The particle invariance in particle physics

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Abstract

Since the particles such as molecules, atoms and nuclei are composite particles, it is important to recognize that physics must be invariant for the composite particles and their constituent particles, this requirement is called particle invariance in this paper. But difficulties arise immediately because for fermion we use Dirac equation, for meson we use Klein-Gordon equation and for classical particle we use Newtonian mechanics, while the connections between these equations are quite indirect. Thus if the particle invariance is held in physics, i.e., only one physical formalism exists for any particle, we can expect to find out the differences between these equations by employing the particle invariance. Using this approach is one of the goals of this paper, consequently, several new relationships between them are found, the most important result is that the obstacles that cluttered the path from classical mechanics to quantum mechanics are found, it becomes possible to derive the quantum wave equations from relativistic mechanics after the obstacles are removed.

Another goal is just to discuss interactions between particles under the particle invariance, several new formulae of interactions are derived and discussed. The new results provide an insight into improving quark model.

1 Introduction

Without doubt, most particles can be regarded as composite particles, such as molecules composed of atoms, atoms composed of electrons and nuclei, nuclei composed of nucleons, so on, it is important to recognize that physics must be invariant for the composite particles and their constituent particles, this requirement is called particle invariance in this paper. But difficulties arise immediately because for fermion we use Dirac equation, for meson we use Klein-Gordon equation and for classical particle we use Newtonian mechanics, while the connections between these equations are quite indirect. Thus if the particle invariance is held in physics, i.e., only one physical formalism exists for any particle, we can expect to find out the differences between these equations by employing the particle invariance. Using this approach is one of the goals of this paper, consequently, several new relationships between them are found, the most important result is that the obstacles that cluttered the path from classical mechanics to quantum mechanics are found, it becomes possible to derive the quantum wave equations from relativistic mechanics after the obstacles are removed.

Another goal is just to discuss interactions between particles under the particle invariance, several new formulae of interactions are derived and discussed. The new results provide an insight into improving quark model.

2 Fermions and Bosons

Fermions satisfy Fermi-Dirac statistics, Bosons satisfy Bose-Einstein statistics, there is a connection between the spin of a particle and the statistics. It is clear that the spin is a key concept for particle physics. In this section we shall show that the spin of a particle is one of the consequences of the particle invariance.

According to Newtonian mechanics, in a hydrogen atom, the single electron revolves in an orbit about the nucleus, its motion can be described by its position in an inertial Cartesian coordinate system $S: (x_1, x_2, x_3, x_4 = ic t)$. As the time elapses, the electron draws a spiral path (or orbit), as shown in FIG.1(a) in imagination.

If the reference frame $S$ "rotates" through an angle about the $x_2$-axis in FIG.1(a), becomes a new reference frame $S'$ (there will be a Lorentz transformation linking the frames $S$ and $S'$), then in the frame $S'$, the spiral path of the electron tilts with respect to the $x_4'$-axis with the angle as shown in FIG.1(b). At one instant of time, for example, $t_4' = t_0$ instant, the spiral path pieces many points at the plane $t_4' = t_0$ , for example, the points labeled $a$, $b$ and $c$ in FIG.1(b), these points indicate that...
the electron can appear at many points at the time \( t_0 \), in agreement with the concept of probability in quantum mechanics. This situation gives us a hint to approach quantum wave nature from relativistic mechanics.

Because the electron pierces the plane \( t_1 = t_0 \) with 4-vector velocity \( u \), at every pierced point we can label a local 4-vector velocity. The pierced points may be numerous if the path winds up itself into a cell about the nucleus (due to a nonlinear effect in a sense), then the 4-vector velocities at the pierced points form a 4-vector velocity field. It is noted that the observation plane selected for the piercing can be taken at an arbitrary orientation, so the 4-vector velocity field may be expressed in general as \( u(x_1, x_2, x_3, x_4) \), i.e. the velocity \( u \) is a function of 4-vector position.

At every point in the reference frame \( S' \) the electron satisfies relativistic Newton’s second law of motion:

\[
m \frac{du_\mu}{dt} = qF_{\mu\nu} u_\nu
\]  

(1)

the notations consist with the convention[1]. Since the Cartesian coordinate system is a frame of reference whose axes are orthogonal to one another, there is no distinction between covariant and contravariant components, only subscripts need be used. Here and below, summation over twice repeated indices is implied in all case, Greek indices will take on the values 1,2,3,4, and regarding the rest mass \( m \) as a constant. As mentioned above, the 4-vector velocity \( u \) can be regarded as a multi-variable function, then

\[
\frac{du_\mu}{d\tau} = \frac{\partial u_\mu}{\partial x_\nu} \frac{dx_\nu}{d\tau} = u_\nu \partial_\nu u_\mu
\]  

(2)

\[
qF_{\mu\nu} u_\nu = qu_\nu (\partial_\mu A_\nu - \partial_\nu A_\mu)
\]  

(3)

Substituting them back into Eq.(1), and re-arranging these terms, we obtain

\[
u_\nu \partial_\nu (mu_\mu + qA_\mu) = u_\nu \partial_\nu (qA_\nu)
\]

\[
u_\nu \partial_\nu (mu_\mu + qA_\nu) = u_\nu \partial_\nu (mu_\mu) - u_\nu \partial_\nu (mu_\nu)
\]

\[
u_\nu \partial_\nu (mu_\mu + qA_\nu) = u_\nu \partial_\nu (mu_\nu + qA_\nu) - \frac{1}{2} \partial_\nu (mu_\nu u_\nu)
\]

\[
u_\nu \partial_\nu (mu_\mu + qA_\nu) = u_\nu \partial_\nu (mu_\nu + qA_\nu) - \frac{1}{2} \partial_\nu (-mc^2)
\]

\[
u_\nu \partial_\nu (mu_\mu + qA_\nu) = u_\nu \partial_\nu (mu_\nu + qA_\nu)
\]  

(4)

Using the notation

\[
K_{\mu\nu} = \partial_\mu (mu_\nu + qA_\nu) - \partial_\nu (ma_\mu + qA_\mu)
\]  

(5)

Eq.(5) is given by

\[
u_\nu K_{\mu\nu} = 0
\]  

(6)

Because \( K_{\mu\nu} \) contains the variables \( \partial_\mu u_\nu, \partial_\mu A_\nu, \partial_\nu u_\mu \) and \( \partial_\nu A_\mu \) which are independent from \( u_\nu \), then a main solution satisfying Eq.(5) is given by

\[
K_{\mu\nu} = 0
\]  

(7)

\[
\partial_\nu (mu_\nu + qA_\nu) = \partial_\nu (ma_\mu + qA_\mu)
\]  

(8)

( In this paper we do not discuss the special solutions that \( K_{\mu\nu} \neq 0 \), if they exist ). According to Green’s formula or Stokes’s theorem, the above equation allows us to introduce a potential function \( \Phi \) in mathematics, further set \( \Phi = -ih \ln \psi \), we obtain a very important equation

\[
(mu_\mu + qA_\mu) \psi = -ih \partial_\mu \psi
\]  

(9)

where \( \psi \) representing wave nature may be a complex mathematical function, its physical meanings is determined from experiments after the introduction of the Planck’s constant \( h \).

The magnitude formula of 4-vector velocity of particle is given in its square form by
\[ -c^2 = u_\mu u_\mu \]  

(10)

which is valid at every point in the 4-vector velocity field. Multiplying the two sides of the above equation by \( m^2 \psi \) and using Eq. (11), we obtain

\[ -m^2 c^2 \psi = m u_\mu \left(-i \hbar \partial_\mu - q A_\mu \right) \psi \]

\[ = (-i \hbar \partial_\mu - q A_\mu)(mu_\mu \psi) - \left[-i \hbar \psi \partial_\mu (mu_\mu) \right] \]

\[ = (-i \hbar \partial_\mu - q A_\mu)(-i \hbar \partial_\mu + q A_\mu) \psi \]

\[ -[-i \hbar \psi \partial_\mu (mu_\mu)] \]  

(11)

According to the continuity condition for the electron motion

\[ \partial_\mu (mu_\mu) = 0 \]  

(12)

we have

\[ -m^2 c^2 \psi = (-i \hbar \partial_\mu - q A_\mu)(-i \hbar \partial_\mu - q A_\mu) \psi \]  

(13)

It is known as the Klein-Gordon equation.

