Delegated RingCT: faster anonymous transactions

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Abstract—We present a modification to RingCT protocol with stealth addresses that makes it compatible with Delegated Proof of Stake based consensus mechanisms called Delegated RingCT.

Our scheme has two building blocks: a customised version of an Integrated Signature and Encryption scheme composed of a public key encryption scheme and two signature schemes (a digital signature and a linkable ring signature); and non-interactive zero knowledge proofs. We give a description of the scheme, security proofs and a prototype implementation whose benchmarking is discussed.

Although Delegated RingCT doesn’t have the same degree of anonymity as other RingCT constructions, we argue that the benefits that the compatibility with DPoS consensus mechanisms brings constitute a reasonable trade-off for being able to develop an anonymous decentralised cryptocurrency that is faster and more scalable than existing ones.

Index Terms—Anonymity, Privacy, Monero, RingCT, Delegated Proof of Stake

I. INTRODUCTION

Bitcoin appeared in 2008 [1] and is widely considered to be the first decentralised cryptocurrency. Its ingenious design, that uses a blockchain as a distributed ledger to store the transactions that happen on the network and the Nakamoto consensus [2] (which centres around the proof-of-work mechanism and the “longest-chain-win” rule) to reach a decentralised consensus about the state of that blockchain, was revolutionary at the time and even today Bitcoin is the most well known and most valuable cryptocurrency.

Since then, the industry has grown and the term cryptocurrency isn’t solely a synonym of currency anymore, but has extended to other use cases (e.g. smart contracts). Still, more than ten years later, we still don’t have a cryptocurrency that is widely used as a currency, instead of just a speculative asset, as Bitcoin was supposed to be per the title of its original paper: a peer-to-peer electronic cash system.

One can argue that this is due to external factors, such as government regulations, lack of knowledge or necessity by societies, ideological motives, etc. But we can also argue that the intrinsic technical limitations of current cryptocurrencies, due to their design, have contributed to this situation. These design flaws include the inability to scale, insufficient maximum throughput, slow confirmation times, ledger size or lack of anonymity.

A. Motivation

In our opinion, the ideal cryptocurrency is decentralised, fast, scalable, anonymous, has a transparent monetary policy and is environmentally friendly. Many cryptocurrencies have been created in the last few years that have tried to fulfil these goals but so far none of them has been able to reach them all.

Some, like Monero [3] and ZCash [4], solve the anonymity issue but still share the same other limitations of Bitcoin. Other cryptocurrencies based on Delegated Proof of Stake (DPoS), like Tezos [5] (Liquid Proof of Stake) and Nano [6] (Open Representative Voting), improve on the maximum throughput and slow confirmation times, but still are only pseudo-anonymous, meaning that anonymity is only maintained as long as a node on the network is not associated to a "real world" identity.

These consensus mechanisms are typically faster than others that use hashrate power competition to select the node that proposes the new transactions, like Bitcoin, allowing for a greater throughput of transactions, and have a much lower carbon footprint.

The goal of this paper is to bring together some of the strengths of these designs and develop a protocol that can be used as a building block for a cryptocurrency with the properties mentioned above, specifically an anonymous decentralised cryptocurrency that is faster and more scalable than the current ones.

B. Contributions

We present an extension to the base protocol of Monero, RingCT with stealth addresses, that makes it compatible with Delegated Proof of Stake, a family of consensus mechanisms where the weight of a node in the consensus for validating transactions is proportional to its delegated stake on the network, called Delegated RingCT.

We first present a generic version of Delegated RingCT constructed from two cryptographic primitives: a customised version of an Integrated Signatures and Encryption scheme (ISE) [7], which is composed of a public key encryption scheme (PKE) and two signature schemes, a digital signature (DS) [8] and a linkable ring signature (LRS) [9]; and non-interactive zero knowledge proofs (NIZK).

We, then, give a concrete efficient instantiation of Delegated RingCT and a prototype implementation whose benchmarking
shows that the scheme can be used to build a faster and more scalable anonymous decentralised cryptocurrency.

Our scheme has some limitations and, despite being secure according to our security model, our definition is somewhat weaker than other RingCT constructions. We argue that the benefits outweigh the cons, as we will discuss later.

C. Overview and Intuition

For completeness, we give a brief summary of the RingCT protocol and the DPoS based consensus mechanisms. We explain the reasoning behind our modifications to combine the two and construct Delegated RingCT.

On a basic level, a transaction has a sender, a receiver and a transferred amount. To achieve anonymity of all three components, RingCT protocol uses the following:

- **Linkable ring signature** to obfuscate the real sender of a transaction within a ring of possible senders and the linkability to detect double spends, since each transaction must have a unique linkable tag (also called key image or serial number).
- **Confidential transactions** to obfuscate the transaction amount, typically using an additive homomorphic commitment scheme like Pedersen commitments [10]. These are used to make a range proof, proving that the balance lies within a certain range, and a balance proof, proving that the total balance of the input accounts spent is equal to the total amount of the created output accounts.
- **Stealth addresses** to obfuscate the receiver of a transaction. Every node has a pair of long term keys (a long term public key and a long term secret key) and every transaction as a one-time pair of keys (a one-time public key and a one-time secret key). The sender can derive a one-time public key and a public auxiliary information from the receiver’s long term public key. The receiver can recover the one-time secret key of the account created in the transaction using his long term secret key and the auxiliary information.

Delegated Proof of Stake is a family of consensus mechanisms that is based on two basic concepts:

- The weight that a node has on the consensus of the network is proportional to his stake (balance) on the network.
- The stake of a node can be delegated to another node, transferring its weight on the consensus to that node.

With this in mind, we first need to introduce the concept of stake delegation in RingCT. We follow the terms used in Nano [6], and call **representative** a node to whom has been assigned some stake by another node. This concept is distinct from the **owner** of an account, but they can be the same.

We do this by switching the representation of a coin in RingCT from a commitment of an additive homomorphic commitment scheme (e.g., Pedersen commitments [10]) to a ciphertext of an additive homomorphic public encryption scheme (e.g., exponential ElGamal [11]). This allows the owner of an account to encrypt its balance with a public key of another node, making him the representative of that account. The representative can then prove by decryption that a certain amount of stake was delegated to him using his long term secret key. A NIZK proof is needed to prove that the encryption is well formed.

Since the PKE is additive homomorphic and the consensus algorithm only needs to know the total amount of delegated stake to a given node, the representative reveals the total amount of delegated balance to him without revealing the individual balances of the accounts that delegated to him, otherwise there would be no obfuscation of balances.

A node can redelegate the stake of an account at anytime if he knows the one-time secret key of that account (digital signature), by reencrypting the balance with a new public key and proving with a NIZK that both ciphertexts are equivalent, i.e., encrypt the same balance.

Transactions are modified from standard ring confidential transactions and because of that, the ciphertext needs to be compatible with the range proof protocol and the balance proof. The linkable ring signatures and the stealth addresses remain the same.

We construct a customised version of an Integrated Signature and Encryption scheme (ISE) to express the fact that the same keys are used for the encryption/decryption, the digital signature and the linkable ring signature.

D. Related Work

Since Monero and ZCash are the two most valuable anonymous cryptocurrencies by market capitalisation, research in this area is mostly divided in two, based on the two different technologies they use.

The base protocol used by Monero was first described in [3]. Since then, it has evolved into a ring of confidential transactions, which combines linkable ring signatures [9], [12] with confidential transactions [13]. [14] gives the first formal syntax of RingCT and improves it with a new version. [15]–[18] improve on the size and the efficiency of the linkable ring signature component and [19] on the range proof. Compatibility with smart-contracts was achieved in [20].

ZCash uses zero-knowledge Succinct Non-interactive Arguments of Knowledge (zk-SNARKs) to construct a decentralised payment scheme [4], but requires a trusted setup. Since then, other zero-knowledge proofs for arithmetic circuits were developed by improving on efficiency, decreasing the amount of "trust" required or increasing the scope of use [21]–[25].

Another relevant approach to anonymous cryptocurrencies is Zerocoin protocol [26], improved in [27].

II. Preliminaries

A. Basic Notation

We use additive notation and define $\mathbb{G}$ as a cyclic group of prime order $p$ in which the discrete logarithm problem is hard and $\mathbb{F}$ as the scalar field of $\mathbb{G}$.

Let $H : 0, 1^* \rightarrow \mathbb{F}$ be a cryptographic hash function. Let $G$ and $H$ be generators of $\mathbb{G}$ with unknown discrete logarithm
relationship and let $N = n^m$ be a size parameter where $n > 1$ and $m > 1$.

A function is negligible in the security parameter $\lambda$, written $\text{negl}(\lambda)$, if it vanishes faster than the inverse of any polynomial in $\lambda$. A probabilistic polynomial time (PPT) algorithm is a randomised algorithm that runs in time poly$(\lambda)$.

In a randomised algorithm $A$, the input randomness $r \in \mathbb{F}$ is explicit and we write $z \leftarrow A(x_1, \ldots, x_n; r)$. We use $x \xleftarrow{} \mathcal{D}$ to denote sampling $x$ uniformly at random from $\mathcal{D}$. For readability we denote the set of elements $\{x_n\}_{n=0}^{N-1}$ by just $x_n$.

