CP Violation in $\bar{B}^0 \to D^{*+} \mu^- \bar{\nu}_\mu$

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ABSTRACT: In order to explain the observed anomalies in the measurements of $R_{D^{(*)}}$ and $R_{J/\psi}$, a variety of new-physics (NP) models that contribute to $b \to c \tau^- \bar{\nu}_\tau$ have been proposed. In this paper, we show how CP-violating observables can be used to distinguish these NP models. Because $\vec{p}_\tau$ cannot be measured (the decay products of the $\tau$ include the undetected $\nu_\tau$), obtaining the angular distribution of $\bar{B}^0 \to D^{*+} \tau^- \bar{\nu}_\tau$ is problematic. Instead, we focus here on $\bar{B}^0 \to D^{*+} (\to D^0 \pi^+) \mu^- \bar{\nu}_\mu$. This process may also receive contributions from the same NP, and LHCb intends to measure the CP-violating angular asymmetries in this decay. There are two classes of NP models that contribute to $b \to c \mu^- \bar{\nu}_\mu$. These involve (i) a $W'$ (two types) or (ii) a leptoquark (LQ) (six types). The most popular NP models predict no CP-violating effects, so the measurement of nonzero CP-violating asymmetries would rule them out. Furthermore these measurements allow one to distinguish the $W'$ and LQ models, and to differentiate among several LQ models.

KEYWORDS: $\bar{B}^0 \to D^{*+} \mu^- \bar{\nu}_\mu$, New Physics, CP Violation

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1 Introduction

At present, there are discrepancies with the predictions of the standard model (SM) in
the measurements of $R_{D^{(*)}} \equiv \mathcal{B}(\bar{B} \rightarrow D^{(*)} \tau^{-} \bar{\nu}_{\tau})/\mathcal{B}(\bar{B} \rightarrow D^{(*)} \ell^{-} \bar{\nu}_{\ell})$ ($\ell = e, \mu$) [1–4] and
$R_{J/\psi} \equiv \mathcal{B}(B_{c}^{+} \rightarrow J/\psi \tau^{+} \nu_{\tau})/\mathcal{B}(B_{c}^{+} \rightarrow J/\psi \mu^{+} \nu_{\mu})$ [5]. The experimental results are shown in Table 1. The deviation from the SM in $R_{D}$ and $R_{D^{*}}$ (combined) is at the 4$\sigma$ level [6–9],
while it is 1.7$\sigma$ in $R_{J/\psi}$ [10]. These measurements suggest the presence of new physics (NP) in $b \rightarrow c\tau^{-}\bar{\nu}$ decays.

| Observable                  | Measurement/Constraint |
|-----------------------------|------------------------|
| $R_{D^{(*)}}^{\tau/\ell}/(R_{D^{(*)}}^{\tau/\ell})_{SM}$ | 1.18 ± 0.06 [1–4]     |
| $R_{D^{(*)}}^{\tau/\ell}/(R_{D^{(*)}}^{\tau/\ell})_{SM}$ | 1.36 ± 0.15 [1–4]     |
| $R_{J/\psi}^{\mu/\ell}/(R_{J/\psi}^{\mu/\ell})_{SM}$ | 1.00 ± 0.05 [6]       |
| $R_{J/\psi}^{\mu/\ell}/(R_{J/\psi}^{\mu/\ell})_{SM}$ | 2.51 ± 0.97 [5]       |

Table 1. Measured values of observables that suggest NP in $b \rightarrow c\tau^{-}\bar{\nu}$.
There have been numerous papers examining the nature of the NP required to explain the above anomalies. These include both model-independent [10–21] and model-dependent analyses [22–49]. There are therefore many possibilities for the NP. In Refs. [12, 50–64], a variety of observables are proposed for distinguishing the various NP explanations. These include the $q^2$ distribution, $D^*$ polarization, the $\tau$ polarization, etc. In this paper, we focus on the measurement of CP-violating observables as a means of differentiating the NP scenarios\(^1\).

All CP-violating effects require the interference of two amplitudes with different weak (CP-odd) phases. The most common observable is the direct CP asymmetry, $A_{\text{dir}}$, which is proportional to $\Gamma(\bar{B}^0 \to D^{*+}\tau^-\bar{\nu}_\tau) - \Gamma(B^0 \to D^{*-}\tau^+\nu_\tau)$. $A_{\text{dir}}$ can be nonzero only if the interfering amplitudes also have different strong (CP-even) phases. Now, strong phases can only arise in hadronic transitions, and here the only such transition is $\bar{B} \to D^*$ (or $b \to c$ at the quark level). Thus, whether the decay proceeds within the SM or with NP, the strong phase will be the same. There is one possible exception: If the NP mediator has colour (e.g., a leptoquark), it can be involved in gluon exchange, leading to additional strong phases. However, strong phases generated in this way cannot be large \(^6\). As a result, though $A_{\text{dir}}$ can be nonzero, we expect it to be small.

The main CP-violating effects in $\bar{B}^0 \to D^{*+}(\to D^0\pi^+)\tau^-\bar{\nu}_\tau$ therefore appear as CP-violating asymmetries in the angular distribution\(^2\). These are kinematical observables, meaning that, in order to generate such effects, the two interfering amplitudes must have different Lorentz structures. This fact allows us to distinguish different NP explanations.

To see this, we note that, in the SM, $b \to c\tau^-\bar{\nu}_\tau$ arises through the exchange of a $W$; the four-fermion effective operator is $(V - A) \times (V - A)$ (LL): $c_{\text{SM}}\bar{c}_L\gamma_\mu b_L\bar{\tau}_L\gamma^\mu\nu_\tau,L$. If the NP coupling is also LL, it simply adds to the SM contribution, so that the full coefficient of the operator is $c_{\text{SM}} + c_{\text{NP}}$. Compared to the SM alone, the correction to the rate is then $2\text{Re}(c_{\text{SM}}c_{\text{NP}}^*) + |c_{\text{NP}}|^2$. On the other hand, if the NP four-fermion effective operator has a Lorentz structure other than LL, there is no SM-NP interference and the correction to the rate is simply $|c_{\text{NP}}|^2$. We generally expect NP effects to be small, i.e., $|c_{\text{NP}}| < |c_{\text{SM}}|$, in which case the largest correction to the rate comes from the SM-NP interference term, $2\text{Re}(c_{\text{SM}}c_{\text{NP}}^*)$. For this reason, scenarios in which the NP four-fermion effective operator is also LL are the preferred explanations. However, in this case, because the SM and NP have the same Lorentz structure, their interference cannot produce CP-violating angular asymmetries. That is, if a nonzero asymmetry were measured, it would rule out NP scenarios with purely LL couplings. Four-fermion effective operators with other Lorentz structures would be required, and these could be distinguished by the different types of CP-violating angular asymmetries that they produce.

In Refs. [15, 51], the decay $\bar{B}^0 \to D^{*+}(\to D^0\pi^+)\tau^-\bar{\nu}_\tau$ was analyzed in the context of an effective Lagrangian containing NP four-fermion operators with all Lorentz structures. The angular distribution was computed, giving the various contributions to the CP-violating

\(^1\)There are also anomalies in various observables involving the decay $b \to s\mu^+\mu^-$, and several different NP explanations have been proposed. In Ref. [66] it is shown that these NP models can be distinguished through the measurement of CP-violating observables in $B \to K^*\mu^+\mu^-$.  
\(^2\)Another possibility is to use excited charm mesons, see Ref. [61].
angular asymmetries. However, there is a practical problem here: the reconstruction of the angular asymmetries requires the knowledge of $\vec{p}_\tau$. But since the $\tau$ decays to final-state particles that include $\nu_\tau$, which is undetected, $\vec{p}_\tau$ cannot be measured.

A complete analysis of CP-violating angular asymmetries in this decay must therefore include information related to the decay products of the $\tau$. One such attempt was made in Ref. [67]. There the decay $\bar{B} \to D\tau^-\bar{\nu}_\tau$ was considered, with $\tau \to V^- (\to \pi^-\pi^0, \pi^-\pi^+\pi^- \text{ or } \pi^0\pi^0\pi^0) \nu_\tau$, and a complicated kinematical CP asymmetry was constructed. Our ultimate goal is to perform a complete angular analysis of $\bar{B}^0 \to D^{*+}(\to D^0\pi^+)\tau^-\bar{\nu}_\tau$, including the angular information from the $\tau$ decay, and compute the NP contributions to all possible CP-violating angular asymmetries. Some work along these lines can be found in Ref. [68].

In this paper, we take a first step towards this goal by examining the NP contribution to the CP-violating angular asymmetries in $\bar{B}^0 \to D^{*+}\mu^-\bar{\nu}_\mu$. There are two reasons for starting here. First, LHCb has announced [69] that it will perform a detailed angular analysis of this decay, with the aim of extracting the coefficients of the CP-violating angular asymmetries. It is important to show exactly what the implications of these measurements are for NP. Second, although the preferred explanation of the $R_{D^{(*)}}$ and $R_{J/\psi}$ anomalies is NP in $b \to c\tau^-\bar{\nu}$, this same NP may well also contribute to $b \to c\mu^-\bar{\nu}$, leading to deviations from the SM in $\bar{B}^0 \to D^{*+}\mu^-\bar{\nu}_\mu$.

We begin in Sec. 2 with a derivation of the angular distribution for $\bar{B} \to D^*(\to D\pi)\ell^-\bar{\nu}_\ell$, both in the SM and with the addition of NP. This angular distribution contains several CP-violating angular asymmetries. In Sec. 3, we describe the various NP models that can contribute to $\bar{B}^0 \to D^{*+}\mu^-\bar{\nu}_\mu$, and compute their contributions to the various CP-violating observables. This provides all the NP implications of the measurement of the CP-violating angular asymmetries. We conclude in Sec. 4.

