BARYON CHIRAL PERTURBATION THEORY IN THE $1/N_c$ EXPANSION

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The chiral Lagrangian for baryons is formulated in an expansion in $1/N_c$. The chiral Lagrangian implements the contracted spin-flavor symmetry of large-$N_c$ baryons as well as nonet symmetry of the leading planar diagrams. Large-$N_c$ consistency conditions ensure that chiral loop corrections are suppressed in $1/N_c$ through exact cancellation of chiral loop graphs to fixed orders in $1/N_c$. Application of $1/N_c$ baryon chiral perturbation theory to the flavor-27 baryon mass splittings and the baryon axial vector currents are considered as examples.

1 Introduction

The realization that large-$N_c$ baryons respect a contracted spin-flavor symmetry has resulted in many important predictions for the spin and flavor properties of baryon amplitudes. The $1/N_c$ corrections to the large-$N_c$ limit have been classified for general spin and flavor representations. Characterization of the $1/N_c$ breakings of large-$N_c$ baryon spin-flavor symmetry has been essential for the application of the $1/N_c$ expansion to the case of physical interest, QCD baryons with $N_c = 3$.

The study of QCD baryons in an expansion in $1/N_c = 1/3$ has been quite successful phenomenologically. $1/N_c$ breaking of contracted $SU(6)$ spin-flavor symmetry is comparable in magnitude to the breaking of $SU(3)$ flavor symmetry. This fact makes it imperative to reformulate baryon chiral perturbation theory in an expansion in $1/N_c$.

In this talk, I will describe the progress which has been made in baryon chiral perturbation theory using the $1/N_c$ expansion. The chiral Lagrangian for baryons is formulated in an expansion in $1/N_c$ and $m_q/\Lambda_{QCD}$ about the double limit $N_c \to \infty$, $m_q/\Lambda_{QCD} \to 0$, with the ratio $\frac{1}{N_c}/(m_q/\Lambda_{QCD})$ held fixed. In this double expansion, the chiral limit cannot be taken independently of the $1/N_c \to 0$ limit, so there is no order-of-limits ambiguity associated with the $1/N_c$ baryon chiral Lagrangian.
2 Heavy Baryon Chiral Perturbation Theory

For large, finite $N_c$, a baryon has mass $M$ which is $O(N_c)$ while mesons have masses which are $O(1)$. Thus, a large-$N_c$ baryon acts as a heavy static fermion in its interactions with pions. The momentum of an on-shell baryon which absorbs a pion carrying momentum $k$ can be written as

$$P^\mu = M v'^\mu = M v^\mu + k^\mu ,$$

where $v'$ and $v$ are the velocities of the initial and final baryons. It readily follows that the baryon velocity is conserved in its interactions with pions,

$$v'^\mu = v^\mu + O(1/N_c) .$$

Thus, it is convenient to formulate Heavy Baryon Chiral Perturbation Theory (HBχPT) in terms of the velocity-dependent baryon field

$$B_v(x) = e^{i M t} v_x^\mu \left( \frac{1 + \not{v}}{2} \right) B(x) .$$

In the rest frame of the baryon, Eq. (3) redefines the baryon field to include the phase factor $e^{i M t}$ and projects onto the particle portion of the Dirac spinor. The baryon propagator in HBχPT is given by

$$i \frac{(P + M)}{P^2 - M^2} \rightarrow i \frac{1 + \not{k}}{k \cdot v} \left( \frac{1 + \not{v}}{2} \right) ,$$

which reduces to $i/E$ in the rest frame of the baryon.

A number of interesting phenomenological results were found in HBχPT: (1) Chiral loop corrections are large if only spin-$\frac{3}{2}$ octet baryons are included in the heavy baryon chiral Lagrangian. (2) Including spin-$\frac{7}{2}$ decuplet baryons results in significant cancellation between loop graphs with intermediate octet and decuplet states. (3) The phenomenological fit to data yields $SU(6)$-like couplings for the parameters of the heavy baryon chiral Lagrangian. All of these features are not consequences of the $SU(3)$ flavor symmetry of the baryon chiral Lagrangian, but are explained by large-$N_c$ spin-flavor symmetry.

