Consistency of Modified Gravity with a decreasing $G_{\text{eff}}(z)$ in a $\Lambda$CDM background

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Recent analyses [1, 2] have indicated that an effective Newton’s constant $G_{\text{eff}}(z)$ decreasing with redshift may relieve the observed tension between the Planck15 best fit $\Lambda$CDM cosmological background (i.e. Planck15/$\Lambda$CDM) and the corresponding $\Lambda$CDM background favored by growth $f\sigma_8$ and weak lensing data. We investigate the consistency of such a decreasing $G_{\text{eff}}(z)$ with some viable scalar-tensor models and $f(R)$ theories. We stress that $f(R)$ theories generically cannot lead to a decreasing $G_{\text{eff}}(z)$ for any cosmological background. For scalar-tensor models we deduce that in the context of a $\Lambda$CDM cosmological background, a decreasing $G_{\text{eff}}(z)$ is not consistent with a large Brans-Dicke parameter $\omega_{BD,0}$ today. This inconsistency remains and amplifies in the presence of a phantom dark energy equation of state parameter ($w < -1$). However, it can be avoided for $w > -1$. We also find that any modified gravity model with the required decreasing $G_{\text{eff}}(z)$ and $G_{\text{eff},0} = G$, would have a characteristic signature in its growth index $\gamma$ with $0.61 \lesssim \gamma_0 \lesssim 0.69$ and large slopes $\gamma_z$, $0.16 \lesssim \gamma'_z \lesssim 0.4$, which is a characteristic signature of a decreasing (with $z$) $G_{\text{eff}}(z) < G$ on small redshifts. This is a substantial departure today from the quasi-static behaviour in $\Lambda$CDM with $(\gamma_0, \gamma'_z) \approx (0.55, -0.02)$.

I. INTRODUCTION

A wide variety of theories [3–11] have been proposed for the description of the observed accelerating expansion of the universe. However, the simplest model (the $\Lambda$CDM model [12–14]) remains consistent with (almost) all cosmological observations [15–26]. The best fit parameter values of this model have been reported by the Planck mission [25, 26] with extreme accuracy and define the concordance Planck15/$\Lambda$CDM model, which is consistent with geometric cosmological observations. Such observations include the Type Ia Supernova data [27–30], the Baryon Acoustic Oscillations data [16, 17] etc.

However, recent analyses [31–35] indicate some tension between the Planck15/$\Lambda$CDM model and some dynamical observations measuring the growth rate of cosmological perturbations. Such observations include Weak Lensing data [36–40] and Redshift Space Distortions (RSD) [1, 2, 31, 41–43]. The robust observable reported by RSD surveys is the product

\[
f\sigma_8(z) \equiv f(z) \cdot \sigma(z) = -(1 + z) \frac{\sigma_{8,0}}{\delta_0} \delta'_m(z) \tag{1.1}
\]

where $f \equiv d\ln \delta_m / d\ln a$ describes the growth of cosmological matter density perturbations $\delta_m = \delta \rho_m / \rho_m$, and a prime stands for the derivative with respect to redshift $z$. The quantity $\sigma_8(z)$ is the rms density fluctuation on comoving scales corresponding to $8h^{-1}\text{Mpc}$ at redshift $z$ while $\sigma_{8,0}$ refers to the present time value of $\sigma_8(z)$.

Since 2006 there has been a significant increase of surveys that measure RSDs leading to a collection of 63 $f\sigma_8$ data points [2]. Despite possible correlations among the data points of this dataset its various subsamples considered in the literature [1, 41, 43, 44] indicate various levels of tension between Planck15/$\Lambda$CDM parameter values and the parameter values favored by the considered $f\sigma_8$ subsample. The level of this tension appears to decrease for more recently published $f\sigma_8$ data [2]. However all considered $f\sigma_8$ subsamples seem to indicate a reduced growth rate compared to the one expected in the context of Planck15/$\Lambda$CDM and GR.

The observed tension [45] could be relaxed following one of the following methods

- Modifying the background, i.e. considering a smaller value for $\Omega_{m,0}$ and/or a smaller value for $\sigma_{8,0}$. Other probes, such as the $\text{WMAP}$, report lower values for both $\Omega_{m,0}$ and $\sigma_{8,0}$ [46].

