Sampling Attacks: Amplification of Membership Inference Attacks by Repeated Queries

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Abstract

Machine learning models have been shown to leak information violating the privacy of their training set. We focus on membership inference attacks on machine learning models which aim to determine whether a data point was used to train the victim model.

Our work consists of two sides: We introduce sampling attack, a novel membership inference technique that unlike other standard membership adversaries is able to work under severe restriction of no access to scores of the victim model. We show that a victim model that only publishes the labels is still susceptible to sampling attacks and the adversary can recover up to 100% of its performance compared to when posterior vectors are provided.

The other sides of our work includes experimental results on two recent membership inference attack models and the defenses against them. For defense, we choose differential privacy in the form of gradient perturbation during the training of the victim model as well as output perturbation at prediction time. We carry out our experiments on a wide range of datasets which allows us to better analyze the interaction between adversaries, defense mechanism and datasets. We find that our proposed fast and easy-to-implement output perturbation technique offers good privacy protection for membership inference attacks at little impact on utility.

1. Introduction

Recent machine learning (ML) methods – especially deep learning approaches – largely owe their success to the availability of big data and the computation power to train huge models with millions of parameters on this training dataset. Often, people might assume that since these models are designed to learn statistical properties of their training dataset, they protect the privacy of the individuals who contribute to these datasets. Unfortunately, this is not the case and the ML models are proven to violate the privacy of this training set in many unintended ways such as memorizing details about the individuals (Song et al., 2017), leaking features of their training set (Melis et al., 2019) and contributing to re-identification problems e.g. via saving users’ geodata (Maouche et al., 2017).

In this study we are concerned with membership inference (MI) attacks. These are attacks carried out by an adversary who has complete or partial data records and access to a trained victim ML model and wishes to know whether that data record has been used to train the victim model. MI attacks pose serious threats on the privacy of the individuals present in databases and have been successfully applied on models trained on image data, medical data, transaction records, etc. (Nasr et al., 2019; Naranyanan & Shmatikov, 2008; Shokri et al., 2017; Song et al., 2019).

MI attacks are carried out in a black-box setting, where adversary has no access to the internal parameters of the victim model. The assumption is that posterior vectors from the victim classifier suffice to reveal the membership status of queried points. For our experiments, we chose two standard MI adversaries: One that learns to distinguish members of the training set from non-members and one with prior belief on their differences (Salem et al., 2018).

We employ differential privacy (DP) (Dwork et al., 2014) to defend against MI attacks. DP is becoming more and more popular as a method to protect the privacy of individuals involved in a machine learning setting. It offers flexible techniques which can be applied locally on the data (Kairouz et al., 2014; Qin et al., 2016), or during the training on...
We also consider the extreme case of protection when the scores are not published by the victim model and the adversary only receives the predicted labels. This results in standard MI attack methods to fail to perform. However, we introduce a new sampling attack model which is constructed to utilize only the labels. We show how in practice this novel method compares with standard MI adversaries.

Our contributions:

- We introduce our novel sampling attack model which performs membership inference under the severe restriction of no access to confidence scores of the attacked classifier.

- We compile a comprehensive list of all the prominent datasets in membership attack studies and compare them under unified metrics and alongside each other. This helps us better understand and analyze membership inference attacks.

- Focusing mainly on the practical implications of differential privacy (DP) (Dwork et al., 2014) rather than the theoretical bounds on the privacy budget, we use DP as a method to defend against membership inference attacks. We show the interplay between datasets, attack models and defenses.

- Considering the practical limitations of applying DP during the training of the models, we formulate an output perturbation method that can be applied on models at prediction time. We also study the effect of this method, which we call DP-Logits, as a defense for membership attacks and contrast it with DP-SGD (Abadi et al., 2016).

The structure of this paper is as follows: We first introduce membership inference attacks in details in Section 2. We then summarize possible defense methods against membership adversaries in Section 3, with a focus on differential privacy as a defense in Section 3.1. We formulate the novel sampling attack and explain the details and practical methods to implement it in Section 4. We finally present our experimental results in Section 5.

2. Membership Inference

Membership inference attack decides whether or not a certain data point is a member of a dataset. The privacy risks of membership inference attacks were first brought into attention by Homer et al. (Homer et al., 2008) when they demonstrated that they could successfully resolve the presence of an individual in a highly-complex DNA mixture. One of their key findings is that publishing only the composite statistics over a collection of genomic data would not protect the privacy of the individuals who are members of that collection. In a follow up paper (Wang et al., 2009) they use more sophisticated test statistics to achieve better results with less prior knowledge on the victims. Similar attacks on other biomarkers such as microRNA have also been successfully performed (Backes et al., 2016).

2.1. Membership Inference on Machine Learning Models

The increased popularity of machine learning models translates to an ever-increasing need for data to train these models. This data often contains sensitive information from individuals and protecting the privacy of it is of great importance. Therefore, our focus in this paper is on membership inference attacks on machine learning models.

