Standard Deviation-Based Quantization for Deep Neural Networks

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Abstract
Quantization of deep neural networks is a promising approach that reduces the inference cost, making it feasible to run deep networks on resource-restricted devices. Inspired by existing methods, we propose a new framework to learn the quantization intervals (discrete values) using the knowledge of the network’s weight and activation distributions, i.e., standard deviation. Furthermore, we propose a novel base-2 logarithmic quantization scheme to quantize weights to power-of-two discrete values. Our proposed scheme allows us to replace resource-hungry high-precision multipliers with simple shift-add operations. According to our evaluations, our method outperforms existing work on CIFAR10 and ImageNet datasets and even achieves better accuracy performance with 3-bit weights and activations when compared to the full-precision models. Moreover, our scheme simultaneously prunes the network’s parameters and allows us to flexibly adjust the pruning ratio during the quantization process.

1. Introduction
Deep neural networks (DNNs) have shown promising results in various applications including Image classification and language processing tasks. However, deployment of the current state-of-the-art (SOTA) DNNs requires expensive and powerful hardware (e.g., GPUs) to perform costly high-precision multiplications (Courbariaux et al., 2015). Moreover, DNNs are generally over-parameterized and utilize large memories, thereby require heavy data transmission between the computation and memory units (You et al., 2020). When it comes to resource-constrained hardware such as mobile devices and embedded systems, the aforementioned requirements make the deployment of such networks a challenging task. Correspondingly, a large amount of effort has gone into developing algorithms and hardware, aiming to reduce the deployment costs of DNNs, whilst maintaining high accuracy. Efficient architecture design (Gholami et al., 2018; Sandler et al., 2018; Howard et al., 2019), pruning (Han et al., 2015; Liu et al., 2018), stochastic computing (Ardakani et al., 2017; Gross & Gaudet, 2019) and spiking neural networks (Ghosh-Dastidar & Adeli, 2009; Smithson et al., 2016) are examples of such efforts.

Another realm of studies focuses on quantization of DNNs where real values are mapped to discrete integer values (Zhou et al., 2016). Using integer values at inference dramatically reduces the computational and memory costs. In this work, we also focus on the quantization of DNNs. Although most existing works have employed uniform quantization methods where discrete levels have the same step size (Choi et al., 2018; Esser et al., 2019; Jung et al., 2019), DNNs can be quantized to non-uniform discrete integers as well (Zhang et al., 2018; Miyashita et al., 2016; Li et al., 2021; Zhou et al., 2017).

In general, the quantization methods mainly fall into three categories. The first category includes the methods trying to minimize the quantization error where real values are mapped to the discrete levels that accurately represents the data (Li et al., 2016; Cai et al., 2017; Wang et al., 2018; Zhang et al., 2018; Zhao et al., 2020). Most recent works that fall into the first category often aim to achieve accurate representation of data by minimizing the quantization error between the quantizer’s input and output. These methods require to constantly monitor weights and activations values (online) or to use the structural characterization of data from the full-precision network (offline) for minimizing the quantization error. For instance, in the HWGQ scheme (Cai et al., 2017), the quantization error is minimized using Lloyd’s algorithm (Lloyd, 1982) and the statistical structure (distribution) of the activations obtained from the full-precision network to fit the quantizer to the data. The main drawback of this method is its offline optimization and that the distribution of the full-precision activations is not necessarily the same as the quantized network. Contrary to HWGQ, LLSQ method uses an online algorithm to minimize the quantization error during the training phase (Zhao et al., 2020). LQ-Net is another online method (Zhang et al., 2018), where a non-uniform quantization scheme was developed that transforms the quantization error to a linear regression problem with a closed form solution. The performance of HWGQ, LLSQ and LQ-Net has been outperformed by other methods.
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(i.e., the second category), empirically showing that minimizing the quantization error is not the best approach to obtain quantized networks.

In the second category, the quantization is treated as an optimization problem that can be simultaneously optimized with the loss function during the training phase. More specifically, the quantizers are redesigned with learnable parameters that can be then used to find the optimal quantization intervals. In this approach, the goal is to find the optimizer that can produce the best accuracy performance, whereas in the first category, the goal is to find the optimizer that represents the data more accurately (less quantization error). PACT (Choi et al., 2018), QIL (Jung et al., 2019) and LSQ (Esser et al., 2019) are the well-known quantization methods from this category.

