Electromagnetically-induced transparency with amplification in superconducting circuits

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We show that electromagnetically-induced transparency and lasing without inversion are simultaneously achieved for microwave fields by using a fluxonium superconducting circuit. As a result of the Δ energy-level structure of this artificial three-level atom, we find the surprising phenomenon that the electromagnetically-induced transparency window in the frequency domain is sandwiched between absorption on one side and amplification on the other side.

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Electromagnetically-induced transparency (EIT) exploits atomic coherence to enable optically-controlled transparency within an absorption line as well as extreme slowing of light [1]. EIT is realized by strong driving of one transition in a three-level atom (3LA) depicted in Fig. 1 which induces a transparency window with bandwidth equal to the effective splitting of the upper energy level. A 3LA also yields a distinct phenomenon known as lasing without inversion (LWI) [2], which corresponds to negative absorption (i.e. amplification) despite the lower-level population exceeding the upper-level population. LWI arises through two interfering excitation pathways thereby suppressing absorption. Here we show that EIT and LWI can be realized simultaneously (as EIT with amplification, or EITA) via a 3LA with all three inter-level transitions being driven in a Δ configuration (a Δ3LA). Furthermore we show that this new phenomenon can be realized with flux [3] or fluxonium [4] artificial atoms, which are solid-state realizations of Δ3LAs [5]. Our theory of EITA predicts asymmetric EIT peaks, commensurate with experimental observations of an anomalous asymmetry of EIT peaks for Rb Δ3LAs [6].

We construct a theory of EITA and build on recent advances with Josephson-junction-based artificial atoms to show how EITA can be achieved and what its experimental signature will be. There are subtleties though in transferring optical atomic experiments to the superconducting circuit domain. In particular the superconducting circuit employs microwave fields that propagate in one dimension in contrast to three-dimensional field propagation and optical frequencies in the atomic experiment. Therefore absorption and transmission spectroscopy translate to reflected and transmitted fields. Also optical experiments employ a large number of 3LAs whereas the superconducting circuit case needs just one 3LA. With EIT and the Autler-Townes splitting having been displayed in experiments, artificial Δ3LAs built with superconducting junctions appear to be good candidates for observing EITA [7, 10].

The Δ3LA depicted in Fig. 1 has three energy levels |i⟩ with frequency differences ω_{ij} and decay rates γ_{ij} between levels |i⟩ and |j⟩ for i, j ∈ {1, 2, 3}. The |i⟩ ↔ |j⟩ transition is driven by a coherent electromagnetic field with electric field E_{ij} and detuning δ_{ij} from the |i⟩ ↔ |j⟩ transition with electric dipole vector d_{ij}. The corresponding (complex) Rabi frequency is Ω_{ij} = d_{ij} · E_{ij} (ℏ ≡ 1). Away from the flux degeneracy point, selection rules do not apply to this one-dimensional superconducting circuit so all dipole transitions can be driven by applying a trichromatic microwave field tuned near each of the ω_{ij} [3].

For δ_{ij} := |i⟩ ⟨j|, the system Hamiltonian is

\[ \hat{H} = \sum_{i=1}^{3} \omega_{i} \hat{\sigma}_{ii} - \frac{1}{2} \sum_{i>j}^{3} (\Omega_{ij} e^{-i(\omega_{ij} + \delta_{ij})t} \hat{\sigma}_{ij} + \text{hc}) , \]  

with hc denoting the Hermitian conjugate. For a rotating frame and taking δ_{12} = δ_{13} − δ_{23}, Eq. 1 is replaced by

\[ \hat{H}_{\text{int}} = -\sum_{i=2}^{3} \delta_{ii} \hat{\sigma}_{ii} - \frac{1}{2} \sum_{i>j}^{3} (\Omega_{ij} \hat{\sigma}_{ij} + \text{hc}) . \]  

Energy relaxation and dephasing caused by coupling to uncontrolled degrees of freedom are described by a
Lindblad-type master equation

\[ \dot{\rho} = -i[H_{\text{int}}, \rho] + \sum_{i<j} \gamma_{ij} D[\sigma_{ij}]\rho + \sum_{i=2}^{3} \gamma_{\phi i} D[\sigma_{\phi i}]\rho =: \mathcal{L}\rho \]

for \( D[c]\bullet := c \bullet - \{c\dagger c, \bullet\}/2 \). Here \( \gamma_{\phi i} \) is a pure dephasing rate for level \( |i\rangle \), which should be negligible for flux \( \Delta 3\Lambda \)s at the flux degeneracy point \( [11] \) and for fluxonium \( \Delta 3\Lambda \)s in a wider range of flux around this degeneracy point \( [1] \).

