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Abstract—Lagrangian descriptions of integer HS representations of the Poincare group subject to a Young
tableaux with two columns are constructed within a metric-like formulation in a d-dimensional flat
space-time on a basis of the BRST approach. A Lorentz-invariant resolution of the BRST complex within
BRST formulations produces a gauge-invariant Lagrangian in terms of the initial tensor field subject
to with an additional tower of gauge parameters realizing the (  - 1)-th stage reducible theory
with a specific dependence on the value . Minimal BRST–BV action is suggested, pro-
viding objects appropriate to construct interacting models with mixed-antisymmetric fields in a general
framework.

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1. INTRODUCTION

Some topical problems of high-energy physics are
related to higher-spin (HS) field theory, being part of
the LHC experiment program. The so-called tension-
less limit of (super)string theory [1], operating an
infinite tower of HS fields with integer and half-inte-
ger generalized spins, incorporates HS field theory
into superstring theory and turns it into a method of
studying the classical and quantum structure of the
latter (for the current status of HS field theory, see the
reviews [2, 3, 4]). This paper examines the construction
of Lagrangian formulations (LFs) and so-called
BRST–BV master action in the minimal sector of the
field-antifield formalism for free integer mixed-anti-
symmetric (MAS) tensor HS fields in a flat -space-time
subject to an arbitrary Young tableaux (YT) with

Irreducible Poincare or (anti)-de-Sitter (A)dS


group representations in constant curvature space-
times may be described by mixed-symmetric (MS) HS
fields subject to an arbitrary YT with rows, 

(antisymmetric basis), determined

time may be described by mixed-symmetric (MS) HS
fields subject to an arbitrary YT with rows, 

and, equivalently, by MAS (spin-)tensor fields subject
to an arbitrary YT, albeit with columns, 

antisymmetric basis), with integers (8, 9) or half-inte-\ngers (antisymmetric basis), having a spin-like interpretation.

MS and MAS HS fields arise in d > 4 space-time
dimensions in addition to totally symmetric and anti-
symmetric irreducible representations of the Poincare
and (A)dS algebras. For these latter, as well as for MS
HS fields, LFs for massless and massive free higher-
spin fields are well-developed (see, refs. in [8]),
[10‒12, 18], including the BRST–BFV approach,
e.g., in [13–17]. For MAS, the problem of field-theo-
retic description has not been solved.

The paper is organized as follows. In Section 2, we
remind the key points of finding BRST–BFV
Lagrangian formulations for MAS HS fields. In Section 3 we
find gauge-invariant Lagrangians in tensor forms. The construction of a minimal BV action and
the possibility to deform its structure by non-qua-
dratic interacting terms with appropriate HS fields in
the BRST–BFV approach are briefly examined in
Section 4.

We use the convention \( \eta_{\mu\nu} = \text{diag}(+,-,\ldots,-) \) for
the metric tensor, with \( \mu, \nu = 0,1,\ldots,d-1 \), the nota-
tion \( \epsilon(A) \), \( [gh_H, gh_L, gh_{tot}](A) \) for the respective values
of Grassmann parity, BFV, \( gh_H \), BV, \( gh_L \) ghost num-ers of a quantity \( A \). The supercommutator \([A, B]\) of
quantities \( A, B \) with definite values of Grassmann par-
ity is given by \( [A, B] = AB - (-1)^{\epsilon(A)\epsilon(B)} BA \).
2. BRST-BFV LAGRANGIAN FORMULATIONS

