One-dimensional dynamic model of cold-formed channel beam with deformed cross-section

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Abstract. In this paper, a one-dimensional high order model based on a set of basic deformation modes of cross section is proposed to analyse one-dimensional dynamic model of cold-formed channel beam considering the deformable cross-section. The model considers the displacement field through the linearly superimposing of 36 basis deformation modes, which stem from the discretization of the cross-section into 9 points and 8 segments. The corresponding deformation function is approximated with Hermite Interpolation. The governing equation is deduced from the principle of Hamilton, then use quadratic Lagrange interpolation for finite element realization. Numerical examples have also been presented and the comparison with ANSYS shell model showed its accuracy, efficiency, and applicability in reproducing three-dimensional behaviors of cold-formed channel beam.

1. Introduction
Cold-formed steel (CFS) is usually manufactured by rolling or pressing thin gauges of steel sheet at room temperature into desired cross-section members[1]. It is light in weight and high in strength, its own weight is only about 1/5~1/4, and the usable area is increased by about 10%~15%, compared with traditional reinforced concrete[2]. The cold-formed channel beam is widely used in the main load-bearing components of factories and civil houses, such as beams, columns, steel frames, etc, due to their advantages of light weight, high strength, simple process and easy erection[3]. The cold-formed channel beams are usually susceptible to cross-sectional deformation because of thin-walled structures. Indeed, dynamic behavior of such elements is strongly affected by the section deformation. Therefore, the influence of cross-sectional deformation should be considered in the cold-formed channel beam dynamic model. In recent years, many scholars have studied the section deformation of cold-formed C-beam structures.

At present, the most common method for modelling, the section deformation of cold-formed thin-walled beams is the Geometrically Accurate Beam Theory (GEBT), and subsequent scholars have improved the beam model by further considering special deformation modes. Capdevielle et al. [4] solved the coupling evolution of warpage and damage based on the local and global equilibrium equations. Li et al. [5] introduced the excessive parameters to investigate the bending, buckling and free vibration problems of an axially loaded Timoshenko beam. Rajasekaran et al. [6] have proposed the formulation for buckling and free vibration analysis of Bi-Directional Functionally Graded (BDFG) Thin-Walled non-prismatic beams of generic open/closed cross section. Karthik et al. [7] systematically analysed the behavior of thirteen different cross-sections under the action of four-point bending. Piana et al. [8] studied the effects of warping and warping con- straints on the yield load and natural frequency of open thin-walled beams through experiments. Bourihan et al. [9] using a high-order implicit
algorithm to analyze the forced nonlinear dynamic behavior of open cross section beam under external dynamic loads. Addessi et al. [10] introduced the out-of-plane warping displacement field to describe the coupling relationship between the stress components. Gatheeshgar et al. [11] simplified the method of predicting the critical elastic yield capacity of MCO beams with a thin-walled beams section moment capacity results database. Basaglia et al. [12] studied the local, distortional and global in-stability behavior of GBT beams. Rong et al. [13] proposed a thin-walled beam model considering warping deformation and Wagner effects. Due to the geometric characteristics of thin-walled beams, they present more complex deformations than solid beams. It is exceedingly difficult to accurately describe the deformation of the in-plane and out-of-plane sections of cold-formed thin-walled beams. For thin-walled beams, one-dimensional models usually give satisfactory results with very low computational cost and modelling time [8].

In this paper, a one-dimensional dynamic model of a cold-formed channel beam structure considering cross-sectional deformation is presented. First, the displacement of discrete points is used to replace the actual displacement, which derive from the discretization of the cross-section into 9 points and 8 segments. Second, the basis deformation modes are linearly superimposed with the propose of reducing the three-dimensional displacement field to one-dimensional. Then, making use of the principle of Hamilton, the governing equation is deduced. And a finite element format is converted because the governing equations are interpolated. Last, a numerical example is used to verify the accuracy of the model with ANSYS shell element.

2. Displacement and Deformation Fields
The cold-formed channel beam is shown in Figure 1. The displacement of a point on the midline of the cross-section is defined in terms of the axial u, tangential v, and normal w components, which are defined to be positive, along the axes of the local coordinate system (n, s, z). The global coordinate system (x, y, z) with its origin located on the center line of the cross-section of the beam end is also shown. The beam length and beam wall thickness are defined as l and t, the midline height and width of the beam section are defined as h and b, the curling length is defined as b/3. There are six natural joints in the cross section, which connect adjacent walls or locates at the free ends. Since the height and width are much larger than the curling length and these beams may greatly vary in length, three artificial nodes 5, 7, 9 are introduced, which will contribute to the capability of capturing cross-section deformation from the view-point of interpolation. The cross-section discretization has defined by the set of natural and intermediate nodes.

