Unitarity effects in DIS

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Abstract

We argue that diffractive DIS, dominated by soft interactions, is probably the unique process which allows us to observe unitarity effects in DIS. Guided by a close analogy between the diffractive dissociation of a highly virtual photon and the elastic scattering of hadrons we propose a specific procedure to analyse the data in order to detect the onset of the unitarity limit. Lacking appropriate data, we use the predictions of a realistic model as an input for our analysis, to demonstrate that the output unitarity signal is sufficiently large to be detectable.
Introduction

Diffractive phenomena, or large-rapidity-gap events in deep-inelastic scattering (DIS) have become one of the central issues of HERA physics. Diffraction, according to its optical analogy, is usually associated with shadowing and is viewed as a domain of soft hadronic interactions. Therefore, one could naively expect that diffraction in DIS vanishes at high \(Q^2\); this is not the case according to recent experimental HERA data [1, 2]. We begin by commenting on the similarities and differences between diffraction in hadronic interactions and in DIS. Since manifestations of saturation of the unitarity limit have already been found long ago in elastic hadronic scattering, one may hope to detect similar effects in diffractive DIS.

The main objective of this paper is to clarify if and how one could observe unitarity effects in DIS. In particular, can the data reveal the onset of unitarity or whether the unitarity limit has already been reached in the diffractive cross section at HERA energies? In DIS, a substitute for the elastic cross section has to be found which enables one to perform an analysis analogous to that of hadronic reactions. We give a specific prescription of how to analyse the diffractive DIS data in order to identify unitarity effects. Modelling the diffraction cross section within a realistic model we demonstrate that the expected signal of unitarity limit is nearly as large as in hadronic elastic scattering.

Diffraction in DIS

DIS is traditionally viewed as a hard probe of the parton distribution in hadrons. However, looking at it in the proton rest frame, which is better suited to discuss diffraction in DIS, one easily realises that the interaction of a virtual photon has a substantial soft contribution even at high \(Q^2\). This argument was first put forward by Bjorken and Kogut [3] with phase space motivations. It was further demonstrated within perturbative QCD [4] that the soft contribution to the total DIS cross section scales with \(Q^2\) and is not a higher twist effect. The reason why the soft component does not vanish at high \(Q^2\) can be made plausible in the following way [5]. Soft hadronic fluctuations of a highly virtual photon are expected to be rare; they are suppressed by a factor \(\sim 1/Q^2\) (for transversely polarized pho-
tons \[4\]) as compared to hard fluctuations. But soft fluctuations have a large cross section, of the order of a typical hadronic cross section. On the other hand, hard fluctuations, due to color screening, have a small size and, therefore, a tiny interaction cross section \(\sim 1/Q^2\). The net result is that the two contributions to the DIS cross section, soft and hard, end up having the same leading twist behaviour. This conclusion is supported by the experimental observation of a rather weak \(Q^2\)-dependence in nuclear shadowing \[5\].

In the case of diffractive scattering, the equilibrium between soft and hard components breaks down. The diffractive cross section is still proportional to the same probability of finding the appropriate fluctuation (hard or soft) within the photon; it is, however, not proportional to the total cross section, but to its square (see \[6\]). As a consequence, hard fluctuations get an extra factor \(1/Q^2\) and become a higher twist effect in diffraction while, on the contrary, soft fluctuations in diffractive DIS remain leading twist.

These considerations are, obviously, very qualitative. The distinction between soft and hard components is quite conventional; in addition, there is an intermediate region of semi-hard fluctuations (whose size is \(Q^2\)-independent although quite small). The question then is whether the above conclusions are confirmed by the data. The answer is yes, and comes from nuclear shadowing in DIS which is closely related to diffraction. The same size, which dominates the photon fluctuations in the diffraction dissociation on a proton, is also responsible for the first order nuclear corrections. The size of nuclear shadowing observed in heavy nuclei at low \(x\) \[7\] requires a cross section of about 12 \(mb\), which corresponds to a \(q\bar{q}\) separation of nearly \(0.6 \div 0.8 \text{ fm}\); this proves that, indeed, soft diffraction is the dominant contribution to diffractive DIS.

