Applied Sciences

Article

Observation of the Velocity Variation of an Explosively-Driven Flat Flyer Depending on the Flyer Width

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Received: 26 October 2018; Accepted: 14 December 2018; Published: 26 December 2018

Abstract: Lim proposed a theoretical study on the velocity profile of an explosively-driven flat flyer affected by the rarefaction (or release wave) intrusion during the metal’s projection. This work shows somewhat reasonable agreement in a given range. However, this work is limited only in the early stage of detonation (~3 µs), and the larger scaled flyer projection (or extended time duration) behavior is needed for an engineering perspective. As continued work originating from this investigation, the velocity profile of explosively-driven flyers with different widths is studied based on multiple different approaches which include hydrocode simulation, the Gurney model, and Baum’s side loss correction (or effective charge mass approach), followed by a series of field experiments. In this study, the focus is on the observation of the flat flyer velocity (or terminal velocity) variation, depending on the width of the flyers which vary from 12, 25, 50, and 75 mm (or 100 mm). The terminal velocity profile variation, depending on the flyer width, is observed, and a general trend is identified.

Keywords: gurney model; flyer projection; flyer width

1. Introduction

The Gurney model provided a great deal of improvement in the understanding of explosively-driven systems, delivering a fundamental insight of the coupling between the explosives and metallic flyer based on the conservation equations. It also introduced a well-known parameter of the mass of the flyer divided by mass of the explosive charge (M/C) ratio, and it became a common gate-way into related research. This approach begins with the following assumptions: (1) the constant output per unit mass of a charge, (2) one-dimensional, linear detonation, gas velocity profile, (3) volume detonation of the given charge in the system, (4) no shock effect in the projection of the flyer, etc. [1]. This approach was further developed by many researchers and expands the application into areas including various system configurations of cylinders, symmetric or non-symmetric sandwiches, spheres, etc. [2]. Despite providing great potential to related research fields, the Gurney model reveals minor limitations because of the assumptions addressed above. For example, the Gurney model only predicts the terminal velocity of the flyer based on the initial system configuration, meaning that the acceleration profile of the explosively-driven flyer is not readily available. If the explosives system functions in an early stage of detonation, then the Gurney model is not suitable because the system is still in an acceleration stage, and it may reach the terminal velocity later. Another limitation of the Gurney model is that the model originated from a one-dimensional point of view, and prohibits a detailed understanding of the side rarefaction (or edge effect) upon detonation of the charge, which eventually will affect the terminal velocity. This is an important subject for applications where the speed of projection is considered as the measure of performance, i.e., shaped charges, fragmentation of...
casing, slapper detonators, etc. It is obvious that the design of engineered explosive devices cannot be a one-dimensional configuration, and this fact may affect the use of the conventional Gurney equations, without the consideration of multi-dimensional configuration, or no consideration of the side rarefaction, delivering inaccuracy. In terms of the side rarefaction in a typical flat flyer projection, there is a study by Baum, to understand the effect from the side rarefaction (or side loss) by applying a new term of "effective charge mass" \( C_e \), and this study is based on a semi-empirical observation delivering more advanced applications in this field [3,4]. In this study, \( C_e \) is calculated based on a cone-shaped charge volume with 60° base angle (dotted line in Figure 1). This approach can be applied to the Gurney model to optimize the charge amount as follows

\[
\frac{v}{\sqrt{2E}} = \left\{ \frac{\left(1 + 2\frac{M}{C_e}\right)^3 + 1}{6\left(1 + \frac{M}{C_e}\right)} + \frac{M}{C_e} \right\}^{-\frac{1}{2}}
\]

where \( M \) is the mass of the flyer, \( \sqrt{2E} \) is the Gurney velocity constant, and \( v \) is the speed of the flyer.

![Figure 1. Baum’s effective charge mass and Gurney model (open face sandwich configuration).](image)

This approach addresses the effect from the side rarefaction (or side loss, Figure 1) in order to calculate the flyer velocity, and the side rarefaction is empirically calculated based on the 60° base angle (or 30° cone angle). The 60° base angle is closely related to the speed of rarefaction coming from the surface of the charge, or a half-speed of detonation velocity in the charge, and this angle is deduced upon consideration of the rarefaction speed. This is a somewhat simplified approach, but it greatly emphasizes the importance of side rarefaction that affects the flyer projection. Later, Lian proposed a 45° base angle based on a series of numerical simulations under the same concept [5].

