Study on the Reason of Static Stiffness of the Whole Machine in Non-Circular Phenomenon When Vertical Machining Center Milling Circle

To cite this article: Erfei Li et al 2018 IOP Conf. Ser.: Mater. Sci. Eng. 452 042038

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Study on the Reason of Static Stiffness of the Whole Machine in Non-Circular Phenomenon When Vertical Machining Center Milling Circle

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Abstract. Aiming at the phenomenon that the tool-work-piece offset causes the milling to be non-circular during the circumferential milling of CNC machine tools, the reason of the static stiffness of the whole machine is studied. In this paper, a vertical machining center is taken as the research object. Based on the multi-body system theory and the homogeneous coordinate transformation, the deformation modeling of the whole machine is carried out. The force deformation in the direction of X, Y and Z is analysed. The static stiffness coefficient method is used to integrate and modify the deformation in the direction of X, Y and Z. Finally, the mathematical model of the static stiffness of the whole machine is derived. After the research, it is found that in the same direction, the static stiffness of the tool branch chain and the work-piece branch chain are different, resulting in different deformation of the whole machine; the static stiffness of the tool branch chain and the work-piece branch chain in the X and Y directions is different, and finally the circumferential milling is non-circular. It lays a certain theoretical foundation for the design optimization of CNC machine tool components and the improvement of the static stiffness of the CNC machine tool. It provides reference for other types of CNC machine tool static stiffness modelling, has a guiding significance.

1. Introduction

With the popularization of CNC machining technology, the application of CNC circumferential milling technology is increasing. Circumferential milling is the milling of circular, cylindrical surfaces such as milling holes and threads and by the circumferential feed motion of the milling cutter. When the CNC machine tool uses the end mill for circumferential milling, the X and Y axes of the CNC machine tool should be linked. In actual production, circumferential milling often results in milling non-circular phenomenon.

There are many factors in the circumferential milling that produce non-circular shape. One of the most important factors is the deformation caused by the milling force of the machine tool, and the force deformation of the machine tool is largely due to insufficient static stiffness. The static stiffness of CNC machine tools has a significant effect on its machining accuracy [1]. The static stiffness of CNC machine tools in the X and Y directions is different, which affects the machining accuracy to varying degrees, and finally causes the work-piece to be non-circular. Improving the static stiffness of machine tools is very effective for improving machining accuracy, efficiency and surface machining.
quality [2]. For the circumferential milling, the tool-work-piece offset causes the milling to be non-circular. This paper studies the cause of the static stiffness of the whole machine.

Based on the analysis of the finite element model of the high-speed vertical milling machine XH786A, Zhu Jun of Shanghai Jiaotong University has optimized the bed and column of machine tool to improve its static performance [3]. The finite element analysis and structural optimization design of the vertical car /milling combined machining center column of Xiao Qi of Dalian University of Technology have established an optimization model of static and dynamic multi-objective and multi-parameters of the column, which improves the static stiffness of the column [4]. However, the above researches were focused on the static stiffness of the machine tool single component. The whole machine was not explicitly studied. The research on the static stiffness characteristics of the whole machine was relatively small, and it was impossible to systematically start from the static stiffness of the whole machine and analyse the cause of non-circular milling in the middle of production. It is necessary to know that there are many components of the CNC machine tool and the structure is complicated, and there are many factors affecting the static stiffness of the whole machine.

The research object of this paper is a three-axis vertical machining center, which is a typical multi-body system [5-6]. In this paper, the static stiffness coefficient method is used to abstract the physical model of the component into a mathematical model, which characterizes the static stiffness of various parts of the machine tool. Based on the multi-body system theory and the homogeneous coordinate transformation, the whole machine deformation model is established. Through the analysis of the force deformation of the main components, combined with the deformation of composite, finally get the mathematical model of static stiffness of the whole machine.

