Does A Narrow Tetraquark $cc\bar{u}\bar{d}$ State Exist?

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Abstract

The existence of a shallow or virtual tetraquark state, $cc\bar{u}\bar{d}$, is discussed. Using the putative masses for the doubly charmed baryons ($ccu/ccd$) from SELEX, the mass of the $cc\bar{u}\bar{d}$ state is estimated to be about $3.9\, GeV$, only slightly above the $DD^*$ threshold. The experimental signatures for various $cc\bar{u}\bar{d}$ masses are also discussed.
I. INTRODUCTION

Multiquark exotic hadrons different from the ordinary mesons or baryons have been discussed and searched for many years [1,2,3,4,5,6,7,8]. Motivated by the possible discovery of doubly charmed $ccar{u}/ccar{d}$ baryons, [9], we reconsider the $ccar{u}ar{d}$ tetraquark—the doubly charmed exotic state. We find that such a $J^P = 1^+$ state is likely to exist below or near the $D^*D$ thresholds, and may be the first stable/narrow exotic state to be discovered.

II. $QQ'\bar{q}\bar{q'}$ STATES IN THE HEAVY QUARK LIMIT

Let $T(QQ'\bar{q}\bar{q'})$ denote a putative tetraquark state (it is denoted as $X_c$ in [1]) consisting of two heavy quarks—$Q, Q' = c$ or $b$—and two light quarks—$q, q' = u, d$. We are interested in genuine four quark “one bag states” with a “connected color network”. Figure 1 indicates one such state where $Q_\alpha Q'_{\beta}$ are combined via $\epsilon_{\alpha\beta\gamma} Q_\alpha Q'_{\beta}$ into $(\bar{3})_\gamma$ color diquark, and $q_\mu q'_\nu$ make an $\epsilon_{\mu\nu\gamma} \bar{q}_\mu \bar{q'}_\nu = (3)_\gamma$—an anti-diquark. The $\bar{3}$ and $3$ combine to give an overall color singlet state. Note that according to the well-known arguments, [13,14], in the large $N_c$ limit such a four-quark state is unstable against a decay into the two mesons, $D$ and $D^*$ for instance. Alternatively, however, one can assume that the state $T(cc\bar{u}\bar{d})$ in the large $N_c$ limit corresponds to the state containing $N_c - 1$ heavy quarks, which combine into $\bar{N}_c$ representation of $SU(N_c)$ color, and $N_c - 1$ light quarks combined into $N_c$. According to the standard large $N_c$ counting rules the binding energy of such a $(2N_c - 2)$-exotic state, [13,14], is of order $O(N_c^0)$, i.e. of the same order as the meson masses.

The identity $\epsilon_{\alpha\beta\gamma} \epsilon_{\sigma\tau\gamma} = \delta_{\alpha\sigma} \delta_{\beta\tau} - \delta_{\alpha\tau} \delta_{\beta\sigma}$ allows one to express the above tetraquark state as a superposition of two meson states, $(Q \bar{q})$ etc., which are separately color singlets.

$$T\{QQ' \rightarrow \bar{3}, \bar{q} \bar{q'} \rightarrow 3\} = |(Q \bar{q})_1 (Q' \bar{q'})_1\rangle - |(Q \bar{q'})_1 (Q' \bar{q})_1\rangle.$$  \hspace{1cm} (1)

1 Much effort was devoted in the past to search for hexaquark and pentaquark states [10,11,12].
FIG. 1. The color coupling pattern adopted here. Two charm quarks $c_1, c_2$ couple antisymmetrically to an intermediate $\bar{3}$ and $\bar{u}, \bar{d}$ couple to $3$.

If the $Q Q'$ (and $\bar{q} q'$) colors are coupled symmetrically to $\bar{6}$ (and $6$) the analog of the Eq. (1) will have a plus relative sign.

In a “string picture” the chromoelectric fluxes are squeezed into thin “vortices” connecting the various (anti)quarks and/or junction points. In this case the various lines in Fig. 1 describe not only the color coupling but also the actual layout of the strings. The transition between the tetraquark (color connected—“one-bag” state) to the two-meson state can be pictorially described by shrinking the string bit connecting the two junctions, and then annihilating them via the above $\epsilon \cdot \epsilon$ contraction (Fig. 4). This naively would suggest the two-meson state, in which the above string bit has been eliminated, is lighter than the tetraquark state rendering the latter unstable since $T(Q Q' \bar{q} q') \rightarrow Q \bar{q} + Q' \bar{q}'$ would be kinematically allowed.

