Annular Cherenkov High Gradient Wakefield Accelerator: Beam-Breakup Analysis and Energy Transfer Efficiency.

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Abstract. In this paper, we give a complete analytical solution for Cherenkov wakefields generated by an azimuthally asymmetric annular beam propagating in a coaxial two-channel dielectric structure. The transformer ratio of this type of structure is dramatically increased in comparison to a cylindrical wakefield accelerating structure. A particle-Green's function beam dynamics code (BBU-3000) to study beam breakup effects has been upgraded to incorporate annular drive beams and coaxial dielectric wakefield accelerating structures. Beam dynamics simulations of the annular drive beam with asymmetric charge distributions have been carried out to determine the sensitivity of this method to beam imperfections.

1. Introduction
A new application of microwave and THz Cherenkov radiation has been proposed and studied in the last decade to be used for high energy physics colliders and X-ray FELs, the Dielectric Wakefield Accelerator, or DWA [1-3].

In a general sense, a high gradient is desirable for a TeV level linear collider design because it can reduce the total linac length and hence the cost. Recently a high energy linear collider based on a short rf pulse (≈22 ns flat top), high gradient (≈267 MV/m loaded gradient), high frequency (26 GHz) dielectric two beam accelerator scheme has been proposed. The major parameters of a conceptual 3-TeV linear collider based on a DWA have been developed and are presented in reference [4].

![Figure 1](image1.png)

**Figure 1.** Partially filled dielectric wakefield accelerating structure excited by an annular drive beam. The dielectric tubes have inner radius $a$ (c) and outer radius $b$ (d). Regions 0 and 2 are vacuum; layers 1 and 3 are dielectric; and the outermost layer (not shown) is metal.
X-ray free-electron lasers (FELs) are expensive instruments and the accelerator contributes the largest portion of the cost of the entire facility. Using a high-energy gain dielectric wakefield accelerator instead of a conventional accelerator may facilitate reduction of the facility size and significant cost savings. It has been shown that a collinear dielectric wake-field accelerator can accelerate low charge and high peak current electron bunches to a few GeV energy with up to 100 kHz bunch repetition rate [5].

Dielectric loaded accelerator (DLA) structures using various dielectric materials [6] and excited by a high current electron beam or an external high frequency high power RF source have been under extensive study recently [1-3]. The basic wakefield RF structure is very simple - a cylindrical, dielectric loaded waveguide with an axial vacuum channel is inserted into a conductive sleeve. Following at a delay adjusted to catch the accelerating phase of the wakefield is a second electron (witness) beam. The witness beam is accelerated to high energy by the wakefield produced by the drive beam [1]. A series of proof of principle experiments have been successfully performed at Argonne’s Advanced Accelerator providing accelerating gradient in the range exceeding 100 MV/m at X-band [1-3,6]. THz wakefields of ~ GV/m magnitude range have been successfully generated by the UCLA-SLAC collaboration as well [7].

Energy transfer efficiency from the drive to witness bunches is a critical issue for wakefield acceleration techniques. The transformer ratio $R$ is defined as the ratio of the maximum energy gain of the witness bunch to the maximum energy loss of the drive bunch. There are two major classes of wakefield accelerator geometries, collinear and two beam. For a collinear wakefield accelerator, $R$ is less than 2 under very general conditions: linear media; a relativistic, longitudinally symmetric drive bunch; and identical paths through the system of both drive and witness beams [8-9]. A number of techniques have been proposed to overcome the transformer ratio limitation. Some of the methods that can be employed to obtain $R>2$ for the dielectric based accelerator include: a triangular longitudinal drive bunch profile [8]; a train of Gaussian drive bunches of progressively increasing charge (ramped bunch train) [10-11]; and use of a proton drive beam so that the particles can change positions within the bunch during deceleration [12].

Another way to achieve high transformer ratio is by designing separate drive and witness beam lines with different shunt impedances. This technique has been proposed in [13] for all-metal structures and initially proposed for dielectric based high gradient accelerator in [14], where a multichannel structure with an annular drive beam propagating through a coaxial outer vacuum channel and a witness beam through a central channel has been considered.

