A Model for Quasi-Spherical Magnetized Accretion Flow

Mohsen Shadmehri

Department of Physics, School of Science, Ferdowsi University, Mashhad, Iran
e-mail: mshadmehri@science1.um.ac.ir

Received ...; accepted ...

Abstract. A model for axisymmetric magnetized accretion flow is proposed. The dominant mechanism of energy dissipation is assumed to be the magnetic diffusivity due to turbulence in the accretion flow. In analogy to the advection-dominated accretion flow (ADAF) solutions, a constant fraction of the resistively dissipated energy is stored in the accreting gas and the rest is radiated. The general self-similar solutions which describe a resistive and nonrotating flow with purely poloidal magnetic field is first introduced. The radial dependence of physical quantities is identical to viscous ADAF solutions. Although the main focus of this study is nonrotating magnetized accretion flow, for rotating flow with both poloidal and toroidal components of magnetic field we find the radial scaling of solutions similar to the nonrotating case. We show that the accretion and the rotation velocities are both below the Keplerian rate, irrespective of the amount of cooling. We show that the set of the equations is reduced to a second order differential equation for a nonrotating flow. The geometrical shape of the disk changes depending on the fraction of the resistively dissipated energy which is stored in the accreting gas. However, there is a hot low-density gas above the disk in almost all cases. The net accretion rate is calculated for a set of illustrative parameters.

Key words. accretion, accretion disks, MHD

1. Introduction

The importance of the accretion processes has been recognized in connection with the physics of many astrophysical systems such as protostellar disks (e.g., Mineshige & Umemura 1997; Tsuribe 1999; Calvet, Hartmann & Strom 2000), disks around black holes or compact objects or disks at center of the active galactic nuclei (e.g., Lynden-Bell & Rees 1971; Kato, Fukue & Mineshige 1998). According to their geometrical shapes, accretion disks are generally divided into thin and thick disks. A mechanism for transport of angular momentum is another key ingredient in theory of accretion processes and many theoretical uncertainties remain about its nature. However, the theory of thin disks is well understood based on a pioneer work by Shakura & Sunyaev (1973). In this theory it is assumed that there is efficient radiation cooling in the flow.

During recent years another type of accretion disks has been studied, in which the energy released through viscous processes in the disk may be trapped within the accreting gas (see, e.g., Ichimaru 1977; Narayan & Yi 1995; hereafter NY). In this kind of flow which is known as advection-dominated accretion flow (ADAF), the accreting gas has a very low density and is unable to cool efficiently. A full analysis of the dynamics of the solution was presented in an important paper by NY. In this kind of flow which is known as advection-dominated accretion flow (ADAF), the accreting gas has a very low density and is unable to cool efficiently. A full analysis of the dynamics of the solution was presented in an important paper by NY. Since both the angular momentum and the energy dissipation in the flow is undertaken by the turbulent viscosity similar to the standard α model, we can call such flows “viscous” ADAF models.

There are growing evidences that the accretion flows in many astrophysical systems involve magnetized plasma. Accretion disks containing magnetic fields have been studied by many authors (e.g., Ogilvie 1997; Hawley 2001; Casse & Keppens 2002). However, the standard viscous ADAF model completely ignores the presence of an ordered global magnetic field and it is assumed the turbulent magnetic field contribute a constant fraction of the total pressure (NY). However, we can expect accretion flows, in which angular momentum is transported by a global magnetic field and the energy is released by Joule heating. This possibility has been addressed by Kaburaki (2000). He presented a set of analytical solutions for a fully advective accretion flow in a global magnetic field and the conductivity is assumed to be constant for simplicity.

As for the accretion flows in presence of magnetic field, Schwartzman (1971) was the first to point out the importance of the magnetic field in an accretion process. He proposed a hypothesis of equipartition between the magnetic and kinetic energy densities and this picture as mentioned above is usually accepted in modern picture of viscous ADAF models (e.g., NY). More accurate study was done by Bisnovatyi-Kogan & Ruzmaikin (1974), where exact nonstationary solution for a field amplification in the radial accretion flow was obtained. Also, Bisnovatyi-Kogan and Lovelace (2000) suggested that recent papers discussing ADAF as a possible solution for astrophysical accretion should be treated with caution, particularly as for ignorance of the magnetic field. While they obtained a solution for time-averaged magnetic field in a quasipsherial
accretion flow, an analysis of energy dissipation and equipartition between magnetic and flow energies has been presented (Bisnovatyi-Kogan and Lovelace 2000). In our study, we follow similar ideas in order to study magnetized accretion flow in which magnetic energy dissipation is stored in the flow.

Dynamics of radiatively inefficient accretion flows has been the subject of many studies during recent years (e.g., Igumenshchev & Abramowicz 1999, 2000; Stone, Pringle & Begelman 1999; McKinney & Gammie 2002). In these viscous accretion models, the behaviour of the flow depends on the standard dimensionless viscosity parameter $\alpha$. On the other hand, as rotation is allowed, such flows become subject to strong convection with completely different flow pattern and radiative properties (Igumenshchev & Abramowicz 1999; Stone, Pringle & Begelman 1999). Numerical MHD simulations of radiatively inefficient flows have been done recently by many authors (e.g., Hawley 2001; Machida, Matsumoto & Mineshige 2001; Casse & Keppens 2002). But most of these studied have neglected the resistive terms in the MHD equations or the resistivity has been considered only in the induction equation without accounting the corresponding dissipation in the energy equation.

In this paper, we want to explore how the structure of a steady-state thick disk depends on its resistivity by solving the two-dimensional MHD equations for disks which have the simpling feature that they are self-similar in radius. We will consider the general problem of accretion flow where the field anihilation is approximated by a finite "turbulent" conductivity and the viscosity of the fluid is completely neglected in order to construct a fully resistive model for accretion within a global magnetic field. We restrict our attention to the nonrotating accretion flow, in which the flow contains a purely poloidal magnetic field, and the general case will be reported in future. However, we will find the radial scaling of self-similar solutions for the general case where the fluid is rotating and it contains both poloidal and toroidal components of the magnetic field.

This paper is organized as follows. In section 2 the general problem of constructing a model for resistively accretion flow is defined. The self-similar solutions are presented in section 3, and the effects of the input parameters are examined. The implications of this analysis are summarized in section 4.