Finally, the third category mainly focuses on the training process of the quantized networks (Zhou et al., 2017; Zhuang et al., 2018). This includes techniques and tricks that are used during the training of the quantized network to improve its performance accuracy. For instance, it has been shown that a progressive training framework with the help of knowledge distillation significantly improves the performance of the DoReFa-Net (Zhou et al., 2016) quantization method on various tasks (Zhuang et al., 2018).

In this work, we aim to close the gap between these methods by exploiting the statistical characterization of weights and activations distribution during the quantization process. In other words, we propose a new quantization scheme that utilizes the standard deviation of weights/activations distribution during the training phase to find the optimal optimizer that produces the best accuracy performance using task loss and back-propagation. We also explain how standard deviation contributes in improving our quantization method in Section 3. Furthermore, in Section 4, we propose two training techniques which are employed in our proposed quantization method to further improve the performance of the quantized networks.

Moreover, unlike the existing methods that have only focused on one type of quantization, that is either uniform or non-uniform quantization, we propose a quantization framework that is capable of both uniform and non-uniform quantization. Our non-uniform quantization method uses power-of-two discrete levels. We show that the proposed non-uniform quantization method outperforms the SOTA results (Li et al., 2021) while having 10× faster convergence.

Quantizers with the discrete intervals that include zero level, inherently benefit from pruning as values that fall into this level during the quantization are set to 0 and can be pruned. This valuable property makes quantization methods more desirable over other cost reduction methods. However, there are only a few work investigating the relation between pruning and quantization, and their impact on each other (Jung et al., 2019; Han et al., 2015). In this work, not only we study the pruning property of quantization in more depth, we also show that our method allows us to flexibly adjust the pruning ratio during the quantization process. For instance, we show that up to 40% of weights in a 3-bit ResNet-18 model can be pruned at the cost of less than 1% accuracy loss when compared to its full-precision counterpart.

2. Standard Deviation-Based Quantization

To make the connection between the existing quantization methods, we try to solve the following question: “How much of the information is important in a quantized network?”. To answer this question, we need to look at the statistical characterization of weights and activations more carefully. It has been shown that the output of the convolution/fully-connected layers has a bell-shaped distribution, especially if followed by a batch-normalization layer (Ioffe & Szegedy, 2015). Besides the activations, the weights of a network regularized with L2-regularization method have a bell-shaped distribution as well. The width of a bell-shaped distribution is defined by its standard deviation. For instance, in a Gaussian distribution, the values within three standard deviation (σ) from the mean (µ) account for 99.73% of the entire data. However, not all of these values are as important as the others in a neural network. Here we propose a standard deviation-based clipping function with a learnable parameter (α) which is capable of determining how much of these values is actually important.

The backbone of our proposed clipping function is the PACT (Choi et al., 2018) method. In our proposed function, we integrate the standard deviation of the weights/activations in to the PACT such that

\[
y(x) = \begin{cases} 
  x & |x| < \alpha \sigma \\
  \text{sign}(x) \cdot \alpha \sigma & |x| \geq \alpha \sigma 
\end{cases},
\]

where α and σ are the quantizer’s learnable parameter and the standard deviation of the weights/activations, respectively. For the activations, we can integrate the ReLU function with our clipping function, i.e.,

\[
y(x) = \begin{cases} 
  0 & x \leq 0 \\
  x & 0 < x < \alpha \sigma \\
  \alpha \sigma & x \geq \alpha \sigma 
\end{cases}.
\]

The proposed clipping function simply discards the “outliers” that are ασ far from the mean of the weights/activations distribution. Note that in our clipping function, the mean value of weights is considered to be zero all the time. This is because in our experiments, the mean value of weights was always close to zero and did not impact the results when it was omitted. The mean of the activations, however, is always zero. Note that in Equation (2), we use ReLU function to assign zeros to all negative values. Consequently,
we should not calculate \( \sigma \) directly from the activations input. Instead, we calculate the \( \sigma \) by removing all negative values from the inputs and then we apply a horizontal mirroring on the remaining values. It is evident that after this step, the mean value of activations distribution is always zero.

