Passive advection in nonlinear medium∗

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Forced advection of passive tracer, \( \theta \), in nonlinear relaxational medium by large scale (Batchelor problem) incompressible velocity field at scales less than the correlation length of the flow and larger than the diffusion scale is considered. Effective theory explaining small scale scalar fluctuations is proven to be linear, asymptotic free (downscales from the scale of the pumping) and universal. Only three parameters are required to describe exhaustively the small scale statistics of scalar difference: two velocity-dependent ones, average and dispersion (\( \bar{\lambda} \) and \( \Delta \) respectively) of the exponential stretching rate of a trial line element, and \( \alpha \), standing for average rate of linear damping of small scale scalar fluctuations. \( \alpha \) is an explicit functional of potential characterized medium nonlinearity and amplitude of \( \theta^2 \) flux pumped into the system. Structure functions show an extremely anomalous, intermittent behavior: 

\[
\langle |\delta \theta|^q \rangle \sim r^{\xi_q}, \quad \xi_q = \min \left\{ q, \sqrt{\left[ \frac{\bar{\lambda}}{2} \right]^2 + \frac{2\Delta}{\bar{\lambda}} - \frac{1}{2}} \right\}.
\]

No dissipative anomaly is found in the problem.

I. INTRODUCTION

Turbulence is very nonequilibrium state of nature, which becomes stationary if energy is supplied permanently at large scales. To construct a theory of turbulence means to describe temporal and spatial distributions of velocity and of variety of different thermodynamic characteristics of the fluid, i.e. density, if turbulence is compressible, temperature, if thermo-advection is applied, relative concentration of components in case of multi-component (color) flow, magnetic field distribution in a conducting fluid etc. Dynamics of different fields describing a real turbulent flow is both nonlocal and nonlinear. We call the general situation active to emphasize the reciprocal character of interaction between velocity field and thermodynamic characteristic(s). However, sometimes the effect of a thermodynamic field on the velocity distribution is suppressed. It takes place, for example, if scales are separated: a typical spatio-temporal scale of velocity is much larger than one of a thermodynamic quantity. The case, when it is theoretically justified to neglect the effect of back reaction of thermodynamic field on velocity field in comparison with ones of advection and nonlinearity is called passive. The passiveness does not necessarily means linearity. Moreover, our objective is to study passive yet nonlinear situation.

We consider dynamics of a thermodynamic quantity \( \theta \),

\[
\frac{d}{dt} \theta = -\frac{\delta H(\theta)}{\delta \theta} + \phi(t; r),
\]

(1)

\[
H\{\theta\} \equiv \int dr \left[ \frac{\kappa}{2} (\nabla \theta)^2 + U(\theta) \right],
\]

(2)

where \( H\{\theta\} \) is a positive definite thermodynamic functional of the system, \( U(\theta) \) is confined (\( U \to +\infty \) at \( \theta \to \pm \infty \)) potential, \( \kappa \) is diffusion coefficient ; \( \phi(t; r) \) stands for statistically steady forcing providing a constant supply of otherwise relaxational \( \theta \)-dynamics at large scales. \( \theta \) is imbedded in a turbulent flow, i.e. the temporal derivative is extended by sweeping term

\[
\frac{d}{dt} = \frac{\partial}{\partial t} + u \nabla r,
\]

(3)

where incompressible velocity field \( u(t; r) \) is prescribed to be known statistically.

We aim at finding the statistics of the passive scalar \( \theta \) fixed by (1) in the inertial interval of scales i.e. for scales that are less than both the velocity correlation scale, \( L_u \), and the scale of the scalar supply, \( L \), and larger than the

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†We will discuss here the simplest case possible, when the thermodynamic field is a scalar. Notice, however, that generalization of the discussed theory for a vector or generally tensorial object is possible.
diffusion scale. Incompressible velocity field at those scales is modeled by the first term of its local expansion in the radius vector connecting a reference point with the current one,
\[ u(t; r) = \dot{\sigma}(t) r, \]
where \( \dot{\sigma}(t) \) is \( d \times d \) traceless random matrix of velocity’s derivatives.

