The Lifetimes of Spiral Patterns in Disc Galaxies

J. A. Sellwood¹*
¹Rutgers University, Department of Physics & Astronomy, 136 Frelinghuysen Road, Piscataway, NJ 08854-8019, USA

17 August 2010

ABSTRACT
The rate of internally-driven evolution of galaxy discs is strongly affected by the lifetimes of the spiral patterns they support. Evolution is much faster if the spiral patterns are recurrent short-lived transients rather than long-lived, quasi-steady features. As rival theories are still advocated based on these two distinct hypotheses, I review the evidence that bears on the question of the lifetimes of spiral patterns in galaxies. Observational evidence from external galaxies is frustratingly inconclusive, but the velocity distribution in the solar neighbourhood is more consistent with the transient picture. I present simulations of galaxy models that have been proposed to support quasi-steady, two-arm spiral modes that in fact evolve quickly due to multi-arm instabilities. I also show that all simulations to date manifest short-lived patterns, despite claims to the contrary. Thus the transient hypothesis is favoured by both numerical results and the velocity distribution in the solar neighbourhood.

Key words: galaxies: evolution – galaxies: kinematics and dynamics – galaxies: spiral

1 INTRODUCTION
After bars, spirals are the most prominent features of galaxy discs. The question of whether the spirals are short- or long-lived features is of major importance to the internal evolution of disc galaxies. In this paper, I evaluate the evidence that spirals are transient features that change within a few dynamical times, but are superseded by fresh patterns in a recurrent manner.

We still lack a complete, and widely accepted, theory for the origin of spiral patterns in galaxies. There is good evidence (e.g. Kormendy & Norman 1979) that many prominent spiral patterns are found in barred galaxies, or are driven by tidal interactions, or perhaps even DM halo substructure (e.g. Dubinski et al. 2003). There seems little doubt that a tidally-driven spiral pattern evolves rapidly (e.g. Salo & Laurikainen 1993; Dobbs et al. 2010), but repeated excitations may be possible (Byrd & Howard 1992). If bars are long-lived features, as is generally believed (for a dissenting view see Bournaud, Combes & Semelin 2005), spirals driven by bars could also be long-lived patterns. However, simulations (Sellwood & Sparke 1988) suggest that spirals and bars could have differing pattern speeds, which seemed to be supported by observational data (Seigar, Chorney & James 2003; Buta et al. 2004), although the latter group recently changed their minds (Salo et al. 2010).

Although many spirals in galaxies could be driven responses, the ubiquity of the spiral phenomenon suggests others are likely to be self-excited features of discs. In particular, all driving agents can be excluded from N-body simulations of isolated stellar discs, which continue to manifest spiral patterns that must be self-excited. A satisfactory theory to account for self-excited spirals has yet to emerge, despite decades of effort (see reviews by Toomre 1977; Athanassoula 1984; Bertin & Lin 1996; Sellwood 2010a). While all agree spirals are gravitationally-driven variations in the surface density of the old stellar disc, there is no consensus even on the expected lifetimes of the patterns.

Here I first explain why the duration of spiral features is an important issue, and then elaborate on the brief discussion of the lifetimes of spirals presented by Binney & Tremaine (2008, hereafter BT08, p. 526). I evaluate four different types of evidence that bear on the question whether spirals are short- or long-lived patterns.

2 INTERNAL EVOLUTION OF GALAXY DISCS
It is not obvious that the rate of evolution of galaxy discs is affected by the lifetimes of the patterns. It may seem that if the time-averaged amplitude and pitch angle of transient spirals does not differ much from those of a steady long-lived pattern, the rate of change in the distribution of angular momentum and other quantities would be similar.

However, the rate of angular momentum transport by spiral waves is given by the rate at which wave action (Lynden-Bell & Kalnajs 1972) is transported at the group velocity (Toomre 1969), which can differ from the gravity torque by the advective Reynolds stress (BT08, Appendix

* E-mail: sellwood@physics.rutgers.edu

© 2010 RAS
J), also known as “lorry transport.” Thus angular momentum transport is substantially reduced for long-lived patterns that invoke feedback via the long-wave branch of the dispersion relation (Mark 1977), where the advective term actually reverses the sign of angular momentum transport (Sellwood 2010a). Transient spirals do not involve the long-wave branch, and therefore redistribute angular momentum more effectively.

Also, Lynden-Bell & Kalnajs (1972) showed that stars are scattered by a slowly changing potential perturbation only near resonances. More precisely, a spiral potential that is the source and sink of angular momentum changes would take a few Hubble times for a spiral of amplitude, open spirals, which therefore cannot last long if its deposition on stars in the outer disc (Lynden-Bell & Kalnajs 1972). Resonant stars of a finite-amplitude wave will experience secular changes through “surfing” on the potential variations at corotation or through a periodic forcing close to their epicyclic frequency at the Lindblad resonances.

Since exchanges between the stars and the wave take place at resonances, large changes induced by a long-lived pattern would be confined to a few narrow resonances, the most important of which, the inner Lindblad resonance, must be “shielded” to avoid fierce damping of the spiral (Mark 1972, Bertin & Lin 1996). The frequency widths of resonances of a steady wave depend only on the amplitude of the perturbation. But resonances are broader for time-dependent waves, and multiple, short-lived disturbances having a range of pattern speeds characterize the angular momenta of stars over large parts of the disc.

Any mechanism to excite spirals relies on the extraction of angular momentum from stars in the inner disc and its deposition on stars in the outer disc (Lynden-Bell & Kalnajs 1972). Resonant stars of a finite-amplitude wave change their orbits as a result of these angular momentum exchanges and a spiral wave must decay once the resonances, which are the sources and sinks that excite it, have been exhausted. Bertin (1983) found order unity angular momentum changes would take a few Hubble times for a spiral of 20% overdensity with a pitch-angle \( \sim 16^\circ \). But resonances will be depleted on a much shorter time-scale for large-amplitude, open spirals, which therefore cannot last long if the pattern speed is constant. Note that the lifetime could be lengthened by a changing pattern speed, which would allow the resonances to sweep over the disc.

