A study of rock response to failure in the context of the bending properties and comparison with uniaxial tensile and compression behaviour

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Abstract. The phenomenon of rock bending occurs during underground exploitation, construction of underground excavations and tunnels, and even rising heading – shafts. It is also common in building engineering, e.g., in the case of floors. Rocks and concretes as granular materials on the aggregate scale are fractured as a result of exceeding shear and tensile strength. In a complex state of stress – bending, crack propagation occurs from tensioned to compressed fibres. Three-point bending tests of medium-grained quasi homogeneous and isotropic sandstone were tested for strength and deformation properties of rocks. The $E$ deformability modules for compressed and tensioned fibres as well as strains at failure were determined. The results of three-point bending were compared with the results of uniaxial compression and direct tension. Clear differences were found in the values of strengths, moduli of deformation and strains at failure. The bending strength $B$ of about 9.5 MPa is almost 3 times greater than the direct tensile strength $\sigma_T$ of about 3.2 MPa and is 1/10 of the ultimate uniaxial compression strength $\sigma_C$. With three-point bending, the values of the moduli E are equal to: for tensioned fibres about 6.7 GPa, for compressed fibres 14.6 GPa; in uniaxial compression tests about 13.0 GPa and in direct tensile tests 4.8 GPa. Rock material was also failure at various strains values at the ultimate strength. In the case of three-point bending tests, the strains at failure were equal to: for tensioned fibres about 0.125%, and for compressed fibres 0.065%; in uniaxial compression tests $\varepsilon_z$ were equal to about 0.63% and in direct tension tests 0.07%.

1. Introduction
A commonly used material constant describing the strength properties of rocks, but also concretes, composite granular materials and cohesive soils is the one- (and three-) axial compression strength $\sigma_C$. The use of this constants as a strength parameter is usually sufficient to solve most problems in the field of geomechanics, geotechnics and strength at the engineering level. However, materials such as rocks and concrete are in fact fractured by shearing and/or tensioning, which already occur at the level of grain aggregates [1].

Compression and shear tests are in common use, while uniaxial tensile tests are much less common. Failure of rocks is often due to flexure. Bending of rock layers occurs in the case of underground chamber roofs, mining excavations, tunnels and even rising headings at high horizontal stress values. Also in building engineering, the problem of bending of floors is key when designing ground-based and underground structures.

Bending tests of rock beams are rare, e.g., [2]. This is primarily due to the difficulty of obtaining and making rock samples with recommended accuracy, as well as the need to compare
with the results of other strength tests, e.g., uniaxial tensile tests. In the case of rocks in Poland, the phenomenon of bending of rocks was analysed only in the context of the bending strength and bending of roof layers near mining excavations. Such analyses were conducted by: in 1955 Salustowicz [3], Salustowicz and Galanka [4]. Borecki and Chudek [5] and later Kidybinski [6]. They gave the values of bending strength of rocks as one of the three basic material constants. In turn, Hobler [7] and Nagaraj [8] write about the tests of bending strength of rocks in the context of equating it with tensile strength.

Although the phenomenon of rock damage and failure occurs in many cases as a result of their flexure, issues related to rock stability are solved primarily on the basis of compression and shear constants. Constants of elasticity are used, e.g., Young’s modulus E, volumetric modulus K and shear modulus G. Similarly, in the numerical modelling the basic constants are already mentioned constants and constants related to shear strength such as cohesion c or angle of internal friction \( \phi \).

Few tests of rocks indicate, however, that bending strength \( \sigma_B \) should not be equated with tensile strength \( \sigma_T \), and \( \sigma_B = (1.42.0)\sigma_T \). According to Kidybiński [6], the \( \sigma_B/\sigma_T \) ratio is equal: for coarse-grained sandstones 1.7, for fine and medium-grained sandstones 1.4, for slates and coals 2.5. It also turns out that the properties of rock material subjected to bending are definitely different from that characteristic of rocks under compression or tension.

2. Materials and methods

Samples of medium-grained homogeneous and isotropic Brenna sandstone were tested. The properties of this sandstone have been well known as a result of research conducted in previous years, including uniaxial compression- and direct tensile-, Brazilian- and shear under compression tests, e.g., [9, 10].

