On the Day-Night Effect and CC to NC Event Rate Ratio Predictions for the SNO Detector

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Abstract

Detailed predictions for the D-N asymmetry for the Super-Kamiokande experiment, as well as for the Full Night and Core D-N asymmetries in the solar neutrino induced CC event rate and the Day, Night and Core ratios of the CC and NC event rates, measured in the SNO experiment, are derived in the cases of the LMA MSW and LOW solutions of the solar neutrino problem. The indicated observables for the SNO experiment are calculated for two values of the threshold (effective) kinetic energy of the final state electron in the CC reaction on deuterium: $T_{\text{e,th}} = 6.75$ MeV and 5.0 MeV. The possibilities to further constrain the regions of the LMA MSW and LOW solutions of the solar neutrino problem by using the forthcoming SNO data on the D-N asymmetry and on the CC to NC event rate ratio are also discussed.

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1 Introduction

The recent SNO results [1], combined with the data from the Super-Kamiokande experiment [2], clearly demonstrate the presence of \( \nu_\mu \) (\( \nu_\tau \)) component in the flux of solar neutrinos reaching the Earth [2]. This represents a compelling evidence for oscillations and/or transitions of the solar neutrinos.

The SNO experiment measured the rate of the charged current (CC) reaction \( \nu_e + D \rightarrow e^- + p + p \) for \( T_e \geq 0.75 \) MeV, \( T_e \) being the (effective) kinetic energy of the final state electron [2]. The reaction is due to the flux of solar \( \nu_e \) from \( ^8 \text{B} \) decay having energy of \( E \approx 8.2 \) MeV. Assuming that the \( ^8 \text{B} \) neutrino energy spectrum is not substantially modified by the solar neutrino oscillations, the SNO collaboration obtained the following value of the solar \( \nu_e \) flux:

\[
\Phi^{CC}(\nu_e) = (1.75 \pm 0.15) \times 10^6 \text{ cm}^{-2}\text{s}^{-1},
\]

where we have added the statistical and systematic errors and the estimated theoretical uncertainty (due to the uncertainty in the CC reaction cross section) given in [1] in quadrature. Utilizing the data on \( \Phi^{CC}(\nu_e) \) and the data on the solar neutrino flux obtained by the Super-Kamiokande experiment, it is possible to deduce [1] (see also [3]) the value of the non-electron neutrino component in the flux of solar neutrinos measured by the Super-Kamiokande collaboration:

\[
\Phi(\nu_{\mu,\tau}) = (3.69 \pm 1.13) \times 10^6 \text{ cm}^{-2}\text{s}^{-1}.
\]

This flux is different from zero at more than 3 s. d.

Global analyses of the solar neutrino data [4, 5, 6, 7], including the SNO results [1] and the Super-Kamiokande data on the \( e^- \)-spectrum and day-night asymmetry, in terms of the neutrino oscillation hypothesis show \( \theta_{12} \approx 0 \), \( \theta_{23} \approx \pi/4 \) and \( \theta_{13} \approx 0 \), characterizing the two-neutrino MSW, the LOW and the quasi-vacuum oscillation (QVO) solutions of the solar neutrino problem with transitions into active neutrinos. In the case of the LMA solution, the range of values of the neutrino mass-squared difference \( \Delta m^2 > 0 \), characterizing the two-neutrino transitions of the solar neutrinos into an active neutrino, \( \nu_e \rightarrow \nu_{\mu(\tau)} \), was found, e.g., in [8] and [9] to extend (at 99% C.L.) to \( \sim 5.0 \times 10^{-4} \text{ eV}^2 \) and \( \sim 8.0 \times 10^{-4} \text{ eV}^2 \), respectively:

\[
\text{LMA MSW : } \quad 2.0 \times 10^{-5} \text{ eV}^2 \lesssim \Delta m^2 \lesssim (5.0 - 8.0) \times 10^{-4} \text{ eV}^2.
\]

The best fit values of \( \Delta m^2 \) obtained in the independent analyses [8, 9, 10, 11] are grouped in the narrow interval \( \Delta m^2)_{BFV} = (4.3 - 4.9) \times 10^{-5} \text{ eV}^2 \). A smaller best fit value was found in [11], \( \Delta m^2)_{BFV} = (3.3 - 3.7) \times 10^{-5} \text{ eV}^2 \), while a larger value was obtained, e.g., in [14]: \( \Delta m^2)_{BFV} = 6.0 \times 10^{-5} \text{ eV}^2 \). Similar results, \( \Delta m^2)_{BFV} = 6.3 \times 10^{-5} \text{ eV}^2 \) and \( 6.1 \times 10^{-5} \text{ eV}^2 \), were obtained in [12] and in [15] by performing a Bayesian analysis of the solar neutrino data. For the mixing parameter \( \sin^2 2\theta \), which controls the oscillations of the solar neutrinos, it was found, e.g., in [8] at 99% C.L.:

\[
\text{LMA MSW : } \quad 0.60 \lesssim \sin^2 2\theta \lesssim 0.99.
\]

The best fit values of \( \sin^2 2\theta \) obtained, e.g., in [8, 9, 10, 11] are confined to the interval \( \sin^2 2\theta)_{BFV} = (0.79 - 0.82) \). Somewhat smaller values were found in [11], [15] and in [10]: \( \sin^2 2\theta)_{BFV} = (0.75 - 0.79) \); 0.76; 0.77, respectively.