### 2 Angular Analysis

In this section we discuss the kinematics of the decay $\bar{B} \to D^*(\to D\pi)\ell^-\bar{\nu}_\ell$ and define the angular observables in the process using transversity amplitudes. The total decay amplitude for this process can be expressed as a sum over several pairs of effective two-body decays. In the most general case, several of these are due to NP, while one arises from the SM. We begin by examining the SM contribution.

#### 2.1 Transversity amplitudes: SM

Following Ref. [70], the decay $\bar{B} \to D^*\ell^-\bar{\nu}_\ell$ is considered to be $\bar{B} \to D^*W^{*-}$, where the on-shell $D^*$ decays to $D\pi$ and the off-shell $W^{*-}$ decays to $\ell^-\bar{\nu}_\ell$. Its amplitude is given by

$$M_{(m,n)}(\bar{B} \to D^*W^*) = \epsilon_{D^*}^\mu(m)M_{\mu\nu}\epsilon_{W^*}^{\nu}(n), \quad (2.1)$$

Note that, since $R_{D^{(*)}}^{\mu/e}$ is $1.00 \pm 0.05$ (Table 1), NP that contributes to $b \to c\mu^-\bar{\nu}$ must also equally affect $b \to \ell^-\bar{\nu}_\ell$.

The angular distributions for semileptonic $B$ decays were also presented in [71].
where \( \epsilon_{\mu}^{\perp}(m) \) is the polarization of a vector particle \((D^* \text{ or } W^*)\). Here \( m, n = \pm 1, 0 \) and \( t \) represent the transverse, longitudinal and timelike polarizations, respectively. (Only the off-shell \( W^+ \) has a timelike polarization.)

In the \( B \)-meson rest frame we write the polarizations of the two vector particles as

\[
\epsilon_{\mu}^{\perp}(\pm) = (0,1,\pm i,0) / \sqrt{2}, \quad \epsilon_{\mu}^{\perp}(0) = (k_z,0,0,k_\perp) / m_{D^*}, \quad \epsilon_{\mu}^{\perp}(t) = q^\mu / \sqrt{q^2},
\]

where \( k^\mu = (k_0,0,0,k_z) \) and \( q^\mu = (q_0,0,0,q_z) \) are the four momenta of the \( D^* \) and \( W^* \), respectively, both written in the rest frame of the \( B \). The polarization vectors of the off-shell \( W^* \) satisfy the following orthonormality and completeness relations:

\[
\epsilon_{\mu}^{\perp}(m)\epsilon_{\nu}^{\perp}(m') = g_{mm'},
\]

\[
\sum_{m,m'} \epsilon_{\mu}^{\perp}(m)\epsilon_{\nu}^{\perp}(m')g_{mm'} = g^{\mu\nu},
\]

(2.3)

where \( g_{mn'} = \text{diag}(+, -, -) \) for \( m = t, \pm 0 \). For the on-shell \( D^* \), these relations are

\[
\epsilon_{\mu}^{\perp}(m)\epsilon_{\nu}^{\perp}(m') = -\delta_{mm'},
\]

\[
\sum_{m,m'} \epsilon_{\mu}^{\perp}(m)\epsilon_{\nu}^{\perp}(m')\delta_{mm'} = g^{\mu\nu} + \frac{k^\mu k^\nu}{m_{D^*}^2}.
\]

(2.4)

Since the \( B \) meson has spin 0, of the 12 combinations of \( D^* \) and \( W^* \) polarizations, only 4 are allowed, producing the following helicity amplitudes:

\[
\mathcal{M}_{(+,+)}(B \to D^*W^*) = A_+, \quad \mathcal{M}_{(-,-)}(B \to D^*W^*) = A_-, \quad \mathcal{M}_{(0;0)}(B \to D^*W^*) = A_0, \quad \mathcal{M}_{(0;\pm)}(B \to D^*W^*) = A_{\pm}.
\]

(2.5)

One may also go to the transversity basis by writing the amplitudes involving transverse polarizations as

\[
A_{||,\perp} = (A_+ \pm A_-) / \sqrt{2}.
\]

(2.6)

The full amplitude for the decay process \( B \to D^* \to D\pi\ell^-\bar{\nu}_\ell \) can now be expressed as

\[
\mathcal{M}(B \to D^* \to D\pi \to \ell^-\bar{\nu}_\ell) \propto \sum_{m,m'=\pm,0} \epsilon_{\mu}^{D^*}(m)(p_D)_\sigma g_{mm'} \epsilon_{\mu}^{D^*}(m') M_{\rho\nu} \sum_{n,n'=t,\pm,0} \epsilon_{\nu}^{W^*}(n') g_{nn'} \epsilon_{\nu}^{W^*}(n)(\bar{u}_\ell \gamma_\mu P_L v_{\nu_\ell}) .
\]

(2.7)

Here we have made explicit use of the fact that \( \epsilon_{\mu}^{D^*}(p_{D^*})_\sigma = \epsilon_{\mu}^{D^*}(p_D + p_\pi)_\sigma = 0 \), so that \( A(D^* \to D\pi) \propto \epsilon_{\mu}^{D^*}(p_D - p_\pi)_\sigma = 2\epsilon_{\mu}^{D^*}(p_{D^*})_\sigma \). In the above amplitude, one can project out
the relevant helicity components to obtain
\[
\mathcal{M}(B \to D^*(\to D\pi)W^*(\to \ell^-\bar{\nu}_\ell)) \\
\propto \sum_{m,m'=\pm,0} \sum_{n,n'=\pm,0} \epsilon_{D^*}''(m)(p_D)_{\sigma} \, g_{mn'} \, \mathcal{M}_{(m',n')} (B \to D^*W^*) \, g_{n'n} \, \epsilon_{W^*}''(n) \, (\bar{u}_\ell\gamma_\mu P_L v_{\bar{\nu}_\ell}) \\
\propto - \sum_{m=\pm,0} \sum_{n=\ell,\pm,0} g_{mn} \mathcal{H}_{D^*}(m) \, \mathcal{M}_{(m,n)} (B \to D^*W^*) \, \mathcal{L}_{W^*}(n) , \quad (2.8)
\]
where
\[
\mathcal{H}_{D^*}(m) = \epsilon_{D^*}(m) \cdot p_D \quad , \quad \mathcal{L}_{W^*}(n) = \epsilon_{W^*}(n)(\bar{u}_\ell\gamma_\mu P_L v_{\bar{\nu}_\ell}) . \quad (2.9)
\]
The notation of Eq. (2.8) can be simplified by defining a timelike polarization for the \( D^* \):
\[
\mathcal{H}_{D^*}(t) \equiv \mathcal{H}_{D^*}(0) . \quad (2.10)
\]
In this case, the helicities of Eq. (2.5) become \( \mathcal{M}_{(m,m)} (B \to D^*W^*) = A_m \) and
\[
\mathcal{M}(B \to D^*(\to D\pi)W^*(\to \ell^-\bar{\nu}_\ell)) \propto \sum_{m=\ell,\pm,0} g_{mm} \, A_m \, \mathcal{H}_{D^*}(m) \, \mathcal{L}_{W^*}(m) . \quad (2.10)
\]

Written in this form, the differential decay rate can now be constructed from the helicity amplitudes and the Lorentz-invariant quantities \( \mathcal{H}_{D^*} \) and \( \mathcal{L}_{W^*} \). The spin-summed square of the amplitude is
\[
|\mathcal{M}|^2 \propto \sum_{m,m'=\pm} g_{mm'} \epsilon_{A_m} \epsilon_{A_{m'}} \, (\mathcal{H}_{D^*}(m)\mathcal{H}_{D^*}(m')) \sum_{\text{spins}} \mathcal{L}_{W^*}(m)\mathcal{L}_{W^*}(m') \quad (2.11)
\]
The leptonic part of the above squared amplitude is given in Eq. (A.2).

2.2 New Physics

From Eq. (2.10), we see that, in the SM, the decay amplitude can be written as the product of a hadronic piece \( \mathcal{H}_{D^*}(m) \), a leptonic piece \( \mathcal{L}_{W^*}(m) \), and a helicity amplitude \( A_m \), summed over all helicities \( m \). As we will see, this same structure holds in the presence of NP. We can consider separately the NP leptonic and hadronic contributions. We begin with the leptonic piece.

