Composite Fermions with a Warped Fermi Contour

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(Dated: December 4, 2014)

Via measurements of commensurability features near Landau filling factor \( \nu = 1/2 \), we probe the shape of the Fermi contour for hole-flux composite fermions confined to a wide GaAs quantum well. The data reveal that the composite fermions are strongly influenced by the characteristics of the Landau level in which they are formed. In particular, their Fermi contour is warped when their Landau level originates from a hole band with significant warping.

At very low temperatures, two-dimensional (2D) carrier systems with high mobility manifest signatures of many-body interaction in the presence of a strong perpendicular magnetic field (\( B_\perp \)). One such phenomenon is the fractional quantum Hall effect (FQHE) which is elegantly explained in the framework of composite fermions (CFs), exotic quasi-particles composed of charged particles bound to an even number of magnetic flux quanta [13]. This flux attachment cancels out the external magnetic field at Landau levels (LL) filling factor \( \nu \) \( \approx \) 1. This result was significantly extended to fillings \( \nu = 1 \) [1], \( \nu = 3 \) [11], \( \nu = 5 \) [76]. At very low temperatures, two-dimensional (2D) carrier systems with high mobility manifest signatures of many-body interaction in the presence of a strong perpendicular magnetic field (\( B_\perp \)). One such phenomenon is the fractional quantum Hall effect (FQHE) which is elegantly explained in the framework of composite fermions (CFs), exotic quasi-particles composed of charged particles bound to an even number of magnetic flux quanta [13]. This flux attachment cancels out the external magnetic field at Landau levels (LL) filling factor \( \nu \) \( \approx \) 1. This result was significantly extended to fillings \( \nu = 1 \) [1], \( \nu = 3 \) [11], \( \nu = 5 \) [76].

The role of anisotropy in FQHE has been featured in many recent studies which focus on fundamental issues, such as the transference of an anisotropic Fermi contour from particles to CFs [9, 10, 13–17]. In the simplest scenario, the band properties of the low-field particles should not be mapped onto CFs because the latter are primarily a product of interaction. However, measurements on AlAs quantum wells (QWs) containing 2D carriers with an elliptical Fermi contour and anisotropic transport have revealed that CFs also exhibit resistance anisotropy [14]. This suggests a possible inheritance of energy band properties by CFs. In this Letter, we report direct measurements of the Fermi contour for hole-flux CFs confined to wide GaAs QWs where the 2D hole Fermi contour is significantly warped. We find that the warping is qualitatively transferred to CFs if the LL in which CFs are formed originates from a hole band with significant warping. Our additional data, taken with an applied parallel magnetic field (\( B_\parallel \)), provide a remarkable confirmation of this conclusion.

We studied 2D hole systems (2DHSs) confined to a 35-nm-wide symmetric GaAs QW, grown by molecular beam epitaxy on a (001) GaAs substrate. The QW, located 131 nm below the surface, is flanked on each side by 95-nm-thick Al0.24Ga0.76As spacer layers and C doped layers. The 2DHS density \( \rho \) at \( T = 0.3 \) K is \( \approx 1.67 \times 10^{11} \) cm\(^{-2}\), and its mobility is \( \approx 10^6 \) cm\(^2\)/Vs. As shown in Fig. 1(a), we fabricated a Hall bar with two perpendicular arms oriented along [110] and [110]. The arms are covered with periodic stripes of negative electron-beam resist which, through the piezoelectric effect in GaAs, produce a density modulation of the same period in the 2DHS [8, 11, 13, 22]. We measured the longitudinal resistances along the two arms in purely perpendicular and also in tilted magnetic fields, with \( \theta \) denoting the angle between the field direction and the normal to the 2D plane (Fig. 1(a)). The sample was tilted around [110] so that \( B_\parallel \) was always along [110].

