Non-thermal hot dark matter in light of the $S_8$ tension

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The $\Lambda$CDM prediction of $S_8 \equiv \sigma_8(\Omega_m/0.3)^{0.5}$ – where $\sigma_8$ is the root mean square of matter fluctuations on a $8\ h^{-1}\text{Mpc}$ scale – once calibrated on Planck CMB data is $2 - 3\sigma$ lower than its direct estimate by a number of weak lensing surveys. In this paper, we explore the possibility that the 'S8-tension' is due to a non-thermal hot dark matter (HDM) fractional contribution to the universe energy density leading to a power suppression at small-scales in the matter power spectrum. Any HDM models can be characterized by its effective mass $m^\text{eff}$ and its contribution to the relativistic degrees of freedom at CMB decoupling $\Delta N^\text{eff}$. Taking the specific example of a sterile particle produced from the decay of the inflaton during a matter dominated era, we find that from Planck only the tension can be reduced below $2\sigma$, but Planck does not favor a non-zero $\{m^\text{sp}, \Delta N^\text{eff}\}$. In combination with a measurement of $S_8$ from KiDS1000+BOSS+2dLenS, the $S_8$-tension would hint at the existence of a particle of mass $m^\text{sp} \approx 0.67_{-0.48}^{+0.26}\text{ eV}$ with a contribution to $\Delta N^\text{eff} \approx 0.06 \pm 0.05$. However, Pantheo and BOSS BAO/$f\sigma_8$ data restricts the particle mass to $m^\text{sp} \approx 0.48_{-0.36}^{+0.17}\text{ eV}$ and contribution to $\Delta N^\text{eff} \approx 0.046_{-0.011}^{+0.004}$. We discuss implications of our results for other canonical non-thermal HDM models – the Dodelson-Widrow model and a thermal sterile particle with a different temperature in the hidden sector. We report competitive results on such hidden sector temperature which might have interesting implications for particle physics model building, in particular connecting the $S_8$-tension to the longstanding short baseline oscillation anomaly.

I. INTRODUCTION

The $\Lambda$ Cold Dark Matter (ACDM) model of cosmology is incredibly powerful at describing a wide variety of observation up to a high degree of accuracy despite the nature of its dominant components - Cold Dark Matter (CDM) and Dark Energy (DE) – still being unknown. Nevertheless, in recent years, a number of intriguing discrepancies has emerged between the values of some cosmological parameter predicted within ACDM – once the model is calibrated onto Planck Cosmic Microwave Background (CMB) data, Baryon Acoustic Oscillation (BAO) and luminosity distance to SuperNovae of type Ia (SNIa) – and their direct measurements.

At the heart of this study is the longstanding tension affecting the determination of the amplitude of the matter fluctuations, typically parameterized as $S_8 \equiv \sigma_8(\Omega_m/0.3)^{0.5}$, where $\sigma_8$ is the root mean square of matter fluctuations on a $8\ h^{-1}\text{Mpc}$ scale, and $\Omega_m$ is the total matter abundance. The latest prediction from Planck CMB data within the ACDM framework is $S_8 = 0.832 \pm 0.013$ [1]. Originally, observations of galaxies through weak lensing by the CFHTLenS collaboration have indicated that the ACDM model predicts a $S_8$ value that is larger than the direct measurement at the $2\sigma$ level [2]. This tension has since then been further established within the KiDS/Viking data [1]–[5], but is milder within the DES data [6]. However, a re-analysis of the DES data, combined with KiDS/Viking, lead to a determination of $S_8$ that is discrepant with Planck at the $3\sigma$ level, $S_8 = 0.755_{-0.021}^{+0.019}$ [5]. Recently, the combination of KiDS/Viking and SDSS data has established $S_8 = 0.766_{-0.014}^{+0.012}$ [7]. Moreover, it is now understood that the tension is driven by a lower matter clustering amplitude $\sigma_8$. This is particularly interesting for model building: resolving the $S_8$ tension requires to decrease the amplitude of matter fluctuations on scales $k \sim 0.1 - 1\ h/\text{Mpc}$, which can be easily achieved in a variety of models often related to new DM properties [8–20], or new neutrino properties [21]–[22].

In this paper, we explore the possibility that the 'S8-tension' is due to the existence of a non-thermal hot dark matter (HDM) particle contributing to a fraction of the dark matter (DM) density in the universe and leading to a power suppression at small-scales in the matter power spectrum. It is well known that just adding a thermal neutrino like radiation $\Delta N^\text{eff}$, together with a non-zero neutrino mass $m_w$, does not resolve the $S_8$ problem [1]–[21]. Here, we explore the consequences of a non-thermal momentum distribution for the hot component (or a temperature different from our visible sector), for the $S_8$-tension. In practice, we consider the momentum distribution associated with sterile particle produced from decays during a matter domination to radiation domination transition of the universe. We dub...
the model $\nu_{NT}\Lambda$CDM. From the point of view of theoretical models, it is natural for the early universe to enter an epoch of matter to radiation transition (this happens during the decay of the inflation or if cold moduli dominates the energy density of the universe at early times). Such particles have a characteristic momentum distribution, that is associated with decays taking place in a matter dominated universe evolving to radiation domination.

