Theoretical investigation of thin-film multiferroic heterostructures based on a width-modulated slot transmission line

A A Nikitin$^{1,2}$, A A Nikitin$^{1,2,3}$, A B Ustinov$^{1,2}$, E Lähderanta$^2$, A Stashkevich$^{14}$

$^1$Department of Physical Electronics and Technology, St. Petersburg Electrotechnical University, St. Petersburg, 197376 Russia
$^2$Department of Mathematics and Physics, Lappeenranta University of Technology, Lappeenranta, 53850 Finland
$^3$International laboratory “MultiferrLab”, ITMO University, St. Petersburg, 197101 Russia
$^4$LSPM (CNRS-UPR 3407), Universite Paris 13, Sorbonne Paris Cite, 93430 Villetaneuse, France

E-mail: alexeynikitin1@gmail.com

Abstract. The microwave properties of multiferroic magnonic crystals based on a width-modulated slot-line were investigated theoretically for the first time. Dispersion characteristics of spin-electromagnetic waves propagating in the heterostructures were calculated. It was shown that band gaps in the spin-electromagnetic wave spectrum are controlled with both electric and magnetic fields.

I. Introduction

One of the most common ways to implement a magnonic crystal (MC) is a periodic modulation of a waveguiding structure. Nowadays an increased interest to study such structures for microwave applications is evident. From one hand they are a convenient model for the investigation of linear and nonlinear wave effects [1]. From other hand new functional microwave devices can be created with MCs. It can be generators [2], phase shifters [3], and other [4].

Development of frequency-agile materials for microwave devices has led to appearance of artificial multiferroics [5]. The multiferroics are usually fabricated by a combination of ferrite and ferroelectric materials so as to obtain structures in the form of multilayers, pillars, spheres, wires and other. A distinctive feature of the multiferroics is dual tunability of their physical properties. The tunability exists basically due to two effects. The first one is magnetoelastic effect based on mechanical interaction between ferrite and ferroelectric crystal lattices [6]. The second one is effect of electrodynamic coupling of spin waves (SWs) and electromagnetic waves (EMWs) in the ferrite-ferroelectric structures [7]. These coupled waves are called hybrid spin-electromagnetic waves (SEWs).

The purpose of the present work is the theoretical investigation of thin-film multiferroic heterostructures based on a width-modulated slot transmission line. This periodic heterostructure can be considered as a dual tunable multiferroic magnonic crystal.

II. Topology of the magnonic crystal
The investigated periodic structure is shown in figure 1. It consists of several layers: a dielectric substrate (layer $j = -2$), a ferroelectric film (layer $j = -1$), a ferromagnetic film (layer $j = 1$) and a dielectric substrate (layer $j = 2$). Thickness and dielectric permittivity of a layer with number $j$ will be designated as $d_j$ and $\varepsilon_j$, respectively. The slot transmission line has the form of a narrow slot between two infinitely thin perfectly conducting metal electrodes. Bias voltage can be applied to these electrodes in order to change the polarization of the ferroelectric film.

As is shown in Fig. 1b a slot width varied with a period $\Lambda$ in order to implement microwave MC. It consists of sections of a length $L_1$ and a width $w_1$, and sections of a length $L_2$ and a width $w_2$. We assume that the SEW propagates along the x-axis in the tangentially magnetized structure.

III. Theoretical model of the thin-film multiferroic magnonic crystals

Development of the theoretical model of the structure was carried out in several steps. As the first step we calculated dispersion characteristics of SEWs in the ferrite-ferroelectric structure based on the slot transmission lines with different width. Dispersion relation for SEWs in the periodic structures formed with multiferroic slot-line sections with different width of the slot were obtained by coupled-mode approach. Following this method, a periodic variation of the slot width is considered as a disturbance. It leads to a coupling between the unperturbed waves. Therefore the SEW dispersion in the magnonic crystal is described by the following expression [8]:

$$\cos KA = \cos k_1 L_1 \cos k_2 L_2 - \frac{k_1^2 + k_2^2}{2k_1 k_2} \sin k_1 L_1 \sin k_2 L_2$$

where $K$ is Bloch wave vector, $k_1$ and $k_2$ are wave numbers of SEWs for wide and narrow sections of the slot transmission line. These wave numbers were derived with a method of approximate boundary conditions developed in the work [9]. This work shows that such a method allows to calculate dispersion characteristics of SEWs in planar multiferroic heterostructures with high accuracy.