![Figure1.global (x, y, z) and local (n, s, z) coordinate systems and discretization of cold-formed channel beam](image)

In order to realize the interpolation process, the node displacement should be defined correctly first. There are four degrees of freedom of each node, including three translations and one rotation about the longitudinal axis. In this cross section, it consists of nine nodes comprising six nodes natural, artificial
three nodes, a total of 36 potential deformation patterns. The basis function is viewed as the mathematical description of a basis deformation mode, the function will then be approximated by an interpolation polynomial, among them, a set of linear Lagrange functions for the axial (out-of-plane) a set of cubic Hermite functions for the normal (in-plane) component and tangential (in-plane) displacements. As is shown in figure 2.

Figure 2. Basis deformation modes of cold-formed channel beam, hollow cross-section with nine discretization node

The displacement field, $\mathbf{u} = [u(s,z), v(s,z), w(s,z)]^T$ of cold-formed channel beam is defined as:

$$
\begin{align*}
\begin{bmatrix}
U_z(s,n,z) \\
U_x(s,n,z) \\
U_a(s,n,z)
\end{bmatrix}
&= \begin{bmatrix}
u(s,z) - n \frac{\partial w(s,z)}{\partial z} \\
v(s,z) - n \frac{\partial w(s,z)}{\partial s} \\
w(s,z)
\end{bmatrix} \\
&= \begin{bmatrix}
u(s,z) - n \frac{\partial w(s,z)}{\partial z} \\
v(s,z) - n \frac{\partial w(s,z)}{\partial s} \\
w(s,z)
\end{bmatrix}
\end{align*}
$$

A set of linearly independent basis functions that defined over the cross-section separately approximate the displacement variables $u(s,z), v(s,z)$ and $w(s,z)$. The displacement field on the cross-section midline, $\mathbf{u} = [u(s,z), v(s,z), w(s,z)]^T$, is written as follows:

$$
\begin{align*}
\mathbf{u}(s,z) &= \mathbf{\Psi}_1 \mathbf{x}, \quad \mathbf{v}(s,z) = \mathbf{\Psi}_2 \mathbf{x}, \quad \mathbf{w}(s,z) = \mathbf{\Psi}_3 \mathbf{x} \\
\mathbf{\Psi}_1 &= [\varphi_1(s), \varphi_2(s), ..., \varphi_{32}(s)], \quad \mathbf{\Psi}_2 = [\psi_1(s), \psi_2(s), ..., \psi_{32}(s)], \quad \mathbf{\Psi}_3 = [\psi_1(s), \psi_2(s), ..., \psi_{32}(s)]
\end{align*}
$$
Where $\psi_1, \psi_2$ and $\psi_3$ correspond to a set of basis functions. The three-dimensional displacement can be presented as:

$$
\begin{align*}
U_z &= \psi_z x - n\psi_1 (\frac{\partial x}{\partial z}) \\
U_x &= \psi_x x - n(\frac{\partial \psi_3}{\partial s}) \\
U_{s} &= \psi_3 x
\end{align*}
$$

(4)

On the assumption of small displacement, neglecting defect and material uncertainty, the deformation and the corresponding stress fields are as follows:

$$
\begin{align*}
\epsilon_{zz} &= \frac{\partial x}{\partial z} - n\psi_3 (\frac{\partial^2 x}{\partial z^2}), \\
\epsilon_{ss} &= \frac{\partial \psi_2}{\partial s} x - n(\frac{\partial^2 \psi_3}{\partial s^2}), \\
\gamma_{zs} &= \frac{\partial \psi_1}{\partial s} x + \psi_2 \frac{\partial x}{\partial z} - 2n \frac{\partial \psi_3}{\partial s} \frac{\partial x}{\partial z}
\end{align*}
$$

(5)

$$
\begin{align*}
\sigma_{zz} &= \epsilon^{*} \epsilon_{zz} + E\nu \epsilon_{ss}, \\
\sigma_{ss} &= \epsilon^{*} \epsilon_{ss} + E\nu \epsilon_{zz}, \\
\tau_{zs} &= G\gamma_{zs}, \\
\epsilon^{*} &= \frac{E}{1-\nu^2}
\end{align*}
$$

(6)

where $G$, $\nu$ and $E$ are the shear modulus, Poisson’s ratio and material Young’s modulus.

3. Governing Equation

Establish the dynamic equation of the cold-formed channel beam based on the Hamilton principle, the strain energy, the potential energy, and the kinetic energy are deduced as follows:

$$
U = \frac{1}{2} \int_{t_1}^{t_2} \epsilon^T \sigma dV, \\
U_p = -\int_{t_1}^{t_2} \int_{t_1}^{t_2} \rho \frac{dU}{dt} dV
$$

(7)

where $\rho$ and $p$ are the material density and the loading vector; $A$, $L$ and $V$ are the sectional area, the length of the structure, and volume. The Hamiltonian principle is applied to the governing equation.

$$
\delta \int_{t_1}^{t_2} L_a dt = 0, \delta x \bigg|_{t_1} = 0, \delta x \bigg|_{t_2} = 0
$$

(8)

where $L_a$ is Lagrangian that defined as $L_a = T - U - U_p$, and $t_1$ and $t_2$ are the start and end times, respectively.