For decay products that thermalize, thermalization leads to loss of all information about the kinematics of the decay process. But in a setting with a large number of hidden sectors, one can expect that some of the species produced during the decay do not thermalize due to very weak interactions. Our scenario belongs to such category where a moduli or inflaton field decays to non-thermal sterile particle. There might be other particle like Feebly interacting massive particle (FIMP) which can also produce non-thermal or partially thermal neutrino like particle [23]. The presence of a non-thermal dark radiation can affect CMB [24] as well as large scale structure in specific ways and can be probed by precision cosmological data. The study of the implications of sterile particles with this momentum distribution for precision cosmology was recently initiated in Ref. [25]. Given that the effect of massive sterile particles on the CMB and matter power spectra is well-known (e.g. [29–31] for reviews), it was anticipated there one might get a strong power suppression in the matter power spectra due to the typical momentum distribution of non-thermal decay product. This power suppression has implications for the $S_8$-tension.

In this article, we perform a comprehensive monte-carlo markov chains (MCMC) analysis against up-to-date data from Planck, BOSS (BAO and $f\sigma_8$) and Pantheon data, with and without the inclusion of a prior on the value of $S_8$ as measured with the KiDS/Viking+BOSS+2dFLens data. We find that the $\nu_{NT}\Lambda$CDM model can indeed alleviate the tension between Planck and $S_8$ measurements, but the success of the resolution is degraded once BOSS and Pantheon data are included in the analysis. To better understand the model features leading to a resolution of the tension, we compare the non-thermal sterile neutrino model to the standard massive neutrino model with extra relativistic degrees of freedom. We find that, for similar effect on the CMB power spectrum, the $\nu_{NT}\Lambda$CDM leads to a much stronger suppression in the matter power spectrum at late-times, and therefore to a more significant decrease in $\sigma_8$. The impact of the $\nu_{NT}\Lambda$CDM is barely visible on the BAO scale and luminosity distance, but does significantly affect $f\sigma_8$ predictions. The model is therefore further constrained by BOSS data. Future measurements of the matter power-spectrum and $f\sigma_8$ at late-times will further test this scenario.

Although the MCMC analysis is carried out for sterile particles with the above described momentum distributions, it has implications for a wide class of models. As is well known (see e.g. [20, 32]), the cosmological implications of a hot a sterile component is captured effectively by just two parameters: a) the contribution of the component to the present day energy density, usually reported in terms of the effective mass parameter: $m_{\text{eff}}$, b) the contribution of the component to the energy density at the time of the CMB decoupling, usually reported in terms of $\Delta N_{\text{eff}}$ [8]. These parameters are determined by the first two moments of the momentum distribution and the mass of the sterile particle. Two models with equal values of $m_{\text{eff}}$ and $\Delta N_{\text{eff}}$ will have the same physics even if the functional form of the associated momentum distributions is different. We translate the results of the analysis for our model parameters to results on the effective parameters. Our results therefore have direct implications for other well motivated momentum distributions such as a thermal distribution with a different temperature from that of Standard Model [20, 35], the Dodelson-Widrow distribution [31] or distributions similar to the Dodelson-Widrow discussed in Refs. [39, 40].

Our paper is structured as follows: In Sec. II, we present our model and the mapping onto generic phenomenological parameters; in Sec. III we perform a MCMC analysis against a suite of up-to-date cosmological data and discuss the extent at which the $\nu_{NT}\Lambda$CDM can resolve the $S_8$-tension; in Sec. IV we draw implications of our results for other HDM models; finally, we conclude in Sec. V.

II. NON-THERMAL HOT DARK MATTER

A. The model

The physics of a constituent species of dark matter depends on its mass, interactions and also on its momentum distribution function. For species that thermalise, the process of thermalisation brings the momentum distribution to the Fermi-Dirac or Bose-Einstein form. On the other hand, for non-thermal constituents the momentum distribution is determined by their production mechanism. Thus, it is important to isolate natural production mechanisms for species that can constitute dark matter, the associated momentum distributions and their implications for cosmology.

In this section, we will review the basics of the production mechanism and the form of the momentum distribution that we will be considering. Our discussion will be

\[ \Delta N_{\text{eff}} \]

\[ S_8 \]
brief, we refer the reader to [28] and the references therein for the details. At early times, the energy density of the universe will be taken to be dominated by cold particles of a species $\varphi$. We will denote the mass of the particles of $\varphi$ by $m_\varphi$ and their decay width to be $\tau$. We will be focusing on the case when the $\varphi$ is the inflaton, with inflation taking place at the GUT scale and decays of the inflaton taking place due to a non-renormalizable interaction at the GUT scale. Thus we will take $m_\varphi \sim 10^{-6}M_{\text{pl}}$ and $\tau \sim 10^9/m_\varphi$. The branching ratio of the $\varphi$ particles to the sterile particles will be taken to be $B_{\text{sp}}$, the sterile particles so produced will be taken not to thermalise. We will assume that the other decay products thermalise, as this sector would contain the Standard Model, we will refer to it as the Standard Model sector. All decay products will be taken to be relativistic at the time of production. As the $\varphi$ particles decay, the universe goes into a matter to radiation epoch, finally becoming fully radiation dominated.

