A robust possibilistic programming model for a responsive closed loop supply chain network design

Alireza Hamidieh¹, Bahman Naderi¹, Mohammad Mohammadi*¹ and Mohamadreza Fazli-Khalaf¹

Abstract: Concerns about the outbreak of perturbations and their major losses have led a lot of researchers to consider reliability while designing supply chain networks. In addition, the inherent uncertainty of input parameters is another important issue in the design of supply chain networks due to its adverse effects on strategic, tactical, and operational decisions. This present paper proposes a new model for designing a sustainable closed-loop single-product multi-component multi-level logistics network under uncertainty conditions. The model is based on a robust possibilistic programming approach. The proposed models not only minimize the total costs but also develop an effective resistant network under disruptions strikes and control the product delivery speed at appropriate safety levels. Finally, the effectiveness and applicability of the model are displayed in a national project with the actual nominal data.

Subjects: Science; Environmental Studies & Management; Mathematics & Statistics; Technology; Computer Science

Keywords: closed-loop supply chain; responsiveness; uncertainty; robust possibilistic programming

ABOUT THE AUTHORS
Alireza Hamidieh is a faculty member of Payamnoor University. He also is a PhD student in industrial engineering at the Kharazmi University of Iran. His research and teaching interests include: business strategic management, mathematical modeling, and supply chain management.

Bahman Naderi is an assistant professor in the Department of Industrial Engineering at the Kharazmi University, Tehran, Iran. His research and teaching interests include: production scheduling, mathematical modeling, and solution algorithms.

Mohammad Mohammadi is an associate professor in the Department of Industrial Engineering at the Kharazmi University, Tehran, Iran. His research and teaching interests include: sequencing and scheduling, production planning, timetables, metaheuristics, and supply chains.

Mohamadreza Fazli-Khalaf is a PhD student at Kharazmi University in the industrial engineering field. His field of study is supply chain network design, network reliability, and different methods of extending models under uncertainty such as possibilistic, robust fuzzy possibilistic, stochastic, and robust programming.

PUBLIC INTEREST STATEMENT
Nowadays, equipping and providing electronic appliances for schools and technical centers is one of the most important challenges of governments and ministry of education of different countries. Paying attention to the sustainability of distribution networks is also a key factor in assessing the government’s social responsibility. In this regard, this paper presents a forward–reverse supply chain network for a case study of equipping schools and technical centers as an Iranian national project. It is strived to minimize costs of network design aside with minimization of delivery time of electronic appliances to schools in different regions of Iran. Another important point is the intrinsic uncertainty of parameters while designing such supply chain networks. In this regard, a hybrid solution method is extended that is capable of controlling the risk-aversion level of output decisions for company managers.
1. Introduction
To deal with the current competitive market and dynamic demands of customers, business firms are integrated in the form of supply chain networks (Baghalian, Rezapour, & Farahani, 2013; Pishvaee, Kianfar, & Karimi, 2009). Moreover, due to increasing environmental concerns in recent years, business firms intend to reuse the defective or second-hand products through the recycling process. This has led to interesting paths for investigation of reversed and closed-loop supply chain network designs (Altmann & Bogaschewsky, 2014). The reasons for this reuse are the prevention of further resource wastes, reduction of environmental pollution and achievement of profitability with regard to social and commercial considerations (El korch & Millet, 2011; Kara & Onut, 2010).

The closed-loop supply chain network design is defined based on both tactical and strategic objectives (Altiparmak, Gen, Lin, & Karaoglan, 2009; Fleischmann et al., 1997). At the strategic level, supply chain planning includes decision-making for the network configuration, for example, the number, location, capacity, and technology of the facilities in the reverse and forward network (Selim & Ozkarahan, 2006). At the tactical level, supply chain planning includes the total and current values of materials for purchasing, processing, and distributing the products; therefore, an efficient supply chain design ensures the success of the whole chain and stakeholders (Inuiguchi & Ramk, 2000; Melo, Nickel, & Saldanha-da-Gama, 2009).

Notably, an important dimension of this practical and national research is a concept known as corporate social responsibility (CSR) in the supply chain. The concept of CSR is defined as the impact of corporates on different social groups, such as environmental protection, employee rights, workplace safety, and the right conditions for employees. However, in the present study, this concept is viewed from a new perspective. Since equipping the intelligent training centers and their preparation for the beginning of the school year is of great importance, and the lack of necessary equipment or even slow supply of equipment may lead to disruptions in the country’s education system and challenge the government’s social responsibility toward the students. The responsiveness and speed of delivering products with the desired safety levels from national pole centers to the intelligent training centers are regarded as important social commitment factors in the supply chain network design.

Despite the importance of the above-mentioned issues, few applied research have been conducted in these areas. Pishvae, Jalai, and Razmi (2009), Fleischmann et al. (1997) performed a study on the use of mathematical modeling for reverse logistics management. Selim and Ozkarahan (2006) designed and optimized a five-level supply chain network. Three objective functions of the above-mentioned problem include minimization of the total cost, minimization of the investments on the establishment of production plants and warehouses and maximization of service levels to the retailers. Xu, Liu, and Wang (2008) proposed a random fuzzy multi-objective programming model to minimize the fix costs of factories and the supply chain’s distribution centers and to maximize customer services. Jayaraman, Patterson, and Rolland (2003) proposed a mixed-integer linear programming model under a pull customized system based on customers demand. Üster, Easwaran, Akçali, and Çetinkaya (2007) designed a closed-loop network in which only the Rehabilitation and reverse Centers are located and the forward and reverse flows are optimized simultaneously. Wang, Lai, and Shi (2011) used the spanning tree approach to design a non-linear model for the closed-loop supply chain and solved it by the genetic algorithms. El-Sayed, Afia, and El-Kharbotly (2010), developed possibilistic programming approaches in the design of multi-objective supply chain networks. Salema, Barbosa-Povoa, and Novais (2010) and Santoso, Ahmed, Goetschalckx, and Shapiro (2005) developed Fleischmann model to a high capacity multi-product logistics network with regard to the uncertainty conditions. Ramezani, Bashiri, and Tavakkoli-Moghaddam (2013) proposed a multi-objective possibilistic model for designing an integrated logistics network under uncertainty conditions. Amin and Zhang (2013) developed a multi-product closed-loop supply chain network under uncertainty conditions and, taking into account the environmental factors, added an environmental function to the problem. Pishvaee and Razmi (2012) investigated a double-objective model to minimize costs and maximize the social effects. According to the literature, the material flow and the capacity
of all levels are not taken into considerations in any of the above-mentioned studies. In addition, none of the above research was concerned with the implementation of a national project with time limits and delivery reliability as a social responsibility indicator.

