New Low-Energy Realization of The Superstring-Inspired $E_6$ Model

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Abstract

The superstring-inspired $E_6$ model is reduced to the supersymmetric standard model extended by a specific $U(1)$ factor. This choice allows for the existence of naturally light singlet neutrinos which also mix with $\nu_e$, $\nu_\mu$, and $\nu_\tau$, thus making it possible to accommodate all present neutrino data. Other consequences of this model are also discussed: oblique corrections from $Z$-$Z'$ mixing, phenomenology of the two-Higgs-doublet sector, and possible scenarios of gauge-coupling unification.

1 Introduction

There are a number of possible low-energy realizations of the superstring-inspired $E_6$ model. Their particle content is often that given by the fundamental 27 representation of $E_6$ and the gauge group is often larger than that of the standard model by at least one $U(1)$ factor. In particular, there are two neutral singlets in each 27 representation, $N$ and $S$, which may be considered as “neutrinos” because they are very weakly interacting. The former ($N$) is the Dirac partner of an ordinary doublet neutrino $\nu$, whereas the latter ($S$) is not. A possible scenario is that both $N$ and $S$ are heavy and that $\nu$ is light via the seesaw mechanism. In that case, the neutrino sector is equivalent to that of the standard model with very small Majorana neutrino masses. On the other hand, there are experimental indications at present for three types of neutrino oscillations: solar, atmospheric, and laboratory, requiring three mass differences which are not possible with only three neutrinos. Adding a fourth
doublet neutrino is not allowed because the invisible width of the $Z$ boson tells us that there should be only three such neutrinos. Hence the idea of one or more naturally light singlet neutrinos should be entertained, and in this talk I will show how a properly chosen extra $U(1)$ factor allows for this possibility\cite{footnote} and discuss also some of the other features of this new model.

### 2 Reduction of the Superstring-Inspired $E_6$ Model

The fundamental $27$ representation of $E_6$ may be decomposed according to its maximum subgroup $SU(3)_C \times SU(3)_L \times SU(3)_R$:

$$27 = (3, 3, 1) + (3^*, 1, 3^*) + (1, 3^*, 3).$$

Under the reductions $SU(3)_L \to SU(2)_L \times U(1)_{Y_L}$ and $SU(3)_R \to U(1)_{T_3R} \times U(1)_{Y_R}$, the individual left-handed fermionic components are defined as follows\cite{footnote}.

$$\begin{align*}
(u, d) &\sim (3; 2, \frac{1}{6}; 0, 0), \quad u^c \sim (3^*; 0, 0; -\frac{1}{2}, -\frac{1}{6}), \quad d^c \sim (3^*; 0, 0; \frac{1}{2}, -\frac{1}{6}), \\
(\nu_e, e) &\sim (1; 2, \frac{1}{6}; 0, -\frac{1}{3}), \quad e^c \sim (1; 0, \frac{1}{3}; \frac{1}{2}, \frac{1}{6}), \quad N \sim (1; 0, \frac{1}{3}; -\frac{1}{2}, \frac{1}{6}), \\
(\nu_E, E) &\sim (1; 2, -\frac{1}{6}; -\frac{1}{2}, \frac{1}{6}), \quad (E^c, N_E^c) \sim (1; 2, -\frac{1}{6}; \frac{1}{2}, \frac{1}{6}), \\
h &\sim (3; 0, -\frac{1}{3}; 0, 0), \quad h^c \sim (3^*; 0, 0; 0, \frac{1}{3}), \quad S \sim (1; 0, \frac{1}{3}; 0, -\frac{1}{3}).
\end{align*}$$

The electric charge is given here by

$$Q = T_{3L} + Y_L + T_{3R} + Y_R,$$

and three families of the above fermions and their bosonic superpartners are assumed.

