High energy astrophysical neutrino flux
and modified dispersion relations

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Abstract

Motivated by the interest in searches for violation of CPT invariance, we study its possible effects in the flavour ratios of high-energy neutrinos coming from cosmic accelerators. In particular, we focus on the effect of an energy-independent new physics contribution to the neutrino flavour oscillation phase and explore whether it is observable in future detectors. Such a contribution could be related not only to CPT violation but also to a nonuniversal coupling of neutrinos to a torsion field. We conclude that this extra phase contribution only becomes observable, in the best case, at energies greater than $10^{16.5}$ GeV, which is about five orders of magnitude higher than the most energetic cosmological neutrinos to be detected in the near future. Therefore, if these effects are present only in the oscillation phase, they are going to be unobservable, unless a new mechanism or source capable to produce neutrinos of such energy were detected.

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I. INTRODUCTION

Experimental evidence has confirmed that flavour transitions are the solution to the former so-called solar and atmospheric neutrino deficit problems [1, 2, 3]. Further evidence was provided by experiments performed with neutrinos generated in particle accelerators and nuclear reactors, such as KamLAND [4] and K2K [5]. The mechanism responsible for these transitions requires neutrinos to be massive: the probability of a flavour transition is oscillatory, with oscillation length \( \lambda^{\text{std}} \equiv \frac{4\pi E}{\Delta m^2} \), where \( E \) is the neutrino energy and \( \Delta m^2 \) is the difference of the squared masses of the different neutrino mass eigenstates. However, even though this mass-driven mechanism is the dominant one in the energy regimes that have been explored experimentally (MeV-TeV), there is still the possibility that alternative mechanisms contribute to the flavour transitions in a subdominant manner, which perhaps can manifest at higher energies.

Although these alternative mechanisms involving new physics (NP) are able to produce flavour transitions, it is known that none of them can explain the combined data from atmospheric, solar, accelerator, and reactor neutrino experiments performed in the MeV-TeV range, unlike the pure \( \Delta m^2 \) oscillation mechanism [6, 7, 8, 9]. Some of these alternatives [10] are the violation of the equivalence principle (VEP), of Lorentz invariance (VLI), of CPT invariance (VCPT), the non-universal coupling of neutrinos to a space-time torsion field (NUCQ), decoherence during the neutrino’s trip, and non-standard interactions (NSI).

Typically, these mechanisms result in oscillation lengths \( \lambda^{NP} \) that have a different dependence on \( E \), usually expressed as a power-law, \( \lambda^{NP} \sim E^n \), with the value of \( n \) depending on which mechanism is being considered: for instance, \( n = 0 \) for VCPT and NUCQ and \( n = 1 \) for VEP and VLI, while with \( n = -1 \) the standard \( \Delta m^2 \) oscillations are recovered. Atmospheric events from Super-Kamiokande (SK) [7] were used to find the value \( n = -0.9 \pm 0.4 \) at 90% C.L., thus confirming the dominance of the \( \Delta m^2 \) oscillation mechanism and forcing any other mechanisms to be subdominant, at least within the energy range and pathlength considered in said analysis.

So far, searches for NP effects in neutrino oscillations have been limited to energies ranging from a few MeV to a few GeV [6, 7, 8, 9] and have turned out negative. However, proposals for analyses of atmospheric neutrinos with energies of up to \( 10^4 \) TeV in second-generation neutrino detectors such as IceCube [13] and ANTARES [14] are being con-
sidered. Due to the energy dependence of the oscillation lengths, the oscillation phases scale as \((2\pi L/\lambda^{NP}) / (2\pi/\lambda^{std}) \sim E^{1-n}\), that is, the relative dominance of the NP contribution grows with the neutrino energy provided that \(n \leq 0\), so that the observation of very energetic neutrinos -such as the ones expected from presumed cosmic accelerators like active galaxies and gamma ray bursts- would offer a means to establish whether the \(\Delta m^2\) oscillation mechanism is still the dominant one at high energies or to otherwise set stronger bounds on the NP parameters.