- Considering modified gravity theories which give a decreasing function of $G_{\text{eff}}(z)$ with $z$.

In this analysis we investigate the consequences of the second case.

The linear evolution of $\delta_m$ is given by the equation

\[
\delta_m + 2H\delta_m - 4\pi G_{\text{eff}} \rho \delta_m = 0 \tag{1.2}
\]

In terms of the redshift, Eq. (1.2) is rewritten as

\[
\delta''_m + \left( \frac{\ln h^2 \prime}{2} - \frac{1}{1 + z} \right) \delta'_m - \frac{3}{2} (1 + z) h^{-2} \frac{G_{\text{eff}}(z, k)}{G} \Omega_{m,0} \delta_m , \tag{1.3}
\]

where $\Omega_{m,0}$ is the present relative matter density, $h \equiv \frac{H_0}{H_0}$ and $H_0$ is the Hubble parameter today, $G_{\text{eff}}(z, k)$ is the effective Newton’s constant which for General Relativity (GR) is the usual Newton’s constant $G$. In general for
modified gravity models $G_{\text{eff}}$ depends both on the redshift $z$ and the scale $k$.

The central quantity $G_{\text{eff}}(z,k)$ comes from a generalization of Poisson’s equation \[47],[48]
\[
\nabla^2 \phi \approx 4\pi G_{\text{eff}} \rho \delta_m, \tag{1.4}
\]
while the potential $\phi$ can be read off the perturbed metric in the longitudinal (Newtonian) gauge
\[
ds^2 = -(1 + 2\phi)dt^2 + a^2(1 - 2\psi)d\vec{x}^2 \tag{1.5}
\]
Solar system constraints \[49],[50] imply that
\[
\left| H_0^{-1} \frac{\dot{G}_{\text{eff},0}}{G} \right| = \left| \frac{G_{\text{eff},0}'}{G} \right| \lesssim 10^{-3} \tag{1.6}
\]
whereas they actually leave the second derivative unconstrained since
\[
\left| \frac{G_{\text{eff},0}''}{G} \right| \lesssim 10^5 \tag{1.7}
\]
Thus an interesting question that arises is the following: “Which modified gravity models are consistent with $G_{\text{eff}}(z)/G < 1$ at low $z$?” A naive answer to this question would indicate that any modified theory of gravity can lead to $G_{\text{eff}}(z)/G < 1$ at low $z$ for some appropriate parameter values. In the present analysis we address this question and argue that this is not so for at least two important and intensively studied examples, the standard, massless scalar tensor gravity model and $f(R)$ models. More specifically, we address the following questions:

1. What is the generic form of $G_{\text{eff}}(z)$ at low $z$ for standard scalar tensor and $f(R)$ theories when one assumes a $\Lambda$CDM background expansion?

2. How do the above answers change for different background expansion rates $H(z)$?

The structure of this paper is the following: In the next section we derive the generic form of $G_{\text{eff}}(z)$ for low $z$ for some modified gravity models. In Sec. III we consider the behaviour of the growth index in these models. Finally in Sec. IV we summarize and discuss our results.

II. $G_{\text{eff}}(z)$ IN SOME MODELS

It is our purpose to investigate whether some modified gravity models allow for a decrease of $G_{\text{eff}}$ below the usual Newton’s constant $G$, its value in GR.

II.1. $f(R)$ modified gravity models

The answer is negative \[51\] for viable $f(R)$ models, see e.g. \[9], \[52\]. This can be seen immediately from the expression of $G_{\text{eff}}$ in these models, viz.\n\[
G_{\text{eff}}(z,k) = \left( \frac{df}{dR} \right)^{-1} \left[ 1 + \frac{(\frac{z}{c})^2}{3 (1 + (\frac{z}{c})^2)} \right], \quad \lambda = \frac{a(t)}{k}. \tag{2.1}
\]
where $\lambda_c$ is a function of $R$ and is the Compton wavelength of the scalaron \[52\]. Eq. (2.1) is the equivalent form of (we set $a_0 = 1$) \[53\]
\[
G_{\text{eff}}(z,k) = \left( \frac{df}{dR} \right)^{-1} \left[ 1 + \frac{4 \left( \frac{df}{d\lambda} \frac{d\lambda}{dR} \right) \cdot k^2 (1 + z)^2}{1 + 3 \left( \frac{df}{d\lambda} \frac{d\lambda}{dR} \right) \cdot k^2 (1 + z)^2} \right] \tag{2.2}
\]
with $\lambda_c^2(R) = \frac{3df}{d\lambda}$. In viable $f(R)$ models, all relevant cosmic scales satisfy $\lambda \gg \lambda_c(R)$, with $\frac{df}{d\lambda} = 1$ to high accuracy, deep in the matter era. Hence the standard growth of perturbations is regained during that era.

