Synthesis and scalarization of hierarchical vector criterion in production management problems

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Abstract. The multicriteria problem has many aspects. The paper is dedicated to one of them, namely: development of the methodology for building and using dynamically changing multi-level vector criteria for optimization of engineering and manufacturing complexes. The approach applied most frequently involves vector to scalar conversion of the optimization criterion. When multi-level criteria are used we have to deal with a system of embedded convolutions forming a hierarchical system of vectors. The current principles of making convolutions for such systems can be implemented in different ways depending on peculiarities of the controlled object and management goals. This study discusses building of hierarchical criteria in mathematical setting up of engineering and manufacturing complex optimization problems. A method of forming and scalarization of a hierarchical vector criterion following a recurrent procedure based on an invariant representing a linear combination of Holder norms of partial criteria is suggested. An example of techniques usage in the development of software for multi-criterial optimization of refinery is given.

1. Introduction
Production management is associated with the vector optimization problem. The orthodox approach according to which the management goal is characterized by one index and conditions – by multiple rigid restrictions prohibiting any violations falls short in real life today [1]. Improvement of the economic status and the level of innovative development of enterprises might be implemented only based on application of a multi-criterial vector optimization and building a systemic-hierarchical model comprising about ten integral groups of indices [2].

Most available methods, especially when we are talking about optimization, involve vector criterion scalarization. At first, all input partial criteria are reduced to a comparable dimensionless form, normalized relative to some reference or plan values. Then, the normalized values, taking into account weight coefficients showing their importance, are convoluted according to some formula and the result obtained is regarded as a generalized criterion of optimization. Usually, convolutions based on a weighted average power are used. In practice, the goal programming method is the most common. It consists in usage of convolution that is some measure of a distance from the vector estimate under analysis to some target value.

An important aspect of the scalarization problem is criteria ranking, which can be performed at least by one of the two methods. The first method implies direct priority assignment; the second – ranking based on their pairwise comparison. In the second case, priorities are later found as eigenvalues of the matrix of pairwise comparisons. Assignment of priorities is a procedure of expertise
nature that is difficult to formalize. Most papers in this field are dedicated to development of the fuzzy set approach [3], [4]. In this approach, subtests are represented in one unified form independent on the essential nature of indices. For such representation, the fuzzy set theory offers the apparatus of desirability functions, which usage allows making no principal difference between criteria and the set of constraints in the mathematical statement of optimization problems, which is quite similar to the situation occurring when penalty function methods are used.

The disadvantage of the fuzzy set method, in the context of the problem under discussion, is that it is more suitable for analysis than for synthesis of optimal solution. The generalized criterion obtained by this method is not integrated into the optimization problem directly – it can be used only to evaluate offered solutions.

Another aspect of the problem is criteria multi-levelness. The task of building and scalarization of multi-level hierarchical criteria arises in connection with the necessity of splitting numerous production indices into groups and subgroups. It is impossible to manage the priorities of a great number of indices without systematizing them. Production indices are classified into technological, economic, organizational, and ecological. Each of these groups is divided into subgroups. As a result, a multi-level classification of criteria reflected as a tree-like graph, its leaves being primary indices, is produced.

Multi-level criteria are formed and scalarized using the MCHP methodology. Within the frames of this methodology, Voronin has suggested a method of multi-level criterion synthesis by forming a system of embedded convolutions [5]. Each convolution is formed from some set of lower-level convolutions subject to priorities of the latter and is itself one of components of a higher-level convolution. The generalized criterion is nothing but a convolution obtained for the highest level. A generalized criterion convolution is presumably computed using some recurrent procedure that represents a sequence of convolutions of vector criteria of each level starting from the lowest.

This method allows multiple implementations. If the root convolution of a multi-level criterion is not represented analytically, it is impossible to apply efficient classical techniques to solve the optimization problem. This explains the fact that multi-goal optimization problems are often considered in connection with usage of various randomized heuristics [6]. The most frequently used algorithms are evolution algorithms [7], [8]. Paper [9] describes a multi-goal optimization system utilizing interactive Pareto domain visualization techniques. The task of a computer is to represent information in a form convenient for the user to make his choice based on his experience and intuition. The heuristic solution search algorithm is included into the system as a supplementary option only.