During the matter to radiation dominated epoch the evolution of the universe is governed by the equations

$$\dot{\rho}_{\text{mat}} + 3H\rho_{\text{mat}} = -\frac{\rho_{\text{mat}}}{\tau},$$

(1)

$$\dot{\rho}_{\text{rad}} + 4H\rho_{\text{rad}} = +\frac{\rho_{\text{rad}}}{\tau},$$

(2)

and

$$H = \left(\frac{\dot{a}}{a}\right) = \sqrt{\frac{\dot{\rho}_{\text{mat}} + \dot{\rho}_{\text{rad}}}{3M_{\text{pl}}^2}}.$$  

(3)

In the above, $\rho_{\text{mat}}$ denotes the energy density in matter and $\rho_{\text{rad}}$ is the energy density in radiation. The energy density in radiation is the sum of the energy densities in the Standard Model sector and the sterile particles (since the sterile particles are highly relativistic at the time of production, thus they contribute to the energy density as radiation during the epoch that the decays take place). It is useful to introduce the dimensionless variables

$$\theta = \frac{\tau}{\hat{\tau}}, \quad \hat{s}(\theta) = a(\tau\theta),$$

$$e_{\text{mat}}(\theta) = \frac{\tau^2\rho_{\text{mat}}(\tau\theta)}{M_{\text{pl}}^2}, \quad e_{\text{rad}}(\theta) = \frac{\tau^2\rho_{\text{rad}}(\tau\theta)}{M_{\text{pl}}^2}.$$  

(4)

Once almost all $\varphi$ particles have decayed, one can take the universe to be composed of a thermal bath (which contains the Standard Model sector) and the sterile particles governed by the standard cosmological evolution equations. In practice, we will start with a matter dominated universe at an ‘initial time’ ($t = \theta = 0$), evolve the universe using the equations (1) and (3) up to a fiducial dimensionless time $\theta^*$ which is large enough so that a good fraction the $\varphi$ particles have decayed by that time (we will choose $\theta^* = 15$ for this) and use the results of this as the initial conditions for the standard cosmological evolution.

The momentum distribution of the sterile particles can be computed using the following: as a result of the decays the co-moving number density of the sterile particles falls off as $N(t) = N(0)e^{-t/\tau}$ (and the branching ratio to the sterile particles is $B_{\text{sp}}$), once produced the sterile particles free-stream. We will be making use of publicly available package CLASS [41–43] to incorporate the effects of the sterile particles, the input required for this the momentum distribution of the sterile particles today. This was obtained in [28] to be

$$f(q) = \frac{32}{\pi\hat{E}^3} \left(\frac{N(0)B_{\text{sp}}}{\hat{s}(\theta^*)}\right) e^{-\hat{s}^{-1}(y)},$$

(5)

where

$$y = \frac{q}{4}\hat{s}(\theta^*),$$

(6)

and the range of $q$ is given by

$$\frac{4}{\hat{s}(\theta^*)} < q < 4.$$  

(7)

where $\hat{E} = m_\varphi/2$, $N(0)$ is the initial number density of the $\varphi$ particles, $\hat{s}(\theta^*)$ the value scale factor[4] at the fiducial dimensionless time $\theta^*$, $\hat{s}^{-1}$ the functional inverse of the scale factor function as a function of the dimensionless time and $\hat{H} = \hat{s}'(\theta)/\hat{s}(\theta)$ the dimensionless Hubble constant. The momentum $q$ in (5) is the magnitude of the momentum in units of the typical momentum of the sterile particles today ($T_{\text{ncdm},0}$). The typical momentum was found to be

$$T_{\text{ncdm},0} = 0.418 \left(\frac{m_\varphi^2\tau^2}{M_{\text{pl}}^4}\right)^{1/2} \frac{T_{\text{cmb}}}{(1 - B_{\text{sp}})^{1/4}} \equiv \zeta T_{\text{cmb}}$$  

(8)

in [28]. The distribution function in (5) is in units of $T_{\text{ncdm},0}^3$. Note that although naively $f(q)$ seems to depend on $N(0)$, the full expression is independent of $N(0)$ as long as we take the universe to be completely matter dominated at the initial times. It is interesting to compare the distribution to a thermal one, as shown in figure [4].

For the same value of $\Delta N_{\text{eff}}$, the distribution is peaked at higher values of the momentum, but is much broader. The mean momentum of sterile particles is greater than that of the CMB by the factor $\zeta$ defined in [5]. For our choice of parameters $\zeta \sim 5$. The sterile particles become non-relativistic when their typical momentum becomes of the order of their mass i.e the temperature of the Standard Model plasma becomes of the order $m_\varphi/5$.