These models are usually trained on a set of data points \( x \) so that the function \( f(x; \theta) \), which is characterized by parameters \( \theta \), is learned. In this context, the goal of the adversary is to determine: given a trained victim model, is a certain data point \( x_i \), a member of the training set of this model? i.e:

\[
A(x; \phi) : X \rightarrow \{0 \text{ (non-member)}, 1 \text{ (member)} \}
\]  

where \( A \) is the adversary, \( \phi \) are the parameters that the adversary utilizes and \( X \) is the space of possible data points.

Now we will explain two adversarial models that cover standard membership attacks. The first adversary requires training with the objective of finding statistical differences between members and non-member data points; whereas the second model has a prior belief on these statistical differences and no learning phase is required.

Learning Based Adversary  
First introduced in (Salem et al., 2018), this adversary relies on training a shadow model and a binary attack model. Shadow models mimic the behavior of the victim model and are in the possession of the adversary. The adversary can freely study the behavior of these models as a surrogate for the victim models with restricted access. The task of the binary attack classifier is to classify its input as member/non-member of the training set.

The following steps are taken by the adversary:
1. **Shadow model training:** Train the shadow model with a set of data points from the same distribution as the training set of the victim model.

2. **Binary classifier training:** Query the trained shadow model with its training set as well as a hold-out test set. Collect the posteriors and pass them to the binary attack classifier.

3. **Attack the victim model:** Query the victim model with the desired data points and use the trained binary classifier to decide the membership status.

The assumption about this attack is that the adversary has a black box access to the victim model and can only study the returned posterior vectors. In this method, only one shadow model and one binary classifier are used. This can be viewed as a relaxed version of multiple shadow models and multiple binary classifiers of Shokri et al. (Shokri et al., 2017).

We will refer to this attack model as **LRN** adversary.

**Learning Free Adversary** The dependence of the **LRN** adversary on the shadow model and the binary classifier as well as data to train these models, is an undesirable factor. For this reason, Salem et al. (Salem et al., 2018) suggest a more versatile model that requires no shadow model or attack binary classifier. A black-box access to the victim model is also assumed for this attack. The posterior vectors from the victim model are inspected and if the maximum element exceeds a calibrated threshold it will be classified as a member, otherwise non-member. This can be summarized in the following function:

\[
A(x; T) = \begin{cases} 
1 & \text{if } \max_y \Pr(y|x) \geq T \\
0 & \text{otherwise} 
\end{cases}
\]  

which is parameterized by a single threshold \(T\). The adversary queries the victim model by data point \(x\) and decides based on the maximum of the returned posterior vector \(\Pr(y|x)\) whether or not it is a member of the training set of the victim model.

The name “learning free” comes from the fact that no shadow or attack classifiers are trained. We will refer to this attack as **LRN-Free** adversary.

**3. Defenses for Membership Inference Attacks**

In the previous section we defined the membership inference attacks and described two generic models to carry out these attacks in practice. Now, we will explore methods to defend against them.

To defend against these attacks, we need to understand what factors make these attacks possible and how we can limit and paralyze the adversary. Most of the previous work on defenses against membership inference attacks can be summarized into two categories:

**Generalization-based techniques** (Shokri et al., 2017) was the first to define the membership inference attacks in a machine learning setting. They also identify the overfitting of the victim model as one of the main culprits for vulnerability to membership inference attacks. They hypothesize that the victim model memorizes its training set such that the posteriors show a statistical difference between the seen and hold-out data. A more comprehensive study about the correlation of overfitting to membership inference attacks can be found in (Yeom et al., 2018).

These findings prompt a line of defense that relies on enforcing generalization on the victim model. (Shokri et al., 2017) suggest using \(L_2\) regularization of the parameters and restricting the number of training epochs. (Salem et al., 2018) use dropout and ensemble learning to train the victim model to help it generalize better. In a slightly different approach, (Nasr et al., 2018) utilizes adversarial training of the victim model in the form of a min-max game to help the model generate indistinguishable predictions on its training set and an unseen dataset.

**Noising-based techniques** Adding randomness to different parameters of the victim model at different stages is one of the most natural ways to confuse any adversary. In fact, the first defenses against membership inference attacks on the genome data (Wang et al., 2009) proposes adding carefully-crafted noise to the published dataset.

Jia et al. (Jia et al., 2019) suggest adding noise to the output of the victim model. They generate specially-composed noise vectors for the victim model’s posteriors such that they act as adversarial examples for the attacker.

In a more formal and mathematics-driven line of work **differential privacy** is leveraged to add noise the gradients during the training of the victim model (Jayaraman & Evans, 2019; Rahman et al., 2018).