The cited paper [14] is obscure now but it is conceptually important to repeat its basic statements here. The structure geometry consists of a cylindrical cavity filled with a dielectric as presented in figure 1 (a). The structure proposed in [14] anticipates wakefield excitation with an annular drive beam, so the dielectric is separated from the wall by a vacuum region in which this beam is propagated. The metal wall is coated with a dielectric liner, and a small vacuum aperture in the center of the dielectric rod allows for the passage of a low current witness beam there.

In [14] it was correctly noted that because of the dielectric properties the Cherenkov radiation generated by the annular beam passing through the waveguide of this geometry will concentrate near the axis of the device. It was clearly stated that without the central rod a large transformer ratio can only be achieved by proper pulse shaping of the drive beam, and that including the rod results in an enhancement of the transformer ratio due to the geometrical effect.

The analytical solutions for the multizone coaxial geometry was obtained in [14] by solving for the fields in the regions 1–3, figure 1(b), and matching the corresponding boundary conditions. A 10 MeV linearly ramped bunch (100 kA, $t = 0.25$ ns) has been used in these simulations. The maximum accelerated field of 225 MV/m has been simulated providing 50 MeV energy gain of the witness beam in 37 cm with
average field of 130 MV/m. It should be noted that the ramped triangular bunch was also considered as a drive beam in reference [14].

Recently the coaxial scheme [14] was considered again for high-gradient wakefield acceleration, and detailed analytical studies for this type of coaxial dielectric-loaded structure have been presented [15–18]. In [15], an analytical solution for wakefields in a coaxial two-channel dielectric structure has been carried out, primarily concentrating on the transformer ratio $R$ optimization in X-band structures. It was found that, by appropriate choice of parameters, $R$ can be much greater than 2, the limiting value for collinear wakefield accelerators. Transformer ratio values as large as $R = 4.5$ were predicted.

In [16], the coaxial scheme initially proposed in [14] has been considered for the THz frequency range and for the SLAC FACET drive bunch parameters: an annular drive beam with the length of 30 μm, 6 nC charge and 5 GeV energy, while the simulated transformer ratio coefficient was found to be close to $R \approx 10$. The structure parameters correspond to THz frequency range wakefields (~ 0.1 mm witness beam aperture) and can be found in reference [16].

2. Analytic results and comparison with numerical simulations

The coaxial geometry proposed in [14] for transformer ratio enhancement of the dielectric based wakefield structures is considered here for GHz Ka-band [15] and THz acceleration [16–18]. At the same time, dynamics of the beam in structure-based wakefield accelerators leads to beam stability issues not ordinarily found in other machines. In particular, the high current drive beam in an efficient wakefield accelerator loses a large fraction of its energy in the decelerator structure, resulting in physical emittance growth, increased energy spread, and the possibility of head-tail instability for an off axis beam, all of which can lead to severe reduction of beam intensity [20]. Beam breakup (BBU) effects resulting from parasitic wakefields provide a potentially serious limitation to the performance of dielectric structure based wakefield accelerators as well [20-21].

Correspondingly, the transverse stability of the annular driver beam can be a critical issue for a coaxial high transformer ratio DWA [17]. We report on beam breakup study results for the coaxial Ka-band DWA currently planned to be tested at Argonne Wakefield Accelerator (AWA). Recently we have developed a particle-Green’s function beam breakup code (BBU-3000) that allows rapid, efficient simulation of beam breakup effects in advanced linear accelerators [21]. The goal of this work is foremost to design mitigation techniques for BBU and to test these concepts as part of an ongoing series of experiments at ANL/AWA, BNL/ATF and SLAC/FACET.

![Figure 2](image-url)

**Figure 2.** Simulations of wakefields generated by a point-like electron beam (table 2 Beam 1) passing through a coaxial structure (table 1 Structure 1). Solid line is CST simulation, dotted line is the BBU3000 simulation using the analytical Green's functions.
Table 1. Parameters of the coaxial waveguides used in the transverse beam dynamic analysis.

| #  | a, cm | b, cm | c, cm | d, cm | ε₁ | ε₂ | f, GHz (TM₀₂) |
|----|-------|-------|-------|-------|----|----|---------------|
| 1  | 0.2   | 0.60  | 1.00  | 1.40  | 4.76| 4.76| 13.84         |
| 2  | 0.2   | 0.32  | 1.35  | 1.41  | 9.80| 9.80| 28.02         |

