Cubic vertices of interacting massless spin 4 and real scalar fields in unconstrained formulation

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Abstract

New method (arXiv:2104.11930) is applied to construct cubic interactions of massless spin 4 and real scalar fields. In contrast with the case of massless spin 3 fields (arXiv:2208.05700, 2209.03678, 2210.02842) the procedure requires to use an unconstrained formulation (arXiv:0702.161) for the Fronsdal theory. It is shown that in the unconstrained formulation there exists a four-parameter family of cubic interactions between massless spin 4 and real scalar fields, which contains four derivatives in vertices and is invariant with respect to the original gauge transformation. These vertices contain cubic interactions between auxiliary and scalar fields too. Eliminating all auxiliary fields from the obtained result using the equations of motion for the initial action gives a one-parameter family of cubic vertices for constrained massless spin 4 and real scalar fields. Such cubic vertices are invariant with respect to constrained gauge transformations of the Fronsdal theory.

Keywords: BV-formalism, deformation procedure, anticanonical transformations, higher integer spin fields

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1 Introduction

The study of interactions among higher spin fields has a long and reach story beginning with famous papers by Fradkin and Vasiliev \[1, 2, 3, 4, 5\]. The list of publications devoted to numerous problems of high spin theory contains hundreds of papers and there is no way to mention them all here. The simplest interactions involving higher spin fields are described by cubic vertices including at least one higher spin field.\(^{2}\) Explicit construction of cubic vertices is usually based on the Noether procedure when an free initial action and gauge transformations are subjected to deformations maintaining the gauge invariance of the deformed action under the deformed gauge transformations in each order in the deformation parameter. In a certain sense, such the scheme can be considered as a deformation procedure built into the Batalin-Vilkovisky (BV) formalism \[12, 13\] and described for the first time by Barnich and Henneaux in the papers \[14, 15\]. The method \[14, 15\] operates infinite number of equations appeared in expansion of solutions to the classical master equation of the BV-formalism with respect to the deformation parameter. Then the system of these equations is analyzed in each order using cohomological methods.

Recently, the new method for the deformation procedure within the BV-formalism has been proposed \[16, 17, 18\]. From a formal point of view, the new approach looks like the summation of the Taylor expansion in exact and closed form for both the deformed action and the deformed gauge symmetry. An informal reason to arrive at the resulting description is related to the invariance of the classical master equation under anticanonical transformations, which make it possible to connect two solutions to each other. In particular, it was allowed to connect an initial gauge theory with the deformed one with the help of special anticanonical transformations acting non-trivially in the sector of initial fields \[16\]. Applications of the BV-formalism for the deformation procedure require an initial action to belong the class of unconstrained theories that means linear independence of initial fields. In the case of the Fronsdal theory \[19\] for free massless integer spin fields it leads to restrictions on value of spins, \(s \leq 3\), when one can use the BV-formalism directly. The action for massless spin 3 field belongs to the class of first-stage reducible gauge theories when the BV-formalism works. Interactions of massless spin 3 field between massive vector and real scalar fields within the new method have been recently studied in papers \[20, 21, 22\]. It was shown that in all considered models cubic vertices containing one massless spin 3 field and invariant under original gauge transformations are forbidden while quartic and quintic vertices can be explicitly constructed. In turn it opened a way in construction of new consistent models with interactions in Quantum Field Theory.

In the present paper, the new method \[16\] is applied to construct cubic interactions of massless spin 4 and real scalar fields. Such application requires to use an unconstrained formulation \[23\] for the Fronsdal theory of massless integer spin fields. It is shown that in the unconstrained formulation there exists a four-parameter family of cubic interactions between massless spin 4 and real scalar fields containing four derivatives in vertices and being invariant under original gauge transformation. These vertices contain cubic interactions between auxiliary and scalar fields too. Eliminating all auxiliary fields from the obtained result using the equations of motion for the initial action gives an one-parameter family of cubic vertices for constrained massless spin 4 and real scalar fields. Such cubic vertices are invariant with respect to constrained gauge transformations of the Fronsdal theory.

