Does quantum mechanics tell an atomistic spacetime?

Hans-Thomas Elze
Dipartimento di Fisica “Enrico Fermi”, Largo Pontecorvo 3, I-56127 Pisa, Italia
E-mail: elze@df.unipi.it

Abstract. The canonical answer to the question posed is “Yes.” – tacitly assuming that quantum theory and the concept of spacetime are to be unified by ‘quantizing’ a theory of gravitation. Yet, instead, one may ponder: Could quantum mechanics arise as a coarse-grained reflection of the atomistic nature of spacetime? – We speculate that this may indeed be the case. We recall the similarity between evolution of classical and quantum mechanical ensembles, according to Liouville and von Neumann equation, respectively. The classical and quantum mechanical equations are indistinguishable for objects which are free or subject to spatially constant but possibly time dependent, or harmonic forces, if represented appropriately. This result suggests a way to incorporate anharmonic interactions, including fluctuations which are tentatively related to the underlying discreteness of spacetime. Being linear and local at the quantum mechanical level, the model offers a decoherence and natural localization mechanism. However, the relation to primordial deterministic degrees of freedom is nonlocal.

1. Quantum features, information loss and spacetime discreteness

Classical ensemble theory with its Liouville equation implies the absence of a stable ground state, when rewritten as a Hilbert space theory with an analogue of the Schrödinger equation [1, 2, 3]. Correspondingly, the analogue of the von Neumann equation obtains an unusual superoperator which couples the Hilbert space and its dual [4], cf. Section 2 below.

These features have presented serious obstacles to a number of recent attempts to understand quantum mechanics as an emergent phenomenon [5, 6, 7, 8, 9, 10], see also Refs. [11, 13, 14]. Such studies are strongly motivated by issues surrounding quantum gravity, in particular, the fact that quantum theory is hardly compatible with the symmetry requirements of general relativity or of other theories describing gravity and spacetime, which are based on general coordinate invariance. Furthermore, despite the great successes in explaining statistical aspects of experiments, the intrinsically indeterministic features of quantum mechanics remain problematic. This is seen, for example, in the unresolved measurement problem, with related issues of wave function collapse or objective reduction [15, 16], and in the quantum mechanics of the universe, considered as a whole, which might be self-contradictory.

Yet there is a proof of existence of deterministic models for quantum mechanical objects, in which dissipation, i.e., a fundamental information loss mechanism, has been an essential ingredient [17, 18].

With these findings in mind, we presently discuss aspects of a dynamical transition from classical to quantum behaviour, assuming that: *Quantum features originate from a dissipative process which affects all physical objects. Dynamics and statistical interpretation of quantum states (Born rule) originate from deterministic rules, such as embodied in classical mechanics, in an ensemble theory.*
We speculate that the atomistic structure of spacetime itself is responsible for effects which are attributed to quantum mechanics, typically operating at length scales very much larger than the Planck length. Here “atomistic” refers to a discrete set of elements with the presence, or absence, of a certain order relation between any two elements. Furthermore, this set, in particular the number of its elements changes dynamically, possibly establishing new order relations, or erasing old ones. In this way, “time happens”.

Similar ideas about the nature of spacetime have been formulated every now and then throughout the history of natural philosophy [19, 20]. However, only recently such general scenario has been elaborated in more detail in the theory of causal sets. In mathematical terms, these are locally finite ordered sets. Their evolution by sequential growth through random (“sprinkling”) appearance of new set elements together with their order relations has been studied [20, 21, 22].

In the absence of an equally elaborate theory of matter in relation to such atomistic structure of spacetime, we can nevertheless state the following, concerning the situation of a typical object. Consider an electron, for example, an object that to highest known precision behaves according to the laws of quantum mechanics. We may term its “situation” a hypothetical complete set of its properties that are accessible by experiments.

Now, first of all, there are interactions between such object and its environment (the “rest of the universe”), last not least gravitational ones. Besides more familiar aspects, however, there must exist a continual information loss about its situation, since the atomistic spacetime beneath evolves. With respect to the latter, quantum theory at present deals with very coarse-grained phenomena, when describing the dynamics of matter. Loosely speaking, in order to fully characterize an electron, the evolution of its causal relations with a continually changing number of spacetime “atoms” has to come into play. Consequently, in a coarse-grained picture where this is not explicitly taken into account, information about an object degrades, due to the evolving – always and everywhere present – environment which is spacetime.