Now, as $\frac{df}{d\lambda} > 0$ \[52\] (which is a crucial assumption for the avoidance of ghost instabilities), the factor in front of the brackets in (2.1) increases when $R$ decreases with the expansion, and thus it is always larger than one. The expression inside the brackets in (2.1) is obviously always larger than one too. So we have for $f(R)$ models that $G_{\text{eff}} > G$ for any scale at any time.

At low redshifts further, as the critical length $\lambda_c$ increases significantly with the decrease of matter density and of the Ricci scalar $R$, the expression inside the brackets can become as large as $\frac{4}{3}$ in the present era on scales $\lambda \ll \lambda_c$. Hence the growth of matter perturbations on these scales will be enhanced compared to the standard growth. Note that this does not exclude the possibility for $G_{\text{eff}}(z)$ to evolve non monotonically as a function of $z$. Indeed, $G_{\text{eff}}(z)$ can, and generically does, increase with $z$ on some interval in the present era, however always satisfying $G_{\text{eff}}(z) > G$. Note that (2.1) uses also $\frac{d\lambda}{dR} > 0$, besides $\frac{df}{d\lambda} > 0$, ensuring the absence of ghost.

It is important to emphasize that the result presented above, i.e. $G_{\text{eff}}(z) > 1$, is independent of the background expansion in contrast to the results we will derive in the next subsection in scalar-tensor gravity models.

II.2. (Massless) Scalar-Tensor Gravity

The action for this family of scalar-tensor (ST) gravity models reads (see e.g. \[48\])
\[
S = \int d^4x \sqrt{-g} \left[ \frac{1}{2} F(\phi) R - \frac{1}{2} Z(\phi) g^{\mu \nu} \partial_\mu \phi \partial_\nu \phi - U(\phi) \right] + S_m \tag{2.3}
\]
where $R$ is the Ricci scalar and $S_m$ is the matter action which does not involve the scalar field $\phi$. Note that the coupling of these matter components to gravity is the same as in GR. In what follows we set $Z(\phi) = 1$ and we consider $U > 0$. This means that we are dealing
with situations where the Brans-Dicke coefficient $\omega_{BD}$ is positive (see below).

We consider the flat Friedmann Lemaître Robertson Walker metric (FLRW), which is given by

$$ds^2 = -dt^2 + a^2(t) \left[ dr^2 + r^2(d\theta^2 + \sin^2 \theta d\phi^2) \right]. \quad (2.4)$$

Then it is straightforward to show that the dynamical equations of the system are

$$3FH^2 = \rho_m + \frac{1}{2} \dot{\phi}^2 - 3H \dot{F} + U \quad (2.5)$$
$$-2F \ddot{H} = \rho_m + \dot{\phi}^2 + \ddot{F} - H \dot{F} \quad (2.6)$$

In Eq. (2.5) and Eq. (2.6), a homogeneous scalar field and a homogeneous comoving perfect dustlike fluid are assumed ($p_m = 0$).

