Analytical predictions of non-Gaussian distribution parameters for stellar plasmas

A.M. Scarfone • P. Quarati • G. Mezzorani • M. Lissia

Abstract Stimulated by the recent debate on the physical relevance and on the predictivity of $q$-Gaussian formalism, we present specific analytical expressions for the parameters characterizing non-Gaussian distributions, such as the nonextensive parameter $q$, expressions that we have proposed for different physical systems, an important example being plasmas in the stellar cores.

Keywords Exact results, Rigorous results in statistical mechanics.

1 Introduction

Many recent experimental measurements of space and momentum distribution functions of particle systems and of physical quantities in several fields of natural and social science show behaviors that can be described with non-Gaussian distributions: from cosmic rays to dark matter, from astrophysical systems to turbulence and quark-gluon plasmas, from biology to finance. More concisely: power laws are detected in several complex systems (Gell-Mann and Tsallis 2004).

Among various non-Gaussian distributions that emerge from consistent thermodynamical and statistical frameworks (Kaniadakis et al. 2004; Kaniadakis et al. 2005; Lavagno et al. 2007), $q$-Gaussians, based on the so-called nonextensive statistical mechanics introduced by Tsallis (1988), are appealing for their simplicity and have found the most applications. A critical debate on its theoretical foundation and on its practical applicability is still going on. In particular, there has been a great effort in this field to construct a statistical mechanics capable of describing systems affected by nonlocal and memory effects. Relevance of these problems in astrophysics and space science has been widely discussed in many papers Saxena et al. (2004,2006) (see also Mathai and Haubold 2007; Haubold and Kumar 2007 for comments on applications of $q$-Gaussians in the above field).

Two important questions regarding $q$-Gaussian distributions have been recently raised by Douxois (2007): if $q$-Gaussian law describes physical phenomena and if the entropic $q$ parameter could be predicted in terms of microscopic parameters. Tsallis (2008), has answered these questions reviewing the present state of nonextensive statistical mechanics and reporting analytical relations for the parameter $q$ that have appeared over the recent years. It is our opinion that natural field of application and validation of $q$-Gaussians is astrophysics and space science.

In this work, we present additional interpretations of the non-Gaussian parameters, mostly in terms of the parameter $q$, with the corresponding analytical expressions, that our group have proposed: a general statistical interpretation based on Fokker-Plank kinetic equations and on Langevin equation, phenomenological interpretations about weakly nonideal plasmas and charged particles in electric and magnetic fields inelas-
tically interacting with a medium and an interpretation based on the quasi-particle description of correlated particles with finite lifetime. This list is not exhaustive, but we believe it offers an important contribution to the scientific debate on the physical interpretation of the deformation and on the possibility of directly linking parameter $q$, or other analogous parameters, to the underlying microphysics; in these specific cases, the link has simple analytical forms. In the cases presented, the deviation of the distributions from the exponential case, $q \neq 1$, is due to the non-trivial spatial correlations among the particles of the system or time correlations of long-living metastable systems. The possibility of a two-parameter distribution that capture the power-law behavior in two different asymptotic regimes (Kaniadakis et al. 2004, 2005) and its application to chaotic systems is also recalled (Tonelli et al. 2006).

The physically important case of plasmas in stellar cores is discussed to clarify the importance of non-Gaussian distributions.

## 2 Non-Gaussian distribution parameters

By solving a nonlinear Fokker-Planck equation with dynamical and friction coefficients given by a polynomial expression with positive coefficients, in 1992 we derived (Kaniadakis and Quarati 1993) a set of stationary non-Maxwellian distributions with depleted and enhanced tails compared to Maxwellian tail.

Analytical forms of the distributions were given for expansion up to the sixth order, with the parameters of the distributions expressed in terms of the derivatives of the dynamical friction and diffusion coefficients. All these distributions deviates from the Maxwellian one, which is recovered in the limit of vanishing nonlinear corrections to the Fokker-Planck equation.

As a specific example, Eq. (30b) of Kaniadakis and Quarati (1993)

$$f(u) = \frac{\Gamma(\mu)}{\sqrt{\pi} \Gamma(\mu - 1/2)} \left(1 + \frac{u^2}{\mu - 1}\right)^{-\mu}$$

with $\mu = 1 + \beta_0/(2\gamma_1)$, where $\beta_0$ and $\gamma_1$ are the coefficients of the leading and subleading term of the expansion in the velocity of the dynamical friction and diffusion terms.