This research [3–5] begins with the side rarefaction near the edge of the flyer, indicating the flyer velocity variation depending on the width of the flyer, and it provides a significant improvement of the projection of the flat flyer.

After Baum’s and other similar approaches, and closer to the topic of this effort, Chanteret proposed an analytical calculation of the flyer velocity variation depending on the width of the flyer [6]. His research specifically describes the rarefaction intrusion to the center of the flyer, and it causes the variation of flyer projection velocity during the detonation gas expansion. This analytical approach is added into the conventional Gurney equation with the use of Gurney velocity \( \sqrt{2E} \), and it is compared to the simulation and provided a favorable result (Figure 2).

Note that the flyer projection velocity varies depending on the flyer width, and this trend is compared to 2D hydrocode simulation results.

In this research, Chanteret mainly used a symmetrical sandwich configuration, and it reveals the flyer velocity variation depending on the width of the flyer [6]. However, this research is built based on the assumption that the rarefaction intrusion from the edge would be the same in the difference configuration (i.e., open-face and symmetric sandwich), and the unknown factor of \( \sqrt{2E} \) (Gurney velocity) is obtained by the use of another set of hydrocode and it may carry some of the minor issues of the Gurney equation.
Figure 2. Flyer velocity variation depending on the width of flyer [6].

More recently, Lim proposed an analytical calculation of the flyer acceleration profile under the influence of the side rarefaction based on the detonation gas isentrope, and Newton’s law of motion [7]. In this approach, the Gurney equation is not applied in a way that would deliver more insight of the projection behavior. Due to the complexity of the given subject during the detonation along with the projection, Lim proposed a new term of pressure release ratio $\eta$, behind the flyer to predict the pressure/acceleration profile and the deformation profile shown by Equation (2) and Figure 3

$$P_{tn}^*=P_{tn-1}^* (\eta)^\gamma$$  \hspace{1cm} (2)

where, $P_{tn}^*$ is the rarefaction pressure behind the flyer at time $t_n$, and $\gamma$ is the specific heat ratio.

The spirit of this approach is similar with Chanteret’s work, but the description of the rarefaction intrusion and the specific velocity profile for each location on the flyer are greatly emphasized to predict the arc deformation of the flyer.

This approach limits its applicability because the time range of investigation is at the immediate stage after detonation (around 3 $\mu$s), and assumes the flyer is under a strong hydrodynamic regime. However, it delivers a way to predict the flyer projection velocity and acceleration profile in addition to the arc deformation. Because this approach is built on top of the side rarefaction, the width of the flyer affects the result as well.

In this context herein, a general characteristic of the typical metallic flat flyer projection behavior driven by an explosive’s detonation is investigated utilizing a series of hydrocode simulation, conventional analytical tools, including the Gurney model and Baum’s approach, and a series of field tests. This paper focuses on the observation and description of the two-dimensional effect of the flyer configuration (i.e., flyer width) to understand and review the limitation and applicability.
of the conventional methods, while also exploring a more advanced approach to obtain an accurate estimation of the flyer projection behavior.

2. Numerical Observation of the Flyer after Detonation

The behavior of a flat metallic flyer during explosively-driven projection must be considered throughout the entire spectrum of dynamic material behavior. This behavior ranges from simple elastic behavior to extreme non-linear dynamic behavior. The extreme non-linear dynamic behavior can also be described as the hydrodynamic regime, where the material’s strength is neglected due to the extremely high pressures of the detonation. Note that ‘hydrodynamic’ in this paper represents the theoretical material behavior under the extreme dynamic pressure loading where the input pressure is well above the material strength and the material strength is negligibly small, assuming the material acts like liquid flow in the theoretical approach. This fact delivers the most challenging aspect of this research because each stage of dynamic behavior needs a different approach, and analytical expression. Most of all, there is no clear definition of each dynamic behavior stage.