Table 2. Parameters of the annular drive beams used in the transverse beam dynamic analysis

| #  | beam type                  | Q, nC | σ_r, cm | σ_α, cm | offset, cm | R, cm | W, MeV |
|----|----------------------------|-------|---------|----------|------------|-------|--------|
| 1  | transversely point-like    | 10    | 0.2     | 0        | 0.8        | -     | 10     |
| 2  | symmetrical annular beam   | 10    | 0.2     | 0.01     | 0.02       | 0.8   | 10     |
| 3  | asymmetrical annular beam  | 10    | 0.2     | 0.01     | 0          | 0.8   | 10     |
| 4  | transversely point-like    | 50    | 0.1     | 0        | 0.8        | -     | 14     |
| 5  | symmetrical annular beam   | 50    | 0.1     | 0.05     | 0.15       | 0.85  | 14     |
| 6  | asymmetrical annular beam  | 50    | 0.1     | 0.05     | 0.1        | 0.85  | 14     |

The coaxial, two-channel DLA structure of figure 1 has an inner vacuum region “0” (r < a) for the witness beam and an outer vacuum region “2” (b < r < c) for the drive beam. The drive beam used to excite the DLA is azimuthally asymmetric; this is necessary to excite dipole modes which cause beam instabilities. In this paper, we will concentrate on the dipole wakefield excitation due to a non-uniform ring of electrons. For an annular electron beam moving at speed v~c, the corresponding wave equations for the longitudinal field components $E_z$ and $H_z$ can be obtained from

$$\left\{ \begin{array}{l} \nabla^2 - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \right\} E_z = -4\pi e \left( \frac{\beta}{c} \frac{\partial n (r)}{\partial t} + \frac{\partial n (r)}{\partial z} \right), \\
\n\left\{ \begin{array}{l} \nabla^2 - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \right\} H_z = 0, \end{array} \right.$$  \hspace{1cm} (1)

where $n = \frac{Q}{e} \frac{\delta (z - v t)}{r} \frac{\delta (r - r_0)}{r} \delta (\theta - \theta_0)$ is the point-like beam charge density. Consequently one can define transverse components of wakefields by solving system equation (1). Using boundary conditions for the tangential components of electric and magnetic fields at the dielectric surfaces (a,b,c,d) we obtain system equations for unknown coefficients $[X]$:

$$[M][X] = [N].$$

Longitudinal components of the electric field generated by the electron beam passing through vacuum region “2” can be written as

$$E_z = \sum_{v=1}^{N_v} \sum_{n=1}^{N_n} \left( A_{v,n} I_v (\chi_2 k_{v,n} r) + B_{v,n} K_v (\chi_2 k_{v,n} r) \right) \cos (\nu (\theta - \theta_0)) \cos (k_{v,n} z)$$  \hspace{1cm} (2)
where $A_{n,m}$, $B_{n,m}$ – coefficients defined by structure geometry, dispersion equation roots, beam charge and transverse position, $k_{n,m}$ – dispersion equation roots $\det(M) = 0$, $\zeta$ – distance behind the bunch. By using Panofsky-Wenzel theorem

$$\frac{1}{r} \frac{\partial}{\partial \theta} E_z = \int F_0 dz, \quad \frac{\partial}{\partial r} E_z = \int F_r dz,$$

one can obtain transverse force components $F_r$ and $F_\theta$

$$F_r = \sum_{n=1}^{N_n} \sum_{m=1}^{N_m} \chi_2 (A_{n,m} I_{n}^\prime (\chi_2 k_{n,m} r) + B_{n,m} K_{n}^\prime (\chi_2 k_{n,m} r)) \cos(v(\theta - \theta_0)) \sin(k_{n,m} \zeta)$$

$$F_\theta = \sum_{n=1}^{N_n} \sum_{m=1}^{N_m} \chi_2 (A_{n,m} I_{n} (\chi_2 k_{n,m} r) + B_{n,m} K_{n} (\chi_2 k_{n,m} r)) \sin(v(\theta - \theta_0)) \sin(k_{n,m} \zeta).$$

The wakefield is a superposition of axisymmetric (TM-modes, $v = 0$) and non-axisymmetric (HEM-modes, $v = 1, 2, 3, \ldots$). The magnitude of wake field is determined essentially by the HEM-mode [20], whose contribution increases with the deflection of the beam from the waveguide axis. A comparison of calculations obtained using the expression (1) with the calculations takes into account the CST Particle Studio HEM-mode up to $v = 6$ (see Table 1). Comparisons were made for a point-like (in the transverse coordinates) bunch passing the structure along region “2”. Figure 2 shows that the results obtained using analytical formulas that are consistent with numerical simulations by CST Particle Studio.

