Reissner-Nordstrom Solution from Weyl Transverse Gravity

Ichiro Oda

Department of Physics, Faculty of Science, University of the Ryukyus,
Nishihara, Okinawa 903-0213, Japan.

Abstract

We study classical solutions in the Weyl-transverse (WTDiff) gravity coupled to an electro-magnetic field in four space-time dimensions. The WTDiff gravity is invariant under both the local Weyl (conformal) transformation and the volume preserving diffeormorphisms (transverse diffeomorphisms) and is known to be equivalent to general relativity at least at the classical level (perhaps even in the quantum regime). In particular, we find that only in four space-time dimensions, the charged Reissner-Nordstrom black hole metric is a classical solution when it is expressed in the Cartesian coordinate system.

1E-mail address: ioda@phys.u-ryukyu.ac.jp
1 Introduction

It is fair to say that the success of modern particle physics has been achieved with the help of the concept of spontaneous symmetry breakdown of local gauge symmetries [1]. However, gravity is outside the purview of this success, and what was worse, one veritable crisis is generated once gravity is introduced into the standard model of particle physics. This crisis is nowadays called the cosmological constant problem [2], and in the context at hand it might be simply phrased as follows: Astronomical observation shows that the cosmological constant at present is many orders of magnitude smaller than that estimated in modern particle physics.

The situation associated with the cosmological constant problem becomes worse when quantum effects are taken into account. Namely, every time the higher-order loop corrections are added in perturbation theory, we need to fine-tuning the value of the cosmological constant. The cosmological constant problem is therefore one of the most difficult hierarchy problems.

A number of theoretical physicists have advocated changing the rules of general relativity in such a way that the cosmological constant emerges as an integration constant which is irrelevant to any parameters in the action. This approach is sometimes called unimodular gravity [3]-[14], and it has been expected that unimodular gravity could suppress the radiative corrections coming from the higher-order loop corrections in perturbation theory although the value of the cosmological constant must be fixed by an initial condition. ²

Recently, a new type of unimodular gravity, which is called the Weyl-transverse (WTDiff) gravity [18]-[24], has been developed where as a new ingredient, the local Weyl (conformal) symmetry is introduced into the original unimodular gravity, which has only the tranverse diffeomorphisms (TDiff), or equivalently, the volume-preserving diffeomorphisms as local symmetries. The Weyl symmetry is known to be the original symmetry of the conventional gauge symmetries and is expected to help us to understand the short distance phenomena for a long time [25]. Furthermore, in order to realize the Weyl symmetry in a gravitational theory without introducing higher-derivative terms, we usually need a scalar field, and just the existence of the scalar field provides us with a possibility such that gravity is also unified in the standard model via spontaneous symmetry breakdown of the Weyl symmetry.

One of the purposes in this article is to study classical solutions in the WTDiff gravity. Even if the WTDiff gravity is equivalent to general relativity at the classical level (perhaps, even at the quantum level [26]-[31]), to our knowledge, nobody has explicitly derived classical solutions within the framework of the WTDiff gravity except our recent work of the Schwarzschild black hole metric [32].

In particular, we wish to investigate whether the charged Reissner-Nordstrom black hole metric is included in classical solutions when the WTDiff gravity is coupled to an electromagnetic field. In reality, a highly charged black hole would be quickly neutralized by interactions with matter in its vicinity and therefore such a solution is not extremely relevant

²Recently, we have established a topological model where the Newton’s constant is determined by an initial condition [15]-[17].
to realistic astrophysical situations. Nevertheless, charged black holes illustrate a number of important features of more general situations \cite{33}, so it is worthwhile to verify its existence in classical solutions of the WTDiff gravity coupled to an electro-magnetic field.

This paper is organised as follows: In Section 2, we review the WTDiff gravity. To get the WTDiff gravity, we start with the conformally invariant scalar-tensor gravity, and then fix the Weyl symmetry by a gauge condition which does not violate the TD iff \cite{32}. In Section 3, we solve the equations of motion of the WTDiff gravity coupled to an electro-magnetic field in the static and spherically symmetric ansatz. We show that the Reissner-Nordstrom metric in the Cartesian coordinate system is in fact a classical solution. The final section is devoted to discussions.