Once the detonation occurs in the explosive behind a flyer in contact, the sudden expansion of the detonation gas along with the strong shockwave provides the extreme pressure loading and acceleration to the flyer, forcing the flyer to stay in the hydrodynamic regime. During this period, the material strength of the flyer is almost negligible due to the fact that the detonation gas loading far exceeds the material strength. The behavior of the metallic flyer gets close to the hydrodynamic behavior, where there is no shear strength of the material. As the flyer moves away from the direct effect of the detonation gas, the flyer reaches another stage of dynamic plastic regime because of the sudden release of the detonation pressure behind the flyer. After this point, the flyer may experience an elastic/plastic regime followed by a simple elastic behavior (Figures 4 and 5).

![Figure 4. Explosively-driven metallic flyer projection and its deformation/behavior path.](image1)

![Figure 5. High speed images of flat flyer projection (1 Million fps). 2 in. by 2 in (25.4 mm by 25.4 mm). 2 mm thick flat copper flyer with 0.5 in. (12.7 mm) thick RDX based sheet explosives behind.](image2)
center of the flyer by the rarefaction in the detonation gas. During this process, the flyer experiences a different projection velocity along the width, which causes the unique arc-shaped deformation of the flyer (Figures 3–5). Figure 6 shows a typical velocity profile of a flat flyer during the projection. This is from a numerical simulation with Autodyn™ \((M/C = 1.12\) and the width of the flyer is 12 mm). See Table A1 for detailed simulation input parameters.

![Figure 6](image)

**Figure 6.** Simulation results of the velocity profile of a 12 mm width flat flyer after detonation (the location of 5 gauges are evenly distributed along the half width of the flyer).

In general, the velocity profile shows three different stages of areas: (1) acceleration stage, (2) velocity adjustment stage, and (3) velocity converging stage, depending on the trend of the velocity profile.

Acceleration stage (Area 1): the gauge near the center of the flyer \((G1)\) experiences delayed rarefaction intrusion, and the flyer segments in this area are with fast and long acceleration duration. However, the gauges near the edge of the flyer \((near \ G5)\) show fast reduction of the speed and short acceleration duration due to the early rarefaction intrusion. It is believed that the duration of acceleration for each flyer segment is strictly dependent on the speed of rarefaction intrusion to the specific location as was addressed previously [7]. In addition, this stage should be under the hydrodynamic regime, in which each flyer segment projects independently without much effect from the nearby segments.

Velocity adjustment stage (Area 2): right after the initial acceleration stage, the flyer segments (with different speeds) try to reach an equilibrium state by eliminating the speed difference. This is because the flyer segments near the center move faster than those near the edge, and this produces a strong tension wave along the flyer. In other words, the flyer segments near the center pull the segments near the edge, and the segment near the edge drags the center segments. This is obvious from the velocity profile in Figure 6, showing that the flyer segments in G4 and G5 are accelerating, and those in G1–3 are slowly decelerating during this stage. The material strength should provide a major role in this stage in order to create the pulling/dragging effect. This means that this stage should not be in the hydrodynamic stage, because if this occurs in hydrodynamic regime, then the pulling/dragging effect cannot exist.

Velocity converging stage (Area 3): after a long period of the velocity adjustment stage with pulling/dragging, the entire flyer segments finally reach to a uniform terminal velocity. In this stage, the pulling/dragging effects with tension stress in the flyer segments are under the elastic limits, creating an equilibrium state, or uniform stress state, in the flyer. This is why there is little, or no velocity difference. Another possibility is that the tension wave propagation inside the flyer equalizes the stress difference in segments after enough wave reverberation time along the width of the flyer. Each segment in the flyer is closely bonded with nearby segments and the pressure from the detonation gas has dissipated significantly. This stage should be where the terminal velocity of the flyer is determined.

Note that during the course of investigation, the effects from the surrounding air on the projecting flyer (i.e., drag, friction, thermal effects, afterburning of detonation gas, etc.) are ignored in order to
simplify and focus more on the given subject of this investigation. We assume that the effects from the surrounding air are negligibly small in the early stages of flyer projection.

