Fitting the Galaxy Rotation Curves: Strings versus NFW profile

Yeuk-Kwan E. Cheung and Feng Xu
Department of Physics, Nanjing University
22 Hankou Road, Nanjing 210098, P. R. China
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Remarkable fit of galaxy rotation curves is achieved using a simple model from string theory. The rotation curves of the same group of galaxies are also fit using dark matter model with the generalized Navarro-Frenk-White profile for comparison. String model utilizes three free parameters vs five in the dark matter model. The average $\chi^2$ of the string model fit is 1.649 while that of the dark matter model is 1.513. The generalized NFW profile fits marginally better at a price of two more free parameters.

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While the Cold Dark Matter cosmology has been accepted by many as the correct theory for structure formation at large scale and the solution to the missing mass problem at the galactic scale there still lacks hard proof for the existence of Dark Matter particles. Until the coming Linear Hadron Collider, and future experiments, tells us definitely what constitute Dark Matter a more natural and universal explanation in lieu of dark matter cannot be excluded. Here we entertain the possibility that an extra centripetal acceleration for matter towards the center of a galaxy. If not properly accounted for, it would be a Lorentz force in four dimensions providing a higher-rank gauge field universally coupled to strings and hence couples uniquely to the worldsheet of the strings, is constant. Because we are no longer approximating strings as point particles, this coupling between the two-form gauge potential $B_{(2)}$ and the two-dimensional worldsheet of the string produces a net force on the string when viewed as a single particle. The center of mass of the closed string executes Landau orbits:

$$a = a_0 + r e^{i \lambda t}$$

where $a$ is the complex coordinate of the plane in which the time-like part of the three-form field has non-zero components. This phenomenon is analogous to the behaviour of an electron in a constant magnetic field.

For the present purposes a slightly different field configuration is required as compared to the exactly solvable Nappi-Witten model. We retain the spatially constant time-like component of the field strength $H_{0ij}$ only, denoted by $H$. We assume that the field is aligned with the plane of the galaxy, and envision that it may be self-consistently produced by the galaxy itself via a dynamo effect. This possibility is favoured from the string theory point of view, because the coupling of the field to matter has universal strength, i.e. all matter is charged, rather than neutral. Therefore if all matter is indeed made up of fundamental strings and hence couples universally to $H_{0ij}$ each star in a galaxy will then execute the circular motion in concentric landau orbits on the galactic plane. Effectively there is an additional Lorentz force term in the equation of motion for a test star:

$$m \frac{v^2}{r} = q H v + m F_s$$

where the field, $H$, is generated by the rotating stellar matter and the halo of gases alike.

To describe the visible mass we use the parametric distribution with exponential fall off in density due to van der Kruit and Searle:

$$\rho(r, z) = \rho_0 e^{-\frac{r}{R_d}} \operatorname{sech}^2\left(\frac{z}{Z_d}\right)$$

with $\rho_0$ being the central matter density, $R_d$ the characteristic radius of the stellar disc and $Z_d$ the characteristic

I. THE STRING MODEL

Consider a four-dimensional string model proposed by Nappi and Witten [1] in which the string theory is exactly integrable to all orders of the string scale. Furthermore all tree-level correlation functions which capture finite-sized effect of the strings, exact to all orders in string scale, have been computed [2]. This is valid for all energy scales as long as the string coupling constant is weak. The three-form background gauge field, $H_{(3)}$, coupled uniquely to the worldsheet of the strings, is constant.
thickness. Following a common practice we choose $Z_d$ to be $\frac{1}{\sigma} R_d$; the dependence of the final results on this choice is very weak.

Gravitational attraction due to the visible matter is given by

$$F_\star(r) = G_N \rho_0 R_d \bar{F}(\bar{r} \equiv \frac{r}{R_d})$$

where

$$\bar{F}(\bar{r}) = \frac{\partial}{\partial \bar{r}} \int_{\text{all space}} \frac{e^{-\bar{r}' \ sech^2(2\bar{r}')}}{|\bar{r}' - \bar{r}|} r' dr' dz' d\phi'$$

is universal function for all galaxies. We are now ready to fit the galaxy rotation curves data with three free parameters, $\Omega$, $R_d$, and $\rho_0$ defined by the following equation:

$$\frac{v^2}{r} = 2 \Omega v + G_N \rho_0 R_d \bar{F}$$

with $\Omega \equiv \frac{\pi H}{2m}$. Together with the fundamental charge-to-mass ratio the strength of the gauge field is encoded in the parameter $\Omega$.

A few remarks are in order. Independent of any galaxy rotation modelling, $\rho$ and $R_d$ can be either fit or cross checked with photometric data and hence are not really free parameters. Since we are only interested in comparing the dark matter model and our string model in fitting the galaxy rotation curves we are treating them as free parameters in both models. We will later on use the best fit values for these two parameters (and three others in the dark matter model) to compute the total mass, as well as the mass-to-light ratios, for these galaxies. These will serve as sanity check for the best fit values of the parameters in both models. We are ignoring the gas contribution from our fitting because putting in more free parameters will no doubt improve the fit for both models. For the same reason we do not allow for any correction for star extinctions and supernova feedback as they would not affect any conclusion we draw concerning the relative quality of the fit. Keeping this simplistic spirit we do not allow for any dark matter component at all in the string model and we also assume that the strength of the string field is constant throughout the span of each galaxy. Back reaction of spacetime to the presence of the string field is also ignored.