Later on, we realized that this specific distribution, which in our paper we called second-order stationary solution of the Fokker-Planck equation, basically coincides with the $q$-distribution obtained independently by Tsallis a few years earlier with an entropic approach (Tsallis 1988). In this case the analytical form of corresponding entropic parameter $q$ is

$$q = 1 + \frac{1}{\mu} = 1 + \frac{2\gamma_1}{2\gamma_1 + \beta_0}.$$  \hspace{1cm} (2.2)

Clearly, non-Gaussian distributions and extended entropies were already known and used in different fields (Mathai and Rathie 1975; Haußold et al. 2007; Kaniadakis and Lissia 2004), however it is only after Tsallis seminal paper that systematic extensions of thermodynamics and statistical mechanics were developed. Such approaches received great attention and are actually used in many fields in science.

In Kaniadakis and Quarati (1997), we have investigated more in detail the relationship between our results and the nonextensive statistical mechanics results, realizing the connection between the physical meaning of some of our results of Kaniadakis and Quarati (1993) and the corresponding Tsallis distribution and entropic parameter $q$.

We have also studied Kaniadakis et al. 2004, 2005) a wider class of generalized two-parameter statistical mechanics that includes not only the $q$-thermostatistics, but also other physically motivated extensions such as the $\kappa$ distribution with its appealing relativistic structure (Kaniadakis 2002, 2005), and the Abe distribution related to quantum group (Abe 1997). The interpretation of the two parameters exist only for selected cases or when there remains only one effective parameter and it is still object of study.

### 2.1 Polynomial expansion of diffusion and drift coefficients

One of the distributions emerging as equilibrium solutions of the nonlinear Fokker-Planck equation for a specific form of the coefficients was first interpreted in term of $q$-distribution in 1997 (Kaniadakis and Quarati 1997), where we gave an analytical expression of $q$ in terms of the ratio of the noise induced drift over total drift for specific model with leading order terms in a velocity expansion:

$$q = \frac{4\theta - 1}{2\theta - 1}$$ \hspace{1cm} (2.3)

where

$$\theta = \frac{J_n(v)}{J_n(v) + J_d(v)},$$ \hspace{1cm} (2.4)

with $J_n(v) = \frac{1}{q}(1 - q)v$, $J_d(v) = \frac{1}{q}(3 - q)v$ and $\gamma$ is a constant. The drift coefficient $J(v)$ is the sum of the deterministic drift $J_d(v)$ plus the induced-noise drift $J_n(v)$. When the noise-induced drift is absent we have $q =$
1 and the $q$ nonextensive distribution reduces to Maxwellian one. The microscopic Langevin equation related to Tsallis distribution is
\[
\frac{dv}{dt} + \gamma \left[ 1 - (1 - q) \left( 1 + \frac{\sigma}{2} \right) \right] v = \left\{ \frac{\gamma}{m \beta} [1 - (q - 1) \beta \gamma] + \frac{\gamma}{2} (q - 1) v^2 \right\}^{1/2} \Gamma(t),
\]
and the solution can easily be written in
\[
v(t) = \frac{1}{\gamma \sqrt{t}} \int \Gamma(t) dt + \arcsinh \left( \frac{v(0)}{\gamma} \right),
\]
where $\Gamma(t)$ is a random Gaussian variable with zero mean value and $\delta$ correlation function, $\gamma = 2 \left[ 1 - (m - 1) \beta \lambda \right] / [ \beta m (m - 1) ]$, $\beta = 2 / [ \gamma (q - 1) ]$, $\gamma = (\sigma + 4) / (\sigma + 2)$ and $\lambda$ is an energy dimensional normalization constant.

### 2.2 Phenomenological derivation for correlated particles in plasmas

Soon after the interpretation in terms of drift and diffusion coefficients, it was possible to derive an analytical expression for $q$ concerning stellar/solar core plasmas, where particles are correlated and entangled to the electromagnetic fields, and their interaction cannot be described by local two-body potentials (Coraddu et al. 1999). This first specific analytical expression of $q$ in terms of microscopic parameters, derived from the solution of a Boltzmann equation valid when deformation of the global thermodynamic equilibrium distribution (Maxwellian) is small (about $0.8 < q < 1.2$), concerns the ionic components of a weakly nonideal neutral plasma, as the solar core, in presence of an electric microfield distribution.

The parameter $q$ can be given in terms of plasma parameter $\Gamma = (Z C_0) n^{1/3} / (kT)$, the ratio of the mean (Coulomb) potential energy and the mean kinetic (thermal) energy, where $n$ is the average density, and of a parameter $\alpha$ of the ion-ion correlation function (whose value is $0.4 < \alpha < 0.9$), as defined by Ichimaru (Yan and Ichimaru 1986; Ichimaru 1992). By fixing $q$, we can describe with a simple expression the distribution of a system of correlated particles interacting elastically not only through a pure Coulomb potential but also, for instance, through a reinforced Coulomb interaction.

