Scaling of hadron masses and widths in thermal models for ultra-relativistic heavy-ion collisions

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By means of a simple rescaling, modifications of hadron masses and widths are incorporated into the thermal analysis of particle ratios in ultra-relativistic heavy-ion collisions. We find that moderate, up to 20%, changes of hadron masses do not spoil the quality of the fits, which remain as good as those obtained without modifications. The fits with the modified masses yield, however, modified values of the optimal temperature and baryon chemical potential. In particular, with decreasing masses of all hadrons (except for pseudo-Goldstone bosons) the fitted values of the temperature and the baryon chemical potential are lowered, with the change approximately proportional to the scaling of masses. In addition, we find that the broadening of the hadron widths by less than a factor of two practically does not affect the fits.

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Recent theoretical studies show that the hadronic yields and ratios in ultra-relativistic heavy-ion collisions can be described well in the framework of simple thermal models. The thermodynamic parameters obtained in this approach define the so-called chemical freeze-out point, i.e., a stage in the evolution of the hadronic system when the “chemical composition” is established. Thermal-model fits to the SPS data show that the temperature at the chemical freeze-out, \( T_{\text{chem}} \), as well as the baryon density, are large. One typically obtains \( T_{\text{chem}} \sim 170 \text{ MeV} \), which is close to the expected critical value for the deconfinement/hadronization phase transition. In this situation one may expect that hadron properties at the chemical freeze-out are strongly modified by the presence of the hadronic environment. Indeed, such modifications are predicted by model calculations, which inter alia helps to

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explain the low-mass dilepton enhancement observed in the CERES \cite{13} and HELIOS \cite{14} experiments. In this paper we incorporate possible modifications of hadron masses and widths into thermal analysis of particle ratios. We generalize the results of Refs. \cite{15,16} where the problem was studied without refitting thermodynamical parameters.

In the first part we include only the mass modifications and calculate the particle densities from the ideal-gas expression

\[ n_i = \frac{g_i}{2\pi^2} \int_0^\infty \frac{p^2 \, dp}{\exp \left[ \frac{\left(E_i^* - \mu_{B_{\text{chem}}}^i - \mu_{S_{\text{chem}}}^i - \mu_{I_{\text{chem}}}^i\right)}{T_{\text{chem}}} \right] \pm 1}, \tag{1} \]

where \( g_i \) is the spin degeneracy factor of the \( i \)th hadron, \( B_i, S_i, I_i \) are the baryon number, strangeness, and the third component of isospin, and \( E_i^* = \sqrt{p^2 + (m_i^*)^2} \) is the energy. The quantities \( \mu_{B_{\text{chem}}}^i, \mu_{S_{\text{chem}}}^i, \mu_{I_{\text{chem}}}^i \) are the chemical potentials enforcing the appropriate conservation laws. In standard thermal-model fits Eq. (1) is used with the vacuum masses, \( m_i^* = m_i \). The in-medium masses, \( m_i^* \), may depend on temperature and density in a complicated way. In order to explore possible different behavior of in-medium masses and, at the same time, keep simplicity, we do our calculations with the meson and baryon masses rescaled by the two universal parameters, \( x_M \) and \( x_B \), namely

\[ m_M^* = x_M m_M, \quad m_B^* = x_B m_B. \tag{2} \]

An exception from this rule are the masses of pseudo-Goldstone bosons (\( \pi, K, \eta \)) which we keep constant.

Equation (1) is used to calculate the “primordial” density of stable hadrons and resonances at the chemical freeze-out. The final (observed) multiplicities receive contributions from the primordial stable hadrons, and from the secondary hadrons produced by decays of resonances after the freeze-out. We include all light-flavor hadrons listed in the newest review of particle physics \cite{19}, with a few exceptions for the cases where the properties of a listed particle are ambiguous or not known sufficiently well. Ref. \cite{19} is also used to determine the branching ratios. We neglect the finite-size and excluded volume corrections: the former are negligible, whereas the latter do not affect the particle ratios \footnote{The use of Eq. (1) is valid when the in-medium hadrons can be regarded as good quasi-particles. A thermodynamically consistent approach has been constructed so far only for the lowest multiplets of hadrons \cite{15,16}. At SPS energies, however, it is crucial to include all hadrons with masses up to (at least) 1.8 GeV. For such a complicated system, a thermodynamically consistent approach is not at hand at the moment.}

\footnote{The last property is due to the fact that the overwhelming majority of hadrons is heavy and may be treated as classical particles. In this case we may replace the...}
Fig. 1. Dependence of $\chi^2$, and the fitted values of the temperature and the baryon density on the scale parameter $x$. The plot is for the Pb + Pb collisions at SPS energies, with 19 particle ratios included in the analysis. Solid lines: all hadron masses (except for Goldstone bosons) are scaled with $x_M = x_B = x$. Dashed lines: only meson masses are scaled, $x_M = x, x_B = 1$. Dotted lines: only baryon masses are scaled, $x_B = x, x_M = 1$. The nuclear saturation density $\rho_0 = 0.17\text{fm}^{-3}$.

