Opportunistic Beamforming with Beam Selection in IRS-aided Communications

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Abstract—In this work, we propose an opportunistic beamforming strategy, which enables beam selection through random-rotations of an intelligent reflecting surface (IRS). To boost performance over a time slot, the proposed scheme splits the training period into multiple mini-slots. In each mini-slot, the access point generates different sets of orthonormal beamforming vectors and the IRS employs random-rotations. We provide an analytical framework for the sum-rate capacity and it is shown that a trade-off between the sum-rate capacity and the length of the training period exists due to the time constraint on the communication process. Based on this, we also derive the optimal number of the training mini-slots. The proposed low-complexity scheme outperforms conventional counterparts (single training slot) and approximates the performance of conventional beamforming (with channel state information) even for small number of users. Finally, by utilizing extreme value theory tools, we analyze the system’s performance under an asymptotic scenario, where the number of the users significantly increases.

Index Terms—Beam selection, extreme value theory, intelligent reflecting surfaces, order statistics, sum-rate capacity.

I. INTRODUCTION

The 5G wireless communication networks have posed increasingly high demands in terms of the system capacity, massive connectivity and energy efficiency. An important enabling technology that can support all these requirements, and can achieve ultra high capacity and reliability with low complexity is the intelligent reflecting surface (IRS) [1]. Specifically, an IRS is composed of a large number of passive reflecting elements, mounted on a planar surface, where each element can induce a phase shift on the incident signal, thus collaboratively altering the propagation channel. [2]. The deployment of IRSs can enhance the wireless communication performance by increasing the energy efficiency due to the passive operation of their elements as well as achieving high spectral efficiency since they operate in full-duplex mode.

Due to these promising features, IRSs have been integrated under several classical wireless communication setups to achieve different communication objectives [3]. As a result, IRS-aided communications have attracted significant attention recently and have been studied under various scenarios [2]–[4]. Specifically, in [2], the authors study a single-cell wireless system where an IRS facilitates the communication between a multi-antenna access point (AP) and multiple single-antenna users. The active beamforming at the AP and the passive beamforming at the IRS, were jointly optimized to minimize the transmit power. Furthermore, a downlink multi-user multiple-input single-output (MISO) communication system is considered in [4], where the energy efficiency of the IRS is maximized by designing the base-station transmit power allocation and the phase shifts of the IRS elements.

However, there are several practical challenges on the road to the successful adoption of this technology, such as channel state information (CSI) acquisition. Most of the existing works that optimize the IRS phase shifts, in order to yield the so-called coherent beamforming gains, assume the availability of perfect CSI. However, this is highly impractical due to the limited signal processing capability at the IRS. The recent works on channel estimation protocols for such systems have shown that the training time required increases proportionally with the number of IRS elements [5], [6]. Therefore, the performance gains that can be achieved from deploying a large number of IRS elements are being compromised due to the prohibitively high feedback overhead.

In order to address these challenges, an IRS-enabled random-rotation scheme is studied in [3], [7], [8], where the IRS elements induce random phase rotations without requiring CSI. By increasing the dynamic range of channel fluctuations, multiuser diversity gain can be enhanced. This is achieved by using the random phase shifts induced by an IRS at each channel use, generating an artificial fast fading channel. Specifically, the work in [7] analyzes the outage probability and energy efficiency of the proposed low-complexity techniques in a single-input single-output (SISO) system, while [3] and [8] develop the sum-rate capacity scaling laws under opportunistic scheduling in a MISO and SISO broadcast channel, respectively. In particular, the work in [3] requires high number of users to approach the full-CSI capacity gains achieved by dirty paper coding scheme.

