Research on Applicability of Sensitivity Table Method in Optical System Alignment

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Abstract. Sensitivity table method (STM), which is a most widely used computer-aided alignment technology in optical system alignment, faces a problem that is how to determine the correctness of the applied object and established sensitivity table. In order to solve the problem, two criteria for applying STM correctly are concluded, one is determining the linearity relation between misalignments and wavefront aberration, the other one is establishing the effective sensitivity table by monitoring its condition number. Two optical systems are tested in this paper, and experimental results show the correctness of the proposed method.

1. Introduction

In the 1980s, Ira M. Egdall [1] put forward the idea of computer-aided alignment, which solves the problem that is how to assembly complex and all-reflecting optical systems faster and accurately. After decades of research and development, many theories and methods of optical system assembly are proposed.

Sensitivity table method (STM) is the most widely used one among computer-aided alignment technologies. There are many research results based on STM, Anastacia M. Hvisc [2] use the method to analyze the aberration characteristics of the misaligned spherical aberration corrector of south Africa large telescope. Sun Jingwei [3] completed the installation and adjustment of the launching telescope under the instruction of STM, and the final RMS of system performance after alignment can reach 0.1467λ. Gong Dun [4] also apply it to an off-axis three-mirror optical system, and experimental results show that the RMS value of the system is lower than 0.04λ after iterations. However, some research [5] point out that the misalignments calculation based on STM are not accurate when the misalignment range are large, it is because that the function relationship between the Zernike coefficient representing the wavefront aberration of system and misalignment is not linear. In order to solve the shortcoming of STM, many other methods are proposed to solve the misalignment, such as merit function regression (MFR) [5, 6], differential wavefront sampling (DWS) [7] and some methods based on nodal aberration theory [8, 9] and artificial neural networks [10]. However, those methods don’t explain the reason to calculate the misalignments inaccurately based on STM in some optical systems.
In this paper, our work will concentrate on three questions: (1) what kind of optical system can be suitable for STM; (2) how to determine the correctness and effectiveness of established sensitivity table; (3) why there need multiple iterations in optical system alignment when using STM. This work will contribute to a deep understanding of the method of STM and reasons for inaccuracy of the misalignment calculation. This paper is organized as follows. Firstly, the concept and theory of STM are introduced and its mathematic problem is analyzed. Secondly, the two criteria of how to use STM are summarized. Thirdly, two cases are introduced and analyzed to verify the correctness of proposed method. Finally, we conclude the paper.

2. Theoretical analysis and mathematical analysis of STM

2.1. Concept of STM

The misalignments are the displacements between actual position and the ideal position of the elements of optical system. The image quality will descend if there are misalignments in system, which will bring the wavefront error (WFE). CAA technology is to research the relation between misalignments and introduced WFE, and then determine the misalignments based on the relation.

STM are proposed based on the assumption that the WFE which usually represented by Zernike coefficients are linear to misalignments, each Zernike coefficient can be derived from the linear combination of different misalignment parameters with relevant Zernike sensitivities as expressed as following.

\[ \Delta F = A \Delta X \]  

Where:

\[ \Delta F = \begin{bmatrix} \Delta Z_1 \\ \vdots \\ \Delta Z_n \end{bmatrix}, \quad A = \begin{bmatrix} \frac{\partial Z_1}{\partial x_1} & \cdots & \frac{\partial Z_1}{\partial x_m} \\ \vdots & \ddots & \vdots \\ \frac{\partial Z_n}{\partial x_1} & \cdots & \frac{\partial Z_n}{\partial x_m} \end{bmatrix}, \quad \Delta X = \begin{bmatrix} \Delta x_1 \\ \vdots \\ \Delta x_m \end{bmatrix} \]

Here, \( \Delta x_i \) (\( i = 1, 2, \cdots, m \)) are the misalignments need to be solved, which represent the amount of disturbances in optical system such as displacement, tilt or decenter, and \( m \) is the total number of misalignment parameters. \( \Delta Z_i \) (\( i = 1, 2, \cdots, n \)) are the differences of Zernike coefficients between the measured and designed WFEs, \( n \) is the total number of Zernike coefficients adopted. \( A \) is the sensitivity table, which can be obtained by simulation in optical design software.

The misalignments can be calculate by solving the linear equation (1) when \( \Delta F \) and \( A \) are obtained, the schematic diagram is concluded as follow figure 2.

There are many methods to solve the Eq. (1), such as damped least square method, singular value decomposition method and pseudo-inverse method. From there, we can answer the first question. STM can only be applied in those cases or conditions where the relations between Zernike coefficients and misalignments are linear or near linear, which is the first criterion. Otherwise, the calculation is not accurate and even wrong if it’s based on STM. However, STM can be used in those cases if the relations become near linear in some conditions, which will be discussed detailly in section 3.1.
2.2. Mathematic analysis of STM

If the linear relations are found, the second question will be faced after a sensitivity table is established. Referring to Eq. (1), STM is modeled by m linear equations with n Zernike coefficient variables. It is known that the system only has a single unique solution if the number of independent equations is equal to the number of unknown variables. However, a big challenge is how to select the Zernike coefficients properly as there are many Zernike coefficients in different fields, which is the key to computer the misalignments correctly.

In order to solve the problem, the second judgement criterion is proposed based on condition number of sensitivity table. The condition number of a matrix measures how much the output value of the function can change for a small change in the input argument. A low condition number means the system established is well-conditioned, and it’s ill-conditioned while the condition number is a large. So, the judgement criterion tells us that we need to find the Zernike coefficients that make the condition number of sensitivity table as small as possible.

