The Physical Essence of Pulsar Glitch

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ABSTRACT

Based on the magnetic dipole radiation from the $^3P_2$ neutron superfluid vortices ($^3P_2$ NSFV) in neutron stars, we propose a model of glitch for young pulsars by oscillation between B phase and A phase of $^3P_2$ Neutron superfluid. The main behavior of glitches of pulsars may be naturally explained by our model. Our results show that the glitch is a repeat phenomena with quasi period $^3P_2$ neutron superfluid B phase $\Rightarrow$ A phase $\Rightarrow$ B phase $\Rightarrow$ many repeated glitches with quasi-period. With repeating of the phase transition, the vortex quantum number, n, of $^3P_2$ NSFV is gradually reduced, and the heating rate $\varepsilon(B)$ in B phase is also getting lower and lower. After a number of glitches, the time intervals of successive glitch will gradually become long, and the glitch amplitude is downward. When the heating rate $\varepsilon(B)$ of the old neutron star becomes lower than the cooling rate of the Direct Urca, which can happen in super strong magnetic field (Peng et al. 2016b), the B phase of the $^3P_2$ NSFV is no longer returned to the A phase state. The phase oscillation of the system is stopped immediately. That means, old pulsars will no longer present the glitch. All of these results are consistent with observations (Lyne et al. 2000). The slowing glitch phenomenon for some older pulsars is a naturally result in our theory. The relationship between Glitch amplitude and the stationary time interval (see Fig.2 from Middleditch et al. (2006)) is also naturally got by our theory.

Subject headings: stars: neutron — pulsars: general — stars: evolution.

1. Introduction

1.1. Observations on glitches of pulsars

The observed pulsar rotation periods often show that some young pulsars experience one or more glitches (or macro jump). The regular pulse signals could be occasionally shortened by glitches at typical amplitude of $\Delta \Omega_0/\Omega_0 \sim (10^{-10} - 10^{-6})$. These glitches are usually accompanied by a spin-down effect at a much larger rate $\Delta \Omega/\Omega \approx (10^{-3} - 10^{-2})$ (Lyne et al. 2000; Chamel & Haensel 2008; Espinoza et al. 2011). The archetypal glitch neutron star is the Vela pulsar, which has exhibited a regular sequence of similar size glitches since the first observed event in 1969. There are 478 glitches detected among 174 pulsars up to date and 120 glitches among them detected in eight of the glitch pulsars are great glitches with $\Delta \Omega_0/\Omega_0 > 10^{-6}$. Eleven glitches with nine grand ones were detected from PSR Vela during 36 years. There are also nineteen smaller glitches detected during 36 years from PSR Crab with smaller magnitudes. In some pulsars, besides such macro glitches, there are detected micro-glitches with jump amplitudes less than $10^{12}$ in greater numbers.

The most important observational statistics of pulsar glitch phenomena up to date are given as follows:

1) There is a rough tendency for both the jump amplitude and the frequency of glitches to decrease with the pulse period as the pulsar ages (Lyne et al. 2000). Up to the present no glitches...
have been detected in pulsars with periods longer than 0.7s.

(2) Pulsar glitch phenomena is primarily concentrated in the group of very young pulsars with strong magnetic fields (Lyne et al. 2000): a) The younger the pulsars the most often the glitch, the larger the amplitude of the glitch becomes. The glitches are less often as the pulsars age and the amplitude of the glitches are lowered. b) The glitches are more often for pulsars with strong magnetic fields. On the other hand, the glitches are less often for pulsars with weak magnetic fields, and the lower the amplitudes of the glitches become (Lyne et al. 2004). These observational facts can not be explained by current pulsar models.

(3) Pulsar glitches are usually of served as a sudden change with very short time scale. However, a slow glitch with long time scale is more than several days in 2005. These observational facts can not be explained by current pulsar models.

(4) Middleditch et al. (2006) discovered that the young pulsar PSR J0573-6910 in the Large Magellanic Cloud (LMC), which has glitch with amplitude change roughly proportional to the time separation between two successive glitches after ten years of observation to monitor the radio pulsar period. This observation can not be explained by current pulsar models.

1.2. Theoretical researches on the glitches of pulsars

The consensus view is that these events are a manifestation of the presence of a superfluid component in the star’s interior (Ruderman 1969). This idea was first put forward by Anderson & Itoh (1975), who envisaged a glitch as a tug-of-war between the tendency of the neutron superfluid to match the spin down rate of the rest of the star by expelling vortices and the impediment experienced by the moving vortices due to pinning to crust nuclei. Strong vortex pinning prevents the neutron superfluid from spinning down, creating a spin lag with respect to the rest of the star (which is spun down electro-magnetically). This situation cannot persist forever. The increasing spin lag leads to a build up in the Magnus force exerted on the vortices. Above some threshold pinning can no longer be sustained, the vortices break free and the excess angular momentum is transferred to the crust. This leads to the observed spin-up.

The current pulsar models for the mechanism to explain glitches are given as follows:

(1) The star quake model (Baym et al. 1969)

This model predicts that the time separation between two successive glitches for the pulsar Vela PSR is roughly 1000 years. This is quite different from the observed facts of 36 years, eleven times.

(2) Vibration model of the neutron star core (Pines et al. 1972)

In this model, glitch may appear every several years with energy release $10^{45}$erg. The neutron star is immediately heated to become a strong X-ray source. However, such strong X-rays has never been observed during or after glitch.

(3) A creep model of vortex filament by an action of shell - superfluid coupling (Alpar et al. 1981; Andersson et al. 2003; Anderson & Itoh 1975)

Pulsar glitches are contributed to the motion of superfluid vortex lines; this lines tend to be pinned to nuclei of stellar crust and sudden, large-scale creep of these lines from one pinning site to another may be responsible for glitches (Link et al. 1993). In this model, the key idea is that the roots of the superfluid vortexes slide randomly in the inner shell, and occasionally they are pinned to the heavy nucleus. This model has been now regarded as the mainstream model by most researchers concerned (Haskell et al. 2015, 2016, 2017; Haskell. 2018; Khomenko et al. 2018). In their review paper (Haskell et al. 2016), Haskell et al. (2016) wrote a following detail comment: Anderson & Itoh (1975) suggested that interactions between vortices and ions in the NS crust can ‘pin’ the vortices and restrict their outward motion. As long as the vortices are pinned, i.e. stay fixed in position, the superfluid does not spin down, storing angular momentum, which is periodically released in glitches. The vortex model has become the standard picture for pulsar glitches”. “Nevertheless several issues still need to be resolved before the vortex picture attains the status of a self-consistent, predictive, falsifiable the-
ory. The trigger for vortex unpinning is still unknown and may be due to vortex accumulation in strong pinning areas, vortex domino effects, hydro-dynamical instabilities or quakes. Recent calculations have also showed that Bragg scattering severely limits the mobility of neutrons in the crust, limiting the amount of angular momentum that can be stored in the crust between glitches. An analysis of the Vela pulsar reveals that in this case it is difficult to accommodate the implied angular momentum during a glitch unless the pulsar has a low mass $\leq 1M_\odot$ or part of the core is involved in the process” (Haskell et al. 2016).

This model may be the mechanism for the production of micro glitches of neutron stars. But it is difficult for this model to explain the grand glitches of Vela PSR. Besides, there are too many free parameters in this model. It is rather difficult to determine all these parameters to explain a series of observational facts of neutron star glitches to be elaborated later.

(4) A model due to the effect of twisted neutron superfluid vortex filament with proton superconducting flux tube (Ruderman et al. 1998).

Although this model may explain the grand glitch ($\Delta \Omega_0/\Omega_0 \geq 10^{-6}$), but it contains too many free parameters to determine, consequently, it is also difficult for this model to explain the observed glitches. Espinoza et al. (2011); Link (2003); Rezania (2003) discovered, in view of the fact that the pulsar spin axis does not coincide with its magnetic axis, the Two-component model (i.e., neutron superfluid vortices tangle with proton superconducting magnetic tulle model) would predict the procession of the neutron star spin axis with periods of several seconds. Analysis of the observed data for PSR B1818-11 gives the procession period of the neutron star spin axis of the order of several years. Thus, they conclude that the neutron superfluid vortices may not coexist with the type II proton superconducting state (superconducting magnetic tube). Although it might appear in the inner core of the neutron star with density in the region of $2 - 3\rho_{\text{nuc}}$ (Haskell et al. 2017), however, the quark model of nuclear matter in this density region may be dominant. Therefore, Link (2003) have series doubts about this model.

(5) Mastrano et al. (2005) investigated Kelvin-Helmholtz instability and circulation transfer at an isotropic-anisotropic superfluid interface in a neutron star. By hydrodynamic method, they suggested that this instability may provide a trigger mechanism for pulsar glitches. However, further explanations of many observed phenomena of pulsar glitches are needed for their theory.

