Control of the Silica Photonic Crystal Fiber Production

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Abstract. This paper deals with the problem of optimal control of the drawing process of special silica optical fibers. The research is based on a nonlinear mathematical model of drawing a silica capillary, which is a constituent unit of a photonic crystal fiber. Under well-grounded assumptions, this model is modified and reduced to a linear model. The problem of optimal stabilizing control of the silica capillary drawing is formulated on the basis of such model. An optimality system is written using variational principles. An algorithm for the numerical solution of the optimization system and the results of numerical simulation are proposed.

1. Introduction

The rapid growth of requirements for the production and improvement of optical fibers has led to describe and create a device for mathematical modeling of the processes that form a complete cycle of manufacturing optical fibers. The complete production cycle of optical fibers from raw material to finished fiber can be represented as a number of independent production stages, among which four main ones can be distinguished: extrusion of silica tubes (preforms) from silica raw materials; alloying of silica pipes; jacketing or scaling of silica pipes; fiber drawing [1,2]. The possibility of control is an important and urgent problem, since in this case we can talk not only about improving the quality of the finished product, but also about economic efficiency due to a decrease in the unjustified consumption of expensive raw materials in the event of a defect.

One type of special silica fiber is the so-called photonic crystal fiber (PCF). There are two fundamentally different technologies for obtaining PCF. The first of them, called “stack and draw”, is associated with the lining of the core with several layers of thin silica capillaries, each of which usually has a circular or 6-sided cross section [3]. The second production method is based on multiple drilling of parallel holes in a solid cylindrical silica preform [4]. The preforms obtained by one of the described methods are then subjected to high-temperature action and drawn into a photonic crystal fiber (PCF). The main quality criterion at the drawing stage is the preservation of all geometric proportions and cross-sectional shapes of the preform and fiber. Fig. 1 shows an example of PCF cross sections made by the first and second methods. The fibers (left and right) have core diameters of about 5 microns.
Since the production of PCFs is a complex process for a number of industries, it is important to imagine how a single capillary (thin hollow silica tube) is drawn out. The purpose of this research is to formulate and solve the problem of optimal stabilizing control of one of the stages of the complete production cycle, which is drawing of a silica capillary (the main unit of the PCF).

2. Statement of the optimal stabilizing control problem

Let us assume that the flow of silica melt is axisymmetric and occurs under the action of a drawing force. Let \( x \) to be an axial coordinate, \( t \) to be the time, \( L \) to be a length of the considered section of the drawing. Also, let \( V_f(t) \) to be a preform feed rate at the initial cross-section points \( x=0 \), \( V_d(t) \) to be the drawing speed at the point \( x=L \). Consider the initial geometric dimensions of the preform (external and internal radii \( R_{01}(t,x), R_{20}(t,x) \)) are known too. The general scheme of the drawing is shown in Fig. 2. The research is based on a modification of the model of silica capillary drawing, considered in [5,6]. This is a one-dimensional nonstationary isothermal model in the form of an initial-boundary value problem with a parabolic differential operator for determining the functions of the transfer velocity of silica particles \( V(t,x) \), as well as the values of the inner and outer radius of the capillary \( R_1(t,x), R_2(t,x) \). Let us linearize this model in the vicinity of a certain steady-state drawing mode. You can choose a stationary solution to the problem as such a mode. The steady-state drawing is called a programmed state during production. The linearization is possible because the actual deviations of the system states are usually insignificant. In this case, we assume that the actual states of the system can be represented in the form \( V(t,x) = \tilde{V}(x)(1+\tilde{V}(x)\tilde{V}(t,x)) \), \( R_1(t,x) = \tilde{R}_1(x)(1+\tilde{R}_1(x)\tilde{R}_1(t,x)) \), \( R_2(t,x) = \tilde{R}_2(x)(1+\tilde{R}_2(x)\tilde{R}_2(t,x)) \), where \( \tilde{V}(t,x), \tilde{R}_1(t,x), \tilde{R}_2(t,x) \) is the initial programmed state, \( V(t,x), R_1(t,x), R_2(t,x) \) are small perturbations. We obtain a differential problem with respect to small perturbations of velocity and radii \( \tilde{V}(t,x), \tilde{R}_1(t,x), \tilde{R}_2(t,x) \) in the following form:
Figure 2. General drawing of a silica capillary.