In the large-$N_c$ limit, the baryon spin-flavor symmetry requires inclusion of the complete spin-flavor multiplet 56 which contains both the spin-$\frac{3}{2}$ octet and spin-$\frac{7}{2}$ decuplet baryons. The decuplet-octet mass splitting

$$\Delta \equiv (m_T - m_B) \sim 1/N_c$$

vanishes in the large-$N_c$ limit, so the spin-$\frac{3}{2}$ and spin-$\frac{7}{2}$ baryons are degenerate. Including only the spin-$\frac{3}{2}$ octet baryons in the chiral Lagrangian breaks large-$N_c$ baryon spin-flavor symmetry explicitly, causing chiral loops to grow with
Figure 1: Diagrams which contribute to the baryon axial-vector pion coupling at one loop. Individual diagrams are order $N_c$ times the tree-level coupling, whereas the sum of all the diagrams is order $1/N_c$ times the tree-level coupling.

powers of $N_c$. In contrast, chiral loops for the 56 are suppressed by powers of $1/N_c$. The proper $1/N_c$ power counting of loop corrections is restored due to exact large-$N_c$ cancellations among different loop diagrams. In addition, the $1/N_c$ expansion provides a quantitative understanding of spin-flavor symmetry for baryon couplings.

3 Chiral Loop Cancellations

The diagrams which correct the baryon axial vector pion-coupling at one loop are displayed in Fig. 1. Each baryon–pion vertex is order $\sqrt{N_c}$, so each loop graph with three vertices is order $N_c^{3/2}$, i.e. a factor of $N_c$ larger than the tree-level vertex. It is possible to show, however, that the sum of the graphs is proportional to a double commutator of three baryon axial vector currents,

$$\left(\sqrt{N_c}\right)^3 \left[ X^{jb}, [X^{jb}, X^{ia}] \right] \leq O\left(\frac{1}{\sqrt{N_c}}\right),$$

which is at least a factor of $1/N_c$ smaller than the tree-level vertex. The large-$N_c$ consistency condition appearing in the one-loop chiral correction is precisely the same consistency condition obtained from the analysis of tree diagrams for $B + \pi \to B' + \pi + \pi$ scattering.

In general, exact large-$N_c$ cancellations ensure that the chiral correction at $L$ loops is of relative order $(1/N_c)^L$, rather than order $(N_c)^L$. Notice that the cancellations are increasingly significant as the number of loops $L$ increases.

4 "Nonet" Symmetry

For finite and large $N_c$, planar diagrams dominate the quark-gluon dynamics. Quark loops are suppressed by one power of $1/N_c$, so there is no quark-antiquark pair creation and annihilation at leading order. The suppression of quark loops in large-$N_c$ QCD implies that planar QCD has the additional flavor symmetry,

$$U(1)_{q_i} \times U(1)_{\bar{q}_i},$$

(7)
which says that the number of quarks and the number of antiquarks of each flavor are separately conserved by the leading planar diagrams. This planar flavor symmetry implies that there are nonets of mesons in large $N_c$, i.e. the $\pi$, $K$, $\eta$ and $\eta'$ form a flavor nonet in the large-$N_c$ limit, as well as Zweig’s rule. The consequences of planar flavor symmetry for baryons, however, have been obtained only recently.\(^5\)

In the large-$N_c$ planar limit, baryon diagrams consist of $N_c$ quark lines connected by planar gluon exchange with no quark loops. The $SU(3)$ flavor symmetry of QCD extends to a $U(3)$ flavor symmetry for large-$N_c$ QCD in the planar limit. For baryons, $U(3)$ planar flavor symmetry implies that baryon amplitudes form representations of $U(3)$ flavor symmetry up to a correction of relative order $1/N_c$. For example, the baryon axial vector flavor-octet currents $A^{i\alpha}$ and the baryon axial vector flavor-singlet current $A^i$ form a flavor nonet at leading order,

$$A^i = A^i_9 + \mathcal{O}\left(\frac{1}{N_c}\right). \quad (8)$$

This nonet symmetry is broken at relative order $1/N_c$ by nonplanar baryon diagrams with a single quark loop, Fig. 2. Note that $U(3)$ flavor symmetry is a symmetry of large-$N_c$ QCD in the leading planar limit only; it is not a symmetry of QCD itself.

