Hydrodynamics of $\mathcal{N} = 6$ Superconformal Chern-Simons Theories at Strong Coupling

Mohammad R. Garousi$^1$ and Ahmad Ghodsi$^2$

Department of Physics, Ferdowsi University of Mashhad, P.O.Box 91775-1436, Mashhad, Iran
and
Institute for Research in Fundamental Sciences, P.O.Box 19395-5531, Tehran, Iran

Abstract

Using the duality conjecture between $\mathcal{N} = 6$ supersymmetric $U(N)_k \times U(N)_{-k}$ Chern-Simons theory and M-theory on $AdS_4 \times S^7/Z_k$, we calculate the corrections to the shear viscosity of the field theory at temperature $T$. At strong 't Hooft coupling and at small $k$ level, we have considered one-loop correction to the M-theory effective action. At large $k$ level, we have considered the $\alpha'$ correction to the type IIA effective action. In both cases the correction to the ratio of shear viscosity to the entropy density is positive.

$^1$garousi@mail.ipm.ir
$^2$ahmad@mail.ipm.ir
1 Introduction

One of the most exciting observations from AdS/CFT correspondence is the universality of the ratio of the shear viscosity $\eta$ to the entropy density $s$, for any gauge theory with an Einstein gravity dual in the limit of large color $N$ and large 't Hooft coupling $\lambda$ [1–4]. It has been conjectured in [4] that this universal ratio is the lower bound of all materials at strong couplings, \( i.e., \)

\[
\frac{\eta}{s} = \frac{1}{4\pi} + \alpha
\]

where $\alpha$ is a positive number.

The Einstein gravity is not renormalizable, hence, it can not be considered as a consistent dual theory for a gauge theory. The candidate theory for quantum gravity, \( i.e., \) string theory/M-theory, contains Einstein gravity as well as the higher derivative corrections resulting from the stringy or loop effects. These higher derivative terms fix the correction $\alpha$ in the corresponding gauge theory. Using the AdS/CFT duality between $\mathcal{N} = 4$ supersymmetric $SU(N)$ Yang-Mills gauge theory and type IIB string theory on $AdS_5 \times S_5$, the first stringy effect to the $\eta/s$ for the $\mathcal{N} = 4$ SYM theory has been calculated in [5], [6], [7]. The correction is positive, consistent with the conjectured bound $\frac{3}{4\pi}$.

Recently another example of AdS/CFT correspondence has been proposed by Aharony-Bergman-Jafferis-Maldacena (ABJM) [9]. They consider a particular brane configuration which preserves $\mathcal{N} = 3$ supersymmetry. At low energies, by integrating out the massive modes of the brane configuration one finds $U(N)_{k} \times U(N)_{-k}$ Chern-Simons conformal field theory which preserves $\mathcal{N} = 6$ supersymmetry. This theory is renormalizable and is consistent even at high energies. By lifting the brane configuration to the M-theory they have shown that the gauge theory is equivalent to the low energy theory of $N$ coincident M2-branes in orbifold $R^8/Z_k$. Using the AdS/CFT correspondence, then they have conjectured that the 3-dimensional $\mathcal{N} = 6$ superconformal $U(N)_{k} \times U(N)_{-k}$ Chern-Simons-matter theory is dual to the M-theory on $AdS_4 \times S^7/Z_k$. See [10] for recent studies in different aspects of this duality.

The 't Hooft coupling of this gauge theory is $\lambda = N/k$. For $k = 1$, theory has no weak coupling regime. The viscosity at large $N$ has been found in [11] to be $\eta = 2^{5/2}\pi N^{3/2}T^2/3^3$. For $k > 1$, however, theory has both weak and strong coupling regimes. At strong couplings, the viscosity becomes $\eta = 2^{3/2}\pi N^2T^2/(3^4\sqrt{\lambda})$. On the other hand, the entropy of this theory at the supergravity level is [9, 12, 13] $S = 2^{7/2}\pi^2 N^2T^2V_2/(3^3\sqrt{\lambda})$ which gives the universal ratio $\eta/s = 1/4\pi$. In this paper, we would like to examine the quantum and stringy corrections to this universal value.

The higher derivative corrections to the supergravity in general have field redefinition freedom [17, 18], so one may choose different scheme for them. The scheme in which the corrections are written in terms of the 11-dimensional Weyl tensor, modifies the maximal supergravity solution $AdS_4 \times S^7$. On the other hand, it has been shown in [19] that the maximal solutions of supergravity are not modified by the higher derivative corrections.

\[^3\text{See [8], for a class of four-dimensional gauge theories in which the conjectured lower bound is violated.}\]
Hence, in this scheme the higher derivative corrections associated with the gauge field \( F_{(4)} \) influences the solution. In the scheme in which the corrections are written in terms of the 4-dimensional Weyl tensor, the maximal solution is not modified. Hence, it has been argued in [20, 21] that this scheme may include all higher derivative corrections associated with the gravity and the gauge field strength \( F_{(4)} \).

An outline of this paper is as follows. In section two we briefly review the Minkowski AdS/CFT prescription for calculating the shear viscosity. In section 3, using the Minkowski AdS/CFT prescription, we calculate the effect of two different schemes of one-loop correction of M-theory effective action to the shear viscosity. In section 4, using the fact that for large \( k \) level the appropriate description of the gauge theory is the type IIA string theory on \( AdS_4 \times CP^3 \), we calculate two different schemes of \( \alpha' \) correction to the shear viscosity. In all cases, the corrections to the classical value of the shear viscosity is positive. Moreover, the corrections to \( \eta/s \) are positive which is consistent with the \( \eta/s \) bound conjecture.

2 Shear viscosity from supergravity

One way of calculating the shear viscosity of a \( p + 1 \) dimensional field theory is to use the Kubo relation

\[
\eta = \lim_{\omega \to 0} \frac{1}{2\omega} \int dt d^{p+1}x \, e^{i\omega t} \langle [T_{x_1 x_2}(x), T_{x_1 x_2}(0)] \rangle ,
\]

which expresses the shear viscosity of a slightly non-equilibrium system in terms of the real-time correlation function of the stress energy tensor \( T_{x_1 x_2} \) computed in an equilibrium thermal ensemble. This relation can be written in terms of retarded Green’s function as

\[
\eta = \lim_{\omega \to 0} \frac{1}{2i\omega} \left[ (G_{x_1 x_2}^{R}(\omega, 0))^* - G_{x_1 x_2}^{R}(\omega, 0) \right] ,
\]

where the momentum space retarded Green’s function is defined as

\[
G_{x_1 x_2}^{R}(\omega, q) = -i \int d^{p+1}x \, e^{-iq \cdot x} \theta(t) \langle [T_{x_1 x_2}(x), T_{x_1 x_2}(0)] \rangle .
\]

Using the Minkowski AdS/CFT prescription [14, 15], one relates the retarded Green’s function to the on-shell bulk action as

\[
G_{x_1 x_2}^{R}(\omega, q) = \lim_{u \to 0} 2\mathcal{F}(\omega, q, u) ,
\]

where \( u = 0 \) is the boundary of \( AdS \) on which the gauge theory lives, and \( \mathcal{F}(\omega, q, u) \) is related to the on-shell action, in which the bulk metric is perturbed by the graviton which couples to the \( T_{x_1 x_2} \) of the gauge theory, i.e.,

\[
S = S_0 + \int \frac{d^{p+1}k}{(2\pi)^{p+1}} \mathcal{F}(\omega, q, u) \bigg|_{u=0}^{u=1} + \cdots
\]
In the above equation, $S_0$ is the part of the action which is independent of the perturbations, and dots refer to the terms that are cubic or higher order of perturbations. Moreover, using the equation of motion for the perturbations, the quadratic order terms in the action can be written as boundary terms, i.e., at horizon of Schwarzschild $AdS$, $u = 1$, and at boundary of $AdS$, $u = 0$.

