Effective Resistance Mismatch and Magnetoresistance of a CPP-GMR system with Current-Confined-Paths

Jun Sato, Katsuyoshi Matsushita and Hiroshi Imamura*
Nanotechnology Research Institute (NRI), Advanced Industrial Science and Technology (AIST), AIST Tsukuba Central 2, Tsukuba, Ibaraki 305-8568, Japan.

Abstract—We theoretically study the magnetoresistance of a CPP-GMR system with current confined paths (CCP) in the framework of Valet-Fert theory. The continuity equations for charge and spin currents are numerically solved with the three-dimensional CCP geometry by use of finite element method. It is confirmed that the MR ratio is enhanced by the CCP structure, which is consistent with the experimental results. Moreover, we find that there exists a certain contact width which maximize the MR ratio. We show that the contact width which maximize the MR ratio is well described by the effective resistance matching.

Index Terms—CPP-GMR, current-confined-path, spin accumulation, nano-oxide-layer

CURRENT-perpendicular-to-plane giant magnetoresistance (CPP-GMR) has attracted much attention for its potential application as a read sensor for high-density magnetic recording [1], [2]. In order to realize a high-density magnetic recording, we need MR devices with high MR ratio and low resistance area product (RA). Although the RA value of a CPP-GMR system is much smaller than that of a tunneling magnetoresistance (TMR) system, the MR ratio of a conventional CPP-GMR system still remains a small value of a few %. Much effort has been devoted to increasing the MR ratio of the CPP-GMR system.

Recently, Fukuzawa et al. reported that they achieved the MR ratio of 10.2 % by CPP-GMR spin-valve with a current-confined-path (CCP) structure made of a nano-oxide-layer (NOL) with a lot of small metallic channels [3], [4]. They showed that the MR ratio increases with increasing the RA value, which implies that the MR ratio is enhanced for the system with narrow metallic channels. The similar enhancement of the MR ratio due to the CCP structure containing a domain wall was also reported by Fuke et al. [5]. The enhancement of the MR ratio due to the CCP structure is theoretically explained by one of the author [6]. However, the analysis in [6] is based on the assumption that the thickness of the non-magnetic layer is zero and little attention is paid to the effect of the nonmagnetic spacer layer on the enhancement of the MR ratio.

In 2000, Schmidt et al. pointed out that the conductivity mismatch between these materials. For conventional CPP-GMR spin-valves without CCP structures, the conductivity mismatch seems not to be important because both of the ferromagnetic electrodes and the nonmagnetic spacer layer are made of metals. However, for the CCP-CPP-GMR system the effective resistance of the contact region increases with decreasing the radius of the contact, which implies that the conductivity mismatch plays an important role in the CCP-CPP-GMR system. Therefore, it is intriguing to ask how the mismatch of the effective resistance affects the MR ratio of the CCP-CPP-GMR system.

In this paper, we analyze the dependence of the MR ratio of the CCP-CPP-GMR spin-valves on the contact radius and resistance area product (RA) in the framework of Valet-Fert theory [8]. The spin-dependent electro-chemical potentials are obtained by numerically solving continuity equations for charge and spin currents with the three-dimensional CCP geometry by use of the finite element method [9], [10]. We show that the CCP-CPP-GMR spin-valve can take a larger MR ratio than that of the conventional CPP-GMR spin-valve. However, the MR ratio is not a monotonic function of the contact radius but takes a maximum value at a certain value of the contact radius. We also show that the contact radius which maximizes the MR ratio can be explained by considering the matching of effective resistances.

I. VALET-FERT THEORY OF CPP-GMR

By starting with the Boltzmann equation, Valet and Fert constructed the macroscopic model of the CPP-GMR[8], on which our calculation is based. Before showing our results, we shall give a brief introduction to the Valet-Fert theory of CPP-GMR.

