Pairing correlations and effective mass

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Abstract

We study the effect of effective mass on pairing correlations in the ground states of superfluid nuclei \textsuperscript{124}Sn and \textsuperscript{136}Sn. Various parameter sets of Skyrme interactions and relativistic Lagrangians are adopted to study pairing correlations across a wide range of effective mass. It is shown that surface-type pairing interaction gives an almost constant pairing gap as a function of the effective mass, while volume-type pairing interaction shows rather strong dependence of the pairing gap upon the effective mass. The local pair potentials of various effective interactions are also examined in relation to the effective mass.

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I. INTRODUCTION

Pairing correlations have been known as one of the most important correlations in finite nuclei and also infinite nuclear matter [1, 2]. Pairing correlations play an especially crucial role in providing realistic descriptions of the saturation properties and the excitation spectra of open shell nuclei. As an effective pairing interaction, volume-type and surface-type pairing interactions have often been used in Hartree-Fock (HF)+BCS and HF+Bogoliubov (HFB) calculations of medium-heavy and heavy nuclei [3, 4]. Recently, these pairing interactions have been applied not only to stable nuclei but also to unstable nuclei far from the stability line. It has been known that the pairing correlations are much affected by the level density around the Fermi energy, which is essentially determined by the effective mass [5]. The level density may also change from stable nuclei to unstable nuclei near the drip lines where new shell closures appear.

Many different parameter sets are available for mean field theories, such as Skyrme Hartree-Fock (SHF) and relativistic mean field (RMF) models. These parameter sets show a large range of the effective mass $m^*/m = (0.4 \sim 1.0)$. However, the effect of effective mass on the pairing correlations has not yet been studied systematically using different mean field models. In this article, we examine how much the pairing correlations are influenced by the effective mass of adopted interaction. To this end, we take two $\delta$-type pairing interactions, volume-type and surface-type, and apply them to Sn isotopes in SHF+BCS and RMF+BCS models. Two isotopes, $^{124}$Sn and $^{136}$Sn, are chosen for detailed study since they are spherical and accurate experimental data of binding energies is available. This paper is organized as follows. We give a brief summary of the mean field models for the study of pairing correlations in Section II. Section III is devoted to discussion of results of numerical calculations. The summary is presented in Section IV.

II. MEAN FIELD MODELS FOR PAIRING CORRELATIONS

We study the relation between the averaged pairing gap energy and the effective mass in the SHF+BCS and RMF+BCS models. The Skyrme interaction $V_{Sky}$ is an effective
zero-range force with density- and momentum-dependent terms [6],

\[
V_{\text{Sky}}(\vec{r}_1, \vec{r}_2) = t_0(1 + x_0 P_\sigma)\delta(\vec{r}_1 - \vec{r}_2) + \frac{1}{2}t_1(1 + x_1 P_\sigma)\left\{ \vec{k}'^2 \delta(\vec{r}_1 - \vec{r}_2) + \delta(\vec{r}_1 - \vec{r}_2)k^2 \right\} \\
+ t_2(1 + x_2 P_\sigma)\vec{k}' \cdot \delta(\vec{r}_1 - \vec{r}_2)k + \frac{1}{6}t_3(1 + x_3 P_\sigma)\rho^a(\vec{r})\delta(\vec{r}_1 - \vec{r}_2) \\
+ iW(\vec{\sigma}_1 + \vec{\sigma}_2) \cdot \vec{k}' \times \delta(\vec{r}_1 - \vec{r}_2)k, \tag{1}
\]

where \( \vec{k} = (\vec{\nabla}_1 - \vec{\nabla}_2)/(2i) \), acting on the right, and \( \vec{k}' = -(\vec{\nabla}_1 - \vec{\nabla}_2)/(2i) \), acting on the left, are the relative-momentum operators, \( P_\sigma \) is the spin exchange operator, \( \vec{\sigma} \) is the Pauli spin matrix, and \( \vec{r} = \frac{1}{2}(\vec{r}_1 + \vec{r}_2) \). The interaction (1) simulates the G-matrix for nuclear Hartree Fock calculations. We use 13 different Skyrme parameter sets (SI, SIII, SIV, SVI, Skya, SkM, SkM*, SLy4, MSkA, SkI3, SkI4, SkX, SGII), taken from Refs. [7–16]. The effective mass of SHF can be expressed analytically as

