1. Introduction

The current economic conditions require for the rail transport of Ukraine an improved efficiency of freight haulage and an ensured sustainable use of resources [1]. The reduction in the traction of the rolling stock fleet and the lack of new locomotives’ replenishment at the JSC Ukrainian Railways (Ukrzaliznytsia, Kyiv, Ukraine) have made the choice of schemes of locomotive circulation in the railway network one of the most important tasks in the railcar traffic organization. The performance efficiency of the working fleet of freight trains’ locomotives in the network predetermines the ability of the JSC Ukrainian Railways to perform the planned haulage volumes under the existing resource constraints [2–4].

The available schemes of providing trains with locomotives and servicing the former by locomotive crews in railway networks are aimed at shorter traction hauls, which were historically used for diesel traction. However, the topology of the railway network of Ukraine is changing, and its intensive electrification [5] has fostered a substantial increase in the traffic volumes and destinations. This requires theoretically grounded approaches to solving the problem of a quick search for rational schemes of locomotive circuity and locomotive crews’ circulation at branched operating sites of the railway network in accordance with the train schedule and the actual operating conditions. The existing approaches to servicing trains with locomotives largely involve the traction haul method at short distances between the nearest maintenance stations. This method is more efficient for servicing local train flows. At the same time, carrying through trains along their routes by a relay method, with stops for re-hooking the locomotives at each technical station, causes undue delays, increases the time of goods delivery, and reduces the competitiveness of Ukrainian railways on the market of transit traffic [6]. Under such conditions, in recent years, there has spread the practice of extending the traction hauls of locomotives with a selection of the required series and types of traction for a more efficient handling of each freight flow. Innovative analytical decision-making under conditions of large-scale computations is necessary on the basis of a combination of different methods of maintaining locomotives and locomotive crews with a simultaneous search for options of traction hauls’ lengthening, efficient train driving...
and increased speed combined with the operation of various locomotive series, technologies of trains’ handling at technical stations, as well as changes in the weight and length of trains within a single network site.

Thus, the task of improving the methods for determining the schemes of locomotives’ circulation at the tactical level of planning with account of the technological features of handling individual railcar flows is quite topical.

2. Literature review and problem statement

The task of selecting schemes of locomotives’ circulation in the railway network is addressed in many studies at all stages of developing the railway transportation system of Ukraine. Past research [7, 8] confirms the complexity of solving this problem due to the many stages and interrelations of the various steps in the process of transportation planning. According to the existing guidance documents on the railways of Ukraine [9], the stage of determining station boundaries for servicing with locomotives and by locomotive crews takes place after developing the plan of rolling stock formation, and it is the starting point in scheduling the movement of trains. This stage is based on technical-economic expert calculations of the recommended distances of rational section lengths for rotation of locomotives, defined for the most common conditions. This approach is flawed because it does not solve the problem comprehensively by making calculations for operating a network site; there has not been any detailed consideration of the nature of railway traffic, and the estimates are not processed automatically yet. Thus, all known studies of Ukrainian railways or their analogues are aimed only at eliminating the above-mentioned drawbacks [10–13].

In study [12], an approach is suggested to solve the problem of choosing the schemes and parameters of traction provision for freight trains in the rail network on the basis of formulating a complex of optimization problems that are solved sequentially, with their results being interrelated. Although this approach automates calculations, the developed methodical approaches and algorithms of forming competitive options for traction systems of freight trains are based on a subjective expert assessment, which reduces the accuracy and speed of solving the problem in comparison with a simultaneous solution of optimization problems contained within a single model of using known optimization methods.

Many studies attempt to solve the problem of choosing the schemes of locomotive circulation at the level of tactical planning for the construction of the transportation system based on schedules of trains. Moreover, this problem is solved to develop strategic plans for rail networks. As an example, in [10, 11], research is described for choosing competitive options in determining the boundaries of locomotives’ circulation with regard to the scope of movement that has become adopted at the tenth year of the site operation. On balance, the variable parameters of the problem are the location and the power of management devices for locomotive maintenance, repair of locomotives, and locomotive crews’ deployment. However, at the level of tactical planning, these parameters are unchanged, which should be taken into account when choosing the schemes of locomotives’ circulation for servicing railcar traffic through an individual approach, on the basis of considering technological peculiarities.

