CLUSTER INTERPRETATION OF PROPERTIES OF
ALTERNATING PARITY BANDS IN HEAVY NUCLEI

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Abstract

The properties of the states of the alternating parity bands in actinides, Ba, Ce and Nd isotopes are analyzed within a cluster model. The model is based on the assumption that cluster type shapes are produced by the collective motion of the nuclear system in the mass asymmetry coordinate. The calculated spin dependences of the parity splitting and of the electric multipole transition moments are in agreement with the experimental data.

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Key words: Cluster states; dinuclear system; parity splitting; multipole moments; actinides
I. INTRODUCTION

The low-lying negative parity states observed in actinides and in heaviest known Ba, Ce, Nd and Sm isotopes are definitely related to reflection–asymmetric shapes \[12\]. There are several approaches to treat collective motion leading to reflection–asymmetric deformations. One of them is based on the concept of a nuclear mean field which has a static octupole deformation or is characterized by large amplitudes of reflection–asymmetric vibrations around the equilibrium shape \[3\]. In this approach the parity splitting is explained by octupole deformation. Another approach \[7,8,10\] is based on the assumption that the reflection–asymmetric shape is a consequence of alpha–clustering in nuclei \[11,12,13\]. In the algebraic model \[7,8,10\] the corresponding wave functions of the ground and excited states consist of components without and with dipole bosons (in addition to the quadrupole bosons), which are related to mononucleus and alpha–cluster components, respectively. The variant of algebraic model including the octupole bosons in addition to the dipole bosons has been applied in \[14,15\] to the description of the low-lying negative parity states in actinides. In \[16,17,18,19\] a cluster configuration with a lighter cluster heavier than \(^4\)He was used in order to describe the properties of the low–lying positive and negative parity states. In both models \[7,8,10\] and \[16,17,18,19\] the relative distance between the centers of mass of clusters at fixed mass asymmetry is the main collective coordinate for the description of the alternating parity bands.

Nuclear cluster effects are mostly pronounced in the light even–even \(N = Z\) nuclei with alpha–particle as the natural building block. There is a nice relationship between alpha–cluster description and deformed shell model \[11\]. It is known from Nilsson–Strutinsky type calculations for light nuclei that nuclear configurations corresponding to the minima of the potential energy contain particular symmetries which are related to certain cluster structures \[20,21,22\]. By using antisymmetrized molecular dynamics approach \[23,24\], the formation and dissolution of clusters in light nuclei, like \(^{20}\)Ne and \(^{24}\)Mg, are described. The idea of clusterization applied to heavy nuclei does not contradict the mean field approach.
The coexistence of the clustering and of the mean field aspects is a unique feature of nuclear many body system. The problem of existence of a cluster structure in a ground state of heavy nuclei has attracted much attention, especially, because of the experimentally observed cluster decay [25]. The available experimental and theoretical results provide the evidence for existence of fission modes created by the clustering of the fissioning nuclei [26]. Indications of clusterization of highly deformed nuclei are demonstrated in [27,28].

The aim of the present paper is a development of the cluster–type model which provides not only a qualitative but also a quantitative explanation of the properties of alternating parity bands. The description of the excitation spectra, $E\lambda$–transition probabilities ($\lambda=1,2,3$) and the angular momentum dependence of the parity splitting [29,30] are the main subjects of this paper. Our model is based on the assumption that the reflection–asymmetric shapes are produced by the collective motion of the nuclear system in the mass asymmetry coordinate [31]. The values of the odd multipolarity transitional moments (dipole and octupole) are strongly correlated with the mass asymmetry deformation of nucleus. In general, the value of the quadrupole moment is related to the degree of the quadrupole correlations (deformation) in nucleus. However, the collective motion in the mass asymmetry degree of freedom simultaneously creates a deformation with even and odd multipolarities. Therefore, the calculations of $E\lambda$–transition moments are of interest in the proposed model. The single particle degrees of freedom are not taken explicitly into consideration since our aim is to show that the suggested cluster model gives a good quantitative explanation of the observed properties of the low–lying negative parity states. If it is so, this model can serve as a good ground for development of an extended model with additional degrees of freedom.

It should be noted that the first results of calculations of the alternating parity spectra for a few actinides within the cluster model are already presented in the Letter [31]. Besides Ra, Th and U isotopes, in the present paper we present the results of calculations of the energies of alternating parity bands in $^{240,242}$Pu, $^{144,146,148}$Ba, $^{146,148}$Ce, and $^{146,148}$Nd. The electromagnetic transitions are described in this paper with the cluster model for many nuclei and the spin dependence of the intrinsic quadrupole transition moment is predicted for $^{238}$U.
Simple analytical expressions obtained for the parity splitting and the spectra of alternating parity bands are useful for the estimations. The dependence of alpha–clusterization in actinides on the angular momentum is shown for the first time.

**II. MODEL**

A. Hamiltonian in mass asymmetry coordinate

Dinuclear systems consisting of a heavy cluster $A_1$ and a light cluster $A_2$ were first introduced to explain data on deep inelastic and fusion reactions with heavy ions [32,33,34]. The mass asymmetry coordinate $\eta$, defined as $\eta = (A_1 - A_2) / (A_1 + A_2)$, ($|\eta| = 1$ if $A_2 = 0$ and $A_1 = A$), which describes a partition of nucleons between the nuclei forming the dinuclear system and the distance $R$ between the centers of clusters are used as relevant collective variables [35]. The wave function in $\eta$ can be thought as a superposition of different cluster–type configurations including the mononucleus configuration with $|\eta| = 1$, which are realized with certain probabilities. The relative contribution of each cluster component in the total wave function is determined by the collective Hamiltonian described below. Our calculations have shown that in the considered cases the dinuclear configuration with an alpha cluster ($\eta = \eta_\alpha$) has a potential energy which is close or even smaller than the energy of the mononucleus at $|\eta| = 1$ [28,31]. Since the energies of configurations with a light cluster heavier than an $\alpha$–particle increase rapidly with decreasing $|\eta|$, we restrict our investigations to configurations with light clusters not heavier than Li ($\eta = \eta_{Li}$), i.e. to cluster configurations near $|\eta| = 1$.

The Hamiltonian describing the dynamics in $\eta$ has the following form

$$H = -\frac{\hbar^2}{2B_\eta} \frac{d^2}{d\eta^2} + U(\eta, I),$$

(1)

where $B_\eta$ is the effective mass and $U(\eta, I)$ is the potential. In order to calculate the dependence of parity splitting on the angular momentum and the electric dipole, quadrupole and octupole transition moments we search for solutions of the stationary Schrödinger equation describing the dynamics in $\eta$:
\[ H \Psi_n(\eta, I) = E_n(I) \Psi_n(\eta, I). \] (2)

The eigenfunctions \( \Psi_n \) of this Hamiltonian have a well defined parity with respect to the reflection \( \eta \rightarrow -\eta \). Before we come to the results of Eq. (2), we discuss the calculation of the potential \( U(\eta, I) \), the mass parameter \( B_\eta \) and the moments of inertia \( \Im(\eta) \) appearing in \( H \).