The thermal $\mathcal{N} = 6$ superconformal $U_k(N) \times U_{-k}(N)$ Chern-Simons matter theory at strong 't Hooft coupling and for small $k$ level is described by 11-dimensional supergravity on Schwarzschild $AdS_4 \times S^7/Z_k$. Using this gravity dual, the shear viscosity becomes [11]

$$\eta = \frac{2^{\frac{3}{2}}}{3^3} \pi T^2 \frac{(kN)^{\frac{3}{2}}}{k} = \frac{2^{\frac{3}{2}}}{3^3} \pi T^2 \frac{N^2}{\sqrt{\lambda}}$$

(2.6)

where the overall factor of $1/k$ is due to the orbifolding the $S^7$ and $N$ is the number of M2-branes. For large $k$, the orbifold circle becomes small, hence, one should reduce the M-theory to a weakly coupled type IIA theory. Using this gravity dual, one recovers again the above shear viscosity.

3 Quantum corrections to shear viscosity

We have seen that the shear viscosity of the $\mathcal{N} = 6$ superconformal $U_k(N) \times U_{-k}(N)$ Chern-Simons matter theory at strong 't Hooft coupling is given by (2.6). For small $k$, the good description is M-theory on $AdS_4 \times S^7/Z_k$. The first correction to the result (2.6) is coming from the one-loop correction to the 11-dimensional supergravity. We consider two different schemes for the one-loop corrections. In the next subsection, we consider the scheme in which the higher derivative terms are written in terms of the 11-dimensional Weyl tensor, and in subsection 3.2 in terms of the 4-dimensional Weyl tensor.

3.1 Eleven-dimensional Weyl tensor

The one-loop corrected action may be given as [16]

$$S = \frac{1}{2\kappa_{11}^2} \int d^{11}x \sqrt{-g} \left( R - \frac{1}{2(4!)} E_{(4)}^2 + \gamma W \right),$$

(3.1)

where $\gamma = 4\pi^2 \kappa_{11}^4/3$ and $W$ in terms of the Weyl tensors is

$$W = C^{\mu\nu\rho\kappa} C_{\mu\nu\rho\kappa} + \frac{1}{2} C^{\mu\nu\rho\kappa} C_{\mu\nu\rho\kappa} C^{\nu\kappa\rho\kappa}.$$  

(3.2)

In this subsection we choose the indices in Weyl tensors to run over the eleven-dimensional coordinates.

The background $AdS_4 \times S^7/Z_k$ is a solution of the 11-dimensional supergravity which has maximal supersymmetry and should not be modified by the higher derivative corrections to
this effective action [19]. In the presence of the above one-loop correction, the Schwarzschild $AdS_4 \times S^7/Z_k$ solution modifies to [13] \[4\]

\[
    ds^2 = S^{2n}(u) \left( \frac{r_0}{Lu} \right)^2 \left[ -K^2(u)dt^2 + P^2(u)du^2 + \sum_{i=1}^{2} dx_i^2 \right] + L^2 S^2(u)ds^2_{S^7/Z_k},
\]

\[
    F_{tx_1x_2u} = - \frac{6r_0^4}{L^6 u^3} K(u)P(u)S^{4n-7}(u),
\]

where $n = -7/2$, and

\[
    K(u) = (1 - u^3)^{\frac{1}{2}} (1 + \gamma X(u)),
\]

\[
    P(u) = \frac{L^2}{2r_0} (1 - u^3)^{-\frac{3}{2}} (1 + \gamma Y(u)),
\]

\[
    S(u) = (1 + \gamma Z(u)).
\]

where $X(u)$ and $Y(u)$ are

\[
    X(u) = \frac{1}{L^6} \left( \frac{3568}{25} u^3 - \frac{3568}{25} u^6 + 88u^9 \right),
\]

\[
    Y(u) = \frac{1}{L^6} \left( \frac{1533}{100} + \frac{3568}{25} u^3 + \frac{2784}{25} u^6 - 536u^9 \right).
\]

The differential equation for $Z(u)$ does not have such a simple solution. For extremal case, $u = 0$, the tree level solution is modified which is not consistent with the observation made in [19]. Hence, the one-loop correction associated with the four-form $F(4)$ which is not included in (3.1) should affect the solution. It has been argued in [21] that the one-loop correction associated with the four-form $F(4)$ are scheme-dependent and thus are adjusted to have the $AdS_4 \times S^7/Z_k$ solution.

One may also expect that the one-loop correction associated with $F(4)$ modifies the differential equation for $Z(u)$ such that as in Schwarzschild $AdS_5 \times S^5$ case, it has a simple power law solution [21]. In [13], it has been shown that the thermodynamic quantities such as the entropy and the temperature do not depend on $Z(u)$. We will see that the hydrodynamical shear viscosity quantity does not depend on $Z(u)$ either. All above quantities, however, do depend on $X(u)$ and $Y(u)$, so they are scheme dependent.

Following the prescription for calculation the shear viscosity [14], [15], [6], one has to perturb the metric by a graviton component that couples to $T_{x_1x_2}$ component of the field theory at the boundary of $AdS$, i.e.,

\[
    ds^2 = S^{2n} \left( \frac{r_0}{Lu} \right)^2 \left[ -K^2 dt^2 + P^2 du^2 + \sum_{i=1}^{2} dx_i^2 + 2\phi x_1 dx_2 \right] + L^2 S^2 ds^2_{S^7/Z_k},
\]

\[4\] Note that in Euclidean space, one should replace this ansatz into the action and then find the equation of motion for $K(u), P(u), S(u)$. However, in the Minkowski space, one has to replace the ansatz into the Legendre transform of the action with respect to the electric field. This is the same description that one uses in entropy function formalism [23].
We set \( L = 1 \). So the perturbation is

\[
h_{x_1x_2} = \frac{r_0^2}{u^2} S^{2n}(u) \varphi(u, t, x_i),
\]

(3.7)

We now perform the Fourier decomposition

\[
\varphi(u, t, x_i) = \int \frac{d^3k}{(2\pi)^3} e^{-i\omega t + iq \cdot x} \varphi_k(u),
\]