Solutions of such problems for rail systems that differ from the principles of operating railway transport in Ukraine are described in studies on the locomotive planning problem or the locomotive scheduling problem and the crew scheduling problem [14–16]. For example, in [12], the locomotive planning problem for U.S. railways is solved by suggesting an optimization mathematical model that is formulated as a task of mixed-integer programming, MIP. A large-scale practical problem is solved by using a model that contains about 197,000 integer variables and 67 thousand constraints. The results for the railway company CSX Transportation, which operates a fleet of more than 3,000 locomotives, make it possible to achieve savings of more than 400 locomotives, which is more than a hundred million dollars a year. However, it should be noted that although this model does not include any restrictions on the maintenance of locomotives, the results confirm the practical effectiveness of solving problems to improve the methods of determining schemes of freight locomotives’ circulation.

Similar problems are solved in [17, 18].

In [19], the research is devoted to solving the task of linking locomotives to trains in terms of how a transport operator performs rail transportation in the Czech Republic. In addition to planning the operation of specific locomotives, the developed mathematical model allows for the efficient use of some locomotive combinations of other companies. In [20], the task of allocating locomotives according to the schedule of trains of various categories is formulated as a problem of routing vehicles through time periods. This problem is solved by using a hybrid genetic algorithm, which has proved to be efficient in terms of quality and time in comparison with the classical method. A successful application of the genetic algorithm is also described in [21]. This confirms the promising use of optimization methods that are based on an evolutionary search in solving problems of planning locomotive operations.

The problem of developing a schedule of locomotive crews’ operation concerns research on devising their work schedule and planning their work modes [22, 23]. The downside of such calculations is the lack of a complex solution in determining the working plan for locomotives and locomotive crews. It should be noted that there are studies of these tasks for the railways of Ukraine or their analogues [4, 24]. These problems are solved during the operational planning phase when the trains’ schedule is already known. Such calculations are usually very detailed, but they do not solve the problem of choosing the schemes of locomotives’ circulation at the level of tactical planning when the schedule of trains is not available.

The above analysis shows that no railway network in Ukraine functions according to the holistic approach to formalizing the procedures for selecting the circulation schemes of locomotives and locomotive crews. Moreover, almost no research has been yet developed on designing circulation schemes of locomotives to address individual applications for individual railcar traffic, particularly in terms of the right to a private traction. It necessitates research in this area, which would be the basis for the automation of planning processes.

3. The purpose and objectives of the study

The research is aimed at improving the methods of determining the schemes of locomotives’ circulation in the
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railway network of Ukraine under the accelerated handling of individual railcar traffic in view of its technological peculiarities.

To achieve this goal, it is necessary to do the following tasks:

– to develop a mathematical model to determine a rational scheme of locomotives’ circulation within a railway network with the possibility of finding the weight of trains on the railcar traffic route, the circulation schemes for locomotives with regard to deploying the fleet of different series at a network site, and the scheme of locomotive crews’ operation;

– to improve the efficiency of solving the existing problems by the developed mathematical model of choosing the schemes of locomotives’ circulation in the railway network due to the optimization method that is based on a genetic algorithm.

4. Improving the method for determining the schemes of locomotives’ circulation with regard to the technological peculiarities of railcar traffic

4. 1. The development of a mathematical model of determining a rational scheme of locomotives’ circulation in a railway network

To increase competitiveness in the market of transit transportation, the JSC Ukrainian Railways needs to improve its strategic railway directions (international corridors) by using an acceleration technology in the traffic handling process for certain specialized railcar traffic. Under such conditions, traffic schemes for traction resources are built not in a general conjunction with all types of railcar traffic but for various specific railcar-traffic groups. The analysis of the practices of independent shipping companies in the railways worldwide allows making a conclusion that when such companies can choose a route for their own railcar traffic in the network with the possibility of traction provided by locomotive depots of different types of ownership, this individual approach to the choice of traction supply for various railcar traffic under the condition of private traction and transportation rights will become increasingly used for traffic on the Ukrainian railways. Thus, the present study suggests formalizing this approach at the level of tactical planning.

