + \frac{\Delta t}{2} \dddot{M}^{-1} \right) \).

Substituting from Eq. (18) into Eq. (17), we have
\[ T_{i+1}^n = \dddot{Y}^{-1} \left( T_n^i + \Delta t \left( \dddot{Y}^{-1} \left( T_n^i + \dddot{R} \dddot{T}^i_{i+1} \right) + \dddot{T}^i_n \right) \right) - \dddot{R} T_{i+1}^n \]
(19)
Substituting \( \dddot{T}_{i+1}^n \) from Eq. (18) into Eq. (10) we obtain
\[ T_{i+1}^n = \dddot{Y}^{-1} \left( T_n^i + \Delta t \left( \dddot{Y}^{-1} \left( T_n^i + \dddot{R} \dddot{T}^i_{i+1} \right) + \dddot{T}^i_n \right) \right) - \dddot{R} T_{i+1}^n \]
(20)
Using the algorithm of Fahmy [16-22], we have the temperature and the displacements.

4. SHAPE DESIGN SENSITIVITY ANALYSIS AND OPTIMIZATION

Thus, the design sensitivities with respect to the design variables \( x_n \) for the displacement and temperature which describe the structural response are performed by implicit differentiation of equations (9) and (10), respectively. Let \( R \) be a region with boundary \( C \) and continuous functions \( m_n \) and \( \omega \) satisfy
\[ \int_R \left( \frac{\partial \omega}{\partial x_1} - \frac{\partial m_n}{\partial x_2} \right) d x_1 d x_2 = \int_C \left( m_n d x_1 + \omega d x_2 \right) \]
(21)
The area \( A = \frac{1}{2} \int d x_1 d x_2 \) of the domain \( R \) can be written over the boundary using the Green’s theorem as [16-18]
\[ A = \frac{1}{2} \int_C \left( x_1 d x_2 - x_2 d x_1 \right) \]
(22)
By discretizing the boundary of the structure into $Q$ quadratic boundary elements, we have the following relation at $b$th element

$$x_m(\xi) = N^b(\xi)x_m^b$$

(23)

Also, the area can be expressed as follows

$$\tilde{A} = \frac{1}{2} \sum_{b=1}^{Q} \int_{-1}^{1} [x_1(\xi)n_1 + x_2(\xi)n_2] J(\xi) d\xi$$

(24)

where $n_1$ and $n_2$ can be written in terms of the Jacobian matrix of the transformation $J(\xi)$ as

$$n_1 = \frac{dx_2}{d\tilde{A}} = \frac{dx_2/d\xi}{dA/d\xi} = \frac{dx_2/d\xi}{J(\xi)}$$

(25)

$$n_2 = -\frac{dx_1}{d\tilde{A}} = -\frac{dx_1/d\xi}{dA/d\xi} = -\frac{dx_1/d\xi}{J(\xi)}$$

(26)

Substitution of equations (25) and (26) into equation (24) yields

$$\tilde{A} = \frac{1}{2} \sum_{b=1}^{Q} \int_{-1}^{1} [x_1(\xi) \frac{dx_2}{d\xi} - x_2(\xi) \frac{dx_1}{d\xi}] d\xi$$

(27)

By differentiating (27) taking into consideration that

$$\frac{\partial}{\partial x_b} \left( \frac{dx_2}{d\xi} \right) = 0$$

(28)

and

$$\frac{\partial}{\partial x_b} \left( x_2(\xi) \right) = 0$$

(29)

Therefore

$$\frac{\partial \tilde{A}}{\partial x_b} = \frac{1}{2} \sum_{b=1}^{Q} \int_{-1}^{1} \left[ x_1(\xi) \frac{dx_2}{d\xi} - x_2(\xi) \frac{dx_1}{d\xi} \right] d\xi$$

(30)

If $x_b$ is the $x_b$ coordinate of a movable node, then

$$\frac{\partial}{\partial x_b} \left( x_1(\xi) \right) = 0$$

(31)

and

$$\frac{\partial}{\partial x_b} \left( x_2(\xi) \right) = 0$$

(32)

Therefore

$$\frac{\partial \tilde{A}}{\partial x_b} = \frac{1}{2} \sum_{b=1}^{Q} \int_{-1}^{1} \left[ x_1(\xi) \frac{dx_2}{d\xi} - x_2(\xi) \frac{dx_1}{d\xi} \right] d\xi$$

(33)

where weight minimization is equivalent to area minimization.