(3.8)

where \( k = (\omega, q) \). Since in the Kubo relation, the spatial momentum is zero, we restrict our calculations to \( q = 0 \). Replacing the perturbed metric in (3.1) and expanding it in terms of powers of \( \varphi_k \), one finds the quadratic order to be

\[
S_2 = \frac{V_7}{2k \kappa^2} \int \frac{d^3k}{(2\pi)^3} \int_0^1 du \left\{ A \varphi''_k(u) \varphi_{-k}(u) + B \varphi'_k(u) \varphi'_{-k}(u) + C \varphi'_k(u) \varphi_{-k}(u) + D \varphi_k(u) \varphi_{-k}(u) + E \varphi''_k(u) \varphi''_{-k}(u) + F \varphi''_k(u) \varphi'_{-k}(u) \right\},
\]

(3.9)

where \( V_7 \) is the volume of the 7-sphere. The equation of motion for the perturbed metric is

\[
A \varphi''_k + C \varphi'_k + 2D \varphi_k - \frac{d}{du}(F \varphi''_k + 2B \varphi'_k + C \varphi_k) + \frac{d^2}{du^2}(2E \varphi''_k + F \varphi'_k + A \varphi_k) = 0,
\]

(3.10)

and the coefficients \( A, ..., F \) are given in the appendix A. In order to have a well-defined variational principle, one must add the following Gibbons-Hawking boundary terms to the action [6]:

\[
\mathcal{K} = -A \varphi_k \varphi'_{-k} - \frac{F}{2} \varphi'_k \varphi'_{-k} + E(p_1 \varphi'_k + 2p_0 \varphi_k) \varphi'_{-k},
\]

(3.11)

where \( p_0, p_1 \) are given by the equation of motion, i.e.,

\[
\varphi''_k + p_1 \varphi'_k + p_0 \varphi_k = O(\gamma),
\]

(3.12)

Using the equation of motion, the on-shell action then becomes the boundary term (2.5) with

\[
\mathcal{F}(\omega, 0, u) = \frac{V_7}{2k \kappa^2} \left\{ (B - A - \frac{F'}{2} + 2p_0 E) \varphi'_{-k} \varphi_{-k} + \frac{1}{2}(C - A') \varphi_k \varphi_{-k} + E p_1 \varphi'_k \varphi'_{-k} - \frac{E'}{2} \varphi''_k \varphi'_{-k} + \frac{E''}{2} \varphi''_k \varphi''_{-k} \right\},
\]

(3.13)

To evaluate the above function at boundary \( u = 0 \), one needs to know the solution of \( \varphi_k \). After replacing the coefficients from appendix A, the equation of motion (3.10) becomes

\[
(u^7 - 2u^4 + u) \varphi''_k + (u^6 + u^3 - 2) \varphi'_k + u \omega^2 \varphi_k = \gamma \frac{1088}{9(-1 + u^3)} \times
\]

\[
\left( \frac{1088}{9(-1 + u^3)} \times \right) \]

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\left( \frac{1088}{9(-1 + u^3)} \times \right)
\]
\[ \times \left[ (-1 + u^3)^4 u^3 \left( u^6 - \frac{14}{255} u^5 + \frac{1673}{2550} \right) \phi'''' + 24 (-1 + u^3) u^5 (u^6 - \frac{46}{85} u^3 + \frac{581}{1700}) \phi'\right] \\
+ \frac{8981}{68} (-1 + u^3)^2 u(u^{12} - \frac{731854}{673575} u^9 + \frac{18019}{44905} u^6 - \frac{5871223}{71848000} u^3 - \frac{456013}{30792000} + \frac{136}{8981} u^8 \omega^2 \\
- \frac{16}{19245} u^5 \omega^2 + \frac{956}{96225} u^2 \omega^2) \phi'' + \frac{25}{34} (-1 + u^3)^2 (u^{12} + \frac{79471}{1875} u^9 + \frac{14}{125} u^6 - \frac{1409143}{400000} u^3 \\
+ \frac{3192091}{600000} + \frac{408}{25} u^8 \omega^2 - \frac{56}{125} u^5 \omega^2) \phi' - \frac{2809}{68} (u^{12} - \frac{323858}{210675} u^9 + \frac{25233}{70225} u^6 - \frac{4767127}{22472000} u^3 \\
+ \frac{2778181}{67416000} - \frac{68}{2809} u^8 \omega^2 + \frac{56}{42135} u^5 \omega^2 - \frac{3346}{210675} u^2 \omega^2) u \omega^2 \phi \right], \] (3.14)

where \( \omega = \omega/2r_0 \). It is interesting to note that while the function \( Z(u) \) appears in coefficients \( C \) and \( D \), the above equation of motion is independent of \( Z(u) \). Hence, in order to find the solution we do not need to know this function. To find a solution for the above equation, one chooses the following ansatz:

\[ \varphi_k(u) = (1 - u)^\beta G_k(u), \] (3.15)

where \( G_k(u) \) should be a regular function at horizon \( u = 1 \). We find \( G_k(u) \) perturbatively as

\[ G_k(u) = G_k^{(0)}(u) + \gamma G_k^{(1)}(u), \] (3.16)

The solution for \( G_k(u) \) to order \( O(\omega^2) \) is regular, i.e.,

\[ \varphi_k(u) = (1 - u)^\beta \left[ 1 + \beta \ln(u^2 + u + 1) - 400 \beta \gamma u^3 (u^6 + \frac{1796}{625} u^3 + \frac{3146}{625}) \right], \] (3.17)

where we have normalized \( \varphi_k(u) \) to one at the boundary \( u = 0 \). The regularity condition for the whole \( G_k(u) \) at supergravity level fixes \( \beta = \pm i \omega/3 \). According to the Minkowski AdS/CFT prescription, one has to choose the incoming wave at the horizon, i.e., \( \beta = -i \omega/3 \). As pointed out in [22], in order to have the regularity at order \( \gamma \), one must check the regularity of \( O(\omega^2) \) terms at this order. By considering \( \beta = -i \omega(1 + \gamma \beta_1)/3 \) the coefficient of \( O(\omega^2) \) terms gives the following relation

\[ \frac{1}{15} \gamma (1383 + 20 \beta_1) r_0^3 \frac{1}{u - 1} + O((u - 1)^0) = 0, \] (3.18)

which gives \( \beta = -i \omega (1 - \gamma 1383/20)/3 \). Using the relation between temperature and \( r_0 \) which has been found in [13], i.e.,

\[ T = \frac{3}{2\pi r_0 (1 + \gamma 1383/20)}, \] (3.19)

