KURTOSIS AND LARGE–SCALE STRUCTURE

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Abstract

We discuss the non–linear growth of the excess kurtosis parameter of the smoothed density fluctuation field \( \delta \), \( S_4 \equiv \left[ \langle \delta^4 \rangle - 3 \langle \delta^2 \rangle^2 / \langle \delta^2 \rangle^3 \right] / \langle \delta^2 \rangle^3 \) in an Einstein–de Sitter universe. We assume Gaussian primordial density fluctuations with scale–free power spectrum \( P(k) \propto k^n \) and analyze the dependence of \( S_4 \) on primordial spectral index \( n \), after smoothing with a Gaussian filter. As already known for the skewness ratio \( S_3 \), the kurtosis parameter is a decreasing function of \( n \), both in exact perturbative theory and in the Zel’dovich approximation. The parameter \( S_4 \) provides a powerful statistics to test different cosmological scenarios.

Subject headings: Galaxies: clustering – large–scale structure of the Universe

1 Introduction

The study of the statistical distribution of the matter in the universe may be a way to address fundamental issues such as the origin and the formation of structures on large scales.

The simplest and most usually accepted hypothesis, supported by the inflationary model, is that the very early distribution is Gaussian. In such a case, the connected \( N \)–point correlations (and the \( N \)–th order connected moments) with \( N > 2 \) are zero. However, even if the primordial fluctuations \( \delta \) are Gaussian, the non–linear time evolution will ensure that the mass density fluctuations become highly non–Gaussian (Peebles 1980; Fry 1984). It is important to understand the nature of the higher moments of the mass density induced by gravity in order to distinguish their effects from those of possible primordial non–Gaussian fluctuations: late–time phase transitions, cosmic string models and global textures are indeed models whose statistics may not be described by a Gaussian distribution (see e.g. Vilenkin 1985; Turok 1989; Scherrer & Bertschinger 1991). Moreover, variations of the inflationary model which lead to non–Gaussian primordial fluctuations have been recently discussed (see e.g. Salopek 1992).

A powerful method to distinguish if the non–Gaussian nature of the matter distribution is intrinsic, is to analyze the growth of higher moments, like the skewness or the kurtosis (third and fourth connected moments, respectively) of the density fluctuation field. Peebles (1980) first showed that gravity induces, for an unsmoothed initial Gaussian density field, a skewness ratio \( S_3 \equiv \langle \delta^3 \rangle / \langle \delta^2 \rangle^2 = 34/7 \), for any primordial power spectrum. Juszkiewicz, Bouchet, & Colombi (1993) find that the filtering operation actually introduces a dependence of \( S_3 \) on the primordial spectral index \( n \); in particular for scale–free spectra, \( S_3 = 34/7 - (n+3) \) for a top–hat window function, and a decreasing trend with \( n \) is recovered also for a Gaussian filter. Coles et al. (1993) investigate, using \( N \)–body simulations, the growth of the skewness ratio to test the hypothesis of Gaussian primordial density fluctuations against possible alternatives. Analytical expressions for \( S_3 \) arising from non–linear evolution in perturbation theory have been worked out for arbitrary non–Gaussian models by Fry & Scherrer (1993) and Catelan & Moscardini
Finally, higher order moments are known to depend extremely weakly on the density parameter \( \Omega \) (see Martel & Freudling 1991; Bouchet et al. 1992; Bernardeu 1992).

Here we analyse the dependence of the induced–by–gravity excess kurtosis of the density field, smoothed with a Gaussian filter, namely the parameter \( S_4 \equiv \left[ \langle \delta^4 \rangle - 3\langle \delta^2 \rangle^2 / \langle \delta^2 \rangle^3 \right] / \langle \delta^2 \rangle \), on an initial (scale–free) power spectrum \( P(k) \). To do this, we take advantage of the exact perturbative technique (Fry 1984; Goroff et al. 1986) and the Zel’dovich approximation (see Grinstein & Wise 1987). The kurtosis describes features such as sharpness or stretchiness of the mass distribution and the extent of its rare–event tail. Moreover, it is possibly related to the initial sign of the skewness – as predicted in some non–Gaussian models (see e.g. Luo & Schramm 1993) – which is important for the final galaxy clustering pattern (Moscardini et al. 1991; Messina et al. 1992; Weinberg & Cole 1992).