Notably, the complex and dynamic nature of supply chains causes a degree of uncertainty in the planning decisions which mainly result in systemic risk and inevitable factors in the supply chain such as disruptions in the supply side, uncertainty of the demanding side, regulatory, legal and administrative changes, infrastructure disruptions, etc (Baghalian et al., 2013; Lu, Ran, & Shen, 2015; Mete & Zabinsky, 2010; Pishvae, Rabbani, & Torabi, 2011). The important point is that many approaches are developed to control uncertainty of input parameters such as stochastic programming, possibilistic programming, and robust programming. Stochastic programming approach controls uncertainty of parameters by applying probability distribution of uncertain parameters (Hatefi & Jolai, 2014; Peng, Snyder, Lim, & Liu, 2011; Shishebori & Yousefi Babadi, 2015). However, this approach has some deficiencies. Firstly, historical data of uncertain parameters should be available to estimate their probability distribution. Besides, the increment of a number of uncertain parameters and their modeling could heighten complexity of the model. To solve aforementioned gaps, possibilistic programming methods are developed by researchers. This method uses knowledge of field experts and incomplete available data to model uncertain parameters by application of possibility distribution. Extended possibilistic programming methods are capable of adjusting satisfaction level of uncertain parameters based on the opinion of decision-makers and company managers (Pishvae et al., 2011). However, these methods do not guarantee optimization of satisfaction levels and reliability of output results. In this regard, robust programming methods which strive to control the performance of the model in a risk-averse manner could help to solve deficiencies of possibilistic programming methods (see Ben-Tal & Nemirovski, 1998, 2000; Bertsimas & Sim, 2004; Inuiguchi & Sakawa, 1995; José Alem & Morabito, 2012; Mulvey, Vanderbei, & Zenios, 1995; Pan & Nagi, 2010; Soyster, 1973). Finally, it is worthy to mention that extending a robust possibilistic programming model could result in the reliability of outcome results and make a reliable decision-making tool for company decision-makers.

Based on the enumerated matters, the main contributions of this paper that differentiate it from other research papers in related literature is as follows:

• Presenting a closed-loop supply chain network design model that optimizes horizontal and vertical decision levels (i.e. operational and tactical decisions) aside with concurrent optimization of forward and reverse supply chain network design
• Extending a multi-objective model minimizing network design costs besides maximizing social responsibility of supply chain network based on its responsiveness to demands
• Proposing a robust possibilistic programming network design model that efficiently copes with uncertainty of parameters and unlike previously extended models enables decision-makers to optimize output results of model based on their level of risk-averseness
• Formulating a closed-loop network model based on the case study of mobilizing Iranian training centers. However, it could be applied as a general model in other industries such as glass production or paper recycling with minor reforms.
• Managing and controlling material flow in the reverse and forward directions of the studied network that provides managers with credible feedback for deciding on the supply of components, and other required equipment for the intelligent training centers.

The rest of the paper is organized as follows. Section 2 presents comprehensive problem definition of the supply chain based on case study. Model formulation and definition of indices, parameters and decision variables is presented in Section 3. Section 4 comprehensively presents the robust possibilistic programming model of the problem and its benefits against previously extended possibilistic programming models. Section 4 uses the generalized Epsilon-constraint method to solve the multi-objective problem. Section 4 outcomes results of the model analyzed based on data extracted...
from the case study. Section 6 finally concludes the paper and presents further research guidelines.

2. Problem definition

The aim of this paper is presenting a multi-objective closed-loop supply chain network design based on case study of the national project of providing a portable notebook for intelligent training centers of the Ministry of Education throughout the country. This device consists of three components: the body with keyboard and LCD monitor attached to it (the first component), mainboard and its attached power module (the second component), the main memory or hard drive with the corresponding connections (the third component). Studied supply chain network consists of forward and reverse network direction for production and recovery of products. Based on the opinion of field experts, notebook consists of three main components. In this regard, suppliers provide different components for factories to assemble and produce final products in the forward direction of the network. Then, final products are transported to educational centers via pole centers that are regarded as a distribution center in the extended network. It is assumed that a previously estimated percentage of products are defective and would be returned to support centers for recovery or disposal from each educational center. Collected products at support centers have different deficiencies. In this regard, they can be classified as: completely useless, software damaged, and hardware damaged. Hardware-damaged notebooks could be classified as: one-component damaged, two-component damaged, and requiring minor recovery. Notebooks needing software and hardware minor recovery are repaired at support centers and are transported to pole centers to meet educational centers’ need aside new notebooks produced at plants. Also, intact components of notebooks with one or two damaged components would be refurbished and transported to plants to produce new notebooks beside provided components supplied by suppliers. Damaged components of useless notebooks beside damaged components of refurbished products would be delivered to disposal centers for safe disposal. In the studied case, number and location of suppliers, disposal, and educational centers are fixed and predetermined. Also, the potential location for plants, pole and support centers should be determined and optimized based on the operational and fix opening cost of noted centers. Demand of educational centers should be completely satisfied. A predetermined capacity level is defined for plants, pole, and support centers. Graphical representation of the studied network is presented in Figure 1.

As the responsiveness of network is very important for the ministry of education as the owner of the project, it is strived to consider different packaging and delivery technologies at pole centers to balance cost efficiency and social responsibility of extended supply chain network. In this regard, extended model minimizes total costs of network design aside with maximizing responsiveness as an indicator of social responsibility. Number and opening location of potential facilities should be optimized via the extended network. Also, components and final products flow at forward and reverse networks should be optimized based on proposed objective function.

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Figure 1. Graphical representation of extended closed-loop Supply chain network.
3. The model formulation

The indices, parameters, and variables used to formulate the concerned closed-loop supply chain network (CLSCND) problem are presented as follows.

Indices:

- \( s \) Index of potential locations for suppliers \( s = 1, \ldots, S \)
- \( k \) Index of potential locations for plants \( k = 1, \ldots, K \)
- \( p \) Index of potential locations for national pole centers \( p = 1, \ldots, P \)
- \( e \) Index of fixed locations of customers \( e = 1, \ldots, E \)
- \( f \) Index of potential locations for support centers \( f = 1, \ldots, F \)
- \( m \) Index of potential locations for disposal centers \( m = 1, \ldots, M \)
- \( c \) Index of components \( c = 1, \ldots, C \)
- \( i \) Index of safety level of pole centers \( i = 1, \ldots, I \)

Parameters:

- \( D_e \) Demand of training centers \( e \)
- \( RD_e \) Percentage of the returned demand of training center \( e \)
- \( FP_{pi} \) Fix cost of opening pole center \( p \) with safety level \( i \)
- \( FF_f \) Fix cost of opening support center \( f \)
- \( FK_k \) Fix cost of opening production center \( k \)
- \( VS_{sc} \) Purchase cost per unit of component \( c \) from supplier \( s \)
- \( TSK_{sc} \) Shipping cost per unit of component \( c \) from supplier \( s \) to plant \( k \)
- \( TKP_{kp} \) Shipping cost per unit of product from plant \( k \) to pole center \( p \)
- \( TPE_{pe} \) Shipping cost per unit of product from pole center \( p \) to training center \( e \)
- \( TEF_{ef} \) Shipping cost per unit of product from training center \( e \) to support center \( f \)
- \( TFP_{fp} \) Shipping cost per unit of product from support center \( f \) to pole center \( p \)
- \( T1F_{fk} \) Packaging and shipping cost of each unit of component 1 from support center \( f \) to plant \( k \)
- \( T2F_{fk} \) Packaging and shipping cost of each unit of component 2 from support center \( f \) to plant \( k \)
- \( T3F_{fk} \) Packaging and shipping cost of each unit of component 3 from support center \( f \) to plant \( k \)
- \( T12F_{fk} \) Packaging and shipping cost of each unit of component 1, 2 from support center \( f \) to plant \( k \)
- \( T13F_{fk} \) Packaging and shipping cost of each unit of component 1, 3 from support center \( f \) to plant \( k \)
- \( T23F_{fk} \) Packaging and shipping cost of each unit of component 2, 3 from support center \( f \) to plant \( k \)
- \( T1M_{fm} \) Packaging cost of each unit of component 1 from support center to disposal center \( m \)
- \( T2M_{fm} \) Packaging cost of each unit of component 2 from support center to disposal center \( m \)
- \( T3M_{fm} \) Packaging cost of each unit of component 3 from support center to disposal center \( m \)
- \( SP_{pi} \) Delivery speed of products at pole Center \( P \) with safety level \( i \)
- \( CK_k \) Maximum capacity of plant \( k \)
**Variables:**

