INTERACTING RARITA–SCHWINGER FIELD AND ITS SPIN-PARITY CONTENT

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Abstract

We obtain in analytical form the dressed propagator of the massive Rarita-Schwinger field and discuss its properties. The calculation of the self-energy contributions demonstrates that besides \(s = \frac{3}{2}\) component the Rarita-Schwinger field contains also two \(s = \frac{1}{2}\) components of opposite parity.

1. The vector-spinor Rarita-Schwinger field \(\Psi^\mu\) \(^[1]\) is used for description of the spin-3/2 particles in QFT. However, in addition to spin-3/2 this field contains extra spin-1/2 components and it generates the main difficulties in its description \([2, 3]\).

There are 10 components in decomposition of propagator so the construction of a dressed propagator is a rather complicated issue and its total expression is unknown up to now. Thus a practical use of \(G^{\mu\nu}\) (e.g. in case of \(\Delta(1232)\) production) needs some approximations in its description. The standard approximation \([4, 5]\) consist in a dressing the spin-3/2 components only while the rest ones can be neglected or considered as bare. Another way to take into account the spin-1/2 components is a numerical solution of the appearing system of equations \([6, 7]\).

Here we derive an analytical expression for the interacting R.–S. field’s propagator with accounting all spin components and discuss its properties. It turned out that the spin-1/2 part of the dressed propagator has rather compact form, and a crucial point for its deriving is the choosing of a suitable basis \([8]\).

2. The Dyson-Schwinger equation for the propagator of the R.–S. field has the following form

\[
G^{\mu\nu} = G_0^{\mu\nu} + G^{\mu\alpha} J^{\alpha\beta} G_0^{\beta\nu}. \tag{1}
\]

Here \(G_0^{\mu\nu}\) and \(G^{\mu\nu}\) are the free and full propagators respectively, \(J^{\mu\nu}\) is a self-energy contribution. The equation may be rewritten for inverse propagators as

\[
(G^{-1})^{\mu\nu} = (G_0^{-1})^{\mu\nu} - J^{\mu\nu}. \tag{2}
\]

If we consider the self-energy \(J^{\mu\nu}\) as a known value (so called ”rainbow” approximation), than the problem is reduced to reversing of relation \((2)\).

The most convenient basis for the spin-tensor \(S^{\mu\nu}(p)\) is constructed by combining 5 well known tensor operators \([9, 10, 4]\)

\[
(P^{3/2})^{\mu\nu} = g^{\mu\nu} - \frac{2}{3} \frac{p^\mu p^\nu}{p^2} - \frac{1}{3} \gamma^\mu \gamma^\nu + \frac{1}{3p^2} (\gamma^\mu p^\nu - \gamma^\nu p^\mu) \hat{p},
\]
\[
(P_{11}^{1/2})_{\mu\nu} = \frac{1}{3} \gamma_{\mu} \gamma_{\nu} - \frac{1}{3} \frac{p_{\mu} p_{\nu}}{p^2} - \frac{1}{3} \frac{p^2 (\gamma_{\mu} p_{\nu} - \gamma_{\nu} p_{\mu}) \hat{p}}{p^2}, \quad (P_{22}^{1/2})_{\mu\nu} = \frac{p_{\mu} p_{\nu}}{p^2},
\]
\[
(P_{21}^{1/2})_{\mu\nu} = \sqrt{\frac{3}{p^2}} \cdot \frac{1}{3 p^2} (-p_{\mu} + \gamma_{\mu} \hat{p}) \, p_{\nu}, \quad (P_{12}^{1/2})_{\mu\nu} = \sqrt{\frac{3}{p^2}} \cdot \frac{1}{3 p^2} p_{\mu} (-p_{\nu} + \gamma_{\nu} \hat{p}) \hat{p}
\]

