Abstract

This is a slightly extended version of a seminar given the 8th of June at the TASI 99 at Colorado University in Boulder. The motivations behind two time theory are explained and the theory is introduced via one of the theory’s easier gauges of a particle on a black hole background. Important results that should be interesting as well in the light of the recent AdS mania will be summarized.

1 Motivations for Two Times

1.1 "I have Two Times"

Here is one common misconception about two time theory: They saw many spatial dimensions being used so they tried many time dimensions. After realizing there would be new negative norm states (ghosts) they looked desperately for a gauge freedom to fix it.

Seen this way, the theory seems artificial, contrived and just not asked for.

This is NOT the way two time theory has been found. I personally like to introduce it in the following way (not quite historically stringent either) - stressing that nature actually tells us that two times are needed:
The huge success of today’s fundamental theories like the standard model and general relativity is due to localization of global gauge freedoms. Say you start out discovering that the world shows Poincare symmetry. Lifting this symmetry to a local one (motto: why should a region here be too rigidly related to a region over there) gives back general relativity. Once we recognize that a global symmetry becomes much more fundamental when localized, nature tells us in this way that it has gravity, photons, and so on. There is one global symmetry that has never been localized. It is the \( \text{Sp}(2) \) symmetry in phase space. The symmetry acts on the doublet

\[
\begin{pmatrix}
\Phi_1 \\
\Phi_2
\end{pmatrix} = \begin{pmatrix} X \\
P \end{pmatrix}
\]

which lets us rewrite the quantization \([X, P] = i\hbar\) as

\[
\Phi_ig^{ij}\Phi_j = i\hbar
\]

where \( (g_{ij}) = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \). Just like nature tells us ”I have gravity” when localizing the Poincare symmetry, it says ”I have two times” when localizing this symmetry. Let us see how so. Write down the easiest Lagrangian that would give us an energy \( \dot{X} \cdot P \) back:

\[
\mathcal{L} = \frac{1}{2} \Phi^\dagger \tau g \Phi = \frac{1}{2} \Phi^\dagger \tau g^{ij} \Phi_j
\]

where \((g_{ij}) = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \) if \( \Phi = \begin{pmatrix} X \\ P \end{pmatrix} \), or for example use the \( Osp(1/2) \) group metric \((g_{ij}) = \begin{pmatrix} i & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix} \) if \( \Phi = \begin{pmatrix} \Psi \\ X \\ P \end{pmatrix} \), that is, if we want to include spin degrees of freedom, or use \( Sp(2N) \) for \( N \) particles and so on. The localization is a standard method. We introduce the covariant derivative \( \ltimes gA \) and get

\[
\mathcal{L} = \frac{1}{2} \Phi^\dagger \ltimes g \Phi = \frac{1}{2} \Phi^\dagger \tau g \Phi - \frac{1}{2} \Phi^\dagger A \Phi
\]

The Euler Lagrange equations give the equations of motion as:

\[
\Phi_{\tau} = gA \Phi \Rightarrow \mathcal{L}^\dagger = \mathcal{L}
\]

\[
\Phi_i \Phi_j = 0 \Rightarrow X \cdot X = 0 = X \cdot P = 0 = P \cdot P
\]
Thus it follows that $X$ and $P$ are two light-like vectors that are not parallel. Therefore they have to have two time-like dimensions and they must have $d + 2$ entries. We write $X^M$ where $M \in \{0, 0', 1, \ldots\}$ and $X^M$ is now a $SO(d, 2)$ vector. Thus we see via the localization that nature insists on two times because this is the only way not to get a trivial theory after gauge fixing all three gauge choices that $Sp(2) \simeq Sl(2, R) \simeq SO(2, 1)$ offers. The three gauge choices (five for $Osp(1/2)$) fix a gauge surface inside the $SO(d, 2)$ symmetric starting space such that we are left with $SO(d-1, 1)$. The third gauge choice fixes the parametrization of the world line $\tau(t)$. Note that we have to have one more time and one more spatial direction - nature is telling us about two times only, not about many times. We write $X^M$ where $M \in \{0, 0', 1, 1', 2, 3, \ldots, d-2\}$. The ”accident” $Sp(2) \simeq SO(2, 1)$ means that we can interpret the above as conformal gravity on the world line. For the $Osp(1/2)$ case it is conformal supergravity on the world line.

1.2 More Motivations for Two Times

Looking at M-theory dualities one immediately bumps into two problems:

Firstly, in order to put all the dualities into one group, we need a bigger group than allowed by a $(d-1, 1)$ signature of the space time. Looking at supergravity is enough to understand this. People tried to find the one group that could give us SUGRA, but $Osp(1/32)$ is too small and $Osp(1/64)$ would lead to particles with spin higher than 2 if realized in the usual signature. In order to embed all we know about M-theory and its dualities, we need $Osp(1/64)$ and therefore we need two times if we want to avoid particles with spin larger than 2. This means we need 13 dimensions to start with, i.e. $SO(11, 2)$ to be gauged down to a $SO(10, 1)$ theory.