| Symbol | Description |
|--------|-------------|
| CP<sub>p</sub> | Maximum capacity of pole center p |
| CF<sub>f</sub> | Maximum capacity of support center f |
| VK<sub>k</sub> | Production cost per unit of product at plant k |
| VP<sub>p</sub> | Inventory holding cost per unit of product at pole center p |
| VFP<sub>f</sub> | Repairing cost of software per unit of product at support center f |
| VFG<sub>f</sub> | Limited hardware repair cost per product at support center f |
| VP1<sub>f</sub> | Processing (operation and isolation) cost of component 1 at support center f |
| VP2<sub>f</sub> | Processing (operation and isolation) cost of component 2 at support center f |
| VP3<sub>f</sub> | Processing (operation and isolation) cost of component 3 at support center f |
| VP12<sub>f</sub> | Processing (operation and isolation) cost of component 1 and 2 at support center f |
| VP13<sub>f</sub> | Processing (operation and isolation) cost of component 1 and 3 at support center f |
| VP23<sub>f</sub> | Processing (operation and isolation) cost of component 2 and 3 at support center f |
| VT1<sub>f</sub> | Quality control cost of component 1 at support center f |
| VT2<sub>f</sub> | Quality control cost of component 2 at support center f |
| VT3<sub>f</sub> | Quality control cost of component 3 at support center f |
| q<sub>1</sub> | Percentage of the returned product with correct component 1 |
| q<sub>2</sub> | Percentage of the returned product with correct component 2 |
| q<sub>3</sub> | Percentage of the returned product with correct component 3 |
| PSR | Percentage of the returned product for software repairing |
| q<sub>12</sub> | Percentage of the returned product with correct components 1,2 |
| q<sub>13</sub> | Percentage of the returned product with correct components 1,3 |
| q<sub>23</sub> | Percentage of the returned product with correct components 2,3 |
| PHR | Percentage of the returned product for hardware repairing |

| Symbol | Description |
|--------|-------------|
| Y<sub>skc</sub> | Quantity of components c supplied by supplier s and shipped to plant k |
| U<sub>kp</sub> | Quantity of products produced at plant k and shipped to pole center p |
| W<sub>pe</sub> | Quantity of products stored at pole center p and shipped to training center e |
| R<sub>ef</sub> | Quantity of products shipped from training center e to support center f |
| O<sub>fp</sub> | Quantity of repaired products owing to software problems and shipped from support center f to pole center p |
| GP<sub>fp</sub> | Quantity of debugged products owing to hardware problems at support center f and shipped to pole center p |
| T<sub>fm</sub> | Quantity of defective components shipped from support center f to disposal center m |
| J<sub>1k</sub> | Quantity of correct components 1 isolated at support center f and shipped to plant k |
| J<sub>2k</sub> | Quantity of correct components 2 isolated at support center f and shipped to plant k |
| J<sub>3k</sub> | Quantity of correct components 3 isolated at support center f and shipped to plant k |
| X12<sub>p</sub> | Quantity of correct components 1, 2 isolated at support center f and shipped to plant k |
| X13<sub>p</sub> | Quantity of correct components 1, 3 isolated at support center f and shipped to plant k |
X23_fk  Quantity of correct components 2, 3 isolated at support center f and shipped to plant k
T1_fm  Quantity of defective components 1 isolated at support center f and shipped to disposal center m
T2_fm  Quantity of defective components 2 isolated at support center f and shipped to disposal center m
T3_fm  Quantity of defective components 3 isolated at support center f and shipped to disposal center m

\[
X_{P_{pi}} = \begin{cases} 
1 & \text{if pole center } p \text{ with safety level } i \text{ is opened} \\
0 & \text{otherwise} 
\end{cases}
\]

\[
X_{F_f} = \begin{cases} 
1 & \text{if support center } f \text{ is opened} \\
0 & \text{otherwise} 
\end{cases}
\]

\[
X_{K_k} = \begin{cases} 
1 & \text{if plant } k \text{ is opened} \\
0 & \text{otherwise} 
\end{cases}
\]

In terms of the above notations, the CLSCND problem can be formulated as follows (model 1):

\[
\text{MIN } W_1 = \sum_{p} \sum_{i} \left( \sum_{f} \left( \sum_{s} \left( T_{P_{sp}} + V_{P_{sp}} \right) \right) \right) U_{sp} + \sum_{p} \sum_{e} \left( \sum_{f} \left( T_{P_{pe}} + V_{P_{pe}} \right) \right) W_{pe} + \sum_{f} \sum_{e} \left( \sum_{s} \left( T_{E_{ef}} + R_{ef} \right) \right)
\]

\[
= \sum_{p} \sum_{i} \left( \sum_{f} \left( T_{P_{sp}} + V_{P_{sp}} \right) \right) U_{sp} + \sum_{p} \sum_{e} \left( \sum_{f} \left( T_{P_{pe}} + V_{P_{pe}} \right) \right) W_{pe} + \sum_{f} \sum_{e} \left( \sum_{s} \left( T_{E_{ef}} + R_{ef} \right) \right)
\]

\[
\text{MIN } W_2 = \sum_{i} \sum_{e} \sum_{p} W_{pei} SP_{pi} \]

\[
\sum_{p} U_{sp} \leq CK_k \quad \forall k,
\]

\[
\sum_{e} W_{pei} \leq \sum_{f} CP_{pi} XP_{pi} \quad \forall f,
\]

\[
\sum_{e} R_{ef} \leq CF_f X_{F_f} \quad \forall f,
\]

\[
\sum_{p} \sum_{i} W_{pei} \geq D_e \quad \forall e,
\]

\[
\sum_{i} XP_{pi} \leq 1 \quad \forall p,
\]
\[ \sum_{f} R_{ef} = RD_e D_e \quad \forall e, \quad (1-8) \]

\[ \sum_{p} \sum_{k} U_{kp} + \sum_{f} O_{fp} = \sum_{p} \sum_{i} W_{pei} \quad \forall p, \quad (1-9) \]

\[ \sum_{f} Y_{skc} + \sum_{f} J_{1f}^k + \sum_{f} X_{12f}^k + \sum_{f} X_{13f}^k = \sum_{p} U_{kp} \quad \forall k, c = 1, \quad (1-10) \]

\[ \sum_{f} Y_{skc} + \sum_{f} J_{2f}^k + \sum_{f} X_{12f}^k + \sum_{f} X_{23f}^k = \sum_{p} U_{kp} \quad \forall k, c = 2, \quad (1-11) \]

