Scaling laws for the oblique impact cratering on an inclined granular surface

Shinta Takizawa¹, Hiroaki Katsuragi¹

Department of Earth and Environmental Sciences, Nagoya University, Nagoya 464-8601, Japan

Abstract

Although a large number of astronomical craters are actually produced by the oblique impacts onto inclined surfaces, most of the laboratory experiments mimicking the impact cratering have been performed by the vertical impact onto a horizontal target surface. In previous studies on the effects of oblique impact and inclined terrain, only one of the impact angle φ or target inclination angle θ has been varied in the experiments. Therefore, we perform impact-cratering experiments by systematically varying both φ and θ. A solid projectile of diameter \( D_p = 6 \) mm is impacted onto a sand surface with the range of impact velocity \( v_i = 7–97 \) m s\(^{-1}\). From the experimental result, we develop scaling laws for the crater dimensions on the basis of \( \Pi \)-groups method. As a result, the crater dimensions such as cavity volume, diameter, aspect ratio, and depth-diameter ratio can be scaled by the factors \( \sin \phi \) and \( \cos \theta \) as well as the usual impact parameters (\( v_i, D_p, \) density of projectile, and surface gravity). The possible application of the obtained scaling laws to the analysis of actual planetary-crater shape is also briefly discussed.

Keywords: Impact processes, Cratering, Experimental techniques

1. Introduction

Rocky astronomical bodies covered with regolith usually have a lot of impact craters on their surfaces. The majority of these craters has almost axisymmetric (circular) cavity structure. However, there are also asymmetric peculiar craters. Possible origins of the peculiar crater shapes are the oblique impact, topography, target heterogeneity, post-impact deformation (tectonic phenomena), or their combinations. Among them, target heterogeneity and tectonic effects are more or less geologic effects. In this study, we are interested in the instantaneous physical effects such as oblique impact onto inclined surface. For example, asymmetric ejecta deposition can be induced by the oblique impact (Melosh 1989, 2011). In most of the natural impact events, the meteorite obliquely collides onto the target surface. However, the population of asymmetric craters produced by oblique impacts on Mars is very limited (Herrick and Hessen 2006). On the Earth, only one elliptical crater probably formed by the oblique impact has been found (Kenkmann et al. 2009). Actually, very shallow-angle impact is necessary to form asymmetric crater by the oblique impact. Moreover, the critical angle to produce an elliptic crater depends on the size of crater (Collins et al. 2011). The effect of target inclination should also be considered to analyze asymmetric craters. Indeed, the non-circular craters have been observed on the inclined terrains of Moon, Mars, and asteroids (Elbeshausen and Wünnemann 2011) Elbeshausen et al. 2012; Jaumann et al. 2012; Plescia 2012; Krohn et al. 2014; Neish et al. 2014). On steeply sloped terrains, topography obviously influences the cratering process by modifying the transient crater shape due to the asymmetric landsliding driven by gravity.

To discuss the impact cratering on the regolith layer, various impact experiments on granular targets have been conducted. Particularly, in the granular physics field, crater morphology and penetration dynamics have been extensively studied by low-speed impact with impact velocity \( v_i \sim 10^0 \) m s\(^{-1}\) (Walsh et al. 2003; Uehara et al. 2003; Katsuragi and Durian 2007; Goldman and Umbanhowar 2008; Seguin et al. 2009; Clark et al. 2014). All of these experiments are the vertical impact onto a horizontal granular surface. In addition, there are some studies investigating the oblique impact onto a granular layer (Nishida et al. 2010; Wang et al. 2012). While these studies have revealed the fundamental nature of granular impact phenomena, the applicability of the obtained physical laws to the astronomical impact cratering is not very clear. Particularly, the impact speed is much slower than the typical astronomical impact cratering. To directly mimic the astronomical oblique impact, Gault and Wedekind (1978) have conducted high-speed impact experiment (\( v_i \sim 10^2 \) m s\(^{-1}\)) by systematically varying the impact angle \( \phi \) using a solid projectile and quartz-sand target. Here, the impact angle \( \phi \) is defined by the angle from the horizontal surface, i.e., \( \phi = 90^\circ \) corresponds to the vertical impact. In Gault and Wedekind (1978), the ricochet of projectile was observed at shallow-angle impacts. The resultant craters possess the approximately circular shape when \( \phi \) is greater than 10°. However, the crater shape is elongated along the impact direction in the range of \( \phi \leq 10^\circ \). The elongation degree depends on the type of projectile and impact velocity \( v_i \). The circularity of the crater shape significantly decreases only in the range of \( \phi \leq 10^\circ \). However, the crater volume clearly depends on \( \phi \) even in relatively large \( \phi \) regime. Specifically, Gault and Wedekind (1978) have reported that the volume of crater cavity is proportional to \( \sin \phi \).

The effect of inclined terrain has also been studied experimentally. In general, astronomical bodies have various sloped
terrains (e.g., large crater wall). Particularly, small bodies such as asteroids show large topographic slope variations. Therefore, the oblique impact experiments using only horizontal target are insufficient to fully understand the general impact cratering phenomena. Recently, solid-projectile-impact experiments using inclined granular target have been conducted (Hayashi and Sumita, 2017; Aschauer and Kenkmann, 2017; Takizawa et al., 2019). Hayashi and Sumita (2017) performed the vertical free-fall-impact experiments in which the inclination angle of the dry granular surface $\theta$ and the impact kinetic energy $E$ are varied. They found that the resultant crater shapes can be divided into three phases. Obviously, a circular crater is formed by the vertical impact onto a horizontal sand target ($\theta = 0^\circ$). The sharp rim structure is clearly left around the cavity (full-rim crater phase) in this type of circular crater. However, as $\theta$ increases, the final crater shape becomes shallower and elongated in the slope direction. In addition, the collapse of crater upper wall is induced at $\theta \geq 22^\circ$ (broken-rim crater phase). At the vicinity of repose angle of target sand ($\theta = 34^\circ$), the crater cavity is almost buried by the large-scale avalanche towards the downslope direction (depression phase). Hayashi and Sumita (2017) also found that the crater shape depends mainly on the inclination angle $\theta$ than the impact energy $E$. In other words, the scale of crater-wall collapse is principally determined by $\theta$. Similar trend has been confirmed by another previous study of the impact on an inclined granular surface (Aschauer and Kenkmann, 2017). Although the range of impact speed is different between Hayashi and Sumita (2017) ($\approx 5$ m s$^{-1}$) and Aschauer and Kenkmann (2017) (180 m s$^{-1}$), their results are very similar. Takizawa et al. (2019) conducted the normal impact experiments on an inclined wet (cohesive) granular target and found that the catastrophic collapse of the slope can be induced by the impact when the target granular layer is cohesive enough and the inclination angle $\theta$ is close to the angle of repose. These previous studies suggest that the collapse of crater wall could significantly modify the crater shape when the target granular layer is inclined. Note that the inclination angle $\theta$ is defined relative to the gravitationally horizontal plane, i.e., horizontal surface corresponds to $\theta = 0^\circ$.