On the condition of non-relativity, Schrödinger equation can be derived from the Klein-Gordon equation (8) (P.469).

However, we must admit that we are careless when we use the continuity condition Eq. (12), because, from Eq. (8) we obtain

\[ \partial_\mu (mu_\mu) = \partial_\mu (-i \hbar \partial_\mu \ln \psi - q A_\mu) = -i \hbar \partial_\mu \partial_\mu \ln \psi \]  

(14)

where we have used Lorentz gauge condition. Thus from Eq. (10) to Eq. (11) we obtain

\[ -m^2 c^2 \psi = (-i \hbar \partial_\mu - q A_\mu)(-i \hbar \partial_\mu - q A_\mu) \psi + \hbar^2 \psi \partial_\mu \partial_\mu \ln \psi \]  

(15)

This is of a complete wave equation for describing the motion of the electron accurately. The Klein-Gordon equation is a linear wave equation so that the principle of superposition is valid, however with the addition of the last term of Eq. (13), Eq. (15) turns to display chaos.

In the following we shall show Dirac equation from Eq. (8) and Eq. (14). From Eq. (14), the wave function can be given in integral form by

\[ \Phi = -i \hbar \ln \psi = \int_{x_0}^{x} (mu_\mu + q A_\mu) dx_\mu + \theta \]  

(16)

where \( \theta \) is an integral constant, \( x_0 \) and \( x \) are the initial and final points of the integral with an arbitrary integral path. Since Maxwell’s equations are gauge invariant, Eq. (14) should preserve invariant form under a gauge transformation specified by

\[ A'_\mu = A_\mu + \partial_\mu \chi, \quad \psi' \leftrightarrow \psi \]  

(17)

where \( \chi \) is an arbitrary function. Then Eq. (14) under the gauge transformation is given by

\[ \psi' = \exp \left\{ \frac{i}{\hbar} \int_{x_0}^{x} (mu_\mu + q A_\mu) dx_\mu + \frac{i}{\hbar} \theta \right\} \exp \left\{ \frac{i}{\hbar} q \chi \right\} \]

\[ = \psi \exp \left\{ \frac{i}{\hbar} q \chi \right\} \]  

(18)

The situation in which a wave function can be changed in a certain way without leading to any observable effects is precisely what is entailed by a symmetry or invariant principle in quantum mechanics. Here we emphasize that the invariance of velocity field is held for the gauge transformation.

Suppose there is a family of wave functions \( \psi^{(j)}, j = 1, 2, 3, ..., N \), which correspond to the same velocity field denoted by \( P_\mu = mu_\mu \), they are distinguishable from their different phase angles \( \theta \) as in Eq. (14). Then Eq. (14) can be given by

\[ 0 = P_\mu P^\mu \psi^{(j)} (\psi^{(j)} + m^2) \psi^{(j)} \]  

(19)

Suppose there are four matrices \( a_\mu \) which satisfy

\[ a_{\mu j} a_{\mu j} + a_{\mu j} a_{\nu j} = 2 \delta_{\mu \nu} \delta_{jk} \]  

(20)

then Eq. (19) can be rewritten as

\[ 0 = a_{\mu j} a_{\mu j} P_\mu \psi^{(j)} (\psi^{(j)} + m^2) \psi^{(j)} \]

\[ + (a_{\mu j} a_{\mu j} + a_{\mu j} a_{\nu j}) P_\mu \psi^{(j)} (\psi^{(j)} + m^2) \psi^{(j)} \]  

\[ + m c \psi^{(j)} ] [a_{\mu j} P_\mu \psi^{(j)} - i \delta_{jk} m c \psi^{(j)}] \]  

(21)

Where \( \delta_{jk} \) is Kronecker delta function, \( j, k, l = 1, 2, ..., N \). For the above equation there is a special solution given by

\[ [a_{\mu j} P_\mu - i \delta_{jk} m c] \psi^{(j)} = 0 \]  

(22)

There are many solutions for \( a_\mu \) which satisfy Eq. (20), we select a set of \( a_\mu \) as

\[ N = 4, \quad a_\mu = \gamma_\mu \quad (\mu = 1, 2, 3, 4) \]  

(23)

\[ \gamma_n = -i \beta \alpha_n \quad (n = 1, 2, 3), \quad \gamma_4 = \beta \]  

(24)

where \( \gamma_\mu, \alpha \) and \( \beta \) are the matrices defined in Dirac algebra[2] (P.557). Substituting them into Eq. (22), we obtain

\[ [ic (-i \hbar \partial_4 - \gamma A_4) + c_\alpha (-i \hbar \partial_4 - \gamma A_4) + \beta mc^2] \psi = 0 \]  

(25)
where $\psi$ is an one-column matrix about $\psi^{(k)}$. Then Eq.(25) is just the Dirac equation.

The Dirac equation is a linear wave equation, the principle of superposition is valid for it. Let index $s$ denote velocity field, then $\psi_s(x)$, whose four component functions correspond to the same velocity field $s$, may be regarded as the eigenfunction of the velocity field $s$ ( it may be different from the eigenfunction of energy ). Because the velocity field is an observable in a physical system, in quantum mechanics we know, $\psi_s(x)$ constitute a complete basis in which arbitrary function $\phi(x)$ can be expanded in terms of them

$$
\phi(x) = \int C(s)\psi_s(x)ds \tag{26}
$$

Obviously, $\phi(x)$ satisfies Eq.(25). Then Eq.(25) is just the Dirac equation suitable for composite wave function.

Alternatively, another method to show the Dirac equation is more traditional: At first, we show the Dirac equation suitable for composite wave function. From the Dirac equation we can predict that a spin state. That is why we want to classify particles into 1/2 spin, in other words, due to the approximation the classification is not necessary. It is noted that Eq.(21) is nonlinear while the Dirac equation is linear, this reminds us that we can never find any precise solutions in a linear equation which satisfy Eq.(21). Therefore, for this problem, a good solution depends on how much precision we can reach for our requirement.

In one hand, it is rather remarkable that Klein-Gordon equation and Dirac equation can be derived from relativistic Newton’s second law of motion approximately, in another hand, all particles, such as fermions, bosons and classical particles, satisfy the relativistic Newton’s second law (it will be further clear later), thus it is a natural choice that only the relativistic Newton’s second law is independent and necessary. Only one formalism is necessary for any particle, this is just the particle invariance, we arrive at the aim.

As mentioned above, the spin is one feature hidden in the relativistic Newton’s second law, but more features will turn out from the relativistic Newton’s second law in the following sections.

## 3 Determining the Planck’s constant

In this section we discuss how to determine the Planck’s constant that emerges in the preceding section.

In 1900, M. Planck assumed that the energy of a harmonic oscillator can take on only discrete values which are integral multiples of $h\nu$, where $\nu$ is the vibration frequency and $h$ is a fundamental constant, now either $h$ or $h = h/2\pi$ is called as Planck’s constant. The Planck’s constant next made its appearance in 1905, when Einstein used it to explain the photoelectric effect, he assumed that the energy in an electromagnetic wave of frequency $\omega$ is in the form of discrete quanta (photons) each of which has an energy $h\omega$ in accordance with Planck’s assumption. From then, it has been recognized that the Planck’s constant plays a key role in quantum mechanics.

