I. INTRODUCTION

Of broad interest in particle and astroparticle physics is the hadronic cross section. The inclusive hadronic cross section is measured over a range of energies in laboratory experiments [1], and in cosmic ray experiments, for example, the recent proton-air cross section measured at $\sqrt{s} = 57$ TeV by the Pierre Auger Observatory [2]. The hadronic cross section is important for the analysis of a range of experiments and it is an essential input to many astroparticle physics models for DIS has virtual photons fluctuate to $q\bar{q}$ dipoles which then scatter with the hadronic target via the dipole cross section $\hat{\sigma}$. In this approach, the structure function $F_i$ comes from the convolution of the virtual photon wave function squared (for splitting into the color dipole) $|\Psi_{L,T}(r, x; Q^2)|^2$ with the dipole cross section $\hat{\sigma}(r, x)$, integrated over the spatial splitting of the dipole $r$ and the longitudinal momentum fraction $x$ of the quark in the dipole. The dipole picture can be useful in many contexts, for example, ultrahigh energy neutrino scattering where one may approach the unitarity limit, and high energy hadronic production of charm, where small $x$ and $Q$ are relevant kinematic variables.

The usual starting point in the color dipole model is to postulate a functional form of the dipole cross section guided by theoretical or phenomenological considerations. Given this parametrization of $\hat{\sigma}$, hadronic processes as well as DIS scattering can be used to determine free parameters in $\hat{\sigma}$. There are a number of models for the dipole cross section [12–21]. The Golec-Biernat-Wüsthoff model [12] explicitly includes saturation and an effect called geometric scaling. Geometric scaling describes the behavior of DIS observables at low $x$, for example $\sigma_{\gamma p}$, depends on a scaling variable $T = Q^2 R_0(x)$, independent of $W^2$, the square of the total energy in the $\gamma^* p$ collision [13]. Other authors, including Soyez [16] have done more elaborate fits to DIS data for dipole cross section parameterizations guided by theory that also in-
clude geometric scaling \cite{14, 15}.

Besides the parameterizations of the dipole cross section, there are direct parameterizations of $F_2(x, Q^2)$ \cite{22–30} based on fits to the DIS data \cite{31, 32}, typically for small $x$ and moderate $Q^2$. The functional form of the $F_2$ fits are postulated independent of any guidance from dipole cross sections.

Our goal here is to examine how a parametrization of $F_2$ translates to a corresponding $\hat{\sigma}$. This phenomenological approach is the inverse of the usual starting point of $\hat{\sigma}$. Eventually, one hopes to explore how different treatments of the small $x$ behavior of $F_2$ guided by unitarity considerations translate to the dipole cross section and vice versa. This paper is a first step in this phenomenological program.

We take advantage of approximate relations between $F_2$ and $\hat{\sigma}$ as discussed by Ewerz, von Manteuffel and Nachtmann in Ref. \cite{37}. As noted by Ewerz et al. \cite{37}, the convolution of the photon wave function and the dipole cross section is simplified in the $m_q = 0$ case. We use Fourier transforms to factorize the convolution for $m_q = 0$ to find an approximate $\hat{\sigma}_0$ from an inverse Fourier transform. This dipole cross section is a starting point for the dipole treatment when $m_q \neq 0$.

In the next section, we review the relevant formulas for the structure function $F_2$ in terms of the dipole cross section and photon wave functions. We show the convenient variables introduced in Ref. \cite{37} make the problem amenable to inversion by Fourier transforms in the case of $m_q = 0$. In Sec. III, we discuss the determination of $\hat{\sigma}_0$ in a toy model with a single massless unit charged quark. Fourier transforms require knowing the structure function at both low and high $Q^2$. For a starting point, we use the Donnachie-Landshoff parametrization \cite{22} of $F_2$ which is defined for the full range of $Q^2$, even if it does not describe DIS data for the full $Q^2$ range. In Sec. IV, we look at the more physical case with 5 massive quarks. We find that with a simple normalization constant, the dipole cross section solution to the toy model does quite well in reproducing the Donnachie-Landshoff structure function. In Sec. V, we compare our results with the GBW \cite{12} and Soyez \cite{16} dipole cross section parameterizations. We compare our numerical results with the small $Q$ and large $Q$ approximations of Ref. \cite{37}. We conclude in Sec. VI.