In the above-mentioned previous works, only one of the inclination angle $\theta$ or the impact angle $\phi$ was varied. The crater formation process and the final crater shape depending on both $\theta$ and $\phi$ have not yet been systematically clarified. In general astronomical impacts, meteorites obliquely collide onto sloped terrains. Nevertheless, $\theta$ and $\phi$ have not been simultaneously varied in any laboratory experiment. Furthermore, the dimensionless scaling laws, which allow us to extrapolate the laboratory-experiment results to the astronomical impacts, have not been developed for the oblique impact onto an inclined surface. Dimensionless scaling called II-groups method (Buckingham, 1914, 1915) is a very powerful methodology to consider the scale-independent cratering dynamics. Indeed, the II-groups method has been applied to various impact-cratering analyses (Schmidt, 1980; Holsapple and Schmit, 1982, 1987; Holsapple, 1993; Holsapple and Housen, 2007; Housen and Holsapple, 2011). However, the II-groups method has not been applied to the oblique impact onto an inclined surface.

Therefore, in this study, we are going to develop the scaling laws for craters produced by the oblique impact onto an inclined granular surface. To clarify the crater formation process and the scaling laws including the effects of $\theta$ and $\phi$, we conduct experiments in which $\theta$ and $\phi$ are systematically and independently varied (Fig. 1). Then, we discuss the guideline for the possible application of the experimentally obtained scaling laws to astronomical impact-cratering analysis.

2. Experiment

To perform the systematic impact experiments, we develop an experimental apparatus that can control both the inclination angle $\theta$ and the impact angle $\phi$. The entire system of the developed experimental apparatus is shown in Fig. 2. The sand vessel is mounted on a tiltable inclination stage driven by a stepping motor (Orientalmotor, AZ98MD-HS100). The resolution of rotation angle is 0.0036° per pulse. Using this inclination stage, $\theta$ can be precisely controlled. The rotatable injection equipment (projectile gun) is also mounted on the inclination stage. The impact angle $\phi$ is controlled by manually rotating the gun relative to the container vessel. Using a spring-based air-compression mechanism, a solid projectile is injected towards the center of sand target. All experiments are performed under the atmospheric pressure condition.

For target, Toyoura sand (TOYOURA KEISEKI KOGYO K.K.) is used. The diameter of sand grains (0.1–0.2 mm) is
to electronic stages (COMS, PM80B-200X and PM80B-100X) to slide the profiler in XY plane. By combining line-profile data series, three-dimensional (3D) surface profile is synthesized. Since these electronic stages are mounted on the inclination stage, the crater profile along the surface of target (in XY plane) can be obtained. The size of measurable XY area is 191×65 mm². The measurement resolution is 50 μm in horizontal (XY) direction and 0.5 μm in vertical (Z) direction. These resolutions are sufficiently smaller than the mean diameter of sand grains 0.15 mm. 3D profiles of the sand surface are measured before and after the projectile impact, and the crater profile is computed by subtracting the before-impact profile from the after-impact profile. Using the obtained crater profiles, we measure the crater dimensions such as diameter, depth, and volume.

3. Results

3.1. θ and φ dependence of crater shape

By the systematic impact experiments, morphology of resultant craters can qualitatively be classified. First, θ dependence of the crater shape is shown in Fig. 4. Figure 4(a) shows a symmetric circular crater formed by almost normal (and vertical) impact onto a horizontal sand surface (φ = 90° and θ = 0°). In Fig. 4(b,c), craters produced by the normal impact (φ = 90°) to the inclined sand surfaces are presented. As can be seen in Fig. 4(b,c), the asymmetry of the crater shape is enhanced as θ increases. This asymmetry mainly originates from the collapse of the wall of transient crater cavity. Particularly, the upper wall significantly collapses when the inclination angle θ is close to angle of repose.

In Fig. 4, high-speed images of the normal impact to (a) horizontal and (b) 30°-tilted sand surfaces are shown. One can confirm that the ejecta splashing right after the impact is almost isotropic in both cases. This means that the transient crater cavity should be almost axisymmetric around the normal axis at the impact point. Thus, to produce asymmetric craters shown in Fig. 4(b,c), the transient crater walls must collapse. Actually, the asymmetric transient-crater-wall collapse can be also observed in another high-speed video data as well. These observations are qualitatively consistent with recent previous ex-

Figure 3: (a) Schematic illustration of the experimental setup. Sand is poured into the container, and its surface is initially set to be parallel to the bottom of container by manual flattening. Then, the inclination angle θ is varied by rotating the stage. The impact angle φ is controlled by rotating the injection unit. A two-dimensional (2D) laser profiler is attached to electronic stages and scans the sand-target surface along the unit. A two-dimensional (2D) laser profiler is attached to electronic stages (COMS, PM80B-200X and PM80B-100X) as shown in Fig. 3(a). This laser profiler, which measures the topography in width of ~ 40 mm, is attached into the container by manual flattening. Then, the inclination angle θ into the container, and its surface is initially set to be parallel to the bottom

Figure 4: Final crater shapes produced by the impact conditions of (a) θ = 0°, φ = 90.2°, v_i = 80.2 m s^{-1}, and m_i = 0.12 g, (b) θ = 20°, φ = 88.5°, v_i = 77.5 m s^{-1}, and m_i = 0.12 g, and (c) θ = 30°, φ = 90.5°, v_i = 84.2 m s^{-1}, and m_i = 0.12 g. Scale bars indicate 10 mm.

