Origin of adiabatic and non-adiabatic spin transfer torques in current-driven magnetic domain wall motion

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A consistent theory to describe the correlated dynamics of quantum mechanical itinerant spins and semiclassical local magnetization is given. We consider the itinerant spins as quantum mechanical operators, whereas local moments are considered within classical Lagrangian formalism. By appropriately treating fluctuation space spanned by basis functions, including a zero-mode wave function, we construct coupled equations of motion for the collective coordinate of the center-of-mass motion and the localized zero-mode coordinate perpendicular to the domain wall plane. By solving them, we demonstrate that the correlated dynamics is understood through a hierarchy of two time scales: Boltzmann relaxation time $\tau_B$, when a non-adiabatic part of the spin-transfer torque appears, and Gilbert damping time $\tau_{DW}$, when adiabatic part comes up.

Spin torque transfer (STT) process is expected to revolutionize the performance of memory device due to non-volatility and low-power consumption. To promote this technology, it is essential to make clear the nature of the current-driven domain wall (DW) motion. Recent theoretical studies have disclosed that the STT consists of two vectors perpendicular to the local magnetization $\mathbf{m}(x)$ and can be written in general as $\mathbf{N} = c_1 \partial_x \mathbf{m} + c_2 \mathbf{m} \times \partial_x \mathbf{m}$. The $c_1$ and $c_2$-terms respectively come from adiabatic and non-adiabatic processes between conduction electrons and local magnetization. Behind appearance of the $c_2$ term is the so called transverse spin accumulation (TSA) of itinerant spins. Now, any consistent theory should explain how the adiabatic and non-adiabatic STT come up starting with microscopic model. In particular, it should be made clear how the TSA caused by the non-adiabatic STT eventually leads to translational motion of the whole DW. In this letter, to solve this highly debatable problem, we propose a consistent theory to describe the correlated dynamics of quantum mechanical itinerant spins and semiclassical local magnetization.

We consider a single head-to-head Néel DW through a magnetic nanowire with an easy $x$ axis and a hard $z$ axis. Conduction electrons travel along the DW axis. We describe a local spin by a semiclassical vector $\mathbf{S} = |\mathbf{n}| = |\sin \theta \cos \varphi, \sin \theta \sin \varphi, \cos \theta|$ where $|\mathbf{S}| = |\mathbf{S}|$ and the polar coordinates $\theta$ and $\varphi$ are assumed to be slowly varying functions of one-dimensional coordinate $x$ [Fig.1(a)]. The DW formation is described by the Hamiltonian (energy per unit area) in the continuum limit,\n
$$\mathcal{H}_{DW} = \frac{JS^2}{2a} \int_{-\infty}^{\infty} dx \left[ (\partial_x n)^2 - \lambda^{-2} \tilde{n}_x^2 + \kappa^{-2} \tilde{n}_z^2 \right], \quad (1)$$

where $a$ is the cubic lattice constant, $J$ is the ferromagnetic exchange strength, $\lambda = \sqrt{J/K}$ and $\kappa = \sqrt{J/K_{\perp}}$ respectively represent the single-ion easy and hard axis anisotropies measured in the length dimension. The stationary Néel wall ($\theta_0 = \pi/2$) is described by $\mathbf{n}_0 = (\cos \varphi_0, \sin \varphi_0, 0)$ with $\varphi_0(z) = 2 \arctan(e^{x/\lambda})$. In the infinite continuum system, the DW configuration has continuous degeneracy labeled by the center of mass position, $X$, of the DW. This degeneracy apparently leads to rigid translation of the DW, i.e., $\mathbf{n}_0(x) \rightarrow \mathbf{n}_0(x - X)$. As explicitly shown below, however, the translation in off-equilibrium accompanies internal deformation of the DW.