For EIT, a strong pump field \( \Omega_{23} \gg \Omega_{13} > 0 \) causes Autler-Townes splitting of level (3) yielding two absorption peaks at \( \delta_{13}/\gamma_{13} = \pm \frac{\Omega_{23}}{2\Gamma_3} \) for \( \Gamma_3 = (\gamma_{13} + \gamma_{23} + \gamma_{\phi 3})/2 \) with a transparency window centered at \( \delta_{13} = 0 \) and full-width at half-maximum FWHM = \( \gamma_{12} + \gamma_{\phi 2} + |\Omega_{23}|^2/2\Gamma_3 \) \([12]\). Optical dispersion and absorption are quantified, respectively, by the real and imaginary parts of the first-order susceptibility \( \chi^{(1)} \propto |d_{13}|^2 \rho_3^{13}/\Omega_{13} \) with \( \rho_{ij} = \langle i|\rho|j\rangle \) the steady-state solution of the master equation. Hence dispersion and absorption are proportional to Re[\( \rho_3^{13} \)] and Im[\( \rho_3^{13} \)], shown in Figs. 2(a,b) for EITA, EIT (\( \Omega_{12} = 0 \)) and LWI (\( \Omega_{13} = 0 \)). As expected, these dispersion and absorption curves are related by the Kramers-Kronig relation.

The EIT absorption curve exhibits a transparency window between two Autler–Townes peaks, and the linear dispersion curve in Fig. 2(b) indicates that the group velocity is constant in this window. The LWI absorption curve shows the characteristic transparency at resonance with absorption in the red-detuned (left) region and amplification (or negative absorption) in the blue-detuned (right) region. EITA exhibits the transparency window characteristic of EIT but with the LWI feature that the window is bounded by an absorption and an amplification peak rather than by two Autler-Townes absorption peaks. Fig. 2(c) confirms that population inversion \( \rho_{11}^{\delta} - \rho_{33}^{\delta} \) is always positive for EIT, LWI and EITA so amplification is not due to population inversion.

In fact, EITA is not a simple combination of EIT and LWI as coherence between each pair of levels adds to the richness of the phenomenon. Due to inter-level coherence, controlling the relative phase of (at least) one field with respect to the other two affects whether amplification is in the red- or blue-detuned region or even whether there is amplification at all, as depicted in Fig. 3. This control becomes evident by taking \( \rho_{33}^{\delta} \approx 0 \) \([13]\):

\[ \rho_{31}^{\delta} = -e^{-i\phi_{13}} \left[ 2i\Omega_{13} (\rho_{11}^{\delta} - \rho_{33}^{\delta}) (i\delta_{13} - \gamma_{12}/2) + \Omega_{23}^2 e^{-i(\phi_{12} + \phi_{23} - \phi_{13})} (\rho_{11}^{\delta} - \rho_{22}^{\delta}) \right]/F, \]

where \( F = 4 (i\delta_{13} - \Gamma_3) (i\delta_{13} - \gamma_{12}/2) + \Omega_{23}^2 \). For \( \Phi := \phi_{12} + \phi_{23} - \phi_{13} \) we observe that the absorption curve of Fig. 2(a) is recovered for \( \Phi = 0 \). EITA is replaced by ordinary absorption for \( \Phi = \pi/2 \) with \( \gamma_{12} \ll \gamma_{13} \), and the mirror image of the \( \Phi = 0 \) absorption curve occurs for \( \Phi = \pi \). For \( \Phi = 3\pi/2 \), the absorption curve corresponds to an EIT profile but with the transparency window replaced by an amplification window accompanied by a linear dispersion profile (not shown) so group velocity is constant for this window.

Our theory of a \( \Delta 3\Lambda \) is applicable to a recent EIT experiment with Rb atomic gas \([2]\), which exhibited both transmission enhancement and asymmetry between the red- and blue-detuned transmission peaks. Their theory explains transmission enhancement but not the observed peak asymmetry. As the lower two levels of their Rb 3LA is driven by a microwave field, their system is the \( \Delta 3\Lambda \) discussed here, and our theory predicts the observed peak asymmetry although, of course, a quantita-
negative analysis is required to see how much of the asymmetry is due to $\Delta$ electronic structure effects as opposed to other reasons. Although our theory also predicts negative absorption (amplification), inhomogeneous broadening and absorption in the gas cell could obscure the amplification signature. An advantage of our proposal to study EITA with superconducting artificial atoms coupled to one-dimensional transmission lines is that EITA can be investigated in a controlled way without some of the complications that arise for gases.

Flux [3] and fluxonium [4] 3LAs closely approximate $\Delta$3LAs away from flux degeneracy [5], hence are natural candidates for realizing EITA. Fig. 4(a) shows the energy levels structure of the fluxonium 3LA and Fig. 4(b) the corresponding transition matrix elements $|t_{ij}|$, both as a function of the externally-applied flux $\Phi_{\text{ext}}/\Phi_0$, with $\Phi_0$ being the flux quantum. Parameters are from Ref. [4]. We suggest biasing the artificial 3LA at $\Phi_{\text{ext}}/\Phi_0 = 0.08$, indicated by a dashed vertical line, where $t_{12} = t_{23}$, optimal for observation of EITA.