The condition number formula of sensitivity table A is listed as following.

$$\text{cond}(A) = \|A\| \|A^{-1}\|$$  \hspace{1cm} (2)

Where $\|A\|$ is the norm of $A$, and $A^{-1}$ is the inverse of $A$.

3. Two simulation cases

In this section, two optical system cases of misalignments calculation are discussed and analyzed to verify the correctness of the methods proposed.

3.1. Case one: a co-axis three-mirror-astigmatism (TMA) optical system

The co-axis TMA a typical three mirrors optical system, which consist of a concave ellipsoid primary mirror (PM), a convex hyperboloid secondary mirror (SM), a flat mirror (FM) that can change the direction of light and a concave ellipsoid tertiary mirror (TM) (See Table 1, Figure 2).

| Surface Type | Type | Conic constant | Radius | Thickness (mm) |
|--------------|------|----------------|--------|----------------|
| PM(stop)     | Conic| -0.9668        | 1501.10| -598.32        |
| SM           | Conic| -2.1872        | 389.53 | 781.32         |
| FM           | Flat | -              | Infinity| -355.25        |
| TM           | Conic| -0.51302       | 521.13 | 639.7          |
Figure 2. The layout of Hilbert telescope

Here we only take SM as an example. So, the misalignments need to be calculate are the
displacement and tilt of SM, and we use XDE, YDE, ZDE represent the decenter of SM along X axis,
Y axis and Z axis respectively, ADE and BDE represent the tilt of SM along x axis, y axis respectively.

Figure 3 shows the relations between misalignment and Zernike coefficient in two different FoVs. Obviously, in on-axis FoV, Zernike coefficient Z5 is quadratic respect to the misalignment ADE, which is not linear. So, we can know the Zernike coefficient Z5 in on-axis FoV is not suitable for STM. However, in an off-axis FoV, it become near linear, which reach the usage condition of STM. According to the first criterion, the off-axis FoV is selected to computer the misalignment. The introduced misalignments are listed in table 2. The change curve of residential misalignments after adjustments is shown in figure 4.

Table 2. Introduced misalignments of SM for TMA.

| XDE/mm | YDE/mm | ZDE/mm | ADE/° | BDE/° |
|--------|--------|--------|-------|-------|
| 0.23   | 0.16   | -0.17  | 0.146 | 0.25  |

Figure 4. The relations between residual misalignments and iterations for STM
From the Figure 4, we can know that the misalignments are eliminated by two iterations. There are still exist some misalignments after first computation. Unlike the MFR method, which can derive the misalignments by only once computation, it needs more steps based on STM. This is because that the established linear model is not accurate based on the near linear relations.

3.2. Case Two: an off-axis solar telescope (ST) optical system
As shown in Fig. 3, a general solar telescope is a typical off-axis optical system. It has a 1000mm aperture stop (located at PM) with a $0.03° \times 0.03°$ field of view (FOV). The optical parameters of ST are represented in Table 3.

![Figure 5. The layout of ST](image)

**Table 3.** Optical parameters for ST.

| Surface     | Type  | Conic constant | Radius (mm) | Thickness (mm) |
|-------------|-------|----------------|-------------|----------------|
| PM (stop)   | Conic | -1             | -3200       | -1847.8106     |
| SM          | Conic | -0.524033      | 427.259654  | 1547.489       |
| Image Plane | Plane | 0              | Infinity    | -              |

Similarly, the misalignments definition is the same with the former case, and the relations between misalignments and Zernike coefficients are near linear in on-axis FoV $(0°,0°)$ and off-axis FoV $(0.025°,0.025°)$. Two sensitivity tables are established, one is only based on one field, and the other is based on two fields. The introduced misalignments are same with TMA shown in table 2. Based on the two matrixes, the calculated misalignments and each condition number are listed in table 4.

**Table 4.** Misalignment calculations and its relative errors based on different sensitivity tables

|        | XDE/mm | YDE/mm | ZDE/mm | ADE/° | BDE/° | Condition number |
|--------|--------|--------|--------|-------|-------|------------------|
| STM1   |        |        |        |       |       |                  |
| Calculated results | -11.772 | 1.740  | 0.195  | 2.919 | 0.247 | 39842            |
| Relative errors  | 5218.45% | -987.8% | 214.78% | 1899.0% | 1.3% |                  |
| STM2   |        |        |        |       |       |                  |
| Calculated results | 0.237 | 0.207  | -0.159 | 0.142 | 0.246 | 1228             |
| Relative errors  | 3.0% | 29.4%  | 6.2%   | 2.9%  | 1.5%  |                  |

Note that STM1 and STM2 are established by the data in one and two FoV respectively.
According to table 4, we can find that the calculations by STM1 are not correct, while it is function well by STM2. This is because the equations based on the aberration in one field are not all independent, which will cause the ill-condition of the equations. It can be seen from the large condition number 39842, while it smaller in STM2, just is 1228. Figure 6 shows the change curve of residential misalignments after adjustments, we can know that the system can be aligned perfectly by two adjustments, which verify the criterion two that is should select the method with a smaller condition number.

4. Conclusion
In this paper, the principle and mathematic problem of STM are introduced and analyzed, two criteria of how to apply STM correctly are concluded: (1) STM should be established under the linear and near linear relations; (2) ST should be monitored by the condition number, which need to be as small as possible. Two cases are simulated to verify the correctness of the proposed method, which can have a great influence in the application of STM in practice.

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