I review some aspects related with the connections between neutrino physics and the thermal leptogenesis mechanism for the generation of the cosmological baryon asymmetry of the Universe. A special attention is devoted to the problem of establishing a bridge between leptonic CP violation at low and high energies.

PACS numbers: 11.30.Fs, 13.35.Hb, 14.60.Pq, 98.80.Cq

1. Introduction

The problem of why our Universe is dominated by matter has been object of intense study in the last decades. In spite of satisfying à priori all the necessary conditions for the implementation of a successful baryogenesis mechanism, the standard model (SM) is unable to provide a plausible explanation for the observed baryon asymmetry of the Universe (BAU). This, and other longstanding theoretical hints in favor of the existence of physics beyond the standard model, have been recently supported by one of the most exciting discoveries of modern particle physics namely, neutrino oscillations. Apart from presenting us with the challenge of unraveling the pattern of neutrino masses and mixing, this discovery may also have a profound impact on our understanding of the Universe.

Among the various models proposed to explain why neutrinos are massive (and much lighter than the other known fermions), the seesaw mechanism [1] has become the most popular due its simplicity and versatility. However, and in spite of all the attempts done in the direction of finding a way to test it, we are still far from unequivocally select the seesaw mechanism as the one behind neutrino mass generation. The most promising way of achieving this goal seems to be the investigation of physical phenomena capable of constraining the seesaw parameter space. A possible indirect
test relies on the fact that the BAU may have been generated through the out-of-equilibrium decays of the seesaw heavy neutrino states via the leptogenesis mechanism [2]. Since both the low-energy neutrino mass and mixing pattern and the value of the BAU depend on the fundamental seesaw parameters at very high scales, one expects that for example the amount of CP violation needed to generate a sufficient BAU is in some sense related with the strength of low-energy CP violation in the leptonic sector. If this is the case, future neutrino oscillation experiments will be crucial to test leptogenesis in the seesaw framework. In this short review I will analyse some aspects related with the connection between neutrino physics and the thermal leptogenesis mechanism for the generation of the BAU.

2. Seesaw neutrino masses

The most economical framework over which the seesaw mechanism [1] can be realized corresponds to the SM extended with $n_R$ right-handed neutrino singlets $\nu_{Rj}$. In the leptonic sector, the $SU(2) \times U(1)$ invariant Yukawa and mass terms are:

$$\mathcal{L} = \bar{\ell}_{Li} (Y_\nu)_{ij} \nu_{Rj} + \bar{\ell}_{Li} \phi (Y_\ell)_{ij} e_{Rj} + \frac{1}{2} \nu_{RI} C (M_R)_{ij} \nu_{Rj} + h.c.,$$

where $\bar{\ell}_{Li}$ and $e_{Ri}$ stand for the left-handed lepton doublets and right-handed charged-lepton singlets while $\phi$ denotes the usual SM lepton Higgs doublet with $\tilde{\phi} = i \tau_2 \phi^*$. From now on, I will consider that the Dirac neutrino Yukawa coupling matrix $Y_\nu$ is defined in the weak basis where the right-handed neutrino mass matrix $M_R$ and the charged lepton Yukawa couplings $Y_\ell$ are diagonal. The right-handed neutrino mass term is $SU(2) \times U(1)$ invariant and, as a result, the typical scale of $M_R$ can be much higher than the electroweak symmetry breaking scale $v = 174$ GeV, thus leading to naturally small left-handed Majorana neutrino masses. The effective light neutrino mass matrix $\mathcal{M}$ is given by the well-known seesaw formula

$$\mathcal{M} = -v^2 Y_\nu M_R^{-1} Y_\nu^T, \quad U^\dagger \mathcal{M} U^* = \text{diag}(m_1, m_2, m_3),$$

where I have denoted the light neutrino masses by $m_i$. The unitary matrix $U$ is the so-called Pontecorvo-Maki-Nakagawa-Sakata (PMNS) leptonic mixing matrix which can be parameterized by three angles and three CP-violating phases in the following way:

$$U = U_\delta (\theta_{12}, \theta_{13}, \theta_{23}, \delta) P(\alpha, \beta), \quad P(\alpha, \beta) \equiv \text{diag} (1, e^{i\alpha/2}, e^{i\beta/2}).$$

Here, $\theta_{ij}$ are the leptonic mixing angles and $\delta$ a Dirac-type CP-violating phase. The unitary matrix $U_\delta$ is a CKM-like mixing matrix and can be
written in the standard form as given for instance in Ref. [3]. Since the light neutrinos are predicted to be Majorana particles in the seesaw scenario, there are two extra Majorana phases $\alpha$ and $\beta$.

The structure of the effective neutrino mass matrix $M$ is dictated by $Y_\nu$ and $M_R$. Although the relation between low and high energy parameters is well established by Eq. (2), it is straightforward to see that there is no one-to-one correspondence between both parameter spaces. Therefore, even if all the entries of $M$ are measured, $Y_\nu$ and $M_R$ cannot be reconstructed.