$$F'' + \left[ (\ln h)' - \frac{4}{1 + z} \right] F' + \left[ \frac{6}{(1 + z)^2} - \frac{2}{1 + z} (\ln h)' \right] F = \frac{6u}{(1 + z)^2} \frac{F_0}{h^2} \Omega_{U,0} + 3(1 + z)h^{-2} F_0 \Omega_{m,0} \quad (2.7)$$

$$\frac{\dot{\phi}^2}{6} = -\frac{F'}{1 + z} + \frac{F}{(1 + z)^2} - \frac{F_0 u}{(1 + z)^2 h^2} \Omega_{U,0} - \frac{F_0 (1 + z)}{h^2} \Omega_{m,0}, \quad (2.8)$$

where a prime stands for a derivative with respect to $z$. The first equation is a second order master equation for the quantity $F$ which is obtained by eliminating the kinetic term of the scalar field $\phi$. The second equation is an algebraic equation for the scalar field kinetic term once the equation for $F$ is solved. For our purposes however, we want rather to eliminate the potential energy $U$ and combining (2.7), (2.8) we easily get the following equation

$$\frac{\dot{\phi}^2}{6} = -F'' - \left[ (\ln h)' + \frac{2}{1 + z} \right] F' + 2 \frac{(\ln h)'}{1 + z} F - 3 \Omega_{m,0} (1 + z) F_0 h^{-2} \quad (2.9)$$

For our later calculations, it is convenient to introduce the quantity $\Delta^2$. Its value today relative to $F_0$ is

$$\Delta^2 \equiv \frac{\dot{\phi}^2}{F_0^2} = 6 \left( \Omega_{DE,0} - \Omega_{U,0} - \frac{F''}{F_0} \right), \quad (2.10)$$

where $\Omega_{DE,0} = 1 - \Omega_{m,0}$. The last equality follows from the Friedmann equations and in particular from Eq. (2.6). In our universe, $\Delta^2$ is a positive quantity so that our notation is not confusing. Indeed, the right-hand side of (2.9) is positive whenever the representation with $Z = 1$ applies ($\omega_{BD} > 0$). This is the case in our universe today and on very low redshifts. Once the background is fixed, (2.9) expresses the kinetic term of the scalar field in terms of $F$ and its derivatives. We return now to the quantity $G_{\text{eff}}$ on which we want to focus. In terms of the redshift $z$, $G_{\text{eff}}$ can be written as

$$G_{\text{eff}} = G_N \frac{2F + 4 \dot{F}^2}{2F + 3 \dot{F}^2} \quad (2.11)$$

and we have set

$$G_N \equiv \frac{1}{8\pi F}. \quad (2.14)$$

Notice that we have considered the massless scalar-tensor gravity model. This means physically that no screening (chameleon) mechanism is at work here, in contrast to the $f(R)$ models considered in the previous subsection. In the $f(R)$ models the mass term is central in the chameleon mechanism where locally $R$ and the mass become very large which enables the model to evade all local constraints. This is not so for our massless ST model, in particular this is why we have $G_{\text{eff},0} = G$ in this case.

Solar system constraints imply today the very strong inequality [49]

$$\omega_{BD,0} = \frac{\Delta^2}{(F''/F_0)^2} > 4 \times 10^4, \quad (2.15)$$
hence we have in particular
\[ G_{\text{eff},0} = G = G_{N,0} , \]  
(2.16)
where \( G \) is the usual Newton’s constant. We see that \( \Delta^2 \) is positive as said above. Let us consider now the evolution of \( G_{\text{eff}} \). On low redshifts, we can write the Taylor expansion
\[ G_{\text{eff}}(z) = G_{\text{eff},0} + G_{\text{eff},0}' z + G_{\text{eff},0}'' \frac{z^2}{2} + \ldots \]  
(2.17)
The systematic expansion at low redshifts of all basic physical quantities in this ST gravity model was performed earlier \[ [49] \] (see also Ref.\[50\]). Here we extend these results by considering their implication for the low \( z \) expansion of the effective gravitational constant \( G_{\text{eff}} \) up to second order (the first order was already derived there).

Before proceeding with the calculation of the coefficients in the expansion (2.17) we return to the consequences of solar system constraints. We have the following expression for \( \omega_{BD,0} \) \[ [49] \]
\[ \omega_{BD,0} = \frac{6(\Omega_{DE,0} - \Omega_{U,0} - \frac{F_0'}{F_0})}{\frac{F_0'}{F_0}} \]  
(2.18)
As we have said above, see (2.15), \( \omega_{BD,0} \) is a very large quantity. Hence solar system constraints imply
\[ \left| \frac{F_0'}{F_0} \right| \lesssim 10^{-2} . \]  
(2.19)
This strong inequality will considerably simplify all calculations and will be assumed everywhere below. Using (2.12), (2.13) and (2.16) the following results are obtained straightforwardly

\[ G_{\text{eff}}(z) = G_{\text{eff},0} + G_{\text{eff},0}' z + G_{\text{eff},0}'' \frac{z^2}{2} + \ldots \]  
(2.17)