As the second step, we calculated frequency response with the transfer-matrix method described in details in the work [10]. The advantage of this method is a possibility of modelling S-parameters for...
the periodic finite-length structure taking into account the losses during the wave propagation. In this method, the travelling waves in a narrow and wide sections of the slot line and reflecting waves at the transition between the sections are described by T-matrices. Propagation of SEWs in the narrow section of the slot-line is described by the matrix:

\[
T_1 = \begin{bmatrix}
\exp[-ik_1 + \alpha_1]d_1 & 0 \\
0 & 1/\exp[(-ik_1 + \alpha_1)d_1]
\end{bmatrix}.
\] (2)

Reflection of wave from the boundary between narrow and wide slot-line sections is described by the matrix:

\[
T_2 = \begin{bmatrix}
1/(1 - G) & G/(1 - G) \\
G/(1 - G) & 1/(1 - G)
\end{bmatrix}.
\] (3)

Propagation of SEWs in the wide section of the slot-line is described by the matrix:

\[
T_3 = \begin{bmatrix}
\exp[-ik_2 + \alpha_2]d_2 & 0 \\
0 & 1/\exp[(-ik_2 + \alpha_2)d_2]
\end{bmatrix},
\] (4)

and reflection of wave from the boundary between wide and narrow sections of the slot-line is described by the matrix:

\[
T_4 = \begin{bmatrix}
1/(1 + G) & -G/(1 + G) \\
-G/(1 + G) & 1/(1 + G)
\end{bmatrix}.
\] (5)

Here \(G\) is a reflection coefficient from the boundary between the sections, \(\alpha_1(\omega)\) and \(\alpha_2(\omega)\) are damping decrements of SEWs in wide and narrow sections, respectively. To determine the damping decrements we use the following approximate expression [11]:

\[
\alpha(\omega) = |\frac{\partial k(\omega)}{\partial H}| \cdot \Delta H + |\frac{\partial k(\omega)}{\partial \varepsilon}| \cdot \varepsilon_{-1} \cdot \tan\delta,
\] (6)

where \(\Delta H\) is the half-width of the ferromagnetic resonance curve, \(\varepsilon_{-1}\) is the permittivity of the ferroelectric layer, and \(\tan\delta\) is the dielectric loss tangent of the ferroelectric.

Propagation of SEWs at the period length \(\Lambda\) can be described by serial multiplication of the four T-matrix:

\[
T_A = T_1 \cdot T_2 \cdot T_3 \cdot T_4.
\] (7)

Therefore, propagation of SEWs at the overall MC with the number of periods \(N\) is described by the following expression:

\[
T_\Sigma = T_A^N.
\] (8)

Finally, the transmission coefficient of the whole MC can be obtained from the following equation:

\[
S_{21} = 20 \cdot \log(1/T_{\Sigma11}).
\] (9)

**IV. Numerical analysis**

Dispersion characteristics of SEWs and transmission characteristics of the thin-film multiferroic periodic structure were investigated with the theoretical model discussed in the previous Section. We assume that the structure consists of an epitaxial yttrium iron garnet (YIG) film, a gadolinium gallium garnet (GGG) substrate, the slot-line electrodes, a polycrystalline barium strontium titanate (BST) film, and a sapphire substrate. The calculations were carried out for the typical parameters of these materials.

For numerical simulation we assumed that the dielectric layer with number "-2" has parameters of the sapphire substrate with thickness \(d_2 = 500\) µm and permittivity \(\varepsilon_{-2} = 10\). The layer "-1" has parameters of the polycrystalline film of a BST with \(d_1 = 2\) µm and \(\varepsilon_{-1} = 1500\). Parameters of a YIG film
with number “1” and a GGG substrate with number “2” are $d_1 = 13.6 \mu m$, $\varepsilon_1 = 14$, and $d_2 = 500 \mu m$, $\varepsilon_2 = 12$, respectively.