\[ \frac{dR_1}{dt} + \alpha_1(x) \frac{dV}{dx} + \alpha_2(x) R_1 + \alpha_3(x) \tilde{R}_2 + \beta_1(x) \frac{\partial \tilde{V}}{\partial x} + \beta_2(x) \tilde{V} = 0, \quad (1) \]

\[ \frac{dR_2}{dt} + \alpha_4(x) \frac{dR_2}{dx} + \alpha_5(x) \tilde{R}_2 + \alpha_6(x) \tilde{R}_1 + \beta_3(x) \frac{\partial \tilde{V}}{\partial x} + \beta_4(x) \tilde{V} = 0, \quad (2) \]

\[ \frac{\partial \tilde{V}}{\partial t} = \nu \frac{\partial^2 \tilde{V}}{\partial x^2} + \beta_5(x) \frac{\partial \tilde{V}}{\partial x} + \beta_6(x) \tilde{V} + \alpha_7(x) \frac{\partial R_1}{\partial x} + \alpha_8(x) \tilde{R}_1 + \alpha_9(x) \frac{\partial R_2}{\partial x} + \alpha_{10}(x) \tilde{R}_2. \quad (3) \]

The initial and boundary conditions are written as:

\[ \tilde{V}(0,x) = \frac{V(0,x)}{V(x)} - 1, \quad \tilde{R}_1(0,x) = \frac{R_1(0,x)}{R_1(x)} - 1, \quad \tilde{R}_2(0,x) = \frac{R_2(0,x)}{R_2(x)} - 1, \quad (4) \]

\[ \tilde{V}(L,t) = \frac{V(L,t)}{V(L)} - 1, \quad \tilde{R}_1(L,t) = \frac{R_1(L,t)}{R_1(0)} - 1, \quad \tilde{R}_2(L,t) = \frac{R_2(L,t)}{R_2(0)} - 1, \quad V(t,L) = \frac{V(t,L)}{V(L)} - 1. \quad (5) \]

So, model (1)-(5) describes the deviations of the actual values of the geometric dimensions of the fiber and the rates of silica particles transfer during the drawing process from the set programmed states. In conditions (4), (5), the functions in the numerators of fractions are the actual values of the radii and velocities. The denominators are programmed stationary values. Therefore, if the actual values exactly coincide with the programmed ones, we have a differential problem with homogeneous initial and boundary conditions, which has the only one zero solution. In practice, this would mean a complete absence of deviations of the geometrical shapes of the silica capillary and the rate of particle transfer from the reference values. However, in real production conditions this is often not the case. Note that in practice, during the implementation of the production process of drawing, such deviations may arise due to various reasons, but most often this is a geometric defect of the original preform, expressed by its discrete or distributed flaw. This can be either a point defect in the material associated
with the presence of an air bubble in silica at the stage of production, or a global violation of the
 cylindricality of the preform, manifested in its taper or the presence of screeds [7]. The shapes of
typical top material defects are shown in Fig. 3.

![figure 3](image)

**Figure 3.** Forms of typical top material defects of preforms:
(a) air bubbles, (b) streaks, (c) inclusions and contaminations, (d) top material lack

Suppose that the defect in the preform is known in advance, the size of the defect is small, so the
preform is not rejected and is suitable for drawing. It is then possible to describe the defect
mathematically as one of the initial or boundary conditions in (4), (5). For example, a defect has an
outer radius of the preform and it is described by the following boundary condition

$$\tilde{R}_2(t,0) = R_{def}(t).$$  \hfill (6)

To eliminate the defect, let us choose the drawing speed of the finished fiber as a control function:

$$\tilde{V}(t,L) = u(t).$$  \hfill (7)

We assume that there are homogeneous initial and boundary conditions (4), (5), except for those
defined by equalities (6) and (7). In this case, the purpose of the control will be to stabilize the
gemetric shape of the fiber, or in other words, to get rid of the defect on the finished fiber. (x=L). So
we describe the goal of the optimal control problem for an integral type functional

$$F(u) = \int_0^\tau \tilde{R}_2(t,L)^2 dt + d\|u(t)\|^2 \rightarrow \min, \alpha > 0,$$ \hfill (8)

Here, \(\tau\) is control time, \(\alpha\) is regularization parameter, known in literary sources as "control cost" [8-
10]. Differential problem (1) - (5), supplemented by conditions (6) and (7), describing the defect in the
geometry of the preform and the type of control action on the system, as well as the objective functional
(8), give a general formulation of the capillary drawing optimal control problem in the production of
PCF.