In this work we mainly focus on differential privacy as a defense since it is a well-defined privacy framework and very flexible with respect to the methods that can be applied to build a differentially-private model. In the rest of this section we introduce differential privacy and explain how it can be utilized in our setting.

**3.1. Differential Privacy**

Differential privacy (DP) (Dwork et al., 2014; 2015; Dwork, 2011; Dwork et al., 2006) is a mathematical definition
We say that the randomized algorithm $M$ where

Now that we are familiar with the basics of differential privacy we can look at some mechanisms that ensure DP.

Properties of differential privacy Next, we will introduce two of the most important properties of differential privacy:

1. Immunity to post processing (Dwork et al., 2014): Any function of the output of the $(\epsilon, \delta)$-DP algorithm $M$ is $(\epsilon, \delta)$-DP. This means that an adversary without any additional knowledge on the dataset, is unable to extract more information about the dataset with further analysis of $M$.

2. Composition (Dwork et al., 2014): The combination of two or more DP algorithms is also a DP algorithm with degraded privacy budgets $\epsilon'$ and $\delta'$. There are various theorems which act under certain conditions to calculate $\epsilon'$ and $\delta'$ given the privacy budgets of each initial mechanism. In the simplest form, the independent use of $k$ mechanisms, each $(\epsilon, 0)$-DP is $(k\epsilon, 0)$-DP.

Some DP Mechanisms Now that we are familiar with the basics of differential privacy we can look at some mechanisms that ensure DP:

1. Randomized response mechanism (Warner, 1965): One of the oldest privacy-ensuring mechanisms, randomized response was first used to protect participants in surveys with a "yes/no" possible answer. Assume we are interested whether a property $P$ is present in participants. Participants are asked to flip a coin then:

   - if tails: respond truthfully
   - if heads: flip a coin again. If heads answer "yes" otherwise "no"

   For this mechanism $\epsilon$ can be calculated as:

   $$e^\epsilon = \frac{\Pr(\text{answer} = \text{yes} | \text{truth} = \text{yes})}{\Pr(\text{answer} = \text{yes} | \text{truth} = \text{no})} = \frac{3/4}{1/4} = 3$$

   So the randomized response mechanism is $(\ln(3), 0)$ differentially private.

2. Gaussian Mechanism (Dwork et al., 2014): Consider a deterministic real-valued function $f : D \rightarrow \mathbb{R}$, we define the $l_2$-sensitivity of $f$ as:

   $$\Delta_2(f) = \max_{d,d' \text{ adjacent}} \|f(d) - f(d')\|$$

   We can approximate $f$ with an $(\epsilon, \delta)$-DP function $M$ by adding Gaussian noise to it:

   $$M(d) = f(d) + \mathcal{N}(0, \sigma^2)$$

   where $\mathcal{N}(0, \sigma^2)$ is a Gaussian distribution with mean $0$ and standard deviation $\sigma \geq c\Delta_2(f) / \epsilon$, and $c^2 > 2 \ln(1.25/\delta)$.

3.1.1. Differential Privacy for Machine Learning

For sophisticated processes which require a combination of multiple functions, we can utilize and combine DP mechanism and use the composition theorems to keep track of the accumulated privacy budget. In this paper we work with deep learning models which are non-linear combination of functions that are trained over multiple iterations. If we aim to make a deep learning model differentially private, we need to be cautious about the DP methods that are applied and how to correctly track the privacy budget. In this section, we introduce two possible DP methods for deep learning:

DP-Stochastic Gradient Descent (DP-SGD) (Abadi et al., 2016) This method achieves privacy by adding noise to the parameters of the model during the training. First, the gradients are clipped then an additive noise is applied to them:

$$\tilde{g}_t(x_i) \leftarrow g_t(x_i) / \max(1, \frac{\|g_t(x_i)\|_2}{C})$$

$$\tilde{g}_t \leftarrow \frac{1}{L} \left( \sum_{i} \tilde{g}_t(x_i) + \mathcal{N}(0, (\sigma CT)^2) \right)$$

where $g_t$ is the gradient vector at epoch $t$, $C$ is the clipping norm, $L$ is the number of samples randomly chosen for calculation of the gradient and $\mathcal{N}(0, \sigma^2)$ is a Gaussian
with standard deviation $\sigma$. This process can be viewed as
Gaussian mechanism on top of the stochastic gradient de-
scent. The reason behind the clipping step is to bound the
$l_2$-sensitivity of the gradients.

The final privacy budget after the training can be calculated
with moments accountant which is a composition method
for a sequence of adaptive mechanisms.

