Calculation the (E2/M1) mixing ratios for 160,162 dysprosium isotopes using an IBM-2.

Rawaa M Obaid and Mohamed A Al Shreefi
Physics Department, College of Science, Babylon University, Hilla, Iraq
ashrawaa@gmail.com

Abstract. The energy levels, B(E2), B(M1) and the mixing ratios (E2/M1) for chosen transitions are calculated for 160,162 Dy isotopes using an IBM-2. The calculations are included for showing the effects of Majorana parameters on the energy of the blending symmetry states. The results obtained for dysprosium isotopes are agreement with the experimental results.

1. Introduction

The nucleus of dysprosium with proton number 66 exactly lie between the closures at 50 and 82, and number of neutron above limit of 90. The chosen 160,162 Dy isotopes are the best candidate to study the collective properties of low lying levels since these nuclei are quite well studied experimentally. These nuclei are part of rotational region due to clear band structure and the ground band energy levels. The deviation from the prediction of the symmetric rotor for high energy states may be explain in terms of band mixing. The recent studies about this nucleus are; Al-Khudair et. al [1] carried out systematic study of the positive states in even-even 152-160 Dy within the framework of the Approximation boson model. Khalaf A.M. et. al [2] used the geometric collective model to describe the nuclear transitions for Gd and Dy isotopes. Salah et. al [3] studied the excited positive and negative parity states, potential energy surfaces, B(E1), B(E2), back bending, staggering effect, ΔI = 1, and electric monopole strength, X(E0/E2) of 154,156 Dy.

The Interacting Boson Model (IBM) has been remarkably successful in describing the low-lying collective states and the electromagnetic transitions in many medium to heavy even- even nuclei. [4-9] In the original version of the IBM (IBM1), nuclei are considered as systems composed of bosons. The new version of this model, later suggested by Iachello and Arima which distinguishes between neutron and proton bosons.[10,11] In the present work on 160-162 Dy we intended to investigate the level structure, quadrupole electric transition probability (BE2), multipole mixing ratio and asymmetry states in these isotopes. However, the main task of this paper is to study the effect of Majorana parameters on the high level energy and magnetic transition probability.

2. The Model

The Hamiltonian of the model is given by:[10-13]

\[ H = \varepsilon_d (\hat{f}_d^T \hat{f}_d) + \kappa (\hat{Q}_\nu \hat{Q}_\pi) + V_{\nu\nu} + V_{\pi\pi} \]

\[ \varepsilon_d \] the energy of d boson, \( n_\rho \) is the number of d bosons, \( \rho \) referred to \( \pi, \nu \) “proton and neutron bosons”, \( \kappa \) the strength interaction of “quadruple - quadruple”, is define as [7]

\[ Q_\rho = [d_\pi^T s_\rho + s_\pi^T d_\rho]^{(2)} + x_\rho [d_\pi^T d_\rho]^2 \]

where \( x_\rho \) shows the deformation parameters.

The terms \( V_{\nu\nu} \) and \( V_{\pi\pi} \) represent the interaction of similar bosons are given as bellow [12]

\[ V_{\rho\rho} = \frac{1}{2} \Sigma_{L=0,2,4} C_L^\rho (\{d_\rho^T d_\rho\}^{(L)} - \{\hat{d}_\rho \hat{d}_\rho\}) \]

The Majorana term \( M_{\pi\pi} \), which consist of three parameters \( \xi_1 \), \( \xi_2 \) and \( \xi_3 \) can be taken as:[11-14]

\[ M_{\nu\pi} = \frac{1}{2} \Sigma_2 \{[s_\nu d_\pi - d_\nu s_\pi]^{(2)} [s_\nu d_\pi - d_\nu s_\pi]^{(2)} \} \]

3. Calculations and Results

3.1. Energy levels
The energies in the model mentioned above were calculated using the “program NPBOS”. The “program NPBTRN” were used to determine the electromagnetic matrix elements.[13] The boson proton number of the $^{160,162}$Dy isotopes $N_p = 8$ and the number neutron boson $N_n$ varied from 6 to 7. The fit for the parameters of is given in table 1 and the calculated levels of energy for isotopes under study are compared with the experimental data taken from reference [14] in figure 1.

