Entropy production due to electroweak phase transition in the framework of two Higgs doublet model

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Abstract

We revisit the possibility of first order electroweak phase transition (EWPT) in one of the simplest extensions of the Standard Model (SM) scalar sector, namely the two-Higgs-doublet model (2HDM). We take into account the ensuing constraints from the electroweak precision tests, Higgs signal strengths, and the recent LHC bounds from direct scalar searches. By studying the vacuum transition in 2HDM, we discuss in detail the entropy released in the first order EWPT in various parameter planes of 2HDM.

1 Introduction

It is a well-established fact that EWPT is either a second order or a smooth crossover in the SM of particle physics. So is the fact that the entropy density in the early universe plasma is conserved in the course of the cosmological expansion if the plasma is in thermal equilibrium state with negligible chemical potential of every species \cite{1,2}. The entropy conservation law is given by

$$s = \frac{P + \rho}{T} a^3 = \text{const.}$$

where $a(t)$ is the scale factor, $T(t)$ is the temperature of the fluid (or plasma), $\rho$ and $P$ are the energy density and pressure of the plasma respectively.

In the early universe, the state of matter is quite close to the equilibrium as the reaction rate $\Gamma \sim n\sigma v$ is much faster than the cosmological expansion rate, i.e., the Hubble parameter $H = \dot{a}/a \propto T^2/m_{\text{Pl}}$. The equilibrium condition $\Gamma > H$ is always satisfied for at temperature $T < \alpha m_{\text{Pl}}$. Here $\alpha$ is the coupling constant of the particle interaction of the order of $\sim 10^{-2}$ and $m_{\text{Pl}}$ is the Planck Mass. Due to the large value of $m_{\text{Pl}}$, thermal equilibrium exists in most of the history of the universe, if $\alpha$ is $<< 1$.

As mentioned above, during thermal equilibrium, the entropy density in the comoving volume is conserved. But there are scenarios where the entropy density is not conserved. For example, if the universe at a certain stage was dominated by primordial blackholes \cite{3}, the entropy production can be very high, high enough to delete the pre-existing baryon asymmetry \cite{4}. In the context of the modern cosmological paradigm of inflationary Universe with baryosynthesis and dark matter/energy, physics beyond the Standard model (BSM) underlying these necessary
elements of the modern cosmological model can provide many examples of various mechanisms of high entropy production (see e.g. [5] for review and references). Taking apart the wide range of various possibilities we consider here the problem of entropy production by minimal extension of SM and start the discussion from the SM predictions for the cosmological entropy production.

A large entropy production could take place during QCD phase transition at \( T \sim 100 - 200 \text{MeV} \). But due to strong technical issues, QCD phase transition in early universe cosmology is not known in details. For reference, please see [6].

Few mechanisms of realistic though very weak entropy production could take place during the freeze-out of dark matter (DM) particles. But usually, the fraction of DM density was quite low at the freezing our temperature and the effect is tiny.

An interesting effect, not covered in this paper, is the formation of bubbles walls that can take place in the early universe. The collision of them can lead to the formation of primordial black holes due to first order phase transition with background gravitational waves [8], [9].

Most probably, the largest entropy production took place considering the SM during the EWPT. The entropy release happened when the universe went from a phase of symmetric electroweak phase to an asymmetric electroweak phase during the universe cooling. In the minimal SM with one Higgs field, the process is a mild crossover and the entropy production is about 13% [17].

According to the electroweak (EW) theory at the temperatures higher than a critical one, \( T > T_c \), the expectation value of the Higgs field, \( \langle \phi \rangle \), in the fluid (plasma) is zero and the universe is in electroweak symmetric phase [7]. When the temperature drops below \( \langle T_c \rangle \), a non-zero expectation value is created, which gradually rises, with decreasing temperature, up to the vacuum expectation value \( \eta \). Such a state does not satisfy the conditions necessary for the entropy conservation and an entropy production is expected.

A huge amount of entropy is released if EWPT is first order, which is the case even with the minimalist extension of standard model namely two-Higgs-doublet Model (2HDM). In what follows, we have considered a real 2HDM and scanned over certain parameter spaces and used numerical analysis to calculate the entropy production for some interesting and unique benchmark points.

The paper is arranged as follows: In the next section details about 2HDM is given along with some LHC constrains followed by the theoretical framework of the process. Due to cumbersome and very difficult analytical calculations, we did numerical analysis of certain parameters using BSMPT package [11] and it followed by a general discussion and conclusion. The paper has 2 Appendixes, giving details about the metric that is being used here and also the masses of the scalar bosons generated by 2HDM.

2 2HDM: A small review

There are two scalar doublets in the framework and they are defined as:

\[
\varphi_I = \left( \frac{1}{\sqrt{2}} (\phi_I^+ + \rho_I) + i \eta_I \right),
\]

with \( I = 1, 2 \). Here \( \phi_I^+, \rho_I, \eta_I \), and \( v_I \) indicate the charged, neutral CP-even and neutral CP-odd degrees of freedom (d.o.f.) and the vacuum expectation value (vev) of the \( I \)-th doublet respectively.