Besides, Chamel (2014, 2016); Delsate et al. (2016) have conducted a series of researches concerning the glitch phenomenal of the Vela pulsar. They considered the coupling of the shell with the superfluid interior of neutron stars. They did not investigate the mechanism for the production of glitches of pulsars.

Up to now, we note that the physical reason for the generation of pulsar glitches is still not clearly understood and it is one of the most difficult puzzling topic in pulsar researches. In this paper, we propose a new physical mechanism for generating Glitch, which is completely different from the existing known models. The main ideas of ours are mentioned in the abstract of the paper.

This paper is organized as follows. In Section 2, we study the anisotropic $^3P_2$ superfluid vortex motion in neutron star interiors. In Section 3, we summarize our researches and discuss the cooling and heating problem in neutron star interiors. In Section 4, we present our model and analyze the properties of pulsar glitch. Some discussions and conclusions are given in Section 5.

2. The anisotropic $^3P_2$ superfluid vortex motion in neutron star interiors

2.1. Two types of neutron superfluid

There are two types of superfluid with different properties between the thin crust and the interior of neutron stars. For densities $10^{11} < \rho (g/cm^3) < 1.4 \times 10^{14}$, the superfluid is isotropic. The energy gap of the Cooper pairs may reach (1-2) MeV in the $^1S_0$ state. The initial surface temperature of the nascent neutron star from the gravitational collapse of the supernovae core may reach $10^{11}K$. In a relating short time scale the temperature is
lowered to roughly $10^6 \text{K}$, for example, the Crab PSR was formed in 1054 during supernovae explosion. The surface temperature of the Crab has lowered to $10^8 \text{K}$ in less than a thousand year. The interior temperature of it was estimated to be $2.0 \times 10^8 \text{K}$. When the interior temperature is lowered critical temperature of the $^1S_0$ state, $T_C(^1S_0) = \Delta(^1S_0)/k \approx 10^{10} \text{K}$ (where $k$ is the Boltzmann constant), the isotropic neutron superfluid will appear. For neutron Cooper pairs coupled by the $^1S_0$ wave interaction, the spins of the two neutrons in the Cooper pair are anti-parallel, so that the total spin of the $^1S_0$ Cooper pair is zero and there is no net magnetic moment. The $^1S_0$ state is isotropic in external magnetic fields. The property of this state is similar to liquid $^4\text{He}$ ($^4\text{He}$ II) approaching near to absolute zero (lower than 0.2K) in the earth’s laboratories. When matter densities in the range $3.3 \times 10^{14} < \rho (\text{g/cm}^3) < 5.2 \times 10^{14}$ (Note that nuclear density $\rho_{\text{nuc}} = 2.8 \times 10^{14} \text{g/cm}^3$), the energy gap of the Cooper pairs in the $^3P_2$ state is roughly 0.045MeV (Elgar et al. 1996). When the interior temperature of neutron state is lowered to below $2.8 \times 10^8 \text{K}$, the neutron fluid in this region will become the anisotropic $^3P_2$ superfluid after a phase transition from the normal fluid. For neutron Coupler Pairs coupled by the $^3P_2$ wave interaction the spins are parallel, so that the total spin is equal to 1 and there is equal a net magnetic is just twice that of the neutron (abnormal magnetic moment).

In the presence of external magnetic field, the magnetic moment of $^3P_2$ neutron Cooper has a tendency to reverse the direction of the external magnetic field (temperature thermal effect makes the magnetic moment of the $^3P_2$ neutron Cooper to be chaotic (i.e., in confusion) in direction due to thermal agitation). Consequently, the $^3P_2$ state is anisotropic superfluid $^3P_2$ state. The properties of this $^3P_2$ state is similar to the liquid $^3\text{He}$ for temperatures approaching absolute zero in the earth’s laboratories (lower than 0.02K).

### 2.2. Neutron superfluid vortex motion in neutron stars

Rotating Superfluid are quantized to become superfluid Vortex flow (or eddy current or whirling fluid) (Feynman 1955) being analogous to type II superconductivity, the neutron superfluid in the interior of neutron stars is in a vortex state, i.e., there are plenty of vortex lines (vortex filament). In general, the vortex filaments are arranged in a systematic lattice, they are parallel to the axis of rotation of the neutron star and as a whole they revolve around the axis of rotation of the neutron star almost rigidly. The circulation of each vortex filament intensity $\Gamma$ is quantized

$$\Gamma = \oint \mathbf{V} \cdot d\mathbf{l} = n\Gamma_0, \quad \Gamma_0 = \frac{2\pi \hbar}{m_n}$$

where $n$ is a circulation quantum number of the vortex, $m_n$ is the mass of a neutron, $\hbar$ is Planck’s constant divided by $2\pi$, $\Gamma_0$ is the intensity of the unit vortex quantum.

It may be supposed that the core of the superfluid vortex is a cylindrical region of normal neutron fluid immersed in the superfluid neutron sea. As a usual, the radius of the core of the vertex, $a_0$, may be simply estimated as follows: in the core of the vortex, the position uncertain of the neutron is $\Delta x \sim a_0$, the momentum uncertain of the neutron is $\Delta p \sim \hbar/a_0$ by the Heisenberg’s principle of uncertainty and then the uncertain for energy of the neutron is about $\Delta E \sim (\Delta p)^2/2m_n \sim \hbar^2/(2m_n a_0^2)$. The neutron fluid in the core of the vortex will be in the normal state (no Cooper pairs), where the energy uncertain of the neutron is greater than the energy gap of the neutron superfluid (or the binding energy for the Cooper pair of neutrons) $\Delta E > \Delta_n$. Thus we have $a_0 \approx h/\sqrt{2m_n \Delta_n}$. For the vortexes of isotropic neutron superfluid $\Delta(^1S_0) \approx 2\text{MeV}$, $a_0 \sim 10^{-12}\text{cm}$ and for the vortexes of isotropic neutron superfluid $\Delta(^3P_2) \approx 0.045\text{MeV}$, $a_0 \sim 10^{-11}\text{cm}$.

Outside the core of the vortex, neutrons are in a superfluid state. The superfluid neutrons revolve round the vortex line with a velocity (Feynman 1955)

$$v_s(r) = \frac{n\hbar}{2m_n r},$$

where $r$ is the distance from the axis of the vortex filament. The distribution of the angular velocity of neutron superfluid revolving around the vortex filament is

$$\omega_s(r) = \frac{n\hbar}{2m_n r^2},$$

Therefore the revolution of superfluid neutrons around the vertex filament is placed in a differ-
potential state. Near $r \sim a_0$, the angular velocity reaches the largest value is given by

$$\omega_{s, \text{max}} = \frac{n\hbar}{2m_n a_0^2}. \quad (4)$$

We have $\omega_{s, \text{max}}(1S_0) \sim 10^{21} n$ and $\omega_{s, \text{max}}(3P_2) \sim 10^{19} n$. Inside the core of the vertexes ($r < a_0$), however, the normal neutron fluid revolves rigidly at angular velocity $\omega_{s, \text{max}}$.

According to (Feynman 1955), the number of superfluid vertex filaments per unit area is $2\Omega/a_G$. Then the order of magnitude of the average separation, $b$, between vertex filaments and the total number, $N_{\text{Vertice}}$, of the superfluid vertexes in the superfluid region are respectively

$$b = \left(\frac{\bar{n}\hbar}{2m_n \Omega}\right)^{1/2}, \quad (5)$$

$$N_{\text{Vertice}} = \frac{2m_n \Omega}{\bar{n}\hbar} R_s^2, \quad (6)$$

where $\Omega$ is the angular velocity of rotation as a whole, $R_s$ is the radius of the region of the superfluid. $\bar{n}$ is the circulation quantum number of each vortex filament on the average.

It is generally believed that the circulation quantum number of each vortex filament for liquid $^4$He and $^3$He in the Earth's low temperature is very low, even in the lowest basic state $n = 1$ for thermal dynamical equilibrium. However, the interior of the nascent neutron star must be in a turbulent vortex state. This is because that neutron stars originated from the collapsed supernovae core during violent supernovae explosion in a very short time less than 10 seconds and it is hard to transport the rotational angular momentum of the collapsed supernovae core outwards. It not only rotates fast (but its angular velocity cannot exceed the critical angular velocity to maintain neutron star stability) but also store considerable part of the stellar angular momentum in the violent turbulent state of the neutron fluid (Peng et al 1980; Peng et al. 1982). The classical circulation of vortex filaments intensity ($\Gamma$) may be very large. As the interior temperature of the neutron star is lowered to below the critical temperature of the isotropic superfluid $1S_0$ state and the anisotropic superfluid $3P_2$ state respectively, the $1S_0$ state is isotropic while the $3P_2$ state becomes anisotropic. At this time, the violent classical turbulent vortex state with very large vortex quantum number. We expect that for very young pulsars, the quantum number $n$ in Eq. (2) may reach above $10^2 - 10^4$.