In contrast to other works, in this paper, we propose an opportunistic beamforming (OBF) scheme, which approaches the full-CSI capacity gains for even small number of users with low feedback requirements. Specifically, inspired by [9], the proposed OBF scheme employs beam selection in a random-rotation-based IRS-aided multiuser MISO communication system, based on a training period consisting of multiple mini-slots. In particular, during the training period over the
current time slot, the AP randomly generates multiple sets of orthonormal beamforming vectors, and the only feedback required from each user is its highest signal-to-interference-plus-noise ratio (SINR) and the corresponding beam index at each mini-slot. Then, the AP employs beam selection based on the highest sum-rate capacity. Although multiuser diversity gain increases with the large number of mini-slots, the sum-rate capacity degrades due to the large training overhead. Therefore, we derive the optimal value of the number of mini-slots for the training and present a complete theoretical framework for the sum-rate capacity of the network. Finally, by utilizing extreme value theory (EVT) tools, we analyze the system’s performance under an asymptotic scenario, where the number of the users is large.

II. SYSTEM MODEL

A. Topology & channel model

We consider an IRS-assisted multiuser MISO communication system, where an $N$-antenna AP communicates with $K$ single antenna users, by deploying an IRS consisting of $L$ elements. The received baseband signal at user $k$ can be expressed as

$$y_k = G_k^H x + n,$$  \hspace{1cm} (1)

where $G_k$ is the overall channel between the AP and user $k$, $x$ is the $N \times 1$ signal vector transmitted by the AP and $n \sim CN(0, \sigma_n^2)$ denotes the additive white Gaussian noise (AWGN) at user $k$ with variance $\sigma_n^2$.

The overall channel of the IRS-aided communication between AP and user $k$ is described by

$$G_k = Q\Theta h_k + g_k,$$  \hspace{1cm} (2)

where $Q \in \mathbb{C}^{N \times L}$ denotes the channel matrix between AP and the IRS, $h_k \in \mathbb{C}^{L \times 1}$ is the channel vector of the IRS-user $k$ link and $g_k \in \mathbb{C}^{N \times 1}$ is the direct channel vector of the AP-user $k$ link. All wireless links are assumed to exhibit Rayleigh block fading; their entries are complex Gaussian random variables with zero mean and unit variance, i.e., $[Q]_{n,l} \sim CN(0,1), \forall n \in \{1, \ldots, N\}$, $\forall l \in \{1, \ldots, L\}$, $[h_k]_{l} \sim CN(0,1)$, $[g_k]_{l} \sim CN(0,1)$. $\Theta = \beta \text{diag}[e^{j\theta_1}, \ldots, e^{j\theta_L}] \in \mathbb{C}^{L \times L}$ represents the IRS response, with $\beta \in [0, 1]$ being the amplitude reflection coefficient and $\theta_l \in [0, 2\pi]$ being the phase shift of the $l$-th reflector of the IRS. The considered system topology is illustrated in Fig. 1.

B. Multiple-mini-slots opportunistic beamforming

In order to enhance the multiuser diversity gain, a beamforming strategy which employs beam selection is of practical interest [10]. In this approach, the AP generates $B$ random beams and each beam is assigned to the user with the highest corresponding SINR. Motivated by this and inspired by [9], we incorporate a multiple-mini-slots opportunistic beamforming (MMOB) scheme.

Time is slotted and the time slot duration is equal to $\tau$ time units. We assume a training period in each time slot, consisting of $M$ mini-slots each of length $t$. Note that the maximum value of $M$ is $\lfloor \frac{\tau}{t} \rfloor$, where $\lfloor \cdot \rfloor$ denotes the floor function. Therefore, the time duration of the communication phase is $\tau - Mt$, as depicted in Fig. 2. During the training period, the AP constructs multiple sets of $B$ orthonormal beamforming vectors. Specifically, at every mini-slot $m$, the $B$ pilot symbols are multiplied by the $B$ respective random beamforming vectors and are transmitted to the $K$ users through the IRS elements. The signal vector transmitted by the AP at the $m$-th mini-slot is given by

$$x_m = \sum_{b=1}^{B} v_{b,m} s_b^m,$$  \hspace{1cm} (3)

where $v_{b,m} \in \mathbb{C}^{N \times 1}$, $m = 1, \ldots, M$ is the $b$-th random beamforming vector at $m$-th mini-slot and $s_b^m$ is the $b$-th transmit pilot symbol.

