Calculation of the real part of the nuclear amplitude at high $s$ and small $t$ from the Coulomb amplitude

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A new method for the determination of the real part of the elastic scattering amplitude is examined for high energy proton-proton at small momentum transfer. This method allows us to decrease the number of model assumptions, to obtain the real part in a narrow region of momentum transfer and to test different models. The possible non-exponential behavior of the real part was found on the base of the analysis of the ISR experimental data.
Let us note that the real part of the Coulomb $pp$ at which we have the remarkable equality

$$\text{size of } F$$

the minimum of $\Delta \exp$ zero at some value $t$ for $p$ known as being positive at relatively high (ISR) energies, it is obvious that $\Delta$ $p$ $\rho$ are the real and imaginary parts of the hadron amplitude; $\rho$ using the experimental data on the differential cross section $s$ we obtain:

$$\rho\text{Coulomb-hadron phase in order to obtain the value of } \Phi$$

the usual, that at high energies and small angles the one-flip and double-flip amplitudes are small with respect to the spin-nonflip ones and that the hadronic contributions to $\Phi_1$ and $\Phi_3$ are the same, as are the electromagnetic ones. Therefore the scattering amplitude can be written as:

$$F(s,t) = F_N + F_C \exp(i\alpha \varphi). \quad (1)$$

In the standard fitting procedure, one neglects the $\alpha^2$ term and the differential cross section has the form:

$$d\sigma/dt = \pi[(F_C(t))^2 + (\rho(s,t)^2 + 1)(ImF_N(s,t))^2] + 2(\rho(s,t) + \alpha \varphi(t))F_C(t)ImF_N(s,t)], \quad (2)$$

where $F_C(t) = \mp 2\alpha G^2(t)/|t|$ is the Coulomb amplitude (the upper sign is for $pp$, the lower sign is for $\bar{p}p$) and $G^2(t)$ is the proton electromagnetic form factor squared; $ReF_N(s,t)$ and $ImF_N(s,t)$ are the real and imaginary parts of the hadron amplitude; $\rho(s,t) = ReF_N(s,t)/ImF_N(s,t)$.

The formula $[2]$ is used for the fit of experimental data in getting hadron amplitudes and the Coulomb-hadron phase in order to obtain the value of $\rho(s,t)$.

Let us note two points concerning the familiar exponential forms of $ReF_N(s,t)$ and $ImF_N(s,t)$ used by experimentalists. First, for simplicity reasons, one makes the assumption that the slope of imaginary part of the scattering amplitude is equal to the slope of its real part in the examined whole interval of momentum transfer.

We define the imaginary part of the scattering amplitude via the usual exponential approximation in the small $t$-region

$$ImF_N(s,t) = \sigma_{tot}/(0.389 \cdot 4\pi) \exp(Bt/2), \quad (3)$$

where 0.389 is the usual converting dimensional factor for expressing $\sigma_{tot}$ in mb.

Let us define the sum of the real parts of the hadron and Coulomb amplitudes as $\sqrt{\Delta R}$, so we can write:

$$\Delta_R^{th}(s,t_i) = [ReF_N(s,t_i) + F_C(t_i)]^2 \geq 0 \quad (4)$$

Using the experimental data on the differential cross sections we obtain:

$$\Delta_R^{exp}(s,t_i) = (1/\pi) d\sigma^{exp}/dt(t_i) - (\alpha \varphi F_C(t_i) + ImF_N(s,t_i))^2 \quad (5)$$

Let us note that the real part of the Coulomb $pp$ scattering amplitude is negative and exceeds the size of $F_N^{pp}(s,t)$ at $t \to 0$, but has a large slope. As the real part of the hadron amplitude is known as being positive at relatively high (ISR) energies, it is obvious that $\Delta_R^{th}$ must go through zero at some value $t = t_{min}^{pp}$ and therefore $\Delta_R^{exp}$ must have a minimum at the same value $t = t_{min}^{pp}$ at which we have the remarkable equality

$$ReF_N^{pp}(t_{min}^{pp}) = -F_C^{pp}(t_{min}^{pp}) \quad (6)$$

The minimum of $\Delta_R^{exp}$ corresponds to a zero in $\Delta_R^{th}$ at some fixed $s$