3. Two-Dimensional Flyer Projection

A general velocity profile along the width of the flyer from the previous section was observed. However, it creates another important question about the relation between the velocity profile and the width of the flyer. As addressed before, if the speed of rarefaction intrusion in the gas, or tension wave in the flyer, is a key to understanding the velocity profile [7], then a flyer with more width should be affected, because more width means more distance to travel for the rarefaction or tension wave. This should be an important clue to answer because it may provide a way to advance the conventional Gurney model into a more realistic, two-dimensional approach. Note that the conventional Gurney model assumes that the flyer/charge system is under a one-dimensional configuration, and disregards the effect from the side rarefaction. In order to address this limitation, Baum proposed an approach with effective charge mass \( \frac{M}{C} \) based on the side rarefaction (Figure 1) [3]. A more detailed description will be followed later in this paper.

In order to answer the question above, a series of hydrocode simulations was accomplished using Autodyn™. For more information regarding the simulation configuration, please refer to Figure A1 and Table A1.

In this series of simulations, four different flyer widths of 12, 25, 50, and 75 (or 100 mm), under varied \( M/C \) are simulated, and the results are tabulated in Figures 7–9, and Table 1. The flyer’s material is a 2 mm thick, copper flyer, and the explosives charge material is a 10 mm thick, C4 charge (see Appendix A, Table A1 for detailed simulation parameters). The simulation was run until the velocity converging stage in order to observe the entire three stages of velocity profiles.

![Typical velocity profile of different widths of flat metallic flyers](image)

**Figure 7.** Typical velocity profile of different widths of flat metallic flyers (refer to Figure 5 for the gauge location).

Note 1: Some gauge points near the edge of flyer were physically separated during the projection and those data points were neglected from these graphs.
Note 2: Due to the plotting limitation on the number of data points in the graph, the graphs for 50 and 75 mm only show up to 0.2 ms after detonation, and the converging points are not shown in the graph (for the converging time, see Table 1).

![Graphs showing terminal velocity comparison for various M/C](image)

**Figure 8.** Terminal velocity (or converging velocity) comparison for various M/C.

**Table 1.** Flyer velocity comparison.

| Thickness of Charge (mm) | Thickness of Flyer (mm) | M/C  | Width of Flyer (mm) | Simulation | Baum's Approach in Velocity (m/s) | Gurney Model (m/s) |
|-------------------------|-------------------------|------|---------------------|------------|-----------------------------------|-------------------|
|                         |                         |      |                     | Converge Time (ms) | Converge Velocity (m/s) ** |                     |
| 5                       | 2                       | 2.23 | 12                  | 0.063      | 501                               | 666               |
|                         |                         |      | 25                  | 0.191      | 656                               | 755               |
|                         |                         |      | 50                  | 0.396      | 743                               | 795               |
|                         |                         |      | 75                  | 0.498      | 777                               | 808               |
| 10                      | 2                       | 1.12 | 12                  | 0.023      | 674                               | 858               |
|                         |                         |      | 25                  | 0.035      | 1061                              | 1156              |
|                         |                         |      | 50                  | 0.307      | 1265                              | 1277              |
|                         |                         |      | 75                  | 0.560      | 1330                              | 1315              |
|                         |                         |      | 100                 | 0.759      | 1365                              | 1334              |
| 15                      | 2                       | 0.74 | 12                  | 0.024      | 726                               | 859               |
|                         |                         |      | 25                  | 0.035      | 1251                              | 1370              |
|                         |                         |      | 50                  | 0.069      | 1584                              | 1597              |
|                         |                         |      | 75                  | 0.379      | 1720                              | 1665              |

