Emergent gravity and ether-drift experiments

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Abstract

According to several authors, gravity might be a long-wavelength phenomenon emerging in some ‘hydrodynamic limit’ from the same physical, flat-space vacuum viewed as a form of superfluid medium. In this framework, light might propagate in an effective acoustic geometry and exhibit a tiny anisotropy that could be measurable in the present ether-drift experiments. By accepting this view of the vacuum, one should also consider the possibility of sizeable random fluctuations of the signal that reflect the stochastic nature of the underlying ‘quantum ether’ and could be erroneously interpreted as instrumental noise. To test the present interpretation, we have extracted the mean amplitude of the signal from various experiments with different systematics, operating both at room temperature and in the cryogenic regime. They all give the same consistent value \( \langle A \rangle = O(10^{-15}) \) which is precisely the magnitude expected in an emergent-gravity approach, for an apparatus placed on the Earth’s surface. Since physical implications could be substantial, it would be important to obtain more direct checks from the instantaneous raw data and, possibly, with new experimental set-ups operating in gravity-free environments.
1. Introduction

According to the generally accepted view, gravitational phenomena are described through the introduction of a non-trivial local metric field $g_{\mu\nu}(x)$ which is interpreted as a fundamental modification of the flat space-time of Special Relativity. By fundamental, one means that, in principle, deviations from flat space might also occur at arbitrarily small scales, e.g. down to the Planck length.

However, it is an experimental fact that many physical systems (moving fluids, condensed matter systems with a refractive index, Bose-Einstein condensates,...) for which, at a fundamental level, space-time is exactly flat, are nevertheless described by an effective curved metric in their hydrodynamic limit, i.e. at length scales that are much larger than the size of their elementary constituents. For this reason, one could try to explore the alternative point of view where the space-time curvature observed in gravitational field emerges in a similar way from hydrodynamic distortions of the same physical, flat-space vacuum viewed as a form of superfluid ether ('emergent-gravity' approach).

In this different perspective, local re-scalings of the basic space-time units could represent the crucial ingredient to generate an effective non-trivial curvature, see e.g. [4, 5], in view of the substantial equivalence with the standard interpretation: "It is possible, on the one hand, to postulate that the velocity of light is a universal constant, to define natural clocks and measuring rods as the standards by which space and time are to be judged and then to discover from measurement that space-time is really non-Euclidean. Alternatively, one can define space as Euclidean and time as the same everywhere, and discover (from exactly the same measurements) how the velocity of light and natural clocks, rods and particle inertias really behave in the neighborhood of large masses" [6].

Although one does not expect to reproduce exactly the same features of classical General Relativity, still there is some value in exploring this possibility. In fact, beyond the simple level of an analogy, there might be a deeper significance if the properties of the underlying ether, that are required by the observed metric structure, could be matched with those of the physical vacuum of electroweak and strong interactions. In this case, the so called vacuum condensates, that play a crucial role for fundamental phenomena such as mass generation and quark confinement, could also represent a bridge between gravity and particle physics.

For a definite realization of this idea, one could then start by representing the physical quantum vacuum as a Bose condensate of elementary quanta and look for vacuum excitations that, on a coarse grained scale, resemble the Newtonian potential. In this case, it is relatively easy [7] to match the weak-field limit of classical General Relativity or of some of its possible variants. The idea that Bose condensates can provide various forms of gravitational dynamics
is not new (see e.g. [8, 9] and references quoted therein) and it is conceivable that the analogy could also be extended to higher orders. Therefore, being faced with two completely different interpretations of the same metric structure, one might ask: could this basic conceptual difference have phenomenological implications? The main point of our paper is that, in principle, a phenomenological difference might be associated with a small anisotropy of the velocity of light in the vacuum and that this tiny effect is within of reach of the present generation of precise ether-drift experiments.

After this general introduction, the plan of the paper is as follows. In Sect. 2, we shall illustrate this theoretical framework in a class of emergent-gravity scenarios, those in which the effective curvature is indeed induced by a re-definition of the basic space-time units and by a non-trivial vacuum refractive index. Although, in many respects, this perspective is equivalent to the conventional point of view, there is however a notable difference: the speed of light in the vacuum might not coincide with the basic parameter $c$ entering Lorentz transformations. On a general ground, this opens the possibility of a non-zero light anisotropy whose typical fractional magnitude, for an apparatus placed on the Earth’s surface, can be estimated to be $O(10^{-15})$.

Checking this expectation requires to get in touch with the ether-drift experiments (whose general aspects will be reviewed in Sect.3) that indeed observe an instantaneous signal of this order of magnitude but have interpreted so far this effect as spurious instrumental noise.

Yet, some arguments might induce to modify this present interpretation, at least in this particular context where one is taking seriously the idea of an underlying superfluid ether. In fact, the traditional analysis of these experiments is based on a theoretical model where the hypothetical, preferred reference frame is assumed to occupy a definite, fixed location in space. However, suppose that the superfluid ether exhibits a turbulent behaviour. On the one hand, this poses the theoretical problem of how to relate the macroscopic motions of the Earth’s laboratory (daily rotation, annual orbital revolution,..) to the microscopic measurement of the speed of light inside the optical cavities. On the other hand, from an experimental point of view, it suggests sizeable random fluctuations of the signal that could be erroneously interpreted as instrumental noise.