Unlike the standard deviation of the weights distribution, the standard deviation of activations can have a significantly different value whenever a new batch of data is introduced to the network. Therefore, we need to average \( \sigma \) for activations distribution during the training phase. Inspired by the batch-normalization technique, we keep track of \( \sigma \) by employing the exponential running average with the momentum of 0.001,

\[
\hat{\sigma}_{\text{new}} = (1 - \text{momentum}) \times \hat{\sigma} + \text{momentum} \times \sigma_t, \quad (3)
\]

where \( \hat{\sigma} \) is the running average and \( \sigma_t \) is the standard deviation of the current batch. It is worth mentioning that the momentum value of 0.001 was empirically obtained from our experiments.

The output of the clipping function can be then quantized to \( L_P + L_N + 1 \) discrete levels (integer values) using \( b \) bits such that

\[
y_d = \text{clip}(y \cdot \frac{L_P}{\alpha \sigma}), \quad (4)
\]

where function \( \lfloor \cdot \rceil \) rounds a to its nearest integer value and \( y_d \in \mathbb{N} \). Variable \( L \) is determined by the quantizer bit-width (\( b \)) and the data type (signed/Unsigned). For unsigned data (activations) \( L_N = 0 \) and \( L_P = 2^b - 1 \) and for signed data (weights) \( L_P = L_N = 2^{b-1} - 1 \). It is worth mentioning that our method quantizes signed values symmetrically. For instance, quantization of signed values using 2 bits results in three discrete values (i.e., ternary discrete values).

Finally, the quantization process is completed by multiplying the quantizer scale value with \( y_d \) such that

\[
y_q = y_d \cdot \frac{\alpha \sigma}{L_P}, \quad (5)
\]

where \( y_q \in \mathbb{R} \). We would like to emphasize that the multiplication of the quantizer scale with \( y_d \) can simply be integrated with the batch normalization layer without any additional costs. As shown in Figure 1, our proposed quantization method automatically prunes the network as well. In fact, any values that falls into the pruning area of the quantizer is pruned (set to zero) during the quantization. From Equation (4), we can observe that

\[
\text{if } |y| < \frac{\alpha \sigma}{2L_P}, \text{ then } y_d = 0,
\]

which shows a direct relation between the pruning ratio and the clipping threshold of the quantizer. In other words, higher clipping thresholds result in greater pruning ratios.

![Figure 1](image.png)

**Figure 1.** An example of a 3-bit quantizer for signed values.

### 2.1. Optimizing \( \alpha \) with Backpropagation

Following the same principles introduced in PACT and using Straight-Through Estimator (ETS) (Bengio et al., 2013), we can derive the gradient for the quantizer parameter \( \alpha \) by

\[
g_\alpha = \begin{cases} 
\sigma \cdot g_y & x \geq \alpha \sigma \\
0 & \text{otherwise}
\end{cases} \quad (6)
\]

where \( g_y \) is the incoming gradient from the front layers. Furthermore, the gradient of the activations input \( g_x \) can be computed using

\[
g_x = \begin{cases} 
g_y & 0 < x < \alpha \sigma \\
0 & \text{otherwise}
\end{cases} \quad (7)
\]

We can add a gradient scale \( s \) and weight decay \( \lambda \) to \( g_\alpha \) and rewrite Equation (6) as

\[
g_\alpha = \begin{cases} 
s \sigma \cdot g_y + \lambda \alpha & x \geq \alpha \sigma \\
0 & \text{otherwise}
\end{cases} \quad (8)
\]

The gradient scale and weight decay were added to \( g_\alpha \) to prevent \( \alpha \) from exploding/vanishing and to control the pruning ratio.

