The Dynamics of COVID-19 spread in Lebanon

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Abstract

We explore the spread of the Coronavirus disease 2019 (COVID-19) in Lebanon by adopting three different approaches, namely the SEIR model, a model of repeated iterations and a Fermi-Dirac-like model. We fit the first seventy five days of available daily data since the first diagnosed case and we forecast possible scenarios of contagion associated with different levels of social distancing measures. We determine the initial reproductive transmission rate in Lebanon and its subsequent dynamics. Our results suggest that preserving severe mitigation measures would halt the spread of the disease. Nevertheless, relaxing measures would trigger a second outbreak of infections within a couple of weeks, with severity depending on the extent of relaxation.

Keywords: Covid-19; SEIR model; Fermi Dirac distribution; Mathematical modeling in epidemiology; Compartmental Models

1 Introduction

The Coronavirus disease 2019 (COVID-19) has been widely spreading worldwide since it appeared in the city of Wuhan, China towards the end of December, 2019. The World Health Organization classified the spread as a pandemic in March 2020 \cite{WHO}. Europe and the United States of America have endured the severest repercussions in terms of number of infections and deaths. The US cases amounted to about one third of total global infections by April 2020 \cite{US}. Governmental and institutional reactions and measures varied across countries with respect to the time of introduction of social distancing measures (SDM, henceforth) and with respect to their degree of severity. In spite of some governments being slower in adopting mitigation measures and endorsing the epidemiological
concept of herd immunity \cite{3} to create a resistance to the contagion in the long run at the expense of short term losses while keeping the economy functional, the majority adopted SDM’s that reached countrywide lockdowns.

A considerable amount of research has been carried out focusing on the dynamics and extent of the pandemic in different countries notably in the countries that witnessed the first cases \cite{4, 5, 6, 7, 8, 9, 10, 11}. In comparison with the most recent deadly mass pandemic of the "Spanish flu" that hit the world after World War I during the years of 1918-1919 and recurred in two waves, and caused the death of tens of millions of people \cite{12, 13, 14, 15}, the extent of the spread of COVID-19 has been far less. The question of the containing COVID-19 and preventing its spread and expansion into a similar deadly pandemic is a key motivation for the study of various models that describe, simulate and forecast epidemics and dynamics of infections under different reproductive rates and mitigation measures.

In Lebanon, the spread of COVID-19 coincided with a period of political turmoil, few months of popular uprising \cite{16, 17} and economic collapse finally depicted by the default on debts in early March 2020 \cite{18}. Additionally, the currency underwent severe depreciation and the economic deterioration was acute before the COVID-19 related lockdown that brought the economy further down \cite{19, 20}.

**Lebanon SDM measure** SDM were adopted at a relatively early stage in Lebanon. While the first confirmed case was recorded on February 21, 2020, all academic institutions, namely schools and universities, were closed starting the first of March. This was followed by a resolution of "Public Mobilization" and ban of public gatherings that imposed closure of all churches, mosques, shops, restaurants, etc. except for grocery stores and drugstores on March 22 \cite{21}. Another subsequent measure consisted in constraining vehicles’ mobility to alternating between odd- and even-ending plate numbers, while fully prohibiting mobility on Sundays. The source of the first infection was documented to be from a traveler coming from Iran \cite{22, 23} where the spread of the virus had started early on \cite{24}. However, it has been discussed that many of the following first cases were transmitted by travelers coming from the Vatican city during the early stages of the pandemic spread there \cite{25}. The efficacy of SDM’s can be examined by discerning the daily rate of infection as shown in the daily data of the first seventy five days \cite{26}. Our results were obtained in relation to data available until May 9, 2020.

**Literature and Methodology** Models exploring contagion and particularly spread of infections were developed and extensively studied in various fields of mathematics, physics, economics and statistics \cite{27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38}. The effect of mitigation measures on the spread of infections was studied in \cite{35, 36, 37, 38}. It was shown that the use of protective masks \cite{39}, combined with social distancing, ensures high levels
of safety and protection. We establish our study of the dynamics of the COVID-19 spread in Lebanon on three models that have been used in analogous studies of infectious diseases including COVID-19 worldwide and in other countries, namely: the SEIR model [40, 41, 42], the repeated iterations model presented in [43] (henceforth the RI model) and the Fermi-Dirac-like model of [44]. We use a daily data of the Coronavirus in Lebanon provided by mainly the Ministry of Public Health [26] over the length of seventy five days.