### B. Integrated Signatures and Encryption Scheme

The concept of combining public key schemes was first introduced by [7], [28], inspired by [29], combines one signature scheme with two signatures schemes, a public key encryption scheme with two signatures schemes, and "1" if they were signed using the same private key. Since each component being individually secure doesn't imply that the composition of all the components is also secure, we need to have a joint security model, i.e., a model that evaluates the security of each component in the presence of the others, which are simulated by oracles. The only component that doesn’t need to be simulated by an oracle in the public key setting is the PKE, since an adversary can easily do it.

**Definition** (Joint Security for ISE). We say an ISE is jointly secure if:

- its PKE component is IND-CPA secure (1-plaintext/2-receipt) in the presence of two signing oracles, one for the DS and the other for the LRS.
- its DS component is EUF-CMA secure in the presence of a signing oracle simulating the LRS component.
- its LRS component is secure, following the security model of [18], in the presence of a signing oracle simulating the DS component.

### C. Non-Interactive Zero-Knowledge Proof

A NIZK proof system in the CRS model consists of the following four PPT algorithms [30]:

- $pp \leftarrow \text{Setup}(\lambda)$: on input a security parameter $\lambda$, outputs public parameters $pp$.
- $\text{KeyGen}$: this algorithm is divided in three steps to capture the concept of **stealth addresses**, in the following way.
  - $(\text{ltpk}, \text{ltsk}) \leftarrow \text{LongTermKeyGen}(pp)$. On input public parameters $pp$, it randomly generates a keypair $(\text{ltpk}, \text{ltsk})$.
  - $(\text{pk}, \text{aux}) \leftarrow \text{OneTimePKGen}(\text{ltpk}; \lambda)$. On input a long term public key $\text{ltpk}$, it outputs a random one-time public key $\text{pk}$ and the auxiliary information $\text{aux}$.
  - $(\text{sk}, \text{aux}) \leftarrow \text{OneTimeSKGen}(\text{pk}, \text{aux}, \text{ltsk})$. On input a one-time public key $\text{pk}$, an auxiliary information $\text{aux}$ and a long term secret key $\text{ltsk}$, it outputs the one-time secret key $\text{sk}$ if $\text{ltsk}$ is valid. If not, it returns $\perp$.
- $c \leftarrow \text{Encrypt}(\text{pk}, m; r)$: on input a public key $\text{pk}$ and a plaintext $m$, it outputs a random ciphertext $c$.
- $m \leftarrow \text{Decrypt}((\text{sk}, c)$: on input a secret key $\text{sk}$ and a ciphertext $c$, it outputs a plaintext $m$.
- $\sigma \leftarrow \text{Sign}(\text{sk}, m)$: on input $\text{sk}$ and a message $m$, output a signature $\sigma$.
- $0/1 \leftarrow \text{Verify}(\text{pk}, m, \sigma)$: on input $\text{pk}$, a message $m$, and a signature $\sigma$, output “1” if the signature is valid and “0” if it is not.
- $\sigma_{\text{ring}} \leftarrow \text{Sign}_{\text{ring}}((\text{sk}, m; R))$: Generates a linkable ring signature $\sigma_{\text{ring}}$ on a message $m$ with respect to a ring $R$ of one-time public keys generated by KeyGen, provided that $\text{sk}$ is a one-time secret key corresponding to some $\text{pk}$ in the ring.
- $0/1 \leftarrow \text{Verify}_{\text{ring}}(\sigma_{\text{ring}}, m; R)$: Verifies a signature $\sigma$ on a message $m$ with respect to a ring of public keys $R$. Outputs “0” if the signature is rejected, and “1” if accepted.
- $0/1 \leftarrow \text{Link}(\sigma, \sigma')$: Determines if signatures $\sigma$ and $\sigma'$ were signed using the same private key. Outputs “0” if the signatures were signed using different private keys and “1” if they were signed using the same private key.

We begin by describing the data structures used by a Delegated RingCT system.

**Blockchain.** A Delegated RingCT protocol operates on top of a blockchain $B$, which is a publicly accessible and append-only database. At any given time $t$, all users have access to $B_t$, which is a sequence of transactions. If $t < t'$, state of $B_t$ is anterior to the state of $B_{t'}$.

**Public parameters.** A trusted party generates the public parameters $pp$, which are used by the protocol’s algorithms. These include the group in which the algorithms perform operations, generators of the group, cryptographic hash functions and parameters regarding transactions, namely:

- $V$, which specifies the maximum possible number of coins that the protocol can handle. Any balance and transfer must lie in the integer interval $V = [0, v_{\text{max}}]$.
- $N$, the maximum size of the ring used in a delegated ring confidential transaction DRingCTx, i.e., the maximum number of input accounts.
- $M$, the maximum number of spend accounts in a DRingCTx, such that $M \subset N$.
- $T$, the maximum number of output accounts of a DRingCTx.
Keys. There are two pairs of keys: long term keys, which are composed of a long term public key \( ltpk \) and a long term secret key \( ltsk \) and are associated with a unique node on the network; and one-time keys, which are composed of a one-time public key \( pk \) and a one-time secret key \( sk \) and are associated with a unique account. One-time keys are derived from long term keys and one node on the network can have multiple accounts.

Account. Each account is associated with a one-time key-pair \((pk, sk)\) and a coin \( c \), which is a ciphertext of an additive homomorphic PKE scheme and encrysts an amount/balance \( a \) with randomness \( k \), also known as coin key. The one-time public key \( pk \) acts as a "stealth address" and can only receive one transaction. The secret key \( sk \) is kept privately and is used to spend once the balance of the account.

Delegated ring confidential transaction. A delegated ring confidential transaction \( DRingCTx \) consists of a ring of input accounts, a linkable ring signature, output accounts with zero-knowledge proofs of encryption and range, and an auxiliary information to help the owners of the destination addresses recover the one-time secret keys of the output accounts. Typically, one of the output accounts belongs to the sender and its balance is the "change" of a transaction.

Change representative transaction. A change representative transaction \( CRx \) consists of a digital signature based on the one-time secret key of the account and a zero-knowledge proof of equivalence of the old ciphertext \( c \) and the new ciphertext \( c' \), assuring that the balance is the same.

B. Algorithms

A Delegated RingCT scheme is a tuple of polynomial-time algorithms defined as below:

- \( pp \leftarrow \text{Setup}(\lambda) \): on input a security parameter \( \lambda \), output public parameters \( pp \).
- \((act, aux) \leftarrow \text{CreateAccount}(a, ltpk)\): on input an amount \( a \) and a long term public key \( ltpk \), outputs an auxiliary information \( aux \) and an account \( act = (pk, c) \), composed of a one time public key \( pk \) and a coin \( c \).
- \( a \leftarrow \text{RevealBalance}(sk, c)\): on input a secret key \( sk \) and a coin \( c \), outputs the balance \( a \) in plaintext. This algorithm is used by the representative of an account.
- \( DRingCTx = (act_n, \sigma_{ring}, aux, act_t, \pi_{range}, \pi_{enc}, \pi_{bal}) \) \( \perp \leftarrow \text{CreateDRingCTx}(act_n, ask_m, a_t, ltpk_i) \): on input a ring of \( N \) input accounts, \( M \) account secret keys corresponding to some of those accounts, \( T \) output accounts and \( T \) destination addresses, it outputs the input accounts \( act_n \), a linkable ring signature \( \sigma_{ring} \), auxiliary information \( aux \), \( T \) output accounts, a range proof \( \pi_{range} \), an encryption proof \( \pi_{enc} \) and a balance proof \( \pi_{bal} \), if the accounts secret keys are valid. It outputs \( \perp \) otherwise.
- \( 0/1/\perp \leftarrow \text{VerifyCTx}(DRingCTx)\): on input a delegated ring confidential transaction \( DRingCTx \), outputs "0" if the transaction is valid, "1" if it is invalid and "\( \perp \)" if the transaction is linked to a previous valid transaction, i.e., if any of \( act_n \) has been spent previously. If \( DRingCTx \) is valid, it is recorded on the blockchain \( B \). Otherwise, it is discarded.

- \( CRx = (\sigma, act', \pi_{equal}) \) \( \perp \leftarrow \text{CreateCRx}(ask, act, pk')\): on input an account \( act \) with the corresponding account secret key \( ask \) and a one-time public key \( pk' \), it outputs a digital signature \( \sigma \), a new account \( act' \) with the same amount \( a \) encrypted with the new \( pk' \) and a proof of equivalence \( \pi_{equal} \), if the account secret keys is valid. It outputs \( \perp \) otherwise.
- \( 0/1 \leftarrow \text{VerifyCRx}(CRx) \): on input a change representative transaction \( CRx \) and a digital signature \( \sigma \), it outputs "0" if its invalid. Otherwise, it outputs "1" and the \( CRx \) transaction is appended to the blockchain \( B \).

C. Correctness

Correctness of Delegated RingCT requires that:

- A valid delegated ring confidential transaction \( DRingCTx \) will always be accepted and recorded on the blockchain \( B \).
- A valid change representative transaction \( CRx \) will always be accepted and recorded on the blockchain \( B \).

D. Security Model

We focus only on the transaction layer of a cryptocurrency, and assume that network-level and consensus-level attacks are out of scope.

Intuitively, a Delegated RingCT protocol should have the following properties.

Unforgeability. This property captures the idea that only someone who knows the account secret key of an account can spend it and change its representative.

