The effect of intermittency and damping on heat transfer of the flowing down turbulent liquid films

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Abstract. The model of calculation of thickness and heat exchange coefficient of turbulent film moving under the action of gravity and tangential friction stress of gas flow is constructed. The purpose of the work is to include additional damping caused by the turbulence intermittency and damping characteristic for small Reynolds numbers in the previously proposed simple model of the turbulent viscosity with the cubic law of attenuation in a viscous sublayer and with logarithmic velocity asymptotic away from the wall. The deviation of the calculation results from the experimental data available in the literature decreases when the additional damping is introduced.

1. Introduction
Flowing of the viscous liquid film on an inclined surface is accompanied by formation of nonlinear three-dimensional waves of rather large amplitude \( \frac{\delta_{\text{max}}}{\delta_{\text{min}}} \sim 5 \). According to Brauer's experiments [1] the transition to the turbulent film flow regime occurs at \( \text{Re} = 400 \). However, as shown in Adomeit, Renz [2], the first turbulent spots appear already at \( \text{Re} = 75 \). Bobylev et al. [3] demonstrate that at \( \text{Re} > 190 \) such turbulent spots are visualized on the film behind the ridges of large waves in the form of small-scale three-dimensional capillary ripple with wave amplitude of about one millimeter. These spots have a lifetime of a few hundredths of a second. Then they fall down to the back front of the big wave and disappear. At \( \text{Re} > 400 \), these spots become more chaotic, and their frequency of occurrence and life time increases [3]. For \( \text{Re} > 400 \) the whole surface of three-dimensional large waves is covered with chaotic capillary ripples reflecting small-scale vortex turbulence in the liquid film, Bobylev et al. [3]. Such transition to the turbulent flow regime in the film resembles a picture of transition to turbulence in the boundary layer through a sequential complication of unstable oscillatory modes: first, Tolmin-Schlichting waves appear at \( \text{Re} \approx \text{Re}_{cr} \), then they develop into three-dimensional nonlinear perturbations at some \( \text{Re} > \text{Re}_{cr} \), then their destruction and transition to localized turbulence spots, Emmons spots, occur [4]. Thus, the transition to turbulence in the flowing films, as in many other liquid flows, is accompanied by such a characteristic phenomenon as intermittency: alternating laminar sections on the film and areas of intensive turbulent fluctuations of velocity, which last for some time from the total time of observation. It should be noted that ripples of turbulent spots and the capillary precursor in front of large waves have the same wavelength (~ 1 mm), so in experiments it is not possible to reliably separate and measure fractions of time of turbulent and laminar mode of film flow.

One of the first models to describe the turbulent film was the Dukler model [5]. In fact, he proposed to transfer the Prandtl-Karman turbulence model [6], developed for pipes and channels to the
film. It is recognized that the Dukler model [5] poorly describes the film thickness and heat exchange in it because it does not take into account the influence of large amplitude waves. One can agree with this, but we believe that a very important disadvantage of the Dukler model [5] is that it ignores intermittency in turbulent exchange. In our previous model of turbulent exchange in a liquid film [7], intermittency was also neglected. This led to overestimating the calculated coefficients of turbulent viscosity and heat conductivity and to higher calculated values of turbulent film thicknesses and heat transfer coefficients in comparison with experimental ones. The purpose of our work is to account for intermittency in turbulent transfer through the introduction of a damping factor in the turbulent viscosity.

2. Model of turbulent viscosity and thermal conductivity

In a large number of works on turbulent films the model of turbulent transfer (turbulent viscosity) is based on Prandtl's concept of "mixing length" l, [6]

\[ \nu_r = \frac{l^2}{\rho} \frac{dU}{dy} \]

where, \( \kappa = 0.4 \) is the Karman constant, and \( A_0 = 27 \) is the Van Drist constant. It should be emphasized that the concept of "stirring path length", despite its wide use in semi-empirical models of turbulence, has no strict justification. It gives the correct behavior of velocity profiles in the wall (logarithmic) zone, but in the viscous sublayer and in the center of the channel calculations by formula (1) it requires correction. For example, according to formula (1), at small \( y_+ \), the coefficient of turbulent viscosity decreases as \( \nu_r \sim y_+^4 \). This contradicts the behavior of velocity pulsations in the viscous sublayer \( (u' \sim y_+, v' \sim y_+^2) \), from which follows

\[ \nu_r = -\bar{u}'\bar{v}' \left( \frac{dU}{dy} \right)^{-1} \sim \nu y_+^3. \]  