The layout of this Letter is as follows: in Section 2 we review the exact perturbative theory and the Zel’dovich approximation; in Section 3 we discuss the induced–by–gravity kurtosis parameter \( S_4 \) of an initial Gaussian density field and its dependence on the primordial spectral index; we state the main conclusions in Section 4.

2 Non–Linear Time Evolution

We assume that present–day structures in the universe formed by gravitational instability from Gaussian fluctuations \( \delta \) in a pressureless fluid with matter density \( \rho = \rho_b[1 + \delta] \), where \( \rho_b \) is the background mean density. The density fluctuation field \( \delta \) may be written as a Fourier integral,

\[
\delta(x, t) = (2\pi)^{-3} \int dk \tilde{\delta}(k, t) e^{ik \cdot x},
\]

where \( x \) and \( k \) are the comoving Eulerian coordinate and wavevector respectively, \( t \) is the cosmic time. The power spectrum \( P(k) \), defined as the 2–point correlation function in Fourier space (at a given time),

\[
\langle \tilde{\delta}(k_1) \tilde{\delta}(k_2) \rangle = (2\pi)^3 \delta_D(k_1 + k_2) P(k_1),
\]

fully determines the statistics of the primordial Gaussian density field, whose variance is given by the expression \( \sigma^2 = (1/2\pi^2) \int_0^\infty dk k^2 P(k) \).

To bridge the gap between observational data and theory, it is necessary to filter the fluctuation field \( \delta \) by means of a window function \( W_R \),

\[
\delta_R(x, t) = \int dy \delta(y, t) W_R(|x - y|).
\]

In the following we will adopt a Gaussian window function. The mass variance on scale \( R \), \( \sigma_R^2 \), is related to the primordial spectrum by \( \sigma_R^2 = (1/2\pi^2) \int_0^\infty dk k^2 P(k) [\tilde{W}_R(k)]^2 \), where \( \tilde{W}_R(k) \) is the Fourier transform of \( W_R(x) \).

2.1 Equations of Motion: Perturbative Theory

The time evolution equations for the matter density fluctuation \( \delta(x, t) \) and the peculiar velocity field \( v(x, t) \) are the Poisson equation, the Euler equation and the continuity equation, i.e.

\[
\nabla^2 \Phi = 4\pi G \rho_b a^2 \delta, \tag{1}
\]

\[
\partial_t v + \frac{1}{a} (v \cdot \nabla) v + \frac{\dot{a}}{a} v = -\frac{1}{a} \nabla \Phi, \tag{2}
\]

\[
\partial_t \rho + \nabla \cdot (\rho v) = 0.
\]
\[ \partial_t \delta + \frac{1}{a} \nabla \cdot (1 + \delta) \mathbf{v} = 0. \]  

(3)

Here \( \partial_t \equiv \partial / \partial t \) and spatial derivatives are with respect to \( \mathbf{x} \). We analyze these equations assuming an Einstein–de Sitter universe with vanishing cosmological constant. In such a model, the scale factor \( a \) is proportional to \( t^{2/3} \) during the matter dominated epoch, and the adiabatic expansion implies that \( 6\pi G \rho_b t^2 = 1 \). The quantity \( \Phi \) is the Newtonian gravitational potential. Combining the divergence of the Euler equation with the continuity equation, a second order differential equation for the density contrast \( \delta \) may be introduced (Peebles 1980)

\[
\partial_t^2 \delta + 2 \frac{\dot{a}}{a} \partial_t \delta - 4\pi G \rho_b \delta = 4\pi G \rho_b \delta^2 + \frac{1}{a^2} \left[ \partial_\alpha \delta \partial_\alpha \Phi + \partial_\alpha \partial_\beta (1 + \delta)v^\alpha v^\beta \right].
\]  

(4)

The first term in the r.h.s. of Eq.(4) corresponds to the spherical collapse of an isolated proto–object, while the “geometrical” term in square brackets describes tidal and shear effects. The linear approximation, adequate when \( \delta^2 \ll 1 \), corresponds to dropping the r.h.s. of Eq.(4). The first order solution has the well–known self–similar form (considering only the growing mode)

\[ \delta^{(1)}(\mathbf{x}, t) = D(t) \delta_1(\mathbf{x}), \]  