From the H1 \[1\] and ZEUS \[2\] experiments at HERA, we know that the fraction of photon diffraction in DIS cross section is about 10 %. At first sight one may think that this is the analogue of what one finds in hadronic interactions, where about the same fraction of total cross section comes from single diffraction. But this is not so. In the virtual photon interaction, diffraction corresponds to both, the elastic and the inelastic diffractive scattering of the fluctuations. For instance, in the photon interaction, the diffractive cross section on the black disk reaches 50 % of the total cross section \[8\]. In hadronic collisions,
the inelastic diffraction would vanish, while the elastic cross section becomes 50 % of the
total cross section. For a meaningful comparison with hadronic interactions we have to look
at the fraction of the total cross section taken by elastic and single inelastic diffraction i.e.,
about 25 % in πp and pp interactions; this is substantially larger than in diffractive DIS.
The reason is clear, while the contribution of hard interaction to diffraction vanishes, it still
provides more than half of the total DIS cross section.

After realising that the diffraction dissociation of a highly virtual photon is predom-
inantly a soft process and that the elastic scattering of the photon fluctuations play an
important role, one may expect that unitarity effects in diffractive DIS could be about
as important as in hadronic reactions. Before addressing the question of the unitarity in
DIS, let us briefly recall some relevant points concerning how the same question has been
approached in soft hadronic physics.

**Unitarity in hadronic reactions**

It is known that a growth of total cross sections as a power of energy would ultimately
lead to a violation of unitarity; the latter, in fact, restricts the rate of growth of total cross
sections to \( \ln^2(1/x) \) according to Froissart’s theorem. In hadronic interactions, the data
are commonly reproduced either using unitarity non violating logarithmic forms or a power
behaviour whose growth is so gentle (\( \sim (s/s_0)^{0.1} \)) that a noticeable violation of the Froissart-
Martin bound is expected only at energies far beyond the range of present accelerators [9].
By contrast, a much steeper growth with energy of the cross section for interaction of highly
virtual photons was discovered at HERA, \( \sigma^{γ∗N}_{tot} \propto (1/x)^{Δ} \), where the power \( \Delta \) reaches much
larger values, 0.3 ÷ 0.4 at high \( Q^2 \). This fact led to the widespread expectation of an earlier
onset of unitarity in DIS as compared with soft hadronic interactions, which is supposed
to show up as a slow down of the observed growth of \( \sigma^{γ∗N}_{tot} \) at large \( 1/x \). It is, however,
not obvious how we could claim to observe such a slow down, since we do not know any
reliable baseline to search for a deviation from, i.e. we do not have any rigorous theoretical
prediction for this \( x \)-dependence to compare with. The power behaviour \( (1/x)^{Δ} \) would
correspond to the Pomeron if it were a Regge-pole, but it is not, since \( Δ \) grows with \( Q^2 \).
violating factorization. Thus, this behaviour has to be regarded merely as a convenient parameterization not dictated by any rigorous theory especially in the HERA energy range. For instance, the double-leading-log approximation \cite{10} predicts that the effective power $\Delta(x) \to 0$ in the limit $x \to 0$.

In addition, locally, unitarity sets restrictions only on the magnitude of the imaginary part of the partial elastic amplitude, not on its variation with energy. No detectable unitarity effect can be unambiguously ascertained if the amplitude is much smaller than one (like in $\gamma^* p$ interaction) irrespective of its actual energy-dependence. The observed steep growth of $\sigma^{\gamma^* N}_{tot}$ at low $x$ is supposed to originate from the dominant photon fluctuations, which have a size $\sim 1/Q^2$ \cite{10}. As already mentioned, however, the cross section for these fluctuations is tiny ($\sim 1/Q^2$) and cannot cause any problem with unitarity, independently of its energy dependence. On the other hand, the soft component of DIS corresponding to large size hadronic fluctuations may need some unitarity corrections, even though it is supposed to have about the same weak energy dependence as in hadronic interactions. We believe that it is hopeless to observe those corrections in the total DIS cross section. Even in hadronic interaction, $\sigma^{pp}_{tot}$ shows no deviation from an effective power energy dependence up to the highest available energies \cite{9}. Unitarity corrections were nevertheless detected and shown to be important in hadronic interactions in the analysis of elastic scattering data over the ISR energy range \cite{11}.

Let us write the hadronic (say $pp$ for definiteness) differential cross section as $\frac{d\sigma}{dp_T^2} = |R(s, p_T^2) + iA(s, p_T^2)|^2$ where $R(s, p_T^2)$ and $A(s, p_T^2)$ are the real and the imaginary parts of the amplitude and $p_T$ is the proton transverse momentum. At high energies we know the real part to contribute a relatively small (say $\leq 1\%$) fraction of the differential elastic cross section at small $p_T^2 \leq 0.5 GeV^2$, where about 99$\%$ of the events are concentrated. Neglecting $R(s, p_T^2)$ one can simply invert the above equation to determine $A(s, p_T^2) \approx \sqrt{d\sigma/dp_T^2}$ and apply a Fourier transform to obtain the approximate partial wave amplitude $F(s, b)$ in the impact-parameter representation, for which the unitarity condition reads

$$0 \leq G_{in}(s, b) = 2ImF(s, b) - |F(s, b)|^2 \leq 1$$
(our notation differs from that of [11] by a factor of $i$ so that what is their Real part becomes our Imaginary one and vice versa). In the literature, $F(s, b)$ and $G_{in}(s, b)$ are also known as the profile function and the inelastic overlap function respectively. As a consequence of the above condition, $0 \leq \text{Im} F(s, b) \leq 1$.

