Roles of crust and core in the tidal deformability of neutron stars

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ABSTRACT

With the recent measurement of GW170817 providing constraints on the tidal deformability of a neutron star, it is very important to understand what features of the equation of state have the biggest effect on it. We therefore study the contribution of the crust to the tidal deformability and the moment of inertia of a neutron star for a variety of well-known equations of state. It is found that the contributions to these quantities from the low density crust are typically quite small and as a result the determination of the tidal deformability provides an important constraint on the equation of state of dense matter.

Keywords: stars: neutron, gravitational waves, equation of state

1. INTRODUCTION

Neutron stars (NS) are of enormous interest as their cores are the repositories of the densest nuclear matter in the Universe. Exploring the nature of nuclear matter at densities many times that of nuclear matter is one of the most fascinating challenges in strong interaction physics, not least because of the uncertainties surrounding the appearance of hyperons, $\Delta$'s and even quark matter. Of course, these dense cores are surrounded by a crust which contains low density nuclear matter which is expected to exhibit fascinating phase changes Weber (2005); Setani et al. (2017); Glendenning (2000). The low density region (LDR), corresponding to the crust, poses challenges with regards to modelling because of these complex phase structures as well as the variety of finite nuclei far from stability that may appear there Chamel & Haensel (2008).

Following the recent GW170817 measurement Abbott et al. (2018a,b), there has been enormous interest in studying the tidal deformability (TD) and the moment of inertia (MoI) of NS Wei et al. (2018); De et al. (2018a); Han & Steiner (2018); Carson et al. (2019); Zhao & Lattimer (2018); Qian et al. (2018); Landry & Kumar (2018), across a variety of equations of state (EoS). Indeed, the TD is the key parameter extracted from the gravity wave observation of the NS merger and the bound produced in Refs. Abbott et al. (2018a,b); De et al. (2018b), namely $70 \leq \Lambda_{1.4} \leq 580$, has been analysed recently Landry & Kumar (2018) via the I-Love-Q relations to produce a constraint on the MoI of $8.82 \leq \bar{I} \leq 14.74$ for a NS of mass $M = 1.338M_\odot$ (where $\bar{I} = I/M^3$).

In this paper, we explore the proportional contribution that the LDR of the EoS makes to the TD and MoI for a selection of commonly used EoS which yield stars with a wide variety of radii and masses. Our conclusion is that the LDR corresponding to the crust makes a surprisingly small contribution to both the TD and the MoI and hence the constraints imposed by the analysis of the gravitational wave observations are of direct relevance to the determination of the EoS of dense matter.

2. THEORETICAL FRAMEWORK

We study eight EoS obtained from the online database in Ref. EoS Database (2018), which details all the models in question, as well as using two EoS calculated within the Quark Meson Coupling (QMC) model, discussed in Refs. Stone et al. (2007); Guichon et al. (1996); Motta et al. (2019). These EoS include examples with and without hyperons and lead to stars with maximum masses varying over the range 1.8 to 2.5$M_\odot$ and with radii between 9 and 14km, as shown in Fig. 1. This broad spread of NS properties gives us confidence that the conclusions drawn from our study are essentially model independent. To calculate the NS profile, ordinarily one would match the model EoS description of the core to some low density EoS which accounts for the NS crust Setani et al. (2017); Chamel & Haensel (2008). In the region of energy density above 100 MeV/fm$^3$, we
have fit each EoS using a polytropic form

\[ P(\epsilon) = A\epsilon^B + C\epsilon^D, \]

where \(A, B, C\) and \(D\) \(\in\) \(\mathbb{R}\). This form continued to zero density defines the EoS without LDR modelling, with the original EoS containing LDR modelling. The choice of 100 MeV/fm\(^3\) as the boundary is typical of the values usually chosen and we have checked that using 50 MeV/fm\(^3\) instead makes no significant difference to our conclusions. As we see in Fig. 1, by comparing solid and dashed curves of the same colour, the inclusion or omission of the LDR region leads to dramatically different mass versus radius curves.

