Modified Schrödinger dynamics with attractive densities

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Abstract

The linear Schrödinger equation does not predict that macroscopic bodies should be located at one place only, or that the outcome of a measurement should be unique. Quantum mechanics textbooks generally solve the problem by introducing the projection postulate, which forces definite values to emerge during measurements; many other interpretations have also been proposed. Here, in the same spirit as the GRW and CSL theories, we modify the Schrödinger equation in a way that efficiently cancels macroscopic density fluctuations in space. Nevertheless, we do not assume a stochastic dynamics as in GRW or CSL theories. Instead, we propose a deterministic evolution that includes an attraction term towards the averaged density in space of the de Broglie-Bohm position of particles, and show that this is sufficient to ensure macroscopic uniqueness and compatibility with the Born rule. The state vector can then be considered as a direct representation of physical reality.

Macroscopic uniqueness is not a natural physical consequence of standard quantum mechanics. This is because the linear Schrödinger equation can lead to situations where the position of macroscopic physical systems (the pointer of a measurement apparatus for instance) have non-zero probabilities to be at the same time at very different points of space. This general difficulty is illustrated by the famous Schrödinger cat thought experiment: the linear evolution of the state vector leads to a state containing at the same time a dead and an alive cat, that is two completely different macroscopic situations. Schrödinger considers this as a “quite ridiculous case” [1, 2, 3]. The problem is that nothing in the dynamical equations can reduce the big fluctuations of the macroscopic density of particles that occur in such situations. But macroscopic physical objects occupying simultaneously macroscopically distinct positions have never been observed; when experiments are performed, a single position of the macroscopic measurement pointer seem to appear for each realization.

Many interpretations of quantum mechanics have been proposed to deal with this apparent contradiction. Historically, the projection postulate was introduced by von Neumann [4] (but not approved by Bohr), who started from an analysis of the measurement process in terms of quantum mechanics. He uses the Schrödinger equation to study the behavior of a chain of measurement apparatuses, and finds that no definite result will ever be obtained, even after a long chain of measurements – this difficulty is known as
the “infinite von Neumann regress”. He solves it by introducing his projection postulate, which assumes that a sudden change of the of the state vector is introduced, in order to update it with the information gained in a measurement. At the other extreme, in the Everett interpretation [5], the problem is solved by considering macroscopic uniqueness is not a physical phenomenon, but a delusion arising from the very functioning of the memory registers of human minds. Numerous other interpretations have been proposed [6]: modal, relational, consistent histories, informational, etc. Most of them do not change the standard equations of quantum mechanics, but focus on the best way to interpret the state vector, its relation with physical reality, information, experimental context, etc. All these interpretations are interesting, but for the moment none has emerged as the universally accepted optimal point of view.

Other families of interpretations consider that the problem of reconciling quantum dynamics with what seems to be a routine observation, namely the uniqueness of the classical world, should be taken seriously: the formalism and equations of quantum mechanics should be adapted to predict this uniqueness without ambiguity. The two best known categories are the de Broglie-Bohm (dBB) interpretation [7, 8, 9] and the spontaneous localization theories, either in the original GRW discontinuous form [10], or in the continuous CSL form [11]. In the dBB theory, particle positions moving in ordinary 3D space are added to the variables of standard quantum mechanics; these positions have uncontrollable random initial values, but then move in a perfectly deterministic way. They are considered as physically real. In the GRW and CSL theories, no additional variables are assumed, but the usual Hamiltonian in the equation of motion of the state vector gets additional stochastic terms; the wave function is considered as a field propagating in configuration space (not ordinary 3D space) under the effect of fundamentally random processes. In these theories, measurement processes are not seen as special events, but just ordinary interaction processes between a measured system and apparatus; the observer is not a necessary ingredient of the theory.

Here we propose a combination of these two theories, where the dBB positions are still part of the dynamical equations: the positions are driven by the wave function (as in the dBB theory), but they also react on it (which does not occur in the dBB theory). The dynamics of this process is very different from that of GRW and CSL theories, since it is deterministic (no Wiener processes are assumed); the random character of a result of measurement is then just a consequence of the random value of the initial positions. The dynamics also suppresses the “empty waves” of the dBB theory, which are known to raise conceptual problems concerning the reality of these waves. It therefore seems to provide a simple and a reasonably plausible mechanism for quantum collapse.

