The Role of Gluon Depletion in $J/\psi$ Suppression

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Abstract

The depletion of gluons as the parton flux traverses a nucleus in a heavy-ion collision can influence the production rate of heavy-quark states. Thus the suppression of $J/\psi$ can be due to gluon depletion in the initial state in addition to nuclear and hadronic absorption in the final state. A formalism is developed to describe the depletion effect. It is shown that, without constraints from other experimental facts beside the $J/\psi$ suppression data in $pA$ and $AB$ collisions, it is not possible to determine the relative importance of depletion vs absorption. Possible relevance to the enhanced suppression seen in the $Pb-Pb$ data is mentioned but not studied.

1 Introduction

The subject of $J/\psi$ suppression in heavy-ion collisions has been extensively investigated ever since its first proposal as a signature of color deconfinement [1]. The recent measurement of enhanced suppression in $Pb-Pb$ collisions by NA50 [2] has added considerable excitement to the possible interpretation of the data as such a signature [3, 4]. Many alternative interpretations of the data have also been proposed [5–7]. While some of them may have inconsistencies with all the $pA$ and nuclear data [10], as pointed out in [3], a definitive interpretation of the $Pb$ data has not yet reached general consensus. It is not the purpose of this paper to add to the controversy; in fact, the anomalous suppression in the $Pb$ data is not our main concern here. We want to point out that there is a loophole in the interpretation of the $pA$ and nuclear data (prior to the NA50 result) that is generally accepted, i.e., the $J/\psi$ suppression is due to the absorption effects of the nuclear (and hadronic) matter that the $c\bar{c}$ system must pass through after it is produced. We investigate the possibility of another mechanism of $J/\psi$ suppression that has not been widely considered. It is the depletion of gluons before the formation of the $c\bar{c}$ state that leads to $J/\psi$. If this new mechanism is found to be relevant to any heavy-ion collisions, including the NA38 experiments using $O$ and $S$ beams [11], then the phenomenology of those past experiments must be re-examined before
a definite conclusion can be reached concerning the anomalous suppression seen in the Pb data.

The essential point to be made in this paper is that what happens to the gluons in the nuclei (apart from shadowing) before the basic subprocess $g + g \to c + \bar{c}$ is as important as what happens to the $c\bar{c}$ state after its formation. Most investigations on the subject concentrate on the latter, but the relevance of the former can easily be seen by considering the following extreme case. Suppose that an ordinary nucleus $A$ collides with an extraordinary target nucleus $B$ which is infinitely large. Clearly, the constituents of $A$ cannot propagate through $B$ indefinitely without momentum degradation and depletion. At some penetration depth the subprocess $g + g \to c + \bar{c}$ just cannot take place. Thus the production rate of $c\bar{c}$ (regardless of its fate afterwards) depends on the size of $B$ and where the production points are. If that is accepted, then the issue becomes only a quantitative matter. What are the sizes of $A$ and $B$ when the initial-state effects are not negligible?

There are two aspects of the initial-state effects on the partons: degradation and depletion. The degradation of parton momenta has been suggested previously \cite{12, 13}. The mechanism of momentum loss relies on the radiation of soft gluons, as the partons pass by scattering centers. However, such processes of multiple emission of soft gluons take time and have been shown to be suppressed by the Landau-Pomeranchuk-Migdal effect \cite{14}, although the energy loss is not entirely negligible \cite{15}. Indeed, the dependence of Drell-Yan production in $p$-$A$ collision is $\propto A^{1/2}$ \cite{16}; it may be taken as evidence of the ineffectiveness of multiple small-angle scatterings of quarks. Gluon depletion, on the other hand, is different. Whereas a quark undergoing scattering must remain as a quark, a gluon can, in addition to emitting gluons, also create $q\bar{q}$ pairs as it interacts with target partons. When that transmutation occurs, the gluon is lost from the beam, and the distribution of the gluons available for $c\bar{c}$ production downstream is thereby altered. For $J/\psi$ production the relevant momenta are $> 1.5$ GeV in the $c\bar{c}$ rest frame. If the subprocess of $g \to q\bar{q}$ involves an energy change of $\Delta E > 0.8$ GeV, then the corresponding $\Delta t (< 0.25$ fm/c) is short enough for the subprocess to be completed in a distance corresponding to a mean free path $\lambda$, i.e., in $\Delta z = \lambda/\gamma$, where $\lambda \approx 2.7$ fm, and $\gamma$ is the Lorentz factor ($\approx 10$) for the CERN-SPS energy. Even if $\Delta E < 0.8$ GeV so that the formation time for $q\bar{q}$ is long, those gluons that produce the $q\bar{q}$ upstream cannot be effective in producing $c\bar{c}$ downstream in the same nucleus. For the dominant soft processes where the $q\bar{q}$ pairs are formed outside the nucleus, those quarks and antiquarks cannot contribute to the production of lepton pairs. Thus it is quite possible that the gluons can be stripped away from the incident gluon flux, as it traverses the target nucleus, without necessitating an enhancement of the dilepton production rate. Gluon depletion is therefore the loss of gluons from the incident beam along its path for any energy change, leading to a suppression of the $g + g \to c + \bar{c}$ subprocess downstream.

