Neutrino masses in theories with dynamical electroweak symmetry breaking

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Abstract

We address the problem of accounting for light neutrino masses in theories with dynamical electroweak symmetry breaking. We discuss this in the context of a class of (extended) technicolor (ETC) models and analyze the full set of Dirac and Majorana masses that arise in such theories. As a possible solution, we propose a combination of suppressed Dirac masses and a see-saw involving dynamically generated $|\Delta L| = 2$ condensates of standard model singlet, ETC-non-singlet fermions. We show how this can be realized in an explicit ETC model. An important feature of this proposal is that, because of the suppression of Dirac neutrino mass terms, a see-saw yielding realistic neutrino masses does not require superheavy Majorana masses; indeed, these Majorana masses are typically much smaller than the largest ETC scale.

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1. Introduction

An understanding of the fermion mass spectrum remains an intriguing challenge for particle physics. The standard model (SM) accommodates quark and charged lepton masses by the mechanism of Yukawa couplings to a postulated Higgs boson, but this does not provide insight into these masses, especially since it requires small dimensionless Yukawa couplings for all of the charged fermions except the top quark, ranging down to $10^{-6} - 10^{-5}$ for the electron and $u$ and $d$ quarks. The standard model has zero neutrino masses, and hence must be modified to take account of the increasingly strong evidence for the very small but non-zero neutrino masses and significant

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lepton mixing from solar and atmospheric data \cite{1,2}, consistent with the K2K accelerator neutrino experiment \cite{3}.

Since masses for the quarks, charged leptons, and observed neutrinos break the chiral gauge symmetry of the standard model, an explanation of these masses necessarily involves a model for electroweak symmetry breaking (EWSB). One possibility is dynamical electroweak symmetry breaking driven by a strongly coupled gauge interaction, associated with an exact gauge symmetry, denoted generically as technicolor (TC) \cite{4–10}. The EWSB arises from the condensation of technifermion bilinears. The generation of realistic masses for the charged leptons and $u$, $d$, $s$, $c$, and $b$ quarks seems attainable in this framework, via extended technicolor, in particular with slowly running (“walking”) technicolor. Although additional ingredients are very likely necessary to explain the large top-quark mass, we explore here the possibility that an ETC model of the above type can yield particularly with slowly running (“walking”) technicolor. Although additional ingredients are very likely necessary to explain the large top-quark mass, we explore here the possibility that an ETC model of the above type can yield

2. Neutrino mass terms in extended technicolor theories

We first present a general discussion taking the technicolor gauge group to be $SU(N_{TC})$. The set of technifermions includes, as a subset, one family, viz., $Q_L = \left( \begin{array}{c} U_L \\ D_L \end{array} \right)$, $L_{TC,L} = \left( \begin{array}{c} N_L \\ U_R, D_R, N_R, E_R \end{array} \right)$ transforming according to the fundamental representation of $SU(N_{TC})$ and the usual representations of $G_{SM} = SU(3) \times SU(2)_L \times U(1)_{Y}$ (color and TC indices are usually suppressed). To satisfy constraints from flavor-changing neutral-current processes, the ETC vector bosons, which can mediate generation-changing transitions, must have large masses. We envision that these arise from self-breaking of the ETC gauge symmetry, which requires that ETC be a strongly coupled, chiral gauge theory. The self-breaking occurs in stages, for example, at the three stages $\Lambda_1 \sim 10^3$ TeV, $\Lambda_2 \sim 50$ TeV, and $\Lambda_3 \sim 3$ TeV, corresponding to the $N_{gen} = 3$ standard-model fermion generations. Then $N_{ETC} = N_{TC} + N_{gen}$.

A particularly attractive choice for the technicolor group, used in the explicit model to be studied here, is $SU(2)_{TC}$, which has the appeal that it minimizes the TC contributions to the $S$ parameter \cite{12} and can yield walking behavior, allowing for realistically large quark and charged lepton masses. With $N_{gen} = 3$, the choice $N_{TC} = 2$ corresponds to $N_{ETC} = 5$. With $N_f = 8$ vectorially coupled technifermions in the fundamental

1 Other data is from the Homestake, Kamiokande, GALLEX, and SAGE experiments. The optimal fit to this data involves $\nu_e$ oscillations into $\nu_x$ and $\nu_x$ with $\Delta m^2_{31} \sim 5 \times 10^{-3}$ eV$^2$, where $\Delta m_{ji}^2 = m_1 v_i^2 - m_j v_j^2$, and a relatively large associated mixing angle.
2 SuperKamiokande and data from Kamiokande, IMB, Soudan-2, and MACRO experiments. This data can be explained by $\nu_x \rightarrow \nu_e$ oscillations with $|\Delta m^2_{31}| \sim 2.5 \times 10^{-3}$ eV$^2$ and a maximal value of the associated mixing angle factor $\sin^2 2\theta_{23}$. (The sign of $\Delta m_{31}^2$, $j = 1, 2$, is not determined by this data.)
3 In general, lepton family number is violated by both Dirac and Majorana mass terms.
4 Although we require our model to yield small $S$, a reanalysis of precision electroweak data is in order in view of the recent measurement of $\sin^2 \theta_W$ by the NuTeV experiment.
representation, studies suggest that this $SU(2)_{\text{TC}}$ theory could have an (approximate) infrared fixed point (IRFP) in the confining phase with spontaneous chiral symmetry breaking but near to the phase transition (as a function of $N_f$ for fixed $N_{\text{TC}}$) beyond which the theory would go over into a non-Abelian Coulomb phase [13]. This approximate IRFP provides the walking behavior, enhancing the technifermion condensates that control the quark and charged lepton masses. The walking can also enhance the masses of pseudo-Nambu–Goldstone bosons, but further ingredients are likely needed to ensure the absence of some massless Nambu–Goldstone bosons.

