Heat Transfer in the Enclosing Structures of a Blast Furnace. Part 2. Solution of boundary-value problems of heat transfer

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Abstract. This work is the second part out of a series of articles with a shared name “Heat transfer in the shielding structure of the blast furnace”. In the first part with the subtitle “Problem Statement and background for calculation” multilayer shielding structures of the blast furnace were examined. A description of the layers that are a part of these structures is presented. The lining layer is focused on. The process of cast iron smelting and the temperature zones of the individual layers of the blast furnace internal environment are briefly described. Based on A.V.Lykov’s theory original equations were analyzed, they describe interconnected heat and mass transfer in a solid object as applied to the given problem – appropriate process depiction aimed at the further rational development work on the multilayer shielding structures of the blast furnace. A priori the shielding mathematically is considered as an unlimited plate. In the second part boundary problems of heat transmission are examined in separate layers of the structure with different boundary conditions, solutions for them are introduced which are the basis of development of the mathematical model of the nonstationary process of heat transfer in the multilayered shielding structure. The mathematical model will be introduced in the third part of the series of articles.

1. Introduction

LIST OF SYMBOLS

- \( t \) – temperature,
- \( \tau \) – time,
- \( \lambda_{q,m} \) – coefficients of heat and mass conductivity,
- \( \alpha_{q,m} \) – coefficients of heat exchange and mass,
- \( \delta \) – thermogradient coefficient, referred to the difference of moisture content
- \( a_q = l_q / c_q \delta_0 \) – the coefficient of potential diffusivity of heat transmission (temperature conductivity),
- \( l_q \) – the coefficient of heat conductivity,
- \( c_q \) – specific thermal capacity,
- \( \delta_0 \) – the density of the dry part of the object,
- \( c_m \) – specific isothermal mass capacity,
2. Methods

Physically the process can be described the following way (Fig. 1a): the multilayered (to simplify matters we examine a three layered structure) wall is stationary so the temperature distribution take the form: \( t_1(x,0) = t_2(x,0) = t_3(x,0) = t_0 \);

At \( \tau = 0 \) from the left side of the wall the heat flow \( q \) is applied, under its effect the first layer starts to warm up (Fig.1b). The change in the temperature field is characterized by curves 1 and 2. Incidentally, the second and third layers stay with the temperature \( t_0 \).

At \( \tau_1^* \) (curve 3) the heat wave reaches the boundary between the first and second layer, and in this place the temperature gradient appears:

\[
\text{grad} t[\delta, \tau_1] = -\frac{\partial t[\delta, \tau_1]}{\partial x},
\]

After this instant of time the temperature field will make way further inside the wall, as curves 4 and 5 show. The temperature of the third layer will still be \( t_0 \) till the moment \( \tau_2^* \) when the heat wave will reach the joint between the second and third layers. And so on.

At \( \tau_3^* \) when the heat wave will reach the external boundary of the last layer of the shielding, all layers of the shielding become involved in the process of heat transfer and this instant of time can be called sufficient for determining the heat transmission resistance (R) when the process is nonstationary. When the duration of the process is long enough (theoretically when \( \tau \to \infty \)) inside the structure will form a stationary temperature field (the zigzag line 7 on fig.1c), the value of which will be used for the calculation of \( R \). For the considered model system this will take the form of the relation:

\[
R = \frac{1}{a_1} + \frac{\delta_1}{\lambda_1} + \frac{\delta_2}{\lambda_2} + \frac{\delta_3}{\lambda_3} + \frac{1}{a_2} \frac{t_{\text{int}} - t_{\text{ext}}}{q}
\]

where \( t_{\text{int}} \) – temperature of the internal environment of the furnace;

\( t_{\text{ext}} \) - temperature of the external environment of the furnace.

The proposed technique allows to directly calculate the value of \( R \) out of the nonstationary temperature field with the method of the solution of the inverse problem.

