Electrokinetic Flow in Microchannels with Finite Reservoir Size Effects

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Abstract. In electrokinetically-driven microfluidic applications, reservoirs are indispensable and have finite sizes. During operation processes, as the liquid level difference in reservoirs keeps changing as time elapses, the flow characteristics in a microchannel exhibit a combination of the electroosmotic flow and the time-dependent induced backpressure-driven flow. In this work, an assessment of the finite reservoir size effect on electroosmotic flows is presented theoretically and experimentally. A model is developed to describe the time-dependent electrokinetic flow with finite reservoir size effects. The theoretical analysis shows that under certain conditions the finite reservoir size effect is significant. The important parameters that describe the effect of finite reservoir size on the flow characteristics are discussed. A new concept denoted as “effective pumping period” is introduced to characterize the reservoir size effect. The proposed model clearly identifies the mechanisms of the finite-reservoir size effects and is further confirmed by using micro-PIV technique. The results of this study can be used for facilitating the design of microfluidic devices.

1. Introduction

Microfluidics refers to devices and methods for controlling and manipulating fluid flows with length scales less than one millimeter, and it has a wide spectrum of applications such as chemical analysis and biomedical diagnosis [1]. Owing to numerous advantages, electroosmotic flow (EOF) is often utilized in microfluidic devices to transport liquid solutions and to manipulate sample solutes. Fundamental understanding of the behavior of fluid flow is thus essential to successful design and optimal control of microfluidic devices.

In electrokinetic applications, such as pumping, mixing and separation, reservoirs are indispensable as the working fluid containers and channel ends. Previous theoretical analyses are based on the assumptions of infinite reservoir size and thus the zero pressure gradient along microchannel. Similarly, in most previous experimental works relevant to EOF, researchers often use different methods to avoid the induced pressure driven flow effects resulted from change of reservoirs’ fluid levels; for example the reservoir size is fabricated as big as possible. From a practical viewpoint, the reservoirs size is finite. The reservoir-size effects will affect fluid flow and thus alter the pumping efficiency in an electrokinetic system.

Few attentions have been paid on the finite reservoir size effects on EOF. Zhang et al. [2]. and Yang et al. [3] investigated on the end effects in entry regions by using analytical and numerical methods respectively. Crabtree et al. [4] studied the Laplace pressure effects on microchip injection and separation and further gave practical methods to avoid unwanted Laplace pressure-driven flow.
However, no study has been reported on systematic analysis of the effect of finite reservoir size on EOF in microchannels.

The assumption of infinite reservoirs is only an ideal condition to simplify theoretical models or simulate the conditions of large-sized reservoirs used. In reality, this assumption is not invalid, because the finite size of the reservoirs would cause a change of the liquid levels in reservoirs, and thus results in the induced back pressure. As a result, the flow in the channel is not only driven by electroosmosis, but also is affected by induced back pressure. This paper presents a theoretical model to analyze the finite-reservoir effects on EOF in a rectangular microchannel. Further, experiments are performed using a micro-PIV system to validate the theory.

2. Mathematical model

2.1. Formulation of the EOF in a rectangular microchannel

As shown in Fig. 1, a rectangular channel has a length \( l \) and a cross section \( 2w \times 2h \), and it is connected with two cylindrical reservoirs initially with same fluid level. The liquid in the microchannel is assumed an incompressible, Newtonian, symmetric electrolyte of constant density, \( \rho \), and viscosity, \( \mu \). The channel wall is uniformly charged with the zeta potential, \( \zeta \). When an external DC electric field \( E \) is applied along the axial direction of the channel, the liquid starts to move as a result of the interaction between the net charge density in the EDL and the applied electric field.

For a microchannel whose length dimension is much larger than its lateral dimensions, the flow in the channel can be regarded as fully-developed and unidirectional. Surface tension effect on liquid level in the reservoir may change the hydrostatic pressure difference \([4]\). However, since we choose identical reservoirs and the Laplace pressure effects on the reservoirs are balanced each other \([5]\). Strictly speaking, there is an initial transient stage for an EOF after an electric field is imposed. This transient process in a typical microfluidic system however is in the order of milliseconds, so we can safely neglect it when studying the normal period of electroosmotic pumping\([6]\). For low Reynolds number flow in microfluidic systems, the flow field is governed by the linear Stokes equation. Thus, we can treat the flow field as a superposition of two parts, which include the electroosmotic flow field and the pressure driven flow field due to the induced back pressure resulted form the liquid level difference in reservoirs, as shown in Fig. 2. Hence we can write the expression for the flow field as

\[
\begin{align*}
  u(y,z,t) &= u_\infty + u_p(y,z,t) \\
\end{align*}
\]

In the modeling, the electroosmotic flow field \( u_\infty \) is considered to remain unchanged, while the back pressure driven flow field \( u_p(y,z,t) \) is time-dependent because of the change of liquid level in the reservoirs with time.