Since physical implications could be substantial, we believe that it could be worth to perform some alternative test to check the validity of the present interpretation. After all, other notable examples are known (e.g. the CMBR) where, at the beginning, an important physical signal was interpreted as a mere instrumental effect. These more technical aspects of our analysis will be discussed in Sects.4 and 5. Finally, Sect.6 will contain a summary and our conclusions.
2. Vacuum refractive index and effective acoustic metric

As anticipated, the emergent-gravity approach derives from the interesting analogies that one can establish between Einstein gravity and the hydrodynamic limit of many physical systems in flat space. However, to compare with experiments in weak gravitational field, one should concentrate on the observed form of metric structure and this restricts somehow the admissible types of theoretical models. Thus, in order to set a definite framework for our analysis, we shall consider the scenario sketched in Sect.1 where curvature arises due to modifications of the basic space-time units. This general picture can be included in the emergent-gravity philosophy provided one adopts some dynamical description where these apparent curvature effects show up for length scales that are much larger than any elementary particle, nuclear or atomic size (e.g. a fraction of millimeter or so as in Ref.[7]). While this approach leads naturally to look for a non-zero light anisotropy, a similar effect might also be expected if other mechanisms are used to generate an effective geometry of the acoustic form.

This idea of curvature as due to modifications of the basic space-time units has been considered by several authors [10, 11, 12] over the years and requires to first adopt a "Lorentzian perspective" [13] where physical rods and clocks are held together by the same basic forces underlying the structure of the 'ether' (the physical vacuum). Thus the principle of relativity means that the measuring devices of moving observers are dynamically affected in such a way that their uniform motions become undetectable. In this representation, a gravitational field is interpreted as a local modification in the state of the ether such that now both the space-time units and the speed of light become coordinate-dependent quantities thus generating an effective curvature [4, 5, 6].

Let us now consider the problem of measuring the speed of light. On a very general ground, to determine speed as (distance moved)/(time taken), one must first choose some standards of distance and time and different choices can give different answers. This is already true in Special Relativity where the universal isotropic value \( c \) entering Lorentz transformations is only obtained when describing light propagation in an inertial frame. However, inertial frames are just an idealization. Therefore, the appropriate realization of this idea is to assume local standards of distance and time such that the speed of light \( c \), is \( c \) when measured in a freely falling reference frame (at least in a space-time region small enough that tidal effects can be neglected). This provides the operative definition of the basic parameter \( c \) entering Lorentz transformations.

With these premises, to describe light propagation in a vacuum optical cavity, from the point of view of an observer \( S' \) sitting on the Earth’s surface, one can adopt increasing degrees of approximations:

i) \( S' \) is considered a freely-falling frame. Here, one starts from the observation that the
free-fall condition (in the gravitational field of the Sun, of the other planets, of the Galaxy,...) represents, up to tidal effects of the external gravitational potential $U_{\text{ext}}(x)$, the best approximation to an inertial frame. In this first approximation one assumes $c_\gamma = c$ so that, given two events which, in terms of the local space-time units of the freely-falling observer, differ by $(dx, dy, dz, dt)$, light propagation is described by the condition (ff='free-fall')

$$(ds^2)_\text{ff} = c^2dt^2 - (dx^2 + dy^2 + dz^2) = 0$$

ii) To a closer look, however, an observer placed on the Earth’s surface can only be considered a freely-falling frame up to the presence of the Earth’s gravitational field. Its inclusion leads to tiny deviations from Eq.(1). These can be estimated by considering again $S'$ as a freely-falling frame, in the same external gravitational field described by $U_{\text{ext}}(x)$, that however is also carrying on board a heavy object of mass $M$ (the Earth’s mass itself) that affects the effective local space-time structure. To derive the required correction, let us again denote by $(dx, dy, dz, dt)$ the local space-time units of the freely-falling observer $S'$ in the limit $M = 0$ and by $\delta U$ the dimensionless extra Newtonian potential produced by the heavy mass $M$ at the experimental set up where one wants to describe light propagation. Light propagation for the $S'$ observer can then be described by the condition [14, 7]

$$(ds^2)_\delta U = \frac{c^2d\hat{d}^2}{N^2} - (d\hat{x}^2 + d\hat{y}^2 + d\hat{z}^2) = 0$$

where, to first order in $\delta U$, the space-time units $(d\hat{x}, d\hat{y}, d\hat{z}, d\hat{t})$ are related to the corresponding ones $(dx, dy, dz, dt)$ for $\delta U = 0$ through an overall re-scaling factor

$$\lambda = 1 + |\delta U|$$

and

$$N' = 1 + 2|\delta U| > 1$$

Therefore, to this order, light is formally described as in General Relativity where one finds the weak-field, isotropic form of the metric

$$(ds^2)_{\text{GR}} = c^2dT^2(1 - 2|U_N|) - (dX^2 + dY^2 + dZ^2)(1 + 2|U_N|) = c^2d\tau^2 - dl^2$$

In Eq.(5) $U_N$ denotes the Newtonian potential and $(dT, dX, dY, dZ)$ arbitrary coordinates defined for $U_N = 0$. Finally, $d\tau$ and $dl$ denote the elements of proper time and proper length in terms of which, in General Relativity, one would again deduce from $ds^2 = 0$ the same universal value $c = \frac{dl}{d\tau}$. This is the basic difference with Eqs.(2)-(4) where the physical unit of length is $\sqrt{d\hat{x}^2 + d\hat{y}^2 + d\hat{z}^2}$, the physical unit of time is $d\hat{t}$ and instead a non-trivial refractive index $N'$ is introduced. For an observer placed on the Earth’s surface, its value is

$$N' - 1 \sim \frac{2G_NM}{c^2R} \sim 1.4 \cdot 10^{-9}$$
\( G_N \) being Newton’s constant and \( M \) and \( R \) the Earth’s mass and radius.

