SUPERSYMMETRY STUDIES AT FUTURE LINEAR $e^+e^-$ COLLIDERS

Howard Baer$^1$, Ray Munroe$^1$ and Xerxes Tata$^2$

$^1$Department of Physics, Florida State University, Tallahassee, FL 32306 USA
$^2$Department of Physics and Astronomy, University of Hawaii, Honolulu, HI 96822 USA

(May 8, 2018)

Abstract

We examine various aspects of supersymmetric particle production at linear $e^+e^-$ colliders operating at a centre of mass energy of $\sqrt{s} = 500$ GeV, and integrated luminosity of $\int L dt = 20 - 50$ fb$^{-1}$. Working within the framework of the minimal supergravity model with gauge coupling unification and radiative electroweak symmetry breaking (SUGRA), we study various signatures for detection of sparticles, taking into account their cascade decays, and map out the regions of parameter space where these are observable. We also examine strategies to isolate different SUSY processes from another. In addition, we perform four detailed SUGRA case studies and examine the detectability of sparticles when several SUSY processes are simultaneously occurring. We show that precision mass measurements of neutralinos, sneutrinos and top-squarks are possible, in addition to previously studied precision mass measurements of sleptons and charginos.
I. INTRODUCTION

The realization that weak scale supersymmetry (SUSY) can stabilize the symmetry breaking sector of the Standard Model (SM) has made the search for supersymmetric particles \([1,2]\) one of the standard items for experiments at high energy colliders. Experiments at the CERN LEP2 \(e^+e^-\) collider will soon probe the existence of charginos, sleptons, squarks and even the lightest of the Higgs scalars of supersymmetric models if they are lighter than about \(\simeq 80 - 95\) GeV \([3]\). Direct searches for gluinos and squarks lighter than about 300 GeV will be carried out at the Fermilab Tevatron Main Injector \(p\bar{p}\) collider, which should commence operation towards the end of the century. Assuming the unification of gaugino masses, experiments at the Main Injector may have their greatest reach for supersymmetry (SUSY) via the clean trilepton channel from \(\tilde{W}_1\tilde{Z}_2 \rightarrow 3\ell\) production, and ought to be able to probe chargino masses ranging from \(\sim 50 - 200\) GeV (corresponding to gluinos as heavy as 500-600 GeV), depending on the values of model parameters \([4]\). The supersymmetry reach of Tevatron experiments is sensitive to the assumption of \(R\)-parity conservation and may be significantly larger (smaller) if the lightest supersymmetric particle (LSP) decays via \(e\) or \(\mu\) (baryon number) violating interactions \([2]\). It thus appears that while LEP2 or the Main Injector could well find a SUSY signal, a decisive exploration of the existence of weak scale supersymmetry would require a direct investigation of the TeV scale. This is possible, for instance, at the CERN Large Hadron Collider (LHC) which can probe gluino and squark masses up to \(\sim 1300 - 2000\) GeV with just 10 fb\(^{-1}\) of integrated luminosity \([5,6]\). We will see later that electron-positron colliders operating at a centre of mass energy of \(\sim 1.5\) TeV would have a similar reach for SUSY discovery as the LHC: a SUSY signal may, of course, be discovered at a collider operating (much sooner) with a lower value of \(\sqrt{s}\).

If signals for New Physics are first found, either at the LHC or at the first phase of an \(e^+e^-\) collider operating at 300-500 GeV, the immediate task at hand would be to establish their origin. The strategy for this would be quite different at the two facilities. Focussing on supersymmetry as the origin of these signals, at the LHC, we would expect signals in various event topologies \((n\text{-jets plus } m\text{ leptons plus } E_T, \text{ with and without } b \text{ tags})\) from several sparticle production processes. It is just this complex plethora of signals that would point to their SUSY origin. In contrast, it is unlikely that (except for anticipated degeneracies, \(e.g.\) for various flavours of sleptons) several SUSY reaction thresholds will be crossed at the relatively low centre of mass energy of the first phase of the Linear Collider, so that signals may be present from just a single source, and in just a few (perhaps, only one) channels. This would, of course, greatly facilitate their interpretation; the price \(vis-a-vis\) the LHC is that only a few sparticles might be kinematically accessible. Having found indications for supersymmetry, the next step would be to sort out the super-particle masses and quantum numbers. The sparticle mass spectrum could provide clues \([7]\) about the mechanism of supersymmetry breaking, and perhaps also provide a window to the symmetries of GUT or Planck scale physics. The detailed measurement of sparticle properties may well be very difficult at a hadron collider, owing to, among other things, large backgrounds, indefinite subprocess collision energy, additional QCD radiation and the fact that, in general, several SUSY reactions may simultaneously contribute to any one signal.

Many groups have been exploring the physics capabilities of a new, high energy linear \(e^+e^-\) collider \([8-10]\) where the cleanliness of the experimental environment makes it relatively
easy to detect signals from the production and subsequent decays of new, heavy particles if sufficient luminosity is available. We will refer to this machine as the Next Linear Collider, or NLC. In its first stage the NLC would operate at a centre of mass energy of $\sqrt{s} \approx 300 - 500$ GeV, and accumulate about $20-50 \text{ fb}^{-1}$ of integrated luminosity over the first several years of operation. It is envisioned that the energy of the machine would be upgraded in stages to the TeV region. In addition, there exists the possibility of longitudinal electron beam polarization, perhaps reaching magnitudes of $\sim 90 - 95\%$ left- or right- polarization.

The SUSY discovery potential of experiments at such a facility has been the subject of many previous studies. In Ref. [11], it has been shown that using relatively simple cuts, $\tilde{W}_1 \tilde{W}_1$ can be detected above backgrounds over almost the entire kinematically accessible regions of parameter space by searching for mixed leptonic/hadronic or purely hadronic decays of the chargino pair. In Ref. [12], it has been shown that sleptons can usually be detected above background for slepton masses up to $\sim 90\%$ of the beam energy. Ref. [13] shows that with the availability of beam polarization, it should be possible to measure squark masses to about $\sim 5$ GeV, even taking their cascade decays into account: in particular, it is possible to obtain the difference between the masses of $\tilde{q}_L$ and $\tilde{q}_R$. Finally, in a pioneering paper, two very detailed case studies have been performed by Tsukanivt et. al. [14], showing that an $e^+e^-$ linear collider operating at $\sqrt{s} \sim 300-500$ GeV can make a variety of precision measurements (which test the assumptions underlying the supergravity GUT framework) of sparticle masses, spins and coupling parameters. This study makes innovative use of the capability for polarization of the electron beam. It is, however, assumed that the beam energy is adjustable so that it is sufficient to focus on signals from a single SUSY reaction, taken to be chargino or slepton pair production — this simplifies the analysis in that the parent sparticle decays directly to the LSP; i.e. there are no complicated cascades to be untangled, and further, there is no need to sort out various SUSY processes from one another. The purpose of this paper is to expand upon these studies in order to attain two broad goals: 1.) to delineate the reach of such a machine for supersymmetry and compare it to the reach of the LHC, and 2.) to ascertain to what extent precision measurements of sparticle masses are possible, even when several SUSY processes are occurring simultaneously, and cascade decays of sparticles are operative.

We work within the framework of the minimal supergravity GUT model (SUGRA) as defined in Ref. [15]. The only role of “supergravity” is to provide a rationale for the universal boundary conditions at an ultra-high scale $M_X$ which we take to be $M_{GUT}$. This model is then completely specified [1] by four SUSY parameters (in addition to SM masses and couplings). A hybrid set consisting of the common mass $m_0$ ($m_{1/2}$) for all scalars (gauginos), a common SUSY-breaking trilinear coupling $A_0$ all specified at the scale $M_X$ together with $\tan \beta$ proves to be a convenient choice. These parameters fix the masses and couplings of all sparticles. In particular, $m_A$ and the magnitude (but not the sign) of $\mu$ are fixed.

The SUGRA framework (and also a SUGRA-inspired MSSM framework without radiative electroweak symmetry breaking [16]) has been incorporated [13] into the event generator program ISAJET 7.16 [17]. All lowest order $2 \rightarrow 2$ sparticle and Higgs boson production mechanisms have been incorporated into ISAJET. These include the following processes [18] (neglecting bars over anti-particles):

$$e^+e^- \rightarrow \tilde{q}_L\tilde{q}_L, \tilde{q}_R\tilde{q}_R.$$
\[ e^+ e^- \rightarrow \tilde{\ell}_L \tilde{\ell}_L, \tilde{\ell}_R \tilde{\ell}_R, \tilde{\ell}_L \tilde{\ell}_R, \]
\[ e^+ e^- \rightarrow \tilde{\nu}_L \tilde{\nu}_L, \]
\[ e^+ e^- \rightarrow \tilde{W}_1 \tilde{W}_1, \tilde{W}_2 \tilde{W}_2, \tilde{W}_1 \tilde{W}_2, \]
\[ e^+ e^- \rightarrow \tilde{\nu}_L \tilde{\nu}_L, \tilde{\nu}_R \tilde{\nu}_R, \tilde{\nu}_L \tilde{\nu}_R, \]
\[ e^+ e^- \rightarrow ZH_1, ZH_2, H_p H_1, H_p H_2, H^+ H^- . \]

In the above, \( \ell = e, \mu \) or \( \tau \). All squarks (and also all sleptons other than stau) are taken to be \( L \) or \( R \) eigenstates except the stops, for which \( \tilde{t}_1 \tilde{t}_1, \tilde{t}_1 \tilde{t}_2 \) and \( \tilde{t}_2 \tilde{t}_2 \) (here, \( \tilde{t}_{1,2} \) being the lighter/heavier of the top squark mass eigenstates) production is included. Given a point in SUGRA space, and a collider energy, ISAJET generates all allowed production processes according to their relative cross sections. The produced sparticles or Higgs bosons are then decayed into all kinematically accessible channels, with branching fractions calculated within ISAJET. The sparticle decay cascade terminates with the (stable) lightest SUSY particle (LSP), taken to be the lightest neutralino (\( \tilde{\chi}_1 \)). Final state QCD radiation is included, as well as particle hadronization. ISAJET currently neglects spin correlations, sparticle decay matrix elements, and also, initial state photon radiation. In the above reactions, spin correlation effects are only important for chargino and neutralino pair production, while decay matrix elements are only important for three-body sparticle decays.

To facilitate investigation of polarized beam effects on signal and background cross sections \[19\], we have recently incorporated polarized beam effects into the ISAJET \( e^+ e^- \) cross sections. The degree of longitudinal beam polarization has been parametrized as

\[ P_L(e^-) = f_L - f_R, \]  
\[ f_L = \frac{n_L}{n_L + n_R} = \frac{1 + P_L}{2} \]  
and
\[ f_R = \frac{n_R}{n_L + n_R} = \frac{1 - P_L}{2} . \]

In the above, \( n_{L,R} \) are the number of left (right) polarized electrons in the beam, and \( f_{L,R} \) is the corresponding fraction. Thus, a 90\% right polarized beam would correspond to \( P_L(e^-) = -0.8 \), and a completely unpolarized beam corresponds to \( P_L(e^-) = 0 \).

In Sec. II of this paper, we make a rough scan of parameter space to delineate the regions where a signal for supersymmetry may be observed above background. We also delineate those regions where signals from different production processes can be sorted out from one other. A complete visual display of the 4+1 dimensional parameter space is of course not possible. Instead, we present results in the \( m_0 \) vs. \( m_{1/2} \) parameter plane, for a low and intermediate value of \( \tan \beta \) (\( \tan \beta = 2 \) and \( 10 \)), and for both \( \mu < 0 \) and \( \mu > 0 \). Variation of the parameter \( A_0 \) mainly leads to changes in third generation squark and slepton masses, especially for the top squarks \( \tilde{t}_1 \) and \( \tilde{t}_2 \). Hence, we do not address in detail consequences of \( A_0 \) variation, although in Sec. 3 we perform a case study where the \( \tilde{t}_1 \) becomes light enough to be accessible. A recent analysis of production and decay rates of third generation sparticles at linear \( e^+ e^- \) colliders can be found in Ref. \[20\]. In most of the work presented here, we set \( A_0 = 0 \).

Our objective in Sec. III can be viewed as a follow-up to reference \[14\]. Here, we perform a number of case studies in some detail to explore the capability of an NLC to perform
precision measurements of sparticle masses and parameters. We allow for the simultaneous production of all sparticles with cross sections and decay patterns as given by the model, and describe our attempts to extract the masses of various sparticles. We examine the following four cases, which have also been the subject of recent studies on supersymmetry at various workshops on linear colliders \[21\]. For each case, we list \((m_0, m_{1/2}, A_0, \tan \beta \text{ and } \text{sign}(\mu))\), with the mass parameters in GeV. We have taken \(m_t = 180 \text{ GeV}\) throughout this paper.

1. \((400, 200, 0, 2, -1)\) (dominantly chargino production),
2. \((100, 300, 0, 2, -1)\) (dominantly slepton production),
3. \((200, 100, 0, 2, -1)\) (mixed chargino/slepton/sneutrino production),
4. \((300, 150, -600, 2, +1)\) (includes \(\tilde{t}_1 \tilde{t}_1\) production),

For each case, we use ISAJET 7.16 to simultaneously produce all allowed SUSY particles as well as SM backgrounds.

We conclude in Sec. IV with a summary of our results and some comparisons between the NLC and the CERN LHC \(pp\) collider. Finally, in an appendix, we collect various expressions for lowest order production cross sections of SM and SUSY particles via polarized beams.

II. REACH OF THE NLC IN SUGRA PARAMETER SPACE

A. Kinematic Reach

To gain some orientation for our study of the SUSY reach of the NLC in the parameter space of the minimal SUGRA model, we begin by examining where in parameter space the various sparticle production mechanisms are kinematically accessible. The lightest SUSY Higgs boson \(H_\ell\) is very special, since in the minimal model, \(m_{H_\ell} \lesssim 120 - 130 \text{ GeV} \[22\], and so, is accessible either via \(e^+e^- \rightarrow ZH_\ell\) or via \(e^+e^- \rightarrow H_pH_\ell\) processes for all values of parameters. Furthermore, if \(|\mu|\) is large as is often the case in the SUGRA framework, its couplings to SM fermions and gauge bosons are expected to be very similar to those of a SM Higgs boson. Recently, detailed studies of the detectability of SM Higgs bosons as well as of the Higgs bosons of the minimal SUSY model at the NLC have been performed by Janot \[23\].

