MEM Analysis of Glueball Correlators at $T > 0$

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Maximum Entropy Method (MEM) is applied to glueball correlators at finite temperature constructed by using 5,500 – 9,900 gauge field configurations generated in SU(3) quenched lattice QCD with the lattice parameter $\beta_{\text{lat}} = 2\beta$, $\gamma = 6.25$ and the renormalized anisotropy $a_s/a_t = 4$. The results support the thermal width broadening of the $0^{++}$ glueball near the critical temperature.

It is believed that, in the neighborhood of the critical temperature $T_c$, the QCD vacuum begins to change its structure such as the reduction of the string tension, the partial restoration of spontaneous chiral-symmetry breaking and so on. Since the hadrons are composites of quarks and gluons, whose interactions depend on the properties of the QCD vacuum, some of the hadrons are expected to change their structure drastically near $T_c$. For instance, effective model studies suggest the pole-mass reductions of charmonium and light $q\bar{q}$ mesons as consequences of the reduction of the string tension and the partial restoration of spontaneous breaking of chiral symmetry, respectively [1,2,3]. In fact, these changes are considered as important precritical phenomena of QCD phase transition in RHIC QGP experiments, and corresponding lattice QCD calculations have been performed at quenched level [4,5]. In this paper, we study the thermal $0^{++}$ glueball, whose mass reduction seems natural in terms of the large difference between the glueball mass $m_G \simeq 1.5\text{GeV}$ and $T_c \simeq 280\text{MeV}$ in quenched QCD [6,7].

To study the pole mass of a hadron in lattice QCD, one has first to construct a temporal correlator as

$$G(\tau) = \langle \phi(\tau)\phi(0) \rangle,$$

and then resorts to its spectral representation as

$$G(\tau) = \int_0^\infty d\omega K(\tau, \omega)A(\omega),$$

where $K(\tau, \omega) = \frac{\cosh(\omega(\beta/2 - \tau))}{\sinh(\omega/2)}$, $\beta \equiv 1/T$, and $A(\omega)$ is the spectral function with its spatial momentum projected to zero, i.e., $A(\omega) = A(\omega, \vec{p} = \vec{0})$. Each peak position of $A(\omega)$ provides us with a pole mass of a hadron at $T > 0$, which can be observed in high-energy experiments.

The most popular way to extract the spectral function $A(\omega)$ from the constructed correlator is to adopt an ansatz for $A(\omega)$, and then to perform the fitting analysis. In the case of glueballs, we first investigated their pole masses using the simple narrow-peak ansatz with $A(\omega) = C \{\delta(\omega - m) - \delta(\omega + m)\}$, where $C$ and $m$ are the strength and the temperature-dependent pole mass, respectively [8].

We then proceeded to an advanced analysis adopting the Breit-Wigner ansatz, a more general and sophisticated ansatz, for the shape of the spectral function as

$$A(\omega) = C \{\delta_\Gamma(\omega - \omega_0) - \delta_\Gamma(\omega + \omega_0)\}$$

expressed with the peak function of

$$\delta_\Gamma(\omega) \equiv \frac{1}{\pi} \frac{1}{\omega^2 + \Gamma^2}.$$  \hspace{1cm} (3)

Here, $C$, $\omega_0$, and $\Gamma$ are the strength, the peak center corresponding to the pole mass, and the thermal width of the glueball peak, respectively. (The thermal width should not be confused with the decay width. It is important to keep in mind that, at $T > 0$, the thermal width is generated even for stable particles through the interaction with the thermally-excited particles.)
The Breit-Wigner analysis indicates that the main thermal effect for the $0^{++}$ glueball appears as the thermal width broadening of about 300 MeV with a reduction in the peak center of about 100 MeV. This Breit-Wigner ansatz for the lowest-lying peak in the spectral function seems natural and solid at relatively low temperature. However, near and above the critical temperature, more complicated structure may appear in the spectral function.

In this point, Maximum Entropy Method (MEM) is quite attractive, because it provides us with a numerical procedure to reconstruct the spectral function. We present our preliminary results on the MEM-reconstructed $0^{++}$ glueball spectrum at $T > 0$.

We first consider the smearing method. Use of the extended operator is one of the most useful techniques to enhance the low-energy spectra. However, it has a disadvantage that it may make the physical interpretation of resulting peak less trivial. It can create an unphysical bump structure even in the spectral functions of non-interacting particles. For instance, we consider the extreme case for glueballs. The naive continuum limit of the $0^{++}$ glueball operator is given as $\phi \propto G_{ij} G_{ij}$. After the smearing, it is spatially extended according to the Gaussian distribution as

$$\phi' \propto \int \frac{d^3y d^3z}{(2\pi)^3/2 \rho^3} \exp \left( -\frac{(\vec{y} - \vec{z})^2}{2\rho^2} \right) G_{ij}(\vec{y}) G_{ij}(\vec{z}),$$

where $\rho$ controls the size of the spatial extension. We adopt here the Coulomb gauge to obtain the continuum expression. We calculate the corresponding two-point functions perturbatively up to $O(\alpha_s^4)$, and extract their spectral functions. While the non-smeared spectral function is given as $A(\omega) \propto \omega^4$, the smeared spectral function acquires an additional Gaussian factor as

$$A(\omega) \propto \omega^4 \exp \left\{ -(\omega \rho)^2/4 \right\},$$

which possesses an unphysical bump at $\omega = 2\sqrt{2}/\rho$. The smearing method suppresses the overlap to the higher spectral components, and, as a consequence, this factor appears.

