The Analytic of Image Processing Smoothing Spaces Using Wavelet

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Abstract. Image analysis took wide areas in many fields, including medicine, physics, and other areas where you need a tool to deal with it smoothly and softly without losing the original image information. Using an image of a sample of a physical atom that was analyzed and highlighting the compression and raising the noise, histogram and statistics the image statistics where the best results were recorded when using a specific threshold i.e. when pressing the methods were used the first has the threshold methods is Balance sparsity-norm, Remove near 0 and Bal-sparsity-norm(sqrt). As for the methods of raising the noise are fixed form thresholding method with soft threshold, penalize high with hard threshold, penalize medium with hard threshold, penalize low with hard threshold, Bal sparsity norm (sqrt) with soft threshold, where image parameters were divided into approximation coefficients and details coefficients. Through the analysis, a suitable threshold value was obtained, which helps to restore energy that leads to the fact that the compressed necessity did not lose much of its original information, which proves the new wavelets in the field of physical and medical imaging.

1. Introduction

Image compression helped to avoid the problem of the large size that the image needs in storage and transportation as it is one of the techniques that achieve reducing the space that the image needs for storage and transportation when analyzing the image and is considered one of the applications that are obtained by using wavelets [1-3]. Other techniques are to raise the noise, image statistics and histogram that are recognized through the use of wavelets after the effect of the wavelets used, where the image is analyzed into coefficients of the approximation and details, and the latter is divided into vertical, diagonal and horizontal [4-7]. Many of the works included types of standard wavelengths that were used in the field of image analysis, where these wavelets bear the characteristic of orthogonally and their affiliation to the fields of linear and non-linear approximation of these common wavelets families are Symlets (Level 2), coiflets (constitute a family of wavelets with an unusual property-Level 2), Daubecheis (Level 2) and Haar [8-10]. Multi Resolution Analyze (MRA) that achieves the wavelets feature of the ability to analyze the wavelets of images, which is the analysis of wavelets into two fields, is the field scaling function and wavelet function, the field to which the wavelet belongs [11-12]. Wavelets that are constructed from the mother wavelet were used, which we obtain expansion and contraction with the help of coefficients s and r such as first, second, third and fourth Chebyshev wavelets, Legendre wavelet Hermite wavelet and Laguerre wavelets [13-22]. Through which the best results were obtained to reach the optimal solution to solve many numerical problems such as variational problems, optimal problems, integral equations Fredholm and volterra and integro differential equations.
In this work, a kind of these wavelengths Second Chebyshev Wavelet Transform (SCWT) have used in the field of image analysis after demonstrating several theories that qualify them to perform this function and a technique has obtained Multi Resolution Analyze (MRA) that obtained wavelet analysis to approximation factors and details. The new wavelet has used to analyze the image that belongs to a sample of a physical atom [23-32], where image statistics have obtained and noise has removed from them using methods the first has the threshold methods are Balance sparsity-norm, Remove near 0 and Bal-sparsity-norm(sqrt) in compression image and in de noising image used methods are soft threshold, penalize high with hard threshold, penalize medium with hard threshold, penalize low with hard threshold, Bal sparsity norm(sqrt) with soft threshold, and get the best results, which shows that the Tables (1-5). The objective of the present work is new wavelets were used for the first time that was used in other works to solve problems numerically. However, in this work they have used to analyze the image as it performed its job in the best image by the filter produced from it before that many new theories have demonstrated to prove its readiness in the field of images. One of these theorems is to prove the affiliation of the new wavelets to the fields of linear and non-linear approximation because of the property that the new wavelets possess, which is the property of orthogonality and their contain Scaling function and the realization of Multi Resolution Analyze (MRA) that achieves the wavelets feature of the ability to analyse the wavelets of images.

2. Suggested Wavelet Conversions

In this section, wavelets that have been used in previous works are chosen for many numerical solutions and to reach the optimum solution or the exact solution compared to other wavelets that have been compared with them where the best is reached. These wavelets will be built on the mother waves in the Eq. 1

$$\Psi_{s,r}(x) = |s|^r \Psi \left( \frac{s-r}{s} \right), s, r \in \mathbb{R}, s \neq 0$$

where:

$$\Psi(t) = [\psi_0(t), \psi_1(t), ..., \psi_{M-1}(t)]$$

The elements $$\psi_0(t), \psi_1(t), ..., \psi_{M-1}(t)$$ are the basis functions, orthogonal on the [0,1].