1 Equations of motion with collapse

The change of the dynamics of the quantum state we propose is less violent than the von Neumann reduction; it is continuous, but nevertheless manages to suppress the cat paradox and the von Neumann’s infinite regress. It then becomes possible to consider that the state vector directly represents physical reality, as in the GRW and CSL theories.
The study of Bohmian positions actually provides a convenient indicator of Schrödinger’s “ridiculous cases”, and therefore suggests a way to avoid them. Consider the Bohmian positions of the atoms contained in the glass bottle containing the poison that may kill the cat. After some time, the linear Schrödinger equation predicts a superposition of states where the bottle is broken and intact; the probability densities of the constituent atoms are spread between different locations in space. By contrast, for each realization of the experiment, the dBB theory predicts that the bottle is either broken or intact; the Bohmian positions of its constituents atoms remain together in only one of the possible locations. This means that, in configuration space, one of the components of the quantum wave function propagates accompanied by Bohmian positions, while the other propagates “alone”; it has become what Bohm calls an “empty wave” [8]. In order to introduce macroscopic uniqueness in the propagation of the wave function, we will therefore introduce a dynamical process that suppresses components that are propagating too far of the Bohmian positions; in other words, we force a better match between the evolution of the wave function and that of the positions.

1.1 Bohmian localization operator $K$

We consider a system of $N$ identical particles associated with a quantum field operator $\Psi (r)$ defined at each point $r$ of ordinary 3D space. When the system is in state $|\Phi \rangle$, the local (number) density $D_\Phi (r)$ of particles at $r$ is:

$$D_\Phi (r) = \frac{\langle \Phi | \Psi^\dagger (r) \Psi (r) |\Phi \rangle}{\langle \Phi |\Phi \rangle} \tag{1}$$

In dBB theory, the local density $D_B (r)$ of Bohmian positions is a sum of delta functions:

$$D_B (r,t) = \sum_{n=1}^{N} \delta (r - q_n) \tag{2}$$

where the sum runs over all $N$ particles with Bohmian position $q_n (t)$. We wish to introduce a dynamics that favors evolutions where $D_\Phi (r)$ is attracted towards regions where $D_B (r)$ is high, with a space averaging suppressing the microscopic fluctuations of $D_B (r)$. For this purpose, we introduce an averaging length $a_L$ and the following integral of $D_B$:

$$N_B (r,t) = \int d^3 r' \ e^{-(r-r')^2/ (a_L)^2} D_B (r',t) = \sum_{n=1}^{N} e^{-(r-q_n)^2/ (a_L)^2} \tag{3}$$

The order of magnitude of $N_B (r,t)$ is the number of Bohmian positions within a volume $(a_L)^3$ around point $r$; we have $0 \leq N_B (r,t) \leq N$. We then introduce the localization operator $L (t)$ by:

$$L (t) = \int d^3 r \ N_B (r,t') \ \Psi^\dagger (r) \Psi (r) = \sum_{p=1}^{N} N_B (R_p, t') \tag{4}$$
(R_p is the position operator of particle p). This operator combines the quantum density operator \( \Psi^\dagger (r) \Psi (r) \) with the classical averaged density \( N_B (r, t') \). It has the form of a single-particle potential energy operator; \( L (t) \) multiplies any wave function \( \Phi (r_1, r_2, \ldots, r_N) \) by the the sum over \( p \) of the individual potentials \( N_B (r_p, t') \). The result depends on the relative positions of all Bohmian positions \( q_n \). If they are at large relative distances from each other (larger than \( a_L \)), the \( r \) dependence of \( N_B \) exhibits a series of bumps centered on each \( q_n \), each of height unity, and separated by intervals where \( N_B \) practically vanishes; if none of the variables \( r_p \) falls inside one of these bumps, the effect of \( L (t) \) merely cancels the wave function. If, at the other extreme, all Bohmian positions are clustered together inside a single volume \( V \) of size smaller than \( a_L^3 \), the \( r \) dependence of \( N_B \) now exhibits a single bump of height \( N \), and the effect of \( L (t) \) is more focussed in space; for instance, in the region of configuration space where all variables \( r_p \) of \( \Phi (r_1, r_2, \ldots, r_N) \) fall inside \( V \), the wave function is multiplied by \( N^2 \). This fast \( N \) dependence plays an important role in the sudden collapse mechanism we discuss below.