Secondly, M-theory provides dualities between theories with differing topologies. This is what two time theory is very strong in. Often different theories are $Sp(2)$ -gauge duals with different topologies of the gauge surfaces. Examples are the massless particle having a world line that goes through the origin of the $X^0 \times X^0'$ -plane and the simple harmonic oscillator having a world line that is a circle in this plane.
2 Remarks to be Noted

Many people used spaces with two time like dimensions in one form or another. There is a neat way to get AdS spaces from embeddings in two times. These are geometrical tricks. The two time theory described here is very different in the following ways:

1) It is a dynamical theory. We start out with an Lagrangian and can generalize to the string case which means it is an interacting theory.

2) It gives well known ghostfree physical systems.

3) It is a consistent and unitary quantum theory. The $SO(d,2)$ covariant quantization gives the Casimir $C_{2[SO(d,2)]} = 4C_{2[Sp(2)]} - \frac{1}{4}(d^2 - 4) = 0 - \frac{1}{4}(d^2 - 4)$ and the canonical or field theoretical quantization after gauge fixing gives $C_2 = -\frac{1}{4}(d^2 - 4)$ also [3].

4) We easily can include spin [3] and supersymmetry [7].

3 Example

It is high time to give an easy example. The example given will be published shortly in a wider context [1]. It is very easy since after gauge fixing we are left with an only $1 + 1$-dimensional system. Write

$$M = (+', -', 0, 1)$$

$$X^M = \begin{pmatrix} 1, \ X', \ t, \ r_* \end{pmatrix} N_{(t,r)}$$

$$P^M = \frac{1}{N}(0, P', pt, p_*)$$

The metric is given by $\eta^{+'} = -1$ and the line element is

$$ds^2 = (dX^M)(dX_M) = N^2[-(dt)^2 + (dr_*^2)]$$
The well known black hole metric

\[ ds^2 = -N^2(dt)^2 + N^{-2}(dr)^2 \]  \hspace{1cm} (14)

requires \( dr_* = drN^{-2} \). Thus we need to integrate to get the right \( r_* \) from the \( N \) that we want. We might like to model a particle on a Reissner-Nordstrom background \( (N = \sqrt{1 - \frac{r_+}{r} \sqrt{1 - \frac{r_-}{r}}} \) and get \( r_* = r + \frac{r_+^2}{r_+ - r_+^2} \ln (r - r_+) + r_-^2 \ln (r - r_-) \).

In order to show the quantum mechanical treatment etc. it is convenient to define \( \sqrt{2u} = t + r_* \) and \( \sqrt{2v} = t - r_* \):

\[
\begin{align*}
M &= (+' -' + -) \\
X^M &= (1, -uv, \ u, \ v) N_{(u,v)} \\
P^M &= \frac{1}{N} (0, \ up_u + vp_v, -p_v, -p_u) \\
dX^M &= (0, -d(\mu v), du, dv) N - \frac{X^M}{N} dN
\end{align*}
\]  \hspace{1cm} (15)

\hspace{1cm} (16)

\hspace{1cm} (17)

The metric is given by \( \eta^{+'-'} = \eta^{+-} = -1 \) and the line element is

\[ (ds)^2 = (dX^M)(dX_M) = -2N^2 du dv \]  \hspace{1cm} (18)

Inserting these forms in the original Sp(2, R) local and SO(2, 2) global invariant Lagrangian \[5\] gives

\[ L = \dot{X} \cdot P - \frac{1}{2} A^{22} P \cdot P - \frac{1}{2} A^{11} X \cdot X - A^{12} X \cdot P \]  \hspace{1cm} (19)

\[ = \dot{u} p_u + \dot{v} p_v + \frac{A_{22}}{N^2} p_u p_v \]  \hspace{1cm} (20)

\[ = -N^2 \frac{A_{22}}{A_{22}} \dot{u} \dot{v} = \frac{1}{2A_{22}} G_{\mu \nu} \dot{x}^\mu \dot{x}^\nu \]  \hspace{1cm} (21)

The metric is recognized in the line element or in the last line \( G_{\mu \nu} = \eta_{\mu \nu} N^2 \), which is obtained by integrating out the momenta.

This form shows that the system has the larger symmetry SO(2, 2) whose generators are the Lorentz generators in the 2+2 dimensional space \( L^{MN} = X^M P^N - X^N P^M \). We quantize via \( [u, p_u] = \hat{i} = [v, p_v] \). In the present gauge these take a form that is quantum ordered already:

\[
\begin{align*}
L^{+'-'} &= up_u + vp_v, \quad L^{+-'} = -p_v \\
L^{+'-} &= -p_u, \quad L^{-'+} = -u^2 p_u \\
L^{-'-} &= -v^2 p_v, \quad L^{+-} = -up_u + vp_v
\end{align*}
\]  \hspace{1cm} (22)

\hspace{1cm} (23)

\hspace{1cm} (24)
Under the $\text{SO}(2,2) = \text{SL}(2,R)_L \otimes \text{SL}(2,R)_R$ the generators may be reclassified in the form

$$
G^L_2 = v p_v, \quad G^L_+ = G^L_0 + G^L_1 = -p_v, \quad G^L_- = G^L_0 - G^L_1 = -v^2 p_v , \quad (25)
$$

$$
G^R_2 = u p_u, \quad G^R_+ = G^R_0 + G^R_1 = -p_u, \quad G^R_- = G^R_0 - G^R_1 = -u^2 p_u . \quad (26)
$$

where $G^L,R_{0,1}$ are the compact generators and $G^L,R_{1,2}$ are the non-compact ones.

