Modelling correlations in heavy ion collisions is reviewed from model independent characterization of the two-particle correlations to Buda-Lund hydro model fits to correlations and spectra in Au + Au collisions at $\sqrt{s} = 130$ AGeV at RHIC.

1 Recent results in modelling correlations

Modelling correlations in heavy ion reactions is a broad and rapidly expanding field that is reviewed here very briefly, due to space limitations. For a more detailed and illustrated version of this talk, see ref. 1. For more detailed recent reviews see also refs. 2, 3.

Although some claims were made in the literature that the shape of the two-particle Bose-Einstein correlation functions has to be a (multi-variate) Gaussian, by now it is clear that these claims were mistaken and even approximately Gaussian correlation functions may contain subleading non-Gaussian corrections, for example oscillatory prefactors, that are particularly important in case of effective binary sources. It is also clear that refined analysis of the two-particle correlation functions indicates significant deviations from Gaussian shapes in elementary particle induced reactions. Sensitive searches for non-Gaussian components have not yet been performed in correlation measurements in heavy ion collisions at CERN SPS and at RHIC, to our best knowledge. A model independent parameterization of the two-particle correlation function is given in terms of the Edgeworth or Laguerre expansions. These expansions were shown to be sensitive to even small deviations from Gaussian or exponential shapes.

The deviation from an exactly Gaussian form of the two-particle correlation function can be caused by the decay products of long-lived resonances. A core-halo picture has been introduced to describe such a situation.
Within the core-halo picture, the intercept parameter $\lambda^*$ of the two-particle correlation function is interpreted as the squared, momentum dependent fraction of particles emitted from the core of the interaction, and the shape of the correlation function is interpreted as a measure of the core part of the interaction region. The core-halo structure of the particle emitting source has recently been observed with the help of a new imaging method, developed by D. A. Brown and P. Danielewicz\textsuperscript{11}, that reconstructs the relative source distribution directly from the measured data points with the help of the known final state interactions of the emitted pair.

Good progress has been made recently in the study of multi-particle Coulomb corrections. A wave-function integration method has been developed for 3 - 5 particles, utilizing a cluster decomposition of the multiparticle final state\textsuperscript{12}. Such Coulomb corrections have to be applied within the context of a core-halo picture\textsuperscript{13}. A modification of the Coulomb interactions (permutations in the charge allocation) has recently been proposed to develop Monte-Carlo event generators with Bose-Einstein correlations\textsuperscript{14}.

An interesting new direction of modelling correlations in heavy ion collisions is the study of back-to-back correlations (BBC) for bosonic\textsuperscript{15,16,17,18} as well as fermionic cases\textsuperscript{19} (bBBC and fBBC). When the thermalized vacuum state decays, it always produces particle - anti-particle pairs with opposite spin and momenta. Hence the strength of BBC is increasing inversely with the single particle spectra and in principle, it can be unlimitedly large\textsuperscript{18,21}. It would be certainly of great interest to observe BBC experimentally. The bosonic BBC in heavy ion physics has a famous analogy in astrophysics, namely the Bekenstein-Hawking radiation of black holes\textsuperscript{22,23}. Both effects are related to squeezing and decay of a modified vacuum to pairs of bosons of asymptotic fields with back-to-back momentum and opposite quantum numbers.

2 Hydro models fit spectra at RHIC

Modelling high energy heavy ion collisions is not only difficult because of the large number of degrees of freedom involved in the process but also because various theoretical concepts and approaches are relevant for different stages of the collision. A comprehensive review of this process has recently been given by S.A. Bass\textsuperscript{24}. During a partonic cascading process (local) thermalization is achieved, that can be maintained for a while due to the intensive collision rate and the time evolution corresponds to nuclear fluid dynamics. As the system rarifies, the collision rate within the “macroscopically infinitesimal” fluid cells decreases and one expects that the hydrodynamical approximation may break down and non-equilibrium hadronic transport models start to play
a role\cite{27,28}. A version of this well established concept has been utilized to describe the single particle spectra and two-particle correlations in 130 AGeV Au + Au collisions at RHIC\cite{29}. Numerical solution of relativistic hydrodynamics was terminated by particle freeze-out and a subsequent resonance decay and hadronic cascading by a code developed by Soff, Bass and Dumitru. The model fitted the single particle spectra measured by the PHENIX and STAR experiments at RHIC\cite{30,31}, but it overestimated, in a statistically unacceptable manner, the experimentally observed longitudinal and out radius components of the Bose-Einstein correlation functions\cite{32,33}. Both the longitudinal and the out radius components are sensitive to the distribution of particle production in time. One of the problems with the calculation could be its utilization of the Gaussian variances\cite{27,28} of the source distribution, that are known to overestimate the HBT radii for core-halo type of systems\cite{34,35}.

