Matter density perturbation and power spectrum in running vacuum model

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ABSTRACT

We investigate the matter density perturbation $\delta_m$ and power spectrum $P(k)$ in the running vacuum model (RVM) with the cosmological constant being a function of the Hubble parameter, given by $\Lambda = \Lambda_0 + 6\sigma H_0 + 3\nu H^2$, in which the linear and quadratic terms of $H$ would originate from the QCD vacuum condensation and cosmological renormalization group, respectively. Taking the dark energy perturbation into consideration, we derive the evolution equation for $\delta_m$ and find a specific scale $\delta_{cr} = 2\pi/k_{cr}$, which divides the evolution of the universe into the sub and super-interaction regimes, corresponding to $k \ll k_{cr}$ and $k \gg k_{cr}$, respectively. For the former, the evolution of $\delta_m$ has the same behavior as that in the ACM model, while for the latter, the growth of $\delta_m$ is frozen (greatly enhanced) when $\nu + \sigma > (\nu - \sigma)$ due to the couplings between radiation, matter and dark energy. It is clear that the observational data rule out the cases with $\nu < 0$ and $\nu + \sigma < 0$, while the allowed window for the model parameters is extremely narrow with $\nu, |\nu| \lesssim O(10^{-7})$.

Key words: Running vacuum energy, matter power spectrum, dark energy

1 INTRODUCTION

It is well-known that the Type-Ia supernova observations (Riess et al. 1998; Perlmutter et al. 1999) have revealed the late-time accelerating expansion of our universe. To realize the accelerating universe, it is necessary to introduce a negative pressure fluid to the gravitational theory, referred to as “Dark Energy” (Copeland et al. 2006), while the negative pressure fluid to the gravitational theory, referred to as “Dark Energy” (Copeland et al. 2006), while the negative pressure fluid to the gravitational theory, referred to as “Dark Energy” (Copeland et al. 2006), while the negative pressure fluid to the gravitational theory, referred to as “Dark Energy” (Copeland et al. 2006), while the negative pressure fluid to the gravitational theory, referred to as “Dark Energy” (Copeland et al. 2006), while the negative pressure fluid to the gravitational theory, referred to as “Dark Energy” (Copeland et al. 2006), while the negative pressure fluid to the gravitational theory, referred to as “Dark Energy” (Copeland et al. 2006). In this model, the cosmological constant evolves in time and decays to radiation and matter in the evolution of the universe, leading to the same order of magnitude for the energy densities of dark energy and dark matter. Its observational applications have been also extensively explored in the literature (España-Bonet et al. 2004; Tamayo et al. 2015). Additionally, it has been shown that the RVM can fit various observational data, indicating that this scenario is good in describing the evolution history of our universe (Sola 2010; Sola et al. 2013; Sola et al. 2014; Sola et al. 2016). In our study, we will concentrate on the specific model with $\Lambda = \sum_i \lambda_i H^i$ (Borges and Carneiro 2005; Borges et al. 2005a; Carneiro et al. 2009; Zimdahl et al. 2011; Sola 2013; Sola and Gomez-Valent 2015), in which the quadratic term, $\lambda_2 H^2$, might come from the quantum effects induced by the cosmological renormalization group (Alcaniz et al. 2003; Costa et al. 2014; Sola 2014; Gomez-Valent et al. 2013), while the linear term, $\lambda_1 H$, would originate from the theory with the QCD vacuum condensation associated with the chiral phase transition (Schutzhold 2002; Banerjee et al. 2005; Klinkhamer and Volovik 2006; Ohta 2011; Cai et al. 2011).

When it comes to the decaying dark energy model, it is reasonable to consider not only the background evolution equations but also the density perturbation of dark energy.
We follow the same method in the references (Fabris et al. (2007); Borges et al. (2008b)) to rewrite dark energy as a function of a Lorentz scalar $\nabla_\mu U^\mu$, where $U^\mu = dx^\mu/\sqrt{-g}$ is the four-velocity. Based on such an expression, we examine the matter density perturbation $\delta_m$ and power spectrum $P(k)$ in the linear perturbation theory of gravity. Note that in the literature (Fabris et al. (2007)), the matter density perturbation evolves from $z = 1100$ (the recombination era) to $z = 0$ (the present), where the initial conditions are taken from the $\Lambda$CDM limit with the BBKS transfer function. However, the density perturbation of the RVM may influence the evolution of the matter density perturbation in the high redshift regime. We take the scale invariance initial conditions at the very early time of the universe, in which all the perturbation modes are at the super-horizon scale with the same behavior as that in the $\Lambda$CDM model. Then, we analyze the properties in the sub and super-interaction scales with the allowed ranges for the model parameters discussed.