2. Formulation of the Problem

As stated in the Introduction, we are interested in constructing a model for describing magnetized accretion disks. The macroscopic behavior of such flows can be studied by MHD approximation. For simplicity, the self-gravity and general relativistic effects have been neglected. The first assumption is true as long as the total mass of the disk is small compared with the product of the mass of central object and the angular thickness of the disk. Thus, the basic equations of our problem are written as follows:

$$\frac{\partial \rho}{\partial t} + \nabla (\rho \mathbf{v}) = 0,$$

$$\rho (\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}) \nabla \mathbf{v}) = -\nabla p - \rho \nabla \Psi + \frac{1}{4\pi} \mathbf{J} \times \mathbf{B},$$

where $\rho$, $\mathbf{v}$, $p$, $\Psi$ are the density, the velocity, the pressure and the gravitational potential due to a central object, respectively. We assume that the mass of central object is $M$ and so, the gravitational potential becomes $\Psi = -GM/r$ where $r$ is the spherical radial coordinate. Also, $\mathbf{J} = \nabla \times \mathbf{B}$ is the current density and $\eta$ represents the magnetic diffusivity. The term on the right hand side of the energy equation, $Q_{\text{diss}}$, is the rate of heating of the gas by the dissipation and $Q_{\text{cool}}$ represents the energy loss through radiative cooling and $\gamma$ is the adiabatic index. We assume that the difference between energy dissipation and radiative cooling is a fraction $f$ of energy dissipation, i.e. $Q_{\text{diss}} - Q_{\text{cool}} = fQ_{\text{diss}}$ (e.g., NY). The parameter $f$ measures the degree the flow is advection-dominated and in general it may depends on the details of the heating and cooling mechanisms and will vary with both $r$ and $\theta$. However, we assume a constant $f$ for simplicity. Clearly, the case $f = 1$ corresponds to the extreme limit of no radiative cooling and in the limit of efficient cooling, we have $f = 0$.

We assume that the disk is stationary and axisymmetric. There are numerous studies, in which magnetized equilibrium structures have been investigated via analytical or numerical methods (e.g., Igumenshchev & Abramowicz 1999, 2000; Stone, Pringle & Begelman 1999; Shalybkov & Rüdiger 2000; Ogilvie & Livio 2001; McKinney & Gammie 2002; Rüdiger & Shalybkov 2002). This paper will be restricted to nonrotating accretion flow under purely poloidal magnetic fields configuration. The whole magnetic field is maintained by the electric current induced in the accretion region, and the solutions are determined only by the assumption of self-similarity. According to the anti-dynamo theorem (e.g., Cowling 1981), such a configuration cannot be maintained in a stationary state by an axisymmetric flow in the presence of the resistive dissipation. We can resolve this criticisms in two forms: For many problems, the magnetic dissipation time is very long, much longer than the age of the system. For such problems, one assumes the field to have been generated in the distant past, and restricts one’s attention to its effect on current dynamics, equilibrium, stability, etc.

For other problems, the dissipation time is not longer than the age of the system. In such cases (as in this study), one is making some implicit assumption about the regeneration of the field by (dynamo) processes outside of the computational regime of the problem at hand (e.g., on a 3-D time-dependent microscale, or beyond the boundaries of the formal computation). Whether these assumptions are consistent with the computational details is, of course, a function of how good is the author’s physical intuition, since such assumptions lie beyond the physical and mathematical equations that are actually posed and solved.

There many studies, in which the time-averaged physical quantities have been analyzed so that one can still talk
about stationary state and this approach has been widely used in literature (e.g., Lovelace, Wang & Sulkainen 1987; Lovelace, Romanova & Newman 1994; Shalybkov & Rüdiger 2000; Ogilvie & Livio 2001; Rüdiger & Shalybkov 2002). In this study the physical quantities such as velocity \( v \), magnetic field \( B \) etc. denote the time-averaged values and in general, the instantaneous value of each physical variable consists of a stationary part and a fluctuating component. The influence of the fluctuating parts is included crudely, by modifying the transport coefficients which would apply in the absence of fluctuations. Thus, we neglect the time average of fluctuating parts of velocity multiply magnetic field and the generation of magnetic field in the disk by dynamo activity is not included (e.g., Pudritz 1981). Since we are interested in nonrotating quasispherical accretion, one may ask about the instantaneous magnetic field lines. Recently, Bisnovatyi-Kogan and Lovelace (2000) proposed a model for magnetized quasispherical accretion, in which a sketch of the instantaneous poloidal magnetic field lines and the time-averaged flow velocity has been shown in Figure 1 of their work. As for the accretion flow within global poloidal magnetic field, we are assuming the same configuration.

We work in spherical polar coordinate \((r, \theta, \phi)\) with the origin on the central object. We assume that the net accretion rate \( \dot{M} = -\int 2\pi r^2 \sin \theta \rho v_r d\theta \) is independent of radius \( r \). Since \( r^2 \rho v_r \) is independent of \( r \) and the disk is stationary and axisymmetric, the continuity equation shows \( v_\theta = 0 \) and takes the form

\[
\frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \rho v_r r^2 \right) = 0. \tag{6}
\]

The condition of constant \( \dot{M} \) is similar to what has been used by NY, however, it is relaxed by some authors and they find solutions with outflows (e.g., Henriksen & Valls-Gabaud 1994). In this paper, we consider the simple case of constant net mass accretion rate at all radii. We can write the components of equation of motion (equation 2) in spherical coordinates:

\[
\rho \frac{v_r}{r} \frac{\partial v_r}{\partial r} = \frac{\partial p}{\partial r} - \frac{\rho GM}{r^2} + \frac{1}{4\pi} (J_\theta B_\phi - J_\phi B_\theta), \tag{7}
\]

\[
-\rho v_r \cot \theta \frac{\partial \rho}{\partial \theta} + \frac{r}{4\pi} (J_\theta B_\phi - J_\phi B_\theta), \tag{8}
\]

\[
\rho v_r \frac{\partial v_\theta}{\partial r} + \frac{\rho v_\phi}{r} = \frac{1}{4\pi} (J_\theta B_\phi - J_\phi B_\theta). \tag{9}
\]

