Abstract: Pixelated phase-mask interferograms have become an industry standard in spatial phase-shifting interferometry. These pixelated interferograms allow full wavefront encoding using a single interferogram. This allows the study of fast dynamic events in hostile mechanical environments. Recently an error-free demodulation method for ideal pixelated interferograms was proposed. However, non-ideal conditions in interferometry may arise due to non-linear response of the CCD camera, multiple light paths in the interferometer, etc. These conditions generate non-sinusoidal fringes containing harmonics which degrade the phase estimation. Here we show that two-dimensional Fourier demodulation of pixelated interferograms rejects most harmonics except the complex ones at \{-3^{rd}, +5^{th}, -7^{th}, +9^{th}, -11^{th}, \ldots\}. We propose temporal phase-shifting to remove these remaining harmonics. In particular, a 2-step phase-shifting algorithm is used to eliminate the \(-3^{rd}\) and \(+5^{th}\) complex harmonics, while a 3-step one is used to remove the \(-3^{rd}\), \(+5^{th}\), \(-7^{th}\) and \(+9^{th}\) complex harmonics.

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OCIS codes: (120.3180) Interferometry; (120.2650) Fringe analysis.

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1. Introduction

One of the most used techniques to demodulate fringe patterns in optical metrology is Phase Shifting Interferometry (PSI), either in the spatial or temporal domain. In the temporal approach, PSI techniques require a mobile piston element in the interferometer to introduce a uniform phase step between successive samples, in order to obtain three or more phase-shifted
interferograms to process. Typically, Phase-Shift Algorithms (PSAs) are constructed on the assumption that fringe patterns present a sinusoidal profile. In practice, non-sinusoidal fringes are due to nonlinear photodetector response, gain saturation, multiple beams interferences, etc. These systematic errors lead to the generation of harmonics of the fundamental signal (distorted fringes) which degrades the quality of the recovered phase map [1].

Many-steps PSAs allow good phase estimations even with non-sinusoidal fringe patterns. However, stability of the experimental set-up usually limits the number of samples that may be obtained. Several authors have described spatial techniques for simultaneous acquisition of phase-shifted interferograms [2–6]. This allows data acquisition faster than one-at-a-time temporal PSI. In the technique proposed by Miller et al. [6], a periodic phase-mask array of micro-polarizers is physically attached to a CCD sensor to obtain a unique phase-shift over each pixel. The physical principles of this approach are review in Ref [6].

A useful mathematical model for a Pixelated Phase-Mask (PPM) interferogram is

$$ I(x, y) = a(x, y) + b(x, y) \cos[\phi(x, y) + pm(x, y)], $$

where $a(x, y), b(x, y), \phi(x, y)$ are respectively the background, local contrast and the searched phase function. The pixelated carrier $pm(x, y)$ is a periodic 2-D phase-array. In the following analysis we work with the so-called “2x2 circular orientation”, illustrated in Fig. 1, however the “stacked” 2x2 spatial configuration [6] may also be analyzed in a similar way.

$$ \begin{array}{c|c}
(2x,2y) & (2x+1,2y) \\
0 & \pi/2 \\
(2x,2y+1) & (2x+1,2y+1) \\
3\pi/2 & \pi \\
\end{array} $$

Fig. 1. Periodic distribution for the basic building cell of the phase-mask $pm(x, y)$.

Fourier phase demodulation of pixelated phase-mask interferograms was shown independently by Servin & Estrada [7] and Kimbrough & Millerd [8], and it is reviewed below.

2. Fourier demodulation of pixelated phase-mask interferograms

In spatial-carrier interferometry, the frequency of the carrier must be higher than the highest frequency component of the signal of interest. If this condition is satisfied the components of the interferogram can be isolated by linear filtering [9]. For PPM interferograms this condition must be satisfied in two dimensions, which may be formally stated as

$$ \|\nabla pm(x, y)\| > \max \|\nabla \phi(x, y)\|, \quad (2) $$

where $\nabla$ is the gradient operator. This is a correction of the one presented in [7].