2. Machining center components

A vertical machining center mainly consists of five components: workbench, cross slide, bed, and column and spindle box. As shown in Figure 1, 0 is the machining center bed; 1 is the cross slide; 2 is the workbench; 3 is the work-piece; 4 is the column; 5 is the spindle box; 6 is the spindle. In order to facilitate the deformation analysis of the machining center and the static stiffness modeling of the whole machine, two branches of the tool and the work-piece are established: "bed-tool" and "bed-work-piece". The moving parts are then sorted according to natural growth.

![Figure 1. Vertical machining center type structure diagram composed member](image)

The static stiffness coefficient of the component represents the physical quantity of the elastic deformation of the component under external force, and its physical meaning is the static stiffness value of some parts. Whether it is a structural part or a functional part, the final deformation will affect the relative deformation of the tool and the work-piece.

The structural component takes the column as an example. The Z-bead screw bears the main Z-direction force. It is not necessary to define the static stiffness coefficient of the Z-direction of the
The static stiffness coefficient of the column is \( k_{x1}, k_{y1}, k_{z1}, k_{x2}, k_{y2}, k_{z2}, k_{x3}, k_{y3}, k_{z3}, k_{x4}, k_{y4}, k_{z4} \). Similarly, the static stiffness coefficient of the bed is \( k_{b1x}, k_{b1y}, k_{b1z}, k_{b2x}, k_{b2y}, k_{b2z}, k_{b3x}, k_{b3y}, k_{b3z}, k_{b4x}, k_{b4y}, k_{b4z} \), the static stiffness coefficient of the cross slide is \( k_{c1x}, k_{c1y}, k_{c1z}, k_{c2x}, k_{c2y}, k_{c2z}, k_{c3x}, k_{c3y}, k_{c3z}, k_{c4x}, k_{c4y}, k_{c4z} \), and the static stiffness coefficient of the spindle box is \( k_{dx}, k_{dy}, k_{dz} \).

In the static stiffness modeling process of the whole machine, the main functional components such as the spindle, ball screw and guide rail-slider are regarded as a whole [7]. The static stiffness coefficient of the main shaft refers to the static stiffness values of the three directions at the bottom end of the spindle, respectively \( k_{x}, k_{y}, \) and \( k_{z} \). The static stiffness coefficient of the guide rail-slider refers to the values of its normal stiffness and tangential stiffness. For example, the X-direction rail-slider, for \( k_{x}, k_{y}, \) and \( k_{z} \). The static stiffness coefficient of the ball screw refers to the axial stiffness value at which the lead screw nut pair is coupled to the front end member. For example, the X-direction ball screw is \( k_{x}. \)

### 3. Machine deformation modeling

Reference the whole machine deformation modeling method of document [8], based on the multi-body system theory and the homogeneous coordinate transformation, describes the multi-body system topology by low-order body sequence, and characterizes the pose transformation of typical body of multi-body system with feature matrix. The functional relationship between the deformation of the component and the offset of the tool-work-piece is established, and the deformation of the whole machining center is modeled. The topology diagram is shown in Figure 2, the low-order array is shown in Table 1.

![Figure 2. CNC machine tool topology](image)

**Table 1. CNC machine tool low-matrix array table**

| Typical body | 1 | 2 | 3 | 4 | 5 | 6 |
|--------------|---|---|---|---|---|---|
| \( L^x(j) \)  | 1 | 2 | 3 | 4 | 5 | 6 |
| \( L^y(j) \)  | 0 | 1 | 2 | 0 | 4 | 5 |
| \( L^z(j) \)  | 0 | 0 | 1 | 0 | 0 | 4 |
| \( L^x(j) \)  | 0 | 0 | 0 | 0 | 0 | 0 |

The final offset in three directions \( \Delta x \), \( \Delta y \), and \( \Delta z \) is,

\[
\begin{align*}
\Delta x &= \left[ L_x \Delta y_{0x} + \Delta x_{0x} + \Delta x_{5x} + S_x \Delta y_{0x} - C_x \Delta y_{0x} - \Delta x_{2x} - \Delta x_{23} \right] \\
\Delta y &= \left[ L_x \Delta x_{0y} + \Delta y_{0y} + \Delta y_{5y} + S_y \Delta x_{0y} - C_y \Delta x_{0y} - \Delta y_{2y} - \Delta y_{23} \right] \\
\Delta z &= \left[ L_x \Delta x_{0z} + \Delta x_{0z} + \Delta x_{5z} + S_z \Delta x_{0z} - C_z \Delta x_{0z} - \Delta x_{2z} - \Delta x_{23} \right]
\end{align*}
\]
4. Force deformation analysis of components