In the anisotropic superfluid region of neutron Cooper pairs. Every $^3P_2$ Cooper pairs consist two neutrons with parallel spins so that the net spin of the $^3P_2$ Cooper pair is equal to 1. Every pair possesses abnormal (anomalous) magnetic moment with magnitude twice as that of the neutron ($\mu_n$). The total number of neutrons contained in the Cooper pairs in the $^3P_2$ anisotropic superfluid in neutron stars is 8.7% of the total of neutrons number in that region (Peng et al. 2016a). At high temperatures with $\mu_n B/kT \ll 1$, in the presence of external magnetic fields, the direction of the magnetic moments of the Cooper pairs in the anisotropic $^3P_2$ neutron superfluid region is very chaotic almost reaching the equal probability state (ESP). The anisotropic superfluid state at this time is called phase A. Here $\mu_n$ is the magnetic moment of neutron, $T$ the interior temperature, $k$ is the Boltzmann constant and $B$ is the external magnetic field strength.

When there exists possible elective cooling process in neutron state, the temperature of the anisotropic $^3P_2$ superfluid region may be lowered to below the Curie temperature ($\mu_n B/kT \gg 1$) the majority moments of the $^3P_2$ Cooper pairs are spontaneously arranged in the same direction as the external strong magnetic field (similar to the formation of magnetic domains in the low temperature laboratories). This anisotropic superfluid state is called phase B. The phase A and B in the anisotropic superfluid in neutron stars are similar to the those of in the anisotropic superfluid $^3$He in low temperature laboratories near absolute zero ($T < 0.02$K). But the phase B of the $^3P_2$ neutron superfluid possesses very strong magnetic fields and the effective magnetic moments of the $^3P_2$ Cooper pairs are also very strong (Peng et al. 2016a,b). According to the theory proposed in our works (Peng et al 1980; Peng et al. 1982), the magnetic moments of the $^3P_2$ Cooper pairs can emit very strong magnetic dipole radiation as the neutron Cooper rotate around the axis of the superfluid vortex. This radiation may be considered as an effective heating mechanism in neutron star interiors. Our present paper is just based on this idea.
3. Our researches on neutron stars and the cooling and heating problem in neutron star interiors

3.1. Our researches on neutron stars

We proposed a theory in 1982 (Peng et al. 1982) that the neutrino radiation by neutron superfluid vortexes of neutron stars is a decisive factor for spin down of pulsars with longer period \( P > 1.0 s \) and the rate of spin down, \( \dot{P} \), is proportional to \( P^2 (P \propto P^2) \), which was repeatedly supported by statistical works of pulsars from Malov. (1985, 1987, 2001). It is also confirmed by the recent observed \( (P - \dot{P}) \) diagram of pulsars (ATNF Pulsar Catalogue, 2016 see Fig.1). (we can see the link of http://www.atnf.csiro.au/research/pulsar/psrcat/) and which deviates seriously from the model of magnetic dipole radiation with the relation \( P \propto P^{-1} \) (the standard model). The observed pulsar \( (P - \dot{P}) \) diagram (ATNF Pulsar Catalogue, 2016) is not only supporting the spin down mechanism by the neutrino radiation from neutron superfluid vortexes of neutron stars, but also is supporting the idea of existence for neutron superfluid vortexes of neutron stars. Therefore, it is also in favor of our another paper (Huang et al. 1982) in which we proposed a pulsar heating mechanism by magnetic dipole radiation from the anisotropic \( ^3P_2 \) neutron superfluid vortex (afterwards \( ^3P_2 \) MDRA). In that paper, however, we calculated only the heating rate in lower magnetic field when \( \mu_n B \ll kT \). In present paper, we will recalculate the heating rate with two different case of lower magnetic field and stronger magnetic field when \( \mu_n B \gg kT \).

In the last decade we have studied the origin of the strong magnetic fields of neutron stars and the origin of the super strong magnetic fields of the magnetars (Peng et al. 2006, 2007). We have systematically considered the properties of magnetars such as the physics of the high X-ray luminosity in terms of principles and methods of condensed matter physics (Peng et al. 2016a,b). We have also investigated weak interaction rates and neutrino energy loss in magnetars (Liu 2013, 2014, 2015; Liu et al. 2017a,b,c, 2018a,b).

The magnetic fields of most pulsars are \( 10^{11} - 10^{13} \) Gauss with typical magnetic field strength \( 10^{12} \) Gauss. The average magnetic field of the sun is one gauss, and the upper half main sequence stars of large mass do not have surface convection, their magnetic fields are not very strong except the Ap stars. The collapse of the central region of stars of large mass during supernova explosion can only produce magnetic fields of \( 10^9 - 10^{11} \) Gauss. In other words, the primary magnetic fields (or the fossil magnetic fields) of neutron star produced by the collapse of the central core during supernova explosion (because the conservative of magnetic induction flux) cannot reach \( 10^{12} - 10^{13} \) Gauss. It is even more difficult to obtain magnetic fields of strength \( 10^{14} - 10^{15} \) Gauss, of the magnetars discovered by astronomical observations during the last decades. Some authors suggested that magnetars may still originals from neutron stars due to the core collapse during supernova explosion. i.e. there may already exist very strong magnetic fields in massive stars even before the collapse of the central core. It seems that there are no convincing observation evidence to support this theory. How to produce the strong pulsar magnetic field? This is a very interesting and important question to astrophysics and astronomers. We make use the methods of the very famous theory in condensed matter physics to study and discuss the origin of the strong pulsar magnetic fields. Similar to the production of the induction magnetic gas in metals in the presence of external magnetic fields, we found that the Pauli Spin paramagnetic effect of the extreme relativity degenerate electron gas in neutron star interiors can amplify the fossil magnetic field \( B^{(0)} (10^9 - 10^{11} \) Gauss) which was originated from the central collapse of supernova explosion) by a factor of 90. The resulting magnetic field may thus reach \( 10^{12} - 10^{13} \) Gauss (Peng et al. 2007).

About the physical origin of the super strong magnetic fields of the magnetars, we have already discussed the Pauli spin paramagnetic effect of the extreme relativistic degenerate electron gas (Peng et al. 2007, 2016a). The main idea is that the magnetic moment of the \( ^3P_2 \) Cooper pairs in the presence of the external strong magnetic fields which are already amplified through the Pauli spin paramagnetic effect, may also show some paramagnetic effect. This is similar to the theory of magnetic domain in condensed matter physics. When the interior temperature is lowered to below the Curie temperature \( (1 - 2) \times 10^7 K \), the magnetic mo-
ments of most of the $^3P_2$ Cooper pairs are spontaneously aligned in the same direction and leading to super-strong magnetic fields of the magnetic (Peng et al. 2016a). Of course, larger is the mass of the $^3P_2$ anisotropic superfluid, stronger the magnetic field.

Since the discovery of pulsars in 1967, almost all the theoretical discussion concerning pulsar physics quoted the density of state for a relativistic electron gas in strong magnetic fields from the popular statistical mechanics text books by Kubo (1965); Pathria (1972); Canuto & Chiu (1971); Haensel et al. (2007); Lai (2001); Lai & Shapiro (1991)) discussed the state equation of electron gas and the properties of magnetars in super-strong magnetic fields in detail. However, their theoretical investigations are not able to explain the astronomical observations such as the mechanism for the production of high X-ray luminosity in magnetars.

We also discussed the properties of magnetars and the mechanism for the production of high X-ray. Firstly, in the presence of super strong magnetic fields ($B > B_{cr}$, $B_{cr} = 4.414 \times 10^{15}$G is Landau critical magnetic field), the Fermi energy of the electron gas increases with the magnetic field strength (Peng et al. 2016b)

$$E_F(n) = 42.9 \left(\frac{B}{B_{cr}}\right)^{1/4} \text{ MeV.} \quad (7)$$

From Eq. (7), we may have two important conclusions (Peng et al. 2016b) as follows. Firstly, the mechanism to generate high X-ray luminosity of magnetar can be naturally explained. Secondly, direct Urca (hereafter DUrca) process ($p + e^- \rightarrow n + \nu_e, n \rightarrow p + e^- + \nu_e$) may happen in super-strong magnetic fields.

In the current model of neutron stars, there are 95% neutrons, 5% protons and the number of electrons are equal to the number of protons so as to maintain charge neutrality. The DUrca process is forbidden in the $\beta$ equilibrium neutron star interior. This is because the conservation of energy and momentum cannot be simultaneously satisfied unless the fraction of protons is more than 9% (It is most believed that the material in the inner core is really made by quarks) (Shapiro et al. 1983; Page et al. 2006).