One can write the incoming wave as \( (1 - u)^{-i \omega/4 \pi T} \). Similar relation has been found in [22] for D3-brane case.
Putting the solution into \( \mathcal{F}(\omega, 0, u) \) one finds the retarded Green’s function (2.4) to be the following value:

\[
G^R_{x_1 x_2, x_1 x_2}(\omega, 0) = \lim_{u \to 0} 2\mathcal{F}(\omega, 0, u) = \lim_{u \to 0} \frac{V_7 r_0^3}{\kappa_{11}^2} \left[ -\frac{2}{u^3} + 2 - i\omega (1 + \frac{233308}{125} \gamma) \right] + \gamma \left( \frac{32193}{250} \frac{1}{u^3} + \frac{107167}{250} - \frac{7}{2} \frac{Z'''}{u} - \frac{7}{u} \frac{Z''}{u} - \frac{7}{u^2} Z' + \mathcal{O}(\omega^2) \right].
\]

The viscosity (2.2) then becomes

\[
\eta = \lim_{\omega \to 0} \frac{V_7 r_0^3 \omega}{\kappa \omega \kappa_{11}^2} \left( 1 + \frac{233308}{125} \gamma \right).
\]

It is important to note that while function \( Z(u) \) appears in the retarded Green’s function, it does not appear in the shear viscosity as we have anticipated before. Using \( \omega = \omega/2r_0, V_7 = \pi^4/3, 1 = (kN)^{3/2} \kappa_{11}^2 \sqrt{2}/\pi^5 \), and the relation between \( r_0 \) and temperature, one finds the viscosity in terms of the temperature to be

\[
\eta = \frac{2\pi^3}{3^3} \frac{T^2 N^2}{\sqrt{\lambda}} (1 + \frac{432041}{250} \gamma).
\]

where the first term is the supergravity result (2.6). Note that the correction is proportional to \( \sqrt{\lambda} \), i.e., \( \gamma = 2^{5/3} \pi^{16/3} \lambda / (3N^2) \). The entropy density to first order of \( \gamma \), has been found in [13], is

\[
s = \frac{2\pi^2}{3^3} \frac{T^2 N^2}{\sqrt{\lambda}} (1 + \frac{41}{250} \gamma).
\]

The ratio of \( \eta/s \) will be

\[
\frac{\eta}{s} = \frac{1}{4\pi} \left( 1 + 1728 \gamma \right),
\]

\[
= \frac{1}{4\pi} \left( 1 + 2^\frac{5}{3} \pi^{16} 1152 \frac{\lambda}{N^2} \right).
\]

The correction is positive which is consistent with the \( \eta/s \) bound [4]. Note that \( \lambda/N^2 = 1/kN \) is a small number. In the next subsection we will consider another scheme for the higher derivative corrections of the supergravity. We will see that even though the temperature, entropy and the shear viscosity all change, the ratio \( \eta/s \) remains the same.

### 3.2 Four-dimensional Weyl tensor

We have seen that the one-loop correction in terms of 11-dimensional Weyl tensor modifies the maximal solution \( AdS_4 \times S^7/Z_k \). This indicates that this scheme does not include all
one-loop corrections associated with the 11-dimensional gravity and the gauge field strength \( F_4 \). In [21], it has been argued that the scheme in which the Weyl tensor is written in terms of only 4-dimensional \( AdS_4 \) coordinates, includes in fact the effect of gravity and the gauge field \( F_4 \). In this subsection, we work in this scheme, so the action is given by (3.1) in which the Weyl tensor (3.2) is four dimensional.

In this scheme, the Schwarzschild \( AdS_4 \times S^7/Z_k \) solution modifies to (3.3) in which \( n = -7/2 \) and \( X(u), Y(u), Z(u) \) are

\[
X(u) = \frac{1}{L^6} \left( -136u^3 - 136u^6 + 88u^9 \right), \\
Y(u) = \frac{1}{L^6} \left( 136u^3 + 136u^6 - 536u^9 \right), \\
Z(u) = \frac{1}{L^6} \left( \frac{32}{27} (u^6 + u^9) \right). 
\]

(3.26)

As we have anticipated before, the differential equation of \( Z(u) \) has the above simple power law solution. For extremal case, \( u = 0 \), the tree level solution is not modified which is consistent with the observation made in [19].

Perturbing the metric as before, one finds the following coefficients for the quadratic order action (3.9):\n
\[
A = -\frac{r_0^3}{u^2} \left[ 4(u^3 - 1) + 64u^3 \gamma (33u^9 - 50u^6 + 17 + 3u^5 \omega^2) \right], \\
B = -\frac{r_0^3}{u^2} \left[ 3(u^3 - 1) + 48u^3 \gamma (39u^9 - 56u^6 + 17 + 8u^5 \omega^2) \right], \\
C = -\frac{r_0^3}{u^3(u^3 - 1)} \left[ 12(u^3 - 1) + \frac{64u^6}{9} \gamma (2976u^9 - 5993u^6 + 3031u^3 - 14 - 162u^5 \omega^2) \right], \\
D = \frac{r_0^3}{u^4(u^3 - 1)^2} \left[ (6u^3 - 6 - u^2 \omega^2)(u^3 - 1) - \frac{16u^5}{3} \gamma (36u^{13} + 460u^{10} - 1000u^7 + 476u^4 \\
+ 28u - 18u^5 \omega^4 - 27u^6 \omega^2 + 240u^6 \omega^2 - 51 \omega^2) \right], \\
E = r_0^3 \left[ 96u^6 \gamma (u^3 - 1)^2 \right], \\
F = 0. 
\]

where \( \omega = \omega/2r_0 \). To evaluate the function (3.13) at boundary \( u = 0 \), one needs to know the solution of \( \varphi_k \). After replacing the above coefficients in the equation of motion (3.10), one finds

\[
(u^7 - 2u^4 + u)\varphi''_k + (u^6 + u^3 - 2)\varphi'_k + u\omega^2 \varphi_k = \]

8
\[
\begin{aligned}
&= \gamma \frac{96 u^3}{(1 + u^3)} \left[ (-1 + u^3)^4 u^6 \varphi'''' + 12 (-1 + u^3)^3 u^5 (2 u^3 - 1) \varphi'''ight. \\
&+ \frac{259}{2} (-1 + u^3)^2 u(u^9 - \frac{832}{777} u^5 + 60 \frac{1}{259} u^3 - \frac{17}{777} + 4 \frac{1}{259} u^5 w^2) \varphi'' \\
&- 25(-1 + u^3)^2 (u^9 - \frac{112}{75} u^5 + 17 \frac{1}{150} - \frac{12}{25} u^5 w^2) \varphi' \\
&- \frac{117}{2} (u^9 - \frac{568}{351} u^5 + 20 \frac{1}{39} u^3 - \frac{17}{351} - \frac{2}{117} u^5 w^2) u w^2 \varphi_k \\&+ 12 \gamma (80 + \beta_1) r_0^3 + O((u - 1)) = 0, \\
\end{aligned}
\]

To find a solution for the above equation, we again choose the following ansatz:

\[
\varphi_k(u) = (1 - u)^\beta G_k(u),
\]

where \(G_k(u)\) should be a regular function at horizon \(u = 1\). We find \(G_k(u)\) perturbatively as

\[
G_k(u) = G_k^0(u) + \gamma G_k^1(u).
\]

The solution for \(G_k(u)\) to order \(O(w^2)\) is regular, i.e.,

\[
\varphi_k(u) = (1 - u)^\beta \left[ 1 + \beta \ln(u^2 + u + 1) - 400 \beta \gamma u^3 (u^6 + 71 \frac{1}{25} u^3 + 5) \right],
\]

where we have normalized \(\varphi_k(u)\) to one at the boundary \(u = 0\). The regularity condition for the whole \(G_k(u)\) at supergravity level fixes \(\beta = \pm i w/3\), and at order \(\gamma\) fixes \(\beta = -i w (1 + \gamma \beta_1)/3\) in which \(\beta_1\) is given by the regularity of the coefficient of \(O(w^2)\), i.e.,

\[
12 \gamma (80 + \beta_1) r_0^3 + O((u - 1)) = 0,
\]

which gives \(\beta = -i w (1 - 80 \gamma)/3\).