In this work, we have introduced the aforementioned new physics through the use of a modified dispersion relation, and focused our analysis on the case of \(n = 0\), corresponding, as we will see, to an energy-independent NP contribution to the neutrino flavour oscillation phase. We have calculated the proportion of each flavour arriving at Earth from distant cosmic accelerators and explored how it is affected by the parameters that control the new physics, and whether these effects are observable at all.

The outline of the paper is as follows. Section II describes how the NP arising from a modified dispersion relation affects the flavour-transition probability for neutrinos that travel cosmological distances. In Section III we explore the case of an energy-independent NP contribution and its effects on the flavour ratios. Finally, in Section IV we present our conclusions.

II. FLAVOUR-TRANSITION PROBABILITY IN THE PRESENCE OF A MODIFIED DISPERSION RELATION

The NP effects can modify flavour transitions in two ways: by transforming both the oscillation length and the neutrino mixing angles or by altering only the first. The former case occurs, for instance, when considering the low energy phenomenological model of string theory, known as 'Standard Model Extension' and has been examined using SK and K2K data. The second case can be achieved by considering a modified dispersion relation which departs from the well-known formula \(E^2 = p^2 + m^2\). Because we wish to explore whether solely effects on the phase are observable at high energies, we follow this second alternative and consider the following modified dispersion relation, which allows us to
study the contributions of NP effects in a model-independent way:

\[ E^2 = p^2 + m^2 + \eta' p^2 \left( \frac{E}{m_P} \right)^\alpha = p^2 + m^2 + \eta p^2 E^\alpha, \]

where \( m_P \simeq 10^{19} \) GeV is the Planck mass, \( \eta' = \eta m_P^\alpha \) is an adimensional parameter that controls the strength of the NP effects and, following the literature, \( \alpha \) has been chosen to be of integer value. Such a dispersion relation assumes that the scale of NP effects is the Planck scale where, according to theories of quantum gravity, space-time might become “foamy”. Eq. (1) was recently used to predict the sensitivity of the ANTARES neutrino telescope to NP effects in the high-energy atmospheric neutrino flux.

We now derive the flavour-transition probability in the presence of NP effects, for neutrinos that propagate over a cosmological distance. Flavour transitions arise as a consequence of the fact that flavour eigenstates \( |\nu_\alpha\rangle \) \( (\alpha = e, \mu, \tau) \) are not also mass eigenstates \( |\nu_i\rangle \) \( (i = 1, 2, 3) \), but rather a linear combination of them, i.e. \( |\nu_\alpha\rangle = \sum_{i=1}^3 U_{\alpha i}^* |\nu_i\rangle \), with \( U_{\alpha i}^* \) elements of the neutrino mixing matrix.

Using the standard dispersion relation, it is a common procedure to derive an approximate expression for the momentum of the \( i \)-th neutrino mass eigenstate,

\[ p_i = \sqrt{E^2 - m_i^2} \simeq E - \frac{m_i^2}{2E}, \]

where \( m_i \) is the mass of the neutrino and \( E \) is its energy, and are such that, at the energies that we have considered, \( m_i \ll E \). From this equation we obtain the usual expression for the momenta difference:

\[ \Delta p_{ij} \equiv p_j - p_i = \frac{\Delta m_{ij}^2}{2E}. \]

In accordance with the latest bounds obtained from global fits, we have set the three mixing angles that parametrise \( U \) to \( \sin^2 (\theta_{12}) = \sqrt{0.304}, \theta_{13} = 0 \) and \( \theta_{23} = \pi/4 \). The mass-squared differences have been set to \( \Delta m_{21}^2 = 8.0 \times 10^{-5} \text{ eV}^2 \) and \( \Delta m_{32}^2 = 2.5 \times 10^{-3} \text{ eV}^2 \), and we have assumed a normal mass hierarchy (i.e. \( m_3 > m_1 \)), so that \( \Delta m_{31} = \Delta m_{32} + \Delta m_{21} \). The probability that a neutrino created with flavour \( \alpha \) is detected as having flavour \( \beta \) after having propagated a distance \( L \) in vacuum is given by

