Evidence for tetrahedral symmetry in $^{16}\text{O}$

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We derive the rotation-vibration spectrum of a 4α configuration with tetrahedral symmetry, $T_d$, and show evidence for the occurrence of this symmetry in the low-lying spectrum of $^{16}\text{O}$. All vibrational states with $A$, $E$ and $F$ symmetry appear to have been observed, as well as the rotational bands with $L^2 = 0^+, 3^-, 4^+, 6^+$ on the $A$ states, and part of the rotational bands built on the $E$, $F$ states. We derive analytic expressions for the form factors and $B(EL)$ values of the ground state rotational band and show that the measured values support the tetrahedral symmetry of this band.

The cluster structure of light nuclei is a long standing problem which goes back to the early days of nuclear physics [1]. Recent experimental developments have shown that the low-lying states of $^{12}\text{C}$ can be described as rotation-vibration of a 3α cluster with $D_{3h}$ symmetry (equilateral triangle) [2,3]. Departures from a rigid cluster structure appear to be moderate in size and can be accounted for by perturbation theory. In this article, we show that the low-lying states of $^{16}\text{O}$ can be described as rotation-vibration of a 4α cluster with $T_d$ symmetry (tetrahedral). The suggestion that $^{16}\text{O}$ has a tetrahedral 4α structure goes back many years [4,5]. However, clear signatures could not be identified. We take advantage of the algebraic cluster model (ACM) [11, 12] to produce the rotation-vibration spectrum of an object with $T_d$ symmetry and compare with the observed spectrum. We also derive an analytic expression for the $B(EL)$ values along the ground state rotational band. A comparison with the experimental values of the energy spectrum and electromagnetic transitions provides strong evidence for tetrahedral symmetry in $^{16}\text{O}$.

The algebraic cluster model is a description of cluster states as representations of a $U(\nu + 1)$ group where $\nu$ is the number of space degrees of freedom [11, 12]. In Ref. [11, 12], we described three-body clusters, where the number of degrees of freedom (after removal of the center of mass) is $\nu = 3n - 3 = 6$, in terms of the algebra of $U(7)$. The space degrees of freedom are there the Jacobi coordinates $\vec{\rho} = (\vec{r}_1 - \vec{r}_2) / \sqrt{2}$ and $\vec{\lambda} = (\vec{r}_3 + \vec{r}_2 - 2\vec{r}_1) / \sqrt{6}$, where $\vec{r}_i$ are the coordinates of the three α particles ($i = 1, 2, 3$). We describe four-body clusters with $\nu = 3n - 3 = 9$ in terms of the algebra of $U(10)$. The space degrees of freedom are here three Jacobi vectors, $\vec{\rho} = (\vec{r}_1 - \vec{r}_2) / \sqrt{2}$, $\vec{\lambda} = (\vec{r}_3 + \vec{r}_2 - 2\vec{r}_1) / \sqrt{6}$ and $\vec{\eta} = (\vec{r}_3 + \vec{r}_2 + \vec{r}_3 - 3\vec{r}_1) / \sqrt{12}$, where $\vec{r}_i$ are the coordinates of the four α particles ($i = 1, \ldots, 4$). The algebra of $U(10)$ is constructed by introducing three vector bosons, $b_\rho$, $b_\lambda$, and $b_\eta$, together with an auxiliary scalar boson, $s$. The bilinear products of creation and annihilation operators generate the algebra $U(10)$

$$b_{\rho,m}^\dagger, b_{\lambda,m}^\dagger, b_{\eta,m}^\dagger, s^\dagger \equiv c_\alpha (m = 0, \pm 1),$$

