Simulation of Plasma Emission in Magnetized Plasmas

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Abstract

The recent Parker Solar Probe observations of type III radio bursts show that the effects of the finite background magnetic field can be an important factor in the interpretation of data. In the present paper, the effects of the background magnetic field on the plasma-emission process, which is believed to be the main emission mechanism for solar coronal and interplanetary type III radio bursts, are investigated by means of the particle-in-cell simulation method. The effects of the ambient magnetic field are systematically surveyed by varying the ratio of plasma frequency to electron gyrofrequency. The present study shows that for a sufficiently strong ambient magnetic field, the wave–particle interaction processes lead to a highly field-aligned longitudinal mode excitation and anisotropic electron velocity distribution function, accompanied by a significantly enhanced plasma emission at the second-harmonic plasma frequency. For such a case, the polarization of the harmonic emission is almost entirely in the sense of extraordinary mode. On the other hand, for moderate strengths of the ambient magnetic field, the interpretation of the simulation result is less clear. The underlying nonlinear-mode coupling processes indicate that to properly understand and interpret the simulation results requires sophisticated analyses involving interactions among magnetized plasma normal modes, including the two transverse modes of the magneto-active plasma, namely, the extraordinary and ordinary modes, as well as electron-cyclotron-whistler, plasma oscillation, and upper-hybrid modes. At present, a nonlinear theory suitable for quantitatively analyzing such complex-mode coupling processes in magnetized plasmas is incomplete, which calls for further theoretical research, but the present simulation results could provide a guide for future theoretical efforts.

Unified Astronomy Thesaurus concepts: Space plasmas (1544); Solar wind (1534)

1. Introduction

Solar radio burst phenomena in the meter wavelengths were discovered in the 1950s and subsequently classified into five categories (Wild & McCready 1950; Wild 1950a, 1950b; Wild et al. 1954), of which the present paper is concerned with some fundamental theoretical aspects undergirding the type III radio bursts. For an excellent review on the subject matter of solar radio emissions up until the mid-1980s, see the collection of articles in the monograph edited by McLean & Labrum (1985). For a more recent review on the subject of solar type III bursts, see, e.g., Reid & Ratcliffe (2014, and references therein). The general theoretical framework most commonly adopted in the literature for type III radio bursts is based upon the pioneering work of Ginzburg & Zheleznyakov (1958), which in a more modern form can be described as a plasma-emission mechanism based on the weak-turbulence paradigm. The plasma weak-turbulence theory (Kadomtsev 1965; Sagdeev & Galeev 1969; Tsytytovich 1970; Davidson 1972; Melrose 1980; Sitenko 1982) found in the literature is of two varieties. One is based upon the intuitive method of the semiclassical (or semi-quantum mechanical) approach, while the other takes the classical nonequilibrium statistical mechanical approach. For a systematic discourse on the plasma-emission theory within the framework of a semiclassical formulation of weak-turbulence theory, see the excellent monograph by Melrose (1980). The semiclassical theory was pioneered by scientists from the former Soviet Union, most notably by Tsytytovich (1970). The statistical method is also available in the literature (see, e.g., Kadomtsev 1965; Davidson 1972; Sitenko 1982; Yoon 2000, 2006; Yoon et al. 2012; Yoon 2019). The nonequilibrium statistical mechanics, or equivalently, kinetic theory, starts from a governing equation of the many-particle system, and systematically introduces perturbation expansion and statistical averages. The semiclassical and statistical theories in the end produce an equivalent set of equations, which can be applied for detailed investigations of solar type III (and type II) radio bursts.

According to the weak-turbulence-based standard theory of plasma emission, active flare events in the solar chromosphere generate bursts of energetic electrons that travel outward along open magnetic field lines and interact with the background plasma via a beam–plasma instability. This bumpy-on-tail instability excites the longitudinal electrostatic Langmuir waves (L), which subsequently undergo nonlinear three-wave decay/coalescence and nonlinear wave–particle scattering, to generate a backward-traveling L mode mediated by the generation of a low-frequency ion-acoustic (S) mode. The merging of the forward- and backward-propagating L modes leads to a radiation emission at twice the plasma frequency (harmonic, or H, emission). The beam-generated L mode also decays into an S mode and a transverse electromagnetic mode (T) at the plasma frequency. This process, which is equivalent to the transformation of the longitudinal mode to radiation, is responsible for the generation of radiation at the fundamental (F) plasma frequency. This standard scenario has been investigated for many decades and various aspects of the plasma-emission process have been studied in the literature (Tsytytovich 1967; Kaplan & Tsytytovich 1968; Melrose 1982;
Goldman & DuBois 1982; Goldman 1983; Melrose 1987; Cairns 1987; Robinson & Cairns 1998a, 1998b; Zlotnik et al. 1998; Kontar 2001a, 2001b, 2001c; Gosling et al. 2003; Li et al. 2005a, 2005b, 2006, 2008, 2011, 2012; Ratcliffe et al. 2012; Reid & Kontar 2017; Krafft et al. 2013; Ziebell et al. 2014, 2015, 2016; Tkachenko et al. 2021).

Weak-turbulence-based plasma emission is the most widely accepted theoretical framework for interpretation of solar type III (and type II) radio bursts. However, there are alternative theories as well. For instance, the dipole radiation emanating from the solitary electrostatic structure, which is a consequence of the so-called strong Langmuir turbulence theory (Zakharyov & Shabat 1972), is an alternative proposed radiation process (Papadopoulos et al. 1974; Papadopoulos & Freund 1978; Goldstein et al. 1979; Smith et al. 1979; Goldman et al. 1980; Goldman 1984; Robinson 1997; Thejappa et al. 2013; Thejappa & MacDowall 2020; Che et al. 2017). A radiation mechanism based on the strong-turbulence processes may be particularly applicable for intense radio bursts. For the fundamental emission, a linear-mode conversion process (Hinkel-Lipsker et al. 1992; Kim et al. 2007) may also be a significant aspect that cannot be overlooked when interpreting spacecraft data, which includes the recent Parker Solar Probe (PSP) data (Krasnoselskikh et al. 2019; Volokitin & Krafft 2020).