Brenna sandstone is a medium-compactness light grey sandstone with a green, fine-grained shade with a random texture. A standard testing machine (compression/tension, 400kN) EDZ-40 by Werkstoffprüfmaschinen Leipzig (DDR) was used for the tests. The piston pressed out from the machine’s working cylinder at a constant velocity of 0.003mm/s. Cuboid samples with dimensions of 1555cm (hab) were placed on a steel plate with hardness above 45HRC on two cylindrical pins. Samples were made in 2017-2019 in accordance with the recommendations of the International Society of Rock Mechanics [11]. They were made in the sample preparation workroom of the Department of Geomechanics and Underground Construction. The distance between the supports was 11.5cm. The sample was loaded halfway through its ball-and-socket joint (figure 1).

Axial (longitudinal) strains \( \varepsilon_z \) were measured by latticed strain gauges type RL350/30/2.15 (made of Spoldzielnia Pracy “Techno-Mechanik”, Gdansk, Poland) and by the 16-channel MGCPplus strain gauge bridge (Hottinger Baldwin Messtechnik, HBM). For the acquisition of measurement data, the Catman v. 5.0 (HBM) program was used. A system of six independently connected strain gauges glued in the middle of the sample height, vertically to its longitudinal axis on the left, right and the bottom side of sample walls, taking into account the position of sample in the device, was used (figure 2).

Some authors, e.g., Hobler, equate bending strength \( \sigma_b \) with tensile strength (in bending test) and state that \( \sigma_B \) is calculated as:

\[
\sigma_T (= \sigma_B) = \frac{2}{3} \frac{F \cdot l}{b \cdot h^2} \quad \text{Pa}
\]

where: \( \sigma_T (= \sigma_B) \) – tensile strength determined in the three-point bending test, \( F \) – point load, \( N \), \( l \) – distance between supports, \( m \), \( h \) – beam height, \( m \), \( b \) – beam base (width), \( m \).

As already mentioned, laboratory tests show, however, that bending strength \( \sigma_B \) should not be equated with tensile strength, and \( \sigma_T = (0.50.7)\sigma_B \).
Figure 1. *Brenna* sandstone samples with strain gauges before testing.

Figure 2. Test stand for three-point bending tests of *Brenna* sandstone beams.

Figure 3. Method and places of gluing electro-resistance strain gauges; *right* side of sample.

Figure 4. *Brenna* sandstone beams after bending tests.
Jastrzebski et al. [12] describe the phenomenon of bending such as “the originally straight axis of the beam is curved, whereby the longitudinal fibres of the beam from the convex side are elongated, and from the concave side they are shortened”. If in the cross section except to the bending moments there are no lateral forces or they are negligible, we can talk about pure bending. Tasks related to the determination of beam deformations and stresses can be easily solved in the case of simple symmetrical bending. “It occurs when all forces act in one plane, called the plane of forces, which is also the plane of symmetry of the beam”. Then, only normal stresses $\sigma_n$ occur in the cross-section of the beam relative to this cross-section. There are no tangential stresses $\tau$. Stresses and strains can be determined strictly only in a few simple load cases; the simplest is three-point bending. Nagaraj [8] writes that in the case of pure bending, by determining the strength and deformation modulus for tensioned fibres of rock beam sample, one can (probably) talk about determining the tensile strength and modulus for tensioning the rocks.

If the elongation (deformation) of longitudinal fibres lying on the tensile bar’s wall (figures 5-6):

$$\varepsilon_x = \frac{z}{\delta},$$

where: $\delta$ – radius of curvature, $m$, $z$ – distance from neutral axis (NA); for rectangular cross-section $z = 0.5h$, $m$. If the maximum normal stresses $\sigma_x$ change linearly at the beam cross-section height and their extreme values are in the fibres clearance line from the neutral axis, we can write that:

$$\varepsilon_x = \frac{E_s z}{\delta},$$

Therefore:

$$\varepsilon_x = \frac{\sigma_x \delta}{z},$$

where: $E_b$ – modulus of elasticity of tensioned fibres, GPa, $\sigma_x = \sigma_B$ – concurrently, (after [12]). Knowing the value of bending strength $\sigma_B(= \sigma_x)$, one can determine the value of the elastic modulus $E_b$ of tensile beam fibres.
3. Results of laboratory tests
The results of the Brenna sandstone bending tests were compared with the results of other tests carried out in previous years. Brenna sandstone has already been subjected to uniaxial compression (figures 7a, 7b and 7c), uniaxial tension (figures 7d) and tensile tests using the Brazilian method (figures 8a and 8b). In the first series of bending tests, conducted 3 years earlier, tests were carried out by measuring the strains of tensioned fibres on the bottom wall of the sample at the failure strength (tensile stresses in tables 1 and 2, [13]).

