Modelling height-diameter relationship of fifteen tree species planted on reclaimed agricultural lands with random species effects

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Abstract. Taiwan has a long history of reforesting degraded forest lands and reclaimed agricultural lands to improve forest ecosystem services. While the effort has been ongoing since 1950’s, very few studies attempted to understand how tree species planted on these lands are growing. To fill the knowledge gap, we established plots across elevation from 250 to 2500 m above sea level on reforested fruit orchards, betel nut farms, and bamboo plantations. Diameter and height of 15 plantation tree species were measured. Mixed effects modelling framework was applied to model tree height and diameter relationship of the 15 species with the Wykoff model. Random effects were added to species so that species with few observations could share information from species with abundant information. Preliminary results show that stand productivity and tree-level competition have potential effects on height-diameter relationship. Studying height-diameter relationship could assist decision making for better management of ecosystem services of reclaimed agricultural lands.

Keywords: Ecosystem services; Wykoff model; Mixed effects modelling; rehabilitation; reforestation.

1. Introduction
Agricultural lands were abandoned due to social-economic and ecological factors such as loss of land productivity [1]. Restoration success depends on soil and site stability, hydrologic function, and biotic integrity [2]. Continuous ground monitoring could help adapting future management decisions for long-term restoration success [2]. As such, after a land is reforested from active tree planting, monitoring tree growth is essential. Tree diameter at breast height (DBH) and total height are two fundamental measurements important to forest management [3]. Total height is less frequently monitored than DBH because it is difficult and costly to measure [4]. Modelling the relationship between tree height and DBH allows us to predict height from DBH, which in turn fills in missing heights in our inventory and improves volume or growth estimation [5].

Taiwan has a long history of national reforestation effort starting as early as 1951. Early effort was to lease heavily degraded national forests to farmers with financial incentives to reforest them [6]. About 75,000 ha of forests have been leased until 1975 [7]. Although the intention was to improve livelihood of rural communities and maintain ecosystem services of watersheds, underperforming local timber market caused many farmers to illegally convert the leasehold lands to agricultural lands or to entirely abandon them [7]. Nonetheless, constant threats from natural disasters prompted government to initiate large-scale reclamation of the leasehold lands.

Height-diameter models have been developed for decades. Usually, some forms of exponential function of DBH are used [8]. Because the relationship is highly species dependent and varied to stand conditions, additional covariates are often incorporated during the model process [9]. Totally, more than
130 tree species have been planted in Taiwan reforestation efforts over the past few decades [7]. Some tree species are planted more often than the others; thus, some tree species are not commonly found on reforested sites. However, all planted tree species are equally important to the success of reforestation programs. Hence, there is a need to model height-diameter relationship even for the rarely planted tree species despite the common practice of excluding them from modelling effort.

The objectives of this study were to: (1) extend the approach developed by [5] to utilize species as random effects under the nonlinear mixed effects modelling framework, and (2) assess the roles of stand productivity and tree-level competition in height-diameter relationship.

2. Materials and methods

2.1 Data

Sampling was conducted in central Taiwan within Dongshih Forestry District, National Taiwan University Experimental Forest, and Nantou Forestry District. A total of 27 0.05 ha plots were established across the elevation from 250 to 2,500 m above sea level. The annual rainfall ranged from 1,650 to 2,931 mm, and mean annual temperature ranged from 12 to 23 °C. The former land use of the 27 plots were fruit orchard, bamboo plantation, and betel nut tree plantation.

In this study, 15 tree species were planted and modelled for height-diameter relationship: (1) *Acacia confusa* Merr. (Acacia), (2) *Acer serrulatum* Hay. (Acer), (3) *Acer morrisonense* Hayata (Acerred), (4) *Cinnamomum camphora* (L.) J.Presl (Camphor), (5) *Chamaecyparis formosensis* Matsum.(Chamaecyparis), (6) *Eriobotrya deflexa* (Hemsl.) Nakai (Eriobotrya), (7) *Eurya chinensis* Brown (Eurya), (8) *Fraxinus griffithii* C. B. Clarke (Fraxinus), (9) *Liquidambar formosana* Hance (Liquidambar), (10) *Michelia compressa* (Maxim.) Sargent (Michelia), (11) *Quercus glauca* Thunb. (Oak), (12) *Pistacia chinensis* Bunge (Pistacia), (13) *Pseudotsuga wilsoniana* Hayata (Psudotsuga), (14) *Taiwania cryptomerioides* Hayata (Taiwania), and (15) *Zelkova serrata* (Thunb.) Makino (Zelkova). A total of 860 sampled trees were measured for diameter and height with minimum and maximum diameter of 1.0 cm and 19.7 cm, respectively.