(5)

where \( D(t) \propto a(t) \) is the time growth factor of the mass fluctuations. Higher order approximations of the solution of Eq.(4) may be recovered if one expands the mass density fluctuation field \( \delta(\mathbf{x}, t) \) about the background solution \( \delta = 0 \) (and \( \mathbf{v} = 0 \)), namely \( \delta = \sum_n \delta^{(n)} \) with \( \delta^{(n)} = O(\delta^n) \), then solving the differential equation for any \( \delta^{(n)} \) (Peebles 1980; Fry 1984). The perturbative expansion for \( \delta \) is (e.g. Goroff et al. 1986)

\[ \delta(\mathbf{x}, t) = \sum_{n=1}^{\infty} [D(t)]^n \delta_n(\mathbf{x}). \]  

(6)

The first term of the expansion corresponds to the linear approximation. Second and third order solutions have been discussed by Peebles (1980) and Fry (1984) respectively. We see that the scale factor \( D(t) \) acts as a coupling constant in this perturbative approach, since \( \delta^{(n)} \propto D^n \).

Here we review the exact perturbative technique to solve approximately the equations of motion of a pressureless gravitational fluid up to third order in the density fluctuation field. In particular, we will use the third order solution to compute the fourth order moment (namely the kurtosis parameter \( S_4 \)). We use explicitly the same notation of Fry (1984).

\( (i) \) Second Order Density Solution

The second order contribution \( \delta^{(2)} \) is related to the second order gravitational potential \( \Phi^{(2)} \) by the Poisson equation \( \nabla^2 \Phi^{(2)} = 4\pi G a^2 \rho_b \delta^{(2)} \); it is solution of the differential equation

\[
\partial_t^2 \delta^{(2)} + 2 \frac{\dot{a}}{a} \partial_t \delta^{(2)} - 4\pi G \rho_b \delta^{(2)} = \left[ 4\pi G \rho_b + \left( \frac{\dot{D}}{D} \right)^2 \right] \delta^{(1)^2} + \]

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where \( \Delta^{(1)} \) is the rescaled linear gravitational potential defined by \( \Delta^{(1)} \equiv \Phi^{(1)}/4\pi G\rho_b a^2 \), for which \( \nabla^2 \Delta^{(1)} = \delta^{(1)} \). Each side of Eq.(7) is homogeneous in powers of \( t \). For an Einstein–de Sitter universe, noting that \( D^2 \propto \delta^{(2)} \equiv D^2(t) \delta_2(\mathbf{x}) \), the solution of Eq.(7) is given by (Peebles 1980, §18)

\[
\delta^{(2)} = \frac{5}{7} \delta^{(1)2} + \partial_\alpha \delta^{(1)} \partial_\alpha \Delta^{(1)} + \frac{2}{7} \partial_\alpha \partial_\beta \Delta^{(1)} \partial_\alpha \partial_\beta \Delta^{(1)}. \tag{8}
\]

Note that \( \langle \delta^{(2)} \rangle = 0 \), i.e. mass is conserved to second order. We see from Eq.(8) that the behaviour of \( \delta \) at second order is non–local: the mass fluctuation at the position \( \mathbf{x} \) depends on initial perturbations at other positions via \( \Delta^{(1)} \). Physically this means that, unlike the linear local case, when density fluctuations grow in amplitude their spatial dependence, in comoving coordinates, changes. Thus, the gravitational field changes direction and particles are not accelerated in a fixed direction, as it occurs in linear regime. The last term in right–hand side of Eq.(8) corresponds to the velocity shear contribution.

We can obtain the Fourier transform \( \tilde{\delta}^{(2)} \) directly from the differential equation (8). Explicitly (time dependence is understood),

\[
\tilde{\delta}^{(2)}(\mathbf{k}) = \frac{5}{7} \tilde{\delta}^{(1)2} + j^{(2)}(\mathbf{k}, \mathbf{k}) \tilde{\delta}^{(1)}(\mathbf{k}) \tilde{\delta}^{(1)}(\mathbf{k} - \mathbf{k}'),
\]

where we have defined the kernel

\[
j^{(2)}(\mathbf{k}, \mathbf{k'}) \equiv \frac{5}{7} + \frac{\mathbf{k} \cdot \mathbf{k'}}{k^{'2}} + \frac{2}{7} \left( \frac{\mathbf{k} \cdot \mathbf{k'}}{k' k} \right)^2.
\]