The creation and annihilation operators for vector bosons $(b_{\rho,m}^\dagger, b_{\lambda,m}^\dagger, b_{\eta,m}^\dagger)$ and $(b_{\rho,m}, b_{\lambda,m}, b_{\eta,m})$ represent the second quantized form of the Jacobi coordinates and their canonically conjugate momenta, while the auxiliary scalar boson is introduced in order to construct the spectrum generating algebra. The energy levels can be obtained by diagonalizing the Hamiltonian $H$. In this article, we consider clusters composed of four identical particles (4α), for which $H$ must be invariant under the permutation group $S_4$. The most general one- and two-body Hamiltonian that describes the relative motion of four identical particles, is a scalar under $S_4$, is rotationally invariant, and conserves parity as well as the total number of bosons is given by [13,14].

$$H = \epsilon_0 s^\dagger s - \epsilon_1 (b_\rho^\dagger \cdot \hat{b}_\rho + b_\lambda^\dagger \cdot \hat{b}_\lambda + b_\eta^\dagger \cdot \hat{b}_\eta)$$
$$+ u_0 s^\dagger s \hat{s} \hat{s} - u_1 s^\dagger (b_\rho^\dagger \cdot \hat{b}_\rho + b_\lambda^\dagger \cdot \hat{b}_\lambda + b_\eta^\dagger \cdot \hat{b}_\eta)$$
$$+ v_0 \left[(b_\rho^\dagger b_\rho + b_\lambda^\dagger b_\lambda + b_\eta^\dagger b_\eta) s^\dagger s + \text{h.c.}\right]$$
$$+ \sum_{L=0}^{2} a_L \left[2b_\rho^\dagger b_\rho + 2\sqrt{2} b_\lambda^\dagger b_\lambda \right](L) \cdot \text{[h.c.]}(L)$$
$$+ \sum_{L=0}^{2} c_L \left[-2\sqrt{2} b_\rho^\dagger b_\rho \eta^\dagger \eta + 2b_\eta^\dagger b_\eta \right](L) \cdot \text{[h.c.]}(L)$$

$$+ \sum_{L=0}^{2} d_L \left(b_\rho^\dagger b_\rho \lambda^\dagger \lambda + b_\lambda^\dagger b_\lambda \eta^\dagger \eta \right) \cdot \text{[h.c.]}(L)$$

with $\hat{b}_{km} = (-1)^{1-m} b_{k-m} (k = \rho, \lambda, \eta)$ and $\hat{s} = s$. The coefficients $\epsilon_0, \epsilon_1, u_0, u_1, v_0, a_0, a_2, c_0, c_2, d_0$ and $d_2$ parametrize the interactions. The Hamiltonian $H$ is diagonalized within the space of the totally symmetric representation $|N\rangle$ of $U(10)$. 

\[ G : G_{\alpha \beta} = \delta_{\alpha \beta} \quad (\alpha, \beta = 1, \ldots, 10) \]

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Associated with the Hamiltonian, $H$, there are transition operators, $T$. Electromagnetic transition rates and form factors can all be calculated by considering the matrix elements of the operator

$$T = e^{-ig\beta D_{m}/X_{D}},$$

$$D_{m} = (\bar{b}_{\eta} \times \bar{s} - s_{\eta} \times \bar{b}_{\eta})^{(1)},$$

which is the algebraic image of the operator $\exp(iqrz)$ obtained from the full operator $\sum_{i=1}^{4} \exp(i\vec{q}_{i} \cdot \hat{r}_{i})$ by choosing the momentum transfer, $\vec{q}$, in the $z$-direction taken perpendicular to the base triangle in the direction of the 4th $\alpha$-particle and considering all particles to be identical (the coefficient $X_{D}$ is a normalization factor).