After some straightforward calculation, using (2.19), we finally obtain to leading order
\[ G_{\text{eff},0} \sim -\frac{F_0'}{F_0} G \ll G \]  
(2.20)
\[ G''_{\text{eff},0} \sim \left[ \frac{F''_0}{F_0} + \frac{F''_0}{\phi'^2} \right] G \]  
(2.21)
Note that the leading order of \( G_{\text{eff},0}' \) is proportional to \( \frac{F_0'}{F_0} \) in agreement with the result obtained in \[ [49] \]. The expression for \( G_{\text{eff},0}'' \) can be further simplified using (2.9) at \( z = 0 \) which takes the form
\[ \phi'^2 = -\frac{F''_0}{F_0} + 2(\ln h)'_0 F_0 - 3\Omega_{m,0} F_0 \]  
(2.22)
When substituted in (2.17), we obtain
\[ G_{\text{eff}}(z) \simeq G \left( 1 + \frac{F''_0}{F_0} \left[ -2 + \Delta^{-2} \left( (\ln h)'_0 - 3\Omega_{m,0} \right) \right] \frac{z^2}{2} \right) \]  
(2.23)
\[ \simeq G \left( 1 + \frac{F''_0}{F_0} \left[ -1 + \frac{3}{2} \Delta^{-2} (1 + w_{DE,0})(1 - \Omega_{m,0}) \right] \frac{z^2}{2} \right) . \]  
(2.24)
Hence the variation of \( G_{\text{eff}} \) on low redshifts, and in particular its departure from \( G \), depends crucially on the magnitude and on the sign of \( \frac{F''_0}{F_0} \). For \( \frac{F''_0}{F_0} \ll 1 \), we have
\[ \frac{F''_0}{F_0} = 3(w_{DE,0} + 1)\Omega_{DE,0} - 6(\Omega_{DE,0} - \Omega_{U,0}) . \]  
(2.25)
When this is substituted in (2.27), we finally obtain
\[ G_{\text{eff}}(z) \simeq G \left( 1 + \left[ 3(w_{DE,0} + 1)\Omega_{DE,0} - 6(\Omega_{DE,0} - \Omega_{U,0}) \right] \left[ -1 + \frac{3}{2} \Delta^{-2} (1 + w_{DE,0})\Omega_{DE,0} \right] \frac{z^2}{2} \right) . \]  
(2.26)
Before proceeding with our investigation, a first important remark is that (2.29) simplifies considerably for a
ACDM background to yield
\[ G_{\text{eff}}(z) \simeq G \left( 1 - \frac{F_0''}{F_0} z^2 \right) \]
\[ \simeq G \left( 1 + 6(\Omega_{DE,0} - \Omega_{U,0}) z^2 \right) \tag{2.30} \]
Two cases can arise depending on the sign of \( \Omega_{DE,0} - \Omega_{U,0} \).

a) The most natural case to consider is
\[ \Omega_{DE,0} - \Omega_{U,0} \gg \frac{F_0''}{F_0} \],
while the solar system constraint (2.15) is satisfied using (2.18), (2.19). In this case we have from (2.10)
\[ \Delta^2 \approx 6(\Omega_{DE,0} - \Omega_{U,0}) \],
and we obtain for a ACDM background from (2.28), (2.30)
\[ \frac{F_0''}{F_0} \simeq -6(\Omega_{DE,0} - \Omega_{U,0}) \approx -\Delta^2 < 0 \tag{2.33} \]
and
\[ G_{\text{eff}}(z) = G \left( 1 + 6(\Omega_{DE,0} - \Omega_{U,0}) z^2 \right) \approx G \left( 1 + \Delta^2 z^2 \right) \tag{2.34} \]
which is a central result of our calculation. Hence for a ACDM background, \( G_{\text{eff}}(z) \) will increase rather than decrease in the past on low redshifts. It is seen from (2.28) that this result applies whenever dark energy (DE) is of the phantom type today and satisfies \( w_{DE,0} < -1 \). It can even hold for some small range of values satisfying \( w_{DE,0} \gtrsim -1 \).

