Spin polarised carrier injection into high $T_c$ superconductors: A test for the superconductivity mechanism?

Satadeep Bhattacharjee and Manas Sardar

Materials Science Division, IGCAR, Kalpakkam 603102, India

We point out in this short communique, that be it a s-wave or d-wave superconductor, in a non equilibrium situation(i.e. in presence of excess unpolarised/polarised quasi-particles maintained by an injection current) the superconducting gap suppression by the presence of same amount excess quasi-particles, can at best differ by a factor of 2, for a conventional BCS superconductor. For the high $T_c$ superconductors on the other hand, there is a huge difference in gap suppression between unpolarised/polarised quasiparticle injection, as observed in the experiments. We argue that this is due to the excess polarised quasiparticles blocking the interlayer pair tunneling process. We also point out that spin polarised quasi-particle injection in a superconductor is a very easy way to distinguish between a s-wave or an anisotropic gap superconductor.

The free energy of the superconductor $F_s$ is given by,

$$F_s = 2 \sum_k (\epsilon_k - \mu)(f_k - 2 f_k u_k^2 + u_k^2) - \sum_{k,k'} V_{k,k'} u_k u_{k'} (1 - f_k) (1 - f_{k'}) - T \sum_k [f_k \log f_k + (1 - f_k) \log (1 - f_k)]$$

(1)

Keyword: High $T_c$ superconductors, Spin polarised tunneling, Interlayer pair tunneling.
where $v_k$ and $v_k$ has its usual meaning, and $f_k = \langle \gamma_{k,\sigma}^\dagger \gamma_{k,\sigma} \rangle$ is the fermi function for the superconducting quasiparticles and $\epsilon_k$ is the dispersion of the electrons in the normal state. However in addition to the usual constrain equation for the total number of electrons $\sum_{k,\sigma} \langle \epsilon_{k,\sigma}^\dagger \epsilon_{k,\sigma} \rangle = N$ which is enforced by proper choice of the chemical potential $\mu$, an additional constrain on quasi-particle excitation number will be imposed, $\sum_{k,\sigma} f_{k,\sigma} = n << N$. This can be done by introducing an extra chemical potential $\mu^*$ so that now, $f_k = [1 + e^{\beta(E_k - \mu^*)}]^{-1}$. The modified BCS gap equation is now,

$$\Delta_k = \sum_{k'} \frac{V_{k,k'} \Delta_{k'}}{2E_{k'}}(1 - f_{k'\uparrow} - f_{k'\downarrow}) = \sum_{k'} \frac{V_{k,k'} \Delta_{k'}}{2E_{k'}} \frac{\tanh \beta}{2}(E_{k'} - \mu^*)$$

(2)

Assuming a momentum independent $V_{k,k'}$ (s-wave superconductor) and going from momentum summation to energy integration with a cut-off $\omega_D$, we get,

$$\frac{1}{N(E_F)V} = \int_{-\omega_D}^{\omega_D} \frac{dE}{2E} \tanh \frac{1}{2} \beta(E - \mu^*)$$

The number of excess quasi-particles is,

$$n = 2 \sum_k [f(E_k - \mu^*) - f(E_k)] = 4N(E_F) \int_0^\infty \frac{dE}{e^{\beta(E - \mu^*)} + 1}$$

(3)

the factor of 2 before the momentum summation is because of two spin species.

Defining $n_0 = \frac{\int dE}{4N(E_F)V} \Delta_0$, where $\Delta_0$ is the gap at $T = 0$ and $\mu^* = 0$, we can easily solve for the gap value at any $T$ and $n$ from the above two equations. In the limit of zero temperature the gap value can be determined from the algebraic equation,

$$\frac{\Delta_0}{\Delta} = \frac{n_0 \Delta_0}{\Delta} + \sqrt{1 + 4n_0^2 \frac{\Delta_0^2}{\Delta^2}}$$

(4)

This is the result of Owen and Scalapino, for excess unpolarised quasiparticles.

