STIRAP schemes for atomic Bose-Einstein condensates

Andreas M. D. Thomasen* and Tim Byrnes**

*,** New York University, 1555 Century Avenue, Pudong, Shanghai 200122, China
** National Institute of Informatics, 2-1-2 Hitotsubashi, Chiyoda-ku, Tokyo 101-8430, Japan
** NYU-ECNU Institute of Physics at NYU Shanghai, 3663 Zhongshan Road North, Shanghai 200062, China
E-mail: andreas.thomasen@nyu.edu

Abstract. In coherent control of Bose-Einstein condensates (BEC) the major limitation to successful high fidelity transfer using optical Raman transitions is spontaneous emission of radiation and the decoherence that this causes. We present a scheme based on stimulated Raman adiabatic passage (STIRAP) designed specifically for Rubidium 87. STIRAP is a method of population transfer that relies on the adiabatic theorem of quantum mechanics. Instead of transferring population between Hamiltonian ground states, one adiabatically evolves the ground states into some desired superposition. We show that one may thereby implement arbitrary rotations of spinor BECs with extremely high fidelities. Our simulations show that given a BEC with $N = 10^4$ atoms, we may do an arbitrary unitary rotation with an infidelity of about $10^{-7}$ that takes approximately 100 ns.

1. Introduction
Two component Bose-Einstein condensates may exist in coherent states of matter that are accurately described by

$$|\alpha, \beta\rangle \equiv \frac{1}{\sqrt{N!}} \left( \alpha a^\dagger + \beta b^\dagger \right)^N |0\rangle. \quad (1)$$

Here $a^\dagger$ and $b^\dagger$ create an atom in the BEC in some internal orthogonal states that are labeled $a$ and $b$ respectively. We propose that this type of state may be accurately controlled by a stimulated Raman adiabatic passage (STIRAP). Furthermore, we do so in the presence of bosonically enhanced spontaneous emission. Similar schemes have been proposed before such as [1, 2], and we base our ideas partly on them.

The main advantage of using STIRAP is that one avoids populating excited internal states of the BEC constituent atoms and thus limits decoherence caused by spontaneous emission [3, 4].

1.1. STIRAP
STIRAP is a thoroughly studied and well-understood method of population transfer in atoms and molecules [5, 6, 7]. A time-dependent Hamiltonian has some set of instantaneous eigenstates $|d_1(t)\rangle, |d_2(t)\rangle, \ldots, |d_n(t)\rangle$. A general quantum state may then be written as

$$|\psi(t)\rangle = \sum_j c_j(t) |d_j(t)\rangle. \quad (2)$$
In the adiabatic approximation each of the $c_j(t) \approx c_j(0)$.

In STIRAP, one typically defines dark states as the eigenstates that contain only stable electronic states, i.e. states that do not spontaneously emit radiation. The ones that do are termed bright states. Consider a superposition of two dark states as the initial state of some atomic system, then an elementary STIRAP procedure is one in which the two dark states replace each other. I.e.

$$|\psi(t)\rangle = c_1(0)|d_1(t)\rangle + c_2(0)|d_2(t)\rangle$$

$$\rightarrow c_1(0)|d_2(t)\rangle + c_2(0)|d_1(t)\rangle.$$

This is typically accomplished by a so-called counter-intuitive sequence of laser pulses, where a Stokes pulse precedes a pump pulse. Thus no Rabi transition occurs and only dark states are ever populated in the adiabatic limit.

As an example of how this occurs, consider the Hamiltonian

$$\hat{H}/\hbar = [\Omega_a(t)\hat{e}^\dagger\hat{a} + \Omega_b(t)\hat{e}^\dagger\hat{b} + \text{H.c.}] - \Delta/2\hat{a}^\dagger\hat{a} - \Delta/2\hat{b}^\dagger\hat{b} + \Delta/2\hat{e}^\dagger\hat{e}.$$  (4)

Here the dark states may be created by the operator

$$\frac{\Omega_a}{\cos \theta(t)} = \frac{\Omega_b}{\sin \theta(t)}, \quad 0 \leq \theta(t) \leq \pi/2.$$  (6)

Then

$$\hat{d}_0^\dagger = \cos \theta(t)\hat{a}^\dagger - \sin \theta(t)\hat{b}^\dagger.$$  (7)

Now the mixing angle defines some adiabatic path taken by the eigenstate, which may transfer population completely from $a$ to $b$ or $b$ to $a$, as well as ending the transfer in some superposition of the two (fractional STIRAP).