The problem \([17,18]\) describes forced advection of a scalar pollutant in the viscous-convective range absorbed or generated depending on the sign of the nonlinear rate \( \partial_u^2 U(\theta) \), for example via a chemical reaction with other species presented in abundance in the flow. The problem is of fundamental importance for geophysical atmospheric turbulence (see [2] for review). Other relevant phenomenon is turbulent thermo-advection in a cell attached to thermal bath (see \([1] \) for review). Other relevant phenomenon is turbulent thermo-advection in a cell attached to thermal bath (see \([1] \) for review). The last (but not the least) situation to mention is a phase ordering in a system described by scalar nonconserved order parameter (a very well known object of the phase transition theory, see \([4–7] \) for reviews) adveected by large scale turbulent flow (see also \([3] \) on some discussion of reciprocal description of advection and critical dynamics).

Our consideration will be based essentially on understanding, results and general terminology emerged from studies of the pure problem of passive scalar advection (no medium effect at all, \( U = 0 \)) having an almost five decades of history (see Obukhov and Corrsin papers \([8,9] \) for the earliest contributions). Batchelor \([11] \) has pioneered study of the smooth velocity field limit \([1]\), which nowadays has grown to be (through important contributions of many people \([12,13]\) one of the most advanced theory in the field. Temporal short-correlated but spatially non-smooth model of velocity, one which gave more than two decades later the first ever analytical evidence of intermittency in turbulence was invented by Kraichnan. Structure functions of scalar difference in the convective range,
\[ S_q(r) = \langle (\theta(t; r) - \theta(t; 0))^q \rangle \sim r^{\xi_q}, \]
became the key object in the intermittency study. The anomalous scaling, \( \Delta_{2n} \equiv n \xi_2 - \xi_{2n} \), describing the law of the algebraic growth with \( L/r \) of the dimensionless ratio, \( S_{2n}(r)/|S_2(r)|^n \), was shown to exist generically \([24,25]\). The anomalous exponents were calculated perturbatively in expansions about three non-anomalous ( \( \Delta_{2n} = 0 \) ) limits, of large space dimensionality \( d \) \([24,27]\), of extremely non-smooth \([24,25]\) and almost smooth \([26]\) velocities respectively. A strong anomalous scaling (saturation of \( \xi_{2n} \) to a constant) was found for the Kraichnan model at the largest \( n \) by a steepest descent formalism \([24,30]\). Although the restricted asymptotic information about anomalous exponents in the model is available a future possibility to establish rigorously complete dependence of \( \xi_{2n} \) on \( n, d \) and degree of velocity non-smoothness seems very unlikely (in a sense, recent Lagrangian numerics \([31]\) compensates the lack of rigorous information).

The problem \([14,15]\) discussed in the present paper is also showing anomalous scaling, \( \xi_{2n} < n \xi_2 \), which is resolved analytically for any kind of temporal correlations in \( \dot{\sigma} \) and an arbitrary (yet confined at \( \theta \to \pm \infty \)) potential \( U(\theta) \). An important circumstance helping to find the general answer is scale separation between scalar fluctuations at the integral scale, \( L \), and in the current one, \( r \), from convective range. For general \( U(\theta), \partial_{\theta} \) obeys the same statistics as one would expect from an auxiliary (linear!) problem with quadratic potential, \( U^*(\theta) = \alpha \theta^2 \), where \( \alpha \) is given by average of \( \partial_\theta^2 U(\theta) \) with respect to single point scalar distribution, \( \mathcal{P}_1 \sim \exp[-U(\theta)/\chi_0] \), with \( \chi_0 \equiv \int_0^\infty dt \langle \delta(\theta(t); 0) \delta(\theta(0); 0) \rangle \). \( \alpha \) is always positive, i.e. at the smallest scales the effect of nonlinearity, generally alternating between damping and acceleration, is reduced to a pure linear damping. Linear problem, by itself, happens to be solvable not only in the case of the short-correlated velocity \([32]\) but for \( \dot{\sigma}(t) \) statistics of a general position (providing correlation functions of \( \dot{\sigma}(t) \) taken at different moments of time decays with the time shift faster than algebraically). Finally, we have found anomalous exponents
\[ \xi_q = \min \left\{ q, \sqrt{\frac{\lambda^2}{\Delta}} + \frac{2aq}{\Delta} - \frac{\lambda}{\Delta} \right\}, \]
where \( \lambda \) and \( \Delta \) are respectively average and dispersion (with respect to \( \dot{\sigma} \) averaging) of the rate of line stretching, \( \lambda(t) = t^{-1} \ln[R(t)/R(0)] \), with \( R(t) \), satisfied to \( \dot{R}(t) = \dot{\sigma}(t) R(t) \). To make the statement we used the method of \([17,18]\).