Figures 1(b) and (c) show the energy band dispersions and the Fermi contours of a 2DHS confined to a wide GaAs QW [23, 24], based on an 8 \( \times \) 8 Kane Hamiltonian [25] which combines the Dresselhaus spin-orbit coupling and the non-parabolicity of the 2D hole bands. As seen in Fig. 1(c), the Fermi contour is significantly warped as a result of severe mixing between the heavy-hole (HH) and light-hole (LH) states [25]. Spin-orbit coupling also causes the contours of two different spin species to split. As a result of warping, the Fermi wave vectors, \( k_F \) for both majority and minority spin contours along [110] and [10] are larger than \( k_F \) of a circular Fermi contour which contains the same number of (spin-unpolarized) 2D holes.

We use commensurability oscillations (COs) to probe the Fermi contour shapes of both holes and hole-flux CFs. The COs are manifested in the magnetoresistance as a minimum whenever the quasi-classical cyclotron orbit diameter \( 2R_c \) of the particles becomes commensurate with the period of the density modulation, \( a \). Since \( 2R_c = 2\hbar k_F/eB_\perp \), the \( B_\perp \)-positions of COs resistance minima provide a direct measure of \( k_F \). For a spin-unpolarized, circular Fermi contour, the expected positions of these minima are given by the electrostatic commensurability condition, \( 2R_c/a = i - 1/4 \) [27, 33], where, \( i \) is an integer and \( k_{F,cir} = \sqrt{2\pi a} \). In Fig. 1(d) we show the low-field magnetoresistance of our 2DHS along [110] which allows us to deduce \( k_F \) along [110] [33]. The red tick-marks in the inset denote the expected \( i = 3, 4, 5 \) \( B_\perp \)-positions for \( k_{F,cir} \) while the solid and dotted black lines indicate the expected positions of the measured COs; the agreement is excellent.
The Fermi contour. (We define the warping as the ratio of observed resistance minima (arrows) in Fig. 1(d) inset do not match the red marks. Each minimum, however, is close to the dotted black tick mark of a given \( i \), suggesting that the Fermi contour is warped. This observation also implies that the Fermi contour agrees better with the minority spin contour which is consistent with previous studies on other 2DHSs [19]. With this interpretation, we deduce a value of \( \sim 20\% \) for the observed warping of the Fermi contour. (We define the warping as the ratio of \( k_T \) along [110] or [\( \overline{1} \)10] over \( k_{F,cir} \).) We note that, generally, warping is significantly more pronounced in wide wells such as those studied here than in narrower QWs studied previously [19].

Having established a significant warping in our 2DHS Fermi contours, we now turn to the Fermi contour of \( \nu = 1/2 \) CFS. As seen in the magnetoresistance data of Fig. 2, there are two pronounced minima on the sides of \( \nu = 1/2 \), flanked by shoulders of rapidly increasing resistance. These two minima correspond to the commensurability condition, \( 2R_c/a = 5/4 \) \([8,11,35-41]\), where \( 2R_c = \frac{2\hbar k_{F,cir}/eB_d^*}{(1-\nu)/\nu} \) is the quasi-classical cyclotron orbit diameter \( 2R_c^* \) with \( a \). Quantitatively, for a circular CF Fermi contour, the positions of these resistance minima are given by the magnetic commensurability condition, \( 2R_c^*/a = 5/4 \) \([8,11,35-41]\), where \( 2R_c^* = \frac{2\hbar k_{F,cir}^*/eB_d^*}{\nu} \) is the quasi-classical cyclotron orbit diameter of CFs at the effective magnetic field \( B_d^* \), \( k_{F,cir}^* = \sqrt{4\pi p^*} \), and \( p^* \) is the CF density. The expression for \( k_{F,cir}^* \) assumes full spin-polarization at high fields of \( \sim 14 \) T. Recent studies have established that, in the vicinity of \( \nu = 1/2 \), \( p^* \) is equal to the minority carrier density, namely \( p^* = p \) for \( B_d^* > 0 \) and \( p^* = [(1-\nu)/\nu]p \) for \( B_d^* < 0 \) \([11]\). In Fig. 2 inset, we mark the expected field positions (red tick-marks) of CF commensurability minima for a circular Fermi contour based on the minority density in the lowest LL. The positions of the observed resistance minima (vertical arrows) are measurably farther from \( B_{\perp,1/2} \) than the red marks, providing clear evidence that CFS have a warped Fermi contour. Based on Fig. 2 data, and also similar data taken on three other samples, we deduce a warping \( (k_T/k_{F,cir}) \) of \( \sim 15\% \) for the CFS. This is comparable to, but somewhat smaller than the warping we measure for the hole Fermi contour (Fig. 1(d)), suggesting that CFS inherit some warping in their Fermi contour from the LL in which they are formed.