large enough to neglect the air-drag and humidity effects (Duran [2010] Andreotti et al. [2013] Katsuragi, 2016). The true density and angle of repose of Toyoura sand are 2.63×10^3 kg m^{-3} and θ_r = 34°, respectively. Thus, θ is varied in the range 0 ≤ θ < θ_r. We pour sand into a container (inner width:200 mm, length:300 mm, height:200 mm). The container width and length are sufficiently large to neglect the container-wall effect on the impact dynamics (Seguin et al., 2008; Nelson et al., 2008). To make frictional boundary, the identical sand grains are glued on the container walls. The thickness and packing fraction of the target sand layer is fixed to 100 mm and 0.55, respectively, in all experiments. First, the target layer is manually flattened at the horizontal position. Then, the target is tilted and the gun is rotated to control θ and φ. Since we use the common rotation axis (Y axis), both angles θ and φ are varied in the same two-dimensional space (XZ plane). The variation of φ in this experiment ranges in 10° ≤ φ ≤ 170°. The XYZ coordinate system is defined as shown in Fig. 3(a). The surface of target layer corresponds to XYZ plane.

The injection gun shoots a spherical projectile with diameter D_p = 6 mm and mass m_i = 0.12 g, 0.25 g, or 0.4 g. The range of impact speed is 7 < v_i ≤ 97 m s^{-1}. The gun muzzle is kept at least 100 mm away from the target surface. The actual impact angle φ and speed v_i are measured by using a high-speed camera (Photron, SA1) placed at the side of container, with a frame rate of 10,000 frames per second (Fig. 3(b)). The crater formation process is taken by another high-speed camera (CASIO, EX-F1) placed in front of the sand surface, with a frame rate of 300 frames per second. The spatial resolutions of the side-view and the front-view images are 0.18 mm per pixel and 0.5 mm per pixel, respectively.

To measure the final crater shape formed by the impact, we use a linear two-dimensional (2D) laser profiler (KEYENCE, LJ-V7080) as shown in Fig. 3(c). This laser profiler, which measures the topography in width of ~ 40 mm, is attached
The shape of ejecta curtain is almost symmetric around the normal axis to the surface. With \( \theta \), \( \phi \) impact conditions: (a) \( \theta = 0^\circ \), \( \phi = 90^\circ \), \( v_1 = 80.2 \text{ m s}^{-1} \), and \( m_i = 0.12 \text{ g} \), and (b) \( \theta = 30^\circ \), \( \phi = 88.5^\circ \), \( v_1 = 77.5 \text{ m s}^{-1} \), and \( m_i = 0.12 \text{ g} \). In both cases, the shape of ejecta curtain is almost symmetric around the normal axis to the surface. Examples of 3D crater profile measured by the laser profiler in \( X \) direction of the crater shape formed by oblique impacts onto horizontal sand surfaces \((\phi = 70^\circ, 10^\circ \text{ and } \theta = 0^\circ)\) are shown. Scale bars indicate 10 mm. Projectiles impact from the right and rebound to the left. (b) The dependence of cross-sectional crater profiles in \( X \) direction. The crater shape becomes asymmetric and the cavity volume becomes small as \( \phi \) decreases.

In Fig. 6(a) the effect of the impact angle \( \phi \) is presented. Figure 6(a) shows crater shapes formed by oblique impacts onto horizontal sand surface \((\phi = 70^\circ \text{ and } 10^\circ \text{ with } \theta = 0^\circ)\). Due to the oblique impact, the crater shapes are slightly elongated. The crater wall seems to be partially removed by the projectile rebound. Note that the projectile comes from the right side and rebounds to the left side in Fig. 6(b). When \( \phi \) is in the range of \( 90 \pm 10^\circ \) (almost normal impact), rebound of projectile does not occur. However, the rebound of projectile can be observed in most of the oblique impacts. The rebound of projectile has also been confirmed in previous study of the oblique impact \( \text{Gault and Wedekind (1978)} \). Figure 6(b) shows cross-sectional profiles in \( X \) direction of the crater shape formed by oblique impact onto horizontal sand surfaces \((\phi = 90, 70, 30, \text{ and } 10^\circ \text{ with } \theta = 0^\circ \text{ and almost identical } v_1)\). The shallow, asymmetric, and small craters are produced by small \( \phi \) impacts.

Figure 5: Sideview images of the impacts taken by a high-speed camera. Impact conditions are: (a) \( \theta = 0^\circ \), \( \phi = 90^\circ \), \( v_1 = 80.2 \text{ m s}^{-1} \), and \( m_i = 0.12 \text{ g} \), and (b) \( \theta = 30^\circ \), \( \phi = 88.5^\circ \), \( v_1 = 77.5 \text{ m s}^{-1} \), and \( m_i = 0.12 \text{ g} \). In both cases, the shape of ejecta curtain is almost symmetric around the normal axis to the surface.

3.2. Phase diagram

By systematically varying \( \theta \) and \( \phi \), we observe three representative phases of the final crater shape (symmetric, asymmetric, and catastrophic collapses). The symmetric-crater phase is defined by the case in which the transient crater cavity formed by the excavation collapses isotropically (symmetrically). This modification is very small so that the initial transient cavity is almost preserved in this phase. On the other hand, asymmetric-crater phase is characterized by the asymmetric collapse of upper wall of the transient crater cavity which results in the asymmetric final crater shape. In addition, the catastrophic collapse phase is defined by the large-scale collapse in which the flow initiated at the upper crater wall reaches the lower crater rim. Figure 7 shows the phase diagram based on the above-mentioned classification of the transient-crater-wall collapse. The phase diagram is independent of \( v_1 \) in the experimented range \( 10–100 \text{ m s}^{-1} \). As confirmed in Fig. 7, the scale of the collapse increases as \( \theta \) increases. The obtained phase diagram is qualitatively consistent with the previous study \( \text{Hayashi and Sumita (2017)} \). Moreover, one can also confirm that the collapse scale is hardly influenced by the impact angle \( \phi \). However, the rebound condition seems to depend on \( \phi \). Black filled circles in Fig. 7(a) indicate no-rebound cases. As shown in Fig. 7(a), most of the oblique impacts result in the projectile rebound. In Fig. 7(b), the typical final crater shapes and the corresponding crater profiles (in \( X \) direction) are shown. As \( \theta \) increases, the asymmetry is enhanced, and finally the crater cavity is almost buried by the significant upper crater-wall collapse at \( \theta = 30^\circ \).

