Adversarial learning in revenue-maximizing auctions

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Abstract

We introduce a new numerical framework to learn optimal bidding strategies in repeated auctions when the seller uses past bids to optimize her mechanism. Crucially, we do not assume that the bidders know what optimization mechanism is used by the seller. We recover essentially all state-of-the-art analytical results for the single-item framework derived previously in the setup where the bidder knows the optimization mechanism used by the seller and extend our approach to multi-item settings, in which no optimal shading strategies were previously known. Our approach yields substantial increases in bidder utility in all settings. Our approach also has a strong potential for practical usage since it provides a simple way to optimize bidding strategies on modern marketplaces where buyers face unknown data-driven mechanisms.

Introduction

Repeated auctions are widely used in modern economic systems to sell a variety of items ranging from wireless spectrum licenses to ad placements on the Internet. In online marketplaces, most auctions are designed using techniques at the junction of classical auction theory \cite{23} and statistical learning theory. Sellers take advantage of the enormous amount of data gathered on buyers’ behavior and strategies - through billions of auctions a day - to learn and implement revenue maximizing auctions on different platforms.

In the case of single-item auctions, the design of an optimal incentive-compatible revenue-maximizing auction is well understood \cite{23}, assuming the seller knows the value distribution of each buyer. Indeed, under this perfect knowledge assumption, she can define the allocation and payment rules maximizing her expected revenue.

The multi-item framework is more intricate. Myerson’s fundamental result has been extended to specific settings depending on the number of objects and on the properties of the bidders’ utility functions \cite{4, 20, 11, 34}. A general and analytical optimal auction in the multi-item framework has yet to be found.

Because of the amazingly large variety of different settings, automatic mechanism design has been introduced to provide a (numerical) framework for learning revenue-maximizing mechanisms satisfying constraints chosen by the designer \cite{3, 1}. This framework was recently complemented by the introduction of neural networks for different instances of the multi-item problem \cite{13, 30, 15}; as expected, it does take advantage of the large expressivity power of neural networks architectures.

This line of research still traditionally assumes that the value distributions of bidders are known to the seller. However, in practice, the seller does not have access to such information and can only statistically estimate these distributions using a finite sample of bids made in past (hopefully truthful) auctions \cite{26, 8, 22, 17, 6}. To simplify the presentation, we can represent this as a two-stage game between a seller and buyers. The first stage consists in a sequence of truthful auctions (say, second price auctions without reserve price or with random reserve prices) where bidders are assumed to bid truthfully. This will provide the seller with a batch of i.i.d. samples from the different value distributions (since, in truthful auctions, observed bids are equal to unobserved values). Under this - quite strong - truthfulness assumption, the seller can compute the empirical revenue-maximizing auction, based on the bid samples collected in the first stage \cite{27, 21, 29, 12}.

However, bidders might have been strategic in the first round, in order to maximize their long-term utility. Indeed, strategically shading bids during the first round can and will induce the seller to select another mechanism for the second stage (the one that maximizes her revenue with respect to the distribution of bids instead of the real distribution of values). This alternative mechanism might actually be better for the seller than the one selected under truthful bidding. The main idea is that a buyer might want to lose some money during the first round to increase - drastically - his revenue during the second one.

Several approaches have been introduced for the seller to disincentive bidders from being strategic. A solution is to compute the reserve price of a bidder using information stemming solely from the other ones \cite{5, 18, 14}. This approach is theoretically sound, but practically limited as it requires that bidders have similar value distributions. For instance, it cannot handle heterogeneous settings with a dominant buyer \cite{14} as the optimal reserve price of the latter can not be computed from bids of the others. Unfortunately, this
is precisely the scenario where revenue optimizing mechanisms unveil their full potential. Another line of research assumes that bidders value the future a lot less than the seller by considering discount factors of different magnitude orders. Although necessary to theory, this technique introduces an artificial asymmetry between bidders and seller in order to force bidders to bid truthfully in the seller learning phase (or at least during a significant fraction of it).

All these limitations were recently overcome by assuming that bidders actually adapt to automatic mechanisms in single-item auctions by being strategic in the first stage. A new class of skewing or shading strategies was introduced for the lazy second price auction with monopoly reserve prices, and for the Myerson auction. In particular, a new variational approach was recently discovered. It unlocks, through numerical optimization, a method to find best-responses to most of the approximated Myerson auction, such as boosted second price auctions. The major and prohibitive drawback of these approaches is that they require that strategic bidders perfectly know the underlying mechanism design problem (i.e. the revenue maximization problem) solved by the seller.

