Spin polarization and g-factor of a dilute GaAs two-dimensional electron system

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(Dated: March 22, 2022)

The effective g-factor \( (g^*) \) of a dilute interacting two-dimensional electron system is expected to increase with respect to its bare value as the density is lowered, and to eventually diverge as the system makes a transition to a ferromagnetic state. We report here measurements of \( g^* \) in dilute \( (\text{density } 0.8 \text{ to } 6.5 \times 10^{10} \text{ cm}^{-2}) \), high-mobility GaAs two-dimensional electrons from their spin polarization in a parallel magnetic field. The data reveal a surprising trend. While \( g^* \) is indeed significantly enhanced with respect to the band g-factor of GaAs, the enhancement factor decreases from about 6 to 3 as the density is reduced.

PACS numbers: 73.50.-h, 71.70.Ej, 73.43.Qt

The ground state of a dilute, interacting electron system has been of interest for decades. It has long been expected that, because of interaction, such a system makes a transition to a ferromagnetic state as the density is reduced below a certain threshold \([1, 2]\). For even lower densities, the system should eventually become an electron solid (Wigner crystal) \( \text{[3]} \). A relevant parameter associated with this evolution is the effective g-factor \( (g^*) \) of the system. In the limit of high density, when the parameter \( r_s \), the average interparticle separation measured in units of effective Bohr radius, approaches zero, \( g^* \) should have the "bare" value determined by the energy band structure of the host material. With decreasing density, \( g^* \) is expected to increase monotonically and diverge at the density below which the electron system enters its ferromagnetic state. For an ideal two-dimensional electron system (2DES), quantum Monte Carlo calculations \( \text{[4]} \) indeed confirm the above trend.

An excellent candidate for testing these predictions is the GaAs 2DES, as it possesses very low disorder combined with a simple band structure. Here we report measurements of the spin polarization of a very high quality, dilute 2DES in a modulation doped GaAs/AlGaAs heterostructure as a function of an in-plane magnetic field. Via transport measurements, we find the magnetic field above which the 2DES becomes fully spin polarized, and from this field we determine \( g^* \). The results reveal a remarkable trend: as the density is lowered from 6.5 to 0.8 \( \times 10^{10} \text{ cm}^{-2} \), corresponding to an increase in \( r_s \) from 2.1 to 6.3, the measured \( g^* \) decreases from 2.7 to 1.3. This implies a substantial overall enhancement of \( g^* \) with respect to the band g-factor of GaAs \( (|g| = 0.44 \text{ in GaAs } \text{[6]}) \). The decrease of \( g^* \) with \( r_s \), however, is unexpected.

We studied a Si-modulation doped GaAs heterostructure grown on a \( (100) \) GaAs substrate. We used a square sample in a Van der Pauw geometry, with a backgate to control the density. We made measurements in a dilution refrigerator at a temperature \( (T) \) of \( \approx 25 \text{ mK} \) and magnetic fields \( (B) \) up to 18T, and in pumped \( ^3 \)He at \( \approx 0.3 \text{ K} \) and fields up to 33T. The sample was mounted on a single-axis tilting stage that can be rotated, using a computer controlled stepper motor, in order to change the angle \( (\theta) \) between the sample plane and the magnetic field. The measurements were done using low-frequency lock-in techniques. At zero gate bias, the sample has a density \( n = 1.4 \times 10^{10} \text{ cm}^{-2} \) and a mobility of 55 m\(^2\)/Vs.

An external magnetic field applied parallel to the 2DES causes a Zeeman splitting of the energy bands. This splitting induces a difference in population of the spin-up and spin-down subbands, which leads to a net spin polarization of the system. If the splitting exceeds the Fermi energy of the system, all spins are aligned and the 2DES is fully spin polarized. Assuming a simple model, in which \( g^* \) is independent of the applied magnetic field, we can write the splitting between the spin-up and spin-down subbands as \( E_Z = |g^*| \mu_B B \), where \( \mu_B \) is the Bohr magneton. In this model the 2DES becomes fully spin polarized at a field \( B_P \), given by

\[
B_P = (h^2/2\pi\mu_B) \cdot (n/m^*g^*)
\]

where \( m^* \) is the effective mass, \( n \) is the total density of the 2DES and \( h \) is Planck’s constant. We emphasize that we measure the effective g-factor defined, as in Eq. (1), by the field at which full polarization is achieved.

In our experiments we measure, via the analysis of Shubnikov-de Haas (SdH) oscillations, the Fermi contours of the two spin subbands. We apply a constant magnetic field parallel to the 2D plane and slowly rotate the sample around \( \theta = 0^\circ \) to induce a small perpendicular field component, \( B_\perp \). We record the sample resistance during the rotation, and Fourier analyze its SdH oscillations with \( B_\perp \) to obtain the populations of the two spin subbands \( \text{[7]} \). Our experiments allow a determination of the field, \( B_P \), above which the minority spin-subband depopulates and the 2DES becomes fully spin polarized. In the range where the Fourier transforms are done the parallel component of the field, \( B_\parallel \), is equal to the total field \( B \) to better than 2%.

