TERAD91:
A Program package for the calculation of
the cross sections of deep inelastic NC and CC
scattering at HERA

Arif A. Akhundov\textsuperscript{1}, Dmitri Yu. Bardin\textsuperscript{2},
L. Kalinovskaya\textsuperscript{3} and T. Riemann\textsuperscript{4}

\textsuperscript{1} Institute of Physics, Azerbaijan Academy of Sciences,
pr. Azizbekova 33, 370143 Baku, Azerbaijan
\textsuperscript{2} Joint Institute for Nuclear Research, Joliot-Curie 6, 141980 Dubna, Russia
\textsuperscript{3} Gomel Polytechnical Institute, October ave. 48, 246746 Gomel, Belarus
\textsuperscript{4} DESY - Institut für Hochenergiephysik, Platanenallee 6, D-15738 Zeuthen, Germany

Abstract: We present a brief description of the recently developed Fortran code TERAD91 for semi-analytical calculations of the double differential cross sections of NC and CC deep inelastic electron-proton scattering and of some related observables. The code is mainly intended for the calculations at HERA or other \textit{ep}-colliders but may be also used for similar processes like muon-proton scattering in fixed-target experiments.

1 Program Summary

Title of the program: TERAD91
Version: version 2.10, 3 Oct. 1991
Computer: IBM-3090, DESY
Program language: FORTRAN-77
Number of program lines: about 11,000
Other programs used: DIVDIF, interpolation routine from the CERN library KERNLIB.
External files needed: None.
Method of solution: Numerical integration.
Typical running time: Depending on the actual requirements like parameters, requested contributions, see text.

2 Introduction

TERAD91 is a software product by the Dubna-Zeuthen Radiative Corrections Group (DZRCG). It is intended for calculations of Deep Inelastic Scattering (DIS) cross sections and related observables (cross section ratios, asymmetries etc.) at HERA. Its typical output is the double differential cross section of DIS with account of QED and electroweak (EW) radiative corrections as chosen by flags:

$$\frac{d^2\sigma}{dxdy}$$

(in nanobarn) with \(x\) and \(y\) being some scaling variables - either leptonic or mixed or hadronic. For a brief description of the basic theoretical issues and definitions see [1].

TERAD91 was created during the 1990-91 Workshop on Physics at HERA. It accumulates about 15 years experience in the field of DIS collected by the DZRCG. Formally, it originates from four different codes: TERAD90 [2], DISEPNC [3], DISEPCC [4] and DIZET [5].

TERAD90 was its prototype, created just before the Workshop. Its roots date back till the mid seventies [6, 7]. In the eighties, the TERAD approach was used for the analysis of muon DIS data by the BCDMS experiment [8, 9].

DISEPNC and DISEPCC are codes developed in the late eighties for the QPM calculations of DIS at HERA [3, 4]. A similar approach was used earlier for the analysis of neutrino DIS data by the CDHSW [10] and CHARM-I [11] experiments.

DIZET is our electroweak library, used before for LEP-I physics [5]. It allows to calculate \(\Delta r\), and the complete \(O(\alpha)\) corrections, with inclusion of some leading higher order terms connected with the top-quark, to \(Z, W\) widths, and to the weak form factors for cross sections in the \(s\)- and \(t\)-channels. These form factors depend on the kinematics \((s, t, u\)-invariants), and on the fermion type of the scattering particles.

Some theoretical basis of the TERAD approach is presented in the contributions of the working group on radiative corrections in these proceedings [1]. Here we concentrate on the description of the code itself assuming that the reader is aware of the relevant physical problems and of the terminology. We present a description of its MAIN routine and of SUBROUTINE SET where all flags are collected which should be set by a user.
3 MAIN program of TERAD91

The **MAIN** routine of **TERAD91** consists of a **CALL** of **SUBROUTINE SET** which sets various user flags followed by a **CALL** activating one of the calculational **chains**, which computes some observable (e.g. cross section) or some other physical quantity (e.g. an electroweak form factor). Normally only one chain is active, **CALLs** to others are commented. We recommend to work only with one chain at a time. Otherwise the running would be too cumbersome as some of the calculations are time consuming and default outputs are lengthy. Moreover, a destructive cross influence of chains is not excluded. Below we briefly describe each **chain**.