Even for the most difficult case, where \(m_{H_\ell} \sim M_Z\), a 5\(\sigma\) signal should be attainable within a month of running at or near design luminosity at a 500 GeV linear collider, assuming a \(b\) tagging efficiency of 50% with a rejection of 97.5% (99.9%) against \(c\bar{c}\) (light quark pairs) is achieved. Detection of \(H_\ell\) should be possible even if it decays invisibly via \(H_\ell \rightarrow \tilde{Z}_1 \tilde{Z}_1\) pairs. Thus, even if the lightest SUSY Higgs boson \(H_\ell\) eludes detection at LEP2, the Tevatron and the LHC, it should certainly be discovered at NLC. Hence, the NLC will be able to exclude the minimal SUGRA model if no \(H_\ell\) signal is seen \[24\]. However, if an \(H_\ell\) signal is the only new signal seen, it may well be difficult \[25\] to distinguish whether one has seen a SUSY or a SM Higgs boson: thus, detection of \(H_\ell\) alone will likely not be definitive evidence for supersymmetry. In the remainder of this paper, we assume the lightest SUSY Higgs boson \(H_\ell\) will be detected at least at the NLC, and so focus our attention on the detectability of super-partners.
In Fig. 1, we show the regions of the $m_0$ vs. $m_{1/2}$ parameter plane where various 2 → 2 SUSY particle processes are kinematically accessible to an $e^+e^-$ linear collider operating at $\sqrt{s} = 500$ GeV, for $\tan \beta = 2$, $A_0 = 0$ and a) $\mu < 0$ and b) $\mu > 0$. In Fig. 2, we show the corresponding results for $\tan \beta = 10$. The regions labelled by TH are excluded by various theoretical constraints [13], while regions labelled EX are excluded by experimental searches for SUSY at the LEP [26] and Fermilab Tevatron colliders as described in Ref. [15] (we do not include constraints from the LEP1.5 run). The regions below the contours labelled by a sparticle pair is where the corresponding reaction is kinematically accessible. The neutralino pair contours are an exception. Since the neutralino production cross section can be strongly suppressed by mixing angle factors, we conservatively show the regions where the production pair contours are an exception. Since the neutralino production cross section can be strongly suppressed by mixing angle factors, we conservatively show the regions where the production cross section $\sigma(\tilde{Z}_1\tilde{Z}_j) > 10$ fb ($i = 1 - 4, j = 2 - 4$). We see from Fig. 1a and b that the outermost boundary of the kinematic reach for SUSY is comprised of three contours: the $\tilde{e}_R\tilde{e}_R$ contour for low $m_0$, the $\tilde{W}_1\tilde{W}_1$ contour for large $m_0$, and a small intermediate region where the $\tilde{Z}_1\tilde{Z}_2$ reaction might be accessible. The situation is similar in Fig. 2, except that an additional sliver of parameter space may be accessible by searching for $\tilde{\tau}_1\tilde{\tau}_1$ pairs (because of $\tau_L - \tau_R$ mixing induced by the tau Yukawa coupling which increases with $\tan \beta$, the lighter of the two staus is lighter than $\tilde{e}_R$) as well; in practice, the detection efficiency is larger [4] for identifying selectron and smuon signals so the $\tau$ channel is unlikely to be relevant for the maximal reach. The smuon contours (as well as stau contours in Fig. 1) essentially overlap with the selectron contours and have not been shown. However, the detection and measurement of their properties is very important for testing slepton universality [14] and the flavour structure of the slepton sector: this could shed light [27] on the nature of physics at the GUT scale. Interesting information about the composition (gaugino versus Higgsino) of $\tilde{Z}_1$ [28] can also be obtained from a detailed study of the stau signal. For parameter space points outside these regions, SUSY particles will be accessible at NLC500 only via higher order processes [29], while inside these regions, at least one and often many SUSY particles might be produced with significant rates.

In the low $m_0$ region, we see that various slepton pair reactions—$\tilde{\ell}_R\tilde{\ell}_R$, $\tilde{e}_R\tilde{e}_R$, $\tilde{\ell}_L\tilde{\ell}_L$ and $\tilde{\nu}_L\tilde{\nu}_L$—can be accessible. The sneutrinos may or may not have visible decay products, depending on how massive they are relative to the charginos and neutralinos. Potentially, the greatest reach for $\tilde{e}_L$ is via the $\tilde{e}_R\tilde{e}_L$ production which occurs via $t$–channel neutralino exchange; the production cross section is, however, significantly smaller [18] than $\sigma(\tilde{e}_R\tilde{e}_R)$ or $\sigma(\tilde{e}_L\tilde{e}_L)$ (if both selectrons have the same mass). Smuons and staus can only be produced via s-channel processes. The regions with $m_0 \lesssim 250$ GeV are also the regions most favored by cosmological neutralino relic density constraints [30], which require $100 \lesssim m_{\tilde{\chi}} \lesssim 250$ GeV to obtain a dark matter relic density in accord with inflationary cosmological models with 1:2 mixed hot to cold dark matter.

In the large $m_0$ region, sfermions are too heavy so that this part of parameter space can best be probed by searching for chargino pairs, and possibly $\tilde{Z}_1\tilde{Z}_2$ or $\tilde{Z}_2\tilde{Z}_2$ pairs. Since $\tilde{Z}_{1,2}$ is gaugino-like for a wide range of SUGRA parameters, these neutralino pair cross sections depend significantly on $t$ or $u$ channel $\tilde{e}_L$ or $\tilde{e}_R$ exchange, so the neutralino pair production rate typically decreases with increasing $m_0$. The large $m_0$ region is difficult to accommodate cosmologically since the annihilation rate of the (stable) LSPs via sfermion exchanges is suppressed; this implies too short an age for our universe unless $\tilde{Z}_1\tilde{Z}_1$ annihilation can efficiently proceed via s-channel $Z$ or $H_\ell$ resonances [30].
In the regions with smaller \(m_0\) and \(m_{1/2}\), the higher mass chargino and neutralino states \(\tilde{W}_2, \tilde{Z}_3, \tilde{Z}_4\) may become accessible to NLC500 searches. For still lower \(m_0\) and \(m_{1/2}\), \(\tilde{t}_1\tilde{t}_1\) production and ultimately \(\tilde{q}\tilde{q}\) production can become accessible. However, for NLC500, \(m_{\tilde{q}}\) must be less than 250 GeV to be at all accessible, so that these strongly interacting states would have been seen much earlier \([4]\) at Tevatron collider experiments. Finally, we see that in the lowest \(m_0\) and \(m_{1/2}\) regions, heavy Higgs bosons such as \(H^\pm\) may become accessible to NLC searches.

By comparing Fig. 1a and b with Fig. 2a and b, we see that although the sparticle accessibility contours can change somewhat with variations in \(\tan \beta\) or \(\text{sign}(\mu)\), the overall qualitative trends are the same: NLC500 is most likely to access the non-colored, charged SUSY particle states and sneutrinos, with little hope of seeing squarks or gluinos, unless they have already been discovered at Tevatron experiments. The exception is the possibility of seeing third generation \(\tilde{t}_1\) or \(\tilde{b}_1\) squarks, which can have significantly lighter masses than the other squarks, which should be nearly mass degenerate in the minimal SUGRA model. Variation of the \(A_0\) parameter mainly affects third generation \(\tilde{t}_i, \tilde{b}_i\) and \(\tilde{\tau}_i\) masses \((i = 1\) or 2\), so that only their reach contours change appreciably with \(A_0\).

B. Results from Event Simulation

We now turn to the evaluation of the reach of the NLC by comparing signal against SM backgrounds using explicit event generation. Since beam polarization has been shown to be a useful tool, as a first step, we show in Fig. 3 various lowest order SM background cross sections, as a function of electron polarization \(P_L(e^-)\) for an unpolarized positron beam. The \(e^+e^- \to e^+e^-\) contribution includes only s-channel contributions. For unpolarized beams \((P_L(e^-) = 0)\), \(WW\) production is the dominant SM process. By tuning \(P_L(e^-)\) to \(\sim -0.9\) (95% right polarized beam), the magnitude of the \(WW\) cross section can be significantly reduced relative to other SM backgrounds, which show only a mild dependence on beam polarization. Since \(WW\) production is a major background for many new physics processes, Fig. 3 suggests that the use of (dominantly) right-handed electron beams would yield a better signal to background ratio, except for those signals whose cross sections become small when \(P_L(e^-) \sim -1\). We have not shown backgrounds from \(2 \to 3\) and \(2 \to 4\) SM processes– these can be reduced by using suitable cuts \([4]\).

Our next step is to generate explicit events for signal and background. We focus on optimizing cuts for \(e_R \bar{e}_R\) and \(\tilde{W}_1\tilde{W}_1\) production, which should give the largest reach into parameter space. We use the ISAJET toy detector ISAPLT with the following characteristics. We simulate calorimetry covering \(-4 < \eta < 4\) with cell size \(\Delta \eta \times \Delta \phi = 0.05 \times 0.05\). Energy resolution for electrons, hadrons and muons is taken to be \(\Delta E = \sqrt{.0225E + (.01E)^2}\), \(\Delta E = \sqrt{.16E + (.03E)^2}\) and \(\Delta p_T = 5 \times 10^{-4} p_T^2\) respectively. Jets are found using fixed cones of size \(R = \sqrt{\Delta \eta^2 + \Delta \phi^2} = 0.6\) using the ISAJET routine GETJET (modified for clustering on energy rather than transverse energy). Clusters with \(E > 5\) GeV and \(|\eta(\text{jet})| < 2.5\) are labeled as jets. Muons and electrons are classified as isolated if they have \(E > 5\) GeV, \(|\eta(\ell)| < 2.5\), and the visible activity within a cone of \(R = 0.5\) about the lepton direction is less than \(\text{max} \left(\frac{E}{10}, 1\text{ GeV}\right)\). Finally, \(b\)-jets are tagged with an efficiency of 50%, while \(c\)-jets are misidentified as \(b\)'s with an efficiency of 3%. Jets with one or three charged prongs are
classified as \( \tau s \) for the purpose of \( \tau \)-veto (see Sec. IIID).

The signature for \( \ell R\ell R \) production is a pair of acollinear same flavor/opposite sign leptons recoiling against \( \mathcal{E}_T \). To search for such a signal, we essentially follow the cuts of Ref. [14] and require: i) \( 5 \text{ GeV} < E(\ell) < 200 \text{ GeV} \), ii) \( 20 \text{ GeV} < E(\text{visible}) < 400 \text{ GeV} \), iii) \( |m(\ell+\ell^-) - M_Z| > 10 \text{ GeV} \), iv) \( |\cos \theta(\ell^\pm)| < 0.9 \), v) \( -Q_\ell \cos \theta_\ell < 0.75 \), vi) \( \theta_{\text{acop.}} > 30^\circ \), vii) \( \mathcal{E}_T > 25 \text{ GeV} \) and viii) \( \theta(\ell^-) \) veto events with any jet activity, where the polar angle is measured from the electron beam, and \( Q \) is the charge of the lepton. Cut iiii) eliminates backgrounds from \( e^+e^- \rightarrow ZZ, \nu\nu Z \) and \( e^+e^-Z \) production, while cuts iv) and v) greatly reduce the backgrounds from \( WW \) and \( e\nu W \) production (we neglect the latter). For unpolarized beams, the resulting background level was 17 fb, while for \( P_L(e^-) = -0.9 \), the background was 2.4 fb. Thus, for the polarized case, a 5\( \sigma \) signal for 20 fb\(^{-1}\) of integrated luminosity requires a signal rate larger than 1.73 fb.

To search for chargino pairs, one may search either for 4-jet events from both \( \tilde{W}_1 \) hadronic decays, or \( 1-\ell +2\text{-jet} \) events from mixed hadronic/leptonic chargino decays. We found that either signature gives a similar reach; we focus on the mixed hadronic/leptonic signature since ultimately it is more useful for chargino mass measurements. Following Ref. [14], we require events with one lepton plus 2 jets, and i) \# of charged tracks > 5, ii) \( 20 \text{ GeV} < E(\text{visible}) < 400 \text{ GeV} \), iii) \( E(jj) > 200 \text{ GeV} \), then \( m(jj) < 68 \text{ GeV} \), iv) \( \mathcal{E}_T > 25 \text{ GeV} \), v) \( |m(\ell\nu) - M_W| > 10 \text{ GeV} \) for a \( W \)-pair hypothesis, vi) \( |\cos \theta(jj)| < 0.9 \), vii) \( |\cos \theta(\ell)| < 0.9 \), viii) \( -Q_\ell \cos \theta_\ell < 0.75 \) and \( Q_\ell \cos \theta(jj) < 0.75 \), viii) \( \theta_{\text{acop.}}(WW) > 30^\circ \) for a \( W \)-pair hypothesis. Although the dominant background from \( WW \) production is smallest for \( P_L(e^-) \) close to -1, the signal cross section also drops rapidly since the chargino is frequently an \( SU(2) \) gaugino, and so, couples only to the doublet electron. Hence, the use of left-handed electron beams is required. For \( P_L(e^-) = +0.9 \), the resultant background level was 155 fb, so that a 5\( \sigma \) signal for 20 fb\(^{-1}\) of integrated luminosity requires a signal rate larger than 14 fb.

In Fig. 4 and Fig. 5, we show the 5\( \sigma \) reach of NLC500 for minimal SUGRA via \( \ell^+\ell^- \) and \( 1-\ell +2\text{-jets} \) searches for \( \tan \beta = 2 \) and \( \tan \beta = 10 \), respectively. Here, we assume an integrated luminosity of 20 fb\(^{-1}\) and compare the reach we obtain with the kinematic reach contours of Figs. 1 and 2, shown as dashed contours for the \( \ell^+\ell^- \) and \( 1\ell +2j \) signals. In Fig. 4a alone, we compare the reach for a polarized \( e^- \) beam with the unpolarized case. The dotted curves correspond to the NLC500 reach using the above cuts with unpolarized beams, while the solid curves correspond to the reach for a 95\% polarized electron beam with dominantly left (right) handed polarization for the chargino (\( \ell_R \)) search. Notice that for \( m_0 \gtrsim 250 \text{ GeV} \), the \( \ell^+\ell^- \) signal (with unpolarized beams) from charginos is observable in between the two dotted curves; below the lowest dotted curve the chargino is rather light, and our cuts are not optimised for their selection. By comparing the dotted and solid curves, we see that there is only a marginal gain in reach using polarized beams with \( P_L(e^-) = -0.9 \) for the slepton signal (to reduce \( WW \) backgrounds), and \( P_L(e^-) = +0.9 \) for the chargino signal (to gain the largest signal cross-section). For this reason, we have chosen not to show the polarization dependence in the other frames in Fig. 4 and Fig. 5. The real power of polarization is for precision measurements of masses and couplings [14][31]. We note that the electron search contours fill most of the region of slepton accessibility, except for a small region around \( (m_0, m_{1/2}) = (100,500) \text{ GeV} \), where \( m_{\tilde{e}_R} \simeq m_{\tilde{\tau}_1} \), and the two leptons have very little visible energy. The \( \ell +2\text{-jets} \) signal from chargino pair production can likewise be seen almost up to the kinematic limit over much of the parameter space;
the exception is around \((m_0, m_{1/2}) = (250, 275)\) GeV, where the \(\tilde{\nu}_L\) becomes light enough to cause a significant drop in the chargino pair production total cross section. For even lower values of \(m_0\), the region of detectable charginos falls off due to rising (diminishing) chargino leptonic (hadronic) branching fractions, as sleptons become very light, in which case a signal may be observable in the acollinear \(e\mu + E_T\) channel.

