Numerical investigation of effectiveness of two seismic dampers on a benchmark structure

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Abstract. The passive energy dissipation devices have been widely applied in seismic control of new constructions and reinforcement of seismic damage structures. The technique of passive energy dissipation has been written into the China Seismic Code for Buildings, which can give an impulse to the use and development of this technique. With the international common performance evaluation platform of structural vibration control — the third stage of benchmark, using three models and some performance evaluation indices of nonlinear vibration control, the control results of buildings with displacement-based and velocity-based dampers were compared. Combining the passive control of structures with the modern control theory, using MATLAB/SIMULINK to establish simulation models of the structures with displacement-based and velocity-based dampers, the responses of three benchmark structure models were calculated under far- and near-field earthquakes. Control effect is compared for different types of dampers on different structures through different evaluation indices, which can provide a certain criterion of reference for the application and design of the passive energy dissipation dampers.

1. Introduction
Over thirty years, many researchers devoted themselves to the study of experiments and theoretical analyses of various passive, active, semi-active and combination strategies for mitigating structural damage against severe earthquakes and strong winds. As different devices and control algorithms have its own merits depending on the desired effect and particular application, researchers set different evaluation criteria to discuss the effectiveness and accuracy. The ASCE Committee on Structural Control recognized the significance of structural control benchmark problems which set the standard to compare the systems using different algorithms and devices. A scale model of a three-storey building employing an active mass driver was studied in the first stage and different control algorithms were compared [1]. Researchers proposed to develop benchmark models to provide systematic and standardized means in the Second International Workshop on Structural Control, held in Hong Kong in 1996. Yang et al. [2] proposed structural control problem against wind. Spencer et al. [3] suggested
selecting a typical structure as the benchmark model. This is the second generation of structural control benchmark problems and the structural model was considered to remain perfectly elastic. Different control devices and algorithms were analyzed and compared according to the same structure, ambient excitation and performance evaluation indices. However, inelastic behaviour may be caused in elements of controlled structures subjected to severe earthquakes, which results in nonlinear responses. Benchmark control problems for seismically excited nonlinear buildings were presented in the third generation. Until now, many scholars [4-5] dedicated themselves to various kinds of active, semi-active and combination strategies. Fukukita et al. [6] discussed the control effect of 20-storey benchmark building using semi-active oil dampers and viscous damping walls.

In China, studies on experiments, theoretical analyses and design methods of various energy dissipation devices have made great development. Active control devices require external power source and the algorithms are usually complicated. The technique of passive energy dissipation has been widely applied in seismic control of new constructions and reinforcement of seismic damage structures and has extensive forms. Contents related to passive energy dissipation has been written into the China Seismic Code (GB 50011-2010) [7]. However, for a variety of forms and different requirements of the projects, how to choose suitable type of dampers will affect the development of the technology during its popularization, which has important realistic meaning.

The main objective of this paper is to compare nonlinear control effect of 20-storey benchmark building with different types of passive dampers according to benchmark evaluation indices to provide a certain criterion of reference for the application and design of passive energy dissipation dampers. Combining the passive control of structures with modern control theory, simulation models were established by using MATLAB/SIMULINK.

2. Benchmark problems
The structural control benchmark problem is a standard for testing and evaluating different systems using different algorithms and devices, which has the same structures, excitation and performance indices. Ohtori and Spencer et al. [8] suggested the 3-, 9- and 20- storey structures to be used as the third generation benchmark problem. These three structures meet seismic code are designed for SAC Phase II Steel Project and represent typical low, medium and high rise buildings.

