Thick brane in $f(R)$ gravity with Palatini dynamics

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This work deals with modified gravity in five dimensional spacetime. We study a thick Palatini $f(R)$ brane, that is, a braneworld scenario described by an anti-de Sitter warped geometry with a single extra dimension of infinite extent, sourced by real scalar field under the Palatini approach, where the metric and the connection are regarded as independent degrees of freedom. We consider a first-order framework which we use to provide exact solutions for the scalar field and warp factor. We also investigate a perturbative scenario such that the Palatini approach is implemented through a Lagrangian $f(R) = R + \epsilon R^n$, where the small parameter \( \epsilon \) controls the deviation from the standard thick brane case.

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I. INTRODUCTION

Investigations dealing with spacetime engendering higher spatial dimensions started in physics soon after the appearance of General Relativity (GR) through the Kaluza-Klein models (see e.g. [1] for a review), aimed to study unification of the electromagnetic interaction with gravity. Nowadays, the presence of higher spatial dimensions is very natural in high energy physics, in string, superstring and other unification and fundamental theories [2]. However, the addition of extra spatial dimensions is in conflict with the natural world which, when probed in any experiment, has only revealed the presence of three spatial dimensions.

To reconcile the constraint of three spatial dimensions of the natural world with the introduction of extra dimensions, important scenarios have been proposed. Here we focus our attention upon the Randall-Sundrum (RS) work [3], where the relevant portion of the higher dimensional spacetime is embedded within a five-dimensional anti-de Sitter (AdS) geometry. This scenario assumes that the (3,1) spacetime that describes the natural world is embedded in an $AdS_5$ warped geometry, with a single extra spatial dimension of infinite extent. This is known as the RS2 braneworld scenario, and the warp factor identifies a thin brane profile, decaying along the extra dimension $y$ in the form $\exp(-2|y|)$.

Soon after the proposed thin braneworld scenario, it was modified with the presence of scalar fields, giving rise to a new, very interesting thick braneworld scenario, in which the warp factor is now described by another function, which depends on the specific scalar field model one considers [4, 5]. The presence of scalar fields brought interesting possibilities, as the appearance of a new feature, the splitting of the brane, which springs under the presence of distinct effects, at finite temperature [6] or with specific scalar field models [7] in the presence of at least one additional parameter, to be used to control the splitting of the brane.

Like the number of spatial dimensions, there are other foundational aspects of the idea of gravitation as a geometric phenomenon which are likely to provide interesting new viewpoints on the fundamental open questions of gravitational physics. In this sense, most extensions of GR adopt the implicit assumption that spacetime is a Riemannian structure completely determined by the metric degrees of freedom (see e.g. [8] for some reviews). However, this is a question that must be determined by observation rather than imposed by convention or selected on practical grounds. For this reason, one can legitimately consider new geometric scenarios and explore their phenomenology to gain insight on the kind of new physics that they could bring about. In this sense, some of us have recently carried out a program where black hole solutions have been obtained within extensions of GR formulated à la Palatini, i.e., assuming that metric and connection are independent geometrical entities. Though in GR metric and Palatini formulations lead to the same field equations, this ceases to be the case as soon as one considers extensions beyond GR. New gravitational physics can thus be found in four-dimensional theories formulated in the Palatini formalism, like $f(R)$ [9], $f(R, R_{\mu\nu}R^{\mu\nu})$ [10], Born-Infeld gravity [11] and also in five-dimensional $f(R)$ gravity [12], which differs from the usual metric formulation of those same theories. A nice feature of these Palatini theories, in particular, is that the point-like black hole singularity found in GR...
is generically replaced by a finite area wormhole structure, with potentially relevant consequences for the understanding of the last stages of black hole evaporation and the phenomenology of black holes at particle accelerators.

