Unparticle constraints from SN 1987A

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The existence of an unparticle sector, weakly coupled to the standard model, would have a profound impact on supernova (SN) physics. Emission of energy into the unparticle sector from the core of SN 1987A would have significantly shortened the observed neutrino burst. The unparticle interaction with nucleons, neutrinos, electrons and muons is constrained to be so weak that it is unlikely to provide any missing-energy signature at colliders. One important exception are models where scale invariance in the hidden sector is broken by the Higgs vacuum expectation value. In this case the SN emission is suppressed by threshold effects.

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I. INTRODUCTION

The notion of unparticles was recently introduced by Georgi [1, 2] as an interesting possibility for physics beyond the Standard Model. In this scenario, there is a hidden sector with a non-trivial infrared fixed point, Λ, below which the sector exhibits scale invariance. At energies above Λ, a hidden-sector operator \( O_{\text{UV}} \) of dimension \( d_{\text{UV}} \) couples to standard model operators \( O_{\text{SM}} \) of dimension \( n \) via the exchange of heavy particles of mass \( M \),

\[
\mathcal{L}_{\text{UV}} = \frac{O_{\text{UV}} O_{\text{SM}}}{M^{d_{\text{UV}}+n-4}}.
\]

The hidden sector becomes scale invariant at Λ. The couplings then become

\[
\mathcal{L}_{U} = C_U \Lambda^{d_{\text{UV}}-d} M^{d_{\text{UV}}+n-4} O_{\text{SM}} O_{U},
\]

where \( O_{U} \) is the unparticle operator of dimension \( d \) in the low-energy limit, and \( C_U \) is a dimensionless coupling constant. Because of scale invariance, the phase space for \( O_{U} \) resembles that of \( d \) massless particles; the salient feature of the unparticle sector is that \( d \) may take on non-integer values. Unparticle phenomenology has been investigated in a large number of recent papers [3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47].

If the unparticle sector indeed appears at low energies in the form of new massless fields coupled very weakly to standard model particles, one expects that the usual stellar energy-loss limits [46, 47], notably from the neutrino burst duration of SN 1987A, will provide much more restrictive limits than any laboratory experiment could achieve. Invisible axions [48, 49, 50, 51, 52] and Kaluza-Klein gravitons [53, 54, 55] are typical examples where astrophysical limits preclude any realistic hope of finding these particles directly in collider experiments, and the same is expected for unparticles as noted by Davoudiasl [13]. We will here elaborate this idea in some detail.

If a scalar unparticle operator of dimension \( d < 2 \) couples to the standard-model Higgs, then scale invariance in the hidden sector is broken once the Higgs acquires a non-zero VEV so that there is no unparticle signature at low energies [13]. In this case the SN 1987A energy-loss argument is moot. Therefore, the astrophysical limit serves to constrain the possible realizations of the unparticle concept that could show up as missing energy at colliders.

The dominant lowest-order process for the emission of new particles \( X \) in a SN core tends to be nucleon bremsstrahlung \( N + N \rightarrow N + N + X \). The nucleon density is huge. Their interaction rate is large, and this process involves the small coupling of the new particles to lowest order. The emission of unparticles will be fully analogous, apart from phase-space modifications. In addition, processes of the form \( \bar{f} + f \rightarrow U \) now have a nonvanishing phase space and provide additional contributions. Therefore, the SN 1987A energy-loss argument provides very restrictive constraints on the unparticle couplings to nucleons, neutrinos, electrons and muons. We note that the SN 1987A emission bound is much stronger than other astrophysical constraints, as is also the case for Kaluza-Klein graviton emission.

Numerical studies of SN energy losses by axions or Kaluza-Klein gravitons reveal that the neutrino burst would have been excessively shortened unless the volume energy-loss rate of the SN core obeys

\[
Q \lesssim 3 \times 10^{33} \text{ erg cm}^{-3} \text{ s}^{-1},
\]

where \( Q \) is to be calculated at the benchmark conditions \( T = 30 \text{ MeV} \) and \( \rho = 3 \times 10^{14} \text{ g cm}^{-3} \). Armed with this simple criterion, all that is needed is an estimate of \( Q \) for unparticle emission.

We begin in Sec. II with the nucleon bremsstrahlung process, stressing its general features for the emission of different types of new radiation. In Sec. III we consider the pair-annihilation process for various particles that are abundant in a SN core. We conclude in Sec. IV.
II. NUCLEON BREMSSTRAHLUNG

A. General approach

A reliable calculation of the bremsstrahlung rate $N + N \rightarrow N + N + X$ in a SN core is not possible because the $NN$ interaction potential is not well known, $NN$ correlation effects are likely strong, and multiple-scattering effects can be important. Therefore, one cannot do much better than an “educated dimensional analysis” based on simple principles.