5  \(1/N_c\) Baryon Chiral Lagrangian

It is natural to formulate the $1/N_c$ baryon chiral Lagrangian in the rest frame of the baryon. In compact notation, the chiral Lagrangian is given in the flavor symmetry limit by

$$\mathcal{L}_{\text{baryon}} = iD^0 - M_{\text{hyperfine}} + \text{Tr} \left(A^i \lambda^a\right) A^{i\alpha} + \text{Tr} \left(A^i \frac{2I}{\sqrt{6}}\right) A^i + \ldots, \quad (9)$$
where each of the operators is understood to act on the baryon $56$, and the $O(N_c)$ flavor-singlet mass of the baryon $56$ has been removed from the chiral Lagrangian by the heavy-baryon field redefinition. The Lagrangian retains the hyperfine mass splitting

$$M_{\text{hyperfine}} = \Delta \frac{1}{N_c} J^2,$$

where the coefficient $\Delta$ is the mass splitting of the spin-$\frac{3}{2}$ and spin-$\frac{1}{2}$ baryons.

The chiral Lagrangian depends on the pion fields through the combinations

$$V_0 = \frac{1}{2} (\xi \partial^0 \xi^\dagger + \xi^\dagger \partial^0 \xi) , \quad A^i = \frac{i}{2} \left( \xi \nabla^i \xi - \xi^\dagger \nabla^i \xi \right) = \nabla^i \Pi/f + \cdots ,$$

which are defined in terms of $\xi = e^{i \Pi/f}$, where $\Pi = \frac{2 \alpha^a}{2} N_c + \frac{\eta'}{\sqrt{6}}$ contains the $\pi$, $K$, $\eta$ and $\eta'$ fields. Eq. (11) states that the flavor-octet axial vector pion current is coupled to the baryon axial vector flavor-octet current $A^a$, whereas the flavor-singlet axial vector $\eta'$ current is coupled to the baryon axial vector flavor-singlet current $A^i$. $U(3)$ flavor symmetry implies that these baryon axial vector currents form a nonet at leading order in the $1/N_c$ expansion and that this flavor-nonet baryon axial vector current couples to the axial vector flavor-nonet current of pseudo-Goldstone bosons involving the pion octet and the $\eta'$ singlet.

Each of the baryon operators in the chiral Lagrangian has a $1/N_c$ expansion in terms of operator products of the baryon spin-flavor generators

$$J^i = q^\dagger \left( \frac{\sigma^i}{2} \otimes 1 \right) q, \quad T^a = q^\dagger \left( 1 \otimes \frac{\lambda^a}{2} \right) q, \quad G^{ia} = q^\dagger \left( \frac{\sigma^i}{2} \otimes \frac{\lambda^a}{2} \right) q .$$

It will be easy to impose the $U(3)$ planar flavor symmetry of baryon amplitudes using the additional generators

$$G^{i9} = q^\dagger \left( \frac{\sigma^i}{2} \otimes \frac{I}{\sqrt{6}} \right) q = \frac{1}{\sqrt{6}} J^i, \quad T^9 = q^\dagger \left( I \otimes \frac{I}{\sqrt{6}} \right) q = \frac{1}{\sqrt{6}} N_c 1 ,$$

which reduce to the baryon spin generator and identity operator, up to normalization factors.

The $1/N_c$ expansion of the axial vector flavor-octet current for QCD baryons is given by

$$A^{ia} = a_1 G^{ia} + b_2 \frac{1}{N_c} J^i T^a + b_3 \frac{1}{N_c^2} \left\{ J^i, \{ J^j, G^{ja} \} \right\}$$

$$+ d_3 \frac{1}{N_c^2} \left( \{ J^2, G^{ia} \} - \frac{1}{2} \left\{ J^i, \{ J^j, G^{ja} \} \right\} \right) ,$$
where the $1/N_c$ expansion extends only to 3-body operators for QCD baryons with three valence quarks. The $1/N_c$ expansion for $A^a$ is given in terms of four unknown coefficients $a_1$, $b_2$, $b_3$ and $d_3$. The baryon axial vector flavor-singlet current has a $1/N_c$ expansion given by