Some additional reach may be gained by looking for \(\tilde{Z}_1\tilde{Z}_2\) production around \((m_0, m_{1/2}) = (250, 320)\) GeV. For the \(\tan\beta = 2\) cases, \(B(\tilde{Z}_2 \rightarrow \tilde{Z}_1 H_{\ell})\) is almost 100\%, so the \(\tilde{Z}_1\tilde{Z}_2\) signature is \(b\bar{b} + E_T\). The physics background consists of \(ZZ\) and \(ZH_{\ell}\) production, which occurs at a much larger rate. For the \(\tan\beta = 10\) cases, \(\tilde{Z}_2 \rightarrow \tilde{Z}_1 Z\) is comparable to \(\tilde{Z}_2 \rightarrow \tilde{Z}_1 H_{\ell}\), so one can also look for \(Z + E_T\) events, where \(Z \rightarrow \ell^+\ell^-\). The \(Z + E_T\) signals also have large backgrounds from \(WW\) production. To determine the viability of the \(\tilde{Z}_1\tilde{Z}_2\) signal in this region, we examined the point \((m_0, m_{1/2}, A_0, \tan\beta, \text{sgn}(\mu)) = (240, 340, 0, 2, +1)\) in parameter space. In our simulation we used \(P_L(e^-) = +0.9\), and assumed 20 \(\text{fb}^{-1}\) of integrated luminosity. To isolate the \(\tilde{Z}_1\tilde{Z}_2\) signal, we require events with 2 tagged \(b\)-jets, \(E_T > 25\) GeV and \(30^\circ < \Delta\phi_{E_T} < 150^\circ\). We also require that the missing mass \(m_H > 340\) GeV since for this case, the two main backgrounds ideally have \(m_H = M_Z\), up to considerable smearing corrections; in contrast, the SUSY signal requires \(m_H > 2m_{\tilde{Z}_1}\). For the particular point we considered for our simulation, we found a total \(\tilde{Z}_1\tilde{Z}_2\) cross section of 21 \(\text{fb}\), with a signal efficiency of 6\%. No SUSY or SM background events were found. We thus conclude that a left-polarized cross section of \(\sigma(\tilde{Z}_1\tilde{Z}_2) \approx 10\) \(\text{fb}\) is needed to achieve a \(\sim 10\) event signal with 20 \(\text{fb}^{-1}\) of data. We show in Figs. 4 and 5 the 10 \(\text{fb}\) contour labelled \(\tilde{Z}_1\tilde{Z}_2\), which represents the rough reach in parameter space for the \(\tilde{Z}_1\tilde{Z}_2\) signal, assuming that \(\tilde{Z}_2 \rightarrow Z H_{\ell}\) is the dominant decay of \(\tilde{Z}_2\) and that the detection efficiency varies slowly in this region of parameter space.

III. SPARTICLE MASS MEASUREMENTS AT LINEAR COLLIDERS: FOUR CASE STUDIES

Our task in this Section is to go beyond the detectability of new particles, and to examine prospects for the determination of their properties. Towards this end, we perform four detailed case studies where we attempt to isolate the different SUSY production processes from one another in order to facilitate the interpretation of our results. We focus here on precision mass measurements, although certainly a wide range of other measurements such as spin quantum numbers, total cross sections, sparticle branching fractions etc. are possible [14]. Such measurements can serve as tests of the underlying framework (the minimal SUGRA model, in our case), and perhaps even help to determine some of its fundamental parameters.

Since SUSY particle decays always terminate in the LSP \(\tilde{Z}_1\), a direct reconstruction of SUSY particle masses via “mass bumps” is not possible. However, the cleanliness of \(e^+e^-\) scattering events, combined with the well-defined initial state, leads to kinematic restrictions which depend directly on sparticle masses. For instance, in the reaction \(e^+e^- \rightarrow p_1 + p_2\), followed by \(p_2 \rightarrow p_3 + p_4\), the energy of particle \(p_3\) is restricted to lie between

\[
\gamma(E_3^* - \beta p_3^*) \leq E_3 \leq \gamma(E_3^* + \beta p_3^*),
\]

(3.1)
where \( E_3^* = (m_2^2 + m_3^2 - m_4^2)/2m_2, \) \( p_3^* = \sqrt{E_3^{*2} - m_3^2}, \) \( \gamma = E_2/m_2, \beta = \sqrt{1 - 1/\gamma^2} \) and \( E_2 = (s + m_2^2 - m_3^2)/2\sqrt{s}, \) up to corrections from energy mis-measurements, particle losses, bremsstrahlung, etc. We will see below that this formula provides a simple yet clean way for the determination of slepton and LSP (or sneutrino and chargino) masses [14] and, with appropriate analysis, also of the chargino mass when the chargino decays via \( W_1 \rightarrow f\bar{f}Z_1 \) (see Sec. IIID).

A. Case 1: Dominantly chargino production

The first case that we examine corresponds to the SUGRA point \((m_0, m_{1/2}, A_0, \tan \beta, sgn(\mu)) = (400, 200, 0, 2, -1), \) (where parameters with mass dimensions are in GeV) whose locus in the \( m_0 \) vs. \( m_{1/2} \) plane is labelled “1” in Fig. 4a. We show in Fig. 6 the total cross sections for accessible 2 \( \rightarrow \) 2 SUSY particle reactions as a function of beam polarization \( P_L(e^-) \). These may be compared directly to various SM cross sections shown in Fig. 3. We also plot the masses of the accessible SUSY particles to help orient the reader. For this case, \( m_{Hi} = 85 \) GeV, so it would likely have been already discovered at LEP2. The \( ZH \) cross section only varies by 50% over the range of \( P_L(e^-) \). The dominant SUSY process is \( W_1\bar{W}_1 \) production \((m_{\bar{W}_1} = 175 \) GeV\) and, because the chargino is essentially an \( SU(2) \) gaugino, its cross section drops rapidly as \( P_L(e^-) \rightarrow -1 \). At high energy \((\sqrt{s} \gg M_Z)\), we may think of just the neutral \( SU(2) \) vector boson exchange contributing in the \( s \)-channel, so that \( s \)-channel amplitudes for right-handed electrons are suppressed; since the sneutrino exchange amplitude always involves just left-handed electrons, the polarization dependence of this cross section is readily understood. The same reasoning explains the behaviour of the \( \tilde{Z}_2\tilde{Z}_2 \) cross section. In the limit that \( \tilde{Z}_1 \) is the bino, the \( t \)-channel selectron amplitude dominates \( \tilde{Z}_1\tilde{Z}_1 \) (and \( \tilde{Z}_1\tilde{Z}_2 \) ) production. The polarization dependence of \( \sigma(\tilde{Z}_1\tilde{Z}_1) \) is readily understood once we recognize that the cross section varies as \( Y^4 \), where \( Y \) is the hypercharge of the selectron exchanged in the \( t \)-channel. Finally, because \( \tilde{Z}_2 \) has suppressed hypercharge gauge couplings, the polarization dependence of \( \sigma(\tilde{Z}_1\tilde{Z}_2) \) follows that of \( \sigma(\tilde{Z}_2\tilde{Z}_2) \).

In this scenario, \( \bar{W}_1 \rightarrow W\tilde{Z}_1 \) with nearly 100% branching ratio, so the \( \bar{W}_1W_1 \) signal should be easily seen above the 5\( \sigma \) level of Sec. 2 in either the 4-jet or 1\( \ell \)+2-jets mode. However, to extract a chargino mass, a clean event sample is needed, and further discrimination of signal from SM (mainly WW) background is necessary. We focus here on the 1\( \ell \)+2-jets signal, for which mass measurements are relatively straightforward. We use unpolarized beams and assume \( \int L dt = 20 \) fb\(^{-1} \). The missing mass, defined by \( \eta_k = \sqrt{E^2 - p^2} \) provides a powerful discriminator. For \( \bar{W}_1W_1 \) production, \( \eta_k \) is constrained to be \( \eta_k > 2m_{\tilde{Z}_1} = 172 \) GeV, while \( WW \) production has no such constraint. We show in Fig. 7a the \( \eta_k \) distribution for both signal and background. In this case, a rather clean SUSY signal can be obtained by requiring \( \eta_k > 240 \) GeV. The distribution of surviving events is plotted as a function of dijet energy \( E_{jj} \) in Fig. 7b. The background level is indicated by the histogram, while the signal cross section is shown by the points with error bars. In this case, \( E_{jj} \simeq E_W \) from the \( \bar{W}_1 \rightarrow W\tilde{Z}_1 \) decay, so the endpoint structure of a 2-body decay discussed above should apply. The tips of the arrows indicate the the theoretically expected endpoints obtained using Eq. (3.1). The distribution has significant smearing (particularly at the low end) due to our calorimeter simulation and use of the cone algorithm for jet finding (which entails
some loss from energy outside the cone), and may well be improved with different jet finding schemes, or by plotting all visible energy aside from the detected lepton. Since Tsukamoto et al. [4] have already shown that a fit to the $E_{jj}$ distribution leads to a mass measurement of $m_{\tilde{W}_1}$ to $\sim 5\%$, we have not made any attempt to improve our jet algorithm and repeat this same analysis here.

Instead we focus on the other interesting possibility which is to isolate a signal from $\tilde{Z}_2$ in order to measure its mass. For our case, $B(\tilde{Z}_2 \rightarrow \tilde{Z}_1 H_\ell) = 99.6\%$, so that $\tilde{Z}_1 \tilde{Z}_2$ production almost exclusively results in $b\bar{b} + E_T$ events. The physics background here is mainly due to $ZZ$ and $ZH_\ell$ production. In this case, we use $P_L(e^-) = +0.9$, and assume 50 fb$^{-1}$ of integrated luminosity. To isolate the $\tilde{Z}_1 \tilde{Z}_2$ signal, we require events with two tagged $b$-jets, $E_T > 25$ GeV and $30^\circ < \Delta \phi_{b\bar{b}} < 150^\circ$. At this point, we can proceed with a plot of $m_k$, which is shown in Fig. 8a. For this case, the two main backgrounds have $m_k = M_Z$, up to considerable smearing corrections, while the SUSY signal requires $m_k > 2m_{\tilde{Z}_1} = 172$ GeV again. In fact, the kinematic endpoints for $m_k$ in the SUSY case are easily calculable, since $m_k = \sqrt{s} - \sqrt{p_T^2} = \sqrt{s - 2sE_{H_{\ell}} + m_{H_{\ell}}^2}$ (with $E_{H_{\ell}}$ bounded as in Eq. (3.1)), and are indicated on the plot. A clean separation between signal and background can be obtained by requiring $m_k > 300$ GeV. If the $H_\ell$ mass is well measured from LEP2 or NLC, then these endpoints can be used to measure $m_{\tilde{Z}_2}$ and $m_{\tilde{Z}_1}$. The dijet invariant mass from $H_\ell \rightarrow b\bar{b}$ may be checked for consistency with a Higgs boson mass hypothesis. We show its distribution in Fig. 8b after the $m_k > 300$ GeV cut. It peaks somewhat below $m_{H_\ell}$ due to imperfect jet reconstruction, energy mismeasurement and energy loss due to neutrinos. Finally, $E_{H_{\ell}} = E_{b\bar{b}}$ may also be used for a determination of $m_{\tilde{Z}_1}$ and $m_{\tilde{Z}_2}$. We show the dijet energy distribution in Fig. 8c along with the expected background (solid histogram). The tips of the arrows denote the theoretically expected end points. Once again, we see that there is considerable smearing at the lower end.

The missing mass distribution in Fig. 8a appears best suited for a mass measurement because missing energy from misimmeasurements or calorimeter losses cancels out to some degree in constructing $m_k$. We perform a fit to the $m_k$ distributions, which depends on $m_{\tilde{Z}_2}$ and $m_{\tilde{Z}_1}$ (assuming $m_{H_\ell}$ is known from LEP2 or NLC), and plot the resulting $\Delta \chi^2 = \chi^2 - \chi^2_{\text{min}} = 2.3$ and 4.6 contours (these correspond to $68\%$, i.e. “1\sigma” and 90% CL error ellipses) for a 50 fb$^{-1}$ “data” sample. The result is shown in Fig. 9a, from which we see that $m_{\tilde{Z}_2} = 176.5 \pm 4$ GeV, and $m_{\tilde{Z}_1} = 86.1 \pm 3$ GeV (1\sigma error bars), to be compared with the input values of $m_{\tilde{Z}_2} = 175.2$ GeV and $m_{\tilde{Z}_1} = 85.9$ GeV. The “data” (points) and the best fit (histogram) are shown below in Fig. 9b.

B. Case 2: Dominantly selectron production

To contrast with Case 1, we now consider a set of parameters for which selectron pair production dominates, and pair production of charginos is kinematically forbidden at NLC500. For Case 2, we choose $(m_0, m_{1/2}, A_0, \tan \beta, \text{sgn}(\mu)) = (100, 300, 0, 2, -1)$. The cross sections for accessible processes are shown versus $P_L(e^-)$ in Fig. 10 together with relevant sparticle masses. The polarization dependence of the slepton pair production cross sections is straightforward to understand — $\ell_R$ pair production is maximized for a right-handed electron beam because it occurs essentially due to hypercharge gauge boson exchange, and $Y(\ell_R) = 2Y(\ell_L)$
(for \( \tilde{e}_R \tilde{e}_R \) production, this effect is further accentuated by the fact that the \( t \)-channel exchange amplitude vanishes for \( P_L(e^-) = 1 \)). The production of \( \tilde{e}_L \tilde{e}_R \) pairs, which occurs only via \( t \)-channel exchange, is independent of beam polarization. Finally, the production of left slepton (or sneutrino) pairs shows the opposite dependence as \( \tilde{\ell}_R \) pair production, but it should be kept in mind that these have both hypercharge and \( SU(2) \) couplings so that the analysis for this reaction is not as simple.