Table 1. The energy levels for $^{160,162}$Dy isotopes calculated in IBM-2

| Isotope | $N_n$ | $E$  | $K$  | $\chi_n$ | $\chi_p$ | $CL_{\nu}(0,2,4)$ | $CL_{\nu}(0,2,4)$ | $\zeta_{1=3}$ | $\zeta_{2}$ |
|---------|-------|------|------|----------|----------|-------------------|-------------------|----------------|------------|
| $^{160}$Dy | 6     | 0.370| -0.042| -1.1     | -1.2     | 0.0, 0.01, 0.12   | 0.0, 0.0, 0.12    | 0.23          | 0.27       |
| $^{162}$Dy | 7     | 0.336| -0.040| -1.1     | -1.2     | 0.0, 0.01, 0.15   | 0.0, 0.0, 0.12    | 0.23          | 0.26       |

Figure 1. Energy levels calculated in IBM-2 and the available experimental data for the $^{160}$Dy isotope.

The $2^+_1$ level concentrated on it for getting an adjusted results, observe that its energy varies with ($\varepsilon_d$ and $\kappa_p$) as shown in figures 2 and 3.

Figure 2. The energy of the $2^+_1$ as a function of $\varepsilon_d$.

Figure 3. The energy of the $2^+_1$ as a function $\kappa_p$. 
For knowing the dynamical symmetry for these isotopes belong to it, the energy ratio of $(4^+_1, 2^+_2)$ states are used for this purpose. The ratio $E4^+_1/E2^+_2$ states have limiting values of (2) for vibrator rotor $U(5)$, (2.5) for gamma-soft $O(6)$ and 3.33 for rotational symmetry $SU(3)$ respectively. [4,5]

Table 2 shows the ratio $E4^+_1/E2^+_2$ which indicate that these isotopes belong to third symmetry group.

**Table 2.** The ratio $E4^+_1/E2^+_2$ for $^{160,162}$Dy isotopes

| Nucleus | $E4^+_1/E2^+_2$ Exp. | $E4^+_1/E2^+_2$ IBM-2 | SU(5) | O(6) | Su(3) |
|---------|---------------------|----------------------|-------|------|-------|
| $^{160}$Dy | 92 | 3.22 | 3.2314 | 2 | 2.5 | 3.33 |
| $^{162}$Dy | 94 | 3.29 | 3.290 | 2 | 2.5 | 3.33 |

**3.2. Electromagnetic Transition**

**3.2.1. E2 transition probability**

For calculating the reduced transition probability, the equation below can be utilized: [16].

\[
T^{(E2)} = e_n Q_n + e_y Q_y
\]  

where $e_n$ and $e_y$ are the effective charges of the boson, the method for determining their parameters are exit in references [15,16]. We obtained that $e_n = 0.153$ e.b. and $e_y = 0.446$ e.b.

The computed and experimental values for electric quadruple probabilities $B(E2)$ of $^{160,162}$Dy isotopes are given in table 3.

