Genetically determined variation in lysis time variance in the bacteriophage φX174.

Baker, Christopher W.*, 1, 2, 3, Craig R. Miller†‡§, Tanayott Thaweethai*, 4, Jeffrey Yuan*, 5, Meghan Hollibaugh Baker*, 6, Paul Joyce†, and Daniel M. Weinreich***

February 20, 2016

Author affiliations:
*Department of Ecology and Evolutionary Biology, Brown University, Providence, RI 02912.
†Department of Mathematics, University of Idaho, Moscow, ID 83844.
‡Department of Biological Sciences, University of Idaho, Moscow, ID 83844.
§Center for Modeling Complex Interactions, University of Idaho, Moscow, ID 83844.
**Center for Computational Molecular Biology, Brown University, Providence, RI 02912.

1 Present address: Department of Molecular and Cellular Biology, Harvard University, Cambridge, MA 02138.
2 These authors contributed equally to this work.
3 Corresponding author (see information on second page)
4 Present address: Harvard School of Public Health, Boston, MA 02115.
5 Present address: Bioinformatics IDP, University of California - San Diego, San Diego, CA 92093.
6 Present address: The George Washington University, Washington, DC 20052.
Short title: Lysis time variance in phage φX174

Key words: lysis time; variance; genetics of adaptation; evolutionary theory; life history

Corresponding author:
Department of Molecular and Cellular Biology
Harvard University
52 Oxford St.
Cambridge, MA 02138
+1 (860) 989-0644
baker03@g.harvard.edu
ABSTRACT

Researchers in evolutionary genetics recently have recognized an exciting opportunity in decomposing beneficial mutations into their proximal, mechanistic determinants. The application of methods and concepts from molecular biology and life history theory to studies of lytic bacteriophages (phages) has allowed them to understand how natural selection sees mutations influencing life history. This work motivated the research presented here, in which we explored whether, under consistent experimental conditions, small differences in the genome of bacteriophage φX174 could lead to altered life history phenotypes among a panel of eight genetically distinct clones. We assessed the clones’ phenotypes by applying a novel statistical framework to the results of a serially-sampled parallel infection assay, in which we simultaneously inoculated each of a large number of replicate host volumes with ∼1 phage particle. We sequentially plated the volumes over the course of infection and counted the plaques that formed after incubation. These counts served as a proxy for the number of phage particles in a single volume as a function of time. From repeated assays, we inferred significant, genetically determined heterogeneity in lysis time and burst size, including lysis time variance. These findings are interesting in light of the genetic and phenotypic constraints on φX174’s single-protein lysis mechanism. We speculate briefly on the mechanisms underlying our results and discuss the potential importance of lysis time variance in viral evolution.
INTRODUCTION

A key opportunity in evolutionary genetics is the decomposition of beneficial mutations into their proximal, mechanistic determinants. To date, much of this work has applied methods from molecular biology to dissect the basis of protein evolution (Harms and Thornton 2013; Dean and Thornton 2007), but life history theory (Stearns 1992; Roff 2002) similarly decomposes population growth rate into constituent life history traits. It can thus predict which mutations influencing life history will be favored by natural selection (i.e., increase population growth rate) (Stearns et al. 2000). The simple life history of the lytic bacteriophage (phage) makes it particularly well suited to this sort of work. Each generation begins with a free phage adsorbing to a host cell and injecting its genome. Transcription and translation facilitate the assembly of progeny phages within the host, and, finally, host cell lysis releases the next generation of free phages. Assuming the density of host cells is not limiting, Wang et al. 1996 provided an expression for phage growth rate in terms of mean burst size and mean time to lysis. Later, Bull et al. 2006 extended this treatment by incorporating adsorption and phage death rates. Other studies have further developed this formalism (Rabinovitch et al. 1999, 2002), with many applying it to explore the mechanistic basis of beneficial mutations in lytic phages (Pepin et al. 2006; Shao and Wang 2008; Bull et al. 2011). Several have demonstrated that a mutation’s fixation probability depends critically on which component of life history it affects (Wahl and DeHaan 2004; Hubbarde et al. 2007; Patwa and Wahl 2008, 2009).

At steady state (i.e., when host cell infections are at their stable age-of-infection distribution), population growth rate is entirely determined by mean life history traits (Bull et al. 2011). But the moment a novel mutation appears, it establishes a lineage that, by definition, is far from this equilibrium. In populations deviating from a stable
age distribution, variance in both lysis time and adsorption rate may play a significant role in determining growth rate (Storms and Sauvageau 2014), since it may affect how long the deviation from stable age distribution persists (Bull et al. 2011).

These findings motivate interest in the possibility of genetic control for higher moments in phage life history traits. Previously, Dennehy and Wang 2011 used a microscopy-based approach to demonstrate heritable variation lysis time mean and variance among a panel of phage λ holin protein mutants.

To explore heritable variation in phage life history traits, including lysis time variance, we studied a panel of eight genetically distinct clones of the lytic bacteriophage φX174. φX174 is an ubiquitous (Afshinneko et al. 2015) and industrially important phage (Labrie et al. 2014). It was chosen for this research in large part due to its small, well-characterized genome (Hayashi et al. 1988), the first of its kind ever sequenced (Sanger et al. 1977). In addition, it infects a well-studied host (Escherichia coli C), has a short generation time, and is robust to a wide range of temperatures, making it amenable to evolution experiments and other lab manipulations (Wichman and Brown 2010).

The gold standard for measuring phage lysis time and burst size is the one-step phage growth assay (Hyman and Abedon 2009). However, this method affords only limited access to information about lysis time variance and the relationship between burst size and time for a given phage. Therefore, the panel of eight clones was assessed using an updated version of a venerable experimental method first proposed by Burnet 1929 and later extended by Delbrück 1945. We simultaneously inoculated each of a large number of replicate host cell volumes with, on average, less than one phage particle of identical haplotype. These volumes were then plated at 15-s intervals on lawns of host cells, and the resulting plaques were counted after incubation.

From this assay, we obtained data that could help determine mean and variance in both lysis time and burst size (Figure 1). However, the information we wanted was
obscured by a suite of hidden variables. In wells showing lysis events, we didn’t know when lysis actually occurred. We also did not know the number of phage particles originally in these wells and, if that number was greater than one, how many had lysed their hosts. To infer lysis time and burst size from the two observed metrics (time of sampling and plaque count), we somehow had to circumvent these hidden variables. We overcame this challenge by developing a novel inferential apparatus that complemented the experimental framework with a maximum likelihood-based approach.

In this article, we describe our finding that, under the tested conditions, genetic heterogeneity could give rise to significant heterogeneity in φX174 lysis time and burst size, including lysis time variance. These findings are interesting in light of the genetic and phenotypic constraints on φX174’s single-protein lysis mechanism, and they point to promising avenues for future theoretical and experimental work.

EXPERIMENTAL METHODS

Media, host cells, and bacteriophages: Difco™ LB Lenox Broth (Detroit, MI), pH 7.50 and supplemented with 2mM CaCl₂ was used throughout. We studied φX174’s performance on E. coli C (DSMZ 13127, kindly provided by Olivier Tenaillion, Institut National de la Santé et de la Recherche Médicale (INSERM), University Denis Diderot Paris 7), the phage’s natural host. A stock of E. coli C was stored in 40% glycerol at -80°. We used this primary stock to seed bacterial colonies on agar plates stored at 4°. Colony plates were created anew every two to four weeks. To generate experimental bacterial cultures, 10 mL LB was inoculated with a single colony and shaken in a 50-mL flask at 200 rpm and 37° for about 16 hours. The resulting stationary-phase cells were then diluted somewhere between 33- and 80-fold into 40 mL LB in a 250-mL flask. The diluted culture was shaken at 37° for about 3 hours until it reached an optical density (OD₆₀₀) of 0.6. Exponential-phase cells prepared in this way were used for each assay.
To synchronize phage infections, we treated cells using a starvation protocol adapted from Denhardt and Sinsheimer 1965 and kindly provided by Darin Rokyta, Florida State University. The treatment starved cells so that they “shut down,” allowing for phage adsorption but not entry. (It is likely that the treatment also had the fortunate side effect of reducing variance in time to cell division compared to an exponential-phase culture, since it was bound to cause a lag in the cell cycle. See DISCUSSION for the cell cycle’s relevance.) First, we transferred 1 mL cells to a 1.5-mL centrifuge tube and pelleted them by spinning in an Eppendorf Centrifuge 5417R for 2 minutes at 20,800 rcf. After aspirating the supernatant, we re-suspended the cells in 1 mL HFB-1 solution (0.06 M NH₄Cl / 0.09 M NaCl / 0.1 M KCl / 0.1 M Tris-HCl (pH 7.4) / 1.0 mM MgSO₄ / 1.0 mM CaCl₂) by vortexing for eight seconds. Then, we centrifuged the solution, removed the supernatant, and re-suspended in HFB-1 twice more. After removing the supernatant from the third and final HFB-1 rinse, we re-suspended cells in 1 mL HFB-2 solution (HFB-1 / 10 mM MgCl₂ / 5 mM CaCl₂). This served as our culture of starved bacterial cells, ready for phage adsorption.

We studied a panel of eight genetically distinct clones, chosen based on reports of lysis profiles different from the wild type’s. These included three Epos mutants carrying point mutations in the E gene, whose product is the chief player in φX174 lysis (pos4b, pos5, and pos6, first described in Bernhardt et al. 2002; kindly provided by Ry Young and Rohit Kongari, Texas A&M University); four carrying point mutations in the D-promoter region, which exerts regulatory control over transcription of the E gene (mut319, mut321, mut323, and mut324, first described in Brown et al. 2010 and Brown et al. 2013; kindly provided by Celeste Brown and Amber Stancik, University of Idaho); and a wild type (also provided by Ry Young and Rohit Kongari) (Table 1, Figure 2). Clones available from DMW on request. Presence of the mutations of interest was verified by Sanger sequencing of the 65-730 base region of each clone’s genome.
(nucleotides numbered as in SANGER et al. 1977). The clones’ sequences in this region (GenBank Accession KU646482 to KU646588) were identical to the wild-type sequence (GenBank Accession J02482.1) apart from the noted mutations. The wild type employed was the ancestor of the Epos mutants, which differed from the wild-type ancestor of the D-promoter mutants (GenBank Accession AF176034) at five sites: G833A (nonsynonymous), G1650A (nonsynonymous), T2811C (synonymous), A4518G (nonsynonymous), and T4784C (synonymous) (PEARSON et al. 1997). The G833A difference entailed an amino acid change in the E (but not the D) gene, though it fell well outside the region coding for the E protein’s essential transmembrane region (Figure 2) (TANAKA and CLEMONS JR. 2012).

Phage titers were determined by combining a known volume of phage stock with 200 µL exponentially growing host cells (described above) and plating in top agar held at 42°, consisting of 7.0 g/L Difco™ BactoAgar (Detroit, MI) added to LB media (as described above). After several minutes on the bench at room temperature, we incubated plates agar-side-up at 37°. Preliminary results (not shown) demonstrated that accurate plaque counts required at least four hours of incubation. Occasionally, for logistical reasons, we interrupted incubation with overnight refrigeration at 4°.

