A new helioseismic constraint on a cosmic-time variation of $G$

Alfio Bonanno$^1$ and Hans-Erich Fröhlich$^2$

$^1$INAF, Osservatorio Astrofisico di Catania, via S. Sofia, 78, 95123 Catania, Italy
$^2$Leibniz Institute for Astrophysics Potsdam (AIP), An der Sternwarte 16, 14482 Potsdam, Germany

Helioseismology can provide strong constraints on the evolution of Newton’s constant over cosmic time. We make use of the best possible estimate of 8640 days of low-$\ell$ BiSON data, corrected for the solar cycle variation, to obtain a new constraint on an evolving gravitational constant. In particular, by means of a Bayesian analysis we conclude that $\dot{G}/G_{\text{today}} = (1.25 \pm 0.30) \times 10^{-13}$ yr$^{-1}$. Our result, a 4-$\sigma$ effect, is more than one order of magnitude stronger than previous constraints obtained with helioseismology. We also take into account possible systematic effects by considering the theoretical uncertainties on the efficiency of the proton-proton (pp) fusion cross-section. We show that models with variable $G$ significantly outclass models with no secular variation of $G$, viz by a Bayes factor exceeding 30.

INTRODUCTION

The idea that the Sun can be considered a laboratory for fundamental physics traces back to the early developments in nuclear physics by contributing to the understanding of the basic nuclear processes involved in stellar nucleosynthesis. In recent times, accurate measurements of acoustic $p$-mode spectrum combined with inversion techniques have further stressed this role \cite{1}. Important examples are the investigation of the equation of state \cite{2}, the discovery of neutrino flavour oscillations \cite{3,4}, the properties of Dark Matter \cite{5,6,7}, the constraints on axions emission \cite{9,10} the properties of the screening of nuclear reaction rates \cite{11,12} and constraints on physical constants \cite{13}.

A fundamental problem that can be tackled by means of helioseismology is the possibility of limiting secular variations of $G$, a possibility argued long ago by Dirac \cite{14} and Milne \cite{15}. This initial intuition has been further elaborated in \cite{16,17} and is nowadays an important ingredient of various scalar-tensor theories \cite{18}, quantum-gravity inspired models of modified gravity \cite{19,20}, and string theory low-energy models \cite{21}.

In this context a widely used approach to promote the gravitational constant to a dynamical variable is to extend the general relativistic framework in which gravity is mediated by a massless spin-2 graviton, to include a spin-0 scalar field which couples universally to matter fields. As the universality of free-fall is maintained theories that predict that the locally measured gravitational constant vary with time often violate the equivalence principle in its strong form. For this reason empirical constraints on $\dot{G}/G_{\text{today}}$, where the dot indicates a derivative with respect to the cosmic time $t$, have been obtained in several contexts \cite{22,23}. Current limits on $\dot{G}/G_{\text{today}}$ span from $\dot{G}/G_{\text{today}} = (4 \pm 9) \times 10^{-13}$yr$^{-1}$ obtained from the Lunar Laser Ranging (LLR) experiment \cite{24}, to $-3 \times 10^{-13} < \dot{G}/G_{\text{today}} < 4 \times 10^{-13}$yr$^{-1}$ from BBN \cite{25}, or $\dot{G}/G_{\text{today}} \sim 10^{-12}$yr$^{-1}$ from white dwarfs \cite{26}.

Helioseismology is able to provide independent constraints on possible time evolution of the gravitational constant $G$ over cosmic time because the stellar luminosity $L$ varies as $\sim G^7$ \cite{27}. For example, a monotonically increasing Newton’s constant must be compensated for a systematic decrease of core temperature and a corresponding change in the hydrogen abundance in order to match $L_\odot$, the solar radius $R_\odot$ and the metal to hydrogen abundance ratio $(Z/X)_\odot$. In \cite{28} a direct comparison of low-degree $p$-modes to GONG data has allowed us to obtain $\dot{G}/G_{\text{today}} \lesssim 1.6 \times 10^{-12}$yr$^{-1}$, assuming a power-law of the type $G(t) \propto t^{-\alpha}$. In this paper we shall present a new limit on $\dot{G}/G_{\text{today}}$ based on a bayesian approach which makes use of the definitive “best possible estimate” of 8640 days of low-$\ell$ frequency BiSON data, corrected for the solar cycle modulation \cite{29}.