2 Brief review of the Weyl-transverse (WTDiff) gravity

We will start with an action of the conformally invariant scalar-tensor gravity in a general n dimensional space-time \cite{34-4}

\[
S = \int d^n x \sqrt{-g} \left[ \frac{n-2}{8(n-1)} \varphi^2 R + \frac{1}{2} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi \right],
\]

which is invariant under not only general coordinate transformation but also Weyl transformation of the metric tensor \( g_{\mu\nu} \) and a ghost-like scalar field \( \varphi \) as

\[
g_{\mu\nu} \rightarrow g'_{\mu\nu} = \Omega^2(x) g_{\mu\nu}, \quad \varphi \rightarrow \varphi' = \Omega^{-\frac{n-2}{2}}(x) \varphi,
\]

where \( \Omega(x) \) is an arbitrary scalar function of space-time coordinates \( x^\mu \).

With a gauge condition for the Weyl transformation

\[
\varphi = 2 \sqrt{\frac{n-1}{n-2}},
\]

the conformally invariant scalar-tensor gravity produces the Einstein-Hilbert action of general relativity. On the other hand, a gauge condition for the longitudinal diffeomorphism

\[
\varphi = 2 \sqrt{\frac{n-1}{n-2}} |g|^{-\frac{n-2}{4n}} ,
\]

\[\text{[We follow notation and conventions by Misner et al.’s textbook \cite{34}, for instance, the flat Minkowski metric } \\
\text{diag}(-, +, +, +), \\
\text{the Riemann curvature tensor } R^\mu_{\nu\alpha\beta} = \partial_\alpha \Gamma^\mu_{\nu\beta} - \partial_\beta \Gamma^\mu_{\nu\alpha} + \Gamma^\mu_{\sigma\alpha} \Gamma^\sigma_{\nu\beta} - \Gamma^\mu_{\sigma\beta} \Gamma^\sigma_{\nu\alpha}, \\
\text{and the Ricci tensor } R_{\mu\nu} = R^\sigma_{\mu\sigma\nu}. \] \\
\text{The reduced Planck mass is defined as } M_p = \sqrt{\frac{c \hbar}{8 \pi G}} = 2.4 \times 10^{18} GeV. \\
\text{Throughout this article, we adopt the reduced Planck units where we set } c = \hbar = M_p = 1. \] \\
\text{In this units, all quantities become dimensionless. Finally, note that in the reduced Planck units, the Einstein-Hilbert Lagrangian density takes the form } L_{EH} = \frac{1}{2} \sqrt{-g} R. \]

\[\text{[This conformally invariant gravity theory has a wide application in phenomenology and cosmology \cite{35-38}.] \]
leads to an action of the WTDiff gravity

\[ S = \frac{1}{2} \int d^n x \left[ R + \frac{(n-1)(n-2)}{4n^2} \frac{1}{|g|^2} g^{\mu\nu} \partial_\mu |g| \partial_\nu |g| \right], \tag{5} \]

where we have defined \( g = \det g_{\mu\nu} < 0 \). The action (5) turns out to be invariant under not the full group of diffeomorphisms (Diff) but only the transverse diffeomorphisms (TDiff) in addition to the Weyl transformation [32]. From this derivation, the WTDiff gravity is found to be at least classically equivalent to general relativity since the both actions are obtained via different choices of gauge condition from the same action (1). We conjecture that this equivalence holds even in the quantum regime [26]-[31]. To put differently, we believe that there are no anomalies associated with the Weyl symmetry and the longitudinal diffeomorphism.

