Conformal Symmetry and A New Gauge in the Matrix Model

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Abstract

We generalize the background gauge in the Matrix model to propose a new gauge which is useful for discussing the conformal symmetry. In this gauge, the special conformal transformation (SCT) as the isometry of the near-horizon geometry of the D-particle solution is directly reproduced with the correct coefficient as the quantum correction to the SCT in the Matrix model. We also present a general argument for the relation between the gauge choice and the field redefinition in the Matrix model.
1 Introduction

It has been established that at low energies D-branes are described effectively by the Born-Infeld action, whose lowest order with respect to the number of derivatives is the Super-Yang-Mills theory (SYM) \[1\]. When M-theory is compactified in the infinite momentum frame, only the degrees of freedom of D-particles remain. It was therefore conjectured that the system is fundamentally described by \(d = 1\) SYM, i.e., the Matrix model \[2, 3\]. The conjecture of the Matrix model has been intensively investigated in many systems and compared to supergravity. Especially, D-particle scatterings with multi-bodies have been checked up to two loops \[4, 5, 6\], though disagreement for higher loops in a more complicated system is suspected \[7\]. Therefore, we have to show the full agreement between the Matrix model and supergravity to confirm the validity of the Matrix model as a non-perturbative definition of M-theory. A possible way to show the full agreement is to rely on symmetries. For example, the supersymmetries put restrictions on the form of the Matrix model effective action in the first few orders of the derivative expansion \[8, 9, 10\].

Conformal symmetry imposes another restriction on the effective action of the SYM, and in particular, the Matrix model. In fact, it is shown in \[11\] that the Born-Infeld action with the background of the near-horizon geometry of the D3-brane solution (that is, the Anti-de-Sitter or AdS space) can be determined exactly by the isometry of the AdS space. The special conformal transformation (SCT) of the isometry of the AdS space differs from the canonical one in SYM by an extra term which vanishes on the boundary. However, interestingly, this extra term can be derived from SYM as a quantum correction \[12\]. In this way, the problem of showing the full agreement between the Born-Infeld action with the AdS background and the effective action of SYM is reduced to showing that the quantum modified symmetry of SYM reproduces exactly the isometry of the AdS space.

Although the maximally SYM in \(d \neq 4\) is not a conformal field theory and the near-horizon geometry of the Dp-brane with \(p \neq 3\) is not of the AdS type, one can generalize the above arguments to Dp-branes by varying also the coupling constant under the dilatation and the SCT in both the SYM and the Dp-brane geometry \[13\]. This time, though the isometry of the near-horizon geometry determines the Born-Infeld action, the SCT derived from the SYM appears to differ from that of the isometry by a numerical factor. The SCT Ward-Takahashi identity still holds only when one keeps the terms proportional to the derivative of the coupling constant in the effective action, because they are also relevant after the transformation \[14, 15\]. It is also shown that the two SCTs (i.e., the isometry and the quantum modified one in SYM) are related by a field redefinition. Though consistent, the notorious numerical factor prevents us from determining the Born-Infeld action directly. Hence, it would be difficult to show the full agreement between the Matrix model and the Born-Infeld theory from the symmetry.
Besides, it is unclear why the field redefinition is necessary in spite of the fact that in other works \[\cite{4, 5, 6}\] studying the relationship between the Matrix model and the supergravity they always agree without any field redefinitions.

In this paper, we present a new gauge in SYM, which is a natural extension of the background gauge adopted in all the previous works \[\cite{13, 14, 15}\]. This new gauge has a marvelous property that the SCT as the isometry of the near-horizon geometry of the Dp-brane solution is correctly reproduced without field redefinitions as the quantum modified SCT of SYM, unlike in the case of the conventional background gauge.

Then, the result in our new gauge raises a question: what is the meaning of the gauge choice in the Matrix model? Therefore, we give a general argument to identify the change of gauge-fixing in the Matrix model as the redefinition of the D-brane coordinates. As a concrete example of this argument, we carry out the calculation in the \( R_\xi \) gauge. We find among other things that the agreement between the Matrix model and the supergravity without any field redefinitions \[\cite{4, 5, 6}\] is merely a special nature of the background gauge.

The organization of the rest of this paper is as follows. In section 2, we summarize the quantum conformal symmetry in SYM. We derive the quantum modified SCT and explain the disagreement of the numerical factor with that in the isometry. In section 3, we present our new gauge and analyze the quantum SCT in this gauge. In section 4, we give a general argument on the relation of changing the gauge in SYM to the field redefinition in the effective action, and in section 5 we analyze the \( R_\xi \) gauge as an example of the general formalism given in section 4. In the final section, we summarize the paper and discuss further directions. In the appendices, we present some technical details used in the text.

\section{Quantum conformal symmetry in SYM}

First, let us summarize the quantum conformal symmetry in SYM \[\cite{13, 12, 14, 15}\] in detail, because the methods needed afterwards are essentially the same. It is well known that the action of \( d = 4 \mathcal{N} = 4 \) SYM has conformal symmetry. This is also the case for the SYM with sixteen supersymmetries in other dimensions if we assign the coupling constant a conformal dimension and vary it under the transformation. We will here consider SYM in the Euclidean formulation:

\[
S_{\text{SYM}} = \int d^{p+1}x \frac{1}{g_{\text{YM}}^2} \text{tr} \left( \frac{1}{4} F_{\mu\nu}^2 + \frac{1}{2} (D_\mu X_m)^2 - \frac{1}{4} [X_m, X_n]^2 + \text{fermionic part} \right). \tag{1}
\]

The action (1) is invariant under both the dilatation and SCT. In particular, the transformation law of the bosonic variables under SCT reads

\[
\delta_{\text{SCT}}^C x_\mu = 2 \epsilon \cdot x x_\mu - \epsilon_\mu x^2, \quad \delta_{\text{SCT}}^C A_\mu = -2 \epsilon \cdot x A_\mu - 2 (x \cdot A \epsilon_\mu - \epsilon \cdot A x_\mu),
\]
\[ \delta_{\text{SCT}}^C X_m = -2\epsilon \cdot x X_m, \quad \delta_{\text{SCT}}^C g_{\text{YM}}^2 = 2(p - 3)\epsilon \cdot x g_{\text{YM}}^2, \]  

where the superscript \( C \) in \( \delta_{\text{SCT}}^C \) is for distinguishing the present classical transformation from the quantum one to be given later.

