Ab initio calculation of the contact operator contribution in the standard mechanism for neutrinoless double beta decay

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Introduction. The neutrinoless double-β (0νββ) decay is a hypothetical weak process that converts two neutrons into two protons, emitting two electrons but no corresponding antineutrinos. The observation of neutrino oscillations confirmed that neutrinos have non-zero masses, which have boosted interest in experimental searches for 0νββ decay. The observation of this decay would confirm the existence of a Majorana mass term for the neutrinos, shedding light on the mechanism of neutrino mass generation, and providing direct evidence of lepton number violation beyond the standard model—a key ingredient for generating the matter-antimatter asymmetry in the universe. Hence, there is a vast interest in this process with multiple large-scale experiments planned or underway searching for it.

An important ingredient used to find suitable candidate nuclei for the search, and to interpret an observed lifetime is the nuclear matrix element (NME) of neutrinoless double-beta decay, assuming a light Majorana neutrino-exchange mechanism. The corresponding low-energy constant (LEC) is determined by fitting the transition amplitude of the $nn \rightarrow ppe^{-}e^{+}$ process to a recently proposed synthetic datum. We examine the dependence of the amplitude on similarity renormalization group (SRG) scale and chiral expansion order of the nuclear interaction, finding that both dependences can be compensated to a large extent by readjusting the LEC. We evaluate the contribution of both the leading-order contact operator and standard long-range operator to the neutrinoless double-beta decays in the light nuclei $^6$He and the candidate nucleus $^{48}$Ca. Our results provide the first clear demonstration that the contact term enhances the NME by 43(7)% in $^{48}$Ca, where the uncertainty is propagated from the synthetic datum.

In this work, we compute the $nn \rightarrow ppe^{-}e^{+}$ transition amplitude using chiral nucleon-nucleon (NN) interactions. We show that the renormalized transition amplitude is robust with respect to changes in the nuclear interaction, making it a reliable starting point for NME calculations in finite nuclei. In particular, we investigate the change of the contact contribution when subjecting the NN interaction to a similarity renormalization group (SRG) transformation, as well as its dependence on the expansion order of a chiral interaction. Finally, we show that the leading-order contact transition operator enhances the NME by 43(7)% in the lightest 0νββ-decay candidate nucleus $^{48}$Ca compared to the recent ab initio calculations with only the standard long-range transition operator [21]. This finding conveys an important positive message for planning and interpreting future experiments.

The 0νββ decay operators. The central object of our investigation is the 0νββ transition operator in the standard light Majorana neutrino-exchange mechanism. Since the transition amplitude is computed in the $^1S_0$ channel, we restrict the discussion of the operators to that channel, which simplifies the resulting expressions. In particular, the only contributing parts of the neutrino potentials are the Fermi (F) and Gamow-Teller (GT) parts. The tensor part does not contribute in this channel.

The leading-order neutrino potentials in the $^1S_0$ channel are
The couplings contain dipole form factors and are given by

\[ V_F(r) = -\frac{g_{\alpha}^2}{4\pi r}, \]

\[ V_{GT}(r) = -\frac{g_{\alpha}^2}{4\pi r} \left[ 3 - e^{-m_\pi r} \left( 1 + \frac{m_\pi r}{2} \right) \right]. \]

We use the axial coupling constant \( g_\alpha = 1.27 \) and the average pion mass \( m_\pi = 138.039 \text{MeV} \). Phenomenological and higher-order corrections can be incorporated in momentum space,

\[ V_i(r) = \frac{1}{2\pi^2} \int_0^\infty dq \, q^2 h_i(q) j_0(qr), \]

where

\[ h_F(q) = -\frac{g_{\alpha}^2(q)}{q^2}, \]

\[ h_{GT}(q) = -3\frac{g_{\alpha}^2(q)}{4m_N^2} \frac{g_{\alpha}(q)gp(q)}{m_N} - \frac{g_M^2(q)}{2m_N^2}. \]

