Time-varying free-surface properties of hydraulic jumps: a comparative analysis of experimental methods

Rui LI¹, Kristen D. SPLINTER² and Stefan FELDER³

¹ PhD Candidate, UNSW Sydney, Water Research Laboratory, School of Civil and Environmental Engineering, 110 King St, Manly Vale, NSW, 2093, Australia
ORCID: 0000-0002-7387-8004

² Senior Lecturer, UNSW Sydney, Water Research Laboratory, School of Civil and Environmental Engineering, 110 King St, Manly Vale, NSW, 2093, Australia
ORCID: 0000-0002-0082-8444

³ Senior Lecturer, UNSW Sydney, Water Research Laboratory, School of Civil and Environmental Engineering, 110 King St, Manly Vale, NSW, 2093, Australia
+61 (2) 8071 9861; s.felder@unsw.edu.au (corresponding author)
ORCID: 0000-0003-1079-6658
Abstract

Hydraulic jumps occur commonly in natural channels and energy dissipation systems of hydraulic structures in the violent transition from supercritical to subcritical flows. Hydraulic jumps are characterised by large flow aeration, high turbulence and strong fluctuations of the free-surface and the jump toe. For free-surface measurements, fast-sampling, fixed-point instruments such as acoustic displacement meters (ADMs) and wire gauges (WGs) have been used previously while LIDAR technology has only recently been applied for the recording of instantaneous free-surface motions. To assess the comparative performances of LIDAR with ADMs and WGs, simultaneous laboratory experiments of aerated hydraulic jumps were conducted. Measurement for all three methods compared well in terms of free-surface elevations and fluctuations, while the ADM provided less distinct peaks in characteristic free-surface frequencies. Free-surface integral time and length scales were considerably larger for the ADMs while WGs and LIDAR compared well. The LIDAR was able to continuously capture the entire free surface profile as well as the jump toe location. Herein the LIDAR had the comparative advantage to assess the mean jump toe location based upon a continuous time series rather than visual observations as traditionally done. Overall, the results validated LIDAR technology as a remote measurement method for highly aerated free surfaces.

Keywords

Acoustic displacement meter, air-water flows, LIDAR, light detecting and ranging, remote sensing, signal processing, wire gauge.
1. Introduction

Hydraulic jumps occur at the violent transition from supercritical to subcritical flows associated with large air entrainment, turbulence and energy dissipation (Rajaratnam 1965, Hager 1992, Chanson 2009). Hydraulic jumps are commonly observed in natural open channels or as energy dissipators in stilling basins (Peterka 1978, Chanson and Carvalho 2015). An example of a highly aerated hydraulic jump in a laboratory flume is illustrated in Figure 1. Air is entrained locally at the impingement point (jump toe) and continuously entrained and detrained along the roughened surface (Ervine 1998, Chanson 1996, Wang et al. 2015). The air-water interface is characterized by entrained bubbles, water droplets and air pockets trapped in the free-surface roughness (Chanson 1996). As shown in Figure 1, the flow is highly three-dimensional, with both fast and slow jump toe motions linked to internal vortex pairing and shedding (Long et al. 1991, Zhang et al. 2013, Wang et al. 2015, Montano et al. 2018).

Figure 1. Side view of an aerated hydraulic jump with $d_1 = 0.028 \text{ m}, Fr_1 = 8, Re = 1.2 \times 10^5$; ADMs (foreground) and WGs (orange boxes with stems in background) installed above the flume.
Previous experiments have mainly focused on the internal flow structures including air-water flow properties (e.g. Resch et al. 1974, Takahashi and Ohtsu 2017, Felder and Chanson 2018), internal pressures and forces (Wang et al. 2015, Montano and Felder 2019) and internal turbulence characteristics (e.g. Rouse et al. 1959, Liu et al. 2004, Zhang et al. 2013, Kramer and Valero 2020).

The free-surface features of hydraulic jumps have also been extensively studied. Traditionally, pointer gauges (e.g. Rajaratnam, 1962, Hager 1993) have been applied to estimate time-averaged free-surface profiles, while phase-detection intrusive probes are able to provide an indirect air-water flow measurement of the time-average free-surface elevation (e.g. Wang et al. 2015, Montano 2019). Advancements in instrumentation have enabled instantaneous measurements of free-surface properties including fluctuations, characteristic frequencies and free-surface integral scales. Table 1 summarises experimental studies of free-surface features in hydraulic jumps focussing on fast-sampling instrumentation applied along the channel centreline including intrusive wire gauges (WGs), non-intrusive acoustic displacement meters (ADMs) and the remote sensing technology LIDAR. While WGs and ADMs can record instantaneous free-surface motions at a single fixed point per instrument, LIDAR technology allows the simultaneous and continuous recording of free-surface motions with high spatial resolution (Table 1).

Image-based techniques can also be applied for free-surface measurements to record continuous free-surface profiles at the sidewall (Long et al. 1991, Zhang et al. 2013,
Kramer and Valero 2020). However, the sidewall dampens the three-dimensional hydraulic jump motions and the free-surface differs from the centreline data.

Previous studies highlighted the complexity and non-stationary three-dimensional structure of the free-surface and the jump toe. The jump toe profiles vary randomly across the width of the flume with average jump toe positions in the centerline further downstream compared to the side walls (Zhang et al. 2013, Felder and Chanson 2018). The jump toe is characterized by fast fluctuations that are related to the inflow conditions showing larger fluctuations with increasing inflow Froude numbers $F_{r1}$ (Long et al. 1991, Wang and Chanson 2015, Montano et al. 2018, Montano and Felder 2020). The dominant frequencies of the fast jump toe motions are in the range of $0.5 - 2$ Hz (Chanson and Gualtieri 2008, Murzyn and Chanson 2009, Montano et al. 2018) while secondary slower oscillating motions may have frequencies in the range of $0.002 - 0.004$ Hz (Wang and Chanson 2015, Montano et al. 2018).

The free-surface of aerated hydraulic jumps varies most strongly in the first half of the jump roller (e.g. Wang and Chanson 2015, Montano et al. 2018) with dominant free-surface frequencies $F_{fs}$ in the range of $0.5 < F_{fs} < 4$ Hz (Murzyn and Chanson 2009, Chachereau and Chanson 2011, Wang and Chanson 2015, Montano et al. 2018). For larger inflow Froude numbers, the free-surface frequencies are smaller (Chachereau and Chanson 2011, Wang and Chanson 2015). Free-surface integral time and length scales
Table 1. Relevant experimental studies of free-surface properties in hydraulic jumps using ADMs, WGs and LIDAR ($d_1$ = inflow depth, $Fr_1$ = inflow Froude number, $Re$ = Reynolds number, $W$ = channel width).