For the parameter choice in Case 2, \( m_{\mu_R} = 93 \) GeV and is very near the limit for observability at LEP2. We see from Fig. 10 that \( \tilde{e}_R \tilde{e}_R \) is the dominant SUSY particle cross section over most of the range of \( P_L(e^-) \). \( \tilde{\nu}_L \tilde{\nu}_L \) production also occurs, but since \( \tilde{\nu}_L \rightarrow \tilde{Z}_1 \tilde{\nu}_L \), this process is invisible. \( \tilde{e}_R \tilde{e}_L \) also occurs at a large rate and can provide an opportunity for \( \tilde{e}_L \) mass measurement (\( \sigma(\tilde{e}_L \tilde{e}_L) \) is an order of magnitude smaller). The cross sections for \( \tilde{\ell}_R \tilde{\ell}_R \) (\( \ell_L \tilde{\ell}_L \)) production shown in the figure are summed over \( \tilde{\mu} \) and \( \tilde{\tau} \), and are smaller than the corresponding selectron production cross sections because there is no \( t \)-channel contribution in their case. Finally, we note that \( \tilde{\ell}_R \rightarrow \ell \tilde{Z}_1 \) and \( \ell_L \rightarrow \ell \tilde{Z}_1 \) are the only allowed two body decays of sleptons, so that the analysis of slepton production which is signalled by like flavour, opposite sign acollinear lepton pairs, is free from complications from cascade decays.

Observation of a signal in the acollinear \( e^+e^- + \not{E_T} \) or \( \mu^+\mu^- + \not{E_T} \) channels without an accompanying signal in the \( e^+\mu^- \) channel would suggest a slepton hypothesis. Focussing, for the moment on the dimuon channel, we can select out \( \tilde{\mu}_R \tilde{\mu}_R \) production by adjusting the beam polarization. Operating with \( P_L(e^-) = -0.9 \) reduces the \( WW \rightarrow \mu\mu \) background to tiny levels, while enhancing production of right sleptons \( \tilde{\mu}_R \). Turning to prospects for the smuon mass measurement, we show in Fig. 11a the \( E_\mu \) distribution in dimuon events after the dilepton cuts of Sec. 2, for an integrated luminosity of 20 fb\(^{-1} \). The background level is denoted by the dashed histogram while the arrows denote the theoretical endpoints. In Ref. \( [14] \), it was shown a mass measurement of \( m_{\tilde{\mu}_R} \) and \( m_{\tilde{Z}_1} \) can be made to \( \sim \pm 1\% \), so we do not repeat the \( \chi^2 \) analysis here.

In Fig. 11b, we show the \( E_{e^+} \) distribution for the same luminosity and polarization. This distribution consists of two components: the solid histogram is from \( \tilde{e}_R \tilde{e}_R \) production, while the dashed histogram is from \( \tilde{\mu}_R \tilde{\mu}_R \) production. (Note: with \( P_L(e^-) = -0.9 \), \( \tilde{\mu}_R \tilde{\mu}_R \) is produced at a much higher rate than \( \tilde{e}_R \tilde{e}_L \).) Since \( m_{e_L} = 238 \) GeV here, in contrast to \( m_{e_R} = 157 \) GeV, the endpoints for the two components differ significantly. Now, because we already know \( m_{\tilde{Z}_1} \) from Fig. 11a, the upper endpoint will yield \( m_{\tilde{\ell}_L} \) to a similar degree of precision (a one-parameter fit may be made). The energy distribution of \( e^- \) is shown in Fig. 11c, again for both \( \tilde{e}_R \tilde{e}_R \) and \( \tilde{\mu}_R \tilde{\mu}_R \) components (notice the different scale from Fig. 11b). The endpoints of \( \tilde{e}_R \tilde{e}_L \) are completely contained within the \( \tilde{e}_R \tilde{e}_R \) distribution. A clean sample of \( \tilde{e}_R \tilde{e}_L \) events can be isolated by requiring \( E_{e^+} > 75 \) GeV from Fig. 11b. Then the subsequent \( E_{e^-} \) distribution can be plotted in Fig. 11d. These endpoints now depend on \( m_{\tilde{\ell}_L}, m_{\tilde{e}_R} \), and \( m_{\tilde{Z}_1} \), so that consistency with the previous mass measurements as well as with the assumed production mechanisms, can be checked.

The other opportunity, in this case, is to identify \( \tilde{Z}_1 \tilde{Z}_2 \) production and measure \( m_{\tilde{Z}_2} \). In this case, \( \tilde{Z}_2 \rightarrow \ell \ell \rightarrow \ell \ell \tilde{Z}_1 \) with a branching fraction of about 5%, so one can search again for acollinear dilepton pairs. SM backgrounds mainly come from \( WW \) production and we have to discriminate the \( \tilde{Z}_2 \) signal from \( \ell \ell \) production processes. We focus here on the dimuon signature, so we can ignore background from \( \tilde{e}_L \tilde{e}_R \) production. In this case,
the $\bar{\mu}_R$ and $\bar{Z}_1$ masses are already well measured. We run with $P_L(e^-) = +0.9$, for a 50 fb$^{-1}$ data sample. To eliminate the $WW$ background, we require $n_k > 250$ GeV, and $\Delta \phi(\mu \mu) < 90^\circ$. In the remaining event sample, the muons from $\bar{\mu}_L$ decay are very hard, so we require $E_\mu(fast) > 75$ GeV, and plot the $E_\mu(slow)$ distribution, which is shown in Fig. 12. A small remaining background from $\bar{\mu}_L\bar{\mu}_L$ production populates the $E_\mu(slow) > 60$ GeV region, while the residual SM background populates the $E_\mu(slow) < 20$ GeV region. The $\bar{Z}_1\bar{Z}_2$ signal gives a distinct upper endpoint, which depends on $\bar{Z}_1$, $\bar{Z}_2$ and $\bar{\mu}_L$ masses. By combining this information with the previous mass measurements, a constraint on $m_{\bar{Z}_2}$ can be obtained.

C. Case 3: Mixed chargino, slepton and sneutrino production

In order to study the additional complications that arise when charginos and sleptons are simultaneously accessible, we are led to consider Case 3 with $(m_0, m_{1/2}, A_0, \tan \beta, sgn(\mu)) = (200, 100, 0, 2, -1)$, for which the spectrum of sparticles and production cross sections are shown in Fig. 13. The polarization dependence of the various cross sections is as in Fig. 6 and Fig. 10 and needs no further discussion.

Clearly, many superpartners would be produced at NLC500. In this case, $m_{H_L} = 69$ GeV, so that the light Higgs boson would presumably have been discovered and studied at LEP2. As can be seen from Fig. 13, $\bar{W}_1$, $\bar{Z}_2$, $\bar{\ell}_L$, $\bar{\ell}_R$, $\bar{\nu}_L$ and possibly even $\bar{Z}_3$, $\bar{Z}_4$ and $\bar{W}_2$ should be accessible. From the cross sections shown in Fig. 13, we see that for right polarized beams, $\bar{e}_R\bar{e}_R$ is the dominant process, while for left polarized beams, in fact $\bar{\nu}_e\bar{\nu}_e$ is dominant, followed closely by $\bar{W}_1\bar{W}_1$ production. In this case, the sneutrinos and sleptons are significantly heavier than the charginos and neutralinos, so that their cascade decays need to be incorporated in the analysis. For instance, the $\bar{\nu}$ with a mass of 207 GeV, decays visibly via $\bar{\nu} \rightarrow \bar{Z}_2\nu$ (32%), and $\bar{\nu} \rightarrow \bar{W}_1\ell$ (61%), with $\bar{\nu} \rightarrow \bar{Z}_1\nu$ making up the balance. The decay pattern of $\bar{\ell}_L$ is similar although its direct decay to the LSP occurs about 20% of the time. In contrast, $\bar{\ell}_R$ which has no $SU(2)$ gauge interactions essentially always decays via $\bar{\ell}_R \rightarrow \ell\bar{Z}_1$. As can be seen from Fig. 13, there are so many SUSY processes taking place at $\sqrt{s} = 500$ GeV that it is potentially difficult to isolate one from another! The ability to tune the beam energy would certainly be desirable in this situation, so that one could sequentially study each SUSY process as one passes production threshold. With $m_{\bar{W}_1} \simeq m_{\bar{Z}_2} \simeq 96$ GeV, however, at least $\bar{W}_1\bar{W}_1$, $\bar{Z}_1\bar{Z}_2$ and $\bar{Z}_2\bar{Z}_2$ would be occurring simultaneously, even running NLC at energies as low as $\sqrt{s} \sim 300$ GeV. Incidentally, it is worth remarking that for this scenario, SUSY would certainly have been discovered \cite{1} at the Main Injector upgrade of the Tevatron, and in several channels, so that it is again reasonable for us to focus our attention on precision measurements of sparticle properties.

With unpolarized beams, one may attempt to measure $m_{\bar{W}_1}$ and $m_{\bar{Z}_1}$ via the $\bar{W}_1\bar{W}_1 \rightarrow \ell\nu\bar{Z}_1q\bar{q}$ mode. We adopt the chargino cuts of Sec. 2, but find substantial background from $\bar{\nu}_L\bar{\nu}_L$ production which distorts the $E_{jj}$ distribution. In this case, running NLC at 400 GeV, below $\bar{\nu}_L\bar{\nu}_L$ threshold allows a clean distribution of $E_{jj}$ to be made. A mass measurement of $m_{\bar{W}_1}$ and $m_{\bar{Z}_1}$ should be possible with a precision of a few percent \cite{1}.

As a second measurement, we run with $P_L(e^-) = -0.9$ in an effort to pick out a $\bar{e}_R\bar{e}_R$ signal. We run with the slepton cuts of Sec. 2, for 20 fb$^{-1}$. The resulting distribution for
A good measurement of $m \chi$ contours of $\Delta m_{b}$ corresponding to data and best fit are shown below in Fig. 16. The energy distribution of the two fastest leptons has endpoints depending on the left selectrons and smuons are mass degenerate). The energy distribution of the two fastest leptons has endpoints determined by $m_{\tilde{\nu}_{e}}$ and $m_{\tilde{W}_{1}}$. We fit an appropriate function to the expected $E_{e}$ distribution, and map out the $\chi^{2}$ values in the $m_{\tilde{\nu}_{e}}$ vs. $m_{\tilde{W}_{1}}$ plane for a data set of 20 fb$^{-1}$ at $P_{L}(e^{-}) = +0.9$. The minimum $\chi^{2}$ is shown in Fig. 15a, along with contours of $\Delta \chi^{2} = 2.3$ and 4.6 from the minimum. We obtain a measured value of $m_{\tilde{\nu}_{e}} = 207.5 \pm 2.5$ GeV and $m_{\tilde{W}_{1}} = 96.9 \pm 1.2$ GeV--a 1% measurement of these masses. The $E_{e}$ distribution from data, along with the best fit, are shown in Fig. 15b. Because our fit to the $E_{e}$ distribution had been done for the same “theory” set, we have double checked our procedure by repeating it for a nearby point in parameter space. The results are shown in Fig. 15c and 15d; we see that we obtain similar precision as in Fig. 15a.

The cascade decays of selectrons and smuons can also lead to unique event signatures from $\tilde{e}_{L}\tilde{e}_{L}$ or $\tilde{\nu}_{L}\tilde{\mu}_{L}$ production (of course, selectron production dominates). One possibility is where each $\tilde{e}_{L} \rightarrow eZ_{2} \rightarrow e + \ell\ell + \tilde{Z}_{1}$, which can give events with 6 leptons plus $E_{T}$. We focus only on this unusual channel, although similar or perhaps even better measurements can possibly be performed in other channels. We require events with six leptons, no jets and $E_{T} > 25$ GeV. We further require the two fastest leptons to be of same flavor/opposite sign, and then plot $E_{\ell}$ of the two hardest leptons. We study a 50 fb$^{-1}$ sample with $P_{L}(e^{-}) = +0.9$, and obtain a sample with 70 events, all from the signal (this implicitly assumes that the left selectrons and smuons are mass degenerate). The energy distribution of the two fastest leptons has endpoints depending on $m_{\tilde{\nu}_{e}}$ and $m_{\tilde{Z}_{2}}$. Again, by fitting this distribution, contours of $\Delta \chi^{2}$ values are calculated, with the result shown in Fig. 16a. We see that a measurement of $m_{\tilde{\nu}_{e}} = 221.6 \pm 6$ GeV and $m_{\tilde{Z}_{2}} = 94.7 \pm 6.5$ GeV is possible. The corresponding data and best fit are shown below in Fig. 16b. We note that the relationship $m_{\tilde{\nu}_{e}}^{2} - m_{\tilde{\nu}_{e}}^{2} = -m_{W}^{2} \cos 2\beta$ depends only on the $SU(2)$ gauge invariance and so is very robust. A good measurement of $m_{\tilde{\nu}_{e}}$ and $m_{\tilde{\nu}}$ can lead to a model-independent determination of the parameter $\tan \beta$. Unfortunately for the present case, $\tilde{\ell}_{L}$ and $\tilde{\nu}$ are very close in mass and a combination of these mass measurements only implies $\tan \beta > 1.8$. A better determination of $\tan \beta$ may be possible for other values of parameters.

Finally, for Case 3, we examine $\tilde{Z}_{1}\tilde{Z}_{2}$ production. We require the dilepton cuts of Sec. 2, but in addition require 2 opposite sign muons with $E_{\text{vis.}} < 200$ GeV and $\phi(\ell\ell) < 90^\circ$. We show the resulting dimuon invariant mass distribution in Fig. 17. The solid histogram denotes the “signal” and the dashed one the background. In this case, the signal consists of 68% $\tilde{Z}_{1}\tilde{Z}_{2}$ production, 21% $\tilde{\nu}\tilde{\nu}$ production and 10% $\tilde{Z}_{2}\tilde{Z}_{2}$ production. All these signal processes lead to leptonically decaying $\tilde{Z}_{2}$ plus missing energy. The $(\mu^{+}\mu^{-})$ plot has a relatively sharp upper cutoff at $m_{\tilde{Z}_{2}} - m_{\tilde{Z}_{1}}$, allowing an independent determination of $m_{\tilde{Z}_{2}}$ given the information on $m_{\tilde{Z}_{1}}$ from $\tilde{e}_{R}$ or $\tilde{W}_{1}$ production processes discussed above.
D. Case 4: Includes signals from top squark production

Up to now, we have only considered cases with \( A_0 = 0 \). The weak scale \( A \)-parameters, of course, do not vanish, and are obtained by renormalization group evolution. The \( A \)-parameters (and hence \( A_0 \)) mainly affect the phenomenology of third generation sfermions (and of gluinos, charginos and neutralinos via modifications of their decay patterns \[^{32}\]) which can be significantly lighter than their first and second generation siblings. We are thus led to consider Case 4 with \((m_0, m_{1/2}, A_0, \tan \beta, sgn(\mu)) = (300, 150, -600, 2, +1)\), chosen so as to lead to a sparticle mass spectrum which contains a light top squark with \( m_{\tilde{t}_1} = 180 \) GeV which essentially always decays via \( \tilde{t}_1 \to b \tilde{W}_1 \). The SUSY particle masses and total cross sections as a function of \( P_L(e^-) \) are shown in Fig. 18. In this case, \( m_{H_L} = 102 \) GeV and \( m_{\tilde{W}_1} = 110 \) GeV, so both are just beyond the reach of LEP2. The novel feature, not encountered previously, is the accessibility of \( \tilde{t}_1 \tilde{t}_1 \) production with a cross section of about 40 fb, independent of \( P_L(e^-) \). A stop with this mass is unlikely to be observable even at the Main Injector upgrade of the Tevatron \[^{33}\].