For spin polarised quasiparticle injection, we introduce a different chemical potential, for only the up spin quasiparticles, i.e. assuming complete spin polarisation of electrons in the ferromagnet. The gap equation (2) will be modified to,

$$\Delta_k = \sum_{k'} \frac{V_{k,k'} \Delta_{k'}}{2E_{k'}} \frac{\tanh \beta}{2}(E_{k'} - \mu^*) + \frac{\tanh \beta}{2}E_{k'}$$

(5)

Corresponding equation for the number of excess quasiparticles will be, same as equation (3) with the factor of 2 missing before the momentum summation, because excess quasiparticles are of only one spin species. In the limit of zero temperature we get the following algebraic equation for the normalise gap value,

$$\frac{\Delta_0}{\Delta} = \frac{2n_0 \Delta_0}{\Delta} + \sqrt{1 + 4n_0^2 \frac{\Delta_0^2}{\Delta^2}}$$

(6)

In figure 1 we show the normalized gap versus extra quasiparticles for a s-wave superconductor(solutions of equations 3 & 4). We find that for, s-wave superconductors, a first order phase transition to normal metal occurs, for $n = 0.15$: $\Delta = 0.63\Delta_0$, and $n = 0.08$: $\Delta = 0.58\Delta_0$ for unpolarised and polarised quasiparticle injections. Beyond this critical concentration of injected carriers, the free energy of the perturbed superconductor becomes larger than normal state free energy. Notice though that, for the same amount of injected carriers, the relative gap suppression in unpolarised/polarised carriers differs by at most by a factor of 2.

Just as at finite temperatures quasi-particle excitations interfere with the pairing interaction and eventually destroy superconductivity at $T_c$, when an excess number of quasiparticles are injected into the superconductor, it basically reduces the phase space for BCS pair scattering process and reduce the gap value. BCS interaction scatters pairs $(k \uparrow, -k \downarrow) \rightarrow (k' \uparrow, -k' \downarrow)$ across the fermi surface. So any excess quasiparticle occupying these states limits the phase space for the BCS interaction. It is obvious that when the injected quasi-particles are polarised(all of one spin) they interfere with BCS interaction more severly and hence the gap value falls faster with quasi-particle over -population, compared to the unpolarised injection current.
We next investigate the effect of quasi-particle injection on superconductors with anisotropic gap, specifically gaps of d-wave symmetry. The dispersion of electrons is chosen to be of the form

$$\epsilon(k) = -2t \left( \cos k_x + \cos k_y \right) + 4t' \cos k_x \cos k_y - 2t'' \left( \cos 2k_x + \cos 2k_y \right),$$

with $t = 0.25$ eV, $\frac{t'}{t} = 0.45$, $\frac{t''}{t} = 0.2$. We also choose $\epsilon_F = -0.45$ eV corresponding to a Fermi surface which is closed around the $\Gamma$ point. These choices are inspired by band structure calculations for the YBCO compound at optimal doping concentration. The cut off the pairing interaction is chosen to be $\omega_D = 30$ meV, and the pairing interaction strength $V_{k,k'} = V_0 f_k f_{k'}$ with $f_k = \cos(k_x) - \cos(k_y)$ and $V_0 = 2.8$ meV. With this choice of parameters the $T_c$ comes out to be $30^0K$.

In figure 2, we plot the normalised momentum averaged value of the superconducting gap magnitude, $\Delta(n_c, T)/\Delta(0, T)$ versus $n$(quasi-particle over population) for different temperatures. There are some interesting features to be noticed here.

1. Notice the crossing of curves, for $T = 5$ and $25^0K$. The origin of this can be seen by looking at the gap equation (2). States of energy $E_k$ less that $\mu^*$ interferes with the pairing process by giving a negative contribution to the binding. At low temperatures and small injection currents, the extra carriers occupy the states near the deep gap nodes, where the denominator $E_k$ is also small giving rise to large reduction in binding energy. At larger temperatures for same amount of injected carriers on the other hand the quasi-particles are distributed over a larger range of energies (larger denominator) leading to smaller suppression of binding (and hence larger gap values). Larger concentration of injected carriers, of course will ultimately be more damaging to superconductivity, because of larger number of thermally excited quasi-particles that are already present. This leads to the crossing of curves as seen in fig 2. This will be a generic feature for any superconductor with deep gap nodes, and should be easily seen in spin injection experiments.

2. Notice also that the critical concentration of excess quasi-particles that destroy superconductivity is not a monotonic function of temperature and goes through a maximum around $T_c/2$.

3. Though it is not shown in the plot, injected current suppresses superconductivity more in the d-wave superconductors, than for a s-wave superconductor having same critical temperature, as one would expect for gap functions with deep nodes.

In figure 3, we have plotted the normalised momentum averaged gap magnitude of a d-wave superconductor, with a $T_c$ of $90^0K$ versus $n$(quasi-particle over population) on the other hand the quasi-particles are distributed over a larger range of energies (larger denominator) leading to smaller suppression of binding (and hence larger gap values). Larger concentration of injected carriers, of course will ultimately be more damaging to superconductivity, because of larger number of thermally excited quasi-particles that are already present. This leads to the crossing of curves as seen in fig 2. This will be a generic feature for any superconductor with deep gap nodes, and should be easily seen in spin injection experiments.