The set of equations (18, 20, 21, 23, 24) [1] is nonlinear and analytically unsolvable. For the problem solution we will use the combined method of solving the boundary problems of heat transmission, which is based on combining elements of analytical and numerical solutions. [See 2… 10]

The idea of the method is that the whole process of heat transmission is divided into a set of small time intervals. Within the limits of each interval we assume that the temperature is constant on the
boundaries II and III and the density of the heat flow through the surfaces that are in contact is also constant i.e. an ideal heat contact.

The general problem is divided into 3 autonomous but interconnected problems.

**Problem 1.** The heat transfer in layer 1 with the boundary conditions of the third kind, which takes into account the convective exchange on the boundary I, and first kind, which characterizes the consistency of temperature on boundary II of the layers 1 and 2.

**Problem 2.** The heat transfer in layer 2 with the boundary conditions of the second kind, which represents the consistency of density of the heat flow through the II boundary, and first kind, which characterizes the consistency of temperature on boundary III.

**Problem 3.** The heat transfer in layer 3 with the boundary conditions of the second kind on the boundary III and boundary conditions of the third kind, which characterizes the heat transfer between the surface of layer 3 on the boundary IV with the external environment under Newton's law.

Each of these problems is solved analytically. The solution to the general problem of nonstationary heat conductivity can be found as a result of linking these analytical solutions on each time interval. This allows for the going from the boundary conditions of the fourth kind to the boundary conditions of the first and second kind on the surfaces of the joints of layers 1 and 2, 2 and 3, which simplifies the solution of the problem.

**Problem 1.** (See fig.2)

Mathematically the problem of heat conductivity for layer 1 can be written down in the form:

\[
\frac{\partial t(x,\tau)}{\partial \tau} = a \frac{\partial^2 t(x,\tau)}{\partial x^2}; \quad (0 \leq x \leq \delta_1) \]

Starting conditions:

\[
t(x,0) = t_0(x) \]

Boundary conditions:

\[
t(\delta_1, \tau) = t_{\delta_1} \]

\[
-\lambda \frac{\partial t(0,\tau)}{\partial x} = \alpha int [t_{ext} - t(0,\tau)] \]

The boundary condition 3 represents that the surface II of joint between layers 1 and 2 has the constant temperature \(t_{\delta_1}\). Condition (4) characterizes the convective heat exchange on the external surface I.

The solution to an analogous problem is presented in [9].

For the convenience of the transformations the system (1)...(4) can be transformed to a dimensionless form:
\[
\frac{\partial T(\bar{x}, F_0)}{\partial F_0} = \frac{\partial^2 T(\bar{x}, F_0)}{\partial \bar{x}^2}; \quad (5)
\]

\[
T(\pi, 0) = T_0; \quad (6)
\]

\[
T(1, F_0) = T_0; \quad (7)
\]

\[
\frac{\partial T(0, F_0)}{\partial \bar{x}} = Bi \cdot T(0, F_0). \quad (8)
\]

The general solution of the problem with the dimensionless variables has the form [9]:

\[
T(\bar{x}, F_0) = \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{Bi \sin(\mu_n (\pi - 1))}{Bi + \cos^2 \mu_n} \exp(-\mu_n^2 F_0) \times \]

\[
\int_{\xi} \left[ \sum_{n=1}^{\infty} \frac{Bi \sin(\mu_n (1 - \xi))}{\mu_n^2} d\xi - \frac{T_0}{\mu_n} \right]; \quad (9)
\]

where:

\[
T(\bar{x}, F_0) = \frac{t(x, \tau) - t_{\delta}}{t_{\delta}}; \quad (10)
\]

\[
\bar{x} = x/\delta \quad - \text{dimensionless coordinate}; \quad (11)
\]

\[
F_0 = at/\delta^2 \quad - \text{Fourier number}; \quad (12)
\]

\[
Bi = a_{\text{int}} \delta/\lambda \quad - \text{Biot number.} \quad (13)
\]

\[
\mu_n \quad - \text{the roots of the characteristic equation } \mu_n = -Bi \tan(\mu_n); \quad (14)
\]

\[
\xi \quad - \text{current dimensionless coordinate.}
\]

Notice: Index (1) that represents the correlation of the computations with the first layer to simplify matters is discarded. Problem 2 (See fig. 3)

Conditions of problem 2, according to the simplifications we introduced earlier, can be stated in the form:

Heat transfer in an unlimited plate with combined boundary conditions of the second kind on the surface II and first kind on the surface III and nonuniform starting conditions.