\[ P_{\nu_\alpha \to \nu_\beta} (E, L) = \delta_{\alpha\beta} - 4 \sum_{i>j} \text{Re} (J_{ij}^{\alpha\beta}) \sin^2 \left( \frac{\Delta p_{ij} L}{2} \right), \]

where \( J_{ij}^{\alpha\beta} \equiv U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^* \). Since \( \theta_{13} = 0 \), \( U \) is a real matrix, independent of the CP-violation phase, \( \delta_{CP} \).
In the case of the modified dispersion relation of Eq. (1) we can also find an expression for the momenta difference. To first order in \( \eta_i \), and discarding terms higher than second power in \( m_i \) or involving \( \eta_i m_i^2 \), we obtain

\[
p_i \simeq E - \frac{m_i^2}{2E} - \frac{\eta_i E^n}{2},
\]
with \( n \equiv \alpha + 1 \), and hence

\[
\Delta \tilde{p}_{ij} = \frac{\Delta m_{ij}^2}{2E} + \frac{\Delta \eta_{ij}^{(n)} E^n}{2},
\]
where \( \Delta \eta_{ij}^{(n)} \equiv \eta_i^{(n)} - \eta_j^{(n)} \), for the NP mechanism with an \( E^n \) energy dependence. Note that it is necessary that the \( \eta_i \) have different values for different mass eigenstates in order to have a nonzero NP contribution to the momenta difference. The corresponding oscillation probability is Eq. (4) with \( \Delta p_{ij} \rightarrow \Delta \tilde{p}_{ij} \); hence, the NP affects solely the oscillation phase, but not its amplitude.

Since \( L \gg 1 \) for high-energy astrophysical neutrinos, \( \sin^2 (\Delta p_{ij} L/2) \) is a rapidly oscillating function and so, due to the limited energy resolution of neutrino telescopes, the average flavour-transition probability is sometimes used instead, i.e.

\[
\langle P_{\nu_\alpha \rightarrow \nu_\beta} \rangle = \sum_i |U_{\alpha i}|^2 |U_{\beta i}|^2.
\]

Let \( \langle P_{\nu_\alpha \rightarrow \nu_\beta} \rangle^{std} \) be the standard average probability, that is, when there are no NP effects present. If the extra term in \( \Delta \tilde{p}_{ij} \) has \( n > 0 \), its effect will be that, at high energies, \( P_{\nu_\alpha \rightarrow \nu_\beta} \) will oscillate even more rapidly with energy, but still around the same mean value \( \langle P_{\nu_\alpha \rightarrow \nu_\beta} \rangle^{std} \). If \( n < 0 \), the oscillations will continue up to high energies, also around the same mean, and at a high enough value, when \( \Delta p_{ij} = \Delta m_{ij}^2 / 2E \rightarrow 0 \), the probability will tend to zero. However, if \( n = 0 \), then the extra term in \( \Delta \tilde{p}_{ij} \) is energy-independent and so when \( \Delta p_{ij} \rightarrow 0 \), \( P_{\nu_\alpha \rightarrow \nu_\beta} \) becomes constant, but different from zero due to the existence of the extra term. Furthermore, when this happens, and depending on the values of the \( \Delta \eta_{ij}^{(0)} \), it is in principle possible for the constant probability to be different from \( \langle P_{\nu_\alpha \rightarrow \nu_\beta} \rangle^{std} \). Such a nonzero, constant probability at high energies could therefore be interpreted as being due to the contribution from energy-independent new physics. We will focus on this possibility.