In order to quantize the system, we have to add the gauge-fixing and the corresponding ghost terms to the original action (1):

\[ S_{gf+gh} = \int d^{p+1}x \, i\delta_{\text{BRST}} \text{tr}(\bar{C}G). \]

In the original work of [14], they adopted the famous background gauge

\[ G = -\partial_\mu A_\mu + i[B_m, Y_m] + \frac{1}{2}g_{\text{YM}}^2 b, \]

where \( Y_m \) is the fluctuation of the scalars \( X_m \) from the diagonal background \( B_m \); \( X_m = B_m + Y_m \), and \( b \) is the auxiliary field for the off-shell closure of the BRST algebra. The BRST transformation of the fields are

\[
\begin{align*}
\delta_{\text{BRST}} A_\mu &= D_\mu C \equiv \partial_\mu C - i[A_\mu, C], \\
\delta_{\text{BRST}} C &= iC^2, \\
\delta_{\text{BRST}} \bar{C} &= i\bar{C}, \\
\delta_{\text{BRST}} b &= 0.
\end{align*}
\]

We assign the SCT of the unphysical fields so that \( \delta_{\text{SCT}}^C \) and the BRST transformation \( \delta_{\text{BRST}} \) are commutative, \([\delta_{\text{SCT}}^C, \delta_{\text{BRST}}] = 0\):

\[
\begin{align*}
\delta_{\text{SCT}}^C C &= 0, \\
\delta_{\text{SCT}}^C \bar{C} &= -2(p - 1)\epsilon \cdot x \bar{C}, \\
\delta_{\text{SCT}}^C b &= -2(p - 1)\epsilon \cdot x b.
\end{align*}
\]

While \( S_{gf+gh} \) with \( G \) of (4) is dilatation invariant, it is not invariant under SCT (2):

\[ \int d^{p+1}x \, i\delta_{\text{BRST}} \lambda[A, \bar{C}], \]

where \( \lambda[A, \bar{C}] \) is given by

\[ \lambda = \frac{p - 1}{2} \cdot (-4) \int d^{p+1}x \, \text{tr} \left( \bar{C}(x) A(x) \cdot \epsilon \right). \]

In general, if a symmetry of the classical action is violated by the gauge-fixing and the ghost terms \( S_{gf+gh} \) and the violation is given as the BRST-exact form \( i\delta_{\text{BRST}} \lambda \), the symmetry can be restored by adding to the original transformation for a generic field \( \phi \) the BRST transformation \(-i\lambda \delta_{\text{BRST}} \phi\) with this (field dependent) transformation parameter \( \lambda \). In fact, the change of the path-integral measure under the added BRST transformation just cancels the violation \( i\delta_{\text{BRST}} \lambda \). Therefore, in the present case, the effective action \( \Gamma[B, g_{\text{YM}}^2] \) of the system satisfies the following SCT Ward-Takahashi identity,

\[ \int d^{p+1}x \, \left( \delta_{\text{SCT}}^C g_{\text{YM}}^2(x) \frac{\delta}{\delta g_{\text{YM}}^2(x)} + \left( \delta_{\text{SCT}}^C + \delta_{\text{SCT}}^Q \right) B_{m,i}(x) \frac{\delta}{\delta B_{m,i}(x)} \right) \Gamma[B, g_{\text{YM}}^2] = 0, \]
where the extra term $\delta_{\text{SCT}}^Q B_{m,i}$ for $B_m = \text{diag} \left( B_{m,i} \right)$ is

$$
\delta_{\text{SCT}}^Q B_{m,i}(x) = 2(p - 1) \langle [C(x), X_m(x)]_{ii} \int d^{p+1}y \, \text{tr}(\bar{C}(y)A(y) \cdot \epsilon) \rangle.
$$  \hspace{1cm} (10)

Note that the total SCT for $B_m$ is now given as a sum of the classical part $\delta_{\text{SCT}}^C B_{m,i} = -2 \epsilon \cdot x B_{m,i}$ and the quantum correction $\delta_{\text{SCT}}^Q B_{m,i}$.

Now let us explicitly calculate $\delta_{\text{SCT}}^Q B_{m,i} \left[ \equiv \right]$. At the 1-loop order it is given by

$$
\delta_{\text{SCT}}^Q B_{m,i}(x) = 2(p - 1) \int d^{p+1}y \left( \langle C_{ij}(x) \bar{C}_{ji}(y) \rangle \langle Y_{m,ji}(x) A_{\mu,ij}(y) \epsilon_\mu \rangle - (i \leftrightarrow j) \right),
$$  \hspace{1cm} (11)

where the free propagators are

$$
\langle C_{ij}(x) \bar{C}_{ji}(y) \rangle = i \langle x | \Delta_{ij} | y \rangle,
$$

$$
\langle Y_{m,ji}(x) A_{\mu,ij}(y) \rangle = -2i g_{\text{YM}}^2 \langle x | \Delta_{ij} (\partial_\nu B_{m,ij}) \Delta_{ij} (\mathcal{M}^{-1})_{\nu\mu} | y \rangle,
$$  \hspace{1cm} (12)

with $\Delta_{ij} \equiv \left( -\partial^2 + B_{ij}^2 \right)^{-1}$, $B_{m,ij} \equiv B_{m,i} - B_{m,j}$ and $\mathcal{M}_{\mu\nu} \equiv \delta_{\mu\nu} - 4(\partial_\mu B_{\ell,ij}) \Delta_{ij} (\partial_\nu B_{\ell,ij}) \Delta_{ij}$. Keeping only the lowest order terms in the derivatives, eq. (11) is reduced to

$$
\delta_{\text{SCT}}^Q B_{m,i}(x) = \sum_j 8(p - 1) g_{\text{YM}}^2 \epsilon \cdot \partial B_{m,ij} \Delta_{ij}^3 = \sum_j \frac{4(p - 1) \Gamma \left( (5 - p)/2 \right) g_{\text{YM}}^2}{(4 \pi)^{(p+1)/2} B_{ij}^{p-2}} \epsilon \cdot \partial B_{m,ij}.
$$  \hspace{1cm} (13)

Restricting ourselves to the source-probe configuration with $N$ Dp-branes as the source at the origin and the probe at $B_m$; $B_m = \text{diag}(0, \cdots, 0, B_m)$, we obtain the final form of the quantum modified SCT $\delta_{\text{SCT}} \equiv \delta_{\text{SCT}}^C + \delta_{\text{SCT}}^Q$ for $x_\mu$, $g_{\text{YM}}^2$ and $U_m \equiv 2 \pi B_m$:

$$
\delta_{\text{SCT}} x_\mu = 2 \epsilon \cdot x x_\mu - \epsilon_\mu x^2,
$$

$$
\delta_{\text{SCT}} g_{\text{YM}}^2 = 2(p - 3) \epsilon \cdot x g_{\text{YM}}^2,
$$

$$
\delta_{\text{SCT}} U_m = -2 \epsilon \cdot x U_m - \frac{p - 1}{2} \frac{k R_p^4}{U^2} \epsilon \cdot \partial U_m,
$$  \hspace{1cm} (14)

with

$$
k \equiv \frac{2}{5 - p}, \quad R_p^2 \equiv \sqrt{d_p g_{\text{YM}}^2 N U^{p-3}}, \quad d_p \equiv 2^{7 - 2p} \pi^{(9 - 3p)/2} \Gamma \left( \frac{7 - p}{2} \right).
$$  \hspace{1cm} (15)