Framework and contributions

Our starting point is a recent end-to-end learning approach which might be first general attempt to learn revenue-maximizing auctions in various multi-item settings. This approach assumes that the seller can somehow generate samples from the value distributions of the bidders to update her mechanism. The mechanism is then parametrized by two neural networks corresponding respectively to the two rules, allocation and payment, that define a mechanism. These networks are trained to maximize the revenue of the seller under the incentive compatibility constraint.

This approach is interesting, from a theoretical perspective, to surmise the potential performance of a revenue-maximizing auction in settings where no optimal mechanisms are known. It can also be used in practice by sellers who have access to a large set of truthful bids to compute an appropriate mechanism.

Here again, the main practical limitation lies in the fact that bidders might be strategic and submit skewed bids, thus altering the distribution of observations of the seller and inherently affecting her mechanism updates (based on those bids), which ultimately leads to a change of the output of the game. Inspired by the recent line of work that focuses on possible adversarial attacks on standard learning systems, we aim at exploring manipulation opportunities for bidders in such learning approaches. The auction framework offers a nice sandbox to test new learning approaches in multi-agent games. From a game-theoretic standpoint, we simply cast the overall interaction between players as a Stackelberg game where bidders play first - hence are leaders - and the seller is the follower, playing second. We emphasize here that we do not assume that bidders know the rules/algorithm/processes used by the seller to optimize her revenue; instead they discover them through a classic explore-exploit trade-off.

Our contributions are the following. We introduce a new numerical framework to study economic interactions when several agents use learning algorithms based on data provided by other rational agents. We focus specifically on how bidders can find good bidding strategies when facing mechanisms such as those introduced by. We improve on recent single-item approach, by removing the prior knowledge on the exact algorithmic procedure used by the seller to optimize her mechanism.

Inspired by reinforcement learning techniques, we introduce an exploration policy corresponding to a distribution over possible strategies and use classical policy optimization algorithms to tune the parameters of our policy. Furthermore, our approach elicits new shading strategies in classical and cutting-edge settings of the multi-item literature. For instance, we obtain a 54% uplift in utility in the 2 bidders and 2 objects framework with one strategic bidder, where bidders have additive valuations and uniform value distributions between 0 and 1. This constitutes a first benchmark of the impact of strategic behavior on multi-item revenue maximizing auctions’ performances. The implementation of our experimental setup in PyTorch is also available on Github for reproducibility concerns.

Auction design and Stackelberg games

Classical mechanism design literature usually studies the Stackelberg game where the seller is the leader, and chooses a mechanism knowing the bidders’ value distributions. We assume that the seller does not have prior knowledge of bidders’ value distributions and consider the reverse Stackelberg game where the bidders are leaders. Bidders are able to choose what value distribution to submit and thus impact the mechanism chosen by the designer.

Notations

We consider the setting with a set of bidders and items with $M = \{1, \ldots, m\}$. We denote by $v_i : \{0, 1\}^M \rightarrow \mathbb{R}_{\geq 0}$ the valuation function of bidder $i$. For any bundle of items $S \subseteq M$, $v(S)$ represents how much bidder $i$ values the bundle $S$. As classically done in the auction literature, we assume that bidders’ valuations are drawn independently from value distributions that we denote by $\{F_i\}_{i \in [1, n]}$. We denote by $F = F_1 \times \cdots \times F_n$ their product distribution.

A bidding strategy $\beta_i : \mathbb{R}^m \rightarrow \mathbb{R}^m_{\geq 0}$ is a mapping from values to bid. We denote $\beta = (\beta_1, \ldots, \beta_n)$ and write $\beta_{-i}$ the set of strategies without that of bidder $i$. The bid distribution $F_{\beta_i}$ is the distribution of bids induced by using $\beta_i$ on $F_i$. We denote by $\vec{b}_i$ the vector of bid submitted by bidder $i$ and $B = \{(b_1, \ldots, b_n), \vec{b}_i \in \mathbb{R}^{2^m}\}$ the set of all possible bid profiles.

