Abstract. We investigate the production of dark matter in a generic bouncing universe framework. Our result shows that, if the future-experimentally-measured cross section and mass of dark matter particle satisfy the cosmological constraint, \( \langle \sigma v \rangle m_X^2 < 1.82 \times 10^{-26} \), it becomes a strong indication that our universe went through a Big Bounce—instead of the inflationary phase as postulated in Standard Big Bang Cosmology—at the early stage of the cosmological evolution.

Keywords: Dark matter, Cross section, Bounce universe

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1 Introduction

Inflation has almost been crowned as it solves the Horizon Problem [1], generates nearly scale-invariant curvature spectrum [2], which agrees well with the current array of observations [3, 4], and predicts the recently observed scalar-tensor ratio [5]. However, it suffers Initial Singularity Problem and Fine-tuning Problem [6], which renders it unreliable on which it stands [7].

To address the Initial Singularity Problem which inflation scenario inevitably suffers, a concordance of effort have made in recent years by utilizing a generic bouncing feature of early universe models inspired by underlying physics [8–15]. And a stable and scale-invariant curvature spectrum, compatible with the current observation, in the bouncing universe scenario rid of Initial Singularity and Fine-tuning Problems is recently obtained [16, 17]. Therefore, we are well motived to turn our attention to the bouncing universe scenario (See [18, 19] for recent reviews.)

However, the previously measured CMB spectra as well as scalar-tensor ratio may not be enough for distinguishing the inflationary and bouncing universe concretely, due to two respects: 1) the well-established duality between inflationary and bouncing universe empowers both of them to generate the stable scale-invariant curvature power spectrum, which is compatible with current observation, with the same probability in the unified parameter space [8, 17, 20–22], therefore, the CMB spectra may not be able to serve as the direct evidence for neither inflationary nor bouncing universe; and 2) so far all of the inflationary and bouncing universe models are utilizing some undetectable fields to drive inflation or big bounce at the early stage of the cosmological evolution. Hence their predictions about CMB spectra and the scalar-tensor ratio, which are built upon the linear perturbation theory of these unconfirmed fields, is still questionable. Therefore, we are motivated furtherly to investigate a concrete way for distinguishing the inflationary and bouncing universe with the recent and near-future experimentally detectable evidences.
Hopefully, the abundance of each subatomic particle and light chemical element would serve as a good candidate for investigating the early history of our universe—enjoying the successful philosophy and experience of Big Bang Theory. However, for these subatomic particles and light chemical elements, their synthesis and thermal decoupling happened below 1 MeV, the energy scale of Big Bang Nucleosynthesis, which is much lower than the typical energy scale of inflation and big bounce [23, 24]. Therefore, they are incapable to give a stringent criteria for distinguishing the inflation and big bounce at the early stage of the cosmological evolution. Fortunately, the dark matter particle makes an exemption for this limitation thank to its small cross section and heavy mass. The idea of using dark matter mass and its cross section as a smoking gun signal of the existence of the bouncing universe was firstly proposed in [25]. However, in that approach, a temperature-independent scattering matrix is adopted, so that the cosmological constraint on the cross section and mass of dark matter is insensitive to the energy scale of the big bounce, which causes that its prediction is narrow on the parameter space and hard to be detected experimentally.

In this paper, we utilize a temperature-dependent scattering matrix, $|\mathcal{M}|^2 \propto T^2$, which renders the thermally averaged cross section of dark matter temperature-independent, $\langle \sigma v \rangle \propto T^0$ \footnote{This type of interaction, $|\mathcal{M}|^2 \propto T^2$, is ubiquitous in Standard model of particle physics [26].}, to study the evolution of dark matter abundance through a generic bounce universe scenario. The result predicts that, if the future-experimentally-measured cross section and mass of dark matter particle satisfy either the condition,

$$\langle \sigma v \rangle m_X^2 < 1.82 \times 10^{-26}, \; m_X \geq 432 \; eV,$$

or the other condition,

$$m_X = 216 \; eV, \; 0.42x_b \times 10^{-28} eV^{-1} \ll \langle \sigma v \rangle \ll 1.68 \times 10^{-28} eV^{-1},$$

it becomes a strong indication that our universe went through a Big Bounce—instead of the inflationary phase as postulated in Standard Big Bang Cosmology—at the early stage of the cosmological evolution.