### B. Mapping onto generic parameters

Our model has four microscopic parameters: $m_\varphi$, $\tau$ (the mass and lifetime of the decaying particle), $B_{\text{sp}}$ (the branching ratio for decay to the sterile particle)
and \(m_{sp}\) (the mass of the sterile particle) in addition to those of \(\Lambda\)CDM. Our choice of the first two parameters \((m_\nu \sim 10^{-6} M_{pl} \text{ and } \tau \sim 10^8/m_\nu)\) is motivated by taking \(\nu\) to be driving inflation at the GUT scale and decaying by GUT scale interactions. On the other hand, the other parameters \(B_{sp}\) and \(m_{sp}\) will be traded for effective parameters more directly connected to observables. Indeed, the physical effects of new sterile particle/species on the cosmological background and perturbation evolution can be completely described by three parameters: \(\Delta N_{\text{eff}}\) (the effective number of relativistic neutrinos at the time of neutrino decoupling), \(w_{sp} \equiv \Omega_{sp} h^2\) (where \(\Omega_{sp}\) is the fractional contribution of the particle to today’s energy density and \(h\) the reduced Hubble parameter, this is often characterised by the effective mass of the particle \(m_{sp}^\text{eff} \approx w_{sp} 94.05 \text{eV}\)) and \(\lambda_F\) (the free-streaming length associated with species). The free-streaming length is determined once the first two quantities are known, hence effectively there are two parameters \([29]\). Here we can take \(\Delta N_{\text{eff}}\) and \(m_{sp}\) as two independent quantities, defined as

\[
\Delta N_{\text{eff}} \equiv \frac{\rho_{\nu}^0}{\rho_{\text{c}}^0} = \left[ \frac{1}{\pi^2} \int dp \, p^3 f(p) \right] / \left[ 7 \frac{\pi^2}{15} T_{\nu}^{3/2} T_{\theta}^{1/2} \right] \tag{9}
\]

with \(T_{\nu}^{3/2} \equiv (4/11)^{1/3} T_\gamma\) and

\[
m_{sp}^\text{eff} = \frac{94.05 \text{eV}}{\sqrt{\frac{\omega_s}{\Omega_{sp} h^2}}} \equiv \omega_s \Omega_{sp} h^2 = \left[ \frac{m_{sp}^\text{eff}}{\Omega_{sp} h^2} \right] \times \left[ \frac{\rho_{\text{c}}^0}{\rho_{\nu}^0} \right] \tag{10}
\]

where \(\rho_{\text{c}}^0\) is the critical density today and \(h\) the reduced Hubble parameter. In our model, the effective parameters \(m_{\text{eff}}\) and \(\Delta N_{\text{eff}}\) in terms of the microscopic parameters are given by \([28]\):

\[
\Delta N_{\text{eff}} = \frac{43}{7} \frac{B_{sp}}{1 - B_{sp}} \left( \frac{g_{s}(T(t_\nu))}{g_{s}(T(t^*))} \right)^{1/3} \tag{11}
\]

and

\[
m_{sp}^\text{eff} = \frac{62.1 m_{sp}}{g_s^{1/4}(T(t_\nu))} \left( \frac{B_{sp}}{1 - B_{sp}} \right)^{3/4} \left( \frac{M_{pl}}{\tau m_{\nu}^2} \right)^{1/2} \tag{12}
\]

where \(g_s(T(t_\nu))\) and \(g_s(T(t^*))\) are the effective number of degrees of freedom at the time of neutrino decoupling and the end of the reheating epoch (we will take the later to be equal to 100). We will thus scan over \(m_{sp}\) and \(B_{sp}\), and use Eqs. \([12]\) and \([11]\) to relate to phenomenological parameters. In sec. \([IV]\) we will then translate our results for two other models of interest.

III. RESOLVING THE S8 TENSION WITH A NON-THERMAL STERILE NEUTRINO

A. Details of the analysis

We perform a comprehensive MonteCarlo Markov Chain (MCMC) analysis and confront the non thermal hot dark matter model to various combination of the following data-sets:

- Planck 2018 measurements of the low-\(\ell\) CMB TT, EE, and high-\(\ell\) TT, TE, EE power spectra, together with the gravitational lensing potential reconstruction \([44]\).
- The BAO measurements from 6dFGS at \(z = 0.106\) \([45]\), SDSS DR7 at \(z = 0.15\) \([46]\), BOSS DR12 at \(z = 0.38, 0.51\) and \(0.61\) \([47]\), and the joint constraints from eBOSS DR14 Ly-\(\alpha\) auto-correlation at \(z = 2.34\) \([48]\) and cross-correlation at \(z = 2.35\) \([49]\).
- The measurements of the growth function \(f \sigma_8(z)\) (FS) from the CMASS and LOWZ galaxy samples of BOSS DR12 at \(z = 0.38, 0.51, \) and \(0.61\) \([47]\).
- The Pantheon SNIa catalogue, spanning redshifts \(0.01 < z < 2.3\) \([50]\).
- The KIDS1000+BOSS+2dILenS weak lensing data, compressed as a split-normal likelihood on the parameter \(S_8 = 0.766^{+0.024}_{-0.014}\) \([4]\).

Our baseline cosmology consists in the following combination of the six \(\Lambda\)CDM parameters \(\{\omega_b, \omega_{\text{cdm}}, 100 \times h, n_s, \ln(10^{10} A_s), \tau_{\text{reio}}\}\), plus two parameters describing the nonthermal hot dark matter, namely \(\{m_{sp}, B_{sp}\}\). We dub this model \(\nu_{\text{NT}}\Lambda\)CDM. Standard model neutrinos are assumed to be massless.