Detailed results were obtained in [8, 9, 10, 11, 12, 13, 15, 16] for the LOW solution as well. The 95% C.L. allowed intervals of values of \( \Delta m^2 \) and \( \sin^2 2\theta \) found in [8], for instance, read:

\[
\text{LOW : } \quad 6.0 \times 10^{-8} \text{ eV}^2 \lesssim \Delta m^2 \lesssim 1.8 \times 10^{-7} \text{ eV}^2, \quad 0.94 \lesssim \sin^2 2\theta \lesssim 1.0.
\]
The best fit values of $\Delta m^2$ and $\sin^2 2\theta$ for the LOW solution, derived, e.g., in \cite{3, 5, 9, 10, 11, 14} are compatible with each other and are all approximately given by $(\Delta m^2)_{BFV} \approx 10^{-7}$ eV$^2$ and $(\sin^2 2\theta)_{BFV} \approx (0.94 - 0.97)$. A substantially different value of $(\Delta m^2)_{BFV}$ was found in \cite{14}: $(\Delta m^2)_{BFV} \approx 5.5 \times 10^{-8}$ eV$^2$ and $(\sin^2 2\theta)_{BFV} \approx 0.99$.

The analyses \cite{3, 8, 9, 10, 11, 12, 16} were based, in particular, on the standard solar model (SSM) predictions of ref. \cite{15} (BP2000) for the different components of the solar neutrino flux ($pp, pe, ^7Be, ^8B, CNO, hep, ^{17}F$). In \cite{3, 5, 9, 10, 11, 12, 14} the published Super-Kamiokande data on the day-night (D-N) asymmetry \cite{2} were used as input in the analyses, while in \cite{16} the latest (preliminary) results on the D-N asymmetry, obtained from the analysis of all currently available Super-Kamiokande solar neutrino data was utilized (see further). The authors of ref. \cite{14} have used in their analysis a new value of the $^8B$ neutrino flux which is suggested by the results of the latest (and more precise) experimental measurement \cite{19} of the cross section of the reaction $p^{+7}Be \rightarrow ^8B + \gamma$. According to the SSM, the $^8B$ is produced in the Sun in the indicated reaction and the $\beta^+$-decay of $^8B$ in the central part of the Sun gives rise to the solar $^8B$ neutrino flux. The results obtained in \cite{13} give a larger $p^{−7}Be$ reaction cross-section (with smaller uncertainty), and correspondingly - a larger astrophysical factor $S_{17}$ (see, e.g., \cite{17}) than the one used in \cite{13}, which implies, in particular, a larger value of the $^8B$ neutrino flux than the value predicted in \cite{13}.

In the global Bayesian analysis performed in \cite{15} the SSM predictions for the solar neutrino fluxes were not used: both the values of the fluxes and of the oscillation parameters were derived from the data.

The best fit values of $\Delta m^2$ found in \cite{3, 8, 9, 10, 11} differ from that derived in \cite{16} essentially due to the difference in the Super-Kamiokande data on the D-N asymmetry used as input in the corresponding analyses: in \cite{16} the latest (preliminary) Super-Kamiokande result implying a smaller mean value of the D-N asymmetry than the published one in \cite{2} was utilized. The smaller possible D-N asymmetry drives $(\Delta m^2)_{BFV}$ to larger (smaller) value in the LMA MSW (LOW) solution region \cite{13}. Although the data on the D-N asymmetry used in \cite{3, 8, 9, 10, 11} and in \cite{14} are the same, the best fit value of $\Delta m^2$ in the LMA MSW solution region found in \cite{14} is smaller than those found in \cite{3, 8, 9, 10, 11} because of the difference between the values of the astrophysical factor $S_{17}$, and thus of the $^8B$ neutrino flux, used in \cite{14} and in \cite{3, 8, 9, 10, 11}.

In the present article we update our earlier predictions \cite{20, 21, 22} for the D-N asymmetry for the Super-Kamiokande and SNO experiments, taking into account the recent progress in the studies of solar neutrinos. The day-night (D-N) effect - a difference between the solar neutrino event rates during the day and during the night, caused by the additional transitions of the solar neutrinos taking place at night while the neutrinos cross the Earth on the way to the detector (see, e.g., \cite{23, 24} and the references quoted therein), is a unique testable prediction of the MSW solutions of the solar neutrino problem. The experimental observation of a non-zero D-N asymmetry

$$A^N_{D−N} \equiv \frac{R_N − R_D}{(R_N + R_D)/2},$$

where $R_N$ and $R_D$ are, e.g., the one year averaged event rates in a given detector respectively during the night and the day, would be a very strong evidence in favor (if not a proof) of an MSW solution of the solar neutrino problem. Extensive predictions for the magnitude of the D-N effect for the Super-Kamiokande and SNO detectors have been obtained in \cite{20, 21, 22, 23, 24, 25, 26, 27, 28}.

High precision calculations of the D-N asymmetry in the one year averaged recoil-electron spectrum measured in the Super-Kamiokande experiment and in the energy-integrated event rates for the $^8B$ neutrino flux predicted in \cite{15} reads $\Phi(B)_{BP2000} = 5.05 \times (1^{+0.20}_{−0.19}) \times 10^6$ cm$^{-2}$s$^{-1}$, while the flux utilized in the analysis performed in \cite{14} is $\Phi(B)_{BP2000} = 5.93 \times (1^{+0.14}_{−0.13}) \times 10^6$ cm$^{-2}$s$^{-1}$.

Let us note that, e.g., in \cite{3, 5, 9, 10, 11} results obtained by treating the $^8B$ neutrino flux as a free parameter in the analysis were also reported. These results were taken into account when we quoted above the $\Delta m^2$ and $\sin^2 2\theta$ best fit values.
two experiments were performed for three event samples, \textit{Night}, \textit{Core} and \textit{Mantle}, in [20, 21, 22, 27]. The night fractions of these event samples are due to neutrinos which respectively cross the Earth along any trajectory, cross the Earth core, and cross only the Earth mantle (but not the core), on the way to the detector.