| Reference                  | $d_1$ [m] | $Fr_1$ [-] | $Re$ [-] | $W$ [m] | Inflow Condition          | Instrument (Sampling Frequency) | No. of simultaneous measurement locations | Sampling duration [s] | Free-surface Parameters | Time and length scales |
|----------------------------|-----------|------------|----------|---------|---------------------------|---------------------------------|---------------------------------|----------------------|------------------------|-----------------------|
| Mouaze et al. (2005), Muryzn et al. (2007) | 0.021-0.059 | 1.9-4.8    | $3.3 \times 10^4 - 8.9 \times 10^4$ | 0.3     | Partially developed       | WG (128 Hz)                     | N/A                             | 5                    | Elevation | x | x | x |
| Murzyn and Chanson (2009)  | 0.018     | 3.1-8.5    | $2.4 \times 10^4 - 6.4 \times 10^4$ | 0.5     | Partially developed       | ADM (50 Hz)                     | 6                               | 600                  | x | x | x |
| Chachereau and Chanson (2011) | 0.038-0.045 | 2.4-5.1    | $6.6 \times 10^4 - 1.3 \times 10^5$ | 0.5     | Partially developed       | ADM (50 Hz)                     | 7                               | 600 / 60             | x | x | x | x |
| Nóbrega et al. (2014)      | 0.027     | 2.4        | $3.3 \times 10^4$            | 0.48    | N/A                       | ADM (25 Hz)                     | 1                               | 120                  | x |    |    |    |
| Wang and Chanson (2015), Wang et al. (2015) | 0.012-0.054 | 3.8-10     | $2.1 \times 10^4 - 1.6 \times 10^5$ | 0.5     | Partially developed       | ADM (50 Hz)                     | 5                               | 540                  | x | x | x |
| Montano et al. (2018)      | 0.032-0.154 | 2.1-4.7    | $8.4 \times 10^4 - 3.9 \times 10^5$ | 0.5 / 0.6 | Fully developed           | LIDAR (35 Hz)                   | 120-195                    | 1800            | x | x | x |
| Stojnic et al. (2019)      | 0.048-0.066 | 6.2-6.6    | $2.0 \times 10^5 - 3.6 \times 10^5$ | 0.5     | N/A                       | ADM (12.5 Hz)                   | 1                               | 328                  | x | x |    |    |
| Montano and Felder (2020)  | 0.02-0.046 | 3.6-10     | $6.2 \times 10^4 - 1.2 \times 10^5$ | 0.6     | Fully developed           | LIDAR (35 Hz)                   | -                              | 1800                 | x | x | x |    |
| Present Study              | 0.028     | 8          | $1.2 \times 10^5$            | 0.6     | Fully developed           | LIDAR (35 Hz)                   | 135-180                        |                       |                       |                       |                       |
|                            | 0.034     | 5          | $1.0 \times 10^5$            |         | Fully developed           | ADM (100 Hz)                    | 6                              | 600-1800             | x | x | x | x | x |
|                            | 0.041     | 3.5        | $9.2 \times 10^5$            |         | Fully developed           | WG (100 Hz)                     | 6                              |                      |                       |                       |                       |
have also been investigated by a number of authors (Mouaze et al. 2005, Murzyn and Chanson 2007, Chachereau and Chanson 2011, Montano and Felder 2020). Montano et al. (2018) showed that the LIDAR measurements of basic free-surface properties including time-averaged free-surface profiles, fluctuations and characteristic frequencies are in general agreement with previous free-surface data recorded with ADMs and WGs. However, Montano and Felder (2018, 2020) discussed discrepancies in free-surface time and length scales linked with the limitations of spacing of fixed-point instrumentation (ADMs and WGs) and effects of data filtering on the free-surface integral scales. Whereas Montano and Felder (2018, 2020) compared previously published data from different experiments, this new study presents simultaneously sampled data from all three instruments, providing unique insights into the effects of instrumentation and signal processing.

The inflow conditions upstream of hydraulic jumps can also affect the internal motions and flow aeration in hydraulic jumps (Takahashi and Ohtsu 2017, Montano and Felder 2019) as well as the free-surface properties (Montano 2019). To better assess the comparative performance of free-surface measurement methods in aerated hydraulic jumps, a direct comparison is needed. Herein simultaneous experiments with LIDAR, ADMs and WGs were conducted in an aerated hydraulic jump for a range of inflow conditions (Table 1). Comparative analyses of basic free-surface properties including elevations, fluctuations and frequencies, as well as advanced parameters including integral free-surface time and length scales were performed to provide for the first time, missing information on the comparative performance of common free-surface measurement methods.
2. Methodology

2.1 Experimental setup and flow conditions

New experiments were conducted in a flume of 40 m length and 0.6 m width at the UNSW Water Research Laboratory. Supercritical flows entered the flume underneath a sluice gate with an upstream rounded corner (Figure 2a). The flow was controlled with an ABB WaterMaster® FET100 electromagnetic flowmeter with an accuracy of ±0.4% of the flow rate. More details on the experimental setup can be found in Montano (2019). The experiments comprised three different hydraulic jumps with fully developed inflow conditions with Froude numbers of $Fr_1 = 3.5$, $5$ and $8$ corresponding to Reynolds numbers of $Re = 9.2 \times 10^4$, $1 \times 10^5$ and $1.2 \times 10^5$ and discharges per unit width of $q = 0.09$, $0.1$ and $0.12$ m$^2$/s, respectively. LIDAR, ADMs and WGs were used to simultaneously sample the free-surface (Figure 2).

The LIDAR was a SICK LMS511 sampling with a frequency of 35 Hz and an angular resolution of 0.25°, which was consistent with Montano et al. (2018) and Montano and Felder (2020). The LIDAR transmitted laser pulses and distances were calculated based on the Time-Of-Flight principle (Amann et al. 2001, SICK 2015). The LIDAR raw data were recorded using the software SOPAS ET of SICK (2015). The LIDAR was positioned 1.5 m above the channel bed and approximately 0.5 m upstream of the jump toe to provide the best possible perspective for the free-surface measurements (Li et al. 2019). As per the manufacturer manual (SICK 2015), the LIDAR had systematic and statistical errors of ±25 mm and ±6 mm, respectively, a laser beam width of < 20 mm albeit laboratory tests with spacers suggested a beam width as low as 5 mm.
Six ADMs (Microsonic™ Mic+35/IU/TC) with an accuracy of ±1% (temperature drift compensated) and a vertical resolution of 0.17 mm for the selected operating range of 65 to 600 mm were used. A cylindrical hollow extension of 4 cm was attached to the ADMs to protect the sensor head from water splashes as well as interference of adjacent sensors as suggested by Kramer and Chanson (2018). As per the manufacturer, the spot size of the ADM within the present experimental setup would be ca. 100 mm. However, considering the cylindrical extension that limited the signal spreading in the upper part and according to Zhang et al. (2018), the actual spot size was more likely in the range of 50 to 80 mm. The relatively large spot size of ADMs may result in erroneous data capture due to water splashing, signal interference by adjacent sensors (Chachereau and Chanson 2011, Bung 2013, Wang and Chanson 2015), or slope effects if the free-surface is greater than 13.5° (Zhang et al. 2018). In aerated hydraulic jumps strong free surface non-stationarities, splashing and slope effects affect the data quality recorded by ADMs.

In addition, six capacitance WGs (Manly Hydraulics Laboratory, Sydney) were used. The WGs consisted of a dielectric coated wire (Ø = 0.2 mm) of 200 mm length supported by a metal frame (Figure 1). Depending upon the length of wire immersed in the water, the resistivity of the WG provides a measure of the flow depth. While little information is known about accuracies of WGs in hydraulic jumps, Mouaze et al. (2005) and Murzyn et al. (2007) suggested that strong turbulence and high aeration may introduce uncertainties. The raw voltage signals of ADMs and WGs were recorded digitally with LabVIEW on the same computer used for LIDAR measurements to ensure synchronisation. ADMs and WGs were simultaneously sampled at 100 Hz to minimize aliasing distortion (Zhang et al. 2018). All instruments were warmed up for at least 1 hour before experiments to
eliminate signal drifting. During data recording, real time data of all instruments were visually monitored. If the raw signal of any of the ADMs indicated a flat signal (corresponding to direct water impact), the recordings of all instruments were terminated and the measurements repeated. The recorded raw data of LIDAR and ADMs/WGs were manually trimmed to the same start time using the computer clock.