**B. Potential energy**

The potential \( U(\eta, I) \) in Eq. (1) is taken as a dinuclear potential energy for \(|\eta|<1\)

\[ U(R, \eta, I) = B_1(\eta) + B_2(\eta) - B + V(R = R_m, \eta, I). \] (3)

Here, the internuclear distance \( R = R_m \) is the touching distance between the clusters and is set to be equal to the value corresponding to the minimum of the potential in \( R \) for a given \( \eta \). The quantities \( B_1 \) and \( B_2 \) (which are negative) are the experimental binding energies of the clusters forming the dinuclear system at a given mass asymmetry \( \eta \), and \( B \) is the binding energy of the mononucleus. The quantity \( V(R, \eta, I) \) in (3) is the nucleus-nucleus interaction potential. It is given as

\[ V(R, \eta, I) = V_{coul}(R, \eta) + V_N(R, \eta) + V_{rot}(R, \eta, I) \] (4)

with the Coulomb potential \( V_{coul} \), the centrifugal potential \( V_{rot} = \hbar^2 I(I+1)/(2\Im(\eta, R)) \) and the nuclear interaction \( V_N \). In the realization of the cluster model developed in this paper, where overlap of clusters is much smaller than in the model of [16], the choice of the relevant cluster configuration follows the minimum of the total potential energy of the system with a cluster-cluster interaction taken additionally into consideration. As the result we describe the same nuclear properties as in [10] with configurations of clusters having larger mass asymmetry and a smaller overlap.

The potential \( V(R, \eta, I) \) and the moment of inertia \( \Im(\eta, R) \) are calculated for special cluster configurations only, namely for the mononucleus (\(|\eta|=1\)) and for the two cluster
configurations with the $\alpha$- and Li - clusters as light clusters, respectively. These calculated points are used later to interpolate the potential smoothly by a polynomial. The energies of the Li-cluster configurations are about 15 MeV larger than the binding energies of the mononuclei considered. Therefore, for small excitations only oscillations in $\eta$ are of interest which lie in the vicinity of $|\eta|=1$, i.e. only cluster configurations up to Li - clusters need to be considered. The potential $V_N$ is obtained with a double folding procedure with the ground state nuclear densities of the clusters. Antisymmetrization between the nucleons belonging to different clusters is regarded by a density dependence of the nucleon–nucleon force which gives a repulsive core in the cluster–cluster interaction potential. Details of the calculation of $V_N$ are given in [36]. The parameters of the nucleon-nucleon interaction are fixed in nuclear structure calculations [37]. Other details are presented in [31].

Our calculations show that the potential energy has a minimum at $|\eta|=\eta_\alpha$ in $^{218,220,222,224,226}$Ra and $^{222,224,226}$Th isotopes. In order to demonstrate the dependence of the potential on the neutron number, we present in Fig. 1 calculated values of $U(\eta_\alpha, I = 0) \equiv U(\eta_\alpha)$ of configurations with an $\alpha$ - cluster taking the long chain of Ba isotopes as an example. In the neutron deficient isotopes $U(\eta_\alpha)$ is smaller than zero and an $\alpha$-clusterization is more likely. When the neutron number approaches the magic value of 82, the nucleus becomes stiffer with respect to vibrations in $\eta$ and $U(\eta_\alpha)$ is larger than zero. The appearance of two neutrons above shell closure is in favor for an $\alpha$ - clusterization. In this case $U(\eta_\alpha)$ drops much and again becomes smaller than zero. Further addition of neutrons increases the nuclear stiffness with respect to $\eta$ vibrations.

C. Moments of Inertia

The calculation of the moment of inertia $\Im(\eta) = \Im(\eta, R_m)$ needed to determine the potential energy at $I \neq 0$ has been described in [31]. For completeness, we repeat in this subsection the most important information. As was shown in [28], the highly deformed states are well described as cluster systems and their moments of inertia are about 85% of
the rigid-body limit. Following this, we assume that the moment of inertia of the cluster configurations with $\alpha$ and Li as light clusters can be expressed as

$$\mathfrak{I}(\eta) = c_1(\mathfrak{I}_1^r + \mathfrak{I}_2^r + m_0 A_1 A_2 R_m^2).$$

(5)

Here, $\mathfrak{I}_i^r, (i = 1, 2)$ are the rigid body moments of inertia for the clusters of the dinuclear system, $c_1 = 0.85$ \cite{28,31} for all considered nuclei and $m_0$ is the nucleon mass.

It should be noted that the angular momentum is treated in this paper as the sum of the angular momentum of the collective rotation of the heavy cluster and of the orbital momentum of the relative motion of the two clusters. Single particle effects, like alignment of the single particle angular momentum in the heavy cluster, are presently disregarded.

For $|\eta| = 1$, the value of the moment of inertia is not known from the data because the experimental moment of inertia is a mean value between the moment of inertia of the mononucleus ($|\eta| = 1$) and the ones of the cluster configurations arising due to the oscillations in $\eta$. We assume that

$$\mathfrak{I}(|\eta| = 1) = c_2 \mathfrak{I}^r(|\eta| = 1),$$

(6)

where $\mathfrak{I}^r$ is the rigid body moment of inertia of the mononucleus with $A$ nucleons calculated with deformation parameters from \cite{38} and $c_2$ is a scaling parameter which is fixed by the energy of the first $2^+$ or other positive parity state, for example $6^+$. The chosen values of $c_2$ vary in the interval $0.1 < c_2 < 0.3$. So, in our calculations there is a free parameter $c_2$. However, this parameter is used to describe the rotational energies averaged over the parity and not the parity splitting studied in this paper.

**D. Mass parameter**

The method of the calculation of the inertia coefficient $B_\eta$ used in this paper is given in \cite{39}. Our calculations show that $B_\eta$ is a smooth function of the mass number $A$. As a consequence, we take nearly the same value of $B_\eta = 20 \times 10^4 m_0 \text{ fm}^2$ for almost all considered
actinide nuclei with a variation of 10%. However, for $^{222}\text{Th}$ and $^{220,222}\text{Ra}$ we varied $B_\eta$ in the range $B_\eta = (10-20) \times 10^4 m_0$ fm$^2$ to obtain the correct value of $E_0(I = 0)$. These variations of $B_\eta$ lead to better results for light Ra isotopes than those in [31], where the obtained values of the parity splitting at the beginning of the alternating parity band are smaller than the experimental ones. Using a smooth mass dependence of $B_\eta$ [39] we get $B_\eta = 4.5 \times 10^4 m_0$ fm$^2$ in the Ba, Ce and Nd region. However, better results we obtain for $B_\eta = 3 \times 10^4 m_0$ fm$^2$.