EITA can be probed by connecting either 3LA to a transmission line supporting traveling modes [14]. The absorption and dispersion profiles can be measured both in transmission and reflection, and a possible setup for homodyne measurement of the reflected signal is illustrated in Fig. 5. To determine how the reflected signal contains information about $\rho_{31}$, we use input-output theory. In the Markov approximation and focusing on the signals centered about the probe ($a$), pump ($b$) and control ($c$) frequencies, the transmission-line free Hamiltonian is

$$\hat{H}_{\text{TL}} = \sum_{\delta \in \{a,b,c\}} \int_{-\infty}^{\infty} d\omega \delta^{\dagger}(\omega)\delta(\omega)$$

with the microwave field annihilation operators $\delta(\omega)$ satisfying $[\delta(\omega), \delta^{\dagger}(\omega')] = \delta_{\omega,\omega'}\delta(\omega - \omega')$.

Treating the transmission-line mode as three commuting quasi-monochromatic modes is valid if separation between the transitions frequencies greatly exceeds the linewidths. In this approximation, the $\Delta$3LA-transmission line interaction Hamiltonian is

$$\hat{H}_{\text{int}} = i \int_{-\infty}^{\infty} d\omega \left[ \sqrt{\frac{\gamma_{13}}{2\pi}} a_{\text{in}}^{\dagger}(\omega)\delta_{13} + \sqrt{\frac{\gamma_{23}}{2\pi}} b_{\text{in}}^{\dagger}(\omega)\delta_{23} + \sqrt{\frac{\gamma_{12}}{2\pi}} c_{\text{in}}^{\dagger}(\omega)\delta_{12} - \text{hc} \right].$$

Using input-output theory, the output field operator centered at the probe frequency is

$$\hat{a}_{\text{out}}(t) = \hat{a}_{\text{in}}(t) + \sqrt{\gamma_{13}}\sigma_{13}(t),$$

with $\hat{a}_{\text{in}}(t)$ the annihilation operator for the input field centered at the probe frequency. With the homodyne setup illustrated in Fig. 5 effectively measuring $\langle \hat{a}_{\text{out}}(t) \rangle = \langle \hat{a}_{\text{in}}(t) \rangle + \sqrt{\gamma_{13}}\rho_{31}(t)$, access to the dispersion and absorption profiles is straightforward.

We propose to bias the fluxonium at $\Phi_{\text{ext}}/\Phi_0 = 0.08$, indicated by a dashed vertical line in Fig. 4, where $t_{12} = t_{23}$ as this choice is optimal for observation of EITA. Contrary to the case of resonators [14], coupling to the transmission line traveling modes exposes the $\Delta$3LA to environmental vacuum fluctuations of the voltage at the
A flux qubit was coupled to a transmission line with \( \gamma_{12}/2\pi = 11 \text{ MHz} \) at zero flux \([14]\). Assuming white noise, the decay rates at \( \Phi_{\text{ext}}/\Phi_0 = 0.08 \) can be estimated using the matrix elements of Fig. 1, yielding \( \gamma_{13}/2\pi = 25 \text{ MHz} \), \( \gamma_{12}/2\pi = 2.6 \text{ MHz} \) and \( \gamma_{23}/2\pi = 2.6 \text{ MHz} \). Figs. 2 and 3 have been obtained using these values, showing that EITA with superconducting ∆3LA should be possible with current experimental parameters.

Another approach to probing EITA with superconducting circuits is by quantum state tomography where the density matrix is fully reconstructed. This can be done, for example, by coupling the ∆3LA to a resonator rather than a transmission line \([17]\), and strong coupling of a flux ∆3LA to a resonator has been studied \([18–20]\). An advantage of this approach is that the resonator will shield the ∆3LA from noise away from the resonator frequency, thereby decreasing significantly the decay rates.

In summary, we have developed the theory of EITA which shows a EIT window sandwiched between an absorption line and an amplification line in a superconducting ∆3LA system. The EITA absorption and dispersion profiles can be controlled by the phase of one of the three microwaves applied to the superconducting atom. We suggest a homodyne measurement scheme for a direct observation of the EITA absorption and dispersion profiles of the probe field where a fluxonium artificial atom is coupled to a one-dimensional transmission line. EITA is exciting as a surprising combination of electromagnetically-induced transparency in a single system, and superconducting artificial atom realizations will enable controlled study of this phenomenon. EITA could be useful for superconducting circuits by enabling slowing and storage of microwave fields, and the amplification effect could be useful for partially offsetting absorption.

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