Mathematical Model of the Deaeration of Finely Dispersed Solid Media in a Spherical Matrix of a Roller-Type Apparatus

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The additional operation of deaeration (compaction) of powders affects the quality of many products of chemical industries, the conditions for their delivery. Otherwise, energy consumption increases significantly. The aim of this work is the modeling of the deaeration of solid finely dispersed media in a gap with perforated hemispherical shapes on the surfaces of the shaft and conveyor belt within the framework of the mechanics of heterogeneous systems. A plane-deformation model is described, neglecting the forces of interphase interaction and taking into account the compressibility of a solid-particle-gas mixture without elastoplastic deformations. The model assumes consideration of the movement of (1) the components of the solid skeleton together with the carrying phase as a whole; (2) gas in an isothermal state through the pores of a finely dispersed material. This work is devoted to the study of part (a), i.e., behavior of the solid particle-gas system as a whole. The efficiency of the seal-deaerator is estimated using the obtained analytical dependencies for the main strength and speed indicators. The change in the degree of compaction of a spherical granule made of kaolin with given strength characteristics is investigated. It is shown that for the initial time interval up to $3.7 \times 10^{-2}$ s, the growth of the porosity value relative to the horizontal coordinate along the conveyor belt is exponential and increases by a factor of 1.1. After eight such time intervals, the porosity values stabilize along the indicated coordinate with an increase of more than 1.4 times from the initial value.

Keywords: model, deaeration (densification), finely dispersed medium, roller, porosity

INTRODUCTION

Preliminary deaeration (compaction) of powder components (Akiyama et al., 1986; Kapranova and Zaitzev, 2011; Francis, 2016), including soot and kaolin, affects the strength characteristics of the finished product, for example, car tires, and other polymer products. Transportation of sealed containers with a powder product with a high content of gas in its pores, in particular, for construction or food purposes, violates the principles of energy saving and energy efficiency. In contrast to the pressing of powders (Pizette et al., 2010; Bayle et al., 2016; Seong et al., 2020) or larger particles (Gai et al., 2005), deaeration refers to its initial stage, when there is no destruction of particles of the compacting medium. If it is necessary to obtain a special structure of a dispersed medium with given strength characteristics, it is advisable to use a mechanical deaeration method (Akiyama et al., 1986; Kapranova and Zaitzev, 2011) in particular, when obtaining granules from bitumen and mineral powder (Zaitsev et al., 2010), dry dye mixtures.
The design of roller devices for the deaeration of dispersed media is associated with the formation of theoretical foundations (Kapranova et al., 2000; Kapranova, 2010; Kapranova et al., 2015) for the engineering calculation of the parameters of these devices (Kapranova et al., 2001; Kapranova et al., 2006a; Kapranova et al., 2006b). For example, this is relevant in the manufacture of granules (Zaitsev et al., 2010) from bitumen (Santos et al., 2014; Fingas and Fieldhouse, 2009) and mineral powder (Renner et al., 2007). For these purposes, as a rule, the mechanics of heterogeneous systems are used (Nigmatulin, 1978; Generalov, 2002). The analytical results (Kapranova et al., 2000; Kapranova, 2010; Kapranova et al., 2015) when describing the behavior of the system solid particles-gas have some advantages over numerical solutions, (Pizette et al., 2010; Bayle et al., 2016; Seong et al., 2020) for example, when choosing rational ranges for changing the main parameters of the compaction process or when evaluating their optimal values (Kapranova et al., 2001; Kapranova et al., 2006a; Kapranova et al., 2006b). The importance of understanding the mechanism of the behavior of compacted materials is obvious for any type of modeling methods: analytical (Kapranova and Zaitzev, 2011; Kapranova et al., 2015; Udalov et al., 2019) or numerical (Khoei, 2005; Pizette et al., 2010; Bayle et al., 2016; Seong et al., 2020).