To better understand how the \(\nu_{\text{NT}}\Lambda\)CDM model can resolve the \(S_8\) tension, we will compare it to the standard \(\Lambda\)CDM model with massless neutrinos, as well as to the \(\Lambda\)CDM model with free neutrino masses \(m_\nu\) and additional relativistic degrees of freedom \(\Delta N_{\text{eff}}\). In that latter case, we assume degenerate neutrino masses and a free-streaming \(\Delta N_{\text{eff}}\). We dub this model \(\nu\Lambda\)CDM. We
TABLE I. The mean (best-fit) ±1σ error of the cosmological parameters in the ΛCDM and ννNTACDM model obtained from the analysis of Planck [44] and Planck+5a [7] data. The definition of $m_{\text{eff}}^{\nu}$ is given in Eq. [12]. Upper limits are given at the 95% C.L.

TABLE II. Same as Tab.I with the addition of 'Ext' data, which refers to the combination BAO/FS+Pantheon.

We run our MCMCs with the Metropolis-Hasting algorithm as implemented in the MontePython-v3 [51] code interfaced with our modified version of CLASS. All reported $\chi^2_{\text{min}}$ are obtained with the python package iMINUIT [52]. We make use of a Choleski decomposition to better handle the large number of nuisance parameters [53] and consider chains to be converged with the Gelman-Rubin convergence criterion $R − 1 ≤ 0.05$ [54].

B. Results

We run two sets of analysis; in the first one, we confront the ΛCDM, $\nu$CDM and ννNTACDM models to Planck only and Planck+5a. In the second one, we add the BAO and Pantheon data to our analysis. Our main results are reported in Tabs. I and II and displayed on Figs. 2 and 3. We report results in the ννNTACDM in terms of $\Delta N_{\text{eff}}$ and $m_{\text{eff}}^{\nu}$, defined in Eqs. [11] and [12]. We give the $\chi^2_{\text{min}}$ per experiment in App. A.

1. Planck only

When the ννNTACDM model is confronted to Planck only, we obtain a bound on the mass $m_{\text{sp}}^{\nu} < 0.93$ eV and $\Delta N_{\text{eff}} < 0.18$. Similarly, in the $\nu$CDM case we obtain $m_{\nu} < 0.073$ eV and $\Delta N_{\text{eff}} < 0.28$ (recall that this limit applies to individual neutrino masses in the degenerate case). The $\chi^2_{\text{min}}$ of Planck in the $\nu$CDM and ννNTACDM scenario is not improved over that of ΛCDM. We note that the ννNTACDM model predicts a lower $S_8$ value than other models. Indeed, we find $S_8(\nu$CDM) = 0.831 ± 0.012 and $S_8(\Lambda$CDM) = 0.832 ± 0.011, to be compared to $S_8(\nu$NTACDM) = 0.816 ± 0.022, i.e., a $\gtrsim 1\sigma$ downward shift. As a result, the $S_8$ tension is alleviated from the $\sim 2.7\sigma$ level to the $\sim 1.9\sigma$ level in the non-thermal HDM model. We note that our constraints on $\Delta N_{\text{eff}}$ in the non-thermal case is stronger than that reported in Ref. [44] (constraints are identical in the thermal case). This likely comes from the impact of running on physical parameters as opposed to phenomenological parameters when exploring the parameter space.

Including the prior on $S_8$, we notice a mild detection of non-zero $m_{\text{sp}}^{\nu} = 0.67^{+0.26}_{−0.48}$ eV and $\Delta N_{\text{eff}} = -0.84^{+0.16}_{−0.72}$.
FIG. 2. Reconstructed 2D posterior distributions of \( \{m_{\text{eff}}^\nu, \Delta N_{\text{eff}}, S_8, \Omega_m\} \) with Planck and Planck+S8 data (left panel) or Planck+BAO+SN1a and Planck+BAO+SN1a+S8 data (right panel).

FIG. 3. Same as fig. 2 in the thermal neutrino case.

0.0614\(^{+0.0052}_{-0.0047}\) in the \( \nu_{\text{NT}}\Lambda\text{CDM} \) model, while the constraints on the thermal neutrino mass simply relaxes to \( m_\nu < 0.1 \) eV. This translates into a reconstructed \( S_8(\nu_{\text{NT}}\Lambda\text{CDM}) = 0.789 \pm 0.016 \) and \( S_8(\nu\Lambda\text{CDM}) = 0.812 \pm 0.011 \), to be compared with the baseline \( S_8(\Lambda\text{CDM}) = 0.814^{+0.011}_{-0.011} \). As a consequence, the \( \chi^2_{\text{min}} \) in the combined analysis is lower in the non-thermal HDM case \( \Delta \chi^2_{\text{min}}(\nu_{\text{NT}}\Lambda\text{CDM}) = \chi^2_{\text{min}}(\Lambda\text{CDM}) - \chi^2_{\text{min}}(\nu_{\text{NT}}\Lambda\text{CDM}) = -4.8 \) than in the thermal neutrino case \( \Delta \chi^2_{\text{min}}(\nu\Lambda\text{CDM}) = \chi^2_{\text{min}}(\Lambda\text{CDM}) - \chi^2_{\text{min}}(\nu\Lambda\text{CDM}) = -1.4 \). If the \( S_8 \) tension worsens in the future, it would be interesting to perform a more complete Bayesian analysis comparing these models. We notice however that the total \( \chi^2_{\text{min}} \) is much less significantly affected by the inclusion of the \( S_8 \) prior in the non-thermal case (+3.6) than in the thermal case (+7.9), which is encouraging and indicates that the \( \nu_{\text{NT}}\Lambda\text{CDM} \) model can potentially alleviate the tension between Planck and KIDS+BOSS. It remains to be seen whether this is robust to additional data sets (and in the future it should be tested against the full KiDS and BOSS likelihoods).

