Study of double parton scattering processes with heavy quarks

Ivan Belyaev† and Daria Savrina‡
Institute for Theoretical and Experimental Physics,
B. Cheremushkinskaya 25, 117218, Moscow, Russia
†Ivan.Belyaev@itep.ru, ‡Daria.Savrina@cern.ch

Abstract

Study of double parton scattering processes (DPS) involving heavy quarks provides the most precise probing of factorization hypothesis for DPS for gluon-mediated processes. The measurements are performed for the different final states, including open and hidden-flavour hadrons and for the different kinematic ranges of incoming gluons.

Contents

1 Introduction 2
2 Studies with open-flavour hadrons 4
3 Double quarkonia studies 8
4 Other DPS measurements involving heavy quarks 17
5 Summary 17

---

1 Prepared for: Multiple Parton Interactions at the LHC, Eds. P. Bartalini and J. R. Gaunt, World Scientific, Singapore.
1 Introduction

The double parton scattering processes in the high-energy hadron-hadron interactions attract \cite{1-4} significant theoretical and experimental interest. While the DPS processes first have been observed thirty years ago by AFS collaboration \cite{5} in the proton-proton collisions at relatively low energy $\sqrt{s} = 63$ GeV, and later studied in the $p\bar{p}$ collisions at $\sqrt{s} = 1.8$ and 1.96 TeV by CDF \cite{6,7} and D0 \cite{8-11} collaborations, the role of DPS processes becomes much more important \cite{12-14} at higher energies, in particular, for $pp$ collisions at $\sqrt{s} = 7, 8$ and 13 TeV at Large Hadron Collider (LHC).

The cross section of DPS processes can be expressed as

$$\sigma_{DPS} = \frac{1}{s} \sigma_A \sigma_B \sigma_{\text{eff}}, \quad (1)$$

where $\sigma_{A,B}$ are the individual cross sections for processes $A$ and $B$, $s = 1, 2$ is a symmetry factor and $\sigma_{\text{eff}}$ is an effective cross section. In the factorization approach the latter is expressed via the integral over the transverse degrees of freedom of the partons in the protons and is an universal process and scale-independent constant. The value of $\sigma_{\text{eff}}$ is expected to be of the order of the inelastic cross section, but currently it can’t be calculated \cite{15-18} from the basic principles. Significant violations \cite{19,20} of the factorization are expected for some kinematic regions and processes, e.g. for the processes with high-$x$ partons.

In addition to DPS $2 \otimes 2$ process, characterized by the independent scattering of two pairs of partons from the incoming protons, Fig. 1(left), one also needs to account for the potentially large additional contribution from the $1 \otimes 2$ process, where a parton from one incoming proton splits at some hard scale \cite{21-24} and creates two partons that further participate in the two independent single-scattering processes, see Fig. 1(right). This contribution is calculated \cite{25-27} in perturbative QCD and provides significant dependency of multi-parton interaction cross sections on the transverse scale. However for small-$x$ processes with relatively small transverse momenta, this large dependence is expected to stabilize \cite{28} by accounting of soft correlations, turning $\sigma_{\text{eff}}$ to be a constant.

Testing the universality of $\sigma_{\text{eff}}$, in terms of its (in)dependency on the process, scale and collision energy, allows to shed light on the role of factorization and contribution of $1 \otimes 2$ processes and provides the unique opportunity to probe the parton-parton correlations in the proton. Good understanding of DPS processes is very important for the search of New Physics (NP) effects at LHC, since for certain final states DPS process mimics \cite{30,32} the production of heavy exotic particles. Studies of DPS processes with heavy-quarks
or quarkonia in the final state are especially important both for QCD tests, mentioned above and for the deep understanding of potentially important background source for NP searches.

For DPS processes producing four heavy quarks in the final state, \( Q_1\bar{Q}_1 Q_2\bar{Q}_2 \), the corresponding elementary subprocesses \( gg \rightarrow Q_1\bar{Q}_1 \) and \( gg \rightarrow Q_2\bar{Q}_2 \) are studied in detail. The production of open-charm hadrons at high-energy hadron collisions has been studied in pp collisions by LHCb collaboration \cite{33,35} at \( \sqrt{s} = 5, 7 \) and 13 TeV, by ATLAS collaboration \cite{36} at \( \sqrt{s} = 7 \) TeV and by ALICE collaboration \cite{37,39} at \( \sqrt{s} = 2.76 \) TeV and 7 TeV, and by CDF collaboration \cite{40} in pp collisions at \( \sqrt{s} = 1.96 \) TeV. The measurements are in reasonable agreement with calculations at the next-to-leading order (NLO) using the generalized mass variable flavour number scheme \cite{41,45} (GMVFNS), Powheg \cite{46}, and fixed order with next-to-leading-log resummation \cite{47,50} (FONLL), see Fig. 2.

Experimentally, the processes with heavy-quarks often could be studied in details up to very low transverse momenta, comparable or even smaller than the masses of the involved heavy-quarks, especially for the final states with the dimuon decays of the \( \J / \Psi \) or \( \Upsilon \) mesons. It opens the unique possibility to explore the low-\( p_T \) region, where DPS contribution \cite{12,13} is not suppressed. Full reconstruction of the final state quarkonia or the open-flavour heavy hadron allow to minimize some large experimental uncertainty, \( e.g. \) related to jet reconstruction or jet-energy calibration. Currently practically all (experimental) knowledge of DPS processes in very interesting and important kinematic region \( x_1, x_2 \gg x_1', x_2' \) comes from the measurements involving heavy-quarks \cite{51,54} in the forward region, where studies with \( e.g. \)
light quarks (jets) and/or the direct photons are experimentally challenging.

2 Studies with open-flavour hadrons

The LHCb experiment studied [52] the associated production of CC and J/ψC combinations, where C stands for D⁰, D⁺, Dˢ⁺ or Λ⁺, in the kinematic region of 2 < yJ/ψ < 4.5, pₜJ/ψ < 10 GeV/c and 3 < p_C < 12 GeV/c using 355 ± 13 pb⁻¹ of data taken in pp collisions at the centre-of-mass energy of √s = 7 TeV. Open-charm hadrons are reconstructed via D⁰ → K⁻π⁺, D⁺ → K⁻π⁺π⁺, Dˢ⁺ → K⁻K⁺π⁺ and Λ⁺ → pK⁻π⁺ modes, while J/ψ mesons are reconstructed in dimuon final state. Charge-conjugated processes are included. Clear high-statistics signals with significance in excess of five standard deviations have been observed for six CC modes, D⁰D⁰, D⁰D⁺, D⁰Dˢ⁺, D⁰Λ⁺, D⁺D⁺ and D⁺Dˢ⁺, and for four J/ψC modes, J/ψD⁰, J/ψD⁺, J/ψDˢ⁺ and J/ψΛ⁺. Large D⁰D⁰ and J/ψD⁰ signals are shown in Fig. 3. The possible backgrounds from hadrons produced in two different pp interactions within
the same bunch crossing (pile-up) and contamination from b-hadrons decays were reduced to a negligible level by imposing cuts based on the consistency of the decay chain.