Next, we will derive the equations of motion for the WTDiff gravity (5). A method of the derivation is to work with the action (1) of the conformally invariant scalar-tensor gravity, derive its equations of motion, and then substitute the gauge condition (4) into them. After some calculations, it turns out that the action (1) produces the equations of motion for \( g_{\mu\nu} \) and \( \varphi \), respectively

\[ \frac{n-2}{8(n-1)} \left[ \varphi^2 G_{\mu\nu} + (g_{\mu\nu} \Box - \nabla_\mu \nabla_\nu)(\varphi^2) \right] = \frac{1}{4} g_{\mu\nu} \partial_\rho \varphi \partial_\rho \varphi - \frac{1}{2} \partial_\mu \varphi \partial_\nu \varphi, \tag{6} \]

and

\[ \frac{n-2}{4(n-1)} \varphi R = \Box \varphi, \tag{7} \]

where \( G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R \) is the Einstein tensor and \( \Box \varphi = g^{\mu\nu} \nabla_\mu \nabla_\nu \varphi \). It is easy to show that the equation of motion (7) for the spurion field \( \varphi \) is not an independent equation, but can be derived from the trace part of Eq. (6).

Then, substituting the gauge condition (4) into Eq. (6) gives us the equations of motion for the WTDiff gravity

\[ R_{\mu\nu} - \frac{1}{n} g_{\mu\nu} R = T_{\mu\nu} - \frac{1}{n} g_{\mu\nu} T, \tag{8} \]

where the energy-momentum tensor \( T_{\mu\nu} \) is defined as

\[ T_{\mu\nu} = \frac{(n-2)(2n-1)}{4n^2} \frac{1}{|g|^2} \partial_\mu |g| \partial_\nu |g| - \frac{n-2}{2n} \frac{1}{|g|} D_\mu D_\nu |g|, \tag{9} \]

where we have defined \( D_\mu D_\nu |g| = \partial_\mu \partial_\nu |g| - \Gamma^\rho_{\mu\nu} \partial_\rho |g| \). Note that Eq. (8) is purely the traceless part of the Einstein field equations. By an explicit calculation, it is easy to verify that the equations of motion (8) are invariant under both the Weyl transformation and the TDiff [32].

Since the whole formalism, that is, the conformally invariant scalar-tensor gravity, is generally covariant, the energy-momentum tensor (9) should satisfy the covariant conservation law

\[ \nabla^\mu T_{\mu\nu} = 0. \tag{10} \]
Of course we can verify the validity of Eq. (10) by a direct calculation, and its calculation is simplified by going to a local Lorentz frame where the affine connection and first derivatives of the metric tensor vanish. Taking the covariant derivative of Eq. (8) and using the Bianchi identity

$$\nabla^\mu (R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R) = 0,$$

(11)

produces

$$\frac{n-2}{2n} \nabla_\mu R = -\frac{1}{n} \nabla_\mu T.$$  

(12)

This equation means that $R + \frac{2}{n-2} T$ is a constant, which we will call $\frac{2n}{n-2} \Lambda$

$$R + \frac{2}{n-2} T = \frac{2n}{n-2} \Lambda.$$  

(13)

Eq. (8), together with Eq. (13), yields the Einstein equations

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} = T_{\mu\nu}.$$  

(14)

Although we have recovered the Einstein equations from the equations of motion of the WTDiff gravity, the cosmological constant $\Lambda$ appears as a mere integration constant and has nothing to do with any terms in the action or vacuum fluctuations. In other words, Eq. (8) does not essentially include the cosmological constant and the contribution from the radiative corrections to the cosmological constant cancels in the RHS of Eq. (8), thereby guaranteeing the stability of the cosmological constant against the quantum corrections. This phenomenon is the most appealing point of the WTDiff gravity and unimodular gravity.