In the SM, we have \( \bar{B} \to D^*W^{*-} \), where the \( W^{*-} \) decays to \( \ell^-\bar{\nu}_\ell \) via a \( (V-A) \) interaction. If NP is present, there are several possible differences. First, there may also be scalar and/or tensor interactions. Second, the decay products may include a \( \bar{\nu} \) of a flavour other than \( \ell \). Finally, a right-handed (RH), sterile neutrino may be produced [42, 43, 48]. In what follows, we assume that neutrinos are left-handed, as in the SM, though we will discuss how our analysis is affected if a RH neutrino is involved. Regarding the \( \bar{\nu} \) flavour, technically we should write \( \bar{\nu}_i \) and sum over all possibilities for \( i \) (since the \( \bar{\nu} \) is undetected). However, this makes the notation cumbersome, and does not change the physics. For this reason, for notational simplicity, we continue to write \( \bar{\nu}_\ell \), though the reader should be aware that other \( \bar{\nu} \) flavours are possible. Thus, in the presence of NP, the relevant two-body processes to consider are \( \bar{B} \to D^*N^{*-} (\to \ell^-\bar{\nu}_\ell) \), where \( N = S-P, V-A, T \) represent left-handed scalar, vector and tensor interactions, respectively. In what follows, we label these \( SP, VA \) and \( T \). (The \( VA \) contribution includes that of the SM.)
Turning to the hadronic piece, we note that the underlying decay is \( b \to c \ell^- \bar{\nu} \). For each of the leptonic \( S^P, \, V^A \) and \( T \) Lorentz structures, we introduce NP contributions to the \( b \to c \) transition. The effective Hamiltonian is

\[
\mathcal{H}_{\text{eff}} = \frac{G_F V_{cb}}{\sqrt{2}} \left\{ \left[ (1 + g_L) \bar{c} \gamma_\mu (1 - \gamma_5) b + g_R \bar{c} \gamma_\mu (1 + \gamma_5) b \right] \bar{\ell} \gamma^\mu (1 - \gamma_5) \nu_\ell \\
+ [g_S \bar{c} b + g_P \bar{c} \gamma_5 b] \bar{\ell} (1 - \gamma_5) \nu_\ell + g_T \bar{c} \sigma^{\mu\nu} (1 - \gamma_5) b \ell \sigma_{\mu\nu} (1 - \gamma_5) \nu_\ell + h.c. \right\} .
\] (2.12)

### 2.3 Transversity amplitudes: NP

Including all possible contributions (SM + NP), the amplitude for the process can be expressed as

\[
\mathcal{M}^{\text{SM+NP}} \propto \sum_{m,m' = \pm, 0} \epsilon^\nu_{D^*} (m) (pD)_\nu g_{nn'} \epsilon^\mu_{D^*} (m') M^{SP}_{\mu} \left( \bar{u}_\ell P_L \nu_\ell \right) \\
+ \sum_{m,m'} \epsilon^\nu_{D^*} (m) (pD)_{\nu} g_{nn'} \epsilon^\mu_{D^*} (m') M^{VA}_{\mu} \sum_{n,n'} \epsilon^\nu_{VA} (n') g_{n'n} \epsilon^\mu_{VA} (n) \left( \bar{u}_\ell \gamma_\mu P_L \nu_\ell \right) \\
+ \sum_{m,m'} \epsilon^\beta_{D^*} (m) (pD)_{\beta} g_{nn'} \epsilon^\gamma_{D^*} (m') M^T_{\rho,\sigma} \\
\times \sum_{n,n'} \epsilon^\alpha_{T^*} (n') g_{n'n} \epsilon^\mu_{T^*} (n) \sum_{p,p'} \epsilon_{T^*} (p') g_{p'p} \epsilon_{T^*} (p) \left( \bar{u}_\ell \sigma_{\mu\nu} P_L \nu_\ell \right). \tag{2.13}
\]

The vector part is identical to the SM with the SM coupling replaced by possible NP couplings in the hadronic amplitudes.

As in the vector-current case, we can define hadronic amplitudes by contracting the currents with polarization vectors of the intermediate states. The scalar, vector, and tensor amplitudes are

\[
\mathcal{M}^{SP}_{(m)} (B \to D^* S^P^*) = \epsilon^\mu_{D^*} (m) M^{SP}_{\mu}, \\
\mathcal{M}^{VA}_{(m,n)} (B \to D^* V^A^*) = \epsilon^\nu_{D^*} (m) M^{VA}_{\nu} \epsilon_{VA} (n), \\
\mathcal{M}^{T}_{(m,n,p)} (B \to D^* T^*) = i \epsilon^\rho_{D^*} (m) M^T_{\rho,\sigma} \epsilon^\gamma_{T^*} (n) \epsilon^\beta_{T^*} (p). \tag{2.14}
\]

Using the above definitions we can now rewrite the total amplitude of Eq. (2.13) as

\[
\mathcal{M}^{\text{SM+NP}} \propto - \sum_{m = \pm, 0} \mathcal{H}_{D^*} (m) \left\{ \mathcal{M}^{SP}_{(m)} \mathcal{L}_{SP} + \sum_{n = t, \pm, 0} g_{nn} \mathcal{M}^{VA}_{(m,n)} \mathcal{L}_{VA} (n) \\
+ \sum_{n,p = t, \pm, 0} g_{nn} g_{pp} \mathcal{M}^{T}_{(m,n,p)} \mathcal{L}_T (n, p) \right\} \tag{2.15}
\]

where the leptonic amplitudes have been defined as

\[
\mathcal{L}_{SP} = \bar{u}_\ell P_L \nu_\ell, \\
\mathcal{L}_{VA} (n) = \epsilon_{VA} (n) \bar{u}_\ell \gamma_\mu P_L \nu_\ell, \\
\mathcal{L}_{T} (n, p) = -i \epsilon^\rho_{T^*} (n) \epsilon^\gamma_{T^*} (p) \left( \bar{u}_\ell \sigma_{\mu\nu} P_L \nu_\ell \right). \tag{2.16}
\]
Since the decaying $B$ meson is a pseudoscalar, conservation of angular momentum leads to the relationships $m = 0$ for the scalar part, $m = n$ for the vector part and $m = n + p$ for the tensor part. In addition, since the tensor current is antisymmetric under the interchange of $n$ and $p$, the amplitudes corresponding to $n = p$ automatically vanish. Thus, similar to Eq. (2.5), the non-zero helicity amplitudes in the full angular distribution are given by

\[
\begin{align*}
M_{(0)}^{SP}(B \to D^*SP^*) &= A_{SP}, \\
M_{(+;+)}^{VA}(B \to D^*VA^*) &= A_+ , \\
M_{(-;-)}^{VA}(B \to D^*VA^*) &= A_- , \\
M_{(0;0)}^{VA}(B \to D^*VA^*) &= A_0 , \\
M_{(0;0)}^{VA}(B \to D^*VA^*) &= A_t , \\
M_{(+;+,0)}^{T}(B \to D^*T^*) &= M_{(+;+,t)}^{T}(B \to D^*T^*) = A_{+,T} , \\
M_{(0;-,+)}^{T}(B \to D^*T^*) &= M_{(0,0;+)}^{T}(B \to D^*T^*) = A_{0,T} , \\
M_{(-;-,0)}^{T}(B \to D^*T^*) &= M_{(-;-,t)}^{T}(B \to D^*T^*) = A_{-,T} .
\end{align*}
\tag{2.17}
\]

The differential decay rate is proportional to the spin-summed amplitude squared. We have

\[
|M^{SM+NP}|^2 = |M_{SP}|^2 + |M_{VA}|^2 + |M_T|^2 + 2\text{Re}[M_{SP}M_{VA}^* + M_{SP}M_{T}^* + M_{VA}M_{T}^*] .
\tag{2.18}
\]

The individual terms are given by

1. \[
|M_{SP}|^2 \propto \sum_{m,m' = \pm,0} M_{(m;0;0)}^{SP} M_{(m';0;0)}^{SP*} H_D^{*}(m) H_D^{*}(m') \sum_{spins} \mathcal{L}_{SP} \mathcal{L}_{SP}^* ,
\]

\[
= |A_{SP}|^2 |H_D^{*}(0)|^2 \sum_{spins} \mathcal{L}_{SP} \mathcal{L}_{SP}^* .
\tag{2.19}
\]

2. $|M_{VA}|^2$ is given in Eq. (2.11).

3. \[
|M_T|^2 \propto \sum_{m,m' = \pm,0} (H_D^{*}(m) H_D^{*}(m')) \sum_{n,n',p,p' = t,\pm,0} g_{nn} g_{n'n'} g_{pp} g_{p'p'}
\]

\[
\times \left( M_{(m,n;p)}^{T} M_{(m'n',p')}^{T*} \right) \sum_{spins} \mathcal{L}_{T}(n,p) \mathcal{L}_{T}(n',p') .
\tag{2.20}
\]

4. \[
M_{SP}M_{VA}^* \propto \sum_{m = \pm,0} H_D^{*}(0) H_D^{*}(m) \sum_{n = t,\pm,0} g_{nn} M_{(0;0)}^{SP}
\]

\[
\times M_{(m;n)}^{VA*} \sum_{spins} \mathcal{L}_{SP} \mathcal{L}_{VA}^*(n) .
\tag{2.21}
\]
5. \[
\mathcal{M}_{SP} \mathcal{M}^*_T \propto \sum_{m=\pm,0} \mathcal{H}_{D^*}(0) \mathcal{H}_{D^*}^*(m) \sum_{n,p=l,\pm,0} g_{nn'} g_{pp'} \mathcal{M}_{SP}^{(0)}(m) \times \mathcal{M}^{T_s}_{(m,n,p)} \sum_{\text{spins}} \mathcal{L}_{SP} \mathcal{L}_T^*(n, p). \tag{2.22}
\]

6. \[
\mathcal{M}_{VA} \mathcal{M}^*_T \propto \sum_{m,m'=\pm,0} \mathcal{H}_{D^*}(m) \mathcal{H}_{D^*}^*(m') \sum_{n,n',p'=l,\pm,0} g_{nn'} g_{pp'} \mathcal{M}_{VA}^{(m,n)} \times \mathcal{M}^{T_s}_{(m',n',p')} \sum_{\text{spins}} \mathcal{L}_{VA}(n) \mathcal{L}_T^*(n', p'). \tag{2.23}
\]

The leptonic contributions to \(|\mathcal{M}^{SM+NP}|^2\) are given in the appendix, Sec. A. The expressions for the helicity amplitudes in terms of form factors are given in the appendix, Sec. B.

The relationships between amplitudes in the helicity and transversity bases are

\[
\mathcal{A}_{||,T} = (\mathcal{A}_{+,T} + \mathcal{A}_{-,T})/\sqrt{2},
\]

\[
\mathcal{A}_{\perp, T} = (\mathcal{A}_{+,T} - \mathcal{A}_{-, T})/\sqrt{2}. \tag{2.24}
\]

(A different choice for the transversity basis is used in Ref. [72]. However, one can show that the two bases are equivalent.)

