A Note on GSO-Free RNS Superstrings and Pure Spinor Constraint

Dimitri Polyakov\(^{(1),(2)}\)

\textit{Center for Theoretical Physics} \(^{(1)}\)
\textit{College of Physical Science and Technology}
\textit{Sichuan University, Chengdu 610064, China}

\textit{Institute for Information Transmission Problems (IITP)}\(^{(2)}\)
\textit{Bolshoi Karetny per. 19/1}
\textit{127994 Moscow, Russia}

Abstract

Using an elementary perturbative open string field theory solution involving a twistor-like parameter, we study the cohomology of new nilpotent BRST charge corresponding to the space-time background defined by this solution. The BRST cohomology of the deformed background automatically cuts off the GSO-odd spectrum in RNS superstring theory and keeps the GSO-even spectrum intact, without a need of GSO-projection. The on-shell constraints in the GSO-even sector get deformed in the new background, corresponding to BRST type transformations of the low-energy effective action with the ghost-like commuting spinor parameter satisfying the pure spinor constraint in \(d = 10\).

August 2015

\(^{(1),(2)}\)\ polyakov@sogang.ac.kr ; polyakov@scu.edu.cn; twistorstring@gmail.com
1. Introduction

One well-known property of the Ramond-Neveu-Schwarz superstring theory [1], [2] is the need to implement the GSO-projection [3], in order to eliminate the tachyon from the spectrum (along with other states which, while being in the BRST cohomology, behave in an unnatural way in terms of the spin-statistics properties). This, in a sense, raises a problem because the physical states of superstring theory are most naturally defined in terms of the BRST cohomology (incorporating all the gauge-theoretic properties of the model), while the GSO-projection is essentially an artificial operation, implemented by hands in order to exclude the unwanted superstring excitation modes. Moreover, this operation breaks the BRST cohomology of the RNS model into two subsectors, GSO even and GSO odd which are not completely independent or separable. Indeed, the on-shell operator product of two arbitrary GSO-even vertex operators doesn’t contain GSO-odd operators, making the GSO-even part formally “insulate”. The inverse, however, is generally not true, and this is why in RNS superstring theory, even with the GSO-projection implemented, the Veneziano amplitudes of all even states would generally get contributions from the odd sector, such as tachyonic poles.

On the other hand, the presence of the GSO-odd states in the BRST cohomology and in the amplitudes is the property of flat space-time background which does not necessarily extend to other target space backgrounds with different cohomologies. One particularly convenient and efficient tools to explore the BRST cohomologies in various backgrounds is string field theory with the equations of motion [4], [5]:

\[ Q\Psi + \Psi \star \Psi = 0 \]  

which is background-independent by construction. That is, assume \( \Psi_0 \) is a solution of the equation (1). Then the form of (1) is invariant under the shift

\[ \Psi \rightarrow \tilde{\Psi} = \Psi + \Psi_0 \]  

with the simultaneous shift of the BRST charge \( Q \rightarrow \tilde{Q} \), so that \( Q^2 = \tilde{Q}^2 = 0 \) and the new nilpotent charge \( \tilde{Q} \) defined according to

\[ \tilde{Q}\Psi = Q\Psi + \Psi_0 \star \Psi + \Psi \star \Psi_0 \]  

for any \( \Psi \). Then the new BRST charge \( \tilde{Q} \) defines the new cohomology, different from that of the original charge \( Q \), corresponding to string theory in a new background, depending on
the structure of $\Psi_0$. The advantage of this approach is that, in principle, it allows to explore the string theory in new geometrical backgrounds (e.g. in a curved geometry, such as AdS) while technically using the operator products of the old string theory (say, in originally flat background) for the vertex operators in the new BRST cohomology, defined by $\hat{Q}$. In this letter we consider an example of an elementary open SFT solution describing simple perturbative background deformation and eliminating the odd sector from the cohomology. The deformation is parametrized by a commuting spinor parameter which has to satisfy the pure spinor constraints in order to ensure the nilpotence of the BRST charge. The deformation of the cohomology results in eliminating the GSO-odd operators from the spectrum and in some modifications of the on-shell constraints for the space-time fields multiplied by the appropriate vertex operators in the GSO-even sector. In terms of the low-energy effective action in space-time, this particularly corresponds to deforming the space-time fermions by the ghost-type bosonic pure spinor variables (BRST-type transformation). In the rest of the note, we shall describe this background deformation and analyze the cohomology change, leading to the GSO-free superstring with the modified even sector.