However, the naive string picture may not apply to the ground state hadron considered here. Indeed we can directly show that $T(Q Q' \bar{q} q')$ is stable in the heavy quark limit ($m_Q, m_{Q'} \rightarrow \infty$). The $Q$ and $Q'$ would then bind into a $\bar{3}$ via the perturbative one gluon exchange. The essentially coulombic interaction yields a binding energy $\mathcal{O}(\alpha_s^2) m_Q / 2$ (for $m_Q = m_{Q'}$). Once $m_Q$ is sufficiently large this binding exceeds hadronic energy scales and possible bindings in the heavy-light $Q \bar{q}$ mesonic systems. This then ensures the stability against decay into two such mesons.

Unfortunately for this mechanism to generate stable tetraquarks $m_Q$ (much) larger than $(m_c) m_b$ is required. Detailed, and to some extent model dependent, considerations are thus
FIG. 2. The color flux vortices and their evolution as the tetraquark state separates into two $D + D^*$ states.

required to motivate a $cc \bar{u} \bar{d}$ state near or below $D^* D$ threshold. An alternative approach, which can directly address the stability of the physical $1^+ (cc \bar{u} \bar{d})$ state against a decay into $DD^*$ or $D^*D^*$, starts with these charmed mesons and attempts to form bound states via the potential due to light mesons, particularly one-pion exchange. Following Törnqvist, we will call such “deutron like” or “molecular” bound states of two mesons, “deusons”. The ranges and strengths of such potentials are independent of $m_Q$ since $DD^* \pi$ or $D^*D^* \pi$ couplings, for instance, depend only on the light quark degrees of freedom. Thus, if the two heavy mesons are attracted by these potentials, binding is again guaranteed in the heavy quark limit as the kinetic energy and centrifugal barriers for $\ell \neq 0$ waves vanish like $1/m_Q$.

III. DO $D^* D$ BOUND DEUSONS EXIST?

Previous calculations utilizing OPEP (one-pion exchange potential) disagree on this issue. Thus Törnqvist [17,18,19] finds $B^* B$, $D^* D$ and even $K^* K$ bound states whereas Manohar and Wise [20] find only $B^* B$ bound states. The difficulty stems from the fact that much of the binding is due to the short range part of the potential. The derivative pion coupling generates in the tensor part of the OPEP a $e^{-\mu r}/r^3$ term which is singular at short distance.

$^2$The central part of the OPEP in the $D^{(*)} D^{(*)}$ channel has the form:

$\frac{(\vec{e}_1 \cdot \vec{q})(\vec{e}_2 \cdot \vec{q}) (\vec{\tau}_1 \cdot \vec{\tau}_2)}{q^2 + \mu^2}$
distance. Cutting off the OPEP at some distance $r_0$ is therefore required. The fact that Manohar and Wise (conservatively (?)) chose $r_0 \approx 1/2m_\pi$ and Törnqvist (boldly (?)) takes $r_0 \approx 1/4m_\pi$ is the likely reason for their disagreement on bound $D^* D$ deusons.

We do not believe that this issue can be settled. Yet the fact that the binding of this system by OPEP alone is not guaranteed may make the problem even more interesting. To decide the issue of a physical bound state with $c\bar{c}u\bar{d}$ flavor we may need the genuine four quark—"one bag" component of the state—where nontrivial aspects of QCD are operative. Indeed, once the distance between the $D$ and $D^*$ mesons defined, say, by the distance between the respective charmed quarks, is smaller than the size of $D$ or $D^*$ we can no longer treat the system as two separate hadrons exchanging light mesons. Rather, we need to revert to the "one bag" tetraquark description. The various quark-(anti)quark interactions in this state may then supply just the extra attractive interaction at (relatively) short distances needed in order to bind the system.

The same physical $c\bar{c}u\bar{d}$ state would then be a $T(c\bar{c}u\bar{d})$ at short distances, $r \leq r_0(\approx 0.7fm)$ and a $D^* D$ deuson for $r \geq r_0$. Neither deusons nor tetraquarks are true ground states. The basic variational principle implies that mixing will lower the true ground state energy below the lowest energies found in each sector separately.