1.2. Internal state manifolds of $^{87}$Rb

We consider specifically $^{87}$Rb BECs. Here the $5^2S_{1/2}$ and $5^2P_{1/2}$ manifolds of hyperfine states are the most relevant optically addressable states. In the rotating frame we may write a Hamiltonian as

$$\hat{H}/\hbar = \left[\Omega_a(t)(\hat{e}_1^\dagger + \hat{e}_2^\dagger)\hat{a} + \Omega_b(t)(\hat{e}_1^\dagger - \hat{e}_2^\dagger)\hat{b} + \Omega_{c_1}\hat{e}_1\hat{c}_1 + \Omega_{c_2}\hat{e}_2\hat{c}_2 + \text{H.c.}\right]$$

$$+ \Delta_1\hat{e}_1\hat{e}_1 + \Delta_2\hat{e}_2\hat{e}_2.$$  (8)

Here $\hat{a}^\dagger$, $\hat{b}^\dagger$, $\hat{e}_1^\dagger$ and $\hat{e}_2^\dagger$ create an atom in the hyperfine ground states $|F = 1, m_F = -1\rangle$, $|F = 2, m_F = 1\rangle$, $|F = 2, m_F = 0\rangle$ and $|F = 1, m_F = 0\rangle$ respectively of the $5^2S_{1/2}$ manifold. The excited states $|F' = 1, m'_F = 0\rangle$ and $|F' = 2, m'_F = 0\rangle$ of the $5^2P_{1/2}$ manifold are created by $\hat{e}_1^\dagger$ and $\hat{e}_2^\dagger$ respectively.

In writing up this Hamiltonian we have used a rotating frame, where some of the off-diagonal elements have terms that rotate with a frequency given by the hyperfine splitting of the ground state. This is approximately 6.8 GHz, which is much faster than the timescales we consider. These terms therefore average to zero approximately.
Figure 1. Here is a level scheme showing the internal electronic levels of Rubidium 87 (a). Also indicated are the STIRAP pulses and their relative timing in our scheme (b). Due to Klebsch Gordan coefficients specific to the D1 transition of $^{87}$Rb the laser amplitudes $\Omega_a$ and $\Omega_b$ cause transitions to $e_1$ and $e_2$ to occur with different phases.

Equation (8) has the following dark states creation operators.

$$\hat{d}_1^{(1)} = \frac{\Omega_{c_1} \Omega_{c_2} \hat{c}^\dagger_1 - \Omega_{c_1} \Omega_{c_2} \hat{c}^\dagger_2 - \Omega_{c_1} \Omega_{c_2} \hat{c}^\dagger_2}{\sqrt{|\Omega_{c_1}|^2 |\Omega_{c_2}|^2 + |\Omega_{c_1}|^2 (|\Omega_{c_1}|^2 + |\Omega_{c_2}|^2)}}$$  (9)

$$\hat{d}_2^{(1)} = \frac{\Omega_{c_1} \Omega_{c_2} \hat{c}^\dagger_1 - \Omega_{c_1} \Omega_{c_2} \hat{c}^\dagger_2 + \Omega_{c_2} \Omega_{c_1} \hat{c}^\dagger_1 + \Omega_{c_2} \Omega_{c_1} \hat{c}^\dagger_2}{\sqrt{|\Omega_{c_1}|^2 |\Omega_{c_2}|^2 + |\Omega_{c_1}|^2 (|\Omega_{c_1}|^2 + |\Omega_{c_2}|^2)}}$$  (10)

We also define an equivalent dark state basis for later convenience. It is defined by the creation operators

$$\hat{d}_1^{(2)} = \frac{-\Omega_{c_1} \Omega_{c_2} \hat{c}^\dagger_1 - \Omega_{c_1} \Omega_{c_2} \hat{c}^\dagger_2 + \Omega_{c_2} \Omega_{c_1} \hat{c}^\dagger_1 + \Omega_{c_2} \Omega_{c_1} \hat{c}^\dagger_2}{\sqrt{|\Omega_{c_1}|^2 |\Omega_{c_2}|^2 + |\Omega_{c_1}|^2 (|\Omega_{c_1}|^2 + |\Omega_{c_2}|^2)}}$$  (11)

$$\hat{d}_2^{(2)} = \frac{-\Omega_{c_1} \Omega_{c_2} \hat{c}^\dagger_1 - \Omega_{c_1} \Omega_{c_2} \hat{c}^\dagger_2 + \Omega_{c_2} \Omega_{c_1} \hat{c}^\dagger_1 + \Omega_{c_2} \Omega_{c_1} \hat{c}^\dagger_2}{\sqrt{|\Omega_{c_1}|^2 |\Omega_{c_2}|^2 + |\Omega_{c_1}|^2 (|\Omega_{c_1}|^2 + |\Omega_{c_2}|^2)}}$$  (12)

