The new limits of the neutrinoless \((\mu^-, e^-)\) conversion branching ratio

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Abstract

The nuclear physics dependence of the exotic \((\mu^-, e^-)\) conversion branching ratio \(R_{\mu e^-}\) for the experimentally most interesting nuclei \(^{208}\)Pb and \(^{48}\)Ti, is investigated in various nuclear models. The results thus obtained are combined with the new experimental limits extracted at PSI for these nuclei to put bounds on the elementary particle parameters entering \(R_{\mu e^-}\) such as intermediate neutrino masses and mixing angles as well as relevant parameters of intermediate supersymmetric particles (masses and mixing of s-fermions and neutralinos).
There is a plethora of processes predicted by many extensions of the standard model like grand unification, supersymmetric theories etc., which violate the lepton family quantum numbers \([1]-[4]\). Among them the neutrinoless conversion of a bound muon to an electron,

\[
(A, Z) + \mu^-_b \rightarrow e^- + (A, Z)^*
\]  

(1)

which violates the muon and electron numbers, stands out as one of the most prominent processes to search for lepton flavour non-conservation if it exists \([5]-[8]\). The recent experiments at PSI \([5, 6]\) and TRIUMF \([7]\), have up to now put only upper limit on the branching ratio \(R_{\mu e^-} = \Gamma(\mu^-, e^-)/\Gamma(\mu^-, \nu_{\mu})\) in the values

\[
R_{\mu e^-}^{Ti} < 4.3 \times 10^{-12}, \quad \text{for } ^{48}Ti \text{ target} \]

\[
R_{\mu e^-}^{Pb} < 4.6 \times 10^{-11}, \quad \text{for } ^{208}Pb \text{ target} \]

We mention that, the limit set this year at PSI by using \(^{208}Pb\) as target \([8]\), improved over the previous limit \([7]\) by an order of magnitude. By progressively improving the experimental sensitivity, it is expected in the next few years these limits to be pushed down by two to three orders of magnitude \([5]-[8]\).

On the theoretical side, up to now the elementary particle physics aspects of the \((\mu^-, e^-)\) conversion amplitudes have been investigated in the context of several models allowing lepton flavour violation \([1]-[4, 9, 10]\). On the other hand, the nuclear form factors, which describe the rates of process (1) have been computed employing various nuclear models \([11]-[16]\). In recent years some refinements of the quasi-particle RPA approximation, which have been popular in evaluating successfully the nuclear transition matrix elements entering processes like double beta decay (see e.g. ref. \([17]\)), double charge exchange \([18]\) etc, have also been employed in the investigation of \((\mu^-, e^-)\) conversion in the presence of nuclei \([12]\).

In the present work, the p-p and n-n quasi-particle RPA is used in evaluating the nuclear matrix elements entering \(R_{\mu e^-}\) for the targets \(^{208}Pb\) and \(^{48}Ti\), for various mechanisms leading to reaction (1) utilizing the method developed in ref. \([12]\). Our goal is, by combining the nuclear calculations with the experimental limits quoted above, to set constraints on the elementary particle parameters entering various models involving intermediate massive neutrinos or other exotic supersymmetric particles (s-leptons and neutralinos) as it will be discussed below. We perform explicit calculations of the nuclear matrix elements of Pb and Ti in the appropriate values of the momentum transfer by taking into account the muon binding energy entering both the phase space and the nuclear transition form factors. Furthermore, we calculate the contributions coming from Z-exchange diagrams which have not been included in previous studies \([1, 12]\). We have paid special attention to the nucleus \(^{208}Pb\), not only because it is currently used as target at PSI but, as we have previously shown \([13]\), the rate of the \((\mu^-, e^-)\) conversion appears to attain a maximum in this region. We mention that, since \(^{208}Pb\) is a doubly closed shell nucleus, it needs a special treatment in the context of quasi-particle RPA \([17]\).

The nuclear calculations performed in this work rely on the effective interaction \((\mu^-, e^-)\) conversion Hamiltonian constructed in the framework of common extensions of the standard model involving mixing of intermediate neutrinos - gauge bosons or neutralinos - s-leptons (supersymmetry). Below we describe in brief the essential formalism required for our study.