The structure of the paper is as follows. In section 2, we present the three models. Results and forecasts are discussed in section 3 and section 4 concludes the paper.

2 Theoretical Framework

We describe here three models that we use to forecast the path of daily cases and to measure the extent of the epidemic in Lebanon. Following an abrupt rise in infected cases at the start, the rate has fallen to a significantly low level after implementation of severe SDM’s, in comparison with other regional and country-level data [45]. Figure 2 illustrates the progression of the number of daily infections over time in different countries: Lebanon, Turkey, Iran, Italy and the USA. Day 0 represents the date of the first reported infection. Lebanon has a relatively low number of infections per capita, despite the fact that the first case was recorded relatively early. The proposed models accommodate for the actual data and allow future predictions.

Figure 1: Daily infections (per million) in Lebanon, Turkey, Iran, Italy and the USA.
2.1 SEIR Model

The study of the spread of epidemics can be executed by the fundamental SIR model first introduced by Kermack and McKendrick [46], that consists of dividing the population into different compartments and investigating the contagion of the disease by examining the rate of change of the sizes of these groups. Later versions were developed to accommodate for different settings and assumptions [47, 48, 49]. We use here the SEIR model described as follows. Consider a population $N$ that we normalize to size 1 and divide into four categories of individuals: susceptible $S$, exposed $E$, infectious $I$ and removed (through recovery or death) $R$. The cumulative number of cases is provided by $C = I + R$, since each recorded case would at a given time be either infectious or recovered or dead. The rates of change of these categories are given by $\frac{dS}{dt}$, $\frac{dE}{dt}$, $\frac{dI}{dt}$ and $\frac{dR}{dt}$ respectively. The model can be formally described using the following differential equations:

$$\frac{dS}{dt} = -\beta_t \frac{S}{N} I$$  \hspace{1cm} (1)

$$\frac{dE}{dt} = \beta_t \frac{S}{N} I - \sigma E$$  \hspace{1cm} (2)

$$\frac{dI}{dt} = \sigma E - \gamma I$$  \hspace{1cm} (3)

$$\frac{dR}{dt} = \gamma I$$  \hspace{1cm} (4)

with $\beta_t = R_t \gamma$ where $\beta_t$ is the rate at which infected individuals bump into others (the $\frac{S}{N}$ susceptibles), $\sigma$ is the rate at which exposed individuals become infected and it is associated to the mean incubation period; $\gamma$ is rate of exit by recovery or death per-day and it is associated to the average illness period. $R_t$ is the ratio of meeting rate to exit rate; it determines the transmission from susceptible to infected and is a proxy for social distancing measures. There are several ways to model $R_t$. It can be taken as a constant parameter in some circumstances, or a time dependent function as in [40].

In our model, using data available from the first 75 days in Lebanon, we assume that $R_t$ can be parameterized by the step function

$$R_t = \begin{cases} 
R_0 & 0 < t < t_1 \\
R_1 & t > t_1 
\end{cases}$$  \hspace{1cm} (5)

after the application of SDM, and before any relaxation. When the measures are loosened after an initial period of SDM, then $R_t$ would be parameterized as follows:
where $R_0$, $R_1$ and $R_2$ are constant parameters that depend on the severity of measures and commitment to those measures, with $R_1 < R_2 < R_0$. $R_0$ is the reproductive transmission rate of the disease in the initial phase, while $R_1$ and $R_2$ are the reproductive transmission factors under strict and relaxed SDM respectively. We also take $\sigma = \frac{1}{5.2}$ in relation to an average period of incubation of 5.2 days, and $\gamma = \frac{1}{20}$ in relation to an average period of recovery (or death) of 20 days.