II. DIPOLE FORMALISM AND THE DIPole CROSS SECTION

The dipole formula for deep inelastic electromagnetic scattering involves the photon wave function and the dipole cross section. Our focus is on the electromagnetic structure function $F_2(x, Q^2)$ in deep inelastic scattering where $ep \rightarrow eX$ involves the subprocess $\gamma^*p \rightarrow X$. In terms of the subprocess cross sections, the dipole picture has \cite{10}

$$F_2(x, Q^2) = \frac{Q^2}{4\pi^2\alpha_e} \left[ \sigma_L^{\gamma p}(\xi, Q^2) + \sigma_T^{\gamma p}(\xi, Q^2) \right]$$

in terms of the virtual photon-proton cross section

$$\sigma_L^{\gamma p}(\xi, Q^2) = \sum_f \int d^2r \int_0^1 da \left| \Psi_L^{(f)}(r, \alpha; Q^2) \right|^2 \times \hat{\sigma}(r, \xi) .$$

The photon wave functions are written in terms of the Bessel functions $K_0$ and $K_1$, and they depend on the quark electric charge $e_f = 2/3, -1/3$ and mass $m_f$ through

$$|\Psi_L^{(f)}(r, \alpha; Q^2)|^2 = e_f^2 \frac{\alpha N_c}{2\pi^2} 4Q^2\alpha^2(1-\alpha)^2 K_0^2(\bar{q}r_f) \quad (3)$$

$$|\Psi_T^{(f)}(r, \alpha; Q^2)|^2 = e_f^2 \frac{\alpha N_c}{2\pi^2} \left( (\alpha^2 + (1-\alpha)^2) K_0^2(\bar{q}r_f) + m_f^2 K_1^2(\bar{q}r_f) \right) \quad (4)$$

for $\bar{q}^2 = \alpha(1-\alpha)Q^2 + m_f^2$. A common choice is to use $m_u = m_d = m_s = 0.14$ GeV, $m_c = 1.4$ GeV and $m_b = 4.5$ GeV, however, when $m_f = 0$, the photon wave functions depend on $Q$ only through $z = Qr$, a considerable simplification. The electromagnetic fine-structure constant is labeled $\alpha_e$ and $N_c = 3$ is the number of colors in eqs. \cite{34, 34}.

The dipole cross section in eq. \cite{3} depends on the dipole transverse size $r$ and $\xi$. Ewerz et al. in Ref. \cite{37} consider $\xi = x$ and $\xi = W^2$. While theoretically $\xi = W^2$ is more justifiable, the choice of $\xi = x$ allows the dipole model to match experimental data over a wider range of $Q^2$ \cite{37}. There are a number of models for the dipole cross section that depend on $x$ rather than $W^2$ \cite{12, 14, 17}. Since our aim is to invert parameterizations of $F_2$ to determine the dipole cross section, our choice of $\xi$ is determined by how $F_2$ is parameterized. For our discussion here, $F_2 = F_2(x, Q^2)$ so $\hat{\sigma}(r, \xi = x)$.

Ewerz et al. in Ref. \cite{37} discuss the convolution formula of eq. \cite{3} and consider limiting regimes for the dipole cross section. They have conveniently rewritten the convolution in terms of smooth functions with properties amenable to treatment with Fourier transforms. First, the integral over $d\alpha$ and the angular integral in $d^2r$ in eq. \cite{3} are performed to write

$$F_2(x, Q^2) = \sum_f Q \int dr h(Qr, m_f r) \frac{1}{r^2} \hat{\sigma}(r, x) ,$$

