ON THE MAGNETISM OF STARS AND PLANETS

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Abstract

The self-consistent statistical approach to the problem of planetary and stellar magnetism is suggested. The mechanism of magnetic fields generation in the astronomical objects, where the existence of fields is associated with the axial rotation of objects, is discussed. In the general case the light pressure, the centrifugal, gravitational and other forces produce partial $\rho$-separation of the charges. As a result of the system rotation, the magnetic fields of the currents of these charges are not compensated. The influence of various factors on the magnetic field of some object is analysed.

1 Introduction

The origin of magnetism in various astronomical objects is of great theoretical and practical interest. The existence of magnetic fields in these cases indicates that currents are circulating inside such objects. What is the nature of these currents?

At the present time there is a great deal of information on this subject. In the literature the existence of magnetic fields is associated with:

1) various dynamo processes: a) the classical kinematic dynamo-models (Busse, 1979; Parker, 1979; Moffat, 1978); b) the kink instability mechanism (Alfven et al., 1974; Alfven, 1981); c) the action of gravitational forces (Hide, 1956); d) the specific models (heat convection, precession) (Urey, 1952; Busse, 1976; Bullard et al., 1971; Runcorn, 1975);

2) or with the remanent magnetization (Sharpe et al., 1976).

These theories encounter some problems: (Runcorn, 1975; Malkus, 1963; Stevenson, 1974); the Earth’s core paradox (Higgins et al., 1971; Kennedy et al., 1973); the energetic contradictions (Jacobs et al., 1972); the contradictions of precession mechanism (Rochester et al., 1975; Busse, 1971) etc.

A variety of approaches to the problem of magnetism of astronomical objects may exist. One of them consists in describing the process of magnetic field birth and generation up to observable values. This approach is undoubtedly important and interesting, from principal positions, to elucidate the mechanism of such a generation. As an example we refer to the "battery theory" for the generation of cosmic magnetic fields (Biermann 1950; Mestel et al., 1962; Dolginov, 1988).
Note, that the paper is concerned with the above mentioned theory (in initial cause). However, some differences exist. The basis for the "battery theory" is the "Biermann effect", where the following term provides the magnetic field generation:

\[ c \nabla \eta \times \nabla T = -\frac{c}{e} \nabla \times \frac{\nabla P_e}{N_e} = \frac{1}{2} M_i \nabla \times \vec{g}_{eff}, \]

where \( \eta \) is the thermopower constant, \( P_e \) is the electron pressure, \( \vec{g}_{eff} = \vec{g} + \vec{\Omega} \times \vec{\Omega} \times \vec{r} \) is the sum of gravitational and centrifugal accelerations (the causes of generation: nonuniform rotation, the inhomogeneous distribution of chemical elements, temperature inhomogeneities). At the same time, however, it has rather essential disadvantages. First, one must proceed from some unknown hypothetic state. Second, for the time during which the magnetic field is increasing, so many various random events take place, that they can not be taken into consideration even in principle. For example, random flows of charged particles (cosmic rays) can violate the 2\(^{nd} \) magnification (as a consequence of long-range Coulomb interactions). Third, even the main factors can be taken into account within the framework of approximate models only (the continuum medium model, for instance). Besides, there exist problems for calculation. At present it is impossible to solve the full system of equations (Dolginov, 1988). The magnitude of the stationary field cannot be determined from the linearized equations (note that the nonlinear terms are the same order of magnitude as the linear ones). But investigation of nonlinear effects are still in their infancy. All these difficulties do not allow quantitative characteristics of the field to be determined. There exists, however, some another approach to this problem. It is as follows. At present, a set of observable characteristics exists for studying astronomical objects. These characteristics (seem to be stable) include such ones, as the frequency of rotation of a system, its composition, electric and magnetic fields, etc. Here, the question arises: is there any connection between these observable quantities, so that, proceeding from some particular observable system’s parameters, one can calculate other parameters.

Many astronomical objects (nebulae, planets, the Sun and other stars) are known to be rotating bodies. The attempt to find the relation between these phenomena is made in this article. Since some astronomical magnetic fields are rather stable, it can be supposed stable or quasistable state exists in the system and the microscopic statistical approach to this problem can be proposed. Developing this approach, an attempt is made in this paper to describe theoretically stable states of the system and the dynamics of fields. In principle, the statistical approach allows the exact solution for the stationary field to be obtained in integral form. Some distinct value of the field (to sufficient accuracy) can be calculated from this solution for all specific values of parameters (nonlinear effects are implicitly taken into consideration).