Figure 2 shows the experimental setup with all instruments. Measurements were conducted along the centreline as well as along two transects with an offset of Δz = ±0.07 m (Figure 2b). In preliminary tests, LIDAR data were recorded along the three cross-sections confirming little transverse variations in free-surface properties within the central part of the hydraulic jump.

The main experiments were conducted in three stages. In the first stage, free-surface measurements with ADMs and LIDAR were simultaneously conducted with the LIDAR in the centreline and the ADMs offset to either side by Δz = ±0.07 m. The longitudinal distance between two consecutive ADM sensors, ΔX_{ADM}, was 0.086 ≤ ΔX_{ADM} ≤ 0.131 m, 0.120 ≤ ΔX_{ADM} ≤ 0.200 m and 0.196 ≤ ΔX_{ADM} ≤ 0.308 m for Fr1 = 3.5, 5.0 and 8.0, respectively. In the second stage of experiments, LIDAR, ADMs and WGs were recorded simultaneously with the LIDAR in the centreline and the ADMs and WGs at either side of the centreline (Figures 1 and 2). The longitudinal measurement locations of ADMs and WGs were identical. For the first two stages, data were recorded for six to seven repeated runs with sampling durations between 10 and 30 minutes for each flow condition.

In the third stage, free-surface integral scales were simultaneously measured with LIDAR, ADMs and WGs. For these experiments, the spacing of ADMs was ΔX_{ADM} =
47.5 mm for the first five sensors and 95 mm for the last sensor. The distance between consecutive WGs was consistently 95 mm due to the sensors electronic box limiting shorter spacing (Figure 1). For each flow condition, the array of ADMs and WGs was placed at one of the six ADM positions from the first and second stages. At each location, all sensors were sampled simultaneously for at least 15 minutes.

![Experimental setup](image)

Figure 2. Experimental setup and positioning of instrumentation (not to scale): (a) Side view; (b) Top view.

2.2 Post-processing of raw data

All signals were post-processed in MATLAB following the methodology shown in Figure 3. For the LIDAR, the raw data were first transferred into a cartesian coordinate system of elevations $y$ and distances $x$ along the flume using the recorded channel bed without water as the reference. Based upon a detailed sensitivity analysis of filtering (Li et al.
the LIDAR data were not filtered along the jump roller \((x/L_r \leq 1)\) where \(L_r\) is the length of the roller measured with the LIDAR (Montano 2019). For \(x/L_r > 1\), LIDAR data were filtered using 3 standard deviations of 12 neighbourhood points in the space domain and 4 standard deviations of 12 neighbourhood points in the time domain (Li et al. 2019). The data quality was overall high, with less than 3% of non-detected or filtered LIDAR data along the hydraulic jump \((x/L_r \leq 1.4)\) for all experiments.

In the next post-processing step, the instantaneous jump toe positions and the mean jump toe location \((X_{toe})\) relative to the start of the sluice gate, were calculated for each experiment. In the fourth step, signals upstream of the jump toe with depth smaller than inflow depth \((y < d_1)\) were removed and replaced with NaN to not bias the signal processing. Note that previous LIDAR studies (Table 1) have replaced depths below the inflow depth with \(d_1\). The LIDAR data were then interpolated with a constant longitudinal distance between consecutive data points between 8 mm \((Fr_1 = 3.5)\) and 12 mm \((Fr_1 = 8)\).

The post-processing of the raw ADM and WG data is also shown in Figure 3. A manual check of raw data showed no outliers in contrast to previous reports by Wang (2014). Using the work of Valero et al. (2019) as a guide, different filter methods were tested for ADM and WG data comprising simple cut-off thresholds based on standard deviations and percentiles, as well as the elliptical bound filter based on sampled flow elevations and its derivative (vertical velocity) using the method of Goring and Nikora (2002). A sensitivity analysis of all WG and ADM data showed that the filtering method had only minor effects on the free-surface observations. Subsequently no filter was applied to avoid potential removal of meaningful data.
To ensure different sampling frequencies of the three instruments did not impact on the overall comparative analyses, both ADM and WG raw data were down sampled to 33.3 Hz, to closely resemble the 35 Hz sampling frequency of the LIDAR. No effects on post-processed free-surface parameters was found between the raw data and the down-sampled data, therefore the raw ADM and WG data sampled at 100 Hz were used in the present study. In the first post-processing step, any ADM and WG data below the inflow depth were removed and replaced with NaN. In the second step, the x coordinates of the ADM and WG data points were adjusted relative to the mean jump toe position $X_{toe}$ recorded with the LIDAR.

Figure 3. Post-processing steps of LIDAR, ADM and WG data.

Figure 4 shows a segment of typical raw data for the three measurement devices as well as the probability mass functions (PMFs) for the entire sampling duration. All raw timeseries data showed strong fluctuations in free-surface elevations of similar magnitude.
and with similar patterns. The raw data for the ADM (Figure 4b) and the WG (Figure 4c) had a more continuous signal compared to the LIDAR (Figure 4a). This observation appears to be linked with the higher sampling frequency for the ADMs and WGs as well as spot size for the ADMs and some smoothing due to wetting and drying times of the intrusive WGs. The PMFs of all three instruments showed similar distributions independent of the measurement locations and flow conditions.

Figure 4. Selected raw data (left hand side) and PMFs for a signal of 1380 s (right hand side) at $x/L_r = 0.37$, $d_1 = 0.028$ m, $Fr_1 = 8$, $Re = 1.2 \times 10^5$: (a) LIDAR; (b) ADM; (c) WG.
Data analysis of the raw data provided basic free-surface properties including the mean, standard deviations, skewness and kurtosis of the free-surface elevations as well as the characteristic frequencies of the free-surface motions. In addition, the LIDAR provided statistics and frequencies of the jump toe motions. For all instruments, the auto- and cross-correlation functions of the free-surface data were also calculated along the jump roller starting from a location downstream of $X_{toe}$ where less than 5% of the data was NaN. To estimate the advective time and length scales of the free-surface structures, the auto-correlation functions were integrated until the first crossing of the x-axis or the minimum auto-correlation if non-zero auto-correlation existed providing the integral auto-correlation time scales ($T_{xx}$) (Chachereau and Chanson 2011, Montano and Felder 2020):

$$ T_{xx} = \int_{\tau=0}^{\tau=R_{xx=R_{xx,\min} \& R_{xx}=0}} R_{xx}(\tau) d\tau $$

(1)

where $\tau$ is the time lag, $R_{xx}$ the auto-correlation function and $R_{xx,min}$ the minimum auto-correlation coefficient. The calculation of the free-surface cross-correlation integral length scales ($L_{xy}$) was based upon the integration of the maximum cross-correlation coefficients $R_{xy,max}$ between two data points with distance $\Delta x$ up to the maximum distance $\Delta x_{max}$ (Chachereau and Chanson 2011, Montano and Felder 2020):

$$ L_{xy} = \int_{x=0}^{x=\Delta x_{max}} R_{xy,max}(x) dx $$

(2)

For the continuous LIDAR data, Montano and Felder (2020) defined $\Delta x_{max}$ as the distance to the first crossing of the x-axis or as the distance with minimum $R_{xy,max}$ if no zero cross-correlation existed. In contrast, the calculation of $L_{xy}$ for ADMs and WGs strongly depended upon the sensor separation distance (Montano and Felder 2020).
3. Basic free-surface characteristics

3.1 Free-surface profiles and fluctuations

Dimensionless mean free-surface profiles \((y/d_1)\) and free-surface fluctuations, represented as standard deviation \((y'/d_1)\) were directly compared for all flow conditions. Figure 5 summarises the results of all experiments including repeated runs for respective flow conditions. For all instruments, the shapes of the free-surface elevations were consistent with previous studies of aerated hydraulic jumps (Murzyn and Chanson 2009, Chachareau and Chanson 2011, Wang and Chanson 2015, Montano et al. 2018) showing a continuous increase in free-surface elevations along the jump roller. The free-surface fluctuations rapidly increased from the most upstream position of the jump toe \((x/L_r \sim -0.2)\) reaching a maximum in \(y/d_1\) in the first part of the roller \((x/L_r < 0.4)\). Further downstream the fluctuations decreased gradually. With increasing \(Fr_1\), the free-surface fluctuations increased (Figure 5), which is consistent with previous studies (Chachereau and Chanson 2011, Wang and Chanson 2015, Montano and Felder 2020).

