Simulation of the in-cylinder working process of an Opposed-Piston Free-Piston Linear Generator

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Abstract. Due to the potential application on hybrid, an opposed-piston free-piston linear generator (OPFPLG) is designed, and its operating principle is described. According to its structural parameters and motion characteristics, the mathematical model is established, including the dynamics, thermodynamics, and control equations. The simulation model based on Matlab/Simulink platform is set up for numerically analyzing its performance. The result of the simulation generally accords with the design expectation, and the in-cylinder characteristics of the OPFPLG is analyzed. The indicate power is 6 kW and the mechanical efficiency is 38.2% in the simulation model. The influence of variable parameters – mass and ignition advance angle – is illustrated and discussed.

Introduction

Internal combustion engine (ICE) is widely used as power device in modern society. However, the ensuing emission problem and the shortage of petroleum call for exploring new vehicle power devices, and the research on free-piston linear generator (FPLG) is emerged. Compared with the traditional ICE, the free-piston engine possesses the characteristics of high efficiency, high power density, variable compression ratio, high fuel flexibility and low NOx formation, so its application on hybrid vehicle is promising [1]. Opposed piston free-piston linear generator (OPFPLG) are essentially two single ended machines placed so that their combustion pistons occupy either end of the same combustion cylinder [2]. OPFPLG has lighter moving mass which results in higher speed and higher power output and the mass of air in the bounce chamber can be varied to help control piston motion. The numerical simulation of the performance of OPFPLG is fundamental when accurately evaluating the system. T.J. Callahan and S.K Ingram in Southwest Research Institute, San Antonio, Texas adopted computer-modeling to evaluate the feasibility of an opposed-piston free-piston engine coupled to a linear generator to serve as an auxiliary power unit for a hybrid vehicle, finding that the efficiency of the linear generator was highly dependent on the design concept [3]. Ling Huang in Nanjing University of Science and Technology has designed and simulated an opposed-piston free-piston linear generator for hybrid electric vehicle. The simulation model proved its feasibility and a 15 kW average output electric power with a generating efficiency of 42.5% can been obtained [4]. Nguyen Ba Hung and Ock Taeck Lim in University of Ulsan studied a two-stroke free piston linear engine by numerical models and simulation. The numerical models consisted of three parts: dynamic model, linear alternator model, and thermodynamic model. The simulation results indicated that by using numerical models as mentioned, the calculation data were closely similar to experimental data [5]. Chang-ping Lee in the University of Michigan developed a
numerical model of the FPE in Matlab/Simulink that can be used to understand the conceptual design and operation, but several sub-models in the engine model can be updated for improvements in the analysis [6]. Terry A. Johnson and Michael T. Leick have developed a prototype free piston engine-linear alternator system at Sandia National Laboratories, and its potential for use in hybrid electric vehicles was comprehensively analyzed by a series of simulation and the corresponding experiments [7]. According to the discussion above, several research groups have made the numerical simulation for the working process. The simulation model is generally based on dynamics, thermal dynamics, fluid dynamics and control equations. Due to the difference between the structural designs, the simulation models are different, especially on the hypotheses introduced and the empirical equations adopted. Based on the structure designed by the author, to accurately analyze its working process, an all-around simulation model is put forward.

1. Structure and working principle of the OPFPLG
The configuration of OPFPLG is showed in figure 1. It adopts the symmetrical structure. There are a couple of moving mass with two different pistons. The permanent magnet and the connecting rod are situated between the pistons. The outside piston sets in the gas spring bounce chamber and the inside pistons share the same combustion chamber, where the air inlets, exhaust vents, an ignition plug and a fuel injector are arranged. The moving mass with permanent magnet moves freely between the top dead center (TDC) and the bottom dead center (BDC) without contacting the surface of the chamber, which is the only significant moving part of the OPFPLG and converts the kinetic energy of moving mass into electric energy of the linear generator.