Finally we introduce the time-integral \( K (t) \) of \( L (t) \):

\[
K (t) = \int_0^t dt' L (t') = \int_0^t dt' \int d^3 r \, N_B (r, t') \, \Psi^\dagger (r) \Psi (r)
\]

This operator is the essential ingredient in the modified dynamics we propose. Both \( L (t) \) and \( K (t) \) are Hermitian operators.

### 1.2 Modified attractive quantum dynamics

The Hamiltonian of the system is \( H (t) \), corresponding to a unitary evolution operator \( U (t) \) for the standard state vector \( |\Phi_S (t)\rangle \):

\[
|\Phi_S (t)\rangle = U (t) |\Phi_S (0)\rangle
\]

We introduce a new state vector \( |\Phi (t)\rangle \) obeying the relation:

\[
|\Phi (t)\rangle = e^{\gamma_L K(t)} U (t) |\Phi (0)\rangle
\]

where \( \gamma_L \) is a constant time rate that will be specified below. The time evolution of \( |\Phi (t)\rangle \) is given by:

\[
\frac{i\hbar}{dt} |\Phi (t)\rangle = \left[ e^{\gamma_L K(t)} \, H (t) \, e^{-\gamma_L K(t)} + i\hbar \gamma_L \, L (t) \right] |\Phi (t)\rangle
\]

We assume that \( H (t) \) is the sum of a kinetic energy operator \( T \), a (possibly time-dependent) potential energy \( V_1 (t) \), and an interaction energy \( W_2 (t) \):

\[
H (t) = T + V_1 (t) + W_2
\]

The operators \( V_1 (t) \), \( W_2 \) and \( K (t) \) are all diagonal in the position representation, and therefore commute. We then have:

\[
e^{\gamma_L K(t)} \, H (t) \, e^{-\gamma_L K(t)} = H (t) + \tilde{T}_L (t)
\]
where $\tilde{T}_L (t)$ is the change of the kinetic energy under the effect of the transformation associated with the exponential $e^{\gamma L K (t)}$:

$$\tilde{T}_L (t) = e^{\gamma L K (t)} T e^{-\gamma L K (t)} - T$$  \hspace{1cm} (11)

Inserting these results into (8), we obtain the time evolution of $|\Phi (t)\rangle$ in the form of a modified Schrödinger equation:

$$i \hbar \frac{d}{dt} |\Phi (t)\rangle = \left[ H (t) + \tilde{T}_L (t) + i \hbar \gamma L L (t) \right] |\Phi (t)\rangle$$ \hspace{1cm} (12)

The modified dynamics we study in this article is defined by this time differential equation.

One immediately notices that non-Hermitian terms appear in the right hand side of equation (8) or (12), so that the modified dynamics does not conserve the norm of $|\Phi \rangle$. This is not a problem if we consider that $|\Phi \rangle$ defines the direction in the space of states (a one-dimension subspace of this space, what von Neumann calls a “ray”), so that its norm is irrelevant. Nevertheless, if desired, one can also obtain a normalized state vector $|\Phi\rangle$, which obeys the following equation of evolution:

$$i \hbar \frac{d}{dt} |\Phi (t)\rangle = \left[ H (t) + \tilde{T}_L (t) + i \hbar \gamma L L (t) + iC (t) \right] |\Phi (t)\rangle$$ \hspace{1cm} (13)

where:

$$C (t) = - \langle \Phi (t) | \left[ \hbar \gamma L L (t) + \frac{1}{2} \left( e^{\gamma L K (t)} T e^{-\gamma L K (t)} - e^{-\gamma L K (t)} T e^{\gamma L K (t)} \right) \right] |\Phi (t)\rangle$$ \hspace{1cm} (14)

