Measurement on the cosmic curvature using the Gaussian process method

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ABSTRACT

Inflation predicts that the Universe is spatially flat. The Planck 2018 measurements of the cosmic microwave background anisotropy favour a spatially closed universe at more than 2σ confidence level. We use model independent methods to study the issue of cosmic curvature. The method reconstructs the Hubble parameter $H(z)$ from cosmic chronometers data with the Gaussian process method. The distance modulus is then calculated with the reconstructed function $H(z)$ and fitted by type Ia supernovae data. Combining the cosmic chronometers and type Ia supernovae data, we obtain $\Omega_{k0} h^2 = 0.102 \pm 0.066$ which is consistent with a spatially flat universe at the 2σ confidence level. By adding the redshift space distortions data to the type Ia supernovae data with a proposed novel model independent method, we obtain $\Omega_{k0} h^2 = 0.117^{+0.058}_{-0.045}$ and no deviation from ΛCDM model is found.

Key words: cosmology: cosmological parameters

1 INTRODUCTION

The Planck 2018 temperature and polarization measurements of the cosmic microwave background anisotropy find that $\Omega_{k0} = -0.044^{+0.018}_{-0.015}$ (Aghanim et al. 2020), an apparent detection of cosmic curvature over 2σ confidence level. A spatially closed universe preferred by the Planck 2018 result is inconsistent with inflationary prediction of a flat universe. Unfortunately, the Planck 2018 result assumes a non-flat ΛCDM model and it is model dependent which limits the validity of the result. Due to the strong degeneracy between the cosmic curvature and dark energy, the value of the cosmic curvature depends on the dark energy model used in fitting the observational data (Wang et al. 2005; Gong & Zhang 2005; Gong et al. 2008; Clarkson et al. 2007; Pan et al. 2010; Gong et al. 2011; Dossett & Ishak 2012; Witzemann et al. 2018; Ryan et al. 2018; Park & Ratra 2019; Ryan et al. 2019). Because the spatial curvature of our Universe has profound consequences for inflation and fundamental physics, it is an outstanding issue in cosmology and it is necessary to determine the cosmic curvature with model independent method so that the issue of the curvature tension can be better understood.

The Hubble expansion rate $H(z)$ depends on both the background geometry and dark energy models, so the determination of the cosmic curvature from the expansion history of the Universe is model dependent. The model dependence can be alleviated if we can determine distances and the Hubble expansion rate directly from observations. Using the Friedmann-Lemaitre-Robertson-Walker (FLRW) metric, a relation between the luminosity distance and the Hubble expansion rate is derived. Since distances depend on both the Hubble expansion rate and the spatial curvature, a null test of the cosmic curvature based on the relation between distances and the Hubble expansion rate was proposed (Clarkson et al. 2007, 2008). The null test of the spatial curvature is independent of cosmological model and gravitational theory. It can determine the spatial curvature, test the FLRW metric and detect the tension between observational data. Due to the singularity of the comoving distance $D(z)$ at the redshift $z = 0$, an alternative null test of the spatial curvature which just tests the flatness of the Universe was proposed (Cai et al. 2016). With the non-parametric method of reconstructing the Hubble expansion rate from the cosmic chronometers (CCH) data and reconstructing distances from type Ia supernovae (SNe Ia) or baryon acoustic oscillation data, the null tests of the spatial curvature were applied to determine the cosmic curvature and test the flatness of the Universe (Cai et al. 2016; Shafieloo & Clarkson 2010; Li et al. 2014; Sapone et al. 2014; L’Huillier & Shafieloo 2017; Yu & Wang 2016; Marra & Sapone 2018; Yu et al. 2018; Li et al. 2019). Instead of reconstructing distances from observational data and determining the cosmic curvature at each redshift, we can obtain the constraint on the cosmic curvature by using the χ² minimization (Wei & Wu 2017; Wei 2018; Wang et al. 2017; Wei & Melia 2020; Wang et al. 2021). However, the value of the Hubble constant $H_0 = 100 h \text{ km/s/Mpc}$ is needed to reconstruct $E(z) = H(z)/H_0$, so the result with this methods depends on the value of the Hubble constant. Interestingly, the reconstructed $E(z)$ can be used to provide model independent direct evidence of cosmic acceleration (Yang & Gong 2020). Model independent estimate of the cosmic curvature by using weak lensing galaxy-shear correlations was proposed in Bernstein (2006). The possibility of using strong lensing systems...
and quasars as standard candles to determine the cosmic curvature was also discussed (Rana et al. 2017; Qi et al. 2019; Wang et al. 2020; Liu et al. 2020). By combining time delays between strongly lensed images of time variable sources and the SNe Ia distance, both the Hubble constant the cosmic curvature were determined model independently and no deviation from a spatially flat universe is detected (Collett et al. 2019). In this paper, we use the Gaussian process (GP) method to reconstruct the Hubble parameter $H(z)$ from CCH data and fit the combined parameter $\Omega_{k0} h^2$ to SNe Ia data so that the issue of the value of the Hubble constant is avoided.

