Control of atomic entanglement by the dynamic Stark effect

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Abstract
We study the entanglement properties of two three-level Rydberg atoms passing through a single-mode cavity. The interaction of an atom with the cavity field allows the atom to make a transition from the uppermost (lowermost) to the lowermost (uppermost) level by emission (absorption) of two photons via the middle level. We employ an effective Hamiltonian that describes the system with a Stark-shifted two-photon atomic transition. We compute the entanglement of formation of the joint two-atom state as a function of Rabi angle. It is shown that the Stark shift can be used to enhance the magnitude of atomic entanglement over that obtained in the resonant condition for certain parameter values. We find that though the two-atom entanglement generally diminishes with the increase of the two-photon detuning and the Stark shift, it is possible to sustain the entanglement over a range of interaction times by making the detuning and the Stark shift compensate each other. Similar characteristics are also obtained for a thermal state cavity field.

1. Introduction

The most interesting idea associated with composite quantum systems is quantum entanglement. In quantum mechanics, a pair of particles is said to be entangled if its state cannot be expressed as a product of the states of its individual constituents. Einstein, Podolsky and Rosen [1] were the first to point out certain non-trivial consequences of entanglement on the ontology of quantum theory. The preparation and manipulation of these entangled states lead to a better understanding of basic quantum phenomena. For example, complex entangled states, such as the Greenberger, Horne and Zeilinger [2] triplets of particles are used for tests of quantum nonlocality [3]. Beyond these fundamental aspects, entanglement has become a fundamental resource in quantum information processing, and there has been a rapid development of this subject in recent years [4, 5].

Cavity QED has been a favourite tool to test the foundations of quantum mechanics including entanglement. Many beautiful experiments have been carried out, and in recent years, entangled states have been created and verified [6]. Maximally entangled states between two modes in a single cavity have been generated using a Rydberg atom coherently interacting with each mode in turn [7]. Practical realization of various features of quantum entanglement is obtained in atom–photon interactions in optical and microwave cavities. Several studies have been performed to quantify the entanglement generated in atom–photon interactions in cavities [8–15].

The above-cavity QED-related investigations involved mostly the absorption or emission of a single photon in an atomic transition. However, involvement of more than one photon, in particular two photons, in the transition between two atomic levels via a non-resonant intermediate level has been known for a long time [16–20]. The output radiation from such interactions exhibits novel nonclassical properties such as sub-Poissonian photon statistics. Needless to say, the idea of squeezed light has originated from a two-photon process [21, 22]. Two-photon processes have also been studied in cavity QED [23–26]. It showed compact and regular quantum revivals in the atomic population in the single atom two-photon cavity QED [23, 24]. Haroche and co-workers have demonstrated experimentally the two-photon maser action in a micromaser cavity [25]. In total, the two-photon process mostly exhibits nonclassical properties compared to the one-photon process [26, 27]. Thus, it would be interesting to study the properties of atom–atom entanglement in the framework of a two-photon process.

The two-photon atomic transition process also introduces a dynamic Stark shift in the atomic transition which is related to the magnitude of the electric field of the radiation inside
the cavity. This non-trivial effect which is naturally present in actual experiments involving two-photon transitions has to be properly accounted for in its theoretical analysis, for example in the two-photon micromaser [25]. Various possibilities of exploiting the Stark effect in quantum optical applications have been noted in recent years. Schemes for applying the dc as well as the ac Stark shifts towards implementation of quantum logic gates and algorithms [28–30] and for the improvement of photon sources for interferometry [31] have been suggested. It would be thus interesting to study if the Stark shift can be utilized to enhance atom–atom entanglement as well.

The purpose of the present paper is to investigate the possibility of controlling atomic entanglement by the Stark shift generated in atomic transitions inside cavities. As stated earlier, there exist several schemes for entangling atoms inside cavities, which may lead to more entanglement creation over the mechanism studied in this work. Note that our motivation here is not to look for creation of greater entanglement between atoms. On the other hand, we would like to investigate entanglement generation in two-photon processes, especially the mechanism studied in this work. Note that our motivation in the two-photon process is that the Stark shift provides an additional parameter to control entanglement. Though there have been some works on entanglement creation in two-photon processes (see, for example, [28]), none of those studies has investigated the effect of Stark shift on atomic entanglement.