1. Intermediate neutrinos - gauge bosons.

It is well known that, within the minimal standard model, the strengths of the flavour changing interactions are related to the neutrino masses. In this spirit, one writes the weak
neutrino eigenstates $\nu_e$, $\nu_\mu$, ... in terms of the mass eigenstates $\nu_j$ (light neutrinos) and $N_j$ (heavy neutrinos) as

$$\nu_e = \sum_{j=1}^{n} U^{(1)}_{ej} \nu_j + \sum_{j=1}^{n} U^{(2)}_{ej} N_j$$

$$\nu_\mu = \sum_{j=1}^{n} U^{(1)}_{\mu j} \nu_j + \sum_{j=1}^{n} U^{(2)}_{\mu j} N_j$$

where $n$=number of generations and $U^{(1)}_{ej}$, $U^{(1)}_{\mu j}$ are the elements of the charged lepton current mixing matrix for light neutrinos $\nu_j$ associated with the electron and the muon (see figs. 1,2). The corresponding quantities with superscript (2) refer to heavy neutrinos $N_j$.

In order to deduce the effective interaction Hamiltonian of the ($\mu^-, e^-$) process at the nuclear level needed for our study, one starts from the weak vector and axial vector quark currents,

$$J_\lambda = \sum_j \bar{q}_j \gamma_\lambda (1 - \gamma_5) q_j, \quad J_\lambda^\pm = \sum_j \bar{q}_j \gamma_\lambda (1 - \gamma_5) \tau^\pm q_j$$

($J_\lambda$ neutral, $J_\lambda^\pm$ charged currents) where $q_j, q'_j = u, d$ quarks (we neglect strange quark contribution into the nucleon and consider only left-handed currents).

The next step is, to write at the nucleon level the hadronic current for each specific mechanism by taking the matrix elements of eq. (4) using the appropriate quark model wave function for the nucleon. In this work we assumed a non-relativistic nucleon wave function and studied the following types of mechanisms:

i) Photonic diagrams: The ($\mu^-, e^-$) conversion can proceed via the diagrams of figs. 1(a) and 2(a), i.e. by exchange of a virtual photon, provided the upper vertex is allowed. The hadronic vertex is the usual electromagnetic coupling. At the nucleon level it can be written as

$$J_\lambda^{(1)} = \bar{N}_p \gamma_\lambda N_p = \bar{N} \gamma_\lambda \frac{1}{2} (1 + \tau_3) N \quad \text{(photonical)}$$

($N_p, N$ represent the proton, nucleon spinors) This current involves equal isoscalar and isovector components.

ii) Non-photonic diagrams: There is a plethora of such diagrams. The most obvious are the one which is mediated by Z-particle exchange shown in figs. 1(a),(b) and 2(a),(b), and the box diagrams involving intermediate massive neutrinos and W-bosons as in fig. 1(c) or mediated by s-leptons and neutralinos as in fig. 2(c). In such cases the hadronic current takes the form

$$J_\lambda^{(2)} = \bar{N} \gamma_\lambda \frac{1}{2} [ (3 + f_V \beta \tau_3) - (f_V \beta'' + f_A \beta' \tau_3) \gamma_5 ] N \quad \text{(non-photonical)}$$

The parameter $\beta''$ takes the value unity except in Z-exchange when it is zero. For the box diagrams we have $\beta = \beta'$. In the case of W-boson exchange we have $\beta = 5/6$ while for intermediate s-leptons we get $\beta = 0.6$. For Z-exchange we find

$$\beta' = \frac{3}{2 \sin^2 \theta_W} = 6.90$$

$$\beta = 3 (1 - \frac{1}{2 \sin^2 \theta_W}) = -3.46$$

In the above equations $\beta = \beta_1/\beta_0$ with $\beta_0$ ($\beta_1$) being the isoscalar (isovector) quantities at the quark level. $f_V, f_A$ are the vector, axial vector static nucleon form factors ($f_A/f_V = 1.24$).
The effective Lagrangian at the nucleon level, which involves both photonic and non-photonic contributions can be cast in the form