\( (ii) \) Third Order Density Solution

The third order approximation \( \tilde{\delta}^{(3)} \) is solution of the differential equation

\[
\partial_\alpha^2 \tilde{\delta}^{(3)} + 2a a^{-1} \partial_\alpha \tilde{\delta}^{(3)} - 4\pi G\rho_b \tilde{\delta}^{(3)} = 8\pi G\rho_b \delta^{(1)} \delta^{(2)} + a^{-2} \left( \partial_\alpha \delta^{(1)} \partial_\alpha \Phi^{(2)} + \partial_\alpha \delta^{(2)} \partial_\alpha \Phi^{(1)} \right) + a^{-2} \partial_\alpha \partial_\beta \left( 2 v^{(1)\alpha} v^{(2)\beta} + \delta^{(1)} v^{(1)\alpha} v^{(1)\beta} \right).
\]

In this case, working directly in Fourier space is much simpler. Since \( \delta^{(3)} \propto D^3 \propto t^2 \), we have \( \partial_\alpha^2 \tilde{\delta}^{(3)} + 2a a^{-1} \partial_\alpha \tilde{\delta}^{(3)} - 4\pi G\rho_b \tilde{\delta}^{(3)} = 4 \tilde{\delta}^{(3)}/t^2 \); by using the second order results it is not difficult to show that (Fry 1984)

\[
\tilde{\delta}^{(3)}(\mathbf{k}) = \frac{d\mathbf{k}_1 d\mathbf{k}_2 d\mathbf{k}_3}{(2\pi)^6} \delta_D(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3 - \mathbf{k}) \ J^{(3)}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) \tilde{\delta}^{(1)}(\mathbf{k}_1) \tilde{\delta}^{(1)}(\mathbf{k}_2) \tilde{\delta}^{(1)}(\mathbf{k}_3), \tag{12}
\]

where the third order kernel is

\[
J^{(3)}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) \equiv J^{(2)}(\mathbf{k}_2, \mathbf{k}_3) \left[ \frac{1}{3} + \frac{1}{3} \frac{\mathbf{k}_1 \cdot (\mathbf{k}_2 + \mathbf{k}_3)}{(\mathbf{k}_2 + \mathbf{k}_3)^2} + \frac{4}{9} \frac{\mathbf{k} \cdot \mathbf{k}_1 \cdot \mathbf{k} \cdot (\mathbf{k}_2 + \mathbf{k}_3)}{k^2 (\mathbf{k}_2 + \mathbf{k}_3)^2} \right] +
\]
\begin{align*}
&- \frac{2}{9} \frac{k \cdot k_1}{k_1^2} \frac{k \cdot (k_2 + k_3)}{(k_2 + k_3)^2} \frac{(k_2 + k_3) \cdot k_3}{k_3^2} + \frac{1}{9} \frac{k \cdot k_2}{k_2^2} \frac{k \cdot k_3}{k_3^2}.
\end{align*}

It is clear from Eq. (11) that, in order to derive the solution \( \delta^{(3)} \), we need to know explicitly the second order velocity \( v^{(2)} \). This is given by

\begin{equation}
\begin{aligned}
v^{(2)} &= a \frac{\dot{D}}{D} \left[ \delta^{(1)} \nabla \Delta^{(1)} - 2 \nabla \Delta^{(2)} \right],
\end{aligned}
\end{equation}

where \( \Delta^{(2)} \equiv \Phi^{(2)}/4\pi G \rho_b a^2 \) is the rescaled second order gravitational potential. [In the solution (14) we neglect an additive homogeneous term whose divergence is zero.] In an Einstein–de Sitter universe \( v^{(2)} \sim t \), slower than \( \delta^{(2)} \sim t^{4/3} \). We stress the fact that \( v^{(2)} \) is not parallel to the second order acceleration \([ \propto -\nabla \Delta^{(2)} ]\): this is a consequence of non–locality. Finally, we note that \( v^{(2)} \) is known only once \( \delta^{(2)} \) is known.