The peculiarities of this work are as follows:
- the mechanism dispenses with the need for initial magnetic fields;
- the mechanism proposed can explain the existence of magnetic fields in a wide majority of astronomical objects, i.e. it is rather universal;

- the mechanism has self-consistent characteristics and leans upon the microscopic properties of matter, rather than upon the continuous medium models;

- the mechanism is simple enough, that is, there is no need to make any additional assumption on properties of the inner region of objects (good conduction, for example) or on processes occurring inside the objects (convection, for example).

Note, that the suggestion of the statistical approach to the problem is the most important part of the article, since there exist a possibility to study magnetic fields without knowledge about the nature of electric double layers: the statistical method can include different mechanisms which can lead to separation of charges (but not only suggested). For example, the reverse problem can be formulated: knowing fields, to find the possible position of electric double layers and theirs magnitudes. After that the discussion of magnetism (concrete mechanism) is more objective. However, it is not our intention to follow this way in the article.

In section 2.1 the separation of charges and the magnetic field appearance are investigated for the rotating plasma model. This model can be applied to some nebulae, for example. In section 2.2 the action of a gravitational force in the compact rotating system is additionally considered. This model can explain the existence of stellar magnetism. In section 2.3 the magnetism of planets (some "cooled" objects) is discussed. In this case the "cooled" (magnetic) materials in the surface layer of a planet (the Earth crust, for example) at temperature less than the Curie temperature can influence the magnetic field. In section 2.4 the general remarks on the Kauling theorem and different mechanisms, and some evaluations are presented. Section 3.1 outlines self-sustained rotation of a system and the equilibrium configuration of electric and magnetic fields. Section 3.2 analyses the influence of various internal and external factors and the dynamics of fields.

2 Models

2.1 The magnetic field in rotating plasma

The mechanism of magnetic field generation in the astronomical objects can be qualitatively explained in terms of this model. The rotating plasma model can be applied for explanation of magnetic fields existence in rotating plasmoids, such as ionized cloud, hot nebulae, etc.

As a consequence of long-range interaction between the charged particles, the plasma essentially differs from gas in some respects. The rotating plasma has some peculiarities as compared to rotating objects involving neutral particles. The existence of the centrifugal force

$$\mathbf{F}_c = m\Omega^2 \rho e \mathbf{e}_\rho,$$  \hspace{1cm} (1)
(where \( \Omega \) is the angular frequency of the system, \( m \) is the particle mass, \( \rho \) is its distance from the axis of rotation) causes different effects on particles of different masses. As a result of these different effects, the \( \rho \)-dependences of particle concentrations are bound to be different for particles of different masses.

According to the Boltzmann distribution,

\[
n_\alpha = n_{0\alpha} \exp \left( -\frac{U_\alpha}{kT} \right),
\]

where \( T \) is the system temperature, \( k \) is the Boltzmann constant, \( n_{0\alpha} \) is the particles concentration on the axis of rotation, \( U_\alpha \) is the potential energy of particles of \( \alpha \)-sort; for neutral particles

\[
U_\alpha = -\frac{m_\alpha \Omega^2 \rho^2}{2}.
\]

Since the particles of different masses in plasma have different charges, this partial separation of particles produces a partial separation of charges, that is, the negatively charged region must lie near the axis of rotation, whereas the positively charged region must lie near the system boundary (the distance \( R_0 \) from the axis of rotation). The electric field \( E_0(\rho, z) \) (in the polar coordinate system) exists as a result of this separation of charges.

This field opposes the considerable separation of charges. Therefore, the effect of partial separation of charges is bound to be often negligible for small systems, whereas the separation of charges can exert considerable influence on some physical characteristics of astronomical objects.

There is some distance from the axis of rotation \( R_1(z) \), where the local charge equals zero (some transverse section to the axis). For \( 0 < \rho < R_1(z) \) the plasma is negatively charged on the average; for \( R_1(z) < \rho \leq R_0 \) the plasma is positively charged. Therefore, the field \( E_0(\rho, z) \) at \( R_1(0) \) is directed towards the axis of rotation.

