DECAY RATE OF A POSITRONIUM.
REVIEW OF THEORY AND EXPERIMENT

VALERI V. DVOEGLAZOV∗,†
Departamento de Física Teórica
Instituto de Física, UNAM, Apartado Postal 20-364
01000 Mexico, D. F. MEXICO

and

RUDOLF N. FAUSTOV
Scientific Council for Cybernetics
Russian Academy of Sciences, Vavilov str., 40
Moscow 117333 RUSSIA

and

YURI N. TYUKHTYAEV‡
Department of Theoretical & Nuclear Physics
Saratov State University, Astrakhanskaya str., 83
Saratov 410071 RUSSIA

Abstract
The present status of theoretical and experimental investigations of the decay rate of a positronium is considered. The increasing interest to this problem has been caused by the disagreement of the calculated value of $\Gamma_3(o-Ps)$ and the recent series of precise experiments. The necessity of new calculations on the basis of the quantum field methods in bound state theory is pointed out with taking into account the dependence of the interaction kernel on relative energies.

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∗ On leave from: Dept. Theor. & Nucl. Phys., Saratov State University and Sci. & Tech. Center for Control and Use of Physical Fields and Radiations Astrakhanskaya str., 83, Saratov 410071 RUSSIA
† Email: valeri@ifunam.ifisicacu.unam.mx
dvoeglazov@main1.jinr.dubna.su
‡ Email: postmaster@ccssu.saratov.su
The quantum-electrodynamic systems, consisting of particle and anti-particle, have specific features. Apart from a scattering channel, the annihilation channel appears in this case. A positronium atom, which is a specimen of these systems, has no stability. The life time of a positronium (or the decay rate) is the subject of precise experimental and theoretical investigations. The charge parity of a positronium, \( C = (-1)^{L+S} \) (\( L \) is the eigenvalue of an angular momentum operator, \( S \) is the eigenvalue of a total spin operator for a system under consideration), is a motion constant. Consequently, all its states are separated into the charge - even states (\( S = 1 \)) and the charge - odd ones (\( S = -1 \)). The positronium total spin is also conserved and the energy levels are classified as the singlet levels (\( S = 0 \), parapositronium) and as the triplet ones (\( S = 1 \), orthopositronium). The \( S \) – state \( (L = 0) \) parapositronium has a positive parity and the \( S \) – state orthopositronium has a negative parity. As a consequence of conservation of a charge parity in electromagnetic interaction a parapositronium is disintegrated into the even number of photons and an orthopositronium is decayed into the odd one.

At the present time, the essential disagreement is turned out between the theoretical and experimental values for the decay rate of an orthopositronium. The theoretical predictions are \([1]-[4]\):

\[
\Gamma_{3}^{\text{theor}}(o - Ps) = \frac{\alpha^6 mc^2}{\hbar} \frac{2(\pi^2 - 9)}{9\pi} \left[ 1 + A_3 \frac{\alpha}{\pi} - \frac{1}{3} \alpha^2 \ln \alpha^{-1} + B_3 \left( \frac{\alpha}{\pi} \right)^2 + \ldots \right] =
\]

\[
= \Gamma_0 + \frac{ma^7}{\pi^2} \left\{ \left( -1.984(2) \right) \right\} + \frac{ma^8}{\pi^2} \ln \alpha^{-1} \left[ -\frac{4}{9} \zeta(2) + \frac{2}{3} \right] + \frac{ma^8}{\pi^3} \chi + \ldots
\]

\[
= \left\{ \begin{array}{c}
7.03862(2) \\
7.03830(7)
\end{array} \right\} \mu s^{-1}, \tag{1}
\]

where

\[
A_3^{[3]} = -10.266 \pm 0.011, \tag{2}
\]

\[
A_3^{[4]} = -10.282 \pm 0.003. \tag{3}
\]

Taking into account the modern value of \( \alpha \), the fine structure constant, the result can be recalculated \([5]\):

\[
\Gamma_{3}^{\text{theor}} = 7.03831(5) \mu s^{-1}. \tag{4}
\]

The last experimental measurements are \([6, 7]\):

\[
\Gamma_{6}^{\exp}(o - Ps) = 7.0514(14) \mu s^{-1} \tag{5}
\]

\[
\Gamma_{7}^{\exp}(o - Ps) = 7.0482(16) \mu s^{-1}. \tag{6}
\]