These fits are then used to analyse the proportional contributions to the TD through a direct comparison of their values. This is appropriate because the TD is defined as a boundary point and not something cumulative (e.g. an integral) Postnikov et al. (2010); Thorne & Campolattaro (1967), so the direct calculation of its LDR contribution is impossible. This is equivalent to modelling a core only NS, containing just degenerate nuclear matter. The LDR contribution for the MoI may be calculated directly as it is defined as an integral Harl
tle (1967); Hartle & Thorne (1968).

The NS masses are obtained by numerically solving the Tolman-Oppenheimer-Volkoff (TOV) equations Oppenheimer & Volkoff (1939)

\[
\begin{align*}
\frac{dP}{dr} &= -\frac{(P(r) + \epsilon(r))(m(r) + 4\pi r^3 P(r))}{r(r - 2m(r))}, \\
\frac{dm}{dr} &= 4\pi r^2 \epsilon(r),
\end{align*}
\]

until the point where the pressure vanishes, \(P(r = R) = 0\) (\(R\) the radius of the NS). This procedure led to the plots of mass versus radius shown in Fig. 1, where we note the characteristic shrinking of the radius of the star at small mass in the case where it is taken to consist of degenerate nuclear matter alone.

We define the variation of a parameter \(\alpha\) as \(\Delta\alpha/\alpha\) (for non-cumulative quantities) in terms of the percentage difference of the parameter when compared to its LDR removed counterpart

\[
\frac{\Delta\alpha}{\alpha} = \frac{\alpha_{\text{with LDR}} - \alpha_{\text{without LDR}}}{\alpha_{\text{with LDR}}}. \quad \tag{4}
\]

Note that in the case of the TD this requires a choice of the parameter which is fixed for the two cases. We choose to compare the TD for stars with the same mass, since that is the physical parameter which is usually best known. The MoI, \(I\), is calculated using the methods described in Refs. Hartle (1967); Hartle & Thorne (1968).

The methods for calculating the tidal Love number, \(k_{2}^{\text{tidal}}\), TD, \(\Lambda\), and binary TD, \(\tilde{\Lambda}\), are those presented in

Figure 1. Mass radius relation for the broad range of EoS studied here. The box represents the GW170817 90% confidence constrained mass region Abbott et al. (2017, 2018a,b).
Figure 2. The percentage of the MoI arising from the LDR is shown versus the NS total MoI. As explained in the text, this is calculated using the full EoS, including the LDR. The enlargement shows the constraint $8.82 \leq \bar{I} \leq 14.74$ Landry & Kumar (2018).

Figure 3. The tidal Love numbers $k_2^{\text{tidal}}$ are shown as a function of the compactness parameters of the stars, keeping same conventions as in Fig. 1.

Figure 4. Illustration of the percentage of the TD arising from the LDR versus the NS total TD for fixed NS mass. The enlargement shows a smaller range of masses, with the black bar representing the bound $70 \leq \Lambda_{1.4} \leq 580$ Abbott et al. (2018a,b).

Figure 5. The I-Love relation is shown for all the EoS considered here, keeping same conventions as in Fig. 1.
We stress that none of the results reported here should be interpreted as suggesting that the LDR is not crucial for a number of NS properties. As discussed in detail in Ref. Piekarewicz & Fattoyev (2019) and illustrated here in Fig. 1, the LDR contributes quite significantly to the radius for a star of a given mass. Furthermore, the imminent measurement by the NICER mission of the radius of a NS reaffirms that good modelling of the LDR will prove incredibly significant in obtaining reliable predictions for such an empirical result.

4. CONCLUSION

The present study of a wide variety of EoS suggests a surprising and important result. That is, while the crust of a NS does make a very important contribution to the radius of a star of given mass and hence to the compactness parameter, its contribution to the tidal deformability and moment of inertia is at the level of 10% and 5% or less, respectively, for stars of mass above $1.4M_\odot$. This leads us to the conclusion that the measurement of binary tidal deformability in gravitational wave measurements does indeed provide a significant constraint on the EoS of dense nuclear matter.

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