We shall explore here the interlayer tunneling model of superconductivity.

We begin by writing the gap equation for interlayer tunneling model with unpolarised injected carriers as

$$\Delta_{k} = T_{J}(k)\frac{\Delta}{2E_k} \tanh \frac{\beta(E_k - \mu^*)}{2} + \sum_{kk'}^{'} V_{kk'} \frac{\Delta_{k'}}{2E_{k'}} \tanh \frac{\beta(E_{k'} - \mu^*)}{2},$$

For polarised carrier injection, this equation has to be corrected as shown in equation (5).

This gap equation can be obtained by considering two close Cu-O layers as in YBCO coupled by a Josephson tunneling term of the form

$$H_J = -\frac{1}{t} \sum_{k} t_{\perp}(k) \left( e_{k,\uparrow}^\dag e_{-k,\downarrow} d_{-k,\downarrow} \right) + \text{h.c.},$$

where $t_{\perp}(k)$ is the bare single electron hopping term between the two coupled layers $c$ and $d$ and $t$ is a band structure parameter in the dispersion of electrons along the Cu-O plane. Finally, $T_{J}(k)$ in the right hand side of equation (1) is given by $T_{J}(k) = \frac{\Delta_{c}}{2E_k} \tanh \frac{\beta(E_k - \mu^*)}{2}$, where $t_{\perp}(k) = \frac{\Delta_{c}}{2E_k}(\cos(kx) - \cos(ky))^2$. The dispersion of electrons along the Cu-O plane is given by equation 5. Note that the Josephson coupling term in $H_J$ conserves the individual momenta of the electrons that get paired by hopping across the coupled layers. This is as opposed to a BCS scattering term which would only conserve the center of mass momenta of the pairs. This is the origin of all features that are unique to the interlayer tunneling mechanism. This term has a local $U(1)$ invariance in $k$-space and cannot by itself give a finite $T_c$. It needs an additional BCS type non local interaction in the planes which could be induced by phonons or residual correlations. Here we assume the inplane pairing interaction to be d-wave kind i.e., $V_{kk'} = V_0 f_k f_{k'}$ with $f_k = \cos(k_x) - \cos(k_y)$. $T_{J}(k)$ can be inferred from electronic structure calculations. As shown in reference, it is adequate to choose $t_{\perp}(k) = \frac{\Delta_{c}}{2}(\cos(k_x) - \cos(k_y))^2$, with $\Delta_{c}/t \equiv 1/3$ to $1/5$. According to Anderson, it is the $k$-space
locality that leads to a scale of $T_c$ that is linear in interlayer pair tunneling matrix element. He finds that in the limit $T_J > V_{kk'}$, $k_B T_c \approx \frac{2}{J}$, and in the other limit, $k_B T_c \approx \hbar \omega_D e^{-\frac{\omega_D}{k_B T}}$, where $\omega_D$, $\rho$ and $V_0$ are Debye frequency, density of states at the fermi energy, and fermi surface average matrix element $V_{kk'}$. The important point being that even with a little help from the in plane pairing interaction the interlayer tunneling term can provide a large scale of $T_c$. In this model the gap value is mainly controlled by the interlayer tunneling amplitude, rather than the inplane pairing interaction, though the symmetry of the gap function is determined overwhelmingly by the inplane BCS kind of interaction strength $V_{kk'}$. and the gap value is very sensitive to the interlayer pair tunneling amplitude $\Delta_{II}$.

In figure 4, we plot the normalised momentum averaged gap values versus both unpolarised/polarised quasi-particle over population up to $n = 0.085$. We find that for $n = 0.085$, the reduction in gap value for unpolarised injection current is about 5%, whereas for the polarised carrier injection is about 35% (that is a factor of 7!).

The large difference between the two situations (as observed in the experiments also) now shows up. We argue that when polarised carriers are injected, then, over and above the usual dynamical pair breaking (due phase space blocking of BCS interaction) in the planes like in usual BCS superconductors, there is an added inhibition of interlayer pair tunneling. This is so, because there are much less number of extra singlets near the fermi surface, to tunnel from plane to plane. This effect is absent when unpolarised quasiparticles enter the planes. In this mechanism, extra quasiparticles directly affect the interlayer pairing tunneling process, which is the main source of superconducting condensation energy gain. This is over and above the interference in the binding process in the individual layers that we have discussed earlier in usual BCS superconductors.

