Phenomenological view on baryon-baryon potentials from lattice QCD simulations

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Abstract. A qualitative discussion on the range of the potentials as they result from the phenomenological meson-exchange picture and from lattice simulations by the HAL QCD Collaboration is presented. For the former pion- and/or η-meson exchange are considered together with the scalar-isoscalar component of correlated ππ/KK exchange. It is observed that the intuitive expectation for the behavior of the baryon-baryon potentials for large separations, associated with the exchange of one and/or two pions, does not always match with the potentials extracted from the lattice simulations. Only in cases where pion exchange provides the lowest ranged contribution, like in the NN system, a reasonable qualitative agreement between the phenomenological and the lattice QCD potentials is found for baryon-baryon separations of r ≥ 1 fm. For the ΛN and ΩΩ interactions where isospin conservation rules out one-pion exchange a large mismatch is observed, with the potentials by the HAL QCD Collaboration being much longer ranged and much stronger at large distances as compared to the phenomenological expectation. This casts some doubts on the applicability of using these potentials in few- or many-body systems.

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1 Introduction

The study of the strong interaction as given by the fundamental theory of QCD on the lattice has made significant progress over the last few years, not least due to the availability of high performance computers and improved algorithms. This progress is documented in several review articles, see, e.g., Ref. [1] as far as the masses of light hadrons are concerned, or Refs. [2] and [3-5] with regard to the scattering of two mesons or two baryons, respectively.

However, in recent times there has been also some discord. This concerns in particular baryon-baryon scattering, where there is an ongoing controversy [6-8,9,10] about the applicability of one of the basic tools of lattice QCD (LQCD), namely the Lüscher finite volume formula [11,12] which is commonly used to relate the energy levels obtained in LQCD simulations to two-body phase shifts. In addition, there have been critical remarks on the so-called HAL QCD method [13], suggested in Ref. [7] as a way to circumvent the difficulties with the Lüscher approach. In this method the Bethe-Salpeter wave function is extracted from the lattice simulation and based on it, a (local) potential is constructed which is then utilized to calculate the phase shifts. This method was called into question very recently, see the discussions in Refs. [14,15,16]. Note that there are actually two different methods proposed and employed by the HAL QCD Collaboration, a time-dependent one and the more recent time-independent one [13], also called imaginary-time HAL QCD method.

In the present work we do not dwell into formal aspects and, specifically, we do not add anything directly to those controversies mentioned above. Rather we would like to concentrate on the intuitive and phenomenological side, namely on the potentials themselves as they emerge from the calculations and publications of the HAL QCD Collaboration. Of course, potentials are not observable physical quantities, see e.g. the discussion in the review [17]. Nonetheless, since the days of Yukawa [18], potentials (in configuration space) have played an important role as an intuitive visual guidance for the interpretation of the reaction dynamics. For example, the nucleon-nucleon (NN) interaction, as the most prominent case, is seen as being composed of a long-range part provided by pion exchange, an attractive intermediate-range part that is due to (correlated) ππ exchange and, finally, a repulsive short-range part that is due to vector-meson exchange, specifically the ω [22].

Certainly, this traditional view might have been one of the reasons why the work of the HAL QCD Collaboration focusses prominently on potentials. But do the potentials extracted from the lattice simulations indeed meet the intuitive expectations formulated above? In particular, do they exhibit the features we would anticipate from the meson-exchange dynamics? These are the questions we want to address in the present study. Thereby,
we concentrate on the long range behavior of the potentials, i.e. on the results for baryon-baryon distances $r \geq 1$ fm. Obviously, only for large separations the dynamics can be expected to be simple enough and accessible for an intuitive interpretation.