**DP-Logits** While DP-SGD provides a strong guarantee,
for practical black-box deployment scenarios, these guar-
antees significantly hinder performance. For example, ap-
plying DP-SGD requires clipping of the gradients in each
step of the training which results in a slow-downed training
process. Also the accumulated value of $\epsilon$ can potentially
grow to very large numbers for complex neural network
architectures that require many iterations of training and
are sensitive to noise. Furthermore, the party in the posses-
sion of a model may not always have access to the training
phase of the model. Consequently, we now work towards
an approach that perturbs the model outputs (rather that
all parameters) to specifically safeguard against black-box
membership inference attacks. This method can be rapidly
applied on top of any trained model.

For this, we use Gaussian mechanism to add noise to $l_2$
normalized logits at prediction time:

$$\tilde{l}(x_i) \leftarrow l(x_i) / \max(1, \frac{\|l(x_i)\|}{S})$$

(8)

$$\tilde{l}(x_i) \leftarrow \tilde{l}(x_i) + \mathcal{N}(0, (\sigma S\|\|)^2)$$

(9)

where $l(x_i)$ indicates the vector of logits for the input $x_i$
and $S$ is the clipping norm. The privacy budget with the
noise scale $\sigma$ and $l_2$ sensitivity $S$ can be calculated as:

$$\sigma \geq \frac{S\sqrt{q}}{\epsilon}\sqrt{2\ln(\frac{1.25}{\delta})} \quad \text{for } \delta = \frac{1}{|d|}$$

(10)

where $|d|$ is the size of the training dataset, and $q$ the number
of queries. As opposed to the DP-SGD method which is
applied during the training and leaves the trained model
immune to post-processing, adding noise to the logits at
the prediction time suffers from degradation by the number
of queries. Especially important is to restrict the number of
times that a same point can be queried, as an adversary
would be able to zero out the noise by repeated querying.
However, this method has the advantage of being fast and
easy to implement.

### 3.2. Argmax Defense

Diverging from differential privacy and the other suggested
defense methods, we can take a step back and tackle this
problem from a different perspective. To date, all the mem-
bership inference adversaries that we are aware of, rely on
and utilize the posterior vectors from the victim model. This
means that if the victim model returns the most confident
'label' $k = \arg \max_y \Pr(y = k|x_i)$ instead of the
full posterior, the adversary is unable to carry out the attack.
We refer to this method as argmax defense.

Note that the argmax defense is not always feasible, mostly
as a result of the problem design setting where the scores
are required and expected by the benign user.

### 4. Sampling Attack

In Section 3.2 we argued that the argmax defense is effec-
tive against all the previously-suggested membership
inference adversaries due to their dependence on the posteri-
ors vectors. Now we will introduce our novel attack method
which is designed to work under this severe restriction.

Attacks under limited access to the posterior vectors have
been studied before, for example, (Dang et al., 2017) pro-
pose evasion attacks when the score of the detector is not
accessible to the adversary. To fool the detector, they gen-
erate perturbations of the malicious sample and attempt to
find the first perturbed sample that traverses the malicious/benign
boundary of the detector. However, their attack does not
directly depend on the posterior vectors and the goal is to
only evade the detector.

Membership inference adversaries are designed to study the
posterior vector, so the idea behind our sampling adversary
is to reconstruct these vectors from the returned labels. We
achieve this by populating a sphere around each data point
with multiple perturbations of it and counting the number of
perturbed samples that fall under each label.

If $x'$ is a perturbation of data point $x$, from probability theory
we have:

$$\Pr(y = c|x = x_i) = \sum_{x' \in \mathcal{X}_i'} \Pr(y = c, x'|x = x_i)$$

(11)

$$= \sum_{x' \in \mathcal{X}_i'} \Pr(y = c|x', x = x_i)\Pr(x'|x = x_i)$$

(12)

$$= \sum_{x' \in \mathcal{X}_i'} \Pr(y = c|x')\Pr(x'|x = x_i)$$

(13)

where $\mathcal{X}_i'$ is the space of possible perturbations of $x_i$.
Line 11 is the marginalization over the perturbed data points
$x'$ and line 12 is the expansion according to the Bayes’ the-
orem. The transition to line 13 comes from the fact that the
victim models decision on the posterior of $x'$ does not de-
depend on the unperturbed point $x_i$. The probability $\Pr(x'|x)$
is the perturbation function acting on $x$. So this can be for-
mulated as a Monte-Carlo integration (MacKay, 1998) in
the form of:

$$\Pr(y = c|x = x_i) = \frac{1}{N} \sum_{x'} \Pr(y = c|x') \quad \text{where } x' \sim \text{pert}(x_i)$$  \hspace{1cm} (14)$$

But we do not have access to the posteriors \( \Pr(y|x') \), so we propose relaxing Eq. 14 and using the returned labels instead. Algorithm 1 demonstrates how the sampling adversary works. The perturbation function \( \text{pert}() \) acts on each data point \( x \) to generate \( N \) perturbed samples and the identity function \( I \) builds histograms over the labels of the perturbed points. We hypothesize that this histogram can be a suitable replacement for the posterior.