An interesting study of the identified particle elliptic flow at RHIC has been reported by the STAR collaboration\cite{36}. From the mass dependence of the second flow coefficient as a function of transverse momentum, a suggestive evidence has been obtained that early local equilibration followed by a hydrodynamic expansion is responsible for the particle spectra in central and mid-peripheral collisions up to $p_t \leq 1.5$ GeV. A similar conclusion has been reached by Kolb, Huovinen, Heinz and Heiselberg, who observed in addition that the very rapid thermalization in Au + Au collisions at RHIC provides a serious challenge for kinetic approaches based on classical scattering of on-shell particles\cite{37}. On the other hand, the high $p_t$ behaviour of the elliptic flow pattern was shown to be a sensitive measure of the initial parton density distribution of an initial quark-gluon plasma phase\cite{38}. Detailed numerical solution of relativistic hydrodynamics has been reported to describe well the single particle spectra, the first and the second flow coefficients for various domains of centrality and transverse momentum\cite{39}; however, this calculation also over-predicted by a factor of 2 the measured, effective HBT radii of the two-particle Bose-Einstein correlation function, possibly indicating a problem with the freeze-out time distribution in this kind of calculations. A good description of various aspects of the measured single particle spectra, and the $m_t$ dependence of the longitudinal HBT radius parameters has been reported by Hirano, Morita, Muroya and Nonaka\cite{40} based on an exact solution of relativistic hydrodynamics. The order of magnitude of the calculated side and out radii were also found to be in agreement with the experimental data, however, the side radius component was slightly underestimated and the out radius component over-estimated by this model, and the observed decrease of the out component with increasing transverse mass has not been reproduced.
3 Buda-Lund hydro fits spectra and HBT radii at RHIC

It has been observed that simple parameterizations of the freeze-out phase space distribution of a locally thermalized, three dimensionally expanding, cylindrically symmetric finite systems describe the preliminary single particle spectra at RHIC reasonably well in the whole available kinematic domain.

The Buda-Lund hydrodynamical parameterization (BL-H) has been developed in refs. 35,36 to describe particle correlations and spectra in heavy ion collisions at CERN SPS energies. The model was formulated on the principle of cylindrical symmetry in central collisions and a relativistic, 3 dimensional flow profile that corresponds to a Hubble type of transverse flow sitting on top of a longitudinal Bjorken flow. Under certain conditions, the BL-H parameterization predicted an $R_t \approx R_s \approx R_o \propto 1/\sqrt{m_t}$ scaling of the HBT radius parameters, as a direct reflection of the cylindrical symmetry of the source.

Both the single particle spectra and the two-particle Bose-Einstein correlation functions are analytically calculated as a function of 8 model parameters plus constants of normalization for the absolutely normalized single particle spectra. The BL-H model fits simultaneously the double differential single particle spectra and the $m_t$ dependence of the radius parameters.
Table 1. Source parameters from simultaneous fittings of CERN SPS (NA49, NA44 and preliminary WA98) and RHIC (preliminary PHENIX and STAR) particle spectra and HBT radius parameters with the Buda-Lund hydrodynamical model.

| BL-H parameters | STAR prel. | PHENIX prel. | (RHIC) | (SPS) |
|-----------------|------------|--------------|--------|-------|
|                 | Value      | Error        | Value  | Error |
| $T_0$ [MeV]     | 143 ± 4    | 140 ± 3      | 142 ± 2 | 139 ± 6 |
| $\langle u_t \rangle$ | 0.76 ± 0.06 | 0.67 ± 0.3 | 0.71 ± 0.05 | 0.55 ± 0.06 |
| $R_G$ [fm]      | 7.5 ± 0.3  | 6.6 ± 0.3    | 7.1 ± 0.7 | 7.1 ± 0.2 |
| $\tau_0$ [fm/c] | 8.8 ± 0.5  | 7.8 ± 0.3    | 8.3 ± 0.7 | 5.9 ± 0.6 |
| $\Delta r$ [fm/c] | 0.01 ± 0.1 | 1.3 ± 0.1    | 1.3 ± 1.0 | 1.6 ± 1.5 |
| $\langle \Delta T_T \rangle_r$ | 1.0 ± 0.9 | 1.5 ± 0.1 | 1.3 ± 0.4 | 2.1 ± 0.4 |
| $\langle \Delta T_T \rangle_t$ | 0.09 ± 0.02 | 0.01 ± 0.02 | 0.05 ± 0.05 | 0.06 ± 0.05 |
| $\chi^2/NDF$    | 46/58 = 0.79 | 45/54 = 0.83 | 0.81 | 1.20 |

in hadron-proton reactions at CERN SPS. The BL-H also describes the NA49, NA44 and the preliminary WA98 data for spectra and correlations at CERN SPS, with a statistically acceptable $\chi^2/NDF$ fit, as summarized in the last column of Table 1 for a comparison with BL-H fits to the preliminary PHENIX and STAR data for Au + Au at $\sqrt{s} = 130$ AGeV at RHIC. We observe that the BL-H fits of these RHIC data are satisfactory. The central value of the freeze-out temperature, $T_0$, is about the same at RHIC as at CERN SPS, and within three standard deviations, most of the fit parameter values are similar. However, the mean value of the freeze-out time $\tau_0$, and the strength of the transverse flow at the geometrical radius, $\langle u_t \rangle$, is significantly larger at RHIC than at CERN SPS. The system at RHIC freezes out later, with a larger transverse flow parameter $\langle u_t \rangle$ than at CERN SPS. The preliminary results indicate that within the presently big errors the transverse geometrical size $R_G$ is about as big at RHIC as at CERN SPS. This research was supported by grants OTKA T026435 T034296 N25487 (with NWO), NSF-MTA-OTKA 0089462 and US DOE DE - FG02 - 93ER40764.

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