Elastic Scattering of Neutrinos off Polarized Electrons

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Abstract

We calculate the cross sections for elastic \(\nu_l + e^- \rightarrow \nu_l + e^-\) and \(\bar{\nu}_l + e^- \rightarrow \bar{\nu}_l + e^-\) scattering \((l = e, \mu \text{ or } \tau)\) in the Born approximation and with exactly fixed polarization states of target and final-state electrons, discussing their sensitivity to the incident (anti)neutrino flavor. We suggest investigation of the flavor composition of a (anti)neutrino beam by a flux-independent analysis of the scattering of its constituents off polarized electrons.

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The main objective of this letter is to present analytic formulae for the differential cross sections of the elastic scatterings $\nu_l + e^- \to \nu_l + e^-$ and $\bar{\nu}_l + e^- \to \bar{\nu}_l + e^-$ ($l = e, \mu$ or $\tau$), and to propose investigation of the flavor of (anti)neutrinos by a flux-independent analysis of their elastic scattering off polarized electrons. We investigate these scattering processes to lowest order in the electroweak interaction with all polarization states specified. We neglect terms of order $r/M^2_W$, where $r$ indicates any of the Mandelstam variables intervening in the scattering process and $M_W$ is the $W$ boson mass. Neutrinos and antineutrinos are considered massless.

Consider the frame of reference in which the electron is initially at rest and let the initial electron polarization vector lie in an arbitrary direction $\hat{n}$. If we choose helicity to describe the polarization of the final-state electron, the Standard Model (SM) prediction for the elastic neutrino–electron differential cross section is

$$\left[\frac{d\sigma}{dE}\right]_{\hat{n}, \pm_{\nu}} = \frac{2mG^2}{\pi} \left[g_L A_{\pm}(E) + g_R B_{\pm}(E) (1 - z)^2 - g_L g_R C_{\pm}(E) \frac{mz}{\nu}\right],$$

where

$$A_{\pm}(E) = \frac{1}{2} (1 + c_\alpha) (1 \mp \cos \theta)$$

$$B_{\pm}(E) = \left[\frac{1}{2} (1 - c_\alpha) + \frac{mzc_\alpha}{2(\nu - T)}\right] \left[1 \pm \left(1 + \frac{m}{T - \nu}\right) \frac{T}{l}\right]$$

$$C_{\pm}(E) = \frac{1}{2} (1 + c_\alpha) \left(1 \pm \frac{T - 2\nu}{l}\right).$$

The upper (lower) sign indicates positive (negative) helicity of the recoil electron and $c_\alpha$ is the cosine of the angle between $\hat{n}$ and the incoming neutrino momentum. $G_\mu = 1.16637(1) \times 10^{-5}$ GeV$^{-2}$ is the Fermi coupling constant, $m$ is the electron mass, $g_L = \sin^2 \theta_W \pm 1/2$ (upper sign for $\nu_e$ and $\bar{\nu}_e$, lower sign for $\nu_{\mu,\tau}$ and $\bar{\nu}_{\mu,\tau}$), $g_R = \sin^2 \theta_W$, and $\sin^2 \theta_W \approx 0.23$ is the squared sine of the weak mixing angle. In this elastic process, $E$, the electron recoil energy, ranges from $m$ to $E_{\text{max}} = [m^2 + (2\nu + m)^2]/[2(2\nu + m)]$, where $\nu$ is the incident neutrino energy in the frame of reference in which the electron is initially at rest. Also, $l = \sqrt{E^2 - m^2}$ and $T = E - m$ are the final electron three-momentum and kinetic energy, $z = T/\nu$, and $\cos \theta = (1 + m/\nu)(T/l)$ is the cosine of the angle between the momenta of the recoil electron and incident neutrino. For simplicity of notation, only the $E$ dependence of the functions $A_{\pm}$, $B_{\pm}$ and $C_{\pm}$ has been explicitly indicated. Note that the differential cross section in eq. (1) includes an
integration over the nontrivial dependence on the azimuthal angle of the final-state electron momentum, with the z-axis taken along the direction of the incoming neutrino momentum.