Charginos are the most copiously produced sparticles if the electron beam is unpolarized or its polarization is dominantly left-handed. We first consider the prospects for measuring \( \tilde{W}_1 \) and \( \tilde{Z}_1 \) masses by analyzing \( \tilde{W}_1 \tilde{W}_1' \) production, using \( P_L(e^-) = +0.9 \). The difference from the chargino mass analysis in Case 1 is that now \( \tilde{W}_1 \) decays into 3-body \( qq'\tilde{Z}_1 \) and \( \ell\nu\tilde{Z}_1 \) final states (with essentially the same branching fractions as for \( W \) decay) so that Eq. (3.1) is not applicable. To isolate the \( \tilde{W}_1 \tilde{W}_1' \) signal from SM background, we use the \( 1\ell + 2\)-jets cuts of Sec. II, and in addition require \( m_\ell \geq 240 \) GeV. The resulting distribution in \( E_{jj} \) is shown in Fig. 19a. The lower end point of this distribution depends on the jet algorithm, while the upper end point is given by Eq. (3.1) with \( m_4 = m_{jj} = 0 \). The mass analysis is thus not as straightforward as in the earlier cases. In this case, we first consider the scatter plot of \( E_{jj} \) vs. \( m_{jj} \) shown in Fig. 19b. For each value of \( m_{jj} \), Eq. (3.1) implies a definite range of \( E_{jj} \). The resulting envelope of the \( (m_{jj} \) dependent) endpoints of the \( E_{jj} \) distribution is shown as the solid contour in the Figure. The chargino signal lies almost entirely within this envelope, while some remaining background events populate the outer regions. For an integrated luminosity of 50 fb\(^{-1}\) the event sample is large enough to artificially force 2-body kinematics onto the \( \tilde{W}_1 \) decay by requiring that the dijet invariant mass \( m_{jj} \) lie within some specified bin. In our analysis, we take four bins each of width 4 GeV, centered at \( m_{jj} = 22, 26, 30 \) and 34 GeV. For each bin, the \( E_{jj} \) distribution follows the form for fitting \( m_{\tilde{W}_1} \) and \( m_{\tilde{Z}_1} \) for the forced two-body kinematics is the same as in previous cases, except that now we fit simultaneously to four different mass bins; the resulting contours of \( \Delta \chi^2 = \chi^2 - \chi^2_{\text{min}} \) are shown in Fig. 20a for a data set of 50 fb\(^{-1}\). We find that \( m_{\tilde{W}_1} = 110.6 \pm 5 \) GeV (a 5% measurement), compared to the input value of \( m_{\tilde{W}_1} = 109.8 \) GeV. Likewise, \( m_{\tilde{Z}_1} = 57.5 \pm 2.5 \) GeV, compared with the input of \( m_{\tilde{Z}_1} = 57 \) GeV. These measurements may be improved by trying different bin choices or different fitting procedures. The \( E_{jj} \) distribution and best fit to the data for the \( m_{jj} \) bin centered at 30 GeV are shown in Fig. 20b. In this mass bin, the function

\[
F(E) = N(1 + \exp((E_{\text{min}} + 21.5 - E)/3.7/\sigma_{E_{\text{min}}}))^{-1}
\times(1 + \exp((-E_{\text{max}} + 24 + E)/1.9/\sigma_{E_{\text{max}}}))^{-1}
\]
(where all energy parameters are in GeV) provides a fit to the theoretical expectation for the $E_{jj}$ distribution.

Next, we focus on $\tilde{Z}_2\tilde{Z}_2$ events with a $P_L(e^-) = +0.9$ beam. The $\tilde{Z}_2$ here decays via the three-body mode, with $B(\tilde{Z}_2 \to e\tilde{e}_L) = 4.5\%$. To obtain a clean event sample, we require four isolated leptons with no jets in each event, and require as well $B_T > 25$ GeV and $20 \text{ GeV} < E_{vis.} < 400$ GeV. We look at only $e^+e^-\mu^+\mu^-$ events, and veto any events with $m(\ell^+\ell^-) = m_Z \pm 10$ GeV. We are left with just 9 signal events for $\int dt = 50$ fb$^{-1}$, with no SM background. The distribution of the two like-flavour dilepton masses in each event is shown in Fig. 21. This dilepton invariant mass is restricted to lie between 0 and $m_{\tilde{Z}_2} - m_{\tilde{Z}_1}$, so that the highest mass value gives a lower bound on this mass difference (the fact that we have a small number of events only means that there is a significant chance that we may not find an event in the highest bin). For $m_{\tilde{Z}_2} - m_{\tilde{Z}_1} = 54$ GeV, we can combine results with the $m_{\tilde{Z}_1}$ measurement from $\tilde{W}_1\tilde{W}_1$ events to deduce that $m_{\tilde{Z}_2} \simeq 109 \pm 3.5$ GeV (where we have neglected the mismeasurement uncertainty on $m(\ell^+\ell^-)$).

Finally, we turn our attention to top-squarks. In this case, $\tilde{t}_1 \to b\tilde{W}_1$, so that stop pair production is signalled by events with two $b$-jets together with additional jets or leptons from the decay products of the charginos and $B_T$. Since the $\tilde{t}_1\tilde{t}_1$ cross section varies hardly at all with $P_L(e^-)$, we run with right polarized beams ($P_L(e^-) = -0.9$) to minimize backgrounds from WW production. We search for events with $\geq 5$ jets, with two tagged as $b$’s and no isolated leptons. We exclude hadronically decaying $\tau$s by vetoing jets with one or three charged particles as discussed in Sec. 1. This veto capability is crucial to reduce large backgrounds from top quark production, where one of the tops decays hadronically and the other decays via $t \to b\tau\nu_\tau$. We also require $m_h > 140$ GeV. For a 50 fb$^{-1}$ sample of data, we are left with a SUSY signal of 286 events, compared with SM background of 36 events. At this point, we can plot the energy distribution of the $b$-jets, which should have endpoints determined by $m_{\tilde{t}_1}$ and $m_{\tilde{W}_1}$. Again, we fit a function depending on these masses to a large sample of generated top-squark pair events, and then obtain contours of $\Delta\nu^2$ for 50 fb$^{-1}$ of data. The results are plotted in Fig. 22a while the resulting $E_b$ distribution of signal plus background is shown in Fig. 22b, along with the best fit histogram. We see that a stop mass measurement of $m_{\tilde{t}_1} = 182 \pm 12$ GeV is obtained, while $m_{\tilde{W}_1} = 113.6 \pm 8$ GeV. This measurement of $m_{\tilde{W}_1}$ is independent of the measurement from $\tilde{W}_1\tilde{W}_1$ production described above and can provide checks of the inferred sources of the signals. Since $m_{\tilde{W}_1}$ is determined to greater precision via the $\tilde{W}_1\tilde{W}_1$ production channel, the precision of the stop mass measurement can be improved by combining the two chargino mass measurements. Since we had used this parameter point also to obtain the theoretical fit to the $E_b$ distribution, we have repeated this exercise for a somewhat different input for the $\tilde{t}_1$ mass in Fig. 21c and Fig. 21d. We see that a similar precision is obtained.

IV. CONCLUDING REMARKS

Electron-positron collisions provide not only a clean facility for the discovery of supersymmetric particles but also provide a unique locale for detailed determination of their properties [3–14]. The discovery of charged sparticles is possible almost all the way up to the kinematic limit for their production unless the mass difference between the parent
particle and the LSP becomes very small, so that the visible decay products become very soft. Their main disadvantages, relative to hadron colliders, are the lower centre of mass energy and generally smaller cross sections so that considerable luminosity is needed for physics. These are balanced by the clean experimental environment, simplicity of the initial state and, at future linear colliders, the availability of longitudinally polarized beams. The differences make for complementary capabilities of hadron and $e^+e^-$ colliders.

In this paper, we have extended the pioneering work of Tsukamoto et. al. [14] and analysed what data from experiments at NLC500 might look like if supersymmetry manifests itself via the minimal SUGRA framework, and sparticles are kinematically accessible. In Sec. II, we have mapped out the discovery reach of NLC500 in the $m_0 - m_{1/2}$ plane for several sets of other parameters. We have incorporated all the cascade decays of the sparticles into our analyses. For the most part, the portion of the SUGRA parameter space that can be probed at the NLC is determined by where $\tilde{e}_R$ and $\tilde{W}_1$ signals are observable, though there is a small additional region that might be probed via signals from $e^+e^- \to \tilde{Z}_1\tilde{Z}_2$ production. While the availability of longitudinally polarized beams is extremely useful for reducing SM backgrounds (mainly from $WW$ production) as well as for isolating various sparticle reactions, we found that the beam polarization does not significantly increase the SUSY reach if we use the $5\sigma$ level as our criterion for observability.

The combined reach of NLC500 (assuming an integrated luminosity of 20 fb$^{-1}$), in the $m_0 - m_{1/2}$ plane for $A_0 = 0$, $\tan \beta = 2$ and 10 and both signs of $\mu$ is shown by the lower solid curve in Fig. 23a-d. These reach contours are not particularly sensitive to the polarizability of the electron beam. The dashed and dashed-dotted curves, respectively, denote the kinematic limits for producing $\tilde{e}_R$ and $\tilde{W}_1$ at Linear Colliders for three different choices of $\sqrt{s}$. The small region where the solid curve extends beyond the “kinematic boundaries” is the region that should be explored via the $\tilde{Z}_1\tilde{Z}_2$ channel. Finally, the upper solid curve denotes the reach of the LHC via the $E_T$ and multilepton channels (the single lepton channel yields the greatest reach over the whole space illustrated in Fig. 23) as obtained in Ref. [6]. We immediately see that as far as, and only as far as, the supersymmetry reach is concerned, the LHC reach would be comparable to that of a linear collider operating at $\sqrt{s} \sim 1500$ GeV. Of course, at this energy SM backgrounds from $2 \to 3$ and $2 \to 4$ (vector boson scattering) processes would be important and need to be analysed before drawing definite conclusions. We should mention though that NLC500 is guaranteed to find the lightest neutral Higgs boson of the minimal model, and so can exclude this framework if no signal is found. In contrast, the discovery of the Higgs boson at the LHC is very difficult, and will certainly require machine and detector performance at their design levels [5].

It might also be worth noting that sparticles cannot be much beyond the weak scale if SUSY is the new physics that stabilizes the symmetry breaking sector of the SM. Several authors [34,35] have attempted to quantify this and obtained upper bounds on sparticle masses; e.g. Anderson and Castaño [35] have argued that the most favoured region from this point of view is where $m_{1/2} \leq 200$ GeV, $m_0 \leq 200 - 300$ GeV. Interestingly, the lightest neutralino is an acceptable mixed dark matter candidate if SUGRA parameters are in this range [30]. While this region can partially be probed even at upgrades of the Tevatron [4], and certainly at NLC500, it is worth keeping in mind that fine-tuning considerations are qualitative, while the cosmological constraints can be simply evaded, for instance, by allowing a small amount of $R$-parity violation which could have no impact for collider searches.
Thus, the larger reach of the LHC or energy upgrades of the NLC provide a safety margin and may prove essential for a conclusive exploration of SUSY. Nonetheless, the capability of NLC500 for discovering the Higgs boson(s), in itself, appears to us sufficient motivation for its construction.

But the Higgs sector aside, there are many other measurements that are possible at Linear Colliders that would be very difficult or impossible at the LHC. These include the precision measurement of sparticle masses, spins and couplings which can lead to incisive tests of the underlying framework [14]. In Sec. III, we have performed four case studies to assess the prospects for measuring various sparticle masses. Our study extends the earlier analysis in that we allow for all accessible sparticle reactions, and attempt to devise strategies to measure slepton masses even when these do not decay directly to the LSP. Also, we demonstrate, for the first time, that at least for favourable values of parameters, experiments at NLC500, with an integrated luminosity of 20-50 fb$^{-1}$, should be able to obtain masses of sneutrinos, scalar top quarks and even the second lightest neutralino with a precision of a few percent. In this analysis, the availability of a polarized electron beam with 95% longitudinal polarization has been assumed. We have also identified strategies (that again make critical use of the polarization capability of the NLC) to isolate various SUSY reactions from one another. This separation facilitates the mass measurements, and also allows for the measurement of SUSY cross sections which would be a first step in the determination of sparticle couplings. While some mass measurements [36] are indeed possible at the LHC, the systematic precision spectroscopy of sparticles appears to be possible only at Linear Colliders.

As emphasized in Ref. [14], the measurement of sparticle masses will allow stringent tests of the assumptions underlying the SUGRA framework, and so provide a window to the symmetries of physics at ultra-high scales. It could be that these assumptions will ultimately prove to be incorrect. For instance, other models where SUSY is broken at a relatively low scale $\sim 10 – 100$ TeV [37] have been proposed. These models can have significantly different mass patterns and can lead to very different phenomenology [38] from what we have considered. Sparticle spectroscopy will provide guidance about the mechanism of supersymmetry breaking. On a more pragmatic note, information about chargino and neutralino masses and couplings obtained from experiments at NLC500 may prove very useful in analysing the complicated cascade decay chains that should be present in the LHC data sample. Indeed if it appears that NLC500 is due to become operational significantly after the LHC, we would advocate archiving the LHC data in a form suitable for subsequent reanalysis in light of new knowledge gained from the NLC.

In summary, we have affirmed that if sparticles are kinematically accessible at NLC500, it will not only be possible to detect the signals, but it will also be possible to measure their masses with a precision of 1-5%, even if they do not directly decay into the LSP. While electron beam polarization is not essential for SUSY discovery, it is a crucial tool [14] for these precision measurements. The complementary capabilities [39] of NLC500 and the LHC cannot be overemphasized [10]. While the LHC essentially probes the complete parameter space of the minimal SUGRA framework (mainly via signals from the cascade decays of gluinos and squarks), the reach of NLC500 (which indeed probes much of the theoretically favoured region) is somewhat smaller. Higgs boson searches, on the other hand, are much simpler at the NLC. Also, precision measurements of sparticle masses and
couplings, which are only possible at the NLC, would be very helpful in disentangling LHC signals from heavy squarks and gluinos (which may not even be accessible at NLC500). Working in tandem, experiments at these facilities may allow us to uncover the mechanism of electroweak symmetry breaking, and perhaps also provide clues about the symmetries of physics at ultra-high energy scales.

ACKNOWLEDGMENTS

We thank H. Murayama, U. Nauenberg, F. Paige and M. Peskin for discussions, and members of the NLC SUSY working group for discussions and motivation to produce this report. XT is grateful to the High Energy Physics Group at Florida State University for hospitality during his sabbatical when this study was initiated. This research was supported in part by the U. S. Department of Energy under grant numbers DE-FG-05-87ER40319 and DE-FG-03-94ER4083.