A rough estimate of the quark and charged lepton masses can be made by considering a one-loop diagram in which a fermion $f_a$ emits a virtual ETC gauge boson, going to a virtual technifermion $F$ which reabsorbs the gauge boson, producing the mass term $m_{fa} f_a L f_a R + \text{h.c.}$ with

$$m_{fa} \sim \frac{\Lambda_a^{2}_{\text{ETC}} \eta_{a} N_{\text{TC}} A_{\text{TC}}^{2}}{4 \pi^2 M_{\text{ETC}}^2} \tag{1}$$

where $M_{\text{ETC}} \sim \sqrt[n_{\text{ETC}}]{A_{\text{ETC}}}$ is the mass of the ETC gauge bosons that gain mass at scale $A_{\text{ETC}}$ and $n_{\text{ETC}}$ is the running ETC gauge coupling evaluated at this scale. In Eq. (1) $\eta_{a}$ is a possible enhancement factor incorporating walking, which can be as large as $\Lambda_{a}/f_{F}$ [6,7] where $f_{F}$ is the technicolor pseudoscalar decay constant (for our purposes we can take $f_{L} \simeq f_{Q} = f_{F}$. We recall that $A_{\text{TC}}$ is determined by using the relation $m_{W}^2 = (g^2/4)(N_{c} f_{Q}^2 + f_{F}^2) = (g^2/4)(N_{c} + 1) f_{F}^2$, which gives $f_{F} \simeq 130$ GeV. In QCD, $f_{\pi} = 93$ MeV and $\Lambda_{\text{QCD}} \sim 170$ MeV, so that $\Lambda_{\text{QCD}}/f_{\pi} \simeq 2$; using this as a guide to technicolor, we infer $A_{\text{TC}} \sim 260$ GeV.

Technicolor models in general also have a set of electroweak-singlet neutrinos, $\chi_{R} = (\chi_{1}, \ldots, \chi_{n_\text{ETC}})_{R}$, some technicolored and some techni-singlets, in addition to the left-handed, weak-isospin-doublet neutrinos and technineutrinos. The contributions to the total neutrino mass matrix, generated by condensates arising at the TC and ETC scales, are then of three types: (i) left-handed Majorana, (ii) Dirac, and (iii) right-handed Majorana. The left-handed Majorana mass terms, which violate $L$ by two units, take the form

$$m_{fa} \sim \frac{\Lambda_a^{2}_{\text{ETC}}}{{\sqrt{n_{a}^{2}}(M_{\text{EL}})_{ij} n_{j L}}} + \text{h.c.}, \tag{2}$$

where $n_{a} = (\nu_{\alpha}, (N)_{L})_{\text{el}}$ includes the electroweak-doublet left-handed neutrinos for $i, j = 1, 2, 3$ and technineutrinos for $i, j = 4, \ldots, N_{\text{ETC}}$; and $C = \gamma_{2} \gamma_{5}$. Left-handed Majorana masses violate the electroweak gauge symmetry, and, for technineutrinos, also the TC symmetry, which is exact. Thus, $(M_{L})_{ij} = 0$ for $i$ or $j = 4, \ldots, N_{\text{ETC}}$. The Dirac mass terms take the form

$$m_{fa} \sim \frac{\Lambda_a^{2}_{\text{ETC}}}{{\sqrt{n_{a}^{2}}(M_{D})_{as} \chi_{s R}}} + \text{h.c.} \tag{3}$$

5 For a vectorial $SU(N)$ theory with $N_{f}$ fermions in the fundamental representation, an IRFP occurs if $N_{f} > N_{f, \text{min,IR}}$, where, perturbatively, $N_{f, \text{min,IR}} \simeq 34N^{2}/(13N^{2} - 3)$. At this IRFP, using the criticality condition (see footnote 6), the theory is expected to exist in a confining phase with $S_{\text{TC}}$ if $N_{f, \text{min,IR}} < N_{f} < N_{f, \text{con}}$, where $N_{f, \text{con}} \simeq (2/5)N(50N^{2} - 33)/(5N^{2} - 3)$ and in a conformal phase if $N_{f, \text{con}} < N_{f} < 11N_{c}/2$. For $N = 2$ we have $N_{f, \text{min,IR}} \sim 5$ and $N_{f, \text{con}} \sim 8$, respectively. For attempts at non-perturbative lattice studies of these properties.

6 In the approximation of a single-gauge-boson exchange, the critical coupling for condensation $R_{1} \times R_{2} \rightarrow R_{c}$ is given by the condition $\frac{2\pi}{\alpha} A_{C_{2}} = 1$, where $A_{C_{2}} = |C_{2}(R_{1}) + C_{2}(R_{2}) - C_{2}(R_{c})|$, and $C_{2}(R)$ is the quadratic Casimir invariant. Instanton contributions are also important [8].

7 Here $n_{\alpha} = \exp \int_{f_{T}}^{f_{E}} (d\mu/\mu)(\gamma(\alpha(\mu))|_{\mu})$, and in walking TC theories the anomalous dimension $\gamma \simeq 1$ so $n_{a} \simeq \Lambda_{a}/f_{F}$.

8 We write SM-singlet neutrinos as right-handed fields $\chi_{i, R}$. 
Finally, the Majorana bilinears with SM-singlet neutrinos are

$$\sum_{s,s'=1}^{n_s} \chi^T_{sR} C(M_R)_{ss'} \chi_{s'R}.$$  

(4)

In (3) and (4) \((M_D)_{ss} = 0\) and \((M_R)_{ss'} = 0\) for technicolor-non-invariant entries.

