Non-singlet baryons in gauge/ gravity duality

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Corfu, September 2012
Motivation:

Non-singlet baryons are predicted in N=4 SYM by the AdS/CFT correspondence.

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Analyze the holographic description in various backgrounds with reduced supersymmetries and/or confining
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Do they also exist in more realistic theories?

How:

Analyze the holographic description in various backgrounds with reduced supersymmetries and/or confining

- Reduced SUSY: $AdS_5 \times Y_5$, Lunin-Maldacena $\beta$ deformed, Frolov multi-$\beta$ deformed

- Confining: Maldacena-Nuñez
Results:

- Non-singlet baryons exist in all these backgrounds
- Same number of quarks in all $AdS_5 \times Y_5$ Einstein manifolds with 5-form flux, independent of SUSY
- More restricted number of quarks in MN
- Stable against fluctuations
- Non-singlet baryons at finite ‘t Hooft coupling
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(Based on arXiv:1203.6817, D. Giataganas, Y.L., M. Picos, K. Siampos, JHEP)
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Baryon vertex in the gravity side: D5-brane wrapped on the 5-sphere (Witten’98):

$$S_{CS} = 2\pi \, T_5 \int_{\mathbb{R} \times S^5} P[F_5] \wedge A = N \, T_{F_1} \int_{\mathbb{R}} dt A_t$$
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$N$ charge cancelled by $N$ F-strings ending on the 5-brane
The strings stretching to the boundary behave as fermions, giving the antisymmetry of the baryon vertex.
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In $AdS_5 \times S^5$: $5N/8 \leq k \leq N$

In $AdS_4 \times CP^3$: $2N/3 \leq k \leq N$ (Y.L., Picos, Sfetsos, Siampos’11)
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\begin{align*}
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What happens in less supersymmetric and/or confining backgrounds?
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What happens in less supersymmetric and/or confining backgrounds?

And at finite ‘t Hooft coupling?
2. Gauge/gravity calculation of the energy

(Brandhuber, Itzhaki, Sonnenschein, Yankielowicz’98; Imamura’98; Maldacena’98)

Consider a uniform distribution of strings on an $M_p$ shell Non-SUSY but we can ignore the backreaction

In the probe brane approach: $S = S_{Dp} + S_{NF1}$:

$$S_{NF1} = -N T_{F1} \int dt \ dr \sqrt{|\det P(G)|}$$

with $\tau = t$ $\sigma = r$ and the AdS direction $\rho = \rho(r)$
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\[ S_{Dp} = -T_p \int_{\mathbb{R} \times M_p} d^{p+1} \xi \sqrt{|\det P(G + 2\pi F - B)|} \]

\textbf{In } AdS_5 \times Y_5:\]

\[ ds^2 = \frac{\rho^2}{R^2} dx_{1,3}^2 + \frac{R^2}{\rho^2} d\rho^2 + R^2 ds_{Y_5}^2 \]

\[ R^4 = \frac{4\pi^4 N g_s}{\text{Vol}(Y_5)} , \quad F_5 = 4 R^4 (1 + \ast) d\text{Vol}(Y_5) \]

\textbf{Bulk equation of motion:} \[ \frac{\rho^4}{\sqrt{\frac{\rho^4}{R^4} + \rho'^2}} = c \]

\textbf{Boundary equation of motion:} \[ \frac{\rho_0'}{\sqrt{\frac{\rho_0^4}{R^4} + \rho_0'^2}} = \frac{T_5 R^4 \text{Vol}(Y_5)}{N T_{F_1}} \]
Define \( \sqrt{1 - \beta^2} = \frac{T_5 R^4 \text{Vol}(Y_5)}{N T_{F1}} \) \text{ with } \beta \in [0, 1]

The two equations can be combined into:

\[
\frac{\rho^4}{\sqrt{\frac{\rho^4}{R^4} + \rho'^2}} = \beta \rho_0^2 R^2
\]

Integrating: \textbf{Size of the configuration:}

\[
L = \frac{R^2}{\rho_0} \int_1^\infty dz \frac{\beta}{z^2 \sqrt{z^4 - \beta^2}}
\]