With the above aim in view, the plan of this paper is as follows. In section 2, we study the properties of the atom–atom entanglement generated through the effective Hamiltonian for the atom–field two-photon interactions. This Hamiltonian is quadratic in the field operators, which is an outcome of the procedure followed in section 3, we show that similar trends are also obtained for a thermal cavity field. We conclude the paper with a summary of our results in section 4.

2. Two-atom entanglement

We consider a ladder system of a three-level Rydberg atom interacting with a single mode of a microwave cavity field. The middle level may be a group of closely spaced levels removed far away from one-photon resonance. Thus the interaction involves simultaneous absorption (or emission) of two photons between the two atomic levels via a group of (or one) intermediate levels. This is called a degenerate process since the two photons are from the same mode of the radiation field. Let us label the lower and the upper level as |g⟩ and |e⟩ respectively and the intermediate levels are labelled as |i⟩ (see figure 1). The interaction is described by an effective Hamiltonian derived in the appendix. It is of the form

\[ H_{\text{eff}} = |\Delta + (\beta_e + \beta_g)a^\dagger a|S_z + \frac{1}{2}(\beta_e - \beta_g)a^\dagger a + g(S^+a^2 + S^-a^2) \]  

(1)

If we take \( \beta_e = \beta_g = \beta \) the Hamiltonian reduces to

\[ H_{\text{eff}} = |\Delta + 2\beta a^\dagger a|S_z + g(S^+a^2 + S^-a^2) \]  

(2)

It may be noted here that \( H_{\text{eff}} \) is a function of the two-photon detuning \( \Delta \) which is an outcome of the procedure followed here. In other words, we need not assume the resonance condition \( \Delta = 0 \) unlike in other methods in the literature [23], but this method gives the two-photon detuning \( \Delta \) as an independent parameter for the analysis. The effective Hamiltonian can easily be contrasted when compared with a Hamiltonian describing the one-photon process [27]. First, the effective Hamiltonian is now cavity photon number dependent and is thus dynamic. Secondly, the Hamiltonian is quadratic in annihilation and creation operators. As mentioned earlier, they are at the root of all the nonclassical behaviours in two-photon processes. Using such a Hamiltonian, we now study the entanglement of two such atoms passing through the cavity one after the other.

The effective Hamiltonian derived above can be written in the matrix form in the basis of \(|e,n⟩, |g,n+2⟩\) states as

\[
H_{\text{eff}} = \begin{pmatrix} 
\frac{\Delta}{2} + \beta n & g\sqrt{(n+1)(n+2)} \\
g\sqrt{(n+1)(n+2)} & -\frac{\Delta}{2} + \beta n + 2\beta 
\end{pmatrix}.
\]  

(3)

The eigenvalues of \( H_{\text{eff}} \) are

\[
\lambda_1 = -\beta + \sqrt{\left(\Delta \over 2 + \beta(n+1)\right)^2 + g^2(n+1)(n+2)},
\]  

(4)

\[
\lambda_2 = -\beta - \sqrt{\left(\Delta \over 2 + \beta(n+1)\right)^2 + g^2(n+1)(n+2)}.
\]  

(5)

The corresponding eigenstates can be written as

\[
|\Psi\rangle_{\lambda_1} = c_1|e,n⟩ + c_2|g,n+2⟩,
\]  

(6)

\[
|\Psi\rangle_{\lambda_2} = c_2|e,n⟩ - c_1|g,n+2⟩,
\]  

(7)

where

\[
c_1 = \frac{\lambda_1 + \left(\Delta \over 2 + \beta n + 2\beta\right)}{\sqrt{g^2(n+1)(n+2) + (\lambda_1 + \left(\Delta \over 2 + \beta n + 2\beta\right)^2}}
\]  

(8)

and

\[
c_2 = \frac{g\sqrt{(n+1)(n+2)}}{\sqrt{g^2(n+1)(n+2) + (\lambda_1 + \left(\Delta \over 2 + \beta n + 2\beta\right)^2}}.
\]  

(9)
We envisage a process in which the two atoms pass through an ideal cavity \((Q = \infty)\) such that there is no overlap of their flights there \([12–15]\). The interaction of each atom in the cavity is described by \(H_{\text{eff}}\) in equation (3). We assume that the two atoms are in their respective upper states \(|e\rangle\) before they enter the cavity empty of photons. After passage of the first atom through the cavity, the joint atom–field state given at any time \(t\) is

\[
|\Psi_1\rangle_t = (r_1 - is_1)|e, n\rangle + (r_2 - is_2)|g, n + 2\rangle,
\]  