where

$$h(Qr, m_f r) = \frac{Q r^3}{2\pi \alpha_e m} \int d\alpha \times \left[ |\Psi_L^{(f)}(r, \alpha; Q^2)|^2 + |\Psi_T^{(f)}(r, \alpha; Q^2)|^2 \right] . \quad (5)$$
Given $F(x, t)$ in Eq. (9) for a wide range of $t$ (namely, a wide range of $Q$), $S(x, t')$ can be found by taking the Fourier transform to factorize the convolution, then inverting the Fourier transform. In the next section, we perform this procedure numerically where we use $F_2(x, Q^2)$ parameterized by Domnachie and Landshoff (1993) $[24]$. Even though this parametrization has a limited range of applicability in its description of the experimental data (e.g., $0.1 \text{ GeV}^2 < Q^2 < 10 \text{ GeV}^2$), we use this simple form to demonstrate the procedure and to interpret the results for $\hat{\sigma}$.

III. MASSLESS, UNIT CHARGE QUARK EXAMPLE

As a starting point for using the Fourier transform to extract $\hat{\sigma}$ from a parametrization of $F_2$, we consider the toy model of a massless quark with unit charge. Using the usual formulas for the Fourier transform and its inverse,

$$f(t) = \int_0^\infty dk \{ a_f(k) \cos kt + b_f(k) \sin kt \}$$

$$a_f(k) = \frac{1}{\pi} \int_{-\infty}^\infty dt \cos kt f(t)$$

$$b_f(k) = \frac{1}{\pi} \int_{-\infty}^\infty dt \sin kt f(t) ,$$

for a completely symmetric function $\kappa(\tau)$, $b_\kappa(k) = 0$. We approximate $b_\kappa(k) \approx 0$ in what follows. We find that the dipole cross section we obtain, when convoluted with the photon wave function and appropriately normalized, is a good approximation to the parameterized structure function.

Because in the massless case $\kappa(\tau)$ depends only on $\tau$, the convolution integral factorizes with Fourier transformed and simplifies to the following when $b_\kappa = 0$:

$$\pi a_F(k) = \int_{-\infty}^\infty dt \cos kt \int_{-\infty}^\infty dt' \kappa(t-t') S(t')$$

$$\pi a_\kappa(k) = \int_{-\infty}^\infty dt \sin kt \int_{-\infty}^\infty dt' \kappa(t-t') S(t')$$

$$\pi b_S(k) = \int_{-\infty}^\infty dt \cos kt \int_{-\infty}^\infty dt' \kappa(t-t') S(t')$$

and

$$\pi b_F(k) = \int_{-\infty}^\infty dt \sin kt \int_{-\infty}^\infty dt' \kappa(t-t') S(t')$$

$$\pi a_\kappa(k) \pi b_S(k) .$$

The Fourier transform of $\kappa$ is straightforward. For the Fourier transform of $F$, as noted above, we need $F_2(x, Q^2) = Q^2 e^{2Q^2}$ for the full range of $Q^2$, in principle between $Q^2 = 0 \rightarrow \infty$. The Domnachie-Landshoff
parametrization \[22\] meets the requirement of being defined for all \(Q^2\), even if not valid for the full range,

\[
F_2(x, Q^2) = Ax^{1-\alpha} \left( \frac{Q^2}{Q^2 + a} \right)^\alpha + Bx^{1-\beta} \left( \frac{Q^2}{Q^2 + b} \right)^\beta
\]

(15)

where

\[
A = 0.324 \quad a = 0.5616 \text{ GeV}^2 \quad \alpha = 1.0808
\]
\[
B = 0.098 \quad b = 0.01114 \text{ GeV}^2 \quad \beta = 0.5475
\]

This is based on a fit to NMC data \[38\] for \(Q^2 < 10\) GeV\(^2\). Our comparison will be restricted to the \(Q^2\) range between 0.1 – 10 GeV\(^2\). We have chosen \(Q_0 = 0.82\) GeV so that \(F(x, t)\) has a maximum at \(t = 0\). While it is not completely symmetric, \(F(x, t)\) has a similar shape for \(t\) less than and greater than zero, for small \(x\).