In the most natural scenario where the SM is extended with three heavy right-handed neutrino singlets, $Y_\nu$ contains 15 parameters and $M_R$ is defined by the three heavy Majorana neutrino masses $M_i$. In total, there are 18 parameters to be confronted with the 9 (3 masses+3 mixing angles+3 phases) of $M$ at low-energies. This ambiguity inherent to the seesaw mechanism can be illustrated taking into account that, for fixed $U$, $m_i$ and $M_i$, the relation

$$M = U d_\nu U^T = -v^2 Y_\nu d_R^{-1} Y_\nu^T ,$$

where $d_\nu = \text{diag}(m_1, m_2, m_3)$ and $d_R = \text{diag}(M_1, M_2, M_3)$, is always satisfied for [4]:

$$Y_\nu = \frac{i}{v} U \sqrt{d_\nu} \mathcal{O} \sqrt{d_R} , \quad \mathcal{O}^T \mathcal{O} = 1 .$$

Since $\mathcal{O}$ can be any orthogonal complex matrix it is clear that there is no univocal correspondence between the high and low-energy neutrino parameters. The meaning of $\mathcal{O}$ can be suggestively interpreted in terms of the different roles played by the heavy neutrinos in the seesaw mechanism. In fact, $\mathcal{O}$ can be viewed as a dominance matrix since it gives the weights of each heavy Majorana neutrino in the determination of the different light neutrino masses $m_i$ [5]. The fact that $\mathcal{O}_{ij}^2$ are weights for $m_i$ is quite obvious due to the orthogonality of $\mathcal{O}$: $m_i = \sum_j m_j \mathcal{O}_{ij}^2$. On the other hand, the single contribution $m_i \mathcal{O}_{ij}^2$ is also given by:

$$m_i \mathcal{O}_{ij}^2 = -\left(\frac{v U^\dagger Y_\nu}{M_j}\right)_{ij}^2 \equiv \frac{X_{ij}}{M_j} .$$

Therefore, once $U$ is settled, each weight $\mathcal{O}_{ij}^2$ just depends on $M_j$ and on its couplings with the left-handed neutrinos $(Y_\nu)_{kj}$. Consequently, the contribution of each heavy neutrino to $m_i$ is well defined and expressed by the weight $\text{Re}(\mathcal{O}_{ij}^2)$. In this sense, one can say that the heavy Majorana neutrino with mass $M_j$ dominates in $m_i$ if

$$\frac{|\text{Re}(X_{ij})|}{M_j} \gg \frac{|\text{Re}(X_{ik})|}{M_k} , \quad k \neq j ,$$

(7)
which leads to $|\text{Re}(O_{ij}^2)| \gg |\text{Re}(O_{ik}^2)|$. So, if one of the heavy Majorana neutrinos gives the dominant contribution to $m_i$, this information is encoded in the structure of $O$.

An alternative way to test the seesaw mechanism relies on the fact that in the supersymmetric seesaw new flavor violating effects appear due to the presence of the Dirac type couplings between the heavy right-handed neutrinos and the ordinary SM lepton doublets. In particular, even in the case where the supersymmetry breaking mechanism is flavor blind, new contributions are generated in lepton flavor violating rare decays like $\ell_i \rightarrow \ell_j \gamma$. However, in this case there is some intrinsic model dependence due to the fact that neither the high-energy neutrino flavor structure nor the true mechanism of supersymmetry breaking is known. Nevertheless, future experimental data will be of extreme value in testing lepton flavor violation in a wide class of models like those based on grand unification. In alternative scenarios where, for instance, the seesaw mediator is a heavy triplet, the results are more predictive since the amount of flavor violation induced by radiative corrections is directly related with the low-energy neutrino data.

3. Neutrinos and the origin of matter: thermal leptogenesis

The most recent WMAP results and BBN analysis of the primordial deuterium abundance imply the following range:

$$\eta_B = \frac{n_B}{n_\gamma} = (6.1 \pm 0.3) \times 10^{-10},$$

for the baryon-to-photon ratio of number densities. Although satisfying the three necessary Sakharov conditions for the generation of a baryon asymmetry, the SM is not able to justify the number given in (8). Among the viable mechanisms to account for the matter-antimatter asymmetry observed in the Universe, leptogenesis has undoubtedly become one of the most appealing ones. Indeed, its simplicity and close connection with low-energy neutrino physics render it an attractive and eventually testable scenario. The crucial ingredient in leptogenesis scenarios is the $CP$ asymmetry generated through the interference between the tree-level and one-loop heavy Majorana neutrino decay diagrams, depicted in Fig. 1. For the decay of the

---

1 For interesting discussions on this subject see Ref. 5.
2 Here I will only concentrate on the case of thermal leptogenesis with $CP$ asymmetries generated in the decays of heavy Majorana neutrinos. For a discussion on alternative leptogenesis scenarios the reader is addressed to Ref. 11.
heavy Majorana neutrino $N_i$, the $CP$ asymmetry can be expressed as \[ \epsilon_i = \frac{\Gamma_i - \bar{\Gamma}_i}{\Gamma_i + \bar{\Gamma}_i} = -\frac{3}{16\pi} \sum_{j \neq i} \text{Im} \left[ \frac{(Y_{\nu}^\dagger Y_{\nu})_{ij}^2}{(Y_{\nu}^\dagger Y_{\nu})_{ii}} \right] \frac{M_i}{M_j} (C_{ij}^V + C_{ij}^S), \] (9)

where $C_{ij}^V$ and $C_{ij}^S$ denote the vertex and self-energy contributions, respectively. Explicitly, these are given by:

\[
C_{ij}^S = \frac{2}{3} \frac{M_j^2 \Delta_{ji}}{\Delta_{ji}^2 + M_i^2 \Gamma_j^2}, \quad C_{ij}^V = \frac{2}{3} M_j^2 \left[ \left(1 + \frac{M_j^2}{M_i^2}\right) \log \left(1 + \frac{M_i^2}{M_j^2}\right) - 1 \right], \]