The result of Ref. \([8]\) has 9.4 standard deviation from the predicted theoretical decay rate and the result of \([9]\) has 6.2 standard deviation. The coefficient \( B_3 = 1 \) in the \( O(\alpha^8) \) term can contribute \( 3.5 \cdot 10^{-5} \mu s^{-1} \) (or 5 ppm of \( \Gamma_3 \)) only. To take off the above disagreement the coefficient \( B_3 \) has to be equal to about \( \simeq 250 \pm 40 \), what is very unlikely.

\[ \text{See Table I for the preceding experimental results.} \]
indeed. However, this opportunity is pointed out in [7] just not to be rejected \textit{a priori}. Therefore, the calculation of $B_3$ coefficient is desirable enough now.

For the first time the main contribution in the orthopositronium decay rate has been calculated in [1]:

$$\Gamma_0(o - Ps) = -2Im(\Delta E_{3\gamma}) = \frac{2}{9\pi}(\pi^2 - 9)m\alpha^6 = 7,211\,17\,\mu s^{-1}. \quad (7)$$

The corrections of the $O(\alpha)$ order of the magnitude to this quantity had been calculating by numerical method [2, 4, 8, 9] at first, but later some of them has been figure out analytically in [3, 13] and [10]-[12] in the Feynman gauge. The corrections, which come from the diagrams with the self-energy and vertex insertions, have been calculated by Adkins [11, 12]:

$$\Gamma_{OV} = \Gamma_0 \frac{\alpha}{\pi} \left\{ D + \frac{3}{4(\pi^2 - 9)} \left[ -26 - \frac{115}{3} \ln 2 + \frac{911}{18} \zeta(2) + \frac{443}{54} \zeta(3) + \frac{3419}{108} \zeta(2) \ln 2 - R \right] \right\} = \Gamma_0 \frac{\alpha}{\pi} \left[ D + 2.971\,138\,5\,(4) \right], \quad (8)$$

$$\Gamma_{SE} = \Gamma_0 \frac{\alpha}{\pi} \left\{ -D - 4 + \frac{3}{4(\pi^2 - 9)} \left[ -7 + \frac{67}{3} \ln 2 + \frac{805}{36} \zeta(2) - \frac{1049}{54} \zeta(3) - \frac{775}{54} \zeta(2) \ln 2 \right] \right\} = \Gamma_0 \frac{\alpha}{\pi} \left[ -D + 0.784\,98 \right], \quad (9)$$

$$\Gamma_{IV} = \Gamma_0 \frac{\alpha}{\pi} \left\{ \frac{1}{2} D + \frac{3}{4(\pi^2 - 9)} \left[ -4 - \frac{34}{2} \ln 2 - \frac{841}{36} \zeta(2) + \frac{1253}{36} \zeta(2) \ln 2 + \frac{1589}{54} \zeta(3) + \frac{17}{40} \zeta^2(2) - \frac{7}{8} \zeta(3) \ln 2 + \frac{5}{2} \zeta(2) \ln^2 2 - \frac{1}{24} \ln^4 2 - a_4 \right] \right\} = \Gamma_0 \frac{\alpha}{\pi} \left[ \frac{1}{2} D + 0.160\,677 \right], \quad (10)$$

where

$$R = \int_0^1 dx \frac{\ln(1-x)}{2-x} \left[ \zeta(2) - Li_2(1 - 2x) \right] = -1.743\,033\,833\,7\,(3), \quad (11)$$

$$a_4 = Li_4\left(\frac{1}{2}\right) = \sum_{n=1}^{\infty} \frac{1}{n^4 2^n} = 0.517\,479\,061\,674, \quad (12)$$

$$\zeta(2) = \frac{\pi^2}{6}, \quad \zeta(3) = 1.202\,056\,903\,2, \quad (13)$$

and

$$D = \frac{1}{2 - w} - \gamma_E + \ln(4\pi) \quad (14)$$

is standard expression of a dimensional regularization ($2\omega$ is a space dimension). The indices "IV" and "OV" designate the insertions in the internal photon-electron vertex and in the outer ones, correspondingly. The above results are co-ordinated with the Stroscio’s result [10] when

$$\Gamma_0 \frac{\alpha}{\pi} \left[ -D - 4 - 2\ln(\gamma^2/m^2) \right] \quad (15)$$
being added to the last one. This is necessary because of the different regularization
procedures which have been used in [10] and in [11, 12], correspondingly.