The origin of coordinates will be set on the joint of the layers i.e. on the surface II.

Mathematically the problem of heat transfer for the second layer can be stated as:

\[
\frac{\partial t(x, \tau)}{\partial \tau} = \frac{a}{\delta^2} \frac{\partial^2 t(x, \tau)}{\partial x^2}; \quad |0 \leq x \leq \delta_2|; \quad (15)
\]

Starting conditions:

\[
t(x, 0) = t_0(x); \quad (16)
\]

Boundary conditions:

\[
-\lambda \frac{\partial t(0, \tau)}{\partial x} = q_0; \quad (17)
\]

\[
t(\delta_2, \tau) = t_{\delta}; \quad (18)
\]

The problem (15)…(18) we will solve through dimensionless variables, for this we will introduce:

\[
T(\bar{x}, F_0) = \frac{t(x, \tau) - t_{\delta}}{t_{\delta}}; \quad (19)
\]

\[
\bar{x} = x/\delta; \quad F_0 = at/\delta^2. \quad (20)
\]

Then out of (15) we will receive:

\[
\frac{\partial}{\partial \tau} \left[ \frac{t(x, \tau) - t_{\delta}}{t_{\delta}} \right] = \frac{a}{\delta^2} \frac{\partial^2 t(x, \tau) - t_{\delta}}{\partial x^2}; \quad (21)
\]

Or regarding (19) and (20):

\[
\frac{\partial T(\bar{x}, F_0)}{\partial \tau} = \frac{a^2 \partial^2 T(\bar{x}, F_0)}{\partial \bar{x}^2}. \quad (22)
\]
The general solution of the problem is acquired in [6] and takes the form:

$$
T(\bar{x}, Fo) = Ki \left[ 1 - \bar{x} \right] - \sum_{n=1}^{\infty} \frac{2 \cos[\mu_n \bar{x}]}{n^2} \exp \left( -\frac{\pi^2 n^2}{4} Fo \right) \int_0^1 T_0(\xi) \cos[\mu_n \xi] d\xi.
$$

(23)

**Characteristic equation** \( \cos \mu_n = 0 \).

**Problem 3.** (see fig.4)

Conditions of problem 2, according to the simplifications we introduced earlier, can be stated in the form: Heat transfer in the unlimited plate with combined boundary conditions of the second kind for the surface III and of the first kind for the surface IV and nonuniform starting conditions.

The origin of coordinates will be set on the joint of the layers i.e. on the surface III.

Mathematically the problem of heat transfer for the third layer can be stated as:

$$\frac{\partial t(x, \tau)}{\partial \tau} = a \frac{\partial^2 t(x, \tau)}{\partial x^2}; \quad 0 \leq x \leq \delta_3$$

(24)

Starting conditions:

$$t(x, 0) = t_0(x)$$

(25)

Boundary conditions:

$$-\lambda \frac{\partial t(0, \tau)}{\partial x} = q_3;$$

$$-\lambda \frac{\partial t(0, \tau)}{\partial x} = B_i T(0, Fo)$$

(26)

(27)

The problem (24)...(27) we will solve through dimensionless variables, for this we will introduce:

Equation (24) in the dimensionless form will take the form:

$$\frac{\partial T(\bar{x}, Fo)}{\partial \tau} = \frac{\partial^2 T(\bar{x}, Fo)}{\partial \bar{x}^2}. \quad (28)$$