Examples of 3D crater profile measured by the laser profiler are shown in Fig. 8. The crater depth \( \Delta Z \) indicates the height difference between before and after the impact. In Fig. 8, \( \theta \) is varied while \( \phi = 90^\circ \) is fixed (i.e., normal impacts). At \( \theta = 0^\circ \), the wall of the transient crater slightly collapses in a symmetric way so that the isotropic rim clearly exists around the crater cavity (Fig. 8(a)). Contrastively, for the cases of \( \theta \geq 20^\circ \) (Fig. 8(b,c)), the upper rim structure is flown out due to the collapse of the upper wall. In addition, the deepest point in the cavity migrates...
downward compared to $\theta = 0^\circ$, causing the asymmetric profiles as also shown in Fig. 7(b). That is, both the rim and cavity structures are modified by the effect of inclination $\theta$. At $\theta = 30^\circ$, the crater cavity has a peculiar (almost flat) shape because the large-scale collapse of the upper crater wall reaches the lower crater rim (Fig. 8(c)). These trends are qualitatively consistent with previous studies (Hayashi and Sumita, 2017; Aschauer and Kenkmann, 2017). The effect of impact angle $\phi$ on the crater asymmetry is actually limited compared to the effect of $\theta$. As shown in Fig. 6, the small $\phi$ results in the small crater cavity rather than the enhancement of asymmetry.

3.3. Definition of crater dimensions

For quantitative analyses of the final crater shape, the following crater dimensions are defined and measured: length $D_{cx}$, width $D_{cy}$, depth $H_c$, and volume $V_c$. Fig. 9 shows an example of (a) 3D crater profile and (b) corresponding cross-sectional profile. A broken circular curve in Fig. 9(a) indicates a contour of $\delta Z = 0$ (around the crater floor) defining the outline of crater cavity. The crater width $D_{cy}$ is the maximum width of the contour ($\delta Z = 0$) in $Y$ direction (perpendicular to the inclination direction). The crater length $D_{cx}$ is the linear dimension of the contour ($\delta Z = 0$) in $X$ direction at the center of $D_{cy}$. Usually, $D_{cx}$ and $D_{cy}$ correspond to the major and minor axes of the crater shape. The crater depth $H_c$ is defined by the largest negative displacement (max $|\delta Z|$) in the profile (Fig. 9(b)). The crater volume $V_c$ is the volume of the crater cavity ($\delta Z \leq 0$).

3.4. Energy dependence of the crater dimensions

To understand the crater formation process, we investigate the relation among impact kinetic energy and the crater dimensions defined above. Here, the impact kinetic energy $E$ is simply defined as

$$E = \frac{1}{2} m v_i^2. \tag{1}$$

Figure 10 shows $E$ dependence of the crater dimensions $D_{cx}$, $D_{cy}$, $H_c$, and $V_c$ for various $\theta$ (with fixed $\phi = 90^\circ$). As seen in Fig. 10 all of these crater dimensions show power-law relations with $E$. The broken lines in Fig. 10 indicate the power-law fittings. The obtained scaling exponents for $D_{cx}$, $D_{cy}$, and $H_c$ almost coincide with each other for all inclination-angle ($\theta$) cases. The value of scaling exponent is approximately obtained as 0.19. The scaling of crater diameter has been studied well by the low-speed granular impact using solid and liquid-drop projectiles. The obtained scaling exponent is basically close to $1/4 = 0.25$ (Walsh et al., 2003; Uehara et al., 2003; Katsuragi, 2010). However, the scaling exponent $1/6$ has also been reported in a droplet impact onto a granular layer (Zhao et al., 2010). The values obtained in this study (0.18–0.19; Fig. 10(a–c)) are close to 1/6. The scaling exponent for crater volume $V_c$ is about 0.54 (Fig. 10(d)). This value can roughly be derived as $V_c \sim D_{cx}D_{cy}H_c \sim E^{0.19+0.18+0.19} \sim E^{0.56}$. Although the scaling exponents seem to be universal, the specific values (scaling coefficients) of $D_{cx}$, $H_c$, and $V_c$ depend on $\theta$. Contrarily, $D_{cy}$ is independent of $\theta$. All of the $D_{cx}$ data of various $\theta$ cases are scaled by the single power-law relation (Fig. 10(b)).
4. Analysis and discussion

4.1. Crater modification process

From these observations, the collapse process of transient crater wall can be qualitatively understood. Figure 11(a) shows a schematic image of the transient crater shape (cross section). The collapse of upper crater wall and deposited ejecta (green and blue regions, respectively, in Fig. 11(a)) is triggered when the slope of upper crater wall is steep enough to be unstable. The scale of collapse is mainly determined by the inclination angle \(\theta\). As a result, \(D_{cy}\) becomes an increasing function of \(\theta\). The final crater shape is formed by the reaccumulation of the collapsed crater wall and ejecta deposit falling back into the crater cavity (Fig. 11(b)). Thus, the above-mentioned collapse mechanism is also consistent with the negative correlation between \(H_c\) and \(\theta\). Furthermore, due to the ejecta volume falling back to the crater cavity, the crater volume \(V_c\) becomes a decreasing function of \(\theta\) (Fig. 11(b)). However, the crater width \(D_{cy}\) is independent of \(\theta\) because \(D_{cy}\) is perpendicular to the collapse direction.

4.2. Scaling of the crater dimensions

For further quantitative understanding of the cratering dynamics, scaling with dimensional analysis is employed here. The appropriate dimensional analysis is necessary to discuss the applicability of the laboratory-experiment results to astronomical-scale phenomena. Here we use the standard \(\Pi\)-groups method to analyze the current experimental results. Because the final crater volume \(V_c\) is determined by the combination of all effects, we first develop the scaling law for \(V_c\). Based on the \(\Pi\)-groups method, we start with two dimensionless numbers:

\[
\pi_v = \frac{\rho_A V_c}{m_1} \text{ and } \pi_2 = \frac{g D_k}{v_i^2},
\]

where \(\rho_A\) and \(g\) are the density of target and the magnitude of surface gravitational acceleration of the target surface, respectively. Since the target consists of cohesionless sand, the cratering is in the gravity-dominant regime. Therefore, \(\pi_2\) is employed as a relevant parameter. \(\pi_v\) of the craters produced by the vertical impact onto a horizontal sand surface is well scaled by

\[\pi_2 (\text{Holsapple and Schmidt, 1987}).\]

Thus, first examine the relation between crater efficiency (normalized crater volume) \(\pi_v\) and the gravity-scaled size \(\pi_2\).