### 2 The Signature of the Big Bounce

To facilitate a model independent analysis of the dark matter production in a generic bouncing universe scenario, we divide the bounce schematically into three stages [25] as shown in Fig. 1:

- **Phases I**: the pre-bounce contraction, in which $H < 0$ and $m_X < T < T_b$ ;
- **Phases II**: the post-bounce expansion, in which $H > 0$ and $m_X < T < T_b$ ;
- **Phases III**: the freeze-out phase, in which $H > 0$ and $m_X > T$ ;

and take a temperature-independent thermally averaged cross section $\langle \sigma v \rangle \propto T^0$, where $m_X$ is the mass of dark matter, $\chi$, and $H$ the Hubble parameters taking positive value in expansion and negative value in contraction. $T$ and $T_b$ are the temperatures of the cosmological background and of the bounce point, respectively. The bounce point, connecting Phases I and II with $T \sim T_b$, is highly model-dependent. The detailed modeling is sub-leading effect to our analysis of dark matter production as long as its time scale is short, and the bounce is assumed to be smooth. Given that the entropy of universe is conserved around the bounce...
Figure 1. The breakdown of the Big bounce period into a pre-bounce contraction (phase I), a post-bounce expansion (phase II), and the freeze-out of the dark matter particles (phase III).

point [27], we, therefore, have a match condition for the relic abundance at the end of the pre-bounce contraction (denoted by $-$) and the initial abundance of the post-bounce expansion (denoted by $+$),

$$Y_-(x^-) = Y_+(x^+), \quad Y \equiv \frac{n_\chi}{T^3}, \quad x \equiv \frac{m_\chi}{T}.$$

(2.1)

where $n_\chi$ is the number density of dark matter particles. In the early stage before Phase I in the bouncing universe scenario, the temperature of background, $T \ll m_\chi$, is too low to produce dark matter particle efficiently, so the number density of dark matter particles can be set to zero at the onset of the pre-bounce contraction phase without of loss generality [25]:

$$Y_-(T \sim m_\chi) = 0.$$

(2.2)

We consider the dark matter particles, $\chi$, interact with the light bosons, $\phi$, which is strongly coupled to thermal background, through the reaction $\chi + \chi \leftrightarrow \phi + \phi$. Therefore, the evolution of dark matter in the bouncing universe scenario can be analyzed using the Boltzmann equation,

$$\frac{d(n_\chi a^3)}{a^3 dt} = \langle \sigma v \rangle \left[ \left( n_\chi^{(0)} \right)^2 - n_\chi^2 \right],$$

(2.3)

where $n_\chi^{(0)}$ is the equilibrium number density of dark matter, $a$ the scale factor of the cosmological background, and $\langle \sigma v \rangle$ the thermally averaged cross section. In accordance with the generic bounce universe scenario, we model the pre-bounce contraction and the post-bounce expansion phases to be radiation-dominated, $H \propto a^{-4}$. Then, in the pre-bounce contraction phase, (Eq. 2.3) is simplified to be,

$$\frac{dY_}{dx} = -f \langle \sigma v \rangle m_\chi x^{-2} (1 - \pi^4 Y_2^2),$$

(2.4)
where \( f \) is constant during the radiation-dominated era, \( f = \frac{m_{\chi}^2}{\langle H \rangle^2} \approx 6.01 \times 10^{26} \) eV, as constrained by observations. Consequently, in the post-bounce expansion phase, (Eq. 2.3) also simplifies

\[
\frac{dY_+}{dx} = f \langle \sigma v \rangle m_{\chi} x^{-2}(1 - \pi^4 Y_+^2) ,
\]

which differs (Eq. 2.4) by an overall sign \( \pm \) due to the signs of Hubble constant in either expansion or contraction.