The aim of this work is to offer a first step to combine the two fundamental questions on the foundations of space-time raised above. We explore the theoretical and phenomenological implications for braneworld scenarios of admitting that the spacetime has independent metric and affine structures, as in the Palatini approach. We shall thus focus on the thick braneworld scenario with a single extra dimension of infinite extent, considering the presence of a real scalar field in the $AdS_5$ geometry and assuming the gravitational dynamics to be described by a Palatini $f(R)$ Lagrangian (see Refs. [16, 17] for reviews), as the simplest extension of GR. See also Ref. [18] for previous studies on thick braneworld scenarios with $f(R)$ dynamics in the metric approach. We note that in the Palatini formulation, the vacuum field equations exactly boil down to the equations of GR with, possibly, a cosmological constant (depending on the particular gravity Lagrangian chosen). Nevertheless, when matter fields are present, modified dynamics arises. Therefore, Palatini theories offer a way to generate new gravitational effects without the need for introducing new dynamical degrees of freedom.

We start in Sec. II by introducing notation and the model to be investigated, which describes a source scalar field minimally coupled to the Palatini $f(R)$ geometry. We then deal with the braneworld scenario in Sec. III and there we write the equations of motion and the scalar field equations in a first-order framework. Also, we verify consistency of the equations of motion that appear in the Palatini braneworld scenario, which further reduce to the equations of motion of the standard thick braneworld scenario when one changes $f(R) \rightarrow R$, as expected. We study a simple example of a Palatini brane in Sec. IV and there we show that the warp factor vanishes asymptotically, much faster than it does in the case of a standard thick brane.

In order to strengthen the scope of the work, in Sec. V we follow another route, and we consider the case with $f(R) = R + \epsilon R^n$, with $n = 2, 3, \ldots$, and with $\epsilon$ very small. We then implement a perturbative procedure, obtaining results valid up to first-order in $\epsilon$. We study two distinct examples, one with the potential of the source scalar field being polynomial, up to the $\phi^4$ power and engendering spontaneous symmetry breaking, and the other nonpolynomial, of the sine-Gordon type. The results are compared with the cases of standard thick branes, with $\epsilon = 0$, and they appear to behave consistently, suggesting that the new braneworld scenarios are robust. In Sec. VI we end the work with some comments and conclusions.

II. FIELD EQUATIONS FOR FIVE-DIMENSIONAL PALATINI $f(R)$ GRAVITY

We start with the action

$$S = \frac{1}{2\kappa^2} \int d^5x \sqrt{-g} f(R) + S_s(g_{\mu\nu}, \varphi),$$  (1)

where $\kappa^2$ is Newton’s gravitational constant in appropriate system of units (in GR, $\kappa^2 = 8\pi G$), $g$ is the determinant of the spacetime metric $g_{\mu\nu}$, $R = g^{\mu\nu} R_{\mu\nu}$ is the curvature scalar constructed with the Ricci tensor $R_{\mu\nu}(\Gamma) = \partial_\lambda \Gamma^\lambda_{\mu\nu} - \partial_\nu \Gamma^\lambda_{\lambda\mu} + \Gamma^\lambda_{\lambda\nu} \Gamma^\lambda_{\mu\nu} - \Gamma^\lambda_{\nu\rho} \Gamma^\rho_{\lambda\mu}$, where the connection $\Gamma \equiv \Gamma^\lambda_{\mu\nu}$ is a priori independent of the metric (Palatini formalism). For simplicity we assume a torsionless scenario, $\Gamma^\lambda_{\mu\nu} = \Gamma^\lambda_{\nu\mu}$ (see Ref. [19] for more details on the role of torsion in Palatini theories). The source contribution $S_s$ is supposed to couple to the metric only, and $\varphi$ denotes collectively the source fields. The spacetime is five-dimensional, so $\mu, \nu, \ldots = 0, 1, ..., 4$; also, we will use latin indices $a, b, \ldots = 0, 1, 2, 3$ to span the four-dimensional spacetime, and denote the fifth dimension $x^5$ by $y$.