Figure 7. The servo-operated research laboratory where uniaxial compression and tensile tests were carried out: laboratory view (a), Brenna sandstone sample before (b) and after (c) the uniaxial compression test and before the uniaxial tensile test (d).
The mean constant values were calculated based on the laboratory tests:

- uniaxial compression strength $\sigma_C = 94.86\, MPa$,
- axial strains at failure under uniaxial compression $\varepsilon_{z\max} = 0.63\%$,
- axial elasticity modulus under uniaxial compression $E_C = 12.99\, GPa$,
- uniaxial tensile strength $\sigma_T = 3.18\, MPa$,
- axial strains at failure under uniaxial tension $\varepsilon_{z\max} = 0.07\%$,
- axial elasticity modulus under uniaxial tension $E_T = 4.81\, GPa$,
- tensile strength under Brazilian test $\sigma_{TB} = 5.68\, MPa$,
- strength under three-point bending $\sigma_B(=\sigma_T) = 9.32\, MPa$,
- axial strains at failure under bending $\varepsilon_{z\max} = 0.17\%$,
- modulus of elasticity of tensioned fibres $E_b = 5.62\, GPa$.

Figure 8. Brazilian test of Brenna sandstone samples: sample in the Brazilian test apparatus and view of the samples after typical tensile failure under lateral compression (Brazilian test).

This means that the fracture of deflectioned rock material occurs at tensile strain values for tensioned fibres almost 1.5 times greater than at failure of rock material under the direct tensile tests (see tables 1 and 2).

The three-point bending tests carried out in the 1st series clearly pointed differences in the constants describing the strength and deformational properties of the rock material in simple (uniaxial compression and uniaxial tension tests) and in the complex state of stress (three-point bending test), both for compressed and tensioned fibres.
In the 1st series of three-point bending, the strains $\varepsilon_x$ was gauged using a system of three in series-connected strain gauges glued to the bottom walls of the samples (similar to G3 and G4 strain gauges, figure 6). It can be said in a simplification that such a strain gauge system measured (as in a serial array) the average strains over the entire width of the sample.

In the case of the 2nd, the latest series of three-point bending tests, tensile strains $\varepsilon_x$ were also measured in the middle bottom part of the sample (strain gauges G3 and G4) and were equal to 0.14%. They were smaller than the average (measured in the 1st series of test) by about 21%, but clearly by about 142% greater than the tensile strains at the failure strength measured in uniaxial tensile tests.

Table 1. Brenna sandstone constants calculated on the basis of uniaxial compression, uniaxial tension and tensile Brazilian tests. Constant values calculated in the three-point bending test: $\varepsilon_{x_{\text{max}}}$ - strains along the $x$ axis (axial) at the failure, $\sigma_B (=\sigma_T)$ - tensile strength under the three-point bending test, $E_b$ - modulus of elasticity of fibres under tensile stress along the $x$ axis (1st series of tests, [13]).

| Uniaxial compression test | Uniaxial tension test | Brazilian test | Three point bending test (1st series) |
|---------------------------|-----------------------|----------------|-------------------------------------|
| $h : d = 2,$              | $h : d = 4,$          | $h : d = 0.5,$ | (                        )          |
| $h = 84\text{mm}$         | $h = 168\text{mm}$    | $h = 21\text{mm}$ | [
| $\sigma_C$                | $\varepsilon_{z_{\text{max}}} E_c$ | $\sigma_T$ | $\varepsilon_{z_{\text{max}}} E_T$ | $\sigma_{TB}$ | $\varepsilon_{x_{\text{max}}}$ | $\sigma_B$ | $E_b$ |
| No. [MPa][%] [GPa]        | No. [MPa][%] [GPa]    | No. [MPa]      | No. [MPa][%] [GPa]               |
| B1 95.80 0.65 12.76       | B20 3.15 0.07 4.89    | B41 5.48       | Bb21 - 8.98 -                   |
| B2 97.40 0.62 13.85       | B21 3.54 0.09 4.13    | B42 6.42       | Bb22 0.16 9.16 5.67            |
| B10 92.30 0.62 12.96      | B23 3.63 0.08 4.46    | B43 5.10       | Bb23 0.17 10.14 5.98           |
| B11 93.70 0.62 12.44      | B30 2.74 0.04 6.65    | B44 5.74       | Bb24 0.16 9.34 5.75            |
| B13 95.10 0.63 12.99      | B31 2.84 0.09 3.93    | B45 5.67       | Bb25 0.17 9.00 5.37            |
| $\mu$ 94.86 0.63 12.99    | $\mu$ 3.18 0.07 4.81  | $\mu$ 5.68    | $\mu$ Bb26 0.17 9.29 5.32     |