The first step to demodulate a pixelated phase-mask interferogram [7,8] is to multiply Eq. (1) by the complex pixelated carrier $\exp[-i pm(x, y)]$,

$$ I(x, y) \exp[-i pm(x, y)] = [a + b \cos(\phi + pm)] \exp(-i pm), $$

$$ = a \exp(-i pm) + (b/2) \{ \exp(i\phi) + \exp[-i(\phi + 2pm)] \}. \quad (3) $$

Applying a low-pass filter $h(x, y)$ to Eq. (3), preferably in the Fourier domain [7,8], the high frequency terms are rejected (henceforth, the “hat” denotes estimation):
\( (1/2)b(x, y)\exp[i\hat{\phi}(x, y)] = \{\exp[-i\ pm(x, y)]\}^*h(x, y). \quad (4) \)

Solving Eq. (4) for the estimated phase \( \hat{\phi}(x, y) \), the spatial PSA results as

\[ \tan[\hat{\phi}(x, y)] = \frac{\text{Im}\{\exp[-i\ pm(x, y)]\}^*h(x, y)}{\text{Re}\{\exp[-i\ pm(x, y)]\}^*h(x, y)}, \quad (5) \]

where the operators \( \text{Im}[\cdot] \) and \( \text{Re}[\cdot] \) take the imaginary and real parts of their argument.

### 3 Fourier demodulation with distorted-fringe pixelated interferograms

Experimental data frequently have systematic errors (nonlinear photodetector response, gain saturation, multiple beams interferences, etc.) which adds harmonics to the signal. A distorted-fringes interferogram may be represented by the following Fourier series,

\[ I(x, y) = a(x, y) + \sum_{n=1}^{\infty} b_n(x, y)\cos\{n[\phi(x, y) + pm(x, y)] + \theta_n\}, \quad (6) \]

where \( b_n(x, y) \) are the fringe contrast for the \( n \)-th harmonics, and \( \theta_n \) are irrelevant angular displacements which arise from the Fourier series expansion of the fringes.

Multiplying the distorted interferogram in Eq. (6) by the complex carrier, one obtains

\[ I(x, y)\exp[-i\ pm] = a\exp[-i\ pm] + \sum_{n=1}^{\infty} (b_n/2)\{\exp[i(n-1)\ pm]\exp[i(n\phi + \theta_n)] + \exp[-i(n+1)\ pm]\exp[-i(n\phi + \theta_n)]\}. \quad (7) \]

Since \( pm(x, y) \) is periodic, its symmetry allows developing \( \exp[\pm i(n+1)\ pm(x, y)] \) for each 2\( \times \)2 fundamental cell to obtain the spectral shift for every harmonic as:

\[
\begin{align*}
  &k = 1, \quad \exp\left[i\left(\begin{array}{c} 0 \\ 3\pi/2 \\ \pi \end{array}\right)\right] = \exp[\pm i\ pm(x, y)], \\
  &k = 2, \quad \exp\left[i\left(\begin{array}{c} 0 \\ 3\pi/2 \\ \pi/2 \end{array}\right)\right] = \exp\left[i\left(\begin{array}{c} 0 \\ \pi \\ 2\pi \end{array}\right)\right] = \exp[\pm i\ (x+y)], \\
  &k = 3, \quad \exp\left[i\left(\begin{array}{c} 0 \\ 3\pi/2 \\ \pi/2 \end{array}\right)\right] = \exp\left[i\left(\begin{array}{c} 0 \\ \pi/2 \\ \pi \end{array}\right)\right] = \exp[\pm i\ pm(x,y)^T], \\
  &k = 4, \quad \exp\left[i\left(\begin{array}{c} 0 \\ 3\pi/2 \\ \pi/2 \end{array}\right)\right] = \exp\left[i\left(\begin{array}{c} 0 \\ 2\pi \\ 4\pi \end{array}\right)\right] = \left[\begin{array}{c} 1 \\ 1 \\ 1 \end{array}\right].
\end{align*}
\]

where \( k \) labels the harmonic carriers in Eq. (7). From Eq. (8), it is clear that for higher than the 4th harmonics the carrier is chosen from the above 4 possibilities using \( k = (n+1)\) mod 4.

Substituting Eq. (8) into Eq. (7), and grouping terms with the same spatial carrier, results

\[ I(x, y)\exp(-i\ pm) = (1/2)\{b_i \exp(i\phi) + b_i \exp(-i3\phi) + b_i \exp(i5\phi) + b_i \exp(-i7\phi) + \ldots\} \\
+ (1/2)\{b_i \exp(-i\phi) + b_i \exp(i3\phi) + b_i \exp(-i5\phi) + \ldots\} \exp(i\pi(x+y)) \\
+ \{a + b_i \cos(2\phi) + b_i \cos(4\phi) + b_i \cos(6\phi) + b_i \cos(8\phi) + \ldots\} \exp(i\ pm) \}. \quad (9) \]

