Neutrinos: Windows to New Physics

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Abstract.
After briefly reviewing how the symmetries of the Standard Model (SM) are affected by neutrino masses and mixings, I discuss how these parameters may arise from GUTs and how patterns in the neutrino sector may reflect some underlying family symmetry. Leptogenesis provides a nice example of how different physical phenomena may be connected to the same neutrino window of physics beyond the SM. I end with some comments on the LSND signal and briefly discuss the idea that neutrinos have environment dependent masses.

1. Symmetries of the Standard Model
The Standard Model (SM) is invariant under local transformations of an $SU(3) \times SU(2) \times U(1)$ group, which is spontaneously broken to the subgroup $SU(3) \times U(1)_{em}$. However, in addition, at the Lagrangian level the SM is invariant under four global $U(1)$ transformations, associated with baryon number $B$ and the individual lepton numbers of each generation of leptons $L_i$. It is convenient to group these symmetries into the set $U(1)_{B+L}; U(1)_{B-L}; U(1)_{L_i-L_j} [i, j = (1, 2, 3)]$, where $L$ is the total lepton number, $L = \Sigma_i L_i$. The last three symmetries are exact, but $U(1)_{B+L}$ has an electroweak anomaly, so it is not a symmetry at the quantum level.

The existence of neutrino masses and mixings, inferred from neutrino oscillation experiments, alters these symmetry patterns and, hence, provides evidence for physics beyond the SM. Neutrino mixings imply that individual lepton number is not conserved. In effect, because of the observed mixings one loses the two SM global symmetries associated with $U(1)_{L_i-L_j}$. The existence of neutrino masses also affects the symmetry structure of the SM, indicating the presence of new interactions which most likely violate $U(1)_{B-L}$.

2. Disquisitions on Neutrino Masses
Since neutrinos have zero charge, besides the usual Dirac (particle-antiparticle) mass neutrinos can have also a Majorana (particle-particle) mass. Using $\nu^c = C\nu^\dagger$, the most general mass term for neutrinos has the structure:

$$L_{mass} = -\frac{1}{2}(\nu_L^c, \nu_R) \begin{pmatrix} m_T & m_T^T \\ m_D & m_S \end{pmatrix} \begin{pmatrix} \nu_L \\ \nu_R \end{pmatrix} + \text{h.c.}$$

It is clear from the above that $m_T$ and $m_S$ are Majorana mass terms, while $m_D$ is a Dirac mass term. These masses break the SM symmetries differently, since $m_T$ is an $SU(2)$ triplet.
with (B-L)-charge \( Q_{B-L} = -2 \); \( m_D \) is an \( SU(2) \) doublet with (B-L)-charge \( Q_{B-L} = 0 \); while \( m_S \) is an \( SU(2) \) singlet with (B-L)-charge \( Q_{B-L} = 2 \). Because of these different transformation properties, each of these mass terms is connected to different physics. What combination of \( m_T \), \( m_S \), and \( m_D \) determines the tiny (\( m_\nu < 1 \text{ eV} \)) neutrino masses observed is a function of what this underlying physics is. Below, I discuss each of these mass terms in turn.

Perhaps the most conservative stance to adopt is to admit the existence of a right-handed neutrino \( \nu_R \), but assume that (B-L) remains a symmetry of nature. Then the only neutrino mass term is \( m_D \), which arises from the Yukawa coupling \( \Gamma_\nu \) of the left-handed lepton doublet to \( \nu_R \) and the Higgs field \( \Phi \). Because the observed neutrino masses are so small, one must then understand what physics gives \( m_D = \Gamma_\nu < \Phi > \ll m_\ell = \Gamma_\ell < \Phi > \).

One interesting possibility is to invoke extra dimensions. \[2\] I want to illustrate this idea by considering a \( d = 5 \) theory compactified to a space of radius \( L \). In this theory, before compactification, the Einstein action in 5-dimensions is characterized by a Planck mass \( M^* \). After compactification the 4-dimensional Planck mass \( M_P \) is related to its 5-dimensional counterpart through the relation \( M_P^2 = M^* L^3 \). In such a 5-dimensional theory, it is natural to assume that while the SM particles live on the \( D3 \) brane, the right-handed neutrinos live in the 5-dimensional bulk. Then, the neutrino action involving \( \nu_R \) reads

\[
S = M^* \int d^4x \int dy \sqrt{-g_{5\nu}} \gamma^\mu \partial_\mu \nu_R + \int d^4x \Gamma_5^{\nu} \Phi \nu_L + h.c. \quad (2)
\]