\[
\frac{\hbar^2}{2m^*_q(r)} = \frac{\hbar^2}{2m} + \frac{1}{8} \left[ t_1(2 + x_1) + t_2(2 + x_2) \right] \rho(r) - \frac{1}{8} \left[ t_1(1 + 2x_1) - t_2(1 + 2x_2) \right] \rho_q(r), \tag{2}
\]

where \( \rho_q(r) \) is the neutron \( (q = n) \) or proton density \( (q = p) \), and \( \rho(r) = \rho_n(r) + \rho_p(r) \). In symmetric nuclear matter, Eq. (2) gives

\[
\frac{m^*_\text{SHF}}{m} = \left[ 1 + \frac{m\rho_0}{8\hbar^2} \left\{ 3t_1 + t_2 (5 + 4x_2) \right\} \right]^{-1}, \tag{3}
\]

where \( \rho_0 \) is the saturation density. As RMF Lagrangians, we adopt the non-linear \( \sigma \) model with parameter sets NLSH, NL3, and NLC [17–19]. The effective mass of RMF can be expressed as [20]

\[
m^*_\text{RMF} = m - \frac{1}{2}(V - S), \tag{4}
\]

where \( V \) and \( S \) are the vector and scalar potentials. For the RMF Lagrangian with \( \sigma-, \omega-, \) and \( \rho- \) meson fields, the scalar potential is given by the \( \sigma- \) potential, while the vector potential depends upon the \( \omega- \), and \( \rho- \), and Coulomb potentials. Note that the effective mass \( m^*_\text{RMF} \) in Eq. (4) is different from so-called “Dirac mass” given by \( m^*_\text{Dirac} = m + S \) [20].

In this study, we adopt two types of pairing interactions, surface- and volume-type, in the mean field calculations. The pairing interaction \( V_\text{pair} \) is generally defined by the equation

\[
V_\text{pair}(r) = -V_0 \left[ 1 - \beta \left( \frac{\rho(r)}{\rho_0} \right) ^\gamma \right], \tag{5}
\]

where \( V_0, \beta \) and \( \gamma \) are parameters to be determined by the model. The value of \( \beta \) is fixed to be 0 and 1 for the volume- and surface-type pairing interactions, respectively. We also use
the values \( \rho_0 = 0.16 \text{ fm}^{-3} \) and \( \gamma = 1 \). The pairing strength \( V_0 \) is chosen to be 999 MeV·fm\(^{-3}\) for the surface-type pairing interaction and 323 MeV·fm\(^{-3}\) for the volume-type [4]. These pairing strength parameters were originally determined to reproduce empirical pairing gaps of medium heavy nuclei with a Skyrme interaction SkI4. We adopt these values as a reference in the following calculations with all adopted Skyrme interactions and also RMF Lagrangians. There is some freedom in the selection of values for \( \beta \) and \( \gamma \). It has been pointed out that the mixed-type pairing with \( \beta = 1/2 \) exhibits good properties to describe the pair density [21]. A weak density dependence is also suggested by the realistic nucleon-nucleon interaction for the pairing channel [22]. Since it is not the main goal of this article to study the effect of parameters \( \beta \) and \( \gamma \) on the pairing field, we adopt the surface- and volume-type pairing interactions which have been widely used in the literature.

The pairing correlations are taken into account by BCS approximation in this study. Since the two-neutron separation energies \( S_{2n} \) of two nuclei \(^{124}\text{Sn} \) and \(^{136}\text{Sn} \) are more than 5 MeV, BCS approximation is considered to be a good model for evaluating the pairing correlations in the ground states [13, 23]. The state-dependent gap strength \( G_{ij} \) is defined by

\[
G_{ij} = -\int d\mathbf{r} \, V_{\text{pair}}(\mathbf{r}) |\phi_i(\mathbf{r})|^2 |\phi_j(\mathbf{r})|^2,
\]

where \( \phi_i \) is a mean field single-particle wave function. The pairing gap energy \( \Delta_i \) is obtained by solving the following BCS gap equation,

\[
\Delta_i = \frac{1}{2} \sum_j f_j \frac{G_{ij} \Delta_j}{\sqrt{(\varepsilon_j - \lambda)^2 + \Delta_j^2}},
\]