**Serially-sampled parallel infection assay:** We began by adding 90 µL starved cells to each well of a TempPlate non-skirted 96-well PCR plate (USA Scientific item no. 1402-9598) at 15°. The PCR plate was mounted on a thermocycler to control its temperature precisely. Using a 12-channel pipettor, we then added 10 µL 30-PFUs/mL phage dilution to each well in each row of twelve wells, allowing for a 30-second interval between rows. PCR plates were held at 15° for at least 20 min, during which time phages could adsorb to the starved cells. We estimate that 10-20% of phage particles adsorbed to hosts during this incubation period (Celeste Brown, personal communication; NEWBOLD and
Then, infections were initiated by raising the PCR plate temperature to 37° and adding 100 µL warm LB (incubated at 37° for at least 45 minutes) to each well in a row with a 12-channel pipettor, again allowing 30-second intervals between rows. Consequently, the infective process in each row was staggered by half a minute, allowing ample time for the sampling procedure that followed. Note that the temperature increase alone did not trigger infection. The addition of rich media such as LB was required for infection to advance (Newbold and Sinsheimer 1970). Based on past reports of φX174 attachment kinetics under conditions similar to our assay (Brown et al. 2013), unattached phage particles likely adsorbed to hosts very quickly at this time, with an average adsorption time of less than 30 seconds (assuming adsorption followed an exponential decay process (Abedon et al. 2001)).

It is important to note that, based on our knowledge of the mutated genes’ functions (Fane et al. 2006) and past results (Brown et al. 2013), the studied mutations likely had little or no effect on phage adsorption rate. This includes the non-synonymous difference (G1650A) between the D-promoter mutants and other clones in coat protein F. While a change in the F protein could theoretically have affected adsorption (Bernal et al. 2004), it is unlikely to have done so here, because the difference was merely between one basic polar residue (His) and another (Arg).

Five minutes after infection was initiated, we began sampling wells at 15-second intervals. Sampling was performed by transferring the entire 200 µL contents of a well to a sterile 96-well PCR plate, held at 4° to halt the infective process. We assume that this sampling procedure did not artificially induce lysis in sampled infections. We staggered sampling between rows such that, factoring in the 30-second gaps in infection between each row, we sampled one well every 15 seconds between $t = 5$ min and $t = 28.75$ min. The endpoint of 28.75 min was chosen on the basis of preliminary experiments.
showing that most infections had ended in lysis by that time. Even though not all phages would lyse their hosts by 28.75 min, we found no evidence that φX174 undergoes T4-like lysis inhibition (Abedon 1992), so we believed that samples taken between 5 and 28.75 min would give us a reasonably complete picture of the clones’ lysis phenotypes.

At the conclusion of this process, the contents of each sampled well – now at 4° – were plated and incubated, as described, and then counted. Typical results are shown in Figure 1. Four to six replicate assays were performed for each clone.

STATISTICAL METHODS

The model: We begin with an overview of the model that we assume generated the data. We then develop methods of estimation from the model and apply them to real experimental data.

Each time an assay was performed, we assumed a small, Poisson-distributed number of phage particles had been added to each well. The mean of the Poisson was allowed to vary between assays, which is to say, we assumed phage concentrations in the stocks differed by clone and could change over time.

Within the wells, we assumed phage particles adsorbed instantaneously to starved cells upon the initiation of infection. (We discuss the rationale underlying this assumption in the RESULTS section). We assumed that each phage particle infected and then lysed a cell $T$ minutes after the addition of warm media, where $T$ could not be less than a biologically imposed minimum (the latent period) and lysis was equally probable every moment thereafter (though we also considered a model in which lysis probability changed with time, as described below). We assumed that burst size grew linearly with lysis time, a relationship generally assumed and supported by past work (Hutchison and Sinsheimer 1966; Abedon 1989; Wang et al. 1996; Abedon et al. 2001; Chao

et al. 2002; Bull et al. 2004; Wang 2006; but see Hutchison and Sinsheimer 1963 and Storms et al. 2014). The fate of each infecting phage particle within a well was assumed to be independent of other particles in that well. Wells were sampled and plated at 15-second intervals from $t = 5$ min through $t = 28.75$ min, and we assumed 100% plating efficiency, implying that the plaque count on a plate revealed the true number of phage particles in a well at the time of sampling. A plaque count of zero meant no particles were in the well, a count of one to a few implied that as many particles as visible plaques were added but that no host cells had been lysed yet, and a count of more than a few represented the sum of the burst sizes of lysis events prior to the sample time, plus any infected yet unlysed cells. Note that, in the third case, we did not know how many phage particles were in the well to begin with, we did not know how many lysis events had occurred by the time of sampling, and, of the lysis event(s) that had occurred, we did not know when they occurred, except that they must have occurred prior to the moment of sampling. From a statistical perspective, this suite of hidden variables made the problem both interesting and challenging.

**Notation:** Developing estimators under this model required substantial notation. Table 2 summarizes this notation for the reader’s reference.

**Number of phage particles per well:** Our first task was to estimate $\beta_a$, the Poisson parameter that governs the probability of having 0, 1, 2, \cdots phage particles in each well for assay $a$. This was somewhat complicated by the fact that, before the latent period ended and lysis began, we observed the actual Poisson counts, but after this period, we only observed the true Poisson counts for “zero” wells and wells that do not show lysis events. In wells showing lysis events, we knew only that the Poisson count must have been greater than zero. To derive an estimator tailored to the situation, we ignored the
time element of the data and defined $p$ as the probability that a randomly selected phage particle from a randomly selected well would not have lysed its host. We let $n$ be the Poisson number of phage particles initially in a well (i.e., before lysis occurs). Then, we noted that each sampled well fell into one of three categories: samples with zero phage particles, samples with phage particles but no lysis events, and samples showing lysis events. The probability of a well belonging to each category was, respectively, $e^{-\beta a}$, $(p/\beta a)^n e^{-\beta a} / n!$, and $1 - e^{-(1-p)\beta a}$.

We defined wells not showing lysis events as those having plaque counts greater than zero and less than or equal to seven. When the Poisson parameter $\beta$ is less than or equal to one, counts greater than seven are highly improbable in wells where lysis has not occurred ($P < 0.001$). Of course, it is possible that wells showing fewer than eight plaques could in fact be the results of infections. But, ultimately, we had to set the threshold for distinguishing lysis events from non-lysis events somewhere. This threshold-setting exercise was not straightforward. Complicating factors included the following:

1. Poisson average numbers of particles per well varied for each assay, so setting the threshold based purely on the Poisson probabilities was not trivial.

2. There was a non-zero chance of slight inaccuracies in some plaque counts. Rare errors could have occurred in the counting of plaques that would have increased counts on average (e.g., plaque-like air bubbles). In addition, certain experimental manipulations (e.g., mechanical stress from pipetting) could feasibly have induced very premature lysis of some cells, resulting in plates with extremely low plaque counts that otherwise would have had just one plaque.

3. The few plaque counts of six and seven, a couple of which were sampled at unusually early time points, could have exerted disproportionate influence on estimates of clones’ lysis phenotypes if included as lysis events. This is because, as discussed
below, an artificially early lysis event would force the value of the $t_0$ parameter to shift to some time before that event.

Therefore, given these intermediate plaque counts’ rarity, their potential origins in experimental error, and their potential to distort our estimates of clones’ lysis phenotypes, we chose to set the threshold for lysis events at greater than or equal to eight. It is worth noting, however, that when our data instead were analyzed using a threshold of greater than or equal to five, our results did not change qualitatively (analysis not shown).

We let the number of wells in assay $a$ be $L_a$ and the number of wells that showed lysis events be $R_a$. The likelihood of the data, then, was

$$L(N_{a1}, N_{a2}, \cdots, N_{aM_a}, R_a) = \frac{\prod_{i=1}^{M_a} (p\beta_a)^{N_{ar}} e^{-\beta_a} N_{ar}!}{(L_a R_a)} \cdot \left(1 - e^{-(1-p)\beta_a} R_a\right)$$

(1)

We then took the log of the likelihood, the partial derivatives with respect to $\beta_a$ and $p$, set each equal to zero, and did a considerable amount of algebra (see Appendix 1 for details). This resulted in the following estimator:

$$\hat{\beta}_a = \bar{N}_a + \ln(M_a + R_a) - \ln(M_a)$$

(2)

We validated this estimator by simulating data under $\beta_a = 0.5, 1, 1.5,$ and $2$, and we compared it to two reasonable alternatives: the “zero-class” estimator ($\hat{\beta} = -\log_e(p_{zero})$, where $p_{zero}$ is the proportion of sampled wells that had plaque counts of zero) and the “mean-count” estimator ($\hat{\beta} = \bar{X}$, where $X$ is the mean among observed plaque counts, applied only to wells sampled before we witnessed any lysis events). The results (which we will not cover in detail) revealed that equation (2) had a very slightly positive bias ($\approx 3.5\%$ when $\beta_a = 0.5$, falling to $\approx 1.5\%$ at $\beta_a = 2$) but a lower mean squared error.
than the other estimators across conditions. Valuing the nearness of the estimate to the truth above all else, we adopted the new estimator of $\beta_a$ throughout.

**Time to lysis:** We considered two nested models describing time to lysis. Both assumed that phages do not lyse cells until the latent period of $t_0$ ends. The exponential model assumed that lysis occurred with equal probability each moment after $t_0$, while the Weibull model allowed the probability of lysis to change with time. Under the Weibull, the time to lysis, $T$, had the following distribution:

$$P(T < t) = w_t = 1 - e^{-(t-t_0)/\lambda^\alpha}$$

(3)

where $\alpha$ was a hazard parameter that dictated how lysis probability changed with time. When $\alpha = 1$, the Weibull model simplified to the exponential model. Because they are nested, we employed a likelihood ratio test to determine if the improved fit under the Weibull justified the additional parameter. To do so, we analyzed each genetically distinct clone separately. We fitted the data to each model using maximum likelihood (as described below), estimated parameters, obtained the log-likelihood, and took the difference: $LR_{obs}$. We then simulated 100 datasets under the null (exponential) parameter estimates. For each simulated dataset, we fitted the data to both models and calculated the difference in their log-likelihoods: $LR_{boot}$. We then approximated the $P$-value by the proportion of $LR_{boot}$ greater than or equal to $LR_{obs}$. Large $P$-values (i.e., $P > 0.05$) would indicate the simpler exponential model described the data sufficiently well. This test returned $P$-values greater than 0.3 for all clones, and we therefore assumed the exponential model for the duration of the study. One important implication of this assumption was that, under the exponential model, the lysis time variance was equivalent to $\lambda^2$. 
Defining $B_{\text{tar}}$ as the number of phages in well $r$, assay $a$ that had lysed their hosts by time $t$, $B_{\text{tar}}$ followed the Poisson distribution with rate $\beta_a w_t$. If we let $Y_{\text{tar}}$ be an indicator function that is one when $B_{\text{tar}}$ is greater than or equal to one and zero when $B_{\text{tar}}$ is zero, and we let $U_{\text{tar}}$ be the number of phages that had not lysed their hosts in well $r$, assay $a$ at time $t$, we show in Appendix 2 that the log-likelihood of the data was proportional to

$$\ln L(\lambda, t_0) \sim \sum_a \sum_t \sum_r -(1 - Y_{\text{tar}}) \beta_d w_t + Y_{\text{tar}} \ln \left(1 - e^{-\beta_d w_t}\right) + (1 - Y_{\text{tar}}) (U_{\text{tar}} \ln((1 - w_t)\beta_d) - \beta_d (1 - w_t)).$$

(4)