The creation operator of a conduction electron is written in a spinor form as $c^\dagger(x) = (c_{1}^\dagger(x), c_{1}^\dagger(x))$. By performing the local gauge transformation $c(x) = \hat{U}(x)c(x)$ with the unitary operator $\hat{U}(x) = e^{i\tilde{\sigma}_x \varphi_0(x)/2}$ ($\tilde{\sigma}_z$ is a block operator), we can work out the zero-mode accumulation (OPZA) of local spins $\mathbf{n}_0$. These magnetic accumulations respectively cause the non-adiabatic torque $\mathbf{T}_{\perp}$ and adiabatic torque $\mathbf{T}_{\parallel}$.

FIG. 1: (a) Stationary configuration of local spins ($\mathbf{n}_0$) associated with a single Néel wall. Labotatory frame $x, y, z$ and local frame $\tilde{x}, \tilde{y}, \tilde{z}$ are indicated. (b) Schematic view of the transverse spin accumulation (TSA) of itinerant spin $\mathbf{s}$ and the out-of-plane ($\theta$) zero-mode accumulation (OPZA) of local spin $\mathbf{n}$. These magnetic accumulations respectively cause the non-adiabatic torque $\mathbf{T}_{\perp}$ and adiabatic torque $\mathbf{T}_{\parallel}$. 

Pauli matrix) the quantization axis becomes parallel to the local spin located at $x$. Assuming $a\partial_x \varphi_0(x) \approx a/\lambda \ll 1$, i.e. wall thickness is much larger than atomic lattice constant, this procedure leads to the single-particle Hamiltonian,

$$H_{el} = \frac{\hbar^2}{2m^* a} \int_{-\infty}^{\infty} dx \left[ \frac{1}{2} [\partial_x \tilde{c}]^2 + i (\partial_x \tilde{c}^\dagger) \tilde{A}_x \tilde{c} \right] + c.c.,$$

where the effective mass of the conduction electron is $m^*$. The SU(2) gauge field is introduced as $\tilde{A}_x = \int_{-\infty}^{\infty} d\eta \tilde{U} [\eta] \tilde{U} = (\partial_x \varphi_0)/\hbar/2$. The conduction electrons are assumed to interact with the local spins by a s-d coupling represented in the form,

$$H_{sd} = -\frac{J_{sd}}{a^3} \int_{-\infty}^{\infty} dx \tilde{s}(x) \cdot \vec{S}(x - X),$$

where $\tilde{s}$ and $\vec{S} = Sn$ are respectively the spins of itinerant and localized electrons. We treat $\tilde{s}(x) = \xi e^V \hat{s}$ as fully quantum mechanical operator, while $n$ is a semiclassical vector.

**Boltzmann relaxation:** let switch on the electric field $E$ at $t = 0$. We introduce the Boltzmann relaxation time $\tau_{el}$ and the number density of the conduction electrons $f_{k\sigma}$ in the state $k, \sigma$. We assume that the deviation from equilibrium Fermi-Dirac distribution $f_0(\varepsilon_{k\sigma}) = \exp[ (\varepsilon_{k\sigma} - \mu)/k_B T ] + 1]^{-1}$ is small, where $\varepsilon_{k\sigma}$ is the single-particle energy, $\mu$ is the chemical potential. Using standard Boltzmann kinetic equation with relaxation time approximation, $\xi_{el}$, the distribution function is written as

$$f_{k\sigma} \approx f_0(\varepsilon_{k\sigma}) + eEr_{el}v_{k\sigma} \frac{\partial f_0(\varepsilon_{k\sigma})}{\partial \varepsilon_{k\sigma}},$$

where the electron charge is $-e$ and the spin-dependent velocity is $v_{k\sigma} = h^{-1} \partial \varepsilon_{k\sigma}/\partial k$. The spin-dependence of $\varepsilon_{k\sigma}$ originates from the SU(2) gauge fields $(\tilde{A}_x)_{1\downarrow}$ and $(\tilde{A}_x)_{1\uparrow}$. In the process of approaching to stationary current flowing state around the time $t \sim \tau_{el}$, as we will show explicitly, the statistical average of the conduction electron’s spin component perpendicular to the local quantization axis accumulates and acquires finite value. As schematically depicted in Fig.1(b), this process is exactly the TSA. The TSA causes an additional magnetic field acting on the local spins and exert the non-adiabatic torque on the local spins.