2.2 Methods

The Wykoff [10] model was chosen for this study because it has been widely applied and has the flexibility of modelling different height-diameter relationship. The full Wykoff model was specified as,

\[
HT = 1.3 + e^{[\beta_1 + h_1 + \beta_2 \log(HMAX)]-[\beta_1 + h_1 + \beta_2 \log(BAL)](DBH+1)}
\]

where, \(\beta_k\) was fixed effects parameters for \(k = 1, ..., 4\), \(HMAX\) was maximum tree height in a plot that represented stand productivity, \(BAL\) was the sum of basal area per ha of trees larger than the subject tree that represented tree-level competition, and \(b_1\) and \(b_2\) were species random effects to be distributed as follows,

\[
\begin{bmatrix}
  h_1 \\
  b_2
\end{bmatrix} \sim MVN
\begin{bmatrix}
  0 \\
  \sigma_{12}^2
\end{bmatrix}
\begin{bmatrix}
  \sigma_1^2 & \sigma_{12} \\
  \sigma_{12} & \sigma_2^2
\end{bmatrix}
\]

The species random effects \(h_1\) and \(b_2\) implied that the parameters \((\beta_1 + h_1)\) and \((\beta_2 + b_2)\) varied by species.

Initial model fitting suggested that the species random effect \(b_2\) was not significant because it had very small standard deviation. Hence, it was removed. Furthermore, there was the issue of heteroscedasticity in the distribution of residuals. Hence, a weighted Wykoff model was fitted with weights specified as an inverse variance function of DBH, \(g(DBH) = DBH^{-p}\), where \(p\) was optimally determined. Thus, the final model was,

\[
HT = 1.3 + e^{[\beta_1 + h_1 + \beta_2 \log(HMAX)]-[\beta_1 + h_1 + \beta_2 \log(BAL)](DBH+1)}
\]
with the random effect as $b_i \sim N\left(0, \sigma_i^2\right)$. The Furnival Index [11] was applied to convert the model residual standard error back to its original scale. Lastly, modelling was carried out with the nlme package [12] in R statistical software [13].

3. Results and discussion

Overall, the model goodness of fit was satisfactory with residual standard error (RSE) of 0.9811, which was less than 1 m (Table 1). With the weighting added to the Wykoff model, heteroscedasticity was removed from the final model. The fixed parameter $\beta_1$ represented asymptote of the Wykoff model, which suggested maximum potential height of a tree. The results suggested that the asymptote varied by tree species with some species having smaller asymptote and some having larger asymptote ($b_1$ ranged from -0.2416 to 0.4151, Table 1). This result is within expectation and is also supported by [5].

Table 1. Parameter estimates of Wykoff mixed-effect model.

| $k$ | $\beta_k$ | se($\beta_k$) | $\sigma$ | Minimum $b_k$ | Maximum $b_k$ | RSE | $p$ |
|-----|-----------|--------------|---------|---------------|---------------|-----|-----|
| 1   | 1.7552    | 0.1355       | 0.2199  | -0.2416       | 0.4151        |     | 0.981 | 0.7475 |
| 2   | 6.2392    | 0.1795       | -       | -             | -             |     |       |     |
| 3   | 0.1729    | 0.0507       | -       | -             | -             |     |       |     |
| 4   | -0.1303   | 0.0207       | -       | -             | -             |     |       |     |

The fixed parameter $\beta_3$ suggested that stand productivity as represented by $HMAX$ had a positive effect on asymptote of the fitted Wykoff model (Table 1). This is reasonable because as a stand is more productive, the trees growing with the stand should be higher. Lastly, the fixed parameter $\beta_4$ suggested that $BAL$ has a negative effect on rate of increase of the Wykoff model (Table 1). This result is also reasonable because a large BAL suggests that a subject tree has lower social position. A tree in lower social position is usually a suppressed tree and often experiences greater competition from its neighbors which limit its growth, and thus, the rate of increase in the Wykoff model.

The predicted height-diameter curves of each tree species were compared to the global height-diameter curve (red dotted vs black solid lines, Figure 1). The predicted height-diameter curve of Acerred and Pistacia mirrored the global curve (Figure 1c,l). This is because both species are rare species with only three and four observations, respectively. Thus, their relationships are most likely driven by the global model. On the other hand, although Chamaecyparis and Taiwania are also rare species, their species height-diameter curves diverged from the global model (Figure 1e,n). This is because both species have data that are more spread out over the DBH range. For the abundant species such as Fraxinus and Zelkova, it was obvious that their species height-diameter curves diverged from the global model (Figure 1h,o).

In short, the Wykoff model was highly flexible in fitting height-diameter relationships of various species. Moreover, this study has demonstrated that the strengths of mixed effects modelling, which is sharing information across species during the modelling process. Mixed effects modelling showed that for rare species with limited data, fitted models could be different depending on spread of data over the range of interests. Thus, mixed effects models accommodate data availability when necessary [12]. Lastly, for abundant species, mixed effects models essentially let the data drive the fitted model to capture the overall observed trends in the data [12]. Hence, this study has shown the flexibility of mixed effects model framework in modelling relationships of rare species. Nonetheless, it is important to note that mixed effects modelling should not be substitute of lack of data, and efforts should still be made to acquire more information on the rare species.
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