With that aim in mind we take a closer look at the potentials in the $NN$, $\Sigma N$, $\Omega N$, and $\Omega\Omega$ channels. For several $S$-wave states of those systems lattice simulations by the HAL QCD Collaboration are available, performed at almost physical masses ($m_{\pi} = 146$ MeV). Pertinent results for the potentials can be found Ref. [23] for $\Sigma N (^{3}S_{1}; I = 0)$, in [24] for $\Omega N (^{3}S_{2})$, and in [25] for $\Omega\Omega (^{1}S_{0})$. Like in case of $NN$, pion exchange provides the long-range contribution to the $\Sigma N$ potential, supplemented by correlated $\pi\pi$ ($\sigma$) exchange at somewhat shorter distances. The situation is different for $QN$ and $\Omega\Omega$. Since the isospin of the $Q$ baryon is $I = 0$, pion exchange is not allowed by conservation of isospin and should be strongly suppressed. The contribution with the longest range should be due to $\eta$ exchange. Also correlated $\pi\pi$ exchange should be suppressed because, again for isospin reasons, the $\pi\pi$ state cannot couple directly to the $Q$ but only via $KK$ and/or $\eta\eta$ [20,26]. This is different from the situation in $\Lambda N$ or $\Delta A$, say, where leading-order pion exchange is likewise not possible. However, two-pion exchange can contribute due to the coupling to $\Sigma N$ or $\Sigma\Sigma$ [29]. In case of $Q\Omega$ there is no other baryon with the same strangeness quantum number that would facilitate such a coupling.

In the present paper we evaluate the potentials for the baryon-baryon channels in question based on pion- or $\eta$ exchange or even shorter-ranged $\pi\pi$ exchange in the scalar-isoscalar (i.e. "$\sigma$") channel with coupling strengths taken from microscopic models [20,26]. The resulting potentials are then confronted with the ones that are extracted by the HAL QCD Collaboration from their lattice simulations. As we will see, there is a reasonable qualitative agreement in case of the $\Sigma N$ interaction. However, for the $Q\Omega$ and $\Omega\Omega$ interaction we observe a striking difference. Here the potentials from lattice QCD are much longer ranged and much stronger for large baryon-baryon separations than what one would expect from the meson-exchange picture.

The paper is structured as following: The basic ingredients of our calculation are summarized in Sect. 2. A brief overview of the evaluation of the potential from correlated $\pi\pi/KK$ exchange via dispersion theory is provided in an appendix. Our results are presented in Sect. 3. We first discuss the interactions in the $NN$ and $\Sigma N$ channels where the long-range part is provided by pion exchange. Then we consider the $\Omega N$ and $\Omega\Omega$ interactions where only $\eta$ exchange or even shorter-ranged contributions are possible. The paper closes with concluding remarks. Some technicalities are relegated to an Appendix.

### 2 Ingredients

The potentials resulting from the exchange of a pseudoscalar (ps) meson and a scalar (s) meson between the baryons $B$ and $\Omega^\prime$ for the $^{1}S_{0}$ ($^{3}S_{2}$) partial wave are given by

\begin{align}
V_{ps} &= \frac{1}{3} f_{BB\Omega} f_{B \bar{B} \bar{\Omega}} \frac{m_{ps} Y(m_{ps} r)}{4 \pi} O,
V_{s} &= \frac{g_{s}^{2}}{4 \pi} m_{s} \left[1 - \frac{m_{s}^{2}}{4 M_{\Omega} M_{\Omega}}\right] Y(m_{s} r),
\end{align}

where $Y(\chi) = e^{i \chi}/\chi$, $m_{ps}$ and $m_{s}$ stand for the masses of the mesons. The potential for ps exchange needs to be multiplied with the expectation values for the appropriate spin ($\sigma_{1} \cdot \sigma_{2}$, $\Sigma_{1} \cdot \sigma_{2}$, $\Sigma_{1} \cdot \Sigma_{2}$) and/or isospin operators, see Ref. [27], indicated in Eq. (1) symbolically by $O$. There is an additional term involving the (irreducible) tensor operator in case of ps exchange, and one involving the spin-orbit operator in case of scalar exchange [22]. But their expectation values vanish for the $S$-wave states considered and, therefore, they are omitted in Eqs. (1) and (2).

With regard to the coupling constants of the ps mesons in Eq. (1) we note that $f_{NN}/m_{s} \approx g_{s}/(2 f_{s})$, where $g_{s}$ is the axial-vector strength and $f_{s}$ the weak pion decay constant. In the actual calculation the values $f_{s} = 92.1$ MeV and $g_{s} = 1.26$ [21] are used. The strength for the other couplings of ps mesons to octet baryons are fixed by imposing SU(3) flavor symmetry [28] based on $f / f_{s} \approx g_{s} / (2 f_{s})$ (see Table 1) substituting, however, the physical $\eta$ decay constant, $f_{\eta} = 1.3 f_{s}$, in the actual calculations.

