Study on Artificial Boundary of Saturated Soil Medium Viscoelastic Dynamic Based on Biot Dynamic Consolidation Equations

Jun Yu\(^*\), Yue He\(^1\), Shuang Zhao\(^1\) and Zhen Li\(^1\)

\(^1\)School of Civil Engineering, Central South University, Changsha, Hunan, 410075, China
\(^*\)Corresponding author’s e-mail: jjyy1017@163.com

Abstract. Based on modification of Biot dynamic consolidation equation, the equations in saturated soil plane normal, out-of-plane tangential and the tangential plane are calculated by method of separation of variables and operator decomposition, analytical solutions of the displacement of the saturated soil skeleton and the pore are obtained under the corresponding conditions. Then the physical equations of Biot saturated porous media are expressed in two parts by combined use of geometric equations, the three-dimensional artificial boundary of saturated soil considering energy dissipation and transformation is established. Finally, numerical examples of classic wave motion problems demonstrate that high precision is achieved by use of three-dimensional visco-elastic boundaries, and that the boundaries can be used in analysis of three-dimensional wave motion problems easily.

1. Introduction

When the fixed boundary is used to calculate the seismic wave, the seismic wave will be reflected on the intercepted side surface, and the displacement field and stress field of the foundation-structure interaction will be changed. For this problem, Liao Zhenpeng et al. carried out the relevant research\(^{[1–3]}\), and pointed out that the introduction of artificial boundary conditions is the most effective way to solve the problem of seismic wave oscillation on the boundary section.

At present, the development of artificial boundary is divided into two categories: ① the global artificial boundary, ② the local artificial boundary\(^{[4–8]}\). These boundaries are mostly for single-phase media derived from the widely available in the natural saturated porous media, generally composed of solid skeleton and solid pore fluid composition. Due to the complexity of saturated porous media, the study on the artificial boundary and numerical methods of saturated porous media at home and abroad is much less than that of single solid media. Liu Guanglei\(^{[9]}\) studied the viscoelastic boundary conditions of the two-dimensional saturated foundation in the \(u-p\) form. The expressions of the pore pressure and stress on the viscoelastic boundary were given based on the cylindrical wave. It was proved that the viscoelastic boundary met the needs of most engineering problems when the permeability coefficient was small and the second Equations of compression waves were neglected.

In this paper, based on the Biot dynamic consolidation equation, the equations in saturated soil plane normal, out of plane tangential and the tangential plane are calculated, and analytical solutions of the displacement of the saturated soil skeleton and the pore are obtained under the corresponding conditions.
2. Derivation of artificial boundary of saturated soil medium viscoelastic dynamic

2.1. Dynamic consolidation equation of saturated soil
In cartesian coordinates, the motion equation of soil skeleton and fluid can be expressed by $u_i$ of skeleton displacement and $w_i$ of fluid relative to skeleton:

\[
\mu \frac{\partial^2 u_i}{\partial t^2} + \left( \lambda + \alpha^2 M + \mu \right) \frac{\partial^2 u_i}{\partial x^2} + \alpha M \frac{\partial^2 w_i}{\partial x^2} = \rho \frac{\partial u_i}{\partial t} + \frac{\partial}{\partial x} \left( \rho \frac{\partial u_i}{\partial x} \right)
\]

\[
\alpha M \frac{\partial^2 u_i}{\partial t^2} + M \frac{\partial^2 w_i}{\partial x^2} = \rho \frac{\partial w_i}{\partial t} + \frac{\partial}{\partial x} \left( \rho \frac{\partial w_i}{\partial x} \right)
\]

Equation (1) and (2) are combined with the saturated soil skeleton and pore fluid radial potential function, the equations are as follows:

\[
(\lambda + 2G + \alpha^2 M) \frac{\partial^2 \varphi_1}{\partial r^2} + \alpha M \frac{\partial^2 \varphi_2}{\partial r^2} + \alpha M \frac{\partial^2 \varphi_3}{\partial r^2} + \rho \frac{\partial v}{\partial r} = 0
\]

\[
\alpha M \frac{\partial^2 \varphi_1}{\partial r^2} + M \frac{\partial^2 \varphi_2}{\partial r^2} + \rho \frac{\partial v}{\partial r} = 0
\]

2.2. In-plane normal viscoelastic boundary conditions
In cylindrical coordinates, the in-plane normal direction only considers $u_r$ and $w_r$. Equation (1) and (2) are combined with the saturated soil skeleton and pore fluid radial potential function, the equations are as follows:

\[
(\lambda + 2G + \alpha^2 M) \frac{\partial^2 \varphi_1}{\partial r^2} + \alpha M \frac{\partial^2 \varphi_2}{\partial r^2} + \alpha M \frac{\partial^2 \varphi_3}{\partial r^2} + \rho \frac{\partial v}{\partial r} = 0
\]

\[
\alpha M \frac{\partial^2 \varphi_1}{\partial r^2} + M \frac{\partial^2 \varphi_2}{\partial r^2} + \rho \frac{\partial v}{\partial r} = 0
\]

Physical equations in saturated two-phase media are:

\[
\sigma_r = \left[ c' a M \left( \frac{1}{2r} - k \right) - c(\lambda + 2G + \alpha^2 M + k' a M) \right] \frac{\partial u_r}{\partial r} + \left[ 2G(-k - \frac{1}{2r}) + (\lambda + \alpha^2 M + k' a M) \right] \frac{1}{2r} - k \right] + c' a M \right] u_r
\]

In which, $k, c, k', c'$ are all real constants.