Now, we consider the following minimization problem

Minimize

$$\tilde{A}(x_b)$$

(34)

Subject to

$$y_m(x_b) \leq 0, \quad m = 1, \ldots, M$$

(35)

$$x^{\text{lb}} \leq x_b \leq x^{\text{ub}}$$

(36)

where $x_b = [x_{b_1}, x_{b_2}, \ldots, x_{b_n}]^T$.

The feasible direction method (FDM) can be successfully applied for solving the current optimization problem using the following iteration process:

$$x_b = x_{b_{-1}} + s_b d_b$$

(37)

Under the following condition

$$\tilde{A}(x_b) - \tilde{A}(x_{b-1}) \leq \varepsilon$$

(38)

where $\delta$, $\varepsilon$, $s_b$, and $d_b$ are respectively iteration number, predefined tolerance, line step parameter, search direction $d_b$ which can be defined as

$$d_b = -H^{\dagger} \nabla \tilde{A}(x_b)$$

(39)

where the inverse Hessian matrix can be approximated in terms of the identity matrix $I$ by

$$H^{\dagger + 1} = \left[ I - \frac{P^T Q^T}{(P^T Q)^T} \right] H^{\dagger} \left[ I - \frac{Q^T (P^T)^T}{(P^T)^T Q} \right] + \frac{P^T Q^T}{(P^T)^T Q}$$

(40)

In which

$$P^T = x_{b+1} - x_b, \quad Q^T = \nabla \tilde{A}(x_{b+1}) - \nabla \tilde{A}(x_b), \quad H^0 = I$$

Using FDM, we have

$$\nabla \tilde{A}(x_b)d_b \leq 0$$

(41)
Now, we want to solve the following search direction problem [19]

Maximize \( \delta \)

Subject to \( d^T\nabla X_m(x_d) + \theta_m \delta \leq 0 \) \hspace{1cm} (43)
\( d^T\nabla A(x_d) + \delta \leq 0 \) \hspace{1cm} (44)
\(-1 \leq d \leq 1 \) \hspace{1cm} (45)

where \( \theta_m \) is the push-off factor which can be written as

\[
\theta_m = \left[ 1 - \frac{x_m(x_d)}{\varepsilon} \right]^2 \theta_0
\]
where \( \varepsilon \) and \( \theta_0 \) are constants.

We will use the preceding formulation when the design is inside the feasible domain. But when the design is outside the feasible domain, we will solve the following search direction problem

Maximize \( \nabla A(x_d) \cdot d + \Phi \delta \)

Subject to \( \nabla X_m(x_d) \cdot d + \theta_m \delta \leq 0, \quad m \in J \) \hspace{1cm} (47)
\( d^T \cdot d \leq 1 \) \hspace{1cm} (48)

where \( J \) and \( \Phi \) are respectively potential constraint set and weighting factor.

5. NUMERICAL RESULTS AND DISCUSSION

In order to illustrate the numerical results of the current study, the following physical constants for material "A" are as follows

Elasticity tensor
\[
C_{ijkl} = \begin{bmatrix}
430.1 & 130.4 & 18.2 & 0 & 0 & 201.3 \\
130.4 & 116.7 & 21.0 & 0 & 0 & 70.1 \\
18.2 & 21.0 & 73.6 & 0 & 0 & 2.4 \\
0 & 0 & 0 & 19.8 & -8.0 & 0 \\
0 & 0 & 0 & -8.0 & 29.1 & 0 \\
201.3 & 70.1 & 2.4 & 0 & 0 & 147.3
\end{bmatrix} \text{ GPa}
\]