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2.1. Second Kind of Chebyshev Wavelet Transform (SCWT)

These wavelets are Second Kind of Chebyshev Wavelet Transform (SCWT) [33-34], which have proven in many actions in the foreground between the sons of their generation from the first, third and fourth wavelets of the worms of Chebyshev, by comparing the results when solving many numerical problems. For example, covariance issues Discrete Second Kind of Chebyshev Wavelet Transform (SCWT), denoted by (t) $$D_{s,r}(t) = D(t, s, r, n)$$, the parameters s, r transformed to discrete wavelet by used x = $$2^{-(n-1)}(2n^t)$$, d = $$2^{-(n-1)}(2s - 1)$$, c = $$2^{-(n-1)}$$ by substituting these parameters in Eq. 1 that have obtained Discrete Second Chebyshev Wavelet Transformations (DSCWT) as shown in Eq. 2,

$$D_{s,r}(t) = \begin{cases} 2^nD_r(2^n t - 2s + 1) & t \in \left[\frac{s-1}{2^n-1}, \frac{s}{2^n-1}\right] \\ 0 & \text{otherwise} \end{cases}$$

where: $$D_r(t) = \sqrt{2 \pi} D_r(t)$$ \quad r = 0, 1, 2, ..., M - 1 for k = 2 with weight function $$w_r(2^n t - 2s + 1) = \sqrt{1 - (2^n t - 2s + 1)^2}$$ on the interval [−1, 1], satisfied the recursive formula

$$D_0 = 1, \quad D_1 = 2t, \quad D_{r+1}(t) = 2tD_r(t) - D_{r-1}(t), \quad r = 1, 2, ..., M - 1$$

$$D_{1,0} = \frac{2\sqrt{2}}{\sqrt{\pi}}$$
\[ D_{1,1} = \frac{2\sqrt{2}}{\sqrt{\pi}} (8t - 2) \]
\[ D_{1,2} = \frac{2\sqrt{2}}{\sqrt{\pi}} (64t^2 - 32t + 3) \]
\[ D_{1,3} = \frac{2\sqrt{2}}{\sqrt{\pi}} (512t^3 - 384t^2 + 80t - 4), \text{ where: } 0 \leq t < \frac{1}{2} \]
\[ D_{2,0} = \frac{2\sqrt{2}}{\sqrt{\pi}} \]
\[ D_{2,1} = \frac{2\sqrt{2}}{\sqrt{\pi}} (8t - 6) \]
\[ D_{2,2} = \frac{2\sqrt{2}}{\sqrt{\pi}} (64t^2 - 96t + 35) \]
\[ D_{2,3} = \frac{2\sqrt{2}}{\sqrt{\pi}} (512t^3 - 1152t^2 + 848t - 204), \text{ where: } \frac{1}{2} \leq t < 1 \]

With respect to SKCWT the same above have get with the coefficients the function approximation
\[ X(t) \cong X_2^{n-1,M-1} = \sum_{s=1}^{2^{n-1}} \sum_{r=0}^{M-1} X_{s,r}D_{s,r}(t) = X^TD_{s,r}(t) \]  \hspace{1cm} (3)

where: C, D_{s,r} are \(2^{n-1}M \times 1\) matrices
\[ D = [D_{1,0}, D_{1,1}, ..., D_{1,M-1}, D_{2,0}, ..., D_{2,M-1}, ..., D_{2^{n-1},0}, ..., D_{2^{n-1},M-1}]^T \]
and
\[ X = [X_{1,0}, X_{1,1}, ..., X_{1(M-1)}, X_{2,0}, ..., X_{2(M-1)}, ..., X_{2^n,0}, ..., X_{2^nM-1}]^T \]

### 2.2. Properties of SCWT and It's Scaling Function

Theories that will be demonstrated adapt the smoothness of the proposed waves in many new uses, for example image processing, such as medical and physical images [35-43], etc. These theories are the most important mathematical aspects in the field of wavelets.

#### Definition 2.1

For every pair of \(s, r \in \mathbb{Z}\), define the period \(1_{s,r}\) by \(1_{s,r} = [2^{-s}r, 2^{-s}(r + 1)]\) which is familiar as dyadic period. The group of all such period is called dyadic sub periods of \(R\).

The collection of functions
\[ \{\alpha_{s,r}(t)\}_{s,r \in \mathbb{Z}} = \frac{1}{2^s}\alpha(2^{-s} - t) \forall s, r \in \mathbb{Z} \]  \hspace{1cm} (4)

#### Definition 2.2

(SCWT scaling function): Eq. 4 is called the scaling function of Second Kind of Chebyshev Wavelet Transform (SCWT), it defines
\[ \alpha(t) = \begin{cases} 
2\sqrt{2} & \text{if } 0 \leq t \leq 1 \\
0 & \text{otherwise} 
\end{cases} \]  \hspace{1cm} (5)

The system of Second Kind of Chebyshev Wavelet Transform (SCWT), \(\forall s, r \in \mathbb{Z}\) define
\[ \{D_{s,r}(t)\}_{s,r \in \mathbb{Z}} = \frac{1}{2^s}\alpha(2^{-s} - t) \forall s, r \in \mathbb{Z} \]  \hspace{1cm} (6)

