GROUP ACTIONS ON CATEGORIES AND ELAGIN’S THEOREM REVISITED

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Abstract. After recalling basic definitions and constructions for a finite group $G$ action on a $k$-linear category we give a concise proof of the following theorem of Elagin: if $C = \langle A, B \rangle$ is a semiorthogonal decomposition of a triangulated category which is preserved by the action of $G$, and $C^G$ is triangulated, then there is a semiorthogonal decomposition $C^G = \langle A^G, B^G \rangle$. We also prove that any $G$-action on $C$ is weakly equivalent to a strict $G$-action which is the analog of the Coherence Theorem for monoidal categories.

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1. Introduction

1.1. The setting of finite groups acting on categories is a well-studied ground, see e.g. [D97, S11, GK14, E12, E14] and references therein. A useful way to define the action is to require for every $g \in G$ an autoequivalence $\rho_g : C \to C$ together with a choice of isomorphisms $\rho_g \rho_h \simeq \rho_{gh}$ satisfying a cocycle condition, see 2.1. One would then study the category of equivariant objects $C^G$, see 2.4.

1.2. For instance, if $C = D^b(X)$ is the derived category of coherent sheaves on a variety $X$ then a $G$-action on $X$ induces a $G$-action on $C$, and furthermore $C^G$ can be interpreted as the derived category of coherent sheaves on the quotient stack $X/G$.

1.3. The main goal of this paper is to give a direct proof of the Theorem of Elagin [E12, E14] stating that if $C = \langle A, B \rangle$ is a semi-orthogonal decomposition of triangulated categories and $G$ is finite group acting on $C$ by triangulated autoequivalences in such a way that the category of equivariant objects $C^G$ is triangulated and preserving $A$ and $B$, then there is a semi-orthogonal decomposition $C^G = \langle A^G, B^G \rangle$, see Theorem 6.2. In the setup of 1.2 this Theorem is often quite useful in constructing semiorthogonal decompositions for the quotient stack $D^b(X/G)$ from semiorthogonal decompositions of $D^b(X)$.

1.4. In our proof we construct the functors $C^G \to A^G$ and $C^G \to A^G$ adjoint to the inclusion functors. The key step in the proof is to show that if $\Phi : A \to C$ is a $G$-equivariant functor which admits a left or right adjoint functor $\Psi$, then $\Psi$ is automatically equivariant: see Proposition 3.13.
1.5. We also prove that every $G$-action on a category $C$ is $G$-weakly equivalent to a strict $G$-action, that is to an action satisfying $\rho_g \rho_h = \rho_{gh}$, see Theorem 5.4. This is analogous to the Coherence Theorem for monoidal categories: every monoidal category is equivalent to a strict monoidal category, see e.g. [L04, 1.2.15].

1.6. In order to formulate and prove these facts we need to develop the language of $G$-functors, $G$-natural transformations and so on. Perhaps relevant definitions and constructions are well-known to experts but we include these for completeness as we could not find the reference that fits our purpose.

1.7. All categories, functors etc are $k$-linear where $\text{char}(k) = 0$. Groups acting on categories are finite and we denote by $1 \in G$ the neutral element of the group.

We use the symbol “$\circ$” to denote vertical composition of natural transformations of functors, the other types of compositions are denoted by concatenation.

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2. $G$-CATEGORIES AND EQUIVARIANT OBJECTS

2.1. By a $G$-action on $C$ we mean the following data [E14, Def. 3.1]:

- For each element $g \in G$ an autoequivalence $\rho_g : C \to C$
- For each pair $g, h \in G$ an isomorphism of functors
  $$\phi_{g,h} : \rho_g \rho_h \cong \rho_{gh}.$$ 

The data must satisfy the following associativity axiom: for all $g, h, k \in G$ the diagram of functors $C \to C$ is commutative:

\[
\begin{array}{ccc}
\rho_g \rho_h \rho_k & \xrightarrow{\phi_{g,h,k}} & \rho_g \rho_{hk} \\
\phi_{g,h} \cdot \phi_{h,k} & | & | \\
\rho_{gh} \rho_k & \xrightarrow{\phi_{gh,k}} & \rho_{ghk}
\end{array}
\]