The paper is organized as follows. In section 2, gauge invariance of initial action of massless spin 4 and real scalar fields is discussed. In section 3, suitable deformations of both initial action and gauge transformations leading to cubic vertices in the deformed action are described. In section 4, gauge invariance of local part of the deformed action in unconstrained formulation is studied. Section 5 is devoted to on-shell limit of local cubic vertices when all auxiliary fields are extracted with the help of corresponding equations of motion. In section 6 we summarize

\(^{2}\)For recent activities in this sphere see, for example, \[6, 7, 8, 9, 10, 11\] and references therein.
the results.

The DeWitt’s condensed notations \[24\] are systematically used. The right functional derivatives are marked by special symbols ” \(←\)”. Arguments of any functional are enclosed in square brackets \([\ ]\), and arguments of any function are enclosed in parentheses, ( ).

2 Initial action of model

The action for massless spin 4 fields within the unconstrained formulation \[23\] has the form

\[
S_0[\varphi, F, \alpha, \lambda(2), \lambda(4)] = \int dx \left[ \varphi_{\mu\nu\lambda\sigma} \Box \varphi^{\mu\nu\lambda\sigma} - 12 F_{\mu\nu} \Box F^{\mu\nu} + 4 \Lambda_{\mu\nu\lambda} \Lambda^{\mu\nu\lambda} + \lambda^{\mu\nu}_2 (\eta^{\lambda\sigma} \varphi_{\mu\nu\lambda\sigma} - 2 F_{\mu\nu} - 2 \partial_\mu \alpha_\nu) + \lambda(4) (\eta^{\mu\nu} F_{\mu\nu} - \partial^\mu \alpha_\mu) \right],
\]

where the notation

\[
\Lambda_{\mu\nu\lambda} = \partial^{\sigma} \varphi_{\sigma\mu\nu\lambda} - \partial_{(\mu} F_{\nu\lambda)}
\]

is used. In \(\Box\) \(\varphi^{\mu\nu\lambda\sigma} = \varphi^{\mu\nu\lambda\sigma}(x)\) is completely symmetric forth rank tensor, \(\Box\) is the D’Alembertian, \(\Box = \partial_\mu \partial^\mu\), \(\eta_{\mu\nu}\) is the metric tensor of flat Minkowski space of the dimension \(d\), and \(F_{\mu\nu} \), \(\alpha_\mu \), \(\lambda^{\mu\nu}_2 \) , \(\lambda(4) \) are auxiliary fields of corresponding ranks coinciding with number of indices.

We are going to study a possibility in construction of interactions of massless spin 4 fields with a real scalar field \(\phi = \phi(x)\). We assume that the initial action has the form

\[
S_0[A] = S_0[\varphi, F, \alpha, \lambda(2), \lambda(4)] + S_0[\phi],
\]

where \(S_0[\phi]\) is the action of a free real scalar field,

\[
S_0[\phi] = \int dx \frac{1}{2} [\partial_\mu \phi \partial^\mu \phi - m^2 \phi^2],
\]

and \(A\) denotes collection of all fields of the model, \(A^i = (\varphi^{\mu\nu\lambda\sigma}, F_{\mu\nu}, \alpha_\mu, \lambda^{\mu\nu}_2, \lambda(4), \phi)\). The action \(3\) is invariant under the gauge transformations\(^3\)

\[
\delta \varphi^{\mu\nu\lambda\sigma} = \partial^{(\mu} \xi^{\nu\lambda\sigma)}, \quad \delta F^{\mu\nu} = \partial_\nu \xi^{\lambda\mu}, \quad \delta \alpha_\mu = \eta_{\mu\nu} \xi^{\lambda\mu}, \quad \delta \lambda^{\mu\nu}_2 = 0, \quad \delta \lambda(4) = 0, \quad \delta \phi = 0,
\]

where \(\xi^{\mu\nu\lambda}\) are arbitrary totally symmetric third rank tensor. Algebra of gauge transformations is Abelian.