Of mine interest here is the fact, that the partial separation of charges gives rise to the \( \rho \)-dependence of charge density, that is, the plasma possessing a given charge density moves round a circle of distinct radius. Therefore, the circulating currents are inside the rotating plasma despite the fact, that all particles revolve with \( \Omega \) on the average (\( \Omega \) has a distinct direction). For \( R_1 < \rho \leq R_0 \) the current flows in the direction of rotation (since the charge is positive), whereas for \( 0 < \rho < R_1 \) the current is opposite to the direction of plasma rotation (since the charge is negative). In the general case the magnetic actions of these currents are not compensated and the magnetic field exists. This is the magnetic field of a solenoid whose axis coincides with the axis of rotation of the system.

It follows from the Biot-Savart law, that the magnetic field is

\[
H = \frac{1}{c} \int_{(V)} \frac{[j' \times R']}{{R'}^3} dv',
\]
where $V$ is the volume of the system, $c$ is the speed of light, $dv'$ is the region with currents, $R'$ is the distance of this region to the point of observation, $j'$ is the current density. We have

$$R' = (R_\rho; R_\phi; R_z) = (\rho - \rho_1 \cos(\varphi_1 - \varphi); \rho_1 \sin(\varphi - \varphi_1); z - z_1);$$

$$dv' = \rho_1 d\rho_1 d\varphi_1 dz_1 , \quad j' = (0; j_\varphi; 0) , \quad j_\varphi(\rho_1, \varphi_1, z_1) = e\Omega_1 q(\rho_1, \varphi_1, z_1).$$

The electric charge density $q$ is

$$q(\rho_1, z_1) = n_0i \exp\left(-\frac{1}{kT}U_i\right) - n_0e \exp\left(-\frac{1}{kT}U_e\right),$$

(5)

For rotating, fully ionized plasma, which consists of the elements with atomic number $N_i$, it follows in polar coordinates $\rho, \varphi, z$ (axis $z$ is in the $\Omega$ direction), that

$$U_e = -\frac{m\Omega^2 \rho_1^2}{2} + e \int_0^{\rho_1} E_0\rho(\rho', z_1)d\rho' + e \int_{z_1}^{z_1} E_0z(\rho_1, z')dz' +$$

$$\int_{\rho_1}^{\rho_1} eH_2 \Omega'\rho' d\rho' - \int_{z_1}^{z_1} eH_2 \Omega_1 d\rho',$$

(6)

$$U_i = -\frac{M\Omega^2 \rho_1^2}{2} - eN_i \int_0^{\rho_1} E_0\rho(\rho', z_1)d\rho' - eN_i \int_{z_1}^{z_1} E_0z(\rho_1, z')dz' -$$

$$\int_{\rho_1}^{\rho_1} eH_2 \rho' d\rho' + \int_{z_1}^{z_1} eH_2 \Omega_1 dz',$$

(7)

where $n_e(\rho, z)$ is the concentration of electrons, $n_i(\rho, z)$ is the concentration of ions, $-e$ is the electron charge ($e > 0$), $m$ is the electron mass, $M$ is the ion mass, the electron and ion temperatures $T_e = T_i = T_e$; the quantities $n_{0a}$ can be found from conditions: $\int_{(V)} n_{a} dv' = N_{\alpha} K_{\alpha}$, where $K_{\alpha}$ is the total number of particles of $\alpha$-sort, $N_{\alpha} = 1$ and $K_{\alpha} = N_i K_i + \Delta$, $\Delta$ is the electron surplus.