\textbf{On-shell energy:}

\[
E = E_{Dp} + E_{NF1} = N T_{F1} \rho_0 \left( \sqrt{1 - \beta^2} + \int_1^\infty dz \frac{z^2}{\sqrt{z^4 - \beta^2}} \right)
\]
Binding energy:

\[ E_{\text{bin}} = NT_{F1} \rho_0 \left( \sqrt{1 - \beta^2} + \int_{1}^{\infty} dz \left[ \frac{z^2}{\sqrt{z^4 - \beta^2}} - 1 \right] - 1 \right) \]

where we have subtracted the energy of the constituents (when the brane is located in \( \rho_0 = 0 \) the strings become radial and correspond to free quarks)
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As a function of \( L \):

\[ E_{\text{bin}} = -f(\beta) \frac{\sqrt{\lambda}}{L} \]

with \( f(\beta) \geq 0 \)
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As a function of \( L \):

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E_{\text{bin}} = -f(\beta) \frac{\sqrt{\lambda}}{L} \quad \text{with} \quad f(\beta) \geq 0
\]

- The configuration is stable
- \( E_{\text{bin}} \sim 1/L \) dictated by conformal invariance
- \( E_{\text{bin}} \sim \sqrt{\lambda} \) non-trivial prediction for the non-perturbative regime of the gauge theory
Universal behavior for all $AdS_5 \times Y_5$ backgrounds, independent of SUSY
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Same thing in beta deformed LM backgrounds and multi beta deformed (Frolov) backgrounds (non-SUSY)

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In $AdS_4 \times CP^3$: $E_{\text{bin}} = -f(\beta) \frac{\sqrt{\lambda}}{L}$ with a different $f(\beta)$

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Confining background?

Maldacena-Nuñez: $E_{\text{bin}} \sim L$

(Loewy, Sonnenschein’01)
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Non-singlets?
3. Reduce the number of quarks

In $AdS_5 \times S^5$: Baryon vertex classical solutions with number of quarks $5N/8 \leq k \leq N$ (non-singlet) (Brandhuber, Itzhaki, Sonnenschein, Yankielowitz’98; Imamura’98)

Stable against fluctuations for $0.813N \leq k \leq N$ (Sfetsos, Siampos’08)
3.1. The classical solution

The boundary equation of motion changes:

\[
\frac{\rho'_0}{\sqrt{\frac{\rho_0^4}{R^4} + \rho'_0^2}} = \frac{T_5 R^4 \text{Vol}(Y_5)}{k T_{F1}} + \frac{N - k}{k} \leq 1
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\[\Rightarrow 5N/8 \leq k \leq N.\]
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Same analysis in Maldacena-Nuñez: \( 3N/4 \leq k \leq N \)
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\[ \Rightarrow \quad \text{Non-singlet states in non-SUSY or confining backgrounds} \]
3.2. Stability analysis

Important in establishing the physical parameter space (Avramis, Sfetsos, Siampos’06-08)
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Ansatz for the fluctuations (for the strings):

$$\delta x^\mu (t, \rho) = \delta x^\mu (\rho) e^{-i\omega t} \quad \text{for} \quad x^\mu = r, \theta, \phi$$
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Expand the Nambu-Goto action to quadratic order and study the zero mode problem $\leftrightarrow$ Critical curve in the parametric space separating the stable and unstable regions

Stability reduced to an eigenvalue problem of the general Sturm-Liouville type

Instabilities emerge from longitudinal fluctuations of the strings
For $AdS_5 \times Y_5$ and beta deformed:

Bound for the number of F-strings coming from stability:

$$k \geq \frac{N}{1 + \gamma_c} \left(1 + \sqrt{1 - \beta^2}\right) \quad \gamma_c = 0.538$$

More restrictive than the bound imposed by the existence of a classical solution:

$$k \geq \frac{N}{2} \left(1 + \sqrt{1 - \beta^2}\right)$$

For MN: Same bound for stability and existence of classical sol.
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For MN: Same bound for stability and existence of classical sol.

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Can we reach the finite ‘t Hooft coupling region?
5. Baryons at finite ‘t Hooft coupling

Generalize the baryon vertex adding a magnetic flux.