The average ($\mu$) values of the axial deformation modulus $E_b$ determined for tensioned fibres for the 1st series of tests (3 strain gauges on the bottom sample wall, table 1) was equal to $5.62\text{GPa}$, and for the 2nd series of tests (2 strain gauges G3BL and G4BL on the bottom sample wall sample, table 2) about 19% higher and equal to $6.7\text{GPa}$. The $E_b$ values determined at bending (tables 1 and 2) for tensioned fibres were definitely higher, by approx. 30% than the $E_T$ values determined under direct tensile tests (table 1, see figure 9).

Comparing the values of strains at the failure strength obtained on the basis of three-point bending tests $\varepsilon_x$ and uniaxial compression and tensile $\varepsilon_z$ (tables 1, 2 and 3), several interesting conclusions can be drawn:

- the absolute value of compressive strains of compressed fibres at the failure strength in the upper part of the sample under bending is lower than the value of tensile strains (tensioned fibres) in the bottom part of the beam; these values are 0.07% and 0.11% respectively it means that in the case of rocks where is no symmetry of deformation (and probably stress
Table 2. Constant values calculated on the basis of three-point bending tests for a system of 2 strain gauges measuring tensile strains of the bottom beam wall (see Figure 2.8, 2nd series of tests): $\varepsilon_{x_{\text{max}}}$ - strains along the $x$ axis (axial) at the failure strength, $\sigma_B$ - bending strength under the three-point bending, $E_b$ - modulus of elasticity of fibres under tensile stress along the $x$ axis.

| Sample No. | Bottom strain gauges (bottom beam wall, tensioned fibres) | $\varepsilon_{x_{\text{max}}} [%]$ | $\sigma_B [\text{MPa}]$ | $E_b [\text{GPa}]$ |
|-----------|----------------------------------------------------------|----------------------------------|---------------------------|------------------|
| 1         | Left G3 BL | 0.147 | 9.02 | 6.14 |
|           | Right G4 BR | 0.141 | 9.02 | 6.40 |
| 2         | Left G3 BL | 0.156 | 8.63 | 5.53 |
|           | Right G4 BR | - | - | - |
| 3         | Left G3 BL | 0.146 | 9.40 | 6.44 |
|           | Right G4 BR | 0.107 | 9.40 | 8.79 |
| 4         | Left G3 BL | 0.172 | 9.49 | 5.52 |
|           | Right G4 BR | 0.139 | 9.49 | 6.83 |
| 5         | Left G3 BL | 0.150 | 9.47 | 6.31 |
|           | Right G4 BR | 0.127 | 9.47 | 7.46 |
| 6         | Left G3 BL | 0.154 | 9.55 | 6.20 |
|           | Right G4 BR | 0.118 | 9.55 | 8.09 |
|           | **Average** | **0.142** | **9.48** | **6.70** |

Figure 9. Stress $\sigma$ - strain $\varepsilon$ characteristics for different parts of the Brenna 1 beam sample under three-point bending.
Table 3. Tensile and compressive strains at the failure strength $\varepsilon_z$ measured on the side walls of the beam. G1LT and G6RT strain gauges measured compressive strains, while G2LB and G5LB strain gauges measured tensile strains. For determined constants: $\sigma_B = 9.26\text{MPa}$ - average bending strength, $\sigma_C = 94.86\text{MPa}$ average uniaxial compression strength, $\varepsilon_z = 0.63\%$ maximum strains at failure under uniaxial compression, $\sigma_T = 4.81\text{MPa}$ average direct tension strength, $\varepsilon_z = 0.07$ average maximum strains at failure under direct tension.