We know that the conversion of gluons to sea quarks must take place efficiently in soft processes, since the gluons that carry roughly half the incident momentum in $pp$ collisions are all turned into soft pions via the enhanced $q\bar{q}$ sea with the same total momentum fraction \cite{17}, while the valence quarks produce the leading baryons, with no detectable glueballs produced. In $pA$ collisions the wounded nucleon model that is successful in describing soft pion multiplicities \cite{18} can be recast in the framework of the parton model, and one obtains a picture that is consistent with gluon depletion in that the incident gluons, once interacted, or converted to $q\bar{q}$ pairs, are ineffectual in producing more pions in subsequent collisions.
The idea of gluon depletion was first applied to the problem of $J/\psi$ suppression in $pA$ collisions by the use of an effective gluon distribution whose deviation from the gluon distribution in the physical nucleon increases at larger $A$ \cite{19}. It is shown that the idea cannot be ruled out by the existing data on $J/\psi$ and $\Upsilon$ production rates in $pA$ collisions. We now want to present a more detailed analysis of the problem for $AB$ collisions.

If gluon depletion is important in $pA$ and $AB$ collisions, then there should be a suppression of open charm production. There are some data on open-charm production, though not abundant. They are all on the single-inclusive production of the $D$ mesons \cite{20}–\cite{23}, none on $D\bar{D}$ pair production. They reveal the nuclear dependences, $A^\alpha$, that are characterized by values of $\alpha$ ranging from 0.81 to 1.02. The uncertainty is too large to be conclusive about gluon depletion. For comparison the observed $A$ dependence for $J/\psi$ suppression corresponds only to $\alpha = 0.92$ \cite{24}. Furthermore, since the single-$D$ inclusive production cross-section can include contribution from the hadronization of the $c\bar{c}$ component in the incident nucleons and from processes not initiated by gluon fusion, those data, even if accurate, cannot provide us with a reliable inference on gluon depletion. We therefore urge dedicated experimental efforts to examine the $A$ dependence of two-particle back-to-back production of $D\bar{D}$. Information acquired in such experiments can provide crucial constraints that can resolve some of the ambiguities uncovered in our study in this paper.

\section{2 Eikonalized Gluon Depletion}

The usual expression for the production of cross section of heavy-quark pairs $Q\bar{Q}$ is

$$\sigma = \sum_{i,j} \int dx_1dx_2F_i(x_1,\mu_F)F_j(x_2,\mu_F)\hat{\sigma}_{ij}(x_1,x_2,\mu_R)$$

where $\hat{\sigma}_{ij}(x_1,x_2,\mu_R)$ is the cross section for the hard subprocess, $i + j \rightarrow Q + \bar{Q}$, $i$ and $j$ being the partons involved and $\mu_R$ being the renormalization scale. $F_i(x_{1,2},\mu_F)$ are the number densities of the partons $i$ and $j$ at momentum fractions $x_1$ and $x_2$ and factorization scale $\mu_F$. Usually, the two scales are set equal to $\mu_R = \mu_F = \mu = 2m_Q$. Equation (1), which is used for hadronic collisions has generally been applied to nuclear collisions also with the appropriate replacement of the parton distributions by $F_{i/A}(x_1)$ and $F_{j/B}(x_2)$ that take into account the shadowing effects in the nuclei $A$ and $B$. Suppression of the detected onium states of $Q\bar{Q}$ due to processes that take place after the production of $Q\bar{Q}$ does not alter (1), which describes only the initial states of the hard subprocesses.

The basic point about gluon depletion is to question the validity of (1) in the gluon sector. More specifically, the challenge is in the factorizability of the nuclear gluon distributions. If $A$ and $B$ are hypothetically large, then factorization cannot be valid on physical grounds. We give below a formulation of its nonfactorizability in terms of a physical cross section.

