Chiral Yukawa models in the planar limit.

by

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Abstract

We consider the most general renormalizable chiral Yukawa model with $SU(3)_{\text{color}}$ replaced by $SU(N_c)$, $SU(2)_L$ replaced by $SU(N_w)$ and $U(1)_Y$ replaced by $U(1)^{N_w - 1}$ in the limit $N_c \to \infty$, $N_w \to \infty$ with the ratio $\rho = \sqrt{\frac{N_w}{N_c}} \neq 0, \infty$ held fixed. Since for $N_w \geq 3$ only one renormalizable Yukawa coupling per family exists and there is no mixing between families the limit is appropriate for the description of the effects of a heavy top quark when all the other fermions are taken to be massless. A rough estimate of the triviality bound on the Yukawa coupling is equivalent to $m_t \leq 1 \, \text{TeV}$. 
In the minimal standard model at energies above 100 GeV only the scalar self-coupling $\lambda$ is completely unknown and might be strong. Skeptics\(^1\) might argue that a strong Yukawa coupling $y$ has not been yet ruled out completely and this note addresses the problem how to do calculations if they are right \([1]\).

Excepting the $\lambda$ and $y$ all other couplings are weak and can be treated perturbatively. The problem then is to set up a computation first in the case where all the weak couplings are set to zero. We have some requirements: Because of “triviality” and “vacuum stability” issues one ought to be able to keep an explicit cutoff in the calculations. The cutoff has to be defined in a reasonable manner, meaning that an expansion in inverse powers of the cutoff can be set up and has the normal structure one would expect of terms induced by a higher embedding theory (for example, a sharp momentum cutoff is not acceptable). We do not want to use the lattice since many of the non–perturbative effects in lattice models have little to do with continuum physics.\(^2\) Intuitively we feel that a sensible approximation should keep more or less a democratic view of fermionic and scalar excitations.

We propose that a good way to achieve the above is to consider a model with a single Higgs multiplet where $SU(3)_{\text{color}}$ is replaced by $SU(N_c)$, $SU(2)_L$ is replaced by $SU(N_w)$ and $U(1)_{Y}$ is replaced by $U(1)^{N_w-1}$ in the limit $N_c \to \infty$, $N_w \to \infty$ with the ratio $\bar{\rho} = \sqrt{\frac{N_w}{2N_c}} \neq 0, \infty$ held fixed. Just letting $N_c(N_w)$ go to infinity with $N_w(N_c)$ fixed suppresses bosons (righthanded fermions) and these limits may distort effects like vacuum destabilization \([2]\) or bag formation \([3]\). In any case more diagrams get summed up when both $N_c$ and $N_w$ go to infinity and all diagrams you would get when only $N_c$ or $N_w$ go to infinity are included.

We introduce the model in Euclidean space; with a finite cutoff the model is non-perturbatively well defined but the continuation to Minkowski space would introduce unitarity violating effects at energies of the order of the cutoff. This drawback is compensated by preserving translational and full rotational invariance with propagators regulated in a Pauli–Villars manner. The action is given by $S$:

\[
-S = \int_x \bar{\psi}_{-}^{ia(r)}\phi^{ia(r)} + \chi_{+}^{a(r)s}h_{\chi}(-\partial^2)\phi_{-}^{a(r)s} + \phi^{is}h_{\phi}(-\partial^2)\phi^{i} - m_0^2\phi^{is}\phi^{i} - \frac{\lambda_0}{4N_w} (\phi^{is}\phi^{i})^2
\]

\[+
\frac{1}{\sqrt{N}} \sum_s \left[ \bar{\psi}_{-}^{ia(r)^s}\phi^{i}A^{(s)}_{rr'} + \chi_{-}^{a(r)^s}\phi^{i}A^{(s)}_{rr'} + \bar{\psi}_{+}^{ia(r')^s}\phi^{i}A^{(s)}_{rr'} \right] \]

\(1\) The assumption that the top Yukawa coupling is relatively weak is consistent with present knowledge indicating that $m_t \leq \sim 200$ GeV.

