QCD in the cores of neutron stars

HOW PQCD CONSTRAINS THE EQUATION OF STATE AT NEUTRON STAR DENSITIES
KOMOLTSEV & AK, PRL128 (2022) 20, 2111.05350

AB-INITIO QCD CALCULATIONS IMPACT THE INFERENCE OF NEUTRON-STAR EQUATION OF STATE
GORDA, KOMOLTSEV & AK, ASTROPHYS.J. 950 (2023), 2204.118

STRONGLY INTERACTING MATTER EXHIBITS DECONFINED BEHAVIOR IN MASSIVE NEUTRON STARS
ANNALA, AK, ET AL 2303.11356, NATURE COMM. IN PRINT
Gravity:

\[ R \sim 2R_s \]

QCD:

\[ n \sim 5 - 8 n_s \sim \frac{\text{baryon}}{\text{fm}^3} \]

\[ \epsilon \sim 1 \frac{\text{GeV}}{\text{fm}^3} \]
Phase diagram of QCD

Heavy-ion collisions

Matter made of quarks and gluons

Matter made of hadrons

Vacuum

Nuclear matter

Neutron stars

Baryon number chemical potential

0-0.94 GeV

0.2 GeV

0.94 -? GeV

CMS Experiment at the LHC, CERN

Data recorded 2010-2013, Run 3, 337.44 fb^{-1} (5.02 TeV, 8 TeV, 13 TeV)

Run/Event: 310094, 290887
Neutron stars are femtoscopes:

Hydrostatic equilibrium couples \textbf{macroscopic} properties of neutron stars to \textbf{microscopic} properties of the matter.

\begin{align*}
  \frac{dP}{dr} &= -\frac{G \epsilon(r) M(r)}{r^2} \left[ 1 + \frac{P(r)}{\epsilon(r)} \right] \left[ 1 + \frac{4\pi r^3 P(r)}{M(r)} \right] \left[ 1 - \frac{2GM(r)}{r} \right]^{-1} \\
  \frac{dM}{dr} &= 4\pi r^2 \epsilon(r)
\end{align*}

\[ \epsilon(P) \Leftrightarrow R(M) \]
Properties of neutron stars reflect properties of dense matter

Green bank telescope

NICER

LIGO/Virgo(+KAGRA)
Properties of neutron stars reflect properties of dense matter

Demorest et al. Nature 467, 1081-1083 (2010);
Antoniadis et al., Science 240, 448 (2013);
Cromartie et al. Nature Astronomy, 439 (2019)
Properties of neutron stars reflect properties of dense matter

\( M_{\text{J1614-2230}} = 1.908(16) \)
\( M_{\text{J0348+0432}} = 2.01(4) \)
\( M_{\text{J0740+6620}} = 2.14(10) \)

Demorest et al. Nature 467, 1081-1083 (2010);
Antoniadis et al., Science 240, 448 (2013)
Cromartie et al. Nature Astronomy, 439 (2019)

Green bank telescope

NICER

LIGO/Virgo(+KAGRA)

Miller et al. APJL 918 (2021)
Riley et al. APJL 918 (2021)
Properties of neutron stars reflect properties of dense matter

$$M_{1614-2230} = 1.908(16)$$
$$M_{1534+402} = 2.01(4)$$
$$M_{10740+620} = 2.14(10)$$

Demorest et al. Nature 467, 1081-1083 (2010);
Antoniadis et al., Science 240, 448 (2013)
Cromartie et al. Nature Astronomy, 439 (2019)

Green bank telescope

$M(M_\odot)$ vs $R_e$(km)

Miller et al. APJL 918 (2021)
Riley et al. APJL 918 (2021)

Abbott+ (LIGO Scientific, Virgo) PRL 119 (2017);
PRL 121 (2018); PRX 9 (2019).
What do we know about the equation of state?
Equation of state, theoretically:

- Perturbative QCD
- Chiral EFT

Tews et al. PRL 110 (2013)
Hebeler, Lattimer et.al. APJ 773 (2013)
Drischler, Furnstahl et.al. PRL 125 (2020)
Keller, et al, PRL 130, 072701 (2023)
Equation of state, pQCD:

EoS $P(\mu) = \text{effective potential with: } \mathcal{L}_{QCD} + \mu \bar{\psi} \gamma_0 \psi$

\[
\frac{1}{p_0^2 + \vec{p}^2} \rightarrow \frac{1}{(p_0 + i\mu)^2 + \vec{p}^2}
\]

\[
\frac{p}{p_0} = 1 + h_1 \alpha_s + h_2 \alpha_s^2 + h_3 \alpha_s^3 + \cdots
\]

