An Efficient OFDM-Based Monostatic Radar Design for Multitarget Detection

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ABSTRACT In this paper, we propose a monostatic radar design for multitarget detection based on orthogonal-frequency division multiplexing (OFDM), where the monostatic radar is co-located with the transmit antenna. The monostatic antenna has the perfect knowledge of the transmitted signal and listens to echoes coming from the reflection of fixed or moving targets. We estimate the target parameters, i.e., range and velocity, using a two-dimensional (2D) periodogram. By this setup we improve the periodogram estimation performance under the condition of low signal-to-noise ratio (SNR) using Zadoff-Chu precoding (ZCP) and the discrete Fourier transform channel estimation (DFT-CE). Furthermore, since the dimensions of the data matrix can be much higher than the number of targets to be detected, we investigate the sparse Fourier transform-based Fourier projection-slice (FPS-SFT) algorithm and compare it to the 2D periodogram. An appropriate system parameterization in the industrial, scientific, and medical (ISM) band of 77 GHz, allows to achieve a range resolution of 30.52 cm and a velocity resolution of 66.79 cm/s.

INDEX TERMS Fourier slice theorem, joint communication and radar sensing (JCAS), monostatic radar, OFDM, Zadoff-Chu precoding.

I. INTRODUCTION

Wireless communication systems and radio sensing systems are two different engineering paradigms that have evolved separately in the past. However, nowadays, with the ever-increasing need for radio resources [1], implementing these two technologies separately leads to an inefficient utilization of the available spectrum. Despite their differences, communication and radar detection systems share many common features, particularly in terms of signal processing and equipment [2]. This has led several researchers to investigate the implementation of a unified system merging both technologies [3], [4], [5], [6], [7], [8]. Several approaches have been proposed in previous years, among which one of the prominent ideas is the coexistence of communications and radar detection [9], [10], [11]. By such a model, the radar and communication systems can be co-located and even physically integrated. However, they transmit two different signals that overlap in the time and/or frequency domains. To minimize the interference between them, both systems need to operate simultaneously by sharing the same resources in a cooperative way [2]. Nevertheless, with this coexistence, managing interference becomes a challenging task [9], [12]. In addition, another approach is to merge the communication and radar subsystems in a single device, using exactly the unified spectral and hardware resources. Such a conception is called in the literature joint communication and radar sensing (JCAS) or radio frequency (RF) convergence [13]. JCAS is significantly different from the aforementioned coexisting communication-radar systems. Instead, by JCAS, the same waveform is used for both communication and sensing. In such a design, many approaches can be considered. In [14], the authors proposed constant modulus waveforms for dual-function radar communication systems based on index modulation. The authors in [15] proposed a scheme that exploits constant-modulus and similarity constraints.
to simultaneously integrate communication and sensing. In addition, in [16], the design proposed aims to simultaneously achieve a desired radar waveform in one spatial direction while transmitting an information-bearing communication signal in another spatial direction using the same antenna array. Another work presented in [17], introduced a novel approach called vehicular behavior-aware integrated sensing and communication. This approach involves steering beams based on vehicular behaviors to aid in driving tasks, while also facilitating spectral-efficient uplink data services through the assistance of a roadside unit.

Moreover, in [18] and [19], the authors demonstrate that WiFi or Zigbee signals could be used for object sensing as well, showing the possibility of exploiting communication signals for sensing (communication-centric design). This design strives to utilize the traditional communication waveforms directly or make certain alterations to achieve sensing capabilities concurrently. In such a design, the sensing information can be efficiently extracted from the echoes of the target, while ensuring the primary performance of the communication functionality [20]. As one of the pioneering works, the authors in [3] put forth a methodology that utilizes modulation symbols for signal processing in an integrated sensing and communication system based on orthogonal frequency division multiplexing (OFDM). In addition, in the approach proposed in [21], it was shown that the allocation of power, the number of OFDM subcarriers, the signal-to-noise ratio (SNR) per subcarrier are parameters that have an impact on sensing performance. In [22], the parameters of OFDM, such as the number of subcarriers, OFDM symbols, subcarrier interval, and pulse repetition interval, were designed to meet diverse sensing and communication requirements. Moreover, an alternative design was proposed in [23] to minimize the transmit power of each subcarrier while considering constraints related to conditional mutual information and data rates.

Furthermore, another design called sensing-centric or radar-centric leverages the sensing waveforms to enable secondary communication functionality concurrently. Radars, especially those used for military purposes are known for their remarkable long-range operation spanning hundreds of kilometers, they present a notable advantage when incorporating communication capabilities. This integration allows for the establishment of long-range communications. In [24], the authors consider the radar user as an active participant in the multiple-access channel. They derive the performance bounds for the coexistence of joint radar communications. In [25], the proposal involves the integration of a communication signal within the radar emission to achieve dual functionality. This is accomplished by selecting the radar transmit waveform from a bank of waveforms, where each waveform corresponds to a specific communication symbol, on a pulse-to-pulse basis. In addition, the study described in [26] emphasizes the utilization of time-modulated arrays presented as a means to achieve a dual-function array. This innovative approach enables the performance of radar functions in the main lobe while facilitating communication capabilities in the sidelobes of the array.

Given that OFDM has been widely adopted in contemporary mobile communication standards [27], [28], employing OFDM waveform for detection/radar purposes has attracted increasing interest in recent years [2], [29], [30], [31], [32]. In such a design, while the communication receiver needs to perform the channel estimation (CE) before decoding the transmitted data, the radar only needs to apply a detection algorithm on the reflected signal to estimate the range and velocity of targets.

In the field of parameter estimation, several detection algorithms have been proposed, among all target detection algorithms, the periodogram technique which is based on the discrete Fourier transform (DFT) was widely investigated [33]. The periodogram algorithm stands as a fundamental technique. In [34], the authors enhance our knowledge of the periodogram by exploring advanced techniques, such as non-uniform sampling and fast Fourier transform (FFT). In addition, the work proposed in [35] shows how the likelihood function can reveal the relationship between the carrier frequency offset and the periodogram of the received signal. This maximum likelihood has been developed based on zero-padded FFT, which is invoked for implementation. The author in [33] investigated the periodogram method, which is fundamentally based on the FFT. He showed that the maximum likelihood estimator could be derived for the case where there is exactly one target, which is anything but trivial. Furthermore, the authors in [36] presented spectral monitoring based on signal detection tools, by which the spectral components are estimated by the averaged periodogram non-parametric approach using an FFT.