Anonymity. This property captures the idea that an outside party, other than the owner or the representative of the sender account, can’t know who is the real sender, who is the receiver or what is the amount of a \( DRingCTx \) transaction.

Linkability. This property captures the idea that you can only spend once from an account, i.e., you can’t double spend.

Non-frameability. This property captures the idea that a malicious party can’t construct a transaction that invalidates a valid transaction.

We formalise the above intuitions into a game-based security model between an adversary \( \mathcal{A} \) and a challenger \( \mathcal{CH} \). The capabilities of the adversary are modelled by the queries that he can make to oracles implemented by the challenger, which are described bellow.

- \( ltpk_i \leftarrow \text{KeyOracle}(i) \): on the \( i \)th query, the challenger \( \mathcal{CH} \) runs \( (ltsk_i, ltpk_i) \leftarrow \text{ISE.LongTermKeyGen}(pp, is_e) \) and returns \( ltpk_i \) to \( \mathcal{A} \).
- \( ltsk_i \leftarrow \text{CorruptOracle}(ltpk_i) \): on input a long term public key \( ltpk \), that corresponds to a query to KeyOracle, it returns the associated long term secret key \( ltsk \).
- \( a_t \leftarrow \text{AccountOracle}(ltpk_i, a) \): the challenger \( \mathcal{CH} \) runs \( a_t \leftarrow \text{CreateAccount}(a, ltpk_i) \) if \( ltpk_i \) was generated by a query to KeyOracle. Returns the \( a_t \) to \( \mathcal{A} \).
- \( DRingCTx \leftarrow \text{TransOracle}(act_n, act_m, a_t, ltpk_i) \): on input a set of \( N \) input accounts, \( M \) spend accounts, \( T \) amounts and \( T \) destination addresses, \( \mathcal{CH} \) runs
DRingCTx ← CreateDRingCTx(\(act_n\), \(ask_m\), \(a_t\), \(ltpk_i\)) and returns DRingCTx to \(\mathcal{A}\).

- CRx ← ChangeOracle(\(act_{t}\), \(ltpk_i\)): on input an account \(act_{t}\) and a \(ltpk_i\), it runs CRx ← CreateCRx(\(ask_{t}\), \(act_{t}\), \(ltpk_i\)) and returns CRx to \(\mathcal{A}\).

**Definition 1.** (Unforgeability) This property requires that:
- The probability of an adversary being able to forge a valid ring confidential transaction without knowing any secret key of the public keys of the ring, is negligible. This part of the property is captured by the following game between a challenger \(\mathcal{CH}\) and a probabilistic polynomial-time adversary \(\mathcal{A}\):
  - The adversary \(\mathcal{A}\) can query all oracles and outputs DRingCTx = (\(act_n\), \(\sigma_{ring}\), \(aux\), \(act_{t}\), \(\pi_{range}\), \(\pi_{enc}\), \(\pi_{bal}\)), such that all of the \(N\) input accounts were generated by queries to AccountOracle and none was used as input to CorruptOracle or TransOracle.
  - \(\mathcal{A}\) wins if \(\Pr[\text{VerifyDRingCTx}(\text{DRingCTx}) = 1] \geq \text{negl}(\lambda)\).

The probability of an adversary being able to forge a valid change representative transaction for an account without knowing its secret key, is negligible. This part of the property is captured by the following game between a challenger \(\mathcal{CH}\) and a probabilistic polynomial-time adversary \(\mathcal{A}\):
- The adversary \(\mathcal{A}\) can query all oracles and outputs CRx = (\(\sigma\), \(act_{t}\), \(\pi_{equal}\)), such that ChangeOracle was not queried with (\(act_{t}, \cdot\)) and \(act_{t}\) wasn’t corrupted by CorruptOracle.
- \(\mathcal{A}\) wins if \(\Pr[\text{VerifyCRx}(CRx) = 1] = 1 \geq \text{negl}(\lambda)\).

If \(\mathcal{A}\) wins both previous games, we say that our scheme is unforgeable.

**Definition 2.** (Anonymity) Anonymity requires that as long as a ring contains two uncorrupted input accounts, an adversary can do no better than guessing at determining the sender of a valid transaction. This property is captured by the following game between a challenger \(\mathcal{CH}\) and a probabilistic polynomial-time adversary \(\mathcal{A}\):
- \(\mathcal{A}\) has access to all oracles. He chooses a ring of input accounts \(act_{n}\), where all the accounts are generated by AccountOracle and at least two of them were not corrupted by CorruptOracle. The challenger \(\mathcal{CH}\) picks an account \(act_{n}\) out of the uncorrupted accounts to be the real sender of the transaction and outputs a DRingCTx. \(\mathcal{A}\) tries to guess which account is the real sender with an index \(n\) and wins if \(|\Pr[n = a^*] - \frac{1}{\lambda(n)}| \geq \text{negl}(\lambda)\), where \(c\) is the number of corrupted accounts in the ring.

**Definition 3.** (Linkability) Linkability requires that an adversary be unable to produce \(N + 1\) non-linked valid transactions from a ring of \(N\) input public keys. This property is captured by the following game between a challenger \(\mathcal{CH}\) and a probabilistic polynomial-time adversary \(\mathcal{A}\):
- \(\mathcal{A}\) has access to all oracles. He chooses \(N\) input accounts and produces \(N + 1\) DRingCTx, which are sent to the challenger. \(\mathcal{A}\) wins if all the \(N + 1\) delegated ring confidential transactions verify with non-negligible probability, i.e.: \(\Pr[\text{VerifyDRingCTx}(\text{DRingCTx}_{a}) = 1] \geq \text{negl}(\lambda)\), where \(n \in [1, N + 1]\).

**Definition 4.** (Non-frameability/Non-slanderability) This property requires that an adversary is unable to generate a valid transaction that links with another previous valid transaction that was generated honestly. This property is captured by the following game between a challenger \(\mathcal{CH}\) and a probabilistic polynomial-time adversary \(\mathcal{A}\):
- \(\mathcal{A}\) has access to all oracles. He chooses \(N\) input accounts and outputs a delegated DRingCTx, such that the input accounts weren’t used as inputs to the TransOracle. \(\mathcal{A}\) wins if DRingCTx links to any of the previous valid transactions that were generated by queries to TransOracle with non-negligible probability, i.e.: \(\Pr[\text{VerifyDRingCTx}(\text{DRingCTx}) = -1] \geq \text{negl}(\lambda)\).

IV. DELEGATED RINGCT PROTOCOL

**A. A Generic Construction**

We present a generic construction of Delegated RingCT from ISE and NIZK, in the following way:
- Let \(\text{ISE} = (\text{Setup}, \text{KeyGen}, \text{Sign}, \text{Sign}_{\text{ring}}, \text{Verify}, \text{Verify}_{\text{ring}}, \text{Enc}, \text{Dec})\) be an ISE scheme whose PKE is additively homomorphic and used to encrypt the balance of an account. The LRS component is used to authenticate a DRingCTx and the DS component to authenticate a CRx.
- Let \(\text{NIZK}_{\text{correct}} = (\text{Setup}, \text{CRSGen}, \text{Prove}, \text{Verify})\) be a NIZK proof system for \(L_{\text{correct}}\). It is used to construct a valid DRingCTx, is composed of:

\[
L_{\text{enc}} = \{(pk, c) | \exists a, k \text{ s.t. } c = \text{ISE.Enc}(pk, a; k)\}
\]

\[
L_{\text{range}} = \{c | \exists a \text{ s.t. } a \in V\}
\]

\[
L_{\text{bal}} = \{(c_m, c_l) | \exists a_m, a_l \text{ s.t. } \sum_{m=0}^{M-1} a_m = \sum_{l=0}^{T-1} a_l\}
\]

- Let \(\text{NIZK}_{\text{equal}} = (\text{Setup}, \text{CRSGen}, \text{Prove}, \text{Verify})\) be a NIZK proof system for \(L_{\text{equal}}\). It is used to make a valid CRx:

\[
L_{\text{equal}} = \{(pk_1, pk_2, c_1, c_2) | \exists a_1, a_2 \text{ s.t. } a_1 = a_2\}
\]

A Delegated RingCT construction is composed of the following algorithms:
- \(pp \leftarrow \text{Setup}(\lambda)\): on input a security parameter \(\lambda\), it runs \(pp_{\text{ise}} \leftarrow \text{ISE.Setup}(\lambda)\), \(pp_{\text{nizk}} \leftarrow \text{NIZK.Setup}(\lambda)\), \(crs \leftarrow \text{NIZK.CRSGen}(pp_{\text{nizk}})\), outputs \(pp = (pp_{\text{ise}}, pp_{\text{nizk}}, crs)\).
- \((act, aux) \leftarrow \text{CreateAccount}(a, ltpk)\). On input a one-time public key \(ltpk\) and an amount \(a\), it runs \((pk, aux) \leftarrow \text{ISE.KeyGen.OneTimePKGGen}(ltpk; r)\) and computes
the coin \( c \leftarrow \text{ISE.Enc}(pk, a; k) \). It outputs the account \( \text{act} = (pk, c) \) and the auxiliary information \( \text{aux} \).