Recall [8] that a massless integer-spin irreducible representation of the Poincare group in Minkowski space $\mathbb{R}^{1,d-1}$ is described by a rank-$ (\hat{s}_1 + \hat{s}_2) $ tensor field $\Phi_{[b_1]_{i_1}^{l_1}b_2^{l_2}} = \Phi_{[b_1_{\mu_1}^{l_1},a_1^{a_1}]^{i_1}}_{[b_2_{\mu_2}^{l_2},a_2^{a_2}]}$ with generalized spin $s = (s_1, s_2, s_{s_1 + 1}, s_{s_2 + 1}, \ldots, s_1, s_2) = (2, 2, \ldots, 2, 1, \ldots, 1)$ (omitting the symbol “m” under $\hat{s}_1$ and $s_1 \geq s_2 > 0, s_1 \leq |d/2|$, subject to a YT, $Y[\hat{s}_1, \hat{s}_2]$) and 2 columns of heights $s_1, s_2$. The field satisfies differential equations (Klein–Gordon and divergentless ones) (1) and algebraic equations (traceless and mixed-antisymmetry ones) (2):

\[
\partial^\mu \partial_\mu \Phi_{[b_1]_{i_1}^{l_1}b_2^{l_2}} = 0, \quad (1)
\]

\[
\partial^{\mu i_1} \Phi_{[b_1]_{i_1}^{l_1}b_2^{l_2}} = 0, \quad \text{for } 1 \leq i_1 \leq s_1, \quad i = 1, 2,
\]

\[
\eta^{\mu_{i_1}i_2} \Phi_{[b_1]_{i_1}^{l_1}b_2^{l_2}} = 0, \quad \text{for } 1 \leq i_2 \leq s_2,
\]

\[
\Phi_{[b_1]_{i_1}^{l_1}b_2^{l_2}}^{\mu_1\mu_2} = 0,
\]

where the bracket means that the indices inside do not take part in antisymmetrization.

Equivalently, the relations

\[
(l_0, l, l_2, l_{12})|\Phi\rangle = \left(\partial^\mu \partial_\mu + i a^{a_1}_\mu \partial^{i_1} - \frac{1}{2} a^{a_1}_\mu a^{a_2}_\mu, a^{a_1}_\mu a^{a_2}_\mu\right)|\Phi\rangle = 0,
\]

\[
\sum_{a_1=0}^{[d/2]} \sum_{a_2=0}^{s_1} \sum_{i_1=1}^{s_1} \sum_{l_2=1}^{2} \sum_{s_2=0}^{s_2} \prod_{i_1=1}^{s_1} a^{\mu_1}_{i_1} |0\rangle,
\]

for all the integer spin MAS ISO(d, 1–1) group irreps with the help of a string-like vector $|\Phi\rangle \in \mathcal{H}_f$ in an auxiliary Fock space $\mathcal{H}_f$, generated by 2 pairs of fermionic oscillators $a^{i_1}_\mu(x), a^{i_1+}_\mu(x)$: $\{a^{i_1}_\mu, a^{i_1+}_\mu\} = -\eta^{\mu_1\mu_2}$ and $|\Phi\rangle = \left[\Phi_{[b_1]_{i_1}^{l_1}b_2^{l_2}}\right]$. To describe the single Poincare group irrep of spin $s = [s_1, s_2]$, we extend (3) by spin relations with the number particle operators $g^{i_i}_0$:

\[
g^{i_i}_0 |\Phi\rangle = \left(s_1 - \frac{d}{2}\right)|\Phi\rangle, \quad \text{for } g^{i_i}_0 = -\frac{1}{2} [a^{i_1+}_\mu, a^{i_1+}_\mu].
\]

The condition that BRST operator be hermitian leads to extending a set of constraints $|q_0\rangle = \{l_0, l, l_1, l_2, l_{12}\}$ by their hermitian conjugates with respect to the scalar product in $\mathcal{H}_f$:

\[
\langle \Psi | \Phi \rangle = \int d^d x \sum_{a_1=0}^{[d/2]} \sum_{a_2=0}^{s_1} \sum_{i_1=1}^{s_1} \sum_{l_2=1}^{2} \sum_{s_2=0}^{s_2} \prod_{i_1=1}^{s_1} a^{\mu_1}_{i_1} |\chi\rangle |\Phi|\langle \chi| |\Phi\rangle - \epsilon_j ne^{j} \langle \partial^{i_1} a^{a_1} \rangle \frac{1}{2} + \text{c.c.}
\]