For very asymmetric dinuclear systems, we can use simple analytical expressions to establish a connection between the relative distance and mass asymmetry coordinates on one side and the multipole expansion coefficients $\beta_2$ and $\beta_3$ on the other side [28]

$$
\beta_2 = \sqrt{\frac{5}{4\pi}} \frac{\eta}{3} \left(1 - \eta^2\right) \frac{R^2}{R_0^2},
$$
$$
\beta_3 = \sqrt{\frac{7}{4\pi}} \frac{\eta(1 - \eta^2)}{3} \frac{R^3}{R_0^3}.
$$

Here, $R_0$ is the spherical equivalent radius of the corresponding compound nucleus. One finds

$$
\frac{d\beta_3}{d\eta} = \frac{\sqrt{7\pi}}{12} \left[(1 + \eta)^{1/3} + (1 - \eta)^{1/3}\right]^3 \times \left[(1 - 3\eta^2) + \eta(1 - \eta^2) \left(\frac{(1 + \eta)^{-2/3} - (1 - \eta)^{-2/3}}{(1 + \eta)^{1/3} + (1 - \eta)^{1/3}}\right)\right].
$$

In the actinide region for an $\alpha$-particle configuration, $\eta \approx 0.96$ and $(d\beta_3/d\eta)^2 \approx 11.25$. Then the mass parameters for $\beta_3$ and $\eta$-variables are related as

$$
B_\eta \approx (d\beta_3/d\eta)^2 B_{\beta_3}.
$$

If we take the value of $B_{\beta_3} = 200\hbar^2$ MeV$^{-1}$ known from the literature [4], then $B_\eta \approx 9.3 \times 10^4 m_0$ fm$^2$. This value is compatible with the one used in our calculations.

**III. INTRINSIC ELECTRIC MULTIPOLe MOMENTS**

Solving the eigenvalue equation (2), we obtain the wave functions of the positive and negative parity states for different values of the quantum number $I$ of angular momentum.
These wave functions are used then to calculate transition matrix elements of the electric multipole operators by integration over $\eta$. The electric multipole operators for a system of a dinuclear shape have been calculated [28] by using the following expression

$$Q_{\lambda \mu} = \sqrt{\frac{16\pi}{2\lambda + 1}} \int \rho^Z(r) r^\lambda Y_{\lambda \mu}(\Omega) d\tau. \quad (10)$$

For slightly overlapping clusters when the intercluster distance $R_m$ is about or larger than the sum of the radii of clusters $(R_1 + R_2)$, the nuclear charge density $\rho^Z$ can be taken as a sum of the cluster charge densities

$$\rho^Z(r) = \rho^Z_1(r) + \rho^Z_2(r). \quad (11)$$

Using (11) and assuming axial symmetry of the nuclear shape, we obtain [28] the following expressions for the intrinsic electric multipole moments

$$Q_{10} = 2D_{10} = e \frac{A}{2} (1 - \eta^2) R_m \left( \frac{Z_1}{A_1} - \frac{Z_2}{A_2} \right), \quad (12)$$

$$Q_{20} = e \frac{A}{4} (1 - \eta^2) R_m^2 \left( (1 - \eta) \frac{Z_1}{A_1} + (1 + \eta) \frac{Z_2}{A_2} \right) + Q_{20}(1) + Q_{20}(2), \quad (13)$$

$$Q_{30} = e \frac{A}{8} (1 - \eta^2) R_m^3 \left( (1 - \eta)^2 \frac{Z_1}{A_1} - (1 + \eta)^2 \frac{Z_2}{A_2} \right) + \frac{3}{2} R_m ((1 - \eta)^2 Q_{20}(1) - (1 + \eta)^2 Q_{20}(2)), \quad (14)$$

where the charge quadrupole moments of clusters $Q_{20}(i) \ (i = 1, 2)$ are calculated with respect to their centers of mass. Effective charges for electric dipole and octupole transitions are used in our calculations in order to take the coupling of the mass–asymmetry mode to the higher–lying giant dipole and octupole excitations [40] effectively into account, which are not present in the model.

The charge to mass ratios $Z_1/A_1$ and $Z_2/A_2$ are functions of $\eta$. For instance, for $|\eta|=1$ (mononucleus) this ratio takes the values 0.3–0.4 for the nuclei considered in the paper. For the $\alpha$–particle this ratio is equal to 0.5. The results for the electric dipole moment are sensitive to the dependence of $Z/A$ on $\eta$. In the calculations we parameterize the $Z_i/A_i$ ratio in the following way. For $\eta_\alpha < |\eta| \leq 1$, the ratio $Z_2/A_2$ for the light cluster takes the same value as for the mononucleus. For a smaller value of $|\eta|$, we set it equal to 0.5 as for the $\alpha$–cluster.
IV. RESULTS OF CALCULATIONS AND DISCUSSION

A. Calculation procedure

As was mentioned in Sect. II.B, our consideration can be restricted to cluster configurations near $|\eta|=1$. Then it is convenient to substitute the coordinate $\eta$ by the following variable

$$x = \begin{cases} 
\eta - 1 & \text{if } \eta > 0, \\
\eta + 1 & \text{if } \eta \leq 0,
\end{cases}$$

and to use the following smooth parameterization

$$U(x, I) = \sum_{k=0}^{4} a_{2k}(I)x^{2k}$$

of the potential $U(\eta, I)$ from Eq. (3). This formula contains five parameters $a_{2k}(I)$. If a minimum of the potential is located at $|x|=x_\alpha$ four parameters are determined by the experimental ground state energy, potential energies $U(x, I)$ for $x = x_\alpha$, $x = x_{Li}$ and by the requirement that the potential has a minimum at $|x|=x_\alpha$. The fifth parameter $a_8$ is necessary to avoid a fall-off of the potential for $|x| \geq x_{Li}$ because of the negative value of $a_6$ needed to describe correctly $U(x_{Li}, I)$. We take the minimal necessary positive value of $a_8$ to guarantee an increase of $U$ for $|x| > x_{Li}$. The ground state energy is obtained by solving the Schrödinger equation. Since the ground state wave function is distributed over $x$, the potential energy at $x = 0$ is not equal to the experimental binding energy of the mononucleus. To reach the correct value of the ground state energy $E_0(I = 0) = 0$, we can vary the potential $U(x = 0, I = 0)$. In the majority of cases this procedure leads to the value of $U(x = 0, I = 0)$ close to $E_0(I = 0)$. The variation of $B_\eta$ is also done in the case of light Ra isotopes to obtain $E_0(I = 0) = 0$. Besides the barrier height, which determines the stiffness of the potential well at $x=x_\alpha$, the ground state energy $E_0(I = 0)$ of Eq. (2) depends also on the frequency of oscillations in $x$. This frequency is ruled by the value of the inertia coefficient $B_\eta$. If the minimum is located at $x = 0$ ($|\eta|=1$), only three parameters $a_0$, $a_2$...
and $a_4$ in Eq. (15) are necessary. Potentials with other parameterizations show almost no difference in the description of the parity splitting in the majority of considered nuclei.