Let us denote the nuclear thickness of nucleus $A$ at impact parameter $b_A$ by

$$T_A(b_A) = \int_{-\infty}^{\infty} dz \rho_A(b_A,z)$$

where $\rho_A(b_A,z)$ is the nuclear density, normalized such that

$$\int d^2b_A T_A(b_A) = A.$$
For $A > 1$, we define

\begin{align}
T_A^-(b_A, z_A) &= \left(1 - \frac{1}{A}\right) \int_{z_A}^\infty dz \rho_A(b_A, z) , \\
T_A^+(b_A, z_A) &= \left(1 - \frac{1}{A}\right) \int_{-\infty}^{z_A} dz \rho_A(b_A, z) ,
\end{align}

so that $\int d^2 b_A [T_A^+(b_A, z_A) + T_A^-(b_A, z_A)] = A - 1$. The variable $z$ is positive in the direction of $A$’s momentum in the cm system. Thus if a hard subprocess occurs at $z_A$, then $T_A^-(b_A, z_A)$ measures the nuclear matter in the path before the interaction point, while $T_A^+(b_A, z_A)$ refers to the matter that trails behind. Clearly, the former is relevant to the initial-state interaction, and the latter the final-state interaction. Similar expressions are defined for $T_B^\pm(b_B, z_B)$, except that $z_B$ is positive in the opposite direction, i.e., in the direction of $B$’s cm momentum. Assuming that the hard subprocess is sufficiently rare so that in any $AB$ collision it can occur at most once, we may identify $z_A$ with $z_B$ at the interaction point, although the two variables are later independently integrated over to account for all possible relative positions in the $A$ and $B$ nuclei.

The average number of inelastic collisions that a nucleon in $A$ at $(b_A, z_A)$ suffers as it traverses $B$ in a straightline path leading to $z_B$ is $\sigma_{in} T_B^-(b_B, z_B)$, where $\sigma_{in}$ is the inelastic $pN$ collision cross section. Strictly speaking, after the first collision of that nucleon (call it $p$) in $A$ with a nucleon in $B$, the former becomes a broken nucleon (call it $p'$) as it proceeds through the remaining part of $B$, and the relevant cross section for the subsequent collisions should be $\sigma_{in}^{p'N}$ rather than $\sigma_{in}^{pN}$ \[25\]. We do not make the distinction here now, and denote it by the generic symbol $\sigma_{in}$. Later, it will be combined with some other unknowns in the problem and become one overall parameter.

The probability that $p$ makes $\nu_1$ collisions in $B$ before getting to $z_B$ is

\begin{equation}
\pi_{\nu_1}(b_B, z_B) = \frac{1}{\nu_1!} \left[ \sigma_{in} T_B^-(b_B, z_B) \right]^{\nu_1} \exp\left[ -\sigma_{in} T_B^-(b_B, z_B) \right] ,
\end{equation}

where a Poisson distribution has been assumed. After a collision the gluon distribution is modified by a factor $h(x_1)$, for which we assume a rather general form

\begin{equation}
h(x_1) = h_0 x_1^\alpha (1 - x_1)^\beta .
\end{equation}

Assuming that the same $h(x_1)$ applies at every collision, the overall modified gluon distribution after $\nu_1$ collisions is then

\begin{equation}
G_{\nu_1}(x_1) = h^{\nu_1}(x_1) g_A(x_1) ,
\end{equation}

where $g_A(x_1)$ is the gluon distribution of a nucleon in $A$ before any collisions but with shadowing effects taken into account. Similar modification takes place for the gluons in $B$ due to collisions with nucleons in $A$, and we have

\begin{equation}
G_{\nu_2}(x_2) = h^{\nu_2}(x_2) g_B(x_2) ,
\end{equation}

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where \( \nu_2 \) is the number of collisions that a nucleon in \( B \) encounters in \( A \) before reaching \( z_B \) with probability \( \pi_{\nu_2}(b_A, z_A) \).

These modified gluon distributions are what must replace \( F_i F_j \) in (11), if gluon depletion is to be taken into account. Thus focusing on the \( gg \rightarrow Q\bar{Q} \) subprocess in (11) we have for a particular interaction point in \( A \) and \( B \)

\[
\bar{\sigma}_{QQ}(b_A, z_A; b_B, z_B) = \sum_{\nu_1=0}^{\infty} \sum_{\nu_2=0}^{\infty} \pi_{\nu_1}(b_B, z_B) \pi_{\nu_2}(b_A, z_A) \]

\[
\int dx_1 dx_2 g_{\nu_1}(x_1) g_{\nu_2}(x_2) \tilde{\sigma}_{gg \rightarrow QQ}(x_1, x_2) . \tag{10}
\]

It is clear from this equation that factorization does not hold. The gluon density at \( x_1 \) (in \( A \)) depends on the path length in \( B \) from \( \infty \) to \( z_B \), contained in \( \pi_{\nu_1}(b_B, z_B) \). The total production cross section of the \( QQ \) state is

\[
\sigma_{QQ} = \int d^2 b d^2 s d z_A d z_B \rho_A(s, z_A) \rho_B(s-b, z_B) \tilde{\sigma}_{QQ}(s, z_A; s-b, z_B) . \tag{11}
\]

where \( b_A = s \) and \( b_B = s-b \). Equations (10) and (11) represent an improvement of the initial-state description of \( AB \) collisions that has hitherto not been considered.