For the coupling of ps mesons to decuplet baryons we proceed in the same way, see Ref. [27]. There are estimates for the corresponding coupling constants $g_{1}$, relevant for $f_{S\Xi T}$, in the non-relativistic quark model [29] and from large $N$ considerations [30,31] which lead to $g_{1} \approx 9/5 g_{s}$, i.e. $g_{1} \approx 2.27$. On the other hand, lattice QCD calculations suggest $g_{1} \approx g_{s}/2 (g_{1} \approx 0.60)$ [32]. Note that the decuplet-decuplet coupling constant $f_{D\bar{D}}$ used in Table 1 is $f_{D\bar{D}} / f_{s} \approx g_{s} / (2 f_{s}) / 9$, due to our normalization of the isospin-3/2 operators [27,33], so that $f_{S\Xi T} = f_{NN}/5$ for the quark-model value.

The coupling constants for the scalar ($\sigma$ or $f_{0}(980)$) exchange are estimated from a microscopic calculation of correlated $\pi\pi/KK$ exchange. A corresponding model study for baryon-baryon ($BB$) systems has been presented in Ref. [20] for the "$\sigma$" (i.e. for the $J = 0$, $I = 0$ channel) as well as for the "$\rho$" (i.e. the $J = 1$, $I = 0$ channel). It is based on dispersion theory and utilizes crossing symmetry to relate the correlated $\pi\pi/KK$ exchange with amplitudes in the crossed channels $BB \rightarrow \pi\pi, \kappa\kappa$. For a comprehensive description of this model we refer the reader to Ref. [20]. A short overview of the basic ingredients and details on how the coupling constants are obtained are provided in the appendix. The effective $\sigma$ coupling strengths we employ are taken from Table 5 of Ref. [20].

The underlying spectral functions $p^{\sigma}$ from which those coupling constants are extracted are reproduced in Fig. 1. The actual value of the coupling constants are $g_{\sigma}^{2}/4\pi = 7.77$ for $NN$ scattering and $g_{\sigma}^{2}/4\pi = 1.52$ for $NN$. They are based on an effective $\sigma$ mass of 550 MeV [20], a choice suggested by the behavior of $|p^{\sigma}(t)|$ for $NN$ which is strongly peaked around this value, cf. Fig. 1.

The effective $\sigma$ coupling strength for the $\Omega\Omega$ channel has not been evaluated in Ref. [20]. Is strength depends crucially on the coupling constants of the ps mesons to the decuplet baryons which are quite uncertain, as mentioned above. In view of that
we refrain from attempting an explicit but tedious evaluation of the contribution for correlated $2\pi$ exchange. Rather we aim for a qualitative estimation of the coupling strength and we focus primarily on the expected range, where pertinent information can be deduced by considering the situation in the $\Lambda\Lambda$ system. For the latter the corresponding spectral function has been calculated in [20] and it is shown in Fig. 1 (dash-dotted line). The deduced effective $\sigma$ coupling strength is $g_{\sigma}^2/4\pi = 2.00$. Indeed, since the small $t$ behavior for $\Xi N$ and $\Lambda\Lambda$ are similar one expects a $\sigma$-like interaction with comparable strength and range in both channels.

When taking those results as guideline for $\Omega\Omega$ one has to keep in mind, however, that the spectral function for $\Lambda\Lambda$ receives contributions from several pieces as depicted in Fig. 2. Some of those involve a coupling to $\pi\pi$ (in combination with $\Sigma, \Sigma^*$ states), and those are the ones which give rise to the long range part of correlated $\pi\pi/K\bar{K}$ exchange. Others involve only a coupling to $K\bar{K}$ (combined with a nucleon or $\Xi, \Xi^*$) and they provide only shorter ranged contributions. In case of correlated $\pi\pi/K\bar{K}$ exchange in $\Omega\Omega$ scattering only kaons in combination with $\Xi, \Xi^*$ states can contribute, see Fig. 2. We can easily simulate this situation by re-calculating the spectral function for $\Lambda\Lambda$ with the $\Sigma, \Sigma^*$ exchanged switched off. The corresponding $\rho^\sigma$ is indicated by the dotted line in Fig. 1. Clearly, now the small $t$ (long range) part is depleted and the spectral function is basically concentrated around the $K\bar{K}$ threshold. Transformed to $r$-space it is best represented by the exchange of the $f_0(980)$ meson whose effective coupling constant can be deduced from the spectral function and amounts to $g_0^2/4\pi = 1.77$. We will use the above strength for the effective $f_0(980)$ exchange as basis for the $\Omega\Omega$ interaction. Actually, one can do a simplistic estimation of what do expect for $\Omega\Omega$ based on the finding that the $f_0(980)$ for $\Lambda\Lambda$ results primarily from the contribution from $N$ exchange. Considering the SU(3) structure and taking the quark-model values for the coupling constants $f_{N\Lambda\Lambda}$ and $f_{\Lambda\Lambda\Lambda}$ one arrives at $f_{\Xi\Xi\Lambda}^2 = 9/2 f_{\Xi\Xi\Lambda}^2$ and $f_{\Xi\Xi\Lambda}^2 = 6/25 f_{\Xi\Xi\Lambda}^2$ which suggests that $\Xi$ exchange should dominate. Since the coupling constants enter with the fourth power into the spectral function/potential the effective $f_0(980)$ coupling strength could be enhanced by as much as $(9/2)^2 \approx 20$ as compared to the $\Lambda\Lambda$ case. Of course, one should not forget that the actual results for $\Omega\Omega$ will not only depend on the relevant ($\Omega\Xi K$, $\Xi\Xi K$) coupling constants, but there is also a different spin-momentum structure at the vertices, a difference in the masses of the involved baryons as compared to the $\Lambda\Lambda$ case, etc.