Thus the question of the lifetimes of spirals is of major importance to our understanding of disc evolution. If, on the one hand, the recurrent transient picture is correct, spirals are the most important agents driving the evolution of galaxy discs: they transport angular momentum, scatter stars into non-circular orbits, and cause radial mixing (see also §6). Long-lived, quasi-steady patterns, on the other hand, which must have moderate amplitude and be tightly wrapped in order to persist, would have a much less significant effect.

3 OBSERVATIONAL EVIDENCE

Near-IR photometric images (Schweizer 1976, Block et al. 1994, Gnedin, Goodman & Frei 1995, Rix & Zaritsky 1995, Seigar & James 1998, Eskridge et al. 2002, Grosbol, Patsis & Pompei 2004) reveal that spiral patterns represent large amplitude variations in the surface brightness of the old stellar disc. Zibetti, Charlot & Rix (2009) attempt to derive a more faithful mass map from their H-band images by applying a pixel by pixel correction based on population synthesis models for the broad-band colours. All this work has confirmed that spiral patterns constitute large-amplitude density variations in the total stellar density in the disc, but says nothing about the lifetimes of the features.

Velocity maps of galaxies that can resolve inter-arm variations reveal that spiral patterns are associated with organized non-axisymmetric streaming flows (e.g. Vissers 1972, Canzian, Allen & Tilanus 1993, Shetty et al. 2007). The amplitude of the non-axisymmetric potential variations can be estimated by modeling such data (e.g. Kranz, Slyz & Rix 2004), confirming that spirals are associated with significant potential variations. But unfortunately, these measurements are again quite insensitive to the lifetimes of the spirals; the flow pattern is essentially the gas response to the instantaneous potential variation, and its qualitative features are independent of the mechanism that created it (BT08, p. 526).

Since resonances are broader for short-lived waves, the quantitative gas response may depend weakly on the duration of the wave, but any attempt to use this as a discriminant between theories would require an accurate independent estimate of the spiral amplitude.

Gnedin, Goodman & Frei (1995) and Foyle, Rix & Zibetti (2010) estimate the gravity torque from the spiral pattern. If this measurement determines the rate of angular momentum transport, then one can use it to estimate the time scale for significant changes to the initial angular momentum distribution. If the spiral is long-lived, the gravity torque must be corrected for the advective term (§2), which can substantially lengthen the time-scale. Advection is far less important for transient spirals. While it is of great interest that the angular momentum evolution time scale could be less than a Hubble time were spirals to be transient, this measurement again does not discriminate between short- and long-lived spiral patterns.

The “density wave” theory of spiral modes (§1.3) requires the outer disc to be dynamically cool, in particular Toomre’s axisymmetric stability parameter should be in the range \( 1 \lesssim Q \lesssim 1.2 \). In principle, a direct measurement of a larger value of \( Q \) would cause difficulties for this theory. A number of attempts have been made to estimate \( Q \) in real discs (e.g. Kormendy 1984, Bottema 1993, Herrmann & Ciardullo 2001), but uncertainties are large because the measurement of the low velocity dispersion in a disc is challenging and the \( Q \) value depends on the disc surface density, which is not tightly constrained. It should also be noted that the raw \( Q \)-value estimated from the stars only may not be the relevant quantity in realistic discs, where the gas component and perhaps also the young stellar population can have a disproportionate effect on the overall responsiveness (Rafikov 2001).

Dobbs & Pringle (2010) suggest that the downstream distribution of star clusters can be used to determine the origin of a spiral pattern. However, their method really is a test only of whether the gas that formed the star clusters streamed through the spiral arm at a speed that differs significantly from the local circular speed – i.e. merely whether a coherent density wave is present – and does not
discriminate between mechanisms for its origin or test for its lifetime. Meidt et al. (2008) generalize the method of Tremaine & Weinberg (1984) to discs that may have multiple pattern speeds. They apply it to CO data on a number of galaxies, finding evidence for multiple patterns in several cases. Other methods to estimate the pattern speed of spiral arms often make the (implicit) assumption that there is a single pattern; for example, the kinematics of stars formed in spiral arms (Bash 1979), the change in radial flow direction across corotation (e.g. Elmegreen, Wicots & Pisani 1998), or the change in the pattern of residual velocities (Canziani 1992), or brute force modelling (e.g. Rautiainen et al. 2008), etc. Once again, however, the results tell us little about the lifetimes of the patterns.

In summary, I concur with BT08 that it is frustratingly difficult to determine the lifetimes of spiral patterns, or the mechanism that gave rise to them, from observations of external galaxies.

However, evidence from the Milky Way seems to favour the transient spiral picture. The in-plane velocity components of stars in the solar neighbourhood as revealed by the HIPPARCOS mission (Dehnen 1998; Nordstrom et al. 2004) do not have the smooth double Gaussian anticipated by K. Schwarzschild (BT08, p. 321), but instead are characterized by a number of separate streams, with essentially no underlying smooth component (Bovy, Hogg & Roweis 2009). The features are too substantial to have simply arisen from groups of stars that were born with similar kinematics (e.g. Eggen 1996), as confirmed in detailed studies (Famaey et al. 2007; Bensby et al. 2007; Bovy & Hogg 2010), and it is clear that the entire DF has been sculptured by dynamical processes.

Individual features in this distribution have been attributed to resonances with the bar (Raboud et al. 1993; Dehnen 2000; Fux 2001), or with a spiral pattern (Quillen & Minchev 2003; Sellwood 2010b), while other models include both bars and spirals (Quillen 2003; Chakrabarty 2007; Antoja et al. 2009). The overall appearance of the velocity distribution was successfully reproduced by De Simone, Wu & Tremaine (2004) who invoked a succession of short-lived spiral transients.