For the absorptive effects on the production of \( J/\psi \) we use the conventional description \(26, 27\). First, let us replace \( \tilde{\sigma}_{gg \rightarrow QQ}(x_1, x_2) \) in (11) by the cross section of the subprocess of \( J/\psi \) production, \( \hat{\sigma}_{gg \rightarrow J/\psi}(x_1, x_2) \), before any absorptive effect, but including intermediate states such as \( c\bar{c}g, \chi, \) etc. \(27\). Next, we take the absorption into account by writing the survival probability in the exponential form

\[
\exp \left\{ -\sigma_a \left[ T_A^+(b_A, z_A) + T_B^+(b_B, z_B) \right] \right\} \tag{12}
\]

where \( \sigma_a \) is the absorption cross section of \( J/\psi \) interacting with the final-state medium (whether hadronic, nuclear or quark-gluon plasma), leading to open charm. Putting all these factors together, we write the final result for \( J/\psi \) production cross section in the following way:

\[
\sigma_{J/\psi} = \int d^2 b d^2 s d z_A d z_B \rho_A(s, z_A) \rho_B(s-b, z_B)
\]

\[\cdot \int dx_1 dx_2 F_A(x_1, s-b, z_B) F_B(x_2, s, z_A) \hat{\sigma}_{gg \rightarrow J/\psi}(x_1, x_2)
\]

\[\cdot \exp \left\{ -\sigma_{in} \left[ T_A^-(s, z_A) + T_B^-(s-b, z_B) \right] - \sigma_a \left[ T_A^+(s, z_A) + T_B^+(s-b, z_B) \right] \right\} . \tag{13}\]

where

\[
F_A(x_1, b_B, z_B) = \sum_{\nu_1=0}^{\infty} \frac{1}{\nu_1!} \left[ \sigma_{in} T_B^-(b_B, z_B) \right]^{\nu_1} G_{\nu_1}(x_1) \tag{14}
\]

\[
F_B(x_2, b_A, z_A) = \sum_{\nu_2=0}^{\infty} \frac{1}{\nu_2!} \left[ \sigma_{in} T_A^-(b_A, z_A) \right]^{\nu_2} G_{\nu_2}(x_2) \tag{15}\]

Equation (14), for example, can be given the interpretation of the (improperly normalized) gluon distribution of \( A \) modified by the depletion effects due to \( B \). The extra normalization factor is the exponential term, which is now included in the last line of (13) for a reason that will become self-evident in the next section.
3 A Simplified Case at $x_F = 0$

In Eq. (13) we have an expression for the $J/\psi$ production cross section, obtained under a rather general description of the gluon depletion process in high-energy nuclear collisions. In principle, if all the parameters controlling different factors in the problem are known, the computation of $\sigma_{J/\psi}$ in accordance to (13) is straightforward. That is especially true if one determines only the suppression effects by computing the ratio

$$S_{J/\psi}^{AB} = \frac{\sigma_{J/\psi}^{AB}}{\sigma_{J/\psi}^{AB(0)}}$$,

where $\sigma_{J/\psi}^{AB(0)}$ is the $J/\psi$ production cross section in $AB$ collisions without absorption or depletion, since in that case the inaccuracies in the leading-order approximation of $\hat{\sigma}_{gg \rightarrow J/\psi}(x_1, x_2)$ cancel in the ratio.

At this point our investigation of the gluon depletion effects is still preliminary, since the various factors involved in the relevant dynamics are poorly understood. A systematic program for its exploration should therefore begin with a simplified calculation that can make transparent the connections between the physics issues and their phenomenological consequences. More detailed calculations can come later when proper focuses can be placed on specific issues, after a general picture becomes clear. Our immediate aim is therefore to capture that general picture and see whether gluon depletion can be relevant to the present and forthcoming experiments in the first place.