**B. Parity splitting**

With Eq. (2) we first calculated the parity splitting for the isotopes of Ra, Th, U, Pu, Ba, Ce and Nd for different values of the angular momentum $I$. The results of calculations are shown in Tables 1-5. As is seen from the Tables, they agree well with the experimental data [41,42,43,44,45,46,47,48,49,52]. The largest deviations of the calculated values from the experimental ones are found at low spins in some of the considered nuclei. A good description of the experimental data, especially of the variation of the parity splitting with $A$ at low $I$ and of the value of the critical angular momentum at which the parity splitting disappears, means that the dependence of the potential energy on $\eta$ and $I$ for the considered nuclei is described correctly by the proposed cluster model. The used value of the inertia coefficient $B_\eta$ is also important.

Of course, other effects related to degrees of freedom, which are not included in the model, like the alignment of the single particle momenta or interaction with other negative parity bands with different $K$ quantum number can contribute as well. However, a general agreement between the experimental data and the results of calculations shows that the simple cluster model used in this paper gives a firm ground for the consideration of the alternating parity bands.

In the considered nuclei the ground state energy level lies near the top of the barrier in $\eta$, if exists, and the weight of the $\alpha$—cluster configuration (Fig. 2) estimated as that contribution to the norm of the wave function which is located at $|\eta| \leq \eta_\alpha$ is about $5 \times 10^{-2}$ for $^{226}$Ra, which is close to the calculated spectroscopic factor [25]. This means that our model is in qualitative agreement with the known $\alpha$—decay widths of the nuclei considered.

The spectra of those considered nuclei whose potential energy has a minimum at the alpha cluster configuration can be well approximated by the following analytical expression
\[ E(I) = \frac{\hbar^2}{2J(I)} I[I + 1], \text{ if } I \text{ is even,} \]
\[ E(I) = \frac{\hbar^2}{2J(I)} I[I + 1] + \delta E(I), \text{ if } I \text{ is odd.} \] (16)

Here, the parity splitting \( \delta E(I) \) is given as
\[ \delta E(I) = \frac{2E_1(I^\pi = 1^-)}{1 + \exp(b_0 \sqrt{B_0 I[I + 1]})} \] (17)

with
\[ B_0 = \frac{\hbar^2}{2} \left( \frac{1}{\Im(\eta = 1)} - \frac{1}{\Im(\eta = \eta_\alpha)} \right). \]

The quantity \( B_0 \) describes the change of the height of the barrier with spin \( I \). The moment of inertia in Eq. (16) is given by the expression
\[ J(I) = w_m(I) \Im(\eta = 1) + [1 - w_m(I)] \Im(\eta = \eta_\alpha) \] (18)
containing a weight function \( w_m(I) \)
\[ w_m(I) = \frac{w_m(I = 0)}{1 + b_1 B_0 I[I + 1]}, \] (19)

which is the probability to find the mononucleus component in the wave function of the state with spin \( I \) of the ground state band. Since \( w_m(I) \) decreases with increasing angular momentum, \( J(I) \) increases with \( I \) in agreement with an experimental tendency. The quantity \( w_\alpha(I) = 1 - w_m(I) \) gives the corresponding probability of the \( \alpha \)-cluster component. A qualitative derivation of the above analytical formulae is given in the Appendix. The constants \( \Im(|\eta| = 1) = 0.3 \times \Im^*(|\eta| = 1) \), \( w_m(I = 0) = 0.93 \), \( b_0 = \pi \text{ MeV}^{-1/2} \) and \( b_1 = 0.2 \text{ MeV}^{-1} \) were obtained by fitting the experimental spectra for the nuclei considered (see Fig. 3).

These formulae clearly demonstrate that there are two important quantities which predetermine a description of the spectra of the alternating parity bands. They are \( E_1(I^\pi = 1^-) \), which is determined by the depth of the minimum of the potential at \( I = 0 \) and by the value of the mass parameter \( B_\eta \), and \( B_0 \), which determines the angular momentum dependence of \( w_m(I) \), i.e. of \( J(I) \) and \( \delta E(I) \).
C. \( E\lambda \)-transitions

With the wave functions obtained, we have calculated the reduced matrix elements of the electric multipole moments \( Q(E1) \), \( Q(E2) \) and \( Q(E3) \). The effective charge for \( E1 \)-transitions has been taken to be equal to \( e_{1}^{\text{eff}} = e(1 + \chi) \) with an average state-independent value of the \( E1 \) polarizability coefficient \( \chi = -0.7 \) [40]. This renormalization takes into account a coupling of the mass–asymmetry mode to the giant dipole resonance in a dinuclear system. In the case of the quadrupole transitions we did not renormalized the charge \( e_{2}^{\text{eff}} = e \) although an effective charge of \( 1.35 \) \( e \) describes the data for actinides better as it is seen from the results of calculations. For octupole transitions our cluster model Hamiltonian includes the octupole mode responsible for description of the shape variation and deformation of the nuclear surface. This is the low–frequency collective octupole mode. However, high–frequency isovector as well as isoscalar octupole modes are not presented in the model Hamiltonian. For example, to simplify the consideration the charge asymmetry coordinate is not independent dynamical one but is rigidly related in our model to the mass asymmetry coordinate. The octupole transition operator is not exhausted by the term produced by the low–frequency octupole degree of freedom and includes also a contribution of the high–frequency octupole modes. For this reason for the octupole transitions the effect of the coupling of the low–frequency octupole mode to the high frequency mode should be taken into account by the octupole effective charge. The estimate of this effective charge is given in [40]. The combined effect gives \( \delta e_{3}^{(\text{pot})} \approx (0.5 + 0.3 \tau_z) e \) [40]. So, we have taken the effective charge to be equal to \( e_{3,\text{proton}}^{\text{eff}} = 1.2 e \) for protons and \( e_{3,\text{neutron}}^{\text{eff}} = 0.8 e \) for neutrons.

The results of these calculations are listed in Tables 6, 7 and shown in Figs. 4–10. The obtained values are in agreement with the known experimental data for \( Q_{\lambda}^{\text{exp}} \) [2, 14, 15, 16, 17, 49, 50, 51, 52, 53]. Only in \(^{224}\text{Ra}\) and \(^{146}\text{Ba}\) (for \( I = 7 \)) the calculated values of \( D_{10} \) are larger by factor of four than the experimental \( D_{10} \). In Th the isotopic dependence of the dipole moment is well reproduced. The higher multipole moments are in agreement with the calculations of Ref. [38]. Taking into account the collective character
of our model and the absence of the parameters to fit the data, the description of the experimental data is rather good. It should be also noted that the experimental data on the dipole moment have some uncertainties.