FIG. 1. 2-dimensional posterior probability distribution. The closed contour with a probability density of 10.3 per cent of the peak density comprises 90 per cent of the total probability. The hatched strip marks the 68.3-per-cent ($\pm 1$-$\sigma$) interval of $\alpha$’s marginal distribution.
SOLAR MODELS AND MODEL UNCERTAINTIES

In this context it is important to reduce as much as possible any source of systematic uncertainties in the input physics of the calibrated solar models in order to obtain a significant constraint on $G/G_{\text{today}}$. From this point of view the main problem is clearly our ignorance of the efficiency of the proton-proton $(pp)$ fusion cross-section for which only theoretical estimates are available. An uncertainty of $\pm 3\%$ on the value of $S_{pp}(0)$, the astrophysical $S$-factor at zero energy, is quoted in \cite{39}, in particular. Therefore both $G/G_{\text{today}}$ as well as $S_{pp}(0)$, have been estimated from the data in a Bayesian manner.

Our solar models are built using the Catania version of the GARSTEC code \cite{31,32}, a fully-implicit 1D code including heavy-elements diffusion and updated input physics. We prescribed the time evolution of the gravitational constant as a power-law \cite{35,33}

$$G(t) = G_0 \left( \frac{t_0}{t} \right)^{\alpha}$$

where $G_0$ is the cosmologically recent value of Newton’s constant according to 2010 CODATA so that $G_0 = 6.67384 \times 10^{-8}$ cm$^3$g$^{-1}$s$^{-2}$ and $t_0 = 13.7$ Gyr is a reference age of the Universe according to most of $\Lambda$CDM estimates. As $G_0 M_{\odot} = 1.3271244 \times 10^{36}$ cm$^3$s$^{-2}$ \cite{34} is fixed, $M_{\odot} = 1.98855 \times 10^{33}$ g is assumed. Irwin’s equation of state \cite{35} with OPAL opacities for high temperatures \cite{30} and Ferguson’s opacities for low temperatures \cite{37} are employed and the nuclear reaction rates are taken from the compilation in \cite{30}.

Our starting models are chemically homogeneous PMS models with $\log L/L_{\odot} = 0.21$ and $\log T_e = 3.638$ K, thus close to the birth line of a $1 M_{\odot}$ object. Initial Helium fraction, $(Z/X)$ and mixing-length parameter are adjusted to match the solar radius $R_\odot = 6.95613 \times 10^{10}$ cm (based on an average of the two values and quoted error bar in Table 3 of \cite{38}), the solar luminosity $L_\odot = 3.846 \times 10^{33}$ erg s$^{-1}$ \cite{34} and the chemical composition of \cite{39} with $(Z/X)_\odot = 0.0245$ at the surface. We also employed the new accurate meteoritic estimate of the solar age of \cite{40}, $t_\odot = 4.567$ Gyr, a value consistent with the helioseismic solar age \cite{41}. We further noticed that models with the so-called “new abundances” for which $(Z/X)_\odot = 0.0178$ \cite{42} would lead to much smaller Bayes factors and we decided not to discuss these models in this work.

In order to define a proper seismic diagnostic we adopted a widely used approach: if $\nu_{n,\ell}$ is the frequency of the mode of radial order $n$ and angular degree $\ell$, the frequency separation ratios

$$r_{\ell,\ell+2}(n) = \frac{\nu_{n,\ell+2} - \nu_{n-1,\ell+1}}{\nu_{n,\ell+1} - \nu_{n-1,\ell+1}}$$

can be shown to be localized in the core and weakly dependent on the complex physics of the outer layers \cite{42,44}. In particular in the limit $n \gg 1$

$$r_{\ell,\ell+2}(n) \approx -(4\ell + 6) \frac{1}{4\pi^2 r_{n,\ell}} \int_{0}^{R_\odot} \frac{d\varepsilon}{dR} \frac{dR}{R}$$

so that a change in temperature $(T)$ and mean molecular weight $(\bar{\mu})$ directly impacts on the $r_{\ell,\ell+2}(n)$ terms as $\delta \varepsilon_s / \varepsilon_s \approx \frac{1}{2} \delta T / T - \frac{1}{2} \delta \bar{\mu} / \bar{\mu}$.

BAYESIAN APPROACH

We consider the following two-dimensional parameter space: $-0.1 \leq \alpha \leq 0.1$ and $0.97 \leq S/S_{pp}(0) \leq 1.03$. The proposed $\alpha$ range generously covers all previous $G/G_{\text{today}}$ limits obtained by independent methods \cite{33}. Moreover, the $S$ interval $0.97$–$1.03$ allows for an up to $\pm 3\%$ deviation from the recommended value $S_{pp}(0) = (4.01 \pm 0.04) \times 10^{-22}$ keV b in \cite{30}.

