NTRUCipher-Lattice Based Secret Key Encryption

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Abstract—NTRU cryptosystem has allowed designing a range of cryptographic schemes due to its flexibility and efficiency. Although NTRU cryptosystem was introduced nearly two decades ago, it has not yet received any attention like designing a secret key encryption. In this paper, we propose a secret key encryption over NTRU lattices, named as NTRUCipher. This NTRUCipher is designed using modification of the NTRU public key encryption. We analyze this cipher efficiency and the space complexity with respect to security aspects, and also show that the NTRUCipher is secured under the indistinguishability chosen plaintext attack.

Key words - NTRUCipher; multiple transmission attack; product form polynomials; secret key encryption.

I. INTRODUCTION

Cryptosystem is classified as the secret key cryptosystem (symmetric key cryptosystem) and public key cryptosystem (asymmetric cryptosystem) based on nature of cryptographic key functions and properties. The secret key cryptosystem could be categorized as the stream cipher and block cipher based on size of the secret key, plaintext and ciphertext. The block ciphers are the most influential cryptographic primitives in designing cryptographic schemes such as encryptions, hash functions, and message authentication codes. In the secret key encryption, communication parties, a sender and receiver share and use a common key, as the secret key, whereas in the public key cryptosystem both the sender and receiver contain their own secret key and associated public key. The secret key cryptosystem provides cryptographic services such as confidentiality, integrity and authentication of a message. The strength of the secret key encryption relies on two parameters: strength of algorithm and length of the secret key. The well known secret key block ciphers are the Advanced Encryption Standard (AES) [10], RC5 [15], Blowfish [3], Data Encryption Standard (DES) [2], and International data encryption algorithm (IDEA) [17]. These ciphers are built using Feistel network except IDEA for encryption and decryption. The Feistel network [7] was designed using XOR operator and Permutation-Box (P-Box) and Substitution-Box (S-Box).

The proposed cipher presented in this paper is designed over finite fields. Most of the promising candidates of the NTRU cryptosystems are NTRUEncrypt [8] and NTRUSign [9] which are resistance to the Shor’s algorithm [14] on quantum computers. Damien Stehle et al. proposed a provable security version of the NTRU public key cryptosystem over the ring, \( R = \mathbb{Z}_q[x]/(x^n + 1) \), where \( n \) is a power of 2 and \( q \) is a prime, which provides encryption [5,6] and digital signature [6]. There is still scope to work further on their contribution in terms of setting up a specific security parameters. Recently, Daniel J Bernstein et al. proposed another variant of the NTRU, named as NTRU Prime [4], over the ring, \( R = \mathbb{Z}_q[x]/(x^n - x - 1) \), where \( n \) is a prime and \( q \) is a power of 2. In this work, we propose lattice based secret key encryption over NTRU lattices, named as NTRUCipher. The framework of the encryption and decryption is the same as NTRUEncrypt [8] and the decryption is a probabilistic like the NTRUEncrypt [8]. Furthermore, we prove the NTRUCipher is secured under the indistinguishability chosen plaintext attack (IND-CPA), and analyze efficiency and the space complexity with respect to security aspects.

The paper is organized as follows: In Section II, we recall the definition of a secret key encryption and adversary model. Section III presents truncated polynomial rings. In Section IV, we propose the NTRUCipher - lattice based secret key encryption and recommend parameters for the cipher. In Section V, we analyze the NTRUCipher with respect to performance, security aspects, and space and time complexity. Finally, we provide conclusion remarks in Section VI.

II. PRELIMINARIES

In this Section, we recall the formal definition of a secret key encryption and its security notations.

