Novel techniques for the analysis of dynamic pressure in penstocks

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Abstract. This paper focuses on the analysis of dynamic pressure fluctuations in hydroelectric power plants. Pressure fluctuations are usually measured with remote sensors and the dynamic behaviour of the remote tubing line modifies the readings of dynamic pressure and thus introduces a bias. A method to characterize the remote tubing line and to correct the actual pressure is proposed. The corrected measurements are compared with those obtained using intrusive flush-mounted sensors as well as non-intrusive sensors (PVDF wires and strain gauges). Then, the technique to separate the forward and backward components of the obtained pressure pulsations is outlined. It is further shown how the pulsations and the hoop stresses can be reconstructed along the penstock. Finally, measurements carried out in a hydropower plant are presented to demonstrate the proposed techniques applicability for the analysis of dynamic pressure in a penstock. The results show that a pressure profile can be mapped along the penstock length which allows a comparison with the direct pressure sensor measurements.

1. Introduction

The dynamic pressure in a penstock is an important parameter for operating a hydro-electric power plant. The dynamics of large-diameter water pipes in hydro-electric power plants has already been studied in depth and reported in papers and textbooks, e.g. [1]. Some recent work on a real penstock has been presented and illustrated [2]. In this paper, emphasis is given to advanced processing techniques aimed at analysing the dynamics of penstocks. Such techniques require accurate measurement of pressure pulsations.

On a prototype, pressure fluctuations are usually measured with remote sensors whereas flush-mounted sensors are used during model tests. The remote sensors can be located several meters from the penstock using a remote tubing line. The dynamic behaviour of a remote line modifies the readings of dynamic pressure and thus introduces a bias. From this outlook, a method to characterize the remote line and to subsequently correct the pressure readings is described in detail. This method constitutes Annex D of IEC TS 62882 (TC4 / WG33).

Based on the modelling of fluid-structure coupling, non-intrusive measurements of pressure pulsations are conceived. The corrected remote measurements are compared with those carried out by intrusive flush-mounted sensors as well as non-intrusive strain sensors (PVDF wires and strain gauges). Furthermore, a technique to reconstruct the pressure profile along the penstock using a three-sensor-array is presented. First, the actual wave speed is measured. Then, the
forward and the backward waves are computationally separated in order to reconstruct the pressure at other locations along the penstock.

The outlined methods are illustrated with measurements carried out on an actual penstock during the commissioning of a new medium head Francis runner in a Hydro-Québec power plant.

![Scheme of the penstock and sensor instrumentation](image)

**Figure 1.** Scheme of the penstock (left) and sensor instrumentation (right).

2. **Remote pipe measurements**

2.1. **Method**

The dynamic pressure is generally measured in a penstock with remote sensors. A 1/2” or 3/4” diameter tube (usually made of steel) connects the penstock pressure tap to the sensor. The remote tubing line can be quite long (15 - 20 meter long for the studied penstock). The dynamic behaviour of the remote tubing line modifies the readings of dynamic pressure and it will be shown that it may introduce a bias. The goal here is to characterize the wave propagation in the tubing line to correct the reading measured by the sensor in order to obtain pressure matching the one measured with a flush-mounted sensor at the penstock pressure tap. The correction is carried out in the frequency domain. Given the small tube diameter, only plane wave propagation has to be considered. The wave can be defined at any position by a dynamic pressure \( P \) and a dynamic particle velocity \( V \). Considering a homogenous tube, the wave propagation is fully characterized by a propagation speed and a decay rate. Although these parameters can be estimated from the pipe properties (material, thickness, diameter) and those of the fluid (bulk modulus, mass density), it seems more straightforward to obtain them experimentally.

The pressure and the particle velocity at the penstock pressure tap are linked to the ones at the sensor location by the following transfer matrix:

\[
\begin{pmatrix}
P_{\text{tap}} \\
V_{\text{tap}}
\end{pmatrix} = \begin{bmatrix}
\cos(k_fL) & jZ_f\sin(k_fL) \\
jZ_f\sin(k_fL) & \cos(k_fL)
\end{bmatrix} \begin{pmatrix}
P_{\text{sensor}} \\
V_{\text{sensor}}
\end{pmatrix},
\]

\( L \) being the length of the pipe, \( k_f \) the characteristic wavenumber and \( Z_f \) the characteristic impedance of the fluid. The wavenumber \( k_f \) is inversely proportional to the speed of wave \( c_f \):
The tube is closed at the sensor location and the particle velocity $V_{sensor}$ can be considered as null. The pressure at the tap is thus related to the one measured by the sensor with the following formula:

$$P_{tap} = \cos(k_f L) P_{sensor}.$$  

(3)

Figure 2. Remote sensor station with air vent valves.