In Fig. 1 we show plots of the sample resistance \( R \) vs. \( B_\perp \), taken at a density \( n = 2.05 \times 10^{10} \text{ cm}^{-2} \), as determined from the positions of quantum Hall states. The top trace was taken in a purely perpendicular field. The Fourier transform (FT), shown on the right, exhibits two peaks, one at 0.85T and another at approximately half this value, 0.42T. The 0.85T frequency, when multiplied by \( (e/h) \), gives 2.05 \( \times 10^{10} \text{ cm}^{-2} \), i.e., the total density...
of the 2DES. The 0.42T peak stems from the spin unresolved SdH oscillations. The rest of the traces shown in Fig. 1 were taken by rotating the sample at the indicated B applied almost parallel to the 2DES. With increasing $B_{\parallel}$, we observe a splitting of the lower FT peak (0.42T) into two peaks. The positions of these two peaks, multiplied by $(e/h)$, give the two spin subband populations. Note that the two populations add up to the total density of the sample. As $B_{\parallel}$ is increased, the majority spin subband peak merges with the total density peak (0.85T) and the minority spin subband peak moves to very low frequencies and is no longer resolved [8].

In Fig. 2 we summarize the positions of the FT peaks corresponding to the majority and minority spin subbands as a function of $B$ for the case examined in Fig. 1. Above a certain field $B_P$ the majority spin subband population saturates at a value which corresponds to the total density of the 2DES. Therefore, $B_P$ marks the onset of full spin polarization. Within the experimental error, the evolution of the FT peak positions as a function of field is linear. This implies that $g^*$ is roughly independent of the applied parallel field.

We also measured the in-plane magnetoresistance (MR), by fixing $\theta$ at 0° and recording the resistance as a function of the applied magnetic field. The MR trace, taken for $n = 2.05 \times 10^{10}$ cm$^{-2}$, is also shown in Fig. 2. This trace exhibits a clear break in the functional form of the MR as it changes from an $\sim e^{B^2}$ dependence at low field to a simple exponential, $\sim e^B$ dependence at higher fields. The data in Fig. 2 demonstrate that the onset of the simple exponential behavior of the in-plane MR coincides with the field $B_P$ above which the spins are fully polarized. Remarkably, the same functional behavior of the in-plane MR is seen in GaAs 2D hole systems [9], and it has been shown [10] that the onset of the simple exponential regime corresponds to the full spin polarization of the 2D system. Several studies on 2D electrons in Si-MOSFETs have also pointed out a correlation between the in-plane MR and the full spin polarization [11, 12, 13, 14].

The strong correlation between the full spin polarization and the onset of exponential MR provides another method of finding the field $B_P$. While this method does not allow a direct measurement of the spin subband populations, it can be useful at lower densities where the SdH method is no longer practical because of a decrease in the number of resistance oscillations. We summarize in Fig. 3 our MR data taken at different densities. At all densities, the MR exhibits an $\sim e^B$ dependence at high fields and the onset of this dependence clearly depends on the density. We emphasize that in several cases where we have made both MR and SdH oscillations measurements in constant field, the values of $B_P$ obtained from the two methods coincide.

Using the measured values of $B_P$ and the relation (1) we determine $g^*$. For $B_P$ we use the saturation field of the majority spin subband FT peak and the onset of the exponential regime of the in-plane MR. Another parameter needed is the effective mass, $m^*$, which also can be different from the band effective mass due to electron-electron interaction [14]. We independently measured $m^*$ from the $T$ dependence of SdH oscilla-

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**Fig. 1**: Resistance vs. perpendicular field at the indicated parallel fields and the Fourier transforms of the SdH oscillations. The total density of the 2DES is $2.05 \times 10^{10}$ cm$^{-2}$.

**Fig. 2**: Summary of FT peak positions as a function of applied magnetic field, along with the in-plane magnetoresistance trace. The dashed line marks the field $B_P$ at which the 2DES becomes fully spin polarized.
tions in a purely perpendicular magnetic field at several densities in the range 1.4 to $3 \times 10^{10}$ cm$^{-2}$. The analysis uses the Dingle formula, $\Delta R/R_0 \sim \xi/\sinh \xi$, where $\Delta R/R_0$ is the normalized amplitude of the SdH oscillations, $\xi = 2\pi^2 k_B T/\hbar \omega_c$ and $\omega_c = eB/m^*$. Fits of the Dingle formula to the data are shown in Fig. 4, together with the values of $m^*$ that provide the best least-squares fit. Within the experimental error (7%), $m^*$ is the same as the band effective mass of electrons in GaAs, $m_b = 0.067 m_e$, where $m_e$ is the free electron mass. For simplicity, in determining $g^*$ we have used the band effective mass.