In the macroscopic model of the CPP-GMR, it is assumed that the conduction electrons with spins $s = \uparrow, \downarrow$ can be characterized by the conductivity $\sigma_s$, spin relaxation rate $\tau_s$, density of states $N_s$, and diffusion constant $D_s$. Since we consider a collinear alignment of the magnetizations we neglect the mixing of spin-up and spin-down bands. The current density for each spin band under an electric field $E$ is given by

$$j_s = \sigma_s E - qD_s \nabla \delta n_s,$$

where $q$ is the charge of an electron and $\delta n_s$ represents the accumulation of the electrons with spin $s$. By using Einstein's
relation \( \sigma_s = q^2 N_s D_s \), we can rewrite (1) as

\[
j_s = -\frac{\sigma_s}{q} \left( -qE + \frac{\nabla \delta n_s}{N_s} \right).
\]

Introducing the deviation of the spin-dependent chemical potential from the equilibrium state \( \delta \mu_s \), the accumulation \( \delta n_s \) can be expressed as \( \delta n_s = N_s \delta \mu_s \). Since the electric field is given by the gradient of the electrostatic potential \( \phi \) as \( E = -\nabla \phi \), it is convenient to define the spin-dependent electro-chemical potential as \( \mu_s = \delta \mu_s + q \phi \). Hence the current density (1) can be written as

\[
j_s = -\frac{\sigma_s}{q} \nabla \mu_s.
\]

In the ferromagnetic materials, the spin-polarization of the conductivity is represented by the parameter \( \beta \) which is defined as \( \beta = (\sigma_\uparrow - \sigma_\downarrow)/ (\sigma_\uparrow + \sigma_\downarrow) \). For example, the spin-polarization parameter \( \beta = 0.36 \sim 0.5 \) for Co.

The equations that determine electro-chemical potentials are derived by the continuity equations for charge and spin. The continuity equation for charge is given by

\[
\nabla \cdot (j_\uparrow + j_\downarrow) = 0,
\]

which means that there is no source for electron charge. The continuity equation for spin is given by

\[
\nabla \cdot (j_\uparrow - j_\downarrow) = q \left( \frac{N_\uparrow}{\tau_\uparrow} \mu_\uparrow - \frac{N_\downarrow}{\tau_\downarrow} \mu_\downarrow \right),
\]

which means that the spin relaxation plays a role of spin source. Substituting (3) into (4) and (5) we obtain the following diffusion equations for electro-chemical potentials

\[
\begin{align*}
\sigma_\uparrow \nabla^2 \mu_\uparrow + \sigma_\downarrow \nabla^2 \mu_\downarrow &= 0, \\
\nabla^2 (\mu_\uparrow - \mu_\downarrow) &= \frac{1}{\lambda^2} (\mu_\uparrow - \mu_\downarrow),
\end{align*}
\]

where \( \lambda \) is the characteristic length of the spin diffusion called “spin diffusion length”. The spin diffusion length is defined as \( \lambda = \sqrt{\tau D} \), where \( \tau \) is the averaged spin relaxation time defined as \( \tau^{-1} = (\tau_\uparrow^{-1} + \tau_\downarrow^{-1})/2 \) and \( D \) is the averaged diffusion constant defined as \( D^{-1} = (N_\uparrow D_\uparrow^{-1} + N_\downarrow D_\downarrow^{-1})/(N_\uparrow + N_\downarrow) \).

By solving (6) and (7) with proper boundary conditions, we obtain the electrochemical potential and therefore the total resistance of the system. The MR ratio is defined by the total resistance for the parallel alignment of the magnetizations \( R_p \), and that for the anti-parallel alignment \( R_{AP} \) as \( MR = (R_{AP} - R_p)/R_p \).

II. CPP-GMR WITHOUT CCP

Let us first consider the MR ratio of the conventional CPP-GMR spin-valves shown in Fig. 1(a). The nonmagnetic (N) spacer layer is sandwiched between two ferromagnetic (F) electrodes. The magnetization of the top electrode is assumed to be fixed at a certain direction and the magnetization of the bottom electrode is aligned to be parallel or anti-parallel to that of the top electrode.

We assume the translational invariance in the \( x- \) and \( y- \) directions and the current is flowing perpendicular to the plane, i.e., in the \( z- \) direction. Hence the equations to be solved reduce to a couple of one-dimensional diffusion equations:

\[
\begin{align*}
\sigma_\uparrow \frac{\partial^2}{\partial z^2} \mu_\uparrow + \sigma_\downarrow \frac{\partial^2}{\partial z^2} \mu_\downarrow &= 0, \\
\frac{\partial^2}{\partial z^2} (\mu_\uparrow - \mu_\downarrow) &= \frac{1}{\lambda^2} (\mu_\uparrow - \mu_\downarrow).
\end{align*}
\]

The general solutions of (8) and (9) are obtained as

\[
\begin{align*}
\mu_\uparrow(z) &= c_1 + c_2 z + \frac{1}{\sigma_\uparrow} \left( c_3 e^{-z/\lambda} + c_4 e^{z/\lambda} \right), \\
\mu_\downarrow(z) &= c_1 + c_2 z - \frac{1}{\sigma_\downarrow} \left( c_3 e^{-z/\lambda} + c_4 e^{z/\lambda} \right),
\end{align*}
\]

where \( c_j \)'s are arbitrary integration constants determined by the proper boundary conditions.