3. Beam dynamics simulations of an annular driving beam in the coaxial DWA structure

BBU-3000 represents an approach to beam dynamics computations which is complementary to the usual electromagnetic PIC (Particle in Cell) approach. Particle pushing is done in the same fashion as a PIC code but the wakefields caused by the charged particles are computed using the known analytic expressions for the Green's functions in a dielectric tube [20]. Details of the scalar code for the Gaussian shaped beam have been published elsewhere [21]. Here we focus on the recent work on the annular electron beam simulation.

It should be noted that for the annular beam the offset value is limited in comparison with ordinary Gaussian beam because of intrinsic large diameter (relatively to a witness beam aperture) of a vacuum channel of the coaxial DWA structure. Consequently, the beam dynamics of the annular drive beam will be dominated by dipole mode fields, or HEM modes with $v=1$ with the field structure presented in figure 3. In this paper, the beam breakup of the annular driving bunches has been studied with respect to its both azimuthal and radial asymmetries.
Figure 3. Electric field patterns of the dipole modes generated by an annular beam passing through coaxial dielectric wakefield structure presented in figure 1, region 2: (a) radial $F_r$ component; (b) azimuthal $F_\theta$ component, region 2.

Figure 4. Beam dynamic simulations of an annular beam passing through coaxial dielectric wakefield structure presented in figure 1, region 2: (a) azimuthally symmetric beam with offset (table 1, structure #1; table 2, beam # 2); (b) azimuthally symmetric beam with no offset (table1,structure # 1; table 2, beam # 3).

Figure 4 shows the results of BBU300 simulation for the azimuthally symmetric annular beam with offset and the beam with no offset but with azimuthal charge asymmetry. The beams pass through the coaxial dielectric wakefield structure presented in figure 1. One can see radial deflection for the azimuthally symmetric beam caused by center of mass shift (offset of the beam) while the azimuthal deflection is still compensated by the charge density symmetry, figure 4 (a). At the same time, the azimuthal asymmetry generates $F_\theta$ components strong enough to increase dramatically the charge density along the line of structure center – maximum of charge density distortion, figure 4. Figure 5 (a) presents the transverse deflecting field vs. radial position of the beam particle: one can see the flat region where there is no force increase. figure 5 (b) shows the deflecting field of the annular beam vs. its radial offset that shows the same behavior as those of an ordinary Gaussian beam in a cylindrical tube-like DWA structure [20].
Figure 5. (a) Transverse deflecting field vs. radial position of the beam particle in region 2 for the structure #1 of table 1, for the beam #4 of table 2; (b) Maximal deflecting field vs. radial offset for the structure #1 of table 1, beam #5 of table 2.

Figure 6. Beam dynamic simulations of an annular beam passing through coaxial structure presented in figure 1, azimuthally asymmetric beam with 0.1 cm offset to the asymmetry direction; structure #2 of table 1, beam #6 of table 2.

Figure 6 presents simulation results for the annular beam passing through coaxial structure of figure 1. It is azimuthally asymmetric beam with 0.1 cm offset to the asymmetry direction; note strong deflection to both inner and outer dielectric surfaces. The same effect is more clearly seen in figure 7, where an annular beam passing the structure of figure 1 with 0.1 cm offset opposite to the asymmetry direction. It should be noticed azimuthal concentration of a charge density and its strong attraction by the inner dielectric rod. The length of the structure with no focusing is limited to 50 cm for both of the structures, figures 6-7.
Figure 7. Beam dynamics simulations of an annular beam passing through coaxial dielectric wakefield structure presented in figure 1, azimuthally asymmetric beam with 0.1 cm offset opposite to the asymmetry direction; structure # 2 of table1, beam # 6 of table2.

Acknowledgments. This work is supported by the SBIR Program of the US Department of Energy and RFBR award.

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