Let \(f(t)\) is defined on \(L^2[0,1]\) has an expansion in terms of Second Kind of Chebyshev functions as follows.
\[ f(t) = \sum_{s=0}^{s} a_{s,r} \alpha_{s,r}(t) + \sum_{s=0}^{\infty} \sum_{r=0}^{2^s} \omega_{s,r}(t) \]
\[ = \sum_{s=0}^{s} a_{s,r} \omega_{s,r}(t) + \sum_{s=0}^{\infty} \sum_{r=0}^{2^s} d_{s,r} \omega_{s,r}(t) \]  \hspace{1cm} (7)

\(d_{s,r}\) is the details of coefficients and \(a_{s,r}\) is the approximate coefficients.
2.3. Multi Resolution Analyses (MRA) With Wavelet

The main coefficients of wavelet DSCWT through a system MRA $L^2(R)$:

$$f = \sum \sum X_{sr} D_{s,r}$$

$$f \in V_s = \{ f(t) | f(t) = \frac{1}{2^{s/2}} h(2^{-s} t) , \ h(t) \in V_0 \} ,$$

$$f(t) = \sum_{s \in Z} < f , \alpha(r) > \alpha(r)$$

Then a MRA of DSCWT on R is a sequence of subspaces $\{ V_s \} s \in Z$ of functions $L^2$ on R, First and foremost, in order for the new wavelets to be ready for the applications that can take place in the next sections, the following characteristics must be found.

For $\forall s, r \in Z , V_s \subseteq V_{s+1}$.

If $f(t)$ is R, then $f(t) \in \text{span}\{V_s\} s \in Z$, with $\epsilon > 0$, there is an $s \in Z$ and a function $g(t) \in V_s$ such that $\| f - g \|_2 < \epsilon$.

$$\cap_{s \in Z} V_s = \{ 0 \} .$$

A function $f(t) \in V_0 \mapsto 2^{-s/2} f(2^{-s} t) \in V_s$.

There exists a function $\alpha(t)$, $L^2$ on R, called the scaling function such that the collection $\alpha(t - s)$ is an orthonormal system of translates and $V_0 = \text{span}\{\alpha(t - s)\}$. 

Definition 2.3

Let $\{ V_s \}$ be an MRA with scaling function $\alpha(t)$ which satisfies (9) and $h(r)$ in this definition are considered calibration colander, where

$$D_{s,r}(t) = \frac{1}{2^s} D(2^{-s} t - r)$$

$$h(r) = < \frac{1}{\sqrt{\pi}} \alpha \left( \frac{1}{2} \right) , \alpha(t - r) >$$

$\alpha(r)$ is the function that has called scaling wavelet

$$g(t) = (-1)^s h(1 - r)$$

DSCWT is defined

$$D(x) = \sum_{\tau \in Z} g(\tau) \frac{1}{\sqrt{\pi}} \alpha(t - r)$$

Then $\{D_{s,r}(t)\} s, r \in Z$ is a Chebyshev wavelet orthonormal standard on R.

Definition 2.4

Obtaining the Eq. 13 representing orthogonal projection if the function $f$ is arbitrary

$$P_s f = \sum_{s \in Z} < f , \alpha_{s,r} > \alpha_{s,r}$$

$f \in L^2$ when it on to $V_s$

3. Suggested Wavelet Conversions With Approximation Spaces [44-55]

In this section, many theories that demonstrate the affiliation of wavelets proposed in this paper have demonstrated approximation spaces.

3.1. Approximation in Space of Square Integrable Functions over R

Let $f \in L^2(R)$ is continuous then the bases of SCWT be a series

$$f = \sum_{s=0}^{\infty} \sum_{r=0}^{R-1} < f , D_{s,r} > D_{s,r}(t)$$

$D_{s,r}$ be in the interval $l_{s,r} = \left[ \frac{r}{2^{-s}}, \frac{r+1}{2^{-s}} \right].$

$$< f , D_{s,r} > = \int f(t) D_{s,r}(t) dt = 2^{s/2} \int_{l_{2^s}} f^{(r+1)2^s} f(t) D(2^s t - r) dt.$$ (15)

Let $S = 2^T$ is the coefficients if $T \in S$ has account in finite sum $s = 0, 1, 2, \ldots, 2^T - 1 = S$, then $\sum_{s=0}^{T} \sum_{r=0}^{s-1} 1 + 2 + 2^2 + \cdots + 2^T - 1 = S - 1$ coefficients of in SCWT that for each $s$ only one of the non-zero coefficients is its size $2^{-s}/2$.

E is the approximate error in $L_2(R)$, which has determined from the Eq. 16
\[\left\| f - \sum_{s=0}^{T-1} \sum_{r=0}^{R-1} < f, D_{s,r} > D_{s,r}(t) \right\|^2_{L_2} = \left\| \sum_{s=T}^{\infty} \sum_{r=0}^{S-1} < f, D_{s,r} > D_{s,r}(t) \right\|^2_{L_2} = \sum_{s=T}^{\infty} \sum_{r=0}^{S-1} \left| < f, D_{s,r} > \right|^2 = \sum_{s=T}^{\infty} 2^{s-T} = 2^{-T} = \frac{1}{S} = \sigma(2^{-s/2}) \quad (16)\]

Not: \(\sigma\) is any approximation result.