The model-independent cross sections for double charm production in the fiducial range were calculated using the yields obtained from the fits of two-dimensional distributions. The dominating systematic uncertainties are related to the luminosity determination, to the C-hadrons branching fractions and to the differences between the data and simulated sample used for the reconstruction efficiency determination. The resulting cross sections for CC and J/ψC events are shown in the left part of Fig. 4 as well as several SPS theoretical estimations \[55–57\] of J/ψC production. The expectations from SPS process are by a factor of about 20 below the measured cross sections. Also the SPS predictions for D^0D^0 are also approximately 30 times smaller \[58, 59\] than the measured production rate. The observed ratio of cross sections for CC and corresponding C\bar{C} processes, proportional to \(\frac{\sigma(c\bar{c}c\bar{c})}{\sigma(c\bar{c})}\), is close to 10%. This value is very large compared with e.g. those measured for \(\frac{\sigma(J/\psi J/\psi)}{\sigma(J/\psi)} = (5.1 \pm 1.0\text{(stat.)} \pm 1.1\text{(syst.)}) \times 10^{-4}\) in the same kinematic region \[51\].

The kinematic properties of CC and J/ψC events were studied. To compare the transverse momentum spectra of J/ψ and open-charm hadrons, the spectra were fit to an exponential function. The resulting slope parameters are shown in Fig. 4(right). The transverse momentum spectra for J/ψ mesons from J/ψC events are significantly harder than those observed in
Figure 4: (left) Measured cross sections $\sigma_{\text{CC}}$, $\sigma_{\overline{c}c}$ and $\sigma_{J/\psi C}$ (points with error bars). For $J/\psi C$ channels, the SPS theory calculations are shown with hatched [55, 56] and shaded [57] areas. The inner error bars indicate the statistical uncertainty whilst the outer error bars indicate the sum of the statistical and systematic uncertainties in quadrature. For the $J/\psi C$ case the outermost error bars correspond to the total uncertainties including the uncertainties due to the unknown polarization of the prompt $J/\psi$ mesons. (right) The slope parameters for transverse momentum spectra of charm hadrons for CC and $J/\psi C$ events.

prompt $J/\psi$ meson production [60] while the spectra for open-charm mesons in $J/\psi C$ case appear to be very similar to those observed [34] for the prompt charm hadrons. Similar transverse momenta for CC and CC events are observed. However these spectra are much harder than those measured [34] for prompt charm events. It indicates large correlations between transverse momenta of two charm hadrons, possibly due to a large role of the gluon splitting process. The rapidity and azimuthal angle distributions, presented in Fig. 5 do not exhibit visible correlations between the two charm hadrons in CC and $J/\psi C$ events and are well consistent with uncorrelated production, supporting the dominant role of DPS contribution for CC and $J/\psi C$ production.

Another study exhibiting a good separation power between the SPS and DPS mechanisms performed by the LHCb experiment is the study [53] of the associated production of $\Upsilon$ and an open-charm meson. For such process non-relativistic QCD (NR QCD) color-singlet (CS) [61] and $k_T$-factorization [56, 62, 69] predict the ratio $R_{\Upsilon c\bar{c}} = \frac{\sigma(\Upsilon c\bar{c})}{\sigma(\Upsilon)}$ to be around $(0.1 - 0.6)\%$ in LHCb acceptance, while the DPS predicts this ratio to be of the order of 10%. The measurement was performed with the combined sample of data collected in pp collisions at the centre-of-mass energies of 7 and 8 TeV. Twelve different combinations of $\Upsilon(nS)$ (n=1,2,3) and open-charm hadrons,
Figure 5: Distributions of the difference in azimuthal angle (left) and rapidity (right) for $J/\psi C$ (top), $D^0D^0$ and $D^0D^+\bar{D}^+$ events (bottom). The dashed line shows the expected distribution for uncorrelated events.

$C$, were studied in the fiducial volume of $2 < \upsilon_T, \upsilon_C < 4.5$, $p_{T}^\psi < 15$ GeV/c and $1 < p_{T}^C < 20$ GeV/c. In five of them signal significance exceeds five standard deviations: $\Upsilon(1S)D^0$, $\Upsilon(1S)D^+$, $\Upsilon(1S)D^+_s$, $\Upsilon(2S)D^0$ and $\Upsilon(2S)D^+$. Two of them with the highest signal yields, $\Upsilon(1S)D^0$ and $\Upsilon(1S)D^+$ were used for the production cross section measurements and DPS studies. The ratios $R_{\Upsilon c\bar{c}}$ were measured to be $(7.7 \pm 1.0)$ % and $(8.0 \pm 0.9)$ % for data sets collected at 7 and 8 TeV, respectively: these significantly exceed SPS calculations, and agrees with DPS expectations.

All differential cross sections for $\Upsilon(1S)D$ events nicely agree with expectations from DPS process. Figure 6(left) shows the distribution of the azimuthal angle difference $\Delta\phi$ for $\Upsilon(1S)D^0$ events together with both SPS and DPS expectations. The transverse momentum and rapidity distributions of $\Upsilon(1S)$ mesons also agree well with SPS predictions, obtained using $k_T$-factorization approach, while the shape of the transverse momentum spectra of $\Upsilon(1S)$ mesons, see Fig. 6(right), disfavours the SPS predictions obtained using the collinear approximation.
Figure 6: Distributions for $|\Delta \phi|/\pi$ (left) and $p_T^{(1S)}$ (right) for $\Upsilon(1S)D^0$ events (solid red circles). Straight blue line in the $|\Delta \phi|/\pi$ plots shows the result of the fit with a constant function. The SPS predictions for the shapes of $\Delta \phi$ distribution are shown with dashed (orange) and long-dashed (magenta) curves for calculations based on the $k_T$-factorization and the collinear approximation, respectively. The transverse momentum spectra, derived within the DPS mechanism using the measured production cross sections [34, 70] for $\Upsilon$ and open-charm mesons, are shown with the open (blue) squares. All distributions are normalized to unity.

3 Double quarkonia studies

Unlike the DPS measurements with open-flavour hadrons, that are unique for the LHCb experiment, several experiments contributed to the study of the double quarkonia production. The dimuon decay of quarkonia combines relatively high branching fraction and experimentally favourable signature together with efficient triggering and low background.