\[ \mathcal{M} = \frac{4\pi \alpha}{q^2} j_{(1)}^\lambda J_{(1)}^\lambda + \frac{\zeta}{m_\mu^2} j_{(2)}^\lambda J_{(2)}^\lambda \]  \tag{9}

where \( \alpha \) is the fine structure coupling constant, \( \zeta \) is given by

\[ \zeta = \frac{G_F m_\mu^2}{\sqrt{2}} (W - \text{boson exchange}) \]  \tag{10}

and \( q \) is the momentum transfer which in a good approximation is written as

\[ q = m_\mu - \epsilon - (E_f - E_{gs}) \]  \tag{11}

with \( \epsilon \) the muon binding energy and \( E_f \) (\( E_{gs} \)) the energy of the final (ground) state of the nucleus.

In eq. (9) \( j_{(1)}^\lambda \) and \( j_{(2)}^\lambda \) represent the leptonic currents corresponding to each mechanism i.e.

i) Photonic mechanism:

\[ j_{(1)}^\lambda = \bar{u}(p_e) \left[ (f_{M1} + \gamma_5 f_{E1}) i \sigma^{\lambda\nu} \frac{q_\nu}{m_\mu} + (f_{E0} + \gamma_5 f_{M0}) \gamma_\nu \left( q^{\lambda\nu} - \frac{q^\lambda q^\nu}{q^2} \right) \right] u(p_\mu) \]  \tag{12}

ii) Non-photonic mechanism:

\[ j_{(2)}^\lambda = \bar{u}(p_e) \gamma^\lambda \frac{1}{2} (f_1 + f_2 \gamma_5) u(p_\mu) \]  \tag{13}

where \( p_e, p_\mu \) the lepton momenta and \( f_{E0}, f_{E1}, f_{M0}, f_{M1}, f_1, f_2 \) parameters depending on the assumed gauge model.

In the last step, the effective interaction Hamiltonian, which converts a muon to an electron in the nucleus \((A, Z)\), is obtained by summing over all type of eq. (9) single nucleon contributions. Behind this summation lies the assumption that the A nucleons interact individually with the muon field (impulse approximation).

2. Intermediate neutralinos - s-leptons (supersymmetric model).

The supersymmetry (SUSY) associates to each particle its superpartner (s-particle). In this way, the fundamental particles of nature are essentially doubled and as a consequence several new mechanisms, some of which violate the lepton flavour conservation, are predicted. Some typical supersymmetric diagrams leading to the \((\mu^-, e^-)\) conversion are shown in fig. 2.

In the effective Hamiltonian of eq. (9), the hadronic current is given by eq. (6) by putting the values of \( \beta \) discussed before. For the leptonic currents, in the simplest version of this model, the photonic and non-photonic amplitudes are simply related [11]. One finds:

\[ (4\pi \alpha) f_{M1} = -(4\pi \alpha) f_{E1} = -\frac{1}{24} f \]  \tag{14}

\[ (4\pi \alpha) f_{E0} = -(4\pi \alpha) f_{M0} = -\frac{1}{72} f \]  \tag{15}

\[ f_1 = -f_2 = \frac{1}{16} \beta_0 f \]  \tag{16}

\[ f = \alpha^2 \frac{m_\mu^2}{m^2} \eta \]  \tag{17}
\[ \tilde{\eta} = \left( \frac{\delta m^2_{\text{ll}}}{m^2} \right)_{12} \]  
\[ \zeta = 1.0 \]  
\[ (18) \]
where \( \bar{m}^2 \) is the average of the square of the masses of the s-fermions entering the loop and \( \beta_0 = 5/9 \). The parameter \( \tilde{\eta} \) depends on the details of the model \([10]\).

The Z-exchange contribution in SUSY models like the one just discussed yields \([20]\)

\[ f_1 = -f_2 = \alpha^2 \xi \tilde{\eta} \bar{m}^2_{\mu} \]  
\[ (20) \]
where the parameter \( \xi \) depends on the details of the model. It vanishes if one ignores the Higgsino components of the neutralinos as e.g. in the case of pure photino. It also vanishes if the probabilities of finding the two Higgsinos in the neutralino are equal. In the model \#2 of ref. \([19]\) it takes the value \( \xi = 2.8 \times 10^{-2} \). Clearly, the Z-exchange is more favored in models in which \( \bar{m}^2 \) is larger than \( m^2_{Z} \). Accurate formulas and details will be presented elsewhere \([20]\).