A mechanism is a pair $m = (a, p)$ consisting of an allocation rule $a_i : B \rightarrow \{0, 1\}^{2^m}$ and a payment rule $p_i : B \rightarrow \mathbb{R}^{2^m}_{\geq 0}$. For bids $\vec{b} = (\vec{b}_1, \ldots, \vec{b}_n)$, $a_i(\vec{b})$ gives the allocation of the items, $p_i(\vec{b})$ the payment for each bidder and $u_i(\vec{b}) = a_i(\vec{b})(x_i - p_i(\vec{b}))$ the utility of bidder $i$. 
The seller’s revenue in an auction \((a, p)\) given bidding strategies \(\{\beta_i\}_{i \in [1, n]}\) is defined as

\[
R(m, \beta) = \mathbb{E}_F \left( \sum_j a_j (\bar{B}_1, \ldots, \bar{B}_n) p_j (\bar{B}_1, \ldots, \bar{B}_n) \right)
\]

where \(\bar{B}_i = \beta_i (\bar{X}_i)\) and \(\bar{X}_i\) is randomly drawn from \(F_i\). The utility of bidder \(i\) is defined as:

\[
U_i(m, \beta) = \mathbb{E}_F \left( \left[ X_i - p_i (\bar{B}_1, \ldots, \bar{B}_n) \right] a_i (\bar{B}_1, \ldots, \bar{B}_n) \right)
\]

We will denote by \(\beta_{Id}\) the truthful strategy corresponding to a player bidding his own valuation.

**Classical seller’s learning problem**

We write the seller’s mechanism optimization problem as a learning problem following methods introduced by the automated mechanism design literature. Indeed, several works investigate methods for learning optimal mechanisms from data sampled from bidders’ true value distributions, using numerical optimization and machine learning techniques.

In the classical framework, the seller seeks to solve the constrained optimization problem consisting of maximizing her revenue under the ex-post incentive compatibility constraint.

**Definition 1 (DSIC)** A mechanism \(m\) is ex-post dominant strategy incentive-compatible (DSIC) if bidding truthfully is a dominant strategy, i.e.,

\[
\forall i \in [1, n], U_i (m, \beta_i, \beta_{-i}) \leq U_i (m, \beta_{Id}, \beta_{-i})
\]

An automatic mechanism design algorithm \(\mathcal{A}\) takes a class of mechanisms \(\mathcal{M}\) and the bidders’ value distributions as inputs, and outputs a mechanism solving a given constrained optimization problem.

The problem of automated mechanism design as first introduced by \([8]\) and implemented in practice by \([26]\) essentially consists in a Stackelberg game where the seller takes bidders’ value distributions as given, and enforces them to bid truthfully by choosing a DSIC mechanism. This first type of Stackelberg game takes the seller as leader.

**Definition 2 (Seller/Bidder Stackelberg game)**

(Stackelberg) Game in which the seller chooses a mechanism among a class of DSIC mechanisms \(\mathcal{M}\) which maximizes her revenue assuming she knows the bidders’ value distributions.

From this game, we can define the seller’s learning algorithm \(\mathcal{A}\) solving this Seller/Bidder Stackelberg game. If we denote by \(\mathcal{F}\) a class of value distributions and \(\mathcal{M}\) a class of mechanisms, a seller’s learning algorithm \(\mathcal{A}\) is defined as

\[
\mathcal{A} : \quad \mathcal{F} \quad \rightarrow \quad \mathcal{M}
\]

\[
F \quad \mapsto \quad m(F) = \operatorname{arg\,max}_{m \in \mathcal{M}} R(m, \beta_{Id})
\]

s.t. \(U_i (m, \beta_i, \beta_{-i}) \leq U_i (m, \beta_{Id}, \beta_{-i}), \quad \forall i \in [1, n], \forall \beta_i, \forall \beta_{-i}\)

By assuming that the bidders’ value distributions are common knowledge, Myerson analytically solved this Stackelberg game in the one-item, multiple bidders setting in his seminal paper \([23]\).

We consider the approach taken by \([13]\) for the implementation of the seller’s optimization process. Their work provides a general algorithmic approach to approximately solve this problem in multi-item, multi-bidder settings. The seller’s auction is parametrized by a weight vector \(\omega\) corresponding to two neural networks which take bids for each item and each player \((n \times m)\) entries as inputs and return respectively the allocation probability \(a_\omega\) of each item, for each player \((n \times m)\) outputs and the payment for each player \(p_\omega\) \((n\) outputs). In the case of combinatorial auctions, bidders would submit a bid for each possible bundle \((n \times 2^m)\) entries.