| Strain gauge | Sample part | Strain/position | $\varepsilon_{b_{max}}$ % | Strain gauge | Sample part | Strains | $\varepsilon_{b_{max}}$ % |
|--------------|-------------|-----------------|-----------------|--------------|-------------|----------------|-----------------|
| G1 LT        | 1 Left upper | Compressive (+) | 0.060 G2        | LB 1 leftlower | Tensile (-) | 0.095 | Average 0.065 |
|              | 2           |                 | 0.070           | 2            | 0.095 | Average 0.107 |
|              | 3           |                 | 0.048           | 3            | n.d. |                      |
|              | 4           |                 | 0.080           | 4            | 0.119 |                      |
|              | 5           |                 | 0.080           | 5            | 0.104 |                      |
|              | 6           |                 | 0.068           | 6            | 0.114 |                      |
| G6 RT        | 1 Right upper | Compressive (+) | 0.064 G5        | RB 1 Right lower | Tensile (-) | 0.107 | Average 0.065 |
|              | 2           |                 | 0.061           | 2            | 0.118 | Average 0.107 |
|              | 3           |                 | 0.053           | 3            | 0.107 |                      |
|              | 4           |                 | n.d.            | 4            | n.d. |                      |
|              | 5           |                 | n.d.            | 5            | n.d. |                      |
|              | 6           |                 | 0.068           | 6            | n.d. |                      |

values; compare Figure 8) relatively to the neutral axis, as described by theoretical solutions based on the mechanics of continuous and elastic media, the neutral axis is probably not the axis of symmetry of beam, and it is displaced upwards;

• all stress $\sigma$ - strain $\varepsilon$ characteristics of samples under bending, for both compression and tension fibres, are non-linear over the entire stress range (figure 10). Such non-linear behaviour was characteristic of direct tensile tests (figure 10a), whereas in the uniaxial compression tests there were intervals of linear (elastic) behaviour (figure 10b).

To sum up, bending tests carried out on well-known homogeneous and isotropic sandstone clearly show significant differences in the values of constants calculated on the basis of bending compared to constants based on compression and tensile tests. These differences relate not only to strength and deformability, but also to strains for which the process of material failure starts.

4. Conclusions
Rocks are granular materials. Mineral grains are bound by a binder. Their behaviour in the field of compressive stress is well known. Shear and tensile tests are also carried out. Tensile tests are
Figure 10. Typical characteristics of normal stress $\sigma_z$ - strain $\varepsilon$ of Brenna sandstone: comparison of the characteristics $\sigma_z = f(\varepsilon)$ for uniaxial compression (bottom) and uniaxial tension (top) (a) and the stress level $\sigma_z/\sigma_C$ and $\sigma_z/\sigma_C(b)$; $\varepsilon_z$ – axial strain, $\varepsilon_o$ – lateral strain, $\varepsilon_V$ – volumetric strain.

It is well-described and known sandstone. In previous years, research was conducted on it, including uniaxial compression, direct tension and shear under compression tests. The presented results of laboratory tests clearly indicate the differences between the values of constants calculated on the basis of three-point bending, direct tension and uniaxial compression tests. For the tested Brenna sandstone, these differences are very clear. The bending strength $\sigma_B$ of about 9.5MPa is almost 3 times greater than the direct tension strength $\sigma_T$ of about 3.2MPa and is 1/10 of the uniaxial compression strength $\sigma_C$.

Differences also occur in the case of deformability moduli $E$. Under three-point bending, $E$ is equal to: for tensioned fibres about 6.7GPa, and for compressed fibres 14.6GPa. For the same sandstone, the $E$ values were equal: in uniaxial compression tests around 13.0GPa, and in direct tensile tests 4.8GPa. The deformability is therefore very different.

Rock material was also failed at various strains values $\varepsilon$. In the case of three-point bending tests, the $\varepsilon_z$ strains were equal: for tensioned fibres about 0.125%, and for compressed fibres 0.065%. In uniaxial compression tests, failure occurred at a $\varepsilon_z$ value of about 0.63%, and in
direct tensile tests 0.07%.

The stress $\sigma$ - strains $\varepsilon$ characteristics for both compressed and tensioned fibres of sandstone samples under bending were non-linear over the entire stress range. Until now, the properties of rocks under bending were most often equated with their properties under tensile conditions. As it results from the conducted tests, rocks in the conditions of a complex state of bending stress show different properties than those in compression and tension, and the failure occurs at different strains values.

These rock material properties are important, especially from the point of view of researching and forecasting the development of damage zones in the excavation ceilings and underground structures, mining excavations and tunnels, and in the description of the rock material during numerical analysis and simulations.

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