$$A^i = c_1 J^i + c_2 \frac{1}{N_c^2} \{ J^2, J^i \}, \quad (15)$$

in terms of two $1/N_c$ operators. In contrast, the parametrization of baryon axial vector flavor-octet current in terms of the SU(3) flavor invariants of the HBχPT Lagrangian is given by

$$2D \text{Tr} BS^\mu \{ A_\mu, B \} + 2F \text{Tr} BS^\mu [A_\mu, B] + C \left( \bar{T}^a A_\mu B + \bar{B} A_\mu T^a \right) + 2H \bar{T}^a S^\nu A_\nu T_\mu , \quad (16)$$

in terms of the four SU(3) parameters $D$, $F$, $C$ and $H$, whereas the SU(3) analysis of the flavor-singlet yields two invariants

$$2S_B \text{Tr} A_\mu \bar{B} \Sigma_v^a B_v - 2S_T \text{Tr} A_\mu \bar{T}^a \Sigma_v^a T_\mu . \quad (17)$$

Notice that there are the same number of coefficients in the $1/N_c$ expansion as there are parameters in the SU(3) analysis; however, the $1/N_c$ expansion is advantageous because it orders the terms in powers of $1/N_c$.

At leading order in the $1/N_c$ expansion, the following truncations are possible, yielding relations between the SU(3) parameters at leading order in the $1/N_c$ expansion:

$$M = m_0 N_c \mathbf{1} \quad \Rightarrow \quad m_T = m_B$$

$$A^ia = a_1 G^ia + b_2 \frac{1}{N_c} J^i T^a \quad \Rightarrow \quad C = -2D, \ H = 3D - 9F \quad (18)$$

$$A^i = c_1 J^i \quad \Rightarrow \quad S_B = \frac{1}{3} S_T .$$

In the planar limit with zero quark loops, $U(3)$ planar flavor implies that the baryon axial vector flavor-singlet current $A^i$ is the ninth component of the flavor-octet currents $A^a$ so

$$A^i = \frac{1}{\sqrt{6}} (a_1 + b_2) J^i + \frac{1}{\sqrt{6}} (2b_3) \frac{1}{N_c^2} \{ J^2, J^i \} , \quad (19)$$

up to a correction of relative order $1/N_c$. Thus, $U(3)$ planar flavor symmetry relates the coefficients of the flavor-singlet current to the coefficients of the corresponding flavor-octet currents at leading order in $1/N_c$. Note that nonet
symmetry is valid for all of the operators in the $1/N_c$ expansion separately, not just the leading ones, since violation of the nonet symmetry only comes from diagrams with quark loops. In terms of the $SU(3)$ parameters, nonet symmetry implies that

$$S_B \to \frac{1}{3} (3F - D), \quad S_T \to - \frac{1}{3} H.$$  \hspace{1cm} (20)

5.1 $SU(3)$ Flavor Symmetry Breaking

A similar analysis can be performed for the flavor symmetry breaking terms of the $1/N_c$ baryon chiral Lagrangian.

The $1/N_c$ baryon chiral Lagrangian with explicit $SU(3)$ flavor symmetry breaking through the quark mass matrix is given to linear order in the quark mass matrix $M$ by

$$L_M = \text{Tr} \left( \left( \mathcal{M} \Sigma + \mathcal{M}^\dagger \Sigma^\dagger \right) \frac{I}{\sqrt{6}} \right) \mathcal{H}^0$$

$$+ \sum_{a=3,8} \text{Tr} \left( \left( \xi^\dagger M \xi + \xi M^\dagger \xi^\dagger \right) \frac{\lambda_a^0}{2} \right) \mathcal{H}^a,$$  \hspace{1cm} (21)

where the baryon flavor-singlet and flavor-octet operators $\mathcal{H}^0$ and $\mathcal{H}^a$ have $1/N_c$ expansions

$$\mathcal{H}^0 = c_0 N_c 1 + c_2 \frac{1}{N_c} J^2$$

$$\mathcal{H}^a = d_1 T^a + d_2 \frac{1}{N_c} \{J^i, G^{ia}\} + d_3 \frac{1}{N_c} \{J^2, T^a\}.$$  \hspace{1cm} (22)