The couplings contain dipole form factors and are given by

\[ g_V(q) = g_V \left( 1 + \frac{q^2}{\Lambda_V^2} \right)^{-2}, \]

\[ g_{\alpha}(q) = g_{\alpha} \left( 1 + \frac{q^2}{\Lambda_A^2} \right)^{-2}, \]

\[ g_M(q) = (1 + \kappa_1)g_V(q), \]

\[ gp(q) = \frac{2m_N g_{\alpha}(q)}{q^2 + m_\pi^2}. \]

Here, \( m_N = 938.919 \text{MeV} \) is the average nucleon mass, the symbol \( \kappa_1 = 3.7 \) is the isovector anomalous magnetic moment of the nucleon. Following Ref. [29], we choose \( \Lambda_V = 850 \text{ MeV} \) and \( \Lambda_A = 1090 \text{ MeV} / c \) for the vector and axial form factors. These corrections modify the potentials at short range only. At longer range, the potentials are identical to the leading-order ones.

The short-range part needed to renormalize the operator is given by a nonlocally regularized contact interaction,

\[ h_S(p, p') = \left( \frac{m_N g_{\alpha}^2}{4f_\pi^2} \right)^2 \exp\left[ -\left( \frac{p}{\Lambda} \right)^{2\alpha_{\text{cusp}}} \right] \exp\left[ -\left( \frac{p'}{\Lambda} \right)^{2\alpha_{\text{cusp}}} \right], \]

with the pion decay constant \( f_\pi = 92.2 \text{MeV} \). The contact interaction can be expressed in coordinate space as

\[ V_S(r, r') = \left( \frac{m_N g_{\alpha}^2}{4f_\pi^2} \right)^2 f_{\Lambda}^{\alpha_{\text{cusp}}} (r)f_{\Lambda}^{\alpha_{\text{cusp}}} (r'), \]

where

\[ f_{\Lambda}^{\alpha_{\text{cusp}}} (r) = \frac{1}{2\pi^2} \int_0^\infty dq \, q^2 \exp\left[ -\left( \frac{q}{\Lambda} \right)^{2\alpha_{\text{cusp}}} \right] j_0(qr). \]

We choose the prefactor similar to Ref. [25], such that the LEC multiplying the contact term becomes dimensionless and of natural size.

The definition of the nuclear part of the \( nn \to ppe^{-}e^{-} \) transition amplitude compatible with Refs. [26, 27] is

\[ \mathcal{A}(p, p') = 4\pi \langle \hat{T}_0(p') \rangle V_F + V_{GT} - 2\hat{g}_\pi \langle \hat{T}_0(p) \rangle. \]

The wavefunctions \( |\hat{T}_0(p)\rangle \) and \( |\hat{T}_0(p')\rangle \) are scattering solutions for neutrons and protons in the \( ^1S_0 \) channel at incoming and outgoing momenta \( p \) and \( p' \), respectively. Analogously, we define amplitudes \( \mathcal{A}_L(p, p') \) using the neutrino potentials from eq. \( (3) \). In that case, we denote the LEC multiplying the short-range operator by \( \hat{g}_\pi \).

**Scattering wavefunctions.** We compute scattering wavefunctions using the R-matrix formalism [30] with the channel radius set to \( a = 15 \text{ fm} \), well beyond the range of the nuclear potential. The wavefunctions are normalized such that the asymptotic form of the radial wavefunction in the \( ^1S_0 \) channel is

\[ u_p(r) = rR(r) \to \frac{1}{p} \sin[pr + \delta(p)], \]

This normalization recovers \( R(r) = j_0(r) \) as free solution, such that the full plane wave is normalized as \( \phi_p(r) = \exp[ip\cdot r]. \) To be consistent with Refs. [26, 27], we omit the Coulomb interaction from all two-body calculations.