Assuming that initially no SDM are applied, $R_0$ represents the transmission of disease with no mitigation measures. In [50] they adopt $R_0 = 3.1$, while in [51] they take values between 2.76 and 3.25, and [40] considers different values between 3 and 1.6. Recent studies reveal that $R_0$ of COVID-19 can assume higher values up to 3.87, 5.7 and 6.47 according to data analyzed from each of Mexico and China [52, 53, 54]. After introduction of measures, $R_t$ can assume values less than 1 in case extremely severe mitigation measures are applied [55]. It was well established that COVID-19 has higher $R_0$ than other infections like SARS [56]. The estimation of $R_0$ and $R_t$ is essential for

Figure 2: Cumulative number of infections according to the SEIR model in Lebanon, with appropriate parameterization with $R_0 = 5.6$ for the first 32 days and $R_1 = 0.65$ afterwards. The actual cases are represented in black and the expected cumulative infections are in brown.

$$R_t = \begin{cases} R_0 & 0 < t < t_1 \\ R_1 & t_1 < t < t_2 \\ R_2 & t > t_2 \end{cases}$$

(6)
forecasting the spread, but their determination depends on the available data and the accuracy of the reporting of initial cases and dates. More accurate values of $R_0$ and $R_t$ usually become available after the maturation of the spread \[57, 58\].

We assume that the initial value of $I$ is $I_0 = \frac{1}{6M}$, in line with the first initial case in Lebanon reported on February 21 and a gross population of 6 million inhabitants. We take $E_0 = 12I_0$ to account for the fact that the initial case registered had been in contact with many people on a flight from Iran which raises the number of initially exposed people. This entails some uncertainty in the initial conditions of the spread. In comparison, in \[40\], they consider a value $I_0 = \frac{1}{10M}$ with 33 initial cases in the United States whose population stands at around 330 millions, and $E_0 = 4I_0$ given 132 individuals were initially carrying but not contagious, acknowledging the considerable uncertainty related to initial cases in the US.

In our model we find that $R_0 = 5.6$ and $R_t = 0.65$ for $t_1 = 32$ days and $t_2 = 80$ days, provide the best prediction for registered cumulative cases, and then we simulate four possible future scenarios with $R_2 = 0.65, 1.3, 1.9$ and 2.5 after $t_2 = 80$ days.

2.2 The RI model

We consider another model that takes into account the most recent available daily data of confirmed and recovered (or dead) cases. Here we implement a variation of the repeated iterations method proposed in \[43\]. Denote the currently infected daily values by $I_i$ where $i$ is the index of days and $i \in [1, n]$. We take the last $m$ values of $I_i$ to determine the average arithmetic gross rate in the last $m$ days according to

$$G_a = \frac{1}{m} \sum_{i=n-m+1}^{n} \left( \frac{I_i}{I_{i-1}} - 1 \right)$$

while the average geometric gross rate of the same set of data is defined by

$$G_g = \left( \frac{I_i}{I_{i-m}} \right)^{\frac{1}{m}}$$

Each of $G_a$ or $G_g$ allows us to forecast the number of people infected for $i > n$ by simply implementing a progression for the following days according to

$$I_{i+1} = I_i (1 + G_a)$$

using the arithmetic gross infection rate $G_a$ or alternatively

$$I_{i+1} = I_i G_g$$

using the geometric gross infection rate $G_g$. 
It is important to note that we also have to take into account the number of people dead or recovered at later dates. To account for this, we denote the death rate by \( p \), the recovery rate by \( 1 - p \), the average number of days needed for recovery by \( h \) and the average number of days between infection and death by \( d \). This means that on day \( i + 1 \), the number of deaths will be \( p (I_{i-d} - I_{i-d-1}) \) where the term in brackets represents the number of people who caught the virus \( d \) days ago. Similarly the number of people recovered would be proportional to the number of people who caught the virus \( h \) days ago, thus it is given by \((1 - p) (I_{i-h} - I_{i-h-1})\). Our recursive relation includes the number of dead or recovered people as predicted from the people who got the infection \( h \) and \( d \) days ago respectively. The values of \( p \) and \( h \) vary in the literature and in the available data from the specific country considered.