\[ \sum_{f} Y_{skc} + \sum_{f} J_{3f}^k + \sum_{f} X_{13f}^k + \sum_{f} X_{23f}^k = \sum_{p} U_{kp} \quad \forall k, c = 3, \quad (1-12) \]

\[ q_1 \sum_{e} R_{ef} = \sum_{k} J_{1f}^k \quad \forall f, \quad (1-13) \]

\[ q_2 \sum_{e} R_{ef} = \sum_{k} J_{2f}^k \quad \forall f, \quad (1-14) \]

\[ q_3 \sum_{e} R_{ef} = \sum_{k} J_{3f}^k \quad \forall f, \quad (1-15) \]

\[ \text{PSR} \sum_{e} R_{ef} = \sum_{p} O_{fp} \quad \forall f, \quad (1-16) \]

\[ q_{12} \sum_{e} R_{ef} = \sum_{k} X_{12f}^k \quad \forall f, \quad (1-17) \]

\[ q_{13} \sum_{e} R_{ef} = \sum_{k} X_{13f}^k \quad \forall f, \quad (1-18) \]

\[ q_{23} \sum_{e} R_{ef} = \sum_{k} X_{23f}^k \quad \forall f, \quad (1-19) \]

\[ \text{PHR} \sum_{e} R_{ef} = \sum_{p} GP_{fp} \quad \forall f, \quad (1-20) \]

\[ q_1 \sum_{e} R_{ef} + q_3 \sum_{e} R_{ef} + q_{23} \sum_{e} R_{ef} + \text{PHR} \sum_{e} R_{ef} = \sum_{m} T_{1fm} \quad \forall f, \quad (1-21) \]

\[ q_1 \sum_{e} R_{ef} + q_3 \sum_{e} R_{ef} + \text{PHR} \sum_{e} R_{ef} + q_{13} \sum_{e} R_{ef} = \sum_{m} T_{2fm} \quad \forall f, \quad (1-22) \]

\[ q_1 \sum_{e} R_{ef} + q_2 \sum_{e} R_{ef} + \text{PHR} \sum_{e} R_{ef} + q_{12} \sum_{e} R_{ef} = \sum_{m} T_{3fm} \quad \forall f, \quad (1-23) \]

\[ XP_{pi}, XF_{fi}, XK_{ik} \in \{0, 1\} \quad \forall p, i, f, k \quad (1-24) \]

\[ U_{kp}, W_{pei}, Y_{skc}, R_{ef}, O_{fp}, GP_{fp}, T_{fm}, J_{1f}^k, J_{2f}^k, J_{3f}^k, X_{12f}^k, X_{13f}^k, X_{23f}^k, T_{1fm}, T_{2fm}, T_{3fm} \geq 0 \quad (1-25) \]

The objective function (1) minimizes the chain costs. These costs include “the fix costs for establishment of facilities in pole centers,” “support centers and manufacturing cost of purchasing, storing and transporting the products in the straight line,” “processing and transportation costs for the
returned products with two intact components,” “processing and transportation costs for the returned products with defective components.” The objective function (2) minimizes the total delivery time of products from the pole center \( P \) opened with safety level \( i \). Constraints (3) to (5) prohibit maximum capacity violation at plants, pole centers, and support centers, respectively. Constraint (3) ensures that the number of product sent from each factory to pole centers is equal or lower than factory’s maximum capacity level. Constraint (4) balances the product shipping from the each pole center to the training centers based on the maximum capacity level of the center poles. Constraint (5) ensures that the number of returned products from the training centers to each support center doesn’t exceed the support center’s maximum capacity level. Constraint (6) ensures that at most one capacity level should be opened for each pole center. Constraint (7) assures the balance between customer demand and product flow from the pole centers. In other words, it ensures full satisfaction of demand of customer zones. Constraint (8) ensures that total number of returned products from each training center to support centers should be equal to a predefined percentage of demand of customer zones. Constraints (9) to (23) present flow balance at different levels of supply chain network. Constraint (9) ensures that the input flow of products to each pole center is equal to its output flow. Constraints (10) to (12) ensure flow balance at each plant. Accordingly, a total number of each kind of component supplied by suppliers and support centers to each production center should be equal to total number of final products shipped to pole centers. Constraints (13) to (15) ensure that the percentage of products with only one intact component (first or second or third component) which are returned from the training centers to each support center is equal to the number of components (first, second, or the third) isolated from the products with only one intact component. Constraint (16) ensures that a total number of returned products to each support center with software problems should be equal to a total number of shipped products from each support center to the pole centers after their problem is solved. Constraints (17) to (19) ensure that a percentage of returned products with two intact components (first and second, or first and third, or second and third components) is equal to the number of separated components (first and second, or the first and third, or second and third) from the products with two intact components. Constraint (20) guarantees that total number of returned products from training centers to each support center with hardware problem should be equal to a number of products transported to pole centers after their problem is solved. Constraint (21) ensures that the number of separated defective components is equal to the independent input flows which include products with intact second Components, intact third Component, and intact second and third Components, as well as products with limited hardware problems. Constraint (22) ensures that the number of separated defective components (second components) is equal to the independent input flows including products with intact first components, intact third components, intact first and third components, as well as products with limited hardware problems. Constraint (23) ensures that the number of separated defective components (the third component) is equal to the independent input flows including products with intact first components, intact second Components, intact first and second components as well as products with the limited hardware problem. Constraints (24) and (25) impose the binary and non-negativity restriction on the corresponding decision variables.

4. Proposed robust possibilistic programming model
There are two kinds of uncertainties in supply chain network design models. First one is associated with the uncertainty of parameters that possibilistic programming models are developed to control the noted type of uncertainties. The second one is related to the uncertainty of constraints and the target value of goals. Flexible programming models are applied to cope with this kind of uncertainty (Bashiri & Sherafati, 2012; El-Sayed et al., 2010; Hatefi & Jolai, 2014). As the scope of studied supply chain network design models belongs to the first type of uncertainty, chance-constrained possibilistic programming model extended by Pishvaee, Razmi, and Torabi (2012) is employed to model ambiguous parameters. However, this approach has some deficiencies and is not capable of controlling risk-aversion of outcome results. In this regard, as the supply chain of equipping training centers is of great importance and the government is trying to adopt a conservative and risk-aversive policy for optimum implementation of its commitments, while taking into account the cost–benefit approach
and flexibility in development of this model, robust optimization model under possibilistic uncertainty conditions can help to optimize confidence level of uncertain parameters and provide efficient outcomes for decision-makers.

4.1. Possibilistic programming

Given that nowadays many parameters are included in the decision-making process and also multiple objectives are used to achieve the desired optimality. Planning to deal with the uncertainty adversely affecting decision-making conditions and required rapid response to customer demand under different conditions of the network is the main approach of logistics management and the main objectives of the present study. However, fuzzy coefficient or parameter uncertainty is a fundamental challenge in the optimal management of the network. In recent years, fuzzy mathematical programming is proposed to deal with the uncertainty of coefficients in objective functions and constraints (Ozgen & Gulsun, 2014). Fuzzy mathematical programming is classified into two classes: possibilistic programming and flexible programming, and provides a framework to cope with variety of uncertainties, including fuzzy coefficients due to lack of knowledge and flexibility of constraints and objectives targeted value simultaneously.

