On the problems of analysis and synthesis of control of boundary layer on permeable surfaces of hypersonic aircraft

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Abstract. The problems of mathematical modeling of effective control of heat and mass transfer and friction on permeable cylindrical and spherical surfaces of hypersonic aircraft are considered. The systems of ordinary differential equations are obtained by A.A.Dorodnicyn generalized integral relations method to approximate the systems of partial differential equations describing laminar boundary layers on permeable cylindrical and spherical surfaces of hypersonic aircraft. The joint systems are applied in the mathematical model. The parameters of the mathematical model at the flow stagnation point are determined from the joint systems of nonlinear algebraic equations. The blowing into boundary layer, temperature factor and magnetic field are used as controls. Integration, observation and control meshes are introduced throughout the entire control segment. The local and integral characteristics of heat transfer and friction computed according to the parameters of the mathematical model corresponding to the given controls are analyzed in direct problems. The controls corresponding to the given characteristics of heat transfer and friction are synthesized in inverse problems. The direct and inverse problems in ordinary and extreme statements are considered throughout the entire control segment. The statements of the hybrid direct extreme problem are given. A hybrid objective function is introduced. The statements of two-dimensional inverse problems that have, in contrast to one-dimensional ones that admit only interpolation and approximation statements, additionally two mixed statements are considered in details. Additional restrictions for inverse problems in the approximation statement are described. Classification of the listed problems is carried out on the entire control segment. The results of the classification are presented in the form of tables.

1. Introduction
To simulate all real conditions of hypersonic aircraft (HA) movement in dense layers of the atmosphere is actually impossible. So, it is necessary to elaborate adequate mathematical models allowing describe various physical and chemical processes (dissociation, ionization) characterizing the HA flight [1]. By means of A. A. Dorodnitsyn [2] generalized integral relations method the PDE systems describing laminar boundary layers (LBL) can be reduced to the approximating ODE systems. This approach is very popular in engineering for compressible gas aerodynamic characteristics computation [3–6]. The approximating ODE systems describing LBL on the segment of control for HA permeable cylindrical and spherical surfaces are obtained in [7, 8]. This paper continues the research [7–19] and considers the questions of analysis and
synthesis of control of LBL on HA permeable cylindrical and spherical surfaces. The classification of problems of LBL control on permeable surfaces of HA for fragments of segment of control are considered in [20].

2. Direct and inverse problems

2.1. Basic notation

Let’s consider the following problem [7–10]. According to preset given controls: \( m(x) \) is the blowing into LBL, \( \tau(x) \) is the temperature factor \( (\tau(x) = T_w(x)/T_{e_0}) \), where \( T_w(x) \) is the wall temperature, \( T_{e_0} \) is the temperature in the flow stagnation point, \( s(x) = \sigma B_0^2(x) \) is the magnetic field, where \( x \in X = [0; 1] \) (the axis \( x \) is directed along the body contour), it is necessary to compute the LBL mathematical model parameters \( \theta_0(x; m, \tau, s), \theta_1(\ldots), \omega_0(\ldots), \omega_1(\ldots) \) from the approximating ODE systems obtained in [7,8,19]. After that it is necessary to determine the local heat flow \( q = q(x; m, \tau, s) \); the local tangent friction \( f = f(x; m, \tau, s) \); the total heat flow \( Q = Q(X; m, \tau, s) \); the total Newton friction force \( F = F(X; m, \tau, s) \); the total blowing system power \( N = N(X; m, \tau, s) \) determined with the use of H. Darcy filtration law, i.e. the direct problem (DP) is

\[
(m, \tau, s) \rightarrow (q, f, \eta; Q, F, N).
\]

Let’s consider some inverse problems (IP) [13,15]: one-dimensional IP on \( m \) and IP on \( \tau \)

\[
q^\sim \rightarrow m^\sim, \quad (m^\sim, \tau, s) \rightarrow (q^\sim \approx q^\sim, f^\sim),
\]

\[
f^\sim \rightarrow m^\sim, \quad (m^\sim, \tau, s) \rightarrow (q^\sim, f^\sim \approx f^\sim),
\]

\[
q^\sim \rightarrow \tau^\sim, \quad (m, \tau^\sim, s) \rightarrow (q^\sim \approx q^\sim, f^\sim),
\]

\[
f^\sim \rightarrow \tau^\sim, \quad (m, \tau^\sim, s) \rightarrow (q^\sim, f^\sim \approx f^\sim)
\]

(6)

and the two-dimensional IP

\[
(q^\sim, f^\sim) \rightarrow (m^\sim, \tau^\sim), \quad (m^\sim, \tau^\sim, s) \rightarrow (q^\sim \approx q^\sim, f^\sim \approx f^\sim)
\]

(6)

further on noted by IP\(^m\), IP\(^f\), IP\(^\tau\), IP\(^f\) and IP\(^{(m,f)}\), respectively.