A non-trivial flux adds lower dim brane charges →

Complementary description of the baryon in terms of D1-branes expanding by Myers dielectric effect
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- Description in terms of the expanded brane (macroscopical)
  valid in the sugra limit: \[ R >> 1 \iff \lambda >> 1 \]
- Micro when \[ \frac{4\pi R^2}{n} << l_s^2 \iff \lambda << n^2 \]
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- Micro when $\frac{4\pi R^2}{n} << l_s^2 \iff \lambda << n^2$

$\Rightarrow$ It allows to explore the region of finite $\lambda$

- Complementary at finite $n$. Should agree at large $n$
The magnetic flux modifies the dynamics:
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In $AdS_5 \times Y^{p,q}$:

$$ds^2 = \frac{\rho^2}{R^2} dx_{1,3}^2 + \frac{R^2}{\rho^2} d\rho^2 + R^2 ds_{Y^{p,q}}^2$$

$$R^4 = \frac{4\pi^4 N g_s}{Vol(Y^{p,q})}$$

$$ds_{Y^{p,q}}^2 = ds^2(M_4) + \left(\frac{1}{3} d\psi + \sigma\right)^2$$

$$k^\mu = \delta^\mu_\psi : \text{Reeb vector}$$

$$M_4 : \text{Local Kähler-Einstein metric}$$
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$$k^\mu = \delta^\mu_\psi : \text{Reeb vector}$$

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Take $F = N J$ with $J = \frac{1}{2} d\sigma$ the Kähler form:

$$\sqrt{1 - \beta^2} = \frac{T_5 R^4 \text{Vol}(Y^{p,q})}{NT_F F_1} \left(1 + \frac{4\pi^2 N^2}{R^4}\right)$$

$$\beta \leq 1 \Rightarrow \mathcal{N}_{\text{max}} \leftrightarrow \beta = 0$$
Binding energy:

\[ E_{\text{bin}} = N T_{F1} \rho_0 \left( \sqrt{1 - \beta^2} + \int_1^\infty dz \left[ \frac{z^2}{\sqrt{z^4 - \beta^2}} - 1 \right] - 1 \right) \]

- \( E_{\text{bin}} \) negative and decreases monotonically with \( \beta \)
- \( E_{\text{bin}} = 0 \) for \( \beta = 0 \) (N free radial strings stretching from \( \rho_0 \) to \( \infty \) plus a Dp-brane at \( \rho_0 \))

\[ N_{\text{max}} \sim \sqrt{\lambda} \]

\[ E_{\text{bin}} = -f(\beta) \frac{\sqrt{\lambda}}{L} \]
Non-singlets:

The bound for the number of quarks is modified (parameter space bounded by the values for which the baryon vertex reduces to free quarks):

\[ k \geq \frac{5}{8}N + \frac{N^2}{8\pi^2} \text{Vol}(Y^{p,q}) \]
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\]

\(k_{\text{min}}\) is maximum for the 5-sphere, the most SUSY case

\[
\pi^3 = \text{Vol}(S^5) > \frac{16}{27}\pi^3 = \text{Vol}(T^{1,1}) > \text{Vol}(Y^{2,1}) \approx 0.29\pi^3
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\(Y^{p,q}\) with largest volume
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Spike solutions non-SUSY for \(T^{1,1}, Y^{p,q}\) and MN

(Areán, Crooks, Ramallo’04; Canoura, Edelstein, Pando Zayas, Ramallo, Vaman’05; Imamura’04)
Important to obtain information at finite ‘t Hooft coupling.
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Micro description:
D1’s expanding into a fuzzy $M_4$. Only known for the $S^5$ (Janssen, Y.L., Rodriguez-Gomez’06) and the $T^{1,1}$.
The fuzzy $T^{1,1}$

The $T^{1,1}$ is a U(1) bundle over $S^2 \times S^2 \Rightarrow$ Take the D1’s wrapped on the U(1) fibre and expanding into a $S_{\text{fuzzy}}^2 \times S_{\text{fuzzy}}^2$
The fuzzy $T^{1,1}$