Before including external data, we comment on the possibility for non-thermal hot dark matter to resolve the
Hubble tension (see e.g. [55–57] for a review). We find that, whether we include the $S_8$ prior or not, the value of $H_0$ is barely affected by the extra $\Delta N_{\text{eff}}$ (in fact, even shifted slightly towards lower $H_0$ due to the well-known anti-correlation with $m_{\nu}^\text{eff}$ [31]). We therefore confirm that these models cannot be responsible for the high-$H_0$ measured with some of the local probes.

2. Planck+BOSS+SN1a

When the BAO/FS and SN1a data are added to the analysis, the constraints on the thermal neutrino mass and non-thermal hot dark matter mass strengthen. We find $m_{\nu}^\text{eff} < 0.73$ eV and $\Delta N_{\text{eff}} < 0.12$ in the $\nu_8\Lambda$CDM model, while we get $m_{\nu} < 0.04$ eV and $\Delta N_{\text{eff}} < 0.27$ in the thermal case. Still, the reconstructed $S_8$ value $S_8(\nu_8\Lambda$CDM) = 0.814$^{+0.017}_{-0.014}$ and $S_8(\nu\Lambda$CDM) = 0.83 ± 0.011 are slightly smaller than in the Planck-only analysis. This is because the reconstructed value of $\omega_{\text{cdm}}$ is slightly smaller in the combined analysis with BAO/FS and SN1a data, regardless of the model.

Once the prior on $S_8$ is added to the analysis, we again find a mild detection of $m_{\nu}^\text{eff} = 0.48^{+0.17}_{-0.36}$ eV and $\Delta N_{\text{eff}} = 0.045^{+0.038}_{-0.031}$ . However, the mean value has decreased by 0.5$\sigma$ due to the inclusion of BAO/FS and SN1a data. This reflects in a slightly larger reconstructed $S_8$ value, $S_8(\nu_8\Lambda$CDM) = 0.795$^{+0.013}_{-0.015}$. A similar pattern is observed in the thermal case, for which the relaxation of the constraint to $m_{\nu} < 0.057$ eV is much milder than without BAO/FS and SN1a data, while the reconstructed $S_8(\nu\Lambda$CDM) = 0.814 ± 0.01 is stable. Looking at $\chi^2_{\text{min}}$, one can see that the non-thermal case still provides a better fit $\Delta \chi^2_{\text{min}}(\nu_8\Lambda$CDM) = −3.7 than the thermal case $\Delta \chi^2_{\text{min}}(\nu\Lambda$CDM) = −1.8. However, the inclusion of the $S_8$ prior as increased the total $\chi^2_{\text{min}}$ by +4.8 in the non-thermal case and +6.9 in the thermal case. We conclude that, while Planck and $S_8$ data can be accommodated in the non-thermal scenario (not in the thermal one), the BAO/FS and SN1a data poses a serious challenge to this model.

C. Understanding the MCMC

In order to understand better the results of the MCMC analyses, we show in Fig. 4 the residuals of the CMB TT, EE, lensing (top panel) and matter (bottom panel) power spectra with respect to $\Lambda$CDM in the bestfit $\nu\Lambda$CDM and $\nu_8\Lambda$CDM models obtained when considering Planck+S8 and Planck+Ext+S8 data. We also show in Fig. 5 the corresponding transverse BAO (top panel), longitudinal BAO (middle panel) and growth factor (bottom panel). The first thing to notice is that, for a similar effect in the CMB power spectra, the corresponding power suppression in the matter power spectrum is much stronger in the $\nu_8\Lambda$CDM than in the $\nu\Lambda$CDM model.

This is the reason why the $\nu_8\Lambda$CDM can perform much better in resolving the $S_8$ tension.

Looking at the BAO and $f\sigma_8$ prediction, one can see that the most important difference is in the latter, which is significantly lower at all $z$ in the $\nu_8\Lambda$CDM because of this power suppression. This explains the small degradation in $\chi^2$ in the combined analysis with $S_8$. Moreover, the reconstructed dark matter density $\omega_{\text{cdm}}$ in the $\nu_8\Lambda$CDM is also shifted by roughly $\sim 1\sigma$ downwards (to compensate for the higher energy density due to the non-relativistic transition of the non-thermal neutrinos), which also leads to a small degradation in the fit to Planck data (hardly visible by eye in CMB power spectra residuals). This small difference in the matter density is also visible in the small-$k$ (large scales) branch of the matter power spectrum, particularly sensitive to $\Omega_m$. [59]. While these differences do not yet unambiguously rule out the $\nu_8\Lambda$CDM as a resolution to the $S_8$ tension, they do provide an interesting avenue to probe the model with future data, in particular through accurate measurements of the matter power spectrum, CMB lensing power spectrum and growth factor $f\sigma_8$. An interesting avenue to improve over the $\nu_8\Lambda$CDM results presented here is to assume that the hot component comes from the decay of a meta-stable cold dark matter species in the late-universe [19–20], instead of being present at
all times. A good fit to all data can then be obtained when the mass-ratio of the mother and daughter particle \( \epsilon \sim 0.007 \) and the CDM lifetime \( \tau \sim 55 \) Gyrs.