There are two possible $SO(10)$ subgroups which also contain the $SU(5)$ subgroup which contains the standard $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge group. The conventional decomposition is

$$E_6 \to SO(10)_A \times U(1)_{\psi_A},$$

such that the $27$ splits up as follows:

$$\begin{align*}
(16, 1)_A &= (u, d) + u^c + e^c + d^c + (\nu_e, e) + N, \\
(10, -2)_A &= h + (E^c, N_E^c) + h^c + (\nu_E, E), \\
(1, 4)_A &= S.
\end{align*}$$
Note that all the usual quarks and leptons are contained in $(16, 1)_A$, and the Higgs bosons are in $(10, -2)_A$. The next step of the decomposition is

$$SO(10)_A \rightarrow SU(5) \times U(1)_{\chi A},$$

such that

$$16_A = (10, 1) + (5^*, -3) + (1, 5), \quad 10_A = (5, -2) + (5^*, 2), \quad 1_A = (1, 0).$$

(12)

Note that $U(1)_{\psi A} \times U(1)_{\chi A}$ breaks down to $U(1)_{\eta}$ via the adjoint $78$, resulting in $Q_{\eta} = T_3 + 5Y_L - Q$.

The alternative decomposition\[7\] is

$$E_6 \rightarrow SO(10)_B \times U(1)_{\psi B},$$

(13)

with

$$
(16, 1)_B = (u, d) + u^c + e^c + h^c + (\nu_E, E) + S, \quad (10, -2)_B = h + (E^c, N_E^c) + d^c + (\nu_e, e), \\
(1, 4)_B = N.
$$

(14)

(15)

(16)

The next step is then

$$SO(10)_B \rightarrow SU(5) \times U(1)_{\chi B},$$

(17)

such that

$$16_B = (10, 1) + (5^*, -3) + (1, 5), \quad 10_B = (5, -2) + (5^*, 2), \quad 1_B = (1, 0).$$

(18)

Note that $S$ is trivial under $U(1)_{\chi A}$, whereas $N$ is trivial under $U(1)_{\chi B}$.

In the $E_6$ superstring-inspired model, the Yukawa terms are supposed to be restricted to only those contained in $27^3 \rightarrow 1$, namely $u^c(uN_E^c - dE^c)$, $d^c(uE - d\nu_E)$, $e^c(\nu_e E - e\nu_E)$, $Shh^c$, $S(EE^c - \nu_E N_E^c)$, and $N(\nu_e N_E^c - eE^c)$. In the following, I also assume a $Z_2$ discrete symmetry where all superfields are odd, except one copy each of $(\nu_E, E), (E^c, N_E^c)$, and $S$, which are even. The bosonic components of the even superfields will serve as Higgs bosons which break the electroweak gauge symmetry. Specifically, $\langle \tilde{N}_E^c \rangle$ generates $m_u, m_D$, and $m_1$; $\langle \tilde{\nu}_E \rangle$ generates $m_d, m_e$, and $m_2$; and $\langle \tilde{S} \rangle$ generates $m_h$ and $m_E$. The mass matrix spanning $\nu_e, N, \nu_E, N_E^c$, and $S$ is then given by

$$
\mathcal{M} = \begin{bmatrix}
0 & m_D & 0 & 0 & 0 \\
0 & m_D & 0 & 0 & 0 \\
0 & 0 & m_E & m_1 & 0 \\
0 & 0 & m_E & m_2 & 0 \\
0 & m_1 & m_2 & 0 & 0
\end{bmatrix}
$$

(19)
3 Naturally Light Singlet Neutrinos

If the $E_6$ superstring breaks only via the flux mechanism, then it is not possible to have only the standard-model gauge group. The latter must be extended by at least the $U(1)_{\eta}$ factor mentioned previously. Under $U(1)_{\eta}$, $N$ and $S$ transform identically and so do $(\nu_e,e)$ and $(\nu_E,E)$ as well as $d^c$ and $h^c$ but nontrivially, hence they are forbidden to have Majorana mass terms $N^2$ and $S^2$. Therefore, in the above mass matrix $M$, $\nu_e$ and $N$ together make up a Dirac neutrino of mass $m_D$, whereas $m_S \sim 2m_1m_2/m_E$. If $U(1)_{\eta}$ is actually broken at some large scale, then $N$ and $S$ may acquire large Majorana masses through gravitationally induced nonrenormalizable interactions. In that case, $N$ and $S$ are superheavy, whereas $m_\nu \sim m_D^2/m_N$.