Recently, the calculations of these corrections have been finished [5] in the Fried –
Yennie gauge:

$$\Gamma_{SE} = \frac{m\alpha^7}{\pi^2} \left[ -\frac{13}{54} \zeta(3) + \frac{461}{108} \zeta(2) \ln 2 - \frac{251}{72} \zeta(2) - \frac{29}{6} \ln^2 2 + \frac{9}{2} \right] =$$

$$= \frac{m\alpha^7}{\pi^2}(-0.007 132 904) = \Gamma_0 \frac{\alpha}{\pi}(-0.036 911 113),$$

$$\Gamma_{OV} = \frac{m\alpha^7}{\pi^2} \left[ -\frac{88}{54} \zeta(3) - \frac{299}{216} \zeta(2) \ln 2 + \frac{49}{18} \zeta(2) + \frac{13}{6} \ln^2 2 - \frac{1}{6} R \right] =$$

$$= \frac{m\alpha^7}{\pi^2}(0.732 986 380) = \Gamma_0 \frac{\alpha}{\pi}(3.793 033 599).$$

(16)

The contributions from the remained diagrams (with a radiative insertion in a vertex of
an internal photon; with two radiative photons spanned; the diagram taking into account
boundary effects and the annihilation diagram, see Fig. I in [5b]), have been calculated
numerically. Totally, the \(O(\alpha)\) corrections are joined to give

$$\frac{m\alpha^7}{\pi^2} [-1.987 84(11)] = \Gamma_0 \frac{\alpha}{\pi} [-10.286 6(6)].$$

(18)

Then we have [6]:

$$\Gamma_{theor}^{(5)}(o - Ps) = 7.038 236(10) \mu s^{-1}.$$  (19)

The above result is the most precise theoretical result available at the present moment.

To solve the existing disagreement between theory and experiment, the 5 – photon
mode of \(o - Ps\) decay and the 4 – photon mode of \(p - Ps\) decay have been under consid-
eration in [14, 15]. The following theoretical evaluations have been obtained:

$$\frac{\Gamma_5^{[14]}(o - Ps)}{\Gamma_3^{(o - Ps)}} = 0.177 (\frac{\alpha}{\pi})^2 \simeq 0.96 \cdot 10^{-6},$$

(20)

$$\frac{\Gamma_4^{[14]}(p - Ps)}{\Gamma_2(p - Ps)} = 0.274 (\frac{\alpha}{\pi})^2 \simeq 1.48 \cdot 10^{-6},$$

(21)

and

$$\Gamma_5^{[15]}(o - Ps) = 0.018 9(11) \alpha^2 \Gamma_0,$$

$$\Gamma_5^{[15]}(p - Ps) = 0.013 89(6) m\alpha^7.$$  (22)

(23)

They are in agreement with each other and with the results of the previous papers [16]:

$$\Gamma_4^{[16]}(p - Ps) = 0.013 52 m\alpha^7 = 11.57 \cdot 10^{-3} s^{-1}.  \quad (24)$$

\(^b\)The uncalculated yet \(O(\alpha^8)\) corrections are not accounted here.

\(^c\) As a consequence of a conservation of an angular momentum and an isotropic properties of coordinate
space an orthopositronium has to decay into the odd number of photons and a parapositronium has to
decay into the even one, as outlined before.

\(^d\)The result [17] is not correct, four times less than the above cited results. The explanation of this
was given in [15].
In the connection with the present situation with respect to the decay rate of $\eta - Ps$ investigations of alternative decay modes for this system (e.g. $\eta - Ps \rightarrow \gamma + a$, $a$ is an axion, a pseudo-scalar particle with mass $m_a < 2m_e$) are of present interest \[18\]-\[23\]. In the article \[21\] the following experimental limits of the branching of decay width have been obtained:

$$Br = \frac{\Gamma(\eta - Ps \rightarrow \gamma + a)}{\Gamma(\eta - Ps \rightarrow 3\gamma)} < 5 \cdot 10^{-6} - 1 \cdot 10^{-6} \ (30 \ ppm),$$

(25)

provided that $m_a$ is in the range 100 – 900 keV. In the case of the axion mass less than 100 keV (which is implied by the Samuel’s hypothesis \[22\]; according to the cited paper $m_a < 5.7 \ keV$, $g_{ae^+e^-} \sim 2 \cdot 10^{-8}$) the limits of $Br$ are the following ones \[23\] :

$$Br = 7.6 \cdot 10^{-6}, \ \text{if} \ m_a \sim 100 \ keV,$$

(26)

$$Br = 6.4 \cdot 10^{-5}, \ \text{if} \ m_a < 30 \ keV.$$  

(27)

These limits are about 2 orders less than the value which is necessary to remove the disagreement.

