Analysing categories of mathematical proficiency based on Kilpatrick opinion in junior high school

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Abstract. There are at least five mathematical proficiencies that student needs to have to be successful in learning math. The five proficiencies are: (1) conceptual understanding; (2) procedural fluency; (3) strategic competence; (4) adaptive reasoning; and (5) productive disposition. The math teacher has to facilitate student to develop these five mathematical proficiency. The purpose of this study is to find out how mathematical proficiency of students in learning to problem solving in public junior high school at North Bandung. The research method used is descriptive research using qualitative approach. The subject in this research was 10 students from 8 classes in junior high schools, in North Bandung, West Java. The instruments used in this research were mathematical proficiency test and interviews. The result of the research can be concluded is that mathematical proficiency in public junior high school students grade VIII at North Bandung are good, that can be seen from the result of the students’ answers that bring out 21 out of 25 categories of mathematical proficiencies. The strategy mostly used by students is a trial and error strategy.

1. Introduction

Students need a lot of things to be successful in learning math. Professional mathematics teacher, a mathematics curriculum that gives students the opportunity to learn important and meaningful mathematical concepts and procedures, interactive mathematics classes which allows each student to have exciting experiences and challenging learning mathematics, and a number of mathematical proficiency that must be had, are some of the many important things that students need to succeed in learning math.

Professional mathematics teachers are required by students as facilitators in achieving optimal performance. Teacher competence includes pedagogic competence, professional competence, personal competence, and social competence. By having these four competencies a teacher will be able to guarantee every student to be able to study and learn from each other in order to achieve their optimal achievement.

The purpose of learning mathematics according to KTSP as written in the Minister of National Education Regulation \cite{2} is for students to have a standard of mathematical proficiency, such as conceptual understanding, procedural fluency, strategic competence, adaptive reasoning and...
productive disposition. The curriculum is a very important element in teaching and learning activities, but a curriculum should be more than just a set of activities. The mathematics curriculum must be coherent, focused on important mathematics, and well-articulated from a grade level to the next class level (NCTM) [6]. This means that the selection of materials, sequences and presentation approaches, as well as instruments for measuring student learning outcomes, should be considerate to the characteristics, needs, and levels of student's thinking.

The interactive math class that allows each student to gain information, exciting and challenging mathematics learning must be the class that every student desires. In such a class, each student will be involved in the entire teaching-learning process designed by the teacher with high enthusiasm, from the first minute to the last minute. There is no boredom, feeling lazy, lacking confidence, or giving up before it works.

Besides the factors outside the student's, such as teachers, curriculum, and class, there is also an important factor in the students that are very important in supporting the success of learning mathematics. The factor is mathematical proficiency. “The first key component of mathematical proficiency is the ability to understand, use, and as necessary, create definitions.” Milgram [5]. Asmara [1] said that “To have the ability think critically, creatively, logically, and systematically students must have mathematical proficiency”

Milgram's opinion is accord with Kilpatrick, et. al. [4] “Mathematical proficiency, as we see it, has five strands: (1) conceptual understanding—comprehension of mathematical concepts, operations, and relations; (2) procedural fluency—skill in carrying out procedures flexibly, accurately, efficiently, and appropriately; (3) strategic competence—ability to formulate, represent, and solve mathematical problems; (4) adaptive reasoning—capacity for logical thought, reflection, explanation, and justification; (5) productive disposition—capacity for logical thought, reflection, explanation, and justification; (5) productive disposition—habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one’s own efficacy.” This mathematical skill should be developed in an integrated and balanced way for students who are learning math.

These five mathematical proficiency must be coherent, as Kilpatrick, et. al. [4] have noted, "Mathematical proficiency is not a one-dimensional trait, and it cannot be achieved by focusing on just one or two of these strands." These five strands of mathematical proficiency are not separate, but intertwined into one that represents different aspects of something complex. A similar statement is expressed by Herman [3] who says that generally mathematical competence (mathematical proficiency) is not a separate domain, but the five are intertwined strands of proficiency that strengthen each other's knowledge, skills, abilities and beliefs of a person.

This mathematical proficiency is not a mere "congenial" skill of the student, but a combination of students' knowledge, skills, abilities and beliefs with the help or support of teachers, curricula, and a reliable (class) learning environment. Therefore, the students' mathematical proficiency needs to be improved and developed, among others, through the assistance or support of their math teachers.

2. Mathematical Proficiency
This paper will focus on what is meant by conceptual understanding, procedural fluency, strategic competence, adaptive reasoning and productive disposition. The description of the links between these five components is described by Kilpatrick, et. al. [4] as follows.

