Ground based 2DoF test for LISA and LISA PF

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Abstract. On-ground tests are required to study the couplings between LISA test masses and the spacecraft that host them. Very interesting and useful results have already been obtained with a 1 DoF torsion pendulum. In order to study couplings that might act between two or more degrees of freedom in measuring the position and acting on the position of each test mass, a many degrees of freedom facility is needed. Here we present a new 2 DoF double torsion pendulum that will be used to test LISA Gravitational Reference Sensor (GRS) on the ground. The facility will be located at INFN Laboratory at Gran Sasso (LNGS), in order to reduce the local ambient noise that limits the sensitivity of the system.

1. Introduction

According to LISA requirements, the residual acceleration noise of the test masses in the free-falling frame must be smaller than $3 \times 10^{-15} \text{m s}^{-2} \text{Hz}^{-1/2} @ 0.1 \text{mHz}$ [1]. The spacecraft that host the test masses are driven by high precision microthrusters to follow the motion of the test masses. The test masses are controlled by capacitive position sensors (GRS, or gravitational reference sensor) that measure the relative distance between spacecraft and test mass. In preparation for LISA, ESA and NASA have planned a risk-reduction mission, LISA Pathfinder, which will test the accuracy of free-fall to (no comma) within one order of magnitude from LISA requirements. In order to make preliminary tests on the behaviour aboard LISA and LISA Pathfinder, development of prototype sensors for the flight model has begun, accompanied by the development of ground based methods to investigate the free-fall condition to the best possible level. Ground tests make use of a
torsion pendulum bench [2], where a lightweight LISA test-mass is suspended by a thin wire (delete comma) that compensates the vertical pull of gravity, and allows rotations along its axis. Rotation is the only soft degree of freedom.

The test mass position and displacement are monitored and controlled by GRS capacitive sensor. The capacitive sensor electrode housing and its electrodes are shown in Figure 1. This test bench allows (delete to) study of the residual acceleration of the test mass along 1 DoF to within a sensitivity two orders of magnitude from the LISA requirement [2], [3], [4].

The limits of the 1 soft DoF torsion pendulum set-up are the sensitivity to environmental noise (such as tilt/twist, where tilt is generated by the local seismic noise coupled to the twist of the thin fiber) [2] and, more important, that it can only test 1DoF. To overcome these limitations, we are following two strategies. First of all, we develop a system which will allow movement of the test mass along 2 soft degrees of freedom: this facility will simulate more closely the interaction between LISA test mass and the host spacecraft. Second, we will reduce the environmental noise, with the choice of a quiet site at INFN Laboratory of Gran Sasso, where we will locate the new test bench.

Figure 1. On the left, the sensor housing. On the right are shown the electrodes that act on each axis; in red are the injection electrodes.

2. The roto-translational pendulum

In the design of a many soft DoF facility, the problem is to compensate for gravity, in order to minimize coupling to the environment, such that the information we can collect from the experiment is significant.

After considering a number of many-DoF geometries (Scott’s linkage, Roberts linkage… [5], [6]), we have chosen a roto-translational pendulum. The 2 sensitive DoFs (see figure 2) are the rotation \( \varphi \) around the central suspension wire, and \( \theta - \varphi \), where \( \theta \) is the rotation around the lateral suspension wire; both \( \varphi \) and \( \theta \) are measured in the laboratory frame. The angle \( \varphi \) translates at the test mass level, to a linear displacement \( y \), in the limit of small angles. With the 2DoF roto-translational pendulum we will be able:

- To measure and control the test mass displacement along 2 DoFs.
- To monitor the effects generated by several sources of noise (thermal, seismic, electrostatic, magnetic, gravitational and thermal gradients), and the induced residual stiffness and cross-stiffness on the system DoF couplings.
- To measure the stiffness and cross-stiffness with closed feedback loops on the test mass position.
- To test actuation cross talk with closed feedback loops: in particular, to measure the residual disturbance along the sensitive translational axis when we close the control loop along the \( \theta - \varphi \) rotation.
- To verify the dc stray voltage compensation technique simultaneously in the two soft DoFs [7], [8].
- To verify the compatibility of the charge measurement by means of a dithering voltage applied in terms of noise induced in \( y \) [7], [8].