The first term of the loss function used to train the network is the negated empirical revenue computed on the dataset of bids \(S = \{\bar{b}^{(1)}, \ldots, \bar{b}^{(L)}\}\)

\[
\mathcal{L}_{\text{Rev}} = - \frac{1}{L} \sum_{k=1}^{L} p_\omega (\bar{b}^{(k)})
\]

To ensure the IC constraint, the authors use two different approaches. The first one is a hard constraint implemented by defining an architecture which is DSIC by design. Myerson’s lemma \([23]\) is used to design the MyersonNet architecture which learns the optimal DSIC auction in the single-item setting. However, for each new setting of the problem, a new architecture must be designed as shown in \([30]\).

Their second approach, the RegretNet architecture, uses a soft constraint in a Lagrangian corresponding to the incentive-compatibility objective. For each bidder, they introduce the empirical ex post regret for bidder \(i\):

\[
\hat{r}_{gt_i}(\omega) = \frac{1}{L} \sum_{k=1}^{L} \max_{b^{mis}_i} u_i^\omega (\bar{b}_i^{mis}, \bar{b}_i) - u_i^\omega (\bar{b}_i, \bar{b}_i^{mis})
\]

This regret is the difference between the maximum utility bidder \(i\) can get by optimizing his misreport \(\bar{b}_i^{mis}\) and the utility he gets when bidding truthfully. They use the augmented Lagrangian method to optimize the Lagrangian function defined as:

\[
\mathcal{L}(\omega, \lambda) = \mathcal{L}_{\text{Rev}} + \sum_{i=1}^{N} \lambda_i \hat{r}_{gt_i}(\omega) + \frac{\rho}{2} \left( \sum_{i=1}^{N} \hat{r}_{gt_i}(\omega) \right)^2
\]

This Lagrangian function is the sum of the negated actual revenue of the mechanism with two penalties which quantify the lack of incentive compatibility, thus insuring that the learned mechanism is approximatively DSIC. The mis-reports \(\bar{b}_i^{mis}\) are not optimized assuming that the bidders consider that their bids have an impact on the mechanism learned by the seller. This is why the networks learned following this approach only satisfies a DSIC constraint and are not robust to an attack from a strategic bidder optimizing his global bid distribution.
Adversarial bidder attack

In practice, the fact that the seller uses past bids to estimate bidders’ value distributions before optimizing her mechanism in repeated auctions provides bidders with the opportunity to design “attacks” in order to find bidding strategies that increase their long-term utility. By strategically adjusting their bids, they are able to control the bidding distributions perceived by the seller and used to optimize her mechanism. This corresponds to a new Stackelberg game in which the bidders are the leaders of the game, which is the focus of our work here.

Definition 3 (Bidder/Seller Stackelberg game) [31, 24, 25]

Stackelberg game in which strategic bidders assume the existence of a seller’s learning algorithm $A$. Each strategic bidder $i$ chooses a strategy $\beta_i$ that induces a (pushforward) bid distribution $F_{B_i} = \beta_i#F_i$ used as input by the seller’s algorithm. The goal of the strategic bidder is to optimize

$$\arg \max_{\beta_i \in B_i} U_i(A(F_{B_i}, F_{-i}), \beta_i)$$

An adversarial “attack” consists in solving the bidder/seller Stackelberg game by finding a bidding distribution that increases bidders’ utility for a specific seller’s learning algorithm. If we denote $B_i$ the set of possible strategies for bidder $i$, an adversarial attack $\Pi_i(A)$ for a seller’s learning algorithm $A$ is defined as following:

$$\Pi_i(A) : \mathcal{F} \rightarrow B_i$$

$$F \rightarrow \Pi_i(F) = \arg \max_{\beta_i \in B_i} U_i(A(F_{B_i}, F_{-i}), \beta_i)$$

Several approaches have already tackled this problem [31, 24, 25]. In all these papers, the authors assume perfect knowledge of the optimization algorithm used by the seller. Our goal is to extend these approaches by getting rid of the assumption that bidders know the seller’s algorithm and by proposing a method that automatically adapts to this new framework. We provide a general method that applies in particular to general multi-item auctions, and hence to cutting edge auction theoretic results. For these auctions, allocation and payment rules are currently available only through numerical methods such as the one developed by [13], which preclude the design of attacks based on analytic understanding of auction rules. We provide a general approach to designing such attacks, proving that the networks introduced by [13] are not robust to adversarial attacks.