There are two sufficiently developed classical approaches to the formation of the initial model for calculating the main indicators of the powder compaction process. In the first method (roller rolling of metal powders) (Generalov, 2002; Wang et al., 2015), the conditions of air outflow from volumes are experimentally investigated depending on the shape of the constituent particles during their granulometric analysis (Vinogradov et al., 1969; Pimenov et al., 2015). In this case, scaling methods are used (Pimenov et al., 2015) within the framework of the Pi-Buckingham theorem (Buckingham, 1915; Annenkov et al., 2005); equilibrium equations (Generalov, 2002; Misic et al., 2010) and the limit state in the linearized representation (Tselikov et al., 1980; Generalov et al., 1984); indicators of changes in the volume of the specified workpiece (Generalov and Chainikov, 1972; Tselikov et al., 1980; Hu et al., 2021). The second method of description (Torner, 1977) does not make it possible to consider the compressibility of the system solid particles - gas when air is removed from the pores.

Two factors here are two factors that determine the modeling approach to modeling based on (Nigmatulin, 1978): (1) a significant content of the carrier phase in the composition of the specified system of solid particles—gas; (2) the maximum possible value of the degree of compaction of the material. This method makes it possible to carry out analytical calculations for the main indicators of the process under study, depending on the coordinates, time, design, and operating parameters, in particular, for the porosity of the mixture of solid particles-gas and the components of the velocity of the phases.

THEORY

Hemispherical shaft and belt surfaces are used to obtain deaerated portions of powder (Figure 1) with radius $r$. The Cartesian coordinate system $Oxy$ and the “inverted” motion method (Kapranova et al., 2000; Kapranova et al., 2015) are used when the horizontal tape appears to be stationary. The planar motion of the surfaces of the shaft forms is assumed when decomposed into translational motion together with the $K_i$ pole (Figure 1).

![Figure 1](image-url) | Conditional scheme for the movement of the compacted finely dispersed medium in the gap of the shaft-conveyor belt: 1—deaerated granule-sphere; 2—solid finely dispersed material; 3—shaft; 4—conveyor belt; 5—hemispherical shapes (cells).
Let the total number of flat cells on the rim of the shaft section be \( N \) and the number of centers belonging to a quarter of the rim is denoted \( n = N/4 \). Then the coordinates of the points \( K_i, D_j, \) and \( O_i \) (Figure 1) are determined by the recurrent formulas (\( i = 1, n_0 - 1 \)).

\[
x_{K_i} = r; y_{K_i} = 2r \sin[(i - 1)\varphi] + I \sum_{j=0}^{i-1} \sin[(2j + 1)y] \quad (1)
\]

\[
x_{D_j} = r(1 - 2\cos i\varphi); y_{D_j} = I \sum_{j=0}^{i-1} \sin[(2j + 1)y] + 2r \sum_{j=0}^{i-1} \sin j\varphi \quad (2)
\]

\[
x_{O_i} = \frac{r}{R + r} y_{O_i}; y_{O_i} = y_{O_0} + \frac{(R + r)}{\sin \alpha} (t_{O_i} - t_0) \quad (3)
\]

Here it is indicated: \( n_0 \) is the number of cells for the section of the shaft, filled with powder; \( \gamma, \varphi \) are characteristic angles; \( t_{O_0} = (x_{O_0} \sin \alpha)/\omega(\rho \omega) \); \( \omega \) is the angular velocity of rotation of the shaft. The values of \( n_0, \gamma, \varphi \) are determined by geometric parameters (linear \( R, r, l, H \)) and angular \( \psi, \beta \) according to Figure 1.

Let for a dispersed system solid particles-gas in further designations the subscript “2” corresponds to the dispersed phase (solid skeleton), the subscript “1”-to the carrier phase. The classical conditions for the proportionality of the reduced \( \rho \) and true \( \rho_{tri} \) values of the phase densities are valid \( \rho_i = \alpha_i \rho_{tri}, i = 1, 2 \) (Nigmatulin, 1978). The developed method for modeling the process of deaeration of finely dispersed media (Kapranova and Zaitzev, 2011), due to its rather slow course, allows us to consider the movements of (1) the components of the solid skeleton of the dispersed medium together with the carrier phase as a whole; and (2) gas in an isothermal state through the pores of a finely dispersed material.

This work is devoted to the study of part (1), i.e., motion at a speed \( v_{12} \) of the solid particles-gas system as a whole, when the following conditions are met: \( v_2 \gg v_1; v_{12} \approx v_2 \) for the velocities of the phases \( v_i, i = 1, 2 \). Part (2) was studied by the authors in (Kapranova et al., 2010; Kapranova et al., 2011).