4 SPIRAL STRUCTURE THEORY

Three distinct mechanisms have been proposed to account for self-excited spiral patterns in disc galaxies. Bertin & Lin (1996) propose that spiral features are manifestations of quasi-steady global modes of the underlying disc. Goldreich & Lynden-Bell (1965) and Toomre (1966) argue that, aside from tidal- and bar-driven cases, most spirals are short-lived transient features that are essentially collective responses to internal density fluctuations within the disc. Sellwood (2000) also supposes that spirals are short-lived, but suggests they result from a recurrent cycle of vigorous large-scale modes. Here I give brief summaries of each in turn. Sellwood (2010a) gives a more detailed review.

4.1 Swing Amplified Noise

Swing amplification was first described by Goldreich & Lynden-Bell (1965) and Julian & Toomre (1966). Toomre (1981) highlighted its importance, which he illustrated with the now classic figure named “dust-to-ashes” reproduced in BT08 (their Fig. 6.19). It shows the dramatic transient trailing spiral that results from a small input leading disturbance. Julian & Toomre (1966) showed that swing-amplification causes the collective response of a disc to a co-orbiting mass clump to be a substantial spiral “wake.”

Goldreich & Lynden-Bell (1965) and Toomre (1990) suggested that a large part of the spiral activity observed in disc galaxies is the collective response of the disc to clumps in the density distribution. As a spiral wake is the collective response of a disc to an individual co-orbiting perturber, multiple perturbers will create multiple responses that all orbit at different rates. The behaviour of this polarized disc reveals a changing pattern of trailing spirals, which can equivalently be regarded as swing-amplified noise. Toomre & Kalnajs (1991) show that amplified noise arising from the massive disc particles themselves can be understood in the shearing sheet, where the resulting spiral amplitudes are linearly proportional to the input level of shot noise.

This could be a mechanism for “a swirling hotch-potch of pieces of spiral arms” (Goldreich & Lynden-Bell 1965) in very gas rich discs, where a high rate of dissipation may be able to maintain the responsiveness of the disc (Toomre 1990) while the clumpiness of the gas distribution may make the seed noise amplitude unusually high. However, the spiral structure should be chaotic, with little in the way of clear symmetry expected. Also, it seems likely that spiral amplitudes (e.g. Zibetti, Chariot & Rix 2008) are too large to be accounted for by this mechanism in most galaxies.

4.2 Recurrent Cycle of Groove Modes

Sellwood & Kahn (1991) showed that discs are destabilized by a deficiency of stars over a narrow range of angular momentum; in a disc without random motion, such a deficiency would be a “groove.” The groove itself is unstable, and the instability becomes a global mode through the vigorous suppression of the surrounding disc, producing a large-scale spiral instability. Sellwood & Kahn (1991) showed that the linear instability, which is driven from corotation, also develops in discs with random motion, while Sellwood & Binney (2002) showed that the amplitude of the mode was limited by the onset of horseshoe orbits.

Sellwood & Kahn (1991), and earlier Lovelace & Hohl (1978), found that almost any narrow feature in the angular momentum density is destabilizing. Thus the common starting assumption of spiral structure studies, that the underlying disc is featureless and smooth, may throw the spiral baby out with the bathwater.

Sellwood & Lin (1989) showed, in simulations of a low mass disc, that the particles were scattered at the Lindblad resonances as each coherent wave decayed, thereby leaving behind an altered distribution function that created the conditions for a new instability. Sellwood (2000) described in more detail that each spiral pattern seems to be a strongly unstable true mode of the disc in which it grows; it lasts for only a few rotations, changing the disc properties as it

© 2010 RAS, MNRAS 000, 1–11
decays so as to provoke new instabilities of the altered disc having different pattern speeds. The discovery of resonance scattering among the stars of the solar neighbourhood (Sellwood 2010) suggests that this mechanism may occur in real galaxies. I am actively engaged in addressing the many details of this picture that remain unclear.

4.3 Long-lived Global Modes

Simple models of disc galaxies support many global linear instabilities (e.g. Kalnajs 1978; Jalali 2007). The bar-forming mode is generally the fastest growing, but has almost no sparsity. These studies are therefore important to understand stability, but do not appear promising for spiral generation.

The “density wave” theory for spiral modes, described in detail by Bertin & Lin (1996), invokes a more specific galaxy model with a cool outer disc and hot inner disc. The local stability parameter, $Q = \sigma_R/\sigma_{R,\text{crit}}$ (Toomre 1964), is postulated to be $Q > 1$ in the outer disc and to rise steeply to $Q > 2$ near the centre. Under these specific conditions, Bertin & Lin find slowly evolving spiral modes that grow by their WASER (Mark 1977) mechanism. They invoke shocks in the gas to limit the amplitude of the slowly growing mode, leading to a quasi-steady global spiral pattern. Lowe et al. (1994) present a model of this kind to account for the spiral structure of M81.

The principal objection to their picture is that it is likely that an outer disc with such a low $Q$ will support other, more vigorous, collective responses that will quickly alter the background state by heating the outer disc, as I now show.

4.4 A Direct Test

Here I report simulations designed to test directly whether a galaxy model of the kind invoked by Bertin et al. (1989; hereafter BLLT) can in fact survive to support the slowly-growing global mode they predict should dominate.

The model I adopt here is from the $\Lambda_1$ survey of BLLT, which is a minor variant of that proposed by Fall & Efstathiou (1980). The surface density of the disc is

$$\Sigma(R) = (1 + \Delta) \frac{M_d e^{R/R_d} f(2R/R_d)}{R_d^2},$$

with

$$f(y) = \begin{cases} 1 - (1 + 4y)(1 - y)^4(1 - e^{-R/R_d}/6), & y < 1 \\ 1 & \text{otherwise.} \end{cases}$$

Here $R_d$ and $M_d$ are respectively the scale length and mass, when $\Delta = 0$, of the unmodified exponential disc. The $f(y)$ factor creates a central dip in the surface density of approximate radius $R_d/2$. The rotation curve of their model has the form

$$V(r) = V_0 \frac{x}{(1 + x^2)^{1/2}} \text{ with } x = 2R/R_d,$$

that is independent of the disc contribution. Because mass removed by the central cutout is effectively replaced by rigid matter, the model can be thought of as including a small central bulge as well as a pseudo-isothermal halo. Here I adopt units such that $G = M_d = R_d = 1$, and also $V_0 = 1$; the orbit period at $R = 2R_d$ is $\sim 13$ time units.