The standard weak-turbulence theory has successfully addressed the plasma-emission problem, as noted above. Recently, complete equations of the electromagnetic weak-turbulence theory have been solved in order to quantitatively investigate the plasma-emission process (Ziebell et al. 2015), and its validity was confirmed by 2.5-dimensional (two-dimensional space and three-dimensional velocity) particle-in-cell (PIC) simulation (Lee et al. 2019; see also Ratcliffe et al. 2014). Note that PIC simulations of the plasma-emission process have been carried out, as evidenced by a number of publications in the literature (Kasaba et al. 2001; Umeda 2010; Karlicky & Vandas 2007; Rhee et al. 2009a, 2009b; Ganse et al. 2012a, 2012b; Thurgood & Tsiklauri 2015; Che et al. 2017; Henri et al. 2019; Li et al. 2021; Krafft & Savoini 2021). It should be noted, however, that a direct and quantitative comparison between the PIC simulation and weak-turbulence theory has rarely been undertaken (Ratcliffe et al. 2014; Lee et al. 2019).

A common thread that connects various works regarding the standard model of the plasma-emission theory, and also the existing PIC simulations thereof, is that the effects of the background quasi-static magnetic field are ignored. The assumption of unmagnetized plasma is valid in interplanetary space sufficiently far away from the solar surface, since the solar wind’s magnetic field decreases quite rapidly as one moves away from the Sun. However, near the solar-active region, which is associated with the type III radio source, the effects of the strong magnetic field can be significant (Morosan et al. 2016). The contemporary PSP measurement of type III radio bursts also leads to open questions regarding the polarization properties (Pulupa et al. 2020). Without the ambient magnetic field, the type III bursts should be unpolarized, but Pulupa et al. (2020) report circular polarizations, which indicate that the type III emissions occurring close to the Sun are influenced by the background magnetic field. In a similar vein, Ma et al. (2021) analyzed the PSP data by surveying the low-frequency cutoffs associated with the type III bursts and speculate that the cyclotron maser emission may also be effective in the radiation process. Further, Chen et al. (2021) modeled the interplanetary type IIIb emissions observed by PSP by means of this cyclotron maser instability mechanism. The cyclotron maser emission mechanism, which requires the presence of a strong ambient magnetic field and a loss-cone feature (or perpendicular population inversion in momentum space) associated with the energetic electrons, was first suggested as an alternative mechanism for type III radio bursts near the source region by Wu et al. (2000), but recent PSP measurements have revived the interest in such an alternative mechanism (Chen et al. 2021; Zhou et al. 2020).

In another development, Ni et al. (2020, 2021) analyzed the possibility of plasma emission by mode coupling among the cold plasma modes in magnetized plasmas, in particular the emission involving whistler- (or electron-cyclotron) mode waves. In the literature, the plasma emissions in magnetized plasmas have been investigated in some detail by Trakhtengerts (1970), Melrose & Sy (1972), Melrose et al. (1978, 1980), Zlotnik (1981), and Willes & Melrose (1997), but in these works the standard plasma-emission mechanism developed for unmagnetized plasmas is extended to the magnetized case, where high-frequency waves undergo decay interaction with low-frequency sonic types of mode (such as the magneto-acoustic mode, which is a thermal mode that does not propagate in cold plasmas). However, Ni et al. (2020, 2021) revived the old idea of plasma emission in magnetized plasmas involving the whistler-mode type wave, which does not require finite-temperature effects. Such an idea was proposed earlier by Chiu (1970) and Chiu (1972), which was subsequently criticized by Melrose (1975), though Ni et al. (2020) and Ni et al. (2021) reinvestigated such a possibility by means of modern PIC simulation.

The contemporary developments and research activities as outlined above show that the venerable theory of plasma emission needs to be extended to include the effects of the ambient magnetic field. While some early works (Trakhtengerts 1970; Melrose & Sy 1972; Melrose et al. 1978, 1980; Zlotnik 1981; Willes & Melrose 1997) have indeed attempted to address such an issue, the problem is far from complete. Unlike the unmagnetized situation, where the standard weak-turbulence theory has advanced to a mature stage so that a direct numerical comparison with PIC simulation is possible (Lee et al. 2019), the situation for the magnetized case is at an infantile stage in comparison. A complete self-consistent set of equations that readily lend themselves to numerical (or theoretical) analysis are not available at this time.

The purpose of the present paper is to address the influence of the ambient magnetic field on plasma emission by carrying out PIC simulation. The present paper performs PIC simulations of electron-beam–plasma instability in a magnetized plasma and the ensuing nonlinear-mode coupling processes, which includes the radiation emissions at the fundamental plasma frequency and/or its harmonics. The PIC simulations are performed by varying the degree of magnetization in the plasma. This is accomplished by choosing differing ratios of plasma-to-gyro frequencies, $\omega_p/\Omega_{ce}$, where $\omega_p = (4\pi n_e e^2/m_e)^{1/2}$ is the plasma oscillation frequency, and $\Omega_{ce} = eB_0/m_e c$ is the electron-cyclotron (or gyro) frequency. When the ratio $\omega_p/\Omega_{ce}$ is high, then the plasma is weakly magnetized, and vice versa. By carrying out simulations with different values of $\omega_p/\Omega_{ce}$ we aim to establish a constraint against which future quantitative theories of plasma emission in magnetized plasma can be tested.
With this aim, we present our findings, which are organized as follows. In Section 2 a brief description of the simulation code is presented. Then, in Section 3 results from the PIC simulations, together with interpretations, are presented. Finally, in Section 4 we summarize the major findings and present our conclusions.