The propagation parameters of the remote tubing line are identified from an impulse response which can be generated by a water hammer while operating the air vent valve (Fig. 2). Although the water hammer is not a perfect pulse, it enables the remote tubing line characterization. The dynamic response of the tube to the water hammer is a damped sinusoid. The characteristic frequency of an open-closed tube corresponds to the quarter wave frequency. Thus a minimization procedure of the following function can be implemented:

$$P(t) = A \cos(2\pi f_{1/4} t + \phi)e^{-\text{dec} \ t}$$

(4)

with $A$ the amplitude, $f_{1/4}$ the quarter wave frequency, $\phi$ the phase and $\text{dec}$ the decay rate.

The quarter wave frequency and the decay rate enable to express the characteristic wavenumber (complex value) of a homogeneous pipe:

$$k_f = \frac{2\pi f}{c_f (1 + j\eta_f)},$$

(5)

with $c_f = 4 f_{1/4} L$ and $\eta_f = \frac{\text{dec}}{2\pi f_{1/4}}$.

In practice, an error can be made on the propagation speed due to the estimation of the tube length $L$. This deviation has no impact on the correction because the $\cos(k_f L)$ function is used and the product $k_f L = 2\pi f/(4f_{1/4})$ does not depend on $L$. A filter is then defined in the time domain from the frequency correction function (Eq. 3) in order to perform the correction directly in the time domain.
2.2. Application

The water hammer response and the estimate using the Eq. 4 are shown in Fig. 3 for the sensor P1 with a 15-meter-long-pipe. The uncorrected (P1) and the corrected (P1_{corr}) pressures, measured by the remote sensor, are then compared to a flush mounted sensor (P1_{local}) in Fig. 4. The peak-to-peak values and the errors on peak-to-peak and root mean square (RMS) values are given in Tab. 1 for four remote sensors located in the same plane section of the penstock (as depicted in Fig. 1) compared to the flush mounted sensor.

Table 1. Peak-to-Peak 97%, Peak-to-Peak 97% error and RMS error between corrected and uncorrected remote sensors P1, P2, P3, P4.

| Capteur | Peak-to-Peak 97% (bar) | Peak-to-Peak 97% Error (%) | RMS Error (%) |
|---------|------------------------|-----------------------------|---------------|
| P_{local} | 2.92 | | |
| P1 | 3.35 | 14.6 | 37.1 |
| P1_{corr} | 2.92 | 0.1 | 6.1 |
| P2 | 3.42 | 17.3 | 38.2 |
| P2_{corr} | 2.92 | 0.2 | 6.7 |
| P3 | 3.39 | 16.0 | 36.4 |
| P3_{corr} | 2.94 | 0.8 | 8.8 |
| P4 | 3.26 | 11.8 | 32.2 |
| P4_{corr} | 2.92 | 0.2 | 6.6 |

Fig. 4 shows a large peak of the uncorrected signal around the quarter wave frequency in the frequency domain and the resulting error in the time domain. This error can be around 15% on the peak-to-peak value and more than 30% on the RMS value. The correction reduces this error by several times.
3. Non-intrusive measurements in pipes

3.1. Method

A pipe is deformable and its total deformation is the combination of several modes of deformation, some of which are illustrated in Fig. 5. The coupling of the deformable wall with the fluid results in a large number of different pipe waves. Some of these are propagating and carry energy; the others, evanescent, do not propagate neither do carry energy. The propagating wave having homogeneous distribution of pressure and deformation, the plane wave, is of prime concern. It is related to phenomena such as water hammer. The dynamic pressure measurement is generally done with intrusive sensors which requires perforating the pipe and stopping the hydraulic system. Moreover, since the measurement is carried out at a single point the sensor responds to all pressure waves. On the contrary a hoop-strain sensor responds to plane wave only if the strain is averaged along the pipe circumference. This can be achieved by using either a ring of strain gauges or a single strain sensor wound around the pipe. The example of the latter type of sensor, a PVDF wire, has been employed successfully by the authors on several penstocks.