* Open-face sandwich configuration; ** Same as the terminal velocity of the given flyer in the simulation; *** Gurney model calculation with the effective charge mass (open-face sandwich configuration).
From the figure above (Figure 7), the general velocity profile bearing the three projection stages, and each velocity profile, is somewhat different depending on the width of the flyer (with constant $(M/C)$). From this result, the following observations are made: (1) As the flyer width gets larger, the terminal velocity (or converging point velocity) of the flyer increases (Table 1 and Figures 8 and 9) and it approaches to the Gurney prediction. This observation is one of the reasons why the conventional Gurney model limits itself, and this originated from the one-dimensional assumption. When the flyer width increases, the rarefaction from the flyer edges needs to travel a long distance (or time) to the center, and this long duration causes a faster projection of the center segments due to the longer acceleration time in the center flyer segments; (2) The velocity converging time shows a very clear trend. As the width of the flyer becomes large, the converging point time and velocity (or terminal velocity) increases. See Figure 7 and Table 1 for comparison. This is somewhat related with the speed of rarefaction, or tension wave speed, in the flyer, and this may provide a clue to understand the material behavior under the strong detonation gas release. However, the converging point data shows somewhat large discrepancies depending on the width of the flyer, and this is believed to originate from the dynamic material behavior where the local speed of sound somewhat decreases as the flyer experiences more deformation. This is out of scope of this paper; (3) The comparison between the velocity converging point in the simulation and Baum’s effective charge mass included calculation delivers an interesting result. When the width of the flyer is large, the two data points show somewhat good agreement (see the values in 50, 75 and 100 mm width flyers) (Figure 8, Table 1). However, when the width of the flyer decreases (see the values in 12, and 25 mm width flyers), the two data points show a large discrepancy. It is believed that this is due to how Baum’s approach is constructed, which is based on the removal of the charge mass affected by the side rarefaction near the edge of the original configuration (Figure 1). This is to calculate the optimum (or effective) charge mass depending on the configuration. However, this approach may provide some discrepancy when the width is varied. For example, if $(M/C)$ is constant, the removed charge from the varied width of flyer/charge system should be the same, and the removed charge mass grows larger (comparatively) as the width of the flyer gets smaller (Figure 10).

![Flyer velocity variation depending on width in simulation.](image)

**Figure 9.** Flyer velocity variation depending on width in simulation.

This creates a large discrepancy in flyers with smaller widths. In other words, when the charge width becomes large, the Baum’s approach may lose its effectiveness because the removed charge

![Baum’s side loss comparison between large and small width.](image)

**Figure 10.** Baum’s side loss comparison between large and small width. The charge removed for the side loss is constant, and the effect (or velocity reduction) from the removed charge decreases as the flyer width gets larger.
weight is always constant, and becomes relatively smaller. (4) As the width of the flyer gets larger, the simulation approach, Baum’s approach, and the Gurney model show a good agreement regarding the flyer velocity (Figure 8). This is an important observation because it may deliver a range where all three approaches are under a reasonable agreement, and an analytical solution may exist. In addition, it was observed that when the \( M/C \) value decreases, the discrepancy between Baum’s, and the Gurney Model, reduces, which represents a general solution tendency for different approaches (Figure 8).

4. Experimentation of Flyer Projection

From above, there is evidence that the flyer velocity variation depends on the width of the flyer. According to the analytical and numerical observations, the flyer projection velocity is closely related with the speed of rarefaction during the detonation gas expansion, and an analytical solution regarding the pressure profile behind the flyer was published [7].

In order to understand how the explosively-driven flyer projection behavior varies (addressed above), a series of field tests were accomplished. Due to the data acquisition difficulty during a flyer projection, a different type of flyer configurations was tested to measure the velocity during projection. A total of 3 different configurations of flat flyers were tested under a flash x-ray from a side-view with 150 kv, and those images were studied to identify the unique behavior of projection. This series of tests is a somewhat indirect measurement of the flyer velocity using the Taylor bending angle. According to the Taylor bending angle calculation with a long flyer, the speed of the flyer is closely related with the bending angle as follows in Equation (3) and (Figure 11) [8].

\[
\theta = 2\sin^{-1}\left(\frac{v}{2D}\right)
\]

where \( \theta \) is the flyer turning angle (or twice of the Taylor bending angle), \( v \) is the flyer velocity, and \( D \) is the detonation velocity.