Prior to spontaneous symmetry breaking (SSB), the tree-level 2HDM Lagrangian, assumes the form

\[
\mathcal{L} = \mathcal{L}_{\text{kin}} + \mathcal{L}_{\text{Yuk}} - V(\varphi_1, \varphi_2) + \mathcal{L}_6,
\]
where,
\[
\mathcal{L}_{\text{kin}} = -\frac{1}{4} \sum_{X=G^0,W^\pm,B} X_{\mu\nu} X^{\mu\nu} + \sum_{l=1,2} |D_\mu \varphi_l|^2 + \sum_{\psi=Q,L,u,d,l} \bar{\psi} i D \psi,
\]
\[
\mathcal{L}_{\text{Yuk}} = \sum_{l=1,2} Y_{l}^{\mu} \bar{e}_l \varphi_l + \sum_{l=1,2} Y_{l}^{d} \bar{d}_l \varphi_l + \sum_{l=1,2} Y_{l}^{u} \bar{u}_l \varphi_l,
\]
\[
V(\varphi_1, \varphi_2) = m_{11}^2 |\varphi_1|^2 + m_{22}^2 |\varphi_2|^2 - (\mu^2 |\varphi_1|^2 + h.c.) + \lambda_1 |\varphi_1|^4 + \lambda_2 |\varphi_2|^4 + \lambda_3 |\varphi_1|^2 |\varphi_2|^2 + \lambda_4 |\varphi_1|^2 |\varphi_2|^2 + \left[ \left( \frac{\lambda_5}{2} \varphi_1^2 \varphi_2 + \lambda_6 |\varphi_1|^2 + \lambda_7 |\varphi_2|^2 \right) \varphi_1 \varphi_2 + h.c. \right],
\]
(4)

In this paper, we assume the CP-conserving 2HDM scenario, and hence $\lambda_{6,7} = 0$. The electroweak symmetry is broken by the vacuum expectation values (vev), namely $v_1$ and $v_2$ corresponding to the two doublets $\varphi_{1,2}$ respectively. This leads to the mixing of same types of degrees of freedom of $\varphi_{1,2}$.

After spontaneous symmetry breaking, the Yukawa sector of the 2HDM can be written as,
\[
-\mathcal{L}_{\text{Yuk}} = \frac{1}{\sqrt{2}} (\kappa_D s_{\beta-\alpha} + \rho_D c_{\beta-\alpha}) \bar{D} D h + \frac{1}{\sqrt{2}} (\kappa_D c_{\beta-\alpha} - \rho_D s_{\beta-\alpha}) \bar{D} D h
\]
\[
+ \frac{i}{\sqrt{2}} (\kappa_s s_{\beta-\alpha} + \rho_s c_{\beta-\alpha}) \bar{U} U h + \frac{i}{\sqrt{2}} (\kappa_s c_{\beta-\alpha} - \rho_s s_{\beta-\alpha}) \bar{U} U h
\]
\[
+ \frac{i}{\sqrt{2}} (\kappa_L s_{\beta-\alpha} - \rho_L c_{\beta-\alpha}) \bar{L} L h + \frac{i}{\sqrt{2}} (\kappa_L c_{\beta-\alpha} - \rho_L s_{\beta-\alpha}) \bar{L} L h
\]
\[
- i \frac{\sqrt{2}}{\sqrt{2}} \bar{U} \gamma_5 \rho_U U A + i \frac{\sqrt{2}}{\sqrt{2}} \bar{D} \gamma_5 \rho_D D A + i \frac{\sqrt{2}}{\sqrt{2}} \bar{L} \gamma_5 \rho_L L A
\]
\[
+ (\bar{U} (V_{\text{CKM}} \rho_D P_R - \rho_U V_{\text{CKM}} P_L) D H^+ + \bar{D} \rho_L P_R H^+ + h.c.).
\]
(5)

The generation indices of the fermionic fields have been suppressed in eq. (5). The limit $\cos(\beta - \alpha) \rightarrow 0$ with heavy scalars can lead back to the standard model scenario.

For type-I 2HDM, where range is allowed to be $|\cos(\beta - \alpha)| \lesssim 0.4$. Among the tree-level couplings, the decays of new scalars, $AZh$ and $H^\pm hW^\mp$ are proportional to $\cos(\beta - \alpha)$, whereas $AZh$ and $H^\pm hW^\mp$ are proportional to $\sin(\beta - \alpha)$. It is possible to realize an exact alignment in the multi-Higgs-doublet models in the framework of certain additional symmetries of the 2HDM potential [30, 31, 32, 33, 34]. The impact of the $\cos(\beta - \alpha)$– $\tan \beta$ plane has been discussed in ref. [35]. A hierarchical spectrum like $m_A > m_H \sim m_{H^\pm} \sim v$ can lead to a first order EWPT providing an explanation for the matter-antimatter asymmetry.