### 3 Reissner-Nordstrom solution

In a recent article [32], we have investigated classical solutions in the WTDiff gravity and found that the Schwarzschild metric is indeed a classical solution to the equations of motion of the WTDiff gravity, (8). A study of the Schwarzschild solution is of importance since the Schwarzschild solution corresponds to the basic one-body problem of classical astronomy, and the reliable experimental verifications of the Einstein equations are almost based on the Schwarzschild line element. Then, it is natural to ask ourselves whether the charged Reissner-Nordstrom black hole metric is also a classical solution to the equations of motion of the WTDiff gravity coupled to an electro-magnetic field or not. In this section, we will see that it is indeed the case.

Before attempting to show that the charged Reissner-Nordstrom metric is a classical solution, we must recall that an action of a $U(1)$ gauge field is invariant under the Weyl transformation only in four space-time dimensions, so henceforth we will confine ourselves to the
specific case of $n = 4$. Then, an action of the WTDiff gravity coupled to an electro-magnetic field $A_\mu$ reads

$$S = \int d^4x \left[ \frac{1}{2} |g|^{1/2} \left( R + \frac{3}{32} \frac{1}{g |g| 2} g^{\mu\nu} \partial_\mu |g| \partial_\nu |g| \right) - \frac{1}{4} |g|^{1/2} g^{\mu\nu} g^{\rho\sigma} F_{\mu\rho} F_{\nu\sigma} \right],$$

(15)

where we have defined the field strength of the $U(1)$ gauge field by $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$.

From the action (15), it is straightforward to derive the equations of motion for the metric tensor $g_{\mu\nu}$ and the gauge field $A_\mu$. The result for the metric variation is given by

$$R_{\mu\nu} - \frac{1}{4} g_{\mu\nu} R = T_{\mu\nu} - \frac{1}{4} g_{\mu\nu} T,$$

(16)

where the energy-momentum tensor $T_{\mu\nu}$ is defined as

$$T_{\mu\nu} = \frac{7}{32} \frac{1}{|g|^{2}} \partial_\mu |g| \partial_\nu |g| - \frac{1}{4} \frac{1}{|g|} D_\mu D_\nu |g| + |g|^{1/2} F_{\mu\alpha} F_{\nu}^\alpha.$$

(17)

And the Maxwell equations take the form

$$\partial_\nu (\sqrt{-g} F^{\mu\nu}) = 0.$$

(18)

Now we wish to look for a gravitational field outside an isolated, static, spherically symmetric object with mass $M$ and electric charge $Q$. In the far region from the isolated object, we assume that the metric tensor is in an asymptotically Lorentzian form

$$g_{\mu\nu} \to \eta_{\mu\nu} + \mathcal{O} \left( \frac{1}{r} \right),$$

(19)

where $\eta_{\mu\nu}$ is the Minkowski metric, and the radial coordinate $r$ is defined as

$$r = \sqrt{(x^1)^2 + (x^2)^2 + (x^3)^2} = \sqrt{(x^i)^2},$$

(20)

with $i$ running over spatial coordinates ($i = 1, 2, 3$).

Let us recall that the most spherically symmetric line element in four space-time dimensions reads [32]

$$ds^2 = -A(r)dt^2 + (dx^i)^2 + B(r)(x^i dx^i)^2,$$

(21)

where $A(r)$ and $B(r)$ are functions depending on only $r$.

From this line element (21), we can read off the non-vanishing components of the metric tensor

$$g_{tt} = -A, \quad g_{ij} = \delta_{ij} + B x^i x^j,$$

(22)

and the components of its inverse matrix are

$$g^{tt} = -\frac{1}{A}, \quad g^{ij} = \delta^{ij} - \frac{B}{1 + Br^2} x^i x^j.$$

(23)
Moreover, using these components of the metric tensor, the affine connection is calculated to be

\[
\Gamma^r_{ti} = \frac{A' x^i}{2A r}, \quad \Gamma^i_{tt} = \frac{A'}{2(1 + Br^2)} x^i, \\
\Gamma^i_{jk} = \frac{1}{2(1 + Br^2)} x^i \left( \frac{2Br \delta_{jk} + B' x^j x^k}{r} \right),
\]

(24)

where the primes on \( A(r) \) and \( B(r) \) denote the differentiation with respect to \( r \), for instance, \( A' = \frac{dA}{dr} \).