Then the repeated iterations model forecasts the net number of infected people by

\[
(I_{i+1})_{\text{net}} = I_{i+1} - p (I_{i-d} - I_{i-d-1}) - (1 - p) (I_{i-h} - I_{i-h-1}) \tag{11}
\]

This model is a general model and not country specific. From available data in Lebanon, it is reasonable to take \( p = 0.04 \), \( h = 20 \) days and \( d = 24 \) days. Note that in this iteration, we use the available data to forecast the next unavailable day, and recalculate \( G \) accordingly, hence recursively predicting the future development of the rate and the number of infected people. Here we will consider \( m = 14 \) previous days, and forecast the next upcoming 14 days as well. The interesting feature of the RI model is that as new daily data is revealed, we can easily update our daily future forecast, hence have a new 14 day future expectation every day.

\( G \) is a dynamic quantity and it will depend on the rapidness and strictness of the public policies of social distancing, ban of public gatherings and curfews, as well as on the commitment of people to those measures and to health measures (sanitation, wearing protective masks, gloves...). This is why, to forecast future situations, we have to take into account different scenarios and possibilities for the progression of \( G \) in relation to those measures and practices. In our simulations, we explore the predictions based on \( G_a, G_g \), the maximum attained value of \( G \) in the past \( m \) days as well as possible absolute increases or decreases in those rates.

### 2.3 Fermi-Dirac like Model

The idea of using a Fermi Dirac like model to study the progression of the cumulative number of infections was proposed by [44]. It is based on the assumption that the maximum number of possible infected people \( P \) is a known parameter, with \( C(t) \) representing the cumulative number of infections at time \( t \). Then the percentage of the people who will be infected later is given by \( \frac{P - C(t)}{P} \). In a time interval \( \Delta t \) and with coefficient \( n \) representing the number of interactions between an infected and non-infected persons,
the number of people who would catch the infection is proportional to \( \left( \frac{P-C(t)}{P} \right) n \Delta t \). We use this to derive the difference equation that determines the number of people catching the infection between times \( t \) and \( \Delta t \).

This is given by:

\[
\frac{C(t + \Delta t) - C(t)}{\Delta t} = D \left( \frac{P - C(t)}{P} \right) C(t)
\]

(12)

where \( D \) represents the frequency of infection and is directly proportional to \( n \). In a short time interval \( dt \), it leads to the differential relation

\[
\frac{dC(t)}{dt} = D \left( \frac{P - C(t)}{P} \right) C(t)
\]

(13)

Integrating equation (13) we can determine the cumulative number of infected people at a time \( t \) by:

\[
\frac{C(t)}{P} = \frac{1}{me^{-Dt} + 1}
\]

(14)

where \( m \) is an integration constant, and \( C(t) \) saturates into \( P \) after enough time \( t \). This can be more accurately applied on data from countries where the disease has already advanced and started reaching some early levels of saturation in order to predict the time remaining for reaching its maximum short term spread and the daily changes until then, with the appropriate parameters \( m \) and \( D \).

The expression in (14) is a distorted variation of the well-known Fermi-Dirac distribution for \( m \neq 1 \) and has a strong resemblance to the FD distribution [59, 60] expressed by

\[
F(\epsilon) = \frac{1}{e^{(\epsilon-\mu)/k_B T} + 1}
\]

(15)

\( D \) plays a role analogous to the energy of a Fermi gas \( (\epsilon - \mu) \), while \( t \) is associated to the Boltzmann factor \( \beta = \frac{1}{k_B T} \), \( T \) is the temperature of the Fermi gas and \( k_B \) is the Boltzmann constant.

3 Results

3.1 SEIR

In our specific parameterization of the SEIR model for the pattern of cases registered in Lebanon, we found that the initial value of the reproductive transmission factor is very high and starts at \( R_0 = 5.6 \) but then significantly decreases to \( R_1 = 0.65 \) at \( t_1 = 32 \) days. This parameterization supplies an accurate fit with the registered cases for the
Figure 3: The reproduction transmission factor $R(t)$ as a function of time $t$ in days. It starts at $R_0 = 5.6$ then falls down at $t = 32$ days to $R_1 = 0.65$ after strong mitigation measures. At $t = 80$ days, we inspect four possible scenarios of $R_2$, with values equal to 0.65, 1.3, 1.9 and 2.5 in black, blue, brown and red respectively.

