under study, which could open the door to studying frustration in quantum spin models in a well-controlled setting.

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NONLINEAR OPTICS

Symmetry breaking in laser cavities

The ability to invoke and switch between asymmetric lasing states in two coupled cavities built in a nonlinear photonic crystal creates opportunities for a new form of optical memory.

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When discussing the propagation of excitations in a physical system, the shape of the trapping potential determines the system’s symmetry. A well-known type is the so-called double-well potential (DWP), which has symmetry due to the use of two adjacent identical wells (Fig. 1). The DWP is one of the most fundamental settings in quantum mechanics, for which it is commonly known that its ground state (GS) is symmetric, whereas the first excited state is antisymmetric, both being single (non-degenerate) states.

Interestingly, there is a far-reaching analogy between the quantum-mechanical states in a DWP and the propagation of light in two coupled waveguides or cavities, which display a similar symmetry and set of modes. The analogy arises because the behaviours of both systems are governed by similar fundamental equations. The propagation of light is derived from Maxwell’s equations in the paraxial approximation (assuming weak diffraction of a relatively broad beam), which reduce to an equation resembling the linear Schrödinger equation in quantum mechanics. In the presence of optical nonlinearity, the analogy still holds with the nonlinear Kerr effect adding a self-focusing cubic nonlinearity to the propagation equation, making it tantamount to the celebrated nonlinear Schrödinger equation.

In the quantum world, a similar situation occurs when the DWP traps an ultracold rarefied atomic gas in the state of the Bose–Einstein condensate (BEC). In the mean-field approximation, which is extremely accurate for rarefied atomic gases, repulsive or attractive collisions between atoms give rise to a cubic nonlinearity that emulates the optical Kerr effect. The outcome in this scenario is that the Gross–Pitaevskii equation replaces the linear Schrödinger equation for describing the system. Importantly, in such systems with nonlinear self-focusing, the GS symmetry follows the symmetry of the underlying DWP structure only in the weakly nonlinear regime. As the strength of the nonlinearity increases, a fundamental phenomenon called spontaneous symmetry breaking (SSB) occurs. In its simplest form, the SSB implies that an asymmetry develops and the probability to find the quantum particle in one well of the DWP structure — or the intensity of the guided light beam in one core of the dual-core waveguide — is larger than in the other. In the case of the defocusing nonlinearity, the GS symmetry remains unbroken, but the nonlinear term breaks the antisymmetry of the first excited state.

The nonlinear asymmetric GS is doubly degenerate: the SSB gives rise to a pair

Figure 1] The simplest example of the double-well potential structure in quantum mechanics. A potential box, $U(x)$, is split by a narrow tall barrier in the middle. Even and odd wavefunctions of the ground and first excited states, $\psi(x)$, are schematically shown by the solid and dashed curves, respectively. In the limit of the infinitely deep potential box, $U_b \rightarrow \infty$, and for the central barrier replaced by the Dirac delta function, $\delta(x)$, where $\epsilon = U_d / a$, the GS energy is $\hbar^2 k^2 / (2m)$, where $m$ is the mass of the quantum particle and $k$ is the smallest root of the equation $\tan(\pi L / 2) = -\hbar^2 k / (\epsilon a)$ and the (larger) energy of the first excited state is $2\pi \hbar^2 / (mL^2)$.

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of two mutually symmetric GSs, with the maximum of the atomic density, or the intensity of the guided light beam, spontaneously emerging in either the left or right potential well or guiding core.

The same nonlinear system still admits a symmetric state coexisting with the asymmetric ones, but, above the SSB point, the asymmetric state no longer represents the GS, being unstable against symmetry-breaking perturbations. Accordingly, in the course of the spontaneous transition from the unstable symmetric state to a stable asymmetric one, the choice between the two mutually degenerate asymmetric states is determined by random perturbations that ‘push’ the system either to the left or to the right.

Similar to the situation taking place in other areas of physics dominated by nonlinear effects, there is a stark imbalance between the number of theoretical and experimental studies on the topic of SSB. Whereas theoretical analyses are numerous and have advanced (see ref. 4 for an overview), there are only a few experimental studies.

An essential step forward in this direction is reported in this issue of Nature Photonics by Hamel et al., who demonstrate and analyse SSB in a dual nanocavity laser embedded into a nonlinear photonic crystal made of a semiconductor material. Such coupled dual cavities are sometimes referred to as a photonic molecule, as the system’s optical modes resemble the electronic states of a diatomic molecule like hydrogen. The two closely spaced laser cavities are formed close to the lasing threshold, by effectively eliminating the carrier-population variables, thus reducing the system to a pair of linearly coupled complex Ginzburg–Landau equations (CGLEs)

Simulations of the model demonstrate close agreement with experimental observations. The basic SSB effect observed and theoretically modelled by Hamel et al., is in the form of a pitchfork bifurcation, with a spontaneous transition from a symmetric mode, with equal powers in the coupled cavities, to a mode that clearly features a larger power in either of the two cavities. A basic pitchfork-bifurcation diagram, which is common for a broad class of nonlinear dual-cavity systems, including the one realized by Hamel et al., is displayed in Fig. 2a. This bifurcation, with two mutually symmetric branches of the asymmetric states going forward from the bifurcation (critical) point, belongs to the supercritical (alias forward) type. In contrast with the subcritical (alias backward) type, it does not admit the coexistence of symmetric and asymmetric states below the critical point. The analysis reported by Hamel et al., demonstrates that the observed SSB scenario is quite robust; in particular, it is not altered if random noise is added to the underlying model.