Mechanical temperature coefficient
\[
\beta_{ij} = \begin{bmatrix}
1.01 & 2.00 & 0 \\
2.00 & 1.48 & 0 \\
0 & 0 & 7.52
\end{bmatrix} \cdot 10^6 \text{ N/Km}^2
\]

Tensor of thermal conductivity
\[
k_{ij} = \begin{bmatrix}
5.2 & 0 & 0 \\
0 & 7.6 & 0 \\
0 & 0 & 38.3
\end{bmatrix} \text{ W/Km}
\]

Mass density \( \rho = 7820 \text{ kg/m}^3 \) and heat capacity \( c = 461 \text{ J/kg K} \).

A prismatic material is taken as material B in the numerical calculations with the following physical constants

Elasticity tensor
\[
C_{ijkl} = \begin{bmatrix}
60.23 & 18.67 & 18.96 & -7.69 & 15.60 & -25.28 \\
18.67 & 21.26 & 9.36 & -3.74 & 4.21 & -8.47 \\
18.96 & 9.36 & 47.04 & -8.82 & 15.28 & -8.31 \\
-7.69 & -3.74 & -8.82 & 10.18 & -9.54 & 5.69 \\
15.60 & 4.21 & 15.28 & -9.54 & 21.19 & -8.54 \\
-25.28 & -8.47 & -8.31 & 5.69 & -8.54 & 20.75
\end{bmatrix} \text{ GPa}
\]

Mechanical temperature coefficient
\[
\beta_{ij} = \begin{bmatrix}
0.002 & 0.004 & 0.003 \\
0.02 & 0.004 & 0.04 \\
0.03 & 0.04 & 0.05
\end{bmatrix} \cdot 10^6 \text{ N/Km}^2
\]

Tensor of thermal conductivity
\[
k_{ij} = \begin{bmatrix}
0.8 & 0.1 & 0.15 \\
0.1 & 0.9 & 0.12 \\
0.15 & 0.12 & 0.7
\end{bmatrix} \text{ W/Km}
\]

Mass density \( \rho = 1600 \text{ kg/m}^3 \) and heat capacity \( c = 0.1 \text{ J/kg K} \).

Also, a monoclinic North Sea sandstone reservoir rock is taken as material C in the numerical computations with the following physical constants

Elasticity tensor
\[
C_{ijkl} = \begin{bmatrix}
0.002 & 0.004 & 0.003 \\
0.02 & 0.004 & 0.04 \\
0.03 & 0.04 & 0.05
\end{bmatrix} \cdot 10^6 \text{ N/Km}^2
\]

Tensor of thermal conductivity
\[
k_{ij} = \begin{bmatrix}
0.8 & 0.1 & 0.15 \\
0.1 & 0.9 & 0.12 \\
0.15 & 0.12 & 0.7
\end{bmatrix} \text{ W/Km}
\]

Mass density \( \rho = 1600 \text{ kg/m}^3 \) and heat capacity \( c = 0.1 \text{ J/kg K} \).
The tensor of thermal conductivity is given by:

\[
\mathbf{k}_{pj} = \begin{bmatrix}
    0.001 & 0.02 & 0 \\
    0.02 & 0.006 & 0 \\
    0 & 0 & 0.05
\end{bmatrix} \cdot 10^6 \text{ N/Km}^2
\]

The mass density is \( \rho = 2216 \text{ kg/m}^3 \) and the heat capacity is \( c = 0.1 \text{ J/kg K} \).