Table 1. Characteristics of the GRS [4]: the
minimum value in sensing and the maximum value in actuation are indicated.

|                     | Rotation                          | Translation                       |
|---------------------|----------------------------------|-----------------------------------|
| Sensing             | $<100nrad Hz^{-1/2}$             | $<1nm Hz^{-1/2}$                  |
| Torque              | $5 \cdot 10^{-8}Nm$              | $2.5 \cdot 10^{-6}N$              |

Starting from the translational and angular sensitivities of the GRS, summarized in table 1 ([2], [4]), we derive the parameters of the 2DoF system, with the requirement of keeping it sensitive to forces within two orders of magnitudes from LISA. We first determined the torsion stiffness needed to meet our requirements, that reproduce on each DoF the results obtained for the 1DoF torsion pendulum [3]:

$$\begin{cases} 
K_1(\theta - \varphi)_{\text{min}} \approx 10^{-13} NHz^{-1/2}, \\
\frac{K_2 y_{\text{min}}}{(d/2)^2} \approx 10^{-13} NHz^{-1/2}, 
\end{cases}$$

(1)

Figure 2. Scheme of the roto-translational pendulum: the test mass hangs from a thin wire; three balancing masses are attached to the cross with rigid rods. The 2 soft DoFs, the translation $y$ and the rotation $\theta - \varphi$, are also indicated.

where $(\theta - \varphi)_{\text{min}}, y_{\text{min}}$ are the smallest rotation and translation which the GRS can appreciate (see table 1), and $K_1, K_2$ are, respectively, the stiffness of the lateral and central wire. The torsion stiffness $K$ is given by:

$$K = \frac{g^2 M^2 (F + \Psi Y)}{2L \pi \Psi^2 Y^2},$$

(2)

where $M$ is the total load, $L$ is the length of the wire, $F$ is the rigidity modulus, $Y$ is the yield point, and $\Psi Y$ is the load, as percentage of the yield point. Choosing a working point at the 70% of the yield, we can directly calculate the parameters of the wires:
\[ r_I = \left( \frac{M_I}{M_T} \right)^{1/2} r_J, \quad \quad K_I = \left( \frac{M_I}{M_T} \right)^{1/2} K_J, \]  

(3)

where \( M_I \) is the mass of the test mass, and \( M_T \) is the total mass. We can determine now the minimum length of the arms of the cross. Imposing also a sag smaller than \( 10 \mu m \) for the arms of the cross, we get, for \( W \) wires 1 m long:

\[
\begin{align*}
\frac{d}{2} &\approx 7 \text{ cm} \quad \Rightarrow r_I = 25 \mu m \quad K_I = 6 \cdot 10^{-9} \text{ Nm} \\
M_T &\approx 1 \text{ kg} \quad \Rightarrow r_{II} = 100 \mu m \quad K_{II} = 5 \cdot 10^{-7} \text{ Nm} \quad \Rightarrow f_I = 2 \text{ mHz} \\
\end{align*}
\]

(4)

With this geometry, we checked the thermal noise relevance to our experiment; the mechanical impedance for the 2 DoFs is:

\[
Z_I(\omega) = \frac{1}{i \omega} \left[ (K_I - \omega^2 I_I) + i K_I \phi_I(\omega) \right],
\]

\[
Z_{II}(\omega) = \frac{1}{i \omega} \left[ (K_{II} - \omega^2 I_{II} + m_{\text{TEST}} \frac{d^2}{4}) + i K_{II} \phi_{II}(\omega) \right],
\]