The electric field can be found from the integral equation

$$E_0(\rho, z) = \int_{(V)} \frac{eR'(\rho', z')q(\rho', z')dv'}{R'^3}$$

Heat movements counteract the compensation of the charge density, the statistical (dynamical) equilibrium is established. In the general case for rotating plasma of complex composition, $j_\varphi \rightarrow \sum_i j_\varphi^i$, $n_{0a} \rightarrow n_{0a}^{(i)}$. In principle, we can introduce the "effective" ion mass $M_i^{eff}$ (in the general case $M_i^{eff} \geq M_i^{av} = (\sum M_i K_i)/(\sum K_i)$) and $N_i^{eff}$. Note, that a small quantity of heavy ions plays an important role. As follows from the derived expressions, the effect is proportional to the particle’s concentration (or, the electron surplus $\Delta$). Since for $\Omega = 0$ and for $\Omega \rightarrow \infty$ the effect vanishes, there exists such a value $\Omega_m$ that $|H_2(\Omega_m)| = max$. Analogously, there exists $T_m$ for which $|H_2(T_m)| = max$. The dependence $H_2(\Omega, T)$ can be represented as a "pit". This is only the discussion of principles of approaches to the magnetic field generation.
problem. Strictly speaking, the model uses the externally given fixed "wall". To use the described model for real objects, the following factors need to be taken into consideration: the gravitational force, if it is important (maintains the system as a whole, for example), or, the expansion of system boundary (the change of integration limits, if the equilibrium distribution has a chance to be set) and the dependence $\Omega(\rho, \varphi, z)$ (nonrigid rotation). Some practical remarks will be given in Section 2.4.

### 2.2 The magnetic fields of rotating stars

The consideration of the starry magnetism differs from the case discussed above both quantitatively ($\Omega$ and $n_{0\alpha}$ may be considerably larger than corresponding quantities for the ionized cloud) and qualitatively. One of such qualitative differences lies in the fact, that the gravitational force must be taken into account (the external boundary doesn’t be fixed, but can be determined in the self-consistent manner):

$$F_g = \gamma \frac{M(r)mr}{r^3},$$

(8)

where $\gamma$ is the gravitational constant, $r$ is the distance from the center of a star, $M(r)$ is the mass of a star part in the $0 \leq r' \leq r$ region of the star (the star is spherically symmetrical).

The gravitational force is directed toward the star center, that is, this force has the $\rho$-projection (toward the axis of rotation). Therefore, this force competes with the centrifugal force (the actions of these forces are opposite) in the processes of separation of charges and generation of electric and magnetic fields. It is clear from the general reasoning, that at the given angular frequency and starry composition the magnetic field reverses its direction at some "critical" mass (or at "critical" $R_{0\alpha}$); or the magnetic field of a certain star reverses its direction at some certain critical angular frequency $\Omega_c$. In the case of gravitational confinement, the (more light) electron cloud is distributed near the boundary (and just near the axis), but the (more heavy) ion layer is distributed near the center (the value $kT$ is the same for all particles and $\approx U_g$).

The potential energy of some particle with mass $m$ is:

$$U_g = \gamma \frac{M(r)m}{r},$$

(9)

where $r = \sqrt{\rho^2 + z^2}$.

In the general case the distribution of charges includes three charged regions. The negatively charged region is placed near the axis of rotation and the system boundary; the positively charged region (torus) is placed near (and far) the radius which can be determined from the condition $F_g = F_c$. The bounds between the charged regions can be found from the equation $n_e(\rho, z) = \sum_i n_i(\rho, z)$. For example, it follows that the equation for plasma consisting of particles with one
number \( N \) is:
\[
n_{0e} \exp \left( \frac{1}{kT} \left[ \int_0^z \frac{eH_p \Omega \rho_1 dz_1}{c} - \int_0^\rho \frac{eH_p \Omega \rho_1 d\rho_1}{c} - e \int E'_0 d\rho' + \frac{m_e \Omega^2 \rho^2}{2} - \frac{\gamma M(r) m_e}{r} \right] \right) = \\
n_{0i} \exp \left( \frac{1}{kT} \left[ \frac{M_i \Omega^2 \rho^2}{2} + Ne \int E'_0 d\rho' + \int_0^\rho \frac{eH_z \Omega \rho_1 d\rho_1}{c} - \int_0^z \frac{eH_p \Omega \rho_1 dz_1}{c} - \frac{\gamma M(r) M_i}{r} \right] \right). \\
\tag{10}
\]

In the general case the components \( E_0(\rho, z) \) can be found from the system of integral equations:
\[
E_{0\rho}(\rho, z) = e \int_{(V)} \frac{q(\rho_1, z_1)}{R'^3} \rho_1 [\rho - \rho_1 \cos(\varphi_1 - \varphi)] d\rho_1 dz_1 d\varphi_1 , \\
\tag{11}
\]
\[
E_{0z}(\rho, z) = e \int_{(V)} \frac{q(\rho_1, z_1)}{R'^3} (z - z_1) \rho_1 d\rho_1 dz_1 d\varphi_1 , \\
\tag{12}
\]
\[
R' = \sqrt{(z - z_1)^2 + \rho^2 + \rho_1^2 - 2\rho \rho_1 \cos(\varphi_1 - \varphi)}
\]
with additional term \( \gamma M(r) M_i / r \) in \( U_i \) and \( \gamma M(r) m_e / r \) in \( U_e \).