As found in [11] for the ISR $pp$ and $\bar{p}p$ data the normalization of $G_{in}$ at $b = 0$ is so close to unity as to essentially saturate unitarity. Therefore, an extrapolation to higher energies of the supercritical Pomeron form $(s/s_0)^\Delta$ would violate unitarity at $b = 0$ already at the $SpS\bar{p}$ energies.

The important result of [11], however, is that if one takes the difference $\Delta G_{in}(s, b)$ (evaluated from the data) of $G_{in}(s, b)$ calculated at the lowest (23 GeV) and at the highest (63 GeV) ISR energies, it is almost entirely concentrated on the periphery at $b \sim 1\text{ fm}$ (see Fig.8 of [11]). This is the crucial observation which we propose as a guide on how to analyze the DIS diffraction data so as to make evident the effects of unitarity.

Procedure of Analysis

The procedure we suggest in order to detect possible unitarity effects from the data on diffractive DIS is essentially the same as we have just described for hadronic reactions. We treat the various channels of diffractive dissociation of a virtual photon as elastic scattering of hadronic fluctuations of the photon in its eigenstate basis (e.g. $|q\bar{q}\rangle$, $|q\bar{q}g\rangle$, etc. with definite transverse separations).

Given the diffractive dissociation cross section $d\sigma_{dd}^{\gamma^*p}(x, Q^2)/dp_T^2\ dM^2$, we sum over all events with different $Q^2$s in order to increase the statistics. This is allowed because there is no substantial $Q^2$ dependence in the diffractive cross section (as expected since diffraction is dominated by soft interactions). Then, we sum over the final states by integrating over $M^2$.

$$
\frac{d\sigma_{dd}^{\gamma^*p}(x)}{dp_T^2}(x) = \frac{M_{\text{max}}^2}{M_{\text{min}}^2} \int_{M_{\text{min}}^2}^{M_{\text{max}}^2} \frac{d\sigma_{dd}^{\gamma^*p}}{dp_T^2\ dM^2} dM^2
$$

One should confine oneself to the mass interval where a $q\bar{q}$ component of the photon dominates, because it is the softest one, and one should cut off the low-mass resonance produc-
tion, which is a hard process. As a suggestion, one can use $M_{\text{min}}^2 \approx 5 \text{ GeV}^2$. The region $M^2 \gg Q^2$ is known to be dominated by the triple-Pomeron term, which corresponds to the higher Fock components in the photon, $|q\bar{q}g\rangle$, $|q\bar{q}2g\rangle$, etc. These fluctuations are expected to have a smaller size because we know from lattice calculations the shortness of the gluon correlation radius ($\sim 0.2 \div 0.3 \text{ fm}$). To suppress this contribution one should restrict the mass interval to $M^2 \sim Q^2$; typically, one can take $M_{\text{max}}^2$ equal to a few units of $Q^2$. The exact value does not seem so important because the high mass tail $d\sigma_{dd}/dM^2 \propto 1/M^2$ does not contribute much. Note that these higher Fock components, or the triple-Pomeron contribution to diffraction is just another aspect of the gluon-gluon fusion mechanism which is a widely recognised manifestation of unitarity.

Next, we take the square-root of the differential cross section (1) and perform a Fourier transformation to the impact-parameter representation

$$F(b, x) = \int d^2 p_T e^{ip_T b} \sqrt{\frac{d\sigma_{dd}^p}{dp_T^2}}$$

Contrary to the hadronic case, we cannot set any unitarity restriction on the value of $F(b, x)$, since it includes factors (the fine structure constant, etc.), which cannot be given in a model-independent way. For this reason we do not care about the normalization of $F(b, x)$. The way we suggest to observe unitarity effects is by studying the $x$-dependence of $F(b, x)$. In strict analogy with the hadronic case, we expect only a slow growth of $F(b, x)$ with $1/x$ at small $b$ and the main increase at large $b$. To check this conjecture, we define the slope of the $x$-dependence in the normalization-free way

$$\Delta_{\text{eff}}(b) = \frac{d\ln[F(b, x)]}{d\ln(1/x)}$$

In the case of elastic hadron scattering $\Delta_{\text{eff}}(b)$ is $b$-independent, the unitarity corrections are neglected, and is related to the Pomeron intercept, $\Delta_{\text{eff}} = \alpha_P(0) - 1$. Since any unitarity correction is expected to slow down the rate of growth of $F(b, x)$ with $1/x$ for central collisions, $\Delta_{\text{eff}}(b)$ should be smaller at small $b$ than at large $b$. This is the signature of unitarity effects we expect.
Theoretical experiment