Secondly, however, common objects are characterized by a certain persistence, which makes them identifiable in experiments. Therefore, the information loss must be a delicate one. It must be compatible with the conservation of probability, which is a basic tenet of a reasonable ensemble theory.

Contrary to measurement processes in quantum mechanics, where information is transferred from microscopic to macroscopic objects, we propose here that matter degrees of freedom are continually “measured” by the evolution of spacetime.

These heuristic considerations lead us to modify the classical ensemble theory in important ways. We will incorporate dissipation into the Liouville equation, however, in such a way that probability conservation remains intact. This provides us with a glimpse of the mechanism that might be responsible for turning the deterministic evolution of classical objects, described here by an ensemble theory, into the Schrödinger evolution of quantum objects.

2. The Liouville equation in terms of Hilbert space operators

It will be shown here how the classical Liouville equation can be written in the form of the quantum mechanical von Neumann equation, generally, incorporating a characteristic extra term.

We consider objects with a single continuous degree of freedom, for simplicity. To begin with, we consider conservative forces, such that the equations of motion are determined by the generic Hamiltonian function:

$$H(x,p) := \frac{1}{2}p^2 + v(x) ,$$  \hspace{1cm} (1)

---

1 In a way which is not understood in all its aspects in theory. For recent discussions, which critically review related ideas, see Refs. [13, 16].

2 The following is easily repeated for few-body systems or fields.
depending on generalized coordinate \( x \) and momentum \( p \), and where \( v \) denotes the true potential. In Section 3, we will come back to the notion of the true potential and distinguish it from a related coarse-grained potential \( V \).

An ensemble of such objects, for example, following trajectories with different initial conditions, is described by a distribution function \( f \) in phase space, i.e., by the probability \( f(x,p,t)\mathrm{d}x\mathrm{d}p \) to find a member of the ensemble in an infinitesimal volume at point \((x,p)\). This distribution evolves according to the Liouville equation:

\[
- \frac{\partial f}{\partial t} = \frac{\partial H}{\partial p} \cdot \frac{\partial f}{\partial x} - \frac{\partial H}{\partial x} \cdot \frac{\partial f}{\partial p} = \{p \partial_x - v'(x) \partial_p\} f ,
\]  

(2)

with \( v'(x) := \frac{dv(x)}{dx} \). We recall that the relative minus sign in the Poisson bracket, or between terms here, reflects a symplectic phase space symmetry. This will translate into the familiar commutator structure in Eq. (5).

A Fourier transformation, \( f(x,p,t) = \int \mathrm{d}y \, e^{-ipy} f(x,y,t) \), replaces the Liouville equation by:

\[
i \frac{\partial f}{\partial t} = \{- \partial_y \partial_x + yv'(x)\} f ,
\]  

(3)

without changing the notation for the distribution function, whenever changing variables. Thus, momentum is eliminated and a doubled number of coordinates results. Finally, with the transformation:

\[
Q := x + y/2 , \quad q := x - y/2 ,
\]  

(4)

we obtain the Liouville equation in the form:

\[
i \frac{\partial f}{\partial t}(Q,q;t) = \left\{ \hat{H}_Q - \hat{H}_q + \mathcal{E}(Q,q) \right\} f(Q,q;t) ,
\]  

(5)

\[
\hat{H}_\chi := -\frac{1}{2} \partial_\chi^2 + v(\chi) , \quad \text{for} \quad \chi = Q,q ,
\]  

(6)

\[
\mathcal{E}(Q,q) := (Q-q)v'(\frac{Q+q}{2}) - v(Q) + v(q) = -\mathcal{E}(q,Q) .
\]  

(7)

Several comments are in order here:

- The present reformulation of classical dynamics in phase space can be carried out rather independently of the number of degrees of freedom and is applicable to matrix or Grassmann valued variables as well; see, for example, Refs. [6, 23]. Gauge theories or, generally, theories with constraints have to be examined carefully.

- Most importantly, the Eq. (5) closely resembles the von Neumann equation for a density operator \( \hat{f}(t) \), considering \( f(Q,q;t) \) as its matrix elements. We automatically recover the Hamiltonian operator \( \hat{H} \) related to the Hamiltonian function, Eq. (1), as in quantum theory. However, an essential difference consists in the interaction \( \hat{E} \) between bra- and ket-states. The Hilbert space and its dual here are coupled by a superoperator.\(^3\)

- Alternatively, the Eq. (5) might be read as the Schrödinger equation for two identical (sets of) degrees of freedom. However, their respective Hamilton operators, \( \hat{H}_{Q,q} \), contribute with opposite sign, which must be traced to the classical symplectic symmetry. Since their

\(^3\) This superoperator is of a very specific form, which leads to the antisymmetry in Eq. (7). It differs from a Lindblad superoperator, often obtained as a symmetric double commutator structure, in the case of open quantum mechanical systems [24]; this is seen, for example, in our Eq. (23) below.
interaction \( \hat{E} \) is antisymmetric under \( Q \leftrightarrow q \), the complete (Liouville) operator on the right-hand side of Eq. (5) has a symmetric spectrum with respect to zero and, in general, will not be bounded below.