Experimentally, the branching ratio \( R_{\mu e^-} \) is quite interesting quantity. From a theoretical point of view, in general it is not easy to formulate an expression containing contributions from both mechanisms, photonic and non-photonic ones. For this reason, the photonic and non-photonic contributions are usually discussed separately.

In the case of the coherent channel, however, one can separate the dependence on the nuclear physics from the leptonic form factors in the branching ratio \( R_{\mu e^-} \), provided that the nuclear vertex is calculated in the Born approximation and the density of the muon bound state inside the nucleus is approximated by a mean wave function (see below). Under these assumptions the muon capture rates can well be described by the Goulard-Primakoff function \( f_{GP}(A, Z) \) \([21]\). In general, it is impossible to separate the nuclear structure aspects from the elementary particle parameters. This can, however, approximately be done in the models discussed above \([10]\). Thus one can write

\[ R_{\mu e^-} = \rho \gamma \]  
\[ (21) \]
The function \( \gamma(A, Z) \) contains all the nuclear dependence of \( R_{\mu e^-} \) and the quantity \( \rho \) is independent on nuclear physics. They are defined as follows:

i) Neutrino mixing models:

\[ \gamma = \frac{E_{e}P_{e}}{m^2_{\mu}} \frac{|ME|^2}{G^2 Z f_{GP}(A, Z)} \]  
\[ (22) \]
\( (G^2 \approx 6, \ p_e = -q) \) where \( |ME| \) represent the nuclear matrix elements given in terms of the proton \( (F_{Z}(q^2)) \) and neutron \( (F_{N}(q^2)) \) nuclear form factors. In the special case of photonic diagrams \( |ME| \) is proportional to the elastic or inelastic form factor \( F_{Z}(q^2) \) depending on which final state is populated. Then, \( \gamma \) takes the form

\[ \gamma_{ph} = \frac{E_{e}P_{e}}{m^2_{\mu}} \frac{Z|F_{Z}(q^2)|^2}{G^2 f_{GP}(A, Z)} \]  
\[ (23) \]
The quantity \( \rho \) in eq. \([21]\), in the case of left-handed currents only, takes the form

\[ \rho = (4\pi\alpha)^2 \frac{|f_{M1} + f_{E0}|^2 + |f_{E1} + f_{M0}|^2}{(G_{F}m^2_{\mu})^2} \]  
\[ = \frac{9\alpha^2}{64\pi^2} \frac{\bar{m}^2_{\nu}}{m^2_{W}} \eta_{\nu} + \eta_{N} \]  
\[ (\text{photonic diagrams}) \]  
\[ (24) \]
\[
\rho = \frac{|\beta_0 f_1|^2 + |\beta_0 f_2|^2}{2} = \frac{9}{64\pi^4} (G_F m_W^2)^2 |\frac{m_e^2}{m_W^2}| \eta_\nu + \eta_N|^2
\]

\[ (\text{non - photonic diagrams}) \quad (25) \]

The lepton violating parameters associated with intermediate light \((\eta_\nu)\) or heavy \((\eta_N)\) neutrinos depend on the gauge model considered. They are given by

\[
\eta_\nu = \sum_j U_{e j}^{(1)} U_{\mu j}^{(1)\ast} \frac{m^2_j}{m_e^2}, \quad \eta_N = \sum_j U_{e j}^{(2)} U_{\mu j}^{(2)\ast} \frac{m_N^2}{M_j^2} \left(-2 \ln \frac{M_j^2}{m_N^2} + 3\right) \quad (26)
\]

ii) Neutralino and s-lepton mixing models: Flavour violation in these models occurs via the s-lepton mixing provided that it is different from the corresponding charged lepton mixing. It also involves intermediate neutralinos. Following [10] we will assume that the dominant contribution comes from the photino (SUSY partner of the photon). Under some plausible assumptions and neglecting the Z-exchange we find that the function \(\gamma(A, Z)\) can be written as

\[
\gamma = \zeta \left(\frac{13}{12} + \frac{1}{2} \frac{N}{Z} \frac{F_N}{F_Z}\right)^2 \gamma_{ph} \quad (27)
\]