The corresponding $SU(3)$ analysis gives the Lagrangian

$$\mathcal{L}^M = \sigma_B \text{Tr} \left( \mathcal{M} (\Sigma + \Sigma^\dagger) \right) \text{Tr} (\mathcal{B} \mathcal{B}) - \sigma_T \text{Tr} \left( \mathcal{M} (\Sigma + \Sigma^\dagger) \right) \mathcal{T}_\mu^\nu T_\mu$$

$$+ b_D \text{Tr} \mathcal{B} \{\{\xi^\dagger M \xi^\dagger + \xi M \xi\}, B\} + b_F \text{Tr} \mathcal{B} \{\{\xi^\dagger M \xi^\dagger + \xi M \xi\}, B\}$$

$$+ c \mathcal{T}_\mu^\nu (\xi^\dagger M \xi^\dagger + \xi M \xi) T_\mu,$$  \hspace{1cm} (23)

in terms of two flavor-singlet parameters and three flavor-octet parameters.

The leading order $1/N_c$ truncations for the flavor-singlet and flavor-octet baryon amplitudes yield

$$\mathcal{H}^0 = c_0 N_c 1 \quad \Rightarrow \quad \sigma_T = \sigma_B$$

$$\mathcal{H}^a = d_1 T^a \quad \Rightarrow \quad b_F = - \frac{1}{3}, \quad b_D = 0.$$  \hspace{1cm} (24)
whereas the $1/N_c$ truncation of the flavor-octet baryon amplitude at first subleading order in $1/N_c$ yields
\[
    \mathcal{H}^a = d_1 T^a + d_2 \frac{1}{N_c} \{ J^i, G^{ia} \} \quad \Rightarrow \quad (b_D + b_F) = -\frac{1}{3} c . \tag{25}
\]
In addition, $U(3)$ planar flavor symmetry implies that the flavor-singlet amplitude $\mathcal{H}^0$ is the ninth component of $\mathcal{H}^a$, \[
    \mathcal{H}^0 = \frac{1}{\sqrt{6}} d_1 N_c 1 + \frac{2}{\sqrt{6}} (d_2 + d_3) \frac{1}{N_c} J^2 , \tag{26}
\]
up to relative order $1/N_c$. The corresponding nonet symmetry relations for the $SU(3)$ parameters are
\[
    \sigma_B \rightarrow \frac{1}{3} (3b_F - b_D) , \quad \sigma_T \rightarrow -\frac{1}{3} c . \tag{27}
\]

6 Calculating Chiral Loop Corrections

The baryon propagator depends on the baryon spin through the hyperfine mass operator. It can be defined in terms of the spin-$\frac{1}{2}$ and $-\frac{1}{2}$ projection operators
\[
    P_{\frac{1}{2}} = -\frac{1}{3} \left( J^2 - \frac{15}{4} \right) , \quad P_{-\frac{1}{2}} = \frac{1}{3} \left( J^2 - \frac{3}{4} \right) \tag{28}
\]
as
\[
    \frac{i P_j}{(k^0 - \Delta_j)} , \quad \Delta_j = \frac{1}{N_c} (j(j + 1) - j'(j' + 1)) \tag{29}
\]
where the mass splitting $\Delta_j$ is given for a propagating spin-$j$ baryon emitted with pions from a spin-$j'$ baryon.

Chiral corrections can be calculated directly as products of $1/N_c$ baryon operators. The advantages of the $1/N_c$ operator expansion are that the group theory and the $1/N_c$ cancellations are explicit. A couple sample calculations will be considered to illustrate the method.