(iii) General Solution

As Goroff et al. (1986) have shown, the (Fourier transformed) \( n \)–th order mass fluctuation term may be represented in integral form as

\begin{equation}
\begin{aligned}
\tilde{\delta}_n(k) &= \left\{ \prod_{h=1}^n \int \frac{d k_h}{(2\pi)^3} \tilde{\delta}_1(k_h) \right\} \left[ (2\pi)^3 \delta_D \left( \sum_{j=1}^n k_j - k \right) \right] J^{(n)}(k_1, \ldots, k_n).
\end{aligned}
\end{equation}

The presence of the Dirac delta function comes from momentum conservation in Fourier space. The kernels \( J^{(n)} \) are symmetric homogeneous (with degree 0) functions of the wavevectors \( k_1, \ldots, k_n \), describing the effects of non–linear collapse (tidal and shear forces). In general \( J^{(n)} \) are very complicated for \( n > 3 \). [A discussion of the properties of the kernels \( J^{(n)} \) is given in Wise (1988). Explicit recursion relations with their Feynman diagrammatic representation are given by Goroff et al. (1986) and Wise (1988).]

2.2 Zel’dovich Approximation

In the Zel’dovich approximation (Zel’dovich 1970) the motion of particles from the initial co-moving (Lagrangian) positions \( q \) is approximated by straight paths. The Eulerian position at time \( t \) is then given by the uniformly accelerated motion

\begin{equation}
\begin{aligned}
x[q, t] &= q + D(t) S(q),
\end{aligned}
\end{equation}

where \( D(t) \) is the growth factor of linear density perturbations and \( S(q) \) is the displacement vector related to the primordial velocity field. The Zel’dovich approximation provides an exact solution of the equations of motion for one–dimensional perturbations, and reduces to the linear approximation when \( \delta \) and \( v \) are small. In general, the Zel’dovich approximation is not an exact solution of the equations of motion, in that in a finite time particles converge into singular regions of infinite density (caustics), and the map in Eq. (16) becomes multi–valued. The treatment of these formal singularities is the main problem to solve during the
highly non–linear stage of structure formation. Smoothing on a suitable scale $R$ partially solves this problem. Grinstein & Wise (1987) give an Eulerian representation of the Zel’dovich approximation by a diagrammatic perturbative approach similar to that of the previous section. They showed that the $n$–th order perturbative corrections $\delta_n(x)$, when the density fluctuation field $\delta$ is evolved according to the Zel’dovich approximation, are such that

$$\delta(x,t) = \sum_{n=1}^{\infty} \frac{(-1)^n}{n!} [D(t)]_n^n \sum_{[h_n]=1}^3 \frac{\partial}{\partial x_h_1} \cdots \frac{\partial}{\partial x_h_n} [S_{h_1} \cdots S_{h_n}] .$$  (17)

Here $\sum_{[h_n]} \equiv \sum_{h_1} \cdots \sum_{h_n}$. Note that the first term recovers the linear approximation, in that $S = v_1$, where $\delta_1(x) = -\nabla \cdot v_1$. The perturbative expansion for $\delta$ of Eq.(17) simply corresponds to calculate different symmetric kernels $J_{ZA}^{(n)}$. These can be written in the following compact form

$$J_{ZA}^{(n)}(k_1, \ldots, k_n) = \frac{1}{n!} \prod_{h=1}^n \frac{k \cdot k_h}{k_h^2} ,$$  (18)

where $k \equiv \sum_{h=1}^n k_h$. Note that, unlike the perturbative case, the kernels $J_{ZA}^{(n)}$ are manifestly symmetric by construction.

3 Kurtosis of the Density Field

In this section, we compute the induced–by–gravity kurtosis parameter $S_4$ of an initial Gaussian density field in a flat universe. The lowest order non–zero contribution to $S_4$ is

$$\langle \delta^{(1)} \rangle^3 S_4 = 6 \langle \delta^{(1)} \delta^{(2)} \rangle + 4 \langle \delta^{(1)} \delta^{(3)} \rangle .$$  (19)

From Eqs.(10) and (13), after tedious but straightforward algebra, one finally gets the integral expression of the kurtosis ratio,

$$S_4 = \frac{24}{\sigma^6} \int \frac{dk_1 dk_2 dk_3}{(2\pi)^9} P(k_1) P(k_2) \left[ P(k_3) J^{(3)}(k_1, k_2, k_3) + 2 J^{(2)}(-k_2, k_2 + k_3) J^{(2)}(k_1, k_2 + k_3) P(|k_2 + k_3|) \right] .$$  (20)

The expression in the Zel’dovich approximation is obtained by substituting the kernels $J^{(n)}$ with the corresponding $J_{ZA}^{(n)}$ in the previous relation. The angular integrations in Eq.(20) may be analytically performed. One obtains $S_4 = 60,712/1,323 \approx 45.9$ in the perturbative case (Fry 1984; Bernardeau 1992) and $S_4 = 88/3 \approx 29.3$ in the Zel’dovich approximation. These unsmoothed–case results are independent of the primordial spectral index, as is the skewness ratio $S_3$.