first 75 days. In addition to absence of any SDM, a relatively high value of $R_0$ could be attributed to late reporting or under reporting of the early cases, hence the fast surge of registered results during the first few days of official testing. $R_0$ here reflects the fast pace of spread of the disease in the early days before taking and implementing social distancing measures. The rapid fall of the rate from $R_0$ to $R_1$ occurs after the implementation and the social commitment to mitigation measures, hence the rapid decrease in the number of daily infections and the slow increase in the cumulative number of infections. The very low rate of $R_1$ sharply diminishes the number of new infections, and the curve of the cumulative number of infections (Figure 2) starts flattening out slowly.

However, if the measures are relaxed at a time $t_2 = 80$ days, we expect a an increase in the reproductive rate from $R_1$ to a another constant value $R_2$. The exact value of $R_2$ will still depend on the extent of relaxation and the public commitment to SDM. We considered here four possible values of $R_2$: Continued strict mitigation measures with $R_2 = R_1 = 0.65$, a continued commitment to measures with weak relaxation is parameterized by $R_2 = 1.3$ while more public social interaction and moderate relaxation is parameterized by $R_2 = 1.9$. The last choice is $R_2 = 2.5$, corresponding to wide relaxation, yet panic among people helps in preventing $R_2$ from returning back to high values as those of the initial rate $R_0$. 
We find out that the cumulative number of infections will rise again at a higher pace as depicted in Figure 4 even for the lowest increase in $R_2$. The number of cumulative infections can reach a total varying between 1791 to more than 8271 infections in July 2020, depending on the extent of relaxation. On the contrary, a continued commitment to SDM measures on the same levels as $R_1$ would lead to a controlled total of 1086 infections by then. This means that the disease can swiftly spread again once the measures are relaxed, and the pace of the spread would depend on the level of official and public relaxation of mitigation measures. This is a result numerically specific to Lebanon, but the general pattern is a universal outcome and means that a second wave of COVID-19 infections is inevitable in absence or weakening of SDM.

### 3.2 RI model

The arithmetic and geometric means proposed in equations (7) and (8) of the repeated iterations model assume that all SDM will be maintained at their current levels in the upcoming $m$ days under consideration. But this is not necessarily the case. In order to take into account possible changes we considered the following scenarios that we simulated in figure 5:

1. We determine the arithmetic and the geometric means of the last 14 days and
Figure 5: Number of currently infected people according to the RI model. The colored lines represent the progression of the number of currently infected people in the next 14 days according to five possible scenarios. The lower pink line corresponds to an absolute decrease of 1% in the current rate of infection. The middle black and brown lines almost coincide and they correspond to the continuation of the current geometric and arithmetic average rate of infection. The red line represents a progression with a rate corresponding to the maximal attained daily rate of the last 14 days, while the upper green line corresponds to an absolute increase of 2% in current average rate.

the corresponding future forecasts are plotted in brown and black respectively. It is clear that with the rate infection registered in Lebanon, and with more people recovered, the number of currently infected people would almost stabilize during the upcoming couple of weeks, and the geometric and arithmetic means considered lead to very similar forecasts.

2. The rate of infection decreases by an absolute value of 2% and the number of recovered people is also on the rise so the total number of currently infected people slowly decreases faster.

3. The rate of progression is defined as the maximum of the rates of increase recorded in the past fourteen days

\[ G_{\text{max}} = \text{Max}\left\{ G_{g_{i-(m-1)}}, G_{g_{i-(m-2)}}, \ldots, G_g \right\} \]

with an increment of 1% in absolute rate. The number of current infections will continue increasing despite of recoveries and deaths.
Figure 6: Cumulative number of infected people according to a Fermi-Dirac like model. The green line represents the actual cumulative number of infections in Lebanon. The black line represents the Fermi-Dirac distribution that fits the cumulative data with $m = 67.7$ and $d = 0.0947$, with a future forecast of 15 upcoming days, assuming all current measures and rates remain constant.

4. The mean rate of infection increases by an absolute rate of 2%, and the number of currently infected people would rise quickly despite recoveries or deaths from previous cases.