**Algorithm 1 Sampling Attack**

**Input:** Data points \( \{x_1, \ldots, x_M\} \), perturbation function \( \text{pert}(x_i; p) \). Parameters: number of perturbations \( N \), perturbation scale \( p \).

1. \( \text{for } i \in [M] \text{ do} \)
2. \( \quad \text{for } n \in [N] \text{ do} \)
3. \( \quad \quad \text{get labels: } l = \text{pert}(x_i; p) \)
4. \( \quad \quad \text{build histograms: } \Pr(y = c|x_i) = \frac{1}{N} \sum I(l = c) \)
5. \( \text{end for} \)
6. \( \text{end for} \)

**Output:** Posterior vectors \( \Pr(y|x) \)

In practice, the following steps are taken by the adversary:

1. **Shadow model training:** Train a shadow model with data from the same distribution as the training set of the victim model.

2. **Sampling on the shadow model:** Execute algorithm 1 for the training set as well as a hold-out test set of the shadow model. Repeat for different perturbation levels \( p \).

3. **Attack the shadow model:** Using the reconstructed posteriors from the previous step, attack the shadow model with one of the conventional adversaries.

4. **Attack the victim model:** Choose the optimum value of \( p \) according to some performance metric of the adversary. Attack the victim model with the chosen \( p \) value and the adversary from step 3.

In Table 1 we compare the sampling attack with the LRN and LRN-Free adversaries. The sampling attack requires the training of a shadow model. If we choose LRN-Free adversary for steps 3 and 4 of the sampling attack, no binary classifier training is required.

## 5. Experiments

In this section we present the experimental results of membership inference attacks and the effectiveness of the suggested defenses against them.

For the first two parts we mostly focus on the interplay of adversaries and defense mechanisms from a practical point of view. To this end we choose a collection of 8 different datasets that were used in the most influential membership inference attack studies (e.g. (Shokri et al., 2017; Salem et al., 2018; Jia et al., 2019; Nasr et al., 2018)) to be able to compare the results with one unified measure.

We choose two differentially-private defenses (see Section. 3.1), one that perturbs the parameters of the model and one that perturbs the outputs, and apply them on the adversaries.

At last, with an insight into how adversaries work and what datasets are worth further investigating, we present the results on our novel sampling attack technique.

### 5.1. Datasets

The datasets that we begin our experiments with, are:

- **MNIST** The MNIST\(^*\) dataset consists of 60,000 training data and 10,000 test data of grayscale images of size 28 × 28. These images depict handwritten digits (0 - 9) and are centered with respect to the frame of the image.

- **FashionMNIST** Created by Zalando\(^1\), this dataset consists of 60,000 and 10,000 training and test set data points, respectively. These images are also 28 × 28 and in grayscale and represent 10 classes of fashion items such as "tops", "trousers", "sneakers", etc

- **CH-MNIST** This preprocessed dataset obtained from Kaggle\(^2\) contains 5000 greyscale images of different types of tissue in colorectal cancer patients. The size of images is 64 × 64 and the task is to classify these images into one of the 8 possible tissue categories.

- **CIFAR10, CIFAR100** We used CIFAR-10 and CIFAR-100\(^3\) for our experiments. Both consists of color images of size 32 × 32 and have 50,000 training data and 10,000 test data. CIFAR-10 has 10 classes such as "air plane", "dogs", "cats", etc.;5000 randomly-selected images per class in its training set and 1000 randomly-selected images per class in its test set. On the other hand, CIFAR-100 has 20 super classes, each containing 5 class (in total 100 classes)

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\(^*\)https://yann.lecun.com/exdb/mnist/

\(^1\)https://www.kaggle.com/zalando-research/fashionmnist

\(^2\)https://www.kaggle.com/khadertem/colorectal-histology-mnist

\(^3\)https://www.cs.toronto.edu/~kriz/cifar.html
of different subjects such as animals, humans and vehicles. Similar to CIFAR-10, it also has 5000 randomly-selected images per class in its training set and 1000 randomly-selected images per class in its test set.

**Purchase100**  Purchase\(^\d\) is dataset of shopping history of several thousand customers and the aim is to classify the customers into \(k\) different classes of shopping styles so that accurate coupon promotions can be suggested to them. This dataset has no ground truth for the labels. Similar to (Salem et al., 2018; Shokri et al., 2017), we use a simplified version of this dataset with \(\sim 200,000\) data points and a 600-dimensional vector of purchases per data point where each element can take a value of either 0 or 1 (present or not present in the shopping history). Afterwards, \(k\)-means clustering algorithm (Lloyd, 1982) is used to cluster these vectors into \(100\) classes. We call this version of the Purchase dataset with 100 classes Purchase100.