The first term in this action, after compactification becomes

\[
M^* \int d^4x \int dy \sqrt{-g_{4\nu}} \gamma^\mu \partial_\mu \nu_R \rightarrow M^* L \int d^4x \sqrt{-g_{4\nu}} \gamma^\mu \partial_\mu \nu_R. \quad (3)
\]

Thus, to get the correct kinetic energy for the right-handed neutrinos, one must rescale the \( \nu_R \) field by a factor \((M^* L)^{-1/2} \equiv M^*/M_P\). This rescaling, effectively replaces the Yukawa coupling \( \Gamma_5^{\nu} \), which one presumes is of the same order of magnitude as \( \Gamma_\ell \), by \( \Gamma_\nu \equiv \frac{M^*}{M_P} \Gamma_5^{\nu} \). Hence, in these theories, one gets naturally a large reduction for the right-handed neutrino Yukawa couplings.

There are many issues, however, which remain open. For instance, why should (B-L) be conserved, given that a right-handed neutrino exists. Given that \( m_S \) breaks no symmetries, besides (B-L), why is this mass term not permitted in the theory? In addition, one must worry about the tower of Kaluza-Klein states associated with the fact that \( \nu_R \) lives in the bulk. What is the effect of this tower of states and are these states dangerous? This suggests, that perhaps a better alternative is to have no \( \nu_R \) states in the theory at all.

If there are no right-handed neutrinos, neutrino masses can still arise from a triplet mass. Because these masses break \( SU(2) \), \( m_T \) is proportional to the VEV of a Higgs field which transforms as an \( SU(2) \) triplet. This field is either elementary, \( \bar{T} \), or composite, \( \bar{T}_{\text{eff}} \equiv \Phi^T C \Phi/\Lambda \), where \( \Lambda \) is a new mass scale. In the first case, small neutrino masses arise either because the triplet Yukawa coupling is very small or because \( \langle \bar{T} \rangle \ll \langle \Phi \rangle = v_F \sim 250 \text{ GeV} \). Because neither possibility is easily explainable, the second alternative is much more natural. \[3\] It ascribes the smallness of neutrino masses to the presence of the large scale \( \Lambda \), associated with (B-L)-breaking, entering in the composite operator \( \bar{T}_{\text{eff}} \). In this case, neutrino masses are given by the seesaw formula

\[
m_\nu = m_T \sim \frac{v_F^2}{\Lambda}. \quad (4)
\]

and their presence reflects physics well beyond the electroweak scale, since \( \Lambda >> v_F \).

One can arrive at similar seesaw formula from singlet breaking, \[4\] provided \( m_S >> m_D \). Because \( m_S \) carries no SM quantum numbers, its scale is unconnected to \( v_F \) and reflects directly...
the scale of $U(1)_{B-L}$ breaking. If a hierarchy exists, $m_S >> m_D$, then the neutrino mass matrix

$$M = \begin{bmatrix} 0 & m_D^T \\ 1 & m_S \end{bmatrix} \tag{5}$$

has both large, $M_N = m_S$ and small eigenvalues $M_\nu = -m_D^T (m_S)^{-1} m_D$.

Let me summarize the principal lessons one has learned from the existence of neutrino masses and mixings. They are two-fold:

i.) There are additional interactions beyond the SM, which break all the global symmetries of the standard model.

ii.) Most likely, the small neutrino masses observed are related to the large scale where $U(1)_{B-L}$ breaks down.

In relation to the second point, it is not really possible to tease apart whether $M_\nu$ arises from a triplet composite operator $[M_\nu = m_T \sim \langle \mathbf{T}^\text{eff} \rangle \sim v_F / \Lambda]$, or large right-handed neutrino masses $[M_\nu = -m_D^T (m_S)^{-1} m_D]$, or a combination of both mechanisms $[M_\nu = m_T - m_D^T (m_S)^{-1} m_D]$.

### 3. Recondite Physics

Looking beyond the SM for hints to the origins of neutrino masses and mixing, there are three fruitful avenues to follow:

i.) A top-down approach, where one looks for possible new symmetries and symmetry breakings associated with giving neutrinos masses.

ii.) A bottom-up approach, where one tries to uncover patterns of observed masses and mixings from what has been observed experimentally.

iii.) A more pragmatic approach, where one looks for connections between physical phenomena.