iii) Differently from General Relativity, in an interpretation where \( \mathcal{N} \neq 1 \), the speed of light in the vacuum no longer coincides with the parameter \( c \) entering Lorentz transformations. Therefore, as a general consequence of Lorentz transformations, an isotropic propagation as in Eq. (2) can only be valid if the Earth were at rest in a preferred frame \( \Sigma \). In any other case, one expects a non-zero anisotropy. To derive its value, one can start from the original derivation of Jauch and Watson [16] who worked out the quantization of the electromagnetic field in a moving medium of refractive index \( \mathcal{N} \). They noticed that the procedure introduces unavoidably a preferred frame, the one where the photon energy does not depend on the direction of propagation, and which is “usually taken as the system for which the medium is at rest”. However, such an identification reflects the point of view of Special Relativity with no preferred frame. More generally one can adapt their results to the case where the angle-independence of the photon energy defines some preferred frame \( \Sigma \). Then, for any non-zero velocity \( \mathbf{V} \) of the Earth’s laboratory, the mass shell condition for the photon energy-momentum 4-vector \( p_\mu \equiv (E/c, \mathbf{p}) \)

\[
p_\mu p_\nu g^{\mu\nu} = 0,
\]

is governed by the effective acoustic metric

\[
g^{\mu\nu} = \eta^{\mu\nu} + \kappa u^\mu u^\nu
\]

where

\[
\kappa = \mathcal{N}^2 - 1
\]

In the above relations, \( \eta^{\mu\nu} \) indicates the Minkowski tensor and \( u^\mu \) the dimensionless Earth’s velocity 4-vector \( u^\mu \equiv (u^0, \mathbf{V}/c) \) with \( u_\mu u^\mu = 1 \). In coordinate space, the analogous condition is

\[
ds^2 = g_{\mu\nu} dx^\mu dx^\nu = 0
\]

with

\[
g_{\mu\nu} = \eta_{\mu\nu} - \frac{\mathcal{N}^2 - 1}{\mathcal{N}^2} u_\mu u_\nu
\]

and we have used the relations \( g^{\mu\lambda} g_{\lambda\nu} = \delta^\mu_\nu \) and \( u_\mu = \eta_{\mu\nu} u^\nu \equiv (u^0, -\mathbf{V}/c) \).

To first order in both \( \mathcal{N} - 1 \) and \( V/c \), the off-diagonal elements

\[
g_{0i} \sim 2(\mathcal{N} - 1) \frac{V_i}{c}
\]

can be imagined as being due to a directional polarization of the vacuum induced by the now moving Earth’s gravitational field and express the general property [15] that any metric, locally, can always be brought into diagonal form by suitable rotations and boosts.
Quantitatively, Eq.(7) gives a photon energy ($u_0^2 = 1 + V^2/c^2$)

$$E(|p|, \theta) = c \frac{-\kappa u_0 \zeta + \sqrt{|p|^2(1 + \kappa u_0^2) - \kappa \zeta^2}}{1 + \kappa u_0^2}$$  \hspace{1cm} (13)

with

$$\zeta = p \cdot \frac{V}{c} = |p| \beta \cos \theta,$$  \hspace{1cm} (14)

where $\beta = |V|/c$ and $\theta \equiv \theta_{lab}$ indicates the angle defined, in the laboratory $S'$ frame, between the photon momentum and $V$. By using the above relation, one gets the one-way speed of light in the $S'$ frame

$$\frac{E(|p|, \theta)}{|p|} = c_{\gamma}(\theta) = c \frac{-\kappa \beta \sqrt{1 + \beta^2 \cos \theta} + \sqrt{1 + \kappa + \kappa \beta^2 \sin^2 \theta}}{1 + \kappa(1 + \beta^2)}$$  \hspace{1cm} (15)

or to $O(\kappa)$ and $O(\beta^2)$

$$c_{\gamma}(\theta) = \frac{c}{N} \left[ 1 - \kappa \beta \cos \theta - \frac{\kappa}{2} \beta^2 (1 + \cos^2 \theta) \right]$$ \hspace{1cm} (16)

Further, one can compute the two-way speed

$$\bar{c}_{\gamma}(\theta) = \frac{2c_{\gamma}(\theta)c_{\gamma}(\pi + \theta)}{c_{\gamma}(\theta) + c_{\gamma}(\pi + \theta)}$$

$$\sim \frac{c}{N} \left[ 1 - \beta^2 \left( \kappa - \frac{\kappa}{2} \sin^2 \theta \right) \right]$$ \hspace{1cm} (17)

and define the RMS [17] anisotropy parameter $B$ through the relation [1]

$$\frac{\bar{c}_{\gamma}(\pi/2 + \theta) - \bar{c}_{\gamma}(\theta)}{\langle \bar{c}_{\gamma} \rangle} \sim B \frac{V^2}{c^2} \cos(2\theta)$$  \hspace{1cm} (18)

with

$$|B| \sim \frac{\kappa}{2} \sim N - 1$$  \hspace{1cm} (19)

1There is a subtle difference between our Eqs.(16) and (17) and the corresponding Eqs. (6) and (10) of Ref. [18] that has to do with the relativistic aberration of the angles. Namely, in Ref.[18], with the (wrong) motivation that the anisotropy is $O(\beta^2)$, no attention was paid to the precise definition of the angle between the Earth’s velocity and the direction of the photon momentum. Thus the two-way speed of light in the $S'$ frame was parameterized in terms of the angle $\theta \equiv \theta_\Sigma$ as seen in the $\Sigma$ frame. This can be explicitly checked by replacing in our Eqs. (16) and (17) the aberration relation $\cos \theta_{lab} = (-\beta + \cos \theta_\Sigma)/(1 - \beta \cos \theta_\Sigma)$ or equivalently by replacing $\cos \theta_\Sigma = (\beta + \cos \theta_{lab})/(1 + \beta \cos \theta_{lab})$ in Eqs. (6) and (10) of Ref. [18]. However, the apparatus is at rest in the laboratory frame, so that the correct orthogonality condition of two optical cavities at angles $\theta$ and $\pi/2 + \theta$ is expressed in terms of $\theta = \theta_{lab}$ and not in terms of $\theta = \theta_\Sigma$. This trivial remark produces however a non-trivial difference in the value of the anisotropy parameter. In fact, the correct resulting $|B|$ Eq. (19) is now smaller by a factor of 3 than the one computed in Ref.[18] by adopting the wrong definition of orthogonality in terms of $\theta = \theta_\Sigma$. 