The quantity \(\rho\) in this model can be written as

\[
\rho = \frac{1}{288} \frac{\alpha^4}{(G_F m^2)^2} |\bar{\eta}|^2 \quad (28)
\]

where \(\bar{\eta}\) is given by eq. (18). In the case of Z-exchange [20] we find

\[
\gamma = 0.053 \left(1 - 14.04 \frac{N}{Z} \frac{F_N}{F_Z}\right)^2 \gamma_{ph} \quad (29)
\]

The quantity \(\rho\) is now given by

\[
\rho = 5.5 \times 10^{-4} \frac{\alpha^4}{(G_F m^2)^2} |\bar{\eta}|^2 \quad (30)
\]

We stress that for the coherent process, in the models discussed above, the only variable of the elementary particle sector, which enters the function \(\gamma\) is the parameter \(\beta = \beta_1/\beta_0\). Once \(\gamma(A, Z)\) is known, e.g. by nuclear model calculations, from eq. (21) one can extract information about the interesting parameter \(\rho\) from the experimental data and compare it with the value predicted for various mechanisms by the gauge models. Such kind of calculations we include below.

The task of studying the nuclear physics aspects of the exotic \((\mu^-, e^-)\) conversion process is to evaluate the function \(\gamma(A, Z)\) of eqs. (23), (25), (27) and (29). In general, the nuclear matrix elements entering \(\gamma(A, Z)\) depend on the final nuclear state populated during process (1). What, however, is more interesting is the coherent process, since then the rate is free of the background from bound muon decay [8]. Furthermore, extensive calculations have shown that the coherent channel dominates the \((\mu^-, e^-)\) conversion throughout the periodic table [13] (for the isotopes studied in the present work see table 2 below).

If we assume an average value for the muon wave function, so that it cancels in the branching ratio, the coherent matrix elements needed in the computation of \(\gamma(A, Z)\) can be written as
\[ M^2_{\text{coh}}(q^2) = \left[ 1 + Q(\beta) \frac{N}{Z} \frac{F_N(q^2)}{F_Z(q^2)} \right]^2 Z^2 F^2_Z(q^2) \]  

(31)

where \( Q(\beta) \), for the models discussed in the present work, takes the values

\[
Q(\beta) = \frac{3 - f_V \beta}{3 + f_V \beta} = \begin{cases} 
0, & \text{photonic diagrams, } \beta = 3 \\
13/23, & \text{W - boson exchange, } \beta = 5/6 \\
2/3, & \text{SUSY Photonic + Box diagrams, } \beta = 3/5 \\
-14.04, & \text{SUSY Z - exchange, } \beta = -3.46 
\end{cases}
\]  

(32)

Notice that, in the photonic case \( M^2_{\text{coh}} = Z^2 F^2_Z(q^2) \) (only protons of the considered nucleus contribute, see eq. (3)).

In table 1, we show the results of the nuclear form factors and the coherent matrix elements for \( ^{208}\text{Pb} \) and \( ^{48}\text{Ti} \), calculated in the context of quasi-particle RPA using the method of ref. |12|. For comparison, in this table the results of ref. |11| (shell model) and those of ref. |13| (local density approximation, LDA) are also presented. The quasi-particle RPA form factors are in good agreement with experimental electron scattering data |23| also listed in table 1.

For the benefit of the reader we provide here a brief description of the main steps followed in the computational procedure. At first, we have chosen an appropriate model space: for \( ^{208}\text{Pb} \) 18 levels above the core \( ^{\text{190}}\text{Sn} \) and for \( ^{48}\text{Ti} \) 16 levels without core (for the calculations of \( ^{48}\text{Ti} \) in ref. |12| we used a smaller model space, consisted of only 10 levels without core). In order to satisfy the convergence of BCS equations in the case of the doubly closed shell nucleus \( ^{208}\text{Pb} \), we determined the strength parameters of pairing for protons and neutrons from the experimental proton and neutron pairing gaps of the \((N - 2, Z + 2)\) neighbor nucleus \( ^{208}\text{Po} \), following the procedure used recently in the study of double beta decay |17|.