The full neutrino mass term is then

$$-\mathcal{L}_m = \frac{1}{2}(\bar{\nu}_L \chi^T L) \begin{pmatrix} M_L & M_D \\ (M_D)^T & M_R \end{pmatrix} \begin{pmatrix} \nu^c_R \\ \chi_R \end{pmatrix} + \text{h.c.}$$  

(5)

Since \((M_L)^T = M_L\) and \((M_R)^T = M_R\), the full \((N_{ETC} + n_s) \times (N_{ETC} + n_s)\) neutrino mass matrix \(M\) in (5) is complex symmetric and can be diagonalized by a unitary transformation \(U^T\) as \(M_{\text{diag}} = U^T M U^T\). This yields the neutrino masses and transformation \(U_{\nu}\) relating the group eigenstates \(\nu_L = (\bar{\nu}, \chi)^T L\) and the corresponding mass eigenstates \(\nu_{m,L}\), according to \(v_{j,L} = \sum_{k=1}^{N_{ETC} + n_s} (U_{\nu})_{jk} \nu_{k,m,L}\), \(1 \leq j \leq N_{ETC} + n_s\) (the elements \((U_{\nu})_{jk}\) connecting techni-singlet and technicolored neutrinos vanish identically). The lepton mixing matrix for the observed neutrinos [14] \(v_{\ell,L} = U_{\nu m,L}\) is then given by

$$U_{\ell k} = \sum_{j=1}^{3} (U_{\ell,L})_{ij} (U_{\nu})_{jk}, \quad 1 \leq i \leq 3, \quad 1 \leq k \leq N_{ETC} + n_s,$$

(6)

where \(U_{\ell k} = U_{\ell k}\), etc., and where the diagonalization of the charged lepton mass matrix is carried out by the bi-unitary transformation \(M_{\text{diag}} = U_{\ell,L} M_{\ell} U_{\ell,R}^T\).

3. Specific extended technicolor model

We next present an analysis of a specific ETC model based on the gauge group \(G = SU(5)_{ETC} \times SU(2)_{HC} \times G_{SM}\). One additional gauge interaction, \(SU(2)_{HC}\), where HC denotes hypercolor, has been introduced along with \(SU(5)_{ETC}\) and \(G_{SM}\). Both the \(SU(2)_{HC}\) and \(SU(5)_{ETC}\) interactions become strong, triggering a sequential breaking pattern. The fermion content of this model is listed below, where the numbers indicate the representations under \(SU(5)_{ETC} \times SU(2)_{HC} \times SU(3)_C \times SU(2)_L\) and the subscript gives the weak hypercharge:

\[
\begin{align*}
(5, 1, 3, 2)_1/3, L & \quad (5, 1, 3, 1)_4/3, R & \quad (5, 1, 3, 1)_{-2/3}, R \\
(5, 1, 1, 2)_{-1}, L & \quad (5, 1, 1, 1)_{-2}, R & \quad (10, 1, 1, 1)_0, R \\
(10, 2, 1, 1)_0, R &
\end{align*}
\]

(7)

Thus the fermions include quarks and techniquarks in the representations \((5, 5, 3, 2)_{1/3}, L, (5, 1, 3, 1)_{4/3}, R\) and \((5, 5, 3, 1)_{-2/3}, R\), left-handed charged leptons and neutrinos and technileptons in \((5, 1, 1, 2)_{-1}, L\) and right-handed charged leptons and technileptons in \((5, 1, 1, 1)_{-2}, R\), together with SM-singlet fermions \(\psi_{ij,R}\) in the antisymmetric tensor representation \((10, 1, 1, 1)_{0}, R\). The unusual assignment of the SM singlets makes the \(SU(5)_{ETC}\) gauge theory chiral. Finally, in order to render the theory anomaly-free and to provide interactions to help trigger the symmetry breaking, one adds the hypercolored fields in the \((10, 2, 1, 1)_{0}, R\), denoted \(\xi_{ij R}^{\mu\alpha}\), where \(ij\) and \(\alpha\) are ETC and HC indices. Thus, \(n_s = 30\). We label the ETC gauge bosons as \((V_{ij}^\mu)_\mu, 1 \leq i, j \leq 5\). To fix the convention for the lepton number assigned to \(\psi_{ij,R}\), we take it to be \(L = 1\) in order that Dirac terms \(\bar{\psi}_{L} \psi_{j,R}\) conserve lepton number.

The lepton number assigned to the \(\xi_{ij R}^{\mu\alpha}\) fields is also a convention; since they have no Dirac terms with observed neutrinos, we leave it arbitrary. We write \(\chi_R = (\psi, \xi)^R\).

Each of the non-Abelian factor groups in \(G\) is asymptotically free. There are no bilinear fermion operators invariant under \(G\) and hence there are no bare fermion mass terms. The \(SU(2)_{HC}\) and \(U(1)_{HC}\) interactions and
the SU(2)TC subsector of SU(5)ETC are vectorial. This model has some features in common with the ETC model, denoted AT94, of [9], but has different gauge groups and fermion content.