Note that $\sum_a$ directed us to sum over all assays of a particular clone. We analyzed each clone separately. We have omitted a clone subscript throughout to avoid further complicating notation. In practice, we obtained joint maximum likelihood estimates of $\lambda$ and $t_0$ by establishing a grid of joint values at narrow intervals, calculating the log-likelihood of each using (3), and selecting those values that together give the best likelihood. Confidence intervals on $\lambda$ and $t_0$ were obtained by parametric bootstrap. To do this, we used $\beta_a$, $\hat{\lambda}$, and $\hat{t}_0$ to simulate one suite of datasets to match (in number and well count) the real datasets for the clone under analysis. We analyzed the simulated suite of datasets using (4) and a grid of potential values to obtain $\hat{\lambda}_{\text{boot}}$ and $\hat{t}_{0,\text{boot}}$. The one condition we imposed in this estimation step was that $\hat{t}_{0,\text{boot}}$ must be less than or equal to the minimum observed lysis time from the real data. If we did not do this, the bootstrap would have returned values and hence confidence bounds for $t_0$ greater than this minimum lysis time, and we knew such values would not be valid confidence bounds when the probability of the real data under them is zero. We repeated 500 bootstrap replicates. We then sorted the $\hat{\lambda}_{\text{boot}}$ values and defined the 95% confidence interval as

15
the central 95% of the values. We independently sorted the $\hat{t}_{0,\text{boot}}$ values and used the central 95% of these. Thus, the confidence intervals on $\hat{\lambda}$ and $\hat{t}_0$ were not joint. We did not validate that these intervals capture the truth greater than or equal to 95% of the time, but we did confirm that the means of $\hat{\lambda}_{\text{boot}}$ and $\hat{t}_{0,\text{boot}}$ were close to $\hat{\lambda}$ and $\hat{t}_0$, respectively, suggesting that the parametric bootstrap was a valid technique.

We also were interested in inferring mean lysis times from our data. Given the unique properties of exponential distributions, we could derive each clone’s mean lysis time simply by summing its estimated $t_0$ and $\lambda$. A confidence interval for this estimate could be obtained by repeating this procedure using the upper and lower bounds on the individual parameter estimates.

We now turn our attention to how to determine whether lysis time parameters $\lambda$ and $t_0$ are different for different clones. We conducted several formal tests for differences between clones using a likelihood ratio framework. To test whether both $\lambda$ and $t_0$ clones, we note that, under the null hypothesis, a single shifted exponential function would describe the data from both clones being compared; the alternative hypothesis asserted the data for each clone came from a distinct function. We calculated the log-likelihood of the data under the null ($L_{\text{Null}}$) by pooling both clones’ data, fitting as just described to obtain $\hat{\lambda}_{\text{Null}}$ and $\hat{t}_{0(\text{null})}$, and then using (4). We calculated the log-likelihood under the alternative ($L_{\text{Alt}}$) by doing the same thing, except we fitted each clone’s datasets separately, calculated their log-likelihoods separately, and then summed. We define $\Lambda_{\text{Obs}} = L_{\text{Null}} - L_{\text{Alt}}$. To determine whether $\Lambda_{\text{Obs}}$ is significantly large, we needed the distribution of $\Lambda$ under the null. We obtained this by simulation. We simulated data analogous in size and sample times to the pooled data using $\hat{\lambda}_{\text{Null}}$ and $\hat{t}_{0(\text{null})}$, repeated the fitting process, and calculated log-likelihoods and their difference to yield $\Lambda_{\text{Sim}}$. We repeated this process 500 times and took the approximate $P$-value of $\Lambda_{\text{Obs}}$ as the proportion of bootstrap simulations where $\Lambda_{\text{Sim}}$ was greater than $\Lambda_{\text{Obs}}$. 

16
To test whether just $\lambda$ differed between pairs of clones, we used a similar likelihood ratio test. Under the null, both clones shared a common value of $\lambda$ but could have different values of $t_0$; under the alternative, both $\lambda$ and $t_0$ could differ between clones. Note that the null did not fix $\lambda$ at a single specific value; rather, it merely stipulated that $\lambda$, whatever value that may be, be shared by the clones being compared. We fitted the data to the null by finding the maximum likelihood estimates for each clone under the constraint of a shared value of $\lambda$ and unconstrained values of $t_0$ and then summing their log likelihoods ($L_{Null}$). We fitted the alternative by imposing no constraints and summing the two log-likelihoods ($L_{Alt}$). We defined $\Lambda_{Obs} = L_{Alt} - L_{Null}$. To obtain the distribution of $\Lambda$ under the null, we simulated 500 datasets using the null parameter estimates, repeated the model-fitting exercise for each, and took the difference in the log-likelihood: $\Lambda_{Boot}$. The proportion of the bootstrap replicates where $\Lambda_{Boot}$ was greater than $\Lambda_{Null}$ was the approximate $P$-value for this comparison. To test for significant differences in $t_0$, we repeated this procedure, except $t_0$ was constrained and $\lambda$ was allowed to vary.

To test whether the distribution of $P$-values calculated for each set of pairwise parameter comparisons diverged from the null expectation (i.e., a uniform distribution), we binned the $P$-values into 4 categories ([0.00,0.25), [0.25,0.50), [0.50,0.75), and [0.75,1.00]) and conducted an exact multinomial test against expected frequencies in each bin under the null. The null bin frequencies were equal across the four categories (i.e., 25% in each bin).

**Burst size as a function of time:** We modeled the potential for burst size to change with time as a linear function, $D_T = \mu + \alpha T + \epsilon$, where $D_T$ was the expected burst size when $T$ minutes had elapsed since $t_0$, $\mu$ was the burst size at $t_0$, $\alpha$ was the rate of increase in burst size (i.e., the slope parameter), and $\epsilon$ captured the normally distributed
variation from expectation (mean 0, variance $\sigma^2$). Note that none of the terms in this model was observable. If a well sampled at time $t$ showed a lysis event, we knew that lysis occurred before $t$, but we did not know exactly when. In addition, we did not know how many phage particles initially were in the well. Thus, both lysis time and burst size were hidden variables. In Appendix 3, we show that the observed count (rather than the burst size) could, like burst size, be expressed as a linear function:

$$C_{\text{tar}} = \alpha X_{\text{tar}} + \mu B_{\text{tar}} + \delta$$

(5)

where $C_{\text{tar}}$ was the count observed in well $r$ sampled at time $t$ from assay $a$, $B_{\text{tar}}$ was the number of phage particles in well $r$ (assay $a$) that had lysed their hosts by time $t$, $X_{\text{tar}}$ was the sum of their lysis times, and $\delta$ was normally distributed with mean zero and variance $B_{\text{tar}}\sigma^2$. Our goal was to estimate the parameters $\mu$, $\alpha$ and $\sigma$, but because (5) depended on the unobservable $X_{\text{tar}}$ and $B_{\text{tar}}$, we could not use standard regression. Instead, we employed an Expectation Maximization (EM) algorithm, through which the problem was circumvented by replacing $X_{\text{tar}}$ and $B_{\text{tar}}$ with their expected values before doing regression and then iterating (Meng and Rubin 1993). More specifically, our EM algorithm involved the following steps: (1) guess values for $X_{\text{tar}}$ and $B_{\text{tar}}$; (2) estimate the parameters $\mu$, $\alpha$, and $\sigma$ by least-squares regression (equivalent to maximum likelihood) assuming the guesses of $X_{\text{tar}}$ and $B_{\text{tar}}$ are correct; (3) assuming the parameter estimates are correct, calculate the expected values of $X_{\text{tar}}$ and $B_{\text{tar}}$ and update the guesses to these values; and (4) repeat steps 2 and 3 until convergence occurs. It turns out that both parameter estimation (step 2) and calculating expectations (step 3) involved a fair amount of math; we present this in Appendix 3.

To obtain confidence intervals and test for differences between clones in the regression parameters, we used a parametric bootstrap approach. For each genetically distinct
clone, we simulated 1000 datasets using the estimated parameters and, each time, estimated $\alpha$, $\mu$ and $\sigma$. We then sorted the bootstrap set of parameter values individually and used the 25th and the 975th values (i.e., the central 95%) to define the confidence intervals. To keep our analysis simpler, we did not conduct formal likelihood ratio tests for significant differences between clones for these parameters. Instead, we compared confidence intervals and applied the conservative criteria that when confidence intervals were non-overlapping, the two clones were significantly different.

Data and code required to repeat this analysis can be found in the supplementary material.

**RESULTS**

Thirty-seven assays were conducted for the eight clones (four to six each). (Raw data from the assays are available in the Supporting Information.) The estimated Poisson parameter $\beta_a$ ranged from 0.20 to 0.91 with a mean of 0.46. It follows that approximately 63% of all wells had no phage particles, 29% had one particle, 6.5% had two particles, 1% had three, and 0.1% had four. Observed plaque counts ranged from zero to 1270, with eight being the smallest plaque count classified as representing a lysis event (see **statistical methods**).

Across the clones, the earliest times at which lysis was observed ranged from 11.0 min ($pos6$) to 14.5 min ($mut321$, $mut323$). Estimates of $t_0$ are shown in Table 3. This range of latent periods was consistent with past results, as reported in Bernhardt et al. 2002 (one-step growth assays in Fig. 2A in Bernhardt et al. 2002, about 12 min for wild type, $pos6$, and $pos4B$ infections, though about 22min for $pos5$) and Brown et al. 2010 (about 12-14 min for the $D$-promoter mutants and wild type).

The smallest observed value of $\lambda$ among the clones was 4.8 min ($mut321$), meaning that the cumulative probability of lysis after $t_0$ rose most rapidly for this clone under
the experimental conditions. The wild type was found to have the largest $\lambda$ (11.2 min). Values of $\lambda$ for other clones, along with confidence intervals, can be found in Table 3.

Figure 3 illustrates how the observed increase over time in the proportion of wells showing lysis events was used to estimate the cumulative lysis probability function for each clone. The figure also includes pairwise comparisons of each clone with the wild type, demonstrating clear differences between them. Note that Figure 3 does not visualize the $\beta$ parameter, which, in addition to the lysis events shown, was important in the estimation of lysis time phenotypes. Taken together, the curves form three rough clusters: (1) $pos6$, (2) wild type and $pos4B$, and (3) the remaining clones. This clustering can be observed clearly when the parameters composing those functions, $\lambda$ and $t_0$, are plotted against each other (Figure 4).

These visual comparisons were formalized into pairwise statistical tests. Along with parameter estimates, Table 3 shows the pairwise $P$-values associated with the null hypothesis that the clones being compared came from the same function (i.e., that jointly, both $\lambda$ and $t_0$ are the same for the clones). If either parameter differed, this test would reject the null. By contrast, Table 4 shows the results of two different sets of tests: the set of pairwise tests of the null in which $\lambda$ was held to be the same between clones but $t_0$ was allowed to differ and a similar test in which $t_0$ was held to be the same but $\lambda$ was allowed to differ. In other words, the first of these two sets of tests gauged differences between clones’ values of $\lambda$, whereas the second set of tests gauged differences between clones’ values of $t_0$.

