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Polarized vortices in optical speckle field: observation of rare polarization singularities

Jan Dupont1,∗ and Xavier Orlik1

1ONERA, Theoretical and Applied Optics Department, 31055 Toulouse, France
*Jan.Dupont@onera.fr

Abstract: Using a recent method able to characterize the polarimetry of a random field with high polarimetric and spatial accuracy even near places of destructive interference, we study polarized optical vortices at a scale below the transverse correlation width of a speckle field. We perform high accuracy polarimetric measurements of known singularities described with an half-integer topological index and we study rare integer index singularities which have, to our knowledge, never been observed in a speckle field.

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1. Introduction

Wave dislocations [1], and the associated phase singularities, are notions which come with diffraction. Indeed, at places where the amplitude of a complex scalar wavefield vanishes, the phase is undetermined. Optical singularities present a growing interest for applications, e.g. metrology for precise displacement measurement [2], or orbital angular momentum modulation for optical communications [3].

Polarization singularities are the vectorial (or topological) counterpart of wave dislocations, and occur in almost all electromagnetic waves [4, 5].

There is a growing interest in better understanding these polarization singularities which seem to be present in a wide range of physics, e.g. molecules arrangement in liquid crystals [6, 7], cells arrangement in formation of biological tissues [8], or in gravitational lensing shear fields [9].

In that paper, we propose the polarimetric study of a speckle field produced by bulk scattering. In order to carry out the polarimetric analysis of polarization singularities in such a field, we use a new method [10] sufficiently sensitive to perform polarimetric measurements near places where wave dislocations occur, i.e. where the intensity vanishes.

Some types of polarization singularities called star, lemon, monstar have been widely studied [11–16], and are described with an half integer index. We point out here high accuracy measurements of these three known types of half integer index singularities in a speckle field. However, we detect other polarimetric structures which exhibit an integer index. Their geometric shapes have already been observed [16] and simulated [17] in maps of instantaneous direction of the transverse electric field, at places where half integer index polarization singularities occur. Here, we detect in a speckle field polarimetric ellipse arrangement along these geometric shapes.

2. Polarimetric analysis by full projection in the Poincaré space

We consider the spatial polarimetric variations of a speckle field in a single transverse plane relatively to the propagation direction. The states of polarization (SOP) are described using the Stokes formalism. We denote two different SOP using the formalism \( S_x, S_y \), while the Stokes components are represented by \( I, Q, U, V \). A classical method used to determine this 2D SOP map consists in the acquisition of 6 intensity images, corresponding to the projection of the field to analyse onto 6 different polarimetric states: linear along the ±45 degrees direction (resp. \( I_{±45} \)), vertically and horizontally polarized (resp. \( I_V, I_H \)), right and left handed circularly polarized (resp. \( I_R, I_L \)). The SOP is then obtained for each pixel of the CCD camera with:
Polarimetric analysis of a speckle field is very challenging. Any wavefront deformation produced by mechanical movement like rotating elements, or non perpendicular incidence on nematic liquid crystals (NLC), will produce spatial intensity variations uncorrelated to the polarimetry. These spatial intensity variations lead to significant errors in the SOP determination if one try to perform polarimetric measurements using the classical method described by Eq. (1), which is based on images subtractions. Moreover, near places of destructive interference, the polarimetric signal is notably affected by noises like dark current or shot noise.

The degree of polarization (DOP) and the polarization ellipse parameters $\alpha$ (azimuth) and $\varepsilon$ (ellipticity), are deduced from the Stokes vector with the following equations:

$$DOP = \frac{\sqrt{Q^2 + U^2 + V^2}}{I}$$

$$\tan(2\alpha) = \frac{U}{Q}$$

$$\sin(2\varepsilon) = \frac{V}{T}$$

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Thus, in order to achieve a better accuracy for the SOP measurements of a speckle field either produced by surface or bulk scattering, we have developed the State Of Polarization Analysis by Full Projection in the Poincaré space (SOPAFP) method [10].

This method consists in taking $n$ intensity images corresponding to projections of the unknown incident SOP ($\vec{S}^\text{in}$) on $n$ distinct states of projection ($\vec{S}_1^\text{out}, \vec{S}_2^\text{out}, \ldots, \vec{S}_n^\text{out}$) performed by two NLCs and a polarizer. These states of projection are chosen regularly spaced on the poincaré sphere surface, and their Stokes components ($Q_{1,\alpha}, U_{1,\alpha}, V_{1,\alpha}$) are calibrated with a 1% accuracy, using a commercial TXP polarimeter from Thorlabs as reference. The signal recorded by each pixel, if we neglect noise sources, is proportional to the scalar products of the unknown incident Stokes vector on the projection Stokes vectors. For the pixel located at the $i^{th}$ line and $j^{th}$ row, and for the $k^{th}$ state of projection, the intensity value is:

$$I(i,j,k) = \frac{1}{2} \vec{S}^\text{in}(i,j) \cdot \vec{S}^\text{out}_k$$

$$= \frac{1}{2} [I^\text{in}(i,j) I^\text{out}_k + Q^\text{in}(i,j) Q^\text{out}_k + U^\text{in}(i,j) U^\text{out}_k + V^\text{in}(i,j) V^\text{out}_k]$$