**Theorem 3.1 (Approximation in \(L_p(R)\))**

Let \(L_p(R)\) be a partial sum of SCWT and \(f \in L_p(R)\) then

\[K = \sum_{s=0}^{T-1} \sum_{r=0}^{S-1} < f, D_{s,r} > D_{s,r}(t) \rightarrow E_{app} = \sigma(2^{-s/2})\]

**Proof**

\(E_{app}\) is a determine the approximate error in \(L_p(R)\)

\[E_{app} = \|f - K\|_{L_p} = \left\| f - \sum_{s=0}^{T-1} \sum_{r=0}^{S-1} < f, D_{s,r} > D_{s,r}(t) \right\|_{L_p} = \|\sum_{s=T}^{\infty} \sum_{r=0}^{S-1} < f, D_{s,r} > D_{s,r}(t)\|_{L_p} = \left(\sum_{s=T}^{\infty} \sum_{r=0}^{S-1} \left| < f, D_{s,r} > \right|^p \right)^{1/p} \]

**Theorem 3.2 (Approximation in iPM \(L(\mu, P)\))**

Let \(P\) is a constant \(1 < P \leq \infty\), \(M > 0\) and if \(f \in L_{pm}(\mu, P)\) then SCWT and \(T \leq S\) then

\[K(t) = \sum_{s=0}^{T-1} \sum_{r=0}^{S-1} < f, D_{s,r} > D_{s,r}(t) \rightarrow E_{app} = \sigma(2^{-s/2})\]

**Theorem 3.3 (Approximation in Besov Space \(L(R)\))**

In SCWT \(K(t) = \sum_{s=0}^{T-1} \sum_{r=0}^{S-1} < f, D_{s,r} > D_{s,r}(t) \in F^p(R)\) then \(T \leq S\rightarrow E_{app} = \sigma(2^{-s/2})\) where \(S = 2^T\) and \(E_{app}\) is the approximate error of in \(F^s L_2(R)\)

**Proof**

\[E_{app} = \|f - K\|_{L_2} = \left\| f - \sum_{s=0}^{T-1} \sum_{r=0}^{S-1} < f, D_{s,r} > D_{s,r}(t) \right\|_{L_2} \leq \left(\sum_{s=T}^{\infty} \sum_{r=0}^{S-1} \left| < f, D_{s,r} > \right|^2 \right)^{1/2} \leq \left(\sum_{s=T}^{\infty} \sum_{r=0}^{S-1} \left| < f, D_{s,r} > \right|^2 \right)^{1/2} \leq \left(2^{-s} \sum_{s=T}^{\infty} \sum_{r=0}^{S-1} \left| < f, D_{s,r} > \right|^2 \right)^{1/2} \leq \left(\sum_{s=T}^{\infty} \sum_{r=0}^{S-1} \left| < f, D_{s,r} > \right|^2 \right)^{1/2} \leq \left(\sum_{s=T}^{\infty} \sum_{r=0}^{S-1} \left| < f, D_{s,r} > \right|^2 \right)^{1/2} \leq \left(\sum_{s=T}^{\infty} \sum_{r=0}^{S-1} \left| < f, D_{s,r} > \right|^2 \right)^{1/2} \leq \left(\sum_{s=T}^{\infty} \sum_{r=0}^{S-1} \left| < f, D_{s,r} > \right|^2 \right)^{1/2} \]

**4. Nonlinear Approximation in \(L(R)\)**

**Theorem 4.1**

If the partial sum of SCWT with \(f\) and let \(f \in L_p(R)\)

\[K = \sum_{s=0}^{T-1} \sum_{r=0}^{S-1} < f, D_{s,r} > D_{s,r}(t), s \in Z \rightarrow E_{app} = \sigma(2^{-s/2})\]

**Proof**

\[E_{app} = \|f - K\|_{L_p} = \left\| f - \sum_{s=0}^{T-1} \sum_{r=0}^{S-1} < f, D_{s,r} > D_{s,r}(t) \right\|_{L_p} = \left\| \sum_{s=T+1}^{\infty} \sum_{r=0}^{S-1} < f, D_{s,r} > D_{s,r}(t) \right\|_{L_p} = \left(\sum_{s=T+1}^{\infty} \sum_{r=0}^{S-1} \left| < f, D_{s,r} > \right|^p \right)^{1/p} \]

\[\left(\sum_{s=T+1}^{\infty} \sum_{r=0}^{S-1} 2^{-TP/2} \right)^{1/p} \leq 2^{-T/2} = \sigma 2^{-T/2}\]
5. New Filter of SCWT

The filter is constructed for the proposed wavelets starting from scaling function for SCWT according to the Eq. 5 with the wavelength function at zero. The following two basic atoms of wavelets called the laguerre wavelet.

\[ D_{s,r}(t) = 2^n D(2^n t - 2s + 1) \quad \text{for } (s, r) \in \mathbb{Z}^2 \]  \hfill (17)

\[ \alpha_{s,r}(t) = 2^n \alpha(2^n t - 2s + 1) \quad \text{for } (s, r) \in \mathbb{Z}^2 \]  \hfill (18)

Eq. 17 is the wavelet equation and Eq. 18 is the scaling equation.