2. Elementary OSFT Solution and Cohomology Change

In our recent previous work \[6\] we particularly pointed out a class of the elementary OSFT solutions describing perturbative background deformations. Namely, Let $V_i(z,p)(i = 1,\ldots)$ be the set of all physical vertex operators in string theory in the cohomology of the original BRST charge $Q$ (primary fields of ghost number 1 and conformal dimension 0) and $\lambda^i(p)$ are the corresponding space-time fields (where $p$ is the momentum in space-time and we suppress the space-time indices for the brevity). Then the string field

$$\Psi_0 = \sum_i \lambda^i V_i$$

is the solution of (1) provided that the zero $\beta$-function conditions:

$$\beta_{\lambda^i} = 0$$

are imposed on the space-time fields in the leading order of the perturbation theory. The related BRST charge deformation is then

$$Q \rightarrow \hat{Q} = Q + \sum \lambda_i V^i$$
with \( \tilde{Q} \) acting on string fields according to the prescription (3), which details we shall also discuss below. One particularly simple OSFT solution, relevant to our discussion is

\[
\Psi_0 = \xi_\alpha c e^{-\frac{1}{2}\phi \Sigma^\alpha} + B_m c e^{-\phi \psi^m} \tag{7}
\]

where \( \phi \) is the bosonized superconformal ghost field, \( \Sigma^\alpha (\alpha = 1, \ldots, 16) \) is the 16-component spin operator for the matter RNS fermions and \( \xi_\alpha, B_m \) are some parameters which, for simplicity, are considered constant in this work (unless specified otherwise). The 16-component spin operator is defined in the standard way. That is, take 10 RNS fermions \( \psi_m (m = 1, \ldots, 10) \), use them to make 5 complex fermions according to

\[
\Lambda_i = \psi_1 + i\psi_2, \ldots, \Lambda_5 = \psi_9 + i\psi_{10}
\]

and bosonize as

\[
\Lambda_i = e^{i\varphi_i} \quad i = 1, \ldots, 5 \tag{8}
\]

Define the 32-component spin operator according to

\[
S_\alpha = \exp\left\{ \frac{1}{2} \sum_{i=1}^{5} \pm i\varphi_i \right\} \tag{9}
\]

with \( \alpha = 1, \ldots, 32 \) corresponding any of 32 possible \( \pm \)-combinations. This 32-component, operator can, in turn, be decomposed into two 16-component operators \( S = \Sigma \oplus \tilde{\Sigma} \) with \( \Sigma^\alpha (\alpha = 1, \ldots, 16) \) corresponding to 16 \( \pm \)-configurations with even number of \( + \)'s and \( \tilde{\Sigma}^\alpha \) corresponding to 16 \( \pm \)-configurations with odd number of \( + \)'s. Such a decomposition can, of course, be viewed as a split of the spin operator \( S \) into GSO-even and odd components, however, note that the string field \( \Psi_0 \) of (7) is the solution of the OSFT equations of motion, invariant under the change \( \Sigma \rightarrow \tilde{\Sigma} \), bearing no reference to the GSO-projection.

For the sake of certainty we shall concentrate on the SFT solution (7) involving the spin operator \( \Sigma \) with the even number of the \( + \)-components. Furthermore, for our purposes we shall mostly ignore the spin 1 component of the solution, concentrating on the spin \( \frac{1}{2} \).