We investigate the Fermi contour warping of CFS further by utilizing the crossing of the two lowest-energy LLs at large \( B_{\perp} \) \([24,44]\). Such a crossing, prevalent in wide QWs where the energy separation between HH and LH subbands is small, can be tuned by either changing the 2DHS density or, at a fixed density, by applying a parallel magnetic field \( B_{\parallel} \) \([24,43]\). Here we present data, taken as a function of \( B_{\parallel} \), demonstrating how the character of the LL in which CFS are formed influences the CFS’ Fermi contour warping. In Figs. 3(a) and (b) we summarize the evolution of the magnetoresistance features near \( \nu = 1/2 \) as a function of \( \theta \). There are pronounced CF commensurability features consistent with a warped Fermi contour at \( \theta = 0^\circ \), along both [110] and [\( \overline{1} \)10]. As \( \theta \) increases to \( \sim 26^\circ \), the resistance near \( \nu = 1/2 \) increases by a factor of \( \sim 2 \), and the magnetoresistance traces become monotonic, losing all commensurability features.
For $\theta > 30^\circ$, however, the resistance near $\nu = 1/2$ again becomes comparable to that of the $\theta = 0^\circ$ trace and, remarkably, the commensurability features around $\nu = 1/2$ reappear and become very pronounced.

To explain the data of Fig. 3, we focus on the nature of the two lowest-energy LLs. Although these LLs originate from states which are HH-like at the subband edge $k = 0$, their exact characteristic is complex due to the admixture of LH and HH states at finite $k$. For simplicity we will refer to these LLs as “LH” and “HH”, respectively. Figure 3(c) shows a qualitative picture for the crossing between “LH” and “HH”, which can be explained by the different in-plane (cyclotron) effective masses of “LH” and “HH”. “HH” has a smaller in-plane effective mass compared to “LH” increases in energy more rapidly with $B_{\perp}$ than “LH”, leading to the crossing at sufficiently large $B_{\perp}$. At $\theta = 0^\circ$, $\nu = 1/2$ starts out to the right of the crossing and the CFs are formed in “LH” (Fig. 3(c) lower panel). As $\theta$ increases, the confining potential due to $B_{\parallel}$ becomes stronger thus lowering “LH” in energy with respect to “HH”. As a result, the crossing position moves closer to $\nu = 1/2$ (Fig. 3(c) middle panel). The increase in resistance near $\nu = 1/2$, when $\theta \sim 26^\circ$ indeed comes about because, when the crossing occurs very close to $\nu = 1/2$, the ground-state at $\nu = 1/2$ becomes insulating; this is best seen in data taken at lower temperatures on another 2DHS confined to a 35-nm-wide GaAs QW. When the sample is tilted to higher $\theta$ so that the crossing moves well to the right of $\nu = 1/2$ (Fig. 3(c) top panel), the resistance near $\nu = 1/2$ decreases by a factor of $\sim 2$, suggesting that the insulating phase has passed and the CFs now form in “HH”.

According to the above discussion, the character of the LL in which CFs are formed changes from “LH” to “HH” in the course of the crossing. This change should affect the CF’s Fermi contour warping. Using the relation, $2\hbar k^*_F/e B^*_\perp = 5/4$, we extract the size of $k^*_F$ along [110] and [110] from the positions of the CF commensurability minima along [110] and [110], respectively. The deduced values of $k^*_F$, normalized to $k^*_F,cir$, and plotted as a function of $B_{\parallel}$ in Figs. 4(a) and 4(b), provide a measure of the CF Fermi contour warping. For $B_{\parallel} = 0$, $k^*_F/k^*_F,cir > 1$ is consistent with warping of the CF Fermi contour. With increasing $B_{\parallel}$, $k^*_F$ increases along both directions until the LL crossing region sets in at $B_{\parallel} \sim 5$ T (see Figs. 4 (a) and 4(b)). Once the commensurability features reappear past the crossing region ($B_{\parallel} \gtrsim 10$ T), $k^*_F/k^*_F,cir$ clearly shows a smaller value than at $B_{\parallel} \sim 5$ T. This drop in $k^*_F$ coincides with the Fermi level at $\nu = 1/2$.  