Finally, a decay $\eta - Ps \rightarrow nothing$ (that is into weak-interacting non-detected particles) \[24\] has been investigated in \[25\]. The obtained result

$$\frac{\Gamma(\eta - Ps \rightarrow nothing)}{\Gamma(\eta - Ps \rightarrow 3\gamma)} < 5.8 \cdot 10^{-4} \ (350 \ ppm)$$

(28)

expels the opportunity that this decay mode is an origin of disajustment between theory and experiment.

The decay of $\eta - Ps$ into two photons, which breaks the CP – invariance, as else mentioned in \[26, 27\], was experimentally rejected in \[28\].

Let us mention, the contribution of weak interaction has been studied in \[30\]. However, because of the factor $m_e^2/M_W^2 \sim G_F \cdot m_e \simeq 3 \cdot 10^{-12}$ it cannot influence final results. In the cited article the weak decay modes are estimated as

$$\frac{\Gamma(p - Ps \rightarrow 3\gamma)}{\Gamma(p - Ps \rightarrow 2\gamma)} \sim \frac{\Gamma(o - Ps \rightarrow 4\gamma)}{\Gamma(o - Ps \rightarrow 3\gamma)} \simeq \alpha(G_Fm_e^2g_V)^2 \simeq 10^{-27},$$

(30)

where $G_F$ is the Fermi constant for weak interaction,

$$g_V = 1 - 4sin^2\Theta_W \simeq 0.08,$$

(31)

\footnote{The proposed values do not influence the agreement of theoretical and experimental results of an anomalous magnetic moment (AMM) of an electron.}

\footnote{Analogously to Glashow’s hypothesis of the decay into invisible ”mirror” particle \[24\].}

\footnote{The physics ground of these speculations is a possible existence of an unisotropic vector field with non-zero vacuum expectation \[29\], with whom an electron and a positron could be interacting}

$$\mathcal{L} = g\bar{\psi}O_{\alpha\beta}\psi A^\alpha\Omega^\beta,$$

(29)

$\mathcal{L}$ is the interaction Lagrangian.
Θ_W is the Weinberg angle. The current experimental limits are [31, 32]:

\[
\frac{\Gamma(p - Ps \rightarrow 3\gamma)}{\Gamma(p - Ps \rightarrow 2\gamma)} \leq 2.8 \cdot 10^{-6}, \\
\frac{\Gamma(o - Ps \rightarrow 4\gamma)}{\Gamma(o - Ps \rightarrow 3\gamma)} \leq 8 \cdot 10^{-6}.
\] (32) (33)

In the Table I all experimental results for the \(o - Ps\) decay rate, known to us, are presented.

Table I. The experimental results for the \(o - Ps\) decay rate.

| Year | Reference | \(\Gamma_3(o - Ps)\), \(\mu s^{-1}\) | Error, ppm | Technique |
|------|-----------|----------------------------------|------------|-----------|
| 1968 | [36]      | 7.262(15)                        | 2070       | gas       |
| 1973 | [37]      | 7.262(15)                        | 2070       | gas       |
| 1973 | [38]      | 7.275(15)                        | 2060       | gas       |
| 1976 | [39]      | 7.104(6)                         | 840        | powder SiO_2 |
| 1978 | [40]      | 7.056(7)                         | 990        | gas       |
| 1978 | [41]      | 7.045(6)                         | 850        | gas       |
| 1978 | [42]      | 7.050(13)                        | 1840       | vacuum    |
| 1978 | [43]      | 7.122(12)                        | 1680       | vacuum    |
| 1982 | [44]      | 7.051(5)                         | 710        | gas       |
| 1987 | [45]      | 7.031(7)                         | 1000       | vacuum    |
| 1987 | [46]      | 7.0516(13)                       | 180        | gas       |
| 1989 | [6]       | 7.0514(14)                       | 200        | gas       |
| 1990 | [7]       | 7.0482(16)                       | 230        | vacuum    |

Regarding the results for the decay rate of a parapositronium, the situation was highly favorable until the last years. The theoretical value, which was found out elsewhere in the fifties [48, 49], is equal to

\[
\frac{\alpha^5 mc^2}{2h} \left[ 1 - \frac{\alpha}{\pi} \left(5 - \frac{\pi^2}{4}\right) \right] = 7.9852 \text{ } ns^{-1}.
\] (34)

