D(p)-Branes Moving at the Speed of Light

Bjørn Jensen
NORDITA,
Blegdamsvej 17, DK-2100 Copenhagen Ø,
Denmark

Abstract

We construct new bosonic boundary states in the light-cone gauge which describe D(p)-branes moving at the speed of light.

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1\textsuperscript{e-mail:}bjensen@gluon.uio.no
2\textsuperscript{On leave from Institute of Physics, University of Oslo, Norway.}
1 Introduction

Static and moving D(p)-branes can either be perceived as spacelike hyper-surfaces on which open strings can end, or they can equivalently be described by D(p)-brane boundary states into which closed strings can disappear. D(p)-branes are characterised by a tension \( T_p \neq 0 \) which can be computed by considering the exchange of closed strings between two D(p)-branes. Because of this non-vanishing tension these branes are either static, or they move with velocities which are less than the speed of light. A natural problem to pose, and one which we will pursue in this paper, is whether it is possible to extend the notion of D(p)-branes above to also include D(p)-branes which move at the speed of light, or (equivalently) D(p)-branes which have a vanishing tension \( T_p = 0 \). In the following we will call D(p)-branes which move at the speed of light tensionless D(p)-branes.

The expression for \( T_p \) in terms of the inverse of the fundamental string tension \( \alpha' \), and the string coupling \( g \), is given by

\[
T_p = 2\pi g^{-1}(2\pi \sqrt{\alpha'})^{-p-1},
\]

where \( p \geq 0 \) is the number of spacelike directions in the brane. When this expression is extrapolated to arbitrarily large values of \( g \), \( T_p \) can attain values which are arbitrarily close to zero. Hence, in the strongly coupled regime D(p)-branes become light states (since their masses \( M_p \) behaves as \( M_p \sim g^{-1} \)), and when \( g = \infty \) the D(p)-branes will naively become tensionless. This behaviour lies at the very foundation of M(atrix)-Theory. However, we will not address the behaviour of D(p)-branes at infinitely strong coupling in this paper. We will at the outset assume that the string coupling constant is relatively small, so that our considerations are constrained to the domain of standard string theory. The question we will address is thus specifically whether it is possible to construct boundary states in a perturbative sector of M-Theory which can fit the role as tensionless D(p)-branes.

Boundary states which describe moving D(p)-branes were derived in \( ^4 \) by acting with a Lorentz boost on the static boundary states which were derived in \( ^6 \). This method of constructing moving D(p)-brane boundary states is necessarily restricted to the construction of boundary states which describe D(p)-branes which move with velocities which are less than the

\(^3\text{Note that this expression is true when we are dealing with static D(p)-branes. When the branes are moving } T_p \text{ receives velocity corrections } \[.\)
speed of light. In the present paper we demonstrate that the moving bosonic 
D(p)-brane boundary states in \[4, 5\] can be constructed via a route which is 
somewhat different from the one followed in those works. Our way of deriving 
these boundary states also allows us to construct bosonic tensionless D(p)-brane boundary states.

Tensionfull D(p)-branes also appear in the super-symmetric type IIA and 
type IIB string theories. The bosonic boundary states which we construct 
in this paper will also appear in these super-symmetric theories, and will 
represent the bosonic pieces of the complete boundary states which describe 
D(p)-branes there. The construction of the complete boundary states which 
describe tensionless D(p)-branes in the super-symmetric string theories is 
straightforward, and will be presented elsewhere.

We have organised the rest of this paper as follows. We next rederive the 
moving bosonic D(p)-brane boundary states in \[4, 5\] (which we will call time-
like D(p)-branes), and discuss briefly the conditions set by the requirement 
of BRST invariance of bosonic D(p)-brane boundary states in general. In 
the third section we construct boundary states which describe D(p)-branes 
which move at the speed of light. In section four we briefly touch upon the 
issue about the nature of the zero-modes of tensionless D(p)-brane bound-
ary states. We summarise our results in the last section. The exposition of 
the subject matter in this paper is, partially for the purpose of future refer-
ence, aimed at displaying the steps in the various derivations in the paper in 
considerable detail.