2.2. It follows from the definition that there is an isomorphism of functors

$$\phi_1 : \rho_1 \cong id$$

obtained by post-composing

$$\phi_{1,1} : \rho_1 \rho_1 \to \rho_1$$

with $\rho_1^{-1}$. That is we have

$$\phi_{1,1} = \rho_1 \phi_1.$$
Furthermore one can show that $\phi_1$ satisfies [GK14, 2.1.1(e)]:

$$
\phi_{g,1} = \rho_g \phi_1: \rho_g \phi_1 \rightarrow \rho_g \\
\phi_{1,g} = \phi_1 \rho_g: \rho_1 \rho_g \rightarrow \rho_g
$$

so that definition 2.1 coincides with that of [GK14, 2.1].

On the other hand if one asks for $\phi_1$ to be the identity transformation, one gets a slightly stronger definition of a $G$-descent datum of [N90, Def. 1.1].

2.3. Using the language of monoidal functors [L04, Def. 1.2.10] one can give a very concise definition of a group acting on a category. For that consider $G$ as a monoidal category: $G$ is discrete as a category and its monoidal structure defined by

$$
g \otimes h = gh \\
\text{id}_g \otimes \text{id}_h = \text{id}_{gh}.
$$

Now a $G$-action on $C$ amounts to the same thing as an action of monoidal category $G$ on $C$ [L04, Ex. 1.2.12], i.e. a weak monoidal functor

$$
\rho: G \rightarrow [C, C]
$$

where on the right is the category of functors $C \rightarrow C$ with monoidal structure given by composing functors.

2.4. One defines the category of $G$ equivariant objects $C^G$ [E14, GK14] as follows: objects of $C^G$ are linearized objects, i.e. objects $c \in C$ equipped with isomorphisms

$$
\theta_g: c \rightarrow \rho_g(c), \ g \in G
$$

satisfying the condition that the diagrams are commutative:

Morphisms of equivariant objects consist of those morphisms of the underlying objects in $C$ which commute with all $\theta_g$, $g \in G$.

3. $G$-functors and $G$-natural transformations

3.1. Given two categories $C$, $D$ with $G$-actions and a functor $\Phi: C \rightarrow D$, $\Phi$ is called a **right lax $G$-functor** if there are given natural transformations

$$
\delta_g: \rho_g \Phi \rightarrow \Phi \rho_g
$$
such that the two natural transformations $\rho_g \rho_h \Phi \to \Phi \rho_g \rho_h$ coincide:

This commutative diagram is called the pentagon axiom.

Similarly $\Phi$ is called a **left lax $G$-functor** if there are given natural transformations $\delta_g: \Phi \rho_g \to \rho_g \Phi$

satisfying the dual pentagon axiom.

A right (or left) lax $G$-functor $\Phi$ is called a **weak $G$-functor** if all $\delta_g$ are isomorphisms.

The following lemma is a useful criterion for a weak $G$-functor.

**3.2. Lemma.** Let $\Phi$ be a right (or left) lax $G$-functor. The following conditions are equivalent:

1. The natural transformation $\delta_1: \rho_1 \Phi \to \Phi \rho_1$ is an isomorphism.
2. $\Phi$ satisfies the identity element axiom:

   $$\Phi \phi_1 \circ \delta_1 = \phi_1 \Phi: \rho_1 \Phi \to \Phi.$$

3. $\Phi$ is a weak $G$-functor.

**3.3. Proof.** Implications $(3) \implies (1), (2) \implies (1)$ are obvious. Let us prove that $(1) \implies (3)$. Consider the case of the right lax $G$-functor. Applying the pentagon axiom to the pair $(g^{-1}, g)$ gives:

$$\delta_{g^{-1}} \rho_g \circ \rho_{g^{-1}} \delta_g = \Phi \phi_{g^{-1}} \circ \delta_1 \circ \phi_{g^{-1}} \Phi.$$

Since the natural transformation on the right-hand side is an isomorphism (note that $\delta_1$ is an isomorphism by the identity element axiom) and $\rho_g, \rho_{g^{-1}}$ are equivalences, it follows that $\delta_{g^{-1}}$ is left invertible and $\delta_g$ is right invertible. Thus we see that all $\delta_g$ are isomorphisms.