\[
\text{with } \epsilon_j = -\epsilon_{j_2}, \epsilon_{j_2} = 1, \text{ a generalized spin operator } \sigma_i.
\]
The Lagrangian formulation in the unconstrained case for an HS field with given spin \([s_1, s_2] = [s]_2\) is determined by a gauge-invariant action and a sequence of \((s_1 + s_2)\)-stage reducible gauge transformations:

\[
\mathcal{S}_{[s_1]} = \int d\eta [\Phi^0] \left( K_{[s_1]} \mathcal{L}_{\Phi^0} [\Phi^0] \right),
\]

\[
\delta \chi^k = \left( K_{[s_1]} \mathcal{L}_{\delta \chi^k} [\chi^k] \right), \quad k = 0, \ldots, \sum s_i,
\]

with a non-gauge \(|\chi^{(i+1)}\rangle_{[s_1]}\), where \(|\chi^0\rangle_{[s_1]}\) = \(|\Phi\rangle_{[s_1]}\), \(|\chi^k\rangle_{[s_1]}\) \(=\) \(|\Phi\rangle_{[s_1]}\), and with the substitutions \((\mathcal{K}, \mathcal{Q})_{h_0 \rightarrow h(s)} = (K_{[s_1]}, Q_{[s_1]})\) made for \(h(s)\) which follows from:

\[
(\mathcal{P} | \chi^0 \rangle, \delta | \chi^k \rangle) = (0, \mathcal{P} | \chi^{k+1} \rangle (1 - \delta_k \sum_{s_{i+1}}) \quad \text{and} \quad [\sigma^i + h^i] | \chi^k \rangle = 0,
\]

\[
| \chi^k \rangle = \sum_{(n_i)_{a=0,|n_i|=0}}^\infty \eta^{n_0 \eta_1 \eta_{i+1}} \phi^{n_0 \eta_1 \eta_{i+1}} \cdot \phi^{n_0 \eta_1 \eta_{i+1}}
\]

\[
\times \prod_{i=1}^2 q_i^{n_0 \eta_1 \eta_{i+1}} \cdot \phi^{n_0 \eta_1 \eta_{i+1}}
\]

therefore \(h'(s) = s_i - \frac{d + 1 - \delta(m, 0) - (-1)'}{2}, (15)\),

in the massless and massive cases, with 2 pairs of additional odd oscillators in \((L_i, L_i^\dagger) = (l_i + mF^l, l_i^\dagger + mF_i^l)\).

In the constrained case, the only differential constraints compose a BRST complex, including a restricted BRST operator and a spin operator \((Q, \sigma') = (Q_{[s_1]}, \sigma')_{[s_1]} = 0\), without conversion and with off-shell BRST-extended traceless and MAS constraints,

which enter into the definition of a constrained LF of \(s_1 + s_2 - 1\)-stage of reducibility (for \(k = 0, \ldots, \sum s_i\)),

\[
S_{[s_1]} = \int d\eta [\Phi^0] \left( \chi^0 \mathcal{L}_{\chi^0} [\chi^0] \right),
\]

\[
(\delta l_{12} + \frac{1}{2} \epsilon_{e} \eta_{p} l_{e} l_{p} + q_{1} \eta_{1}^p + q_{1}^p \eta_{1}^p) \chi^k_{12} \right) = (Q, | \chi^k_{12} \rangle, 0).
\]

Note, without off-shell constraints, we have obtained from (16) a generalized triplet formulation, which describes reducible Poincare group representations with different spins.