Take the column as an example for force analysis. When the size is unit force, the cutting force with a direction of +X acts on the tip of the tool, equivalent to a force and a torque acting on the column. The torque is the multiplication of the force and the distance from the tip point to the column, equivalent to a pair of couples, which act at position 1-4 as shown in Figure 3. After equivalent, position 1, 2, 3, and 4 bear tangential forces. In addition, position 1 and 3 are subjected to pressure. Position 2 and 4 withstand pull.

\[ \text{Figure 3. Analysis of force acting on column when subjected to X-direction force} \]

Suppose the force is \( F_i^X \), the torque is \( M_j^X \), the distance is \( L_k \), a pair of couples is \( \pm \left( \frac{M_j}{W_k} \right) \), the tangential force is \( \left( \frac{F_i}{4} \right) \), the pressure is \( \left( -\frac{F_j L_k}{2W_k} \right) \) and the pull is \( \left( \frac{F_j L_k}{2W_k} \right) \). By these equivalent forces available:

\[ -M_j^X = F_i^X \times L_k = -W_j^X \times \left[ \left( \frac{M_j}{W_k} \right) \right] = -W_j^X \times 2 \left( \frac{F_j L_k}{2W_k} \right) \]

(1)

\[ Q_i^X = Q_j^X = Q_k^X = F_i^X / 4 \]

(2)

\[ N_s^X = N_s^X = -\frac{F_j L_k}{2W_k} \]

(3)

\[ N_s^X = N_s^X = \frac{F_j L_k}{2W_k} \]

(4)

In the formula, \( W_j \) is the X-direction distance between the two sliders of the Z-direction rail, and \( L_k \) is the Y-direction distance between the tool tip and the column.

Refer to the above force analysis to get \( Q_i^X \), \( Q_j^X \), \( Q_k^X \), \( Q_s^X \), \( Q_i^Y \), \( Q_j^Y \), \( Q_k^Y \), \( N_s^X \), \( N_s^X \), \( N_s^X \), \( N_s^X \), \( N_s^X \), \( N_s^X \), \( N_s^X \), \( N_s^X \) and \( N_s^X \).

According to the force analysis of the column, the deformation include: under the X-direction force, bending deformation along the X direction and torsional deformation around the Z axis, under the Y-direction force, bending deformation along the Y direction and torsional deformation around the X-axis, under the Z-direction force, torsion deformation around the Z-axis. The column bending...
deformation along the X direction under the X-direction force will be described as an example. When the machining center is in the middle position, the four sliders of the column are bent in the X direction under the X-direction force and deformed into the rigid body translation, and the deformation $\Delta x_a$ is basically the same, that is, the static stiffness coefficient $k_{a_1x} = k_{a_2x} = k_{a_3x} = k_{a_4x}$. From the small deformation theory:

$$\Delta x_a = F_s/4k_{a_{1x}} = F_s/4k_{a_{2x}}$$ (5)

Refer to the above deformation analysis to get $\Delta y_{a1}, \Delta y_{a2}, \Delta a_{a1}, \Delta a_{a2}, \Delta x_b, \Delta y_b, \Delta a_b, \Delta z_b, \Delta x_c, \Delta y_c$ and $\Delta z_c$.