The Hamiltonian of Eq. (1) with an appropriate choice of parameters can describe any dynamics of four-particle systems. In two cases, corresponding to the dynamic symmetries $U(10) \supset U(9)$ (harmonic oscillator) and $U(10) \supset SO(10)$ (deformed oscillator) the eigenvalues of the Hamiltonian $H$ of Eq. (1) can be obtained analytically. Here we discuss another situation, namely that of four particles at the vertices of a tetrahedron with $T_d$ symmetry. The spectrum of a tetrahedral configuration can be obtained from the Hamiltonian of Eq. (1) by setting some coefficients equal to zero and taking specific linear combinations of others.

$$H = \xi_1 (R^2 s_{1} \cdot s_{1} - b_{\rho} \cdot b_{\rho} - b_{\alpha} \cdot b_{\alpha} - b_{\eta} \cdot b_{\eta}) \text{(h.c.)}$$

$$+ \xi_2 \left[ (-2\sqrt{2} b_{\rho} \cdot b_{\rho} + b_{\alpha} \cdot b_{\alpha} - b_{\eta} \cdot b_{\eta} \right) \text{(h.c.)}$$

$$+ (2\sqrt{2} b_{\rho} \cdot b_{\rho} + b_{\alpha} \cdot b_{\alpha} - b_{\eta} \cdot b_{\eta}) \text{(h.c.)}$$

$$+ (2\sqrt{2} b_{\rho} \cdot b_{\rho} + b_{\alpha} \cdot b_{\alpha} - b_{\eta} \cdot b_{\eta}) \text{(h.c.)}$$

$$+ (2\sqrt{2} b_{\rho} \cdot b_{\rho} + b_{\alpha} \cdot b_{\alpha} - b_{\eta} \cdot b_{\eta}) \text{(h.c.)}$$

$$+ \xi_3 \left[ (2b_{\rho} \cdot b_{\rho} + b_{\alpha} \cdot b_{\alpha} - b_{\eta} \cdot b_{\eta}) \text{(h.c.)}$$

$$+ (2b_{\rho} \cdot b_{\rho} + b_{\alpha} \cdot b_{\alpha} - b_{\eta} \cdot b_{\eta}) \text{(h.c.)}$$

$$+ (2b_{\rho} \cdot b_{\rho} + b_{\alpha} \cdot b_{\alpha} - b_{\eta} \cdot b_{\eta}) \text{(h.c.)}$$

$$+ \kappa_1 \vec{L} \cdot \vec{L} + \kappa_2 (\vec{L} \cdot \vec{L} - \vec{I} \cdot \vec{I})^2 \right].$$

Here $\vec{L}$ denotes the angular momentum in coordinate space $(x, y, z)$ and $\vec{I}$ the angular momentum in the so-called ‘index’ space $\rho, \alpha, \eta$.

The eigenvalues of $H$ of Eq. (3), given in terms of five parameters $\xi_1$, $\xi_2$, $\xi_3$, $\kappa_1$, $\kappa_2$ and the rigidity parameter $R^2$, cannot be calculated analytically. However, an approximate energy formula can be obtained by semiclassical methods ($N \to \infty$ in $U(10)$). A tetrahedral configuration has three vibrational modes $v_1$, $v_2$ and $v_3$ labeled by their $T_d$ symmetry. The vibration $v_1$ is the symmetric stretching (breathing mode) with $A$ symmetry. The vibration $v_2 = v_{2a} + v_{2b}$ is the doubly degenerate vibration with $E$ symmetry and $a, b$ components. The vibration $v_3 = v_{3a} + v_{3b} + v_{3c}$ is the triply degenerate vibration with $F$ symmetry and $a, b, c$ components. Since the tetrahedral group $T_d$ is isomorphic to the permutation group $S_4$, the vibrations can also be labeled by representations of $S_4$.
derived closed forms of these in the $U(9)$ and $SO(10)$
dynamic symmetries and in the large $N$ limit for the
spherical top with tetrahedral symmetry. This constitutes an
important new result of the ACM. In the spherical top
case discussed here, the form factors for transitions along
the ground state band $(0,0,0)A$ are given by spherical
Bessel functions, $F_L(0^+ \rightarrow L^P; q) = c_L L_L(q \beta)$. The coefficients
$c_L$ for the first few states are $c_0^2 = 1$, $c_2^2 = 35/9$, $c_4^2 = 7/3$ and $c_6^2 = 32/81$ for the $L^P = 0^+$, $3^-$, $4^+$ and $6^+$, respectively. The transition probabilities $B(EL)$ can be extracted from the form factors in the long wavelength limit

$$B(EL; 0 \rightarrow L) = \left( \frac{Ze\beta L}{4} \right)^2 \frac{2L+1}{4\pi} \left[ 4 + 12P_L(-\frac{1}{3}) \right]$$