With the wavefunctions obtained from the R-matrix formalism, we compute the long- and short-range parts of the amplitude,

\[ \mathcal{A}_L(p, p') = 4\pi \int_0^\infty dr u_p(r) [V_F(r) + V_{GT}(r)] u_p(r) \]

\[ \mathcal{A}_S(p, p') = 4\pi \int_0^\infty dr' \int_0^\infty dr'' \int_0^\infty dr''' V_S(r, r', r''') u_p(r'), \]

such that

\[ \mathcal{A}(p, p') = \mathcal{A}_L(p, p') - 2g \mathcal{A}_S(p, p'). \]

We obtain the value of the LEC \( g \) by requiring that the total amplitude matches the synthetic datum

\[ \mathcal{A}(p = 25 \text{ MeV} / c, p' = 30 \text{ MeV} / c) = -0.0195(5) \text{ MeV}^{-2} \]

given by Refs. [26, 27]. We validate our calculation of the amplitudes and extraction of the LEC against the results shown in Ref. [27]. See the supplemental material [31] for details.

**Nucleon-nucleon interactions.** For the purpose of this work, we employ three different interactions, all derived from chiral effective field theory. First, we investigate the effect of an SRG transformation on the transition amplitude, employing the N^3LO interaction by Entem and Machleidt [32], which we denote by “EM”. Next, we perform an analysis of the convergence behavior of the amplitude with respect to the chiral order of the interaction. For this, we use the family of interactions from Entem et al. [33], called “EMN” in the following, which provides interactions from LO to N^3LO. Finally, we consider the AN^2LOQO(394) [34] Hamiltonian, a low-cutoff NN+3N interaction whose construction accounts for Δ isobars and whose parameters are constrained by \( A \leq 4 \) few-body data as well as nuclear matter properties. With these
interactions, we make the connection to the \textit{ab initio} calculations of the $0\nu\beta\beta$ NME in light nuclei \cite{15} and the candidate $^{48}\text{Ca}$ \cite{16, 17}.

\textbf{SRG scale dependence.} In order to accelerate convergence of many-body calculations, the nuclear Hamiltonian is usually preprocessed via unitary transformations that reduce the coupling between low and high momenta. One choice is the similarity renormalization group \cite{18, 19, 20}. The continuous unitary SRG transformation introduces a scale $\lambda$ to the Hamiltonian that controls its bandwidth in momentum space. The transformation preserves the eigenvalues of $H$ but changes its eigenstates. Thus, all other observables in principle have to be subject to the same transformation.

Instead of evolving the $0\nu\beta\beta$ operator exactly, we try to absorb the effect of the SRG by readjusting the contact LEC. To this end, we calculate the long and short-range amplitudes at the kinematic point using wavefunctions of the Entem and Machleidt interaction at different SRG scales\footnote{Consistent with the regulator for the LO part of the EM interaction, we set $n_{\text{exp}} = 3$ when regularizing the contact.}.

The results are shown in fig. 1(a). The long-range part of the amplitude (with or without higher-order corrections) shows a very mild dependence on the SRG scale while the short-range part initially changes by 18\%. The change at lower SRG scales is smaller. This confirms the intuition that the SRG mainly affects short-range operators.

The total amplitudes adjusted to the synthetic datum also change by less than a percent over the range of flow parameters shown. Overall, the short-range operator enhances the transition amplitude by approximately 22\% at the kinematic point. The similar momentum dependence, shown in fig. 1(b), implies that the short-range amplitude just acquires a scale-dependent factor $Z(\lambda)$ during the SRG evolution, $A(\lambda) = Z(\lambda)A_{\infty}(\lambda = \infty)$. This scaling factor can be compensated by a change in the LEC, resulting in a total amplitude that is virtually independent of the SRG scale once the LEC has been fixed to the synthetic datum.

\textbf{Convergence of the chiral expansion.} Next, we consider the dependence of the $0\nu\beta\beta$ amplitude on the order of the chiral interaction employed. To this end we use the EMN family of interactions from LO up to N$^3$LO \cite{15}.