**DXYLEP**

**SUBROUTINE DXYLEP** calculates double differential cross sections in terms of usual leptonic scaling variables \( x, y \). For leptonic type variables, **TERAD91** is mostly elaborated. It calculates both NC and CC cross sections in both TERAD and DISEP approaches (we will refer to them as two different **branches**). For this reason the output of **DXYLEP** contains a lot of cross sections (Born, corrected by EW and/or QED) and corrections calculated by both branches. The resulting cross section and correction for NC reaction is a mixture of numbers, calculated by both TERAD and DISEP branches. Because of two-fold numerical integrations, involved in the calculation of cross sections in this case, the running with **DXYLEP** is rather slow and takes about 70 minutes for a calculation in 96 kinematical points (default option) on the DESY IBM.

**AXYLEP**

**SUBROUTINE AXYLEP** calculates a double differential polarization asymmetry in leptonic variables \( x, y \). All calculations again are performed by both branches; since now one has to calculate each cross section twice, for two polarizations, the calculational time needed is approximately doubled. When using this chain, one should have in mind that initialization of polarizations is done inside **AXYLEP**. Also electroweak parameters could be re-initialized there in order to investigate the sensitivity of the asymmetry to the EW parameters.

**DXYMIX**

**SUBROUTINE DXYMIX** calculates double differential cross sections in terms of mixed scaling variables \( x_m, y_m \).

**DXYHAD**

**SUBROUTINE DXYHAD** does the same in terms of hadronic variables \( x_h, y_h \) (for their definitions see [1, 2]).

For mixed and hadronic variables, the **TERAD91** package contains only the TERAD approach for only the NC reaction. For this reason the outputs of **DXYMIX** and **DXYHAD** are very short: only the NC Born cross section and total correction are printed for each chosen \((x, y)\) pair. As only one numerical integration is left for the calculation of the cross section, these chains are much faster as compared to the first two. Each of them require less than 1 minute for the calculation in 96 default kinematical points.

The three subroutines **DXYLEP**, **DXYMIX** and **DXYHAD** are the main interfaces to the core of the **TERAD91** code - which realizes some formulae describing differential DIS cross sections with account of QED and EW Radiative Corrections (EWRC) in some scaling variables \( x, y \), see Section 3.2.1 of [1] and [2]. But, **TERAD91** can easily also be used to calculate many other observables, related to differential cross sections, and the discussed above **AXYLEP** gives an example of a user interface for the calculation of another observable,
namely polarization asymmetry \( A \). Similar to \textsc{dylep}, \textsc{dymix} and \textsc{dxyhad}, \textsc{axylep} also returns a double differential in \( x \) and \( y \) quantity.

The three subsequent interfaces exhibit examples of the calculation of integrated quantities: cross section ratio \( R = \text{NC}/\text{CC} \) (\textsc{rmin1} and \textsc{rmin2}) and again the polarization asymmetry \( A \) (\textsc{amin1}) in a kinematical region restricted by some cuts. All these subroutines deal with definitions of cross sections in terms of leptonic variables and use the DISEP approach. In \textsc{rmin1,2} and \textsc{amin1}, corresponding observables are constructed out of the Improved Born Approximated (IBA) DIS cross sections, i.e. with EW but without QED radiative corrections. This is done to study their sensitivities to the variation of the EW parameters. Due to additional integrations over the restricted by cuts kinematical region, these interfaces are rather slow. It is another reason why we use the DISEP approach there and restrict ourselves to the IBA.

\textsc{rmin1}
\textsc{amin1}

\textsc{Subroutines} \textsc{rmin1} and \textsc{amin1} calculate \( R \) and \( A \) with cuts in \( x \), \( y \) and \( Q^2 \).