This paper is organized as follows: We briefly introduce the running vacuum model in Sec. 2. We derive the linear perturbation equations with the synchronous gauge and the evolution property of the matter density perturbation in Sec. 3. In Sec. 4, we show the evolutions of $\delta_m$ and $P(k)$. Our conclusions are presented in Sec. 5.

2 RUNNING COSMOLOGICAL CONSTANT MODEL

We start from the Einstein equation with $\kappa^2 = 8\pi G = 1$.

$$R_{\mu\nu} - \frac{\Lambda g_{\mu\nu}}{2} + \Lambda g_{\mu\nu} = T^\mu_{\mu\nu},$$

where $R = g^{\mu\nu} R_{\mu\nu}$ is the Ricci scalar, $\Lambda$ is the time-dependent cosmological constant, and $T^\mu_{\mu\nu}$ is the energy-momentum tensor of matter and radiation. In the Friedmann-Lemaître-Robertson-Walker (FLRW) metric, we obtain

$$ds^2 = -dt^2 + a^2(t) \left[ \delta_{ij} dx^i dx^j \right],$$

$$H^2 = \frac{1}{3} (\rho_M + \rho_\Lambda),$$

$$H = -\frac{1}{2} \left( \rho_M + P_M + \rho_\Lambda + P_\Lambda \right),$$

where $H = \dot{a}/a$ presents the Hubble parameter, $\rho_M$ ($P_M$) corresponds to the total energy density (pressure) of matter and radiation, and $\rho_\Lambda$ ($P_\Lambda$) is the energy density (pressure) of the cosmological constant, derived from Eq. (1).

$$\rho_\Lambda = -P_\Lambda = \Lambda H.$$

In the running cosmological constant model, $\Lambda(H)$ taking to be a function of the Hubble parameter $H$ (Basilakos et al. (2009); Gomez-Valent and Sola (2013); Gomez-Valent et al. (2015)), given by

$$\Lambda(H) = \Lambda_0 + 6cH_0 (H - H_0) + 3\upsilon (H^2 - H_0^2),$$

where $\nu$, $\sigma$ and $\Lambda_0$ are free parameters, while $H_0$ is the Hubble parameter at the present. We note that the linear and quadratic terms in Eq. (6) could originate from two possible physical sources of the QCD vacuum condensation associated with the chiral phase transition (Schutzhold (2003); Borges and Carneiro (2008)) and the quantum effect induced by the cosmological renormalization group running of the vacuum energy in curved space-time (Sola (2013)), respectively. Substituting Eq. (6) into the conservation equation $\nabla_\nu (T^\mu_{\mu\nu} + T^\mu_\nu) = 0$, we obtain

$$\dot{\rho}_\Lambda + 3H(1 + \nu \Lambda) \rho_\Lambda = \rho_\Lambda,$$

$$\dot{\rho}_M + 3H \rho_M = -R_\nu \delta_\nu,$$

$$\dot{\rho}_\Lambda + 4H \rho_\Lambda = -R_\nu \delta_\nu,$$

where $R_\nu$ represents the interaction rate between radiation (matter) and dark energy with

$$R_\nu = \frac{\rho_{(\nu M)} + P_{(\nu M)}}{\rho_M + P_M},$$

respectively, where $\rho_M = \sum_{i=0}^\infty \rho_i$ and $P_M = \sum_{i=0}^\infty P_i$. Note that the signs for the model parameters $\nu$ and $\sigma$ will be carefully examined.

In order to investigate the dynamics of dark energy, the cosmological constant in Eq. (6) should be represented as a function of a Lorentz scalar. By taking the FLRW metric, the covariant derivative of the four-velocity $U^\mu \equiv dx/\sqrt{-g}$ is given by

$$\nabla_\mu U^\mu = 3H,$$

As a result, $\Lambda(H)$ can be rewritten as

$$\Lambda = \Lambda_0 - 3(2\nu + \sigma) H_0^2 + 2\sigma \nabla_\nu U^\mu + \frac{\nu}{3} (\nabla_\mu U^\mu)^2.$$ (12)

Even though the expression for the Hubble parameter is not unique, the relation in Eq. (11) is the simplest way to rewrite the Hubble parameter to be a Lorentz scalar (Fabris et al. (2007); Borges et al. (2008b); Velasquez-Toribio (2012)).