2 Timelike D(p)-Branes

We will first rederive the moving bosonic D(p)-brane boundary states in \[4, 5\]. 
The boundary state in question is assumed to move in the \(X^{(p+1)}\) target-space 
direction. We will furthermore assume that the classical position of the D(p)-brane in the spatial directions perpendicular to the brane, is at the origin of 
the global coordinate system. The coordinates perpendicular to the brane, 
extcept for the target-space time-coordinate \(X^{(0)}\) and the \(X^{(p+1)}\)- coordinate, 
are

\[ A = \{X^{(A)}; A \in \{p + 2, \ldots, 25\}\}, \]

and the spacelike coordinates in the brane are

\[ B = \{X^{(A)}; A \in \{1, \ldots, p\}\}. \]
Introduce two new coordinates $Z$ and $Y$ defined by
\[
\begin{pmatrix}
  Z \\
  Y
\end{pmatrix} = \begin{pmatrix}
  1 & -v \\
  -v & 1
\end{pmatrix} \begin{pmatrix}
  X^{(p+1)} \\
  X^{(0)}
\end{pmatrix},
\]
where $v$ will turn out to be the constant physical linear velocity of the state ($|v| \leq 1$). The $Z$-coordinate has been used in previous works in the context of open strings [7, 8]. Our choice of the $Z$ and $Y$ coordinates is guided by the fact that they will give rise to the correct form of the zero-modes below. The vectors associated to $Z$ and $Y$ have the properties
\[
\begin{cases}
  \partial_2 Z = -\partial_2 Y = 1 - v^2 \equiv \gamma^2, \\
  \partial Z \cdot \partial Y = 0.
\end{cases}
\]
Note that the linear transformation in eq.(2) together with a normalisation of the $\partial Z$ and $\partial Y$ operators will constitute a Lorentz boost.

We decompose the closed string target-space coordinates into the canonical form
\[
X^{(A)}(\sigma, \tau) = x^{(A)} + p^{(A)} \tau + \frac{i}{2} \sum_{n \neq 0} \frac{1}{n} (\alpha^{(A)}_n e^{-2i(n-\sigma)} + \tilde{\alpha}^{(A)}_n e^{-2i(n+\sigma)})
\]
for any $A \in \{0, \ldots, 25\}$. The $\alpha_n$’s are related to the conventionally normalised harmonic oscillator operators by $\alpha^{(A)}_m = \sqrt{ma^{(A)}_m}$ and $\alpha^{(A)}_{-m} = \sqrt{ma^{(A)}_m}$ ($m > 0$). The, at the outset, uncoupled $\alpha^{(A)}_m$ and $\tilde{\alpha}^{(A)}_m$ operators are normalised to the flat Minkowski metric $\eta^{AB}$ with signature $(- + +)$, i.e.
\[
\begin{align*}
[\alpha^{(A)}_m, \alpha^{(B)}_n] &= [\tilde{\alpha}^{(A)}_m, \tilde{\alpha}^{(B)}_n] = m \delta_{m+n} \eta^{AB}, \\
[\alpha^{(A)}_m, \tilde{\alpha}^{(B)}_n] &= 0.
\end{align*}
\]

We will consider the following mixed set of Dirichlet and Neumann boundary conditions on the closed string target-space coordinates
\[
\begin{align*}
I & : (\partial_\sigma X^{(A)})|_{\tau=0} B; p, v \rangle = 0, \quad X^{(A)}|_{\tau=0} B; p, v \rangle = 0; \quad A \in \mathcal{A}, \\
II & : (\partial_\tau X^{(A)})|_{\tau=0} B; p, v \rangle = 0; \quad A \in \mathcal{B}, \\
III & : (\partial_\sigma Z)|_{\tau=0} B; p, v \rangle = 0, \quad Z|_{\tau=0} B; p, v \rangle = 0, \\
IV & : (\partial_\tau Y)|_{\tau=0} B; p, v \rangle = 0.
\end{align*}
\]
$|B; p, v \rangle$ denotes the state-vector of a D(p)-brane boundary state with intrinsic spatial dimensionality $p$, and velocity $v$. $\sigma$ and $\tau$ are the spacelike and
timelike coordinates in the closed string world-sheet, respectively. We will let lower-case Greek letters denote the world-sheet coordinates in the following. We have also assumed, without any loss of generality, that the closed strings are absorbed (or emitted) by the boundary state at world-sheet time $\tau = 0$.

