Application of planar curve offset compensation in path modification

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Abstract. Offset compensation is an indispensable functional module in the current CNC system. But offset compensation is not only used to improve machining accuracy, but also can be used to modify path. This paper first discusses the origin of the curve offset in the CNC system, and points out that the essence of the offset curve is a method of path modification. Then the general formula for offset compensation of plane curve is obtained through normalization. Then the classification of offset compensation is explained. Finally, line segments, arcs and B-spline curves are taken as examples to verify the feasibility and practicability of the offset compensation method applied to path processing. The offset compensation method is a flexible and effective path modification method.

1. Introduction
There are always errors in parts machining, and these errors are caused by the machining conditions that cannot reach the ideal state. In order to reduce machining errors, scholars have carried out research on reducing machining errors. Some scholars concentrate on the error measurement technology in processing, and some scholars use the phenomenon in machining to build a model of error to predict and eliminate errors.

Gu [1] presented a Global Offset compensation method utilizing the the computed deviation between the measured and nominal dimensions of the part. Narendra[2], Liu[3] and Blaster[4] studied the strategies of thermal error compensation in various machining conditions. Chen[5] used error prediction model in thin-walled parts processing. Eldessouky[6], Cho[7] and Xu[8] adopted on-machine method for error compensation. Chen[9] and Zhao[10] proposed a method to decompose the error to compensate. Zuo[11] proposed an integrated model of machining error. Checchi[12] tried to use error compensation on portable machine tools.

The foci of the above literatures are mainly on the generation, measurement and prediction of errors, and there is little research on the path after error compensation. There is no relevant research on the geometric relationship between the path after error compensation and the original path. Error compensation is actually a technique for modifying the path.

There are many sources of error in machine tool machining, including the geometric accuracy of the machine tool and the workpiece, the response delay of the control system, the thermal deformation caused by the temperature rise, and the local deformation of the cutting force of the tool and the rigidity of the workpiece. Since error compensation appears as offset on the plane, it also becomes offset compensation. The path error during machining is shown in figure 1, where C0 indicates the target path, C1 indicates the path after adding tool radius compensation, C2 indicates the path deformation caused by vibration and deformation during machining.
Error offset compensation is used to improve machining accuracy, but it can also be used in path modification. This paper is to study the application of offset compensation in path modification. The rest content of this paper is arranged as follows. Section 2 introduces the preprocessing of planar curve for offset compensation. Section 3 presents offset compensation in different situations. Section 4 proves the effect of offset compensation in path modification through an example of B-spline curve machining.

2. The preprocessing of planar curves

The expressions of plane curves commonly include explicit equations, implicit equations and parametric equations. Because parameter expression has the advantages of easy determination of boundaries, easy transformation and geometric invariance, it is most used in path planning.

Parameters have different meanings in various curve equations. The parameters in the standard straight line equation represent the distance from the current point to the starting point. The parameter in the circle equation represents the central angle. The parameters in the parabolic equation are proportional to the X coordinate. The parameters in the spline curve have no specific meaning, and the range is [0,1]. These parameters have different meanings and different value ranges, and it will be quite difficult to handle them according to different conditions. These parameters have different meanings and different value ranges, and it will be quite difficult to handle them according to different conditions. A unified parameter range can be used to process different curves in the same way. Therefore, it is preferred to unify the parameter range.

2.1. Normalization of parameter ranges

To unify the parameter range to [0,1], firstly the parameter unification process of the spline curve can be omitted, and secondly, this range is also very easy to understand. Therefore, the unified parameter range is the normalization of the parameters of the curve equation. The normalized formula is shown in equation (1)

\[ u = (1 - t)a + bt, \quad u: a \rightarrow b, \quad t \in [0,1] \]  

where \( u \) represents the original parameter, and the value ranges from \( a \) to \( b \), and \( b \) can be smaller than \( a \); letter \( t \) represents the normalized parameter, and the parameter value is increased from 0 to 1. Substituting equation 1 into the parameter expressions of various curves, the normalized parameter curve equations can be obtained.

2.2. General equation for offset of plane curve

The parameter equation of the plane parameter curve after normalization is shown in Equation (2).

\[
\begin{align*}
\{x &= f(t) \quad t \in [0,1] \\
y &= g(t) \quad t \in [0,1]
\end{align*}
\]  

Curve offset means that the point on the curve is offset by a distance along the normal direction. The relationship between the offset curve and the original curve is shown in figure 2. Here is an example of a 90° counterclockwise rotation that is left offset.
Figure 2. Planar curve offset

The tangent vector of a point A on the curve can be expressed as $\vec{MT}(x', y')$, and the normal vector of length $r$ can be expressed as $r/l(-y', x')$. The equation for the offset distance $r$ of any point on the curve along the normal is shown in equation (3).

\[
\begin{align*}
    x &= f(t) - r \times g'(t)/L \\
    y &= g(t) + r \times f'(t)/L
\end{align*}
\]

where $f(t)$ and $g(t)$ denote the original curves' function. $f'(t)$ and $g'(t)$ denote the first-order derivative of the basis curve. $r$ denotes the offset value. $L$ denotes the length of the tangent vector, which is calculated by the equation $L = \sqrt{f'(t)^2 + g'(t)^2}$. In this way, we get the general formula for the plane offset curve.