3 EWPT theory in 2HDM

The lagrangian density of the Electroweak theory (discussed in details in the previous section) in 2HDM can be expressed as [12]
\[
\mathcal{L} = \mathcal{L}_f + \mathcal{L}_{\text{Yuk}} + \mathcal{L}_{\text{gauge,kin}} + \mathcal{L}_{\text{Higgs}}
\]
(6)

The first term on the right hand side, $\mathcal{L}_f$, is the kinetic term for the fermion-fields
\[
\mathcal{L}_f = \sum_j i \left( \bar{\Psi}_L^{(j)} i D \Psi_L^{(j)} + \bar{\Psi}_R^{(j)} i D \Psi_R^{(j)} \right)
\]
(7)
\[
= i \bar{\Psi}_L \gamma^\mu (\partial_\mu + ig W_\mu + ig Y_L B_\mu) \Psi_L
\]
\[
+ i \bar{\Psi}_R \gamma^\mu (\partial_\mu + ig W_\mu + ig Y_R B_\mu) \Psi_R
\]
(8)
Thus from Eq. 11, we get
\[ y_e e_R \Phi_e^L L_L + y_e L_L \Phi_e^R e_R + \cdots \]  
where \( y_e \) is a complex dimensionless constant, \( \Phi_a \) (\( a = 1, 2 \)) is a \( SU(2)_L \) doublet and for the Lagrangian to be gauge invariant it is coupled with another \( SU(2)_L \) fermion \( L_L \). \( e_R \) is the right chiral electron field and the same goes for other fermions like quarks, neutrinos, etc.

The second term of Eq. 6, Yukawa interaction term (for details, see previous section), \( L_{Yuk} \) is [13]

\[ L_{Yuk} = \frac{1}{4} G_{\mu\nu} G^{\mu\nu} - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \]  
where \( G_{\mu\nu} = \partial_{\mu} \Phi_{\nu} - \partial_{\nu} \Phi_{\mu} - g\epsilon^{ijk} W^i_{\mu} W^j_{\nu} \) and \( F_{\mu\nu} = \partial_{\mu} B_{\nu} - \partial_{\nu} B_{\mu} \).

The lagrangian desity for the doublet Higgs bosons is given by

\[ L_{Higgs} = (D^\mu \Phi_1)^\dagger (D_\mu \Phi_2) + (D^\mu \Phi_1)^\dagger (D_\mu \Phi_2) - V_{\text{tot}}(\Phi_1, \Phi_2) = \{(\partial_{\mu} + igT^i W^i_{\mu} + ig' Y_{B_{\mu}}) \Phi_1\}^\dagger \{(\partial_{\mu} + igT^i W^i_{\mu} + ig' Y_{B_{\mu}}) \Phi_2\} \\
+ \{(\partial_{\mu} + igT^i W^i_{\mu} + ig' Y_{B_{\mu}}) \Phi_1\}^\dagger \{(\partial_{\mu} + igT^i W^i_{\mu} + ig' Y_{B_{\mu}}) \Phi_2\} - V_{\text{tot}}(\Phi_1, \Phi_2, T) \]  

We define
\[ W_{\mu} = g T^i W^i_{\mu} + g' Y_{B_{\mu}} \]  
Thus from Eq. 11 we get

\[ L_{Higgs, \text{kin}} = (D^\mu \Phi_a)^\dagger (D_\mu \Phi_a) - i (W^\mu \Phi_a)^\dagger (D_\mu \Phi_a) + i (D^\mu \Phi_a)^\dagger W_{\mu} \Phi_a + (W^\mu \Phi_a)^\dagger W_{\mu} \Phi_a \]  

The standard CP-conserving 2HDM potential \( V_{\text{tot}}(\Phi_1, \Phi_2, T) \) consists of tree level potential \( V_{\text{tree}}(\Phi_1, \Phi_2) \)

\[ V_{\text{tree}}(\Phi_1, \Phi_2) = m_1^2 \Phi_1^\dagger \Phi_1 + m_2^2 \Phi_2^\dagger \Phi_2 - \left[ m_{12}^2 \Phi_1^\dagger \Phi_2 + m_{12}^2 \Phi_2^\dagger \Phi_1 \right] + \frac{1}{2} \lambda_1 \left( \Phi_1^\dagger \Phi_1 \right)^2 \\
+ \frac{1}{2} \lambda_2 \left( \Phi_2^\dagger \Phi_2 \right)^2 + \lambda_3 \left( \Phi_1^\dagger \Phi_1 \right) \left( \Phi_2^\dagger \Phi_2 \right) + \lambda_4 \left( \Phi_1^\dagger \Phi_2 \right) \left( \Phi_2^\dagger \Phi_1 \right) \\
+ \frac{1}{2} \lambda_5 \left( \Phi_1^\dagger \Phi_2 \right)^2 + \frac{1}{2} \lambda_5 \left( \Phi_2^\dagger \Phi_1 \right)^2 \]  