In addition to the periodogram, there are several detection algorithms available for signal processing tasks. These algorithms offer different approaches and capabilities for parameter estimation and signal detection. One notable algorithm is the multiple signal classification (MUSIC) algorithm, which is based on eigenanalysis of the signal correlation matrix. MUSIC is particularly effective in identifying the frequencies of multiple sinusoidal components in a signal [37]. Another widely used algorithm is the estimation of signal parameters via rotational invariance technique (ESPRIT), which exploits the rotational invariance property of a uniform linear array to estimate the signal parameters [38]. Both MUSIC and ESPRIT are parametric methods that can provide good resolution in parameter estimation. However, it is important to consider that these algorithms typically require a large number of samples to achieve robust results [37], [38]. This requirement can pose challenges in scenarios where limited data is available or real-time processing is needed. Moreover, compressive sensing (CS) is another approach that has gained popularity in recent years [39], [40]. CS aims to reconstruct sparse signals from a small number of non-adaptive linear measurements. It offers an intriguing alternative to traditional signal processing.
methods by exploiting the sparsity of signals in certain domains. However, the success of CS heavily relies on the sparsity assumption, and its performance may degrade if the underlying signal does not exhibit sufficient sparsity. Furthermore, the matrix pencil method can provide accurate results in some cases [41], it faces challenges when matching frequency pairs, which can lead to errors in parameter estimation. Some proposed techniques, such as the one described in Step 3 of Subalgorithm 1 in [42], aim to address this limitation by refining the matching process. However, further research and improvements are still needed to enhance the robustness and accuracy of the matrix pencil method in complex scenarios.

It is worth noting that despite their strengths, algorithms like MUSIC and ESPRIT suffer from high computational complexity. This computational burden can limit their practical applicability, particularly in real-time systems or resource-constrained environments. Additionally, the performance of these algorithms is heavily influenced by the SNR [43].

In this work, we propose a new design to integrate sensing to OFDM-based communications using co-located monostatic radars. In terms of sensing, we adopt the periodogram. The simplicity and computational efficiency of the periodogram have made it a widely used method in various signal processing applications. The periodogram algorithm is mainly computed using DFT and inverse discrete Fourier transform (IDFT). Ranges and velocities are contained in channel characteristics, which means that a precise estimation of these parameters by the radar system requires accurate channel state information. However, in the low SNR region, achieving an error-free channel estimate is a daunting task.

To overcome this challenge, we introduce the DFT-CE approach to reduce the false positive error in target selection. Additionally, we use Zadoff-Chu precoding (ZCP) and show that it can improve the estimation performance by ameliorating the SNR. Furthermore, even though using FFT and inverse fast Fourier transform (IFFT) to compute the periodogram is generally efficient, FFT/IFFT do not take into account the signal structure whereas a plethora of algorithms can be even faster by considering the signal sparsity [44], [45], [46], [47]. Consequently, we investigate the Fourier projection-slice (FPS)-sparse Fourier transform (SFT) [45] and discuss the trade-off to be taken between the estimation time and the estimation accuracy.

In sum, our main contributions in this paper are summarized as follows:

- By precoding the OFDM symbols with a Zadoff-Chu code matrix, we first reduce the peak-to-average power ratio (PAPR) of the OFDM signal and then, decrease the estimation error of ranges and velocities in the low SNR region.
- Furthermore, DFT-CE is a channel estimation algorithm used to reduce the amount of noise in the frequency bin single-tap channel estimate. In this work, we adapt it to filter false positive targets. By combining it with ZCP, we come up with a better periodogram to accurately estimate targets’ range and velocity.
- Finally, knowing that in practice the number of effective targets is smaller compared to the whole frame dimension, the signal is sparse in the frequency domain. We take advantage of this characteristic and apply the FPS-SFT to reduce the computational complexity along with the number of signal samples needed for estimation. By comparing it with the two-dimensional (2D) periodogram, we observe a compromise between the execution time and the accuracy in the low SNR region.

The remainder of this paper is organized as follows. In Section II, we introduce the problem statement, the system model, and the periodogram-based radar processing. Afterwards, we theoretically demonstrate the improvements, using the DFT-CE combined to ZCP and applying the FPS-SFT approach for reducing the computational complexity associated with the 2D periodogram processing in Section III. Simulation results and discussions are presented in Section IV. Finally, Section V concludes the work and sets forth our perspectives.

II. PROBLEM STATEMENT AND SYSTEM MODEL
A. PROBLEM STATEMENT

We consider a wireless communication system consisting of a communication antenna Tx co-located with a monostatic radar as depicted in Fig. 1. In the downlink, the signal emitted from the communication subsystem, is known to the radar, and is reflected by a certain number of targets characterized by their ranges and velocities. OFDM is one of the leading technologies used in contemporary wireless communication systems. Considerable attention has been given to OFDM for its performance advantages, such as its ability to mitigate inter carrier interference (ICI) and inter symbol interference (ISI) by making suitable use of a cyclic prefix (CP), as well as its robustness against frequency selective fading in addition to its efficient spectral utilization. In this regard, we adopt OFDM as the multiplexing scheme since using a single signal for communication and sensing is strongly dependent on the data structure [29]. The transmitted signal consists of pilots used for channel estimation and net information.

It is important to mention that channel estimation needs to be performed for both the communication and the radar subsystems, but in different ways and for different purposes. At the communication receiver, as usual, the received frequency-domain pilots are used to perform channel estimation using either least square (LS)-CE, minimum mean square error (MMSE)-CE, or any other channel estimation algorithms. Depending on the type of pilot arrangements, either frequency or time domain interpolation (or both) can be performed to infer the channel on the non-pilot subcarriers of each symbol [33], [48]. At the radar, which we treat in this work, the entire grid of the transmitted signal is used to perform the channel estimation [33], [48] since
it knows the transmitted frames. The problem consists of estimating the channel to efficiently approximate the ranges and velocities of the targets using the reflected signals as shown in Fig. 1, and further, improve channel estimation. In this work, we focus on the downlink for simplicity, whereas most results and insights can be easily extended to the uplink. We also hypothesize that the interference between the radar and the communication antenna is negligible.

