An Adhesive Wear Model Based on a Complete Contact Model for a Fractal Surface

Xin Li¹* and Bingbing Wang²

¹ School of Mechanical Engineering, Anhui Polytechnic University, Wuhu, China
² School of Chemical and Environmental Engineering, Anhui Polytechnic University, Wuhu, China

*Corresponding author email: lixin@ahpu.edu.cn

Abstract. An adhesive wear model based on a complete contact model for a fractal surface is presented in this work. A contact model which contains effect of adhesion is firstly presented based on ME model. A complete contact model is then proposed. Finally, an adhesive wear model based on this model is given. The results suggest that the maximum contact area increases firstly and then decreases as fractal dimension increases. The percentage of plastic contact area increases with increase of the fractal dimension. And the experimental results for wear volume have shown a good consistency with the results calculated by the wear model.

Keywords: Adhesion; Wear; Fractal surface; Complete contact model.

1. Introduction

Wear is a complicated phenomenon. It plays an important role in mechanical industry. A severe wear will reduce the life of mechanical parts greatly. So learning the mechanism of wear will be helpful for engineers to improve the condition of lubrication between parts with relative motion and prolong the life of mechanical parts. Sometimes, even though we cannot change the lubrication condition, we will predict when mechanical parts should be replaced according to an appropriate wear model.

Theory of contact is the base for studying the mechanism of wear. Majumdar A and Bhushan B [1] presented a contact model (namely MB contact model) for a fractal surface. Some researchers investigated the behaviours of tribology based on MB contact model. Dong Guan studied the tangential contact of spherical pump according to MB model [2]. Ji CC [3] revised the elastic-plastic contact model for fractal surface with MB contact model. Li G [4] predicted the wear behaviour of surface of involute gear through MB contact model. Gong WJ [5] proposed a fractal model to study characteristics of adhesive and abrasive wear for carbon steel spool valve. Yin X [6] proposed an adhesive wear model for a fractal surface with MB contact model. However, there was an obvious shortcoming in MB contact model. According to Majumdar A and Bhushan B, the asperity deformed plastically when the contact area was below the critical contact area and elastically when the contact area was above the critical contact area. Therefore, an asperity would deform plastically firstly and then elastically as the normal load was increased. This is not consistent with the classical theory of contact. But there are still researchers applying MB contact model to carry out their research. To correct the contact model, Morag Y and Etsion I [7] presented a revised contact model (namely ME contact model) which brought in an additional variable independent of the height of asperity. This model was consistent with the classical theory of contact. But the model did not contain the effect of adhesion which should not be ignored when studying behaviour of wear. Besides, this model was only considered for a single asperity. A complete contact model was not given in the work.
Considering these aspects, an adhesive wear model based on a complete contact model for a fractal surface is presented in this work. A contact model which contains effect of adhesion is firstly presented based on ME contact model. A complete contact model is then presented. Finally, an adhesive wear model based on this model is given.

2. Contact Model for A Single Asperity Considering Adhesion Effect

The profile of a single asperity is always expressed with a cosine function written as:

$$z(x) = G^{D-1} \gamma^{-2} \cos(\pi \gamma^{n} x)$$

(1)

where \( G \) is the roughness coefficient, \( D \) represents fractal dimension, \( n \) is index of frequency and \( \gamma \) is always evaluated with a constant, 1.5.

The profile is displayed in Figure 1. The height of the single asperity, \( \delta \), is given as:

$$\delta = z(0) = G^{D-1} \gamma^{-2} \left(2^{D} \right)$$

(2)

![Image of Figure 1: Profile for one asperity.](image)

The curvature radius of the asperity is defined as:

$$R = \left(\frac{\pi^{2} G^{D-1} \gamma^{Dn}}{\pi^{2} G^{D-1} \gamma^{D}} \right)^{-1}$$

(3)

The critical height is given:

$$\omega_{c} = \frac{(KH)^{2}}{2E} \frac{1}{G^{D-1} \gamma^{D}}$$

(4)

Then, we can have an expression of the critical contact area:

$$a_{c} = \pi R \omega_{c} = \frac{1}{\pi} \left(\frac{KH}{2EG^{D-1} \gamma^{D}} \right)^{2}$$

(5)

We can see from Eq.(4) and Eq.(5) that not only the critical height but also the critical contact area is related to the length scale of the asperity. For asperities deforming elastically, the height of deformation is relevant with contact area as:

$$a = \pi R \omega$$

(6)