The quantum part $\delta_{\text{SCT}}^Q U_m$ of SCT has an extra numerical factor $(p - 1)/2$ compared to the isometry of the near-horizon geometry of the Dp-brane solution [14]. Since it is the isometry of the near-horizon geometry that determines the Born-Infeld action with the background, one may wonder if the SCT derived in SYM is consistent with the Ward-Takahashi identity. However, it was pointed out in [14] that, since the derivative of the coupling constant $\eta_\mu \equiv \partial_\mu g_{\text{YM}}^2/g_{\text{YM}}^2$ transforms under SCT as

$$
\delta_{\text{SCT}} \eta_\mu = 2(p - 3) \epsilon_\mu + O(\eta),
$$  \hspace{1cm} (16)
we have to keep terms linear in \( \eta \) in the calculation of the 1-loop effective action to confirm the validity of the Ward-Takahashi identity.\footnote{We consider the lowest non-trivial order in \( \eta \) and hence put \( \eta = 0 \) after the SCT.} Let us check it for the D-particle case. For this purpose, we need the quadratic parts of the action:

\[
\mathcal{L}_{YY} = \frac{1}{2g_{YM}^2} Y_{m,ij}(-\partial^2 + \eta \partial + B_{ij}^2) Y_{m,ji}, \quad \mathcal{L}_{AA} = \frac{1}{2g_{YM}^2} A_{ij}(-\partial^2 + \eta \partial + B_{ij}^2) A_{ji},
\]

\[
\mathcal{L}_{YA} = \frac{2i}{g_{YM}^2} V_{m,ij} Y_{m,ji} A_{ji}, \quad \mathcal{L}_{CC} = -i\bar{C}_{ij}(-\partial^2 + B_{ij}^2) C_{ji}, \quad \mathcal{L}_{\theta\theta} = \frac{1}{2} \theta_{\alpha,ij}(-\delta_{\alpha\beta} \partial + \gamma_{m\alpha\beta} B_{m,ij}) \theta_{\beta,ji},
\]

with \( V_{m,ij} \equiv \dot{B}_{m,ij} - \frac{1}{2} \eta B_{m,ij} \). We can read off the 1-loop effective action for the source-probe situation from (17) as

\[
\Gamma_{1\text{-loop}} = N \text{Tr} \left\{ 10 \ln \left( -\partial^2 + \eta \partial + B^2 \right) + \ln \left( 1 - 4V_m \Delta^\eta V_m \Delta^\theta \right) - 2 \ln \left( -\partial^2 + B^2 \right) - 4 \sum_{\pm} \ln \left( -\partial^2 + B^2 \pm \dot{B} \right) \right\},
\]

(18)

where \( \text{Tr} \) denotes the trace over the functional space of \( \tau \), and \( \Delta^\eta \) is defined by \( \Delta^\eta \equiv (-\partial^2 + \eta \partial + B^2)^{-1} = \Delta - \Delta \eta \partial \Delta + \mathcal{O}(\eta^2) \) with \( \Delta \equiv (-\partial^2 + B^2)^{-1} \). In (18), the first term is the contribution of \( \mathcal{L}_{YY} \) and \( \mathcal{L}_{AA} \), the second term is due to the mixing between \( Y_m \) and \( A \), the third term is the ghost loop and the last term is from \( \mathcal{L}_{\theta\theta} \). Keeping only terms independent of and linear in \( \eta \), we have

\[
\Gamma_{1\text{-loop}} = N \text{Tr} \left\{ 8 \ln \left( -\partial^2 + B^2 \right) + \ln \left( 1 - 4\dot{B}_m \Delta \dot{B}_m \Delta \right) - 4 \sum_{\pm} \ln \left( -\partial^2 + B^2 \pm \dot{B} \right) + 10\eta \partial \Delta + 4\eta \left( B_m \Delta \dot{B}_m \Delta + 2\partial \Delta \dot{B}_m \Delta \dot{B}_m \Delta \right) \right\},
\]

(19)

Note that the SCT of the terms of the form \( \int d\tau \eta(\tau) \times (\text{total derivative terms}) \) in the effective action vanish if we put \( \eta = 0 \) after the transformation. Then, since we have \( \langle \tau | \partial \mathcal{O} | \tau \rangle = (1/2)\partial_\tau \langle \tau | \mathcal{O} | \tau \rangle \) for \( \mathcal{O} = \Delta \) and \( \Delta \dot{B}_m \Delta \dot{B}_m \Delta (1 - 4\dot{B}_i \Delta \dot{B}_i \Delta)^{-1} \) due to \( \langle \tau_1 | \Delta | \tau_2 \rangle = \langle \tau_2 | \Delta | \tau_1 \rangle \), the terms proportional to \( \eta \) and relevant for SCT are

\[
\text{Tr} \left\{ 4 \left( \eta B_m \Delta \dot{B}_m \Delta \right) \left( 1 - 4\dot{B}_i \Delta \dot{B}_i \Delta \right)^{-1} \right\} = \text{Tr} \left\{ 4\eta B_m \Delta \dot{B}_m \Delta + 16\eta B_m \Delta \dot{B}_m \Delta \dot{B}_i \Delta \dot{B}_i \Delta \right\},
\]

(20)

where we have kept only the terms with the number of derivatives less than or equal to four. Note that we cannot adopt the eikonal approximation and drop the acceleration terms here \footnote{We consider the lowest non-trivial order in \( \eta \) and hence put \( \eta = 0 \) after the SCT.}, because by integration by parts, they can be converted to terms without the accelerations. A method to evaluate it was given in our previous work \footnote{We consider the lowest non-trivial order in \( \eta \) and hence put \( \eta = 0 \) after the SCT.}: we took polynomial forms for the background \( B_m(\tau) \), calculated the 1-loop effective action, and identified the result as a
functional of $B_m(\tau)$. An equivalent, but more refined method was presented in [10]: all the terms are rearranged into the forms of $f(\tau)\partial^n \Delta^n$, which are calculated using the proper-time representation. Here we adopt the more convenient method of [10]. Using the formulas presented in the appendix, the terms linear in $\eta$ in $\Gamma_{1\text{-loop}}$ are calculated to give

$$N \int d\tau \left\{ \frac{15}{16} \frac{(\dot{B}^2)^2}{B^5} + \frac{15}{8} \frac{\dot{B}^2 \dot{B} \cdot B}{B^5} + \text{total derivative terms} \right\}. \quad (21)$$