The magnetic field can be obtained from the following expressions:
\[
H_\rho = \frac{1}{c} \int_{(V)} \frac{j_\varphi(\rho_1, z_1)}{R'^3} (z - z_1) \rho_1 d\rho_1 dz_1 d\varphi_1 , \\
\tag{13}
\]
\[
H_z = \frac{1}{c} \int_{(V)} \frac{j_\varphi(\rho_1, z_1)}{R'^3} \rho_1 [\rho_1 \cos(\varphi_1 - \varphi) - \rho] d\rho_1 dz_1 d\varphi_1 , \\
\tag{14}
\]

In the case of unsteady rotation (\( \Omega^2 \rho^2 / 2 \to \int_0^\rho \Omega(\rho') \rho' d\rho' \)).

As a first approximation, the effect is proportional to the particle’s concentration (or, the ion surplus \(-\Delta\)). There exists such a value \( T_m \), that \( |H_z(T_m)| = max \). At first, the value \( H_z \) increases (to a maximum \( H_z(\Omega_m^1) > 0 \)) with increasing the value \( \Omega \) to \( \Omega_m^1 \); furthermore, the value \( H_z \) decreases (to a minimum \( H_z(\Omega_m^2) < 0 \)) with increasing the value \( \Omega \), and, finally, the value \( H_z \) tends to zero with further increasing \( \Omega \). We can write approximately
\[
\frac{\partial H_z}{\partial T} = \frac{H_z}{2T} - \frac{\Omega}{2T} \frac{\partial H_z}{\partial \Omega}.
\]

The dependence \( H_z(\Omega, T) \) can be represented as "a hill passing to a pit". Analogously, the dependence \( H_z(M, T) \) can be described as "a pit passing to a hill". However, in the real case the value \( T \) can increase with increasing the body mass \( M \), and the value \( H_z \) can be finite (\( \neq 0 \)) with increasing the value \( M \).
There is another qualitative difference between the cases of star and rotating plasma. The star is a nonequilibrium system. At some point of the star the radiation from and to the center of the star are not compensated, thus causing the light pressure other than zero to exist. The action of this light pressure for particles depends on the effective section of interaction of these particles. Therefore, the light pressure force for ions is larger than that for electrons, and this influence on the charge separation process is opposite to the gravitational force action.

Given the effective sections of interactions $\sigma_e$ and $\sigma_i$ and the light pressure function $P(\rho, z)$, the field $H$ can be obtained in terms of this effect. To do this, the following substitutions need to be done:

$$-\frac{m_e\Omega^2 \rho^2}{2} \rightarrow -\frac{m_e\Omega^2 \rho^2}{2} - \sigma_e \int_{(0,0)}^{(\rho, z)} P(\rho', z')d\rho';$$

$$-\frac{M_i\Omega^2 \rho^2}{2} \rightarrow -\frac{M_i\Omega^2 \rho^2}{2} - \sigma_i \int_{(0,0)}^{(\rho, z)} P(\rho', z')d\rho'.$$

Some important remarks will be made in section 2.4.

2.3 The planetary magnetism

In this case the radiation can be ignored. In some instances the gravitational force can be ignored only when the of charge separation region is small, or, more precisely, its action is reduced to the action of pressure on the boundary of this region. $\Omega$ and $N_i$ quantities for the planets may be larger than those for the other astronomical objects. The distribution of electron (or ion surplus) is determined by characteristics of planetary system (Boltzmann factor depends on the solar mass, chemical composition etc). For some planets with strong magnetic field the radiation belts (the Earth radiation belt, for example, which is the rotating plasma substantially) have influence on the magnetic field. Note, that radiation belts can be considered as a separate system (with their own boundary conditions), for which all derived formulae are applicable.