5. Component deformation synthesis and correction

According to the analysis of the force deformation of the above main components, the deformation synthesis and correction of components are carried out. Taking the X-moving components as an example, the deformation of the X-moving components includes: rotation around the X-axis, translation along the X direction, rotation around the Y-axis, and translation along the Z direction. The rotation around the X-axis includes: rotation of the Y-direction guide-lead screw around the X-axis and rotation of the cross slide around the X-axis, the rotation of which is $\Delta x_{a_1}$, it can be obtained that:

$$\Delta x_{a_1} = F_s S_d / W_t^2 (1/k_{a_{1x}} + 1/k_{a_{1y}})$$ (6)

In the previous derivation process, it is assumed that the deformation of the components is the translation and rotation of the rigid bodies of the four sliders that are independent of each other. However, the component is flexible body [9]. When a slider produces force deformation, it will inevitably affect the force deformation of other sliders. Therefore, the influence factor is introduced to correct the functional relationship between component deformation and static stiffness coefficient. Correcting $\Delta x_{a_1}$, it can be obtained that:

$$\Delta x_{a_1} = F_s S_d / W_t^2 (1/k_{a_{1x}} + 1/k_{a_{1y}}) + \delta = F_s S_d / W_t^2 (1/k_{a_{1x}} + 1/k_{a_{1y}})(1 + 1/h_s)$$ (7)

Where, $\delta = 1/h_s[(1/k_{a_{1x}} + 1/k_{a_{1y}})F_s S_d / W_t^2]$, $1/h_s$ is the influence factor of the normal stiffness of the X-moving component.

Refer to the above deformation synthesis and correction to get $\Delta x_{a_1}, \Delta z_{a_1}, \Delta y_{a_1}, \Delta x_{a_2}, \Delta z_{a_2}, \Delta y_{a_2}, \Delta x_{b1}, \Delta z_{b1}, \Delta y_{b1}, \Delta x_{b2}, \Delta z_{b2}, \Delta y_{b2}, \Delta a_{b1}, \Delta a_{b2}, \Delta y_{b3}, \Delta z_{b3}, \Delta a_{b3}$ and $\Delta y_{b4}$.

6. Whole machine static stiffness modeling

In the bed-tool branch chain, including the bed, column, spindle box, spindle and tool, combined with the functional relationship between component deformation and static stiffness coefficient, it can be obtained that:

$$\begin{bmatrix}
1 \\
1/k_{a_{1z}} \\
1/k_{a_{2z}} \\
1/k_{a_{3z}} \\
1/k_{a_{4z}}
\end{bmatrix}
= \begin{bmatrix}
1 & 1 & 1 & 1 & 1 \\
1 & k_{a_{1x}} & k_{a_{1y}} & k_{a_{1z}} & k_{a_{1y}} \\
1 & k_{a_{2x}} & k_{a_{2y}} & k_{a_{2z}} & k_{a_{2y}} \\
1 & k_{a_{3x}} & k_{a_{3y}} & k_{a_{3z}} & k_{a_{3y}} \\
1 & k_{a_{4x}} & k_{a_{4y}} & k_{a_{4z}} & k_{a_{4y}}
\end{bmatrix}
$$
Where,

\[
\frac{1}{k_{\alpha z z}} = \frac{1}{4k_{\alpha z z}} \left( 1 + \frac{1}{I_z} \right), \quad \frac{1}{k_{\beta z z}} = \frac{1}{4k_{\beta z z}} \left( 1 + \frac{1}{I_z} \right)
\]

\[
\frac{1}{k_{\alpha x y}} = \frac{L_x^2}{k_{\alpha x y} W_x} \left( 1 - \frac{1}{I_y} \right), \quad \frac{1}{k_{\beta x y}} = \frac{L_y^2}{k_{\beta x y} W_y} \left( 1 - \frac{1}{I_y} \right)
\]

\[
\frac{1}{k_{\alpha y y}} = \frac{1}{4k_{\alpha y y}} \left( 1 + \frac{1}{I_y} \right), \quad \frac{1}{k_{\beta y y}} = \frac{1}{4k_{\beta y y}} \left( 1 + \frac{1}{I_y} \right)
\]

\[
\frac{1}{k_{\alpha z y}} = \frac{L_y^2}{k_{\alpha z y} L_h} \left( 1 - \frac{1}{I_y} \right), \quad \frac{1}{k_{\beta z y}} = \frac{L_y^2}{k_{\beta z y} L_h} \left( 1 - \frac{1}{I_y} \right)
\]

\[
\frac{1}{k_{\alpha z x}} = \frac{L_x^2}{k_{\alpha z x} L_h} \left( 1 + \frac{1}{I_h} \right), \quad \frac{1}{k_{\beta z x}} = \frac{L_x^2}{k_{\beta z x} L_h} \left( 1 + \frac{1}{I_h} \right)
\]