As a double check of our result, we use the fact that the incoming wave should be written as \((1 - u)^{-i w/4 \pi T}\) [22]. Using the gravity solution (3.26), one finds the temperature to be

\[
T = \frac{3}{2 \pi} r_0 (1 + 80 \gamma),
\]

So the incoming wave can be written as \((1 - u)^{-i w/4 \pi T}\).

Putting the solution (3.30) into \(F(\omega, 0, u)\) one finds the retarded Green’s function (2.4) to be the following value:

\[
G_{R_{x_1 x_2 x_1 x_2}}^R(\omega, 0) = \lim_{u \to 0} 2 F(\omega, 0, u)
\]

\[
= \lim_{u \to 0} \frac{V_7 r_0^3}{k \kappa_{11}^2} \left[ -\frac{2}{u^3} + 2 - i w (1 + 1920 \gamma) + \gamma (544 - \frac{448}{9} u^3) + O(w^2) \right].
\]

The viscosity (2.2) then becomes

\[
\eta = \lim_{\omega \to 0} \frac{V_7 r_0^3 w}{k \omega \kappa_{11}^2} (1 + 1920 \gamma)
\]

\[
= \frac{2 \pi}{3} T^2 \frac{N^2}{\sqrt{\lambda}} (1 + 1760 \gamma).
\]

(3.33)
where the first term is the supergravity result (2.6).

Using the gravity solution (3.26), one can calculate the entropy from the Wald formula or from the free energy as in [13]. The result is

\[ s = \frac{27\pi^2}{3^3} T^2 \frac{N^2}{\sqrt{\lambda}} (1 + 32\gamma). \] (3.34)

The ratio of \( \eta/s \) then becomes

\[ \frac{\eta}{s} = \frac{1}{4\pi} (1 + 1728\gamma), \] (3.35)

\[ = \frac{1}{4\pi} (1 + 2\pi^{16} 1152 \frac{\lambda}{N^2}). \] (3.36)

which is exactly the same value that we have found in previous section. In that section we have considered only the higher derivative terms associated with the 11-dimensional gravity, whereas, in the present section by working with four dimensional Weyl tensor one may expect that all higher derivative terms are include. Hence, the above result may indicate that the higher derivative corrections associated with the gauge field strength \( F(4) \) in eleven dimension have no contribution to \( \eta/s \). It would be interesting to check this explicitly. Similar calculation has been done in [7] for D3-brane case.

## 4 Stringy corrections to shear viscosity

The shear viscosity of the \( \mathcal{N} = 6 \) superconformal \( U_k(N) \times U_{-k}(N) \) Chern-Simons matter theories at strong 't Hooft coupling is given by (2.6). For large \( k \), the appropriate description is type IIA string theory on \( AdS_4 \times CP^3 \). The correction to the result (2.6) is coming from one-loop correction and/or \( \alpha' \)-correction to the 10-dimensional supergravity. Using the dimensional analysis, one observes that the one-loop correction to viscosity in type IIA and in M-theory has the same dependency on \( N \) and \( \lambda \). So, the one-loop correction should be the same as the one-loop correction in M-theory case that we have found in the previous section. In this section, we are interested in calculating the \( \alpha' \)-correction to the shear viscosity. We consider two different schemes for the \( \alpha' \) corrections. In the next subsection, we consider the scheme in which the higher derivative terms are written in terms of the 10-dimensional Weyl tensor, and in subsection 4.2 in terms of the 4-dimensional Weyl tensor.

### 4.1 Ten-dimensional Weyl tensor

The \( \alpha' \)-corrected type IIA supergravity may be given as [24]

\[ S = -\frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{g} \left\{ e^{-2\phi} (\mathcal{R} + 4(\partial\phi)^2) - \frac{1}{24!} F_{(4)}^2 - \frac{1}{22!} F_{(2)}^2 + \gamma e^{-2\phi} W \right\}, \] (4.1)

where \( \gamma = \frac{1}{8}\zeta(3)(\alpha')^3 \). The indices in the Weyl tensors run over the ten dimensional coordinates. So the scheme here is similar to the one considered in section 3.1.
The background Schwarzschild $AdS_4 \times CP^3$ is a solution of the 10-dimensional type IIA supergravity. In the presence of the above $\alpha'$ corrections, the solution modifies to [13]

\[
    ds^2 = \frac{L}{kR_{11}} \left( S^{-6}(u) \left( \frac{r_0}{Lu} \right)^2 \left( -K^2(u) dt^2 + P^2(u) du^2 + \sum_{i=1}^{2} dx_i^2 \right) + L^2 S^2(u) ds_{CP^3}^2 \right),
\]

\[
    F_{txx} = -\frac{6r_0^4}{L^5 u^4} K(u) P(u) S^{-18}(u),
\]

\[
    F_{(2)} = R_{11} k d\omega,
\]

where $R_{11} = g_s^{2/3} l_p$ is the radius of the eleventh direction, and

\[
    \omega = \frac{1}{2} \left( \cos^2 \xi - \sin^2 \xi \right) d\psi + \frac{1}{2} \cos^2 \xi \cos \theta_1 d\phi_1 + \frac{1}{2} \sin^2 \xi \cos \theta_2 d\phi_2,
\]

\[
    ds_{CP^3}^2 = d\xi^2 + \cos^2 \xi \sin^2 \xi (d\psi + \cos \frac{\theta_1}{2} d\phi_1 - \cos \frac{\theta_2}{2} d\phi_2)^2
\]

\[
    + \frac{1}{4} \cos^2 \xi (d\theta_1^2 + \sin^2 \theta_1 d\phi_1^2) + \frac{1}{4} \sin^2 \xi (d\theta_2^2 + \sin^2 \theta_2 d\phi_2^2).
\]

and $K(u), P(u), S(u)$ are those appear in (3.4) in which $X(u)$ and $Y(u)$ are

\[
    X(u) = \frac{1}{L^6} \left( \frac{k R_{11}}{L} \right)^3 \left( -136 u^3 - 136 u^6 + 88 u^9 \right)
\]

\[
    Y(u) = \frac{1}{L^6} \left( \frac{k R_{11}}{L} \right)^3 \left( 4 + 136 u^3 + 136 u^6 - 536 u^9 \right).
\]

The differential equation for $Z(u)$ does not have a power law solution. Moreover, for the extremal case, $u = 0$, the supergravity solution is modified. This indicates that the higher derivative corrections associated with the gauge fields $F^{(4)}$ and $F^{(2)}$ which are not included in action (4.1) affects the solution.