The field equations for the action (1) are obtained by independent variation with respect to the metric and connection. The details of this derivation can be found in Ref. [12] and therefore we bring here the final results:

$$f R_{\mu\nu} - \frac{1}{2} f g_{\mu\nu} = \kappa^2 T_{\mu\nu},$$  (2)

$$\nabla^\lambda (\sqrt{-g} f R g^{\mu\nu}) = 0,$$  (3)

where $T_{\mu\nu} = -\frac{1}{2\kappa^2} \delta S_s / \delta g_{\mu\nu}$ is the energy-momentum tensor of the matter, and $\nabla^\lambda$ denotes the covariant derivative with respect to the independent connection $\Gamma$. To solve these equations we introduce an auxiliary metric $h_{\mu\nu}$, so that Eq. (3) can be formally written as

$$\nabla^\lambda (\sqrt{-h} h^{\mu\nu}) = 0$$  (4)

Comparison between these two equations leads to

$$h_{\mu\nu} = f_2^{1/2} g_{\mu\nu}; \quad h^{\mu\nu} = f_R^{-2/3} g^{\mu\nu},$$  (5)

which puts forward that the two metrics are conformally related. We note that these equations imply that the independent connection $\Gamma$ is metric-compatible with $h_{\mu\nu}$ (but not with $g_{\mu\nu}$), which implies that $\Gamma^\lambda_{\mu\nu}$ is the Levi-Civita connection of $h_{\mu\nu}$ (see Ref. [15] for details). The introduction of this auxiliary metric greatly simplifies the explicit expression of the metric field equations (2), which read

$$R^\nu_{\mu\nu}(h) = \frac{\kappa^2}{f_R^{5/3}} \left( \frac{f}{2\kappa^2} T^\nu_{\mu\nu} + T^{\nu}_{\mu\nu} \right),$$  (6)

where $T^\nu_{\mu\nu} = T_{\mu\alpha\beta\gamma} g^{\alpha\beta\gamma}$. We note from Eq. (5) that for $f(R) = R$, the new metric $h_{\mu\nu}$ coincides with $g_{\mu\nu}$, and
we then get back to GR. It is worth mentioning that i) the second-order character of the field equations (11) and ii) the fact that in vacuum, $T_{\mu \nu} = 0$, they boil down to the equations of GR plus a cosmological constant term (depending on the explicit functional form of the $f(R)$ Lagrangian chosen), which implies absence of new propagating degrees of freedom in the spectrum of the theory.

Further information on this problem can be obtained by taking the trace in Eq. (2), which yields

$$R f_R - \frac{5}{2} f = \kappa^2 T.$$  \hspace{1cm} (7)

We note that this is not a differential equation, but just an algebraic nonlinear relation between the gravity and source field equations (6), together with the formulation of the field equations (5), which implies absence of new propagating degrees of freedom in the spectrum of the theory.

Putting these elements into the field equations (6) we get

$$R_{\mu}^{\nu}(h) = \frac{1}{2 f_R^{5/3}} \begin{pmatrix} (f - \kappa^2 (\phi_y^2 + 2V)) I_{4 \times 4} & \hat{0} \\ \hat{0} & f + \kappa^2 (\phi_y^2 - 2V) \end{pmatrix}.$$  \hspace{1cm} (13)

On the other hand, the source scalar field equation reads

$$\phi_{yy} + 4 A_y \phi_y = V_\phi.$$  \hspace{1cm} (14)

We note that the system of equations with the symmetries of the problem gives rise to three equations to be solved, though they are not fully independent and one of them can be deduced from the other two. This gives consistency to the problem since we have two independent variables, $A(y)$ and $\phi(y)$. This issue also appears in the standard braneworld scenario.

Using the fact that the coefficients of the independent connection $\Gamma$ correspond to the Christoffel symbols of the metric $h_{\mu \nu}$ (recall Eq. (11)) we obtain, after standard calculations, the components of the Ricci tensor

$$R_i^j(h) = -\frac{1}{3 f_R^{5/3}} \left( 7 A_y \frac{f_{R,y}}{f_R} + 3 (4 A_y^2 + A_{yy}) + \frac{f_{R,yy}}{f_R} \right)$$  \hspace{1cm} (15)

$$R_4^4(h) = -\frac{4}{3 f_R^{2/3}} \left( -f_{R,y}^2 + 3 (A_y^2 + A_{yy}) + A_y \frac{f_{R,y}}{f_R} + \frac{f_{R,yy}}{f_R} \right).$$  \hspace{1cm} (16)