Figure 2 shows dispersion characteristics for thin-film multiferroic structures based on a slot-line without modulation. These dependences were calculated for slot-lines having different width of the slot $w$ of 50 and 75 $\mu m$, a magnetization $4\pi M_0 = 1750$ Oe, and a magnetic field $H = 1350$ Oe.

![Figure 2](image2.png)

**Figure 2.** Dispersion characteristics of SEWs for slot-lines having different width of the slot $w$ as indicated.

![Figure 3](image3.png)

**Figure 3.** SEW dispersion characteristics (top graphs) and the amplitude-frequency characteristics (bottom graphs) for different number of the period $N = 10$ (a) and 20 (b).

Dispersion characteristics of SEWs and amplitude-frequency characteristics (AFCs) of the multiferroic magnonic crystal for the slot width of $w_1=50 \mu m$, $w_2=75 \mu m$ are shown in figure 3. The numerical simulation was carried out for the fixed value of period $\Lambda = 500 \mu m$ but for different number
of the periods, namely $N = 10$ (Fig. 3(a)) and $N = 20$ (Fig. 3(b)). The results show that an increase in the number of the period (or in another words in the length of the structure) results in the increasing of transmission losses at the frequencies of the band gaps. For example, if the structure consists of 20 periods, the first band gap provides losses of -69 dB. Thus, one can obtain different AFCs with the necessary attenuation of SEWs by changing the length of the multiferroic MC.

Electric tuning of frequency responses are shown in figure 4. An electric field applied to the slot-line electrodes produces a reduction of the ferroelectric film permittivity $\varepsilon_{\perp}$. It provides an increase in group velocity of the electromagnetic wave in the slot-line. Calculation parameters are the same like in previous case except that permittivity $\varepsilon_{\perp}$ was changed from 1500 (solid line) to 1000 (dashed line).

![Figure 4](image)

**Figure 4.** Amplitude-frequency characteristics for different ferroelectric permittivity.

As is seen from figure 4, a decrease in $\varepsilon_{\perp}$ leads to the shift of the band gap toward the higher frequencies. A width of band gaps reduces with frequency increasing. This behaviour is determined by the efficiency of hybridization electromagnetic and spin waves. The difference between transmission spectra shown in Fig. 4 reaches values of 880 kHz with the ferroelectric film permittivity varying from 1500 to 1000. Such a change in permittivity can be obtained by applying control voltage about 100 V to the slot-line electrodes. In practice, we can detect this electrical tuning range by the vector network analyzer. It allows to measure the transmission spectra with an accuracy of 1 Hz.

![Figure 5](image)

**Figure 5.** Amplitude-frequency characteristics for different bias magnetic fields.

Similar characteristics were simulated for different values of the external magnetic field $H$ of 1345 Oe (solid line) and 1350 Oe (dashed line). They are shown in figure 5. An increase of the external magnetic field $H$ leads to the shift of the SEWs spectrum toward higher frequencies. Therefore, the area of the effective hybridization of EMW and SW demonstrates an up-frequency shift also.
It was shown that a significant advantage of the multiferroic MCs is a possibility to control dispersion and transmission characteristics by both electric and magnetic fields. The advantage of electrical tuning is high speed of operation. It means that this method allows to shift band gaps for a few MHz (Fig.4). If it is necessary to control the transmission spectra in relatively wide ranges of frequencies magnetic tunability can be used, as it was shown in Fig. 5.

V. Conclusion

Transmission coefficients of thin-film ferrite-ferroelectric structure with the width-modulated slot-line were analyzed. It was shown that there are band gaps in transmission coefficient for such structures. Location of these gaps can be tuned by the external electric and magnetic fields. It can be concluded that investigated structures could find application for development of new tunable microwave devices.

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