![Fig. 1. Geometry of the rectangular channel.](image)
In microfluidic systems, due to very thin thickness of the EDL, the electroosmotic velocity profile inside the EDL region becomes insignificant, and thus EOF can be considered as the flow induced by a moving wall with a velocity (slip velocity) given by Smoluchowski equation [7]

$$u_{eo} = -\frac{\varepsilon_0 \varepsilon_r \zeta E}{\mu} \tag{2}$$

Using the Cartesian coordinates system in Fig. 1, with assumption of the pseudo-static pressure driven flow, the flow velocity profile is described by the well-known Poiseuille flow which in a rectangular channel is expressed as [8]

$$u_p(y, z, t) = -\frac{h^2}{2\eta} \frac{\Delta p(t)}{l} \left[ 1 - \left( \frac{z}{h} \right)^2 + d \sum_{k=1}^{\infty} \frac{(-1)^k}{\alpha_k} \cosh \left( \frac{\alpha_k y}{h} \right) \cosh \left( \frac{\alpha_k z}{h} \right) \right]$$

$$\tag{3}$$

where $\alpha_k = (2k-1)\frac{\pi}{2}$, and $k = 1, 2, \cdots$, and $\Delta p(t)$ is the time-dependent pressure induced by change of the liquid levels in the reservoirs.

Similarly, the total flow rate in the channel consists of two parts, which are electroosmotic flow rate and the back pressure-driven flow rate. Then the total flow rate can be expressed as

$$q(t) = q_0 - q_p(t) \tag{4}$$

where $q_0$ is the initial total flow rate, and is expressed as

$$q_0 = u_{eo} A_{res} = -\frac{\varepsilon_0 \varepsilon_r \zeta E}{\mu} \frac{4wh}{4} \tag{5}$$

Integrating Eq. (3) over the cross-section area of rectangular microchannel gives the volumetric flow rate in the induced pressure driven flow

$$q_p(t) = \frac{4wh^2}{3\mu l} \Delta p(t) \left[ 1 - \frac{6h}{w} \sum_{k=1}^{\infty} \frac{1}{\alpha_k} \tanh \left( \frac{\alpha_k w}{h} \right) \right] \tag{6}$$

For two identical reservoirs, the radii $r_1 = r_2 = r$, and the cross section area of the reservoirs $A = \pi r^2$. Thus, the induced pressure drop resulted from the change of fluid levels in the reservoirs is be related to the total quantity of liquid transported during the period of time zero to time $t$, and the relevant correlation can be expressed as

$$\Delta p(t) = \rho g \frac{2Q(t)}{A} \tag{7}$$

where $Q(t)$ is the total quantity of liquid transported at given time $t$ and can also be expressed as

$$Q(t) = \int_0^t q(\tau) d\tau \tag{8}$$

From Eqs. (6) and (7), we can obtain...
Introducing a parameter,

\[ C = \frac{8wh^3 \rho g}{3\mu AL} \left[ 1 - \frac{6h}{w} \sum_{k=1}^{\infty} \frac{1}{a_k^2} \tanh \left( \frac{a_k w}{h} \right) \right] \]

we can write

\[ q_p(t) = CQ(t) \] 

According to Eqs. (4), (8) and (11), we can show the expression for the total flow rate

\[ q(t) = q_0 - C \int_0^t q(\tau) d\tau \] 

Eq. (12) is an integral equation. With the initial condition of \( q(t=0) = q_0 \), we can solve it and obtain the time dependent flow rate as

\[ q(t) = q_0 \exp(-Ct) \] 

Substituting Eq. (13) into Eq. (8) yields an expression for the total quantity of liquid transported during the period of time zero to time \( t \):

\[ Q(t) = \frac{q_0}{C} \left[ 1 - \exp(-Ct) \right] \]

Substituting Eq. (14) into Eq. (7), we can show that the pressure gradient along the channel can be expressed as

\[ \frac{\Delta p(t)}{l} = \frac{2\rho q_0}{CAL} \left[ 1 - \exp(-Ct) \right] \]

2.2. Simulation results and discussion

For analysis, we choose the following parameters: the half height and half width of the rectangular channel \( w = h = 150 \mu m \), the channel length is \( l = 4 \) cm, the electrolyte solution with zeta potential \( \xi = -62.3 \) mV, the electric strength \( E = 100 \) V/cm, and the reservoir radius \( r_1 = r_2 = 1 \) mm.

Fig. 3 presents the volumetric flow rate and the pressure gradient versus time. It can be observed that both the volumetric flow rate and the pressure gradient undergo a transient process and finally reach to a steady state. We can clearly observe the transient process from initial state to steady state by viewing the evolution of the volumetric flow rate in the channel. As shown in Fig. 3, there is a flat part of the curve for volumetric flow rate. During this period of the flat part, the change of the fluid level in the two reservoirs is small and the induced pressure gradient is negligible, so the flow field is dominated by EOF. Since the velocity profile can keep a “plug flow”, during this period, we define it as the effective pumping period. After the effective pumping period, the volumetric flow rate decreases and the induced pressure gradient increases until they reach steady state.