After making further the identification of the total derivative terms, we find the final expression of $\Gamma_{1\text{-loop}}$:

$$\Gamma_{1\text{-loop}} = N \int d\tau \left\{ -\frac{15}{16} \frac{(\dot{B}^2)^2}{B^5} + \eta \left( \frac{15}{8} \frac{\dot{B}^2 \dot{B} \cdot B}{B^5} + \text{total derivative terms} \right) \right\}. \quad (22)$$

The problem of the extra factor $(p-1)/2$ in $\delta_{\text{SCT}}^Q U_m$ (14) is now resolved by taking into account the SCT of the term $\eta(15/8)\dot{B}^2 \dot{B} \cdot B/B^7$ in (22). Moreover, this $\eta$-dependent term in (22) can be eliminated by making the field redefinition $B_m \rightarrow \tilde{B}_m$ with

$$\tilde{B}_m = B_m - g^2_{\text{YM}} N \frac{3}{4} \eta \dot{B}_m \frac{1}{B^5}, \quad (23)$$

in $\Gamma_{\text{tree}} = \int d\tau \dot{B}^2_m / 2g^2_{\text{YM}}$. Then, the SCT for the new variable $\tilde{B}_m$ is that of the isometry without the extra factor $(p-1)/2$.

### 3 A new gauge

Instead of the usual background gauge (4), here we propose a bit different gauge function useful for discussing the conformal symmetry in the SYM. In fact, we shall find that the notorious numerical factor $(p-1)/2$ does not appear this time. The gauge function of our new gauge is

$$G = -\partial_\mu A_\mu + \eta_\mu A_\mu + i[B_m, Y_m] + \frac{1}{2} g^2_{\text{YM}} b, \quad (24)$$

which has the additional term $\eta_\mu A_\mu$ compared to the old one (4). As in the previous case, the SCT symmetry broken by $S_{gf+gh}$ can be restored by adding to the classical SCT the BRST transformation with a field dependent parameter $\lambda$. In our new gauge, $\lambda[C, A]$ is

$$\lambda = -4 \int d^{p+1}x \text{tr} \left( C(x) A(x) \cdot \epsilon \right). \quad (25)$$

Note that in the usual background gauge (4) the factor $(p-1)/2$ in $\delta_{\text{SCT}}^Q U_m$ (14) originates in (8). However, it is missing in (25). This implies that we can derive the SCT of the isometry with the correct factor in our new gauge, namely, $\delta_{\text{SCT}}^Q U_m$ in the new gauge is given by

$$\delta_{\text{SCT}} U_m = -2 \epsilon \cdot x U_m - \frac{kR_p}{U^2} \epsilon \cdot \partial U_m, \quad (26)$$
instead of that in (14).

Since now the isometry that determines the Born-Infeld action has been reproduced with the correct coefficient, we can expect that the $\eta$-dependent terms such as $\eta(15/8)\dot{B}^2\dot{B}/B^2$ in (22) are missing from the one-loop effective action for the D-particle. Let us explicitly check it in the rest of this section. The quadratic parts of the D-particle action in the new gauge are

$$L_{YY} = \frac{1}{2g_{YM}^2} Y_{m,ij} (-\partial^2 + \eta \partial + B_{ij}^2) Y_{m,ji}, \quad L_{AA} = \frac{1}{2g_{YM}^2} A_{ij} (-\partial^2 - \eta \partial + B_{ij}^2) A_{ji},$$

$$L_{YA} = \frac{2i}{g_{YM}^2} \dot{B}_{m,ij} Y_{m,ji}, \quad L_{\bar{C}C} = -i \bar{C}_{ij} (-\partial^2 + \eta \partial + B_{ij}^2) C_{ji},$$

and $L_{\dot{\theta}\theta}$ is the same as (17) in the old gauge. The one-loop effective action for the source-probe configuration is given by

$$\Gamma_{1\text{-loop}} = N \text{Tr} \left\{ 9 \ln \left( -\partial^2 + \eta \partial + B^2 \right) + \ln \left( -\partial^2 - \eta \partial + B^2 \right) \right. $$

$$\left. + \ln \left( 1 - 4 \dot{B}_m (-\partial^2 + \eta \partial + B^2)^{-1} \dot{B}_m (-\partial^2 - \eta \partial + B^2)^{-1} \right) -2 \ln \left( -\partial^2 + \eta \partial + B^2 \right) - 4 \sum_{\pm} \ln \left( -\partial^2 + B^2 \pm \dot{B} \right) \right\},$$

where the origin of the respective terms are the same as before (18). The $\eta$-independent term in $\Gamma_{1\text{-loop}}$ is the same as in (22), while the term linear in $\eta$ is seen to be expressed as

$$\text{Tr} \{ \eta \partial O \} = \int d\tau \, \eta(\tau) \frac{1}{2} \partial_\tau \langle \tau | O | \tau \rangle,$$

in terms of a symmetric $O$ satisfying $\langle \tau_1 | O | \tau_2 \rangle = \langle \tau_2 | O | \tau_1 \rangle$. Eq. (29) is SCT invariant by putting $\eta = 0$ after the transformation.

Let us summarize our findings in this section. In the usually adopted background gauge (4), the quantum SCT has an extra numerical factor $(p-1)/2$. However, if we adopt the new gauge (24), this extra factor disappears and we can derive directly the conformal symmetry that determines the full Born-Infeld action. Accordingly, the $\eta$-dependent terms in $\Gamma_{1\text{-loop}}$ is also missing up to total derivative terms in our new gauge.

The calculation in this section shows that the form of quantum SCT depends crucially on the choice of gauge. Therefore one may question what the role of the gauge function is. We can find a clue in (14), where it is shown that the SCT with the extra factor $(p-1)/2$ can be related to the SCT without it by the field redefinition (23). Therefore it seems that the role of the gauge function is to choose the definition of the fields in the Born-Infeld action side. In the following sections, we shall study the relationship between the gauge choice and the redefinition of the fields.
4 Gauge shifts and field redefinitions

Here we give some general arguments on the relations between the change of the gauge function and the field redefinition. The following is essentially the reproduction of the old arguments about the independence of the physical S-matrices in the Yang-Mills theory on the choice of gauge [16].

Let us consider the partition function,

\[ Z_G[J] = \int D\phi e^{-S+J \cdot \phi}, \]

where \( \phi \) denotes collectively all the fields in the system and \( J \) is the corresponding source. The action \( S \) consists of the gauge-fixing and the ghost terms as well as the gauge invariant one; \( S = S_{\text{SYM}} + i\delta_{\text{BRST}}(\bar{C} \cdot G) \). The dots in \( J \cdot \phi \) and \( \bar{C} \cdot G \) denote both the integration over the coordinates and the trace over the group indices.