$$\Delta_R^{th}(s,t_{min}^{pp}) = 0 \quad (7)$$

Numerous discussions of the $\rho$-parameter measured by the UA4$[4]$ and UA4/2 Collaboration in $pp$ scattering at $\sqrt{s} = 541$ GeV have revealed the ambiguity in the definition of this semi-theoretical parameter. As a result, it has been shown that one has some trouble in extracting from experiment the total cross sections and the value of the forward ($t = 0$) real part of the scattering amplitudes $[3,4]$. The standard procedure to extract the magnitude of the real part of the hadron elastic scattering includes a fit to the experimental data by minimizing the $\chi^2$ function. We assume, as usual, that at high energies and small angles the one-flip and double-flip amplitudes are small with respect to the spin-nonflip ones and that the hadronic contributions to $\Phi_1$ and $\Phi_3$ are the same, as are the electromagnetic ones. Therefore the scattering amplitude can be written as:

$$F(s,t) = F_N + F_C \exp(i\alpha \varphi). \quad (1)$$

In the standard fitting procedure, one neglects the $\alpha^2$ term and the differential cross section has the form:

$$d\sigma/dt = \pi[(F_C(t))^2 + (\rho(s,t)^2 + 1)(ImF_N(s,t))^2] + 2(\rho(s,t) + \alpha \varphi(t))F_C(t)ImF_N(s,t)], \quad (2)$$

where $F_C(t) = \mp 2\alpha G^2(t)/|t|$ is the Coulomb amplitude (the upper sign is for $pp$, the lower sign is for $\bar{p}p$) and $G^2(t)$ is the proton electromagnetic form factor squared; $ReF_N(s,t)$ and $ImF_N(s,t)$ are the real and imaginary parts of the hadron amplitude; $\rho(s,t) = ReF_N(s,t)/ImF_N(s,t)$.

The formula $[2]$ is used for the fit of experimental data in getting hadron amplitudes and the Coulomb-hadron phase in order to obtain the value of $\rho(s,t)$.

Let us note two points concerning the familiar exponential forms of $ReF_N(s,t)$ and $ImF_N(s,t)$ used by experimentalists. First, for simplicity reasons, one makes the assumption that the slope of imaginary part of the scattering amplitude is equal to the slope of its real part in the examined range of momentum transfer, and, for the best fit, one should take the interval of momentum transfer sufficiently large. Second, the magnitude of $\rho(s,t)$ thus obtained corresponds to the whole interval of momentum transfer.

We define the imaginary part of the scattering amplitude via the usual exponential approximation in the small $t$-region

$$ImF_N(s,t) = \sigma_{tot}/(0.389 \cdot 4\pi) \exp(Bt/2), \quad (3)$$

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Let us note that the real part of the Coulomb $pp$ scattering amplitude is negative and exceeds the size of $F_N^{pp}(s,t)$ at $t \to 0$, but has a large slope. As the real part of the hadron amplitude is known as being positive at relatively high (ISR) energies, it is obvious that $\Delta_R^{th}$ must go through zero at some value $t = t_{min}^{pp}$ and therefore $\Delta_R^{exp}$ must have a minimum at the same value $t = t_{min}^{pp}$ at which we have the remarkable equality

$$ReF_N^{pp}(t_{min}^{pp}) = -F_C^{pp}(t_{min}^{pp}) \quad (6)$$

The minimum of $\Delta_R^{exp}$ corresponds to a zero in $\Delta_R^{th}$ at some fixed $s$

$$\Delta_R^{th}(s,t_{min}^{pp}) = 0 \quad (7)$$
Let us finally note that our method gives a powerful test for the exponential forms of \( \text{Re} F_N^{pp}(s,t) \) and \( \text{Im} F_N^{pp}(s,t) \). Namely, in the case of these exponential forms, we have

\[
\rho^{pp}(s,t) = \frac{\text{Re} F_N^{pp}(s,t)}{\text{Im} F_N^{pp}(s,t)} = \rho^{pp}(s,0) = \text{const} = \rho^{pp}(s,t_{\text{min}}). \tag{8}
\]

However, our method gives the possibility to extract \( \rho^{pp}(s,t_{\text{min}}) \) without assuming the exponential form for \( \text{Re} F_N^{pp}(s,t) \), from eqs. 4 and 5. If this numerical value of \( \rho^{pp}(s,t_{\text{min}}) \) is significantly different from the value \( \rho^{pp}(s,0) \) extracted by a given experiment, this means that the exponential form of \( \text{Re} F_N^{pp}(s,t) \), supposed to be identical to that of \( \text{Im} F_N^{pp}(s,t) \), is doubtful.