In the limit $\omega \rightarrow 0$, where $\omega$ is the frequency of the emitted radiation, bremsstrahlung factorizes into an essentially classical radiation process and the ordinary collision. The advantage of this approach is that for nucleons one can estimate the collision rate from measured cross sections or phase shifts even without a detailed model of the interaction potential [51, 54].

Assuming weakly interacting radiation we can employ lowest-order perturbation theory for the interaction between the radiation and the medium. The emitted power is then a phase-space integral over the dynamical structure function $S(\omega, \mathbf{k})$ of the medium. Here, $S$ is the thermal average of a correlator of the operator to which the radiation couples. While we do not know the dynamical structure function for a strongly interacting medium, we can still take advantage of its general properties. In particular, the principle of detailed balancing tells us that the probability for emitting a quantum of energy $\omega$ from a thermal medium is $e^{-\omega/T}$ times the probability of absorbing one.

These ideas suggest that the volume energy-loss rate is roughly given by

$$Q \approx \langle \sigma v \rangle n_B^2 \int_0^\infty d\omega \frac{dI}{d\omega}|_{\text{low-}\omega} e^{-\omega/T},$$

where $\sigma$ is the nucleon scattering cross section, $v$ their relative velocity, and $n_B$ their number density (the baryon density). The dimensionless quantity $dI/d\omega$ is the spectrum of energy emitted in a single collision and includes all coupling constants and the radiation phase space.

Typically $dI/d\omega$ is of the power-law form $G^2|\omega|^p$ where $G$ is a coupling constant of dimension (energy)$^{-p/2}$ and includes numerical factors. Therefore, we estimate

$$Q \approx \langle \sigma v \rangle n_B^2 G^2 T^{p+1},$$

where we have ignored a numerical factor $\Gamma(p+1)$ that plays little role as long as $p$ is not too large.

B. Graviton Bremsstrahlung

The usual axion constraints from SN 1987A were derived along these lines [52]. Moreover, two of us have carried out this exercise in detail, including all numerical factors, for the case of graviton bremsstrahlung [55]. In this case the classical emission rate is

$$\frac{dI}{d\omega}|_{\text{low-}\omega} = \frac{8}{5\pi} (G_N m^2) v^4 \sin^2 \Theta_{\text{CM}},$$

where $G_N$ is Newton’s constant, $m$ the nucleon mass, $v$ the velocity of the colliding nucleons, and $\Theta_{\text{CM}}$ the scattering angle in the center-of-mass frame. The quantity $G_N m^2$ plays the role of the square of a dimensionless coupling constant and the $v^4$ behavior reflects the quadrupole nature of the radiation. Assuming that $NN$ scattering is isotropic with a total cross section $\sigma$, after performing the nucleon phase-space integral as well as the $d\omega$ integration it was found that

$$Q = \left[ \frac{512 \ln 2}{3^{3/2} 5^{5/2}} (G_N m^2) T \right] \times \left[ \sigma n_B^2 \left( \frac{3T}{m} \right)^{5/2} \right].$$

Of course it is somewhat arbitrary how to group the numerical factors. We note that the average velocity of thermal nucleons is given by $\langle v^2 \rangle = 3T/m$ so that $\langle v^2 \rangle = 3T/m$, motivating the use of $3T$ rather than $T$ in the last term that derives from the nucleon velocity factors. (Four powers of velocity come from the quadrupole nature of the radiation, and another factor from the $NN$ relative velocity.) In this way the overall numerical coefficient is 0.818 and thus very close to unity. Graviton bremsstrahlung has a flat spectrum so that we have $p = 0$ in the spirit of Eq. (4), implying a simple factor $T^{p+1} = T$ within the first term.

C. Unparticle emission

In the simplest case unparticles couple via a vector current to nucleons, an assumption made in all previous studies. The structure of the coupling is

$$\mathcal{L}_{UN} = C_{\text{UN}} \frac{A_{\text{UV}} - d}{M_{\text{UV}} + 1} \bar{N} \gamma_\mu N O_U^\mu.$$

Nucleon bremsstrahlung would then seem to be normal dipole emission similar to electromagnetic bremsstrahlung. However, if the interaction with all nucleons is the same, bremsstrahlung is suppressed in the nonrelativistic limit, just as it is suppressed in the nonrelativistic limit for $e^- + e^- \rightarrow e^- + e^- + \gamma$. Therefore, we expect quadrupole radiation to dominate, involving a factor $v^4$ as in graviton bremsstrahlung.