### 2.2. Base-2 Logarithmic Quantization

We can use the clipping function described in Equation (1) to quantize weights to power-of-two discrete values, i.e., \( y_{p2} \in \{0, \pm 2^k\} \), where \( k \in \mathbb{Z}_+ \). With this new power-of-two quantized weights, we can replace the expensive multiplications with simple shift and addition/subtraction operations. To quantize weights to power-of-two discrete values, we first modify Equation (4) to produce discrete integers (\( y_{\text{int}} \in \mathbb{N} \)) such that

\[
y_{\text{int}} = \left\lfloor \log_2(|y| \cdot \frac{L_{p2}}{\alpha \sigma}) \right\rfloor, \quad (9)
\]

where \( L_{p2} = 2^{2^{b-1}} - 2 \). We then transform \( y_{\text{int}} \) to discrete power-of-two integers including zero (\( y_{p2} \in \{0, \pm 2^k\} \)) us-


Inclusivity: Unlike PACT where the gradients of the quantizer parameter only rely on the outliers, the gradients in our method are sensitive to the entire data. More specifically, we introduce the standard deviation of weights and activations distribution as a new feature that appears in the task loss and is used to scale the quantizer parameter.

Adaptive gradient scale factor: The impact of the gradient scale has been investigated in LSQ. It was shown that large gradient scales (e.g., $L_{p2}$) produce weak accuracy results. Looking at Equation (8), the standard deviation in our method can also be interpreted as an adaptive scaling factor for the quantizer parameter $\alpha$. In most of our experiments, the gradient scale was initially set to 1 and then adjusted to a proper value that achieves the best accuracy performance. In fact, the role of standard deviation as the adaptive gradient scale enables us to trade-off between the accuracy performance and the pruning ratio by treating the gradient scale as a hyper-parameter. Using a small gradient scale forces $\alpha$ to converge to smaller values and consequently, less data is pruned. On the other hand, setting the gradient scale to larger values allows us to prune more data, though at the cost of losing accuracy performance. We investigate this property in Section 5.5.

Faster convergence: In the previously-proposed methods, the clipping threshold of weights and activations solely relies on the quantizer parameter $\alpha$. Consequently, if the distribution changes rapidly, it may take a longer time for the clipping threshold to catch up with the new distribution of data. In our proposed quantizer, the clipping threshold relies on both the $\alpha$ and the standard deviation of weights/activations distribution. As a result, even without updating $\alpha$, the clipping threshold can follow the new distribution effortlessly as illustrated in Figure 2.

4. Quantization Techniques

In this section, we propose two quantization techniques that can be employed to further improve the performance of our quantization method.

4.1. Improved Progressive Training

Arguably, any neural network quantized to extremely low bit-widths suffers from accuracy loss due to poor data representation. However, the inaccurate representation of the weights/activations is not the only factor, negatively impacting neural networks. In fact, the gradient vanishing problem as a result of intense pruning during the quantization process is a major issue causing significant performance degradation (Li et al., 2021). Furthermore, intense pruning reduces the learning capacity of neural networks since a huge portion of the weights are removed (set to zero). Previous studies tried to address this issue by proposing different progressive training methods (Zhou et al., 2017; Zhuang et al., 2018). A recent method, for instance, proposed to quantize higher values of weights first while keeping the lower values in full-precision (Zhou et al., 2017). This
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Figure 3. Quantization levels (intervals) of a 3- and 2-bit quantizers with the same \( \alpha \). The blue line shows the intervals of the 3-bit quantizer, whereas the red line shows the intervals for the 2-bit quantizer. The pruning area of the 2-bit quantizer (rectangle with the shade of red) is 3 times wider than the 3-bit quantizer (rectangle with the shade of blue).

method allows the gradient to back propagate through the weights with lower values which would have been pruned due to quantization. In another method, it has been shown that a progressive training (e.g., training 2-bit width networks using the trained parameters from the 3-bit width networks) improves the training capacity and the accuracy performance of the quantized networks. However, by just transferring the learned parameters, we are changing the learned quantization intervals according to Equation (4). Consequently, a huge portion of the network parameters are pruned during the transformation from higher bit-width networks to lower bit-width networks due to the larger pruning area as illustrated in Figure 3. By changing the quantization intervals, the learning capacity of the quantized network will be reduced even more, not only because of the poor representation of data due to low bit-width quantization, but also as a result of the pruning.