According to the previous section, no matter how to move or when to move in Minkowski’s space, the motion of a particle is governed by a potential function $\Phi$ as

$$
mu_\mu + qA_\mu = \partial_\mu \Phi \tag{27}
$$

For applying Eq.(27) to specific applications without loss of generality, we set $\Phi = -i\kappa\psi$, then Eq.(27) is rewritten as

$$
(mu_\mu + qA_\mu)\psi = -i\kappa \partial_\mu \psi \tag{28}
$$

the coefficient $\kappa$ is subject to the interpretation of $\psi$. 

\[4\]
There are three mathematical properties of \( \psi \) worth recording here. First, if there is a path \( l_i \) joining initial point \( x_0 \) to final point \( x \), then
\[
\psi_i = e^{\frac{i}{\hbar} \int_{x_0(i_1)}^{x} (\mu \psi + q A_c) dx_c}
\] (29)
Second, the integral of Eq. (29) is independent from the choice of path. Third, the superposition principle is valid for \( \psi_i \), i.e., if there are \( N \) paths from \( x_0 \) to \( x \), then
\[
\psi = \sum_{i=1}^{N} \psi_i
\] (30)
where \( m \overline{\mu} \psi = \sum_{i=1}^{N} m \mu \psi_i / \sum_{i=1}^{N} \psi_i \) (31)

\[
(m \overline{\mu} + q A_c) \psi = -i \kappa \partial_\mu \psi
\] (32)

where \( m \overline{\mu} \) is the average momentum.

To gain further insight into physical meanings of this equations, we shall discuss two applications.

### 3.1 Two slit experiment

As shown in FIG. 2, suppose that the electron gun emits a burst of electrons at \( x_0 \) at time \( t = 0 \), the electrons arrive at the point \( x \) on the screen at time \( t \). There are two paths for the electron to go to the destination, according to our above statement, \( \psi \) is given by

\[
\psi = e^{\frac{i}{\hbar} \int_{x_0(i_1)}^{x} (\mu \psi + q A_c) dx_c}
\]
\[
+ e^{\frac{i}{\hbar} \int_{x_0(i_2)}^{x} (\mu \psi + q A_c) dx_c}
\]

(33)
where we use \( l_1 \) and \( l_2 \) to denote the paths \( a + b \) and \( c + d \) respectively. Multiplying Eq. (33) by its complex conjugate gives

\[
W = \psi(x) \psi^*(x)
\]
\[
= 2 + e^{\frac{i}{\hbar} \int_{x_0(i_1)}^{x} (\mu \psi + q A_c) dx_c} - \frac{1}{\kappa} \int_{x_0(i_1)}^{x} q A_c dx_c
\]
\[
+ e^{\frac{i}{\hbar} \int_{x_0(i_2)}^{x} (\mu \psi + q A_c) dx_c} - \frac{1}{\kappa} \int_{x_0(i_2)}^{x} q A_c dx_c
\]
\[
= 2 + 2 \cos[\frac{\hbar}{\kappa}(l_1 - l_2)]
\]
(34)

where \( p \) is the momentum of the electron. We find a typical interference pattern with constructive interference when \( l_1 - l_2 \) is an integral multiple of \( \kappa / p \), and destructive interference when it is a half integral multiple. This kind of experiments have been done since a long time ago, no matter what kind of particle, the comparison of the experiments to Eq. (34) leads to two consequences: (1) the complex function \( \psi \) is found to be probability amplitude. i.e., \( \psi(x) \psi^*(x) \) expresses the probability of finding a particle at location \( x \) in the Minkowski’s space. (2) \( \kappa \) is the Planck’s constant.

The integral of time component in the above calculation has been automatically canceled because the experimental pattern is stable.

### 3.2 The Aharonov-Bohm effect

Let us consider the modification of the two slit experiment, as shown in FIG. 3. Between the two slits there is located a tiny solenoid S, designed so that a magnetic field perpendicular to the plane of the figure can be produced in its interior. No magnetic field is allowed outside the solenoid, and the walls of the solenoid are such that no electron can penetrate to the interior. Like Eq. (33), the amplitude \( \psi \) is given by

\[
\psi = e^{\frac{i}{\hbar} \int_{x_0(i_1)}^{x} (\mu \psi + q A_c) dx_c} + e^{\frac{i}{\hbar} \int_{x_0(i_2)}^{x} (\mu \psi + q A_c) dx_c}
\]
and the probability is given by

\[
W = \psi(x) \psi^*(x)
\]
\[
= 2 + e^{\frac{i}{\hbar} \int_{x_0(i_1)}^{x} (\mu \psi + q A_c) dx_c} - \frac{1}{\kappa} \int_{x_0(i_1)}^{x} q A_c dx_c
\]
\[
+ e^{\frac{i}{\hbar} \int_{x_0(i_2)}^{x} (\mu \psi + q A_c) dx_c} - \frac{1}{\kappa} \int_{x_0(i_2)}^{x} q A_c dx_c
\]
\[
= 2 + 2 \cos[\frac{\hbar}{\kappa}(l_1 - l_2)]
\]
(35)
\[ = 2 + 2 \cos\left(\frac{\phi}{\kappa}\right) \]  
(36)

where $\overline{l_2}$ denotes the inverse path to the path $l_2$, $\phi$ is the magnetic flux that passes through the surface between the paths $l_1$ and $\overline{l_2}$, and it is just the flux inside the solenoid.

In the present paper, Eq.(39) has been elevated to an essential requirement for definition of force, which brings out many new aspects for Coulomb’s force and gravitational force.

\[ u_\mu f_\mu = u_\mu m \frac{du_\mu}{d\tau} = \frac{m}{2} \frac{d(u_\mu u_\mu)}{d\tau} = 0 \]  
(39)

This simple inference clearly tells us that the forces are not centripetal or centrifugal forces about their sources, even if in 3-dimensional space [see Eq.(47)], this character provides a internal reason for accounting for the quantum behavior of particle or chaos. Thus the derivations in terms of 4-vector velocity field in the preceding section become reasonable.

In this section we shall correct a mistake about Coulomb’s force and gravitational force in physical education, which cluttered the path from classical mechanics to quantum mechanics. We also shall discuss Maxwell’s equations in detail.

In the world, almost every young person was educated to know that the Coulomb’s force and gravitational force act along the line joining a couple of particles, but this knowledge is incorrect in the theory of relativity.

In relativity theory, the 4-vector velocity $u$ of a particle has components $u_\mu$, the magnitude of the 4-vector velocity $u$ is given by

\[ |u| = \sqrt{u_\mu u_\mu} = \sqrt{-c^2} = ic \]  
(38)

The above equation is valid so that any force can never change $u$ in the magnitude but can change $u$ in the direction. We therefore conclude that the Coulomb’s force and gravitational force on a particle always act in the direction orthogonal to the 4-vector velocity of the particle in the 4-dimensional space-time, rather than along the line joining a couple of particles. Alternatively, any 4-vector force $f$ satisfy the following perpendicular or orthogonal relation

\[ u_\mu f_\mu = u_\mu m \frac{du_\mu}{d\tau} = \frac{m}{2} \frac{d(u_\mu u_\mu)}{d\tau} = 0 \]  
(39)

4.1 Coulomb’s force and Lorentz force

We assume that Coulomb’s law remains valid only for two particles both at rest in usual 3-dimensional space. Suppose there are two charged particle $q$ and $q'$ locating at positions $x$ and $x'$ in a Cartesian coordinate system $S$ and moving at 4-vector velocities $u$ and $u'$ respectively, as shown in FIG.4, where we use $X$ to denote $x - x'$. The Coulomb’s force $f$ acting on particle $q$ is perpendicular (orthogonal) to the velocity direction of $q$, as illustrated in FIG.4 like a centripetal force, the force $f$ should make an attempt to rotate itself about its path center, the center may locate at the front or back of the particle $q'$, so the force $f$ should lie in the plane of $u'$ and $X$, then

\[ f = Au' + BX \]  
(40)

where $A$ and $B$ are unknown coefficients, the possibility of this expansion will be further clear in the next subsection in where the expansion is not an assumption [see Eq.(51)]. Using the relation $f \perp u$, we get

\[ u \cdot f = A(u \cdot u') + B(u \cdot X) = 0 \]  
(41)

we rewrite Eq.(40) as

\[ f = \frac{A}{u \cdot X}[(u \cdot X)u' - (u \cdot u')X] \]  
(42)

