Elastic-plastic analysis of reinforced concrete frame buildings: Developing a new software package

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Abstract. In this work we will explain a new methodology that have been utilized in a new software package (program) to determine the lateral loads at which a 3D reinforced concrete frame building collapses. The solution is represented by forming the capacity curve (force–displacement) of the structure taking into account the elastic-plastic behavior of the element materials represented by plastic hinges positioned at the ends of the linear elements. This work takes into account only the plastic hinges caused by bending moments. The program is based on finite elements method and capable of analyzing and designing three dimensions buildings. The program provides a drawing environment (3D view, plan view, elevation view) with tools to create the model and to perform the structural analysis and design.

1. Introduction
The issue of evaluating the strength of existing buildings subjected to lateral loads is at present an urgent issue from a social and scientific point of view to maintain the safety of people. The development of engineering sciences, global codes and computer technology enables the simulation of the structural behavior and strength analysis according to clear steps. In order to optimize the structure from the point of view of its strength and stiffness, inelastic behaviour of structural elements after yielding point must be studied. This requires the development of structural analysis methods, and particularly, the finite elements method to perform plastic analysis.

There are many commercial finite elements softwares (for example: Computers and structures, Inc (CSI) softwares). These programs provide a schematic representation of the capacity curve (force-displacement) of the structure using nonlinear static pushover analysis (NSPA). They are based on specific codes that are involved in the analysis and design processes, that creates some difficulties and limitations when trying to adapt them for use according to working conditions outside the scope of what is presented within these programs. Therefore, we have developed a new program completely independent of other programs.

2. The solution methodology
The solution method starts from the same considerations adopted by structural analysis programs that use NSPA, which take into account the nonlinear properties of the element materials and their elastic-plastic behaviour. It is based on representing the response of the multi degrees of freedom (MDOF) of the structure with an equivalent response to the single degree of freedom (ESDOF). This includes applying a lateral load distributed along the height of the structure. This load is increased by a
steady step gradually until the translation in the monitored joint (it is identified as a joint at the top of the structure) reaches a specific translation called the target displacement (which is determined from the standards). Each step is represented by a point on a curve that links the lateral force to the associated translation in the monitored joint. This curve is called the capacity curve (force–displacement). The plastic behavior of the element is modeled by finite plastic hinges. The moment hinges are assigned to the ends of the beams because the higher moment values will be occurred at these ends as a result of the lateral loads. The behavior of the plastic hinge is represented by introducing the effect of materials nonlinear properties. The program initially defines concrete and steel materials, creates a stress-strain curves for these materials, and stores them as a series of set points (by default 20 points), the program calculates the Mander confined and unconfined concrete stress-strain curves [1], and the Park stress-strain curve for steel [2].

Each linear element in space is a reinforced concrete element having a cross section of certain dimensions (rectangular section with height and width) and a specific distribution of reinforcing bars (defined by their materials and size). Accordingly, the program creates a moment–curvature curves for all sections, taking into account the confinement effect. It calculates the negative and positive moments and curvatures corresponding to yielding, occurrence of modified–plastic moment, and failure. These curves are stored as a series of points (by default 20 points).

After creating the 3D structure model, the analysis process takes place. The program assembles the general stiffness matrix and calculates the internal forces in elements.

During the lateral load increasing phases, the program checks if plastic hinges occur. The entire section of the element in the hinge is assumed to remain elastic until the yield strength of the section is reached, at which the section transfers from elastic to plastic state and continues its ductile behavior until its failure.

If the moment value is in the plastic region, the plasticity effect is added by modifying the element’s stiffness matrix by reducing a moment stiffness at this end using massless spring systems, so that the moment in the hinge remains equal to the plastic moment. In the event when the curvature value exceeds the maximum value, the entire moment will be released so that it is equal to zero. This affects the stiffness of the whole structure, which requires (at each step) recalculating the structure stiffness and re-performing the analysis. If the target displacement value is exceeded, the analysis will be stopped. At each step the values of force and displacement are saved to form the capacity curve at the end.