(10)

where $\Delta_{ji} = M_j^2 - M_i^2$ and $\Gamma_j = (Y_{\nu}^\dagger Y_{\nu})_{jj} M_j / (8\pi)$ is the tree-level decay width of $N_j$. If the heavy Majorana neutrino masses are such that $M_1 \ll M_2, M_3$ only the decay of the lightest Majorana neutrino $N_1$ is relevant for the lepton asymmetry. This is a reasonable assumption if the interactions of the lightest Majorana neutrino $N_1$ are in thermal equilibrium at the time of the $N_2, 3$ decays, so that the asymmetries produced by the heaviest neutrino decays are erased before the lightest one decays, or if $N_2, 3$ are too heavy to be produced after inflation \[13\]. In this case, $\epsilon_1$ reduces to

\[ \epsilon_1 \simeq -\frac{3}{16\pi} \sum_{j \neq 1} \text{Im} \left[ \frac{(Y_{\nu}^\dagger Y_{\nu})_{ij}^2}{(Y_{\nu}^\dagger Y_{\nu})_{ii}} \right] \frac{M_1}{M_j}. \]

(11)

As expected, in order to produce a lepton asymmetry, $CP$ and lepton number must be violated by the interactions of the heavy Majorana neutrinos. The out-of-equilibrium condition for the decay of $N_1$ is attained if $\Gamma_1(M_1) < H(M_1)$, where $H(M_1)$ is the Hubble parameter evaluated at $T = M_1$. It is instructive to define a decay parameter $K = \Gamma_1(M_1)/H(M_1)$ in such a way that the out-of-equilibrium condition is translated into $K < 1$.

The produced lepton asymmetry $Y_L$ is converted into a net baryon asymmetry $Y_B$ through the $(B + L)$-violating sphaleron processes. By considering
the interactions in thermal equilibrium in the thermal plasma and the correspon
dent balance between the chemical potentials of the different particle
species one can obtain the following relation between the $B$, $B - L$ and $L$
number-to-entropy density ratios \[ Y_B = a Y_{B-L} = \frac{a}{a-1} Y_L , \quad a = \frac{8 N_f + 4 N_H}{22 N_f + 13 N_H}, \] (12)
where $N_f$ and $N_H$ are the number of fermion families and complex Higgs
doublets, respectively. Taking into account that $N_f = 3$ and $N_H = 1$ for
the SM, one gets $a \approx 1/3$ and $Y_B \approx -0.5 Y_L$. Alternatively, one can use
the number-to-photon density ratios $\eta_X = n_X/n_\gamma$ which, in the case where
entropy is conserved, are related to $Y_X$ by $\eta_X = 7.04 Y_X$. Departing from the
scenario where no asymmetry is present, the final value of $\eta_B$ can be simply
expressed by the relation: $\eta_B \approx -10^{-2} \kappa \varepsilon_1$, where $\kappa$ is the leptogenesis
efficiency factor. The computation of $\kappa$ requires the numerical solution of
the Boltzmann equations which take into account the different processes in
which the heavy Majorana neutrinos are involved. The result depends on
the interplay between the terms which produce the lepton asymmetry, and
those which tend to erase it. Exhaustive discussions on these matters can
be found for instance in Refs. [15, 16, 17], where simple fits which allow to
calculate $\kappa$ in some regimes are also presented.

In a theoretical framework where neutrino masses are generated through
the seesaw mechanism, leptogenesis arises as the natural candidate to ex-
plain the baryon asymmetry of the Universe. Therefore, it is natural to ask
whether leptogenesis is capable of constraining the high and/or low-energy
parameter spaces. In the limit where the heavy Majorana neutrino masses
are hierarchical, two important bounds have been obtained for the light
effective neutrino masses \[ m_3 \lesssim 0.15 \text{ eV} , \quad M_1 \gtrsim (10^8 - 10^9) \text{ GeV}. \] (13)
In particular, the upper bound on $m_3$ is of crucial importance since future
experiments will be sensitive to neutrino masses of the order of 0.1 eV. An
example of such an experiment is KATRIN [20] which will probe neutrino
masses down to 0.35 eV. Therefore, a positive signal at KATRIN would ex-
clude the most simple thermal leptogenesis scenario. Upcoming neutrinoless
double beta decay setups will be important as well, since they can provide
relevant information about neutrino masses [21].

4. Flavor, CP violation and leptogenesis

From the point of view of model-building, the constraints presented
above constitute a necessary condition to be fulfilled by any seesaw model
(with hierarchical heavy Majorana neutrinos) which aims at explaining the value of the BAU. This fact has led many authors to investigate whether there is a connection between flavor, leptonic CP violation and leptogenesis can be established or not [22]. A very simple example of such interplay has been presented in Ref. [23] where the flavor structure of the high-energy neutrino sector is similar to the quark one. The Dirac neutrino Yukawa coupling matrix reads:

$$v Y_\nu = V_L^\dagger d U_R, \quad d = \text{diag}(m_u, m_c, m_t),$$

(14)

where $m_{u,c,t}$ are the up, charm and top quark masses. The matrix $V_L$ is analogous to the CKM matrix of the quark sector. In the case where the small mixing in $V_L$ is neglected, the heavy Majorana masses are approximately given by

$$M_1 \simeq \frac{m_u^2}{s_{12}^2 \sqrt{\Delta m_{21}^2} + \sqrt{\Delta m_{32}^2} s_{13}^2} \simeq 3.3 \times 10^5 \left( \frac{m_u}{1 \text{MeV}} \right)^2 \text{GeV}$$

(15)

for a hierarchical spectrum of the light neutrinos ($m_1 \to 0$). It is clear that, for an up-quark mass $m_u(M_1) \simeq 1$ MeV, this value is far below the lower bound given in (13) for $M_1$. In this case, $\eta_B$ has the maximum value:

$$\eta_{B\text{max}} \simeq 9.5 \times 10^{-13} \left( \frac{m_u}{1 \text{MeV}} \right)^2,$$

(16)

which is three orders of magnitude below the experimental value shown in [25]. For an inverted-hierarchical neutrino mass spectrum, Eq. (15) remains valid, but the maximum value of $\eta_B$ is decreases by two orders of magnitude. A similar situation occurs in a model based in the assumption of democratic flavor structures in the leptonic sector [24]. The above discussion shows that the hierarchy in the Dirac neutrino Yukawa couplings has to be weaker than the one observed in the quark sector in order for leptogenesis to be successful.