3 LINEAR PERTURBATION THEORY

Since the model with the strong couplings between radiation/matter and $\Lambda$, corresponding to $\nu, \sigma \sim O(1)$ is unable to fit the current astrophysical and cosmological observations (Gomez-Valent and Sola (2013); Gomez-Valent et al. (2015)), we only focus on the small ones with $\nu, \sigma \ll 1$. Following the standard procedure (Ma and Bertschinger (2015)), we only focus on the small ones with $\nu, \sigma \ll 1$. Following the standard procedure (Ma and Bertschinger (2015)), we calculate the evolutions of linear perturbation equations under the synchronous gauge with the perturbed dark energy density. The metric perturbation is given by

$$ds^2 = -dt^2 + a^2(t) \left[ \delta_{ij} + h_{ij} \right] dx^i dx^j,$$

with

$$h_{ij} = \int d^3 k e^{ik \hat{r}} \left[ \delta_{ij} h(k, \hat{r}) + 6 \left( \delta_{i3} + \delta_{j3} - \frac{1}{3} \delta_{ij} \right) \eta(k, \hat{r}) \right],$$ (14)

where $i, j = 1, 2, 3, h$ and $\eta$ are two scalars in the synchronous gauge, and $\hat{k}$ is the $k$-space unit vector. From the relation

$$\nabla_\mu U^\mu = 3H + \left( \frac{\hat{h}}{2} \right),$$ (15)

the density perturbation of dark energy is given by

$$\delta \rho_\Lambda = 2(\sigma H_0 + \nu H) \left( \frac{\hat{h}}{2} \right),$$ (16)

where $\eta \equiv \nabla_\mu U^\mu$ is the momentum perturbation. From the conservation equation $\nabla_\nu (T^\mu_{\mu\nu} + T^\mu_\nu + T^\nu_\nu) = 0$ with $\delta T_\nu^\nu = 0$, we can rewrite the conservation equation $\nabla_\nu (T^\mu_{\mu\nu} + T^\mu_\nu + T^\nu_\nu) = 0$ as

$$\nabla_\nu (T^\mu_{\mu\nu} + T^\mu_\nu + T^\nu_\nu) = 0.$$ (17)

with $\eta \equiv \nabla_\mu U^\mu$ is the momentum perturbation. From the conservation equation $\nabla_\nu (T^\mu_{\mu\nu} + T^\mu_\nu + T^\nu_\nu) = 0$ with $\delta T_\nu^\nu = 0$, we can rewrite the conservation equation $\nabla_\nu (T^\mu_{\mu\nu} + T^\mu_\nu + T^\nu_\nu) = 0$ as
The evolutions of the density perturbation \( \delta_t \equiv \partial \rho_t / \rho_t \) and momentum perturbation \( \theta_t \equiv \partial \nu_t^\ell / \nu_t^\ell (\ell = r \ or \ m) \) can be derived. Explicitly, one gets

\[
\dot{\delta}_t = -(1 + w_f) \left( \dot{\nu}_t + \frac{h}{2} \right) + R_1 \left[ \frac{\rho_\Lambda}{\rho_t} \delta_t - \left( \frac{\rho_\Lambda \delta_\Lambda + \rho_\Lambda \delta_\Lambda}{\rho_t} \right) \right],
\]

\[
\dot{\theta}_t = -H (1 - 3w_f) \theta_t + \frac{w_f}{1 + w_f} \frac{k^2}{a^2} \delta_t
\]

where \( w_f \equiv P_f / \rho_f = \delta P_f / \delta \rho_f \) and \( w_f = 0 \) are used. In addition, the evolution of the synchronous scalar \( h(a, k) \) is given by,

\[
\ddot{h} + 2H \dot{h} = -\sum_{\ell=0}^\infty (1 + 3w_f) \delta \rho_t
\]

from the field equation in Eq. 1.