2. Arbitrary rotation of coherent spinors

We now may introduce a scheme that uses STIRAP for arbitrary rotation of BECs. This scheme will consist of two STIRAPs, where the first one takes the condensate into a coherent spinor composed of the intermediate $\hat{c}^\dagger_1$ and $\hat{c}^\dagger_2$ states. After this the second STIRAP will take the
condensate back into the original subspace. This scheme is heavily inspired by Ref. [2]. In summary

\[
|\alpha_0, \beta_0\rangle = \frac{1}{\sqrt{N!}}(\alpha_0 a^\dagger + \beta_0 b^\dagger)^N |0\rangle
\]  

\[
\rightarrow \frac{1}{\sqrt{N!}}(\alpha_1 c_1^\dagger + \beta_1 c_2^\dagger)^N |0\rangle
\]  

\[
\rightarrow \frac{1}{\sqrt{N!}}(\alpha_2 d^\dagger + \beta_2 f^\dagger)^N |0\rangle.
\]  

We may define the total transformation as an SU(2) rotation. Then

\[
\begin{bmatrix}
\alpha_2 \\
\beta_2
\end{bmatrix} = U \begin{bmatrix}
\alpha_0 \\
\beta_0
\end{bmatrix}
\]  

The main result of this work is that we have found a way to perform arbitrary rotations. It is well known that a general SU(2) rotation may be written as

\[
U = e^{i\alpha} \begin{bmatrix}
\cos \frac{\gamma}{2} & -e^{i\beta/2}\sin \frac{\gamma}{2} \\
e^{i\beta/2}\sin \frac{\gamma}{2} & \cos \frac{\gamma}{2}
\end{bmatrix}.
\]  

We now prove that this is possible using a pair of STIRAPs on the internal state manifold of \(^{87}\)Rb. Firstly, we define a unimodal function \(f(t)\) whose maximum is \(f(0) = 1\). In STIRAP this function is usually taken to be Gaussian. We now define \(T_\pm\) as the time difference between a Stokes and pump pulse and \(\Delta T\) the time difference between the two STIRAP procedures. We then define explicitly the pulses

\[
\Omega_a^{(1)} = e^{i\theta_a^{(1)}} f(t - T_- / 2 + T_+ / 2), \quad \Omega_a^{(2)} = e^{i\theta_a^{(2)}} f(t + T_- / 2 - T_+ / 2)
\]  

\[
\Omega_b^{(1)} = e^{i\theta_b^{(1)}} f(t - T_- / 2 + T_+ / 2), \quad \Omega_b^{(2)} = e^{i\theta_b^{(2)}} f(t + T_- / 2 - T_+ / 2)
\]  

\[
\Omega_c^{(1)} = e^{i\theta_c^{(1)}} f(t - T_- / 2 + T_+ / 2), \quad \Omega_c^{(2)} = e^{i\theta_c^{(2)}} f(t - T_- / 2 - T_+ / 2)
\]  

\[
\Omega_d^{(1)} = e^{i\theta_d^{(1)}} f(t + T_- / 2 + T_+ / 2), \quad \Omega_d^{(2)} = e^{i\theta_d^{(2)}} f(t + T_- / 2 - T_+ / 2).
\]  

Here the \(\Omega_a^{(1,2)}\) are the pulses belonging to the first and second STIRAP respectively.

The dark states have the following limits in the first STIRAP.

\[
\tilde{d}_1^{(1)} \rightarrow \begin{cases}
\frac{e^{i(\theta_a^{(1)} + \theta_c^{(1)})} a^\dagger}{\sqrt{2}} & \text{for } t \rightarrow -\infty \\
\frac{-e^{-i(\theta_a^{(1)} + \theta_c^{(1)})} c_1^\dagger - e^{-i(\theta_a^{(1)} + \theta_c^{(1)})} c_2^\dagger}{\sqrt{2}} & \text{for } t \rightarrow \infty
\end{cases}
\]  

\[
\tilde{d}_2^{(1)} \rightarrow \begin{cases}
\frac{e^{i(\theta_b^{(1)} + \theta_d^{(1)})} b^\dagger}{\sqrt{2}} & \text{for } t \rightarrow -\infty \\
\frac{-e^{-i(\theta_b^{(1)} + \theta_d^{(1)})} d_1^\dagger + e^{-i(\theta_b^{(1)} + \theta_d^{(1)})} d_2^\dagger}{\sqrt{2}} & \text{for } t \rightarrow \infty
\end{cases}
\]  