The non-intrusive measurement is based on the measurement of the homogeneous pipe
Figure 5. Pipe deformation depending on the circumferential mode $n$.

deformation. A pressure variation in the fluid filling the pipe induces a strain in the shell wall. The knowledge of the fluid-structure coupling enables to link the dynamic pressure in the fluid to the strain measurement of the shell pipe \([3][4][5]\). The relationship between the pressure $P_n$ and the strain $\varepsilon_n$ of the pipe is given for each circumferential mode $n$:

$$P_n = \varepsilon_n \frac{Eh}{r(1 - \nu^2)} \Gamma_n (f),$$

(6)

with $E$ the Young modulus of the pipe, $\nu$ its Poisson ratio, $r$ its radius, $h$ its thickness, $\Gamma_n$ the frequency dependent coupling factor due to the fluid structure coupling.

The relationship between the strain and the radial displacement $W_n$ writes:

$$\varepsilon_n = \frac{W_n}{r}.$$  

(7)

In practice, a PVDF wire is mounted around the pipe with an integer number of spires to measure only the breathing ($n = 0$) deformation of the pipe (see Fig. 1). One can thus deduce the radial displacement and the dynamic pressure in the fluid responsible of this deformation. The breathing mode $n = 0$ is the most energetic mode that can propagate in the fluid and therefore the most critical mode to take into account in fatigue calculations.

Another way to measure this breathing deformation would be to use several intrusive sensors placed around the circumference of the pipe and to average their responses. This method would require more sensors and more perforations. Note that the strain can also be measured with strain gauges directly on the pipe.

3.2. Application to penstocks

Two PVDF wires $P_{\text{wire}}$ and a strain gauge $P_{\text{gauge}}$ were located close to the flush mounted sensor $P_{\text{local}}$ (see Fig. 1). Results are compared in both the frequency and the time domains in Fig. 6. A very good correlation between the measured pressures is observed up to 30 Hz. The time domain signals are mainly controled by the low frequency content and are thus very close. The strain gauge presents a worse signal-to-noise ratio than other sensors and is less sensitive given the higher background noise. The PVDF wires seem more sensitive than the intrusive local sensor.

4. Wave separation and pressure profile

4.1. Actual speed of plane waves in a pipe

The study of the wave propagation in a pipe requires the knowledge of the wave speed. The pipe is deformable and coupled to the fluid. This implies that the actual speed of wave in a flexible pipe is lower than in an infinite fluid. The wave speed can be estimated theoretically from the fluid-structure coupling model. The speed of waves in the fluid decreases as the elasticity of the pipe increases, i.e. as the diameter of the pipe increases or its thickness decreases. The actual speed of wave can be measured. The actual speed is a crucial parameter for the analysis of pressure wave along the pipe.
Figure 6. Comparison of intrusive and non-intrusive measurements a) in the frequency domain, b) in the time domain with strain gauge, c) in the time domain without strain gauge.

Considering plane waves in a pipe, the pressure can be written, at each frequency, as a superposition of a forward wave and a backward one, with amplitudes $A_+$ and $A_-$ and using $+j\omega t$ time convention:

$$p(x) = A_+ e^{-jk_+x} + A_- e^{jk_-x},$$

with $k_+ = \frac{\omega}{c_f + v}$, $k_- = \frac{\omega}{c_f - v}$, $v$ the flow speed, $\omega = 2\pi f$ the angular frequency and $c_f$ the speed of longitudinal waves in the fluid.

Figure 7. Forward and backward wave pattern in a duct.

Pavić [6] has introduced a method to determine the actual speed of wave using an invariant function $\Theta_{ab}$ defined as:

$$\Theta_{ab} = \frac{\Im \left[ S_{ab} e^{jM(x_b - x_a)} \right]}{\sin (k(x_a - x_b))},$$
with $S_{ab} = p(x_A)^*p(x_B)$ the cross-spectrum between points A and B, $k = \frac{\omega}{c_f}$ the wavenumber, $M = \frac{c_f}{v}$ the Mach number and $\Im$ represents the imaginary part. This function depends on the distance between points A and B, the flow speed $v$ (generally known or estimated) and the wave speed in the fluid $c_f$.