Table 1: SU(3) relations for the relevant coupling constants. For the $F/(F + D)$-ratio we adopt the value $\alpha = 0.4$.

| Baryons | $f_{NN\Lambda}$ | $f_{\Xi\Xi\Lambda}$ | $f_{NN\eta}$ |
|---------|----------------|---------------------|--------------|
| Octet   | $f_{NN\Lambda} = f$ | $f_{\Xi\Xi\Lambda} = -(1 - 2\alpha)f$ | $f_{NN\eta} = (4\alpha - 1)f$ |
| Decuplet| $f_{\Lambda\Lambda\Lambda} = f_{DD}$ | $f_{\Xi\Xi\Lambda} = -\sqrt{12}f_{DD}$ |

3 Results

3.1 Results for $NN$ and $\Xi N$

Let us start with a pedagogical case, namely the $NN$ potential in the $^1S_0$ partial wave. In this case the wealth of scattering data
has allowed to pin down the phase shifts rather precisely and, as a consequence, the r-dependence of (local) potentials that reproduce those shifts is rather well constrained. This is illustrated in Fig. 3 where results for the Reid [34] (dotted line) and Argonne V18 [35] (dash-double-dotted line) potentials are presented. Both potentials include the contribution from one-pion exchange (OPE) that provides the longest ranged piece of the interaction. However, it is obvious from the figure that in the range of $r \approx 1 - 2$ fm where the essential intermediate attraction comes from, the contribution from OPE is basically negligible. That large attractive contribution is usually attributed to (correlated) $\pi \pi$ exchange, often represented by $\sigma$ exchange. The dash-dotted line in Fig. 3 is the contribution from correlated $\pi \pi$ exchange based on the effective $NN\sigma$ coupling deduced in Ref. [28], cf. Table 5. Note that the value suitable for OBE exchange is taken. The solid line is the sum of $\pi$ and $\sigma$ exchange. One can see that this sum explains rather well the behavior of the potentials obtained from fitting to the phase shifts in the range $r \approx 1 - 2$ fm. This concerns not only the magnitude (strength) but also the shape (range), which is quite remarkable. At shorter internuclear distances additional dynamics becomes relevant so that deviations have to be expected. Note also that no cutoff or regularization is applied to the $\pi$ and $\sigma$ exchange potentials shown.

In Fig. 4 corresponding results for the $\Xi N$ interaction are presented. There is no quantitative experimental information on $\Xi N$ scattering and, therefore, no empirical constraint on the potential. However, an effective $\Xi N$ has been published by the HAL QCD Collaboration [23] based on a lattice simulation close to the physical point which is included in the figure.

Pion exchange provides again the longest ranged contribution. The curve shown in Fig. 4 is based on a coupling constant fixed via SU(3) flavor symmetry which implies that the strength is only about 20% of that in the $NN$ channel. There is also a contribution from correlated $2\pi$ exchange where again the effective $\sigma$ coupling strength can be taken from Ref. [20]. The emerging picture is similar to that in $NN$. Specifically, at distances around $r \approx 1$ fm correlated $2\pi$ exchange provides the dominant contribution while that of OPE is small. Interestingly, the computed potential agrees fairly well with that extracted from the lattice simulations, at least on a qualitative level. Indeed, by re-adjusting the strength of the effective $\sigma$ exchange by a factor of roughly two the two potentials practically coincide in the region of $r \approx 1 - 2$ fm, suggesting that the ranges are indeed compatible.