It follows from the direction of Eq.(42) that the unit vector of the Coulomb’s force direction is given by

\[ \hat{f} = \frac{1}{c}[(u \cdot X)u' - (u \cdot u')X] \]  
(43)

because
\[ \hat{f} = \frac{1}{c^2 r^2} [(u \cdot X)u' - (u \cdot u')X] \]
\[ = \frac{1}{c^2 r^2} [(u \cdot R)u' - (u \cdot u')R] \]
\[ = -[(\hat{u} \cdot \hat{R})\hat{u}' - (\hat{u} \cdot \hat{u}')\hat{R}] \]
\[ = -\hat{u}' \cosh \alpha + \hat{R} \sinh \alpha \]  
\hspace{1cm} (44)

Where \( \alpha \) refers to the angle between \( u \) and \( R \), \( R \perp u' \), \( r = |R| \), \( \hat{u} = u/|c| \), \( \hat{u}' = u'/|c| \), \( \hat{R} = R/|r| \). Suppose that the magnitude of the force \( f \) has classical form \[ |f| = kqq' \]  
\hspace{1cm} (45)
Combination of Eq. (40) with (43), we obtain a modified Coulomb’s force

\[ f = \frac{kqq'}{c^2 r^2} [(u \cdot X)u' - (u \cdot u')X] \]
\[ = \frac{kqq'}{c^2 r^2} [(u \cdot R)u' - (u \cdot u')R] \]  
\hspace{1cm} (47)
This force is in the form of Lorentz force for the two particles, relating with the Ampere’s law and Biot-Savart-Laplace law.

\[
\begin{align*}
\partial_{\mu} A_{\nu} &= \frac{kq'u'_\mu}{c^2} \partial_{\nu} \left( \frac{1}{r} \right) = -\frac{kq'u'_\mu}{c^2} \left( \frac{R_{\nu}}{r^3} \right) = 0 \tag{52}
\end{align*}
\]

It is known as the Lorentz gauge condition.

**4.3 Maxwell’s equations**

To note that \( R \) has three degrees of freedom on the condition \( R \perp u' \), so we have
\[
\partial_{\rho} R_{\mu} = 3 \tag{53}
\]
From Eq. (49), by exchanging the indices and taking the summation of them, we have
\[
\partial_{\nu} F_{\mu\nu} = \partial_{\nu} \partial_{\mu} A_{\nu} - \partial_{\nu} \partial_{\mu} A_{\mu} = -\partial_{\nu} \partial_{\mu} A_{\mu} = \frac{kq'u'_\mu}{c^2} \partial_{\nu} \partial_{\nu} \left( \frac{1}{r} \right) = \frac{kq'u'_\mu}{c^2} 4\pi \delta(R) \tag{54}
\]

where we define \( J'_\nu = q'u'_\nu \delta(R) \). From Eq. (49), by exchanging the indices and taking the summation of them, we have
\[
\partial_{\lambda} F_{\mu\nu} + \partial_{\mu} F_{\nu\lambda} + \partial_{\nu} F_{\lambda\mu} = 0 \tag{56}
\]
The Eq. (55) and (56) are known as the Maxwell’s equations. For continuous media, they are valid as well.

**4.4 Lienard-Wiechert potential**

From the Maxwell’s equations, we know there is a retardation time for action to propagate between the two particles, the retardation effect is measured by the distance \( r = c\Delta t = c\frac{q'O}{ic} \)
\[
= c\frac{\hat{u}' \cdot X}{ic} = \frac{u'_\nu(x'_\nu - x_\nu)}{c} \tag{57}
\]
as illustrated in FIG. 4. Then
\[
A_{\mu} = \frac{kq'u'_\mu}{c^2 r} = \frac{kq'}{c} \frac{u'_\mu}{c} (x'_\nu - x_\nu) \tag{58}
\]
Obviously, Eq. (58) is known as the Lienard-Wiechert potential for a moving particle.
The above formalism clearly shows that Maxwell’s equations can be derived from the classical Coulomb’s force and the perpendicular (orthogonal) relation of force and velocity. In other words, the perpendicular relation is hidden in Maxwell’s equation. Specially, Eq. (52) directly accounts for the geometrical meanings of curl of vector potential, the curl contains the perpendicular relation. Since the perpendicular relation of force and velocity is one of the consequences from relativistic Newton’s second law, it is also one of the features from the particle invariance.

4.5 Gravitational force

The above formalism has a significance on guiding how to develop the theory of gravity. In analogy with the modified Coulomb’s force of Eq. (47), we directly suggest a modified universal gravitational force as

\[
f = \frac{-Gmm'}{\varepsilon_0^2r^3}[(u \cdot X)u' - (u \cdot u')X]
\]

for a couple of particles with masses \(m\) and \(m'\) respectively.

Comparing with some incorrect statements about Coulomb’s force and gravitational force in most textbooks, and for emphasizing its feature, the perpendicular (orthogonal) relation of force and velocity was called the direction adaptation nature of force in the author’s previous paper [11].

4.6 The Magnet-like components of the gravitational force

We emphasize that the perpendicular relation of force and velocity must be valid if gravitational force can be defined as a force. It follows from Eq. (52) that we can predict that there are gravitational radiation and magnet-like components for the gravitational force. Particularly, the magnet-like components will act as a key role in the physics and atmosphere physics.

If we have not any knowledge but know there exists the classical universal gravitation \(f\) between two particles \(m\) and \(m'\), what form will take the 4-vector gravitational force \(f'\)? Suppose that \(m'\) is at rest at the origin, using \(u = (u, u_4)\), \(u' = (0, 0, 0, ic)\) and \(u \cdot f = 0\), we have

\[
f = \frac{u_4 f_4}{u_4} = \frac{u \cdot f}{u_4} - \frac{u \cdot f}{u_4} = \frac{u \cdot f}{u_4}
\]

where \(R \perp u\), \(R = (R, 0)\). If we "rotate" our frame of reference to make \(m'\) not to be at rest, Eq. (59) will still be valid because of covariance. Then we find the 4-vector gravitational force goes back to the form of Eq. (59), like Lorentz force, having the magnet-like components.

It is noted that the perpendicular relation of force and velocity is valid for any force: strong, electromagnetic, weak and gravitational interactions, therefore there are many new aspects remaining for physics to explore.

5 Interactions between particles

Under the invariance of particle, the most simple model of particle is that all particles are composed of identical constituents, the constituent is regarded as "the most elementary and most small particle" in the world. Since quarks have never been observed, our speculation leads us to propose a better model to organize known data. For this challenging purpose, in the present paper, we introduce a fictitious elementary particle, given a name "Dolland" for our convenience, to assemble other particles such as fermions, mesons or classical particles, the Dollon is regarded as "the most elementary and most small particle" in the world. Our work focuses on conceptual development.

5.1 Basic force

Consider a Dollon moving in Minkowski’s space \((x_1, x_2, x_3, x_4 = ic)\) with 4-vector velocity \(u = (u, u_4)\), the motion of the Dollon satisfies the magnitude formula of 4-vector velocity of particle

\[
u_\mu u_\mu = -c^2
\]

Differentiating the above equation with respect to the proper time interval \(d\tau\) of the Dollon gives

\[
\frac{du}{d\tau} = f
\]

where the result has been written in the two parts by defining a 3-dimensional vector \(f\). Defining a 4-vector

\[
f = (f, -\frac{u \cdot f}{u_4})
\]

then from Eq. (62) we have readily
\begin{equation}
\frac{du}{d\tau} = f \quad u \cdot f = u_\mu f_\mu = 0 \quad (65)
\end{equation}

It means that \( u \) and \( f \) are orthogonal with each other.