Figure 12(a) shows \(\pi_2\) dependence of \(\pi_v\). All experimental data with various \(\theta\), \(\phi\), \(v_i\), and \(m_1\) are plotted in Fig. 12. Although the negative correlation between \(\pi_v\) and \(D_2\) can clearly be confirmed, the data in Fig. 12(a) show considerable scattering. The reason for this data scattering is rather obvious. We varied both \(\theta\) and \(\phi\) in this experiment whereas these factors are not taken into account in the dimensionless numbers \(\pi_v\) and \(\pi_2\). To improve the quality of data collapse in the scaling plot, we have to modify the dimensionless numbers by considering \(\theta\) and \(\phi\). First, we consider the effect of \(\phi\). According to Gault and Wedekind (1978), the crater volume \(V_c\) decreases as the impact angle \(\phi\) becomes small, obeying the factor \(\sin \phi\). Therefore, we modify \(\pi_2\) by using the factor \(\sin \phi\) as

\[\pi_2' = \frac{g D_k}{v_i^2 \sin \phi}.
\]

Figure 12(b) shows the relation between \(\pi_v\) and \(\pi_2'\). Although the data of identical \(\theta\) seem to collapse onto scaling relations (straight lines in log-log plot), \(\theta\) dependence of \(\pi_v\) remains as data scattering. However, one can confirm that the scaling exponent (slope in Fig. 12(b)) is almost independent of \(\theta\). Namely, the scaling law can be written as

\[f(\theta) = 0.14 (\cos \theta)^{1.5}.
\]

Here, we use \(\cos \theta\) because it becomes unity in the standard case \(\theta = 0^\circ\). Based on this relation, \(\pi_v\) is modified using the factor \(\cos \theta\) as

\[\pi_v' = \frac{\rho_A V_c}{m_1 (\cos \theta)^{1.5}}.
\]
Figure 12(c) shows the relation between $\pi'_2$ and $\pi'_1$. Finally, all data are collapsed onto a unified scaling relation. The scaling law including the effects of $\sin \phi$ and $\cos \theta$ is finally written as
\[
\frac{\rho_i V_c}{m_i} = 0.14 (\cos \theta)^{4.5} \left[ \frac{gD_s}{v_i^2 \sin \phi} \right]^{-0.52}.
\] (7)

The relation between $\pi'_2$ and $\pi'_1$ obeys a power-law form with nontrivial scaling exponent 0.52 and scaling coefficient 0.14$(\cos \theta)^{4.5}$. Eq. (7) can be rewritten as $\pi'_1 = 0.14(\cos \theta)^{4.5}(\sin \theta)^{0.52}\pi'_2^{0.52}$. In this analysis, the common exponent value 0.52 is used for both $\pi_2$ and $\sin \phi$ to reduce the number of free fitting parameters in the scaling model. Nevertheless, the quality of data collapse by the scaling is excellent. Alternatively, the vertical component of impact velocity $v$ strongly depends on target properties such as porosity and internal friction (Wunnemann et al., 2006). For example, the momentum transfer could be more important than the energy-governing limit ($\mu \pi v^2$) at $v_1 = 1$ m s$^{-1}$.

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Actually, the form of Eq. (7) is similar to the scaling obtained by the high-speed impact ($v_1 \sim 10^3$ m s$^{-1}$) to sand target. Schmidt (1980) performed the experiment of vertical impact onto a horizontal surface under the vacuum condition. This situation simply corresponds to $\sin \phi = \cos \theta = 1$ in Eq. (7). Figure 13 shows the consistency between the current result and the previous study (Schmidt, 1980). In Fig. 13, the data reported in Schmidt (1980) are plotted as well as the current experimental results. In addition, one data point of a very low-speed ($v_1 \sim 1$ m s$^{-1}$) impact (the largest $\pi'_2$ data) is added in this plot (not shown in the previous plots). Although this data point comes from the free-fall impact to the flat surface, it completely obeys the scaling. The excellent data collapse shown in Fig. 13 suggests that Eq. (7) is a universal scaling relation independent of the experimental conditions such as impact-velocity range and ambient pressure. The weak point of this study is the too low impact speed to mimic large-scale astronomical impacts. However, the current result is fully consistent with the high-velocity impact experiment. The scaling law obtained by Schmidt (1980) is very robust. In this study, we expand the scaling form by using two factors: $\cos \theta$ and $\sin \phi$.

By the same protocol, crater diameter $D_{cy}$ can also be scaled. Since $D_{cy}$ is independent of $\theta$, we can directly scale the non-dimensionalized crater radius,
\[
\pi_R = \frac{D_{cy}}{2} \left[ \frac{\rho_i}{m_i} \right]^{1/3},
\] (9)

using $\pi'_2$ as shown in Fig. 14. We can clearly confirm the scaling relation,
\[
\frac{D_{cy}}{2} \left[ \frac{\rho_i}{m_i} \right]^{1/3} = 0.53 \left[ \frac{gD_s}{v_i^2 \sin \phi} \right]^{-0.19},
\] (10)

where the coefficient 0.53 and exponent 0.19 are computed by the least square fitting to all data. In Fig. 14 data from Schmidt (1980) are also plotted just like Fig. 13. Again, the excellent agreement between this study and Schmidt (1980) can be confirmed.