The contact load which contains the effect of adhesion has been presented in [8]:

$$P_{e}(r) = \frac{4Er^{3}}{3R} - \left(8\pi W_{AB} Er^{3} \right)^{1/3}$$

(7)

where \( r \) is radius of contact area. \( W_{AB} \) is adhesion work of contact surfaces:

$$W_{AB} = c_{m} c_{l} \left(\Gamma_{A} + \Gamma_{B} \right)$$

(8)

$$P_{e}(a) = 2.36 EG^{D-1} \gamma^{D} a^{3} - \sqrt{4.51 W_{AB} E a^{3}}$$

(9)

where \( c_{m} \) is index of material compatibility. \( c_{l} \) is the lubrication compatibility index. \( \Gamma_{A} \) and \( \Gamma_{B} \) are surface energies. Substitute Eq.(3), Eq.(6) into Eq.(7) and the relationship between contact area and contact load will be obtained.

For asperities deforming plastically, relationship between contact load and contact area is[9]:

$$P_{p}(a) = Ha - 2\pi W_{AB} R$$

(10)
Substitute Eq.(3) into the expression above and we get:

$$P_p(a) = Ha - \frac{0.64W_{db}}{G^{D-1} \gamma_{Dn}}$$  \hspace{1cm} (11)

3. Wear Model for A Complete Contact Model

3.1. Real Contact Area and Contact Load

The contact area of asperities at different length scales distributes randomly. Dimension distribution of contact area, \(n(a)\), is:

$$n(a) = \frac{D}{2} \frac{a^{\frac{D}{2}}}{a^{\frac{D-1}{2}}}$$  \hspace{1cm} (12)

where \(a_{\text{max}}\) is the maximum contact area. Total contact area can be written with \(n(a)\) as follows:

$$A_p = \int_{a_{\text{min}}}^{a_{\text{max}}} n(a) \, da = \frac{D}{2-D} a_{\text{max}}$$  \hspace{1cm} (13)

where \(a_{\text{min}}\) is the minimum contact area. \(a_{\text{min}}\) is always taken as zero.

During \(n_{\text{pc}}<n<n_{\text{max}}\), the plastic contact area is:

$$A_{p_{pp}} = \sum_{n=n_{\text{pc}}}^{n=n_{\text{max}}} \int_{a_{(\text{n})}}^{a_{(\text{n})}} n(a) \, da = \frac{D}{2-D} \left( \frac{\sqrt{\pi}}{2} \right)^{2-D} a_{\text{max}}^{2-D} \sum_{n=n_{\text{pc}}}^{n=n_{\text{max}}} \left[ \frac{1}{\gamma^n} \right]^{2-D} \left( \frac{1}{\gamma^{n+1}} \right)^{2-D}$$  \hspace{1cm} (14)

During \(n_{\text{ec}}<n<n_{\text{pc}}\), the plastic contact area is:

$$A_{p_{pep}} = \sum_{n=n_{\text{ec}}}^{n=n_{\text{pc}}} \int_{a_{(\text{n})}}^{a_{(\text{n})}} n(a) \, da = \frac{D}{2-D} \left( \frac{\sqrt{\pi}}{2} \right)^{2-D} \frac{D}{a_{\text{max}}} \sum_{n=n_{\text{ec}}}^{n=n_{\text{pc}}} \left[ \frac{1}{\gamma^n} \right]^{2-D} \left( \frac{KH}{\pi EG^{D-1} \gamma_{Dn}} \right)^{2-D}$$  \hspace{1cm} (15)

Therefore, total plastic contact area is:

$$A_p = A_{p_{pp}} + A_{p_{pep}}$$  \hspace{1cm} (16)

Total elastic contact area can be expressed as:

$$A_{e} = A_p - A_{p}$$  \hspace{1cm} (17)

Based on Eq.(11), Eq.(14) and Eq.(15), total plastic contact load is:

$$P_p = \sum_{n=n_{\text{pc}}}^{n=n_{\text{max}}} \int_{a_{(\text{n})}}^{a_{(\text{n})}} P_p(a) n(a) \, da + \sum_{n=n_{\text{ec}}}^{n=n_{\text{pc}}} \int_{a_{(\text{n})}}^{a_{(\text{n})}} P_p(a) n(a) \, da$$  \hspace{1cm} (18)

