Using historical perspective in designing discovery learning on Integral for undergraduate students

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Abstract. In the course of Integral Calculus, to be able to calculate an integral of a given function is becoming the main idea in the teaching beside the ability in implementing the application of integral. The students tend to be unable to understand the conceptual idea of what is integration actually. One of the promising perspectives that can be used to invite students to discover the idea of integral is the History and Pedagogy Mathematics (HPM). The method of exhaustion and indivisible appear in the discussion on the early history of area measurement. This paper study will discuss the designed learning activities based on the method of exhaustion and indivisible in providing the undergraduate student's discovery materials for integral using design research. The designed learning activities were conducted into design experiment that consists of three phases, i.e., preliminary, design experimental, and teaching experiment. The teaching experiment phase was conducted in two cycles for refinement purpose. The finding suggests that the implementation of the method of exhaustion and indivisible enable students to reinvent the idea of integral by using the concept of derivative.

1. Introduction

The fact that lecturers, in teaching calculus, tend to focus only on the procedural ability is undeniable. Calculus is treated as a tool in which the materials only cover symbolic manipulation of function [2–4]. In addition, calculus textbooks do not support fully about the conceptual understanding of calculus [2]. Students’ understanding of calculus may appear as a procedural method only. This understanding could restrict the concept of calculus developed from the idea of the infinitesimal.

Misunderstandings on the concept of calculus lead to students’ difficulties in working in integral calculus problems. In general, students would encounter difficulties in learning calculus. Limit and infinitesimal concept produce several cognitive difficulties, such as; (1) terminology conflict of terms involved (“approaching”, ”arbitrarily small”, ”reach infinity”) and (2) inability in estimating what happen in infinity [5]. In detail, the students’ difficulties can be stated as (1) wrong interpretation of notational meaning – Leibniz notation, (2) difficulties in translating real world by using suitable representation into calculus formulation that restrict imagery of a function mentally, and (3) lack of
algebraic manipulation that makes the application of procedural method override the conceptual understanding.

Several approaches are proposed to help students in the beginning of their calculus class. Several correctives strategies can be proposed such as: (1) avoiding early reference to the language of limit, derivative, and integration; (2) confronting students with the continuous properties and infinitesimal concept as a new concept for the students; (3) emphasizing only in procedural aspect; (4) using formal definition axiomatically; etc. [1,6]. One of the perspectives that can be used to support the students in calculus is what is called History and Pedagogical in Mathematics [7–10]. Using historical perspective to support the students’ mathematics learning development has been spreading in the recent decades. In this perspective, the relationship between ontogenesis (development of the individual) and phylogenesis (development of mankind) in the context of mathematics concept development has been proposed lately [11]. Table 1 shows how the development of mathematics through the millennia is parallel with the cognitive development in students.

| Development of Mathematics through History | Mathematical Content | Development of Mathematics in Individual Progression |
|------------------------------------------|----------------------|--------------------------------------------------|
| Primitive Man, Primary and derivative occupation | Geodesy Egyptian and Babylonian | Astronomy and Mechanics Greek, Hindu and Arabic | Physic, Europe |
| Separate unit represented by symbols | Origin of Rhetorical Algebra | Practical Trigonometry | Rhetorical Algebra |
| Counting (Gesture or Language Symbols) | Experimental Geometry (Inductive and approximate – empirical and intuitional evidence) e.g. Pythagorean Theorem | Deductive Arithmetic | Arithmetic (Zero and Place Value) |
| | Arithmetic (Development of symbols) | Deductive Mechanics (Archimedes) | Trigonometry, Decimals (Arithmetic Development) |
| | Additive Principle for collective units e.g | Deductive Geometry (Theory) | Logarithm, Symbolic Algebra |
| | | | Analytical Geometry |
| | | | Differential Calculus and Theoretical Mechanics, Integral Calculus |
| | | | Higher Arithmetic, Function Theory |
| | | | Universal Algebra, Probability |

The development of mathematics through ages can be seen similar to the mathematics developed in individual progression. Table 1 shows that the early development of mathematics started as practical geometry and its basic counting. That mathematical content used by primitive man. The same mathematical content also appeared in the infancy stage of development of mathematics in individual progression. In the next stages, the development of mathematics became more complex especially after Renaissance, Europe Era. This is in line with the individual development as they enter college
stage. This point of view considers several objections to the implementation of historical perspective in the teaching of mathematics [11].