Then, in every mini-slot $m$, each user $k$ computes the values of the following $B$ SINRs by assuming $s_b^m, b = 1, \ldots, B$ as the desired pilot symbol

$$\text{SINR}_{k,b,m} = \frac{|G_k^H v_{b,m}|^2}{\gamma + \sum_{i \neq b} |G_k^H v_{i,m}|^2},$$  \hspace{1cm} (4)

where $\gamma = \frac{P}{\sigma_n^2}$ denotes the input signal-to-noise ratio (SNR), where $P$ is the transmit power. Then, each user feeds back its maximum SINR $\text{SINR}_{k,b,m}$ with the corresponding index $b$. It is worth noting that the probability that user $k$ achieves the highest SINR on more than one beam is negligible when $K >> B$. Then, the set of $B$ users is determined by the AP based on the achieved sum-rate at each mini-slot $m$ given by

$$R_m = \sum_{b=1}^{B} \log \left(1 + \max_{1 \leq k \leq K} \text{SINR}_{k,b,m}\right).$$  \hspace{1cm} (5)
In other words, after the training period consisting of $M$ mini-slots, there are $M$ sets constituting $B$ users each, and the AP selects the set of $B$ users that achieves the highest sum-rate $R_m$ along with the corresponding beams. Therefore, the data symbols $s_b$ are transmitted through the IRS to the corresponding users. Specifically, they are multiplied by the $B$ random beamforming vectors $v_b$, and thus, the transmitted signal vector $x$ can be expressed as

$$x = \sum_{b=1}^{B} v_b s_b,$$  \hspace{1cm} (6)

where $s_b \sim CN\left(0, \frac{\gamma}{2}\right)$ is the $b$-th data symbol and $v_b \in \mathbb{C}^{N \times 1}$ is the $b$-th orthonormal beamforming vector. Since the communication between the AP and the users is achieved through the $B$ selected beams, the overall sum-rate capacity can be written as

$$R_c = (\tau - Mt) \max_{1 \leq m \leq M} R_m.$$  \hspace{1cm} (7)

### III. SUM-RATE CAPACITY ANALYSIS

In this section, we derive an analytical expression for the sum-rate capacity and determine the optimal number of mini-slots. Specifically, although multiuser diversity gain increases with the large number of mini-slots, the sum-rate degrades due to the large training overhead. Therefore, the optimal value of the number of mini-slots $M^*$ that maximizes the $R_c$ is

$$M^* = \arg \max_{M=1, \ldots, [\tau]} \mathbb{E}[R_c],$$  \hspace{1cm} (8)

where

$$\mathbb{E}[R_c] = (\tau - Mt) \mathbb{E}\left[\max_{1 \leq m \leq M} R_m\right].$$  \hspace{1cm} (9)

Since it is hard to obtain closed-form expression for $\mathbb{E}[R_c]$, based on the theory of order statistics, we consider an upper bound for $\mathbb{E}\left[\max_{1 \leq m \leq M} R_m\right]$, which can be expressed as [11]

$$\mathbb{E}\left[\max_{1 \leq m \leq M} R_m\right] \leq \mathbb{E}[R_m] + \frac{M - 1}{\sqrt{2M - 1}} \sqrt{\text{Var}[R_m]},$$  \hspace{1cm} (10)

where $R_m$ is defined in (5). In order to compute $\mathbb{E}[R_m]$ and $\text{Var}[R_m]$, we calculate the probability density function (PDF) of $r_{b,m} = \max_{1 \leq k \leq K} \text{SINR}_{k,b,m}$ such as [11]

$$f_{r_{b,m}}(x) = K f_{\text{SINR}_{k,b,m}}(x) F_{\text{SINR}_{k,b,m}}(x)^{K-1},$$  \hspace{1cm} (11)

where $f_{\text{SINR}_{k,b,m}}(x)$ is the PDF of a random SINR$_{k,b,m}$. In the following proposition, we derive a closed-form expression for the PDF of the highest SINR$_{k,b,m}$.