Solving (Eq. 2.4) and (Eq. 2.5) with the initial condition (Eq. 2.2) and (Eq. 2.1) directly, we obtain the complete solution of the abundance of dark matter until the ending of the post-bounce expansion phase,

\[
Y_+ = \frac{1 - e^{2\pi^2 f \langle \sigma v \rangle m_{\chi} \left( \frac{1 + x_b^{-2}}{x_b} \right)}}{\left( 1 + e^{2\pi^2 f \langle \sigma v \rangle m_{\chi} \left( \frac{1 + x_b^{-2}}{x_b} \right)} \right) \pi^2}.
\]

At the end of dark matter production, \( T \approx m_{\chi} \), this complete solution can be categorized in two limits, “Thermal Production” and “Non-thermal Production”:

\[
Y_+ \bigg|_{x=1} = \begin{cases} \pi^{-2}, & 4\pi^2 f \langle \sigma v \rangle m_{\chi} x_b^{-1} \gg 1 \\ 2f \langle \sigma v \rangle m_{\chi} x_b^{-1}, & 4\pi^2 f \langle \sigma v \rangle m_{\chi} x_b^{-1} \ll 1 \end{cases}.
\]

As depicted in Fig. 2, in Thermal Production case in which \( 4\pi^2 f \langle \sigma v \rangle m_{\chi} x_b^{-1} \gg 1 \), dark matter is produced thermally to be in fully thermal equilibrium and the abundance tracks the equilibrium values; whereas in the case of “Non-thermal Production”, \( 4\pi^2 f \langle \sigma v \rangle m_{\chi} x_b^{-1} \ll 1 \), the production of dark matter is insufficient to reach thermal equilibrium, and the information of the cosmological evolution of the bouncing universe, \( 2f x_b^{-1} \), is carried on its outcome. If such information can survive after the freeze-out of dark matter, it would become a smoking gun signature of the bounce universe scenario. As universe cools sufficiently in the radiation dominated expansion with \( T < m_{\chi} \), dark matter undergoes a thermal decoupling and then freeze-out. In this freeze-out phase, (Eq. 2.3) simplifies

\[
\frac{dY}{dx} = f \langle \sigma v \rangle m_{\chi} \left( \frac{\pi}{8} xe^{-2x} - \pi^4 \frac{Y^2}{x^2} \right),
\]

where the first term on the right hand side of (Eq. 2.8) is subdominant and hence discarded for \( x > 1 \). Integrating (Eq. 2.8) from \( x = 1 \) to \( x \to \infty \), we obtain the relic abundance of dark matter after freeze-out,

\[
Y_f \equiv Y \bigg|_{x \to \infty} = \frac{1}{\pi^4 f \langle \sigma v \rangle m_{\chi} + (Y_+ \bigg|_{x=1})^{-1}}.
\]

The relic abundance of dark matter, (Eq. 2.9), implies two distinctive patterns for the freeze-out process [25]:

**Strong freeze-out** According to (Eq. 2.7), the maximal abundance of dark matter produced before the thermal decoupling is \( \pi^{-2} \), \( Y_+ \bigg|_{x=1} \leq \pi^{-2} \). Therefore, if the cross section
and mass of dark matter is large enough, $\langle \sigma v \rangle m_\chi \gg \pi^{-2} f^{-1} = 1.68 \times 10^{-28} eV^{-1}$, the dark matter freeze-out strongly,

$$Y_f = \frac{1}{\pi^4 f \langle \sigma v \rangle m_\chi} = \frac{0.17 \times 10^{-28} eV^{-1}}{\langle \sigma v \rangle m_\chi}, \quad (2.10)$$

where the relic abundance of dark matter is independent of $Y_+|_{x=1}$ and inverse to the cross section. Hence the information of the early universe dynamics carried in $Y_+|_{x=1}$ is almost washed out, as it is also the case in Standard Cosmology where the strong freeze-out condition is always assumed [23, 28, 29]: the well-known WIMP miracle [30], WIMP-less miracle [31] all share this property.

Weak freeze-out On the other hand, if, however, the cross section and mass of dark matter is small, $\langle \sigma v \rangle m_\chi \ll 1.68 \times 10^{-28} eV^{-1}$, the relic abundance of dark matter after freeze-out is just the initial abundance at onset of the freeze-out process,

$$Y_f = Y_+|_{x=1}, \quad (2.11)$$

in which the information of the cosmological evolution of the bouncing universe could be carried on $Y_+|_{x=1}$, and then $Y_f$ to present.

Relic Abundance Finally, we obtain three outcomes of relic abundance of dark matter in all four combined possibilities, thermal or non-thermal production and strong or weak freeze-out, as summarized in Table 1.