With $I^\text{out}_k = 1$, if we assume that the intensity only varies in function of the polarimetric projection. As the $\vec{S}^\text{out}_k$ components ($Q^\text{out}_k, U^\text{out}_k, V^\text{out}_k$) are known, by non-linear regression using the $\vec{S}^\text{in}(i,j)$ components ($I^\text{in}(i,j), Q^\text{in}(i,j), U^\text{in}(i,j), V^\text{in}(i,j)$) as adjustable variables in Eq. (5), we deduce the incident Stokes vector for each pixel intensity variation. One can notice that the $\vec{S}^\text{in}$ components are obtained in numerical counts, so we normalize each vector by its intensity
value $I^m(i, j)$. Then, we can compute the DOP and the ellipse parameters with Eqs. (2), (3), (4). We also compute the difference between the best adjusted intensity variation and the experimental one, a low value ensures that the recorded intensity variation is representative of a polarimetric state. Thus, all SOPAFP measurements are obtained pairwise with a 2D trust map.

To resume, the interest of SOPAFP method is that we only extract the polarimetric-dependant part of the experimental signal. Even if it is perturbed by random spatial intensity variations coming from interferences, wavefront variations, or by noises like dark current or non-linearity of the CCD, we are able to find the unique SOP which polarimetric projections are closest to the experimental intensity variation.

Thus, the method is sufficiently sensitive to perform polarimetric analysis of speckle fields either produced by surface or bulk scattering, even near places of destructive interferences.

3. Optical vortices and classification of polarization singularities

The places of destructive interferences in speckle patterns are places where real and imaginary parts of the field equals zero, at these points the phase is locally undetermined. Phase fields exhibit a $2\pi$ symmetry, a charge $k$ is defined: in a closed path around the singularity, the phase will rotate from 0 to $2k\pi$. These structures are called vortices in scalar optics [1], the majority of vortices in speckle fields are of charge $k = 1$, but rarely vortices of charge $k = 2$ have been observed [18].

Space variant polarimetric fields are described with azimuth (Eq. (3)) and ellipticity (Eq. (4)). As we can see in these two equations, they have a $\pi$ rotational symmetry. For example the azimuth is represented by an undirected line rather than a vector.

For these two parameters $\alpha$ and $\varepsilon$, in a single transverse plane the singularities are [5]:

- The C points, where the SOP is circular (right or left handed), and the azimuth can’t be defined.
- The L lines, where the SOP is linear and the ellipticity handedness is not defined. Regions of opposite handedness are separated by L lines.

For a polarization singularity, one can define the polarimetric index $m$: Around the singularity, any state of polarization will repeat itself $2m$ times.

The classical morphology describes the polarization singularities in three different shapes, according to the geometry of the polarization streamlines: lemon, star, monstar. Their properties are described by three parameters, with x, y subscripts indicating the spatial derivatives. The sign of $D_I$ is representative of the index ($m$) of the C points [11]:

$$D_I = Q_x U_y - Q_y U_x$$ (6)

If $D_I < 0$ the singularity is a star, and $m = -\frac{1}{2}$. If $D_I > 0$ the singularity is a lemon or monstar, and $m = \frac{1}{2}$.

The line classification distinguishes lemon from star and monstar by the sign of $D_L$. If $D_L > 0$, the singularity is a lemon, if $D_L < 0$, the singularity is a star or monstar [11]:

$$D_L = [(2Q_y + U_x)^2 - 3U_y (2Q_x - U_y)] [(2Q_y - U_x)^2 + 3U_x (2Q_x + U_y)] - (2Q_x Q_y + Q_y U_x - Q_x U_y + 4U_x U_y)^2$$ (7)

Each of these singularities (lemon, star, monstar) can be of elliptic ($D_C > 0$) or hyperbolic ($D_C < 0$) type [11], depending on the sign of the contour parameter $D_C$.
\[ D_C = (Q_x U_y - Q_y U_x)^2 - (Q_x I_y - Q_y I_x)^2 - (I_x U_y - I_y U_x)^2 \]  

\[ (8) \]

A recently developed method called monstardom classify the half-integer singularities by their coordinates in a parametric space. These two coordinates are the polarization azimuth (Eq. (3)), and the normalized anisotropy index [19]:

\[ \Upsilon = \frac{2(Q_x U_y - Q_y U_x)}{Q_x^2 + Q_y^2 + U_x^2 + U_y^2} \]  

\[ (9) \]

Hajnal was the first to observe these three types of singularities, in the interference field produced by four microwaves beams (Fig. 10(a) in [16]).