\(2\) The lattice introduces in a smooth manner a finite bottom to the Dirac sea and this is particularly disturbing to chiral fermions. One could however use our technique to study non–perturbative effects in lattice models and hopefully sort out the situation there.
\[ N = \sqrt{N_w N_c}, \quad \phi_+ = \sigma_\mu \partial_\mu, \quad \phi_- = \bar{\sigma}_\mu \partial_\mu \]

\[ \bar{\sigma}_1 = \sigma_1, \quad \bar{\sigma}_2 = \sigma_2, \quad \bar{\sigma}_3 = \sigma_3, \quad \bar{\sigma}_4 = -\sigma_4 = -i \]

\[ i = 1, \ldots, N_w \quad a = 1, \ldots, N_c \quad r = 1, \ldots, n_f \quad s = 1, \ldots, N_w \]

The \( \sigma_{1,2,3} \) are Pauli matrices. The functions \( h_\chi \) and \( h_\phi \) can be chosen in many ways and introduce the regularization. Ultraviolet divergences can be cured by just regularizing the \( \chi \) and \( \phi \) propagators because there are no loops consisting of only \( \psi \) propagators. When in need of an explicit form we shall use \( h_\chi(p^2) = h_\phi(p^2) = 1 + (p^2/\Lambda^2)^n \) with a sufficiently large \( n \).

The symmetry properties are as follows: The space–time group is \( SU(2) \times SU(2) \) and a spinor with subscript \(+\) transforms as \((\frac{1}{2},0)\) while one with subscript \(−\) transforms as \((0,\frac{1}{2})\). \( \bar{\psi}, \psi, \bar{\chi}, \chi \) are two component Grassmann spinors and complex conjugation does not act on them. Under \( SU(N_c) \times SU(N_w) \) the representations are \( \bar{\psi}_- \sim (\bar{N}_c, \bar{N}_w), \quad \psi_+ \sim (N_c, N_w), \quad \bar{\chi}_- \sim (\bar{N}_c, 1), \quad \chi_+ \sim (N_c, 1), \quad \phi^* \sim (1, \bar{N}_w), \quad \phi \sim (1, N_w) \). This representation content is independent of the indices \( r \) and \( s \) whenever they appear; the index \( r \) runs over families and \( s \) over the number of fermionic weak singlets (and color multiplets) per family. The representations under the remaining \( U(1) \)'s are chosen subject to anomaly considerations when gauging is contemplated and eliminate all but one of the matrices \( A^{(s)} \). The non–vanishing matrix is chosen to be \( A^{(1)} \) and the remaining \( \chi \) fields with index \( s \geq 2 \) decouple at zero gauge couplings. When \( N_w = 2 \) an additional set of Yukawa couplings is allowed by \( SU(N_c) \times SU(N_w) \) and also by the \( U(1) \)'s. These couplings do not generalize to \( N_w \geq 3 \) and will henceforth be ignored. The matrix \( A^{(1)}_{\psi \phi^*} \) can be made diagonal with positive entries by a bi–unitary transformation, decoupling the families. In conclusion one can ignore the \( r \) and \( s \) indices, which is just as well because we want to keep only one potentially heavy “top” color multiplet. The model then becomes [1]:

\[
-S = \int_x \bar{\psi}_-^a \phi_+^a + \bar{\chi}_-^a h_\chi (-\partial^2) \phi_-^a + \phi^* h_\phi (-\partial^2) \phi^* - m_0^2 \phi^* \phi^* - \frac{\lambda_0}{4N_w} (\phi^* \phi^*)^2 \\
+ \frac{g_0}{\sqrt{N}} [\bar{\psi}_-^a \chi_-^a \phi^i + \bar{\chi}_+^a \psi_+^a \phi^i]; \quad g_0 > 0.
\]

When \( N \to \infty \) with \( \bar{\rho} \equiv \frac{N_c}{\sqrt{2N_w}} \) held fixed the dominating diagrams are planar. Renormalizability allows us to get away with not having to introduce four Fermion interactions and with them an insoluble planar diagram problem, at no cost to generality.\(^3\) Thus the model is manageable being similar diagrammatically to two dimensional QCD with matter at large \( N_c \). The large \( N \) limit can be found by looking at the structure of planar Feynman graphs: essentially one sums over all “cactuses of bubbles” and all “rainbows”. For future work it is somewhat preferable to use functional integral manipulations to the same end.