Notice that we have not corrected for multiple tests. Our $P$-values are based on 500-1,000 bootstrap replicates, whereas the amount of replication needed to achieve significance under a Bonferroni correction would be in the 5,000-10,000 range and computationally infeasible. We note, however, that if the joint null hypothesis were universally true (i.e., all clones followed the same function), we would expect the $P$-values of the 28
pairwise tests to follow a uniform distribution, and we would expect just one or two false positives \((28 \times 0.05 = 1.4)\). Indeed, we would be unlikely to have more than three false positives, since the binomial probability of greater than three false rejections is 0.0491. Instead, the distribution of \(P\)-values for the joint \(\lambda-t_0\) comparisons skewed strongly from a uniform distribution (exact multinomial test, \(P < 0.0001\) (McDonald 2014)), and we observed 13 pairwise results less than or equal to 0.05 (Table 3). (The binomial probability of 13 or more false rejections is \(\sum_{i=13}^{28} \binom{28}{i} 0.05^i (1-0.05)^{28-i} = 2.24 \times 10^{-10}\.) We suggest, therefore, that most though perhaps not all of the differences that appeared as significant were truly significant. Similarly, the tests for pairwise differences in only \(\lambda\) and only \(t_0\) yielded 10 and 13 values less than or equal to 0.05, respectively, which is far more than expected under the null of a uniform distribution (exact multinomial test, \(P = 0.0068\) and \(P < 0.0001\), respectively). (The probability of 10 or more false rejections is \(5.56 \times 10^{-7}\), and the probability of 13 or more is \(2.24 \times 10^{-10}\); Table 4.)

We also note that our omission of adsorption time from our statistical model (see METHODS) had little effect on these findings. By ignoring adsorption time, we treated the time to lysis as a single exponential process, when in reality it involves the sum of two exponential waiting times (Abedon et al. 2001). However, additional analysis (not shown) demonstrated that, because adsorption happens at least an order of magnitude faster than lysis (Brown et al. 2013), the distribution of lysis time under the one-exponential model is very similar to that generated by a more complex two-exponential model. Furthermore, the variance of our one-exponential model is slightly larger than that of the two-exponential model, which implies that tests underlying the pairwise phenotypic comparisons were conservative. To see why, recall that the variance of an exponential with mean \(\lambda\) is \(\lambda^2\). The variance of the sum of two independent exponentials with means \(\lambda_{\text{lysis}}\) and \(\lambda_{\text{adsorb}}\) is the sum of their individual variances: \(\lambda_{\text{lysis}}^2 + \lambda_{\text{adsorb}}^2\). But the variance of a single exponential with mean \(\lambda = \lambda_{\text{lysis}} + \lambda_{\text{adsorb}}\) is \((\lambda_{\text{lysis}} + \lambda_{\text{adsorb}})^2 = \)
\[ \lambda_{\text{lysis}}^2 + \lambda_{\text{adsorb}}^2 + 2\lambda_{\text{lysis}}\lambda_{\text{adsorb}}. \]

Therefore, although we might have gained marginally more power in our phenotypic estimates by adopting a more complex two-exponential statistical model (supplemented with data on adsorption) or following the pre-attachment protocols described in Brown et al. 2013 and Cherwa Jr. et al. 2009, the methods we employed were adequate.

The regression analysis of burst size by time revealed that burst size increased with time for all clones (Figure 5). The parameter estimates for each clone are provided in Table 5 along with confidence intervals. Figure 5 illustrates these regression parameter estimates graphically, showing that clones fell into four clusters of decreasing slope: (1) pos6, (2) pos4B and pos5, (3) wild type and mut319, and (4) mut321, mut323, and mut324. Formal testing of parameter differences was not conducted. However, where confidence intervals were non-overlapping, there was good evidence for a significant difference. For slope, the confidence intervals within the four clusters were generally overlapping while intervals between clusters were not.

Table 6 summarizes the calculated mean and variance in lysis time for each clone.

DISCUSSION

In recent years, it has been found that the probability a beneficial mutation will fix in a lytic phage population depends sensitively on the life history parameter it affects (Wahl and DeHaan 2004; Hubbarde et al. 2007; Patwa and Wahl 2008, 2009). Furthermore, analytical work has demonstrated that variance in life history traits, not just the trait means, can influence growth rate predictions, particularly in iteroparous populations far from a stable age distribution (Bull et al. 2011). Inspired by this body of work, we wished to learn more about whether bacteriophage life history traits, including higher moments (e.g., lysis time variance), could be subject to the phage’s
genetic control.

To this end, we used a serially-sampled parallel infection assay to interrogate the lysis time and burst size phenotypes of a panel of eight genetically distinct φX174 clones. Among these clones were four with mutations in the D promoter that have been reported to downregulate the transcription of several genes, including the lysis gene (E); three Epos clones with mutations in E that have been found to upregulate its expression; and the wild-type ancestor of the Epos mutants. (As noted in the EXPERIMENTAL METHODS, these mutations were introduced on slightly different genetic backgrounds.)

Using a novel statistical approach that allowed us to overcome the challenge of hidden variables, we found significant variation in both lysis time and burst size among these clones, including evidence that φX174 exerts genetic control over lysis time variance under our experimental conditions.

The inferred differences among the clones’ burst size phenotypes are generally consistent with our limited understanding of the mutations’ biological effects (Table 5, Figure 5). In accordance with their depressed rates of transcription at the D promoter (BROWN et al. 2010), burst size increased more slowly than the wild type in three of the four D-promoter mutants. (Burst size increased with lysis time at about the same rate as the wild type in the fourth D-promoter mutant, mut319.) By contrast, burst size increased more quickly than the wild type in the three Epos mutants. This may be related to BERNHARDT et al. 2002’s finding that the R3H and L19F mutations lead to markedly faster E protein synthesis than the wild type, perhaps by increasing mRNA translatability. Given φX174’s compact genome and E’s location within the D gene (Figure 2), it is possible that these mutations somehow upregulate gene expression more globally, thereby precipitating more rapid accumulation of progeny virions and larger burst sizes.

As with burst size, we inferred significant variation in lysis time phenotypes among the eight clones under the tested experimental conditions. The clones seem to differ
in their overall cumulative lysis probability functions (Table 3) and in both \( t_0 \) and \( \lambda \) individually (Table 4).

Within these differences, we observe an inverse relationship between \( t_0 \) and \( \lambda \) (Figure 4): under the tested conditions, the mutations appear to delay the onset of lysis while causing the cumulative lysis probability to rise more quickly compared to the wild type (Figure 3). (The most noteworthy exception is \( \text{pos6} \), the only clone with a value of \( t_0 \) less than that of the wild type.) A consequence of this trend is that, despite substantial differences in \( t_0 \) and \( \lambda \), the clones’ mean lysis times (computed as the sum of \( t_0 \) and \( \lambda \)) are rather similar (Table 6). A mechanistic investigation of the apparent maintenance of mean lysis time is beyond the scope of this study, but we speculate that it may reflect constraints on lysis timing imposed by the host’s cell cycle (for \( \phi X174 \): Witte et al. 1998; Young et al. 2000; and other phages as well: Hadas et al. 1997; Storms et al. 2014). While mutations in \( E \) and related genes could potentially bias lysis timing earlier or later or make the process more or less noisy, the host’s physiology and cell cycle may prevent extreme divergence from a common mean.

In fact, given the genetic and phenotypic constraints on \( \phi X174 \) life history, it is notable that burst size and lysis time phenotypes appear to diverge as much as they do. A primary genetic constraint is the phage’s compact genomic architecture. The \( E \) gene is entirely embedded within the structurally essential \( D \) gene (Barrell et al. 1976; Figure 2), and the \( D \) promoter simultaneously regulates the transcription of \( E, D \), and four other structural genes downstream (i.e., \( J, F, G, H \)) (Fane et al. 2006). The work presented here stands with and expands upon previous studies (e.g., Brown et al. 2010; Brown et al. 2013; Bernhardt et al. 2002) in demonstrating that, despite the relatively limited genotype space accessible to the organism, \( \phi X174 \)’s genomic architecture permits a significant degree of flexibility in lysis time and burst size phenotypes. Still, the genetic constraints on these phenotypes are clearly visible in the form of pleiotropic...
effects on burst size and lysis time in our data.

The inferred disparity among our clones’ lysis time parameters is also interesting given the aforementioned phenotypic constraints imposed by the host’s cell cycle and physiology. Whereas λ and many other larger DNA phages effect lysis via a tightly regulated, two-protein lysis strategy, ϕX174 and other Microviridae lyse their hosts by way of a single protein (Young et al. 2000; Young and Wang 2006). ϕX174’s lysis timing is thought to be dictated largely by the timing of host cell division and therefore, compared with phages like λ, even less subject to the phage’s genetic control (Witte et al. 1998; Young et al. 2000; Zheng et al. 2008; Dennehy and Wang 2011; Singh and Dennehy 2014; Storms et al. 2014). But our demonstration of genetic control of both lysis time mean and variance hints at the possibility of as-yet-unknown mechanisms exerting some control over phage lysis timing. Indeed, researchers have long speculated that protein E may mediate lysis timing not just through its interactions with MraY, but also through interactions with the host protein SlyD (Young and Wang 2006). SlyD plays an important role in cell division and cell cycle regulation (Roof et al. 1997) and is required for stability of wild-type E protein (Bernhardt et al. 2002).

**Strengths and limitations of our experimental and statistical approach**

The strength of the experimental and statistical approach taken in this study lies in its power and applicability. It is powerful in that, more effectively than most other methods, it simultaneously captures information related to lysis time variance and burst size as a function of time. The approach also has potential application to a range of microbiological questions. The principles and tools underlying the assay and inferential apparatus could be applied usefully to other cases in which one seeks to make inferences about dynamic, Poisson-distributed processes that are sampled destructively over a time course (e.g., tracking the fates of individual tagged mutants in a dynamic population of
However, we concede our approach has limitations. There is increasing evidence that host cells’ physiology plays a vital role in determining phage infection dynamics (Hadas et al. 1997; Rabinovitch et al. 2002; Storms et al. 2014; Bull et al. 2014). Yet in synchronizing and rapidly sampling infections, our assay exposed the cells to temperature changes and nutritional regimes on a short timescale, which may have altered cells’ physiology in ways important for the measured phenotypes (van Elsas et al. 2011). While these sorts of manipulations are not uncommon in the literature (e.g., Wang 2006; Cherwa Jr. et al. 2009), they limit the generality of some conclusions drawn from this work. The unique conditions of the assay complicate comparisons to previous estimates of the clones’ burst size and lysis time phenotypes, and they make it impractical to validate our methods against traditional single-step growth experiments with much confidence.

By the same token, the lysis times and burst sizes inferred in this work may differ from those φX174 exhibits in nature. Environmental differences in factors like host density and diversity, as well as temperature changes and nutritional regimes, could possibly lead to differences between phenotypes measured in the assay and in the wild. That said, it is unclear how well any particular assay approximates the phage’s natural environment, which remains largely uncharacterized (Michel et al. 2010). Further study of φX174’s ecological niche would be necessary to clarify which environmental differences between our assay and the wild are most relevant to the phage’s life history phenotypes.

Finally, our assumption of a linear increase in burst size with time, though substantiated by significant past work with φX174 and other phages (Hutchison and Sin-sheimer 1966; Wang et al. 1996; Wang 2006), may be subject to debate (Storms et al. 2014). We hope further research will help better evaluate the appropriateness of this assumption.
When it comes to achieving the core aims of this work, we believe the strengths of our experimental and statistical approach outweigh its limitations. It allowed us to demonstrate that, under at least one set of conditions held constant for eight similar but distinct \( \phi X174 \) clones, burst size and lysis time phenotypes (including lysis time variance) differed significantly. Furthermore, it is reasonable to assume that the degree of genetic variability among the clones is comparable to that which is the substrate for natural selection acting in the wild.