3.2 Results for $\Omega N$ and $\Omega \Omega$

For $\Omega N$ scattering a full-fledged meson-exchange potential is available in the literature [26]. Therefore, it is preferable to utilize that one directly for the comparison with the potential extracted from the lattice simulations, specifically because that potential is constructed in such a way that it reproduces the HAL QCD results at low energies (scattering length, bound state) in the relevant $^3S_1$ partial wave. The potential includes $\eta$ exchange and the possible coupling of $\Omega N$ to the $\Lambda\Xi$, $\Sigma\Xi$, and $\Lambda\Xi'$ channels [26]. Moreover, an elaborate evaluation of the contribution from correlated $2\pi$ exchange to the $\Omega N$ potential has been performed, which involves besides $\pi\pi$ and $K\bar{K}$ correlations also those from the $\eta\eta$ channel. These components constitute the long-range part of the potential, indicated by the dotted line in Fig. 5. In addition a contact term is included to parameterize the short-range physics, whose range is determined by the form factor and, specifically, by the chosen cutoff mass of $\Lambda = 1$ GeV. Its contribution is indicated by the dashed
line in Fig. 5 whereas the total potential is given by the solid line. Evidently, the so-called long-range and short-range components give rise to rather similar contributions in the region of $r \approx 0.7 - 1.5$ fm. But this can be understood if one recalls that the ranges are set by roughly two times the $K$ or $\eta$ mass for the meson-meson correlations and by the cutoff mass for the contact term, which are both in the order of 1 GeV. Much more conspicuous is the difference to the potential extracted from the lattice simulations. The latter is clearly longer ranged and there is also more strength located at large distances.

This peculiar feature is incorporated in the parameterization of the $\Omega N$ potential in Ref. [24] by including a term with the range $2m_\eta$, called (Yukawa) $^2$. As motivation for that a possible OZI violating vertex is quoted that should allow two pions to couple to the $\Omega$. However, in our opinion to speak only of OZI violation in this context might be somewhat misleading. First and foremost it is a violation of isospin symmetry which is required for coupling a pion (or two) to the $\Omega$. Indeed, the mixing of $\eta$ and $\pi^0$ provides such a contribution but, of course, it is expected to be rather small. For example, utilizing the electromagnetic mass matrix,

$$\langle \pi^0 | \Delta m^2 | \eta \rangle = [m_{\pi^0}^2 - m_{\eta}^2 + m_K^2 - m_{\pi^0}^2] / \sqrt{3},$$

as a measure for the mixing strength (see Ref. [26]) one obtains

$$f_{\Omega \Omega \pi} = \frac{\langle \pi^0 | \Delta m^2 | \eta \rangle}{m_{\eta}^2 - m_{\pi^0}^2} f_{\Omega \Omega \eta} \approx 0.0106 f_{\Omega \Omega \eta}.$$  \hspace{1cm} (3)

Results for the $\Omega \Omega$ potential in the $^1S_0$ partial wave are presented in Fig. 6. Since the coupling constants of pseudoscalar mesons to decuplet baryons is not constrained empirically, we consider two cases: (a) The coupling constant from lattice simulations [23] which indicate that $f_{\Omega \Lambda \Lambda} \approx f_{\Omega N \eta}/20$ for the normalization of the spin and isospin operators used by us [27].

(b) The coupling strength that results from the non-relativistic quark model (or from large $N_c$ arguments) which implies $f_{\Omega \Lambda \Lambda} \approx f_{\Omega N \eta}/5$. The value for the $\Omega \Omega \eta$ coupling, relevant here, is obtained from the standard SU(3) relations [27], see Table 1 under the assumption that $\eta \approx \eta_\Lambda$.