**Texas100**  This includes patients’ data published by the Texas Department of State Health Services\(^\d\). This dataset contains 6,169 binary features of 67,330 patients, such as diagnosis of various diseases, procedures performed on the patient and other properties of each patient. We use a preprocessed version obtained from (Jia et al., 2019). Given the input data of each patient, the task is to choose among the most suitable procedure among the available 100 most frequent ones.

**Location**  This dataset is the binary representation of 446 locations\(^**\) (either visited or not visited) by users. This comes with 5,010 data points and the task is to classify the datapoint into one of the 30 possible classes. We obtained a preprocessed version from (Jia et al., 2019).

### 5.2. Membership Inference Attacks on All Datasets

Throughout our studies we encountered many papers on membership inference attacks and/or defenses against them which each used different datasets and different metrics to evaluate the success of the attacks and defenses. This motivated us to first combine all these datasets under a unified metric to be able to compare and better understand the results. We chose LRN and LRN-Free (see Section 2) as our conventional adversaries. Below we explain the details of our experiments:

**Evaluation metrics**  To evaluate the performance of the adversary we chose Area Under the ROC Curve (AUC) since it is independent of the threshold that the adversary chooses to distinguish members from non-members and gives a better overview of the performance of the attacker. An AUC value of 0.5 means random guessing and implies completely unsuccessful attack, whereas AUC value of 1.0 implies a perfect attack.

**Data splits**  We divide each dataset into 4 equal parts \(D_{\text{train}}\), \(D_{\text{test}}\), \(D_{\text{shadow}}\) and \(D_{\text{test}}\) for our LRN adversary. For this purpose, all of the training and test sets of MNIST, FashionMNIST, CH-MNIST, CIFAR10, CIFAR100 and Location datasets are combined and used. For Purchase100 and Texas100 we use a total of 80,000 and 40,000 randomly selected points, respectively.

**LRN adversary**  We use \(D_{\text{train}}\) to train the shadow model. After training we query the shadow model by its training set and the unseen data points of \(D_{\text{test}}\) and use these posterior predictions to train the binary classifier. We then test the performance of the binary classifier on the posterior prediction of the victim model when queried by its training set \(D_{\text{train}}\) and the unseen set \(D_{\text{test}}\). To avoid choosing a decision threshold, at this stage we only take the output of the sigmoid function of the binary classifier. This allows us to calculate the AUC values over decisions of the binary classifier.

**LRN-Free adversary**  For a fair comparison with the LRN adversary we only use \(D_{\text{train}}\) and \(D_{\text{test}}\) for our learning-free method. We query the trained victim model by its training data \(D_{\text{train}}\) and the hold-out \(D_{\text{test}}\) and take the maximum element of the returned posterior vectors. Similar to the LRN adversary, we refrain from choosing a decision threshold and calculate AUC values over all the possible thresholds.

### Table 1. Comparison of LRN and LRN-Free to the sampling adversary. Full circles mean that the condition is required and the half full circle means flexibility in terms of training.

| adversary | shadow model | binary classifier | data distribution access | posterior access | training |
|-----------|--------------|-------------------|--------------------------|-----------------|---------|
| LRN       | ●            | ●                 | ●                        | ●               | ●●     |
| LRN-Free  | -            | -                 | -                        | ●               | -      |
| sampling  | ●            | -                 | ●                        | -               | ●      |

\(^\d\)https://www.kaggle.com/c/acquire-valued-shoppers-challenge/data

\(^\d\)https://www.dshs.texas.gov/THCIC/Hospitals/Download.shtm

\(^**\)https://sites.google.com/site/yangdingqi/home/foursquare-dataset
We noticed that some datasets that were well-protected in
we use a fully connected neural network with
layer sizes [256, 128, 128, 30]. For Texas 100 and Pur-chase100 we use a fully connected neural network with
layer sizes [512, 256, 128, 100]. We train these models with
AdamOptimizer with learning rate of 0.001. The max-
imum number of epochs is set to 50 but an early stopping
criterion is also set.

Shadow model We use the same structures as the vic-
tim model for the shadow model and train the model with D_{\text{shadow}} using the same procedure as the victim model.

Attack binary classifier For the attack classifier we use a neural network with one 64-unit hidden layer and a sigmoid output for the final binary classification.

Results and discussion Table. 2 demonstrates the evalu-
ated performance of the LRN and LRN-Free adversaries
on each dataset. We observe that the adversaries are more
successful (higher AUC) on datasets with lower test accu-
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Figure 1. The structure of the CNN used for all the image datasets. All the red squares indicate $3 \times 3$ convolution filters and the blue squares show $2 \times 2$ pooling with stride of 2. In the end there are two fully-connected (FC) layers.