After transforming conditions (26) and (27) to the dimensionless form, we will receive:

$$-\frac{\partial T(0, Fo)}{\partial \bar{x}} = Ki,$$

where

$$Ki = \frac{q_3 \delta_3}{\lambda_3 t_s}$$

Kirpichev number;

$$-\frac{\partial T(0, Fo)}{\partial \bar{x}} = B_i T(0, Fo),$$

where

$$B_i = \alpha_{ext} \delta / \lambda_3$$

Biot number;

(29)

(30)
Thus a solution is necessary for the problem (28) with boundary conditions (29) and (30) and the starting conditions:

\[ T(\bar{x},0) = T_0(\bar{x}) = t(\bar{x},0) = \frac{t(x,0) - t_{\text{ext}}}{t_{\text{ext}}}; \]

(31)

The general solution is acquired in [11] and has the form:

\[ T(\bar{x},Fo) = -K_i \sum_{n=1}^{\infty} \frac{2Bi \cos(\mu_n \bar{x})}{\mu_n^2 Bi + \sin^2 \mu_n} \exp(-\mu_n^2 Fo) \times \]

\[ + \sum_{n=1}^{\infty} \frac{2Bi \cos(\mu_n \bar{x})}{\mu_n^2 Bi + \sin^2 \mu_n} \exp(-\mu_n^2 Fo) \int_0^1 T_0(\xi) \cos(\mu_n \xi) d\xi. \]

(32)

Characteristic equation \( \mu_n = Bi \ctg(\mu_n); \)

We will determine the value of the temperature gradient differentiating (9) with respect to \( \bar{x} : \)

\[ \frac{\partial T(\bar{x},Fo)}{\partial \bar{x}} = T_0 \left( \frac{Bi}{Bi + 1} \right) + 2 \sum_{n=1}^{\infty} \frac{Bi \mu_n \cos(\mu_n(1 - \bar{x}))}{Bi + \cos^2 \mu_n} \exp(-\mu_n^2 Fo) \times \]

\[ \int_0^1 T_0(\xi) \sin(\mu_n(1 - \bar{x}))) d\xi \frac{T_0}{\mu_n}. \]

(33)

We will determine the value of the temperature gradient differentiating (23) with respect to \( \bar{x} : \)

\[ \frac{\partial T(\bar{x},Fo)}{\partial \bar{x}} = -K_i + K_i \sum_{n=1}^{\infty} \frac{8\mu_n \sin(\mu_n \bar{x})}{\pi \mu_n^2} \exp\left(-\frac{\pi^2 \mu_n^2}{4} Fo\right) \times \]

\[ - \sum_{n=1}^{\infty} \frac{2\mu_n \sin(\mu_n \bar{x})}{\mu_n^2 Bi + \sin^2 \mu_n} \exp\left(-\frac{\pi^2 \mu_n^2}{4} Fo\right) \int_0^1 T_0(\xi) \cos(\mu_n \xi) d\xi. \]

(34)

We will determine the value of the temperature gradient differentiating (32) with respect to \( \bar{x} : \)

\[ \frac{\partial T(\bar{x},Fo)}{\partial \bar{x}} = +K_i \sum_{n=1}^{\infty} \frac{2Bi \mu_n \sin(\mu_n \bar{x})}{\mu_n^2 Bi + \sin^2 \mu_n} \exp(-\mu_n^2 Fo) \times \]

\[ - \sum_{n=1}^{\infty} \frac{2Bi \mu_n \sin(\mu_n \bar{x})}{\mu_n^2 Bi + \sin^2 \mu_n} \exp(-\mu_n^2 Fo) \int_0^1 T_0(\xi) \cos(\mu_n \xi) d\xi. \]

(35)

3. Results

Equations (9, 23, 32, 33, 34, 35) are basis of the calculations of the nonstationary heat transfer in multilayered structures.

In the third part of this research a computation algorithm for the nonstationary process of heat transfer through the multilayered shielding structure of the blast furnace will be introduced and in the fourth part the specific examples of calculations and recommendations on designing.

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