THE DIMENSION OF A VARIETY

EWA GRACZYŃSKA

Opole University of Technology
Institute of Mathematics
Luboszycka 3, 45–036 Opole, Poland

e-mail: egracz@po.opole.pl
http://www.egracz.po.opole.pl/

AND

DIETMAR SCHWEIGERT

Technische Universität Kaiserslautern
Fachbereich Mathematik
Postfach 3049, 67653 Kaiserslautern, Germany

Abstract

Derived varieties were invented by P. Cohn in [4]. Derived varieties of a given type were invented by the authors in [10]. In the paper we deal with the derived variety $V_{\sigma}$ of a given variety, by a fixed hypersubstitution $\sigma$. We introduce the notion of the dimension of a variety as the cardinality $\kappa$ of the set of all proper derived varieties of $V$ included in $V$.

We examine dimensions of some varieties in the lattice of all varieties of a given type $\tau$. Dimensions of varieties of lattices and all subvarieties of regular bands are determined.

Keywords: derived algebras, derived varieties, the dimension of a variety.

2000 Mathematics Subject Classification: Primary: 08B99, 08A40; Secondary: 08B05, 08B15.

The results of this paper were presented during the Workshop AAA71 and CYA21 in Będlewo, Poland on February 11, 2006.
References

[1] G. Birkhoff, *On the structure of abstract algebras*, J. Proc. Cambridge Phil. Soc. **31** (1935), 433–454.

[2] P.M. Cohn, *Universal Algebra*, Reidel, 1981 Dordrecht.

[3] K. Denecke and J. Koppitz, *M-solid varieties of algebras*, Advances in Mathematics, Vol. 10, Springer 2006.

[4] K. Denecke and S.L.Wismath, *Hyperidentities and Clones*, Gordon & Breach, 2000, ISBN 90-5699-235-X. ISSN 1041-5394.

[5] T. Evans, *The lattice of semigroups varieties*, Semigroup Forum **2** (1971), 1–43.

[6] Ch.F. Fennemore, *All varieties of bands*, Ph.D. dissertation, Pennsylvania State University 1969.

[7] Ch.F. Fennemore, *All varieties of bands I*, Mathematische Nachrichten **48** (1971), 237–252.

[8] J.A. Gerhard, *The lattice of equational classes of idempotent semigroups*, J. of Algebra **15** (1970), 195–224.

[9] E. Graczynska, *Universal algebra via tree operads*, Opole 2000, ISSN 1429-6063, ISBN 83-88492-75-6.

[10] E. Graczynska and D. Schweigert, *Hyperidentities of a given type*, Algebra Universalis **27** (1990), 305–318.

[11] E. Graczynska and D. Schweigert, *Derived and fluid varieties*, in print.

[12] G. Grätzer, *Universal Algebra*. 2nd ed., Springer, New York 1979.

[13] R. McKenzie, G.F. McNulty and W. Taylor, *Algebras, Lattices, Varieties*, vol. I, 1987, ISBN 0-534-07651-3.

[14] J. Płonka, *On equational classes of abstract algebras defined by regular equations*, Fund. Math. **64** (1969), 241–247.

[15] J. Płonka, *Proper and inner hypersubstitutions of varieties*, pp. 106–116 in: “Proceedings of the International Conference Summer School on General Algebra and Ordered Sets”, Olomouc 1994.

[16] D. Schweigert, *Hyperidentities*, pp. 405–506 in: Algebras and Orders, I.G. Rosenberg and G. Sabidussi, Kluwer Academic Publishers, 1993, ISBN 0-7923-2143-X.

[17] D. Schweigert, *On derived varieties*, Discussiones Mathematicae Algebra and Stochastic Methods **18** (1998), 17–26.

Received 23 February 2006
Revised 10 June 2006