We next analyze the stages of symmetry breaking. We envision that as \( E \sim A_1 \sim 10^3 \text{ TeV}, \) \( \sigma_{\text{ETC}} \) is sufficiently large to produce condensation in the attractive channel (\( \mathbf{T}, 1, 1, 1)_0, R \times (\mathbf{T}, 1, 1, 1)_0, R \rightarrow (5, 1, 1)_0, R \), breaking SU(5)ETC \( \rightarrow \) SU(4)ETC. In the most attractive channel (MAC) analysis this is a highly attractive channel, with \( \Delta C_2 = 24/5 \), although it is not the MAC itself. The MAC is \( \mathbf{T}, 1, 1, 1)_0, R \times (10, 2, 1, 1)_0, R \rightarrow (1, 2, 1, 1)_0, R \), with \( \Delta C_2 = 36/5 \); this is undesired since it would break SU(2)HC. The desired condensation channel is nearly as strong and is just as probable within the uncertainties of MAC analyses. With no loss of generality, we take the breaking direction in SU(5)ETC as \( i = 1 \); this entails the separation of the first generation of quarks and leptons from the components of SU(5)ETC fields with indices lying in the set \{2, 3, 4, 5\}. With respect to the unbroken SU(4)ETC, we have the decomposition (\( \mathbf{T}, 1, 1, 1)_0, R \rightarrow (4, 1, 1, 1)_0, R + (6, 1, 1, 1)_0, R \). We denote the \((4, 1, 1, 1)_0, R \) and antisymmetric tensor representation \((6, 1, 1, 1)_0, R \) as \( \alpha_{\Delta \psi C} = \psi_{i, R} \) for \( 2 \leq i \leq 5 \) and \( \xi_{i, j, R} = \psi_{i, j, R} \) for \( 2 \leq i, j \leq 5 \). The associated SU(5)ETC-breaking, SU(4)ETC-invariant condensate is then

\[
\langle \epsilon_{1ijk} \xi^{ijT}_R \bar{C} \xi_R^{kT} \rangle = 4 \langle \epsilon_{\alpha \beta}^{2345T} \bar{C} \xi_R^{25} - \xi_R^{24T} \bar{C} \xi_R^{35} + \xi_R^{25T} \bar{C} \xi_R^{34} \rangle. \tag{8}
\]

This condensate and the resultant dynamical Majorana mass terms for the six components of \( \xi \) in Eq. (8) violate total lepton number as \( |\Delta L| = 2 \). The dynamical formation of Majorana mass terms and violation of total lepton number is an important feature of these models, providing a necessary ingredient for a (dynamical) see-saw mechanism.\(^9\)

At lower scales, depending on relative strengths of couplings, different symmetry-breaking sequences occur. One plausible sequence, denoted \( G_a \), is as follows: at \( A_2 \sim 10^2 \text{ TeV}, \) SU(4)ETC and SU(2)HC couplings are sufficiently large to lead together to the condensation \((4, 2, 1, 1)_0, R \times (6, 2, 1, 1)_0, R \rightarrow (4, 1, 1, 1)\), breaking SU(4)ETC \( \rightarrow \) SU(3)ETC [9]. This condensate is

\[
\langle \epsilon_{\alpha \beta} \epsilon_{12} \xi_R^{ijT} C \xi_R^{kT} \rangle = 4 \langle \epsilon_{\alpha \beta} \xi_R^{12, \alpha T} \bar{C} \xi_R^{25, \beta} \rangle = 4 \langle \epsilon_{\alpha \beta} \xi_R^{12, \alpha T} \bar{C} \xi_R^{25, \beta} + \xi_R^{15, \alpha T} \bar{C} \xi_R^{24, \beta} \rangle. \tag{9}
\]

and the twelve \( \xi_R^{ij, \alpha} \) fields in this condensate gain masses \( \sim A_2 \). Both the SU(4)ETC and SU(2)HC interactions are strongly attractive in this channel, together making the channel an example of the big-MAC of Ref. [9]. The fact that the neutrino-like fields \( \alpha_{\Delta \psi C} \) transform as a \( 24 \) of SU(4)ETC, while the left-handed neutrinos and technineutrinos transform as a \( 4 \), will lead to a strong suppression of relevant entries in the Dirac submatrix \( M_D \) [5,9].

In the \( G_a \) symmetry-breaking sequence, at the lowest ETC scale, \( A_3 \sim 3 \text{ TeV}, \) the \((3, 2, 1, 1)_0, R, \xi_R^{2j, \alpha}, j = 3, 4, 5, \) from the \((6, 2, 1, 1)_0, R \) is assumed to condense as \((3, 2, 1, 1)_0, R \times (3, 2, 1, 1)_0, R \rightarrow (3, 1, 1, 1)\), breaking SU(3)ETC \( \rightarrow \) SU(2)TC [9]. The condensate is \( \langle \epsilon_{\alpha \beta} \xi_R^{24, \alpha T} \bar{C} \xi_R^{25, \beta} \rangle \). This breaking again involves the combination of attractive ETC and HC interactions [9]. Further, we expect that at a scale \( \sim A_3 \) the HC interaction produces the condensate \( \langle \epsilon_{\alpha \beta} \xi_R^{12, \alpha T} \bar{C} \xi_R^{25, \beta} \rangle \). This condense again involves the combination of attractive ETC and HC interactions [9].

A different sequence of condensations, denoted \( G_b \), can occur if the SU(2)HC coupling is somewhat smaller. At a scale \( \Lambda_{\text{BHC}} \lesssim A_1 \) (BHC = broken HC), the SU(4)ETC interaction produces a condensation in the channel \((6, 2, 1, 1)_0, R \times (6, 2, 1, 1)_0, R \rightarrow (1, 3, 1, 1)\). With respect to ETC, this channel has \( \Delta C_2 = 5 \) and is, hence, slightly more attractive than the initial condensation (8) with \( \Delta C_2 = 24/5 \), but it can occur at the somewhat lower scale

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\(^9\) Since ETC is a chiral gauge theory which is strongly interacting at its self-breaking scale (here, \( A_1 \)) and since \( \pi_3(SU(N)) = \mathbb{Z} \), the associated ETC instantons will generically violate total lepton number \( L \). Because \( \epsilon_{\text{ETC}} \sim O(1) \) at \( A_1 \), these instantons are not suppressed by small \(-\alpha_{\text{ETC}}^{-1/2}\) factors, in contrast to the situation at zero temperature in the weak SU(2) sector. However, the resultant effective multifermion operators are of quite high dimension and are thus suppressed at low energies.
\( A_{\text{BHC}} \) because it is repulsive with respect to hypercolor. With no loss of generality, one can orient \( SU(2)_{\text{HC}} \) axes so that the condensate is

\[
\{\epsilon_{ij}C_{ij}^{1,2T}C_{ij}^{1,2}\} + (1 \leftrightarrow 2).
\]