6
From the previous analysis, by replacing the value of the refractive index \( \text{Eq.}(6) \) and adopting, as a rough order of magnitude, the typical value of most cosmic motions \( V \sim 300 \text{ km/s} \), one expects an average fractional anisotropy

\[
\langle |\Delta \vec{c}_\theta| \rangle \sim |B| \frac{V^2}{c^2} = \mathcal{O}(10^{-15})
\]  

(20)

that could finally be detected in ether-drift experiments by measuring the beat frequency \( \Delta \nu \) of two orthogonal cavity-stabilized lasers.

3. Ether-drift experiments in a superfluid vacuum

In ether-drift experiments, the search for time modulations of the signal that might be induced by the Earth’s rotation (and its orbital revolution) has always represented a crucial ingredient for the analysis of the data. For instance, let us consider the relative frequency shift of two optical resonators for the experiment of Ref.\[19\]

\[
\frac{\Delta \nu(t)}{\nu_0} = S(t) \sin 2\omega_{\text{rot}} t + C(t) \cos 2\omega_{\text{rot}} t
\]  

(21)

where \( \omega_{\text{rot}} \) is the rotation frequency of one resonator with respect to the other which is kept fixed in the laboratory and oriented north-south. If one assumes that, for short-time observations of 1-2 days, the time dependence of a hypothetical physical signal can only be due to (the variations of the projection of \( \mathbf{V} \) in the interferometer’s plane caused by) the Earth’s rotation, \( S(t) \) and \( C(t) \) admit the simplest Fourier expansion (\( \tau = \omega_{\text{sid}} t \) is the sidereal time of the observation in degrees) \[19\]

\[
S(t) = S_0 + S_{s_1} \sin \tau + S_{c_1} \cos \tau + S_{s_2} \sin(2\tau) + S_{c_2} \cos(2\tau)
\]  

(22)

\[
C(t) = C_0 + C_{s_1} \sin \tau + C_{c_1} \cos \tau + C_{s_2} \sin(2\tau) + C_{c_2} \cos(2\tau)
\]  

(23)

with time-independent \( C_k \) and \( S_k \) Fourier coefficients. Therefore, by accepting this theoretical framework, it becomes natural to average the various \( C_k \) and \( S_k \) over any 1-2 day observation period. By further averaging over many short-period experimental sessions, the general conclusion \[20 \ 21\] is that, although the typical instantaneous signal is \( \mathcal{O}(10^{-15}) \), the global averages \( (C_k)_{\text{avg}} \) and \( (S_k)_{\text{avg}} \) for the Fourier coefficients are at the level \( \mathcal{O}(10^{-17}) \) or smaller and, with them, the derived parameters entering the SME \[22\] and RMS \[17\] models.

However, by taking seriously a flat-space origin of curvature from the distortions of an underlying, superfluid quantum ether, there might be different types of ether-drift where this straightforward averaging procedure is not allowed. Then, the same basic experimental data might admit a different interpretation and a definite instantaneous signal \( \Delta \nu(t) \neq 0 \) could become consistent with \( (C_k)_{\text{avg}} \sim (S_k)_{\text{avg}} \sim 0 \).
For this reason, we believe that, by accepting the idea that there might be a preferred reference frame, which is the modern denomination of the old ether, before assuming any definite theoretical scenario, one should first ask: if light were really propagating in a physical medium, an ether, and not in a trivial empty vacuum, how should the motion of (or in) this medium be described? Namely, could this relative motion exhibit variations that are not only due to known effects as the Earth’s rotation and orbital revolution?

Without fully understanding the nature of that substratum that we call physical vacuum, it is not possible to make definite predictions. Still, according to present elementary-particle theory, this physical vacuum is not trivially empty but is filled by particle condensates and therefore it becomes natural to represent the vacuum as a superfluid medium, a quantum liquid. By further considering the idea of a non-zero vacuum energy, this physical substratum could also represent a preferred reference frame. In this picture, the standard assumption of smooth sinusoidal variations of the signal, associated with the Earth’s rotation and its orbital revolution, corresponds to describe the superfluid flow in terms of simple regular motions.

However, visualization techniques that record the flow of superfluid helium show the formation of turbulent structures with a velocity field that fluctuates randomly around some average value. In our case, the concept of turbulence arises naturally if one takes seriously the idea of the vacuum as a quantum liquid, i.e. a fluid where density and current satisfy local uncertainty relations, as suggested by Landau to explain the phenomenon of superfluidity. According to this quantum-hydrodynamical representation, a fluid whose density is exactly known at some point becomes, at that same point, totally undetermined in its velocity. Therefore a (nearly) incompressible quantum liquid should be thought as microscopically turbulent.

While this provides some motivation to look for an ultimate quantum origin of turbulence and for the striking similarities between many aspects of turbulence in fluids and superfluids, this picture of the vacuum allows to establish a link with the old perspective where the ether was providing the support for the electromagnetic waves. In fact, it is known that one can establish a formal equivalence between the propagation of small disturbances in an incompressible turbulent fluid and the propagation of electromagnetic waves as described by Maxwell equations. To this end, one has to decompose all basic quantities of the fluid (pressure, velocity, density) into an average background and fluctuating components and then linearize the hydrodynamical equations. By using this method, Puthoff has been able to extend the analogy to general relativity by deriving effective metric coefficients of the type needed to account for ‘gravitomagnetic’ and ‘gravitoelectric’ effects. For all these reasons, the idea of a turbulent quantum ether becomes a natural representation of the physical vacuum.