3. Offset compensation in different situations

Some of these offsets are constant and some are variable. For the convenience of research, these offsets are divided into constant, linear, fitted parametric curves and piecewised.

3.1. Constant and linear offset compensation

Constant offset compensation is the earliest and most widely used. Due to the size of the tool itself, after the size of the workpiece is obtained, it must be converted into the path of the tool. The constant offset path is shown in figure 3(a). The constant offset compensation expression is shown in equation (4)

\[ r = C \]

Constant offset compensation is based on the assumption that the tool does not occur wear. This assumption is applicable in general. The change of tool wear is divided into three stages. Normal machining is generally in the second stage, that is, the amount of wear and the amount of machining have a linear relationship. The machining wear of a single part is generally small. However, for materials with large machining volume, high hardness or difficult to process, the wear during machining cannot be ignored. In this case, linear offset compensation is required to reduce machining errors.

Linear offset compensation usually requires two pieces of information: machining path information and the corresponding offset compensation expression. The information of linear offset compensation is shown in figure 3(b). The linear offset compensation expression is shown in equation (5)

\[ r = Bt + C \]
For straight lines and arcs, the parameter \( t \) is proportional to the machining amount, so the compensation amount can be calculated in proportion. For the curve whose parameter \( t \) is not proportional to the arc length, equation (2) cannot calculate the accurate compensation amount. However, if equation (2) is substituted into the calculation of the offset curve, a smooth offset curve can be obtained.

### 3.2. Fitted parametric curve offset compensation

Except for tool wear, the effects of other parameters on tool position are non-linear. In this case, curve fitting is usually used. The fitting curve compensates the path based on the error of measurement or prediction. Polynomial curve is the most used for fitting curve. As shown in Figure 4, a quadratic polynomial curve offset compensation is performed. The curvature of the middle of the curve is completely changed.

![Figure 4. A quadratic polynomial offset compensation](image)

### 3.3. Piecewised offset compensation

In a complex machining process, different tools or machining conditions will be encountered in a single machining path. Different processing conditions use different error models, so it is necessary to segment a single path, and then use different error compensation. This type of compensation belongs to piecewised offset compensation.

The offset value of the segment can be continuous, such as a ladder diagram, as shown in figure 5. Offset value \( r \) is divided into three intervals according to different offset forms: \([0,a],[a,b],[b,1]\), the first and third intervals are linear offsets but the trends are different, the second interval is constant offset. The offset values of the three intervals are continuous, and the finally obtained curve presents different forms in different intervals.

![Figure 5. A piecewised offset compensation](image)

### 4. Applications

According to the theory of offset compensation described above, it can be seen that offset compensation is a way to modify the path. This method can be used for machining error compensation, or for path modification. Table 1 compares the characteristics of path modification with and without offset compensation.
Table 1. Comparison of using and not using offset compensation in path modification

| Items          | With offset compensation | Without offset compensation |
|----------------|--------------------------|-----------------------------|
| Original data  | Retain the original data | Replace the original data   |
| Flexibility    | High                     | High                        |
| Complexity     | Depend on the offset type| Depend on the curve type    |
| Calculation    | Depend on the offset type| Depend on the curve type    |

As can be seen from table 1, offset compensation and non-offset modification have their own advantages, so offset compensation can be used as a supplement to path modification.

This section takes B-spline curves as examples to illustrate the characteristics and feasibility of offset compensation.

Usually the modification of the spline curve is mainly by modifying the control point coordinates and node vector values. This method cannot retain the original path information. Although it is locally modifiable, it cannot achieve overall modification for the B-spline curve with many data points. However, the offset compensation method can be used for partial modification and overall modification on the premise of saving the original data. An example of offset modification of B-spline curve is shown in figure 6. The figure shows the offset function, graphic display and processing effect of the three offset situations of the B-spline curve. The geometric information of the closed B-spline curve is as follows:

- Control points: (55.287,48.199) (58.757, 45.343) (62.413, 43.361) (67.918, 42.322) (74.237, 47.458) (76.67, 38.854) (71.8, 34.412) (73.332, 29.536) (76.794, 25.803) (65.139, 29.425) (59.91, 30.143) (53.027, 21.824) (51.683, 33.373) (59.102, 37.927) (52.898, 42.505) (55.287,48.199) (58.757, 45.343) (62.413, 43.361)

Figure 6. An example of offset modification: (a) constant offset (b) piecewised offset (c) polynomial offset
Node vector: [-0.1573, -0.0792, -0.0408, 0, 0.0422, 0.1199, 0.189, 0.2536, 0.3394, 0.4043, 0.4136, 0.5061, 0.6077, 0.6736, 0.7661, 0.8427, 0.9208, 0.9592, 1, 1.0422, 1.1199, 1.189]

The machining result shows that the curve path can be flexibly modified using offset compensation.

5. Conclusions
According to the machining method and application of offset compensation, we can see the flexibility of offset compensation method in path modification. Offset compensation can be used not only for overall modification, but also for partial modification. Offset compensation has obvious advantages compared with traditional path modification methods. Motion offset compensation to modify the path can greatly improve the flexibility and diversity of parts machining.

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