**Evolutionary implications of heritable variation in lysis time variance**

Our finding that variance in \( \phi X174 \)'s lysis time can be subject to at least some genetic control inspired us to ask, “What, then, could be the evolutionary implications of mutations affecting variance in phage lysis time?” Trivially, if burst size grows more slowly than exponentially once a cell has been infected, then a mutation increasing lysis time variance will be selectively favored, because early lysis events contribute more to population growth than late lysis events detract (Bull et al. 2011; but see Singh and Dennehy 2014). Empirically, phage burst size is thought to increase linearly (Hutchinson and Sinsheimer 1966; Wang et al. 1996; Wang 2006; but see Storms et al. 2014), immediately suggesting one mode of selection. Mutations increasing lysis time variance in a given environment likely would fix quickly under this mode of selection.

In order to control for the influence of this mode of selection, we recently have explored a model in which burst size increases exponentially in time since infection (Weinreich, Brown and Wahl, unpublished). This has allowed us to infer the intrinsic selective consequences of mutations that influence lysis time variance. As expected, in phage populations growing at their stable age-of-infection distribution in unlimited host cells (Bull et al. 2011; Bull 2006), we found such mutations to be selectively neutral, since by construction the contribution of early bursts to population growth is now exactly
balanced by (exponentially larger) late bursts.

However, the moment a mutation appears, the mutant subpopulation is by definition not at stable age distribution, and indeed we have found in simulations that mutations increasing variance in lysis time are transiently enriched in the population (Weinreich and Brown, unpublished). This effect follows from the fact that the frequency of phages with high lysis time variance exceeds (sometimes considerably) the frequency of their low-variance brethren early in each lysis cycle. While the reverse is true late in the cycle, the effect is of smaller magnitude and much shorter lived. Finally, overlaying periodic demographic bottlenecks (e.g., as employed in lab conditions but also surely in nature), we observed that mutations increasing lysis time variance can enjoy a dramatic increase in population frequency, despite their neutrality when they finally reach their stable age-of-infection distribution (Weinreich and Brown, unpublished). We therefore suggest a second mode of selection acting on mutations affecting variance in lysis time.

**Future directions**

The findings reported here suggest two complementary avenues for future investigation. One is microbiological, and it involves exploring φX174 lysis in relation to its genetic determinants and its host’s biology. It would be interesting to explore the mutants’ performance in a range of assays, such as those involving thermally-regulated microscopy (Dennehy and Wang 2011), single-cell microfluidics-based techniques (Yin and Marshall 2012), or periodic filtration and sampling (Hutchison and Sinsheimer 1963). (The last and oldest of these techniques could prove especially fruitful, since it would allow direct access to both lysis time and burst size.) In addition, it would be highly informative to use methods like those designed by Storms et al. 2014 to study how φX174 mutants’ burst sizes and lysis times vary as a direct function of the host’s cell cycle. These types of experiments could yield valuable insight into the regulation and
host-dependence of $\phi$X174 life history phenotypes.

Theoretical evolutionary problems present another opportunity for future research. Does natural selection act on lysis time variance in nature? What is the fixation probability of a mutation that affects lysis time variance, and how do populations carrying such mutations evolve in the face of periodic bottlenecks? Existing computational models could be strengthened by more thoroughly exploring the models’ parameter sensitivity and investigating how mutations that pleiotropically influence adsorption, lysis time, and burst size affect predicted population dynamics. In addition, the models must be integrated with other theoretical work exploring the fixation probabilities of mechanistically distinct beneficial mutations in lytic phages (Wahl and DeHaan 2004; Hubbarde et al. 2007; Patwa and Wahl 2008, 2009). Once elaborated and refined, models could be tested empirically through both “single-sampling” assays and longer-term evolution experiments. The former approach would assess the probability of loss of lysis time variants introduced at very low frequency in replicate phage populations, while the latter would explore whether real evolutionary outcomes match our theoretical predictions.

ACKNOWLEDGMENTS

We thank Olivier Tenaillion, Amber Stancik, Celeste Brown, Rohit Kongari, and Ry Young for providing the bacterial and phage strains used in this study. Additionally, we thank Darin Rokyta for providing us with the $E.\ coli$ starvation protocol and Sohini Ramachandran for offering feedback on an early draft of this manuscript.

We also are grateful for support from the Brown Division of Biology and Medicine to DMW, from the Brown Dean of the College to JY, and from the NSF (G-K12 Award 0638688 to T. Herbert) to support MBH. CRM and PJ were supported by grant number R01-GM076040 from the National Institutes of Health.
This research is based in part upon work conducted using the Rhode Island Genomics and Sequencing Center which is supported in part by the National Science Foundation under EPSCoR Grants Nos. 0554548 & EPS-1004057.
| Type      | Clone  | Base change(s) | Amino acid change(s) | Lysis time on E. coli C (37°C) | Via transfected plasmid |
|-----------|--------|----------------|---------------------|-------------------------------|-------------------------|
| D-Promoter | mut319 | G319T*         | V63F                | +2min                         | N/A                     |
|           | mut321 | T321C          | None                | +2min                         | N/A                     |
|           | mut323 | A323G          | N64G                | +0-2min                       | N/A                     |
|           | mut324 | C324T          | None                | +0-2min                       | N/A                     |
| Epos      | pos5   | G624T          | L19F                | +10min                        | Slightly faster         |
|           | pos6   | G575A          | R3H                 | Same                          | Much faster             |
|           | pos4B  | G575A, G624T   | R3H, L19F           | Same                          | Much faster             |

Table 1 Characteristics of φX174 D-promoter and Epos mutants.

All non-synonymous D-promoter changes are within the C gene (Brown et al. 2010, Brown et al. 2013). L19F is an amino acid change in the transmembrane region of the E protein, while R3H is a change in E’s periplasmic N-terminus. The same mutation that changes leucine to phenylalanine in E confers a A79S change in the D protein. The five background nucleotide differences between the D-promoter and Epos mutants are not included in this table (see Experimental Methods). The rightmost column refers to the bulk lysis kinetics of the mutated E genes after they had been cloned onto plasmids and inserted into E. coli K-12 slyD+ cells (Bernhardt et al. 2002).

* indicates a change adjacent to the D promoter’s sigma factor binding site. The three other D promoter changes are within the binding site.

Relevant accession numbers are provided in the Experimental Methods.
| Symbol | Meaning |
|-------|---------|
| $\beta_a$ | The number of phages in a well from assay $a$ is Poisson with mean $\beta_a$. |
| $p$ | Probability a random phage in a randomly selected well in the experiment has not lysed its host |
| $L_a$ | # of wells examined in assay $a$ |
| $R_a$ | # of wells examined in assay $a$ that show lysis events |
| $M_a$ | # of wells examined in assay $a$ that do not show lysis events but have counts $\geq 1$ |
| $Z_a$ | # of wells examined in assay $a$ that have zero phages in them |
| $N_{ar}$ | # of phages in well $r$ from assay $a$. We reorder wells such that $r = 1, 2, \cdots, Z_a$ have 0 phages, wells $r = Z_a + 1, Z_a + 2, \cdots, Z_a + M_a$ do not show lysis events (but are non-zero), and $r = Z_a + M_a + 1, Z_a + M_a + 2, \cdots, L_a$ are wells showing lysis events |
| $\bar{N}_a$ | Mean count in wells not showing lysis events; $\sum_{r=Z_a+1}^{M_a} N_{ar}/M_a$ |
| $T_{iar}$ | Time to lysis of phage $i$ in well $r$ from assay $a$ |
| $D_T$ | Burst size of a phage that lysed its host at time $T$ |
| $C_{tar}$ | Observed count in well $r$ sampled in assay $a$ at time $t$ |
| $B_{tar}$ | The number of phages in well $r$, assay $a$, that have lysed their hosts by time $t$ when well is sampled; $B_{tar} = \sum_{i=1}^{N_{ar}} I\{T_{iar} \leq t\}$ |
| $Y_{iar}$ | An indicator function that is 1 when $B_{tar} > 0$ and 0 when $B_{tar} = 0$ |
| $U_{tar}$ | The number of phages in well $r$, assay $a$, yet to lyse their hosts when sampling occurs at time $t$; $U_{tar} = N_{ar} - B_{tar}$ |
| $X_{tdr}$ | Sum of lysis times for all phages in well $r$, assay $a$, that have lysed their hosts prior to sampling at time $t$; $X_{tdr} = \sum_{i=1}^{N_{ar}} I\{T_{iar} \leq t\}$ |
| $\alpha, \mu, \sigma$ | Parameters describing the assumed linear relationship between burst size and time $C_T = \mu + \alpha T + \epsilon$ where $\epsilon \sim N(0, \sigma^2)$ |
| $t_0$ | Latent period. For $t < t_0$, the lysis rate = 0; for $t > t_0$ it is constant per unit time |
| $w_t$ | The probability a phage will have lysed its host by time $t$ |
| $\lambda$ | Parameter relating lysis probability to time; $w_t = 1 - e^{-(t-t_0)/\lambda}$ |

Table 2 Summary of notation used in statistical modeling.
Table 3 Estimates of $\lambda$ and $t_0$ for each clone.

95% confidence intervals are in parentheses. The diagonal matrix to the right gives the $P$-values (based on 500 bootstrap replicates) of tests of the null hypothesis, which holds that the clones being compared have the same joint $\lambda$ and $t_0$ values. $P$-values less than 0.050 are shaded black; between 0.051 and 0.150 are shaded grey; greater than 0.151 are shaded white.
Table 4 Approximate *P*-values of pairwise comparisons of individual parameters in lysis probability by time functions.

Above the diagonal are results of tests of the null in which λ is held to be the same between clones but *t₀* is allowed to differ. Below the diagonal are results of similar tests in which *t₀* is held to be the same but λ is allowed to differ. In other words, the first of these two sets of tests gauges differences between clones’ values of λ (above the diagonal), whereas the second set of tests gauges differences between clones’ values of *t₀* (below the diagonal). Based on 500 bootstrap replicates each. *P*-values less than 0.050 are shaded black; between 0.051 and 0.150 are shaded grey; greater than 0.151 are shaded white.
Table 5 Parameter estimates and confidence intervals for burst size by time regression analysis.

Each confidence interval based on 1000 bootstrap replicates.

| Clone | Slope ($\hat{\alpha}$) (PFU/min) | Intercept ($\hat{\mu}$) (PFU) | $\hat{\sigma}$ (PFU) |
|-------|----------------------------------|-----------------------------|---------------------|
| WT    | 16.7 (15.1, 18.4)                | 28.4 (19.9, 36.5)           | 19.9 (15.6, 23.5)   |
| 319   | 19.3 (19.0, 19.6)                | 12.6 (11.4, 13.8)           | 3.2 (2.6, 3.7)      |
| 321   | 11.3 (10.7, 12.0)                | 13.9 (11.6, 16.1)           | 7.4 (6.2, 8.3)      |
| 323   | 12.4 (11.7, 13.1)                | 14.8 (12.1, 17.5)           | 7.2 (5.8, 8.3)      |
| 324   | 8.7 (7.6, 9.8)                   | 27.5 (23.0, 32.3)           | 13.0 (10.7, 15.2)   |
| 5     | 26.7 (25.0, 28.3)                | 34.6 (28.1, 41.0)           | 17.5 (14.4, 20.2)   |
| 6     | 31.6 (31.1, 32.2)                | 8.0 (5.3, 10.7)             | 7.2 (5.8, 8.4)      |
| 4B    | 25.1 (24.5, 25.6)                | 16.7 (13.9, 19.2)           | 6.6 (5.3, 7.7)      |

Table 6 Estimates of lysis time mean and variance of $\phi X 174$ clones.