where \( \varepsilon_j \) and \( \lambda \) are the single particle and the Fermi energy, respectively, and \( f_j \) is a smooth cut-off factor for the continuum states, defined by

\[
f_j = \frac{1}{1 + e^{(\varepsilon_j - \lambda - \mu)/\Delta E}},
\]

where \( \mu \) and \( \Delta E \) are fixed to be 5.0 and 0.5, respectively. The occupation probability \( v_i^2 \) is defined by

\[
v_i^2 = \frac{1}{2} \left[ 1 - \frac{\varepsilon_i - \lambda}{\sqrt{(\varepsilon_i - \lambda)^2 + \Delta_i^2}} \right].
\]

We show the correlation between the effective mass and the averaged neutron pairing gap energy \( \bar{\Delta} \) of \(^{124}\text{Sn} \) and \(^{136}\text{Sn} \) in Figs. 1 and 2, respectively. There are two definitions of the
averaged pairing gap energy $\bar{\Delta}$,

$$\bar{\Delta}_{v^2} = \frac{\sum_{i\in n} v_i^2 \Delta_i}{\sum_{i\in n} v_i^2}, \tag{10}$$

$$\bar{\Delta}_{uv} = \frac{\sum_{i\in n} u_i v_i \Delta_i}{\sum_{i\in n} u_i v_i}, \tag{11}$$

where $u_i = \sqrt{1 - v_i^2}$. In the next section, we discuss the averaged pairing gap energies $\bar{\Delta}_{v^2}$ and $\bar{\Delta}_{uv}$ for neutrons.

### III. RESULTS OF SHF+BCS AND RMF+BCS CALCULATIONS

As shown in Fig. 1(a), the averaged neutron pairing gap energies $\bar{\Delta}_{v^2}$ and $\bar{\Delta}_{uv}$ with the surface-type pairing interaction are almost independent of the effective mass with only minor variation. Contrarily, the volume-type pairing shown in Fig. 1(b) has a strong effective mass dependence. Specifically, Fig. 1(b) reveals a correlation between the effective mass and the gap energy, i.e., a larger effective mass yields a larger averaged neutron pairing gap energy. We can see also that, in the surface-type pairing interaction, the averaged neutron pairing gap energies $\bar{\Delta}_{uv}$ are, on the average, 300 keV larger than $\bar{\Delta}_{v^2}$. In contrast, the pairing gap energies $\bar{\Delta}_{v^2}$ are 100 keV larger than $\bar{\Delta}_{uv}$ in the volume-type pairing interaction. As shown in Fig. 2, the results of $^{136}$Sn have almost the same characteristic features as those of $^{124}$Sn. Quantitatively, the dependence of the averaged neutron pairing gap upon the effective mass in $^{136}$Sn is somewhat stronger than that of $^{124}$Sn. No substantial difference is observed between SHF and RMF in Figs. 1 and 2 regarding the correlation between $\bar{\Delta}$ and the effective mass.

Figures 3 and 4 show the local pairing potentials of neutrons $\Delta_n(r)$ for $^{124}$Sn and $^{136}$Sn, respectively, in the coordinate space. The local pair potential of neutrons is given by

$$\Delta_n(r) = -V_{\text{pair}}(r) \sum_{i\in \Omega_n, i>0} u_i v_i |\phi_i(r)|^2. \tag{12}$$

The upper panels show the pair potentials of surface-type pairing, while the lower panels show those of volume-type pairing with the Skyrme interactions SVI ($m^*/m = 0.949$), MSkA ($m^*/m = 0.794$), and SIV ($m^*/m = 0.471$). These interactions are denoted by the numbers 4 (SVI), 9 (MSkA), and 3 (SIV), respectively, in Figs. 1 and 2. The local pair potential
\( \Delta_n(r) \) gives the pairing gap energy \( \Delta_i \) for a single particle state \( i \) as

\[
\Delta_i = \int d\mathbf{r} \phi_i^\dagger(\mathbf{r}) \Delta_n(\mathbf{r}) \phi_i(\mathbf{r}).
\] (13)