We now manipulate these equations to put them in a more amenable form. Consider the combination $4 R_i^j - R_4^4$, which yields

$$\left( A_y + \frac{1}{3} \frac{f_{R,y}}{f_R} \right)^2 = -\frac{1}{8 f_R} \left( f - \kappa^2 \left( 2V + \frac{5}{3} \phi_y^2 \right) \right),$$  \hspace{1cm} (17)

where the object $f_{R,y}/f_R$ can be computed as

$$\frac{f_{R,y}}{f_R} = \frac{2 \kappa^2 f_{RR}}{f_R (f_{RR} - \frac{3}{2} f_R)} (6 A_y \phi_y - V_\phi) \phi_y.$$  \hspace{1cm} (18)

Note that in the GR limit, $f_R = 1$, Eq. (16) nicely recovers the right expression, $A_y^2 = \frac{f_{RR}}{17} (\phi_y^2 + 2V)$ (with $\kappa^2 = 2$). See, e.g., Refs. [3].

On the other hand, the combination $R_i^i - R_4^4$ yields

$$3 \left( A_{yy} + \frac{1}{3} \frac{f_{R,yy}}{f_R} - \frac{1}{3} \left( \frac{f_{R,y}}{f_R} \right)^2 \right) = \frac{f_{R,y}}{f_R} \left( A_y + \frac{1}{3} \frac{f_{R,y}}{f_R} \right) - \frac{\kappa^2 \phi_y^2}{f_R}.$$  \hspace{1cm} (19)

We then define

$$\theta \equiv A_y + \frac{1}{3} \frac{f_{R,y}}{f_R},$$  \hspace{1cm} (20)

and using the trace equation, we can turn (16) into

$$\theta^2 = \frac{2 \kappa^2 \phi_y^2}{15 f_R} - \frac{R}{20}.$$  \hspace{1cm} (21)

On the other hand, (18) can be rewritten as

$$\theta_y = \frac{f_{R,y}}{3 f_R} \theta - \frac{\kappa^2 \phi_y^2}{3 f_R}.$$  \hspace{1cm} (22)
A. Consistency of the model

So far we have obtained the metric field equations (20) and (21). Conservation of energy-momentum, which follows from the metric field equations, must imply on consistency grounds the scalar field equation (14). We now proceed to verify this point. We begin by taking a derivative of (20) with respect to $y$ and then using (21) to replace $\theta_y$. This leads to

$$
\frac{2 f_R \phi_y}{3 f_R} - 2 \kappa^2 \left( \theta - \frac{1}{5} f_R \phi_y \right) \frac{\phi_y}{20} + \frac{R_\phi}{5 f_R} = \frac{4 \kappa^2}{15 f_R} \phi_{yy}.
$$

Replacing $\theta$ in this equation with (20) and then using (21), we get

$$
A_y \phi_y + \frac{1}{20 \kappa^2} \left( R f_R \phi - \frac{3}{2} R_\phi f_R \right) = -\frac{2}{5} \phi_{yy}.
$$

Here is where we can use the scalar field equation. The trick is to use $A_y \phi_y = -(\phi_{yy} - V_\phi)/4$, which leads to

$$
5 \kappa^2 V_\phi + 3 \kappa^2 \phi_{yy} = -\left( R f_R \phi - \frac{3}{2} R_\phi f_R \right).
$$

Given that $\phi_{yy} = -\frac{1}{2} \frac{d^2 \phi}{d \phi^2}$ and that $R f_R \phi - \frac{3}{2} R_\phi f_R = \frac{d}{d \phi} \left( R f_R - \frac{5}{2} f \right)$, we can rewrite (23) in the form

$$
- \kappa^2 \frac{d^2 \phi}{d \phi^2} (5 V + \frac{3}{2} \phi_\phi^2) = \frac{d}{d \phi} \left( R f_R - \frac{5}{2} f \right),
$$

which is just the derivative with respect to $\phi$ of the trace equation (7) with the matter described by (12), for which $T = -(5 V + \frac{3}{2} \phi_\phi^2)$. This verifies the consistency of the metric and scalar field equations.