To exploit the possible implications for ether-drift experiments, let us first recall the general aspects of any turbulent flow. This is characterized by extremely irregular variations of the
velocity, with time at each point and between different points at the same instant, due to the formation of eddies [33]. For this reason, the velocity continually fluctuates about some mean value and the amplitude of these variations is not small in comparison with the mean velocity itself. The time dependence of a typical turbulent velocity field can be expressed as [33]

\[ v(x, y, z, t) = \sum_{p_1p_2...p_n} a_{p_1p_2...p_n}(x, y, z) \exp(-i \sum_{j=1}^{n} p_j \phi_j) \quad (24) \]

where the quantities \( \phi_j = \omega_j t + \beta_j \) vary with time according to fundamental frequencies \( \omega_j \) and depend on some initial phases \( \beta_j \). As the Reynolds number \( \mathcal{R} \) increases, the total number \( n \) of \( \omega_j \) and \( \beta_j \) increases. In the \( \mathcal{R} \to \infty \) limit, their number diverges so that the theory of such a turbulent flow must be a statistical theory.

Now, due to the presumably vanishingly small viscosity of a superfluid ether, the relevant Reynolds numbers are likely infinitely large in most regimes and we might be faced precisely with such limit of the theory where the temporal analysis of the flow requires an infinite number of frequencies and the physical vacuum behaves as a stochastic medium. In this case random fluctuations of the signal, superposed on the smooth sinusoidal behaviour associated with the Earth’s rotation (and orbital revolution), would produce deviations of the time dependent functions \( S(t) \) and \( C(t) \) from the simple structure in Eqs. (22) and (23) and an effective temporal dependence of the fitted \( C_k = C_k(t) \) and \( S_k = S_k(t) \). In this situation, due to the strong cancellations occurring in vectorial quantities when dealing with stochastic signals, one could easily get vanishing global inter-session averages

\[ (C_k)^{\text{avg}} \sim (S_k)^{\text{avg}} \sim 0 \quad (25) \]

Nevertheless, as it happens with the phenomena affected by random fluctuations, the average quadratic amplitude of the signal could still be preserved. Namely, by defining the positive-definite amplitude \( A(t) \) of the signal

\[ \frac{\Delta \nu(t)}{\nu_0} = A(t)e^{i\Phi(t)} \quad (26) \]

where

\[ A(t) = \sqrt{S^2(t) + C^2(t)} \quad (27) \]

a definite non-zero \( \langle A \rangle \) might well coexist with \( (C_k)^{\text{avg}} \sim (S_k)^{\text{avg}} \sim 0 \). Physical conclusions would then require to compare the obtained value of \( \langle A \rangle \) with the short-term, stability limits of the individual resonators.
4. Noise or stochastic turbulence?

To provide some evidence that indeed, in ether-drift experiments, we might be faced with stochastic fluctuations of a physical signal, we have first considered the experimental apparatus of Ref. [34] where, to minimize all sources of systematic asymmetry, the two optical cavities were obtained from the same monolithic block of ULE (Ultra Low Expansion material). In these conditions, due to sophisticated electronics and temperature controls, the short-term (about 40 seconds) stability limits for the individual optical cavities are extremely high. Namely, for the non-rotating set up, by taking into account all possible systematic effects, one deduces a stability of better than $\pm 0.05$ Hz for the individual cavities and thus better than $\pm 2 \cdot 10^{-16}$ in units of a laser frequency $\nu_0 = 2.82 \cdot 10^{14}$ Hz. This is of the same order of the average frequency shift between the two resonators, say $(\Delta \nu)^{\text{avg}} \lesssim \pm 0.06$ Hz, when averaging the signal over a very large number of temporal sequences (see their Fig.9b).

However, the magnitude of the instantaneous frequency shift is much larger, say $\pm 1$ Hz (see their Fig.9a), and so far has been interpreted as spurious instrumental noise. To check this interpretation, we observe that, in the absence of any light anisotropy, the noise in the beat frequency should be comparable to the noise of the individual resonators. Instead, for the same non-rotating set up, the minimum noise in the beat signal was found to be 10 times bigger, namely $1.9 \cdot 10^{-15}$ (see Fig.8 of Ref. [34]). Also the trend of the noise in the beat signal, as function of the averaging time, is different from the corresponding one observed in the individual resonators thus suggesting that the two types of noise might have different origin.

The authors tend to interpret this relatively large beat signal as cavity thermal noise and refer to [35]. However, this interpretation is not so obvious since the same noise in the individual cavities was reduced to a much lower level.

Similar conclusions can be obtained from the more recent analysis of Ref. [21] where the stability of the individual resonators is at the same level $10^{-16}$. Nevertheless, the typical $C(t)$ and $S(t)$ entering the beat signal are found in the range $\pm 10^{-15}$ (see their Fig.4a) and are again interpreted in terms of cavity thermal noise.

In any case, as an additional check, one can always compare with other experiments performed in the cryogenic regime. If this typical $O(10^{-15})$ beat signal reflects the stochastic nature of an underlying quantum ether (and is not just an instrumental artifact of the resonating cavities) it should be found in these different experiments as well.
5. An alternative analysis of the data

Motivated by the previous arguments, we have decided to explore the idea that the observed beat signal between vacuum optical resonators could be due to some form of turbulent ether flow. This poses the problem to relate the macroscopic motions of the Earth’s laboratory (daily rotation, annual orbital revolution,...) to the microscopic nature of the measurement of the speed of light inside the optical cavities. For very large Reynolds numbers, some macroscopic directional effects can be lost in the reduction process as energy is transferred to smaller and smaller scales. Thus, even though the relevant Earth’s cosmic motion corresponds to that indicated by the anisotropy of the CMBR (V~370 km/s, angular declination γ ~ −6 degrees, and right ascension α ~ 168 degrees) it might be difficult to detect these parameters in the laboratory.

In this perspective, one should abandon the previous type of analysis based on assuming a fixed preferred reference frame and extract the amplitude $A(t)$ of the signal from the instantaneous data obtained from a few rotations of the interferometer before any averaging procedure. As anticipated, by inspection of Fig.4a of Ref.[21] the typical $C(t)$ and $S(t)$ entering the beat signal are found in the range $\pm 12 \cdot 10^{-16}$ and this fixes the typical size of $A(t)$. However, the instantaneous values cannot be extracted from the figure. Therefore, in this condition, to obtain a rough estimate, we shall try to evaluate $\langle A \rangle$ from the $C_k$ and $S_k$ Fourier coefficients obtained after averaging the signal within each short-period session.

