New Universality Class at the Superconductor–Insulator Transition

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We study dynamic properties of thin films near the superconductor - insulator transition. We formulate the problem in a phase representation. The key new feature of our model is the assumption of a local ohmic dissipative mechanism. Coarse graining leads to a Ginzburg-Landau description, with non-ohmic dynamics for the order parameter. For strong enough damping a new universality class is observed. It is characterized by a non-universal d.c. conductivity, and a damping dependent dynamical critical exponent. The formulation also provides a description of the magnetic field-tuned transition. Several microscopic mechanisms are proposed as the origin of the dissipation.

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Granular superconductors and Josephson junction arrays behave similarly in many respects. This is because the key degrees of freedom are thought to be exclusively bosonic in nature, the Cooper pairs of the underlying electronic problem. Quantum phase transitions are present in the ordered arrays, when superconductivity gives way to a gapped (Mott-) insulator. This is a direct consequence of the uncertainty relation between phase and charge degrees of freedom [4]. When disorder is present, an additional insulating phase may appear, exhibiting glassy behaviour [5]. In two dimensions, early experimental studies reported the conductivity σ, to assume a universal value [6]. A theoretical explanation of this universality was proposed based on scaling considerations [7]. However, subsequent measurements observed critical resistivities, differing as much as tenfold [5]. The main motivation of our work is to develop an understanding of this apparent non-universality.

In this paper we develop a Ginzburg-Landau-Wilson (GLW) formulation for the superconducting order-parameter. We emphasize the importance of electronic excitations to account for the experiments. Our main result is the observation of a new universality class at the superconductor - insulator transition. The conductivity at criticality is found to be

\[ \sigma = \frac{J}{U} \]  

for strong enough dissipation. The smallness of the grains manifests itself in a charging energy \( U \) which is the energy cost of transferring an extra Cooper pair on the island. The Josephson coupling between neighboring islands is denoted by \( J \). If the low lying electronic excitations have a finite density of states, then the damping will be ohmic; its Fourier transform is given by \( \alpha(\omega) = \alpha_0|\omega_\mu|/2\pi \). The locality of the damping is reflected in the fact that the dissipative term in Eq. (1) couples the phase in a single island at different times. Additional non-local charging or dissipative terms do not change qualitatively the results and for simplicity will be dropped.

A Hubbard-Stratonovich transformation is employed to decouple the Josephson term [8]. This introduces the complex order-parameter field \( \psi \) such that its expectation value is proportional to that of \( \exp(i\varphi) \). This yields the GLW action which in two dimensions takes the form

\[ S[\varphi] = \int_0^\beta d\tau \left[ \sum_i \dot{\varphi}_i(\tau)^2 - J \sum_{i<j} \cos[\varphi_i(\tau) - \varphi_j(\tau)] \right] 
+ \frac{1}{2} \int_0^\beta d\tau d\tau' \sum_i \alpha(\tau - \tau') (\varphi_i(\tau) - \varphi_i(\tau'))^2. \tag{1} \]

The correlation function \( \langle \exp(i\varphi(\tau) - i\varphi(0)) \rangle \) is given as:

\[ g(\tau) = \langle \exp(i\varphi(\ tau) - i\varphi(0)) \rangle_0, \text{ where } \langle \cdot \rangle_0 \text{ is an expectation value taken with respect to the single site Gaussian part of the action of Eq. (1). In the presence of local damping } g(\tau) \text{ decays algebraically in time } (\sim \tau^{-2/\alpha_0}). \]

The Fourier transform for small frequencies reads

\[ g(\omega_\mu) = \frac{1}{2J} \left( g_0 - \eta |\omega_\mu|^s - \zeta \omega_\mu^2 \right); \quad s = \frac{2}{\alpha_0} - 1, \tag{3} \]

where \( \eta = -4J(\tau - \cos(s\pi/2)\left(4U/\alpha_0\right)^{s+1}, \quad g_0 = 16J/[U(2 - \alpha_0)]. \) To summarize the low frequency form of the GLW action reads:
\[ F[\psi, \psi] = \frac{1}{2\beta JN} \sum_{k, \omega \mu} \left[ \epsilon + \frac{\kappa^2}{4} + \eta|\omega\mu|^s + \zeta \omega^2 \right] |\psi(k, \omega \mu)|^2 + \kappa \int d^2r d\tau |\psi(r, \tau)|^4, \]

where \( \epsilon = 1 - g_0 \). Clearly the model exhibits a non-ohmic dissipative dynamics, reducing to ohmic damping only in the special case \( s = 1 \). Surprisingly, the ohmic damping in the quantum phase model of Eq. (4) yields a non-ohmic dynamics for the coarse-grained order-parameter. The quadratic frequency dependence dominates over the dissipative dynamics for \( s > 2 \), or equivalently \( \alpha_0 < 2/3 \).