B. First-order equations

We now go further into the above Palatini thick brane scenario and, in analogy with the standard GR problem, we search for first-order differential equations able to solve the equations of motion. In order to extend the procedure of Ref. [29] to nonlinear $f(R)$ Lagrangians, we introduce two new functions $W = W(\phi)$ and $\alpha = \alpha(\phi)$, and we write $\theta$ as

$$
\theta = -\frac{1}{3} \alpha(\phi) W(\phi),
$$

with $\alpha(\phi)$ an unspecified function of $\phi$. Computing $\theta_y$ and inserting the result in (21), we get

$$
W \left( \frac{\alpha f_R \phi}{3 f_R} - \alpha \phi \right) + \kappa^2 \phi_y = \alpha W_\phi.
$$

We note that for $f = R$, we get from (19) that $\theta = A_y$, and the above equation becomes

$$
\phi_y = \frac{1}{2} W_\phi,
$$

with the choices $\kappa^2 = 2$ and $\alpha = 1$. This is the expected result, which recovers the standard braneworld scenario of GR. Inspired on this, we now make a different choice, considering $\alpha = f_R^{1/3}$. In this case, we get a very natural extension of the standard results, using

$$
\theta = -\frac{1}{3} f_R^{1/3} W.
$$

The first-order equations are

$$
\phi_y = \frac{f_R^{1/3}}{\kappa^2} W_\phi,
$$

$$
A_y = -\frac{1}{3} f_R^{1/3} \left( W + \frac{1}{\kappa^2} f_R W_\phi \right)
$$

To go further, we insert the results (24) and (30) into (20) to get

$$
R = \frac{f_R^{1/3}}{3} \left( \frac{2}{5 \kappa^2} f_R W^2 - \frac{1}{3} W^2 \right)
$$

We note now that by specifying the functions $W(\phi)$ and $f_R(\phi)$, one automatically gets $R(\phi)$ from (29). It is then immediate to obtain $f(\phi) = \int f_R(\phi) dR(\phi)$. This yields a parametric representation for $f[R(\phi)]$.

With the above results it is easy to verify that for $f = R$, $\alpha = 1$, and $\kappa^2 = 2$ the results of GR [20] are nicely recovered. In fact, from (30) one obtains (28), and from (31) the first-order equation

$$
A_y = -\frac{1}{3} W.
$$

Also, we make simple algebraic manipulations to write

$$
V(\phi) = \frac{1}{8} W_\phi^2 - \frac{1}{3} W^2.
$$

These results lead to the correct first-order Eqs. (25) and (33), with the potential (34), as they appear in the standard GR scenario.

IV. EXAMPLE

Let us now work out a simple example of a Palatini brane. We first use (31) to have the equation for the warp function in the form

$$
A(\phi) = -\frac{1}{3} \ln f_R - \frac{\kappa^2}{3} \int d\phi f_R^{-1} W - A_0
$$

where $A_0 = A(0)$. Now, we suppose that the scalar field obeys

$$
\phi_y = b \cos(b \phi),
$$

where $b$ is real parameter. This equation can be integrated to give the solution

$$
\phi(y) = \frac{1}{b} \arcsin(\tanh(b^2 y)).
$$
Also, we suppose that
\[ f_R = (1 + ab^2 \cos^2(b\phi))^{-3/4}, \]
where \( a \) is another real parameter. Thus, from Eq. (30) we get
\[ W(\phi) = \kappa^2 \sin(b\phi) \left( \left(1 + \frac{2ab^2}{3}\right) + \frac{ab^2}{3} \cos^2(b\phi) \right). \]

Thus, the warp function (35) becomes
\[ A(y) = -A_0 + \ln(u) + \frac{2\kappa^2}{27b^2} u^3 - \frac{\kappa^2}{3b^2} \left(1 + \frac{2ab^2}{3}\right) \left( \text{arctanh}(1/u) + \text{arctan}(u) \right), \]
where \( u(y) = (1 + ab^2 S(y))^{1/4} \) and \( S(y) = \text{sech}^2(b^2 y) \). Moreover, the scalar curvature can be written as
\[ R(y) = \frac{20}{3} u^4 \left( \frac{2b^2}{5} S^2 u^5 - \frac{1}{3} \kappa^2 \left(1 + \frac{2ab^2}{3}\right) \right) \left(1 + \frac{ab^2}{3}\right) S^2 - \frac{ab^2}{3} S^4 \right). \]

Figure 1. Profile of the warp function \( e^{2A(y)} \), where \( A(y) \) is given by Eq. (40). We take \( b = 1, \kappa^2 = 2, \) and \( a = 0 \) (solid line), 0.125, 0.25, 0.5 and 1.0 (dashed lines).