For our analysis, we have first re-written Eq.(21) as

$$\frac{\Delta \nu(t)}{\nu_0} = A(t) \cos(2\omega_{rot} t - 2\theta_0(t))$$

(28)

with

$$C(t) = A(t) \cos 2\theta_0(t) \quad S(t) = A(t) \sin 2\theta_0(t)$$

(29)

$\theta_0(t)$ representing the instantaneous direction of the ether-drift effect in the plane of the interferometer. In this plane, the projection of the full $\textbf{V}$ is specified by its magnitude $v = v(t)$ and by its direction $\theta_0 = \theta_0(t)$ (counted by convention from North through East so that North is $\theta_0 = 0$ and East is $\theta_0 = \pi/2$). If one assumes Eqs.(22) and (23), then $v(t)$ and $\theta_0(t)$ can be obtained from the relations [36, 37]

$$\cos z(t) = \sin \gamma \sin \phi + \cos \gamma \cos \phi \cos(\tau - \alpha)$$

(30)

$$\sin z(t) \cos \theta_0(t) = \sin \gamma \cos \phi - \cos \gamma \sin \phi \cos(\tau - \alpha)$$

(31)

$$\sin z(t) \sin \theta_0(t) = \cos \gamma \sin(\tau - \alpha)$$

(32)

$$v(t) = V \sin z(t),$$

(33)
where $\alpha$ and $\gamma$ are respectively the right ascension and angular declination of $\mathbf{V}$. Further, $\phi$ is the latitude of the laboratory and $z = z(t)$ is the zenithal distance of $\mathbf{V}$. Namely, $z = 0$ corresponds to a $\mathbf{V}$ which is perpendicular to the plane of the interferometer and $z = \pi/2$ to a $\mathbf{V}$ that lies entirely in that plane. From the above relations, by using the $\mathcal{O}(v^2/c^2)$ relation $A(t) \sim \frac{v^2(t)}{c^2}$, the other two amplitudes $S(t) = A(t) \sin 2\theta_0(t)$ and $C(t) = A(t) \cos 2\theta_0(t)$ can be obtained up to an overall proportionality constant. By using the expressions for $S(t)$ and $C(t)$ reported in Table I of Ref. [19] (in the RMS formalism [17]), this proportionality constant turns out to be $\frac{1}{2}|B|$ so that we finally find the basic relation

$$A(t) = \frac{1}{2}|B|\frac{v^2(t)}{c^2}$$

(34)

where $B$ is the anisotropy parameter entering the two-way speed of light Eq. (18). It is a simple exercise to check that, by using Eqs. (29), Eqs. (30)-(34) and finally replacing $\chi = 90^o - \phi$, one re-obtains the expansions for $C(t)$ and $S(t)$ reported in Table I of Ref. [19].

We can then replace Eq. (33) into Eq. (34) and, by adopting a notation of the type in Eqs. (22)-(23), express the Fourier expansion of $A(t)$ as

$$A(t) = A_0 + A_1 \sin \tau + A_2 \cos \tau + A_3 \sin(2\tau) + A_4 \cos(2\tau)$$

(35)

where

$$\langle A \rangle = A_0 = \frac{1}{2}|B|\frac{v^2(t)}{c^2} = \frac{1}{2}|B|\frac{V^2}{c^2} \left(1 - \sin^2 \gamma \cos^2 \chi - \frac{1}{2} \cos^2 \gamma \sin^2 \chi\right)$$

(36)

$$A_1 = -\frac{1}{4}|B|\frac{V^2}{c^2} \sin 2\gamma \sin \alpha \sin 2\chi$$

$$A_2 = -\frac{1}{4}|B|\frac{V^2}{c^2} \sin 2\gamma \cos \alpha \sin 2\chi$$

(37)

$$A_3 = -\frac{1}{4}|B|\frac{V^2}{c^2} \cos^2 \gamma \sin 2\alpha \sin^2 \chi$$

$$A_4 = -\frac{1}{4}|B|\frac{V^2}{c^2} \cos^2 \gamma \cos 2\alpha \sin^2 \chi$$

(38)

and we have denoted by $\langle \ldots \rangle$ the daily average of a quantity (not to be confused with the intersession experimental averages denoted by $\langle \ldots \rangle_{\text{avg}}$).

To obtain $A_0$ from the $C_k$ and $S_k$, we observe that by using Eq. (35) one obtains

$$\langle A^2(t) \rangle = A_0^2 + \frac{1}{2}(A_1^2 + A_2^2 + A_3^2 + A_4^2)$$

(39)

On the other hand, by using Eqs. (22), (23) and (29), one also obtains

$$\langle A^2(t) \rangle = \langle C^2(t) + S^2(t) \rangle = C_0^2 + S_0^2 + Q^2$$

(40)

where

$$Q = \sqrt{\frac{1}{2}(C_{11}^2 + S_{11}^2 + C_{22}^2 + S_{22}^2)}$$

(41)

and

$$C_{11} \equiv \sqrt{C_{s1}^2 + C_{c1}^2} \quad C_{22} \equiv \sqrt{C_{s2}^2 + C_{c2}^2}$$

(42)
\[ S_{11} \equiv \sqrt{S_{s1}^2 + S_{c1}^2} \quad S_{22} \equiv \sqrt{S_{s2}^2 + S_{c2}^2} \]  

(43)

Therefore, one can combine the two relations and get

\[ A_0^2(1 + r) = C_0^2 + S_0^2 + Q^2 \]  

(44)

where

\[ r \equiv \frac{1}{2A_0^2}(A_1^2 + A_2^2 + A_3^2 + A_4^2) \]  

(45)

By computing the ratio \( r = r(\gamma, \chi) \) with Eqs. (36)-(38), one finds

\[ 0 \leq r \leq 0.4 \]  