Therefore, we do not use the "mixing length" to solve the closure problem, but construct a model of turbulent exchange on a simple expression for the turbulent viscosity introduced in [7].

\[ \frac{\nu_r}{\nu} = \frac{y_+^3}{A + B y_+^2} \]  

with constants \( A = 1015 \), \( B = 11/\kappa = 2.5 \). This formula gives a true description in the viscous sublayer and logarithmic zone and then it will be modified near the free surface.

Formula (3) does not take into account intermittency or turbulence damping at the intermediate \( \text{Re} \). Therefore let us introduce the damping factor into the constant \( A \). The new formula has the form

\[ \frac{\nu_r}{\nu} = \frac{y_+^3}{A/ D + B y_+^2} \]  

For the factor \( D \), we enter the expression

\[ D = 1 - \exp \left[ -\left( \frac{\delta_0 - \delta_0}{\delta_0} \right)^2 \right] \] for \( \delta_0 > \delta_0 \), and \( D = 0 \) for \( \delta_0 < \delta_0 \),

where \( \delta_0 = \delta_{u'}/\nu \), \( \delta_0 \), \( \delta_0 \) are some constants, \( \nu = \sqrt{\tau_u/\rho} \) is the friction velocity, \( \tau_u \) is the tangential friction stress on the wall and \( \rho \) is the liquid density. Values are offered for constants: \( \delta_0 = 10 \), \( \delta_0 = 80 \). As will be shown below, they lead to correct calculations of film thickness. Note that according to formulas (4)-(5) at \( \delta_0 < \delta_0 \) the coefficient of turbulent viscosity becomes zero, that is, for very thin films a pure laminar flow exists. For \( \delta_0 > \delta_0 \) the value \( A/ D \), standing in the denominator of the formula (4) and responsible for the thickness of the viscous sublayer increases, since \( D < 1 \).
In practice, the case of motion of the liquid film driven with the gas flow is important. The influence of the moving gas flow will be described through the tangential friction stress \( \tau_L \) on the surface of the film or through the friction factor \( F = \tau_L / (\rho \cdot g \cdot \delta) \), where a characteristic viscosity-gravity scale \( \delta = (\nu^2 / g)^{1/3} \) is used. After integration by \( y \) the averaged equation for momentum conservation law takes the form

\[
(v_L + v_r) \frac{du}{dy} = \frac{\tau_L}{\rho_L} + g \left( \delta - y \right).
\]

The dimensionless velocity profile is obtained from (6) as an integral.

\[
U_+ = \frac{u}{v_+} = \int_0^y \left( \frac{1 - \Delta \cdot y_+ / \delta_+}{1 + \varphi(y_+)} \right) dy_+,
\]

where \( \Delta = \frac{\delta}{F + \delta}, \quad \delta_+ = \frac{\delta}{\delta_+}, \quad \varphi = v_r / v \).

Here, further modification of formula (4) for dimensionless turbulent viscosity can be considered. In experiments [8], near the free surface of liquid films flowing down an inclined plane for turbulent thermal conductivity, a linear attenuation with distance to the free surface was found.

\[
a_t / a = 1 - y_+ / \delta_+,
\]

where \( a_t, a \) are the turbulent and molecular thermal diffusivity, respectively. It was assumed that linear attenuation (8) near the film surface is also true for turbulent viscosity, [9]. However, this ratio was applied only for films without a gas flow. In the case of a non-zero friction on the film surface, we propose to introduce the relative friction factor (proposed also in [10] for the "mixing length") in expression (4) and to consider that near the film surface

\[
\frac{v_r}{v} = 1 - \frac{\tau(y)}{\tau_w} = 1 - \frac{y_+}{\delta_+} \Delta,
\]

that gives at \( F = 0 \) a linear attenuation of type (8), and for non-zero friction factor it gives a non-zero turbulent viscosity at the free surface. Thus, for films driven by the gas flow there is no similarity in the turbulent transfer of heat and momentum near the free surface of the liquid. It should be noted that in the viscous sublayer near the solid wall, the turbulent Prandtl number may also depend on the thermal properties of the wall, as shown in paper [11]. In the logarithmic layer there is also no complete similarity in the turbulent transfer of heat and momentum, because here, according to many papers, \( \text{Pr}_T = v_r / a_t = 0.9 \). Further we use this value in the calculation of heat transfer in the film.