Now we prove $(1) \implies (2)$. Consider the natural transformation

$$\varepsilon = \Phi \phi_1 \circ \delta_1 \circ \phi_{1}^{-1} \Phi: \Phi \to \Phi.$$

We are given that $\varepsilon$ is an isomorphism and we need to prove that $\varepsilon$ is in fact an identity.

We use Lemma 3.4 applied to the trivial group $H := \{1\}$ and the composition

$$(\mathcal{C}, id) \to (\mathcal{C}, \rho_1) \to (\mathcal{D}, \rho_1) \to (\mathcal{D}, id)$$
which gives a lax $\mathcal{G}$-functor

$$(\mathcal{C}, \text{id}) \xrightarrow{(\Phi, \varepsilon)} (\mathcal{D}, \text{id}).$$

The pentagon axiom for this functor yields

$$\varepsilon^2 = \varepsilon$$

and we deduce that $\varepsilon = \text{id}$.

3.4. **Lemma.** If $(\Phi, \delta^\Phi): \mathcal{C} \to \mathcal{D}$, $(\Psi, \delta^\Psi): \mathcal{D} \to \mathcal{E}$ are right/left/weak $\mathcal{G}$-functors, then their composition $(\Psi \Phi, \Phi \delta^\Psi \circ \delta^\Phi \Psi)$ is a right/left/weak $\mathcal{G}$-functor.

For the proof one needs to check that the composition satisfies the pentagon and/or the identity element axioms; this is a straightforward check.

3.5. **Lemma.** A weak $\mathcal{G}$-functor $\Phi: \mathcal{C} \to \mathcal{D}$ induces a functor on the categories of equivariant objects

$$\Phi^G: \mathcal{C}^G \to \mathcal{D}^G$$

such that the following diagram is commutative

\[\begin{array}{ccc}
\mathcal{C}^G & \xrightarrow{\Phi^G} & \mathcal{D}^G \\
\downarrow \Phi & & \downarrow \Phi \\
\mathcal{C} & \xrightarrow{\Phi} & \mathcal{D}
\end{array}\]

3.6. **Proof.** For $(c, \theta) \in \mathcal{C}^G$ we define linearization on $\Phi(c)$ as a composition of isomorphisms

$$\Phi(c) \to \Phi \rho_g(c) \to \rho_g \Phi(c)$$

of $\Phi \theta_g$ with $\delta_g$. It is now a standard check that $\Phi(c)$ becomes an equivariant object and that $\Phi^G$ is a functor.

3.7. **Definition.** A natural transformation between two weak $\mathcal{G}$-functors $\mu: \Phi_1 \to \Phi_2: \mathcal{C} \to \mathcal{D}$ is called a $\mathcal{G}$-natural transformation if for every $g \in \mathcal{G}$ the following diagram commutes:

\[\begin{array}{ccc}
\rho_g \Phi_1 & \xrightarrow{\rho_g \mu} & \rho_g \Phi_2 \\
\delta_{1,g} & & \delta_{2,g} \\
\Phi_1 \rho_g & \xrightarrow{\mu \rho_g} & \Phi_2 \rho_g.
\end{array}\]

3.8. **Lemma.** A $\mathcal{G}$-natural transformation $\mu$ between two weak $\mathcal{G}$-functors $\Phi_1, \Phi_2: \mathcal{C} \to \mathcal{D}$ induces a natural transformation $\mu^G: \Phi_1^G \to \Phi_2^G$. 
Proof. To prove that \( \mu \) descends to a natural transformation \( \mu^G : \Phi_1^G \to \Phi_2^G \), we check that for every \((c, \theta) \in C^G\) the morphism \( \mu : \Phi_1(c) \to \Phi_2(c) \) commutes with linearizations:

\[
\begin{array}{c}
\rho_g \Phi_1(c) \\
\delta_1 \downarrow \leftarrow \delta_2 \\
\Phi_1 \rho_g (c) \\
\Phi_1 \theta_g \\
\Phi_1(c)
\end{array} \xrightarrow{\mu(c)} \\
\begin{array}{c}
\rho_g \Phi_2(c) \\
\Phi_2 \rho_g (c) \\
\Phi_2 \theta_g \\
\Phi_2(c)
\end{array}
\]

The transformation \( \mu^G \) is natural since the original transformation \( \mu \) is natural and the forgetful functor \( C^G \to C \) is faithful.