Combining the measurements of distances and the growth of large structure, we can estimate the cosmic curvature model independently (Mortonson 2009). The growth rate of matter perturbation can distinguish modified gravity and dark energy models. In a spatially curved universe, a good approximation of the growth factor is $f(z) = \Omega_{k0} + (\gamma - 4/3) \Omega_k$ (Gong et al. 2009), where the growth index $\gamma$ is a indicator of the underlying model. For the $\Lambda$CDM model, $\gamma \approx 0.545$. For the Dvali-Gabadadze-Porrati (DGP) brane-world model (Dvali et al. 2000), $\gamma \approx 0.6875$. Armed with the analytical expression for the growth factor, we propose a model independent method to use the observations of the redshift space distortions (RSD) to constrain the spatial curvature. With the reconstructed smooth function $H(z)$ from CCH data, we can reconstruct the growth factor $f(z)$ and the function $f \sigma_8(z)$. The cosmic curvature and the growth index are then determined by fitting the reconstructed $f \sigma_8(z)$ to the RSD data. In the reconstruction process, no cosmological model or gravitational theory is used, so the measurements of the cosmic curvature and the growth index from RSD data are model independent. The combined CCH, SNe Ia and RSD data are used to determine the cosmic curvature and the growth index.

The paper is organized as follows. In section 2, we apply the model independent method of null tests of the spatial curvature to test the flatness by using the CCH and SNe Ia data. In section 3, we introduce model independent methods of determining the cosmic curvature. A model independent method of using the observations of RSD to constrain the spatial curvature is proposed. The model independent method probes not only the geometry of the Universe but also the underlying theory. The CCH, SNe Ia and RSD data are then used to constrain the cosmic curvature. The paper is concluded in section 4.

2 THE NULL TESTS OF COSMIC CURVATURE

From FLRW metric,
\begin{equation}
    ds^2 = -dt^2 + a^2(t) \left[ \frac{dr^2}{1-Kr^2} + r^2(d\theta^2 + \sin^2 \theta d\phi^2) \right],
\end{equation}
we get the luminosity distance
\begin{equation}
    d_L = \frac{1+z}{H_0 \sqrt{-\Omega_{k0}}} \sin \left[ \sqrt{-\Omega_{k0}} \int_0^z \frac{dz'}{H(z')} \right],
\end{equation}
where $K$ denotes the spatial curvature and $\Omega_{k0} = -K/H_0^2$. If $\Omega_{k0} > 0$, then the function $\sin(z)$ becomes $\sinh(z)$. From the definition of the luminosity distance (2), the spatial curvature can be written as (Clarkson et al. 2007, 2008)
\begin{equation}
    \Omega_{k0} = \frac{\left( E(z) D'(z) \right)^2 - 1}{D(z)^2},
\end{equation}
where the dimensionless comoving distance $D(z)$ is related with the luminosity distance $d_L$ as $D(z) = H_0 d_L/(1+z)$, $E(z) = H(z)/H_0$ and the prime refers to the derivative with respect to the redshift $z$. Once we know the Hubble rate $H(z)$ and the distance $D(z)$ at a redshift $z$, then from Eq. (3) we can obtain the cosmic curvature $\Omega_{k0}$. Because Eq. (3) relies on the FLRW metric only and it is independent of cosmological model and gravitational theory, so it not only determines the value of $\Omega_{k0}$ model independently, the constancy of $\Omega_{k0}$ is also a model independent test of FLRW metric (Clarkson et al. 2008) and a consistency check for the observational data. Since $D(z=0)=0$ brings a singularity at $z=0$, we can use an alternative model independent null test (Cai et al. 2016)
\begin{equation}
    \Omega_{k0} = E(z) D'(z) - 1.
\end{equation}
A flat universe implies $\Omega_{k0} = 0$, so it can be used to test the flatness of the universe, but the value of $\Omega_{k0}$ cannot be directly determined from this null test and it cannot be used to test the FLRW metric and the consistency of observational data.

