Stronger Hardy’s quantum nonlocality: theory and simulation

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Depending on the way one measures, quantum nonlocality might manifest more visibly. Using nonlocal hidden variable theory leads to a paradox. Extended from the original work, we introduce a quantum nonlocal scheme for n-particles ensembles using two distinct approaches. First, a theoretical model is derived with analytical results for nonlocal conditions and probability. Second, a quantum simulation using quantum circuits is constructed that matches very well to the analytical theory. Executing this experiment on real quantum computers for n = 3, we obtain reasonable results compared to theory. As n grows, the nonlocal probability asymptotes 15.6%, implying that nonlocality might persist even at macroscopic scales.

Introduction.— Through gedanken experiment on particle-antiparticle interactions inside two intertwined Mach–Zehnder interferometers, Hardy delivered a contextual nature of nonlocal quantum mechanics without using inequalities [1, 2], an all-versus-nothing criterion for local hidden variable (LHV) theory. Due to the nonlocal nature of quantum states, it is possible to argue the pair met without an annihilation event, causing a paradox. Suppose we have two physical observables \( U_i = |u_i⟩⟨u_i| \) and \( D_i = |d_i⟩⟨d_i| \) with their basis transformation

\[
|u_i⟩ = A|c_i⟩ - B|d_i⟩, \quad |v_i⟩ = B^*|c_i⟩ + A|d_i⟩,
\]

\[
|c_k⟩ = A|u_k⟩ + B|v_k⟩, \quad |d_k⟩ = -B^*|u_k⟩ + A^*|v_k⟩,
\]

where \( |A|^2 + |B|^2 = 1 \). With N being the normalization factor, the quantum state writes in equivalent forms:

\[
|Ψ⟩ = N(|c_1⟩|c_2⟩ - A^2|u_1⟩|u_2⟩), \quad (1)
\]

\[
= N(AB|u_1⟩|v_2⟩ + AB^*|v_1⟩|u_2⟩ + B^2|v_1⟩|v_2⟩), \quad (2)
\]

\[
= N(|c_1⟩(A|u_2⟩ + B|v_2⟩) - A^2(|c_1⟩|c_2⟩ - B|d_1⟩|u_2⟩)), \quad (3)
\]

\[
= N((A|u_1⟩ + B|v_1⟩)|c_2⟩ - A^2|u_1⟩(A^*|c_2⟩ - B|d_2⟩)), \quad (4)
\]

\[
= N(|c_1⟩|c_2⟩ - A^2(A^*|c_1⟩ - B|d_1⟩)(A^*|c_2⟩ - B|d_2⟩)). \quad (5)
\]

Hardy’s original argument can be divided into three set of nonlocal conditions. Firstly in [1 and 2], the two particles start as a separable state \( |c_1⟩|c_2⟩ \), but are entangled by an interaction which effectively quench the \( |u_1⟩|u_2⟩ \) state when measuring \( |Ψ⟩ \) in \( \{ |u_i⟩, |v_i⟩ \} \) basis. In other words, the probability to find \( U_1 = 1 \) and \( U_2 = 1 \) simultaneously is zero: \( P(U_1U_2) = 0 \). Here and throughout the paper, we write \( P(U_1U_2 \cdots) = 0 \) as a shorthand for \( P(U_1U_2 \cdots = 1) = 0 \). Secondly in [3], LHV-correlations between \( U_i \) and \( D_i \) are established by mixing measurement basis for different particles. Using conventional notations for conditional probability, the probability that \( U_2 = 1 \) under the condition of \( D_1 = 1 \) is \( P(U_2|D_1) = 1 \). Similarly, \( P(U_1|D_2) = 1 \). Finally in [4], \( P(D_1D_2) > 0 \), when combined with the secondly established probabilities would imply \( P(U_1U_2) > 0 \). This last nonzero probability directly contradicts the first nonlocal condition, which is \( P(U_1U_2) = 0 \).

This elegant approach was quickly extended to many aspects of 2-particle states [3, 4], 3-particle states [10, 12], or arbitrary n-particle states [13, 17]. It is found that Hardy-type nonlocality proof can be formed from GHZ states [11, 16, 17], graph states [18, 19], symmetric states [20], W and Dicke states [21], and Wigner’s argument [22]. Its strength and visibility compared to other nonlocality proof such as Bell’s theorem, GHZ and CHSH are explored [23, 24], and there are several attempts at unification [27, 28]. The nonlocal visibility can also be amplified using “ladder” logic [29, 31], graph-theoretic logic [32, 33], and high-dimensional systems [34–37]. Meanwhile, the correlation between nonlocality and entanglement is at the spotlight in quantum foundation [3, 38–43]. More importantly, there are experimental evidences in photons [44, 45], atoms [46, 52], superconducting circuits [53, 54], and generic quantum computers [55, 56].