The $\Omega \Omega$ potential that results from $\eta$ exchange with the coupling constant from LQCD is presented in Fig. 6 by the dashed line. The potential from the effective $f_0(980)$ exchange (dash-dotted line) is the one with the strength deduced from the correlated $KK$ exchange in the $\Lambda \Lambda$ system, cf. Sect. 2. Adding the two contributions and assuming that the actual $f_0$ coupling in the $\Omega \Omega$ case could be larger by a factor of up to $(9/2)^2$, as discussed in Sect. 2 leads to the band confined by the two solid lines. Thus, considering the sizable uncertainty in the effective $f_0$ coupling constant, the strength of the potential from LQCD at $r \approx 1$ fm can be roughly reproduced. However, it is obvious that the $r$-dependence deduced from the lattice simulation and the one which follows from $\eta + f_0(980)$ exchange are quite different. The potential for $\eta$ exchange with the coupling constant from the quark model is shown by the dotted line. Adding here $f_0$ exchange would lead to an overestimation of the LQCD result, with again a mismatch as far as the $r$-dependence is concerned.

Finally, in Fig. 7 we display the potentials from the HAL QCD Collaboration for $\Xi N$ ($^1S_0$, $I = 0$) [24], $\Omega N$ ($^3S_2$) [24], and $\Omega \Omega$ ($^1S_0$) [25] together. The most striking feature is that they more or less coincide in the region $r \approx 1 - 2$ fm, say. This
We do not have a ready explanation for the discrepancies in those channels where the dynamics is expected to be governed by short-distance physics. After all, according to Yukawa, the ranges for the $\Omega N$ and $\Omega\Omega$ interactions should correspond roughly to the inverse mass of the $\eta$ and/or $f_0(980)$ mesons. One plausible reason could be, of course, that non-local effects become much more important in cases where pion exchange is absent and even correlated $\pi\pi$ exchange is suppressed. Then, the attempt to represent such a possibly highly non-local interaction by a local potential, as done by the HAL QCD Collaboration, could lead to artifacts which manifest themselves in form of a long range and a sizeable strength. Indeed, such a trend has been seen in applications of inverse scattering theory to cases like $\pi N$ or $KK$ scattering [37].

It is possible that such potential artifacts have no or only minor consequences for the predicted phase shifts in the $\Omega N$ and $\Omega\Omega$ channels. In this context, see the critical remarks and discussions in Refs. [14, [15, [16]. However, it remains unclear what is going to happen when those potentials are used in calculations of few- or many-body systems. Then short-ranged but non-local interactions lead most likely to different results than their local and long-ranged counterparts, despite yielding the same phase shifts on the two-body level. In any case, the most important lesson is certainly that one has to be somewhat cautious in the perception of those potentials and, specifically, one should not take them too seriously as far as their physical interpretation is concerned.

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4 Conclusions

In this paper we presented a qualitative discussion on the range of the potentials as they result from the phenomenological meson-exchange picture and from lattice QCD simulations by the HAL QCD Collaboration. For the former pion- and/or $\eta$-meson exchange are considered together with the exchange of a scalar- or isoscalar meson ($\sigma$ or $f_0(980)$), where scalar-meson exchange is viewed as being due to correlated $\pi\pi/K\bar{K}$ exchange and its actual strength and range has been inferred from a pertinent microscopic model [20].

It turned out that the intuitive expectation for the behavior of the baryon-baryon potentials for large separations, associated with the exchange of one and/or two pions, does not always match with the potentials extracted from the lattice simulations. For the $\Xi N$ channel, where pion exchange provides the longest ranged contribution, there is a reasonable qualitative agreement between the phenomenological and the LQCD potentials, for the considered $^1S_0$ partial wave within isospin $I = 1$ and for separations $r \gtrsim 1$ fm. On the other hand, for the $\Omega\Omega$ channel where isospin conservation rules out one-pion exchange we observe a large mismatch, with the potential from LQCD being much longer ranged and much stronger at large distances as compared to the phenomenological expectation. The same is also the case for the $\Omega N$ interaction, where the comparison was done for a meson-exchange potential from the literature [26].

A Potential from correlated $\pi\pi$ and $K\bar{K}$ exchange

A detailed derivation of the baryon-baryon interactions in the $J^P = 0^+$ ($\sigma$) and $1^-$ ($\rho$) channels from correlated $\pi\pi$ and $K\bar{K}$ exchange can be found in Ref. [20]. Here we summarize only the essential steps and restrict ourselves to the “$\sigma$” case. Assuming analyticity for the amplitudes dispersion relations can be formulated for the baryon-baryon amplitudes, which connect physical amplitudes in the $s$-channel with singularities and discontinuities of these amplitudes in the pseudophysical region of the $t$-channel processes for the $J^P = 0^+$ ($\sigma$) channel:

$$V^{(0^+)}_{B_1B_2 \to B'_1B'_2}(t) \propto \int_{4m_\sigma^2}^{\infty} \frac{\text{Im}V^{(0^+)}_{B_1,B_2 \to B'_1,B'_2}(t')}{t' - t}, \quad t < 0.$$  \hspace{1cm} (4)