The anomalous behavior in the model under consideration differs from one perceived in the Kraichnan model. First of all, \( \xi_{2n} \) as a function of \( n \) does not saturate to a constant at the largest \( n \) but keeps growing with \( n \) as \( \sqrt{n} \). Second (and major) difference is associated with the concept of dissipative anomaly. It is generally accepted to talk about dissipative anomaly if some object calculated at zero dissipation (\( \kappa = 0 \)) does not coincide with its \( \kappa \to 0 \) counterpart.
In the Kraichnan model the anomalous scaling coexists with dissipative anomaly \cite{23}. However, the nonlinear problem \cite{3-4}, as well as its linear descendant, shows no dissipative anomaly while the anomalous scaling is present. We base the important conclusion on the following no anomaly criterium (which, we believe, is general): if zero dissipation analysis produces normalizable and everywhere positive solution for the Probability Density Functional (PDF) of fluctuated field (θ in our case) then the dissipative anomaly is absent.

The problem is formulated in Section II. To describe the scalar fluctuations at a current scale from convective range we show how to integrate out the large scale contribution in Section III. The scale separation results in suppression of nonlinearity. The effective small scale theory appears to be a linear one with uniform damping. All the final answers emerging from study of the linear problem are presented in Section IV. The last Section V is reserved for conclusions.

II. FORMULATION OF THE PROBLEM

\cite{3} describe advection of a passive scalar θ(t; r) by the smooth incompressible velocity field \cite{4}. The scalar is forced by random field φ(t; r), which for a sake of simplicity is considered to be Gaussian thus fixed unambiguously by

$$\langle \phi(t_1; r_1) \phi(t_2; r_2) \rangle = \chi(|r_1 - r_2|) \delta(t_1 - t_2),$$

where the function χ(r) decays fast enough if r exceeds the integral scale L ≤ L_u. χ₀ = χ(0) is the flux of θ² pumped into the system. δ is a random in time matrix process described by its PDF, Φ{δ(t)}, which is supposed to be known. Diffusion is supposed to be small, such that the range of scales in between r_d = √κ/[S/τ]¹/₄ (S and τ are typical values of the strain and velocity correlation time respectively) and L is sufficiently large, L/r_d ≫ 1.

We will be mainly aiming to find the two-point scalar PDF,

$$\mathcal{P}_2(x_+, x_- | r) \equiv \langle δ(x_+ - θ(t; 0) + θ(t; r)) \delta(x_- - θ(t; 0) - θ(t; r)) \rangle,$$

and the scalar structure functions,

$$\mathcal{S}_{2n}(r) \equiv \langle [θ(t; r) - θ(t; 0)]^{2n} \rangle,$$

where averaging, ⟨⋯⟩, with respect to both δ(t) and φ(t; r), is assumed.

Other important objects used in the course of the forthcoming calculations will be the two point scalar PDF, measured for particular δ(t) configuration,

$$\mathcal{G}_2(x_1, x_2 | r_1, 2; t; {δ(t')}; -∞ ≤ t' ≤ t) \equiv \langle δ(x_1 - θ(t; r_1)) \delta(x_2 - θ(t; r_2)) \rangle_φ,$$

and the single point scalar PDF

$$\mathcal{P}_1(x) \equiv \langle δ(x - θ(t; r)) \rangle.$$

Notice, that the last object does not depend on the velocity field statistics because of spatial homogeneity assumed.

Deep inside the convective range (at L ≫ r₁₂), |θ₁ - θ₂| ≪ |θ₁ + θ₂|, and \cite{4} can be decomposed into the product

$$\mathcal{G}_2(θ₁, θ₂ | r_1, 2; t; {δ(t')}; -∞ ≤ t' ≤ t) = \mathcal{P}_1(θ₁) * \mathcal{G}_-(θ₁ - θ₂ | r_1 - r₂; t; {δ(t')}; -∞ ≤ t' ≤ t).$$

The average of \cite{12} over δ reads as

$$\mathcal{P}_2(θ₁, θ₂ | r₁₂) = \mathcal{P}_1(θ₁) * \mathcal{P}_-(θ₁ - θ₂ | r₁₂).$$

The assumption on the absence of the dissipative anomaly in the case of a very small diffusion lies in the core of our consideration. The formal consequence of the statement is the possibility to drop off the dissipative κ-dependent term from \cite{3} already on the dynamical (yet unaveraged) level. The no-anomaly assumption will be justified by the positivity and normalizability of the derived answers for PDFs.
III. REDUCTION OF THE NONLINEAR PROBLEM TO A LINEAR ONE