**IV. THE $c\bar{c}u\bar{d}$ TETRAQUARK**

Lacking a consistent first principle computational framework we appeal to the vast existing lore and literature. Thus to approach the problem of “color connected single bag” states one utilizes:

\[
\mu^2 = m_\pi^2 - (m_{D^*} - m_D)^2 \quad \text{in } D^* - D \quad \text{and} \quad \mu^2 = m_\pi^2 \quad \text{in } D^* - D^*.
\]

Averaging over the polarizations we get an expression like $q^2/(q^2 + \mu^2) \approx 1 - \mu^2/(q^2 + \mu^2)$. The second piece becomes negligible as $\mu \to 0$ (except at $q = 0$) and the first piece contributes in configuration space a $\delta$-function which clearly cannot be utilized in a reliable way for binding an extended object of interest.
i) “constituent” massive quarks $m_u \approx m_d \approx 350 \, \text{MeV}$, $m_c \approx 1.6 \, \text{GeV}$;

ii) appropriate $q/\bar{q} - q$ long range interactions, or alternatively, an overall bag which confines the quarks into a single state;

iii) the chromomagnetic hyperfine pairwise interactions:

$$H.F. \equiv \mathcal{H}_{ij} \approx -\frac{1}{m_i m_j} \langle |\Psi_{ij}(0)|^2 \rangle (\bar{\sigma}_i \cdot \bar{\sigma}_j) (\bar{\lambda}_i \cdot \bar{\lambda}_j),$$

where $\sigma_i/\lambda_i$ are the spin/color matrices of the (anti)quarks, and $|\Psi_{ij}(0)|^2$ is the relative wave function at zero separation for a $q\bar{q}_j$ meson and more generally the probability of overlap of the two (anti)quarks considered.

Rather than attempting an ab initio calculation we adopt a more phenomenological approach. It utilizes known masses and insight from successful past calculations instead based on (i)-(iii) above in order to extrapolate to the $cc\bar{u}\bar{d}$ mass.

We focus on the $cc\bar{u}\bar{d} \, I = 0$ state (rather then $cc\bar{u}\bar{u} \, (cc\bar{d}\bar{d}) \, I = 1$ states) since in both the tetraquark and deuson approach it is more strongly bound. The $H.F.$ hyperfine interaction, Eq. (2), strongly favors $1S\bar{u}\bar{d}$ which is a 3 of color. The resulting anti-symmetry in color and separately in spin does then require flavor antisymmetry, i.e. isosinglet $\bar{u}\bar{d}$ state, in order to maintain overall Fermi statistics. Note that the $c_1 c_2$ quarks, which are in 3 of color, must then form a spin triplet. This suggests an overall $s$-wave $1^+$ singlet $cc\bar{u}\bar{d}$ state whose lightest putative decay channel is indeed $D D^*$. Also in the deuson approach the attractive OPEP is three times as strong in the $I = 0$ than in the $I = 1$ channel.

We thus consider the following putative double difference relation,$$
\left( m_{(cc\bar{u}\bar{d})1^+} - m_{(ccu)1/2^+} \right) - (m_{\Lambda_c} - m_{D^0}) = 0 .
$$

It assumes that the extra energy required for replacing a $u$ (or $\bar{u}$) quark by a $u d$ ($\bar{u} \bar{d}$) diquark in the presence of a $c$ quark or $(cc)_3$ diquark is the same. It is clearly true in the heavy quark limit. Indeed in this limit the compact tightly bound $QQ$ pair is like a heavier antiquark flavor. Equation (3) would then be analogous to the relation inspired by the heavy quark symmetry:
\[(m_{\Lambda_b} - m_B) - (m_{\Lambda_c} - m_D) = 0, \tag{4}\]

which holds very well.

To further motivate Eq. (3) we note that quark masses and almost all pairwise interactions cancel out via the double difference. The interactions between the \(c_1\) and \(c_2\) quarks in the \(cc = \bar{3}\) diquark in \(cc\bar{u}\bar{d}\) and in \(ccu\) match as well as the interactions between the \(\bar{u}\bar{d} = \frac{\bar{3}}{2}\) (1s of spin) in \(cc\bar{u}\bar{d}\) and the similar \(ud\) pair in \(cud\).