Different earthquake records, even though similar intensities, lead to widely varying responses. Four ground acceleration records are adopted in this research. Two near-field historical records, El Centro and Hachinohe, are selected and 0.5, 1.0 and 1.5 times the magnitude of the absolute peak acceleration are considered. The absolute peak accelerations are modulated 0.5 and 1.0 time of two far-field history records, Northridge and Kobe. Ten records are considered totally for analyzing as the table 1 shown.

| Number | Earthquake records | Site | Component | Peak (m/s²) |
|--------|--------------------|------|-----------|-------------|
| 1 | Imperial Valley (1940/5/18) | El Centro array #9,180 | NS | 1.709 |
| 2 | Imperial Valley (1940/5/18) | El Centro array #9,180 | NS | 3.417 |
| 3 | Tokachi-oki (1968/5/16) | Hachinohe city | NS | 5.126 |
| 4 | Tokachi-oki (1968/5/16) | Hachinohe city | NS | 1.125 |
| 5 | Northridge (1994/1/17) | Sylmar County | NS | 2.250 |
| 6 | Northridge (1994/1/17) | Sylmar County | NS | 3.375 |
| 7 | Hyogoken Nanbu (1995/1/17) | Kobe Japanes | NS | 4.134 |
| 8 | Hyogoken Nanbu (1995/1/17) | Kobe Japanes | NS | 8.268 |
| 9 | Hyogoken Nanbu (1995/1/17) | Kobe Japanes | NS | 4.089 |
| 10 | Hyogoken Nanbu (1995/1/17) | Kobe Japanes | NS | 8.178 |
The evaluation indices of benchmark problem are divided into four categories: building responses and damage indices, control devices and strategy indices. There are seventeen indices totally. The main task of this paper is to compare the responses of buildings with different dampers and only the building response indices are calculated. This kind of index contains two types: the peak value and root mean square value of responses. The smaller the value is, the better the control performance is. The peak value indices, interstory drift ratio \( J_1 \), level acceleration \( J_2 \) and base shear \( J_3 \) are as follows.

\[
J_1 = \max \left\{ \frac{\max_{i,j} d_i(t)}{h_i} \right\} \quad \text{(1)}
\]

\[
J_2 = \max \left\{ \frac{\max_{i,j} \ddot{x}_a(t)}{\ddot{x}_a^{\max}} \right\} \quad \text{(2)}
\]

\[
J_3 = \max \left\{ \frac{\sum m_ia_i \ddot{x}_a(t)}{F_b^{\max}} \right\} \quad \text{(3)}
\]

where \( d_i(t) \) represents the interstory deformation of the \( i \)-storey above ground level due to earthquakes; \( h_i \) is the height of the \( i \)-storey; \( \ddot{x}_a^{\max} \) represents the maximum storey-drift angle of buildings with no control devices, which is calculated by \( \max_{i,j} \ddot{x}_a(t) \); \( \ddot{x}_a(t) \) is the acceleration of the \( i \)-storey for the building with control devices; \( \ddot{x}_a^{\max} \) represents the maximum acceleration of buildings with no control devices; \( m_i \) is the mass of the \( i \)-storey above ground level; \( F_b^{\max} \) represents the maximum base shear of the building with no control devices due to earthquakes.

In order to evaluate the building damage, mean square root is usually considered to represent the energy level of random variable, which is the supplyment of the peak value. The normed interstory drift ratio \( J_4 \), normed level acceleration \( J_5 \) and normed base shear \( J_6 \) are as follows.

\[
J_4 = \max \left\{ \frac{\max_i \ddot{d}_i(t)}{h_i} \right\} \quad \text{(4)}
\]

\[
J_5 = \max \left\{ \frac{\max_i \ddot{x}_a(t)}{\ddot{x}_a^{\max}} \right\} \quad \text{(5)}
\]

\[
J_6 = \max \left\{ \frac{\sum m_i \ddot{x}_a(t)}{F_b^{\max}} \right\} \quad \text{(6)}
\]
where $\|\varepsilon\|_{\text{max}} = \max_j \|\varepsilon_j(t)/h\|$ represents the mean square root value of the maximum storey-drift angle of buildings with no control devices due to earthquakes; $\|\ddot{x}_{\text{w}}\|_{\text{max}}$ and $\|F_{b_{\text{max}}}\|$ represent the mean square root value of the maximum level acceleration and base shear for the building with no control devices subjected to earthquakes, respectively.