First of all, we shall specify the related deformation of the BRST charge. Again, for our purposes it is sufficient to limit ourselves to string fields constrained to combinations of the on-shell operators, i.e. the dimension 0 primaries. Using (9), it is straightforward to check that the nilpotence of the new BRST charge:

\[
\tilde{Q} = Q + \xi_\alpha c e^{-\frac{1}{2}\phi \Sigma^\alpha} \tag{10}
\]
implying
\[ \tilde{Q}\Psi = Q\Psi + \xi_\alpha (ce^{-\frac{1}{2}\phi} \Sigma^\alpha \Psi - \Psi \Psi - ce^{-\frac{1}{2}\phi} \Sigma^\alpha) \] (11)

requires the constraint
\[ \xi_\alpha \xi_\beta [ce^{-\frac{1}{2}\phi} \Sigma^\alpha \Psi - ce^{-\frac{1}{2}\phi} \Sigma^\beta \Psi] = 0 \] (12)

for some string field \( \Psi \). This constraint requires the vanishing of the worldsheet correlators
\[ \xi_\alpha \xi_\beta \ll \Psi, ce^{-\frac{1}{2}\phi} \Sigma^\alpha \Psi - ce^{-\frac{1}{2}\phi} \Sigma^\beta \Psi \gg \]
\[ = \xi_\alpha \xi_\beta \ll h \circ f^3 \circ \Psi(0) h \circ f^3 \circ (ce^{-\frac{1}{2}\phi} \Sigma^\alpha)(0) h \circ f^3 \circ (ce^{-\frac{1}{2}\phi} \Sigma^\beta)(0) \gg \] (13)

where the conformal transformations
\[ f^3_k(z) = e^{\frac{2\pi i (k-1)}{3}} \left( \frac{1 - i z}{1 + i z} \right)^{\frac{3}{4}} \]
\[ k = 1, 2, 3 \] (14)

map the string fields living on three separate worldsheets to three wedges of a single disc, while the transformation
\[ h(z) = \frac{1 - z}{1 + z} \]

further maps this disc to the halfplane (e.g. see [7] for explanations on the star product details in SFT). Although the action of the conformal transformations (14) is complicated for generic \( \Psi \), it acts trivially on dimension 0 on-shell primaries in the three-point correlator, simply shifting them from 0 to the points \( \sqrt{3}, 0, -\sqrt{3} \) respectively. Therefore the value of this correlator is given simply by the structure constant in front of the \( (z - w)^0 \)-term in the operator product of \( \xi_\alpha ce^{-\frac{1}{2}\phi} \Sigma^\alpha(z) \) with itself at \( w \). The operator product is easily evaluated as
\[ \xi_\alpha ce^{-\frac{1}{2}\phi} \Sigma^\alpha(z) \xi_\beta ce^{-\frac{1}{2}\phi} \Sigma^\beta(w) \sim (z - w)^0 \xi_\alpha \gamma^m_{\alpha\beta} \xi_\beta \partial cce^{-\phi} \psi_m + O(z - w) \] (15)

so the vanishing of the star product (13) to ensure the nilpotence of \( \tilde{Q} \) requires
\[ \xi \gamma^m \xi = 0 \] (16)

i.e. the pure spinor constraint on \( \xi \). Note that the similar constraint could have been obtained by requiring the nilpotence of the charge:
\[ Q + \xi_\alpha \int dz e^{-\frac{1}{2}\phi} \Sigma^\alpha \] (17)
with the second term in (17) being structurally reminiscent the BRST sharge \( \sim \oint \lambda^\alpha d_\alpha \) with the Green-Schwarz variable \( d_\alpha \) corresponding to the space-time supercurrent at picture \(-\frac{1}{2}\). There is some subtlety here. The operator \( \oint dze^{-\frac{1}{2}\phi \Sigma^\alpha(z)} \) is known to be the operator of the space-time charge. This operator is an anticommuting space-time spinor. The anticommutation property can be easily seen from analyzing the midpoint OPE of the integrand with itself which only contains the odd powers of \( z - w \). On the other hand, the corresponding unintegrated operator, \( ce^{-\frac{1}{2}\phi \Sigma^\alpha(z)} \), is a commuting space-time spinor since its midpoint OPE only contains the even powers of \( z - w \). (note that the unintegrated operator is, strictly speaking, not related to the space-time supercharge, as its OPE with itself does not contain a momentum generator). In the on-shell string theory this spin-statistics change is not of significance, since the unintegrated and integrated vertices are related by the \( b - c \) picture-changing transformation, defined by the nonlocal BRST-invariant operator \( Z = b\delta(T) \) where \( T \) is the full matter+ghost worldsheet stress-energy tensor. The \( Z \)-operator, particularly mapping unintegrated on-shell vertex operators into integrated ones, is the \( b - c \) analogue of the usual picture-changing operator for the \( \beta - \gamma \) system and, being an anticommuting fermion of conformal dimension 0, twists the spin-statistics, when applied to the on-shell operators. In string field theory, however, the \( Z \)-transformation becomes ill-defined and it is essential that the star product operates with the unintegrated vertices only. For this reason, it is essential that the \( \xi_\alpha \)-parameter in the deformed BRST charge (10) is a commuting space-time spinor, i.e. a twistor-like parameter. The nilpotence of the BRST charge thus entails the pure spinor constraint (17) on this parameter. The essential property of the deformation (10) of the BRST charge is that it eliminates all the GSO-odd modes from the spectrum since the OPE of the deformation term with any GSO-odd operator contains a branch point. For example, for the tachyon one gets