**Proposition 1.** The PDF of the highest SINR$_{k,b,m}$ can be expressed as

$$f_{r_{b,m}}(x) = K \sum_{w=0}^{K-1} \left(\frac{K - 1}{w}\right) (-1)^w \exp\left(-\frac{x + 1}{(L + 1)\gamma}\right) \left(B - 1 + \frac{x + 1}{(L + 1)\gamma}\right),$$  \hspace{1cm} (12)

where $\phi(B, w) = B - w(1 - B)$.

**Proof.** See Appendix A.

Then, by making use of the PDF $f_{r_{b,m}}(x)$ given by (12), we obtain analytical expressions for $\mathbb{E}[R_m]$ and $\text{Var}[R_m]$ in the proposition below. Note that since $\text{Var}[R_m]$ does not have closed-form expression, we assume an upper bound for it.

**Proposition 2.** The $\mathbb{E}[R_m]$ and $\text{Var}[R_m]$ can be expressed as

$$\mathbb{E}[R_m] = \frac{BK}{\ln 2} \sum_{w=0}^{K-1} \left(\frac{K - 1}{w}\right) (-1)^w \exp\left(-\frac{w + 1}{(L + 1)\gamma}\right) \times \left((B - 1)G_{2,3}^{3,0}\left(\frac{\phi(B, w)}{0,\phi(B, w)}, -1, \phi(B, w) - 1, \left|\frac{w + 1}{(L + 1)\gamma}\right|\right) + \frac{1}{(L + 1)^{\gamma}} G_{2,3}^{3,0}\left(\frac{\phi(B, w) - 1, \phi(B, w) - 1, \left|\frac{w + 1}{(L + 1)\gamma}\right|\right),$$  \hspace{1cm} (13)

where $G_{p,q}^{r,s}(x)$ is the Meijer $G$ function, and

$$\text{Var}[R_m] \leq B \log^2 \left(1 + K \sum_{w=0}^{K-1} \left(\frac{K - 1}{w}\right) (-1)^w (B - 1) \times \mathbb{E}(B, L, \gamma, w, 1) + \frac{\mathbb{E}(B, L, \gamma, w, 2)}{(L + 1)^2} \right) - \frac{(\mathbb{E}[R_m])^2}{B},$$  \hspace{1cm} (14)

where

$$\mathbb{E}(B, L, \gamma, w, q) = \exp\left(-\frac{w + 1}{(L + 1)\gamma}\right) \frac{\phi(B, w) - q + \frac{w + 1}{(L + 1)\gamma} - 1}{\phi(B, w) - q + \frac{w + 1}{(L + 1)\gamma} - 1},$$

$$w(\phi(B, w))$$ is given by Proposition 1 and $E_n(z)$ is the exponential integral function [12].

**Proof.** See Appendix B.

By combining (9) and (10), the upper bound on the sum-rate capacity can be expressed as

$$\mathbb{E}[R_c] \leq (\tau - Mt) \left[\mathbb{E}[R_m] + \frac{M - 1}{\sqrt{2M - 1}} \sqrt{\text{Var}[R_m]}\right] + \frac{M - 1}{\sqrt{2M - 1}} \sqrt{\text{Var}[R_m]},$$  \hspace{1cm} (15)

where $\mathbb{E}[R_m]$ and $\text{Var}[R_m]$ can be substituted by (13) and (14), respectively. Therefore, in order to obtain the optimal value of the number of mini-slots $M^*$, we substitute (15) in (8), as such [9]

$$M^* = \arg \max_{M=1, \ldots, [\tau]} \left[(\tau - Mt) \left[\mathbb{E}[R_m] + \frac{M - 1}{\sqrt{2M - 1}} \sqrt{\text{Var}[R_m]}\right] + \frac{M - 1}{\sqrt{2M - 1}} \sqrt{\text{Var}[R_m]}\right].$$  \hspace{1cm} (16)

The sum-rate capacity as well as the optimal number of mini-slots are also obtained for the special case where the input SNR is high ($\gamma \rightarrow \infty$), i.e., interference-limited system, in the following remark.