The form factors and $B(EL)$ values only depend on the
parameter $\beta$, the distance of each $\alpha$ particle from the
center of the tetrahedral configuration., and on the $T_d$
symmetry which gives the coefficients $c_L$. By extracting
the value of $\beta$ from the elastic form factor measured in
electron scattering, one can thus make a model independent
test of the symmetry.

Whereas $L$ is an exact symmetry of $H$, $I$ is not. If $L \neq I$, perturbations must be added. The algebraic
model allows one to study these perturbations quantita-
tively by diagonalizing the Hamiltonian $H$ of Eq. (3) in an
appropriate basis. A convenient basis to construct states
with good permutation symmetry $S_4$ is the 9-dimensional
harmonic oscillator basis \textbf{[16]} corresponding to the reduc-
tion $U(10) \supset U(9) \supset U(3) \otimes U(3) \otimes U(3)$. We have con-
structed a set of computer programs to calculate energies and
electromagnetic transition rates in this basis.

Our derivation of the spectrum of clusters with $T_d$ sym-
metry can be used to study cluster states in $^{16}$O. The ob-
served experimental spectrum of $^{16}$O is shown in Fig. 2.
It appears that a rotational ground state band with an-
gular momenta $L^P = 0^+$, $3^-$, $4^+$, $6^+$ has been observed with moment of inertia such that $\kappa_1 = 0.511$ MeV. It
appears also that all three vibrations, $A$, $E$ and $F$, have
been observed with comparable energies, $\sim 6$ MeV, as
one would expect from Eq. (4) if $\xi_1 = \xi_2 = \xi_3$. A rota-
tional band with $0^+$, $3^-$, $4^+$, $6^+$ appears also to have been observed for the $A$ vibration $(1,0,0)$ (breathing mode).
This band is similar in nature to the band built on the
Hoyle state in $^{12}$C and recently observed \textbf{[2,4]}. It has a moment of inertia such that $E = 0.463 L(L + 1)$ MeV.
The moment of inertia of the $A$ vibration is larger than
that of the ground state due to its nature (breathing vi-
bration). The situation is summarized in Fig. 3. The
observed spectrum has perturbations. The most notable
perturbation is the splitting of the $2\pm$ states of the $E$ vi-
bration. This cannot be simply described by the formula $E \propto L(L + 1)$ and requires a diagonalization of the full Hamiltonian.

Having identified the cluster states, one can then test
the $T_d$ symmetry by means of the electromagnetic form
factors and $B(EL)$ values. We extracted the value of $\beta$
from the first minimum in the elastic form factor \textbf{[18]},
obtaining $\beta = 2.0$ fm. Table \textbf{[1]} shows the results for the
$B(EL)$ values. The $T_d$ symmetry appears to be unbro-
ken in the ground state band of $^{16}$O. We also investi-
gated the electromagnetic decays of the vibrational bands
$(1,0,0)A$, $(0,1,0)E$ and $(1,0,0)F$. For these bands the

![FIG. 2: The observed spectrum of $^{16}$O \textbf{[17]}. The levels are
organized in columns corresponding to the ground state band
and the three vibrational bands with $A$, $E$ and $F$ symmetry of
a spherical top with tetrahedral symmetry. The last column
shows the lowest non-cluster levels.]

![FIG. 3: The excitation energies of cluster states in $^{16}$O plotted
as a function of $L(L + 1)$: closed circles for the ground state band,
closed squares for the $A$ vibration, open circles for the $E$
vibration and open triangles for the $F$ vibration.]