1.3 Coupled evolutions

The positions $q_n$ evolve according to the usual Bohmian equation of motion:

$$\frac{dq_n}{dt} (t) = \hbar \nabla_n \xi$$ \hspace{1cm} (15)

where $\xi (r_1, r_2, ..., r_N)$ is the phase of the wave function $\Phi (r_1, r_2, ..., r_N)$. Since $K$ is diagonal and real in the position representation, this operator does not change the phase of the wave function; as a consequence, relation (7) shows that the phase of the wave function $\langle r_1, r_2, ..., r_N | \Phi (t) \rangle$ is the same as that resulting from standard Schrödinger dynamics for the wave function $\langle r_1, r_2, ..., r_N | e^{-iHt/\hbar} |\Phi (0)\rangle$. Therefore, this modified dynamics changes the modulus of the wave functions but not its phase; if $|\Phi (0)\rangle = |\Phi_S (0)\rangle$, the Bohmian trajectories remain exactly the same as in standard theory. We then conserve a key property of Bohmian theory: when a series of experiments is performed, if we assume a distribution of the initial values of the $q_n$’s at $t = 0$ that it is the standard “quantum equilibrium” distribution (the square modulus of the normalized wave function in configuration space), an average over the initial positions reproduces exactly the usual predictions at any further time $t$ of quantum mechanics with an initial state $|\Phi (0)\rangle$.

In each region of configuration space, the effect of the localization operator $e^{\gamma L K}$ in (7) is to multiply the wave function $\Phi (r_1, r_2, ..., r_N)$ by the time integral of $N_B (r, t')$. 5
The density of Bohmian positions play the role of an attractor for the various variables of the wave function, with a sort of attractive force that tends to localize them in regions where $N_B(r)$ is maximal. The localization length $a_L$ introduces a space averaging in this process: what really drives the field is only variations of densities occurring over distances exceeding $a_L$; no microscopic change is introduced into the wave function, but only a slow amplitude change. Equation (8) is linear but time-dependent, even if the Hamiltonian is time independent in general, since the Bohmian positions and therefore $N_B(r)$ depend on time; moreover, the norm-conserving version is non-linear because coefficient $C$ depends on $D_\Phi(r')$ depends on the state vector $|\Phi\rangle$.

When $\gamma_L$ is sufficiently small, one can expand definition (11) to first order in $\gamma_L$ and obtain:

$$\tilde{T}_L \simeq \gamma_L \left[K, T\right] = \gamma_L \int d^3r \left[N_B(r) \Psi^\dagger(r) \Psi(r), T\right]$$

$$= i\hbar \frac{\gamma_L}{2m} \sum_n \left[\vec{\nabla} N_B(R_n) \cdot P_n + P_n \cdot \vec{\nabla} N_B(R_n)\right]$$

(16)

Here $R_n$ and $P_n$ are the position and momentum operators associated with particle $n$.

A general remark is that, in the limit $a_L \to \infty$, the localization term has no effect: in (3), $N_B(r)$ then becomes equal to the number of particles $N$ (a constant) and, in (4) and (5), $L(t)$ and $K(t)$ respectively become the products $N\tilde{N}$ and $N\tilde{N}t$ (where $\tilde{N}$ is the operator associated with the total number of particles). Since $\tilde{N}$ commutes with $T$, in (11) $\tilde{T}_L$ then vanishes; in the right hand side of (13) the term in $L(t)$ and $C(t)$ cancel each other, and only the standard Hamiltonian $H$ remains; nothing is then changed with respect to standard Schrödinger dynamics.

## 2 Collapse in small or large systems

We now assume investigate the dynamics of physical systems obeying the modified Schrödinger equation (12). As in GRW and CSL theories, our purpose is to check that it is possible to find values of the two parameters $\gamma_L$ and $a_L$ for which no contradiction occurs with the enormous body of experimental data agreeing with quantum mechanics, sometimes with an incredible precision ($10^{-12}$ in a few cases!). What is needed is a compromise between different requirements: a localization dynamics that has fantastically small effects on microscopic systems, but nevertheless produces a fast collapse of superpositions of macroscopically different states. We will choose values inspired by those often chosen in GRW and CSL theories, namely:

$$\gamma_L = 10^{-16}$$

$$a_L \simeq 10^{-6} \text{ m}$$

(17)

Other values can be selected, but our purpose here is not to find accurate values of these constants; we just wish to show that there is a wide range of acceptable values that are acceptable with the above criterion.
2.1 Microscopic system

Consider first a microscopic system, atom, molecule or nucleus, with a wave function of all the constituent particles extending over a range $a_0 \ll a_L$. Since the $q_n$'s can never reach regions of space where the wave function vanishes, they also remain localized in a region of space of dimension $a_0$. This corresponds to the case mentioned above, where $N_B(\mathbf{r})$ is of the order of the total number of particles $N$ in a domain of size $a_L$ centered on the atom, and tends rapidly to zero outside of this domain. In the limit $a_0/a_L \rightarrow 0$, we have seen that $K(t) \rightarrow N\tilde{N}t$, so that in (7) the localization term $e^{\gamma tK(t)}$ has no effect on the wave function (except a multiplication by an overall factor without physical consequence).