**Remark 1.** At the high-SNR regime, the sum-rate capacity can be expressed as

$$\mathbb{E}[R_c] \leq (\tau - Mt) \frac{B}{(B - 1)\ln 2} H_2 + \frac{M - 1}{\sqrt{2M - 1}} \times \frac{B (\pi^2 - 6\phi(1) (K + 1))}{6 (B - 1)^2 (\ln 2)^2}.$$  \hspace{1cm} (17)
where $H_K = \sum_{k=1}^{K} \frac{1}{k}$ is the $K$-th harmonic number and $\psi^{(r)}(z) = \frac{d^r}{dz^r} \psi(z)$ is the $r$-th derivative of the psi function $\psi(z)$ \cite{12}.

**Proof.** See Appendix C.

Note that the optimal number of mini-slots for high-SNR regime can be easily obtained by substituting (17) in (8).

**IV. EVT-BASED PERFORMANCE ANALYSIS**

In this section, we examine the asymptotic performance of the system in terms of the number of the users, i.e., $K \to \infty$. We analyze the performance of the system by deriving the limiting distribution of the highest SINR$_{k,b,m}$ and evaluate the asymptotic sum-rate through EVT tools. It is worth noting that in this section, the sum-rate analysis is based on a single mini-slot ($M = 1$), since for high number of users, the use of multiple mini-slots would not affect the system’s performance.

Based on order statistics theory, by assuming $\eta$ and $\xi$ as normalizing constants we can easily prove that for $K \to \infty$, the limiting distribution of the $r_b$, where $r_b$ corresponds to the max$_{1 \leq k \leq K}$ SINR$_{k,b}$ ($m$ is dropped since $M = 1$), is of the Gumbel type with CDF given by \cite{11}

$$G(x) = \exp(-\exp(-x)), \quad -\infty < x < \infty. \quad (18)$$

The normalizing constants $\xi > 0$ and $\eta$ satisfy the following condition $\lim_{K \to \infty} F_{r_b}(\xi x + \eta) = G(x)$, where $F_{r_b}(\cdot)$ is the CDF of $r_b$. These constants can be obtained by solving the equations $1 - F_{\text{SINR}_{k,b}}(\eta) = \frac{1}{K}, 1 - F_{\text{SINR}_{k,b}}(\eta + \xi) = \frac{1}{e}$, where $e$ is Euler’s number.

The asymptotic sum-rate capacity in terms of the number of the users is given by the following proposition.

**Proposition 3.** The sum-rate capacity for high number of users ($K \to \infty$) can be approximated as

$$E[R^K_e] \leq (\tau - t) B \log(1 + \eta + \xi \gamma_e), \quad (19)$$

where $\eta$ and $\xi$ denote the normalizing constants and $\gamma_e$ denotes the Euler-Mascheroni constant.

**Proof.** See Appendix D.

**V. NUMERICAL RESULTS**

In this section, we validate the derived analytical expressions with Monte Carlo simulations. The analytical results are illustrated with lines and the simulation results with markers. The performance of the system without IRS is used as a benchmark and is represented by the dashed lines. For the sake of presentation, we consider $L = 20$, $B = N = 5$, $t/\tau = 5\%$, $P = 10$ dB, $\sigma_n^2 = 0$ dB.

Fig. 3 plots the sum-rate capacity achieved by the MMOB scheme in an IRS-aided communication system versus the number of mini-slots for different numbers of the users. The analytical sum-rate capacity curve corresponds to the upper bound described by (15) which is computed analytically. We observe that it closely follows the curve obtained from (9) through simulations, especially for small $M$ and high number of users $K$. Moreover, the sum-rate capacity increases with the number of the mini-slots until a certain point due to the beam selection benefit. A further increase of $M$, i.e., $M > M^*$, degrades significantly the sum-capacity due to the large training overhead. Furthermore, it is worth noting that $M^*$ derived by simulations coincides with that of the analytical curves. We also observe that the sum-rate capacity achieved by the MMOB scheme in an IRS-aided communication system is significantly better than that in a system without IRS for any number of users or mini-slots.