Consider two particles "Bob" and "Alice" located at \( x \) and \( x' \) in the 4-dimensional space respectively, they are composed of many Dollons, in Alice is \( M \), and in Bob is \( m \), when Bob and Alice move with 4-vector velocities \( u \) and \( u' \) respectively, following Eq.(63), they can be assigned two sets of motion equations as

\begin{equation}
Bob: M \frac{du}{d\tau} = Mf \quad m \frac{du}{d\tau} = -m \frac{u \cdot f}{u_4} \quad (66)
\end{equation}

\begin{equation}
Alice: M \frac{du'}{d\tau'} = Mf' \quad M \frac{du'}{d\tau'} = -M \frac{u' \cdot f'}{u_4'} \quad (67)
\end{equation}

Now we have a question: what is the interaction between Bob and Alice? Obviously, the form of Eq.(66) seems to be relativistic Newton second law for Bob, \( f \) seems to be a 3-vector force, \( u \cdot f \) seems to be the rate at which the force does work on Bob. For seeking for further answers, we need to recall the Newton’s first law of motion, the law is valid in theory of relativity and reads

First Law: An object at rest will remain at rest and an object in motion will continue to move in a straight line at constant speed forever unless some net external force acts to change this motion.

If the object is a composite system composed of many Dollons, then we can understand the First Law with three consequences.

Consequence 1: Let \( S \) denote the number of Dollons in a composite system, the average velocity of the system is defined as

\begin{equation}
u_c = \frac{1}{S} \sum_{i} u^{(i)} \quad (68)
\end{equation}

where \( u^{(i)} \) is the 4-vector velocity of the \( i \)th Dollon. The average velocity represents the motion of the center of the system. The First Law only means that the center of the system remains at rest or in motion, i.e., rotation about its center is permitted.

Consequence 2: The total number of Dollons in the system must be unchanged, i.e., the conservation of Dollon number must be held, otherwise any creation or annihilation of Dollon will lead to a sudden shift of the center of the system.

Consequence 3: When two bodies are seperated from an infinite distance, the interaction between them must vanish. Otherwise, no body can be at rest, because a rest body will always be affected by the motion of a far distance body, whereas the far distance bodies are innumerable as a background.

Now we go back to consider the whole system composed of Bob and Alies, without loss of generality, suppose that the center is at rest at the origin of the frame of reference, then the center has a 4-vector velocity \( u_c = (0, 0, 0, u_{4c}) \), the "at rest" refers to being at rest in usual 3-dimensional space. From Eq.(69), the quantity \( f \) must be orthogonal with the 4-vector velocity \( u \) of Bob, likewise for Alice, we have

\begin{align}
Bob: & \quad u \cdot f = u_\mu f_\mu = 0 \quad (69) \\
Alice: & \quad u' \cdot f' = u'_\mu f'_\mu = 0 \quad (70)
\end{align}

They set up a rule for the interaction between Bob and Alice in the composite system. We specially choose to study the interaction which happens at such instant that the position vector \( X \) of Bob with respect with Alice (i.e., \( X = x - x' \)) is orthogonal to \( u \) and \( u' \) simultaneously.

\begin{align}
Bob: & \quad u \cdot X = 0 \quad (71) \\
Alice: & \quad u' \cdot X = 0 \quad (72)
\end{align}

The existence of such instant of time will become clear in the subsection 5.3. From Eq.(69) and Eq.(71), we get parallel relations

\begin{align}
Bob: & \quad f \propto X \quad (73) \\
Alice: & \quad f' \propto X \quad (74)
\end{align}

For Bob, using notation \( X = (r, X_4) \), \( r = |r| \), vector-multiplying Eq.(66) by \( r \), because \( f \) parallels \( r \), we have

\begin{equation}
0 = r \times (M \frac{du}{d\tau}) = M \frac{d(r \times u)}{d\tau} = m r \times f = 0 \quad (75)
\end{equation}

It means

\begin{equation}
r \times u = h = const. \quad (76)
\end{equation}

where \( h \) is an integral constant. Likewise for Alice. From Eq.(73) we can expand \( f/\mu_4 \) in a Taylor series in \( 1/r \), this gives

\begin{equation}
f = \frac{r}{r} \left[b_0 + b_1 \frac{1}{r} + b_2 \frac{1}{r^2} + b_3 \frac{1}{r^3} + \ldots\right] \quad (77)
\end{equation}

From Eq.(73) we obtain

\begin{align}
u_4 = & \quad \int (-\frac{u \cdot f}{u_4}) d\tau = -\int \frac{|f|}{u_4} dr \\
= & \quad \varepsilon - b_0 r - b_1 \ln r + b_2 \frac{1}{r} + b_3 \frac{1}{r^2} + \ldots \quad (78)
\end{align}

where \( \varepsilon \) is an integral constant. Now consider Eq.(76), it means that Bob moves around Alice (no matter by attractive or repulsive interaction), when \( h \to 0 \), Bob
may access Alice as close as possible at perihelion point, at the perihelion point we find
\[ h^2 = |\mathbf{r} \times \mathbf{u}|^2 = r^2 u^2_{\text{perihelion}} = r^2 (-c^2 - u^2_{\text{perihelion}}) \]
where we have used point about Alice with a distance \( d \mathbf{r} \). Substituting Eq.(80) and Eq.(82)
we get
\[ \frac{du}{dt} = \frac{icb}{M} = - \frac{ica}{m} = K \quad (86) \]
This equation leads to
\[ \frac{d}{dt} \left( \frac{mM \mathbf{r}}{r^3} \right) = -K \frac{mM \mathbf{r}}{r^3} \quad (87) \]
If \( K \) takes a negative constant, then, the above equations show that Bob is attracted by Alice with Newton’s universal gravitation force. But we do not want to make this conclusion at once, because there are still a few problems among them.

### 5.2 Coulomb’s force

In this subsection, we study Coulomb’s force by based on our the most simple model: all particles are composed of identical constituents—Dollons.

From the above subsection, now we can manifestly interpret the quantity \( f \) as the 4-vector force exerting on a Dollon of Bob. It is a natural idea to think of that Dollon has two kinds of charges: positive and negative. If Bob and Alice are separated by a far distance, and \( f \) is the force acting on a positive Dollon in Bob, then \(-f\) is the force acting on a negative Dollon in Bob. Regardless of the internal forces in Bob, it follows from Eq.(82) that the motion of the \( i \)th Dollon is governed by
\[ \frac{df^{(i)}}{dt^{(i)}} = f^{(i)} \quad \text{or} \quad \frac{df^{(i)}}{dt^{(i)}} = \frac{icf^{(i)}}{u^{(i)}} \quad (89) \]
where \( df^{(i)} \), \( u^{(i)} \) and \( f^{(i)} \) denote the proper time interval, 4-vector velocity and 4-vector force acting on the \( i \)th Dollon, respectively. Taking sum over all Dollons in Bob, we get
\[ \sum_{i=1}^{m} \frac{du^{(i)}}{dt^{(i)}} = \frac{d}{dt} \left( \sum_{i=1}^{m} u^{(i)} \right) = \frac{d(mu_{e})}{dt} \quad (90) \]
\[ \sum_{i=1}^{m} \frac{icf^{(i)}}{u^{(i)}} \simeq \frac{icf_{e}}{u_{e4}} \quad (91) \]
where \( u_{e} \) is the 4-vector velocity of the center of Bob, \( u_{e4} \) denotes its 4th component, \( q \) denotes the net charges of Bob, \( f_{e} \) denotes the 4-vector force acting on the Dollon located at the center of Bob (this Dollon may be virtual one because it features the average action). Combining Eq.(90) and Eq.(91) with Eq.(89), we obtain
\[ \frac{d(mu_{e})}{dt} = q \frac{icf_{e}}{u_{e4}} \quad \text{or} \quad \frac{d(mu_{e})}{d\tau_{e}} = q f_{e} \quad (92) \]
where we neglect the approximation in Eq. (91).

