The criticality of the Hantavirus infected phase at Zuni

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A preliminary analysis of the temporal evolution of a population of Peromyscus maniculatus infected with Hantavirus Sin Nombre is made. Ecological and epidemiological parameters are derived from the data, and they are used as inputs for the analytical model presented in [Abramson and Kenkre, Phys. Rev. E 66, 011912 (2002)]. A prediction of the critical carrying capacity and its associated critical mouse density is made, and the time series is analyzed under the light of these. It is found that the sporadic disappearances and reappearances of the infected phase correspond to the bifurcation predicted by the model.

This report contains a first attempt to test, in a quantitative way, the predictions of the analytical model for the Hantavirus epizootic proposed by Abramson and Kenkre [1, 2]. The analysis is based on ecological and epidemic data published by Yates et al. [3], consisting of a time series of mice populations at two sites near Zuni, New Mexico, covering monthly a period from 1995 to 2001 (78 months). The population data are displayed in Fig. 1, where the total, susceptible and infected populations of Peromyscus maniculatus, the main host of Hantavirus Sin Nombre, are shown as \( N, N_S \) and \( N_I \) respectively. The period does not include the outbreak of 1993, when the virus was discovered to be the cause of the severe Hantavirus Pulmonary Syndrome (HPS). The great peak that can be seen around month 40 in Fig. 1 corresponds to a population explosion following the Niño event of 1997-1998, that was in turn followed by a small outbreak of \( N_I \) and HPS in 1999 (see Fig. 2). Observe also that the vertical scale is logarithmic to facilitate the visualization of small values. It can be seen that \( N_I \) drops to 0 several times, and the corresponding line is discontinuous at these values because of the logarithm (not because of a lack of data). These are the sporadic disappearance of the infected phase, reported in [3, 4, 5], and that are accounted for as bifurcations controlled by the environmental carrying capacity in the model of Ref. [1]. The assumptions of the present analysis are big but the results seem encouraging and support the analytical results of Ref. [1].

The basic model, mean-field-like and not extended in space, for the dynamics of the mice populations is:

\[
\frac{dN_S}{dt} = bN - cN_S - \frac{N_S N}{K} - aN_S N_I, \quad (1)
\]

\[
\frac{dN_I}{dt} = -cN_I - \frac{N_I N}{K} + aN_S N_I, \quad (2)
\]

where \( b \) is the birth rate, \( c \) is the death rate, \( a \) is the contagion rate, and \( K \) is the carrying capacity. The interested reader may consult Refs. [1, 2] for a detailed discussion on the motivation and the implications of this model, as well as for a discussion of infection waves observed in the spatially extended model.

Let us make certain assumptions that will allow us to derive values for the parameters from the data. The first assumption is that \( b, c \) and \( a \) are constant, determined by biological properties of the agents involved, and independent of anything that is changing in the system and that makes the populations grow and decline. This role is reserved solely for \( K = K(t) \). This is, indeed, the same assumption made in Refs. [1, 2], to analyze the bifurcation and to simulate a time dependent scenario.

A further assumption is to use simplified models to extract the parameters from restricted regions of the graphs. For example, observe the best visible population explosion of \( N \), from \( t = 30 \) to 40. It is a very well

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FIG. 1: Population of \( P. \) maniculatus at the Zuni capture site (New Mexico), from [3]. Total, susceptible and infected populations are shown, as indicated in the legend. Reference [3] provides data for the total and the seropositive populations, here represented as \( N \) and \( N_I \) respectively. The susceptible population \( N_S \) is defined as \( N - N_I \).
The exponential growth of $N$ is the nicest, because there are several relatively broad regimes of such behavior. The two other processes, decay and infection, are less clear cut, because both are derived from the same $N_I(t)$, and also because the regimes are narrower.

