Quantum Properties of General
Gauge Theories
with Composite and External Fields

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The generating functionals of Green’s functions with composite and external fields are considered in the framework of BV and BLT quantization methods for general gauge theories. The corresponding Ward identities are derived and the gauge dependence is investigated.

1. Introduction

The most general rules for manifestly covariant quantization of gauge theories in the path integral approach are provided by the BV formalism [1], based on the principle of BRST symmetry [2], as well as by its Sp(2)-covariant version, the BLT quantization scheme [3], based on the principle of extended BRST symmetry [4]. These methods currently underly the study of quantum properties of arbitrary (general) gauge theories in the Lagrangian formalism, either immediately providing the corresponding basis (e.g. derivation of the Ward identities, study of renormalization and gauge dependence [5], analysis of unitarity conditions [6]) or playing a key role in the interpretation of alternative quantization methods (e.g. triplectic [7], superfield [8], osp(1,2)-covariant quantization [9]). In particular, the methods [1, 3] have been used to analyse the quantum structure of general gauge theories with composite fields [10, 11] (in the BV and BLT formalisms, respectively); for these theories the corresponding Ward identities were derived and the related issue of

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gauge dependence was investigated. Also based on the use of the quantization methods [1, 3] was the recent study of Ref. [12], which carried out an investigation of the Ward identities and gauge dependence for general gauge theories with external fields.

In particular, the studies of Refs. [11, 12] revealed the fact that the gauge dependence for theories with composite or external fields is described with the help of certain fermionic operators in the BV formalism, and doublets of fermionic operators in the BLT formalism. At the same time, a remarkable feature, commonly shared by both types of theories, consists in the property of (generalized) nilpotency of the fermionic operators in question.

The purpose of this paper is to provide an extension of the studies of Refs. [10, 11, 12], which is aimed at incorporating composite fields, simultaneously with external ones, into arbitrary quantum gauge theories. The reason to consider this very general setting consists in the following. The procedure for constructing the effective action with composite fields [13] (see also Ref. [14]) offers a wide range of applications to quantum field theory models. Among the most important of them we find the study of such phenomenologically relevant theories as the Standard Model [15] and models of the inflationary Universe [16], as well as SUSY theories [17] and theory of strings [18]. Recent activities in supersymmetric YM theories and other gauge models have signalled the relevance of their extension to the case of composite fields on external background. The external field approach is applied mainly to handle certain difficulties in the path integral formulation of quantum theories by means of lifting the functional integration from a part of the variables, which are afterwards considered as external parameters. In the absence of a consistent theory of quantum gravity, this approach currently has the status of an indispensable tool providing insight into numerous problems which arise in the physics of black holes, as well as permitting to incorporate gravitational effects into the cosmology of the early Universe.

In this paper, the generating functionals of Green’s functions with composite and external fields combined are considered in the framework of the BV (section 2) and BLT (section 3) quantization methods for general gauge theories. For these functionals we derive the corresponding Ward identities and investigate the most general form of gauge dependence. It is revealed that combining composite and external fields into one scheme gives rise to a radical change in the character of both the Ward identities and gauge dependence (as compared to those obtained in Refs. [11, 12]). In particular, it
is shown that in the most general case of a quantum theory with composite and external fields the operators describing the gauge dependence suffer from the violation of (generalized) nilpotency.

We use De Witt’s condensed notations [19] as well as notations adopted in Refs. [1, 3]. The invariant tensor of the group Sp(2), which is a constant antisymmetric second-rank tensor, is denoted as $\varepsilon^{ab} \ (a = 1, 2)$, with the normalization $\varepsilon^{12} = 1$. Symmetrization over Sp(2) indices is denoted as $A_{\{ab\}} = A^{ab} + A^{ba}$. Derivatives with respect to sources and antifields are understood as acting from the left, and those to fields, as acting from the right (unless otherwise specified); left-hand derivatives with respect to the fields are labelled by the subscript “l” ($\delta_l/\delta\phi$ stands for the left-hand derivative with respect to the field $\phi$).

2. Quantum Gauge Theories with Composite and External Fields in the BV Formalism

We first recall that the quantization of a gauge theory within the BV approach [1] requires introducing a complete set of fields $\phi^A$ and a set of corresponding antifields $\phi^*_A$ (which play the role of sources of BRST transformations), with Grassmann parities

$$\varepsilon(\phi^A) \equiv \varepsilon_A, \quad \varepsilon(\phi^*_A) = \varepsilon_A + 1.$$ 

The content of the configuration space of the fields $\phi^A$ (composed by the initial classical fields, the (anti) ghost pyramids and the Lagrangian multipliers) is determined by the properties of the original classical theory, i.e. by the linear dependence (for reducible theories) or independence (for irreducible theories) of the generators of gauge transformations.

In terms of the variables $\phi^A$ we define composite fields $\sigma^m(\phi)$, i.e.

$$\sigma^m(\phi) = \sum_{k \geq 2} \frac{1}{k!} \Lambda^m_{A_1...A_k} \phi^{A_1}...\phi^{A_k}, \quad \varepsilon(\sigma^m) \equiv \varepsilon_m,$$ 

where $\Lambda^m_{A_1...A_k}$ are some field-independent coefficients.

The extended generating functional $Z(J, L, \phi^*)$ of Green’s functions with composite fields is constructed within the BV quantization approach by the
rule (see, for example, Ref. [10])

\[ Z(J, L, \phi^*) = \int d\phi \exp \left\{ \frac{i}{\hbar} \left( S_{\text{ext}}(\phi, \phi^*) + J_A \phi^A + L_m \sigma^m(\phi) \right) \right\}, \]  

(1)

where \( J_A \) are the usual sources of the fields \( \phi^A \), \( \varepsilon(J_A) = \varepsilon_A \); \( L_m \) are the sources of the composite fields \( \sigma^m(\phi) \), \( \varepsilon(L_m) = \varepsilon_m \); and \( S_{\text{ext}} = S_{\text{ext}}(\phi, \phi^*) \) is the gauge-fixed quantum action defined in the usual manner as

\[ \exp \left\{ \frac{i}{\hbar} S_{\text{ext}} \right\} = \exp \left( \hat{T}(\Psi) \right) \exp \left\{ \frac{i}{\hbar} S \right\}. \]  

(2)

In eq. (2), \( S = S(\phi, \phi^*) \) is a bosonic functional satisfying the equation

\[ \frac{1}{2} (S, S) = i\hbar \Delta S, \]  

(3)

or equivalently

\[ \Delta \exp \left\{ \frac{i}{\hbar} S \right\} = 0, \]  

(4)

with the boundary condition

\[ S|_{\phi^* = \h = 0} = S, \]

where \( S \) is the original gauge-invariant classical action. At the same time, the operator \( \hat{T}(\Psi) \) has the form

\[ \hat{T}(\Psi) = [\Delta, \Psi], \]  

(5)

where \( \Psi \) is a fermionic gauge-fixing functional.

In eqs. (3)–(5) we use the standard definition of the antibracket, given for two arbitrary functionals \( F = F(\phi, \phi^*) \), \( G = G(\phi, \phi^*) \) by the rule

\[ (F, G) = \frac{\delta F}{\delta \phi^A} \frac{\delta G}{\delta \phi^*_A} - (-1)^{(\varepsilon(F)+1)(\varepsilon(G)+1)} \frac{\delta G}{\delta \phi^A} \frac{\delta F}{\delta \phi^*_A}, \]

as well as the usual definition of the operator \( \Delta \)

\[ \Delta = (-1)^{\varepsilon_A} \frac{\delta}{\delta \phi^A} \frac{\delta}{\delta \phi^*_A}, \]
possessing the property of nilpotency $\Delta^2 = 0$.