In this note we wish to show feasibility within a simplest nontrivial example. We set our modest goal to compute a \( \beta \)–function associated with the Yukawa coupling constant defined in some simple manner and ignoring cutoff effects. This can be done with relative

\(^3\) Except for possible cutoff effects.
ease because even at infinite order in the couplings the flow of this Yukawa coupling is unaffected by $\lambda_0$, similarly to the finite $N$ form at one loop order. From this calculation a rough estimate for the “triviality” bound on the top mass can be obtained.\footnote{In practice this bound might be somewhat uninteresting because it is likely to be higher than the vacuum stability bound for reasonable Higgs masses.}

To find $\beta$ functions it suffices to solve the model at criticality when all masses vanish. We start in the symmetric phase and pick a mass independent renormalization scheme to make the zero mass limit smooth.

We introduce bilocal fields,

\begin{equation}
S_{\alpha,\beta}(x,y) = \frac{1}{N^c} \bar{\chi}_{\alpha}(x) \chi_{-\beta}(y), \quad K(x,y) = \frac{1}{N^w} \phi^*(x)\phi(y),
\end{equation}

where $\alpha, \beta$ are space–time spinor indices, into the functional integral by writing representations for the appropriate $\delta$–functions with the help of two auxiliary bilocal fields $\lambda_{\alpha,\beta}(x,y)$ and $\mu(x,y)$. We can integrate out now all the original fields, including the zero mode of the scalar field because we assumed that we are in the symmetric phase. This makes the $N$ and $\bar{\rho}$ dependence explicit and, when $N \to \infty$, the integral over $S, \lambda, K$ and $\mu$ is dominated by a saddle point which has to satisfy the following equations:

\begin{align}
K(x,y) &= \left\{-h_{\phi}(-\partial^2)\partial^2 + m_0^2 + \mu \right\}_{y,x} \\
S_{\alpha,\beta}(x,y) &= \left\{h_{\chi}(\partial^2)\hat{\phi} + \lambda \right\}_{y,\alpha} \\
\lambda_{\alpha,\beta}(x,y) &= -g^2 \bar{\rho} \sqrt{2} \{\phi^+\}_{x,\alpha,y,\beta} K(x,y) \\
\mu(x,y) &= \frac{g^2}{\bar{\rho} \sqrt{2}} \sum_{\alpha,\beta} \{\phi^+\}_{x,\alpha,y,\beta} S_{\alpha,\beta}(x,y) + \frac{\lambda_0}{2} K(x,x) \delta_{x,y}
\end{align}

We go to Fourier space and use translational and rotational invariance to write

\begin{equation}
\hat{\lambda}_{\alpha,\beta}(p) = ip_\mu \tilde{\sigma}_\mu \hat{\lambda}(p^2) \quad \hat{\mu}(p) = \hat{\mu}(p^2)
\end{equation}

Let us introduce $Z_\chi$ and $Z_\phi$, wave function renormalization constants, and new functions $f = Z_\chi[\hat{\lambda}(p^2) + h_{\chi}(p^2)]$ and $g(p^2) = Z_\phi[p^2 h_{\phi}(p^2) + m_0^2 + \hat{\mu}(p^2)]$. Since at the planar level there are no vertex corrections to the Yukawa coupling\footnote{This is also true at one loop order at finite $N$.} it makes sense to define a renormalized Yukawa coupling by $g_R^2 = Z_\phi Z_\chi g_0^2$, ($\alpha \equiv m_0^2/16\pi^2$). We also set $\lambda_0 = \lambda_0 Z_\phi^2$ but, unlike $\alpha$, do not expect $\lambda_0$ to stay finite when $\Lambda \to \infty$. $Z_\chi$ and $Z_\phi$ are defined in terms of the bare couplings by $f(\mu^2) = g(\mu^2)/\mu^2 = 1$.