Although our analysis of NP effects using Eq. (1) is model-independent, \( \Delta \eta_{ij}^{(0)} \) takes a different form depending on the particular mechanism being considered\([6, 13]\). In the energy-independent oscillation mechanism that we are focusing on, the extra contribution could be
due to VCPT through Lorentz invariance violation, in which case

\[ \Delta \eta_{ij}^{(0)} = b_i - b_j \equiv b_{ij}, \]  

(8)

with \( b_i \) the eigenvalues of the Lorentz-violating CPT-odd operator

\[ \bar{\nu}_L b^{\alpha\beta} \gamma_\mu \nu_\nu^\beta. \]

Alternatively, the contribution could be due to NUCQ, and in this case we would consider different couplings, \( k_i \neq k_j \) (for mass eigenstates \( i \) and \( j \)), to a torsion field \( Q \), so that

\[ \Delta \eta_{ij}^{(0)} = Q (k_i - k_j) \equiv Qk_{ij}. \]

(9)

Strict bounds have been set on the parameters that control the energy-independent NP mechanism using data from atmospheric and solar neutrinos, as well as SK and K2K, with energies up to about 1 TeV:

\[ b_{21} \leq 1.6 \times 10^{-21} \text{ GeV} \quad , \quad b_{32} \leq 5.0 \times 10^{-23} \text{ GeV} . \]

(10)

Because the relative dominance of the NP energy-independent phase over the standard oscillation phase increases with neutrino energy, i.e. \( (\Delta p_{ij} - \Delta p_{ij}) / \Delta p_{ij} \sim E \), we would like to look at the most energetic neutrinos available. Hence, we will consider neutrinos originating at cosmic accelerators, such as active galactic nuclei, where it is presumed that they are created with energies of up to \( 10^{11} \) GeV. Because the typical distance to these accelerators is in the order of hundreds of Mpc, we must include in the flavour-transition probability the effect of cosmological expansion. Hence, instead of the argument that appears in the sine of Eq. (4), we define an accumulated phase\(^{19}\) \( \phi_{ij} \) as follows:

\[ \phi_{ij} (t_f, t_i) = \int_{t_i}^{t_f} \Delta p_{ij} (\tau) d\tau = \int_{t_i}^{t_f} \frac{\Delta m_{ij}^2}{2E_\circ} \left( \frac{\tau}{t_\circ} \right)^{2/3} d\tau = \frac{3}{10} \frac{\Delta m_{ij}^2 t_\circ}{E_\circ} \left[ \left( \frac{t_f}{t_\circ} \right)^{5/3} - \left( \frac{t_i}{t_\circ} \right)^{5/3} \right] \]

(11)

where \( t_i \) and \( t_f \) are the times at which the neutrino was produced and detected, respectively; \( t_\circ = 13.7 \text{ Gyr} \) is the age of the Universe\(^{20}\); and we have used the relation between the energy at detection \((E_\circ)\) and production epochs \((E)\), in an adiabatically expanding universe, \( E (\tau) = E_\circ (t_\circ/\tau)^{2/3} = E_\circ (1 + z) \). Considering the detection time \( t_f \) in the present epoch, \( t_f = t_\circ \), we obtain the accumulated phase

\[ \phi_{ij} (E_\circ, z) = 1.97 \times 10^{23} \frac{\Delta m_{ij}^2}{E_\circ [\text{GeV}]} \left[ \left[ 1 - (1 + z)^{-5/2} \right] \right] , \]

(12)

where we have made use of the relation \( t_i/t_\circ = (1 + z)^{-3/2} \).
By replacing the momenta difference $\Delta p_{ij}$ with $\Delta \tilde{p}_{ij}$, we obtain, correspondingly,

$$
\tilde{\phi}_{ij} (E_o, z) = \phi_{ij} (E_o, z) + \frac{\Delta \eta^{(n)}_{ij} E_o n_o}{2} \left[ 1 - (1 + z)^{n-3/2} \right] \equiv \phi_{ij} (E_o, z) + \xi^{(n)}_{ij} (E_o, z) ,
$$