and other correction terms \( V_{\text{CW}}(v_1, v_2) \) and \( V_T \). The correction terms are defined as [14] [15]

\[ V_{\text{CW}}(v_1 + v_2) = \sum_i \frac{n_i}{64\pi^2} (-1)^{2s_i} m_i^4 (v_1, v_2) \left[ \log \left( \frac{m_i^2 (v_1, v_2)}{\mu^2} \right) - c_i \right] \]  

\[ V_T = \frac{T^4}{2\pi^2} \left( \sum_{i=\text{bosons}} n_i J_B \left[ \frac{m_i^2 (v_1, v_2)}{T^2} \right] + \sum_{i=\text{fermions}} n_i J_F \left[ \frac{m_i^2 (v_1, v_2)}{T^2} \right] \right) \]  

where \( \mu \) is the renormalisation scale which we take to be 246 GeV.

The potential dependent mass of fermions and bosons \( m_i (v_1 + v_2) \) and the corresponding \( n_i, s_i, \) and \( c_i \) are discussed in details in Appendix [13].

\( J_B \) and \( J_F \) are approximated Landau gauge up to leading orders as following

\[ J_B = \frac{1}{2} \lambda_1 \left( \Phi_1^\dagger \Phi_1 \right)^2 + \frac{1}{2} \lambda_2 \left( \Phi_2^\dagger \Phi_2 \right)^2 + \lambda_3 \left( \Phi_1^\dagger \Phi_1 \right) \left( \Phi_2^\dagger \Phi_2 \right) + \lambda_4 \left( \Phi_1^\dagger \Phi_2 \right) \left( \Phi_2^\dagger \Phi_1 \right) + \frac{1}{2} \lambda_5 \left( \Phi_1^\dagger \Phi_2 \right)^2 + \frac{1}{2} \lambda_5 \left( \Phi_2^\dagger \Phi_1 \right)^2 \]  

\[ J_F = \frac{1}{2} \lambda_1 \left( \Phi_1^\dagger \Phi_1 \right)^2 + \frac{1}{2} \lambda_2 \left( \Phi_2^\dagger \Phi_2 \right)^2 + \lambda_3 \left( \Phi_1^\dagger \Phi_1 \right) \left( \Phi_2^\dagger \Phi_2 \right) + \lambda_4 \left( \Phi_1^\dagger \Phi_2 \right) \left( \Phi_2^\dagger \Phi_1 \right) + \frac{1}{2} \lambda_5 \left( \Phi_1^\dagger \Phi_2 \right)^2 + \frac{1}{2} \lambda_5 \left( \Phi_2^\dagger \Phi_1 \right)^2 \]
\[ T^4 J_B \left[ \frac{m^2}{T} \right] = -\frac{\pi^4 T^4}{45} + \frac{\pi^2}{12} T^2 m^2 - \frac{\pi}{6} T (m^2)^{3/2} - \frac{1}{32} m^4 \ln \frac{m^2}{a_b T^2} + \cdots \]  

\[ T^4 J_F \left[ \frac{m^2}{T} \right] = \frac{7 \pi^4 T^4}{360} - \frac{\pi^2}{24} T^2 m^2 - \frac{1}{32} m^4 \ln \frac{m^2}{a_f T^2} + \cdots \]  

where \( a_b = 16a_f = 16\pi^2 \exp(3/2 - 2\gamma_E) \) with \( \gamma_E \) being the Euler-Mascheroni constant.

When the temperature of the universe drops down to the critical temperature \( T_c \), a second local minimum appears with the same height of the global minimum situated at \( \langle \Phi_1 \rangle = \langle \Phi_2 \rangle = 0 \) \([16]\). The critical temperature can be obtained using the following expression:

\[ V_{tot} (\Phi_1 = 0, \Phi_2 = 0, T_c) = V_{tot} (\Phi_1 = v_1, \Phi_2 = v_2, T_c) \]  

During Electroweak Phase Transition (EWPT), if \( \rho \) and \( P \) are respectively the energy density and pressure of the fluid determining the course of evolution of the early universe, then from \([5]\)

\[ \rho = \rho_f + \rho_{gauge, kin} + \rho_{Higgs} - g^{00} \mathcal{L}_{Yuk} \]  

\[ P = P_f + P_{gauge, kin} + P_{Higgs} - \frac{1}{3} g^{ii} \mathcal{L}_{Yuk} \]  

We have assumed that dark matter and other components might have been present but they did not contribute much to the energy density of the universe during the particular epoch of EWPT which happened in radiation domination. The expressions for \( \rho_f, P_f \) and \( \rho_{gauge, kin}, P_{gauge, kin} \) appear solely from fermionic and gauge sectors and their interactions. The Stress-energy tensor for the above quantities is mentioned in the Appendix [A].