The actual low-lying glueball is not a perturbative system but a definite bound state/resonance in quenched QCD below $T_c$. Note that, in order to study the mass and the width of a definite resonance, the problem of unphysical bump is not serious, because the pole position in the complex $\omega$ plane is unaffected by the smearing. In this paper, we use the smearing method to enhance the lowest-lying $0^{++}$ glueball peak, which is negligibly small in the non-smeared spectral function.

We use the SU(3) anisotropic lattice plaquette action as

$$S_G = \frac{\beta_{lat}}{N_c} \frac{1}{\gamma_G} \sum_{s,i,j \leq 3} \text{ReTr} \left\{ 1 - P_{ij}(s) \right\} + \frac{\beta_{lat}}{N_c} \gamma_G \sum_{s,i,j \leq 3} \text{ReTr} \left\{ 1 - P_{34}(s) \right\},$$

where $P_{\mu\nu}(s) \in SU(3)$ denotes the plaquette operator in the $\mu\nu$-plane. The lattice parameter and the bare anisotropic parameter are fixed as $\beta_{lat} \equiv 2N_c/g_c^2 = 6.25$ and $\gamma_G = 3.2552$, respectively, so as to reproduce the renormalized anisotropy as $a_s/a_t = 4$. The scale unit is introduced from the on-axis data of the static interquark potential with the string tension $\sqrt{\sigma} = 440$ MeV. The resulting lattice spacings are given as $a_t^{-1} = 9.365(66)$ GeV and $a_s^{-1} = 2.341(16)$ GeV. The critical temperature is estimated as $T_c \simeq 280$ MeV from the susceptibility of the Polyakov loop. We generate 5,500–9,900 gauge configurations to construct the glueball correlators, where statistical data are divided into bins of the size 100 to reduce possible auto-correlations near the critical temperature. We adopt an appropriate smearing to enhance the low-energy spectrum.

To reconstruct the spectral function $A(\omega)$, we apply MEM to the appropriately smeared glueball correlator normalized according to $G(\tau = 0) = 1$. As the practical numerical procedure, we in principle follow Ref. We adopt the following Shannon-Jaynes entropy as

$$S \equiv \int_0^\infty \left[ A(\omega) - m(\omega) - A(\omega) \log \left( \frac{A(\omega)}{m(\omega)} \right) \right],$$

where $m(\omega)$ is a real and positive function referred to as the default model function. $m(\omega)$ is required to mimic the asymptotic behavior of
A(ω) as ω → ∞. As m(ω), we adopt Eq. (8), i.e., the perturbative spectral function up to O(α_s^2) as

\[ m(\omega) = N \omega^4 \exp \left\{ -\frac{(\omega \rho)^2}{4} \right\}, \]  

(8)

where the normalization factor N is determined so as to mimic \( G(\tau = 0) = 1 \), i.e.,

\[ 1 = \int_0^{\infty} d\omega K(\tau = 0, \omega)m(\omega). \]  

(9)

In Fig. 1, we show the reconstructed spectral functions of the lowest 0++ glueball from the temporal correlators at \( T = 130, 253, 275 \) MeV. Since the error bar estimated by following Ref. [11] appears to be unreasonably small, we do not put it in Fig. 1 to avoid unnecessary confusion. For a reasonable estimate, it seems necessary to use the jackknife error estimate [14]. In Fig. 1, we see the tendency that the peak becomes broader with increasing temperature.

To summarize, we have applied Maximum Entropy Method (MEM) to appropriately smeared glueball correlators constructed with the anisotropic SU(3) lattice QCD at finite temperature below \( T_c \), and have presented our preliminary results. We have seen the tendency that the peak becomes broader with increasing temperature, which supports the thermal width broadening [9] of the glueball near the critical temperature. It is interesting to apply MEM to the glueball correlator above \( T_c \), where it becomes less trivial to find a proper ansatz for the spectral function. Note that there may survive some non-perturbative effects even above the critical temperature. In fact, it was suggested, in σ-π sector, that a strong correlation may still survive above the critical temperature [15]. To investigate the correlation above \( T_c \) with MEM, it would be necessary to analyze the non-smeared correlators.

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