As shown in Figs. 3 and 4, the graph of \( \Delta_n(r) \) with the volume-type interaction has a plateau inside the nuclear surface similar to the neutron density \( \rho_n(r) \), whereas the graph of \( \Delta_n(r) \) with the surface-type pairing interaction has a bump at the nuclear surface. In general, the value of \( u_i v_i \) reaches a maximum for the single particle states close to the Fermi level, whereas the value of \( v_i^2 \) is unity for the deeply bounded single particle states below the Fermi level and decreases near the Fermi level, eventually, vanishing in the continuum. The averaged neutron pairing gap energies \( \bar{\Delta}_{uv} \) are obtained by averaging \( \Delta_i \) with the factor \( v_i^2 (u_i v_i) \) over all single-particle states in Eq. (10) (Eq. (11)). In the case of the surface-type pairing interaction, the pairing gap \( \Delta_i \) is larger for the single-particle state near the Fermi surface since the overlap of the surface peaked \( \Delta_n(r) \) and the wave function \( \phi_i \) is larger. Then, the averaged neutron pairing gap energy \( \bar{\Delta}_{uv} \) is larger than \( \bar{\Delta}_{v^2} \) because the product \( u_i v_i \) reaches its maximum for the single-particle states near the Fermi level. In contrast, in the case of volume-pairing interaction, the well-bound single-particle states below the Fermi level have large pairing gaps \( \Delta_i \) because the interior part of \( \Delta_n(r) \) has a large overlap with the well-bound wave functions. Then, for the volume-type pairing interaction, \( \bar{\Delta}_{v^2} \) becomes larger than \( \bar{\Delta}_{uv} \) because the occupation probability \( v^2 \) is larger for the well-bound states.

The neutron densities of \(^{124}\text{Sn}\) and \(^{136}\text{Sn}\) are a half of the respective averaged central densities at \( r = 5.7 \) and 5.9 fm, respectively, irrespective of the Skyrme interactions SVI, MSkA and SIV. We refer to these radii as the surface positions of two nuclei. Let us now examine the magnitude of the pair potential at this surface position. In the case of surface-type pairing interaction shown in Fig. 3, the difference of the local pair potentials at \( r = 5.7 \) fm is only 19% between SVI and SIV for \(^{124}\text{Sn}\), while it is 33% in the volume-type pairing interaction. The local pair potential of MSkA stays between those of SVI and SIV. It turns out that the difference of \( \bar{\Delta}_{uv} \) between SIV and SVI is only 9% in the surface-type pairing interaction, while it is 39% in the volume pairing interaction, as shown in Fig. 1. Figure 4 for \(^{136}\text{Sn}\) shows that, in the case of surface-type pairing interaction, the difference of the local pair potentials at \( r = 5.9 \) fm is 36% between SVI and SIV, while it is 65% in the volume-type pairing interaction. The corresponding difference appears also in Fig. 2: the difference of \( \bar{\Delta}_{uv} \) between SIV and SVI is 32% in the surface-type pairing interaction while it is 72% in
the volume-type pairing interaction. Thus, the variation of the averaged neutron pairing gap energy for different effective masses is much smaller for the surface-type pairing interaction than for the volume-type pairing interaction, as seen in Figs. 1, and 2. This phenomenon can be understood by the following discussion.

The single-particle states near the Fermi level claim the dominant contribution to the averaged pairing gap energy in the case of surface-type pairing, while the deeply-bound single-particle states also have substantial contributions in the case of volume-type pairing. Since the single-particle levels in $^{124}$Sn near the Fermi level are not much affected by change of effective mass, the averaged pairing gap energies in Fig. 1(a) show only small variation. In contrast, the single-particle energies of the well-bound states are very much affected by the effective mass. Consequently, the average pairing gaps in Fig. 1(b) show a clear effective mass dependence due to the well-bound states. These arguments apply qualitatively in the case of $^{136}$Sn, as shown in Fig. 2. However, one can see larger variation in the unstable nucleus $^{136}$Sn, whose single-particle states are more sensitive to change of effective mass than the stable nucleus $^{124}$Sn.