**IV. IMPLICATIONS FOR OTHER NON-THERMAL HOT DARK MATTER MODELS**

As discussed in the introduction and section II B, any distribution with the same values of \( \Delta N_{\text{eff}} \) and \( m_{\text{eff}} \) as ours should also relax the \( \sigma_8 \) tension. Our results can thus be used to extract implications for the microscopic parameters of models which have momentum distributions different from the one we have used. Here, we present such results for two models:

a) Sterile particles at a different temperature from that of the Standard Model neutrinos. In this model, sterile neutrinos follow a thermal Fermi-Dirac Distribution,

\[
f(p) = \frac{1}{e^{p/T_s} + 1}
\]

where \( T_s \) is the temperature of sterile particles. For a thermal sterile particle with a Fermi-Dirac distribution and a different temperature \( T_s \), the quantities \( \Delta N_{\text{eff}} \) and \( \omega_s \) become

\[
\Delta N_{\text{eff}} = \left( \frac{T_s}{T_{\nu}} \right)^4, \quad \omega_s = \frac{m_{sp}}{94.05} \left( \frac{T_s}{T_{\nu}} \right)^3.
\]

b) The Dodenson-Widrow distribution [54]

\[
f(p) = \frac{\chi}{1 + e^{p/T_{\nu}}}
\]

where \( T_{\nu} \) is the temperature of the neutrinos today, \( \chi \) is a parameter related to the phenomenological parameters as [29]:

\[
\Delta N_{\text{eff}} = \chi, \quad m_{\text{eff}} = m_{sp} \times \chi,
\]

and \( m_{sp} \) is the individual neutrino mass in the model. We report the best-fit value of the model parameters in

**FIG. 6.** Residuals of Matter power spectra (top panel) and \( C_l \) TT, TE and EE power spectra (bottom panel).
| Model                  | Non-thermal | Thermal | Dodelson Widrow |
|------------------------|-------------|---------|-----------------|
| Data set               | m$_{\nu}$ [eV] | $B_{\nu}$ | $\frac{\Delta N_{\nu}}{\Delta N_{\nu}}$ | $m_{\nu}$ [eV] | $\chi$ |
| Planck                 | 0.05        | 0.01    | 0               | 0.40          | 0.03 |
| Planck+i+S$_8$         | 3.62        | 0.012   | 11.36           | 0.43          | 26.43 | 0.03 |
| Planck+i+Ext           | 18.98       | 0.01    | 0.45            | 0.36          | 12.85 | 0.02 |
| Planck+i+Ext+S$_8$     | 39.81       | 0.01    | 11.75           | 0.43          | 27.49 | 0.03 |

TABLE III. Best-fit values of the physical parameters in the non-thermal, thermal and Dodelson-Widrow sterile neutrino models derived from our analyses.

Tab. III, obtained from translating our constraints on $\Delta N_{\nu}$ and $m_{\nu}$. We also show in Fig. 6 the residuals of the CMB TT, TE, EE and matter power spectra between our best-fit non-thermal HDM model and these two models. This explicitly demonstrates our claim that, once $\Delta N_{\nu}$ and $m_{\nu}^{\text{eff}}$ are fixed, observables are indistinguishable. We note that the residuals between the thermal neutrino model at different temperature and our non-thermal HDM model are of the order of the sensitivity of future LSS experiments such as EUCLID and LSST, and therefore this simple mapping might become limited in the future. Note that, to avoid biasing constraints due to prior effects, we refrain from translating our reconstructed posterior on $\Delta N_{\nu}$ and $m_{\nu}^{\text{eff}}$ into the model parameters.

The values we report in Tab. III have direct implication for thermalized hidden sector from both particle physics [60] and cosmological perspective [61, 62]. Interestingly, the main parameter for building a thermal hidden sector model is the temperature ratio $\xi = \frac{T_{\nu}}{T}$, which received a competitive constraint (though it depends on the model) from our analysis and it may have strong implications for light sterile neutrino [61] or other hidden sector particle physics models [63, 64]. If the thermal hidden particle interacts with dark matter or other particle in dark sector, the coupling and other particle physics parameters can be constrained from our result [63].

It is tantalizing to connect the hot dark matter discussed here to the longstanding (and debated) short base line (SBL) anomalies [66, 67] (see [68, 69] for recent reviews). Concretely, within the so called “3+1” neutrino scenario, those can be explained by a sterile neutrino with $m_s \approx \sqrt{\Delta m_{41}^2}$ 1eV and a mixing angle leading to $\Delta N_{\nu} \approx 1$. However, we find that the sterile particles required by the S8-tension hints to a somewhat higher mass range $m_s \sim O(10)$ eV (see tab. III), and an almost negligible $\Delta N_{\nu}$. Our constraints, whether we include the $S_8$-prior or not, thus further confirm that a viable sterile neutrino solutions to the SBL anomalies would require some additional mechanism to prohibit large $\Delta N_{\nu}$ production (see e.g. [70, 73] for examples). Nevertheless, it could be interesting to perform analysis including results short base line neutrino oscillation (e.g. with an additional prior as in Ref. [61]). This is beyond the scope of this paper and is kept for future study.