2. Particle-in-cell Simulation of Plasma Emission in Magnetized Plasma

The electromagnetic PIC simulation adopted in the present paper is briefly described herewith. The numerical program is a two-dimensional relativistic and electromagnetic PIC simulation code. The simulation box size is $L_x = 64.76c/\omega_{pe}$ for all four cases and $L_y = L_x = 64.70c/\omega_{pe}$ for $\omega_{pe}/\Omega_{ce} = 100$ and 10, but $L_y = 2L_x = 129.53$ for $\omega_{pe}/\Omega_{ce} = 3$ and 1 for higher resolution of $k_x$, where $c$ is the speed of light, $V_{th}$ is the electron thermal speed and $\omega_{pe}$ is the electron plasma frequency. The number of grid points is $N_x \times N_y = 1024 \times 1024$ for $\omega_{pe}/\Omega_{ce} = 100$ and 10, and $N_x \times N_y = 1024 \times 2048$ for $\omega_{pe}/\Omega_{ce} = 3$ and 1, so that the grid size is $\Delta x = \sqrt{2} \lambda_D$. The simulation step size is $\Delta t = \Delta x / 2c$, which satisfies the Courant–Friedrichs–Lewy condition, where $\Delta t$ is the time step.

The number of particles is 200 per grid per species, and we use protons and electrons. Electrons are further subdivided into 99.9% background electrons and 0.1% beam electrons. The proton-to-electron mass ratio is realistic, $m_p/m_e = 1836$, where $m_p$ is the proton mass and $m_e$ is the electron mass. The electron thermal speed is $V_{th} = 4.0 \times 10^{-3}$. The electron temperature is seven times higher than the proton temperature, $T_e/T_i = 1/7$, where $T_i$ is the ion (proton) temperature and $T_e$ is the electron temperature. The ambient magnetic field is directed along the $x$-axis.

The beam consists of 0.1% of the total electron content, namely, $n_b/n_0 = 10^{-3}$, where $n_b$ is the beam number density and $n_0$ is the electron number density. The Maxwellian beam temperature is the same as the background electron temperature, $T_b = T_e$, where $T_b$ is the electron beam temperature. The plasma-to-cyclotron frequency ratio is systematically varied from $\omega_{pe}/\Omega_{ce} = 100$, to 10, to 3, to 1. The boundary conditions are periodic for both the $x$- and $y$-directions.

The above-described parameters are chosen as in the paper by Ziebell et al. (2015). In all our simulations, we adopted the same average beam drift speed, $V_b/V_{th} = 8$. Again, such choices reflect the same initial condition as those considered by Ziebell et al. (2015). The PIC code simulation is initiated with the distributions of electrons and protons that preserve a current-free condition so that the background electrons have a slight negative drift against the background of protons. The initial velocity-diffusion function for the electrons is specified according to

\[
F_e(V,0) = \left(1 - \frac{n_b}{n_0}\right) \left(\frac{m_i}{2\pi T_e}\right)^{3/2} e^{-\frac{m_e V^2}{2T_e}} \left(1 - \frac{n_b (V_+)}{n_0} \left(\frac{m_e}{2\pi T_e}\right)^{3/2} e^{-\frac{m_e V^2}{2T_e}} + \frac{n_b}{n_0} \left(\frac{m_e}{2\pi T_b}\right)^{3/2} e^{-\frac{m_e V_+^2}{2T_b}} \right)
\]

3. Numerical Results

We begin by presenting a two-dimensional phase-space plot of the electron velocity distribution function (VDF) at $\omega_{pe} = 2000$. Figure 1 shows four different cases: (A) $\omega_{pe}/\Omega_{ce} = 100$, (B) $\omega_{pe}/\Omega_{ce} = 10$, (C) $\omega_{pe}/\Omega_{ce} = 3$, and (D) $\omega_{pe}/\Omega_{ce} = 1$. Case (A) is almost identical to the unmagnetized situation (Lee et al. 2019) in that the Maxwellian core is not much affected by the beam-plasma instability, while the beam electrons are shown to have undergone not only a plateau formation along $V_{th}$, but also a perpendicular spreading along $V_{\perp}$. Note that the present PIC simulation, as well as that of Lee et al. (2019), is carried out over fully three-dimensional velocity space, but the spatial dimension is in two-dimensional space. In comparison, the quasilinear calculation carried out by Ziebell et al. (2015) was done in two-dimensional velocity space as well as in two-dimensional wavenumber space. Recent papers by Harding et al. (2020) and Melrose et al. (2021) discuss some issues associated with the velocity-space diffusion in one-dimensional versus three-dimensional configurations. The quasilinear calculation by Ziebell et al. (2015) (not shown in the present paper) is the intermediate two-dimensional case, which, upon comparison with the simulated VDF in three dimensions in the paper by Lee et al. (2019), was shown to be in good qualitative agreement.