The case of planets has some qualitative difference from the other cases. The planets are relatively cold objects. Therefore, two remarks are in order. First, not all particles are involved in the processes of charge separation and of the magnetic field appearance. The neutral particles influence on the electric field $E_0(\rho, z)$. To take into account this influence, the following substitution needs to be done in the denominators of equations (11), (12), (13), (14): $R^3 \rightarrow R^3 \varepsilon(\rho, z)$, where $\varepsilon(\rho, z)$ is the permittivity. The polarization of nonmagnetic materials can influence on the magnetic field.

Second, the influence of magnetic materials in the surface layer of a planet on the magnetic field at temperature lower than the Curie temperature must be taken into account:

$$B = \mu H + 4\pi M_0,$$
where $\mathbf{H}$ is the magnetic field from (13) or (14), $\mu$ is the permeability, $M_0$ is the remanent magnetization.

The latter factor can become the only one which maintains the existence of the magnetic field as the axial rotation of a planet is decelerated. The qualitative picture, described above in previous subsection, is valid for the charge distribution and the dependence of magnetic field on different factors. Note, that (an agreement with this paper) there exists the electron surplus near the Earth’s surface (see The Physical Values, 1991). The most significant remarks with practical viewpoint is made in the next subsection.

## 2.4 Some remarks and estimations

First of all we note that the Kauling theorem does not valid for models described. The electric field possesses the $E_{0\varphi}$ and $E_{0z}$ components only ($E_{0\varphi} = 0$). However, the nature of currents does not determined by the magnetic force $\frac{1}{c}[\mathbf{v} \times \mathbf{B}]$. The existence of currents is the consequence of the system rotation: portions of charged particles (which form charged regions) are involved in the rotation. In spite of particles movement (rotation) with the same $\Omega$ (without friction), currents can have the mutually reverse directions. There is a surface with $j = 0$; the field $\mathbf{B}$ can be equal zero on some surface; $\text{rot} \mathbf{B}$ can also be equal zero. Thus, the principal conclusion consists in the fact, that the magnetic field can origin in a system with cylindrical symmetry.

Second remark needs to be made in connection with conditions of application of models. It is easy to make an estimate of the field produced by the effect described for $T = 0$. The magnetic field (in Gaussian units) is $H \sim RJ/c$, here $R$ is the system radius, $J \sim \rho v$. If the gravitational force is most important, then $eE \sim M_ig', \rho \sim M_ig'/(eR)$ here $g' \sim j\rho_bR$, where $j$ is the gravitational constant, $\rho_b$ is the body density. Substituting typical numbers ($M_i \sim 10^{-23}$ gram, $v/c \sim 10^{-4}$, $e \sim 10^{-10}$ esu, $\rho_b \sim 10^{-3}$ gram/cm$^3$), we have $H \sim 10^{-4}$ Gauss for $R \sim 10^9$ a.u. However, the model is statistical (principally): statistical (dynamical) equilibrium holds by heat movements, and the originated electric force cannot compensate the charge density (for example, the noncompensated force $F_g$ exists, but the earth atmosphere doesn’t ”fall” on the Earth). The estimations needs to be corrected for stellar and planetary magnetism. To do this, two ways exist. 1) The above estimations suppose that the magnetic field is produced by a charge surplus. Note, that if for an earthlike body $H \sim 1 - 10^{-4}$ Gauss, then the surplus charge density $\rho$ needs to be equal $\rho \sim 10^{-4} - 10^{-8}$ esu/cm$^3$, that is the number of charged particles is $10^2 - 10^6$ per cm$^3$. It is possible for the earth phenomena (see The Physical Values, 1991). 2) The ”elucidation” of the gravitational force for system neutral as a whole does not meant that the body will scatter under influence of electric force. These forces do not compensate. Besides, any liquid or rigid body can possess rather great portion of charges without interruption. The ”capacitor” (described above the double layer) discharges. To account the latter effect, one can use some potential (barrier maintaining a ”contact potential
difference”) which counteracts the discharge process. Note as a remark, that Earth structure (in chemical and material composition) can be determined by self-consistent manner (using the chemical potential \( \mu \)). The presence of charged particles can be described as a ”dilute solution”, and the equilibrium condition for different phases is: \( \mu + U = \text{const} \).