![Figure 11. Taylor bending angle [8].](image)

If the flyer turning angle during the flyer projection, and the detonation velocity of the charge behind the flyer are known, then one can obtain the velocity of the flyer. If the velocity of the flyer gets affected by the width of the flyer, then the Taylor turning angle should be affected by this as well. In order to see the flyer velocity variation depending on the flyer width change, the following tests with 9 in. (228 mm) long copper flyers (density 8.9 g/cm\(^3\)), with three different flyer widths (24, 37 and 50 mm.), were accomplished (Figure 12). All of the X-ray shot images were taken 35 \( \mu s \) after the initiation in order to see the entire projection of the flyers. The charge behind the flyer is multi-layers of RDX based sheet-explosives with a measured detonation velocity of approximately 7.12 km/s, and a density of 1.44 g/cm\(^3\). Note that the type of charges used in the simulation and the field-test do not match due to the technical limitations in achieving a uniform/precise charge density in the field-test configuration and within the given budget range.

After three field tests, the Taylor bending angle from each flyer is measured in order to calculate the speed of projection for different width flyers, which are tabulated for comparison in Table 2 (Figure 13).

Note that the Taylor bending angles from the X-ray images above are measured directly from the images. In order to prevent any perspective errors during this process, the entire test configurations (including the test samples, X-ray heads and X-ray films) were aligned in a way to provide the accurate angles measurement (against the X-ray head and films) (refer to Figure 12 for the test configuration).
Figure 12. Schematic diagram of test configurations with side cladding added (initiated from the bottom).

Figure 13. X-ray test results.

The large white block at the left side of the images is from a protection device preventing the fragments from impacting the X-ray head. The curved liner near the bottom of the images is from the initial run-distance. The large opening at the initiation location caused issues during the detonation which decreased the effectiveness of the liner projection, and this curved liner is out of scope of this work and it is neglected in this paper. Note that the initiation point is at the bottom of the charge.
Due to the fact that the detonation front sweeps through the length of charge, it was able to observe the Taylor bending angle. From the images, it was identified that the flyer during projection remains straight, representing that the speed of projection already reaches the terminal velocity at this time stage, and the flyer width affects the projection velocity.

Table 2. Flyer velocity comparison of field test.

| Thickness of Charge (mm) | Thickness of Flyer (mm) | M/C * | Width of Flyer (mm) | Terminal Velocity * (m/s) | Gurney Model ** (m/s) |
|-------------------------|------------------------|-------|---------------------|---------------------------|-----------------------|
| Field Test              |                        |       |                     |                           |                       |
| 12                      | 1.6                    | 0.82  | 24                  | 1624                      | 1703                  |
| 12                      | 1.6                    | 0.82  | 37                  | 1980                      | 1703                  |
| 26                      | 1.6                    | 0.38**| 50                  | 2219                      | 2292                  |

* Symmetric sandwich with side cladding added; ** Symmetric sandwich configuration; *** This configuration is different than M/C = 0.82, and it is not included in the analysis.

In order to observe the similar behavior from other references, the analytical calculation results from Chanteret are tabulated in Table 3. This is from a symmetrical configuration with two 10 mm thick steel plates and 20 mm thick EL506A charge in the middle, and it was able to observe the same behavior of the flyer projection velocity variation depending on the flyer width.

Table 3. Flyer velocity comparison from Chanteret [6].

| Thickness of Charge (mm) | Thickness of Flyer (mm) | M/C * | Width of Flyer (mm) | Terminal Velocity * (m/s) |
|-------------------------|------------------------|-------|---------------------|---------------------------|
| Analytical calculation  |                        |       |                     |                           |                       |
| 20                      | 10                     | 2.72  | 25                  | ~220**                    |
| 20                      | 10                     | 2.72  | 100                 | ~530**                    |

* Type of charge: EL506A, density: 1.48 g/cm\(^3\), Flyer: Steel, density 8.05 g/cm\(^3\), Symmetric sandwich configuration [6]; ** These values are from the graph in the publication [6] and this is an approximated value.

Note that the results comparison between the field test and Chanteret’s analytical calculation was not accomplished due to the lack of information of Gurney velocity constant \(\sqrt{2E}\) which is necessary for the given change type.