A. Notations

A real valued function \( \epsilon(c) < c^{-\lambda} \) is negligible if for every \( \lambda > 0 \) there exists \( c_{\lambda} > 0 \) such that \( \epsilon(c) < c^{-\lambda} \) for all \( c > c_{\lambda} \). A probabilistic polynomial time algorithm is said to be efficient if its running time is polynomial in its input length. We represent \( x \) a random variable sampled from the probability distribution \( D \) as \( x \leftarrow D \). The effectiveness of an algorithm to distinguish between
two probability distributions $D_0$ and $D_1$ is measured by its distinguishing advantage, defined by $|Pr_{x \in D_0}[A(x) = 1] - Pr_{x \in D_1}[A(x) = 1]|$. We say that a decision problem is hard if there does not exist an efficient algorithm for it that has a non-negligible advantage in $\lambda$. The statistical distance $\Delta(D_0; D_1)$ between two distributions $D_0, D_1$ on some countable domain $X$ is defined as $\Delta(D_0; D_1) = |D_0(x) - D_1(x)|$.

B. Secret Key Encryption

The goal of the secret key encryption is to furnish confidentiality of communications of two or more parties. In the secret key encryption, a common secret key is shared among the communication parties, before decryption of the ciphertext, to furnish confidentiality of the plaintext. The following definitions are acquired from [12, 13]. For further details, the reader is referred to [12, 13].

Definition 1. A secret key encryption $SE = (KeyGen, Enc, Dec)$ consists of the following three algorithms:

(i) Key Generation: $KeyGen(1^\lambda)$ is a randomized algorithm that outputs a random key. When the algorithm is run, a different key is generated every time. Note that in this case, the input for the algorithm is a null.

(ii) Encryption: $Enc(\mu, k)$ is a randomized algorithm that takes a plaintext $\mu$ and the secret key $k$ as input, and outputs a ciphertext $c$.

(iii) Decryption: $Dec(c, k)$ is a deterministic algorithm.

Correctness of the secret key encryption works as follows: We say that a secret key encryption $SE = (KeyGen, Enc, Dec)$ is correct, if it holds for every plaintext $\mu$ that $Pr[Dec(Enc(\mu), k) \neq \mu] < \text{negl}(\lambda)$.

C. Security Notations

The aim of an adversary $A$ is to capture the secret key of the secret key encryption and then perceive the plaintext corresponding to the ciphertext. We assume that the adversary does not have prior knowledge of the secret key $k$. The indistinguishability under chosen-plaintext attack or IND-CPA security is defined as follows:

Definition 2. Let $A$ be an adversary. Let $SE = (KeyGen, Enc, Dec)$ be the secret key encryption. Let us define the following experiment between a challenger and $A$:

1. The challenger runs $k \leftarrow KeyGen(1^\lambda)$.
2. $A$ outputs a pair of plaintexts $(\mu_0, \mu_1)$ of the same length and sends $(\mu_0, \mu_1)$ to the challenger.
3. The challenger computes $c^* \leftarrow Enc(\mu, k)$ and then sends $c^*$ to the adversary.
4. $A$ continues its computation and outputs $b'$.
5. Output 1 if $b = b'$, and 0 otherwise.

Definition 3. A secret key encryption $SE = (KeyGen, Enc, Dec)$ is indistinguishable under chosen plaintext attack, if it holds for all probabilistic polynomial time adversary $A$ that

$Adv^{IND\text{-}CPA}_{SE}(A) = |Pr[Exp^{IND\text{-}CPA}_{SE}(A) = 1] - \frac{1}{2}| \leq \text{negl}(\lambda)$.

III. TRUNCATED POLYNOMIAL RINGS

Let $q \in \mathbb{N}$ be a prime. We write $\mathbb{Z}_q$ for the integer modulo $q$ and represents this set by integers in the range $\left(-\frac{q}{2}, \frac{q}{2}\right)$. The truncated polynomial ring $\mathbb{Z}_{n,q} = \mathbb{Z}_q[x]/(x^n + 1)$ consists of all polynomials with coefficients in $\mathbb{Z}_q$ and degree less than $n$. An element $f \in R_q$ is represented as a polynomial,

$f = \sum_{i=0}^{n-1} f_i x^i = [f_0, f_1, \ldots, f_{n-1}]$.