\[
: ce^{-\frac{1}{2}\phi \Sigma^\alpha} : (z) : (p\psi)e^{ipX} : (w) \sim \sqrt{z - w}e^{-\frac{1}{2}\phi (\gamma^m p_m)_{\alpha\beta} \Sigma^\beta} e^{ipX}(w)
\]

and similarly for any other GSO-odd operator. For the GSO-even sector, the deformation preserves analiticity, however, the on-shell BRST-invariance constraints are modified in the new cohomology. Consider the operator for the supermultiplet including the photon:

\[
V_{ph} = A_m(p) \times \{ c(\partial X^m + i(p\psi)\psi^m) + \frac{1}{2} \gamma^m \} e^{ipX} \\
+ u_\alpha(p)ce^{-\frac{1}{2}\phi \Sigma^\alpha} e^{ipX}
\]
where \( A_m(p) \) is the photon’s polarization vector and \( u_\alpha(p) \) is the commuting space-time spinor and calculate its commutator with the modified BRST charge (10). Note that, although this operator appears to have an uncertain spin-statistics, with the first term being odd and the second even, this uncertainty is merely the artefact of our \( b - c \) picture choice (with the both of the operators being at unintegrated picture). Indeed, all the NS operators are even at the integrated picture and odd at the unintegrated one, while the Ramond operators are odd at the integrated picture and even at the unintegrated one. Therefore the spin-statistics of the operator (19) can be made certain (even) by a picture-changing transformation, bringing the first term to the integrated picture, and leaving the second unintegrated. Changing the \( b - c \) pictures of the operators also entails switching from anti-commutators to commutators (or vice versa) in the commutation relations with the BRST charge. In particular, with the picture choice in (19) this implies the anticommutation of \( Q \) with the first term and the commutation with the second.

Straightforward calculation using (10), (11) gives:

\[
\{\tilde{Q}, V_{ph}(p)\} = \partial c c e^{-i \phi \psi_m e^{ipX}(p^2 A^m(p) + \xi \gamma^m u)} + \partial c c e^{-i \phi \Sigma^\alpha (p^2 u_\alpha + \xi \beta \gamma_{\alpha \beta} F_{mn})} + \frac{i}{2} c c e^{\frac{1}{2} \phi - \chi \Sigma^\alpha \gamma^m (p_m u_\beta(p) + \xi \beta A_m)} - \frac{i}{2} c c e^{i \chi e^{ipX}(pA(p))}
\]

with \( F_{mn} = p_{[m} A_{n]}(p) \) and the BRST-invariance imposing the on-shell constraints:

\[
\begin{align*}
p^2 A^m(p) + \xi \gamma^m u &= 0 \\
\gamma_{\alpha \beta}^m (p_m u_\beta(p) + \xi \beta A_m) &= 0
\end{align*}
\]

with the Lorenz gauge condition

\[
pA(p) = 0
\]

Here the \( \{ \} \) symbol stands for the anticommutator of the BRST with the first term and the commutator with the second, as explained above. Note that the \( \{ \} \rangle -operation ensures that the on-shell constraints (20) have the definite (even) spin-statistics.