The boundary conditions I-IV above translate into the following set of operator relations when we substitute the expansion in eq.(4) into eq.(7-10), and equate coefficients of $e^{im\sigma}$

\begin{align}
I : \quad & \alpha_m^{(A)} = \tilde{\alpha}_m^{(A)} , \quad x^{(A)} = 0 ; \quad A \in \mathcal{A} , \quad (11) \\
II : \quad & \alpha_m^{(A)} = -\tilde{\alpha}_m^{(A)} , \quad p^{(A)} = 0 ; \quad A \in \mathcal{B} , \quad (12) \\
III : \quad & \alpha_m^{(p+1)} + v\alpha_m^{(0)} = \tilde{\alpha}_m^{(p+1)} + v\tilde{\alpha}_m^{(0)} , \quad x^{(p+1)} = v x^{(0)} , \quad (13) \\
IV : \quad & \alpha_m^{(p+1)} + \alpha_m^{(0)} = -(v\tilde{\alpha}_m^{(p+1)} + \tilde{\alpha}_m^{(0)}) , \quad v p^{(0)} = p^{(p+1)} . \quad (14)
\end{align}

The structure of the resulting operator relation in I and the resulting operator relation in II is that a right moving creation operator equals a left moving annihilation operator (say). Clearly, the operator relations in III and IV also have this structure, but in order to make this feature more transparent we define the following new set of operators using an obvious notation

\begin{align}
A_r \equiv & \quad \alpha_r^{(p+1)} + v\alpha_r^{(0)} , \quad \tilde{A}_r \equiv \tilde{\alpha}_r^{(p+1)} + v\tilde{\alpha}_r^{(0)} ; \quad r \neq 0 , \quad (15) \\
B_s \equiv & \quad v\alpha_s^{(p+1)} + \alpha_s^{(0)} , \quad \tilde{B}_s \equiv v\tilde{\alpha}_s^{(p+1)} + \tilde{\alpha}_s^{(0)} ; \quad s \neq 0 . \quad (16)
\end{align}

In terms of these operators the operator relations in III and IV become

\begin{equation}
A_r = \tilde{A}_{-r} , \quad B_r = -\tilde{B}_{-r} . \quad (17)
\end{equation}

Clearly, we have that $A_{-r} = A_r^\dagger , \quad B_{-s} = B_s^\dagger (r, s > 0)$. The new operators $A, B, \tilde{A}$ and $\tilde{B}$ can easily be seen to satisfy the following uncoupled, and closed algebras

\begin{align}
\left\{ \begin{array}{l}
\{ A_r , B_s \} = 0 , \\
\{ A_r , \tilde{A}_s \} = -[B_r , B_s] = r(1 - v^2)\delta_{r+s} ,
\end{array} \right. \quad (18)
\end{align}

and

\begin{align}
\left\{ \begin{array}{l}
\{ \tilde{A}_r , B_s \} = 0 , \\
\{ \tilde{A}_r , \tilde{A}_s \} = -[\tilde{B}_r , \tilde{B}_s] = r(1 - v^2)\delta_{r+s} .
\end{array} \right. \quad (19)
\end{align}

It is evident that these algebras have the same structure as the algebra in eq.(5-6). With $v = 0$ we find that $A_r = \alpha_r^{(p+1)}$ and $B_s = \alpha_s^{(0)}$. This suggests that we in general (provided that $|v| < 1$, of course) can interpret the $A_r$
operators as corresponding to spacelike excitations, while the $B_s$ operators can be looked upon as timelike excitations. The same reasoning also holds in the tilded sector. Clearly, the signs of the commutators in eq. (18) and eq. (19) are consistent with our interpretation of these operators. The algebra in eq. (18) has the same structure as the algebra in eq. (19). Hence, the right moving sector of operators is just a copy of the left moving sector of operators on this algebraic level.

The way we have organised the operators in eq. (11), eq. (12) and eq. (17) suggests to use the operators $\alpha_m^{(A)}$ with $A \neq (0, p + 1)$, as well as the $A_r$ and $B_s$ operators as a basis for a canonical coherent state construction of the moving D(p)-brane boundary states. In a canonical coherent state construction which involves an infinite number of bosonic degrees of freedom, one assumes the existence of an infinite set of annihilation and creation operators $d_k, d_{-k}$ ($k, l : \text{positive integers}$) which are normalised to $(\pm)$ unity (i.e., $[d_k, d_{-l}] = \pm \delta_{k-l}$), and the existence of a vacuum state $|0\rangle \in \mathcal{F}$ on which the creation operators can act so as to produce the usual Fock space representation of the canonical commutation relations. Let $\{z_k\}$ denote an arbitrary sequence of complex $c$-numbers such that $\sum_k |z_k|^2 < \infty$. $\{z_k\}$ defines an element in an infinite dimensional Hilbert space $\mathcal{H}$. To each such element in $\mathcal{H}$ we define a unit vector $|\{z_k\}\rangle \in \mathcal{F}$. One can then construct an over-complete set of basis vectors for $\mathcal{F}$ with each of the vectors given by