Note that the terms in the 1\( \text{st} \) row do not undergo any frequency shift; they are all located at the spectral origin along with \( \exp[i\phi(x, y)] \). The terms in the 2\( \text{nd} \) row are multiplied by \( \exp[i\pi(x+y)] \) and displaced to the spectral corners (\( \pm \pi, \mp \pi \)). The terms in the 3\( \text{rd} \) row are shifted by the complex carrier \( \exp[i\ pm(x,y)] \) to the cardinal points (0, \( \pm \pi \)) and (\( \pm \pi, 0 \)).
Applying a low-pass filter \( h(x,y) \) in the Fourier domain, the terms in the 2\(^{nd} \) and 3\(^{rd} \) rows in Eq. (9) are supressed:

\[
\begin{align*}
A \exp[i \hat{\phi}(x,y)] &= \exp[i \phi(x,y)] \ast h(x,y), \\
&= b_1 \exp[i \phi(x,y)] + \sum_{n=1}^{\infty} b_{2n+1} \exp[i(-1)^n(2n+1) \phi(x,y)].
\end{align*}
\] (10)

where \( A \) is a proportionality constant and \( \hat{\phi}(x,y) \) is our single-image phase estimation. To our knowledge, no prior harmonic analysis has been made on (distorted-fringe) pixelated interferogram demodulation. Therefore, Eq. (10) represents our first contribution in this paper.

In Fig. 2 and Fig. 3 we illustrate several stages of this demodulation process. (The spectral plots are in pseudo-color logarithmic intensity scale with the spectral origin at the center).

Fig. 2. (Color online) (a) Non-sinusoidal fringe pattern having many harmonics overlapped at the spectral origin. (b) The same measuring phase but now modulated by the pixelated phase-mask over the CCD. As shown in Eqs. (6-9), some harmonics are spectrally displaced.

Fig. 3. (Color online) (a) Multiplying the pixelated interferogram by the complex carrier leave the complex harmonics \( \{-3^{rd}, 5^{th}, -7^{th}, 9^{th}, \ldots \} \) at the spectral origin distorting \( \exp[i \hat{\phi}(x,y)] \). (b) The phase estimation obtained by filtering-out outside the circle shown in (a), (Eq. (10)).

As can be seen in Fig. 3b and Eq. (10), the complex harmonics not-rejected by the low-pass filtering \( h(x,y) \) are \( \{-3^{rd}, 5^{th}, -7^{th}, 9^{th}, -11^{th}, \ldots \} \). Please note that, these harmonics coincides with those not-rejected in a standard 4-steps temporal least-squares PSA [10]. The next section shows our second contribution, namely how to remove some of these complex harmonics using few-steps temporal PSI techniques (a quadrature temporal filter).

4. Spatio-temporal demodulation of non-sinusoidal pixelated interferograms

Whenever possible, one may introduce a linear phase-step \( \omega_0 \) (radians/step) between successive samples of a distorted-fringes pixelated interferogram (Eq. (6)):

\[
I(x,y,t) = a(x,y) + \sum_{n=1}^{\infty} b_n(x,y) \cos\left\{ n[\phi(x,y) + pm(x,y) + \omega_0 t] + \theta_n \right\}, \quad t = 0,1,2,\ldots \] (11)
Applying the spatial demodulation in Section 2, to these phase-shifted distorted-fringes interferograms (Eq. (11)), the following 3 single-image phase estimations are obtained:

\[
\exp[i \hat{\phi}] = b_1 \exp(i \phi) + b_2 \exp(-i3\phi) + b_3 \exp(i5\phi) + b_4 \exp(-i7\phi) + \ldots
\]

\[
\exp[i (\hat{\phi} + a_\omega)] = b_1 \exp(i (\phi + a_\omega)) + b_2 \exp[-i3(\phi + a_\omega)] + b_3 \exp[-i5(\phi + a_\omega)] + \ldots
\]

\[
\exp[i (\hat{\phi} + 2a_\omega)] = b_1 \exp[i (\phi + 2a_\omega)] + b_2 \exp[-i3(\phi + 2a_\omega)] + b_3 \exp[-i5(\phi + 2a_\omega)] + \ldots
\]

(12)

Which are the phase-shifted versions of Eq. (10). Then, one forms the temporal signal:

\[
s(t) = \exp[i \hat{\phi}] \delta(t) + \exp[i (\hat{\phi} + a_\omega)] \delta(t-1) + \exp[i (\hat{\phi} + 2a_\omega)] \delta(t-2). \tag{13}
\]