- \( \text{CRx} = (σ, \pi, \text{equal})/ \perp \leftarrow \text{CreateCRx}(\text{act}, sk, \text{ltpk}) \). On input an account \( \text{act} \), the corresponding account secret key \( \text{ask} \) and the long term public key \( \text{ltpk} \) as the new representative, it outputs:
  - \( σ \leftarrow \text{ISE.Sign}(sk) \), which outputs a digital signature proving the knowledge of \( sk \).
  - \( \pi, \text{equal} \leftarrow \text{NIZK.Prove} \), which outputs a proof of equivalence of ciphertexts.
  - it returns \((σ, π, \text{equal})\).

- \( 1/0 \leftarrow \text{VerifyCRx}(\text{CRx}) = (σ, \pi, \text{equal}) \).
  - check if \( \text{ISE.Verify}(σ) = 1 \) and \( \text{NIZK.Verify}(π) = 1 \).
  - if both the above tests pass, return 1.
  - else, return 0.

- \( a \leftarrow \text{RevealBalance}(sk, c) \). On input a ciphertext \( c \) and a long time secret key \( \text{ltsk} \), it runs:
  - \( a \leftarrow \text{ISE.Decrypt}(sk, c) \).

- \( \text{DRingCTx} = (r, \text{aux}, act, \text{range}, \pi, \text{enc}, \pi, \text{bal}) \leftarrow \text{CreateDRingCTx}(act, \text{ask}, \text{ltsk}, \text{ltpk}) \). On input a ring of \( N \) accounts, \( M \) spend accounts, \( T \) amounts and \( T \) long term public keys, it creates a ring confidential transaction via the following steps:
  - run \((act, \text{aux}) \leftarrow \text{CreateAccount}(act, \text{ltpk})\) to generate the output accounts.
  - run \( π, \text{correct} \leftarrow \text{NIZK.Prove} \) for all output accounts to generate a zero-knowledge proof \( π, \text{correct} \) for \( L, \text{correct} \).
  - compute \( σ, \text{ring} \leftarrow \text{ISE.Sign}(σ, \text{ring}) \).
  - output the delegated ring confidential transaction \( \text{DRingCTx} \).

- \( 1/0/ − 1 \leftarrow \text{VerifyDRingCTx}(\text{DRingCTx}, act) \). On input a \( \text{DRingCTx} \) and the corresponding ring of input accounts, it outputs “1” if both the above algorithms output “1”. It outputs “−1” if \( \text{ISE.Link} \) outputs “1” and “0” otherwise.
  - \text{ISE.Verify}(\( σ, m, pk_n \)): on input a \( \text{LRS} \) \( σ, m \), \( N \) input public keys, it outputs “1” if the signature is valid and “0” if it is invalid.
  - \( \text{NIZK.Prove} \) on input \( crs, x, \pi, \text{correct} \): on input \( crs, x \) and a proof \( \pi, \text{correct} \), it outputs “1” if the proof is valid and “0” if it is invalid.

1) Analysis: Correctness of our generic Delegated RingCT construction follows from the correctness of ISE and the completeness of all the NIZK used and security is captured by the following theorem and lemmas.

**Theorem 1.** Assuming ISE’s security and the zero-knowledge property of NIZK, our Delegated RingCT construction is secure.

**Proof.** We prove this theorem via the following four lemmas.

**Lemma 1.** Assuming the unforgeability property of ISE’s linkable ring signature component, the zero-knowledge property of NIZK and the EUF-CMA security of ISE’s digital signature component, our Delegated RingCT construction satisfies unforgeability.

**Lemma 2.** Assuming the anonymity property of ISE’s linkable ring signature component and the zero-knowledge property of NIZK, our Delegated RingCT construction satisfies anonymity.

**Lemma 3.** Assuming the linkability property of ISE’s linkable ring signature and the zero-knowledge property of NIZK, our Delegated RingCT construction satisfies linkability.

**Lemma 4.** Assuming non-frameability property of ISE’s linkable ring signature and the zero-knowledge property of NIZK, our Delegated RingCT construction satisfies non-frameability.

We present the security proofs for the above lemmas in appendix A and we give an intuition for them here.

As defined in our security model, our Delegated RingCT is secure if it satisfies four properties: unforgeability, anonymity, linkability and non-frameability.

In order to prove that the scheme satisfies each one of these properties, we assume the security of ISE and the zero-knowledge of NIZK. We then simulate a game in which an adversary \( A \) tries to break the property in question. This game can be simulated by another adversary \( B \), which acts as the challenger in \( A \)’s game.

\( B \) himself wants to break the security of ISE in its own game, so he only needs to make sure that \( A \)’s response to the challenge can be used as an answer to \( B \)’s challenge. If \( A \) has a non-negligible advantage in his game, then \( B \) will have as well. However, we assumed that the ISE is secure, so, we prove by contradiction that \( A \) can’t have a non-negligible advantage in his game and that the property of Delegated RingCT he wants to break is satisfied.

**B. A Concrete Instantiation**

We use the following instantiations for ISE and NIZK components of Delegated RingCT:

- **Setup:** On input security parameter \( 1^{λ} \), outputs public parameters \( pp \). The cyclic group \( G \) used is the elliptic curve Curve25519 \([31]\) and the corresponding scalar field \( \mathbb{F} = \mathbb{Z}_p \), where \( p = 2^{255} − 19 \). The hash function used is SHA-3 and \( v_{\max} = 2^{32} \).
- **ISE.KeyGen:** We use the instantiation of \([15]\).
  - **LongTermPKGen:** The user picks his long term secret key \( \text{ltsk} = (x_1, x_2) ∈ \mathbb{Z}_p^2 \) and computes his long term private key \( \text{ltpk} = (x_1G, x_2G) \).
  - **OneTimePKGen:** On input a long term public key \( \text{ltpk} = (x_1G, x_2G) \), it picks a random \( r ∈ \mathbb{Z}_p \) and computes a one-time public key \( pk = x_1G · H(x_2G)G \). It outputs \( pk \) and the auxiliary information \( R = rG \).
- **OneTimeSKGen:** On input a one-time public key \( pk \), an auxiliary information \( R \) and a long term secret key \( \text{ltsk} = (x_1, x_2) \), it checks if \( pk = x_1G · H(x_2R)G \). If it is correct, then it outputs the one-time secret key \( sk = x_1 + H(x_2R) \).
  - **ISE.PKE:** Twisted ElGamal \([28]\).
  - **ISE.DS:** Schnorr \([32]\).
• ISE.LRS: Triptych [18].
• NIZK for $L_{enc}$: $\pi_{enc}$ of [28].
• NIZK for $L_{bal}$: Balance proof from [15].
• NIZK for $L_{range}$: Bulletproofs [19].
• NIZK for $L_{equal}$: $\pi_{equal}$ of [28].

1) Analysis: Correctness follows from the correctness of ISE’s components instantiations and the completeness of NIZK’s instantiations, and security is captured by the following theorem and lemmas.

Theorem 2. The obtained ISE scheme is jointly secure if the twisted ElGamal is IND-CPA secure (1-plaintext/2-recipient), the Schnorr signature is EUF-CMA secure and the triptych LRS is secure.

Lemma 5. The PKE component is IND-CPA secure in the presence of two signing oracles, one for the Schnorr signature and the other for the triptych LRS.

Lemma 6. The DS component is EUF-CMA secure in the presence of a signing oracle for the triptych LRS.

Lemma 7. The LRS component is secure in the presence of a signing oracle for the Schnorr signature.

We present the security proofs for our ISE instantiation in appendix B and give an intuition about them here.

We want to prove the security of each component of the ISE instantiation in the presence of one or two signing oracles (the PKE oracle can be easily simulated by the adversary in the public setting).

First, we assume the standalone security of that component. Then, we construct a challenger $\mathcal{CH}$ that is able to simulate the signing oracle(s) without the knowledge of the secret keys and an adversary $\mathcal{A}$ that can query the oracle(s) and tries to break the security of the ISE.

Then, we construct another adversary $\mathcal{B}$ that acts as the challenger $\mathcal{CH}$ in $\mathcal{A}$’s game and wants to break the standalone security of one of the components of the ISE in his own game. $\mathcal{B}$ can use the attempt of $\mathcal{A}$ for his own attempt, and, so, he will have the same advantage as $\mathcal{A}$. If $\mathcal{A}$ has a non-negligible advantage in winning his game, $\mathcal{B}$ will have too.

However, we assume that the component in question is secure, thus, we prove by contradiction, that the game $\mathcal{A}$ is trying to break is also secure, which is indistinguishable from the real experiment.

2) Performance: We implemented DelegatedRingCT in Ubuntu 18.04, Intel Core i7-4790 3.60GHz, 16GB RAM. We used the dalek cryptography repository ¹, which has implementations for Curve 25519, Bulletproofs and ed25519 signatures in Rust, and where each element in $G$ and $Z_p$ are represented by 32 bytes.

The benchmarking results for the proving time, verification time and size² of DRingCTx are in Figures 1, 2 and 3, respectively. CRx has a size of 352 bytes, a verification time of 660 us and a proving time of 261.89 us.

¹https://github.com/dalek-cryptography
²We used offsets, like Monero, for the input accounts and assumed a size of 3 bytes each.

V. DISCUSSION AND FUTURE WORK

On one hand, our Delegated RingCT can vastly improve on the verification times of a transaction and, consequently, on the scalability of a cryptocurrency that implements this protocol, as shown in the previous section.

Even though we haven’t implemented any distributed consensus mechanism, validation times aren’t expected to increase too much and will mostly depend on the type of ledger used and the blocks time interval (asynchronous vs synchronous).