\[ \rho_{H, F, G} = \left[ \{ \partial^\alpha \Phi^\dagger_a - i(\mathcal{W}^\alpha \Phi_a)^\dagger \} \partial^\beta \Phi_a + \{ \partial^\beta \Phi_a + i\mathcal{W}^\beta \Phi_a \} \partial^\alpha \Phi^\dagger_a \right] \]  

\[ -g^{00} \left[ (\partial^\alpha \Phi^\dagger_a)(\partial_\alpha \Phi_a) - i(\mathcal{W}^\alpha \Phi_a)^\dagger(\partial_\alpha \Phi_a) + i(\partial^\alpha \Phi_a)^\dagger \mathcal{W}_\alpha \Phi_a + (\mathcal{W}^\alpha \Phi_a)^\dagger \mathcal{W}_\alpha \Phi_a \right] \]  

\[ -g^{00} [V_{tot}(\Phi_1, \Phi_2, T)] \]  

\[ P_{H, F, G} = \left[ \{ \partial^\alpha \Phi^\dagger_a - i(\mathcal{W}^\alpha \Phi_a)^\dagger \} \partial^\beta \Phi_a + \{ \partial^\beta \Phi_a + i\mathcal{W}^\beta \Phi_a \} \partial^\alpha \Phi^\dagger_a \right] \]  

\[ -g^{ii} \left[ (\partial^\alpha \Phi^\dagger_a)(\partial_\alpha \Phi_a) - i(\mathcal{W}^\alpha \Phi_a)^\dagger(\partial_\alpha \Phi_a) + i(\partial^\alpha \Phi_a)^\dagger \mathcal{W}_\alpha \Phi_a + (\mathcal{W}^\alpha \Phi_a)^\dagger \mathcal{W}_\alpha \Phi_a \right] \]  

\[ -g^{ii} [V_{tot}(\Phi_1, \Phi_2, T)] \]  

The early universe was flat, hence the metric \( g_{\mu\nu} = (+, - , - , -) \). And hence \( \rho_{H, F, G} + P_{H, F, G} \) becomes:

\[ \rho_{H, F, G} + P_{H, F, G} = \partial^\beta \Phi_a \partial^\beta \Phi^\dagger_a + 2 (\mathcal{W}^\alpha \Phi_a)^\dagger \mathcal{W}_\alpha \Phi_a \]  

\[ + \left[ (\partial^\beta \Phi^\dagger_a)(\partial_\beta \Phi_a) - i(\mathcal{W}^\beta \Phi_a)^\dagger(\partial_\beta \Phi_a) + i(\partial^\beta \Phi_a)^\dagger \mathcal{W}_\beta \Phi_a \right] \]  

\[ - [V_{tot}(\Phi_1, \Phi_2, T) + \mathcal{L}_{Yuk}] \]  

where the explicit expression for \( \rho_{H, F, G} \) is given in the Eq.25

\[ \rho_{H, F, G} = \partial^\beta \Phi_a \partial^\beta \Phi^\dagger_a + (\mathcal{W}^\alpha \Phi_a)^\dagger \mathcal{W}_\alpha \Phi_a \]  

\[ - [V_{tot}(\Phi_1, \Phi_2, T) + \mathcal{L}_{Yuk}] \]  

\[ \]
The oscillations of the Higgs fields around minimum after it appeared in the course of the phase transition are damped due to particle production by the oscillating field. The characteristic time is equal to the decay width of the Higgses and it is large in comparison with the expansion rate and the universe cooling rate. So we may assume that Higgses essentially live in the minimum of the potential. In principle, it can be calculated numerically by the solution of the corresponding Klein-Gordon equation with damping induced by the particle production. \[17\].

With the above assumption

\[\rho = \frac{\dot{\Phi}^2}{2} + V_{\text{tot}}(\Phi_1, \Phi_2, T) + \frac{g_s \pi^2}{30} T^4\]  \hspace{1cm} (26)

The last term in Eq. 26 arises from the Yukawa interaction between fermions and Higgs bosons and from the energy density of the fermions, the Gauge bosons and the interaction between the Higgs and Gauge bosons. This is the energy density of the relativistic particles which have not gained mass till the moment of EWPT.

Since for relativistic species \(P = (1/3)\rho\), we can write

\[P = \frac{\dot{\Phi}^2}{2} + \frac{1}{3} \frac{g_s \pi^2}{30} T^4\]  \hspace{1cm} (27)

The oscillations of scalar fields around their minima are quickly damped, so we take the time derivative of the fields equivalent to the their derivative around the minima, and neglect higher order terms of their time derivative \(\dot{\phi}^2\) and so on. And as a result the evolution of the minima induced by the expansion of the universe is very slow.