However, it is totally different in the presence of super-strong magnetic field. Really the Fermi energy of the relativistic degenerate electron gas in the neutron star interiors increases with the magnetic field in the super-strong magnetic field $B > B_{cr}$ (Peng et al. 2016b). The Fermi energy of the electrons is apparently exceed the Fermi energy of the neutrons of the non-relativistic degenerate neutron gas. Therefore the current abstinence rule above is broken in the superstrong magnetic field.

We note that the DUrca process in strong magnetic field had been discussed in detail by Ruderman et al. (1998). In the abstract of their paper, however, they declared that in the case of superstrong magnetic fields, such that e and p populate only the lowest Landau levels is briefly outlined. Their idea is not suitable for neutron star research evidently. The reason is following: Their idea is just suitable for the Boltzmann’s classical gas really. But in the neutron stars, the highly degenerate quantum charged Fermi gas (e.g. e and p) in the superstrong magnetic fields, the filling of e and p is totally different with ones for the Boltzmann’s classical gas due to both the Pauli exclusion principle and the number of the electrons in a unit volume is finite (See the Fig.1 and Fig.2 of Peng et al. (2016b)). Besides, It is showed that the largest magnetic field of magnetars is about $3 \times 10^{15}$ G (Peng et al. 2016a), which is really a magnetic domain of the $3P2$ neutron anisotropic superfluid under Curie temperature. The case with higher magnetic fields ($\geq 10^{18}$G) in the paper by Ruderman et al. (1998) is only an assumption without physical reason. Therefore, we discuss it further according to our own idea regardless of their work.

From this it can be concluded that the DUrca process is allowed in the presence of super strong magnetic fields. The DUrca process necessarily leads to the following two important consequences. (1) The DUrca process can supply the most effective cooling mechanism to the young pulsars. (2) In addition, the DUrca process can also supply the effective cooling mechanism to the anisotropic $^3P_2$ superfluid region in neutron star interiors in super-strong magnetic fields.

For magnetic fields not very strong we have
proposed a pulsar heating mechanism, magnetic dipole radiation emitted by the magnetic moments of the Cooper pairs with parallel spins in the anisotropic $^3P_2$ superfluid vortex motion in neutron stars (Huang et al. 1982; Peng et al. 1980). In this paper, we will generalize the heating mechanism mentioned to the case of super strong magnetic fields. We also combine the generalized heating mechanism (in super-strong magnetic fields) with effective cooling mechanism in super-strong magnetic fields (i.e., the DURCA process) to investigate the phase oscillation between phase A and phase B in the anisotropic $^3P_2$ superfluid similar to the phase A and phase B in the liquid $^3$He superfluid. It is expected that this phase oscillation may be the desired mechanism to explain pulsar glitch. This is a theoretical basis of this paper.

3.2. The cooling and heating problem in neutron star interiors

3.2.1. The Cooling mechanism in neutron star interiors

The cooling problem in neutron star interiors is an important and difficult problem in the last fifty years. Neutron stars are born as the remnant of supernova explosion. Initial temperature of the nascent neutron stars is roughly $10^{11}$ K. But the observed surface temperature of young pulsars is roughly only $10^6$ K. For example the Crab nebula pulsar (PSR0531) is the remnant of the supernova explosion during 1054 with surface temperature less than $10^6$K. What is the cooling mechanism that cooled the nascent neutron star very rapidly in less than a thousand year. The general relativistic effect of gravitational radiation can be effective only in the initial few hours. This effect can cool the neutron star by at most two orders of magnitude. In the two decades between 1970 and 1990, it is hoped that the so called $\pi$–condensation in nuclear physics may be effective to cool the nascent neutron star. However, the $\pi$–condensation has never been discovered in nuclear physics experiment. Thus, the theoretical assumption of the $\pi$–condensation is unreliable.

On the other hand, if the direct Urca process is possible in neutron stars, it is a very effective cooling mechanism because energy is very rapidly carried away by the emitted ($\nu_e, \bar{\nu}_e$) pairs. Thus, it is still generally expected that the possible DURca in neutron stars is the effective cooling mechanism. The neutron energy loss rate (cooling rate) in the DURca process is

$$\epsilon_{\nu}^{\text{DURca}} = E_{\nu}^{\text{DURca}} = 10^{12} \Re \rho_{\text{neu}} T_8^6 \text{ eV cm}^3 \text{ s}^{-1},$$

where $\Re$ is the superfluid suppress factor. For $T \ll T_{cr}$, $\Re \sim (T/T_{cr})^2$, otherwise $\Re \sim (T/T_{cr})$. This superfluid suppress factor is not the usual Boltzmann suppress factor $e^{-(k_B T/\hbar c)} \sim (0.1 \sim 0.2)$ for $T_8 = 1.0$, and $\Re \approx (0.01 \sim 0.05)$ for $T_8 = 0.2$. As we noted before that the direct Urca (DURca) process is forbidden in normal neutron stars because conservation of energy and momentum cannot be simultaneously satisfied (Shapiro et al. 1983). The neutrino emission rate (cooling rate) of the modified Urca process with the suppress factor due to the neutron superfluid ($n + p + e^- \rightarrow n + n + \nu_e$, or MURca process) is

$$E_{\nu}^{\text{MURca}} = 7.4 \times 10^{12} (\rho/\rho_{\text{neu}})^{2/3} T_8^8 e^{-T/T^*} \text{ ergs cm}^3 \text{ s}^{-1},$$

where $T^* = \Delta(3P_2)/2k \approx 2.8 \times 10^6$K.

This is a high order weak interaction of six particles so it is a much less effective process than the direct Urca process as a cooling mechanism. Another important cooling mechanism is the so-called PBF (pain breaking and formation) neutrino emission mechanism. When the temperature of the neutron star reaches the transition temperature so that normal neutrons transform to superfluid neutrons (i.e., the two free neutrons with adverse direction at the surface of the Fermi sphere form Cooper pairs), the weak current leads to the emission of neutrino pairs $n + n \rightarrow [n, n] + \nu \bar{\nu}$. Thermal agitation or other heating mechanism can supply energy to break the Cooper pairs. This cyclic process may then lead to neutron star cooling. The neutrino energy emission rate (i.e., the cooling rate) is

$$E_{\nu}^{\text{PBF}} = A_{\text{PBF}} T_8^7 \text{ eV cm}^3 \text{ s}^{-1},$$

where $A_{\text{PBF}} = 10^{15}$. This process can happen only during the phase transition from normal neutrons to superfluid neutrons. For superfluid without normal neutrons (for temperatures obviously lower than the phase transition temperature) this PBF cooling mechanism is ineffective.
The most difficult problem to overcome in the cooling of neutron star interior is whether or not the direct Urca process is possible. It is impossible in the current theory. This is because the core region of neutron stars is in a quark state and outside the core the proton component is about 5%. According to the current theory the direct Urca process is impossible to occur. However, this taboo is broken by our recent paper in 2016 (Peng et al. 2016b). We pointed out that in the presence of super-strong magnetic fields of $10^{15}$ Gauss of the anisotropic superfluid $^3P_2$ state in the young pulsars (and especially magnetars $B > B_{cr}$), the direct Urca process is possible. This is because the Fermi energy of the electron gas increases with the magnetic field strength in the presence of super-strong magnetic fields in magnetars (see Eq.(7)). The Fermi energy of the relativistic degenerate electron gas exceeds the Fermi energy (about 60 MeV) of non-relativistic degenerate neutron gas. Thus, the protons near the non-relativistic degenerate proton surface may join the electrons near the relativistic degenerate electron Fermi surface to form neutrons $p + e^- \rightarrow n + \nu_e$; the energy of the emitted neutrons exceed the Fermi energy of the non-relativistic degenerate neutron gas. The emitted high energy neutrons may emit electrons via beta decay $n \rightarrow p + e^- + \bar{\nu}_e$. In this way the direct Urca process is possible.

The direct Urca process may then lead two important effects. Firstly, it can provide an effective quick cooling mechanism for the young pulsars. Secondly, the direct Urca process may lead to effective cooling in the neutron anisotropic superfluid region. In addition, we have also proposed that the heating mechanism due to the magnetic dipole radiation of the magnetic moments of the anisotropic $^3P_2$ superfluid vortex motion is very effective (Huang et al. 1982; Peng et al 1980). The combined effects of the cooling mechanism and heating mechanism in the anisotropic $^3P_2$ neutron superfluid may then lead to the phase oscillation between phase A and phase B in neutron stars similar to the phase A and phase B in neutron stars similar to the phase A and phase B in liquid superfluid $^3$He. This phase oscillation can naturally explain pulsar glitch.

Here we would like to point out that the magnetic field in the region of the anisotropic neutron superfluid $^3P_2$ state will arrive at above $10^{15}$ G. The reason is as follows: The observed magnetic field on the polar region of young pulsars are about $(2 - 5) \times 10^{12}$ G usually. The anisotropic neutron superfluid $^3P_2$ state is about in the region $2Km < r(3P_2) < 5Km$. Due to dipole magnetic fields decrease with $B(r) \propto r^{-3}$, we may suppose that the strength of the magnetic field in the region of the anisotropic neutron superfluid $3P2$ state will arrive at above $10^{15}$ G. Therefore we may take the DUrca process as the effective cooling in the neutron anisotropic superfluid region.