#### 2.2.1 The effects of random microfields in plasmas.

The deviation from Gaussian distribution in plasmas (Coraddu et al. 1999) has been recently generalized (Ferro and Quarati 2005) to consider the presence of a microfield distribution of a random force $F$. The expression of $q$ valid for small deformations, $0.8 < q < 1.2$ is
\[
q = 1 \mp \frac{F^2}{k \mu^2 n^2} \frac{\alpha_1^2}{\alpha_0^2}
\]
where the two signs indicates subdiffusion or superdiffusion, $k$ is the energy transfer coefficient, $n$ is the number density, $\mu$ is the reduced mass, $\alpha_0$ and $\alpha_1$ are dimensional constants related to the particle interactions and correlations through the cross sections $\alpha_0(v) = \alpha_0 / v$ and $\alpha_1(v) = \alpha_1$ and therefore to the collision frequencies $\nu_0$ and $\nu_1(v)$. If $F$ is proportional to the electric field $E$ ($F = e E$) and $E$ is greater than a given critical value $E_{\text{crit}}$, then we obtain
\[
q = 1 \pm 2 \delta, \quad \text{valid for } \delta < 1/2
\]
with $\delta$ the above mentioned correlation parameter.

More concisely, we can derive that the correction $\delta$ is the square of the ratio between the thermal energy density over the reinforced energy density times the energy transfer coefficient.

For stellar cores, considering the different ionic components one finds that very small corrections to the value $q = 1$ of the order, let us say, of a few percents can sensibly change the thermonuclear fusion rates with important consequences on the processes occurring in the core and leaving unchanged the bulk properties of the star (Coraddu et al. 1999). In the case of the sun, helioseismology can test the proton distribution function (Degl’Innocenti et al. 1998).

#### 2.2.2 Electromagnetic fields in two-energy-level medium.

Rossani and Scarfone (2000) have self-consistently derived from a linear Boltzmann equation an analytical expression of the $q$ parameter as an explicit function of the electric and magnetic field when charged particles are inelastically interacting with a medium endowed with two energy levels. Concisely
\[
q = 1 + \frac{4}{3} kT f(e, b),
\]
with
\[
f(e, b) = \frac{1}{(\Delta E)^2} \left( \frac{e \cdot b}{c_0} \right)^2 + c_0 \left( e^2 + b^2 \right)
\]
where \( e = qE/m \) and \( b = qB/m \) are the rescaled electric and magnetic fields, \( c_0 \) and \( \epsilon_\infty \) are constant coefficients related to the microscopic cross section (due to the inelastic contributes), and \( \Delta E \) is the energy gap between the excited and the fundamental level of the medium.

### 2.2.3 Particle correlation function and phase space cell

The expression of the square fractional deviation of the number density from a Maxwell-Boltzmann distribution of a system of correlated particles by means of the evaluation of the phase space volume occupied by a nonextensive system of \( N \) classical particles that slightly deviates from Maxwell-Boltzmann phase space volume gives the possibility to derive an analytical expression of \( q \) in terms of the radial correlation function \( g(r) \) (Quarati and Quarati 2003).

By considering the square fractional deviations from Maxwell-Boltzmann distributions defined by \( \sum_i \frac{(\delta n_i)^2}{n_{i0}} \), where \( \delta n_i \) is the variation of the number distribution and \( n_{i0} \) is the number distribution proportional to Boltzmann factor and using the relation (Goodstein 1975)

\[
\sum_i \frac{(\delta n_i)^2}{n_{i0}} = 1 + \frac{N}{V} \int [g(r) - 1] \, dr , \tag{2.12}
\]

where the sum is extended over all energy levels, neglecting large deviations, we have derived the expression:

\[
q = 1 - \frac{1}{9} \left\{ \frac{1}{N} + \frac{1}{V} \int [g(r) - 1] \, dr \right\} . \tag{2.13}
\]

Therefore, when \( g(r) \rightarrow 1 \) and \( N \rightarrow \infty \) we have that \( q \rightarrow 1 \). Otherwise if \( N \rightarrow \infty \) but the particles are correlated because we take \( g(r) \neq 1 \) we obtain

\[
q = 1 - \frac{1}{9V} \int [g(r) - 1] \, dr , \tag{2.14}
\]

and if the particles are not correlated \([g(r) = 1]\) but the number \( N \) is small and finite we have

\[
q = 1 - \frac{1}{9N} . \tag{2.15}
\]