Table 3. Optimal values of noise multipliers and corresponding privacy budgets, for both methods

| Method         | CIFAR10 | CIFAR100 | Purchase100 | Texas100 | Location |
|----------------|---------|----------|-------------|----------|----------|
| $m^\epsilon_{\text{DPSGD}}$ | 0.001   | 0.005    | 0.05        | 0.01     | 0.5      |
| $\epsilon_{\text{DPSGD}}$   | $\sim 10^9$ | $\sim 5 \times 10^7$ | $\sim 10^5$ | $\sim 10^6$ | $\sim 10^2$ |
| $m^\epsilon_{\text{DP-logits}}$ | 0.01    | 0.001    | 0.001       | 0.005    | 0.005    |
| $\epsilon_{\text{DP-logits LRN-Free}}$ | $\sim 500$ | $\sim 5000$ | $\sim 5000$ | $\sim 10^4$ | $\sim 10^1$ |
| $\epsilon_{\text{DP-logits LRN}}$ | $\sim 10^7$ | $\sim 10^8$ | $\sim 10^8$ | $\sim 10^8$ | $\sim 10^6$ |

increasing order of the value of noise. We observe that as the noise level rises the accuracy of the model as well as the performance of the adversary drop. What we are looking for in these plots are operating points near the grey "ideal defense" bubble. This indicates an area where the accuracy of the model does not suffer while the adversary is unable to perform a successful attack.

Table 3 summarizes the (rounded) optimal values of noise multiplier $m$ and $\epsilon$ for DP-SGD and DP-Logits. We found these by inspecting the plots for the largest $m$ value (highest privacy) that falls within 80% of the initial accuracy. We found that LRN and LRN-Free adversaries perform on the same level and even for the DP-Logits method with very different $\epsilon$ values, neither outperforms the other. This can be due to the fact that LRN is not designed to learn the noise structure of the DP-Logits.

We also observe that the privacy budget for the DP-Logits method for LRN-Free adversary is generally lower than the DP-SGD method. This was expected as the DP-SGD is designed to protect the whole model including the internal parameters and to achieve better test accuracy for the more complex victim models that are applied on more advanced datasets we need to allow a significant privacy budget. We see that these models are highly sensitive to noise and high values of noise that guarantees meaningful differential privacy bounds, completely destroys the utility of the model. For LRN adversary repeated queries to the victim model degrades the privacy budget significantly, however, as mentioned we do not find any difference in performance of these adversaries.

But perhaps the most curious finding is that despite the very large values of $\epsilon$, our DP methods offer good protection against membership inference attacks. This is most likely due to the fact that these adversaries utilize very little information from the models, for example no information from the internal parameters, and the budget of a DP method is defined to guarantee privacy under any adversarial technique and any auxiliary information.

5.4. Novel Sampling Attack and Defenses for It

With an understanding of how the conventional membership inference attacks perform in practice and how successful different DP mechanisms are to defend against them, we can start our experiments on the sampling attack. Similar to Section 5.3 we choose the datasets that show meaningful weaknesses towards adversaries.

The structure of the victim and shadow models are the same as the previous sections.

Here also we decided to use two DP mechanisms for defense against the sampling adversary. We choose DP-SGD as a method that is applied during the training and randomized response that can be applied at prediction time (see Section 3.1 and 3.1.1).
Figure 2. Results of application of DP-SGD and DP-logits for different noise levels. The accuracy of the victim model is shown on the x-axis. Each dot represents a noise level and they are connected with an increasing level of noise.
**Sampling adversary** For image datasets (i.e. CIFAR10/100) we choose \( \text{pert}(x; p) = x + \mathcal{N}(0, p^2) \) which is an additive Gaussian noise to each pixel of the image in each channel. We chose \( p = \{ i \times 0.01 | 0 \leq i \leq 20, i \in \mathbb{N} \} \).

For binary datasets (Location, Texas100, Purchase100) \( \text{pert}(x; p) = \text{flip}(x) \). So we flip the values of each dimension with a probability \( p \) to the other value, that is, we flip 1 to 0 and 0 to 1. The steps of perturbation are chosen such that \( p = \{ i \times 0.005 | 0 \leq i \leq 20, i \in \mathbb{N} \} \).

For all of the datasets, we chose to generate \( N = 100 \) perturbed samples for the attack.

For steps 3 and 4 of the sampling adversary (see Section 4) We choose LRN-Free as the conventional adversary. This is based on our findings from the previous section that showed no significant difference between the performance of LRN and LRN-Free. LRN-Free is more versatile and reduces one extra step of the training an attack binary classifier for the sampling adversary. The structure of the LRN-Free is the same as the previous sections.

**DP-SGD** We use DP-SGD as the parameter perturbation mechanism against this adversary. We pick the optimum value of noise level from Table. 3 and run the sampling attack on the model trained with this noise level of DP-SGD.

**Randomized response** We choose randomized response (see Section 3.1) on the returned labels, assuming that a trained model that follows \( \text{argmax} \) protocol only allows adjusting of the labels and not the logits. Since we have more than two possible answers (labels), we modify the randomized response with a fair coin, in the following way:

- **if tails:** reveal the returned label
- **if heads:** toss the fair coin again. If heads reveal the returned label; otherwise uniformly at random choose one of the remaining classes.