BLLT also specify the radial variation of Toomre’s local axisymmetric stability parameter $Q(R) = Q_{\text{OD}}(1 + 1.5e^{2R/R_d})$; i.e. $Q = 2.5Q_{\text{OD}}$ at the centre and decreases rapidly to $Q = Q_{\text{OD}}$ over most of the outer disc. Because BLLT do not supply stellar dynamical distribution functions, I set the initial velocities to create a rotationally supported disc having the desired $Q$ profile, using the Jeans equations in the epicyclic approximation to determine the azimuthal dispersion and asymmetric drift (BT08, eq. 4.228).

I have chosen the case with $Q_{\text{OD}} = 1$ and $\Delta = -0.35$. The upper panel of Fig. 1 shows the rotation curve, which indicates that this model has a strongly sub-maximal disc (Sackett 1997), i.e. the disc contributes merely $\sim 25\%$ of the central attraction at $R = 2R_d$. The lower panel shows the initial $Q$ profile. Because the disc is so light, and the disc surface density dips towards the centre, the initial random velocities are small enough that the epicycle approximation is adequate and the Jeans equations lead to a quite respectable equilibrium model.

BLLT predict that this model has the slowly-growing, tightly-wrapped, bi-symmetric spiral mode illustrated second from left in the bottom row of their Fig. 3, but they do not give the frequency of the mode.

Here I report the results of two series of simulations with this disc model. In the first series, I restrict disturbance forces to $m = 2$ only, as do BLLT in their mode calculations. In the second, I allow sectoral harmonics $2 \leq m \leq 8$ to contribute. In neither case do I include disturbance forces from

Figure 1. Top panel shows the rotation curve of the model discussed in this section in units given in the text. The solid line is the full circular speed, the dotted line shows the contribution from the modified exponential disc, and the dot-dashed line that from the halo. The lower panel shows the initial radial variation of $Q$. 

© 2010 RAS, MNRAS 000, 000
m = 0 & 1. Excluding m = 1 avoids unbalanced disturbance forces that could arise from a lop-sided mass distribution in a rigid halo; there are no external perturbations to excite such waves, which are also most unlikely to be self-excited in this low mass disc (see below). The unchanging axisymmetric central attraction maintains the rotation curve shown in Fig. 4.

The simulations employ the grid code that has been shown to reproduce global modes predicted by linear theory for a number of stellar dynamical models (Sellwood & Athanassoula 1986; Sellwood 1989; Earn & Sellwood 1995; Sellwood & Evans 2001). The particles move over a 2D polar grid having 65 x 96 mesh points and their motion is integrated with a time step of 0.05. Direct tests reveal that results are insensitive to these numerical parameters. The particles interact with forces derived from a Plummer softening kernel with scale 0.05Rd.

With m = 2 only disturbance forces, a simulation with N = 200 000 particles barely evolved. No non-axisymmetric features were visible in the first 40 disc rotations and the Q-profile remained unchanged. There is a hint in the power spectrum shown in Fig. 2 of a coherent wave in the inner disc rotating at the angular rate mΩp ≃ 0.8, but it is barely stronger than the noise peaks along the mΩ curve. Without the predicted eigenfrequency, I am unable to confirm whether it corresponds to the WASER mode predicted by BLLT. Nevertheless, it is clear that the model does not possess any rapidly-growing bi-symmetric instabilities.

The outcome is completely different when disturbance forces from higher sectoral harmonics are included, as shown in Fig. 3. Irrespective of the number of particles, or whether the particles are positioned carefully on rings (a quiet start) or placed at random, the model quickly develops multi-arm spiral patterns that heat the disc, as shown in Fig. 4.

The reason for this different behaviour when higher sectoral harmonics are included is clearly related to swing-amplification, which is strongly dependent on the value of $X = R_{\text{crit}}/m$ (Julian & Toomre 1966; Toomre 1981; BT08), where $k_{\text{crit}} \equiv \kappa^2/(2\pi G\Sigma)$, the smallest wavenumber of axisymmetric Jeans instabilities (Toomre & Kalnajs 1991). In the present model, selected from BLLT, this parameter rises steadily with radius from $X \sim 8/m$ at $R = 1$ to $X \sim 20/m$ at $R = 3$, continuing to higher values farther out. Since the swing-amplifier is vigorous only when $1 \leq X \leq 2.5$ in a flat rotation curve, as here, it is pretty much dead at all radii for $m = 2$ waves, but vigorous responses are expected for $m \geq 4$, as we observe.

Fig. 4 shows the time evolution of Q at $R = 3R_d$ in three simulations with differing numbers of particles showing that heating of the disc, which coincides with the occurrence of visible spiral patterns, is increasingly delayed as larger numbers of particles are employed. The principal effect of increasing N is to reduce the amplitude of initial density fluctuations. The swing-amplifier quickly polarizes the disc, creating trailing spiral responses having an amplitude proportional to the input noise signal (Toomre & Kalnajs 1991). The larger the particle number the smaller the initial amplitude, and it is clear from Fig. 5 that the swing-amplified noise has too small an amplitude to cause heating at first when $N = 20M$. We do not expect global instabilities in
a smooth disc for \( m > 2 \) because small-amplitude disturbances at most reasonable pattern speeds will be damped at an inner Lindblad resonance (Mark 1974). Thus the later increase in the spiral amplitude must be a non-linear effect, perhaps related to the recurrent instability cycle reported by Sellwood & Lin (1989). Whatever the cause, the similar heating rate and final \( Q \) value is consistent with spiral amplitudes that are independent of \( N \) at later times. The origin of multi-arm spiral waves will be followed up in future work.

Fig. 5 shows that even with \( N = 20M \), heating begins after just 15 disc rotations. This should be contrasted with the case where forces were restricted to \( m = 2 \) where no visible bi-symmetric features appeared in 40 rotations even with the ten times larger seed amplitude implied by 100 times fewer particles. It should be further noted that real spiral galaxy discs have large star clusters and giant molecular clouds that imply much larger seed density fluctuations than arise from \( N = 20M \) randomly-placed, equal-mass particles.