\textsc{rmin2}

\textsc{Subroutine} \textsc{rmin2} calculates \( R \) with a cut in \( p_{h \perp}^2 \) and \( Q_h^2 \).

All of them return \( R \) and \( A \) as functions of varying top-quark and Higgs masses. Similar interfaces can be readily written by a user to calculate other integrated observables.

The three last interfaces are intended for a calculation of purely EW effects. They are indeed interfaces only to the EW library \textsc{dizet}. They were used to produce some Tables and Figures for the Workshop proceedings and are left in the \textsc{terad91} as further examples of interfacing.

\textsc{dizetf}

\textsc{Subroutine} \textsc{dizetf} returns a Table for the static EW parameters, \( M_W \) and \( \sin^2 \theta_W \).

\textsc{formfs}
\textsc{formft}

\textsc{Subroutines} \textsc{formfs} and \textsc{formft} return some tables illustrating the \( s \)- and \( t \)-dependences of EW form factors. We will not describe them in detail here.

\section{SUBROUTINE SET}

In this \textsc{subroutine} we collected all flags used to initialize the \texttt{chains}. The main interfaces \textsc{dylep}, \textsc{dymix} and \textsc{dxyhad} are initialized by \textsc{set} completely. Other interfaces may contain some additional initialization inside them; \textsc{set} prints also all flags.

In \textsc{terad91} there are flags which act in both the TERAD and DISEP \texttt{branches}; but there are also flags inherent to only one of the branch. The notion of default flags is not supported in \textsc{terad91}. Flag setting should be controlled by the user for each specific run.

\subsection{TERAD - flags}

\texttt{model}

\texttt{model=1}: quark distributions are used for the calculation of the structure functions,
(presently the Duke and Owens [12] parametrization),

**MODEL=0**: in principle any parametrization can be used for the eight structure functions $F_1, F_2, G_1, G_2, G_3, H_1, H_2, H_3$. The user can substitute instead of using **SUBROUTINE SFF1** (for $F_1$) etc. any parametrization available to him/her. Now we use a merge of Brasse [13], Stein [14] and Duke-Owens [12] fits (see [15] for more details).

**IVAR**

**IVAR=0**: the quark distributions are not modified in the low $Q^2$ region,

**IVAR=1,2**: they are made vanishing at low $Q^2$ through two different modifications. They, as well as related physics, are described in [16].

**ITERAD**

**ITERAD=0**: the quark distributions and $\alpha_{QED}$ (if running, see below the description of flag **IVPOL**) in leptonic bremsstrahlung are artificially chosen to depend on $Q^2$ as defined from lepton momenta. This option was implemented to have a possibility to compare results calculated by both TERAD and DISEP branches.

**ITERAD=1**: they depend on $Q^2_h$ as defined from the hadronic momenta ($Q^2_h$ is integration variable). This option must be used for realistic calculations.

**IHCUT**

**IHCUT=0**: no cut on the hadronic final state,

**IHCUT=1**: rejects hadronic final states with $Q^2_h \leq TCUT$ (GeV$^2$) and invariant hadronic mass (photon not included) $W^2_h \leq AMF2CT$ (GeV$^2$). In this case the two desired cut values $TCUT$ and $AMF2CT$ are set.

**IGCUT**

**IGCUT=0**: no cut on photon variables,

**IGCUT=1**: rejects all final states with a photon energy $E_\gamma \geq E_{cut}$ (GeV) and with photon angle in the interval $(\theta_{cut2}, \theta_{cut1})$, angles in radians ($0, \pi$) with respect to the electron beam.

These two cuts (**IHCUT** and **IGCUT**) are implemented only in **DXYLEP**.

**IWEAK**

This flag governs how EWRC are applied in the TERAD branch.

**IWEAK=0**: they are applied only via $\sin^2 \theta_W$,

**IWEAK=1**: they are applied via the full set of the EW form factors; a QPM treatment is used for the structure functions $G_i$ and $H_i$ in this case.