The $T^{1,1}$ is a U(1) bundle over $S^2 \times S^2$ ⇒ Take the D1’s wrapped on the U(1) fibre and expanding into a $S^2_{\text{fuzzy}} \times S^2_{\text{fuzzy}}$

Substituting in Myers action for D1-branes:

$$S_{DBI} = - \int \text{STr} \left\{ e^{-\phi} \sqrt{\left| \text{det} \left( P [E_{\mu\nu} + E_{\mu i} (Q^{-1} - \delta)^i_j E^{jk} E_{k\nu}] \right) \right|} \right\}$$

$$Q^i_j = \delta^i_j + \frac{i}{2\pi} [X^i, X^k] E_{kj}$$
The fuzzy $T^{1,1}$

The $T^{1,1}$ is a U(1) bundle over $S^2 \times S^2 \Rightarrow$ Take the D1's wrapped on the U(1) fibre and expanding into a $S^2_{\text{fuzzy}} \times S^2_{\text{fuzzy}}$

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$$E_n D_1 = \frac{N \rho_0}{8\pi} \frac{(m + 1)^2}{m(m + 2)} \left( 1 + \frac{36\pi^2 m(m + 2)}{R^4} \right)$$
The fuzzy $T^{1,1}$

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$$E_{nD1} = \frac{N \rho_0}{8\pi} \frac{(m + 1)^2}{m(m + 2)} \left( 1 + \frac{36\pi^2 m(m + 2)}{R^4} \right)$$

where $X^i = \frac{1}{\sqrt{m(m + 2)}} J^i$ for each $S^2$
The fuzzy $T^{1,1}$

The $T^{1,1}$ is a U(1) bundle over $S^2 \times S^2$ \Rightarrow Take the D1's wrapped on the U(1) fibre and expanding into a $S^2_{\text{fuzzy}} \times S^2_{\text{fuzzy}}$

Substituting in Myers action action for D1-branes:

$$S_{DBI} = - \int \text{STr} \left\{ e^{-\phi} \sqrt{|\det \left( P[E_{\mu\nu} + E_{\mu i}(Q^{-1} - \delta)^i_j E^{jk} E_{k\nu}] \right) |} \det Q| \right\}$$

$$Q^i_j = \delta^i_j + \frac{i}{2\pi} [X^i, X^k] E_{k j}$$

$$E_{nD1} = \frac{N \rho_0}{8\pi} \frac{(m + 1)^2}{m(m + 2)} \left( 1 + \frac{36\pi^2 m(m + 2)}{R^4} \right)$$

where

$$X^i = \frac{1}{\sqrt{m(m + 2)}} J^i$$

Agreement with macro descrip. in large m
The F1 in the micro description

\[ S_{CS} = \int STr \left\{ P \left( e^{i \frac{2\pi}{i} (i_X i_X)} \sum_{q} C_q e^{-B_2} \right) e^{2\pi F} \right\} \]

includes

\[ S_{CS} = \frac{1}{\pi} \int dt STr \left[ (i_X i_X)^2 i_k F_5 \right] A_t \]
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\[ = N \frac{(m + 1)^2}{m(m + 2)} \int dt A_t \]
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The F1 in the micro description

\[ S_{CS} = \int S\text{Tr}\left\{ P\left( e^{\frac{i}{2\pi}(iX_iX)} \sum_q C_q e^{-B_2} \right) e^{2\pi F} \right\} \]

includes

\[ S_{CS} = \frac{1}{\pi} \int dt \ S\text{Tr}\left[ (iX_iX)^2 i_F i_{F_5} \right] A_t \]
\[ = N \frac{(m + 1)^2}{m(m + 2)} \int dt \ A_t \quad \rightarrow \quad N \int dt \ A_t \]

Similar micro analysis in MN in terms of D1’s expanding into a fuzzy \( S^2 \)
6. Conclusions

- Non-singlet baryons are predicted by gauge/gravity duality in less supersymmetric and/or confining backgrounds

- They are stable against fluctuations

- At finite ‘t Hooft coupling:
  
  Microscopical description of the vertex

  Complete this analysis with $\alpha'$ corrections to the NG action of the strings (or microscopic spike), $\alpha'$ corrections to the background
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Thanks!