![Graph of \(\hat{\sigma}_0\) as a function of \(r\) given a unit charged, massless quark, for \(x = 10^{-5}, x = 10^{-4}\) and \(x = 10^{-3}\) using the Donnachie-Landshoff parametrization of \(F_2\). The vertical dotted line shows \(r_0 = 2.93\) GeV\(^{-1}\) = 0.58 fm.](image)

![Graph of \(F_2(x, Q^2)\) as a function of \(Q^2\) for \(x = 10^{-5}, x = 10^{-4}, x = 10^{-3}\) using the dipole cross section shown in Fig. 2 (dashed) and the Donnachie-Landshoff parametrization for Eq. (15).](image)

TABLE I: The values of the parameter in the dipole cross section formula, with \(s_0 = 27.95\) mb and \(r\) is in units of GeV\(^{-1}\).

| \(r [\text{GeV}^{-1}]\) | \(a\) | \(b\) | \(c\) | \(d\) | \(e\) |
|---|---|---|---|---|---|
| \(r < 1\) | 0.02315 | 3.34 | | | |
| \(1 < r < 2.9\) | 0.9943 | 7.599 \times 10^{-3} | 4.3028 | 2.469 \times 10^{-2} | 4.0702 |
| \(2.9 < r < 5\) | 2.7866 | -1.762 \times 10^{-2} | 4.1789 | 6.4517 | -0.8983 |
| \(r > 5\) | 35.2489 | -2.62 | | | |

The resulting dipole cross section \(\hat{\sigma}_0\) from the Fourier transform and inversion of \(F\) and \(k\), for \(m_f = 0\) with a unit charged quark, is shown in Fig. 2. Three values of \(x\) are shown, \(x = 10^{-3}, 10^{-4}\) and \(10^{-5}\) in increasing order. The dipole cross section can be parameterized as

\[
\hat{\sigma}_0 = s_0 \left( \frac{x}{10^{-4}} \right)^{1-\alpha} (a r^b),
\]

(16)

for \(r < 1\) GeV\(^{-1}\), \(r \geq 5\) GeV\(^{-1}\)

\[
\hat{\sigma}_0 = s_0 \left( \frac{x}{10^{-4}} \right)^{1-\alpha} (\pm 1 - a e^{br^c} + dr^e),
\]

(17)

where \(s_0\) is 27.95 mb and \(r\) is in units of GeV\(^{-1}\). The other parameters are presented in Table I. The \(x\) dependence follows directly from the \(x\) dependence of \(F_2\) in the Donnachie-Landshoff parametrization, where for this range of \(x\) and \(Q^2 > 0.1\) GeV\(^2\), the first term in Eq. (15) dominates.

Fig. 3 shows the result for \(F_2\) when the dipole cross section shown in Fig. 2 is convoluted with the massless unit-charged quark wave function. It matches well with the original parametrization up to \(Q \approx 2 - 3\) GeV. At \(Q^2 = 0.1\) GeV\(^2\), the dipole formula with \(\hat{\sigma}_0\) is about 7% larger than the Donnachie-Landshoff parametrization of \(F_2\), while for \(Q^2 = 10\) GeV\(^2\), the dipole formula is about 11% below the D-L parametrization. We do not get an exact match to \(F_2\) in our inversion because we have made the approximation \(b_h(k) = 0\) and some numerical approximations in our integration at large \(k\). Nevertheless, the procedure works reasonably well. As we show below, when \(m_f \neq 0\), this approximate \(\hat{\sigma}_0\) is sufficient.

IV. MASSIVE QUARKS

For massive quarks, eq. (22) involves a sum over flavors and the introduction of an \(r\) dependence in \(h(z = Qr, m_f r)\). We will rewrite \(m_f r = m_f z/Q\). Fig. 4 shows a comparison of \(k\) for \(m_f = 0\) (dashed line) and \(k\) for five flavors with \(m_f \neq 0\) and \(Q = 1\) GeV. To find \(\hat{\sigma}\), we take as our starting point \(\hat{\sigma}_0\). Since the \(\gamma^5 p\) center of mass energy squared \(W^2 \simeq Q^2/x\), for \(x < 5 \times 10^{-3}\) we have
shows the ratio \( (22) \) of the evaluation of \( F_{\sigma} \) of the Donnachie-Landshoff parametrization is in the range of validity of the DL parametrization.