(13)

with $\xi^{(n)}_{ij}$ the contribution to the phase due to the NP effects. For $n = 0$,

$$
\xi^{(0)}_{ij} (E_o, z) = 3.28 \times 10^{41} b_{ij} [\text{GeV}] \left[ 1 - (1 + z)^{-3/2} \right] .
$$

(14)

Hence, instead of the traditional expression in Eq. (4) for $P_{\nu_\alpha \to \nu_\beta}$, we will employ

$$
P_{\nu_\alpha \to \nu_\beta} (E_o, z) = \delta_{\alpha\beta} - 4 \sum_i \text{Re} (J_{\alpha\beta}^{ij}) \sin^2 \left( \frac{\phi_{ij}}{2} \right) ,
$$

(15)

where the explicit expression for $\phi_{ij} \equiv \phi_{ij} (E_o, z)$ is either of Eqs. (12) or (13), depending on which dispersion relation is being considered.[25]

### III. OBSERVABILITY OF THE NP EFFECTS IN THE HIGH-ENERGY NEUTRINO FLAVOUR RATIOS

Using the flavour-transition probability obtained in the previous section, Eq. (15), we can calculate the ratio of neutrinos of each flavour to the total number of neutrinos that arrive at the detector from a source with redshift $z$. For $\alpha$-flavoured neutrinos ($\alpha = e, \mu, \tau$) with energy $E_o$, this is

$$
\Upsilon^{D}_{\nu_\alpha} (E_o, z) = \sum_{\beta=e,\mu,\tau} P_{\nu_\beta \to \nu_\alpha} (E_o, z) \Upsilon^{S}_{\nu_\beta} ,
$$

(16)

where $\Upsilon^{D}_{\nu_\alpha}$ is the ratio at the detector and $\Upsilon^{S}_{\nu_\beta}$ is the ratio at the source. The latter is estimated assuming that neutrinos are secondaries of high-energy proton-proton or proton-photon collisions, which produce pions that decay into neutrinos and muons which decay into neutrinos too[21, 22, 23]: $\pi^+ \to \mu^+ \nu_\mu \to e^+ \nu_e \overline{\nu}_\mu \nu_\mu$, $\pi^- \to \mu^- \overline{\nu}_\mu \to e^- \overline{\nu}_e \nu_\mu \overline{\nu}_\mu$. It is easy to see that $\Upsilon^{S}_{\nu_e} : \Upsilon^{S}_{\nu_\mu} : \Upsilon^{S}_{\nu_\tau} = 1/3 : 2/3 : 0$. (Actually, $\nu_\tau$ are expected to be produced through the decay of $D_{s}^{\pm}$ charmed mesons generated also in $pp$ and $p\gamma$ collisions. However, $D_{s}^{\pm}$ production is strongly suppressed[23] and $\Upsilon^{S}_{\nu_\tau} < 10^{-5}$.)

The $\Upsilon^{D}_{\nu_\alpha}$ are very rapidly oscillating functions of energy. Taking into account the limited energy sensitivity of current and envisioned neutrino telescopes (AMANDA-II, for instance, had an energy resolution of 0.4 in the logarithm of the energy of the $\nu_\mu$-spawned muon[24]),
FIG. 1: (left) Eigenvalues $b_{21}$ and $b_{32}$ as functions of $E^{NP}$, the energy at which the standard and NP energy-independent oscillation phases become comparable, i.e. $\phi_{ij} \sim \xi_{ij}^{(0)}$, according to Eq. (18). The redshift $z = 1$. The current upper bounds are plotted as horizontal lines. Notice that, due to these bounds, $E^{NP}$ cannot be lower than about 1 GeV. (right) Standard oscillation phase $\phi_{21}$ and phase including the energy-independent contribution, $\tilde{\phi}_{21}$, as functions of neutrino energy. The redshift $z = 1$. Note that the phases start to differ at $E^{NP} = 10^6$ GeV, which corresponds to $b_{21} = 6.1 \times 10^{-29}$ GeV and $b_{32} = 1.9 \times 10^{-27}$ GeV. Below this energy, they are indistinguishable.