We focus here, in particular, on providing detailed predictions for the D-N asymmetry for the LMA MSW and the LOW solutions of the solar neutrino problem, which are favored by the current solar neutrino data. We will consider in what follows the \textit{Night} (or \textit{Full Night}) and the \textit{Core} D-N asymmetries, $A^N_{D-N}$ and $A^C_{D-N}$. The current Super-Kamiokande data [2] do not contain evidence for a substantial D-N asymmetry: the latest published result on $A^N_{D-N}$ reads

$$A^N_{D-N}(SK) = 0.033 \pm 0.022 \text{ (stat.)} +^{0.013}_{-0.012} \text{ (syst.)},$$

while the result of the latest analysis of \textit{all currently available Super-Kamiokande solar neutrino data} gives even smaller mean value [16]

$$A^N_{D-N}(SK) = 0.021 \pm 0.022 \text{ (stat.)} +^{0.013}_{-0.012} \text{ (syst.)}. \quad (8)$$

Adding the errors in eqs. (7) and (8) in quadrature, one finds that at 1.5 (2.0) s.d., $A^N_{D-N}(SK) < 0.072 \ (0.085)$ and $A^N_{D-N}(SK) < 0.060 \ (0.073)$, respectively.

We give in the present article also detailed predictions for another important observable - the ratio of the event rates of the CC reaction $\nu_e + D \rightarrow e^- + p + p$, $R_{SNO}(CC)$, and of the neutral current (NC) reaction $\nu + D \rightarrow \nu + n + p$, $R_{SNO}(NC)$, induced by the solar neutrinos in SNO,

$$R^{SNO}_{CC/NC} \equiv \frac{R_{SNO}(CC)}{R_{SNO}(NC)}, \quad (9)$$

which is normalized above to the value of the same ratio in the absence of oscillations of solar neutrinos, $R^{0}_{SNO}(CC)/R^{0}_{SNO}(NC)$. First results on the D-N asymmetry and on the CC to NC event rate ratio $R^{SNO}_{CC/NC}$ are expected to be published in the near future by the SNO collaboration. We discuss as well the possibilities to further constrain the regions of the LMA MSW and LOW-QVO solutions of the solar neutrino problem by using the forthcoming SNO data on the D-N asymmetry $A^N_{D-N}$ and on the CC to NC event rate ratio $R^{SNO}_{CC/NC}$.

Updated predictions for the \textit{Night} D-N asymmetry and the average CC to NC event rate ratio for the SNO experiment were derived after the publication of the first SNO results also in [11, 14]. However, our study overlaps little with those performed in [11, 14].

2 The LMA MSW and LOW Solutions and the D-N Asymmetry for Super-Kamiokande and SNO Experiments

Our predictions for the \textit{Full Night} D-N asymmetry in the regions of the LMA MSW and LOW solutions of the solar neutrino problem for the Super-Kamiokande and SNO experiments, $A^N_{D-N}(SK)$ and $A^N_{D-N}(SNO)$, are shown in Figs. 1 (upper panel) and 2, respectively, while in Figs. 1 (lower panel) and 3 we show predictions for the \textit{Core} D-N asymmetry for the two detectors [14]. $A^C_{D-N}(SK)$ and $A^C_{D-N}(SNO)$. The calculations of $A^{N,C}_{D-N}(SNO)$ have been performed by taking into account, in particular, the energy resolution function of the SNO detector [14]. The effect of the energy resolution function on the values of the \textit{Full Night} and \textit{Core} D-N asymmetries, $A^{N,C}_{D-N}(SNO)$, as our results show, is negligible for values of the asymmetries $A^{N,C}_{D-N}(SNO) \geq 0.01$. In Figs. 2 and

\footnote{The calculations of the D-N effect for the Super-Kamiokande and SNO detectors performed in the present article are based on the methods developed for our earlier studies of the D-N effect for these detectors, which are described in detail in [20, 21, 22, 27]. Here we use the BP2000 SSM [13] predictions for the electron number density distribution in the Sun.}
3 we show contours of constant $A_{D-N}^N(SNO)$ and $A_{D-N}^{NC}(SNO)$ in the $\Delta m^2 - \tan^2 \theta$ plane for two values of the threshold kinetic energy of the final state electron $T_{e,\text{th}} = 6.75$ MeV (upper panels) and 5.0 MeV (lower panels). The published SNO data were obtained using the first value $[4]$, while the second one is the threshold energy planned to be reached at a later stage of the experiment. A comparison of the upper and lower panels in Fig. 2 shows that the Full Night D-N asymmetry $A_{D-N}^N(SNO)$ decreases somewhat in the LMA MSW solution region - approximately by $\sim (8 - 10\%)$, when $T_{e,\text{th}}$ is decreased from 6.75 MeV to 5.0 MeV. The change in the LOW solution region is opposite and larger in magnitude than in the LMA MSW solution region: $A_{D-N}^N(SNO)$ increases by about $\sim (15 - 20\%)$ when $T_{e,\text{th}}$ is reduced from 6.75 MeV to 5.0 MeV. The above results imply that for given $\sin^2 2\theta$, the same values of the asymmetry at $T_{e,\text{th}} = 5.0$ MeV occur in both the LMA MSW and LOW solution regions at smaller values of $\Delta m^2$ than for $T_{e,\text{th}} = 6.75$ MeV. Qualitatively similar conclusions are valid for the Core D-N asymmetry $A_{C-D-N}^C(SNO)$ (Fig. 3).