### 2.2.4 Fluctuations of collective effective parameter.

Correlations among particles and fluctuations of intensive parameters, such as the inverse Debye-Hückel radius, \( 1/R_{DH} \), produce nonlinear effects inducing the system in a stationary metastable state that can be described by a \( q \)-Gaussian distribution. It is possible to show (Quarati and Scarfone 2007) that, following the development usually adopted in superstatistics for the inverse temperature (Beck 2001; Wilk and Wlodarczyk 2007), a neutral astrophysical plasma, as the one of a stellar core, has a parameter \( q \) defined by

\[
q = 1 - \left[ \frac{\Delta(1/R_{DH})}{1/R_{DH}} \right]^2 , \tag{2.16}
\]

where \( \Delta(1/R_{DH}) \) is the fluctuation of the inverse Debye-Hückel radius. If we know the equation of state of the system the correction factor can be expressed in terms of temperature fluctuation.

### 2.3 Quasi-particle life-time and non-Gaussian momentum distribution

Many properties of a system of interacting particles can often be described by weakly interacting excitations or quasi-particles.

Let us define

\[
\Sigma(\omega, \mathbf{p}^2) = \Sigma_R + i \Sigma_I , \tag{2.17}
\]

the self energy of the one-particle propagator, where \( \omega \) and \( \mathbf{p} \) are related by \( \omega = \mathbf{p}^2/2m + \Sigma(\omega, \mathbf{p}^2) \).

When the imaginary part of the self-energy cannot be disregarded (\( \Sigma_I > 0 \)) the momentum distribution deviates from a Maxwellian distribution, even if the energy itself is Gaussian. By limiting ourselves to the case of \( q > 1 \) for small \( \Sigma_I > 0 \) we have derived (Coraddu et al. 2000) a phenomenological interpretation of the parameter \( q \) as

\[
q = 1 + C \left( \frac{\Sigma_I}{\epsilon_p} \right)^2 , \tag{2.18}
\]

where \( C \) is a constant, \( \epsilon_p = \mathbf{p}^2/2m^* + \Sigma_R \) and \( m^* = m[1 - (\partial \Sigma_R/\partial \omega)]/[1 + 2m(\partial \Sigma_R/\partial \mathbf{p}^2)] \).

Spatial and temporal correlations among ions have large effects on thermomolecular reactions that occur between high-energy ions tunneling the Coulomb barrier. Quark-gluon plasma is also formed in heavy-ion collision and similar plasma effects are important in the evaluation of reaction rates. This non-Gaussian momentum distribution arising from the finite lifetime of the quasi-particles should always be taken into account when studying nuclear processes in any plasma, the more so when the tail of the momentum distribution is important.

### 3 Two-parameter statistical mechanics: \( \alpha \) and \( \beta \)

Entropies or power-law distributions of many anomalous system entropies might be more completely de-
scribed by a two-parameter class of logarithms (Kaniadakis et al. 2004, 2005; Tonelli et al. 2006). In fact, the two parameters basically characterize the possibly different power-law behaviors of the distribution for large positive and negative arguments.

As an example, at the onset of chaos the logistic map shows an universal power-law behavior that fixes one of the exponents. This same exponent characterizes the time dependence of the entropy and the sensitivity to initial conditions: one measures one of the two and can predict the behavior of the other. The second exponent can be related to the ratio of growth of entropies in different formulations (Tonelli et al. 2006).

This generalized entropies and distributions can be introduced by means of a corresponding two-parameter class of logarithms

$$\log(\xi) \equiv \frac{\xi^\alpha - \xi^{-\beta}}{\alpha + \beta},$$

and an infinity of one-parameter deformed logarithms can be obtained as particular cases by imposing relationships between the two parameters.

In some cases the emerging deformed parameter can be related to the microscopic quantities of the system under inspection. The $q$-statistical mechanics is recovered, as one-parameter subclass of the two-parameter statistical mechanics, for $\alpha = 1 - q$ and $\beta = 0$ whilst, for instance, for $\alpha = \beta = \kappa$, we obtain a new one-parameter subclass of deformed logarithms (Kaniadakis 2005). Therefore, the question of the analytical derivation of the deformed parameter arise also in other context than the $q$-statistics. To cite an example, the deformed parameter $\kappa$ has been determined implicitly in Kaniadakis (2002, 2005) by considering the black-hole entropy, obtaining the relation

$$\ln_{(\kappa)}(M) = \frac{2}{3},$$

where $M$ is the mass of the black-hole. A similar relation has also been derived in the case of the $q$-statistics.