The angular momentum dependence of the reduced matrix elements of the electric dipole operator is presented in Figs. 4 and 7 for $^{226}$Ra and $^{148}$Nd, respectively. The calculations qualitatively reproduce the angular momentum dependence of the experimental matrix elements \[13,52\]. The same is true for the reduced matrix elements of the electric quadrupole and octupole operators (Figs. 5,6 and 8,9).

Fig. 10 illustrates the angular momentum dependence of the calculated intrinsic transition quadrupole moment. It is interesting that the cluster model shows an increase of the quadrupole moment with angular momentum in the transitional nucleus $^{226}$Ra and a constant dependence in the well deformed isotope $^{238}$U. Staggering shown in Fig. 10 for both $^{226}$Ra and $^{238}$U nuclei is explained by the higher weight of the $\alpha$–cluster component in the wave functions of odd $I$ states (see Fig. 2). This cluster configuration has larger quadrupole and octupole deformations.

The calculated results for the E3–reduced matrix elements in $^{148}$Nd exceed the experimental data for transitions to the ground band [45]. This can be explained as follows. The experimental E3 matrix elements connecting the negative parity states of $^{148}$Nd to the $\beta$ band are unexpectedly large, about 70% of the matrix elements within the ground band [54]. This shows a considerable fractionation of the E3 strength among the $K=0$ bands. In our model the $\beta$ degree of freedom is absent and all the E3 strength is concentrated in the transitions to the ground band. The nucleus $^{148}$Nd is transitional in its collective properties between spherical and deformed nuclei and the $\beta$ anharmonicity is quite large. This explains a fractionation of the E3 strength between the ground and the $\beta$ bands. The summed E3 strength for $^{148}$Nd corresponds to the intrinsic transitional octupole moment of $\sim 2000e$ fm$^3$ (instead of $\sim 1500e$ fm$^3$ for the transitions in the ground band) which agrees with the calculated value.
V. SUMMARY

We suggest a cluster interpretation of the properties of the alternating parity bands in heavy nuclei assuming collective oscillations in mass asymmetry degree of freedom. The existing experimental data on the angular momentum dependence of parity splitting and on multipole transition moments are quite well reproduced. This supports the idea that cluster type states exist in heavy nuclei. The characteristics of the Hamiltonian used in the calculations were determined by investigating a completely different phenomenon, namely, heavy ion reactions at low energies. Due to this fact, a predictive power of the suggested model is quite high. The proposed analytical expression (16) for $E(I)$ can be applied to estimate the position of the low–lying states which are not yet measured. The calculated staggering behavior of alpha–clusterization can be verified by measuring the angular momentum dependence of the width of the alpha–decay.

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APPENDIX A

Let us assume that the barrier at $x=0$ separates two minima of the potential (3): the minimum with reflection asymmetric deformation (minimum of the potential at $x = x_{\alpha}$) and its mirror image. The nonzero penetration through this barrier lowers the energy of the levels with even $I$ with respect to the energy of the levels with odd $I$. With increasing spin $I$ the barrier between two minima, which is equal to $U_b(I) = U_b(I = 0) + B_0 I[I + 1]$, becomes higher and the penetration probability goes to zero. According to standard WKB-analysis (with higher order corrections) the transmission probability (per tunneling event) for the
potential barrier described by the inverted oscillator with frequency $\hbar \omega_b(I)$ for the energy $\hbar \omega_m(I)/2$ above the potential minimum is given by \[ P(I) = \frac{1}{1 + \exp(2\pi \frac{U_b(I) - \hbar \omega_m(I)/2}{\hbar \omega_b(I)})}, \] (A1)

and $\hbar \omega_b(I) \sim \sqrt{U_b(I)}$ at fixed $x_\alpha$. Here, $\omega_m(I)$ is the frequency of the harmonic oscillator which approximates the potential $U$ around the $\alpha$-cluster minimum. Within the semiclassical approximation one can neglect the $I$ dependence of $\omega_m$ taking $\omega_m(I) \approx \omega_m(I = 0)$. This frequency then essentially determines the rate $\omega_m/\pi$ at which the wave packet strikes the barrier, so that the effective coherent tunneling frequency, i.e. the shift of the negative parity states with respect to the positive parity ones, is given by

$$\delta E(I) = \frac{\hbar \omega_m}{\pi} P(I).$$  

(A2)

Assuming that $(U_b(I = 0) - \frac{1}{2} \hbar \omega_m)$ is small compared to $\hbar \omega_b(0)$ (this case is realized in nuclei considered in the paper), we obtain $\hbar \omega_m/2\pi = E(I^\pi = 1^-)$. For those values of $I$ at which $U_b(I = 0)$ is much smaller than $B_0 I(I + 1)$ we obtain Eq. (17) with the fitting parameter $b_0$. However, we found numerically that Eq. (17) works quite well also at low $I$.

The weight $w_m(I)$ of the mononucleus component in the wave function of the state with spin $I$ can be expressed through the ratio of characteristic times $\tau_m(I)$ and $\tau_b(I)$ which a system spends in the minima and at the barrier, respectively

$$w_m(I) = \frac{\tau_b(I)}{\tau_b(I) + \tau_m(I)}.$$  

(A3)

The mononucleus configuration is located at the top of the barrier. We neglect below the $I$ dependence of the $\tau_m(I)$. Denote by $\tau_b(0)$ the value of $\tau_b(I)$ at $I=0$. At very high angular momentum when the barrier height is mainly determined by the rotational energy and is equal to $B_0 I(I + 1)$ the value of $\tau_b(I)$ can be determined from the time–energy uncertainty relation

$$\tau_b(I \gg 1) = \frac{\hbar}{B_0 I(I + 1)}.$$  

(A4)
To combine the two limits at $I=0$ and for $I \gg 1$, we use the following expression

$$\tau_b(I) = \frac{\hbar}{\hbar/\tau_b(0) + U_b(I) - \hbar \omega_m/2} = \frac{\tau_b(0)}{1 + \tau_b(0)B_0 I[I + 1]/\hbar}.$$ (A5)

Substituting this result into (A3), we obtain

$$w_m(I) = \frac{\tau_b(0) / (\tau_b(0) + \tau_m)}{1 + \frac{\tau_b(0)\tau_m}{\hbar(\tau_b(0)+\tau_m)}B_0 I[I + 1]}.$$ (A7)

The last expression can be rewritten as

$$w_m(I) = \frac{w_m(0)}{1 + b_1 B_0 I(I + 1)},$$ (A8)

where $w_m(0) = \tau_b(0) / (\tau_b(0) + \tau_m)$ and $b_1 = \frac{1}{\hbar} \tau_b(0)\tau_m / (\tau_m + \tau_b(0))$. 
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FIGURES

FIG. 1. Potential energy (full circles) of the $\alpha$–cluster configuration $U(\eta_{\alpha}) \equiv U(\eta_{\alpha}, I = 0)$ as a function of the neutron number in Ba isotopes.

FIG. 2. Calculated probability of the $\alpha$–cluster component in the wave function of the state with spin $I$ of the alternating parity band for $^{224,226}$Ra and $^{226,232}$Th.