In this case, the privacy budget can be calculated as:

\[
e^\epsilon = \frac{\Pr(\text{revealed} = k | \text{returned} = k)}{\Pr(\text{revealed} = k | \text{returned} = k^*)} = \frac{0.75}{0.25/(C - 1)} = 3/(C - 1)
\]

where \( C \) is the total number of classes in the labels. So this mechanism is \((\ln(3C - 3), 0)\) differentially private.

The expected accuracy of the model after using this method can be calculated as:

\[
\text{accuracy} = \frac{n_T}{n_T + n_F}
\rightarrow \mathbb{E}[\text{accuracy}_{DP}] = \frac{0.75 \times n_T}{n_T + n_F} + \frac{0.25/(C - 1) \times n_F}{n_T + n_F}
= \frac{0.75 \times \text{accuracy} + 0.25}{C - 1}(1 - \text{accuracy})
\]

where \( n_T \) and \( n_F \) indicate the number of points which correctly match the ground truth labels and the number of points which deviate from the ground truth labels, respectively. And \( n_T + n_F \) is the total number of data points that the model was tested on.

**Results and discussion** In Figure. 3 we show the performance of the sampling adversary for different perturbation levels and compare it to the horizontal baseline of the LRN adversary when the full posterior access is allowed.

We observe that the sampling adversary is able to gain up to...
50% of its initial performance for image datasets and 100% of the performance for binary datasets. The best result is achieved for Location dataset and this could correlate with the fact that location has the fewest number of features among all datasets and the perturbations around each data point are more meaningful and building the histograms over the labels is more representative of the true posteriors.

Table 4 includes the best perturbation scales $p$ with the highest AUC of adversary, found for each dataset. We see that this value is very small and particularly in the case of the binary datasets $a p < 0.015$ works best. After this range, further perturbation results in random and noisy behavior of the sampling attack.

Next, we pick $p^*$ and attack the victim model. Table 5 shows the results for the LRN with access to posteriors, the sampling attack AUC and sampling attack when DP-SGD and randomized response are used. Both DP-SGD and randomized response mitigate the risks of such attacks, however DP-SGD seems to protect better and the attacker’s performance drops to almost chance level, for most datasets.

We then study the effect of sampling number $N$ on the performance of the adversary on the victim model. We choose $N \in \{10, 100, 1000\}$ and calculate the AUC of the adversary for the optimum $p$ values from Table 4. The results are shown in Table 6. As expected, higher number of queries improve the attack. This is because according to Equation. 14 our estimate for the posterior limits to the true posterior when $n \rightarrow \text{inf}$. It is important to mention that querying a victim model multiple times is not in general a desirable action. A victim model can detect and block multiple queries. We also observe that the increase in performance is more noticeable between $N = 10 - 100$ than $N = 100 - 1000$, so for these datasets $N = 100$ is an acceptable and safe value.

At last, we also show the AUC values of the attack when the optimum $p$ of another dataset is used to attack. Table 7 shows the AUC of adversary on the victim model for different $p^*$ of datasets. We transfer the best perturbation scale among image datasets and binary datasets, separately. These results show us that an acceptable attack performance can be achieved even when the adversary trains the shadow model for a different dataset. With this strategy, the attacker is able to train the shadow model once on a similar type of dataset and carry out attacks on other data and save on the training time.

6. Discussion and Conclusion

By evaluating membership inference attacks over a large scope of different dataset, we highlight issues with the rapidly developing research thread of membership inference. Often attack performance is assessed with different type of models, while such performance cannot be seen in isolation of data and training procedure. We urgently need more transparency in reporting membership attack performance in order to be really in a position to compare and measure progress in this area. While a large fraction of the work is focused on attacks, we also provide defense evaluation. We investigate “standard” DP-SGD, but also look at effective “post-hoc” defenses, that might be a lot more practical (e.g. faced with “legacy” models) as they can operate on already trained models and provide strong protection.

Most notably, we investigate the $\text{argmax}$ defense, which previously was thought to prevent membership inference attacks. While for previous attacks this is true, we show a new sampling attack that attracts by repeated query of the model surrogate information of the model that is able to recover a large fraction of the attack performance. In turn, we also present a modification of the randomized response defense, that is in part capable of mitigating the new attack.
vector.

At time of publication, we will make all our attacks and defenses publicly available – which constitute the largest assembly of techniques in the scope of membership inference attacks that we know of. We believe that this is a contribution that the community will also benefit from. We will also publish our victim models and encourage to re-use these as a point of reference, but advocate transparency and advise to publish more details on training procedures in future work as otherwise comparable results and ultimately the scientific progress in this area is compromised.

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