3.2.2. The heating mechanism in neutron star interiors

We have proposed that the heating mechanism due to the magnetic dipole radiation of the magnetic moments of the anisotropic $^3P_2$ superfluid vortex motion is very effective (Huang et al. 1982; Peng et al 1980). The combined effects of the cooling mechanism and heating mechanism in the anisotropic $^3P_2$ neutron superfluid may then lead to the phase oscillation between phase A and phase B in neutron stars similar to the phase A and phase B in neutron stars similar to the phase A and phase B in liquid superfluid $^3$He. This phase oscillation can naturally explain pulsar glitch. We reinvestigated and improved our early model above (Huang et al. 1982; Peng et al 1980). The magnetic dipole radiation of the anisotropic neutron superfluid in neutron stars is recalculated. We now elaborate in more detail this heating mechanism. The magnetic moments of the Cooper pairs in the anisotropic $^3P_2$ neutron superfluid will produce magnetic dipole radiation as the Cooper pairs rotate around the axis of the superfluid vortex. In the strong external magnetic field, the number of the $^3P_2$ Cooper pairs with magnetic moments anti-parallel to the external magnetic field is more than the number of the $^3P_2$ Cooper pairs with magnetic moments parallel to the external magnetic field. This leads to the parallel magnetic moments as the temperature is lowered. In particular, magnetic moments are strengthened at low temperatures. These superfluid vortex neutrons
rotate with high angular velocity around the superfluid axis. The closer the neutrons from the vortex axis the larger the angular velocity (inversely proportional to the square of the distance from the vortex axis, the highest angular velocity may reach above $10^{20}$ s$^{-1}$). We note that the magnetic dipole radiation from the magnetic moments of the $^3P_2$ Cooper pairs is operating via the Ekman pump cycle (Huang et al. 1982). This would lead to the motion of the normal neutrons toward the deep interior in neutron stars. The thermal X-ray photons due to the magnetic dipole radiation from the magnetic moments of the $^3P_2$ Cooper pairs are quickly absorbed by matter because of the high opacity in the stellar interior and then become a heating mechanism.

In our previous work (Huang et al. 1982; Peng et al. 1980), we actually discussed only the case for magnetic fields not very strong and at relatively high temperatures. We are now considering superstrong magnetic fields ($B > B_c$). In this case the anomalous moments of the anisotropic $^3P_2$ superfluid neutron Cooper pairs tend spontaneously to become anti-parallel to external magnetic field (and thermal agitation tend to make the direction of the magnetic moments chaotic). This is similar to the magnetic moments of the triaxial electron in metals tend to become the Pauli paramagnetism and can produce induction magnetic moments. The total induction magnetic moments of the $^3P_2$ neutron superfluid is (Peng et al. 2016a)

$$\mu_{\text{pair}}^{\text{tot}}(^3P_2) = \mu_n q N_A m(^3P_2) f(\mu_n B/kT),$$  \quad (11)

The average effective magnetic moment of each neutron Cooper pair in $^3P_2$ neutron superfluid is

$$\bar{\mu}_{n}^{\text{eff}} = \mu_n q N_A m(^3P_2) f(\mu_n B/kT),$$  \quad (12)

$$q = 3\left(\frac{\Delta(^3P_2(n))}{E_F(n)}\right)^{1/2} \approx 8.7\%,$$  \quad (13)

Where $q$ is the fraction of the neutrons that combined into the $^3P_2$ Cooper pairs. $\Delta(^3P_2(n))$ is the energy gap of the superfluid. $f(\mu_n B/kT)$ is the Brillouin function.

$$f(x) = \frac{2\sinh(2x)}{1 + 2\cosh(2x)},$$  \quad (14)

when $x \ll 1$, $f(x) \approx 4x/3$, otherwise, $f(x) \to 1$.

When the magnetic moment of the Cooper pair rotate around the axis with angular velocity, the power emitted by magnetic dipole radiation is

$$w(n) = \frac{2\omega^4}{3c^2} \bar{\mu}_{n}^{\text{eff}} \sin^2 \alpha,$$  \quad (15)

where $\alpha$ is the angle between the direction of the magnetic moment and the direction of the axis of rotation.

The axis of every superfluid vortex is parallel to the axis of rotation of the neutron star, and the length of the vortex can be approximately attained as the radius $R_P$ of the $^3P_2$ superfluid region. The angular velocity of the $^3P_2$ Cooper pair at the distance $r$ from the axis is $\omega(r)$, and the wavelength of the emitted magnetic dipole radiation from the magnetic moment is $\lambda(r) = 2c/\omega(r)$. Consider now a series of cylindrical region along the vortex axis with height $\eta \lambda(r)$ ($\eta \approx 1\%$) and radius $r \to r + dr$. The Cooper paint rotate around the superfluid vortex axis with the angular velocity $\omega(r) = nh/4\pi m_n r^2$. The magnetic dipole radiation emitted by these Cooper paint all has wavelength $\lambda(r)$. The phases of the magnetic dipole cyclotron radiation emitted by the rotating magnetic moments of the $^3P_2$ neutron Cooper pairs around the vortex filament axis are very close in this small cylinder. Therefore, the amplitudes of these radiation are added. The magnetic dipole cyclotron radiation intensity is proportional to the square of the number of Cooper paint in this region, i.e., these electromagnetic radiations are interfering with each other. The power of the magnetic dipole radiation emitted by the neutron superfluid in these interfering region is

$$W(n) = \frac{2\omega^4}{3c^2} M_{\Delta V}^2 \sin^2 \alpha,$$  \quad (16)

where $M_{\Delta V}$ denotes the total magnetic moment in $\Delta V$

$$M_{\Delta V} = dN_{\Delta V} \bar{\mu}_{n}^{\text{eff}},$$  \quad (17)

$$dN_{\Delta V} = \frac{\rho_n(r)}{m_n} \eta \lambda_s(r) 2\pi r dr,$$  \quad (18)

$$\lambda_s(r) = \frac{2\pi c}{\omega_s(r)} = \frac{4\pi c m_n r^2}{nh} \quad (r > a_0),$$  \quad (19)

where $dN_{\Delta V}$ are the number of neutron in $\Delta V$, and $\lambda_s(r)$ is the magnetic dipole radiation photon wavelengths from the vortex motion of $^3P_2$
neutron superfluid. \( a_0 \) is the core radius of the \( ^3P_2 \) superfluid vortex with normal neutrons, \( a_0 \sim (10 - 100) \text{fm} \).

The radiation power of one \( ^3P_2 \) superfluid vortex is

\[
W_1^s = \int_{a_0}^{b_0} R_\text{vrt} \sin^2 \alpha \frac{2 \omega^3}{m_n} \frac{\partial^2 \Psi_n}{\partial \phi^2} d\phi d\Delta V = A \int_{a_0}^{b_0} \int_{a_0}^{b_0} \omega^3(r') r'^2 \delta(\omega - r'^2)(dr')^2,
\]

where

\[
A = \frac{2\omega^3}{m_n} \left( \frac{\eta_0}{\eta} \right)^2 \times R_\text{vrt} \left( \frac{\eta}{0.01} \right)^2 \left( \frac{a_0}{10 \text{fm}} \right)^2 \sin^2 \alpha \times R_{p,5} [f(\mu_n B/kT)]^2 \text{ ergs s}^{-1},
\]

where \( R_{p,5} \equiv R(\text{ ^3P_2 })/(1 \text{Km}) \). Making use of Eq.(6), the total power of the emitted magnetic dipole radiation from all the \( ^3P_2 \) anisotropic superfluid vortex is given by

\[
W = \frac{1}{3} N_{\text{Vertice}} = A R_p^2 m_n \Omega \left( \frac{\sin^2 \alpha}{\eta_0} \right)
= 20 \times 10^{30} R_\text{p,5}^3 \left( \frac{n}{10^{-2}} \right) \left( \frac{\eta_0}{0.01} \right) \left( \frac{a_0}{10 \text{fm}} \right)^2 \times \left( \frac{P_{SF}(^3P_2)}{\text{1ms}} \right)^{-1} [f(\mu_n B/kT)]^2 \text{ ergs s}^{-1}.\]

(22)

The heating rate (i.e., the radiation power per unit volume) due to the magnetic dipole radiation from the \( ^3P_2 \) neutron superfluid vortex in neutron star is

\[
\varepsilon \approx 20 \times 10^{24} \left( \frac{n}{10^{-2}} \right) \left( \frac{\eta_0}{0.01} \right) \left( \frac{a_0}{10 \text{fm}} \right)^2 \sin^2 \alpha \times \left( \frac{P_{SF}(^3P_2)}{\text{1ms}} \right)^{-1} [f(\mu_n B/kT)]^2 \text{ ergs cm}^{-3} \text{ s}^{-1},\]