In Figure 5, the LIDAR data are shown as a continuous line, while the ADM and WG data are shown as symbols as per their fixed measurement locations. Note that all data were adjusted to the mean jump toe position \(X_{toe}\) measured with the LIDAR (Figure 3). The ADM and WG data are shown for all experiments representing some scatter between repeated runs (Figure 5). Using the adjustment with \(X_{toe}\), clear trends in both elevations and fluctuations were observed for both WG and ADM data despite some scatter. For all LIDAR experiments, the mean free-surface elevations and fluctuations are shown including error bars representing the 5\(^{th}\) and 95\(^{th}\) percentiles. The largest difference between 5\(^{th}\) and 95\(^{th}\) percentiles of all LIDAR data was observed close to the jump toe.
(x/L_r ~ 0) with y/d_1 = 0.03 and y'/d_1 = 0.03 irrespective of the flow conditions. The uncertainty in the recorded data included the effects of experimental repeatability, measurements at different longitudinal cross-sections with Δz = ±70 mm, as well as potential effects of simultaneous measurements with the intrusive wire gauges. Mean elevations and free-surface fluctuations were affected by the jump toe motions for x/L_r < 0.2 resulting in maximum differences of up to 50% and 20% in y/d_1 and y'/d_1, respectively.

The comparative analyses of the three instrumentations showed close agreement of the overall distribution and shape of both mean elevations and free-surface fluctuations, while distinct differences in magnitudes were observed (Figure 5). The elevations recorded with the ADMs were similar to the LIDAR data at the start of the hydraulic jump (0 < x/L_r < 0.2), while y/d_1 for the ADMs were consistently above the LIDAR in the latter part irrespective of the flow conditions. Maximum differences in mean elevations between LIDAR and ADMs of 10% were observed in the centre of the roller for 0.4 < x/L_r < 0.6, while the differences in all other flow regions were 5% on average corresponding to an average dimensional difference of 8.5 mm for the three present flow conditions. The mean elevations measured by WGs were consistently above the LIDAR and ADM data in the first half of the roller (x/L_r ≤ 0.5), while the differences decreased towards the end of the roller (x/L_r > 0.5) with differences less than 10%. The maximum differences in elevations between LIDAR and WG were observed at approximately x/L_r = 0.1 with differences of 23%, 30% and 35% for Fr_1 = 3.8, 5 and 8 respectively. On average, the WG elevations exceeded the LIDAR data by 17 mm for the three flow conditions. While this is larger than a comparison of LIDAR and WG in waves (Blenkinsopp et al. 2010), the hydraulic jumps’ free-surface is significantly more complex.
Figure 5. Mean free-surface elevations and free-surface fluctuations in aerated hydraulic jumps measured with LIDAR, ADM and WG: (a) $d_1 = 0.041$ m, $Fr_1 = 3.5$, $Re = 9.2 \times 10^4$; (b) $d_1 = 0.034$ m, $Fr_1 = 5$, $Re = 10^5$; (c) $d_1 = 0.028$ m, $Fr_1 = 8$, $Re = 1.2 \times 10^5$. 
The free-surface fluctuations ($y'/d_1$) for WGs and LIDAR were in close agreement along the jump roller with average differences of 8% for $x/L_r \leq 0.4$ for all flow conditions and average differences of 12% for $Fr_1 = 3.5$ and 5% for $Fr_1 = 5$ and 8 for $x/L_r > 0.4$. The comparison of ADM and LIDAR data showed an average 9% larger values of $y'/d_1$ for the ADMs for $x/L_r < 0.4$, while the LIDAR free-surface fluctuations were comparatively larger for $0.4 < x/L_r < 0.8$ with maximum differences of 32%, 25% and 18% for $Fr_1 = 3.5$, 5 and 8, respectively for $x/L_r \approx 0.7$. Further downstream ($x/L_r > 0.8$), the relative differences were less than 14%.

The differences in recordings of elevations and free-surface fluctuations appear to be linked with the operation principles of each instrument. The strong free-surface aeration in the present hydraulic jumps promoted signal reflection of the LIDAR. However, when the flow aeration is low, the LIDAR is unable to record meaningful elevations (Rak et al. 2018, Li et al. 2019). In the present experiments, the LIDAR data were meaningful along the full jump roller up to $x/L_r = 1.4$. The LIDAR beam penetrated into the air-water flows and was reflected at the location of strongest air-water interactions corresponding to local time-averaged void fractions of ca. 50% (Montano 2019, Kramer et al. 2020).

The comparatively larger spot size of the ADMs and the tendency to record the first returned signal when multiple echoes are returned from an aerated free-surface (Zhang et al. 2018) explains the relatively higher elevations which correspond to the upper free surface region in hydraulic jumps with void fraction of 60 to 80% (Murzyn and Chanson 2009, Chachereau and Chanson 2011). The vertical positioning of the ADMs along the sloping free-surface of the jump roller combined with significant water splashing and
ejections resulted in higher free-surface profiles for $0.3 < x/L_r < 0.6$ (Figure 5). The larger spot size may also explain the comparatively lower standard deviations of ADMs within the region with the strongest free-surface slope ($0.4 < x/Lr < 0.8$). The WGs measured the flow depth intrusively resulting in local bulking of water in front of the WG stem, which was most pronounced in regions of largest flow velocities. In addition, the wetting and drying processes of the WGs, as well as impacts of free-surface splashing onto the wires may result in larger free-surface elevations in the first part of the roller.

3.2 Free-surface skewness and kurtosis

Skewness and kurtosis provide additional insights into the free-surface data distributions along the hydraulic jump (Figure 6). With increasing Froude numbers, the overall magnitude of skewness increased irrespective of the instrumentation. This is a direct result of more water ejections and splashes resulting in a skewness towards higher recorded free-surface elevations for higher Froude numbers. The skewness distributions decreased with increasing $x/L_r$ for all instruments and this trend was steeper for higher Froude numbers. This decreasing trend in skewness along the jump is a result of less ejections and splashing away from the jump toe. The data distribution at the end of the roller ($x/L_r = 1$) was close to normal with Skew = 0 irrespective of flow conditions. For $Fr_1 = 8$, both ADMs and WGs showed a rmse of 0.24 compared to LIDAR. For lower Froude numbers, there was better agreement between WG and LIDAR (rmse = 0.19) compared to ADM and LIDAR (rmse = 0.3).

In examining excess kurtosis (i.e. kurtosis - 3) to determine the heaviness of the tails in the distributions, there was a strong influence due to inflow conditions. For the lowest Froude number, excess kurtosis ~ 0, suggesting the data was well represented by a normal
distribution with very few outlier data points. In contrast, as Froude numbers increased, excess kurtosis was as high as 7 ($Fr_1 = 8$), suggesting extreme outliers in the data. As discussed above, these outliers are most likely a result of splashing and ejections by the more violent flow conditions. Additionally, for the higher Froude numbers, excess kurtosis showed a strong decreasing trend along the length of the hydraulic jump. Overall there was good agreement between LIDAR and WG ($rmse = 0.8$) compared to LIDAR and ADM ($rmse = 1.2$).