To address this, we propose to re-scale the quantizer parameter \( \alpha \) so that the lower bit-width network starts with the same quantization intervals of the higher bit-width network by applying

\[
\alpha_b = \alpha_{b+n} \times \frac{L_b}{L_{b+n}},
\]

where \( \alpha_b \) and \( L_b \) are the quantizer parameter and the discretization level of the quantized networks with \( b \) bits, respectively.

Our proposed progressive training method offers two advantages: improving the training capacity of the quantized network and preventing the quantizer from pruning the parameters further. Consequently, our method reduces the impact of gradient vanishing problem. More importantly, this allows us to limit the search space for the quantizer parameter \( \alpha \), forcing the network to find optimal intervals near the ones found in networks with higher bit-widths. This can be done by using smaller gradient scale values \( s \) in Equation (8) which allows the weight decay to prevent the quantizer parameter \( \alpha \) from getting larger.

4.2. Two-Phase Training

It has been shown that a huge portion of the weights resides near the quantization intervals boundaries (transition boundaries) (Jung et al., 2019). Our experimental result also shows the same patterns across different networks as illustrated in Figure 4. Based on these observations, we suspected that jointly optimizing the network parameters along with the quantizer’s could negatively impact the performance of the network since a small change in quantization intervals can possibly result in a dramatic change in the quantized weights, especially for 2-bit quantized networks where there are fewer discrete levels. In addition, it is more difficult for weights near the transition boundaries to converge to the optimal quantization intervals when the transition boundaries are constantly fluctuating. Furthermore, while we do not see the same pattern for the quantized activations (because of the batch normalization), this fluctuation could still hurt the quantization of the activation but with less severity. Furthermore, the gradients of the quantized networks with learnable parameter are affected by that parameter when passing through the layers. Looking at Equation (7), it is evident that both \( \alpha \) and \( \sigma \) are controlling the gradients passing to the previous layers. Since \( \alpha \) and \( \sigma \) are both getting updated continuously during the training, we suspected that this might introduce some noise to the gradients. To investigate this hypothesis, we retrained our quantized networks keeping \( \alpha \) and \( \sigma \) frozen to their optimal value from the first training. Interestingly, our empirical results show constant improvement across different networks and datasets, which highlights the importance of our proposed two-phase training method when training parameterized quantized networks.

Figure 4. Distribution of a convolution layer from ResNet-18 model trained with our quantization method using 3 bits. The spikes in the distributions are the transition boundaries of the 3-bit quantizer.
Table 1. Quantization accuracy performance on CIFAR10 dataset with ResNet-20 model (FP accuracy: 91.74%). Methods included in this table are LQ-Net (Zhang et al., 2018), DSQ (Gong et al., 2019) and PACT (Choi et al., 2018)

| Quantization method | Training technique | Accuracy @ precision (A and W) |
|---------------------|--------------------|--------------------------------|
|                     |                    | 5   | 4   | 3   | 2   |
| DSQ                 | DSQ               | NA  | –   | –   | 90.11 |
| LQ-Net              | LQ-Net            | NA  | –   | 91.6 | 90.2  |
| PACT                | DoReFa            | From scatch | 91.7 | 91.3 | 91.1 | 89.7 |
| Ours                | DoReFa            | From scatch | 91.75 | 91.63 | 91.25 | 89.77 |
|                     | DoReFa            | Progressive | 91.90 | 91.81 | 91.68 | 89.74 |
|                     | DoReFa            | Two-phase training | 92.03 | 92.00 | 91.65 | 90.32 |
| Ours                | Ours              | Progressive with re-scaling and two-phase training | **92.27** | **92.28** | **92.23** | **90.77** |

5. Experiments

To validate our proposed quantization method, we conduct several experiments on CIFAR10 (Krizhevsky et al., 2009) and ImageNet (Russakovsky et al., 2015) datasets on various architecture. Furthermore, we present several ablation studies to evaluate the impact of the hyper-parameters and the proposed quantization techniques on the performance of the quantized networks. In all experiments, we use the network’s weight decay value for the quantizer’s weight decay. Furthermore, we use the same data augmentation proposed in (He et al., 2016) for both CIFAR10 and ImageNet datasets.