Figure 2. Profile of the curvature \( R(y) \) given by Eq. (41). We take \( b = 1, \kappa^2 = 2, \) and \( a = 0 \) (solid line), 0.125, 0.25, 0.5 and 1.0 (dashed lines).

We depict the warp factor (40) in Fig. 1 and the scalar curvature (41) in Fig. 2. We note from the warp factor, that for \( b = 1 \) and \( \kappa = 2 \), the value \( a = 0 \) reproduces the GR result, and the increasing of the parameter \( a \) tends to localize further the extra dimension. This is an interesting effect, not found in the standard thick brane scenario.

\[ W(\phi) = \kappa^2 \sin(b\phi) \left( \left(1 + \frac{2ab^2}{3}\right) + \frac{ab^2}{3} \cos^2(b\phi) \right). \]

V. PERTURBATIVE APPROACH

In order to further probe the Palatini brane scenario, in this section we study models that can be seen as small fluctuations around the standard thick brane scenario. We take units \( \kappa^2 = 2 \) and investigate Palatini brane, governed by a real, very small parameter \( \epsilon \), which in the limit \( \epsilon \to 0 \) leads us back to the case of a standard thick brane. This possibility is implemented using
\[ f(R) = R + \epsilon R^n, \]
with \( n \) integer, \( n = 2, 3, \ldots \). This model, with \( n = 2 \), was considered in [21] to study charged black hole (Reissner-Nordström-like) solutions and also to study inflation (see e.g. [21]) and non-singular cosmologies [22]. Here, this example will allow us to implement a standard perturbative approach, going up to first order in \( \epsilon \).

With this in mind, we use (22) to obtain
\[ R = R_0(\phi) - \epsilon c(n) R_0^{n-1}(\phi) W^2 W_\phi^2, \]
where
\[ R_0(\phi) = \frac{4}{3} W_\phi^2 - \frac{20}{9} W^2, \quad c(n) = \frac{7n}{405}. \]
The potential takes the form
\[ V(\phi) = \frac{1}{8} W_\phi^2 - \frac{1}{3} W^2 - \epsilon V_\epsilon(\phi), \]
with the contribution \( V_\epsilon \) given by
\[ V_\epsilon(\phi) = \frac{1}{20} \left( (2n - 5) R_0 + (4 + 3c(n)W^2) W_\phi^2 R_0^{n-1} \right). \]

Moreover, from Eq. (30) we have
\[ \phi_y = \frac{1}{2} W_\phi \left( 1 + \frac{4}{3} \epsilon R_0^{n-1} \right), \]
and so
\[ 2 \frac{d\phi}{W_\phi} \left( 1 - \frac{4}{3} \epsilon R_0^{n-1} \right) = dy, \]
or better
\[ 2 \int \frac{d\phi}{W_\phi} = y + \frac{8}{3} \int \phi_0 R_0^{n-1} W_\phi d\phi = \tilde{y}. \]
The case of \( \epsilon = 0 \) gives
\[ 2 \int \frac{d\phi}{W_\phi} = y \rightarrow \phi = \phi_0(y), \]
where \( \phi_0(y) \) is the solution for \( \epsilon = 0 \), the unperturbed solution. Then, for \( \epsilon \neq 0 \), from (19) we have
\[ 2 \int \frac{d\phi}{W_\phi} = \tilde{y} \rightarrow \phi = \phi_0(\tilde{y}) = \phi_0 \left( y + \frac{8}{3} \int \phi_0 R_0^{n-1} W_\phi d\phi \right). \]
Expanding up to first-order in $\epsilon$ we obtain

$$\phi(y) = \phi_0(y) + \frac{8}{3} \epsilon \frac{d\phi_0}{dy} \int_{y_0}^{y} \frac{R_0^{n-1}}{W_0} d\phi,$$

or

$$\phi(y) = \phi_0(y) + \epsilon \phi_\epsilon(y),$$

with

$$\phi_\epsilon(y) = \frac{4}{3} \frac{d\phi_0}{dy} \int R_0^{n-1}(\phi_0) dy.$$

Now, we use (31) to write

$$A_y = -\frac{1}{3} W(\phi_0) - \frac{1}{3} \epsilon Q(\phi_0) - \frac{1}{3} \epsilon \phi_\epsilon(y) \frac{d}{dy}(W(\phi_0)),$$

where

$$Q = \frac{1}{3} W R_0 + 20m(n-1) \left( \frac{W_0}{15} - \frac{W}{9} \right) W_\phi^2 R_0^{n-2}. $$