To derive a Hardy’s type paradox, one starts with choosing a specific quantum state, such as GHZ or W. After establishing a set of nonlocal conditions for the chosen state, the nonlocal probability is calculated verifying these assumptions. In this work, we follow this strategy and find a particular version of Hardy’s paradox that yields a very high nonlocal probability that approaches 15.6% as the ensemble size \( n \) grows. Besides the theoretical description, we provide a quantum simulation by using quantum circuits on any simulators and any available quantum hardware. We perform this experiment on IBM quantum computers and obtain data that match reasonable well to theory.

n-particle Hardy’s paradox— Consider an ensemble of \( n \) qubits labeled by \( Ω = \{ 1, \cdots, n \} \). For each qubit
For the term without $|u_1 u_2 \cdots u_n\rangle$.

For the next $n$ conditions, $|\Psi_n\rangle$ is examined in mixed bases. Measuring particle $k^{th} \in \Omega$ in $\{|c_k\}, |d_k\rangle\}$ while others are in $\{|u\}, |v\rangle\}$, we have

$$|\Psi_n\rangle = N \left[ |B_k|^2 A^*_{c_k} |u_1 \cdots u_{k-1} c_k u_{k+1} \cdots u_n\rangle + \sum_{\alpha \in P(n) \setminus \{n\}} A_{c_\alpha} B_{\alpha} \bigotimes_{i \in \alpha} |u_i\rangle \otimes \bigotimes_{j \in \pi \setminus \alpha} |v_j\rangle \otimes |c_k\rangle + B_k A_{\Omega} |u_1 \cdots u_{k-1} d_k u_{k+1} \cdots u_n\rangle \right].$$

(12)

Here, $k = \{k\}$, then $\pi = \Omega \setminus k$. The second set of $n$ nonlocal conditions writes as conditional probabilities

$$P(D_k | D_l) = P(U_\Omega), \quad \forall k, l \in \Omega, l \neq k.$$  

Therefore, measuring $|\Psi_n\rangle$ should not yield $D_k D_l = 1, l \neq k$. However, the third set of conditions writes

$$P(D_\alpha) > 0, \forall \alpha \in P(\Omega), |\alpha| \geq 2,$$  

which we call the nonlocal probabilities, resulting in $P(U_\Omega) > 0$, which contradicts Eq. (11). We emphasize that there are more than one condition in Eq. (15).

In $\{|c_k\}, |d_k\rangle\}|^{\otimes n}$ basis, $|\Psi_n\rangle$ from Eq. (8) is rewritten

$$|\Psi_n\rangle = N \left[ |c_1 \cdots c_n\rangle - A_{\Omega} \left( A^*_{c_k} |c_k\rangle - B_k |d_k\rangle \right) \right]^{\otimes n} \in \{|c_k\}, |d_k\rangle\}|^{\otimes n}$$  

(16)

The tensor product is ordered by Latin indices with normalization constant $N = \left[ 1 - |A_{\Omega}|^2 \right]^{-1/2}$.

Then, measuring $|\Psi_n\rangle$ in $\{|u\}, |v\rangle\}|^{\otimes n}$ yields the first nonlocal condition

$$P(U_\Omega) \equiv P(U_1 U_2 \cdots U_n) = 0,$$  

(11)

which establishes an LH-v correlation that $D_k = 1 \Rightarrow U_\Omega \equiv U_1 U_2 \cdots U_{k-1} U_{k+1} \cdots U_n = 1$. Every time we measure $|\Psi\rangle$, if $D_k = 1$ then $U_\Omega = 1$, or $P(D_k) \leq P(U_\Omega)$. Hence, if 2 particles simultaneously yield $D_k D_l = 1, l \neq k$, then $U_\Omega \equiv U_1 U_2 \cdots U_n = 1$. With $x = \Omega \setminus \{l\}$, the second set of conditions Eq. (13) also writes

$$P(D_k D_l | U_\Omega) = P(U_\Omega), \quad \forall k, l \in \Omega, l \neq k.$$  

Here, $k = \{k\}$, then $\pi = \Omega \setminus k$. The second set of $n$ nonlocal conditions writes as conditional probabilities

$$P(D_k | D_l) = P(U_\Omega), \quad \forall k, l \in \Omega, l \neq k.$$  

Therefore, measuring $|\Psi_n\rangle$ should not yield $D_k D_l = 1, l \neq k$. However, the third set of conditions writes

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In $\{|c_k\}, |d_k\rangle\}|^{\otimes n}$ basis, $|\Psi_n\rangle$ from Eq. (8) is rewritten

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The tensor product is ordered by Latin indices with normalization constant $N = \left[ 1 - |A_{\Omega}|^2 \right]^{-1/2}$.