![Figure 6. Skewness and Excess Kurtosis of the free-surface in aerated hydraulic jumps measured with LIDAR, ADM and WG (error bars indicate the 5th and 95th percentiles of repeated runs for LIDAR): (a) $d_1 = 0.041$ m, $Fr_1 = 3.5$, $Re = 9.2\times10^4$; (b) $d_1 = 0.028$ m, $Fr_1 = 8$, $Re = 1.2\times10^5$.]

### 3.3 Characteristic free-surface frequencies

The characteristic free-surface frequencies were analysed at each measurement location using Fast Fourier Transforms (FFT). The peak of the FFT was selected as the characteristic dominant free-surface frequency. In cases with a non-distinct peak the dominant frequency was determined as the average frequency within a range of 1 Hz before the sharp decay in the frequency spectrum. No secondary frequencies were considered in the present study. Typical FFT distributions are shown for the three
instrumentations at an example position along the hydraulic jump (Figure 7). The example FFTs for the LIDAR, ADM and WG show similar distributions with a dominant frequency $F_{fs} \approx 2.0$ Hz. Independent of the measurement position and flow conditions, frequency analysis of the LIDAR and WGs presented more distinct peaks compared to the ADMs (Figure 7). The FFT data for the LIDAR were consistent with previous studies (Montano et al. 2018, Li et al. 2019), while the FFT for the ADMs were comparable to data of Wang and Chanson (2015), but had less distinct peaks compared to data presented by Murzyn and Chanson (2009). The differences between Murzyn and Chanson (2009) and this study may be due to spot size, but this requires further investigation.

Figure 7. FFT analysis representing typical characteristic free-surface frequencies in aerated hydraulic jumps for $x/L_r = 0.39$, $d_1 = 0.028$ m, $Fr_1 = 8$, $Re = 1.2\times10^5$: (a) LIDAR; (b) ADM; (c) WG.

Figure 8 shows all distinct and indistinct frequency peaks for LIDAR, ADM and WG data along the jump roller. The frequencies are shown as Strouhal number $F_{fs}d_1/v_1$, where $v_1$ is the cross-sectional average inflow velocity. The magnitude and distributions of characteristic frequencies were similar irrespective of the instrumentation, with $0.2 < F_{fs}$.
< 3.7 Hz. While characteristic frequencies for all instruments showed some data scatter, the characteristic frequencies close to the jump toe \((x/L_r < 0.2)\) resembled frequencies of the jump toe movement \((0.8 – 1 \text{ Hz})\). These findings, as well as a decrease in Strouhal numbers with increasing \(Fr_1\) were consistent with previous studies (Murzyn and Chanson 2009, Chachereau and Chanson 2011, Wang and Chanson 2015, Montano et al. 2018, Montano and Felder 2020).

Figure 8. Characteristic dimensionless free-surface frequencies along the jump roller for LIDAR, ADMs and WGs: (a) \(d_1 = 0.041 \text{ m}, Fr_1 = 3.5, Re = 9.2 \times 10^4\); (b) \(d_1 = 0.028 \text{ m}, Fr_1 = 8, Re = 1.2 \times 10^5\).

4. Free-surface integral time and length scales

4.1 Free-surface auto-correlation functions and integral time scales

The auto-correlation functions and auto-correlation integral time scales were calculated for all experiments (Eq. 1). Typical auto-correlation functions for the three instruments are shown in Figure 9 for simultaneously sampled signals. The auto-correlation function patterns for \(Fr_1 = 3.5\) and 5 were similar and no crossing of the x-axis was observed for any of the instruments for \(x/L_r < 0.8\) (Figure 9a), while further downstream a crossing of
the x-axis occurred (Figure 9b). In the most strongly aerated hydraulic jump with $Fr_1 = 8$ the crossing of the x-axis occurred earlier, i.e. for $x/L_r > 0.22$ for LIDAR and WGs (Figure 9c) and for $x/L_r > 0.35$ for ADMs (Figure 9d).

The comparison of the auto-correlation functions for the three instruments revealed strong similarity in $R_{xx}$ for the LIDAR and WGs independent of the flow condition and the location along the jump roller. While the overall patterns of the auto-correlation functions were similar for the ADMs, the values of $R_{xx}$ were consistently above the values of the LIDAR and WGs (Figure 9). This was most pronounced in the first half of the jump roller for the less violent hydraulic jumps with $Fr_1 = 3.5$ and 5 (Figure 9a).

It appears that higher auto-correlations determined from the ADM data are based upon two factors: (a) the intensity of the free-surface motions including droplet ejections and splashes; and (b) the spot size of the ADM. The hydraulic jumps with the lower Froude numbers were characterised by overall less intense free-surface motions in the first part of the jump roller with less intense free-surface fluctuations $y'$ (Figure 5) and less ejected droplets and spray compared to the hydraulic jump with $Fr_1 = 8$. It appears that a less fragmented free-surface provided stronger correlation between the free-surface data at a given location resulting in larger auto-correlation functions for $Fr_1 = 3.5$ and 5. The much larger spot size for the ADMs compared to LIDAR and WGs allowed for repeat capture of the free-surface motions, possibly including distinct free-surface patterns several times leading to higher auto-correlation functions. Additionally, as shown in Figure 8, the ADM did not produce distinct peaks in the characteristic frequency also suggesting some form of smoothing may have occurred due to the larger spot size. This was most pronounced
for the hydraulic jumps with lower Froude numbers since the free-surface motions were less fragmented compared to the hydraulic jump with $Fr_1 = 8$ and any distinct free-surface patterns may be more recognisable in the free-surface time series of any instrument.

Figure 9. Auto-correlation functions of simultaneously sampled free-surface data in aerated hydraulic jumps with LIDAR, ADM and WG: (a) $Fr_1 = 5$, $x/L_r = 0.32$; (b) $Fr_1 = 5$, $x/L_r = 0.92$; (c) $Fr_1 = 8$, $x/L_r = 0.23$; (d) $Fr_1 = 8$, $x/L_r = 0.51$.

Figure 10 shows dimensionless integral time scales $T_{xx}(g/d_1)^{0.5}$ for all instruments. For all data, $T_{xx}$ was analysed starting from the location, $x/L_r$, with less than 5% of data being
NaNs to eliminate the effect of jump toe motions on the free-surface integral scales (Montano and Felder 2020). Overall, the patterns in $T_{xx}$ were similar for all instruments and flow conditions with slightly lower dimensionless auto-correlation time scales for $Fr_1 = 3.5$. The shapes and magnitudes of $T_{xx} \times (g/d_1)^{0.5}$ for the LIDAR data were consistent with the observations of Montano and Felder (2020) with a small peak in $T_{xx} \times (g/d_1)^{0.5}$ at $x/L_r \approx 0.7$. While the free-surface auto-correlation times scales for the LIDAR and WGs were in close agreement with relative differences of less than 18%, $T_{xx} \times (g/d_1)^{0.5}$ measured with the ADMs were consistently larger for $x/L_r < 0.8$ (Figure 10). These observations were consistent with observations of the auto-correlation functions (Figure 9) and the lack of distinct characteristic frequencies found by the ADM (Figure 8). As discussed above, increased auto-correlation time scales measured with the ADMs may be a function of spot size, whereby an increased spot size records a stronger connection between the free-surface motions and smoothing of the frequencies (Figure 7) leading to a potential overestimation of the true characteristic time scales of the free-surface.