The problem here is that we extract a small quantity – the real part of the hadron elastic amplitude – affected by large errors. In order to minimize these errors we need a very high-precision experiment. The only \( pp \) data we did find in literature, satisfying our criterion, are those at \( \sqrt{s} = 52.8 \text{ GeV} \). In Fig. 1a we plot \( \Delta_{\text{th}}^{R}(s,t_i) \) as given by eqs. 5, with \( \sigma_{\text{th}}^{pp} = 42.38 \text{ mb} \) and \( B^{pp} = 12.87 \text{ (GeV)}^{-2} \). The error bars of the \( \Delta_{\text{exp}}^{R} \) points are calculated from the errors bars of \( d\sigma^{\text{exp}}/dt \) points. We also plot on the same figure \( \Delta_{\text{th}}^{R}(s,t_i) \) as given by Eq. 4, where

\[
\text{Re} F_N^{pp}(s,t) = (\rho^{pp} \cdot \sigma_{\text{th}}^{pp})/(0.389 \cdot 4\pi) \exp(B^{pp}t/2), \tag{9}
\]

with \( \rho^{pp} = 0.077 \). We see from Fig. 1a that there is a clear disagreement between \( \Delta_{\text{th}}^{R}(s,t_i) \) and \( \Delta_{\text{exp}}^{R}(s,t_i) \) in the region \( 0.03 < -t < 0.06 \text{ GeV}^2 \). Namely, \( \Delta_{\text{th}}^{R}(s,t_i) \) goes through zero at \( -t \approx 0.024 \text{ GeV}^2 \) while \( \Delta_{\text{exp}}^{R}(s,t_i) \) goes through a minimum at a very different value of \( t \). Moreover, the values of the two quantities are very different in the region \( -t > 0.03 \text{ GeV}^2 \). In fact the entire shape of \( \Delta_{\text{th}}^{R} \) in the above region of \( t \) is not consistent with the shape of \( \Delta_{\text{exp}}^{R} \). As it can be seen from Fig. 1a, \( \Delta_{\text{th}}^{R} \) rises very slowly, while \( \Delta_{\text{exp}}^{R} \) shows a rapid rise in this region.

The result of the polynomial fit, with \( \chi^2/pt \) value of 0.73, is shown in Fig. 1b. The corresponding value of \( t_{\text{min}}^{pp} \) is \( t_{\text{min}}^{pp} = 0.0325 \pm 0.0025 \text{ GeV}^2 \), significantly different from the value \( t = -0.024 \text{ GeV}^2 \) where \( \Delta_{\text{th}}^{R}(s,t_i) \) goes through zero. We can therefore evaluate, from Eq. 4, \( \text{Re} F_N^{pp}(\sqrt{s} = 52.8 \text{ GeV}, t = t_{\text{min}}^{pp}) = 0.375 \pm 0.037 \text{ (GeV)}^{-2} \) and, from Eq. 3, \( \text{Im} F_N^{pp}(\sqrt{s} = 52.8 \text{ GeV}, t = t_{\text{min}}^{pp}) = 7.027\text{ (GeV)}^{-2} \). Therefore \( \rho^{pp}(\sqrt{s} = 52.8 \text{ GeV}, t = t_{\text{min}}^{pp}) = 0.053 \pm 0.005 \), a value which is somewhat different (\( \sim 2 \) standard deviations) from the value given in Ref. [5]: \( \rho^{pp}(\sqrt{s} = 52.8 \text{ GeV}, t = t_{\text{min}}^{pp}) = 0.077 \pm 0.009 \). The difference in \( \rho \)-values is not highly significant, but it shows the power of our method in the case of high-precision experimental data. The calculation presented here points out toward a real new effect revealed by our method. This new effect might simply mean that \( \rho \) is not a constant but a function of \( t \), as well as \( B \) might not be a constant but also a function of \( t \). In others words one must make the analysis of the experimental data with more sophisticated analytic forms of the scattering amplitude that the exponential one.