(10)

where

\[
r_1 = \left[ c_1^2 \cos(\lambda_1 t) + c_2^2 \cos(\lambda_2 t) \right],
\]

(11)

\[
s_1 = \left[ c_1^2 \sin(\lambda_1 t) + c_2^2 \sin(\lambda_2 t) \right],
\]

(12)

\[
r_2 = c_1 c_2 \left[ \cos(\lambda_1 t) - \cos(\lambda_2 t) \right],
\]

(13)

\[
s_2 = c_1 c_2 [\sin(\lambda_1 t) - \sin(\lambda_2 t)].
\]

(14)

The second atom then interacts with the cavity field modified by the passage of the first atom. Assuming the flight time of the two atoms through the cavity to be the same, the joint state of the two atoms and the cavity after the second leaves the cavity is given by

\[
|\Psi_{12}\rangle_t = (r_1 - is_1)^2|e_1, e_2, n\rangle + (r_2 - is_2)|g_1, g_2, n + 2\rangle + (r_2 - is_2)(r'_1 - is'_1)|g_1, e_2, n + 2\rangle + (r_2 - is_2)(r'_2 - is'_2)|g_1, g_2, n + 4\rangle,
\]

(15)

where \(r'_1 = r_1^{\text{ann} = 2}, s'_1 = s_1^{\text{ann} = 2}, r'_2 = r_2^{\text{ann} = 2}\) and \(s'_2 = s_2^{\text{ann} = 2}\).

Next we calculate the two-atom mixed state taking trace over the field variables. The joint two-atom mixed state density matrix in the basis of \(|e_1, e_2\rangle, |e_1, g_2\rangle, |g_1, e_2\rangle\) and \(|g_1, g_2\rangle\) states is given by

\[
\rho_{12} = \begin{pmatrix}
\alpha & 0 & 0 & 0 \\
0 & \gamma & \epsilon & 0 \\
0 & \epsilon^* & \delta & 0 \\
0 & 0 & 0 & \eta
\end{pmatrix},
\]

(16)

where

\[
\alpha = (r_1^2 + s_1^2)^2,
\]

\[
\gamma = (r_1^2 + s_1^2)(r_2^2 + s_2^2),
\]

\[
\delta = (r_2^2 + s_2^2)(r_1^2 + s_1^2),
\]

\[
\eta = (r_1^2 + s_1^2)(r_2^2 + s_2^2)
\]

and

\[
\epsilon = (r_2^2 + s_2^2)(r_1 - is_1)(r'_1 + is'_1).
\]

We compute the two-atom entanglement using the well-known measure of the entanglement of formation \([32, 33]\) given by

\[
E_F(\rho) = h \left( \frac{1 + \sqrt{1 - C^2(\rho)}}{2} \right),
\]

(17)

where \(C\) is called the concurrence defined by the formula

\[
C(\rho) = \max(0, \sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4})
\]

(18)
between the two atoms decreases with the increase of $\Delta$ or $\beta$.

The more interesting case is obtained when the detuning and the Stark shifts are of opposite signs. This is revealed in figure 3 where the two-atom entanglement $E_F$ versus $gt$ for $\Delta = \beta = 0$ is compared with $E_F$ when $\beta = -\Delta = 1$. We see that the magnitude of the entanglement between the two atoms increases when $\beta = -\Delta = 1$ with respect to the case when $\Delta = \beta = 0$ for certain values of the Rabi angle, e.g., $gt \approx 1$. Such enhancement of entanglement is observed in varying measures for other values of the Rabi angle also. Its origin lies in the presence of the photon number operator $a\dagger a$ in the effective Hamiltonian in equation (14) which makes the ‘effective’ two-photon detuning $\Delta + 2\beta a\dagger a$ dynamic as the cavity photon number oscillates in time. One sees over a range of interaction times that the atomic entanglement is in general sustained with variations in oscillatory behaviour with respect to $gt$, if the two-photon detuning $\Delta$ is compensated by the Stark shift. Or in other words, the maximum entanglement that can be obtained by varying $gt$ over a range of interaction times remains similar to that in the resonant case. This is in striking contrast to the case displayed in figure 2 where the entanglement reduces substantially with the increase of $\Delta$ and $\beta$, both having the same sign. It is interesting to note that the entanglement between two atoms is also preserved if we interchange the sign of $\Delta$ and $\beta$ but with the condition $\Delta + \beta = 0$. Overall, we find that the Stark shift acts as a control parameter for the atom–atom entanglement.