In the second STIRAP we have

\[
\tilde{d}_1^{(2)} \rightarrow \begin{cases}
\frac{e^{i(\theta_a^{(2)} + \theta_c^{(2)})} a^\dagger}{\sqrt{2}} & \text{for } t \rightarrow -\infty \\
\frac{-e^{-i(\theta_a^{(2)} + \theta_c^{(2)})} c_2^\dagger}{\sqrt{2}} & \text{for } t \rightarrow \infty
\end{cases}
\]  

\[
\tilde{d}_2^{(2)} \rightarrow \begin{cases}
\frac{e^{i(\theta_b^{(2)} + \theta_d^{(2)})} b^\dagger}{\sqrt{2}} & \text{for } t \rightarrow -\infty \\
\frac{-e^{-i(\theta_b^{(2)} + \theta_d^{(2)})} d_1^\dagger + e^{-i(\theta_b^{(2)} + \theta_d^{(2)})} d_2^\dagger}{\sqrt{2}} & \text{for } t \rightarrow \infty
\end{cases}
\]
Figure 2. (Color online). This figure shows an interpretation of the overall scheme as a pair of complete transitions back and forth between two Bloch spheres. The rotation $U$ is the net effect of these two transitions, each of which contribute to $U$ with the partial rotations $\mathcal{E}^{(1,2)}$. The wavy magenta arrows represent the action of the actual lasers in the STIRAP scheme, which is to make a transition happen from one Bloch sphere to another.

We now examine the first STIRAP given some initial state $|\alpha_0, \beta_0\rangle$. 

\[ |\alpha_0, \beta_0\rangle = \frac{1}{\sqrt{N!}} \left( \alpha_0 e^{-i(\theta_1^{(1)}+\theta_1^{(2)})} d_1^{(1)\dagger} + \beta_0 e^{-i(\theta_1^{(1)}+\theta_2^{(1)})} d_2^{(1)\dagger} \right)^N |0\rangle \]

\[ = \frac{1}{\sqrt{N!}} \left( \alpha_0 e^{-i(\theta_1^{(1)}+\theta_1^{(2)})} d_1^{(1)\dagger} e^{i(\theta_1^{(1)}+\theta_1^{(2)})} c_1^{\dagger} + \beta_0 e^{-i(\theta_1^{(1)}+\theta_2^{(1)})} d_2^{(1)\dagger} e^{i(\theta_1^{(1)}+\theta_2^{(1)})} c_2^{\dagger} \right)^N |0\rangle \]

\[ = |\alpha_1, \beta_1\rangle \]  

(24)  

(25)  

(26)

Here the coherent spinor is with respect to the intermediate states $c_1$ and $c_2$. Rearranging the terms of equation (26) according to equation (15), we obtain a matrix transformation between $[\alpha_0, \beta_0]^T$ and $[\alpha_1, \beta_1]^T$ given by

\[ \mathcal{E}^{(1)} = \frac{1}{\sqrt{2}} \begin{bmatrix} -e^{i(\theta_1^{(1)}-\theta_1^{(2)})} & -e^{i(\theta_1^{(1)}-\theta_1^{(2)})} \\ -e^{i(\theta_2^{(1)}-\theta_2^{(2)})} & e^{i(\theta_2^{(1)}-\theta_2^{(2)})} \end{bmatrix} . \]

(27)

Now performing the second STIRAP procedure, the intermediate states move back into the
original storage basis again.

\[
|\alpha_1, \beta_1\rangle = \frac{1}{\sqrt{N!}} \left( \alpha_1 e^{-i(\theta_1^{(2)}+\theta_0^{(2)})} a_1^{\dagger} + \beta_1 e^{-i(\theta_0^{(2)}+\theta_2^{(2)})} a_2^{\dagger} \right)^N |0\rangle
\]

\[
N = \frac{1}{\sqrt{2}} \left( \alpha_1 e^{-i(\theta_1^{(2)}+\theta_0^{(2)})} e^{i(\theta_1^{(2)}+\theta_0^{(2)})} a_1^{\dagger} - e^{i(\theta_2^{(2)}+\theta_0^{(2)})} b_1^{\dagger} a_1^{\dagger} + e^{-i(\theta_2^{(2)}+\theta_0^{(2)})} b_2^{\dagger} a_1^{\dagger} \right)^N |0\rangle
\]

\[
= |\alpha_2, \beta_2\rangle.
\]