Appendix: Production Cross-sections with Polarized Beams

We present in this appendix various SM and SUSY cross sections retaining information on the polarization of the incoming beams. The notation used is that of Ref. [18]. First, we present lowest order SM cross sections for R or L polarized incoming electrons and positrons. For SM fermion and gauge boson pair-production, we have:

\[
\frac{d\sigma}{dz}(e_R \bar{e}_L \rightarrow f \bar{f}) = \frac{N_f}{4\pi} \frac{p}{E} \Phi_{f_R}^L(z)
\]

where \( z = \cos \theta \) (\( \theta \) is the angle between incoming and outgoing fermions) \( p \) and \( E \) are the momentum magnitude and energy of either final state particle, \( f = \mu, \tau, \nu_\mu, \nu_\tau, \) and \( q \), and:

\[
\Phi_{f_R}^L(z) = e^4 \left[ \frac{q_f^2}{s^2} \left( E^2(1 + z^2) + m_f^2(1 - z^2) \right) \right.
\]

\[
+ \frac{(\alpha_e \pm \beta_e)^2}{(s - M_Z^2)^2 + M_Z^4 \Gamma_Z^2} \left( [(\alpha_f^2 + \beta_f^2)(E^2 + p^2 z^2) \pm \right.
\]

\[
4\alpha_f \beta_f E p z + (\alpha_f^2 - \beta_f^2) m_f^2] \right)
\]

\[
- \frac{2(\alpha_e \pm \beta_e)(s - M_Z^2)q_f}{s[(s - M_Z^2)^2 + M_Z^4 \Gamma_Z^2]} \left( \alpha_f \left[ E^2(1 + z^2) + m_f^2(1 - z^2) \right] \pm 2\beta_f E p z \right) \]

\[
\left. \right] \left( \alpha_f \left[ E^2(1 + z^2) + m_f^2(1 - z^2) \right] \pm 2\beta_f E p z \right) \]

where \( s, t(z), \) and \( u(z) \) are the Mandelstam variables.

\[
\frac{d\sigma}{dz}(e_R \bar{e}_L \rightarrow ZZ) = \frac{d^4}{dz^4} \left[ \frac{q_f^4}{s^2} \left( E^2(1 + z^2) + m_f^2(1 - z^2) \right) \right.
\]

\[
+ \frac{(\alpha_e \pm \beta_e)^2}{(s - M_Z^2)^2 + M_Z^4 \Gamma_Z^2} \left( [(\alpha_f^2 + \beta_f^2)(E^2 + p^2 z^2) \pm \right.
\]

\[
4\alpha_f \beta_f E p z + (\alpha_f^2 - \beta_f^2) m_f^2] \right)
\]

\[
- \frac{2(\alpha_e \pm \beta_e)(s - M_Z^2)q_f}{s[(s - M_Z^2)^2 + M_Z^4 \Gamma_Z^2]} \left( \alpha_f \left[ E^2(1 + z^2) + m_f^2(1 - z^2) \right] \pm 2\beta_f E p z \right) \]

\[
\left. \right] \left( \alpha_f \left[ E^2(1 + z^2) + m_f^2(1 - z^2) \right] \pm 2\beta_f E p z \right) \]

where \( s, t(z), \) and \( u(z) \) are the Mandelstam variables.
where:
\[
\Phi_{WWR}(z) = \frac{4(\alpha_e + \beta_e)^2 \tan^2 \theta_W |D_Z|^2}{s^2} \left[U_T(z)(p^2 s + 3M_W^4) + 4M_W^2 p^2 s^2 \right]
\]
and
\[
\Phi_{WWL}(z) = \frac{U_T(z)}{s^2} \left[3 + 2(\alpha_e - \beta_e) \tan \theta_W(s - 6M_W^2)ReD_Z \right.
\]
\[+ 4(\alpha_e - \beta_e)^2 \tan^2 \theta_W (p^2 s + 3M_W^4)|D_Z|^2 + \frac{U_T(z)}{t^2(z)} + \]
\[+ 8(\alpha_e - \beta_e) \tan \theta_W M_W^2 ReD_Z + 16(\alpha_e - \beta_e)^2 \tan^2 \theta_W M_W^2 p^2 |D_Z|^2 \right]
\[+ 2 \left[1 - 2(\alpha_e - \beta_e) \tan \theta_W M_W^2 ReD_Z \right]\frac{U_T(z)}{st(z) - 2M_W^2} \left]\left]\frac{2M_W^2}{t(z)} \right] \]

where \(U_T(z) = u(z)t(z) - M_W^4\), and \(D_Z = (s - M_Z^2 + iM_Z \Gamma_Z)^{-1}\).

The expressions for lowest order MSSM Higgs Production include:

\[
\frac{d\sigma}{dz}(e_R \bar{e}_L \rightarrow ZH_l) = \frac{p^3}{16\pi \sqrt{s} \tan^2 \theta_W \cos^2 \theta_W} \frac{e^4 \sin^2(\alpha + \beta)}{s^2} \frac{(\alpha_e \pm \beta_e)^2}{(s - M_Z^2)^2 + M_Z^2 \Gamma_Z^2} \left(M_Z^2 + E_Z^2 - p^2 s^2 \right)
\]

For \(ZH_h\) production, replace \(\sin^2(\alpha + \beta)\) with \(\cos^2(\alpha + \beta)\).

\[
\frac{d\sigma}{dz}(e_R \bar{e}_L \rightarrow H_l H_p) = \frac{p^3}{16\pi \sqrt{s} \tan^2 \theta_W \cos^2 \theta_W} \frac{e^4 \cos^2(\alpha + \beta)}{s^2} \frac{(\alpha_e \pm \beta_e)^2}{(s - M_Z^2)^2 + M_Z^2 \Gamma_Z^2} (1 - z^2)
\]

For \(H_l H_p\) production, replace \(\cos^2(\alpha + \beta)\) with \(\sin^2(\alpha + \beta)\).

\[
\frac{d\sigma}{dz}(e_R \bar{e}_L \rightarrow H^+ H^-) = \frac{e^4}{4\pi} \frac{p^3}{\sqrt{s}} (1 - z^2) \left[\frac{1}{s^2} + \left(\frac{2 \sin^2 \theta_W - 1}{2 \cos \theta_W \sin \theta_W} \right)^2 \frac{(\alpha_e \pm \beta_e)^2}{(s - M_Z^2)^2 + M_Z^2 \Gamma_Z^2} \right]
\]
\[+ \frac{1}{s} \left(\frac{2 \sin^2 \theta_W - 1}{\cos \theta_W \sin \theta_W} \right) \frac{(\alpha_e \pm \beta_e)(s - M_Z^2)}{(s - M_Z^2)^2 + M_Z^2 \Gamma_Z^2} \]

For sfermion pair production (\(\tilde{f}_L \tilde{f}_R\), with \(f = \mu, \tau, \nu_\mu, \nu_\tau, u, d, c, s, b\) and \(i = L\) or \(R\)), we find:

\[
\frac{d\sigma}{dz}(e_R \bar{e}_L \rightarrow \tilde{f}_L \tilde{f}_R) = \frac{N_f \ p^3}{256\pi E^3} \Phi_{\tilde{f}_L}(z)
\]

where
where $A_{f_{L,R}} (t) = 2(\alpha_f \mp \beta_f)$. For the special case of $\bar{t}_1 \bar{t}_1$ production, we have $A_{t_1} = 2(\alpha_f - \beta_f) \cos^2 \theta_t + 2(\alpha_f + \beta_f) \sin^2 \theta_t$; for $\bar{t}_2 \bar{t}_2$ production, simply switch $\cos^2 \theta_t$ with $\sin^2 \theta_t$. Also,

$$
\frac{d\sigma}{dz}(e_R \bar{e}_L \rightarrow \bar{t}_1 \bar{t}_2) = \frac{48\pi \alpha^2 (\alpha_e \pm \beta_e)^2 \beta_t^2 \cos^2 \theta_t \sin^2 \theta_t}{s} \left[ (s - M_Z^2)^2 + M_Z^2 \Gamma_Z^2 \right] p^3 (1 - z^2)
$$

For selectron pair production, we find

$$
\frac{d\sigma}{dz}(e_R \bar{e}_L \rightarrow \tilde{e}_L \tilde{e}_L) = \frac{1}{256\pi E^3} \Phi_{\tilde{e}_L R} (z)
$$

where

$$
\Phi_{\tilde{e}_L R} (z) = \Phi_{\tilde{e}_L R} (z)
$$

and

$$
\Phi_{\tilde{e}_L L} (z) = \Phi_{\tilde{e}_L L} (z) + \sum_{i=1}^{4} \frac{\left| A^e_{\tilde{Z}_i} \right|^4 s (1 - z^2)}{2E (E - pz) - m^2_{\tilde{e}_L} + m^2_{\tilde{Z}_i}} \times \left[ 1 + \frac{(\alpha_e - \beta_e)^2 s (s - M^2_{\tilde{Z}_i})}{(s - M^2_{\tilde{Z}_i})^2 + M^2_{\tilde{Z}_i} \Gamma^2_{\tilde{Z}_i}} \right] + \sum_{i<j=1}^{4} \frac{\left| A^e_{\tilde{Z}_i} \right|^2 \left| A^e_{\tilde{Z}_j} \right|^2 s (1 - z^2)}{2E (E - pz) - m^2_{\tilde{e}_L} + m^2_{\tilde{Z}_i} [2E (E - pz) - m^2_{\tilde{e}_L} + m^2_{\tilde{Z}_j}]
$$

Similarly,

$$
\frac{d\sigma}{dz}(e_R \bar{e}_L \rightarrow \tilde{e}_R \tilde{e}_R) = \frac{1}{256\pi E^3} \Phi_{\tilde{e}_R R} (z)
$$

where

$$
\Phi_{\tilde{e}_R L} (z) = \Phi_{\tilde{e}_R L} (z)
$$

and $\Phi_{\tilde{e}_L L} (z) \rightarrow \Phi_{\tilde{e}_R R} (z)$ with the substitutions: $A^e_{\tilde{Z}_i} \rightarrow B^e_{\tilde{Z}_i}$, $m_{\tilde{e}_L} \rightarrow m_{\tilde{e}_R}$, and $(\alpha_e - \beta_e) \rightarrow (\alpha_e + \beta_e)$. Also,

$$
\frac{d\sigma}{dz}(e_R \bar{e}_L \rightarrow \tilde{e}_L \tilde{e}_R) = \frac{d\sigma}{dz}(e_R \bar{e}_R \rightarrow \tilde{e}_R \tilde{e}_L) = 0,
$$

while

$$
\frac{d\sigma}{dz}(e_R \bar{e}_L \rightarrow \tilde{e}_L \tilde{e}_R) = \frac{1}{32\pi E} \left[ \sum_{i=1}^{4} \frac{\left| A^e_{\tilde{Z}_i} \right|^2 \left| B^e_{\tilde{Z}_i} \right|^2 m^2_{\tilde{Z}_i}}{[E_{\tilde{e}_L} - pz + a_{\tilde{Z}_i}]^2} + \sum_{i<j=1}^{4} \frac{2m_{\tilde{Z}_i} m_{\tilde{Z}_j} Re(A^e_{\tilde{Z}_i} A^e_{\tilde{Z}_j} B^e_{\tilde{Z}_i} B^e_{\tilde{Z}_j})}{[E_{\tilde{e}_L} - pz + a_{\tilde{Z}_i}] [E_{\tilde{e}_L} - pz + a_{\tilde{Z}_j}]^2} \right]
$$
where $a_{\tilde{Z}_i} = \frac{m_{\tilde{Z}_i}^2 - m_{\tilde{e}_R}^2}{2E}$. Also,

$$\frac{d\sigma}{dz}(e_R \tilde{e}_R \to \tilde{e}_R \tilde{e}_L) = \frac{1}{32\pi s E} \left[ \sum_{i=1}^{4} \frac{|A_{\tilde{Z}_i}^e|^2 |B_{\tilde{Z}_i}^e|^2 m_{\tilde{Z}_i}^2}{|E_{\tilde{e}_R} - pz + a_{\tilde{Z}_i}|^2} + \sum_{i<j=1}^{4} 2m_{\tilde{Z}_i}m_{\tilde{Z}_j} \text{Re}(A_{\tilde{Z}_i}^e A_{\tilde{Z}_j}^e B_{\tilde{Z}_i}^e B_{\tilde{Z}_j}^e) \right]$$

where now $a_{\tilde{Z}_i} = \frac{m_{\tilde{Z}_i}^2 - m_{\tilde{e}_R}^2}{2E}$.

For $\tilde{\nu}_e$ pair production, we find

$$\frac{d\sigma}{dz}(e_R \tilde{e}_L \to \tilde{\nu}_e \tilde{\nu}_e) = \frac{d\sigma}{dz}(e_R \tilde{e}_L \to \tilde{\nu}_e \tilde{\nu}_\mu)$$

while

$$\frac{d\sigma}{dz}(e_L \tilde{e}_R \to \tilde{\nu}_e \tilde{\nu}_e) = \frac{p^3 E}{8\pi} (1 - z^2)$$

$$\times \left[ \frac{4e^4 (\alpha_\nu - \beta_\nu)^2 (\alpha_\nu - \beta_\nu)^2}{(s - M_{\tilde{Z}}^2)^2 + M_{\tilde{Z}}^2 \Gamma_{\tilde{Z}}^2} + \frac{g^4 \sin^4 \gamma_R}{[2E(E - pz) + m_{\tilde{e}_L}^2 - m_{\tilde{e}_R}^2]^2} \right]$$

$$\times \left[ \frac{g^4 \cos^4 \gamma_R}{[2E(E - pz) + m_{\tilde{e}_L}^2 - m_{\tilde{e}_R}^2]^2} + \frac{4e^2 g^2 (\alpha_\nu - \beta_\nu)(\alpha_\nu - \beta_\nu)(s - M_{\tilde{Z}}^2) \sin^2 \gamma_R}{[(s - M_{\tilde{Z}}^2)^2 + M_{\tilde{Z}}^2 \Gamma_{\tilde{Z}}^2][2E(E - pz) + m_{\tilde{e}_L}^2 - m_{\tilde{e}_R}^2]} \right]$$