Like that in the above subsection, the First Law must be valid for the composite system of Bob and Alice, in other words, when they are separated from a infinite distance they are isolated, whereas they go to nearest points they should not touch each other, these requirements lead to

$$\text{Bob} : \frac{f_c}{u_{c4}} = \frac{b}{r^3} \quad u_{c4} = \varepsilon + \frac{b}{r} \quad (93)$$

$$\text{Alice} : \frac{f'_c}{u'_{c4}} = \frac{a}{r^3} \quad u'_{c4} = \varepsilon' + \frac{a}{r} \quad (94)$$

where $\varepsilon, \varepsilon', b$ and $a$ are coefficients. Without loss of generality, we have

$$\frac{du_c}{dt} = 0 = \left( \frac{ic}{u_{c4}} \frac{d(mu_c)}{dr_c} + \frac{ic}{u'_{c4}} \frac{d(Mu'_{c})}{dr'_{c}} \right) / (m + M) \quad (95)$$

Substituting Eq. (93) and Eq. (94) into Eq. (95), we get

$$\frac{ic}{u_{c4}} qf_c + \frac{ic}{u'_{c4}} q'f'_c = \frac{br}{r^3} + \frac{i cq'ar}{r^3} = 0 \quad (96)$$

where $q'$ denotes the net charges of Alice. This equation leads to

$$\frac{icb}{q'} = - \frac{ica}{q} = k \quad (97)$$

where $k$ is a constant. Then the motions of Bob and Alice are governed by

$$\text{Bob} : \quad m \frac{du}{dt} = kqqr \quad (98)$$

$$\text{Alice} : \quad M \frac{du'}{dt} = -kqqr \quad (99)$$

The 4th component equations corresponding to the above equations express the energy change rates of Bob and Alice, they are not independent components.

The Eq. (98) and Eq. (99) are known as the Coulomb’s forces. That the net force acting on Bob is proportional to the number of Dollons in Bob, Eq. (111) reads

$$\sum_{i=1}^{m} \frac{icf^{(i)}}{u_{4i}^3} = g \frac{icm_{f_c}}{u_{c4}} \quad (100)$$

where $g$ is a very very small proportional coefficient. Then the motion of Bob is given by

$$\frac{d(mu)}{dt} = g \frac{icm_{f_c}}{u_{c4}} \quad \text{or} \quad \frac{d(mu)}{dt} = gm_{f_c} \quad (101)$$

In analogy with the above subsections, we may obtain the motion equations of Bob and Alice, they are governed by

$$\text{Bob} : \quad m \frac{du}{dt} = -G \frac{mMr}{r^3} \quad (102)$$

$$\text{Alice} : \quad M \frac{du'}{dt} = G \frac{mMr}{r^3} \quad (103)$$

where $G$ is a constant proportional to $g$.

The $m$ and $M$ has been identified or defined as the masses by employing Dollon mass as a unit when we count the Dollon numbers in Bob or Alice. The Eq. (102) and Eq. (103) are known as the Newton’s universal gravitational forces.

Why is the net force of Bob attractive? This may be explained as that electrons with light masses move always around massive nuclei, the attraction is a little bigger than the repulsion between two atoms separated by a far distance. In this formulation, the gravitational force possesses statistic meanings.

### 5.4 Nuclear force

We use the most simple model— all particles are composed of identical Dollons— to study nuclear force, to fulfill the conceptual development boosted by the Newton’s first law of motion.

Now consider that Bob and Alice are two nucleons composed of Dollons. If Bob and Alice go closely in a distance comparable with the sizes of them, then it is clear that Eq. (111) turns to be inadequate, their polarization can provide a strong interaction, while the effect of net charges between their centers becomes to be trivial. The strong interaction is regarded as the nuclear force in this paper. Therefore, the strong nuclear force is charge-independent, it only comes into play when the nucleons are very close together, and it drops rapidly to Coulomb’s force for far distance, we know from experiments that the sensitive distance is about $10^{-15}m$.

As mentioned above, the ith Dollon in Bob is governed by
\[ \frac{d\mathbf{u}^{(i)}}{dt^{(i)}} = \mathbf{f}^{(i)} \]
\[ \frac{d\lambda u^{(i)}}{dt^{(i)}} = -\lambda \frac{\mathbf{u}^{(i)} \cdot \mathbf{f}^{(i)}}{u_{4}^{(i)}} \]  
(104)

Then the motion of Bob is given by
\[ \frac{d(m\lambda u)}{dt} = i\mathbf{c} \sum_{i=1}^{m} \frac{\mathbf{f}^{(i)}}{u_{4}^{(i)}} \]  
(105)
\[ \frac{d(m\lambda u_{4})}{dt} = -i\mathbf{c} \sum_{i=1}^{m} \frac{\mathbf{u}^{(i)} \cdot \mathbf{f}^{(i)}}{|u_{4}^{(i)}|^2} \]  
(106)

where
\[ m\lambda u_{4} = \sum_{i=1}^{m} u^{(i)} \]  
(107)

where \( u_{c} \) can be understood as the velocity of momentum center (see Eq.\[\text{107}\]), but \( u_{c} \) is not the relativistic velocity of the geometrical center of Bob, the relativistic velocity of the geometrical center of Bob is defined by using its geometrical center proper time, i.e., \( u_{\text{center}} = \frac{d\mathbf{x}_{\text{center}}}{d\tau_{\text{center}}} \), thus we have to establish their relation by introducing a correcting factor \( \lambda \) so that
\[ u_{c} = \lambda u_{\text{center}} \text{, i.e.,} \]
\[ \sum_{i=1}^{m} u^{(i)} = m\lambda = \lambda m u_{\text{center}} \]  
(108)

In the following we drop the subscript "center" when without confusion, then above equations can be rewritten as
\[ \frac{d(m\lambda \mathbf{u})}{dt} = i\mathbf{c} \sum_{i=1}^{m} \frac{\mathbf{f}^{(i)}}{u_{4}^{(i)}} \]  
(109)
\[ \frac{d(m\lambda u_{4})}{dt} = -i\mathbf{c} \sum_{i=1}^{m} \frac{\mathbf{u}^{(i)} \cdot \mathbf{f}^{(i)}}{|u_{4}^{(i)}|^2} \]  
(110)

To note that the right side of Eq.\[\text{110}\] is the rate at which the forces do works on Bob, then the quantity \( m\lambda u_{4} \) in the left side should be "energy", thus we can define the energy as
\[ E = -i\mathbf{c} \lambda m u_{4} = m_{r} \lambda c^{2} \]  
(111)
\[ m_{r} = \frac{m}{\sqrt{1-u^{2}/c^{2}}} \]  
(112)

where \( u_{4} = i\mathbf{c}/\sqrt{1-u^{2}/c^{2}} \), \( v \) is the classical speed of the geometrical center of Bob, \( m_{r} \) is the relativistic mass, while \( m \) is the rest mass. Eq.\[\text{111}\] is known as the energy mass relationship. but Eq.\[\text{111}\] has a little difference from Einstein’s mass-energy relationship. Our energy formula contains a factor \( \lambda \) that represents the internal motion of Dollons in Bob, obviously, \( \lambda \geq 1 \), this can be seen clearly from Eq.\[\text{108}\], in other words, even if the center is at rest, the internal constituents can still have relativistic energies.

In dealing with nuclear reaction, in many textbooks, mass defect is understood as the decrease in total relativistic mass, even if all nuclei seem to be at rest before or after the nuclear reaction—the total relativistic masses should not have apparent change. We have been puzzled by these statements for a long time. Now the reasons are clear, no relativistic masses change but \( \lambda \) changes in these cases, in other words, the internal energy of particle has changed. \( \lambda \) is a physical quantity sensitive to the internal structure of a particle, is a criteria for particle being elementary or not.