Central to the Bayesian hypothesis testing is the likelihood. In the following, a Gaussian has been assumed,

$$L(\alpha, S) = \prod_{i=1}^{N=17} \frac{1}{\sqrt{2\pi \sigma_i}} \exp \left( -\frac{(d_i - m_i(\alpha, S))^2}{2\sigma_i^2} \right) ,$$

where $d_i = r_{02}(n)$ are the observed data $(n = i + 8, i = 1 \ldots N, N = 17)$, $m_i$, the theoretical model values, and $\sigma_i$ the errors (see also \cite{41} for an application of this likelihood to the helioseismic determination of the solar age). All 17 contributions enter the likelihood with the same weight.

The posterior probability distribution is the likelihood weighted with a prior distribution. Obviously, this prior distribution should be a flat one compared to $\alpha$. Concerning $S$ we decided to take a conservative point of view, i.e. that nothing is known about $S_{pp}(0)$. In that case we are on the safe side and the only eligible prior distribution is a flat one over the logarithm, $\log(S)$.

In the end two hypotheses have to been compared: $H_1 = H_\odot(-0.1 \leq \alpha \leq 0.1, 0.97 \leq S/S_{pp}(0) \leq 1.03)$ vs. our zero hypothesis $H_0(\alpha = 0, 0.97 \leq S/S_{pp}(0) \leq 1.03)$.

RESULTS

The posterior probability distribution is indistinguishable from a two-dimensional Gaussian (Fig. 1). The reason is that the theoretical models $m_i(\alpha, \log(S))$ are linearly dependent on both $\alpha$ and $\log(S)$ (cf. \cite{45}) as we checked in all our models. From $\alpha$’s marginal distribution one reads its mean value and standard deviation: $\langle \alpha \rangle = -0.0017 \pm 0.0004$. Formally, this is a $4\sigma$ effect. With $t_0 = 13.7$ Gyr this translates to $G/G_{\text{today}} = (1.25 \pm 0.30) \times 10^{-13}$ yr$^{-1}$. As a by-product
one gets \( \log(S/S_{pp}(0)) = 0.011 \pm 0.008 \) and a correlation coefficient of -0.62. An enhanced \( S \) goes with a reduced \( \alpha \). However, the indicated slight enhancement of Adelberger et al. \cite{20} \( pp \) cross-section by 1% proves insignificant. Our result is one order of magnitude stronger than the limit obtained in \cite{28} and comparable in precision to those obtained with LR \cite{24} or BBN \cite{25}.

Integrating the posterior over the whole parameter space or subsections of it, respectively, one gets the required evidences. The evidence in favour of a hypothesis is the prior-weighted mean of the likelihood over parameter space. The ratio of the evidences, \( E(H_1)/E(H_0) \), the so-called Bayes factor amounts to 34.0. (If one trusts the 

Acknowledgements. — We acknowledge L. Santagati for careful reading of the manuscript.