Since this is an adjoint representation of hypercolor, it breaks \( SU(2)_{\text{HC}} \rightarrow U(1)_{\text{HC}} \). We let \( \alpha = 1, 2 \) correspond to \( Q_{\text{HC}} = \pm 1 \) under the \( U(1)_{\text{HC}} \). This gives dynamical masses \( \sim A_{\text{BHC}} \) to the twelve \( \epsilon_{ij}^{\alpha} \) fields involved.

At a lower scale, \( A_{23} \), in the \( G_b \) sequence, we envision that a combination of the \( SU(4)_{\text{ETC}} \) and \( U(1)_{\text{HC}} \) attractive interactions produces the condensation \( 4 \times 4 \rightarrow 6 \) with condensate \( \langle c_{\alpha\beta}C_{ij}^{12,\alpha}C_{ij}^{13,\beta} \rangle \), which then breaks \( SU(4)_{\text{ETC}} \rightarrow SU(2)_{\text{ETC}} \) and is \( U(1)_{\text{HC}} \)-invariant. Thus, the sequence \( G_b \) has only two ETC breaking scales, \( A_1 \) and \( A_{23} \); additional ingredients are needed to obtain the requisite range of SM fermion masses. Here we take \( A_{23} \sim 10 \text{ TeV} \). Although there is a residual \( U(1)_{\text{HC}} \) gauge interaction in these models, its effects are shielded since it does not couple directly to SM particles. Finally, for both \( G_a \) and \( G_b \), at the still lower scale \( A_{\text{TC}} \sim f_F \), technifermion condensation takes place, breaking \( SU(2)_L \times U(1)_Y \rightarrow U(1)_{\text{em}} \).

4. Calculations and results

The mass matrix \( M \) of neutrino-like (colorless and electrically neutral) states in Eq. (5) has \( N_{\text{ETC}} = 5 \) and \( n_s = 30 \). Since the hypercolored fields do not form bilinear condensates and resultant mass terms with hypercolor singlets, \( M \) is block-diagonal, comprised of a \( 15 \times 15 \) block \( M_{\text{HCS}} \) involving hypercolor-singlet neutrinos and a \( 20 \times 20 \) block \( M_{\text{HC}} \) involving the hypercolored fermions, \( M_{\text{HCS}} \). The entries in the matrix \( M \) arise as the high-energy physics is integrated out at each stage of condensation from \( A_1 \) down to \( A_{\text{TC}} \). Composite operators of various dimension are formed, with bilinear condensation then leading to the masses. The non-zero entries of \( M \) arise in two different ways: (i) directly, as dynamical masses associated with various condensates, and (ii) via loop diagrams involving dynamical mass insertions on internal fermion lines and, in most cases, also mixings among ETC gauge bosons on internal lines. Since the ETC gauge boson mixing arises at the level of one or more loops, most graphs for non-zero type-(ii) elements of \( M \) arise at the level of at least two-loop diagrams. The different origins for the elements of \( M \) give rise to quite different magnitudes for these elements; in particular, there is substantial suppression of most type-(ii) entries. This suppression is not primarily due to the ETC gauge couplings, which are strong, but to the fact that the diagrams involve ratios of small scales such as \( A_{\text{TC}} \) and lower ETC scales to larger scales such as \( A_1 \). The \( 20 \times 20 \) matrix \( M_{\text{HC}} \) involving the \( \{10, 2, 1, 1\}_R \) fermions contains dynamical fermion mass entries resulting from the hypercolor condensates and has \( \text{Tr}(M_{\text{HCS}}) = 0 \).

The matrix of primary interest, \( M_{\text{HCS}} \), is given by the operator product

\[
-\mathcal{L}_{\text{HCS}} = \frac{1}{2} \bar{\xi} \langle \eta_{\alpha,\beta} \eta^\dagger_{\gamma,\delta} \rangle \begin{pmatrix} M_L & (M_D)_\alpha \xi & (M_D)_\alpha \xi & n^\alpha_R \\ (M_D)^T_{\alpha \xi} & (M_R)_{\alpha \alpha} & (M_R)_{\alpha \alpha} & (n^\alpha_R) \xi_R \\ (M_D)^T_{\alpha \xi} & (M_R)_{\alpha \alpha} & (M_R)_{\alpha \alpha} & (n^\alpha_R) \xi_R \\ n^\alpha_R & (M_R)_{\alpha \alpha} & (M_R)_{\alpha \alpha} & (n^\alpha_R) \xi_R \end{pmatrix} + \text{h.c.} \tag{11}
\]

The five-component \( n^\alpha_R \), the four-component \( \alpha_R \), and the six-component \( \xi_R \) each contain TC singlets as well as non-singlets. One of the two Dirac submatrices is

\[
(M_D)_{\alpha i} = \begin{pmatrix} b_{12} & b_{13} & 0 & 0 \\ b_{22} & b_{23} & 0 & 0 \\ b_{32} & b_{33} & 0 & 0 \\ 0 & 0 & 0 & c_1 \\ 0 & 0 & -c_1 & 0 \end{pmatrix}.
\tag{12}
\]

The vanishing entries are zero because of exact technicolor gauge invariance. The entry \( c_1 \) represents a dynamical mass directly generated by technicolor interactions corresponding to \( \sum_{i, j = 4, 5} \epsilon^{ij} \langle \eta_{i, L} \alpha_{j, R} \rangle \), so that \( |c_1| \sim A_{\text{TC}} \).
the loop momenta in Fig. 1 are cut off far below \( \Lambda_b \) entries in generation, incorporating the mixing of the weak eigenstates of these fermions to form mass eigenstates. The function as \( \Sigma \) behavior the \( Gb \epsilon_{ij} \) Note that this involves the antisymmetric, \( \epsilon \) contraction of \( SU(2)_{TC} \) indices and thus makes crucial use of the fact that the technicolor group is \( SU(2) \) rather than \( SU(N) \) with \( N \geq 3 \).