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**Figure 2.** A schematic plot of the time evolution of dark matter in a generic bounce universe scenario. Three outcomes producing dark matter for satisfying current observations are illustrated.
Table 1. Relic abundance of dark matter after freeze-out

| Condition | Thermal Production | Non-thermal Production |
|-----------|--------------------|------------------------|
| Strong Freeze-out | $4\pi^2 f(\sigma v)m_\chi x_b^{-1} \gg 1$ | — |
| $\langle \sigma v \rangle m_\chi \gg 1.68 \times 10^{-28} eV^{-1}$ | $A : Y_f = \frac{0.17 x(10^{-28} eV)}{1} m_\chi > 216 eV$ | |
| Weak Freeze-out | $4\pi^2 f(\sigma v)m_\chi x_b^{-1} \ll 1$ | $B : Y_f = 1.2(\sigma v)m_\chi x_b^{-1} \times 10^{24} eV^{-1}$ |
| $\langle \sigma v \rangle m_\chi \ll 1.68 \times 10^{-28} eV^{-1}$ | $C : \langle \sigma v \rangle m_\chi \ll 1$ | $\langle \sigma v \rangle m_\chi \ll 0.42 x_b \times 10^{-28} eV^{-1}$ |

These three avenues of the production and freeze-out of dark matter in the bouncing universe scenario, as illustrated in Fig. 2, indicate three distinctive relations between the cross section and relic density of dark matter, $\Omega_\chi$, :

- $\Omega_\chi \propto \langle \sigma v \rangle^{-1}$, in which the dark matter produced in thermal production freezes out strongly, marked branch $A$;
- $\Omega_\chi \propto \langle \sigma v \rangle$, in which the dark matter produced in non-thermal production undergoes the weak freeze-out, marked branch $B$;
- $\Omega_\chi \propto \langle \sigma v \rangle^0$, in which the dark matter produced in thermal production freezes out weakly, marked branch $C$.

The branch $A$, $\Omega_\chi \propto \langle \sigma v \rangle^{-1}$, is indistinguishable with standard cosmology [23, 28], because the production of dark matter is in fully thermal equilibrium and get strong freeze-out. As the abundance tracks the equilibrium until freeze-out, all information of the early universe is washed out.

Encouragingly, the signature of big bounce arises in the branch $B$ and branch $C$, because both of them freeze-out weakly, so that the information of the cosmological evolution of the bouncing universe scenario encoded in $Y_+|_{x=1}$ survives to present:

The signature of the Big Bounce  Imposing the currently observed value of $\Omega_\chi$ [23],

$$\Omega_\chi = 1.18 \times 10^{-2} eV \times m_\chi Y_f = 0.26 \ , \quad (2.12)$$
the cosmological constraints on $\langle \sigma v \rangle$ and $m_\chi$ for branch $B$ and $C$ are obtained, as listed in Table 2 and plotted in .

Table 2. Cosmological constraints on $\langle \sigma v \rangle$ and $m_\chi$ in the Bounce Universe Scenario.

| Condition | $\langle \sigma v \rangle$, $m_\chi$ |
|-----------|----------------------------------|
| Branch $A$ : | $\langle \sigma v \rangle = 0.31 \times 10^{-39} cm^2$, $m_\chi \gg 216 eV$ |
| Branch $B$ : | $\langle \sigma v \rangle = 1.82 \times 10^{-26} m_\chi^2 x_b$, $m_\chi \gg 432 eV$ |
| Branch $C$ : | $m_\chi = 216 eV$, $0.42 x_b \times 10^{-28} eV^{-1} \ll \langle \sigma v \rangle \ll 1.68 \times 10^{-28} eV^{-1}$ |

The concrete relation, $\langle \sigma v \rangle = 1.82 \times 10^{-26} m_\chi^2 x_b$, which is predicted by the branch $B$ in Table 2 for satisfying the current observations of $\Omega_\chi$, is a falsifiable signature of the bounce.

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2 A similar relation but within reheating framework has been analyzed in [32–34]
universe scenario. Moreover, with $x_b \leq 1$, we can conclude that, if experimentally measured $m_\chi$ and $\langle \sigma v \rangle$ satisfy following condition, the shaded region in Fig. 3,

$$\langle \sigma v \rangle m_\chi^2 < 1.82 \times 10^{-26}, \quad m_\chi \geq 432 \text{ eV},$$

it strongly indicates that our universe went through a Big Bounce–instead of the inflationary scenario as postulated in Standard Cosmology–at the early stage of the cosmological evolution.