II. THE DARK MATTER MODEL

According to the Cold Dark Matter (CDM) paradigm there is approximately ten times more dark matter than visible matter. The fluctuations of the primordial density perturbations of the universe get amplified by gravitational instabilities. Hierarchical clustering models furthermore predict that dark matter density traces the density of the universe at the time of collapse and thus all dark matter halos have similar density. Baryons then fall into the gravitational potential created by the clusters of dark matter particles, forming the visible part of the galaxies. In a galaxy the dark matter exists in a spherical halo engulfing all of the visible matter and extending much further beyond the stellar disc. To describe the dark matter component we use the generalized Navarro-Frenk-White profile [3]:

$$\sigma = \frac{\sigma_0}{(\frac{r}{r_s})^\alpha (1 + \frac{r}{r_s})^{3-\alpha}}$$

where $\alpha = 1$ corresponds to the NFW profile. $r_s$ is the characteristic radius of the dark matter halo. In the fitting routine we allow the $r_s$ to vary from $3R_d$ to $30R_d$. We further require that the dark matter density be strictly smaller than the visible mass density, $\sigma_0 < \rho_0$. (The data can in fact be fit equally well with the roles of dark matter and visible matter inverted.) Here we treat $\sigma_0$, $r_s$ and $\alpha$ as free parameters. Together with $\rho_0$ and $R_d$ from the visible component, the Dark Matter model utilizes five free parameters. All in all the rotation velocity of a test star is given by

$$\frac{v^2}{r} = F_\star + F_{DM}$$

in the Dark Matter model.

III. ANALYSIS

The rotation velocity of a given test star is solved from equations (5) and (7), respectively, for the string and the dark matter model. The best fit parameters of each model are obtained by minimizing the $\chi^2$ functionals. We obtained our rotation curve data of the twenty-two galaxies in the SINGS sample from the FaNToM website.

In Figure 1 rotation curve of galaxy NGC2403 fitted using string model (left) and NFW profile (right) are plotted side by side for comparison. Squares with error bars are observational data. Theoretical predictions are indicated by the solid lines with stars in the string fit (left) and with triangles in the NFW fit (right). The string model clearly gives a better fit. The $\chi^2$-squared value, per degree of freedom, from string fit is 4.304 while that from dark matter model is 4.515. The X-axis is in kpc and Y-axis in kms$^{-1}$. 

![FIG. 1: Rotation curve of NGC2403 fit with the string model (left) and with the dark matter model (right). The $\chi^2$-squared value per degree of freedom from string fit is 4.304 while that from dark matter model is 4.515. The X-axis is in kpc and Y-axis in kms$^{-1}$.](image-url)
whereas that from the NFW fit is 4.515. Overall string model gives a \( \chi^2 \) value of 1.656 averaged over the 22 galaxies while the dark matter model gives 1.594. So we can see that the NFW profile fits marginally better at a price of two more free parameters.

After we obtain the best fit values for the free parameters we can compute the (total) masses for the galaxies.

**String Model:** For this model there is only visible matter whose mass can be straightforwardly computed by integrating \( \rho \) with the best fit values of \( R_{\text{d}} \) and \( \rho_0 \) for each galaxy.

**NFW profile:** Matter in this model consists of the visible matter, same as that in the string model, and the dark matter which assumes the generalized NFW density profile \( \rho(r) \). The NFW profile gives divergent mass if the radius is integrated to infinity. We therefore adopt the usual cutoff and compute the mass only up to the virial radius within which the average density is 200 times the critical density for closure.

### Visible Mass to Light vs B-magnitude

Using the measured B-band absolute magnitudes we compute the visible mass to light ratios for the galaxies. In string model these ratios fall between 0.11 \( \sim \) 5.6 and centred around 1. The same ratios from the NFW model span five orders of magnitude (see Fig. 2) with a lot of them falling much below 1. For the NFW model we also compute the percentage of baryonic matter in the total mass. According to the CDM paradigm this number should be around 10%. However the actual results come with a wild scatter. The scatter in the mass-to-light ratios and the baryon fractions clearly indicate that NFW profile is not capturing the underlying physics correctly.