Given the limitations of time and space I will not attempt to be comprehensive here, but will illustrate each of these approaches by means of some relevant examples. [5]

#### 3.1. Extended Symmetries

Interesting insights on neutrinos emerge in Grand Unified Theories (GUTs), [6] where quarks and leptons are in the same multiplet(s). In the simplest GUT $SU(5)$ the right-handed neutrino transforms as a singlet under the group. Using that $\psi^c_R$ transforms as a left-handed state, in $SU(5)$ the left-handed quark and lepton multiplets are organized in the following multiplets:

$$10 = \{ Q_L, u^c_R, \ell^c_R \} \ ; \quad 5 = \{ d^c_R, L_L \} \ ; \quad 1 = \{ \nu^c_R \} \tag{6}$$

As a result, because $\nu_R$ is a singlet, there are no constraints on the mass term $m_S$. Hence, there is no direct connection between the scale of $SU(5)$-breaking, $M_{GUT}$, and that of $(B-L)$-breaking, given by $m_S$.

Matters are different in $SO(10)$, where all quarks and leptons, including $\nu_R$, are in the same representation:

$$16 = \{ Q_L, u^c_R, d^c_R, L_L, \ell^c_R, \nu^c_R \} \tag{7}$$

Because $U(1)_{B-L}$ is contained in $SO(10)$, for this GUT now $U(1)_{B-L}$ is a local, not a global, symmetry. Thus the singlet mass $m_S$ of right-handed neutrinos is naturally related to the GUT breaking scale, if $(B-L)$ is broken at that scale. However, in $SO(10)$ there can be an initial symmetry breaking of the GUT group which preserves $(B-L)$, with this symmetry then being broken at an intermediate scale. An example is provided by the Pati-Salam [7] symmetry breakdown chain $SO(10) \rightarrow SU(3) \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \rightarrow SU(3) \times SU(2) \times U(1)$. Hence, in $SO(10)$ models it is possible to contemplate a range of masses for $m_S$, ranging from perhaps $10^{10}$ GeV to $10^{16}$ GeV.
Because the product of two 16-dimensional representations in \( SO(10) \) contains a 10, a symmetric 126, and an antisymmetric 120 representations, one can consider Yukawa interactions containing \( SO(10) \) Higgs fields in these representations. In particular the Higgs fields transforming according to the 126 representation, \( 126_H \), contains a SM singlet field carrying (B-L) charge, so that naturally \( m_S \sim <126_H> \). The simplest \( SO(10) \) model contains just a 10\(_H\) and a 126\(_H\) and, using either \( U(1)_{\text{PQ}} \) or supersymmetry to eliminate any \( \overline{10}_H \) couplings, has just two Yukawa coupling terms leading to quite a restrictive mass spectrum, so that

\[
L_{\text{Yukawa}} = \Gamma_D 16.16.10_H + \Gamma_S 16.16.126_H, \tag{8}
\]

with \( m_D = \Gamma_D <10_H> \sim <m_S = \Gamma_S <126_H> \).

A different class of \( SO(10) \) models, rather than using an elementary 126\(_H\), replaces this Higgs field by a composite Higgs term made up of low-dimensional fields: \( 126_H \to (16_H \ast 16_H)/M \). Then, even if \( U(1)_{\text{B-L}} \) is broken at the GUT scale, \( m_S \) is at an intermediate scale: \( m_S \sim M_{\text{GUT}}^2/M \). Irrespective of whether \( 126_H \) (and other Higgs) are composite or not, to obtain a realistic spectrum and mixings for the charged fermions and neutrinos, in general one imposes some additional flavor symmetry on the models - typically \( U(1) \times D \), where \( D \) is some discrete symmetry. The remnants of these symmetries should then be visible in the mass matrices. Some examples of \( SO(10) \) models which produce realistic quark, charged lepton, and neutrino masses are discussed by Mohapatra \[10\] in this meeting.

### 3.2. Patterns in the Neutrino Sector

A different approach (more bottom-up) is to try to divine from the data on neutrino masses and mixings the structure of the underlying theory. In a 3 neutrino framework, ignoring the LSND \[11\] result for now, one has \[12\]

\[
|\Delta m_{23}^2| = \Delta m_{\text{atmos}}^2 = (2.4 \pm 0.3) \times 10^{-3} \text{eV}^2, \tag{9}
\]

\[
|\Delta m_{21}^2| = \Delta m_{\text{sol}}^2 = (7.9 \pm 0.4) \times 10^{-5} \text{eV}^2. \tag{10}
\]

These mass squared differences are consistent both with a hierarchical spectrum of masses, \( m_3 >> m_2 >> m_1 \) (or an inverted hierarchy), and other patterns (e.g. \( m_3 >> m_2 \approx m_1 \)).