2.4 Angular Distribution

In the previous subsection, we computed the square of the full amplitude for \(\bar{B} \to D^* (\to D\pi) \ell^\mp \nu_\ell\). Using Sec. B in the appendix, this can be expressed as a function of the final-state momenta. In this section, we obtain the angular distribution of the decay.

To this end, we use the formalism of helicity angles defined in the rest frames of the intermediate particles, as shown in Fig. 1. We have chosen the \(z\)-axis to align with the direction of the \(D^*\) in the rest frame of the \(B\). With this choice of alignment, the helicity angles \(\theta^*\) and \(\pi - \theta\) respectively measure the polar angles of the \(D\) and the charged lepton in the rest frames of their parent particles (\(D^*\) and \(N^*\), respectively), and \(\chi\) is the azimuthal angle between the decay planes of the two intermediate states. For the CP-conjugate decay, the helicity angles are defined in the same way. Thus, in comparing the decay and the CP-conjugate decay, \(\bar{\theta}^* = \theta^*, \bar{\theta}_\ell = \theta_\ell\), and \(\bar{\chi} = \chi\).

Using the above definitions we can express the four momenta of the \(D\) and the \(\ell^-\) in the rest frames of their respective parent particles as follows:

\[
p_D^\mu = (\bar{E}_D, |\vec{p}_D| \sin \theta^*, 0, |\vec{p}_D| \cos \theta^*),
\]

\[
p_\ell^\mu = (\bar{E}_\ell, |\vec{p}_\ell| \sin \theta_\ell \cos \chi, |\vec{p}_\ell| \sin \theta_\ell \sin \chi, -|\vec{p}_\ell| \cos \theta_\ell), \tag{2.25}
\]
where $E_X$ and $\vec{p}_X$ ($X = D, \ell$) represent the energy and the three-momentum of $X$ in its parent rest frame. The complete angular distribution can then be written as

$$
\frac{d^4\Gamma}{dq^2 \, d\cos \theta_\ell \, d\cos \theta^* \, d\chi} = \frac{3}{8\pi} \frac{G_F^2 |V_{cb}|^2 (q^2 - m_\ell^2)^2 |p_{D^*}|}{2^5 \pi^3 m_B^2 q^2} \times B(D^* \to D\pi) \left( N_1 + \frac{m_\ell}{\sqrt{q^2}} N_2 + \frac{m_\ell^2}{q^2} N_3 \right), \quad (2.26)
$$

where $q = p_\ell + \bar{p}_\ell$, and $|p_{D^*}| = \sqrt{\lambda(m_B^2, m_{D^*}^2, q^2)/(2m_B)}$, with $\lambda(a, b, c) = a^2 + b^2 + c^2 - 2ab - 2ac - 2bc$, is the 3-momentum of $D^*$ in the $B$-meson rest frame. For $N_1$, $N_2$ and $N_3$, the angular functions associated with the various (combinations of) helicity amplitudes are given in Tables 2, 3 and 4, respectively. The angular distribution derived here can be compared with that given in Ref. [51]. There are some sign differences, but these are just conventions – if everything is written in terms of form factors, the two angular distributions agree.

### 2.5 CP Violation

The components in the angular distribution that particularly interest us are those whose coefficients are $\text{Im}(A_i A_j^*)$, where $A_{i,j}$ are two different helicity amplitudes. These are the terms that are used to generate CP-violating asymmetries. Note that they are all proportional to $\sin \chi$.

Technically, these angular components are not, by themselves, CP-violating observables. Suppose that the helicity amplitudes $A_i$ and $A_j$ had the same weak phase but different strong phases. $\text{Im}(A_i A_j^*)$ would then be nonzero, but this would not indicate CP violation, since the weak-phase difference vanishes. This would be a fake signal. Suppose instead that $A_i$ and $A_j$ had the same strong phase but different weak phases. $\text{Im}(A_i A_j^*)$ would again be nonzero, and in this case it would be a true CP-violating signal. In order
Angular Function

| Amplitude in $N_1$ | Angular Function |
|-------------------|------------------|
| $|A_0|^2$          | $4\sin^2\theta_1\cos^2\theta^*$ |
| $|A_1|^2$          | $2\sin^2\theta^*(\cos^2\chi + \cos^2\theta_1\sin^2\chi)$ |
| $|A_{||,T}|^2$     | $2\sin^2\theta^*(\cos^2\theta_1\cos^2\chi + \sin^2\chi)$ |
| $|A_{\perp, T}|^2$| $32\sin^2\theta_1\sin^2\theta^*\cos^2\chi$ |
| $|A_{0,T}|^2$      | $32\sin^2\theta_1\sin^2\theta^*\sin^2\chi$ |
| $|A_{SP}|^2$       | $64\cos^2\theta_1\cos^2\theta^*$ |
| $\text{Re}(A_1A_1^*)$ | $-4\cos\theta_1\sin^2\theta^*$ |
| $\text{Re}(A_0A_0^*)$ | $-\sqrt{2}\sin 2\theta_1\sin 2\theta^*\cos\chi$ |
| $\text{Re}(A_0A_1^*)$ | $2\sqrt{2}\sin\theta_1\sin 2\theta^*\cos\chi$ |
| $\text{Re}(A_{||,T}A_{||,SP}^*)$ | $8\sqrt{2}\sin\theta_1\sin 2\theta^*\cos\chi$ |
| $\text{Re}(A_{0,T}A_{||,SP}^*)$ | $16\sqrt{2}\sin 2\theta_1\sin 2\theta^*\cos\chi$ |
| $\text{Re}(A_{0,T}A_{0,T}^*)$ | $32\cos\theta_1\cos^2\theta^*$ |
| $\text{Im}(A_{1,0}^*)$ | $-\sqrt{2}\sin 2\theta_1\sin 2\theta^*\sin\chi$ |
| $\text{Im}(A_{0}A_1^*)$ | $2\sin^2\theta_1\sin^2\theta^*\sin 2\chi$ |
| $\text{Im}(A_{SP}A_{\perp, T}^*)$ | $-8\sqrt{2}\sin\theta_1\sin 2\theta^*\sin\chi$ |
| $\text{Im}(A_{0}A_0^*)$ | $-2\sqrt{2}\sin\theta_1\sin 2\theta^*\sin\chi$ |

Table 2. Terms in the $N_1$ part of the angular distribution.

| Amplitude in $N_2$ | Angular Function |
|-------------------|------------------|
| $\text{Re}(A_0A_{0,T}^*)$ | $-32\cos^2\theta^*$ |
| $\text{Re}(A_{0,T}A_{1}^*)$ | $32\cos\theta_1\cos^2\theta^*$ |
| $\text{Re}(A_{0}A_{||,SP}^*)$ | $-8\cos\theta_1\cos^2\theta^*$ |
| $\text{Re}(A_{1}A_{||,SP}^*)$ | $8\cos^2\theta^*$ |
| $\text{Re}(A_{||,T}A_{1}^*)$ | $16\cos\theta_1\sin^2\theta^*$ |
| $\text{Re}(A_{\perp, T}A_{1}^*)$ | $16\cos\theta_1\sin^2\theta^*$ |
| $\text{Re}(A_{||,T}A_{||,T}^*)$ | $-16\sin^2\theta^*$ |
| $\text{Re}(A_{\perp, T}A_{\perp, T}^*)$ | $-16\sin^2\theta^*$ |
| $\text{Re}(A_{0}A_{\perp, T}^*)$ | $-8\sqrt{2}\sin\theta_1\sin 2\theta^*\cos\chi$ |
| $\text{Re}(A_{0,T}A_{\perp, T}^*)$ | $-8\sqrt{2}\sin\theta_1\sin 2\theta^*\cos\chi$ |
| $\text{Re}(A_{1}A_{\perp, T}^*)$ | $8\sqrt{2}\sin\theta_1\sin 2\theta^*\cos\chi$ |
| $\text{Im}(A_{0}A_{SP}^*)$ | $-2\sqrt{2}\sin\theta_1\sin 2\theta^*\sin\chi$ |
| $\text{Im}(A_{0}A_{0, T}^*)$ | $-8\sqrt{2}\sin\theta_1\sin 2\theta^*\sin\chi$ |
| $\text{Im}(A_{1}A_{1, T}^*)$ | $-8\sqrt{2}\sin\theta_1\sin 2\theta^*\sin\chi$ |
| $\text{Im}(A_{1}A_{SP}^*)$ | $-2\sqrt{2}\sin\theta_1\sin 2\theta^*\sin\chi$ |

Table 3. Terms in the $N_2$ part of the angular distribution. These are suppressed by $m_\ell/\sqrt{q^2}$.
to distinguish true and fake signals, one must compare the same quantity in the decay and the CP-conjugate decay. For a true signal, the angular component will be the same in both decays. This is because, in going from process to antiprocess, the weak phases change sign and the azimuthal angle $\chi \to -\chi$. A fake signal will be indicated if the angular component changes sign. Thus, in the general case, to obtain a true CP-violating signal, one must add the angular distributions for the decay and the CP-conjugate decay. (Even though we are adding the distributions, these are referred to as CP-violating asymmetries.) Triple-product asymmetries [73, 74] exhibit a similar behaviour. Indeed, the above angular asymmetries are a generalization of triple products.