FIG. 3. Comparison of experimental (solid points) and calculated with Eqs. (16)-(19) (solid lines) energies of states of the alternating parity bands in $^{220,224}$Ra and $^{222}$Th. The fitting parameters are the same for all nuclei (see text). Experimental data are taken from [41,42].

FIG. 4. Angular momentum dependence of the calculated reduced matrix elements of the electric dipole operator (solid curve) in $^{226}$Ra. The experimental data (squares) are taken from [52].

FIG. 5. The same as in Fig. 4, but for the quadrupole operator

FIG. 6. The same as in Fig. 4, but for the octupole operator

FIG. 7. Angular momentum dependence of the calculated reduced matrix elements of the electric dipole operator (solid curve) in $^{148}$Nd. The experimental data (squares) are taken from [45].

FIG. 8. The same as in Fig. 7, but for the quadrupole operator

FIG. 9. The same as in Fig. 7, but for the octupole operator

FIG. 10. Angular momentum dependence of calculated intrinsic quadrupole transition moments in $^{226}$Ra and $^{238}$U.
TABLE I. Comparison of experimental ($E_{\text{exp}}$) and calculated ($E_{\text{calc}}$) energies of states of the alternating parity bands in $^{232-222}\text{Th}$. Energies are given in keV. Experimental data are taken from [41,42].

| $I^\pi$ | $^{232}\text{Th}$ | $^{230}\text{Th}$ | $^{228}\text{Th}$ | $^{226}\text{Th}$ | $^{224}\text{Th}$ | $^{222}\text{Th}$ |
|---------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
|         | $E_{\text{exp}}$ | $E_{\text{calc}}$ | $E_{\text{exp}}$ | $E_{\text{calc}}$ | $E_{\text{exp}}$ | $E_{\text{calc}}$ | $E_{\text{exp}}$ | $E_{\text{calc}}$ | $E_{\text{exp}}$ | $E_{\text{calc}}$ |
| 1$^-$   | 714             | 693             | 508             | 485             | 328             | 350             | 230             | 254             | 251             | 204             | 250             | 195             |
| 2$^+$   | 49              | 49              | 53              | 53              | 58              | 58              | 72              | 72              | 98              | 98              | 183             | 183             |
| 3$^-$   | 774             | 761             | 572             | 557             | 396             | 423             | 308             | 340             | 305             | 311             | 467             | 366             |
| 4$^+$   | 162             | 160             | 174             | 172             | 187             | 177             | 226             | 238             | 284             | 296             | 440             | 461             |
| 5$^-$   | 884             | 882             | 687             | 684             | 519             | 549             | 451             | 490             | 465             | 494             | 651             | 616             |
| 6$^+$   | 333             | 330             | 357             | 354             | 378             | 391             | 447             | 475             | 535             | 563             | 750             | 760             |
| 7$^-$   | 1043            | 1051            | 852             | 859             | 695             | 748             | 658             | 698             | 700             | 739             | 924             | 920             |
| 8$^+$   | 557             | 553             | 594             | 589             | 623             | 634             | 722             | 761             | 834             | 868             | 1094            | 1077            |
| 9$^-$   | 1249            | 1263            | 1065            | 1075            | 921             | 971             | 923             | 958             | 998             | 1036            | 1255            | 1258            |
| 10$^+$  | 827             | 822             | 880             | 869             | 912             | 919             | 1040            | 1079            | 1174            | 1202            | 1461            | 1430            |
| 11$^-$  | 1499            | 1511            | 1322            | 1326            | 1190            | 1229            | 1238            | 1263            | 1347            | 1384            | 1623            | 1624            |
| 12$^+$  | 1137            | 1130            | 1208            | 1215            | 1239            | 1235            | 1395            | 1424            | 1550            | 1564            | 1851            | 1815            |
| 13$^-$  | 1785            | 1792            | 1615            | 1629            | 1497            | 1517            | 1596            | 1609            | 1739            | 1772            | 2016            | 2019            |
| 14$^+$  | 1482            | 1470            | 1573            | 1565            | 1605            | 1572            | 1781            | 1796            | 1959            | 1966            | 2260            | 2226            |
| 15$^-$  | 2101            | 2099            | 1946            | 1941            | 1838            | 1823            | 1989            | 2002            | 2165            | 2194            | 2432            | 2450            |
| 16$^+$  | 1858            | 1841            | 1971            | 1935            | 1993            | 1918            | 2196            | 2200            | 2398            | 2405            | 2688            | 2663            |
| 17$^-$  | 2445            | 2449            | 2310            | 2274            | 2209            | 2154            | 2413            | 2429            | 2620            | 2651            | 2873            | 2906            |
| 18$^+$  | 2262            | 2229            | 2398            | 2318            | 2406            | 2281            | 2635            | 2640            | 2864            | 2880            | 3134            | 3128            |
| 19$^-$  | 2813            | 2794            | 2703            | 2624            | 2861            | 2890            | 3341            | 3380            |                  |                  |                  |                  |
| 20$^+$  | 2691            | 2633            | 2850            | 2709            | 3097            | 3115            | 3596            | 3621            |                  |                  |                  |                  |
TABLE II. Comparison of experimental ($E_{\text{exp}}$) and calculated ($E_{\text{calc}}$) energies of states of the alternating parity bands in $^{220-226}\text{Ra}$ and $^{240,242}\text{Pu}$. Energies are given in keV. Experimental data are taken from [41,42,43]. For $^{220,222}\text{Ra}$, the parameter $c_2$ was adjusted to the $6^+$ state.