As far as mixing goes \[12\], two of the angles in the leptonic mixing matrix \( U_{\text{PMNS}} \) are large, \( s_{23} \approx 1/\sqrt{2}, s_{12} \approx 1/2 \) (actually, \( s_{12} \approx 0.56 \)), while the third is bound at \( 3\sigma \) by \( s_{13} < 0.22 \) and so could be a small angle. Thus, approximately, letting \( s_{13} = \epsilon \), one has

\[
U_{\text{PMNS}} \approx \begin{pmatrix}
\frac{\sqrt{2}}{2} & \frac{1}{\sqrt{2}} & \epsilon e^{-i\delta} \\
-\frac{1}{2\sqrt{2}} & \frac{\sqrt{3}}{2} & \frac{1}{\sqrt{2}} \\
\frac{1}{2\sqrt{2}} & -\frac{\sqrt{3}}{2} & \frac{1}{\sqrt{2}}
\end{pmatrix}. \tag{11}
\]

where \( \delta \) is a, not yet determined, CP-violating phase.

It is often assumed \[13\] that the patterns seen reflects some (perhaps approximate) family symmetry in the neutrino mass matrix

\[
M_\nu = U_{\text{PMNS}}^* m_\nu \text{ diag } U_{\text{PMNS}}^\dagger \tag{12}
\]

where \( m_\nu \text{ diag} = \text{ diag } [m_1, m_2 e^{i\alpha_2}, m_3 e^{i\alpha_3}] \). Here \( \alpha_2 \) and \( \alpha_3 \) are two other unknown (so called, Majorana) CP-violating phases. Because the uncertainty in the neutrino masses \( m_i \) is large, and the Majorana phases \( \alpha_i \) are unknown, it is difficult to reconstruct the matrix \( M_\nu \). It is better, instead, to focus on the mixing angles.
In this respect, an interesting starting point is provided by matrices $M_\nu$ which produce $s_{13} = 0$ and give maximal mixing $s_{23} = 1/\sqrt{2}$. It is easy to see that matrices $M_\nu$ that are 2-3 symmetric

$$M_\nu = \begin{bmatrix} X & A & A \\ A & B & C \\ A & C & B \end{bmatrix}$$

(13)

precisely accomplish this. This 2-3 permutation symmetry can be part of a larger discrete or continuous symmetry. Many models exist, based on the groups: $A_4$, $S_3$, $Z_4$, $D_4$. [5] Because this 2-3 symmetry is approximate in nature (after all $m_\mu \neq m_\tau$), the way that the symmetry is broken determines the correlation among the mixing angles. Thus, $s_{23}^2 - \frac{1}{2} = F(s_{13})$, where the model dependent function $F(s_{13})$ is normalized so that $F(0) = 0$.

A different bottom-up approach pursued by Smirnov and collaborators [15] uses the observation that $\theta_{12} + \theta_{12}^{\text{had}} \simeq \theta_{23} + \theta_{23}^{\text{had}} \simeq \frac{\pi}{4}$. (14)

to build models that display this quark-lepton complementarity (QLC). It may well be that the above relation is a coincidence, but it is an interesting avenue to pursue.

I will illustrate these models with an example due to Minakata and Smirnov. [16] Recall that the mixing matrices are obtained after combined unitary transformations of the leptons and quarks. Namely, $U_{\text{PMNS}} = U_\ell^\dagger U_\nu$, while $U_{\text{CKM}} = U_u^\dagger U_d$. Imagine that mixing in the doublet Higgs sector comes only from the "down" side (thus $U_u = 1$) and that some GUT forces $U_\ell = U_d = U_{\text{CKM}}$. Then one gets an interesting QLC relation if the seesaw mechanism forces the neutrino unitary matrix $U_\nu$ to be of the bi-maximal form:

$$U_\nu = U_{\text{bm}} = \frac{1}{2} \begin{bmatrix} \sqrt{2} & \sqrt{2} & 0 \\ -1 & 1 & \sqrt{2} \\ 1 & -1 & \sqrt{2} \end{bmatrix}$$

(15)

If this is so, then $U_{\text{PMNS}} = U_{\text{CKM}}^\dagger U_{\text{bm}}$.