In the bed-work-piece branch chain, it is equally acceptable,

\[
\begin{bmatrix}
\frac{1}{k_{\alpha x}} \\
\frac{1}{k_{\beta x}} \\
\frac{1}{k_{\alpha y}} \\
\frac{1}{k_{\beta y}} \\
\frac{1}{k_{\alpha z}}
\end{bmatrix} = \begin{bmatrix}
\frac{1}{k_{\alpha x x}} + \frac{1}{k_{\alpha x y}} + \frac{1}{k_{\alpha y y}} + \frac{1}{k_{\alpha z z}} + \frac{1}{k_{\alpha x z}} + \frac{1}{k_{\alpha y z}} + \frac{1}{k_{\alpha z y}} + \frac{1}{k_{\alpha z x}} \\
\frac{1}{k_{\beta x x}} + \frac{1}{k_{\beta x y}} + \frac{1}{k_{\beta y y}} + \frac{1}{k_{\beta z z}} + \frac{1}{k_{\beta x z}} + \frac{1}{k_{\beta y z}} + \frac{1}{k_{\beta z y}} + \frac{1}{k_{\beta z x}} \\
\frac{1}{k_{\alpha x x}} + \frac{1}{k_{\alpha x y}} + \frac{1}{k_{\alpha y y}} + \frac{1}{k_{\alpha z z}} + \frac{1}{k_{\alpha x z}} + \frac{1}{k_{\alpha y z}} + \frac{1}{k_{\alpha z y}} + \frac{1}{k_{\alpha z x}} \\
\frac{1}{k_{\beta x x}} + \frac{1}{k_{\beta x y}} + \frac{1}{k_{\beta y y}} + \frac{1}{k_{\beta z z}} + \frac{1}{k_{\beta x z}} + \frac{1}{k_{\beta y z}} + \frac{1}{k_{\beta z y}} + \frac{1}{k_{\beta z x}} \\
\frac{1}{k_{\alpha z x}} + \frac{1}{k_{\alpha z y}} + \frac{1}{k_{\alpha z z}} + \frac{1}{k_{\alpha x z}} + \frac{1}{k_{\alpha y z}} + \frac{1}{k_{\alpha z y}} + \frac{1}{k_{\alpha z x}} + \frac{1}{k_{\alpha z x}}
\end{bmatrix}
\]

Where,

\[
\frac{1}{k_{\alpha x x}} = \frac{S_x^2}{k_{\alpha x x} L_x} \left( 1 + \frac{1}{h_x} \right), \quad \frac{1}{k_{\beta x x}} = \frac{S_x^2}{k_{\beta x x} L_x} \left( 1 + \frac{1}{h_x} \right)
\]

\[
\frac{1}{k_{\alpha x y}} = \frac{1}{4k_{\alpha x y}} \left( 1 - \frac{1}{h_x} \right), \quad \frac{1}{k_{\beta x y}} = \frac{1}{4k_{\beta x y}} \left( 1 - \frac{1}{h_x} \right)
\]

\[
\frac{1}{k_{\alpha y y}} = \frac{1}{k_{\alpha y y}} \left( 1 + \frac{1}{h_y} \right), \quad \frac{1}{k_{\beta y y}} = \frac{1}{k_{\beta y y}} \left( 1 + \frac{1}{h_y} \right)
\]

\[
\frac{1}{k_{\alpha z z}} = \frac{1}{k_{\alpha z z}} \left( 1 + \frac{1}{h_z} \right), \quad \frac{1}{k_{\beta z z}} = \frac{1}{k_{\beta z z}} \left( 1 + \frac{1}{h_z} \right)
\]