The entropy conservation law holds when the plasma (assumed to be an ideal fluid) was in thermal equilibrium with negligible chemical potential. But as the temperature went below \(T_c\), EWPT happened and the universe went into a thermally non-equilibrium state. It is to be noted that one of the main consequences of EWPT is Electroweak baryogenesis and following Sakharov’s principle, out of equilibrium process is a necessary condition for successful baryogenesis.

As a result of this deviation from thermal equilibrium, the entropy conservation law is no more valid during EWPT and hence a rise in the entropy production can be noticed significantly during this process.

To calculate this production, it is necessary to solve the evolution equation for energy density conservation

\[\dot{\rho} = -3H(\rho + P)\]  \hspace{1cm} (28)

From hence forward computational analysis was used for further calculations which are discussed in the next section.

4 Entropy release in 2HDM scenarios

At very early time when the temperature of universe \(T \gg T_c\), the universe was in thermal equilibrium and also was dominated by relativistic species. Almost all of the fermions and bosons were massless and contribution from those who were already massive (e.g. DM) to the total energy density of the universe was insignificant. During that epoch the chemical potential of the massless bosons was zero and with the assumption that chemical potential of the fermions was negligible, the entropy density per comoving volume was conserved and is given by

\[s \equiv \frac{\rho_r + P_r}{T} a^3 = \text{const.}\]  \hspace{1cm} (29)
where the subscript $r$ is used to indicate relativistic components. For our scenario

$$\rho_r + P_r \sim g_* T^4$$  \hfill (30)$$
g_* is not constant over time; it depends on the components of the hot primordial hot soup. Those two equations (Eq. (29) and Eq. (30)) implies

$$T \sim a^{-1}$$  \hfill (31)$$
As long as the thermal equilibrium were maintained, $s$ remained constant. If the thermal equilibrium was not remained at some epoch at later time, the value of $s$ and thus $g_* (T) a^3 T^3$, might have increased as entropy can only either increase or remain constant.

As temperature decreased to $T_c$ Higgs potential got degenerate minima. Later temperature dropped down more. If the temperature dropped to the mass of any component of the relativistic plasma, that component gained mass and became non-relativistic and decoupled from relativistic fluid. We are assuming this process was instantaneous and the universe was not in thermal equilibrium. There is change in $g_*$ of the relativistic plasma. This led to increase of $s$. If this decoupling process was in thermal equilibrium, it would cause a sudden increase of temperature of the universe.

Suppose at $T_c$, the scale factor was $a_c$ and $g_* \equiv g_{*,tot} = 110.75$ for our 2HDM model and thus at that moment $s_c \sim g_{*,tot} a_c T_c$. While temperature dropped to $T \sim T_x$, the component '$x$' would decoupled and thus $g_*$ of the relativistic plasma would decrease. If the instantaneous decoupling process was occurred at equilibrium, it would increase the temperature of the photons. However, Why are we considering as we are considering non-equilibrium case, $s$ of the universe was increased. If $g_{*,+}$ and $g_{*,-}$ be the $g_*$-factor before and after the decoupling of the '$x$', then change in $s$ relative to the time of critical temperature

$$\frac{\delta s}{s_c} = \frac{(g_{*,+} a_x T_x)^3 - (g_{*,tot} a_c T_c)^3}{(g_{*,tot} a_c T_c)^3}$$  \hfill (32)$$
BSMPT is a C++ package which deals with various properties and features related to 2HDM and baryon asymmetry. In this case, the package was used to calculate the critical temperature for $T_c$ and the vacuum expectation value vev and also $V_{eff}(T)$ for each benchmark points. The calculation was repeated for 4 parameter space, the first one being the benchmark points provided in the BSMPT manual. The differential equation Eq.28 was solved numerically by interpolating the data for $V_{eff}(T)$ for all the benchmark points and the entropy release was calculated for the same.

For 4 different benchmark points, as mentioned in Table the entropy release has been calculated with the assumption of $a_c T_c \sim 1$ and it is presented in Fig. 1.

As it is seen from Fig. 1, the amount of entropy release increases as the critical temperature for EWPT increases. For example, the entropy production for $T_c = 139.5 GeV$ is $\sim 41\%$, for $T_c = 151 GeV$ is $\sim 52\%$, for $T_c = 173.5 GeV$ is $\sim 63\%$ and for $T_c = 255.5 GeV$ is $\sim 73\%$. All these results are way higher than the entropy release by EWPT in SM which is $\sim 13\%$.

The main reason for this excess in the production of entropy is the extra scalar bosons produced in 2HDM which contributes the most to the process. It is to be noted that the contributions from lighter particles like electrons and neutrinos are similar to that of SM.

5 Conclusion

It is shown and calculated in this paper that the total entropy release due to EWPT is very large even in the framework of minimal extension of SM of particle physics namely 2HDM compared to minimal SM of physics. It is a proven fact that unlike SM where EWPT is of second order, in the mere extension of SM EWPT becomes a first order phase transition. An interesting fact is
Figure 1: Entropy production for various benchmark points as given in Table-1.