For the purpose of numerical calculations of materials A, B and C, we considered the following constants:

- \( H_0 = 1000000 \text{ Oersted} \), \( \mu = 0.5 \text{ Gauss/Oersted} \), \( r_0 = 0.5 \text{ m} \), \( \Delta r = 0.0001 \text{ m} \), \( M_0 = 0.5 \), \( 
\nu = 1 \) :

It can be noticed from numerical results that the DRBEM results are in very good agreement with those obtained using the finite element method (FEM) of Gao and Yao [23]. The right half of the link plate shown in Fig. 1a and design variables shown in Fig. 1b are considered. The optimum shapes of the considered structure for selected anisotropic materials produced from the current study are shown in Fig. 2. It can be seen that the weight and the maximum stress have increased. Fig. 3 shows the iteration history for elastic compliance of the link plate for selected materials A, B and C. It can be seen that the weight and the maximum stress of the link plate have been decreased (see Table 1). Figures 4 and 5 show the sensitivities of the displacement distributions. Also, figure 6 shows the sensitivity of the temperature distribution to demonstrate the accuracy of the current technique (see Table 2). For further finite difference method details, we refer the reader to [24-29]. Also, for more boundary element method details we refer the reader to [30-49].

REFERENCES

[1] D. Nardini, and C. A. Brebbia, A new approach to free vibration analysis using boundary elements. Boundary elements in engineering, ed. C. A. Brebbia, Springer, Berlin; 1982.
[2] P. W Partridge, C. A. Brebbia and L. C. Wrobel, The dual reciprocity boundary element method, Computational Mechanics Publications, Southampton; 1992.
[3] P. W. Partridge and L. C. Wrobel, The dual reciprocity boundary element method for spontaneous ignition. Int. J. Numer. Methods Eng., vol. 30, 1990, pp. 953–963.
[4] P. W. Partridge and C. A. Brebbia, Computer implementation of the BEM dual reciprocity method for the solution of general field equations. Commun. Appl. Numer. Math. 1990;6:83–92.
[5] C. A. Brebbia, J. C. F. Telles, and L. Wrobel, Boundary element techniques in Engineering, Springer-Verlag, New York; 1984.
[6] L. Gaul, M. Kögl and M. Wagner, Boundary element methods for engineers and scientists, Springer-Verlag, Berlin; 2003.
[7] M. A. Fahmy, A time-stepping DRBEM for magneto-thermo-viscoelastic interactions in a rotating nonhomogeneous anisotropic solid, International Journal of Applied Mechanics. 2011;3:1-24.
[8] M. A. Fahmy, A time-stepping DRBEM for the transient magneto-thermo-visco-elastic stresses in a rotating non-homogeneous anisotropic solid, Engineering Analysis with Boundary Elements. 2012;36:335-345.
[9] M. A. Fahmy, Transient magneto-thermoviscoelastic plane waves in a non-homogeneous anisotropic thick strip subjected to a moving heat source, Applied Mathematical Modelling. 2012;36:4565-4578.
[10] M. A. Fahmy, The effect of rotation and inhomogeneity on the transient magneto-thermoviscoelastic stresses in an anisotropic solid, ASME Journal of Applied Mechanics. 2012; 79:1015.
[11] M. A. Fahmy, Transient magneto-thermo-elastic stresses in an anisotropic viscoelastic solid with and without a moving heat source, Numerical Heat Transfer, Part A: Applications. 2012; 61:633-650.
[12] M. A. Fahmy, Implicit-Explicit time integration DRBEM for generalized magneto-thermoelasticity problems of rotating anisotropic viscoelastic functionally graded solids, Engineering Analysis with Boundary Elements. 2013; 37:107-115.
[13] M. A. Fahmy, Generalized magneto-thermo-viscoelastic problems of rotating functionally graded anisotropic plates by the dual reciprocity boundary element method, Journal of Thermal Stresses. 2013; 36:1-20.
[14] M. A. Fahmy, A three-dimensional generalized magneto-thermo-viscoelastic problem of a rotating functionally graded anisotropic solids with and without energy dissipation, Numerical Heat Transfer, Part A: Applications. 2013; 63:713-733.
[15] M. A. Fahmy, A Computerized DRBEM model for generalized magneto-thermo-visco-elastic stress waves in functionally graded anisotropic thin film/substrate structures, Latin American Journal of Solids and Structures. 2014;11:386-409.
[16] M. A. Fahmy, A Predictor-corrector Time-Stepping Drbem for Shape Design Sensitivity and Optimization of Multilayer FGA Structures, Transylvanian Review. 2017;XXV:5369-5382.