First observed by the NA3 experiment in pion-nucleon [71] and proton-nucleon [72] interactions, the $J/\psi$ pair production was later studied both in $p\bar{p}$ collisions at the Tevatron by D0 collaboration and in $pp$ collisions at different energies at the LHC [51, 54, 73, 74] by ATLAS, CMS and LHCb collaborations.

The first measurement in the $pp$ collisions [51] was made by the LHCb experiment. The oppositely charged tracks identified as muons were combined to obtain pairs of $J/\psi$ candidates ($J/\psi_1$ and $J/\psi_2$). To determine the signal event yield the invariant mass of the second muon pair was plotted in bins
of the first pair invariant mass, see Fig. 7(left). Fit to this distribution was performed with a function including a signal component for $J/\psi$ and a component for the combinatorial background. Other sources of background were found to be negligible. In total $141 \pm 19$ $J/\psi$ pairs were found with the signal significance exceeding 6 standard deviations.

The total cross section in the region $2 < y_{J/\psi} < 4.5$, $p_T^{J/\psi} < 10$ GeV/c was found to be

$$\sigma^{J/\psi,J/\psi} = 5.1 \pm 1.0(\text{stat.}) \pm 1.1(\text{syst.}) \text{ nb}.$$ 

This value is not precise enough to distinguish between the SPS and DPS contributions. The SPS contributions are calculated to be $4.0 \pm 1.2$ nb and $4.6 \pm 1.1$ nb in the leading-order\cite{75,78,79} NR QCD CS approach, and $5.4^{+2.7}_{-1.1}$ nb using complete\cite{79} NLO CS approach. The DPS contribution is expected to be $3.8 \pm 1.3$ nb. The differential production cross section as a function of the mass of the $J/\psi J/\psi$ system together with expectations from leading-order (LO) CS calculations is presented in Fig. 7(right). The statistics didn’t allow to study the kinematic properties of these events and make firm conclusions.

Another measurement at $\sqrt{s} = 7$ TeV was performed\cite{74} by the CMS collaboration, which included the kinematic regions where the color-octet (CO) models could play a more significant role in the double heavy quarkonium
Figure 8: Differential cross section for prompt J/ψ pair production as a function of the absolute rapidity difference between J/ψ mesons (left) and the mass of J/ψ-J/ψ system (right). The shaded regions represent the statistical uncertainties only, and the error bars represent the statistical and systematic uncertainties added in quadrature.

production. After excluding possible sources of background, they observed 446 ± 23 J/ψ pairs produced promptly in the same pp collision. The cross section in the fiducial region was measured to be

$$\sigma^{J/\psi} = 1.49 \pm 0.07(\text{stat.}) \pm 0.13(\text{syst.}) \text{nb.}$$

The differential cross sections as a function of the rapidity difference between the two J/ψ mesons, |Δy|, and \(M_{J/\psi-J/\psi}\), the mass of J/ψ-J/ψ system, are shown in Fig. 8. These distributions are very sensitive to the DPS effects \([76,77,80,81]\) and non-vanishing cross sections for large |Δy| and \(M_{J/\psi-J/\psi}\) bins could be considered as a sign of DPS mechanism.

These distributions have been analyzed \([81]\) against incomplete NLO* CS predictions. From this analysis Lansberg&Shao \([81]\) concluded the importance of \(\alpha_s^5\) contributions at medium and large transverse momenta and dominance of DPS mechanism at large |Δy| and large \(M_{J/\psi-J/\psi}\). No significant CO contribution was found. The extracted value of \(\sigma_{\text{eff}}\) parameter has been found \([81]\) to be \(\sigma_{\text{eff}} = 11.0 \pm 2.9 \text{mb.}\)

A measurement with higher centre-of-mass energy of 8 TeV has been performed by the ATLAS collaboration. In total they observed 1160 ± 70 promptly produced J/ψ pairs. The total cross section in the fiducial region
Figure 9: The DPS and total differential cross sections as a function of (left) the azimuthal angle between the two J/ψ mesons, and (right) the difference in rapidity between the two J/ψ mesons. Shown are the data as well as the DPS and NLO* SPS predictions. The DPS predictions [82] are normalized to the value of $f_{DPS}$ found in the data and the NLO* SPS predictions are multiplied by a constant feed-down correction factor. The data-driven DPS-weighted distribution and the total data distribution are compared to the DPS theory prediction and the total SPS+DPS prediction.

was measured to be

$$\sigma^{J/\psi J/\psi} = 160 \pm 12 \pm 14 \pm 2 \pm 3 \text{ pb},$$

where the first uncertainty is statistical, the second one is systematic mainly due to the trigger efficiency determination. The third and fourth uncertainties are coming correspondingly from the known J/ψ → μμ branching fraction uncertainty and the luminosity determination uncertainty. A data-driven model-independent technique was used to separate the DPS and the SPS contributions. The overall differential cross section and the differential cross section for DPS contribution are shown in Fig. 9.

The measured DPS-sensitive distributions were compared and found in a good agreement with the DPS predictions [82] while the SPS distributions have shown some discrepancy with the NLO* predictions [81,83] which however could be explained by feed-down from higher charmonium states. The DPS fraction was found to be $f_{DPS} = (9.2 \pm 2.1(\text{stat.}) \pm 0.5(\text{syst.}))\%$, resulting in

$$\sigma_{\text{eff}} = 6.3 \pm 1.6 \pm 1.0 \pm 0.1 \pm 0.1 \text{ mb},$$
Figure 10: Comparisons between measurements \cite{54} and theoretical predictions \cite{79,81,83–87} for the differential cross sections as a function of (left) $p_T^{J/\psi J/\psi}$ and (right) $|\Delta y|$. The (black) points with error bars represent the measurements.

where the first uncertainty is statistical, the second one is systematic, the third one comes from the $J/\psi \rightarrow \mu\mu$ branching fraction and the fourth is an uncertainty on the luminosity determination.

One more study of the double $J/\psi$ production was performed by the LHCb collaboration \cite{54} at the centre-of-mass energy of 13 TeV. They observed $(1.05 \pm 0.05) \times 10^3$ signal $J/\psi$ pairs. The total cross section in the fiducial region was measured to be

$$\sigma^{J/\psi J/\psi} = 15.2 \pm 1.0(\text{stat.}) \pm 0.9(\text{syst.}) \text{ nb.}$$

A large statistics of $J/\psi$ pairs allows to study differential cross sections and compare them with different theory models. Figure 10 shows the normalized differential cross sections as a function of the transverse momentum of $J/\psi J/\psi$ system and the rapidity difference between two $J/\psi$ mesons.