- Finally, the following observation will be important:

  \[
  \hat{E} \equiv 0 \iff \text{true potential } v(x) \text{ is constant, linear, or harmonic.} \tag{8}
  \]

  The analogous vanishing of \( \hat{E} \) in a field theory is equivalent with having massive or massless free fields, with or without external sources, and with or without bilinear couplings. Generally, in all these cases, anharmonic forces or interactions are absent.

  We emphasize that the coupling of the Hilbert space and its dual and the related lacking of a stable ground state, in general, show that our reformulation of Hamiltonian dynamics does not qualify as a proper quantum theory. However, we will motivate certain modifications from which dynamical aspects of quantum mechanics seem to emerge after all.

3. Spatiotemporal discreteness and fluctuations it may cause

Here we present a simple-minded argument indicating that the discreteness of spacetime may be relevant for the emergence of quantum mechanical phenomena, turning the Liouville equation into the von Neumann - Lindblad equation, in particular.

Given that spacetime is discrete, there must be a characteristic length scale, the Planck length, where the continuum description of all phenomena breaks down. This implies that one might overlook important traces of this atomistic structure by employing continuum quantities which allow to arbitrarily extrapolate their functional dependence to scales \( l \approx l_{Pl} \), where \( l_{Pl} \) denotes the Planck length.

Instead of having a \textit{coarse-grained potential} \( V \), for example, the \textit{true potential} \( v \) should become piecewise defined somehow, when approaching smaller and smaller scales in the continuum picture. Thus, the function \( V \) is an approximation to \( v \) and the difference between the two must give rise to local \textit{fluctuations} \( \delta V \). Therefore, we set:

\[
v(x) = V(x) + \delta V(x), \tag{9}
\]

in order to relate the short distance behaviour to its coarse-grained description.

Two sources of fluctuations can enter here: First, the spatiotemporal discreteness \[25\]. Second, the possibly discrete nature of interactions or matter. The latter might go beyond what is usually seen as \textit{effects} of quantum mechanics. This has not been explored in parallel with the causal set theory of the deep structure of spacetime and will not be further discussed at present.

3.1. “Asymptotic freedom” on causal sets, piecewise linear potentials and emergence of the von Neumann equation

However, there is an important “asymptotic freedom” effect, caused by spatiotemporal discreteness: on a (background) causal set, cross sections must fall to zero when the center of mass energy of two scattering particles reaches the Planck scale \[26\].

This is based on

---

4 I thank Rafael Sorkin for the following illuminating description of this “No Interaction Theorem”: Imagine each particle or wave-packet as a “world tube” in Minkowski space. In the center of mass frame, the two tubes make an “X”. The center of the X, where the tubes meet, is the interaction region. Now boost each particle to a very high energy. Because of the Lorentz contraction, the tubes become so flattened that the interaction region shrinks down to less than a Planck volume. Hence there is (very likely) no element of the causet to represent this interaction region, whence there is no interaction.
viewing the causal set as if generated by a Poisson process, “sprinkling” set elements with an average density of one element per Planckian volume, \((l_{Pl})^4\) – for example, into Minkowski space, if this is considered as the continuum limit.

The “asymptotic freedom” effect suggests to consider the true potential function \(v\) as piecewise linear, with the pieces – simplices in higher dimensions – characterized by short “linearity lengths” \(\delta_n\). They must be, however, sufficiently larger than the Planck length, \(\delta_n > l_{Pl}\), such that the continuum description is meaningful.