The ranges of $\pi'_2$ shown in Figs. 13 and 14 correspond to those for typical astronomical impacts on planets (or satellites like Mars and Moon), or asteroids (like an asteroid 162173 Ryugu). The value of $\pi'_2$ ranges in $10^{-7}$–$10^{-6}$ when we consider the gravity corresponding to Moon or Mars ($g \sim 10^2$ m s$^{-2}$), $D_s = 0.1–10$ km, and $v_1 = 1–10$ km s$^{-1}$. On the other hand, $\pi'_2 = 10^{-13}$–$10^{-7}$ is obtained when we consider small-asteroids-level gravity $g = 10^{-4}$ m s$^{-2}$, $D_s = 0.1–10$ m, and $v_1 = 0.1–10$ km s$^{-1}$. As shown in Figs. 13 and 14 the current experimental result corresponds to the relatively larger-scale impacts while the previous study (Schmidt, 1980) actually simulates the asteroid-scale impacts. Namely, in terms of dimensional analysis, the obtained scaling law covers from asteroid scale to planet...
scale. The scaling law is satisfied over eight orders of magnitude in $n^2$ as shown in Figs. 13 and 14. Although we have assumed the scale-free nature of the cratering phenomena based on the similarity law, the absolute impact speed used in this experiment is small. However, the excellent data collapse shown in Figs. 13 and 14 strongly suggests the wide applicability of the scaling laws.

### 4.3. Scaling of the crater aspect ratios

Next, two kinds of aspect ratios $D_{cx}/D_{cy}$ and $H_i/D_{cy}$ are analyzed. Here we assume that these aspect ratios are independent of the scale of impact, i.e., the crater morphology should be similar independent of $E$. Then, the ratios should depend only on $\theta$ and $\phi$. In Fig. 15(a), the relation between $D_{c3}/D_{cy}$ and $\cos \theta$ is shown. As seen in Fig. 15(a), $D_{c3}/D_{cy}$ decreases with $\cos \theta$ for each $\phi$. This trend is qualitatively consistent with previous studies (Hayashi and Sumita 2017; Aschauer and Kenkmann 2017). In addition, we can confirm that $D_{c3}/D_{cy}$ satisfies the relation $\frac{D_{c3}}{D_{cy}}(\phi) = \frac{D_{c3}}{D_{cy}}(180^\circ - \phi)$. Namely, $\sin \phi$ is a relevant parameter to analyze the data. From Fig. 15(a), an empirical scaling form $\frac{D_{c3}}{D_{cy}} = \gamma(\sin \phi)(\cos \theta)^{\beta}$ is obtained. Here, $\gamma$ is a certain dimensionless function of $\sin \phi$. To determine the form of $\gamma(\sin \phi)$, the relation between $\gamma$ and $\sin \phi$ is shown in Fig. 15(b). As a result, we empirically obtain $\gamma = [\sin \phi]^{-0.2}$. Thus, we finally obtain the relation,

$$\frac{D_{c3}}{D_{cy}} = [\sin \phi]^{-0.2} [\cos \theta]^{-1}. \tag{11}$$

The corresponding normalized scaling relation is plotted in Fig. 15(c).

Similarly, the depth-diameter ratio $H_i/D_{cy}$ is scaled by $\cos \theta$ and $\sin \phi$. The relation between $H_i/D_{cy}$ and $\cos \theta$ is displayed in Fig. 16(a). Although $H_i/D_{cy}$ seems to be scaled by $\cos \theta$, both the scaling coefficient and exponent depend on $\sin \phi$. The corresponding scaling form is written as $H_i/D_{cy} = \alpha(\sin \phi)[\cos \theta]^{\alpha(\sin \phi)} (\alpha$ and $\beta$ are dimensionless functions of $\sin \phi$). Therefore, the relations between $\alpha$ vs $\sin \phi$ and $\beta$ vs $\sin \phi$ are plotted in Figs. 16(b) and (c), respectively. As a consequence, we empirically obtain two relations: $\alpha = 0.2[\sin \phi]^{-0.3}$ and $\beta = 10 \sin \phi$. Thus, the scaling for $H_i/D_{cy}$ is written as,

$$\frac{H_i}{D_{cy}} = 0.2[\sin \phi]^{-0.3}[\cos \theta]^{10\sin \phi}. \tag{12}$$

The corresponding scaling plot is shown in Fig. 16(d).

These scaling laws are obtained by a highly empirical way. Moreover, the effects of $\theta$ and $\phi$ are not independent in the scaling of $H_i/D_{cy}$ (Eq. 12). Whereas these relations are dimensionless, physical basis on these scaling laws is not very firm. Furthermore, the scale-free (similarity) assumption for the aspect ratios might not be held in large scale. The relative population of the elliptic craters actually depends on the scale of craters (Collins et al. 2011). Therefore, the validity of these relations has to be checked by observational data. This is a crucial future work.

### 4.4. Possible application to astronomical impact cratering

Thus far, we have derived scaling laws for the crater dimensions and aspect ratios (Eqs. 7, 10, 11, and 12). Since these relations are dimensionless, they are scale free and basically applicable to large-scale phenomena (astronomical impact cratering). Therefore, in this section, we consider a possible way to estimate the impact conditions such as impact angle, slope of target terrain, impact velocity, etc. from the observable crater dimensions. The parameters included in the scaling laws are $D_i$, $\rho_i$, $v_i$, $\rho_g$, $g$, $\theta$, and $\phi$. Among them, $D_i$, $\rho_i$, and $v_i$ can be combined to a coupling parameter $C_p$ (Eq. 8). In addition, $\rho_i$ and $g$ can be estimated from the observation of the
target body. Therefore, we have three unknown parameters: $C_p$, $\theta$, and $\phi$. And, we have four scaling laws (Eqs. (7), (10), (11), and (12)). Namely, from the four observables, $V_c$, $D_{cy}$, $D_{cx}$, and $H_c$, one can obtain the three value, $C_p$, $\theta$, and $\phi$. For fresh craters, $\theta$ can also be estimated from the observation. Then, four parameters, $D_c$, $p_i$, $v_i$, and $\phi$ (three ingredients of $C_p$ and the impact angle) can be computed. However, we have to be careful when the scaling laws are used to analyze the astronomical impact craters. For instance, crater relaxation caused by impact-induced seismic shaking should affect the shape of old craters on small bodies (Richardson, 2004; Richardson et al., 2005; Katsuragi, 2016; Tsuji et al., 2015). For small bodies like asteroids, impact-induced resurfacing (Yamada et al., 2005; Katsuragi, 2016; Tsuji et al., 2018) becomes significant in very large craters. Particularly, this effect becomes significant in very large craters (Yamada et al., 2016). Besides, small impacts impinging the wall of a large crater also relaxes the crater shape on relatively large target bodies (Soderblom, 1970). Actually, the relaxation of lunar craters have been analyzed by this type of gradual erosion (Fassett and Thomson, 2014). Therefore, the scaling laws can only be applied to fresh craters. In addition, the aspect ratio $D_{cx}/D_{cy}$ could depend on the crater scale. Particularly, this effect becomes significant in very large craters (Collins et al., 2011). Besides, we neglect the effect of melting and shock wave propagation induced by the hypervelocity impact.