we see that they are sensitive not to the instantaneous value of the ratios, $\Upsilon^D_{\nu\alpha}(E_o, z)$, but rather to the energy-averaged flavour ratios

$$
\langle \Upsilon^D_{\nu\alpha}(E_o, z) \rangle = \frac{1}{\Delta E_o} \int_{E_o^{\text{min}}}^{E_o^{\text{max}}} \Upsilon^D_{\nu\alpha}(E'_o, z) \, dE'_o,
$$

where $E_o^{\text{min}} = E_o - \delta E_o$, $E_o^{\text{max}} = E_o + \delta E_o$ and $\Delta E_o \equiv E_o^{\text{max}} - E_o^{\text{min}} = 2\delta E_o$, with $\delta E_o$ a small energy displacement. Without the NP effects, the high-energy neutrino flux from a distant astrophysical source is equally distributed among the three flavours, i.e. $\langle \Upsilon^D_{\nu_e} \rangle : \langle \Upsilon^D_{\nu_\mu} \rangle : \langle \Upsilon^D_{\nu_\tau} \rangle = 1/3 : 1/3 : 1/3$.

In the presence of NP effects, however, the detected flavour ratios might be modified. Given that the relative dominance of the energy-independent NP phase $\xi_{ij}^{(0)}$ over the standard phase $\phi_{ij}$ grows with energy, i.e. $\xi_{ij}^{(0)}/\phi_{ij} \sim E_o$, we would expect that any modifications became more pronounced in the UHE range, PeV–EeV, or higher. As explained in Section II while the NP phase remains constant in energy, the standard phase decreases and, as a consequence, beyond a certain threshold (determined by the values of the $b_{ij}$), the detected ratios $\Upsilon^D_{\nu\alpha}$ would acquire a constant nonzero value, which might differ from the standard
FIG. 2: Energy-averaged detected $\nu_\mu$ ratio $\Upsilon^D_{\nu_\mu}$, Eq. (17), as a function of neutrino energy $E_o$, for different values of the $b_{ij}$. Note that the ratio becomes constant only for unrealistically high energies: $\sim 10^{16.5}$ GeV in the best case, when the $b_{ij}$ are set to their upper bounds. For lower values of the $b_{ij}$, the energy at which the ratio becomes constant is higher. The neutrino flux from cosmic accelerators is predicted to span up to about $10^{11}$ GeV; hence, the regime of constant $\Upsilon^D_{\nu_\mu}$ due to an energy-independent contribution to the oscillation phase would not be observable.

ratios $1/3 : 1/3 : 1/3$, thus providing a distinct phenomenological signature of a possible energy-independent contribution to the oscillation phase.

As a means of estimating the values of the $b_{ij}$ for which the NP phase starts to be of importance, we can demand that $\xi^{(0)}_{ij} \sim \phi_{ij}$. From this requirement, we can calculate, for given values of the $b_{ij}$, the energy $E^{NP}$ above which the NP effects are expected to become increasingly more dominant in the oscillation. Doing this, we obtain

$$E^{NP} [\text{GeV}] = 6 \times 10^{-19} \frac{\Delta m^2_{ij} [\text{eV}^2]}{b_{ij} [\text{GeV}]} \frac{1 - (1 + z)^{-5/2}}{1 - (1 + z)^{-3/2}}.$$  \quad (18)

The left panel of Fig. 2 shows a plot of $b_{21}$ and $b_{32}$ as functions of $E^{NP}$. The current upper bounds are shown as horizontal lines. The lower the value of $E^{NP}$, the earlier the NP effects would manifest. Notice that, due to the current bounds, $E^{NP}$ cannot be lower than about 1 GeV. The plots have been generated for a fixed $z = 1$; for lower values of $z$ we will have a
higher value of $b_{ij}$ (30% if we take $z = 0.03$), while we will obtain a decrease in the values of $b_{ij}$ for large $z$ (a 20% decrease for $z = 6$). The right panel of Fig. 1 shows the standard and the energy-independent NP phases, $\phi_{21}$ and $\tilde{\phi}_{21}$, respectively, as functions of neutrino energy, assuming that $E^{NP} = 10^6$ GeV. Notice that the phases start to differ precisely at this energy.