and off-shell projection operators \( \Lambda^\pm = (1 \pm \hat{p}/\sqrt{p^2})/2 \). Ten elements of this basis look as
\[
P_1 = \Lambda^{+} P_3^{3/2}, \quad P_3 = \Lambda^{+} P_1^{1/2}, \quad P_5 = \Lambda^{+} P_2^{2/2}, \quad P_7 = \Lambda^{+} P_1^{1/2}, \quad P_9 = \Lambda^{+} P_1^{1/2},
P_2 = \Lambda^{-} P_3^{3/2}, \quad P_4 = \Lambda^{-} P_1^{1/2}, \quad P_6 = \Lambda^{-} P_2^{2/2}, \quad P_8 = \Lambda^{-} P_2^{2/2}, \quad P_{10} = \Lambda^{-} P_2^{2/2},
\]

where tensor indices are omitted. We will call (4) as the \( \Lambda \)-basis.

Decomposition of a spin-tensor in this basis has the following form:
\[
S_{\mu\nu}(p) = \sum_{i=1}^{10} P_i^{\mu\nu} \bar{S}_i(p^2).
\]

The \( \Lambda \)-basis has very simple multiplicative properties which are represented in the Table 1.

|                  | \( P_1 \) | \( P_2 \) | \( P_3 \) | \( P_4 \) | \( P_5 \) | \( P_6 \) | \( P_7 \) | \( P_8 \) | \( P_{10} \) |
|------------------|----------|----------|----------|----------|----------|----------|----------|----------|----------|
| \( P_1 \)       | 1        | 0        | 0        | 0        | 0        | 0        | 0        | 0        | 0        |
| \( P_2 \)       | 0        | 0        | 0        | 0        | 0        | 0        | 0        | 0        | 0        |
| \( P_3 \)       | 0        | 0        | 0        | 0        | 0        | 0        | 0        | 0        | 0        |
| \( P_4 \)       | 0        | 0        | 0        | 0        | 0        | 0        | 0        | 0        | 0        |
| \( P_5 \)       | 0        | 0        | 0        | 0        | 0        | 0        | 0        | 0        | 0        |
| \( P_6 \)       | 0        | 0        | 0        | 0        | 0        | 0        | 0        | 0        | 0        |
| \( P_7 \)       | 0        | 0        | 0        | 0        | 0        | 0        | 0        | 0        | 0        |
| \( P_8 \)       | 0        | 0        | 0        | 0        | 0        | 0        | 0        | 0        | 0        |
| \( P_9 \)       | 0        | 0        | 0        | 0        | 0        | 0        | 0        | 0        | 0        |
| \( P_{10} \)    | 0        | 0        | 0        | 0        | 0        | 0        | 0        | 0        | 0        |

Table 1: Multiplicative properties of the \( \Lambda \)-basis

Let us denote the inverse dressed and free propagators by \( S_{i}^{\mu\nu} \) and \( S_{0}^{\mu\nu} \) respectively. Decomposing the \( S_{i}^{\mu\nu} \), \( S_{0}^{\mu\nu} \) and \( J^{\mu\nu} \) in \( \Lambda \)-basis according to (5) we reduce the equation (2) to set of equations for the scalar coefficients
\[
\bar{S}_i(p^2) = S_{0i}(p^2) - \bar{J}_i(p^2), \quad i = 1 \ldots 10.
\]

After it the reversing of the \( S_{i}^{\mu\nu} \) leads to equations for the coefficients \( \bar{G}_i \):
\[
\left( \sum_{i=1}^{10} P_i^{\mu\nu} \cdot \bar{G}_i(p^2) \right) \cdot \left( \sum_{k=1}^{10} P_k^{\mu\nu} \cdot \bar{S}_k(p^2) \right) = \sum_{i=1}^{6} P_i^{\mu\nu},
\]