3 Deformations of initial theory

Now, we consider possible deformations of initial action using the procedure which is ruled out by the generating functions \(\hat{h}(A)\) \[16\]. Here, we restrict ourself by the case of anticanonical transformations acting effectively in the sector of fields \(\varphi^{\mu\nu\lambda\sigma} \) and \(F^{\mu\nu}\) of the initial theory. It means the following structure of generating functions \(\hat{h}(A) = (\hat{h}^{\mu\nu\lambda\sigma}(\phi), \hat{h}^{\mu\nu}(\phi), 0, 0, 0, 0)\). In construction of suitable generating functions \(\hat{h}^{\mu\nu\lambda}(\phi)\), we have to take into account the dimensions of quantities involved in the initial action \(S_0[\varphi, F, \alpha, \lambda(2), \lambda(4)]\) \(\Box\),

\[
\dim(\varphi^{\mu\nu\lambda\sigma}) = \dim(F^{\mu\nu}) = \dim(\phi) = \frac{d - 2}{2},
\]

\[
\dim(\xi^{\mu\nu\lambda}) = \frac{d - 4}{2}, \quad \dim(\partial_\mu) = 1, \quad \dim(\Box) = 2.
\]

\(^3\)The symbol (\(\cdots\)) means the cycle permutation of indexes involved.
The generating functions $h_{(\phi)}^{\mu\nu\lambda\sigma}$ and $h_{(F)}^{\mu\nu}$ should be symmetric and non-local with the dimension equals to $-(d+2)/2$. The non-locality will be achieved by using the operator $1/\Box$. To construct cubic vertices $\sim \varphi \phi \phi$, $h_{(\phi)}^{\mu\nu\lambda\sigma}$ should be at least quadratic in fields $\phi$. The tensor structure of $h_{(\phi)}^{\mu\nu\lambda\sigma}$ is obeyed by using partial derivatives $\partial_\mu$ and the metric tensor $\eta_{\mu\nu}$. The minimal number of derivatives equals to 4. Therefore, the more general form of $h_{(\phi)}^{\mu\nu\lambda\sigma} = h_{(\phi)}^{\mu\nu\lambda\sigma}(\phi)$ satisfying requirements listed above reads

\[ h_{(\phi)}^{\mu\nu\lambda\sigma} = a_0 \frac{1}{\Box} \left( c_0 \partial_\mu \partial_\nu \partial_\lambda \partial_\sigma \phi + c_1 \partial_{(\mu} \partial_\nu \partial_\lambda \partial_{\sigma)} \phi + c_2 \partial_{(\mu} \partial_\nu \partial_\lambda \partial^\rho \partial_{\sigma)} \phi + 
\]

\[ + c_3 \eta^{(\mu\nu} \partial_\lambda \partial_\rho \partial_{\sigma)} \phi + c_4 \eta^{\mu\nu\rho\sigma} \partial_\phi \partial_\rho \partial_\phi + c_5 \eta^{(\mu\nu} \partial_\lambda \partial_\rho \partial_{\sigma)} \phi + 
\]

\[ + c_6 \eta^{\mu\nu\lambda\sigma} \partial_\phi \partial_\rho \partial_\phi \right), \tag{7} \]

where $a_0$ is the coupling constant with $\dim(a_0) = -d/2$ and $c_i, i = 0, 1, ..., 7$ are arbitrary dimensionless constants. By the same reason the generating function $h_{(F)}^{\mu\nu}$ is chosen in the form

\[ h_{(F)}^{\mu\nu} = a_0 \frac{1}{\Box} \left( d_0 \Box \partial_\mu \partial_\nu \phi + d_1 \partial_\mu \partial_\nu \partial_\rho \partial_\phi + d_2 \Box (\partial_\mu \partial_\nu \partial_\phi) + d_3 \partial_\mu \partial_\nu \partial_\phi \partial_\rho + 
\]

\[ + d_4 \partial_\mu \partial_\nu \partial_\rho \partial_\phi + d_5 \eta^{\mu\nu\rho\sigma} \partial_\phi \partial_\rho \partial_\phi + 
\]

\[ + d_6 \eta^{\mu\nu\lambda\sigma} \partial_\phi \partial_\rho \partial_\phi \right), \tag{8} \]

where $d_i, i = 0, 1, ..., 8$ are dimensionless constants.