The \(H.F.\) interaction between the \(c\) quark and the \(u, d\) quarks in \(\Lambda_c = (cud)\) cancel out since \(\vec{\sigma}_u = -\vec{\sigma}_d\) implies

\[
(\vec{\sigma}_u \cdot \vec{\sigma}_c) (\vec{\lambda}_u \cdot \vec{\lambda}_c) + (\vec{\sigma}_d \cdot \vec{\sigma}_c) (\vec{\lambda}_d \cdot \vec{\lambda}_c) = (\vec{\sigma}_u \cdot \vec{\sigma}_c) (\vec{\lambda}_u - \vec{\lambda}_d) \cdot \vec{\lambda}_c) = 0, \tag{5}\]

where we utilized the color neutrality condition, \(\vec{\lambda}_u + \vec{\lambda}_d + \vec{\lambda}_c = 0\), for \(\Lambda_c\).

Similarly the \(H.F.\) interactions between the charmed quarks and the \(\bar{u}, \bar{d}\) quarks in \(T(cc\bar{u}\bar{d})\) cancel. Apart from an overall common factor \(|\Psi_{uc}(0)|^2/m_\Lambda m_c\) we have, using again \(\vec{\sigma}_u = -\vec{\sigma}_d\),

\[
(\vec{\sigma}_{c1} \cdot \vec{\sigma}_{\bar{u}}) (\vec{\lambda}_{c1} \cdot \vec{\lambda}_{\bar{u}}) + (\vec{\sigma}_{c1} \cdot \vec{\sigma}_{\bar{d}}) (\vec{\lambda}_{c1} \cdot \vec{\lambda}_{\bar{d}}) + (\vec{\sigma}_{c2} \cdot \vec{\sigma}_{\bar{u}}) (\vec{\lambda}_{c2} \cdot \vec{\lambda}_{\bar{u}}) + (\vec{\sigma}_{c2} \cdot \vec{\sigma}_{\bar{d}}) (\vec{\lambda}_{c2} \cdot \vec{\lambda}_{\bar{d}}) = (\vec{\sigma}_{c1} \cdot \vec{\sigma}_{\bar{u}}) (\vec{\lambda}_{\bar{u}} - \vec{\lambda}_{\bar{d}}) \cdot \vec{\lambda}_{c1} + (\vec{\sigma}_{c2} \cdot \vec{\sigma}_{\bar{u}}) (\vec{\lambda}_{\bar{u}} - \vec{\lambda}_{\bar{d}}) \cdot \vec{\lambda}_{c2} = \hat{O}_A. \tag{6}\]

The last expression is an operator \(\hat{O}_A\) anti-symmetric under the exchange of the color degrees of freedom of \(\bar{u}\) and \(\bar{d}\). In the tetraquark state the \(\bar{u}\) and \(\bar{d}\) colors are also coupled anti-symmetrically in forming the \(3\) (anti)diquark. As a result the total \(H.F.\) interaction energy between the light and the heavy quarks in the tetraquark has the form \(\langle T_A |\hat{O}_A| T_A \rangle\) which vanishes since upon an exchange of color indices of \(\bar{u}\) and \(\bar{d}\) this matrix element changes sign.

To complete motivating Eq. (3) we still need to show that the \(cu\) (and \(c\bar{u}\) \(H.F.\) interactions in \(\Xi_{ccu}(1/2^+)\) and in \(D^0(0^-)\) match. The latter is:
\[- \frac{1}{m_u m_c} |\Psi_{uc}(0)|^2 (\vec{\sigma}_c \cdot \vec{\sigma}_u) (\vec{\lambda}_c \cdot \vec{\lambda}_u) \approx -\frac{3}{4} \lambda^2 \frac{|\Psi_{uc}(0)|^2}{m_u m_c}, \quad (7)\]

where we used \( \vec{\sigma}_c + \vec{\sigma}_u = 0 \) and \( \vec{\lambda}_c + \vec{\lambda}_u = 0 \).