Panel (B), which is for $\omega_{pe}/\Omega_{ce} = 10$, shows only a minimal difference when compared with the case for $\omega_{pe}/\Omega_{ce} = 100$. On the other hand, when $\omega_{pe}/\Omega_{ce}$ is reduced to 3 (panel (C)), one begins to see that the diffusion along $V_{\perp}$ space is restricted. The trend is further enhanced when $\omega_{pe}/\Omega_{ce}$ is reduced even further to 1. In panel (D), the electron-beam VDF shows that the diffusion along $V_{\perp}$ is markedly limited when compared with the previous cases. The reason for such a difference may be understood from the velocity-diffusion coefficient for the magnetized case. When one includes the $B$-field effects, then the velocity-diffusion coefficient includes the factor

\[
\sum_{n=\infty}^{0} J_n^2 \left(\frac{k_1 V_{\perp}}{\Omega_{ce}}\right) \delta(\omega - k_1 V_{\perp} - n\Omega_{ce}).
\]

For a sufficiently strong magnetic field, the diffusion along $V_{\perp}$ space will be severely limited by the Bessel-function factor, since $J_n^2(k_1 V_{\perp}/\Omega_{ce})$ decreases in magnitude as $V_{\perp}$ increases. Note, that for $n = 0$, $J_0(x)$ is maximum at $x = 0$ and then decreases with increasing $x$, which conforms with the above statement. For $n \neq 0$, $J_n(x)$ is zero at $x = 0$ but peaks at some finite $x$. However, the peak intensity for each $n$ decreases monotonically as $n$ increases. As a consequence, the dominant term is $J_0^2(x)$. This may explain the limited diffusion characteristics along $V_{\perp}$ in a qualitative sense, but in order to fully characterize the velocity-space diffusion behavior, one must actually solve the diffusion equation, which is beyond the scope of the present paper.

Figure 2 displays the longitudinal electric-field spectrum in two-dimensional space, $|E_k|$, in a logarithmic colormap scale. Positive $k_1$ corresponds to a parallel wavevector component along the beam-propagation direction (i.e., forward propagation), while $k_1 < 0$ corresponds to the antiparallel direction (i.e., backward propagation). $k_1$ denotes the wave-number perpendicular to the beam-propagation direction (and also perpendicular to the ambient magnetic field). For case (A),
which corresponds to $\omega_{pe}/\Omega_{ce} = 100$, the spectral pattern is virtually identical to the unmagnetized case (Lee et al. 2019) in that the forward-propagating (or $k_\parallel > 0$) mode, which can be interpreted as quasi-Langmuir waves (here, we use the term “quasi-” since for magnetized plasmas, the high-frequency longitudinal mode is Langmuir wave-like for the quasi-parallel propagation direction, but gradually turns into an upper-hybrid wave for oblique-to-quasi-perpendicular angles of propagation), has undergone a significant backscattering via combined decay and induced scattering by the time the beam–plasma instability has progressed to $\omega_{pe}t = 2000$. The backward-propagating waves are those corresponding to $k_\parallel < 0$. Also

Figure 1. Electron velocity distribution function at $\omega_{pe}t = 2000$ for (A) $\omega_{pe}/\Omega_{ce} = 100$, (B) $\omega_{pe}/\Omega_{ce} = 10$, (C) $\omega_{pe}/\Omega_{ce} = 3$, and (D) $\omega_{pe}/\Omega_{ce} = 1$.

Figure 2. Longitudinal mode spectra, $|E_x(k)|$, at $\omega_{pe}t = 2000$ for (A) $\omega_{pe}/\Omega_{ce} = 100$, (B) $\omega_{pe}/\Omega_{ce} = 10$, (C) $\omega_{pe}/\Omega_{ce} = 3$, and (D) $\omega_{pe}/\Omega_{ce} = 1$. 
notice the long-wavelength modes broadly distributed near $k \sim 0$, which is due to Langmuir condensation effects. For case (B), which corresponds to $\omega_{pe}/\Omega_{ce} = 10$, the spectrum is qualitatively similar to panel (A). Recall that the electron VDFs for (A) and (B) are also quite similar to the unmagnetized case. Thus, the situation with the spectra is consistent with that of the electrons.

For case (C), for which $\omega_{pe}/\Omega_{ce} = 3$, the longitudinal electric-field spectrum appears to be shrunk along the $k_{\parallel}$ direction. Again, this can be understood in a qualitative sense. As with the particle-diffusion coefficient, the linear growth rate of the longitudinal mode also has the same Bessel-function factor (2). As such, for sufficiently high $k_{\perp}$, the factor $J_2^2(k_{\perp}V_{th}/\omega_{pe})$ decreases in magnitude, thus preventing the wave growth at a high-$k_{\perp}$ regime. Such a feature is also reflected in the backscattered ($k_{\parallel} < 0$) mode as well as the condensate ($k \sim 0$) mode. In spite of such a qualitative explanation, the actual reconstruction of the wave spectrum requires numerical solution of the underlying equations of weak-turbulence theory in magnetized plasmas, which is not available yet. For case (D), with $\omega_{pe}/\Omega_{ce} = 1$, the wave dynamics in both the linear and nonlinear stages appear almost one-dimensional. For strongly magnetized plasmas, the electron gyroradius around the ambient magnetic field can be quite small while the field-aligned motion is much less restricted. As a result, the wave growth, saturation, and nonlinear-mode coupling all appear to be confined to quasi-one-dimensional space. Again, to fully characterize this one must actually solve the fundamental equations of magnetized plasma weak-turbulence theory.