After performing the angular part of the momentum integrals one gets when the products in the saddle point equations for $x,y$ dependent quantities are converted into
convolutions in momentum space one finds:

\[ f(p^2) = Z_\chi h_\chi(p^2) + \sqrt{2\alpha \bar{\rho}} \left[ \frac{1}{2} \int_{p^2}^\infty \frac{dk^2}{g(k^2)} + \frac{1}{p^2} \int_0^{p^2} \frac{k^2 dk^2}{g(k^2)} - \frac{1}{2p^2} \int_0^{p^2} \frac{k^4 dk^2}{g(k^2)} \right] \]

\[ g(p^2) = Z_\phi p^2 h_\phi(p^2) + \sqrt{2\alpha \bar{\rho}} \left[ \frac{p^2}{2} \int_{p^2}^\infty \frac{dk^2}{f(k^2)} + \int_0^{p^2} \frac{dk^2}{f(k^2)} - \frac{1}{2p^2} \int_0^{p^2} \frac{k^2 dk^2}{f(k^2)} \right] + m^2 \]  

(7)

The massless case is obtained by adjusting \( m_0^2 \) so that \( m^2 = 0 \) and then it makes sense to define \( h(p^2) = \frac{f(p^2)}{p^2} \) leading to a set of equations of a more symmetric appearance:

\[ h(p^2) = Z_\phi h_\phi(p^2) + \sqrt{2\alpha \bar{\rho}} \mathcal{L}[f](p^2) \]

\[ f(p^2) = Z_\chi h_\chi(p^2) + \sqrt{2\alpha \bar{\rho}} \mathcal{L}[h](p^2) \]  

(8)

Note the interesting bosonic–fermionic symmetry at \( \bar{\rho} = 1 \). We now rescale the functions and the momentum variable by:

\[ f(u\mu^2) = \sqrt{2\alpha \bar{\phi}}(u) \quad h(u\mu^2) = \sqrt{\alpha \bar{\psi}}(u) \]  

(9)

and obtain:

\[ \bar{\psi}(u) = \frac{1}{\sqrt{\alpha}} + \frac{Z_\phi}{\sqrt{\alpha}}[h_\phi(u\mu^2) - h_\phi(\mu^2)] + \frac{1}{\bar{\rho}} \int_u^1 \frac{dv}{v} \int_0^1 \frac{(1-x)dx}{\phi(xv)} \]

\[ \bar{\phi}(u) = \frac{1}{2\sqrt{\alpha}} + \frac{Z_\chi}{2\sqrt{\alpha}}[h_\chi(u\mu^2) - h_\chi(\mu^2)] + \bar{\rho} \int_u^1 \frac{dv}{v} \int_0^1 \frac{(1-x)dx}{\psi(xv)} \]  

(10)

It is clear that when \( n \) is large enough \( h \) and \( f \) are dominated by the \( h \) functions in the ultraviolet and therefore the large cutoff behavior of the wave function renormalization constants is such that in the infinite cutoff limit at \( u \) and \( \mu^2 \) fixed \( Z_\phi \) and \( Z_\chi \) simply disappear from the above equation leaving us with

\[ \bar{\psi}(u) = \frac{1}{\sqrt{\alpha}} + \frac{1}{\bar{\rho}} \int_u^1 \frac{dv}{v} \int_0^1 \frac{(1-x)dx}{\bar{\phi}(xv)} \]

\[ \bar{\phi}(u) = \frac{1}{2\sqrt{\alpha}} + \bar{\rho} \int_u^1 \frac{dv}{v} \int_0^1 \frac{(1-x)dx}{\bar{\psi}(xv)} \]  

(11)
On physical grounds it is obvious that we wish that $\psi$ and $\phi$ be positive. The equations above show that if this is true in some interval $(0, u_*)$ then the functions will be monotonically decreasing there. However, the equation do not admit an asymptotic behavior as $u \to \infty$ with both functions approaching non-negative limits so the positivity requirement must get violated somewhere in the ultraviolet.\footnote{A zero of $\bar{\psi}$ or $\bar{\phi}$ at a positive $u$ corresponds to a pole in a two-point function at an Euclidean momentum; the “particle” associated with this pole would be tachyonic.} Starting at the scale where the violation first occurs cutoff effects cannot be neglected anymore. This “unphysical” scale is the usual Landau pole, this time appearing in a nonperturbative approximation.

