Snow Load Modeling on Different Surfaces

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Abstract. The snow load distribution model on the buildings and structures coatings is considered. The main factors affecting the snow load formation on the coatings of random shape are revealed. The dependence of the snow thickness on the coating on the angle of the roof inclination, the coating curvature radius, and the snow friction coefficient on the surface of the roof is determined. The level of individual factors influence on the snow load distribution is revealed. Recommendations on the coating structures calculation are given. These allow taking into account possible unfavorable loading options during the highest snowfall period.

1. Introduction
There are collapse of buildings and structures coatings by the reasons of the snow load despite the improvement of the calculation and design methods, the development of the materials quality and the introduction of modern construction technologies. A variety of factors affecting the snow accumulation leads to a variation of snow load parameters and complicates the choice of a design model for coatings with a non-standard configuration [1], roof surface roughness, etc. The main factors determining snow accumulation on the coating are the coating outline, the snow density, the snow friction forces on the roof surface, and the wind direction. The indicators diversity affecting the nature of the snow formation often lead to a mismatch in the loading scheme calculation, the actual snow load for a given shape of the coating surface [2].

The collapse of the arched structure in Krasnoyarsk occurred on May 10 (!) 2016 is a significant example. There wasn’t winter snow on the roof. It is noted that "that day it was wet snow, the arched structure roof did not cope with the load, the snow stuck to the metal structures on the principle of a snowman, and the arch fell" [3].

Monitoring shows that fresh snow has an increased ability to hold onto various surfaces including sloping surfaces. In addition, it has a large dimensional weight due to the high humidity. It should be considered that the snow load decrease due to snow taking away by wind is not significant and also snow does not melt on unheated roofs during heavy snowfall.

Despite the snow may fall during the whole winter period and has not yet accumulated on the surface, so the loading variant after the snowfall may be a certain danger and lead to irreversible consequences.

Thus, it is necessary to consider the conditions that promote the formation of special loading options on coatings where there is an accumulation of snow that differs from ordinary situations.
2. Theoretical part
We consider the coating arched shape with the main snow load on the roof (Fig. 1) in order to make a universal model of snow load. Let us cut a basic section of snow with length $dS$ of unit width and thickness $h$. Its position is determined by the angle of inclination of the coating curvature radius $R$ to the vertical $\phi$ (Fig. 2). Snow weight $dG$ affects on this fragment and the friction force $T$ on the surface of the coating. It is known that the snow friction coefficient depends on the surface roughness of the coating, temperature conditions, roofing material and varies over a rather wide range [4,5,6].

The gravity of the snow fragment $dG$ with a volume weight $\rho$ and a length of the segment $dS = Rd\phi$ is $dG = \rho R d\phi$. We consider the weight of the snow directed vertically downwards into a sloping $T$ and a normal $dP$ components. The slope component equal to $dG \sin \phi = \rho h R \sin \phi$ is tangential to the roof surface and tends to move the snow down, and the normal component equal to $dP = dG \cos \phi = \rho h R \cos \phi$ creates friction forces to hold the snow on the roof.

Snow sliding is also hampered by the resistance force to stretching of the snow cover. And its magnitude depends on the snow strength under tension $\sigma$ [4].

This force will be $\sigma h$ at equal distribution of stresses over the thickness of the snow cover $h$.

The sum forces that stretch the snow lead to the additional friction forces, due to the presence of the radial component $dN = T d\phi$ determined by the curvature of the coating [7].

We make the equilibrium condition of the chosen snow cover fragment projecting all the acting forces on the tangent to the roof axis. We will take the positive direction toward the increasing of the inclination angle $\phi$.