**Local spin dynamics:** next we formulate dynamics of the local spins coupled with the conduction electrons. We introduce the $\delta \theta(x,t)$ (out-of-plane) and $\delta \varphi(x,t)$ (in-plane) fluctuations of the local spins around the stationary DW configuration $n_0(x)$. We say “out-of-plane” and “in-plane” with respect to the DW plane. The fluctuations are spanned by the orthogonal basis functions $v_q$ and $u_q$ as $\varphi(x) = \varphi_0(x - X) + \delta \varphi(x - X)$ and $\theta(x) = \pi/2 + \delta \theta(x - X)$, where

$$\delta \varphi(x) = \int_{-\infty}^{\infty} dq \eta_q(t)v_q(x), \quad \delta \theta(x) = \int_{-\infty}^{\infty} dq \xi_q(t)u_q(x).$$

At this stage, $X$ is not a dynamical variable, but just a parameter. The basis functions obey the Schrödinger equations, $(JS^2/2) (\partial^2_x - 2x^2 \sin^2 \varphi_0 + \lambda^2) v_q(x) = \varepsilon^q \eta_q(x)$ and $(JS^2/2) (\partial^2_x - 2x^2 \sin^2 \varphi_0 + \lambda^2 + \kappa^2) u_q(x) = \varepsilon^q \xi_q(x)$. Both $\theta$ and $\varphi$ modes consist of a single bound state (zero mode) and continuum states (spin-wave modes). The dimensionless zero mode wave functions are given by $u_0(x) = v_0(x) = \Phi_0(x)$, where

$$\Phi_0(x) \equiv \sqrt{\frac{a^2}{2}} \frac{\partial_x \varphi_0(x)}{\partial x},$$

with the corresponding energies respectively given by $\varepsilon^0 = JS^2/(2\kappa^2)$ and $\varepsilon^0_\sigma = 0$. The normalization is given by $a^{-1}\int_{-\infty}^{\infty} dx |\Phi_0(x)|^2 = 1$. Although to excite the out-of-plane $(\theta)$ zero mode costs finite energy gap $\varepsilon_\theta^0$, coming from the hard-axis anisotropy, we still call this “zero mode.” The spin-wave states have energy dispersions given by $\varepsilon^q_\theta = \frac{1}{2} JS^2(q^2 + \lambda^2 + \kappa^2)$ and $\varepsilon^q_\varphi = \frac{1}{2} JS^2(q^2 + \lambda^2)$. Because the zero mode and the spin-wave states are orthogonal to each other and separated by the anisotropy gaps, the spin-wave modes are totally irrelevant to a low energy effective theory. Therefore, we ignore the spin-wave modes from now on.

**Out-of-plane zero-mode(OPZ) coordinate $\xi_0$:** in order to obtain the correct form of the dynamical Hamiltonian, one has to regard the variable $X$ as a dynamical variable $X(t)$ and replace the zero mode coordinate $\xi_0$ with $X(t)$. Following this idea, the zero-mode fluctuations should be given by

$$\varphi(t,x) = \varphi_0(x - X(t)),$$

$$\theta(t,x) = \pi/2 + \xi_0(t) \Phi_0(x - X(t)).$$

Eq. (5) is a key ingredient of this letter, which has never been explicitly treated so far.[13] That is to say, we naturally include the out-of-plane(OPZ) zero-mode, in addition to the in-plane $(\varphi)$ zero-mode replaced by $X(t)$. The zero-mode wave function $\Phi_0(x - X(t))$ serves as the basis function of the $\theta$-fluctuations localized around the center of the DW and $\xi_0(t)$ is the OPZ coordinate. Now, our effective theory is fully described by two dynamical variables $X(t)$ and $\xi_0(t)$ which naturally give physical coordinates along the Hilbert space of orthogonal $\theta$ and $\varphi$ fluctuations. As we will see, we have $\xi_0(t) \neq 0$ only for inequilibrium current flowing state under $E \neq 0$ [Fig. 2(a)].