95% confidence intervals are in parentheses. Mean lysis time = $t_0 + \lambda$. Variance in lysis time = $\lambda^2$.  

| Lysis time | Clone | Mean (min) | Variance (min$^2$) |
|------------|-------|------------|--------------------|
|            | WT    | 23.4 (18.5, 28.4) | 125.4 (56.3, 231.0) |
|            | 319   | 18.5 (16.0, 20.8) | 33.6 (16.0, 60.8) |
|            | 321   | 19.3 (17.5, 21.0) | 23.0 (12.3, 38.4) |
|            | 323   | 20.1 (17.7, 22.7) | 38.4 (20.3, 72.3) |
|            | 324   | 19.4 (16.7, 21.7) | 36.0 (17.6, 67.2) |
|            | 5     | 18.9 (16.5, 21.0) | 32.5 (16.0, 56.3) |
|            | 6     | 18.9 (15.2, 23.0) | 79.2 (38.4, 148.8) |
|            | 4B    | 22.2 (18.8, 25.5) | 90.3 (49.0, 156.3) |
Here, sampling was performed at 15-second intervals between 5 and 28.75 minutes. At each
time point, three outcomes were possible: zero plaques, a “few” plaques (here, fewer than eight;
see statistical methods), and more than a few. Assuming 100% plating efficiency, these
outcomes implied respectively that no phage particles were added to the corresponding host
cell volume, that the visible number of phage particles was added but none lysed its host by the
time of sampling, and that at least one infected cell had been lysed by the time of sampling.
In these data, the first sample with more than a few plaques occurred at 15.5 min, giving an
upper bound for the time to lysis ($t_0$). Inset illustrates plaque counts prior to the first observed
lysis event. The absence of corresponding stratification among plaque counts after 15.5 min
implied high variance in burst size. Similarly, the two plaques observed at 23.5 min (arrow)
corresponded to two phage particles that hadn’t yet lysed their hosts by that time, implying
high variance in lysis time. Thus, these data contain information about higher moments in both
burst size and lysis time.
Figure 2 Positions of mutations in φX174 genome.

The figure shows a 832-bp region of interest, including genes C, D, E, and J. Gene C is involved in DNA replication, gene D is an external scaffolding protein required for procapsid morphogenesis, gene E is responsible for lysis of the host, and gene J is required for DNA packaging (Fane et al. 2006). Gene E is shown in three sections, corresponding to three domains of the protein. The transmembrane domain binds the E protein’s substrate, MraY (Mendel et al. 2006; Zheng et al. 2009). Mutations are indicated with pins at the position, and the site and nucleotide change are indicated to the right of the pin head. The clone(s) carrying each mutation is indicated in parentheses. Note that one of the five background nucleotide differences between the Epos and D-promoter mutants (G833A) is included in this figure.
Figure 3 (Caption on next page.)
Figure 3 (Previous page.)

Estimated cumulative lysis probability by time functions, pairwise comparisons of each clone with the wild type.

The wild type panel (top left) provides an example of fitting a cumulative lysis probability by time function to wild-type data. The grey line is the best-fit curve. Dotted lines represent 100 of the 500 bootstrap replicates. Each bubble’s position shows the proportion of wells sampled at a given time in which we observe a lysis event; the size of each bubble shows the number of samples on which the proportion is based. In all other panels, the wild type is represented with a grey line and grey bubbles.
Figure 4 Estimates of parameters $\lambda$ and $t_0$ that define the lysis probability function for wildtype and each mutant.

Bars represent the approximate 95% confidence intervals for each parameter independent of the other (based on 500 bootstrap replicates).
Figure 5  
**Estimated burst size by time relationships for wild type and each mutant.**
LITERATURE CITED

ABEDON, S. T., 1989 Selection for bacteriophage latent period length by bacterial density: a theoretical examination. Microb. Ecol. 18: 79–88.

ABEDON, S. T., 1992 Lysis of lysis-inhibited bacteriophage T4-infected cells. Journal of Bacteriology 174: 8073–8080.

ABEDON, S. T., T. D. HERSCHLER and D. STOPAR, 2001 Bacteriophage latent-period evolution as a response to resource availability. Appl. Environ. Microbiol. 67: 4233–4241.

AFSHINNEKOO, E., C. MEYDAN, S. CHOWDHURY, D. JAROUDI, C. BOYER et al., 2015 Geospatial resolution of human and bacterial diversity with city-scale metagenomics. Cell Systems 1: 1–15.

BARRELL, B. G., G. M. AIR and C. A. H. III, 1976 Overlapping genes in bacteriophage φX174. Nature 264: 34–41.

BERNAL, R. A., S. HAFENSTEIN, R. ESMERALDA, B. A. FANE and M. G. ROSSMANN, 2004 The φX174 protein J mediates DNA packaging and viral attachment to host cells. Journal of Molecular Biology 337: 1109–1122.

BERNHARDT, T. G., W. D. ROOF and R. YOUNG, 2002 The Escherichia coli FKBP-type PPIase SlyD is required for the stabilization of the E lysis protein of bacteriophage φX174. Molecular Microbiology 45: 99–108.

BROWN, C. J., A. D. STANCIK, P. ROYCHOUDHURY, S. M. KRONE et al., 2013 Adaptive regulatory substitutions affect multiple stages in the life cycle of the bacteriophage φX174. BMC Evolutionary Biology 13: 1–12.
BROWN, C. J., L. ZHAO, K. J. EVANS, D. ALLY and A. D. STANCIK, 2010 Positive selection at high temperature reduces gene transcription in the bacteriophage φX174. BMC Evolutionary Biology 10: 378.

BULL, J. J., 2006 Optimality models of phage life history and parallels in disease evolution. Journal of Theoretical Biology 241: 928–938.

BULL, J. J., R. H. HEINEMAN and C. O. WILKE, 2011 The phenotype-fitness map in experimental evolution of phage. PLoS One 6: e27796.

BULL, J. J., J. MILLSTEIN, J. ORCUTT and H. WICHMAN, 2006 Evolutionary feedback mediated through population density, illustrated with viruses in chemostats. The American Naturalist 167: E39–E51.

BULL, J. J., D. W. PFENNIG and I.-N. WANG, 2004 Genetic details, optimization and phage life histories. Trends in Ecology and Evolution 19: 76–82.

BULL, J. J., C. S. VEGGE, M. SCHMERER, W. N. CHAUDHRY and B. R. LEVIN, 2014 Phenotypic resistance and the dynamics of bacterial escape from phage control. PLoS One 9: e94690.

BURNET, F., 1929 A method for the study of bacteriophage multiplication in broth. British Journal of Experimental Pathology 10: 109–115.

CHAO, L., C. U. RANG and L. E. WONG, 2002 Distribution of spontaneous mutants and inferences about the replication mode of the RNA bacteriophage φ6. Journal of Virology 76: 3276–3281.

CHERWA Jr., J. E., P. SANCHEZ-SORIA, MEMBERS OF THE UNIVERSITY OF ARIZONA VIROLOGY LABORATORY COURSE 2007, H. A. WICHMAN and B. A. FANE, 2009
Viral adaptation to an antiviral protein enhances the fitness level to above that of the uninhibited wild type. Journal of Virology 83: 11746–11750.

Dean, A. M. and J. W. Thornton, 2007 Mechanistic approaches to the study of evolution: the functional synthesis. Nature Reviews Genetics 8: 675–688.

Delbrück, M., 1945 The burst size distribution in the growth of bacterial viruses (bacteriophages). Journal of Bacteriology 50: 131–135.

Denhardt, D. T. and R. L. Sinsheimer, 1965 The process of infection with bacteriophage φX174. III. Phage maturation and lysis after synchronized infection. Journal of Molecular Biology 12: 641–646.

Dennehy, J. J. and I.-N. Wang, 2011 Factors influencing lysis time stochasticity in bacteriophage λ. BMC Microbiology 11: 174.

Fane, B. A., K. L. Brentlinger, A. D. Burch, M. Chen, S. Hafenstein et al., 2006 φX174 et al., the Microviridae. In R. Calendar and S. T. Abedon, editors, The Bacteriophages, chapter 11. Oxford University Press, 2nd edition, 129–145.

Hadas, H., M. Einav, I. Fishov and A. Zaritsky, 1997 Bacteriophage T4 development depends on the physiology of its host Escherichia coli. Microbiology 143: 179–185.

Harms, M. J. and J. W. Thornton, 2013 Evolutionary biochemistry: revealing the historical and physical causes of protein properties. Nature Reviews Genetics 14: 559–571.

Hayashi, M., A. Aoyama, D. L. Richardson and M. N. Hayashi, 1988 Biology of the bacteriophage φX174. In R. Calendar, editor, The Bacteriophages, volume 2. Plenum Press, New York, N. Y., 1–71.
Hubbarde, J. E., G. Wild and L. M. Wahl, 2007 Fixation probabilities when generation times are variable: the burst-death model. Genetics 176: 1703–12.

Hutchison, C. A. and R. L. Sinsheimer, 1963 Kinetics of bacteriophage release by single cells of φX174-infected E. coli. Journal of Molecular Biology 7: 206–208.

Hutchison, C. A. and R. L. Sinsheimer, 1966 The process of infection with bacteriophage φX174: X. Mutations in a φX lysis gene. Journal of Molecular Biology 18: 429–447.

Hyman, P. and S. T. Abedon, 2009 Practical methods for determining phage growth parameters. In M. R. J. Clokie and A. M. Kropinski, editors, Bacteriophages: Methods and Protocols, Volume 1: Isolation, Characterization, and Interactions, volume 501 of Methods in Molecular Biology, chapter 18. Humana Press, 175–202.

Labrie, S. J., M.-E. Dupuis, D. M. Tremblay, P.-L. Plante, J. Corbeil et al., 2014 A new microviridae phage isolated from a failed biotechnological process driven by Escherichia coli. Applied Environmental Microbiology 80: 6992–7000.

McDonald, J., 2014 Handbook of Biological Statistics. Sparky House Publishing, Baltimore, MD, 3rd edition.

Mendel, S., J. M. Holburn, J. A. Schouten and T. D. H. Bugg, 2006 Interaction of the transmembrane domain of lysis protein E from bacteriophage φX174 with bacterial translocase MraY and peptidyl-prolyl isomerase SlyD. Microbiology 152: 2959–2967.

Meng, X.-L. and D. B. Rubin, 1993 Maximum likelihood estimation via the ECM algorithm: A general framework. Biometrika 80: 267–268.
MICHEL, A., O. CLERMONT, E. DENAMUR and O. TENAILLON, 2010 Bacteriophage φX174's ecological niche and the flexibility of its Escherichia coli lipopolysaccharide receptor. Applied and Environmental Microbiology 76: 7310–7313.

NEWBOLD, J. E. and R. L. SINSHEIMER, 1970 Process of infection with bacteriophage φX174, XXXII. Early steps in the infection process: attachment, eclipse, and DNA penetration. Journal of Molecular Biology 49: 49–66.