Figure 3 plots the wave spectra for the same four cases except that the electric-field spectra correspond to the transverse polarization, $|E_z(k)|$. In the simulation it is not so easy to separate the radiation modes from the transverse-polarized modes associated with nonescaping plasma normal modes. In the cold magnetized plasma the extraordinary ($X$) and ordinary ($O$) modes are transverse radiations that can escape to free space in an inhomogeneous medium. On the other hand, the slow-extraordinary, or the $Z$ mode, and the whistler, or $W$, mode are normal modes trapped within the plasma that cannot escape to free space. The transverse-wave spectra shown in Figure 3 are projections of all these modes, regardless of the characteristic wave frequency associated with each mode, onto two-dimensional wavenumber space. As such, interpretation of the spectra requires some further analyses, which we will get to shortly. For the moment, paying attention to panel (A), the most prominent features are the concentric circles (the radius of the central circle is so small that the inner circle appears as a dot). These correspond to the fundamental and harmonic plasma emissions, with the inner circle being the fundamental emission with $\omega \sim \omega_{pe}$, while the outer circle depicts the second-harmonic emission with $\omega \sim 2\omega_{pe}$. Such a concentric, ring-like spectral feature is highly reminiscent of the unmagnetized case (Lee et al. 2019). Note that the circular plasma-emission pattern is consistently present in all four cases, and, in fact, the intensity becomes slightly higher as $\omega_{pe}/\Omega_{ce}$ decreases in magnitude. For case (D), in particular, the third-harmonic ($\omega \sim 3\omega_{pe}$) emission is also present, and even what appears to be a faint fourth harmonic. Moreover, the ring associated with the second-harmonic emission has increased in width. This has to do with the modification of the dispersion relation in the presence of a strong magnetic field, which we will come back to later. Returning to case (A), one may notice that a faint quadrupole pattern of enhanced wave intensity is present, which seems to connect the two plasma-emission inner and outer rings. This superposed pattern becomes more evident.
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in panel (B), as $\omega_{pe}/\Omega_{ce}$ is reduced to 10. For $\omega_{pe}/\Omega_{ce} = 3$ (panel C), the quadrupole pattern gets not only distorted but, also, a new enhanced wave intensity appears outside the outer ring of the second-harmonic emission (the rabbit-ear-like protrusion). This was also present, albeit very weakly, in panel (B), but now it becomes quite evident. We will investigate this in more detail later. For the fourth case of strongly magnetized plasma, case (D), with $\omega_{pe}/\Omega_{ce} = 1$, the quadrupole pattern between the fundamental emission (inner dot) and second-harmonic emission (outer ring) is now more or less merged with the second-harmonic outer ring to the extent that it is difficult to separate the two. Notice, also, a narrow jet-like feature, which is present along the positive $k_{\parallel}$ axis. This is probably the rabbit-ear-like protrusion that is apparent in case (C), which has now collapsed into a narrow, quasi-one-dimensional structure. A salient feature associated with the transverse spectrum is that, regardless of the strength of the background magnetic field, the effects of the Bessel-function factor are not apparent. That is, for the velocity distribution and the longitudinal mode, an increasing value of $\omega_{pe}/\Omega_{ce}$ has the effect of restricting the perpendicular dynamics so that both the velocity distribution and the longitudinal mode spectrum have tendencies to become quasi-one-dimensional. In contrast, the perpendicular mode structure associated with the transverse mode spectrum remains unchanged. This seems to imply that the transverse mode generation is unaffected by the Bessel-function factor $J_2(k_{\parallel}V_{e}/\Omega_{ce})$, which, in turn, could mean that the transverse mode generation processes are inherently nonlinear.

The complex behavior and morphology associated with the transverse-wave spectrum as shown in terms of the projections onto two-dimensional $k$ space can only provide a limited understanding. In order to further analyze the situation, we consider the full three-dimensional structure associated with the transverse mode spectrum. Thus, in the remainder of the present paper, we revisit certain aspects associated with the wave spectra that the simple projection onto two-dimensional wavenumber space could not elucidate. Let us first pay attention to panel (D) of Figure 3, where we have noted that the second-harmonic, ring-like spectrum has an increased width and that the intermediate intensity that connects the inner ring and outer ring appeared to merge with the outer ring. These features are the result of two-dimensional projection. In Figure 4, we plot a three-dimensional spectrum including the characteristic frequency associated with each point in the two-dimensional $k$ spectrum. We accomplish this by plotting the wave spectral intensity as a three-dimensional scatter plot in $(k_{\parallel}, k_{\perp}, \omega)$ space. As Figure 4 makes evident, the second-harmonic ring is associated with the X mode. For $\omega_{pe}/\Omega_{ce} = 1$, the X and O modes are clearly separated, the X mode dispersion surface being situated above the O mode surface. The O and Z mode surfaces are intertwined near the cutoff frequencies ($k \sim 0$) so that the two modes undergo switchovers. The low-frequency W mode surface lies underneath the other three dispersion surfaces. The modification and broadening of the X mode dispersion surface near the cutoff lead to an increase of width associated with the harmonic emission spectral ring.

It is quite clear from Figure 4 that the Z mode intensity is rather high, which obscures the second-harmonic ring from the Z mode wave intensity when projected onto two-dimensional $k$ space. However, when the spectrum is separated in frequency one may glean that the second-harmonic ring and the broad Z mode spectrum belong to separate branches. Note that the jet-like enhanced wave intensity along the positive $k_{\parallel}$ axis (the jet-like protrusion) is associated with the Z mode, which the present viewing angle makes it difficult to visualize, though when plotted with a different viewing angle, such a feature becomes more evident (not shown).

For the present case of $\omega_{pe}/\Omega_{ce} = 1$, the radiation emission at the fundamental plasma frequency is in the sense of the O mode, but since the O mode and Z mode exist in close vicinity of each other in both frequency and wavenumber, it is difficult to determine which mode is which. Judging from the enhanced W mode intensity, the radiation emission at the fundamental plasma frequency might be mediated by decay processes involving W modes, as discussed by Chiu (1970), Chin (1972) and Ni et al. (2020, 2021) in spite of the negative assessment by Melrose (1975). Note that we have imposed an upper limit on the frequency at $3\omega_{pe}$, so that fourth harmonic is not shown, but one can see, albeit with some difficulties, that the transverse mode intensity is somewhat enhanced in the vicinity of $3\omega_{pe}$. However, it is difficult to visually discern whether the third-harmonic emission is in the sense of X or O mode polarization due to the fact that at such a frequency the X and O mode dispersion surfaces are in close proximity, and also because of the fact that the third-harmonic emission is quite weak in intensity.