To find the shear viscosity, one has to perturb the metric as

\[
    ds^2 = ds^2|_{\text{unperturb}} + \frac{L}{kR_{11}} S^{-6}(u) \frac{r_0^2}{L^2 u^2} 2 \varphi dx_1 dx_2.
\]

By Fourier expanding the perturbation as before, one finds the following expression for $\mathcal{F}(\omega, 0, u)$:

\[
    \mathcal{F}(\omega, 0, u) = \frac{V_{CP^3}}{2\kappa_{10}^2} \left[ (B - A) - \frac{F'}{2} + 2p_0 E) \varphi_k' \varphi_{-k} + \frac{1}{2} (C - A') \varphi_k \varphi_{-k} + E p_{1} \varphi_k' \varphi_{-k}'
    
    - E' \varphi_k' \varphi_{-k}' - E \varphi_k' \varphi_{-k}' - E \varphi_k'' \varphi_{-k}' \right],
\]

where the coefficients $A, ..., F$ are given in the appendix B. Replacing them into (3.10), one finds the following equation of motion

\[
    (u^7 - 2u^4 + u)\varphi_k'' + (u^6 + u^3 - 2)\varphi_k' + u\varphi_k = \gamma \frac{k^3 R_{11}^3}{L^9} \frac{120}{u^3 - 1} \times
\]

\[
    \text{[continued on the next page]}
\]
\[ \times \left[ (u^3 - 1)^4 (u^6 + \frac{3}{5} u^3 \varphi''_k + 24 (u^3 - 1)^3 (u^6 - \frac{1}{2} u^3 + \frac{3}{10}) u^5 \varphi''_k \right] \\
+ 132(u^3 - 1)^2 u^{12} - \frac{521}{495} u^7 + \frac{119}{330} u^6 - \frac{209}{3960} u^3 - \frac{19}{1320} + \frac{1}{66} \varphi''_k \frac{1}{3} u^2 u^8 + \frac{1}{110} \varphi''_k \right] \\
+ \frac{532}{15} (u^3 - 1)^2 (u^9 - \frac{6}{133} u^6 - \frac{83}{1064} u^3 + \frac{3}{28} + \frac{45}{133} \varphi'_k) \\
- \frac{209}{5} u \varphi''_k (u^{12} - \frac{88}{19} u^9 - \frac{89}{418} u^6 + \frac{5}{114} - \frac{5}{209} \varphi''_k - \frac{3}{209} \varphi''_k \right), \quad (4.7) \\
\]

where \( \varphi = \frac{\omega^2}{2r_0} \). We again solve the equation of motion perturbatively and find the following result

\[ \varphi_k(u) = \frac{L^3}{k R_{11}} (1 - u)^\beta \left[ 1 + \beta \ln(u^2 + u + 1) - 400 \beta \gamma k^3 R_{11}^3 L^9 (u^6 + \frac{71}{25} u^3 + 5) \right], \quad (4.8) \]

In this case the regularity condition at supergravity level gives \( \beta = -i \varphi/3, \) and regularity of \( \gamma \) order terms computes the correction to \( \beta. \) If we consider \( \beta = -i \varphi (1 + \beta_1 \gamma k^3 R_{11}^3/L^9)/3 \) then

\[ \frac{1}{12} \beta_0^3 \gamma (76 + \beta_1 \gamma k^3 R_{11}^3/L^9) = -i \varphi (1 + \beta_1 \gamma k^3 R_{11}^3/L^9)/3 \]

which gives rise to \( \beta = -i \varphi (1 - 76 \gamma k^3 R_{11}^3/L^9)/3. \) Putting the solution into the \( F(\varphi, 0, u) \) one finds the following value for the retarded Green’s function:

\[ G_{x_1, x_2, x_1, x_2}^R (\omega, 0) = \lim_{u \to 0} 2F(\omega, 0, u) \\
= \frac{V_{CP}^3 r_0^3}{\kappa_{10}^2} \frac{16 L^3}{k R_{11}} \left[ \frac{1}{8 u^3} + \frac{1}{8} - \frac{1}{16} i \varphi \left( 1 + 1840 \gamma k^3 R_{11}^3 L^9 \right) \right] \quad (4.10) \]

The shear viscosity [22] then becomes

\[ \eta = \lim_{\omega \to 0} \frac{V_{CP}^3 r_0^3 \varphi}{\omega \kappa_{10}^2} \frac{L^3}{k R_{11}} (1 + 1840 \gamma k^3 R_{11}^3 L^9) \]. \quad (4.11) \]

The temperature in terms of \( r_0 \) is [13]

\[ T = \frac{3 r_0}{2 \pi L^2} \left( 1 + 76 \frac{\gamma}{L^6} \left( \frac{k R_{11}}{L} \right)^3 \right), \quad (4.12) \]

The shear viscosity in terms of temperature becomes

\[ \eta = \frac{2 \pi^2 L^9 V_{CP} T^2}{9 k R_{11} \kappa_{10}^2} \left( 1 + 1688 \frac{\gamma}{L^6} \left( \frac{k R_{11}}{L} \right)^3 \right). \quad (4.13) \]
Using the relations $\frac{2\pi R_{11}}{\kappa_{11}} = \frac{1}{\kappa_{10}}, \ 2\pi \text{Vol}(CP^3) = \text{Vol}(S^7), l_{11} = g_{11}^{1/3} l_s$ and $2\kappa_{11}^2 = (2\pi l_{11})^9 / 2\pi$, one can write the viscosity in terms of 't Hooft coupling and the number of colors as

$$\eta = \frac{2^2 \pi^2 N^2}{3^3} T^2 \left( 1 + \frac{1688 \zeta(3)}{\pi^3 2^{3/2} \lambda^{3/2}} \right).$$  \hspace{1cm} (4.14)

The first term is the supergravity result and the second term is its $\alpha'$ correction. The entropy density to first order of $\gamma$ has been found in [13] to be

$$s = \frac{2^7 \pi^2 N^2}{3^3} T^2 \left( 1 - \frac{5 \zeta(3)}{\pi^3 2^{3/2} \lambda^{3/2}} \right).$$  \hspace{1cm} (4.15)

The ratio of $\eta/s$ in this case is

$$\frac{\eta}{s} = \frac{1}{4\pi} \left( 1 + 27 - \frac{\zeta(3)}{\pi^3 2^{3/2} \lambda^{3/2}} \right).$$  \hspace{1cm} (4.16)

Again the correction is positive which is consistent with the $\eta/s$ bound [4]. The above results do not include the higher derivative terms associated with the gauge fields $F_{(4)}$ and $F_{(2)}$. In the next subsection, we consider another scheme which may include the important part of these higher derivative terms that one needs for calculating the temperature, entropy and the shear viscosity.