TABLE I: Comparison of theoretical and experimental $B(EL)$ values in $e^2\text{fm}^{-4}$ and $E_\gamma$ values in keV, along the ground state band. The theoretical $B(EL)$ values are obtained from Eq. (3), and the $E_\gamma$ values are obtained from $E = 0.511 \times (L+1) \text{ MeV}$. The experimental values are taken from [12].

| $B(EL; L^P \rightarrow 0^+$) | Th | Exp | $E_\gamma(L^P)$ | Th | Exp |
|-----------------------------|----|-----|-----------------|----|-----|
| $B(E3; 3^+ \rightarrow 0^+_1)$ | 181 | 205 ± 10 | $E_\gamma(3^+_1)$ | 6132 | 6130 |
| $B(E4; 4^+ \rightarrow 0^+_1)$ | 338 | 378 ± 133 | $E_\gamma(4^+_1)$ | 10220 | 10356 |
| $B(E6; 6^+ \rightarrow 0^+_1)$ | 8245 | | $E_\gamma(6^+_1)$ | 21462 | 21052 |

$T_d$ symmetry appears to be broken and, in addition, they decay mostly by $E2$ quadrupole transitions. For $E2$ transitions the simple analytic formula of Eq. (3) does not apply, since the $E2$ operator is not in the representation $A$ of $T_d$ as the $E3$, $E4$ and $E6$ operators and hence can connect different representations. A full account of these transitions will be given in a forthcoming longer publication [19].

Cluster states represent only a portion of the full spectrum of states. They are obtained by assuming that the $\alpha$ particles have no internal excitation. At energies of the order of the shell gap, $\sim 16$ MeV in $^{16}\text{O}$, one expects to have non-cluster states, and thus the spectrum to be composed of cluster states immersed into a bath of non-cluster states. Assigning states to cluster or non-cluster above this energy is a difficult task. We note, however, that the tetrahedral structure in Fig. 1 has no $0^-$ state and only one $1^-$ state in the $F$-vibration. Thus $0^-$ states are clearly non-cluster states. Also with $\alpha$ particles one cannot form $T = 1$ states. These states are obviously non-cluster. In Fig. 2 we have assigned the states $L^P = 1^−, 0^−$ ($T = 0$) at $E = 9.585 \text{ MeV}$ and $10.957 \text{ MeV}$ and $L^P = 0^−, 1^−$ ($T = 1$) at $E = 12.796 \text{ MeV}$ and $13.090 \text{ MeV}$, as the shell model configuration $1p_{1/2}2s_{1/2}$. The shell model states $1p_{1/2}1d_{5/2}$ with $L^P = 2^−, 3^−$ and $T = 0$ and $T = 1$ can also be easily identified but they are not shown in Fig. 2 not to overcrowd the figure. For the same reason, we do not show in Fig. 2 other states with $L^P = 4^±, 5^−, 6^±, \ldots$ which can be assigned to cluster configurations.

An important question is the shell-model description of cluster states. It was suggested long ago [20] that the state at 6.049 MeV is a $4p – 4h$, while the state at 7.116 MeV is a $5p – 5h$. In view of the recent developments of large-scale shell model calculations and of the no-core shell model it would be interesting to study once more the shell model description of the states in Fig. 2.

Very recently, also an ab initio lattice calculation of the spectrum and structure of $^{16}\text{O}$ has been reported [21]. This calculation confirms the tetrahedral structure of the ground state of $^{16}\text{O}$ in agreement with our findings. For the excited states, $0^+_2$ and $2^+_1$ instead, a square configuration is suggested. This would imply a large breaking of the $T_d$ symmetry for the vibrations in Fig. 2. Although we expect the $T_d$ symmetry to be broken for the vibrational states due to the near degeneracy of them, $\xi_1 = \xi_2 = \xi_3$, i.e. even a small breaking term in $H$ may cause a large mixing, we nonetheless feel at this stage that our interpretation of the excited states of $^{16}\text{O}$ as vibrations provides a good starting point for further studies. Algebraic methods are quite general, and as shown in Ref. [23], they can accommodate all sorts of configurations of four particles, including configurations with $T_d$, $D_{3h}$ and $D_{4h}$ (square) symmetry. In connection with tetrahedral configurations in nuclei, we mention here also the work of [24] in light nuclei and [25] in heavy nuclei for which, however, there is no experimental confirmation.