Here \( q^\sim \) and \( f^\sim \) are observed values, \( q^\sim \) and \( f^\sim \) are computed ones, \( m^\sim \) and \( \tau^\sim \) are restored ones. Thus, the listed problems are realized depending on which two of the four parameters \( m, \tau, q, f \) are free (“0” in table 1), and which ones are given (“1” in table 1). In particular, the characteristic of DP is the absence of observed \( q, f \), and IP characterizes by the presence of at least one of them.

2.2. Meshes, elements, restrictions

In [13,15] the controls \( m, \tau, s \) are given and \( m^\sim, \tau^\sim \) are sought as elements [18] of the mesh of control

\[
X^\wedge: \quad x_0^\wedge = 0 < x_1^\wedge < \ldots < x_{n^\wedge}^\wedge = 1
\]

(7)

for some \( n^\wedge \geq 1 \); the observed values \( q^\sim = (q_j^\sim)_{j=0,...,n^\wedge}, f^\sim = (f_j^\sim)_{j=0,...,n^\wedge} \) are given and the values \( q^\sim = (q_j^\sim)_{j=0,...,n^\wedge}, f^\sim = (f_j^\sim)_{j=0,...,n^\wedge} \) are computed for all nodes of the mesh of observation

\[
X^\vee: \quad x_0^\vee = 0 < x_1^\vee < \ldots < x_{n^\vee}^\vee = 1
\]

(8)

for some \( n^\vee \geq 1 \). As in [13,15] let’s assume \( X^\wedge \) is the mesh of integration that

\[
X^\wedge \subseteq X^\vee \subseteq X^\wedge,
\]

(9)
and the restrictions [18] on $m^\sim$ and on $\tau^\sim$ for $x \in [x_{j-1}; x_j^\sim]$, $j = 1, \ldots, n^\sim$ have the form

\[(m^\sim)^{(k)}(x) \in I_{j,k}^m \quad \text{for} \quad k = 0, \ldots, \nu^m; \quad (10)\]

\[(\tau^\sim)^{(k)}(x) \in I_{j,k}^m \quad \text{for} \quad k = 0, \ldots, \nu^\tau; \quad (11)\]

for some $\nu^m \geq 0$, $\nu^\tau \geq 1$ and $I_{j,k}^m = [b_{j,k}^m; t_{j,k}^m]$, $I_{j,k}^\tau = [b_{j,k}^\tau; t_{j,k}^\tau]$.

2.3. One-dimensional IP statements

One-dimensional IP (ODIP) (2), (3) have two possible statements [13–15].

2.3.1. Interpolation statement. In the case of an interpolation statement of ODIP (ODIPIS) for given $p \in [1; +\infty]$ and $\varepsilon > 0$ it is required to find the control $c = m^\sim$ so that for the given local parameter ($y = q$ or $y = f$) the following proximity condition is satisfied

\[R_p(y^\sim; y^\sim) \leq \varepsilon, \quad (12)\]

where

\[R_\infty(y^\sim; y^\sim) = \max_{j=0,\ldots,n^\sim} |y_j^\sim - y_j^\sim| \quad \text{for} \quad p = +\infty, \quad (13)\]

\[R_p(y^\sim; y^\sim) = \left( \sum_{j=0}^{n^\sim} |y_j^\sim - y_j^\sim|^p \right)^{1/p} \quad \text{for} \quad p \in [1; +\infty). \quad (14)\]

2.3.2. Approximation statement. In the approximation statement of ODIP (ODIPAS) for given $p \in [1; +\infty]$ it is required to find the control $c = m^\sim$ as an approximate solution of the problem

\[\inf_c R_p(y^\sim; y^\sim). \quad (15)\]