3. Atomic entanglement mediated by the thermal field

The thermal field is the most easily available radiation field, and so, its influence on the entanglement of spins is of much interest. The atomic entanglement mediated by the thermal field through single-photon processes has been studied earlier [10, 11, 15]. Since the thermal field is related to the temperature of the medium, photons of this field are naturally present inside the cavity. So it is not out of place to include the Bose statistics for the thermal field in our analysis. The field at thermal equilibrium obeying Bose–Einstein statistics has an average photon number at temperature $TK$, given by

$$\langle n \rangle = \frac{1}{e^{\beta n/TK} - 1}. \quad (20)$$

The photon statistics is governed by the distribution $P_n$ given by

$$P_n = \frac{\langle n \rangle^n}{(1 + \langle n \rangle)^{n+1}}. \quad (21)$$

This distribution function always peaks at zero, i.e., $n_{peak} = 0$. For a field in a thermal state, the joint two-atom-cavity state is obtained by summing over all $n$ and is given by

$$|\Psi_{12}\rangle = \sum_n A_n [(r_1 - is_1)^2|e_1, e_2, n) + (r_1 - is_1)(r_2 - is_2)|e_1, g_2, n + 2) + (r_2 - is_2)(r_1' - is_1')|g_1, e_2, n + 2) + (r_2 - is_2)(r_1' - is_1')|g_1, g_2, n + 4)], \quad (22)$$

where $P_n = |A_n|^2$ is the photon distribution function of the thermal field. The reduced mixed density matrix of the two atoms after passing through the thermal cavity field in the basis of $|e_1, e_2), |e_1, g_2), |g_1, e_2)$ and $|g_1, g_2)$ states is given by

$$\rho_{12} = \begin{pmatrix} \alpha_1 & 0 & 0 & 0 \\ 0 & \gamma_1 & \epsilon_1 & 0 \\ 0 & \epsilon_1^* & \delta_1 & 0 \\ 0 & 0 & 0 & \eta_1 \end{pmatrix}. \quad (23)$$

where

$$\alpha_1 = \sum_n P_n (r_1^2 + s_1^2)^2,$$

$$\gamma_1 = \sum_n P_n (r_1^2 + s_1^2)(r_2^2 + s_2^2),$$

$$\delta_1 = \sum_n P_n (r_2^2 + s_2^2)(r_1^2 + s_1^2),$$

$$\eta_1 = \sum_n P_n (r_2^2 + s_2^2)(r_1^2 + s_1^2)^2$$

and

$$\epsilon_1 = \sum_n P_n (r_1^2 + s_1^2)(r_1 - is_1)(r_1' + is_1').$$

We again compute the entanglement of formation of the joint two-atom state after it emerges from the cavity. Similar to the case of the vacuum cavity field considered in section 2, the thermal field mediates entanglement between the two atoms even though there is no direct interaction between them. This feature was also observed earlier in the context of the one-photon atomic transition process [10, 11, 15]. The variation of the magnitude of the two-photon entanglement versus the Rabi angle is displayed in figures 4, 5 and 6.

Our purpose here is to investigate the effect of Stark shift on atomic entanglement, and to this end we plot in figures 4 and 5 the entanglement of formation as a function of the Rabi angle $gt$ for different combinations of the two-photon detuning $\Delta$ and the Stark shift parameter $\beta$ when the thermal field has an average photon number $\langle n \rangle = 0.1$. We note that the variation in $E_F$ as a
The average thermal photon number \( \langle n \rangle \) is given by

\[
\langle n \rangle = \frac{\beta}{g}
\]

These characteristics are still noted for higher values of the Rabi angle. Figure 5 shows that if the two-photon detuning is compensated by the Stark shift (figure 5), the atomic entanglement can be increased compared to the resonant case with the increase of the Rabi angle. Figure 6 shows the possibility of using the dynamical Stark shift in controlling the atomic entanglement mediated both by the vacuum cavity field and by the thermal cavity field.

![Figure 5](image1.png)

**Figure 5.** \( E_F \) is plotted versus the Rabi angle \( g t \) for (i) \( \Delta / g = 0 \) and \( \beta / g = 0 \) (solid line), (ii) \( \Delta / g = -1 \) and \( \beta / g = 1 \) (dotted line). The average thermal photon number \( \langle n \rangle = 0.1 \).