It is possible however to get a decreasing \( G_{\text{eff}}(z) \) if we move away from ACDM towards higher values of \( w_{DE,0} \) satisfying \( w_{DE,0} > -1 \). By inspection of (2.29), this is the case if the following inequality holds
\[ \Delta^2 < 3(w_{DE,0} + 1)\Omega_{DE,0} < 2\Delta^2 \tag{2.35} \]
The inequality (2.35) can be easily satisfied for a large number of parameter values as we show on Figure 1. We conclude that, for case a), \( w_{DE,0} > -1 \) is necessary in order to have a decreasing \( G_{\text{eff}}(z) \) on low redshifts.

b) In principle, there is also the possibility \( \Omega_{DE,0} - \Omega_{U,0} < 0 \). In that case however we have from (2.18)
\[ |\Omega_{DE,0} - \Omega_{U,0}| < \left| \frac{F_0''}{F_0} \right| \lesssim 10^{-4} \tag{2.36} \]
and also
\[ \Delta^2 < \left| \frac{F_0''}{F_0} \right| \lesssim 10^{-4} \tag{2.37} \]
In (2.28) we should now discard the second term on the right hand side as we have done for all terms proportional to \( \frac{F_0''}{F_0} \) and we simply write
\[ \frac{F_0''}{F_0} = 3(w_{DE,0} + 1)\Omega_{DE,0} \tag{2.38} \]
and (2.29) becomes
\[ G_{\text{eff}}(z) \simeq G \left( 1 + \left[ 3(w_{DE,0} + 1)\Omega_{DE,0} \right] \left[ -1 + \frac{3}{2} \Delta^2 (1 + w_{DE,0})\Omega_{DE,0} \right] z^2 \right) \tag{2.39} \]

We illustrate these results with Fig. 1. Clearly, \( G_{\text{eff},0}'' < 0 \) (blue regions) can only be achieved for \( w > -1 \). This behavior remains valid for different values of \( \Omega_{m,0} \). The results presented in this section assume that \( G_{\text{eff},0}'' \simeq 0 \) (or equivalently \( F_0'' \simeq 0 \)) due to solar system constraints. In the presence of screening this assumption may not be necessary as in that case the cosmological behavior of \( G_{\text{eff}} \) gets decoupled from the corresponding behaviour in the solar system where the mean curvature and density are significantly larger than in cosmological scales. However, as we have seen with \( f(R) \) models, this does not necessarily imply that a decreasing \( G_{\text{eff}}(z) \) is allowed and actually in these models, it is not allowed.
consistent with zero or positive values at a level more cated by areas where \( G \) is nearly constant in GR \( \gamma \approx 0.55 \) [54]. For many modified theories, \( \gamma \) departs from this quasi-constant behaviour [55] and can be written at small \( z \) as \( \gamma = \gamma_0 + \gamma'_0 z \).

Using Eqs. (3.2), (3.4), we have

\[
2 \ln \Omega_m \frac{d^2 \gamma}{d \ln a} + (2 \gamma - 1) \frac{d \ln \Omega_m}{d \ln a} + 1 + 2 \Omega_m \gamma_m = 0
\]

So if we know the background expansion and \( \Omega_{m,0} \), as well as the behaviour of \( G_{\text{eff}} \), we can calculate \( \gamma \) [56]. Assuming a Planck15/ΛCDM background while \( G_{\text{eff}} \) is of the form (3.1) we are left with a first order differential equation for \( \gamma \). We fix the initial condition in the past in order to find \( \gamma(z) \), and therefore \( \gamma_0 \equiv \gamma(0) \) and \( \gamma'_0 \equiv \gamma'(0) \), for each \((g_a, n)\) (see Fig. 3). Notice that initial conditions (in the past) are essentially irrelevant at the present time because of the presence of an attractor so we get the same behaviour at late time.

For the case \( n = m \) in Eq. (3.1), our result are consistent with previous results derived in [56]: a weaker gravitational constant \( (G_{\text{eff}} < G) \) implies \( \gamma_0 > \gamma_0^{\Lambda \text{CDM}} \) for a given background while a stronger gravitational constant \( (G_{\text{eff}} > G) \) implies \( \gamma_0 < \gamma_0^{\Lambda \text{CDM}} \).