By integrating the velocity profile (7) with the film thickness we obtain the fluid flow rate and the Reynolds number

\[
\text{Re} = \int_0^y U_+ dy_+ = \int_0^y \int_0^y \left( \frac{1 - \Delta \cdot y_+ / \delta_+}{1 + \varphi(y_+)} \right) dy_+.
\]

Conversion of double integral (10) into single integral gives the formula

\[
\text{Re} = \delta_+^3 \left[ (1 - \Delta) f_1(\delta_+) + \Delta f_2(\delta_+) \right],
\]

where two functions are introduced

\[
f_1 = \int_0^1 \frac{(1-z)}{1 + \varphi(\delta_+ z)} \, dz, \quad f_2 = \int_0^1 \frac{(1-z)^2}{1 + \varphi(\delta_+ z)} \, dz,
\]

defined by dimensional turbulent viscosity

\[
\varphi(y_+) = \frac{y_+^3}{A / D + B y_+^2} \left( 1 - \frac{\Delta y_+}{\delta_+} \right),
\]

with consideration of damping factor \( D \) and attenuation of turbulent transfer near the free surface.
Let us consider the heat transfer in the film, arising from evaporation or condensation, when the heat flow across the cross section of the film is constant, and the temperature on the surface of the film is equal to the saturation temperature $T_s$. The average equation of heat transfer after integration by $y$ has the form

$$
(a + a_r) \frac{dT}{dy} = \frac{q(y)}{\rho c_p} = \frac{q_w}{\rho c_p},
$$

\hspace{1cm} (14)

where $c_p$ is the heat capacity of the fluid. Introducing the characteristic temperature scale $\theta_s = q_w/(\rho c_p v_s)$ and dimensionless temperature $T_e = T/\theta_s$, we convert the previous equation to the following form

$$
(\Pr^{-1} + \Pr_T^{-1} \varphi_T(y_s)) \frac{dT}{dy_s} = 1,
$$

\hspace{1cm} (15)

where $\varphi_T(y_s) = \frac{y_s^3}{A + D + B y_s} \left(1 - \frac{y_s}{\delta_s}\right)$.

The integration of equation (15) in terms of film thickness gives the full temperature difference between the wall and the film surface

$$
\Delta T_s = \frac{T_w - T_s}{\theta_s} = \int_0^{\delta_s} \left(\Pr^{-1} + \Pr_T^{-1} \varphi_T(y_s)\right)^{-1} dy_s = \delta_s \Pr f_T(\delta_s, P),
$$

\hspace{1cm} (16)

where $f_T(\delta_s, P) = \frac{1}{\delta_s} \int_0^{\delta_s} \frac{dz}{(1 + P \varphi_T(\delta_s, z))}$.

In (17) the symbol $P$ indicates the ratio of molecular and turbulent Prandtl numbers $P = \Pr/\Pr_t$. Now we introduce the Nusselt criterion (dimensionless heat transfer factor).

$$
Nu_s = \frac{q_w}{(T_w - T_s)} \frac{\delta_s}{\lambda} = \frac{1}{\delta_s f_T(\delta_s, P)},
$$

\hspace{1cm} (18)

where the thermal conductivity of the fluid is $\lambda = \rho c_p a$.

Note that we use two dimensionless film thicknesses $\delta_0 = \delta/\delta_s$ and $\delta_s = \delta v_s/v$. They are connected by a simple ratio arising from the definition of these values:

$$
\delta_0^2 = \frac{\delta^2}{v^2} \left(\frac{\tau_e}{\rho g_s} \delta + \delta\right) = \delta_0^2 \left(F + \delta_0\right)
$$

\hspace{1cm} (19)

or $\delta_0 = \delta_0 \sqrt{F + \delta_0}$.