(46)

for the latitude of the laboratories in Berlin [19] and Düsseldorf [38] in the full range \( 0 \leq |\gamma| \leq \pi/2 \). We can thus define an average amplitude, say \( \hat{A}_0 \), which is determined in terms of \( Q \) alone as

\[ \hat{A}_0 \equiv \frac{Q}{\sqrt{1 + r}} \sim (0.92 \pm 0.08)Q \]  

(47)

where the uncertainty takes into account the numerical range of \( r \) in Eq. (46). This quantity provides, in any case, a lower bound for the true experimental \( \langle A \rangle \) since

\[ \langle A \rangle = A_0 = \sqrt{C_0^2 + S_0^2 + Q^2 \geq \frac{Q}{\sqrt{1 + r}} = \hat{A}_0} \]  

(48)

At the same time \( Q \) is determined only by the \( C_{s1}, C_{c1}, ... \) and their \( S \)-counterparts. According to the authors of Refs. [19, 20], these coefficients are much less affected by spurious effects, as compared to \( C_0 \) and \( S_0 \), and so will be our amplitude \( \hat{A}_0 \).

By starting from the basic data for the \( C_k \) and \( S_k \) reported in in Fig.2 of Ref.[20] we have thus computed the \( Q \) values for the 27 short-period experimental sessions. Their values are reported in Table I. These data represent, within their statistics, a sufficient basis to deduce that a rather stable pattern is obtained. This is due to the rotational invariant character of \( Q \) in the 8-th dimensional space of the \( C_{s1}, C_{c1}, ..., S_{s2}, S_{c2} \) so that variations of the individual coefficients tend to compensate. By taking an average of these 27 determinations one finds a mean value

\[ (Q)^{\text{avg}} = (13.0 \pm 0.7 \pm 3.8) \cdot 10^{-16} \quad \text{Ref. [20]} \]  

(49)

where the former error is purely statistical and the latter represents an estimate of the systematical effects.

As anticipated, for a further control of the validity of our analysis, we have compared with the cryogenic experiment of Ref. [38]. In this case, we have obtained the analogous value

\[ Q = (13.1 \pm 2.1) \cdot 10^{-16} \quad \text{Ref. [38]} \]  

(50)
from the corresponding $C_k$ and $S_k$ coefficients. Thus, by using Eq.(47) and the two values of $Q$ reported above, we obtain

$$\langle \hat{A}_0 \rangle_{\text{avg}} = (12.0 \pm 1.0 \pm 3.5) \cdot 10^{-16} \quad \text{Ref.[20]}$$

(51)

$$\hat{A}_0 = (12.1 \pm 1.0 \pm 2.1) \cdot 10^{-16} \quad \text{Ref.[38]}$$

(52)

where the former uncertainty takes into account the variation of $r$ in Eq.(46) and the latter is both statistical and systematical.

We emphasize that this stable value of about $10^{-15}$ is unlikely to represent just a spurious instrumental artifact of the optical cavities as that discussed in Ref.[35]. In fact, the estimate of Ref.[35] is based on the fluctuation-dissipation theorem, and therefore there is no real reason that both room temperature and cryogenic experiments exhibit the same experimental noise.

In conclusion, our alternative analysis confirms an average experimental amplitude

$$\langle A \rangle_{\text{exp}} \sim 10^{-15}$$

(53)

that can be compared with our theoretical prediction based on Eqs.(6), (19) and (34)

$$\langle A \rangle_{\text{th}} = \frac{1}{2}(N - 1)\frac{\langle v^2(t) \rangle}{c^2} \sim 7 \cdot 10^{-10} \frac{\langle v^2(t) \rangle}{c^2}$$

(54)

Therefore, by assuming the typical speed $\sqrt{\langle v^2(t) \rangle} \sim 300$ km/s of most cosmic motions, one predicts a theoretical value

$$\langle A \rangle_{\text{th}} \sim 7 \cdot 10^{-16}$$

(55)

in good agreement with the experimental result Eq.(53).

6. Summary and conclusions

In the framework of the so-called emergent-gravity approach, the space-time curvature observed in a gravitational field is interpreted as an effective phenomenon originating from long-wavelength fluctuations of the same physical, flat-space vacuum, viewed as a form of superfluid quantum ether. Thus, this view is similar to a hydrodynamic description of moving fluids where curvature arises on length scales that are much larger than the size of the elementary constituents of the fluid. This is the basic difference with the more conventional point of view where, instead, curvature represents a fundamental property of space-time that can also show up at arbitrarily small length scales.

In principle, being faced with two different interpretations of the same, observed metric structure, one might ask if this basic conceptual difference could have phenomenological implications. We have argued in Sects. 1 and 2 that, in an approach where an effective curvature emerges
from modifications of the basic space-time units, it is a pure experimental issue whether the velocity of light $c_γ$, which is measured inside a vacuum optical cavity, coincides or not with the basic parameter $c$ entering Lorentz transformations. Thus, it makes sense to consider a scenario where $c_γ \neq c$ and the idea of an angular anisotropy of the two-way speed $\langle \Delta c_θ \rangle$. This could be detected through the frequency shift

$$\frac{\Delta \nu(t)}{\nu_0} = S(t) \sin 2\omega_{\text{rot}} t + C(t) \cos 2\omega_{\text{rot}} t$$  \hspace{1cm} (56)$$

of two rotating optical resonators in those ether-drift experiments that represent the modern version of the original Michelson-Morley experiment. As indicated at the end of Sect.2, for an apparatus placed on the Earth’s surface, this fractional asymmetry is expected to be $\frac{\langle \Delta c_θ \rangle}{c} = \mathcal{O}(10^{-15})$ and this should be compared with the experimental data.