3.3. Connecting Physical Phenomena

A third approach for exploring the neutrino window for physics beyond the SM is by looking for physical interrelations among phenomena. The best example of this is provided by leptogenesis, the generation of the Universe’s matter-antimatter asymmetry from a primordial lepton asymmetry. The ratio of baryon to photon density in the Universe now, $\eta = n_B/n_\gamma$, is a measure of the primordial matter-antimatter asymmetry in the Universe. [18] This ratio is well determined by WMAP [19] and by Big Bang Nucleosynthesis (BBN), [20] and one finds $\eta = (6.097 \pm 0.206) \times 10^{-10}$.

The heavy neutrinos needed for the seesaw provide a nice mechanism for generating $\eta$. In this leptogenesis scenario [17] a primordial lepton-antilepton asymmetry is generated from out of equilibrium decays of the heavy Majorana neutrinos $[N \rightarrow \ell \Phi; N \rightarrow \ell^\dagger \Phi^\dagger]$. However, because the (B+L)-current is anomalous, [11] sphaleron processes [21] transmute this lepton number asymmetry into a baryon number asymmetry. [22] In the SM one can show [23] that about one-third of the produced lepton asymmetry becomes a baryon asymmetry. Thus one needs to generate $\eta_L \simeq 2 \times 10^{-9}$ to produce the desired value for $\eta$.

The leptonic asymmetry produced from heavy neutrino decays in the early Universe is given by the formula [18]

$$\eta_L \simeq \frac{7 e\kappa}{g^*}.$$  

(16)
Here the factor of 7 relates the entropy density to the photon density, while $\epsilon$, $\kappa$, and $g^*$, respectively, are a measure of the CP-asymmetry in the decays of the heavy neutrinos, take into account of a possible washout of the asymmetry by equilibrium processes, and count the effective degrees of freedom at the time of leptogenesis ($g^* \sim 100$). Both $\epsilon$ and $\kappa$ depend on properties of the light neutrino spectrum.

Assuming hierarchal heavy neutrinos, $\epsilon$ is determined by the decays of the lightest heavy neutrino, of mass $M_1$, and is given by the formula

$$
\epsilon = -\frac{3M_1}{16\pi^2 v_F^2} \frac{\text{Im} \Gamma^\nu \Gamma^\nu_{\nu}}{(\Gamma^\nu \Gamma^\nu_{\nu})_{11}}.
$$

(17)

Leptogenesis occurs at temperatures $T$ of $O(M_1)$ and for $T \sim M_1 \sim 10^{10}$ GeV, one needs very light neutrino masses $(m_\nu \leq eV)$ to obtain the typical parameters needed for the CP asymmetry ($\epsilon \sim 10^{-6}$) and the washout ($\kappa \sim 10^{-2}$) to get $\eta_L \sim 2 \times 10^{-9}$.

In fact, Buchmüller, Di Bari, and Plumacher have shown that the washout factor $\kappa$ is independent of initial abundances (and of any pre-existing asymmetry) if the effective neutrino mass parameter $\tilde{m}_1$ is in the range $10^{-3}$eV $\leq \tilde{m}_1 \leq 1$eV, precisely the range of the observed masses for neutrinos. Furthermore, $\kappa$ gets smaller as the neutrino masses increase, because the washout rate is proportional to the sum squared of the neutrino masses, $W \sim \Sigma m_i^2$. As a result, successful leptogenesis provides an upper bound on the light neutrinos masses

$$
m_i \leq 0.1 \text{ eV}.
$$

(18)

This result is perfectly consistent with observations (excluding the LSND results) on light neutrinos.

Nevertheless, there are some clouds on the horizon. Because $\epsilon$ cannot be to small for leptogenesis to work, this provides a lower bound on the mass of the lightest heavy neutrino, $M_1$. For $\tilde{m}_1 \geq 10^{-3}$eV, where the result for $\eta_L$ is independent of pre-existing conditions, one finds

$$
M_1 \geq 2 \times 10^9 \text{ GeV}.
$$

(19)

However, the fact that leptogenesis occurred at temperatures $T \sim M_1 > 2 \times 10^9$ GeV has significant import for supersymmetric theories. It turns out that if the reheating temperature after inflation $T_R$ is too high one overproduces gravitinos. This has catastrophic consequences, since the decay products produced in gravitino decays destroy the light elements produced in Big Bang Nucleosynthesis. To avoid troubles, one must require a much lower reheating temperature $T_R \leq 10^7$ GeV. But leptogenesis argues $T_R \geq M_1 \geq 2 \times 10^9$ GeV! There are solutions to the gravitino problem, but these in general alter the “normal” SUSY expectations coming from supergravity. For example, one needs either very heavy gravitinos $(m_{3/2} \geq 100$ TeV) or very light gravitinos $(m_{3/2} \leq 1$ GeV), so that the gravitino is the LSP.