Two polynomials $f, g \in R_q$ are multiplied by the ordinary convolution,

$(f \ast g)_k = \sum_{i+j=k \text{ mod } n} (f_i, g_j), k = 0, 1, \ldots, n-1$, which is commutative and associative. The convolution product is represented by $\ast$ to distinguish it from the multiplication in $\mathbb{Z}_q$. We define a center $l_2$-norm of an element $f \in R_q$ by

$||f||_2 = \sum_{i=0}^{N-1} f_i x^i/||f||_\infty = \max_{0 \leq i \leq n-1} ||f_i||_\infty$.

Lemma 1[14, 15]. For any $f, g \in R_{n,q} = \mathbb{Z}_q[x]/(x^n + 1)$, $||f \ast g||_\infty \leq \sqrt{n} ||f||_\infty ||g||_\infty$ and $||f \ast g||_\infty \leq n. ||f||_\infty ||g||_\infty$.

A. Cryptographic Assumptions

In this Subsection, we define the NTRUCipher ciphertext cracking problem for which the parameters are chosen as recommended in the table 1. The search and decision ciphertext cracking problems are defined as follows:

1. Search NTRUCipher Ciphertext Cracking Problem: Given $c = r \ast k^{-1} + \mu (mod q) \in R_{n,q}$, with $r \leftarrow \mathbb{P}_n(a_1, a_2, a_3)$, $k \leftarrow \mathbb{P}_n(a_1, a_2, a_3)$, compute $(r, k, \mu)$.

2. Decision NTRUCipher Ciphertext Cracking Problem: Given $c = r \ast k^{-1} (mod q) \in R_{n,q}$, distinguish whether $c$ is sampled from the distribution $D_0 = \{c = r \ast k^{-1} (mod q) : r \leftarrow \mathbb{P}_n(a_1, a_2, a_3), k \leftarrow \mathbb{P}_n(a_1, a_2, a_3)\}$ or from the uniform distribution $D_1 = U(R_{n,q})$.

We assume that the decision NTRUCipher Ciphertext Cracking Problem is hard to indistinguish computationally.
IV. NTRUCipher

In this Section, we propose NTRUCipher which is drawn from NTRUEncrypt [24] by modification. In this cipher, we use a ring of \( R_{n,q} = \mathbb{Z}/[x]/(x^n + 1) \) and propose NTRUCipher-lattice based secret key encryption.

**System Parameters:** The cipher would have three integer parameters, \( n, p \) and \( q \). The integer \( n \) has to be \( 2^i \), \( p \) is a small prime, and \( q \) is large prime such that \( \gcd(p, q) = 1 \), and \( p < q \).

| Table I |
|------------------|
| NTRUCipher - System Parameters |
| 1. \( n \) - Degree Parameter |
| 2. \( q \) - Large Modulus |
| 3. Ring Parameters, \( R_{n,q} = \mathbb{Z}/[x]/(x^n + 1) \) |
| 4. \( p \) - Plaintext space modulus |
| 5. \( a_1, a_2, a_3 \) - Non-zero coefficient counts for product form polynomial terms. |
| 6. \( \mu \) - Plaintext |
| 7. \( c \) - Ciphertext |
| 8. \( k \) - Secret key |
| 9. \( r \) - Ephemeral key |
| 10. \( D_k \) - Ciphertext space |
| 11. \( D_k \) - Ephemeral key space |
| 12. \( B_n \) - (Binary Polynomials) |
| 13. \( T_n \) = (Ternary Polynomials) |
| 14. \( P_n(a_1, a_2, a_3) \) = (Product form of polynomials \( A_1 \ast A_2 + A_3 : A_i \in \mathbb{T}_n(a_1, a_2, a_3) \)) |

(i) **Key Generation:** 1. The secret key \( k \) is a polynomial of the form \( p \cdot k' + 1 \), where \( k' \) is generated by product form of polynomials, \( P_n(a_1, a_2, a_3) \). Note that this form ensures that \( k \) has inverse 1 modulo \( p \).