Then, measuring $|\Psi_n\rangle$ in $\{|u\}, |v\rangle\}|^{\otimes n}$ yields the first nonlocal condition

$$P(U_\Omega) \equiv P(U_1 U_2 \cdots U_n) = 0,$$  

(11)
the transmission coefficients of the beamsplitters. The red curve projects $P_{\text{nonlocal}}$ maximal value for each $n$. It indicates that $P_{\text{nonlocal}}$ asymptotes 15.6% as $n$ grows. Dots are results from a quantum simulation that runs on a virtual machine, as presented in the following section.

Quantum simulation—The above analysis can be realized on quantum hardware. Here, we implement these results to measure the nonlocality of the quantum state $|\Psi_n\rangle$ on a quantum circuit. Figure 2 illustrates our approach for $|\Psi_3\rangle$, i.e. $n = 3$. The left column contains three quantum circuits that are consistent with the sets of nonlocal conditions Eq. (11), (13), (15). The right column shows their result for specific value $A = 0.9$. Rotation gates $R_Y(\theta)$ transform each of them from XY plane to XZ plane, corresponding to a basis transformation from $\{|e\rangle, |d\rangle\}$ to $\{|u\rangle, |v\rangle\}$ basis. The transformation coefficient $A$, therefore, relates to $\theta$ as $A_k = \sin(\theta_k/2)$. State $|\Psi_n\rangle$ is constructed using post-selection scheme 15, 16, 58 with the help of a Toffoli gate targeting an auxiliary qubit. It is straightforward to generalize this quantum circuit to the case of an arbitrary $n$. In this case, the Toffoli gate is instead controlled by all $n$ qubits.

The first condition Eq. (11) is satisfied by a state preparation with “zero-probability substates” in it that excludes $|u_k\rangle \otimes n$. We begin with $|\Psi_{\text{start}}\rangle = |c_k\rangle \otimes n$, then a set of rotation gates $R_Y(\theta_k)$ transforms $|\Psi_{\text{start}}\rangle$ into $|\{u_k\}, |v_k\rangle\rangle \otimes n$ basis. The Toffoli gate marks $|u_k\rangle \otimes n$ to help post-selecting $|\Psi_n\rangle$, following Eq. (11). For $n = 3$, the circuit in Fig. 2a satisfies the first set of nonlocal conditions and yields $\langle u_1 u_2 u_3 | \Psi_3 \rangle = 0$, which is confirmed in Fig. 2b with $P(111) = 0$.

The second set of conditions Eq. (13) is tested by measuring the $k^{\text{th}}$ qubit in $|c_k\rangle, |d_k\rangle$ basis, while leaving others in their $|\{u_k\}, |v_k\rangle\rangle$ bases. This is done on the circuit by applying a single $R_Y(-\theta)$ on the $k^{\text{th}}$ qubit. In Fig. 2, if the measurement yields $D_k = 1$, it ensures $U_1 = U_3 = 1$. This means $P(U_1 D_2 U_3) = P(111) = 0.215 > 0$, while $P(V_1 D_2 U_3) \equiv P(011) = 0$, $P(U_1 D_2 V_3) \equiv P(110) = 0$, and $P(V_1 D_2 V_3) \equiv P(010) = 0$, as shown in Fig. 2f. It is straightforward to verify that $R_Y(-\theta)$ has a similar effect when applied on $|q_1\rangle$ or $|q_3\rangle$, or the case of generalized $n$. In other words, the same correlation is found when measuring $D_1 U_2 U_3$ or $U_1 U_2 D_3$.

The third set of conditions Eq. (15) is a nonzero total probability of measurements that contradicts previous conditions, generally tested in $|\{c_k\}, |d_k\rangle\rangle \otimes n$. To check, we apply $R_Y(-\theta)$ on all qubits and measure them in $|\{c_k\}, |d_k\rangle\rangle \otimes n$ basis. For $n=3$, these states are $|d_1 d_2 c_3\rangle$, $|d_1 c_2 d_3\rangle$, $|c_1 d_2 d_3\rangle$, and $|d_1 d_2 d_3\rangle$. In our quantum circuit, the corresponding states are $|110\rangle$, $|101\rangle$, $|011\rangle$, and $|111\rangle$, which yield a combined probability of 0.107 as shown in Fig. 2g, in agreement with theoretical result in Eq. (18) for $A = 0.9$ and $n = 3$. Henceforth, all nonlocal conditions are satisfied, and a Hardy-type paradox is established.