The complex features associated with the various modes necessitate further theoretical analysis, which will be the subject of future research. Even with the three-dimensional spectral scatter plot, the best we can do is to provide verbal descriptions based on visual examination. However, with a full development of weak-turbulence theory in magnetized plasmas, the future goal is to replicate these various features by theoretical means. Only then could one truly understand the effects of the ambient magnetic field on the plasma-emission process.

It is interesting to note that the simulated wave spectrum, when plotted as a three-dimensional scatter plot, traces the magneto-ionic dispersion surfaces rather accurately. To see this, we have plotted, in Figure 5, the cold-plasma-mode dispersion relations as four separate manifolds (or surfaces) in $\omega$ versus $(k_{\parallel}, k_{\perp})$ space. The nested surfaces are, from top, the fast-extraordinary (or X) mode, the ordinary (or O) mode, which is coupled with the slow-extraordinary (or Z) mode near the plasma frequency $\omega_{pe}$ or upper-hybrid frequency $\omega_{wh} = (\omega_{pe}^2 + \Omega_{pe}^2)^{1/2}$ regime, and, underneath these, the whistler (or W) mode dispersion surface. Upon comparison with the three-dimensional scatter plot of simulated wave intensity, it becomes quite evident that the magneto-ionic theory of waves is quite valid as far as the mode configuration in $\omega$ versus $k$ space is concerned. That is, the weakly turbulent processes taking place in strongly magnetized plasma ($\omega_{pe}/\Omega_{ce} = 1$) seem to involve linear and nonlinear wave–particle and wave–wave interactions among particles and linear eigenmodes of the magnetized plasma, but the eigenmodes themselves are well described by the linear wave theory. Note that for warm plasmas, besides the cold magneto-ionic modes, thermal acoustic (or sonic) modes are also excited, but as such modes are low-frequency modes in the vicinity of ion-plasma or ion-cyclotron frequency, it is difficult to set the genuine thermal sonic modes apart from numerical noise-like fluctuations according to the present PIC simulation. Recall that the traditional approach to interpreting plasma emission in
magnetized plasmas has been to invoke these thermal sonic modes as being more important than W mode (Trakhtengerts 1970; Melrose & Sy 1972; Melrose et al. 1978, 1980; Zlotnik 1981; Willes & Melrose 1997).

We repeat the three-dimensional scatter plot of the wave spectrum for case (C), which corresponds to \( \omega_{pe}/\Omega_{ce} = 3 \) and which is plotted in Figure 6. In this case, the X and O mode dispersion surfaces are very close to each other at \( \omega \sim 2\omega_{pe} \). As
such, the harmonic emission, which is seen as the ring-like enhancement, might have a mixed X/O mode polarization, but visual inspection is insufficient to ascertain this. It is interesting to note that the fundamental emission is buried within a broad swath of the enhanced Z mode spectrum, which makes it difficult to distinguish one from the other. The peculiar feature in Figure 3(D), namely, the quadrupole pattern of enhanced intensity in between the inner and outer ring, as well as the pair of rabbit-ear-like protrusions outside the outer ring, can now reasonably be identified with the Z mode wave intensity. When the spectrum is projected onto two-dimensional space, one cannot tell apart all these features from one another, but separating the spectrum in $\omega$ space facilitates the delineation of different modes. Note that the low-frequency W mode is associated with a weak but visible wave intensity, which, again, points to the cold plasma W mode participating in the plasma emission (Chiu 1970; Chin 1972; Ni et al. 2020, 2021). Again, even lower-frequency thermal sonic modes could not be distinguished from numerical noise in a meaningful way in the present simulation.

We have not repeated the same scatter plots for cases (A) and (B), but shown in Figure 7 is the analog of Figure 5 except that magneto-ionic dispersion surfaces corresponding to $\omega_{pe}/\Omega_{ce} = 3$ are plotted. The comparison between the scatter plot of wave intensity in Figure 6 and the magneto-ionic dispersion relation again confirms that the basic eigenmodes excited in the simulation are those that can be described by the linear theory of plasma waves. However, the linear plasma modes undergo linear and nonlinear interactions among themselves as well as with the particles. This is the basic picture implicit in the weak-turbulence theory, which shows that the plasma-emission problem is not only relevant for solar radio burst phenomena but is also an ideal problem to test and validate the weak-turbulence theory.

Despite the good agreement between Figures 6 and 7, we note that both the scatter plot and the dispersion surface plot obscure the complicated mode structure near the X and O mode cutoff regions (near $k \sim 0$). In order to furnish additional information on the mode structure near the $k \sim 0$ region, we plot in Figure 8 a two-dimensional cut associated with the wave spectrum in $\omega$ versus $k_\parallel V_{th}/\omega_{pe}$ space, for the case of $\omega_{pe}/\Omega_{ce} = 3$ (case (C)), which is generated for fixed $k_\perp V_{th}/\omega_{pe} = 0.028$. The simulated transverse electric-field energy spectrum is shown as a colormap. Superposed with the thin curves are the theoretical dispersion relations. As one may appreciate, the theoretical magneto-ionic dispersion curves fit almost perfectly with the simulated spectrum. Note the well-defined second-harmonic emission near 2$\omega_{pe}$ and a faint but identifiable third-harmonic emission at 3$\omega_{pe}$. Near the fundamental plasma frequency, however, the enhanced wave spectrum has a complex frequency structure. Both the X and O modes have enhanced intensity, which means the fundamental emission for type III radio sources characterized by $\omega_{pe}/\Omega_{ce} = 3$ could have a mixed polarization. The second-harmonic emission could also have a mixed polarization, but, as noted before, because of the close proximity of the two dispersion curves, it is difficult to distinguish the two modes. Near $\omega \sim \omega_{pe}$, the Z mode has a high intensity, and the enhanced Z mode spectrum extends beyond the infinitely narrow normal mode dispersion curve. As noted above, the W mode shows a slightly enhanced intensity, which could be the result of decay instability involving the beam-generated Z mode, or the electron beam directly exciting the W mode by a linear-instability mechanism. Again, all of these multi-step processes must be investigated by a first-principle theory in the future.