Finally, the energy density $T_{00}$ can be written as

$$\rho(y) = -\epsilon e^{2A(y)} \mathcal{L}(y),$$

with $\mathcal{L}(y)$ given by

$$\mathcal{L}(y) = -\frac{1}{2} \phi_0'^2 - V(\phi_0) - \epsilon \left( \phi_0 + \frac{d}{dy} V(\phi_0) \right).$$

These are the general results, which we now illustrate with two distinct examples.

The first example is described by $W(\phi) = 2\phi - 2\phi^3/3$ and $n = 2$. The potential (45) is illustrated in Fig. 3. The static solution is given by (53), where $\phi_0 = \tanh(y)$ and

$$\phi_\epsilon(y) = \frac{16}{729} \operatorname{sech}^2(y) \left( 141 \tanh(y) - 52 \tanh^3(y) + 3 \tanh^5(y) - 60 \right).$$

This solution is depicted in Fig. 4. The warp factor $e^{2A(y)}$ is obtained integrating numerically Eq. (55), is shown in Fig. 5. From this figure we see that the warp factor contributes to further localize the extra dimension, as we noted before in the example depicted in Fig. 4. We have plotted the corresponding energy density in Fig. 6.

The second example is described by $W(\phi) = 2a^2b \sin(\phi/a)$ and $n = 2$. The potential (56) is illustrated in Fig. 7. The static solution around $V(\phi = 0)$ is given by (57), where $\phi_0 = a \arcsin(\tanh(y))$ and

$$\phi_\epsilon(y) = \frac{16}{27} a^3 b^2 \operatorname{sech}(by) \left( (3 + 5a^2) \tanh(by) - 5a^2 b \right).$$

which is depicted in Fig. 8. Again, the warp factor is obtained after integrating numerically Eq. (55), and it is displayed in Fig. 9. Finally, in Fig. 10 we depict the respective energy density. Once again, we note that the warp factor depicted in Fig. 9 shows the tendency to localize the extra dimension, as we move away from the standard scenario.
VI. COMMENTS AND CONCLUSIONS

In this work we have investigated braneworld models in an $AdS_5$ warped geometry with a single extra spatial dimension of infinite extent. We implemented the study under the Palatini approach, in which both the metric and connection are treated as independent degrees of freedom. As the Palatini formulation for GR turns out to be equivalent to the standard metric approach (where the connection is taken a priori to be compatible with the metric), we had to replace the Einstein-Hilbert Lagrangian, $R$, by a nonlinear $f(R)$ Lagrangian, as the simplest extension of GR. The source action, which is responsible for the generation of the brane, was taken as that of a single scalar field in analogy with the standard thick brane scenario of GR.

We worked out the equations of motion for arbitrary $f(R)$ Lagrangian, which are second-order differential equations, and proved their consistency. A first-order framework to solve the equations of motion has been obtained by generalizing the approach used in the GR case. We then investigated a simple (and exact) example of a Palatini brane and showed that the warp factor engenders an interesting effect, the tendency to localize the extra dimension due to the nonlinear corrections.

We also studied another example of brane in which a small parameter is introduced to control the departures from the dynamics of GR. The small parameter was used to carry out a perturbative investigation. Although the perturbative approach cannot be used to probe the model in full detail, it has been useful to show the robustness of the brane solutions against Palatini $f(R)$ perturbations in the dynamics. As in the exact model studied in Sec. IV in the two perturbative examples that we investigated in Sec. V, we also noted that the warp factor tends to localize the extra dimension. We then conjecture that the thick braneworld scenario developed in the Palatini approach contributes to localize the extra dimension, an effect which is not undesirable and deserves further attention. In this context, it would be interesting to study further extensions of GR including additional curvature invariants, as suggested by the quantization of fields in curved space-times [23], and in other braneworld scenarios. These investigations are currently underway.

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