Also, the three components of induction equation are:

\[
\frac{\partial}{\partial \theta} \left( r \sin \theta (v_r B_\theta - \eta J_\phi) \right) = 0, \tag{10}
\]

\[
\frac{\partial}{\partial r} \left( r v_r B_\theta - \eta J_\phi \right) + \frac{\partial}{\partial \theta} \left( v_\theta B_\phi - \frac{\partial}{\partial \theta} (\eta J_\phi) \right) = 0. \tag{12}
\]

In equations 10, 11 and 12, the terms without \( \eta \) represent the advection of the magnetic field by the flow motion, while the terms containing \( \eta \) account for the diffusion of the field. From equations 10 and 11 one can simply deduce that

\[
v_r B_\theta - \eta J_\phi = 0. \tag{13}
\]

This important equation can be understood easily in this way: In a stationary, axisymmetric configuration, the electric field can be written as a gradient of a scalar potential, and its \( \phi \) component which is in proportion to \( v_r B_\theta - \eta J_\phi \) in our notation, should vanish identically. This expression appears in the parentheses in equations 10 and 11. So, these equations reduce into one equation, i.e. equation 13, as we showed above. We will find self-similar solutions so that satisfy into equation 13.

The energy equation and the magnetic flux conservation give:

\[
\rho \frac{v_r}{\gamma - 1} \frac{\partial (p \rho)}{\partial r} - \frac{p}{\rho^2} \frac{\partial p}{\partial r} v_r = f Q_{\text{diss}}, \tag{14}
\]

\[
\frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 B_\theta \right) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} (B_\phi \sin \theta) = 0, \tag{15}
\]

where in the above equations the components of the current density \( J \) are

\[
J_r = \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} (B_\phi \sin \theta), \quad J_\theta = -\frac{\partial}{\partial r} (r B_\phi), \quad J_\phi = \frac{1}{r} \left[ \frac{\partial}{\partial r} (r B_\theta) - \frac{\partial B_\theta}{\partial \theta} \right]. \tag{16}
\]

To close the system of equations, we should specify the functional forms of the magnetic diffusivity \( \eta \) and the energy dissipation \( Q_{\text{diss}} \). In this study, we have completely neglected the effects of viscous processes and the attention has been restricted only to the resistive dissipation. In other words, the accretion flow of our model is not infinitely conducting. For simplicity, we could assume the resistivity to be constant (see, e.g., Kaburaki 2000). But, we assume the magnetic diffusivity is due to turbulence in the accretion flow and it is reasonable to express this parameter in analogy to the \( \alpha \)-prescription of Shakura and Sunyaev (1973) for the turbulent viscosity,

\[
\eta = \eta_0 \frac{c_s^2}{\Omega_K} = \eta_0 \frac{p}{\rho \Omega_K}, \tag{16}
\]

where \( \Omega_K = \sqrt{GM/r^3} \) is Keplerian angular velocity, and \( c_s \) is the isothermal sound speed. NY applied similar functional form for kinematic coefficient of viscosity, i.e. \( \nu = \alpha (p/\rho \Omega_K) \). Exactly in analogy to NY’s prescription, we are using the above equation as for magnetic diffusivity \( \eta \). Note that magnetic diffusivity \( \eta \) is not constant and depends on the physical variables of the flow and \( \eta \) in our self-similar solutions, as we will show, scales with radius as \( r^{1/2} \). The magnetic diffusivity has the same units as kinematic viscosity and we assume that the magnitude of \( \eta \) is comparable to that of turbulent viscosity (Bisnovatyi-Kogan and Ruzmaikin, 1976; Parker , 1979 ), although we have neglected kinematic viscosity. This form
of scaling for diffusivity has been widely used by many authors (e.g., Lovelace, Wang & Sulkanan 1987; Lovelace, Romanova & Newman 1994; Ogilvie & Livio 2001; Ridiğer & Shalybkov 2002). Due to arbitrary \( \eta_0 \), it is not so important which velocity is used for the scaling. For example, in some studies Alfvén velocity is used for the velocity in the above scaling (e.g., Fendt & Cemeljic 2002), while in other works still sound speed is applied even in the magnetized case (e.g., Lovelace, Romanova & Newman 1994; Ridiğer & Shalybkov 2002). Nevertheless, the sound speed, it seems, has an advantage; because it is the largest intrinsic velocity parameter in the turbulent plasma, and may be used universally, with or without magnetic field. In this study, following many previous studies, we use the above parameterized form for the diffusivity.

For the heating term, \( Q_{\text{diss}} \), we may have two sources of dissipation. Magnetic reconnection may lead to energy release. Also, we can consider the viscous and resistive dissipations due to a turbulence cascade. In this study, only the resistive dissipation has been considered as the dominant process: \( Q_{\text{diss}} = \frac{\eta}{4\pi} J^2 \).

Now, we have constructed our model and the main equations of the model are equations (6), (7), (8), (12), (13), (14) and (15). In the next section, we will present self-similar solutions of these equations.

### 3. Analysis

#### 3.1. Self-Similar Solutions

To better understand the physics of our accretion flow in a global magnetic field, we seek self-similar solutions of the above equations. Of course, this method is familiar from its wide applications to the full set of equations of MHD in many research fields of astrophysics. As long as we are not interested in boundaries of the problem, such solutions can accurately describe the behavior of the solutions in an intermediate region far from the radial boundaries.

Before presenting the self-similar solutions, we introduce a convenient functional form for the magnetic field. Generally, we may have both poloidal and toroidal components of the magnetic field. We can define a positive-definite magnetic flux function \( \Phi(r, \theta) \) from which we may obtain the poloidal components of the magnetic field:

\[
B = \frac{1}{2\pi} \nabla \times \left( \frac{\Phi}{r \sin \theta} e_{\phi} \right) + B_\phi e_{\phi}.
\]

We can easily show that by integrating the above equation over the circular area \( \pi r^2 \) at plane of \( \theta = \pi/2 \), the result, i.e., \( \Phi(r, \pi/2) \), is the defined magnetic flux. This equation automatically satisfies the equations (15).