With these remarks in mind, we reconsider the force term \(\propto v'\) in Eq. (5). We rewrite this equation, admitting more than one spatial dimension, more simply as:

\[
i\partial_t f(Q, q; t) = [\hat{H}_0, \hat{f}](Q, q; t) + (Q - q) \cdot \nabla v(\frac{Q + q}{2}) f(Q, q; t)
\]

(10)

where \(\hat{H}_0 := -\frac{1}{2} \Delta\), with \(\Delta\) denoting the Laplacian, in an obvious generalization. Now, the term involving the derivative of the potential is related to the would-be quantum mechanical term – the potential difference between points \(Q\) and \(q\) – as a linear mid-point approximation to an integral:

\[
(Q - q) \cdot \nabla v(\frac{Q + q}{2}) \approx \int_Q^q ds \cdot \nabla v(s) = v(Q) - v(q).
\]

(11)

The approximate relation becomes an exact equality in the cases covered by observation (8).

More generally, the integral – which yields exactly the quantum mechanical term – can be decomposed into a sum of integrals along straight line segments:

\[
v(Q) - v(q) = \sum_n \int_{s_n}^{s_{n+1}} ds \cdot \nabla v(s) = \sum_n (s_{n+1} - s_n) \cdot \nabla v\left(\frac{s_n + s_{n+1}}{2}\right)
\]

(12)

determined by a set of positions \(\{s_n\}\) which match the piecewise linearity of the true potential with the path of integration from \(q\) to \(Q\), i.e., with the linearity lengths, \(\delta_n = |s_{n+1} - s_n|\).

We conclude that within small volumes, the linear size of which is determined by the linearity length \(\delta\) of the potential there, the Liouville equation of classical statistical mechanics, in the form of Eq. (5) or Eq. (10), is indistinguishable from the corresponding quantum mechanical von Neumann equation, which both refer to the continuum description.

Furthermore, we learn from Eqs. (10)–(12) that for larger distances, covering several linearity lengths \(\delta_n\), the potential (commutator) term of the von Neumann operator is obtained as a sum of classical contributions. This summation is natural in view of the fact that each term adds an appropriate amount of energy to the potential difference \(v(Q) - v(q)\), with one contribution per interval over which the potential is linear.

In this way, we obtain the Hamilton operator for arbitrarily extended, piecewise linear potentials:

\[
\hat{H} = -\frac{1}{2} \partial^2_x + v(x)
\]

(13)

in the coordinate representation, and readily infer the von Neumann equation in operator form:

\[
\partial_t \hat{f} = -i[\hat{H}, \hat{f}]
\]

(14)

However, following the argument from Eq. (10) to Eq. (14), there is a subtlety behind the last two equations here, concerning the composition of the simplices.

Such considerations may deviate from the strategy to adapt continuum quantities to a discrete spacetime – for example, by approximating a quartic potential by a quartic expression with support on the elements of a causal set. It seems a valuable alternative to look for a model of matter degrees of freedom and their interactions which is formulated in terms innate to a causal set, as far as possible. For free particles, work in this direction has been reported in Ref. [27].
In the continuum picture, each simplex forms a tiny “box” with a \textit{sharply localized} boundary condition, which prohibits the leaking of probability to the outside – as long as they are sufficiently separated. Bringing two such boxes next to each other, such that a common border region arises which is less than a Planck length “thick”, the confining boundary condition will dissolve into a free boundary condition, no matter how it originated in the first place. This is another manifestation of the “asymptotic freedom” property, caused by the discrete spacetime structure. Thus, we imagine that the range of variables entering the matrix elements of \( \hat{f} \) can adiabatically increase to the full range over both simplexes.

While this “Gedankenexperiment” might seem plausible, a serious justification of the extension of the range of validity of Eq. (14) over more than one linearity length seems necessary. We intend to come back to this important point elsewhere.

### 3.2. Coarse-graining, decoherence and localization mechanism

In order to extend our considerations further to reach the scales where quantum mechanics is known to work, we have to address also the coarse-graining that must be involved. This cannot be analyzed rigorously without understanding how the common forces / interactions that we encounter relate to phenomena at much shorter distance scales, such as the linearity scale.

However, qualitatively, we expect the coordinates to \textit{fluctuate}, which enter the continuum description, in particular, when we write down the von Neumann equation with the potential terms obtained in Eq. (12). – Indeed, let us consider, for example, an event determined in the continuum picture. The position ascribed to it in a fixed Lorentz frame cannot be exactly maintained with time, if Minkowski space is a \textit{continuum approximation} to an underlying discrete structure: to the future of this event there likely will be Planck scale size (or larger) patches of Minkowski space which contain no element of the approximated causal set. It appears to us that the event, respectively its successors, must change position by small amounts with time (“swerve”), effectively performing a random walk within its future lightcone \cite{27}.