The following assumptions are made: \( \rho_i \ll \rho_2 \); there is no sliding of the dispersed medium on the surfaces of hemispherical shapes. The gravitational and inertial forces are neglected in comparison with the action of surface forces. The flow of the medium in the specified gap is laminar and one-dimensional with significant compressibility and gas permeability in contrast (Akiyama et al., 1986) to the models of the motion of polymer compositions (Nigmatulin, 1978; Generalov, 2002). Let there be a linear relationship between changes in the velocity components of the rigid skeleton in coordinates and shear stresses. Similar to the generalized Hooke’s law (Alcoverro, 2003) the linear dependence between the components of the averaged effective stress tensor \( \sigma_x, \sigma_y \) and of the averaged strain tensor deformations \( \varepsilon_{x2}, \varepsilon_{y2} \) of the dispersed phase is reflected by expressions according to the form

\[
\sigma_x = a_x [\lambda (\varepsilon_{x2} + \varepsilon_{y2}) + 2\mu \varepsilon_{x2}], \sigma_y = a_x [\lambda (\varepsilon_{x2} + \varepsilon_{y2}) + 2\mu \varepsilon_{y2}], \quad (4)
\]

where \( a_x \) is the porosity of the powder, and \( \lambda, \mu \) are the Lamé coefficients. Additionally, the condition of limiting equilibrium is assumed (Kapranova and Zaitzev, 2011; Kapranova et al., 2015). According to Kapranova and Zaitzev (2011), neglecting the deformations of the dispersed medium along the \( z \) coordinate, the following representations are used for the equation of porosity change and the relation for shear stresses, respectively.

\[
\alpha_2 = \frac{\alpha_{20}}{1 + \varepsilon_{x2} - \varepsilon_{y2}} \quad (5)
\]

\[
\tau_{xy} = \frac{\alpha_2 (\alpha_2 - \alpha_{20})}{1 + \varepsilon_{x2} - \varepsilon_{y2}} \quad (6)
\]

where \( \zeta_0 \equiv \lambda + 2\mu \). Here \( \alpha_{20} \) is the initial value of the porosity of the powder. The last relation (6) was obtained from the conditions \( \alpha_2 = (\sigma_x + \sigma_y)/2; \tau_{xy} = \zeta_0 (\varepsilon_{x2} + \varepsilon_{y2}) \).

In addition, for shear stresses, the vertical component of the velocity of the solid skeleton along the \( x \) coordinate is neglected, i.e., communication is performed

\[
\tau_{xy} = \mu_0 \frac{\partial v_{2x}}{\partial y} \quad (7)
\]

where coefficient \( \mu_0 \) is determined from the condition of adhesion of the compacted material to the surface of the hemispherical matrix.

The system of Equations 4–6 in Cartesian coordinates is supplemented by the equations of motion of the medium with the true density of the solid phase \( \rho_{tri} \) taking into account external pressure \( P \).

\[
\frac{\partial P}{\partial x} = -\frac{\partial \tau_{xy}}{\partial y} - \rho_{tri} a_x v_z \frac{\partial v_{2x}}{\partial y} \quad (8)
\]

\[
0 = -\frac{\partial \tau_{xy}}{\partial x} - \rho_{tri} a_y v_z \frac{\partial v_{2y}}{\partial y} \quad (9)
\]

and the following equation of continuity of the solid

\[
\frac{\partial v_{2x}}{\partial x} + \frac{\partial v_{2y}}{\partial y} = 0 \quad (10)
\]

System (8)–(10) allows you to obtain analytical approximations for the main indicators of the process. The applied solution methods include a combination of the method of model equations and the method of substitution of constants instead of variable parameters (Kapranova et al., 2010; Kapranova et al., 2011; Kapranova et al., 2009).