With Eq.(9), the elastic contact load during \(n_{\text{min}}+1<n<n_{\text{ec}}\) is:

$$P_{e} = \sum_{n=n_{n_{\text{ec}}}}^{n=n_{\text{pc}}} \int_{a_{(\text{n})}}^{a_{(\text{n})}} P_e(a) n(a) \, da$$  \hspace{1cm} (19)

Similarly, during \(n_{\text{ec}}<n<n_{\text{pc}}\), the elastic contact load is:

$$P_{pe} = \sum_{n=n_{\text{ec}}}^{n=n_{\text{pc}}} \int_{a_{(\text{n})}}^{a_{(\text{n})}} P_{pe}(a) n(a) \, da$$  \hspace{1cm} (20)

So the total contact load is given with Eq.(18) and Eq.(21):

$$P = P_p + P_{e} + P_{pe}$$  \hspace{1cm} (21)

3.2. Wear Model with Adhesive Effect

A dominant wear mechanism is adhesive wear. There are asperities in contact when two surfaces are squeezed together. Thus, some asperities bear huge load which exceeds the yield strength of material. These asperities will deform plastically and will be taken away by the opposite surface during the movement. The adhesive wear model was presented in [10]:

...
where $W$ is the wear volume. $\lambda$ is a constant equal to 9 according to [11]. $s$ is the sliding distance. $k_e$ and $k_p$ are probability coefficient of elastic wear and plastic wear, respectively. Since $s=vt$, the rate of wear volume can be written as:

$$\frac{dW}{dt} = \left(1 + \lambda \mu \right)^2 \left( k_e A_{re} + k_p A_{rp} \right) \nu$$

(23)

From Eq.(13) to Eq.(21), one can obtain total elastic contact area $A_{re}$ as well as total plastic contact area $A_{rp}$ according to normal load $P$. Therefore, the rate of wear volume will be worked out.

4. Experiment and Results

4.1. Experiment

To validate the results calculated by the model presented by this paper, an experiment of wear is performed. The samples are machined as plates. They are fixed to the fixtures, respectively. The lower fixture is still while the upper fixture moves with a reciprocating motion. A specified normal load is set on the upper fixture. No lubricant is applied between the interfaces of the samples. The relative velocity of the fixtures is set as 2m/s. The experiment is stopped in some time and the profile of the surface of the sample is measured through surface profiler. The wear volume can be calculated through dividing the weight of loss of material by the density of the sample.

4.2. Results and Discussions

Results of both calculation and experiment are displayed in this section. Parameters are displayed in Table 1 [12,13]. The law of the maximum contact area, $a_{max}$, varying with the fractal dimension, $D$, is displayed in Figure 2. It is proved that the maximum contact area increases with increase of the fractal dimension when fractal dimension is small. However, the maximum contact area decreases when fractal dimension is close to two. Under the same normal load, the maximum contact area of the contact pair Al$_2$O$_3$/TiC is always smaller than that of the contact pair of Al$_2$O$_3$/CrN.

| Contact pairs   | Materials | $\Gamma$(mN/m) | E(GPa) | $\nu$ | H(GPa) | $E^*$(GPa) | $H^*$(GPa) |
|-----------------|-----------|----------------|--------|-------|--------|------------|------------|
| Al$_2$O$_3$/TiC | Al$_2$O$_3$ | 740            | 307    | 0.25  | 27.6   | 192        | 23.5       |
|                 | TiC       | 900            | 450    | 0.18  | 23.5   | 84         | 14.8       |
| Al$_2$O$_3$/CrN | Al$_2$O$_3$ | 740            | 307    | 0.25  | 27.6   | 84         | 14.8       |
|                 | CrN       | 44.3           | 103    | 0.3   | 14.8   | 84         | 14.8       |

The ratio of plastic contact area to real contact area is shown in Figure 3. We can find that the ratio increases rapidly when fractal dimension is close to two. It illustrates that plastic contact area expands quickly when fractal dimension is increased. We can explain it as follows. The surface is smoother if the fractal dimension is closer to two. In other words, the difference of the heights of asperities is smaller. Thus, the load which is larger than the yield strength will be set on more asperities instead of only a few asperities under a smaller fractal dimension because of a larger difference of heights of asperities. Due to the properties of materials, the percentage of plastic contact area of the contact pair Al$_2$O$_3$/TiC is slightly higher than that of the contact pair of Al$_2$O$_3$/CrN.