Writing the equations in a non-dimensional form, that is, scaling all the physical variables by their typical values, brings out the non-dimensional variables. We can simply show that a solution of the following form satisfies the equations of our model:

\[
p(r, \theta) = \rho_0 \frac{R(\theta)}{(r/r_0)^{3/2}},
\]

\[
v_r(r, \theta) = r \Omega_K(r) V(\theta),
\]

\[
v_\phi(r, \theta) = r \Omega_K(r) \Omega(\theta),
\]

\[
B_r(r, \theta) = \frac{B_0}{2\pi} \frac{d \phi(\theta)}{d \theta} \frac{1}{(r/r_0)^{3/2}},
\]

\[
B_\theta(r, \theta) = -\frac{B_0}{8\pi} \frac{d \phi(\theta)}{d \theta} \frac{1}{(r/r_0)^{3/2}},
\]

\[
B_\phi(r, \theta) = \frac{b(\theta)}{(r/r_0)^{3/2}},
\]

where \( \rho_0, p_0, B_0 \) and \( r_0 \) provide convenient units with which the equations can be written in non-dimensional forms and \( \Omega_K \) denotes the Keplerian angular velocity, viz. \( \Omega_K(r) = \sqrt{GM/r^3} \). Also, we obtain the following equations for the components of current density:

\[
J_r(r, \theta) = \frac{B_0}{r_0} \frac{1}{\sin \theta d \theta} \frac{d (b(\theta) \sin \theta)}{d \theta} \frac{1}{(r/r_0)^{3/4}},
\]

\[
J_\theta(r, \theta) = \frac{B_0}{4\pi} \frac{b(\theta)}{(r/r_0)^{3/2}},
\]

\[
J_\phi(r, \theta) = \frac{B_0}{2\pi r_0^2} \left[ \frac{3 \phi(\theta)}{16 \sin \theta} - \frac{d}{d \theta} \left( \frac{1}{\sin \theta} \frac{d \phi(\theta)}{d \theta} \right) \right] \frac{1}{(r/r_0)^{3/4}},
\]

and considering equation (16) magnetic diffusivity \( \eta \) becomes

\[
\eta = \eta_0 \frac{\rho_0}{\rho_0} \frac{P(\theta)}{R(\theta)} \frac{r}{r_0}^{1/2}.
\]

Substituting the above solution in the momentum, induction and energy equations, we obtain a set of coupled ordinary differential equations. Note that the above solutions automatically satisfy continuity equation (6). Equations of motion (7), (8) and (9) become

\[
R(-\frac{V^2}{2} - \Omega^2) = \frac{5}{2} A_1 P - R + \frac{A_2}{16\pi} \left( b^2 + \frac{3 \phi}{16 \sin \theta} - \frac{d}{d \theta} \left( \frac{1}{\sin \theta} \frac{d \phi}{d \theta} \right) \right),
\]

\[-R \Omega^2 \cot \theta = -A_1 \frac{dp}{d \theta} + \frac{A_2}{4\pi} \frac{1}{\sin \theta d \theta} \frac{d \phi}{d \theta} \times \left[ \frac{3 \phi}{16 \sin \theta} - \frac{d}{d \theta} \left( \frac{1}{\sin \theta} \frac{d \phi}{d \theta} \right) \right] - \frac{b}{\sin \theta d \theta} \left( \frac{d \phi}{d \theta} \right),
\]

\[
RV \Omega = -\frac{A_2}{4\pi \sin \theta} \left( b \frac{d \phi}{d \theta} + \frac{3 \phi}{4 \sin \theta d \theta} \right).
\]

Equations (12) gives

\[
2\pi V b - \frac{d}{d \theta} \left( \Omega \phi \right) - \frac{\Omega}{\sin \theta} \frac{d \phi}{d \theta} + \frac{\eta_0 \rho_0 \Phi b}{2} \frac{R}{R} + \frac{8 \pi \eta_0 \rho_0}{3} \frac{d}{d \theta} \left( \frac{P}{R \sin \theta d \theta} \frac{d \phi}{d \theta} \right) = 0,
\]
and equation (33) is cast into this form
\[ \frac{d}{d\theta} \left( \frac{1}{\sin \theta} \frac{d\phi}{d\theta} \right) - \frac{3\phi}{16 \sin \theta} - \frac{3}{4\rho_0 A_1 \rho \sin \theta} V \phi = 0, \] (33)
and finally the energy equation (14) becomes
\[ RV \frac{3\gamma - 5}{2(\gamma - 1)} = f_{r0} A_2 \left( \frac{1}{4\pi^2} - \frac{3\phi}{16 \sin \theta} - \frac{1}{d\theta} \left( \frac{1}{\sin \theta} \frac{d\phi}{d\theta} \right)^2 \right) + \frac{b^2}{16} + \left( \frac{1}{\sin \theta} \frac{d}{d\theta} (b \sin \theta) \right)^2, \] (34)
where
\[ A_1 = \frac{p_0}{\rho_0 (GM/r_0)} = \left( \frac{\epsilon_0}{v_0} \right)^2, \] (35)
\[ A_2 = \frac{B_0^2}{\rho_0 (GM/r_0)} = 4\pi \left( \frac{\epsilon_0 a}{v_0} \right)^2, \] (36)
where \( v_0 \) is the Keplerian velocity, and \( \epsilon_0 a = B_0 / \sqrt{4\rho_0} \) is Alfvén velocity.

We can define the ratio of the thermal to the magnetic pressures \( \beta \) as
\[ \beta(\theta) = \beta_0 \frac{4\pi^2 P \sin^2 \theta}{27 \phi_0^2 / 16 + (d\phi/d\theta)^2 + 4\pi^2 b^2 \sin^2 \theta}, \] (37)
where \( \beta_0 = \frac{p_0}{B_0^2 / 8\pi} = 8\pi A_1 / A_2^2 \).

Equations (29)-(34) constitute a system of ordinary non-linear differential equations for the six similarity variables \( \phi(\theta), b(\theta), R(\theta), P(\theta), V(\theta) \) and \( \Omega(\theta) \).

Indeed, the behaviour of the solution depends on boundary conditions which are supposed based on some physical assumptions such as symmetry with respect to the equatorial plane. Ogilvie (1997) studied equilibrium of magnetized non-accreting disks by assuming self-similarity in spherical radial coordinate. If we neglect resistivity, our solution reduces to the Ogilvie’s solution, i.e. the model describes a non-accreting disk containing poloidal and toroidal components of magnetic field.