The model exhibits a phase transition at \( \epsilon(\alpha_0, J, U) = 0 \). The ordered phase supports long-range superconducting order, whereas the disordered phase is an insulator. The phase diagram for \( T = 0 \) is displayed in the inset of Fig. 1. Increasing damping shifts the phase boundary to lower values of \( J \). The insulating phase disappears for \( \alpha_0 > 2 \). In this region the dissipative processes completely suppress the quantum fluctuations, in analogy with the dissipative phase transition, discussed in Ref. [9].

\[ \sigma(\omega) = \frac{\sigma_Q}{16\pi^2 \omega} \int_{-\infty}^{\infty} \frac{d\zeta}{1 - e^{-\beta \zeta}} \int_0^{\infty} dk k^3 \left[ G_R(k, z) - G_A(k, z) \right] \times \left[ G_R(k, z) + G_A(k, z + \omega) - G_A(k, z - \omega) \right], \]

where \( \sigma_Q = h/4e^2 \). The advanced and retarded Greens functions are given by

\[ G^A(k, \omega) = \epsilon + \frac{k^2}{4} - \zeta \omega^2 + \eta|\omega|^s \left[ \cos \frac{s\pi}{2} \pm i \text{sign}(\omega) \sin \frac{s\pi}{2} \right], \]

with \( \epsilon \geq 0 \). Without damping the frequency dependence exhibits a gap of the size \( \omega_0 = \sqrt{4\epsilon / \zeta} \), thus this disordered phase is a Mott-insulator. With increasing damping the Mott gap is smeared out. For \( s < 2 \) and low frequencies \( \omega \ll \omega_0 = \sqrt{4\epsilon / \zeta} \) one finds

\[ \text{Re} \sigma(\omega) = \sigma_Q \frac{\eta^2 \sin^2(\frac{s\pi}{2})}{6\pi^2} \left[ \frac{\Gamma(1 + s)^2}{\Gamma(2 + 2s)} \right] |\omega|^{2s}. \]

The conductivity shows a power-law behaviour at low frequency, where the power depends on the strength of the dissipation for \( s \leq 2 \). These two types of behaviours are displayed in Fig. 2.

At \( T = 0 \) a quantum phase transition takes place which is characterized by a dynamical critical exponent \( z \). In the limit of weak damping, the critical behaviour is that of the non-dissipative models, \( z = 1 \). In the general case it is given by \( z = \max(1, 2/s) \), and the critical behaviour is that of a \((2 + z)\)-dimensional XY-model. Therefore our results establish the existence of a new universality class of the superconductor - insulator transition for \( s < 2 \).

This quantum phase transition attracted intense interest because of the claim of a universal conductivity at criticality [10]. To address the issue of universality we calculate the conductivity in terms of two and four point Green’s functions [11]. In the Gaussian approximation the four point function factorizes. An analytic continuation \( i\omega_0 \rightarrow \omega + i\delta \) yields the dependence on the real-frequency [4].

\[ \text{Re} \sigma(\omega) = \sigma_Q \frac{\eta^2 \sin^2(\frac{s\pi}{2})}{6\pi^2} \left[ \frac{\Gamma(1 + s)^2}{\Gamma(2 + 2s)} \right] |\omega|^{2s}. \]
attraction. In the region of $0 < \alpha_0 < 2/3$ the dissipation is an irrelevant operator. It is characterized by $z = 1$ and a universal critical conductivity. For strong enough coupling to localized fermions $\alpha_0 > 2/3$ a new universality class is present, with damping dependent $\sigma^*$ and $z$. This is the central result of our paper.

Renewed interest in the superconductor - insulator transition was generated by the early experimental report of a universal conductivity at criticality, with the value $(\sigma^*)^{-1} = 6.5 k\Omega$. However, subsequent measurements on superconducting films and on Josephson junction arrays reported resistivity values distributed in the broad range of 2-20 k$\Omega$. Our emphasis on the electronic degrees of freedom offers a possible explanation of this lack of universality.

In many experiments the transition is tuned by a magnetic field $\vec{B}$, which we therefore discuss next. The magnetic field is incorporated by introducing a vector potential $\vec{A}(x, \tau)$ in Eq. (2). For weak frustration $f < 1$ the lattice structure can be neglected. The magnetic field leads to the formation of Landau levels. The longitudinal conductivity is given by

$$\sigma(\omega) = \sigma_0 \frac{\omega^2}{2\omega_c^2} \sum_{n=0}^{\infty} \sum_{n'} (n + 1) \left[ 2G_{\omega,n} G_{\omega,n+1} - G_{\omega+n,n} G_{\omega,n+1} - G_{\omega+n,n} G_{\omega+n+1} \right], \quad (8)$$

where $G_{\omega,n} = \left( \frac{\omega - \omega_c(n+1/2)}{\omega_c} + \eta \frac{\omega^2}{\omega_c^2} + \zeta \omega_n^2 \right)^{-1}$, with the dimensionless cyclotron frequency $\omega_c = \pi f$. The magnetic frustration $f = \Phi/\Phi_0$ is proportional to the flux through a unit cell. In the mean-field approximation the phase boundary is given by $e + \omega_c/2 = 0$. The fact that the location of the phase boundary is magnetic field dependent demonstrates, that our formulation is capable of capturing the field-tuned transition.