B. SYSTEM MODEL

In this section, we describe the communication and radar channel model and explain how the radar performs the target parameter estimation. We consider a total bandwidth $B$ which can be divided into $N$ small bands with central frequencies $f_0, f_1, \ldots, f_{N-1}$ such that $\Delta f = B/N$. An OFDM symbol is a packet of $N$ modulated symbols transmitted at the same time on $f_0, f_1, \ldots, f_{N-1}$. The OFDM symbol duration $T$ is thus given by $T = 1/\Delta f$. After modulating bits by quadrature amplitude modulation (QAM), IFFT is applied to the QAM data symbols to generate OFDM waveforms in the time domain. Subsequently, a CP is added between consecutive symbols to mitigate ISI. Several OFDM waveforms are summed up to obtain an OFDM frame. The signal goes through a high power amplifier (HPA) and the communication antenna transmits it. The channel is a multi-path channel, to which an additive white Gaussian noise (AWGN) is added. At the radar, inverse operations are executed. First, the CP is removed, and then FFT is performed on the OFDM bandpass signals. Finally, after the spectral division, the target detection algorithm is applied. The holistic process of transmission and detection is depicted in Fig. 2.

We assume that the complex symbols $\{a_k,l\}$ are generated after QAM modulation. Taking IFFT on the zeroth OFDM symbol $a_k,0$, the OFDM symbol sampled at sampling time $T_0 = T/N$ can be represented as [33]:

$$x[n] = x(nT_0) = \sum_{k=0}^{N-1} a_{k,0} e^{j2\pi nk/N}, \quad 0 \leq n \leq N - 1, \quad (1)$$

which is the IFFT of the QAM symbols.

Taking into account the CP transmission time $T_{cp}$, an OFDM symbol transmission time $T_s = T + T_{cp}$. In terms of the number of symbols, we have $N_s = N + N_{cp}$, where $N_{cp}$ is the number of complex symbols transmitted during $T_{cp}$. Assuming that an OFDM frame is composed of $M$ OFDM symbols, the transmitted signal can be represented as:

$$x[n] = \sum_{l=0}^{M-1} \sum_{k=0}^{N-1} a_{k,l} e^{j2\pi nk/N}. \quad (2)$$

1) RADAR CHANNEL MODEL

We consider a baseband signal $x(t)$ with carrier frequency $f_c$. The transmitted passband signal is $x_{pb}(t) = x(t) e^{j2\pi f_c t}$. For a given reflecting target at distance $d$ from the transmitter and moving at velocity $v$, the received passband signal is impacted by [33]:

- Attenuation factor $g$ which depends on distance $d$, radar cross section (RCS) $\sigma_{RCS}$, carrier frequency $f_c$, and speed of light $c$, and by using the Friis equation of transmission, we obtain:

$$g = \frac{c \sigma_{RCS}}{(4\pi)^3 d^2 f_c^2}. \quad (3)$$

- Signal delay $\tau$ caused by the round-trip, and $\tau = 2d/c$.
- Doppler-Shift $f_D$ caused by the velocity of the target, and $f_D = \frac{2v}{c} f_c$.
- Random rotation phase $\varphi$ introduced when hitting the target.
- AWGN $z(t)$, and $z(t) \sim \mathcal{N}(\mu, \sigma^2)$.

Assuming a total number of $N_t$ reflecting moving targets, and taking into account all the previous constraints, received passband signal $y_{pb}(t)$ is written as follows [48], [49]:

$$y_{pb}(t) = \sum_{p=0}^{N_t-1} g_p x(t - \alpha_p(t)) e^{j2\pi f_c (t - \tau_p)} e^{j\varphi_p} + z_{pb}(t), \quad (4)$$

where $\alpha_p(t) = 2\frac{d_p}{c} + \frac{v_p t}{c} = \tau_p + B_p t$ and $B_p = \frac{2v_p}{c}$.

At the radar, the received baseband signal $y(t)$ is obtained by demodulating $y_{pb}(t)$ as $y(t) = y_{pb}(t) e^{-j2\pi f_c t}$. Thereafter, (4) can be rewritten as

$$y(t) = \sum_{p=0}^{N_t-1} g_p x(t - \alpha_p(t)) e^{-j2\pi f_c t_p} e^{j2\pi f_D t_p} e^{j\varphi_p} + z(t). \quad (5)$$

Eq. (5) is the received signal containing attenuation $g_p$, channel delays $\tau_p$, Doppler effects $f_D$, and time-scale factor.
$B_p$. In essence, $y(t)$ is the filtered signal of $x(t)$ with the channel impulse:

$$h(t) = \sum_{p=0}^{N_z-1} g_p e^{-j2\pi f_p t} e^{j2\pi f_D nT_0} e^{j\phi_p} \delta \left((1-B_p)t - \tau_p\right).$$  \hspace{1cm} (6)

where $\delta(t)$ is the Dirac function. A discrete-time counterpart considering a sample time $T_0$ is given by

$$h[n] = h(nT_0) = \sum_{p=0}^{N_z-1} g_p e^{-j2\pi f_p \frac{n}{N}} e^{j2\pi f_D nT_0} e^{j\phi_p} \times \delta \left((1-B_p)n - \tau_p/T_0\right).$$  \hspace{1cm} (7)

The discrete form of (5) can thus be written as

$$y[n] = x[n] \otimes h[n] + z[n],$$  \hspace{1cm} (8)

where $\otimes$ is the convolution operation.

Knowing that $T_0 = \frac{1}{\Delta f N}$, the discrete-time counterpart of (5) can be obtained by introducing (7) and (2) in (8) [48]:

$$y[n] = \sum_{p=0}^{N_z-1} \sum_{k=0}^{M-1} g_p e^{j\Delta f_k n} e^{j\phi_p} e^{-j2\pi f_D k \frac{n}{N}} e^{j2\pi f_D nT_0} e^{j\phi_p} + z[n].$$  \hspace{1cm} (9)

The signal for the $l$th OFDM symbol is expressed as

$$y[n, l] = \sum_{p=0}^{N_z-1} \sum_{k=0}^{M-1} g_p e^{j\Delta f_k n} e^{j\phi_p} e^{-j2\pi f_D k \frac{n}{N}} e^{j2\pi f_D nT_0} e^{j\phi_p} + z[n].$$  \hspace{1cm} (10)

According to [33], a large sub-carrier distance heavily alleviates the de-orthogonalizing effect of a frequency offset. Therefore, it must be ensured that $\Delta f$ is larger than the Doppler shift caused by the object with the maximum relative velocity $v_{\text{max}}$, i.e.,

$$v_{\text{max}} \ll \frac{\Delta f}{2c},$$  \hspace{1cm} (11)

which depends on the wave parameterization.