4.5. Future problems

In order to verify the validity of the obtained scaling laws, statistical analysis of a large number of astronomical craters could be useful. Specifically, the effect of the inclination angle $\theta$ on the crater shapes ($D_c/D_{cy}$ and $H_c/D_{cy}$) could be statistically verified by examining the relation between fresh crater shapes and the local inclination angles on e.g., Moon or Mars on which abundant crater records can be found. As already mentioned before, the relations of Eqs. (13) and (12) are still highly speculative. The statistical analysis could be helpful to improve these relations by combining experimental and observational results.

The current oblique-impact experiment neglects the effect of inclination around X axis. In this study, all the angle variations are defined around Y axis. This is the reason why $D_{cy}$ is independent of $\theta$. To consider the truly 3D oblique impact, the incident angle around X axis should also be taken into account. Furthermore, spin of projectile could also affect the cratering dynamics. However, these effects are probably limited since the former could only affect the principal axis directions of the elongated crater shape and the latter could only increase the effective impact energy. Thus, we focus on the simpler case in this study.

In addition, to reveal the crater formation process in more details, it is necessary to develop the scaling laws for the ejecta velocity and the timescale of crater formation. The detail analysis of rebound velocity and the timescale of rebound are also interesting future problems. By the preliminary analysis, we find that the rebound timescale significantly depends on $\phi$. The rebound dynamics depending on $\phi$ could be a key factor to understand the physics of general impact cratering phenomena.

5. Conclusion

In this study, oblique-impact experiments onto an inclined granular layer were performed for understanding the crater formation process and obtaining the scaling laws including the effects of both impact angle $\phi$ and inclination angle of the target surface $\theta$. From the phase diagram based on the type of crater-wall collapse, we found that the scale of collapse of upper wall on transient crater cavity depends mainly on $\theta$. As a consequence of the collapse, the final crater dimensions differently depend on the impact kinetic energy $E$. While the crater length $D_{cx}$ is an increasing function of $E$, crater width $D_{cy}$ is independent of $E$. And, the crater depth $H_c$ and volume $V_c$ decrease as $E$ increases. To obtain the universal scaling for the crater dimensions, parameters were non-dimensionalized and analyzed on the basis of $\Pi$-groups method. In addition to the conventional dimensionless numbers ($\pi_v = \rho V_c/m_i$, $\pi_R = (D_{cy}/2)(\rho_i/m_i)^{1/3}$, and $\pi_2 = g D_f/v_c^2$), we took into account the effects of two angle factors: $\sin \phi$ and $\cos \theta$. As a result, we found that the normalized crater volume $\pi^* = \rho V_c/m_i(\cos \theta)^{0.5}$ and $\pi_R$ are scaled by the modified gravity parameter $\mu = g D_f/(v_c^2 \sin \phi)$ as written in Eqs. (7) and (10). Besides, the crater aspect ratios $D_{cx}/D_{cy}$ and $H_c/D_{cy}$ were scaled by $\sin \phi$ and $\cos \theta$ (Eqs. (11) and (12)). Finally, a possible guideline of application of the scaling laws for estimating the parameters characterizing actual impact craters was briefly discussed.

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Data availability

All the experimental raw data are tabulated in the supplementary file (all-data.csv). The file includes the data set of impact conditions $\theta$, $\phi$, $v_i$ and $m_i$, and the measured crater dimensions: $D_{cx}$, $D_{cy}$, $H_c$, and $V_c$.

References

Andreotti, B., Forterre, Y., Pouliquen, O., 2013. Granular Media : Between Fluid and Solid. Cambridge University Press.

Aschauer, J., Kenkmann, T., 2017. Impact cratering on slopes. Icarus 290, 89–95.

Buckingham, E., 1915. The principle of similitude. Nature 96, 396–397.

Clark, A.H., Pettersen, A.J., Behringer, R.P., 2014. Collisionsal model for granular impact dynamics. Phys. Rev. E 89, 012201–12.

Collins, G., Elbeshausen, D., Davison, T., Robbins, S., Hynek, B., 2011. The size-frequency distribution of elliptical impact craters. Earth and Planetary Science Letters 310, 1 – 8.

Duran, J., 2010. Sands, powders, and Grains : An introduction to the Physics of Granular Materials. Springer.
Elbeshausen, D., Wünnemann, K., 2011. THE EFFECT OF TARGET TOPOGRAPHY ON THE IMPACT ANGLE ON CRATER FORMATION - INSIGHT FROM 3D NUMERICAL MODELLING. Proc. Lunar Planet. Sci. 42, 1778.

Elbeshausen, D., Wünnemann, K., Sierks, H., Vincent, J.B., Oklay, N., 2012. THE EFFECT OF TOPOGRAPHY ON THE IMPACT CRATERING PROCESS ON LUTETIA. Proc. Lunar Planet. Sci. 43, 1867.

Fassett, C.I., Thomson, B.J., 2014. Crater degradation on the lunar maria: Topographic diffusion and the rate of erosion on the Moon. J. Geophys. Res. Planets 119, 2255–2271.

Gault, D.E., Sonett, C.P., 1982. Laboratory simulation of pelagic asteroidal impact: Atmospheric injection, benthic topography, and the surface wave radiation field. Geological Society of America Special Paper 190, 69–92.

Herrick, R.R., Hessen, K.K., 2006. The planforms of low-angle impact craters in the northern hemisphere of Mars. Meteoritics & Planetary Science 41, 1483–1495.

Holsapple, K.A., 1993. THE SCALING OF IMPACT PROCESSES IN PLANETARY SCIENCES. Annu. Rev. Earth Planet. Sci. 21, 333–373.