Obtaining identical result for $P_{\text{nonlocal}}$, but our analytical calculation and the quantum simulation are different approaches. While the first part is a theoretical model of nonlocality, the second part is an experimental protocol to probe nonlocality that can be implemented on real quantum hardware. For example, realizing the circuit on an optical quantum computer would produce $n$ intertwined Mach–Zehnder interferometers. To demonstrate its practical aspect, we execute the quantum circuit in Fig. 2e on IBM’s quantum computers Nairobi and Quito 59 and obtain results shown on Fig. 2g, averaging from 2 million shots each, equally divided into 100 different runs. Apparently, the systematic errors on Quito are slightly larger than those on Nairobi. These are the best results we obtain from IBM’s transmon processors. Any experiment for $n > 3$ yields result with big error, as compared to IBM’s own QASM simulator.
Nonlocality and entanglement—The connection between entanglement and quantum nonlocality is one of contentious nature [39–42]. To further study between entanglement and quantum nonlocality is one of the optimum $A$ (open diamonds), 0.5 (open circles), 0.7 (open triangles), and 0.9 (open squares). The solid circles are data obtained at the optimum $A$, i.e., at the maximum value of $P_{\text{nonlocal}}$.

Nonlocality at the macroscopic scale: (a) $P_{\text{nonlocal}}$ for $|\Psi_n\rangle$ per Eq. (18) at two values: $A = 0.9$ and $A$ optimum are directly compared to results for generalized GHZ state in Eq. (28), Ref. [16] and Eq. (2), Ref. [17]. (b) The total nonlocal probability $P$ quickly reduces to zero as $n$ grows in a log-log plot.

FIG. 3. Entanglement entropy: von Neumann entropy $S(\rho_{[n/2]})$ calculated at half-chain bipartition is shown on the left axis in blue. In direct comparison, $P_{\text{nonlocal}}$ per Eq. (18) is shown on the right axis in red. (a) $S(\rho_{[n/2]})$ and $P_{\text{nonlocal}}$ as a function of the transformation coefficient $A$ at $n = 2, 5,$ and 11. (b) $S(\rho_{[n/2]})$ and $P_{\text{nonlocal}}$ as a function of $n$ at $A = 0.1$ (open diamonds), 0.5 (open circles), 0.7 (open triangles), and 0.9 (open squares). The solid circles are data obtained at the optimum $A$, i.e., at the maximum value of $P_{\text{nonlocal}}$.

As $n$ grows, $P_{\text{nonlocal}}$ quickly reduces to zero at fixed $A$, as shown in Fig. 4a for a representative value $A = 0.9$. This observation matches with previous work on generalized GHZ states. At $n = 2$, $P_{\text{nonlocal}} = 9\%$ because they are all equal to the original Hardy’s result. The fast reduction of $P_{\text{nonlocal}}$ at large $n$ in Ref. [60] might relates to the nature of GHZ states. Although maximally entangled, GHZ states do not necessarily reach maximum in nonlocality [39–42]. There is an optimum value of $A = |A_k|$ for all $k \in \Omega$ that $P_{\text{nonlocal}}$ is maximized whereas the entanglement entropy $S(\rho_{[n/2]})$ behaves monotonically.

However, it is difficult to detect this nonlocal probability of 15.6% at the macroscopic scale. The total nonlocal probability of an $n$-ensemble can be calculated from the area of the $P_{\text{nonlocal}}(A)$ curve $P = \int_0^1 P_{\text{nonlocal}}(A) dA$. As seen in Fig. 4b, $P$ quickly reduces to zero as $n$ grows. This is also evident from Fig. 1, as the probability curves become narrower with larger $n$. A small change in the transformation coefficient $A$ leads to a large drop in $P_{\text{nonlocal}}$. Still, interestingly, there is a high chance nonlocality persists even at macroscopic scales.

In summary, we have extended the original Hardy’s paradox to a general case and obtained a nonlocality with probability approaching 15.6% as the ensemble size grows. The nonlocal conditions and nonlocal probability are derived analytically. A quantum simulation is proposed that matches well to the theory, especially when tested on real quantum computers.

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Discussion and Conclusion.—The connection between entanglement and quantum nonlocality is one of contentious nature [39–42]. To further study between entanglement and quantum nonlocality is one of the optimum $A$, i.e., at the maximum value of $P_{\text{nonlocal}}$.
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