Fig. 4 depicts the sum-rate capacity versus the number of the users $K$. It is worth noting that the sum-rate capacity improves with $K$ due to the multi-user diversity effect. In addition, as it is mentioned before, the sum-rate gain achieved by the proposed scheme by using $M = 2$ is noticeable for any number of users. Furthermore, we observe that by deploying the MMOB scheme in an IRS-aided communication network, even for low number of users, it outperforms the conventional OBF strategy. Note that the conventional scheme uses one slot for training of duration $t$. Furthermore, the asymptotic performance of the network obtained through EVT tools in Section IV is also plotted in Fig. 4, where the sum-rate
The derived analytical framework provides useful insights for by employing EVT, the system’s performance was evaluated. Furthermore, we considered capacity of the network and obtained the optimal number of an IRS-assisted communication network was presented. In training period with multiple mini-slots and achieves sum-rate communication system. The beamforming strategy is based on a scheme in a random-rotation-based IRS-aided MISO com-
microphone. Since the random phases have no effect on the channel gain, \( V_k = \) a unitary matrix 

\( h_{k,b,m} \), \( Q_{b,m} \) and \( g_{k,b,m} \) are the elements of \( h_k^H \), \( Q^H \) and \( g_k^H \) respectively. 

By making use of the central limit theorem [7], we can state that 

\[
\sum_{l=1}^{L} h_{k,l} Q_{l,n} + g_{k,n} \sim CN(0, L).
\]

Therefore, the overall \( G_k^H v_{b,m} \sim CN(0, L+1) \) is a zero-mean complex Gaussian random variable. 

Let \( Z = |G_k^H v_{b,m}|^2 \) and \( Y = \sum_{i \neq b} |G_i^H v_{i,m}|^2 \). Then, \( Z \) is exponentially distributed with parameter \( \frac{1}{L+1} \), i.e. \( f_Z(z) = \frac{1}{L+1} \exp(-\frac{1}{L+1}z) \) and as a result, \( Y \) follows Erlang distribution with shape and scale parameter \( B = L+1 \). Therefore, the corresponding PDF of \( Y \) can be approximated as 

\[
f_Y(y) = \left( \frac{1}{L+1} \right)^{B-1} B^{-2} y^{B-2} \exp\left(-\frac{1}{L+1}y\right) \left( B-2! \right).
\]

In order to derive the PDF of the SINR\(_{k,b,m}\), we rewrite (4) in the following form 

\[
\text{SINR}_{k,b,m} = \frac{|G_k^H v_{b,m}|^2}{\sum_{i \neq b} |G_i^H v_{i,m}|^2} = \frac{Z}{\frac{B}{L+1} + Y}.
\]

Hence, the PDF of \( \text{SINR}_{k,b,m} \) can be expressed as 

\[
f_{\text{SINR}_{k,b,m}}(x) = \int_0^\infty f_{\text{SINR}_{k,b,m}}(y) f_Y(y) dy.
\]

The PDF of the highest \( \text{SINR}_{k,b,m} \) is given by (11), where \( f_{\text{SINR}_{k,b,m}}(x) \) is the PDF of a random \( \text{SINR}_{k,b,m} \) and is given below by substituting (23) in (25) such as 

\[f_{\text{SINR}_{k,b,m}}(x) = \left( \frac{1}{L+1} \right)^B \frac{1}{(B-2)!} \int_0^\infty \left( \frac{1}{\gamma} + y \right) y^{B-2} \times \exp\left(-\frac{1}{L+1}\left( \frac{x}{\gamma} + xy + y \right) \right) dy.
\]

which, after some algebraic manipulations, is simplified to 

\[f_{\text{SINR}_{k,b,m}}(x) = \exp\left(-\frac{x}{L+1}\right) \left( B - 1 + \frac{x+1}{L+1}\gamma \right).
\]

Therefore, substituting (27) to (11) and by using the binomial theorem \( (x+y)^{n} = \sum_{m=0}^{n} C_{n}^{m} x^{n-m} y^{m} \), the PDF of the highest \( \text{SINR}_{k,b,m} \) can be reduced to (12). 