In general, the two tests with constant M/C = 0.82, with different flyer widths, show a general trend of increasing flyer velocity when the width of flyer increases. This is an interesting observation that the flyer projection velocity is somewhat related with the width of the flyer, and the fundamental reason behind of this occurrence is related with the rarefaction intrusion.

5. Discussion and Conclusions

A series of simulations and field tests reveal the unique characteristics of the flyer projection velocity driven by an explosive detonation. The flyer projection speed varies depending on the flyer width. An analytical analysis for this occurrence is out of the scope of this report because the only focus of this paper is on the observation of such an occurrence. Certainly, the rarefaction intrusion during the projection can be a reason for this occurrence as is suggested in a previous publication [6,7] and shown in numerical simulations in the sections above. More detailed and accurate investigation would be available with the use of more sophisticated equipment like multi-points Photon Doppler Velocimetry (PDV) system to measure the velocity profile of the flyer during projection.

The difference in the projection velocity originates from the rarefaction (or release wave) beginning from the edge of the flyer and intruding to the center of the flyer, which reduces the projection velocity. The conventional Gurney approach lacks this aspect due to the fact that it is based on simple, one-dimensional analysis. Conventional analytical tools have delivered a great deal of improvement, but also reveal some limitations by the one-dimensional assumption. The most important outcome from this observation is the understanding of the velocity profile on varied flyer widths. The properties
of the detonation gas during acceleration is the key information to study the flyer behavior because the explosively-driven flyer is significantly affected by the detonation gas. Finally, because any realistic flyers will have finite dimensions, this effect cannot be neglected, and a more detailed investigation should continue to advance the conventional theoretical approaches.

**Author Contributions:** Lim leads the research project and technical/theoretical approaches, and Baldovi contributes the completion of the detailed subtasks and technical analysis.

**Funding:** This research was partially funded by Office of Naval Research, grant number: N00014-17-1-2181, and the APC was funded by New Mexico Tech, Socorro, NM, USA.

**Conflicts of Interest:** The authors declare no conflicted of interest.

**Appendix A**

The series of simulations in this paper is based on the similar size (but not identical) of flyers that have tested in the field. This is to identify the possible variations of projection speed depending the varied M/C ratios as reference points. The flyers are 2 mm thick and varied width in Lagrange cells, and 5 cells per 2 mm in linear length, totaling 30 × 5 cells in the 12 mm width flyer and 63 × 5 cells in the 25 mm width flyer. Note that the recommended number of cells in this type of thin-shelled structure is at least 3 cells per thickness by the manufacturer. The explosives charge in the simulation is with Eulerian cells with 1 cell per 1 mm in linear length. The Lagrangian-Eulerian coupled method structure is at least 3 cells per thickness by the manufacturer. The explosives charge in the simulation in the 25 mm width flyer. Note that the recommended number of cells in this type of thin-shelled structure is at least 3 cells per thickness by the manufacturer. The explosives charge in the simulation is with Eulerian cells with 1 cell per 1 mm in linear length. The Lagrangian-Eulerian coupled method is used for the simulation. Cell convergence study has been accomplished in this configuration, and it was identified that as long as the number of cells in thickness wise is over 3 cells, there was no visible difference.

![Figure A1. Simulation configuration for 12 mm width and 2 mm thick flyer (bottom with five gauges points), and 10 mm thick C4 (top). The five gauges points in the flyer are added.](image)