If $a_0/a_L \ll 1$, the exponential in (3) can be approximated by $1 - c(a_0/a_L)^2$, where the term in 1 does not contribute (this is the limit $a_L \rightarrow \infty$), and where $c \simeq 1$ (the exact value of $c$ depends on the Bohmian positions). So, retaining only the term in $(a_0/a_L)^2$, we see that the parts of the wave function at the periphery of the atom are reduced at a rate $\gamma$ given by:

$$\gamma \lesssim \gamma_L \left(\frac{a_0}{a_L}\right)^2 N^2 \quad (18)$$

while the parts at the center of the atom remain unaffected.

For a small atom (Hydrogen or Helium for instance), with values (17), $a_0/a_L \simeq 10^{-4}$ so that $\gamma \lesssim 10^{-24}N^2$, where $N$ is a few units; this rate is clearly extremely low and undetectable. For a molecule, a size of 10 nm is already large, which corresponds to $a_0/a_L \simeq 10^{-2}$ and to $\delta \lesssim 10^{-20}N^2$; even with a number of constituents (protons, neutrons) of the order of $10^4$, we still obtain an extremely small rate.

Now consider an interference experiment made with the same microscopic system. In the interferometer, its wave function is localized at the same time in very different regions of space; in one of these regions, $N_B(\mathbf{r})$ is as above equal to $N$, but in the other it is zero. This clearly introduces an imbalance between the full wave, which increases at a rate $\gamma_L N^2$, and the empty wave, which remains constant. Nevertheless, the rate of growth of this imbalance is:

$$\gamma \simeq \gamma_L N^2 \quad (19)$$

Therefore, even for a long experiment lasting one second, if $N < 10^7$, the localization rate remains negligible, and the interference takes place as in standard Schrödinger dynamics; but, for larger values of $N$, this dynamics predicts that the contrast of fringes should decrease and vanish in the limit $N \gg 1/\sqrt{\gamma_L t}$.

2.2 Macroscopic system

The orders of magnitude are completely different for macroscopic systems. Consider for instance the pointer of a measurement apparatus, which after measurement may reach (for instance) two different positions that are 10 microns apart from each other. The solution of the linear Schrödinger equation has components where the particles of the pointer are, either in one region of space, or in another; in other words, big fluctuations of the local density of particles take place. Nevertheless, in a given realization of the experiment, the corresponding Bohmian variables all remain clustered in the vicinity of only one of these
two positions. They remain together because the state vector does not have components where some of the pointer particles are in one site, some in the other: this is forbidden by the cohesion forces inside the material forming the pointer. So, when the measurement is performed, one component of the wave function resides in the same region of configuration space as many Bohmian variables (this is a “full wave”), but the other component in region where the density of Bohmian positions is zero (this is an empty wave).

For the “full wave”, in the localization term in the right hand side of (5), the relevant values of \( r \) in the integral are those in the region of space where this wave propagates. Since \( N_B \) has significant values in this region, the integral over \( d^3r' \) is relatively large, and this term increases the modulus of the full wave. For the “empty wave”, the relevant values of \( r \) are those in the region of space where \( N_B \) is zero, and the localization term \( e^\gamma L K(t) \) does not have any effect. When the measurement result is fully registered in the position of the particles of the pointer, the relative weight of the full wave is therefore increased exponentially with a time rate \( \gamma \) of the order of:

\[
\gamma \simeq \gamma_L N_B N_P
\]

where \( N_P \) is of the order of the number of particles in the pointer and \( N_B \), the number of its particles in volume \((a_L)^3\). If we choose conservatively small values \( N_P = 10^{20} \), \( N_B = 10^{11} \), we obtain a very fast rate \( \gamma \simeq 10^{15} \). Clearly, in this case the dynamical equation leads to an extremely fast collapse of the wave function!