The obtained cross section value is interpreted as a sum of the SPS and DPS contributions, which were separated by a study of differential cross sections. Fit to the data was performed with a function, corresponding to the sum of an SPS and DPS model predictions. The fraction $f_{\text{DPS}}$ was treated as free parameters of the fit. Examples of DPS fit to $m(J/\psi J/\psi)$ and $|\Delta y|$ distributions using full NLO CS calculations for SPS model are shown in Fig. 11. For all fits the fractions of the DPS contribution were found to be higher than 50%, and for many cases compatible with 100%.
Figure 11: Result of templated DPS fit for $\frac{d\sigma}{dm}$ (left) and $\frac{d\sigma}{d|\Delta y|}$ (right). The points with error bars represent the data. The total fit result is shown with the thick (red) solid line, the DPS component is shown with the thin (orange) solid line and the SPS component (full NLO CS) is shown with small (red) crosses.

The obtained values of $\sigma_{\text{DPS}}$ and the known $[^{88}]$ J/ψ production cross section were used to calculate $\sigma_{\text{eff}}$. The obtained $\sigma_{\text{eff}}$ values are shown in Table 1 depending on the choice of the SPS model used. They turned out to be slightly higher than values measured from the central J/ψ pair production. In spite of the large difference between different SPS models, the values of $\sigma_{\text{eff}}$ exhibit only modest model dependency.

Table 1: Summary of the $\sigma_{\text{eff}}$ values (in mb) from DPS fits $[^{54}]$ for different SPS models. The uncertainty is statistical only. The common systematic uncertainty of 12%, accounting for the systematic uncertainty of $\sigma(J/\psi J/\psi)$ and the total uncertainty for $\sigma(J/\psi)$, is not shown.

| Variable | LO $k_T$ | NLO∗ CS | NLO CS |
|----------|----------|----------|--------|
| $p_{T}^{J/\psi J/\psi}$ | $^{89}$ | $^{81}$ | $^{79}$ |
| $y^{J/\psi J/\psi}$ | $^{11.9 \pm 7.5}$ | $^{10.0 \pm 5.0}$ | — |
| $m^{J/\psi J/\psi}$ | $^{10.6 \pm 1.1}$ | $^{10.2 \pm 1.0}$ | $^{10.4 \pm 1.0}$ |
| $|\Delta y|$ | $^{12.5 \pm 4.1}$ | $^{12.2 \pm 3.7}$ | $^{11.2 \pm 2.9}$ |

The D0 collaboration has for the first time observed the pair J/ψ production in $p\bar{p}$ collisions at $\sqrt{s} = 1.96$ TeV at Tevatron, see Fig. 12 (left).
Figure 12: (left) Dimuon invariant mass distribution in data for two muon pairs. (right) The distribution of the rapidity difference between two J/ψ candidates in data after background subtraction. The distributions for the SPS and DPS templates are shown normalized to their respective fitted fractions. The uncertainty band corresponds to the total systematic uncertainty on the sum of SP and DP events.

After the selection 242 pairs of J/ψ mesons were found, about 40% of which were determined to be prompt signal J/ψ pairs. The cross section in the fiducial region was measured to be

$$\sigma_{J/\psi} = 129 \pm 11\text{(stat.)} \pm 37\text{(syst.)}\text{ fb.}$$

In order to study the SPS and DPS contributions separately, they were distinguished by fitting the $|\Delta y|$ distribution by a sum of SPS and DPS templates with the corresponding fractions used as the free parameters, see Fig. 12(right). The DPS fraction $f_{DPS}$ was found to be $(42 \pm 12)\%$, leading to the following values of the cross sections: $59 \pm 6\text{(stat.)} \pm 22\text{(syst.)}\text{ fb}$ and $70 \pm 6\text{(stat.)} \pm 22\text{(syst.)}\text{ fb}$ for SPS and DPS processes, respectively. The corresponding $\sigma_{\text{eff}}$ was measured to be $4.8 \pm 0.5\text{(stat.)} \pm 2.5\text{(syst.)}\text{ mb.}$

The measurements of associated productions involving bottomonia resonances were performed by D0 and CMS collaborations. The D0 collaboration observed the associated J/ψ and Υ production using 8 fb$^{-1}$ of data collected at $\sqrt{s} = 1.96$ TeV in pp collisions. This combination is expected to be produced predominantly through DPS mechanism with SPS contribution suppressed by an additional powers of $\alpha_s$. It results in the hierarchy of expected SPS production cross sections $\sigma_{J/\psi\Upsilon}^{\text{SPS}} < \sigma_{\Upsilon\Upsilon}^{\text{SPS}} < \sigma_{J/\psi J/\psi}^{\text{SPS}}$ that is op-
Figure 13: (left) Dimuon invariant mass distribution in data for two muon pairs together with the two-dimensional fit surface. The scaling factor is applied so that the height of the peak bin is the number of observed events in that bin. (right) The distribution of the azimuthal angle between the J/ψ and Υ candidates in data after background subtraction. Also shown the expectations from DPS and SPS processes in arbitrary units.

posite to more intuitive DPS hierarchy $\sigma_{DPS}^{ΥΥ} < \sigma_{DPS}^{J/ψΥ} < \sigma_{DPS}^{J/ψJ/ψ}$. That makes the observation of J/ψ and Υ pair production and the precise measurement of the hierarchy of the production cross sections very important.

Dimuon invariant mass distribution in data for two muon pairs in the fiducial region of $p_T^μ > 2$ GeV/c and $|\eta^μ| < 2$ is shown in Fig. 13(left) together with the two-dimensional fit surface. In total $12 \pm 3.8\,\text{(stat.)}\pm 2.8\,\text{(syst.)}$ promptly produced J/ψ Υ pairs are observed, corresponding to the significance of $3.2\sigma$.

The distribution of azimuthal angle between J/ψ and Υ mesons is presented in Fig. 13(right). This distribution showed a good agreement with the dominance of DPS mechanism. It allows to translate the measured production cross section for J/ψ Υ pairs

$$\sigma^{J/ψΥ} = 27 \pm 9\,\text{(stat.)} \pm 7\,\text{(syst.)}\,\text{fb}$$

into the effective cross section $\sigma_{eff} = 2.2 \pm 0.7\,\text{(stat.)} \pm 0.9\,\text{(syst.)}\,\text{mb}$. The obtained value of $\sigma_{eff}$ is below the previous measurements involving heavy quarkonium.

The CMS collaboration made a first observation [90] of double bottomonium production using 20.7 fb$^{-1}$ of data collected in pp collisions at centre-of-mass energy of 8 TeV. The dimuon invariant mass distribution in
Figure 14: (left) Dimuon invariant mass distribution in data for two muon pairs. (right) Invariant mass distributions of the lower-mass muon pair (top) and the higher-mass muon pair (bottom). The data are shown by the points. The different curves show the contributions of the various event categories from the fit.

data for two muon pairs in the fiducial region of $|y^\gamma| < 2$ is presented in Fig. 14 (left). A signal yield of $38 \pm 7$ pairs of $\Upsilon(1S)$ mesons is determined from two-dimensional fit. The projections of two-dimensional fit are shown in Fig. 14 (right). The significance of the signal exceeds $5\sigma$.