**IZERO**

a flag to switch on/off a time consuming CALL of the TERAD branch in **DXYLEP**.

**IZERO=0**: it is switched off; then the numbers of the TERAD branch are set to zero,

**IZERO=1**: it is switched on.

### 4.2 DISEP - flags

**INORM**

**INORM=0**: the nine different QED corrections for the NC reaction [3] are all normalized by the $\gamma$-exchange Born cross section,

**INORM=1**: they are normalized by the complete Born cross section, $\gamma + Z$-exchange,

**INORM=2**: each QED correction is normalized by its corresponding Born contribution, three with $\gamma$- exchange, three with $\gamma Z$-interference and three with $Z$-exchange (three means: lepton, lepton-quark interference, and quark bremsstrahlung).
IEWRC=0: no weak loop corrections are taken into account,
IEWRC=1: they are taken into account.

IBOXF
IBOXF=1: the contributions from ZZ- and WW-boxes are included,
IBOXF=0: they are not included.
(The IBOXF flag acts only if IEWRC ≠ 0.)

ICONV
ICONV=1: weak form factors are running,
ICONV=0: they do not run. The recommended value of this flag is 0.
(The ICONV flag is active only for IEWRC=1).

IQ20
IQ20=0: the light quark masses will be replaced by a $Q_0^2$, which is recommended to be chosen equal to the $Q_0^2$ which enters the QCD evolution equations,
IQ20=1: then the light quark masses are used.
The first choice is recommended, the second is supported for the sake of comparison.

4.3 Flags common both to TERAD and DISEP branches

IEXP
IEXP=1: the soft photon part of leptonic QED corrections is exponentiated,
IEXP=0: it is not exponentiated.

IVPOL
IVPOL=0: no running of $\alpha_{QED}$, $\alpha_{QED} = \alpha$,
IVPOL=1: running $\alpha_{QED}$.

IMOMS
This flag defines the calculational scheme used, i.e. which EW parameters are considered as input quantities and which one is calculated (iterated).
IMOMS=1: input - $\alpha, G_\mu, M_Z, m_t, M_H$; $M_W$ is iterated,
IMOMS=2: input - $\alpha, G_\mu, M_W, m_t, M_H$; $M_Z$ is iterated,
IMOMS=3: input - $\alpha, M_W, M_Z, m_t, M_H$; $G_\mu$ is calculated.
It is recommended to use the IMOMS=1 option. Other options are introduced for some tests.

There are also some additional parameters which could be set by the user. For the TERAD branch one can set parameters GG,GZ,ZZ (=0,1) which switch on/off the contributions due to $\gamma$-exchange, $\gamma Z$-interference, Z-exchange, respectively, in cross sections in any variable in the TERAD branch. (Together with INORM=2 one must fix GG=GZ=ZZ=1.)

There are two parameters W2TR and TTR used to subdivide the 2-dimensional integration region in all DXY...'s into up to three parts. For W2TR=TTR=0, there is only one region. Choosing the parameter TTR (in GeV$^2$) > 0, one splits the region into two parts, one of them with small $Q^2_h$ where the structure functions should have a special behaviour. In addition, this region of small $Q^2_h$ can be split into two subregions by a choice of the parameter W2TR (in GeV$^2$) > $M_{proton}^2$. The latter was used for a proper treatment of the resonance region. Such subdivision of the kinematical region is used for MODEL=0. For more details about the low $x$ region see [15].

Two more parameters are: ALAM - the lepton beam polarization (in the interval $(-1, +1)$), and LEPCH - the lepton beam charge, $-1$ for electron, $+1$ for positron beams respectively.
In **SUBROUTINE SET** one sets also some constants like $\pi$, proton and electron masses, electromagnetic coupling constant $\alpha$, Fermi coupling $G_\mu$, and some combinations of them.

Then goes a part which initializes the calculation of electroweak parameters. It needs an additional setting of flags (they are stored in the NPAR-array) and constants for initialization of the EW library **DIZET**. Since this library was originally designed for LEP-I physics some flags and parameters have not too much meaning at HERA, therefore for some of them we simply report the recommended value avoiding further explanation.