Under an infinitesimal change of the gauge function, \( G \rightarrow G + \Delta G \), we have

\[ Z_{G+\Delta G}[J] - Z_G[J] = \int D\phi \delta_{\text{BRST}}(-i\bar{C} \cdot \Delta G) e^{-S+J \cdot \phi} \]

\[ = \int D\phi (-i\bar{C} \cdot \Delta G)(J \cdot \delta_{\text{BRST}} \phi)e^{-S+J \cdot \phi}, \]

where in the last equality we have used the BRST Ward-Takahashi identity. Therefore, the partition function with the gauge function \( G + \Delta G \) is expressed as

\[ Z_{G+\Delta G}[J] = \int D\phi \left( 1 + (-i\bar{C} \cdot \Delta G)(J \cdot \delta_{\text{BRST}} \phi) \right)e^{-S+J \cdot \phi} \]

\[ = \int D\phi \exp \{ -S + J \cdot \left( \phi + i\delta_{\text{BRST}} \phi (\bar{C} \cdot \Delta G) \right) \}. \]

This is nothing but the partition function for the field \( \phi + i\delta_{\text{BRST}} \phi (\bar{C} \cdot \Delta G) \) in the original gauge \( G \).

Our next task is to restate the property (32) in terms of the effective action \( \Gamma_G[\varphi] \) which is related to \( \ln Z_G[J] \) by the Legendre transformation. In appendix B, we show that (32) implies the following relation between \( \Gamma_{G+\Delta G} \) and \( \Gamma_G \):

\[ \Gamma_{G+\Delta G}[\varphi] = \Gamma_G[\varphi] + \langle i\delta_{\text{BRST}} \phi (\bar{C} \cdot \Delta G) \rangle_{\varphi}, \]

where \( \langle O \rangle_{\varphi} \) denotes the generating functional of the 1PI Green’s function with an insertion of the operator \( O \) in the original gauge.

Let us apply the general arguments given above to the relation between our new gauge (24) and the old one (4) and reproduce the field redefinition (23) in the D-particle case. Using
with \( \varphi = B_m \) and \( \Delta G = \eta A \), and (12), we find that
\[
\bar{B}_m(\tau) = B_m(\tau) + \langle [X_m, C]_{N+1,N+1}(\tau) \int d\tau_0 \operatorname{tr} \tilde{C} \eta A(\tau_0) \rangle \\
= B_m - \int d\tau_0 \eta(\tau_0) \left( \langle C_{N+1,i}(\tau) i C_{i,N+1}(\tau_0) \rangle \langle Y_{m,i,N+1}(\tau) A_{N+1,i}(\tau_0) \rangle - (i \leftrightarrow N + 1) \right) \\
= B_m - 4g^2_{YM} N \langle \tau | \Delta \bar{B}_m \Delta \eta \Delta | \tau \rangle = B_m - g^2_{YM} \frac{3}{4} \eta B_m \frac{\tau}{B^5},
\]
where we have kept only those terms with the number of derivatives less than or equal to two. Note that \( \Delta G = \eta A \) can be regarded as infinitesimal since we are interested only in the lowest non-trivial order in \( \eta \). The result (34) agrees exactly with (23) proposed by [14].

5 Matrix model in the \( R_\xi \) gauge

In this section, we shall consider the Matrix model in another gauge. This will give a support for the validity of the general arguments of the previous section through a non-trivial calculation. It will also give a lesson about the structures of the field redefinitions and the non-renormalization theorem. The gauge we take here is the \( R_\xi \) gauge with \( |1 - \xi| \ll 1 \):
\[
G = -\partial A + i \xi [B_m, Y_m] + \frac{1}{2} \xi g^2_{YM} b,
\]
which has the properties that for \( \xi = 1 \) it reduces to the original background gauge, and that the \( Y-A \) mixing is given by \( \mathcal{L}_{Y_A} \) of (17) independently of the value of \( \xi \). For an infinitesimal \( \alpha \equiv 1 - \xi \), the difference from the original background gauge is
\[
\Delta G = \alpha \left( -i [B_m, Y_m] - \frac{1}{2} g^2_{YM} b \right) = \frac{\alpha}{2} \left( -\partial A + i [B_m, Y_m] \right).
\]
From the general formula (33) of the previous section, we can obtain the field redefinition relating the \( R_\xi \) gauge and the original background gauge (\( \xi = 1 \)). The calculation is carried out by using the free propagators for \( \xi = 1 \),
\[
\langle C_{ij}(\tau_1) \tilde{C}_{ji}(\tau_2) \rangle = i \langle \tau_1 | \Delta_{ij} | \tau_2 \rangle , \\
\langle Y_{m,ji}(\tau_1) A_{ij}(\tau_2) \rangle = -2i \langle \tau_1 | \Delta^\eta_{ij} V_{m,ij} \Delta^\eta_{ij} (1 - 4 V_{n,ij} \Delta^\eta_{ij})^{-1} g^2_{YM}(\tau) | \tau_2 \rangle , \\
\langle Y_{m,ij}(\tau_1) Y_{n,ji}(\tau_2) \rangle = \delta_{mn} \langle \tau_1 | \Delta^\eta_{ij} (1 - 4 V_{\ell,ij} \Delta^\eta_{ij})^{-1} g^2_{YM}(\tau) | \tau_2 \rangle ,
\]
and the formulas given in the appendix A, and taking into account the derivatives of the coupling \( g^2_{YM} \) carefully. We find
\[
\bar{B}_m = B_m + \alpha g^2_{YM} N \left( \frac{1}{4} \frac{B_m}{B^3} - \frac{1}{16} \frac{\bar{B}_m}{B^3} + \frac{5}{8} \frac{\bar{B} \cdot B \bar{B}_m}{B^7} - \frac{5}{16} \frac{\bar{B} \cdot B \cdot B_m}{B^7} \right) \\
- \frac{5}{16} \frac{\bar{B}^2 B_m}{B^7} + \frac{35}{32} \frac{\bar{B} \cdot B \cdot B_m}{B^7} - \frac{5}{16} \frac{\eta \bar{B}_m}{B^7} - \frac{5}{8} \frac{\eta \bar{B} \cdot B \cdot B_m}{B^7} + \frac{3}{16} \frac{\eta \bar{B}_m}{B^5}. \quad (37)
\]
To confirm the field redefinition (37), let us next consider the 1-loop effective action in the $R_{\xi}$ gauge:

$$\Gamma_{\text{1-loop}} = N \left\{ \ln \left( -\partial^2 + \eta \partial + B^2 \right) \delta_{mn} - (1 - \xi) B_m B_n \right\} + \ln \left( -\partial^2 + \eta \partial + \xi B^2 \right)$$

$$+ \ln \left( 1 - 4V_m \left( \Delta^a B_m \Delta^a \right) V_n \xi \left( -\partial^2 + \eta \partial + \xi B^2 \right)^{-1} \right)$$

$$- 2 \ln \left( -\partial^2 + \xi B^2 \right) - 4 \sum_{\pm} \ln \left( -\partial^2 + B^2 \pm \dot{B} \right) \right\}, \quad (38)$$

where $\text{Tr}$ for the first term implies also the trace operation with respect to $(m, n)$. Keeping only those terms proportional to $\alpha$ and at most linear in $\eta$, and further with number of derivatives less than or equal to four, we get after tedious but straightforward calculations the following result for the shift of the effective action in the $R_{\xi}$ gauge from that in the $\xi = 1$ gauge:

$$\int d\tau \alpha N \left\{ \frac{1}{4} \frac{\dot{B}^2}{B^3} - \frac{3}{4} \frac{(\dot{B} \cdot B)^2}{B^5} - \frac{1}{16} \frac{\ddot{B} \cdot \dot{B}}{B^5} - \frac{5}{16} \frac{\dddot{B} \cdot B \cdot \dot{B}}{B^7} - \frac{5}{8} \frac{\dddot{B} \cdot \dot{B} \cdot \dot{B}}{B^7} \right.$$  

$$+ \frac{35}{8} \frac{\dddot{B} \cdot B}{B^9} + \frac{105}{32} \frac{\dddot{B}^2 (\dot{B} \cdot B)^2}{B^9} - \frac{315}{32} \frac{(\dot{B} \cdot B)^4}{B^{11}} + \frac{1}{4} \frac{\dddot{B} \cdot B}{B^3} \right.$$  

$$+ \frac{3}{16} \frac{\dddot{B} \cdot B}{B^5} + \frac{7}{16} \frac{\dddot{B} \cdot \dot{B}}{B^5} - \frac{5}{8} \frac{\dddot{B} \cdot B \cdot \dot{B}}{B^7} - \frac{5}{16} \frac{\dddot{B}^2 \cdot \dot{B} \cdot B}{B^7} + \frac{35}{32} \frac{(\dot{B} \cdot B)^3}{B^9} \right\}. \quad (39)$$

In obtaining (39), we used the cyclicity of the trace to put all the terms coming from the expansion of (38) into the standard forms of $\eta B_m \Delta \cdots \Delta f(\tau) \Delta$ or $\eta \partial \Delta \cdots \Delta f(\tau) \Delta$, and applied the formulas in appendix A.

Then, our next task is to determine the redefinition $B_m \rightarrow \tilde{B}_m$ in such a way that the sum of the kinetic term $\dot{B}^2/2g_{YM}^2$ and the shift of $\Gamma_{\text{1-loop}}$ (39) is identified as the kinetic term of the new field $\tilde{B}_m$. The condition for the identification is apparently overdetermined. For example, the four-derivative terms independent of $\eta$ in (39) must correspond to the shift of $B_m$ by some two-derivative terms. There are 5 kinds of such terms in the shift of $B_m$: $\tilde{B}_m/B^5$, $\tilde{B}_m \dot{B} \cdot B/B^7$, $\cdots$, $B_m (\dot{B} \cdot B)^2/B^9$. On the other hand, there are 11 kinds of four-derivative terms independent of $\eta$ in the 1-loop effective action; $\dddot{B} \cdot B/B^5$, $\dddot{B} \cdot \dot{B} /B^5$, $\cdots$, $(\dot{B} \cdot B)^4/B^{11}$. Besides, there are 5 kinds of total derivative terms as the ambiguity of the effective action; $d/d\tau [\dddot{B} \cdot B/B^5]$, $d/d\tau [\dddot{B} \cdot \dot{B} /B^5]$, $\cdots$, $d/d\tau [(\dot{B} \cdot B)^3/B^9]$. Therefore, we have 11 equations with only 5+5 unknowns. However, there is a solution and it coincides with (37) obtained from the general formula (33).

We can also calculate the quantum SCT in the $R_{\xi}$ gauge to check the consistency of the field redefinition (37). Note the expression for the quantum SCT in the $R_{\xi}$ gauge is not changed form (10), and we obtain

$$\delta_{\text{SCT}}^Q B_m = -g_{YM}^2 N \left\{ \left( \frac{3}{2} + \alpha \right) \frac{\epsilon \tilde{B}_m}{B^5} + \frac{5}{4} \alpha \frac{\epsilon \tilde{B} \cdot B B_m}{B^7} \right\}. \quad (40)$$
One can easily see that the two quantum SCTs, (40) and (13) with $p = 0$, are consistently related by the redefinition (37).

Finally in this section we shall give some comments. First, although it might seem strange that there appear two-derivative terms (the first two terms) in (39), it does not contradict the non-renormalization theorem of [8]. The statement of [8] is that if we remove the $(\dot{B} \cdot B)^2/B^5$ term by a suitable coordinate transformations, the other term $\dot{B}^2/B^3$ will automatically disappear. Actually, the $B_m/B^3$ term in (37) removes the two terms in (39) simultaneously.

Our second comment is on the meaning of the field redefinitions. The redefinition (23) is simply a field dependent shift of the world-volume coordinate $\tau$ and hence is interpretable as a change of reparametrization gauge in the Born-Infeld action. However, the redefinition (37) contains, besides the terms interpretable as the target space coordinate transformation (the $B_m/B^3$ term) and the world-volume reparametrization (terms containing $\dot{B}_m$), all kinds of terms with a given dimension.

6 Conclusions and further directions

In this paper, we proposed a new gauge which is useful for discussing the conformal symmetry in the Matrix model. The form of the quantum modified SCT depends crucially on the choice of gauge in SYM, and our new gauge reproduced exactly the SCT of the isometry. We also gave a general argument on the relation between the change of gauge and the field redefinition in the effective action. Then, we examined the $R_\xi$ gauge as an example and reconfirmed the special nature of the background gauge and our new gauge. Namely, the diagonal elements of the Higgs fields in SYM correspond directly to the target space coordinates in the Born-Infeld action in the static gauge only when we take the background gauge or our new gauge in SYM.