Lepton flavor violation (LFV) provides another example of tension between SUSY and leptogenesis. For example, predictions for radiative muon decays, $\mu \rightarrow e\gamma$, in SUSY theories, although model dependent, are sensitive to the mass of the heavy neutrinos. As a result these LFV processes constrain the heavy neutrino spectrum from above. Obviously, future experimental information will be crucial. Indeed, asking compatibility between SUSY and leptogenesis may lead to testable experimental predictions and insights into neutrino physics.

4. Wild Speculations
4.1. LSND Musings
There well may be more surprises in the neutrino sector. A prime example is provided by data from the LSND experiment. This experiment reported a positive signal of $\nu_\mu \rightarrow \nu_e$.
oscillations for neutrino mass squared differences $\Delta m^2 \sim eV^2$ and mixing $\sin^2\theta \sim 10^{-3}$. This result, obviously, cannot be reconciled in a three-neutrino framework, as we already have two other distinct mass squared differences from solar and atmospheric neutrino oscillation experiments. We need to await results from the Mini BooNe experiment \[31\] for confirmation, but if the LSND phenomena is true it requires different physics than the one we have been discussing up to now, involving sterile neutrinos or CPT violation.

It is easy to find candidates for sterile neutrinos in string/GUT theories. For example, if there is an underlying $E_6$ symmetry, \[32\] in its 27-dimensional representation, which contains the quarks and leptons, one has nine states which are color singlets. Among these nine states there are two $SU(2) \times U(1)$ singlets. One of them is the right-handed neutrino, but there is also an additional state in each 27-dimensional multiplet with the same SM properties as the $\nu_R$. Obviously such states, or mixtures of these states with the $\nu_R$ states, can play the role of sterile neutrinos.

The challenge, however, is not finding schemes where there are sterile neutrinos but to get the mass of these states to be of order $m_{\text{st}} \sim 1$ eV. The normal trick is to use some discrete symmetry to set $m_{\text{st}} = 0$ and then get a small mass from the breaking of this symmetry. For example, Mohapatra et al \[33\] use a 2-3 symmetry for these purposes. However, in my view, these models are quite complex and not that well motivated. Furthermore, the phenomenology of, so called, 3+1 models with one additional sterile neutrino is shaky, \[34\] but 3+2 models might provide a better fit. \[35\].

Invoking CPT violation to account for the LSND result is a much bolder suggestion. \[36\] If CPT is broken in the neutrino sector, one expects differences between $\nu_{\mu} \rightarrow \nu_e$ and $\bar{\nu}_{\mu} \rightarrow \bar{\nu}_e$ oscillations. In particular, there is no reason why the mass squared differences between particles and antiparticles be the same. So, one can fit the LSND result in a three neutrino context, since there are now 4 possible mass squared differences. Unfortunately, as Gonzalez- Garcia, Maltoni and Schwetz showed, \[37\] the phenomenology does not work better in this case either. Keeping all mass squared differences free, a global fit of all data except that of LSND is in agreement with the CPT conserving solution $\Delta m^2 = \Delta m^2$. Thus only the LSND result requires CPT violation. However, a recent paper by Barenboim and Mavromatos \[38\] claims that matters are improved by including quantum decoherence effects. This is not unexpected, since these effects essentially add more parameters to the fit. Obviously, more data is badly needed!

4.2. Mass Varying Neutrinos

Recently, Fardon, Nelson and Wiener (FNW) \[39\] suggested that neutrinos may have environment-dependent masses which may play a role in helping to explain the dark energy in the Universe. The existence of dark energy, which acts as a fluid with negative pressure, was first inferred from supernova data indicating that the Universe’s expansion was accelerating. \[40\] This result has now been confirmed by data from WMAP. \[19\] If one combines WMAP data with that from the Supernova Legacy Survey \[11\] one can determine the dark energy equation of state $w = p/\rho$ and one finds a result which is consistent with the dark energy being a cosmological constant ($w_a = -1$).