In our case, it is necessary to take into account that the production process optimization problems are described by models handling sometimes thousands of variables. In most cases, such problems are solved with the help of linear programming. Hence, the hierarchical criterion intended for use in production optimization models must feature capability of being integrated into a linear programming problem.

The third aspect is related to automation issues. Currently, production processes management is carried out using domain-driving design systems. These are complexes including, in addition to programs realizing optimization algorithms, tools for visualization of conditions and computation results, as well as model building means. The concept is that all functions that can be formalized are assigned to a computer. According to this concept, the user states a problem in terms of object domain categories. In particular, in handling criteria, an informal part pertaining to expert evaluation must be reserved for the user. Processing of expert evaluations and hierarchical criterion building must be done by a computer. That is why the synthesis and scalarization of criteria must be reduced to a unified procedure.

This study is aimed at the development of techniques for the formation and usage of hierarchical vector criteria for the production process management purposes on the basis of linear optimization models. The techniques must include 1) building a hierarchical vector criterion, 2) formation of its convolution, 3) integration into a linear optimization problem.
2. Materials and Methods

Without loss of generality, it can be assumed that target indices are specified by their lower bounds. Let us consider the multi-criteria optimization problem looking like as follows:

\[
\Phi(p, \delta) \rightarrow \min,
\]

\[
\begin{align*}
Ax + g\delta^T &\geq g, \\
Bx &\leq b, \\
x, \delta &\geq 0,
\end{align*}
\]

where \( x \) is the vector of variables; \( g \) is the vector of target values of indices; \( A \) is the matrix of linear conversion of the vector of variables into the vector of indices; \( B \) is the matrix of production and technological parameters; \( b \) is the vector of production and technological constraints; \( \delta \) is the vector of relative deviations of estimates of indices from target values; \( p \) is the vector of indices’ priorities; \( \Phi(p, \delta) \) is the vector criterion convolution.

The convolution determines the measure of approximation of indices to target values. Traditionally, Holder norms are used as such measure, as was already mentioned. Norms of the second and higher orders are inacceptable in this case because they introduce non-linearity into a problem. When norms of the first order are used, a situation might occur when the results based on some components of a criterion are achieved at the expense of diminishment of results based on some other components. The best option in such case would be to use a convex linear combination of norms of the first and infinite orders.

When a convolution is formed directly based on the values of primary indices without taking into account the hierarchy, calculation is carried out according to formula:

\[
\Phi(p, \delta) = \alpha \sum_{i \in C} p_i \delta_i + (1-\alpha) \max \left\{ p_i \delta_i, i \in C \right\},
\]

where \( \alpha \) is the adjustable parameter; \( C \) is the set of controlled primary indices.

The action of this criterion is similar to that of square. Though being non-linear itself, it is nevertheless introduced into a problem in such a way that the problem remains linear:

\[
\begin{align*}
\Delta &\rightarrow \min, \\
Ax + g\delta^T &\geq g, \\
Bx &\leq b, \\
p\delta^T - I\delta &\leq 0, \\
\alpha(p, \delta) + (1-\alpha)\hat{\delta} - \Delta &\leq 0, \\
x, \delta &\geq 0,
\end{align*}
\]

where \( \hat{\delta} \) is the variable equal to the maximum coordinate of vector \( \delta \); \( \Delta \) is the sole and simultaneously root convolution of the criterion; \( I \) is the vector of units.

In this instance, the hierarchical criterion is formed when target indices included in the problem are grouped and subgrouped at several levels. The result is a treegraph with \( L \) number of levels. Its leaves are components of vector \( \delta \). Intermediate nodes of the graph are criteria corresponding to groups and subgroups.

When a multi-level criterion is used, the target function of a problem is determined through a system of intermediate convolutions calculated by formula:

\[
\Delta_v = \alpha_v \sum_{i \in C_v} p_n^{-1} \Delta_n^{-1} + (1-\alpha_v) \max \left\{ p_n^{-1} \Delta_n^{-1}, n \in C_v \right\},
\]
where $\Delta'_v$ is the $v$-th criterion of the $\ell$-th level; $C'_v$ is the set of criteria of the $(\ell-1)$-th level involved in formation of the $v$-th criterion of the $\ell$-th level; $p_n^{v-1}$ is the priority of the $n$-th criterion of the $(\ell-1)$-th level; $\alpha'_v \in [0,1]$ is the adjustable parameter of the convolution; $\Delta^0 = \delta$.