For neutralino pair production, we find

$$\frac{d\sigma}{dz}(e_L e_L \to \tilde{Z}_i \tilde{Z}_j) = \frac{k}{8\pi s \sqrt{s}} \left( M_{\tilde{e}_e \tilde{e}_L} + M_{ZZ \tilde{L}} + M_{ZZ \tilde{L}} \right)$$

where

$$M_{\tilde{e}_e \tilde{e}_L} = 2|B_{\tilde{Z}_i}^e|^2 |B_{\tilde{Z}_j}^e|^2 G_t(m_{\tilde{Z}_i}, m_{\tilde{Z}_j}, m_{\tilde{e}_R}, z)$$

$$M_{\tilde{e}_e \tilde{e}_L} = 2|A_{\tilde{Z}_i}^e|^2 |A_{\tilde{Z}_j}^e|^2 G_t(m_{\tilde{Z}_i}, m_{\tilde{Z}_j}, m_{\tilde{e}_L}, z)$$

$$M_{ZZ \tilde{L}} = \frac{4e^2 |W_{ij}|^2 (\alpha_\nu \pm \beta_\nu)^2}{(s - M_{\tilde{Z}}^2)^2 + M_{\tilde{Z}}^2 \Gamma_{\tilde{Z}}^2} \left[ s^2 - (m_{\tilde{Z}_i}^2 - m_{\tilde{Z}_j}^2)^2 \right]$$

$$- 4(1)^{\theta_i + \theta_j} s m_{\tilde{Z}_i}^2 m_{\tilde{Z}_j}^2 + 4s k^2 z^2$$
\[ M_{\tilde{e}R} = \frac{-e(-1)^{\theta_i + \theta_j + 1}(\alpha_e + \beta_e)(s - M_{\tilde{Z}}^2)}{2[(s - M_{\tilde{Z}}^2)^2 + M_{\tilde{Z}}^2 \Gamma_{\tilde{Z}}^2]} \]

\times \left[ Re(W_{ij} B_{Zi}^e B_{Zj}^e) G_{st}(m_{\tilde{Z}_i}, m_{\tilde{Z}_j}, m_{\tilde{e}_R}, z) \\
+ (-1)^{\theta_i + \theta_j} Re(W_{ij} B_{Zi}^e B_{Zj}^e) G_{st}(m_{\tilde{Z}_i}, m_{\tilde{Z}_j}, m_{\tilde{e}_R}, -z) \right] \]

and

\[ M_{\tilde{e}L} = \frac{-e(\alpha_e - \beta_e)(s - M_{\tilde{Z}}^2)}{2[(s - M_{\tilde{Z}}^2)^2 + M_{\tilde{Z}}^2 \Gamma_{\tilde{Z}}^2]} \]

\times \left[ Re(W_{ij} A_{Zi}^e A_{Zj}^e) G_{st}(m_{\tilde{Z}_i}, m_{\tilde{Z}_j}, m_{\tilde{e}_L}, z) \\
+ (-1)^{\theta_i + \theta_j} Re(W_{ij} A_{Zi}^e A_{Zj}^e) G_{st}(m_{\tilde{Z}_i}, m_{\tilde{Z}_j}, m_{\tilde{e}_L}, -z) \right]. \]

The functions \( G_t \) and \( G_{st} \) are defined in Ref. [18].

For chargino pairs, we have

\[
\frac{d\sigma}{dz}(e_L \tilde{e}_R \rightarrow \tilde{W}_1 \tilde{W}_1) = \frac{1}{64\pi s} \frac{p}{E} \left( M_{\gamma L} + M_{ZZL} + M_{\gamma ZL} \right) + M_{\tilde{e}L} + M_{\gamma \tilde{e}L} + M_{Z\tilde{e}L}
\]

and

\[
\frac{d\sigma}{dz}(e_R \tilde{e}_L \rightarrow \tilde{W}_1 \tilde{W}_1) = \frac{1}{64\pi s} \frac{p}{E} \left( M_{\gamma R} + M_{ZZR} + M_{\gamma ZR} \right)
\]

where

\[
M_{\gamma L} = M_{\gamma R} = \frac{16e^4}{s} \left[ E^2(1 + z^2) + m_{\tilde{W}_1}^2(1 - z^2) \right]
\]

\[
M_{ZZL} = \frac{16e^4 \cot^2 \theta_W s}{(s - M_{\tilde{Z}}^2)^2 + M_{\tilde{Z}}^2 \Gamma_{\tilde{Z}}^2} \left[ (x_c^2 + y_e^2)(\alpha_e \pm \beta_e)^2[E^2(1 + z^2) + m_{\tilde{W}_1}^2(1 - z^2)] - 2y_e^2(\alpha_e \pm \beta_e)^2m_{\tilde{W}_1}^2 \mp 4x_ey_e(\alpha_e \pm \beta_e)^2Epz \right]
\]

\[
M_{\gamma ZL} = \frac{-32e^4(\alpha_e \pm \beta_e) \cot \theta_W (s - M_{\tilde{Z}}^2)}{(s - M_{\tilde{Z}}^2)^2 + M_{\tilde{Z}}^2 \Gamma_{\tilde{Z}}^2} \left[ x_e[E^2(1 + z^2) + m_{\tilde{W}_1}^2(1 - z^2)] \mp 2y_eEpz \right]
\]

\[
M_{\tilde{e}L} = \frac{2e^4 \sin^4 \gamma_R}{\sin^4 \theta_W} \frac{s(E - pz)^2}{[E^2 + p^2 - 2Epz + m_{\tilde{p}}^2]^2}
\]

\[
M_{\gamma \tilde{e}L} = \frac{-8e^4 \sin^2 \gamma_R}{\sin^2 \theta_W} \left[ \frac{(E - pz)^2 + m_{\tilde{p}}^2}{[E^2 + p^2 - 2Epz + m_{\tilde{p}}^2]} \right]
\]
and
\[
M_{Z\tilde{\nu}L} = \frac{8e^4(\alpha_e - \beta_e) \cot \theta_W \sin^2 \gamma_R s(s - M_Z^2)}{\sin^2 \theta_W (s - M_Z^2)^2 + M_Z^4 \Gamma_Z^2} \times \left[ \frac{(x - y)^2((E - pz)^2 + m_{\tilde{W}_1}^2) + 2y_m^2 m_{\tilde{W}_1}}{E^2 + p^2 - 2Epz + m_{\tilde{\nu}}^2} \right].
\]

For $\tilde{W}_2\tilde{W}_2$ production, replace $x_\nu$ with $x_\chi$, $y_\nu$ with $y_\chi$, $\sin \gamma_R$ with $\cos \gamma_R$ and $m_{\tilde{W}_1}$ with $m_{\tilde{W}_2}$. Finally,
\[
\frac{d\sigma}{dz}(e_L\tilde{e}_R \rightarrow \tilde{W}_1\tilde{W}_2) = \frac{e^4}{64\pi E} m_{Z\nu} \left[M_{Z\nu L} + M_{\nu\nu L} + M_{Z\tilde{\nu}L}\right]
\]
and
\[
\frac{d\sigma}{dz}(e_R\tilde{e}_L \rightarrow \tilde{W}_1\tilde{W}_2) = \frac{e^4}{64\pi E} m_{Z\nu} \left[M_{Z\nu L} + M_{\nu\nu L} + M_{Z\tilde{\nu}L}\right]
\]
where
\[
M_{Z\nu L} = \frac{4(\alpha_e \pm \beta_e)^2(\cot \theta_W + \tan \theta_W)^2}{(s - M_Z^2)^2 + M_Z^4 \Gamma_Z^2} \left[ (x^2 + y^2)(E^2 + p^2 z^2) - \Delta^2 - \Delta^2 - \Delta^2 + 2x^2 \xi m_{\tilde{W}_1} m_{\tilde{W}_2} \pm 4xy Epz \right]
\]
\[
M_{\nu\nu L} = \frac{2 \sin^2 \gamma_R \cos^2 \gamma_R}{\sin^4 \theta_W} \frac{[(E - pz)^2 - \Delta^2]}{[2E(E - \Delta) - 2Epz + m_{\tilde{\nu}}^2 - m_{\tilde{W}_1}^2]^2}
\]
\[
M_{Z\tilde{\nu}L} = \frac{-4\theta_y(\alpha_e - \beta_e)(\cot \theta_W + \tan \theta_W) \sin \gamma_R \cos \gamma_R (s - M_Z^2)}{\sin^2 \theta_W [(s - M_Z^2)^2 + M_Z^4 \Gamma_Z^2]]} \times \left[ \frac{(x - y)^2((E - pz)^2 - \Delta^2 - \Delta^2 - \Delta^2 - \Delta^2 + 2x^2 \xi m_{\tilde{W}_1} m_{\tilde{W}_2})}{2E(E - \Delta) - 2Epz + m_{\tilde{\nu}}^2 - m_{\tilde{W}_1}^2} \right]
\]

The final cross section can be calculated from
\[
\sigma = f_L(e^-)f_L(e^+)\sigma_{LL} + f_L(e^-)f_R(e^+)\sigma_{LR} + f_R(e^-)f_L(e^+)\sigma_{RL} + f_R(e^-)f_R(e^+)\sigma_{RR},
\]
where $f_L$ and $f_R$ are defined in Sec. I, and $\sigma_{ij}$ ($i, j = L, R$) refers to the cross section from $e_i^- e_j^+$ annihilation. The above formulae have been incorporated into the event generator ISAJET \[17\].
REFERENCES

[1] For phenomenological reviews of SUSY, see H. P. Nilles, Phys. Rep. 110, 1 (1984); H. Haber, Lectures presented at TASI92, University of Colorado, Boulder, [hep-ph/9306207]; X. Tata, in The Standard Model and Beyond, p. 304, edited by J. E. Kim, World Scientific (1991); V. Barger and R. J. N Phillips, in The Woodlands 1993 Proceedings: Recent Advances in the Superworld, MAD/PH/765 (1993); R. Arnowitt and P. Nath, Lectures presented at the VII J. A. Swieca Summer School, Campos do Jordao, Brazil, 1993 CTP-TAMU-52/93; Properties of SUSY Particles, L. Cifarelli and V. Khoze, Editors, World Scientific (1993); X. Tata, Lectures presented at TASI95, University of Colorado, Boulder, [hep-ph/9510287].

[2] For a recent phenomenological review, see H. Baer et. al., in Electroweak Symmetry Breaking and Physics Beyond the Standard Model, edited by T. Barklow, S. Dawson, H. Haber and J. Siegrist, (World Scientific, to be published), FSU-HEP-950401 [hep-ph/9503479].

[3] C. Dionisi et. al. Proceedings of ECFA Workshop on LEP 200; C. Dionisi and M. Dittmar, Proc. of the Workshop on Physics at Future Colliders, La Thuile, CERN Report 87-07 (1987); J-F. Grivaz in Properties of SUSY Particles; M. Chen, C. Dionisi, M. Martinez and X. Tata, Phys. Rep. 159, 201 (1988); J. Lopez, D. Nanopoulos, H. Pois, X. Wang and A. Zichichi, Phys. Rev. D48, 4062 (1993); H. Baer, M. Brklik, R. Munroe and X. Tata, Phys. Tev. D52, 5031 (1995); see also G. F. Giudice et. al., hep-ph/9602207 (1995) for a recent review.

[4] T. Kamon, J. Lopez, P. McIntyre and J. T. White, Phys. Rev. D50, 5676 (1994); H. Baer, C. H. Chen, C. Kao and X. Tata, Phys. Rev. D52, 1565 (1995); S. Mrenna, G. Kane, G. Kribs and J. Wells, Phys. Rev. D53, 1168 (1996); H. Baer, C. H. Chen, F. Paige and X. Tata, FSU-HEP-960415 (1996) [hep-ph/9604403].

[5] W. W. Armstrong et. al., ATLAS Technical Proposal, CERN/LHCC/94-43 (1994); G. L. Bayatian et. al., CMS Technical Proposal, CERN/LHCC/94-38 (1994).

[6] H. Baer, C. H. Chen, F. Paige and X. Tata, Phys. Rev. D52, 2746 (1995) and Phys. Rev. D53, 6241 (1996).

[7] M. Peskin, presented at the Proceedings of the Yukawa International Seminar, Kyoto, Japan, August, 1995, SLAC-PUB-7133 (1995) [hep-ph/9604333].

[8] Proceedings of Workshop on Physics and Experiments with Linear e+e- Colliders, Saariselkä, Finland, ed. by R. Orava, P. Eerola and M. Nordberg, (World Scientific, 1992)

[9] Proceedings of Workshop on Physics and Experiments with Linear e+e- Colliders, Waikaloa, Hawaii, ed. F. A. Harris, S. L. Olsen, S. Pakvasa and X. Tata, (World Scientific, 1993).

[10] Proceedings of Workshop on Physics and Experiments with Linear e+e- Colliders, Morioka-Appi, Iwate, Japan, September 1995 (to be published).

[11] J.-F. Grivaz, “Prospects for Supersymmetry Discoveries at a 500 GeV in Proceedings of Physics and Experiments with Linear Colliders, Saariselkä, Finland, Sep 9-14, 1991, ed. by R. Orava, P. Eerola, and M. Nordberg, World Scientific, Singapore, 1992.