| Amplitude in $N_3$ | Angular Function |
|---------------------|-------------------|
| $|A_t|^2$            | $4 \cos^2 \theta^*$ |
| $|A_0|^2$            | $4 \cos^2 \theta \cos^2 \theta^*$ |
| $|A_{\parallel T}|^2$ | $2 \sin^2 \theta \sin^2 \theta^* \sin^2 \chi$ |
| $|A_{\perp T}|^2$    | $2 \sin^2 \theta \sin^2 \theta^* \cos^2 \chi$ |
| $|A_{\parallel T}|^2$ | $32 \sin^2 \theta^* (\cos^2 \theta \cos^2 \chi + \sin^2 \chi)$ |
| $|A_{\perp T}|^2$    | $32 \sin^2 \theta^* (\cos^2 \chi + \cos^2 \theta \sin^2 \chi)$ |
| $|A_{0,T}|^2$        | $64 \sin^2 \theta \cos^2 \theta^*$ |
| $\Re(A_0 A_0^*)$    | $-8 \cos \theta \cos^2 \theta^*$ |
| $\Re(A_0 A_0^*)$    | $\sqrt{2} \sin 2\theta \sin 2\theta^* \cos \chi$ |
| $\Re(A_{\parallel T} A_{\parallel T}^*)$ | $2 \cos^2 \theta \sin 2\theta \sin 2\theta^* \cos \chi$ |
| $\Re(A_{\perp T} A_{\perp T}^*)$ | $-16 \sqrt{2} \sin 2\theta \sin 2\theta^* \cos \chi$ |
| $\Re(A_{0,T} A_{0,T}^*)$ | $-64 \cos \theta \sin^2 \theta^*$ |
| $\Im(A_0 A_0^*)$    | $-2 \sin^2 \theta \sin^2 \theta^* \sin 2\chi$ |
| $\Im(A_0 A_0^*)$    | $2 \sqrt{2} \sin \theta \sin 2\theta^* \sin \chi$ |
| $\Im(A_{\parallel T} A_{\parallel T}^*)$ | $2 \sqrt{2} \sin \theta \sin 2\theta \sin 2\theta^* \sin \chi$ |
| $\Im(A_{\perp T} A_{\perp T}^*)$ | $\sqrt{2} \sin 2\theta \sin 2\theta^* \sin \chi$ |

**Table 4.** Terms in the $N_3$ part of the angular distribution. These are suppressed by $m_{t}^2/q^2$.

Now, as argued in the introduction, in the case of $\bar{B} \to D^*(\to D\pi)\ell^- \bar{\nu}_\ell$, the SM and NP contributions all basically have the same strong phase. That is, there is no strong-phase difference between any pair of transversity amplitudes. In this case, the angular components whose coefficients are $\Im(A_i A_i^*)$ are signals of CP violation.

In Tables 2, 3 and 4, one finds, respectively, four, three and four of these CP-violating observables. However, one must be careful here. These do not all involve different factors of $\Im(A_i A_i^*)$ — some combinations of helicity amplitudes appear in more than one Table. Also, these observables involve only three angular functions, so there can be a number of different contributions to a single observable. In addition, the angular components listed in the three Tables are not all the same size. Compared to Table 2, the observables in Tables 3 and 4 are suppressed by $m_{t}/\sqrt{q^2}$ and $m_{t}^2/q^2$, respectively. Typically, one has $q^2 = O(m_{t}^2)$, so these suppression factors are significant. However, if the angular distribution can be measured in that region of phase space where $q^2 = O(m_{t}^2)$, useful
In this case, the measurements are dominated by the unsuppressed contributions of Table 5. 

Table 5. The CP-violating terms in the angular distribution, their corresponding NP couplings, and the angular functions to which they contribute.

| Not suppressed | Coupling | Angular Function |
|----------------|----------|------------------|
| \( \im(\mathcal{A}_L\mathcal{A}_0^\dagger) \) | \( \im[(1 + g_L + g_R)(1 + g_L - g_R)] \) | \(- \sqrt{2} \sin 2\theta_\ell \sin 2\theta^* \sin \chi \) |
| \( \im(\mathcal{A}_0\mathcal{A}_0^\dagger) \) | \( \im[(1 + g_L - g_R)(1 + g_L + g_R)] \) | \( 2 \sin^2 \theta_\ell \sin^2 \theta^* \sin 2\chi \) |
| \( \im(\mathcal{A}_{SP}\mathcal{A}_{\perp,T}^\dagger) \) | \( \im(g_P g_T^* \) | \(- 8 \sqrt{2} \sin \theta_\ell \sin 2\theta^* \sin \chi \) |
| \( \im(\mathcal{A}_0\mathcal{A}_0^\dagger) \) | \( \im[(1 + g_L - g_R)(1 + g_L + g_R)] \) | \(- 2 \sqrt{2} \sin \theta_\ell \sin 2\theta^* \sin \chi \) |

| Suppressed by \( m_\ell/\sqrt{q^2} \) | Coupling | Angular Function |
|----------------|----------|------------------|
| \( \im(\mathcal{A}_0\mathcal{A}_0^\dagger) \) | \( \im[(1 + g_L - g_R)g_T^* \) | \( 8 \sqrt{2} \sin \theta_\ell \sin 2\theta^* \sin \chi \) |
| \( \im(\mathcal{A}_0\mathcal{A}_0^\dagger) \) | \( \im[(1 + g_L - g_R)g_T^* \) | \(- 8 \sqrt{2} \sin \theta_\ell \sin 2\theta^* \sin \chi \) |
| \( \im(\mathcal{A}_0\mathcal{A}_0^\dagger) \) | \( \im[(1 + g_L + g_R)g_T^* \) | \(- 8 \sqrt{2} \sin \theta_\ell \sin 2\theta^* \sin \chi \) |
| \( \im(\mathcal{A}_0\mathcal{A}_0^\dagger) \) | \( \im[(1 + g_L + g_R)g_T^* \) | \(- 2 \sqrt{2} \sin \theta_\ell \sin 2\theta^* \sin \chi \) |

| Suppressed by \( m_\ell^2/q^2 \) | Coupling | Angular Function |
|----------------|----------|------------------|
| \( \im(\mathcal{A}_0\mathcal{A}_0^\dagger) \) | \( \im[(1 + g_L + g_R)(1 + g_L - g_R)] \) | \( -2 \sin^2 \theta_\ell \sin^2 \theta^* \sin 2\chi \) |
| \( \im(\mathcal{A}_0\mathcal{A}_0^\dagger) \) | \( \im[(1 + g_L + g_R)(1 + g_L - g_R)] \) | \( 2 \sqrt{2} \sin \theta_\ell \sin 2\theta^* \sin \chi \) |
| \( \im(\mathcal{A}_0\mathcal{A}_0^\dagger) \) | \( \im[(1 + g_L + g_R)(1 + g_L - g_R)] \) | \( 2 \sqrt{2} \sin \theta_\ell \sin 2\theta^* \sin \chi \) |

Information can be obtained from the CP-violating observables in these Tables. Finally, the helicity amplitudes all get contributions from the NP operators in Eq. (2.12), so if a particular NP operator is nonzero, several helicity amplitudes may be affected.

In Table 5 we present all the information about the CP-violating angular observables: the contributing helicity amplitudes, the angular functions, the suppression factor, and the NP couplings probed. This allows us to interpret possible future measurements.

For example, suppose that the angular distribution is measured using the full data set. In this case, the measurements are dominated by the unsuppressed contributions of Table 2. This angular distribution contains both CP-conserving and CP-violating pieces, and both can be affected by NP. We focus on the CP-violating observables of Table 5.

- Suppose that the angular distribution is found to include the component \( \sin 2\theta_\ell \sin 2\theta^* \sin \chi \). This indicates that \( \im(\mathcal{A}_\perp\mathcal{A}_0^\dagger) \neq 0 \), which implies that \( g_R \neq 0 \), and that it has a different (weak) phase than \( (1 + g_L) \). In this case, one expects to also observe nonzero coefficients for the other two angular functions in Table 5, \( \sin^2 \theta_\ell \sin^2 \theta^* \sin 2\gamma \) and \( \sin \theta_\ell \sin 2\theta^* \sin \chi \).

- The third angular function, \( \sin \theta_\ell \sin 2\theta^* \sin \chi \), receives an additional contribution from \( \im(\mathcal{A}_{SP}\mathcal{A}_{\perp,T}^\dagger) \). But if it has been established that \( g_R \neq 0 \), one cannot tell if \( g_P \) and \( g_T \) are also nonzero. This is where the CP-conserving observables come into play. From Table 2, we see that both \( |\mathcal{A}_{SP}|^2 \) and \( |\mathcal{A}_{\perp,T}|^2 \) can be determined from the angular distribution, so in principle we will know if they are nonzero (though we will have no information about their phases).

- If it is found that the coefficients of the first two angular functions are \( \sim 0 \), this implies that \( g_R \approx 0 \) (or that its phase is the same as that of \( (1 + g_L) \)). In this case,
the measurement of a nonzero coefficient of the third angular function will point clearly to \( \text{Im}(A_{SP}A^*_{TP}) \neq 0 \).

Finally, suppose that the angular analysis reveals no unsuppressed CP-violating observables. To probe other such observables, it will now be necessary to reconstruct the angular distribution for the data with \( q^2 = O(m^2) \). If this is possible, one can see if the angular function \( \sin \theta \sin 2\theta^* \sin \chi \) has a nonzero coefficient in the data suppressed by \( m_t/\sqrt{q^2} \). If it does, this indicates that \( g_T \) or \( g_P \) (or both) is nonzero. As noted above, one can perform a cross-check by measuring CP-conserving observables. In particular, from Table 2, we see that the angular distribution can give us information about new tensor and scalar interactions.