Consider the predictions for the D-N asymmetry in the case of the LMA MSW solution. As Figs. 1 - 3 show, at $\Delta m^2 > 1.5 \times 10^{-4}$ eV$^2$ both $A_{D-N}^N(SK)$ and $A_{D-N}^{NC}(SNO)$ are smaller than 1%. For given $\Delta m^2 \lesssim 10^{-4}$ eV$^2$ and $\sin^2 2\theta$ from the LMA solution region we have $[22]$

$$A_{D-N}^N(SNO) \cong (1.5 - 2.0)A_{D-N}^N(SK),$$

The difference between $A_{D-N}^N(SK)$ and $A_{D-N}^N(SNO)$ in the indicated region is due to i) the contribution of the NC $\nu_{\mu(\tau)} - e^-$ elastic scattering reaction (in addition to that due to the $\nu_e - e^-$ elastic scattering) to the solar neutrino event rate measured by the Super-Kamiokande experiment, and ii) to the relatively small value of the solar $\nu_e$ survival probability in the Sun, $P \sim 0.3$. The indicated NC contribution to the Super-Kamiokande event rate tends to diminish the D-N asymmetry. Obviously, there is no similar contribution to the SNO CC event rate.

Thus, in the case of the LMA MSW solution of the solar neutrino problem, the D-N asymmetry measured in SNO can be considerably larger than the D-N asymmetry measured in the Super-Kamiokande experiment $[22]$. The 2 s.d. upper limit on the D-N asymmetry, $A_{D-N}^N(SK) < 8.5\% (7.3\%)$, following from the Super-Kamiokande data, eq. (7) (eq. (8)), for instance, does not exclude a D-N asymmetry in the SNO CC event rate as large as $\sim (10 - 15)\%$. As Fig. 2 shows, $A_{D-N}^N(SNO)$ can reach a value of $\sim 20\%$ in the 99\% C.L. region of the LMA MSW solution, eq. (3). In the 95\% C.L. LMA solution region of $[14]$ one has $A_{D-N}^N(SK) \approx 13\%$. In the best fit point of the LMA MSW solution, found in $[3, 8, 9, 11]$, we get for $T_{e,\text{th}} = 6.75$ MeV (5.0 MeV),

$$(A_{D-N}^N(SNO))|_{BF1}^{LMA} \cong 7.3 (6.6)\%.$$ Even larger value of the asymmetry $A_{D-N}^N(SNO)$ corresponds to the best fit point obtained in $[14]$: $(A_{D-N}^N(SNO))|_{BF2}^{LMA} \approx 10.1 (9.3)\%$. At the same time, one finds a considerably smaller value of $A_{D-N}^N(SNO)$ in the best fit point derived in $[14]$: $(A_{D-N}^N(SNO))|_{BF3}^{LMA} \approx 5.0 (4.6)\%$. The Full Night D-N asymmetry in the Super-Kamiokande detector in the indicated three different best fit points found in $[3, 8, 9, 11]$, $[14]$ and $[16]$ for $T_{e,\text{th}} = 5.0$ MeV read, respectively: $(A_{D-N}^N(SK))|_{BF}^{LMA} \approx 3.9\%; 5.4\%; 2.6\%$.

Actually, as it is not difficult to show, the following approximate relation between $A_{D-N}^N(SK)$ and $A_{D-N}^N(SNO)$ holds for fixed $\Delta m^2$ and $\sin^2 2\theta$ from the region of the LMA MSW solution:

$$A_{D-N}^N(SNO) \cong A_{D-N}^N(SK) \left[ 1 + \frac{r}{(1-r)P} \right],$$

where $r = \sigma(\nu_{\mu(\tau)}e^-)/\sigma(\nu_e e^-)$, $\sigma(\nu_e e^-)$ being the $\nu_1 - e^-$ elastic scattering cross section, $l = e, \mu, \tau$, and $P$ is the average probability of solar $\nu_e$ survival in the Sun. For the solar neutrino energies of interest one has $r \approx 0.155$. For $\Delta m^2$ and $\sin^2 2\theta$ from the LMA MSW solution region, the transitions of the solar ($^{8}$B) neutrinos with energies $E \gtrsim 5.0$ MeV are adiabatic and in a relatively large sub-region one finds $P \approx \sin^2 \theta$. We would like to emphasize that the relation $[14]$ is not very precise and can serve only for rough estimates.

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$^6$The results of our calculations show that in the LMA MSW and LOW solution regions the predicted Mantle D-N asymmetry in the CC event rate at SNO $[22]$ practically coincides with the Full Night D-N asymmetry.
As a comparison of Figs. 2 and 3 indicates, for given $\Delta m^2$ and $\sin^2 2\theta$ from the LMA MSW solution region, the Core D-N asymmetry in the SNO detector is predicted to be larger than the Full Night D-N asymmetry typically by a factor of $\sim 1.2$ [22]: $A_{D-N}^C(SNO) \approx 1.2A_{D-N}^N(SNO)$.

The predicted values of $A_{D-N}^N(SK)$ and $A_{D-N}^N(SNO)$ in the LOW solution region differ less than in the case of the LMA MSW solution since the average survival probability $P$ is typically by a factor of $\sim 1.5$ larger for the LOW solution than in the case of the LMA MSW solution. As it follows from Fig. 2, in the LOW solution region given by eq. (5) one has $(A_{D-N}^N(SNO))^\text{LOW} \approx (A_{D-N}^N(SNO))^\text{LOW}$.

In what regards the most recent calculations of the CC and NC reaction cross sections [35], they lead to a value of the ratio $A_{D-N}^N(SNO) \approx 10\%$ of interest is practically the same when it is calculated within a given theoretical model, ref. [31] or ref. [32], for the CC and NC reaction cross sections: $R_{CC}^N(SNO) \approx 1.2$ (1.5)%.