Our method uses a given model for \( \text{Im} F_N^{pp} \) which is supposed to describe well the experimental data. We know (e.g. from the Regge model) that the forward hadron scattering amplitude is predominantly imaginary. Therefore a model which describes well the experimental \( dN/dt \) data necessarily has a good \( \text{Im} F_N(s,t) \) for high \( s \) and small \( t \), even if its real part \( \text{Re} F_N(s,t) \), as a small correction, could be wrong. In other words, our method is quasi model-independent: different models for \( \text{Im} F_N(s,t) \) lead to a quite restricted range of values of \( t_{\text{min}}^{pp} \). This is explicitly shown in Fig. 1c, where we plot \( \Delta_{\text{exp}}^{R}(\sqrt{s} = 52.8 \text{ GeV}, t_i) \) computed from a model dynamically different from the exponential form, the Gauron-Leader-Nicolescu (GLN) model\(^{27}\). This model builds the scattering amplitudes from the asymptotic theorems constraints as a combination of Bessel functions and Regge forms, embodies the Heisenberg-Froissart \( \ln^2 s \) behavior for \( \sigma_T \) and includes the maximal Odderon.\(^{27}\) In this case, \( \rho(s,t) \) at a given \( s \) is no more a constant but varies with \( t \). This dynamical characteristics are translated through the fact that \( \Delta_{\text{R}}^{GLN} \), as it can be seen from Fig. 1c, has a fast increase in the region \( 0.03 < -t < 0.06 \text{ GeV}^2 \), in agreement with
the increase shown by $\Delta_{R}^{\exp}$. The disagreement between $\Delta_{R}^{\theta}$ and $\Delta_{R}^{\exp}$ is seen also through the values of $\chi^2/pt$. The overall $\chi^2/pt$ value is comparable with the one in the exponential model case: $2.3/pt$ for a total of 34 points.

We conclude that neither the exponential model nor the GLN model can reproduce entirely the effect discussed in the present paper: the disagreement between $\Delta_{R}^{\theta}$ and $\Delta_{R}^{\exp}$. However, the stability of the value $t_{pp}^{\min}$ extracted from $\Delta_{R}^{\exp}$ is remarkable: in both models examined in the present paper this this value is perfectly compatible with the value obtained by polynomial fit.

In conclusion, we did find a new method for the determination of the real part of the elastic proton-proton amplitude at high $s$ and small $t$ at a given point $t_{pp}^{\min}$ near $t = 0$. The real part of the hadron amplitude is computed, at $t = t_{pp}^{\min}$, from the known Coulomb amplitude. This method provides a powerful consistency check for the existing models and data and has a predictive power for the future measurements of the $\rho$-parameter at LHC.

Our method requires high-precision data and a large number of experimental points. We illustrated how our method works by using the data at $\sqrt{s} = 52.8$ GeV (Ref. [5]).

As a byproduct of our method we discovered two new effects in the data at $\sqrt{s} = 52.8$ GeV:

1. the significant discrepancy between $\Delta_{R}^{\theta}$ as defined in Eq. (4) and $\Delta_{R}^{\exp}$ as defined in Eq. (5), $\Delta_{R}^{\theta}$ involving $ReF_N$ while $\Delta_{R}^{\exp}$ involves $ImF_N$; 2. $\Delta_{R}^{\exp}$ goes through a minimum around a $t$-value $|t| \simeq 0.030 - 0.035$ GeV$^{-2}$ and has a sharp increase after this $t$-value.

The dynamical origin of these effects is still obscure. Maybe they are a result of oscillations in the very small $t$ region. In order to clarify their dynamical origin, high-precision experimental data at a high energy other than $\sqrt{s} = 52.8$ GeV are needed. In principle, the experiments which will be performed at LHC could explore this problem.

Let us note that our method can be easily extended (with minor changes) to proton-antiproton scattering, by observing that, in this case, it is the combination $ReF_{\bar{N}}^{pp} - E_{\bar{C}}^{C}$ which must go through zero at some value $t = t_{pp}^{\min}$. The method described in the present paper could be therefore used to analyze the UA4 data at $\sqrt{s} = 541$ GeV[2], a complex work which will be done and presented in a separate paper. Of course, in general, one expects that $t_{pp}^{min} \neq t_{pp}^{\min} \bar{t}$ at fixed $s$. Our method could be also extended to the case of proton-nucleus scattering at high energies.

Figure 1: a) $\Delta_{R}^{\theta}$ (the solid curve) and $\Delta_{R}^{\exp}$ (the triangle points) for $pp$ scattering (Eqs. (4) and (5)) at $\sqrt{s} = 52.8$ GeV as a function of $t$, computed with the exponential form of the amplitude (Eqs. (3) and (9)). b) $\Delta_{R}^{\exp}$ fitted by the polynomial form (the solid curve); c) $\Delta_{R}^{\exp}$ (the solid curve) and $\Delta_{R}^{\exp}$ (the triangle points) for $pp$ scattering (Eqs. (4) and (5)) at $\sqrt{s} = 52.8$ GeV as a function of $t$, computed within the GLN model Ref. [6]
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