In recent years, many articles have been presented based on the possibilistic programming methods in order to deal with fuzzy coefficients of the objective function and constraints (Bairamzadeh, Pishvaee, & Saidi-Mehrabad, 2016; Chopra & Saxena, 2014; Luhandjula, 1987). Chance-constrained programming approach has been used in some of these articles which enable decision-makers to adjust the risk-averse performance of extended model based on uncertainty of parameters.

In this study, uncertainty is modeled through the possibilistic programming method and parameter uncertainty is provided through triangular fuzzy sets. Chance-constrained programming among the possibilistic programming approaches is applied to develop a robust possibilistic programming model (Peidro, Mula, Jiménez, & del Mar Botella, 2010). Notably, chance-constrained programming approach maximizes (minimizes) objective function based on the expected target values of uncertain parameters while considering a certain degree of satisfaction for constraints containing uncertain parameters (Dubois, Fargier, & Fortemps, 2003).

The overall compact structure of a closed-loop supply chain network model with one objective function (i.e. cost minimization) is presented as follows:

\[
\begin{align*}
\text{MIN} & \quad z = (f_{\text{co}})y + (p_{\text{c}} + s_{\text{c}})x \\
\text{s.t.} & \quad Ax \geq d \\
& \quad Bx = 0 \\
& \quad Hx \leq Ny \\
& \quad Tx \leq 1 \\
& \quad y \in \{0, 1\}, \quad x \geq 0
\end{align*}
\]  

(2)

In the model 2, the fco vector (fix costs of opening the facilities), pc (variable costs of production), sc (variable costs of transportation), and d (demand of training centers) are uncertain parameters. Matrices T, N, H, B, A are the constraints coefficients. The vectors y and x are the binary and continuous variables of the model. In the above general model, the second objective function is not taken into consideration owing to importance of cost minimization objective, but it can be treated just like the first objective function. It should also be noted that determining the basic relationships between two objective functions is also very effective and will be accounted for in the next section.

Assume that sc, pc, d and fco and the coefficient matrix N are uncertain and fuzzy in the closed-loop supply chain model. Triangular possibility distribution is chosen to model fuzzy parameters and define their three main points (i.e. \( r = (r_{1}, r_{2}, r_{3}) \) (Figure 2).
Based on the above-mentioned points, the equivalent crisp model, according to the triangular possibility distribution for the fuzzy parameters is presented as follows (model 3):

$$\text{MIN } \hat{Z}_{exp} = \left( \frac{f_{co_1} + f_{co_2} + f_{co_3}}{3} \right) y + \left[ \left( \frac{sc_1 + sc_2 + sc_3}{3} \right) + \left( \frac{pc_1 + pc_2 + pc_3}{3} \right) \right] x$$

s.t.  
$$Ax \geq (1 - \alpha)d_2 + \alpha d_3$$
$$Bx = 0,$$
$$Hx \leq (1 - \beta)N_2 + \beta N_1 y$$
$$Tx \leq 1$$
$$y \in \{0, 1\}, x \geq 0$$

The above Model is known as the basic possibilistic chance-constrained programming model. In this approach, system administrators determine the minimum confidence level for the constraints including uncertain parameters through parameters $\alpha$ and $\beta$ (i.e. $0.5 < \alpha, \beta \leq 1$). In fact, in order to put outputs of proposed network at the risk-averse mode, and be able to make the network responsive in the pessimistic conditions (i.e. in maximum demand and minimum capacity) confidence levels could be increased to their maximum value.

In this approach, the expert or company manager determines the minimum confidence level for constraints comprising uncertain parameters based on his/her experience, knowledge, and subjective analyses in a random way. In fact, he/she determines several initial values for $\alpha$ and $\beta$ to determine the best solutions based on the output value of objective function. Also, the objective function is modeled regarding expected value of uncertain parameters. There are some fundamental problems in this approach that could be mentioned as follows:

1. There is no guarantee that the obtained values for confidence levels are the best choice owing to their subjective adjustment
2. In the proposed supply chain network, little changes in the confidence levels can impose harmful consequences on the entire network and output decisions
3. With an increase in the number of chance constraints in the model, the number of the tests required to obtain the appropriate confidence levels will increase as well, therefore the expert will go through a costly, time-consuming path with so many errors
4. The above model is not sensitive to the deviation of the objective function value over or under its expected value, thus it leads to the development of unpredictable conditions in the system and output results would not be reliable for decision-makers.

However, to evaluate the performance of large-scale supply chain networks which face with the uncertainty of effective parameters of the model, it is important to apply the appropriate value of
confidence levels that guarantee effective performance of the extended model. In fact, we seek output efficient solutions that reflect the overall optimum performance of the system. Robust programming could help to solve aforementioned problems of proposed chance-constrained possibilistic programming model.

4.2. The proposed robust possibilistic programming model

Here, the robust possibilistic programming model will be provided according to the model obtained from the previous stage. The fco, pc, d and sc vectors and the coefficient matrix N will be considered uncertain. Therefore, the robust model of extended possibilistic chance-constrained programming model could be presented as follows:

\[
\begin{align*}
\text{MIN } & Z_{\text{exp}} + \gamma \left[ |fco_1 \cdot y + (sc_1 + pc_1) \cdot x| - |fco_1 \cdot y + (sc_1 + pc_1) \cdot x| + \delta (d_1 - (1 - \alpha) \cdot d_2 - \alpha \cdot d_3) + \pi \beta N_1 (1 - \rho N_2 - N_3)\right] y \\
\text{s.t. } & Ax \geq (1 - \alpha) d_2 + \alpha d_3 \cdot y \\
& Bx = 0 \\
& Hx \leq (1 - \rho N_2 + \rho N_3) y \\
& Tx \leq 1 \\
& y \in \{0, 1\}, x \geq 0, 0.5 < \alpha, \beta \leq 1.
\end{align*}
\]

(4)

In the objective function of model 4, the \( Y \left[ |fco_1 \cdot y + (sc_1 + pc_1) \cdot x| - |fco_1 \cdot y + (sc_1 + pc_1) \cdot x| \right] \) provides the difference between the two maximum and minimum values of objective function. Based on (5) and (6), the Maximum and minimum values of \( Z_{\text{exp}} \) include:

\[
\begin{align*}
Z_{\text{exp}}^{\text{max}} &= fco_3 \cdot y + (sc_3 + pc_3) \cdot x \\
Z_{\text{exp}}^{\text{min}} &= fco_4 \cdot y + (sc_4 + pc_4) \cdot x
\end{align*}
\]

(5) (6)

Parameter \( \gamma \) shows the weight of optimality robustness against other terms of the objective function. In addition, this term ensures that the gap between the maximum and minimum value of the objective function will be minimized. In fact, this term controls the optimality robustness of the solution vector and keeps value of objective function almost near optimal.

The third term of the objective function \( |d_1 - (1 - \alpha) \cdot d_2 - \alpha \cdot d_3| \) shows the difference between the worst possible value of uncertain parameter and its value in the chance constraint, and the confidence level of each chance constraint. Also, parameter \( \delta \) is the penalty cost regarded for each unit of possible violation of constraints containing uncertain parameters. In fact, a penalty is considered for any violation of the allowed values. This section controls the feasibility robustness in the solution vector and optimizes confidence levels based on the penalty cost of violations in chance constraints. It should be noted that in a real-world supply chain networks, the amount of \( \delta \) can be used as penalty for unsatisfied demand, product shortages, lack of timely responsiveness, and decline in the delivery speed.