**Table 3.** The E2 transition of $^{160,162}$Dy isotopes in units (eb)$^2$

| Transition | $^{160}$Dy | $^{162}$Dy |
|------------|----------|----------|
|           | Exp. | IBM-2 | Exp. | IBM-2 |
| $2^{-}1^{-}\rightarrow0^{-1}$ | 1.00 | 1.22 | 1.1 | 1.23 |
| $4^{-1^{-}\rightarrow2^{-1}$ | 1.466 | 1.72 | 1.51 | 1.54 |
| $2^{-}2^{-}\rightarrow0^{-1}$ | 0.03 | 0.026 | 0.004 | 0.0042 |
| $2^{-}2^{-}\rightarrow2^{-1}$ | 0.043 | 0.06 | 0.04 | 0.09 |
| $2^{-}0^{-}\rightarrow0^{-1}$ | 0.023 | 0.013 | 0.03 | 0.005 |
| $2^{-}4^{-}\rightarrow4^{-1}$ | ---- | 0.003 | ---- | 0.101 |
| $2^{-}1^{-}\rightarrow0^{-1}$ | 0.020 | ---- | ---- | 0.054 |
| $2^{-}1^{-}\rightarrow2^{-1}$ | 0.026 | 0.037 |
| $2^{-}1^{-}\rightarrow2^{-1}$ | 0.004 | 1.33 |
| $2^{-}2^{-}\rightarrow2^{-1}$ | 0.006 | 0.001 |
| $3^{-}2^{-}\rightarrow2^{-1}$ | 0.019 | 0.189 |

In tables 3 It can observed that the $B(E2; 2^{-}\rightarrow0^{-1})$ and $B(E2; 4^{-1}\rightarrow2^{-1})$ have large values which are common while the $B(E2; 2^{-}\rightarrow2^{-1})$ has a small value because it contains admixture of $M_1$. The value of $B(E2; 2^{-}\rightarrow0^{-1})$ is small because this transition is from the second band to the first state band (cross over transition). It can be interpreted these behaviour results from the fact which starts that the transitions probabilities E2 between low-lying states due to mainly the mixing of bands, which belong to fully symmetric states, while the magnitude of the E2 transition rate due to mixed symmetric states is shown to be very smaller than that due to the mixing of bands that belong to fully symmetric states.

**3.2.2. M1 transition probability**

The magnetic dipole moment operator $T(M1)$ were calculated using the equation below[17]:

\[
T^{(M1)} = 0.77 \left( \langle d^\dagger d^- \rangle^{(1)}_\pi - \langle d^\dagger d^- \rangle^{(3)}_\gamma \right) \left( g_\pi - g_\gamma \right) \]  

(6)
the \( g_\pi, g_\gamma \) which appear in equation 6 are gyromagnetic factors of nuclear proton and neutron separately,[18] the values calculated for \(^{160,162}\text{Dy}\) isotope \( g_\pi = -0.222 \, \mu_N \) and \( g_\gamma = 0.8 \, \mu_N \).

The calculated and an experimental estimates for M1 transitions of \(^{160,162}\text{Dy}\) isotopes are given in table 4.

| Transition | \(^{160}\text{Dy}\) Exp. | \(^{160}\text{Dy}\) IBM-2 | \(^{162}\text{Dy}\) Exp. | \(^{162}\text{Dy}\) IBM-2 |
|------------|----------------|----------------|----------------|----------------|
| \( 1^+_1 \rightarrow 0^+_1 \) | 0.443 | 0.54 |
| \( 1^+_2 \rightarrow 0^+_2 \) | 0.206 | 0.258 |
| \( 3^+_1 \rightarrow 2^+_2 \) | 0.0002 | 0.0001 |
| \( 2^+_2 \rightarrow 1^+_2 \) | 8.5E-05 | 6.21E-05 | 6.00E-06 | 3.0E-06 |
| \( 2^+_2 \rightarrow 1^+_1 \) | 0.0035 | 0.001 | 0.000 |
| \( 2^+_2 \rightarrow 2^+_1 \) | 0.0026 | 6.00279E-07 |
| \( 2^+_1 \rightarrow 2^+_1 \) | 2.85E-06 | 7.16366E-06 |

In table 4, the M1 transition between two states of full symmetric states vanishes, so the M1 transitions from mixed-symmetry states to totally symmetric states are small.