### 3. METRIC-LIKE COMPONENT LAGRANGIANS

The deduction of component Lagrangians (for \(m = 0\)) entirely in terms of the initial HS tensor field \(\Phi_{[s_1, s_2, l_1]}\) is based on a partial gauge-fixing procedure for unconstrained LF (see [9] for details):

1. removing the dependence on the auxiliary oscillators \(b_i^+, i = 1, 2\) in \(\chi^k_{[s_1]}\) by means of the gauge transformations (13) resulting in the gauge conditions:

\[
(b_{2b}^+, b_{1b}^+, \lambda_{1b}^+, \lambda_{2b}^+, \chi^k_{12}) = 0, \quad k = 0, \ldots, \sum s_i;
\]

2. removing the dependence on the ghosts \(\theta_{1b}^+, \eta_{1b}^+\) in \(\chi^k_{[s_1]}\) via the residual gauge transformations (13) and the equations of motion, which leads to the vanishing of \(\chi^k_{12} = 0, l = s_1 + 1, \ldots, \sum s_i\) and to algebraic relations on the final field and gauge vectors, so that the surviving \(\chi^k_{12} = 0, k = 0, \ldots, s_i\) depend entirely on the single (restricted only by partial mixed symmetry) component tensor field.

Calculating the scalar products for the resulting Lagrangian action

\[
\mathcal{S}_{[s_1]} = (-1) \sum_{[s_1], [s_2]} \langle \Phi | \sum_{r=0}^{s_2} \frac{(-1)^r}{2^r} \frac{2^r}{r!} (r + 1)! | (l_{12})^r \rangle \left[ l_0 - \frac{r}{2} l_1 + \frac{1}{2} l_1^2 + \frac{2}{r + 2} l_{12}^2 l_{12}^2 \right] | \Phi \rangle_{[s_1]}
\]

and expressing the tensor components in the remaining tower of gauge transformations leads to the final LF for the field \(\Phi_{[s_1, s_2]}\) subject to \(Y_{[s_1, s_2]}\) of spin \([s]_2\):

\[
\mathcal{S}_{[s_2]}(\Phi) = \int d^d x \sum_{r=0}^{s_2} \frac{(-1)^r s_i s_j}{s_i - r} (s_i - r)^{r+1} \left( \mathcal{T} \mathcal{R}^\dagger \Phi \right)_{[s_1]_{12} - [s_2]_{12} - [s_i - r]_{12} - [s_j]_{12}} \left( \mathcal{R}^\dagger \mathcal{T} \Phi \right)_{[s_1]_{12} - [s_2]_{12} - [s_i - r]_{12} - [s_j]_{12}}
\]

\[
- (s_i - r) \left[ l_0 - \frac{r}{2} l_1 + \frac{1}{2} l_1^2 + \frac{2}{r + 2} l_{12}^2 l_{12}^2 \right] \left( \mathcal{T} \mathcal{R}^\dagger \Phi \right)_{[s_1]_{12} - [s_2]_{12} - [s_i - r]_{12} - [s_j]_{12}}
\]

\[
- (s_i - r - 1) \eta^{[s_1]_{12} - [s_2]_{12} - [s_i - r]_{12} - [s_j]_{12}} \left( \mathcal{T} \mathcal{R}^\dagger \Phi \right)_{[s_1]_{12} - [s_2]_{12} - [s_i - r]_{12} - [s_j]_{12}} \left( \mathcal{R}^\dagger \mathcal{T} \Phi \right)_{[s_1]_{12} - [s_2]_{12} - [s_i - r]_{12} - [s_j]_{12}}
\]
with the notation for a multiple trace, 
\begin{equation}
(\text{Tr}\Phi)_{[\mu_1,\ldots,\mu_{n-k}],[\nu_1,\ldots,\nu_k]} \equiv \Phi_{[\mu_1,\nu_1,\ldots,\nu_k],[\nu_1,\ldots,\nu_k]} \equiv \prod_{i_1=1}^{n-k} \Phi_{[\mu_1,\ldots,\mu_{i_1}],[\nu_1,\ldots,\nu_{k+i_1}]},
\end{equation}
and for the gauge-independent \(s_i\)-level gauge tensor parameter \(\phi_{s_i,1}^{n-k}\) in terms of 
\begin{equation}
[\phi_s(a^+)]_{[0,s_2]} \quad \text{with } s_2 = s_1 - l:
\end{equation}
\begin{equation}
\delta [\phi_s^{n-k}(a^+)]_{[k,s_1]} = -((s_1 - k + 1)\partial_{[k_i]} \phi^{n-k+l+1}_{s_i,1} + (-1)^k \partial_{[k_i]} (Y \phi^{n-k+l+1}_{s_i,1})_{[k_i],[s_i],[s_i],[l_i]]},
\end{equation}
\begin{equation}
\text{for } \begin{cases}
(Y_{s_2} \phi_s^{n-k})_{[l_i],[k_i],[s_i],[l_i]}) \in Y[s_2 + k,0], & k = 1,\ldots,l, \\
(Y_{s_1-k} \phi_s^{n-k})_{[l_i],[k_i],[s_i],[l_i]}) \in Y[s_1,s_2 - s_1 + k], & k = l+1,\ldots,s_1
\end{cases}
\end{equation}
where \((Y \phi_s^{n-k})_{[l_i],[k_i],[s_i],[l_i]}) = -s_2 \phi^{n-k}_{s_2,1},\) 