The production cross section of $\Upsilon(1S)$ pairs was measured to be

$$\sigma^{\Upsilon\Upsilon} = 68.8 \pm 12.7 \pm 7.4 \pm 2.8 \text{ pb},$$

where the first uncertainty is statistical, the second one is systematic and the third one comes from the known value of $\Upsilon \rightarrow \mu^+\mu^-$ branching fraction. The DPS contribution in this channel is expected to be small relative to the SPS one. Using a conservative estimate of $f_{\text{DPS}} = 10\%$, the effective cross section was estimated to be $6.6 \text{ mb}$. On the other hand the SPS prediction with feed-down from higher states gives a value of $\sigma_{\text{SPS}}^{\Upsilon\Upsilon} = 48 \text{ pb}$ which combined with the $\sigma^{\Upsilon\Upsilon}$ measured in data leads to $f_{\text{DPS}} = 30\%$ and, correspondingly $\sigma_{\text{eff}} = 2.2 \text{ mb}$. Both estimates are in line with values obtained from other heavy quarkonium measurements. Higher statistics would allow to make a more precise conclusions.
4 Other DPS measurements involving heavy quarks

Besides the studies described above, there are other processes involving heavy quarks which can tell a lot about charm parton distribution inside the proton, charm production mechanism and the DPS. Good examples of such studies are measurements of an associated production of charm and a W or a Z boson. It should be noted that for these final states one probes the PDFs in the region of relatively high-$x$ partons, where significant violation \cite{19,20} of factorization hypothesis is expected.

The CMS and ATLAS collaborations studied \cite{93, 94} an associated production of a W boson with open-charm considering both a case of explicitly produced D meson and a case of jet initiated by the $c$ quark. Each experiment used about 5 fb$^{-1}$ of data collected in pp collisions at the centre-of-mass energy of $\sqrt{s} = 7$ TeV. The total and differential cross sections of these processes were measured and found to be consistent between the two experiments and with theoretical predictions.

The LHCb experiment observed \cite{95} the associated production of a Z boson and an open-charm meson ($D^0$ or $D^+$) in the forward region using data collected at $\sqrt{s} = 7$ TeV. Seven candidate events for associated production of a Z boson with a $D^0$ meson and four candidate events for a Z boson with a $D^+$ meson were observed with a combined significance of 5.1$\sigma$. The fiducial cross sections of these processes were measured, but the lack of statistics didn’t allow to disentangle the DPS and SPS contributions.

With higher statistics these studies promise to give more interesting and useful information for understanding the heavy quark production mechanisms and in particular the DPS studies.

5 Summary

Nowadays a study of DPS using heavy quarks provides the most precise determination of the parameter $\sigma_{\text{eff}}$. Figure 15 summarizes all available measurements of $\sigma_{\text{eff}}$ with heavy quarks.

There are no clear patterns in the measured values of $\sigma_{\text{eff}}$, however the values of $\sigma_{\text{eff}}$ parameter measured with the double quarkonia final state are a bit lower than the reference value \cite{7} of $\sigma_{\text{eff}} = 14.5 \pm 1.7^{+1.7}_{-2.3}$ mb measured in multi-jet events at the Tevatron. This could be a sign that the spatial region occupied by gluons within the proton is smaller than that occupied by quarks. On the other hand, the measurements \cite{52} with two open-flavour hadrons give values of $\sigma_{\text{eff}}$ larger than the reference value. These values are in
very good agreement \cite{27} with calculations. The measurements with quarkonia and open-flavour hadrons are well consistent with the reference value of $\sigma_{\text{eff}}$ and all other measurements \cite{2,96} of $\sigma_{\text{eff}}$ using multi-jet, di-jet+W and $\gamma$+3-jets processes. It should be noted that for many measurements the uncertainties are large, and formally only one measurement \cite{11} is not consistent with the reference value of $\sigma_{\text{eff}}$. Better precision is needed to conclude on universality of $\sigma_{\text{eff}}$. For large part of measurements, especially for production cross section of $\Upsilon\Upsilon$ and $J/\psi\Upsilon$ pairs, the precision is limited by the statistics. Currently none of the LHC experiments used the full Run-I data sample for DPS studies. Extending these analyzes to full Run-I and subsequent larger Run-II data sets will allow significant improvement of the precision of $\sigma_{\text{eff}}$ measurements and it will allow a definite conclusion on the universality of $\sigma_{\text{eff}}$. Also the analysis of larger data sets will allow to study the DPS processes with open-beauty hadrons in the final state. A huge statistics of events with pairs of open-charm hadrons at LHCb could allow the precise measurement of $\sigma_{\text{eff}}$ separately in the different kinematic regions, providing the ultimate test for the universality of $\sigma_{\text{eff}}$. It is worth to mention that for the precise measurements of DPS processes in pp collisions at $\sqrt{s} = 13$ TeV one will need to account for the contribution from the triple parton scattering process \cite{97}. 
Figure 15: Summary of $\sigma_{\text{eff}}$ measurements with heavy quarks. The inner error bars indicate the statistical uncertainty whilst the outer error bars indicate the sum of statistical and systematic uncertainties in quadrature. The hatched area shows the reference value \cite{7} of $\sigma_{\text{eff}} = 14.5 \pm 1.7^{+1.7}_{-1.3} \text{mb}$ measured in multi-jet events at the Tevatron.
References

[1] H. Abramowicz et al., Summary of the workshop on multi-parton interactions (MPI@LHC 2012), arXiv:1306.5413.

[2] S. Bansal et al., Progress in double parton scattering studies, in Workshop on multi-parton interactions at the LHC (MPI@LHC 2013) Antwerp, Belgium, December 2-6, 2013, 2014. arXiv:1410.6664.

[3] R. Astalos et al., Proceedings of the sixth international workshop on multiple partonic interactions at the Large Hadron Collider, arXiv:1506.05829.

[4] H. Jung, D. Treleani, M. Strikman, and N. van Buuren, eds., Proceedings, 7th International workshop on multiple partonic interactions at the LHC(MPI@LHC 2015), 2016.

[5] AFS collaboration, T. Åkesson et al., Double parton scattering in pp collisions at $\sqrt{s} = 63$ GeV, Z. Phys. C34 (1987) 163.