It is well-known that the gauge-fixing (2), (5) represents a particular case of transformation corresponding to any fermionic operator (chosen for $\Psi$) and describing the arbitrariness in solutions of eq. (3) (or eq. (4)), namely,

$$\Delta \exp \left\{ \frac{i}{\hbar} S_{\text{ext}} \right\} = 0.$$  \hfill (6)

In what follows we consider the most general case of gauge-fixing, corresponding to an arbitrary operator-valued fermionic functional $\Psi$ (clearly, it should be chosen in such a way as to ensure the existence of the functional integral).

Let us now consider the following representation of the generating functional $Z(J, L, \phi^*)$ in eq. (1):

$$Z(J, L, \phi^*) = \int d\psi \ Z(J, L, \psi, \phi^*) \exp \left( \frac{i}{\hbar} \mathcal{Y} \psi \right),$$

where

$$Z(J, L, \psi, \phi^*) = \int d\phi \ \exp \left\{ \frac{i}{\hbar} \left( S_{\text{ext}}(\varphi, \psi, \phi^*) + J \varphi + L \sigma(\varphi, \psi) \right) \right\}. \hfill (7)$$

Given this, we have assumed the decomposition

$$\phi^A = (\varphi^i, \psi^\alpha), \quad J^A = (J_i, \mathcal{Y}_\alpha),$$

$$\varepsilon(\varphi^i) \equiv \varepsilon_i, \quad \varepsilon(\psi^\alpha) \equiv \varepsilon_\alpha.$$  \hfill

In what follows we refer to $Z = Z(J, L, \psi, \phi^*)$ as the extended generating functional of Green’s functions with composite fields on the background of external fields $\psi^\alpha$. Clearly, the validity of the functional integral in eq. (1) implies the existence of the integral in eq. (7) without any restriction on the structure of the subspace $\psi^\alpha$. At the same time, the composite fields $\sigma^m(\varphi, \psi)$, according to their original definition, may be considered as given by

$$\sigma^m(\varphi, \psi) = \sum_{k \geq 2} \frac{1}{k!} \Lambda_{i_1 \ldots i_k}^m(\psi) \varphi^{i_1} \ldots \varphi^{i_k},$$

with the arising coefficients now depending on the external fields $\psi^\alpha$. This, as will be shown more explicitly below, leads to additional peculiarities, which are absent in the studies of Ref. [10-12].
The Ward identities for a general gauge theory with composite and external fields considered in the BV quantization scheme can be obtained as a direct consequence of equation (6) for the gauge-fixed quantum action \( S_{\text{ext}} \).

Namely, integrating eq. (6) over the fields \( \varphi^i \) with the weight functional

\[
\exp \left\{ \frac{i}{\hbar} \left[ J_i \varphi^i + L_m \sigma^m(\varphi, \psi) \right] \right\},
\]

we have

\[
\int d\varphi \exp \left[ \frac{i}{\hbar} \left( J_i \varphi^i + L_m \sigma^m(\varphi, \psi) \right) \right] \Delta \exp \left\{ \frac{i}{\hbar} S_{\text{ext}}(\varphi, \psi, \phi^*) \right\} = 0.
\] (8)

Next, performing in eq. (8) integration by parts, with allowance for the relation

\[
\exp \left[ \frac{i}{\hbar} \left( J_i \varphi^i + L_m \sigma^m(\varphi, \psi) \right) \right] \Delta =
\]

\[
\left( \Delta - \frac{i}{\hbar} J_i \frac{\delta}{\delta \varphi^i} - \frac{i}{\hbar} L_m \sigma^m_{\alpha i A}(\varphi, \psi) \frac{\delta}{\delta \phi^*_{A, i}} \right) \exp \left\{ \frac{i}{\hbar} \left( J_i \varphi^i + L_m \sigma^m(\varphi, \psi) \right) \right\},
\]

\[
\sigma^m_{\alpha i A}(\varphi, \psi) \equiv \frac{\delta}{\delta \phi^*_{A}} \sigma^m(\varphi, \psi),
\]

we arrive at the following Ward identities for \( Z = Z(\mathcal{J}, L, \psi, \phi^*) \):

\[
\hat{\omega} Z = 0,
\] (10)

where \( \hat{\omega} \) stands for the operator

\[
\hat{\omega} = i \hbar \Delta_{\psi} + J_i \frac{\delta}{\delta \varphi^i} + L_m \sigma^m_{\alpha i A} \left( \frac{\hbar}{i \delta \mathcal{J}}, \psi \right) \frac{\delta}{\delta \phi^*_{A}} \Delta_{\psi} \equiv (-1)^{\xi_{\alpha}} \frac{\delta L_{m}}{\delta \psi_{\alpha}} \frac{\delta}{\delta \psi_{\alpha}}.
\] (11)

At the same time, the fact that the composite fields \( \sigma^m(\varphi, \psi) \) now depend on the external fields \( \psi^\alpha \) implies that the nilpotency of \( \hat{\omega} \) becomes in general violated, i.e.

\[
\hat{\omega}^2 = i \hbar (-1)^{\xi_i} L_m \sigma^m_{\alpha i A} \left( \frac{\hbar}{i \delta \mathcal{J}}, \psi \right) \frac{\delta}{\delta \psi_{A}} \frac{\delta}{\delta \phi^*_{i}}.
\]

where

\[
\sigma^m_{\alpha i A} \left( \frac{\hbar}{i \delta \mathcal{J}}, \psi \right) \equiv \frac{\delta}{\delta \psi_{\alpha}} \sigma^m_{\alpha i A}(\varphi, \psi) \bigg|_{\varphi = \psi = \phi^*}.
\]
In terms of the generating functional \( \mathcal{W} = \mathcal{W}(\mathcal{J}, L, \psi, \phi^*) \),

\[
Z = \exp \left\{ \frac{i}{\hbar} \mathcal{W} \right\},
\]

of connected Green’s functions with composite and external fields, the identities (10) take on the form

\[
\hat{\Omega} = \frac{\delta \mathcal{W}}{\delta \psi^\alpha} \frac{\delta \mathcal{W}}{\delta \bar{\psi}^\alpha},
\]

\[
\hat{\Omega} = i \hbar \Delta_\psi + \mathcal{J}_i \frac{\delta}{\delta \varphi^i} + L_m \sigma^m_A \left( \frac{\delta \mathcal{W}}{\delta \mathcal{J}} + \frac{\hbar}{i} \frac{\delta}{\delta \mathcal{J}} \psi \right) \frac{\delta}{\delta \phi^*_A}.
\]

In order to introduce the (extended) generating functional of 1PI vertex functions with composite and external fields we make use of the standard definition [13] of the generating functional of vertex functions with composite fields, which admits of a natural generalization to the case of external fields. Namely, let us introduce the generating functional in question by means of the following Legendre transformation with respect to the sources \( \mathcal{J}_i, L_m \):

\[
\Gamma(\varphi, \Sigma, \psi, \phi^*) = \mathcal{W}(\mathcal{J}, L, \psi, \phi^*) - \mathcal{J}_i \varphi^i - L_m \left( \Sigma^m + \sigma^m(\varphi, \psi) \right),
\]

where

\[
\varphi^i = \frac{\delta \mathcal{W}}{\delta \mathcal{J}_i}, \quad \Sigma^m = \frac{\delta \mathcal{W}}{\delta L_m} - \sigma^m \left( \frac{\delta \mathcal{W}}{\delta \mathcal{J}}, \psi \right).
\]