As for the electro-magnetic field \( A_\mu(x) \), we assume that it has a static, spherically symmetric electric potential,

\[
A_t = -\phi(r), \quad A_i = 0,
\]

(25)

where \( \phi(r) \) is a function of \( r \).

Next, in order to show that the Reissner-Nordstrom line element is a classical solution to the field equations (16) and (18), we would like to take a gauge condition for the Weyl transformation

\[
g = -1,
\]

(26)

which is nothing but the unimodular condition so that this gauge condition does not break the transverse diffeomorphisms (TDiff). The reason why we pick up this gauge condition (26) is the following: Look at our Einstein equations (16) where the energy-momentum tensor \( T_{\mu\nu} \) receives the contribution of the determinant of the metric tensor in addition to the electro-magnetic \( U(1) \) gauge field. The Reissner-Nordstrom metric is a solution of the Einstein equations in the vacuum in the sense that the energy-momentum tensor \( T_{\mu\nu} \) is vanishing except the part of the electro-magnetic field. The gauge condition (26) then satisfies this requirement.

With the metric tensor ansatz (21), the gauge condition (26) is reduced to the form

\[
A(1 + Br^2) = 1.
\]

(27)

Using this gauge condition (27) and Eqs. (22)-(24), the Ricci tensor and the scalar curvature can be easily calculated to be

\[
R_{tt} = \frac{1}{2} A(A'' + \frac{2}{r} A') + \frac{1}{r^2} \left[ \frac{1}{r^2} (A - 1) - \frac{A'}{r} \right] \delta_{ij} + \frac{1}{r^2} \left[ \frac{1}{r^2} (A - 1) + \frac{A'}{r} A - \frac{A''}{2 A} \right] x^i x^j,
\]

\[
R_{ij} = -A'' - \frac{4}{r} A' - \frac{2}{r^2} (A - 1),
\]

(28)
These results are used to evaluate the non-vanishing components of the traceless Einstein tensor defined as $G^T_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{4} g_{\mu\nu} R$

\[ G^T_{tt} = \frac{1}{4} A \left( A'' - \frac{2}{r^2} (A - 1) \right), \]
\[ G^T_{ij} = \frac{1}{4} \left( \delta_{ij} - \frac{A + 1}{A} x^i x^j \right) \left[ A'' - \frac{2}{r^2} (A - 1) \right]. \] (29)

First, let us solve the Maxwell equations (18). With the ansatzes (21) and (25) and the gauge condition (27), the Maxwell equations (18) are cast to a single equation

\[ \frac{d}{dr} (r^2 \phi') = 0, \] (30)

which is easily integrated to be

\[ \phi(r) = \frac{\sqrt{2} Q}{r} + c, \] (31)

where $Q$, which corresponds to an electric charge, and $c$ are integration constants. To fix the constant $c$, we will impose a boundary condition

\[ \lim_{r \to \infty} \phi(r) = 0, \] (32)

which determines $c = 0$. Thus, we obtain the final expression for $\phi(r)$

\[ \phi(r) = \frac{\sqrt{2} Q}{r}. \] (33)

Next, let us try to solve the traceless Einstein equations (16). For this purpose, we will calculate the traceless energy-momentum tensor defined as $T^T_{\mu\nu} \equiv T_{\mu\nu} - \frac{1}{4} g_{\mu\nu} T$ whose result is summarized as

\[ T^T_{tt} = A \frac{Q^2}{r^4}, \]
\[ T^T_{ij} = \left( \delta_{ij} - \frac{A + 1}{A} x^i x^j \right) \frac{Q^2}{r^4}. \] (34)