As Fig. 2 indicates, an observation of a non-zero D-N asymmetry which is definitely greater than 1%, $A_{D-N}^N(SNO) > 1\%$, would rule out the QVO solution which requires values of $\Delta m^2$ from the interval $\Delta m^2 \approx (5 \times 10^{-10} - 5 \times 10^{-8})$ eV$^2$ and $\sin^2 2\theta \approx (0.70 - 1.0)$ for a discussion of the QVO oscillations of solar neutrinos and of the QVO solution see, e.g., \cite{3, 8, 9, 11, 12, 29}.

3 Predictions for $R_{CC/NC}^{SNO}$

The importance of the measurement of the CC to NC solar neutrino event rate ratio in the SNO experiment, $R_{CC/NC}^{SNO}$, for determining the correct solution of the solar neutrino problem has been widely discussed (see, e.g., \cite{3, 8, 9, 11, 12, 29}) and the references quoted therein. We have performed a high precision calculations of the ratio $R_{CC/NC}^{SNO}$, in particular, for three different CC reaction cross-sections which were taken from \cite{30, 2, 22}, and using the electron number density distribution in the Sun from \cite{18}. The differences in the results obtained for $R_{CC/NC}^{SNO}$ using the three cross sections are negligible in the regions of the LMA MSW and LOW solutions of the solar neutrino problem of interest. We have found that the effect of the SNO energy resolution function on the predictions for $R_{CC/NC}^{SNO}$ is negligible as well.

The SNO experiment will measure the CC and NC average event rates, $R_{SNO}^{CC}(CC)$ and $R_{SNO}^{NC}(NC)$. In order to compare these results with the predictions for the double ratio $R_{CC/NC}^{SNO}$, eq. \cite{1}, one has to use as a normalization factor the theoretically calculated (in the absence of solar neutrino oscillations) value of the ratio $R_{SNO}^0(C)/R_{SNO}^0(NC)$. The ratio $R_{SNO}^0(C)/R_{SNO}^0(NC)$ of interest is practically the same when it is calculated within a given theoretical model, ref. \cite{31} or ref. \cite{2}, for the CC and NC reaction cross sections: for $T_{e,th} = 5.00$ (6.75) MeV we find $R_{SNO}^{01}(C)/R_{SNO}^{01}(NC) = 1.927 (1.232)$ using the CC and NC cross sections derived in ref. \cite{31}; utilizing the results of ref. \cite{32} we get: $R_{SNO}^{02}(C)/R_{SNO}^{02}(NC) = 1.933 (1.235)$. This is not the case, however, if one calculates the ratio of interest by taking the CC reaction cross section from ref. \cite{31} and the NC reaction cross section from ref. \cite{32} and vice versa - the CC cross section from ref. \cite{32} and the NC cross section from ref. \cite{31}. One finds for $T_{e,th} = 5.00$ (6.75) MeV in the two cases, respectively: $R_{SNO}^{01}(C)/R_{SNO}^{02}(NC) = 2.049 (1.310)$ and $R_{SNO}^{02}(C)/R_{SNO}^{01}(NC) = 1.818 (1.161)$. Now the relative difference between the two calculated ratios is $\sim (10 - 15)\%$ and cannot be neglected. This suggests, in particular, that the ratio $R_{SNO}^{01}(C)/R_{SNO}^{01}(NC)$ should be calculated within a given theoretical model for the CC and NC reaction cross sections. In what regards the most recent calculations of the CC and NC reaction cross sections \cite{22}, they lead to a value of the ratio $R_{SNO}^0(C)/R_{SNO}^0(NC)$ which does not differ from that obtained in ref. \cite{31} by more than 1%.

Similarly, although the effect of the SNO energy resolution function on the predictions for the double ratio $R_{CC/NC}^{SNO}$ is negligible, it is not negligible in the case of the ratio $R_{CC}^{SNO}(CC)/R_{NC}^{SNO}(NC)$. For $T_{e,th} = 6.75$ MeV, for instance, we find that the value of $R_{CC}^{SNO}(CC)/R_{NC}^{SNO}(NC)$ calculated...
using the results of ref. [11] (or of ref. [12]) assuming ideal resolution, is by a factor of 1.033 bigger than the value obtained by taking the SNO resolution function into account.

Our predictions for the average ratio during the day (Day ratio), $R_{CC/NC}^{SNO}(D)$, during the night (Full Night ratio), $R_{CC/NC}^{SNO}(N)$, and for the case of the CC event rate produced at night by solar neutrinos which cross the Earth core on the way to SNO (Core ratio), $R_{CC/NC}^{SNO}(C)$, are shown respectively in Figs. 4 - 6. Results for each of the three ratios were obtained for two values of the (effective) kinetic energy threshold of the detected $e^-$ in the CC reaction: for $E_{e,th} = 6.75$ MeV (upper panels) and $E_{e,th} = 5.00$ MeV (lower panels).

A comparison of Fig. 4 and Figs. 5 - 6 shows that CC to NC ratio increases substantially during the night for values of $\Delta m^2$ and $\sin^2 2\theta$ from the region $2 \times 10^{-7} \text{ eV}^2 \leq \Delta m^2 \leq 2 \times 10^{-5} \text{ eV}^2$, $10^{-2} \leq \sin^2 2\theta \leq 0.98$, which, however, is not favored by the current solar neutrino data. The increase is due to the Earth matter effect. The difference between Night and Core ratio in the indicated region is essentially caused by the Earth mantle-core interference effect [36]. For $\Delta m^2 > 2 \times 10^{-5} \text{ eV}^2$ in the LMA solution region, and in all the LOW solution region, the difference between the Core and and Night ratios is negligible, $R_{CC/NC}^{SNO}(N) \approx R_{CC/NC}^{SNO}(C)$. At $\Delta m^2 \approx 8 \times 10^{-5} \text{ eV}^2$ in the LMA region, and at $\Delta m^2 \approx 10^{-7} \text{ eV}^2$ in the LOW-QVO region, the Day and the Night ratios practically coincide, $R_{CC/NC}^{SNO}(D) \approx R_{CC/NC}^{SNO}(N)$.