Our system parameterization satisfies (11), and, consequently, $(e^{-j2\pi f_D n}, e^{-j2\pi f_D nT_0}, e^{-j2\pi f_D n}) \rightarrow (1, 1, 1)$. Therefore, (10) can be reduced to a simpler form infra:

$$y[n, l] = \sum_{p=0}^{N_z-1} \sum_{k=0}^{M-1} g_p e^{j\Delta f_k n} e^{j\phi_p} e^{-j2\pi f_D k \frac{n}{N}} e^{j2\pi f_D nT_0} e^{j\phi_p} + z[n].$$  \hspace{1cm} (12)

2) RADAR PROCESSING

The main purpose of using the CP is to avoid ISI. For that, the maximum channel delay $\tau_{\text{max}}$ should be less than the cyclic delay $\frac{N_z-1}{2}$.

FIGURE 2. OFDM-based co-located monostatic radar transmission and detection process.

FIGURE 3. 2D periodogram computation process: The vertical $\delta$ and horizontal $\Rightarrow$ arrows indicate FFT over columns and rows, respectively.
prefix transmission time, i.e.,

$$\max_{0 \leq p \leq N-1} \{ T_p \} \leq T_{cp}. \quad (13)$$

We regulate that range $d_{un}$ (respectively a velocity $v_{un}$) is unambiguous if two targets positioned at $d$ and $d + d_{un}$ (respectively at moving velocities $v$ and $v + v_{un}$) cannot be distinguished [33], given

$$d_{un} = \frac{c}{2\Delta f}, \quad \text{and} \quad v_{un} = \frac{c}{2f_c T_s}. \quad (14)$$

A distance $\Delta d$ (respectively velocity $\Delta v$) is called the radar resolution if it is the lowest distance (respectively velocity) such that two targets positioned at $d$ and $d + \Delta d$ (respectively moving at velocities $v$ and $v + \Delta v$) can still be distinguished [33], i.e.,

$$\Delta d = \frac{c}{2N\Delta f}, \quad \text{and} \quad \Delta v = \frac{c}{2Mf_c T_s}. \quad (15)$$

The maximum range and the maximum velocity are directly defined by the parameters of the waveform as follows [33], [50]:

$$d_{max} = \frac{c \times T_{cp}}{2}, \quad \text{and} \quad v_{max} \leq \frac{c}{2f_c T_s}. \quad (16)$$

The first step of the estimation process is the spectral division. Since the radar knows the transmitted frames, the channel information can be retrieved by calculating the ratio of the whole received signal over the whole transmitted one, which results in the LS-CE estimate of the channel. Then, estimation matrix $H$ has entries as follows:

$$h_{k,l} = \frac{y_{k,l}}{a_{k,l}} = \sum_{p=0}^{N-1} b_p e^{j2\pi \frac{IN_{dp}}{NM}} e^{-j2\pi k \Delta f \tau_p} e^{j\Phi} + z_{k,l},$$

(17)

where $h_{k,l}$, $y_{k,l}$, $a_{k,l}$, and $z_{k,l}$ are the entry at the $k$th row and the $l$th column in matrix $H$, received frame matrix $Y$, transmitted frame matrix $A$ and noise matrix $Z$, respectively; $\Phi$ is the phase obtained after the element-wise division. A periodogram is an estimate of the spectral density of a signal. Since in our case $H$ is a two-dimensional signal, the corresponding periodogram can also be written as [13]

$$P(s, r) = \frac{1}{NM} \left| \sum_{k=0}^{N'-1} \left( \sum_{l=0}^{M'-1} h_{k,l} w_{k,l} e^{-j2\pi \frac{r \Delta f \tau_s}{N}} \right)^2 \right|^2,$$

(18)

where $r = 0, \ldots, N'-1$ and $s = \left\lfloor \frac{-M'}{2} \right\rfloor, \ldots, \left\lfloor \frac{M'}{2} \right\rfloor - 1$.

Here, $\left\lfloor \frac{M'}{2} \right\rfloor$ indicates the floor of $\frac{M'}{2}$, and the negative values of $r$ allow estimating negative velocities. The problem here can be formulated as finding the optimal $s$ and $r$ in $P$, by which the dominant frequencies are present. Those frequencies will represent the reflection points which are effective targets. In (18), $w_{k,l}$ is the value at the $k$th row and $l$th column in matrix $W$. It is a window function that reduces the side-lobe levels of each dominant frequency. $N'$ and $M'$ are the extended values of $N$ and $M$ such that $N' \geq N$ and $M' \geq M$. $N'$ and $M'$ can improve the precision of the estimation but do not have any effect on the radar resolution. As such, (18) is equivalent to taking an $M'$-FFT of each column of $H$ then an $N'$-IFFT of each row of the previously resulting matrix as depicted in Fig. 3. Consequently, (18) outputs dominant peaks, where targets are supposed to be located. Due to the whiteness of the noise, the detection threshold is equal to $\sigma^2 \ln(P_{fa})$ [33]. More explicitly, as established in (19), any point $(s, r)$ such that $P(s, r) \geq \sigma^2 \ln(P_{fa})$ is considered as a target, otherwise it is regarded as noise, i.e., a false target, where $P_{fa}$ is the desired probability of false alarm. Mathematically, we have

$$P(s, r) \begin{cases} \geq \sigma^2 \ln(P_{fa}), & \text{target} \\ \leq \sigma^2 \ln(P_{fa}), & \text{noise only} \end{cases}.$$ (19)

Once we obtain the list of estimates $\hat{s}$ and $\hat{r}$, the corresponding target range and velocity values are deduced as follows:

$$\hat{d}_p = \frac{c \hat{s}_p}{2N' \Delta f}, \quad \text{and} \quad \hat{v}_p = \frac{c \hat{r}_p}{2f_c M'T_0}. \quad (20)$$

III. SENSING PERFORMANCE ENHANCEMENT

A. RADAR CHANNEL ESTIMATION USING DFT-CE

As presented previously, since targets’ velocities and ranges can be extracted from the estimated channel information, the overall detection precision depends on the quality of $H$ predicted using (17). The intuition is that an accurate estimation of $H$ would output a precise parameter estimate. The approach that we propose below relies on the channel estimates using DFT-CE.

It is worth mentioning that the CP is longer than the maximum channel delay. As such, each path delay of the multipath channel is lower than the time required to transmit the CP. More explicitly, to estimate ranges and velocities,
3 steps [51]:

- Transform the frequency-domain channel $H$ to the time domain using IFFT:

$$
\hat{h}(n) = \frac{1}{K} \sum_{k=0}^{K-1} H(k)e^{j2\pi \frac{nk}{K}}, \quad 0 \leq n \leq K - 1.
$$

- Restrain the effect of noise by keeping only the cyclic prefix equivalent part of the signal:

$$
\hat{h}_c(n) = \begin{cases} 
\hat{h}(n), & 0 \leq n \leq N_{cp} - 1 \\
0, & \text{otherwise}
\end{cases}
$$

- Convert the channel estimate back to the frequency domain using FFT:

$$
\hat{H}'(k) = \frac{1}{K} \sum_{k=0}^{K-1} \hat{h}_c(n)e^{-j2\pi \frac{nk}{K}}, \quad 0 \leq n \leq K - 1.
$$

The operations of converting the frequency-domain channel to the time domain, then the time-domain channel back to the frequency-domain counterpart are fast since the IDFT and DFT are implemented through IFFT and FFT, respectively. This improves the efficiency of channel estimation with low complexity.