4. Experimental results

In the following section, we present our SOPAFP SOP measurements for a speckle field produced by bulk scattering, and more precisely the SOP variations at places where the intensity vanishes.

We used the experimental setup presented in Fig. 1 and detailed in [10], with a laser diode at the wavelength 532nm illuminating a sheet of paper. The output of the diode which is TEM\(_{00}\) is sent through a vertical polarizer and a quarter wave plate to generate a right handed circular illumination SOP. The beam is then focused through a pinhole and colimated by two lenses. Thus, the incident beam on the scatterer is fully polarized, the wavefront is quasi-flat and spatially coherent.

The CCD camera is composed of \(10^3 \times 10^3\) pixels, and the size of the PSF is increased by reducing the pupil diameter with a pinhole of diameter 0.5mm, placed at 12cm of the scatterer and 17cm of the imaging lenses (focal length: 108mm). Thus the surface of a speckle grain is sampled with roughly 200 pixels, and we detect more than \(2.10^3\) speckle grains.

![Fig. 1. Experimental setup used to compute the 2D SOP map of a speckle field produced by bulk scattering. A laser diode of wavelength 532nm illuminates a sheet of paper with right handed circular SOP, obtained with a linear polarizer (P\(_{lin}\)) oriented at 45° with respect to the fast axis of the quarter-wave plate (\(\lambda/4\)). The scattered field is projected on 300 different SOP by two NLCs (CL\(_1\), CL\(_2\)) and a polarizer (P\(_{lin}\)), and the resulting intensity is recorded by a \(10^3 \times 10^3\) pixels CCD camera. The pupil diameter is reduced by a pinhole (P) in order to increase the number of pixels per speckle grain.](image)

We record the scattered intensity field on 300 polarimetric projections, performed by two NLCs and a polarizer. We compute the incident SOP on each pixel of the CCD using the SOPAFP method presented above, then ellipses parameters i.e. azimuth and ellipticity are obtained with Eqs. (3), (4).
The computed intensity map ($I^{\text{in}}$ in Eq. (5)) for an area of 250 × 250 pixels is displayed in Fig. 2(a). In that area, we can see more than 100 speckle grains. The associated azimuthal polarization streamlines are represented in Fig. 2(b), they are lines of slow varying SOP, computed by only taking into account its azimuthal variations. These lines are a guide in order to easily see the geometry of the polarization singularities. Moreover, at each streamlines intersection there is a polarization singularity.

In Fig. 3(a), we can observe two polarization singularities. As an example, the corresponding trust map given by SOPAFP method, as explained in first section, is represented in the inset. For that area, the mean intensity deviation per polarimetric projection varies from 3.5% to 17%, the highest deviation values lying at the center of the singularities. Obviously, we can’t rely on the experimental intensity variations at the center of optical singularities, indeed as the field vanishes, the signal is quasi only noise. Thus one shouldn’t focus on the central SOP, and its associated DOP which is detected to be very low. However, the SOP at the singularities surroundings are characterised with high reliability. As the error map has the same order of magnitude for all the singularities presented in the following of the document, we choose to show the error map corresponding to that area only.

We have calculated the parameters $D_I$, $D_L$, $D_C$, as well as the $[\Upsilon; \alpha]$ coordinates in the monstardom parametric space, corresponding to the singularities of Fig. 3(a), resp. with Eqs. (6)–(8), (3), (9). The one at the right (1) with $D_I < 0$ and $[\Upsilon; \alpha] = [-0.05; -0.56]$ is a star, and the one at the left (2) with $D_L < 0$ and $[\Upsilon; \alpha] = [0.97; -0.43]$ is a lemon. They exhibit a positive $D_C$, so they are of elliptic type. For convenience, we have displayed an area where these two types of singularities are present, outside of the 250 × 250 pixels area displayed in Fig. 2(a), but in the same 1000 × 1000 pixels speckle image. In Fig. 3(b), we have represented the polarimetric ellipses corresponding to the 20 × 20 pixels area marked A in Fig. 2, which are oriented along a monstar singularity, with $[\Upsilon; \alpha] = [0.28; -0.22]$.

We can notice that in a closed path around these three singularities (star, lemon, monstar), any SOP repeat itself only one time. Thus they present a $\pm \frac{1}{2}$ polarimetric index $m$, depending on the sign of $D_I$.