[6] CDF collaboration, F. Abe et al., Study of four jet events and evidence for double parton interactions in pp collisions at $\sqrt{s} = 1.8$ TeV, Phys. Rev. D47 (1993) 4857.

[7] CDF collaboration, F. Abe et al., Double parton scattering in pp collisions at $\sqrt{s} = 1.8$ TeV, Phys. Rev. D56 (1997) 3811.

[8] D0 collaboration, V. M. Abazov et al., Double parton interactions in $\gamma + 3$ jet events in pp collisions at $\sqrt{s} = 1.96$ TeV, Phys. Rev. D81 (2010) 052012, arXiv:0912.5104.

[9] D0 collaboration, V. M. Abazov et al., Double parton interactions in $\gamma + 3$ jet and $\gamma + b/c$ jet+2 jet events in pp collisions at $\sqrt{s} = 1.96$ TeV, Phys. Rev. D89 (2014) 072006.

[10] D0 collaboration, V. M. Abazov et al., Observation and studies of double $J/\psi$ production at the Tevatron, Phys. Rev. D90 (2014) 111101, arXiv:1406.2380.

[11] D0 collaboration, V. M. Abazov et al., Evidence for simultaneous production of $J/\psi$ and $\Upsilon$ mesons, Phys. Rev. Lett. 116 (2016), no. 8 082002, arXiv:1511.02428.

[12] M. Diehl and A. Schafer, Theoretical considerations on multiparton interactions in QCD, Phys. Lett. B698 (2011) 389, arXiv:1102.3081.
[13] M. Diehl, D. Ostermeier, and A. Schafer, *Elements of a theory for multiparton interactions in QCD*, JHEP 03 (2012) 089, arXiv:1111.0910

[14] A. Szczurek, *A short review of some double-parton scattering processes*, Acta Phys. Polon. Supp. 8 (2015), no. 2 483, arXiv:1505.04067

[15] N. Paver and D. Treleani, *Multi-quark scattering and large $p_T$ jet production in hadronic collisions*, Nuovo Cim. A70 (1982) 215

[16] G. Calucci and D. Treleani, *Minijets and the two-body parton correlation*, Phys. Rev. D57 (1998) 503, arXiv:hep-ph/9707389

[17] G. Calucci and D. Treleani, *Proton structure in transverse space and the effective cross section*, Phys. Rev. D60 (1999) 054023, arXiv:hep-ph/9902479

[18] A. Del Fabbro and D. Treleani, *Scale factor in double parton collisions and parton densities in transverse space*, Phys. Rev. D63 (2001) 057901, arXiv:hep-ph/0005273

[19] V. L. Korotkikh and A. M. Snigirev, *Double parton correlations versus factorized distributions*, Phys. Lett. B594 (2004) 171, arXiv:hep-ph/0404155

[20] J. R. Gaunt, C.-H. Kom, A. Kulesza, and W. J. Stirling, *Same-sign $W$ pair production as a probe of double parton scattering at the LHC*, Eur. Phys. J. C69 (2010) 53, arXiv:1003.3953

[21] B. Blok, Yu. Dokshitzer, L. Frankfurt, and M. Strikman, *The four jet production at LHC and Tevatron in QCD*, Phys. Rev. D83 (2011) 071501, arXiv:1009.2714

[22] B. Blok, Yu. Dokshitzer, L. Frankfurt, and M. Strikman, *pQCD physics of multiparton interactions*, Eur. Phys. J. C72 (2012) 1963, arXiv:1106.5533

[23] B. Blok, Yu. Dokshitzer, L. Frankfurt, and M. Strikman, * Origins of parton correlations in nucleon and multi-parton collisions*, arXiv:1206.5594.

[24] B. Blok, Yu. Dokshitzer, L. Frankfurt, and M. Strikman, *Perturbative QCD correlations in multi-parton collisions*, Eur. Phys. J. C74 (2014) 2926, arXiv:1306.3763
[25] B. Blok and P. Gunnellini, *Dynamical approach to MPI four-jet production in PYTHIA*, Eur. Phys. J. C75 (2015), no. 6 282 arXiv:1503.08246.

[26] B. Blok and P. Gunnellini, *Dynamical approach to MPI in W+dijet and Z+dijet production within the PYTHIA event generator*, Eur. Phys. J. C76 (2016), no. 4 202, arXiv:1510.07436.

[27] B. Blok and M. Strikman, *Open charm production in double parton scattering processes in the forward kinematics*, Eur. Phys. J. C76 (2016), no. 12 694, arXiv:1608.00014.

[28] B. Blok and M. Strikman, *Interplay of soft and perturbative correlations in multiparton interactions at central rapidities*, Phys. Lett. B772 (2017) 219 arXiv:1611.03649.

[29] J. R. Gaunt, R. Maciul, and A. Szczurek, *Conventional versus single-ladder-splitting contributions to double parton scattering production of two quarkonia, two Higgs bosons and cccbar*, Phys. Rev. D90 (2014), no. 5 054017 arXiv:1407.5821.

[30] M. W. Krasny and W. Placzek, *The LHC excess of four-lepton events interpreted as Higgs-boson signal: Background from double Drell-Yan process?*, Acta Phys. Polon. B45 (2014), no. 1 71 arXiv:1305.1769.

[31] M. W. Krasny and W. Placzek, *On the contribution of the double Drell-Yan process to WW and ZZ production at the LHC*, Acta Phys. Polon. B47 (2016) 1045, arXiv:1501.04569.

[32] M. W. Krasny and W. Placzek, *In search of elementary spin 0 particles*, Annals Phys. 352 (2015) 11.

[33] LHCb collaboration, R. Aaij et al., *Measurements of prompt charm production cross-sections in pp collisions at \( \sqrt{s} = 5 \) TeV*, JHEP 06 (2017) 147 arXiv:1610.02230.

[34] LHCb collaboration, R. Aaij et al., *Prompt charm production in pp collisions at \( \sqrt{s} = 7 \) TeV*, Nucl. Phys. B871 (2013) 1 arXiv:1302.2864.

[35] LHCb collaboration, R. Aaij et al., *Measurements of prompt charm production cross-sections in pp collisions at \( \sqrt{s} = 13 \) TeV*, JHEP 03 (2016) 159 Erratum ibid. 09 (2016) 013 arXiv:1510.01707.
[36] ATLAS collaboration, G. Aad et al., Measurement of $D^{*\pm}$, $D^{\pm}$ and $D_s^{\pm}$ meson production cross sections in pp collisions at $\sqrt{s} = 7$ TeV with the ATLAS detector, Nucl. Phys. B907 (2016) 717, arXiv:1512.02913

[37] ALICE collaboration, B. Abelev et al., Measurement of charm production at central rapidity in proton-proton collisions at $\sqrt{s} = 2.76$ TeV, JHEP 07 (2012) 191, arXiv:1205.4007.