In the infrared there is no Landau pole problem and cutoff effects are indeed negligible. To investigate the behavior there we set $x = -\log(u)$ and $\hat{\psi}(x) = \bar{\psi}(u)$, $\hat{\phi}(x) = \bar{\phi}(u)$ and derive:

\[ \begin{align*}
D\hat{\psi} &= \frac{1}{\bar{\rho}\bar{\phi}} \\
D\hat{\phi} &= \frac{\bar{\rho}}{\bar{\psi}} \\
D &= d^3 - 3\frac{d^2}{dx^2} + 2\frac{d}{dx} \left( \frac{d}{dx} - 1 \right) \left( \frac{d}{dx} - 2 \right)
\end{align*} \tag{12} \]

These equations allow us to extract the infrared behavior of the solution of the integral equations. We ended up with only differential equations (rather than integral) reflecting that according to the Renormalization Group one needs only very limited information at a given scale in order to derive the behavior at a scale close by; therefore a differential equation must show up eventually, its order less the number of asymptotic conditions determining the amount of information at a given scale that is needed to determine the behavior at the next scale. Our choice of regularization was made so that even at finite cutoff $\bar{\phi}$ and $\bar{\psi}$ obey a set of purely differential equations of a structure similar to (12).

Since the fixed point governing the infrared behavior is the free field fixed point the form of the solutions in the infrared simply embodies the two anomalous dimensions associated with the $\chi$ and $\phi$ fields when expanded in $\alpha$ around $\alpha = 0$. These anomalous dimensions as a function of the coupling also determine the $\beta$ function and from the asymptotic series of $\hat{\psi}(x)$ and $\hat{\phi}(x)$ at $x \to \infty$ we can get the contributions to the $\beta(\alpha)$ function ordered in the number of loops.

The differential equations lead to:

\[ \begin{align*}
\hat{\psi} &\sim \psi_0 x^b [1 + c \frac{\log x}{x} + c' \frac{1}{x} + \cdots] \\
\hat{\phi} &\sim \phi_0 x^a [1 + d \frac{\log x}{x} + d' \frac{1}{x} + \cdots]
\end{align*} \tag{13} \]
where
\[ a = b \bar{\rho}^2 = \frac{\bar{\rho}^2}{1 + \bar{\rho}^2} \]
\[ d = c \bar{\rho}^2 = -\frac{3\bar{\rho}^4}{(1 + \bar{\rho})^3} \] (14)
\[ \frac{d'}{\bar{\rho}^2} - c' = \frac{3(1 - \bar{\rho}^2)}{2(1 + \bar{\rho}^2)^2}; \quad \psi_0\phi_0 = \frac{\bar{\rho} + \bar{\rho}^{-1}}{2} \]

Only the product \( \psi_0\phi_0 \) and a particular linear combination of \( d' \) and \( c' \) get determined because the equations are invariant under a field rescaling \( \hat{\psi} \rightarrow A^{-1}\hat{\psi}, \hat{\phi} \rightarrow A\hat{\phi} \) reflecting a change in the finite parts of the wave function renormalization constants and a shift \( x \rightarrow x + x_0 \) reflecting a change in \( \mu^2 \). The field–rescaling invariance disappears in the product \( r(u) \equiv \sqrt{2}\bar{\psi}(u)\bar{\phi}(u) \) and this combination is indeed special because it satisfies a Renormalization Group equation in which only the \( \beta \)–function appears (the sum of the two anomalous dimensions rather than each individual one):

\[ \left[ -\frac{\partial}{\partial t} + \beta(\alpha) \frac{\partial}{\partial \alpha} \right] r(e^{2t}, \alpha) = 0 \] (15)

This equation is exact in our limit. From it we derive another exact relation between \( r \) and \( \beta \):

\[ \beta(\alpha(u\mu^2)) = -2[\alpha(u\mu^2)]^2 \frac{dr(u)}{d(\log u)} \bigg|_{r(u) = \frac{1}{\alpha(u\mu^2)^2}} \] (16)

The argument of \( \alpha \) in the equation above can of course be ignored.