For given values of $x_M$ and $x_B$ we fit the temperature, $T_{\text{chem}}$, and the baryonic chemical potential, $\mu_{\text{chem}}^B$, by the minimization the expression $\chi^2 = \sum_k \left( R_k^{\exp} - R_k^{\text{therm}} \right)^2 / \sigma_k^2$, where $R_k^{\exp}$ is the $k$th measured ratio, $\sigma_k$ is the corresponding error, and $R_k^{\text{therm}}$ is the same ratio as determined from the thermal model. The strangeness chemical potential, $\mu_{\text{chem}}^S$, and the isospin chemical potential, $\mu_{\text{chem}}^I$, are determined from the requirements that:

1) the Fermi-Dirac (Bose-Einstein) distribution function in (1) by the Boltzmann distribution function. Thus, for equal excluded volumes of all hadrons the excluded volume corrections factorize and cancel out in the ratios. We have checked that the use of the classical statistics changes our results by a few percent only.
initial strangeness of the system is zero, and ii) the ratio of the baryon number to the electric charge is the same as in the colliding nuclei.

In Fig. 1 we plot our results obtained for the experimental ratios for Pb + Pb collisions at SPS, as compiled in Ref. [3]. In the case $x_M = x_B = 1$ we find: $T_{\text{chem}} = 169$ MeV, $\mu^B_{\text{chem}} = 250$ MeV, $\mu^S_{\text{chem}} = 65$ MeV, and $\mu^I_{\text{chem}} = -9$ MeV. These values are in good agreement with those found in Ref. [3]: $T_{\text{chem}} = 168$ MeV, $\mu^B_{\text{chem}} = 266$ MeV, $\mu^S_{\text{chem}} = 71$ MeV, and $\mu^I_{\text{chem}} = -5$ MeV. In Fig. 1 a) we give our values of $\chi^2$. One can observe that a small decrease of the meson and baryon masses, $x_M = x_B \sim 0.9$, leads to a slightly better fit with the corresponding smaller values of the temperature and the baryon density, as shown in Figs. 1 b) and 1 c). To check the significance of this result we have also analyzed Si + Au collisions at AGS, and S + Au collisions at SPS. In these two cases $\chi^2$ has a flat minimum at $x_M \approx x_B \approx 1$. We thus conclude that moderate modifications of hadron masses, say by 20%, do not spoil the quality of thermal fits. However, with the modified masses the thermodynamic parameters characterizing the fits do change. For example, if we rescale both meson and baryon masses (except for Goldstone bosons) in the same way, $x = x_M = x_B$, the temperature and the chemical potentials are to a very good approximation also rescaled by $x$. This follows from the fact that we study a system of equations which is invariant under rescaling of all quantities with the dimension of energy. If we allowed also for the changes of the masses of the Goldstone bosons, the thermodynamic parameters would scale exactly as $T_{\text{chem}}(x) = x T_{\text{chem}}(x = 0)$ and $\mu_{\text{chem}}(x) = x \mu_{\text{chem}}(x = 0)$. In this case $\chi^2$ remains constant, independently of $x$. For fixed values of the Goldstone-boson masses, the scale invariance is broken, and $\chi^2$ varies weakly with $x$, as shown in Fig. 1 a).

To account for finite in-medium widths, $\Gamma_i^*$, of the resonances we generalize Eq. (1) to the formula [20,21,22,23]

$$n_i = \int_{M_0^2}^{\infty} dM^2 \int_0^\infty dp \frac{1}{\pi N (M^2 - (m_i^*)^2)^2 + (m_i^* \Gamma_i^*)^2} \times \frac{p^2}{2\pi} \exp \left[ \frac{(\sqrt{M^2 + p^2} - \mu^B_{\text{chem}} B_i - \mu^S_{\text{chem}} S_i - \mu^I_{\text{chem}} I_i)}{T_{\text{chem}}} \right] \pm 1, \quad (3)$$

where $N$ is the normalization of the relativistic Breit-Wigner function, $N = \frac{1}{2} + \frac{1}{\pi} \arctan \left[ (m_i^*)^2 - M_0^2 \right] / (m_i^* \Gamma_i^*).$ The integral over $M^2$ is taken to start at the threshold $M_0^2$ corresponding to the dominant decay channel. In the limit $\Gamma_i^* \rightarrow 0$ Eq. (3) obviously reduces to formula (1).