We neglect the effect of interfacial scattering for simplicity. Then we adopt the boundary conditions such that both \( \mu_\uparrow \) and \( j_s \) are continuous at the interface. After some algebra, we can show that the total resistance for parallel alignment of the magnetizations \( R_p \), and that for the anti-parallel alignment \( R_{AP} \), have the form

\[
R_{AP} = R_0 + \Delta R_{AP},
\]

where \( R_0 = (2t_F \rho_F + t_N \rho_N)/S \), \( t_F(N) \) is the thickness of the F(N) layer, \( S \) is the cross-section area of the system, and \( \rho_F(N) \) is the resistivity of the F(N) layer defined as

\[
\rho_F = \frac{1}{\sigma_F + \sigma_p}, \quad \rho_N = \frac{1}{\sigma_N + \sigma_p}.
\]

Here \( \sigma_F^* \) is the conductivity for electrons with spin \( s \) in the F (N) layer. \( R_0 \) is the total resistance of the system.
in the absence of the spin accumulation and independent of the magnetization configuration. The additional resistance $\Delta R$ originates from the voltage drop at the interfaces due to the spin accumulation, which has the form

$$\Delta R = 2 \left\{ r_F^{-1} + r_N^{-1} \right\}^{-1},$$

(14)

where $r_F$ and $r_N$ are the effective resistances for F and N layers defined as

$$r_F = \frac{\beta^2 \rho_F \lambda_F}{S (1 - \beta^2)}, \quad r_N = \frac{\beta^2 \rho_N \lambda_N}{S} f_{RAP} \left( \frac{t_N}{2\lambda_N} \right).$$

(15)

The function $f_{RAP}$ is given by $f_p(x) = \tan h(x)$ and $f_{AP}(x) = \coth(x)$. From (14), $\Delta R$ is considered as a parallel circuit of the effective resistances $r_F$ and $r_N$ as shown in Fig. 1(b).

In Fig. 1(c), we plot the MR ratio against the resistivity of the non-magnetic layer $\rho_N$. We assume that the F layer is made of CoFe and the N layer is made of Cu. For numerical calculation, we use the following values: $\rho_F = 160 \, \Omega \, \text{nm}$, $t_F = 45 \, \text{nm}$, $t_N = 2 \, \text{nm}$, $\lambda_F = 15 \, \text{nm}$, $\lambda_N = 500 \, \text{nm}$, $\beta = 0.7$. The thickness of the F-layer $t_F$ is taken to be thick enough compared with the spin diffusion length $\lambda_F$ in order to eliminate the spin accumulation at the bottom and the top electrodes.

One can obviously see that the MR ratio takes its maximum value at a certain value of $\rho_N$ and vanishes in both limits of $\rho_N \rightarrow \infty$ and $\rho_N \rightarrow 0$. In the limit of $\rho_N \rightarrow \infty$, the effective resistance of N-layer $r_N$ is much larger than that of F-layer $r_F$ and the interface resistance $\Delta R$ equals to $2\beta^2 r_F$ for both P and AP cases, which means that the MR ratio goes to zero. Conversely, in the limit of $\rho_N \rightarrow 0$, $r_N$ is much smaller than $r_F$ and the interface resistance $\Delta R$ goes to zero. Therefore, the MR ratio vanishes also in this limit.

In Fig. 1(d), the MR ratio is plotted against the ratio of the effective resistance $r_F/\bar{r}_N$, where $\bar{r}_N = \rho_N \lambda_N$ is the effective resistance of N-layer for an F/N bilayer system. As shown in Fig. 1(d), the MR ratio is maximized if the effective resistances $r_F$ and $\bar{r}_N$ are balanced with each other, i.e., $r_F \sim \bar{r}_N$. A similar idea is also applicable to the CCP-CPP-GMR system as will be discussed in the next section.