5.1. The Coefficients of SCWT

In this section, the proposed wavelet coefficients are created. The wavelet coefficients of a signal \( S \), in Egs. 19 and 20 the wavelet coefficients and scaling coefficients

\[ \delta_{s,r} = \int S(t) D_{s,r}(t) \, dt \quad \text{in } \mathbb{R} \]  \hfill (19)

\[ \varphi_{s,r} = \int S(t) \alpha_{s,r}(t) \, dt \quad \text{in } \mathbb{R} \]  \hfill (20)

When using the parameters, the signal is rebuilt

\[ S(t) = \sum_{s \in \mathbb{Z}} \sum_{r \in \mathbb{Z}} \delta_{s,r} D_{s,r}(t) \]  \hfill (21)

From MRA the space of scaling function \( V_0 \)

\[ V_0 = \{ f \in L^2(R) | f(t) = \varphi_{0,0} \alpha(2^n t - 2n + 1) \in L^2(\mathbb{Z}) \} \] is span \( V_1, V_2, \ldots, V_s = V_{s+1} + W_{s+1} \)

\( W_0 \) the space of wavelet function

Wavelet coefficients, when analyzed, have divided into approximate coefficients and details coefficients of SCWT and scaling function of SCWT Eqns. 22 and 23 show these coefficients from Eq. 20 to obtain Eq. 22

\[ a_s(t) = \sum_{r \in \mathbb{Z}} \varphi_{s,r} \alpha_{s,r}(t) \quad \text{(Approximate coefficients)} \]  \hfill (22)

\[ d_s(t) = \sum_{r \in \mathbb{Z}} \delta_{s,r} D_{s,r}(t) \quad \text{(Details coefficients)} \]  \hfill (23)

In level \( s \), then the coefficients whence \( s = 0 \) is \( \{ \varphi_{0,r} \} \)

5.2. Decision Tree of SCWT

The signal \( S \) found is from level 0 to \( a_0 = S \), in level 1 \( d_1 = a_0 - a_1 \) then \( S = d_1 \) + \( a_1 \) and in level 2

\[ d_2 = a_1 - a_2 \]

\[ = S - d_1 - a_2 \]

\[ = d_2 + d_1 + a_2 \]

From Eq. 3; if \( f_{s,r} = S \)

\[ S = \sum_{s=1}^{n-1} \sum_{r=0}^{M-1} \delta_{s,r} D_{s,r}(t) = \delta^T D_{s,r}(t) \]  \hfill (24)

In general

\[ S = a_N + \sum_{s \leq N} d_s \]  \hfill (25)

and \( a_{N-1} = a_N + d_N \)

5.3. Formation Packets of SCWT

By Multi Resolution Analysis with the equation if \( e_r, f_r \in L^2 \)

\[ \frac{1}{2} \alpha_{s}^2 = \sum_{t \in \mathbb{Z}} e_r \alpha(t - r) \in L^2 \]  \hfill (26)

\[ D_{s,0,r} = D(t-r) \quad \text{D } \in W_0 \text{ such that} \]

\[ \frac{1}{2} D_{s,0,r}^2 = \sum_{s \in \mathbb{Z}} f_r \alpha(t - r) \in L^2 \]  \hfill (27)

In the norm space the interval \([0,1) \rightarrow \{ D_{s,0,r} \}_{s,r \in \mathbb{Z}} \in L^2 \)

The following Algorithm stages explain the construction Packets steps

Algorithm 5.1: construction Packets of SCWT

Step 1: Start with the candidates \( u_r, v_r \) the orthogonal of SCWT and scaling function of SCWT with MRA of \( L^2(R) \) has been the basis of sequences \( \{ e_r \}_{r \in \mathbb{Z}} \) and \( \{ f_r \}_{r \in \mathbb{Z}} \) in \( L^2 \)

Step 2: The sequence of functions \( (D_s)_{s \in \mathbb{Z}} \) \( (L_n)_{n \in \mathbb{N}} \) the \( D_0 = \alpha \) the scaling function
\[
\begin{align*}
D_{2r}(t) &= \frac{2\sqrt{2}}{\sqrt{\pi}} \sum_{r=0}^{\infty} v_r D_r(2t - s) \\
D_{2r+1} &= \frac{2\sqrt{2}}{\sqrt{\pi}} \sum_{r=0}^{\infty} u_r D_r(2t - s)
\end{align*}
\]

(28)

\[
r = 0 \rightarrow v_0 = v_1 = \frac{2\sqrt{2}}{\sqrt{\pi}}, \quad u_0 = -u_1 = -\frac{2\sqrt{2}}{\sqrt{\pi}}
\]

(29)

Step 3: The wavelet SCWT in \( [0,1) \) in \( r = 1 \) and scaling function in \( r = 0 \)

\[
D_{2r}(t) = D_r(2t) + D_r(2t - 1)
\]

(30)

\[
D_{2r+1}(t) = D_r(2t) - D_r(2t - 1)
\]

(31)

Step 4: Two copies of \( D_r \) in \( [0, 0.5) \) for \( D_r(2t) \) obtain \( D_{2r} \) and in \( [0.5, 1) \) obtain \( D_{2r+1} \) from \( D_r(2t - 1) \).