Algorithm 1 NTRUCipher-Key Generation

**Input:** A set of system parameters

1: \( \text{repeat} \)
2: \( k' \leftarrow P_n(a_1, a_2, a_3) \)
3: \( k = 1 + p \cdot k' \in R_{n,q} \)
4: until \( k \) is invertible in \( R_{n,q} \)

**Output:** Secret key \( k \).

(ii) **Encryption:**

1. To encrypt a plaintext \( \mu \in \{-(p-1)/2, \ldots, (p-1)/2\}^n \) with the secret key \( k \), first a polynomial \( r \) is randomly sampled in \( P_n(a_1, a_2, a_3) \) such that \( r \ast k'\) \( (mod \ q) \in R_{n,q} \).
2. Compute ciphertext \( c = (p \cdot r \ast k' + \mu)(mod \ q) \).

Algorithm 2 NTRUCipher-Encryption

**Input:** Secret key \( k \), message \( \mu \in \{-(p-1)/2, \ldots, (p-1)/2\}^n \), and set a parameter, \( p = 3 \).

1: \( \text{repeat} \)
2: \( r \leftarrow P_n(a_1, a_2, a_3) \)
3: \( c = (p \cdot r \ast k' + \mu)(mod \ q) \in R_{n,q} \)

**Output:** Ciphertext \( c \)

(iii) **Decryption:** To decrypt the ciphertext \( c \) with respect to the secret key \( k \),

1. Compute first ciphertext, \( c' = c \ast k(mod \ q) \), and center the coefficient of \( c' \) in \((-q/2, q/2)\).
2. Then, compute \( \mu' = c' (mod \ p) \), and center the coefficient in \((-p/2, p/2)\) to get the plaintext \( \mu \).

Algorithm 3 NTRUCipher-Decryption

**Input:** Secret key \( k \), Ciphertext \( c \in R_{n,q} \), and set a parameter, \( p = 3 \).

1: \( c' = c \ast k(mod \ q) \in R_{n,q} \)
2: Center the coefficients of \( c' \) in \((-q/2, q/2)\)
3: \( \mu' = c' (mod \ p) \)
4: Center the coefficients of \( \mu' \) in \((-p/2, p/2)\)
5: Result = \( \mu \)

**Output:** Plaintext \( \mu \)

**Completeness:** To decrypt the ciphertext \( c \) with respect to the secret key \( k \),

1. Compute first ciphertext, \( c' = c \ast k(mod \ q) = (p \cdot r + \mu \ast k)(mod \ q) \), and center the coefficient of \( c' \) in \((-q/2, q/2)\).
2. Then, compute \( \mu' = c' (mod \ p) \), and center the coefficient in \((-p/2, p/2)\) to get the plaintext \( \mu \).

A. Probability of decryption failure

In this subsection, we estimate decryption failure of the NTRUCipher in terms of probability. For a successful decryption of the ciphertext \( c \) for the plaintext \( \mu \) using the given secret key \( k \), the coefficient of \( c' = [p \cdot (r + \mu \ast k') + \mu](mod \ p) \) must be in the range of less than \( \frac{q}{2} \). By the triangle inequality, the following relation holds:

\[
\|c'\|_{\infty} \leq \|p\|_{\infty} \|r\|_{\infty} + 1 + \|\mu\|_{\infty} \|k'\|_{\infty} + 1 \leq \frac{q}{2}.
\]

We assume that \( r \) and \( k' \) are chosen in the product form and the plaintext \( \mu \) is a ternary polynomial. Note that the decryption failure can be avoided by ensuring \( q > 8p(2a_1a_2 + a_3) + 2 \). We hereby set a probabilistic bound to estimate the probability.