We shall discuss some further directions of our work. First, the higher loop analysis of the Matrix model in our new gauge is an interesting subject. As the isometry of the near-horizon geometry of the D-brane solution determines the Born-Infeld action and we can derive the isometry directly from the 1-loop calculation in SYM in our new gauge, we expect that the quantum modified SCT in SYM is essentially 1-loop exact. The analysis of the quantum modified SCT in higher loops rather than that of the effective action would be a cleverer way to show the full agreement between the Matrix model and supergravity. Our new gauge would also be useful for analyzing more general multi-D-particle systems than the simple source-probe configuration of this paper.

The new gauge we proposed in this paper may be important even conceptually. The fact that the signs of the $\eta \partial$ in the kinetic terms (27) are opposite between $Y_m$ (the coordinates perpendicular to the branes) and $A_\mu$ (the coordinates parallel to the branes) reminds us of the
spacetime uncertainty principle proposed by \cite{17}. This would be a clue for the understanding of the deep meaning of our new gauge.

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**Appendices**

**A Useful formulas**

Here we present some useful formulas for calculating the expectation values using the method of \cite{10}. Making iterative use of the commutation relations,

\begin{align}
[\partial, f] &= \dot{f}, \\
[\Delta, f] &= \Delta(\dot{f} + 2\dot{\partial})\Delta, \\
[\Delta, \partial] &= 2\Delta(\dot{B} \cdot \dot{B})\Delta,
\end{align}

with $f$ being an arbitrary function of $\tau$, we get

\begin{align}
\Delta f &= f\Delta + 2\dot{f}\partial\Delta^2 + 4\dot{f}\dot{B} \cdot \dot{B}\Delta^3 + \dot{f}\Delta^2 + 4\dot{\partial}\Delta^3, \\
\Delta g\Delta f &= gf\Delta^2 + (2\dot{g}f + 4gf)\partial\Delta^3 + (4\dot{g}f + 12g\dot{f})\dot{B} \cdot \dot{B}\Delta^4 \\
&\quad+ (\dot{g}f + 22g\dot{f})\dot{\partial}\Delta^3 + (4\dot{g}f + 12g\dot{f} + 12\dot{g}\dot{f})\dot{\partial}^2\Delta^4, \\
\Delta h\Delta g\Delta f &= hgf\Delta^3 + (2hg\dot{f} + 4hgf + 6h\dot{g}f)\partial\Delta^4 + (4hgf + 12h\dot{g}f + 24h\dot{g}\dot{f})\dot{B} \cdot \dot{B}\Delta^5 \\
&\quad+ (\dot{h}gf + 2h\dot{g}f + 3h\dot{g}\dot{f} + 2h\dot{g}f + 2h\dot{g}\dot{f})\dot{\partial}\Delta^4 \\
&\quad+ (4\dot{h}gf + 12h\dot{g}f + 24h\dot{g}\dot{f} + 12h\dot{g}\dot{f} + 16h\dot{g}\dot{f} + 32h\dot{g}f)\dot{\partial}^2\Delta^5, \\
\partial\Delta f &= f\partial\Delta + \dot{f}\Delta + 2\dot{f}\partial\Delta^2 + 4\dot{f}\dot{B} \cdot \partial\Delta^3 + 3\dot{f}\partial\Delta^2 + 4\dot{\partial}\Delta^3 \\
&\quad+ (4\dot{f}\dot{B} \cdot \dot{B} + 4\dot{f}\dot{B} \cdot \dot{B})\Delta^3 + (8\dot{f}\dot{B} \cdot \dot{B} + 8\dot{f}\dot{B} \cdot \dot{B} + 24\dot{f}\dot{B} \cdot \dot{B})\dot{\partial}^2\Delta^4 \\
&\quad+ \ddot{f}\Delta^2 + 8\ddot{f}\partial\Delta^3 + 8\ddot{f}\partial\Delta^3, \\
\partial\Delta g\Delta f &= gf\partial\Delta^2 + (\dot{g}f + g\dot{f})\Delta^2 + (2\dot{g}f + 4gf)\partial\Delta^3, \\
\partial\Delta h\Delta g\Delta f &= hgf\partial\Delta^3 + (hgf + h\dot{g}f + h\dot{g}f)\Delta^3 + (2\dot{h}gf + 4h\dot{g}f + 6h\dot{g}f)\dot{\partial}^2\Delta^4,
\end{align}