Also in accordance with [57], we found that \( \gamma_0 \) is linearly related to \( \gamma_0 \) for different values of the free parameters of the model, see Fig. 2. In fact, considering Eq. (3.5) at \( z = 0 \), we have

\[
\gamma'_0 = \frac{1}{2 \ln \Omega_{m,0}} \left[ (2 \gamma_0 - 1) 3 w_{DE,0} (1 - \Omega_{m,0}) + 1 + 2 \Omega_{m,0}^{\gamma_0} - 3 \frac{G_{\text{eff},0}}{G} \Omega_{m,0}^{1 - \gamma_0} \right].
\]

In our case we have by construction \( \frac{G_{\text{eff},0}}{G} = 1 \) for all other words, independently of the specific modified gravity model that produces (3.1) and the Planck15/ΛCDM background expansion, we can find the resulting growth index.

In particular the quantity \( f(z) \) obeys the following equation

\[
\frac{df}{d \ln a} + f^2 + \frac{1}{2} \left( \frac{d \ln \Omega_m}{d \ln a} \right) f = \frac{3 G_{\text{eff}}}{2 G} \Omega_m,
\]

where \( \Omega_m = \frac{\Omega_{m,0}}{H_0^2 / H_0^2} = \frac{\Omega_{m,0}}{H^2(a)} \) and \( \delta \) can be obtained directly through

\[
\delta(a) = \delta_i \exp \left[ \int_{\ln a_i}^{\ln a} f d(\ln a') \right].
\]

The growth rate \( f \) can always be written as

\[
f = \Omega_m^{\gamma_0}.
\]

where \( \gamma \) is nearly constant in GR \( \gamma \approx 0.55 \) [54]. For many modified theories, \( \gamma \) departs from this quasi-constant behaviour [55] and can be written at small \( z \) as \( \gamma = \gamma_0 + \gamma'_0 z \). Using Eqs. (3.2), (3.4), we have

\[
2 \ln \Omega_m \frac{d^2 \gamma}{d \ln a} + (2 \gamma - 1) \frac{d \ln \Omega_m}{d \ln a} + 1 + 2 \Omega_m \gamma_m = 0
\]

Also in accordance with [57], we found that \( \gamma_0 \) is linearly related to \( \gamma_0 \) for different values of the free parameters of the model, see Fig. 2. In fact, considering Eq. (3.5) at \( z = 0 \), we have

\[
\gamma'_0 = \frac{1}{2 \ln \Omega_{m,0}} \left[ (2 \gamma_0 - 1) 3 w_{DE,0} (1 - \Omega_{m,0}) + 1 + 2 \Omega_{m,0}^{\gamma_0} - 3 \frac{G_{\text{eff},0}}{G} \Omega_{m,0}^{1 - \gamma_0} \right].
\]

In our case we have by construction \( \frac{G_{\text{eff},0}}{G} = 1 \) for all parameters \( n, m, g_a \). Hence the relation \( \gamma'_0 = f(\gamma_0) \) from
are shown in the $g_0$ values shown for (Planck15/$\Lambda$CDM). This relation is independent of $g_0$ because we have $G_{\text{eff},0} = G$ by construction for all parameter values $g_0$ and $n$.

(3.6) is the same as shown in Fig. 2.

We have also considered constraints from SNIa data and we find that these do not significantly favor $g_0 < 0$ (see Fig. 3). The distance modulus for the SNIa data can be written as [58]

$$\mu = \mu_{\text{ACDM}} + \frac{15}{4} \log \frac{G_{\text{eff}}}{G_{\text{eff},0}},$$

where the additional term comes from the modification of the luminosity distance as a result of modified gravity. In our analysis we use the latest Pantheon Sample [59] of 1048 SNIa ranging from $0.01 < z < 2.3$.

Finally, as we have stressed earlier, $f(R)$ models always satisfy $G_{\text{eff}} > G$. Therefore, for all background evolutions that would produce $\gamma_0 \approx 0.55$ inside GR, the value of $\gamma_0$ obtained in $f(R)$ models will satisfy $\gamma_0 \lesssim 0.55$ in accordance with [60].