Step 5: The result of smoothed relation for regular function in the interval \( [0, 2M - 1] \)

\[
D_r\left(2^j t - 2s + 1\right)
\]

Let the \((j, k)\) is the SCWT packet \( j = 0,1,2,3,\ldots \), \( k = 0,1,\ldots,2^j - 1 \)

\[
j = 0, \quad \alpha_{0,0}(t) = \frac{\sqrt{2}}{\sqrt{\pi}} D_0(t) = 1 \quad \text{and} \quad j = 1, \quad \alpha_{1,0}(t) = \frac{\sqrt{2}}{\sqrt{\pi}} D_0(t) = \frac{2\sqrt{2}}{\sqrt{\pi}} D_1(2t - 1) = \frac{\sqrt{2}}{\sqrt{\pi}} [1 - (2t - 1)]
\]

\[
\alpha_{0,0}(t) = D_{1,0}(t) = \frac{\sqrt{2}}{\sqrt{\pi}} D_0(t)
\]

The norm in this filter is \( \frac{\sqrt{2}}{\sqrt{\pi}} \) will be obtained high filter and low filter then

High pass filter is \( \frac{\sqrt{2}}{\sqrt{\pi}} [2 - 2] \) and low pass filter is \( \frac{\sqrt{2}}{\sqrt{\pi}} [2 - 2] \) this filter will be used in the image analyses after add this filter in MATLAB.

W divided into low pass filter will symbolize the filter \( L_0 \cdot D \) and high pass filter \( H_0 \cdot D \) they are used in decomposition step and reconstructed with invers SCWT filter denoted by \( R \) they are \( L_{i-1} \cdot R \) and \( H_{i-1} \cdot R \) with SCWT the figure displays that

![Figure 1. The four filters in second Chebyshev Wavelet Transform.](image)

6. Applied SCWT With Image Processing

In this section, various applications in image processing such as compression and noise removal will be identified.

6.1. SCWT filter effect on analytic of image

In this section, the proposed wavelets are affected by the color image analysis. Where a physical image \( 256 \times 256 \) was used for a specific atom, the color image is converted into a gray image. The image is divided into approximation coefficients, and the detail coefficients by which it is divided into three vertical, horizontal, and diagonal sections, so the image is divided into four blocks each Block size \( (16 \times 16) \), Figure 2 shows the effect of SCWT filter on the proposed image, which is the physical image [37].
6.2. The most Important Operations on the Physical Images
After analyzing the image using SCWT, the new wavelet faces four important operations, statistics, histograms, compress and de-noise. Figure 3 shows the statistics operation of image physical using SCWT in level 2 and the norm in statistics in level 1 is $8.91 \times 10^{-8}$ and in level 2 $3.22 \times 10^{4}$ max norm 256.

6.2.1. The Statistics Image by SCWT
In this section, the effect of SCWT on the image will be displayed in terms of statistics with respect to the coefficients of approximation factors and details that are divided into vertical, horizontal and diagonal. Details are shown where the Figure 3 will display the original image with SCWT, Figure 4 shows how the effect Wavelets on approximation and rebuilding coefficients. As for the figure 5, it shows details of coefficients with rebuilding, showing the differences between them, Approximation Coefficients (AC), Approximation Reconstructed (AR), Details Horizontal Coefficients (DHC), Details Vertical Coefficients (DVC), Details Diagonal Coefficients (DDC), Details Horizontal Reconstructed (DHR), Details Vertical Reconstructed (DVR), Details Diagonal Reconstructed (DDR).
6.2.2. The histogram Image by SCWT

This section shows one of the effects of the new wavelet SCWT on the physical sample, where the signal represents and is equal to $a_0$, see section 5.2. The Figure 6 shows the approximation coefficients in level 1 and 2, and Figure 7 shows the details of coefficients horizontal, vertical and diagonal in level 1 and 2.
6.2.3. The Compression Image by SCWT

The appropriate technique for transferring image information without any losing of its information. New waves will be used to compress the image. In Matlab program, the compression process is carried out using two methods, global thresholding and by level thresholding, the first has the threshold methods is Balance sparsity-norm, Remove near 0 and Bal-sparsity-norm (sqrt). Figure 8 represents the process of compression using the first method with SCWT, while Table 1 shows the results that were reached through the first method with the wavelets proposed. The second is with the methods of threshold scarce high, scarce medium, scarce low, balance sparsity-norm, remove near zero, and bal sparsity norm(sqrt), are shown in Figures 9 and 10. The results are identified from Table 2.