Given this, we have

\[
\mathcal{J}_i = - \frac{\delta \Gamma}{\delta \varphi^i} + \frac{\delta \Gamma}{\delta \Sigma^m} \sigma^m_{\varphi^i}(\varphi, \psi), \quad L_m = - \frac{\delta \Gamma}{\delta \Sigma^m}.
\]

From the definition of the Legendre transformation it follows that

\[
\begin{align*}
\frac{\delta}{\delta \mathcal{J}_i} \bigg|_{\mathcal{J}, \psi, \phi^*} & = \frac{\delta \varphi^i}{\delta \mathcal{J}_i} \bigg|_{\psi, \phi^*} + \frac{\delta \Sigma^m}{\delta \mathcal{J}_i} \bigg|_{\varphi^i, \phi^*}, \\
\frac{\delta}{\delta L_m} \bigg|_{\mathcal{J}, \psi, \phi^*} & = \frac{\delta \varphi^i}{\delta L_m} \bigg|_{\psi, \phi^*} + \frac{\delta \Sigma^m}{\delta L_m} \bigg|_{\varphi^i, \phi^*}, \\
\frac{\delta}{\delta \psi^\alpha} \bigg|_{\mathcal{J}, \phi^*} & = \frac{\delta \varphi^i}{\delta \psi^\alpha} \bigg|_{\phi^*, \phi^*} + \frac{\delta \Sigma^m}{\delta \psi^\alpha} \bigg|_{\varphi^i, \phi^*} + \frac{\delta \Sigma^m}{\delta \psi^\alpha} \bigg|_{\varphi^i, \phi^*}, \\
\frac{\delta}{\delta \phi^*_A} \bigg|_{\mathcal{J}, \psi} & = \frac{\delta \varphi^i}{\delta \phi^*_A} \bigg|_{\psi, \phi^*} + \frac{\delta \Sigma^m}{\delta \phi^*_A} \bigg|_{\varphi^i, \phi^*} + \frac{\delta \Sigma^m}{\delta \phi^*_A} \bigg|_{\varphi^i, \phi^*}.
\end{align*}
\]
Then, by virtue of eq. (12) and the relations
\[
\frac{\delta W}{\delta \psi^\alpha} = \frac{\delta \Gamma}{\delta \psi^\alpha} - \frac{\delta \Gamma}{\delta \Sigma^m} \sigma^m_{\alpha}(\varphi, \psi), \quad \frac{\delta W}{\delta \phi_A^*} = \frac{\delta \Gamma}{\delta \phi_A^*},
\]
we arrive at the following Ward identities for the functional \(\Gamma(\varphi, \Sigma, \psi, \phi^*)\):
\[
\frac{1}{2} (\Gamma, \Gamma) + \frac{\delta \Gamma}{\delta \Sigma^m} \left( \sigma^m_{\alpha}(\hat{\varphi}, \psi) - \sigma^m_{\alpha}(\varphi, \psi) \right) \frac{\delta \Gamma}{\delta \phi_A^*} =
\]
\[
= i \hbar \left\{ \Delta \Gamma - (-1)^\varepsilon_{\alpha\rho} (G'' - 1)^{\rho\sigma} \left[ \frac{\delta \Gamma}{\delta \phi^i} \left( \frac{\delta \Gamma}{\delta \psi^\alpha} - \frac{\delta \Gamma}{\delta \Sigma^m} \sigma^m_{\alpha}(\varphi, \psi) \right) \right] \frac{\delta \Gamma}{\delta \phi^i} \right\}
\]
\[
+ (-1)^{\varepsilon_{\alpha\mu} + \varepsilon_{m+1}} (G'' - 1)^{\mu\sigma} \left[ \frac{\delta \Gamma}{\delta \phi^i} \right] \frac{\delta \Gamma}{\delta \Sigma^m} \left[ \sigma^m_{\alpha}(\varphi, \psi) \right] \frac{\delta \Gamma}{\delta \phi^i} \left( \delta \Sigma^m \delta \psi^\alpha \right)
\]
\[- (-1)^{\varepsilon_{\alpha\mu}} \sigma^m_{\alpha}(\varphi, \psi) \left( \delta \Sigma^m \right) \frac{\delta \Gamma}{\delta \phi^i} \left( \delta \Sigma^m \delta \psi^\alpha \right). \tag{13}
\]

In eq. (13), we have assumed the notation
\[
\frac{\delta \Gamma}{\delta \phi^A} \equiv \left( \frac{\delta \Gamma}{\delta \phi^i}, \frac{\delta \Gamma}{\delta \psi^\alpha} \right)
\]
and introduced the operator \(\hat{\phi}^i\)
\[
\hat{\phi}^i = \phi^i + i \hbar (G'' - 1)^{i\sigma} \frac{\delta \Gamma}{\delta \phi^i},
\]
where
\[
\frac{\delta \Gamma}{\delta \phi^i} \equiv - (G'')_{i\rho}, \quad (G'' - 1)^{\rho\delta} (G'')_{\delta\sigma} = \delta_{\sigma}^\rho,
\]
\[
\Phi^\sigma = (\phi^i, \Sigma^m), \quad N^\sigma = \left( - \frac{\delta \Gamma}{\delta \phi^i} + \frac{\delta \Gamma}{\delta \Sigma^m} \sigma^m_{\alpha}(\varphi, \psi), - \frac{\delta \Gamma}{\delta \Sigma^m} \right).
\]

The reader may profit by considering the above results in the particular cases of theories where either composite or external fields alone are present.

Let us first turn to a quantum theory with composite fields only, which corresponds to the generating functional \(Z = Z(J, L, \phi^*)\) of Green's functions in eq. (1).

The Ward identities for the functional (1) can be derived from eqs. (10), (11) which determine the Ward identities for the generating functional (7)

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of Green’s functions with composite and external fields combined. Thus, we have

\[ J_A \delta Z \frac{\delta Z}{\delta \phi_A^*} + L_m \sigma_m^A \left( \frac{\hbar}{i} \frac{\delta}{\delta J} \right) \frac{\delta Z}{\delta \phi_A^*} = 0. \]

The above relation is obtained by integrating eq. (10) over the external fields \( \psi^\alpha \) with the weight functional \( \exp \{ i/\hbar \mathcal{Y}_\alpha \psi^\alpha \} \).

The corresponding Ward identities for the generating functional \( W = W(J, L, \phi^*) \) of connected Green’s functions are consequently given by

\[ J_A \frac{\delta W}{\delta \phi_A^*} + L_m \sigma_m^A \left( \frac{\delta W}{\delta J} + \frac{\hbar}{i} \frac{\delta}{\delta J} \right) \frac{\delta W}{\delta \phi_A^*} = 0, \]

which, in turn, implies the following Ward identities for the generating functional \( \Gamma = \Gamma(\phi, \Sigma, \phi^*) \) of vertex functions:

\[ \frac{1}{2} \langle \Gamma, \Gamma \rangle + \frac{\delta \Gamma}{\delta \Sigma^m} \left( \sigma_m^A(\hat{\phi}) - \sigma_m^A(\phi) \right) \frac{\delta \Gamma}{\delta \phi_A^*} = 0. \]

Here, \( \hat{\phi}^A \) stand for the operators

\[ \hat{\phi}^A = \phi^A + i\hbar(Q''^{-1})^A_p \frac{\delta L}{\delta F^p}, \]

defined as

\[ \frac{\delta_l E_q}{\delta F^p} = -(Q'')_{pq}, \quad (Q''^{-1})^{pr} (Q'')_{rq} = \delta^{pr}_{rq}, \]

\[ F^p = (\phi^A, \Sigma^m), \quad E_p = \left( -\frac{\delta \Gamma}{\delta \phi^A} + \frac{\delta \Gamma}{\delta \Sigma^m} \sigma_m^A(\phi), -\frac{\delta \Gamma}{\delta \Sigma^m} \right). \]

Note that the above Ward identities for a quantum theory with composite fields coincide with the results obtained in Ref. [10].