Consider that a hadron possesses net charge \( q \), we can naturally image that the charge distributes in several parts inside the hadron, assuming three parts, the three parts have net charges denoted by \( I_{q} \), \( B_{q} \), and \( S_{q} \) respectively, then
\[ q = I_{q} + B_{q} + S_{q} \]  
(113)
Comparing with the Gell-Mann-Nishijima relation
\[ q = I_{3} + \frac{B + S}{2} \]  
(114)
we can understand the conservations of isospin \( I_{3} \), baryon number \( B \) and strangeness number \( S \) with four remarks: (1) the three parts inside the hadron are insulated from one another, no charge transports from one to another. (2) during collision of hadrons, only identical parts impact or touch each other, with exchanging net charges. (3) the mass of the hadron seems to depend primarily on the masses of the parts inside the hadron, weakly on the net charges of the parts. (4) if we assign the quantum states of quarks \( u, s \) and \( d \) to the three parts, the quark model seems to be improved in a manner that we can avoid the fractional charges of the quarks.

5.5 Determining the 4-vector \( X \)

In the preceding subsections, we have mentioned that the interaction between Bob and Alice we studied happens at such instant that their relative position in the Minkowski’s space is denoted by a 4-vector \( X = (r, X_{4}) = (\mathbf{r}, ic\Delta t) \), \( X \) satisfies the orthogonal relation simultaneously
\[ u \cdot X = 0 \quad u' \cdot X = 0 \]  
(115)
The purpose of choosing this instant is to meet the convenience that \( X \) parallels to \( f \) and \( f' \) simultaneously, because
\[ Bob \quad : \quad u \cdot f = u_{\mu}f_{\mu} = 0 \]  
(116)
\[ Alice \quad : \quad u' \cdot f' = u'_{\mu}f'_{\mu} = 0 \]  
(117)
See Eq.(69)-(74). Eq.(113) can be rewritten in the form of inner product of two vectors as

\[ |u| \cdot |X| \cosh(u, X) = |u'| \cdot |X| \cosh(u', X) = 0 \quad (118) \]

This leads to two solutions given by

\[ |X| = \sqrt{r^2 - (c\Delta t)^2} = 0 \quad (119) \]
\[ \cosh(u, X) = \cosh(u', X) = 0 \quad (120) \]

Eq.(118) again leads to two solutions given by

\[ r = c\Delta t \quad r = -c\Delta t \quad (121) \]

The first solution expresses that the force acting on Bob is retarded by time \( \Delta t = r/c \), the second one expresses that the action is preceded. Our choice is the first one which gives an effect that follows the cause. We know, this retarded time is just the time needed for the propagation of interaction from Alice to Bob, the propagation speed is \( c \), no matter what kind of interaction.

Eq.(120) represents the orthogonal relationship.

Therefore, the interaction happens at such instant that either in retarded state or in orthogonal state, or mixture.

6 Minkowski's space

In preceding sections, we have realized that relativistic Newton's second law and forces can be derived from Newton's first law and the magnitude formula of 4-vector velocity of particle. The formula is given by

\[ u_{\mu}u_{\mu} = -c^2 \quad (122) \]

in a Minkowski's space. It is noted that all particles satisfy the above equation, then it is regarded as the origin of the particle invariance. We wonder at what is the essence of the Minkowski's space. In this section we shall discuss the Minkowski's space, for this purpose we need to establish a standard method for describing the motion of particle in space-time. Our construction follows four steps.

6.1 First Step: we are lazy

Suppose Alice is a pretty girl being famous for her fast running records, we state some her records here in a story (in imagination).

(1) Jan., 1, 2001, 10:00 am, sportsground in BUAA, Beijing. In a time interval \( \Delta t = 10s \) Alice ran a straight line distance \( \Delta l_1 = 100m \) at a constant speed \( v_1 = 10m/s \).

This data can be given in physical terms by

\[ \Delta l_1 = v_1\Delta t \quad (123) \]

It can be rewritten either as

\[ \Delta x_1^2 + \Delta y_1^2 = (v_1\Delta t)^2 \quad (124) \]

or as

\[ \Delta x_1^2 + \Delta y_1^2 - (v_1\Delta t)^2 = 0 \quad (125) \]

where \( x \) and \( y \) denote the coordinate system fixed at the sportsground. By defining a imaginary quantity \( w_1 = iv_1t \), the data is given by

\[ \Delta x_1^2 + \Delta y_1^2 + \Delta w_1^2 = 0 \quad (126) \]

We appreciate the simplicity and beauty of its form.

It is also our favorite manner to mark the running process in a graph with three mutually perpendicular axes \( x, y \) and \( w = iv_1t \). The distance from the starting point to the final point in this coordinate system equals to zero because of Eq.(124). This graph we called as "20010101 Graph".

(2) Jan., 2, 2001, 10:00 am, sportsground in BUAA, Beijing. In a time interval \( \Delta t = 10s \) Alice ran a straight line distance \( \Delta l_2 = 90m \) at a constant speed \( v_2 = 9m/s \).

This data is given in physical terms by

\[ \Delta x_2^2 + \Delta y_2^2 + \Delta w_2^2 = 0 \quad \Delta w_2 = iv_2\Delta t \quad (127) \]

We directly mark this day running process in the yesterday's 20010101 Graph, we are lazy to draw a new graph.

(3) Jan., 3, 2001, 10:00 am, sportsground at BUAA, Beijing. In a time interval \( \Delta t = 10s \) Alice ran a straight line distance \( \Delta l_3 = 95m \) at a constant speed \( v_3 = 9.5m/s \).

We also directly mark the running process in the 20010101 Graph.

Bob was also a good runner, in a time interval \( \Delta t = 10s \) Bob ran a straight line distance \( \Delta l_5 = 105m \) at a constant speed \( v_5 = 10.5m/s \).

We also directly mark the running process in the 20010101 Graph.

In fact, their running records all are marked in the 20010101 Graph.

6.2 Second Step: Establishing a temporary standard frame

Because of laziness, we only use the 20010101 Graph to record the running data, it has actually become a temporary standard frame, all motions can be marked or calculated in the Graph, it is much convenient for describing any movement.

To note that \( w = iv_1t \), the \( w \) axis in the Graph has involved the speed \( v_1 \) created by Alice on Jan 1, 2001. Thus we find that the geometrical distance \( \Delta s_2 \) from the
It is clear after comparing with Eq. (127). So do for Bob, the distance \( \Delta s_b \) for Bob in the 20010101 Graph is given by

\[
\Delta s_b = \Delta x_b^2 + \Delta y_b^2 + \Delta w_b^2 = v_b^2 \Delta t^2 - v_b^2 \Delta t^2 + v_b^2 \Delta t^2 - v_b^2 \Delta t^2 = -v_b^4 \Delta t^2 (1 - v_b^2/v_1^2) 
\]

Dividing the two sides of the above equation by \( \Delta t^2 (1 - v_b^2/v_1^2) \), we get

\[
-v_1^2 = \left( \frac{\Delta x_b/\Delta t}{\sqrt{1 - v_b^2/v_1^2}} \right)^2 + \left( \frac{\Delta y_b/\Delta t}{\sqrt{1 - v_b^2/v_1^2}} \right)^2 + \left( \frac{\Delta w_b/\Delta t}{\sqrt{1 - v_b^2/v_1^2}} \right)^2 
\]

Defining modified velocity

\[
u_x = \frac{v_x}{\sqrt{1 - v^2/v_1^2}}, \quad \nu_y = \frac{v_y}{\sqrt{1 - v^2/v_1^2}}, \quad \nu_w = \frac{iv_1}{\sqrt{1 - v^2/v_1^2}}
\]

where \( v^2 = v_x^2 + v_y^2 + v_w^2 \), we have dropped the subscript \( b \) that indicates Bob, then Eq. (131) is given by

\[
u_x^2 + \nu_y^2 + \nu_w^2 = -v_1^2
\]

The modified velocity of Bob in the 20010101 Graph is based on the Alice’s best speed record \( v_1 \). In fact, Any one, any body or any particle, their modified velocity in the 20010101 Graph satisfies Eq. (135).