Using now the asymptotic expansion of the exact solution in the infrared we get:

\[ \beta(\alpha) = \alpha^2[\sqrt{2}(\bar{\rho} + \bar{\rho}^{-1}) - 3\alpha + O(\alpha^2)] \] (17)

The one loop result when expressed in a conventional variable, \( y(t) \) (in the standard model the top mass at tree level is \( m_t = y(\sim 0) \ 246 \ [GeV] \)), is, with the true value \( \bar{\rho} = \frac{1}{\sqrt{3}} \), \( \frac{dy^2}{dt} = \frac{y^4}{\pi^2} \) which is close to the finite \( N \) result, \( \frac{dy^2}{dt} = \frac{9y^4}{8\pi^2} \). The difference comes from the wave function renormalization of the lefthanded \( \psi \) field which is suppressed at leading order in \( \frac{1}{N} \). For arbitrary \( N_c \) and \( N_w \) we would have gotten at one loop order \( \frac{dy^2}{dt} = \frac{(2N_c+N_w+1)y^4}{8\pi^2} \) and we see that the relative magnitude of the \( 1/N \) correction is much smaller than in the scalar case [4].

To get the full \( \beta \)–function at infinite \( N \) we solve for \( \hat{\psi} \) and \( \hat{\phi} \) numerically by iterating an integral form of the equations based on (7) but with the normalization conditions at \( \mu^2 \) incorporated; this avoids the appearance of double integrals like in (11-12). The procedure is probably safer than using the third order differential equations and trying to enforce boundary conditions at \( x \rightarrow \infty \) that eliminate the exponentially growing components \( e^x \) and \( e^{2x} \) (see the last line in equation (12)). The result for \( \beta \) is shown in the figure and one again sees explicit evidence for the nonperturbative existence of a Landau pole. It is educational to compare the Landau pole energy obtained from the exact result to the
one loop estimate. For example, with $\alpha(\mu^2) = 1.365$ we get $\Lambda_{\text{Landau}}^2 = 2\mu^2$ while at one loop we get $\Lambda_{\text{Landau}}^2 = 1.565\mu^2$. The value of $\alpha(\mu^2)$ we chose is very high because cutoff effects ought to be substantial when $\mu$ is so close to $\Lambda_{\text{Landau}}$. A top quark corresponding to such a strong coupling would have a mass of $m_t = 4\pi\sqrt{\alpha_{12}246} \text{[GeV]} \approx 1043 \text{GeV}$. We see that “triviality bounds” on the top will likely come out close to the perturbative unitarity bounds of Chanowitz et al. [5] a property of our approximation that we view as an improvement on the work of Einhorn and Goldberg [6] who obtained 5600 GeV. Our estimate is close to that obtained from simple variants of large $N_w(N_c \text{ fixed})$ or large $N_c(N_w \text{ fixed})$ expansions [7]; this agreement holds essentially because a truncation of the set of planar diagrams to one loop turns out to be relatively acceptable numerically. From our non-perturbative result in the figure one sees that the one and two loop errors in $\beta$ are about comparable for $\alpha \approx 0.4$ (roughly of order 20 percent) and that using the one loop results all the way out to infinity is not very harmful, but the two loop result would be completely misleading if extrapolated beyond its region of validity, to $\alpha \approx 1$. A top mass of 200 GeV would correspond to $\alpha \approx 0.05$, well within the perturbative domain.

We hope to work out a more comprehensive analysis of the $N = \infty$ limit in the future, starting with a calculation of a $\beta$–function associated with $\lambda_0$.

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Figure Caption: The nonperturbatively determined $\beta$–function for the Yukawa coupling at $N_c, N_w \to \infty$ with $\frac{N_c}{N_w} = \frac{3}{2}$. At tree level the top quark mass is given by $m_t = 4\pi\sqrt{\alpha_{12}246} \text{[GeV]}$; $\beta(\alpha) = \frac{d\alpha}{dt}$ where $t$ is the logarithm of the energy scale.

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