Figure 10. Free-surface integral time scales in aerated hydraulic jumps measured with LIDAR, ADMs and WGs: (a) $d_1 = 0.034$ m, $Fr_1 = 5$, $Re = 10^4$; (b) $d_1 = 0.028$ m, $Fr_1 = 8$, $Re = 1.2 \times 10^5$. 
4.2 Free-surface cross-correlation functions and integral length scales

Cross-correlation analysis was performed simultaneously with three instruments between two sampling points separated by distance $\Delta x$. In general, the LIDAR and WG cross-correlation functions were comparable for all locations and Froude numbers. ADM data showed larger $R_{xy}$, as well as a slight time-lag in the location of the peak $R_{xy}$ for low Froude numbers but was in better agreement with LIDAR and WG for high Froude numbers. Representative cross-correlation plots are provided in Figure 11. For $Fr_1 = 3.5$, the ADM data showed larger $R_{xy}$ across all time-lags compared to LIDAR and WG in the first half of the jump ($x/L_r < 0.5$) (Figure 11a). Moving towards the downstream end of the roller ($x/L_r > 0.6$), $R_{xy}$ of ADM data more closely agreed with LIDAR and WG and in some instances showed lower maximum cross-correlation $R_{xy,max}$ (Figure 11b). The WG tended to obtain a slightly larger $R_{xy,max}$ (less than 30%) compared to the LIDAR in the second half of the roller ($x/L_r > 0.5$) (Figure 11b). The $R_{xy}$ for $Fr_1 = 5$ (not shown) presented similar trend to $Fr_1 = 3.5$ but showed slightly better comparison between instruments. For the largest Froude number ($Fr_1 = 8$, Figure 11c and d), the three instruments were in good agreement with $R_{xy}$ along the hydraulic jump for all distance $\Delta x/L_r$ between sampling points.

As discussed in Section 4.1, there are several possible reasons for the larger cross-correlation functions derived from the ADM compared to LIDAR and WG, particularly for lower Froude numbers. These include the intensity of the free surface motions and the ADM spot size. These larger surface features may result in higher correlation between two closely spaced ADM signals. The larger spot size also means that the data are possibly closer together as the first return can be from anywhere within the target area,
whereas $\Delta x$ is measured from the centerline of the instrument. For high Froude numbers, free-surface fluctuations are more vigorous and chaotic with larger rates of ejections that may result in lower maximum cross-correlations between two closely spaced sensors and better agreement between the three instrument types presented.

![Figure 11. Cross-correlation functions of free-surface data sampled at two locations separated by a distance $\Delta x$ measured with LIDAR, ADM and WG: (a) $Fr_1 = 3.5$, $x/L_r = 0.25$, $\Delta x/L_r = 0.15$; (b) $Fr_1 = 3.5$, $x/L_r = 0.68$, $\Delta x/L_r = 0.15$; (c) $Fr_1 = 8$, $x/L_r = 0.23$, $\Delta x/L_r = 0.14$; (d) $Fr_1 = 8$, $x/L_r = 0.74$, $\Delta x/L_r = 0.14$.](image-url)
Free-surface integral length scales \((L_{xy})\) were analysed based on the maximum cross-correlation \(R_{xy,\text{max}}\) and the distance between two sampling points \((\Delta x)\) (Eq. 2). Figure 12 shows an example comparison of dimensionless free-surface length scale \(L_{xy}/d_1\) between LIDAR, ADM and WG for \(Fr_1 = 3.5\) and 8 for \(\Delta x_{\text{max}}\). High spatial resolution of LIDAR measurements provided the opportunity to define \(\Delta x_{\text{max}}\) manually to match with the maximum measurement range of ADM (285 mm) and WG (475 mm). The continuous length scale with error bars for LIDAR measurements in Figure 12 was the average of the six cross-correlation tests for each hydraulic jump. Overall higher Froude numbers resulted in larger \(L_{xy}/d_1\) for \(x/L_r > 0.3\). However, trends in the integral length scales along the roller depended on the Froude number (Figure 12), with lower (higher) Froude numbers resulting in a decreasing (increasing) trend with \(x/L_r\).

Comparison of the integral length scales derived from the different instruments revealed an overall good agreement. Maximum difference between LIDAR and WG was 20% for all experiments. Free-surface length scales measured by LIDAR and ADM showed an average difference of 18% for all flow conditions (Figure 12a). However, larger difference up to 45% was found between LIDAR and ADM near the roller (Figure 12a) due to lower \(R_{xy,\text{max}}\) of ADM data for \(x/L_r > 0.5\).

The results presented here are consistent with previous results. Free-surface length scales derived from the ADM were consistent with Chachereau and Chanson (2011) when the same high-pass filtering (0.1 Hz) was applied. Maximum distance between two sampling points \((\Delta x_{\text{max}})\) is an important factor for the free-surface integral length scale as it defines the limit of integration in the calculation (Equation 2). Montano and Felder (2020) used
the distance of the first zero crossing $R_{xy} = 0$ or the minimum cross-correlation $R_{xy,min}$ as the integration distance $Δx_{max}$. Similarly, the integral length scales of LIDAR measurements agreed with Montano and Felder (2020) when the integration distance $Δx_{max}$ was defined as the distance of the first zero crossing $R_{xy} = 0$ or the minimum cross-correlation $R_{xy,min}$.

Figure 12. Free-surface integral length scales in aerated hydraulic jumps measured with LIDAR, ADM and WG: (a) comparison between LIDAR and ADM; (b) comparison between LIDAR and WG.

5. Discussion

The comparative analysis of free-surface properties measured with LIDAR, ADMs and WGs provided novel insights into the comparative performance of these instruments. The overall distribution patterns were similar for all investigated free-surface properties. While key free-surface elevations and fluctuations were similar for the three instruments, it must be emphasised that the distributions of all instruments were adjusted relative to the mean jump toe position ($X_{toe}$) recorded with the LIDAR. An example time series of the instantaneous jump toe positions $x_{toe}$ recorded with the LIDAR is shown in Figure 13.
(note that the instantaneous jump toe positions were adjusted so that $x_{toe} = 0$ corresponded to $X_{toe}$).

![Graph](image)

Fig 13. Segment of instantaneous jump toe positions recorded with the LIDAR for $d_1 = 0.034$, $Fr_1 = 5$, $Re = 10^5$.

Traditionally the average jump toe location has been assessed based upon visual observations (e.g. previous studies in Table 1). Fixed point measurements of free-surface properties, as well as internal air-water flow properties and pressures in hydraulic jumps were then reported relative to the visually determined mean jump toe position $X_{toe,\text{visual}}$. However, due to the extensive jump toe motions (Figure 13) as well as the time-varying and non-uniform jump toe perimeter with an average convex shape (Zhang et al. 2013, Felder and Chanson 2018, Montano et al. 2018), it is challenging to determine $X_{toe,\text{visual}}$ with high consistency and accuracy. To overcome this challenge, the locations of the fixed-point instruments (ADM’s and WGs) as well as the LIDAR data were aligned
relative to the mean jump toe as measured from the LIDAR \((X_{\text{toe}})\) for each experiment. This alignment improved the matching of the experimental data for the three instruments. Figure 14 shows typical PMFs of instantaneous jump toe positions \(x_{\text{toe}}\) relative to the mean jump toe \(X_{\text{toe}} = 0\). With increasing Froude numbers, the PMFs flattened and widened due to stronger jump toe motions. Note the range of the jump toe positions of up to 0.6 m for \(Fr_1 = 8\) (Figure 14c). Figure 14 emphasises potential difficulties in visually determining the mean jump toe position which becomes harder with increasing \(Fr_1\).