5.1. ResNet-20 on CIFAR10

We demonstrate the effectiveness of our proposed method by quantizing ResNet-20 model (He et al., 2016) on CIFAR10 dataset. To have a fair comparison with PACT, we conduct two different experiments based on the quantization method applied on weights. In the first experiment, we quantize ResNet-20 model progressively with re-scaling of the clipping threshold and applying the proposed two-phase training technique. In this experiment, both weights and activations are quantized using our proposed quantization scheme. The gradient scale value (s) is set to \{1, 0.1, 0.01\} for the \{5, 4, 3, 2\}-bit quantized ResNet-20, respectively. It is worth mentioning that the gradient scale values were adjusted for each bit-width after performing hyper-parameter search (see Section 5.6).

In the second experiment, we employ the same weight quantization method used in PACT. We also quantize ResNet-20 from scratch, and provide the accuracy results separately in Table 1. Finally, we employ the progressive method without re-scaling and our proposed two-phase training technique step by step to demonstrate their impact on the performance of the quantized network. Similar to PACT, we do not quantize the first layer and the last layer of ResNet-20. In this experiment, the gradient scale used to quantize activations is set to 1 regardless the bit-widths.

From Table 1, we observe that our quantization method achieves a higher accuracy compared to PACT even when only the activations are quantized with our method. Furthermore, we achieve a higher accuracy when our two-phase training technique is applied over the progressive training method, showing its effectiveness in reducing the gradient noise. In addition, we achieve the best performance when both weights and activations are quantized using our method across all the quantization bit-widths.

5.2. SmallVGG on CIFAR10

We also evaluate our method on CIFAR10 dataset by quantizing the SmallVGG network (Louizos et al., 2018) under three different setups:

**Setup-1** We use our quantization method to quantize both weights and activations of SmallVGG network with 2 bits. In this setup, all layers except the first convolution layer and the fully-connected layer are quantized. The network’s parameters are initialized with the full-precision parameters. We train the network for 300 epochs using gradient scale of 0.001. As shown in Table 2, our method outperforms the state-of-the-art and even achieves a higher accuracy performance compared to the full-precision model.

**Setup-2** In this setup, we quantize activations to 2 bits (similar to the first experiment), but we binarize weights (i.e., quantize with 1 bit) using the sign function as described in (Courbariaux et al., 2016). Interestingly, our method still manages to outperform SOTA and full-precision while using binarized weights and 2-bit activations during inference.

**Setup-3** We repeat the same experiment described in setup-1 to quantize all layers of SmallVGG network. It should be noted that only the weights of the first layer are quantized while its inputs are kept in full-precision. Similar to the first experiment, our method outperformed SOTA and full-precision models. The results from these three experiments show quantization can have a regularization effect as long as the capacity of the quantized network is not decreased due to quantization. Furthermore, we can conclude that Small-
IVGG network is over-parameterized for small datasets like CIFAR10.

Table 2. Comparison with SOTA methods on CIFAR10 dataset using SmallVGG network (FP accuracy: 93.66%). Methods included in this table are LQ-Net (Zhang et al., 2018), HWGQ (Cai et al., 2017), LLSQ (Zhao et al., 2020) and RQST (Louizos et al., 2018).

| Method    | All layers | Accuracy @ precision (A/W) |
|-----------|------------|---------------------------|
| LQ-Net    | No         | 93.40 93.50               |
| HWGQ      | No         | 92.51 NA                   |
| LLSQ      | No         | NA 93.31                   |
| Ours      | No         | 93.88 94.36               |
| RQST      | Yes        | NA 90.92                   |
| LLSQ      | Yes        | NA 93.12                   |
| Ours      | Yes        | NA 93.90                   |

5.3. AlexNet on ImageNet

We test our quantization method on ImageNet dataset using modified AlexNet (Krizhevsky et al., 2012) where a batch normalization layer was added after convolution and fully-connected layers except for the last layer. We use the progressive training method with re-scaling when training the 3-bit and the 2-bit networks. The quantized network is optimized with cosine annealing scheduler with the initial learning rate of 0.001 for 70 epochs. Following the common practice from previous methods, all layers are quantized except for the first layer and the last layer. As shown in Table 3, our proposed method outperforms the previously-proposed quantization methods.