To solve the outlined problem, it is necessary to present a railway network as an undirected graph \( G(V,E) \), where \( V \) is a set of vertices that conform to the technical stations of the origin and fading of railcar traffic at the network site, \( v \in V, v=\{1,...,n\} \). \( E \) is a set of edges that denotes a railway station between two maintenance stations, \( e \in E, e=\{1, ... , b\} \). Let us assume that a \( \text{SCD}_k \) denotes the value of a railcar traffic \( k (k=\{1,...,K\}) \) that has been determined in advance according to the development of a plan of forming freight trains. Each railcar flow \( k \) is characterized by a fixed route \( \mu_k \), which is a simple chain of vertices in the graph \( G \) between the stations of the origin \( v=u \) and fading \( v=t \) of a railcar traffic at the site of the railway network, \( u, t \in V \). Fig. 1 shows an arbitrary site of a rail network as a graph \( G(V,E) \) and a direction diagram for railcar flows.

Depending on the role in providing traction service, technical train stations are divided into stations with the main depot, with a turnover depot or a site of locomotives’ turnover, and with a change point for locomotive crews \( [10] \). To display the spatial availability of stations with fixed and transit locomotive depots, the set \( V \) can be divided into the following subsets: vertices \( V_1 \), which simulate those stations that have the main locomotive depot \( V_0 \subset V \); vertices \( V_2 \), which denote those stations that allow a turnover of locomotives \( V_2 \subset V \); and vertices \( V_3 \), which reflect those stations where there is a point for changing locomotive crews \( V_3 \subset V \).

Let us assume that \( V_1 \), \( V_2 \) denotes a set of all possible stations on the route of a railcar flow \( k \), where there is the main locomotive depot, \( \nu_k \in V_0 \). Then \( V_1 \) can denote a plurality of stations on the route of the railcar flow \( k \), where it is possible to have a turnover of a locomotive, \( \nu_k \in V_2 \). To simplify the perception, all the options can be marked with the index \( j \). This indexing allows writing the number of the main depot station as \( i \), where \( i \in V_0 \). Then all possible hauls for locomotive servicing of trains on the route \( \mu_k \) can be described by the parameter \( a^k_{ij} \in \Lambda \), where

\[
a^k_{ij} = \begin{cases} 1, & \text{if the depot } i \text{ provides with the haul } j \text{ to the station } \nu_k^j; \\
0, & \text{otherwise}. \end{cases}
\]

An example of the graphical representation of the incidence matrix \( A \) for the railcar flow route \( \text{SCD}_{k=4} \) in the form of an undirected graph is shown in Fig. 2.

Let us introduce the variable \( X^k_s \), which will simulate the presence of the main depot station \( i, i \in V_1 \) and the haul \( j \) for the movement of the railcar traffic \( k \) on the route \( \mu_k \), where \( \nu_k \in \{1,0\} \). The main condition for choosing the haul for servicing locomotives is the presence of all stations on the route for consistent progress of the railcar traffic. To formalize this condition, it is necessary to write a matrix \( B, B^k \in B \), whose elements reflect what stations \( s \in S \) on the railcar traffic route \( k \) belong to the traction haul \( j \), where

\[
b^k_{ij} = \begin{cases} 1, & \text{includes the station } s; \\
0, & \text{otherwise}. \end{cases}
\]

Thus, the logical condition under which the service zone includes traction hauls of all stations on the route \( \mu_k \) of the railcar traffic can be written as follows:

\[
\sum_j a^k_{ij} X^k_s = b^k_{ij}.
\]
The choice of a traction haul from the station with the main locomotive depot is always limited by the series of an assigned fleet of freight train locomotives (for example, VL82m, VL11, 2EL4, etc.), which is essential for calculations. To simulate the series of a locomotive \( p = \{1, \ldots, P\} \), which is used in the haul \( j \) of the railcar traffic \( k \), we suggest introducing the following variable:

\[
Y^{bp}_{jp} = \begin{cases} 
1, & \text{if the locomotive of the series } p \text{ is used in the haul } j; \\
0, & \text{otherwise.}
\end{cases}
\]  