To obtain accurate channel state information, a better approach is to use MMSE-CE, which performs better than both LS-CE and DFT-CE. However, MMSE-CE is limited by its high complexity and also by the fact that we need real-time channel statistics (such as the covariance matrix), which are hard to know in practice. Even though in [51] the authors proposed MMSE based DFT-CE, which is faster than the MMSE-CE, its complexity remains very high and prohibitive for some applications when handling large matrices.

Fig. 4 depicts the estimated mean square error (MSE) of LS-CE, linear minimum mean square error (LMMSE) based LS-CE, DFT-CE, and LMMSE based DFT-CE. As shown in the figure, it is clear that LMMSE based DFT-CE outperforms the others followed by DFT-CE. In contrast, DFT-CE provides a good trade-off between desirable estimation performance (compared to LS-CE) and low complexity (compared to LMMSE based DFT-CE).

Fig. 5 and Fig. 6 present the levels of peaks according to OFDM subcarriers index for a five-target case. When SNR is high, all five targets are detectable, as shown in Fig. 5. However, knowing that targets can be located at subcarrier indexes $i$ such that $0 \leq i \leq N_{cp} - 1$, when SNR becomes very low, for instance SNR = $-25$ dB as presented in Fig. 6, some or all targets peaks are confused with the noise. Subsequently, they can be incorrectly selected in subcarrier index $i$, such that $i \geq N_{cp} - 1$, which increases the detection errors as shown in Fig. 6. By applying DFT-CE, however, targets can still be erroneously selected in the right subcarrier index range, i.e., $0 \leq i \leq N_{cp} - 1$, hence reducing the estimation errors as depicted in Fig. 6b.

### B. ZADOFF-CHU PRECODING

Zadoff-Chu sequences are known as a polyphase complex-valued sequences with constant amplitude zero auto-correlation waveform (CAZAC). Because of their good auto-correlation properties, these sequences have many applications especially in 3GPP Long Term Evolution (LTE) for synchronization of mobile phones with base stations.

A Zadoff-Chu sequence of length $L$ can be defined as:

$$
Z_{seq}(k') = \begin{cases} 
e^{j2\pi r'i(\frac{k'}{L} + q'k' + iD)}, & \text{when $L$ is even} \\
e^{j2\pi (\frac{k'}{L} + q'k' + iD)}, & \text{when $L$ is odd},
\end{cases}
$$

where $k' = 0, 1, \ldots, L - 1$; $q' \in \mathbb{Z}$; $r'$ is an arbitrary integer relatively prime to $L$. From a Zadoff-Chu sequence of length $L$, we construct a square Zadoff-Chu matrix $Z_m$ such that:

$$
Z_{seq}(k') = Z_{seq}(iD + j) = Z_m(i, j),
$$

$$
0 \leq i, j \leq D - 1, D = \sqrt{L}.
$$

At the transmitter, we precode the QAM symbols by multiplying them with the matrix $Z_m$. The precoded signal $A_p$ is therefore $A_p = Z_mA$. At the receiver, the precoding can be discarded by multiplying the precoded received signal by $(Z_m)^{-1}$. It is important to mention that the precoding preserves the power of the signal $A$ as we theoretically demonstrated in Appendix A. In addition, the IFFT operation...
which follows the precoding converts the frequency-domain data into the time-domain representation and ensures that the subcarriers remain orthogonal to each other in the time domain. We provided mathematical proof of the power conservation after the IFFT operation using Parseval’s energy conservation theorem in Appendix B. The PAPR of the signal obtained after this block is expressed as

$$PAPR = \max \|x\|^2 / E[\|x\|^2]$$

(26)

where $x$ is the output signal.

One of the major drawbacks of OFDM is its high PAPR. An OFDM signal with high PAPR is highly sensitive to nonlinear distortion caused by an HPA. This distortion increases the adjacent channel interference (ACI) [52], [53]. Therefore PAPR reduction techniques may be employed to help cut it down. As demonstrated in [52], ZCP is used as a PAPR shrinking technique. Hence, from [52], we have the PAPR inequality expressed as

$$PAPR_{\text{IFFT}}(A_p) \leq PAPR_{\text{IFFT}}(A)$$

(27)

The CP adding operation which involves adding a copy of the end part of each OFDM symbol to the beginning of the symbol maintains the PAPR inequality at the output of the CP block. We provided these theoretical derivations in Appendix C. Furthermore, OFDM signals typically require high power levels for effective transmission, especially in scenarios with long-range or high-rate requirements. HPAs are utilized to amplify the OFDM signal to the desired power level for transmission. However, nonlinear HPAs can offer higher power efficiency compared to linear amplifiers. They can amplify the OFDM signal with less power consumption, which is advantageous in terms of energy efficiency and reducing operating costs.

In Fig. 7, we compute the spectrum of one QAM symbol in the 128-OFDM signal with $B = 20$ MHz, with an OFDM signal going through a nonlinear HPA with an output:

$$f_x = \frac{\|x\|}{\sqrt{1 + (\frac{\|x\|}{E(\|x\|) \times 10^{10}})^2}}$$

(28)

with state level $q$; $\|x\|$ and $E(\|x\|)$ are the element-wise modulus of $x$ and the mean of $\|x\|$, respectively; $\Delta f = \frac{B}{128}$. Here, each data symbol is within $[-\Delta f / 2, \Delta f / 2]$. As a result, the ACI of each modulated symbol in the OFDM-based ZCP, represented by the red curve is low compared to the ACI of the OFDM without ZCP illustrated by the yellow curve. Going through such an HPA maintains the PAPR inequality, which implies a better SNR in the radar channel. We thoroughly demonstrated this claim in Appendix D. In this work, we aim to analyze the impact of ZCP on the range and velocity estimations. By applying the previous ZCP on the OFDM signal, we end up with a signal with lower PAPR, lower ACI, high SNR and consequently, improved estimation of the range and velocity.