### 3. Design of damper parameters

In order to compare the efficacy of control results for displacement-based and velocity-based dampers, two kinds of dampers offer the same additional equivalent damping ratio. The metallic yielding dampers (MD) and viscoelastic dampers (VED) are selected to illuminate.

Considering normal work condition, the additional damping ratio of structure is 6 to 15 percent with VEDs. The loss factor that provides a measure of the energy dissipation capability of the VE material is 0.9 to 1.3 [9]. The loss factor and shear modulus can be determined according to the performance curve of actual material at design frequency and a given temperature.

On the principle of modal strain energy method, the relation between the stiffness of VED and floor stiffness can be written as [10]:

$$k_{d,i} = \frac{2\xi}{\eta - 2\xi} k_{s,i}$$

where $k_{d,i}$ represents the additional stiffness of VED; $k_{s,i}$ represents the lateral stiffness of each floor; $\eta$ is the loss factor of dampers; $\xi$ represents the target value of additional damping ratio.

It is supposed that MD offer the same additional damping ratio. Based on the formula for calculating additional effective damping ratio of control devices [7], the relation between two type dampers can be obtained,

$$\frac{\pi}{4} \eta k' u_0 = F_y (1 - \alpha) \left(1 - \frac{1}{\mu}\right)$$

where $\alpha$ is the second stiffness coefficient of MD; $\mu$ represents the ductility factor of dampers; $u_0$ is the relative horizontal displacement between each end of the dampers; $k'$ represents the storage stiffness; $F_y$ is the yield force. The additional stiffness of MD is

$$K_{d,j} = \frac{F_{y,j} [1 + \alpha_j (\mu_j - 1)]}{u_{o,j}}$$

where the subscript $j$ represents the $j$-th damper.

### 4. Passive damped structures

Passively damped structure is treated as a feedback control system. The seismic performance of the structure with VEDs is described by state-space method.

$$\dot{Z}(t) = AZ(t) + BU(t) + DF(t)$$

$$x(t) = CZ(t)$$

where $Z(t) = \begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix}$ is the state vector of the system.

$$A = \begin{bmatrix} 0 & I \\ -M_s^{-1}K_s & -M_s^{-1}C_s \end{bmatrix} ; \quad B = \begin{bmatrix} 0 \\ I \end{bmatrix} ; \quad C = \begin{bmatrix} I & 0 \end{bmatrix}; \quad D = \begin{bmatrix} 0 \\ -M_s^{-1} \end{bmatrix} ; \quad U(t) = -[I] \{\ddot{x}_g\}$$

where $M_s, K_s$ and $C_s$ represent, respectively, the mass, stiffness and inherent structural damping matrices of the structure.

When performing non-linear step-by-step time history analyses for passive energy dissipation structures, a number of displacement-based dampers are installed on the 20-storey benchmark structure. Because different phases and stiffness transition conditions for each damper should be taken...
into account, the use of numerical procedures is required and it is an involved problem for the bilinear model. In this paper, a continuous Bouc-Wen’s model is adopted to characterize the hysteretic force-deformation of the MDs, which is in the form of a differential equation and can be conveniently coupled with the equations to describe the motion of the structure. An especially attractive feature of the Bouc-Wen’s model is that the same equation governs the behaviour in different stages of the inelastic cyclic response of the device \[11\]. So the numerical calculation is simplified for no work on the procedure of transition points.