### IV. SUMMARY AND DISCUSSION

A $G_{\text{eff}}(z) < G$ at low redshifts could alleviate the tension between Planck15/$\Lambda$CDM and the growth data $f\sigma_8$. In this work we have studied the implications of such a $G_{\text{eff}}(z)$ for two classes of modified gravity DE models.

The $f(R)$ DE models cannot produce such a behaviour. More generally they cannot allow for $G_{\text{eff}}(z) < G$ irrespective of the background expansion [51]. We have further shown that in (massless) scalar tensor theories, a decreasing $G_{\text{eff}}(z)$ at low redshifts is not possible for a $\Lambda$CDM background. However this behaviour is possible if we consider $w_{DE,0} > -1$, and a substantial decrease of $G_{\text{eff}}(z)$ requires a substantial departure from $w_{DE,0} = -1$.

We have further shown that any model with the required behaviour of $G_{\text{eff}}(z)$ in a $\Lambda$CDM background will exhibit a characteristic signature of its growth index $\gamma$, with $0.61 \lesssim \gamma_0 \lesssim 0.69$ and a non-negligible slope $\gamma_1$ at $z = 0$, $0.16 \lesssim \gamma_1 \lesssim 0.4$. Once redshift space distortion data become more accurate, it will be possible not only to discriminate between these models and $\Lambda$CDM, but also

-0.3 at the 3$\sigma$ level for $n = 2$. For higher values of $n$ however, significantly lower values of $g_0$ are allowed. Similar results were obtained for the CMB data (ISW effect) in Ref. [1]. These results indicate that the tension of the growth data with Planck15/$\Lambda$CDM can only be partially physical. At least part of this tension is probably due to statistical and/or systematic effects of the growth data. However this tension points to a mildly decreasing $G_{\text{eff}}(z)$ rather than to an increasing, or even a constant, $G_{\text{eff}}(z)$.

To complete this section, we provide the values of $(\gamma_0, \gamma_0')$ corresponding to parameters $(g_0, n)$ favored by the $f\sigma_8$ data (see Table I). For each $n$, the best value of $g_0$ and therefore $G_{\text{eff}}(z)$ was obtained in [1] as shown in Table I.

| $n$ | $g_0$ | $\gamma_0$ | $\gamma_0'$ |
|-----|-------|------------|-------------|
| 0.343 | -1.200 | 0.686 | 0.398 |
| 2 | -1.156 | 0.629 | 0.219 |
| 3 | -1.534 | 0.620 | 0.189 |
| 4 | -2.006 | 0.619 | 0.174 |
| 5 | -2.542 | 0.612 | 0.165 |
| 6 | -3.110 | 0.611 | 0.160 |

TABLE I. Corresponding values of $(\gamma_0, \gamma_0')$ for various $(n, g_0)$ favored by $f\sigma_8$ data alone. The behaviour of $\gamma$ is a characteristic signature for a decreasing $G_{\text{eff}} < G$ on low redshifts ($G_{\text{eff},0} = G$). We remind that all values $n \leq 5$ are ruled out by SNIa data.

FIG. 2. The linear relation between $\gamma_0'$ and $\gamma_0$ is shown for any values of $g_0$ and $n$ and a fixed background (Planck15/$\Lambda$CDM). This relation is independent of $g_0$ because we have $G_{\text{eff},0} = G$ by construction for all parameter values $g_0$ and $n$.

FIG. 3. Constraints at 1$\sigma$, 2$\sigma$, 3$\sigma$ level from the SNIa data are shown in the $g_0, n$ plane. The dashed curves correspond to couples with the same value $\gamma_0$. The corresponding value of $\gamma_0'$ is easily obtained from Fig. 2.
to confirm or to rule out the decreasing $G_{\text{eff}}(z)$ which is required to explain the data.

While it is known that some modified gravity DE models can have $G_{\text{eff}}(z) < G$ in principle [61], it is interesting that two prominent representatives of viable modified gravity DE models cannot produce such a behaviour. If this behaviour plays a role in the solution to the existing tension in the data between Planck15/ΛCDM and the redshift space distortion data, our results imply that more elaborate modified gravity models are required.

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