Parameter Estimation of Muskingum Model based on Whale Optimization Algorithm with Elite Opposition-based Learning

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Abstract. In order to solve the problem of complexity and poor accuracy for parameter estimation of Muskingum model, the Whale Optimization Algorithm with Elite Opposition-based Learning (EOWOA) was established to estimate the parameters of Muskingum model. EOWOA used the elite opposition-based learning to improve the whole search capability of WOA, and the evolution process can be quickened, moreover the convergence speed and accuracy was also improved. And the benchmark function tests demonstrated that EOWOA outperforms DE, PSO and WOA. Finally, the results of application showed that IWOA can effectively estimate the parameters of Muskingum model, and the precision of this method wins great satisfaction, thus to provide a new way in the field of channel flood routing.

1. Introduction
River flood routing is a flood forecasting method which deduces the downstream water regime according to the upstream water regime of the river. It can be divided into hydraulic and hydrological methods. The hydraulic method is based on the solution of St Vennant's equations. It has become an important method for river sections with more accurate river topography and river bed observation data [1]. However, when the data conditions are scarce and fast calculation is required, it is simple. The easy and accurate hydrological method is also an important method for flood calculation. In hydrology, the Muskingum model is the most commonly used method in river flood routing. Its parameters have an important influence on the practical application of the method. Therefore, the first task of applying the Muskingum model is to estimate the model parameters quantitatively. Traditional methods for determining parameters include trial-and-error method and least squares method [2]. However, due to the approximation of flood simulation and the limitation of traditional methods, it is difficult to obtain the optimal parameters. The optimal estimation of parameters in Muskingum model is actually a non-linear parameter optimization problem.

Whale optimization algorithm (WOA) is a new swarm intelligence optimization algorithm proposed by Seyedali Mirjalili and Andrew Lewis in 2016 [3]. Its inspiration comes from the observation and Simulation of whale population predation behaviour. WOA has many advantages, such as simple structure, easy realization, few control parameters and fast searching speed. But in practical application, when particle information and individual extreme value information are dominant, WOA is also easy to fall into local optimal solution. In this paper, elite opposition-based learning [4] is introduced to update the individuals trapped in local optimal, and whale optimization algorithm with elite opposition-based learning (EOWOA) is proposed. The proposed algorithm is applied to the parameters estimation of Muskingum model. Finally, the case study shows that EOWOA is able to effectively estimate the parameters of Muskingum model.
2. Muskingum model
Muskingum model is an effective and widely used channel storage function in flood routing [2]. Its basic equation is as follows:

\[
\begin{align*}
\frac{dW}{dt} &= I - Q \\
W &= KQ' = K[xI + (1 - x)Q]
\end{align*}
\]

where \( W \) is the tank storage, \( I \) and \( Q \) are the inflow and outflow, \( Q' \) is the storage flow, \( x \) is the specific gravity factor of the river, \( K \) is the storage constant and has the dimension of time. Based on the Equation (1), the Muskingum flow calculating equation can be obtained as follows:

\[
Q_2 = C_0 \cdot I_2 + C_1 \cdot I_1 + C_2 \cdot Q_1
\]

In the formula, \( Q_1 \) and \( Q_2 \) are respectively the outflow at the beginning and end of the period, \( I_1 \) and \( I_2 \) are respectively the inflow at the beginning and end of the period, and \( C_0, C_1 \) and \( C_2 \) are the flow calculating coefficients as follows:

\[
C_0 = \frac{1}{2} \Delta t - Kx, \quad C_1 = \frac{1}{2} \Delta t + Kx, \quad C_2 = \frac{K - Kx - \frac{1}{2} \Delta t}{K - Kx + \frac{1}{2} \Delta t}
\]

\[
C_0 + C_1 + C_2 = 1
\]

From the Equation (2), we can see that the parameters determined by Muskingum model are \( C_0, C_1 \) and \( C_2 \). So in this paper, the flow calculating coefficients are optimized directly. Based on the minimum square sum of the deviation between the calculated and the actual outflow, the optimal parameters can be obtained, and then \( K \) and \( x \) can be calculated back to calculate different periods. Hence, the optimization objective function for parameter estimation of Muskingum model is shown as follows:

\[
\min f = SSQ = \sum_{i=2}^{n} \tilde{Q}_i - (C_0 \cdot I_1 + C_1 \cdot I_{i-1} + C_2 \cdot Q_{i-1})
\]

where \( \tilde{Q}_i \) is the measured flow at the period time \( i, i=1,2,\ldots,n \), and \( n \) is the total number of calculation periods. Considering Equation (2) is a constrained complex nonlinear continuous variable optimization problem, here we use the proposed algorithm to solve the problem and compare the results with other methods.