We consider the full amplitude as a function of incoming and outgoing relative momenta for different chiral orders. For incoming momenta up to 375 MeV/$c$, the range up to which the potentials are fitted, we notice a sizable dependence on the chiral order, which is shown in fig. 2(a). The total amplitude computed with the LO interaction drops by more than 60\% compared to N$^3$LO, but systematically converges to the N$^3$LO result with increasing order. The variation in the low-momentum region [cf. fig. 2(b)] is less than 1\% and also rapidly converging.

Finally, we investigate the effect of including beyond-LO terms into the $0\nu\beta\beta$ operator by employing the neutrino potentials from eq. \cite{3}. The phenomenological corrections added there only modify the potential at short range. At distances $r > 1.5$ fm they are virtually indistinguishable from the LO...
ones. Since we use NN interactions with relatively low cut-offs, the total amplitudes are fairly insensitive to the short-range modifications. The relative difference between them is below 0.5 % for momenta within the range of applicability of the respective NN interaction. For the LO NN interaction the difference may reach 3 % at incoming momenta exceeding 300 MeV/c [cf. fig. 2(c)]. The difference between both amplitudes at low momenta, shown in fig. 2(d), is negligible. Table I shows the long-range amplitudes and LECs $A, g$ and $\tilde{A}, \tilde{g}$ associated with the LO long-range operator and its extension, respectively.

**Application to finite nuclei.** While suitable for generating the synthetic datum, a scattering state of neutrons is not ideal for observing $0\nu\beta\beta$ decay in experiment. For that, we need to move to finite nuclei for which the single-$\beta$ decay is energetically forbidden. Due to the long lifetime any competing decay would drown out the $0\nu\beta\beta$-decay signal. A few candidate nuclei that fulfill this requirement have been identified, some of which can even be used to build an active detector.

Previous calculations of the NME in finite nuclei only considered the long-range part of the operator. With the LEC of the short-range part of the operator adjusted to the synthetic datum, we can now calculate its effect and provide a first result renormalized to leading order. Here, we revisit our benchmark calculations for light nuclei [35], as well as the candidate pair $^{48}\text{Ca}$ and $^{48}\text{Ti}$ [21]. The interaction used in these studies is the so-called EM1.8/2.0 [39], which consists of the EM interaction SRG-evolved to a scale $\lambda = 1.8$ fm$^{-1}$ augmented by an unevolved N$^2$LO three-nucleon interaction. To estimate the dependence of the NME on SRG scale and chiral order, we additionally consider Hamiltonians based on the EM interaction with a local-nonlocal 3N force [40], called “LNL” here, one that combines the EMN N$^2$LO with an N$^3$LO 3N interaction [41] (designated there as N$^3$LO$^*$), and the AN$^2$LO$_{CO}(394)$ NN+3N Hamiltonian. The LECs for each of the NN interactions are shown in table I.

![FIG. 2.](image-url)

The NME for finite nuclei is defined as

$$M^{0\nu} = \frac{4\pi R}{g_A^2} (\langle Z + 2 | \hat{V}_F + \hat{V}_{GT} + \hat{V}_T - 2 \hat{g} \hat{\bar{\psi}}_s | V^A Z \rangle, \quad (17)$$

with the empirical nuclear radius $R = R_0 A^{1/3}$ and $R_0 = 1.2$ fm. The operator $\hat{V}_T$ contains the tensor part of the decay operator. With this definition, $M^{0\nu}$ is dimensionless.

First, we investigate the NME in the pairs of light nuclei $^6\text{He} - ^8\text{Be}$ and $^8\text{He} - ^8\text{Be}$ as examples of $\Delta T = 0$ and $\Delta T = 2$ transitions with the importance-truncated no-core shell model (IT-NCSM) [42]. The results are summarized in fig. 3. We note that the contact operator increases the NME by a factor ranging from 11 % to 17 % for $\Delta T = 0$ transition in $^6\text{He}$. Transitions with $\Delta T = 2$ have a node in the transition density that leads to a cancellation between short and long distances. This cancellation affects the long-range part more strongly than the contact, leading to small overall NMEs and relatively larger contributions of the contact term. Thus, the contact increases the $\Delta T = 2$ transition in $^8\text{He}$ by 92 % to 172 %. Overall, SRG-transforming the AN$^2$LO$_{CO}$ as well as switching to the LNL Hamiltonian barely changes the NME.