The corresponding formula for the elastic antineutrino–electron differential cross section is

$$\frac{d\sigma}{dE}_{\bar{\nu}, \pm} = \frac{2 m G^2 \mu}{\pi} \left[ g_L^2 A_{\pm}(E) (1 - z)^2 + g_R^2 B_{\pm}(E) - g_L g_R C_{\pm}(E) \frac{m z}{\nu} \right],$$

where

$$A_{\pm}(E) = \left[ \frac{1}{2} \left( 1 + c_\alpha \right) - \frac{m z c_\alpha}{2(\nu - T)} \right] \left[ 1 + \left( 1 + \frac{m}{T - \nu} \right) \frac{T}{T} \right]$$

$$B_{\pm}(E) = \frac{1}{2} \left( 1 - c_\alpha \right) (1 \pm \cos \theta)$$

$$C_{\pm}(E) = \frac{1}{2} \left( 1 - c_\alpha \right) \left( 1 + \frac{T - 2\nu}{l} \right).$$

We refer the reader to ref. [1] for radiative corrections (RC) to unpolarized neutrino–electron scattering. RC are not included in the results of the present paper.

If we sum over the helicities of the final-state electron, eq. (1) immediately leads us to (the superscript $s$ indicates the sum over the helicities)

$$\frac{d\sigma}{dE}_{\bar{\nu}, s} = \frac{2 m G^2 \mu}{\pi} \left[ (1 + c_\alpha) \left( g_L^2 - g_L g_R \frac{m z}{\nu} \right) \right.

+ \left. g_R^2 \left( 1 - z \right)^2 \left( 1 - c_\alpha + \frac{m z c_\alpha}{\nu - T} \right) \right],$$

in agreement with ref. [2]. As we can see from eq. (2), the corresponding formula for antineutrino–electron scattering is simply obtained by interchanging $g_L$ and $g_R$ and inverting the sign of $c_\alpha$ in eq. (1). Furthermore, if the initial-state electrons are not polarized, then we must also average over their polarizations, thus obtaining the well-known SM result for the unpolarized $\nu_l - e^-\bar{e}^-$ elastic scattering computed long ago by 't Hooft [3] (the superscript $a$ indicates the average over the initial polarizations of the electron)

$$\frac{d\sigma}{dE}_{\nu, a} = \frac{2 m G^2 \mu}{\pi} \left[ g_L^2 + g_R^2 (1 - z)^2 - g_L g_R \frac{m z}{\nu} \right].$$

Once again, for $\bar{\nu}_l - e^-\bar{e}^-$ scattering we should just interchange $g_L$ and $g_R$.

Irrespective of the final helicity of the electron, we note that when the initial-state electron is polarized towards the incoming neutrino (i.e. when
\( c_{\alpha} = -1 \), the differential cross section eq. (1) does not depend on \( g_L \) and is therefore independent of the neutrino flavor. If, on the contrary, we reverse the polarization of the initial-state electron (i.e. when \( c_{\alpha} = +1 \), eq. (1) depends both on \( g_L \) and on \( g_R \), and is different for \( \nu_e \) and \( \nu_{\mu,\tau} \). This can be seen in the upper panel of fig. 1, where we have plotted the differential cross sections for polarized \( \nu_l - e^- \) elastic scattering for an incident neutrino energy \( \nu = 0.862 \text{ MeV} \), summing over the helicities of the final-state electron.

(This value of \( \nu \) was chosen for its relevance in the study of solar neutrinos: \( \nu = 0.862 \text{ MeV} \) is the energy of the monochromatic neutrinos produced by electron capture on \( ^7\text{Be} \) in the solar interior.) Solid (dotted) lines indicate \( \nu_e \) (\( \nu_{\mu,\tau} \)) and their thickness represents the two different initial-state electron polarizations \( c_{\alpha} = +1 \) and \( c_{\alpha} = -1 \) (thick and thin, respectively). The thin solid and thin dotted curves overlap, while the thick ones differ significantly. The lower panel of fig. 1 shows the corresponding differential cross sections for polarized \( \bar{\nu}_l - e^- \) elastic scattering, with the same value \( \nu = 0.862 \text{ MeV} \) for reasons of comparison.