4 Application to stellar plasma and equilibrium time

Because many nuclear reactions in the stellar burning core proceed by way of quantum penetration of a high Coulomb barrier, their cross sections grow exponentially with energy. Therefore, thermal averages do not probe the average energy of the distribution ($kT$), but its high-energy tail. Consequently, rates are very sensitive to a relatively small part of the distribution.

The reasons of such deviations from the exponential tail have been discussed elsewhere (Coraddu et al. 1999; Ferro and Quarati 2005) and include the presence of distributions of random microfields larger than a critical value and the quantum uncertainty between energy and momentum of effective particles which leads to a non-exponential momentum distribution even in presence of an exponential energy distribution. Analytical expressions for these effects have already been presented in this paper.

In addition, screening affects fusion rate in stars not only by lowering the Coulomb barrier and, therefore, increasing the penetration, but also by modifying the distribution. In fact, screening induces effective non-local and retarded interactions between colliding particles, which can not be treated as pure Coulomb collisions. In this respect, since the number of particles inside the Debye-Hückel sphere is too small one should also introduce modifications of the usual Debye-Hückel screening.

Finally, there could be further non-Gaussian effects due to the fact that distributions can be out of equilibrium. Indeed, nuclear burning in stellar cores is a non-equilibrium process with the corresponding ion distributions that, in principle, depart from exponential. However, the deviation from the pure exponential due to the reacting ions is completely negligible for stellar structure. As Bahcall et al. (2002) have argued, this effect is proportional to the ratio of the collision and nuclear reaction times

$$\delta = \frac{\tau_{\text{Coul}}}{\tau_{\text{nucl}}},$$

which is usually very small; for instance, this ratio can be estimated of the order of $\delta \simeq 10^{-20}$ for protons in the sun. Minor and faster reactions can yield larger ratios.

This line of arguing is based on the assumption that only two timescales are relevant for the distribution. In fact, collective modes on several scales are present in nuclear plasmas that have different timescales. For instance, the ratio $\nu_1/\nu_{\text{Coul}}$, where $\nu_1$ is the collision frequency from the so-called reinforced Coulomb scattering, is not at all negligible, possibly leading to non-Gaussian behavior.

Therefore, one should carefully assess what is the equilibrium relevant to the specific problem at hand (nuclear fusion, radiative recombination, diffusion, etc.) and also consider, as also argued in Lavagno and Quarati (2000, 2006), that non-Gaussian behavior in plasmas arises from all collective behaviors discussed above not only from non-equilibrium.

We remind that astrophysical and heavy-ion plasmas require relativistic extensions of the $q$-formalism
The reader might be also interested to some recent applications of nonextensive statistical mechanics, where a physical meaning of $q$ is introduced: to astrophysics by Jiulin and Yeli (2008a, 2008b) to heavy ions by Wilk and collaborators (Osada and Wilk 2007).

5 Conclusion

Non-Gaussian distribution functions appear in interacting particle systems with nonlinear, nonlocal, metastable, or long-memory systems; plasmas are typical and important example of physical context where such distributions appear.

Modified statistical mechanics, such as the one introduced by Tsallis, give consistent frameworks that naturally yield distributions that deviate from the Maxwellian. For many purposes a single effective parameter, such as the entropic parameter $q$, is sufficient to characterize the non-Gaussian behavior of the system. This parameter can always, in principle, be derived from the underlying microscopical dynamics, even if complete dynamic calculations are very difficult for most of the systems.

Even when the calculation from first principle is possible, the answer does not need to be a simple analytical formula, but one often can find numerically the parameter(s) that describe(s) the distribution; nevertheless the result is predictive, since the same number applies to many distributions for similar systems.

In this paper, we have reported a few cases where there exist an analytical formula for the entropic parameter $q$ in terms of properties of the systems. All these cases involve particles that are properly correlated. Other approaches are certainly possible, but nonextensive $q$ statistical mechanics appears particularly well-suited to describe such complex systems, particularly astrophysical and space systems.

At last, we recall that $q$-Gaussian distributions, in addition to provide a very powerful effective description of complex systems, have also a predictive power. For instance, Lutz (2003) proposed an experiment on optical lattices; such an experiment was later performed (Douglas et al. 2006) confirming the expectation. We have also proposed (Maero et al. 2006) an experiment to look for a cut-off in the energy distribution of gammas in electron recombination that could confirm deviations and possibly measure the relevant parameter of the distribution. To obtain predictive results in the astrophysics and space science field is the challenge of the next future.
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