When training the 2-bit AlexNet with the gradient scale of 1 (i.e., $s = 1$), we observed that the training loss starts to increase after several epochs and the network converges to a poor local minima due to extreme pruning, even with re-scaling of the clipping threshold. To address this issue, we changed $s$ to 0.01 to force the 2-bit AlexNet use the intervals found from the 3-bit network with less weight pruning. Furthermore, we trained the 2-bit AlexNet with the intervals obtained from the 4-bit network with $s = 0.01$. Finally, we used our double training method for the 2-bit AlexNet which increased the accuracy by 0.2%. As shown in Table 4, the 2-bit AlexNet with 3-bit intervals has the best performance accuracy while the one trained with 4-bit intervals also outperforms the QIL method.

Table 3. Comparison with the existing methods on AlexNet (FP accuracy: 61.8%). Methods included in this table are QIL (Jung et al., 2019), LQ-Net (Zhang et al., 2018) and TSQ (Wang et al., 2018).

| Method    | Top-1 accuracy @ precision (A and W) |
|-----------|-------------------------------------|
| Ours      | 62.5 62.2 59.2                      |
| QIL       | 62 61.3 58.1                        |
| LQ-Net    | – – 57.4                            |
| TSQ       | – – 58                              |

5.4. ResNet on ImageNet

We evaluate our proposed base-2 logarithmic quantization method described in Section 2.2 on ImageNet dataset using ResNet-18 and ResNet-50 models (He et al., 2016). The under-test models are specifically chosen to fairly compare the accuracy performance of our method with SOTA shift-add methods. We employ previously described two-phase training technique. Both models are initialized with the pre-trained parameters from the Pytorch model zoo (Paszke et al., 2019). We train the models for 70 epochs with cosine learning scheduler. The results summarized in Table 5 show that our method outperforms the SOTA results for both models. In addition to having higher accuracy performance, our method has several advantages over the Sign-Sparse-Shift (Li et al., 2021) method. First, the full-precision parameters of the pre-trained models can be used to initialize the quantized models. Consequently, the quantized models can converge to an acceptable accuracy much faster than the ones initialized randomly. In fact, the Sign-Sparse-Shift method requires 200 epochs to reach the results provided in Table 5, whereas our method achieves a higher accuracy after only 20 epochs (10× faster convergence). Furthermore, the Sign-Sparse-Shift method requires 4 times the number of parameters of its full-precision model during the training phase, which significantly increases the training hardware memory footprint and utilization. Contrary, our method only adds a small number of parameters and registers i.e., $\alpha$ and the standard deviation of the weights.

Table 4. Top-1 accuracy of 2-bit AlexNet under different training setup.

| Setup                          | Gradient scale | Top-1 Acc. (%) |
|--------------------------------|----------------|---------------|
| No re-scaling                  | 1              | 54.4          |
| Re-scaled from 4-bit           | 0.01           | 58.76         |
| Re-scaled from 3-bit           | 0.01           | 59.01         |
| Re-scaled from 3-bit + two-phase training | 0.01 | 59.24 |

5.5. Pruning Ratio and Accuracy Trade-off

As mentioned previously, pruning is an inherit outcome of quantization. A well-pruned network can significantly reduce the size of the memory. However, too much pruning has a negative impact on the model learning capacity. In our quantization method, we can control the pruning ratio indirectly by adjusting the gradient scale ($s$ in Equation (8)). We evaluate this property by training the same 3-bit quantized ResNet-18 model from previous experiment (Section 5.4)
1 and 0.001 are shown in Figure 5 to better demonstrate how the clipping thresholds and the pruning rates of each layers

### Table 5. Comparison between existing shift-add networks on ImageNet dataset. Methods included in this table are DeepShift (El-houshi et al., 2021), INQ (Zhou et al., 2017) and Sign-Sparse-Shift ($S^3$) (Li et al., 2021).

| Model     | Method       | Width | Top-1/Top-5 Acc. (%) |
|-----------|--------------|-------|----------------------|
| FP        | 32           | 69.76/89.08 |
| DeepShift | 5            | 69.56/89.17 |
| INQ       | 3            | 68.08/88.36 |
| $S^3$     | 3            | 69.82/89.23 |
| Ours      | 3            | **70.04/89.14** |
| INQ       | 4            | 68.89/89.01 |
| $S^3$     | 4            | 70.47/89.93 |
| Ours      | 4            | **70.70/89.62** |