The remaining constraint:

\[
p^2 u_\alpha + \xi \beta \gamma_{\alpha \beta}^{mn} F_{mn} = 0
\]

simply follows from the second equation in (21) modulo the gauge condition). The equations (21), (22) are equivalent to the the Euler-Lagrange equations for the low-energy
effective action in the position space:

\[ S = \int d^{10}X \left\{ \frac{1}{4} F_{mn} F^{mn} + \gamma_{\alpha\beta}^m (u^\alpha \partial_m u^\beta(x) + \xi^\alpha D_m u^\beta) \right\} \]

\[ D_m = \partial_m + A_m \]

(24)

If \( \xi \) is a pure spinor parameter (also satisfying the on-shell Dirac constraint), the action (24) is gauge-invariant and can be obtained from the standard QED by the BRST-type transformation \( u_\alpha \rightarrow u_\alpha + \xi_\alpha \) with \( \xi_\alpha \) playing the role of the ghost-like variable. Note that this is formally reminiscent of the BRST transformation \( \delta \theta_\alpha = \xi_\alpha \) in the pure spinor formalism [8], [9] where \( \theta \) is the Green-Schwarz variable [10]. The cohomology of \( \tilde{Q} \) is thus GSO-projected by construction, just like in Green-Schwarz or pure spinor formalisms. However, the cohomology change due to the deformation of the BRST charge in the RNS formalism is that Ramond and NS sectors are no longer separable. That is, in order to obtain a physically meaningful low-energy limit, in the new cohomology the physical operators always have to combine NS and Ramond pieces. For example, if one considers a photon operator \( \sim \left\{ c(\partial X^m + i(p\psi^m)\psi^m) + \frac{1}{2} \gamma \psi^m \right\} e^{ipX} \) of the old cohomology, without adding the Ramond piece, its commutator with \( \tilde{Q} \) would lead to extra constraint \( F_{mn} = 0 \) which would ruin all the physical content. In other words, in the new cohomology the physical operators have a form, manifestly consistent with the space-time supersymmetry.

**Conclusion**

In this note we have considered the deformation of the BRST charge by the Ramond current, consistent with elementary solution in open superstring field theory. The deformation is parametrized by the commuting spinor \( \xi_\alpha \) (which, for simplicity, we considered constant in this work, however, it is in general sufficient that it satisfies the Dirac on-shell constraint). The nilpotence of the new BRST charge imposes the pure spinor constraint on the deformation parameter, and the cohomology of the new BRST charge automatically cuts off the GSO-odd sector of superstring theory, effectively making the RNS formalism GSO-free, on equal footing with Green-Schwarz or pure spinor formalisms. In the new cohomology, NS and Ramond sectors essentially get intertwined, with the physical operators mixing the NS and Ramond ingredients (unlike the original BRST cohomology where these ingredients can be separated). It is known that the GSO-projections, while formally eliminating the tachyon and other odd states from the spectrum, still cannot eliminate the tachyonic poles and the poles from other odd states from the amplitudes. It is natural to expect that the scattering amplitudes of the vertex operators from the new...
cohomology should have the tachyonic and GSO-odd related poles eliminated from their structure. Roughly speaking, the cancellation should occur as a result of the contributions from the NS and Ramond ingredients of the operators. In the present note, we basically didn’t concentrate on the mutual normalizations of the NS and Ramond parts, choosing it arbitrarily by hands in (19). Such a normalization, however, must be significant for the cancellation of the poles from the GSO-odd sector and must be fixed in the process of calculating some concrete scattering amplitudes. This shouldn’t be a hard calculation and we hope to demonstrate it soon explicitly in the future work.
References

[1] P. Ramond, Phys. Rev. D3 (1971) 2415
[2] A. Neveu, J. Schwarz, Nucl. Phys. B31 (1971) 86
[3] F. Gliozzi, Joel Scherk, David I. Olive, Nucl.Phys.B122:253-290, (1977)
[4] E. Witten, Nucl.Phys. B268 (1986) 253
[5] E. Witten, Phys.Rev. D46 (1992) 5467-5473
[6] D. Polyakov, arXiv:1507.06220
[7] I. Bars, Y.Matsuo, Phys.Rev. D66 (2002) 066003
[8] N. Berkovits, JHEP 0004 (2000) 018
[9] N. Berkovits, JHEP 09 (2004) 047
[10] M. Green, J. Schwarz, Phys.Lett. B109 (1982) 444-448