The deformed action has the following explicit and closed form

\[ \tilde{S}[A] = S_0[A] + S_{int}[A], \tag{9} \]

where

\[ S_{int}[A] = 2a_0 \int dx \left[ \varphi_{\mu\nu\lambda\sigma} K^{\mu\nu\lambda\sigma} - 12 F_{\mu\nu} N^{\mu\nu} + 
\]

\[ + 4 \Lambda_{\mu\nu} \frac{1}{\Box} (\partial_\sigma K^{\sigma\mu\nu\lambda} - 2 \partial_{(\mu} N^{\nu\lambda)}) + \frac{1}{2} \lambda^{\mu
u(2)} \frac{1}{\Box} (\eta^{\lambda\sigma} K_{\lambda\sigma\mu\nu} - 2 N_{\mu\nu}) + 
\]

\[ + \frac{1}{2} \lambda^{(4)} \frac{1}{\Box} \eta^{\mu\nu} N_{\mu\nu} + \frac{1}{2} a_0 K_{\mu\nu\lambda\sigma} \frac{1}{\Box} K^{\mu\nu\lambda\sigma} - 6 a_0 N_{\mu\nu} \frac{1}{\Box} N^{\mu\nu} + 
\]

\[ + 2 a_0 \frac{1}{\Box} (\eta^{\lambda\sigma} K_{\lambda\sigma\mu\nu} - 2 N_{\mu\nu}) \frac{1}{\Box} (\eta_{\rho\beta} K^{\rho\beta\mu\nu} - 2 N^{\mu\nu}) \right] \tag{10} \]

and presentation of generating functions

\[ h_{(\phi)}^{\mu\nu\lambda\sigma} = a_0 \frac{1}{\Box} K^{\mu\nu\lambda\sigma}, \quad h_{(F)}^{\mu\nu} = a_0 \frac{1}{\Box} N^{\mu\nu} \tag{11} \]

is used. Notice that $S_{int}[A]$ does not depend on fields $\alpha_\mu$.

For Abelian algebras the deformation of gauge symmetry is defined only by the matrix $M^{-1}(A)$ being inverse to $M(A) = I + h(A) \partial_A$. In the sector of fields $(\varphi^{\mu\nu\lambda\sigma}, F^{\mu\nu}, \phi)$ the structure of $M(A)$ looks like

\[ \begin{pmatrix}
E_{\rho\beta}^{\mu\nu\lambda\sigma} & 0 & h_{(\phi)}^{\mu\nu\lambda\sigma}(\phi) \frac{\partial}{\partial \phi} \\
0 & E_{\rho\beta}^{\mu\nu} & h_{(F)}^{\mu\nu}(\phi) \frac{\partial}{\partial \phi} \\
0 & 0 & 1
\end{pmatrix}, \tag{12} \]
In the case of anticanonical transformations described by generating functions (7) and (8) the inverse matrix \( M^{-1}(A) \) can be explicitly found and in the sector of fields \((\varphi^{\mu\nu\lambda\sigma}, F_{\mu\nu})\) it reads

\[
\begin{pmatrix}
E_{\rho\beta\gamma\delta}^{\mu\nu\lambda\sigma} & 0 & -h_{(\varphi)}^{\mu\nu\lambda\sigma}(\phi) \partial_{\phi} \\
0 & E_{\rho\beta}^{\mu\nu} & -h_{(F)}^{\mu\nu}(\phi) \partial_{\phi} \\
0 & 0 & 1
\end{pmatrix}.
\]

(13)

Here, \( E_{\rho\beta\gamma\delta}^{\mu\nu\lambda\sigma} \) and \( E_{\rho\beta}^{\mu\nu} \) are elements of the unit matrix in the space of forth and second rank symmetric tensors, respectively. In particular, it means that in process of deformation of the initial action the gauge transformations (5) do not deform. In turn, the deformed action

\[
\delta \tilde{S}[A] = 0
\]

(14)

should be invariant under original gauge transformations (5). As a result, for the model under consideration we found explicit description of deformed action and gauge symmetry.