To test the flatness of the Universe with the null tests (3) and (4), we need to know the functions $E(z)$, $D(z)$ and $D'(z)$. We use the GP method to reconstruct $E(z)$ from CCH data and reconstruct $D(z)$ and $D'(z)$ from the Pantheon sample of SNe Ia data. The $H(z)$ data can be obtained with the CCH method using the differential redshift time derived from the spectroscopic differential evolution of passively evolving galaxies (Jimenez & Loeb 2002). In this paper we use the 31 CCH data points (Yang & Gong 2020) which were measured by assuming the BC03 stellar population synthesis model (Simon et al. 2005; Stern et al. 2010; Zhang et al. 2014; Moressco et al. 2012; Morescio 2015; Morescio et al. 2016; Ratsimbazafy et al. 2017). These data cover the redshift range up to $z \sim 2$ and assume no particular cosmological model. We use the public available python package GaPP (Seikel et al. 2012) for GP method to reconstruct the Hubble parameter $H(z)$ and the results are shown in Fig. 1. The reconstructed $H(z)$ is in good agreement with previous results (Cai et al. 2016; Wei & Wu 2017; Yu et al. 2018; Gómez-Valent & Amendola 2018; Pinho et al. 2018; Haridasu et al. 2018; Yang & Gong 2020). From this model independent reconstruction, we get the Hubble constant $H_0 = 67.46 \pm 4.75$ km/s/Mpc and this value will be used for the null tests below.

The comoving distance can be measured from SNe Ia observations. The latest Pantheon sample (Sc Foley et al. 2018) includes 1048 spectroscopically confirmed SNe Ia with the redshift up to $z \sim 2.3$. It consists of 279 spectroscopically confirmed SNe Ia with redshift 0.03 < $z$ < 0.68 discovered by the Pan-STARRS1 Medium Deep Survey (Rest et al. 2014), samples of SNe Ia from the Harvard Smithsonian Center for Astrophysics SN surveys (Hicken et al. 2009), the Carnegie SN Project (Stritzinger et al. 2011), the Sloan digital sky survey (Kessler et al. 2009) and the SN legacy survey (Conley et al. 2011), and high-z data with the redshift $z > 1.0$ from the Hubble space telescope cluster SN survey (Suzuki et al. 2012), GOODS (Riess et al. 2007) and CANDELS/CLASH survey (Rudney et al. 2014; Graur et al. 2014). The calibration systematics is reduced substantially by cross-calibrating all of the SN samples. We reconstruct the comoving distance $D(z)$ from the 1048 Pantheon sample of SNe Ia with the GP method, and the results are shown in Fig. 2.
3 MODEL INDEPENDENT MEASUREMENT ON THE COSMIC CURVATURE

In this section, we use the reconstructed Hubble parameter and the observational data of SNe Ia and redshift space distortions to constrain the cosmic curvature. With the smooth function $H(z)$ reconstructed from the CCH data, we use a simple trapezoidal rule (Holanda et al. 2013) to calculate the proper distance,

$$d_p(z) = \int_0^z \frac{dz'}{H(z')} \approx \frac{1}{2} \sum_{i=0}^n (z_{i+1} - z_i) \left[ \frac{1}{H(z_{i+1})} + \frac{1}{H(z_i)} \right].$$

(5)

We checked the accuracy of the above approximation with ΛCDM model and found good agreement. Using the standard error propagation formula, we obtain the error of the proper distance

$$\sigma_{dp}^2 = \sum_{i=0}^n \sigma_i^2,$$

(6)

where

$$\sigma_i = \frac{1}{2} (z_{i+1} - z_i) \left( \frac{\sigma_{H_{i+1}}^2}{H_{i+1}^4} + \frac{\sigma_{H_i}^2}{H_i^4} \right)^{1/2}.$$

(7)

Through the GP method and the above integration, we obtain the smooth function of the proper distance $d_p(z)$ and its error $\sigma_{dp}(z)$ from the CCH data. Note that no specific cosmological model is assumed, so the reconstructed $d_p(z)$ is model independent.
3.1 The measurement form SNe Ia data

Using the reconstructed result for \( d_p(z) \), we calculate the luminosity distance

\[
d_L(z) = \begin{cases} 
\frac{c(1+z)}{H_0\sqrt{1-\Omega_{K0}}} \sin[H_0\sqrt{-\Omega_{K0}}d_p(z)], & \Omega_{K0} < 0, \\
\frac{c(1+z)}{H_0\sqrt{\Omega_{K0}}} \sin[H_0\sqrt{\Omega_{K0}}d_p(z)], & \Omega_{K0} = 0, \\
\frac{c(1+z)}{H_0\sqrt{\Omega_{K0}}} \sin[H_0\sqrt{\Omega_{K0}}d_p(z)], & \Omega_{K0} > 0.
\end{cases}
\]

The reconstructed luminosity distance now takes the role of a theoretical model. The parameters \( \Omega_{K0} \) and \( H_0 \) appear in the combination of \( \Omega_{K0}H_0^2 \) in the luminosity distance, so they are degenerate and we can only constrain the parameter \( \Omega_{K0}h^2 \).