(23)

The heating power of the dipole radiation from the \( ^3P_2 \) neutron superfluid vortex in phase B, with \( \mu_n B/kT \gg 1 \) is \( (^3P_2 \text{MDRA}) \)

\[
\varepsilon(B) \approx 20 \times 10^{24} \left( \frac{n}{10^{-2}} \right) \left( \frac{\eta_0}{0.01} \right) \left( \frac{a_0}{10 \text{fm}} \right)^2 \sin^2 \alpha \times \left( \frac{P_{SF}(^3P_2)}{\text{1ms}} \right)^{-1} \text{ ergs cm}^{-3} \text{ s}^{-1},\]

(24)

The heating rate for phase A with \( \mu_n B/kT \ll 1 \) is the same as our results \( (^3P_2 \text{MDRA}) \) in 1982 (e.g., Huang et al. (1982))

\[
\varepsilon(A) \approx 20 \times 10^{18} \left( \frac{n}{10^{-2}} \right) \left( \frac{\eta_0}{0.01} \right) \left( \frac{a_0}{10 \text{fm}} \right)^2 \sin^2 \alpha \times \left( \frac{P_{SF}(^3P_2)}{\text{1ms}} \right)^{-1} \left( \frac{B_{12}}{T_8} \right)^2 \text{ ergs cm}^{-3} \text{ s}^{-1},\]

(25)

where \( B_{12} = B/10^{12} \text{Gauss}, T_8 = T/10^8 \text{K} \).

When we compare Eq.(24) and Eq.(25) with Eq.(8) for the cooling mechanism DUca, we obtain

\[
\varepsilon(B) \gg \varepsilon(DUca) \gg \varepsilon(A).\]

(26)

4. Our model and the properties pulsar glitch

In 2006, we proposed a phase oscillation model between normal neutron fluid and \( ^3P_2 \) neutron superfluid vortex state (Peng et al. 2006) to explain the sudden change of pulsar periods (Glitch) which is a difficult puzzle in pulsar physics. Although this model can be used to explain the grand glitches of very young pulsars but because there are too many undetermined free parameters in the model, the important physical quantities and time scales are very difficult to estimate, thus, these are the serious defect of the model.

The main objective is to extend our 2006 model to a new model that can explain the more general aspect of the observed pulsar glitch phenomena on the basis of Eq. (26). We propose a new oscillation model between phase A and phase B to reinterpret the pulsar glitch phenomena

4.1. The phase oscillation of the anisotropic \( ^3P_2 \) neutron superfluid in neutron stars

When the temperature in neutron stars in lowered to below the Curie temperature \( (\mu_n B/kT \gg 1) \), the majority of the \( ^3P_2 \) neutron superfluid Cooper pairs tend spontaneously to orient against the external magnetic field (It is noted that the neutrons and \( ^3P_2 \) neutron Cooper pairs have abnormal magnetic moments). This phase B, \( ^3P_2 \) neutron superfluid state has very strong magnetic field and the effective magnetic moments of the \( ^3P_2 \) neutron Cooper pairs are rather strong. The magnetic moment of Cooper pairs will generate very strong dipole radiation with the heating rate \( \varepsilon(B) \) (see
Fig. 1.— The observed $(P - \dot{P})$ diagram of pulsars with period longer than $P > 1.25$ s (ATNF Pulsar Catalogue, 2016) http://www.atnf.csiro.au/research/pulsar/psrcat/.
Fig. 2.— The relationship between Glitch amplitude and the stationary time interval for the young pulsar PSR J0537-6910. The time bounds drawn for the points are equal (up and down) and each is a quarter of the sum of two time intervals bounding the data segment. The slope of the dashed line fitted to the points and through the origin is 6.4394 days $\mu$Hz$^{-1}$, or 399.37 days ppm$^{-1}$ (Middleditch et al. 2006).
Eq. (24)) as the $^3P_2$ Cooper pairs rotate around the superfluid vortex axis. As we noted before, this is an effective heating mechanism in neutron stars. When the heating rates of this mechanism exceeds the cooling rates of the possible cooling mechanism exist in neutron stars (such as the DUrca process), the temperature in neutron stars would rise very quickly, the resulting thermal agitation would destroy the orderly arrangement of the magnetic moments of the $^3P_2$ neutron Cooper pairs. Once the temperature rises to $\mu_n B/kT \ll 1$, the magnetic moments of the $^3P_2$ Cooper pairs become completely chaotic, the $^3P_2$ superfluid then transforms back phase A (ESP state, i.e., equal probability state). When the $^3P_2$ neutron superfluid transforms to phase A, the strong induced magnetic moments of phase B would also disappear. For $^3P_2$ neutron superfluid in phase A, the effective magnetic moment of the $^3P_2$ Cooper pairs are very weak, and the resulting magnetic dipole radiation emitted by these $^3P_2$ Cooper pairs are also very weak (see $\varepsilon^A$ from Eq.(25)). From Eq.(26), it can be seen that the heating rate of this heating mechanism in the phase A is far less than the cooling rate of the cooling process in neutron stars (mainly the DUrca process). The temperature in neutron stars would gradually be lowered. When the thermal energy $kT$ is lowered to below the energy of the magnetic moments of the $^3P_2$ Cooper pairs ($\mu_n B/kT \gg 1$), then, the magnetic moments of the $^3P_2$ Cooper pairs once again to spontaneously arrange themselves anti-parallel to the external magnetic field, and phase B is recovered. According to Eq.(26), the resulting magnetic dipole radiation is very strong. The heating rate or this heating mechanism is much higher than the cooling rate of the DUrca process. This is the formation of the phase oscillation between phase A and phase B in the $^3P_2$ neutron superfluids.

During this heating process with the heating rate $\varepsilon^B$, the thermal energy supplied from the $^3P_2$MDRA would perturb the arrangement of the direction of the magnetic moments of the $^3P_2$ Cooper pairs, and making them chaotic. Besides, the thermal agitation due to the $^3P_2$MDRA causes some parts (denoted by $\zeta$) of the $^3P_2$ neutron Cooper pairs to break. Every broken $^3P_2$ Cooper pairs release two normal neutrons. When the number of such released normal neutrons increases to certain extent, the slowly rotating crust will be driven by the much more fast rotation of the core $^3P_2$ superfluid due to the strong coupling (by the nuclear force) of such normal neutrons with normal protons in the anisotropic superfluid region. It is mentioned in Section 1.2 that the neutron superfluid vortices may not coexist with the proton super-conducting state (Link 2003) and through electromagnetic coupling of protons in the $^3P_2$ superfluid core with electrons in the crust of the neutron star. We know that the electromagnetic interaction is an interaction through long distance. This process thus generates the pulses glitch. Once the $^3P_2$ superfluid in phase B transforms to phase A, the heating mechanism disappears. The cooling mechanism then quickly cause the neutron superfluid to deviate from the ESP state once more and phase B is recovered from phase A. The induction magnetic moment of the $^3P_2$ neutron superfluid reappears again and generating very strong induction magnetic fields.

4.2. Pulsar Glitch generated by phase oscillation of the $^3P_2$ neutrons superfluid phase A and phase B

4.2.1. Phase oscillation of the $^3P_2$ neutrons superfluid phase A and phase B

The temperature of the $^3P_2$ neutron superfluid is conspicuously lower than the phase transition temperature ($T_{tr} = 2.8 \times 10^8 K$). When the $^3P_2$ neutron superfluid is in phase B, there are no normal neutrons except in the region of the superfluid vortex core. During the phase transition from phase B to phase A, it is mainly the competing processes between the cooling mechanism of the DUrca processes for neutrino emission and the heating mechanism due to the magnetic dipole radiation from the $^3P_2$ neutron superfluid vortex motion. Since the heating rate due to the $^3P_2$MDRA from the magnetic moments of the $^3P_2$ neutron cooper pairs of phase B is much more than the cooling rate by the DUrca process ($\varepsilon^B \gg \varepsilon^{(DUrca)}$), thermal energy is supplied to the system. This causes the directions of the magnetic moments of the $^3P_2$ neutron Cooper pairs to gradually become completely chaotic and recover phase A. This period of time is the heating time scale.

For the phase oscillation between phase A and phase B, once the $^3P_2$ neutron superfluid phase...
B transformed into phase A, the original strong heating rate \(\varepsilon^{(B)}\) of phase B (B-3\(P_2\)MDRA) becomes very weak \(\varepsilon^{(A)}\) of phase A (A-3\(P_2\)MDRA), the DUcar cooling mechanism dominates and the \(3P_2\) neutron superfluid deviates again from the ESP state after a period of the cooling time scale. The \(3P_2\) neutron superfluid phase A transformed to phase B again. The \(3P_2\) neutron superfluid again induce magnetic moments and generate corresponding induction magnetic fields. This is the oscillation between phase A and phase B.