### 4 Gauge invariance of local sector of deformed theory

Now we are in position to study a local sector of the deformed theory. We begin with the local part of the deformed action

\[
S_{\text{loc}}[A] = S_0[A] + S_{1\text{loc}}[\varphi, F, \phi]
\]

(15)

where \( S_{1\text{loc}} = S_{1\text{loc}}[\varphi, F, \phi] \),

\[
S_{1\text{loc}} = 2a_0 \int dx \left[ \varphi^{\mu\nu\lambda\sigma} K_{\mu\nu\lambda\sigma}^{\mu\nu\lambda\sigma} - 12 F_{\mu\nu} N^{\mu\nu} \right].
\]

(16)

Due to the locality of original gauge transformations and the symmetry property of initial action \( S_0[A] \), from gauge invariance of the deformed action (14) it follows that the action \( S_{1\text{loc}}[\varphi, F, \phi] \) should be gauge invariant as well,

\[
\delta S_{1\text{loc}}[\varphi, F, \phi] = 0.
\]

(17)

The equation (17) rewrites in the form

\[
\int dx \xi_{\nu\lambda\sigma} \left[ \partial_{\mu} K_{\mu\nu\lambda\sigma}^{\mu\nu\lambda\sigma} - 3 \partial^{\nu} N^{\lambda\sigma} \right] = 0,
\]

(18)

where \( \xi_{\nu\lambda\sigma} \) are independent gauge parameters. Therefore, the equation (18) is equivalent to

\[
\partial_{\mu} K_{\mu\nu\lambda\sigma}^{\mu\nu\lambda\sigma} - \partial^{\nu} N^{\lambda\sigma} = 0.
\]

(19)

Analysis of the last equation leads to the following relations between constants \( c_i \), \( i = 0, 1, \ldots, 7 \) and \( d_j \), \( j = 0, 1, \ldots, 8 \)

\[
c_2 = -\frac{1}{4} c_0 + c_1, \quad c_5 = 2 c_3, \quad c_7 = c_3 + c_4 - c_6, \\
d_0 = \frac{1}{3} (c_0 + 2 c_3), \quad d_1 = \frac{1}{3} (c_0 + c_1 + 2 c_4), \quad d_2 = -\frac{1}{6} (c_0 - 3 c_1 - 4 c_3), \\
d_3 = \frac{1}{3} (c_1 + 2 c_6), \quad d_4 = -\frac{1}{3} (c_0 - 3 c_1 + 2 c_3 - 2 c_6), \quad d_5 = \frac{2}{3} c_3, \\
d_6 = \frac{2}{3} (2 c_3 + c_4), \quad d_7 = \frac{1}{3} (c_3 + c_6), \quad d_8 = \frac{1}{3} (c_3 + 2 c_4 - c_6).
\]

(20)

Therefore, there exists the four-parameter family of local functionals describing cubic interactions between massless spin 4 and scalar fields and being invariant under original gauge transformations of the initial free model. These functionals are responsible as well for local cubic interactions of auxiliary fields \( F_{\mu\nu} \) with scalar field \( \phi \).
5 On-shell limit for auxiliary fields

Next step of our study is to consider eliminating all auxiliary fields from the obtained results using the equations of motion for them. Following [23] we extract $\alpha_{\mu}$ with the help of requirement $\alpha_{\mu} = 0$. Then, from the equations of motion deriving with the help of the action $S_0[A]$ we find

$$F_{\mu\nu} = \frac{1}{2} \eta^{\lambda\sigma} \varphi_{\lambda\sigma\mu\nu}, \quad \eta^{\mu\nu} \eta^{\lambda\sigma} \varphi_{\mu\nu\lambda\sigma} = 0, \quad \eta_{\mu\nu} \xi^{\mu\nu\lambda} = 0, \quad \lambda_{(2)} = 0, \quad \lambda_{(4)} = 0. \quad (21)$$

In the limit (21) the action (1) reduces to the Fronsdal action for massless spin 4 fields, $S_0[\varphi, F, \alpha, \lambda_{(2)}, \lambda_{(4)}] \rightarrow S_0[\varphi]$,

$$S_0[\varphi] = \int dx \left[ \varphi_{\mu\nu\lambda\sigma} \Box \varphi^{\mu\nu\lambda\sigma} - 6 \eta^{\mu\nu} \eta_{\rho\sigma} \varphi_{\mu\nu\lambda\delta} \Box \varphi^{\rho\sigma\lambda\delta} - 4 \varphi_{\mu\nu\lambda\sigma} \partial_{\mu} \varphi^{\rho\mu\lambda\sigma} + 12 \eta^{\mu\nu} \varphi_{\mu\nu\lambda\rho} \partial_{\sigma} \varphi^{\delta\lambda\rho\sigma} - 6 \eta^{\mu\nu} \eta_{\beta\gamma} \varphi_{\mu\nu\lambda\sigma} \partial_{\rho} \varphi^{\rho\beta\gamma\sigma} \right]. \quad (22)$$