To obtain naturally light singlet neutrinos, it is now obvious that $U(1)_{\eta}$ should be replaced with $U(1)_N (\equiv U(1)_{\chi_B})$ under which

$$N \sim 0, \quad S \sim 5, \quad (u,d), \quad u^c, \quad e^c \sim 1, \quad d^c, \quad (\nu_e,e) \sim 2, \quad h, \quad (E^c,N^c_E) \sim -2, \quad h^c, \quad (\nu_E,E) \sim -3.$$  

(20)

Now it is possible to have both light doublet neutrinos: $m_\nu \sim m_D^2/m_N$, and light singlet neutrinos: $m_S \sim 2m_1m_2/m_E$. In addition, the $\nu_e N^c_E - eE^c$ mass term allows the two types of neutrinos to mix. Note that

$$Q_N = 6Y_L + T_{3R} - 9Y_R.$$  

(21)

4 Oblique Corrections from Z-Z' Mixing

The proposed symmetry breaking is achieved through a combination of the adjoint $78$ which breaks $E_6$ down to $SU(3)_C \times SU(2)_L \times U(1)_{Y_L} \times U(1)_{T_{3R}} \times U(1)_{Y_R}$ and a pair of superheavy $27$ and $27^c$ which break it down to $SO(10)_B$, resulting in the intermediate gauge group

$$G = SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_N.$$  

(22)

Then $U(1)_N$ is broken by $\langle \tilde{S} \rangle \equiv u$, and $SU(2)_L \times U(1)_Y$ breaks down to $U(1)_Q$ via $\langle \tilde{\nu}_E \rangle \equiv v_1$, and $\langle \tilde{N}^c_E \rangle \equiv v_2$. Let

$$v^2 = v_1^2 + v_2^2, \quad \tan \beta = \frac{v_2}{v_1}, \quad g_Z^2 = g_1^2 + g_2^2.$$  

(23)
then the observed $Z$ boson is a linear combination of $Z_1$ from $SU(2)_L \times U(1)_Y$ and $Z_2$ from $U(1)_N$: $Z = Z_1 \cos \theta + Z_2 \sin \theta$, where

$$M_Z^2 \simeq \frac{1}{2} g_Z^2 v^2 \left[ 1 - \left( \frac{\sin^2 \beta - \frac{3}{5}}{\frac{1}{5}} \right)^2 \frac{u^2}{v^2} \right], \quad \theta \simeq -\sqrt{\frac{2}{5}} \frac{g_Z}{g_N} \left( \sin^2 \beta - \frac{3}{5} \right) \frac{v^2}{u^2}. \quad (24)$$

The deviations from the standard model may be expressed in terms of the conventional oblique parameters:

$$\epsilon_1 = \alpha T = \left( \sin^4 \beta - \frac{9}{25} \right) \frac{v^2}{u^2}, \quad (25)$$

$$\epsilon_2 = -\frac{\alpha U}{4 \sin^2 \theta_W} = \left( \sin^2 \beta - \frac{3}{5} \right) \frac{v^2}{u^2}, \quad (26)$$

$$\epsilon_3 = \frac{\alpha S}{4 \sin^2 \theta_W} = \frac{2}{5} \left( 1 + \frac{1}{4 \sin^2 \theta_W} \right) \left( \sin^2 \beta - \frac{3}{5} \right) \frac{v^2}{u^2}. \quad (27)$$

The present precision data from LEP at CERN have errors of order a few $\times 10^{-3}$. This means that $u \sim \text{TeV}$ is allowed.

## 5 Phenomenology of the Two-Higgs-Doublet Sector

At the electroweak energy scale, there are only two Higgs doublets in this model, but they differ from the ones of the minimal supersymmetric standard model (MSSM). The reason is that the superpotential has the term $f(\nu E N^c_E - E E^c)S$ which has no analog in the MSSM. Let

$$\Phi_1 \equiv \left( \begin{array}{c} \phi^0_1 \\ -\phi^-_1 \end{array} \right) \equiv \left( \begin{array}{c} \tilde{\nu}_E \\ \tilde{E} \end{array} \right), \quad \Phi_2 \equiv \left( \begin{array}{c} \phi^+_2 \\ \phi^0_2 \end{array} \right) \equiv \left( \begin{array}{c} \tilde{E}^c \\ \tilde{N}^c_E \end{array} \right), \quad \chi \equiv \tilde{S}, \quad (28)$$

then the quartic terms of the Higgs potential are given by the sum of

$$V_F = f^2[ (\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1) + (\Phi_1^\dagger \Phi_1 + \Phi_2^\dagger \Phi_2)(\chi \chi) ], \quad (29)$$