We next consider the case of a quantum theory with external fields only, evidently corresponding to the assumptions \( \sigma^m(\phi) = 0, L = 0 \). Within these restrictions, the generating functional (7) of Green’s functions, obviously, reduces to \( \mathcal{Z} = \mathcal{Z}(J, \psi, \phi^*) \), while its Ward identities, determined by eqs. (10), (11), accordingly take on the form

\[ \left( i\hbar \Delta_\psi + J_i \frac{\delta}{\delta \phi_i^*} \right) \mathcal{Z} = 0. \]
This implies the following Ward identities for the generating functional $\mathcal{W} = \mathcal{W}(J, \psi, \phi^*)$ of connected Green's functions:

$$
\left(i\hbar \Delta_{\psi} + J_{i} \frac{\delta}{\delta \phi_{i}^*}\right) \mathcal{W} = \frac{\delta \mathcal{W}}{\delta \psi^{\alpha}} \frac{\delta \mathcal{W}}{\delta \psi_{\alpha}^*}.
$$

Finally, one readily establishes the fact that the Ward identities for the corresponding generating functional $\Gamma = \Gamma(\phi, \psi, \phi^*)$ of vertex functions can be represented as

$$
\frac{1}{2} (\Gamma, \Gamma) = i\hbar \Delta_{\psi} \Gamma - i\hbar (\Gamma''^{-1})^{ij} \left( \frac{\delta}{\delta \phi^j} \frac{\delta \Gamma}{\delta \psi^\alpha} \right) \left( \frac{\delta}{\delta \psi_{\alpha}^*} \frac{\delta \Gamma}{\delta \phi^j} \right),
$$

where

$$(\Gamma''^{-1})^{ik} (\Gamma'')_{kj} = \delta^i_j, \quad (\Gamma'')_{ij} \equiv \frac{\delta \Gamma}{\delta \phi^j} \frac{\delta \Gamma}{\delta \phi^i}.
$$

The above Ward identities for a quantum theory with external fields coincide with the results of Ref. [12].

Consider now the change of the generating functionals $\mathcal{Z}$, $\mathcal{W}$, $\Gamma$ with composite and external fields under a variation of the gauge fermion $\Psi$ chosen in the most general form of an operator-valued functional, i.e.

$$
\delta \Psi \left( \phi^A, \phi^*_A; \frac{\delta}{\delta \phi^A}, \frac{\delta}{\delta \phi^*_A} \right) = \delta \Psi \left( \phi^i, \psi^\alpha, \phi^*_A; \frac{\delta}{\delta \phi^i}, \frac{\delta}{\delta \psi^\alpha}, \frac{\delta}{\delta \phi^*_A} \right).
$$

Clearly, the gauge enters the generating functional $\mathcal{Z}$ only in the gauge-fixed quantum action $S_{\text{ext}}$, eq. (2), whose variation reads

$$
\delta \left( \exp \left\{ \frac{i}{\hbar} S_{\text{ext}} \right\} \right) = \hat{T}(\delta X) \exp \left\{ \frac{i}{\hbar} S_{\text{ext}} \right\},
$$

where $\delta X$ is related to $\delta \Psi$ through a certain linear operator-valued transformation. The explicit form of $\delta X$ is not essential for the following treatment; nevertheless it is always possible to choose the operator ordering in such a way that

$$
\delta X \left( \phi^i, \psi^\alpha, \phi^*_A; \frac{\delta}{\delta \phi^i}, \frac{\delta}{\delta \psi^\alpha}, \frac{\delta}{\delta \phi^*_A} \right) = \delta X^{(0)} \left( \phi^i, \psi^\alpha, \phi^*_A; \frac{\delta}{\delta \psi^\alpha}, \frac{\delta}{\delta \phi^*_A} \right)
$$

$$
+ \sum_{N=1} \frac{\delta}{\delta \phi^i} \cdots \frac{\delta}{\delta \phi^*_N} \delta X^{(i_1 \ldots i_N)} \left( \phi^i, \psi^\alpha, \phi^*_A; \frac{\delta}{\delta \psi^\alpha}, \frac{\delta}{\delta \phi^*_A} \right). \tag{14}
$$
With allowance for eqs. (5)–(7), the variation of the functional $Z(J, L, \psi, \phi^*)$ reads
\[
\delta Z(J, L, \psi, \phi^*) = \int d\varphi \exp \left( \frac{i}{\hbar} J_i \varphi^i + L_m \sigma^m \varphi, \psi \right) \times \Delta \left( \delta X \exp \left\{ \frac{i}{\hbar} S_{\text{ext}} (\varphi, \psi, \phi^*) \right\} \right). \tag{15}
\]
Then, performing in eq. (15) integration by parts and taking the relations (9), (11), (14) into account, we transform the variation of the functional $Z(J, L, \psi, \phi^*)$ into the form
\[
\delta Z = \frac{1}{i\hbar} \hat{\omega} \delta \tilde{X}, \tag{16}
\]
where
\[
\delta \tilde{X} = \delta X \left( \frac{\hbar}{i} \frac{\delta}{\delta J_i}, \psi^\alpha, \phi^*_A; (-1)^{\varepsilon_i} \frac{1}{i\hbar} J_i, \frac{\delta}{\delta \psi^\alpha} + \frac{1}{i\hbar} (-1)^{\varepsilon_a} L_m \sigma^m \left( \frac{\hbar}{i} \frac{\delta}{\delta J_i}, \psi \right), \frac{\delta}{\delta \phi^*_A} \right).
\]
In terms of the generating functional $W(J, L, \psi, \phi^*)$ eq. (16) can be repre-\[
\delta W = -\hat{Q} \langle \delta \tilde{X} \rangle, \tag{17}
\]
where $\langle \delta \tilde{X} \rangle$ is the vacuum expectation of the operator-valued functional $\delta \tilde{X}$
\[
\langle \delta \tilde{X} \rangle = \delta X \left( \frac{\delta W}{\delta J_i} + \frac{\hbar}{i} \frac{\delta}{\delta J_i}, \psi^\alpha, \phi^*_A; (-1)^{\varepsilon_i} \frac{1}{i\hbar} J_i, \frac{i}{\hbar} \frac{\delta}{\delta \psi^\alpha} + \frac{\delta}{\delta \phi^*_A}, \frac{i}{\hbar} \frac{\delta}{\delta \phi^*_A} \right),
\]
and $\hat{Q}$ stands for an operator given by the rule
\[
\hat{Q} = \exp \left\{ -\frac{i}{\hbar} \hat{\omega} \right\} \hat{\omega} \exp \left\{ \frac{i}{\hbar} \hat{\omega} \right\}. \tag{18}
\]
From eq. (18) it follows, in particular, that the breakdown of nilpotency in the case of $\hat{\omega}$ is also inherited by $\hat{Q}$, i.e.
\[
\hat{Q}^2 = i\hbar (-1)^{\varepsilon_i} L_m \sigma^m_{i\alpha} \left( \frac{\delta W}{\delta J_i} + \frac{\hbar}{i} \frac{\delta}{\delta J_i}, \psi \right) \left[ \left( \frac{\delta}{\delta \psi^\alpha} + \frac{i}{\hbar} \frac{\delta W}{\delta \psi^\alpha} \right) \left( \frac{\delta}{\delta \phi^*_i} + \frac{i}{\hbar} \frac{\delta W}{\delta \phi^*_i} \right) \right].
\]
By virtue of the Ward identities (12) for the functional $\mathcal{W}(\mathcal{J}, L, \psi, \phi^*)$, eq. (18) admits of the representation

$$\hat{Q} = \hat{\Omega} - \frac{\delta \mathcal{W}}{\delta \psi^\alpha} \frac{\delta}{\delta \psi^*_\alpha} - (-1)^{\epsilon_\alpha} \frac{\delta \mathcal{W}}{\delta \psi^*_\alpha} \frac{\delta}{\delta \psi^\alpha}. \quad (19)$$