Given that for the time being we are not aware of any data that allow us to perform this analysis (these data may, however, well have already been collected), the rest of this paper will be devoted to check our prediction of unitarity effects with a model calculation. It is not only a problem of statistics (which such an analysis needs) which one may have to worry about, but also of other complications which are absent in elastic hadronic scattering and which could well wash away the signal in the diffractive dissociation of a highly virtual photon. Firstly, there is a background of hard and semi-hard photon fluctuations, from which we do not expect any unitarity effect. In addition, it is not obvious how the interplay between many channels in the diffractive interaction of the photon’s fluctuations may affect unitarity.

To check the possibility of observing an onset of unitarity we suggest a theoretical experiment, i.e. we calculate the differential cross section \(d\sigma_{\gamma^*p}^{\gamma^*p}/dp_T^2\) in a crude but realistic model based on pQCD, and then we use the result as an input for our analysis as if it were experimental data.

In the light-cone formalism the photon diffractive dissociation cross section due to \(q\bar{q}\) fluctuations can be written as [4],

\[
\frac{d\sigma_{\gamma^*p}^{\gamma^*p}(x)}{dp_T^2} = \int_{0}^{1} d\alpha W_{\gamma^*}(Q^2, \alpha, \rho) \int d\rho^2 \frac{d\sigma_{el}^{(q\bar{q})p}(\rho, x)}{dp_T^2} \approx K \int_{1/Q^2}^{1/\mu^2} \frac{d\rho^2}{\rho^4} \frac{d\sigma_{el}^{(q\bar{q})p}(\rho, x)}{dp_T^2} \tag{4}
\]

Here \(\rho\) is the transverse separation in the \(q\bar{q}\) fluctuation, \(\alpha\) is the fraction of the photon light-cone momentum carried by the quark and \(W_{\gamma^*}(Q^2, \alpha, \rho)\) is the probability of such a \(q\bar{q}\) fluctuation in the photon. The integration over \(\alpha\) has been performed using the fact [12] that the function \(\rho^4 \int_{0}^{1} d\alpha W_{\gamma^*}(Q^2, \alpha, \rho)\) is approximately constant in the interval \(1/Q^2 < \rho^2 < 1/\mu^2\), where \(\mu\) is a hadronic scale mass parameter, of the order of \(\Lambda_{QCD}\). \(K\) is a normalization factor, irrelevant for further considerations.

The elastic differential cross section \(d\sigma_{el}^{(q\bar{q})p}(\rho, x)/dp_T^2\) for the \(q\bar{q}\) pair with separation \(\rho\) can be estimated using a factorized dipole partial-wave amplitude (a bare Pomeron to be
further unitarized), of the form,

\[ f^{(q\bar{q})p}(b, \rho, x) = \frac{i}{2} \sigma(\rho) \left( \frac{1}{x} \right)^{\Delta_0} \exp \left[ -\frac{b^2}{2B(\rho, x)} \right] , \tag{5} \]

(where we neglect the small real part) which depends on \( b \) (the relative impact parameter between the center of gravity of the \( q\bar{q} \) pair and the proton), on the transverse separation \( \rho \) inside the \( q\bar{q} \) pair and on \( x \). The universal dipole cross section \( \sigma(\rho) \) [6], which vanishes \( \propto \rho^2 \) at small \( \rho \), was estimated in [13]. The exponent \( \Delta_0 \approx 0.1 \), has a value typical for soft hadronic interactions, due to the fact that diffraction is soft dominated (this is confirmed by the analyses [1, 2] of the experimental data). The slope parameter \( B(\rho, x) \) is a smooth function of \( \rho \) and \( x \). Its actual value is not so important for detecting unitarity effects; for simplicity, we assume it to be a constant, \( B(\rho, x) \approx 5 \, GeV^{-2} \) [2].