On the other hand, anonymity of Delegated RingCT is not as strong as in RingCT, since if you delegate an account to a representative, a malicious actor can easily know the balance of that individual account by decrypting the ciphertext, so some amount of trust is required in the representative.

Besides that, we don’t capture in our security model the possibility of the receiver changing the representative of the output accounts of a transaction. If the representative is the
same as of the sender account, he would be able to decrypt both accounts and see that balances match, revealing the real sender of the transaction.

However, even if the representative knows which account is the real sender, he will not know what other accounts belong to the same master address or what is the master address.

Moreover, delegation is optional and you can delegate to yourself, increasing your anonymity, or even not delegate at all (even though your stake will not contribute to the consensus). Nevertheless, further investigation and formal security proofs are needed regarding anonymity against own representative.

Another limitation of our scheme is that the balance size can’t be too large if using a variation of additive ElGamal, because after decryption, the discrete logarithm problem must be solved to reveal the balance.

We used 32 bits in our implementation, which seems reasonable for the total amount of coins of a cryptocurrency, but can be insufficient if the market capitalisation becomes too high. However, if needed, two concatenated ciphertexts, instead of just one, can be used, which come at a cost of a greater size of transaction and higher verification times. Still, this approach is more efficient than other additive homomorphic public key encryption schemes.

Future work will focus on choosing a Delegated Proof of Stake consensus mechanism, as well as the ledger structure, and investigate how and when the stake weights will be updated. Too frequent can hinder anonymity. Too infrequent can create new attack vectors.

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Theorem 1. Assuming the security of ISE and NIZK, the above Delegated RingCT construction is secure.

Proof. We prove this theorem via the following four lemmas.

Lemma 1.1. Assuming the unforgeability property of ISE’s linkable ring signature component, the zero-knowledge property of NIZK and the EUF-CMA security of ISE’s digital signature component, our Delegated RingCT construction satisfies unforgeability.

Proof. We proceed via two sequences of games, one for each part of the unforgeability property, as defined in the security model.

Sequence 1.

Game 1.0. The real experiment. \( CH \) interacts with \( A \) as below.

1) Setup: \( CH \) runs the setup algorithms and sends the public parameters to \( A \).

2) Queries: Throughout the experiment, \( A \) can make queries to all oracles. The challenger \( CH \) answers these queries as defined in the security model.

3) Forge: \( A \) outputs a ring confidential transaction \( DRingCTx \) such that all the input accounts are generated by the AccountOracle and uncorrupted (\( A \) doesn’t know any of the accounts secret keys) and weren’t used as a query to TransOracle. \( A \) wins if the transaction is valid with non-negligible probability.

Game 1.1. Same as Game 0, except \( CH \) makes guess of \( N \) random indexes out of \( Q \), the number of queries made by \( A \) to the GenOracle. If \( A \) picks an account with a different index or queries the CorruptOracle with any of the input accounts, \( CH \) aborts.

Let \( E \) be the event that \( CH \) does not abort and \( S_0 \) the event that \( A \) wins in Game 0. The probability that \( E \) occurs is one out of a combination of \( N \) accounts in of all the \( Q \) queries to the GenOracle, like this: \( Pr[E] \geq \frac{N!(Q-N)!}{Q!} \), where \( N \) is the size of the ring of the DRingCTx. Conditioned on \( CH \) does not abort, \( A \)’s view in Game 0 is identical to that in Game 1. Therefore, we have:

\[ Pr[S_1] \geq Pr[S_0] \cdot \frac{N!(Q-N)!}{Q!} \]

Game 1.2. Same as Game 1.1, except that \( CH \) uses a simulator to generate the necessary zero-knowledge proofs for the TransOracle queries without knowing any of the accounts secret keys, which are indistinguishable from the real ones. By a direct reduction to the zero-knowledge property of the underlying NIZK, we have:

\[ |Pr[S_2] - Pr[S_1]| \leq negl(\lambda) \]

We now argue that no PPT adversary has non-negligible advantage in Game 1.2.

Claim. Assuming the unforgeability of the ISE’s linkable ring signature component, \( Pr[S_2] \leq negl(\lambda) \) for all PPT adversary \( A \).

Proof. Suppose there exists a PPT adversary \( A \) that has non-negligible advantage in Game 1.2. We can build an adversary \( B \) that breaks the security of ISE’s linkable ring signature component with the same advantage. Given the public parameters \( \{pp_{ise}\} \) by its challenger, \( B \) simulates Game 1.2 as follows:

1) Setup: \( B \) runs all the setup algorithms and gives the public parameters to \( A \). \( B \) randomly picks \( N \) indices from \( |Q| \) corresponding to \( \{pk_j\}^N_{j=0} \).

2) Queries: \( A \) can query all oracles adaptively and \( B \) answers them in the following way:

- KeyOracle(i): \( B \) answers the same way as defined in the security model.
- CorruptOracle: if \( A \) queries the oracle with any \( act_n \), \( B \) aborts. Otherwise, it responds as defined in the security model.
- AccountOracle(ltpk, a): if \( i = j \), \( B \) runs \( c_j \leftarrow (pk_j, a; k) \) responds with \( act_j = (pk_j, c_j) \). Otherwise, it responds as defined in the security model.
- TransOracle: if \( A \) queries the oracle with any \( act_j \), \( B \) aborts. Otherwise, it responds as defined in the security model.
- ChangeOracle(act_j, ltpk): \( B \) answers the same way as defined in the security model.

3) Forge: \( A \) submits a ring confidential transaction \( DRingCTx \). If the ring of input accounts of the DRingCTx contains a different public key corresponding to the indices chosen by \( B \), it aborts. Otherwise, \( B \) forwards the linkable ring signature to its own challenger and wins with the same advantage as \( A \).
Throughout the experiment. C\(\mathcal{H}\) interacts with \(A\) as below.

**Sequence 2.**

**Game 2.0.** The real experiment. \(C\mathcal{H}\) interacts with \(A\) as below.

1) Setup: \(C\mathcal{H}\) runs the setup algorithms and sends the public parameters to \(A\).
2) Queries: Throughout the experiment, \(A\) can query any oracles, and \(C\mathcal{H}\) answers these queries as described in the security model.
3) Forge: \(A\) submits a CR\(x\) for an uncorrupted account that was generated by AccountOracle. \(A\) wins if CR\(x\) transaction is valid with non-negligible probability.

**Game 2.1.** The same as Game 0, except \(C\mathcal{H}\) makes a random guess for the index of target act\(_j\), at the beginning, i.e., randomly picks an index \(j \in Q\), being \(Q\) the number of queries that \(A\) makes to the AccountOracle. If \(A\) submits a CR\(x\) with pk\(*\) \(\neq\) pk\(_j\), \(C\mathcal{H}\) aborts.

Let \(E\) be the event that \(C\mathcal{H}\) does not abort and \(S\) the event that \(A\) wins in Game 0. It is easy to see that \(Pr[E] \geq 1/Q\).

Conditioned on \(C\mathcal{H}\) does not abort, \(A\)'s view in Game 0 is identical to that in Game 1. Therefore, we have:

\[
Pr[S_1] \geq Pr[S_0] \cdot \frac{1}{Q}
\]

We now argue that no PPT adversary has non-negligible advantage in Game 1.

**Claim.** Assuming the EUF-CMA security of ISE's digital signature component, \(Pr[S_2] \leq negl(\lambda)\) for all PPT adversary \(A\).

**Proof.** Suppose there exists a PPT adversary \(A\) that breaks the security of ISE's digital signature component with the same advantage. Given the challenge (pp\(_{\text{sig}},\) pk\(*\)), \(B\) simulates Game 1 as follows:

1) Setup: \(B\) runs pp\(_{\text{pick}} \leftarrow\) NIZK.Setup(\(\lambda\)), (crs, \(\theta\)) \(\leftarrow\) \(\mathcal{S}_1\).Setup(pp\(_{\text{pick}}\)), sends pp = (pp\(_{\text{sig}},\) pp\(_{\text{pick}},\) crs) to \(A\). \(B\) randomly picks an index \(j \in [Q]\).
2) Queries: Throughout the experiment, \(A\) can query any oracles and \(B\) answers the following way:
   - KeyOracle: \(B\) answers the same way as defined in the security model.
   - CorruptOracle: \(B\) answers the same way as defined in the security model.
   - AccountOracle: if \(i = j\) and ltpk\(_i\) is uncorrupted (if not, abort), \(B\) runs \(c = \text{Enc}(pk_j, a; k)\) and answers with act\(_n\) = (pk\(_n\), c). Otherwise, it answers the same way as defined in the security model.
   - TransOracle: \(B\) answers the same way as defined in the security model.
   - ChangeOracle: \(B\) aborts if it is queried with act\(_n\). Otherwise, it answers the same way as defined in the security model.
3) Forge: \(A\) submits a change representative transaction CR\(x\). If the digital signature of the CR\(x\) doesn't correspond to the same public key \(B\) wants to forge in its own game, \(B\) aborts. Otherwise, \(B\) forwards the digital signature to its own challenger and wins with the same advantage as \(A\).

B's simulation for Game 2.1 is perfect. The claim immediately follows and, thus, proves Sequence 2. Together with the proof of the Sequence 1, they prove Lemma 1.1.

**Lemma 1.2.** Assuming the anonymity property of ISE's linkable ring signature component, the zero-knowledge property of NIZK, our Delegated RingCT construction satisfies anonymity.