The above value, confirmed in [50, 51], coincides with the direct experimental result up to the good accuracy:

\[
\Gamma_{exp}^{[45]}(p - Ps) = 7.994 \pm 0.011 \text{ } ns^{-1}.
\] (35)

The experimental values of the parapositronium decay rate are showed in the Table II.

\[^{b}\text{The results of the papers [33, 34] and [35] can be accounted as rough estimations only.}\]

\[^{c}\text{The branching of the decay rates of a para- and an orthopositronium \(\Gamma_3(o - Ps)\) had been measuring in the experiments of 1952 and 1954 only. The presented results are recalculated by means of the first direct experimental value \(\Gamma_3(o - Ps) = 7.262(15) \mu s^{-1}\) [36].}\]
Table II. The experimental results for the $p - Ps$ decay rate.

| Year | Reference | $\Gamma_2(p - Ps)$, ns$^{-1}$ | Error, % | Technique |
|------|-----------|-------------------------------|----------|-----------|
| 1952 | [52]      | 7.63(1.02)                    | 13       | gas       |
| 1954 | [53]      | 9.45(1.41)                    | 15       | gas       |
| 1970 | [54]      | 7.99(11)                      | 1.38     | gas       |
| 1982 | [45]      | 7.994(11)                     | 0.14     | gas       |

In the articles [3, 51] it was pointed out that it is necessary to add the logarithmic corrections on $\alpha$ to the Harris and Brown's result. In the article [13a] these corrections to the $\Gamma_3(o - Ps)$ and $\Gamma_2(p - Ps)$ have been calculated again, with the result of the decay rate of a parapositronium differing with the one found out before [3, 51]:

$$\Gamma_2^{[13]}(p - Ps, \alpha^2\ln\alpha) = \frac{m\alpha^5}{2} \cdot 2\alpha^2\ln\alpha^{-1},$$  \hspace{1cm} (36)

$$\Gamma_2^{[51]}(p - Ps, \alpha^2\ln\alpha) = \frac{m\alpha^5}{2} \cdot \frac{2}{3}\alpha^2\ln\alpha^{-1}.$$  \hspace{1cm} (37)

Finally, the quite unexpected (and undesirable) result, presented in the Remiddi’s (and collaborators) talk [53] should be mentioned. The calculations, carried out by the authors of cited paper, lead to the additional contribution:

$$\Gamma_2^{[55]}(p - Ps, \alpha\ln\alpha) = \frac{m\alpha^5}{2} \cdot \frac{\alpha}{\pi}2\ln\alpha,$$  \hspace{1cm} (38)

which is explained by authors to appear as a result of taking into account the dependence of an interaction kernel on the relative momenta.

The above-mentioned shows us at the necessity of a continuation of calculations of the decay rates of an orthopositronium as well as a parapositronium employing more

\[ \text{Let us mark, the additional contributions, which are similar to the ones of Ref. [53], appeared in the calculations of the hyperfine splitting (HFS) of the ground state of two-fermion system by the quasipotential method [56, 57] when taking into account the dependence of the interaction kernel on the relative energies [58, 59]. For example, the } O(\alpha^2\ln\alpha) \text{ correction to the HFS, obtained from one-photon diagram, is equal to} \]

$$\Delta E_{tr}^{\text{HFS}}(\alpha^2\ln\alpha) = E_F \frac{\mu^2\alpha^2}{m_1m_2} \left( \frac{m_1}{m_2} + \frac{m_2}{m_1} + 2 \right)\ln\alpha^{-1},$$  \hspace{1cm} (39)

when using the version of the quasipotential approach based on the amplitude $\tau = (\hat{G}_0^+)^{-1}G_0\hat{T}^\ast G_0^\ast (\hat{G}_0^+)^{-1}$. Here $\hat{G}_0^+$ is the two-time Green’s function projected onto the positive-energy states, $m_1$ and $m_2$ are the masses of the constituent particles, $E_F$ is the Fermi energy. The index "tr" designates that the diagram of one-transversal-photon exchange is under consideration.

Using the kernel constructed from the on-shell physical amplitude we can find out that the last term in the brackets disappears. However, the total result to the HFS accounting all diagrams is the same in both of versions of quasipotential approach.