As part of determining the task, it is necessary to comply with a logical condition under which the haul should be serviced only with one series of freight locomotives:

\[
\sum_{p} X^{bp}_{jp} = 1.
\]  

The formalization of constructing a service circuitry of traction hauls for locomotive crews requires that the model is supplemented with the third variable \( Z^{kg}_{r} \in \{1,0\} \), which allows simulating the inclusion of the traction traffic \( k \) of the traction haul \( j \) into the circulation of locomotive crews with a haul \( g \), \( g = T, G \). It is important to comply with the condition under which the hauls of locomotive crews’ operation \( g \) with the inclusion of each locomotive haul should cover all stations \( s \) on the route \( \mu_s \):

\[
\sum_{j} Z^{kg}_{r} b_{s,j}^{kp} = 1.
\]  

To solve the complex problem of traction, it is necessary to determine the weight standard for a freight train of each railcar flow \( k \) to specify the speed, the rolling stock length, the number of trains on a railway route and, consequently, the arrangement of traction servicing of railcar traffic. Under such conditions, the mathematical model in the present study should also include a fourth variable \( Q^{gr}_{k} \), which denotes the gross weight of the whole train for the railcar traffic \( k \) to roll. To reduce the options for sorting, it is necessary to choose a limited set of gross weight options \( Q^{gr}_{k} \in \{Q^{gr}_{1}, Q^{gr}_{2}, \ldots, Q^{gr}_{r}\} \), taking into account the technical and technological constraints on the route followed by the relevant railcar traffic volume \( k \). For example, on the route of the railcar traffic \( SCD_{k-4} \) (Fig. 2), the traction support and the restrictions on the length of the admitting-dispatching tracks allow the following variations in the rolling stock weight \( Q^{str}_{kp} \in \{4600 m, 5200 m, 6000 m, 6300 m\} \). Moreover, some of the options allow using double traction at some stations \( s \) on the route. To take account of this technological feature, the model also includes an incidence matrix \( C \), \( c_{s,j,q}^{k} \in \mathbb{C} \), where

\[
c_{s,j,q}^{k} = \begin{cases} 
1, & \text{if the haul } j \text{ of the station } s \text{ has a double traction at } Q_{kp}; \\
0, & \text{otherwise.}
\end{cases}
\]

Given that a traction arrangement is built for each individual railcar flow \( k \), the costs of moving the rolling stock and performing station operations can be calculated only for the route direction of a particular railcar flow; which makes it possible to fully evaluate the effectiveness of the selected option and to simplify calculations. Refusal to calculate costs for the opposite direction can be explained by the immutability of these costs, as the locomotive fleet, after the main technological process of transporting, can be used for servicing local traffic volumes \( N^{m} \), which in turn will cover the cost of its turnover in the predetermined pattern. However, it is important for the calculations to take into account the condition of a deviation from the constant costs in the reverse (unloaded) direction due to the presence of expenses for a reserve run of locomotives in the absence of the required number of local trains for a specific number of locomotives returning from the turn-around sites after servicing the railcar traffic \( k \) and after sending such locomotives to return-point sites. The described peculiarity is important to consider when choosing a rational scheme of servicing the traction at a site of a railway network.

Taking into account the above-described conditions, the target function of the mathematical model to determine a rational scheme of using traction resources in a railway network can be implicitly written as follows:

\[
F(Q^{k}_{kp}, X^{bp}_{jp}, Y^{bp}_{jp}, Z^{kg}_{r}) = \sum_{k} \left[ SCD_{k} \sum_{i} X^{bp}_{jp} \sum_{s} b_{s,j}^{kp} \left( \sum_{p} Y^{bp}_{jp} \left( E^{m}_{ij,p} + E^{c}_{ij,k,p} \right) \right) + \sum_{s,j,k} Z^{kg}_{r} E^{c}_{ij,k,rev} + E^{c}_{ij,k,rev} \right] + \sum_{m} S_{ij,m} E^{TMD}_{ij,k} + \min, \tag{4}
\]