Freedman, McLerran, PRD 16 (1977)
Quark masses: Kurkela et al. PRD 81 (2010)
Equation of state, pQCD:

\[
\frac{p}{p_0} = 1 + h_1 \alpha_s + h_2 \alpha_s^2 + h_3 \alpha_s^3 + \cdots
\]
Equation of state, pQCD:

\[
\frac{p}{p_0} = 1 + h_1 \alpha_s + h_2 \alpha_s^2 + h_3 \alpha_s^3 + \cdots + s_2 \alpha_s^2 + s_3 \alpha_s^3 + \cdots + \cdots
\]
Equation of state, pQCD:

\[ \frac{p}{p_0} = 1 + h_1 \alpha_s + h_2 \alpha_s^2 + h_3 \alpha_s^3 + \cdots \]

\[ + s_2 \alpha_s^2 + s_3 \alpha_s^3 + \cdots \]

Medium polarization:

\[ \Pi \sim \alpha_s \int \mu \frac{d^3p}{p} \sim \alpha_s \mu^2 \]
Equation of state, pQCD:

\[
\frac{p}{p_0} = 1 + h_1 \alpha_s + h_2 \alpha_s^2 + h_3 \alpha_s^3 + \cdots + s_2 \alpha_s^2 + s_3 \alpha_s^3 + \cdots
\]

Medium polarization:

\[\Pi \sim \alpha_s \int \frac{d^3p}{p} \sim \alpha_s \mu^2\]

If \(k^2 \sim \alpha_s \mu^2\) correction not small:
Equation of state, pQCD:

\[
\frac{p}{p_0} = 1 + h_1 \alpha_s + h_2 \alpha_s^2 + h_3 \alpha_s^3 + \cdots
\]

+ \[s_2 \alpha_s^2 + s_3 \alpha_s^3 + \cdots\] + ...

Medium polarization:

\[\Pi \sim \alpha_s \int \mu \frac{d^3p}{p} \sim \alpha_s \mu^2\]

If \(k^2 \sim \alpha_s \mu^2\) correction not small:

\[\cdots\]

If \(k^2 \ll \mu^2\): Hard-thermal loop EFT
Equation of state, pQCD:

\[
\frac{p}{p_0} = 1 + h_1 \alpha_s + h_2 \alpha_s^2 + h_3 \alpha_s^3 + \cdots \\
+ s_2 \alpha_s^2 + s_3 \alpha_s^3 + \cdots
\]

Medium polarization:

\[
\Pi \sim \alpha_s \int d^3p \frac{\mu^3}{p} \sim \alpha_s \mu^2
\]

If \( k^2 \sim \alpha_s \mu^2 \) correction not small:

\[
\Pi \sim \alpha_s \int d^3p \frac{\mu^3}{p} \sim \alpha_s \mu^2
\]

If \( k^2 \ll \mu^2 \): Hard-thermal loop EFT

\[
\Pi^{\mu\nu}(K) = \mathcal{P}_T^{\mu\nu}(\hat{K}) \Pi_T(K) + \mathcal{P}_L^{\mu\nu}(\hat{K}) \Pi_L(K)
\]

\[
\Pi_L(P) = 2m^2 \frac{P^2}{|p|^2} \left[ 1 - \frac{iP^0}{2|p|} \ln \frac{iP^0 + |p|}{iP^0 - |p|} \right]
\]
Equation of state, pQCD:

\[ \frac{p}{p_0} = 1 + h_1 \alpha_s + h_2 \alpha_s^2 + h_3 \alpha_s^3 + \cdots + s_2 \alpha_s^2 + s_3 \alpha_s^3 + \cdots \]

\[ \sim \int g_\mu d^3 p \sim \alpha_s^2 \]
Equation of state, pQCD:

\[
\frac{p}{p_0} = 1 + h_1 \alpha_s + h_2 \alpha_s^2 + h_3 \alpha_s^3 + \cdots \\
+ s_2 \alpha_s^2 + s_3 \alpha_s^3 + \cdots
\]

\[\sim \int^{g_{\mu}} d^3 p p \sim \alpha_s^2\]
Equation of state, pQCD:

\[
\frac{p}{p_0} = 1 + h_1 \alpha_s + h_2 \alpha_s^2 + h_3 \alpha_s^3 + \cdots + s_2 \alpha_s^2 + s_3 \alpha_s^3 + \cdots
\]
Is pQCD of any use in Neutron Stars?
Equation of state, theoretically:

\[ P \] perturbative QCD

Chiral effective theory

Freedman, McLerran, PRD 16 (1977)
Kurkela et al. PRD 81 (2010)
Kurkela, Vuorinen, PRL 117 (2016)
Gorda, Kurkela et al. PRL 121 (2018)
Gorda, Kurkela et al. PRD 104 (2021)
Gorda, Kurkela et al. PRL 127 (2021)
Gorda, Kurkela et al. PRD 107 (2023)

Tews et al. PRL 110 (2013)
Hebeler, Lattimer et.al. APJ 773 (2013)
Drischler, Furnstahl et.al. PRL 125 (2020)
Robust EoS constraints:

General considerations:

• Mechanical stability: $c_s^2 > 0$
• Causality: $c_s^2 < 1$

Rhoades & Ruffini, Phys.Rev.Lett. 32 (1974)
Lope-Oter, Windisch, Llanes-Estrada, Alford, J. Phys. G (2019)
Lope-Oter, Llanes-Estrada, EPJA 58 (2022)
Robust EoS constraints:

General considerations:

- Mechanical stability: \( c_s^2 > 0 \)
- Causality: \( c_s^2 < 1 \)
- Consistency:

\[
P(\varepsilon) \ vs. \ \Omega(\mu)
\]

Reduced EoS \hspace{1cm} \text{Full EoS}

Information of \{P, \varepsilon, n\}
EoS constraints:

- **Stability**
  \[ \partial_\mu^2 \Omega(\mu) \leq 0 \quad \Rightarrow \quad \partial_\mu n(\mu) \geq 0 \]

- **Causality**
  \[ c_s^{-2} = \frac{\mu}{n} \frac{\partial n}{\partial \mu} \geq 1 \quad \Rightarrow \quad \partial_\mu n(\mu) \geq \frac{n}{\mu} \]

- **Consistency**
  \[ \int_{\mu_{CET}}^{\mu_{QCD}} n(\mu) d\mu = p_{QCD} - p_{CET} = \Delta p \]
Constraints for fixed $n$ on $\varepsilon - p$ -plane

![Graph showing constraints for fixed $n$ on $\varepsilon - p$ -plane.](image)

- **Integral constraints**
- **Causality constraints**
- **Excluded by pQCD**

**Energy density** $\varepsilon$ [MeV/fm$^3$]

**Pressure** $p$ [MeV/fm$^3$]

Komoltsev & AK, PRL128 (2022)
pQCD and Equation-of-State inference
Nuclear theory

low-energy nuclear experiments

X-ray astronomy

Gravitational waves

Radio astronomy

Pressure: $p$ [GeV/fm$^3$]

Energy density: $\varepsilon$ [GeV/fm$^3$]

$\sim 40 n_s$

$\sim 1.1 n_s$

Maximal central densities

Hebeler, Lattimer, et al., ApJ. 773, 11 (2013)
AK, Fraga, et al., ApJ. 789, 127 (2014)
Gorda, Komoltsev, AK, ApJ. (2023)
Effect of QCD:

QCD input complements NS observations

Gorda, Komoltsev & AK APJ (2023)
QCD responsible for *softening*:

![Graph showing the effect of QCD on pressure and energy density.](image-url)
Is there quark matter in neutron stars?
EoS tells about phases of matter: HIC

Matter described by hadronic mass scale

Hadrons

Quarks and gluons

Approximately conformal matter, with large number of d.o.f.

Gardim et al Nature Physics 16, 615-619 (2020)
Borsanyi et al PLB 730, 99 (2014)
EoS tells about phases of matter:

A clear **non-coformal** to **conformal** transition within cores of neutron stars.

Matter described by Hadronic mass scale

Approximately conformal matter, with large number of d.o.f.
Conclusions

• QCD meets gravity in neutron stars in a very concrete way

• Physics of neutron-star cores can be probed by pQCD computations.

• Neutron stars are non-perturbative but **causality, stability** and **consistency** contrain the EoS

• pQCD responsible for the **conformalization** that has natural interpretation as the onset of **Quark Matter**

• Improvement through completing the **N3LO** computation
Constraints in $\{p, \varepsilon, n\}$ - space

Models from CompOSE database

Komoltsev & AK, PRL128 (2022)
QCD likelihood function:

- **Machine learning** based Bayesian interpretation of **Scale variation** and **Missing Higher Order** errors
  - Cacciari & Houdeau, JHEP 09, (2011), Duhr et al. JHEP 122, (2021), Gorda, Komoltsev, AK, Mazeliauskas JHEP in print, 2303.02175
- Perturbative series modelled as draw from statistical model that is trained with the available terms
Implementing pQCD to EoS inference:

• Bayesian inference setup:

\[ P(\text{EoS} \mid \text{data}) = \frac{P(\text{EoS}) \cdot P(\text{data} \mid \text{EoS})}{P(\text{data})} \]
Implementing pQCD to EoS inference:

- **Gaussian-process** based inference:

  Similar to Landry & Essick PRD 99 (2019), but for function of $n$ instead of $\varepsilon$

\[ P(\text{EoS} \mid \text{data}) = \frac{P(\text{EoS}) P(\text{data} \mid \text{EoS})}{P(\text{data})} \]
Setup:

\[ P(\text{EoS} \mid \text{data}) = \frac{P(\text{EoS})P(\text{data} \mid \text{EoS})}{P(\text{data})} \]

\[ P(\text{data} \mid \text{EoS}) = P(\text{QCD} \mid \text{EoS})P(\text{Mass} \mid \text{EoS})P(\text{NICER} \mid \text{EoS})P(\tilde{\Lambda}, \text{BH} \mid \text{EoS}). \]
Implementing pQCD to EoS inference:

- Inference setup where QCD can be turned on/off
- Easily implemented to any other extrapolation setup

Extrapolate to here and use this area to condition the extrapolation.