In Fig. 1 we show graphs that could yield the \( b_i \)'s. Here the \( \times \) on the fermion line represents the dynamical mass corresponding to a technicolor condensate. Each graph requires non-diagonal insertions on the internal ETC gauge bosons lines. We find that the requisite ETC gauge boson mixings occur to leading (one-loop) order in the \( G_\alpha \) sequence for (i) \( b_{13} \), which involves \( V^4_3 \leftrightarrow V^3_1 \) and \( V^5_1 \leftrightarrow V^4_2 \), and (ii) \( b_{22} \), which involves \( V^4_2 \leftrightarrow V^3_L \) and \( V^3_L \leftrightarrow V^2_4 \). For example, for \( G_\alpha \), we show in Fig. 2 the one-loop graphs contributing to \( V^4_2 \leftrightarrow V^3_5 \). In each respective case, \( G_\alpha \) and \( G_b \), the other \( b_i \)'s are produced by higher-loop diagrams. For example, starting from Fig. 1 for \( b_{23} \) in case \( G_b \), one can construct diagrams in which the incoming \( \alpha_{13, R} \) or the virtual \( \alpha_{14, R} \) or \( n^2_L \) emits a virtual \( V^k \) ETC gauge boson with \( k \in \{1, 2, 3\} \) which, via mixing, becomes \( V^i_j \), which is then absorbed by the \( n^2_L \) to yield an outgoing \( n^i_L \), \( i = 1, 3 \). Other similar graphs involving a triple ETC gauge-boson vertex along with mixing also contribute in this way. These generate \( b_{13} \) and \( b_{23} \) at a level suppressed relative to \( b_{22} \). The \( V^k_2 \rightarrow V^2_j \) mixing arises generically from loop graphs in which at least one internal fermion line is a standard model quark or charged lepton with a mass insertion that is non-diagonal in generation, incorporating the mixing of the weak eigenstates of these fermions to form mass eigenstates. The entries \( b_{12} \) and \( b_{22} \) are generated in a similar way. We next estimate the leading \( b_i \) entries. For either breaking sequence, we denote the ETC gauge boson 2-point function as

\[
\frac{k}{2 \pi^2} \Pi^\mu_\nu(q)_{\mu \lambda} = \int \frac{d^4k}{(2\pi)^4} e^{-iq \cdot x} T \left[ \left\{ (V^i_\alpha)_\mu (x/2)(V^j_\nu)_{-\mu/2} \right\}_0 \right].
\]  

After some manipulations (and Wick rotation), the graph in Fig. 1 yields

\[
g_{ETC}^2 \left[ \bar{b}_{i, L}(p) \gamma_\mu \gamma_\lambda \alpha \eta_{1, L, R}(p) \right] \int \frac{d^4k}{(2\pi)^4} \frac{k^2 \Sigma_{TC}(k)}{(k^2 + \Sigma_{TC}(k))^2} \left\{ (p - k)^2 + M^2 \right\} \left\{ (p - k)^2 + M^2 \right\}.
\]  

where \( \Sigma_{TC}(k) \) is the dynamical technicolor mass associated with the transition \( \alpha_{14, R} \rightarrow n^2_L \). This mass has the behavior \( \Sigma_{TC}(k) \sim \Lambda_{TC} \) for \( k^2 \ll \Lambda_{TC}^2 \), while for \( k^2 \gg \Lambda_{TC}^2 \), (i) \( \Sigma_{TC}(k) \sim \Lambda_{TC}^2 / k \) for a walking theory [6], (ii) \( \Sigma_{TC}(k) \sim \Lambda_{TC}^2 / k^2 \) in a QCD-like theory. Hence, we need \( \frac{k^2}{(2\pi)^4} \Pi^\mu_\nu((p - k)^2)_{\mu \lambda} \) only for \( (p - k)^2 / \Lambda_{TC}^2 \ll 1 \), since the loop momenta in Fig. 1 are cut off far below \( \Lambda_3 \) (at \( \Lambda_3 \) for \( G_\alpha \) or \( \Lambda_{23} \) for \( G_b \)). In Eq. (14), \( M_j \) denotes the mass of the ETC gauge boson that picks up mass at \( \Lambda_j \).
In the sequence $G_b$, for $q^2 \ll \Lambda^2_1$, we estimate
\[ \left[ \frac{5}{2} \Pi_{23}(q) \right]_{\mu\lambda} \sim \frac{\pi_{ETC} \Lambda_{TC}^2}{2\pi^2} g_{\mu\lambda}, \]
where we have assumed a walking behavior of the TC theory up to $\Lambda_{23}$. For $i, j = 2, 3$ and 3, 2, adding the other graph with $4 \leftrightarrow 5$ in Fig. 1, we find
\[ |b_{23}| = |b_{32}| \sim \frac{\pi_{ETC} \Lambda_{4}^2 \Lambda_{23}}{2\pi^4\Lambda_{23}^2} \quad \text{for } G_b, \]
where we have again assumed the above walking TC behavior. For sequence $G_a$, we estimate, using similar methods,
\[ |b_{13}| \sim \frac{\Lambda_{2}^2 \Lambda_{3}}{2\pi^4\Lambda_{1}^2} \quad |b_{22}| \sim \frac{\Lambda_{2}^2 \Lambda_{4}^2 \Lambda_{23}}{2\pi^4\Lambda_{23}^2} \quad \text{for } G_a. \]