![Figure 8. The compressed by global thresholding with SCWT.](image)

| Method               | Original image | Compressed image | The Results |
|----------------------|----------------|------------------|-------------|
| Balance sparsity-norm|                |                  |             |
| Remove near 0        |                |                  |             |
| Bal-sparsity-norm(sqrt)|            |                  |             |

![Table 1. The results compressed of Global thresholding method with SCWT.]

| Method                        | Global thresholding | Remove near zero | Bal-sparsity-norm |
|-------------------------------|---------------------|------------------|-------------------|
| Thresholding                  | 6.76                | 6.76             | 5                 |
| Return energy (Information)   | 99.69%              | 99.69%           | 99.98             |
| Number zero                   | 73.38%              | 73.38%           | 65.42             |
| L2Norm                        | 1.5                 | 1.5              | 2.4               |
| L1Norm                        | 681                 | 681              | 104               |
| Max norm                      | 9                   | 9                | 14                |

![Figure 9. Details coefficients and reconstruct.](image)
Figure 10. Threshold compressed by level method with SCWT.

Table 2. Results compressed by level thresholding with SCWT.

| Thresholding method | Scarce high | Scarce medium | Scarce low | Balance sparsity-norm | Remove near zero | Bal sparsity norm(sqrt) |
|---------------------|-------------|---------------|------------|------------------------|-----------------|------------------------|
| Thresholding        | L₁          | 10.5          | 9          | 6                      | 325.8           | 1.5                    | 18.05                  |
|                     | L₂          | 0             | 0          | 0                      | 47.3            | 1.5                    | 18.05                  |
| Return energy (Information) | 99.97% | 99.99% | 99.98% | 99.92% | 100% | 96.21% |
| Number zero         | 20          | 22.74%        | 25.69%     | 30.85%                 | 14.55%          | 35.70%                 |
| L₂Norm              | 575         | 6.646e+04     | 504.5      | 2.197e+05              | 40.14           | 1.004e+06              |
| L₁Norm              | 1.273e+05   | 326.6         | 1.108      | 961.5                  | 63.36           | 62.46                  |
| Max norm            | 9           | 5             | 8          | 17                     | 1               | 100                    |

6.2.4. De-Noising Image by SCWT

Adding new noise to the image or signal helps reduce and restore the noise of the color image or signal, the simple statistically significant relationship to noise reduction

Let F is the image to be de noised, the image to be restored, e is the noise and θ²1 expressed in the following equation

\[F_{(r,s)} = l_{(s,r)} + e\]  \hspace{1cm} (32)

The removal of noise by wavelets consists of three stages

At the level in which the wavelets decompose the image from which the noise is to be removed.

In three directions the choice of threshold from the detail coefficients is with the absolute threshold dependent on θ.

This stage is to reconstruct the image after analysis and be dependent on the approximation coefficients.

The de noising image by SCWT selected thresholding method

1. Fixed form thresholding method with soft threshold.
2. Penalize high with hard threshold.
3. Penalize medium with hard threshold.
4. Penalize low with hard threshold.
5. Bal sparsity norm (sqrt) with soft threshold.

Select noise structure

1. Unscaled white noise
2. Scale white noise
3. Non scale white noise
Figure 11. De-noising with soft threshold by SCWT.

Figure 12. De-noising with hard threshold by SCWT.

Table 3. The results de noising soft threshold methods with SCWT.

|                      | Fixed form Thresholding | Bal Sparsity Norm(sqrt) |
|----------------------|-------------------------|-------------------------|
|                      | Un scaled white noise   | scale white noise       | non scale white noise |
| Soft threshold       | 4.158                   | 9.247                   | 43.15                 |
|                      | 1.431e+05               | 2.791e+05               | 7.668e+05             |
|                      | 605.7                   | 116.2                   | 290.8                 |
| Max norm             | 6                       | 12                      | 21                    |

Table 4. The results de noising soft threshold methods with SCWT.

|                      | Penalize High | Penalize Medium | Penalize Low |
|----------------------|---------------|-----------------|--------------|
| Hard threshold       | 8.5           | 4.75            | 4.25         |
|                      | 1.068e+05     | 5.071e+04       | 4.344e+04    |
| L1 norm              | 488.4         | 265             | 238          |
| Max norm             | 8             | 5               | 4            |

Table 5. Results of three different methods with the fixed form thresholding.

| Method                | Fixed Form Thresholding |
|-----------------------|-------------------------|
| Hard threshold        | Threshold level 1       | Threshold level 2 |
| Un Scaled white noise |                         |                  |
|                       | Horizontal              | Diagonal         | Vertical      |
|                       |                         |                  |              |
| Scale white noise     |                         |                  |              |
7. Discussion of The Results

In this work, unlike the usual work, a sample of a physical image of an atom was taken to shed the proposed wavelets and was used for the first time in a physical image analysis for analysis based on the wavelet image algorithm analysis, where the image was analyzed and its statistics, pressure, and the noise raising process have obtained. Good results were compared. The standard wavelets in the second level and the tables in the above sections indicate that and compared with the results obtained, they will be shown in the Table 4, where this table shows the efficiency of the new wavelets in this research.