![Figure 1. The configuration of OPFPLG](image)

1 Bounce piston; 2 Stator secondary coil of linear generator; 3 Dynamic piston; 4 Spark plug; 5 Exhaust vent; 6 One-way intake valve of bounce chamber; 7 Permanent magnets; 8 Air intake

At the beginning of the compression stroke, the moving mass stays at the BDC and the gas spring drives the moving mass to the TDC. The air inlet and the exhaust vent close in turn, and the mixture in the combustion chamber is compressed afterwards. When moving at the ignition advance angle, combustion begins as well as the ensuing expansion stroke. During expansion, the high pressure mixture pushes the moving mass to the BDC, and the air in bounce chamber is compressed. The exhaust vent and air inlet open in sequence to exchange the mixture. Finally, moving mass arrives at the BDC and the new working cycle starts.

2. Mathematical model of OPFPLG

2.1. Dynamic model
Based on the working principle, the mathematical model can be established. The force and kinetics analysis of the moving mass is illustrated in figure 2. Since the OPFPLG is of symmetrical structure and
the two moving masses are synchronous. Select the left group of moving mass as reference and build coordinate system. The coordinate origin is at the TDC and the positive direction is left.

![Figure 2. Force and kinetics analysis of the moving mass](image)

Based on the Newton’s second Law, the dynamic equation of the moving mass can be expressed as:

\[ m\ddot{a} = F_{\text{combustion}} + F_{\text{bounce}} + F_{\text{friction}} + F_{\text{electricity}} \]  

(1)

where, \( m \) is the mass of the moving mass, \( a \) is the acceleration of the moving mass, \( F_{\text{combustion}} \) is the gas force of the combustion chamber, \( F_{\text{bounce}} \) is the gas force of the bounce chamber, \( F_{\text{friction}} \) is the friction, \( F_{\text{electricity}} \) is the electric resistance of the linear generator.

The force of in-cylinder is the pressure of in-cylinder pressure

\[ F_{\text{combustion}} = p_{\text{combustion}}A_{\text{combustion}} \]  

(2)

\[ F_{\text{bounce}} = p_{\text{bounce}}A_{\text{bounce}} \]  

(3)

Where, \( p_{\text{combustion}} \) is the pressure of the combustion chamber, \( A_{\text{combustion}} \) is the piston area of the combustion chamber, \( p_{\text{bounce}} \) is the pressure of the bounce chamber, \( A_{\text{bounce}} \) is the piston area of the bounce chamber.

\[ A_{\text{combustion}} = \frac{\pi D_{\text{combustion}}^2}{4} \]  

(4)

\[ A_{\text{bounce}} = \frac{\pi D_{\text{bounce}}^2}{4} \]  

(5)

Where, \( D_{\text{combustion}} \) is the diameter of the combustion chamber, \( D_{\text{bounce}} \) is the diameter of the bounce chamber.

As to the pressure of the bounce chamber, it complies to the isentropic process.

\[ p_{\text{bounce}}V_{\text{bounce}}^\gamma = C \]  

(6)

Where, \( V_{\text{bounce}} \) is the volume of the bounce chamber, \( \gamma \) is the adiabatic index, \( C \) is a constant.

So the differential equation of the pressure of the bounce chamber is deduced as

\[ \frac{dp_{\text{bounce}}}{dr} = -\frac{p_{\text{bounce}}}{V_{\text{bounce}}} \frac{dV_{\text{bounce}}}{dr} \]  

(7)

Where, \( r \) is the time.

The friction force and the electricity force are approximately in proportion to the velocity

\[ F_{\text{friction}} = C_f v \]  

(8)
\[ F_{electricity} = C_e v \]  

(9)

Where, \(C_f\) is the friction load coefficient, \(v\) is the velocity of the moving mass, \(C_e\) is the electric load coefficient.

2.2. Thermodynamic model

The thermodynamic process happens in the combustion chamber. The analysis is based on the zero-dimensional model, ignoring the influence of the shape of the combustion chamber, oil droplet evaporation and the air distribution and assuming that the gas mixture abides to the ideal gas equation, that is

\[ pV = mR_g T \]  

(10)

Where, \(m\) is the mass of the gas, \(R_g\) ideal gas constant, \(T\) is the temperature.