B. Proof of Proposition 2 

In order to evaluate \( \mathbb{E}[R_m] \), we first compute \( \mathbb{E}[R_m] \) as follows 

\[\mathbb{E}[R_m] = B \int_0^\infty \log(1+x) f_{r_{x,m}}(x) dx,
\]

and by substituting (12) in (28), a closed-form expression is obtained in (13). Regarding the \( \text{Var}[R_m] \), since it is not possible to derive a closed-form expression, we compute the upper bound on \( \text{Var}[R_m] \) by using Jensen’s inequality as follows
Var\[R_m\] = B \left( \int_0^\infty (\log(1 + x))^2 f_{r_m}(x) dx \right.
\left. - \left( \int_0^\infty \log(1 + x) f_{r_m}(x) dx \right)^2 \right) \quad (29)

\leq B \log^2 \left( 1 + \int_0^\infty x f_{r_m}(x) dx \right) - \frac{(E[R_m])^2}{B}, \quad (30)

and by substituting (12) in (30) and letting \( \phi(B, w) = B - w(1 - B) \) we obtain

\begin{align*}
\text{Var}[R_m] & \leq BK \sum_{w=0}^{K-1} \left( K - 1 \right) \left( -1 \right)^w \\
& \times \log^2 \left( 1 + \int_0^\infty \frac{x}{(x + 1)^{\phi(B, w)}} \right) \\
& \times \left( B - 1 + \frac{x + 1}{(L+1)\gamma} \right) - \frac{(E[R_m])^2}{B}, \quad (31)
\end{align*}

which is deduced to (14).

C. Proof of Remark 1

For high-SNR regime, we substitute \( \gamma \to \infty \) in (4), and therefore SINR \( k, b, m \) follows Beta prime distribution \( \beta'(1, B - 1) \). Accordingly, the PDF of the PDF of the SINR \( k, b, m \) is given by

\[
f_{\text{SINR}}^\infty(k, b, m)(x) = (B - 1)(x + 1)^{-B}, \quad (32)
\]

and therefore, the PDF of the maximum SINR \( k, b, m \) can be expressed as

\[
f_{\text{SINR}}^\infty(x) = K(B - 1)(x + 1)^{-B} - (1 - (x + 1)^{-B})^{K-1}. \quad (33)
\]

By following the same analytical steps with the previous subsection, we obtain an upper bound on the sum-rate capacity for high-SNR regime as follows

\[
E[R_\infty] = (\tau - Mt)E \left[ \max_{1 \leq m \leq M} R_m \right] \\
\leq (\tau - Mt) \left[ E[R_m^\infty] + \frac{M - 1}{\sqrt{2M - 1}} \sqrt{\text{Var}[R_m^\infty]} \right], \quad (34)
\]

which is deduced to (17), where \( E[R_m^\infty] \) and \( \text{Var}[R_m^\infty] \) are computed by substituting the PDF given by (33) in (28) and (29), respectively, and after algebraic manipulations can be expressed as

\[
E[R_m^\infty] = \frac{B}{(B - 1)} \ln^2 H_K, \quad (35)
\]

and

\[
\text{Var}[R_m^\infty] = \frac{B}{6(2B - 1)^2(\ln 2)^2} \left( \frac{E[R_m^\infty]^2}{B} - \frac{E[R_m^\infty]}{B} \right), \quad (36)
\]

respectively, where \( H_K = \sum_{k=1}^{K} \frac{1}{k} \) is the \( K \)-th harmonic number and \( \psi^{(r)}(z) = \frac{d^r}{dz^r} \psi(z) \) is the \( r \)-th derivative of the psi function \( \psi(z) \) [12].