As you can see, the proposed model is a nonlinear programming model. As the technological factors (in the above model N) have been regarded as uncertain, its satisfaction level has turned in above model into a decision variable and therefore the model is transformed into a nonlinear one. Here, a new variable is introduced for using the advantages of linear programming and several constraints are added to the model to change the model into a linear one. In this case, the variable \( \rho \) is added to the model 4 as an auxiliary variable as follows (7):

\[
\rho = \beta \cdot y
\]

(7)
Thus, the extended robust possibilistic chance-constrained programming model could be reformulated as follows:

\[
\min \hat{Z}_{\text{exp}} + \gamma \left[ f_{\text{co}} \cdot y + (s_{\text{c}} + p_{\text{c}}) \cdot x \right] - \left[ f_{\text{co}} \cdot y + (s_{\text{c}} + p_{\text{c}}) \cdot x \right] + \delta \left[ d_{\text{f}} - (1 - \alpha) \cdot d_{\text{f}} - \alpha \cdot d_{\text{f}} \right] + \pi \rho N_{\text{i}} \left( y - \rho \right) \left( N_{\text{i}} - N_{\text{i}}y \right)
\]

s.t.

\[
\begin{align*}
Ax & \geq (1 - \alpha) d_{\text{f}} + \alpha d_{\text{f}} \\
Bx & = 0, \\
Hx & \leq (y - \rho) N_{\text{i}} + \rho N_{\text{i}} \\
\rho & \leq M y, \\
\rho & \leq \beta, \\
\rho & \geq M (y - 1) + \beta \\
T_x & \leq 1
\end{align*}
\]

\[y \in (0, 1), \ x, \ \rho \geq 0, \ 0.5 < \alpha, \ \beta \leq 1 \quad (8)\]

In the above model (8), M is a very large number and the constraints added to the model ensure that with changes in the binary variable (from zero to one) the variable \( \rho \) would change (to zero and \( \beta \), respectively). The notable point about the present real case study is that the financial investor of the project (Ministry of education) aims to investigate the sensitivity of the objective function with respect to the deviation of the objective function over its expected value to obtain an appropriate analysis of the optimal investment scenarios. Therefore, the above objective function could be redefined as follows (9):

\[
\min \hat{Z}_{\text{exp}} + \gamma \left[ f_{\text{co}} \cdot y + (s_{\text{c}} + p_{\text{c}}) \cdot x \right] - \left[ f_{\text{co}} \cdot y + (s_{\text{c}} + p_{\text{c}}) \cdot x \right] + \delta \left[ d_{\text{f}} - (1 - \alpha) \cdot d_{\text{f}} - \alpha \cdot d_{\text{f}} \right] + \pi \rho N_{\text{i}} \left( y - \rho \right) \left( N_{\text{i}} - N_{\text{i}}y \right)
\]

In fact, the in the new formulation, a strict compensation structure is applied between the expected value and the worst value of the objective function, which is controlled by parameter \( \gamma \) as representative of optimality robustness controller. With any decrease in the strictness of the compensation structure, the following model (model 10) will be achieved:

\[
\min \hat{Z}_{\text{exp}} + \gamma \left[ f_{\text{co}} \cdot y + (s_{\text{c}} + p_{\text{c}}) \cdot x \right] + \delta \left[ d_{\text{f}} - (1 - \alpha) \cdot d_{\text{f}} - \alpha \cdot d_{\text{f}} \right] + \pi \rho N_{\text{i}} \left( y - \rho \right) \left( N_{\text{i}} - N_{\text{i}}y \right)
\]

\[5. \text{Implementation and evaluation}\]

In this section, the performance of the extended models is evaluated based on the data extracted from a case study. Considering the importance of the present research at the national level and fuzziness of the problem parameters, a team of staff and operational managers was established for the above supply chain planning problem. They estimated the possibility distribution of the problem parameters and determined the outstanding amounts of triangular fuzzy numbers. Notably, there are suppliers, plants pole centers and training centers in the forward direction of supply chain network. Also, support centers and disposal centers are regarded in reverse direction of the extended network. The number of facilities extracted from a real case study in the proposed model is discussed in Table 1.

Formulation of the described network includes a large number of certain and uncertain parameters. Therefore, due to space limitations, it is impossible to cover and present all the parameters here. Accordingly, due to the importance of uncertain parameters, some of them (e.g. the demands of provincial training centers and the cost of establishing provincial pole centers) are presented in Tables 2 and 3.

The fixed cost of opening the network pole centers with three safety levels of \( \bar{i}_{\text{f}}, \bar{i}_{\text{f}}, \bar{i} \) is presented in Table 3.
To test the performance of the chance-constrained possibilistic programming model different values of confidence levels ($\alpha, \beta$) (i.e. 0.51, 0.6, 0.7, 0.75, 0.8, and 0.9) are used. The values of two objective functions under different confidence levels are provided in Figures 3 and 4. The presented results show that increasing confidence levels, has led to increasing values of the objective functions. The increase in the value of the objective function is because of the fact that meeting the needs of training centers in the country (customers) as well as collecting the returned components at the higher confidence levels, would require more resources such as components, final products, the higher

### Table 1. The number of facilities in real supply chain network

| Facilities                       | Counter |
|----------------------------------|---------|
| Supplier                         | 2       |
| Plant                            | 3       |
| Pole center                      | 3       |
| Provincial training center (customer) | 32     |
| Safety level                     | 3       |
| Support center                   | 5       |
| Disposal center                  | 3       |

### Table 2. Demand of provincial training centers

| Province | Demand ($D_{\tau}^1$) | Demand ($D_{\tau}^2$) | Demand ($D_{\tau}^3$) |
|----------|-----------------------|-----------------------|-----------------------|
| 1        | 1,316                 | 1,560                 | 1,720                 |
| 2        | 1,046                 | 1,460                 | 1,740                 |
| 3        | 1,658                 | 1,976                 | 2,256                 |
| 4        | 3,980                 | 4,240                 | 4,356                 |
| 5        | 5,780                 | 6,140                 | 6,576                 |
| 6        | 1,100                 | 1,380                 | 1,714                 |
| 7        | 1,460                 | 1,768                 | 1,978                 |
| 8        | 1,100                 | 1,380                 | 1,514                 |
| 9        | 350                   | 680                   | 842                   |
| 10       | 218                   | 540                   | 770                   |
| 11       | 3,620                 | 4,180                 | 4,532                 |
| 12       | 560                   | 1,080                 | 1,330                 |
| 13       | 2,360                 | 2,780                 | 2,924                 |
| 14       | 934                   | 1,110                 | 1,272                 |
| 15       | 380                   | 736                   | 1,048                 |
| 16       | 2,000                 | 2,180                 | 2,296                 |

### Table 3. Opening fixed cost of pole center ($\times 10^9$)

| Pole center | $i_1$ | $i_2$ | $i_3$ |
|-------------|-------|-------|-------|
| FP$_{\tau}^1$ | 3.4 | 5.4  | 4.15  |
| FP$_{\tau}^2$ | 3.6 | 5.65 | 4.47  |
| FP$_{\tau}^3$ | 3.75| 5.82 | 4.66  |
| FP$_{\tau}^1$ | 1.8 | 3.8  | 4.02  |
| FP$_{\tau}^2$ | 1.92| 3.96 | 2.69  |
| FP$_{\tau}^3$ | 2.1 | 4.02 | 2.74  |
| FP$_{\tau}^1$ | 2.05| 2.75 | 2.98  |
| FP$_{\tau}^2$ | 2.22| 2.88 | 3.04  |
| FP$_{\tau}^3$ | 2.31| 2.9 | 3.1   |
capacity of facilities, and transportation capacities. In fact, with any increase in confidence level, the model would perform in a risk-aversive manner.