For reference, we illustrate the range of \( r \sim 1 \) GeV from lowest to highest curve. The rising behavior at low \( r \) is characteristic of other parametrizations of the dipole cross section, but the falling behavior is not. In this section, we compare \( \hat{\sigma} \) with two examples of dipole cross sections, the Golec-Biernat and Wüsthoff cross section [12] and the Soyez parametrization.

\[ \hat{\sigma} = 1.57\sigma_0 = N_\sigma\hat{\sigma}_0. \] (18)

Here, \( N_\sigma \) is the factor that indicates the effect of the charge and the masses of quarks, and this is expressed as

\[ N_\sigma^{-1} = (e_u^2 + e_d^2 + e_s^2)\eta_m (u,d,s) + e_c^2\eta_{m_c} + e_b^2\eta_{m_b}, \] (19)

where \( \eta_m = 0.9292, \eta_{m_c} = 0.0376, \) and \( \eta_{m_b} = 7.959 \times 10^{-4}. \) These values of \( \eta \) account for the effects of the quark masses in the photon wave functions. Using \( \hat{\sigma}_0, \) we found \( \eta_m \) from convoluting \( \hat{\sigma}_0 \) with \( h(z, m_f z/\text{GeV}) \) compared to with \( h(z, 0), \) so the normalization factor depends on the input \( F_2 \) which in turn determines \( \hat{\sigma}_0. \)

With the inclusion of masses for five quark flavors, we show \( F_2(x, Q^2) \) versus \( Q^2 \) for \( x = 10^{-3}, 10^{-4}, 10^{-5} \) in Fig. 5. The agreement between the original Donnachie-Landshoff parametrization shown by the solid lines and the structure function evaluated using \( \hat{\sigma} \) is quite good for the range of validity of the DL parametrization.

For reference, we illustrate the range of \( r \) most relevant for the evaluation of \( F_2. \) Fig. 5 shows the ratio

\[ f(r_{\text{max}}) = \frac{F_2(x, Q^2, r_{\text{max}})}{F_2(x, Q^2, r_{\text{max}} \to \infty)} \] (20)

for \( F_2(x, Q^2) = F_2(x, Q^2, r_{\text{max}} \to \infty) \) evaluated by eq. (2). When \( Q^2 = 10 \) GeV\(^2\), approximately 10\% of the structure function comes from low \( \hat{\sigma} \) at low \( r, r \leq 1 \) GeV\(^{-1}\), and approximately 10\% comes from \( r \geq 4 \) GeV\(^{-1}\). For \( Q^2 = 0.1 \) GeV\(^2\), 80\% of the evaluation of \( F_2(x, Q^2) \) comes from \( r \approx 2 - 5 \) GeV\(^{-1}\). Thus, the bulk of the evaluation of \( F_2 \) for the range of \( Q^2 \) of interest for the Donnachie-Landshoff parametrization is in the range of \( r \approx 1 - 5 \) GeV\(^{-1}\) where the dipole cross section makes the transition from the low \( r \) to large \( r \) forms.

V. DISCUSSION

The parameters for fits to the dipole cross section extracted from the Donnachie-Landshoff \( F_2 \) structure functions are shown in Table 1. To first approximation, a simple power law at low \( r \) and high \( r \) is approximately

\[ \hat{\sigma} \sim r^{3.34} \quad r \sim 0.2 - 1 \text{ GeV}^{-1} \] (21)

\[ \hat{\sigma} \sim r^{-2.62} \quad r \sim 5 - 7 \text{ GeV}^{-1}. \] (22)
In terms of the scaling variable \( N Q r \), the vertical dotted line shows Fourier transform inversion (solid), the Soyez parametrization (dashed) and GBW dipole formula (dot-dashed) for \( x = 10^{-4} \). The approximate form for \( \kappa \) of the dipole cross section based on the Iancu-Itakura-Munier form \[15\].