As the target function of a goal, a convolution corresponding to the root node of a graph is used. The end and intermediate convolutions are determined through a system of additional equations and inequations introduced into the matrix of conditions of the initial problem:

$$
\begin{cases}
-\Delta'_v + \alpha'_v \sum_{n \in C'_v} p_n^{v-1} \Delta_n^{v-1} + (1-\alpha'_v) \cdot \Delta_n^{v-1} = 0, \quad v \in C', \quad \ell = 1,2,...,L, \\
-\Delta_n^{v-1} + p_n^{v-1} \Delta_n^{v-1} \leq 0, \quad n \in C'_v, \quad v \in C', \quad \ell = 1,2,...,L,
\end{cases}
$$

where $C'$ is the set of criteria of the $\ell$-th level; $\Delta_n^{v-1} = \max\{p_n^{v-1} \Delta_n^{v-1}, n \in C'_v\}$.

Priorities, regardless of the method used to obtain them, are normalized in each section of each level so that their sum would be equal to one. This is essential in forming a multi-level criterion. In each section, the priorities of component criteria are determined only relative to each other. The priority of the section itself is formed in the group of criteria of the next level.

The invariance of the criteria formation procedure at each level is reflected on the structure of the block of additional constraints. Criterion $\Delta^{v+1}_v$ is determined through underlying criteria by adding the following block of conditions

$$
\begin{pmatrix}
-1 & (1-\alpha'_v) & w'_v & 0 \\
0 & -I'_v & P'_v & 0 \\
0 & 0 & E'_v & D'_v
\end{pmatrix}
\begin{pmatrix}
\Delta^{(v+1)}_v \\
\Delta'_v \\
\Delta'_n
\end{pmatrix}
\leq
\begin{pmatrix}
0 \\
0 \\
0
\end{pmatrix},
$$

where $w'_v = (\alpha'_v^{v+1} p_n^{v+1}, n \in C^{v+1}_v)$ is the vector of weight coefficients; $I'_v$ is the column of units; $P'_v$ is the diagonal matrix which elements are priorities $p_n^{v+1}, n \in C^{v+1}_v$; $\Delta'_v = (\Delta'_n, n \in C'_v)^T$ is the vector of criteria forming convolution $\Delta^{(v+1)}_v$; $\Delta_n^{v-1} = (\Delta_n^{v-1}, n \in C'_v)^T$ are components of convolutions involved in the formation of vector $\Delta'_v$; $E'_v$ and $D'_v$ are block-diagonal matrices.

The diagonal blocks of matrix $E'_v$ are columns $(1,0)^T$. This matrix links the current matrix with daughter matrices that are input to form criteria of the underlying level and are diagonal blocks of matrix $D'_v$. By denoting elements of set $C'_v$ by sequential numbers $n = 1, 2, ..., \tilde{v}$, where $\tilde{v} = |C'_v|$, we can write

$$
D'_v = \begin{pmatrix}
M_{1}^{v-1} & 0 & ... & 0 \\
0 & M_2^{v-1} & ... & 0 \\
... & ... & ... & ... \\
0 & 0 & ... & M_{\tilde{v}}^{v-1}
\end{pmatrix},
$$

$$
M_{n}^{v-1} = \begin{pmatrix}
-1 & (1-\alpha_n^{v-1}) & w_n^{v-1} & 0 \\
0 & -I_n^{v-1} & P_n^{v-1} & 0 \\
0 & 0 & E_n^{v-1} & D_n^{v-1}
\end{pmatrix}.
$$
As we see, the structure of daughter matrices fully matches the structure of the parent matrix. Matrices $\mathbf{M}$ represent a structural matrix invariant and are a kind of building blocks. The hierarchy of such blocks repeats the hierarchy of the criterion (figure 1). This makes it possible to use a simple unified recurrent procedure in the process of forming a matrix of conditions of a multi-criterial problem.

![Figure 1. Representation of a hierarchical criterion as a matrix.](image)

3. Results and Discussion

The techniques described has been used by the author in the development of software for multi-criterial optimization of refinery [10], [11].

Refinery is a totality of continuous technological processes organized as a single production line. This explains stringent requirements to high and stable performance of business units. At the same time, multivariance and variability are inherent in refinery. Same commodity yield can be achieved with the help of different process flow designs.