C. COMPLEXITY REDUCTION

The DFT is one of the most important and widely used algorithms in computational tasks, such as in signal processing, communications, and audio/image/video compression. It is often implemented through FFT which computes the DFT of an n-dimensional signal with complexity $O(n \log n)$. FFT does not make any assumption about the structure of the signal. However, in many applications, the signal is highly sparse in the frequency domain as depicted in Fig. 8. A signal $x$ is exactly $K$-sparse (or approximately $K$-sparse) in the frequency domain if its Fourier transform contains exactly $K$ non-zero values and the others are zeros. That is, the Fourier transform contains only $K$ dominant values and the others are close to zero. In radar applications, only a few targets are of interest, such as buildings, trucks, and walls. Thus, the final output is often sparse.

Due to the importance of processing sparse data in many areas, several algorithms have been proposed to reduce not only the sampling complexity but also the computational complexity of FFT. A detailed study of such algorithms can be found in [44], [45], [46], and [47]. Some of these algorithms can even reach a computational complexity of $O(K \log K)$ for exactly $K$-sparse signals. The basic principle of all SFT algorithms is to reduce the number of involved signal samples and the computational complexity. Thus, the significant frequencies in the signal are first localized and then estimated, either iteratively or simultaneously. However, most of these algorithms are only proposed for one-dimensional signals, and the extension of these algorithms to multi-dimensional signals is not straightforward. When dealing with multi-dimensional signals, it is required to create a large one-dimensional vector containing the entire signal. After the processing step, in order to return to the initial dimensions to determine the index of dominant frequencies. To alleviate many of the drawbacks brought by the aforementioned techniques, in [45], Wang et al. proposed FPS-SFT, which is a multi-dimensional and iterative SFT algorithm. It processes a multi-dimensional sparse signal with low complexity and small samples under both noise-free and noisy conditions. Consequently, since the signals interested in this paper are highly sparse ($K \ll N \times M$), in this subsection, we take a critical look at FPS-SFT algorithm in terms of its complexity and accuracy. The details of the algorithm can be found in [45]. Since FPS-SFT is iterative and the frequencies recovered in a given iteration are passed to the next iteration, in low SNR region, a frequency recovery error caused in a given iteration is carried through the next ones. Let us assume that the FPS-SFT algorithm executes $I$ iterations and $Q$ is the least common multiple of the signal dimensions $N$ and $M$. For a general case of 2D signals, each iteration uses $3Q$ samples, since it is required that 3 $Q$-length slices are extracted to decode the two frequency components of a 2D signal in the frequency domain. Thus, the sampling complexity of FPS-SFT is $O(3IQ) = O(IQ)$. The core processing of FPS-SFT is the $Q$-point single-dimensional DFT, which can be implemented by the FFT with the computational complexity of $O(Q \log Q)$. In addition to FFT, each iteration needs to evaluate up to $Q$ samples corresponding to different frequencies. Hence, the computational complexity of FPS-SFT is $O(I(Q \log Q + Q)) = O(IQ \log Q)$. If we let the iteration size $I$ equal $I_{max}$ which is sufficiently large so that FPS-SFT converges for a given $K$-sparse signal, the sample and the computational complexity of FPS-SFT become $O(Q)$ and $O(Q \log Q)$, respectively. For $K = O(Q)$, FPS-SFT achieves the lowest sample and computational complexity, i.e., $O(K)$ and $O(K \log K)$, respectively, among all considered SFT algorithms [45].

IV. SIMULATION RESULTS AND DISCUSSION

We evaluate the performance of the radar estimate in the industrial scientific and medical (ISM) band of 77 GHz for a total bandwidth of $B = 491.52$ MHz using the proposed algorithms. The simulation parameters are inspired by the

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5G New Radio (NR) specifications [54] and are summarized in Table 1. In 5G NR, each frame duration is 10 ms, corresponding to 10 subframes. The 240 KHz subcarrier spacing configuration contains 16 slots per subframe and 14 OFDM symbols by slot. Consequently, this gives a total of 2,240 OFDM symbols per frame with $T_{\text{cp}} = 7\% \times T$ (normal CP). This CP length is very small and can allow the radar to achieve only up to the coverage of $d_{\text{max}} = 43.73$ m. For the simplicity of our simulations, we stipulate 4 slots per subframe along with a CP such that $T_{\text{cp}} = 25\% \times T$, which results in a total of 560 OFDM symbols per frame and $d_{\text{max}} = 156.25$ m. In our simulations, we are assuming that all $N$ OFDM subcarriers carry useful information except the first OFDM symbol of each frame, which contains block-type pilots for channel estimation at the communication receiver. We also assume that the multipath channel has a sufficiently long coherence time, which makes it unchanged during the transmission of an OFDM frame. The radar cross section is assumed to be unity. For the sake of simplicity, we further set $N'$ and $M'$ to be $N$ and $M$, respectively. With $N = 2,048$ and $M = 560$. A Hamming window is used as the window function in (18). This configuration allows us to achieve a range resolution of 30.52 cm and a velocity resolution of 0.67 m/s. Such a radar can distinguish very close targets in terms of ranges and velocities. Thus, a good parameterization of the radar and wave characteristics is crucial to achieve good target detection performance.

Fig. 9 shows the ordinary periodogram and the FPS-SFT estimates of a five-sparse-signal case at SNR = 20 dB. We plot only the useful part of the signal where targets are located. From Fig. 9, we can observe that FPS-SFT and periodogram estimates perfectly match each other, which means that FPS-SFT can correctly estimate the dominant frequencies.

Fig. 10 depicts the RMSE of the targets’ range and velocity estimation using FPS-SFT and 2D periodogram. From the figure, we observe that the 2D periodogram is more accurate than FPS-SFT in low SNR regions. However, both methods converge in the high SNR region.

Fig. 11 shows the execution time comparison of FPS-SFT and the periodogram. From the figure, we can notice that
FIGURE 13. Velocity RMSE as a function of SNR for ZCP based LS-CE, LS-CE and DFT-CE.

FIGURE 14. Range RMSE as a function of SNR for Boxcar, Bartlett, Hanning, Hamming, and Blackman window functions in comparison to the proposed scheme.

Fig. 12 and Fig. 13 show the RMSE of the range and velocity estimation for the standard periodogram using LS-CE and DFT-CE with and without ZCP. From these figures, we can observe that LS-CE, and DFT-CE using ZCP outperforms the case without ZCP. Moreover, applying DFT-CE based ZCP yields better estimates than LS-CE based ZCP. However, all the algorithms converge in higher SNR regions.

FIGURE 15. Velocity RMSE as a function of SNR for Boxcar, Bartlett, Hanning, Hamming, and Blackman window functions in comparison to the proposed scheme.