The above table will be compared between one of the standard wavelets in level 8, (Symlet 2) has been compared with the new proposed wavelets. Compresses and compare the image with the results obtained by the method of Set Partitioning in Hierarchical Tree and relying on the measures obtained. Table 4 shows the compassion results of wavelets according to the following

1. Peak signal to noise ratio (PSNR): \[ \text{PSNR} = 10 \log_{10} \left( \frac{255^2}{\text{MSE}} \right) \]  

2. Compression ratio (CR): \[ \text{CR} = \frac{\text{Original image size in bit}}{\text{Compressed image size in bit}} \]  

3. Bits per pixel (BPP): \[ \text{BPP} = \frac{\text{Compressed image size in bits}}{\text{Total number of Pixel in the image}} \]  

4. Mean Squared Error (MSE): \[ \text{MSE} = \sum_{(j,k)} (I(j,k) - G(j,k))^2 \]  

| Loops | MSE  | PSNR  | CR    | BPP            | MSE  | PSNR  | CR    | BPP            |
|-------|------|-------|-------|----------------|------|-------|-------|----------------|
| 1     | 4.07e+04 | 2.025 | 0.03% | 0.007815       | 4.07e+04 | 2.025 | 0.03% | 0.007815       |
| 2     | 9928   | 8.162 | 0.03% | 0.007815       | 9928   | 8.162 | 0.03% | 0.007815       |
| 3     | 9928   | 8.162 | 0.03% | 0.007815       | 9928   | 8.162 | 0.03% | 0.008178       |
| 4     | 4324   | 11.77 | 0.04% | 0.008056       | 5045   | 11.1  | 0.04% | 0.0083008      |
| 5     | 3342   | 12.89 | 0.04% | 0.008911       | 3029   | 13.32 | 0.04% | 0.009511       |
| 6     | 1896   | 15.35 | 0.06% | 0.013306       | 1460   | 16.49 | 0.06% | 0.014282       |
| 7     | 1179   | 17.24 | 0.09% | 0.020508       | 1079   | 17.8  | 0.08% | 0.019409       |
| 8     | 622    | 20.19 | 0.17% | 0.04138        | 702.7  | 19.06 | 0.15% | 0.03686        |
| 9     | 337    | 22.85 | 0.35% | 0.083252       | 351.8  | 22.67 | 0.37% | 0.087769       |
| 10    | 154.3  | 26.25 | 0.77% | 0.18384        | 149.2  | 26.39 | 0.76% | 0.18188        |
| 11    | 55.93  | 30.65 | 1.53% | 0.3667         | 66.34  | 29.91 | 1.35% | 0.32446        |
| 12    | 23.65  | 34.35 | 2.42% | 0.58179        | 30.82  | 33.24 | 2.44% | 0.56545        |
| 13    | 10     | 38.13 | 3.98% | 0.95422        | 13.52  | 36.82 | 4.37% | 1.04930        |
| 14    | 4.526  | 41.57 | 6.43% | 1.5495         | 5.433  | 40.78 | 7.83% | 1.8795         |
| 15    | 2.334  | 44.45 | 10.46%| 2.5096         | 2.054  | 45    | 12.72 | 3.0525         |
| 16    | 1.483  | 46.42 | 16.14%| 3.8745         | 1.108  | 47.69 | 18.12%| 4.3616         |

The above table was compared with the results. It was observed that the difference in favor of the proposed new wavelet starts from step 5, 6 and 7, and this difference is repeated 10, 15 and 16.
This means the result obtained with the new wavelet SCWT is better than the result obtained from using the standard wavelet Symlet 2.

8. Conclusion
In this work, wavelets were relied on and were used in many businesses to find the best numerical solutions. In this work, I found a suitable filter from wavelets SCWT through scaling function and wavelet function find high pass filter and low pass filter used it to analyze the image where it was obtained. Good efficiency results obtained using new wavelets compared to the results obtained from the standard wavelets. Using an image of a sample of a physical atom that was analyzed and highlighting the compression and raising the noise, histogram and statistics the image statistics where the best results were recorded when using a specific threshold i.e. when pressing the methods were used the first has the threshold methods is Balance sparsity-norm, Remove near 0 and Bal-sparsity-norm (sqrt). As for the methods of raising the noise are fixed form thresholding method with soft threshold, penalize high with hard threshold, penalize medium with hard threshold, penalize low with soft threshold, Bal sparsity norm (sqrt) with soft threshold, where image parameters were divided into approximation coefficients and details coefficients. Through the analysis, a suitable threshold value was obtained, which helps to restore energy that leads to the fact that the compressed necessity did not lose much of its original information; which proves the new wavelets in the field of physical and medical imaging.

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