Modeling of ecosystem dynamics: nonlinearity and synergetics

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Abstract. Mathematical modeling and computational experiment of the soil-plant systems (SPS) ecology are forming the basis of analysis and prediction scientific paradigm of more complex ecosystems. The dynamics of processes occurring in open biological systems and phenomena occurring in inorganic environments are comparing in this research. The behavior of systems of biotic and abiotic origin has bifurcation points associated with the nonlinear nature of interaction between the external environment and the elements inside, predetermining the synergy of evolution. During the studying SPS using the example of the Black Lands of the Republic of Kalmykia, a mathematical model of multichannel transitions between elements was developed using the Markov process mechanism. The dynamics of the SPS ecotope destruction and recovery was studied in the approximation of homogeneous Markov chains. Stationary final distributions of the comparison of classes derived from matrices with high degrees revealed bifurcation points, the nonlinearity of the succession process. Mathematical modeling gave the lifetime of an intermediate states stable existence. As a result, the methodology for assessing the ecological status of arid ecosystems using Markov chains has been proposed. The analogy with inorganic systems allowed detailing the process of the origin of bifurcation points. Comparison of the dynamics of open systems «living» and «inanimate» nature has led to their synergistic unity. Nonlinear processes in inorganic environment are qualitatively similar to the dynamics of macroscopic systems in ecology.

1. Introduction
The resources of nature are limited, and their unreasonable exploitation by man leads to disruption of dynamic equilibrium, functioning and self-regulation. As a result, the stability of the biosphere as a whole is endangered.

The results of the analysis of changes in the composition of segetal plant communities (CPC) by edaphoclimatic gradient (ECG) and agrocenotic gradient (ACG) were evaluated by Khasanova G. R., Lebedeva M. V., Mirkin B. M., Naumova L. G. The CPC in the composition of agrocenoses is the r-model of phytocenoses organization. The authors argue that the main factor in their formation is the permanent disruption of communities during tillage, and the constancy of the composition of the CPC is maintained through the soil bank of seeds and vegetative buds of weeds. Analysis of the patterns of the dynamics of the CDS revealed their similarity with the initial stages of the restoration of natural vegetation in the deposits, during which there is a change of segetal species ruderal, and then meadow and steppe [1].

Currently, the problem of the relationship between society and the environment is paramount and
obvious, since before our eyes there are catastrophic changes in climate and biosphere, which previously were visible only to specialists [2]. According to Török P, Ambarli D, Kamp J, Wesche K, Dengler J., pasture, savanna and shrub ecosystems are exposed to a high level of threat around the world (from Central and Eastern Europe to Northern China in the temperate zone of Eurasia) and are of great importance for biodiversity. As a result of high rates of anthropogenic pressure and widespread degradation, the Palaearctic steppes require the restoration and preservation of biodiversity [3]. In order to implement a set of measures to halt the processes of desertification, improve the economic condition and increase land fertility in drylands, environmental projects are being developed and implemented. The large project «General scheme for combating desertification of the Black Lands and Kizlyar pastures» was implemented over an area of more than 7 thousand km², where agroforestry-melioration works were carried out, more than 4 thousand km² of mobile sands were fixed and more than 2.5 thousand km² of pastures were restored. If the measures provided by the scheme are observed, the ecological disaster zones will continue to decrease in this territory [4].

Evolution requires an understanding of the main points and ways in which technology develops. A better understanding of this may come about through the use of synergistic approaches [5, 6]. The ecological scientific knowledge about the laws of biotic and technogenic systems interaction allows us to competently and effectively manage biospheric processes [7]. The system of agroecology is open dissipative structures with non-linear response to external influences, leading to increased sensitivity to small perturbations at bifurcation points and subsequent irreversibility of behavior. The task of such systems programmed control is analytically unsolvable. The only means of studying them is mathematical models.

The aim of the work was to identify the causes of the degradation of arid ecosystems based on a mathematical model with various inter-element coupling patterns. Mathematical modeling in the ecology of soil-plant systems (SPS) is the basis for the analysis and prediction of other more complex systems. This allows us to consider the previously obtained results in a new way in order to clarify the theoretical concepts and the nature of ecosystem functioning. The invariance of the mathematical description of processes of different nature (physical, chemical, biological, economic, social, and so on) allows us to use the mathematical images to establish a synergistic commonality of the such systems behavior.

2. Materials and methods

The dynamics of processes occurring in open biological systems and a phenomena in inorganic environments are comparing in the research. The processes of corrosion of metals in aggressive environment, as well as temperature transformations in oxide systems [8], are qualitatively similar in their behavior to the dynamics of ecological macrosystems.

Modeling on self-similar differential equations and/or partial differential parabolic equations pointed to the synergistic nature of the systems functioning when determining allowable loads or optimal parameters of their dynamic stability. The overlap of the processes nature in systems with different levels of organization makes it possible to clarify the situation in one area on the basis of an analysis of the events and their consequences in another.