![Figure 6](image2.png)

**Figure 6.** \( E_F \) is plotted versus the Rabi angle \( g t \) for \( \Delta / g = -2 \) and \( \beta / g = 2 \).

function of \( g t \) is similar (barring differences in magnitudes) to the case of the vacuum cavity field. From figure 4, we note again that as long as \( \Delta \) and \( \beta \) are of same sign, the magnitude of entanglement between the two atoms decreases compared to the resonant case with the increase of \( \Delta \) or \( \beta \). But, as seen from figure 5, atomic entanglement can be increased for particular values of \( g t \) by choosing \( \beta \) to be of opposite sign as \( \Delta \). Again, similar to the case of the vacuum cavity field, we find that if the two-photon detuning is compensated by the Stark shift (figure 5), the atomic entanglement can be sustained on average over a range of values of the Rabi angle. These characteristics are still noted for higher values of the average thermal photon number \( \langle n \rangle \). As seen from figure 6, only the magnitude of \( E_F \) is reduced with increase of average thermal photons. Thus, the Stark shift can be used to control the atomic entanglement mediated by the thermal field by preserving the maximum magnitude of entanglement obtained in a large range of atom–cavity interaction times.

It may be noted that figures 1–6 have cusps at the maximum entanglement. These are due to the typical dynamics of two-photon processes which may be inferred from the well-known quantum revivals in the atomic population that are rather sharp and compact [23–25]. In our scheme the atoms pass through the cavity one after the other and are thus affected by the above dynamics. Since the created atomic entanglement results from the interaction of successive atoms with the cavity photons, but without any direct interaction between the atoms themselves, the features of the above single-atom dynamics are reflected in the dynamics of atomic entanglement. This is the reason for the presence of sharp cusps in the figures.

4. Conclusions

In this paper we have investigated the possibility of the control of atomic entanglement by the Stark shift generated in atomic transitions inside cavities. To this end, we have considered a degenerate two-photon process in a ladder system and obtained an effective Hamiltonian which describes the interaction efficiently [24, 34]. The two-atom entanglement is shown to be mediated by the cavity field through which the two atoms pass successively without any spatial overlap between them [12–15]. We are able to use the two-photon detuning which comes out naturally in the method presented in section 2, to compensate the dynamic Stark shift to get the atom–atom entanglement. Through this method we have shown the possibility of using the dynamical Stark shift in controlling the atomic entanglement mediated both by the vacuum cavity field and by the thermal cavity field.

We have shown that the entanglement between the two atoms depends on the two-photon detuning and the Stark shift parameter. The magnitude of atomic entanglement quantified by the entanglement of formation diminishes with the increase of the detuning and the Stark shift. However, interestingly, we have found that such a trend could be reversed if the values of the detuning and the Stark shift are made to compensate each other. In the latter case the entanglement could even be enhanced compared to the resonant situation for particular values of the atom–photon interaction time. More generally, it has been shown that the maximum magnitude of entanglement generated over a range of values of the Rabi angle is nearly sustained if we set the values of the two-photon detuning and the Stark shift to be equal and opposite in sign.

The effects of photon statistics of the thermal field on the mediated entanglement [10, 11, 15] have also been studied. We have shown that the general characteristics of the atomic entanglement as a function of the photon detuning and the Stark shift parameter are maintained for the case of the thermal field inside the cavity. The above results are however subject to reservoir effects. Our earlier studies [12–15] have clearly reinforced the importance of taking into account dissipative processes in the context of generation of atomic entanglement in single-photon processes. However, inclusion of decoherence here in the context of two-photon processes involves a few technical complexities and is beyond
the scope of the present paper. We plan to address this aspect in a future study. The use of Stark shifts in some quantum information protocols has been suggested recently [28–31], and an experiment to demonstrate the enhancement of Rydberg atom interactions has actually been performed [35]. Our present study should motivate further investigations on the feasibility of using Stark shifts in the practical manipulation of quantum entanglement.