III. CCP-CPP-GMR

Next we move onto the CCP-CPP spin-valves, where the spacer layer is made of an NOL with a lot of nanometer-size metallic channels. The system we consider is schematically shown in Fig. 2(a). A narrow non-magnetic metallic contact is sandwiched between two ferromagnetic electrodes. The cross-sectional view of the system is shown in Fig. 2(b). The curve of the contact region is modeled by the half ellipse, which is tangent to the ferromagnetic layer. The length of the major axis of the ellipse, which is the same as the thickness of the contact, is taken to be 2 nm. The length of the semi-minor axis of the ellipse is 0.5 nm. The F electrodes are assumed to be 40 nm x 40 nm x 45 nm rectangles.

In our calculation we keep the shape of the ellipse fixed and vary the size of the contact by changing the contact radius $d$.

The obtained MR ratios of the CCP-CPP-GMR system are plotted against the contact radius $d$ in Fig. 3. One can immediately see that the MR ratio increases with decreasing the resistivity of Cu, $\rho_{Cu}$. It has been considered so far that the MR ratio shows monotone increasing with decreasing the contact radius both theoretically [6] and experimentally [4]. We note that there is a contact radius that maximizes the MR ratio, which is indicated by a circle in Fig. 3 for given values of $\rho_{Cu}$ and the MR ratio vanishes in the limit of $d \rightarrow 0$. Fig. 3 also shows that the value of the contact radius that maximizes the MR ratio decreases with decreasing the resistivity of Cu.

Strictly speaking, the Boltzmann approach loses its validity in the limit $d \rightarrow 0$, especially in the region $d < 1$ nm. In Ref. [12], the electron transport across a narrow metallic channel with the width of a single atom is studied based on the Landauer formalism, where it is concluded that MR ratio is diminished by the quantum interference. This does not contradict our results that MR goes to zero in the limit $d \rightarrow 0$. 

![Fig. 2. (a) A CCP-CPP-GMR spin-valve is schematically shown. The non-magnetic (N) contact region is sandwiched between two ferromagnetic (F) electrodes. (b) The cross-sectional view of the CCP-CPP-GMR spin-valve is schematically shown. The curve of the contact region is modeled by the half ellipse, which is tangent to the ferromagnetic layer. In our calculation we keep the shape of the ellipse fixed and vary the size of the contact by changing the contact radius $d$.]

![Fig. 3. MR ratios of CCP-CPP-GMR spin-valves are plotted against the contact radius $d$. The resistivity of Cu in the contact is varied from 17 $\Omega \, \text{nm}$ to 300 $\Omega \, \text{nm}$. The MR ratio takes a maximum value at a certain value of the contact radius which is indicated by a circle.]

where $j_{\text{resistivity}}$ resistivity spin-dependent interfacial resistance potentials at the interface located at is realized by introducing fictitious layers with thickness balanced with each other, i.e., effective resistances in Fig. 4 (b), we plot the MR ratio against the ratio of the estimations:

$$r_F(r) = \rho_N/N_z(2d^2)$$

and $r_N(P) \sim \infty$, where $d$ is the contact radius. Based on this consideration, in Fig. 4 (b), we plot the MR ratio against the ratio of the effective resistances $r_F/r_N$. It is observed that the MR ratio is maximized when the effective resistances $r_F$ and $r_N$ are balanced with each other, i.e., $r_F \sim r_N$.

Finally we shall discuss the effect of the spin-dependent interfacial scattering which is represented by the spin-dependent interfacial resistance $r_s$ in the microscopic model of the CPP-GMR. The boundary conditions for electro-chemical potentials at the interface located at $z = 0$ are given by

$$\mu_s(z = 0 + 0) - \mu_s(z = 0 - 0) = r_s j_s^z (z = 0),$$

where $j_s^z$ is the $z$-component of the current density and the spin-dependent interfacial resistance $r_s$ are defined as

$$r_s^{-1} = \frac{(1 + \gamma)\rho_b^{-1}}{2}, \quad r_s^{-1} = \frac{(1 - \gamma)\rho_b^{-1}}{2},$$

where $\gamma$ is the interfacial spin asymmetry coefficient and $\rho_b$ is the total interfacial resistance.

In our calculation, the spin-dependent interfacial scattering is realized by introducing fictitious layers with thickness $t_{\text{Ir}}$ resistivity $\rho_{\text{Ir}} = \rho_b/t_{\text{Ir}}$, spin polarization of the conductivity $\beta_{\text{Ir}} = \gamma$, and with the infinite spin diffusion length $\lambda_{\text{Ir}} \rightarrow \infty$.