### 4.5 Discussion

While these simulations explicitly test just one of the many models presented in BLLT, all the cases that they regard of most interest have strongly sub-maximal discs in order that swing amplification is ineffective for \( m = 2 \); this aspect is essential so that the model possess only slowly-growing, tightly-wrapped, bi-symmetric spiral modes. However, the vigour of the swing-amplifier for \( m > 2 \) makes it inevitable that every one of their galaxy models with a low-mass disk will be subject to stronger activity due to disturbance forces with \( m > 2 \), which will quickly heat the outer disc and destroy the conditions they require, as just demonstrated. Thus the “basic state” invoked by BLLT for the modes they favour to account for bi-symmetric spiral patterns could not survive in real discs that permit disturbances of all sectoral harmonics.

As this conclusion depends largely on the results from simulations, one must worry whether they can be trusted. The principal source of concern is that the origin of the multi-arm patterns that heat the disc remains obscure. Simulations of this kind over many years (e.g. Sellwood & Carlberg 1984, Fujii et al. 2010) have manifested recurrent multi-arm spiral patterns, and the behaviour has not changed as numerical quality has risen and the codes have passed many tests. However, the possibility that the behaviour could result from some artefact in the simulations cannot be excluded altogether until a satisfactory explanation is provided. Note that to doubt the result on these grounds calls into question all simulations of isolated discs over the past 40 years, as well as those that model the formation of disc galaxies.

Other possible criticisms, such as the simulations could be unreliable because of gravity softening, particle noise, or the restriction to 2D, are more readily rebutted. First, the very same code has been shown to reproduce modes predicted from linear stability analyses of other models, as cited above. Also, Plummer softening in simulations with particles confined to a plane provides a reasonable allowance for finite disc thickness and anyway the same behaviour persists in simulations of discs with finite thickness (Roskar et al. 2008, Fujii et al. 2010). Further, since the only variation in the behaviour as \( N \) is increased 100-fold is an increasing delay due to a decreasing seed amplitude, with no other qualitative differences, an argument that simulations cannot be trusted because \( N \) is too small is somewhat threadbare.

One might also worry that the stellar dynamical realization I have created differs from the model that BLLT analyzed in the hydrodynamic approximation – essentially I have replaced pressure in their hydrodynamic calculations with velocity spreads; a more direct test would require a prediction of a slowly-growing spiral in a stellar-dynamical model. However, it is hard to see why this minor difference should matter, especially in this case where the dispersion is a small fraction of the orbit speed almost everywhere; the exception is at the centre where \( Q \) is so large that the disc is (by design) dynamically inert.

The simulations here have not, of course, made any allowance for the influence of the gas component, which can offset the heating effects from transient spiral patterns to some extent. However, Sellwood & Carlberg (1984) found that simulations that included a very crude form of cooling still settled to \( 1.5 \lesssim Q \lesssim 2 \).

### 5 Simulations

Short-lived recurrent spiral patterns have developed spontaneously in simulations of isolated galaxies, from the first studies by Miller, Prendergast & Quirk (1970) and Hockney & Brownrigg (1974), right through to modern simulations that include more sophisticated physical processes (e.g. Roskar et al. 2008, Agertz, Teyssier & Moore 2010).

Claims of long-lived spiral waves (e.g. Thomasson et al. 1993) have mostly been based on simulations of short duration. For example, Elmegreen & Thomasson (1993) presented a simulation that displayed spiral patterns for \( \sim 10 \) rotations, but the existence of some underlying long-lived wave is unclear because the pattern changed from snapshot to snapshot. Other claims are equally doubtful, as I show next.

#### 5.1 Direct Tests

As Donner & Thomasson (1994, hereafter DT94) and Zhang (1996, hereafter Z96) have presented evidence for long-lived...
spiral in the same model, I have chosen to try to reproduce their results here. I first summarize the model they employed and then report my own analysis of the similar results I obtain when I reproduce their simulations.

DT94 adopted the disc surface density distribution (Rohlis & Kreitschmann 1980)

$$\Sigma(R) = \frac{2M_d}{3\pi d^3} \left[ e^{-R/d} - e^{-2R/d} \right].$$

Here $R_d$ is the scale length of the outer exponential disc and $M_d$ is the disc mass. DT94 and Z96 chose $M_d = 0.5 M_t$, where $M_t$ is the total mass of the model, and employed two additional mass components to represent a central bulge and a halo, both of which exert the central attraction in the mid-plane of a razor-thin simple exponential disc. The masses and scale lengths were respectively 0.1$M_t$, 0.1$R_d$ for the bulge and 0.4$M_t$, 0.5$R_d$ for the halo. The rotation curve of this model is shown in Fig. 6, which compares well with that shown in Fig. 1 of DT94. These authors set the initial velocities in the disc such that $Q = 1$ at all radii.

Here I recreate this model, and compute its evolution using essentially the same 2D polar grid code, but with a larger number of particles. The disc has $N = 2M$ particles that move over a grid having $100 \times 128$ mesh points. As DT94 and Z96, I use a Plummer softening law with a length scale 0.15$R_d$ to compute forces between particles. However, I adopt a more physically motivated set of units for which $G = M_t = R_d = 1$. For comparison with the previous results, it should be noted that $R_d = 10$ in their units, and one rotation period at $R = 2R_d$, which takes $2\pi R/V_c = 18.5$ of my time units (925 time steps), is 314 time steps in DT94 and 628 time steps in Z96.

Fig. 6 shows the evolution computed here, which should be compared with that shown in Fig. 2 of Z96. Since the ten times larger number of particles used here lowers the seed amplitude, a little more evolution is needed for the spiral to grow. To make the closest possible comparison, I therefore show snapshots that are spaced at the same time interval, but are shifted later by a little over one disc rotation from the start. The overall appearance is quite similar; a strong $m = 2$ spiral is developing by time 72 and is perhaps more persistent than that in Zhang’s calculation, where the spiral has faded more by the last two times.