where \( SCD_{k} \) is the number of threads in the train schedule for admitting a train of the railcar flow \( k \); \( \phi_{i} \) is the ratio of the average net weight to the gross weight of a train on the route of the railcar flow \( k \); \( E^{m}_{ij,p} \) is the cost of traction for a train of the railcar traffic \( k \) at a station \( s \); \( UAH; E^{c}_{ij,k,p}, E^{c}_{ij,k} \) are the daily operating costs to satisfy the need for, respectively, locomotives and locomotive crews to service one train of the railcar traffic \( k \) on the traction haul \( j \) of the station \( s \); \( UAH; E^{rev}_{ij,k} \) is the cost of a multiple traction of a train of the railcar traffic \( k \) at the station \( s \) with a selected weight of the train \( Q_{kp} \); \( UAH; E^{m}_{ij} \) is the cost of the stops made by a train weighing \( Q_{kp} \) due to being outdriven and intersected by trains of a higher priority and other categories; \( UAH; E^{TMD}_{ij,m} \) is the cost of labour of maintenance crews at traction maintenance depots (TMDs) \( m \), \( m = \{M\} \), where there are
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stops on each traction haul $j$ of the route $μ_k$ to serve trains of the railcar flow $k$, UAH; $E_{rev,loc}$ is the daily service cost of a reverse reserve run of locomotives at the station $s$ of the traction haul $j$ for the railcar traffic $k$, UAH; $δ_{m}^{st}$ is a supporting Boolean function to model the presence of a stop to maintain a train at the maintenance station $m$, meV for changing locomotives or locomotive crews $δ_{m}^{st} \in \{0, 1\}$.

Function (4) can explicitly be written as follows:

$$F(Q^{gr}_{jk}, X^{gr}_{jk}, Y^{up}_{jk}, Z^{up}_{jk}) =$$

$$= \sum_{s} SCD_{s} \Phi_{Q^{gr}_{jk}} \sum_{j} \sum_{r} \left[ \sum_{s} \left( \sum_{l} \left( Y^{up}_{jk} \left( 1 - Q^{gr}_{jk} \right) + 24 \cdot K^{inbound}_{max} \cdot c_{loc} \right) + 24 \cdot \sum_{r} Z^{up}_{jk} \cdot K^{inbound}_{inertia} \cdot c_{inertia} + 24 \cdot c^{k}_{jk} \cdot \left( K^{inbound}_{inertia} \cdot c_{inertia} + K^{inbound}_{inertia} \cdot c_{inertia} \right) + k_{s}^{stop}(Q^{gr}_{jk}) \left( t_{lock} \cdot c_{lock} + c^{stop}_{jk} \right) \right] + \sum_{i} \delta_{i}^{st} \cdot S_{TMD}^{m} \cdot c_{TMD} \cdot c_{TMD} \right]$$

$$+ \sum_{j} X^{up}_{jk} \sum_{s} \left( \max \left( 0, \left( SCD_{s} \Phi_{Q^{gr}_{jk}} - N^{st} \right) \right) \right) b_{jk}^{st} \cdot c_{rev, loc} \cdot L^{st} \right] \rightarrow \min,$$

where $e^{k}_{jk}(Q^{gr}_{jk})$ is the cost of traction for a train that weighs $Q^{gr}_{jk}$, has a locomotive of a $p$-series, and moves in the freight hauling direction at the $s$-th station (is determined through traction calculations), UAH; $K^{inbound}_{inertia}$ is the station factor of the need for locomotives per one train in the freight hauling direction $k$, which is measured in locomotives per day and is suggested to be determined as half a factor of the need for locomotives to serve a couple of trains.

$$K^{inbound}_{inertia} = 0.5 \cdot K^{inbound} = 0.5 \cdot \frac{\theta_{inbound}}{24},$$