$$|\{z_k\}\rangle = e^{(-\frac{i}{2} \sum_{k=1}^{\infty} |z_k|^2)\mu(S) \sum_k \pm z_k d_{-k})} |0\rangle,$$

where we must use + in the last sum when $d_k$ is normalised to unity, while we must use the minus sign when this operator is normalised to minus one. A key property of the canonical coherent states is that

$$d_k|\{z_k\}\rangle = z_k|\{z_k\}\rangle.$$

In order to go through with a coherent state construction of D(p)-brane boundary states, we may construct the boundary states using right-moving operators, e.g.. Seen from the perspective of the right-moving sector left-moving operators appear as c-numbers, and will thus naturally play the role as the $\{z_k\}$-sequences above. We can of course swap the roles played by the right and left-moving sectors in the construction of the boundary states. It is thus in some sense natural and “democratic” to normalise the $\alpha_m^{(A)}$ ($A \neq 0, p+1$), $A_r$ and $B_s$ operators, as well as their eigenvalues $\tilde{\alpha}_m^{(A)}, \tilde{A}_r$ and
\( \tilde{B}_{-s} \), to unity, i.e. we should rescale these operators according to \((m, r, s > 0)\)

\[
\alpha_m^{(A)} \to \frac{1}{\sqrt{m}} \alpha_m^{(A)}, \quad A_r \to \frac{1}{\sqrt{r} \gamma} A_r, \quad B_s \to \frac{1}{\sqrt{s} \gamma} B_s,
\]

\( (22) \)

\[
\tilde{\alpha}_{-m}^{(A)} \to \frac{1}{\sqrt{m}} \tilde{\alpha}_{-m}^{(A)}, \quad \tilde{A}_{-r} \to \frac{1}{\sqrt{r} \gamma} \tilde{A}_{-r}, \quad \tilde{\tilde{B}}_{-s} \to \frac{1}{\sqrt{s} \gamma} \tilde{\tilde{B}}_{-s}.
\]

\( (23) \)

This corresponds to introducing conventionally normalised harmonic oscillators \( a_m^{(A)} \), as well as normalising the vectors \( \partial_Z \) and \( \partial_Y \) to unity. With this rescaling we find that the oscillator part of a moving bosonic D(p)-brane boundary state is given by

\[
|B; p, v \rangle \sim \exp \left( + \sum_{m=1}^{\infty} \frac{1}{m} \left( \sum_{A=p+2}^{25} \alpha_m^{(A)} \tilde{\alpha}_{-m}^{(A)} - \sum_{D=1}^{p} \alpha_m^{(D)} \tilde{\alpha}_{-m}^{(D)} + \frac{1}{\gamma^2} \left( A_m \tilde{\alpha}_{-m} + B_m \tilde{\tilde{B}}_{-m} \right) \right) \right) |0 \rangle \otimes |B; p \rangle_{\text{ghost}} = \\
= \exp \left( + \sum_{m=1}^{\infty} \left( \sum_{A=p+2}^{25} \alpha_m^{(A)} \tilde{\alpha}_m^{(A)} - \sum_{D=1}^{p} \alpha_m^{(D)} \tilde{\alpha}_m^{(D)} \right) \right) \times \\
\times \exp \left( + \sum_{m=1}^{\infty} \left( a_m^{(0)} \tilde{a}_m^{(0)} + a_m^{(p+1)} \tilde{a}_m^{(p+1)} \right) \right) M(v) \left( \begin{array}{c} \tilde{a}_m^{(0)} \\ \tilde{a}_m^{(p+1)} \end{array} \right) |0 \rangle \otimes |B; p \rangle_{\text{ghost}}, \]

\( (24) \)

where we in the last two lines have switched to the conventionally normalised harmonic oscillator operators in order to facilitate the comparison with previous results. In eq.(24) \(|0\rangle\) denotes the bosonic closed string tachyon vacuum, \(|B; p\rangle_{\text{ghost}}\) denotes the ghost part of the full boundary state, and the matrix \( M(v) \) is given by

\[
M(v) = \begin{pmatrix}
\frac{1+v^2}{1-v^2} & \frac{+2v}{1-v^2} \\
+2v & \frac{1+v^2}{1-v^2}
\end{pmatrix} = \frac{1}{\gamma^2} \begin{pmatrix} 1 & +2v \\ +2v & 1 + v^2 \end{pmatrix} \equiv \frac{1}{\gamma^2} L(v). \]

\( (25) \)