[1] S. Basu, Living Reviews in Solar Physics 13, 2 (2016), arXiv:1606.07071 [astro-ph.SR]
[2] S. Basu, W. Däppen, and A. Nayfonov, ApJ 518, 985 (1999), astro-ph/9810132
[3] Y. Fukuda et al. (Super-Kamiokande), Phys. Rev. Lett. 81, 1562 (1998), arXiv:hep-ex/9807003 [hep-ex]
[4] Q. R. Ahmad et al. (SNO), Phys. Rev. Lett. 89, 011301 (2002), arXiv:nucl-ex/0204008 [nucl-ex]
[5] I. P. Lopes, J. Silk, and S. H. Hansen, MNRAS 331, 361 (2002), astro-ph/0111530
[6] I. Lopes, P. Panci, and J. Silk, ApJ 795, 162 (2014), arXiv:1402.0982 [astro-ph.SR]
[7] I. Lopes and J. Silk, ApJ 757, 130 (2012), arXiv:1209.3631 [astro-ph.SR]
[8] A. C. Vincent, P. Scott, and A. Serenelli, Physical Review Letters 114, 081302 (2015), arXiv:1411.6626 [hep-ph]
[9] H. Schlattl, A. Weiss, and G. Raffelt, Astroparticle Physics 10, 353 (1999), hep-ph/9807476
[10] N. Vinyoles, A. Serenelli, F. L. Villante, S. Basu, J. Redondo, and J. Isern, JCAP 10, 015 (2015), arXiv:1501.01639 [astro-ph.SR]
[11] G. Fiorentini, B. Ricci, and F. L. Villante, Letters B 503, 121 (2001), astro-ph/0011130
[12] A. Weiss, M. Flaskamp, and V. N. Tsytsyev, A&A 371, 1123 (2001), astro-ph/0102353
[13] J. Christensen-Dalsgaard, M. P. Di Mauro, H. Schlattl, and A. Weiss, MNRAS 356, 587 (2005)
[14] P. A. M. Dirac, Proceedings of the Royal Society of London Series A 165, 199 (1938)
[15] E. A. Milne, Nature 139, 409 (1937)
[16] C. Brans and R. H. Dicke, Physical Review 124, 925 (1961)
[17] P. G. Bergmann, International Journal of Theoretical Physics 1, 25 (1968)
[18] Y. Fujii and K. Maeda, The Scalar-Tensor Theory of Gravitation Cambridge Monographs on Mathematical Physics (Cambridge University Press, 2003).
[19] A. Bonanno, G. Esposito, and C. Rubano, Classical and Quantum Gravity 21, 5005 (2004) gr-qc/0403115
[20] L. Smolin, Class. Quant. Grav. 33, 025011 (2016), arXiv:1507.01229 [hep-th]
[21] M. Gasperini, in String Theory and Fundamental Interactions, Lecture Notes in Physics, Berlin Springer Verlag, Vol. 737, edited by M. Gasperini and J. Maharana (2008) p. 787, hep-th/0702166
[22] J.-P. Uzan, Living Reviews in Relativity 14, 2 (2011), arXiv:1009.5514
[23] P. J. Edwin Peebles, European Physical Journal H (2016), 10.1140/epjh/e2016-70034-0, arXiv:1603.06174
[24] J. G. Williams, S. G. Turyshhev, and D. H. Boggs, Physical Review Letters 93, 261101 (2004) gr-qc/0411113
[25] C. J. Copi, A. N. Davis, and L. M. Krauss, Physical Review Letters 92, 171301 (2004) astro-ph/0311334
[26] E. García-Berro, P. Lorenz-Aguilar, S. Torres, L. G. Althaus, and J. Isern, JCAP 5, 021 (2011), arXiv:1105.1992 [gr-qc]
[27] S. degi Innocenti, G. Fiorentini, G. G. Raffelt, B. Ricci, and A. Weiss, A&A 312, 345 (1996), astro-ph/9509000
[28] D. B. Guenther, L. M. Krauss, and P. Demarque, ApJ 498, 871 (1996)
[29] A.-M. Broomhall, W. J. Chaplin, G. R. Davies, Y. Elsworth, S. T. Fletcher, S. J. Hale, B. Miller, and R. New, MNRAS 396, L100 (2009), arXiv:0903.5219 [astro-ph.SR]
[30] E. G. Adelberger et al., Rev. Mod. Phys. 83, 195 (2011), arXiv:1004.2318 [nucl-ex]
[31] A. Bonanno, H. Schlattl, and L. Paternò, A&A 390, 1115 (2002), astro-ph/0204331.
[32] A. Weiss and H. Schlattl, Ap&SS 316, 99 (2008)
[33] J.-P. Uzan, Reviews of Modern Physics 75, 403 (2003), hep-ph/0205340
[34] A. N. Cox, Allen’s Astrophysical Quantities (Springer, 2000).
[35] S. Cassisi, M. Salaris, and A. W. Irwin, ApJ 588, 862 (2003), astro-ph/0301378
[36] C. A. Iglesias and F. J. Rogers, ApJ 464, 943 (1996)
[37] J. W. Ferguson, D. R. Alexander, F. Allard, T. Barman, J. G. Bodnarik, P. H. Hauschildt, A. Heffner-Wong, and A. Tamanai, ApJ 623, 585 (2005), astro-ph/0502045
[38] M. Haberreiter, W. Schmutz, and A. G. Kosovichev, ApJ 675, L53 (2008)
[39] N. Grevesse and A. Noels, in Origin and Evolution of the Elements, Conference Series, Vol. 245, edited by N. Prantzos, E. Vangion-Flam, and M. Casse (1993) p. 14.
[40] J. N. Connelly, M. Bizzarro, A. N. Krot, Å. Nordlund, D. Wieland, and M. A. Ivanova, Science 338, 651 (2012)
[41] A. Bonanno and H.-E. Fröhlich, A&A 580, A130 (2015), arXiv:1507.05847 [astro-ph.SR]
[42] M. Asplund, N. Grevesse, A. J. Sauval, and P. Scott, A&A 47, 481 (2000), arXiv:0909.0948 [astro-ph.SR]
[43] I. W. Roxburgh and S. V. Vorontsov, A&A 411, 215 (2003)
[44] H. Ort Floranes, J. Christensen-Dalsgaard, and M. J. Thompson, MNRAS 356, 671 (2005)
[45] A. O’Hagan and J. J. Forster, ”Kendall’s advanced theory of statistics, volume 2b: Bayesian inference, second edition” (2001) Cambridge University Press.
"edition." (2004).