Via unitarity relations the singularity structure of the baryon-baryon amplitudes for $\pi\pi$ and $K\bar{K}$ exchange are fixed by and
can be written as products of the $B\bar{B} \to \pi\pi, \, K\bar{K}$ amplitudes

$$
P^\alpha_{B_1B_2\to B_1'B_2'}(t') \equiv \text{Im} V^\alpha_{B_1B_2\to B_1'B_2'}(t') \propto \sum_{\alpha=\pi,\bar{K}} T^{(0)\alpha}_{B_1'B_1\to B_2'B_2} T^{(0)*}_{B_1B_1\to B_2B_2}$$

(5)

which are then inserted into dispersion integrals to obtain the (on-shell) baryon-baryon interaction. The ingredients that enter into $T^{(0)*}_{B_1B_1\to B_2B_2}$ are, cf. Fig. 3 for a graphical representation, are the meson-meson correlations (Fig. 9) and a Born term $V_{B\bar{B}\to \sigma}$ where we show only the ones relevant for the present study in Fig. 2, namely those for $\Lambda\Lambda$ and $Q\bar{Q}$.

The spectral functions characterize both the strength and range of the interaction. Clearly, for sharp mass exchanges the spectral function becomes a $\delta$-function at the appropriate mass. Indeed, for convenience the authors of Ref. [20] have presented their results in terms of effective coupling strengths, by parameterizing the correlated processes by (sharp mass) $\sigma$ and $\rho$ exchanges. The interaction potential resulting from the exchange of a $\sigma$ meson with mass $m_\sigma$ between two $j^P = 1/2^+$ baryons $A$ and $B$ has the structure [22]:

$$
V^\sigma_{A,B\to A',B'}(t) = g_{A\sigma}g_{B\sigma} \frac{F^2_\sigma(t)}{1 - m_\sigma^2},
$$

(6)

where a form factor $F_\sigma(t)$ is applied at each vertex, taking into account the fact that the exchanged $\sigma$ meson is not on its mass shell. This form factor is parameterized in the conventional monopole form,

$$
F_\sigma(t) = \frac{\Lambda_\sigma^2 - m_\sigma^2}{\Lambda_\sigma^2 - t},
$$

(7)

with a cutoff mass $\Lambda_\sigma$ assumed to be the same for both vertices. The correlated potential as given in Eq. (4) can now be parameterized in terms of $t$-dependent strength functions $G^{\sigma}_{B_1'B_1\to B_2'B_2}(t)$, so that

$$
V^\sigma_{A,B\to A',B'}(t) = G^\sigma_{AB\to A'B'}(t) F^2_\sigma(t) \frac{1}{1 - m_\sigma^2}.
$$

(8)

The effective coupling constants are then defined as:

$$
g_{A\sigma}g_{B\bar{\sigma}} \to G^\sigma_{AB\to A'B'}(t) = \frac{(t - m_\sigma^2)}{\pi F_\sigma^2(t)} \int_0^\infty \frac{P^\alpha_{A'B'\to AB}(t')}{t' - t} \, dt'.
$$

(9)

The parameterization above does not involve any approximations as long as the full $t$-dependence of the effective coupling strengths is taken into account. In Ref. [20] it was attempted to minimize that $t$-dependence so that the effective coupling strengths are basically coupling constants, see Fig. 20 in that work. This could be achieved by setting the masses $m_\sigma$ and $m_\rho$ of the exchanged particles to the values used in the Bonn-Jülich models of the NN [22] and YY [33] interactions, $m_\sigma = 550\, \text{MeV}$, $m_\rho = 770\, \text{MeV}$, and choosing appropriate values for the cutoff masses $\Lambda_\sigma$ and $\Lambda_\rho$. The resulting values, $\Lambda_\sigma = 2.8\, \text{GeV}$, $\Lambda_\rho = 2.5\, \text{GeV}$, are quite large and, thus, imply that modifications of the potential from the form given in Eq. (2) take place only at rather short distances. Accordingly, in the present study we simply take over the effective coupling constants $G^\rho_{AB\to A'B'}(t = 0)$ deduced in Ref. [20] and summarized in that work in Table 5, and use it in Eq. (2).

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