In the absence of diffusion (13) can be integrated along the Lagrangian trajectories (characteristics)

\[
\frac{d}{dt} \theta(t'; \rho(t')) = - \frac{dU}{d\theta} \Big|_{\theta(t'; \rho(t'))} + \phi(t'; \rho(t')) ,
\]

\[
\frac{d}{dt} \rho(t') = \dot{\sigma}(t') \rho(t') , \quad \rho(t) = r , \quad -\infty < t' < t .
\]

Notice, that the nonlinearity leads to scalar generation in the region of convex (\(\partial_{\theta} U > 0\)) potential while it dumps scalar fluctuations if \(\partial_{\theta} U < 0\). Fokker-Planck equations (see [33] for similar calculations) derived out of (14,15) by means of direct averaging over the Gaussian noise, \(\phi\), are

\[
\left[ \dot{\theta} + \sum_{i=1,2} \left( \sigma^{\mu \nu}(t) r_i^\mu \partial_r^\nu \theta - \partial_{\theta_i} \frac{dU(\theta_i)}{d\theta} \right) - \sum_{i,j=1,2} \chi(r_i - r_j) \partial_{\theta_i} \partial_{\theta_j} \right] G_2 = 0 ,
\]

where \(G_2\) is not stationary, since it does depend on time explicitly through \(\dot{\sigma}(t)\). Integrating \(G_2\) with respect to \(\theta = \theta_1 + \theta_2\) and assuming that the integral is formed at \(|\theta_1 - \theta_2| \ll |\theta_1 + \theta_2|\), where (22) is valid, we arrive at the close equation for the scalar difference PDF,

\[
\{ \dot{\theta} + (\sigma^{\mu \nu}(t) r_i^\mu \partial_r^\nu \theta - \alpha \partial_x x) - 2 [\chi(0) - \chi(r)] \partial^2 \theta \} G_-(x; t; \{\dot{\sigma}(t'); -\infty \leq t' \leq t \}) = 0 ,
\]

where \(\alpha\) is defined as the following average over the large scale \(\theta\) statistics

\[
\alpha \equiv \left( \frac{d^2 U(\theta)}{d\theta^2} \right)_{LS} \equiv \int \limits_{-\infty}^{\infty} d\theta \frac{d^2 U(\theta)}{d\theta^2} P_1(\theta) .
\]

The normalized and everywhere positive solution of (16) is

\[
P_1(\theta) = \frac{\exp[-U(\theta)/\chi_0]}{\int \limits_{-\infty}^{\infty} d\theta \exp[-U(\theta)/\chi_0]} .
\]

Substitution of (20) into (19) gives

\[
\alpha = \left( \frac{dU(\theta)}{d\theta} \right)_{LS}^{2} \chi_0 = \int \limits_{-\infty}^{\infty} d\theta \left( \frac{dU(\theta)}{d\theta} \right)^2 \exp[-U(\theta)/\chi_0] \chi_0 \int \limits_{-\infty}^{\infty} d\theta \exp[-U(\theta)/\chi_0] ,
\]

i.e. \(\alpha\) is principally positive constant, does not matter what is the form of the potential \(U(\theta)\) be (provided it grows with \(\theta\) at \(\theta \to \pm \infty\)). Therefore, we have found that at the smallest scales regions of scalar generation are suppressed statistically.

On the basis of (16) and (18) we conclude that, from the point of view of the small scale statistics of scalar difference our problem is equivalent to the linear one, with \(dU(\theta)/d\theta\) being replaced just by \(\alpha \theta\). In other terms, we may proceed averaging the linear dynamical equation

\[
\partial_t \theta + \sigma^{\mu \nu}(t) r_i^\mu \nabla_r^\nu \theta = -\alpha \theta + \phi(t; r) ,
\]

instead of the original nonlinear one. The steady distribution of the scalar difference underlined (22) was the subject of the recent paper [32], the method and results of which will be briefed and generalized (for the case of finite correlated velocity) in the next Section.
IV. VELOCITY AVERAGING. ANOMALOUS SCALING.