The statements of ODIP (4), (5) are similar: in conditions (11) it is required to find $c = \tau^\sim$. Variants of the IPIS ($\delta^i = 1$ and $\delta^a = 0$) and IPAS ($\delta^i = 0$ and $\delta^a = 1$) are presented in table 2. Note that in tables 1, 2

\[\delta^i = \delta^{i,q} \lor \delta^{i,f}, \quad \delta^a = \delta^{a,q} \lor \delta^{a,f}, \quad (16)\]

\[\delta^q = \delta^{i,q} \lor \delta^{a,q}, \quad \delta^f = \delta^{i,f} \lor \delta^{a,f}, \quad (17)\]

the symbol “$\lor$” means logical disjunction, the logical value “true” is associated with the integer value “1”, the logical value “false” is the integer value “0”.

2.4. Two-dimensional IP statements

Two-dimensional IP (TDIP) (6) has following statements.

2.4.1. Interpolation statement. In the interpolation statement IP$_{(i,q; i,f)}^{(m,\tau)}$ [15] (TDIPIS) for given $p \in [1; +\infty]$ and $\varepsilon > 0$ in conditions (10) and (11) it is required to find the controls $c = (m^\sim, \tau^\sim)$ so that the following proximity condition is satisfied

\[R_p((q^\sim; f^\sim); (q^\sim; f^\sim)) \leq \varepsilon, \quad (18)\]

where (with allowance for the dimensionlessness of $q$ and $f$)

\[R_\infty((q^\sim; f^\sim); (q^\sim; f^\sim)) = \max \{ R_\infty(q^\sim; q^\sim); R_\infty(f^\sim; f^\sim) \} \quad \text{for} \quad p = +\infty, \quad (19)\]

\[R_p((q^\sim; f^\sim); (q^\sim; f^\sim)) = \left( R_p^p(q^\sim; q^\sim) + R_p^p(f^\sim; f^\sim) \right)^{1/p} \quad \text{for} \quad p \in [1; +\infty), \quad (20)\]

where (13) and (14) are used in the right-hand sides of (19) and (20).
2.4.2. Approximation statement. In the approximation statement IP_{(a,q,a,f)} [(15)] (TDIPAS) for a given \( p \in [1; +\infty] \) in conditions (10) and (11) it is required to find the controls \( c = (m^\sim, \tau^\sim) \) as an approximate solution of the problem

\[
\inf_c R_p ((q^\sim; f^\sim); (q^\sim'; f^\sim')).
\] (21)

2.4.3. Mixed statement. In the mixed statement IP_{(i,q,a,f)} (IPMS) for given \( p \in [1; +\infty] \) and \( \varepsilon > 0 \) in conditions (10) and (11) it is required to find the controls \( c = (m^\sim, \tau^\sim) \) as an approximate solution of the problem (15) so that the condition of proximity (12) holds for \( q \). \( \text{IP}_{(m,\tau)} \) statement is analogous: the proximity condition (12) must be satisfied for \( f \).

Variants of the TDIP statements are presented in table 5.

2.5. Note
As in [14] we assume that (10), (11) and conditions (10)-(16) from [18] guarantee non-empty sets of admissible controls \( m^\sim \) and \( \tau^\sim \). If the restrictions (10), (11) for free controls \( m^\sim \) and/or \( \tau^\sim \) are such that

\[
b^{m_{j,0}} = t^{m_{j,0}} \quad \text{for} \quad j = 1, \ldots, n^\wedge
\] (22)

and/or

\[
b^{\tau_{j,0}} = t^{\tau_{j,0}} \quad \text{for} \quad j = 1, \ldots, n^\wedge,
\] (23)

then TDIP degenerates into DP or in ODIP, and ODIP degenerates into DP with

\[
m_j = b^{m_{j,0}} \quad \text{and/or} \quad \tau_j = b^{\tau_{j,0}} \quad \text{for} \quad j = 1, \ldots, n^\wedge.
\] (24)

In the case of degeneracy into DP the initial IP will have solutions (24) only if \( q^\sim' \approx q \) and/or \( f^\sim' \approx f \) for \( q \) and/or \( f \) from (1).