Regarding the connection between the CP violation needed for leptogenesis and that potentially measured in neutrino experiments one can see that, while leptogenesis only depends on the phases of the matrix $V_L$ shown in Eq. (14), the low-energy phases $\delta, \alpha$ and $\beta$ of Eq. (3) depend on both $V_L$ and $U_R$. Using the Casas-Ibarra parametrization defined in Eq. (5) one can show that only the phases of $O$ are relevant for leptogenesis since

$$\Im[(Y_i^\dagger Y_j)^2] = \frac{M_i M_j}{v^4} \sum_{a,b} m_a m_b \Im(O_{ia}^\dagger O_{aj} O_{ib}^\dagger O_{bj}).$$

(17)

Some exceptions can be found if one considers special relations between some elements of the effective neutrino mass matrix [26]. In this case, $M_1 \simeq M_2$, leading to an enhancement of the CP asymmetries in the decays of $N_1$ and $N_2$. 




In particular, this means that one may have viable leptogenesis even in the limit where there are no CP-violating phases (neither Dirac nor Majorana) in the PMNS mixing matrix $U$ and hence, no CP violation at low energies \[26\]. Explicitly defining $O \equiv |O_{ij}|e^{i\varphi_{ij}}/2$, the CP-asymmetry $\varepsilon_1$ reads \[27\]

$$
\varepsilon_1 \simeq \frac{3}{16\pi} \frac{M_1 \Delta m^2_{32} |O_{31}|^2 \sin \varphi_{31} - \Delta m^2_{21} |O_{11}|^2 \sin \varphi_{11}}{m_1 |O_{11}|^2 + m_2 |O_{21}|^2 + m_3 |O_{31}|^2}, \tag{18}
$$

where $\Delta m^2_{21} \simeq 8 \times 10^{-5}$ eV$^2$ and $\Delta m^2_{32} \simeq 2 \times 10^{-3}$ eV$^2$ \[28\] are the neutrino mass squared differences measured in neutrino oscillation experiments.

The above equation recovers what one would have expected by intuition, namely that the physical quantities involved in determining $\varepsilon_1$ are just $M_1$, the spectrum of the light neutrinos, $m_i$, and the first column of $O$, which expresses the composition of the lightest heavy Majorana neutrino in terms of the light neutrino masses $m_i$. Therefore, in general it is not possible to establish a link between low-energy CP violation and leptogenesis. This connection is model dependent: it can be drawn only by specifying a particular ansatz for the fundamental parameters of the seesaw.

In terms of CP-violating invariants, it has been shown \[29\] that the strength of CP violation at low energies, observable for example through neutrino oscillations, can be obtained from the following low-energy weak-basis (WB) invariant:

$$
\mathcal{T}_{CP} = \text{Tr} [\mathcal{H}, H_\ell]^3 = 6i \Delta_{21} \Delta_{32} \Delta_{31} \text{Im} [\mathcal{H}_{12} \mathcal{H}_{23} \mathcal{H}_{31}], \tag{19}
$$

where $\mathcal{H} = \mathcal{M} \mathcal{M}^\dagger$, $H_\ell = Y_\ell Y_\ell^\dagger$ and $\Delta_{21} = (y_\mu^2 - y_e^2)$ with analogous expressions for $\Delta_{31}, \Delta_{32}$. This relation can be computed in any weak basis. The low-energy invariant in Eq. (19) is sensitive to the Dirac-type phase $\delta$ and vanishes for $\delta = 0$. On the other hand, it does not depend on the Majorana phases $\alpha$ and $\beta$ appearing in the leptonic mixing matrix. The quantity $\mathcal{T}_{CP}$ can be fully written in terms of physical observables once

$$
\text{Im} [\mathcal{H}_{12} \mathcal{H}_{23} \mathcal{H}_{31}] = -\Delta^2_{m^2_{21}} \Delta^2_{m^2_{31}} \Delta^2_{m^2_{32}} \mathcal{J}_{CP}, \tag{20}
$$

where the $\Delta^2_{m^2_{ij}}$’s are the usual light neutrino mass squared differences and $\mathcal{J}_{CP}$ is the imaginary part of an invariant quartet of $U$ appearing in the difference of the CP-conjugated neutrino oscillation probabilities $P(\nu_e \rightarrow \nu_\mu) - P(\bar{\nu}_e \rightarrow \bar{\nu}_\mu)$. One can conveniently write \[27\]

$$
\mathcal{J}_{CP} = -\frac{\text{Im} [\mathcal{H}_{12} \mathcal{H}_{23} \mathcal{H}_{31}]}{\Delta^2_{m^2_{21}} \Delta^2_{m^2_{31}} \Delta^2_{m^2_{32}}}, \tag{21}
$$

which allows the computation of the low-energy CP invariant without resorting to the mixing matrix $U$. 