Even though neutrinos are a subdominant component of the Universe’s energy density now, contributing less than 1.5%, what Fardon et al suggested is that $\rho_{\text{dark energy}}$ tracks the energy density in neutrinos, $\rho_{\nu}$. More precisely, in their picture neutrinos and dark energy are assumed to be coupled. In the non relativistic regime they examined, the total dark sector energy density has two components. A neutrino piece and a dark energy piece which is coupled to the neutrinos since $\rho_{\text{dark energy}}$ depends on the mass of the neutrinos. Considering for simplicity just one generation of neutrinos, one has

$$\rho_{\text{dark}} = m_{\nu} n_{\nu} + \rho_{\text{dark energy}}(m_{\nu}),$$

(20)
where \( n_\nu \) is the neutrino density. Because the two components are coupled, neutrino masses are determined dynamically by minimizing the above equation. As a result, in the FNW model, neutrino masses depend on density, \( m_\nu = m_\nu(n_\nu) \).

The equation of state for the dark sector follows from the energy conservation equation and in NR limit one finds \[ w + 1 = \frac{m_\nu n_\nu}{\rho_{\text{dark}}} = \frac{m_\nu n_\nu}{m_\nu n_\nu + \rho_{\text{dark energy}}}. \] If \( w \approx -1 \) the neutrino contribution to \( \rho_{\text{dark}} \) is a small fraction of \( \rho_{\text{dark energy}} \). Furthermore, one can show that, in its simplest version, the FNW model is equivalent to having a cosmological constant, which runs with the neutrino mass. \[ \rho_{\text{dark energy}} = -p_{\text{dark energy}} \equiv V(m_\nu) \]

\[ n_\nu + \frac{\partial V(m_\nu)}{\partial m_\nu} = 0 \]

A typical example of a potential which satisfies the above equation is \( V(m_\nu) \sim m_\nu^\alpha \). In this case, \( \alpha \) is determined by the value of \( w \) now, \( \alpha = -(1 + w_o)/w_o \) and the neutrino mass drops rapidly with increasing density, so that neutrinos of mass \( m_\nu^o = 3 \text{ eV} \) now are relativistic \( (m_\nu = T) \) already at temperatures of order \( T \approx 3 \times 10^{-3} \text{ eV} \). For \( w_o = -0.9 \), the equation of state has \( w \approx w_o \) up to a redshift of order \( z \approx 10 \), but by \( z = 20 \) one has \( w \approx -0.25 \). At high redshift, eventually \( w \rightarrow 1/3 \), as the dark energy density reduces to that of relativistic neutrinos.

There are many issues one can raise concerning neutrino models of dark energy. For instance, what physics fixes the running cosmological constant potential \( V(m_\nu) \)? Or, what dynamical principle demands that \( \rho_{\text{dark}} \) be stationary with respect to variations in the neutrino mass? In addition, the simplest version of mass varying neutrino models described above leads to a dynamical instability, since the speed of sound squared, \( c_s^2 = \partial p/\partial \rho \), is negative in the late stages of the Universe’s evolution. Nevertheless, the idea of variable mass neutrinos is intriguing since it associates the dark energy sector, through the seesaw mechanism, to the \( SU(3) \times SU(2) \times U(1) \) singlet sector connected with heavy neutrinos. Because this sector is difficult to probe, it is easy to imagine that the physics which determines the Universe’s late dynamics lurks there.

5. Concluding Remarks

Neutrino physics has opened windows into phenomena beyond the SM, associated in the first instance with (B-L)-breaking. In my view, the smallness of the masses of neutrinos relative to those of quarks and leptons is a reflection of the hierarchy between the scale of (B-L) breaking and the Fermi scale \( v_F \) embodied in the seesaw mechanism. Attempts to understand the details of neutrino masses and mixings have provided hints of possible flavor symmetries and of unification, although no unequivocal theoretical direction has yet surfaced. Deeper puzzles and mysteries may well surface in the future, involving neutrinos and their role in the Universe. We await more experimental data, particularly connected with CP-violation in the neutrino sector and on the behavior of dark energy with increasing redshift.

Acknowledgments

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1 This equation is written still in the NR limit, but it can be generalized to neutrinos of arbitrary velocity.  
2 This problem can be avoided in more elaborate models.
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