Figure 14. PMFs of the instantaneous jump toe positions in aerated hydraulic jumps: (a) \(Fr_1 = 3.5\); (b) \(Fr_1 = 5\); (c) \(Fr_1 = 8\).

To better quantify potential differences between \(X_{\text{toe,visual}}\) and \(X_{\text{toe}}\), the mean jump toe positions for both methods were systematically observed and the differences between the two methods \((\Delta X_{\text{toe}} = X_{\text{toe}} - X_{\text{toe,visual}})\) calculated. Figure 15 shows the box and whisker plot of \(\Delta X_{\text{toe}}\) for the present experiments providing important guidance on the accuracy of visual mean jump toe positioning. While the median of \(\Delta X_{\text{toe}}\) was close to 0 for all flow conditions, the visual observations may have differed from the LIDAR data by up to 0.055 m, 0.06 m and 0.07 m for \(Fr_1 = 3.5, 5\) and 8, respectively. Such difference is significant considering that the roller length of hydraulic jumps in laboratory conditions is often in the range of 0.3 to 1.4 m. For measurements of flow properties just downstream of the
jump toe, imprecise determination of the mean jump toe can lead to significant over or underestimation of air-water flow properties and other internal flow properties. Differences in reported findings and large data scatter between previous studies may be (partially) explained by the imprecise positioning of the mean jump toe position. It is therefore recommended that any future measurements of flow properties in hydraulic jumps should simultaneously measure the instantaneous hydraulic jump toe position. It appears that LIDAR is a suitable instrument for this allowing the remote recording of $x_{toe}$ and $X_{toe}$ respectively, while the full range of free-surface properties can be simultaneously and accurately recorded with high spatial and temporal resolution.

![Figure 15. Differences between visual observations and LIDAR measurements of the mean jump toe position; Box and Whisker plot of all present experiments (12 runs for $Fr_1 = 3.5$; 12 runs for $Fr_1 = 5$; 14 runs for $Fr_1 = 8$).](image)

6. **Conclusion**

Free-surface properties simultaneously measured with ADMs, WGs and LIDAR were for the first time compared in aerated hydraulic jumps with Froude numbers between 3.5 and
8.0. All instrumentation provided similar distribution patterns in all investigated free-surface properties and all data were consistent with previous studies of free-surface parameters in comparable hydraulic jumps. Small differences in magnitudes were observed in basic free-surface properties including mean free-surface elevations, standard deviations, and characteristic frequencies, while some larger differences were observed in terms of free-surface time and length scales. Elevations measured with ADMs were on average 6% corresponding to a dimensional distance of 9.5 mm above the LIDAR data, while the fluctuations for the LIDAR were on average 12% lower (higher) for $x/L_r < (>) 0.4$. WG data were on average 13% above the LIDAR data (16 mm for $Fr_1 = 3.5$ and 5 and 20 mm for $Fr_1 = 8$) in terms of elevations and 10% higher (lower) for $x/L_r < (>) 0.4$ in terms of fluctuations. The characteristic free-surface frequencies measured with all instruments matched well albeit with less distinct frequency peaks for the ADMs that is likely linked to the larger spot size of the ADMs compared to LIDAR and WGs. The larger auto-correlation of ADMs resulted in higher auto-correlation integral time scale of 42% on average, while the integral time scale of LIDAR and WGs compared well with an average difference of 7%. The difference of cross-correlation integral length scale between LIDAR and ADM (18% on average) was also larger than the difference between LIDAR and WG (10% on average).

Observed differences appear to be linked with spot sizes and measurement principles of the instruments. While the WGs measured the free-surface properties intrusively with a comparable small footprint, the impact of splashes and the wetting and drying time of the wire may have affected the observations. Both ADMs and LIDAR measured the free-surface properties non-intrusively. The ADMs were positioned vertically relative to the
channel bed at fixed locations with comparatively large spot sizes leading to some smoothing of free-surface properties. In contrast, the LIDAR measured the continuous free-surface from a single location with a smaller spot size leading to more distinct and continuous free-surface properties.

All data were adjusted relative to the mean jump toe position measured with the LIDAR showing good agreement in the longitudinal distribution of all properties. The comparison of the mean jump toe positions measured with the LIDAR with visual observations highlighted the strength of the LIDAR to provide more consistent jump toe positions and distributions of free-surface parameters. Future studies of any flow properties of hydraulic jumps should simultaneously measure the jump toe location to report more accurate and consistent results relative to the mean jump toe. Considering the comparatively similar performance of the LIDAR and the ability to measure the free-surface properties continuously along a hydraulic jump with both high spatial and temporal resolution as well as the ability to also provide the instantaneous jump toe position, suggests that the LIDAR might be the comparatively best suited instrument for free-surface measurements of aerated hydraulic jumps.

7. Acknowledgements
The authors thank Rob Jenkins (Water Research Laboratory, UNSW Sydney) for the technical assistance. They thank Dr Laura Montano (Water Research Laboratory, UNSW Sydney) and Dr Matthias Kramer (UNSW Canberra) for fruitful discussions.

References
Amann, M.-C., Bosch, T. M., Lescure, M., Myllylae, R. A. & Rioux, M. (2001). Laser Ranging: A Critical Review of Unusual Techniques for Distance Measurement. *Optical engineering*, 40(1), 10-20.

Blenkinsopp, C. E., Mole, M. A., Turner, I. L. & Peirson, W. L. (2010). Measurements of the Time-Varying Free-Surface Profile across the Swash Zone Obtained Using an Industrial Lidar. *Coastal Engineering*, 57(11-12), 1059-1065.

Bung, D. B. (2013). Non-intrusive detection of air–water surface roughness in self-aerated chute flows. *Journal of Hydraulic Research*, 51(3), 322-329.

Chachereau, Y. & Chanson, H. (2010) *Free-surface turbulent fluctuations and air–water flow measurements in hydraulics jumps with small inflow Froude numbers*. Hydraulic Model Report No. CH78/10, School of Civil Engineering, The University of Queensland, Brisbane, Australia.

Chachereau, Y., & Chanson, H. (2011). Free-surface fluctuations and turbulence in hydraulic jumps. *Experimental Thermal and Fluid Science*, 35(6), 896-909.

Chanson, H. (1996) *Air bubble entrainment in free-surface turbulent shear flows*. Academic, London.

Chanson, H. (2009). Current knowledge in hydraulic jumps and related phenomena. A survey of experimental results. *European Journal of Mechanics-B/Fluids*, 28(2), 191-210.

Chanson, H. (2010). Convective transport of air bubbles in strong hydraulic jumps. *International Journal of Multiphase Flow*, 36, 798–814.

Chanson, H., & Gualtieri, C. (2008). Similitude and scale effects of air entrainment in hydraulic jumps. *Journal of Hydraulic Research*, 46(1), 35-44.
Ervine, D. A. (1998). Air entrainment in hydraulic structures: A review. In *Proceedings of the Institution of Civil Engineers-Water Maritime and Energy*, 130(3), 142-153.

Felder, S., & Chanson, H. (2018). Air–Water Flow Patterns of Hydraulic Jumps on Uniform Beds Macroroughness. *Journal of Hydraulic Engineering*, 144(3), 04017068.

Hager, W. H. (1992). *Energy dissipators and hydraulic jump*. Netherland. Kluwer Academic Publishers.

Hager, W. H. (1993). Classical hydraulic jump: free surface profile. *Canadian Journal of Civil Engineering*, 20(3), 536-539.

Heritage, G. L. and Large, A. R. G. (2009). *Laser Scanning for the Environmental Sciences*. West Sussex, UK. Wiley-Blackwell.