The first step in our simplification is to consider $J/\psi$ in a narrow region around $x_F = 0$. For $\sqrt{s} \simeq 20$ GeV that means $x_1 \simeq x_2 \simeq M_{J/\psi}/\sqrt{s} \simeq 0.15$ or slightly higher for the production of $c\bar{c}$ state that can lead to $J/\psi$ by soft gluon emission. In the approximation that the integrations over $x_1$ and $x_2$ in (13) need only be extended over the narrow range between 0.15 and, say, 0.18, beyond which $\hat{\sigma}_{gg \rightarrow J/\psi}(x_1, x_2)$ is negligible, we may replace the integrals by evaluating the integrand at $x_1 = x_2 = 0$.

Using (17) and ignoring $c$, we have

$$\int dx_1 dx_2 G_{\nu_1}(x_1)G_{\nu_2}(x_2)\hat{\sigma}_{gg \rightarrow J/\psi}(x_1, x_2) \simeq cD^{\nu_1+\nu_2}$$,

where $c$ and $D$ are some constants. Actually $c$ can depend on the nucleon numbers $A$ and $B$ on account of nuclear shadowing, but it will be canceled in the ratio $S_{J/\psi}^{AB}$. On the other hand, $D$ represents the effect of gluon depletion and is raised to the power $\nu_1 + \nu_2$, thereby contributing a factor of crucial importance to us.

Using (17) and ignoring $c$, we have

$$\int dx_1 dx_2 F_A(x_1, b_B, z_B) F_B(x_2, b_A, z_A)\hat{\sigma}_{gg \rightarrow J/\psi}(x_1, x_2)$$

$$= \sum_{\nu_1, \nu_2} \frac{1}{\nu_1! \nu_2!} \left[ \sigma_{in}T^-_B(b_B, z_B) \right]^{\nu_1} \left[ \sigma_{in}T^-_A(b_A, z_A) \right]^{\nu_2} D^{\nu_1+\nu_2}$$

$$= \exp \left\{ \sigma_{in}D \left[ T^-_A(b_A, z_A) + T^-_B(b_B, z_B) \right] \right\}$$.

Combining this simple result with the exponential factor in the integrand in (13) yields the probability factor

$$P = \exp \left\{ -\sigma_d \left[ T^-_A(s, z_A) + T^-_B(s - \vec{b}, z_B) \right] - \sigma_a \left[ T^+_A(s, z_A) + T^+_B(s - \vec{b}, z_B) \right] \right\}$$.
where

\[ \sigma_d = \sigma_{in}(1 - D) \]  \hspace{1cm} (20)

We may call this the depletion cross section, inasmuch as \( \sigma_a \) is called the absorption cross section. Note that \( \sigma_d \) is an overall parameter, summarizing a number of imprecisely known factors. Among them is \( D \). If there is no gluon depletion, then in (1) \( h_0 \) would be 1 and \( \alpha = \beta = 0 \). In that case we would have \( D = 1 \) and \( \sigma_d = 0 \). In past investigations of \( J/\psi \) suppression it is universally assumed that \( \sigma_d = 0 \). We now see how nonvanishing values of \( \alpha \) and \( \beta \) can have phenomenological consequences.

For phenomenology it is sufficient at first to start with (19), which involves just two parameters, \( \sigma_d \) and \( \sigma_a \). The thickness functions \( T_{A,B}^\pm \) accompanying \( \sigma_d \) and \( \sigma_a \) are just what they should be. \( T_A^- \) is the nuclear thickness of \( A \) before the interaction point, and \( T_B^- \) is that for \( B \), while \( T_{A,B}^+ \) are the respective thicknesses after the interaction point. The symmetry of the suppression mechanisms is now complete: depletion during the pre-interaction phase and absorption during the post-interaction phase. Without further study it is not obvious which is more important.

Integration over the geometrical variables can be significantly simplified without much sacrifice in accuracy, if we approximate the nuclear density by a constant value \( \rho_0 \), i.e., \( \rho_{A,B}(r) = \rho_0 \Theta(R_{A,B} - r) \) where \( R_A \) and \( R_B \) are the radii of \( A \) and \( B \), respectively. In that approximation the nuclear thicknesses are

\[ T_A^\pm = \rho_0^A(L_A \pm z_A) \hspace{1cm} \rho_0^A = (1 - 1/A) \rho_0 \]  \hspace{1cm} (21)

and similarly for \( T_B^\pm \), where

\[ L_A = (R_A^2 - s^2)^{1/2} \hspace{1cm} L_B = (R_B^2 - |\vec{s} - \vec{b}|^2)^{1/2} \]  \hspace{1cm} (22)