However, in the case of resistive accretion flow, the scalings of quantities with spherical radius are still similar to those presented by Ogilvie (1997) and some authors (e.g., Blandford \& Payne 1982), i.e. \( \rho \propto r^{-3/2}, p \propto r^{-5/2} \) and \( B \propto r^{-5/4} \).

The main focus of this study is nonrotating accretion flow. But an interesting feature of the solutions is worth emphasizing. Using equation (38), we can rewrite equation (29) as
\[ R(1 - \frac{V^2}{2} - \Omega^2)^2 = \frac{5}{2} A_1 P + A_2 (b^2 - \frac{9RV^2}{16\pi^2 \rho_0 P \sin^2 \theta}). \] (38)

Since all the similarity variables are positive except for \( V \), we see that the right hand side of the equation (38) is positive for all \( \theta \) except for \( \theta = 0 \) which is not evident from the equation. For nonrotating flow within purely poloidal magnetic field (i.e., \( \Omega = 0 \) and \( b = 0 \)) which is our interest in this paper, if we substitute from equations (33) and (34) into the last term of right hand side of the above equation, it is positively well-defined for all \( \theta \), even in the limit of vanishing \( \theta \). However, if we consider both the toroidal and the poloidal components of magnetic field, we can still say that when \( \theta \) tends to zero, the last term of right hand side of equation (38) even if tends to a very large value, it would be negative (because \( V \) is negative, the rest of the variables are positive) and so, the right hand side of the equation is positive (there is minus sign before the last term of right hand side). Thus, we should have
\[ \frac{V^2}{2} + \Omega^2 \leq 1. \] (39)

This inequality implies rotation velocities at below Keplerian rate for the solutions of the model, irrespective of the amount of cooling or value of resistivity \( \eta_0 \). Also, we can say as the value of angular velocity increases, the accretion velocity decreases. In other words, in this model, the high accretion rate corresponds to low rotation rate. In this study, we shall restrict our attention to non-rotating accretion flow, i.e. \( \Omega = 0 \).

3.2. Nonrotating accretion Solutions

What we have is a set of complicated differential equations which must be solved under appropriate boundary conditions. Although a full numerical solutions to the equations (29)-(34) would now be possible, it is more instructive to proceed by analyzing the model in some restrictive conditions such as nonrotating case. To derive this solution we set \( \Omega = 0 \) and \( b = 0 \), which the later condition means that we consider purely poloidal magnetic fields. However, to further simplify the problem, we can not impose spherical symmetry, i.e. \( d/d\theta \equiv 0 \). Because if we impose spherical symmetry, we can not identify the actual magnetic field lines, nor can we account for non-radial forces. But we know that in resistive accretion flow the topology of magnetic field has a vital role.

The same as NY we are defining parameters \( \epsilon = \frac{5/3 - \gamma}{\gamma - 1} \) and \( \epsilon_1 = \frac{\epsilon}{f} \). As we will see the parameter \( \epsilon_1 \) plays an important role in the behaviour of the solutions. Obviously, when we set \( \Omega = 0 \) and \( b = 0 \), equations (31) and (32) then immediately drop out and the rest of equations, i.e. equations (29), (40) and (34) can be written as
\[ R(1 - \frac{V^2}{2}) = \frac{5}{2} A_1 P - \frac{3A_2}{64\pi} \times \] (40)
\[ \frac{\phi}{\sin^3 \theta} \frac{d}{d\theta} \left( \frac{1}{\sin \theta} \frac{d\phi}{d\theta} \right) - \frac{3\phi}{16 \sin \theta} - \frac{3}{4\rho_0 A_1} \rho \phi, \]
\[ A_1 \frac{dP}{d\theta} + \frac{A_2}{16\pi^3 \sin^3 \theta} \left( \frac{1}{\sin \theta} \frac{d\phi}{d\theta} - \frac{3\phi}{16 \sin \theta} \right) \frac{d\theta}{d\phi} = 0, \] (41)
\[ \frac{1}{\sin \theta} \frac{d}{d\phi} \left( \frac{1}{\sin \theta} \frac{d\phi}{d\theta} \right) - \frac{3\phi}{16 \sin \theta} \frac{d\theta}{d\phi} + \frac{24\pi^2 \epsilon_1 RV }{\eta_0 A_1} = 0, \] (42)
and equation (33) remains unchanged. Now, the set of equations reduces to four equations (35), (40), (41) and (42) which we want to solve.

Clearly the expressions inside the brackets in equations (40), (41) and (42) are similar and by substituting from equation (35), these three equations become
\[ R(1 - \frac{V^2}{2}) = \frac{5}{2} A_1 P - \frac{9A_2}{256\pi^3 \eta_0 A_1} \frac{RV \phi^2}{P \sin^2 \theta}, \] (43)
and (50), we can rewrite equation (33) as

\[ A_1 \frac{dP}{d\theta} + \frac{3A_2}{64\pi^3 \eta_0 A_1} \frac{RV \phi}{P \sin \theta} \frac{d\phi}{d\theta} = 0, \tag{44} \]

\[ \frac{RV \phi^2}{P^2 \sin \theta} + \frac{128\pi^3 \eta_0 \epsilon_i A_i^2}{3A_2} = 0. \tag{45} \]

From equation (45), we obtain

\[ RV = -\frac{128\pi^3 \eta_0 \epsilon_i A_i^2}{3A_2} \phi^2 \sin^2 \theta \phi^2, \tag{46} \]

and by substituting RV from the above equation into equations (43) and (44), we have

\[ R(1 - \frac{V^2}{2}) = \frac{5}{2} 3 \epsilon_i A_1 P, \tag{47} \]

\[ 1 \frac{dP}{P} + 2\epsilon_i \frac{d\phi}{\phi} = 0. \tag{48} \]