At still higher powers (typically exceeding the critical value at the SSB point by a factor of around 1.4), the asymmetric lasing states observed by Hamel et al., demonstrate a secondary instability, accounted for by the Hopf bifurcation, which transforms the stationary states into a regime of ultrafast Josephson oscillations between the two cavities, at frequencies of ~150 GHz (Josephson oscillations are closely related to Josephson junctions). These results clearly demonstrate the complexity of the behaviour of nonlinear dissipative systems.

The coexistence of the two robust asymmetric states, which are mirror images of each other, suggests a prospect for using them as elements of a binary-code optical memory. This potential application raises an important question: is it possible to switch one state into another using a control signal? Hamel et al., investigated such a possibility by using a short (~100 ps) control laser pulse to selectively illuminate the cavity with the larger lasing intensity in the asymmetric state (the main pump beam with a central spot size of 2.2 μm was applied symmetrically). The action of the control pulse on carriers in the stronger-excited cavity leads to lasing saturation, switching the excitation into the other cavity.
The results reported by Hamel et al.5 provide interesting insights into the behaviour of bifurcations. In particular, it is well known from theoretical studies that the SSB bifurcations feature two generic forms, as shown in Fig. 2. These forms are super- and subcritical ones, alias forward and backward bifurcations (which are tantamount, respectively, to the phase transitions of the second and first kind in statistical physics). The subcritical bifurcation features bistability of the symmetric and asymmetric states in a narrow region of the power, N, below the critical point. A supercritical bifurcation may be transformed into a subcritical one by a change of the underlying nonlinearity (for example, by a transition from self-focusing to a combined focusing-defocusing nonlinearity).

However, a more relevant option for the expansion of the studies of the SSB phenomenology is to extend the effectively zero-dimensional dual-cavity setting (with each cavity treated as a single quantum dot) to the one-dimensional geometry, in which the two cavities embedded into the photonic crystal are elongated, in the form of parallel stripes devoid of holes, thus establishing a transverse direction, and adding transverse-diffraction terms to the underlying system of coupled CGLEs11.

In this new geometry, it is predicted that various species of two-component solitons, coupled by super- and subcritical SSB bifurcations, should exist11. The creation and experimental observation of such solitons is the next experimental challenge for research in the area. Beyond the framework of photonics, it would be very interesting to see the SSB experimentally realized in a BEC trapped in a dual-core cigar-shaped configuration, as an extension of the single-cigar set-up where matter-wave solitons have been created11.

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NONLINEAR OPTICS
Nonlinear virtues of multimode fibre

The finding that multimode optical fibres support a rich and complex mix of spatial and temporal nonlinear phenomena could yield a plethora of promising applications.

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Supercontinuum generation — the extreme spectral broadening of laser light (a span from the ultraviolet to the mid-infrared is possible) — is a fascinating process that takes place in a dispersive and strongly nonlinear optical medium. For example, a supercontinuum can be created by the propagation of intense ultrashort pulses in a short section of glass, or less powerful pulses in a long optical fibre. High spectral brightness supercontinuum light sources have widespread applications, including use in optical metrology, environmental and biomedical spectroscopy, medical imaging, gas sensing and communications.

This growing list of applications is fuelling intense development of such sources of broadband coherent radiation. Indeed, a resurgence of interest in supercontinuum studies has followed the development of photonic crystal fibres (PCFs)7. PCFs are highly beneficial for nonlinear experiments as they can be engineered to have a reduced core size, which substantially increases the beam intensity, and the fibre’s dispersion profile can be tailored to match virtually any pump laser source. Supercontinuums generated in a single-mode optical fibre have superior beam quality with respect to bulk media. However, single-mode fibres have an inherent major drawback, which is the relatively low energy (typically less than 20 μJ) that can be collected at their output due to their micrometre-scale core size. Thus, supercontinuum sources based on single mode fibres are of limited use for applications that require high-energy sources, such as airborne remote sensing.

Now, as they report in Nature Photonics, Logan Wright and co-workers9 provide the first demonstration of an alternative, all-fibre route to the development of high-energy supercontinuum light sources based on spatiotemporal nonlinear effects in a multimode fibre. Indeed, since the advent of fibre amplifiers, the power of fibre lasers has been increasing significantly, up to megawatt peak values for lasers that operate in the pulsed regime. These developments require fibres with larger mode areas that can guide higher injected pump powers. As it is increasingly difficult to meet such requirements with conventional single mode fibres, it is natural to replace them with multimode fibres for high-power applications.

From a fundamental viewpoint, the experiments of Wright et al.9 involve a range of remarkable nonlinear effects whose complexity requires a much deeper understanding of spatiotemporal nonlinear pulse propagation in multimode fibres. A variety of applications will also likely benefit from the development of novel high-energy, short-pulse broadband and versatile fibre sources, inspired by the results described by Wright and colleagues9.

The findings may also help to improve optical signal processing techniques for spatial division multiplexing, whereby the individual modes of a multimode fibre are exploited to define separate spatial channels for information transmission4. In addition, the observations by Wright et al.9 reveal that multimode fibres can easily be used to generate intense mid-infrared radiation in a conceptually simple experimental arrangement — a feature of great interest for many sensing and spectroscopy applications, as most molecules display fundamental vibrational absorptions in this domain.