and

$$V_D = \frac{1}{8} g_2^2[ (\Phi_1^\dagger \Phi_1)^2 + (\Phi_2^\dagger \Phi_2)^2 + 2(\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) - 4(\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1) ]$$

$$+ \frac{1}{8} g_1^2[ (\Phi_1^\dagger \Phi_1)^2 + (\Phi_2^\dagger \Phi_2)^2 - 2(\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) ]$$

$$+ \frac{1}{80} g_N^2[ 9(\Phi_1^\dagger \Phi_1)^2 + 4(\Phi_2^\dagger \Phi_2)^2 + 12(\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) - 30(\Phi_1^\dagger \Phi_1)(\chi \chi)$$

$$- 20(\Phi_2^\dagger \Phi_2)(\chi \chi) + 25(\chi \chi)^2 ]. \quad (30)$$
Let $\langle \chi \rangle = u$, then $\sqrt{2} \text{Re} \chi$ is a physical scalar boson with $m^2 = (5/4)g_N^2 u^2$, and couples to $\Phi_1^\dagger \Phi_1$ with strength $\sqrt{2}u(f^2 - (3/8)g_N^2)$. The effective $(\Phi_1^\dagger \Phi_1)^2$ coupling $\lambda_1$ is then given by

$$\lambda_1 = \frac{1}{4}(g_1^2 + g_2^2) + \frac{9}{40}g_N^2 - \frac{2(f^2 - (3/8)g_N^2)^2}{(5/4)g_N^2}. \quad (31)$$

The other quartic self-couplings of the reduced Higgs potential involving only $\Phi_1$ and $\Phi_2$ have similar additional contributions. Consequently,

$$\lambda_1 = \frac{1}{4}(g_1^2 + g_2^2) + \frac{6}{5}f^2 - \frac{8f^4}{5g_N^2}, \quad (32)$$

$$\lambda_2 = \frac{1}{4}(g_1^2 + g_2^2) + \frac{4}{5}f^2 - \frac{8f^4}{5g_N^2}, \quad (33)$$

$$\lambda_3 = -\frac{1}{4}g_1^2 + \frac{1}{4}g_2^2 + f^2 - \frac{8f^4}{5g_N^2}, \quad (34)$$

$$\lambda_4 = -\frac{1}{2}g_2^2 + f^2. \quad (35)$$

The MSSM is recovered in the limit of $f = 0$ as expected. Otherwise, the two-Higgs-doublet structure is different. In particular,

$$\left(m_h^2\right)_{\text{max}} = 2v^2\left[\frac{1}{4}g_2^2 \cos^2 \beta + f^2\left(\frac{3}{2} + \frac{1}{5}\cos 2\beta - \frac{1}{2}\cos^2 \beta\right) - \frac{8f^4}{5g_N^2}\right] + \epsilon, \quad (36)$$

where $\epsilon$ comes from radiative corrections, the largest contribution being that of the top quark:

$$\epsilon \simeq \frac{3g_2^2m_t^4}{8\pi^2M_W^2}\ln\left(1 + \frac{\tilde{m}}{m_t}\right). \quad (37)$$

Normalizing $g_N^2$ to be equal to $(5/3)g_1^2$ and varying $f^2$,

$$\left(m_h^2\right)_{\text{max}} < M_Z^2 \left[\cos^2 \beta + \frac{25}{24}\sin^2 \theta_W \left(\frac{3}{2} + \frac{1}{5}\cos 2\beta - \frac{1}{2}\cos^2 \beta\right)^2\right] + \epsilon. \quad (38)$$

Numerically, the maximum value occurs at $\beta = 0$, which corresponds to $m_h \simeq 140$ GeV, as compared to 128 GeV in the MSSM.

### 6 Possible Scenarios of Gauge-Coupling Unification

Consider now the issue of gauge-coupling unification. The evolution equations of $\alpha_i \equiv g_i^2/4\pi$ are generically given to two-loop order by

$$\frac{\partial \alpha_i}{\partial \mu} = \frac{1}{2\pi} \left[ b_i + b_{ij} \alpha_j(\mu) \right] \alpha_i^2(\mu), \quad (39)$$
where $\mu$ is the running energy scale and the coefficients $b_i$ and $b_{ij}$ are determined by the particle content of the model. To one loop, the above equation is easily solved:

$$\alpha_i^{-1}(M_1) = \alpha_i^{-1}(M_2) - \frac{b_i}{2\pi} \ln \frac{M_1}{M_2}. \quad (40)$$