\[\text{Prob}(a \text{ given coefficients of } r + \mu \ast k') \text{ has absolute value } \geq B).\]

We choose \( r \) and \( k' \) in the product form such that \( r = r_1 \ast r_2 + r_3,k' = k_1 \ast k_2 + k_3 \), where each \( r_i \) and \( k_i \) has exactly \( a_i \) coefficients equal to 1, \( a_i \) coefficients equal to -1 and the rest of the coefficients equal to 0. When the coefficients of the plaintext \( \mu \) are chosen from \( \{-(p-1)/2, \ldots, (p-1)/2\}^n \), the probability of taking 0 as coefficients of plaintext \( \mu \) is \( \frac{a_0}{\alpha_0} \), and taking \( \pm1 \) as coefficients is \( (1 - \frac{a_0}{\alpha_0}) \). The coefficients of \( r + \mu \ast k' \) are expected to be distributed according to the convolution of normal distribution with standard deviation, \( \sigma = \sqrt{(4a_1a_2 + 2a_3)(2 - \frac{a_0}{\alpha_0})} \), which is computed by adopting technique of section 6 of [11].

The probability that a normally distributed random variable with mean 0 and standard deviation \( \sigma \) exceeds \( B \) in absolute value is given by the complementary error function, \( erfc(B/\sqrt{2\sigma}) \). Thus, the probability that any of the \( n \) coefficients of \( r + \mu \ast k' \) is greater than \( B \) is bounded by
$n.er.f(c(B/(\sqrt{2}\sigma))$. With respect to security parameter $\lambda$ this imposes the constraint $n.er.f(c(q - 2)/(2\sqrt{2}\sigma)) < 2^{-\lambda}$, where $\sigma = \sigma(n, a_1, a_2, a_3, a_\mu)$.

B. Parameter Sets and Sample space

In this Subsection, we give set of parameters for the NTRUCipher. We discuss how the NTRUCipher parameters are chosen.

1) Binary polynomials: Binary polynomials $B_n$ are used in this cipher to generate product form of polynomials. These can be easy to implement in software and hardware. A disadvantage is that binary polynomials are by definition unbalanced. Therefore, when $k(1) \neq 0$, as a consequence information on the plaintext $\mu$, namely $\mu(1)$ leaks.

2) Ternary polynomials: We define $T_n$ as the set of all ternary polynomials, a particular case $T_n(a,e)$ with $a$ coefficients are $1, e$ coefficients are $-1$ and rest of the coefficients are $0$. These ternary polynomials are used to make product form of polynomials.

3) Product form of polynomials: Product form of polynomials $P_n$ are generated by $P_n(a_1, a_2, a_3) = \{a_1a_2+a_3 : a_1 \in T_n(a,e), a_2 \in T_n(a,e), a_3 \in T_n(a,e)\}$. These polynomials are used in this cipher to choose the secret key $k$ and ephemeral key $r$. For instance, the secret key $k$ is chosen of the form $k = 1+p.k'$ , where $k' \in P_n(a_1, a_2, a_3)$. The number of non-zero coefficients in $k'$ and $r$ are crucial for the performance of the encryption. Note that convolution can be faster if there are a small number of non-zero elements in the polynomial. An advantage of the product form of polynomials is that they allow for exceptionally fast convolution without Fourier transforms.

4) Secret Key Space: The space of secret key $D_k$ consists of all polynomials that are derived from $P_n(a_1, a_2, a_3)$. If $n$ and $q$ are fixed in advance, we can choose $a_1, a_2$ and $a_3$ with $\pm 1$’s for the secret key $k$. When we take $a_1 \approx a_2 \approx a_3$, the expected number of non-zero coefficients in $k$ are $4a_1a_2+2a_3 \approx 2a_3$ which is an optimal for key selection. The space complexity of the secret key is $O((2||k||_{\infty}+1)^n)$. The size of the secret key is $n[log(2||k||_{\infty}+1)]$.