We can also come across with the similar situation in calculations of the anomalous corrections of $O(\alpha\ln\alpha)$ order, which do not appear in the method on the mass shell. However, the version, based on the two-time Green’s function, shows the contraction of these anomalously large terms, taking into account two-photon exchange diagrams [59].
accurate relativistic methods, one of which is the quasipotential approach in quantum field theory \cite{56, 57}, giving the opportunity taking into account of binding effects.

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References

[1] A. Ore and J. L. Powell, Phys. Rev. 75, 1696 (1949).
[2] W. E. Caswell, G. P. Lepage and J. R. Sapirstein, Phys. Rev. Lett. 38, 488 (1977).
[3] W. E. Caswell and G. P. Lepage, Phys. Rev. A20, 36 (1979).
[4] G. S. Adkins, Ann. Phys.(USA) 146, 78 (1983).
[5] G. S. Adkins, A. A. Salahuddin and K. E. Schalm, Phys. Rev. A45, 3333 (1992); ibid, 7774 (1992).
[6] C. I. Westbrook, D. W. Gidley, R. S. Conti and A. Rich, Bull. Am. Phys. Soc. 32, 1051 (1987); Phys. Rev. A40, 5489 (1989).
[7] J. S. Nico, D. W. Gidley, A. Rich and P. W. Zitzewitz, Phys. Rev. Lett. 65, 1344 (1990).
[8] P. Pascual and E. de Rafael, Lett. Nuovo Cim., 4, 1144 (1970).
[9] M. A. Stroscio and J. M. Holt, Phys. Rev. A10, 749 (1974); M. A. Stroscio, Phys. Lett. A50, 81 (1974); Phys. Rev. A12, 338 (1975).
[10] M. A. Stroscio, Phys. Rev. Lett. 48, 571 (1982).
[11] G. S. Adkins, Phys. Rev. A27, 530 (1983).
[12] G. S. Adkins, Phys. Rev. A31, 1250 (1985).
[13] I. B. Khriplovich and A. S. Yelkhovsky, Phys. Lett. B246, 520 (1990); I. B. Khriplovich, A. I. Milstein and A. S. Yelkhovsky, Phys. Lett. B282, 237 (1992).
[14] G. P. Lepage, P. B. Mackenzie, K. H. Streng and P. M. Zerwas, Phys. Rev. A28, 3090 (1983).
[15] G. S. Adkins and F. R. Brown, Phys. Rev. A28, 1164 (1983).
[16] I. Muta and T. Niuya, cited in \cite{31}; A. Billoire, R. Lacaze, A. Morel and H. Navelet, Phys. Lett. 78B, 140 (1978).
[17] G. C. McCoyd, Ph.D. Thesis, St. John’s University, 1965.
[18] J. Cleymans and P. S. Ray, Lett. Nuovo Cim. 37, 569 (1983).
[19] G. Carboni and W. Dahme, Phys. Lett. B123, 349 (1983).
[20] V. Metag et al., Nucl. Phys. A409, 331c (1983) — Proc. of the Int. Conf. on Heavy-Ion Physics and Nuclear Physics. Catania, Italy, 21-26 March 1983.
[21] U. Amaldi, G. Carboni, B. Jonson and J. Thun, Phys. Lett. B153, 444 (1985).
[22] M. A. Samuel, Mod. Phys. Lett. A3, 1117 (1988).
[23] S. Orito et al., Phys. Rev. Lett. 63, 597 (1989).
[24] S. L. Glashow, Phys. Lett. B167, 35 (1986).
[25] G. S. Atoyan, S. N. Gnimenko, V. I. Razin and Yu. V. Ryabov, Phys. Lett. B220, 317 (1989).
[26] L. D. Landau, Doklady AN SSSR 60, 207 (1948), in Russian.
[27] C. N. Yang, Phys. Rev. 77, 242 (1950).
[28] A. P. Mills, Jr. and D. M. Zuckerman, Phys. Rev. Lett. 64, 2637 (1990).
[29] P. R. Phillips, *Phys. Rev.* **139**, 491 (1965).
[30] W. Bernreuter and O. Nachtman, *Zeit. Phys.* **C11**, 235 (1981).
[31] A. P. Mills, Jr. and S. Berko, *Phys. Rev. Lett.* **18**, 420 (1967).