The in-cylinder thermodynamic process is illustrated in figure 3.

**Figure 3.** In-cylinder thermodynamic process

According to the First Law of Thermodynamics, the Law of Conservation of Mass and the Law of Conservation of Energy, the thermodynamic model can be obtained as (omitting the subscript ‘combustion’ in this section)

\[
\frac{dU}{dt} = \frac{dQ_c}{dt} - \frac{dQ_h}{dt} - p \frac{dV}{dt} + \sum_i H_i - \sum_o H_o - \sum_l H_l
\]  

(11)

Where, \(U\) represents the total internal energy of the combustion chamber, \(Q_c\) represents the combustion heat release of the fuel, \(Q_h\) represents the heat transfer of the combustion chamber, \(H_i\) represents the total enthalpy of the air intake, \(H_o\) represents the total enthalpy of the air exhaust, \(H_l\) represents the total enthalpy of the air leakage.

Based on equation (10) and (11), the differential equation of the in-cylinder pressure of the combustion chamber is

\[
\frac{dp}{dt} = \gamma - 1 \left( \frac{dQ_c}{dt} - \frac{dQ_h}{dt} \right) - p \frac{dV}{dt} + \gamma \left( \frac{dm}{dt} - \frac{dm_o}{dt} - \frac{dm_l}{dt} \right)
\]  

(12)

The heat release rate is expressed by Webie Function as
\[
\frac{dQ}{dt} = a \frac{b+1}{t_d} \left( t - t_0 \right)^b \exp \left( -a \left( t - t_0 \right)^{b+1} \right) Q_n
\]  

(13)

Where, \( a \) is the fuel consumption percentage, \( b \) is the combustion quality factor, \( t_d \) is the combustion duration, \( Q_n \) is the total heat value of the fuel.

\[
Q_{\text{in}} = m_{\text{air}} \times AFR \times LHV
\]  

(14)

Where, \( AFR \) is the air-fuel ratio, \( LHV \) is the low heat value of the fuel.

Then, adopt the Hohenberg empirical formula [8]

\[
\frac{dQ_{ht}}{dt} = 130V^{-0.06} \left( \frac{p}{1 \times 10^5} \right)^{0.8} \left( \bar{v} + 1.4 \right)^{0.8} A_w \left( T - T_w \right)
\]  

(15)

Where, \( \bar{v} \) is the average velocity of the moving mass, \( A_w \) is the heat transfer area of the combustion chamber, \( T_w \) is the temperature of the wall of the combustion chamber.

As to the enthalpy change caused by the air intake, exhaust and leakage, applying the gas dynamics formula [9]

\[
\frac{dm}{dt} = \left\{ \begin{align*}
C_d \frac{Ap_h}{\left( R_g T_{\text{ph}} \right)^{\frac{1}{2}}} & \left( \frac{p_i}{p_h} \right)^{\frac{2}{\gamma}} \left[ 1 - \left( \frac{p_i}{p_h} \right)^{\frac{\gamma+1}{\gamma}} \right]^{\frac{1}{2}} \left( \frac{p_i}{p_h} \right) > \left[ 2 / (\gamma + 1) \right]^{\frac{\gamma}{\gamma-1}} \\
& \left( \frac{2}{\gamma + 1} \right)^{\frac{\gamma+1}{\gamma-1}} \left( \frac{p_i}{p_h} \right) \leq \left[ 2 / (\gamma + 1) \right]^{\frac{\gamma}{\gamma-1}}
\end{align*} \right.
\]  

(16)

Where, \( C_d \) is the discharge coefficient, \( p_i \) is the pressure of the low pressure end, \( p_h \) is the pressure of the high pressure end, \( T_{\text{ph}} \) is the temperature of the high pressure end.

\[
C_d = 0.85 - 0.25 \left( \frac{p_i}{p_h} \right)^2
\]  