3. Whale optimization algorithm with elite opposition-based learning (EOWOA)

3.1. whale optimization algorithm (WOA)
WOA is a new proposed stochastic optimization algorithm, which utilizes a population to determine the global optimum for optimization, and the main difference between WOA and other algorithms is the operations that improve the solutions in each step of optimization. In fact, WOA mimics the hunting behaviour of hump back whales in finding and attacking preys called bubble-net feeding behaviour. The population of WOA contains \( N \) \( D \)-dimensions real-valued parameter vectors in the population. In \( g \) generation, the position of \( i \) individual is the vector \( X^g_i = (x^g_{i,1}, x^g_{i,2}, \ldots, x^g_{i,D}) \), and
the global optimal location in population so far is expressed as $X_{\text{best}}^g = (x_{\text{best},1}^g, x_{\text{best},2}^g, \ldots, x_{\text{best},D}^g)$. And the main mathematical equation in WOA is as following:

$$
X_{i}^{g+1} = \begin{cases} 
X_{\text{best}}^g - A \cdot |X_{\text{best}}^g - X_i^g|, & p < 0.5, |A| \geq 1 \\
X_{\text{rand}}^g - A \cdot |X_{\text{rand}}^g - X_i^g|, & p < 0.5, |A| < 1 \\
D' \cdot e^{bl} \cdot \cos(2\pi L) + X_{\text{best}}^g, & p \geq 0.5
\end{cases}
$$

(6)

where $p$ is a random number between 0 and 1, $A = 2a \cdot r - a$, $C = 2 \cdot r$, $a$ linearly decreases from 2 to 0 over the course of generation, and $r$ is as random vector between 0 and 1, $X_{\text{rand}}^g$ is a randomly selected whale position vector. When $|A|$ is more than 1, the algorithm randomly selects a search agent to update the location of other whales according to the randomly selected whale position, so as to find a more suitable prey, which can strengthen the exploration ability of the algorithm for global search. Moreover, $D' = |X_{\text{best}}^g - X_i^g|$ and indicates the distance of $i$ individual, $b$ is a constant for defining the shape of the logarithmic spiral, and $L$ is a random number between -1 and 1. For more information about WOA and detailed descriptions, it can be refer to Reference [3].

3.2. Elite opposition-based learning

According to the probability theory, the generated solution compared with the opposite solution, has a probability of 50% away from the optimal solution of the problem. The opposition-based learning strategy by math and proved the opposite candidate solution is close to the global optimal solution. So a new algorithm named whale optimization algorithm with elite opposition-based learning is proposed, so as to keep the advantage information and increase the local exploration ability.

Assuming in the $g$ generation, for $x_{i,j}^g$, its opposition-based number can be defined as by

$$
\bar{x}_{i,j}^g = a_j + b_j - x_{i,j}^g
$$

(7)

where $a_j$ and $b_j$ is the maximum and minimum value for the whales in the $j$ dimensional at the $g$ generation during the global search.

Of course, the idea of elite opposition-based learning on the basis of general opposition-based learning strategy is proposed for elite whales. The comparative experiments on a series of function optimization tests show that elite opposition-based learning can greatly enhance the performance. From the process of elite opposition-based learning, on the one hand, it uses the population transformation mechanism of opposition-based learning strategy to enhance the diversity of the population and reduce the probability of falling into the local optimal value; on the other hand, it fully absorbs the useful search information of elite individuals in the current population, and can accelerate the convergence speed. Thus, elite opposition-based learning strategy can enhance the performance of traditional WOA.

3.3. Flowchart of EOWOA

With the basic WOA and the improvements by elite opposition-based learning, the flowchart of EOWOA proposed is as follows:

**Step 1:** Initialization. Set generation $g = 0$, and initialize the population space.

**Step 2:** Implement the WOA operation for each individual with Equation (6) in population space.

**Step 3:** If the generation reaches the specified item, elite opposition-based learning strategy is carry out for elite individual.