which are easy to solve due to simple multiplicative properties of \( P_i^{\mu\nu} \):
\[
\bar{G}_1 = 1/\bar{S}_1, \quad \bar{G}_3 = \bar{S}_6/\Delta_1, \quad \bar{G}_5 = \bar{S}_4/\Delta_2, \quad \bar{G}_7 = -\bar{S}_7/\Delta_1, \quad \bar{G}_9 = -\bar{S}_9/\Delta_2,
\]
\[
\bar{G}_2 = 1/\bar{S}_2, \quad \bar{G}_4 = \bar{S}_5/\Delta_2, \quad \bar{G}_6 = \bar{S}_3/\Delta_1, \quad \bar{G}_8 = -\bar{S}_8/\Delta_2, \quad \bar{G}_{10} = -\bar{S}_{10}/\Delta_1.
\]
where $\Delta_1 = \bar{S}_3 S_6 - \bar{S}_7 S_{10}$, \(\Delta_2 = \bar{S}_4 S_5 - \bar{S}_8 S_9\).

The $G_1, G_2$ terms which describe the spin-3/2 have the usual resonance form, the $G_3 - G_{10}$ terms correspond to the spin-1/2 contributions.

3. The obtained dressed propagator of the R.–S. field has rather unusual structure, so we would like to clarify its physical meaning. We suggest to consider the dressing of Dirac fermions with aim to find some analogy for R.–S. field case. The use of the projection operators $\Lambda^\pm$ is very convenient here.

3.1 The dressed fermion propagator $G(p)$ is solution of the Dyson-Schwinger equation

$$G(p) = G_0 + G \Sigma G_0, \quad (8)$$

where $G_0$ is the bare propagator and $\Sigma$ is the self-energy contribution.

Decomposition of any matrix $4 \times 4$, depending on one momentum $p$, has the form:

$$S(p) = \sum_{M=1}^{2} P_M S^M, \quad P_1 = \Lambda^+, \quad P_2 = \Lambda^- \quad (9)$$

Dyson-Schwinger equation in this basis takes the form:

$$\bar{G}^M = \bar{G}_0^M + \bar{G}^M \Sigma^M \bar{G}_0^M, \quad M = 1, 2. \quad (10)$$

Let us look at the self-energy contribution $\Sigma(p)$. As an example we will consider the dressing of baryon resonance $N'$ $(J^P = 1/2^\pm)$ due to interaction with $\pi N$ system. Interaction lagrangian is of the form

$$L_{int} = g \overline{\Psi}'(x)\gamma^5 \Psi(x) \cdot \phi(x) + h.c. \quad \text{for} \quad N' = 1/2^+ \quad (11)$$

and

$$L_{int} = g \overline{\Psi}'(x)\Psi(x) \cdot \phi(x) + h.c. \quad \text{for} \quad N' = 1/2^- \quad (12)$$

Isotopical indexes are irrelevant here and omitted.

Positive parity baryon resonance

$$\Sigma(p) = ig^2 \int \frac{d^4k}{(2\pi)^4} \frac{1}{\bar{p} + \bar{k} - m_N} \gamma^5 \frac{1}{k^2 - m^2_N} = I \cdot A(p^2) + \hat{p}B(p^2) \quad (13)$$

Let us calculate the loop discontinuity through the Landau-Cutkosky rule:

$$\Delta A = -\frac{ig^2m_N}{2(2\pi)^2} I_0, \quad \Delta B = \frac{ig^2}{2(2\pi)^2} I_0 \frac{p^2 + m^2_N - m^2_\pi}{2p^2}. \quad (14)$$

Here $I_0$ is the base integral

$$I_0 = \int d^4k \delta(k^2 - m^2_\pi) \delta((p + k)^2 - m^2_N) = \theta(p^2 - (m_N + m_\pi)^2) \frac{\pi}{2} \sqrt{\lambda(p^2, m^2_N, m^2_\pi)/(p^2)^2}.$$

and $\lambda(a, b, c) = (a - b - c)^2 - 4bc$.