**Proof.** We proceed via a sequence of games.

**Game 0.** The real experiment. \(C\mathcal{H}\) interacts with \(A\) as below.

1) Setup: \(C\mathcal{H}\) runs the setup algorithms and sends the public parameters to \(A\).
2) Pre-challenge queries: Throughout the experiment, \(A\) can query any oracles. \(B\) answers them as defined in the security model.
3) Challenge: \(A\) picks a ring of input accounts \(\{\text{act}_j\}_{n=0}^{N-1}\) and sends it to \(B\), such that at least two of those accounts are uncorrupted. \(B\) chooses an uncorrupted account act\(_j^*\) to be the sender and constructs a DRingCT\(x\), which it sends to \(A\).
4) Post-challenge queries: \(A\) can query any oracles in the same way as in the pre-challenge stage.
5) Guess: \(A\) guesses which account is the real sender of the DRingCT\(x\) with i\(*\) and wins if: \(Pr[i^* = j^*] - \frac{1}{N^*} \leq negl(\lambda)\).

**Game 1.** Same as Game 0, except \(C\mathcal{H}\) makes a ring guess of random \(N\) indexes out of \(Q\) queries made by \(A\) to the GenOracle. If \(A\) picks an account with a different index \(n\) for the ring of DRingCT\(x\), \(C\mathcal{H}\) aborts.

Let \(E\) be the event that \(C\mathcal{H}\) does not abort and \(S\) the event that \(A\) wins in Game 0. The probability that \(E\) occurs is one out of a combination of \(N\) accounts in all the \(Q\) queries to the GenOracle, like this: \(Pr[E] \geq \frac{N!(Q-N)!}{Q!}\), where \(N\) is the size of the ring of the DRingCT\(x\). Conditioned on \(C\mathcal{H}\) does not abort, \(A\)'s view in Game 0 is identical to that in Game 1. Therefore, we have:

\[
Pr[S_1] \geq Pr[S_0] \cdot \frac{N!(Q-N)!}{Q!}
\]

We now argue that no PPT adversary has non-negligible advantage in Game 1.

**Game 2.** Same as Game 1, except that \(C\mathcal{H}\) generates the necessary zero-knowledge proofs for the TransOracle queries without knowing the secret keys via running the simulator. By a direct reduction to the adaptive zero-knowledge property of the underlying NIZK, we have:

\[
|Pr[S_2] - Pr[S_1]| \leq negl(\lambda)
\]
Throughout the experiment, negligible advantage in Game 2, we can build an adversary $A$ of ISE’s linkable ring signature component with the same advantage. Given the challenge $(pp_{ise}, LRS)$, $B$ simulates Game 2 as follows:

1) Setup: $B$ runs $pp_{nizk} \leftarrow NIZK.Setup(1^\lambda)$, $(crs, \theta) \leftarrow S_1.Setup(pp_{nizk})$, sends $pp = (pp_{ise}, pp_{nizk}, crs)$ to $A$. $B$ randomly picks $N$ indices such that $\{pk_j\}_{n=0}^{N-1}$.

2) Pre-challenge queries: Throughout the experiment, $A$ can query all oracles. $B$ answers them as defined in the security model, except for:

- AccountOracle$(ltpk_i, a)$: if $i = j$, $B$ runs $c_j \leftarrow (pk_j, a; k)$ responds with $act_j = (pk_j, c_j)$. Otherwise, it responds as defined in the security model.

3) Challenge: $A$ picks a ring of input accounts $\{act_i\}_{i=0}^{N-1}$. If any of those accounts contains a different public key than the ones chosen by $B$ or at least two of them aren’t uncorrupted or have not been used as spend accounts in queries to TransOracle, he aborts. Otherwise, $B$ uses the LRS given by its challenger to construct the delegated ring confidential transaction $DRingCTx$ and sends it to $A$.

4) Post-challenge queries: $A$ can query all oracles in the same as in the pre-challenge stage.

5) Guess: $A$ guesses which account is the real sender of the $DRingCTx$ and $B$ forwards the guess to its own challenger to try to guess which $pk$ is the real signer of the linkable ring signature. $B$ wins with the same advantage as $A$.

It is easy to see that $B$’s simulation for Game 2 is perfect. The claim immediately follows and, thus, proves Lemma 1.2.

**Lemma 1.3.** Assuming the linkability property of ISE’s linkable ring signature component and the zero-knowledge property of NIZK, our Delegated RingCT construction satisfies linkability.

**Game 0.** The real experiment. $CH$ interacts with $A$ as below.

1) Setup: $CH$ runs the setup algorithms and sends the public parameters to $A$.

2) Queries: Throughout the experiment, $A$ can query all oracles and $CH$ answers as defined in the security model.

3) Forge: $A$ outputs $N + 1$ ring confidential transactions $\overline{DRingCTx}$ from a ring of $N$ input accounts. $A$ wins if all $N + 1$ are valid with non-negligible probability.

**Game 1.** Same as Game 0, except $CH$ makes a ring guess of random $N$ indexes out of $Q$ queries made by $A$ to the GenOracle. If $A$ picks an account with a different index $n$ for the ring of $DRingCTx$ or queries the CorruptOracle with $act_n$, $CH$ aborts.

Let $E$ be the event that $CH$ does not abort and $S_1$ the event that $A$ wins in Game 0. Conditioned on $CH$ does not abort, $A$’s view in Game 0 is identical to that in Game 1. Therefore, we have:

$$Pr[S_1] \geq Pr[S_0] \cdot \frac{N!(Q-N)!}{Q!}$$

**Game 2.** Same as Game 1, except that $CH$ generates the necessary zero-knowledge proofs for the TransOracle queries via running the simulator $\mathcal{S} = (S_1, S_2)$. More precisely, $CH$ runs $(crs, \tau) \leftarrow S_1(pp_{nizk}, \text{and})$ and $S_2(crs, \tau, \text{memo})$ to generate $\pi_{\text{correct}}$ for memo $\in L_{\text{correct}}$. By a direct reduction to the adaptive zero-knowledge property of the underlying NIZK, we have:

$$|Pr[S_2] - Pr[S_1]| \leq negl(\lambda)$$

We now argue that no PPT adversary has non-negligible advantage in Game 2.

**Claim.** Assuming the unlinkability property of the ISE’s linkable ring signature component, $Pr[S_2] \leq negl(\lambda)$ for all PPT adversary $A$.

**Proof.** Suppose there exists a PPT adversary $A$ has non-negligible advantage in Game 2, we can build an adversary $B$ that breaks the anonymity property of ISE’s linkable ring signature component with the same advantage. Given $pp_{ise}$, $B$ simulates Game 2 as follows:

1) Setup: $B$ runs $pp_{nizk} \leftarrow NIZK.Setup(1^\lambda)$, $(crs, \theta) \leftarrow S_1.Setup(pp_{nizk})$, sends $pp = (pp_{ise}, pp_{nizk}, crs)$ to $A$. $B$ randomly picks $N$ indices such that $\{pk_j\}_{n=0}^{N-1}$.

2) Queries: Throughout the experiment, $A$ can query all oracles. $B$ answers them in the same way as in the security model, except for:

- AccountOracle$(ltpk_i, a)$: if $i = j$, $B$ runs $c_j \leftarrow (pk_j, a; k)$ responds with $act_j = (pk_j, c_j)$. Otherwise, it responds as defined in the security model.

3) Challenge: $A$ outputs two ring confidential transactions $\overline{DRingCTx}$ and $\overline{DRingCTx}$ such that all the input accounts are generated
by the AccountOracle and weren’t corrupted by the CorruptOracle. \(A\) wins if the transactions are linked with non-negligible probability.

**Game 1.** The same as Game 0, except \(CH\) makes a random guess for the index of target \(act_j\) at the beginning, i.e., randomly picks an index \(j \in Q\), being \(Q\) the number of queries that \(A\) makes to the AccountOracle. Let \(E\) be the event that \(CH\) does not abort and \(S_0\) the event that \(A\) wins in Game 0. It is easy to see that \(Pr[E] \geq 1/|Q|\). Conditioned on \(CH\) does not abort, \(A\)'s view in Game 0 is identical to that in Game 1. Thereby, we have:

\[
Pr[S_1] \geq Pr[S_0] \cdot \frac{1}{Q}
\]

**Game 2.** Same as Game 1, except that \(CH\) generates the necessary zero-knowledge proofs for the TransOracle queries via running \(S = (S_1, S_2)\) in the simulation mode. More precisely, \(CH\) runs \((crs, \tau) \leftarrow S_1(pp_{nizk, correct})\) and \(S_2(crs, \tau, memo)\) to generate \(\pi_{correct}\) for \(memo \in L_{correct}\). By a direct reduction to the adaptive zero-knowledge property of the underlying NIZK, we have:

\[
|Pr[S_2] - Pr[S_1]| \leq \text{negl}(\lambda)
\]

We now argue that no PPT adversary has non-negligible advantage in Game 2.

**Claim.** Assuming the non-frameability property of the ISE’s linkable ring signature component, \(Pr[S_2] \leq \text{negl}(\lambda)\) for all PPT adversary \(A\).