**Step 4:** If $g$ equal the given maximum generation, export the optimal solution, otherwise $g = g + 1$ and turn **Step 2**.
4. Benchmark Function Tests
In order to demonstrate the effectiveness of EOWOA, four well-known Benchmark functions with their global optimum value are mentioned in Table 1. Additionally, WOA, DE [5] and PSO [6] and are used to show the improvement of the proposed algorithm.

| Function   | Expression                                                                 | Dim | Shift position   | Optimal value |
|------------|-----------------------------------------------------------------------------|-----|------------------|---------------|
| Sphere     | \( f_1(x) = \sum_{i=1}^{D} x_i^2 \)                                                                                         | 30  | [-100,100]       | 0             |
| Schwefel 1.2 | \( f_2(x) = \sum_{i=1}^{D} \left( \sum_{j=1}^{i} x_j \right)^2 \)                                                          | 30  | [-100,100]       | 0             |
| Schwefel 2.22 | \( f_3(x) = \sum_{i=1}^{D} |x_i| + \prod_{i=1}^{D} |x_i| \)                                                                      | 30  | [-5.12,5.12]     | 0             |
| Step       | \( f_4(x) = \sum_{i=1}^{D} (x_i + 0.5)^2 \)                                                                              | 30  | [-600,600]       | 0             |

The parameters of WOA and EOWOA are \( NP=50, D=30, G_{max}=1000 \), and elite opposition-based learning strategy is carried out every 5 generations. The parameters of DE is \( F=CR=0.4 \). The parameters of PSO is \( w_{\text{max}} = 0.9, \ w_{\text{min}} = 0.2, \ c_1 = c_2 = 2 \). The test results are shown in table 2 and each function is independently processed 40 times. And the convergence curves for each function are respectively shown in Fig.1-Fig.4.

Seen from Table 2 and Fig.1-Fig.4, EOWOA outperforms DE, PSO and WOA in solving all functions, and EOWOA can avoid premature effectively and get better convergence precision.
Table 2 Test function results for DE, PSO, WOA and EOWOA

| Function   | Algorithm | Best       | Worst      | Average    | Standard Deviation |
|------------|-----------|------------|------------|------------|-------------------|
| Sphere     | DE        | 3.7280e-19 | 1.8601e-18 | 1.1541e-18 | 7.1841e-19        |
|            | PSO       | 2.4123e-13 | 5.9875e-12 | 3.6149e-12 | 5.5198e-13        |
|            | WOA       | 3.2343e-158| 1.8840e-155| 9.0390e-156| 1.0308e-155       |
|            | EOWOA     | 2.6539e-248| 2.6478e-237| 1.0321e-237| 0                 |
| Schwefel 1.2| DE       | 1.7946e+04 | 2.7462e+04 | 2.2835e+04 | 4.9195e+03        |
|            | PSO       | 2.7843     | 10.9282    | 6.8686     | 3.4712            |
|            | WOA       | 6.0151     | 2.6648e+04 | 1.6105e+04 | 1.1342e+04        |
|            | EOWOA     | 1.2763e-14 | 5.3773e-11 | 1.3523e-11 | 1.3523e-11        |
| Schwefel 2.22| DE       | 1.6730e-11 | 2.4128e-11 | 1.9242e-11 | 3.4317e-12        |
|            | PSO       | 1.0652e-06 | 10.1254    | 4.0120     | 5.1248            |
|            | WOA       | 3.1932e-90 | 7.5380e-87 | 1.9080e-87 | 3.7534e-87        |
|            | EOWOA     | 4.2252e-146| 9.3989e-141| 2.4370e-141| 4.6441e-141       |
| Step       | DE        | 1.4972e-18 | 6.0812e-18 | 3.0670e-18 | 2.6114e-18        |
|            | PSO       | 3.0564e-13 | 1.4236e-11 | 5.1212e-12 | 7.8977e-12        |
|            | WOA       | 0.0106     | 0.0064     | 0.0039     | 0                 |
|            | EOWOA     | 9.6454e-21 | 4.6622e-20 | 2.7616e-20 | 1.8510e-20        |

5. Case study of Parameter Estimation of Muskingum Model via EOWOA
An example of the 1960 flood in the south canal of Haihe River Basin is presented in this section [2], and the flow calculation coefficients in the Muskingum model are optimized by using Trial method (TM), Direct selection method (DSM), DE, and the proposed method EOWOA. We denoted the inflow as $I$, observed outflow as $Q$, and the computed outflow through Muskingum model with Equation (5) via different methods, i.e. TM, DSM, AADE and EOWOA is $\hat{Q}_1$, $\hat{Q}_2$, $\hat{Q}_3$, and $\hat{Q}_4$, respectively. The comparisons of observed and computed outflow as well as the optimal parameter estimation results based on these four methods are presented in Table 3 and Table 4.