These adversarial “attacks” could be called Stackelberg responses to black-box automatic mechanism design. They exploit a conceptual opening in most automatic mechanism design works, i.e. the breakdown of incentive compatibility for the buyer when the seller optimizes over incentive compatible auctions. As such, they differ from standard adversarial attacks in e.g. computer vision, which generally rely on the lack of local robustness of a classifier. Two other features are notable: these “attacks” do not necessarily yield lower revenues for the seller [24]; and they are also part of a dynamic game between buyers and seller and as such have a dynamic component that is absent from classical and static machine learning frameworks, such as image classification.

The single-item auction

We show how to numerically solve the bidder/seller Stackelberg game where one strategic bidder is the leader. They use a zero-order optimization algorithm enabling them to optimize their utility without needing to know or access the optimization algorithm used by the seller to learn her mechanism.

Our architecture

For the single-item setting, we consider the MyersonNet approach introduced in [13]. The allocation rule is defined as an invertible neural network parametrizing a transformation of the bid. The payment rule is obtained in such a way that the auction is DSIC following the Myerson lemma. This provides a first benchmark on how seller learning algorithms are sensitive to adversarial attacks. We focus on one specific bidder and assume that the strategies of other bidders are fixed. We show how the strategic bidder can optimize an exploration bidding policy to increase his utility when the seller is using a MyersonNet-type architecture to optimize her selling mechanism.

Definition 4 (Exploration bidding policy) We consider a set of possible bidding strategies $B$. An exploration bidding policy $\pi$ is a distribution over this set of strategies.

We first consider the case where $B$ is the set of linear bidding strategies because of their simplicity and wide use in modern industrial bidding engines. To parametrize our exploration bidding policy, we use a normal distribution such that

$$\lambda \sim \mathcal{N}(\mu, \sigma^2) = \pi(\mu, \sigma^2)$$
with corresponding bidding strategy $\beta_\lambda(x) = \lambda x$ for the strategic bidder. We do not require any assumption on the other bidders’ behavior.

According to the exploration policy, we sample several shading parameters $\lambda$ which are used as bid multipliers by the strategic bidder. The goal of the strategic bidder is to optimize the parameters $\mu$ and $\sigma^2$ to maximize his utility when the seller is using the MyersonNet architecture. A representation of the global architecture is provided in Figure 1. 

We use the classical Reinforce algorithm \cite{Williams1992} to optimize the parameters $\mu$ in the exploration bidding policy of the seller. For the setting with three bidders, we get an uplift of 20% of the thresholded exploration policy. To compute strategic bidder’s utility, we take a gradient step according to:

$$\nabla_\mu U(\mu) = E_{\Lambda \sim \pi(\mu, \Sigma)} \left( U(\beta_\lambda) \frac{\sum p_{\mu, \Sigma}(\Lambda)}{p_{\mu, \Sigma}(\Lambda)} \right).$$

with $p_{\mu, \Sigma}$ the probability density function (henceforth pdf) corresponding to $\pi(\mu, \Sigma)$. To compute $U(\beta_\lambda)$ we run a full training of the MyersonNet architecture. The full procedure is presented in Algorithm 1. All our implementations are provided in Pytorch.

**Algorithm 1:** Adversarial training for revenue-maximizing auctions

**Input:** Distributions $F_1, \ldots, F_n$, seller’s learning mechanism $A$

Initialize $\mu_1, \Sigma$;

for $t = 1$ to $T$ do

Sample $\Lambda_1, \ldots, \Lambda_q \sim N(\mu_1, \Sigma)$;

for $l = 1$ to $L$ do

Run subroutine $A$ to optimize seller’s mechanism on bids induced by $\beta_\lambda$ for bidder $i$ and $\beta_\lambda = \beta_{1d}, \forall j \neq i$;