The Golec-Biernat and Wüsthoff color dipole cross section is \[12\]

\[
\hat{\sigma}_{\text{GBW}} = \sigma_{\text{GBW}}^0 \left[ 1 - \exp \left( -\frac{r^2}{4r_c^2} \frac{x_0}{x} \right) \right] 
\]

where \( \sigma_{\text{GBW}}^0 = 23 \text{ mb} \), \( \lambda = 0.29 \), \( x_0 = 3 \times 10^{-4} \) and \( r_c = 1 \text{ GeV}^{-1} \). This is often written in terms of an \( x \) dependent saturation scale \( Q_s(x) = Q_0(x_0/x)^{\lambda/2} \) where \( Q_0 = 1/r_c = 1 \text{ GeV} \). The approximate functional form incorporates the observed geometric scaling behavior and depends only on the one combined variable \( T_r = r Q_s \).

The limiting behavior at small \( r \) is therefore \( \hat{\sigma}_{\text{GBW}} \propto \sigma_{\text{GBW}}^0 (T_r/2)^2 \), while at large \( r \), \( \hat{\sigma}_{\text{GBW}} \to \sigma_{\text{GBW}}^0 \).

In Ref. \[16\], Soyez writes the color dipole cross section

\[
\hat{\sigma}_S = \sigma_S^0 N(r Q_s, Y) \]

\[
N(r Q_s, Y) = \begin{cases} 
\mathcal{N}_0 \left( \frac{T_r}{2} \right)^{2\gamma_{\text{eff}}(x, r)} & \text{for } T_r < 2 \\
1 - \exp \left[ -a \ln^2 (b T_r) \right] & \text{for } T_r > 2 
\end{cases} 
\]

in terms of the scaling variable \( T_r \), the saturation scale \( Q_s(x) \) and where \( Y = \ln(1/x) \). In this parametrization, \( \mathcal{N}_0 = 0.7 \), \( \gamma_s = 0.738 \), \( \lambda = 0.220 \), \( x_0 = 1.63 \times 10^{-5} \) and \( \sigma_0^S = 27.3 \text{ mb} \). The "effective anomalous dimension" in eq. \[24\] is

\[
\gamma_{\text{eff}}(x, r) = \gamma_s + \frac{\ln(2/T_r)}{\kappa \lambda Y} \]

with \( \kappa = 9.94 \) \[39\]. For \( r = 0.3 \text{ GeV}^{-1} \) and \( x = 10^{-4} \), \( \gamma_{\text{eff}} \approx 0.84 \), so \( \hat{\sigma} \sim r^{1.68} \) at small \( r \) for this value of \( x \), a slower rise with \( r \) than either the GBW dipole or our \( \hat{\sigma} \).

The low \( r \) and large \( r \) approximate forms of the dipole cross section and their relation to \( F_2 \) are discussed in Ref. \[37\]. Our dipole cross section does not precisely match these limiting forms. For small \( r \), Ewerz et al. have shown that \[37\],

\[
\hat{\sigma}(r, \xi) \simeq \pi^3 r^2 Q^2 \frac{\partial}{\partial Q^2} F_2(\xi, Q^2) |_{Q^2 = (z_0/r)^2} , 
\]

for \( r \ll 0.3 \text{ GeV}^{-1} \). \( \hat{\sigma} \)

This relation relies on approximations including that \( \hat{\sigma}(r, x)/r^2 \) is slowly varying for small \( r \) and on the approximate relation that \( Q r \approx z_0 \). At large \( Q^2 \) (equivalent to small \( r \) ), \( Q^2 \partial F_2/\partial Q^2 \sim 1/Q^2 \), so \( \hat{\sigma} \sim r^2/Q^2 \sim r^4/z_0^2 \). This dependence on \( r \) is contrary to the assumption that \( \hat{\sigma} \sim r^2 \) as \( r \to 0 \), e.g., the low \( r \) form of the GBW dipole cross section. The dipole cross section we obtain scales approximately as \( \hat{\sigma} \sim r^{3.34} \) around \( r \approx 1 \text{ GeV}^{-1} \). Given that \( \hat{\sigma}(r, x)/r^2 \) is not slowly varying for small \( r \), one would not expect a precise agreement between our low \( r \) parametrization and Eq. \[26\].