A general chiral loop calculation involves a nonanalytic function $F(m, \Delta)$ of the meson masses $m$ and the baryon hyperfine mass splitting $\Delta$. First consider the leading order contribution to the flavor-$27$ baryon mass splittings obtained from the baryon wavefunction renormalization diagram. The nonanalytic function can be decomposed into flavor-singlet, octet and $27$ components. The flavor-$27$ component is given by
\[
    \Pi_{27}^{ab} = \left( \frac{1}{3} F(\pi) - \frac{4}{3} F(K) + F(\eta) \right) \left( \delta^{ab} d^{888} - \frac{1}{8} g^{ab} - \frac{3}{5} g^{ab} d^{888} \right) . \tag{30}
\]
The wavefunction diagram yields a flavor-27 correction given by
\[ \frac{1}{N_c} \sum_{j=\frac{1}{2}, \frac{3}{2}} (A^{ia} P_j A^{jb}) \Pi_{27}^i(\Delta_j) . \] (31)

The flavor-27 Gell-Mann–Okubo mass combination, \( \frac{3}{4} \Lambda + \frac{5}{4} \Sigma - \frac{1}{2} (N + \Xi) \), is given by projecting this chiral loop contribution onto the spin-\( \frac{1}{2} \) baryons
\[ \frac{1}{N_c} \left[ P_\frac{3}{2} A^{ia} P_\frac{3}{2} A^{ia} P_\frac{3}{2} F_{27}(m, 0) + P_\frac{1}{2} A^{ia} P_\frac{1}{2} A^{ia} P_\frac{1}{2} F_{27}(m, \Delta) \right] \] (32)
whereas the flavor-27 spin-\( \frac{3}{2} \) baryon mass splitting \( -\frac{4}{7} \Delta + \frac{5}{7} \Sigma^* + \frac{2}{7} \Xi^* - \frac{3}{7} \Omega \), is given by the projection onto spin-\( \frac{3}{2} \) baryons
\[ \frac{1}{N_c} \left[ P_\frac{3}{2} A^{ia} P_\frac{3}{2} A^{ia} P_\frac{3}{2} F_{27}(m, -\Delta) + P_\frac{3}{2} A^{ia} P_\frac{3}{2} A^{ia} P_\frac{3}{2} F_{27}(m, 0) \right] . \] (33)

These complicated expressions reduce to linear combinations of the flavor-27 1/\( N_c \) mass operators
\[ \frac{1}{N_c} \left\{ T^8, T^8 \right\} + \frac{1}{N_c^2} \left\{ T^8, \{ J^i, G^{ia} \} \right\} . \] (34)

Next, consider the one-loop chiral correction \( \delta A^{ia} \) to the baryon axial vector flavor-octet current given by
\[ \frac{1}{2} F^{(1)}(m_b, 0, \mu) [A^{ib}, [A^{ja}, A^{ia}]] - \frac{1}{2} F^{(2)}(m_b, 0, \mu) \{ A^{ib}, [A^{ja}, [M, A^{ib}]] \} \] up to corrections with more insertions of the mass operator \( M \), where
\[ F^{(n)}(m_b, \Delta, \mu) = \frac{\partial^n F(m_b, \Delta, \mu)}{\partial \Delta^n} . \] (36)

The large-\( N_c \) cancellations for the axial vector currents occur only in the terms with zero or one power of \( M \). Thus, a hybrid approach is suggested in which the nonanalytic function is expanded in a Taylor series about \( \Delta = 0 \),
\[ F(m_b, \Delta) = F(m_b, 0) + F^{(1)}(m_b, 0) \Delta + \frac{1}{2} F^{(2)}(m_b, 0) \Delta^2 + \tilde{F}(m_b, \Delta), \] (37)
and the leading terms are kept as 1/\( N_c \) operators while the remainder \( \tilde{F}(m, \Delta) \) can be treated as a nonanalytic function in standard HB\( \chi \)PT since it contains no large-\( N_c \) cancellations.
7 Conclusions

A 1/$N_c$ chiral Lagrangian for baryons has been formulated based on large-$N_c$ QCD symmetries: $SU(3)_L \times SU(3)_R \times U(1)_V \times U(1)_A$ chiral symmetry and contracted $SU(6)$ baryon spin-flavor symmetry. Baryon chiral perturbation theory using the 1/$N_c$ chiral Lagrangian has a well-defined large-$N_c$ limit and the chiral loop expansion satisfies the expected 1/$N_c$ power counting for quantum loops because of large-$N_c$ cancellations. Moreover, the spin, flavor and 1/$N_c$ structure of the baryon chiral expansion is explicit in the 1/$N_c$ operator expansion.

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