and antisymmetrization in \(\partial_{[l_i]} \phi^{n-k}_{s_1,1},\)
\begin{equation}
(Y \phi_s^{n-k})_{[l_i],[k_i],[s_i],[l_i]}) = \frac{1}{k} \partial_{[l_i]} (\phi^{n-k}_{s_1,1})_{[l_i],[k_i],[s_i],[l_i]})
\end{equation}
contains the factor \(1/k \) \((1/s_2)\) . The resulting LF is a gauge theory of 
\((s_1 - l)-1\)-th stage of reducibility, which describes the 
free dynamics of a massless Bose-particle of spin 
\([s_1,s_2] \), with the single off-shell restriction of Young 
symmetry on the field \(\Phi \equiv \phi^0\) and the gauge parameters \(\phi_1,\ldots,\phi^h\). If we derive a component LF without 
auxiliary tensor fields, starting from the constrained 
BRST LF \((16)\), we shall come to the same result 
\((18), (21)\).

4. MINIMAL BRST-BV ACTIONS
AND INTERACTING PROBLEM

For simplicity, we consider a component LF and introduce a total set of minimal ghosts and their anti-fields, according to the rule (for the vanishing value of \(gh_t\)):
\begin{equation}
\phi^{n-k}_{[l_i],[k_i],[s_i],[l_i]} \rightarrow C^{s_i-k}_{[l_i],[k_i],[s_i],[l_i]} : (\epsilon, gh_t) C^{n-k}_{[l_i],[k_i],[s_i],[l_i]},
\end{equation}
\begin{equation}
C^{s_i-k}_{[l_i],[k_i],[s_i],[l_i]} \rightarrow C^{s_i-k}_{[l_i],[k_i],[s_i],[l_i]} : (\epsilon, -gh_t) C^{n-k}_{[l_i],[k_i],[s_i],[l_i]},
\end{equation}
with an odd \(\Phi^0 = C^0_{[l_i],[k_i],[s_i],[l_i]} \equiv C^0_{[l_i],[k_i],[s_i],[l_i]} \quad \text{and } C^0 \equiv \phi^0 = \Phi\).
The minimal BRST–BV action
\begin{equation}
\delta [\phi_s^{n-k}(a^+)]_{[k,s_1]} \in Y[s_2 + k,0] \\
k = 1,\ldots,l,
\end{equation}
\begin{equation}
t_{l_2}^{s_2-k} \phi_s^{n-k}(a^+)]_{[k,s_1]} \in Y[s_1,s_2 - s_1 + k] \\
k = l + 1,\ldots,s_1,
\end{equation}
where the fact of belonging to the YT is understood for 
component tensor gauge parameters \(\phi^{n-k}_{[l_i],[k_i],[s_i],[l_i]} \) of rank
\((k + s_2) \) in \([\phi_s^{n-k}(a^+)]_{[k,s_1]} \) (with the structure being as in \((4)\)) after respective multiple application of the Young antisymmetrization realized by \(t_{l_2}^{s_2-k}\). In 
terms of the tensor relations we represent \((19), (20)\) as