Figure 6. Scatter plot of the wave spectrum for moderately magnetized case of $\omega_{pe}/\Omega_{ce} = 3$. 

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Figure 7. Magneto-ionic dispersion surfaces for $\omega_{pe}/\Omega_{ce} = 3$.

Figure 8. The two-dimensional section of the simulated transverse-wave spectrum for $\omega_{pe}/\Omega_{ce} = 3$. The thick black curves depict the magneto-ionic dispersion relation.
4. Summary and Conclusions

To summarize and conclude the present paper, we have carried out a series of PIC simulations of the plasma-emission process in magnetized plasma. Plasma emission, or the emission of radiation at the fundamental plasma frequency and/or its harmonics, is widely believed to be the essential radiation-emission mechanism responsible for solar coronal and interplanetary type II and type III radio bursts. The plasma-emission problem is also an ideal platform to test and validate plasma weak turbulence from the perspective of fundamental plasma physics. In the literature the plasma-emission problem has been studied by means of theory and simulations, but, almost invariably, the effects of the ambient magnetic field have been ignored. In contrast, in the present paper, we have considered the effects of the background magnetic field by systematically varying the frequency ratio, $\omega_{pe}/\Omega_{ce}$. When $\omega_{pe}^2/\Omega_{ce}^2 = 4\pi n_0 m_e c^2/\varepsilon_0^2$ is high, then the plasma can be considered as weakly magnetized. If this ratio is low, however, then the plasma is strongly magnetized. 

We consider the weak-turbulence theory developed under the unmagnetized condition, which must be done in the framework of diagnostic tools in order to delineate and comprehend the underlying physical processes one must formulate and solve the basic equations of weak-turbulence theory in magnetized plasmas, which must be done in the future. The present simulation results, however, will be valuable in that our work can provide a constraint against such future theories could be tested and validated.