Holsapple, K.A., Housen, K.R., 2007. A crater and its ejecta: An interpretation of Deep Impact. Icarus 187, 345–356.

Holsapple, K.A., Schmidt, R.M., 1982. On the scaling of crater dimensions: 2. Impact processes. J. Geophys. Res. 87, 1849–1870.

Holsapple, K.A., Schmidt, R.M., 1987. Point source solutions and coupling parameters in cratering mechanics. J. Geophys. Res. 92, 6350–6376.

Housen, K.R., Holsapple, K.A., 2011. Ejecta from impact craters. Icarus 211, 856–876.

Jaumann, R., Williams, D.A., Buczkowski, D.L., Yingst, R.A., Preusker, F., Hiesinger, H., Schmedemann, N., Kneissl, T., Vincent, J.B., Blewett, D.T., Buratti, B.J., Carney, V., Denevi, B.W., De Sanctis, M.C., Garry, W.B., Keller, H.U., Kersten, E., Krohn, K., Li, J.Y., Marchi, S., Matz, K.D., McCord, T.B., Saur, J., Mittlefehldt, D.W., Mottola, S., Nathues, A., NEUKUM, G., O'Brien, D.P., Pieters, C.M., Prettyman, T.H., Raymond, C.A., Roatsch, T., Schenk, P., Schmidt, B.E., Sholl, S., Stephane, K., Sykes, M.V., Tricarico, P., Wagner, R., Zuber, M.T., Sierks, H., 2012. Vesta's Shape and Morphology. Science 336, 687–690.

Katsuragi, H., 2010. Morphology Scaling of Drop Impact onto a Granular Layer. Phys. Rev. Lett. 104, 218001–4.

Katsuragi, H., 2016. Physics of Soft Impact and Cratering. volume LNP 910. Springer.

Katsuragi, H., Durian, D.J., 2007. Unified force law for granular impact cratering. Nat. Phys. 3, 420–423.

Kenkmann, T., Artemieva, N.A., Wünnemann, K., Poelchau, M.H., Elbeshausen, D., 2009. The Carancas meteorite impact crater, Peru: Geologic surveying and modeling of crater formation and atmospheric passage. Meteoritics & Planetary Science, 985–1000.

Krohn, K., Jaumann, R., Elbeshausen, D., Kneissl, T., Schmedemann, N., Wagner, R., Voigt, J., Otto, K., Matz, K.D., Preusker, F., Roatsch, T., Stephane, K., Raymond, C.A., Russell, C.T., 2014. Asymmetric craters on Vesta: Impact on sloping surfaces. Planet. Space Sci. 103, 36–56.

Melosh, H.J., 1989. Impact Cratering. Oxford University Press.

Melosh, H.J., 2011. Planetary Surface Processes. Cambridge University Press.

Neish, C.D., Madden, J., Carter, I.M., Hawke, B.R., Giguerre, T., Bray, V.J., Oskinia, G.R., Cahill, J.T.S., 2014. Global distribution of lunar impact melt flows. Icarus 239, 105–117.

Nelson, E.L., Katsuragi, H., Mayor, P., Durian, D.J., 2008. Projectile Interactions in Granular Impact Cratering. Phys. Rev. Lett. 101 , 068001–4.

Nishida, M., Okumura, M., Tanaka, K., 2010. Effects of density ratio and diameter ratio on critical incident angles of projectiles impacting granular media. Granular Matter 12, 337–344.

Plescia, J.B., 2012. Impacts on Sloping Surfaces: Lunar Examples. Meteoritics and Planetary Science Supplement 75, 5318.

Richardson, J., Melosh, H., Greenberg, R., Obrien, D., 2005. The global effects of impact-induced seismic activity on fractured asteroid surface morphology. Icarus 179, 325–349.

Richardson, J.E., 2004. Impact-Induced Seismic Activity on Asteroid 433 Eros: A Surface Modification Process. Science 306, 1526–1529.

Schmidt, R.M., 1980. Meteor Crater: Energy of formation - Implications of centrifuge scaling, in: Bedini, S.A. (Ed.), Lunar and Planetary Science Conference Proceedings, pp. 2099–2128.

Seguin, A., Bertho, Y., Gondret, P., 2008. Influence of confinement on granular penetration by impact. Phys. Rev. E 78, 010301–4.

Seguin, A., Bertho, Y., Gondret, P., Crassous, J., 2009. Sphere penetration by impact in a granular medium: A collisional process. EPL 88, 44002–7.

Soderblom, L.A., 1970. A model for small-impact erosion applied to the lunar surface. J. Geophys. Res., 2655–2661.

Takazawa, S., Niita, H., Tanabe, T., Nishimori, H., Katsuragi, H., 2019. Impact-induced collapse of an inclined wet granular layer. Physica D: Nonlinear Phenomena 386-387, 8–13.

Tsui, D., Otsuki, M., Katsuragi, H., 2018. Relaxation Dynamics of a Granular Pile on a Vertically Vibrating Plate. Phys. Rev. Lett. 120, 128001.

Uehara, J.S., Ambrosio, M.A., Ohja, R.P., Durian, D.J., 2003. Low-Speed Impact Craters in Loose Granular Media. Phys. Rev. Lett. 90, 194301–4.

Walsh, A.M., Holloway, K.E., Habdas, P., de Bruyn, J.R., 2003. Morphology and Scaling of Impact Craters in Granular Media. Phys. Rev. Lett. 91, 104301–4.

Wang, D., Ye, X., Zheng, X., 2012. The scaling and dynamics of a projectile obliquely impacting a granular medium. The European Physical Journal E 35, 7–12.

Wünnemann, K., Collins, G.S., Melosh, H.J., 2006. A strain-based porosity model for use in hydrocode simulations of impacts and implications for transient crater growth in porous targets. Icarus 180, 514–527.

Yamada, T.M., Ando, K., Morota, T., Katsuragi, H., 2016. Timescale of asteroid resurfacing by regolith convection resulting from the impact-induced global seismic shaking. Icarus 272, 165–177.

Zhu, R., Zhang, Q., Tjugito, H., Cheng, X., 2015. Granular impact cratering by liquid drops: Understanding raindrop imprints through an analogy to asteroid strikes. Proceedings of the National Academy of Sciences 112, 342–347.