In order to derive the form of gauge dependence of $\Gamma = \Gamma(\varphi, \Sigma, \psi, \phi^*)$, we observe that $\delta \Gamma = \delta \mathcal{W}$. Hence,

$$\delta \Gamma = -\hat{q} \langle \langle \delta \hat{X} \rangle \rangle, \quad (20)$$

where $\langle \langle \delta \hat{X} \rangle \rangle$ and $\hat{q}$ are the values related through the Legendre transformation to $\langle \delta \hat{X} \rangle$ and $\hat{Q}$ in eq. (17). Namely, the functional $\langle \langle \delta \hat{X} \rangle \rangle$ has the form

$$\langle \langle \delta \hat{X} \rangle \rangle = \delta X \left( \varphi^\dagger, \psi^\alpha, \phi^*; \frac{i}{\hbar} (-1)^{\epsilon_\alpha} \left( \frac{\delta \Gamma}{\delta \varphi^\dagger} - \frac{\delta \Gamma}{\delta \Sigma^m} \sigma^m_{,\alpha}(\varphi, \psi) \right) \right),$$

$$\frac{\delta l}{\delta \psi^\alpha} + \frac{i}{\hbar} (-1)^{\epsilon_\alpha} \left( \frac{\delta \Gamma}{\delta \psi^\alpha} - \frac{\delta \Gamma}{\delta \Sigma^m} \sigma^m_{,\alpha}(\varphi, \psi) \right) \right) \delta l \over \delta \Phi^\rho \right)$$

$$\frac{\delta}{\delta \phi^*_{\alpha}} + \frac{i}{\hbar} \frac{\delta \Gamma}{\delta \phi^*_{\alpha}} - (-1)^{\epsilon_\alpha \epsilon_{\rho}} (G''^*)^{-1,\alpha \rho} \left( \frac{\delta l}{\delta \Phi^\sigma} \frac{\delta \Gamma}{\delta \psi^\alpha} - \frac{\delta \Gamma}{\delta \Sigma^m} \sigma^m_{,\alpha}(\varphi, \psi) \right) \right) \sigma^m_{,\alpha}(\varphi, \psi) \delta l \over \delta \Sigma^m \right)$$

Meanwhile, the operator $\hat{q}$ admits of the representation

$$\hat{q} = i \hbar \left\{ (-1)^{\epsilon_\alpha} \frac{\delta l}{\delta \psi^\alpha} - (-1)^{\epsilon_\alpha \epsilon_{\rho}} \right\} \sigma^m_{,\alpha}(\varphi, \psi) \delta l \over \delta \Sigma^m \right)$$

$$\frac{\delta}{\delta \phi^*_{\alpha}} + \frac{i}{\hbar} \frac{\delta \Gamma}{\delta \phi^*_{\alpha}} - (-1)^{\epsilon_\alpha \epsilon_{\rho}} (G''^*)^{-1,\alpha \rho} \left( \frac{\delta l}{\delta \Phi^\sigma} \frac{\delta \Gamma}{\delta \psi^\alpha} - \frac{\delta \Gamma}{\delta \Sigma^m} \sigma^m_{,\alpha}(\varphi, \psi) \right) \right) \sigma^m_{,\alpha}(\varphi, \psi) \delta l \over \delta \Sigma^m \right) \right.$$
At the same time, the algebraic properties of $\hat{Q}$, eq. (18), obviously imply the breakdown of nilpotency for the operator $\hat{q}$ in the general case of a theory with composite and external fields.

### 3. Quantum Theories with Composite and External Fields in the BLT Approach

Consider now the quantum properties of general gauge theories with composite and external fields in the framework of the BLT formalism [3]. For this purpose we remind that the quantization rules [3] imply introducing a set of fields $\phi^A$ and a set of the corresponding antifields $\phi^*_{Aa}$, $\bar{\phi}_A$, with

$\varepsilon(\phi^A) = \varepsilon_A, \quad \varepsilon(\phi^*_{Aa}) = \varepsilon_A + 1, \quad \varepsilon(\bar{\phi}_A) = \varepsilon_A.$

The doublets of antifields $\phi^*_{Aa}$ play the role of sources of BRST and antiBRST transformations, while the antifields $\bar{\phi}_A$ are the sources of mixed BRST and antiBRST transformations. The structure of the configuration space of the
fields $\phi^A$ in the BLT approach is identical (for any given gauge theory) with that of the BV formalism. Given this, when considered in the framework of the BLT method, the fields are combined into irreducible, completely symmetric, $\text{Sp}(2)$-tensors \[3\].

The extended generating functional $Z(J, L, \phi^*, \bar{\phi})$ of Green’s functions with composite fields is constructed within the BLT formalism by the rule (see, for example, Ref. \[11\])

$$Z(J, \phi^*, \bar{\phi}) = \int d\phi \exp \left\{ \frac{i}{\hbar} \left( S_{\text{ext}}(\phi, \phi^*, \bar{\phi}) + J_A \phi^A + L_m \sigma^m(\phi) \right) \right\}, \quad (22)$$

where $S_{\text{ext}} = S_{\text{ext}}(\phi, \phi^*, \bar{\phi})$ is the gauge-fixed quantum action defined as

$$\exp \left\{ \frac{i}{\hbar} S_{\text{ext}} \right\} = \exp \left( -i \hbar \hat{T}(F) \right) \exp \left\{ \frac{i}{\hbar} S \right\}. \quad (23)$$

In eq. (23), $S = S(\phi, \phi^*, \bar{\phi})$ is a bosonic functional satisfying the equations

$$\frac{1}{2} (S, S)^a + V^a S = i \hbar \Delta^a S, \quad (24)$$

or equivalently

$$\bar{\Delta}^a \exp \left\{ \frac{i}{\hbar} S \right\} = 0, \quad \bar{\Delta}^a \equiv \Delta^a + \frac{i}{\hbar} V^a, \quad (25)$$

with the boundary condition

$$S|_{\phi^* = \phi = \hbar = 0} = S$$

(again $S$ is the classical action). At the same time, $\hat{T}(F)$ is an operator of the form

$$\hat{T}(F) = \frac{1}{2} \varepsilon_{ab}[\Delta^b, [\Delta^a, F]_-]_+, \quad (26)$$

where $F$ is a bosonic (generally, operator-valued) gauge-fixing functional.