Definition. Two weak \( G \)-functors \( \Phi : C \to D \), \( \Psi : D \to C \) are called \( G \)-adjoint if they are adjoint and the unit \( \varepsilon : id \to \Phi \Psi \) and counit \( \eta : \Psi \Phi \to id \) of the adjunction are \( G \)-natural transformations.

Lemma. A \( G \)-adjoint pair of functors \( \Phi, \Psi \) induces an adjoint pair \( \Phi^G, \Psi^G \) between the categories of equivariant objects.

Proof. From 3.8 it follows that we have natural transformations \( \varepsilon^G : id \to \Phi^G \Psi^G, \eta^G : \Phi^G \Psi^G \to id \). The condition for \( \Psi \) and \( \Phi \) to be adjoint is that two compositions

\[ \Phi \eta \circ \varepsilon \Phi : \Phi \to \Phi \Psi \Phi \to \Phi \]

and

\[ \eta \Phi \circ \Psi \varepsilon : \Psi \to \Psi \Phi \Psi \to \Psi \]

are identities. Since the forgetful functor \( C^G \to C \) is faithful, the same holds for \( \Phi^G, \Psi^G \).

Proposition. A left or right adjoint \( \Psi \) to a weak \( G \)-functor \( \Phi \) can be made into a weak \( G \)-functor in such a way that \( \Psi \) and \( \Phi \) become \( G \)-adjoint.

Proof. Let \( \Psi \) be the left adjoint to \( \Phi : C \to D \). We construct the structure of a left lax \( G \)-functor on \( \Psi \) using the structure of a right lax \( G \)-functor on \( \Phi \).

Let \( \varepsilon : id \to \Phi \Psi \) and \( \eta : \Psi \Phi \to id \) be the unit and the counit of the adjunction.

Given a right lax \( G \)-structure \( \delta_g : \rho_g \Phi \to \Phi \rho_g \) on \( \Phi \) we define the left lax \( G \)-structure \( \delta'_g : \rho_g \Psi \to \rho_g \Psi \) on \( \Psi \) as a mate of \( \delta_g \) with respect to the adjunction [KS74, Prop. 2.1], [L04, pp. 185–186], i.e.

\[ \delta'_g = \eta \rho_g \Psi \circ \Psi \delta_g \Psi \circ \Psi \rho_g \varepsilon : \Psi \rho_g \to \Psi \rho_g \Phi \Psi \to \Psi \Phi \rho_g \Psi \to \rho_g \Psi. \]

The pentagon axiom can be expressed as an equality of certain compositions in the double category of [KS74, p.86], hence is preserved under taking mates by [KS74, Prop. 2.2]. Checking the identity axiom for \( \delta'_1 \) is straightforward.
Now by 3.2 $\Psi$ becomes a weak $G$-functor. The proof for right adjoints is analogous.

We now need to prove that the unit and counit transformations $\varepsilon, \eta$ are $G$-natural. We do the proof for the unit $\varepsilon$. We need to check that the following diagram commutes:

$$
\begin{array}{c}
\rho_g id & \varepsilon & \rho_g \Phi \Psi \\
 \downarrow & \downarrow & \downarrow \delta_{\Phi \rho_g} \\
 id \rho_g & \varepsilon & \Phi \Psi \rho_g,
\end{array}
$$

Here $\delta_{\Phi \Psi}$ is defined using 3.4. Unraveling the definitions we are left with checking the diagram (where we use simplified notation for the natural transformations to denote the obvious compositions)

$$
\begin{array}{c}
\rho_g \varepsilon & \rho_g \Phi \Psi \delta_g \simeq & \Phi \rho_g \Psi \\
 \varepsilon & \varepsilon & \varepsilon \eta \\
 \Phi \Psi \rho_g \varepsilon & \Phi \Psi \rho_g \Phi \Psi \delta_g \simeq & \Phi \Psi \Phi \rho_g \Psi
\end{array}
$$

which is easily seen to commute.