Equation (10) is the in-plane normal boundary condition equation of the viscoelastic artificial boundary of the saturated soil. The plane normal wave action in the far-field and the near-field of outside truncated artificial boundary, is simulated by a series of spring-damper systems in the plane in the plane of the truncated artificial boundary, as shown in figure 1. At the artificial boundary of the radius R, the mechanical parameters of the equivalent soil spring damper system are used to simulate the normal action of the far field medium:
2.3. Out-of-plane tangential viscoelastic boundary conditions

When the saturated soil vibrates in the vertical direction, that is, the z direction, the energy spreads to the surrounding in the form of shear waves. The derivation is similar to the previous one, here is not much. The out-of-plane tangential boundary condition equation of the viscoelastic artificial boundary of the saturated soil is as follows:

\[
\tau_{zr} = G \frac{\partial u_z}{\partial r} = G(-ic)u_z + G(-k - \frac{1}{2r})u_z = -\frac{Gc}{\omega} \frac{\partial u_z}{\partial t} + G(-k - \frac{1}{2r})u_z
\]  

(12)

The equation (12) can be modeled by a spring-damper which is similar to the in-plane normal boundary conditions of the viscoelastic artificial boundary of saturated soil, as shown in Fig.2. So the mechanical parameters of the equivalent spring-damper system are presented:

\[
K = G(-k - \frac{1}{2R}); \quad C = \frac{Gc}{\omega}
\]

(13)

Fig.2 Out-of-plane tangential boundary condition of two phase medium in saturated soil

2.4. Tangential plane viscoelastic boundary conditions

When the saturated soil vibrates in the ring direction, that is, the θ direction, assuming that the pore fluid is inviscid. This is similar to the case where saturated soil under vertical vibration, and Derivation process is also similar, here is not much. The tangential plane boundary condition equation of the viscoelastic artificial boundary of the saturated soil is as follows:

\[
\tau_{\theta\theta} = G(\frac{\partial u_{\theta}}{\partial r} - \frac{u_{\theta}}{r}) = G(-ic)u_{\theta} + (-k - \frac{3}{2r})u_{\theta} = -\frac{Gc}{\omega} \frac{\partial u_{\theta}}{\partial t} + G(-k - \frac{3}{2r})u_{\theta}
\]  

(14)
The equation (14) can be modeled by a spring-damper which is similar to the in-plane normal boundary conditions of the viscoelastic artificial boundary of saturated soil, as shown in Fig.3. So the mechanical parameters of the equivalent spring-damper system are presented:

\[
K = G\left(-k - \frac{3}{2R}\right); \quad C = \frac{Gc}{\omega}
\]

(15)

Fig.3 Tangential plane boundary condition of two phase medium in saturated soil

3. Examples of three-dimensional problems

Based on the above theoretical results, with the help of the ADINA finite element program, the three-dimensional numerical simulation of a certain permeability coefficient is carried out. The parameters of Table 1 are used for the analysis.

| Parameters | \( K_s \) (GPa) | \( K_f \) (GPa) | \( v \) | \( G \) (MPa) | \( \rho_f \) (kg/m\(^3\)) | \( \rho_s \) (kg/m\(^3\)) | \( \lambda \) (MPa) | \( n \) | \( \omega \) (Hz) |
|------------|----------------|----------------|------|-------------|----------------|----------------|-------------|-----|-------------|
| Numerical value | 36 | 2 | 0.3 | 40 | 1000 | 2700 | 60 | 0.375 | 5 |

The calculational range is length 10m, width 10m, and height 5m, the size of element is 1m*1m*1m, computational time step is 0.01s. The vertical load acting on the surface is 4*4m in the center of the earth surface, \( P(t) = P_0 \ast \cos(\omega t) \), \( P_0 = 1kN \). The distance from the source to the artificial boundary is averaged by \( R = 5m \), The surface of the model is set to be free draining, and the bottom and the periphery are set as the undrained boundary. ADINA is used to establish the 3D model as shown in Figure 4. Points A and B are the monitoring points in the model.

Fig.4 ADINA3D model
Under the action of cosine periodic load when the permeability coefficient is $10^{-10}$ m/s we can obtain the time history curves of vertical displacement at different monitoring points are plotted, which compared with other artificial boundaries and fixed boundaries, the results are as follows:

![Time history curves of vertical displacement at monitoring points A(m)](image_url)

**Fig. 5 Time history curves of vertical displacement at monitoring points A(m)**

4. Conclusion

(1) The equations in saturated soil plane normal, out-of-plane tangential and the tangential plane are calculated by method of separation of variables and operator decomposition, analytical solutions of the displacement of the saturated soil skeleton and the pore are obtained under the corresponding conditions.

(2) The simulation of 3D numerical examples under various permeability coefficients which about saturated soil dynamic viscoelastic artificial boundary are analyzed by using spring damping element in ADINA. The results showed that the saturated soil viscoelastic boundary of this paper are in good agreement with the saturated soil viscoelastic boundary of Liu Guanglei.

(3) A large number of calculation examples show that the viscoelastic dynamic artificial boundary of saturated soil in this paper is suitable for all kinds of permeability coefficient, it can be applied to the dynamic analysis of saturated soil and structure.

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