Furthermore, the branch $C$ also predicts a new falsifiable signature of the bounce universe scenario for satisfying the current observations,

$$m_\chi = 216 \text{ eV}, \quad 0.42 x_b \times 10^{-28} \text{eV}^{-1} \ll \langle \sigma v \rangle \ll 1.68 \times 10^{-28} \text{eV}^{-1}. \quad (2.14)$$

This serves as a strong indication for the bouncing universe scenario as well and is plotted as the orange dot-dashed curve in Fig. 3.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure3.pdf}
\caption{Cosmological constraints on $\langle \sigma v \rangle$ and $m_\chi$ in the Bounce Universe Scenario. If experimentally measured cross section and mass of dark matter fall in the shaded region predicted by Branch $B$ or on the orange dot-dashed curve predicted by Branch $C$, it strongly indicates that the universe went through a big bounce at the early stage of the cosmological evolution.}
\end{figure}

### 3 Summary

In this paper, we study the production and freeze-out of dark matter in a generic bouncing universe framework. We discover two new possible signatures for the bouncing universe scenario using dark matter mass and its cross section. If future-experimentally-measured cross section and mass of dark matter particle satisfy either

$$\langle \sigma v \rangle m_\chi^2 < 1.82 \times 10^{-26}, \quad m_\chi \geq 432 \text{ eV},$$

the universe went through a Big Bounce at the early stage of the cosmological evolution.
or
\[ m_\chi = 216 \text{ eV} , \quad 0.42 x_6 \times 10^{-28} \text{ eV}^{-1} \ll \langle \sigma v \rangle \ll 1.68 \times 10^{-28} \text{ eV}^{-1} , \quad (3.2) \]
it becomes a strong indication that our universe went through a Big Bounce—instead of the inflationary phase as postulated in Standard Big Bang Cosmology—at the early stage of the cosmological evolution.

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5 Appendix: Cosmological Constraints on the Energy Scale of Big Bounce

In Table 2, the cosmological constraints on the energy scale of big bounce, the cross section and mass of the dark matter for satisfying the current observation, $\Omega = 0.26$, are obtained. In this appendix, we plot the constraints on the energy scale of big bounce and the cross section of dark matter for various $m_\chi$ in accordance with the predication of Branch $B$ in Fig. 4.

\[<\sigma v>/\text{cm}^2\]

![Figure 4](image)

**Figure 4.** Cosmological constraints on $\langle \sigma v \rangle$ and $x_b$ for various mass of dark matter particle—1 KeV (the blue solid line), 1 MeV (the orange dashed line), 1 GeV (the green dot-dashed line) and 1 TeV (the red dashed line)—in the Bounce Universe Scenario.
References

[1] A. H. Guth, *The Inflationary Universe: A Possible Solution to the Horizon and Flatness Problems*, Phys.Rev. D23 (1981) 347–356.

[2] V. F. Mukhanov, H. Feldman, and R. H. Brandenberger, *Theory of cosmological perturbations. Part 1. Classical perturbations. Part 2. Quantum theory of perturbations. Part 3. Extensions*, Phys.Rept. 215 (1992) 203–333.

[3] WMAP Collaboration Collaboration, E. Komatsu et al., *Seven-Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Cosmological Interpretation*, Astrophys.J.Suppl. 192 (2011) 18, [arXiv:1001.4538].

[4] Planck collaboration Collaboration, P. Ade et al., *Planck 2013 results. XV. CMB power spectra and likelihood*, arXiv:1303.5075.

[5] BICEP2 Collaboration Collaboration, P. Ade et al., *BICEP2 I: Detection Of B-mode Polarization at Degree Angular Scales*, arXiv:1403.3985.

[6] A. R. Liddle and D. Lyth, *Cosmological inflation and large scale structure*,

[7] A. Borde and A. Vilenkin, *Eternal inflation and the initial singularity*, Phys.Rev.Lett. 72 (1994) 3305–3309, [gr-qc/9312022].

[8] D. Wands, *Duality invariance of cosmological perturbation spectra*, Phys.Rev. D60 (1999) 023507, [gr-qc/9809062].

[9] J. Khoury, B. A. Ovrut, P. J. Steinhardt, and N. Turok, *The Ekpyrotic universe: Colliding branes and the origin of the hot big bang*, Phys.Rev. D64 (2001) 123522, [hep-th/0103239].