We carried out the same calculations as shown in Figs 3 taking into account of the spin-dependent interfacial scattering at both sides of the contact. The parameters are taken to be $\rho_{\text{Ir}} = 200 \Omega \text{nm}$ and $\gamma = 0.62$. The results are qualitatively same as those shown in in Figs 3. However, the contact radius that maximize the MR ratio decreases due to the spin-dependent interfacial scattering. For $\rho_{\text{Cu}} = 17 \Omega \text{nm}$, for example, the contact radius that maximize the MR ratio decreases as $3.0 \text{nm} \rightarrow 2.3 \text{nm}$.

IV. CONCLUSION

In conclusion, we theoretically study the magnetoresistance of a CCP-CPP-GMR system by numerically solving the diffusion equations for spin-dependent electro-chemical potentials in three-dimensional CCP geometry. We show that the MR ratio is not a monotone function of the contact radius but takes a maximum value at a certain value of the contact radius. We also show that the value of contact radius that maximize the MR ratio is qualitatively understood by considering matching of the effective resistances.

ACKNOWLEDGEMENT

The authors thank M. Sahashi, M. Doi, H. Iwasaki, M. Takagishi, Y. Rikitake and K. Seki for valuable discussions. The work has been supported by The New Energy and Industrial Technology Development Organization (NEDO).

REFERENCES

[1] W. P. Pratt, Jr., S.-F. Lee, J. M. Slaughter, R. Loloee, P. A. Schroeder, and J. Bass, “Perpendicular giant magnetoresistances of Ag/Cu multilayers,” Phys. Rev. Lett. 66, 3060 (1991).

[2] A. C. Reilly, W. Park, R. Slater, B. Ouaglal, R. Loloee, W. PrattJr. and J. Bass, “Perpendicular giant magnetoresistance of Co$_3$Fe$_7$/Cu exchange-biased spin-valves: further evidence for a unified picture”, J. Magn. Magn. Mater. 195, L269 (1999).

[3] H. Fukuzawa, H. Yuasa, S. Hashimoto, K. Koi, H. Iwasaki, M. Takagishi, Y. Tanaka, and M. Sahashi, “MR Ratio Enhancement by NOL Current-Confined-Path Structures in CPP Spin Valves”, IEEE Trans. Magn. 40, 2236 (2004).

[4] H. Fukuzawa, H. Yuasa, S. Hashimoto, H. Iwasaki, Y. Tanaka, and M. Sahashi, “Large magnetoresistance ratio of 10% by Fe$_{9}$Co$_{91}$/Cu layers for current-confined-path current-perpendicular-to-plane giant magnetoresistance spin-valve films”, Appl. Phys. Lett. 87, 082507 (2005).

[5] H. N. Fuke and S. Hashimoto, M. Takagishi, H. Iwasaki, S. Kawasaki, K. Miyake, and M. Sahashi, “Magnetoresistance of FeCo Nanocontacts With Current-Perpendicular-to-Plane Spin-Valve Structure”, IEEE Trans. Magn. 43, 2848 (2007).

[6] H. Imamura, “Spin accumulation and magnetoresistance of a CPP-GMR system with a current confined path,” PHYSICA STATUS SOLIDI (B) 244, 4394 (2007).

[7] G. Schmidt, D. Ferrand, L. W. Molenkamp, A. T. Filip and B. J. van Wees, “Fundamental obstacle for electrical spin injection from a ferromagnetic metal into a diffusive semiconductor”, Phys. Rev. B 62, R4790 (2000).

[8] T. Valet and A. Fert, “Theory of the perpendicular magnetoresistance in magnetic multilayers”, Phys. Rev. B 48, 7099 (1993).

[9] L. Ramdas Ram-Mohan, “Finite element and Boundary Element Applications in Quantum Mechanics”, Oxford University Press(2002)

[10] M. Ichimura, J. Ieda, H. Imamura, S. Takahashi and S. Maekawa, "Numerical analysis of spin accumulation due to a domain wall,” Journal of Magnetism and Magnetic Materials, Volume 310, Issue 2, Part 3, March 2007, Pages 2055-2057

[11] N. F. Mott and H. Jones, “The THEORY of the PROPERTIES of METALS and ALLOYS”, Chap. VII, Dover Publications Inc. New York (1958).

[12] X. Chen, R.H. Victoria, “Effect of pinholes in magnetic tunnel junctions”, Appl. Phys. Lett. 91, 212104 (2007).