Nonlinear Dynamics of Biomimetic Micro Air Vehicles

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Abstract. Flapping-wing micro air vehicles (FMAV) are new conceptual air vehicles that mimic the flying modes of birds and insects. They surpass the research fields of traditional airplane design and aerodynamics on application technologies, and initiate the applications of MEMS technologies on aviation fields. This paper studies a micro flapping mechanism that based upon insect thorax and actuated by electrostatic force. Because there are strong nonlinear coupling between the two physical domains, electrical and mechanical, the static and dynamic characteristics of this system are very complicated. Firstly, the nonlinear dynamic model of the electromechanical coupling system is set up according to the physical model of the flapping mechanism. The dynamic response of the system in constant voltage is studied by numerical method. Then the effect of damping and initial condition on dynamic characteristics of the system is analyzed in phase space. In addition, the dynamic responses of the system in sine voltage excitation are discussed. The results of research are helpful to the design, fabrication and application of the micro flapping mechanism of FMAV, and also to other micro electromechanical system that actuated by electrostatic force.

1. Introduction

Flapping-wing micro air vehicles (FMAV) are new conceptual air vehicles that mimic the flying modes of birds and insects. Compared with fixed wing air vehicle and rotary wing air vehicle, the FMAV integrated lifting, thrusting and hanging function into a flapping wing system, and has the ability to cruise a long distance without energy supply, at the same time FMAV has high maneuverability and flexibility. The flapping wing mode of flying creatures in nature gives us edification that flapping wing mode is superior to fixed wing and rotary wing modes when the wingspan is less than 15cm, and the results of bionics and aerodynamics prove it\(^[1]\).

The characteristics of flapping wing mode lie in the lift and thrust is produced by downstroke and upstroke of the wings, so the design of efficient and reliable flapping mechanism is more important. For FMAV which size is similar to flies and bees, the design, fabrication and study of dynamic characteristics of micro flapping mechanism based on MEMS technology are key questions.

2. The Design of Micro Flapping Mechanism

The millimeter-sized insects are our models to develop insect based FMAV. New concepts should be applied in the design including flexible structures and frictionless joints. The special features of flying

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insects such as external skeletons, elastic joints, deformable thorax and contracting-relaxing muscles provide clues for designing FMAV\cite{2,3}.

Figure 1 shows the cross section of an insect thorax, illustrating the flapping mechanism of the wing. The upward movement of the wing results from distortions of the thorax produced by muscles not directly attached to the wing. Elasticity of the thorax plays an important role in the friction free high-speed wing movement. In most insects beating of the wings is controlled by a nervous stimulus, using on-off control. However, in some small sized insects, such as bees and flies, the beating frequency is much higher than the repetition rate of nervous stimuli. It is instead determined by the natural frequency of the mechanical system formed by the muscles, elastic hinges, and thorax.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure1.png}
\caption{Cross section of an insect thorax.}
\end{figure}

For millimeter-sized FMAV, electrostatic actuator, piezoelectric actuator, and electromagnetic actuator can be used in the flapping mechanism\cite{4,5}. Among those, electrostatic micro actuator is simple in work principle, convenient in realization, small in energy consumption, and easy in concentration, so it has the predominant position in the study and development of micro actuator.

Based on former reasons, the flapping mechanism mimics the thorax structure of insects, and is actuated by electrostatic force. The basic model of two degrees flapping mechanism is shown in figure 2. The system is mainly composed of two parallel electric boards, one of them is fixed on base, and the other is movable and connected with the linkage to actuate the wing beating upward and downward for both sides. The whole structure has no bearing and rotary axis, and all the pivots adopt flexible hinges. When a varying voltage is applied to the two boards, the varying electrostatic force will actuate the wings beating. When different voltage is applied in the left and right of the board, corresponding wing will have different flapping amplitude, so the lift and thrust produced by the wing is different, and make the whole air vehicle turn around.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure2.png}
\caption{Micro electrostatic flapping mechanism based upon insect thorax.}
\end{figure}

3. Coupling Dynamic Model of Actuator Mechanism

For the mechanism illustrated in figure 2, suppose the movable board is rigid. The linkages support and constrain the board and play the role of spring and damper. Suppose the distortion of the flexible hinges and linkages are within their elasticity limit and the equivalent stiffness is linear along the move direction, so the system dynamic model is shown in figure 3.

When a voltage is applied to the two boards, the electrostatic force pulls the movable board toward the fixed board. The distance between the two boards affects the electrostatic force, and at the same time the change of the electrostatic force affects the distance between the two boards, that is to say, there is coupling action between electrostatic field and mechanical field. When the voltage is beyond the critical voltage, the movable board will be pulled in the fixed board.
In figure 3, k is spring stiffness, \(d_0\) is initial distance, \(L\) and \(a\) are width and length of board, \(x\) and \(\theta\) are translation and rotation displacement.