Consider now a Bose-Einstein condensate that is partly reflected by Bragg scattering on a laser standing wave [12]. A matter wave is then split into two coherent parts, which can propagate at macroscopic distances and interfere again if recombined. We must check that the localization term does not destroy the coherence, which would be in contradiction with the experimental observations. The major difference with the preceding case is that the atoms in the condensate propagate almost freely, and that no process forces all of them to go in the same direction; there is no reason to find all atoms in the same output beam. The distribution of Bohmian positions then closely follow the quantum distribution (a Poisson distribution of populations in the two output atomic clouds). In other words, none of the two waves becomes empty; each of them travels accompanied by a Bohmian density that is proportional to its intensity. Moreover, the number of atoms involved in these experiments is of the order of \( 10^5 \), in a volume that is comparable with \( a_L \). So, even in the absence of Bohmian density, relation (20) would lead to \( \gamma \simeq 10^{-6} \), still a very small rate. The collapse predicted in the previous case does not take place here, and the two waves propagate as coherent classical waves.

### 2.3 Measurement, Born rule

A soon as a microscopic quantum system \( S \) interacts with a measurement apparatus \( M \), it becomes entangled with some of its particles: each measurement eigenstate \( |s_i \rangle \) of \( S \) becomes associated with different states of \( M \). As we have seen, the usual Schrödinger dynamics is not affected until a large number \( N_m \) of particles of \( M \) is involved in this process. But, since the very purpose of a measurement apparatus is to transfer information to a macroscopic scale, entanglement progresses rapidly within the measurement apparatus (it
propagates by “contagion” between mutually interacting neighbor particles, as discussed in [13]; it quickly reaches a mesoscopic and then macroscopic scale. In standard quantum mechanics, this phenomenon occurs in parallel in all “measurement channels” (values of the index $i$) and various branches of the state vector develop, each associated with one of the measurement eigenstates $|s_i\rangle$. In Bohmian mechanics, this propagation induces a motion of the Bohmian positions; nevertheless, for each realization of the experiment, the motion is only that associated with one measurement channel (all the other channels correspond to empty waves that have no effect on the Bohmian positions). The changes of the values of $N_B(r,t)$ are therefore those associated with a single branch of the state vector. In other words, for each realization of the measurement, the time dependence of $N_B(r,t)$ is different and, in [5], the operator $K(t)$ tends to localize the wave function in a different way.

Consider now a time when the values of $N_B(r,t)$ have become significantly different for each value of $i$ in macroscopic regions of space; by the process discussed above for macroscopic systems, the effect of the localization process on the wave function is then very fast, and selects in the wave function the component associated with a single value of $i$ (one result of measurement). State vector reduction has then taken place. As a whole, the localization process is similar to fast freezing of a liquid: the suppression of the “empty” components of the wave function occurs with a time constant that is initially completely negligible, but grows faster than linearly in time (exponentially at the beginning of propagation of entanglement).

Now, instead of a single realization of the experiment, consider an ensemble of realizations. We know that, when the initial Bohmian positions of the particles are chosen randomly according to the quantum distribution, the dBB theory exactly reproduces the Born rule of standard quantum mechanics. Therefore, just before the localization process of the wave function takes place, the standard quantum probability gives the proportion of realizations where the $N_m$ Bohmian positions of the measurement apparatus have reached the region of space associated with a measurement eigenstate $|s_i\rangle$. For the wave function, this region determines the branch that will be enhanced by the localization process, while all the empty waves are cancelled. As a consequence, the collapse of the wave function takes place randomly towards one of the regions of space associated with the various possible results of measurement, with a probability given by the standard Born rule. Without conflict with the predictions of quantum mechanics, we can then consider the field associated with $|\Phi(t)\rangle$ (within a phase factor) as representing physical reality directly (in configuration space).

3 Discussion

The localization process contained in equation (8) occurs in ordinary 3D space, as in CSL theory, but is also very different. CSL introduces the simultaneous action of an infinite number of random processes modifying the wave function in all points of space, and described by Wiener processes. A “probability rule” ensures that the processes cooperate in space in order to conserve the maximum possible norm to the state vector, which leads
to the usual Born rule. Gisin [14] also assumes the presence of an additional random term in the Schrödinger equation; the evolution of the state vector depends on a stochastic Wiener process which, conversely, has an evolution that depends on the state vector.

Here, no stochastic process takes place during the evolution, which is entirely deterministic. Randomness arises only from the initial distribution of the dBBohr positions, which ensures the validity of the Born rule at all subsequent times. The time dependence of the modified Schrödinger equation arises only from the time dependence of the Bohmian density $N_B(r,t)$; it is different in different realizations of the same experiment. The dynamics is such that, if no important fluctuation of the Bohmian density occurs, the time-independent terms remain completely negligible; but, in measurement-like situations, they are extremely large.