Table 1: 2HDM Benchmark points for entropy production

|                | $m_h$ | $m_H$ | $m_{H^\pm} = m_A$ | $\tan \beta$ |
|----------------|-------|-------|-------------------|--------------|
| Benchmark-I    | 125   | 500   | 500               | 10           |
| Benchmark-II   | 125   | 400   | 500               | 10           |
| Benchmark-III  | 125   | 90    | 400               | 10           |
| Benchmark-IV   | 125.09| 228.17| 233               | 6.94         |

that as $g_*$ decreases as the temperature falls down. But as we go to a very low temperature scale, the minimum temperature ($T_{min}$) takes the value of the particle mass and their contribution remains the same like that of SM.

There are two points which should be noted. Firstly, the benchmark points are unique and they were calculated using the BSMPT package. BSMPT gives the results for those benchmark points which satisfies the condition $\text{vev}/T_c > 1$. All the benchmark points used here satisfy this condition. Secondly in this paper we have considered only the real sector of 2HDM. If other extensions of 2HDM like complex 2HDM are considered, there might be considerable change in the entropy production.

In passing by, two effects are needed to be mentioned even though they are beyond the scope of this paper. Firstly, the entropy release due to EWPT can considerably reduce the abundance of Dark matter present in the universe before EWPT. Detailed calculations of this dilution factor for Standard model are done in [4]. Secondly, bubble walls that were formed might collide and may produce primordial black holes and might lead to a sufficient entropy production. The bubble collisions are also the source of primordial gravitational wave background. These will be studied in the subsequent papers.

Author Contribution
Article by A.C. and M.K.. The authors contributed equally to this work. All authors have read and agreed to the published version of the manuscript.
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A Appendix: Energy Momentum Tensor

\[
T_{\mu\nu} = \sum_j i \left( \bar{\Psi}_L^{(j)} \gamma_\mu \partial^\nu \Psi_L^{(j)} + \bar{\Psi}_R^{(j)} \gamma_\mu \partial^\nu \Psi_R^{(j)} \right) - g^{\mu\nu} L_f \tag{33}
\]

\[
T_{\mu\nu}^{\text{gauge,kin}} = \left[ F^{\mu\alpha} \partial_\nu B_\alpha - \frac{1}{4} g^{\mu\nu} F^{\alpha\beta} F_{\alpha\beta} \right] + \left[ G^i_{\alpha} \partial_\nu W_\alpha - \frac{1}{4} g^{\mu\nu} G^i_{\alpha\beta} G^i_{\alpha\beta} \right] - g \epsilon^{ijk} \left( W^{ij}_{\alpha} \partial^\nu W_\alpha - W^{ij}_{\alpha} W^{\nu k} \partial^\mu W_\alpha \right) \tag{34}
\]

B Appendix: Masses of new Scalars

\[
c_i = \begin{cases} 
\frac{5}{6}, & (i = W^\pm, Z, \gamma) \\
\frac{3}{2}, & \text{otherwise} 
\end{cases} \tag{35}
\]

| Bosons | \(n_i\) | \(s_i\) | \(m(v)^2\) |
|--------|-------|-------|-----------|
| \(h\)  | 1     | 1     | eigenvalues of 47 Higgs |
| \(H\)  | 1     | 1     | eigenvalues of 47 Higgs |
| \(A\)  | 1     | 1     | eigenvalues of 47 Higgs |
| \(G^0\)| 1     | 1     | eigenvalues of 47 Goldstone |
| \(H^\pm\)| 2   | 1     | Eq. 39 Charged Higgs |
| \(G^\pm\)| 2   | 1     | Eq. 40 Charged Goldstone |
| \(Z_L\)| 1     | 2     | Eq. 37 Higgs |
| \(Z_T\)| 2     | 2     | Eq. 37 Higgs |
| \(W_L\)| 2     | 1     | Eq. 36 Higgs |
| \(W_T\)| 4     | 2     | Eq. 36 Higgs |
| \(\gamma_L\)| 1 | 2     | Eq. 38 |
| \(\gamma_T\)| 2   | 2     | Eq. 38 |

\[
m_W^2 = \frac{g^2}{4} v^2. \tag{36}
\]

\[
m_Z^2 = \frac{g^2 + g'^2}{4} v^2. \tag{37}
\]

\[
m_\gamma^2 = 0. \tag{38}
\]

\[
\bar{m}_{H^\pm}^2 = \frac{1}{2} (M_{11}^C + M_{22}^C) + \frac{1}{2} \sqrt{4 \left( (M_{12}^C)^2 + (M_{13}^C)^2 \right) + (M_{11}^C - M_{22}^C)^2}. \tag{39}
\]

\[
\bar{m}_{G^\pm}^2 = \frac{1}{2} (M_{11}^C + M_{22}^C) - \frac{1}{2} \sqrt{4 \left( (M_{12}^C)^2 + (M_{13}^C)^2 \right) + (M_{11}^C - M_{22}^C)^2}. \tag{40}
\]