[17] M. A. Fahmy, Computerized boundary element solutions for thermoelastic problems: Applications to functionally graded anisotropic structures, LAP Lambert Academic Publishing, Saarbrücken, Germany; 2017.
[18] M. A. Fahmy, Boundary Element Computation of Shape Sensitivity and Optimization: Applications to Functionally Graded Anisotropic Structures, LAP Lambert Academic Publishing, Saarbrücken, Germany; 2017.
[19] M. A. Fahmy, Shape design sensitivity and optimization for two-temperature generalized magneto-thermoelastic problems using time-domain DRBEM, Journal of Thermal Stresses. 2018;41:119-138.
[20] M. A. Fahmy, Shape design sensitivity and optimization of anisotropic functionally graded smart structures using bicubic B-splines DRBEM, Engineering Analysis with Boundary Elements. 2018;87:27-35.
[21] M. A. Fahmy, Modeling and Optimization of Anisotropic Viscoelastic Porous Structures Using CQBEM and Moving Asymptotes Algorithm, Arabian Journal for Science and Engineering, 2018 (In Press)
[22] M. A. Fahmy, Boundary Element Algorithm for Modeling and Simulation of Dual Phase Lag Bioheat Transfer and Biomechanics of Anisotropic Soft Tissues, International Journal of Applied Mechanics, vol. 10, no. 10, 2018 (In Press)
[23] J. Gao and W. Yao, (2017). Thermal stress analysis for bi-modulus foundation beam under nonlinear temperature difference. International Journal of Computational Methods, vol. 14, 2017, pp. 1750024.
[24] M. R. Akbari and J. Ghanbari, Analytical Solution of Thermo-elastic Stresses and Deformation of Functionally Graded Rotating Hollow Discs with Radially Varying Thermo-mechanical Properties under Internal Pressure, CMC, vol.45, no.3, pp.187-201, 2015
[25] M. A. Fahmy, Thermal Stresses in a Spherical Shell Under Three Thermoelastic Models Using FDM, International Journal of Numerical Methods and Applications, vol. 2, 2009, pp. 123–128.
[26] A. M. El-Naggar, A. M. Abd-Alla, M. A. Fahmy and S. M. Ahmed, Thermal Stresses in a rotating non-homogeneous orthotropic hollow cylinder, Heat and Mass Transfer, Vol. 39, 2002, pp. 41-46.
[27] A. M. Abd-Alla, A. M. El-Naggar and M. A. Fahmy, Magneto-thermoelastic problem in non-homogeneous isotropic cylinder, Heat and Mass Transfer, Vol. 39, 2003, pp. 625-629.
[28] A. M. El-Naggar, A. M. Abd-Alla and M. A. Fahmy, The propagation of thermal stresses in an infinite elastic slab, Applied Mathematics and Computation, Vol. 12, 2003, pp. 220-226.
[29] M. A. Fahmy, Finite difference algorithm for transient magneto-thermo-elastic stresses in a non-homogeneous solid cylinder, International Journal of Materials Engineering and Technology, Vol. 3, 2010, pp. 87-93.
[30] A. M. Abd-Alla, T. M. El-Shahat and M. A. Fahmy, Thermal stresses in a rotating non-homogeneous anisotropic elastic multilayered solids, Far East Journal of Applied Mathematics, Vol. 27, Issue 2, 2007, pp. 223-243.
[31] A. M. Abd-Alla, T. M. El-Shahat and M. A. Fahmy, Effect of inhomogeneity on the thermoelastic stresses in micro-engineering anisotropic solid, Far East Journal of Applied Mathematics, Vol. 27, Issue 2, 2007, pp. 245-264.
[32] A. M. Abd-Alla, T. M. El-Shahat and M. A. Fahmy, Thermoelastic stresses in inhomogeneous anisotropic solid in the presence of body force, International Journal of Heat & Technology, Vol. 25, Issue 1, 2007, pp. 111-118.
[33] A. M. Abd-Alla, M. A. Fahmy and T. M. El-Shahat, Magneto-thermo-elastic stresses in inhomogeneous anisotropic solid in the presence of body force, Far East Journal of Applied Mathematics, Vol. 27, Issue 3, 2007, pp. 499-516.
[34] A. M. Abd-Alla, M. A. Fahmy and T. M. El-Shahat, Transient piezothermoelastic stresses in a rotating non-homogeneous composite structure, Far East Journal of Applied Mathematics, Vol. 27, Issue 3, 2007, pp. 489-497.
[35] M. A. Fahmy, Effect of initial stress and inhomogeneity on magneto-thermo-elastic stresses in a rotating anisotropic solid, JP Journal of Heat and Mass Transfer, Vol. 1, 2007, pp. 93-112.
[36] A. M. Abd-Alla, M. A. Fahmy and T. M. El-Shahat, Magneto-thermo-elastic problem of a rotating non-homogeneous anisotropic solid cylinder, Archive of Applied Mechanics, Vol. 78, Issue 2, 2008, pp. 135-148.
[37] M. A. Fahmy, Thermoelastic stresses in a rotating non-homogeneous anisotropic body, Numerical Heat Transfer, Part A: Applications, Vol. 53, Issue 9, 2008, pp. 1001-1011.
[38] M. A. Fahmy and T. M. El-Shahat, The effect of initial stress and inhomogeneity on the thermoelastic stresses in a rotating anisotropic solid, Archive of Applied Mechanics, Vol. 78, Issue 6, 2008, 431-442.