In systems of non-biotic origin: the dynamics of corrosion and temperature transformations [8] are described in the same way as in systems containing biotic components. The description of both types of systems in the language of mathematics leads to the same constructions in the form of differential relations. At the same time, the models qualitative conformity has a quantitative difference.

In the study of SPS, a paradigm of multichannel transitions (successions) between elements was developed according to the Markov processes mechanism. The objects of research were the ecosystems of the North-Western Caspian Sea within the semi-desert regions of the Caspian Lowland in the form of pastures of the Black Lands in Kalmynia. The dynamics of the destruction and restoration of ecosystem classes was studied in the approximation of homogeneous Markov chains (MC). Comparison of the stationary distributions of systems classes obtained from matrices with high degrees made it possible to reveal the nonlinearity of the process and the points of bifurcations. Now it is possible to
calculate the lifetime of a intermediate classes stable existence.

The study of nonlinear abiotic systems was carried out on the example of iron hydroxides formed in the process of coding agricultural mechanisms.

3. Results and Discussion
The study of complex natural processes is difficult because of the complexity and multidimensionality of the biosphere ecosystems structures. The need to observe the nature as a whole and to grasp the main features of the surrounding world behavior require ignoring particulars and limit ourselves to the necessary level of detailing, as the paradigm of system analysis requires. Mathematical methods and models provide a formal answer to the question available for solving [9].

Linking the problems of mathematical modeling in a systems of animate and inanimate nature, it should be noted that the former behave physically incorrect in some cases. This is caused by limited physical concepts of the processes occurring in complex biotic systems. Mathematically, this is manageable through regularization methods. However, in biological tasks due to the emergence of a “threshold”, when minor changes in any system parameter leads to a qualitative restructuring of the entire process, the application of such methods is problematic.

From the point of view of synergy, self-organization is always accompanied by positive feedback leading to nonlinear dynamics. Thus arise phenomena with an exacerbation, when for a limited period of time the process develops abnormally quickly, reflecting the general trend of the development of open systems, often leading to disaster [10].

SPS are vital in various aspects, such as the preservation of a sustainable landscape, the reproduction of the biosphere, the prevention of the agriculture collapse and animal husbandry, and so on. In addition, SPS are a classical ecosystem in the study of the biocenosis stability with the help of a computational experiment.

Ecosystems of the North-Western Pre-Caspian region in the form of pastures of the Black lands in Kalmykia are the standards for the studied natural areas [4]. The developed methods can be extrapolated to other territories of Russia. Virtually the entire soil-plant system of the region has been degraded in recent years due to the fact that this region has been intensively developed in oil and gas fields, with the simultaneous development of agricultural production and the exacerbation of the situation by natural and climatic conditions. In some areas where deflation acquired an avalanche-like character, an environment of ecological disaster developed. This is evidenced by the data of the aerospace monitoring of the Black lands (1954-1993). They serve as the starting material for mathematical modeling and prediction.

In mathematical modeling of successions between the ecotones of the soil-plant systems, matrices of mutual transitions were used, describing direct (destructive) and reverse (recovery) transitions. Four main classes with varying degrees of degradation were identified: $S_1$ - initial pastures, $S_2$ - wormwood and ebelek, $S_3$ - weed-annual pastures, $S_4$ - broken sands. Markov chains in the form of a digraph with a transition probability matrix $P$ were the original mathematical model. The multiplication of the initial state vector $S_0$ by the transition matrix to the power $t$ determines the final state vector $S = P^t \cdot S_0$, which allows forecasting.

The analysis of the transition matrix dynamics showed the nonlinear behavior of the ecosystem in the early stages of the Markov process. However, after a certain number of transitions, the process stabilizes and the matrix ceases to change, determining the final state in the form of a stationary vector. With an increase in the observation time, the transition dynamics changes qualitatively: the transition of the system to a stationary state occurs differently for different observation periods. Due to the fact that the processes occurring in real systems are inhomogeneous, and the MCs are represented by nonlinear matrices, it is necessary to introduce the characteristic of MC deviations from homogeneous ones.

The nonlinearity of the chain will be described using the parameter $\delta (t)$, which determines the degree of ecosystem deviation from the equilibrium state and is a characteristic of the deviation from the steady state. The destabilization parameter $\delta (t)$ depends on the degree of pasture exploitation determined by the value of $\gamma (t)$. The change in the parameter $\delta (t)$ is non-monotonic and indicates instability at the
break points of the $\delta(t)$ curve, while $\gamma(t)$ is monotonic, Figure 1.

This led to the conclusion that the bifurcation points are caused by the mutual dynamic exchange between ecotones and are described by a complex Markov chain. In practice, the functional relationship between the conditional load and the destabilization parameter makes it possible to determine the ultimate load that ensures self-healing and the greatest economic effect in the operation of pastures. It is of fundamental importance to determine the points that distinguish the regimes of the pasture ecosystem sustainable dynamics. As a result of modeling using the MC, we obtain the possibility of determining the duration of the existence of a particular state. Markov chains allow to create a simple predictive model of stable dynamic equilibrium.