In eqs. (24)–(26) we use the extended antibrackets, defined for two arbitrary functionals $F = F(\phi, \phi^*, \bar{\phi}), G = G(\phi, \phi^*, \bar{\phi})$ by the rule \[3\]

$$(F, G)^a = \frac{\delta F}{\delta \phi^A} \frac{\delta G}{\delta \phi^*_A} - (-1)^{(\varepsilon(F)+1)(\varepsilon(G)+1)} \frac{\delta G}{\delta \phi^A} \frac{\delta F}{\delta \phi^*_A},$$
as well as the operators $\Delta^a$, $V^a$

$$
\Delta^a = (-1)^{\varepsilon_A} \frac{\delta_i}{\delta \phi^A} \frac{\delta}{\delta \phi^A_{a}}, \quad V^a = \varepsilon^{ab} \phi^{*b} \frac{\delta}{\delta \phi^A}
$$

with the properties

$$
\Delta^{(a} \Delta^{b)} = 0, \quad V^{(a} V^{b)} = 0, \quad \Delta^{(a} V^{b)} + V^{(a} \Delta^{b)} = 0.
$$

From the above algebraic properties, obviously, follow the identities $[\hat{T}(F), \Delta^a] = 0$, which imply that the functional $S_{\text{ext}}$ in eq. (23) satisfies equations of the same form as in (25), i.e.

$$
\bar{\Delta}^a \exp \left\{ \frac{i}{\hbar} S_{\text{ext}} \right\} = 0. \quad (27)
$$

Consider now the following representation of the generating functional $Z(J, L, \phi^*, \bar{\phi})$ in eq. (22):

$$
Z(J, L, \phi^*, \bar{\phi}) = \int d\psi \ Z(J, L, \psi, \phi^*, \bar{\phi}) \exp \left( \frac{i}{\hbar} \gamma \psi \right),
$$

where $Z(J, L, \psi, \phi^*, \bar{\phi})$ is the (extended) generating functional of Green’s functions with composite fields on the background of external fields $\psi^a$

$$
Z(J, L, \psi, \phi^*, \bar{\phi}) = \int d\varphi \ \exp \left\{ \frac{i}{\hbar} \left( S_{\text{ext}}(\varphi, \psi, \phi^*, \bar{\phi}) + J \varphi + L \sigma(\varphi, \psi) \right) \right\} \quad (28)
$$

with

$$
\phi^A = (\varphi^i, \psi^\alpha), \quad J_A = (J_i, \mathcal{Y}_\alpha).
$$

In order to obtain the Ward identities for a general gauge theory with composite and external fields in the BLT quantization formalism, we apply a procedure quite similar to that presented in the case of the BV approach. Namely, by virtue of eq. (27) for the gauge-fixed quantum action $S_{\text{ext}}$ (23), we have

$$
\int d\varphi \ \exp \left[ \frac{i}{\hbar} \left( J_i \varphi^i + L_m \sigma(\varphi, \psi) \right) \right] \Delta^a \exp \left\{ \frac{i}{\hbar} S_{\text{ext}}(\varphi, \psi, \phi^*, \bar{\phi}) \right\} = 0. \quad (29)
$$

As a result of integration by parts in eq. (29), with allowance for the relations

$$
\exp \left\{ \frac{i}{\hbar} \left( J_i \varphi^i + L_m \sigma^m(\varphi, \psi) \right) \right\} \Delta^a = \quad (30)
$$

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we find that the Ward identities for the functional $Z = Z(J, L, \psi, \phi^*, \bar{\phi})$
have the form

$$\hat{\omega}^a Z = 0,$$

(31)

where

$$\hat{\omega}^a = i\hbar \Delta^a \psi - V^a + J_i \frac{\delta}{\delta \varphi^*_{ia}} + L_m \sigma^m_{,A} \left( \frac{\hbar}{i} \frac{\delta}{\delta \varphi^*_{ia}}, \psi \right) \frac{\delta}{\delta \varphi^*_{Aa}},$$

(32)

$$\Delta^a \psi \equiv (-1)^{\varepsilon\alpha} \frac{\delta}{\delta \psi^*} \frac{\delta}{\delta \psi^*_{aa}}.$$ 

Note that the operators $\hat{\omega}^a$ satisfy the relations

$$\hat{\omega}^a (\hat{\omega}^b) = i\hbar (-1)^{\varepsilon\alpha} L_m \sigma^m_{,ia} \left( \frac{\hbar}{i} \frac{\delta}{\delta \varphi^*_{ia}}, \psi \right) \frac{\delta}{\delta \psi^*_{a(a \delta \varphi^*_{ib})}},$$

which imply the violation of the property $\hat{\omega}^a (\hat{\omega}^b) = 0$ of generalized nilpotency.

In terms of the generating functional $W = W(J, L, \psi, \phi^*, \bar{\phi})$,

$$Z = \exp \left\{ \frac{i}{\hbar} W \right\},$$

of connected Green’s functions, the Ward identities (31) are represented as

$$\hat{\Omega}^a W = \frac{\delta W}{\delta \psi^*} \frac{\delta W}{\delta \psi^*_{aa}},$$

(33)

$$\hat{\Omega}^a = i\hbar \Delta^a \psi - V^a + J_i \frac{\delta}{\delta \varphi^*_{ia}} + L_m \sigma^m_{,A} \left( \frac{\delta W}{\delta \varphi^*_{ia}}, \psi \right) \frac{\delta}{\delta \varphi^*_{Aa}}.$$ 

Consequently, the Ward identities for the generating functional of 1PI vertex functions

$$\Gamma(\varphi, \Sigma, \psi, \phi^*, \bar{\phi}) = W(J, L, \psi, \phi^*, \bar{\phi}) - J_i \varphi^i - L_m \left( \Sigma^m + \sigma^m(\varphi, \psi) \right),$$

$$\varphi^i = \frac{\delta W}{\delta J_i}, \quad \Sigma^m = \frac{\delta W}{\delta L_m} - \sigma^m \left( \frac{\delta W}{\delta J}, \psi \right)$$

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have the form

\[
\frac{1}{2} (\Gamma, \Gamma)^a + V^a \Gamma + \frac{\delta \Gamma}{\delta \Sigma^m} \left( \sigma^m_A(\hat{\phi}, \psi) - \sigma^m_A(\phi, \psi) \right) \frac{\delta \Gamma}{\delta \phi^*_A} =
\]

\[
i\hbar \left\{ \Delta_\\psi^a \Gamma - (-1)^{\epsilon_a \epsilon_\sigma} (G''^a)^{\rho \sigma} \left[ \frac{\delta \Gamma}{\delta \phi^a} \left( \frac{\delta \Gamma}{\delta \psi^\alpha} - \frac{\delta \Gamma}{\delta \Sigma^m \sigma^m_A(\phi, \psi)} \right) \right] \frac{\delta \Gamma}{\delta \phi^a} \frac{\delta \Gamma}{\delta \psi^\alpha} 
\right. 
\]

\[
+ (-1)^{\epsilon_a \epsilon_{\sigma \psi}} (G''^a)^{\sigma \psi} \frac{\delta \Gamma}{\delta \phi^a} \left( \frac{\delta \Gamma}{\delta \psi^\alpha} - \frac{\delta \Gamma}{\delta \Sigma^m \sigma^m_A(\phi, \psi)} \right) \frac{\delta \Gamma}{\delta \phi^a} \frac{\delta \Gamma}{\delta \psi^\alpha} 
\left. 
\right. 
\]

\[
- (-1)^{\epsilon_a \epsilon_\sigma} \sigma^m_A(\phi, \psi) \frac{\delta \Gamma}{\delta \Sigma^m} \frac{\delta \Gamma}{\delta \psi^\alpha} 
\}
\]  \tag{34}

As in the previous section, let us consider the particular cases corresponding to theories with either composite or external fields only.