The rate of wear volume for different contact pairs is displayed in Figure 4 and Figure 5, respectively. The results with compatibility indexes, $c_{1cm}$, are given. As we know, the better the lubricated condition is (namely the smaller the value of $c_{1cm}$ is), the smaller the rate of wear volume gets. It can be found that the rate of wear volume increases firstly and then decreases as fractal dimension increases. And
under the same environment, the rate of wear volume of the contact pair Al₂O₃/TiC is slightly smaller than that of the contact pair Al₂O₃/CrN.

Figure 2. Fractal dimension vs the maximum contact area.

Figure 3. Fractal dimension vs the ratio of the plastic contact area to the real contact area.

Figure 4. Fractal dimension vs the rate of wear volume for contact pair of Al₂O₃ and TiC.

Figure 5. Fractal dimension vs the rate of wear volume for contact pair of Al₂O₃ and CrN.

Figure 6. Fractal dimension varying with time.

Figure 7. Wear volume varying with time.

The tendency of fractal dimension vs time can be obtained through experiment as shown in Figure 6. According to the rate of wear volume, the wear volume can be calculated. Both the calculated results and the experimental results are displayed in Figure 7. It is found that the experimental results meet well with the calculated results during the early period. But the error becomes larger during the later period. The reason for this is that in fact, particles will emerge between the interfaces during the later period and the wear volume will be increased. The wear model proposed in this paper does not take
the effect of abrasive particles. Therefore, the calculated results are smaller than the experimental results in the end of the experiment. However, the early results of the experiment confirm the correctness of the model.

5. Conclusion
An adhesive wear model based on a complete contact model for a fractal surface is presented in this work. A contact model which contains effect of adhesion is firstly presented based on ME contact model. A complete contact model is then presented. Finally, an adhesive wear model based on this model is given. The results suggest that the maximum contact increases firstly and then decreases as fractal dimension increases. The percentage of plastic contact area is increased with increase of fractal dimension. And the experimental results for wear volume have shown a good consistency with the results calculated by the wear model.

Acknowledgments
Thanks for the Fund of Introducing Talent People of Anhui Polytechnic University (No.2020YQQ007) and University Synergy Innovation Program of Anhui Province (No.GXXT-2019-048).

References
[1] Majumdar A and Bhushan B. Role of fractal geometry in roughness characterization and contact mechanics of surfaces, Trans. ASME J. Tribol. 1990, 112 (2): 205-216.
[2] Guan D, Jing L, Harry H H, Gong J J and Yang Z W. Tangential contact analysis of spherical pump based on fractal theory, Tribology International 2018, 119: 531-538.
[3] Ji C C and Jiang W. Revising Elastic-Plastic Contact Models of Fractal Surfaces, 2016, 5th International Conference on Measurement, Instrumentation and Automation (ICMIA 2016).
[4] Li G, Wang Z H and Zhu W D. Prediction of surface wear of involute gears based on a modified fractal method, Journal of Tribology 2018, doi:10.1115/1.4041587.
[5] Gong W J, Chen Y X, Li M W and Kang R. Coupling fractal model for adhesive and three-body abrasive wear of AISI 1045 carbon steel spool valves. Wear 2019, 418-419: 75-85.
[6] Yin X and Komvopoulos K. An adhesive wear model of fractal surfaces in normal contact. International Journal of Solids and Structures 2010, 47: 912-921.
[7] Morag Y and Etsion I. Resolving the contradiction of asperities plastic to elastic mode transition in current contact models of fractal rough surfaces. Wear 2007, 262: 624-629.
[8] Johnson K L, Kendall K and Roberts A D. Surface energy and the contact of elastic solids. Proceedings of the Royal Society of London Series A 1971, 324: 301-313.
[9] Chowdhury S K R and Pollock H M. Adhesion between metal surfaces: the effect of surface roughness. Wear 1981, 66: 307-321.
[10] Zhou G Y, Leu M C and Blackmore D. Fractal geometry model for wear prediction, Wear 1993, 170: 1-14.
[11] Halling J. Principles of Tribology. Macmillan Press, London, 1975: 81-83.
[12] Suh N P. Tribophysics. Prentice-Hall, Englewood Cliffs, NJ. 1986: 66.
[13] Komvopoulos K. Effects of multi-scale roughness and frictional heating on solid body contact deformation. Comptes Rendus Mécanique 2008, 336: 149-162.