Compute strategic bidder’s utility:

$$U_{\Lambda_i} = U(A(F_{B_{\Lambda_i}}, F_{-i}), \beta_{\Lambda_i});$$

Compute gradient:

$$\nabla U(\mu_i) = \frac{1}{\beta} \sum_{i=1}^{L} U_{\Lambda_i} \log \left( f_{N(\mu, \Sigma)}(\Lambda_i) \right);$$

Update: $\mu_{t+1} \leftarrow \mu_t - \rho_t \nabla U(\mu_t)$;

**Experimental results**

We consider the uniform distribution on $[0, 1]$ since this is the standard textbook example in auction design. We can easily extend our approach to any other distributions. We use $\sigma^2 = 0.005$ in our experiment to learn the linear shading and $\Sigma = \text{diag}(0.005, 0.005, 0.005)$ to learn the parameters of the thresholded exploration policy. To compute strategic bidder’s utility and seller’s revenue, we sample bidding strategy parameters according to the exploration bidding policy. Our result are reported on Table 1. For the setting with three bidders, we get an uplift of 20% in terms of utility for the strategic bidder and a decrease of 9% in seller’s revenue with a simple linear shading policy. It shows as expected that the MyersonNet architecture is not robust to adversarial attacks from a strategic bidder. Interestingly, with the thresholded strategies in the case of two bidders, the exploration bidding policy leads to both a higher utility for the strategic bidder and a higher revenue for the seller than when using linear shading. This is the illustration that the auction game is not a zero-sum game.

We compare the performance of our approach with several natural baselines. The VCG auction corresponds to the second-price auction without reserve price. This is a welfare-maximizing auction. It is possible to get a higher utility for a strategic bidder when seller is using a revenue-maximizing auction rather than a welfare-maximizing auction. Indeed, Myerson reduces the competition when all the other bidders are bidding below their re-
serve price. The strategic bidder takes advantage of this reduction of competition to increase his utility. We also compare our method with one recently introduced by [25] for the Myerson shading method. Our approach in the present paper brings three conceptual improvements. First, we are able to extend our approach to the multi-item setting, which is not possible using [25]’s approach. Moreover, our bidding policy is randomized as our goal is to be robust to any change in the seller’s behavior. Finally, we do not assume exact knowledge of the mechanism which was the case in [25]. This explains why we do not recover exactly the performance of [25], as they take advantage of the knowledge of the exact implementation of the Myerson auction.

We only provide experiments for less than 4 bidders since the interest of revenue-maximizing auctions both in terms of utility and revenue decreases dramatically with the number of bidders when they all have symmetric value distributions. This provides a first benchmark to design adversarial attacks against sellers’ learning algorithms. This benchmark could be extended in the near future by testing new seller algorithms and new architecture to learn strategic behaviors. We now show how our approach performs on more complex settings from the multi-item auction literature where no good shading strategies were previously known.

### Extension to various multi-item settings

We present our experimental pipeline for designing adversarial attacks in the multi-item setting.

### Settings

We assume that the seller is using the RegretNet architecture as introduced previously. We benchmark the impact of a linear exploration policy on the RegretNet architecture and see how the seller’s revenue is impacted by a strategic bidder in the multi-item setting. We consider two classical settings of the multi-item literature. We denote by Setting I the setting with two items and two bidders with additive valuations and uniform value distribution $F_1 = F_2 = U([0, 1]^2)$; and by Setting II the setup with two objects and three bidders with additive valuations and value distribution $F_1 = F_2 = F_3 = U([0, 1]^3)$. We consider the uniform value distribution as this is the standard textbook example. Our approach can be extended to any other distributions.

### Experimental results

We implemented Algorithm 1 initializing $\mu_{1}$ to be an array of $m$ ones (corresponding to the truthful strategy), and $\sigma_{k}^{2} = 0.05$ for all $k \in M$. We run $T = 150$ adversarial training epochs, and sample $q = 12$ lambdas per epoch. We optimize the seller mechanism every 3 adversarial epoch by training the RegretNet architecture. We implement the RegretNet architecture in PyTorch by using two neural networks with $H = 2$ hidden layers of size $h = 30$.

Our experimental results are reported in Table 2. We observe substantial improvements in bidders’ utility, with a 108% uplift for Setting I and a 54% uplift for Setting 2. Shading bids in the considered settings also generated important revenue decreases for the seller: a 21% decrease for setting I and a 5% one for setting II. This is the performance of the exploration and it would be possible to improve the strategic bidder’s utility by decreasing the variance of the exploration policy at the cost of not being robust to changes of the learning mechanism.