In the matter dominated era, as the radiation density is sub-dominated to the universe, i.e. \( \rho_r \ll \rho_m \), from Eqs. (16)-(19), we find that

\[
\delta_m^0 \equiv \frac{d\delta_m}{dN} = - \left[ 1 - 4 \left( \frac{k^2}{a^2H^2} \left( \nu + \sigma \frac{H_0}{H} \right) \left( \delta_m + \frac{h'}{2} \right) \right) \right] - \left( \frac{2\nu - \sigma \frac{H_0}{H}}{H} \right) \delta_m,
\]

\[
\theta_m^0 \equiv \frac{d\theta_m}{dN} = - \theta_m - \frac{2k^2}{3a^2H^2} \left( \nu + \sigma \frac{H_0}{H} \right) \theta_m - \frac{h'}{2},
\]

where \( h' \equiv dh/dN \) and \( N = \ln a \) with the higher order terms of \( \nu \) and \( \sigma \) neglected. Combining Eqs. (19)-(21), we obtain the second order derivative equation of the matter density perturbation to be

\[
\delta_m'' + \frac{1}{2} + \frac{2k^2}{3a^2H^2} \delta_m' - \frac{3}{2} - \frac{2\nu^2}{a^2H^2} \delta_m = 0,
\]

where

\[
k^2 = \left( \nu + \sigma \frac{H_0}{H} \right)\frac{k^2}{a^2H^2}.
\]

From Eq. (22), we see that the behaviors of the matter density perturbation at the sub and super-interaction scales, corresponding to \( |k^2|/a^2 \ll H^2 \) and \( |k^2|/a^2 \gg H^2 \), respectively, are quite different. At the sub-interaction scale, Eq. (22) reduces to the \( \Lambda \)CDM case,

\[
\delta_m'' + \frac{1}{2} \delta_m' - \frac{3}{2} \delta_m = 0,
\]

and the growth of \( \delta_m \) increases during the expansion of the universe, \( \delta_m \propto a \). On the other hand, when the perturbation mode \( (k, a) \) enters the super-interaction regime, Eq. (22) becomes

\[
\delta_m'' + \frac{2k^2}{3a^2H^2} \delta_m' + \frac{2\nu^2k^2}{a^2H^2} \delta_m = 0,
\]

which implies that \( \delta_m \) is suppressed (enhanced) by the dark energy perturbation with \( \nu > 0 \) (\( \nu < 0 \)). As a result, we can define the super-interaction divide \( k_{cr} = 2\pi/d_{in} \), indicating that the \( \Lambda \)CDM initial condition is applicable to the RVM. In this section, we analyze the evolution of \( \delta_m \) from Eq. (25) for

\[
\begin{align*}
&\text{Figure 1. The critical scale factor } a_{cr} \text{ as a function of the wavenumber } k \text{ with } \sigma = 0 \text{ and } \nu = 10^{-2} \text{ (solid line), } 10^{-3} \text{ (dashed line) and } 10^{-6} \text{ (dotted line), where the boundary conditions of } \Omega_m = 0.26 \text{ and } \Omega_\Lambda = 8.4 \times 10^{-5} \text{ are used.}
\end{align*}
\]

\[k_{cr}^2 = \frac{\alpha^2H^2}{|\nu + \sigma H_0/H|}.\]
two cases: (i) \( \sigma = 0 \) and (ii) \( \sigma \neq 0 \), and numerically solve \( \delta_m(a) \) and \( P(k) \).

(ii) \( \sigma = 0 \):

At the super-interaction regime \( (k \gg k_s) \), the friction in Eq. (25) plays the most important role in the evolution history of \( \delta_m \). Thus, with \( 0 < \nu < 1 \), the growth of \( \delta_m \) is frozen with \( \delta_m'' = \delta_m' = 0 \), and the matter power spectrum is suppressed by the dark energy perturbation. From Eqs. (17) and (15), one can derive that \( \ddot{h} = -2\dot{\theta}_m < 0 \) and \( \dot{\theta}_m \approx 0 \) at \( k \ll k_s \), showing that \( \delta_P = \rho_s \delta_m < 0 \). When entering the super-interaction scale, the second term in the RHS of Eq. (24) dominates, and the momentum perturbation of \( \theta_m \) becomes the same order of \( |h/2| \), implying that the RVM heats up the cold dark matter for a large value of \( k \). As a result, the particles propagate inside the super-interaction divid, and the growth of \( \delta_m \) is frozen.