Parity conservation tells us that in the transition $N'(1/2^+) \rightarrow N(1/2^+) + \pi(0^-)$ the $\pi N$ pair has the orbital momentum $l = 1$. But according to threshold quantum-mechanical
theorems, the imaginary part of a loop should behave as $q^{2l+1}$ at $q \to 0$, where $q$ is momentum of $\pi N$ pair in CMS. However, we see that this property does not hold for $A, B$ components. But calculating the imaginary part of $\Sigma^M$ components

$$ Im \bar{\Sigma}^1 = Im (A + \sqrt{p^2}B) \sim q^3, \quad Im \bar{\Sigma}^2 = Im (A - \sqrt{p^2}B) \sim q^1, $$

we can see that the $\bar{\Sigma}^1$ demonstrates the proper threshold behavior.

**Negative parity baryon resonance**

$$ \bar{\Sigma}(p) = ig^2 \int \frac{d^4k}{(2\pi)^4} \frac{1}{k + \hat{p} - m_N} \cdot \frac{1}{k^2 - m^2_{\pi}} = IA(p^2) + \hat{p}B(p^2), $$

$$ \Delta A = -i\frac{g^2 m_N}{(2\pi)^2} I_0, \quad \Delta B = -\frac{ig^2}{(2\pi)^2} \frac{p^2 + m^2_N - m^2_{\pi}}{2p^2} $$

Imaginary parts of $\bar{\Sigma}^{1,2}$ again exhibit correct threshold behavior

$$ Im \bar{\Sigma}^1 \sim q^1, \quad Im \bar{\Sigma}^2 \sim q^3. $$

The considered examples show that only $\bar{\Sigma}^1$ component, which has the pole $1/\sqrt{p^2 - m}$ demonstrates the proper parity. Another component $\Sigma^2$, which has the pole $1/(-\sqrt{p^2 - m})$, demonstrates the opposite parity (antifermion!).

3.2 Let us consider the nearest analogy to the R.–S. field: the joint dressing of two fermions of opposite parity $1/2^\pm$. We will suppose that interaction conserves the parity. Now the Dyson-Schwinger equation has the matrix form

$$ G_{ij} = (G_0)_{ij} + G_{ik}\Sigma_{kl}(G_0)_{lj}, \quad i, j, k, l = 1, 2. $$

Every element in this equation has $\gamma$-matrix indexes which are omitted. Decomposition of propagator now is of the form (compare with (9))

$$ S(p) = \sum_{M=1}^4 \mathcal{P}_M \bar{S}^M, \quad \mathcal{P}_1 = \Lambda^+, \quad \mathcal{P}_2 = \Lambda^-, \quad \mathcal{P}_3 = \Lambda^+\gamma^5, \quad \mathcal{P}_4 = \Lambda^-\gamma^5. $$

Multiplicative properties of this basis are seen from Table 2. The Dyson-Schwinger equation (17) reduces for equation on the coefficients $\bar{G}^M$:

$$ \left( \sum_{M=1}^4 \mathcal{P}_M \bar{G}^M \right) \left( \sum_{L=1}^4 \mathcal{P}_L \bar{S}^L \right) = \mathcal{P}_1 + \mathcal{P}_2, $$

where $\bar{G}_M, \bar{S}_L$ are the matrices $2 \times 2$. It leads to matrix equations:

$$ G_1S_1 + G_3S_4 = E_2, \quad G_2S_2 + G_4S_3 = E_2, $$

$$ G_1S_3 + G_3S_2 = 0, \quad G_4S_1 + G_2S_4 = 0, $$

where $E_2$ is the unit matrix $2 \times 2$. Solutions:

$$ G_1 = \left[ S_1 - S_3(S_2)^{-1}S_4 \right]^{-1}, \quad G_2 = \left[ S_2 - S_4(S_1)^{-1}S_3 \right]^{-1}, $$

$$ G_3 = -\left[ S_1 - S_3(S_2)^{-1}S_4 \right]^{-1}S_3(S_2)^{-1}, \quad G_4 = -\left[ S_2 - S_4(S_1)^{-1}S_3 \right]^{-1}S_4(S_1)^{-1}. $$
4. Comparing Tables [1] and [2] one can conclude that presence of the nilpotent operators \( P_7 - P_{10} \) in decomposition (3) is an indication for the transitions between components of different parity \( 1/2^\pm \). To make sure in this conclusion, we can calculate the R-S. self-energy. As an example we will take the standard interaction lagrangian

\[
L_{int} = g_{\pi N \Delta} \overline{\Psi}^\mu(x)(g^{\mu\nu} + a\gamma^\mu\gamma^\nu)\Psi(x) \cdot \partial_\nu \phi(x) + h.c.,
\]

(22)

where \( a \) is some arbitrary parameter.