3. Direct extreme problems
3.1. Direct extreme problems of basic type
DP (1) is a particular case (for (22) or (23)) of the direct extreme problem (DEP) considered in [9,11]. In the basic DEP \( Q \) for given controls \( \tau \) and \( s \) it is required in condition (10) to find the controls \( m^\sim, \tau^\sim \) and the values

\[
Q^\sim = Q(X; m^\sim, \tau, s),
\] (25)

\[
\bar{Q}^\sim = Q(X; m^\sim, \tau, s)
\] (26)

as an approximate solution of the problems

\[
\inf_{m^\sim} Q(X; m^\sim, \tau, s),
\] (27)

\[
\sup_{m^\sim} Q(X; m^\sim, \tau, s),
\] (28)

respectively. The basic DEP \( F \) are formulated similarly. The above mentioned DEP are characterized by the absence of observed parameters (both \( q \) and \( f \)), the presence of one (\( m \) or \( \tau \)) given ("1" in table 3) and one free ("0" in table 3) control parameter, and one (\( Q \) or \( F \)) of the minimized or maximized functional ("–1" or "+1" in table 3).
3.2. Additional restrictions

In [9,11] the basic \( \text{DEP}_{m}^{Q}, \text{DEP}_{m}^{F}, \text{DEP}_{\tau}^{Q}, \text{DEP}_{\tau}^{F} \) for a given segment \( I^{N} = [N; \bar{N}] \) are considered in additional restrictions

\[
N^{-} \in I^{N},
\]

where \( N^{-} = N(X; m^{-}, \tau, s) \) for \( \text{DEP}_{m}^{Q}, \text{DEP}_{m}^{F} \) and \( N^{-} = N(X; m, \tau^{-}, s) \) for \( \text{DEP}_{\tau}^{Q}, \text{DEP}_{\tau}^{F} \).

In addition to (29) \( \text{DEP}_{m}^{Q}, \ldots, \text{DEP}_{\tau}^{F} \) for given segments \( I^{Q} = [Q; \bar{Q}] \), \( I^{F} = [F; \bar{F}] \) can be supplemented by the restrictions

\[
Q^{-} \in I^{Q},
\]

\[
F^{-} \in I^{F},
\]

where \( Q^{-} \) and \( F^{-} \) in the corresponding problems are computed similarly to \( N^{-} \).

Restrictions can be imposed on the computed local parameters \( q, f, \eta \) similarly to (10) and (11): for \( x \in [x_{j-1}^{*}; x_{j}^{*}] \), \( j = 1, \ldots, n^{*} \) they have the form

\[
(q)^{(k)}(x) \in I_{j,k}^{q} \quad \text{for} \quad k = 0, \ldots, \nu^{q};
\]

\[
(f)^{(k)}(x) \in I_{j,k}^{f} \quad \text{for} \quad k = 0, \ldots, \nu^{f};
\]

\[
(\eta)^{(k)}(x) \in I_{j,k}^{\eta} \quad \text{for} \quad k = 0, \ldots, \nu^{\eta}
\]

for some \( \nu^{q} \geq 0, \nu^{f} \geq 0, \nu^{\eta} \geq 0 \) and \( I_{j,k}^{q} = [b_{j,k}^{q}; t_{j,k}^{q}] \), \( I_{j,k}^{f} = [b_{j,k}^{f}; t_{j,k}^{f}] \), \( I_{j,k}^{\eta} = [b_{j,k}^{\eta}; t_{j,k}^{\eta}] \). Note that choice of \( I^{Q}, I^{F}, I^{N}, I^{q}, I^{f}, I^{\eta} \) must be compatible with given (10) and (11).

3.3. Hybrid DEP with constant \( \varphi \)

In [11] the hybrid \( \text{DEP}_{m}^{(Q,F)} \) (HDEP) with constant \( \varphi \) is considered: for given \( \tau, s, I^{N} \) and \( \varphi \in [0; 2\pi] \) it is required to find the control \( m^{-} \) and the value

\[
\Psi^{-} = \Psi(X; m^{-}, \tau, s; \varphi)
\]

in the conditions (10) and (29) as an approximate solution of the problem

\[
\inf_{m^{-}} \Psi(X; m^{-}, \tau, s; \varphi)
\]

of minimization of the functional (taking into account the dimensionlessness of \( Q \) and \( F \))

\[
\Psi(X; m, \tau, s; \varphi) = \cos(\varphi) \cdot Q + \sin(\varphi) \cdot F.
\]

The statement of the HDEP with constant \( \varphi \) is similar: in conditions (11) and (29) it is required to find the control \( \tau^{-} \). Particular cases of HDEP, where \( c = m \) or \( c = \tau \) for \( \varphi \in \{0; \pi/2; \pi; 3\pi/2\} \) are presented in table 4. The values of \( \cos(\varphi) \) and \( \sin(\varphi) \) in (37) are a generalization of the numbers \( -\Delta_{Q}^{F} \) and \( -\Delta_{F}^{F} \) from tables 3, 4. A HDEP with occurrence \( N \) (with allowance for its dimensionlessness) in a functional similar to (37) is considered in [11].