FIGURE 16. Radar image map for 5 targets within the ranges of 50 m, 51 m, 52 m, 53 m, 54 m and moving at the velocities of -5 m/s, 10 m/s, 15 m/s, 20 m/s, 21 m/s, with RCSs of 100 m², 350 m², 120 m², 150 m², 50 m², respectively, given SNR = 5 dB.

From Fig. 10 and Fig. 11, we conclude that a compromise is to be taken between the accuracy and the complexity of both 2D periodogram and FPS-SFT depending on the type of application.

The periodogram execution time depends on the OFDM size, whereas FPS-SFT is almost constant only depending on the sparsity order $K$ of the signal while drastically reducing the execution time.

From Fig. 10 and Fig. 11, we conclude that a compromise is to be taken between the accuracy and the complexity of both 2D periodogram and FPS-SFT depending on the type of application.

Fig. 12 and Fig. 13 show the RMSE of the range and velocity estimation for the standard periodogram using LS-CE and DFT-CE with and without ZCP. From these figures, we can observe that LS-CE, and DFT-CE using ZCP outperforms the case without ZCP. Moreover, applying DFT-CE based ZCP yields better estimates than LS-CE based ZCP. However, all the algorithms converge in higher SNR regions.

Fig. 14 and Fig. 15 show the RMSE of the range estimation for the classical periodogram using Boxcar, Bartlett, Hanning, Hamming, Blackman window functions and our proposed scheme. The figures depict the superior performance of our scheme compared to the classical window functions.

Fig. 16 shows a radar range-Doppler map, also called a radar image for five targets $T_1$, $T_2$, $T_3$, $T_4$, $T_5$ within the ranges of 50 m, 51 m, 52 m, 53 m, 54 m and moving at the velocities of -5 m/s, 10 m/s, 15 m/s, 20 m/s, 21 m/s, respectively, when SNR = 5 dB. According to [55], the RCS of a car at 77 GHz in decibel square meters is about 20 dBsm, which is equivalent to 100 m². Based on this value, we generated the five targets with RCSs equal to 100 m², 350 m², 120 m², 150 m², 50 m², respectively. These values align with potential representations of cars, trucks, and compact vehicles. The strongest reflecting target is $T_2$, because of its high RCS (more than three times the RCS of a car), which is similar to the signal reflected
less than the achievable range resolution means that the radar can not see them as different targets in main lobes on the range axis, they are perfectly aligned, which indistinguishable in terms of range. In fact, by projecting their respective velocities is less than the achievable velocity. \( \Delta v_{T_1, T_4} = 20.5 \text{ m/s} - 20 \text{ m/s} = 5 \text{ cm/s} \), which is less than \( \Delta v = 66.79 \text{ cm/s} \). In addition, \( T_1 \), \( T_2 \), \( T_3 \), and \( T_4 \) are distinguishable since the differences of their respective velocities are all greater than \( \Delta v \).

V. CONCLUSION

In this work, we proposed an improved target detection method using several emergent techniques for the periodogram estimation proceeding. The periodogram algorithm outputs estimation in the limit of the radar resolution which depends on the signal parameterization. Through our simulations, we established that when dealing with low SNRs, the estimation using LS-CE performs poorly since the targets’ peaks are confused with noise. By using DFT-CE, we improved the channel estimation and reduced the target estimation error by filtering some false positive targets. Moreover, we concluded that ZCP, in addition to its important applications in OFDM transmission for PAPR reduction and ICI alleviation, can also improve target estimation performance in low SNR region. Considering that received signals are often sparse, FPS-SFT effectively reduces the estimation complexity but has poor accuracy in low SNR regions. In our future works, we aim to apply a noise reduction method to Zadoff-Chu precoded signals to provide more accurate channel estimates at low SNR. An extended version of those algorithms could be applied to networks of unmanned aerial vehicles where targets are moving in three dimensions. Furthermore, the overall estimation performance can also be improved using machine learning, which could be a worthwhile research direction in the future.

APPENDIX A

PROOF OF THE POWER CONSERVATION BY THE ZADOFF–CHU PRECODING

Let us consider the OFDM QAM matrix \( X = (x)_{i,j} \), the precoded signal \( X_p = (x_p)_{i,j} \) is obtained by multiplying \( X \) with Zadoff-Chu matrix \( Z = (z)_{i,k} \) such as \( X_p = ZX \), \( 1 \leq i, k \leq N \) and \( 1 \leq j \leq M \). We denote by \( \| \cdot \| \) the Euclidean norm and by \( x_{ij}, x_{pj}, y_{ij}, y_{pj} \), the ithub row and ithub column entry of matrices \( (x)_{i,j} \) and \( (z)_{i,j} \), respectively; \( z_{ik} \) is the ithub row and ithub column entry of matrix \( (z)_{i,k} \), and, we have \( \| z_{ik} \| = \frac{1}{\sqrt{N}} \).

We consider that \( Z \) is of dimension \( (N, N) \) and we denote by \( X^A \) with dimension \( (N, M) \), the signal at the point \( A \) in Fig. 18 (the signal before the IFFT). The precoding matrix is such that \( X^A_p = ZX^A \). The ithub row and ithub column entry of matrix \( X^A_p \) can be written as \( x^A_{pik} = \sum_{k=1}^{N} z_{ik} x^A_{kj} \).
Because of ZCP, the information on each subchannel is spread across all subchannels. It means that the power along the jth column of the matrix \((x)_{i,j}\) is the same as the one along the jth column of \((x_p)_{i,j}\). Therefore, this is mathematically expressed as
\[
\lim_{v \to N} \sum_{i=1}^{v} \|x_p^A\|^2 = \lim_{v \to N} \sum_{i=1}^{v} \|x_p^B\|^2. \tag{29}
\]
The mean power of \(X^A\) and \(X_p^A\) are respectively expressed as follows
\[
P_{X^A} = \frac{\sum_{j=1}^{N,M} \|x_p^A\|^2}{N \times M}, \quad \text{and} \quad P_{X_p^A} = \frac{\sum_{j=1}^{N,M} \|x_p^B\|^2}{N \times M}. \tag{30}
\]
By rewriting the expression of \(P_{X_p^A}\) in (30) and substituting using the equation in (29), we obtain:
\[
P_{X_p^A} = \frac{\sum_{j=1}^{N,M} \|x_j^B\|^2}{N \times M} = \frac{\sum_{j=1}^{N,M} \|x_p^B\|^2}{N \times M} = P_{X^A}. \tag{31}
\]
Hence, the signal power remains the same by precoding \(X^A\) by Zadoff-Chu matrix \(Z\).