**IHVP** says how to calculate the hadronic vacuum polarization (HVP) contribution to the running $\alpha_{QED}$:
- IHVP=1: HVP of Jegerlehner, [17],
- IHVP=2: HVP is made of some effective quark masses,
- IHVP=3: HVP of Burkhardt et al., [18],
- IHVP=3: recommended value.

**IAMT4** says how to apply some leading higher order corrections, connected with the top-quark, to $Z, W$ widths and EW form factors:
- IAMT4=0: they are neglected,
- IAMT4=1,2,3: different variants of their treatment,
- IAMT4=3: recommended value.

**IQCD** says how to apply QCD $\mathcal{O}(\alpha_s)$ corrections:
- IQCD=0: they are neglected,
- IQCD=3: they are included approximately (applicable only at LEP-I),
- IQCD=4: they are calculated completely,
- IQCD=4: recommended value (with a subsequent redefinition, see below).

**IMASS**
- IMASS=0: recommended value.

**IALST**
- IALST=1: recommended value.

**IQCD3**
- IQCD3=1: recommended value.

The $Z$-boson mass $AMZ$, top-quark mass $AMT$, Higgs boson mass $AMH$ and $VARQCD = \alpha_s(M_Z^2) (=0.12$ recommended value) should be also set for the subsequent call of the EW library, **CALL DIZET**. For **IMOMS=1**, **DIZET** calculates the $W$-boson mass $AMW$, but having in mind **IMOMS $\neq 1** options, the $W$-mass can also be set. In the **ZPAR**-array, **DIZET** returns many useful static EW quantities. We mention only two: $\Delta r$ is stored in **ZPAR(1)** and $\sin^2 \theta_W$ in **ZPAR(3)**.

After the **DIZET** call one could re-initialize, if needed, the EW parameters. To do that it is sufficient to redefine $AMZ$ and $AMW$. There is then one more important redefinition. As the **IQCD=4** option is very time consuming, we recommend to retain after the call of **DIZET** two statements:
- **IQCD** = 0
- **NPAR(3) = IQCD**

which will result in applying $\mathcal{O}(\alpha_s)$ corrections only in $\Delta r$ but not in the EW form
factors. For applications at HERA this is a very good approximation. The subsequent
\texttt{CALL CONSTQ} initializes all fermion (lepton and quark) masses and charges.

Then one sets some other constants like \( Q_E = 2I_3^e \), twice the weak isospin of the
electron, \( G_V \) and \( G_A \), vector and axial vector electron couplings to the \( Z \)-boson, and the
pion threshold \( W^2_{\text{PIT}} = (M_{\text{proton}} + M_{\text{pion}})^2 \).

\texttt{SUBROUTINE SET} ends with the calculation of the \( s \)-invariant at HERA and the cre-
ation of a lattice in \( x \) and \( y \) over the kinematic domain in which DIS cross sections will
be calculated. The version described creates the \( 6(x)*16(y)=96 \) points defined by

\[ x = .0001, .001, .01, .1, .5, .9 \]

and

\[ y = .01, .02, .05, .1, .15, .2, .3, .4, .5, .6, .7, .8, .9, .95, .98, .99. \]

5 Further developments of the code

The code \texttt{TERAD91} was frozen in October '91 and used to produce numbers for various
Figures and Tables of the Workshop Proceedings. Last bug fixes were made on February
10th of 1992, for this reason it is recommended to use a version saved after that date.
Nowadays only one new calculation is in progress: the DZRCG creates a DISEP branch
for the NC reaction in mixed and hadronic variables. Later it is planned to add a DISEP
branch for the CC reaction in hadronic variables. Further efforts in the future are not
excluded if some new results will be requested by HERA experiments.

The \texttt{TERAD91} package consists of about 11150 lines of coding (including about 4000
lines of weak library \texttt{DIZET}).

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