The following remark concerns the role of electromagnetic forces for Earth phenomena. As a consequence of long-range (rather strong) electric interaction, some small charge surplus is sufficient for comparability the electric energy with the motion energy in atmospheric (different winds) and hydrodynamical (sea currents) phenomena. To account this, one must additionally write: 1) the motion equation for charged particles, and, 2) the equation for the interaction of charged particles and neutral gas (or liquid) masses. Cosmic rays influence on the earth activity (earthquake, vulcan activity), since the presence of magnetic materials (hysteresis) leads to heat production in inner earth regions.

3 Magnetic fields and some factors

3.1 Self-sustained equilibrium rotation

It is shown above, that in a system possessing some fraction of free charges a partial separation of charges occurs under an effect of centrifugal and gravitational forces \( n_e(\rho, z) \) does not coincide with \( n_i(\rho, z) \) and, as a result, the electric field \( E(\rho, z) \) arises. Due to system’s rotation the magnetic effect of currents of these charged regions is not mutually compensated, and the magnetic field \( H \) arises. In this case the rotation frequency \( \Omega(\rho, z) \) was assumed to be specified.

The statement of the problem may be changed. For this purpose we notice that the field \( H \) is directed along the axis of rotation (axis \( z \)), and the field \( E \) has two components \( E_\rho \) and \( E_z \) only, i.e. it has a component directed towards the system’s axis of rotation. As a result, the drift of particles will take place in cross fields \( E_{0\rho}e_\rho \) and \( He_z \). This drift motion will occur over circles around the axis of rotation. Hence, it is worthwhile to find such a rotation rate, that resulting electric and magnetic fields lead to a drift rotation with the same rate, i.e. one should find stable rotational states of a charged particles in a self-consistent manner. Note, that the (averaged) movement of charged particles must be close to the rotation of the system as a whole (a consequence of the system stability).

From condition \( v_d = \Omega \rho \) and the expression for drift velocity

\[ v_d = \frac{e_\rho cE_{0\rho}(\rho, z)}{H(\rho, z)} \]

the unknown rate is determined as

\[ \Omega = \frac{cE_{0\rho}}{\rho[H_0(\rho, z) + H_1]}, \quad (15) \]
where \( \rho \) is the distance to the axis, \( c \) is the light velocity, \( H_1 \) is an outer or intrinsic magnetic field \( (H_1 > H_0) \), and quantities \( E_0\rho \) and \( H_0 \) can be taken from previous section.

### 3.2 Dynamics of magnetic fields

Let us imagine that the external magnetic field \( H_1 \) suddenly increases: \( H_1 \to H_1 + H_1' \). As a result, as follows from (15), the equilibrium frequency of rotation of the object (or a fraction of charged particles) decreases in this case. This decrease of \( \Omega(\rho, z) \) results in decreasing the field \( H_0 \to H_0 - H_0' \) and, hence, the resulting field slightly lowers: \( H = H_0 + H_1 + H_1' - H_0' \). This is a consequence of the general Le Chatelier principle. Similarly, the weakening of the external magnetic field \( H_1 \) leads to increasing the drift rotation rate, which, in its turn, causes the increase of the field \( H_0 \), i.e. the processes weakening the external action occur again (the system tends to conserve its equilibrium state).

Note that at equilibrium either \( \Omega = const \), or \( \Omega = \Omega(\rho, z) \):

\[
\frac{\Omega^2 \rho^2}{2} \to \int_0^\rho \Omega'(\rho', z)\rho' d\rho',
\]

(16)

for charged particles and the source of energy and the mechanism of compensation of heat losses (for friction) must exist. If the state of a system differs from equilibrium one by any reason (for example, the external forces are acting, or some processes occur inside the object, which lead to redistribution of charges), then, in principle, one can find the forces, tending to bring the system into equilibrium, and describe the dynamics (of fields, of rotation rate).