The analytic continuation to determine the conductivity as a function of real frequencies follows the same lines as in the zero field case. We explicitly evaluate the real part of the conductivity as shown in Fig. 3. The frequency dependence reflects the underlying Landau-level structure. It is smeared due to the influence of the damping. The d.c. conductivity at the transition is determined by the lowest Landau level $n = 0$. In the absence of dissipation it diverges. Again we find that for strong enough damping, $\alpha_0 > 1$, the critical conductivity $\sigma^*$ becomes finite, with the damping dependent value, as shown in Fig. 3. For $\alpha_0 \to 1$ it assumes the asymptotic form $\sigma^* \sim 1/(\alpha_0 - 1)$.

The previous analysis showed that the effect of the local damping for the phases has a great impact on the properties of the system. In the second part of this paper we discuss various scenarios which can give rise to local dissipation.

The key feature giving rise to dissipation is the presence of gapless degrees of freedom. These electronic degrees of freedom can either be of intrinsic origin, or they can result from a coupling to the substrate. We argue that both scenarios are viable candidates for the description of the experimental data.

Before reviewing the microscopic motivation we point out the important difference between the local dissipation, investigated here, and the non-local forms, studied in Refs. [9]. When local damping is present, the number of Cooper pairs is not conserved: they can decay into a pool of normal electrons. On the contrary, if the damping is non-local, e.g. it is induced by inter-grain electron transfer, then it conserves the charge within the film. Because of this essential difference it can be expected that genuinely new physics arises in the present model.

In the remainder of the paper we discuss possible origins of the local damping. The two necessary ingredients for the proposed mechanism are local pools of gapless electrons with a temperature independent density, and a coupling term between these electrons and the phase of the order parameter, which leads to ohmic dissipation. Once gapless electrons are present, the Andreev process satisfies these criteria. It describes the conversion of normal electrons into Cooper pairs at superconductor - normal (SN) interfaces. In the absence of a tunnel barrier the charge transfer across the interface is continuous. This results in a fully ohmic dynamics for the superconducting phase. Therefore the proper description of this process is exactly the dissipative term in Eq. (8). For the case of two superconducting islands, connected by a normal metal, a microscopic derivation of the ohmic dynamics was provided in Ref. [14]. Adapting their derivation for the present case of an SN structure confirms our statement explicitly. In the ideal case the value of the Andreev resistance is half that of the normal state resis-
tance, consequently the arising local dissipation can be large. We emphasize that this process involves pools of localized electrons which are therefore not part of the conducting path. Transport across the sample is dominated by tunneling between grains, resulting in a much higher subgap resistance.

The remaining question is the origin of the gapless electrons. In order to give rise to ohmic dynamics, the level spacing of their spectrum has to be small compared to all other energy scales in the problem. This necessitates a relatively large spatial extent of their wavefunction, which can be estimated to be in the range of hundreds of Ångstroms. An intrinsic scenario can be suggested by recalling that in the absence of superconductivity the system is an insulator, otherwise the experiments would display a superconductor - metal transition. On the other hand, in order to be able to support superconductivity, it has to be close to the metal - insulator transition. In this region the combined effects of disorder and enhanced Coulomb repulsion act as very effective mechanisms to break up the Cooper pairs into normal electrons. In the vicinity of the transition the localization length of the electrons is strongly enhanced. Thus their spatial extent is large, leading to a considerable density of states at the Fermi energy. The existence of such low energy excitations with a density comparable to the normal state density was observed in tunneling experiments.

An extrinsic origin of the local damping mechanism can be due to a conducting substrate. In this case the unpaired electrons of the ground plane constitute the reservoir. It is well established that in proximity coupled junction arrays the metallic substrate indeed is a source of localized dissipation. Superconducting films are commonly grown on disordered semiconductor substrates. As was suggested in Ref. the semiconductor substrate might be able to support metallic transport on short length scales. It is possible that the deposited metal dopes the amorphous semiconductor substrate, giving rise to a temperature independent density of states at the Fermi level. An additional remarkable possibility was raised in connection to the study of Ag/Ge, where the metal - semiconductor interface itself seems to support metallic transport.

In sum we developed a modified understanding of the superconductor - insulator transition. We emphasized the importance of localized gapless electronic excitations. They give rise to a local ohmic dissipation for the phase of the superconducting order parameter. A new universality class of the transition was identified for strong enough dissipation. It is characterized by a non-universal value of the critical conductivity and a damping dependent dynamical critical exponent. The dependence of the above results on a magnetic field was also analyzed. We reviewed several possible microscopic origins of the gapless electrons, and identified the Andreev process as a likely source of the ohmic dissipation.

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