### 3 New-Physics Models

In Sec. 2.4, we derived the angular distribution for \( \bar{B} \to D^* (\to D\pi)\ell^-\bar{\nu}_\ell \) in the presence of NP. This applies to \( \ell = e, \mu, \tau \). However, in this paper we focus specifically on \( \bar{B}^0 \to D^+\mu^-\bar{\nu}_\mu \), as LHCb intends to perform a detailed angular analysis of this decay, and measure the CP-violating observables [69]. In this section, we examine the NP models that can generate nonzero CP-violating observables in \( \bar{B}^0 \to D^+\mu^-\bar{\nu}_\mu \).

In the SM, the decay \( b \to c\ell^-\bar{\nu}_\ell \) is due to the tree-level exchange of a \( W \). In order to generate a significant discrepancy with the SM, the NP contributions to this decay must also take place at tree level. There are three classes of NP models in which this can occur. The NP mediating particle can be a charged Higgs \( \tan \beta \) [22, 23, 25, 29, 35–37, 39, 41, 44–46], a \( W' \) boson [24, 28, 31–33, 38, 40, 42, 43, 47, 48], or a leptoquark (LQ) [26, 27, 30].

In Ref. [34], it was pointed out that there are important constraints on NP explanations from the \( B^-_c \) lifetime. In particular, NP models with a \( H^\pm \) are disfavoured. Below we examine whether CP-violating observables can be generated in models with a \( W'^\pm \) or a LQ. Specifically, in each NP model, we determine which of the NP parameters \( g_{L,R,S,P,T} \) [Eq. (2.12)] can be generated.

We stress that our main goal in this paper is to examine the implications of the measurement of CP-violating observables in \( \bar{B}^0 \to D^+\mu^-\bar{\nu}_\mu \). As such, these \( W'^\pm \) and LQ models are not complete. That is, there may be constraints from other measurements that are not taken into account here. For example, it was pointed out in the introduction that, because \( R_{D^0}^{\mu/e}/(R_{D^0}^{\nu/\tau})_{\text{SM}} = 1.00 \pm 0.05 \) (Table 1), any NP that contributes to \( b \to c\mu^-\bar{\nu}_\mu \) must equally affect \( b \to c\ell^-\bar{\nu}_\ell \). But it is well known that a LQ that couples to both \( \mu \) and \( \ell \) will be constrained by \( \mu \to e\gamma \) and \( b \to se\mu \) [75]. Should a CP-violating observable be measured in \( \bar{B}^0 \to D^+\mu^-\bar{\nu}_\mu \) suggesting the presence of LQs, these constraints must be taken into account at the model-building stage.

#### 3.1 \( W'^\pm \) Models

The \( W' \) is a vector boson, so it can contribute only to \( g_L \) and/or \( g_R \) of Eq. (2.12). Two classes of \( W' \) models have been proposed. In the first [28, 31–33, 38, 47], the \( W' \) is SM-like, coupling only to left-handed fermions. Thus, this \( W'_L \) contributes only to \( g_L \), which means that no CP-violating effects can be generated.
The second class uses LR models: one has a right-handed \( W'_R \), and the decay involves a sterile RH neutrino. The \( W'_R \) couples only to right-handed fermions and so contributes to neither \( g_L \) nor \( g_R \) (since these operators involve a left-handed neutrino). One can allow for NP that couples to a RH neutrino by adding the following NP operators to Eq. (2.12):

\[
\mathcal{H}'_{\text{eff}} = \frac{G_F V_{cb}}{\sqrt{2}} \left\{ \left[ g'_L \bar{c} \gamma_\mu (1 - \gamma_5) b + g'_R \bar{c} \gamma_\mu (1 + \gamma_5) b \right] \bar{\mu} \gamma^\mu (1 + \gamma_5) \nu + \left[ g_S \bar{c} b + g_P \bar{c} \gamma_\mu b \right] \bar{\mu} (1 + \gamma_5) \nu + g_T \bar{c} \sigma_{\mu\nu} (1 + \gamma_5) b \bar{\mu} \sigma_{\mu\nu} (1 + \gamma_5) \nu + h.c. \right\}. \tag{3.1}\]

Just as in Table 5, CP-violating observables can be produced due to the interference of any two of these NP operators. However, the \( W'_R \) contributes only to \( g'_R \), so that, once again, no CP-violating effects can be generated.

CP-violating observables can be generated if the \( W' \) contributes to both \( g_L \) and \( g_R \) (or \( g'_L \) and \( g'_R \) if the neutrino is RH). This can occur in the LR model if the SM \( W \) mixes with the \( W'_R \). However, constraints from \( b \to s \gamma \) force this mixing to be small, \( \lesssim O(10^{-3}) \) [40], which means that any CP-violating effects are tiny.

Thus, the only way to generate sizeable CP-violating effects is if there is a \( W'_L \) and a \( W'_R \), both with large contributions to \( b \to c \ell^- \bar{\nu} \), and there is significant mixing. Such a model has not yet been proposed, but it is a possibility.

### 3.2 Leptoquark Models

There are ten models in which the LQ couples to SM particles through dimension \( \leq 4 \) operators [76]. These include five spin-0 and five spin-1 LQs. Six of these can contribute to \( b \to c \mu^- \bar{\nu}_\mu \) [27]. Three have fermion-number-conserving couplings and three have fermion-number-violating couplings. The interaction Lagrangian that generates the contributions to \( b \to c \mu^- \bar{\nu}_\mu \) is given by

\[
\mathcal{L}^{\text{LQ}} = \mathcal{L}^{\text{LQ}}_{\bar{F}=0} + \mathcal{L}^{\text{LQ}}_{\bar{F}=-2} : \\
\mathcal{L}^{\text{LQ}}_{\bar{F}=0} = (h_{1L}^i \bar{Q}_{iL} \gamma^\mu L_{jL} + h_{1R}^i \bar{d}_{iR} \gamma^\mu \ell_{jR}) U_{1\mu} + h_{3L}^i \bar{Q}_{iL} \bar{\sigma} \gamma^\mu L_{jL} \cdot \bar{U}_{3\mu} \\
+ (h_{2L}^i \bar{u}_{iR} L_{jL} + h_{2R}^i \bar{Q}_{iL} \sigma_2 \ell_{jR}) R_2 + h.c., \\
\mathcal{L}^{\text{LQ}}_{\bar{F}=-2} = (g_{1L}^i \bar{Q}_{iL} \gamma_2 L_{jL} + g_{1R}^i \bar{d}_{iR} \ell_{jR}) S_1 + (g_{3L}^i \bar{Q}_{iL} \sigma_2 \bar{\sigma} L_{jL}) \cdot \bar{S}_3 \\
+ (g_{2L}^i \bar{d}_{iR} \gamma_\mu L_{jL} + g_{2R}^i \bar{Q}_{iL} \gamma_\mu \ell_{jR}) V_{1\mu} + h.c. \tag{3.2}\]

Here \( Q \) and \( L \) represent left-handed quark and lepton \( SU(2)_L \) doublets, respectively; \( u, d \) and \( \ell \) represent right-handed up-type quark, down-type quark and charged lepton \( SU(2)_L \) singlets, respectively. The indices \( i \) and \( j \) are the quark and lepton generations. \( \psi^c = C \bar{\psi}^T \) is a charge-conjugated field.

For all six models, we integrate out the LQ to form four-fermi operators. We then perform Fierz transformations to put these operators in the form of Eq. (2.12). In this way, we determine which LQs contribute to which \( g_{L,R,S,P,T} \) coefficients.

\[
U_1: \\
\mathcal{L}_{\text{LQ}} \supset (h_{1L}^{22} \bar{c}_L \gamma^\mu \nu_{\mu L} + h_{1L}^{32} \bar{b}_L \gamma^\mu \mu_{L} + h_{1R}^{32} \bar{b}_R \gamma^\mu \mu_{R}) U_{1\mu} + h.c. \tag{3.3}\]
Four-fermion operators:

\[ \mathcal{L}^{\text{eff}} = -\frac{1}{M_{U_1}} \left[ h_{1L}^{\alpha_1} h_{1L}^{\alpha_2} (\bar{c}_L \gamma^\mu \nu_L)(\bar{\mu}_L \gamma_\mu b_L) + h_{1R}^{\alpha_1} h_{1R}^{\alpha_2} (\bar{c}_L \gamma^\mu \nu_L)(\bar{\mu}_R \gamma_\mu b_R) \right] + \text{h.c.} \quad (3.4) \]

Fierz transformation:

\[ \mathcal{L}^{\text{eff}} = -\frac{1}{M_{U_1}} \left[ h_{1L}^{\alpha_1} h_{1L}^{\alpha_2} (\bar{c}_L \gamma^\mu \nu_L)(\bar{\mu}_L \gamma_\mu b_L) - 2h_{1L}^{\alpha_1} h_{1R}^{\alpha_2} (\bar{c}_L b_R)(\bar{\mu}_R \nu_L) \right] + \text{h.c.} \quad (3.5) \]

\[ U_3: \]

\[ \mathcal{L}_{LQ} \supset (h_{2L}^{\alpha_2} \bar{c}_L \gamma^\mu \nu_L - h_{3L}^{\alpha_2} \bar{b}_L \gamma^\mu \mu_L)U_{3\mu} + \text{h.c.} \quad (3.6) \]

Fierz transformation:

\[ \mathcal{L}^{\text{eff}} = \frac{1}{M_{U_3}^2} h_{2L}^{\alpha_2} h_{3L}^{\alpha_2} (\bar{c}_L \gamma^\mu \nu_L)(\bar{\mu}_L \gamma_\mu b_L) + \text{h.c.} \quad (3.7) \]