Combined with (5)-(8), the governing equation of the cold-formed channel beam is derived as:

$$
\int_{t_1}^{t_2} \delta x^T H^T \eta H \frac{\partial^2 x}{\partial t^2} dV + \int_{t_1}^{t_2} \delta x^T H^T c^T Ec H x dV + \int_{t_1}^{t_2} \delta x^T H^T p dV = 0
$$

(9)

where $H$, $c$ and $E$ are defined as:

$$
\begin{align*}
U_z(s,n,z) &= H_z u \\
U_x(s,n,z) &= H_x u \\
U_{s}(s,n,z) &= H_{s} u
\end{align*}
$$

(10)

$$
\epsilon = cU = \begin{bmatrix} \frac{\partial}{\partial z} & 0 & 0 \\ 0 & \frac{\partial}{\partial s} & 0 \\ \frac{\partial}{\partial s} & \frac{\partial}{\partial z} & 0 \end{bmatrix}
$$

U, \sigma = E\epsilon = \begin{bmatrix} E & E & 0 \\ \frac{1-\nu^2}{1-\nu^2} & \frac{1-\nu^2}{1-\nu^2} & 0 \\ \frac{E}{1-\nu^2} & \frac{E}{1-\nu^2} & 0 \end{bmatrix}
$$

(11)

4. Finite Element Implementation

In order of ease of computation, this paper applies the finite element method, the quadratic interpolation function is used to approximate the axial displacement field within an element.
where $N$ and $X$ represent the shape function matrix and the nodal generalized displacement vector, The Equation (11) could be written as follows:

$$N_1 = (2z^2 / l^2 - 3z / l + 1)I_{32}, \ N_2 = (4z / l - 4z^2 / l^2)I_{32}, \ N_1 = (2z^2 / l^2 - z / l)I_{32}$$

(13)

where $z$ and $l$ are the coordinate variables in a unit and the length of the unit.
According to (9) and (12), the finite element form is given as follows:

$$m \ddot{X} + kX = f$$

(14)

where $m$, $k$, and $f$ are the consistent element mass matrix, the element stiffness matrix and the element force matrix.

5. Numerical Example
In order to verify the accuracy of the new model, numerical calculations are applied to cold-formed channel beams. Related parameters include $\rho = 7800 \text{Kg/m}^3$, $\nu = 0.3$, $E = 200 \text{GPa}$, $h = 0.12 \text{m}$, $b = 0.06 \text{m}$, $t = 0.0025 \text{m}$, $l = 0.6 \text{m}$.

36 kinds of deformations are used to construct high-order models, which to study the free vibration of cold-formed channel beams. The unconstrained modal analysis of ANSYS shell 181 elements is used to obtain the first 8 natural frequencies. The model is discretized with 1680 ANSYS shell 181 elements and 60 elements along the length direction. The natural frequencies of the ANSYS model are expressed by $f_i$ and the present models are expressed by $f$. The first 8 natural frequency comparison is shown in Table 1.

**Table 1. Comparison of the first 8 natural frequencies of the cold-form channel beam**

| Mode | $f_i$(Hz) | $f$(Hz) | Relative errors (%) |
|------|-----------|---------|---------------------|
| 1st  | 70.306    | 70.330  | -0.03               |
| 2nd  | 175.79    | 175.67  | 0.07                |
| 3rd  | 203.06    | 203.19  | -0.06               |
| 4th  | 354.14    | 352.40  | 0.49                |
| 5th  | 358.52    | 360.70  | -0.61               |
| 6th  | 371.10    | 370.12  | 0.26                |
| 7th  | 459.29    | 474.61  | -3.34               |
| 8th  | 660.57    | 649.41  | 1.69                |
As expected, the results in Table 1 show that the natural frequencies obtained by the present model were very close to those from the ANSYS shell theory, with relative differences smaller than 3.5%. So as to further express the mechanical properties of the cold-formed channel beam, the first 8 modal shapes calculated by the present model and ANSYS were shown in figure 3, respectively, which also reconfirmed the better compliance with the ANSYS shell theory.

6. Summary
A new one-dimensional model for cold-formed channel beam has been proposed that considering the cross-sectional deformation.

(i). The model considers the displacement field through the linearly superimposing of 36 basis deformation modes, which stem from the discretization of the cross-section into 9 points and 8 segments.

(ii). The natural frequencies obtained by the present model were very close to those from the ANSYS shell theory, with relative differences smaller than 3.5%.
(iii). The numerical example shows good agreement of the proposed one-dimensional model with two-dimensional shell element and improves the calculation efficiency to a certain extent, which also proves the potential of capturing three-dimensional deformation of the cold-formed channel beam.

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