![Fig. 3. Volumetric flow rate and induced pressure gradient versus time.](image1)

![Fig. 4. Volumetric flow rate versus time for different reservoir radii.](image2)
We also carried out parametric studies using the model developed in this work. Fig. 4 demonstrates the volumetric flow rate versus time for different reservoir radii. By increasing the cross-sectional area of the reservoirs, the period of “plug flow” (the flat part of the curve) can be prolonged. Fig. 5 shows the flow rates for different channel cross sections. Obviously, the smaller the cross section, the longer the effective pumping period, but at the cost of reduction in flow rate. Fig. 6 shows the flow rates for different channel lengths. It can be seen that the longer the channel length, the longer the effective pumping period, with the maximum flow rate unchanged.

![Fig. 5. Volumetric flow rate versus time for different channel cross sections (2w x 2h).](image)

![Fig. 6. Volumetric flow rate versus time for different channel lengths.](image)

The evolution of the velocity distribution with the time is simulated, and the simulation results are shown in Fig. 7. It is noted that the velocity field keeps the “plug-flow” during the period of effective pumping. Afterwards the flow profile begins to fall down due to the induced back pressure resulted from the difference of fluid level in the two reservoirs. In Fig. 7(d), it is observed that the flow velocity field finally reaches a steady state, showing a counterbalance between the electric field induced EOF upward in the near-wall region and the induced back pressure driven flow downward in the central region of the channel.

![Fig. 7. Time evolution of EOF velocity distributions due to finite-reservoir effects.](image)
3. Experimental study

3.1. Experimental setup and procedure
In the present work, a micro-PIV system was used for imaging the velocity field. A schematic of the measurement cell is shown in Fig. 8. The rectangular microchannel (VitroCom, ST8330) is made of borosilicate glass and with a 300×300 μm inner dimension and a 4cm length. The microchannel was embedded in a polymer holder with two cylindrical reservoirs machined in it. Radius of both reservoirs is 1 mm. The working fluid is deionized water. Prior to loading the measurement cell with the particle suspension, the cell was cleaned in an ultrasonic cleaner with deionized water. Fluorescent polystyrene particles of diameter 930nm (Duke Scientific Co.) were used. The seeding particle number concentration in the suspension was approximately 2×10^9 particles/ml.

A DC electric field was applied using platinum electrodes inserted into both reservoirs at the ends of the microchannel cell. A high-voltage power supply (PS350, Stanford Research) was used to apply 400 volts to the electrodes, so the electric field is 100V/cm. Focusing the objective lens on the mid-plane of the rectangular channel, we measured the velocity distributions at four different times: 1s, 10s, 30s, and 100s. The images obtained in measurements were evaluated with PIVview software (PivTec GmbH).

![Fig. 8. Schematic of the experimental setup.](image)

3.2. Experimental results and comparison with simulation
Micro-PIV technique utilizes tracer particles that are usually charged in liquids. As a result, the velocity field obtained is a combination of the electrophoretic velocity of the tracer particles and the flow field. To obtain the flow field, the electrophoretic component has to be subtracted from the measured particle velocity. Using a transient micro-PIV technique reported elsewhere [9], we obtain the zeta potentials of both the channel wall (-62.3 mV) and the tracer particles (-30.4 mV) in DI water. With the images processed, the flow velocity field can be obtained by removing the electrophoretic component of the tracer particles’ velocity measured by micro-PIV directly. The flow velocity distributions at the four different times are shown in Fig. 9.
Brownian motion of the tracer particles can give an explanation on the errors in the measurement. The Brownian velocity can be estimated as \[ u_B = \sqrt{\frac{2D}{\Delta t_{\text{PIV}}}} \] (16)

The diffusion coefficient \( D \) of the particles is determined by \[ D = \frac{k_B T}{(3\pi\eta d_p)} \] (17)

where \( d_p \) is the particle diameter, \( k_B \) is the Boltzmann’s constant, and \( T \) is the absolute temperature of the liquid. For the tracer particles of 930-nm diameter, the time interval is \( 0.5 \text{ ms} < \Delta t_{\text{PIV}} < 2 \text{ ms} \) and the room temperature is \( 25 \degree C \), the Brownian velocity is estimated in the range of \( 30 \mu \text{m/s} < u_B < 46 \mu \text{m/s} \).

4. Concluding remarks
We have developed a theoretical model to assess the finite reservoir size effect. The proposed model has been verified by experiment using micro-PIV technique. Several factors that affect the EOF due to the finite-reservoir effects are analyzed including the dimension of the reservoir’s cross section, the length of the microchannel, and the cross section of the microchannel. From the simulations, we can conclude that when the dimensions of a channel are fixed, the only factor that will affect the EOF is the area of the reservoir’s cross section. Moreover, we also note that the size of the channel’s cross-section significantly affects the effective pumping period. In addition, because the flow rate is proportional to the electric field strength, the flow rate can be improved by increasing the electric field strength applied.

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