[32] K. Marko and A. Rich, *Phys. Rev. Lett.* **33**, 980 (1974).
[33] F. F. Heymann, P. E. Osmon, J. J. Veit and W. F. Williams, *Proc. Phys. Soc. (London)* **78**, part 5(ii), 1038 (1961).
[34] B. G. Duff and F. F. Heymann, *Proc. Roy. Soc. (London)* **A270**, 517 (1962).
[35] J. D. McNutt, V. B. Summerour, A. D. Ray and P. H. Huang, *J. Chem. Phys.* **62**, 1777 (1975).
[36] R. H. Beers and V. W. Hughes, *Bull. Am. Phys. Soc.* **13**, 633 (1968); V. W. Hughes, in *Proc. of the First Int. Conf. on Atomic Physics, New York, June 3-7, 1968*, ed. B. Bederson, V. W. Cohen and F. M. J. Pichanchick (New York: Plenum Press, 1969), p. 15.
[37] P. G. Coleman and T. C. Griffith, *J. Phys.* **B6**, 2155 (1973).
[38] V. W. Hughes, in *Physik 1973. Plenarvortung Physikertagung 37te*, (Physik Verlag, Weinheim, Germany, 1973), p. 123.
[39] D. W. Gidley, K. A. Marko and A. Rich, *Phys. Rev. Lett.* **36**, 395 (1976).
[40] D. W. Gidley, P. W. Zitzewitz, K. A. Marko and A. Rich, *Phys. Rev. Lett.* **37**, 729 (1976).
[41] D. W. Gidley, A. Rich, P. W. Zitzewitz and D. A. L. Paul, *Bull. Am. Phys. Soc.* **22**, 586 (1977); *Phys. Rev. Lett.* **40**, 737 (1978).
[42] T. C. Griffith and G. R. Heyland, *Nature* **269**, 109 (1977); T. C. Griffith, G. R. Heyland, K. S. Lines and T. R. Twomey, *J. Phys.* **B11**, L743 (1978).
[43] D. W. Gidley and P. W. Zitzewitz, *Phys. Lett.* **A69**, 97 (1978).
[44] K. F. Canter, B. O. Clark and I. J. Rosenberg, *Phys. Lett.* **A65**, 301 (1978).
[45] D. W. Gidley, A. Rich, E. Sweetman and D. West, *Phys. Rev. Lett.* **49**, 525 (1982).
[46] P. Hasbach, G. Hilkert, E. Klempt and G. Werth, *Nuovo Cim.* **A97**, 419 (1987).
[47] C. I. Westbrook, D. W. Gidley, R. S. Conti and A. Rich, *Phys. Rev. Lett.* **58**, 1328 (1987).
[48] P. A. M. Dirac, *Proc. Cambridge Phil. Soc.* **26**, 361 (1930).
[49] I. Harris and L. Brown, *Phys. Rev.* **105**, 1656 (1957).
[50] V. K. Cung, A. Devoto, T. Fulton and W. W. Repko, *Phys. Lett.* **B78**, 116 (1978); *Phys. Rev.* **A19**, 1886 (1979).
[51] Y. Tomozawa, *Ann. Phys. (USA)* **128**, 463 (1980).
[52] J. Wheatley and D. Halliday, *Phys. Rev.* **88**, 424 (1952).
[53] V. W. Hughes, S. Marder, C. S. Wu, *Phys. Rev.* **95**, 611 (1954); ibid, **98**, 1840 (1955).
[54] E. D. Theriot, Jr., R. H. Beers and V. W. Hughes, *Bull. Am. Phys. Soc.* **12**, 74 (1967); *Phys. Rev. Lett.* **18**, 767 (1967); E. D. Theriot, Jr., R. H. Beers, V. W. Hughes and K. O. H. Ziock, *Phys. Rev.* **A2**, 707 (1970).
[55] A. Hill, F. Ortolani and E. Remiddi, in *The Hydrogen Atom*, ed. G. F. Bassani et al. (Berlin – Heidelberg: Springer – Verlag, 1989), p. 240.
[56] A. A. Logunov and A. N. Tavkhelidze, *Nuovo Cim.* **29**, 380 (1963).
[57] V. G. Kadyshensky, *Nucl. Phys.* **B6**, 125 (1968).
[58] Yu. N. Tyukhtyaev, Doctor of Science’s Dissertation, Saratov State University, 1985.
[59] N. A. Boikova, V. V. Dvoeglazov, Yu. N. Tyukhtyaev and R. N. Faustov, *Theor. Math. Phys.* **89**, 1174 (1991).