[39] M. A. Fahmy and S. A. Salama, Boundary element solution of steady-state temperature distribution in non-homogeneous media, Far East Journal of Applied Mathematics, Vol. 43, Issue 1, 2010, pp. 31-40.
[40] M. A. Fahmy, Application of DRBEM to non steady-state heat conduction in non-homogeneous anisotropic media under various boundary elements, Far East Journal of Mathematical Sciences, Vol. 43, Issue 1, 2010, pp. 83-93.
[41] M. A. Fahmy, Influence of inhomogeneity and initial stress on the transient magneto-thermo-visco-elastic stress waves in an anisotropic solid, World Journal of Mechanics, Vol. 1, 2011, pp. 256-265.
[42] M. A. Fahmy, A time-stepping DRBEM for magneto-thermo-viscoelastic interactions in a rotating nonhomogeneous anisotropic solid, International Journal of Applied Mechanics, Vol. 3, 2011, pp. 1-24.
[43] M. A. Fahmy, Saleh Manea Al-Harbi and Badr Hamedy Al-Harbi, Implicit Time-Stepping DRBEM for Design Sensitivity Analysis of Magneto-Thermo- Elastic FGA Structure Under Initial Stress, American Journal of Mathematical and Computational Sciences, Vol. 2, 2017, pp. 55-62.
[44] M. A. Fahmy, The Effect of Anisotropy on the Structure Optimization Using Golden-Section Search Algorithm Based on BEM, Journal of Advances in Mathematics and Computer Science, Vol. 25, 2017, pp. 1-18.
[45] M. A. Fahmy, DRBEM Sensitivity Analysis and Shape Optimization of Rotating Magneto-Thermo-Viscoelastic FGA Structures Using Golden-Section Search Algorithm Based on Uniform Bicubic B-Splines, Journal of Advances in Mathematics and Computer Science, vol. 25, Issue 6, 2017, pp. 1-20.
[46] M. A. Fahmy, The DRBEM solution of the generalized magneto-thermo-viscoelastic problems in 3D anisotropic functionally graded solids. I. Idelsohn, M. Papadrakakis, B. Schrefler (eds.), 5th International conference on coupled problems in science and engineering (Coupled Problems 2013), Ibiza, Spain, June 17-19, 2013, pp. 862-872.
[47] M. A. Fahmy, Boundary Element Solution of 2D Coupled Problem in Anisotropic Piezoelectric FGM Plates. Proceedings of the, B. Schrefler, E. Oñate and M. Papadrakakis (eds.), 6th International Conference on Computational Methods for Coupled Problems in Science and Engineering (Coupled Problems 2015), Venice, Italy, May 18-20, 2015, pp. 382-391.
[48] M. A. Fahmy, 3D DRBEM Modeling for Rotating initially stressed Anisotropic functionally graded piezoelectric Plates, 7th European Congress on Computational Methods in Applied Sciences and Engineering (ECCOMAS 2016) M. Papadrakakis, V. Papadopoulos, G. Stefanou, V. Plevris (eds.), Crete Island, Greece, June 5–10, 2016, 7640-7658.
[49] M. A. Fahmy, A time-stepping DRBEM for 3D anisotropic functionally graded piezoelectric structures under the influence of gravitational waves, Rodrigues H, Elnashai A, Calvi GM (eds.), Facing the Challenges in Structural Engineering, Sustainable Civil Infrastructures, Proceedings of the 1st GeoMEast International Congress and Exhibition (GeoMEast 2017), Sharm El Sheikh, Egypt, July 15-19, 2017, pp. 350-365.
[50] M. A. Fahmy, A. M. Salem, M. S. Metwally and M. M. Rashid, Computer Implementation of the DRBEM for Studying the Generalized Thermoelectric Responses of Functionally Graded Anisotropic Rotating Plates with One Relaxation Time, International Journal of Applied Science and Technology, Vol. 3, Issue 7, 2013, pp. 130-140.
[51] M. A. Fahmy, A. M. Salem, M. S. Metwally and M. M. Rashid, Computer Implementation of the DRBEM for Studying the Classical Uncoupled Theory of Thermoelectricity of Functionally Graded Anisotropic Rotating Plates, International Journal of Engineering Research and Applications, Vol. 3, Issue 6, 2013, pp. 1146-1154.
[52] M. A. Fahmy, A. M. Salem, M. S. Metwally and M. M. Rashid, Computer Implementation of the DRBEM for Studying the Classical Coupled Thermoelectric Responses of Functionally Graded Anisotropic Plates, Physical Science International Journal, Vol. 4, Issue 5, 2014, pp. 674-685.
[53] M. A. Fahmy, A. M. Salem, M. S. Metwally and M. M. Rashid (2014), Computer Implementation of the DRBEM for Studying the Generalized Thermo Elastic Responses of Functionally Graded Anisotropic Rotating Plates with Two Relaxation Times, British Journal of Mathematics & Computer Science, Vol. 4, Issue 7, 2014, pp. 1010-1026.
Table 1. Optimization analysis for considered materials.