Namely, for a quantum theory with composite fields, considered in the absence of external fields, we obtain, by virtue of eq. (31), (32), the following Ward identities for the generating functional

\[
Z = Z(J, L, \phi^*, \bar{\phi})
\]

\[
\frac{\delta Z}{\delta \phi^*_A} + L m \sigma^m_A \left( \frac{\hbar}{i} \frac{\delta}{\delta J} \right) \frac{\delta Z}{\delta \phi^*_A} - V^a Z = 0.
\]

Again, they can be obtained by integrating eq. (31) over the external fields \(\psi^\alpha\) with the weight functional \(\exp \{ i/\hbar \mathcal{Y}_\alpha \psi^\alpha \} \).

The corresponding generating functional \(W = W(J, L, \phi^*, \bar{\phi})\) of connected Green’s functions satisfies the Ward identities of the form

\[
\frac{\delta W}{\delta \phi^*_A} + L m \sigma^m_A \left( \frac{\delta}{\delta J} + \frac{\hbar}{i} \frac{\delta}{\delta \phi^*_A} \right) \frac{\delta W}{\delta \phi^*_A} - V^a W = 0.
\]

At the same time, the Ward identities for the generating functional \(\Gamma = \Gamma(\phi, \Sigma, \phi^*, \bar{\phi})\) of vertex functions are given by

\[
\frac{1}{2} (\Gamma, \Gamma)^a + V^a \Gamma + \frac{\delta \Gamma}{\delta \Sigma^m} \left( \sigma^m_A(\hat{\phi}) - \sigma^m_A(\phi) \right) \frac{\delta \Gamma}{\delta \phi^*_A} = 0.
\]

Note that the above Ward identities for a quantum theory with composite fields considered in the BLT formalism coincide with the results obtained in Ref. [11].
Next, for a theory with external fields in the absence of composite fields 

\( \sigma^m(\phi) = 0, \ L_m = 0 \) the generating functional (28) of Green’s functions evidently reduces to 

\[ Z = Z(\mathcal{J}, \psi, \phi^*, \bar{\phi}) \],

which satisfies, by virtue of eqs. (31), (32), the following Ward identities:

\[ \left( i\hbar \Delta_\psi^a + \mathcal{J}_i \frac{\partial}{\partial \phi^*_{ia}} - V^a \right) Z = 0. \]

Meanwhile, the Ward identities for the corresponding generating functional 

\( \mathcal{W} = \mathcal{W}(\mathcal{J}, \psi, \phi^*, \bar{\phi}) \) of connected Green’s functions, accordingly, take on the form

\[ \left( i\hbar \Delta_\psi^a + \mathcal{J}_i \frac{\partial}{\partial \phi^*_{ia}} - V^a \right) \mathcal{W} = \frac{\delta \mathcal{W}}{\delta \psi^\alpha} \frac{\delta \mathcal{W}}{\delta \psi^*_{\alpha a}}. \]

Finally, in the case of the generating functional 

\( \Gamma = \Gamma(\mathcal{F}, \psi, \phi^*, \bar{\phi}) \) of vertex functions the Ward identities in question can be represented as

\[ \frac{1}{2}(\Gamma, \Gamma)^a + V^a \Gamma = i\hbar \Delta^a_\psi \Gamma - i\hbar (\Gamma'^{\alpha-1})^{ij} \left( \frac{\delta}{\delta \phi^j} \frac{\delta}{\delta \psi^\alpha} \right) \left( \frac{\delta}{\delta \psi^*_{\alpha a}} \frac{\delta}{\delta \phi^a} \right). \]

The above Ward identities for a quantum theory with external fields considered in the framework of the BLT formalism coincide with the corresponding results of Ref. [12].

Let us now study the gauge dependence of the above generating functionals with composite and external fields, considered in the BLT quantization approach, under the most general variation

\[ \delta F \left( \varphi^i, \psi^\alpha, \phi^*_{Aa}, \bar{\phi}_A; \frac{\delta}{\delta \varphi^i}, \frac{\delta}{\delta \psi^\alpha}, \frac{\delta}{\delta \phi^*_{Aa}}, \frac{\delta}{\delta \bar{\phi}_A} \right) \]

of the gauge boson. From eq. (23) it follows that

\[ \delta \left( \exp \left\{ \frac{i}{\hbar} S_{\text{ext}} \right\} \right) = -i\hbar \hat{\mathcal{T}}(\delta Y) \exp \left\{ \frac{i}{\hbar} S_{\text{ext}} \right\}, \]

where \( \delta Y \), related to \( \delta F \) through a linear (operator-valued) transformation, always admits of the representation

\[ \delta Y \left( \varphi^i, \psi^\alpha, \phi^*_{Aa}, \bar{\phi}_A; \frac{\delta}{\delta \varphi^i}, \frac{\delta}{\delta \psi^\alpha}, \frac{\delta}{\delta \phi^*_{Aa}}, \frac{\delta}{\delta \bar{\phi}_A} \right) = \delta Y^{(0)} \left( \varphi^i, \psi^\alpha, \phi^*_{Aa}, \bar{\phi}_A; \frac{\delta}{\delta \varphi^i}, \frac{\delta}{\delta \psi^\alpha}, \frac{\delta}{\delta \phi^*_{Aa}}, \frac{\delta}{\delta \bar{\phi}_A} \right) \]

\[ + \sum_{N=1} \frac{\delta l_1}{\delta \phi_1^i} \cdots \frac{\delta l_N}{\delta \phi_N^i} \delta Y^{(i_1 \cdots i_N)} \left( \varphi^i, \psi^\alpha, \phi^*_{Aa}, \bar{\phi}_A; \frac{\delta}{\delta \varphi^i}, \frac{\delta}{\delta \psi^\alpha}, \frac{\delta}{\delta \phi^*_{Aa}}, \frac{\delta}{\delta \bar{\phi}_A} \right). \]
Taking eqs. (27), (28) and (35) into account, we have

\[ \delta Z(J, L, \psi, \phi^*, \bar{\phi}) = \frac{i}{2} \hat{\omega}^a \hat{\omega}^b \delta Y \delta Z, \]  

and therefore, with allowance for eqs. (30), (32), (35), integration by parts in eq. (36) yields

\[ \delta Z = \frac{i}{2\hbar} \epsilon_{ab} \hat{\omega}^a \hat{\omega}^b \delta Y \delta Z, \]  

where

\[ \delta Y = \delta Y \left( \frac{\hbar}{i \delta \mathcal{J}_i} \frac{\delta}{\psi^a}, \phi^*_A, \bar{\phi}_A; (-1)^{\epsilon_i} \frac{1}{i \hbar} \mathcal{J}_i, \right) \]

Transforming eq. (37) in terms of the generating functional of connected Green’s functions \( W(J, L, \psi, \phi^*, \bar{\phi}) \), we arrive at the relation

\[ \delta W = \frac{1}{2} \epsilon_{ab} \hat{Q}^a \hat{Q}^b \langle \delta Y \rangle, \]  

where

\[ \langle \delta Y \rangle = \delta Y \left( \frac{\delta W}{\delta \mathcal{J}_i} \frac{\hbar}{\delta \psi^a}, \phi^*_A, \bar{\phi}_A; (-1)^{\epsilon_a} \frac{1}{i \hbar} \mathcal{J}_i, \right) \]

while the operators \( \hat{Q}^a \) are defined by

\[ \hat{Q}^a = \exp \left\{ -i \frac{\hbar}{\delta W} \right\} \hat{\omega}^a \exp \left\{ i \frac{\hbar}{\delta W} \right\} \]

and admit of the representation

\[ \hat{Q}^a = \hat{\Omega}^a - \frac{\delta W}{\delta \psi^a} \frac{\delta}{\psi^*_A} - (-1)^{\epsilon_a} \frac{\delta W}{\delta \psi^*_A} \frac{\delta}{\psi^a}. \]  