In the expression for the boundary states we have omitted an overall, but for our purposes, unimportant normalisation constant, as well as neglected the contribution from the zero-modes. We return to the form of the zero-modes near the end of this paper. When we deal with a D(p)-brane boundary state which has no spatial extension, i.e. a D-particle, the sum over \( D \) is absent in eq.(24). The quantum states which are described by eq.(24) are exactly equal to the bosonic D(p)-brane boundary states which were constructed in \[5\]. Note that the letter \( v \) in \[5\] denotes the rapidity of the boundary state.
When we follow the prescription for the construction of a bosonic coherent (boundary) state we are, strictly speaking, only forced to normalise the untilded (say) sector, i.e. we must either enforce eq.(22) or eq.(23) but we must not necessarily enforce both sets of rescalings. If we only impose eq.(22) or eq.(23), the boundary state will still have the same form as above, but with \( M(v) \) given by \( M(v) = \gamma^{-1}L(v) \). We have (a priori) no other arguments at this level other than “naturalness”, for why we should choose \( M(v) \) to have the form in eq.(25) rather than the alternative expression for this matrix.

Let us briefly look into the question of BRST invariance of D(p)-brane boundary states in some generality. The BRST charge in the untilded sector is defined by

\[
Q = \sum_{n=-\infty}^{\infty} : L_{-n}c_n : - \frac{1}{2} \sum_{m,n=-\infty}^{\infty} (m-n) : c_m c_n b_{m+n} : -c_0 ,
\]

with a corresponding expression in the tilded sector. \( : \) denotes normal ordering of the operators. The \( c_n \) and \( b_n \) operators represent the ghost and anti-ghost operators respectively, and the \( L_n \)'s are the Virasoro operators defined by

\[
L_m = \frac{1}{2} \sum_{n=-\infty}^{\infty} \alpha^{(A)}_{m-n} \alpha^{(A)}_n .
\]

The BRST invariance condition is simply

\[
(Q + \tilde{Q}) |B; p, v \rangle = 0 .
\]

The nil-potency of the BRST charge holds, of course, by construction, since we are working with critical bosonic string theory.

Let us consider the BRST invariance of the static boundary states \( |B; p, 0 \rangle \). From the first part in the BRST charge above (which contains the Virasoro operators), and the corresponding part in the tilded sector, we get the condition

\[
\frac{1}{2} \sum_{m,n=-\infty}^{\infty} : \tilde{\alpha}^{(A)}_{m+n} \tilde{\alpha}^{(A)}_n : (c_m + \tilde{c}_{-m}) |B; p, 0 \rangle = 0 ; \quad A \in \{0, ..., 25\}
\]

after having imposed the boundary conditions in eq.(11) and eq.(12). Hence, we find that we must set

\[
c_m = -\tilde{c}_{-m}
\]
in order to make the expression in eq.(29) vanish. Similarly, the second term
in the BRST charge implies that (after having imposed the conditions in
eq.(11) and eq.(12))

\[-\frac{1}{2} \sum_{m,n=-\infty}^{\infty} (m - n) : (\bar{c}_m \tilde{c}_n \tilde{b}_{m+n} - \tilde{c}_m \bar{c}_n b_{-(m+n)}) : |B; p, 0\rangle = 0 , \tag{31}\]
such that we likewise find that we must impose

\[\tilde{b}_m = b_{-m} . \tag{32}\]

The last term in the BRST charge does not give rise to any additional
conditions. The conditions in eq.(30) and in eq.(32) were also derived in \[3].
Clearly, these conclusions are invariant under Lorentz transformations. Since
the moving boundary states \(|B; p, 0 < v < 1\rangle\) can be obtained from the static
states via a boost transformation in the \(p+1\) target-space direction, it follows
that these moving states are physically acceptable BRST invariant states. It
also follows that we must choose \(M(v)\) as in eq.(25), and not the alternative
expression for this matrix, since the form of this matrix in eq.(25) also
follows from boost transforming the static boundary states.