For large \( r \), the small \( Q^2 \) regime, the approximate behavior of \( \hat{\sigma} \) is related to the logarithmic derivative of the virtual photon cross section. To get the correct physical behavior of \( \sigma_{\gamma p} \) for \( Q^2 \to 0 \), one needs to fix \( W^2 \approx Q^2/x \) in \( \sigma_{\gamma p} \propto F_2/W^2 \). We therefore write,

\[
\hat{\sigma}(r, x) \simeq -\frac{\pi}{\alpha_c} Q^2 \frac{\partial}{\partial Q^2} \left[ \sigma_T(W^2, Q^2) + \sigma_L(W^2, Q^2) \right] 
\]

with \( Q^2 = (z_0/r)^2 \), \( W^2 \) fixed. \[27\]

The Donnachie-Landshoff parametrization of \( F_2 \) together with this large \( r \) approximation leads to \( \hat{\sigma} \sim r^{-2} \). The dipole cross section we determined has \( \hat{\sigma} \sim r^{-2.62} \) for \( r \approx 5 \text{ GeV}^{-1} \). Eq. \[27\] also relies on \( \hat{\sigma} \) being a slowly varying function of \( r \) at large \( r \), so the fact that our approximate power law behavior deviates from \( \hat{\sigma} \sim r^{-2} \) is again not surprising. As remarked in Ref. \[37\], the falling dipole cross section as a function of large \( r \) is correct when one uses the standard perturbative photon wave function for all \( Q^2 \), even for very low \( Q^2 \). The GBW dipole has \( \hat{\sigma} \to \sigma_c \), a constant value. With the standard perturbative photon wave function, the GBW form of the dipole cross section yields a virtual photon-proton cross section \( \sigma_T(W^2, Q^2) + \sigma_L(W^2, Q^2) \) that does not have the correct \( Q^2 \) behavior without a modification of the low \( Q^2 \) photon wave function. Our approach here is to use the standard perturbative wave function even at the lowest \( Q^2 \) values. The result is that \( \hat{\sigma}(r, x) \) decreases at large \( r \).

VI. CONCLUSIONS

In summary, we have used the simplified form of the convolution formula in the color dipole formalism with \( m_f = 0 \) to extract the dipole cross section from a parametrization of the structure function \( F_2(x, Q^2) \). As a demonstration of the method, we have used the Donnachie-Landshoff parametrization because its simple
form illustrates some of the limiting behavior discussed in Ref. 37. We find that consistent with the Ewerz et al. discussion in Ref. 37, unless the perturbative photon wave function is modified for low $Q^2$, the dipole cross section should fall as $r$ becomes large, contrary to what is assumed in many models. The low and high $r$ relationships between $F_2$ and dipole cross section discussed in Ref. 37 are only approximate according to our dipole extraction.

Our approach is complementary to much of the current literature in which a theoretical form of dipole cross section is postulated and its parameters fit to $F_2$ data. Our dipole formulas in eqs. (16)-(18) reproduce the Donnachie-Landshoff results for $F_2(x,Q^2)$ for a range of $x$ and for 0.1 GeV$^2 < Q^2 < 10$ GeV$^2$. The dipole does not exhibit geometric scaling in $T_0$ because this parametrization of $F_2$ does not exhibit that same scaling behavior in $T_0$. Work is in progress to extract $\hat{T}$ from a range of other parameterizations of $F_2$. This is not completely straightforward because many parameterizations of $F_2$ do not permit the required Fourier integration from $Q'^2 = 0 \to \infty$. In particular, low $Q$ extrapolations are required. Ultimately, one would like to find a range of dipole cross sections that represent a range of postulated $x$ and $Q$ dependencies in parameterizations of $F_2(x,Q^2)$, e.g., as in Refs. 22, 24, 30.

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