In order to analyze the effect of broadening of hadron widths we introduce the parameter $y$ in such a way that

$$\Gamma_i^* = y \Gamma_i. \quad (4)$$
Here $\Gamma_i$ are the vacuum widths, hence the case $y = 1$ corresponds to the physical widths as measured in the vacuum, and the case $y = 0$ represents the situation when the widths are neglected (our previous analysis based on Eq. \eqref{eq:1}). In Fig. 2 we show the results of our fitting procedure. We observe that the inclusion of the vacuum widths does not change the value of $\chi^2$, and the values of $T_{\text{chem}}$ and $\rho_{\text{chem}}^B$. An increase of the widths by a factor of 2 has also little effect. Only for larger modifications of the widths the fit gets worse.

In conclusion we state that the thermal analysis of particle ratios allows for scaling of hadron masses. Such changes, however, result in modifications of the thermodynamical parameters for which the fits are optimal. In particular, lowering of all the masses leads to a smaller values of $T_{\text{chem}}$ and $\rho_{\text{chem}}^B$. This is a desired effect, since $T_{\text{chem}} \sim 170$ MeV is large and may correspond to quark-gluon plasma rather than to a hadron gas. Our study of the modifications of the hadron widths shows that they have small impact on the ratios.
References

[1] P. Braun-Munzinger, J. Stachel, J.P. Wessels, and N. Xu, Phys. Lett. B344 (1995) 43; Phys. Lett. B365 (1996) 1
[2] J. Cleymans, D. Elliott, H. Satz, and R. L. Thews, Z. Phys. 74 (1997) 319
[3] P. Braun-Munzinger, I. Heppe, and J. Stachel, Phys. Lett. B465 (1999) 15
[4] G. D. Yen and M. I. Gorenstein, Phys. Rev. C59 (1999) 2788
[5] F. Becattini, J. Cleymans, A. Keranen, E. Suhonen, and K. Redlich, hep-ph/0002267 and hep-ph/0011322
[6] G. Brown and M. Rho, Phys. Rev. Lett. 66 (1991) 2720
[7] T. Hatsuda and S. H. Lee, Phys. Rev. C46 (1993) R34
[8] T. Hatsuda, H. Shioomi, and H. Kuwabara, Prog. Theor. Phys. 95 (1996) 1009
[9] F. Klingl, N. Kaiser, and W. Weise, Nucl. Phys. A624 (1997) 527
[10] R. Rapp, G. Chanfray, and J. Wambach, Nucl. Phys. A617 (1997) 472
[11] M. Urban, M. Buballa, R. Rapp, and J. Wambach, Nucl. Phys. A641 (1998) 433
[12] R. Rapp and J. Wambach, Eur. Phys. J. A6 (1999) 415
[13] CERES Collab., G. Agakichiev et al., Phys. Rev. Lett. 75 (1995) 1272
[14] HELIOS/3 Collab., M. Masera et al., Nucl. Phys. A590 (1995) 93c
[15] W. Florkowski and W. Broniowski, Phys. Lett. B477 (2000) 73
[16] W. Florkowski and W. Broniowski, Proceedings of the International Workshop XXVIII on Gross Properties of Nuclei and Nuclear Excitations, Hirschegg, Austria, 2000, p. 275
[17] D. Zschiesche, L. Gerland, S. Schramm, J. Schaffner-Bielich, H. Stoecker, and W. Greiner, Nucl. Phys. A681 (2001) 34
[18] G.E. Brown, M. Rho, and C. Song, Nucl.Phys. A690 (2001) 184
[19] Particle Data Group, Eur. Phys. J. C15 (2000) 1
[20] E. Beth and G.E. Uhlenbeck, Physica 3 (1936) 729; Physica 4 (1937) 915
[21] R. Dashen, S.-K. Ma and H.J. Bernstein, Phys. Rev. 187 (1969) 345
[22] W. Weinhold, Zur Thermodynamik des πN-Systems, Diplomarbeit, GSI, Sept. 1995
[23] W. Weinhold, B. Friman, and W. Nörenberg, Acta Phys. Pol. B27 (1996) 3249; Phys. Lett. B433 (1998) 236