Kramer, M., Chanson, H., & Felder, S. (2020). Can we improve the non-intrusive characterization of high-velocity air–water flows? Application of LIDAR technology to stepped spillways. *Journal of Hydraulic Research*, 58(2), 350-362.

Kramer, M., & Valero, D. (2020). Turbulence and self-similarity in highly-aerated shear flows: the stable hydraulic jump. *International Journal of Multiphase Flow*, 129, 103316.

Kucukali, S., & Chanson, H. (2008). Turbulence measurements in the bubbly flow region of hydraulic jumps. *Experimental Thermal and Fluid Science*, 33(1), 41-53.

Leutheusser, H. J., & Kartha, V. C. (1972). Effects of inflow conditions on hydraulic jump. *Journal of the Hydraulics Division*, 98(8), 1367-1385.

Li, R., Montano, L. & Felder, S. (2017). LIDAR Measurements of Free-Surface Characteristics in a Hydraulic Jump. In *Proceedings of the 13th Hydraulics in Water Engineering Conference*, Sydney, Australia, 238-246.
Li, R., Montano, L., Splinter, K., & Felder, S. (2019) Opportunities of LIDAR measurements in air-water flows. In Proceedings of the 38th IAHR World Congress, September 1-6, Panama City, Panama.

Liu, M., Rajaratnam, N., & Zhu, D. Z. (2004). Turbulence structure of hydraulic jumps of low Froude numbers. Journal of Hydraulic Engineering, 130(6), 511-520.

Long, D., Rajaratnam, N., Steffler, P. M., & Smy, P. R. (1991). Structure of flow in hydraulic jumps. Journal of Hydraulic Research, 29(2), 207-218.

Montano, L., & Felder, S. (2018). Effect of inflow conditions on the air-water flow properties in hydraulic jumps. In Proceedings of the 21st Australasian fluid mechanics conference, December 10-13, Adelaide, Australia.

Montano, L. (2019). An experimental study of free surface dynamics and internal motions in fully aerated hydraulic jumps. Ph.D. thesis. School of Civil and Environmental Engineering, University of New South Wales, Sydney, Australia.

Montano, L., & Felder, S. (2019). Measuring internal forces in a hydraulic jump with a load cell. In Proceedings of the 38th IAHR World Congress, September 1-6, Panama City, Panama.

Montano, L., & Felder, S. (2020). LIDAR Observations of free-Surface time and length scales in hydraulic jumps. Journal of Hydraulic Engineering, 146(4), 04020007.

Montano, L., Li, R., & Felder, S. (2018). Continuous measurements of time-varying free-surface profiles in aerated hydraulic jumps with a LIDAR. Experimental Thermal and Fluid Science, 93, 379-397.

Mossa, M., & Tolve, U. (1998). Flow visualization in bubbly two-phase hydraulic jump. Trans-ASME Journal of Fluids Engineering, 120, 160-165.
Mouaze, D., Murzyn, F., & Chaplin, J. R. (2005). Free surface length scale estimation in hydraulic jumps. *Journal of Fluids Engineering, 127*(6), 1191-1193.

Murzyn, F., & Chanson, H. (2009). Free-surface fluctuations in hydraulic jumps: Experimental observations. *Experimental Thermal and Fluid Science, 33*(7), 1055-1064.

Nóbrega, J. D., Schulz, H. E., & Zhu, D. Z. (2014). Free surface detection in hydraulic jumps through image analysis and ultrasonic sensor measurements. In *5th International Symposium on Hydraulic Structures*, June 25-27, Brisbane, Australia, 245-252.

Rajaratnam, N. (1962) Profile equation for the hydraulic jump. *Water Power*, 14, 324-327.

Rajaratnam, N. (1965). The Hydraulic Jump as a Well Jet. *Journal of the Hydraulics Division, 91*(5), 107-132.

Rak, G., Hocevar, M. & Steinman, F. (2017). Measuring Water Surface Topography Using Laser Scanning. *Flow Measurement and Instrumentation*, 56, 35-44.

Resch, F.J., Leutheusser, H.J., and Alemu, S. (1974). Bubbly two-phase flow in hydraulic jump. *Journal of the Hydraulics Division*, 100 (1), 137–149.

Rouse, H., Siao, T. T., & Nagaratnam, S. (1959). Turbulence characteristics of the hydraulic jump. *Journal of the Hydraulics Division, 84*(1), 1-30.

Stojnic, I., Pfister, M., Matos, J., & Schleiss, A. J. (2019). Hydraulic jump downstream of a stepped chute. In *Proceedings of the 38th IAHR World Congress*, September 1-6, Panama City, Panama.

Takahashi, M., and Ohtsu, I. (2017). Effects of inflows on air entrainment in hydraulic jumps below a gate. *Journal of Hydraulic Research, 55*(2), 259–268.
Valero, D., Chanson, H., & Bung, D. B. (2019). Robust estimators for turbulence properties assessment. arXiv:1906.06112

Wang, H. (2014). Turbulence and air entrainment in hydraulic jumps. Ph.D. thesis. School of Civil Engineering. University of Queensland, Australia.

Wang, H., & Chanson, H. (2015). Air entrainment and turbulent fluctuations in hydraulic jumps. *Urban Water Journal*, 12(6), 502-518.

Wang, H., Murzyn, F., & Chanson, H. (2015). Interaction between free-surface, two-phase flow and total pressure in hydraulic jump. *Experimental Thermal and Fluid Science*, 64, 30-41.

Zhang, G., Wang, H., & Chanson, H. (2013). Turbulence and aeration in hydraulic jumps: free-surface fluctuation and integral turbulent scale measurements. *Environmental fluid mechanics*, 13(2), 189-204.

Zhang, G., Valero, D., Bung, D. B., & Chanson, H. (2018). On the estimation of free-surface turbulence using ultrasonic sensors. *Flow Measurement and Instrumentation*, 60, 171-184.

**List of Symbols**

- $d_1$ inflow depth [m]
- $F_{fs}$ characteristic free-surface frequency [Hz]
- $Fr_1$ inflow Froude number [-]
- $g$ gravity acceleration constant [m$^2$/s]
- $L_r$ roller length of the hydraulic jump [m]
- $L_{xy}$ free-surface cross-correlation integral length scale [m]
- $q$ discharge per unit width [m$^2$/s]
\( Re \) Reynolds number [-]
\( R_{xx} \) auto-correlation function [-]
\( R_{xx,min} \) minimum auto-correlation coefficient [-]
\( R_{xy} \) cross-correlation function [-]
\( R_{xy,max} \) maximum cross-correlation coefficient [-]
\( R_{xy,min} \) minimum cross-correlation coefficient [-]
\( T_{xx} \) free-surface auto-correlation integral time scale [s]
\( t \) sampling time [s]
\( v_1 \) depth average inflow velocity [m/s]
\( W \) width of the channel [m]
\( X_{toe} \) mean jump toe position measured with the LIDAR [m]
\( X_{toe,visual} \) visually determined mean jump toe position [m]
\( x \) longitudinal distance relative to the mean jump toe position [m]
\( x_{toe} \) instantaneous jump toe position measured with the LIDAR [m]
\( y \) vertical distance above the channel bed [m]
\( z \) transverse distance relative to the centerline of the channel [m]
\( \Delta X_{ADM} \) longitudinal distance between consecutive ADMs [m]
\( \Delta X_{toe} \) difference between the mean jump toe position observed visually and measured with the LIDAR [m]
\( \Delta x \) distance between sampling points [m]
\( \Delta x_{\text{max}} \) maximum integration distance for free-surface integral length scales [m]
\( \Delta z \) transverse distance between instruments [m]
\( \nu \) water kinematic viscosity \([\text{m}^2/\text{s}]\)