As in the SEIR future forecasts, the different scenarios depend directly on the official measures as well as on public behavior and social distancing. If social life returns back to a more normal situation and the precautions are diminished, the number of infections will be on the rise again according to scenarios 3 or 4. The continued enforcement of prevailing measures will help implement scenario 1 or more optimistically scenario 2 in case of more public commitment (Figure 5).

3.3 Fermi-Dirac model

Using the Fermi-Dirac distribution function provided in equation (14), and fitting with $m = 67.7$ and $d = 0.0947$, we obtain a fit for the available actual data from Lebanon, with saturation attained at $P = 900$ total cases within two upcoming weeks. The curve allows us to forecast the progression of the cumulative number of cases until saturation, provided that the infection rate continues with the same dynamics. The curve in figure 6 implies similar conclusions to what we confer from the SEIR and the RI models in the case of the continuation of current SDM, with a slow and controlled increase in the cumulative number of infections. The fast increase in the rate of infection followed by the fast flattening of the curve also confirms the results obtained from the SEIR model under similar conditions.
Figure 7: The figure shows the number of daily tests conducted per 1 million inhabitants for each of Italy (Blue), Turkey (Brown), Lebanon (Green), Iran (Black) and USA (Red).

3.4 Discussions

The low infection rate in Lebanon in the last weeks of the COVID-19 pandemic could be attributed to public commitment to social distancing and the strict mitigation measures applied. The number and the distribution of tests conducted could also be a factor in revealing or hiding the real scope and rate of infections.

To assess this factor, we compare the number of daily tests conducted per million inhabitants, in Lebanon and several other countries that witnessed stronger spreads like Italy, USA, Iran and Turkey. We can find that the percentage of people tested in Lebanon is similar to that conducted in Iran, and tends to rise up with time but it is significantly less than the other countries as Figure 7 shows. This could in principle lower the detected infections especially among people with very mild symptoms who might opt not to test themselves, despite being positive.

To further assess this effect, it is important to check the ratio of the cumulative number of infections with respect to the cumulative numbers of tests conducted. This criterion would give the rate of infected out those of tested, hence eliminates the uncertainty related to under-testing. A inspection of the publically available data reveals that the cumulative rate of infection among those who were tested in Lebanon was 1.65% on May 9th, 2020, compared to 14.56% in the USA, 8.54% in Italy, 10.12% in Turkey and 18.34% in Iran [2]. This is an assertion that the low rate of infection is much more related to the mitigation measures applied in the country, as rates in more affected countries are larger by several orders of magnitude.

However, the relaxation of measures and the rise of the reproductive rate of infections
due to increased interaction can easily bring the country back to a high rate of infections within a couple of weeks as the forecast presented in Figure 4 and Figure 5 reveal. This is confirmed by our future simulations on both of the SEIR and the RI models, following two different methodologies with several possible scenarios. The aforementioned models both forecast a resumption of a quick spread of the disease and an increase in the number of infected people once the SDM are reduced or abandoned. The severity of the spread depends on the extent of the relaxation. The currently achieved pattern of slow and controlled spread can slide into a swift and wider spread under looser conditions.

Under all circumstances, the continued SDM are essential to keep COVID-19 under control until the introduction of effective medications or vaccines, which is estimated to take at least between 12 to 18 months [61, 62], despite ongoing medical and clinical research around the globe [63, 64, 65, 66].

4 Conclusion

This work presented three different models used in the simulation of the spread of infectious diseases which are the SEIR model, the RI model and a Fermi-Dirac model. We developed some variations to the models and adjusted the parameters to fit the available data from Lebanon. We analyzed in detail the current spread and different forecasts for future developments. We find out that the rate of infection and the number of infected people fell down quickly due to the rapid fall in the reproductive number $R$ due to strong mitigation measures. However, relaxing the measures and the resumption of social activity and interaction would swiftly put the infections on a rapid rise again, thus reversing the temporary success in limiting the spread of COVID-19. Attacking the disease from different angles allows us to show in different ways that the temporary official and public SDM have succeeded in halting the spread of the disease, but the resumption of business and life as usual will put the spread back on track of fast growth. Consequently, SDM should be maintained in order to safely guarantee controlled spread.

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