Consequently, the traceless Einstein equations (16) reduce to an equation

\[ A'' - \frac{2}{r^2} (A - 1) = \frac{4Q^2}{r^4}, \] (35)

which can be rewritten in a more manageable form

\[ \frac{1}{r} \frac{d}{dr} \left[ r \left( A - 1 - \frac{Q^2}{r^2} \right) \right] - \frac{2}{r^2} \frac{d}{dr} \left[ r \left( A - 1 - \frac{Q^2}{r^2} \right) \right] = 0. \] (36)
By performing an integration, \( A(r) \) is given by
\[
A(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2} + ar^2, \tag{37}
\]
where \( M \) and \( a \) are integration constants. From the boundary condition (19), we have to choose \( a = 0 \), and we can obtain the expression for \( B(r) \) in terms of the gauge condition (27). As a result, we reach the expressions for \( A(r) \) and \( B(r) \)
\[
A(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2}, \quad B(r) = \frac{2Mr - Q^2}{r^2(r^2 - 2Mr + Q^2)}. \tag{38}
\]
Then, the line element is of form
\[
ds^2 = -\left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right)dt^2 + (dx^i)^2 + \frac{2Mr - Q^2}{r^2(r^2 - 2Mr + Q^2)}(x^i dx^i)^2. \tag{39}
\]
Accordingly, we have succeeded in showing that the Reissner-Nordstrom metric in the Cartesian coordinate system is a classical solution in the WTDiff gravity coupled to the Maxwell field.

Here we should refer to an important remark. The Reissner-Nordstrom metric (39) in the Cartesian coordinate system can be rewritten in a more familiar form in the spherical coordinate system
\[
ds^2 = -\left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right)dt^2 + \frac{1}{1 - \frac{2M}{r} + \frac{Q^2}{r^2}} dr^2 + r^2 d\Omega^2, \tag{40}
\]
where
\[
d\Omega^2 = d\theta^2 + \sin^2 \theta d\varphi^2. \tag{41}
\]
However, this expression (40) is not a classical solution in the WTDiff gravity plus the Maxwell field. This is a notable feature of the WTDiff gravity where there is no the full group of diffeomorphisms, but only the TDiff.

4 Discussions

In this article, it is shown that the Weyl-transverse (WTDiff) gravity plus the Maxwell action has the charged Reissner-Nordstrom black hole metric as a classical solution when the metric is expressed in the Cartesian coordinate system. It is of interest to note that the conventional form of the Reissner-Nordstrom metric in the spherical coordinate system is not a classical solution. The same statement is also valid in case of the Schwarzschild black hole metric presented in our previous article [32]. This dependence on the coordinate systems of the classical solutions in the WTDiff gravity is a characteristic feature since the TDiff are defined
as a subgroup of the full diffeomorphisms such that the determinant of the transformation matrix is the unity

\[ J \equiv \det J_{\mu}^{\alpha} \equiv \det \frac{\partial x^{\alpha}}{\partial x'_{\mu}} = 1. \] (42)

When we transform the Reissner-Nordstrom metric from the Cartesian coordinate system to the spherical one, we encounter the non-trivial value of \( J \)

\[ J = r^2 \sin \theta, \] (43)

by which the Reissner-Nordstrom metric in the spherical coordinate system is not a solution. Actually, if we rewrite a flat Minkowski space-time in the spherical coordinate system, the resultant line element is not a classical solution owing to the non-trivial Jacobian factor even if it is a classical solution in the Cartesian coordinate (Here we assume \( Q = 0 \)).

It is straightforward to generalize the Reissner-Nordstrom black hole solution to a more general static, spherically symmetric solution where a black hole carries both electric and magnetic charge. A nontrivial generalization is to obtain the Reissner-Nordstrom black hole solution in arbitrary space-time dimensions except four dimensions since the conventional Maxwell action is not invariant under the Weyl transformation in such dimensions. Another interesting problem is to look for classical solutions with the property of \( g \neq -1 \). We wish to return to these problems in future.