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References

Cairns, I. H. 1987, JPhPh, 38, 169
Chen, E., Goldstein, M. L., Diamond, P. H., & Sagdeev, R. Z. 2017, PNAS, 114, 1502
Chen, L., Ma, B., Wu, D., et al. 2021, ApJL, 915, L22
Chin, Y.-C. 1972, P&SS, 20, 711
Chiu, Y. T. 1970, SoPh, 13, 420
Davison, R. C. 1972, Methods in Nonlinear Plasma Theory (New York: Academic)
Ganse, U., Kliian, P., Spanier, F., & Vainio, R. 2012a, ApJ, 751, 145
Ganse, U., Kliian, P., Vainio, R., & Spanier, F. 2012b, SoPh, 280, 551
Ginzburg, V. L., & Zheleznyakov, V. V. 1958, SvA, 2, 653
Goldman, M. V. 1983, SoPh, 89, 403
Goldman, M. V. 1984, RVMP, 56, 709
Goldman, M. V., & DuBois, D. F. 1982, PhFl, 25, 1062
Goldman, M. V., Reiter, G. F., & Nicholson, D. R. 1980, PhFl, 23, 388
Goldstein, M. L., Smith, R. A., & Papadopoulos, K. 1979, ApJ, 234, 683
Gosling, J. T., Skoug, R. M., & McComas, D. J. 2003, GeoRL, 30, 1697
Harding, J. C., Cairns, I. A., & Melrose, D. B. 2020, PhPl, 27, 020702
Henri, P., Sgattoni, A., Briand, C., Amiranoff, F., & Riconda, C. 2019, JGRA, 124, 1475
Hinton, R. L., Dries, B. D., & Morales, G. J. 1992, PhFlB, 4, 1772
Kadomtsev, B. B. 1965, Plasma Turbulence (New York: Academic)
Kaplan, S. A., & Tsytovich, V. N. 1968, SvA, 11, 956
Karlický, M., & Vandas, M. 2007, P&SS, 55, 2336
Kasaba, Y., Matsumoto, H., & Omura, Y. 2001, JGR, 106, 18903
Kim, E., Cairns, I. H., & Robinson, P. A. 2007, PhRvL, 99, 015003
Kontar, E. P. 2001a, PCCP, 3, 1139
Kontar, E. P. 2001b, SoPh, 202, 131
Kontar, E. P. 2001c, A&A, 375, 629
Krafft, C., & Savoini, P. 2021, ApJL, 917, L23
Krafft, C., Volokitin, A. S., & Krasnoselskikh, V. V. 2013, ApJ, 778, 111
Krasnoselskikh, V., Voshchepnyats, A., & Maksimovic, M. 2019, ApJ, 879, 51
Lee, S.-Y., Ziebell, L. F., Yoon, P. H., Gueler, R., & Lee, E. 2019, ApJ, 871, 74
Li, B., Cairns, I. H., & Robinson, P. A. 2008, JGRA, 113, A06104
Li, B., Cairns, I. H., & Robinson, P. A. 2011, ApJ, 730, 20
Li, B., Cairns, I. H., & Robinson, P. A. 2012, SoPh, 279, 173
Li, B., Robinson, P. A., & Cairns, I. H. 2006, PhPl, 13, 092902
Li, B., Willes, A. J., Robinson, P. A., & Cairns, I. H. 2005a, PhPl, 12, 012103
Li, B., Willes, A. J., Robinson, P. A., & Cairns, I. H. 2005b, PhPl, 12, 052324
Li, C., Chen, Y., Ni, S., et al. 2021, ApJL, 909, L5
Ma, B., Chen, L., Wu, D., & Bale, S. D. 2021, ApJL, 913, L1
McLean, D. J., & Labrum, N. R. 1985, Solar Radiophysics (Cambridge: Cambridge Univ Press)
Melrose, D. B. 1975, AuPh, 28, 101
Melrose, D. B. 1980, Plasma Astrophysics: Vol. 2 (New York: Gordon and Breach)
Melrose, D. B. 1982, AuPh, 35, 67
Melrose, D. B. 1987, SoPh, 111, 89
Melrose, D. B., Dulk, G. A., & Gary, D. E. 1980, PASAu, 4, 50
Melrose, D. B., Dulk, G. A., & Smerd, S. F. 1978, A&A, 66, 315
Melrose, D. B., Harding, I. & Cairns, I. H. 2021, PhPl, 296, 42
Melrose, D. B., & Sy, W. 1972, ApSS, 17, 343
Morosan, D. E., Zucca, P., Bloomfield, D. S., & Gallagher, P. T. 2016, A&A, 589, L8
Ni, S., Chen, Y., Li, C., et al. 2020, ApJL, 891, L25
Ni, S., Chen, Y., Li, C., et al. 2021, PhPl, 28, 040701
Papadopoulos, K., & Freeman, H. P. 1978, GeoJet, 5, 881
Papadopoulos, K., Goldstein, M. L., & Smith, R. A. 1974, ApJ, 190, 175
Pulupa, M., Bale, S. D., Badman, S. T., et al. 2020, PhPl, 264, 49
Ratcliffe, H., Bian, N. H., & Kontar, E. P. 2012, PhPl, 21, 122104
Ratcliffe, H., Brady, C. S., Che, Rozenan, M. B., & Nakariakov, V. M. 2014, PhPl, 21, 122104
Reid, H. A. S., & Kontar, E. P. 2017, A&A, 598, A44
Reid, H. A. S., & Ratcliffe, H. 2014, RAa, 14, 773
Rhee, T., Ryu, C.-M., Woo, M., et al. 2009a, ApJ, 694, 618
Rhee, T., Woo, M., & Ryu, C.-M. 2009b, JPKS, 54, 313
Robinson, P. A. 1997, RVMP, 69, 507
Robinson, P. A., & Cairns, I. H. 1998a, SoPh, 181, 363
Robinson, P. A., & Cairns, I. H. 1998b, SoPh, 181, 395
Sagdeev, R. Z., & Galeev, A. A. 1969, Nonlinear Plasma Theory (New York: Benjamin)
Sitenko, A. G. 1982, Fluctuations and Nonlinear Wave Interactions in Plasmas (New York: Pergamon)
Smith, R. A., Goldstein, M. L., & Papadopoulos, K. 1979, ApJ, 234, 348
Thejappa, G., & MacDowall, R. J. 2020, JGRA, 125, e2019JA027714
Thejappa, G., MacDowall, R. J., & Bergamo, M. 2013, JGRA, 118, 4039
Thurgood, J. O., & Tsiklauri, D. 2015, A&A, 584, A83
Tkachenko, A., Krasnoselskikh, V., & Voshchepynets, A. 2021, ApJ, 908, 126
Trakhtengerts, V. Y. 1970, R&QE, 13, 697
Tsytovich, V. N. 1967, SvPhU, 9, 805
Tsytovich, V. N. 1970, Nonlinear Effects in Plasma (New York: Plenum)
Umeda, T. 2010, JGRA, 115, A01204
Volokityn, A. S., & Krafft, C. 2020, ApJL, 893, L47
Wild, J. P. 1950a, AuSRA, 3, 399
Wild, J. P. 1950b, AuSRA, 3, 541
Wild, J. P., & McCready, L. L. 1950, AuSRA, 3, 387
Wild, J. P., Murray, J. D., & Rowe, W. C. 1954, AuPh, 7, 439
Willes, A. J., & Melrose, D. G. 1997, SoPh, 171, 393
Wu, C. S., Yoon, P. H., & Li, Y. 2000, ApJ, 540, 572
Yoon, P. H. 2000, PhPl, 7, 4858
Yoon, P. H. 2006, PhPl, 13, 022302
Yoon, P. H. 2019, Classical Kinetic Theory of Weakly Turbulent Nonlinear Plasma Processes (Cambridge: Cambridge Univ. Press)
Yoon, P. H., Ziebell, L. F., Gaelzer, R., & Pavan, J. 2012, PhPl, 19, 102303
Zakharov, V. E., & Shabat, A. B. 1972, JETP, 34, 62
Zhou, X., Muñoz, P. A., Büchner, J., & Liu, S. 2020, ApJ, 891, 92
Ziebell, L. F., Petruzzellis, L. T., Yoon, P. H., Gaelzer, R., & Pavan, J. 2016, ApJ, 818, 61
Ziebell, L. F., Yoon, P. H., Gaelzer, R., & Pavan, J. 2014, ApJl, 795, L32
Ziebell, L. F., Yoon, P. H., Petruzzellis, L. T., Gaelzer, R., & Pavan, J. 2015, ApJ, 806, 237
Zlotnik, E. Y. 1981, A&A, 101, 250
Zlotnik, E. Y., Klassen, A., Klein, K. L., Aurass, H., & Mann, G. 1998, A&A, 331, 1087