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**FIG. 2.** (color online) Magnetoresistance trace from the [110] Hall bar for a GaAs QW of width $W = 35$ nm. The two prominent minima near $\nu = 1/2$ are signatures of commensurability of CF cyclotron orbit diameter with the period ($a = 200$ nm) of the density modulation. Inset: Enlarged trace near $\nu = 1/2$ shows that the positions of the minima are measurably farther from $B^*_\perp = 0$ than expected for a circular Fermi contour of fully spin-polarized CFs (marked by vertical red tick-marks). For comparison, we also include data (dashed blue trace) from a narrower 2DHS ($W = 17.5$ nm) [8][9][11] where we do not expect significant warping, and the commensurability minima near $\nu = 1/2$ indeed show much better agreement with the red tick-marks.

**FIG. 3.** (color online) (a),(b) Evolution of the magnetoresistance in the vicinity of $\nu = 1/2$ measured along [110] and [110], respectively. Traces are shifted vertically for clarity and tilt angle $\theta$ is given for each trace. The vertical red lines mark the expected positions of the CF commensurability resistance minima if the CF cyclotron orbit were circular. (c) Crossing between the LLs with LH and HH character as a function of $\theta$. Note that the lowest LL, in which $\nu = 1/2$ CFs are formed, changes from “LH” to “HH” as $\theta$ increases.
having moved from the “LH” to the “HH” LL.

After the crossing, for $B_{||} > 10$ T, $k_F^*$ increases along [110], but decreases along [110] as a function of $B_{||}$, implying that the CF Fermi contour is becoming anisotropic. This anisotropy, which is summarized in Fig. 4(c) plot, is a result of the coupling between the out-of-plane (orbital) motion of the holes to $B_{||}$ and is qualitatively consistent with previous findings [9-11]. However, unlike in previous studies, when we plot the geometric mean of $k_F^*$ along [110] and $k_F^*$ along [110], normalized to $k_{F,cir}^*$, we find significant deviations from unity (Fig. 4(d)), implying that the Fermi contour is not elliptical and is severely warped. Figure 4(d) also shows that the warping is more severe just to the left of the crossing region compared to the right. This observation suggests a more severe warping when the CFs are formed in “LH” than in “HH”. We conclude that the CFs do inherit characteristics, such as Fermi contour warping, of the LL in which they are formed.

We acknowledge support through the DOE BES (DEFG02-00ER45841) for measurements, and the Gordon and Betty Moore Foundation (Grant No. GBMF4420), Keck Foundation, and the NSF (DMR-1305691 and MRSEC DMR-0819860) for sample fabrication. Work at Argonne was supported by DOE BES (DE-AC02-06CH11357). Our work was partly performed at the National High Magnetic Field Laboratory (NHMFL), which is supported by NSF (DMR-1157490), the State of Florida, and the DOE. We thank S. Hannahs, T. Murphy, and A. Suslov at NHMFL for valuable technical support.

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**FIG. 4.** (color online) (a),(b) Measured values of the CF Fermi wave vectors $k_F^*$ along [110] and [110], normalized to $k_{F,cir}^*$, as a function of $B_{||}$ for both positive and negative $B_{||}^*$. Data shown with filled squares (black) and filled circles (red) are from measurements in two different systems. (c) Relative anisotropy of the CF Fermi contour is plotted as the ratio of $k_F^*$ along [110] and [110]; data for $B_{||}^* > 0$ were used. (d) The geometric mean of the measured $k_F^*$ along [110] and [110], divided by $k_{F,cir}^*$, as a measure of how much the Fermi contour deviates from an ellipse. The yellow region in all four graphs signifies the LL crossing.

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