Even though we don’t know the exact value of \( \Omega_{K0} \), the value of \( \Omega_{K0}h^2 \) is enough to tell us whether the Universe is spatially flat, open or closed and the conclusion is independent of the value of the Hubble constant. The error \( \sigma_{dL} \) of the reconstructed luminosity distance is

\[
\sigma_{dL}(z) = \begin{cases} 
\frac{c(1+z)}{H_0\sqrt{1-\Omega_{K0}}} \sigma_{d_p}(z), & \Omega_{K0} < 0, \\
\frac{c(1+z)}{H_0\sqrt{\Omega_{K0}}} \sigma_{d_p}(z), & \Omega_{K0} = 0, \\
\frac{c(1+z)}{H_0\sqrt{\Omega_{K0}}} \sigma_{d_p}(z), & \Omega_{K0} > 0.
\end{cases}
\]

and the error \( \sigma_\mu \) of the reconstructed distance modulus \( \mu_{gp} = 5\log_{10}(d_L/\text{Mpc}) + 25 \) is

\[
\sigma_\mu = \frac{5}{10} \frac{\sigma_{dL}}{d_L}. \tag{10}
\]

The error is added to the observational error in quadrature and the total error of the distance modulus is \( \sigma_\mu = \sigma_{\mu_{obs}} + \sigma_\mu \).

Now we fit the only parameter \( \Omega_{K0}h^2 \) to the 1048 Pantheon sample of SNe Ia by minimizing the \( \chi^2 \) function

\[
\chi^2_{SN} = \Delta \mu^T \cdot \Sigma_{\mu}^{-1} \cdot \Delta \mu, \tag{11}
\]

where \( \Delta \mu = \mu_{obs} - \mu_{gp} \) and the result is \( \Omega_{K0}h^2 = 0.102 \pm 0.066 \). The same approach was used in Wei & Wu (2017), but they reconstructed \( E(z) \) and \( H_0d_p \), so the value of the Hubble constant plays a significant role in the determination of the cosmic curvature. Due to the uncertainty or the model dependence of the value of \( H_0 \) and the arbitrary normalization in the SNe Ia data, as well as the degeneracy between \( \Omega_{K0} \) and \( H_0 \), the approach in Wei & Wu (2017) has drawbacks like model dependence. However, the model independent reconstruction of \( H(z) \) and \( d_p(z) \) presented above and the constraint on \( \Omega_{K0}h^2 \) obtained here do not suffer the \( H_0 \) problem.

3.2 The measurement from redshift space distortions

The growth of large structure can not only probe the background evolution of the Universe, but also distinguish modified theories of gravity. In this subsection, we propose a model independent method to use the growth rate data measured from RSD to constrain the spatial curvature.

To the linear order of perturbation, the matter density perturbation \( \delta = \delta \rho_m / \rho_m \) satisfies the following equation

\[
\ddot{\delta} + 2H \dot{\delta} - 4\pi G_{eff} \rho_m \delta = 0, \tag{12}
\]

where \( \rho_m \) is the background matter density and \( G_{eff} \) denotes the effect of modified gravity. For Einstein’s general relativity, \( G_{eff} \) is Newton’s gravitational constant \( G \). Using the growth factor \( f(a) = d\ln \delta / d\ln a \), a good approximated solution to Eq. (12) is (Gong et al. 2009)

\[
f(z) = \Omega_m(z)^3 + (\gamma - 4/7)\Omega_k(z), \tag{13}
\]

where

\[
\begin{align*}
\Omega_m(z) &= \Omega_{m0}(1+z)^3 \frac{(H/H_0)^2}{H^2(z)}, \quad \Omega_k(z) = \Omega_{k0}(1+z)^2 \frac{(H/H_0)^2}{H^2(z)},
\end{align*}
\]