It is here important to note an essential difference between Tatara and Khono’s approach[8] and ours. Tatara and Khono used $X(t)$ and the weighted average, $\delta_0(t) = \int_{-\infty}^{\infty} dx \theta(x,t) \sin^2 \varphi(x - X(t))$, as dynamical variables. Later, they systematically used complex coordinate $\xi = e^{i\varphi} \tan(\theta/2)$ and described the fluctuations in the form $\xi = e^{-i\varphi} \varphi_0(t) + i \xi_0(t)$.[9] (their notation is reproduced by putting $\theta \rightarrow \pi/2 - \theta$, $\varphi \rightarrow \varphi$ in our notation). In our understanding, these descriptions inevitably cause redundant coupling between $u$ and $v$ modes in Eq. (5). Actually, our natural choice of the dynamical variables is essential to appropriately derive relaxational dynamics described by the following equations of motion given by [123] and [125].

**Equations of motion of the DW:** now, we construct an effective Lagrangian $\mathcal{L} = \mathcal{L}_{DW} + \mathcal{L}_{sd}$ to describe the
Using (7) and (8), the local spin counterpart is given by
\[ \mathcal{L}_{\text{DW}} = \hbar S \int_{-\infty}^\infty dx (\cos \theta - 1) \dot{\varphi} - \mathcal{H}_{\text{DW}} \]
explicitly written as
\[ \mathcal{L}_{\text{DW}} = \frac{\hbar S}{a^3} \left( \sqrt{\frac{2a}{\lambda}} \xi_0 + \pi \right) \dot{X} - \frac{JS}{2\hbar^2} \xi_0^2. \] (9)

To understand the effect of the s-d coupling, it is useful to note \( n_\theta \theta + 3\delta \theta, \varphi_0 + 3\delta \varphi \sim n_0 - e_2 \theta / t - n_0 \delta \theta^2 / 2 \), where we dropped \( \delta \varphi \) because this degree of freedom is eliminated by the global gauge fixing. We have thus s-d Lagrangian,
\[ \mathcal{L}_{\text{sd}} = -J_{\text{sd}} (F_0 - S_1 \xi_0^2 / 2), \] (10)
where \( F_0 [X(t)] = \int_{-\infty}^\infty dx \dot{n}_0 [x - X(t)] \cdot \langle \sigma(x,t) \rangle \) and \( S_1 [X(t)] = \int_{-\infty}^\infty dx \{ \Phi_0 [x - X(t)] \}^2 n_0 [x - X(t)] \cdot \langle \sigma(x,t) \rangle \). Finally, to account for dissipative dynamics, we use the Rayleigh dissipation function \( W_{\text{Rayleigh}} = \frac{\hbar S}{2a^3} \int_{-\infty}^\infty dx \ddot{n}_0 \dot{\varphi} \) explicitly written as
\[ W_{\text{Rayleigh}} = \frac{\alpha \hbar S}{2a^3} (a_0^2 + \frac{2}{\lambda} x^2), \] (11)