The equations of motion of an N degree of freedom building with MDs subjected to earthquake can be written as a set of first-order differential equations of the following form \[12\]:

\[
\begin{bmatrix}
\ddot{x}(t) \\
\dot{x}(t) \\
\dot{h}(t)
\end{bmatrix} = g\left[ x(t), \dot{x}(t), h(t), \dot{x}_g(t), t \right]
\] (11)

The previous equation constitutes a set of three coupled non-linear differential equations in the following explicit forms:

\[
\ddot{x}(t) = -M_s^{-1} \left[ C_s \dot{x}(t) + \left( K_s + \alpha \sum_{d=1}^{nl} r_d S R_d k^d_s \right) x(t) + \left( 1 - \alpha \right) \sum_{d=1}^{nl} r_d S R_d k^d_s \Delta_y^d h_d(t) \right] - I \dot{x}_g(t)
\] (12)

\[
\dot{h}_d(t) = \frac{1}{\Delta_y^d} \left[ \gamma r_f^d \dot{x}(t) - \gamma \dot{h}_d(t) \right] \left[ h_d(t) \right]^{\gamma - 1} - \beta r_f^d \dot{x}(t) \left[ h_d(t) \right]^{\gamma}
\] (14)

where \( S R_d \) represents the stiffness ratio of the damper-bracing assembly with the floor of uncontrolled structure; \( \Delta_y^d \) is the yield deformation of the \( d \)th damper; \( \alpha = 0.02, \beta = 0.1, \gamma = 0.9, \zeta = 25 \), they are all model parameters.

5. Numerical analysis

5.1. Dynamic simulation
The 20-storey benchmark building is chosen to be analyzed with mechanical properties given in table 2. In order to understand the characteristic of the two type dampers, simulation models are created and dynamic simulation of passive energy dissipation structures is carried out through the SIMULINK toolbox in MATLAB. According to the analytical model of dampers and system equations introduced previously, the responses of controlled structures are considered as the output datas and hysteresis loops of dampers is the state feedback. The SIMULINK block diagrams for non-linear system with velocity-based dampers are shown in figure 1. Because different types of dampers are comprised of different modules, the system diagrams of hysteresis loops of dampers are not shown here one by one for the limitation of space.

5.2. Parameters of dampers
The additional equivalent damping ratio is 5 percent. The parameters of two type dampers are designed according to the previous section 3. The typical VED with two viscoelastic layers is designed with such parameters: the shear storage modulus is \( 1.4 \times 10^7 \) N/m²; the shear loss modulus is \( 1.54 \times 10^7 \) N/m²; the area and thickness of the VE material are \( 0.5 \times 10^{-2} \) m² and \( 1 \times 10^{-2} \) m, respectively. The work temperature is 25 °C. The initial stiffness of the metallic damper-bracing assembly is \( 2.40 \times 10^7 \) N/m; the second stiffness coefficient is 0.1; the ductility factor is 3; the yield deformation of dampers is taken as 4mm. These dampers are placed on each floor uniformly.

5.3. Calculation process and result analysis
Considering the benchmark 20-storey building, performance evaluation indices are calculated by using the analysis program of dynamic simulation for structures with different dampers as the table 3 and 4 shown.

### Table 2. Parameters of the 20-storey benchmark structure

| Floor | Height (m) | Mass (Kg)   | Stiffness (N/m) | Floor | Height (m) | Mass (Kg)   | Stiffness (N/m) |
|-------|------------|-------------|-----------------|-------|------------|-------------|-----------------|
| 1     | 5.49       | 2.82×10^5  | 1.007×10^8      | 11    | 3.96       | 2.76×10^5  | 1.093×10^8      |
| 2     | 3.96       | 2.76×10^5  | 1.358×10^8      | 12    | 3.96       | 2.76×10^5  | 1.054×10^8      |
| 3     | 3.96       | 2.76×10^5  | 1.338×10^8      | 13    | 3.96       | 2.76×10^5  | 1.036×10^8      |
| 4     | 3.96       | 2.76×10^5  | 1.33×10^8       | 14    | 3.96       | 2.76×10^5  | 9.08×10^7       |
| 5     | 3.96       | 2.76×10^5  | 1.228×10^8      | 15    | 3.96       | 2.76×10^5  | 8.97×10^7       |
| 6     | 3.96       | 2.76×10^5  | 1.248×10^8      | 16    | 3.96       | 2.76×10^5  | 8.82×10^7       |
| 7     | 3.96       | 2.76×10^5  | 1.237×10^8      | 17    | 3.96       | 2.76×10^5  | 7.98×10^7       |
| 8     | 3.96       | 2.76×10^5  | 1.223×10^8      | 18    | 3.96       | 2.76×10^5  | 7.34×10^7       |
| 9     | 3.96       | 2.76×10^5  | 1.208×10^8      | 19    | 3.96       | 2.76×10^5  | 5.98×10^7       |
| 10    | 3.96       | 2.76×10^5  | 1.187×10^8      | 20    | 3.96       | 2.92×10^5  | 4.49×10^7       |
### Table 3. Evaluation indices of 20-storey benchmark structure with VEDs