5) Ephemeral key space: The space of ephemeral key $D_r$ consists of all polynomials that are derived from $P_n(a_1, a_2, a_3)$. The space complexity of the ephemeral key is $O((2||r||_{\infty}+1)^n)$. The size of the ephemeral key is $n[log(2||r||_{\infty}+1)]$.

6) Plaintext space: The plaintext space $D_\mu$ is defined as $D_\mu = \{\mu \in R_n,q/\mu \text{ has coefficients in } (-\frac{a_1}{2}, \frac{a_1}{2})\}$, assuming $p$ odd primitive. The recommended parameter, $p = 3$. The space complexity of the plaintext space is $O(p^n)$. The size of the plaintext is $n[log_2 q]$.

7) Ciphertext space: The ciphertext space $D_\mu$ is defined as $D_\mu = \{c \in R_n,q/c \text{ has coefficients in } (-\frac{a_1}{2}, \frac{a_1}{2})\}$, assuming $q$ odd primitive. The complexity of the ciphertext space is $O(q^n)$. The size of the ciphertext is $n[log_2 q]$.

C. Efficiency

We provide efficiency of the NTRUCipher in terms of $O$ notations. In this NTRUCipher, we can perform addition and multiplication of two polynomials over the ring, $R_n,q = \mathbb{Z}_q[x]/(x^n + 1)$ in $O(n)$ and $O(n^2[log_2 q])$ bits operations respectively. Thus, it follows that the cost of encryption and decryption in both cases is $O(n^2[log_2 q])$ bit operations.

D. Concrete Parameters Set

Our recommendations for parameters of the NTRUCipher suggests taking $n = 256$, $p = 3$, $q = 1087$ , $a_1 = 5$, $a_2 = 5$, $a_3 = 5$, and $a_\mu = 102$ so that we will ensure upper bound on the security parameter that our decryption failure probability is less than $2^{-80}$.

V. Security Analysis

1) Brute-force attack: Brute-force attack is one of the generic cryptographic attacks in which one could try for every possible key permutation until it finds the secret key. The feasibility to find out the secret key in brute force attack relies on key space. In this cipher, the key space order for $n$ bit length is $O((2||k||_{\infty}+1)^n)$. In the proposed cipher, we use minimum key length of 256 bits. Therefore, one should try $2(||k||_{\infty}+1)^{256}$ bit permutations to find the secret key which is large enough for brute force attack. The plaintext $\mu$ is chosen as a polynomial with ternary coefficients of degree 256 in the NTRUCipher, one should try $2^{256}$ bit permutations to find the plaintext $\mu$ on the brute force attack. According to knowledge of the author, currently complexity order $2^{80}$ is considered as the lower bound of security for brute force attack.

2) Multiple transmission attack: In this NTRUCipher, one would send a single plaintext $\mu$ multiple times using the same secret key $k$ and different ephemeral keys, $r$’s. In this scenario, one would transmit $t$ ciphertexts such that $c_i = p.r_i \ast k^{-1} + \mu(mod q)$ for $i = 1, 2, 3, ...., t$. The adversary can then compute $(c_i - c_1) = (r_i - r_1) \ast k^{-1}(mod q)$. Note that $c = r \ast k^{-1}(mod q)$, where $c = c_i - c_1$ and $r = r_i - r_1$, has same structure as the public key generation of the NTRUEncrypt [2,4]. Since $c = r \ast k^{-1}(mod q)$, there exists $u \in R_n,q$ with $u = \frac{c}{r + k^{t+1}}$ such that $[k, u]\Sigma_c = [k, r]$, where $\Sigma_c = \begin{bmatrix} 1 & c \\ 0 & q \end{bmatrix}$. By choosing the appropriate parameters for $[k, r]$, one can find the secret key $k$ by solving SVP (approximate SVP) in $\Sigma_c$.