Here the matrix that transforms $[\alpha_1, \beta_1]^{T}$ into $[\alpha_2, \beta_2]^{T}$ is given by

\[
\mathcal{A}^{(2)} = \frac{1}{\sqrt{2}} \begin{bmatrix}
-e^{i(\theta_1^{(2)}-\theta_0^{(2)})} & -e^{i(\theta_2^{(2)}-\theta_0^{(2)})} \\
-e^{-i(\theta_1^{(2)}-\theta_0^{(2)})} & e^{i(\theta_2^{(2)}-\theta_0^{(2)})}
\end{bmatrix}.
\]

We may now construct the total operation on the BEC as

\[
U = \mathcal{A}^{(2)} \mathcal{A}^{(1)}.
\]

By choosing laser phase angles $\theta_1^{(1)} = \theta_2^{(1)} = -\alpha/2$, $\theta_1^{(2)} = \alpha/2 + \gamma/2$ and $\theta_2^{(2)} = \alpha/2 - \gamma/2$, $\theta_0^{(1)} = -\delta/2$, $\theta_0^{(2)} = \delta/2 + \pi/2$, we obtain the arbitrary SU(2) rotation of equation (18).

Spin coherent states may be given a Bloch sphere representation by using the Schwinger Boson operators. They are

\[
\hat{S}_x = \hat{a}^{\dagger} \hat{b} + \hat{b}^{\dagger} \hat{a}, \quad \langle \hat{S}_x \rangle = N (\alpha^* \beta + \beta^* \alpha)
\]

\[
\hat{S}_y = -i \hat{a}^{\dagger} \hat{b} + i \hat{b}^{\dagger} \hat{a}, \quad \langle \hat{S}_y \rangle = N (-i \alpha^* \beta + i \beta^* \alpha)
\]

\[
\hat{S}_z = \hat{a}^{\dagger} \hat{a} - \hat{b}^{\dagger} \hat{b}, \quad \langle \hat{S}_z \rangle = N (|\alpha|^2 - |\beta|^2).
\]

We may similarly define $\hat{J}_x = \hat{c}_2^{\dagger} \hat{c}_2 + \hat{c}_1^{\dagger} \hat{c}_1$, etc. Then an interpretation of the two STIRAP procedures as jumps from one Bloch sphere to another may be visualized as in figure 2.

3. Simulations

We have carried out a simulation in which

\[
f(t) = e^{-it^2/T^2}.
\]

This has been done for various values of $N$, $\Omega_0$, $t_{\text{end}}$, where $t_{\text{end}} = 8 T_- = 8 T_+/3$. In each case, for a given set of those parameters, we have done the simulation in two steps. The first is an optimization procedure based on golden section search, in which an optimum value of the pulse width $T$ is determined. In the second step, the simulation is carried out 240 times with this optimum $T$. Each time the initial state of the condensate is randomized in terms of $\alpha_0$ and $\beta_0$. The angles of rotation in equation (18) are randomized as well. Each time a simulation finishes, the resulting final state $\rho_{\text{final}}$ is compared to the target state $\rho_{\text{target}}$ that one would have gotten simply by applying the unitary directly. The trace distance between the two is recorded and after all 240 simulations have been carried out, their average is saved for that combination of $N$, $\Omega_0$, $t_{\text{end}}$. We note that in our simulation the master equation is procedurally evaluated from $t = 0$ to $t = t_{\text{end}}$. We also note that both $T_-$ and $T_+$ are fixed with respect to $t_{\text{end}}$. 


3.1. Master equation

We use the master equation

\[
\frac{d\rho}{dt} = \frac{i}{\hbar} [\rho, \hat{H}] - \frac{\Gamma}{2} (|e_1^i e_1^i + |p e_1^i e_1^i|) - \frac{\Gamma}{2} (|e_2^i e_2^i + |p e_2^i e_2^i|).
\]

Here \( \Gamma \) is the rate of spontaneous decay of the \( 5^2P_{3/2} \) excited state. We do not directly time-evolve the entire density matrix, as the size of the Fock-state space grows very fast with the number of particles, especially since we consider six interacting hyperfine states. Generally the number of Fock states equals the number of terms in an \( N \)'th order hexanomial expansion, which for all practical purposes is impossible to simulate in a reasonable amount of time with \( N > 1000 \).