Fortunately, equation (48) is integrable and gives

\[ P = \phi^n, \tag{49} \]

where \( n = 2 \epsilon_i \). Now, considering the above result, we can rewrite equations (46) and (47) as

\[ RV = -\frac{128\pi^3 \eta_0 \epsilon_i A_i^2}{3A_2} \phi^{2n-2} \sin^2 \theta \phi^2, \tag{50} \]

\[ R(1 - \frac{V^2}{2}) = \frac{5}{2} 3 \epsilon_i A_1 \phi^n. \tag{51} \]

From these two equations, one can easily calculate \( V \) and \( R \) as functions of \( \phi \) as follows

\[ V(\theta) = 1 - \sqrt{1 + 2n^2 \phi^{2n-2} \sin^4 \theta}, \tag{52} \]

where

\[ a = \frac{32\pi^2 \beta_0 \rho_0 \epsilon_i}{3 \sqrt{5 + 3 \epsilon_i}}, \tag{53} \]

and by substituting \( V \) as function of \( \phi \) into equations (50) or (51), we can find \( R \) as function of \( \phi \). Thus, if we can find \( \phi \), all physical variables are known. Equation (53) is the last equation which gives us \( \phi \) as a function of \( \theta \). Considering equations (49) and (50), we can rewrite equation (53) as

\[ \frac{d}{d\theta} \left( \frac{1}{\sin \theta} \frac{d\phi}{d\theta} \right) - \frac{3\phi}{16 \sin \theta} + 4n^2 \beta_0 \epsilon_i \phi^{n-1} \sin \theta = 0 \tag{54} \]

or,

\[ \frac{d^2 \phi}{d\xi^2} - \frac{3\phi}{16 (1 - \xi^2)} + 4n^2 \beta_0 \epsilon_i \phi^{n-1} = 0, \tag{55} \]

where \( \xi = \cos \theta \). Now, we have a second order ordinary differential equation for \( \phi(\theta) \) which can be solved using a numerical method (in Appendix we can find an analytical solution for \( n = 1 \)). One boundary condition can be specified on the equatorial plane, by assuming that field lines thread the equator vertically:

\[ \frac{d\phi}{d\theta}|_{\theta=\pi/2} = 0. \tag{56} \]

The second boundary condition can be specified by demanding the magnetic flux enclosed by the polar axis to vanish:

\[ \phi(0) = 0. \tag{57} \]

Thus, the complicated set of differential equations is reduced to a boundary value problem of a second order differential equation. We have obtained numerical solutions of the above equation for different values of \( n \) and \( \eta_0 \). For our illustrative parameters, we assume \( r_0 = 1 \text{pc} \), \( B_0 = 10^{-8} \text{G} \), \( M = 10^6 \) \( M_{\odot} \), \( \beta_0 = 0.1 \) and \( M_0 = 2 \pi \rho_0 \sqrt{GM/r_0} = 1.2 \times 10^{-3} \) \( M_{\odot} \)year which gives \( \rho_0 = 2 \times 10^{-20} \text{g/cm}^3 \). Figure 1 shows some particular solutions corresponding to \( \eta_0 = 0.1 \), \( n = 4/3 \) and different values of the advected energy \( f \). For a fixed value of \( \gamma \), the solutions imply increasing cooling. This general behavior is the same as NY for viscous ADAF.

The behaviour of the solutions, as the six panels in Figure 1 show, is very sensitive to the value of \( f \) or \( \epsilon_i \). The top panels show variations of magnetic flux function \( \phi(\theta) \) as a function of \( \theta \). While this similarity magnetic flux is zero on the polar axis (this is a boundary condition), it has a maximum value on the equator. The maximum value of \( \phi(\theta) \) on the equator increases with decreasing \( f \). This growing flux contrast between the equatorial region and the polar region implies that at a fixed
radius on the equator, the magnetic flux increases with decreasing $\gamma$ (or, equivalently, increasing cooling).

The middle panels show similarity function of radial velocity $V(\theta)$ (see equation (52)). We see that the maximum accretion velocity is at equatorial region and on the polar axis there is no mass inflow. As we expected the accretion velocity is sub-Keplerian. Although for our representative set of parameters the variation of $V(\theta)$ as a function of $\theta$ is decreasing from equatorial region to polar region, we can recognize two distinct regions in the general profile of $V(\theta)$ depending on the value of the velocity. The bulk of accretion occurs from the equatorial plane at $\theta = \pi/2$ to a surface at $\theta = \theta_i$, inside of which the velocity $V(\theta)$ is zero. Note that all of our solutions have well-behaved nonsingular $V(\theta)$ at all $\theta$ and the radial velocity does not diverge at $\theta = \theta_i$. Also, for the flows which extend from the equatorial plane to a surface at $\theta = \theta_i$ where $\theta_i < \theta_c < \pi/2$, the accretion velocity is decreasing. With decreasing $f$, the region with maximum accretion velocity becomes thin, i.e. $\theta_i$ decreases.

In the bottom panels of Figure 1, we plot the density similarity function $R(\theta)$. The density contrast between equatorial and polar regions increases with decreasing $f$. The same as the magnetic flux function $\phi(\theta)$, as $f$ decreases, the density grows and becomes concentrated toward the equatorial plane. As already mentioned, the small $f$ solutions correspond to efficient cooling for a fixed $\gamma$. Thus, the $R(\theta)$ profile clearly shows what we expect with increasing cooling, or equivalently $\epsilon_i$. More interestingly, although the solutions for the large $\epsilon_i$ implies the bulk of the accretion occurs in equatorial region (i.e., like a thin disk configuration), in all cases there is a low density with higher temperature above the disk as the bottom panels of Figure 1 shows. While bulk of the accreting gas along the equatorial plane has approximately constant temperature, this tenuous region also has a constant higher temperature. As cooling increases, the temperatures of both regions decrease. Note that at tenuous region, there is no accretion.

From these Figures we see for fixed values of $\gamma$ and $\eta_0$, as the advected energy decreases and more energy radiates out of the system, magnetic flux function $\phi(\theta)$ increases. It means magnetic field lines can penetrate through the disk easier, when we have cooling. Thus, while thermal pressure decreases due to the cooling, the magnetic pressure increases and we have lower values for the ratio of these two which seems to be important in analyzing the dynamics of the flow. All solutions correspond to poloidal magnetic field configuration, in which the field lines bend only once when passing through the equatorial plane.