It follows from (15), that as the rotation frequency of an astronomical object changes adiabatically, the electric and magnetic fields also change:

\[
\dot{\Omega} = \frac{c}{\rho} \frac{\dot{E}_0\rho}{H} - \frac{E_0\rho H'}{H^2},
\]

As the first approximation, using

\[
\dot{E}_{0\rho} = \frac{\partial E_{0\rho}}{\partial \Omega} \dot{\Omega} + \frac{\partial E_{0\rho}}{\partial \Delta} \dot{\Delta} + \frac{\partial E_{0\rho}}{\partial T} \dot{T}, \quad \frac{\partial E_{0\rho}}{\partial \Omega} = -2T \frac{\partial E_{0\rho}}{\partial T},
\]

we have

\[
\dot{H} = H \left( \frac{\dot{\Delta}}{\Delta} - \frac{\dot{\Omega}}{\Omega} \right) + \frac{H}{E_{0\rho}} \frac{\partial E_{0\rho}}{\partial T} \left( \dot{T} - 2T \frac{\dot{\Omega}}{\Omega} \right).
\]

(17)

The value \( \frac{\partial E_{0\rho}}{\partial T} \) must be evaluated from the self-consistent system of equations (from Sec.2). Since \( H = H_0 + H_1 \), the hysteresis influences on the field dynamics. The change of the magnetic field polarity may occur (without changing the direction of astronomical object’s rotation), when the direction of the electric field \( E_{0\rho} \) inside the system changes. The change of polarity for a system with magnetic
materials (there exists some threshold due to hysteresis) requires long-lasting influence (cosmic rays can change the total charge of the system, for example).

In principle, one can obtain the solution to the problem of fields dynamics, in the case of "sufficiently slow" variation of various external and internal factors. It is apparent that the dynamics of $T, \rho, (V)$ and $\Omega$ determines the dynamics of electric and magnetic fields by formulae (15) - (17). Therefore, to finally determine the fields dynamics and the rotation rate, one should also add the following equations:

1. the moment of external forces acting on a system determines the additional change of rotation frequency $\Omega$ (if the object does not possess central symmetry); this factor may cause the magnetic field precession;
2. the law of variation of object’s form $(V)$ (this factor is insignificant for stars and planets);
3. the law of variation of density $\rho(R, t)$;
4. the law determining the temperature variation $T(R, t)$;
5. the law of (external) change of the total charge (this factor is the most important one).

Now it is clear that the characteristic times of variation of all these quantities must be higher than characteristic times of diffusion for a fraction of charged particles participating in the fields generation. In the opposite case one must take into account diffusion terms in equations for $\rho, T, \Omega$ and $(V)$ (i.e. the equation of diffusion should be written separately for a fraction of charged particles). In this case the hysteresis phenomena are characteristic of the fields variation.

How the object driven from the state of equilibrium will behave? From general considerations, the transition into a new equilibrium state (or into in old one, if the factor acts for a limited time) may occur either asymptotically, or in an oscillational manner (with polarity change, in particular), depending on the fact, whether the characteristic time of system oscillation $(\Omega^{-1})$ is higher (or, respectively, considerably lower) than that of system’s energy dissipation.

In the case, if the rotation frequency inside an astronomical object is lower than the equilibrium frequency possible for the given system, then the new equilibrium state will correspond to the absence of rotation. Because of the dependence $\Omega(\rho, z) \neq \text{const}$ (caused by the fields) the friction will arise between different sections of the object and, as a result of energy dissipation, the rotation of an astronomical object will be slowed down. This is just an internal mechanism of slowing down the rotation of astronomical objects (and warming-up of inner regions).

## 4 Conclusions

The general statistical approach to the magnetism problem is introduced. It can help to obtain the value of the magnetic field to sufficient accuracy (from the exact integral solution). The mechanism of the magnetic field generation in
astronomical objects (nebulae, stars, planets) can be explained in the following way: in the general case the centrifugal force, the gravitational force and the light pressure act on dissimilar particles in a different manner, causing different $\rho$-distributions of these particles and giving rise to the partial separation of charges; the electric field can be found from formulae (11), (12) (see the substitutions in the text); the rotation of these objects produces (from these charges) circulating currents, the magnetic action of which is not compensated, and the magnetic field can be found from formulae (13), (14) (see the substitutions in the text). Generally speaking, the existence of the magnetic field dispenses with the need for conductive regions; the availability of some portion of free charges would be sufficient for this mechanism.

The equilibrium frequency of rotation of an astronomical object (see (15)) may be found in a self-consistent manner. Knowing the factors, which influence the density, temperature, rotation of an object and external fields, one can determine in this case the dynamics of system’s magnetic field.

The further applications of formulae obtained might be associated with numerical methods or with some simplifications which take into account some specific data.

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