3.4. Hybrid objective function with piecewise-continuous bounded \( \varphi \)

The objective function in the form

\[
\Psi(X; m, \tau, s; \varphi) = \int_{X} \left( \cos(\varphi(x)) \cdot \frac{dQ}{dx} + \sin(\varphi(x)) \cdot \frac{dF}{dx} \right) \cdot dx
\]

can be used instead of (37) in the HDEP and HDEP, where \( \varphi(x) \) is a piecewise-continuous (with possible discontinuities at the points \( X^{\wedge} \)) bounded on \( X \) and

\[
Q(x) = Q([0; x]; m, \tau, s), \quad F(x) = F([0; x]; m, \tau, s).
\]
4. Inverse extreme problems
4.1. Basic inverse extreme problems
In basic inverse extreme problems (IEP) in the interpolation statement IEP\(^{(q,F)}\)\(_{(m,\tau)}\), IEP\(^{(q,F)}\)\(_{(m,\tau)}\) two control parameters (both \(m\) and \(\tau\)) are free, and from the observed local parameters \(q\) is given and \(f\) is free. It is required for given \(s, \varepsilon > 0\), \(p \in [1; +\infty]\) in conditions (10), (11) to find the controls \((m^\sim_{F}, \tau^\sim_{F})\), \((m^\sim_{F}, \tau^\sim_{F})\) and the values \(F^\sim = F(X; m^\sim_{F}, \tau^\sim_{F}, s)\), \(\bar{F}^\sim = F(X; m^\sim_{F}, \tau^\sim_{F}, s)\) as an approximate solution of the problems

\[
\begin{align*}
\inf_{m^\sim, \tau^\sim} F(X; m^\sim, \tau^\sim, s), \\
\sup_{m^\sim, \tau^\sim} F(X; m^\sim, \tau^\sim, s)
\end{align*}
\]  

if condition (12) for \(q^\sim\) is satisfied. The statements of the basic IEP\(^{(Q,f)}\)\(_{(m,\tau)}\), IEP\(^{(Q,f)}\)\(_{(m,\tau)}\) are similar. These cases are presented in table 6.

In IEP instead of \(\Psi = (-\Delta^Q) \cdot Q + (-\Delta^F) \cdot F\) with \(\Delta^Q\) and \(\Delta^F\) from table 6 we can use (38).

In contrast to HDEP the function \(\varphi(x)\) must be such that the set of solutions

\[
\sin(\varphi(x)) = 0 \quad \text{for} \quad \text{IEP}\(^{(q,F)}\)\(_{(m,\tau)}\); \quad \text{IEP}\(^{(Q,f)}\)\(_{(m,\tau)}\),
\cos(\varphi(x)) = 0 \quad \text{for} \quad \text{IEP}\(^{(Q,f)}\)\(_{(m,\tau)}\),
\]

is finite.

4.2. Additional restrictions in IEP
Note that the information specified in the IEP\(^{(q,F)}\)\(_{(m,\tau)}\), IEP\(^{(Q,f)}\)\(_{(m,\tau)}\) is not enough for solving both one-dimensional (no control is specified) or two-dimensional (the second observed parameter is not specified) IP. In the IEP\(^{(q,F)}\)\(_{(m,\tau)}\) for a given \(q^\sim\) the approximate value of \(Q^\Sigma\) can be found with an accuracy determined by the mesh of observation (8) and the choice of the quadrature formula:

\[|Q - Q^\Sigma| < \varepsilon^\Sigma.\]  

In these conditions it is not permissible to impose additional restrictions in form (30) and/or (32) for small \(\varepsilon^\Sigma\) in (43) and \(\varepsilon\) in (12). For the remaining integral characteristic \(F\) in conditions of solving problems (39) or (40) additional restrictions in form (31) can either not affect the solution, or the extreme value coincides with the restriction, or the domain of admissible controls is emptied. Restrictions in IEP\(^{(q,F)}\)\(_{(m,\tau)}\) on \(N\) in form (29) are admissible, and these on \(f\) or \(\eta\) in form (33) or (34) are admissible, but can turn it into a TDIP. In IEP\(^{(Q,f)}\)\(_{(m,\tau)}\) it realizes similarly.