Knowing the pressure at three points A, B and C, two functions $\Theta_{ab}$ and $\Theta_{ac}$ can be expressed and a minimization of these functions $\Theta_{ab} - \Theta_{ac}$ can be used to determine the actual speed of wave $c_f$. The frequency representation enables to write this relationship at each frequency and thus to determine the actual speed of wave from multiple frequency data.

### 4.2. Wave separation and extrapolation

We usually focus on the pressure plane wave propagation in pipes. As previously mentioned, the pressure can be written, at each frequency, as a superposition of a forward wave and a backward one.

The wavenumbers $k_+$ et $k_-$ depend on the frequency, the flow speed and the wave speed. Once the flow and wave speeds are known, predicting the pressure at any position $x$ of the pipe requires the knowledge of the amplitudes of forward $A_+$ and backward $A_-$ waves. Two pressure measurements, at two different positions, are required to extract the forward amplitude and the backward one.

Knowing the pressure at two points A and B, the speed of waves $c_f$ and the flow speed $v$, the following set of equations can be written:

\[
\begin{align*}
 p(x_A) &= A_+ e^{-jk_+x_A} + A_- e^{jk_-x_A} \\
p(x_B) &= A_+ e^{-jk_+x_B} + A_- e^{jk_-x_B}
\end{align*}
\] (10)

These two equations enable to identify the two unknowns $A_+$ and $A_-$ and thus to estimate the pressure at another point along the pipe. In practice, three measurement points are used to avoid the singularity due to the distance between two measurement positions. Moreover, three points are necessary to measure the wave speed.

### 4.3. Application

Three additional flush-mounted sensors were located along the penstock at 3, 11 and 25 meters from the reference section. First the speed of waves is measured according to Section 4.1 and then forward and backward amplitudes are estimated according to Section 4.2. Three sensors Plocal ($x=0$ m), Pemb$3$m ($x=3$ m) and Pemb$11$m ($x=11$ m) are used for the identification and the pressure is then reconstructed at the 4th position ($x=25$ m). Fig. 8 shows the forward and backward amplitudes, the reference pressure measured by the sensor Pemb$25$m ($x=25$ m) and the reconstructed one in both the time and the frequency domain. A very good match is observed between the reference and the reconstructed pressures. It is also interesting to note that the backward amplitude is of the same order of magnitude than the forward one, especially at low frequencies where most of the energy is contained. The RMS pressure profiles and time domain comparisons are also shown for two different operating conditions.

#### 4.4. Operating condition 1

The first pressure profile along the penstock shows a usual profile mainly controlled by the quarter wave mode. The pressure is maximum at the bottom of the penstock and minimum at the water intake. The reconstructed pressures are successfully compared to the measured ones at other positions +25 m and -29 m. The sensor located at +25 m is a flush-mounted and the one located at -29 m is remote. The measured pressure of this remote sensor was corrected according to Section 2. The results for this first operating condition are shown in Fig. 9.
Figure 8. Time signal (left) and frequency spectrum (right) reconstruction of the dynamic pressure from the forward and the backward waves.

Figure 9. Operating condition 1: RMS pressure profile (a) and reconstructed time signals x=+25 m (b) and x=-29 m (c).
4.5. Operating condition 2

A second operating condition is presented because the pressure profile along the penstock can be different from a quarter wave profile as shown in Fig. 10. Note that the comparisons of time domain signals at +25 m and -29 m are still in good agreement.

Figure 10. Operating condition 2: RMS pressure profile (a) and reconstructed time signals x=+25 m (b) and x=-29 m (c).

5. Conclusion

This paper presented the bias introduced by remote pressure sensors and a method to correct it. Based on the fluid-structure coupling, it confirmed that the use of non-intrusive sensors is an interesting method for dynamic pressure measurements. Finally, it demonstrated that a three-sensor-array can be used to estimate the actual speed of waves and to reconstruct the pressure along a penstock using the wave separation technique.

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