In this limit the local part of the deformed action should be specified by the following restrictions on parameters of generating functions $K^{\mu\nu\lambda\sigma}$ and $N^{\mu\nu}$

$$c_2 = -\frac{1}{4} c_0 + c_1, \quad c_3 = c_4 = c_5 = c_6 = c_7 = 0,$$

$$d_0 = \frac{1}{3} c_0, \quad d_1 = \frac{1}{3} (c_0 + c_1), \quad d_2 = \frac{1}{2} d_4 = -\frac{1}{6} (c_0 - 3 c_1), \quad d_3 = \frac{1}{3} c_1. \quad (23)$$

Therefore, the action $S_{1\ loc}[\psi, \phi]$,

$$S_{1\ loc}[\psi, \phi] = 2a_0 \int dx \varphi_{\mu\nu\lambda\sigma} \left[ \partial^\mu \partial^\nu \partial^\lambda \partial^\sigma \phi \phi + 4 c_1 \partial^\mu \partial^\nu \partial^\lambda \phi \partial^\sigma \phi - \left(1 - 4 c_1\right) \partial^\mu \partial^\nu \phi \partial^\lambda \partial^\sigma \phi - 2 \Box \partial^\mu \partial^\nu \phi \phi \eta^{\lambda\sigma} - 2 \left(1 + c_1\right) \partial^\mu \partial^\nu \partial^\rho \phi \phi \eta^{\rho\sigma} - 2 c_1 \partial^\mu \partial^\nu \phi \Box \phi \eta^{\lambda\sigma} + 2 \left(1 - 3 c_1\right) \partial^\rho \phi \partial^\mu \phi \partial^\nu \phi \eta^{\rho\sigma} \right], \quad (24)$$

describes the one-parameter family of local actions for massless constrained spin 4 field interacting with real scalar field. These actions are invariant under original gauge transformations in the Fronsdal theory [19].

6 Discussion

In the present paper, the new approach [16] was applied to study interactions between massless spin 4 ($\varphi^{\mu\nu\lambda\sigma}$) and real scalar ($\phi$) fields. In contrast with the case of massless spin 3 field [20, 21, 22] it required to use an unconstrained formulation for free massless integer spin fields of the Fronsdal theory [19]. A few years ago such formulation has been proposed [23]. Then the initial action was chosen as the sum of the unconstrained action for massless spin 4 field and the free action for a real scalar field. The unconstrained action includes additionally in comparison with the Fronsdal action a set of four auxiliary fields of which two are gauge fields. In the space of all fields the initial action is gauge invariant with unconstrained gauge parameters $\xi^{\mu\nu\lambda}$. The gauge algebra is Abelian.

The simplest interactions are cubic ones. In the present study we were interested in cubic vertices containing interactions of the form $\sim \varphi \phi \phi$. To construct such kind of vertices using the new method [16] a more general non-local anticanonical transformation acting non-trivially
in the space of massless spin 4 field and gauge auxiliary field and containing four partial derivatives in vertices was proposed \cite{8}, \cite{9}. The proposed deformation of initial action leaves the gauge symmetry non-deformed. Then the deformed non-local action has been described explicitly in the form of functional of fourth order in fields \cite{10}, \cite{11}. It was shown that the deformed action contains a local part \cite{15}, \cite{16} which should be invariant under original gauge transformations \cite{17}. This requirement led to a four-parameter family of cubic vertices containing, among other things, terms of the form $\phi\phi\phi$. In fact, for the first time in the unconstrained approach to massless integer higher spin fields the family of local gauge invariant actions with interactions between fields has been constructed in a closed and explicit form of fourth-order functionals with respect to fields.

We analyzed as well on-shell limit for auxiliary fields in the local deformed unconstrained action constructed in Section 4. It was found an one-parameter family of cubic vertices invariant under constrained gauge transformations similarly to gauge invariance of the Fronsdal theory. Again, we can claim the appearance due to the new method \cite{16} new examples of consistent local gauge invariant models with higher spin interactions formulated in terms of constrained fields of the Fronsdal theory.

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