Increasing satisfaction level of uncertain parameters leads to enhancement of flow of products and raw materials between different consecutive echelons of supply chain network. In this regard, there is a need to opening new facilities at echelons of the network to manage the flow of materials and respond to consumers demand. Therefore, based on presented results in Figure 3, it could be deduced that some facilities should be opened in some confidence levels that needs a great value of the investment to establish new infrastructures. The noted matter is a factor affecting the value of the first objective function which results in the mutation of the value of the objective function in some cases.

Another important change resulting from increasing confidence levels is the behavior of the pole centers with different safety levels. In the confidence level of $\alpha, \beta = 0.51$, the 2 and 3 pole centers are opened with the $i_1$ safety level. Increased confidence level to 0.8 causes the pole centers 1, 2, 3 to open with safety level $i_1, i_2, i_1$, respectively. Noted matter shows the proper functioning of the model and selection of higher safety levels for better fulfilment of customer needs.

In addition, with increasing customer demands under nominal data, more facilities are opened to provide customer needs with services. Figures 5 and 6 show the increasing trend of objective
functions with regard to increasing delivery speed of products from pole centers to training centers and increasing demands of training centers, respectively.

Figure 5 is representative of increasing delivery speed of products from pole centers to training centers and its result on total costs of network design (i.e. first objective function). As it could be seen an increase of total delivery speed of pole centers has led to increase of total costs of network design. In other words, the increase of time of delivering products in some cases leads to opening new potential facilities in different echelons of the network to lower total delivery time of products by pole centers. Therefore, a big amount of cost would be added to total costs of network design in some cases owing to opening new facilities. Also, Figure 6 corresponds to increasing demand of customer zones and its result on total fixed and variable costs of supply chain network. Increasing demand of customer zones acquires opening new facilities in some cases which result in enhancement of total cost of network design and especially, the total opening cost of facilities.

The total cost of transportation which is displayed under different confidence levels in Figure 7 shows that increasing confidence levels leads to increased transportation costs in the forward and reverse the direction of the network. This reflects the perfect performance of the model.

Extended model in this paper is a multi-objective supply chain network design model and proposed objective functions are conflicting (i.e. cost minimization and minimizing total delivery time of products). Therefore, an effectual multi-objective programming method is selected from related
literature to solve multi-objective model and find non-dominated optimal solutions regarding objective functions. The most important feature of the epsilon-constraint method is its ability to create Pareto-optimal solutions based on the opinion of field experts and company managers. In other words, decision-makers could change the value of right-hand side of epsilon-constraint to find best non-dominating solutions based on their opinion and regarding conflict of objective functions. To solve the multi-objective model using generalized Epsilon-constraint method is applied. Assume a multi-objective mathematical programming problem with p objective functions which must be all minimized. In the Epsilon-constraint method, first, an objective function with the highest priority would be regarded as the primary objective function and is optimized. Based on the value of other objective functions will be added to the feasible solution space X as constraints.

\[
\begin{align*}
\text{MIN} & \quad f_1(x) \\
& \quad f_2(x) \leq e_2, f_3(x) \leq e_3, \ldots, f_p(x) \leq s.t. \\
& \quad x \in X,
\end{align*}
\]  

(11)

Pareto-optimal solutions will be regarded as new constraints using the right parameter variables \((e_1, e_2, \ldots, e_p)\). To apply the Epsilon-constraint method, the changing range of each objective function should be available. These scopes are used to determine the points for evaluating the effect of objective functions on each other. Yield table is the most common method used to calculate the scope of objective functions. For the development of yield tables for each multi-objective problem, first each objective function \(f_i\) is solved separately with the main constraints and their optimal solution is calculated. The optimal value of \(f_i\) is denoted by \(f_i(x^*)\) where \(x^*\) is the vector of decision variables that optimizes the objective function \(f_i\). Then vector of decision variables is used to calculate the values of other objective functions that are denoted by \(f_1(x^*_i), f_2(x^*_i), \ldots, f_p(x^*_i)\). After finding the range of all objective functions, the yield table of the Epsilon-constraint technique is used to divide the range of \(p - 1\) objective functions \((f_2, f_3, \ldots, f_p)\) into \(q_2, q_3, \ldots, q_p\) equal distances. These distance are formed of \((q_2 - 1), (q_3 - 1), \ldots, (q_p - 1)\) equidistant central points. Considering the maximum and minimum extreme points of objective functions, the total number of points for each objective function \(f_i\) is equal to \((q_i + 1)\), therefore we should solve \(\prod_{i} = 2^{(q+1)}\) optimization sub-problems. By solving each optimization problem, a Pareto-optimal solution can be obtained. The Pareto-optimal solution with the highest priority will be selected by the decision-makers as the final decision for long-term performance of a company. It should be noted that the Epsilon-constraint method has two main weaknesses: First, the range of the objective functions obtained from the yield table cannot be applied on the Pareto-efficient sets. Second, this method does not guarantee that the optimal solutions are optimal and efficient. To solve the first problem, the lexicograph technique is used to calculate the yield table. In this technique, the original objective function is solved with the problem constraints. We assume that the optimal value of the objective function is equal to \(f_i = z_i^*\). Then, \(f_i = z_i^*\) should be added to the constraints of the problem in order to maintain optimal
solutions to the first optimization sub-problem and solve the second objective function with added constraints to obtain the second objective function values. The generalized Epsilon-constraint method is used to meet the second weakness (Mavrotas, 2009). First, the inequalities of the constraints associated with the objective function are transformed into equations using slack variables, and then these slack variables are regarded as the second part of the objective function. The noted process provides efficient solutions for company managers.

In this regard, the following model (12) could be applied that parameter \( r_i \) is the range of the \( i \)th objective function.

\[
\begin{align*}
\text{Min} & \quad f_1(x) - \text{eps} \cdot \left[ \frac{s_2}{r_2} + \frac{s_3}{r_3} + \ldots + \frac{s_p}{r_p} \right] \\
\text{s.t.} & \quad f_2(x) + s_2 = e_2 \\
& \quad \ldots f_p(x) + s_p = e_p, \quad x \in X, \quad s \in \mathbb{R}^+.
\end{align*}
\]

(12)

The cost objective function is regarded as the main objective function and the second objective function will be added to the feasible solution space as constraints. After the models with nominal data were solved, the Pareto solutions will be obtained. Results of solving multi-objective model are presented in Table 4.

For example, at a confidence level 0.9, when the objective functions are \( z_1 = 36,649,631,988 \) and \( z_2 = 20,100 \) two factories and two pole centers with safety levels of \( i_1 \) are opened. Also, with regard to the Pareto-optimal solutions with objective functions of \( z_1 = 36,543,837,137 \) and \( z_2 = 21,460 \), the three factories and three pole centers with safety levels of \( i_1, i_1, i_1 \) were opened.