where $\theta_{inbound}$ is the local circulation of a locomotive through each depot on the traction haul $j$, hours; $K^{inbound}$ is the station factor of the need for locomotive crews per one train in the freight hauling direction $k$ in terms of locomotive crews per day; $K^{inbound}_{inertia}$, $K^{inbound}_{inertia}$ is the factor of a need for locomotives and locomotive crews to service one train in the freight hauling direction $k$, which is respectively measured in the number of locomotives and locomotive crews per day; $c_{loc}$ is the cost of one locomotive-hour (excluding investment in the locomotive fleet depot), UAH; $c_{inertia}$ is the cost of one crew-hour, UAH; $c_{inertia}$, and $c_{inertia}$ are the costs of, respectively, one locomotive-hour and one crew-hour of a multiple traction, UAH; $S_{TMD}^{m}$ is the number of TMD crews at a maintenance station $m$; $e^{in}_{TMD}$ is the cost of keeping one TM crew in the admitting depot of a maintenance station $m$, UAH; $e^{in}_{TMD}$ is the number of passenger train stops at the station $s$, caused by handling a train with a weight of $Q^{gr}_{jk}$. $c_{lock}$ is the cost of one train-hour of a passenger train, hours; $c^{stop}_{jk}$ is the cost of a passenger train stop while handling the railcar traffic $k$, UAH; $c_{rev, loc}$ is the cost of a locomotive-km of a reserve run at the station $s$ [15], UAH; $L^{st}$ is the length of the station $s$, km.

In addition to the logical constraints (1)-(3) and the objective function (5), the complete formulation of the mathematical model should also include technological restrictions in determining a rational arrangement of traction resources in a railway network:

1. a limit on the maximum distance of extending traction hauls for locomotives, which is predetermined by the established standards for the frequency of maintenance operations MO-2 for each series of traction rolling stock:

$$X^{up}_{jk} \sum_{s} b^{st}_{jk} \leq L^{max},$$

where $\sum_{s} b^{st}_{jk}$ is the length of the traction haul $j$, km;

$$L^{max}_{st} = V^{st}_{mo} \left( T^{mo}_{2} - \sum_{i} t_{s}^{i} \right)$$

is the locomotive run between MO-2, km; $V^{st}_{mo}$ is the local speed of a train of the railcar traffic $k$, km/h; $T^{mo}_{2}$ is the frequency of maintenance operations MO-2 for a traction rolling stock of a $p$-series, $T^{mo}_{2} \in [24+72$ hours$]$; $t_{s}^{i}$ is the average total time of a locomotive staying at turn-over sites, hours;

2. a limit on the maximum possible distance between the points of changing locomotive crews:

$$X^{up}_{jk} \sum_{s} b^{st}_{jk} \leq L^{max}_{st},$$

where $L^{max}_{st}$ is the actual haul for a locomotive crew operation under a selected mode of servicing trains, km; $L^{max}_{st} = t_{term} \cdot V^{st}_{mo} \cdot t_{term}$, where $t_{term}$ is the permissible time of train driver's work under the labour laws of Ukraine, which should not exceed 12 hours according to the order 40-C of March 10, 1994; $V^{st}_{mo}$ is the average service speed through a station on a given traction haul, km/h;

3. a limit on the size of the operational fleet of locomotives of each $p$-series, which is assigned to the main depot station:

$$\sum X^{up}_{jk} \sum_{s} b^{st}_{jk} \leq N^{p}_{t},$$

where $M^{p}_{t}$ is the operational fleet of locomotives of the $p$-series, stationed at the main station $i$ that has the main depot, loc.;

4. a limit on the effective traffic capacity $N^{t}_{t}^{p}$ for handling the railcar flow at each station $s$ on the route $μ_k$ [27]:

$$\sum X^{up}_{jk} b^{st}_{jk} \leq N^{p}_{t},$$

Thus, we have developed a mathematical model that allows determining the weight of trains on railcar traffic routes, locomotive circulation schemes with regard to the fleet deployment for various series within a network site, as well as locomotive crews' work arrangements in terms of the existing technical and technological possibilities of a locomotive facility and the railway infrastructure management.

4.2. The results of using the mathematical model to determine the rational schemes of locomotives' circulation based on a genetic algorithm with an integer coding

The above-described mathematical model with the objective function (5) and limitations (1)-(3) and (6)-(9) in
general is a set coverage problem [27] in which a set of all possible options of traction and predetermined train weight is used to choose an optimal subset so that every railway station on the route of a given railcar traffic \( k \) could belong to a haul of maintaining locomotives of a particular series by relevant locomotive crews. Even in general terms, the coverage problem refers to the NP-complete complex type [28], and so far there has been no polynomial algorithm to solve it. Therefore, this study suggests using an approximate method of solution on the basis of a genetic algorithm (GA) [29].