and the subscript 0 represents the current value of the variables and the growth index \( \gamma \) depends on the model and gravitational theory. Note that there are three parameters \( \Omega_{m0}h^2 \), \( \Omega_{k0}h^2 \) and \( \gamma \) in Eq. (13). Since the matter density \( \Omega_m(z) \) and the cosmic curvature \( \Omega_k(z) \) can be obtained from the smooth function \( H(z) \) reconstructed from the CCH data, so we can combine the parametrization (13) and the growth rate data to constrain the present cosmic curvature. Note that this method constrains not only the cosmic curvature but also the growth index \( \gamma \) which is an indicator of underlying theory or model. We use 35 RSD data points compiled in Zhang & Li (2018) to constrain the cosmic curvature. The RSD data measure \( f\sigma_\delta(z) \),

\[
f\sigma_\delta(z) = f(z)\sigma_\delta(z)/\delta_0, \tag{16}
\]

where \( \sigma_\delta(z) \) is the matter power spectrum normalization on the scale of 8h⁻¹Mpc. Substituting Eqs. (13), (14), (15) into Eq. (16), we get

\[
\begin{align*}
\frac{1}{\delta_0} f\sigma_\delta(z) &= \frac{\Omega_{m0}}{H(0)} \left[ \left( \frac{H(z)}{H_0} \right)^2 \frac{ \Omega_{m0}(1+z)^3 }{H(z)^2} \right]^{1/2} \frac{1}{\delta_0} \left[ \int_0^z \frac{\Omega_{m0}(1+z)^3}{H(z)^2} \right]^{1/2} \right]^{1/2} \\
&\times \exp \left( -\int_0^z \frac{\Omega_{m0}(1+z)^3}{H(z)^2} \right) dz'.
\end{align*}
\]

Following the same error propagation procedure discussed in the previous section, we estimate the error \( \sigma_f \) on the reconstructed \( f\sigma_\delta \) from the error in \( H(z) \) and added it to the observational error in quadrature. The three parameters \( \Omega_{K0}h^2, \Omega_{m0}h^2 \) and \( \gamma \) are then fitted to the 35 RSD data points with the \( \chi^2 \) minimization. The results are shown in Fig. 5 and Table 1. Due to the large uncertainty, both the flat \( \Lambda \)CDM and DGP models are consistent with the combined RSD and CCH data.

Finally, we fit the three parameters \( \Omega_{K0}h^2, \Omega_{m0}h^2 \) and \( \gamma \) to the combined SNe Ia, RSD and CCH data and we obtain \( \Omega_{m0}h^2 = 0.124^{+0.052}_{-0.068} \), \( \Omega_{k0}h^2 = 0.117^{+0.055}_{-0.045} \) and \( \gamma = 1.06^{+0.7}_{-0.52} \). The results are shown in Fig. 6 and Table 1. With the addition of SNe Ia data, the constraint on the cosmic curvature becomes tighter and the combined data favor a spatially open universe at almost 2\( \sigma \) confidence level. However, no evidence of deviation from general relativity is detected although DGP model is also consistent with the combined data.

4 CONCLUSIONS

Based on the relation between distances and the Hubble expansion rate derived from the background FLRW metric, the null test (3) of \( \Omega_{K0} \) not only determines the cosmic curvature of the Universe but also provides a consistency test for the FLRW metric, while the null test (4) probes the flatness of
without reconstructing the luminosity distance we can determine $\Omega_{k0}$ from SNe Ia data by using the $\chi^2$ minimization. This way of determining the cosmic curvature relies on the FLRW metric only and therefore is model independent. Combining the SNe Ia and CCH data, we find that $\Omega_{k0}h^2 = 0.102 \pm 0.066$ and this result is consistent with a spatially flat universe at the 2$\sigma$ confidence level. Since we don’t use the value of the Hubble constant, this conclusion avoids the problem of the Hubble constant. In addition to the distance data which depends on the background geometry, we also propose a novel model independent method to use the RSD data which measure the growth of large structure to determine the cosmic curvature. This method constrains not only the cosmic curvature but also the growth index $\gamma$, so we can also distinguish the underlying theory. The constraint from the combined CCH, SNe Ia and RSD data is $\Omega_{k0}h^2 = 0.117^{+0.058}_{-0.045}$ and $\gamma = 1.06^{+0.27}_{-0.066}$. While no evidence of cosmic curvature is found from the combined CCH and RSD data, we find a deviation from a spatially flat universe at almost 2$\sigma$ confidence level from the combined CCH, SNe Ia and RSD data. The results suggest that SNe Ia data prefers an open universe. No deviation from ΛCDM model is found from the the combined CCH, SNe Ia and RSD data and the DGP model is also consistent with the combined observational data. More accurate data in the future may help resolve the issue of the spatial curvature of the Universe.

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**DATA AVAILABILITY**

The data underlying this article are available in the paper and the cited references.

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