The dynamics of the studied ecotones of the soil-plant systems obtained from the MC is presented in Figure 2. It is clearly visible non-monotony in some periods of observations (1954-1958 and 1958-1964). In 1954-1958 system dynamics is monotonous, where each ecotone is stabilized, moving to the final state. In the period 1958-1964 the intensity of deflation of the initial non-broken ($S_1$) and weakly-broken ($S_2$) pastures increases significantly, by 1964 decreasing to catastrophic sizes. The course of the curves for 1970–1979 states a critical pasture disintegration by 1979, if the transition matrix remains the same. Thus, the analysis of the Markov process curves for different periods allows to track the dynamics of the ecosystem, make predictions and take appropriate management for the conservation of pastures, Figure 2.

**Figure 1.** Polynomial regression of temporal dependencies: 1 – destabilization parameter $\delta(t)$, 2 – pasture load $R(t)$. 

![Polynomial regression of temporal dependencies](image-url)
Figure 2. Markov chains of deflation processes. The initial state vectors are taken according to the space monitoring data; classes: 1) $S_1$, 2) $S_2$, 3) $S_3$, 4) $S_4$.

The study of nonlinear abiotic systems was carried out on the example of iron hydroxides formed in the process of agricultural mechanisms coding. Basic composition changes of the corrosion products was investigated in order to identify the mechanism of corrosion, as a dynamic transition of corrosion products from one oxide fraction to another.

It has been established that a change in the structure of a corrosive surface is accompanied by an increase in the size of amorphous hydroxide (FeOOH) particles with the subsequent transition to crystals of $\text{Fe}_2\text{O}_3$ (hematite). The gamma-resonance spectra indicated that with an increase in the exposure time, the hematite fraction increases due to a decrease in the fine component from iron hydroxide.

The ratio of the source material volume to the volume of corrosion products is described by the exponent, indicating a non-linear process. Mathematical modeling of the structural steel corrosion
process dynamics has come to describing the development rate of the corrosion layer consisting of many particles growing with an exposure according to the law, reflecting the change in the thickness ($L$) of the affected layer. The mathematical model was defined as an evolutionary synergistic equation:

$$\frac{dC}{dt} = C_s (L - C), \quad \frac{dC}{dt} = aC^{2/3} (L - \beta C) \quad (1)$$

where $C$ – is the corrosion volume, $C_s$ – is the total surface of spherical oxide particles, $L$ – is the initial volume of the material. The surface and volume of the particle are related by the dependence $C_s = \alpha C^{2/3}$. The coefficients $\alpha$ and $\beta$ describe the intensity of the oxidation process and are obtained experimentally.

A regression analysis of the experimental data was performed, where the logistic function was chosen as the theoretical curve:

$$C(\tau) = \frac{a_0}{1 + a_1 \cdot \exp(-a_2 \tau)}, \quad (2)$$

The logistic function (2) is a solution to an autonomous differential equation (1). A sharp increase in the rate of corrosion and subsequent slowing down is a phenomenon characteristic of open non-linear systems, where an explosive nature sometimes takes on a hyperbolic relationship with an asymptote leading to infinity.

The process of surface oxidation under atmospheric conditions generates new systems of oxide particles of different composition, which due to mutual penetration exchange matter, varying qualitatively (internal structure, phase composition) and quantitative (dimensions). Such a phenomenological description does not contradict the physics of these phenomena and is confirmed by the considered mathematical model.

An example of nonlinear phenomena in the abiotic case are also nonlinear processes associated with temperature transformations of oxide iron compounds. Various modifications of iron hydroxides (FeOOH) when heated transfer to a stable fraction in the form of $\alpha$-Fe$_2$O$_3$ [8]. The dynamics of these processes is also non-linear with a clearly defined logistic regularity.

4. Conclusion

Arid ecosystems are functioning in conditions of aggravation in biological synergetics. Modeling the dynamics of the behavior of SPS in zones of aggravation is of great fundamental importance both for understanding the processes of self-organization and for practical applications. Self-regulation processes also exist in inanimate nature. Nonlinear processes in inorganic environment are qualitatively similar to the dynamics of macroscopic systems in ecology and are described by similar models in the form of synergistic structures. System self-organization occurs independently of the systems functional purpose. Comparison of the open systems dynamics by “living” and “inanimate” nature leads to their synergistic unity.

Accounting the ergodicity of processes, Markov chains have proven effectiveness in managing pasture ecosystems to regulate its bio-stocks and can be useful in solving common environmental management problems. The dynamic model based on ergodic matrices provided an opportunity to make long-term forecasts and determine the limits of pasture load. Studies have shown that the process of pasture ecosystems destruction has a complex picture of ecotopes successions with characteristic bifurcation points.

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