As the results exhibited in Figs. 4 - 6 indicate, when $E_{e,th}$ is decreased from 6.75 MeV to 5.0 MeV, the three ratios $R_{CC/NC}^{SNO}(X)$, $X = D, N, C$, change little and only in relatively small sub-regions of the LMA MSW and of the LOW solution regions (compare the upper and lower panels in each of Figs. 4 - 6). In the best fit points of the LMA MSW and LOW solutions, the three ratios $R_{CC/NC}^{SNO}(X)$, $X = D, N, C$, do not change when the threshold energy is reduced from 6.75 MeV to 5.0 MeV (see further).

In the 99% C.L. LMA MSW solution region, eqs. [3] and [4], we find that each of the Day, Night and Core ratios $R_{CC/NC}^{SNO}(X)$, $X = D, N, C$, can take values in the interval $R_{CC/NC}^{SNO}(X) \approx (0.20 - 0.65)$ for both $E_{e,th} = 6.75$ MeV and $E_{e,th} = 5.00$ MeV. If $\Delta m^2 \approx 2 \times 10^{-4} \text{ eV}^2$, which corresponds to the 95% C.L. solution’s region of ref. [3], we have $R_{CC/NC}^{SNO}(X) \approx (0.20 - 0.45)$, $X = D, N, C$. In the best fit points in the LMA MSW solution region, obtained in [3] and [14], we get, respectively, $R_{CC/NC}^{SNO}(D) \approx 0.29$; 0.28; 0.27, $R_{CC/NC}^{SNO}(N) \approx 0.31$; 0.29; 0.29, $R_{CC/NC}^{SNO}(C) \approx 0.31$; 0.30; 0.30, for both values of $E_{e,th}$, $E_{e,th} = 6.75$ MeV; 5.00 MeV. The “best fit” ratios are very sensitive to the best fit value of $\sin^2 2\theta$.

In the case of the LOW solution, the interval of possible values of $R_{CC/NC}^{SNO}(X)$, $X = D, N, C$, is much narrower if $\Delta m^2$ and $\sin^2 2\theta$ lie within the region given by eq. (3): $R_{CC/NC}^{SNO}(X) \approx (0.38 - 0.45)$. Somewhat larger values of $R_{CC/NC}^{SNO}(X)$ - up to $\sim 0.55$, are possible in the 99.73% C.L. LOW solution regions derived in refs. [3] and [14]. In the LOW solution best fit points found in [3] and [14], we obtain for $E_{e,th} = 6.75$ MeV (5.00 MeV), respectively, $R_{CC/NC}^{SNO}(D) \approx 0.44 (0.43)$; 0.49, $R_{CC/NC}^{SNO}(N, C) \approx 0.45 (0.44)$; 0.49.

If the average probability of survival of the solar ($^8$B) $\nu_e$ with energy $8.2$ MeV $\lesssim E \lesssim 14.0$ MeV (the flux of which was measured by the SNO experiment [1]) does not exhibit i) a strong dependence on the neutrino energy and ii) a large day-night variation, we have, as it is not difficult to show,

$$R_{CC/NC}^{SNO} \approx \frac{\Phi_{CC}(\nu_e)}{\Phi_{CC}(\nu_e) + \Phi(\nu_{\mu,\tau})} \approx 0.32 \pm 0.07,$$

where $R_{CC/NC}^{SNO}$ is the averaged ratio over the period of SNO data-taking [1], and we have used eqs. [1] and [4]. Taking into account an uncertainty corresponding to “ 1 standard deviation” and to “2 standard deviations”, we find from eq. (11), respectively, $0.25 \leq R_{CC/NC}^{SNO} \leq 0.39$ and
0.18 \leq R_{CC/NC}^{SNO} \leq 0.46. As Figs. 4 - 5 indicate, an upper limit \( R_{CC/NC}^{SNO} \) of 0.45 would imply that in the cases of the LMA MSW and of the LOW solutions one has \( \Delta m^2 \leq 2 \times 10^{-4} \text{eV}^2 \) and \( \Delta m^2 \approx 6 \times 10^{-8} \text{eV}^2 \), respectively.

4 Constraining the Solar Neutrino Oscillation Parameters

It should be clear from the discussions in Sections 2 and 3 that a measured value of \( A_{D-N}(SNO) > 1.0\% \) and/or of \( R_{CC/NC}^{SNO} \approx 0.45 \) in the SNO experiment can strongly diminish the regions of the allowed values of \( \Delta m^2 \) and \( \tan^2 \theta \) of the LMA MSW and of the LOW solutions of the solar neutrino problem. As it follows from the results shown graphically in Fig. 2 and in Figs. 4 - 5, an experimental upper limit on \( A_{D-N}(SNO) \) in the case of the LMA MSW (LOW) solution would imply a lower (upper) limit on \( \Delta m^2 \). At the same time, an experimental upper limit on \( R_{CC/NC}^{SNO} \) would lead to an upper (lower) limit on \( \Delta m^2 \). Thus, even upper limits on \( A_{D-N}(SNO) \) of the order of 10\% and on \( R_{CC/NC}^{SNO} \) of the order of 0.45 can significantly reduce the LMA MSW and the LOW solution regions.