Pareto-optimal solutions of robust possibilistic programming models 2 and 3 have been presented in Figures 8 and 9. As it can be seen the pace of objective functions changes in the third model is lower compared to the second model. In addition, in the third model, the growth of cost increase compared to the reduction of product delivery time in the pole centers is lower than that in the second model. This reflects the robust performance of the robust model (10). In addition, the generalized Epsilon-constraint method provided more efficient solutions for the extended supply chain network design problem.

To evaluate the utility and reliability of the results obtained from the proposed model against the results of the proposed deterministic model, the results were analyzed under 5 Series of data which were randomly generated for uncertain parameters. It should be noted that in the deterministic model listed under the obtained nominal data which was solved by the mean fuzzy numbers, the value of \( \tilde{r} = r_1, r_2, r_3 \) was also regarded as an uncertain parameter with a triangular possibility distribution. Thus, a uniform random number is generated between the lower and upper extreme points of the function of uncertain parameters for realization purposes (i.e. \( r_{\text{rand}} = [r_l, r_u] \)).

Then, the decision variables obtained from solving the proposed robust possibilistic programming model and the deterministic model are separately used in linear models to analyze data. In this

| Table 4. The performance of proposed basic possibilistic chance constrained multi-objective programming model under nominal data |
|---------------------------------------------------------------|
| \( \alpha = \beta = 0.7 \) | \( \alpha = \beta = 0.8 \) | \( \alpha = \beta = 0.9 \) |
| \( Z_1 \) | \( Z_2 \) | \( Z_1 \) | \( Z_2 \) | \( Z_1 \) | \( Z_2 \) |
| 34,650,364,293 | 20,387.3 | 35,640,183,524 | 21,951 | 36,649,631,988 | 20,100 |
| 34,646,540,974 | 20,778.3 | 37,533,397,166 | 27,754 | 38,582,807,480 | 20,484 |
| 34,644,181,178 | 21,168 | 37,500,791,948 | 33,558 | 39,561,143,531 | 20,644 |
| 34,642,388,171 | 21,559 | 36,917,407,020 | 39,361 | 39,826,908,935 | 20,972 |
| 34,640,183,524 | 21,951 | 36,055,325,420 | 46,164 | 39,943,837,137 | 21,460 |
model, a variable is defined for the constraints of the proposed model consisting of uncertain parameters, because considering the uncertainty of parameters, violation of constraints is quite probable. In addition, a penalty will be considered for added variable in the objective function. Finally, the mean and standard deviation of the models' objective functions will be used under random data to compare the models results. The performance of the proposed models under nominal data is investigated using the following mathematical model.

\[
\begin{align*}
\text{Min} & \quad f_{co_{\text{real}}} y^* + (p_{c_{\text{real}}} + s_{c_{\text{real}}}) x^* + \delta R^d + \pi R^c \\
\text{s.t.} & \quad Z_2 \geq \epsilon, \\
& \quad Ax^* + R^d \geq d_{\text{real}}, \\
& \quad BX^* = 0, \\
& \quad Sx^* \leq N_{\text{real}} y^* + R^c, \\
& \quad Tx^* \leq 1, \\
& \quad R^c, R^d \geq 0,
\end{align*}
\]

In the above model (12), \(R^c, R^d\) are decision variables that determine the violation of chance constraints in each phase under realizations.

Table 5 shows the results of the first proposed model under confidence level 0.9 compared to 0.8 and 0.7. These results indicate the high level of confidence due to risk-averseness of the results, low violation of constraints and consequently high operating costs. In fact, increasing amounts of confidence level \(\alpha\) lead to the further realization of constraints and reduction of the solution space, which will in turn worsen the optimal solution. On the other hand, in the first model, we get a lower standard deviation with regard to confidence level 0.8. In addition, in the robust models (models 8 and 10), the second one (10) has a lower standard deviation. It means that the second robust model (10) has been more conservative and risk-aversive.
Finally, it is worthy to mention that based on presented output results and comments of company decision-makers it could be noted that the extended model is a reliable tool for long-term planning of supply chains. Also, it could be applied in other industries by some minor revisions.

6. Conclusion

The proposed model based on case study of equipping Iranian training centers is used to concurrently optimize the design of the forward and reverse supply chain networks and also effectively provides a highly responsive and cost-effective network. Due to the uncertainty of input parameters, a robust possibilistic chance-constrained programming model is developed to effectively control uncertainty of parameters. Notably, the extended model is capable of optimizing satisfaction level of uncertain parameters on the basis of risk-averseness of company decision-makers. Accordingly, two robust possibilistic programming models with strict compensation structure and controllable optimality and feasibility robustness coefficients were introduced. The generalized Epsilon-constraint method is applied to optimize multi-objective programming model that is capable of producing effective conflicting solutions. Given that the robust models in this study were designed to be implemented in a national project, the effectiveness and quality of the model are tested based on real data extracted from a case study.

Notably, in robust models, we deal with linear stable models which allow for automatic adjustment of confidence levels according to the extent of parametric uncertainty. In fact, given the importance and sensitivity of the national project and the parametric uncertainty, the decision-maker can achieve the desired confidence levels by controlling the feasibility and optimality robustness coefficients. The results obtained by solving the extended models show that the objective function average in the chance-constrained possibilistic programming model approach is lower than that in the robust models. However, if we consider that the national project should be implemented at higher risk-aversion levels, the third robust model would be more suitable.

The suggestions for the further research studies are provided as follows:

- The multi-objective closed-loop supply chain problem solving under uncertainty conditions lies in the category of NP-HARD problems. Accordingly, the problem solving time increases exponentially in accordance with the problem dimensions. Therefore, it’s better to apply a heuristic or metaheuristic algorithm to reduce the problem solving time and compare the obtained results with the results of the detailed method used in this study.

- Some other criterions such as social evaluation indicators can be considered as second objective function in such problems in order to optimize the model aside with regarding economic aspects.

### Table 5. Performance of the proposed models under realizations

| No of realization | Basic possibilistic chance-constrained programming model | Model 8 | Model 10 |
|-------------------|--------------------------------------------------------|--------|---------|
|                   | α = β = 0.7 | α = β = 0.8 | α = β = 0.9 |         |
| 1                 | 35,347,245,231 | 36,522,198,583 | 36,677,483,285 | 90,417,256,714 | 93,528,580,213 |
| 2                 | 34,976,213,329 | 35,842,633,764 | 36,121,237,620 | 89,437,876,763 | 92,691,264,845 |
| 3                 | 35,614,713,628 | 36,114,645,512 | 36,472,172,295 | 92,946,214,237 | 97,756,990,236 |
| 4                 | 36,234,788,541 | 36,934,239,107 | 37,361,546,312 | 94,817,496,582 | 98,478,347,108 |
| 5                 | 36,634,778,675 | 39,712,692,431 | 41,184,371,523 | 95,176,623,748 | 103,649,427,218 |
| Average           | 35,761,547,881 | 37,025,281,879 | 37,563,362,211 | 92,559,093,609 | 97,220,921,924 |
| SD                | 950,241,156   | 425,388,984   | 1,075,365,457  | 1,973,726,443  | 1,752,569,844  |
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