In conclusion, we have introduced an algebraic model capable of describing the full dynamics of four-body clusters. Within this model we have rederived the spectrum of a spherical top with tetrahedral symmetry, and confirmed the evidence for the occurrence of this symmetry in the low-lying spectrum of $^{16}\text{O}$ presented long ago by Kamyen [6] and Robson [10]. An analysis of the $B(EL)$ values along the ground state band provide an even stronger evidence for $T_d$ symmetry than the energies. Another crucial aspect is the development of the $U(10)$ ACM for four-body clusters which allows a detailed description of energies, electromagnetic transition rates, form factors and $B(EL)$ values. We hope that the results in this paper will stimulate further experimental work on the structure of $^{16}\text{O}$. Finally, the results presented here in conjunction with those in $^{12}\text{C}$ emphasize the occurrence of $\alpha$-cluster states in light nuclei.

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[1] J.A. Wheeler, Phys. Rev. 52, 1083 (1937).
[2] M. Itoh et al., Phys. Rev. C 84, 054308 (2011).
[3] M. Freer et al., Phys. Rev. C 86, 034320 (2012).
[4] W.R. Zimmerman et al., Phys. Rev. Lett. 110, 152502 (2013).
[5] D.J. Marin-Lambardi et al., in preparation (2014).
[6] D.M. Dennison, Phys. Rev. 96, 378 (1954).
[7] S.L. Kameny, Phys. Rev. 103, 358 (1956).
[8] D.M. Brink, Int. School of Physics “Enrico Fermi”, Course XXXVI, 247 (1965).
[9] D.M. Brink, H. Friedrich, A. Weiguny and C.W. Wong, Phys. Lett. B 33, 143 (1970).
[10] D. Robson, Nucl. Phys. A 308, 381 (1978); D. Robson, Phys. Rev. Lett. 42, 876 (1979); D. Robson, Phys. Rev. C 25, 1108 (1982).
[11] R. Bijker and F. Iachello, Phys. Rev. C 61, 067305 (2000).
[12] R. Bijker and F. Iachello, Ann. Phys. (N.Y.) 298, 334 (2002).
[13] R. Bijker, AIP Conf. Proc. 1323, 28 (2010).
[14] R. Bijker, J. Phys.: Conf. Ser. 380, 012003 (2012).
[15] G. Herzberg, *Molecular Spectra and Molecular Structure. II. Infrared and Raman Spectra of Polyatomic Molecules*, Krieger, Malabar, Florida (1991).
[16] P. Kramer and M. Moshinsky, Nucl. Phys. **82**, 241 (1966).
[17] D.R. Tilley, H.R. Weller and C.M. Cheves, Nucl. Phys. A **564**, 1 (1993).
[18] I. Sick and J.S. McCarthy, Nucl. Phys. A **150**, 631 (1970).
[19] R. Bijker and F. Iachello, in preparation (2014).
[20] H. Feshbach and F. Iachello, Phys. Lett. B **45**, 7 (1973).
[21] G.E. Brown and A.M. Green, Nucl. Phys. **75**, 401 (1966).
[22] E. Epelbaum *et al.*, Phys. Rev. Lett. (2014), in press [arXiv:1312.7703v1].
[23] D. Larese, M.A. Caprio, F. Pérez-Bernal and F. Iachello, J. Chem. Phys. **140**, 014304 (2014).
[24] J. Zhang, W.D.M. Rae and A.M. Merchant, Nucl. Phys. A **575**, 61 (1994).
[25] J. Dudek *et al.*, Phys. Rev. Lett. **88**, 252502 (2002); J. Dudek *et al.*, Phys. Rev. Lett. **97**, 072501 (2006).