Fierz transformation:

\[ \mathcal{L}^{\text{eff}} = \frac{1}{M_{U_3}^2} h_{2L}^{\alpha_2} h_{3L}^{\alpha_2} (\bar{c}_L \gamma^\mu \nu_L)(\bar{\mu}_L \gamma_\mu b_L) + \text{h.c.} \quad (3.8) \]

\[ R_2: \]

\[ \mathcal{L}_{LQ} \supset (h_{2L}^{\alpha_2} \bar{c}_R \nu_L - h_{2R}^{\alpha_2} \bar{b}_L \mu_R)R_2 + \text{h.c.} \quad (3.9) \]

Fierz transformation:

\[ \mathcal{L}^{\text{eff}} = \frac{1}{M_{R_2}^2} h_{2L}^{\alpha_2} h_{2R}^{\alpha_2} (\bar{c}_R \nu_L)(\bar{\mu}_R b_L) + \text{h.c.} \quad (3.10) \]

\[ S_1: \]

\[ \mathcal{L}_{LQ} \supset (g_{1L}^{\alpha_2} \bar{c}_L \mu_L - g_{1L}^{\alpha_2} \bar{b}_L \nu_L + g_{1R}^{\alpha_2} \bar{c}_R \mu_R)S_1 + \text{h.c.} \quad (3.12) \]

\[ \text{Four-fermion operators:} \]

\[ \mathcal{L}^{\text{eff}} = \frac{1}{M_{S_1}^2} \left[ g_{1L}^{\alpha_2} g_{1L}^{\alpha_2} (\bar{c}_L \nu_L)(\bar{\mu}_L c_L) + g_{1R}^{\alpha_2} g_{1L}^{\alpha_2} (\bar{c}_R \nu_L)(\bar{\mu}_R c_R) \right] + \text{h.c.} \quad (3.13) \]

Fierz transformation:

\[ \mathcal{L}^{\text{eff}} = \frac{1}{M_{S_1}^2} \left[ 4g_{1L}^{\alpha_2} g_{1L}^{\alpha_2} (\bar{c}_L \gamma^\mu \nu_L)(\bar{\mu}_L \gamma_\mu b_L) - 4g_{1R}^{\alpha_2} g_{1L}^{\alpha_2} (\bar{c}_L b_L)(\bar{\mu}_R \nu_L) + g_{1R}^{\alpha_2} g_{1L}^{\alpha_2} (\bar{c}_R \gamma^\mu \nu_L)(\bar{\mu}_R \gamma_\mu b_L) \right] + \text{h.c.} \quad (3.14) \]
we summarize the contributions of all the LQs to the
Four-fermion operator:
\[
\mathcal{L}_{\text{eff}} = - \frac{1}{M_{S_3}^2} g_{3L}^{22} \bar{g}_L \epsilon_{\mu L} (b_L \nu_{\mu L}) + h.c.
\] (3.16)
Fierz transformation:
\[
\mathcal{L}_{\text{eff}} = - \frac{1}{2M_{S_3}^2} g_{3L}^{22} (\bar{c}_L \gamma^\mu b_L) (\bar{\mu}_L \gamma_\mu \nu_{\mu L}) + h.c.
\] (3.17)
V2:
\[
\mathcal{L}_{\text{eff}} = (g_{2L}^{32} \bar{b}_R \gamma_\mu \nu_{\mu L} + g_{2L}^{22} \bar{c}_L \gamma_\mu \nu_{\mu L}) V_2^\mu + h.c.
\] (3.18)
Fierz transformation:
\[
\mathcal{L}_{\text{eff}} = \frac{2}{M_{V_2}^2} g_{2L}^{32} (\bar{c}_L \gamma_\mu b_R) (\bar{\mu}_R \gamma_\mu \nu_{\mu L}) + h.c.
\] (3.19)
In Table 6 we summarize the contributions of all the LQs to the \(g_{L,R,S,P,T}\) coefficients of Eq. (2.12).

| Model | \(g_L\) | \(g_R\) | \(g_S\) | \(g_P\) | \(g_T\) |
|-------|--------|--------|--------|--------|--------|
| U1    | \(-b_{1L}^{22} b_{1L}^{32}\) | 0      | \(-b_{1L}^{32} b_{1L}^{12}\) | \(-b_{1L}^{32} b_{1L}^{22}\) | 0      |
| U3    | \(-b_{3L}^{22} b_{3L}^{32}\) | 0      | 0      | 0      | 0      |
| R2    | 0      | 0      | 0      | 0      | 0      |
| S1    | \(-b_{1L}^{22} b_{1L}^{22}\) | 0      | \(-b_{1L}^{22} b_{1L}^{12}\) | \(-b_{1L}^{22} b_{1L}^{22}\) | 0      |
| S3    | \(-b_{3L}^{22} b_{3L}^{22}\) | 0      | 0      | 0      | 0      |
| V2    | 0      | 0      | 0      | 0      | 0      |

Table 6. Contributions of the various LQs to the \(g_{L,R,S,P,T}\) coefficients of Eq. (2.12). All entries must be multiplied by \(1/(\sqrt{2} G_F V_{cb} M_{LQ}^2)\).

### 3.3 CP Violation

As shown in Table 5, the CP-violating observables involve any pair of \{(1+\(g_L\)), \(g_R\), \(g_P\), \(g_T\)\}. Above we have seen that the \(W^\prime\) and most LQ models contribute to \(g_L\). It must be pointed out that, in \(b \rightarrow c \mu^- \bar{\nu}_\mu\), \(g_L\) cannot be large. This is because it is the coefficient of the \((V-A) \times (V-A)\) operator \(\bar{c}_\mu (1-\gamma_5) b \bar{\mu} i \gamma^\nu (1-\gamma_5) \nu_\mu\), which is related by \(SU(2)_L \times U(1)_Y\)
to the $b \rightarrow s \mu^+ \mu^-$ operator $\bar{s} \gamma_\mu (1 - \gamma_5) b \mu \gamma_\mu (1 - \gamma_5) \mu$ [77]. In order to explain the anomalies in the $b \rightarrow s \mu^+ \mu^-$ observables, we require [78]
\[
 g_L = \frac{\alpha}{2\pi} (-0.68 \pm 0.12) = O(10^{-3}).
\] (3.21)

In $(1 + g_L)$, this is negligible.

Going beyond $g_L$, we note that $g_R$ can only be due to a $W'$, and $g_P$ and $g_T$ can only be generated in LQ models. Furthermore, not all $W'$ models lead to a nonzero $g_R$. And not all LQ models produce $g_P$ and/or $g_T$. Putting all of this together, if NP is present in $b \rightarrow c \mu^- \bar{\nu}_\mu$, we see that the measurement of CP-violating observables can give us a great deal of information as to its identity.

First of all, most NP models proposed to explain the $R_{D(\ast)}$ and $R_{J/\psi}$ experimental data contribute only to $g_L$ (in $b \rightarrow c \tau^- \bar{\nu}_\tau$). As such, they predict no CP-violating effects. Should a nonzero CP-violating observable be measured, this would rule out these models, or at least force them to be modified.

Conclusions about the type of NP present depend on which nonzero observables are measured:

- If the angular distribution is found to include the components $\sin 2 \theta_L \sin 2 \theta^* \sin \chi$ and $\sin^2 \theta_L \sin^2 \theta^* \sin 2 \chi$ (the top two entries in Table 5), this requires a nonzero $g_R$. This can only arise in a $W'$ model, and so excludes all LQ models. And note: this even excludes the standard $W'$ models, with only a $W'_L$ or a $W'_R$. In this case, an unusual model, including both $W'_L$ and $W'_R$, is required.

- If the $\sin 2 \theta_L \sin 2 \theta^* \sin \chi$ and $\sin^2 \theta_L \sin^2 \theta^* \sin 2 \chi$ components do not appear in the angular distribution, but $\sin \theta_L \sin 2 \theta^* \sin \chi$ (the third entry in Table 5) does, this indicates that $g_P$ and $g_T$ are nonzero, and that they have a relative phase. This can only occur in a model with two LQs. $g_T$ can come from a $R_2$ or $S_1$ LQ, while $g_P$ can be due to a $U_1$, $R_2$, $S_1$ or $V_2$ LQ (but the two LQs must be different).

- If none of the above three angular functions are present in the angular distribution, this implies that $g_R$ and one of $g_P$ and $g_T$ are zero (or that there is no phase difference). There can still be a CP-violating observable in the data suppressed by $m_\ell/\sqrt{q^2}$ (entries 5-8 in Table 5). If this is found to be nonzero, it does, this indicates that one of $g_T$ or $g_P$ (or both, if they have the same phase) is nonzero. The $g_P$ option is particularly interesting. The $U_1$ LQ is a very popular NP choice (for example, see Ref. [47]), and it can generate $g_P$, but not $g_T$. If this is the only nonzero CP-violating observable found, this would be strong support for the $U_1$ LQ.

- There is also information from the CP-conserving observables. The full angular distribution has components proportional to $|A_{\parallel,T}|^2$, $|A_{\perp,T}|^2$, $|A_{0,T}|^2$ and $|A_{SP}|^2$. Measurements of these quantities also gives information about which of $g_T$ and/or $g_P$ is or is not nonzero.
4 Conclusions

At the present time, the anomalies in the measurements of $R_{D^(*)}$ and $R_{J/ψ}$ suggest the presence of new physics in $b \to cτ^-\bar{ν}$ decays. A number of different NP explanations have been proposed, as well as several methods for differentiating these NP models. In this paper, we explore the possibility of using CP-violating observables to distinguish the various NP scenarios.