### 4.2.2. The release of normal neutron

As we mentioned before, the thermal agitation due to the \(3P_2\)MDRA causes some parts (\(\zeta\)) of the \(3P_2\) neutron Cooper pairs to break during the heating period. That means during the heating period some nasal neutrons frozen in the Cooper pairs with fraction \(\zeta\) are released to become normal neutrons during the heating period. In other words, at the same time as the \(3P_2\) neutron superfluid transformed back to phase A, there are \(\zeta N(3P_2)\) neutron Cooper pairs are broken to become normal neutrons (total numbers are \(2\zeta N(3P_2)\)). The fraction of the normal neutrons released is \(\zeta q\) (where the ratio of the number of neutrons in all the Cooper pairs to the total number of neutrons is \(q\)).

### 4.2.3. The appearance of glitch

Although the type II superconducting protons (with magnetic tubes) might appear (Haskell et al. 2017), but based on the arguments of Link (2003) that neutron superfluid vortex region may be no coexist with the type II superconducting region, we may regard that the protons are normal Fermi fluids in the neutron superfluid vortex region. The protons are tightly coupled to the electrons in neutron star interiors via coulomb interaction and they are rotation with the observed pulsar angular velocity \(\Omega = 2\pi/P\) around the axis of rotation of the neutron stars. The strongly interacting protons with the electrons are basically decoupled from the neutron superfluid vortex region in neutron stars. The protons can only interact with the small amount of normal neutrons in the neutron superfluid vortex core via nuclear interaction, while the election can only interact via the very weak electron magnetic moment interactions.

But the number, \(\zeta q\), of the normal neutrons released from the broken neutron superfluid Cooper pairs due to heating process can strongly coupled to the normal protons via strong nuclear interaction. This strong coupling can cause the fast rotation of the neutron superfluid core to drive the slowly rotating outer crust, such that the rotation of the whole magnetosphere including the outer crust to suddenly rotate much faster, leading to the appearance of glitch. In other words, glitch is the result of the sudden increase of rotation velocity of the slowly rotating outer crust driven by the fast rotation of the neutron superfluid core. There are suddenly changes during glitch. For instance, pulsar periods are suddenly changed. The angular momentum are transported to glitch. For instance, pulsar periods are suddenly changed. The angular momentum are transported to glitch. For instance, pulsar periods are suddenly changed. The angular momentum are transported to glitch. For instance, pulsar periods are suddenly changed.
ing may recover to become the superfluid neutron Cooper pairs again through the DUrca neutrino emission processes. This leads to the decoupling of the neutron superfluid interior from the outer crust again. The time required for decoupling is determined by the cooling time scale.

4.2.5. Repeated glitches of young pulsars $^3P_2$NSV phase A and phase B repeated oscillations

When the superfluid transformed from phase B to phase A (the ESP state), the induced magnetic moments become very weak as the state phase B disappears ($\varepsilon^{(B)} \gg \varepsilon^{(DURCA)} \gg \varepsilon^{(A)}$). The heating rate $\varepsilon^{(A)}$ of the $^3P_2$MDRA is far lower than the cooling rate of the DUrca, the temperature of the $^3P_2$ neutron superfluid is lowered. When the temperature lowered to below the Curie comparative, the majority of the magnetic moments of the $^3P_2$ neutron Cooper pairs tend to spontaneously parallel magnetic moments reappear. The system returns to phase B again. This is of course just the phase oscillation between phase A and phase B. The phase oscillation just mentioned may happen repeatedly many times and this is indeed the mechanism for repeated glitches (quasi-periodicity). The time interval, $\Delta t_{\text{persist}}$, for the appearance of normal neutron fluids intermediate between phase A and B denoted persist generally very short and is very difficult to monitor. Exact calculation of $\Delta t_{\text{persist}}$ is very complicated because the relevant physics is involved. But the duration of $\Delta t_{\text{persist}}$ is very important for the conservation of different pulsars. We may consider that $\Delta t_{\text{persist}} \approx t_{\text{heat}}$ (the heating time scale).

4.2.6. The disappearance of pulsar glitch

After repeated glitches the superfluous vortex quantum number is gradually lowered and the heating rate of the $^3P_2$MDRA also gradually lowered (see the Eq.(24)). The time interval between successive glitches becomes gradually longer and the amplitude tends to decrease. There is no definite relationship between the amplitude and the time interval of glitches by some unknown random chance. Once the heating rate $\varepsilon^{(B)}$ is lowered to follow the cooling rate $\varepsilon^{(DURCA)}$ of neutron stars, the neutron $^3P_2$ Cooper pairs can no longer be broken and then the $^3P_2$ neutron superfluidity can no longer recover the normal neutron fluid state mentioned before the phase oscillation between phase A and phase B is immediately stopped. This leads to the disappearance of the glitches of the old pulsars. factually, the observation evidence indicates that no glitch was observed for pulsars with periods $P > 0.7\text{s}$ (Lyne et al. 2000). The prediction of the our model is consistent with observation.

4.3. The estimates of the relevant time scales

4.3.1. Heating time scale and duration time scale of glitch

During the heating process, the origin orderly arrangement of the magnetic moments of the $^3P_2$ neutron Cooper pairs becomes completely random due to the $^3P_2$ neutron Cooper pairs with the fraction $\zeta$ absorbed the heat energy which is supplied by $\varepsilon^{(B)}$ of the $^3P_2$MDRA from phase B. This heat energy is very much larger than the cooling rate $\varepsilon^{(DURCA)}$ of neutron stars. The heat energy required during the heating process is

$$Q = \zeta \cdot \Delta N_{\pi} \cdot 2\mu_n B,$$

(27)

where $\Delta N_{\pi}$ is the difference of the number density of $^3P_2$ neutron Cooper pairs with paramagnetic and diamagnetic moment (Peng et al. 2016a)

$$\Delta N_{\pi} = n^{(3P_2-\text{pair})} f(\mu_n B/kT) = (q/2)N_A f(\mu_n B/kT).$$

(28)

Thus we have

$$Q = \zeta q N_A \mu_n B m(3P_2) \approx 4.0 \times 10^{38} (\frac{\zeta}{10^{-8}}) (\frac{m(3P_2)}{0.1M}) B_{15} \text{ ergs}$$

(29)

$$t_{\text{heat}} = \frac{Q}{(4\pi/3)R_p^2 \varepsilon^{(B)}}$$

$$\approx 0.2 (\frac{n^{(3P_2)}}{10^{21}}) (\frac{\zeta}{10^{-8}}) (\frac{\sin^2 \alpha}{0.1}) (\frac{P_{SF}(3P_2)}{1\text{ms}})^{-1}$$

$$\times (\frac{m(3P_2)}{0.1M}) B_{15} \text{ s,}$$

(30)

The heating time scale is also the characteristic time scale for the growth of the number of normal neutrons in the $^3P_2$ neutron superfluid region. It is also the duration time scale of the glitch

$$\Delta t_{\text{persist}} \approx t_{\text{heat}} \approx 0.2 (\frac{n^{(3P_2)}}{10^{21}}) (\frac{\zeta}{10^{-8}}) (\frac{\sin^2 \alpha}{0.1}) (\frac{P_{SF}(3P_2)}{1\text{ms}})^{-1}$$

$$\times (\frac{m(3P_2)}{0.1M}) B_{15} \text{ s,}$$

(31)
For young pulsars, the superfluous vortex quantity number is still very high, and \( n^3/P \gg (10^2 - 10^4) \). Thus, \( \Delta t_{\text{persist}} \) is very small and then it is difficult to discover.

### 4.3.2. The cooling time scale of glitch

During the glitch, the phase B of the \(^3P_2\) neutron superfluity immediately transformed to phase A. The heating rate \( \varepsilon^{(A)} \) is much lower than the cooling rate of the direct Urea process. During the periled of the phase transition from phase B to phase A, only a fraction \( \zeta \) of the \(^3P_2\) neutron Cooper pairs are broken to become normal neutrons. Therefore, during the cooling process, all the normal neutrons can form \(^3P_2\) cooper pairs again, the neutron superfluous recourse phase B again. The time scale required for thin transformation is the cooling time scale, which can be estimated as follows:

The total energy released by the broken fraction \( \zeta \) of the \(^3P_2\) neutron Cooper pairs to become normal is \( \Delta E = \zeta q N_A \Delta \lambda P_2 \cdot m(^3P_2) \). This energy is lost at the cooling rate \( \varepsilon^{(\text{Duca})} \). The cooling time scale is then

\[
t_{\text{cool}} = \frac{\Delta E}{4\pi/3}R_P^3 \varepsilon^{(\text{Duca})} \\
\approx 4.6 \times 10^9 \left( \frac{R}{0.01}\right) \left( \frac{\zeta}{10^{-8}} \right) \left( \frac{R_P}{5 \text{Km}} \right)^3 \left( \frac{m(^3P_2)}{0.01} \right) \text{s},
\]

(32)

Since \( t_{\text{heat}} \ll t_{\text{cool}} \), \( \Delta t_{\text{interval}} = t_{\text{heat}} + t_{\text{cool}} \approx t_{\text{cool}} \). The time interval between successive glitches is same as Eq.(32), and the time scale is about a month.