We can get a feeling for the linear size \( \Delta r \) of these swerves by estimating the probability to have a spherical void of volume \( \propto \Delta r^3 \) that lasts for a Planck time interval. For the Poisson process mentioned earlier and with \( \Delta r \gg l_{Pl} \), we find this to be \( e^{-\left(\Delta r/l_{Pl}\right)^3} \). Therefore, uncertainties \( \Delta r \) of the order of a typical distance pertaining to the Standard Model, for example, are extremely improbable. Nevertheless, at the linearity scale they may play an essential role in washing out the piecewise linear potentials.

By experience, smooth polynomial interactions play an essential role, say in the Standard Model and, to be derived from there, in phenomenological forces of physics at still lower energies. Since details of the coarse-graining are unknown, in particular those leading to highly constrained gauge symmetries, we now consider the phenomenological \textit{Ansatz} of Eq. (9), which connects \textit{short distance behaviour at the linearity scale} with its \textit{coarse-grained description}. In particular, this replaces a true, piecewise linear potential \( v \) by a coarse-grained potential \( V \) plus local fluctuations \( \delta V \).

The fluctuations are treated as white noise with mean and correlation, respectively, given by:

\[
\langle \delta V(x) \rangle = 0 ,
\]

\[
\langle \delta V(x) \delta V(y) \rangle = \nu^2(x) \delta(x - y) / \delta(0) ,
\]

where \( \nu(x) \) describes the width of the local distribution of fluctuations. This distribution cannot be assessed without further insight into how forces arise at small spacetime scales.

\footnote{Similarly, there, they will have an effect on the kinetic terms of the Liouville or von Neumann operator; we simply take them as they are, since they yield the appropriate quantum mechanical operator. However, a detailed study of the emergence of such terms from a causal set perspective, of the wave operator in particular, has recently been performed \cite{25}, eventually to be combined with considerations along the lines presented here.}
We now implement Eq. (9), \( v(x) = V(x) + \delta V(x) \), in the von Neumann equation following from Eqs. (10)–(12), with piecewise linear potential. This yields the von Neumann equation with the coarse-grained potential \( V \) and additional fluctuating terms:

\[
\partial_t \hat{f} = -i[\hat{H}_{\text{eff}}, \hat{f}],
\]

(17)

where the effective Hamilton operator is now given by:

\[
\hat{H}_{\text{eff}} := \hat{H} + \delta V
\]

(18)

\[
:= -\frac{1}{2} \partial_x^2 + V(x) + \delta V(x),
\]

(19)

in the coordinate representation, cf. Eq. (14).

In order to evaluate the influence of the fluctuations, we iterate Eq. (17) once:

\[
\partial_t^2 \hat{f} = -i[\hat{H}, \partial_t \hat{f}] + O(\delta V) - [\delta V, \{\hat{f}, \hat{f}\}].
\]

(20)

Integrating once, with the initial condition:

\[
\partial_t \hat{f}|_{t=0} = -i[\hat{H}, \hat{f}]|_{t=0} + O(\delta V),
\]

(21)

and averaging over the fluctuations, with the help of Eqs. (15)–(16), leads to:

\[
\partial_t \hat{f} = -i[\hat{H}, \hat{f}] - \int_0^t dt'[\{\hat{v}^2, \hat{f}\} - 2\hat{v}\hat{f}\hat{v}],
\]

(22)

with the matrix elements \( \langle x|\{\hat{v}^2, \hat{f}\}|y \rangle := \nu^2(x)f(x,y) + f(x,y)\nu^2(y) \) and \( \langle x|\hat{v}\hat{f}\hat{v}|y \rangle := \nu^2(x)f(x,y)\delta(x - y)/\delta(0) \). For sufficiently short times and slowly varying \( \hat{f} \), we may use the approximation:

\[
\partial_t \hat{f} \approx -i[\hat{H}, \hat{f}] - t(\{\hat{v}^2, \hat{f}\} - 2\hat{v}\hat{f}\hat{v}),
\]

(23)

Thus, we arrive at a Markovian master equation with a dissipative \textit{Lindblad term,} in addition to the leading commutator, which is responsible for unitary quantum evolution à la von Neumann in the absence of dissipation.

Linearity in the density matrix is an important feature of our resulting equations. In particular, in the form of Eqs. (22)–(23), the master equation preserves the normalization of \( \hat{f} \), say \( \text{Tr} \hat{f} = 1 \), which expresses the conservation of probability.