[12] R. Becker and C. Van der Velde, “Aspects of Scalar Lepton Search”, in proceedings of European Meeting of the Working Groups on Physics and Experiments at Linear e+e- Colliders, ed. by P.M. Zerwas, DESY-93-123C.
[13] J. L. Feng and D. E. Finnell, Phys. Rev. D49, 2369 (1994).
[14] T. Tsukamoto, K. Fujii, H. Murayama, M. Yamaguchi and Y. Okada, Phys. Rev. D51, 3153 (1995); see also JLC-1, KEK Report 92-16 (1992).
[15] H. Baer, C. H. Chen, R. Munroe, F. Paige and X. Tata, Phys. Rev. D51, 1046 (1995).
[16] L. Ibáñez and G. Ross, Phys. Lett. B110, 215 (1982); L. Ibáñez, Phys. Lett. B118, 73 (1982); J. Ellis, D. Nanopoulos and K. Tamvakis, Phys. Lett. B121, 123 (1983); L. Alvarez-Gaumé, J. Polchinski and M. Wise, Nucl. Phys. B121, 495 (1983); G. Gamberini, G. Ridolfi and F. Zwirner, Nucl. Phys. B331, 331 (1990).
[17] F. Paige and S. Protopopescu, in Super collider Physics, p. 41, ed. D. Soper (World Scientific, 1986); H. Baer, F. Paige, S. Protopopescu and X. Tata, in Proceedings of the Workshop on Physics at Current Accelerators and Supercolliders, ed. J. Hewett, A. White and D. Zeppenfeld, (Argonne National Laboratory, 1993) (hep-ph/9305342).
[18] H. Baer, A. Bartl, D. Karatas, W. Majerotto and X. Tata, Int. J. Mod. Physics, A4, 4111 (1989).
[19] P. Chiapetta, J. Soffer, P. Taxil, F. Renard and P. Sorba, Nucl. Phys. B259, 365 (1985) and Nucl. Phys. B262, 495 (1985); S. Bilenki and N. Nedelcheva, Nucl. Phys. B283, 295 (1987); E. Christova and N. Nedelcheva, Int. J. Mod. Physics, A5, 2241 (1990); M. Jimbo, Tokyo Management College preprint TMCP-92-1 (1992) and Proceedings of the Nagoya Spin Workshop, 1992, p. 657 (1992).
[20] A. Bartl, H. Eberl, S. Kraml, W. Majerotto, W. Porod and A. Sopczak, UWThPh-1996-20 (1996) (hep-ph/9604221).
[21] Studies on NLC physics have been performed at workshops at Estes Park, Fermilab, SLAC and BNL.
[22] See e.g. A. Djouadi, Int. J. Mod. Phys. A10,1 (1995) for a recent review. See also M. Carena, M. Quiros and C. Wagner, Nucl. Phys. B461, 407 (1996).
[23] P. Janot, Ref. [4].
[24] The existence of a light Higgs boson is generic to all models where the Higgs sector remains perturbative up to a large scale $\sim M_{\text{GUT}}$ as pointed out by G. Kane, C. Kolda and J. Wells, Phys. Rev. Lett. 70, 2686 (1993) and J. Espinosa and M. Quiros, Phys. Lett. B302, 51 (1993). J. Kamoshita, Y. Okada and M. Tanaka, Phys.Lett. B328, 67 (1994) have argued that if the couplings of the lightest Higgs boson are strongly suppressed by mixing so that its signals are unobservable at the NLC, the second lightest Higgs boson will have a mass and coupling such that its signals will be observable.
[25] H. Haber, presented at the Fourth International Conference on Physics Beyond the Standard Model, Lake Tahoe, CA, USA, December 1994, SCIPP 95/15 (1995) (hep-ph/9505240).
[26] L. Montanet et. al., Phys. Rev D50, 1173 (1994).
[27] N. Arkani-Hamed, H.-C. Cheng and L. Hall, Berkeley preprint LBL-37894 (1996); N. Arkani-Hamed, H.-C. Cheng, J. L. Feng and L. Hall, Berkeley preprint LBL-38534 (1996).
[28] M. Nojiri, Phys. Rev. D51, 6281 (1995).
[29] M. Jimbo, T. Kon and T. Ochiai, Phys. Rev. D37, 441 (1988); T. Kon and H. Fujisaki, Europhys. Lett. 7, 101 (1988); T. Kon, Prog. Theor. Phys. 79, 159 (1988) and 79 1009 (1988); M. Jimbo and M. Katuya, Europhys. Lett. 16, 243 (1991).
[30] See, for example, M. Drees and M. Nojiri, Phys. Rev. D47, 376 (1993); G. Kane,
C. Kolda, L. Roszkowski and J. Wells, Phys. Rev. D\textbf{49}, 6173 (1994); H. Baer and M. Brhlik, Phys. Rev. D\textbf{53}, 597 (1993).

[31] J. Feng, H. Murayama, M. Peskin and X. Tata, Phys. Rev. D\textbf{52}, 1418 (1995).

[32] H. Baer, X. Tata and J. Woodside, Phys. Rev. D\textbf{45}, 142 (1992); H. Baer, M. Bisset, X. Tata and J. Woodside, Phys. Rev. D\textbf{46}, 303 (1992); M. Drees and M. Nojiri, Nucl. Phys. B\textbf{369}, 54 (1992); H. Baer, M. Drees, C. Kao, M. Nojiri and X. Tata, Phys. Rev. D\textbf{50}, 2148 (1994); A. Bartl, W. Majerotto and W. Porod, Z. Phys. C\textbf{64}, 499 (1994).

[33] S. Mrenna \textit{et. al.}, Ref. \cite{ref1}.

[34] R. Barbieri and G. Giudice, Nucl. Phys. B\textbf{306}, 63 (1988).

[35] G. Anderson and D. Castaño, Phys.Rev. D\textbf{53}, 2403 (1995).

[36] R. M. Barnett, J. Gunion and H. Haber, Phys. Lett. B\textbf{315}, 349 (1993); H. Baer, C. H. Chen, F. Paige and X. Tata, Phys. Rev. D\textbf{50}, 4508 (1994); H. Baer, C. H. Chen, F. Paige and X. Tata, Ref. \cite{ref1}.

[37] M. Dine, A. Nelson and Y. Shirman, Phys. Rev. D\textbf{51}, 1362 (1995); M. Dine, A. Nelson and Y. Nir and Y. Shirman, Phys. Rev. D\textbf{53}, 2658 (1996).

[38] S. Dimopoulos, M. Dine, S. Raby and S. Thomas, Phys. Rev. Lett. \textbf{76}, 3494 (1996); S. Dimopoulos, S. Thomas and J. D. Wells, SLAC-PUB-7148 (1996) (\texttt{hep-ph/9604452}).

[39] M. Peskin in \textit{Proc. of 22nd INS International Symposium: Physics with High Energy Colliders}, ed. S. Yamada and T. Ishii, World Scientific (1995).

[40] Our considerations here have been confined to the minimal SUGRA framework with $R$-parity conservation. The complementarity of the LHC and NLC extends well beyond this. For instance, if $R$-parity is violated by baryon number non-conserving operators, the LSP (and, perhaps also other sparticles) will have new hadronic decay modes. The $E_T$ signature at the LHC will thus no longer be viable, and multilepton event cross sections will be reduced. Although no detailed studies of these scenarios exist either for the LHC or for NLC, it seems reasonable to expect that new particle production at the NLC will be readily detectable via an excess of spherical events, while at the LHC, the signal will be harder to detect (particularly if sparticles other than the LHC also decay via $R$-violating operators with significant branching fractions). On the other hand, it should be remembered that we do not have any real theory of SUSY breaking parameters, and it could be that, fine-tuning considerations notwithstanding, none of the sparticles may be accessible at NLC500 or even NLC1000.
FIGURES

FIG. 1. Regions of the $m_0$ vs. $m_{1/2}$ plane where various sparticle pair production reactions are kinematically accessible. For $\tilde{Z}_i \tilde{Z}_j$ pairs, instead we plot the 10 fb cross section contours. We take $A_0 = 0$, $\tan \beta = 2$ and $m_t = 180$ GeV. In a), we take $\mu < 0$, while in b) we take $\mu > 0$. The regions denoted by TH are excluded by theoretical constraints, while the region labelled EX is excluded by experimental constraints.

FIG. 2. Same as Fig. 1, except for $\tan \beta = 10$.

FIG. 3. Various lowest order SM cross sections (in fb) for NLC at $\sqrt{s} = 500$ GeV, as a function of $e^-$ polarization parameter $P_L(e^-)$.

FIG. 4. Regions of the $m_0$ vs. $m_{1/2}$ plane where selectrons, charginos and neutralinos are accessible at the 5$\sigma$ level above SM backgrounds, after various cuts discussed in the text. The $\tilde{Z}_1 \tilde{Z}_2$ contour denotes the boundary of the added region where $\geq 10$ $b\bar{b} + E_T$ events (10 fb cross section) should be obtained with $P_L(e^-) = 0.9$ above negligible background. We have taken $\sqrt{s} = 500$ GeV and have assumed 20 fb$^{-1}$ of integrated luminosity. The SUGRA parameters taken are the same as for Fig. 1. In frame a), we show the reach for both polarized and unpolarized beams, and in addition, the location of case study points 1, 2 and 3.

FIG. 5. Same as Fig. 4, except $\tan \beta = 10$.

FIG. 6. Various lowest order SUSY cross sections (in fb) for NLC at $\sqrt{s} = 500$ GeV, as a function of $e^-$ polarization parameter $P_L(e^-)$, for case study point #1. We also show on the left the masses of only the accessible superpartners and Higgs bosons.

FIG. 7. a) Histograms of events after cuts versus missing mass $m_\ell$ for chargino pair events and also for SM and SUSY backgrounds. In b), we plot the $E_{jj}$ distribution after requiring $m_\ell > 240$ GeV; the arrows denote the kinematic endpoints which are related to the $\tilde{W}_1$ and $\tilde{Z}_1$ masses.

FIG. 8. In a), we plot the $m_\ell$ distribution after cuts to isolate the $\tilde{Z}_1 \tilde{Z}_2 \rightarrow b\bar{b} + E_T$ signal from SM and SUSY backgrounds. The arrows denote the expected kinematic endpoints of the signal distribution. After a cut of $m_\ell > 300$ GeV, we plot $m_{jj}$ for the two detected $b$-jets; the distribution is related to the light Higgs boson $H_\ell$ mass, denoted by the arrow. In c), we plot the $E_{jj}$ distribution from the two $b$-jets; the kinematic endpoints, which depend on $m_{\tilde{Z}_2}$, $m_{\tilde{Z}_1}$ and on $m_{H_\ell}$, are denoted by the tips of the arrows.
FIG. 9. In a), comparison of data to a theory fit depending on $m_{\tilde{Z}_1}$ and $m_{\tilde{Z}_2}$ yields the minimum $\chi^2$ value shown, and also the $\Delta \chi^2 = 2.3$ and 4.6 contours. In b), a 50 fb$^{-1}$ ‘data’ sample is shown as well as the best fit theory distribution, for the $\eta$ distribution for $\tilde{Z}_1\tilde{Z}_2$ search using case study point 1.

FIG. 10. Same as Fig. 6, except for case study point #2.

FIG. 11. For 95% right polarized $e^-$ beams, we show in a) the $E_\mu$ distribution after cuts for case study point #2. The endpoints, denoted by the tips of the arrows, depend on $m_{\mu_R}$ and $m_{\tilde{Z}_1}$. In b), we show the $E_{e^+}$ distribution which has two main components from $\tilde{e}_R\tilde{e}_R$ production and from $\tilde{e}_R\tilde{e}_L$ production. The corresponding endpoints are shown as well. In c), the $E_{e^-}$ distribution has a different structure than that shown in frame b). Finally, after requiring $E_{e^+} > 75$ GeV in d), the $E_{e^-}$ distribution from $\tilde{e}_R\tilde{e}_L$ is cleanly isolated as discussed in Sec. IIIB of the text.

FIG. 12. The $E_\mu^{\text{slow}}$ distribution from $\tilde{Z}_1\tilde{Z}_2$ production (after requiring $E_\mu^{\text{fast}} > 75$ GeV) shown along with SM and SUSY backgrounds. The upper endpoint of the solid histogram yields information on $m_{\tilde{Z}_2}$, given knowledge of $m_{\tilde{Z}_1}$ and $m_{\tilde{\nu}_L}$.

FIG. 13. Same as Fig. 6, except for case study point #3.

FIG. 14. Distribution in $E_{e^-}$ from $\tilde{e}_R\tilde{e}_R$ signal and SM and SUSY backgrounds, for case study point #3, using a right polarized $e^-$ beam. The endpoints yield information on $m_{\tilde{e}_R}$ and $m_{\tilde{Z}_1}$.

FIG. 15. The distribution of $E_e$ in $ee\mu+\text{jets}+E_T$ events from $\tilde{\nu}_e\tilde{\nu}_e$ production yields information on $m_{\tilde{\nu}_e}$ and $m_{\tilde{W}_1}$. After comparing ‘data’ to a theoretical fit, we plot the location of the minimum $\chi^2$ value, and contours of $\Delta \chi^2 = 2.3$ and 4.6 in frame a). In b), we show the data compared to the best theory fit. In c), we again plot $\Delta \chi^2$ contours for a nearby point in parameter space. Similar precision to frame a) is attained. In d), we show the $E_e$ distribution for data and best fit for the point taken in frame c).

FIG. 16. The $E_\ell$ distribution from the two hardest leptons in $\tilde{e}_L\tilde{e}_L \rightarrow 6\ell$ events leads to a mass measurement of $m_{\tilde{e}_L}$ and $m_{\tilde{Z}_2}$. In a), we plot the minimum $\chi^2$ and also contours of $\Delta \chi^2 = 2.3$ and 4.6. In b), we show a sample of ‘data’ and also the best fit obtained from frame a).

FIG. 17. Distribution in dimuon invariant mass showing the $\tilde{Z}_1\tilde{Z}_2 \rightarrow \mu^+\mu^- + E_T$ contribution and SM and SUSY backgrounds. The upper endpoint of the signal distribution is bounded by $m_{\tilde{Z}_2} - m_{\tilde{Z}_1}$, and leads to a measurement of $m_{\tilde{Z}_2}$.

FIG. 18. Same as Fig. 6, except for case study point #4.
FIG. 19. In frame a), we show the $E_{jj}$ distribution from $\tilde{W}_1 \tilde{W}_1 \to \ell + jj + E_T$ events along with SM and SUSY backgrounds. No distinctive endpoints are evident, due to the three-body chargino decay kinematics. In b), we show a scatter plot of $E_{jj}$ vs. $m_{jj}$. The signal is kinematically constrained to lie below the solid contour.

FIG. 20. By requiring $m_{jj}$ to lie within 4 narrow mass bins, we force 2-body kinematics onto the signal distribution. Then, for each bin, the endpoints in the $E_{jj}$ distribution depend on $m_{\tilde{W}_1}$ and $m_{\tilde{Z}_1}$. In a), we plot the minimum $\chi^2$ value, along with contours of $\Delta \chi^2$, after performing a common fit to the four $m_{jj}$ bins mentioned in the text. In frame b), we show the ‘data’ sample from the $m_{jj} = 30$ GeV bin, compared to the best fit theory distribution.

FIG. 21. $\tilde{Z}_2 \tilde{Z}_2$ production leads to a clean sample of $e^+e^-\mu^+\mu^- + E_T$ events. A plot of $m_{e^+e^-}$ yields a distribution bounded by $m_{\tilde{Z}_2} - m_{\tilde{Z}_1}$, which gives an estimate of $m_{\tilde{Z}_2}$.

FIG. 22. $\tilde{t}_1 \tilde{t}_1$ production leads to events containing $\geq 5$ jets+$E_T$, where two of the jets are tagged as $b$-jets. A plot of $E_b$ yields information on $m_{\tilde{t}_1}$ and $m_{\tilde{W}_1}$. In frame a), we plot minimum $\chi^2$ and contours of $\Delta \chi^2$ from a comparison of theory to data. In frame b), we show the ‘data’ sample, and the best fit theory distribution. In c), a similar analysis is made for a nearby point in parameter space, and similar precision is obtained. Frame d) shows the associated best fit to the ‘data’ distribution.

FIG. 23. The $m_0$ vs. $m_{1/2}$ plane is illustrated for $A_0 = 0$, and for $\tan \beta = 2$ and 10, and $\mu < 0$ and $\mu > 0$. The regions denoted by TH and EX are excluded by theoretical and experimental constraints, respectively. The dashed lines show contours of $m_{\tilde{e}_R}$ while the dot-dashed lines are contours of $m_{\tilde{W}_1}$, and roughly denote the kinematic reach of NLC500, NLC1000 and NLC1500 via selectron and chargino pair production channels. The lower solid contour shows the reach of NLC500 in parameter space as obtained using our simulation. The upper solid contour shows the reach of LHC with 10 fb$^{-1}$ of integrated luminosity, in the $1\ell$+jets+$E_T$ channel.