The kurtosis of the smoothed density field $\delta_R$ in the exact perturbative case is

$$S_4(R) = \frac{24}{\sigma_R^6} \int \frac{dk_1 dk_2 dk_3}{(2\pi)^9} \tilde{W}_R(k_1) \tilde{W}_R(k_2) \tilde{W}_R(k_3) \tilde{W}_R(|k_1 + k_2 + k_3|) \times$$
\[
P(k_1) P(k_2) \left[ P(k_3) J^{(3)}(k_1, k_2, k_3) + 2 J^{(2)}(-k_2, k_2 + k_3) J^{(2)}(k_1, k_2 + k_3) P(|k_2 + k_3|) \right] ;
\]

(21)

again the substitution \( J^{(n)} \rightarrow J_{ZA}^{(n)} \) leads to the corresponding expression in the Zel’dovich approximation. Note that \( P(k) \) completely describes the process of growth of mass fluctuations from Gaussian initial perturbations.

4 Discussion and Conclusions

We calculate the previous integrals, by an Adaptive Multidimensional Monte Carlo Integration subroutine, for scale–free power spectra \( P(k) \propto k^n \) with \( n \) in the range \(-3 \leq n \leq 1\). Due to the assumed primordial scale–invariance, \( S_4 \) only depends on the primordial spectral index \( n \), and not on the scale \( R \). In Figure 1, we plot the kurtosis ratio \( S_4 \) of the Gaussian–smoothed density field versus the spectral index \( n \), for both the perturbative and Zel’dovich approximations.

It clearly appears that \( S_4 \) strongly depends on the primordial spectral index \( n \). In particular the kurtosis parameter is a decreasing function of \( n \). This is also confirmed by Bernardeau (1993), who, applying the exact perturbative technique, finds a similar trend in the case of top–hat filtering. Anticorrelation with the amount of small–scale power has also been found for the skewness parameter \( S_3 \) (Fry 1984; Juszkiewicz, Bouchet, & Colombi 1993), and it seems therefore a general property of higher order moments of the smoothed matter distribution: larger values of \( n \) correspond to higher power on small scales, where the filtering operation acts.

Furthermore, the Zel’dovich approximation underestimates the induced–by–gravity \( S_4 \) w.r.t. the rigorous perturbative one, as already noted by Grinstein & Wise 1987 (although in the framework of the “standard” cold dark matter model). This is hardly surprising, since the Zel’dovich approximation fails to fully describe the gravitational effects causing particle trajectories to depart from their original directions. It results that the Zel’dovich approximation makes higher orders in the perturbative series for \( \delta \) smaller on large scales than they actually are (see e.g. Wise 1988).

We stress that, due to the importance of the smoothing procedure, but also to its arbitrariness, one must be cautious about making quantitative comparisons between our results and both observational and N–body data. A Gaussian filter is used for instance by Saunders et al. (1991), who however estimated only the skewness in the QDOT–IRAS catalog. The only observational estimates up–to–now of the kurtosis of the galaxy distribution are obtained by counts–in–cells, corresponding to a top–hat filtering (Gaztañaga 1992; Bouchet et al. 1993; Fry & Gaztañaga 1993); a similar method is applied by Lahav et al. (1993) and Lucchin et al. (1993) in N–body simulations with scale–free power spectra. Our results can be used to constrain both the probability distribution and the power spectrum of primordial density fluctuations and thus help to select the most reliable mechanism for the generation of large–scale structures.
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Figure caption

Figure 1. The kurtosis ratio $S_4$ of the density field for power-law spectra $P(k) \propto k^n$ and Gaussian filter versus the primordial spectral index $n$, for both the perturbative (squares and solid line) and Zel’dovich approximations (triangles and dotted line). Note that the values at $n = -3$ correspond to the unsmoothed cases. Error bars refer to the associated uncertainty estimate from the Monte Carlo Integration.