With the numerical inputs given above, we get $|b_{23}| = |b_{32}| \sim O(1)$ KeV for $G_b$ and $|b_{13}| \sim O(1)$ KeV and $|b_{22}| \sim O(10)$ eV for $G_a$. Because the ETC and TC theories are strongly coupled, these estimates based on perturbative expansions in powers of $\alpha$ involve an obvious uncertainty. For each case, the other $b_{ij}$'s are generated at smaller levels. These calculations show how this aspect—suppressed Dirac neutrino masses—of our proposal are realized in an explicit model. While the specific results for the various $b_{ij}$'s are dependent on the model and symmetry breaking pattern, one can infer that this type of suppression can be achieved in a general class of ETC models where Dirac mass terms are generated in a similar manner.

The second Dirac submatrix in Eq. (11) is
\[ (M_D)_{\xi\xi} = \begin{pmatrix} d_{123} & d_{145} & 0 & 0 & 0 & 0 \\ d_{232} & d_{245} & 0 & 0 & 0 & 0 \\ 0 & 0 & c_2 & 0 & c_3 & 0 \\ 0 & 0 & -c_2 & 0 & -c_3 & 0 \end{pmatrix}. \]
Again, the zeros are exact and follow from technicolor invariance. Because the $\xi$ fields decouple from the theory at scales below $\Lambda_1$, the non-zero elements of $(M_D)_{\xi\xi}$ arise indirectly, via loop diagrams and are highly suppressed. These elements of $(M_D)_{\xi\xi}$ have only a small effect on the neutrino eigenvalues because in the characteristic polynomial $P(x)$ they occur as corrections to much larger terms involving $\Lambda_{1}$.

In $M_R$ the $6 \times 6$ submatrix $(M_R)_{\alpha\alpha}$ has six non-zero entries that are dynamical mass terms of order $\Lambda_1$ arising directly from the condensate (8). These are important since they are $|\Delta \psi_L| = 2$ operators, and they, in turn, induce the $(M_R)_{\alpha\alpha}$ Majorana mass terms which play a central role in the see-saw. Thus the $(M_R)_{\beta\xi}$ entries are the underlying seed for the Majorana mass terms involving the observed neutrinos. Note that $\text{Tr}(M_R) = 0$.

The submatrix $(M_R)_{\alpha\alpha}$ has the form
\[ (M_R)_{\alpha\alpha} = \begin{pmatrix} r_{22} & r_{23} & 0 & 0 \\ r_{23} & r_{33} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}. \]
As before, the zeros are exact and are due to technicolor invariance. If the $2 \times 2$ $r_{ij}$ submatrix has maximal rank, this can provide a see-saw which, in conjunction with the suppression of the Dirac entries $b_{ij}$ discussed above, can yield adequate suppression of neutrino masses. The submatrix $r_{ij}$, $2 \leq i, j \leq 3$, produces this see-saw because $\alpha_{12,R}$ and $\alpha_{13,R}$ are the electroweak-singlet techni-singlet neutrinos that remain as part of the low-energy effective field theory at and below the electroweak scale.
Consider the sequence $G_b$. In Fig. 3 we show graphs contributing to $r_{23}$ for this case. These depend on the $V_1^4 \leftrightarrow V_5^3$ ETC gauge mixing produced by the graphs in Fig. 4. From these we calculate

$$r_{23} \sim \frac{A_{BHC}^2 A_{23}^2}{2\pi^4 A_1^3} \text{ for } G_b,$$

where here we have assumed a walking behavior of the ETC theory below $\Lambda_{BHC}$. The entries $r_{22}$ and $r_{33}$ are generated by higher-loop diagrams starting from the graphs in Fig. 3 for $r_{23}$ in a manner similar to that whereby subdominant $b_{ij}$ are generated starting from Fig. 1 for $b_{23}$ and $b_{32}$. Numerically, with the above inputs, $|r_{23}| \sim O(0.1)$ GeV, with smaller values for $r_{ii}, i = 2, 3$. For sequence $G_a$ we find that the $r_{ij}$ entries are generated via higher-loop diagrams analogous to those for $r_{22}$ and $r_{33}$ in sequence $G_b$ and hence are smaller than Eq. (20). In the estimates to follow we concentrate on the sequence $G_b$ since it yields a phenomenologically more successful see-saw, although this sequence has only two ETC breaking scales.

In the $4 \times 6$ submatrix $(M_R)_{ij}$ the entries are either exactly zero by technicolor invariance or are non-zero but highly suppressed because the $\xi$ fields decouple from the effective theory below $\Lambda_1$. The non-zero entries do not have an important effect on the masses of neutrino-like states because of the way that they enter in the characteristic polynomial (similar to the elements of $(M_D)_{ij}$).

We next summarize the above discussion from the viewpoint of effective field theory. At energy scales below $\Lambda_{TC}$, in either the breaking sequence $G_a$ or $G_b$, the sector of neutrino-like states consists of the techni-singlet components $i = 1, 2, 3$ of $n_i^L$, and the techni-singlet components $\alpha_{1i}, R, i = 2, 3$; other fields have gained masses at higher scales and have been integrated out. The effective theory comprised of these degrees of freedom involves bilinear (mass) operators along with a tower of higher-dimension operators. The mass operators are either of the Dirac type (the $b_{ij}$ terms of Eq. (11)) or of the Majorana type (the $r_{ij}$ of Eq. (19)). They form a $5 \times 5$ submatrix of $M_{HCS}$, and their magnitudes, which depend on the specific breaking sequence, are $\ll \Lambda_{TC}$.