Combining (13), (16) and (19) we have for the suppression factor

\[ S_{J/\psi}^{AB} = N_{AB}^{-1} \int d^2 b d^2 s U(\vec{b}, \vec{s}) \]  \hspace{1cm} (23)

where

\[ U(\vec{b}, \vec{s}) = \int_{-L_A}^{L_A} dz_A \int_{-L_B}^{L_B} d z_B \exp \left\{-\sigma_d \left[ \rho_0^A (L_A - z_A) + \rho_0^B (L_B - z_B) \right] - \sigma_a \left[ \rho_0^A (L_A + z_A) + \rho_0^B (L_B + z_B) \right] \right\} \]

\[ = \left( e^{-2\sigma_a \rho_0^A L_A} - e^{-2\sigma_d \rho_0^A L_A} \right) \left( e^{-2\sigma_a \rho_0^B L_B} - e^{-2\sigma_d \rho_0^B L_B} \right) / \left[ \rho_0^A \rho_0^B (\sigma_d - \sigma_a)^2 \right], \]  \hspace{1cm} (24)

\[ N_{AB} = 4 \int d^2 b d^2 s L_a(s) L_B \left( |\vec{s} - \vec{b}| \right) \]  \hspace{1cm} (25)

The symmetry of the problem under the interchange of \( \sigma_a \) and \( \sigma_d \) is now explicit. Furthermore, if \( \sigma_d = 0 \), (23) and (24) agree with the corresponding formula derived by Gerschel and H"ufner [26].
Since we know very little about the dynamics of gluon depletion, we have no reliable information on the magnitude of $\sigma_d$. However, we do know that the Gerschel and H"ufner formula can fit the heavy-ion data on $J/\psi$ suppression, excluding the $Pb-Pb$ collision result \[2\], by use of $\sigma_a = 6-7$ mb (and, of course $\sigma_d = 0$) \[3\]. We therefore can expect that when the effects of gluon depletion are considered, the combined suppression mechanisms would have roughly the same overall cross section. Nevertheless, we use the combined cross section defined by

$$\sigma_c = \sigma_a + \sigma_d$$

as a free parameter to fit the pre-$Pb$ data. The ratio $\eta \equiv \sigma_d/\sigma_a$ can still vary between 0 and 1. The range $\eta > 1$ leads to no new result because of the $\sigma_a \leftrightarrow \sigma_d$ symmetry of $U(\vec{b}, \vec{s})$. Thus the measurement of $S_{J/\psi}^{AB}$ cannot resolve this ambiguity at this level of consideration.

For definiteness we shall examine the range $0 < \eta < 1$. In that range it is not obvious by inspecting (23)-(25) how $S_{J/\psi}^{AB}$ depends on $A$ and $B$. A numerical computation is therefore necessary.

### 4 Some Numerical Results

A crude but quick estimate of (23) without doing the integrations is to replace $L_A$ and $L_B$ in the integrands by their averages, $\frac{3}{4}R_A$ and $\frac{3}{4}R_B$, respectively \[24\], and $\rho_{A,B}^0$ by $\rho_0$ for $A$ and $B > 1$. Denoting the resultant approximation of $S_{J/\psi}^{AB}$ by $\bar{S}_{J/\psi}^{AB}$, one obtains the analytic form

$$\bar{S}_{J/\psi}^{AB} = \frac{4}{9R_AR_B(\lambda_d^{-1} - \lambda_a^{-1})^2} \left( e^{-\frac{3R_A}{\lambda_a}} - e^{-\frac{3R_A}{\lambda_d}} \right) \cdot \left( e^{-\frac{3R_B}{\lambda_a}} - e^{-\frac{3R_B}{\lambda_d}} \right),$$

(27)

where $\lambda_a$ and $\lambda_d$ are the mean free paths

$$\lambda_a = (\sigma_a\rho_0)^{-1}, \quad \lambda_d = (\sigma_d\rho_0)^{-1}.$$ \hspace{1cm} (28)

For $pA$ collisions \[27\] becomes

$$\bar{S}_{J/\psi}^{pA} = \frac{2}{3R_A(\lambda_d^{-1} - \lambda_a^{-1})} \left( e^{-\frac{3R_A}{\lambda_a}} - e^{-\frac{3R_A}{\lambda_d}} \right).$$

(29)

Using $R_A = 1.2A^{1/3}$ fm and $\rho_0^{-1} = \frac{4}{3}\pi(1.2)^3$ fm$^3$, \[27\] and \[29\] can be calculated as a function of $A$ and $B$ for various values of $\eta = \sigma_d/\sigma_a$ subject to the constraint (24). The numerical result for that will be given and discussed below.