Lead by these simple considerations, we introduce \( P_{fh}^h(\nu, c_{\alpha}) \), the polarization asymmetry defined as

\[
P_{fh}^h(\nu, c_{\alpha}) = \frac{[\sigma]_{f}^{\hat{n},h} - [\sigma]_{f}^{-\hat{n},h}}{[\sigma]_{f}^{\hat{n},h} + [\sigma]_{f}^{-\hat{n},h}},
\]

where the superscript \( h \) can be either \( \pm \) or \( s \) (we remind the reader that the superscript \( \pm \) indicates positive or negative helicity of the final-state electron, while \( s \) denotes their sum), \( f = \nu_l \) or \( \bar{\nu}_l \) (\( l = e, \mu \) or \( \tau \)), and \([\sigma]_{f}^{\hat{n},h}\) is the total cross section

\[
[\sigma]_{f}^{\hat{n},h} = \int_{E_{\text{max}}}^{E_{\text{max}}} \left[ \frac{d\sigma}{dE} \right]_{f}^{\hat{n},h} dE
\]

\([\sigma]_{f}^{-\hat{n},h}\) is the corresponding total cross section when the initial electron polarization vector lies in the direction \( -\hat{n} \). Similarly, employing differential cross sections rather than total ones, one can also define the polarization asymmetry parameter \( p_{fh}^h(\nu, E, c_{\alpha}) \).

The polarization asymmetry provides a sensitive tool to investigate the flavor of the incoming (anti)neutrinos. In the upper panel of fig. 2 we plotted \( P_{fh}^h(\nu, c_{\alpha}) \) for incident neutrinos of energy \( \nu \in [1 \text{ KeV}, 10 \text{ MeV}] \) and \( c_{\alpha} = 1 \); the optimal value \( c_{\alpha} = 1 \) was chosen to maximize the flavor sensitivity of the asymmetry parameter. In this panel, the three thick (thin) lines indicate \( \nu_e \) (\( \nu_{\mu,\tau} \)); plots for positive and negative helicity of the recoil electron have
been drawn dashed and dotted, respectively, while solid lines have been employed for final helicity sums. If we focus on the solid lines, we notice their dependence on the incident neutrino flavor. We also note that, for neutrino scattering, the dependence on the flavor would be increased if the final helicity could be measured, and attention limited to events with positive helicity (upper panel, dashed lines). The lower panel of fig. 2 shows the corresponding plots of $P_{j}^{h}(\nu, 1)$ for antineutrinos.

The flavor composition of a $\nu_l$ (or $\bar{\nu}_l$) beam can be investigated in a flux-independent way by measuring the number of scattering events off electrons polarized along and opposite to the direction of the incoming neutrino (or antineutrino) momentum. The comparison of this experimental measurement with the SM prediction [obtained via eqs. (5) and (6)] provides us with the flavor content of the incident $\nu_l$ (or $\bar{\nu}_l$) beam, i.e. the fraction of $\nu_e$ ($\bar{\nu}_e$) vs. $\nu_{\mu,\tau}$ ($\bar{\nu}_{\mu,\tau}$) in the incident flux. Conversely, if the incoming (anti)neutrino flavor is known, this comparison provides the fraction of neutrinos vs. antineutrinos.

The use of the polarization asymmetry parameter for (anti)neutrino-electron scattering was advocated long ago to test the predictions of different models for a unified theory of electromagnetic and weak interactions [4]. More recently, the scattering of $\nu_e$ and $\bar{\nu}_e$ on a polarized electron target has been suggested as a test of the neutrino magnetic moment [2]. The analyses of this note provide new and explicit motivation for the idea of scattering (anti)neutrinos off polarized electrons.

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References

[1] Radiative corrections to neutrino-electron scattering have been investigated by several authors. We refer the reader to J. N. Bahcall, M. Kamionkowski and A. Sirlin, Phys. Rev. D 51 (1995) 6146, for simple and compact results, M. Passera, Phys. Rev. D 64 (2001) 113002 and BUTP 2001/02 [hep-ph/0102212], to appear in Comments Nucl. Part. Phys., for recent results on QED corrections, and to references therein.

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Figure 1: Polarized differential cross sections for $\nu_l - e$ (upper panel) and $\bar{\nu}_l - e$ (lower panel) elastic scatterings. $\nu = 0.862$ MeV. Solid (dotted) lines indicate $\nu_e (\nu_{\mu,\tau})$. Thick and thin lines represent, respectively, $c_{\alpha} = +1$ and $c_{\alpha} = -1$. 
Figure 2: $P^h_\nu(\nu, c_\alpha = 1)$ for incident neutrinos (upper panel) and antineutrinos (lower panel) of energy $\nu \in [1 \text{ KeV}, 10 \text{ MeV}]$. Thick (thin) lines correspond to $\nu_e$ and $\bar{\nu}_e$ ($\nu_{\mu,\tau}$ and $\bar{\nu}_{\mu,\tau}$). Dashed (dotted) lines represent polarization asymmetries for positive (negative) helicity of the final-state electron, while solid curves indicate helicity sums.