(5)

and from the theorem of fluctuation and dissipation we have for the thermal noise torque power spectral density at the resonance:

\[
\begin{align*}
S_{T_{\text{phi}}}/T_{\text{phi}}^{1/2} &= \left( 4K_B T \frac{I_I}{\tau_I} \right)^{1/2} \approx 10^{-15} \text{ NmHz}^{-1/2}, \\
S_{T_{\text{phi}}}/T_{\text{phi}}^{1/2} &= \left( 4K_B T \frac{I_{II}}{\tau_{II}} \right)^{1/2} \approx 3 \cdot 10^{-14} \text{ NmHz}^{-1/2},
\end{align*}
\]

(6)

**Figure 3.** The facility: final design

**Figure 4.** The model
where \( \tau_{1,n} = \frac{Q}{2.\pi f_{1,n}} \) where \( Q \) is the torsional quality factor of the fibers. We have used \( Q = 1680 \). With the geometry we have chosen, and with our test mass, \( S^{1/2} \tau_p \) translates to an acceleration \( a_p \approx 3 \cdot 10^{-13} \text{ms}^{-2} \text{Hz}^{-1/2} \), within 2 order of magnitude from LISA requirements. While the roto-translation pendulum is under construction (in figure 3 an outline of the facility is shown), we are presently testing some of its features with a scaled model (figure 4).

We are also now implementing a linear control system, starting from the linear equations of motion [9]:

\[
\begin{align*}
\dot{u}_f(s) &= C_f(s)(\theta_f - \varphi_f) - (\theta - \varphi) , \\
\dot{u}_H(s) &= C_H(s)(\varphi_f - \varphi) ,
\end{align*}
\]

that will allow us to control the rotation and the displacement of the test mass. Given the linearity of the equations, it is possible to act directly on \( \theta \) and \( \varphi \), even if our orthogonal coordinates are \( \varphi, \theta - \varphi \).

3. Environmental noise

According to experience [2] environmental noise is a limiting factor to torsion pendulum performance: a quiet site would significantly increase the probability to achieve the best performances and would simplify the efforts needed to reduce the environmental disturbances in term of shielding (temperature, magnetic field) and in terms of active control (temperature, floor tilt), and could result in a better design with a high degree of immunity to Newtonian noise. The most relevant environmental noise sources are:

- **Seismic noise**: microseism of laboratory floor can induce a stray torque on the pendulum through several mechanisms [2]. Any tilt motion of laboratory floor can induce a torque on the pendulum through any position dependent torque induced by the GRS itself, or the effect of linear cross-coupling of suspension point tilt into pendulum twist. Linear microseismic of the suspension point of a torsion pendulum may couple swinging modes to torsional modes. Also vertical microseismic noise can couple to pendulum twist via nonlinearities in the wire response to vertical spring-type modes of the pendulum [2],[7],[10].

- **Gravity gradient noise**: a source of torque noise for a torsion pendulum is the coupling of mass multipole moments of the pendulum to gravity gradient fields. As these effects are related to changing ambient mass distribution, one expects that lower seismic noise will produce a more benign gravitational ambient [11],[12],[13]. Operating the roto-translational pendulum at a quiet site will give us the opportunity to measure the relevance of this source of noise.

- **Temperature**: variations of the temperature and/or temperature gradients of the 2Dof pendulum can generate displacement of the test mass noise at a low frequency. Even temperature variations on a daily timescale can alter the geometry of the building and itself induce a tilt of the ground and gravity gradient noise.

Since the best approach to eliminate environmental disturbances in a measurement requires reducing the effect where it originates, the choice of the experimental site is important and must be made with care. Low microseismic noise and stability in temperature and air pressure on time scales exceeding a day qualify the site for the experiment. The environmental stability of an underground site has been already identified as a promising option for gravitational wave detectors and torsion pendulum for experimental gravity. Laboratori Nazionali del Gran Sasso is a good candidate, if adequate attention is given to isolation from human activity. Temperature and pressure are stable on long timescales, and measurements over many years show that, at low frequencies, tilt microseismic noise is at least 5 times
lower than in a ground floor laboratory [14]. In figure 5 we present the tilt measured along two horizontal axes during a recent site test campaign at LNGS.

![Figure 5. Seismic measurements at LNGS](image-url)

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