A direct computation of $S_{J/\psi}^{AB}$ by carrying out the integrations in (23) is, of course, straightforward. Since the measurable total transverse energy $E_T$ depends on $b$, the integration in $\vec{b}$ can be suspended by plotting the suppression factor against the mean longitudinal length $L(b)$ at fixed $b$, which in turn can be related to the $E_T$. $L(b)$ is defined by

$$L(b) = \langle L_A + z_A + L_B + z_B; \vec{s} \rangle \bar{s}_{A;A,\vec{s}}$$

$$= \frac{\int_0^{R_A} ds \int_0^{2\pi} d\theta L_A(s)L_B(\vec{b} - \vec{s}) \left[ L_A(s) + L_B(\vec{b} - \vec{s}) \right]}{\int_0^{R_A} ds \int_0^{2\pi} d\theta L_A(s)L_B(\vec{b} - \vec{s})},$$

(30)
where

\[ L_B(\vec{b} - \vec{s}) = \left( R_B^2 - b^2 - s^2 + 2bs \cos \theta \right)^{1/2} \Theta \left( R_B - |\vec{b} - \vec{s}| \right) , \tag{31} \]

\( \Theta \) being the step function. Parenthetically, we remark that the value mentioned earlier for the average \( L_A \) is

\[ \langle L_A \rangle_{s,z} = \left( \frac{4}{3} \pi R_A^3 \right)^{-1} \int d^2 s \int_{-L_A}^{L_A} dz_A \left[ L_A(s) + z_A \right] = \frac{3}{4} R_A . \tag{32} \]

Thus the approximation made in (27) is effectively

\[ \langle L(b) \rangle \simeq \langle L_A \rangle_{b_A,z_A} + \langle L_B \rangle_{b_B,z_B} . \tag{33} \]

The calculated results on \( S_{J/\psi}^{AB} \) obtained from the use of (27) or (29), as the case may be, and on the more precise \( S_{J/\psi}^{AB} \) from (23)-(25) turn out to be very nearly the same with differences being at the level of \(<1\%\). We therefore present a single figure to represent both. It is shown in Fig. 1. The triangles are the calculated points for \( \sigma_c = 6 \) and 7 mb, and \( \eta = \sigma_d/\sigma_a = 0.05 \). As expected, they compare very well with the data \([2]\) as shown in open squares, except for the \( Pb-Pb \) point. The solid straight lines are best fits of the triangles, and provide adequate fits of the data up to \( SU \). That is the known result from earlier work before the \( Pb \) data, and is based on no gluon depletion. Now we fix \( \sigma_c \) at 7 mb and increase \( \eta \) to 1. The result is shown by the circles, which are fitted by the dotted line according to a quadratic formula of the form \( a + bx + cx^2 \). Although the result does not differ too much from the \( \eta = 0.05 \) case, there is nevertheless a perceptible bend downward at higher values of \( AB \). We find that to be a very encouraging sign for the possible interpretation of the \( Pb \) data as being a manifestation of the gluon depletion effect, provided that an enhancement at high values of \( AB \) can be incorporated in an improved description of the effect.

In Fig. 2 we show the results for \( S_{J/\psi}^{AB} \) calculated at specific values of \( b \), but are plotted against \( L(b) \) using (30). The same values of \( \sigma \), and \( \eta \) as in Fig. 1 are used. The general agreement with the data is again very similar to that in Fig. 1, as it should. The fact that the \( \eta = 1 \) points get within the error bars of two of the three \( Pb-Pb \) data points provides further motivation to take the possibility of gluon depletion seriously.

The most striking feature in those Figs. 1 and 2 is that the dependences on \( \eta \) are very small. Without further physics inputs it is not possible to extract from the data the realistic value of \( \eta \). A conservative conclusion is therefore that the possible contribution of gluon depletion to \( J/\psi \) suppression cannot be ruled out. This statement is already of considerable importance in our view because firstly it means that a loophole has been found in the conventional approach to \( J/\psi \) suppression and secondly with a crack opened in this new way of considering the suppression mechanisms it is possible to imagine enhanced suppression at large \( A \) and \( B \), as will be discussed in the following section. The downturn of the \( Pb \) data requires an increase of either \( \sigma_a \) or \( \sigma_d \). Any speculation on the increase of \( \sigma_d \) is not the purpose of this paper. However, until that possibility is firmly ruled out, the increase of \( \sigma_a \) as the explanation of the \( Pb \) data should only be held as tentative, albeit a very attractive one.
5 Comments

We have raised the issue of gluon depletion in heavy-ion collisions, developed a formalism to describe its effects on $J/\psi$ production, performed numerical computation to examine its consequences, and shown that the present data cannot exclude its possible contribution to $J/\psi$ suppression. The combined cross section is found to be $\sigma_c \simeq 7\,\text{mb}$, but the ratio $\eta = \sigma_d / \sigma_a$ is undetermined because of the insensitivity of the suppression factor to $\eta$.