The central point of this procedure is unitarization of the input partial amplitude (the bare Pomeron) [3]. It may exceed unity and violate unitarity at small \( x \) (mainly at large \( \rho \) and small \( b \)). We use the popular eikonal form for the unitarized amplitude [14],

\[ Im \, F^{(q\bar{q})p}(b, \rho, x) = 1 - \exp \left[ -Im \, f^{(q\bar{q})p}(b, \rho, x) \right] , \]

which is exact in our case since a \( q\bar{q} \) pair with a fixed separation \( \rho \) is an eigenstate of the interaction, i.e. no off diagonal intermediate states are possible. A nice property of the eikonalized amplitude is that it guarantees that unitarity is not violated, \( Im \, F(b) \leq 1 \). Going to lower and lower \( x \) values, we expect it first to stop growing at small \( b \), then this effect spreads to larger \( b \)'s [14]. This is exactly the signature of unitarity limit we are proposing to observe.

Once we have the partial wave amplitude, we can Fourier transform it to calculate the differential elastic cross section

\[ \frac{d\sigma_{el}^{(q\bar{q})p}(\rho, x)}{dp_T^2} = \frac{1}{4\pi} \left| \int d^2b \, e^{i\vec{p}_T\vec{b}} \, F^{(q\bar{q})p}(b, \rho, x) \right|^2 \]

This differential cross section explicitly obeys the unitarity restrictions. However, it should be averaged over \( \rho \) using (4) to produce an observable diffractive dissociation cross section, summed over final state. Doing this we arrive at the goal of these calculation, the differential cross section of diffractive dissociation which can be used until the data become available as an input to eqs. (2)-(3). The result for the \( b \)-dependent \( \Delta_{eff}(b) \) is shown by the solid
curve in Fig. 1. It indeed demonstrates what we had anticipated, a suppressed value of $\Delta_{\text{eff}}$, \textit{i.e.} a substantially less steep $x$-dependence, at small $b$. At $b > 1 \text{ fm}$ the result of the Fourier transform is extremely sensitive to the details of the model, which can even cause oscillations of the partial amplitude. However the amplitude itself is tiny at these impact parameters, and we do not expect experimental data to provide a reasonable accuracy in this region. Notice that ZEUS and H1 spectrometers are blind at $p_T^2 < 0.1 \text{ GeV}^2$ \cite{??}, however, we checked that this may affect $\Delta_{\text{eff}}$ only at $b > 1 \text{ fm}$. The signal of unitarity limit predicted at $b < 1 \text{ fm}$ is unambiguous.

![Predicted $b$-dependence of the effective exponent $\Delta_{\text{eff}}(b)$ defined in (3) for diffractive DIS (solid curve) and for elastic pion-nucleon scattering (dashed curve). The dotted line $\Delta_{\text{eff}}(b) = \Delta_0$ corresponds to the same analyses, but without unitarity corrections.](image)

Figure 1: Predicted $b$-dependence of the effective exponent $\Delta_{\text{eff}}(b)$ defined in (3) for diffractive DIS (solid curve) and for elastic pion-nucleon scattering (dashed curve). The dotted line $\Delta_{\text{eff}}(b) = \Delta_0$ corresponds to the same analyses, but without unitarity corrections.

Notice that if the non unitarized amplitude (3) is used as an input for the analysis, the resulting exponent $\Delta_{\text{eff}}(b)$ is $b$-independent and is equal to $\Delta_0$.

To obtain the feeling of how the suppression of $\Delta_{\text{eff}}(b)$ at small $b$ in diffractive DIS looks compared with that in hadronic interaction we have performed the same analysis for
pion-proton elastic scattering. The photon wave function is replaced with the pion one which we model by a Gaussian with the appropriate charge radius and with a $\delta(\alpha - 1/2)$ distribution over $\alpha$. We use the same universal $\rho$-dependent amplitude (5), but the larger slope parameter $B \approx 8 \text{ GeV}^{-2}$, corresponding to the experimentally observed slope in the pion-proton elastic scattering. The result for $\Delta_{eff}(b)$ is shown by the dashed curve in Fig. 1. Amazingly, the indication is that the unitarity limit effects in diffractive dissociation of a highly virtual photon are not much smaller than in pion-proton elastic scattering, in spite of the many corrections which could have diminished the effect.

*Summarizing*, we suggest that effects of unitarity in DIS can indeed be detected in the diffractive DIS data. We believe that adapting the same procedure of analysis which has been proved effective in elastic hadron-hadron scattering [11] is probably the unique way to pin down unitarity effects in diffractive data and we have given a detailed recipe on how to proceed in this analysis. Lacking appropriate data, we have demonstrated that the procedure works quite effectively within a specific but, we believe, realistic model.

An experimental verification of the impact parameter dependence of the diffractive cross section would be a clean manifestation of the dominance of the soft interaction in these processes.

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