We have also evaluated the incoherent rate, i.e. the matrix elements from the initial (ground) state (a \( 0^+ \) state) to every excited state \(|f\rangle \) included in the chosen model space of the studied nucleus, by calculating explicitly the contribution of each individual channel, for photonic and non-photonic diagrams. These results are denoted as \( M^2_{\text{inc}} \) and are shown in table 2 separately for the vector \( (M_V) \) and axial vector \( (M_A) \) components of the hadronic current eq. (6) \( (M^2_{\text{inc}} = M_V + M_A) \). Obviously, in the photonic mechanism only the vector component contributes, but in the non-photonic one both the vector and axial vector components give non-zero contributions. For each isotope studied we found that, the main contribution to the incoherent rate comes from the low-lying excited states and that high-lying excited states contribute negligible amounts. Such state-by-state calculations of the incoherent channel for a set of nuclei throughout the periodic table are discussed in ref. |12|, |22|.

From the coherent and total matrix elements, \( M^2_{\text{tot}} = M^2_{\text{coh}} + M^2_{\text{inc}} \) another useful quantity of the \((\mu^-, e^-)\) conversion, the ratio \( \eta = M^2_{\text{coh}}/M^2_{\text{tot}} \) which expresses the portion exhausted by the coherent rate in the total branching ratio \( R_{\mu^-, e^-} \), can be obtained (see table 2). We found that, for both isotopes and all mechanisms studied, \( \eta \geq 90\% \), which implies that the coherent rate dominates the \((\mu^-, e^-)\) conversion process. In earlier calculations the ratio \( \eta \) was estimated |14| to be \( \eta \approx 83\% \) in Cu region and smaller in lead region while in ref. |13| we found \( \eta \geq 90\% \) throughout the periodic table. Our present results agree well with those of ref. |13|.

We should note that, in the present study the muon binding energy \( \epsilon_b \) in eq. (14) has been taken into consideration. This property affects significantly the nuclear form factors, especially for heavy nuclei like \( ^{208}\text{Pb} \). In light nuclei \( \epsilon_b \) is negligible (for \( ^{48}\text{Ti} \) \( \epsilon_b = 1.450 \)), but in heavy elements it becomes significant (for \( ^{208}\text{Pb} \) \( \epsilon_b = 10.450 \)). In addition, the factor \( E_{\nu}p_{\nu}/m^2_\mu \) in eq. (22), which takes into account the phase space in the transition matrix elements, depends on
the muon binding energy. By ignoring $\epsilon_b$, this factor becomes unity. For $^{48}\text{Ti}$ this factor is equal to 0.97 and the neglect of $\epsilon_b$ is a good approximation. For $^{208}\text{Pb}$, however, this factor is equal to 0.81 which means that, in heavy nuclei the dependence on $\epsilon_b$ of the phase space cannot be ignored.

The values for $\gamma(A,Z)$ calculated from the coherent QRPA matrix elements are shown in table 3 and compared with the results of refs. [11, 13]. We mention that, the variation of $\gamma(A,Z)$ through the periodic table studied in ref. [11] exhibits a strong dependence on the neutron excess ($N - Z$) which mainly reflects the dependence on ($N, Z$) of the total muon capture rate.

By putting the experimental limits [5, 6] in eq. (21) we determine upper bounds on the parameter $\rho$ for photonic, Z-exchange and W-box diagrams both in conventional extensions of the standard model as well as SUSY theories (see table 3). We note that the values of $\rho$ for $^{48}\text{Ti}$ are slightly improved over those of table 2 of ref. [12], but those of $^{208}\text{Pb}$ are appreciably smaller than those of sect. 5.1 of ref. [11]. This big difference is due to the fact that, in the present work we have used the experimental limit of ref. [6]. This experiment at PSI improved on the previous limit of $^{208}\text{Pb}$ [7], which was used in ref. [11], by an order of magnitude. For both nuclei we found that, the limits obtained from various nuclear models do not significantly differ from each other (we should stress that, the different limits for $^{208}\text{Pb}$ in the shell model results of table 3, are due to the neglect of $\epsilon_b$ in eq. (11) when calculating the nuclear form factors as mentioned above; its consideration gives similar results to those of LDA and QRPA for both mechanisms). This implies that all nuclear models studied here give about same values for $\rho$.

