Light-front holographic QCD with generic dilaton profile

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We generalize the soft-wall and hard-wall models to a light-front holographic QCD model with a generic dilaton profile. The effective potential induced by a higher power dilaton profile is interpreted as a stronger color confinement at long distance, and it gradually evolves to the hard-wall model when the power increases to infinity. As an application, we investigate the exotic meson states recently discovered in experiments in the generic soft-wall model with a higher power dilaton profile, and the results are in agreement with the spectra of the exotic mesons. Our calculation indicates a weaker interaction at short distance and a stronger interaction at large distance for the components in the exotic mesons. The generic dilaton profile deserves further scrutiny for understanding the strong interaction and for applications.

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QCD provides a fundamental description of the strong interaction in terms of quark and gluon degrees of freedom in the standard model. The asymptotic freedom property of QCD allows perturbative calculation to be validly applied. However, the nonperturbative and nonlinear nature of QCD at the low energy scale makes it a challenging task to directly derive all hadronic properties from first principles. During the last decade, the light-front holographic approach [1], which is oriented from the light-front quantization [2] and the correspondence between the gravity in the anti-de Sitter (AdS) spacetime and the conformal field theory (CFT) [3] in the physical spacetime, has proven successful in explaining many hadronic properties [4] and thus has become an alternative nonperturbative method to study hadron physics. Although the conformal symmetry of the classical QCD Lagrangian with massless quarks is broken by the quantum effects, its asymptotic freedom at high energy and the infrared (IR) fixed point inspired by lattice simulation [5], the Dyson-Schwinger equation [6], and the empirical effective charge [7] indicate that QCD is nearly conformal at some region. Recently, a remarkable relation between the light-front dynamics and the conformal quantum mechanics has also been established [8] through the de Alfaro-Fubini-Furlan (dAFF) mechanism [9]. Therefore, the AdS/CFT correspondence can be used as a tool to obtain the first approximation to QCD.

In the light-front holographic framework, the fifth-dimensional coordinate $z$ in the AdS space is exactly mapped to the Lorentz invariant variable $\zeta$, which measures the separation of the constituents in the hadron [10]. As a direct result of the confinement, an IR cutoff should be imposed on the coordinate $z$. This cutoff is also understood as an energy scale, which breaks the conformal symmetry and induces the discrete spectra in QCD. At present, there are two methods, referred to as the hard-wall model [11] and the soft-wall model [12], to introduce the IR cutoff. In the former method, a sharp cutoff is imposed at large distance, and one considers a slice of the AdS space $0 \leq z \leq z_0$ with some boundary conditions at the IR border. In the latter method, a dilaton background is introduced as a deformation of the AdS space and effectively a smooth cutoff at large distance, but its profile is largely unspecified. Practically, a quadratic form is usually adopted. It can reproduce the Regge trajectory [12–14] and the massless pion in the chiral limit [8].

In this paper, we prove that the hard-wall model can be viewed as the limit of the soft-wall model with the power of the dilaton profile tending to infinity. Then, as an application, we investigate the exotic mesons in the generic soft-wall model with a dilaton beyond the quadratic form, and the results are in good agreement with the experimental data of the exotic meson spectra. Therefore, the soft-wall model with a generic dilaton background is not only of theoretical interest but also of physical significance to understand the strong interaction.

Hadrons are eigenstates of the QCD light-front Hamiltonian $H_{\text{LF}} = P_\mu P^\mu = 2P_+^2 P_- - P_\perp^2$ with the invariant mass square as the eigenvalues,

$$H_{\text{LF}}|\psi_H\rangle = M_H^2|\psi_H\rangle. \quad (1)$$

Quantized at fixed light-front time $\tau = (t + z)/\sqrt{2}$, a hadron state can be expanded on the Fock state bases as

$$|\psi_H\rangle = \sum_n \int [dx]\left[d^2k_\perp\right] \psi_{n/H}(x_i, k_{\perp,i})|n : x_i, k_{\perp,i}, \lambda^i\rangle, \quad (2)$$

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where the integral measures are
\[
\int [dx] = \prod_{i=1}^{n} \int dx_i \delta (1 - \sum_{j=1}^{n} x_j),
\]
\[
\int \left[ d^2 k_\perp \right] = \prod_{i=1}^{n} \int \frac{d^2 k_\perp}{(2\pi)^2} 16\pi^3 \delta^{(2)} \left( \sum_{j=1}^{n} k_\perp j \right).
\]

The variables \( x \) and \( k_\perp \) are respectively the light-front momentum fraction and the intrinsic transverse momentum carried by the component. The coefficients \( \psi_{n/H}(x_i, k_\perp i) \) are the light-front wave functions (LFWFs), which are frame independent. They encode all partonic information in the hadron. Then the mass square of the hadron can be expressed in terms of the LFWFs as
\[
M_H^2 = \int [dx] \left[ d^2 k_\perp \right] \sum_{i=1}^{n} \left( \frac{k_\perp^2 + m_i^2}{x_i} \right) |\psi(x_i, k_\perp i)|^2 + \psi^*(x_i, k_\perp i) U \psi(x_i, k_\perp i),
\]
where \( U \) is an effective potential. With a Fourier transformation, Eq. (5) can be expressed in the coordinate space, which is more convenient to investigate the potential. One main purpose of the light-front holographic approach is to obtain the effective potential.