The H.F. interactions in the doubly charmed baryon are however:

\[
\begin{align*}
- \frac{1}{m_u m_c} |\Psi_{uc}(0)|^2 \left\{ (\vec{\sigma}_u \cdot (\vec{\sigma}_{c1} + \vec{\sigma}_{c2})) (\vec{\lambda}_u \cdot (\vec{\lambda}_{c1} + \vec{\lambda}_{c2})) + (\vec{\sigma}_u \cdot (\vec{\sigma}_{c1} - \vec{\sigma}_{c2})) (\vec{\lambda}_u \cdot (\vec{\lambda}_{c1} - \vec{\lambda}_{c2})) \right\} \\
= \frac{1}{2} \frac{|\Psi_{uc}(0)|^2}{m_u m_c} \left\{ (\vec{\sigma}_u \cdot (\vec{\sigma}_{c1} + \vec{\sigma}_{c2})) (\vec{\lambda}_u \cdot (\vec{\lambda}_{c1} + \vec{\lambda}_{c2})) + (\vec{\sigma}_u \cdot (\vec{\sigma}_{c1} - \vec{\sigma}_{c2})) (\vec{\lambda}_u \cdot (\vec{\lambda}_{c1} - \vec{\lambda}_{c2})) \right\}. \quad (8)
\end{align*}
\]

The second term above vanishes since, again, \( \vec{\sigma}_u \cdot (\vec{\sigma}_{c1} + \vec{\sigma}_{c2}) \approx \lambda_{c1}^2 - \lambda_{c2}^2 = 0 \), so that we obtain:

\[
- \frac{1}{2} \frac{|\Psi_{uc}(0)|^2}{m_u m_c} (\vec{\sigma}_u \cdot (\vec{\sigma}_{c1} + \vec{\sigma}_{c2})) \cdot \lambda^2 = -\frac{1}{2} \lambda^2 \frac{|\Psi_{uc}(0)|^2}{m_u m_c}, \quad (9)
\]

where in the last step we used \((\vec{\sigma}_u + \vec{\sigma}_{c1} + \vec{\sigma}_{c2})^2 = 3/4 \) (since the lightest charmed baryon has spin 1/2) and \((\vec{\sigma}_{c1} + \vec{\sigma}_{c2})^2 = 2 \) (since the two charmed quarks are in \( 3S \) state).

We find that the hyperfine (attractive) interaction in \( \Xi_{cuc}(1/2^+) \) and in \( D^0(0^-) \) do not exactly match but are slightly stronger in \( D^0 \) by

\[
\text{H.F.}' = \frac{1}{4} \left( m_{D^*} - m_D \right). 
\]

Thus by subtracting the physical lighter \( D \) mass we are actually causing an imbalance in Eq. (3) and an overestimate of \( m_{T(ccbar)} \). A corrected equation should therefore read:

\[
m_{T(ccbar)} = m_{\Xi_{cuc}} + m_{\Lambda_c} - m_{D^0} - \frac{1}{4} \left( m_{D^*} - m_D \right). \quad (10)
\]

While the masses of \( \Lambda_c, D^0 \) and \( D^{0*} \) are well known, this is definitely not the case for the doubly charmed baryons. The lowest SELEX peak appears in \( ccu^{(++)} \) at 3460 MeV. Using this value in Eq. (10) yields

\[
m_{T(ccbar)} \approx 3.845 \text{ GeV}, \quad (11)
\]
which is about 25 MeV below the $D^* D$ threshold.

Unfortunately the lowest peak in $cc d^{(+)}$ is at 3.52 GeV and not degenerate with the $cc u$ peak as it should be by isospin invariance. We therefore choose the $cc d^{(+)}$ peak (which indeed coincides with a (relatively small) enhancement in the $cc d^{(+)}$ SELEX data) as representing the true value of the lowest charmed baryon. This is clearly a more conservative choice as both SELEX peaks are lower than most previous theoretical predictions. Using this we obtain,

$$m_{T(cc \bar{u} \bar{d})} \approx 3.905 \text{ GeV}$$

some 35 MeV above the $D^* D$ threshold.