[38] ALICE collaboration, B. Abelev et al., Measurement of charm production at central rapidity in proton-proton collisions at $\sqrt{s} = 7$ TeV, JHEP 01 (2012) 128, arXiv:1111.1553.

[39] ALICE collaboration, B. Abelev et al., $D^+_s$ meson production at central rapidity in proton-proton collisions at $\sqrt{s} = 7$ TeV, Phys. Lett. B718 (2012) 279, arXiv:1208.1948.

[40] CDF collaboration, D. Acosta et al., Measurement of prompt charm meson production cross sections in $p\bar{p}$ collisions at $\sqrt{s} = 1.96$ TeV, Phys. Rev. Lett. 91 (2003) 241804, arXiv:hep-ex/0307080.

[41] B. A. Kniehl, G. Kramer, I. Schienbein, and H. Spiesberger, Inclusive charmed-meson production at the CERN LHC, Eur. Phys. J. C72 (2012) 2082, arXiv:1202.0439.

[42] B. A. Kniehl, G. Kramer, I. Schienbein, and H. Spiesberger, Inclusive $D^{*\pm}$ production in $p\bar{p}$ collisions with massive charm quarks, Phys. Rev. D71 (2005) 014018, arXiv:hep-ph/0410289.

[43] B. A. Kniehl, G. Kramer, I. Schienbein, and H. Spiesberger, Reconciling open charm production at the Fermilab Tevatron with QCD, Phys. Rev. Lett. 96 (2006) 012001, arXiv:hep-ph/0508129.

[44] T. Kneesch, B. A. Kniehl, G. Kramer, and I. Schienbein, Charmed-meson fragmentation functions with finite-mass corrections, Nucl. Phys. B799 (2008) 34, arXiv:0712.0481.

[45] B. A. Kniehl, G. Kramer, I. Schienbein, and H. Spiesberger, Open charm hadroproduction and the charm content of the proton, Phys. Rev. D79 (2009) 094009, arXiv:0901.4130.

[46] R. Gauld, J. Rojo, L. Rottoli, and J. Talbert, Charm production in the forward region: constraints on the small-$x$ gluon and backgrounds for neutrino astronomy, JHEP 11 (2015) 009, arXiv:1506.08025.
[47] M. Cacciari, M. L. Mangano, and P. Nason, *Gluon PDF constraints from the ratio of forward heavy-quark production at the LHC at \( \sqrt{s} = 7 \) and 13 TeV*, Eur. Phys. J. C75 (2015), no. 12 610, arXiv:1507.06197.

[48] M. Cacciari, M. Greco, and P. Nason, *The \( p_T \) spectrum in heavy flavor hadroproduction*, JHEP 05 (1998) 007, arXiv:hep-ph/9803400.

[49] M. Cacciari and P. Nason, *Charm cross-sections for the Tevatron Run II*, JHEP 09 (2003) 006, arXiv:hep-ph/0306212.

[50] M. Cacciari, P. Nason, and C. Oleari, *A study of heavy flavored meson fragmentation functions in \( e^+e^- \) annihilation*, JHEP 04 (2006) 006, arXiv:hep-ph/0510032.

[51] LHCb collaboration, R. Aaij *et al.*, *Observation of \( J/\psi \) pair production in \( pp \) collisions at \( \sqrt{s} = 7 \) TeV*, Phys. Lett. B707 (2012) 52, arXiv:1109.0963.

[52] LHCb collaboration, R. Aaij *et al.*, *Observation of double charm production involving open charm in \( pp \) collisions at \( \sqrt{s} = 7 \) TeV*, JHEP 06 (2012) 141, arXiv:1205.0975 [Addendum: JHEP03,108(2014)].

[53] LHCb collaboration, R. Aaij *et al.*, *Production of associated \( \Upsilon \) and open charm hadrons in \( pp \) collisions at \( \sqrt{s} = 7 \) and 8 TeV via double parton scattering*, JHEP 07 (2016) 052, arXiv:1510.05949.

[54] LHCb collaboration, R. Aaij *et al.*, *Measurement of the \( J/\psi \) pair production cross-section in \( pp \) collisions at \( \sqrt{s} = 13 \) TeV*, JHEP 06 (2017) 047, Erratum ibid. 10 (2017) 068, arXiv:1612.07451.

[55] A. V. Berezhnoy, V. V. Kiselev, A. K. Likhoded, and A. I. Onishchenko, *Doubly charmed baryon production in hadronic experiments*, Phys. Rev. D57 (1998) 4385, arXiv:hep-ph/9710339.

[56] S. P. Baranov, *Topics in associated \( J/\psi + c + \bar{c} \) production at modern colliders*, Phys. Rev. D73 (2006) 074021.

[57] J.-P. Lansberg, *On the mechanisms of heavy-quarkonium hadroproduction*, Eur. Phys. J. C61 (2009) 693, arXiv:0811.4005.

[58] M. Luszczak, R. Maciula, and A. Szczurek, *Production of two \( c\bar{c} \) pairs in double-parton scattering*, Phys. Rev. D85 (2012) 094034, arXiv:1111.3255.
[59] R. Maciula and A. Szcurek, *Single and double charmed meson production at the LHC*, EPJ Web Conf. **81** (2014) 01007.

[60] LHCb collaboration, R. Aaij *et al.*, *Measurement of J/ψ production in pp collisions at $\sqrt{s} = 7$ TeV*, Eur. Phys. J. **C71** (2011) 1645, arXiv:1103.0423.

[61] A. V. Berezhnoy and A. K. Likhoded, *Associated production of Υ and open charm at LHC*, Int. J. Mod. Phys. **A30** (2015) 1550125, arXiv:1503.04445.

[62] L. V. Gribov, E. M. Levin, and M. G. Ryskin, *Semihard Processes in QCD*, Phys. Rept. **100** (1983) 1.

[63] E. M. Levin and M. G. Ryskin, *High-energy hadron collisions in QCD*, Phys. Rept. **189** (1990) 267.

[64] S. P. Baranov, *Highlights from the $k_T$ factorization approach on the quarkonium production puzzles*, Phys. Rev. **D66** (2002) 114003.

[65] Small x collaboration, B. Andersson *et al.*, *Small-x phenomenology: Summary and status*, Eur. Phys. J. **C25** (2002) 77, arXiv:hep-ph/0204115.

[66] J. R. Andersen *et al.*, *Small-x phenomenology: Summary and status*, Eur. Phys. J. **C35** (2004) 67, arXiv:hep-ph/0312333.

[67] J. R. Andersen *et al.*, *Small-x phenomenology: Summary of the 3rd Lund Small-x Workshop in 2004*, Eur. Phys. J. **C48** (2006) 53, arXiv:hep-ph/0604189.