The genetic algorithm belongs to applies heuristic methods of problem-solving, and the main difficulty in using it is that it is necessary to construct a chromosome coding system to reflect variables of mathematical models and, consequently, the result of solving a given task. To solve the mathematical model developed in the present study, we assume that the chromosome \( H_h \) is represented by a combination of genes consisting of \( K \) parts of \( H_h=(SG_1, SG_2, SG_3, ..., SG_K) \) each of which simulates traction support for the \( k \)-th railcar traffic, \( k=1, K \). Each \( SG_k \) part of the chromosome is a genetic combination of the following type:

\[
SG_k = \left\{ h_{i_1}^{d_1}, h_{i_2}^{d_2}, ..., h_{i_d}^{d_k}, h_{g_1}^{d_{g_1}}, h_{g_2}^{d_{g_2}}, ..., h_{g_v}^{d_{g_v}}, h_{p_1}^{d_{p_1}}, h_{p_2}^{d_{p_2}}, ..., h_{p_{d_{p}}}^{d_{p_{d_{p}}}} \right\}
\]

where \( h_i^d \) is a \( d \)-th gene that models an option of the code of implementing the traction haul \( j \) at a station with the main locomotive depot \( i \); \( h_i^d \) is a \( d \)-th gene in the first part of the chromosome that corresponds to the station index \( v_i \) on the route of the railcar traffic \( k \) and makes it possible only to turn the locomotive around; \( d=1, D \), where \( D \) is the number of stations with the main and transfer depots on the route \( \mu_k \). The combination of genes is written by a rule of a consistent recording of stations in the route direction of the railcar flow \( k \) under the developed rules that are decoded here below. The gene \( h_i^d \) takes the value of \( h_i^d = \{1,2,3\} \), where the value of 1 is suggested to be decoded as a sign of a station with the main locomotive depot \( i \) of an arrangement that begins the traction haul \( j \); 2 denotes that a station has the main locomotive depot \( i \) of an arrangement that ends (turns around) the traction haul \( j \); 3 denotes that a station has the main locomotive depot \( i \) of an arrangement that begins and ends (turns around) the traction haul \( j \) and \( j+1 \). The gene \( h_i^d \) takes the values of \( h_i^d = \{1,2\} \), which are decoded as follows: if the value is 1, the station \( v_i^d \) does not have the reversion of the locomotive haul \( j \), but if the value is 2, there is such a turnaround. It should be noted that if the decoding for each station sequentially eliminates those options of the traction haul system that logically do not coincide with the beginning of the current arrangement at the final station, the rules entail the end of the traction haul to lock the current scheme of the locomotive circulation.

The second part – \( SG_k \) is a coding of locomotive series that are used on the traction hauls \( j \). Thus, for each station \( i \) with the main locomotive depot, \( h_{g_i}^{d_{g_i}} \) is a gene that models the locomotive series \( p \) for every possible initial schemes of the traction hauls \( j \) (the number of the destinations from the station \( i \) coincides with the number of \( d \)-indexed genes). The set of the genes’ values \( h_{g_i} = \{1,...,p\} \) is different for each station with the main locomotive depot, depending on the locomotive series \( p \) stationed at them.

The part \( SG_k \) simulates the presence or absence of turn-over points for locomotive crews at each station \( g_i \) of \( h_{g_i} = \{1,0\} \). The service circuity of locomotive crews is decoded sequentially for each built traction haul \( j \) within which there are stations \( g \) for which the gene has taken the value of 1, so they are perceived as stations for crew changes within this haul. In other cases, the haul of the locomotive crews’ operations coincides with the locomotive traction haul.

The last set of genes of the part \( SG_k \) models the option of the accepted weight norm for a rolling stock \( h_{p_i} = \{1, r,.., R\} \) within each railcar flow \( k \). For a practical implementation of the suggested approach, the MATLAB environment is supplied with a software code, using standard genetic algorithm functions with integer variables [30]. The problem solution results are visualized in a random graph \( G \) shown in Fig. 3.