Figure 2 shows the ratio of the thermal to magnetic pressures. We see that this ratio decreases by decreasing the amount of the advected energy $f$. Figure 3 shows magnetic field lines for a typical self-similar solution with $\eta_0 = 0.1$ and $f = 0.84$. For other input parameters, we can obtain the same configuration. Ogilvie (1997) studied nonaccreting magnetized disk by considering a polytropic equation of state. However, the magnetic field configuration in his study is similar to what has been obtained in this study for resistive accretion flow. If we neglect turbulent diffusivity and the energy equation, our solutions reduce to Ogilvie’s solutions. But, in our scenario, two main factors in dynamics of the flow are the diffusion mechanism of magnetic field lines and the advected energy. For example, the net rate of accretion depends on the enclosed flux function $\phi(\theta)$. We can write

$$\dot{M} = -2M_0 \int_0^\theta \sin \theta R(\theta) V(\theta) d\theta. \quad (58)$$

Considering equations (42) and (49), we can rewrite the above equation as

$$\dot{M} = \frac{16}{3} \beta_0 \eta_0 A_1 \dot{M}_0 \int_0^\theta \phi^{2n-2} \sin^3 \theta d\theta. \quad (59)$$

Although this kind of magnetized accretion flow is different from Bondi (1952) accretion in various aspects, we can define Bondi accretion rate for the illustrative parameters as $M_{\text{Bondi}} = 4\pi G^2 M^2/\rho_0 c_s^2$. Thus, $M_{\text{Bondi}} = 2(v_{\text{in}}/c_s)^3 M_0$. Assuming $A_1 = 1$, we can write the net accretion rate as follows

$$\dot{M} = \frac{8}{3} \beta_0 \eta_0 M_{\text{Bondi}} \int_0^\theta \phi^{2n-2} \sin^3 \theta d\theta. \quad (60)$$
For the flows which we have at hand, the mass accretion rate is at below Bondi accretion rate for wide range of input parameters (e.g., $\epsilon_f$ and $\eta_0$). Figure 4 shows the net mass accretion rate $M/M_{\text{Bondi}}$ as a function of the advected energy $f$ corresponding to $f = 0.84, 0.79, 0.75, 0.71, 0.48, 0.4, 0.3, 0.2$ and $0 < \eta < 0.1$. As the above equation shows the mass accretion rate changes in proportion to $\eta_0$. For these parameters, we have fitted an approximate function as $M/M_{\text{Bondi}} \approx 0.19 - 0.38 f + 0.19 f^2$. The radial structure of the resistive radiatively inefficient accretion flow is dramatically modified from Bondi profile, and the mass accretion rate is decreased significantly by increasing the advected energy $f$. We see for a fixed value of $\eta_0$, the enclosed flux function $\phi(\theta)$ increases as $\epsilon_f$ increases (i.e., cooling increases) and the net mass accretion rate increases, as well. Thus, in our model, the accretion rate is always suppresses comparing to the Bondi rate, although the precise amount depends on the cooling and the resistivity.

There are many numerical simulations which show the infall of non-rotating or slowly-rotating magnetized gas with reduced $M$ (e.g., Igumenshchev et al. 2000; Hawley, Balbus & Stone 2001). However, our simple model for magnetized accretion flow shows this behavior clearly.

4. Discussion and Conclusions

In this paper, we have obtained axisymmetric magnetized self-similar advection-dominated flow solutions by considering an ordered magnetic field and a resistive dissipation of the energy of the flow. The radial dependence of the solutions has the same behaviour as ADAF solutions (NY), but the angular parts are obtained by solving a set of ordinary differential equations. In the model, the viscosity of the flow is completely neglected in order to make the contrast between the standard viscous and resistive ADAF solutions clear. We showed that the radial and the rotational velocities are well below the Keplerian velocity, and this is independent of the resistivity and the amount of cooling. However, the main aim of this study was to study the nonrotating quasi-spherical magnetized flow directly just by solving the relevant MHD equations. The same problem has been addressed by many authors, under the assumption that the field will achieve equipartition of magnetic and kinetic energies (see, e.g., Shvartsman 1971; NY; Bisnovatyi-Kogan and Lovelace 2000). Bisnovatyi-Kogan and Lovelace (2000) have discussed about equipartition between magnetic and kinetic energies using time-averaged magnetic field solution. They argued for such an equipartition half of the dissipated energy of the accretion flow results from the destruction of the magnetic field. In this paper we showed that the equipartition depends on the input parameters, more importantly the amount of advected energy ($\epsilon_f$) and the resistivity $\eta_0$. Also, the unmagnetized problem is described by pure hydrodynamics and it was solved by Bondi (1952) and has been widely applied. Note that our solutions differ in various aspects from Bondi (1952) spherical accretion. While the magnetic field diffusivity is the main factor in accretion in our model, the energy equation has been considered as well. In Bondi accretion, there is no mechanism for energy transport. Note that in pure Bondi spherical flow, a self-similar accretion is allowed only for a single value of $\gamma$. However, magnetized radiatively inefficient accretion flow is possible for a range of values of $\gamma$. The other interesting feature is that the dynamics of such flow depends not only on magnetic diffusivity $\eta_0$, but also the fraction of advected energy $f$ is important.

For a nonrotating resistive accretion flow, the system of equations simplified to a second order differential equation. For given $\epsilon_f$ and $\eta_0$, this equation determines the magnetic field and the other physical variables can be found using a set of obtained algebraic equations. Although, we have neglected ingredients such as angular momentum and viscosity, the general properties of our solutions are similar to ADAF solutions. More importantly, the geometrical shape of the flow is determined by the amount of the advected energy (i.e., $\epsilon_f$) for a given $\eta_0$. While solutions with efficient cooling resemble to thin disks, the radiatively inefficient accretion describe nearly quasi-spherical flows.

The bulk of accretion with nearly constant velocity occurs in a region which extends from the equatorial plane to a surface at $\theta_1$, inside of which the accretion velocity gradually decreases to zero. Thus, the accretion velocity is constant in the region $\pi/2 < \theta < \theta_1$. As $\epsilon_f$ decreases, the size of this region increases and the accretion tends to a quasi-spherical flow. The temperature of this region is constant as well and is surrounded by a region with low-density and higher temperature. This hot gas corona is similar to what has been discussed in viscous ADAF solutions (NY). As the cooling increases and the flow tends to disk like configuration, the size of this corona increases.