Despite using the same NN interaction at a similar SRG scale as the LNL, the EM1.8/2.0 produces systematically smaller NMEs than the other interactions. The EMN + N$^3$LO$^*$ Hamiltonian yields a smaller NME in $^6\text{He}$ than the LNL while the $^8\text{He}$ NME is larger. Both are driven by the long-range part, the short-range contribution is of similar size compared to the LNL Hamiltonian. This shows that there is still some uncertainty stemming from the Hamiltonian, in particular the 3N interaction, which needs to be quantified further.

For the lightest $0\nu\beta\beta$-decay candidate nucleus $^{48}\text{Ca}$, the short-range operator increases the NME by 43(7) %. With this contribution, the value of $M^{0\nu}$ is 0.875(40) for $^{48}\text{Ca}$ from the in-medium generator coordinate method (IM-GCM) [21] calculation, the uncertainty of which is from the LEC $\tilde{g}$ of the short-range transition operator.

**Conclusions and outlook.** In this work, we present a determination of the LEC of a contact operator that enters the $0\nu\beta\beta$ operator at leading order for a set of chiral interactions, which are used in *ab initio* calculations of nuclei. To fix the LEC, we take the synthetic datum provided by Cirigliano et al. [26, 27], which contains the effect of light Majorana-neutrino exchange. We investigate the dependence of the $nn \to pp ee^-$ amplitude on the SRG scale and order of the interaction. We find that a change in the SRG scale can be compensated by readjusting the LEC, leading to only very small changes in the total amplitude. The dependence on the order of the interaction can be sizable for low-order interactions at high momenta beyond the range applicability of
FIG. 3. The NMEs $M^{\nu\beta\phi}$ of isospin-conserving ($\Delta T = 0$) transition $^8{\text{Be}} \rightarrow ^6{\text{Be}}$, and isospin-nonconserving ($\Delta T = 2$) transitions $^3{\text{He}} \rightarrow ^3{\text{Be}}$ and $^{40}{\text{Ca}} \rightarrow ^{48}{\text{Ti}}$, calculated with different chiral nuclear forces and with both long- and short-range transition operators.

the respective interaction. However, the total amplitude converges quickly when going beyond N$^2$LO over the full momentum range to which the potential is fitted. The robustness of the amplitude shows that the two-body system is under control and any changes in the momentum dependence will come from subleading terms in the operator. Moreover, the changes will likely be small because including beyond-LO terms in the long-range part barely changes the result, apart from a change in the LEC.

The contact operator turns out to increase significantly the NME of isospin-changing transition in finite nuclei. For the lightest candidate nucleus $^{48}{\text{Ca}}$, the NME is enhanced by $43(7)$ %. This enhancement is also found in the ab initio calculations of light nuclei $^{6,8}{\text{He}}$ using the three families of chiral interactions with a low-scale regulator. It indicates that the contact operator will generally enhance the NMEs predicted by ab initio many-body calculations using these interactions and this effect should be taken into account in the future ab initio calculations. The extension of current studies to the NMEs of $0\nu\beta\beta$ decays in heavier candidate nuclei is highly interesting.

We note that the present work relies on the synthetic datum, the uncertainty of which is dominated by neglected inelastic contributions, and hopefully is to be reduced in a future lattice QCD calculation. Nevertheless, apart from the total NMEs all the findings presented here are independent of the concrete value of the synthetic datum. The availability of a more precise datum will just result in a shift of the total amplitudes, and we provide separate short- and long-range parts to enable matching to an updated value.

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The dimensionless LEC $C$ to the neutron-proton scattering length.