Finally, we also note that including data from the Bicep2/Kek array [74, 75], SPT-3G [76] or ACT [77] could help further constraining the sterile neutrino parameters thanks to higher accuracy measurement of the CMB damping tail and lensing spectrum. We also keep that for a future study, but refer to Refs. [76, 78] for examples (constraints typically increases by $\sim 10\%$, without considering a prior on $S_8$).

V. CONCLUSIONS

In this paper, we have explored the possibility that the ‘$S_8$-tension’, the long-standing discrepancy between the determination of the amplitude of the matter fluctuations from local [247] and cosmological [11] probes, is due to the existence of a non-thermal HDM contributing to a fraction of the DM density in the universe and leading to a power suppression at small-scales in the matter power spectrum. Concretely, we have considered non-thermal HDM produced as decay products of the inflaton. Such particles have the momentum distribution associated with decays taking place in a matter dominated universe evolving to radiation domination, as shown in [28]. However, we have argued that any model leading to the same $\Delta N_{\nu}$ and $m_{\nu}^{\text{eff}}$ as our model (barring additional new physics ingredients) would lead to similar effects on cosmological observables, and therefore our constraints generically apply to any HDM models.

We have performed a comprehensive monte-carlo markov chains (MCMC) analysis against up-to-date data from Planck, BOSS (BAO and $f_{S_8}$) and Pantheon data, with and without the inclusion of a prior on the value of $S_8$ as measured with the KiDS/Viking+BOSS+2dFLens data. Our findings can be summarized as follows:

- the $\nu_{\text{NT}}\Lambda\text{CDM}$ model can indeed alleviate the tension between Planck and $S_8$ measurements, but the success of the resolution is degraded once BOSS and Pantheon data are included in the analysis.

- Compared to standard thermal neutrinos, the $\nu_{\text{NT}}\Lambda\text{CDM}$ leads to a much stronger suppression in the matter power spectrum at late-times for similar effect on the CMB power spectrum, and therefore to a more significant decrease in $\sigma_8$.

- The impact of the $\nu_{\text{NT}}\Lambda\text{CDM}$ is barely visible on the BAO scale and luminosity distance, but does significantly affect $f\sigma_8$ predictions. The model is therefore constrained by BOSS growth factor measurements, and future measurements of the matter...
power-spectrum and $f\sigma_8$ at late-times will further test this scenario.

- We further discussed the connection between our model and generic phenomenological parameters constrained by the data that can be easily used to translate our constraints onto other similar models. Especially, we put constraints on other non-thermal HDM models—like the Dodelson-Widrow models or on a thermal sterile particle with different temperature in hidden sector. We report competitive constraint on the hidden sector temperature and DW scaling parameter on the hidden sector temperature in hidden sector. We report competitive constraint on the hidden sector temperature and DW scaling parameter which can have interesting particle physics implications, for instance in the context of SBL anomalies [66–69].

Lyman-α forest flux power spectrum data [10, 79–85] and future high accuracy measurement of the matter power spectrum at small scales by upcoming surveys such as Euclid [80], LSST [87], and DESI [88] can further test these models as a resolution to the $S_8$-tension.

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Appendix A: $\chi^2_{\text{min}}$ per experiment

We report $\chi^2_{\text{min}}$ per experiment in each of the analysis performed.

| Experiment | $\chi^2$ |
|------------|---------|
| Planck high $-\ell$ TT,EE | 2346.7 |
| Planck low $-\ell$ EE | 396.3 |
| Planck low $-\ell$ TT | 23.3 |
| Planck lensing | 8.8 |
| Pantheon | − |
| BAO/FS BOSS DR12 | − |
| BAO BOSS low $-z$ | − |
| KiDS/BOSS/2dFGS | − |
| **total** | **2774.9** |

TABLE IV. Best-fit $\chi^2$ per experiment (and total) in the $\Lambda$CDM model.

| Experiment | $\chi^2_{\text{min}}$ |
|------------|------------------|
| Planck high $-\ell$ TT,EE | 2345.98 |
| Planck low $-\ell$ EE | 396.54 |
| Planck low $-\ell$ TT | 23.3 |
| Planck lensing | 9.03 |
| Pantheon | − |
| BAO/FS BOSS DR12 | − |
| BAO BOSS low $-z$ | − |
| KiDS/BOSS/2dFGS | − |
| **total** | **2774.9** |

TABLE V. Best-fit $\chi^2$ per experiment (and total) in the model with massive thermal neutrinos and additional relativistic degrees of freedom.

| Experiment | $\chi^2_{\text{min}}$ |
|------------|------------------|
| Planck high $-\ell$ TT,EE | 2346.7 |
| Planck low $-\ell$ EE | 396.3 |
| Planck low $-\ell$ TT | 23.1 |
| Planck lensing | 8.8 |
| Pantheon | − |
| BAO/FS BOSS DR12 | − |
| BAO BOSS low $-z$ | − |
| KiDS/BOSS/2dFGS | − |
| **total** | **2773.0** |

TABLE VI. Best-fit $\chi^2$ per experiment (and total) in the non-thermal sterile neutrino model.

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