2.1. Moment-Curvature curve
The moment-curvature (M-Φ) curves define the maximum capacity of structural elements that are primarily subjected to bending moments and therefore their failure mode is flexural. The moment-curvature curves are also used to assess the ductility of structural elements and they are very important for the determination of the amount of plastic energy a structural element can absorb. In order to calculate the moment curvature curve and to introduce the confinement effect into the calculation, the program uses fiber analysis. The program divides the reinforced concrete section into rectangular parts and calculates there areas and centroids and defines which of them are inside the concrete core or outside it. For a specific angle α and axial load P, the program calculates the M-Φ curve [3,4], then calculates the idealized moment curvature-curve to find yield moment M-Yield, plastic moment MP, maximum moment Mmax, yield curvature, plastic curvature, and maximum curvature.

2.2. Formation of the stiffness matrix of fixed beam elements
In order to calculate the internal forces (moments, shear forces, axial forces) in the structural elements using finite elements method, the program first calculates the 3D elastic stiffness matrix [k] for frame elements including shear and bending effects in local coordinates [5], and then assemble the general stiffness matrix of the structure according to Klaus Bathe finite element procedure [6]. Using direct solver (sparse Cholesky skyline decomposition [7]), we solve the system of linear equations to calculate the displacements.
2.3. Formation of the stiffness matrix, consistent load vector, and displacement of a spring-beam element

It is assumed that all deteriorations of an element are represented by massless spring systems [8] which allow flexibilities at the element ends. Having formed the matrix \([k]\), the program uses procedure [8] which finds the stiffness matrix \([k']\), consistent load vector \([p]\), and displacement vector \([d']\) of the spring-beam element \((i'–j')\) in the local principal coordinates. In figure 1, the joints \((i')\) and \((j')\) are the nodal joints in the system, and the joints \((i)\) and \((j)\) are the internal joints of the element.

Figure 1. Representation of a partly connected element in a structural system.

2.4. Formation of the capacity curve

After analysis, the program checks all the elements if plastic hinges occur there. If the moment value is in the plastic region and more than plastic moment \(MP\) of their sections, it modifies the element’s stiffness matrices using the spring systems. So it releases the bending moment about the section local axis 3 \((M33)\) by a value \((MP / M33)\) to get the moment in the hinge equal to the plastic moment. The program changes the stiffness reduction values based on many iterations and do analysis until the values of moments in all the elements are less or equal to the plastic moment \(MP\). The need to perform many iterations is explained by the requirement that the remaining moment should be redistributed to the other elements of the structure. In the event that the curvature value exceeds the maximum curvature, the entire moment will be released at the end so that it is equal to zero. Figure 2 shows the calculation scheme followed during the elastic-plastic analysis of the structure.

Figure 2. The calculation scheme.
3. Calculation example

3.1. Performing analysis Calculation of the capacity curve

For example, we consider simple six-storey building shown in figure 3.

![Figure 3. Model of six-storey building within the program interface.](image)

The structural beams are of rectangular sections (height = 50cm, width = 30cm), upper reinforcement consists of five bars with a diameter of φ14mm, lower reinforcement consists of four bars of the diameter of φ14mm, tie bar diameter is 8mm, clear cover to the tie from all the sides is 2.5cm (figure 4). The structural columns are square sections (width = 40cm), with distributed reinforcement. (The reinforcement details are not important for the columns at this time as the hinges are only assigned to beams). The defined materials details are as follows. (A) Unconfined concrete: compressive strength $f_c' = 27.58$MPa, the concrete strain at $f_c'$ is 0.002219, ultimate strain capacity is 0.005, modulus of elasticity is 24855.58MPa. (B) Confined concrete: compressive strength $f_{cc} = 29.31$MPa, concrete strain at $f_{cc}$ is 0.00292, ultimate compressive strength is 13.57MPa, ultimate concrete strain capacity is 0.01859. (C) Steel: rebar yield stress $f_y = 413.69$MPa, rebar ultimate stress capacity is 620.53MPa, steel modulus of elasticity is 199947.98MPa, rebar strain at $f_y$ is 0.0021, strain in rebar at the onset of strain hardening is 0.01, rebar ultimate strain capacity is 0.09.