| $\pi$ | $^{226}\text{Ra}$ | $^{224}\text{Ra}$ | $^{222}\text{Ra}$ | $^{220}\text{Ra}$ | $^{242}\text{Pu}$ | $^{240}\text{Pu}$ |
|-------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
|      | $E_{\text{exp}}$ | $E_{\text{calc}}$ | $E_{\text{exp}}$ | $E_{\text{calc}}$ | $E_{\text{exp}}$ | $E_{\text{calc}}$ | $E_{\text{exp}}$ | $E_{\text{calc}}$ | $E_{\text{exp}}$ | $E_{\text{calc}}$ |
| 1$^-$ | 254 | 254 | 216 | 193 | 242 | 224 | 413 | 385 | 781 | 778 |
| 2$^+$ | 68 | 68 | 85 | 85 | 111 | 96 | 179 | 125 | 45 | 45 |
| 3$^-$ | 322 | 327 | 291 | 282 | 317 | 324 | 474 | 509 | 832 | 843 |
| 4$^+$ | 212 | 206 | 251 | 253 | 302 | 287 | 410 | 375 | 147 | 146 |
| 5$^-$ | 447 | 455 | 433 | 434 | 474 | 486 | 635 | 709 | 927 | 958 |
| 6$^+$ | 417 | 414 | 480 | 482 | 550 | 550 | 688 | 688 | 306 | 304 |
| 7$^-$ | 627 | 635 | 641 | 642 | 703 | 728 | 873 | 962 | 1122 | 945 |
| 8$^+$ | 670 | 668 | 756 | 747 | 843 | 843 | 1001 | 1016 | 518 | 514 |
| 9$^-$ | 858 | 862 | 907 | 901 | 992 | 1014 | 1164 | 1252 | 1329 | 1145 |
| 10$^+$ | 960 | 954 | 1069 | 1046 | 1173 | 1166 | 1343 | 1356 | 779 | 773 |
| 11$^-$ | 1134 | 1134 | 1222 | 1215 | 1331 | 1346 | 1496 | 1568 | 1578 | 1392 |
| 12$^+$ | 1282 | 1270 | 1415 | 1378 | 1537 | 1525 | 1711 | 1706 | 1084 | 1077 |
| 13$^-$ | 1448 | 1449 | 1578 | 1558 | 1710 | 1722 | 1864 | 1904 | 1863 | 1677 |
| 14$^+$ | 1629 | 1621 | 1789 | 1745 | 1933 | 1924 | 2106 | 2067 | 1431 | 1421 |
| 15$^-$ | 1797 | 1810 | 1970 | 1944 | 2125 | 2140 | 2263 | 2257 | 2181 | 1994 |
| 16$^+$ | 1999 | 2003 | 2189 | 2153 | 2359 | 2366 | 1816 | 1800 | 1816 | 1800 |
| 17$^-$ | 2175 | 2220 | 2389 | 2372 | 2570 | 2602 | 2526 | 2526 | 2526 | 2526 |
| 18$^+$ | 2236 | 2210 | 2236 | 2210 | 2236 | 2210 |
| 19$^-$ | 2894 | 2894 | 2894 | 2894 | 2894 | 2894 |
| 20$^+$ | 2686 | 2646 | 2686 | 2646 | 2686 | 2646 |
TABLE III. Comparison of experimental ($E_{\text{exp}}$) and calculated ($E_{\text{calc}}$) energies of states of the alternating parity bands in $^{238-232}\text{U}$. Energies are given in keV. Experimental data are taken from [41].

| $\Gamma^\pi$ | $^{238}\text{U}$ & $^{236}\text{U}$ & $^{234}\text{U}$ & $^{232}\text{U}$ |
|-------------|-----------------|-----------------|-----------------|-----------------|
|             | $E_{\text{exp}}$ | $E_{\text{calc}}$ | $E_{\text{exp}}$ | $E_{\text{calc}}$ | $E_{\text{exp}}$ | $E_{\text{calc}}$ |
| 1$^-$       | 680             | 675             | 688             | 644             | 786             | 778             | 563             | 583             |
| 2$^+$       | 45              | 45              | 45              | 45              | 44              | 44              | 48              | 48              |
| 3$^-$       | 732             | 744             | 744             | 713             | 849             | 846             | 629             | 653             |
| 4$^+$       | 148             | 156             | 150             | 154             | 143             | 155             | 157             | 158             |
| 5$^-$       | 827             | 863             | 848             | 831             | 963             | 963             | 747             | 774             |
| 6$^+$       | 307             | 316             | 310             | 313             | 296             | 314             | 323             | 320             |
| 7$^-$       | 966             | 1025            | 1000            | 992             | 1125            | 1122            | 915             | 938             |
| 8$^+$       | 518             | 520             | 522             | 516             | 497             | 517             | 541             | 527             |
| 9$^-$       | 1150            | 1222            | 1199            | 1189            | 1336            | 1316            | 1131            | 1138            |
| 10$^+$      | 776             | 759             | 782             | 753             | 741             | 754             | 806             | 768             |
| 11$^-$      | 1378            | 1448            |                 |                 | 1391            | 1366            |                 |                 |
| 12$^+$      | 1077            | 1025            |                 |                 |                 |                 | 1112            | 1036            |
TABLE IV. Comparison of experimental (\(E_{\text{exp}}\)) and calculated (\(E_{\text{calc}}\)) energies of states of the ground state alternating parity bands in \(^{144−148}\)Ba. Energies are given in keV. Experimental data are taken from [41,44]. The parameter \(c_2\) was adjusted to the \(6^+\) state.

| \(I^\pi\) | \(^{148}\)Ba | \(^{146}\)Ba | \(^{144}\)Ba |
|---|---|---|---|
| \(E_{\text{exp}}\) | \(E_{\text{calc}}\) | \(E_{\text{exp}}\) | \(E_{\text{calc}}\) | \(E_{\text{exp}}\) | \(E_{\text{calc}}\) |
| 1\(^-\) | 623 | 739 | 664 | 759 | 607 |
| 2\(^+\) | 142 | 124 | 181 | 143 | 199 | 157 |
| 3\(^-\) | 775 | 771 | 821 | 818 | 838 | 763 |
| 4\(^+\) | 423 | 400 | 514 | 469 | 530 | 505 |
| 5\(^-\) | 963 | 1018 | 1025 | 1078 | 1039 | 1026 |
| 6\(^+\) | 808 | 808 | 958 | 958 | 961 | 961 |
| 7\(^-\) | 1256 | 1342 | 1349 | 1424 | 1355 | 1375 |
| 8\(^+\) | 1265 | 1273 | 1483 | 1491 | 1471 | 1496 |
| 9\(^-\) | 1645 | 1731 | 1778 | 1841 | 1772 | 1796 |
| 10\(^+\) | 1768 | 1788 | 2052 | 2028 | 2044 | 2005 |
| 11\(^-\) | 2117 | 2181 | 2293 | 2323 | 2278 | 2285 |
| 12\(^+\) | 2304 | 2327 | 2632 | 2574 | 2667 | 2546 |
| 13\(^-\) | 2877 | 2871 | 2863 | 2843 |
| 14\(^+\) | 3193 | 3166 | 3321 | 3146 |
| 15\(^-\) | 3524 | 3489 | 3519 | 3473 |
| 16\(^+\) | 3737 | 3823 | 3992 | 3815 |
| 17\(^-\) | 4242 | 4179 |
TABLE V. Comparison of experimental ($E_{\text{exp}}$) and calculated ($E_{\text{calc}}$) energies of states of the ground state alternating parity bands in $^{146,148}\text{Ce}$ and $^{146,148}\text{Nd}$ isotopes. Energies are given in keV. Experimental data are taken from [41,45,46,47,48,49]. The parameter $c_2$ was adjusted to the $6^+$ state.