It is also possible to write WB invariants which are particularly useful to leptogenesis \cite{29}. The requirement of CP invariance implies the vanishing of the following WB invariants

\[ I_1 \equiv \text{Im} \left[ \text{Tr} \left( H_\nu H_R M_R^* H_\nu^* M_R \right) \right], \]
\[ I_2 \equiv \text{Im} \left[ \text{Tr} \left( H_\nu H_R^2 M_R^* H_\nu^* M_R \right) \right], \]
\[ I_3 \equiv \text{Im} \left[ \text{Tr} \left( H_\nu H_R^3 M_R^* H_\nu^* M_R H_R \right) \right], \]

(22)

where \( H_\nu \equiv Y_\nu^\dagger Y_\nu \), \( H_R \equiv M_R^\dagger M_R \). Unless any of the \( M_i \) vanish or one is in the case of right-handed neutrino degeneracy, the conditions \( I_k = 0 \) require the trivial solution \( \text{Im}\left[\left(H_\nu\right)_{ij}^2\right] = 0 \). In terms of \( I_1 \), \( I_2 \) and \( I_3 \) one has

\[ \text{Im}\left[\left(H_\nu\right)_{12}^2\right] = \frac{I_3 - I_2 M_3^2 + I_1 M_3^4}{M_1 M_2 \Delta_{21} \Delta_{31} \Delta_{32}}, \]
\[ -\text{Im}\left[\left(H_\nu\right)_{13}^2\right] = \frac{I_3 - I_2 M_2^2 + I_1 M_2^4}{M_1 M_3 \Delta_{21} \Delta_{31} \Delta_{32}}, \]
\[ \text{Im}\left[\left(H_\nu\right)_{23}^2\right] = \frac{I_3 - I_2 M_1^2 + I_1 M_1^4}{M_2 M_3 \Delta_{21} \Delta_{31} \Delta_{32}}. \]

(23)

In the limit of hierarchical \( M_i \), the CP-asymmetry \( \epsilon_1 \) can be expressed as a function of the three invariants \( I_i \) and of the masses \( M_i \)

\[ \epsilon_1 \simeq -\frac{3}{16 \pi \left(H_\nu\right)_{11}} \frac{I_3 - I_2 M_3^2 + I_1 M_3^4}{M_2^2 M_3^4 M_1^4} \cdot \frac{\tan^2 \beta}{\left(M_X/M\right) \log \left(M_X/M\right)} \]

(24)

A very simple reasoning shows that a general relation between lepton flavor violation in rare decays of the type \( \ell_i \rightarrow \ell_j \gamma \) and leptogenesis does not exist. Taking as an example the minimal supergravity case, the branching ratios of these processes can be approximated by \cite{4}:

\[ \text{BR}(\ell_i \rightarrow \ell_j \gamma) \simeq \frac{\alpha^3}{G_F^2 m_S^2} \frac{3 m_0^2 + A_0^2}{8 \pi^2} \left(Y_\nu^\dagger Y_\nu\right)_{ij} \log \left(\frac{M_X}{M}\right) \left(\frac{m_X}{M}\right)^2 \tan^2 \beta, \]

(25)

where \( m_0 \) and \( A_0 \) are the universal soft mass and trilinear coupling at a high scale \( M_X \), and \( m_S \) is an average slepton mass. For simplicity I have taken the case of degenerate heavy Majorana neutrinos. From the above equation, it is straightforward to see that the flavor violation effects induced in the slepton mass matrix are sensitive to the left-handed rotation \( V_L \) given in Eq. (14). Thus, leptogenesis depends on \( U_R \), one immediately concludes that a general connection between the LFV decay rates and the value of the BAU cannot be established. This fact is illustrated by the numerical computations presented in Refs. \cite{30} for both the cases of hierarchical and quasi-degenerate heavy Majorana neutrino mass spectra. Nevertheless, in some interesting cases some relation between both phenomena may be observed \cite{31}. 


5. Radiative leptogenesis

In supersymmetric theories, the constraints on $M_1$ shown in (13) may be in conflict with the upper bound on the reheating temperature of the Universe, which can be as low as $10^6$ GeV [32]. One of the possible solutions to this problem is to consider the case of quasi-degenerate heavy Majorana neutrinos, which can be perfectly reconciled with the available neutrino data [33]. In such scenarios, leptogenesis may work with heavy neutrino masses as low as 1 TeV, due to an enhancement of the $CP$-asymmetries generated in the $N_i$-decays [34].

A natural way to generate the small heavy Majorana mass splittings is through renormalisation group effects [35]. In order to illustrate how this mechanism works, let me consider the case of two $N_i$. At a scale $\Lambda_D$ the heavy neutrinos are degenerate, i.e. $M_1 = M_2 = M$, with $M < \Lambda_D$. In this limit, $CP$ is not necessarily conserved. This is supported from the fact that the weak-basis invariant
\[ J_1 = M^{-6} \text{Tr} \left[ Y_\nu Y_\nu^T Y_\ell Y_\ell^T Y_\nu^* Y_\nu^* Y_\ell^* Y_\ell^T \right]^3, \]
which is not proportional to $M_2^2 - M_1^2$, does not vanish in the exact degeneracy limit. On the other hand, a non-zero leptonic asymmetry can be generated if and only if the $CP$-odd invariant
\[ J_2 = \text{Im} \text{Tr} \left[ H_\nu M_R^\dagger M_R M_R^T H_\nu^T \right] = M_1 M_2 (M_2^2 - M_1^2) \text{Im} \left[ (H_\nu)^2_{12} \right], \]
does not vanish [34]. The condition $J_2 \neq 0$ requires both $M_1 \neq M_2$ and $\text{Im} \left[ (H_\nu)^2_{12} \right] \neq 0$, at the leptogenesis scale $M$. These requirements are guaranteed by the running of $M_R$ and $Y_\nu$ from $\Lambda_D$ to $M$. The renormalisation group equations (RGE) for $Y_\nu$, $H_\nu$ and the heavy neutrino masses $M_i$ are [36]:

\[
\frac{dY_\nu}{dt} = k Y_\nu + \left[ -a Y_\ell Y_\ell^T - b Y_\nu Y_\nu^* \right] Y_\nu + Y_\nu T, \tag{27}
\]

\[
\frac{dH_\nu}{dt} = 2k H_\nu - 2b Y_\nu^2 - 2a Y_\nu Y_\ell Y_\ell^T Y_\nu + [H_\nu, T], \tag{28}
\]

\[
\frac{dM_i}{dt} = 2c M_i \left( (H_\nu)_{ii} \right) + [H_\nu, T] = H_\nu T - T H_\nu, \tag{29}
\]

where $k$ is a function of $\text{Tr}(Y_X Y_X^T)$ and the gauge couplings [37]. The factors $a$, $b$ and $c$ are $a_{\text{SM}} = -b_{\text{SM}} = 3/2$, $b_{\text{MSSM}} = 3a_{\text{MSSM}} = -3$, $c_{\text{MSSM}} = 2c_{\text{SM}} = 2$ for the SM and MSSM cases. The anti-Hermitian matrix $T$ encodes the effects of rotation to the basis where $M_R$ is diagonal. Defining the degree of degeneracy between $M_1$ and $M_2$ through the parameter $\delta_N \equiv M_2/M_1 - 1$ one has:

\[
T_{12} = \frac{2 + \delta_N}{\delta_N} \text{Re} [H_{12}] + i \frac{\delta_N}{2 + \delta_N} \text{Im} [H_{12}], \quad T_{ii} = 0. \tag{30}
\]
From Eq. (30) on can see that if \(\delta_N = 0\) at a given scale \(\Lambda_D\), then the RGE in Eqs. (27) and (28) become singular, unless one imposes \(\text{Re} (H_{12}) = 0\). This can be achieved by rotating the heavy fields by an orthogonal transformation \(O\), being the rotation angle \(\theta\) such that \(\tan 2\theta = 2\text{Re}[H_{12}]/(H_{22} - H_{11})\). Under this transformation, \(Y_\nu \rightarrow Y'_\nu = Yo\) and \(H_\nu \rightarrow H'_\nu = Y'_\nu Y'^*_\nu = O^\dagger H_o O\). The scale dependence of the degeneracy parameter \(\delta_N\) is governed by

\[
\frac{d\delta_N}{dt} = 2c (\delta_N + 1)[(H_\nu)_{22} - (H_\nu)_{11} + 2\Delta],
\]

with \(\Delta \equiv \tan \theta \text{Re}[H_{12}]\). In the limit \(\delta_N \ll 1\), the leading-log approximation for \(\delta_N(t)\) can be easily found to be

\[
\delta_N(t) \simeq 2c [(H_\nu)_{22} - (H_\nu)_{11} + 2\Delta] t.
\]

For quasi-degenerate Majorana neutrinos the CP-asymmetries generated in their decays are approximately given by

\[
\varepsilon_j \simeq \frac{\text{Im} \left[ H'_{j2} \right]}{16 \pi \delta_N H'_{jj}} \left(1 + \frac{\Gamma_i^2}{4M^2\delta_N^2} \right)^{-1}, \quad i, j = 1, 2 (i \neq j),
\]

where the \(\Gamma_i\) are the heavy Majorana decay widths introduced in the previous section. The above equation shows that

\[
\varepsilon_i(t) \propto \text{Im} \left[ (H'_{i2}) \right] \text{Re} \left[ (H'_{i2}) \right], \quad i = 1, 2.
\]

Therefore, a necessary condition to have a nonzero CP-asymmetry at a given \(t\) is that \(\text{Re} [(H'_{i2})] \neq 0\). Since \(\text{Re} [(H'_{i2})] = 0\), one has to rely on running effects to generate a nonzero \(\text{Re} [(H'_{i2})]\). From Eqs. (28) and (30) and taking into account that \(\text{Re} [(H'_{i2})] = 0\),

\[
\text{Re} [(H'_{i2})] \simeq -\frac{a y_{j2}^2}{16\pi^2} \text{Re} [Y_{31}^* Y_{32}'] t.
\]

The radiatively generated \(\varepsilon_{1,2}\) can be computed from Eqs. (33) and (35).

In the following I will illustrate how the mechanism described above works for a specific example. It is convenient to define the \(3 \times 3\) seesaw operator \(\kappa\) at \(\Lambda_D\), \(\kappa = Y_\nu Y'^T/M\), where \((Y_\nu)_{ij} = y_0 y_{ij}\) is a \(3 \times 2\) complex matrix. In order to reconstruct the high energy neutrino sector in terms of the low energy parameters, I choose \(y_{12} = 0\). The effective neutrino mass matrix \(\mathcal{M}\) is

\[
\mathcal{M} = m_3 U \text{diag}(0, \rho e^{i\alpha}, 1) U^T, \quad \rho \equiv m_2/m_3, \quad \rho = \sqrt{\frac{\Delta m^2_{31}}{\Delta m^2_{32}}}.
\]
Fig. 2. (a) Maximum $CP$ asymmetries $\varepsilon_{1,2}$ as a function of $s_{13}$ in the SM case. (b) Baryon asymmetry as a function of $s_{13}$ in the MSSM case for $\tan\beta = 5, 10$. The dotted (solid) line refers to the result using the one-loop (two-loop) RGE while the dashed line corresponds to the case where the analytical expressions for the $CP$-asymmetries are used. The horizontal line indicates the mean experimental value for $\eta_B$.