Taking into account that the addition of the inclination angle on $d\phi$ leads to a decrease of the snow cover thickness at the unit width by the value of $dh$, we get

$$\sigma (h - dh) - \sigma h = f\rho h R \cos \phi d\phi + 1\rho R \sin \phi d\phi - f\sigma h d\phi = 0. \quad (1)$$

![Figure 1. Geometric coating model diagram.](image)
After changes we will have
\[ \sigma dh = \rho Rh \sin \varphi d\varphi - f \rho h \cos \varphi d\varphi + f \sigma h d\varphi. \]  
(2)

We will get linear differential equation when sharing variables
\[ \frac{dh}{h} = \left( \frac{\rho R}{\sigma} \sin \varphi - \frac{f \rho R}{\sigma} \cos \varphi - f \right) d\varphi. \]  
(3)

The decision of the equation will depend on the radius of axis curvature of cover \( R \) which is defined by configuration of the roof. Radius will be constant for the outline of the arch performed around the circle. Presenting the integration constant as \( \ln C \), we will get the decision of the equation as
\[ \ln h = -\frac{\rho R}{\sigma} \cos \varphi - \frac{f \rho R}{\sigma} \sin \varphi - f \varphi + \ln C. \]  
(4)

When
\[ \frac{h}{C} = \exp \left( -\frac{\rho R}{\sigma} \cos \varphi - \frac{f \rho R}{\sigma} \sin \varphi - f \varphi \right). \]  
(5)

Unknown integration constant is defined from frontier conditions of the task. It is known that the height of snow at \( \varphi = 0 \) will be equal to the snow height on horizontal section of flat cover \( h_0 \). In view of the foregoing, we will have at \( \varphi = 0 \)
\[ h_0 = C \exp \left( -\frac{\rho R}{\sigma} \right), \]  
(6)

where the integration constant will be as
\[ C = h_0 \exp \left( \frac{\rho R}{\sigma} \right). \]  
(7)

Indicating the ratio of the snow height with the current coordinate depending from the inclination angle, to height of snow on the top of cover \( \frac{h}{h_0} = \mu \), we will receive final expression of mathematical model of height of snow
\[ \mu = \exp \left( \frac{\rho R}{\sigma} (1 - \cos \varphi - f \sin \varphi) - f \varphi \right). \]  
(8)

3. Analysis
It can be seen that the snow cover thickness is determined by such parameters as the angle of the roof inclination, the friction coefficient, the curvature radius, and the snow volume weight from the obtained relation (8). The friction coefficient influence on the relative height of snow cover decreases with increasing slope (Fig. 3). The decrease intensity of snow accumulation increases as the friction coefficient reduction. On roofs with increased roughness by a friction coefficient \( f \) greater than 0.15, snow can be held on surfaces with large inclines.
Figure 3. Dependence of the relative snow covers thickness on the roof inclination angle.

A more intensive increase in the snow cover thickness as the friction coefficient increases is observed at smaller inclination angles (Fig. 4).

Figure 4. Dependence of the relative snow covers thickness on the friction coefficient.

It can be seen that fresh snow can be hold on any surfaces even with a significant incline from the obtained dependences. It creates conditions for increasing the roof load. The snow cover thickness grows with the increasing of surface roughness of the roof. Such snow accumulation options are also observed in some curvilinear shapes in real conditions (Fig. 5). It is recommended to choose the design scheme with caution for sensitive structures to overloads. And especially those structures where own weight is comparable or less than the accumulated snow weight. All possible loading options should be considered [8]. It is necessary to take into account the orientation of the structure longitudinal axis relative to the “wind” in order to identify the possibility of the snow formation from the leeward side when choosing unfavorable loading schemes. This can lead to asymmetrical loading that represents a danger for arch coverings.
Figure 5. The snow accumulation variant on the arch cover.

4. Conclusion
Distribution of snow load on buildings and structures coverings of curvilinear shape submits to exponential dependency.

Weight of accumulated snow depends on volume weight, roof inclination, radius of curvature, roof covering roughness.

It can be observed increased level of snow accumulation on curvilinear shape of roof covering for a short time during heavy snow.

It is recommended to consider the option of the cover fully load with fresh snow taking into account the possible formation of snow deposits on the leeward side of the structure in order to prevent possible construction damages besides calculations in accordance with the current regulatory document SP 20.13330.2011 “Loads and exposures” for sensitive structures to snow load.

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