Since DT94 and Z96 claim that the $m = 2$ features are a long-lived spiral, I examine them more closely here. Fig. 7 shows that in my simulation they appear to be the superposition of several waves having differing pattern speeds. This figure should be compared with Fig. 5 of DT94, which presents a similar analysis for their model.

Although I employed 40 times the number of particles used by DT94, the power spectrum is still quite noisy. However, the lowest panel has three or more horizontal ridges that are caused by coherent waves extending roughly from the inner Lindblad resonance to a little outside the outer Lindblad resonance in each case. The lowest frequency peak, which is the farthest out in the disc, has the largest relative amplitude, which is simply a reflection of the fact that the disc surface density decreases outward. Forming separate power spectra on the first and second halves of the evolution (top two panels of Fig. 8) reveals that the separate patterns reach peak amplitude in sequential order, with the fastest rotator ($m\Omega_p = 0.42$) developing and decaying first – there is no significant power at that frequency in the second half of the run. At least two waves co-exist at significant amplitude for most of the evolution.

A least-squares fit to these data (Sellwood & Athanassoula 1986), as well as to an expansion of the particle distribution in logarithmic spirals, finds at least four coherent waves with $m\Omega_p \approx 0.42, 0.30, 0.25 & 0.12$, in rough agreement with the locations of the peaks in the lowest panel of Fig. 8. These values need to be multiplied by $18.5/314 \approx 0.06$ to convert to the frequency unit used by DT94, and these authors plot pattern speed $\Omega_p$ and not $m\Omega_p$ that I measure; thus the pattern speeds I observe are 0.013, 0.009, 0.007 & 0.004 in the units shown in Fig. 5 of DT94, where three waves can also be discerned. The fastest and most slowly rotating waves in DT94 have similar frequencies to those I observe, but neither intermediate wave corresponds to a feature in their plots. Exact correspondence should not be expected as all these waves are seeded by particle noise.

Fig. 8 confirms that the amplitude of bi-symmetric disturbances rises in the inner disc (dotted line) earlier than

---

**Figure 6.** The rotation curve of the initial model adopted by DT94 and Z96 in units given in the text. The solid line is the full circular speed, the dotted line is the speed due to the modified exponential disc, and the dashed and dot-dashed lines show the speed due the bulge and halo respectively.

**Figure 7.** Evolution of the model run to reproduce that reported by DT94 and Z96. The six snapshots, which include one particle in 40, should be compared with those shown in Fig. 2 of both papers. The circle is drawn at $R = 8R_d$, although the grid extends to $12.7R_d$. 

---

© 2010 RAS, MNRAS 000.
J. A. Sellwood

Figure 8. Contours of power as a function of radius and frequency computed from the relative bi-symmetric density variations (\(\Sigma_2/\Sigma_0\)) on the rings of the simulation grid. Peaks indicate coherent features with pattern speed \(m\Omega_p\) that last for a significant fraction of the simulation. The solid curve shows the circular angular frequency \(m\Omega\), and the dashed curves \(m\Omega \pm \kappa\). The top panel is computed from the first half of the simulation, the middle panel from the second half, while the third panel shows the spectrum from both halves combined.

in the outer disc (dot-dash line). Furthermore, the potential variations fluctuate in amplitude, especially in the inner disc, in a manner characteristic of beats between waves rotating at different pattern speeds, as to be expected from the power spectra shown in Fig. 8. Were the evolution dominated by a single \(m = 2\) spiral mode, the temporal evolution in this Figure would be smooth and vary synchronously at each radius.

Thus “the bisymmetric spiral” in this simulation is not a single long-lived pattern, but the superposition of three, or more, waves that each grow and decay. Zhang (1998) argues that this model has a tendency to form a bar, which I have also noticed, and she therefore presents another simulation in which the disc mass is reduced to 0.4\(M_t\) and the halo mass increased to 0.5\(M_t\) in order to weaken the tendency to form a bar. As she again claims that this new model supports a long-lived spiral, I have also tried to reproduce this slightly different simulation. As above, I find that the strong, and visually impressive, bi-symmetric disturbance is the superposition of more than two patterns that each do not last long. In this case the first wave to appear has frequency \(2\Omega_p \approx 0.29\), but later the two dominant waves have \(2\Omega_p \approx 0.36\) & \(0.19\), with also some significant amplitude at \(m = 3\).

6 INDIRECT EVIDENCE

Here I summarize three further arguments that bear indirectly on the question of spiral lifetimes. All three are related to the effect of spiral activity on the orbits of stars in the disc.

6.1 Importance of Gas

Transient spiral activity scatters disc stars away from circular orbits (Barbanis & Woltjer 1967; Carlberg & Sellwood 1984; Binney & Tremaine 1988). As random motion of the disc stars rises, the collective behaviour that produces spiral patterns is weakened. Thus, in the absence of cooling, spiral activity is self-limiting – it heats the disc to a certain level at which the disc can no longer support spiral patterns.

Sellwood & Carlberg (1984) showed this explicitly in their simulations, and went on to show that a moderate amount of cooling in the form of fresh stars added to the disc on circular orbits enabled spiral activity to continue “indefinitely.” Subsequent work on isolated discs (Carlberg & Freedman 1985; Toomre 1990; Roskar et al. 2008a), as well as modern galaxy formation simulations (Agertz, Teyssier & Moore 2010, and many others), has confirmed this result in simulations with ever greater realism. Physically, the random motions gained by gas clouds are dissipated in collisions so that they keep moving on near circular orbits. Stars that form from the clouds therefore have similar motions, and a continuous supply of fresh stars on near-circular orbits maintains a responsive stellar distribution that allows spiral activity to continue. Quantitatively, fresh stars added to a disc at the rate of a few every year is sufficient for this purpose.

Thus the transient spiral picture offers a natural explanation for the absence of spiral patterns in S0 disc galaxies that have little or no gas and the long-noted correspondence that disc galaxies with significant gas components also manifest spiral patterns. Note that this appealing argument again does not uniquely favour short-lived waves, since Bertin & Lin (1996) also invoke a gas component both to limit the
amplitude of their mildly growing modes and to maintain the dynamically cool outer disc.