We instead evaluate the expectation values of bosonic compound operators using the above master equation, i.e. \( \langle \hat{a}^\dagger \hat{a} \rangle, \langle \hat{a}^\dagger \hat{b} \rangle, \ldots, \langle \hat{e}_2^i \hat{e}_2^i \rangle \). These form a self-consistent set of 16 coupled differential equations regardless of \( N \). We have used a mean-field approximation, where terms such as \( \langle \hat{a}^\dagger \hat{a} \hat{b}^\dagger \hat{b} \rangle \approx \langle \hat{a}^\dagger \hat{a} \rangle \langle \hat{b}^\dagger \hat{b} \rangle \). This approximation works well for spin coherent states, as the variance of spin operators diminishes with \( 1/N \), while the right hand graphs have a vanishing overlap which depends exponentially on \( N \).

After deriving all relevant differential equations, we numerically solve it by using an ordinary differential equation (ODE) solver.

3.2. Evaluating fidelity

We use the trace distance as a measure of infidelity. We define the trace distance between two quantum states \( \rho \) and \( \sigma \) as

\[
D(\rho, \sigma) = \left[ (\hat{S}_x)_\rho - (\hat{S}_x)_\sigma \right]^2 + \left[ (\hat{S}_y)_\rho - (\hat{S}_y)_\sigma \right]^2 + \left[ (\hat{S}_z)_\rho - (\hat{S}_z)_\sigma \right]^2 / (4N^2).
\]

Then \( F(\rho_{\text{target}}, \rho_{\text{final}}) \equiv 1 - D(\rho_{\text{target}}, \rho_{\text{final}}) \). Our justification for this choice is that spin coherent states have a vanishing overlap which depends exponentially on \( N \). This is given by

\[
\left| \left\langle \cos \frac{\theta}{2} e^{i\phi} \sin \frac{\theta}{2} \left| \cos \frac{\theta}{2} e^{i\phi} \sin \frac{\theta}{2} \right\rangle \right|^2 = \cos^2 \left( \frac{\theta - \theta'}{2} \right)
\approx e^{-\frac{N(\theta' - \theta)^2}{4}}.
\]

Although the two spin coherent states may be very close together on the Bloch sphere, the overlap still vanishes. If a conventional approach was used where the fidelity is defined in terms of density matrix products, such expressions will inevitably appear.

3.3. Example Simulation

Figure 3 shows the time-evolution of two different spin coherent states. In the example shown, the detuning has been set to \( \Delta_+ = 0 \). The two graphs on the left show the time-evolution of a BEC with \( N = 10^4 \), while the right hand graphs have \( N = 10^5 \). We see that the BEC arrives at its target state in graph (a), whereas graph (b) shows some infidelity. This is because when the average detuning \( \Delta_+ \equiv (\Delta_2 - \Delta_1)/2 = 0 \). When lasers are near resonance, transitions to excited states are much more likely. Consequently we see a dependence on \( N \) emerge as superradiance becomes the main contributor to decoherence. This trend is mirrored in graphs (c) and (d). After the full procedure, no population is left in the intermediate states in (c), whereas in (d) some finite amount is left over. This shows directly that significant spontaneous emission has taken place.

This example shows that the counterintuitive pulse ordering of STIRAP effectively prevents excited states from being populated and thus overcomes enhanced spontaneous emission.
enhanced spontaneous emission. In the following simulations we shall include higher values of 
therefore be transitions to unwanted states. Therefore the above results should only be thought
included in the Hamiltonian of equation (8). In a situation with low detuning, there will generally
\begin{equation}
\tau = 27(4)\text{\;ns. The simulation ends after one t/\tau.}\end{equation}
rotation of a BEC. The simulations have been carried out using data recorded for the D1
transition in [9], and the basic time unit is the decay time of the \(^{87}\text{Rb}\) D1 transition, i.e.
\begin{equation}
\tau = 27.0(4)\text{\;ns. The simulation ends after one t/\tau. All plots are for the case where the initial spin coherent state is |cos 3\pi/8, sin 3\pi/8e^{i5\pi/8}\rangle and the target rotation is the operation U = R_z(\pi/4)R_y(3\pi/4)R_z(\pi/4). The target state after the rotation is indicated with the horizontal lines marked as \langle \hat{S}_{x,y,z}\rangle_{\text{targ}}. The pulses have a Gaussian shape, with T_e = t_{\text{end}}/8, T_+ = 3t_{\text{end}}/8. After pulse width optimization we obtain for (a) (c) N = 10^4 a width of T = 0.1324t_{\text{end}} (FWHM = 0.1835t_{\text{end}}) and for (b) (d) N = 10^5 we obtain T = 0.0877t_{\text{end}} (FWHM = 0.1216t_{\text{end}}). Subfigures (a) (b) show the the evolution of the expectation values of the Schwinger boson operators, \langle \hat{S}_{x,y,z}\rangle. Subfigures (c) (d) depict the occupation of intermediate and excited states.