5 Conclusions

In the present article we have derived detailed predictions for the D-N asymmetry in the solar neutrino induced CC event rate in the SNO detector for the LMA MSW and the LOW solutions of the solar neutrino problem, which are favored by the current solar neutrino data. We have obtained results for the Night (or Full Night) and the Core D-N asymmetries for SNO, \( A_{D-N}^{C}(SNO) \) and \( A_{D-N}^{C}(SNO) \), which are presented in the form of iso-(D-N) asymmetry contour plots in the \( \Delta m^2 - \tan^2 \theta \) plane in Figs. 2 - 3. Detailed predictions for the Night and Core D-N asymmetries for the Super-Kamiokande detector, \( A_{D-N}^{N,C}(SK) \), were also derived (Fig. 1). The high precision calculations of \( A_{D-N}^{NC}(SNO) \) have been performed by taking into account, in particular, the energy resolution function of the SNO detector. Our results show, however, that the effect of the energy resolution function on the predicted values of the Full Night and Core D-N asymmetries is negligible when \( A_{D-N}^{NC}(SNO) \) is greater than 0.01. The asymmetries \( A_{D-N}^{NC}(SNO) \) are calculated for two values of the threshold (effective) kinetic energy of the final state electron, \( T_{\text{e,th}} = 6.75 \text{ MeV} \) and 5.0 MeV. The published SNO data were obtained using the first value, while the second one is the threshold energy planned to be reached at a later stage of the experiment.

The Full Night D-N asymmetry in the CC event rate in the SNO detector, \( A_{D-N}^{N}(SNO) \), can be in the LMA MSW solution region by a factor of \((1.5 - 2.0)\) bigger than the Full Night D-N asymmetry in the solar neutrino induced event rate in the Super-Kamiokande detector \[22\]:

\[
(A_{D-N}^{N}(SNO))^{LMA} \approx (1.5 - 2.0)(A_{D-N}^{N}(SK))^{LMA}.
\]

The asymmetry \( A_{D-N}^{N}(SNO) \) measured in the SNO experiment can be as large as \((15 - 20)\)%. A value of \( A_{D-N}^{N}(SNO) \approx 15\% \), for instance, cannot be excluded by the 95\% C.L. (2 s.d.) upper limit on \( A_{D-N}^{N}(SK) \) following from the Super-Kamiokande data on the D-N effect \[2\]. In the best fit point of the LMA MSW solution region found in \[\#3\] and in \[\#4\] we get for \( T_{\text{e,th}} = 6.75 \text{ MeV} \) (5.0 MeV), \( (A_{D-N}^{N}(SNO))^{BF1} \approx 7.3 \) (6.6)\% and \( (A_{D-N}^{N}(SNO))^{BF2} \approx 10.1 \) (9.3)\%, respectively. At the same time, one finds a considerably smaller value of \( A_{D-N}^{N}(SNO) \) in the LMA solution best fit point obtained in \[\#10\]:

\[
(A_{D-N}^{N}(SNO))^{BF3} \approx 5.0 \text{ (4.6)\%}.
\]

In the LMA MSW solution region, the Core D-N asymmetry in the SNO detector is predicted to be larger than the Full Night D-N asymmetry typically by a factor of \( \sim 1.2 \):

\[
(A_{D-N}^{C}(SNO))^{LMA} \approx 1.2(A_{D-N}^{N}(SNO))^{LMA}.
\]

In the case of the LOW solution of the solar neutrino problem one has (Figs. 1 and 2) in the region where \( A_{D-N}^{N}(SK) > 1\% \): \( A_{D-N}^{N}(SNO) \approx (1.2 - 1.4)A_{D-N}^{N}(SK) \). In the solution region given by eq. (5) we find \( (A_{D-N}^{N}(SNO))^{LOW} \approx (1.0 - 7.5)\% \). In the region under discussion, \( (A_{D-N}^{C}(SNO))^{LOW} \approx (A_{D-N}^{N}(SNO))^{LOW} \). In the best fit point of the LOW solution found in \[\#3\] and in \[\#4\] we get \( (A_{D-N}^{N}(SNO))^{BF1} \approx 3.8 \) (4.2)\% for \( T_{\text{e,th}} = 6.75 \text{ MeV} \) (5.0 MeV), while in the best fit point obtained in \[\#10\] one has \( (A_{D-N}^{N}(SNO))^{BF3} \approx 1.2 \) (1.5)\%. An observation
of $A^N_{D-N}(SNO) \gtrsim 10\%$ will strongly disfavor the LOW solution of the solar neutrino problem, while an observation of $A^N_{D-N}(SNO) > 1\%$ would rule out the QVO solution.

We have derived also detailed predictions for the ratio of the event rates of the CC reaction $\nu_e + D \rightarrow e^- + p + p$, $R_{SNO}(CC)$, and of the neutral current (NC) reaction $\nu + D \rightarrow \nu + n + p$, induced by the solar neutrinos in SNO during the day, $R_{CC/NC}^{SNO}(D)$, during the night, $R_{CC/NC}^{SNO}(N)$, and for the case of the CC event rate produced at night by solar neutrinos which cross the Earth core, $R_{CC/NC}^{SNO}(C)$ (Figs. 4 - 6). The predictions were obtained for $T_{e,th} = 6.75$ MeV and 5.0 MeV. We find that in the LMA MSW solution region given by eqs. (3) and (4), we get $R_{CC/NC}^{SNO}(X) \approx (0.20 - 0.65)$, $X = D, N, C$; for $\Delta m^2 \lesssim 2 \times 10^{-4}$ eV$^2$ from this region we have $R_{CC/NC}^{SNO}(X) \approx (0.20 - 0.45)$. In the LOW solution region given by eq. (5) we obtain $R_{CC/NC}^{SNO}(X) \approx (0.38 - 0.45)$. In the LMA solution best fit points (see the text) we get $R_{CC/NC}^{SNO}(X) \approx (0.27 - 0.31)$, while in the two LOW solution best fit points discussed in the text we find approximately $R_{CC/NC}^{SNO}(X) \approx 0.44$ and 0.49.