**APPENDIX B**

**PROOF OF THE POWER CONSERVATION BY THE IFFT**

After the IFFT (at point B in Fig. 18), the energy conservation claim can be proven by using Parseval’s energy conservation theorem. In the following, we denote
\[
F = \frac{1}{\sqrt{N}} \begin{bmatrix}
1 & 1 & \cdots & 1 \\
1 & W & \cdots & W^{N-1} \\
1 & W^2 & \cdots & W^{2(N-1)} \\
1 & W^3 & \cdots & W^{3(N-1)} \\
\vdots & \vdots & \ddots & \vdots \\
1 & W^{N-1} & \cdots & W^{(N-1)(N-1)}
\end{bmatrix},
\]
where \(W = e^{-j(2\pi/N)}\). In addition, \(X^B\) and \(X_p^B\) are the signals obtained after adding CP to \(X^B\) and \(X_p^B\), respectively. The CP adding operation consists of adding the last \(N_{cp}\) part of each OFDM symbol (each column of \(X^B\) and \(X_p^B\), respectively) at the beginning of the symbol. In addition, from \(X^B\) and \(X_p^B\) with the same dimension \((N, M)\), we end up with \(X^C\) and \(X_p^C\) with dimensions \((N + N_{cp}, M)\).

- The inequality in (37) holds at C since the CP-adding operation extends the signal with elements selected from the same sample. Therefore,
\[
\max_{i,j} \|x_p^C\|^2 \leq \max_{i,j} \|X^C\|^2. \tag{38}
\]
- The mean power of \(X^C\) is expressed as
\[
E[\|X^C\|^2] = \frac{N \times M}{(N + N_{cp}) \times M} \sum_{i=1}^{N,M} \|x^C_{i,j}\|^2
+ \frac{N_{cp} \times M}{(N + N_{cp}) \times M} \sum_{i=1}^{N+M+N_{cp}} \|x^C_{i,j}\|^2
\]
\[
= \frac{N \times M}{(N + N_{cp}) \times M} E[\|X_p^C\|^2]
+ \frac{N_{cp} \times M}{(N + N_{cp}) \times M} \sum_{i=1}^{N+M+N_{cp}} \|x^C_{i,j}\|^2.
\]
In the same way, the mean power of $X_p^C$ is expressed as

$$E[\|X_p^C\|^2] = \frac{N \times M}{(N + N_{cp}) \times M} \sum_{j=1}^{N,M} \|x_{pj}^B\|^2 + \frac{N_{cp} \times M}{(N + N_{cp}) \times M} \sum_{j=N+1}^{N+N_{cp},M} \|x_{pj}^B\|^2$$

$$= \frac{N \times M}{(N + N_{cp}) \times M} E[\|X_p^B\|^2] + \frac{N_{cp} \times M}{(N + N_{cp}) \times M} \sum_{j=N+1}^{N+N_{cp},M} \|x_{pj}^B\|^2$$

$$= \frac{N \times M}{(N + N_{cp}) \times M} E[\|X_p^B\|^2] + \frac{N_{cp} \times M}{(N + N_{cp}) \times M} \sum_{j=N+1}^{N+N_{cp},M} \|x_{pj}^B\|^2.$$

(40)

From (28), we have

$$\frac{\sum_{i=1}^{N+1} \|x_{pi}^C\|^2}{N_{cp} \times M} \approx \frac{\sum_{i=N+1}^{N+N_{cp},M} \|x_{pi}^C\|^2}{N_{cp} \times M}.$$

(41)

and from (36), the first terms of (39) and (40) are equal. Therefore, it can be approximated that

$$E[\|X_p^C\|^2] \approx E[\|X_p^C\|^2].$$

(42)

Hence, by combining (38) and (42), we end up with

$$\mathcal{P}(X_p^C) \leq \mathcal{P}(X^C).$$

(43)

APPENDIX D

ANALYSIS OF PAPR INEQUALITY THROUGH AN HPA

We consider $X_p^D$ and $X_p^C$ as the response of the HPA (at the point D in Fig. 18) taking $X_p^C$ and $X_p^C$ as the input, respectively. From (28), we have

$$X_p^D = \frac{\|x_p\|}{\sqrt{1 + \frac{\|x_p\|^2}{E[\|x_p\|^2 \times 10^{Lq/10}]}}}.$$  

(44)

The state level of the HPA $q$ is selected such that the output is the same as in Fig. 19. Therefore the HPA is a monotonic increasing function. Hence, for any entry $x_{pi}^C$ and $x_{pj}^C$, if $\|x_{pi}^C\| \leq \|x_{pj}^C\| \Rightarrow \|x_{pi}^D\| \leq \|x_{pj}^D\|$, from (42), we can deduce that

$$E[\|X_p^D\|^2] \approx E[\|X_p^C\|^2].$$

(45)

Similarly, from (38), we can deduce that

$$\max_{i,j} \|x_{pi}^D\|^2 \leq \max_{i,j} \|X_p^D\|^2. $$

(46)

Hence finally, from (45) and (46), we have

$$\frac{\max_{i,j} \|x_{pi}^D\|^2}{E[\|X_p^D\|^2]} \leq \frac{\max_{i,j} \|X_p^D\|^2}{E[\|X_p^D\|^2]} \Leftrightarrow \mathcal{P}(X_p^D) \leq \mathcal{P}(X^D).$$

(47)

We normalize the transmitted signal with the maximum amplitude in each signal matrix. Such a normalization operation does not change the PAPR inequality. Consequently, for a given noise variance $\sigma^2$, we denote $\tilde{X}_p^D$ and $\tilde{X}^D$ the normalized signals. By definition, we have

$$\text{SNR}(\tilde{X}_p^D) = \frac{E[\|\tilde{X}_p^D\|^2]}{\sigma^2}. $$

(48)

In addition, we can write

$$\mathcal{P}(\tilde{X}_p^D) = \max_{i,j} \|\tilde{x}_{pi}^D\|^2 = \frac{1}{E[\|X_p^D\|^2]}.$$  

(49)

From (48) and (49), we have

$$\text{SNR}(\tilde{X}_p^D) = \frac{1}{\sigma^2 \mathcal{P}(\tilde{X}_p^D)},$$

(50)

and knowing the inequality in (47), we obtain

$$\frac{1}{\sigma^2 \mathcal{P}(\tilde{X}_p^D)} \geq \frac{1}{\sigma^2 \mathcal{P}(X^D)} \Rightarrow \text{SNR}(\tilde{X}_p^D) \geq \text{SNR}(X^D).$$

(51)

Hence, ZCP increases the SNR in the radar channel.

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