Supplemental Material

Computational details. Since the long-range neutrino potential exhibits $1/r$ behavior and contributes at distances $r \gg a$, we split the amplitude integral at the $R$-matrix channel radius. In the exterior region we use the asymptotic form of the wavefunction. This approximation is exact in the $1S_0$ channel without Coulomb interaction because the free solutions of the radial Schrödinger equation are proportional to sine and cosine functions. For channels with higher angular momentum or ones that include the Coulomb interaction the difference between the full and asymptotic forms is negligible for our choice of the channel radius $a = 15$ fm.

Validation. We use a leading-order isospin-symmetric potential to verify our calculation against the results shown in [27]. In the $1S_0$ channel, the momentum-space form of this potential reads

$$V_{LO}(p, p') = \frac{1}{2\pi^2} \frac{g^2}{4f^2_s} \left( C - \frac{m^2}{4pp'} \log \left[ 1 + \frac{4pp'}{m^2 + (p - p')^2} \right] \right) \times \exp \left[ -\left( \frac{p}{\Lambda} \right)^{2n_{exp}} \right] \exp \left[ -\left( \frac{p'}{\Lambda} \right)^{2n_{exp}} \right].$$

(18)

The dimensionless LEC $C$ is adjusted to reproduce the neutron–proton scattering length $a_{np} = -23.74$ fm.

To validate our calculations, we generate a set of interactions with cutoffs $\Lambda$ from 2 fm$^{-1}$ to 20 fm$^{-1}$, setting the regulator exponent to $n_{exp} = 2$ and 4. The phase shifts up to a relative momentum of 200 MeV/c are shown in fig. 4. It is shown that the phase shifts are only weakly dependent on the regulator cutoffs in the low-momentum region with $p < 50$ MeV/c, consistent with the findings in Ref. [25].

Figure 5 shows the nucleon-nucleon wavefunction for different regulator cutoffs. At short ranges the wavefunctions exhibit a clear cutoff dependence, rising more quickly for higher cutoffs. Beyond a relative distance of 3 fm they all collapse to the same curve, because the phase shift at this momentum is approximately cutoff independent.

With the scattering wavefunctions calculated from this set of interactions, we compute the long- and short-range amplitudes at the kinematic point $p = 25$ MeV/c, $p' = 30$ MeV/c [26,27], shown in fig. 6(a). The long-range part shows a logarithmic dependence on the regulator cutoff $\Lambda$, while the combination $C^2A_s$, computed using the same regulator parameters as the interaction, is virtually independent of it.

By requiring that the total amplitude matches the synthetic datum, this implies that the ratio of LECs $g/C^2$ exhibits the same logarithmic scale dependence. The dependence is shown in fig. 6(b). There is a small discrepancy between our LECs and the ones taken from Ref. [27] for $n_{exp} = 2$, which might be attributable to the different solution methods for the scattering problem or the choice of momentum- or coordinate-space grids. The discrepancy becomes smaller with increasing regulator power $n_{exp}$, in particular for the values $n_{exp} = \{3, 4\}$ used in the remainder of this work.
Order-by-order momentum dependence. Figure 7 shows the amplitudes for the different chiral orders. The long-range part of the amplitude converges quickly beyond LO: the variation between orders over the momentum range shown is less than 3%, and it decreases when going to higher orders. The short-range part shows a much larger variation of 40%, but that dependence is again order-dependent scaling factor that can be absorbed into the LEC. Thus, the total amplitude is essentially converged at leading order for low momenta.

The long-range part of the LO interaction is larger in magnitude than the higher-order results. This difference is because the LO interaction breaks charge symmetry with significantly different \( n \) and \( p \) scattering lengths (even without the Coulomb interaction).

Amplitude tables. Table I and table II show a compilation of the long- and short-range amplitudes at the kinematic point \( p = 25 \text{ MeV/c} \), \( p' = 30 \text{ MeV/c} \) for the EMN and \( \Delta N^2 \text{LO}_{\text{GQ}} \) interactions. The amplitudes are computed at different chiral orders (for the EMN) and various SRG scales. The LECs \( g, \tilde{g} \) are obtained by imposing the synthetic datum \( \tilde{A} = -0.195(5) \text{ MeV}^{-2} \). The LEC uncertainty \( \Delta g \) reflects the uncertainty in the synthetic datum and is the same for \( g \) and \( \tilde{g} \).