3) Chosen Plaintext attack: In this NTRUCipher attack, the adversary can randomly choose plaintexts to be encrypted and gets corresponding ciphertexts. The goal of the attack is to get access to the information that reduces the security of the secret key encryption. We hereby prove the NTRUCipher is indistinguishability chosen plaintext attack secure. We use the following game IND-CPA between a challenger and an adversary.

Theorem 1. The NTRUCipher-Lattice based secret key encryption is INA-CPA Secure if the decision NTRUCipher ciphertext cracking problem is hard.

Proof: Let $A$ be an adversary. $A$ is given random oracle access to $\text{NTRUCipher} - \text{Enc}(\cdot)$ and outputs two plaintexts $\mu_0, \mu_1$ of equal length of $n$ and sends to a challenger. The challenger runs an algorithm $C$ and picks $b \leftarrow \{0, 1\}$,
and computes challenging ciphertext \( c^* = (p, r^* \cdot k^* + \mu)(\text{mod } q) \) instead of \( c = (p, r \cdot k + \mu)(\text{mod } q) \). Then, the challenger sends the challenging ciphertext \( c^* \) to the adversary \( A \). Eventually, when the adversary \( A \) outputs his guess \( b^* \) for \( b \), the algorithm \( C \) outputs 1 if \( b = b^* \), and 0 otherwise.

We now compute advantage function of the adversary under indistinguishability (IND) chosen plaintext attack (CPA) experiment: \( \text{Adv}_{\text{IND-CPA}}^{NTRUCipher}(A) := \Pr[\text{Exp}_{\text{IND-CPA}}^{NTRUCipher}(A) = 1] - \frac{1}{2} \). We say that the NTRUCipher satisfies IND-CPA, if the advantage \( \text{Adv}_{\text{IND-CPA}}^{NTRUCipher}(A) \) is negligible for any polynomial time adversary \( A \).

If we fix \( k^* \in R_q \), the encrypted oracle uses the value \( r^* \) to answer at least one of the \( n \) queries of \( A \). Let \( q_n \) denote the total number of queries made by \( A \). In this experiment, \( A \) can succeed by one of the two possibilities. If \( r^* \) is drawn from \( D_0 \) for some queries of \( A \), then \( A \) succeeds and the algorithm \( C \) returns 1 with probability is at most \( \frac{1}{2} + \frac{\mu}{|R_q|} \). If \( r^* \) is drawn from \( D_1 \) for some queries of \( A \), then \( A \) succeeds only by guessing \( b \), and the algorithm \( C \) returns 1 with probability \( \frac{1}{2} \). Thus, we have \( \Pr[\text{Exp}_{\text{IND-CPA}}^{NTRUCipher}(A) = 1] \leq \frac{1}{2} + \frac{\mu}{|R_q|} \). The advantage of the adversary of the NTRUCipher, \( \text{Adv}_{\text{IND-CPA}}^{NTRUCipher}(A) := \Pr[\text{Exp}_{\text{IND-CPA}}^{NTRUCipher}(A) = 1] - \frac{1}{2} \leq \frac{\mu}{|R_q|} \leq \frac{\mu}{q} \leq \frac{\mu}{q} \leq \text{negl}(\lambda) \). Therefore, the NTRUCipher is IND-CPA secure.

VI. CONCLUSION

In this paper, we have proposed a secret key encryption which is based on truncated polynomials over NTRU lattices instead of classical well known Feistel structure [7]. Attacks such as brute force attack, multiple transmission attack and IND-CPA have been exposed against the proposed NTRUCipher. We have also recommended a set of parameters and analyzed security aspects and efficiency. A disadvantage of the NTRUCipher is that size of ciphertext is very larger compared to other existing secret key block ciphers [2,10]. Further work is required for designing the NTRUCipher based homomorphic secret key encryption and message authentication code.

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