The linear analog of (14) is

\[ \theta(t; r) = \int_0^\infty dt' \exp[-\alpha t'] \phi(t'; \rho(t-t')). \]  

(23)

For the purpose of the 2n-th structure function calculation it is utmost enough to consider the simultaneous product, \( F_{1 \cdots 2n} = \langle \theta_1 \cdots \theta_{2n} \rangle \), which is according to (23) is

\[ F_{1 \cdots 2n} = \sum_{\{i_1, \cdots, i_{2n}\}} \langle \prod_{k=1}^n dt_k e^{-\alpha t_k} \chi \left[ \hat{W}(t_k; r_{i_k; i_{k+1}}) \right] \rangle, \]

(24)

\[ \hat{W}(t) = T \exp \left[ \int_0^t dt' \hat{\sigma}(t') \right], \quad \frac{d\hat{W}(t)}{dt} = \hat{\sigma}(t)\hat{W}(t). \]

(25)

Calculation of \( F_{1 \cdots 2n} \) is essentially simplified for the collinear configuration, \( r_i = n \) \( r_i \), when the \( 2n \times (d-1) \) parametric average (24) is reduced to the following single-parametric one

\[ F_{1 \cdots 2n} = \sum_{\{i_1, \cdots, i_{2n}\}} \langle \prod_{k=1}^n dt_k e^{-\alpha t_k} \chi \left[ e^{\eta(t_k)} r_{i_k; i_{k+1}} \right] \rangle, \]

(26)

with \( r_{ij} \equiv |r_i - r_j| \). Here, the longitudinal stretching rate, \( \eta(t) \equiv \ln|\hat{W}(t)n| \), is the only fluctuating quantity left. The \( \alpha = 0 \) version of (26) was calculated in [17] for the \( d = 2 \) case and generalized for any \( d \geq 2 \) in [18] via a change of variables and further straightforward transformation of the path integral standing for the average over \( \hat{\sigma}(t) \). It is shown [17,18] that at the largest times \( \eta \)-measure is a shifted Gaussian one,

\[ D\eta(t) \exp \left[ -\int_0^\infty dt \frac{(\eta - \bar{\lambda})^2}{2\Delta} \right], \]

(27)

characterized by two parameters only: average, \( \bar{\lambda} \), and dispersion, \( \Delta \), of the Lyapunov exponent, \( \int_0^t dt' \eta(t')/t \). (27) applied to (26) produces

\[ \frac{F_{1 \cdots 2n}}{n!} = \int \left[ \prod_{i=1}^n dt_i d\eta_i \right] \exp \left[ \frac{\bar{\lambda}}{\Delta} \eta_i - \frac{\bar{\lambda}^2}{2\Delta} t_i \right] \sum_{\{k_1, \cdots, k_{2n}\}} \prod_{i=1}^n \left( e^{2\alpha t_i} \chi \left( e^{\eta_i} r_{k_2, k_{2n}} \right) G(t_{i-1,i}; \eta_{i-1,i}) \right), \]

(28)

where \( \eta_i (i \leq n) \) integrations are not restricted, \( 0 \leq t_n \leq \cdots \leq t_1 \leq \infty, t_{n+1} = \eta_{n+1} = 0, t_{i,k} \equiv t_i - t_k \) (with equivalent notations for \( \eta \)) and

\[ G(t; \eta) \equiv \frac{\exp \left[ -\frac{\eta^2}{2\Delta} \right]}{\sqrt{2\pi\Delta}}. \]

(29)

Both \( \bar{\lambda} \) and \( \Delta \) are unambiguously fixed by \( \Phi\{\hat{\sigma}(t)\} \). The integrand of (28) decays exponentially in time with the major contribution into the integral formed at \( t_i \sim 1/\alpha \). The leading term does not depend on any \( r_{ij} \) and gives no contribution into \( 2n \)-th order structure function. The first actual \( r \)-dependent contribution stems from \( n-1 \) temporal integrals formed at \( \tau \sim 1/\alpha \), and one at \( t_i \sim \tau \sim \ln[L/r]/\max[\alpha, D] \). This special integration brings a spatial dependence into the object, therefore on a single distance. Generally, there exists a variety of terms with all the possible combinations, like term with \( k \) integration formed at \( \tau \), while \( n-k \) ones at \( \tau_r \), and therefore dependent explicitly on \( 2(n-k) \) spatial points. However, we are looking exclusively for a term dependent on all the \( 2n \) points since only such a term contributes \( S_{2n}(r) \). It is really simple to calculate the scaling of this term making use of the temporal separation, \( \tau_r \gg \tau \). Indeed, the large time contribution may be extracted out of (28) in a saddle-point
calculation. Variation of all the exponential terms in (28) with respect to $t_i$ gives a chain of saddle equations. The $\chi$ functions in the integrand of (28) limits the $\eta$ integrations from above by $\ln [L/r]$. Therefore, the desirable $2n$-points contribution forms at $t_i = \sqrt{\lambda/ [\Delta (2\alpha n \Delta + \lambda^2/2)]} \ln [L/r]$, and $\eta_l = \ln [L/r]$, where it is assumed that in the leading logarithmic order there is no need to distinguish between contributions of different separations $r_{ij}$. Substituting the saddle-point values of $t_i$ and $\eta_l$ into (28) one arrives at the anomalous part of (31), with $q = 2n$. The normal-scaling counterpart of (31) originates from expansion of $\chi(r)$ from (28) in a regular series in $r^2$.