The history of integral calculus is evolved on the idea of measuring non-rectilinear figure. The first important and famous problem in the beginning of calculus was the circle squaring problem. The aim of this problem was to find a square that has an equal area with the circle. For the time being, the problem developed into the problems of finding the area of cycloid and even arbitrary figures. The progress of the method, which was started by the method of exhaustion and the method of indivisible, can be adapted and implemented to the learning activities of learning integral calculus. In designing learning activities using historical perspective, ones need to link a mathematical concept with the corresponding authentic experience. One of the approaches that can be used in designing such activities is Realistic Mathematics Education (RME). The development of the activities using RME should involve the learning activities by the designed parts which are constructed from guided reinvention, emergent modelling, and didactical phenomenology [12]. By experiencing those stages, students would gain conceptual understanding as the activities of learning.

This paper proposes an alternative approach in teaching integral using historical perspective for undergraduate students of the mathematics department. In fact, in higher education, calculus course had been taught persistently in a non-meaningful way. Hence, after analyzing the history of the development of calculus, we would suggest the use of appropriate big ideas that they can help the students understanding of integral calculus based on the historical perspective of integral. RME proposes a guided reinvention as one of a fundamental idea on how history can be integrated into the teaching of mathematics along with didactical phenomenology and emergent modelling [4]. The designed learning activities for the first implementation is elaborated in 3 parts which are the goals of learning. Therefore, this paper discusses how to design discovery learning using historical perspective on Integral Calculus course for undergraduate students.

2. Methods

The main concern of the researchers is to help students to overcome their difficulties on the topic of integral. Here, we not only can elicit the way of the methods work well, but also we can understand why they work well. The design would be carried out in the two different classes of undergraduate students of mathematics department who take the Integral Calculus course as two consecutive cycles, which are preliminary teaching phase and teaching experiment phase. The preliminary teaching was carried out on the class of eight students, while the teaching experiment was carried out on the class of 24 students.

In designing meaningful learning activities, the use of historical perspective could help the students to focus more on the underlying concept rather than procedural manipulation symbols. The big idea on indefinite integral concept emerges from missing function context into anti-derivative context. While the big idea in the definite integral is to develop from the relation between the area under the curve and the function of the curve to Riemann Sum that is formalized of the Fundamental Theorem of Calculus.

The implementation of the learning activities would be applied and revised accordingly. The resulting outcome of this study was a sequence of learning activities which are crucial to the learning of integral [13]. There are three phases of conducting design experiment [14]. These phases are (1) preliminary phase, (2) design experimental phase, and (3) teaching experiment. The preliminary phase had a goal to formulate hypothesized local instruction theory in the form of a sequence of learning activities. These learning activities were then carried out and tested during the design experiment [14]. The design experimental phase was aimed to carry out the learning design. In this phase, the learning sequence of 4 lessons that were developed in the previous phase was carried out in teaching experiment.

The teaching experiment was conducted in 2 cycles. In the first cycle, the experiment was implemented in a class of eight students with one of the researchers himself as the teacher. The researcher’s observations of the lessons and the written works of the students were documented in
reports. The reports emphasized on classroom discussions, reflections and interesting individual contributions by students to support whether the conjectures really fulfilled or some new issues come out and were used for revising the learning activities in some extent. The second cycle was the actual classroom for the design experiment. It was a class of 24 students. The learning activities sequence was implemented by the teacher. In the class, researchers try to capture the classroom activities of the whole class. At the same time, the researchers also have a focus group to be analysed along with the whole class situation. The data collected from this second cycle will be analysed and used as the final revision of the learning activities to answer our research question.

3. Results and Discussion

This study had designed the historically based learning activities on the course of integral calculus for undergraduate students. The design was implemented and revised during the preliminary teaching and the teaching experiment. The result will be considered as historical based learning activities and the analysis on how the activities can help the students’ difficulties is elicited.