The auxiliary film thickness $\delta_0$ is included in the turbulent transfer model. A dimensionless film thickness $\delta_0$ can be calculated from equation (19) via the Cardano formula, but this solution of the cubic equation (19) can also be presented with an accuracy of 1% by an approximation proposed in [7]

$$
\delta_0 = \delta_0 \left[\left(\delta_0 + F \delta_0^{(2)}\right)^2 + F^2\right]^{-1/6}.
$$

\hspace{1cm} (20)

We now have a complete system of equations to calculate dimensionless thickness $\delta_0(Re, F)$ from equation (11), thickness $\delta_0(Re, F)$ from formula (21) and the Nusselt criterion $Nu_s(Re, F, P)$ from expression (18).

3. Calculated film thickness and heat transfer coefficient. Comparison with experimental data.

In Figs. 1 and 2, the calculated dimensionless thickness as a function of $Re$ and $F$ is compared with the experimental data of works [12-14]. The dashed lines here and further show the calculations of dimensionless thickness for cases without turbulence damping when $D=1$. 

4
It should be noted that the above definition of the Nusselt criterion is directly applicable to the case of evaporation of a liquid heated to the saturation temperature of the surrounding steam, as in experiments of [15]. In experiments for liquid film heating and steam condensation on the films [12,14] researchers used other definitions of the Nusselt criterion to represent a dimensionless heat transfer coefficient. When heating a film with a constant heat flux density, the right side of the heat balance equation (15) is changed: now instead of a unit on the right there is a dimensionless ratio of heat flux density expressed as \( q(y) / q_w = 1 - \psi(y_\infty) / \psi(\delta) \), where \( \psi(y_\infty) = \int_0^{y_\infty} u_y(y)dy \) is a dimensionless stream function. This expression should be inserted in the right side of equation (15). Therefore a weighted average fluid temperature \( T \) is introduced into the definition of the Nusselt criterion for heated film:

\[
\overline{Nu}_h = \frac{q_w \delta_e}{\overline{T}_f \lambda} = \frac{1}{\delta_e} \frac{1}{\bar{f}_f(\delta_e, P)}
\]

(22)

where \( \bar{f}_f(\delta_e, P) = \frac{1}{\delta_e} \int_0^{\delta_e} \left[ \frac{1 - \psi(y_\infty) / \psi(\delta)}{1 + \varphi(y_\infty)} \right] dy \).

(23)

For the case of vapor condensation the average Nusselt number is determined through averaged heat flux density and averaged temperature difference and brings us to harmonic average of the local Nusselt number, expressed by formula (18)

\[
\overline{Nu}_s = \frac{\int_0^L q_s dx \delta_e}{\int_0^L \left[ T_w - T_s \right] dx \lambda} = \frac{\Re_{\text{max}}}{\int_0^L d \Re / Nu_s}
\]

(24)

**Figure 1.** Comparison of dimensionless thicknesses for different factors \( F \) with experimental data of Ueda, Tanaka [12] and Alekseenko et al. [13]. The dashed lines show the case without damping when \( D=1 \).

**Figure 2.** Comparison of dimensionless film thickness with experimental data of Ueda, Kubo, Inoue [14]. The dashed lines show the case without damping when \( D=1 \).
In Figs. 3 and 4 the calculated Nusselt numbers are shown in comparison with experimental data of [12,14] for heating and condensation, respectively. In Fig. 5 and 6 the calculated Nusselt numbers are compared with experimental data [15] for film evaporation at $F=0$. In Fig. 6 the Nusselt numbers are calculated for the changed characteristic value $\delta_*$. Now instead of $\delta_* = 80$ it is dependent on the Kapitsa number $Ka = \frac{(\sigma / \rho)^3}{g v^4}$ via the formula $\delta_* = 0.07 Ka^{3/11}$. This substitution improves the comparison of the calculated and experimentally found data and reflects the effect of liquid properties (surface tension $\sigma$) on the waves in the film, which shifts the transition point to the turbulent flow regime.

**Conclusion**

The previously proposed model of turbulent exchange in the flowing film of liquid has been modified, namely, damping of turbulence in the area of transition to the turbulent flow regime has been taken into account and damping factors of attenuation near the free surface of the film have been introduced into the turbulent viscosity and thermal conductivity. As a result, a decrease in film thickness and heat transfer coefficient and a better agreement with the experimental data known from the literature have been obtained.
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