where \( \alpha \) is the Gilbert damping parameter. It is simple to write down the Euler-Lagrange-Rayleigh equations, \( d(\partial L/\partial \dot{q}_1)/dt - \partial L/\partial q_1 = -\partial W/\partial \dot{q}_1, \) for the dynamical variables \( q_1 = X \) and \( q_2 = \xi_0 \). We obtain the EOMs which contain the dynamical variables in linear order,
\[ \hbar \sqrt{\frac{2a}{\lambda}} \dot{\xi}_0 + J_{\text{sd}} T_\perp = -2a \frac{\hbar}{\lambda} \dot{X}, \] (12a)
\[ -h \sqrt{\frac{2a}{\lambda}} \dot{X} + \left( \frac{a^2 JS}{\kappa^2} + J_{\text{sd}} S_\parallel \right) \dot{\xi}_0 = -\alpha a \dot{\xi}_0, \] (12b)
where the quantities
\[ T_\perp \equiv -\frac{\partial F_0}{\partial X} = \int_{-\infty}^\infty dx \partial_x \varphi_0 [x - X(t)] \langle \dot{s}_y(x) \rangle, \] (13a)
\[ S_\parallel \equiv \int_{-\infty}^\infty dx \Phi_0^2 [x - X(t)] \langle \dot{s}_x(x) \rangle, \] (13b)
respectively give the non-adiabatic STT and longitudinal spin accumulation. The statistical average of the conduction electron’s spin component is denoted by \( \langle \cdot \cdot \cdot \rangle \). The gauge-transformed spin variables are introduced by \( \hat{s}(x) = \hat{U}^{-1} [x - X(t)] \hat{s}(x) \hat{U} [x - X(t)] \) which has local quantization axis tied to the local spin at the position of \( x - X(t) \). To obtain Eq. (13a), we used relations \( \partial_x n_0 [x - X(t)] = -\partial_x \varphi_0(x) e_2 X n_0 [x - X(t)] \) and \( \langle \dot{s}_y \rangle = -\langle \dot{s}_x \rangle \sin \varphi_0 + \langle \dot{\varphi}_0 \rangle \cos \varphi_0 \). The relation implies that the translation of the DW \( (x \to x - X) \) naturally gives rise to the TSA, \( \langle \dot{s}_y \rangle \), along the local \( y \) axis. The appearance of \( \langle \dot{s}_y \rangle \) causes local magnetic moment which triggers the local spins to precess around the local \( y \) axis and consequently produce finite deviation of the polar angle \( \delta \theta = \theta - \theta_0 \). It is seen that upon switching the external electric field, the deviation \( \delta \theta \) relaxes to finite magnitude in the stationary current-flowing state, i.e., the OPZ coordinate \( \xi_0(t) \) accumulates and reaches finite terminal value \( \xi^0_0 \). We call this process out-of-plane zero-mode accumulation (OPZA) as schematically depicted in Fig. 1(b). This effect is physically interpreted as appearance of demagnetization field phenomenologically introduced by Döring, Kittel, Becker [13], and Slonczewski [1]. It is also to be noted that we ignored the term \( \partial S_\parallel / \partial X \). This simplification is legitimate for the case of small sd-coupling.

**Gilbert relaxation**: coupled equations of motion [12a] and [12b] are readily solved to give relaxational solutions,
\[ \xi_0 = \xi^0_0 (1 - e^{-t/\tau_{\text{DW}}}), \quad V \equiv \dot{X} = V^*(1 - e^{-t/\tau_{\text{DW}}}), \] (14)
where the OPZA reaches the terminal value,
\[ \xi^0_0 = -\frac{1}{\alpha} \sqrt{\frac{a^2 JS}{\kappa^2} + J_{\text{sd}} S_\parallel} \approx -\alpha^{-1} \sqrt{\frac{\lambda}{2a}} \left( \frac{\kappa}{a} \right)^2 \frac{J_{\text{sd}}}{JS} T_{\perp}, \] (15)
and correspondingly the terminal velocity of the DW reaches \( \dot{V}^* = -\lambda \frac{\alpha S_\parallel}{2a \hbar} J_{\text{sd}} T_{\perp} \). The relaxation time of the DW magnetization, \( \tau_{\text{DW}} \), is given by
\[ \tau_{\text{DW}} = \frac{\hbar a}{\kappa^2 JS + J_{\text{sd}} S_\parallel} \approx \alpha^{-1} \left( \frac{\kappa}{a} \right)^2 \frac{\hbar}{JS}. \] (16)

This result clearly shows that the DW magnetization try to relax through the Gilbert damping toward the direction of the newly established precession axis. We stress that without the OPZ coordinate \( \xi_0 \) in Eqs. [12a] and [12b], only the terminal velocity is available and the transient relaxational dynamics is totally lost.