![Fig. 3. The results of solving the developed mathematical model to determine a rational arrangement of locomotives’ circulation in the railway network: a][]{image}

The fitness function of the GA produces the above-described decoding process; the model variables

\[
(Q_{ij}, X_{ij}, Y_{ij}, Z_{ij}, w_{ij})
\]

are converted to calculate the target function (5). The model limitations (1)–(3) and (6)–(9) are estimated by an approach based on an unconditional optimization in which each of the restrictions is assessed through penalty functions. For example, Fig. 4 shows the result of the values of the chromosome part \( HG_k \) and its decoding rule after solving the task of providing traction at an arbitrary site of the graph \( G \) (Fig. 1).

![Fig. 4. The results of solving the problem in the arbitrary graph G, using a genetic algorithm with its own system of coding variables, were subjected to expert evaluation of the accuracy of developing the circuity for locomotives and locomotive crews, which has confirmed the adequacy of the developed mathematical model.](image)
6. Discussion of the results of improving the method for determining the circuitry of locomotives in view of technological peculiarities of railcar traffic

The results of this study are based on a mathematical model that has been implemented by using a genetic algorithm with integer encoding. We have proved that it is possible to determine simultaneously the normative gross weight of trains and circuitry of locomotives and locomotive crews, taking into account the effect of the existing technical and technological limitations on the technology of handling railcar traffic. According to all previously known results, including those published in [7, 8, 10, 11], the set of the above-mentioned parameters for providing traction in the railway network were determined only gradually; making it impossible to obtain comprehensive results that would reflect the interdependence of these parameters. Moreover, previous studies were devoted to constructing schemes of locomotives’ circulation in general, for all types of railcar traffic, whereas the present study determines the locomotive circuitry on the basis of an individual approach to choosing traction support for various types of railcar traffic. Taking into account technical peculiarities is one of the main features of the formulated method.

Given that the presented task is formulated as a coverage problem, which belongs to the NP-complete type, our optimization method of the problem solving involves a genetic algorithm, which is known to have been efficiently used in many studies [31, 32]. It should be noted that there have been studies [16, 19, 33] devoted to solving the problem of scheduling locomotives’ work at the operational level. Although the problem solved there differs from the one that is raised in the present study, it is similar in the complexity, and although the results are also different, examples are given of using methods for solving large-scale problems. The absence of any previous example of using our suggested method to solve large-scale problems does not underestimate the results that we have obtained because our main goal was to achieve a solution that could be double-tested by means of an expert assessment of the results’ adequacy. However, since real problems are of a large scope, it is still furthermore necessary to test the effectiveness of the developed optimization method for problems of the large-scale type.

The practical significance of the study is in improving the suggested mathematical model so that it could be used in software automation of the process of identifying locomotive circuitry, taking into account technological peculiarities of railcar traffic that requires special conditions of handling in the railway network of the JSC Ukrainian Railways. Moreover, the suggested improvement of the method of determining the scheme of locomotives’ circulation with regard to the technological features of railcar traffic makes it possible to estimate the necessary traffic traction supply by independent shipping companies in view of a further reform of the railway industry in Ukraine. The results of the study are preliminary. It is still necessary to develop a formulation of the mathematical model that would take into account the existence of different locomotive fleet owners in the railway transportation network. Further research is also essential for proving the possibility of using the suggested algorithm for solving large-scale problems.

7. Conclusions

1. A mathematical model has been formed to determine locomotives’ circuitry, taking into account a complex set of technical and technological factors inherent in the transportation process in the railway network in Ukraine. This allows automation of the planning process for handling separate freight flows on the basis of an individual approach to choosing the traction supply at the tactical level of railway transportation.

2. The optimization problem of choosing the schemes of locomotives’ circulation has been solved by using an integer genetic algorithm with its own system of coding the variables of the mathematical model. The results confirm the adequacy of the developed mathematical model. The implemented mathematical model on the basis of the integer genetic algorithm can be useful for improving the operations of the railway network of Ukraine because it can help automate the complex process of determining the schemes of locomotives’ circulation with regard to the technological peculiarities of railcar traffic and, consequently, improve the accuracy and speed of decision-making for servicing individual applications for route transportation of freight.
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