6.2 Scattering of Stars

It has been clear for some time that the velocity dispersion of disc stars in the solar neighbourhood rises with age (Wieland 1977; Nordström et al. 2004) and also, for main sequence stars, with colour (Aumer & Binney 2009), which is a surrogate for mean age. The highest velocities cannot be produced by cloud scattering (Lacey 1993; Hänninen & Flynn 2002) and some other accelerating agent, such as transient spirals, seems to be required. Long-lived spirals, which do not heat the disc nearly as effectively, could not achieve the requisite high velocities.

6.3 Radial Mixing

Studies of the metallicities and ages of nearby stars (Edwardsson et al. 1992; Nordström et al. 2004; Reid et al. 2007; Holmberg, Nordström & Andersen 2007) find that older stars tend to have lower metallicities on average. As it is difficult to estimate the ages of individual stars, the precise form of the relation is still disputed. However, there seems to be general agreement that there is a spread of metallicities at each age, which is also supported by other studies (Chen et al. 2003; Haywood 2008; Stanghellini & Haywood 2010). A metallicity spread amongst coeval stars is inconsistent with a simple chemical evolution model in which the metallicity of the disc rises monotonically in each annular bin, without mixing in the radial direction. Again Sellwood & Binney (2002) and Roškar et al. (2008ab) showed that when the disc supports recurrent transient spirals, the needed radial mixing arises naturally through angular momentum changes at co-rotation that do not heat the disc. Schönrich & Binney (2009) developed the first chemical evolution model for the Milky Way disc to include this radial churning.

The transient nature of the patterns is an essential aspect to produce efficient churning. The horse-shoe orbits near co-rotation of a large-amplitude spiral cause a single change in the angular momentum if the spiral grows and decays in less than half the (long) orbit period in the rotating frame. Were the spiral to persist for longer than this, then the exchanges a star experiences at one arm would be undone when the star encounters the next arm, and the star would merely take periodic, equal inward and outward steps in radius leading to no lasting change. Minchev et al. (2010) report substantial mixing due to combined influence of a bar and spirals, but their simulations were of short duration and it is likely the effect is large only as the bar forms. Only recurrent transient patterns, with co-rotation radii spread at random locations, can cause the home radii of stars to “diffuse” across the disc throughout its life.

1 In fact, Sellwood & Binney (2002) argue that the spiral mode grows until it reaches the amplitude at which the periods of horse-shoe orbits become short enough for this to happen for many stars, which causes the disturbance to disperse.

6.4 Discussion

Tremaine (private communication) pointed out that transient spirals could co-exist with long-lived patterns. This possibility could be consistent, for example, with the change in the appearance of spiral patterns between the blue and near-IR passbands (e.g. Block & Wainscoat 1991; Block et al. 1994), although these data contain no information about the lifetimes of features of any colour. The co-existence of short-lived spirals seems inconsistent with the specific mode theory for long-lived patterns proposed by Bertin & Lin (1996), since transient waves would heat the outer disc to an extent that must render their proposed mechanism unworkable, as shown in §4. However, some other possible (as yet unknown) mechanism for long-lived spirals might remain viable.

An observational test for the coexistence of both short- and long-lived patterns would be a daunting challenge. For a well-constructed sample of galaxies, one would have to determine the mean amplitude needed to produce the desired heating and churning by the short-lived patterns, and then show that the mean observed spiral amplitudes, estimated somehow from the passband dependent photometry, would or would not allow an additional significant long-lived spiral component.

7 CONCLUSIONS

The question of the lifetimes of spiral patterns is important because galaxy discs evolve to a much greater extent if spirals are transient than if they are quasi-steady (§2). In this paper, I have summarized the evidence that bears on the question; were any one piece decisive, there would have been no need to write this paper. However, I find that the transient picture is favoured by the velocity structure in the solar neighbourhood (§3) and by the behaviour of simulations (§§4 & 5).

Direct observational evidence from external galaxies (§3) is frustratingly inconclusive, largely because our single snapshot view can tell us little about the lifetimes of the patterns or, if self-excited, the mechanism that caused them. However, the overall appearance of the velocity distribution of stars in the solar neighbourhood is most naturally accounted for by a succession of transient spirals, with individual features perhaps being attributable to resonances.

Theoretical work (§4) over many decades has pursued the two mutually exclusive views. In their efforts to gain the upper hand, the rivalry has spurred great improvements in our understanding of density wave mechanics. However, neither side has been able to land a knockout blow.

Simulations have long manifested short-lived transient features, and I have presented two new results here. The first, in §4.4, shows that galaxy models with cool, low-mass discs of the type proposed by Bertin et al. (1989) as likely to manifest long-lived, bi-symmetric spiral patterns will instead heat quickly as a result of short-lived transient spiral patterns with $m > 2$. While the behaviour in the simulations is not fully understood, swing-amplification gives a clear reason for the preference for higher sectoral harmonic disturbances in low-mass discs. Naturally, my demonstration that the most carefully worked-out theory for long-lived
modes may not be viable, does not exclude the possibility of long-lived modes altogether.

I have also shown (Fig. 1) that the most clearly presented case of a long-lived spiral in a simulation is not, in fact, a single spiral mode, but a sequence of a few strong patterns that each have short lifetimes. Thus I am unaware of a credible case of a long-lived spiral in any simulation, and therefore simulation results provide strong support for the transient hypothesis. The principal reason this is not decisive is that we still do not fully understand the origin of spiral patterns in the simulations.

Indirect evidence (Fig. 2) suggests that three quite diverse properties of galaxies are naturally explained by transient spiral waves, but an underlying long-lived wave is a possibility that would be very difficult to exclude observationally.

To hold that spirals are long-lived, one must also argue, (1) that significant long-lived spirals underlie the transients that appear to be needed to explain the velocity structure in the solar neighbourhood, the high random motions of old disc stars and radial mixing in discs, (2) that the long-lived pattern adjusts its rotation rate slowly over time, so that fresh stars become the sources and sinks of angular momentum to drive the pattern, and (3) that no simulation has yet been able to capture the correct dynamical behaviour of spiral patterns.