As proved in the conformal limit [10], the fifth-dimensional coordinate \( z \) in the AdS space exactly corresponds to a Lorentz invariant light-front variable \( \zeta \), which measures the separation of the constituents in the hadron. For a two-body system, it is expressed as
\[
\zeta^2 = x(1-x)b_\perp^2,
\]
where \( b_\perp \) is the Fourier conjugate of the intrinsic momentum \( k_\perp \). Then, the eigenequation is expressed as
\[
\left[ -\frac{d^2}{d\zeta^2} - \frac{1 - 4L^2}{4\zeta^2} + U(\zeta) \right] \phi(\zeta) = M_H^2 \phi(\zeta),
\]
where the masses of the constituents are neglected. The \( \phi(\zeta) \) is the transverse mode of the LFWF:
\[
\psi(x_i, \zeta, \theta) = e^{iL\theta} X(x) \frac{\phi(\zeta)}{\sqrt{2\pi z_0}},
\]
where \( L \) is the spin of the hadron. In practice, a quadratic dilaton profile \( \varphi(z) = \kappa^2 z^2 \) is usually adopted due to its success in phenomenology. However, the dilaton in the soft-wall approach is viewed as a smooth cutoff in the IR limit. Therefore, it should reduce to the hard-wall situation at some limit. For this purpose, a generic dilaton profile should be introduced. For simplicity but still universal, we choose a dilaton profile as
\[
\varphi_p(z) = \kappa^p z^p.
\]

In Fig. 1, we plot the dilaton profile with different \( p \). Then, the effective potential is correspondingly expressed as
\[
U_p(z, J) = \frac{p^2 + 2(J-2)p_\kappa p_\varphi z^p - 2}{2} p_\kappa^2 z^2 p_\varphi^2 - \frac{p^2}{4} \kappa^2 z^{2p-2} + \frac{p^2}{4} \kappa^2 \left( \frac{z}{z_0} \right)^{2p-2},
\]
where \( z_0 \) is a typical length scale defined as \( z_0 = 1/\kappa \). One may observe from Fig. 2 that the potential in the short distance region \( z < z_0 \) decreases to zero with respect to an increasing value of \( p \), while in the large distance region \( z > z_0 \) it gradually increases to infinity.

In the limit of \( p \to \infty \),
\[
\lim_{p \to \infty} e^{-\kappa^2 z^p} = \theta(1 - \kappa^2 z^2),
\]
where \( \theta(x) \) is the Heaviside step function with the convention \( \theta(0) = 1/e \). Correspondingly, a positive dilaton
that the separation between the adjacent eigenvalues in-
tance region and no interaction in the short distance re-
time. This is exactly the picture of the hard-wall model.

The value at $g = 0$ enhances to the hard-wall situation, when the power

tivation. In the limit of $p \to \infty$, the eigenvalues converge to those calculated from the hard-wall approach with a
cutoff at $g$. Simultaneously, the eigenfunctions converge to the Bessel form $\phi(\tilde{z}) \sim \sqrt{\tilde{z}} J_L(\beta_{L,n} \tilde{z})$, which is the so-

lution in the hard-wall model. Thus, we prove that the soft-wall model with the
generic dilaton background reduces to the hard-wall situation, when the power $p$ in
the profile tends to infinity. In other words, both the hard-wall and the soft-wall approaches are unified into a
generalized soft-wall model.

As an application, we investigate the recently discov-
ered charmoniumlike charged meson states, often referred

to as Z states [16–26], together with their isospin part-
ners, in the generalized soft-wall model with a dilaton
distantly beyond the quadratic form. We choose the dilaton profile

$U_0(\tilde{z}, J) = 6(J + 1)\kappa^2\tilde{z}^4 + \kappa^2\tilde{z}^{10}$. (17)

Then, the invariant mass squares are solved from Eq. (16). We should note that the charmoniumlike Z
meson states are usually interpreted as the tetraquark states, which means their lowest Fock states contain four
quarks, i.e., $ccqq\bar{q}$, where $q$ represents the light quark. In this case, we cannot simply neglect the mass of the
constituents, as the charm quark mass is comparable to a
hadron mass. Thus, a mass shift $\Delta m$ is required. At
present, there are many phenomenological models aiming
to unravel the structure of these states, such as the
molecule model [27], the hadroquarkonium model [28],
and the diquark-antidiquark model [29, 30]. In our calcu-
lation, the meson states are regarded as two-body sys-
tems, which means that the tetraquark state is divided
into two clusters. Here, we treat each cluster as a two-
quark system. Then, the mass shift is obtained from the
mass of the cluster. We adopt the mass values of the
c$\bar{q}$ system calculated in the soft-wall model [31] as