An independent experimental constraint is necessary to determine $\eta$. We suggest the $A$ dependence of back-to-back correlated production of $D\bar{D}$ in $pA$ collisions. For $D\bar{D}$ production near threshold the formalism that we have developed is applicable, if $\sigma_a$ is set to zero. Thus any observation of $\sigma_{D\bar{D}} \propto A^\alpha$ with $\alpha < 1$ would be a signature of gluon depletion.

It is also possible to get extra information from $J/\psi$ suppression if we examine the $x_F$ dependence. The constraint to $x_F = 0$ in Sec. 3 simplifies the problem so that the suppression factor $S_{J/\psi}^{AA}$ does not depend on the detailed gluon distributions. However, for $x_F \neq 0$, the details of all the factors in (10) will become relevant, and the effect of gluon depletion cannot be described by one collective parameter $\sigma_d$. Thus the subject has the potential of developing into a fertile field of phenomenology.

A more pressing question is perhaps inescapable: does gluon depletion have any relevance to the more urgent issue of enhanced suppression observed in NA50? We have avoided addressing that issue in order to be clear about what constraints the pre-NA50 data can place on gluon depletion, the pertinence of which in heavy-ion collisions should be investigated independent of the NA50 data. We now ask whether there is any chance that the downturn of the suppression factor in the $Pb-Pb$ data can be due to some aspects of the gluon depletion process. The formalism described in Secs. 2 and 3 does not lead to any prominent nonlinear behaviors in Figs. 1 and 2. However, it should be noted that the modified gluon distribution in (8) is obtained under the assumption that the same depletion factor $h(x_1)$ applies at each of the $\nu_1$ collisions. That is a reasonable first-try to estimate the effect of multiple collisions, but it does not follow from any careful dynamical consideration. In the absence of a workable nonperturbative QCD calculation one can envisage a study in which gluon depletion is viewed as a gain-loss evolution process where gluons in a cell of momentum fraction $x_1 \sim 0.15$ are lost from the cell due to the $g \rightarrow q\bar{q}$ subprocess, but the gain comes from higher-$x_1$ cells due to $g \rightarrow gg$, for example. Since the initial gluon distribution $g_A(x_1)$ behaves roughly as $(1 - x_1)^5$, there are less gains than losses as $\nu_1$ increases. Thus it is conceivable that the modification factor in front of $g_A(x_1)$ in (8) may increase as a nonlinear power of $\nu_1$, resulting in an enhanced effect at higher $\nu_1$. The overall suppression when combined with absorption may show a break from linearity if below the crossover point the absorption is dominant, while above it the depletion is more important. These are speculations that need concrete calculations to gain substance. Nevertheless, unless such possibilities are excluded, there is no water-tight argument in favor of any other interpretation of the enhanced suppression.

Despite the alluring challenges posed by the NA50 data, we feel that at this point it is more important to pin down the extent of gluon depletion in a systematic way, with emphases on $pA$ collisions, correlated $D\bar{D}$ production, and the longitudinal momentum dependence of $J/\psi$ production.
Acknowledgments

We have benefitted from discussions with R. Lietava, H. Satz and X.-N. Wang. This work was supported, in part, by the U. S. - Slovak Science and Technology Program, the National Science Foundation under Grant No. INT-9319091 and by the U. S. Department of Energy under Grant No. DE-FG03-96ER40972.

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**Figure Captions**

**Fig. 1** The suppression factor $S_{J/\psi}^{AB}$, abbreviated as $S$, is plotted against $AB$ for various combinations of the values of the combined cross-section $\sigma_c$ and the ratio $\eta = \sigma_d/\sigma_a$. The data are from [2].

**Fig. 2** Same as in Fig. 1, but plotted against $L(b)$. 

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\[ \sigma_c = 6 \text{ mb}, \eta = 0.05 \]
\[ \sigma_c = 7 \text{ mb}, \eta = 0.05 \]
\[ \sigma_c = 7 \text{ mb}, \eta = 1.00 \]

Data [2]

Fig. 1
\[ \sigma_c = 6 \text{ mb}, \quad \eta = 0.05 \]
\[ \sigma_c = 7 \text{ mb}, \quad \eta = 0.05 \]
\[ \sigma_c = 7 \text{ mb}, \quad \eta = 1.00 \]

Data [2]

Fig. 2