The angular distribution in $\bar{B}^0 \to D^{*+}(\to D^0π^+)τ^-\bar{ν}_τ$ can be used to provide CP-violating asymmetries. Now, the reconstruction of this angular distribution requires the knowledge of the 3-momentum of the $τ$. The problem here is that $\vec{p}_τ$ cannot be measured since its decay products include $ν_τ$, which is undetected. Thus, while our ultimate goal is to compute the complete angular distribution, including information related to the decay products of the $τ$, in this paper we take a first step by focusing on the decay $\bar{B}^0 \to D^{*+}μ^-\bar{ν}_μ$. Here $\vec{p}_μ$ is measurable, so the angular distribution can be constructed. In addition, NP that contributes to $b \to cτ^-\bar{ν}$ may well also affect $b \to cμ^-\bar{ν}$. Finally, LHCb has announced that it intends to measure the CP-violating angular asymmetries in $\bar{B}^0 \to D^{*+}μ^-\bar{ν}_μ$, and we want to examine what the implications of these measurements are for NP.

In the SM, the hadronic $b \to c$ current is purely LH. In the presence of NP, there can be additional contributions to this LH current, parametrized by $g_L$, as well as other Lorentz structures: RH $(g_R)$, scalar $(g_S)$, pseudoscalar $(g_P)$ and tensor $(g_T)$ currents. We compute the angular distribution of $\bar{B}^0 \to D^{*+}ℓ^-\bar{ν}_ℓ$ in terms of the helicity amplitudes $A_i$, both in the SM and with NP. We identify the CP-violating angular asymmetries, proportional to $\text{Im}[A_iA^*_j]$, and show how all CP-violating observables depend on any pair of $(1 + g_L) g_R, g_P, g_T$.

We then examine the models that contribute to $b \to cμ^-\bar{ν}_μ$. There are two classes, involving (i) a $W'$ (two types) or (ii) a LQ (six types). While most models contribute to $g_L$, $g_R$ can only arise in $W'$ models, and $g_P$ and $g_T$ can only be generated due to LQ exchange. Furthermore, not all $W'$ models lead to a nonzero $g_R$, and not all LQ models produce $g_P$ and/or $g_T$.

The most popular explanations of the B anomalies involve NP that contributes only to $g_L$. Should any nonzero CP-violating observable be measured, this would rule out these models, or at least require them to be modified. In addition, there are CP-violating asymmetries that depend on $(1 + g_L) g_R, g_P g_T, (1 + g_L + g_R) g_P$ and $(1 + g_L - g_R) g_T$ interference. By measuring all of these, along with the CP-conserving components of the angular distribution, it will be possible to distinguish the $W'$ and LQ models, and to differentiate among several LQ models.

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A. $|\mathcal{M}^{\text{SM+NP}}|^2$: leptonic contributions

1. $|\mathcal{M}_{SP}|^2$:

$$\sum_{\text{spins}} \mathcal{L}_{SP} \mathcal{L}_{SP}^* = \text{Tr}[(\bar{p}_\ell + m_\ell) P_L \bar{p}_{\nu_R} P_R], \quad (A.1)$$

where $q = p_\ell + p_{\bar{\nu}_\ell}$.

2. $|\mathcal{M}_{VA}|^2$:

$$\sum_{\text{spins}} \mathcal{L}_{VA}(n) \mathcal{L}_{VA}^*(n') = \epsilon_{VA}^\mu(n) \epsilon_{VA}^{\nu*}(n') \text{Tr} \left[ \bar{u}_\ell \gamma_\mu P_L v_{\nu_R} \bar{v}_{\nu_R} \gamma_\nu P_L u_\ell \right]. \quad (A.2)$$

3. $|\mathcal{T}|^2$:

$$\sum_{\text{spins}} \mathcal{L}_{T}(n,p) \mathcal{L}_{T}^*(n',p') = \text{Tr} \left[ (\bar{p}_\ell + m_\ell) \sigma_{\mu\nu} P_L \bar{p}_{\nu_R} \sigma_{\alpha\beta} P_R \right] \times \epsilon_{T}^\mu(n) \epsilon_{T}^{\nu*}(p) \epsilon_{T}^{\alpha*}(n') \epsilon_{T}^{\beta*}(p'). \quad (A.3)$$

4. $\mathcal{M}_{SP} \mathcal{M}_{VA}^*$:

$$\sum_{\text{spins}} \mathcal{L}_{SP} \mathcal{L}_{VA}^*(n) = \text{Tr}[(\bar{p}_\ell + m_\ell) P_L \bar{p}_{\nu_R} \gamma_\mu P_L] \epsilon_{VA}^{\mu*}(n). \quad (A.4)$$

5. $\mathcal{M}_{SP} \mathcal{M}_{T}^*$:

$$\sum_{\text{spins}} \mathcal{L}_{SP} \mathcal{L}_{T}^*(n,p) = i \text{Tr} \left[ (\bar{p}_\ell + m_\ell) P_L \bar{p}_{\nu_R} \gamma_\mu P_L \right] \epsilon_{T}^{\mu*}(n) \epsilon_{T}^{\nu*}(p). \quad (A.5)$$

6. $\mathcal{M}_{VA} \mathcal{M}_{T}^*$:

$$\sum_{\text{spins}} \mathcal{L}_{VA}(n) \mathcal{L}_{T}^*(n',p') = i \text{Tr} \left[ (\bar{p}_\ell + m_\ell) \gamma_\mu P_L \bar{p}_{\nu_R} \sigma_{\alpha\beta} P_R \right] \times \epsilon_{VA}^{\mu*}(n) \epsilon_{T}^{\alpha*}(n') \epsilon_{T}^{\beta*}(p'). \quad (A.6)$$

The CP-violating angular asymmetries that appear in Tables 2, 3 and 4 have two things in common: they are all proportional to $\sin \chi$, and their coefficients are of the form $\text{Im}(a_i a_j^*)$, $i \neq j$. These can be understood from the above traces. First, in the momenta, the only element that contains $\sin \chi$ is the $y$-component of $p_\ell$ [Eq. (2.25)]. Second, in the evaluation of the traces, some terms contain a factor $i$, so that $\text{Re}(a_i a_j^*) \propto \text{Im}(a_i a_j^*)$. These terms come in two types. (i) In the $\perp$ polarizations, the $y$-component includes an $i$ [e.g., see Eq. (2.2)], so that $p_\ell \cdot \epsilon_{N*}(n)$ contains $i \sin \chi$. (ii) Traces involving $\gamma_5$ lead to
terms of the form $i\epsilon_{\mu\nu\rho\sigma}\partial_\mu V_1^\nu\partial_\rho V_3^\sigma$, where the $V_i$ are all different and are $\in \{q, \epsilon_N, (n)\}$ (these lead to triple-product asymmetries). If $\mu = 2$, the factor $i \sin \chi$ is generated.

Eq. (A.4) contains a term of type (i) (with $N = VA$), and leads to $\text{Im}(A_{i\lambda}A_{i\lambda}^*)$. Eq. (A.5) contains a term of type (ii) (with $V_1 = q, V_2 = \epsilon_I^*(n), V_3 = \epsilon_I^*(n')$), and leads to $\text{Im}(A_{SP,A_{i\lambda}^*})$. Eq. (A.2) contains both type (i) (with $N = VA$), leading to $\text{Im}(A_{0i\lambda}A_{0i\lambda}^*)$, and type (ii) (with $V_1 = q, V_2 = \epsilon_{VA}(n), V_3 = \epsilon_I^*(n')$), leading to $\text{Im}(A_{0i\lambda}A_{i\lambda}^*)$ and $\text{Im}(A_{i\lambda}A_{i\lambda}^*)$. Eq. (A.6) contains both type (i) (with $N = T$), leading to $\text{Im}(A_{i\lambda}A_{i\lambda}^*)$ and $\text{Im}(A_{i\lambda}A_{i\lambda}^*)$, and type (ii) (with $V_1 = \epsilon_{VA}(m), V_2 = \epsilon_T^*(n), V_3 = \epsilon_T^*(n')$), leading to $\text{Im}(A_{i\lambda}A_{i\lambda}^*)$.

B Helicity amplitudes in terms of form factors

Using the definitions for the $B \to D^*$ form factors given in Refs. [27, 79], we can find the hadronic helicity amplitudes [Eq. (2.17)]:

\[
A_{SP} = -g_P \frac{\sqrt{\lambda(m_B^2, m_D^*, q^2)}}{m_b + m_c} A_0(q^2),
\]

\[
A_0 = -(1 + g_L - g_R) \frac{(m_B + m_D^*)^2 - m_B^2 - \lambda}{2m_D^* \sqrt{q^2}} A_1(q^2) + (1 + g_L - g_R) \frac{\lambda}{2m_D^* (m_B + m_D^*) \sqrt{q^2}} A_2(q^2),
\]

\[
A_\pm = (1 + g_L - g_R) (m_B + m_D^*) A_1(q^2) - (1 + g_L + g_R) \frac{\sqrt{\lambda(m_B^2, m_D^*, q^2)}}{m_B + m_D^*} V(q^2),
\]

\[
A_{0,T} = g_T \frac{1}{2m_D^*(m_B^2 - m_D^2)} \left((m_B^2 - m_D^2)(m_B^2 + 3m_D^2 - q^2)T_2(q^2) - \lambda(m_B^2, m_D^*, q^2)T_3(q^2)\right),
\]

\[
A_{\pm,T} = g_T \frac{\sqrt{\lambda(m_B^2, m_D^*, q^2)} T_1(q^2) \pm (m_B^2 - m_D^2) T_2(q^2)}{\sqrt{q^2}} (B.1)
\]

where $\lambda(a, b, c) = a^2 + b^2 + c^2 - 2ab - 2ac - 2bc$.

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