That type of polarization singularities are well-known, and have been predicted [11] and observed [14–16].

If we look at the areas designated by B and C on Fig. 2, we can see the singularities repre-
Fig. 3. (a) SOP variations on a $20 \times 20$ pixels area with 2 singularities, $n^1$ is a star and $n^2$ is a lemon. The ellipse colour is representative of the DOP. Polarization streamlines are superimposed to the polarimetric ellipses. The black stripes represent the areas of left handed ellipticity, the other states are right handed ones. There are L lines at each demarcation between areas of opposite handedness. The inset in the upper-left corner is the error map varying from 3.5% to 17%. (b) The same representation is used for the SOP variations in the area designated by $A$ in Fig. 2, we can see a monstar type singularity.

presented in Fig. 4(a) and Fig. 4(b). Outside of the $250 \times 250$ pixels area displayed in Fig. 2, but in the same $1000 \times 1000$ pixels speckle image, we have detected the singularity represented in Fig. 5.

These three geometric arrangements in Fig. 4(a), Fig. 4(b) and Fig. 5 have been respectively called node, spiral and saddle in [14]. In this last paper, spiral, node and saddle were observed in Stokes fields computed from measurement of half-integer index polarization singularities. Thus, it does not correspond to our measurements as we observe these geometric arrangements directly in the SOP map, but we choose to use the same terminology. Moreover, they have the same geometry than the disclinations found by Hajnal in the direction of the instantaneous transverse electric field [16]. In the same way, the disclinations that Hajnal found was corresponding to half integer index polarization singularities.

In the node, saddle and spiral singularities observed here, any SOP repeats itself 2 times with a central symmetry. Thus, they exhibit a polarimetric index $m$ equal to 1. Otherwise, in a clockwise rotation around the node and spiral type singularities, the ellipse azimuth rotates clockwise, while it rotates anticlockwise around the saddle one. Thus node and spiral have a positive index ($m = 1$), and saddle has negative one ($m = -1$).

Qualitatively, if we look at Fig. 2(b), we can see that the integer index singularities are uncommon compared to half integer index ones. Thus, in order to detect integer index singularities, we have to study a large number of speckle realization.

Pancharatnam has generalized the theory of interference by taking into account the variations of polarisation, which produce a phase shift called topological phase [20]. Thus, by analogy with an optical phase field which exhibits a $2\pi$ symmetry and an optical charge $k$, we can define the topological charge $l = 2m$ for polarimetric fields. Half-integer topological index singularities ($m = \pm \frac{1}{2}$) represent $l = \pm 1$ charge vortex, while integer index ones ($m = \pm 1$) represent $l = \pm 2$ charge vortex. Topological phase (or Pancharatnam-Berry phase) rotates from 0 to $2\pi$ around a $m = \pm \frac{1}{2}$ singularity, and from 0 to $4\pi$ around a $m = \pm 1$ singularity. That effect has already been used by space variant polarimetric elements oriented along a monstar ($m = \frac{1}{2}$) and spiral ($m = 1$) polarization singularity, producing helical beams with a topological charge of respectively $l = 1$ and $l = 2$ [21]. Vortices of charge $k = \pm 2$ are rare in speckle fields [18],
Fig. 4. (a) SOP variations on the 20 × 20 pixels area designated by B in Fig. 2. The ellipse colour is representative of the DOP. Polarization streamlines are superimposed to the polarimetric ellipses. The black stripes represent the areas of left handed ellipticity, the other states are right handed ones. We can see that the ellipses major axis are oriented along lines crossing at the singularity. (b) SOP variations on the 20 × 20 pixels area marked C in Fig. 2. The same representation is used, the ellipses major axis are oriented along a spiral centered on the singularity. Two L lines are crossing close to the center of the singularity, demarcating two areas of opposite handedness.

Fig. 5. SOP variations on a 20 × 20 pixels area, around a saddle type singularity. The ellipses colour is representative of the DOP. Polarization streamlines are superimposed to the polarimetric ellipses. The black stripes distinguish areas of left handed ellipticity from the right handed ones.

this is relevant with our observations: singularities with topological charge \( l = \pm 1 \) are very frequent compared to those with topological charge \( l = \pm 2 \).

5. Conclusion

We have presented our experimental SOP measurements in a random speckle field produced by bulk scattering. Using SOPAFP method, we have achieved a sufficient accuracy to analyse the polarimetric behaviour of optical singularities in such a field. Thus, we have presented high accuracy measurements of the three known polarization singularities: star, lemon, monstar. Moreover, by performing measurements on a large number of speckle grains, we have also provided experimental detection of rare integer index singularities: node, saddle, spiral. To our
knowledge, these integer singularities have never been observed in a speckle field produced by the scattering of an uniformly polarized incident beam.

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