The explanations for each component of the mathematical proficiency are as follows: Conceptual understanding is the understanding or mastery of student’s concepts, operations, and mathematical relationships. Indicators that can be used to find out whether a student has conceptual understanding are able to: (a) reiterated the concept that has been learned, (b) classify objects based on whether or not the requirements are forming the concept, (c) give examples or non-examples of learned concepts, (d) presenting concepts in various forms of mathematical representation, (e) linking concepts, and (f) developing the necessary conditions and/ sufficient terms of a concept.

According to Kilpatrick, et. al. [4] a significant indicator of conceptual understanding is the ability to present the mathematical situation in different ways and to know how different representations can be useful for various purposes.
Figure 1. Intertwined Strands of Proficiency

The level of students’ conceptual understanding relates to the wealth and breadth of connections they can make. Ways are done to support the occurrence of conceptual understanding such as mathematical modeling, mathematical vocabulary, and concept mapping.

Procedural fluency refers to the knowledge of the procedure, when and how to use it appropriately, and the skills of performing the procedure flexibly, accurately and efficiently. Therefore, the indicators for the continuity of this procedure include the students being able to: (a) use the procedure, (b) utilize the procedure, (c) select procedures, (d) estimate the outcome of a procedure, (e) modify or refine the procedure, (f) develop procedures. By studying the algorithm as a general procedure, students can gain knowledge that mathematics is structured and can solve routine problems.

Strategic competence refers to the ability to formulate, present, and solve math problems. Therefore, an indicator to know whether a student has strategic competence is if student’s is able to: (a) understand the problem by explaining what is known and what is asked, (b) presenting a problem mathematically in various forms (numerical, symbolic, verbal, or graphical), (c) choosing the right formula, approach or method for solving the problem, and (d) checking the correctness of the problem solved. Kilpatrick, et. al [4] argues that the fundamental characteristic required during the troubleshooting process is flexibility. Flexibility of a person’s can develop through the extension of the knowledge necessary to solve problems that are not routine. For example, a sixth grader will be asked to answer questions about school cafeteria topics. Some may be asked if lunch in the cafeteria is too expensive or the most favorite lunch in the cafeteria. Others may ask how many trays are used or how many cartons of milk are sold. Others may ask how the canteen's layout can be repaired. With the problem formulated, the first step of the student in solving the problem is to make the representation mathematically in several modes, either numerically, symbolically, orally, or graphically.

Adaptive reasoning refers to the capacity to think logically about relationships between concepts and situations, the ability to think reflectively, the ability to explain, and the ability to justify. Indicators for this skill include, if the student is able to: (a) make conjectures, (b) give reasons or evidence to the truth of a statement, (c) draw conclusions from a statement, (d) check the validity of an argument, and (e) finding patterns on a mathematical phenomenon.

The productive disposition deals with the tendency to have productive habits, to see mathematics as possible, useful, meaningful, and valuable, and to have the confidence and persistence in learning mathematics. Therefore, the indicators for this productive disposition include students in learning mathematics: (a) excited, (b) not easily give up, (c) confidence, and (d) have curiosity. As Kilpatrick, et. al [4] argues that a student with a high productive disposition tends to be able to develop their mathematical proficiency in terms of conceptual understanding, procedural fluency, strategic competence, and adaptive reasoning. Otherwise, those who have skills in conceptual understanding, procedural fluency, strategic competence, and adaptive reasoning tend to develop their productive dispositions.

It can be seen that the development of these five components of mathematical proficiency cannot be emphasized on only one component because they are all one.
3. Research Methods and Designs

The method that is used to analyze the mathematical proficiency of public junior high school students is descriptive research with qualitative approach. Descriptive research is a study to describe a phenomenon, events and fact that occur by focusing on actual problems as they are when the research takes place without giving special treatment to the event. Relationship between descriptive research with qualitative approach is because it describes the research activities conducted on certain objects clearly and systematically, by exploring to explain and predict a symptom that occurs on the basis of qualitative data obtained in the field.

The subject in this research was 10 students from 8 classes in junior high schools, in North Bandung, West Java, selected by applying purposive sampling technique. This technique is chosen with a specific purpose and reason based on the research. Participants to be interviewed and observed in this study were selected because they were the people who were directly involved with the research studied in the school.