As depicted in Fig. 2(a), the OPZA [Eq. [15]] gives rise to finite out-of-plane \( \langle z \rangle \) component of the local spin,
\[ n_z(x, t) = \cos \theta \simeq \frac{1}{2\alpha} \left( \frac{\kappa}{a} \right)^2 \frac{J_{\text{sd}}}{JS} \cosh \left( [(x - X(t))/\lambda] \right) T_{\perp}. \] (17)

The resultant local spin \( S_\perp = S e_3 n_2(x, t) \) gives the demagnetization field phenomenologically treated by Slonczewski and gives rise to the adiabatic torque \( T_1 = c_1 \partial_x n(x) = c_1 \left( \partial_x \varphi_0 \right) (\sin \varphi_0, \cos \varphi_0, 0) \). At the interface of the DW boundary, \( \varphi_0 = \pi / 2 \) and \( T_1 = c_1 \left( \partial_x \varphi_0 \right) (-1, 0, 0) \), i.e., the adiabatic torque rotate the local spin to counterclockwise direction when the electric
current flows in the (1,0,0)-direction. As is clear from the above discussion, this adiabatic torque is established after the stationary current-flowing \( |j| = (ne^2 \tau_0/m^*)E \) state establishes the non-adiabatic torque, \( T_\perp \). Around the time scale of \( t \approx \tau_0 + \tau_{DW} \), the whole system (including conduction electrons and DW) reaches non-equilibrium but stationary state. In this state, the DW magnetizations continuously feel the OPZA and macroscopically rotate around it. This process exactly corresponds to stationary translation of the DW.

**Computation of** \( T_\perp \): the final step is to compute an explicit form of \( T_\perp \). By taking Fourier transform \( \tilde{c}_k(t) = \frac{1}{\sqrt{L}} \sum_k e^{i kx} \tilde{c}_k(x,t) \), and retaining only the momentum conserving process, we have

\[
T_\perp = \frac{1}{2} \int_{-\pi/a}^{\pi/a} dk \text{ Re} G_{k_1,k_1}^<(t,t), \quad \text{(18a)}
\]

\[
S_\parallel = \frac{\alpha}{2\pi} \int_{-\pi/a}^{\pi/a} dk \text{ Im} G_{k_1,k_1}^<(t,t). \quad \text{(18b)}
\]

Here, the expectation values are computed by using the lesser component of the path-oriented Green function \( G_{k_\sigma,k_\sigma'}^<(t',t) = i \langle \psi^\dagger_{k_\sigma}(t') \psi_{k_\sigma}(t) \rangle \), where \( t \) (\( t' \)) is defined on the upper (lower) branch of Keldysh contour. Since \( S_\parallel \) does not play an essential role, we pay attention to an essential quantity \( T_\perp \). To evaluate the Green functions, we perturbatively treat the s-d coupling and write down the Dyson equation. Then, we truncate the Dyson equation by using the Born approximation including the s-d coupling in linear order which causes a single spin flip process [Fig.2(c)] and gives rise to off-diagonal component in spin space,

\[
G_{k_1,k_1}^<(t,t) = -i \frac{J_{sd}}{2} \frac{f_{k_1} - f_{k_1}}{\varepsilon_{k_1} - \varepsilon_{k_1} - i0}. \quad \text{(19)}
\]

To obtain the explicit form of \( \varepsilon_{k_\sigma} \), we write the single-particle Hamiltonian \( H_0 \) in Fourier space and obtain \( H_\text{tot} = H_0 + H_\text{gauge} \), where \( H_0 \) represents free conduction and \( H_\text{gauge} \) comes from the second term in Eq. (2). By retaining only momentum conserving process, we have \( H_\text{tot} = \sum_{k_\sigma} \varepsilon_{k_\sigma} \tilde{c}_k \tilde{c}_\sigma \), where \( \varepsilon_{k_1} = \hbar^2 k^2/(2m^*) \), where the shift of the Fermi wave numbers due to the background DW is given by \( \delta k = \pi/(2a) \).