### 4.3.3. The amplitude of glitch (\( \Delta \Omega/\Omega \)) and the correlation of the glitch duration with amplitude

The angular momentum of neutron star crust is \( J_{\text{crust}} = I_{\text{crust}} \Omega \), and the angular momentum of the superfluid core of the neutron star is \( J_{\text{core}} = I_{\text{core}} \Omega_{\text{core}} \). The change of the core angular momentum during glitch is not only proportional to the angular momentum of the neutron star superfluity core and the number of normal neutrons in the superfluity core \( \zeta q N_A m(^3P_2) \), but also proportional to the heating rate \( \varepsilon^{(B)} (R(^3P_2))^3 \) of \(^3P_2\)MDRA heating process of phase B, where the factor \( \varepsilon^{(B)} (R(^3P_2))^3 \) represents the efficiency of the glitch and \( \varepsilon^{(B)} \) is given by Eq.(24). The change of the pulsar angular frequency is then proportional to the following factors,

\[
\frac{\Delta \Omega}{\Omega} \propto \frac{I_{\text{core}}}{I_{\text{crust}}} \frac{\Omega_{\text{core}}}{\Omega} \zeta q m(^3P_2) \varepsilon^{(B)} (R(^3P_2))^3.
\]

(33)

It follows that for old pulsars, \( n^3/P \) transparently decrease in amplitude, the period \( P_{\text{SF}}(^3P_2) \) grows very rapidly and \( \varepsilon^{(B)} \) also decreases conspir-acy. Thus, the amplitude of the glitch also apparently decrease. From Eqs.(24, 33), we obtain

\[
\frac{\Delta \Omega}{\Omega} \propto \frac{I_{\text{core}}}{I_{\text{crust}}} \frac{\Omega_{\text{core}} \left[ \frac{n^3}{10^{-3}} \left( \frac{P_{\text{SF}}(^3P_2)}{1 \text{ms}} \right)^{-1} \right]}{10^2},
\]

(34)

For the same young pulsar, comparing the Eq.(32), we may find an important statistical formula concerning the glitch amplitude \( \Delta \Omega/\Omega \) and the time interval between successive glitches that can be motored

\[
\frac{\Delta \Omega}{\Omega} \propto \frac{I_{\text{core}}}{I_{\text{crust}}} \frac{\Omega_{\text{core}}}{\Omega} \left( \frac{n^3}{10^{-3}} \right)^{-1} \left( \frac{P_{\text{SF}}(^3P_2)}{1 \text{ms}} \right)^{-1} \times \Delta t_{\text{interval}},
\]

(35)

This statistical relation (see Fig.2) is actually consistent with the radio pulsar period of the young pulsar PSR J0537-6910 (LMC) in Magel-lican clouds observed after a long period of 10 years monitoring (Middleditch et al. 2006). This is the strong sensational support to our theory.

We get the Eq.(35) to explain the observational phenomena. Dr. Wang (He is the second author of the paper (Middleditch et al. 2006)) claim to me that the observational data of monitoring of pulsar PSR J0573-6910 are the most complete for the pulsar glitches. our model may explain this phenomena that the pulsar PSR J0573-6910 has glitches with amplitude change roughly proportional to the time separation between two successive. As to most pulsars with glitches, the question whether they have the similar rule is still open due to the monitoring data of the glitches of most pulsars.

#### 4.3.4. Slow glitch phenomena

According to Eq.(31) the duration time scale for glitch, we know that for older pulsars, \( n^3/P \approx
(1 − 10), their superfluid vortex quantum numbers decrease very fast. As $n^3/\pi$ decreases and $P_{\text{SF}}(3P_2) \to 0.1s$, duration time scale $\Delta_t_{\text{persist}}$ may be as long as $(10^4 \sim 10^8)s$ (possibility longer than some days). This may correspond to slow glitch phenomena. Thus, slow glitch is a natural result by our theory. It is also an important observational evidence in favor of our theory.

5. Discussions and conclusions

Based on the recent works on the magnetars (Peng et al. 2016a,b) and on magnetic dipole radiation from the $3P_2$ neutron superfluid vortices ($3P_2$MDRA) in neutron stars (Peng et al. 1982), we propose a new model of glitch for young pulsars by oscillation between B and A phase of $3P_2$ neutron superfluid. The main ideas are given following:

1) On the direct Urca (DUrca) process:

For the young neutron stars, the magnetic field may be ultra strong ($B \gg B_{cr} = 4.4 \times 10^{13}\text{ Gauss}$). The DUrca process of the neutron star cooling may happen in the ultra strong magnetic field, although it is prohibited in the neutron stars with weaker magnetic field ($B \ll B_{cr}$). This result is derived in our paper, which is due to the Fermi energy of the electrons increasing with the magnetic field ($E_F(\nu) \propto (B/B_{cr})^{1/2}$) (Peng et al. 2016a).

2) A-B Phase oscillation of the $3P_2$ neutron superfluid:

When the $3P_2$ neutron superfluid is in the B-phase in the ultra magnetic field, the heating rate by ($3P_2$MDRA) is far more than the cooling rate by the DUrca process. By the heating process, the ordered magnetic moments with the direction of the magnetic field are gradually transformed into fully chaotic ESP state (A phase, i.e., equal probability phase). Once the B phase of the $3P_2$ neutron superfluid is translated into the A phase, the $3P_2$MDRA heating mechanism becomes more weaker than the direct Urca cooling process. The cooling rate will make the $3P_2$ neutron superfluid again soon deviate from the ESP state, the A phase is translated into to the B phase again. $3P_2$ neutron superfluid again induced magnetic moment, resulting in the corresponding induce magnetic field. This is just A-B Phase oscillation.

3) Glitch mechanism:

During the heating process, some $3P_2$ neutron Cooper pairs (the fraction is $\zeta$, and $\zeta \ll 1$) are also disintegrated by heating process and they will be broken up into normal neutrons at the same time. That is, when the $3P_2$ neutron superfluid is translating towards The A phase, the fraction $\zeta$ of $3P_2$ Cooper pairs are split into the normal neutrons simultaneously. The total number of being disintegrated $3P_2$ neutron Cooper pairs is $\zeta N (3P_2)$. For the total number of the normal neutrons, the fraction of them is about $\zeta q(q \approx 0.087)$. When the normal neutron component is accumulated to some amount (for an example, the fraction reaches at $\zeta \sim 10^{-7}$), they are strongly coupled with the normal protons by a nuclear force, which are strongly coupled with the electrons in the crust and the shell of the neutron star also by the Coulomb interaction. Due to these strong couplings, the slowly rotating crust (and the shell) will be suddenly driven by the internal rapidly rotating neutron superfluid. That is, the suddenly strong couplings make the crust (and the shell) suddenly rotating faster, or suddenly accelerating. This is just the Glitch. The Glitches are a repeat phenomena with quasi period $3P_2$ neutron superfluid B phase $\Rightarrow$ A phase $\Rightarrow$ B phase $\Rightarrow$ Many repeated Glitches with quasi-period. With the repeating phase transition processes, the vortex quantum number, $n$, of $3P_2$NSV is gradually reduced, and the heating rate $\varepsilon_{\nu}^{(B)}$ is also getting lower and lower. After a number of glitch, the time intervals of successive Glitch will gradually become long, and the amplitude of the Glitch is downward. But there is no strict rule or following the periodic or quasi periodic relation due to some random factor. The heating rate $\varepsilon_{\nu}^{(B)}$ of the old neutron star decrease lower and lower with decreasing of the vortex quantum number, $n$, and with longing of the rotating period of the $3P_2$ superfluid core. When the heating rate $\varepsilon_{\nu}^{(B)}$ of the old neutron star becomes lower than the cooling rate of the DUrca, the $3P_2$NSV state is no longer returned to the normal neutron fluid state. The phase oscillation of the system is stopped immediately. That means that old pulsars will no longer present the glitch.

4) Comparing with observation: the observation for the pulsars with the period $P \sim 0.7s$, and the glitch no has not been detected (Lyne et al. 2000, 2004). With the pulsar period in-
creasing, the amplitude of glitches decreases, and with the magnetic field weakening, the amplitude of glitches decreases. The slowing glitch phenomenon for some older pulsars is a naturally result in our theory. The relationship between Glitch amplitude and the stationary time interval (See the Fig.2, referenced from Middleditch et al. (2006)) is naturally got by our theory.

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