One can use the limits of $\rho$ to parametrize the muon number violating quantities entering eqs. (24), (25), (28) and (30) and also to compare them directly to the value given from gauge models. As an example, we quote the value of $\rho = 8.2 \times 10^{-18}$ obtained in the supersymmetric model of ref. [11] discussed above. This prediction of $\rho$ is considerably smaller compared to the values listed in table 3, which were extracted from experiment.

In summary, in the present letter we have studied the nuclear physics part of the branching ratio of the exotic $\mu^- \rightarrow e^-$ conversion $R_{\mu e^-}$ in $^{208}\text{Pb}$ and $^{48}\text{Ti}$ nuclei. These two isotopes are the most interesting nuclear targets to search for lepton flavour violation. $^{208}\text{Pb}$ is currently used at PSI in the SINDRUM II experiment.

We have calculated in a reliable way the appropriate nuclear matrix elements entering the branching ratio $R_{\mu e^-}$ in the context of the quasi-particle RPA and compared them with the results given from other nuclear models. We found that, the coherent $(\mu^-, e^-)$ rate, which is measured from experiments, dominates the branching ratio $R_{\mu e^-}$ for both isotopes but it is more pronounced in the heavy nucleus $^{208}\text{Pb}$.

From the calculations of the nuclear part of the branching ratio $R_{\mu e^-}$, using the new experimental limits, especially those for $^{208}\text{Pb}$, we were able to estimate upper limits for the elementary sector part of the $\mu^- \rightarrow e^-$ rate in common extensions of the standard model, which allow lepton flavour violation. These limits are very useful to fix the lepton violating parameters and test the various gauge models.

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Table 1. Quasi-particle RPA results for proton, neutron nuclear form factors and coherent nuclear matrix elements squared for the four cases discussed in the text: photonic and non photonic W-boson exchange as well as the two SUSY cases (photonic + box and Z-exchange). For comparison the experimental charge form factors ($F_{Z}^{exp}$) [23] and the calculations of shell model [11] and LDA [13] are shown. Note that in the Z-exchange the factor 0.053 of eq. (29) has not been included.

| (A,Z) | Shell Model | LDA | Exper. | Quasi-particle RPA Results |
|------|-------------|-----|--------|----------------------------|
|      | $F_{Z}$     | $F_{N}$ | $F_{Z}$ | $F_{Z}^{exp}$ | $F_{N}$ | $F_{N}$ | $M_{ph}^{2}$ | $M_{W-ex}^{2}$ | $M_{s-lep}^{2}$ | $M_{Z-ex}^{2}$ |
| $^{48}Ti$ | .543    | .528 | .528 | .506 | .532 | .537 | .514 | 139.6 | 375.2 | 429.6 | 30918.0 |
| $^{208}Pb$ | .194 | .139 | .250 | .220 | .242 | .271 | .214 | 494.7 | 1405.2 | 1618.8 | 127214.1 |

Table 2. The ($\mu^{-}, e^{-}$) conversion matrix elements (coherent, incoherent, total) and the ratio $\eta = M_{coh}^{2}/M_{tot}^{2}$ for photonic and W-exchange (non-photonic) mechanisms involving intermediate neutrino mixing. Results for the experimentally most interesting nuclei, $^{208}Pb$ and $^{48}Ti$, are presented.