The ratio of the thermal to the magnetic pressure $\beta(\theta)$ depends on the input parameters $\epsilon_f$ and $\eta_0$. In the limit of large $\epsilon_f$, the value of $\beta(\theta)$ is lower than one for all $\theta$. It means that the thermal pressure is not in equipartition with the magnetic pressure and more precisely, the magnetic pressure is stronger than thermal pressure in this regime. On the other hand, for inefficient cooling flows, we found that the value of the ratio exceeds than one for a range of $\theta$. Still there is no equi-partition between the thermal and the magnetic pressures except for a surface at a specific angle which can be determined.
As for the rate of heating $Q_{\text{diss}}$, the resistive dissipation at small scales as a result of a turbulence cascade has been assumed as the dominant mechanism. However, there are other sources such as energy release through magnetic reconnection and more importantly viscous dissipation. Phenomenological considerations may lead us to define $Q_{\text{diss}}$ as a fraction of the gravitational potential energy of the accreting gas. This simple definition allows one to consider all the dissipation processes without having detailed models for them. Preliminary results for this form of $Q_{\text{diss}}$ shows that the scalings of quantities with radius still remains similar to our model, but the other properties of the solutions should be studied.

Since we completely neglected the viscous dissipation, the turbulent magnetic Prandtel number of our model is zero. This unsatisfactory aspect can be removed by considering both the viscosity and the resistivity simultaneously. Fortunately, the radial scaling of the physical variables in the resistive case, as we showed, is similar to the viscous case. By dimensional analysis, one can simply show that self-similarity is possible as long as $\eta$ (and viscosity $\nu$) scales with radius as $r^{1/2}$. Thus, it is still possible to find self-similar solutions in the general case. Given the importance of the value of the turbulent magnetic Prandtel number, we can construct a model for viscous-resistive accretion flow, in which this number is a free parameter.

References

Bisnovatyi-Kogan, G. S., Ruzmaikin, A. A., 1976, AS&SS, 42, 401
Bisnovatyi-Kogan, G. S., Lovelace, R. V. E., 2000, ApJ, 529, 978
Blandford, R. D., Payne, D. G., 1982, MNRAS, 199, 883
Bondi, H., 1952, MNRAS, 112, 195
Calvet, N., Hartmann, L., Strom, S. E.: 2000, in Mannings V., Boss A. P., Russell S. S., eds., Protostars and Planets IV, University of Arizona Press, Tucson, p. 377
Cowling, T. G., 1981, ARA&A, 19, 115
Casse, F., Keppens, R., 2002, ApJ, 581, 988
Fendt, C., Cemeljic, M., 2002, A&A, 395, 1045
Hawley, J. F., 2001, ApJ, 554, 534
Hawley, J. F., Balbus, S. A., Stone, J. M., 2001, ApJ, 554, L49
Henriksen, R. N., Valls-Gabouj, D., 1994, MNRAS, 266, 681
Ichimaru, S., 1977, ApJ, 214, 840
Igumenshchev, I. V., Abramowicz, M. A., 1999, MNRAS, 303, 309
Igumenshchev, I. V., Abramowicz, M. A., 2000, ApJ, 537, L27
Kaburaki, O., 2000, ApJ, 531, 210
Kato, S., Fukue, J., Mineshige, S., 1998, Black-Hole Accretion Disks, Kyoto University Press, Kyoto
Lovelace, R. V. E., Wang, J. C. L., Sulkanen, M. E., 1987, ApJ, 315, 504
Lovelace, R. V. E., Romanova, M. M., Newman, W. I., 1994, ApJ, 437, 136
Lynden-Bell, D., Rees, M. J., 1971, MNRAS, 152, 461
Machida, M., Matsumoto, R., Mineshige, S., 2001, PASJ, 53, L1
McKinney, J. C., Gammie, C. F., 2002, ApJ, 573, 728
Mineshige, S., Umemura, M., 1997, ApJ, 480, 167
Narayan, R., Yi, I., 1995, ApJ, 444, 231 (NY)
Ogilvie, G. I., 1997, MNRAS, 288, 63
Ogilvie, G. I., Livio, M., 2001, ApJ, 553, 158
Parker, E. N., 1979, Cosmical Magnetic Fields (Oxford: Clarendon Press), chap. 17
Pudritz, R. E., 1981, MNRAS, 195, 881
Rdiger, G., Shalybkov, D. A., 2002, A&A, 393, L81
Shakura, N. I., Sunyaev, R. A., 1973, A&A, 24, 337
Shalybkov, D., Rdiger, G., 2000, MNRAS, 315, 762
Shvartsman, V. F., 1971, Soviet Astron., 15, 377
Stone, J. M., Pringle, J. E., Begelman M. C., 1999, MNRAS, 310, 1002
Tsuribe, T., 1999, ApJ, 527, 102

Appendix A:

Equation (55) can be solved analytically for $n = 1$. Assuming

$\phi(\xi) = \frac{16}{35} \pi^2 \beta_0 \xi (1 - \xi^2) + y(\xi),$ \hspace{1cm} (A.1)

equation (55) reduces to

$\frac{d^2 y}{d\xi^2} - \frac{3 y}{16 (1 - \xi^2)} = 0.$ \hspace{1cm} (A.2)

By changing the variables as

$z = \frac{1 + \xi}{2}, y = z Y$ \hspace{1cm} (A.3)

this equation becomes

$z(1 - z) \frac{d^2 Y}{dz^2} + (2 - 2z) \frac{dY}{dz} - \frac{3 z}{16} = 0.$ \hspace{1cm} (A.4)

The general solution of this hypergeometric equation is

$Y(z) = C_1 F\left(\frac{3}{4}, \frac{1}{4}; 2|z|\right) + C_2 F\left(\frac{3}{4}, \frac{1}{4}; 2|z|\right) \times \int \frac{dz}{[z F\left(\frac{1}{4}, \frac{1}{4}; 2|z|\right)]^2},$ \hspace{1cm} (A.5)

where $C_1$ and $C_2$ are arbitrary constants and $F$ is hypergeometric function.