It seems worth while to point out that we arrive at the standard, i.e., in particular \textit{local,} quantum mechanical evolution equation – even if modified by a less standard yet welcome Lindblad term. However, we emphasize that the relation to the primordial deterministic degrees of freedom, which involves a Fourier transformation (cf. Section 2), is highly nonlocal. This may ease the tension created by Bell’s theorem, when it comes to deterministic (“hidden”) variables.

The Lindblad term in Eq. (23) implies an interesting \textit{decoherence and continuous localization mechanism,} which causes the decay of spatial superpositions (“Schrödinger cat states”). While the diagonal matrix elements of \( \hat{f} \) are not affected by the Lindblad term, the off-diagonal matrix elements decay:

\[
f(x,y; t) = f(x,y; 0) e^{-\frac{1}{2}t(\nu^2(x)+\nu^2(y))},
\]

(24)

where we neglected the effect of \( \hat{H} \), for simplicity, which cannot stop the decay. Ultimately, spacetime discreteness produces this mechanism – via the induced fluctuations in interactions.

Since there is no theory yet to tell us about the correlation function \( \nu^2 \), it may suffice to point out that considerations of stochastic effects on quantum mechanics – as it is – have quite
a history, see, for example, Refs. [15, 16, 28, 29] and the literature cited there. This is related to attempts to solve the measurement problem and to account for the apparent absence of spatial superposition states of macroscopic objects [30, 31]. Also quantum gravity is conjectured to lead to characteristic stochastic effects [32, 33, 34]. In this context, various proposals for the analogue of our Lindblad operator \( \hat{\nu} \) in Eq. (23) have been made as well and it has similarly been concluded that spatial superposition states decohere and decay.

While these issues are not settled, our heuristic arguments suggest that if quantum mechanical behaviour indeed emerges dynamically, then the resolution of the measurement problem may be linked to this. ‘Prequantum’ dynamics may account for an objective selection mechanism in accordance with the observed wave function collapse in measurement outcomes.

Generally, Lindblad master equations present a large class of linear Markovian master equations, which are usually derived to describe open quantum systems that interact with particular environments [24, 35, 36]. Here spacetime itself forms the universal environment that “measures” all physical objects.

We have derived this quantum mechanical master equation, beginning with classical statistical mechanics, by incorporating several assumptions about the atomistic nature of spacetime, as well as about the nature of forces acting on matter.

4. Conclusions
We have presented a heuristic discussion to the effect that quantum mechanics – together with a natural decoherence and continuous localization mechanism – is a consequence of classical statistics and concerns dynamics with respect to an atomistic spacetime that is observed with low resolution, i.e., at large distance scales. The spatiotemporal discreteness results in permanent information loss affecting all matter, when described in the coarse-grained way that is appropriate with distances much larger than the Planck scale. We were motivated here by the theory of causal sets, where such a picture of spacetime does not follow from a quantization of gravity but is assumed as primary feature.

There are a number of topics for further study. Work is in progress making newly use of the formalism adopted here, of using Hilbert space operators to describe classical statistics, and going beyond the early suggestions [1, 2].

Furthermore, as pointed out in Ref. [4], the problem of negative probabilities needs to be resolved, which is met when carrying the notion of probability density from the classical phase space theory over to the emergent quantum mechanical one, as we do.

The generalization of the ideas sketched here to field theories has to face the fundamental problem to understand the nature of matter fields and the emergence of (the symmetries of) their interactions with decreasing spatiotemporal resolution. Our approach, paying special attention to piecewise linear potentials at sufficiently small scales, is in accordance with the suggestion of noninteracting fields at the Planck scale. There, the atomistic structure of spacetime becomes effective and – for an underlying causal set – induces such an “asymptotic freedom property”.

Acknowledgments
I am grateful to Lajos Diósi, Jose Isidro, Andrei Khrennikov and Rafael Sorkin for discussions or correspondence.

References
[1] Koopman B O 1931 Proc. Nat. Acad. Sci. (USA) 17 315
[2] von Neumann J 1932 Ann. Math. 33 587, 789
[3] ’t Hooft G 2007 J. Phys.: Conf. Ser. 67 012015 (Preprint arXiv:quant-ph/0604008)
[4] Elze H-T 2009 Int. J. Qu. Inf. (IJQI) 7 83-96 (Preprint arXiv:0806.3408); do. 2009 Symmetry aspects in emergent quantum mechanics Proceedings Discrete’08, Valencia (Spain), December 11-16, 2008, J. Phys.: Conf. Ser. to appear