In Fig. 5 we plot the values of $g^*$, normalized to the GaAs band $g$-factor, as a function of density and $r_s$. The open and closed symbols denote the two methods used to determine $B_P$: their overlap confirms that the onset of exponential regime of the MR marks the full spin polarization. For the lowest density traces, we have data available only at 0.3K. However, $T$ dependence of the in-plane MR, measured at $n = 1.4 \times 10^{10}$ cm$^{-2}$, showed only a small variation of $B_P$ with $T$ in the range 25mK to 0.5K. The larger error bars for the lowest density points in Fig. 5 take into account such variation with $T$.

Figure 5 data highlight the main finding of our work: while the measured values of $g^*$ are three to six times larger than the GaAs band $g$-factor, this enhancement decreases with increasing $r_s$. This trend is unexpected and at variance with theoretical calculations which predict that, for an ideal 2DES, $g^*$ should increase as $r_s$ increases and the interaction becomes more important. While we do not have an explanation for this discrepancy, we point out that at least three factors distinguish our samples from ideal 2D systems: finite layer thickness, disorder, and the spin-orbit interaction.

For a 2DES with finite layer thickness, even in a single-electron picture, an in-plane magnetic field can affect the energy bands and therefore both the effective mass and the $g$-factor. Indeed, band calculations performed for the 2DES studied in our work reveal some enhancement of the effective mass and, to a lesser degree, a reduction of the $g$-factor with in-plane field. However, these modifications of the band parameters appear to be too small to explain the data of Fig. 5.

The effect of disorder is more subtle. As the density is reduced, the disorder potential can play a more dominant role than the electron-electron interaction, and may lead to an inhomogeneous spatial distribution of the electrons in the sample. If the electrons become localized in the potential minima, it is possible that the behavior of the 2DES reverts to that of a single-particle system and $g^*$ decreases. Such a scenario is consistent with the data of Fig. 5: for $n = 0$, $g^*/g_b$ extrapolates to a value close to unity. On the other hand, we have made $g^*$ measurements on a sample with a mobility three times lower than the present specimen, and obtained similar results as those shown in Fig. 5. Moreover, our sample exhibits no signs of strong electron localization: it exhibits SdH oscillations, and its resistivity is less than $h/e^2$.

An examination of the results of several studies which have reported measurements of $g^*$ in Si-MOSFET 2DES provides further argument against disorder being responsible for the anomalous behavior we observe in Fig. 5. These studies have generally reported an enhancement of $g^*$ with increasing $r_s$ in much the same range of $r_s$ that we have examined. The mobility of our 2DES, however, is about a factor of 10 larger than in Si-MOSFETs even though our densi-
FIG. 5: The effective g-factor normalized to the GaAs band

the corresponding values of $\Delta /E_F$. On the right scale we indicate

the lower density data points. On the right scale we indicate

the difference between the band Fermi and Zeeman energies,

the magnetic fields above which the 2D holes are fully

be intrinsic to low disorder GaAs 2D electrons and holes.

The spin-orbit interaction, present in both GaAs 2D

electron and hole systems, but nearly absent in Si-

MOSFETs, may also play a role here. Since $g_b$ is

significantly influenced by the spin-orbit interaction, one

may expect that this interaction should modify $g^*$ in a

many-body picture also. It is not clear, however, how the

spin-orbit interaction would explain the trend in Fig. 5.

Finally, in Fig. 5 we provide a measure of the interaction

energy for the spin polarization of our 2DES. We

introduce the enhancement energy $\Delta E$, defined as the

difference between the band Fermi and Zeeman energies,

both evaluated at the in-plane field where the 2DES be-

comes fully spin polarized, i.e. $\Delta E = (2\pi\hbar^2/m_0)n - g_b\mu_B B_P$. The energy $\Delta E$, measured in units of the Fermi

energy is then simply equal to $(1 - g_b/g^*)$. This quantity,

which is indicated on the right scale of Fig. 5, reiterates

the main result of our study. The measured $\Delta E/E_F$ de-

creases with increasing $r_s$, while theoretical calculations

predict the opposite: $\Delta E/E_F$ should be zero at

$r_s = 0$, increase monotonically with $r_s$, and reach unity

for $r_s$ larger than a critical value of $\approx 30$.

We thank E.P. De Poortere, D.M. Ceperley, A.H. Mac-

Donald, B.L. Altshuler and R. Winkler for fruitful discus-

sions. This work was supported by the DOE, NSF

and the von Humboldt foundation. Part of the work was

done at NHMFL which is supported by NSF; we also

thank T. Murphy and E. Palm.

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