### Discussion

Using simple linear shadings in multi-item settings yielded considerable improvements in bidders’ utility. This suggests that even better improvements in utility could be found using more complex bidding strategies in the spirit of the thresholded-virtual-value strategy introduced by [24] for the single-item framework. Our work thus opens the door to several natural extensions such as using neural networks to parametrize more complex bidding strategies, or studying other bidder types, valuation distributions and auctions such as the combinatorial auction. However, training neural networks to learn the exploration policy would increase the running time of the procedure, which is already substantial for linear shading strategies. We plan to consider this line of improvements in future work. Another way to increase the speed of our algorithm would be to assume a model on the black-box used by the seller. Assuming this prior would enable first-order method to increase the speed of convergence. We could for instance design a bidding engine combining our approach and the one from [25] to be both robust to changes in the selling mechanism and quickly learn a good strategy when sellers use classical selling mechanisms such as the Myerson auction or the eager second price auction with monopoly price.

Our numerical results give a concrete sense of how much strategic bidders can improve their utility in comparison to when they bid truthfully. This reinforces the idea that the
Table 2: The strategic bidder is using a linear bidding exploration policy with parameter $\alpha^2 = 0.05$. The seller is using the RegretNet architecture as selling mechanism. We run $T = 150$ adversarial training epochs, and base our evaluation on averaging over $q = 12$ strategies from the exploration policy each epoch.

| Setting            | VCG          | RegretNet (truthful) | RegretNet (adversarial) |
|--------------------|--------------|----------------------|-------------------------|
|                    | utility | revenue | utility | revenue | utility | revenue |
| Setting I: two bidders, two objects uniform value distribution | 0.336 | 0.666 | 0.149 | 0.882 | 0.306 | 0.696 |
| Setting II: three bidders, two objects uniform value distribution | 0.166 | 1.000 | 0.096 | 1.034 | 0.148 | 0.985 |

This is a first benchmark of how simple learning approaches could help design adversarial attacks in revenue-maximizing auctions. Future works may involve more complicated shading strategies and exploration policies, minimizing for instance the risk of exploration as considered in the safety-exploration learning framework.

A new framework to design robust mechanisms

A natural extension to the design of adversarial attacks against data-driven automated selling mechanisms is the design of learning algorithms which are robust to adversarial attacks. This line of work has been initiated by [2], who find mechanisms which maximize the seller’s revenue against the worst bid distribution in a certain class. To avoid dealing with worst-case scenarios, an intermediate approach would be to consider mechanisms robust to a class of bidding strategies and a class of initial value distributions.

Definition 5 ($\epsilon$-adversarially-robust learning algorithm)

A selling learning algorithm is said to be $\epsilon$-adversarially-robust for this class of value distributions, if for any value distributions in this class, for any adversarial attack $\Pi$

$$R(\Pi) \leq R(\beta_{PA}) + \epsilon$$

This leads to a new definition of incentive compatible learning algorithms where bidders have an incentive to bid truthfully even if the seller is using past bids to optimize her mechanism. Designing such new robust selling mechanisms yields to solve a new Stackelberg game: the seller is the leader and assumes a certain class of possible bidding strategies for strategic agents to find the optimal learning mechanism against such strategic agents.

A follow up on our work could be to investigate feasibility of such robust mechanisms by adding a constraint to an augmented Lagrangian method similar to that used by [13].

Our approach is the first necessary step in the design of such robust mechanisms since it computes how the seller’s revenue is impacted when using a certain selling mechanism. Training such meta-mechanisms will enable to quantify the price for seller to use such $\epsilon$ adversarially-robust learning algorithms.

Conclusion

We present a new way to design adversarial attacks against cutting-edge automatic mechanism design algorithms. Our approach yields very substantial utility gains for the strategic bidder in our numerical experiments. This allows buyers to quantify the price of revealing information about their values in repeated auctions. This strengthens the balance of power between buyers and seller on modern marketplaces since our practical approach can easily be plugged in modern bidding engines. From a theoretical standpoint, this offers a new tool to study economics interactions through an algorithmic lens and represents a new step to reinterpret economics problems as algorithmic learning problems between strategic agents.

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