The results are the following:

$$b - c = 0.21, \quad c = 0.75, \quad a = 0.77, \quad (6)$$

in units of month$^{-1}$ ($c$ and $b - c$) and (mice$\times$month)$^{-1}$ ($a$). It is imperative to take this values cum grano salis. Besides the assumptions mentioned above, there are further uncertainties. In the average $a$, for example, a value of 17 was discarded because it is so bigger than all the others that is inevitable to suspect some artifact of the time series that, at this point, cannot be clarified. Let us carry on, nevertheless, and from (6) and Ref. [1] conclude that

$$K_c = \frac{b}{a(b - c)} \approx 6 \text{ mice } \times \text{ month.} \quad (7)$$

This value of $K_c$ is the main quantitative result of the present analysis. It is a prediction of the model, based on the numerical values of the parameters as can be estimated from the dynamics of the populations. How does it compare with features of the time series? If we observe that the equilibrium solution of the logistic equation satisfied by $N$, when $K$ is constant, is $N^* = K(b - c)$, there is a “critical density” $N_c = K_c(b - c)$ with which the actual density can be compared. When not growing or decaying, $N(t)$ will be more or less at equilibrium during the periods that $K(t)$ remains constant. $N_c = 2$ mice, approximately corresponding to the calculated parameters, is shown as a black horizontal line in Fig. 1.

Let us analyze the time series under the light of this result. At $t < 15$ the population is above critical, and there is, correspondingly, a positive infected phase. During this regime, it is conceivable that $K(t)$ has some time dependence, but that it keeps it above critical. Then, around $t = 15$, $K(t)$ drops to some value below critical, and so does $N^*$, and in consequence $N(t)$ drops, trying to reach an equilibrium which is now below $N_c$. The drop is not monotonous, there seems to be some discrete steps. What is the result? Shortly after the decline of $N$ begins (the indication that $K$ has gone subcritical) the infected phase begins to disappear sporadically. A few infected mice may be entering by migration, or marginal susceptible mice might be being infected, but it is clear that the infection is disappearing from the site. After $t = 30$ a steady population explosion starts, indicating that $K$ has increased, and the observation that the population grows beyond $N_c$ indicates that the system is supercritical again. A recovering of the infected phase is to be expected. It takes time, however, just as in the model (see Fig. 2 in Ref. [1]), and not before $t = 40$ do we see a positive $N_I$ again. After this, the population remains above critical, so $K(t)$ must be critical most of the time, and the infection persists. A brief excursion of $N$ below

FIG. 2: The connection between the environment, in particular weather, and incidence of HPS becomes apparent in this figure, that shows an average sea surface temperature (SST) of the East Pacific Ocean, average precipitation (PPT) in Southwest North America, and cases of HPS in the USA (total and in the Four Corners region, near Zuni). Sources: SST has been calculated from the explosions of $N$ that constitutes the fourth assumption. That is the infection, so in the explosions of $N_I$ we suppose the simplified equation:

$$\frac{dN_I}{dt} = -cN_I, \quad (4)$$

whence the parameter $c$ can be obtained.

Now, the only source of $N_I$ is the infection, so in the explosions of $N_I$ we suppose the simplified equation:

$$\frac{dN_I}{dt} = aN_I N_S, \quad (5)$$

that constitutes the fourth assumption.

The three Eqs. (3), (4) and (5) can be solved (this last only approximately) and the solutions fitted to the data.
$N_c$ at $t \approx 70$ might be the beginning of a new extinction event, and indeed $N_f$ reaches its lowest values since $t = 40$. But shortly after this the time series ends and the analysis can not be carried further. Observe that the drop in $N$ (so in $K$) takes place in 2001, a year that was particularly dry, as can be seen in the precipitation data in Fig. 2.

In summary, even if there are a number of uncertainties, the analysis fits nicely in the picture given by the model of Ref. [1], namely that there is a critical point and that the system lives close to it. It encourages to attempt a more detailed study, in particular involving bigger data sets of the same system.

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