The basic physics of nonzero $\xi_{2n}$ (means deviating from the naive balance of pumping and advection) and generally anomalous ($\xi_{2n} < n\xi_2$) scaling at $\alpha > 0$ can be stated quite clearly. According to (23) the advection changes scales but not amplitude, while the amplitude of injected scalar field decays exponentially from the time of injection at the constant rate $\alpha$. The temporal integrals in (28) forms at the mean time to reach a scale which is proportional to the negative log of the scale. However, the effective spread in the factor by which amplitude has decayed, upon reaching a given scale, increases as scale decreases. It is why $\xi_{2n} > 0$. Also there is more room for fluctuations about the mean time due to the interference of the scalar amplitude and fluctuations of the stretching rate $\eta$. Thus intermittency increases with a scale size decrease.

Another way to derive (31) out of (23) is to construct $\delta_{2n}(r)$ directly. It is easy to check that the structure functions of different orders are produced by the PDF satisfied to

$$\tilde{\lambda} 1^{-2\lambda/\Delta} \partial_r r^{1-2\lambda/\Delta} \partial_r \mathcal{P}_- + \alpha \partial_x (x \mathcal{P}_-) + [\chi_0 - \chi(r)] r^2 \partial_x^2 \mathcal{P}_- = 0,$$

(30)

The solution of (30), in the regime where you can neglect the $\chi$-dependent term is

$$\mathcal{P}_-(x | r) = \frac{1}{2\pi i \theta L} \int_{0^+ + \infty} ds \left[ \frac{\theta s}{x} \right]^{s+1} \left[ \frac{r}{L} \right]^\sqrt{d^2/4 + \alpha s/[D(d-1)] - d/2} a_s. \quad (31)$$

Here, $a_s$ is a function fixed by matching at the integral scale, roughly, $\mathcal{P}_-(x | L) \sim \mathcal{P}_1(x)$, where $\mathcal{P}_1(x)$ is given by (24). The PDF (31) is positive and normalizable, that, therefore, confirms the initial hypothesis on the absence of dissipative anomaly. Also, (31) shows that (31) holds for general (not only even integer) positive $q$.

V. CONCLUSION

We have shown that the nonlinear problem (14) is reduced to a linear one at the smallest (still from convective, not dissipative, range) scales. The asymptotic theory remembers about initial nonlinearity through the effective damping coefficient $\tilde{\lambda}$. The linear problem was solved for the general case of arbitrary correlated in time large scale velocity field.

The most important feature of the problem appears to be the absence of dissipative anomaly, the point which was guessed initially. Selfconsistency of the hypothesis was confirmed afterwards by checking positivity and normalizability of the final expression (31) for PDF. Of course, the absence of dissipative anomaly is, by no means, a common situation in turbulence. It is however suggestive to start analyzing any new turbulent problem from the simple "no anomaly" test.

We discussed only the Batchelor case of large scale velocity. The simplification appears to be a very important both for the fact of absence of dissipative anomaly and solvability of the problem. The Batchelor case is very special, since the Lagrangian dynamics of $n$ particles, generally described by $n(d-1)$ degrees of freedom, is reduced to dynamics of $d - 1$ eigenvalues of stretching matrix. This lies in the core of the Batchelor problem’s solvability. Also, in the Batchelor case scaling dimension of eddy diffusivity operator coincides with one of the $\alpha$ (damping) dependent term. The coincidence of exponents explains the anomalous scaling, particularly the continuous dependance of the exponents on $\alpha$. Any multiscale velocity field (say taken from the Kraichnan model) leads, first, to appearance of the dissipative anomaly already on the medium free ($U = 0$) level, and second to dis-balance of the scaling dimensions of advective and medium-originated contribution into the eddy-diffusivity operator, resulting in complete screening of any medium effect in the convective range. We conclude by this guess, which is rather brave (and, of course, is not rigorous at all). More studies, first of all on the nature of dissipative anomaly, are required in this direction.

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