**Proof.** Suppose there exists a PPT adversary \(A\) has non-negligible advantage in Game 2, we can build an adversary \(B\) that breaks the security of ISE’s linkable ring signature component with the same advantage. Given the pp_{ise}, \(B\) simulates Game 2 as follows:

1) **Setup:** \(B\) runs \(pp_{nizk} \leftarrow \text{NIZK.Setup}(1^\lambda)\), \((crs, \theta) \leftarrow S_1.pp_{nizk. setup}\), sends \(pp = (pp_{ise}, pp_{nizk}, crs)\) to \(A\). \(B\) randomly picks an index \(j \in [Q]\).

2) **Queries:** Throughout the experiment, \(A\) can query all oracles. \(B\) answers them in the following way:
   - **KeyOracle(i):** \(B\) answers the same way as defined in the security model.
   - **CorruptOracle:** if \(A\) queries the oracle with \(act_j\), \(B\) aborts. Otherwise, it responds as defined in the security model.
   - **AccountOracle:** if \(i = j\) and \(ltpk_i\) is uncorrupted (if not, abort), \(B\) runs \(c = \text{Enc}(pk_j; a; k)\) and answers with \(act_j = (pk_n, c)\). Otherwise, it answers the same way as defined in the security model.
   - **TransOracle:** if \(A\) queries the oracle with \(act_j\) for the second time, \(B\) aborts. Otherwise, it responds as defined in the security model.
   - **ChangeOracle:** \(B\) answers the same way as defined in the security model.

3) **Forge:** \(A\) submits two linkable ring confidential transaction \(\text{DRingCTx}\) with the same spend account. If these contain a different public key from the one chosen by \(B\), it aborts. Otherwise, \(B\) forwards the two linkable ring signatures to its own challenger and wins with the same advantage as \(A\).

It is easy to see that \(B\)’s simulation for Game 2 is perfect. The claim immediately follows and, thus, Lemma 1.4.

**APPENDIX B**

**SECURITY PROOFS OF CONCRETE DELEGATED RINGCT**

**Theorem 2.** The obtained ISE scheme is jointly secure if the twisted ElGamal is IND-CPA secure (1-plaintext/2-reipient), the Schnorr signature is EUF-CMA secure and the triptych LRS is secure.

**Proof.** We prove this theorem via the following three lemmas.

**Lemma 2.1.** The PKE component is IND-CPA secure in the presence of two signing oracles, one for the Schnorr signature and the other for the triptych LRS.

**Proof.** We prove via a sequence of games.

**Game 0.** The real security experiment for ISE’s PKE component. Challenger \(CH\) interacts with \(A\) as below:

1) **Setup:** \(CH\) runs \(pp_{ise} \leftarrow \text{Setup}(1^\lambda)\) and sends it to \(A\).

2) **Queries:** the adversary \(A\) can make queries to the following oracles:
   - **KeyOracle(i):** \(CH\) runs \((ltpk_i, ltsk_i) \leftarrow \text{LongTermKeyGen}(pp)\) and \((pk_i, aux_i) \leftarrow \text{OneTimePKGen}(ltpk_i; r_i)\) and returns \(pk_i\) to \(A\).
   - **CorruptOracle(pk_i):** \(CH\) runs \((sk_i) \leftarrow \text{OneTimeSKGen}(pk_i, aux_i, ltsk_i)\) and returns \(sk_i\) to \(A\).
   - **HashOracle(data):** \(CH\) emulates a random oracle by using the lazy sampling technique. He maintains an initially empty list \(L\), an on a given query with some \(data\), if there is an entry \((data, e)\) in the list, \(CH\) returns \(e\). Else, \(CH\) picks \(e \leftarrow Z_p\) and inserts \((data, e)\) in \(L\), then returns \(e\).
   - **SignOracle(pk_i, m):** on input a public key \(pk_i\) and a message \(m\), \(CH\) runs ISE.Sign\((sk_i, m) \rightarrow \sigma\) and returns \(\sigma\) to \(A\).
   - **Sign_{ring}Oracle(pk_i, m, R):** on input a public key \(pk_i\), a message \(m\) and a ring \(R\) of public keys generated by the KeyOracle, \(CH\) runs ISE.Sign\(_{ring}\)(\(sk_i, m, R\) \leftarrow \sigma_{ring}\) and returns \(\sigma_{ring}\) to \(A\).

3) **Challenge:** \(A\) submits two uncorrupted public keys \(pk_1\) and \(pk_2\) and two messages \(m_1\) and \(m_2\). \(CH\) picks a random bit \(\beta\) and randomness \(r\), computes \(X_1 = pk_1^r, X_2 = pk_2^r, Y = g^{r \cdot m_3}\), sends \(C = (X_1, X_2, Y)\) to \(A\).

4) **Guess:** \(A\) outputs its guess \(\beta'\) for \(\beta\) and wins if \(\beta' = \beta\).

According to the definition of Game 0, we have:

\[
\text{Adv}_A(\lambda) = Pr[S_0] - 1/2
\]

**Game 1.** The same as Game 0 except that \(CH\) simulates signing oracle by programming a random oracle \(H\), rather than using the real secret keys, in the following way.

1) **Setup:** \(CH\) runs \(pp_{ise} \leftarrow \text{Setup}(1^\lambda)\) and sends it to \(A\).
2) Queries: the adversary $A$ can make queries to the following oracles:
- $pk_i \leftarrow \text{KeyOracle}(i)$: $CH$ picks picks random $j_i = j,G$ (such that $j_i$ is known), $\xi_i$, $\{P_j\}_{j=0}^{m-1}$ and $z_i$ and computes $pk_i = (z_iG + \sum_{j=0}^{m-1} P_j \xi_j) \cdot \xi_i^{-m}$. Finally, he sends $pk_i$ to $A$.
- $sk_i \leftarrow \text{CorruptOracle}(pk_i)$: $CH$ runs $(sk_i) \leftarrow \text{OneTimeSKGen}(pk_i, aux_i, ltsk_i)$ and returns $sk_i$ to $A$.
- $\xi \leftarrow \text{HashOracle}(\text{data})$: $CH$ emulates a random oracle by using the lazy sampling technique. He maintains an initially empty list $L_{\text{hash}}$, an on a given query with some data, if there is an entry $(\text{data}, \xi)$ in $L_{\text{hash}}$, $CH$ returns $\xi$. Else, $CH$ picks $e \leftarrow Z_p$ and inserts $(\text{data}, \xi)$ in $L_{\text{hash}}$, then returns $\xi$.
- $\sigma \leftarrow \text{SignOracle}(pk_i, m)$: on input a public key $pk_i$ and a message $m$, $CH$ runs ISE.$\text{Sign}(sk_i, m)$ σ and returns σ to $A$. Else, $CH$ aborts to avoid possible inconsistency in programming.
- $\sigma_{\text{ring}} \leftarrow \text{Sign_{ring Oracle}}(pk_i, m, R)$: on input a public key $pk_i$, a message $m$ and a ring $R$ of public keys generated by the KeyOracle, $CH$ runs ISE.$\text{Sign}_{\text{ring}}(sk_i, m, R)$ $\sigma\text{ring}$ and returns $\sigma_{\text{ring}}$ to $A$.

3) Forge: $A$ outputs a signature $\sigma$ and wins if $\text{Verify}(pk,m,\sigma) = 1$.

Game 1. The same as Game 0 except that $CH$ simulates the signing oracle by using a random oracle $H$, rather than using the real secret keys, in the following way:

1) Setup: $CH$ runs $pp_{ise} \leftarrow \text{Setup}(1^\lambda)$ and sends it to $A$.

2) Queries: the adversary $A$ can make queries to the following oracles:
- $\text{KeyOracle}(i)$: $CH$ runs $(lt_{pk_i}, ltsk_i) \leftarrow \text{LongTermKeyGen}(pp)$ and $(pk_i, aux_i) \leftarrow \text{OneTimePKGen}(lt_{pk_i}, r_i)$ and returns $pk_i$ to $A$.
- $\text{CorruptOracle}(pk_i)$: $CH$ runs $(sk_i) \leftarrow \text{OneTimeSKGen}(pk_i, \text{aux}_i, ltsk_i)$ and returns $sk_i$ to $A$.
- $\text{HashOracle}(\text{data})$: $CH$ emulates a random oracle by using the lazy sampling technique. He maintains an initially empty list $L_{\text{hash}}$, an on a given query with some data, if there is an entry $(\text{data}, \xi)$ in $L_{\text{hash}}$, $CH$ returns $\xi$. Else, $CH$ picks $\xi \leftarrow Z_p$ and inserts $(\text{data}, \xi)$ in $L_{\text{hash}}$, then returns $\xi$.
- $\text{Sign_{ring Oracle}}(pk_i, m, R)$: on input a public key $pk_i$, a message $m$ and a ring $R$ of public keys generated by the KeyOracle, $CH$ runs ISE.$\text{Sign}_{\text{ring}}(sk_i, m, R)$ $\sigma\text{ring}$ and returns $\sigma_{\text{ring}}$ to $A$.

Denote the event that $CH$ aborts in Game 1 by E. Conditioned on E does not occur, $A$’s view in Game 0 and Game 1 are identical. This follows from the fact that $CH$ perfectly mimics the hash oracle and signing oracle. Let $Q_{\text{hash}}$ and $Q_{\text{sign}}$ be the maximum number of hash queries and signing queries that $A$ makes during security experiment. By the union bound, we conclude that $Pr[E] \leq (Q_{\text{hash}}Q_{\text{sign}}Q_{\text{sign_{ring}}})/p$, which is negligible in $\lambda$. In summary, we have:

$$|Pr[S_1] - Pr[S_0]| \leq Pr[E] \leq negl(\lambda)$$

We now argue that no PPT adversary has non-negligible advantage in Game 1.