However, in general there are more excited states in the \(5^2P_{1/2}\) manifold of states than we’ve included in the Hamiltonian of equation (8). In a situation with low detuning, there will generally therefore be transitions to unwanted states. Therefore the above results should only be thought of as illustrative of the STIRAP procedure and the dependence of the fidelity on N due to
enhanced spontaneous emission. In the following simulations we shall include higher values of \(\Delta_+\) to avoid these transitions entirely.
there is a trade-off between overall decrease in fidelity with increasing numbers of particles. Thus we find in general that $N$ are eight data-series in total with overcome the problem of spontaneous emission even very close to resonance.

The expectation values of section 3.1 is procedurally evaluated from as in figure 3, but with randomized initial states and unitary operations. The set of coupled equations (34) is obtained by a golden section search procedure. We note also that in each case, the detuning has been set to $\Delta_+ = 0$. This is to support our claim that STIRAP really does overcome the problem of spontaneous emission even very close to resonance.

On figure 3.4, which plots the infidelity vs $t_{end}$, several data-series have been plotted. There are eight data-series in total with $N = 10^3, 10^4, 10^5, 10^6$ and $\Omega_0 = 10^2 \Gamma, 10^3 \Gamma$. Each data-series shows, as an overall trend, that allowing a longer $t_{end}$ increases the fidelity. There is also an overall decrease in fidelity with increasing numbers of particles. Thus we find in general that there is a trade-off between $N$ and $t_{end}$ for some minimum required $F$. Most of the data-series with $\Omega_0 = 10^2 \Gamma$ have infidelities close enough to 1 that most practical applications are difficult to justify. When $N = 10^3$ we do observe that infidelities go to below $10^{-4}$ for adequately long $t_{end}$. More interesting are the results for $\Omega_0 = 10^3 \Gamma$. Here we obtain infidelities below $10^{-4}$ for any number of atoms we have tested except $N = 10^6$. Here we even see infidelities as low as $10^{-8}$. We note that the two data-series for $N = 10^3$ and $N = 10^4$ coincide. This is because in this regime, the STIRAP procedure is so efficient that virtually no excited state population occurs. The main contributor to decoherence in this case is incomplete STIRAP processes, where the condensate does not reach the exact target state. To verify this claim, we have reevaluated these two data-series with spontaneous emission turned off and obtained data that coincides with these two series.

Figure 3.4 shows our main results for a range of different values of the parameters $N$, $\Omega_0$, and $t_{end}$. For any given set of parameters the time-evolution is carried out 240 times, each time as in figure 3, and saved for that particular configuration of $N$, $\Omega_0$, and $t_{end}$. This process is for each set of parameters preceded by an optimization step in which the optimum width of the pulses $T$ (see equation (34)) is obtained by a golden section search procedure. We note also that in each case, the detuning has been set to $\Delta_+ = 0$. This is to support our claim that STIRAP really does overcome the problem of spontaneous emission even very close to resonance.

Figure 4. (Color online). Here the trace distance $D$ is plotted as a function of $t_{end}$. This is done for eight data-series with different combinations of the parameters $N = 10^3, 10^4, 10^5, 10^6$ and $\Omega_0 = 10^2 \Gamma, 10^3 \Gamma$. For each combination of these values a datapoint has been computed at each $t_{end}$ by averaging over the results of 240 randomized rotations and initial states.

3.4. Main results

Figure 3.4 shows our main results for a range of different values of the parameters $N$, $\Omega_0$, and $t_{end}$. For any given set of parameters the time-evolution is carried out 240 times, each time as in figure 3, but with randomized initial states and unitary operations. The set of coupled expectation values of section 3.1 is procedurally evaluated from $t = 0$ to $t = t_{end}$ using an ODE solver. After the final time-step of the evolution has been carried out using this method, we characterize the state by evaluating the expectation values of equation (33). The trace distance is then taken to the target state, and all of the 240 values thus obtained are averaged over and saved for that particular configuration of $N$, $\Omega_0$, and $t_{end}$. This process is for each set of parameters preceded by an optimization step in which the optimum width of the pulses $T$ (see equation (34)) is obtained by a golden section search procedure. We note also that in each case, the detuning has been set to $\Delta_+ = 0$. This is to support our claim that STIRAP really does overcome the problem of spontaneous emission even very close to resonance.