Let us now probe the possibility to construct \(D(p)\)-brane boundary states
\(|B; p, 1\rangle\) which move at the speed of light from the tensionfull states. The
timelike states \(|B; p, |v| < 1\rangle\) are not defined in the limit when \(|v| \to 1\) due
to the normalisation factor \(\gamma\) which we introduced in eq.(22) and eq.(23).
Hence, we cannot derive tensionless states \(|B; p, 1\rangle\) by just taking the naive
\(|v| \to 1\) limit of the timelike \(D(p)\)-brane states. On a more fundamental level
it is clear that the tensionfull and tensionless \(D(p)\)-brane sectors are well
separated, since when we set \(|v| = 1\) we have that \(A_r = B_r\) and \(\tilde{A}_r = \tilde{B}_r\),
which also implies that the algebras in eq.(18) and eq.(19) degenerate. This
algebraic degeneration reflects the fact that the conditions on the oscillators
which stem from eq.(9) and eq.(10) almost coincide on the light-cone. It is
also clear that eq.(17) implies that we cannot impose both eq.(9) and eq.(10)
when \(|v| = 1\), i.e. not both of these conditions can be continued to the light-
cone. We furthermore note another obstruction to a “naive” construction of
light-like boundary states: \(A_r, A_{-r}, \text{ and } \tilde{A}_r, \tilde{A}_{-r}\) commute when \(|v| = 1\).

\(^4\)Clearly, since the ghost fields do not carry Poincare indices the \(|B; p\rangle_{\text{ghost}}\) states have
the same form for all values of \(v\). Hence, the ghost part of a boundary state does not
carry a specific velocity label. The author thanks Prof. Paolo Di Vecchia for this simple
argument.
It follows that these operators cannot be used in a canonical coherent state construction of tensionless D(p)-brane boundary states, since such a scheme presupposes that the operators are normalised to unity.

When $|v| = 1$, and after having imposed the conditions in eq.(11), eq.(12), eq.(30) and eq.(32), we find that the BRST condition in eq.(28) is reduced to

$$ (Q + \tilde{Q})|B; p, 1\rangle = \frac{1}{2} \sum_{m=-\infty}^{\infty} (\tilde{l}_m - l_{-m})\tilde{c}_m : |B; p, 1\rangle = 0 \quad (33) $$

In this equation we have introduced the new operators $l_m$ and $\tilde{l}_m$ which have the same form as the Virasoro operators above, but where the target-space index only runs over 0 and $p + 1$. Hence, we have to impose the following extra constraint on the $|B; p, 1\rangle$ states

$$ l_m|B; p, 1\rangle = \tilde{l}_{-m}|B; p, 1\rangle \quad (34) $$

In the next section we will turn to the light-cone gauge in which all our problems sofar with the construction of tensionless D(p)-brane boundary states will vanish.

3 Tensionless D(p)-Branes

When one deals with D(p)-branes which move at the speed of light it is natural to consider the construction of the corresponding boundary states in an adapted frame, i.e. in the light-cone gauge. In the following we will use the letters $\tau$ and $\sigma$ to denote the world-sheet coordinates also in the light-cone gauge. The light-cone coordinates $X^{(\pm)}$ are defined by

$$ X^{(\pm)} = \frac{1}{\sqrt{2}}(X^{(0)} \pm X^{(p+1)}) \quad (35) $$

We choose $X^{(+)}$ as the “time-coordinate” such that we consistently can set all the $\alpha_m^{(+)}$ and $\tilde{\alpha}_m^{(+)}$ oscillators to zero. $X^{(+)}$ will thus correspond to the $Y$ coordinate in the previous section. We then have that $X^{(+)} = x^{(+)} + p^{(+)}\tau$, where $p^{(+)}$ is the light-cone energy. The $X^{(-)}$-coordinate is, from the string equations of motion, constrained to satisfy the relation

$$ p^{(+)}\partial_\tau X^{(-)} = \partial_\tau X^{(I)}\partial_\sigma X^{(I)} \quad (36) $$
Hence, eq.(7) and eq.(8) imply that $X^(-)$ should satisfy the Dirichlet condition

$$\partial_\tau X^(-)|_{\tau=0} | B; p, 1) = 0.$$  \tag{37}$$

This condition coincides with the condition in eq.(9). It did also appear in [9] in their discussion of static D(p)-brane boundary states in type II superstring theory in the light-cone gauge. In order to constrain the zero-modes to lie on the light-cone, we also impose the second constraint in eq.(9), i.e.

$$X^(-)|_{\tau=0} | B; p, 1) = 0,$$  \tag{38}$$
i.e. the $X^(-)$-coordinate plays the same role as the $Z$-coordinate in the previous section. When we insert the expansion in eq.(4) into eq.(37) and eq.(38) and equate coefficients of $e^{im\sigma}$, we find that ($m \neq 0$)

$$\alpha_m^(-) = \tilde{\alpha}_{-m}^(-),$$  \tag{39}$$

$$x^(-) = x^{(0)} - x^{(p+1)} = 0.$$  \tag{40}$$

In the light-cone gauge there are no ghosts manifestly present. In order to construct tensionless D(p)-brane boundary states the relation in eq.(39) is thus the only additional constraint on the oscillators we have to take into account in addition to the constraints in eq.(11) and eq.(12).