where

\[
c_1 = \frac{1}{48} \left( 12 \lambda_1 + 8 \lambda_3 + 4 \lambda_4 + 3 (3g^2 + g'^2) \right) \tag{41}
\]

\[
c_2 = \frac{1}{48} \left( 12 \lambda_2 + 8 \lambda_3 + 4 \lambda_4 + 3 (3g^2 + g'^2) + \frac{24}{v_2^2} m_b^2 (T = 0) \right) + \frac{1}{2v_2^2} m_b^2 (T = 0) \tag{42}
\]
Table 2: Field dependent mass of all fermions

| Fermions | \( n_i \) | \( s_i \) | \( m_f(T = 0) \) |
|----------|----------|---------|----------------|
| \( e \)  | 4        | \( \frac{3}{2} \) | \( \frac{\mu_i}{\sqrt{2}} v_k \) lepton |
| \( \mu \) | 4        | \( \frac{1}{2} \)  | \( \frac{\nu_{\mu}}{\sqrt{2}} v_k \) lepton |
| \( \tau \) | 4        | \( \frac{1}{2} \)  | \( \frac{\nu_{\tau}}{\sqrt{2}} v_k \) lepton |
| \( u \)  | 12       | \( \frac{3}{2} \) | \( \frac{\nu_u}{\sqrt{2}} v_k \) quark |
| \( c \)  | 12       | \( \frac{1}{2} \)  | \( \frac{\nu_c}{\sqrt{2}} v_k \) quark |
| \( t \)  | 12       | \( \frac{1}{2} \)  | \( \frac{\nu_t}{\sqrt{2}} v_k \) quark |
| \( d \)  | 12       | \( \frac{1}{2} \)  | \( \frac{\nu_d}{\sqrt{2}} v_k \) quark |
| \( s \)  | 12       | \( \frac{1}{2} \)  | \( \frac{\nu_s}{\sqrt{2}} v_k \) quark |
| \( b \)  | 12       | \( \frac{1}{2} \)  | \( \frac{\nu_b}{\sqrt{2}} v_k \) quark |

where \( m_l(T = 0) = 172.5\text{Gev} \) and \( m_b(T = 0) = 4.92\text{GeV} \). For our case \((v_3 = 0)\),

\[
\mathcal{M}_{11}^C = m_{i1}^2 + \lambda_1 \frac{v_1^2}{2} + \lambda_3 \frac{v_2^2}{2} \quad (43)
\]

\[
\mathcal{M}_{22}^C = m_{i2}^2 + \lambda_2 \frac{v_2^2}{2} + \lambda_3 \frac{v_1^2}{2} \quad (44)
\]

\[
\mathcal{M}_{12}^C = \frac{v_1 v_2}{2} (\lambda_4 + \lambda_5) - m_{12}^2 \quad (45)
\]

\[
\mathcal{M}_{13}^C = 0 \quad (46)
\]

Masses of \( h, H \) and \( A \) are the eigen values of the matrix

\[
\mathcal{M}^N = (\mathcal{M}^N) \quad (47)
\]

For our case \((v_3 = 0)\),

\[
\mathcal{M}_{11}^N = m_{i1}^2 + \frac{3\lambda_1}{2} v_1^2 + \frac{\lambda_3 + \lambda_4}{2} v_2^2 + \frac{1}{2} \lambda_5 v_2^2 \quad (48)
\]

\[
\mathcal{M}_{22}^N = m_{i2}^2 + \frac{\lambda_1}{2} v_1^2 + \frac{\lambda_3 + \lambda_4}{2} v_2^2 - \frac{1}{2} \lambda_5 v_2^2 \quad (49)
\]

\[
\mathcal{M}_{33}^N = m_{i3}^2 + \frac{3\lambda_2}{2} v_2^2 + \frac{1}{2} (\lambda_3 + \lambda_4 + \lambda_5) v_1^2 \quad (50)
\]

\[
\mathcal{M}_{34}^N = m_{i4}^2 + \frac{\lambda_2}{2} v_2^2 + \frac{1}{2} (\lambda_3 + \lambda_4 - \lambda_5) v_1^2 \quad (51)
\]

\[
\mathcal{M}_{12}^N = 0 \quad (52)
\]

\[
\mathcal{M}_{13}^N = -m_{12}^2 + (\lambda_3 + \lambda_4 + \lambda_5) v_1 v_2 \quad (53)
\]

\[
\mathcal{M}_{14}^N = 0 \quad (54)
\]

\[
\mathcal{M}_{23}^N = 0 \quad (55)
\]

\[
\mathcal{M}_{24}^N = -m_{12}^2 + \lambda_5 v_1 v_2 \quad (56)
\]

\[
\mathcal{M}_{34}^N = 0 \quad (57)
\]

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