We apply fiber analysis in beams sections to calculate moment–curvature curve and to find yield and plastic moments and yield, plastic, and maximum curvatures.

![Figure 4. Rectangular reinforced concrete section and its representation as a massive of fibers.](image)

The program calculates two moment–curvature curves for each section. The first of them represents the results for positive moments, and the second represents the results for negative moments. For the section presented in figure 4, the positive moment $MP$ is equal to 148.87kN.m, and the negative moment $MP$ is equal to 183.03kN.m.
The gravity loads on beams are dead load (10kN/m in addition to self weight) and live load (5kN/m). The lateral loads are applied in the structure left side joints with values of 10kN (see green arrows in figure 5). The load pattern to be increased is the lateral loads. The gravity loads are applied without increasing with a scale factor is 1.4 for dead loads and 1.6 for live loads. The target displacement is assumed to be 500mm in global X direction (shown in figure 5). The hinges are assigned to all beams ends. The program performs the elastic-plastic analysis described above, calculates the capacity curve, and stores results for each point from this curve.

For comparison we created the same (as above) model in the commercial program ETABS (which is a software from CSI). ETABS provide a schematic representation of the capacity curve of the
structure using NSPA. Figure 8 shows a great convergence between the resulted capacity curves in these two programs.

![Figure 8. Capacity curves built by using our program and ETABS.](image)

3.2. Designing using the program

In order to compare the amount of reinforcement required in the case of elastic design with the amount of reinforcement required in the case of elastic-plastic design, beam B1 (the selected beam in figure 3) was chosen to design. Our program designs the frame elements according to American concrete institute (ACI) [3, 9] using the load and resistance factor design (LRFD). Beams elements are designed to resist the shear force along the section local axis 2, and the bending moment M33.

To perform the design, we have to choose the point from the capacity curve, for example, the point 14 on figure 9, after the hinges are formed in the elements. At this point, the displacement is 67mm and the lateral load scale factor is 9.5.

The value of the lateral load corresponding to the selected point (9.5 × 10 = 95kN) was applied in addition to the fixed gravity loads (with the scale factor 1.4 for dead loads and 1.6 for live loads) to perform an elastic analysis of the structure.

![Figure 9. The capacity curve representing the relationship between the displacement in the monitored joint and the applied lateral load scale factor.](image)

We performed the elastic-plastic analysis for the beams reinforced with upper reinforcement consisting of five bars of φ14mm and lower reinforcement consisting of four bars of φ14mm. As follows from the elastic-plastic analysis, the building remains stable (despite the formation of the plastic hinges at the ends of the beam B1) at the reinforcement amount of 155m length and φ14mm. At the same time, the amount of reinforcement required in beam B1 in the case of performing the elastic
analysis (not allowing the formation of the plastic hinges) was 218m length and ϕ14mm (figure 10), which was 1.4 times greater than in the previous case.

![Diagram showing beam B1 reinforcement](image)

**Figure 10.** The reinforcement in beam B1 calculated in the program after performing elastic analysis.

4. Conclusion

- This work provides workable software for three-dimensional structural analysis and design, based on finite elements method. The program shows a great match with the results of well-known commercial software. The program performs elastic-plastic analysis of 3D frame structures to calculate the capacity curve.
- In this work, only plastic moment hinges have been developed. The next work will include axial force and uniaxial bending moment hinges.
- When designing structures subjected to lateral loads, the codes allow the possibility of forming plastic hinges in the structural elements while a collapsing mechanism of the whole structure is not formed. The analysis revealed that the elastic design is considered to be financially costly when the structure is designed to resist lateral forces with a small occurrence probability.
- The program can be downloaded for free from this site (https://www.amsprogram.ru/).

References

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