![Fig. 2: The visible-mass-to-light ratios derived from the best fit values and the measured B-band luminosity of the 22 galaxies in string model (left) and in dark matter model (right). Both plots are in log scale for easy comparison.](image)

### Tully-Fisher relation

A Tully-Fisher relation can be derived from the string model which relates the rotation velocity in the “flat” region of the rotation curves to the product of the total luminous mass, \( M_* \), and the parameter, \( \Omega \),

\[
v_0^3 = GM_*\Omega .
\]

From the equation of motion (2) we solve for \( v \),

\[
v = \Omega r + \sqrt{\Omega^2 r^2 + F_* r} .
\]

We then look for a balance of falling Newtonian attraction and rising Lorentz force, resulting in \( \frac{\partial v}{\partial r} \sim 0 \). Because we know that the turning point is at \( r \sim 2.2 R_d \), setting \( \frac{\partial v}{\partial r} \sim 0 \) yields a relation between \( r_d \) and \( \Omega \):

\[
8\Omega^2 \sim \frac{GM_*}{r_d^3} .
\]

Inside the orbit \( r \sim 2.2 R_d \) lies most of the visible mass. We have therefore used the point-mass approximation in computing \( F_* \) and \( \frac{\partial F_*}{\partial r} \). Upon substituting (10) into (9) to eliminate \( r_d \) our Tully-Fisher relation follows. The string model therefore provides a dynamic origin of this well-tested rule of thumb. We plot our best fit values of \( GM_*\Omega \) against \( v \) in Fig. 3. The representative velocity, \( v_0 \), is selected to be the maximal observed velocity in the entire curve for each galaxy, to eliminate man-made bias. This no doubt introduces more scatter than necessary. Despite that the data obey the relation reasonably well.

![Fig. 3: The luminous mass and velocity relation of the 22 galaxies fit by the Tully-Fisher relation derived from the string model.](image)

### IV. DISCUSSION

The original appeal of the NFW profile based on the ideas of hierarchical clustering was its universality. One simple NFW profile was expected to explain structure formation, rotation curves of galaxies, giant or dwarf, from high to low surface brightness. This promise has been undermined by the cusp and core debate in dwarf galaxies as well as in the low surface brightness galaxies (see for example [4]). The fact that light does not follow dark matter—well established by detailed observation and analysis in the Milky Way (see [5] for a summary), in addition to a clear deficit of satellite galaxies in MW have only served to thicken the plot. While we are not claiming that our string toy model can answer all these questions in one stroke we merely show that it pans out just as well as the Dark Matter model in fitting the galaxy rotation curves while using two fewer parameters. Moreover by tuning the ratio of the strength of the string
field to stellar mass density galaxies with a wide range of surface brightness and sizes can be accommodated. We have one dwarf and several LSB galaxies in our sample. At the same time the model, based as it is on a tractable physical principle consistent with laws of mechanics and special relativity, does not suffer from the arbitrariness and puzzling inconsistencies of MOND.

In order to describe a universal galaxy rotation curve [6,7] one at most needs three parameters to specify the initial slope, where it bends, and the final slope. Any more parameter is redundant. In this regard the string model utilizes just the right number of them. The fact that it fits well on par with the dark matter model which employs two extra parameters should be taken seriously. However one should guard against reading too much into the game of fitting. For example, one cannot obtain a unique decomposition of the mass components of a galaxy using the rotation curve data alone, a difficulty having been encountered in the context of comparing different dark matter halo profiles. Acceptable fits (defined as $\chi < \chi_{\text{min}} + 1$ [8]) can be gotten with dark matter alone without any stellar matter in CDM model. And the roles of dark matter and stellar matter can be completely reverted in the fitting routine. The physical difference is, on the other hand, dramatic. This degeneracy is less severe in the string model in the sense that string field cannot be completely traded off in favour of stellar matter, or vice versa. However there still exists a range of values for $R_d, \rho_0$ and $H$, where “acceptable” fits can be obtained. Therefore given the quality of the presently available data rotation curve fitting alone cannot distinguish between dark matter and string field in galaxies.

However precision measurements extended to radii $r \sim 20R_d$ can distinguish string model from the other models: a gently rising rotation curve in this region is a signature prediction of this string toy model. At this moment we are, nevertheless, encouraged by this inchoate results to pursue further. In a separate article we shall subject our string model to other reality checks, and we shall report on how this simple string model accounts for gravitational lensing which is often quoted as another strong evidence for the existence of dark matter at intergalactic scales.

At this point it is worth mentioning that a critical reanalysis of available data on velocity dispersion of F-dwarfs and K-giants in the solar neighbourhood, with more plausible models, performed by Kuijken and Gilmore concluded that the data provided no robust evidence for the existence of any missing mass associated with the galactic disc in the neighbourhood of the Sun [9]. Instead a local volume density of $\rho_0 = 0.10M_{\text{sun}}p^{-3}$ is favoured, which agrees with the value obtained by star counting. Dark matter would have to exist outside the galactic disk in the form of a gigantic halo. Their pioneer work was later corroborated by [10,11,12,13] using other sets of A-star, F-star and G-giant data. Note that this observation can be nicely explained by our model as the field only affects the centripetal motion on the galactic plane but does not affect the motion perpendicular to the galactic plane.

We presented a simple string toy model with only one free parameter and we showed that it can fit the galaxy rotation curves equally well as the dark matter model with the generalized NFW profile. The latter employs two more free parameters compared with the string model. The string model respects all known principles of physics and can be derived from first principle using string theory, which in turn unifies gravity with other interactions. Our model has an unambiguous prediction concerning the rotation dynamics of satellites and stars far away from the center of the (host) galaxy. The ability to test the validity of string theory as a description of low energy physics makes the exercise worthwhile.

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