Note that the boundary states $|B; p, 1)$ are positioned at a fixed “time” $X^(+)$ since

$$X^(+)|_{\tau=0} | B; p, 1) = x^+ = \text{constant}, \quad \partial_\sigma X^(+)|_{\tau=0} | B; p, 1) = 0.$$  \tag{41}$$

The resulting kinematics really describe $(p + 1)$-instanton states rather than D(p)-brane states, since $X^+$ satisfies a Dirichlet boundary condition [9]. The D(p)-brane states are related to the instanton states by a double Wick rotation [9]. When we use the words “D(p)-brane” in the following it is really this rotated version we are referring to. Clearly, mapping $X^+$ on a lightlike circle of radius $R$, the dual lightlike circle has radius $\alpha'/R$. Let the dual circle be coordinatised by the coordinate $X^+_{\text{Dual}}$. The general relation $\partial_\mu X^{(A)} = \epsilon_{\mu\nu} \partial^\nu X^{(A)}_{\text{Dual}}$ then implies that

$$\partial_\sigma X^+|_{\tau=0} | B; p, 1) = 0 \Rightarrow \partial_\tau X^+_{\text{Dual}}|_{\tau=0} | B; p, 1) = 0.$$  \tag{42}$$

Hence, in the dual space the boundary states satisfy a set of boundary conditions which are exactly analogous to the ones in eq.(7-10).
In the light-cone gauge we only deal with the transversal degrees of freedom. It follows that the condition in eq.(39) can be expressed entirely in terms of the $\alpha^{(I)}_m$ operators, which at the outset satisfy eq.(11) and eq.(12). Hence, the relation in eq.(39) may potentially induce further relations among, or conditions on, the transversal oscillators. However, since we from eq.(36) have that

$$\alpha^{(-)}_m = \frac{1}{2p^+} \left( \sum_I \sum_{r=-\infty}^{\infty} : \alpha^{(I)}_m \alpha^{(I)}_r : - \delta_m \right), \quad (43)$$

$$\tilde{\alpha}^{(-)}_m = \frac{1}{2p^+} \left( \sum_I \sum_{r=-\infty}^{\infty} : \tilde{\alpha}^{(I)}_m \tilde{\alpha}^{(I)}_r : - \delta_m \right), \quad (44)$$

it is easy to show that eq.(39) reduces to an identity relation when we impose eq.(11) and eq.(12) on the right or the left hand side of eq.(39).

It is now straightforward to construct an explicit expression for the oscillator part of the tensionless D(p)-brane boundary states in the light-cone gauge, since they are only determined by the boundary conditions in eq.(11) and eq.(12). It follows that these states then can be written as

$$|B; p, 1 \rangle \sim \exp \left( + \sum_{m=1}^{25} \sum_{A=p+2}^{A=p+2} a^{(A)}_m \tilde{a}^{(A)}_m - \sum_{D=1}^{p} a^{(D)}_m \tilde{a}^{(D)}_m \right) |0 \rangle, \quad (45)$$

where we again have switched to conventionally normalised operators.

## Zero-Modes

We have so far in our treatment not discussed how we should treat the zero-modes of the D(p)-brane boundary states. The zero-modes describe the classical position of a D(p)-brane. Naively we should therefore expect that it is sufficient to include $\delta$-functions in the expressions for the boundary states in order to take these modes properly into account. Hence, the $|B; p, |v| < 1 \rangle$ states should naively be multiplied with

$$N_p \delta(x^{(p+1)} - vx^{(0)}) \prod_{J \in A} \delta(x^{(J)}) , \quad (46)$$

where $N_p$ is an overall normalisation constant. The tensionless states should likewise be multiplied with a similar kind of expression, but with $v$ set to unity, i.e.

$$M_p \delta(x^{(-)}) \prod_{J \in A} \delta(x^{(J)}) , \quad (47)$$

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where $M_p$ is another overall normalisation constant. However, in [5] it was emphasised that eq.(46) is not the completely correct expression, since a Lorentz boost on $\delta(x^{(p+1)})$ results in $\sqrt{1 - v^2}\delta(x^{(p+1)} - vx^{(0)})$, i.e. a Born-Infeld type factor should also be included in the complete specification of the boundary states. That the extra multiplicative term indeed is the effective Born-Infeld action for the D(p)-brane in question was shown in [4]. Hence, in order to derive the complete expression for the zero-mode part of the tensionless boundary states, we should probably first derive the effective action for tensionless D(p)-branes. This will not be attempted in this paper.