Various explicit and implicit assumptions were made in order to obtain Eq. (10) making for a theoretical uncertainty in the predicted value of $m_{T(cc \bar{u} \bar{d})}$ beyond the 60 MeV experimental uncertainty discussed above.

a) We used a common overlap probability $|\Psi_{ij}(0)|^2$ for all $c \bar{q}$ or $c q$ pairs which corresponds to a “universal bag radius”. In reality the latter could change as we go from two to three to four quark systems. The successful phenomenology of baryon/meson hyperfine splitting suggests that this may be a weak effect. Furthermore a systematic change will largely cancel in the double difference relation in Eq. (3).

b) We have restricted our discussion to diquarks coupled to $\bar{3}$ (or 3) of color only. However in the tetraquark system we encounter (for the first time) the possibility of coupling $c_1 c_2$ (and $\bar{u} \bar{d}$) to 6 (6̄) of color—a coupling, which is clearly not allowed in a baryon. The restriction to the $\bar{3} - 3$ pattern may be well justified by the fact that it has a lower energy than the $6 - 6\bar{6}$. Nonetheless these channels can in principle mix, and allowing for such admixture to optimize the binding will lower the mass of the physical state.

c) The admixture of the “one bag” tetraquark state above and the “two bag” deuson state (which again occurs first in four-quark exotic states) will also, by the above mentioned variational argument, tend to lower the energy.

All the above suggests that if the SELEX peak at 3460 MeV is indeed the lightest
double charmed baryon, we have a $T(c\bar{c}\bar{u}\bar{d})$ state slightly below or slightly above threshold. Specifically, if

$$\epsilon = m_{T(c\bar{c}\bar{u}\bar{d})} - m_{D^*} - m_D,$$

we expect

$$|\epsilon| \leq 30 \div 60\,\text{MeV}.$$  

V. PRODUCTION AND DECAYS OF $T(c\bar{c}\bar{u}\bar{d})$

We would next like to argue that if $T(c\bar{c}\bar{u}\bar{d})$ has the above mass it may well be the first narrow exotic hadron to be discovered. The potential discovery depends jointly on

(i) the rate of $c\bar{c}\bar{u}\bar{d}$ production [22], and

(ii) the existence of decay modes which can provide a unique signature.

The production rates (at hadronic or $e^+e^-$ colliders) of the state $c\bar{c}\bar{u}\bar{d}$ of interest are very small as all the following conditions should be met:

(a) Two pairs of charmed quarks ($\bar{c}_1 c_1$) and ($\bar{c}_2 c_2$) need to be produced.

(b) These pairs should be close spatially. Also $c_1$ from the first pair, and $c_2$ from the second should have small relative momenta in order for a $c_1 c_2$ diquark to form.

(d) Finally the $c_1 c_2$ diquark should pick up a $\bar{u}\bar{d}$ (anti)diquark to form $c\bar{c}\bar{u}\bar{d}$.

The first two factors (a) and (b) also suppress the production rate of doubly charmed baryons $c\bar{c}u/c\bar{c}d$. The only further suppression of the $c\bar{c}\bar{u}\bar{d}$ production rate is due to the need to pick up a $\bar{u}\bar{d}$ diquark instead of merely just one $u$ (or $d$) in the case of $c\bar{c}u/c\bar{c}d$.

This suggests the following “double ratio” relation:

$$\left(\frac{R(c\bar{c}\bar{u}\bar{d})}{R(c\bar{c}u)}\right) : \left(\frac{R(c\bar{u}d)}{R(c\bar{u})}\right) = 1,$$

which is analogous to the double difference relation in Eq. (3).\footnote{This analogy is very natural in a “statistical model”, where the production rate of any hadronic}
The ratio of the charmed baryon, $\Lambda_c$, production and that of $D$'s is roughly the same as in the case of strange baryons/mesons:

$$R_{(c\bar{u}d)/R_{(c\bar{u})}} \approx R_{(s\bar{u}d)/R_{(s\bar{u})}} = R_{(\Lambda)/R_{(K)}} \approx \frac{1}{10}. \quad (15)$$

Hence we expect from Eq. (14)

$$R_{(c\bar{c}\bar{u}\bar{d})} \approx \frac{1}{10}R_{(c\bar{c}u)}, \quad (16)$$

namely that the $T(c\bar{c}\bar{u}\bar{d})$ production rate is about $1/10$ that of charmed baryons.

If $T(c\bar{c}\bar{u}\bar{d})$ is to be discovered this tiny production rate needs to be compensated by striking decay signatures. The decay modes critically depend on $m_{T(c\bar{c}\bar{u}\bar{d})}$ or $\epsilon$, the separation between $m_{T(c\bar{c}\bar{u}\bar{d})}$ and the $m_D + m_{D^*}$ threshold, defined in Eq. (13).