where $m_3$ is the mass of the heaviest neutrino and $\alpha$ is a Majorana phase. In the present case, the maximum $CP$-asymmetry $\varepsilon_1$ is approximately given by

$$\varepsilon_1^\text{max} \simeq -\frac{3y_{\tau}^2 c_{12} (1 + \rho)}{128\pi (1 - \rho)} \simeq -10^{-6},$$

(37)

which is in perfect agreement with the result shown in Fig. 2 (a). After taking into account the washout effects, the final value of the baryon asymmetry is $\eta_B^\text{max} \simeq 3 \times 10^{-10}$, which is smaller than the experimental result by a factor of two. In Fig. 2 (b) $\eta_B$ is computed for the MSSM case. Besides the factor of two which has to be included in Eq. (33) due to the presence of supersymmetric particles in the decays, we one expects an extra enhancement factor of $(1 + \tan^2 \beta)$ in the MSSM respective to the SM case since $\varepsilon_{1,2,} \propto y_{\tau}^2$ (see also Ref. 38). It can be seen that, depending on the value of $\tan \beta$, the maximum of $\eta_B$ can be far above the experimental value. An interesting feature of radiative leptogenesis is that the BAU is practically independent from the gap between $\Lambda_D$ and $M$ which means that $M$ can be as low as 1 TeV.

---

4 The SM result can be reconciled with the experimental by introducing a third heavy Majorana neutrino much heavier than $N_{1,2}$ 39
6. Concluding remarks

The idea that the origin of the matter-antimatter asymmetry of the Universe may be related with the mechanism through which neutrinos become massive has led to an intense investigation in the last few years. Although it will be difficult to establish leptogenesis as being responsible for the generation of the BAU, future neutrino experiments will be able to rule out its simplest variant. This would be the case if the absolute neutrino mass scale happens to be above $\sim 0.2$ eV. On the other hand, positive signals of heavy Majorana neutrinos in future colliders \[40\] could open the window to low-scale leptogenesis mechanisms \[11\].

Upcoming neutrino oscillation experiments will, under certain conditions, be sensitive to $CP$-violating effects in the leptonic sector. Moreover, an experimental indication in favor of neutrinoless double beta decays processes would reveal the Majorana nature of neutrinos \[12\] and possibly give some information about the Majorana phases \[13\]. Even in the most favorable scenario where the strength of leptonic $CP$ violation is measured, it seems to be difficult to conclude whether this $CP$-violating effects are relevant for leptogenesis or not. This stems from the fact that high and low-energy neutrino parameters are not connected in a model-independent way. Obviously, such statements are based in our present understanding of these phenomena. Hopefully, future data from several particle and cosmological experiments, as well as novel theoretical approaches, may reveal unequivocal signals confirming leptogenesis as the mechanism responsible for the generation of the BAU.

Acknowledgements

I am grateful to the Organisers of this Symposium for the invitation to contribute to these Proceedings and to all my collaborators with whom part of the work presented here has been done. In this special occasion, I devote a special word of gratitude to Gustavo Branco wishing him many more successes for the future.

This work was supported by Fundação para a Ciência e Tecnologia under the grant SFRH/BPD/14473/2003.