In Fig. 2a, we show the evolution of \( \delta_m/\delta_m^{\Lambda C D M} \) as a function of the scale factor \( a \) with \( k = 0.25 \text{[h/Mpc]} \) and \( \nu = 0 \) (solid line), \( 10^{-6} \) (dashed line) and \( 10^{-4} \) (dotted line), where \( \delta_m^{\Lambda C D M} \) is the matter density perturbation with \( \nu = \sigma = 0 \), i.e. the \( \Lambda C D M \) limit, at \( z = 0 \) and the boundary conditions are taken to be the same as Fig. 1 with \( k = 0.25 \text{[h/Mpc]} \).

In Fig. 2b, we show the evolution of \( \delta_m/\delta_m^{\Lambda C D M} \) with \( \nu = 0 \) (solid line), \( 10^{-6} \) (dashed line) and \( 10^{-4} \) (dotted line), respectively, where \( \delta_m^{\Lambda C D M} \) is the matter density perturbation with \( \nu = \sigma = 0 \), i.e. the \( \Lambda C D M \) limit, at \( z = 0 \) and the boundary conditions are taken to be the same as Fig. 1 with \( k = 0.25 \text{[h/Mpc]} \).

Figure 2. Evolution of the matter density perturbation \( \delta_m \) as a function of the scale factor \( a \) with \( \sigma = 0 \) and (a) \( \nu = 0 \) (solid line), \( 10^{-6} \) (dashed line) and \( 10^{-4} \) (dotted line), and (b) \( \nu = 0 \) (solid line), \( -2 \times 10^{-6} \) (dashed line) and \( -10^{-5} \) (dotted line), respectively, where \( \delta_m^{\Lambda C D M} \) is the matter density perturbation with \( \nu = \sigma = 0 \), i.e. the \( \Lambda C D M \) limit, at \( z = 0 \) and the boundary conditions are taken to be the same as Fig. 1 with \( k = 0.25 \text{[h/Mpc]} \).

The matter power spectrum \( P(k) \) as a function of the wavelength \( k \) with \( \sigma = 0 \) and (a) \( \nu = 0 \) (solid line), \( 10^{-6} \) (dashed line), \( 10^{-5} \) (dotted line) and \( 10^{-4} \) (dash-dotted line), and (b) \( \nu = 0 \) (solid line), \( 10^{-7} \) (dashed line), \( 10^{-6} \) (dotted line) and \( 10^{-5} \) (dash-dotted line), together with the SDSS LRG DR7 data points, where the boundary conditions are taken to be the same as Fig. 1 respectively. On the other hand, the friction term in Eq. (25) turns into negative if \( \nu < 0 \), and the growth of the matter density perturbation sharply increases. In Fig. 3, we plot \( \delta_m/\delta_m^{\Lambda C D M} \) with \( \nu = 0 \) (solid line), \( 2 \times 10^{-6} \) (dashed line) and \( 10^{-5} \) (dotted line). From the figure, we see that the matter density perturbation \( \delta_m \) deviates from that in \( \Lambda C D M \) around \( a_r \approx 3.4 \times 10^{-2} \) \( (\nu = 2 \times 10^{-6}) \) and \( 1.5 \times 10^{-2} \) \( (\nu = 10^{-5}) \). One can observe that the RVM scenario with \( \nu < 0 \) is extremely different from that with \( \nu > 0 \). Moreover, since the RHS of Eq. (24) is negative, the momentum perturbation has no lower bound, causing \( \theta_m \rightarrow -\infty \). Consequently, the super-interaction scale in both positive and negative \( \nu \) cases results in the consistent outcomes as those in Fig. 1.

In Fig. 3, we show the matter power spectrum \( P(k) \) as a function of the wavenumber \( k_{[h/\text{Mpc}]} \) with \( \nu = 0 \) (solid line), \( 10^{-6} \) (dashed line), \( 10^{-5} \) (dotted line) and \( 10^{-4} \) (dash-dotted line). Our results in the figure are similar to those in the literature (Fabris et al. 2007). The data points in Fig. 3 come from the SDSS LRG DR7 (Reid et al. 2010). In this plot, we observe that the suppressed behaviors for \( \nu = 10^{-4}, 10^{-3} \) and \( 10^{-2} \) become important at the order of \( k \gtrsim 0.02, 0.06 \) and \( 0.2 \text{[h/Mpc]} \), respectively, which also support our results in Fig. 1. Clearly, the larger \( k \) is, the earlier the \( \delta_k \) mode enters the super-interaction regime.
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In Fig. 4, we demonstrate the matter power spectrum $P(k)$ as a function of the wavenumber $k$ with (a) $(v, \sigma) = (0, 0)$ (solid line), $10^{-5}$ (dashed line), $10^{-6}$ (dotted line) and $10^{-7}$ (dash-dotted line). As discussed early in this section, when the scale enters the super-interaction regime with $v < 0$, the evolution of $\delta_m$ acts as an increasing mode, leading to the divergence of $P(k)$ at $k \to \infty$. This phenomenon clearly illustrates that the negative $v$ case fails in describing the evolution of our universe.