We will next discuss the different $m_{T(c\bar{c}\bar{u}\bar{d})}$ ranges starting with the most strongly bound case.

(a) $m_{T(c\bar{c}\bar{u}\bar{d})} \leq 2m_D$ or $\epsilon \leq m_D - m_{D^*} \approx -140$ MeV.

In this case—which the above discussion suggests to be unlikely—we can only have two consecutive weak decays. Strictly speaking only the second vertex involves an on-shell reconstructible $D^+$ or $D^0$. However, up to small binding effects, $2m_D - m_{T(c\bar{c}\bar{u}\bar{d})}$, the tracks emerging from the first decay may well correspond to another $D$.

There is some probability that the charm quark surviving after the first decay may be inside a $D^*$ so that an extra, slow pion emitted from the first decay vertex can combine with the 4-momenta of the second vertex particles to form a $D^*$. Finally all particles from the two decay vertices should reconstruct a narrow $T(c\bar{c}\bar{u}\bar{d})$ state below $2m_D$.

(b) $m_{D^*} + m_D \geq m_{T(c\bar{c}\bar{u}\bar{d})} > 2m_D$ or $0 > \epsilon \geq -140$ MeV.

In this case we will have an electromagnetic decay:

$$T(c\bar{c}\bar{u}\bar{d})^+ \rightarrow D^+ + D^0 + \gamma, \quad (17)$$

state $X$ is suppressed by a Boltzmann factor proportional to $\exp (-m_X/T)$. Equation (14) results then as the exponential of Eq. (3).
with two weak decay vertices due to $D^+$ and $D^0$ decays. While the state will be narrow, reconstructing the invariant mass, $m_{D^+ D^0}$, and looking for a sharp peak requires identifying a relatively soft photon ($E_\gamma \leq 140\ MeV$ in the $T(\bar{c}c \bar{u}d)$ rest frame) emerging from the primary decay vertex.

\[(c)\ m_{T(\bar{c}c \bar{u}d)} > m_{D^*} + m_D \text{ or } \epsilon \geq 0\]

Here we clearly have $T(\bar{c}c \bar{u}d) \to D^{*0} + D^+ \,(\text{or } D^{*+} + D^0)$. The $D^*$ will decay into $D + \pi$ at the primary vertex and the two $D$'s will next decay weakly at separate vertices. Thus we attempt to reconstruct the two $D$'s and the $D^*$ and finally look for an overall peak in the $D^* D$ invariant mass distribution. This last constraint may not be very helpful (and the $T(\bar{c}c \bar{u}d)$ can be altogether missed) if its width $\Gamma(T(\bar{c}c \bar{u}d))$ (substantially) exceeds the experimental resolution which we optimistically take to be $\mathcal{O}(10\ MeV)$.

Since $T(\bar{c}c \bar{u}d, 1^+) \to D^* D$ is an $S$-wave decay and no new quark pairs need to be created (as in $K^* \to K \pi$, for instance) one might expect a large decay width, $\Gamma \approx 300\ MeV$, as is the case with $\bar{q}_i q_j \bar{q}_l q_k$ exotics made of light quarks. Two factors may, however, reduce $\Gamma(T(\bar{c}c \bar{u}d))$. First we have a two-body decay phase space which is proportional to $\beta$ or $\beta^*$, the velocity of $D$ or $D^*$ in the $T(\bar{c}c \bar{u}d)$’s rest frame, which unlike in the decays of light exotics may be significantly less than one:

$$\beta_D \approx \beta_{D^*} \approx m_D \sqrt{1 - \frac{\epsilon}{m_D}} \approx 0.13 \left(\frac{\epsilon}{30\ MeV}\right)^{1/2}. \quad (18)$$

Second, it may well be that the physical $1^+$ hadron of interest has a relatively small deuson component, $|\alpha|^2$:

$$|\Psi_{\bar{c}c \bar{u}d} (\text{physical})\rangle = \sqrt{1 - |\alpha|^2} |T(\bar{c}c \bar{u}d)\rangle + |(D^* D) \text{ deuson}\rangle,$$  

with $T(\bar{c}c \bar{u}d)$ being the “one bag” genuine four quark state. Since only the deuson component readily falls apart into $D^* + D$ we may then have a further suppression by a factor of $|\alpha|^2$. If $|\alpha|^2 \leq \frac{1}{3} - \frac{1}{4}$ the joint $|\alpha|^2 \beta$ effect reduces the decay rate from $\Gamma \approx 300\ MeV$ to $\Gamma \approx 13 - 19\ MeV$.