By Equations 8–10, taking into account the slow nature of powder deaeration, we have

\[
a^{(1)}_2(x, y, t) = a_{20} + \frac{\mu_0}{\zeta_0} [h_2 y_1(t) + h_1 (x, y) y_2(t)] \quad (11)
\]

\[
\tau_{xy}^{(1)}(x, y, t) = \zeta_0 [a^{(0)}_2 (x, y, y) - a_{20}] \quad (12)
\]

\[
v^{(1)}_{2y}(x, y, t) = S_1(t) + h_0 (x, y) S_2(t) \quad (13)
\]

\[
v^{(1)}_{2y}(x, y, t) = \Omega_3 (y) y_2(t) v^{(2)}_{2y}(x, y, t) \quad (14)
\]

Here are the first approximations for tangential stresses \( \tau_{xy}^{(1)} \), velocity components \( v^{(1)}_{2y} \) for the rigid skeleton. Expressions (11), (13) contain a function \( h_0 (x, y) \) that is determined by integration \( h_1 (x, y) \) over the \( y \)-coordinate. Dependencies \( S_1(t), S_2(t) \) from Eq. (13) are set according to the assumption that there is no movement of the powder at the bottom of the cell cross-section \( DK \) (Figure 1), then

\[
v^{(1)}_{2y}(x, y, 0) = v_{2xy}^{(1)}(x, y, 0) = 0 \quad (15)
\]
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The auxiliary functions included in Expressions (11)–(14) are

\[ y_1(t) = \frac{\exp\left(\frac{1}{\tau_n} - 1\right) - \exp(1)}{1 - \exp(1)}, \quad y_2(t) = \frac{\exp\left(\frac{1}{\tau_n}\right) - \exp(1)}{1 - \exp(1)}, \]

\[ S_1(t) = \frac{\nu_{2x}y_2(t)\mu_0}{\rho_0} - \frac{c_0\rho_0}{\nu_{2x}y_1(t)}, \quad S_2(t) = -c_0\nu_{2x}\mu_0, \]

\[ S_0(t) = \frac{\nu_2(t)\mu_0}{\rho_0} - \frac{c_0\rho_0}{\nu_2(t)}; \]

\[ h_0(x, y) = a_1\beta_1(x)\Omega_1(y); \quad h_1(x, y) = a_2\beta_1(x)\Omega_2(y); \]

\[ \Omega_1(y) = C_1\theta_1(y) - \frac{1}{c_0}; \quad \Omega_2(y) = C_2\theta_2(y) - \frac{1}{c_0}; \]

\[ \theta_1(x) = \exp(C_1x); \quad \theta_2(y) = \exp(C_2y). \]

In this case, the constants \( c_0, C_1, C_2, a_1, h_n \) are set by the values of the coordinates of the points \( K_1, D_1 \) from Equations (1–3) and the characteristics of the physical and mechanical properties of the compacted material, including the angle of friction of the dispersed medium \( \rho \) (Kapranova and Zaitzev, 2011; Kapranova et al., 2015) and the adhesion coefficient of the material \( H = k_p \) (Kapranova and Zaitzev, 2011; Kapranova et al., 2015), where \( k_p \) is the parameter of caking (adhesion).

Thus, expressions (11)–(14) can be used to form engineering methods for calculating the swath device (Kapranova et al., 2010; Kapranova et al., 2011; Kapranova et al., 2009).

RESULTS AND DISCUSSION

The calculation of the basic characteristics of the process of mechanical compaction of a dispersed medium \( W_b = \{a_{2x}, y_{2x}\} \) when receiving granules-spheres in a roller device (Figure 1) is carried out using the example of deaeration of kaolin GOST 21235–75 (Figures 2A,B) according to (11) and (13). Additionally, the dependence \( y_{2x}(x, y, t) \) was analyzed using expression (15) (Figure 2C). The values of the main parameters are: \( a_{20} = 2.8 \times 10^{-2}; \quad \rho = 7.0 \times 10^{-2} \text{ m}; \quad r = 5.0 \times 10^{-2} \text{ m}; \quad \omega = 0.524 \text{ s}^{-1}; \quad H_0 = 2.0 \times 10^{-2} \text{ m}; \quad \rho_n = 2.6 \times 10^3 \text{ kg/m}^3; \quad \lambda = 5.1 \times 10^4 \text{ Pa}; \quad \mu = 3.1 \times 10^5 \text{ Pa}; \quad k_p = 2.65 \times 10^4 \text{ Pa}; \quad \rho = 0.471 \text{ rad} \) according to (Kapranova and Zaitzev, 2011) using the techniques (Andrianov, 1982; Bessonov et al., 2001; Kapranova and Zaitzev, 2011).