Below $M_{SUSY}$, assume the standard model with 2 Higgs doublets, then

$$b_1 = \frac{21}{5}, \quad b_2 = -3, \quad b_3 = -7. \quad (41)$$

Above $M_{SUSY}$ in the MSSM,

$$b_1 = 3(2) + \frac{3}{5}(4) \left(\frac{1}{4}\right), \quad b_2 = -6 + 3(2) + 2 \left(\frac{1}{2}\right), \quad b_3 = -9 + 3(2). \quad (42)$$

It is now well-known that for $M_{SUSY} \sim 10^4$ GeV, the gauge couplings do in fact unify at $M_U \sim 10^{16}$ GeV.

In the present model with $u \sim M_{SUSY}$,

$$b_1 = 3(3) + ?, \quad b_2 = -6 + 3(3) + ?, \quad b_3 = -9 + 3(3) + ?, \quad b_N = 3(3) + ? \quad (43)$$

There are two possible scenarios for gauge-coupling unification. The first is an analog of the MSSM. Add one extra copy of $(\nu_e, e)$ and $(E_6^c, N_{E_6}^c)$ [having $\sum Q_N = 0$ so that the theory remains anomaly-free], then

$$\Delta b_1 = \frac{3}{5}, \quad \Delta b_2 = 1, \quad \Delta b_3 = 0, \quad \Delta b_N = \frac{2}{5}. \quad (44)$$

This again leads to $M_U \sim 10^{16}$ GeV, which is actually not so good because the string scale is an order of magnitude higher. Also, it is hard to understand theoretically why the chosen superfields are light but their companions in the same $E_6$ multiplet are superheavy. On the other hand, this is no worse than the usual assumptions taken in SUSY $SU(5)$ or SUSY $SO(10)$.

The second scenario is to insist on having $M_U \sim 5 \times 10^{17}$ GeV, and allow some components of the superheavy 27 and 27* multiplets to be somewhat lighter than the others. In particular, take 3 copies of $(u, d) + (u^*, d^*)$ and $(\nu_e, e) + (\nu_e^*, e^*)$ with $M'$ much below $M_U$, then between $M'$ and $M_U$,

$$\Delta b_1 = 3 \times \left(\frac{1}{5} + \frac{3}{5}\right) = \frac{12}{5}, \quad \Delta b_2 = 3 \times (3 + 1) = 12, \quad \Delta b_3 = 3 \times (2 + 0) = 6, \quad (45)$$

$$\Delta b_N = 3 \times \left(\frac{3}{10} + \frac{2}{5}\right) = \frac{21}{10}. \quad (46)$$

As a result, gauge-coupling unification at the string scale is achieved with an intermediate scale of $M' \sim 10^{16}$ GeV.
7 Conclusions

In conclusion, the superstring-inspired E\(_6\) model provides a framework for accommodating naturally light singlet neutrinos as well as naturally light doublet neutrinos which also mix with each other. The key is the reduction

\[ E_6 \rightarrow SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_N, \]

under which \(N\) is trivial, but \(S\) is not. Hence \(N\) may acquire a large Majorana mass \(m_N\), and the mass matrix \(\mathcal{M}\) of Eq. (19) becomes

\[
\mathcal{M} = \begin{bmatrix}
0 & m_D & 0 & m_3 & 0 \\
0 & m_D & m_N & 0 & 0 \\
0 & 0 & 0 & m_E & m_1 \\
m_3 & 0 & m_E & 0 & m_2 \\
0 & 0 & m_1 & m_2 & 0
\end{bmatrix}.
\]

This means that the doublet neutrinos obtain very small masses from the usual see-saw mechanism: \(m_\nu \sim m_D^2/m_N\), whereas the singlet neutrinos \(S\) get theirs from an analogous \(3 \times 3\) mass matrix: \(m_S \sim 2m_1m_2/m_E\).

Other properties of this model include: (1) the two-Higgs-doublet structure at the electroweak energy scale is not that of the minimal supersymmetric standard model; (2) an additional neutral gauge boson (\(Z'\)) is possible at the TeV energy scale; and (3) gauge couplings may unify at the string compactification scale if there can be variations of masses within some superheavy supermultiplets.

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