### Table 6. Top-1 accuracy and pruning ratio for various gradient scale factors.

| Gradient scale | 1 | 0.1 | 0.01 | 0.001 |
|----------------|---|-----|------|-------|
| Top-1 Acc. (%) | 68.92 | 69.59 | 69.70 | 70.04 |
| Pruning ratio (%) | 40.21 | 29.74 | 24.97 | 21.57 |

### Table 7. Accuracy performance of ResNet-20 on CIFAR10, quantized using 2 and 3 bits with different clipping threshold initialization and gradient scale values.

| Bit-width | Clipping Threshold Initialization | Gradient scale |
|-----------|----------------------------------|----------------|
| 2         | From 3                           | 1              |
| 2         | From 4                           | 0.1            |
| 2         | From 4                           | 0.01           |
| 2         | From 4                           | 0              |

Figure 5. The clipping thresholds and the pruning rates of the quantized ResNet-18 weights on ImageNet dataset with various gradient scale factors. Each model was trained for 20 epochs. The results provided in Table 6 show that we can achieve slightly better accuracy performances at the cost of losing a significant amount of pruning ratio. More specifically, we can achieve 1.12% higher accuracy if we accept 18.64% less pruning. It is up to the users to decide whether the benefits of networks with higher pruning ratios outweigh their slight drop in accuracy. It should be noted that this trade-off can only be considered in a situation where several gradient scale values result in a good convergence and accuracy performance. Furthermore, the clipping thresholds and the pruning rates of each layer for the quantized networks with the gradient scale values of 1 and 0.001 are shown in Figure 5 to better demonstrate how the gradient scale affects the pruning ratio. As expected, the quantized network with the smaller gradient scale value has smaller clipping thresholds. Furthermore, higher clipping thresholds result in greater pruning rates.

### 5.6. Progressive Training and Re-scaling

We evaluate the effectiveness of our proposed progressive training, i.e., the re-scaling of the clipping threshold for the weights quantizer, by training ResNet-20 on CIFAR10 dataset. In this experiment, ResNet-20 model is quantized to 2 and 3 bits. The 2-bit quantized model is initialized and re-scaled from the 3-bit and 4-bit models, and the 3-bit model is initialized and re-scaled using the 4-bit model. From Table 7, we observe that the 2-bit model yields a higher accuracy when the clipping threshold is re-scaled from the 3-bit model. Interestingly, both the 2-bit and the 3-bit models produce comparable accuracy even when the quantizer parameter ($\alpha$) is not updated (the gradient scale is 0). In other words, we can use the same quantization intervals, found from the higher-bit networks, in the lower-bit networks and still achieve acceptable performance. This observation shows the importance and effectiveness of our proposed re-scaling. Furthermore, from the columns where the gradient scale is 0.1 and 0.01, we observe that updating $\alpha$ with a smaller gradient scale does not necessarily achieve a better performance all the time.

### 6. Conclusion

We proposed a new quantization method that takes advantage of the knowledge of weights and activations distribution during the quantization process. Using the standard deviation of weights and activation, our proposed method outperforms the previous works on various image classification tasks. Furthermore, we proposed two training techniques to further improve the performance of our quantization scheme. We introduced a two-phase training technique to address the gradient noise and fluctuation of the quantizer’s transition boundaries caused by the joint optimization of the network’s and the quantizer’s parameters. In addition, we presented a novel re-scaling technique to improve the training of quantized networks by reducing the impact of gradient vanishing problem. Finally, we proposed a novel non-uniform quantization framework (base-2 logarithmic quantization) where weights are quantized to power-of-two discrete intervals to
replace expensive multipliers with simple shift-add operations. The proposed base-2 logarithmic quantization method converges $10\times$ faster than the SOTA method and outperforms existing work. Our proposed quantization method provides the flexibility to trade-off between accuracy and network size by controlling the pruning ratio. In future work, we plan to investigate the compatibility of our method with transformers.

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