The Casimir is \( C_2 [ \text{SO}(d,2)] = 0 \) as it should be in two dimensions since \( C_2 = 1 - \frac{d^2}{4} \).

### 4 Results

Two time theory led to some astonishing results already. Often the results are "just" a better understanding or interpretation of known methods or facts. Among these results fall the understanding of the origin of the \( \text{SO}(d,2) \)-symmetry that occurs in certain spectra as being there from the very start in the two time description.

There is the more natural interpretation of the standard method in order to go from global AdS to the black hole solution. One has had to identify and complexify because both are gauges in a space time with two times.

And for example the always mysterious kappa-symmetry has bosonic partners and becomes understandable. Let us divide the main results into two categories:

#### 4.1 Dualities

In the point particle two time theory it was shown that the massless, the massive, the relativistic, the non-relativistic particle, the simple harmonic oscillator, the particle in an arbitrary central potential, the particle on an \( \text{AdS} \)-background, on an \( \text{AdS} \times S \)-background, on an BTZ background, on an Robertson-Bertotti background and others are all \( \text{Sp}(2) \)-gauge dual theories; meaning they all derive from the same two time theoretical starting point via different choices of the three gauge parameters. For the generalization to the string case where only the tensionless and the rigid string have been found yet, this could mean that the tensionless and tensionful and IIA and IIB etc. strings can all be gotten in a similar way from one common unifying theory with two times.
4.2 Hidden Symmetries

The space-time before gauging shows $SO(d,2)$-symmetry. This symmetry is still there in the gauged theory but it is hard to see since it is non-linearly realized. This was known only for the hydrogen atom and the superparticle [11]. In [7] it was shown how the action of the superparticle that has hidden superconformal symmetries derives from the two time starting point where these symmetries are manifest and still linear before projection onto the gauge surface. Two time theory has shown that these hidden symmetries are inside all of the spectra of all the gauge dual theories. That means that all the mentioned systems have these hidden symmetries. "Hidden" means that the symmetries are symmetries of the action and not of the Hamiltonian. Symmetries hidden in the action can be as vital as the Lorentz boost symmetry for a free particle: A Lagrangian like $\mathcal{L} = -m\sqrt{1 - \dot{r}^2}$ (or better the Hamiltonian $H = \sqrt{p^2 + m^2}$) does not reveal it but surely one would question someone’s understanding of the system at hand if that somebody were not to realize that this expression describes a relativistic particle.

In the gauged SUGRA $AdS_n \times S^m$ the bosonic subgroup is $SO(n-1,2) \times SO(m+1)$ which is smaller than the full $SO(n+m,2)$. As observed in [5] already: The $AdS \times S$ discussion may benefit from this.

5 Outlook

There are very many things that one might try to put into a two time description. A few that have not been attempted yet are:

- The tensionless string gauge being found there should be a string with tension. This is similar to the massless point particle gauge that was a precursor for the massive one. Then of course it needs a generalization to p-branes.

- The introduction of background fields is expected to lead to an electromagnetic theory that is manifestly dual between electrical and magnetic phenomena and still $SO(d,2)$ covariant of course as well.

- Only two time theory can give a large enough supergroup to give M-theory, thus two time theory should lead to M-theory but there is only a toy-model [10] yet.
5.1 How to get further into the subject

This subject has been developed over the past few years and all the recent papers by the pioneer of two time physics Itzhak Bars are recommendet. As a further introduction all those papers can be a little too much though, therefore there will be a different and self contained approach from first principles given in [1].

References

[1] S. Vongehr, ”Black Holes In Two Times” hep-th/9907nnn
[2] M. Banados, C. Teitelboim and J. Zanelly, Phys.Rev.Lett. 69 (1992) 1849-1851
[3] I. Bars and C. Deliduman,”Gauge symmetry in phase space with spin” hep-th/9806085
[4] S. Deser, R. Jackiw and G’t Hooft, Ann.Phys. 152 (1984) 220
[5] I. Bars, ”Two-Time Physics” hep-th/9809034
[6] I. Bars, ”Hidden Symmetries, AdS_D x S^n, and the lifting of one-time-physics to two-time-physics” hep-th/9810025
[7] I. Bars, C. Deliduman, D. Minic, ”Supersymmetric Two-Time Physics” hep-th/9812161
[8] I. Bars, C. Deliduman, D. Minic, ”Strings, Branes and Two-Time Physics” hep-th/9906223
[9] J.H. Schwarz, Nucl. Phys.B 185 (1981) 221
[10] I. Bars, C. Deliduman, D. Minic,”Lifting M-Theory to Two-Time Physics” hep-th/9904063