[10] M. Gasperini and G. Veneziano, *The Pre - big bang scenario in string cosmology*, Phys.Rept. 373 (2003) 1–212, [hep-th/0207130].

[11] P. Creminelli, M. A. Luty, A. Nicolis, and L. Senatore, *Starting the Universe: Stable Violation of the Null Energy Condition and Non-standard Cosmologies*, JHEP 0612 (2006) 080, [hep-th/0606090].

[12] Y.-F. Cai, T. Qiu, Y.-S. Piao, M. Li, and X. Zhang, *Bouncing universe with quintom matter*, JHEP 0710 (2007) 071, [arXiv:0704.1090].

[13] Y.-F. Cai, T.-t. Qiu, R. Brandenberger, and X.-m. Zhang, *A Nonsingular Cosmology with a Scale-Invariant Spectrum of Cosmological Perturbations from Lee-Wick Theory*, Phys.Rev. D80 (2009) 023511, [arXiv:0810.4677].

[14] D. Wands, *Cosmological perturbations through the big bang*, arXiv:0809.4556.

[15] K. Bhattacharya, Y.-F. Cai, and S. Das, *Lee-Wick radiation induced bouncing universe models*, arXiv:1301.0661.

[16] C. Li, L. Wang, and Y.-K. E. Cheung, *Bound to bounce: a coupled scalar-tachyon model for a smooth cyclic universe*, arXiv:1101.0202.

[17] C. Li and Y.-K. E. Cheung, *The Scale-invariant Power Spectrum of Primordial Curvature Perturbation in CSTB Cosmos*, arXiv:1401.0094.

[18] M. Novello and S. P. Bergliaffa, *Bouncing Cosmologies*, Phys.Rept. 463 (2008) 127–213, [arXiv:0802.1634].

[19] R. H. Brandenberger, *The Matter Bounce Alternative to Inflationary Cosmology*, arXiv:1206.4196.

[20] F. Finelli and R. Brandenberger, *On the generation of a scale invariant spectrum of adiabatic fluctuations in cosmological models with a contracting phase*, Phys.Rev. D65 (2002) 103522, [hep-th/0112249].
[21] L. A. Boyle, P. J. Steinhardt, and N. Turok, A New duality relating density perturbations in expanding and contracting Friedmann cosmologies, Phys.Rev. D70 (2004) 023504, [hep-th/0403026].

[22] C. Li and Y.-K. E. Cheung, Dualities between Scale Invariant and Magnitude Invariant Perturbation Spectra in Inflationary/Bouncing Cosmos, arXiv:1211.1610.

[23] S. Dodelson, Modern cosmology. 2003.

[24] V. Mukhanov, Physical foundations of cosmology. 1990.

[25] C. Li, R. H. Brandenberger, and Y.-K. E. Cheung, Big Bounce Genesis, arXiv:1403.5625.

[26] M. E. Peskin and D. V. Schroeder, An Introduction to quantum field theory. 1995.

[27] Y.-F. Cai, R. Brandenberger, and X. Zhang, Preheating a bouncing universe, Phys.Lett. B703 (2011) 25–33, [arXiv:1105.4286].

[28] E. W. Kolb and M. S. Turner, The Early Universe, Front.Phys. 69 (1990) 1–547.

[29] P. Gondolo and G. Gelmini, Cosmic abundances of stable particles: Improved analysis, Nucl.Phys. B360 (1991) 145–179.

[30] R. J. Scherrer and M. S. Turner, On the Relic, Cosmic Abundance of Stable Weakly Interacting Massive Particles, Phys.Rev. D33 (1986) 1585.

[31] J. L. Feng and J. Kumar, The WIMPless Miracle: Dark-Matter Particles without Weak-Scale Masses or Weak Interactions, Phys.Rev.Lett. 101 (2008) 231301, [arXiv:0803.4196].

[32] D. J. Chung, E. W. Kolb, and A. Riotto, Production of massive particles during reheating, Phys.Rev. D60 (1999) 063504, [hep-ph/9809453].

[33] D. J. Chung, E. W. Kolb, and A. Riotto, Nonthermal supermassive dark matter, Phys.Rev.Lett. 81 (1998) 4048–4051, [hep-ph/9805473].

[34] M. Drees, H. Imininiyaz, and M. Kakizaki, Abundance of cosmological relics in low-temperature scenarios, Phys.Rev. D73 (2006) 123502, [hep-ph/0603165].