For concreteness, we will study the detected ratio of $\nu_\mu$ defined in Eq. (17), since our conclusions are independent of the chosen flavour. Fig. 2 shows the predicted ratio calculated for different values of $b_{21}$ and $b_{32}$ (we have assumed that $b_{31} = b_{32} + b_{21}$) as a function of $E_o$.

Fig. 2 shows that $\Upsilon_{\nu_\mu}^{D}$ indeed becomes constant and different from $1/3$ after a certain energy threshold. This occurs when the standard phase $\phi_{ij} \rightarrow 0$, so that, effectively, the oscillation phase is reduced to the energy-independent NP contribution, i.e. $\tilde{\phi}_{ij} \rightarrow \xi_{ij}^{(0)}$, and the transition probabilities become constant. Note, however, that $\Upsilon_{\nu_\mu}^{D}$ is constant only for $E_o \gtrsim 10^{16.5}$ GeV in the most promising case, that is, when the $b_{ij}$ equal their current upper bounds. This is about five orders of magnitude higher than the energy of the most energetic neutrinos expected from cosmic accelerators. For smaller values of these parameters, the energy at which the ratio becomes constant is even higher. Using closer or more distant sources, effectively decreasing or increasing $z$, does not affect the energy threshold, but only modifies the constant value reached by $\Upsilon_{\nu_\mu}^{D}$. Therefore, we conclude that, given the current upper bounds on the $b_{ij}$, an energy-independent NP contribution to neutrino oscillations would be visible in the high-energy astrophysical neutrino flux only if it modifies the oscillation amplitude (i.e. the mixing angles), as well as the phase.

In light of this conclusion, within the formalism used in the present work, a comparative calculation, with and without NP effects, of high-energy astrophysical neutrinos detected at a second-generation neutrino telescope such as IceCube becomes unnecessary.

IV. SUMMARY AND CONCLUSIONS

We have considered the effect of a modified dispersion relation on the detected flavour ratios of high-energy neutrinos from cosmic accelerators. In the scenario of new physics that we have explored, the flavour oscillation phases are modified by the addition of energy-independent terms which depend on the parameters $b_{ij}$. This contribution could correspond to a violation of CPT symmetry or to a nonuniversal neutrino coupling to a torsion field.
The current upper bounds on the $b_{ij}$ are strict: $b_{21} \leq 5.0 \times 10^{-23}$ GeV and $b_{32} \leq 1.6 \times 10^{-21}$ GeV.

At sufficiently high energies, the oscillation phases are dominated by the energy-independent terms and the flavour ratios become constant and, possibly (depending on the values of the $b_{ij}$) different from the average value of the ratios in the standard oscillation case, when new physics effects are absent. We have found, however, that even in the best case, when the $b_{ij}$ are set to their upper bounds, the ratios are constant only for energies above $10^{16.5}$ GeV, about five orders of magnitude higher than the most energetic neutrinos that are expected from cosmic accelerators. Lower values of the $b_{ij}$ will only result in higher energy thresholds for the ratios to become constant.

Therefore, we conclude that, even though there could be, in principle, a clear signature of the presence of energy-independent contributions to the neutrino flavour oscillations, these are not detectable in the flavour ratios of high-energy neutrinos from cosmic accelerators if they affect solely the oscillation phases.

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[25] In the limit of very small z, Eq. (15) reduces to Eq. (4), the expression for the transition probability for neutrinos that travel much less than cosmological distances, i.e. solar, atmospheric
and reactor neutrinos. This can be seen by making $t_i = t_o - \Delta t$ in Eq. (11), with $\Delta t \ll 1$, and discarding terms of order $(\Delta t)^2$ and higher.