In designing meaningful learning activities, the use of historical perspective could help the students to focus more on the underlying concept rather than procedural manipulation symbols. Table 2 summarizes the design of the learning activities in the course of integral calculus. The big idea on indefinite integral concept emerges from missing function context into anti-derivative context. While the big idea in the definite integral is to develop from the relation between the area under the curve and the function of the curve to Riemann sum which is formalized of the fundamental theorem of calculus.

| No. | Subtopics | Learning Activities |
|-----|-----------|---------------------|
| 1   | Indefinite Integral | The problem discussed in this phase is about finding the function which is given as anti-derivative and a point. The students should find the correspondence function then determine the function whose graph passing through the given point. |
| 2   | Initial Value Problem | The problem discussed in this phase is about Free Fall Movement. The students are given an initial condition of where the object starts to fall and how it would fall. By finding the anti-derivate of the function of free fall movement, the students would find the initial function. |
| 3   | Using previously activity, the students will try to: Find the anti-derivative function and Initial Position in Free Fall Movement Determine the velocity in a given point, And so on |
| 4   | Derivative of linear and quadratic function | The problem discussed is determining graph and its given functions (derivatively related). The students then are asked to compare the area under the given linear function and its anti-derivate function values in the interval values. The area under linear curve is equal to the difference between the anti-derivative function value in the edges of the interval (state1) |
| 5   | Definite Integral | The problem discussed in this phase is still about the area under a graph of the linear function and how to extend this idea to more general function. The students will be asked to discussed what would happen if the function is not linear, whether the relation still holds |
| 6   | The students would be introduced the method of exhaustion and the method of indivisible. The demonstration of the method of exhaustion would be presented by the teacher. The students, then, try to experience the method of indivisible in determining the area of the given ellipse |

Table 2. Historical Based Learning Activities on the Topic of Integral
The idea of infinity and small partition are introduced

| Area | Riemann Sum Partition method |
|------|-----------------------------|
| 7    | The problem discussed is about determining the area of the given function graph. The students are asked to make equal partition using rectangular grid and determine the approximation area from each partition. Various solution from different students should be discussed on how small we can use in the partition. |
| 8    | The problem discussed is various type of graphs and the area under the graph. The students would be asked to apply limit in approximating the value of area. In the end, the relation (The area under linear curve is equal with the difference between the anti-derivative function value in the edges of the interval) is carried out as the concept of fundamental Theorem of calculus. |

The historical aspects of mathematics used in the learning activities, which become the big ideas, are the Method of Exhaustion, the Method of Indivisible, and Riemann Sum. The activities illustrated in Figure 1 and 2 are aimed to reinvent the idea of integral as using the approach of infinitesimal concept. Following these activities, a reinvention activity on Riemann Sum approach. In designing meaningful learning activities, the use of historical perspective could help the students to focus more on the underlying concept rather than procedural manipulation symbols. Table 2 summarizes the design of the learning activities on the topic of integral. The development of the big ideas was necessary for the reinvention of the Fundamental Theorem of Calculus.

![Figure 1](image1.png)  ![Figure 2](image2.png)

**Figure 1.** Activities on the Method of Exhaustion  **Figure 2.** Activities on the Method of Indivisible

The Riemann Sum activity invited the students to discover the idea for finding the area under the given curve using partitions of equal width by applying the method of indivisible into the area (see Figure 3).
Moreover, making the partition width to be as small as possible (approaching 0 as applying the method of exhaustion) lead the students to the development of the definite integral concept resulted from Riemann Sum (see Figure 4)

In short, from the discussed results, big ideas on the topics of indefinite integral and definite integral in the learning activities were adjusted accordingly. The context used in the lesson is made clearer to avoid students’ misunderstanding and difficulties in using the sign of object movement orientation. The big idea of relation area under the curve and the introduction to Riemann Sum were
improved by the use of Cavalieri’s Principle. This improvement was required to bridge the gap where the students failed to understand the role of the method of indivisible in the reinvention of Riemann Sum. The students used the Cavalieri’s Principle as an assisting material that leads the students in understanding the idea of the infinitesimal.

4. Conclusions

The learning activities in the integral course emerged from its relation to the derivative concept. In addition, the developed learning activities are parallel with the finding of Dubinsky and Schwingendorf [15]. The implementation of graph-based learning activities helped the students in developing their understanding of the concept of integral calculus [15]. This big idea involved the development of indefinite integral and its relation to function. Given the derived function as the known fact here that must be used for finding the function itself. One critical big idea was that the stage when the students must determine the connection between the area under a certain curve and the derivative of the curve. The conceptual learning happened when the students extended that big idea to a non-general function. Finally, the formulation of the Fundamental Theorem of Calculus involved the use of two big ideas of the method of exhaustion and the method of indivisible. In addition, the use of historical perspective could motivate the students during the course and could give an insight on how the mathematics concept emerged and used. The historical perspective could not be used in the classroom as it is. An adequate modification is needed based on the learning objectives. This view is in line with Savizi claims [16]. The correct interpretation is required to present the former historical problems into classroom materials and activities.

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