The focus of the case study in this research is the students of class VIII, one of the public junior high school in North Bandung who is assumed to have mathematical proficiency because in the public junior high school has applied various models and approaches to innovative and student centered learning. The applied learning model/approach involve actively as a subject in learning, and are considered appropriate in an effort to improve students' mathematical proficiency. The student has good grades in the lessons other than mathematics, this can be seen from the results of student report cards in the previous semester and the result of the observations from the mathematics teacher at the school. Result of student answer sheet will be analyzed by using mathematical proficiency indicator with score one if there is in student answer.

The instruments used in this research were mathematical proficiency test and interviews. Mathematical proficiency test uses essay form that consists of 5 questions. Indicators which covered in the test are: (1) conceptual understanding—comprehension of mathematical concepts, operations, and relations; (2) procedural fluency—skill in carrying out procedures flexibly, accurately, efficiently, and appropriately; (3) strategic competence—ability to formulate, represent, and solve mathematical problems; (4) adaptive reasoning—capacity for logical thought, reflection, explanation, and justification; (5) productive disposition—habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one’s own efficacy.

4. Results and Discussion

One example of the results is as follows:

![Figure 2. First Student Subject Test](image-url)
From the written data presented in the figure obtained: (a) restate the concept that has been studied correctly, (b) classify the objects based on whether or not the requirements of forming the concept correctly, (c) provide examples of the concepts studied correctly, (d) present the concept in various forms of mathematical representation correctly, (e) connect the concepts correctly, and (f) develop the necessary terms and/or requirements adequately concepts correctly.

Based on the description of data analysis above and associated with mathematical proficiency indicators it can be concluded that the students reach level 6 with complete and correct category.

**Figure 3. Results of Second Subject Student Test**

From the written data presented in the figure obtained: (a) re-state the concept that has been studied correctly, (b) classify the objects based on whether or not the requirements of forming the concept are almost correct, (c) provide examples of the concepts studied correctly, (d) present concepts in various forms of mathematical representation correctly, (e) associate nearly true concepts, and (f) develop the necessary terms and/or requirements adequately of a concept correctly.

Based on the description of the data analysis above and associated with Mathematical Proficiency Indicators it can be concluded that the students reach level 5 with almost complete and correct category. Description of the results of students' mathematical proficiency obtained from the six items can be written into percentages as shown in the table below:

**Table 1. Results of Analyzing Mathematical Proficiency**

|                                | Maximum Score | Student Score |
|--------------------------------|---------------|---------------|
| Conceptual Understanding       | 6             | 6             |
| Procedural Fluency             | 6             | 6             |
| Strategic Competence           | 4             | 4             |
| Adaptive Reasoning             | 5             | 4             |
| Productive Disposition         | 4             | 1             |

Based on Table 1 above we can see that category of mathematical understanding, procedural fluency and strategic competence, the indicator that show when students completed the mathematical proficiency is 100%. All indicators of mathematical understanding, procedural fluency, and strategic competence arise when students solve problems. Judging from the analysis of data obtained this shows that students are able to solve problems correctly this is seen from the indicators that appear when students solve the problem.

For the category of adaptive reasoning, the indicator that arises when students complete the problem of mathematical proficiency is 80%. In adaptive reasoning indicators that appears only 4 indicators out of 5.

Lastly on the productive disposition of indicators that appear when students solve the problem of mathematical proficiency only 25%. This means there is only 1 indicator that appears when students solve the problem of mathematical proficiency. So it can be concluded that student's difficulty in solving the problem of mathematical proficiency with indicators of productive disposition.
5. Conclusions
The conclusions can be made about this research is as follows: 1). The mathematical proficiency of grade VIII, students of public junior high school in North Bandung are good, as evidenced by the students' ability to produce 21 out of 25 predefined mathematical proficiency indicators. The occurrence of the indicator varies among each student; 2). From five strands of mathematical proficiency, the productive disposition, especially in the excited, not easily gives up, confident, and curiosity indicator is a difficult skill the students realize when completing the matter of mathematical proficiency; 3). There are various strategies used by students in solving mathematical proficiency on each completed question, example reverse working strategy, creating tables, drawing, experimenting, and making open sentences. However, the strategy most often used by students is a trial and error strategy. Students had difficulties when solving the problem of mathematical proficiency: (a) some students have difficulty in understanding the problems. This is because the ability to understanding the problems still lack, (b) the students have difficulty in choosing the strategy that will be used to solve the problem, (c) the students have difficulty in implementing the solution plan, it is because the students misinterpret the problem condition, (d) students have difficulty in bringing alternative answers in different ways to the problem.

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