![Figure 3. Dynamic model of flapping mechanism.](image)

When the voltage \(U_L\) and \(U_R\) are added to left and right of the board, the movable board will reach an equilibrium point at \((x, \theta)\) as shown in figure 3. Then the electrostatic force in vertical direction is given by

\[
F_{el} = \int_0^L \frac{\varepsilon W U_L^2}{2(d_0 - x - l\theta)^2} dl = \frac{\varepsilon W U_L^2}{2\theta} \left( \frac{1}{d_0 - x - L\theta} - \frac{1}{d_0 - x} \right)
\]

\[
F_{eR} = \int_0^L \frac{\varepsilon W U_R^2}{2(d_0 - x + l\theta)^2} dl = \frac{\varepsilon W U_R^2}{2\theta} \left( \frac{1}{d_0 - x} - \frac{1}{d_0 - x + L\theta} \right)
\]

Where \(F_{el}\) and \(F_{eR}\) are electrostatic forces that added on left and right of the upper board.

The electrostatic torque of the upper board is given by

\[
T_{el} = \int_0^L \frac{\varepsilon W U_L^2}{2(d_0 - x - l\theta)^2} l dl = \frac{\varepsilon W U_L^2}{2\theta^2} \left[ \ln \left( \frac{1}{d_0 - x - L\theta} \right) - \frac{1}{d_0 - x} \right] - 1
\]

\[
T_{eR} = \int_0^L \frac{\varepsilon W U_R^2}{2(d_0 - x + l\theta)^2} l dl = \frac{\varepsilon W U_R^2}{2\theta^2} \left[ \ln \left( \frac{1}{d_0 - x + L\theta} \right) + \frac{1}{d_0 - x} \right] - 1
\]

Where \(T_{el}\) and \(T_{eR}\) are electrostatic torques that added on left and right of the upper board.

The spring force and torque acting on the upper board are

\[
F_s = k(x + L\theta) + k(x - L\theta) = 2kx
\]

\[
T_s = kL(x + L\theta) - kL(x - L\theta) = 2kL^2\theta
\]

According to equation (1), (2) and (3), we can get the coupling nonlinear dynamic equations of micro flapping mechanism

\[
\begin{align*}
\dot{m}\ddot{x} + c_1\dot{x} + 2kx &= F_{el} + F_{eR} \\
J\ddot{\theta} + c_2\dot{\theta} + 2ka\dot{\theta} &= T_{el} + T_{eR}
\end{align*}
\]
Where $c_t$ and $c_r$ are translation and rotation movement damping. Because the FMAV fly in low Reynolds number, the air damping force of the movable board can be expressed as the linear function of the velocity approximately.

For equation (4), if $y = x / d_0$, $\delta = \theta / \theta_{\text{max}} \approx \theta_0 / (d_0 / L) = L \theta_0 / d_0$, the primitive equation is as follows:

$$
\begin{align*}
\frac{m}{2} \ddot{y} + c_t \dot{y} + 2k y &= \frac{c W a}{2d_0 \delta} \left[ U_1^2 \left( \frac{1}{1 - y - \delta} - \frac{1}{1 - y} \right) + U_2^2 \left( \frac{1}{1 - y} - \frac{1}{1 - y + \delta} \right) \right] \\
J \ddot{\delta} + c_r \dot{\delta} + 2k L^2 \delta &= \frac{c W a^3}{2d_0^2 \delta^2} \left[ U_1^2 \left( \ln(1 - \frac{\delta}{1 - y}) + \frac{\delta}{1 - y - \delta} \right) - U_2^2 \left( \ln(1 + \frac{\delta}{1 - y}) - \frac{\delta}{1 - y + \delta} \right) \right] 
\end{align*}
$$

(5)

Define dimensionless time variant $\tau = \omega_0 t$, where $\omega_0 = \sqrt{k / m}$ is nature frequency, and define damping ratio $\zeta_t = c_t / 2m \omega_0$, $\zeta_r = c_r / 2J \omega_0$, so equation (5) can be simplified as

$$
\begin{align*}
\frac{d^2 y}{d \tau^2} + 2\zeta_r \frac{dy}{d \tau} + 2y &= \frac{c W L}{2k d_0^2 \delta} \left[ U_1^2 \left( \frac{1}{1 - y - \delta} - \frac{1}{1 - y} \right) + U_2^2 \left( \frac{1}{1 - y} - \frac{1}{1 - y + \delta} \right) \right] \\
\frac{d^2 \delta}{d \tau^2} + 2\zeta_r \frac{d\delta}{d \tau} + 6\delta &= \frac{3c W L}{2k d_0^2 \delta^2} \left[ U_1^2 \left( \ln(1 - \frac{\delta}{1 - y}) + \frac{\delta}{1 - y - \delta} \right) - U_2^2 \left( \ln(1 + \frac{\delta}{1 - y}) - \frac{\delta}{1 - y + \delta} \right) \right] 
\end{align*}
$$

(6)

So this is the nonlinear dynamic equations of the coupling system.