PATWA, Z. and L. M. WAHL, 2008 Fixation probability for lytic viruses: The attachment-lysis model. Genetics 180: 459–470.

PATWA, Z. and L. M. WAHL, 2009 The impact of host-cell dynamics on the fixation probability for lytic viruses. Journal of Theoretical Biology 259: 799–810.

PEARSON, W., T. WOOD, Z. ZHANG and W. MILLER, 1997 Comparison of DNA sequences with protein sequences. Genomics 46: 24–36.

PEPIN, K. M., M. A. SAMUEL and H. A. WICHMAN, 2006 Variable pleiotropic effects from mutations at the same locus hamper prediction of fitness from a fitness component. Genetics 172: 2047–2056.

RABINOVITCH, A., I. FISHOV, H. HADAS, M. EINAV and A. ZARITSKY, 2002 Bacteriophage T4 development in Escherichia coli is growth rate dependent. Journal of Theoretical Biology 216: 1–4.

RABINOVITCH, A., H. HADAS, M. EINAV, Z. MELAMED and A. ZARITSKY, 1999 Model for bacteriophage T4 development in Escherichia coli. Journal of Bacteriology 181: 1677–1683.

ROFF, D., 2002 Life History Evolution. Sinauer, Sunderland, MA.
Roof, W., H. Fang, K. Young, J. Sun and R. Young, 1997 Mutational analysis of SlyD, an Escherichia coli gene encoding a protein of the FKBP immunophilin family. Molecular Microbiology 25: 1031–1046.

Sanger, F., G. M. Air, B. G. Barrell, N. L. Brown, A. R. Coulson et al., 1977 Nucleotide sequence of bacteriophage φX174 DNA. Nature 265: 687–695.

Shao, Y. and I.-N. Wang, 2008 Bacteriophage adsorption rate and optimal lysis time. Genetics 180: 471–482.

Singh, A. and J. J. Dennehy, 2014 Stochastic holin expression can account for lysis time variation in the bacteriophage λ. Journal of the Royal Society Interface 11.

Stearns, S., 1992 The Evolution of Life Histories. Oxford University Press, Oxford.

Stearns, S., M. Ackermann, M. Doebeli and M. Kaiser, 2000 Experimental evolution of aging, growth, and reproduction in fruitflies. PNAS 97: 3309–13.

Storms, Z. J., T. Brown, D. G. Cooper, D. Sauvageau and R. L. Leask, 2014 Impact of the cell life-cycle on bacteriophage T4 infection. FEMS Microbiology Letters 353: 63–68.

Storms, Z. J. and D. Sauvageau, 2014 Evidence that the heterogeneity of a T4 population is the result of heritable traits. PLoS One 9: e116235.

Tanaka, S. and W. M. Clemons Jr., 2012 Minimal requirements for inhibition of MraY by lysis protein E from bacteriophage φX174. Molecular Microbiology 85: 975–985.

Van Elsas, J. D., A. V. Semenov, R. Costa and J. T. Trevors, 2011 Survival of Escherichia coli in the environment: fundamental and public health aspects. The ISME Journal 5: 173–183.
WAHL, L. M. and C. S. DEHAAN, 2004 Fixation probability favors increased fecundity over reduced generation time. Genetics 168: 1009–18.

WANG, I.-N., 2006 Lysis timing and bacteriophage fitness. Genetics 172: 17–26.

WANG, I.-N., D. E. DYKHIUZEN and L. B. SLOBODKIN, 1996 The evolution of phage lysis timing. Evol. Ecol. 10: 545–558.

WICHMAN, H. A. and C. J. BROWN, 2010 Experimental evolution of viruses: Microviridae as a model system. Phil. Trans. R. Soc. B 365: 2495–2501.

WITTE, A., E. BRAND, P. MAYRHOFER, F. NARENDJA and L. WERNER, 1998 Mutations in cell division proteins FtsZ and FtsA inhibit φX174 protein-E-mediated lysis of Escherichia coli. Archives of Microbiology 170: 259–268.

YIN, H. and D. MARSHALL, 2012 Microfluidics for single cell analysis. Current Opinion in Biotechnology 23: 110–119.

YOUNG, R. and I.-N. WANG, 2006 Phage lysis. In R. Calendar and S. T. Abedon, editors, The Bacteriophages, chapter 10. Oxford University Press, 2nd edition, 104–125.

YOUNG, R., I.-N. WANG and W. D. ROOF, 2000 Phages will out: strategies of host cell lysis. Trends in Microbiology 8: 120–128.

ZHENG, Y., D. K. STRUCK, C. A. DANKENBRING and R. YOUNG, 2008 Evolutionary dominance of holin lysis systems derives from superior genetic malleability. Microbiology 154: 1710–1718.

ZHENG, Y., D. K. STRUCK and R. YOUNG, 2009 Purification and functional characterization of φX174 lysis protein E. Biochemistry 48: 4999–5006.
1 Appendix on Number of Phages per Well Poisson Estimator

The counts from an assay place each well into one of three categories: empty, not showing lysis events but non-zero, and showing lysis events. The probability of a well being empty is $e^{-\beta_a}$. The probability that a well has $n$ phages is $\beta_a^ne^{-\beta_a}/n!$. The probability that all $n$ phages fail to lyse their host is $p^n$ leading the to probability of $n$ in a well showing no lysis events to be $(p\beta_a)^ne^{-\beta_a}/n!$. Finally, the probability of a well showing one or more lysis events is one minus the probability of a well being empty plus the sum of all the probabilities associated with wells that don’t show lysis events, which turns out to be $1 - e^{-(1-p)\beta_a}$. The categorization of wells into three types leads to the following likelihood.

$$L(N_{a1}, N_{a2}, \cdots, N_{aM_a}, R_a) = \prod_{r=1}^{M_a} \frac{(p\beta_a)^{N_{ar}}e^{-\beta_a}}{N_{ar}!} \cdot \left(\frac{L_a}{R_a}\right) (1 - e^{-(1-p)\beta_a})^{R_a}$$

(6)

Therefore,

$$\ln L = \sum_{r=1}^{M_a} N_{ar}(\ln \beta_a + \ln p) - M_a \beta_a - \sum_{r=1}^{M_a} \ln N_{ar}! + \ln \left(\frac{L_a}{R_a}\right) + R_a \ln (1 - e^{-(1-p)\beta_a})$$

(7)

The maximum likelihood estimator is obtained by first calculating two partial derivatives,

$$\frac{\partial}{\partial \beta_a} \ln(L) = \sum_{r=1}^{M_a} \frac{N_{ar}}{\beta_a} - M_a + \frac{R_a(1 - p)e^{-(1-p)\beta_a}}{1 - e^{-(1-p)\beta_a}}$$

(8)
\[
\frac{\partial}{\partial p} \ln(L) = \sum_{r=1}^{M} N_{ar} \frac{R_{a}\beta_{a}e^{-(1-p)\beta_{a}}}{1 - e^{-(1-p)\beta_{a}}} \tag{9}
\]

and then setting both \(\frac{\partial}{\partial \beta_a} \ln(L) = 0\) and \(\frac{\partial}{\partial p} \ln(L) = 0\). Denote the solution by \(\hat{\beta}_a\) and \(\hat{p}\) respectively. The solution associated with equation (8) leads to

\[
\hat{\beta}_a = \bar{N}_a \left( \bar{N}_a + R_{a} \frac{\hat{\beta}_a e^{-(1-p)\hat{\beta}_a}}{1 - e^{-(1-p)\hat{\beta}_a}} \right) \tag{10}
\]

and the solution associated with equation (9) gives

\[
\frac{\bar{N}_a}{\hat{p}} = \frac{R_{a} \hat{\beta}_a e^{-(1-p)\hat{\beta}_a}}{M_{a} 1 - e^{-(1-p)\hat{\beta}_a}}. \tag{11}
\]

Now multiply both sides of the above equation by \(1 - \hat{p}\)

\[
\frac{\bar{N}_a(1 - \hat{p})}{\hat{p}} = \frac{R_{a} \hat{\beta}_d (1 - \hat{p}) e^{-(1-p)\hat{\beta}_a}}{M_{a} 1 - e^{-(1-p)\hat{\beta}_a}} \tag{12}
\]

Note that second term on the right side of equation (10) is equivalent to the left side of equation (12). This leads to the following simple relationship between \(\hat{\beta}_a\) and \(\hat{p}\):

\[
\hat{\beta}_a = \bar{N}_a + \bar{N}_a \frac{1 - \hat{p}}{\hat{p}} \tag{13}
\]

Thus \(\hat{p} = \bar{N}_a / \hat{\beta}_a\) and \((1 - \hat{p})\hat{\beta}_a = \hat{\beta}_a - \bar{N}_a\). We can eliminate \(\hat{p}\) from equation (10) by substituting \(\hat{\beta}_a - \bar{N}_a\) for \((1 - \hat{p})\hat{\beta}_a\) to get
\[
\hat{\beta}_a = \bar{N}_a + \frac{R_a}{M_a} \frac{(\hat{\beta}_a - \bar{N}_a)e^{-(\hat{\beta}_a - \bar{N}_a)}}{1 - e^{-(\hat{\beta}_a - \bar{N}_a)}}
\]

\[
\hat{\beta}_a - \bar{N}_a = \frac{R_a}{M_a} \frac{(\hat{\beta}_a - \bar{N}_a)e^{-(\hat{\beta}_a - \bar{N}_a)}}{1 - e^{-(\hat{\beta}_a - \bar{N}_a)}}
\]

\[
\frac{M_a}{R_a} = \frac{e^{-(\hat{\beta}_a - \bar{N}_a)}}{1 - e^{-(\hat{\beta}_a - \bar{N}_a)}}
\]

\[
\frac{M_a}{R_a} \frac{e^{-(\hat{\beta}_a - \bar{N}_a)}}{1 + \frac{M_a}{R_a}} = e^{-(\hat{\beta}_a - \bar{N}_a)}
\]

Solving (14) for \(\hat{\beta}_a\) gives the estimator,

\[
\hat{\beta}_a = \bar{N}_a + \ln(M_a + R_a) - \ln(M_a)
\]

(15)

2 Appendix on Time to Lysis

Recall that we model the probability of a phage lysing its host by time \(t\) with a shifted exponential, \(P(T < t) = w_t = 1 - e^{-(t - t_0)/\lambda}\). Define \(B_{tar}\) as the number of phages in well \(r\), assay \(a\), that have lysed their hosts by time \(t\) (when sampling occurs). \(B_{tar}\) is Poisson with mean \(\beta_d w_t\). Let \(Y_{tar}\) be an indicator with value 1 if \(B_{tar} > 0\) and 0 otherwise. Let \(U_{tar}\) be the the number of phages that have not lysed their hosts in well \(r\), assay \(a\), at time \(t\) (\(U_{tar} = N_{tar} - B_{tar}\)). Based on standard theory \(U_{tar}\) is independent of \(B_{tar}\) and \(U_{tar}\) is also Poisson distributed but with mean \(\beta_a (1 - w_t)\). However, \(U_{tar}\) is only directly observable when no phages in the well have lysed their hosts (i.e., when \(B_{tar} = 0\)). We
now show how to use the binary $Y_{tar}$ together with Poisson distributed $U_{tar}$ to estimate $\lambda$ and $t_0$. First, the likelihood of the lysis data is given by,

$$L(Y) = \prod_a \prod_t \prod_r \left( e^{-\beta_a w_t} \right)^{1 - Y_{tar}} \left( 1 - e^{-\beta_a w_t} \right)^{Y_{tar}}$$