3.15. **Corollary.** Let $\Phi: \mathcal{C} \to \mathcal{D}$ be a weak $G$-functor. Then the following conditions are equivalent:

(a) $\Phi$ is an equivalence of categories

(b) There exists a weak $G$-functor $\Psi: \mathcal{D} \to \mathcal{C}$ and $G$-natural isomorphisms $\Psi \circ \Phi \simeq id_\mathcal{C}, \Phi \circ \Psi \simeq id_\mathcal{D}$.

In this case we will call $\Phi$ a **weak $G$-equivalence**.

3.16. **Proof.** We only need to prove $(a) \implies (b)$ as the opposite implication is trivial. Let $\Psi: \mathcal{D} \to \mathcal{C}$ be the quasi-inverse functor to $\Phi$. In particular $\Psi$ and $\Phi$ are adjoint (both ways) so that by 3.13 $\Psi$ has a structure of a weak $G$-functor with compositions $G$-isomorphic to identity functors.

4. **Example: $G$-actions on the category of vector spaces**

4.1. In this section we review a well-known example of how equivalence classes of $G$-actions on the category of $k$-vector spaces correspond bijectively to cohomology classes $H^2(G, k^*)$.

4.2. Let $\mathcal{C} = \text{Vect}_k$ be the category of $k$-vector spaces, and let $\rho$ be the $G$-action on $\text{Vect}_k$. As every autoequivalence of $\mathcal{C}$ is isomorphic to the identity functor, let us assume $\rho_g = id$ for every $g \in G$. In this setup the data of the $G$-action $\rho$ defined in 2.1 is equivalent to specifying a cocycle $\phi \in Z^2(G, k^*)$. 

4.3. Consider two $G$-actions on $\text{Vect}_k$ given by cocycles $\phi, \phi' \in Z^2(G, k^*)$. For the $G$-actions to be equivalent there needs to exist a weak $G$-functor

$$\Phi: (\text{Vect}_k, \phi) \to (\text{Vect}_k, \phi')$$

which is an equivalence of categories. Then the pentagon axiom 3.1 requires existence of an element $\delta = (\delta_g)_{g \in G} \in Z^1(G, k^*)$ such that $\phi'_{g,h} = \delta_g \delta_h \delta_{gh}^{-1} \phi_{g,h}$ for all $g, h$. Thus $G$-categories $(\text{Vect}_k, \phi)$ and $(\text{Vect}_k, \phi')$ are equivalent if and only if $[\phi] = [\phi'] \in H^2(G, k^*)$.

4.4. The category of equivariant objects $(\text{Vect}_k, \phi)^G$ is the category of $\phi$-twisted $G$-representations with objects given by vector spaces $V$ together with isomorphism $\theta_g: V \to V$ satisfying $\theta_{gh} = \phi(g, h)\theta_g\theta_h$ and $G$-equivariant morphisms. In particular, if $\phi$ is the trivial cocycle, so that $G$-action on $\text{Vect}_k$ is trivial, $\text{Vect}_k^G$ is the category of $G$-representations.

5. Strictifying $G$-actions

5.1. Let $\Omega(G)$ denote the category with one object for every element $g \in G$ with $\text{Hom}(g, g) = k$ and $\text{Hom}(g, h) = 0$ for $g \neq h$.

5.2. Let $\mathcal{C}$ be a category with a $G$-action. Consider the category of weak $G$-functors and $G$-natural transformations from $\Omega(G)$ to $\mathcal{C}$

$$\mathcal{C}' = \text{Hom}_G(\Omega(G), \mathcal{C}).$$

We endow $\mathcal{C}'$ with the strict $G$-action induced by the $G$-action on $\Omega(G)$.

5.3. Explicitly the objects of $\mathcal{C}'$ consist of families $(c_g \in \mathcal{C})_{g \in G}$ together with isomorphisms $\delta_{h,g}: \rho_h c_g \simeq c_{hg}$ satisfying the cocycle condition that two ways of getting an isomorphism $\rho_k \rho_h c_g \simeq c_{khg}$ coincide. The morphisms from $(c_g)_{g \in G}$ to $(d_g)_{g \in G}$ are morphisms $f_g: c_g \to d_g$ satisfying the condition that the two natural ways of forming a morphism $\rho_h c_g \to d_{hg}$ coincide.