| $\Gamma^\pi$ | $^{148}\text{Ce}$ | $^{146}\text{Ce}$ | $^{148}\text{Nd}$ | $^{146}\text{Nd}$ |
|------------|------------------|------------------|------------------|------------------|
|            | $E_{\text{exp}}$ | $E_{\text{calc}}$ | $E_{\text{exp}}$ | $E_{\text{calc}}$ | $E_{\text{exp}}$ | $E_{\text{calc}}$ |
| 1$^-$      | 760              | 714              | 925              | 776              | 1023              | 734              | 896              |
| 2$^+$      | 159              | 134              | 259              | 195              | 302              | 279              | 454              | 327              |
| 3$^-$      | 841              | 851              | 961              | 956              | 999              | 943              | 1190             | 1202             |
| 4$^+$      | 453              | 424              | 668              | 614              | 752              | 776              | 1042             | 993              |
| 5$^-$      | 1084             | 1183             | 1259             | 1242             | 1261             | 1518             | 1684             |
| 6$^+$      | 840              | 840              | 1171             | 1171             | 1280             | 1280             | 1780             | 1780             |
| 7$^-$      | 1400             | 1550             | 1660             | 1645             | 1647             | 2029             | 2264             |
| 8$^+$      | 1290             | 1289             | 1737             | 1756             | 1856             | 1788             | 2594             | 2510             |
| 9$^-$      | 1790             | 2019             | 2138             | 2132             | 2084             | 2706             | 2889             |
| 10$^+$     | 1792             | 1793             | 2552             | 2345             | 2472             | 2286             | 3320             | 3195             |
| 11$^-$     | 2246             | 2562             | 2681             | 2677             | 2573             | 3501             | 3544             |
| 12$^+$     | 2328             | 2334             | 3013             | 2953             | 3107             | 2819             | 3998             | 3879             |
| 13$^-$     | 2769             | 3163             | 3286             | 3265             | 3120             | 4295             | 4235             |
| 14$^+$     | 2888             | 2919             | 3603             | 3603             | 4694             | 4594             |
| 15$^-$     | 3358             | 3827             | 3954             | 5058             | 4970             |
| 16$^+$     | 3464             | 3554             |                 | 5461             | 5356             |
| 17$^-$     |                 | 4013             |                 |                 |                 |
| 18$^+$     |                 | 4065             | 4243             |                 |                 |
| 19$^-$     |                 | 4735             |                 |                 |                 |
| 20$^+$     |                 | 4685             | 4983             |                 |                 |
TABLE VI. Calculated and experimental intrinsic multipole transition moments. The values of the dipole moment $D_{10}$ are given for those values of the nuclear spin $I$ for which there are experimental data. These values of $I$ are shown in the second column. The experimental data are taken from \cite{2,41,42,43,52,53}.

| Nucleus | $D_{10}$ | $Q_{20}(0^+ \rightarrow 2^+)$ | $Q_{20}(0^+ \rightarrow 2^+)$ | $Q_{30}(0^+ \rightarrow 3^-)$ | $Q_{30}(0^+ \rightarrow 3^-)$ |
|---------|---------|------------------|------------------|------------------|------------------|
|         | (e fm)  | (e fm)           | (e fm$^2$)       | (e fm$^2$)       | (e fm$^3$)       | (e fm$^3$)       |
|         | calc.   | exp.             | calc.             | exp.             | calc.             | exp.             |
| $^{220}$Ra | 0.28 (I=7) | 0.27 | 397 | 558 | 3167 |
| $^{222}$Ra | 0.30 (I=7) | 0.27 | 395 | 675 | 3064 |
| $^{224}$Ra | 0.133 (I=3) | 0.028 | 510 | 633 | 2889 |
| $^{226}$Ra | 0.111 (I=1) | 0.06–0.10 | 574 | 718 | 2611 | 2861 |
| $^{222}$Th | 0.29 (I=6) | 0.38 | 397 | 548 | 3632 |
| $^{224}$Th | 0.312 (I=10) | 0.52 | 495 | 2985 |
| $^{226}$Th | 0.223 (I=8) | 0.30 | 561 | 830 | 2672 |
| $^{228}$Th | 0.151 (I=8) | 0.12 | 653 | 843 | 2255 |
| $^{230}$Th | 0.054 (I=6) | 0.04 | 666 | 899 | 1935 | 2144 |
| $^{232}$Th | 0.007 (I=1) | | 719 | 966 | 1616 | 1969 |
| $^{234}$U | 0.004 (I=1) | | 758 | 1035 | 1541 | 1895 |
| $^{236}$U | 0.004 (I=1) | | 786 | 1080 | 1433 | 1951 |
| $^{238}$U | 0.004 (I=1) | | 818 | 1102 | 1417 | 2041 |
TABLE VII. Calculated and experimental intrinsic multipole transition moments for Ba, Ce and Nd isotopes. The values of the dipole moment $D_{10}$ are given for those values of the nuclear spin $I$ for which there are experimental data. These values of $I$ are shown in the second column.

The experimental data are taken from [44,45,46,48,50,51].

| Nucleus | $D_{10}$ | $D_{10}$ | $Q_{20}(0^+ \rightarrow 2^+)$ | $Q_{20}(0^+ \rightarrow 2^+)$ | $Q_{30}(0^+ \rightarrow 3^+)$ | $Q_{30}(0^+ \rightarrow 3^+)$ |
|---------|---------|---------|-------------------------------|-------------------------------|---------------------------|---------------------------|
|         | (e fm)  | (e fm)  | (e fm$^2$)                    | (e fm$^2$)                    | (e fm$^3$)                | (e fm$^3$)                |
|         | calc.   | exp.    | calc.                         | exp.                         | calc.                     | exp.                      |
| $^{144}$Ba | 0.194 ($I=7$) | 0.071(10) | 250                          | 321                          | 1295                      |                          |
|         | 0.209 ($I=8$) | 0.14(3)   |                              |                              |                          |                          |
| $^{146}$Ba | 0.055 ($I=3$) | 0.06(4)   | 286                          | 368                          | 1147                      |                          |
|         | 0.170 ($I=7$) | 0.037(3)  |                              |                              |                          |                          |
| $^{148}$Ba | 0.095 ($I=7$) | 0.306     |                              |                              |                          |                          |
| $^{146}$Ce | 0.121 ($I=7$) | 0.11(2)   | 313                          | 305                          | 1669                      |                          |
|         | 0.160 ($I=11$) | 0.20(2)   |                              |                              |                          |                          |
| $^{148}$Ce | 0.152 ($I=7$) | 364       | 436                          | 1771                         |                          |                          |
| $^{146}$Nd | 0.071 ($I=11$) | 0.17(2)   | 264                          | 276                          | 1627                      |                          |
|         | 0.115($I=1$)  | 0.24(6)   | 370                          | 400                          | 2161                      | 1500                      |
| $^{148}$Nd | 0.222 ($I=8$) | 0.24(3)   |                              |                              |                          |                          |
Ba

$U(\eta_a) \text{ (MeV)}$

$N$

60 64 68 72 76 80 84 88 92
$\langle I \| Q(E1) \| I+1 \rangle$ (e fm) for $^{226}\text{Ra}$.
\langle I \parallel Q(E2) \parallel I+2 \rangle (\text{e fm}^2)$

\[ ^{226}\text{Ra} \]