We saw that in case of Dirac fermions the propagator decomposition in basis of projection operators demonstrates the definite parity. We can expect the similar property for R.–S. field in \( \Lambda \)-basis. Calculation of self-energy contribution [11] leads to

\[
\Delta \bar{J}_1 = \Delta J_1 + E\Delta J_2 \sim q^3, \quad \Delta \bar{J}_2 = \Delta J_1 - E\Delta J_2 \sim q^5, \quad \Delta \bar{J}_3 = \Delta J_3 + E\Delta J_4 \sim q^3, \quad \Delta \bar{J}_4 = \Delta J_3 - E\Delta J_4 \sim q,
\]

\[
\Delta \bar{J}_5 = \Delta J_5 + E\Delta J_6 \sim q, \quad \Delta \bar{J}_6 = \Delta J_5 - E\Delta J_6 \sim q^3.
\]

Such behavior indicates that the components \( \bar{J}_1, \bar{J}_2 \) exhibit the spin-parity \( 3/2^+ \), while the pairs of coefficient \( \bar{J}_3, \bar{J}_4 \) and \( \bar{J}_5, \bar{J}_6 \) correspond to \( 1/2^+, 1/2^- \) contributions respectively.

5. Thus we obtained the simple analytical expression (7) for the interacting R.–S. field propagator which accounts for all spin components. To derive it we introduced the spin-tensor basis (4) with very simple multiplicative properties.

The obtained dressed propagator (7) solves an algebraic part of the problem, the following step is renormalization. Note that the investigation of dressed propagator is the alternative for more conventional method based on equations of motion (see, e.g. Ref. [12] and references therein).

We found that the nearest analogy for dressing of the \( s = 1/2 \) sector is the joint dressing of two Dirac fermions of different parity. Some hint for such spin-parity content may be seen from algebraical properties of \( \Lambda \) basis (4) with presence on nilpotent operators. Calculation of the self-energy contributions in case of \( \Delta \) isobar confirms it: in the Rarita-Schwinger field besides the leading \( s = 3/2 \) contribution there are also two \( s = 1/2 \) components of different parity.

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References

[1] W. Rarita and J. Schwinger, Phys. Rev. 60, 61 (1941).
[2] K. Johnson and E. C. G. Sudarshan, Ann. Phys. (N.Y.) 13, 126 (1961).
[3] G. Velo and D. Zwanziger, Phys. Rev. 186, 267, 1337 (1969).
[4] V. Pascalutsa and O. Scholten, Nuc. Phys. A591, 658 (1995).
[5] S. Kondratyuk and O. Scholten, Phys. Rev. C62, 025203 (2000).
[6] C. L. Korpa, Heavy Ion Phys. 5, 77 (1997).
[7] A. N. Almaliev, I. V. Kopytin and M. A. Shehalev, J. Phys. G. 28, 233 (2002).
[8] A.E. Kaloshin and V.P. Lomov, Mod.Phys.Lett. A19, 135 (2004).
[9] P. van Nieuwenhuizen, Phys. Rep. 68, 189 (1981).
[10] M. Benmerrouche et al., Phys. Rev. C39, 2339 (1989).
[11] A.E. Kaloshin and V.P. Lomov, [hep-ph/0409052] to appear in Yad. Fiz.
[12] V. Pascalutsa and R. Timmermans, Phys.Rev. C60, 042201 (1999).