### 4.2 Four-dimensional Weyl tensor

In this subsection, we consider the scheme in which the higher derivative terms are given by action (4.1) in which the indices on the Weyl tensor run over the 4-dimensional $AdS_4$ part of the 10-dimensional space. Using the ansatz (4.2), the solution for $X(u)$ and $Y(u)$ are

$$X(u) = \frac{1}{L^6} \left( \frac{kR_{11}}{L} \right)^3 \left( -136u^3 - 136u^6 + 88u^9 \right)$$

$$Y(u) = \frac{1}{L^6} \left( \frac{kR_{11}}{L} \right)^3 \left( 136u^3 + 136u^6 - 536u^9 \right).$$  \hspace{1cm} (4.17)

However, the differential equation for $Z(u)$, i.e.,

$$2u^2(u^3 - 1)Z''(u) + 2u(u^3 + 2)Z'(u) + 35Z(u) - 216 \frac{1}{L^6} \left( \frac{kR_{11}}{L} \right)^3 u^{12} = 0.$$

does not have such a simple power law solution. For extremal case, $u = 0$, the solution for $X$ and $Y$ does not modify the supergravity solution. This may indicate that this scheme includes the effect of gravity as well as the gauge fields. On the other hand, one may expect that when all higher derivative terms are included, the differential equation of $Z(u)$ should then have simple power law solution as in $AdS_5 \times S^5$ and $AdS_4 \times S^7$. One may conclude this
scheme does not include all the higher derivative terms. However, those higher derivative terms that are not included in this scheme may modify only the differential equation of \(Z(u)\) to have power law solution. Since the quantities \(T, s, \eta\) are independent of \(Z\), one may expect that this scheme produces correctly these quantities.

To find the shear viscosity, one has to perturb the above metric as in (4.19). Doing the same steps as before, one finds the following coefficient for the function (4.6):

\[
A = - \frac{L^3}{kR_{11}} \frac{r_0^3}{u^2} \left[ \frac{(u^3 - 1)}{4} + 4u^3 \gamma \frac{k^3 R_{11}^3}{L^9} (33u^9 - 50u^6 + 17 + 3u^3w^2) \right],
\]
\[
B = - \frac{L^3}{kR_{11}} \frac{3r_0^3}{u^2} \left[ \frac{(u^3 - 1)}{16} + u^3 \gamma \frac{k^3 R_{11}^3}{L^9} (39u^9 - 56u^6 + 17 + 8u^3w^2) \right],
\]
\[
C = - \frac{3}{4} \frac{L^3}{kR_{11}} \frac{r_0^3}{u^3(u^3 - 1)} \left[ (u^3 - 1) + u^3 \gamma \frac{k^3 R_{11}^3}{L^9} (1776u^{14} - 3568u^{11} + 1792u^8 \right.
- 96w^2u^{10} - (u^3 - 1)^2Z') \right],
\]
\[
D = \frac{1}{16} \frac{L^3}{kR_{11}} \frac{r_0^3}{u^4(u^3 - 1)^2} \left[ (6u^3 - 6 - 2u^2w^2)(u^3 - 1) - 2u^3 \gamma \frac{k^3 R_{11}^3}{L^9} (432u^{17} + 480u^{14} - 2256u^{11} + 1344u^8 - 48w^2u^9 - 72w^2u^{13} + 640w^2u^{10} - 136w^2u^4 + 3(u^3 - 1)^3Z'' + 3(u^3 - 1)^2Z') \right],
\]
\[
E = 6u^6 r_0^3 k^2 R_{11}^2 \frac{L^2}{L^9} (u^3 - 1)^2,
\]
\[
F = 0,
\]

where \(w = \omega \frac{L^2}{2r_0}\). Replacing them into (3.10), one finds the following equation of motion

\[
(u^7 - 2u^4 + u)\varphi_k'' + (u^6 + u^3 - 2)\varphi_k' + uw^2 \varphi_k =
\]
\[
= \gamma \frac{k^3 R_{11}^3}{L^9} \frac{96u^3}{(-1 + u^3)^2} \left[ (-1 + u^3)^4 u^6 \varphi_k''' + 12 (-1 + u^3)^3 u^5 (2u^3 - 1) \varphi_k''
+ \frac{259}{2} (-1 + u^3)^2 u(u^9 - \frac{832}{777} u^6 + \frac{60}{259} u^3 - \frac{17}{777} + \frac{4}{259} u^5 w^2) \varphi_k''
- 25(-1 + u^3)^2 (u^9 - \frac{112}{75} u^6 + \frac{17}{150} - \frac{12}{25} u^5 w^2) \varphi_k'
- \frac{117}{2} (u^9 - \frac{568}{351} u^6 + \frac{20}{39} u^3 - \frac{17}{351} - \frac{2}{117} u^5 w^2) u w^2 \varphi_k' \right],
\] (4.19)

We again solve the equation of motion perturbatively and find the following result

\[
\varphi_k(u) = \frac{L^3}{kR_{11}} (1 - u) \beta \left[ 1 + \beta \ln(u^2 + u + 1) - 400\beta^2 \gamma \frac{k^3 R_{11}^3}{L^9} u^3(u^6 + \frac{71}{25} u^3 + 5) \right],
\] (4.20)
In this case the regularity condition at supergravity level gives \( \beta = -i \omega / 3 \), and regularity of \( \gamma \) order terms computes the correction to \( \beta \). If we consider \( \beta = -i \omega (1 + \beta_1 \gamma k R_{11} / L^9) / 3 \) then

\[
\frac{3}{4} r_0^3 \gamma (80 + \beta_1) + O((u - 1)) = 0 ,
\]

which gives rise to \( \beta = -i \omega (1 - 80 \gamma k R_{11} / L^9) / 3 \). Putting the solution into the \( \mathcal{F}(\omega, 0, u) \) one finds the following value for the retarded Green’s function:

\[
G_R^{\mathcal{F}}(\omega, 0, u) = \lim_{u \to 0} V_{CP3} r_0^3 \left[ \frac{1}{16} - \frac{1}{8} - \frac{1}{16} i \omega (1 + 1920 \gamma k R_{11} / L^9) \right] (4.22)
\]

\[
(\omega, 0) = \lim_{u \to 0} 2 \mathcal{F}(\omega, 0, u)
\]

The shear viscosity then becomes

\[
\eta = \lim_{\omega \to 0} \frac{2 \pi^2 L^9 V_{CP3} r_0^3 \omega}{9 k R_{11} \kappa_{10}^2} \left( 1 + 1920 \gamma k R_{11} / L^9 \right).
\]

Using the solution (4.17), one finds the temperature to be

\[
T = \frac{3 r_0}{2 \pi L^2} \left( 1 + 80 \gamma \frac{k R_{11}}{L} \right)^3.
\]