Now, the present experiments indeed observe an instantaneous signal that has precisely this order of magnitude but have interpreted so far this effect as spurious instrumental noise. The point is that the traditional analysis of the data is based on a theoretical model where the hypothetical preferred reference frame is assumed to occupy a definite, fixed location in space. Thus a true physical signal has always been searched through smooth, sinusoidal modulations associated with the Earth’s rotation (and its orbital revolution).

However, we have also argued in Sect.3 that, in this particular context, where one is taking seriously the idea of an underlying superfluid quantum ether, one might also consider unconventional forms of ether-drift and alternative interpretation of the experimental data. For instance, some theoretical arguments suggest that the superfluid ether might be in a turbulent state of motion thus making non-trivial to relate the macroscopic motions of the Earth’s laboratory (daily rotation, annual orbital revolution,...) to the microscopic measurement of the speed of light inside the optical cavities. In this scenario, where the physical vacuum behaves as a stochastic medium, a true physical signal might exhibit sizeable random fluctuations that could be erroneously interpreted as instrumental noise.

For this reason, by following the analysis of our Sects. 4 and 5, we propose a consistency check of the present interpretation of the data. This alternative analysis requires to first introduce the instantaneous magnitude $v = v(t)$ and direction $\theta_0 = \theta_0(t)$ of the hypothetical ether-drift effect (projected in the plane of the interferometer). In terms of these two basic parameters, one can re-write the two amplitudes $C(t)$ and $S(t)$ as

$$C(t) = A(t) \cos 2\theta_0(t) \hspace{1cm} S(t) = A(t) \sin 2\theta_0(t)$$  \hspace{1cm} (57)$$

where

$$A(t) = \frac{1}{2} |B| \frac{v^2(t)}{c^2}$$  \hspace{1cm} (58)$$
and $B$ is the anisotropy parameter entering the two-way speed of light Eq. (18). By first concentrating on the amplitude

$$A(t) = \sqrt{S^2(t) + C^2(t)}$$

(which is less dependent on the fluctuating directional aspects of the signal) one should compare its experimental value with the typical short-term stability of the individual resonators. Since this individual stability, in today’s most precise experiments, is at the level $10^{-16}$ the actual experimental value $\langle A(t) \rangle_{\text{exp}} \sim 10^{-15}$ is about ten times larger and might not be a spurious instrumental effect. Moreover, this measured value is completely consistent with the average theoretical expectation Eqs. (54)-(55), namely $\langle A(t) \rangle_{\text{th}} \sim 7 \cdot 10^{-16}$, for the typical speed $\sqrt{\langle v^2(t) \rangle} \sim 300$ km/s of most cosmic motions.

Thus we look forward to a new analysis of the raw data, before any averaging procedure, that one could start, for instance, by considering the daily plots of $A(t) = \sqrt{S^2(t) + C^2(t)}$ in terms of the 13384 individual determinations of $S(t)$ and $C(t)$ reported in Fig. 4a of Ref. [21] (that, in their present form, cannot be used by the reader). In the end, from a new set of precious, combined informations, the observed frequency shift, rather than being spurious noise of the underlying optical cavities, might turn out to reflect two basic properties of the physical vacuum. Namely, this could be a polarizable medium responsible for the apparent curvature effects seen in a gravitational field and, at the same time, a stochastic medium, similar to a superfluid in a turbulent state of motion, responsible for the observed strong random fluctuations of the signal. All together, the situation might resemble the discovery of the CMBR that, at the beginning, was also interpreted as mere instrumental noise.

After this first series of checks, further tests could be performed by placing the interferometer on board of a spacecraft, as in the OPTIS proposal [40]. In this case where, even in a flat-space picture, the vacuum refractive index $N$ for the freely-falling observer is exactly unity, the typical instantaneous $\Delta \nu$ should be much smaller (by orders of magnitude) than the corresponding $\mathcal{O}(10^{-15})$ value measured with the same interferometer on the Earth’s surface. Such a substantial reduction of the instantaneous signal, in a gravity-free environment, would be extremely important for our understanding of gravity.

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Table 1: We report the various values of $Q$, their uncertainties $\Delta Q$ and the ratio $R = Q/\Delta Q$ for each of the 27 experimental sessions of Ref.\[20\]. These values have been extracted, by using Eqs. (41), (42) and (43) and according to standard error propagation for a composite observable, from the basic $C_k$ and $S_k \equiv B_k$ coefficients reported in Fig.2 of Ref.\[20\].

| $Q \times 10^{-16}$ | $\Delta Q \times 10^{-16}$ | $R = Q/\Delta Q$ |
|---------------------|-----------------------------|-------------------|
| 13.3                | 3.4                         | 3.9               |
| 14.6                | 4.8                         | 3.0               |
| 6.6                 | 2.6                         | 2.5               |
| 17.8                | 2.8                         | 6.3               |
| 14.0                | 5.8                         | 2.5               |
| 11.1                | 4.2                         | 2.6               |
| 13.0                | 4.2                         | 3.1               |
| 19.2                | 6.1                         | 3.1               |
| 13.0                | 4.7                         | 2.8               |
| 12.0                | 3.5                         | 3.4               |
| 5.7                 | 2.4                         | 2.4               |
| 14.6                | 5.2                         | 2.8               |
| 16.9                | 3.3                         | 5.1               |
| 8.3                 | 2.4                         | 3.4               |
| 27.7                | 4.5                         | 6.2               |
| 28.3                | 5.7                         | 5.0               |
| 12.7                | 2.5                         | 5.1               |
| 12.1                | 5.3                         | 2.3               |
| 13.7                | 6.0                         | 2.3               |
| 23.9                | 5.7                         | 4.2               |
| 28.9                | 4.3                         | 6.7               |
| 18.4                | 5.1                         | 3.6               |
| 19.2                | 6.2                         | 3.1               |
| 11.9                | 2.7                         | 4.4               |
| 18.1                | 5.4                         | 3.3               |
| 4.2                 | 2.9                         | 1.4               |
| 31.6                | 7.9                         | 4.0               |