5 Summary and Conclusion

Our way of constructing tensionfull D(p)-brane boundary states is divided into two steps:

(1) the choice of coordinates in eq.(2) together with the set I-IV of boundary conditions in eq.(7-10), and

(2) the set of rescalings in eq.(22) and eq.(23) which are necessary to impose in order to represent D(p)-brane boundary states as canonical coherent states.

We noted the existence of an ambiguity in the necessary number of rescalings, which made the particular choice of the form of the matrix $M$ in eq.(24) somewhat unclear. The purpose of the reanalysis of the construction of moving tensionfull D(p)-brane boundary states was to see in detail what “goes wrong” when one attempts to extend the notion of D(p)-branes to also cover the case when these states move at the speed of light. Clearly, step (2) above cannot be part of such a scheme due to diverging normalisation factors. However, even though one overlooks this problem (since the normalisation is linked to the wish of representing the states as canonical coherent states) it is nevertheless inconsistent to try to impose the set I-IV in eq.(7-10) directly when $|v| = 1$. It was specifically noted that it is inconsistent, due to eq.(17), to impose both the condition III and the condition IV in eq.(9-10) at the same time when $|v| = 1$.

In section 3 we attacked the problem of constructing D(p)-brane boundary states from a radically different perspective than the one used in the previous section. We showed that imposing only the conditions in I, II and III (in
eq.(7-9)) leads to a set of boundary conditions which are consistent with the string equations of motion in the light-cone gauge, and which after a duality transformation describe D(p)-brane boundary states which move at the speed of light. Hence, we discovered that the tensionless states in the dual space are completely determined by, and can be constructed entirely from, a set of boundary conditions which are exactly analogous to the ones in I-IV in eq.(7-10). In this way we managed to show that it is possible to derive boundary states which describe D(p)-branes moving at any physical velocity in a “unified” manner.

The derivation of, as well as the actual expression in eq.(45) for, the tensionless D(p)-brane states are remarkably simple. We note in particular that the explicit expressions for the $|B; p, 1\rangle$ vectors are (implicitly) independent of the $X^{(0)}$ and $X^{(p+1)}$-oscillators. It follows that the formal expressions for the tensionless states are unaltered under any duality transformation involving the $X^{(0)}$ and $X^{(p+1)}$-coordinates. This points to a crucial difference between tensionless and tensionfull D(p)-branes when we compare the behaviour of the $|B; p, |v| < 1\rangle$ states under T-duality with the corresponding properties of tensionless D(p)-branes. In the dual expression for the tensionfull states the momentum carried by a moving D(p)-brane is transmuted into an electric field (in a static D(p+1)-brane) in the dual (super-) Yang-Mills description when we dualise on the $X^{(p+1)}$-coordinate. When we let $|v| \to 1$ this electric field becomes super-critical, and it is expected to decay via the usual quantum mechanical mechanisms for decay of strong fields \cite{7}. This prohibits on physical grounds the possibility to boost a $|B; p, |v| < 1\rangle$ state into a (near) $|B; p, 1\rangle$ state. Correspondingly, we pointed out that it is inconsistent on an algebraic level to try to smoothly relate a $|B; p, |v| < 1\rangle$ state to a tensionless D(p)-brane state. Hence, from the tensionfull D(p)-brane side both physical as well as more formal algebraic arguments exist which together effectively prohibits a direct and smooth relation between tensionfull and tensionless D(p)-brane boundary states. The corresponding argument from the tensionless D(p)-brane side is the observation above that the states in eq.(45) are insensitive to any duality transformation in the $X^{(0)}$ and $X^{(p+1)}$ directions. Hence, a $|B; p, 1\rangle$ state cannot in particular be related to a static $D(p + 1)$-brane via duality.

Let us conclude this paper. We have shown that tensionless D(p)-brane states exist in string theory. This tensionless sector is analogous to the massless sector in point-particle theories, and it is effectively separated from the tensionfull D(p)-brane sector. We plan to return to a number of questions
which are raised by this work together with the construction of tensionless D(p)-brane boundary states in super-string theory elsewhere. Of particular further interest is to understand the role played by tensionless D(p)-branes in string perturbation theory, and to construct and investigate the super-Yang-Mills theory which may be associated to tensionless D(p)-branes in super-string theory.

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