NMEs for finite nuclei. Figure 8 shows the dependence of the \( ^8\text{He} \rightarrow ^8\text{Be} \) NME on the contact LEC \( \tilde{g} \) from the IT-NCSM calculation. The value of the contact is similar for the different interactions employed and indicated by the vertical band. The contact term is slightly larger for the LNL Hamiltonian than for the others, leading to a steeper dependence on \( \tilde{g} \). The total NMEs \( M_{L+S}^{\text{by}} \) are very similar, except that the curve for the EMN(2.0) is shifted up due to a larger long-range part. A similar plot is shown in fig. 9 showing the \( ^{48}\text{Ca} \) NME for the EM1.8/2.0 Hamiltonian with \( e_{\text{Max}} = 8 \), and \( h\omega = 16 \text{ MeV} \) from the IM-GCM calculation [21]. Here, the long- and short-range parts are separated. The blue line shows the LEC dependence of the short-range part, whose intersection with the vertical band, showing the uncertainty in \( \tilde{g} \), yields an uncertainty band (green) for the contact contribution. The dashed line marks the contribution of the long-range part, and the gray band shows their sum. The detailed values of the NME for \( ^8\text{He} \rightarrow ^6\text{Be}, ^8\text{He} \rightarrow ^8\text{Be} \) and \( ^{48}\text{Ca} \rightarrow ^{48}\text{Ti} \) are given in Table IV. It is shown that the NMEs by different interactions overlap with each other.
TABLE II. Amplitudes and LECs for the EMN family of interactions at different orders of fm$^{-1}$, amplitudes in units of MeV$^{-2}$. The contact term is regularized using $\Lambda = 500$ MeV/c and $n_{\exp} = 3$. The quantities with a tilde incorporate beyond-LO effects in the operator.

| Order | $\lambda$ | $10^3A_L$ | $10^3A_L$ | $10^3A_L$ | $\tilde{g}$ | $g$ | $\Delta g$ |
|-------|-----------|------------|------------|------------|----------|-----|---------|
| LO    | $\infty$  | -17.653    | -18.034    | 8.8484     | 0.109    | 0.086| 0.029   |
|       | 2.50      | -17.440    | -17.752    | 7.6803     | 0.135    | 0.115| 0.033   |
|       | 2.24      | -17.353    | -17.643    | 7.2423     | 0.148    | 0.128| 0.035   |
|       | 2.20      | -17.338    | -17.625    | 7.1811     | 0.151    | 0.131| 0.035   |
|       | 2.00      | -17.241    | -17.508    | 6.7840     | 0.166    | 0.147| 0.037   |
|       | 1.88      | -17.168    | -17.421    | 6.4956     | 0.180    | 0.160| 0.038   |
|       | 1.80      | -17.111    | -17.354    | 6.2779     | 0.190    | 0.171| 0.040   |
| NLO   | $\infty$  | -15.951    | -16.020    | 2.6403     | 0.672    | 0.659| 0.095   |
|       | 2.50      | -16.005    | -16.111    | 2.9479     | 0.593    | 0.575| 0.085   |
|       | 2.24      | -16.006    | -16.117    | 2.9982     | 0.583    | 0.564| 0.083   |
|       | 2.20      | -16.005    | -16.117    | 3.0032     | 0.582    | 0.563| 0.083   |
|       | 2.00      | -15.994    | -16.108    | 3.0146     | 0.582    | 0.563| 0.083   |
|       | 1.88      | -15.980    | -16.094    | 3.0033     | 0.586    | 0.567| 0.083   |
|       | 1.80      | -15.966    | -16.080    | 2.9855     | 0.592    | 0.573| 0.084   |
| N$^3$LO | $\infty$ | -16.020    | -16.131    | 3.3533     | 0.519    | 0.502| 0.075   |
|       | 2.50      | -16.007    | -16.132    | 3.3458     | 0.522    | 0.503| 0.075   |
|       | 2.24      | -15.991    | -16.117    | 3.3136     | 0.529    | 0.510| 0.075   |
|       | 2.20      | -15.988    | -16.114    | 3.3067     | 0.531    | 0.512| 0.076   |
|       | 2.00      | -15.964    | -16.088    | 3.2524     | 0.544    | 0.524| 0.077   |
|       | 1.88      | -15.942    | -16.065    | 3.2027     | 0.555    | 0.536| 0.078   |
|       | 1.80      | -15.923    | -16.045    | 3.1600     | 0.566    | 0.547| 0.079   |
| N$^4$LO | $\infty$ | -15.857    | -15.903    | 2.8386     | 0.765    | 0.755| 0.105   |
|       | 2.50      | -15.931    | -16.026    | 2.7781     | 0.642    | 0.625| 0.090   |
|       | 2.24      | -15.939    | -16.043    | 2.8565     | 0.623    | 0.605| 0.088   |
|       | 2.20      | -15.939    | -16.044    | 2.8658     | 0.621    | 0.603| 0.087   |
|       | 2.00      | -15.934    | -16.043    | 2.9031     | 0.614    | 0.595| 0.086   |
|       | 1.88      | -15.924    | -16.034    | 2.9078     | 0.615    | 0.596| 0.086   |
|       | 1.80      | -15.913    | -16.023    | 2.9005     | 0.618    | 0.599| 0.086   |