| Material | Iterations | Percentage change between final and initial value | Maximum Stress | Reduction of Compliance |
|----------|------------|---------------------------------------------------|----------------|------------------------|
| A        | 12         | 65%                                               | 0.411          | 92.40                  |
| B        | 12         | 65%                                               | 0.390          | 90.87                  |
| C        | 12         | 73%                                               | 0.223          | 91.10                  |

Table 2. Comparison of computer resources needed for FEM and DRBEM modelling of the right half of the link plate design.

|                         | FEM     | DRBEM   |
|-------------------------|---------|---------|
| Number of elements      | 12980   | 48      |
| CPU-Time [min.]         | 190     | 3       |
| Memory [Mbyte]          | 140     | 0.6     |
| Disc space [Mbyte]      | 200     | 0       |
| Accuracy of results [%] | 2.3     | 1.3     |

Fig. 1. a) Geometry, b) Boundary element model for the link plate.
Fig. 2. Optimum shape design for the link plate.

Fig. 3. Compliance iteration history for the link plate.

Fig. 4. Variation of the temperature $T$ sensitivity with time $\tau$. 
Fig. 5. Variation of the displacement $u_z$ sensitivity with time $\tau$.

Fig. 6. Variation of the displacement $u_z$ sensitivity with time $\tau$. 