3. Optimal control problem solution. Results and discussion

The optimal control problem (1)-(8) is a problem with a compromise boundary control [10], as well as
with boundary observation. Objective functional (8) is convex and coercive due to the presence of the
second term in it. These properties (8) allow us to assert that a solution to the minimization problem
exists [11]. Moreover, the optimal control function will be found explicitly if we write down the
necessary optimality conditions for (1)-(8) in the form of an initial-boundary value problem for the
states \(\tilde{V}(t,x), \tilde{R}_1(t,x), \tilde{R}_2(t,x)\), as well as their dual states. We obtain the necessary optimality
conditions proceeding from the variational principles, in particular, using the variational equality

$$\left\langle F'(u_0), \partial u_0 \right\rangle = 0,$$

where \(u_0\) is minimizing element, ",\,'" is Gateaux differentiation operator, \(\partial u_0\) is
variation of an element \(u_0\). Despite a number of technical transformations similar to those carried out
by the author for the asymptotic cases in [12-15], describing the control of the drawing of a solid fiber, we obtain the general form of the optimality system:

\[
\begin{align*}
\frac{\partial R}{\partial t} + q_1(x)\frac{\partial R}{\partial x} + q_3(x)\frac{\partial R^2}{\partial x} + q_5(x)\frac{\partial \tilde{V}}{\partial x} + q_4(x)\frac{\partial R}{\partial x} + q_6(x)\frac{\partial \tilde{R}}{\partial x} + q_7(x)\frac{\partial \tilde{V}}{\partial x} + q_8(x)\frac{\partial \tilde{R}}{\partial x} &= 0, \\
\frac{\partial \tilde{V}}{\partial t} + v_1(x)\frac{\partial \tilde{V}}{\partial x} + v_3(x)\frac{\partial \tilde{R}}{\partial x} + v_5(x)\frac{\partial \tilde{V}}{\partial x} + v_6(x)\frac{\partial \tilde{R}}{\partial x} + v_7(x)\frac{\partial \tilde{V}}{\partial x} + v_8(x)\frac{\partial \tilde{R}}{\partial x} &= 0, \\
\frac{\partial \tilde{R}}{\partial t} + a_1(x)\frac{\partial \tilde{R}}{\partial x} + a_3(x)\frac{\partial \tilde{R}}{\partial x} + a_5(x)\frac{\partial \tilde{V}}{\partial x} + a_6(x)\frac{\partial \tilde{V}}{\partial x} + a_8(x)\frac{\partial \tilde{R}}{\partial x} + a_9(x)\frac{\partial \tilde{R}}{\partial x} &= 0, \\
\frac{\partial \tilde{R}}{\partial t} + \beta_1(x)p - \beta_3(x)(\tilde{V}) &= 0, \\
\frac{\partial \tilde{R}}{\partial t} + \beta_2(x)p - \beta_4(x)(\tilde{R}) &= 0.
\end{align*}
\]

The optimization system is a boundary value problem consisting of six partial differential equations, for some of which conditions are set at the initial time \( t = 0 \) (initial conditions), and for the rest were conditions at the final moment of time \( t = \tau \). Such inconsistency of the equations of the system in time is a distinctive feature of optimization systems for problems in such formulations [16-20]. Optimality conditions (9) in the form of a differential problem are convenient for use, since in this case the optimal control function is determined from its solution through the following relation:

\[
u(t) = \tilde{V}(t, L) = \frac{1}{\alpha} \left( \beta_3 q(t, L) + 3v \frac{\partial \tilde{V}}{\partial x} (t, L) \right).
\]

The numerical implementation of the optimality system has been done using the finite element method. The software implementation was carried out in Comsol Multiphysics. The inconsistency of the equations of the system in time created certain difficulties in the joint solution of the equations. Additional iterative procedures were used to overcome them. The numerical simulation results are shown in Fig. 4.