(16)

The likelihood for the number of phages in wells not showing lysis events is given by

$$L(U|Y = 0) = \prod_a \prod_t \prod_r \left( (1 - w_t)^{Y_{tar}} \right)^{U_{tar}} U_{tar}! e^{-\beta_d(1 - w_t)}$$

(17)

However, to account for the fact that $U_{tar}$ is only observable when no lysis occurs (i.e., $Y_{tar} = 0$), we must adjust the likelihood for wells showing plaques but not showing lysis events as follows.

$$L(U|Y) = \prod_a \prod_t \prod_r \left( (1 - w_t)^{Y_{tar}} \right)^{U_{tar} (1 - Y_{tar})} U_{tar}! e^{-\beta_d(1 - w_t)(1 - Y_{tar})}$$

(18)

The combined likelihood is,

$$L(Y, U) = \prod_a \prod_t \prod_r \left( e^{-\beta_a w_t} \right)^{1 - Y_{tar}}$$

$$\left( 1 - e^{-\beta_a w_t} \right)^{Y_{tar}} \left( (1 - w_t)^{Y_{tar}} \right)^{U_{tar} (1 - Y_{tar})} U_{tar}! e^{-\beta_d(1 - w_t)(1 - Y_{tar})}$$

(19)

Dropping factorial terms that do not involve parameters and taking the log, the log-likelihood is proportional to

$$\ln L(Y, U) \sim \sum_a \sum_t \sum_r -(1 - Y_{tar}) \beta_a w_t + Y_{tar} \ln \left( 1 - e^{-\beta_a w_t} \right)$$

$$+ (1 - Y_{tar}) \left( U_{tar} \ln \left( (1 - w_t)^{\beta_a} \right) - \beta_a (1 - w_t) \right)$$

(20)
As discussed in the main text, we estimate $\lambda$ and $t_0$ by brute force where we calculate the likelihood using (3) and (20) for a large array of joint values, taking the largest as the (approximate) maximum likelihood estimates.

3 Appendix on Burst size as a Function of Size

The count observable in well $r$ from assay $a$ taken at time $t$, $C_{tar}$, can be expressed as,

$$C_{tar} = \alpha \sum_{i=1}^{N_d} T_{iar} I\{T_{tar} < t\} + \mu \sum_{i=1}^{N_d} I\{T_{iar} < t\} + \sum_{i=1}^{N_d} \epsilon_i I\{T_{iar} < t\}$$

(21)

where $T_{iar}$ is the lysis time of phage $i$ in well $r$ of assay $a$ and $I$ represents an indicator function that takes value 1 when the condition that follows is true and 0 otherwise. This simplifies to, $C_{tar} = \alpha X_{tar} + \mu B_{tar} + \delta$ where $B_{tar}$ is the number of phages initially in well $r$ of assay $a$ sampled at time $t$, $X_{tar}$ is the sum of their lysis times, and $\delta$ is normally distributed with mean zero and variance $B_{tar}\sigma^2$. Our goal is to then estimate $\mu$, $\alpha$ and $\sigma$. Because $X_{tar}$ and $B_{tar}$ are unobservable but are needed to estimate the parameters, we employ the following EM algorithm which replaces them with their expected value. Note that these steps summarize the process; we provide derivations and details in subsections below.

**EM Algorithm**

1. Begin with initial guesses for $B_{tar}^{(0)}$ and $X_{tar}^{(0)}$ for each well. Let $j = 0$. 

53
2. Given these values, estimate \( \alpha \) and \( \mu \) using least-squares.

\[
\min_{\alpha, \mu} \sum_{t,a,r} \left( C_{tar} - \alpha X_{tar}^{(j)} - \mu B_{tar}^{(j)} \right)^2 = \sum_{t,d,r} \left( C_{tar} - \hat{\alpha}_j X_{tar}^{(j)} - \hat{\mu}_j B_{tar}^{(j)} \right)^2
\]  

(22)

3. Use the least-squares estimates \( \hat{\alpha}_j \) and \( \hat{\mu}_j \) to estimate \( \sigma^2 \)

\[
\hat{\sigma}^2 = \frac{1}{n} \sum_{t,a,r} \frac{(C_{tar} - \hat{\alpha}_j X_{tar}^{(j)} - \hat{\mu}_j B_{tar}^{(j)})^2}{B_{tar}^{(j)}}
\]  

(23)

4. Impute the values \( X_{tar}^{(j)} \) and \( B_{tar}^{(j)} \) using their posterior expected values. Details below.

\[
X_{tar}^{(j+1)} = E(X_{tar}^{(j)}|C_{tar}, \hat{\alpha}_j, \hat{\mu}_j, \hat{\sigma}^2)
\]  

(24)

\[
B_{tar}^{(j+1)} = E(B_{tar}^{(j)}|C_{tar}, \hat{\alpha}_j, \hat{\mu}_j, \hat{\sigma}^2)
\]  

(25)

5. Return to step 2, replacing \( j \) with \( j + 1 \), until convergence occurs.

**Least squares estimates for \( \alpha \) and \( \mu \) (step 2 in EM Algorithm)** Because the error is normal, values of \( \mu \) and \( \alpha \) that minimize the sum of squared differences between observed and expected counts provide the maximum likelihood estimates. The function that expresses this sum of squares is,

\[
g(\alpha, \mu) = \sum_{t,a,r} \frac{(C_{tar} - \alpha X_{tar} - \mu B_{tar})^2}{B_{tar}}
\]  

(26)

Then

\[
\frac{\partial}{\partial \alpha} g(\alpha, \mu) = \sum_{t,a,r} \frac{-2X_{tar}}{B_{tar}} (C_{tar} - \alpha X_{tar} - \mu B_{tar})
\]  

(27)

\[
\frac{\partial}{\partial \mu} g(\alpha, \mu) = \sum_{t,a,r} -2(C_{tar} - \alpha X_{tar} - \mu B_{tar})
\]
Set $\frac{\partial}{\partial \alpha} g(\alpha, \mu) = 0$ and $\frac{\partial}{\partial \mu} g(\alpha, \mu) = 0$ and solve. The solutions are the least squares estimates denoted by $\hat{\alpha}$ and $\hat{\mu}$ and satisfy the following set of linear equations.

\[
\sum_{t,a,r} \frac{X_{tar} C_{tar}}{B_{tar}} = \hat{\alpha} \sum_{t,a,r} \frac{X_{tar}^2}{B_{tar}} + \hat{\mu} \sum_{t,a,r} X_{tar}
\]

(28)

\[
\sum_{t,a,r} C_{tar} = \hat{\alpha} \sum_{t,a,r} X_{tar} + \hat{\mu} \sum_{t,a,r} B_{tar}
\]

For notational convenience let

\[
A = \sum_{t,a,r} \frac{X_{tar}^2}{B_{tar}} \quad B = \sum_{t,a,r} X_{tar}
\]

\[
C = \sum_{t,a,r} \frac{X_{tar} C_{tar}}{B_{tar}} \quad E = \sum_{t,a,r} B_{tar}
\]

(29)

\[
F = \sum_{t,a,r} C_{tar}
\]

The estimates are then,

\[
\hat{\alpha} = \frac{CE - BF}{AE - B^2}
\]

(30)

and

\[
\hat{\mu} = \frac{CB - AE}{B^2 - AE}
\]

(31)

**Conditional expectations (step 4 in EM Algorithm)** Now we will calculate the expected number of phages conditional on the size of the burst $E(B_{tar}|C_{tar}, \hat{\alpha}, \hat{\mu}, \hat{\sigma}^2)$ as well as the expected sum of lysis times given the burst size $E(X_{tar}|C_{tar}, \hat{\alpha}, \hat{\mu}, \hat{\sigma}^2)$. Throughout this section, assay $a$ and well $r$ will be fixed; for notational convenience we will drop those subscripts and just denote $C_{tar}, B_{tar}, T_{iar}, N_{ar},$ and $X_{tar}$ by $C_t, B_t, T_i, N, \text{and} X_t$, respectively. Because we will be simulating the joint distribution of $B_t, X_t$ conditional on $C_t$ we will sometimes use a second subscript. In this case $B_{ij}, X_{ij}, T_{ij}, N_j$ will represent
the \( j \)th simulated value. To do this, we will first need the joint distribution of \( X_t \) and \( B_t \) which we obtain through the following simulation algorithm.

1. Simulate \( N_j \) from a Poisson distribution with mean \( \beta_a \). If \( N_j = 0 \) repeat, otherwise proceed.

2. simulate \( T_{1j}, T_{2j}, \cdots, T_{N_j} \) from a shifted exponential with mean \( \hat{\lambda} \), and shift \( \hat{t}_0 \) estimated using the likelihood equation (4).

3. Calculate \( B_{tj} = \sum_{i=1}^{N_j} I\{T_{ij} \leq t\} \) and \( X_{tj} = \sum_{i=1}^{T_j} I\{T_{ij} \leq t\} \) for each value of \( t \) in the assay. Round values of \( X_{tj} \) to the nearest tenth of a minute (for values < 1 minute), the nearest quarter minute for values between 1 and 5 minutes, and the nearest minute for values between 5 and 100 minutes. Store.

4. Repeat 1 through 3 for \( j = 1, 2, \cdots, M \) where we set \( M = 100,000 \).

5. For each value of \( t \), convert the observed joint values of \((B_{tj}, X_{tj})\) to proportions.

The above algorithm generates the empirical distribution \( f_{B_t, X_t}(b_t, x_t|\lambda, t_0) \). We note that the distribution for \( C_t \) conditional on \( B_t \) and \( X_t \) is normally distributed with mean \( \alpha X_t + \mu B_t \) and variance \( B_t \sigma^2 \). Denote this distribution by

\[
f_{C_t}(c_t|b_t, x_t, \alpha, \mu, \sigma^2) = \frac{1}{\sqrt{2\pi b_t \sigma}} \exp\left\{ \frac{(c_t - \alpha x_t - \mu b_t)^2}{2b_t \sigma} \right\} \tag{32}\]

Then

\[
E(X_t|C_t, \alpha, \mu, \sigma^2) = \sum_{x_t} \sum_{b_t} \frac{x_t f_{B_t, X_t}(b_t, x_t|\lambda, t_0) f_{C_t}(c_t|b_t, x_t, \alpha, \mu, \sigma^2)}{f_{C_t}(c_t)} \tag{33}\]

and

\[
E(B_t|C_t, \alpha, \mu, \sigma^2) = \sum_{x_t} \sum_{b_t} \frac{b_t f_{B_t, X_t}(b_t, x_t|\lambda, t_0) f_{C_t}(c_t|b_t, x_t, \alpha, \mu, \sigma^2)}{f_{C_t}(c_t)} \tag{34}\]
where
\[
f_{C_t}(c_t) = \sum_{x_t} \sum_{b_t} f_{C_t}(c_t | b_t, x_t, \alpha, \mu, \sigma^2) f_{B_t,X_t}(b_t, x_t | \lambda, t_0)
\]  \hspace{1cm} (35)

Using (33) and (34) we can calculate the expected values of $X_{tar}$ and $B_{tar}$ for use in the EM algorithm.