Managerial decisions have to be taken in the environment of continuously changing external and internal factors. External factors include, first of all, variable composition of raw material and changing market situation. Internal factors are changes taking place is running process equipment, routine preventing maintenance and emergency repairs. Besides, contemporary refineries experience permanently undergoing reconstruction and modernization. So, in a refinery, changes take place all the time that require fast decision-making, while the objectives and criteria might be absolutely different depending on the current situation.

The software is a domain-specific optimization system designed to work out managerial decisions in given circumstances. It calculates the optimal operation modes of plants, formulations to produce

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commodity mixes, input-output table of the enterprise, and also solves a number of other interrelated problems.

The software consists of three parts: a shell, a data base, and a problem solver (figure 2). The shell lets the user work with the software or, in other words, provides the user interface – a system of rules and tools regulating software interaction with the user. It is a totality of interrelated screen forms containing tools to 1) describe an object in terms of the object domain categories, 2) state a problem, 3) analyze the results, 4) create reports.

![Diagram of software classes](image)

**Figure 2.** Diagram of software classes.

Class DBASE contains a complete set of methods required to work with the database within this application and to access external data. The client side of the application interacts with the database not directly, but through a system of SQL enquiries that are used to view, analyze, and modify data.

Class SOLVER is a solver built on the basis of LINDO. The class realizes the following functions: 1) receives input data from the client side of the application; 2) formulates the optimization problem in a regulated form; 3) solves the optimization problem; 4) furnishes the solution outcome to the client side of the application.

SOLVER is not directly connected with the client-side program. It is not connected in the sense that it does not depend on specifics of its realization. This approach is explained by the fact that the most variable part of the application is everything related to the user interface, which, in turn, is closely connected with data storage and usage arrangements. The interface between the solver and the database is provided by block INTER.

The SOLVER object model is shown on figure 3. The attributes and methods of classes are used to create fragments of the general model. Class CONMATRIX serves to form blocks that are matrix representations of nodal convolutions of a generalized hierarchical criterion and include them into a problem. It receives the convolution node coordinates and presents the result in the so-called MPS-format.
The MPS-format has been chosen because it 1) is a de facto standard for input files of mathematical programming packages to the present day; 2) allows obtaining easily the file of the general problem from files containing separate blocks of the matrix of conditions.

It is a common fact that an MPS-file consists of a totality of lines grouped in sections. Each section and each fragment thereof can be formed independently. Fragments corresponding to matrices of nodal convolutions are linked with fragments corresponding to matrices of daughter convolutions introduced to form lower-level criteria with the help of a single, within a given model, system of coding columns and lines. Thanks to this, the file of general problem conditions becomes a simple concatenation of sections of individual MPS-files.

As an example, figure 4 shows a block fragment created by the CONMATRIX module, which had to be introduced into the matrix of problem conditions to let the model take into account just one aggregated index. To the right of the window, this fragment is represented in the usual form – as columns and lines; and to the left of the window – in the MPS-format.

Figure 3. SOLVER object model.
As we see, employment of hierarchical vector criteria might be connected with a significant increase of the problem dimension. Therefore, the expediency of including some groups of criteria or other into a model is directly dependent on the capabilities of software utilized.

4. Conclusion
A technique of synthesis of hierarchical vector criteria in production process optimization problems is suggested. A recurrent procedure of hierarchical vector criterion scalarization and a method of its integration into the matrix of linear optimization problem conditions have been developed. Program realization of the technique is discussed.

The approach offered allows reducing formation of the vector criterion convolution and adjusting variable parameters to a unified procedure. Tuning of parameters in each compartment of each level is determined separately, independently on others, allowing usage of different methods to estimate and rank input and intermediate criteria.

Determination of the root convolution of hierarchical criterion does not require to perform the sequence of calculations of intermediate convolutions from lower levels to the highest level. The root convolution is determined with the help of a system of linear relations input directly into the matrix of optimization problem conditions. This makes it possible to apply efficient optimization methods, specifically – simplex methods, to address the vector optimization problems.

The hierarchical vector criterion is integrated into the matrix of conditions as a system of embedded into each other structurally similar matrices – matrix invariants. It allows automation of the process of building and utilizing hierarchical criteria employed for optimal production planning.

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