On figure 4.4, which plots the infidelity vs $t_{end}$, several data-series have been plotted. There are eight data-series in total with $N = 10^3, 10^4, 10^5, 10^6$ and $\Omega_0 = 10^2 \Gamma, 10^3 \Gamma$. Each data-series shows, as an overall trend, that allowing a longer $t_{end}$ increases the fidelity. There is also an overall decrease in fidelity with increasing numbers of particles. Thus we find in general that there is a trade-off between $N$ and $t_{end}$ for some minimum required $F$. Most of the data-series with $\Omega_0 = 10^2 \Gamma$ have infidelities close enough to 1 that most practical applications are difficult to justify. When $N = 10^3$ we do observe that infidelities go to below $10^{-4}$ for adequately long $t_{end}$. More interesting are the results for $\Omega_0 = 10^3 \Gamma$. Here we obtain infidelities below $10^{-4}$ for any number of atoms we have tested except $N = 10^6$. Here we even see infidelities as low as $10^{-8}$. We note that the two data-series for $N = 10^3$ and $N = 10^4$ coincide. This is because in this regime, the STIRAP procedure is so efficient that virtually no excited state population occurs. The main contributor to decoherence in this case is incomplete STIRAP processes, where the condensate does not reach the exact target state. To verify this claim, we have reevaluated these two data-series with spontaneous emission turned off and obtained data that coincides with these two series.

4. Conclusion and outlook

We have here proposed a novel scheme for population transfer in $^{87}$Rb BECs which allows for the implementation of arbitrary SU(2) rotations. We have done so using optical pulses and
STIRAP, whereby we successfully mitigate decoherence caused by spontaneous emission. This has been confirmed by our results which show $10^{-7}$ infidelities for arbitrary rotations that take even less than 100 ns.

Potential applications include quantum metrology schemes that rely on extremely accurate preparation of BECs. In quantum magnetometry for instance [10], a BEC is typically prepared in some coherent superposition of magnetic sublevels, whose Larmor precession reveals the strength of some applied magnetic field. Conventionally experimenters here use radiofrequency and microwave pulses to create these desired superpositions. Here our method offers a very high fidelity alternative, that have a number of practical advantages. First and foremost, there appears to be no intrinsic limit to the fidelities, but rather a trade-off between fidelity and speed. Thus, the fidelity gained may be orders of magnitude closer to unity. Secondly, we find that our method is extremely fast, allowing full rotations to occur in less than 100 ns. As a second application, one may consider the growing field of BEC quantum information processing [11, 12]. This has several advantages over conventional quantum computing. One of which is the recent discovery, that one may accurately know the quantum state of a BEC with a minimum of disturbance, using phase-contrast imaging [13]. There are also some works which show that certain interactions between BECs are enhanced by the number of particles present, and schemes to produce useful entanglement have been studied theoretically [14]. As already discussed here, this bosonic enhancement is also a drawback, as it produces increased decoherence rates, but we have successfully demonstrated here that they are no longer an issue for single qubit gates based on STIRAP. As our scheme allows us to use optical pulses, the lasers may be tightly focused thus allowing several BECs to exist in close vicinity to each other. This enables scalable configurations where each spinor BEC can be addressed individually on e.g. an atom chip or similar device [8].

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References
[1] Issoufia Y, Messikh A, Wahiddin M, Umarov B and Gharib M 2013 Information and Communication Technology for the Muslim World (ICT4M) pp 1–6
[2] Kis Z and Renzoni F 2002 Phys. Rev. A 65(3) 032318
[3] Ivanov P A, Vitanov N V and Bergmann K 2005 Phys. Rev. A 72(5) 053412
[4] Görlitz A, Chikkatur A and Ketterle W 2001 Physical Review A 63 041601
[5] Petrosyan D and Mølmer K 2013 Physical Review A 87 033416
[6] Møller D, Madsen L B and Mølmer K 2007 Physical Review A 75 062302
[7] Kis Z and Stenholm S 2002 Journal of Modern Optics 49 111–124
[8] Pyrkov A N and Byrnes T 2013 New Journal of Physics 15 093019
[9] Steck D A 2001 Rubidium 87 d line data
[10] Vengalattore M, Higbie J, Leslie S, Guzman J, Sadler L and Stamper-Kurn D 2007 Physical review letters 98 200801
[11] Byrnes T, Wen K and Yamamoto Y 2012 Phys. Rev. A 85(4) 040306
[12] Byrnes T 2012 World Acad. Sci. Eng. Technol 63 542
[13] Ilo-Okeke E O and Byrnes T 2014 Physical review letters 112 233602
[14] Idlas S, Domenzain L, Spreeuw R and Byrnes T 2016 Physical Review A 93 022319