\begin{equation}
\delta [\phi_s^{n-k}(a^+)]_{[k,s_1]} = (s_1 - k + 1)\partial_{[k_i]} \phi^{n-k+l+1}_{s_i,1} + (-1)^k \partial_{[k_i]} (Y \phi^{n-k+l+1}_{s_i,1})_{[k_i],[s_i],[s_i],[l_i]]},
\end{equation}
\begin{equation}
\text{for } \begin{cases}
(Y_{s_2} \phi_s^{n-k})_{[l_i],[k_i],[s_i],[l_i]}) \in Y[s_2 + k,0], & k = 1,\ldots,l, \\
(Y_{s_1-k} \phi_s^{n-k})_{[l_i],[k_i],[s_i],[l_i]}) \in Y[s_1,s_2 - s_1 + k], & k = l+1,\ldots,s_1
\end{cases}
\end{equation}
\begin{equation}
\text{where } (Y \phi_s^{n-k})_{[l_i],[k_i],[s_i],[l_i]}) = -s_2 \phi^{n-k}_{s_2,1},\) 

and antisymmetrization in \(\partial_{[l_i]} \phi^{n-k}_{s_1,1},\)
\begin{equation}
(Y \phi_s^{n-k})_{[l_i],[k_i],[s_i],[l_i]}) = \frac{1}{k} \partial_{[l_i]} (\phi^{n-k}_{s_1,1})_{[l_i],[k_i],[s_i],[l_i]})
\end{equation}
contains the factor \(1/k \) \((1/s_2)\) . The resulting LF is a gauge theory of 
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\([s_1,s_2] \), with the single off-shell restriction of Young 
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BRST LF \((16)\), we shall come to the same result 
\((18), (21)\).
(18), (21) may be deformed to describe dynamic of MAS HS field with spin $[s]_2$ on the AdS(d) space.

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REFERENCES

1. A. Sagnotti and M. Tsulaia, Nucl. Phys. B 682, 83–116 (2004).
2. M. Vasiliev, Fortsch. Phys. 52, 702–717 (2004).
3. M. Vasiliev, Lect. Notes Phys. 892, 227–264 (2015).
4. A. Fotopoulos and M. Tsulaia, Int. J. Mod. Phys. A 24, 1–60 (2008).
5. E. S. Fradkin and G. A. Vilkovisky, Phys. Lett. B 55, 224–226 (1975).
6. J. M. F. Labastida, Nucl. Phys. B 322, 185–209 (1989).
7. R. R. Metsaev, Phys. Lett. B 354, 78–84 (1995).
8. A. A. Reshetnyak, TSPU Bull. 12, 211–216 (2014); arXiv:1412.0200[hep-th].
9. C. Burdik, N. Boyarintceva, and A. A. Reshetnyak, to be published.
10. C. Fronsdal, Phys. Rev. D: Part. Fields 18, 3624–3640 (1978).
11. V. E. Lopatin and M. A. Vasiliev, Mod. Phys. Lett. A 3, 257–270 (1988).
12. R. R. Metsaev, Class. Quantum Grav. 22, 2777–2796 (2005); hep-th/0412311.
13. C. Burdik, A. Pashnev, and M. Tsulaia, Mod. Phys. Lett. A 16, 731–746 (2001).
14. I. Buchbinder, V. Krykhtin, and H. Takata, Phys. Lett. B 656, 253–264 (2007).
15. P. Yu. Moshin and A. A. Reshetnyak, J. High Energy Phys. 10, 040 (2007).
16. I. L. Buchbinder and A. A. Reshetnyak, Nucl. Phys. B 862, 270–326 (2012).
17. A. A. Reshetnyak, Nucl. Phys. B 869, 523–597 (2013).
18. E. D. Skvortsov, Nucl. Phys. B 808, 569–591 (2009).
19. Yu. M. Zinoviev, Nucl. Phys. B 826, 490–510 (2010).