Appendix. Derivation of the effective Hamiltonian

The atom initially in the lower state $|g\rangle$ absorbs a photon and jumps to one of the intermediate levels $|i\rangle$ and from there moves to the upper level $|e\rangle$ by absorbing another photon. The return path of the atom from $|e\rangle$ to $|g\rangle$ is again via $|i\rangle$ by the emission of two photons to the same mode. To make the equations look simpler let us consider only one intermediate level $|i\rangle$. A microscopically correct Hamiltonian describing the above process can be written as

$$H = H_0 + H_1$$

where the interaction Hamiltonian $H_1$ is of the order of one-photon dipole interaction strength. The $H_0$ and $H_1$ can be written in operator form as

$$H_0 = \omega_e |e\rangle\langle e| + \omega_i |i\rangle\langle i| + \omega_g |g\rangle\langle g| + \omega_\alpha a^\dagger a$$

and

$$H_1 = g_1 \left(S'_e a + S'_g a^\dagger\right) + g_2 \left(S'_i a + S'_e a^\dagger\right)$$

respectively. $g_1$ and $g_2$ are coupling constants (in dipole approximation) for the one-photon interactions responsible for the transitions between the level $|g\rangle$ and $|i\rangle$ and that between $|e\rangle$ and $|i\rangle$ respectively. The atomic operators are given by $S'_e = |e\rangle\langle g|$ and $S'_g = |g\rangle\langle i|$ and similar definitions go for the operators involving the upper level $|e\rangle$. In the basis of the states $|e, n\rangle, |i, n + 1\rangle$ and $|g, n + 2\rangle$, allowed by the rotating-wave approximation, we can write the Hamiltonian in the form of a $3 \times 3$ matrix. The density matrix of the atom–field system obeys the equation

$$i \dot{\rho} = [H, \rho].$$

The equation of motion can be solved by one of the usual known methods, but, the derivations get more and more tedious as the number of intermediate levels increases. However, as these levels are removed far away from one-photon resonance, we can use this to obtain an effective Hamiltonian which efficiently describes the two-photon process. In doing so, we follow the method outlined in [24, 34]. We start by making a canonical transformation [24, 34] $\rho_e = e^{-iK} \rho e^{iK}$ where $K$ is time-independent and Hermitian. $\rho_e$ obeys the equation of motion

$$i \dot{\rho}_e = [H_{\text{eff}}, \rho_e]$$

where

$$H_{\text{eff}} = e^{-iK} He^{iK}$$

$$= H - i[K, H] - \frac{1}{2}[K, [K, H]] + \cdots$$

We know that the probability of one-photon transition is inversely proportional to the one-photon detuning $\omega_{ki} - \omega (k = e, g)$. Since this detuning is large, the one-photon transition probabilities are very small. In this situation, it is safe to retain terms up to second order in one-photon coupling constants. Then, retaining terms of the order of square of the coupling constants in the above expression, we have

$$H_{\text{eff}} = H_0 + H_1 - i[K, H_1] - \frac{1}{2}[K, [K, H_0]].$$

Since $K$ is arbitrary, we choose that

$$[K, H_0] = -iH_1,$$

from which we determine all the elements of $K$. This reduces the effective Hamiltonian to

$$H_{\text{eff}} = H_0 - i/2[K, H_1].$$

Written back in operator form, $H_{\text{eff}}$ takes the form

$$H_{\text{eff}} = [\Delta + (\beta_e + \beta_g)a^\dagger a]|S_e\rangle + \frac{1}{2}(\beta_e - \beta_g)a^\dagger a$$

$$+ g(S^+ a^2 + S^- a^2)$$

where the spin operators are $S_+ = |e\rangle\langle e| - |g\rangle\langle g|$, $S^- = |g\rangle\langle e|$ and $S^0 = |e\rangle\langle g|$. $\Delta$ is the two-photon detuning and is given by $\Delta = \omega_e - \omega_g - 2\omega$ and $g$ is the two-photon coupling constant having the form

$$g = \sum_i \frac{g_1g_2}{2} \sqrt{(n+1)(n+2)} \left[ \frac{1}{\omega_{ei} - \omega} - \frac{1}{\omega_{gi} - \omega} \right]$$

where $\omega_{ei} = \omega_e - \omega_\alpha$ and $\omega_{gi} = \omega_i - \omega_\alpha$. The Stark shifts associated with the levels $e$ and $g$ are, respectively,

$$\beta_e = \sum_i \frac{g_1^2}{\omega_{ei} - \omega}$$

and

$$\beta_g = \sum_i \frac{g_2^2}{\omega_{gi} - \omega}.$$

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