D. Proof of Proposition 3

The asymptotic sum-rate capacity in terms of the number of the users, with \( M = 1 \), can be approximated as follows

\[
E[R_c^K] = (\tau - t)E[R_c^m] \\
= (\tau - t)B \int_0^\infty \log(1 + x) g \left( \frac{x - \eta}{\xi} \right) dx, \quad (37)
\]

where \( g(x) = \exp(-x - \exp(-x)) \) denotes the PDF of the Gumbel distribution and the normalized constants are given by

\[
\eta = (L + 1)(B - 1) \gamma W \left( \frac{(-x^{1/(1+\gamma)}) - \frac{1}{1+\gamma}}{B} \right) - 1, \quad (38)
\]

and

\[
\xi = (L + 1)(B - 1) \gamma W \left( \frac{(-x^{1/(1+\gamma)}) - \frac{1}{1+\gamma}}{B} \right) - 1, \quad (39)
\]

where \( W(\cdot) \) denotes the Lambert function [12]. By using Jensen’s inequality and since \( x \) follows the Gumbel distribution, an upper bound on asymptotic sum-rate capacity can be obtained as follows

\[
E[R_c^\infty] = (\tau - t)B \int_0^\infty \log(1 + x) g \left( \frac{x - \eta}{\xi} \right) dx \\
\leq (\tau - t)B \log(1 + E[x]), \quad (40)
\]

and taking into consideration that the mean of a Gumbel distributed random variable is \( \eta + \gamma \xi_e \), where \( \gamma_e \) is the Euler-Mascheroni constant, (40) is reduced to (19).

REFERENCES

[1] E. Basar, M. Di Renzo, J. de Rosny, M. Debbah, M.-S. Alouini, and R. Zhang, “Wireless communications through reconfigurable intelligent surfaces,” IEEE Access, vol. 7, pp. 116753-116773, Aug. 2019.

[2] Q. Wu and R. Zhang, “Intelligent reflecting surface enhanced wireless network via joint active and passive beamforming,” IEEE Trans. Wireless Commun., vol. 18, no. 11, pp. 5394-5409, Nov. 2019.

[3] Q. -U. -A. Nadeem, A. Zappone and M. Debbah, “Reconfigurable surface enabled random rotations scheme for the MISO broadcast channel,” IEEE Trans. Wireless Commun., vol. 20, no. 8, pp. 5226-5242, Aug. 2021.

[4] C. Huang, A. Zappone, G. C. Alexandropoulos, M. Debbah, and C. Yuen, “Reconfigurable intelligent surfaces for energy efficiency in wireless communication,” IEEE Trans. Wireless Commun., vol. 18, no. 8, pp. 4157-4170, Aug. 2019.

[5] B. Zheng and R. Zhang, “Intelligent reflecting surface-enhanced OFDM: Channel estimation and reflection optimization,” IEEE Wireless Commun. Lett., vol. 9, no. 4, pp. 518-522, April 2020.

[6] Z. He and X. Yuan, “Cascade channel estimation for large intelligent metasurface assisted massive MIMO,” IEEE Wireless Commun. Lett., vol. 9, no. 2, pp. 210-214, Feb. 2020.

[7] C. Psomas and I. Krikidis, “Low-complexity random rotation-based schemes for intelligent reflecting surfaces,” IEEE Trans. Wireless Commun., vol. 20, no. 8, pp. 5212-5225, Aug. 2021.

[8] Q. -U. -A. Nadeem, A. Zappone and M. Debbah, “Reconfigurable surface assisted multi-user opportunistic beamforming,” in Proc. IEEE Int. Symp. Inf. Theory, Los Angeles, CA, USA, June 2020, pp. 2971-2976.

[9] W. Choi, A. Forenza, J. G. Andrews and R. W. Heath, “Opportunistic space–division multiple access with beam selection,” IEEE Trans. Wireless Commun., vol. 55, no. 12, pp. 2371-2380, Dec. 2007.

[10] M. Sharif and B. Hassibi, “On the capacity of MIMO broadcast channels with partial side information,” IEEE Trans. Inf. Theory, vol. 51, no. 2, pp. 506-522, Feb. 2005.

[11] H. A. David, H. N. Nagaraja, Order Statistics. Wiley Series in Probability and Statistics, 2004.

[12] I. S. Gradshteyn and I. M. Ryzhik, Table of integrals, series, and products, 7-th Ed., Elsevier Ac. Press, 2007.