4. The Analysis of System Response Characteristics

4.1. System Nonlinear Dynamic Characteristics in Phase Space

If define $y_1 = y$, $y_2 = \dot{y}$, $\delta_1 = \delta$, $\delta_2 = \dot{\delta}$, then system equation changed into phase space form. The Runge-Kutta numerical method is used to solve the equation and analyze the dynamic characteristics of coupling system in phase space. The phase space orbits in different voltage and damping conditions are as figure 4.
From figure 4 we can draw the conclusions:

1. There is an equilibrium point in the $y_1$ axis ($y_2 = 0$), and this equilibrium point is stable focus. The orbit near the equilibrium position forms a helix toward inner, and in fact, the upper board stabilizes in an equilibrium position gradually. When the voltage is larger than the pull-in voltage, the phase orbit becomes emanative, and the upper board is pulled to fixed board.

2. In the scope of pull-in voltage, the kinematical center move right along with augment of voltage.

3. The augment of damping make the critical pull-in voltage bigger correspondingly.

4. When the initial values are different, the properties of phase space orbit are different. Initial value will affect the amplitude of pull-in voltage.

4.2. System Response in Sine Signal Voltage

In actual work, the time-varying voltage that applied to the two boards makes the wing stroke up and down. For real flapping-wing micro air vehicle, we hope to produce larger flapping angle by lower voltage because of the constraints of weight and voltage promoting circuit. When the excitation frequency is equal to the natural frequency of the system, the sympathetic vibration is occurred and the large flapping movement amplitude is acquired.

When the time-varying excitation voltage is sine signal as $U(t) = U \sin(\omega t)$, and

$$(U \sin(\omega t))^2 = \frac{U^2(1 - \cos(2\omega t))}{2} = \frac{U^2}{2} \left(1 - \cos \frac{2\omega}{\omega_0} \tau\right)$$

(8)

Substitute (7) into equation (6), and define $\gamma = \omega/\omega_0$, solve the differential equations with numerical method. The system response in different frequency ratio $\gamma$ is shown in figure 6. The calculation parameters are $U_L=40V$, $U_R=20V$, $\zeta_r = 0.1$, $\zeta_s = 0.1$, initial condition is $y = 0$, $\dot{y} = 0$, $\delta = 0.01$, $\dot{\delta} = 0$.

In figure 5, $\tau - y_1$ and $\tau - \delta_1$ are the curves of translation and rotation displacement to time, $y_1 - y_2$ and $\delta_1 - \delta_2$ are the phase space orbit of translation and rotation movement, respectively.
From figure 5 we can draw the conclusions:

1. The responses of system of sine excitation voltage include transient response and steady response. When excitation voltage is less than pull-in voltage, the response of the system is periodic, and the response frequency is two times of excitation frequency.

2. From figure when $\gamma = 0.5$ the displacement is maximum. The right item of the equation can be expressed as function of $\cos(2\gamma \tau)$, so the system resonates at $\gamma = 0.5$, and now the movable board has the maximum amplitude. In FMAV, in order to actuate flapping mechanism by as small voltage as impossible, the excitation frequency should be closer to resonance frequency.

5. Conclusions
The design and dynamic characteristics of micro flapping mechanism are the keys of FMAV design. This paper designs an electrostatic actuated micro flapping mechanism based upon insect thorax and flapping movement. The nonlinear static and dynamic characteristics are studied by numerical calculation and analysis. The results of research are helpful to the design, fabrication and application of the micro flapping mechanism of FMAV, and also to other micro electromechanical system that actuated by electrostatic force.

References
[1] Shyy W, Berg M and Ljungqvist D 1999 Prog. in Aerosp. Sci. 35 455
[2] Shimoyama I, Miura H and Suzuki K 1993 IEEE Contr. Syst. Mag. 2 37
[3] Suzuki K and Shimoyama I 1994 J. Microelectromech. Syst. 3 4
[4] Fischer M, Gioussouf M and Schaepperle J 1998 Sensor. Actuat. A 67 89
[5] Chu P B, Nelson P R and Tachiki M L 1996 Sensor. Actuat. A 52 216