(ii) $\sigma \neq 0$:

In Fig. 4b, we display the matter power spectrum $P(k)$ with $(v, \sigma) = (0, 0)$ (solid line), $10^{-5}$ (dashed line), $10^{-6}$ (dotted line) and $10^{-7}$ (dash-dotted line). In Eq. (23), the suppression or enhancement of the matter density perturbation inside the super-interaction regime is controlled by the factor $v + \sigma H_0/H$. Accordingly, it is expected that the suppression of $\delta_m$ is strengthened when $\sigma > 0$, while $P(k)$ is further suppressed.

In Fig. 4b, we exhibit $P(k)$ with $(v, \sigma) = (0, 0)$ (solid line), $10^{-5}$ (dashed line), $10^{-6}$ (dotted line) and $10^{-7}$ (dash-dotted line). This figure shows that the suppression of $P(k)$ is alleviated with $\sigma < 0$ at the late-time of the universe. If $v + \sigma H_0/H \to 0$, the frozen mode of the matter density perturbation melts, and $\delta_m$ starts growing.

However, when $v + \sigma H_0/H < 0$, the growth of $\delta_m$ turns into the extreme enhancement mode, resulting in the divergence of $P(k)$ at $k \to \infty$. As a result, we conclude that the RVM in Eq. (24) with $v + \sigma < 0$ should be also ruled out by the instability problem.

In Fig. 5 we demonstrate the evolution of the density perturbation as a function of the scale factor $a$ with $(v, \sigma) = (0, 0)$ (solid line), $10^{-4}$ (dashed line) and $10^{-5}$ (dotted line). With $\Omega_m = 0.26$ and $(v, \sigma) = (10^{-4}, 10^{-4})$, the turning point of the frozen mode to the increasing one can be estimated from the relation,

$$v + \frac{\sigma H_0}{H} = 0 \quad \Rightarrow \quad a = \left[ \frac{\Omega_m}{(\nu/\sigma)^2 + \Omega_m - 1} \right] \simeq 0.60, \quad (27)$$

which is compatible to the result in Fig. 5.

Finally, we take the massive neutrinos into consideration with

$$\Omega_m h^2 = \sum \frac{m_n}{94} \text{eV}.$$

(28)

In Fig. 6, we plot the matter power spectrum as a function of the wavenumber $k$ with selected parameter sets $(\sum m_n/\text{eV}, v, \sigma) = (0, 0, 0)$ (thin solid line), $(0.2, 0, 0)$ (thick solid line), $(0.5, 2 \times 10^{-7}, 0)$ (thick dashed line), $(0.5, 2 \times 10^{-7}, 5 \times 10^{-7})$ (thick dotted line). The matter power spectrum $P(k)$ is suppressed by the free-streaming massive neutrinos in the subhorizon scale and further frozen due to the decaying dark energy process in the super-interaction regime. As we can see, the values of $P(k)$ are overlapped for $(v, \sigma) = (5 \times 10^{-7}, 5 \times 10^{-7})$ and $(3 \times 10^{-7}, 0)$; the negative $v$ can alleviate the suppression of $P(k)$ from dark energy, but the effect is limited. In order to fit the observational data, we claim that the allowed window for model parameters should be tiny with $v, |\sigma| \lesssim O(10^{-7})$.

5 CONCLUSIONS

We have studied the matter density perturbation $\delta_m$ and matter power spectrum $P(k)$ in the RVM with $\Lambda = \Lambda_0 + \frac{\tau}{\sqrt{2}}$, where $\tau$ is a parameter that measures the degree of super-interaction.

\[\]
be a non-zero value, indicating that the massive cold dark matter is heated up by the decaying dark energy. This kind of the enhancement of $\theta_{n}$ might significantly increase the velocity of dark matter. To realize this effect, we have to further investigate physics at the scale of the dark matter halo, at which the linear perturbation theory is no longer valid, and the non-perturbative calculation is needed.

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