Therefore, we scale the emission rate from Eq. (7), keeping the second factor that encodes the quadrupole nature and nucleon phase space, while adapting the first factor to the unparticle case. From dimensional analysis we thus expect

$$Q = C_{\text{UN}}^2 \frac{A_{\text{UV}} - d}{M_{\text{UV}} + 1} T^{2d-1} \sigma n_B^2 \left( \frac{3T}{m} \right)^{5/2}.$$

To evaluate this rate we assume $\sigma = 25 \times 10^{-27}$ cm$^2$ that is typical for the relevant conditions and was rec-
The emissivity from pair annihilation of neutrinos is roughly estimated by

$$Q \sim C^{2}_{U\nu} \frac{\Lambda^{2(d_{UV} - d)}}{M^{2(d_{UV} - 1)}} T^{2d + 3},$$

which leads to the constraints, for $C_{U\nu} = 1$,

$$\Lambda \lesssim \begin{cases} 
& 20 \text{ GeV} \quad d = 1 \\
& 500 \text{ GeV} \quad d = 2
\end{cases}$$

assuming $d_{UV} = 3$ and $M = 1000$ TeV as before.

For $d = 3/2$ and 2 these bounds are almost identical to those deduced from bremsstrahlung in Eq. (11), although they are based on different physics. The reason is that the smaller neutrino density is compensated by the fact that the $\nu\bar{\nu}$ channel is not suppressed by the $v^4$ factor. For different values of $M$ the bounds on $\Lambda$ scale in the same way as those from the bremsstrahlung calculation so that the results of Fig. 1 apply also in this case. No bound exists for $d = 1$ from $f + f \to U$, since this case corresponds to annihilation into a single massless particle, and its rate must vanish exactly because of energy-momentum conservation.

The bounds Eq. (12) were calculated assuming non-degenerate, relativistic fermions. This approximation is very good for $\nu_e$ and $\nu_\mu$. However, electron neutrinos are degenerate with a chemical potential $\mu$ typically of order 150–200 MeV, implying a degeneracy parameter around 5–7 for $T = 30$ MeV. The presence of a chemical potential reduces the number of pairs, i.e., the quantity $n_{\nu_e} n_{\bar{\nu}_e}$ is largest for a vanishing chemical potential. However, for degeneracy factors up to roughly 8 the suppression of the annihilation rate is less than an order of magnitude (for illustration see Fig. 2 of Ref. [56]). The degeneracy factor for electrons is only slightly larger so that the suppression of $e^+e^-$ annihilation is only slightly worse.

Because of its high temperature the hot proto-neutron star is also abundant in muons. The muon rest mass of 106 MeV does not lead to a very substantial difference in number density relative to massless fermions. At $T = 30$ MeV the number density of muons is suppressed only by a factor $\sim 2.5$ relative to massless fermions, always ignoring chemical potentials that are small for muons in a SN core. An additional suppression factor of about 0.5 comes from the relative velocity of muons so that the annihilation rate overall is roughly 0.2 times that of a massless fermion species.

Lastly, we note that a bound on the $d = 1$ scenario can still be obtained from pair annihilation of charged leptons based on the process $e^+e^- \to U + \gamma$ or $\mu^+\mu^- \to U + \gamma$ which is not phase-space suppressed. Since the squared matrix element is suppressed only by $O(\alpha)$ relative to $f f \to U$, the bound for $d = 1$ can be estimated from taking the emissivity as $\alpha$ times the naive rate given in Eq. (12), and taking into account the suppression from degeneracy (electrons) or the mass threshold (muons). Note that for $d = 1$ the suppression factors enter as a
fourth root, giving an estimated bound for \( d = 1 \) of \( \Lambda \lesssim 20 - 30 \) GeV for \( M = 1000 \) TeV.

Given the overall numerical uncertainties of our limits, we conclude that the pair annihilation and bremsstrahlung limits are nearly identical for all cases, i.e., apart from possible difference in the overall coefficients \( C_U \) to different particle species, the interaction with nucleons, neutrinos of all flavors, electrons and muons are constrained by the limits shown in Fig. 1.

IV. DISCUSSION

We have applied the well-known energy-loss limit based on the SN 1987A neutrino burst duration to Georgi’s new idea of unparticles that can manifest themselves as missing energy in collider experiments with a peculiar phase-space behavior. As expected, the SN limits are very restrictive as long as the unparticle radiation can be emitted without threshold at the relatively low energies prevalent in the SN context. Our approximate constraints shown in Fig. 1 apply without significant modifications to nucleons, neutrinos of all flavors, electrons and muons.

Unparticle signatures can still be detected at colliders in models where scale invariance in the hidden sector is broken by the Higgs vacuum expectation value. In this case the SN emission is suppressed by threshold effects. Thus our astrophysical limits provide a severe restriction on the type of unparticle models that can be detected at colliders.

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