Table 3 Comparisons of the Observed and Computed Outflows by different methods

| Time(h) | $I$ | $Q$ | $\hat{Q}_1$ | $\hat{Q}_2$ | $\hat{Q}_3$ | $\hat{Q}_4$ | $(\hat{Q}_1 - Q)^2$ | $(\hat{Q}_2 - Q)^2$ | $(\hat{Q}_3 - Q)^2$ | $(\hat{Q}_4 - Q)^2$ |
|---------|----|----|-------------|-------------|-------------|-------------|---------------------|---------------------|---------------------|---------------------|
| 0       | 75 | 75 | 75          | 75          | 75          | 75          | 0                   | 0                   | 0                   | 0                   |
| 6       | 407| 80 | 152         | 114         | 117         | 108         | 5184                | 1156                | 1376                | 770                 |
| 12      | 1693| 440| 645         | 521         | 529         | 498         | 42025               | 6561                | 7850                | 3378                |
| 18      | 2320| 1680| 1596        | 1618        | 1606        | 1596        | 7056                | 3844                | 5550                | 6974                |
| 24      | 2363| 2150| 2164        | 2236        | 2223        | 2209        | 196                 | 7396                | 5314                | 3440                |
| 30      | 1867| 2280| 2206        | 2293        | 2280        | 2261        | 5476                | 169                 | 0                   | 364                 |
| 36      | 1220| 1680| 1796        | 1845        | 1844        | 1819        | 13456               | 27225               | 26962               | 19347               |
| 42      | 830| 1270| 1263        | 1254        | 1260        | 1231        | 49                  | 256                 | 98                  | 1525                |
| 48      | 610| 880| 879         | 858         | 864         | 841         | 1                   | 484                 | 262                 | 1497                |
| 54      | 480| 680| 642         | 626         | 630         | 614         | 1444                | 2916                | 2510                | 4313                |
| 60      | 390| 550| 497         | 488         | 490         | 479         | 2809                | 3844                | 3588                | 5009                |
| 66      | 330| 450| 400         | 395         | 397         | 388         | 2500                | 3025                | 2841                | 3789                |
| 72      | 300| 400| 339         | 335         | 336         | 329         | 3721                | 4225                | 4122                | 5062                |
| 78      | 260| 340| 300         | 300         | 300         | 295         | 1600                | 1600                | 1592                | 2034                |
| 84      | 230| 290| 262         | 261         | 262         | 257         | 784                 | 841                 | 790                 | 1077                |
| 90      | 200| 250| 230         | 230         | 231         | 227         | 400                 | 400                 | 369                 | 541                 |
| 96      | 180| 220| 202         | 201         | 202         | 198         | 324                 | 361                 | 328                 | 478                 |
| 102     | 160| 200| 180         | 180         | 181         | 177         | 400                 | 400                 | 376                 | 509                 |

| SSQ     | 87425 | 64703 | 63928 | 60107 |
Table 4 Comparisons of the optimal parameter estimation results by different methods

| Methods | $C_0$  | $C_1$  | $C_2$ | SSQ  |
|---------|--------|--------|-------|------|
| TM      | 0.1175 | 0.7554 | 0.1271| 87425|
| DSM     | 0.2310 | 0.5380 | 0.2310| 64703|
| DE      | 0.1269 | 0.7296 | 0.1435| 63922|
| EOWOA   | 0.1019 | 0.7698 | 0.1139| 60107|

Seen from Table 3 and Table 4, TM, DSM, DE and EOWAO can be used to estimate the parameters of Muskingum model, and it can be seen that the optimal objective function value obtained by EOWOA is obviously smaller than that by other methods, so as to get the best computed outflow results, so as to provide a more effective and simple method for parameter estimation of Muskingum model. Moreover, EOWOA is suitable for the numerical solution of high-dimensional and multi-parameter complex problems. It can be widely used in the optimization of other complex non-linear models. Of course, its applications in other areas will be the direction of further research.

6. Conclusions

In this paper, whale optimization algorithm with elite opposition-based learning (EOWOA) is proposed by using elite opposition-based learning to improve the global search capability of WOA. Benchmark function tests prove that EOWOA has excellent ability and strong robustness. And case study in the south canal of Haihe River Basin indicates that EOWOA provides a new and effective way for parameter estimation of Muskingum model.

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