Claim. Assuming the IND-CPA security (1-plaintext/2-recipent) of twisted ElGamal PKE, $Pr[S_1]$ is negligible in $\lambda$ for any PPT adversary $A$.

Proof. We prove this claim by showing that if there exists a PPT adversary $A$ has non-negligible advantage in Game 1, we can build a PPT adversary $B$ that breaks the IND-CPA security (single-message, two-recipent) of twisted ElGamal PKE with the same advantage, since $CH$ can simulate the signing oracles without using the secret keys. Thereby, given $(pp, pk_1, pk_2)$, $B$ can perfectly simulate Game 1 by forwarding the encryption challenge to its own challenger. This proves the claim.

Lemma 2.2. The DS component is EUF-CMA secure in the presence of a signing oracle for the triptych LRS.

Proof. We prove via a sequence of games.

Game 0. The real security experiment for ISE’s DS component. Challenger $CH$ interacts with $A$ as below:

1) Setup: $CH$ runs $pp_{ise} \leftarrow \text{Setup}(1^\lambda)$ and sends it to $A$.  

$$|Pr[S_1] - Pr[S_0]| \leq Pr[E] \leq negl(\lambda)$$
We now argue that no PPT adversary has non-negligible advantage in Game 1.

**Claim.** Assuming the EUF-CMA security of Schnorr digital signature, $Pr[S_1]$ is negligible in $\lambda$ for any PPT adversary $A$.

**Proof.** We prove this claim by showing that if there exists a PPT adversary $A$ has non-negligible advantage in Game 1, we can build a PPT adversary $B$ that breaks the the EUF-CMA security of Schnorr digital signature with the same advantage, since $CH$ can simulate the signing oracle for the LRS component without using the secret keys. Thereby, given $(pp, pk)$, $B$ can perfectly simulate Game 1 by forwarding the signature forgery to its own challenger. This proves the claim.

**Lemma 2.3.** The LRS component is secure in the presence of a signing oracle for the Schnorr signature.

**Proof.** We prove via a sequence of games.

**Game 0.** The real security experiment for ISE's LRS component. Challenger $CH$ interacts with $A$ as below:

1) **Setup:** $CH$ runs $pp_{sec} \leftarrow Setup(1^\lambda)$ and sends it to $A$.

2) **Queries:** the adversary $A$ can make queries to the following oracles:

- **KeyOracle(i):** $CH$ runs $(ltpk_i, ltsk_i) \leftarrow LongTermKeyGen(pp)$ and $(pk_i, aux_i) \leftarrow OneTimePKGen(ltpk_i, r_i)$ and returns $pk_i$ to $A$.

- **CorruptOracle(pk_i):** $CH$ runs $(sk_i) \leftarrow OneTimeSKGen(pk_i, aux_i, ltsk_i)$ and returns $sk_i$ to $A$.

- **HashOracle(data):** $CH$ emulates a random oracle by using the lazy sampling technique. He maintains an initially empty list $L_{hash}$, an on a given query with some data, if there is an entry $(data, \xi)$ in $L_{hash}$, $CH$ returns $\xi$. Else, $CH$ picks $\xi \leftarrow Z_p$ and inserts $(data, \xi)$ in $L_{hash}$, then returns $\xi$.

- **SignOracle(pk_i, m):** on input a public key $pk_i$ and a message $m$, $CH$ runs ISE.Sign($sk_i, m$) $\rightarrow \sigma$ and returns $\sigma$ to $A$.

3) **Forge:** $A$ outputs a signature $\sigma_{ring}$ wins if $Verify(pk, m, \sigma_{ring}) = 1$.

**Game 1.** The same as Game 0 except that $CH$ simulates signing oracle by programming a random oracle $R$, rather than using the real secret keys, in the following way.

1) **Setup:** $CH$ runs $pp_{sec} \leftarrow Setup(1^\lambda)$ and sends it to $A$.

2) **Queries:** the adversary $A$ can make queries to the following oracles:

- **pk_i:** $CH$ picks random $j_i = j \in G$ (such that $j_i$ is known), $\xi_i \left\{ P_{j_i} \right\}_{j_i=0}^{m-1}$ and $z_i$ and computes $pk_i = (z_i + \sum_{j_i=0}^{m-1} P_{j_i}) \cdot e_i^{-m}$. Finally, he sends $pk_i$ to $A$.

- **sk_i:** $CH$ runs $(sk_i) \leftarrow OneTimeSKGen(pk_i, aux_i, ltsk_i)$ and returns $sk_i$ to $A$.

- **ξ:** $CH$ runs $HashOracle(data)$: $CH$ emulates a random oracle by using the lazy sampling technique. He maintains an initially empty list $L_{hash}$, an on a given query with some data, if there is an entry $(data, \xi)$ in $L_{hash}$, $CH$ returns $\xi$. Else, $CH$ picks $\xi \leftarrow Z_p$ and inserts $(data, \xi)$ in $L_{hash}$, then returns $\xi$.

- **σ:** $CH$ runs $SignOracle(pk_i, m)$: on input a public key $pk_i$ and a message $m$, $CH$ runs ISE.Sign($sk_i, m$) $\rightarrow \sigma$ and returns $\sigma$ to $A$. Else, $CH$ aborts to avoid possible inconsistency in programming.

Denote the event that $CH$ aborts in Game 1 by $E$. Conditioned on $E$ does not occur, $A$'s view in Game 0 and Game 1 are identical. This follows from the fact that $CH$ perfectly mimic the hash oracle and signing oracle. Let $Q_{hash}$ and $Q_{sign}$ be the maximum number of hash queries and signing queries that $A$ makes during security experiment. By the union bound, we conclude that $Pr[E] \leq (Q_{hash}Q_{sign})/p$, which is negligible in $\lambda$. In summary, we have:

$$|Pr[S_1] - Pr[S_0]| \leq Pr[E] \leq negl(\lambda)$$

We now argue that no PPT adversary has non-negligible advantage in Game 1.

**Claim.** Assuming the security of triptych LRS, $Pr[S_1]$ is negligible in $\lambda$ for any PPT adversary $A$.

**Proof.** We prove this claim by showing that if there exists a PPT adversary $A$ has non-negligible advantage in Game 1, we can build a PPT adversary $B$ that breaks the security of triptych LRS with the same advantage, since $CH$ can simulate the signing oracle for the DS component without using the secret keys. Thereby, given $(pp, pk_1, ..., pk_n)$, $B$ can perfectly simulate Game 1 by forwarding the signature forgery to its own challenger. This proves the claim.

Putting all the above together, Theorem 2 immediately follows.

**Appendix C:**

**Triptych LRS**

Sign($sk, m, R$):
- Let $R = pk_0, ..., pk_{n-1}$, such that $pk_1 = skG$.
- Compute $J = sk^{-1}U$.
- Select random $r_A \in \mathbb{F}$ and $\{a_{j,i}\}_{i=1,j=0}^{n-1,m-1} \subset \mathbb{F}$. Set $\{a_{j,i}\}_{j=0}^{m-1} \equiv -\sum_{i=1}^{n-1} a_{j,i}$ and define $A \equiv \text{Com}(a, r_A)$.
- Define $\{\sigma_{j,i}\}_{i,j=0,j=0}^{n-1,j=1} \subset \mathbb{F}$ such that $\sigma_{j,i} \equiv \delta (l_j, i)$ (evaluates to 1 if $i = j$ and 0 otherwise) and choose random $r_B \in \mathbb{F}$. Define $B \equiv \text{Com}(\sigma, r_B)$.
- Select random $r_C \in \mathbb{F}$, and define $C \equiv \text{Com}(a(1 - 2\sigma), r_C)$.
- Select random $r_D \in \mathbb{F}$, and define $D \equiv \text{Com}(-a^2, r_D)$.
- Define coefficients $\{p_{k,j}\}_{k,j=0}^{N-1,m-1}$ such that

$$p_k(x) \equiv \prod_{j=0}^{m-1} (\sigma_{j,k}x + a_{j,k}) \equiv \delta(l, k)x^m + \sum_{j=0}^{m-1} p_{k,j}x^j$$

for all $k \in \{0, N\}$ (using our decomposition notation).
- Select random $\{p_{j}\}_{j=0}^{m-1} \subset \mathbb{F}$.
- For $0 \leq j < m$, let $f_{j,0} \equiv \xi - \sum_{i=1}^{n-1} f_{j,i}$. 

 Verify($\sigma_{ring}, m, R$):
• Compute $\xi = H(m, R, A, B, C, D, X, Y)$.
• Accept if and only if:

\[
A + \xi B = \text{Com}(f, z_A)
\]

\[
\xi C + D = \text{Com}(f(\xi - f), z_C)
\]

\[
\sum_{k=0}^{N-1} pk_k \left( \prod_{j=0}^{m-1} f_{j,k_j} \right) - \sum_{j=0}^{m-1} \xi^j X_j - zG = 0
\]

\[
U \sum_{k=0}^{N-1} \left( \prod_{j=0}^{m-1} f_{j,k_j} \right) - \sum_{j=0}^{m-1} \xi^j Y_j - zJ = 0
\]