[68] S. P. Baranov, *Associated Υ + b+ b̄ production at the Fermilab Tevatron and CERN LHC*, Phys. Rev. **D74** (2006) 074002.

[69] S. P. Baranov, *Prompt Υ(nS) production at the LHC in view of the $k_T$-approach*, Phys. Rev. **D86** (2012) 054015.

[70] LHCb collaboration, R. Aaij *et al.*, *Forward production of Υ mesons in pp collisions at $\sqrt{s} = 7$ and 8 TeV*, JHEP **11** (2015) 103, arXiv:1509.02372.

[71] NA3 collaboration, J. Badier *et al.*, *Evidence for ΨΨ production in π⁻ interactions at 150 GeV/c and 280 GeV/c*, Phys. Lett. **B114** (1982) 457.
[72] NA3 collaboration, J. Badier et al., $\psi\psi$ production and limits on beauty meson production from 400GeV/c protons, Phys. Lett. B158 (1985) 85

[73] ATLAS collaboration, M. Aaboud et al., Measurement of the prompt $J/\Psi$ pair production cross-section in pp collisions at $\sqrt{s} = 7$ TeV with the ATLAS detector, Eur. Phys. J. C77 (2017), no. 2 76 arXiv:1612.02950

[74] CMS collaboration, V. Khachatryan et al., Measurement of prompt $J/\Psi$ pair production in pp collisions at $\sqrt{s} = 7$ TeV, JHEP 09 (2014) 094 arXiv:1406.0484

[75] A. V. Berezhnoy, A. K. Likhoded, A. V. Luchinsky, and A. A. Novoselov, Double $J/\Psi$-meson production at LHC and 4c-tetraquark state, Phys. Rev. D84 (2011) 094023 arXiv:1101.5881

[76] S. P. Baranov, A. M. Snigirev, and N. P. Zotov, Double heavy meson production through double parton scattering in hadronic collisions, Phys. Lett. B705 (2011) 116 arXiv:1105.6276

[77] S. P. Baranov et al., Interparticle correlations in the production of $J/\Psi$ pairs in proton-proton collisions, Phys. Rev. D87 (2013), no. 3 034035 arXiv:1210.1806

[78] C.-F. Qiao, L.-P. Sun, and P. Sun, Testing charmonium production mechanism via polarized $J/\Psi$ pair production at the LHC, J. Phys. G37 (2010) 075019 arXiv:0903.0954

[79] L.-P. Sun, H. Han, and K.-T. Chao, Impact of $J/\Psi$ pair production at the LHC and predictions in nonrelativistic QCD, Phys. Rev. D94 (2016), no. 7 074033 arXiv:1404.4042

[80] C. H. Kom, A. Kulesza, and W. J. Stirling, Pair production of $J/\Psi$ as a probe of double parton scattering at LHCb, Phys. Rev. Lett. 107 (2011) 082002 arXiv:1105.4186

[81] J.-P. Lansberg and H.-S. Shao, $J/\Psi$-pair production at large momenta: indications for double parton scatterings and large $\alpha_s^2$ contributions, Phys. Lett. B751 (2015) 479 arXiv:1410.8822

[82] C. Borschensky and A. Kulesza, Double parton scattering in pair production of $J/\Psi$ mesons at the LHC revisited, Phys. Rev. D95 (2017), no. 3 034029 arXiv:1610.00666
[83] J.-P. Lansberg and H.-S. Shao, *Production of $J/\psi + \eta_c$ versus $J/\psi + J/\psi$ at the LHC: importance of real $\alpha_s^2$ corrections*, Phys. Rev. Lett. 111 (2013) 122001, arXiv:1308.0474.

[84] A. K. Likhoded, A. V. Luchinsky, and S. V. Poslavsky, *Production of $J/\psi + \chi_c$ and $J/\psi + J/\psi$ with real gluon emission at LHC*, Phys. Rev. D94 (2016) 054017, arXiv:1606.06767.

[85] H.-S. Shao, *HELAC-ONIA: an automatic matrix element generator for heavy quarkonium physics*, Comput. Phys. Commun. 184 (2013) 2562, arXiv:1212.5293.

[86] H.-S. Shao, *HELAC-ONIA 2.0: an upgraded matrix-element and event generator for heavy quarkonium physics*, Comput. Phys. Commun. 198 (2016) 238, arXiv:1507.03435.

[87] J.-P. Lansberg and H.-S. Shao, *Double-quarkonium production at a fixed-target experiment at the LHC (AFTER@LHC)*, Nucl. Phys. B900 (2015) 273, arXiv:1504.06531.

[88] LHCb collaboration, R. Aaij et al., *Measurement of forward $J/\psi$ production cross-sections in pp collisions at $\sqrt{s} = 13$ TeV*, JHEP 10 (2015) 172, arXiv:1509.00771.

[89] S. P. Baranov, *Pair production of $J/\psi$ mesons in the $k_t$-factorization approach*, Phys. Rev. D 84 (2011) 054012.

[90] CMS collaboration, V. Khachatryan et al., *Observation of $\Upsilon(1S)$ pair production in proton-proton collisions at $\sqrt{s} = 8$ TeV*, JHEP 05 (2017) 013, arXiv:1610.07095.

[91] A. Novoselov, *Double parton scattering as a source of quarkonia pairs in LHCb*, arXiv:1106.2184.

[92] A. V. Berezhnoy, A. K. Likhoded, and A. A. Novoselov, *$\Upsilon$-meson pair production at LHC*, Phys. Rev. D87 (2013), no. 5 054023, arXiv:1210.5754.

[93] CMS collaboration, S. Chatrchyan et al., *Measurement of associated W + charm production in pp collisions at $\sqrt{s} = 7$ TeV*, JHEP 02 (2014) 013, arXiv:1310.1138.
[94] ATLAS collaboration, G. Aad et al., Measurement of the production of a W boson in association with a charm quark in pp collisions at $\sqrt{s} = 7$ TeV with the ATLAS detector, JHEP 05 (2014) 068, arXiv:1402.6263.

[95] LHCb collaboration, R. Aaij et al., Observation of associated production of a $Z$ boson with a $D$ meson in the forward region, JHEP 04 (2014) 091, arXiv:1401.3245.

[96] ATLAS collaboration, M. Aaboud et al., Study of hard double-parton scattering in four-jet events in pp collisions at $\sqrt{s} = 7$ TeV with the ATLAS experiment, JHEP 11 (2016) 110, arXiv:1608.01857.

[97] R. Maciuela and A. Szczurek, Can the triple-parton scattering be observed in open charm meson production at the LHC?, Phys. Lett. B772 (2017) 849, arXiv:1703.07163.