**Table A1. Simulation input parameters.**

| Material: Copper [9] | Shock | Material: C4 [10] | Equation of State |
|----------------------|-------|-------------------|-------------------|
| Reference density    | 8.9000 × 10^8 (g/cm³) | Reference density | 1.60100 × 10^9 (g/cm³) |
| Gruneisen coefficient| 2.0000 × 10^8 (none)  | Parameter A       | 6.06377 × 10^9 (kPa) |
| Parameter C1 (Bulk speed of sound) | 2.95800 × 10^6 (m/s)  | Parameter B       | 1.29500 × 10^6 (kPa) |
| Parameter S1 (Material constant) | 1.49700 × 10^6 (none) | Parameter R1      | 4.40000 × 10^6 (none) |
| Parameter Quadratic S2 | 0.00000 × 10^6 (s/m)  | Parameter R2      | 1.40000 × 10^6 (none) |
| Relative volume, VE/V0 | 0.00000 × 10^6 (none) | Parameter W       | 2.50000 × 10^6 (none) |
| Relative volume, VB/V0 | 0.00000 × 10^6 (none) | C-J Detonation velocity | 8.19300 × 10^3 (m/s) |
| Reference Temperature | 3.00000 × 10^6 (K)    | C-J Energy / unit volume | 9.00000 × 10^7 (kJ/m³) |
| Specific Heat         | 0.00000 × 10^6 (J/kgK) | C-J Pressure      | 2.60000 × 10^7 (kPa) |
| Thermal Conductivity  | 0.00000 × 10^6 (J/mKs) | Burn on compression fraction | 0.00000 × 10^6 (none) |
Table A1. Cont.

| Material: Copper [9] | Material: C4 [10] |
|----------------------|-------------------|
| **Equation of State** | **Shock** | **Equation of State** | **JWL** [11] |
| **Strength** | **Piecewise JC** | Pre-burn bulk modulus | $0.00000 \times 10^9$ (kPa) |
| Shear Modulus | $4.64000 \times 10^7$ (kPa) | Adiabatic constant | $0.00000 \times 10^9$ (none) |
| Yield Stress (zero plastic strain) | $1.20000 \times 10^7$ (kPa) | Auto-convert to Ideal Gas | Yes |
| Eff. Plastic Strain #1 | $3.00000 \times 10^{-1}$ (none) | Additional Options (Beta) | None |
| Eff. Plastic Strain #2 | $1.00000 \times 10^{20}$ (none) | Strength | None |
| Yield Stress #1 | $4.50000 \times 10^5$ (kPa) | Failure | None |
| Yield Stress #2 | $4.50000 \times 10^5$ (kPa) | Erosion | None |
| Thermal Softening Exponent | $1.00000 \times 10^0$ (none) | **Material Cutoffs** | - |
| Melting Temperature | $1.00000 \times 10^{20}$ (K) | Maximum Expansion | $1.00000 \times 10^{-1}$ (none) |
| Eff. Strain Rate (/s) | $1.00000 \times 10^0$ (none) | Minimum Density Factor | $1.00000 \times 10^{-6}$ (none) |
| **Failure** | **Johnson Cook** | Minimum Density Factor (SPH) | $2.00000 \times 10^{-1}$ (none) |
| Damage Constant, D1 | $1.01000 \times 10^{20}$ (none) | Maximum Density Factor (SPH) | $3.00000 \times 10^5$ (none) |
| Damage Constant, D2 | $1.01000 \times 10^{20}$ (none) | Minimum Soundspeed | $1.00000 \times 10^{-6}$ (m/s) |
| Damage Constant, D3 | $1.01000 \times 10^{20}$ (none) | Maximum Soundspeed (SPH) | $1.01000 \times 10^{20}$ (m/s) |
| Damage Constant, D4 | $1.01000 \times 10^{20}$ (none) | Maximum Temperature | $1.01000 \times 10^{20}$ (K) |
| Melting Temperature | $0.00000 \times 10^0$ (K) | **Erosion** | - |
| Eff. Strain Rate (/s) | $1.00000 \times 10^0$ (none) | Maximum Expansion | $1.00000 \times 10^{-1}$ (none) |
| **Material Cutoffs** | - | Minimum Density Factor | $1.00000 \times 10^{-4}$ (none) |
| Maximum Expansion | $1.00000 \times 10^{-1}$ (none) | Minimum Density Factor (SPH) | $2.00000 \times 10^{-1}$ (none) |
| Minimum Soundspeed | $1.00000 \times 10^6$ (m/s) | Maximum Soundspeed (SPH) | $1.01000 \times 10^{20}$ (m/s) |
| Maximum Temperature | $1.01000 \times 10^{20}$ (K) | Minimum Soundspeed | $1.00000 \times 10^{-6}$ (m/s) |
| Erosion | - | Maximum Temperature | $1.01000 \times 10^{20}$ (K) |

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