| Evaluation indices | Factors of earthquake | Values                |
|--------------------|-----------------------|-----------------------|
|                    |                       | El Centro | Hachinohe | Northridge | Kobe | Maximum |
| $J_1$              | 0.5                   | 0.6274    | 0.8733    | 0.5975     | 0.3916 | 0.8763  |
|                    | 1.0                   | 0.6277    | 0.8735    | 0.6173     | 0.3357 |         |
|                    | 1.5                   | 0.6275    | 0.8763    | —          | —     |         |
|                    | 0.5                   | 0.8500    | 0.8181    | 0.8741     | 0.7515 | 0.8741  |
| $J_2$              | 1.0                   | 0.8505    | 0.8185    | 0.8485     | 0.7192 |         |
|                    | 1.5                   | 0.8504    | 0.8184    | —          | —     |         |
|                    | 0.5                   | 0.8392    | 0.8452    | 0.7958     | 0.6036 | 0.8455  |
| $J_3$              | 1.0                   | 0.8394    | 0.8454    | 0.8330     | 0.7458 |         |
|                    | 1.5                   | 0.8394    | 0.8455    | —          | —     |         |
|                    | 0.5                   | 0.6103    | 0.8561    | 0.5224     | 0.4859 | 0.8562  |
| $J_4$              | 1.0                   | 0.6106    | 0.8562    | 0.6196     | 0.4821 |         |
|                    | 1.5                   | 0.6107    | 0.8335    | —          | —     |         |
|                    | 0.5                   | 0.5700    | 0.7576    | 0.5391     | 0.5200 | 0.7579  |
| $J_5$              | 1.0                   | 0.5705    | 0.7578    | 0.7216     | 0.6306 |         |
|                    | 1.5                   | 0.5702    | 0.7579    | —          | —     |         |
|                    | 0.5                   | 0.7011    | 0.7251    | 0.6409     | 0.6688 | 0.7438  |
| $J_6$              | 1.0                   | 0.7013    | 0.7255    | 0.6921     | 0.7438 |         |
|                    | 1.5                   | 0.7013    | 0.7255    | —          | —     |         |