5.4. Theorem. The functor $\Phi: \mathcal{C}' \to \mathcal{C}$ sending $(c_g)_{g \in G}$ to $c_1$ is a weak $G$-equivalence. Hence, every $G$-action is weakly equivalent to a strict $G$-action.

5.5. Proof. We need to check that $\Phi$ has a structure of a weak $G$-functor and that $\Phi$ is fully faithful and essentially surjective.

The structure of a weak $G$-functor on $\Phi$ is in fact simply given by the structure maps $\delta_{h,g}$. That is we have functorial isomorphisms

$$\rho_g \Phi(c) = \rho_g(c_1) \delta_g^{-1} c_g = \Phi \rho_g(c)$$

and the pentagon axiom follows from the cocycle condition on $\delta$.

To check that $\Phi$ is essentially surjective, one checks that for any $c \in \mathcal{C}$ the family $(\rho_g(c))$ has a structure of an object from $\mathcal{C}$. Furthermore, one can see that any object $(c_g)_{g \in G}$ is isomorphic to $(\rho_g(c_1))_{g \in G}$.

Thus to check that $\Phi$ is fully faithful, we may take two objects $(c_g)_{g \in G} = (\rho_g(c_1))$ and $(d_g)_{g \in G} = (\rho_g(d_1))$ and a morphism $f_g: c_g \to d_g$ between them.
It is then easy to see that \( f_\varphi = \rho(f_1) \) and that conversely for any \( f_1 : c_1 \to d_1 \), the collection \( \rho_\varphi(f_1) \) defines a morphism between \( c \) and \( d \).

6. Elagin’s Theorem

6.1. If \( \mathcal{C} \) is a triangulated category and \( G \) acts by triangulated autoequivalences, then \( \mathcal{C}^G \) is endowed with a shift functor and a set of distinguished triangles: these are the triangles that are distinguished after applying the forgetful functor \( \mathcal{C}^G \to \mathcal{C} \). Furthermore under some mild technical assumptions this gives \( \mathcal{C}^G \) the structure of a triangulated category \( \text{[E14, Theorem 6.9]} \), for instance existence of a dg-enhancement of \( \mathcal{C} \) is a sufficient condition for \( \mathcal{C}^G \) to be triangulated \( \text{[E14, Corollary 6.10]} \).

6.2. Theorem. Let \( \mathcal{C} = \langle \mathcal{A}, \mathcal{B} \rangle \) be a semi-orthogonal decomposition of triangulated categories. Let \( G \) act on \( \mathcal{C} \) by triangulated autoequivalences which preserve \( \mathcal{A} \) and \( \mathcal{B} \). Assume that the equivariant category \( \mathcal{C}^G \) is triangulated with respect to triangles coming from \( \mathcal{C} \). Then \( \mathcal{A}^G, \mathcal{B}^G \subset \mathcal{C}^G \) are triangulated and there is a semi-orthogonal decomposition

\[
\mathcal{C}^G = \langle \mathcal{A}^G, \mathcal{B}^G \rangle.
\]

6.3. Proof. The existence of an adjoint pair between \( \mathcal{C} \) and \( \mathcal{C}^G \) \( \text{[E14, Lemma 3.7]} \) implies that \( \mathcal{B}^G = \perp \mathcal{A}^G \) and \( \mathcal{A}^G = \mathcal{B}^G \perp \). In particular \( \mathcal{A}^G \) and \( \mathcal{B}^G \) are triangulated subcategories of \( \mathcal{C}^G \).

Now in order to establish the semi-orthogonal decomposition \( \mathcal{C}^G = \langle \mathcal{A}^G, \mathcal{B}^G \rangle \) it suffices to show that the embedding \( i^G : \mathcal{A}^G \to \mathcal{C}^G \) has a left adjoint \( \text{[BK89, 1.5]} \). This holds true by \( 3.13, 3.11 \): the functor \( i : \mathcal{A} \to \mathcal{C} \) is (strictly) \( G \)-equivariant, hence its left adjoint \( p : \mathcal{C} \to \mathcal{A} \) induces an adjoint \( p^G \) to the embedding \( i^G : \mathcal{A}^G \to \mathcal{C}^G \).

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