REFERENCES

[1] P. Minkowski, Phys. Lett. B 67 (1977) 421; M. Gell-Mann, P. Ramond and R. Slansky, in Supergravity, eds. P. van Nieuwenhuizen et al., (North-Holland, 1979), p. 315; S.L. Glashow, in Quarks and Leptons, Cargèse, eds. M. Lévy et al., (Plenum, 1980), p. 707; T. Yanagida, in Proceedings of the Workshop on the Unified Theory and the Baryon Number in the Universe, eds. O. Sawada
et al., (KEK Report 79-18, Tsukuba, 1979), p. 95; R. N. Mohapatra and G. Senjanovic, Phys. Rev. Lett. 44 (1980) 912.
[2] M. Fukugita and T. Yanagida, Phys. Lett. B 174 (1986) 45.
[3] S. Eidelman et al. [Particle Data Group], Phys. Lett. B 592 (2004) 1.
[4] J. A. Casas and A. Ibarra, Nucl. Phys. B 618, 171 (2001).
[5] S. F. King, Nucl. Phys. B 562, 57 (1999); S. F. King, Nucl. Phys. B 576, 85 (2000); S. Lavignac, I. Masina and C. A. Savoy, Nucl. Phys. B 633, 139 (2002).
[6] S. Davidson, arXiv:hep-ph/0409339; S. Davidson and A. Ibarra, JHEP 0109, 013 (2001).
[7] F. Borzumati and A. Masiero, Phys. Rev. Lett. 57 (1986) 961.
[8] For some analysis see e.g. A. Masiero, S. K. Vempati and O. Vives, New J. Phys. 6 (2004) 202; J. Hisano, T. Moroi, K. Tobe and M. Yamaguchi, Phys. Rev. D 53 (1996) 2442; I. Masina and C. A. Savoy, Nucl. Phys. B 661 (2003) 365; and references therein.
[9] A. Rossi, Phys. Rev. D 66, 075003 (2002).
[10] D. N. Spergel et al. [WMAP Collaboration], Astrophys. J. Suppl. 148 (2003) 175.
[11] For a discussion of several leptogenesis mechanisms see, T. Hambye, Talk given at SEESAW25: International Conference on the Seesaw Mechanism and the Neutrino Mass, Paris, France, 10-11 Jun 2004. Published in Paris 2004, Seesaw 25, p. 151-168, hep-ph/0412053.
[12] L. Covi, E. Roulet and F. Vissani, Phys. Lett. B 384, 169 (1996); W. Buchmüller and M. Plümacher, Phys. Lett. B 431, 354 (1998).
[13] G. F. Giudice, M. Peloso, A. Riotto and I. Tkachev, JHEP 9908 (1999) 014.
[14] J. A. Harvey and M. S. Turner, Phys. Rev. D 42 (1990) 3344; S. Y. Khlebnikov and M. E. Shaposhnikov, Nucl. Phys. B 308 (1988) 885.
[15] R. Barbieri, P. Creminelli, A. Strumia and N. Tetradis, Nucl. Phys. B 575, 61 (2000).
[16] W. Buchmuller, P. Di Bari and M. Plumacher, Nucl. Phys. B 643, 367 (2002).
[17] G. F. Giudice, A. Notari, M. Raidal, A. Riotto and A. Strumia, Nucl. Phys. B 685, 89 (2004).
[18] T. Hambye, Y. Lin, A. Notari, M. Papucci and A. Strumia, Nucl. Phys. B 695, 169 (2004).
[19] K. Hamaguchi, H. Murayama and T. Yanagida, Phys. Rev. D 65 (2002) 043512; T. Hambye, Nucl. Phys. B 633, 171 (2002); S. Davidson and A. Ibarra, Phys. Lett. B 535, 25 (2002).
[20] A. Osipowicz et al. [KATRIN Collaboration], arXiv:hep-ex/0109033.
[21] F. Feruglio, A. Strumia and F. Vissani, Nucl. Phys. B 637, 345 (2002) [Addendum-ibid. B 659, 359 (2003)]; S. Pascoli, S. T. Petcov and W. Rodejohann, Phys. Lett. B 558 (2003) 141; F. R. Joaquim, Phys. Rev. D 68 (2003) 033019; J. N. Bahcall, H. Murayama and C. Pena-Garay, Phys. Rev. D 70 (2004) 033012; G. L. Fogli, E. Lisi, A. Marrone, A. Melchiorri, A. Palazzo,
P. Serra and J. Silk, Phys. Rev. D 70 (2004) 113003; S. Choubey and W. Rodejohann, Phys. Rev. D 72 (2005) 033016.

[22] See e.g., G. C. Branco and M. N. Rebelo, New J. Phys. 7 (2005) 86.

[23] G. C. Branco, R. González Felipe, F. R. Joaquim and M. N. Rebelo, Nucl. Phys. B 640, 202 (2002).

[24] E. K. Akhmedov, G. C. Branco, F. R. Joaquim and J. I. Silva-Marcos, Phys. Lett. B 498, 237 (2001).

[25] E. K. Akhmedov, M. Frigerio and A. Y. Smirnov, JHEP 0309, 021 (2003) arXiv:hep-ph/0305322.

[26] M. N. Rebelo, Phys. Rev. D 67, 013008 (2003).

[27] G. C. Branco, R. González Felipe, F. R. Joaquim, I. Masina, M. N. Rebelo and C. A. Savoy, Phys. Rev. D 67, 073025 (2003).

[28] A. Strumia and F. Vissani, Nucl. Phys. B 726 (2005) 294.

[29] G. C. Branco, T. Morozumi, B. M. Nobre and M. N. Rebelo, Nucl. Phys. B 617, 475 (2001).

[30] J. R. Ellis and M. Raidal, Nucl. Phys. B 643 (2002) 229; J. R. Ellis, M. Raidal and T. Yanagida, Phys. Lett. B 546 (2002) 228.

[31] S. Pascoli, S. T. Petcov and W. Rodejohann, Phys. Rev. D 68 (2003) 093007.

[32] M. Kawasaki, K. Kohri and T. Moroi, Phys. Lett. B 625 (2005) 7.

[33] R. González Felipe and F. R. Joaquim, JHEP 0109 (2001) 015.

[34] For a recent analysis see A. Pilaftsis and T. E. J. Underwood, Nucl. Phys. B 692, 303 (2004).

[35] R. González Felipe, F. R. Joaquim and B. M. Nobre, Phys. Rev. D 70 (2004) 085009; F. R. Joaquim, Nucl. Phys. Proc. Suppl. 145 (2005) 276.

[36] J. A. Casas, J. R. Espinosa, A. Ibarra and I. Navarro, Nucl. Phys. B 556, 3 (1999).

[37] See for example: P. H. Chankowski and S. Pokorski, Int. J. Mod. Phys. A 17, 575 (2002); S. Antusch, J. Kersten, M. Lindner and M. Ratz, Phys. Lett. B 538 (2002) 87.

[38] K. Turzynski, Phys. Lett. B 589 (2004) 135.

[39] G. C. Branco, R. González Felipe, F. R. Joaquim and B. M. Nobre, arXiv:hep-ph/0507092.

[40] F. del Aguila and J. A. Aguilar-Saavedra, JHEP 0505 (2005) 026; S. Bray, J. S. Lee and A. Pilaftsis, arXiv:hep-ph/0508077.

[41] A. Pilaftsis and T. E. J. Underwood, [arXiv:hep-ph/0506107]

[42] J. Schechter and J. W. F. Valle, Phys. Rev. D 25 (1982) 2951.

[43] S. Pascoli, S. T. Petcov and T. Schwetz, arXiv:hep-ph/0505226.