### Table 4. Evaluation indices of 20-storey benchmark structure with MDs

| Evaluation indices | Factors of earthquake | Values                |
|--------------------|-----------------------|-----------------------|
|                    |                       | El Centro | Hachinohe | Northridge | Kobe | Maximum |
| $J_1$              | 0.5                   | 0.9713    | 1.0000    | 1.0002     | 0.9483 | 1.0002  |
|                    | 1.0                   | 0.9714    | 1.0001    | 0.9622     | 0.9509 |         |
|                    | 1.5                   | 0.9712    | 1.0001    | —          | —     |         |
|                    | 0.5                   | 1.0028    | 1.0438    | 0.9984     | 0.9850 | 1.1109  |
| $J_2$              | 1.0                   | 1.0026    | 1.0440    | 0.9793     | 1.1109 |         |
|                    | 1.5                   | 1.0025    | 1.0444    | —          | —     |         |
|                    | 0.5                   | 1.0128    | 1.0886    | 1.0000     | 1.0870 | 1.0891  |
| $J_3$              | 1.0                   | 1.0140    | 1.0890    | 1.0386     | 1.0100 |         |
|                    | 1.5                   | 1.0144    | 1.0891    | —          | —     |         |
|                    | 0.5                   | 0.8100    | 0.9445    | 0.9481     | 0.9705 | 0.9906  |
| $J_4$              | 1.0                   | 0.8102    | 0.9448    | 0.9906     | 0.9790 |         |
|                    | 1.5                   | 0.8106    | 0.9449    | —          | —     |         |
|                    | 0.5                   | 0.8625    | 1.0390    | 0.9640     | 1.0141 | 1.0392  |
| $J_5$              | 1.0                   | 0.8627    | 1.0392    | 1.0336     | 1.0229 |         |
Figure 2 to 7 show the comparisons of evaluation indices between two type dampers subjected to different earthquakes. The dark column represents the value of VED and the light one represents the value of MD. The analyses and these results indicate that VEDs are more effective on controlling the maximum storey-drift angle of buildings for far-field earthquake records. The control effect is about 38 to 66 percent for far-field earthquakes and 12 to 37 percent for near-field earthquakes. As far as the MDs are concerned, the control effect is only about 5 percent. Because the additional stiffness of MD is less than VED’s and the ratio of damper’s stiffness to lateral stiffness of storey is smaller, the control effect on storey-drift angle is limit as the additional equivalent damping ratio is the same for two type dampers. The same control effect with VED requires more additional stiffness for MD. Considering the control effect on maximum level acceleration of the structure, it’s about 23 to 28 percent for VED, but it’s not obvious for MD. Taking the base shear of structure into consideration, the control effect on far-field earthquakes is better than near-field ones for VED and the value is about 17 to 40 percent. However, the base shear of structures with MDs maybe enlarged 10 percent for various records.

As far as normed interstory drift ratio is concerned, the value decrease about 24 to 39 percent for VED subjected to near-field records and 38 to 51 percent to far-field records. For MD, the control effect on near-field records is better than far-field ones and it’s about 10 percent decrease. Considering the normed level acceleration, the value decrease about 34 to 48 percent for VED, and this is better for various records than the control effect on the maximum level acceleration. However, the control effect of MD is influenced by the earthquake records. The installation of MDs may enlarge the normed level acceleration of structures subjected to earthquakes. For normed base shear, the control effect of structures with VED on far-field records is better than near-field ones and it is about 26 to 36 percent. The value is decreased for structure with MD subjected to El Centro earthquake record, but it is enlarged about 1 to 6 percent for other records. In general, the installation of MDs has no influence on

| $J_e$ | 0.5 | 0.8628 | 1.0392 | — | — |
|-------|-----|--------|--------|---|---|
|       | 0.5 | 0.9719 | 1.0430 | 0.9702 | 1.0576 | 1.0576 |
| 1.0   | 0.9730 | 1.0433 | 1.0073 | 1.0390 |
| 1.5   | 0.9733 | 1.0438 | — | — |

Fig. 2 Comparison of $J_1$ for two type dampers

Fig. 3 Comparison of $J_2$ for two type dampers
the normed base shear if the additional stiffness is small and the control effect has relationship with the characteristic of earthquake records.

6. Conclusion
Combining the passive control of structures with the modern control theory, using MATLAB/SIMULINK to establish simulation models of the structures with displacement-based and velocity-based dampers, the responses of 20-storey benchmark structure model were calculated subjected to far- and near-field earthquakes. Control effects are compared between different types of
The placement of VEDs has a significant control effect on all six indices. The control of structures subjected to far-field earthquakes is better than near-field ones. However, the control effect is unstable for different structures with MDs subjected to various earthquakes, because the additional stiffness of dampers have great influence on the main frequency of structures and the frequency are different for various earthquake records. The fundamental frequency of structures almost has no change with addition of VEDs, so the control effect of structural responses is equivalent for various earthquake ground motions.

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