| (A,Z) | Photonic $\mu^{-} \rightarrow e^{-}$ Mechanism | W-exchange $\mu^{-} \rightarrow e^{-}$ Mechanism |
|------|---------------------------------|---------------------------------|
|      | $M_{coh}^{2}$ | $M_{inc}^{2}$ | $M_{tot}^{2}$ | $\eta$ (%) | $M_{coh}^{2}$ | $M_{inc}^{2}$ | $M_{tot}^{2}$ | $\eta$ (%) |
|      | $M_{V}$ | $M_{A}$ | $M_{V}$ | $M_{A}$ |
| $^{48}Ti$ | 139.6 | 14.1 | - | 153.7 | 90.8 | 375.2 | 10.7 | 2.6 | 388.5 | 96.6 |
| $^{208}Pb$ | 494.7 | 14.0 | - | 508.7 | 97.2 | 1405.2 | 19.0 | 6.8 | 1431.0 | 98.2 |
Table 3. The new limits on the elementary sector part of the exotic $\mu - e$ conversion branching ratio extracted by using eq. (21) and the recent experimental data for the nuclear targets $^{208}Pb$, $^{48}Ti$, [5, 6]. The nuclear part of the branching ratio, described by the function $\gamma(A, Z)$ of eqs. (22), (23), (27) and (29), is also shown.

| Method | Mechanism          | $^{48}Ti$          | $^{208}Pb$          |
|--------|---------------------|--------------------|--------------------|
|        | $\gamma(A, Z)$     | $\rho$            | $\gamma(A, Z)$     | $\rho$            |
| QRPA   | Photonic            | 9.42 $\leq 4.6 \times 10^{-13}$ | 17.33 $\leq 2.7 \times 10^{-12}$ |
|        | W-boson exchange    | 25.31 $\leq 1.7 \times 10^{-13}$ | 49.22 $\leq 0.9 \times 10^{-12}$ |
| SUSY s-leptons | 25.61 $\leq 1.7 \times 10^{-13}$ | 49.50 $\leq 0.9 \times 10^{-12}$ |
| SUSY Z-exchange | 110.57 $\leq 0.4 \times 10^{-13}$ | 236.19 $\leq 0.2 \times 10^{-12}$ |
| LDA    | Photonic            | 9.99 $\leq 4.3 \times 10^{-13}$ | 17.84 $\leq 2.6 \times 10^{-12}$ |
|        | W-boson exchange    | 26.60 $\leq 1.6 \times 10^{-13}$ | 55.53 $\leq 0.8 \times 10^{-12}$ |
| SM     | Photonic            | 9.74 $\leq 4.4 \times 10^{-13}$ | 10.42 $\leq 4.4 \times 10^{-12}$ |
|        | W-boson exchange    | 26.50 $\leq 1.6 \times 10^{-13}$ | 27.43 $\leq 1.7 \times 10^{-12}$ |

Figure Captions

**Figure 1.** Typical diagrams entering the neutrinoless $(\mu^-, e^-)$ conversion: photonic 1(a), Z-exchange 1(a),(b) and W-boson exchange 1(c), for the specific mechanism involving intermediate neutrinos.

**Figure 2.** SUSY diagrams leading to the $(\mu^-, e^-)$ conversion: photonic 2(a), Z-exchange 2(a),(b) and box diagrams 2(c), in a supersymmetric model with charged s-lepton and neutralino mixing are shown. Note that, the Z-exchange in 2(b) as well as in fig. 1(b), comes out of electrically neutral particles (photon exchange does not occur in these diagrams). These Z-exchange diagrams may be important (those of figs. 1(a) and 2(a) are suppressed by $m_{\mu}^2/m_Z^2$).
\( \mu^- \) \( \gamma, \tilde{Z} \) \( e^- \)

(a)

\( \mu^- \) \( \gamma, \tilde{Z} \) \( U_{\mu j} \) \( \tilde{l}_j^- \) \( \tilde{l}_j^- \) \( e^- \)

(b)

\( \mu^- \) \( \gamma, \tilde{Z} \) \( U_{\mu j} \) \( U_{e j} \) \( Z \)

(c)

\( \mu^- \) \( \tilde{l}_j^- \) \( x \) \( \tilde{l}_j^- \) \( e^- \)

\( \gamma, \tilde{Z} \)

\( \gamma, \tilde{Z} \)

\( u (d) \) \( \tilde{d} (\bar{u}) \) \( u (d) \)