The surfaces shown in Figure 2 correspond to a fixed point in time \( 3.65 \times 10^{-2} \text{ s} \) for the position of the form \( K_2D_2 \) (Figure 1).

According to the results obtained for the porosity function from Equation (11) (Figure 2A), the process of deaeration of the powder in the specified gap begins from the area surrounding the point \( K_3, s_n = n_0 - 1 \) at \( t_0 = 0 \) or point \( A_1 \) (Figure 1). Stabilization of \( x_{2x}(t) \) along the indicated coordinates occurs at the last stage of closing hemispherical shapes on the shaft and conveyor (see arc section \( KD \), Figure 1). Thus, for the initial time interval \( (t_0 < t < t_1 = 3.7 \times 10^{-2} \text{ s}) \), the growth of \( x_{2x}(t) \) (Figure 2A) relative to the horizontal coordinate along the conveyor belt is exponential and increases by 1.1 times. After eight such time intervals (at \( t_3 = 3.21 \times 10^{-2} \text{ s} \)), the porosity values...
stabilize along the indicated coordinate with an increase of more than 1.4 times from the initial value of $\alpha_{0x}$.

Analysis of surfaces for $v_2^{(1)}(x, y, t)$, $v_3^{(1)}(x, y, t)$ (Figures 2B, C) from Equations (13) and (14) during its deaeration in the spherical matrix of the described apparatus (Figure 1) showed the presence of a shift of the layers of the compacted material, starting from the time $t_0 = 0$; $v_2^{(1)}(0, 0, t)/v_2^{(1)}(0, r, t) = 0.5$; $\Delta v_2^{(1)} = v_2^{(1)}(x, y, t) - v_2^{(1)}(x, y, t)$ $\leq 2 \times 10^{-11}$ m/s.

**KEY FINDINGS AND RESULTS**

- The plane-deformation modeling of the movement of the solid skeleton of the dispersed medium together with the carrier phase as a whole in the working volume of the specified roller apparatus is carried out, as part 1 for the complete deaeration model (Kapranova and Zaitzev, 2011). The description of the movement of gas in an isothermal state through the pores of a finely dispersed material, as part (2) of this model, is discussed in the works of the authors (Kapranova et al., 2010; Kapranova et al., 2011; Kapranova et al., 2009).

- The theoretical substantiation of the possibility of realizing deaeration of dispersed media in a roller device with a spherical matrix on the surfaces of the shaft and conveyor belt is obtained based on the results of the performed simulation.

- The proposed plane-deformation model contributes to the development of methods for modeling the behavior of dispersed media in the working volumes of seals-deaerators, identifying the main information variables of the deaeration process, for example, according to the approaches, tested for the processing of solid dispersed materials (Kapranova et al., 2020a; Kapranova et al., 2020b) or when transporting liquid media (Kapranova et al., 2020c; Kapranova et al., 2020d).

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**CONCLUSION**

An analytical method is proposed for assessing the efficiency of the deaeration process of solid dispersed components in a gap with perforated hemispherical shapes on the surfaces of the shaft and conveyor belt within the framework of the mechanics of heterogeneous systems taking into account the compressibility. It is noted that the porosity of the finished granule-sphere at the final stage of deaeration in the described gap of the conveyor shaft with a spherical matrix almost uniformly reaches its limiting value. In this case, the difference between the maximum and minimum porosity values does not exceed $2 \times 10^{-12}$. Up to time values of $3.7 \times 10^{-2}$ s, the increase in porosity concerning the horizontal coordinate along the conveyor belt exponentially with an increase of 1.1 times in comparison with the initial value of this indicator. After eight such time intervals (when reaching $3.21 \times 10^{-1}$ s), the porosity values stabilize along the indicated coordinate with an increase in this characteristic of the deaeration process by more than 1.4 times from its initial value. So, the proposed method for modeling the compaction process of solid dispersed components provides a theoretical justification for the possibility of implementing this technological operation in a gap with perforated hemispherical shapes on the surfaces of the shaft and the conveyor belt.

**DATA AVAILABILITY STATEMENT**

The original contributions presented in the study are included in the article/Supplementary Material, further inquiries can be directed to the corresponding author.

**AUTHOR CONTRIBUTIONS**

All authors contributed to manuscript revision, and read and approved the submitted version.
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**Conflict of Interest:** The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.