### 3.2.3. Mixing Ratios

For calculating the blending ratio, we used the relation which define by: [16,19]

\[
\delta(E2/M1) = 0.835 \, \text{E} \frac{|\langle f | P^{E2}| i \rangle|}{|\langle f | P^{M1}| i \rangle|} \tag{7}
\]

\( \text{E} \) is the gamma energy which release when transitions occur between two levels. The multiple mixing ratios of some transitions for the isotopes under study are determined and are compared with some previous experimental outcomes and are tabulated in Table 5. There is a good agreement with the experimental data obtained, it can be estimated from calculations, that there is a small disagreement in the mixing ratios of some transitions, it is because the effect of a very small value of magnetic matrix elements.

| Nucleus | Transition | \( E_\gamma \) (Mev) | \( \delta(E2/M1) \) ( eb/\( \mu_N \) ) | Exp. | IBM-2 |
|---------|------------|----------------|-------------------------------|-----|-----|
| \(^{160}\text{Dy}\) | \( 2^+_1 \rightarrow 2^+_1 \) | 0.879 | -16.6(5) | -20.8 |
| | \( 2^+_2 \rightarrow 2^+_1 \) | -1.5\(^{+7}_{-20}\) | 2.9\(^{+3}_{-10}\) | 1.52 |
| | \( 4^+_1 \rightarrow 2^+_1 \) | 1.431 | 6.86 |
| \(^{162}\text{Dy}\) | \( 2^+_1 \rightarrow 2^+_1 \) | 0.807 | 57 | 44 |
| | \( 2^+_3 \rightarrow 2^+_1 \) | 1.732 | +0.40 (15) | 13 |
| | \( 2^+_4 \rightarrow 2^+_1 \) | 1.647 | -0.2\(^{+15}_{-10}\) | -0.157 |
| | \( 2^+_5 \rightarrow 2^+_1 \) | 1.702 | 1.29 |

### Mixed symmetry states

When the motions of the proton and neutrons are in opposite phase, these states may be hopped and originate from blending two wave functions for the proton and neutron noticed in even even nuclei of mass number \( A = 50 \) to 240. The Mss states can be assessed by their M1 decay to a fully-symmetric state which are usually strong. The energy of these state are governed by the Majorana parameters term which shift the states energies with asymmetric states with respect to the symmetric states.[12,17] A symmetry states composed by Na proton bosons and Nv neutron bosons which have F-spin quantum number \( F = F_{\text{max}} \) are called the Full Symmetry states, all others \( F < F_{\text{max}} \) in IBM-2 are mixed symmetry states and include the \( 1^+ \) and \( 2^+ \) states. In more vibrational and gamma soft nuclei, the lowest MSS is with \( I^+ = 2^+ \) state, while in rotational nuclei observed as \( I^+ = 1^+ \) state. [10]
The Majorana term plays an important part in raising the energy of states containing an antisymmetric two bosons. Majorana parameter (ζ1,3, ζ2) have effect on mixed symmetry states, where the ζ1,3 has different nature from the other parameters of Majorana components, When study the effect of Majorana parameter (ζ1,3,ζ2) on the calculated excitation energy level, we allowed the ζ2 to vary with the ζ1,3 at their best fit value. The levels of 2+ of totally symmetry are reaching the saturation vastly with increasing the ζ2, the energy of 1+ ms increase linearly with the ζ2, these results are noted in figure 4. It shows that the changing in energies levels with varying the ζ2 is a referring to that the levels of 2+ and 1+ are mixing symmetry states and that refer to this way for finding the location of mixed symmetry states.

Figure 4. Variety of energy of levels vs of the Majorana parameter ζ2

4. Conclusion
In the current work, some properties of the nuclear structure for (160Dy and162Dy) are investigated. From the results for energy of levels and ratios of energies to indicate the dynamical symmetry that these nuclei to which belong. It is found these isotopes belong to rotational limit. The E2, M1 transitions and blending ratios are evaluated and indicated a very well. The calculations are extended to found the location of the blending states. It is found that the J+ = 2+3, 2+4, 2+5, 3+2, 1+1 levels in these isotopes are very responsible to the increasing of Majoran term, these confirm their belonging to the mixed symmetry.

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