Using Eqs. (4), (19), and (18a), we finally obtain the STT which points in the z-direction, \( T_1 = T_\perp e_z \), where its magnitude is given in a form,

\[
T_\perp = \frac{1}{4} \frac{J_{sd}}{k_B T} \cos^2 \left( (\varepsilon_0 - \mu) / (2k_B T) \right) j. \quad \text{(20)}
\]

where \( j_0 = 4n_0 e / (\pi a^*) \) and \( \varepsilon_0 = \hbar^2 k^2/(8m^* a^2) \) corresponds to the chemical potential at half-filling. We have a master formula which gives relation between the current density and the terminal velocity of the DW,

\[
V^* = -\frac{1}{8\alpha} \frac{\lambda J_{sd}}{\hbar} \frac{J_{sd}}{k_B T} \cos^2 \left( (\varepsilon_0 - \mu) / (2k_B T) \right) j. \quad \text{(21)}
\]

As shown in Fig.2(b), we see there is no threshold for the velocity, which is consistent with the result obtained by Thiaville et al.\[10\]. Standard choice of parameters, \( j_0 \approx 10^{16} [\text{A-m}^{-2}] \), \( \lambda = 10^{-8} [\text{m}] \), \( \alpha = 10^{-2} \), \( j \approx 10^{14} [\text{A-m}^{-2}] \) give a rough estimate \( V^* \approx -100 (J_{sd}/k_B T)^2 [\text{m/s}] \). Of course, to pursue more quantitative result needs numerical estimation of \( T_\perp \) taking account of real band structure.

It is essential that the Gilbert damping coefficient, \( \alpha \), enters Eq. (20). The relaxation process of the DW dynamics is governed by the Boltzmann relaxation followed by the Gilbert damping in hierarchical manner. As summarized in Figs.2(a) and (b), in our treatment, it is crucial to recognize that the OPZ coordinate \( \xi_0 \) acquires finite value (i.e., accumulation) only for the current flowing state which is non-equilibrium but stationary. This is the case where dynamical relaxation leads to finite accumulation of physical quantities which are zero in equilibrium. Although essential role of the sliding mode to describe localized spin dynamics was pointed out before\[8, 12\] and importance of out-of-plane canting of the local spins was stressed\[1, 8\], the OPZA presented in this letter has not been discussed before. For example, the sliding motion in Ref.\[12\] does not contain internal deformation of the DW. The OPZA is an outcome of time-reversal-symmetry breaking by electric current. This interpretation seems natural because current-flowing state is off equilibrium.

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[1] J.C. Slonczewski, J. Magn. Magn. Mat. 159 L1 (1996).
[2] L. Berger, Phys. Rev. B 54, 9535 (1996).
[3] M. D. Stiles and A. Zangwill, Phys. Rev. B66, 014407 (2002).
[4] Ya.B. Bazaliy, B.A. Jones, and S.-C. Zhang, Phys. Rev. B 57, R3213 (1998).
[5] S. Zhang and Z. Li, Phys. Rev. Lett. 93, 127204 (2004).
[6] J. Xiao, A. Zangwill and M. D. Stiles, Phys. Rev. B73, 054428 (2006).
[7] S. Zhang, P.M. Levy and A. Fert, Phys. Rev. Lett. 88, 236601 (2002).
[8] G. Tataria, H. Kohn, Phys. Rev. Lett. 92, 086601 (2004).
[9] G. Tataria, H. Kohn and J. Shibata, Phys. Rep. 468, 213 (2008).
[10] A. Thiaville, et al., Europhys. Lett. 69, 990 (2005).
[11] S. Petit, et al., Phys. Rev. Lett. 98, 077203 (2007); Z. Li, et al., Phys. Rev. Lett. 100, 246602 (2008).
[12] S. E. Barnes and S. Maejima, Phys. Rev. Lett. 95, 107205 (2005).
[13] G. E. Volovik, J. Phys. Condens. Matter 20, 83(1987).
[14] The out-of-plane zero-mode was discussed in the context of chiral helimagnet by the present authors: I. G. Bostrom, J. Kishine, and A. S. Ovchinnikov, Phys. Rev. B77, 132405 (2008); Phys. Rev. B78, 064425 (2008); I.G. Bostrom, J. Kishine, R. V. Lavrov, A.S. Ovchinnikov, Phys. Lett. A 373, 558(2009).
[15] W. Dorring, Zeits. f. Naturforschung 3a, 374 (1948); R. Becker, Proceedings of the Grenoble Conference, July (1950); C. Kittel, Phys. Rev. 80, 918 (1950).