In the case of the LMA and LOW solutions, the value of $A^N_{D-N}(SNO)$ is very sensitive to the value of $\Delta m^2$, while $R_{CC/NC}^{SNO}$ exhibits a very strong dependence on $\tan^2 \theta$. A measured value of $A^N_{D-N}(SNO) > 1.0\%$ and/or of $R_{CC/NC}^{SNO} \lesssim 0.45$ in the SNO experiment can strongly diminish the regions of the allowed values of $\Delta m^2$ and $\tan^2 \theta$ of the LMA MSW and of the LOW-QVO solutions of the solar neutrino problem. An upper limit on $A^N_{D-N}(SNO)$ in the case of the LMA MSW (LOW-QVO) solution would imply a lower (upper) limit on $\Delta m^2$. At the same time, an experimental upper limit on $R_{CC/NC}^{SNO}$ would lead to an upper (lower) limit on $\Delta m^2$. Thus, even upper limits on $A^N_{D-N}(SNO)$ of the order of 10% and on $R_{CC/NC}^{SNO}$ of the order of 0.45 can significantly reduce the LMA MSW and the LOW-QVO solution regions.

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Figure 1: Iso-(D-N) asymmetry contour plot for the Super-Kamiokande experiment for $T_{e,th} = 5.0$ MeV, $T_{e,th}$ being the detected $e^-$ kinetic energy threshold. The contours correspond to values of the Full Night (upper panel) and Core (lower panel) asymmetries $A_{D-N}^{N,C}(SK) = -0.02, -0.01, 0.01, 0.02, 0.03, 0.04, 0.05, 0.06, 0.08, 0.10, 0.12, 0.14, 0.16, 0.18, 0.20, 0.25, 0.30, 0.45$. 
Figure 2: Iso-(D-N) asymmetry ($A_{D-N}^N(SNO)$) contour plot for the SNO experiment for $T_{e, th} = 6.75$ MeV (upper panel) and $T_{e, th} = 5.00$ MeV (lower panel), $T_{e, th}$ being the (effective) kinetic energy threshold of the detected $e^-$ in the CC reaction. The contours correspond to values of the Full Night asymmetry in the CC event rate $A_{D-N}^N(SNO) = -0.01, 0.01, 0.02, 0.03, 0.04, 0.05, 0.06, 0.08, 0.10, 0.12, 0.14, 0.16, 0.18, 0.20, 0.25, 0.30, 0.45.
Figure 3: Iso-(D-N) asymmetry ($A_{D-N}^{SNO}$) contour plot for the SNO experiment for $T_{e,th} = 6.75$ MeV (upper panel) and $T_{e,th} = 5.00$ MeV (lower panel), $T_{e,th}$ being the (effective) kinetic energy threshold of the detected $e^-$ in the CC reaction. The contours correspond to values of the Core asymmetry in the CC event rate $A_{D-N}^{SNO} = -0.03, -0.02, -0.01, 0.01, 0.02, 0.03, 0.04, 0.05, 0.06, 0.08, 0.10, 0.12, 0.14, 0.16, 0.18, 0.20, 0.25, 0.30, 0.45$. 
Figure 4: Iso-$R_{CC/NC}^{SNO}(D)$ contour plots for $T_{e,th} = 6.75$ MeV (upper panel) and $T_{e,th} = 5.00$ MeV (lower panel): each contour corresponds to a fixed value of the Day ratio of the CC and NC solar neutrino event rates measured in the SNO experiment. In the case of absence of oscillations of solar neutrinos $R_{CC/NC}^{SNO}(D) = 1$. The contours shown are for $R_{CC/NC}^{SNO}(D) = 0.10, 0.20, 0.30, 0.40, 0.45, 0.50, 0.60, 0.70, 0.80, 0.90$. 

\[ \Delta m^2 (\text{eV}^2) \]

\[ \tan^2 \theta \]
Figure 5: The same as in Fig. 4 for the Night ratio of the CC and NC solar neutrino event rates, $R_{CC/NC}^{SNO}(N)$, measured in the SNO experiment. In the case of absence of oscillations of solar neutrinos $R_{CC/NC}^{SNO}(N) = 1$. The contours shown are for $R_{CC/NC}^{SNO}(N) = 0.10, 0.20, 0.30, 0.40, 0.45, 0.50, 0.60, 0.70, 0.80, 0.90$. 

CC/NC for SNO, Full Night, $T_{e, \text{Th}} = 6.75$ MeV

CC/NC for SNO, Full Night, $T_{e, \text{Th}} = 5.00$ MeV
Figure 6: The same as in Fig. 4 for the Core ratio of the CC and NC solar neutrino event rates, $R_{CC/NC}^{SNO}(C)$, measured in the SNO experiment. In the case of absence of oscillations of solar neutrinos $R_{CC/NC}^{SNO}(C) = 1$. The contours shown are for $R_{CC/NC}^{SNO}(C) = 0.10, 0.20, 0.30, 0.40, 0.45, 0.50, 0.60, 0.70, 0.80, 0.90$. 

\[ \Delta m^2 (\text{eV}^2) \]

\[ \tan^2 \theta \]