Note, since the state of interest is an $S$-wave of the $D + D^*$ we may lack any repulsive interaction—akin to Coulomb repulsion in fission or $\alpha$ decays—to generate a resonance in the
first place \[^{23}\]. Rather we may have, as in the \( I = 1 \) \( S \)-wave nucleon-nucleon scattering, a “virtual bound state”. The experimental manifestation of the latter—a strong enhancement at the \( DD^* \) threshold, may be still sufficiently striking. The above lifetime estimates would then still pertain to the width of this enhancement.

In principle we can also have \( c \bar{u} \bar{c} d \) tetraquark (and/or meson-antimeson, \( D^* \bar{D}, D \bar{D}^* \) deusons) with “hidden” charm. Such combinations will be much more easier to produce as only one \( \bar{c} c \) pair needs to be created. Naively one would expect such states to decay very quickly into \( J/\Psi + \pi^+ \). Törnqvist, whose main concern was actually meson-antimeson binding did, however, suggest that a primary extended deuson \( D^* \bar{D} \) state would have little overlap with the \( J/\Psi \) (compact \( \bar{c} c \)) state. This would then make for a relatively narrow \( D^* \bar{D} \) state which still could be nicely identified via its unique \((J/\Psi + \pi^+)\) decay mode at a specific \( J/\Psi + \pi^+ \) invariant mass.

The \( ccu/ccd \) double charmed baryons have presumably been seen at FNAL \[^{21}\]. The \( T(c \bar{c}u\bar{d}) \) is most likely produced there with high lab momenta. The proximity of the \( T(c \bar{c}u\bar{d}) \) mass to the \( D^* D \) threshold cause the two subsequent weak decay vertices to be very precisely aligned with the initial interaction vertex—a feature which will be most helpful in a tetraquark hunt.

The much lower energy \( e^+ e^- \) colliders can also serve as promising search grounds with a much cleaner environment. Thus at \( \sqrt{s} \approx 10.7 \text{GeV} \) (\( \approx m_{\Upsilon(4s)} \)) the primary virtual photon interaction yields in about 25\% of all cases a \( \bar{c} c \) pair. This then will vastly increase the number of events in which we have two charmed quark pairs to be \( \mathcal{O}(1 - 0.25) \), i.e. \( \mathcal{O}(10^6) \) events \[^{21}\]. The events in which \( (J/\Psi) \) will not form will have typical \( D^{(*)} \bar{D}^{(*)} D^{(*)} \bar{D}^{(*)} \) final states (\( D^{(*)} \) indicates \( D \) or \( D^* \)) with a few extra particles due to the limited phase space (\( 4m_{D^*} \approx 8.4 \text{GeV}! \)). The systematic search of peaks in \( D^* D^{(*)} \) or \( \bar{D}^{(*)} \bar{D} \) invariant mass distribution could thus be feasible despite the large combinatorial background.

Peaks in \( D^* \bar{D}^{(*)} \) mass distribution could also occur. These should manifest in the much cleaner \( (J/\Psi + \pi) \) channel. Hopefully by \( D^* \) (or \( D^{**} \)) cascade decays and by \( K^+, K^- \) separation one will be able to distinguish \( D^{(*)} D^{(*)} \) from \( D^{(*)} \bar{D}^{(*)} \) pairs.
Finally we note that the Brown-Hanbury-Twiss effect favoring $D^0 D^0$ (and $D^+ D^+$) pairs with small relative momenta should not be operative here as the two pseudoscalar $D$'s do not emerge from the same vertex (even the $D$'s from the $D^* \to D \pi$ decay would emerge several hundred Fermies away from the primary vertex because of the narrow $D^*$'s width).

VI. SUMMARY AND CONCLUSIONS

We have presented various estimates pertaining to a possible tetraquark $T(c c \bar{u} \bar{d})(1^+)$ state. If the SELEX second peak corresponds to the lightest doubly charmed baryon, then our estimates of the mass of $T(c c \bar{u} \bar{d})(1^+)$ are close to the $D D^*$ threshold. We have also discussed signatures for various possible masses.

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