Figure 4. Estimated values of deviations of the outer and inner radii in the modes of optimal control and without it, the function of optimal control.
The calculations were carried out for the size of the defect in the geometric shape of the preform (function \( R_{\text{def}}(t) \)) in 5\%. Such choice is explained by the fact that this value is critical for production and preforms with defects exceeding this level are usually rejected. The figure shows the profiles of the deviations of the outer and inner radii \( \tilde{R}_2(t,L) \), \( \tilde{R}_1(t,L) \) in observation point \( x = L \). It is shown that deviations from the programmed value in uncontrolled drawing mode will be approximately 7\% and 2\% (green and purple colors, respectively). The propagation of the defect into the fiber and its very noticeable effect on the inner top of the capillary is easily explained. The fact is that the cross-sectional size of the fiber is only a few microns, while the same size for the preform is a few centimeters. The zone of active deformation, the so-called “necking region” (Fig. 2) is also relatively small (about 5 cm). Thus, the system is very sensitive to any perturbation, as demonstrated by model calculations. In the controlled mode, when adjusting the drawing speed according to law (10), the same initial defect can be significantly transformed. In this case, we reduce the defect value on the outer surface of the fiber from 7\% to 1.5\% on average. Of course, the transformation cannot but touch the inner top of the capillary. The calculation of the model shows that almost complete suppression of the “bulging”; defect on the outer wall will lead to an insignificant “retraction” effect on the inner wall (blue and red). The law for adjusting the drawing speed (black color) is also given here. According to it, during the first 120 s of the process after the detection of a defect, a smooth transition is required to increase the programmed speed by 20\%, then to decrease by 10\%. A smooth transition to zero is required between 100 s and 120 s, this means defect suppression.

We also present the analysis of the values of the regularization parameter \( \alpha \).

![Figure 5. Analysis of the values of the regularization parameter.](image)

The calculation showed that the effective values of the regularization parameter \( \alpha \) are in the range from 5E-9 to 1E-6. It was shown that an increase in the values \( \alpha \) to values exceeding 1E-6 is equivalent to no control. Lowering values below the 5E-9 level is also ineffective, since the solutions obtained in this case approach each other infinitely (yellow and purple). The compromise option in this calculation is values close to 1E-7. The results calculated specifically for this value are shown in Fig. 4. In this case, it is possible to significantly reduce the defect on the outer wall of the capillary, while obtaining a not so significant effect of “retraction” on the inner wall.

We present the values of the parameters adopted in the model: the length of the investigated section of the drawing, \( L, [m] \) is 0.5; preform inner radius, \( R_{10}, [m] \) is 0.007; preform outer radius, \( R_{20}, [m] \) is 0.01; silica density, \( \rho, [kg/(m^3)] \) is 2200; silica preform feed rate, \( V_f, [m/s] \) is 5E-04; drawing speed, \( V_d, [m/s] \) is 0.1; gravitational constant, \( g, [m/s^2] \) is 9.8; specific heat capacity of the melt, \( C_p, [J/(kg*K)] \) is 1500; characteristic temperature \( T_0, [K] \) is 2000; dynamic melt viscosity, obtained from
"Poise law", $[\text{Pa}^*\text{s}]$ is 4.0411E5; the difference in pressure acting on the inner and outer tops of the capillary, $p_0$ [Pa] is 0; kinematic viscosity $v=\mu/\rho$, [m$^2$/s] is 17.871; control time $\tau$, [s] is 200.

4. Brief conclusions

On the basis of the linearized model, the capillary drawing was formulated. The problem of optimal stabilizing control of the silica capillary drawing process in the production of PCF was solved. The linearized model allows calculating deviations from real geometrical dimensions and real speed regimes from programmed states. Solving optimal control problems allows to adjust one of the parameters. An example of boundary control and boundary observation is considered. The drawing speed is controlled in order to ensure the top defect of the silica preform. The obtained necessary conditions are optimal in the form of an initial-boundary value problem. The numerical implementation of the optimal control problem on the example of one of the typical production options is given.

5. References

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