The local pair potentials of RMF+BCS model are shown in Figs. 5 and 6 for $^{124}$Sn and $^{136}$Sn, respectively, with the RMF Lagrangians NL3 ($m^*/m=0.634$) and NLSH ($m^*/m=0.635$). General features of RMF+BCS results are similar to those of SHF+BCS in Figs. 3 and 4: the pair potential peaks at the surface in the case of surface-type pairing interaction, while the potential plateaus within the nuclear surface for the volume-type pairing. Because of the thick neutron skin, the surface peak is more extended in $^{136}$Sn, as seen in the upper panel of Fig. 6. Comparing the two RMF Lagrangians NL3 and NLSH, the results appear very close to each other for the surface-type pairing, while NL3 has a slightly larger pair potential than NLSH for the volume-type pairing. The averaged pairing gap potentials show a smooth trend as a function of the effective mass in Figs. 1 and 2 where both RMF and SHF results are plotted. This is due to the similarities of the local pair potentials between the two models.

IV. SUMMARY

We studied pairing correlations in the ground states of typical superfluid nuclei $^{124}$Sn and $^{136}$Sn in relation to the effective mass in SHF+BCS and RMF+BCS models. We adopted
various parameter sets of Skyrme interactions and relativistic Lagrangians together with the surface- and volume-type pairing interactions to study pairing correlations across a wide range of effective mass. We showed that the surface-type pairing interaction gives an almost constant pairing gap, independent of the effective mass, while the volume-type pairing interaction shows rather strong dependence of the pairing correlations upon the effective mass. The local pair potential in the case of surface-type pairing interaction has a peak at the surface of neutron density, while the potential in the case of volume-type pairing interaction has a plateau inside the nuclear surface. These features of the local pair potentials cause a difference between the surface- and volume-type pairing interactions regarding effective mass dependency of the averaged neutron pairing gap energy. We also pointed out that the effect of effective mass on the averaged pairing gap energies is essentially the same between RMF and SHF models since the local pair potentials have similar characteristic features in the two models.

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FIG. 1: The averaged neutron pairing gap energy $\bar{\Delta}$ and the effective mass in $^{124}\text{Sn}$. The left panel (a) is for surface-type pairing interaction, while the right panel (b) is for volume-type pairing interaction in SHF+BCS and RMF+BCS models. Open circles (squares) and filled circles (squares) are obtained by averaging with $v^2$ and $uv$ factors in Eqs. (10) and (11), respectively, in SHF(RMF)+BCS model. The numbers denote the different parameter sets: 1 for SI, 2 for SIII, 3 for SIV, 4 for SVI, 5 for Skya, 6 for SkM, 7 for SkM*, 8 for SLy4, 9 for MSkA, 10 for SkI3, 11 for SkI4, 12 for SkX, 13 for SGII, 14 for NL3, 15 for NLC, and 16 for NLSH.
FIG. 2: The averaged neutron pairing gap energy $\bar{\Delta}$ and the effective mass in $^{136}\text{Sn}$. The left panel (a) is for surface-type pairing and the right panel (b) is for volume-type pairing interaction in SHF+BCS and RMF+BCS models. Open circles (squares) and filled circles (squares) are obtained by averaging with $v^2$ and $uv$ factors in Eqs. (10) and (11), respectively, in SHF(RMF)+BCS model. The numbers denote the different parameter sets: 1 for SI, 2 for SIII, 3 for SIV, 4 for SVI, 5 for Skya, 6 for SkM, 7 for SkM*, 8 for SLy4, 9 for MSkA, 10 for SkI3, 11 for SkI4, 12 for SkX, 13 for SGII, 14 for NL3, 15 for NLC, and 16 for NLSH.
FIG. 3: Local pair potentials of $^{124}\text{Sn}$ in SHF+BCS model. (a) with surface-type and (b) with volume-type pairing interactions. Solid, dotted, and dashed lines are results for SVI, MSkA and SIV parameter sets, respectively. See the text for details.
FIG. 4: Local pair potentials of $^{136}$Sn in SHF+BCS model. (a) with volume-type and (b) with surface-type pairing interactions. Solid, dotted, and dashed lines are results for SVI, MSkA, and SIV parameter sets, respectively. See the text for details.
FIG. 5: Local pair potentials of $^{124}$Sn in RMF+BCS model. (a) with surface-type and (b) with volume-type pairing interactions. Solid and dotted lines are the results for NL3 and NLSH parameter sets, respectively. See the text for details.
FIG. 6: Local pair potentials of $^{136}$Sn in RMF+BCS model. (a) with volume-type and (b) with surface-type pairing interactions. Solid and dotted lines are the results for NL3 and NLSH parameter sets, respectively. See the text for details.