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Note that the operators $\hat{Q}^a$ satisfy the relations
\[
\hat{Q}^{\langle a} \hat{Q}^{b \rangle} = i\hbar (-1)^{\epsilon_1} L_m \sigma^m_{\langle a} \left( \frac{\delta W}{\delta \mathcal{A}} + \frac{\hbar}{i} \frac{\delta}{\delta \mathcal{A}}, \psi \right) \times \\
\times \left[ \left( \frac{\delta}{\delta \psi_{\alpha}} + \frac{i}{\hbar} \frac{\delta W}{\delta \psi_{\alpha}}(a) \right) \left( \frac{\delta}{\delta \varphi_{\hat{b}}} + \frac{i}{\hbar} \frac{\delta W}{\delta \varphi_{\hat{b}}} \right) \right],
\]
which follow from the properties of $\hat{\omega}^a$ and imply, in particular, the breakdown of generalized nilpotency also in the case of $\hat{Q}^a$.

Finally, with allowance for eq. (38), the variation of the generating functional $\Gamma = \Gamma(\varphi, \Sigma, \psi, \phi^*, \phi)$ of vertex Green’s functions is given by
\[
\delta \Gamma = \frac{1}{2} \varepsilon_{ab} \hat{q}^b \langle \langle \delta \hat{Y} \rangle \rangle. \tag{40}
\]
The functional $\langle \langle \delta \hat{Y} \rangle \rangle$ has the form
\[
\langle \langle \delta \hat{Y} \rangle \rangle = \delta Y \left( \hat{\varphi}^i, \psi^\alpha, \phi^*_{\alpha a}, \tilde{\phi}_A; \frac{i}{\hbar} (-1)^{\epsilon_1} \left( \frac{\delta \Gamma}{\delta \phi^*_{\alpha}} - \frac{\delta \Gamma}{\delta \Sigma^m} \sigma^m_{\langle a} (\varphi, \psi) \right) \right),
\]
\[
\frac{\delta \Gamma}{\delta \psi_{\alpha}} + \frac{i}{\hbar} (-1)^{\epsilon_{\alpha}} \left( \frac{\delta \Gamma}{\delta \psi_{\alpha}} - \frac{\delta \Gamma}{\delta \Sigma^m} \sigma^m_{\langle a} (\varphi, \psi) \right)
\]
\[-(-1)^{\epsilon_{\alpha}} (G''-1)^{\rho \sigma} \left[ \left( \frac{\delta \Gamma}{\delta \phi^*_{\rho}} - \frac{\delta \Gamma}{\delta \Sigma^m} \sigma^m_{\langle a} (\varphi, \psi) \right) \right] \frac{\delta \Gamma}{\delta \phi^*_{\rho}}
\]
\[-(-1)^{\epsilon_{\alpha}} (G''-1)^{\sigma \alpha} \sigma^m_{\langle a} (\varphi, \psi) \frac{\delta \Gamma}{\delta \Sigma^m}
\]
\[+(-1)^{\epsilon_{\alpha} + \epsilon_1} (G''-1)^{\rho \sigma} \left( \frac{\delta \Gamma}{\delta \phi^*_{\rho}} - \frac{\delta \Gamma}{\delta \Sigma^m} \sigma^m_{\langle a} (\varphi, \psi) \right) \frac{\delta \Gamma}{\delta \phi^*_{\rho}}
\]
\[+ \frac{\delta}{\delta \phi^*_{\alpha a}} + \frac{i}{\hbar} \frac{\delta \Gamma}{\delta \phi^*_{\alpha a}} - (-1)^{\epsilon_{\rho}} (G''-1)^{\rho \sigma} \left( \frac{\delta \Gamma}{\delta \phi^*_{\rho}} - \frac{\delta \Gamma}{\delta \Sigma^m} \right) \frac{\delta \Gamma}{\delta \phi^*_{\rho}}
\]
\[+ \frac{\delta}{\delta \phi^*_{\alpha a}} + \frac{i}{\hbar} \frac{\delta \Gamma}{\delta \phi^*_{\alpha a}} - (-1)^{\epsilon_{\rho}} (G''-1)^{\rho \sigma} \left( \frac{\delta \Gamma}{\delta \phi^*_{\rho}} - \frac{\delta \Gamma}{\delta \Sigma^m} \right) \frac{\delta \Gamma}{\delta \phi^*_{\rho}}
\]
\[+ (-1)^{\epsilon_{\alpha} + \epsilon_m + 1} (G''-1)^{\rho \sigma} \left( \frac{\delta \Gamma}{\delta \phi^*_{\rho}} - \frac{\delta \Gamma}{\delta \Sigma^m} \right) \frac{\delta \Gamma}{\delta \phi^*_{\rho}}
\]
At the same time, the operators $\hat{q}^a$ can be represented as
\[
\hat{q}^a = i\hbar \left( (-1)^{\epsilon_{\alpha}} \frac{\delta \Gamma}{\delta \psi_{\alpha}} - (-1)^{\epsilon_{\alpha} \epsilon_m} \sigma^m_{\langle a} (\varphi, \psi) \right) \frac{\delta \Gamma}{\delta \Sigma^m}
\]
Clearly, the above values $\hat{q}^a$ and $\langle\delta\hat{Y}\rangle$ are the Legendre transforms of the corresponding values $\hat{Q}^a$ and $\langle\delta\hat{Y}\rangle$ in eq. (38). As far as the operators $\hat{q}^a$ are concerned, this implies, in particular, the violation of their generalized nilpotency in the general case of a theory with composite and external fields.

4. Conclusion

In this paper we have presented an extension of the studies of arbitrary quantum gauge theories with composite and external fields to the case of theories with composite and external fields combined. Namely,
we considered the generating functionals of Green’s functions with composite and external fields in the framework of the BV [1] and BLT [3] quantization methods for general gauge theories. For these functionals we obtained, using the technique developed in refs. [11, 12], the corresponding Ward identities, eqs. (10), (12), (13), (31), (33), (34), and derived the explicit dependence on the most general form of gauge-fixing, eqs. (16), (17), (20), (37), (38), (40). The gauge dependence is described with the help of fermionic operators (11), (19), (21) in the BV method, as well as with the help of doublets of fermionic operators (32), (39), (41) in the framework of the BLT formalism, which bears remarkable similarity to the cases where only composite [11] or external [12] fields were present.

At the same time, it should be noted that the most general scheme combining composite and external fields proves to present some essentially new features, which set it apart from the previous cases [11, 11, 12]. First of all, we refer to the fact that the form of the Ward identities and gauge dependence admits, in the case under consideration, an entirely different character and does not follow from those derived earlier. (The latter, naturally, can be obtained from the general case as the corresponding limits.) Another notable evidence is provided by the fact that in the most general case of composite and external fields combined, the operators describing gauge dependence do not possess the algebraic properties of (generalized) nilpotency which hold for their counterparts [11, 12]. The lack of generalized nilpotency has been traced back to expressions including \( \sigma_{\alpha\alpha}^\mu(\varphi, \psi) \), i.e. to the explicit dependence of the composite fields upon the external ones. Of course, this fact deserves further considerations, especially with respect to the effect it may have on the unitarity of the theory. This, however, has to be postponed to another paper.

Let us finally remark that — despite being much more complicated — an extension of these considerations to the manifest osp(1,2)-covariant quantization [9] would be desirable.

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