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References

[1] Y. Nambu and G. Jona-Lasinio, Phys. Rev. 122 (1961) 345.
[2] S. Weinberg, Rev. Mod. Phys. 61 (1989) 1.
[3] A. Einstein, in ”The Principle of Relativity”, by A. Einstein et al., Dover Publications, New York, 1952.
[4] J. L. Anderson and D. Finkelstein, Am. J. Phys. 39 (1971) 901.
[5] J. van der Bij, H. van Dam and Y. J. Ng, Physica 116A (1982) 307.
[6] W. Buchmuller and N. Dragon, Phys. Lett. B 207 (1988) 292.
[7] M. Henneaux and C. Teitelboim, Phys. Lett. B 222 (1989) 195.
[8] W. Buchmuller and N. Dragon, Phys. Lett. B **223** (1989) 313.

[9] W. G. Unruh, Phys. Rev. D **40** (1989) 1048.

[10] Y. J. Ng and H. van Dam, J. Math. Phys. **32** (1991) 1337.

[11] E. Alvarez and A. F. Faedo, Phys. Rev. D **76** (2007) 064013.

[12] E. Alvarez, A. F. Faedo and J. J. Lopez-Villarejo, JHEP **0810** (2008) 023.

[13] G. F. R. Ellis, H. van Elst, J. Murugan and J.-P. Uzan, Class. Quant. Grav. **28** (2011) 225007.

[14] A. Padilla and I. D. Saltas, Eur. Phys. J. C **75** (2015) 561.

[15] I. Oda, Adv. Studies in Theor. Phys. **10** (2016) 319, arXiv:1602.00851 [gr-qc].

[16] I. Oda, arXiv:1602.03478 [gr-qc], PTEP (in press).

[17] I. Oda, arXiv:1603.00112 [gr-qc], Int. J. Mod. Phys. D (in press).

[18] K-I. Izawa, Prog. Theor. Phys **93** (1995) 615.

[19] C. Barcelo, R. Carballo-Rubio and L. J. Garay, Phys. Rev. D **89** (2014) 124019.

[20] E. Alvarez, S. Gonzalez-Martin, M. Herrero-Valea and C. P. Martin, JHEP **1508** (2015) 078.

[21] E. Alvarez, S. Gonzalez-Martin, M. Herrero-Valea and C. P. Martin, Phys. Rev. D**92** (2015) 061502.

[22] E. Alvarez, S. Gonzalez-Martin, M. Herrero-Valea and C. P. Martin, Phys. Rev. D**93** (2016) 123018.

[23] E. Alvarez, S. Gonzalez-Martin and C. P. Martin, arXiv:1604.07263 [hep-th].

[24] I. Oda, arXiv:1606.01571 [gr-qc].

[25] G. ’t Hooft, arXiv:1410.6675 [gr-qc].

[26] F. Englert, C. Truffin and R. Gastmans, Nucl. Phys. B **117** (1976) 407.

[27] M. Shaposhnikov and D. Zenhausern, Phys. Lett. B **671** (2009) 162.

[28] R. Percacci, New Jour. of Phys. **13** (2011) 125013.

[29] F. Gretsch and A. Monin, Phys. Rev. D **92** (2015) 045036.

[30] R. Carballo-Rubio, Phys. Rev. D **91** (2015) 124071.
[31] D. Benedetti, Gen. Rel. Grav. 48 (2016) 68.

[32] I. Oda, arXiv:1607.06562 [gr-qc].

[33] S. M. Caroll, "Spacetime and Geometry", Addison Wesley, 2004.

[34] C. W. Misner, K. S. Thorne and J. A. Wheeler, "Gravitation", W H Freeman and Co (Sd), 1973.

[35] I. Oda, Phys. Rev. D 87 (2013) 065025, arXiv:1301.2709 [hep-ph].

[36] I. Oda, Phys. Lett. B 724 (2013) 160, arXiv:1305.0884 [hep-ph].

[37] I. Oda, Adv. Studies in Theor. Phys. 8 (2014) 215, arXiv:1308.4428 [hep-ph].

[38] I. Oda and T. Tomoyose, JHEP 09 (2014) 165, arXiv:1407.7575 [hep-th].