The shear viscosity in terms of temperature then becomes

\[
\eta = \frac{2 \pi^2 L^9 V_{CP3} T^2}{9 k R_{11} \kappa_{10}^2} \left( 1 + 1760 \frac{\gamma}{L^6} \left( \frac{k R_{11}}{L} \right)^3 \right)
\]

\[
= \frac{2 \pi^2 N^2}{3^3 \sqrt{\lambda}} T^2 \left( 1 + \frac{1760 \zeta(3)}{\pi^3 2^2 \lambda^{3/2}} \right).
\]

The first term is the supergravity result and the second term is its \( \alpha' \) correction. Using the solution (4.17), one finds the entropy to the first order of \( \gamma \) to be

\[
s = \frac{2 \pi^2 N^2}{3^3 \sqrt{\lambda}} T^2 \left( 1 + \frac{32 \zeta(3)}{\pi^3 2^3 \lambda^{3/2}} \right).
\]

The ratio of \( \eta/s \) is then

\[
\frac{\eta}{s} = \frac{1}{4 \pi} \left( 1 + 1728 \frac{\zeta(3)}{\pi^3 2^2 \lambda^{3/2}} \right).
\]

We expect that the above result includes effect of all higher derivative terms. The above result is not the same as the result in (4.16) which includes only the 10-dimensional gravity effects. On the other hand, as we have argued before in section 3.2, we expect that the higher derivative terms associated with the gauge field \( F_{(4)} \) have no contribution to \( \eta/s \). Hence, the higher derivative terms associated with the gauge field strength \( F_{(2)} \) in ten dimension should have contribution to \( \eta/s \) in (4.16). It would be interesting to perform these calculations explicitly along the line of [7].
Acknowledgment

A. G. would like to thank K. Bitaghsir for discussion.

A. Coefficients in M-theory

The coefficients of equation of motion (3.10) are

\[
A = -\frac{r_0^3}{u^2} \left[ 4(u^3 - 1) + \frac{\gamma}{3375} (7128000u^{12} - 10437120u^9 - 453600u^6 + 2893509u^3 \right.
\]
\[
+ 869211 + 480000u^8 w^2 + 44800u^5 w^2 - 626080u^2 w^2 \bigg],
\]
\[
B = -\frac{r_0^3}{u^2} \left[ 3(u^3 - 1) + \frac{\gamma}{13500} (2493600u^{12} - 36453760u^9 - 3780000u^6 + 4665327u^3 \right.
\]
\[
+ 5184000u^8 w^2 - 362880u^2 w^2 - 2038127 \bigg],
\]
\[
C = -\frac{r_0^3}{u^3(u^3 - 1)} \left[ 12(u^3 - 1) + \frac{\gamma}{3375} (7192800u^{15} - 143052480u^{12} + 70489440u^9 \right.
\]
\[
+ 635040u^6 - 2607633u^3 + 2607633 - 4056000u^{11} w^2 - 291200u^8 w^2 - 218960u^5 w^2 
\]
\[
- 1161440u^2 w^2 - 47250Z'u^7 + 94500Z'u^4 - 47250Z'u \bigg],
\]
\[
D = \frac{r_0^3}{u^4(u^3 - 1)^2} \left[ (6u^3 - 6 - u^2 w^2)(u^3 - 1) - \frac{\gamma}{13500} (11664000u^{18} - 103680u^{15} \right.
\]
\[
- 27708480u^{12} + 9072000u^9 + 12291426u^6 - 10430532u^3 + 5215266 - 1632000u^{10} w^4 
\]
\[
+ 89600u^7 w^4 - 1070720u^4 w^4 - 1944000u^{14} w^2 + 17159040u^{11} w^2 - 151200u^8 w^2 
\]
\[
- 4671621u^5 w^2 + 455301u^2 w^2 - 94500u^{11} Z'' + 283500u^8 Z'' - 283500u^5 Z'' 
\]
\[
+ 94500u^2 Z'' - 94500Z'u^{10} + 283500Z'u^4 - 189000Z'u \bigg],
\]
\[
E = r_0^3 \left[ \frac{32}{675} \gamma(u^3 - 1)(2550u^6 - 140u^3 + 1673) \right],
\]
\[
F = r_0^3 \left[ \frac{448}{9u} \gamma(u^3 - 1)(u^3 + 2)(u^6 - \frac{4}{15}u^3 + \frac{1037}{300}) \right].
\]
B. Coefficients in type IIA theory

The coefficients are

\[ A = \frac{-16 L^3 r_0^3}{k R_{11} u^2} \left[ \frac{(u^3 - 1)}{4} + \gamma \frac{k^3 R_{11}^3}{L^9} (132 u^{12} - 200 u^9 + 47 u^3 + 21 + 9 w^2 u^8 - 9 w^2 u^2) \right] , \]

\[ B = \frac{-4 L^3 \gamma r_0^3}{k R_{11} u^2} \left[ \frac{(u^3 - 1)}{4} + \gamma \frac{k^3 R_{11}^3}{L^9} (154 u^{12} - 232 u^9 - 14 u^6 + 23 u^3 - 3 + 32 w^2 u^8) \right] , \]

\[ C = \frac{-4 L^3 \gamma r_0^3}{k R_{11} u^3(u^3 - 1)} \left[ (u^3 - 1) + \gamma \frac{k^3 R_{11}^3}{L^9} (1776 u^{15} - 3568 u^{12} + 1792 u^9 - 84 u^6 - 100 w^2 u^{11} - 8 w^2 u^8 - 12 w^2 u^5 - 24 w^2 u^2 - u^7 Z' + 2 u^4 Z' - u Z') \right] , \]

\[ D = \frac{L^3}{k R_{11} u^4(u^3 - 1)^2} \left[ (6 u^3 - 6 - u^2 w^2)(u^3 - 1) \right. \]

\[ - 2 \gamma \frac{k^3 R_{11}^3}{L^9} (432 u^{18} + 480 u^{15} - 2256 u^{12} + 1344 u^9 + 252 u^6 - 504 u^3 + 252 \]

\[ - 60 w^4 u^{10} - 36 w^4 u^4 - 72 w^2 u^{14} + 640 w^2 u^{11} - 174 w^2 u^5 + 38 w^2 u^2 \]

\[ - 3 u^{11} Z'' + 9 u^8 Z'' - 9 u^5 Z'' + 3 u^2 Z'' - 3 u^{10} Z' + 9 u^4 Z' - 6 u Z') \right] , \]

\[ E = 16 r_0^3 \left[ \frac{3}{2} \gamma \frac{k^2 R_{11}^2}{L^6} (u^3 - 1)^2 (5 u^6 + 3) \right] , \]

\[ F = 16 r_0^3 \left[ \frac{3}{u} \gamma \frac{k^2 R_{11}^2}{L^6} (u^3 - 1) (u^3 + 2) (u^6 + 3) \right] . \]

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