### 6.3 Third Step: Standard Graph based on the light speed

Because of convenience, it has become a habit for us to use the 20010101 Graph to mark the all motions of any body. It is sure that not all scientists in the world like Alice, then we gradually recognize that we need a permanent runner for establishing a standard graph. Now we had to face a new task: to look for a new hero.

It was said that the light, an element of the nature, is the fastest runner, whenever and whereever its speed is \( 3 \times 10^8 \text{m/s} \). We do not hesitate to use the light speed to replace Alice’s speed, and setup a new frame called “Standard Graph”, the Standard Graph contains four mutually perpendicular axes \( x, y, z \) and \( w =ict \). (we can draw several partial frames to assemble the whole frame). From then, any motions can be described in the Standard Graph with the space-time \( (x, y, z,ict) \) or \( (x_1, x_2, x_3, x_4 = ict) \). In analogy with Eq. (128)-(135), defining modified velocity

\[
u_x = \frac{v_x}{\sqrt{1 - v^2/c^2}}, \quad \nu_y = \frac{v_y}{\sqrt{1 - v^2/c^2}}, \quad \nu_z = \frac{v_z}{\sqrt{1 - v^2/c^2}}, \quad \nu_w = \frac{ic}{\sqrt{1 - v^2/c^2}}
\]

where \( v^2 = v_x^2 + v_y^2 + v_z^2 \), we obtain

\[
u_x^2 + \nu_y^2 + \nu_z^2 + \nu_w^2 = -c^2 \quad u_\mu u_\mu = -c^2
\]

The Standard Graph is just the Minkowski’s space, the 4-vector velocity \( u = \{u_\mu\} \) is known as the relativistic velocity.

### 6.4 Fourth Step: Transformations

We immediately recognize that physics holds its validity only in the Standard Graph (involving with the light speed), rather than in the 20010101 Graph (involving with the Alice’s speed), this situation can be explained by the fact that all physical quantities and their measurements are defined on facilities whose principles are based on the light directly or indirectly, for example, the "meter" and "second" are defined on the light speed directly.

If we do not hope that one graph has advantage over another, then the transformation between the Standard Graph and 20010101 Graph will be given by

\[
x = x_1, \quad y = y_1, \quad z = z_1, \quad t = \frac{v_1}{c}t_1
\]

where the subscript 1 denotes in the 20010101 Graph. It means we need to redefine all physical quantities such as rod and clock in the 20010101 Graph, do not use the light.

### 7 Dynamics in the Hilbert space

A complete inner product space is called a Hilbert space. Our experience in the preceding sections tells us that it
is an easy thing to put dynamics into the Hilbert space if we have an invariant quantity. The formalism of the interaction can be derived from some basic laws, it is strongly based on concrete instances.

8 Discussion

In the section 5.1, the Newton’s first law of motion means that the 4-vector average velocity of an isolated system remains at rest or in motion. This explanation is based on the definition of average velocity given by

$$u_c = \frac{1}{S} \sum_{i} u^{(i)}$$

(140)

where $S$ denotes the number of Dollons in the system as in Eq. (88). The Newton’s first law of motion becomes a sort of strong constraint, inevitably leads to action reaction law or momentum conservation law being valid inside the system, for example, for a rest system composed of two Dollons Alice and Bob we have

$$u_a + u_b = \frac{dx_a}{dt_a} + \frac{dx_b}{dt_b} = 0 \quad u_{4a} + u_{4b} = \text{const}$$

(141)

The above equation means that the action and reaction are equal in magnitude and reverse in directions on the line joining the two particle. But we immediately wonder at that Alice and Bob have to adjust their proper times $dt_a$ and $dt_b$ from time to time to meet the requirement of Eq. (141). That is why we say the First Law is a strong constraint for the system.

Obviously, the geometrical center of a system is defined by

$$x_{\text{center}} = \frac{1}{S} \sum_{i} x^{(i)}$$

(142)

The relativistic 4-vector velocity of the geometrical center of the system is given by

$$u_{\text{center}} = \frac{dx_{\text{center}}}{d\tau_{\text{center}}}$$

(143)

As mentioned in the section 5.4, $u_{\text{center}} \neq u_c$. Immediately, we find the Newton’s first law of motion can be newly explained by based on the relativistic 4-vector velocity of the geometrical center of the system, i.e., the Newton’s first law of motion means that the 4-vector velocity of the geometrical center of an isolated system remains at rest or in motion. This new explanation implies that the action reaction law for the relativistic 4-vector forces inside the system are not held (comparing to Eq. (141) ), but the following expansions for Alice and Bob become possible.

$$Bob : \quad f_b = AX + Bu_a$$

(144)

$$Alice : \quad f_a = CX + Du_b$$

(145)

where $A$, $B$, $C$ and $D$ are unknown coefficients. All conclusions we obtained in the preceding sections can be retained or modified by retracing the route of the paper, in accordance with the section 4. The new explanation seems to be much reasonable, but it is worth further studying the action reaction law and momentum conservation law which are confronting with serious troubles, they need special treatment like that for Ampere’s force in electromagnetism.

Another topic we would like to discuss briefly is SU(n) group. Each infinitesimal transformation of the SU(n) group can be written in the form

$$U = 1 + if_k H_k$$

(146)

As usual, repeated indices must be summed over. Where the real parameter $f_k$ are treated as small quantities, $U$ and $H_k$ are matrices which satisfy the definition of the group

$$UU^* = (1 + if_k H_k)(1 - if_k H_k^*) = 1$$

(147)

We recall from Eq. (29) that Dirac equation was derived from the following equation

$$[a_{\nu lj} P^\nu \psi^{(l)} + i \delta_{l j} mc \psi^{(l)}][a_{\mu kj} P^\mu \psi^{(k)} - i \delta_{j k} mc \psi^{(k)}] = 0$$

(148)

It is much impressive that Eq. (147) and Eq. (148) have a similar form, especially when we let a matrix $E$ to absorb the right side of Eq. (147), i.e.

$$(1 + iE + if_k H_k)(1 - iE^* - if_k H_k^*) = 0$$

(149)

From this comparison we may understand why the SU(n) group could embed in quantum mechanics in a obscure way. This situation arouses our interest to measure a new group whose matrices satisfy

$$Z_j = 1 + if_k Y_{jk} \quad Z_j Z_j^* = 0$$

(150)

We believe this new group has even more direct relations with quantum mechanics.

9 Conclusions

It is important to recognize that physics must be invariant for composite particles and their constituent particles, only one physical formalism exists for any particle, this requirement is called particle invariance.

Under the particle invariance, it is rather remarkable to find that Klein-Gordon equation and Dirac equation
can be derived from the relativistic Newton’s second law of motion on different conditions respectively, thus only one formalism is necessary for particle, the relativistic Newton’s second law is regarded as one which suitable for any kinds of particles.

We point out that the Coulomb’s force and gravitational force on a particle always act in the direction orthogonal to the 4-vector velocity of the particle in 4-dimensional space-time, rather than along the line joining a couple of particles. This inference is obviously supported from the fact that the magnitude of the 4-vector velocity is kept constant. Maxwell’s equations can be derived from classical Coulomb’s force and the magnitude formula of 4-vector velocity of particle.

Our speculation on the quarks model leads to introduce a new elementary particle called Dollon to assemble particles such as baryons, mesons and other composite particles. Instead of quark model, the Dollon model is better in organizing known data, specially in modelling interactions. It is found that relativistic Newton’s second law and various interactions can be derived from the Newton’s first law of motion and the magnitude formula of 4-vector velocity of particle.

The structure of Minkowski’s space is discussed in detail, it indicates that the magnitude formula of 4-vector velocity of particle is only a geometrical distance formula (Pythagoras’ theorem), so that it is completely free from any particle property. Any dynamics or dynamical characteristics originated from the magnitude formula of 4-vector velocity of particle will completely preserve the particle invariance, i.e., the dynamics do not distinguish particle species. Thus the magnitude formula of 4-vector velocity of particle is regarded as the origin of the particle invariance.

Dynamics in Hilbert space can be established in the same way.

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