$m = 1.86$ GeV for a scalar state and $m = 2.02$ GeV
for a vector state. In our scheme, $Z_c(3900)$ [16–19],
$Z_c(4020)$ [20, 21], $Z_c(4200)$ [22], and $Z(4430)$ [23–25]
are categorized into a series of radial excitations with $J = 1,$
while $Z_1(4050)$ and $Z_2(4250)$ [26] are arranged into the
excitations with $J = 2$. In Table II, we compare the
theoretical calculations with the data of the spectra. In
Fig. 3, we show the tendency of the mass spectra with re-
spect to the radial quantum number $n$. Our results are in
good agreement with the experimental data. Therefore,
the generalized soft-wall model with a dilaton beyond
the quadratic form is not only of theoretical interest to unify
the soft-wall and the hard-wall models but also of phys-
ical significance for phenomenological applications. This
will enrich our understanding of the strong interaction.

We should emphasize that our study does not refute
the remarkable success of the soft-wall model with the
quadratic dilaton profile. The choice of the quadratic
form can reproduce the Regge trajectory and the mass-
less pion in the chiral limit. It is also supported by the
dAFF mechanism and the superconformal algebra [32].
TABLE II. The mass spectra of the $Z$ mesons.

| Exotic mesons | $M_{th}$ (MeV) | $M_{ex}$ (MeV) | Ref. |
|---------------|----------------|----------------|-----|
| $Z_c(3900)$   | 3917.5         | 3899.0 ± 3.6 ± 4.9 | [16]|
| $Z_c(3900)$   | 3917.5         | 3894.5 ± 6.6 ± 4.5 | [17]|
| $Z_c(3900)$   | 3917.5         | 3894.8 ± 2.4 ± 3.2 | [18]|
| $Z_c(3900)$   | 3917.5         | 3883.9 ± 1.5 ± 4.2 | [19]|
| $Z_c(4020)$   | 4043.3         | 4022.9 ± 0.8 ± 2.7 | [20]|
| $Z_c(4020)$   | 4043.3         | 4026.3 ± 2.6 ± 3.7 | [21]|
| $Z_c(4200)$   | 4224.1         | 4196.3 ± 11 ± 17  | [22]|
| $Z_{(4430)}$  | 4445.8         | 4443.1 ± 12 ± 13  | [23]|
| $Z_{(4430)}$  | 4445.8         | 4485.1 ± 22 ± 28  | [24]|
| $Z_{(4430)}$  | 4445.8         | 4475 ± 7 ± 25     | [25]|
| $Z_{(4050)}$  | 4079.7         | 4051 ± 14 ± 20    | [26]|
| $Z_{(4250)}$  | 4208.4         | 4248 ± 44 ± 180   | [26]|

FIG. 3. The mass spectra of the $Z$ mesons. The solid curve is the model results with $\kappa = 223$ MeV. The data are taken from Refs. [16–26].

Thus, the effective potential induced by the quadratic dilaton profile provides a good description of the interaction between a quark and an antiquark or that between an active quark and a cluster carrying the quantum numbers of an effective spectator.

However, the conformal property of QCD is valid in the limit of zero quark mass. The finite quark mass inevitably breaks the strictly conformal symmetry, but it is important to reproduce the baryon properties, such as the spin structure [33] for the realistic hadron world. In this sense, some kind of generalization of the quadratic dilaton profile seems to be optional in some specific situations for describing the strong interaction in the AdS/QCD framework. Furthermore, the exotic mesons or the tetraquark states do not obey the Regge trajectory; nor are they viewed as Goldstone bosons in the chiral perturbative theory. Thus, these phenomenological results are no longer constraints on the choice of the dilaton profile when one investigates the exotic states. On the other hand, the effective potential induced by a higher power dilaton profile is interpreted as the interaction between two clusters, each containing two quarks inside. Hence, the effective potentials utilized here do not conflict with the usual one, as they describe different situations.

We also point out that the confinement potential from a higher power dilaton leads to a weaker interaction in the short distance region but a stronger interaction in the large distance region. This provides a physical picture that the clusters feel relatively weak interaction in the hadron, but when the separation increases, they meet a much stronger confinement. Thus, the generic dilaton profile meets the QCD basic properties of asymptotic freedom at short distance and color confinement at large distance.

In summary, we generalize the light-front holographic soft-wall model with a generic dilaton profile. We prove that the generalized soft-wall model reduces to the hard-wall model in the limit of an infinite power. In other words, the soft-wall and hard-wall approaches are unified into one general model. As an application, we investigate the exotic meson states in the generalized soft-wall model with a higher power form of the dilaton background. The calculated results are consistent with the experimental data of exotic meson spectra.

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