The signal in Eq. (13) still contains the remaining complex harmonics \{-3^{rd}, 5^{th}, -7^{th}, 9^{th}, \ldots\}, but we can use temporal quadrature filters (PSAs) to reject them [1,10,11]. Specifically, we propose the following 2-steps PSA to reject the \(-3^{rd}\) and \(5^{th}\) complex harmonics:

\[
H_2(\omega) = 1 - \exp[i(\omega + 3a_\omega)],
\]

\[
h_2(t) = F^{-1}[H_2(\omega)] = \delta(t) - \exp(i3a_\omega) \delta(t + 1); \quad a_\omega = \pi / 4. \tag{14}
\]

Similarly, the following 3-steps PSA rejects the \(-3^{rd}, 5^{th}, -7^{th},\) and \(9^{th}\) complex harmonics:

\[
H_3(\omega) = [1 - \exp[i(\omega + 3a_\omega)]][1 - \exp[i(\omega - 5a_\omega)]],
\]

\[
h_3(t) = F^{-1}[H_3(\omega)] = \delta(t) + [1 - \exp(i a_\omega)] \delta(t + 1) + \exp(-i2a_\omega) \delta(t + 2); \quad a_\omega = \pi / 3. \tag{15}
\]

where \(F^{-1}[\cdot]\) is the inverse Fourier transform operator. The normalized spectral plots of these two quadrature filters (Eqs. (14,15)) are shown in Fig. 4.

Using the 2-step PSA (Eqs. (13,14)) the estimated phase \(\hat{\phi}_2(x, y)\) is given by:

\[
\hat{H}_2(a_\omega) \exp(i \hat{\phi}_2) = \{s(t) * h_2(t)\}_{t=\pm 2} = \exp(i \hat{\phi}_2) - e^{-i2a_\omega} \exp[i (\hat{\phi}_2 + a_\omega)]; \quad a_\omega = \pi / 4. \tag{16}
\]

Similarly, the 3-step PSA (Eqs. (13,15) with \(a_\omega = \pi / 3\)) gives the estimation \(\hat{\phi}_3(x, y)\):

\[
\hat{H}_3(a_\omega) \exp(i \hat{\phi}_3) = \{s(t) * h_3(t)\}_{t=\pm 3}
\]

\[
= \exp(i \hat{\phi}_3) + (1 - e^{-i m}) \exp[i (\hat{\phi}_3 + a_\omega)] + e^{-i2a_\omega} \exp[i (\hat{\phi}_3 + 2a_\omega)]. \tag{17}
\]

Finally, the most general expression for our spatio-temporal phase-demodulation algorithms is given by

\[
A \exp[i \hat{\phi}_O(x, y)] = \{ \exp[-i pm(x, y)] [I(x, y, t) * h_N(t)]_{t=1} \} * h(x, y), \tag{18}
\]

where \(A\) is a proportionality constant. The convolutions order in Eq. (18) may be interchanged, but for numerical efficiency we suggest to first apply the temporal convolution.
Figure 5 shows the application of our spatio-temporal algorithms (Eqs. (16,17)) to the non-sinusoidal pixelated fringe pattern presented in Fig. 2(b).

For comparative purposes, Fig. 6 shows slices of the estimated phase maps using single-image demodulation, and spatio-temporal demodulation with 2, and 3 steps PSAs.

These simulations were done using a resolution of 512x512 pixels, but higher resolution provides greater spectral separation allowing better phase-demodulation. Today, pixelated phase-mask interferograms have resolutions almost sixteen times higher (4 mega-pixels).

5 Conclusions

We have shown that single-image Fourier demodulation of 2x2 pixelated interferogram leaves the complex harmonics \{-3^{rd}, +5^{th}, -7^{th}, +9^{th}, \ldots\} overlapping with the fundamental signal \( \exp[i \phi(x, y)] \), distorting it. Considering this, we proposed few-steps temporal PSI techniques to reject some of these remaining complex harmonics. Specifically, we used a 2-step temporal PSA designed to remove (at least) the \(-3^{rd}\), and \(+5^{th}\) complex harmonics, and a 3-step one to remove (at least) the \(-3^{rd}\), \(+5^{th}\), \(-7^{th}\), and \(+9^{th}\) complex harmonics. Note that temporal demodulation of \(N\)-step, 2x2 pixelated interferograms \((N=1,2,3,\ldots)\) rejects the same harmonics as a temporal \(4N\)-step least-squares PSA. In other words, \(N\)-steps temporal interferometry of 2x2 pixelated fringe patterns is 4 times faster, preserving the harmonics rejection robustness of a \(4N\)-step least-squares PSA.

Acknowledgements

The authors wish to thank the useful reviewers’ comments and the financial support of the Mexican National Council for Science and Technology (CONACyT).