However, the entire question of spiral lifetimes will not be settled until a theory of spiral pattern generation, which is supported by observational data and consistent with simulation results, gains wide acceptance.

ACKNOWLEDGMENTS

I thank the referee (Gene Byrd), Xiaolei Zhang, and especially Giuseppe Bertin for comments on a draft of this paper that helped to clarify some issues. I also thank Scott Tremaine for encouragement and for a number of perceptive remarks.

REFERENCES

Agertz, O., Teysier, R. & Moore, B. 2010, arXiv:1004.0005
Antoja, T., Valenzuela, O., Pichardo, B., Moreno, E., Figueras, F. & Fernández, D. 2009, ApJL, 700, L78
Athanassoula, E. 1984, Phys. Rep., 114, 321
Aumer, M. & Binney, J. J. 2009, MNRAS, 397, 1286
Bash, F. N. 1979, ApJ, 233, 524
Barbanis, B. & Wolfjger, J. E. 1967, ApJ, 150, 461
Bensby, T., Oey, M. S., Feltzing, S. & Gustafsson, B. 2007, ApJL, 655, L89
Bertin, G. 1983, A&A, 127, 145
Bertin, G. & Lin, C. C. 1996, Spiral Structure in Galaxies (Cambridge: The MIT Press)
Bertin, G., Lin, C. C., Lowe, S. A. & Thurstans, R. P. 1989, ApJ, 338, 78
Binney, J. J. & Lacey, C. G. 1988, MNRAS, 230, 597
Binney, J. & Tremaine, S. 2008, Galactic Dynamics 2nd Ed. (Princeton: Princeton University Press), (BT08)
Block, D. L. & Wainscoat, R. J. 1991, Nature, 353, 48
Block, D. L., Bertin, G., Stockton, A., Grosbol, P., Moorwood, A. F. M. & Peletier, R. F. 1994, A&A, 288, 365
Bottema, R. 1993, A&A, 275, 16
Bournaud, F., Combes, F. & Semelin, B. 2005, MNRAS, 364, L18
Bovy, J. & Hogg, D. W. 2010, ApJ, 717, 617
Bovy, J., Hogg, D. W. & Roweis, S. T. 2009, ApJ, 700, 1794
Buta, R. J., Knappen, J. H., Elmegreen, B. G., Salo, H., Lairikainen, E., Elmegreen, D. M., Puerari, I. & Block, D. L. 2009, AJ, 137, 4487
Byrd, G. G. & Howard, S. 1992, AJ, 103, 1089
Cantizan, B. 1993, ApJ, 414, 487
Cantizan, B., Allen, R. J. & Tilanus, R. P. J. 1993, ApJ, 406, 457
Carlberg, R. G. & Freedman, W. L. 1985, ApJ, 298, 486
Carlberg, R. G. & Sellwood, J. A. 1985, ApJ, 292, 79
Chakrabarty, D. 2007, A&A, 467, 145
Chen, L., Hou, J. L. & Wang, J. J. 2003, AJ, 125, 1397
Dehnen, W. 1998, AJ, 115, 2384
Dehnen, W. 2000, AJ, 119, 800
De Simone, R. S., Wu, X. & Tremaine, S. 2004, MNRAS, 350, 627
Dobbs, C. L. & Pringle, J. E. 2010, MNRAS, to appear (arXiv:1007.1399)
Dobbs, C. L., Theis, C., Pringle, J. E. & Bate, M. R. 2010, MNRAS, 403, 625
Donner, K. J. & Thomasson, M. 1994, A&A, 290, 475
Dubinski, J., Gauthier, J.-R., Widrow, L. & Nickerson, S. 2008, in Formation and Evolution of Galaxy Disks, ed. J. G. Funes SJ & E. M. Corsini (San Francisco: ASP 396), p. 321
Earn, D. J. D. & Sellwood, J. A. 1995, ApJ, 451, 533
Eavardsson, B., Andersen, B., Gustafsson, B., Lambert, D. L., Nissen, P. E. & Tomkin, J. 1993, A&A, 275, 101
Eggen, O. J. 1996, AJ, 112, 1595
Elmegreen, B. G. & Thomasson, M. 1993, A&A, 272, 37
Elmegreen, B. G., Wilcots, E. & Pisano, D. J. 1998, ApJL, 494, L37
Eskridge, P. B., et al. 2002, ApJS, 143, 73
Fall, S. M. & Efstathiou, G. 1980, MNRAS, 193, 189
Famaey, B., Pont, F., Luri, X., Udry, S., Mayor, M. & Jorissen, A. 2007, A&A, 461, 957
Foyles, K., Rix, H.-W. & Zibetti, S. 2010, MNRAS, to appear
Fujii, M. S., Baba, J., Saitoh, T. R., Makino, J., Kokubo, E. & Wada, K. 2010, arXiv:1006.1228
Fux, R. 2001, A&A, 373, 511
Gnedin, O. Y., Goodman, J. & Frei, Z. 1995, AJ, 110, 1105
Goldreich, P. & Lynden-Bell, D. 1965, MNRAS, 130, 25
Grosbol, P., Patsis, P. A. & Pompei, E. 2004, A&A, 423, 849
Hänninen, J. & Flynn, C. 2002, MNRAS, 337, 731
Haywood, M. 2008, MNRAS, 388, 1175
Herrmann, K. A. & Ciardullo, R. 2009, ApJ, 705, 1686
Hockney, R. W. & Brownrigg, D. R. K. 1974, MNRAS, 167, 351
Holmberg, J., Nordström, B. & Andersen, J. 2007, A&A, 475, 519
Jalali, M. A. 2007, ApJ, 669, 218
Julian, W. H. & Toomre, A. 1966, ApJ, 146, 810
Kalnajs, A. J. 1978, in IAU Symposium 77 Structure and Properties of Nearby Galaxies eds. E. M. Berkhuysen & R. Wielebinski (Dordrecht:Reidel) p. 113

© 2010 RAS, MNRAS