The introduction of an extra chemical potential $\mu^*$ to tackle the nonequilibrium superconductivity, in presence of artificially maintained quasiparticles, is a reasonable starting point, when the excess quasiparticles thermalize with low energy phonons more often than they recombine into pairs. In these limit the quasiparticles are in steady state at $T$ but have an excess number denoted by the increased chemical potential $\mu^*$. One important ingredient of Anderson et al’s mechanism is that, in the normal state the electrons are spin-charge separated and no quasiparticles in the fermi liquid sense exists. We have assumed here that, below the superconducting $T_c$ somehow spin-charge separation is absent. This seems to be the case from photoemission experiments which show clear signal of well defined quasiparticles on the fermi surface. In the superconducting state how exactly it happens, is still not clear.

At the present moment all that we can say is that, because of spin-charge separation in the normal state, the single particles (which are not even well defined properly) cannot tunnel from plane to plane in a coherent fashion. On the other hand there are indications from some earlier theoretical attempts that electrons can tunnel in pairs. In other words spin-charge separation itself is responsible for pair tunneling. We believe that when some BCS type of pairing interaction is introduced, this anomolous self energy actually leads to weakening of spin charge separation, and electrons recover fermi liquid type quasi-particles below $T_c$, as it is observed in the photoemission experiments. We have implicitly assumed this while working with the interlayer tunneling mechanism. Clearly a more carefull analysis of this problem is needed within this mechanism.

Throughout this analysis, we have assumed that, (1) the London penetration depth is lesser that the superconducting film thickness, so that direct magnetic field of the ferromagnetic metal does not penetrate the superconductor much. (2) The spin diffusion length is much larger that the SC film thickness, so that no spatial variation of the gap has to be taken into account. This is certainly true when there are no magnetic impurity or for small spin orbit interaction, for the then extra spin density relaxes slowly.

We have also not taken into account the case of finite recombination time for the excess quasiparticles.

It would be interesting to see if, the presence of extra spin polarised quasiparticles will give rise to polarisation field, that will be felt by the nuclear magnetic moment through the contact hyperfine interaction. This will significantly affect the NMR relaxation rate and ESR linewidths.

Recently N. C. Yeh et al have observed an initial, actual increase in critical current for low enough injection currents, when the insulating barrier thickness is small. They argue that, some up spin quasiparticles in the superconductors can diffuse into the magnetic materials, creating spin imbalance in the superconductor (more of down spin quasiparticles). On the other hand when the injection current is switched on, then up spin electrons starts coming into the superconductor, nullifying the spin imbalance in the superconductor. That is why $T_c$ increases for small injection currents. For larger injection currents of course $T_c$ falls drastically as is seen in experiments, and as we have argued to be natural within the Interlayer tunneling mechanism of Anderson et al.

While this maybe the case, we would also like to point out, that this initial increase can also happen, when there are some (1) magnetic impurity inside the superconductor or (2) spin-orbit interaction is important. Because the injected polarised carriers will give an internal magnetic field which will polarise the magnetic impurities, so that, they become less effective in breaking pairs. Spin orbit interaction can similarly be counteracted by the internal magnetic field due to spin imbalance in the material, though the spin orbit interaction might be absent in the cuprate superconductors.
FIG. 1. $\Delta(T = 0, n_0)/\Delta(T = 0, n_0 = 0)$ versus $n_0$ (the quasi-particle over population, as defined in the text) for s-wave superconductor (the solutions of equations 3 & 4).

FIG. 2. The normalised momentum averaged gap $\Delta(T, n)/\Delta(T, n_0 = 0)$ of a d-wave superconductor, with $T_c = 30^0K$ versus injected carrier concentration. Solid line is for $T = 5^0K$, long dashes $T = 10^0K$, short dashes $T = 15^0K$, dotted line $T = 20^0K$ and the dash-dotted line os for $T = 25^0K$.

FIG. 3. The normalised momentum averaged gap $\Delta(T, n)/\Delta(T, n_0 = 0)$ of a d-wave superconductor, with $T_c = 90^0K$ versus injected carrier concentration at $T = 10^0K$. The solid and dashed lines are for unpolarised and polarised injected carriers.

FIG. 4. The normalised momentum averaged gap $\Delta(T, n)/\Delta(T, n_0 = 0)$ of a (d-wave superconductor + interlayer tunneling), with $T_c = 90^0K$ versus injected carrier concentration at $T = 10^0K$. The solid and dashed lines are for unpolarised and polarised injected carriers.
normalised gap vs quasi-particle over population
normalised gap vs quasi-particle over population