where we have dropped the higher derivative terms on the right hand sides.
Then, we need to calculate \( \langle \tau | \partial^m \Delta^n | \tau \rangle \). First we express it in the proper-time representation as
\[
\langle \tau_1 | \Delta^n | \tau_2 \rangle = \frac{1}{[-\partial^2_{\tau_1} + B(\tau_1)^2]^n} \delta(\tau_1 - \tau_2) = \frac{1}{\Gamma(n)} \int_0^\infty d\sigma \sigma^{n-1} e^{-\sigma[-\partial^2_{\tau_1} + B(\tau_1)^2]} \delta(\tau_1 - \tau_2). \tag{49}
\]
This can be evaluated using
\[
e^{-\sigma[-\partial^2_{\tau_1} + B(\tau_1)^2]} = \left[ 1 - \sigma^2(\dot{B} \cdot B + \dot{\dot{B}}^2) - 2\sigma^2(\dot{B} \cdot B) \partial_{\tau} - \frac{8}{3} \sigma^3(\dot{B} \cdot B)^2 - \frac{4}{3} \sigma^3(\dot{B} \cdot B + \dot{\dot{B}}^2) \partial_{\tau}^2 + 2\sigma^4(\dot{B} \cdot B)^2 \partial_{\tau} + \text{higher derivative terms} \right] e^{-\sigma B(\tau)^2} e^{\sigma \partial^2_{\tau_1}}, \tag{50}
\]
obtained from the Baker-Campbell-Hausdorff’s formula, and
\[
e^{\sigma \partial^2_{\tau_1}} \delta(\tau_1 - \tau_2) = \frac{1}{\sqrt{4\pi\sigma}} \exp \left[ -\frac{1}{4\sigma}(\tau_1 - \tau_2)^2 \right]. \tag{51}
\]
Hence, we get the following derivative expansions for \( \langle \tau | \Delta^n | \tau \rangle \):
\[
\langle \tau | \Delta^2 | \tau \rangle = \frac{1}{4 B^3} \left( \frac{5}{16} \frac{\dot{B} \cdot B}{B^7} - \frac{5}{16} \frac{\dot{\dot{B}}^2}{B^7} + \frac{35}{32} \frac{B \cdot B^2}{B^9} \right),
\]
\[
\langle \tau | \Delta^3 | \tau \rangle = \frac{3}{16} \frac{B \cdot B^5}{B^7} - \frac{35}{64} \frac{B \cdot B^2}{B^9} - \frac{35}{64} \frac{B \cdot B}{B^9} + \frac{128}{128} \frac{B^{11}}{B^{11}},
\]
\[
\langle \tau | \Delta^4 | \tau \rangle = \frac{5}{32} \frac{B} {B^7}, \quad \langle \tau | \Delta^5 | \tau \rangle = \frac{35}{256} \frac{1}{B^9}, \quad \langle \tau | \Delta^6 | \tau \rangle = \frac{63}{512} \frac{1}{B^{11}}.
\tag{52}
\]
As for \( \langle \tau | \partial^m \Delta^n | \tau \rangle \), we have, using \( \partial^2 = -\Delta^{-1} + B^2 \) and \( \langle \tau | \partial \Delta^n | \tau \rangle = \frac{1}{2} \partial_{\tau} \langle \tau | \Delta^n | \tau \rangle \),
\[
\langle \tau | \partial \Delta^2 | \tau \rangle = -\frac{3}{8} \frac{\dot{B} \cdot B}{B^5} - \frac{5}{32} \frac{\dot{B} \cdot B}{B^7} - \frac{15}{32} \frac{\dot{\dot{B}} \cdot B \cdot B}{B^7} + \frac{35}{16} \frac{\dot{B} \cdot B \cdot B}{B^9} + \frac{35}{16} \frac{\dot{\dot{B}} \cdot B \cdot B}{B^9} - \frac{315}{64} \frac{B \cdot B^5}{B^9} - \frac{35}{64} \frac{B \cdot B^2}{B^9} - \frac{35}{64} \frac{B \cdot B}{B^9} - \frac{315}{64} \frac{B \cdot B^3}{B^9},
\]
\[
\langle \tau | \partial \Delta^3 | \tau \rangle = -\frac{15}{32} \frac{\dot{B} \cdot B}{B^7}, \quad \langle \tau | \partial \Delta^4 | \tau \rangle = -\frac{35}{64} \frac{B \cdot B}{B^9},
\]
\[
\langle \tau | \partial^2 \Delta^3 | \tau \rangle = -\frac{1}{16} \frac{B \cdot B^3}{B^7} - \frac{15}{64} \frac{B \cdot B^2}{B^7} - \frac{15}{64} \frac{B^2}{B^7} + \frac{128}{128} \frac{B^9}{B^9},
\]
\[
\langle \tau | \partial^2 \Delta^4 | \tau \rangle = -\frac{1}{32} \frac{1}{B^5}, \quad \langle \tau | \partial^2 \Delta^5 | \tau \rangle = -\frac{5}{256} \frac{1}{B^7},
\]
\[
\langle \tau | \partial^3 \Delta^4 | \tau \rangle = \frac{15}{64} \frac{\dot{B} \cdot B}{B^7}, \quad \langle \tau | \partial^3 \Delta^5 | \tau \rangle = \frac{1}{32} \frac{1}{B^3}, \quad \langle \tau | \partial^4 \Delta^5 | \tau \rangle = \frac{3}{256} \frac{1}{B^5}.
\tag{53}
\]

### B Derivation of eq. (33)

In this appendix we present the derivation of eq. (33) for the effective action from eq. (32) for the partition function \( Z_G[J] \). For this purpose, let us introduce the generating functional of
the connected Green's function \( W[J, K] \) in the gauge \( G \) with sources for \( \Delta \phi \equiv \delta_{\text{BRST}} \phi (\bar{C} \cdot \Delta G) \) as well as for \( \phi \):

\[
e^{W[J,K]} = \int \mathcal{D}\phi \, e^{-S + J \cdot \phi + K \cdot \Delta \phi}.
\] (54)

Then, \( W_{G+\Delta G}[\tilde{J}] \equiv \ln Z_{G+\Delta G}[\tilde{J}] \) is expressed in terms of \( W[J, K] \) as

\[
W_{G+\Delta G}[\tilde{J}] = W[\tilde{J}, \tilde{J}].
\] (55)

Hence, \( \Gamma_{G+\Delta G}[\varphi] \), which is the Legendre transformation of \( W_{G+\Delta G}[\tilde{J}] \), is expressed using (54) and the Taylor expansion \( W[J, K] = W[J, 0] + K \cdot (\delta W[J, K]/\delta K)_{K=0} \), as

\[
\Gamma_{G+\Delta G}[\varphi] = \tilde{J} \cdot \varphi - W[J, 0] - \tilde{J} \cdot \delta_{\delta K} W[\tilde{J}, K] \bigg|_{K=0},
\] (56)

where the relation between \( \varphi \) and \( \tilde{J} \) is given by

\[
\varphi = \frac{\delta}{\delta \tilde{J}} W[\tilde{J}, \tilde{J}].
\] (57)

On the other hand, let us define \( \Gamma[\varphi, K] \) which is the Legendre transformation of \( W[J, K] \) with respect only to \( J \):

\[
\Gamma[\varphi, K] = J \cdot \varphi - W[J, K],
\] (58)

where \( \varphi \) and \( J \) are related by

\[
\varphi = \frac{\delta}{\delta J} W[J, K].
\] (59)

The precise meaning of \( \langle \delta_{\text{BRST}} \phi (\bar{C} \cdot \Delta G) \rangle_\varphi = \langle \Delta \phi \rangle_\varphi \) in (33) is in fact \( (\delta \Gamma[\varphi, K]/\delta K)_{K=0} \).

Therefore, the right hand side of (33) is Taylor expanded as

\[
\Gamma_G[\varphi + \langle \Delta \phi \rangle_\varphi] = \Gamma_G[\varphi] + \frac{\delta}{\delta \varphi} \Gamma[\varphi, 0] \frac{\delta}{\delta K} \Gamma[\varphi, K] \bigg|_{K=0},
\] (60)

where we have used \( \Gamma_G[\varphi] = \Gamma[\varphi, 0]. \)

For a given \( \varphi \), let \( J \) be defined by (59) with \( K = 0 \), and hence \( J = \delta \Gamma[\varphi, 0]/\delta \varphi \). In view of the definition (57) for \( \tilde{J} \), the difference between \( \tilde{J} \) and \( J \) is infinitesimal of order \( \Delta G \). Plugging \( \tilde{J} = J + \Delta J \) into (54) and Taylor expanding with respect to infinitesimal \( \Delta J \), we get

\[
\Gamma_{G+\Delta G}[\varphi] = J \cdot \varphi - W[J, 0] - J \cdot \frac{\delta}{\delta K} W[J, K] \bigg|_{K=0} + \Delta J \cdot \left( \varphi - \frac{\delta}{\delta J} W[J, 0] \right).
\] (61)

This agrees with (60) since we have \( \delta W[J, K]/\delta K = -\delta \Gamma[\varphi, K]/\delta K \) obtained from the derivative of (58). Note that the explicit expression of \( \Delta J \) is unnecessary here.
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