Integrating out the $\alpha_{1i}, R$ and $\alpha_{13}, R$ fields then yields the lowest-scale effective field theory, in which there are three light fermions, $n_i^L$. The mass terms in this theory correspond to elements of $M_L$, and there are also higher-dimension operators involving the $n_i^L$. With respect to the mass terms, this procedure corresponds to a block diagonalization (“block-see-saw”) of the $5 \times 5$ submatrix of $M_{HCS}$, keeping only the light, $M_L$ matrix. Its dominant terms arise in this manner; other, smaller entries are generated via higher-loop diagrams involving higher-dimension operators, for example, induced by the exchange and mixing of ETC gauge bosons. The final step in
the effective-field-theory approach is to diagonalize this $3 \times 3$ matrix, leading to the neutrino mass eigenvalues and mixing angles. Equivalently, one can think in terms of diagonalizing the full $M_{\text{FCS}}$-matrix in one fell swoop.

To be specific, we focus on the $G_b$ sequence since it most clearly yields a see-saw. The largest $M_L$ entry is $(M_L)^{23}$ (since $M_L = M_L^T$, we take $i \leq j$), and other, smaller terms arise from higher dimension operators. The electroweak-non-singlet neutrinos are, to very good approximation, linear combinations of three mass eigenstates, of which the heaviest is $v_3$ or $v_2$ and has a mass

$$m_{\nu,\text{max}} \sim \frac{|b_{23} b_{32}|}{|r_{23}|} \sim \frac{A_{\text{BHC}}^2 A_{\text{TC}}^3}{2\pi^4 A_{22}^2 A_{\text{BHC}}}.$$  \hspace{1cm} (21)

With the above-mentioned numerical values and $A_{\text{BHC}} \simeq 0.3 A_1$, we find $m_{\nu,\text{max}} \simeq 0.05$ eV, consistent with experimental indications [2] based on a hierarchical spectrum, in which $m_{\nu,\text{max}} \simeq \sqrt{\Delta m_{\text{sol}}^2}$. The model naturally yields large $\nu_i - \nu_\tau$ mixing because of the leading off-diagonal structure of the $b_{ij}$ and $r_{ij}$ with $ij = 23$ and 32. The value of $|\Delta m_{\text{sol}}^2|$ depends on details of the model but is on the low side of the experimental range. The lightest neutrino mass, $m(v_1)$, arises from the subdominant terms in $M_L$, and is therefore predicted to be considerably smaller than $m(v_i)$, $i = 2, 3$. The group eigenstates involved in these (Majorana) mass eigenstates are $n_{i,R}$, $i = 1, 2, 3$, and $\alpha_{1i,R}$, $j = 2, 3$. This model thus exhibits our proposed explanation for light neutrino masses incorporating highly suppressed Dirac neutrino mass entries, $|\Delta L| = 2$ neutrino condensates and associated dynamical Majorana mass terms, and a resultant see-saw.

The model also yields the following mass eigenvalues and corresponding eigenvectors for the other neutrino-like states: (i) linear combinations (LC’s) of components of the six $\xi_{i,j,R}$ with $2 \leq i, j \leq 5$ get masses $\sim A_1$; (ii) LC’s of the $\zeta_{i,j,R}^{1,a}$ with $2 \leq i, j \leq 5$ get masses $\sim A_{\text{BHC}}$; (iii) LC’s of the $\zeta_{i,j,a}^{1,a}$ with $j = 1, 2$ get masses $\sim A_{22}$; (iv) for technicolor non-singlets, LC’s of the $\zeta_{i,j,a}^{1,a}$ with $j = 4, 5$ and LC’s of $n_i,R$ and $\alpha_{1i,R}$, $i = 4, 5$ get masses $\sim A_{\text{TC}}$; (v) LC’s of $\alpha_{1R}$ with $i = 2, 3$ get masses $\sim r_{23}$. These masses are (nearly) Dirac.

Not only are the $m_R$ entries responsible for the see-saw not superheavy masses; they are actually much smaller than the ETC scales $\Lambda_i$. A generic prediction of ETC models with the proposed see-saw is that some components of SM-singlet neutrino group eigenstates comprise dominant parts of mass eigenstates with masses given by the elements in $M_R$ that are involved in the see-saw (here, $r_{23}$). A condition to fit current limits on the emission of massive neutrinos, via lepton mixing, in particle decays would be that the $|U_{\nu k}|^2$, $|U_{\nu k}|^2 \lesssim 10^{-2}$ for $k > 3$ [16,17], which can be met while also maintaining sufficiently short lifetimes to satisfy astrophysical constraints.

### 5. Conclusions

In summary, we have given a general analysis of neutrino masses in the context of dynamical electroweak symmetry breaking theories, taking account of both Dirac and Majorana mass terms. We proposed a possible solution to the problem of obtaining light neutrino masses in this class of theories. This solution involves two main parts: (i) strong suppression of Dirac neutrino masses, and (ii) dynamical formation of bilinear Majorana neutrino condensates at ETC scales and resultant Majorana masses violating total lepton number as $|\Delta L| = 2$, and consequently a see-saw mechanism. We have shown how this proposal can be realized in an explicit ETC model. While further work is needed to obtain the detailed structural features needed to fit current indications for neutrino masses and lepton mixing [15], we believe that our proposal contains key ingredients for a solution to this problem in the context of theories with dynamical electroweak symmetry breaking. An important aspect of this suggestion is that it does not need any superheavy scale for a viable see-saw; indeed, the relevant Majorana masses may be much smaller than the highest ETC scale.
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