TABLE III. Same as table II but for the $\Delta N^2 LO_{GO}(394)$ interaction. The contact term is regularized using $\Lambda = 394$ MeV/c and $n_{\exp} = 4$.

| Decay | Interaction | $\lambda$ (fm$^{-1}$) | $M^\nu_{L}$ | $M^\nu_{S}/2\tilde{g}$ | $M^\nu_{LIS}$ |
|-------|-------------|------------------------|-------------|------------------------|--------------|
| $^6$He $\rightarrow$ $^6$Be | EM | 1.8 | 4.017 | 0.426 | [4.468, 4.613] |
|       | EMN | 2.0 | 4.101 | 0.510 | [4.640, 4.888] |
|       | LNL | 2.0 | 4.218 | 0.538 | [4.788, 4.971] |
|       | $\Delta N^2 LO_{GO}$ | 2.0 | 4.340 | 0.538 | [4.927, 5.107] |
|       | $\Delta N^2 LO_{GO}$ | $\infty$ | 4.221 | 0.506 | [4.732, 4.894] |
| $^4$He $\rightarrow$ $^4$Be | EM | 1.8 | 0.129 | 0.158 | [0.297, 0.350] |
|       | EMN | 2.0 | 0.225 | 0.166 | [0.401, 0.458] |
|       | LNL | 2.0 | 0.165 | 0.176 | [0.352, 0.412] |
|       | $\Delta N^2 LO_{GO}$ | 2.0 | 0.176 | 0.158 | [0.349, 0.402] |
|       | $\Delta N^2 LO_{GO}$ | $\infty$ | 0.168 | 0.157 | [0.327, 0.377] |

| Decay | Interaction | $\lambda$ (fm$^{-1}$) | $M^\nu_{L}$ | $M^\nu_{S}/2\tilde{g}$ | $M^\nu_{LIS}$ |
|-------|-------------|------------------------|-------------|------------------------|--------------|
| $^{48}$Ca $\rightarrow$ $^{48}$Ti | EM ($\epsilon_{\text{Max}} = 6$) | 1.8 | 1.03 | 0.357 | [1.407, 1.529] |
|       | EM ($\epsilon_{\text{Max}} = 8$) | 1.8 | 0.78 | 0.281 | [1.077, 1.173] |
|       | EM ($\epsilon_{\text{Max}} = 10$) | 1.8 | 0.66 | 0.231 | [0.905, 0.983] |
|       | EM(extra.) | 1.8 | 0.61 | - | [0.836, 0.915] |