\[
\frac{1}{k_{\alpha x x}} = \frac{1}{4k_{\alpha x x}} \left( 1 + \frac{1}{h_x} \right), \quad \frac{1}{k_{\beta x x}} = \frac{1}{4k_{\beta x x}} \left( 1 + \frac{1}{h_x} \right)
\]

\[
\frac{1}{k_{\alpha y y}} = \frac{C_y^2}{k_{\alpha y y} W_y} \left( 1 + \frac{1}{i_y} \right), \quad \frac{1}{k_{\beta y y}} = \frac{C_y^2}{k_{\beta y y} W_y} \left( 1 + \frac{1}{i_y} \right)
\]

\[
\frac{1}{k_{\alpha z z}} = \frac{C_z^2}{k_{\alpha z z} L_z} \left( 1 + \frac{1}{i_z} \right), \quad \frac{1}{k_{\beta z z}} = \frac{C_z^2}{k_{\beta z z} L_z} \left( 1 + \frac{1}{i_z} \right)
\]

\[
\frac{1}{k_{\alpha z x}} = \frac{C_z^2}{k_{\alpha z x} L_x} \left( 1 + \frac{1}{i_z} \right), \quad \frac{1}{k_{\beta z x}} = \frac{C_z^2}{k_{\beta z x} L_x} \left( 1 + \frac{1}{i_z} \right)
\]

Finally, the static stiffness model of the machining center is,

\[
\begin{bmatrix}
\frac{1}{k_x} \\
\frac{1}{k_y} \\
\frac{1}{k_z}
\end{bmatrix} = \begin{bmatrix}
\frac{1}{k_{th}} + \frac{1}{k_{gh}} \\
\frac{1}{k_{ty}} + \frac{1}{k_{gy}} \\
\frac{1}{k_{tz}} + \frac{1}{k_{gz}}
\end{bmatrix}
\]

In the formula, \( L_x \) and \( W_x \) are the X and Y distances between the two sliders of the X-direction rail, \( W_y \) and \( L_y \) are the X and Y distances between the two sliders of the Y-direction rail. \( S_d \) is the Z-direction distance between the tool tip and the cross slide, \( C_z \) is the Z-direction distance between the
tool tip and the bed. $l/b_s$ is the influence factor of the tangential stiffness of X-moving components, $l/l_n$, $l/t_l$ is the influence factor of normal stiffness and tangential stiffness of Y-moving components, $l/l_n$, $l/t_l$ is the influence factor of normal stiffness and tangential stiffness of Z-moving components. $k_{dx}$, $k_{dy}$, $k_{dz}$ is the static stiffness of the X-direction, Y-direction and Z-direction in the bed-tool branch chain, respectively. $k_{nx}$, $k_{ny}$, $k_{nz}$ is the static stiffness of the X-direction, Y-direction and Z-direction in the bed-work-piece branch chain, respectively.

By establishing the static stiffness model of the whole machining center, the static stiffness of the whole machine is analyzed and compared, and the reason for the non-circular milling of the circumferential milling is studied. Taking the static stiffness of the tool end in the X and Y directions as an example, during circumferential milling, the milling force in the X and Y directions is equal and larger than that in the Z direction, that is $F_x = F_y > F_z$. At this point for the spindle, the spindle box and the column, $l/k_{dx} = 1/k_{dy}$, $l/k_{dx} = 1/k_{dy}$, $l/k_{dx} = 1/k_{dy}$, $l/k_{dx} = 1/k_{dy}$ and $l/l_n = 1/l_n$, however $W_F = L_F$, $l/k_{dx} > 1/k_{dy}$, therefore $1/l_{dx} > 1/l_{dy}$.

7. Conclusion

By establishing the static stiffness model of the whole machine, the causes of non-circularity of circumferential milling are explored from the perspective of static stiffness, which lays a certain theoretical foundation for further improving the static stiffness of the whole machine and designing optimized components. This method also provides reference for other types of CNC machine tool static stiffness modeling, which has important guiding significance.

Acknowledgments

This work was financially supported by Tianjin Natural Science Foundation Project (No. 18JCYBJC20100).

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