4.3. Admissible restrictions in non-extreme IP
In IPIS\(^q\)\(_m\) (or IPIS\(^q\)\(_\tau\)) for the same reasons the imposition of restrictions on \(Q\) is unacceptable. Unlike IEP the second (both local and integral) characteristic (i.e. \(f^\sim\) and \(F^\sim\)) after finding free control \(m^\sim\) (or \(\tau^\sim\)) for a given \(\tau\) (or \(m\)) is determined by integrating the ODE system. Therefore, the imposition of restrictions on \(f\) or on \(F\) is also inadmissible. It does similarly in IPIS\(^f\)\(_m\), IPIS\(^f\)\(_\tau\), IPIS\(^{(q,f)}\)\(_{(m,\tau)}\). In IPAS\(^q\)\(_m\), IPAS\(^q\)\(_\tau\) imposing of restrictions on \(Q\), \(F\), \(N\) is permissible, and the restrictions on \(f\) or on \(\eta\) are admissible, but can turn them into a IPMS analog with one free control. It does similarly in IPAS\(^f\)\(_m\), IPAS\(^f\)\(_\tau\). In IPAS\(^{(q,f)}\)\(_{(m,\tau)}\) the restrictions on \(Q\), \(F\), \(N\) are admissible. In IPMS\(^{(q,a,f)}\)\(_{(m,\tau)}\) for identical reasons, the imposition of additional restrictions on \(Q\) is unacceptable, and for \(F\) and \(N\) it is possible, but for \(F\) it is inadmissible.
5. Tables

5.1. One-dimensional case

Table 1.

|     | m | τ | q | f |
|-----|---|---|---|---|
| DP  | 1 | 1 | 0 | 0 |
| IPₘ | 0 | 1 | 1 | 0 |
| IPₖ | 0 | 1 | 0 | 1 |
| IPₖ | 1 | 0 | 1 | 0 |
| IPₖ | 1 | 0 | 0 | 1 |
| IPₙ | 0 | 0 | 1 | 1 |

Table 2.

|     | m | τ | q | f |
|-----|---|---|---|---|
| IPₙ | 0 | 1 | 0 | 0 |
| IPₙ | 1 | 0 | 0 | 0 |
| IPₙ | 0 | 1 | 0 | 1 |
| IPₙ | 0 | 1 | 0 | 0 |
| IPₙ | 1 | 0 | 0 | 0 |
| IPₙ | 0 | 1 | 0 | 0 |
| IPₙ | 0 | 1 | 0 | 1 |

Table 3.

|     | m | τ | Q | F |
|-----|---|---|---|---|
| DEPₘ | 0 | 1 | ±1 | 0 |
| DEPₖ | 0 | 1 | 0 | ±1 |
| DEPₖ | 1 | 0 | ±1 | 0 |
| DEPₖ | 1 | 0 | 0 | ±1 |

Table 4.

|     | ϕ | cos(ϕ) | sin(ϕ) | HDEP(Q,F) |
|-----|---|--------|--------|-----------|
|     | 0 | +1     | 0      | DEPₙ      |
|     | π/2 | 0   | +1   | DEPₙ      |
|     | π | −1    | 0      | DEPₙ      |
|     | 3π/2 | 0 | −1    | DEPₙ      |

Table 5.

|     | m | τ | q | f |
|-----|---|---|---|---|
| IPₙ | 0 | 0 | 1 | 0 |
| IPₙ | 0 | 0 | 0 | 1 |
| IPₙ | 0 | 0 | 1 | 1 |
| IPₙ | 0 | 0 | 1 | 0 |

Table 6.

|     | q | f | Q | F |
|-----|---|---|---|---|
|     | 0 | 1 | +1 | 0 |
|     | 1 | 0 | 0 | +1 |
|     | 0 | 1 | −1 | 0 |
|     | 1 | 0 | 0 | −1 |

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