The equations obey Galilean relativity (absolute time), which is logically consistent for a modification of the Schrödinger equation; it would nevertheless be interesting to extend them to Einsteinian relativity with a time delayed density localization term.

One may wonder why, in the modified Schrödinger equation (12), two terms $\tilde{T}_L(t)$ and $i\hbar\gamma(t)L(t)$ are added to the usual Hamiltonian instead of a single one, while the only term $L(t)$ seems to be perfectly sufficient to produce collapse when necessary. The reason why the term in $\tilde{T}_L(t)$ appears in the equations is that we chose to introduce new wave function that has exactly the same phase as the standard wave function (at each point of the configuration space): this was obtained by equation (7), which implies that this phase is strictly equal to that obtained in standard theory by propagating the ket $|\Phi_S(0)\rangle = |\Phi(0)\rangle$ from time $t = 0$ to time $t$. It is nevertheless tempting to suppress the term in $\tilde{T}_L(t) = 0$ from equation (12): this modified dynamics is simpler, more pleasing esthetically, and preserves full time invariance. But the consequences of such a simplification are not straightforward to assess, because of the changes of the Bohmian trajectories induced by a change of the phase. In fact, the phase of the wave function is not directly modified by $L(t)$, but the kinetic energy operator transforms this spatial dependence into a spatial dependence of the phase. With such a dynamics, the trajectories of Bohmian positions are therefore slightly changed with respect to those of standard theory. Even if it seems likely that these changes should remain sufficiently tiny to preserve the validity of the above discussion, this is not straightforward to prove rigorously, because of the non-linear character of the equations and their possible sensitivity to initial conditions. We therefore leave the possibility of this simpler dynamics as an open question.

Another model where empty waves disappear has been proposed by Allori et al. [15]; in this model, the waves that have disappeared may reappear later, when necessary to create interference. This does not ensure macroscopic uniqueness; the model is also different from ours because it depends only on one parameter (the localization length $a_L$), instead of two, and because the averaging takes place in configuration space (not ordinary space).

**Conclusion:** The equations of evolution proposed here imply that, in practice, no significant collapse occurs unless a macroscopic fluctuation of density exists in ordinary space, meaning that entanglement must have propagated from a microscopic to a macroscopic level. This is precisely, by construction, what a measurement apparatus is supposed to do with a microscopic system. Collapse occurs when quantum macroscopic density fluctuations occur within the pointer of a measurement apparatuses, but neither measurements
nor observers play any special role.

Different attitudes are possible to interpret such dynamical equations. One is to consider that the positions are the essence of reality, as in the dBB theory. The advantage of the proposed dynamics is then to get rid of the “empty waves”, which persist forever in the dBB theory, while they are supposed to play no physical role whatsoever; their existence complicates the discussion of the physical reality of the waves. Another attitude is to consider that the directly observable component of physical reality is the field (the wave function); the other component of reality would then be the attractive density $D_B(r)$, acting on the field but not directly observable (a sort of “dark density”). Bohmian positions may then be considered as the tool to generate the propagation in real space of an attractive density $D_B(r)$; this is slightly reminiscent of de Broglie’s ideas on singularities associated with the wave function and propagating with it (theory of the double solution). The propagation in space of the field is then free of the counter-intuitive aspects attached to Bohmian positions (bouncing in free space for instance). Nevertheless, in this view, a naive realism is not restored: the field remains very different from a classical field, since it does not propagate in ordinary space.

The main purpose of the present work can be seen as a proof of existence: as GRW and CSL theories have already shown, it is not impossible to build a simple theory where waves directly represent physical reality, while remaining compatible with present experimental data. Our contribution is to show that introducing a stochastic dynamics is not a necessary condition. Macrorealism [16] can indeed emerge from the dynamics, without elaborate mathematics in the equations of motion. Needless to say, it remains perfectly legitimate to invoke esthetical or philosophical reasons to maintain the Schrödinger dynamics unchanged, and adhere to one of the interpretations that are available. But one can also prefer to change the dynamics to obtain a completely unified dynamics including measurement processes; for the moment, this choice remains a matter of personal preference.

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