(17)

2.3. Control model

To accurately control the compression ratio (CR), this research adopts the variable alternator load strategy based on proportional-integral (PI) controller to maintain a stable CR. The controller weight is allowed to fluctuate within an upper or lower values. The proportional part corrects the controller weight based on the error between the actual CR and the target CR. The integral part integrates the error over the time and add it to the proportional result. By using such controller, even small error will increase in the integral part and such response will accumulate over time as long as the error is not zero. Eventually, the CR is maintained at the pre-designed value. The controller can be described in the mathematical form as [10]

\[
F_c = W_c \cdot C_e \cdot \nu
\]  

(18)

\[
W_c = 1 - K_p \cdot \text{error} - K_i \cdot \int \text{error} \cdot dt
\]  

(19)

\[
\text{error} = CR_{\text{Target}} - CR_{\text{Actual}}
\]  

(20)

Where, \( W_c \) is the controller weight, \( K_p \) is the proportional component, \( K_i \) is the integral component, \( CR_{\text{Target}} \) is the target CR, \( CR_{\text{Actual}} \) is actual CR.

3. Simulation and result analysis

3.1. Simulation parameters
For the mathematical model above, it can be solved using a numerical simulation program in Matlab/SIMULINK and some of the parameters are defined according to a OPFPLG prototype. Before starting the simulation program, the geometric dimensions of the engine, the initial conditions and the initial values of some parameters need to be determined. Some of the detailed specifications and initial parameters are given in table 1.

| Parameters                          | Value   |
|-------------------------------------|---------|
| Bore of combustion chamber          | 56.5mm  |
| Bore of bounce chamber              | 115mm   |
| Stroke length                       | 55mm    |
| Mass of moving mass                 | 4kg     |
| Friction load coefficient           | 80-90   |
| Electrical load coefficient         | 210-230 |
| Intake pressure                     | 1bar    |
| Piston initial position             | 45mm    |
| Piston initial velocity             | -0.001m/s |
| Air fuel ration                     | 14.8    |
| Compression ratio                   | 10      |

3.2. Results
Figure 4 shows the piston velocity-displacement curve from the initial condition to stable condition. The overall simulation time is 0.3s. Different from the traditional engine, the velocity is asymmetrical with respect to the TDC, where the velocity is faster after the TDC, so the expansion stroke is faster than the compression stroke. Due to such characteristics, the piston stays shorter at TDC, which reduces the production of high temperature pollutants, like NOx. The outermost cycle is bigger than the overlapped stable cycle, because of the overshoot of the PI controller. Once coming into stable condition, the piston can reach at the TDC accurately, where the displacement is zero on the coordinate.

![Figure 4. Velocity-displacement curve of the moving mass](image-url)
Figure 5 shows the pressure of the combustion chamber and the bounce chamber. The overshoot of the controller in the first cycle is more apparent in this figure. Because the initial velocity is nearly zero, the first cycle cannot reach the TDC at once and cause large error for the controller, so the overshoot takes place. When the OPFPLG works stably, the peak pressure of combustion chamber is 9 bar, which will not affect the reliability of the structure, and the pressure of bounce chamber also tends to be stable. The peaks of the two chambers are alternate, which reflects the working principle of the OPFPLG. Because the boundary condition of air exchange is not exact in the zero-dimensional simulation, the curve of this process is not smooth enough.

![Figure 5. Pressure of combustion and bounce chambers](image)

Figure 6 shows the eventual CR of both controlled and uncontrolled conditions. The target CR of design is 10. Because the initial electrical load consumes much more energy of the gas spring, if without controller, the eventual CR is only 8.5. When leading into the controller, after 3-cycle regulation, the electrical load decreases and the piston can finally reach the target CR. Therefore, the validity of the PI controller is proved. By adjusting the initial condition and the control parameters of the controller, the overshoot can be prevented and the regulation time can be reduced.

![Figure 6. CR under both controlled and uncontrolled conditions](image)
Figure 7 illustrates the indicate power of the linear generator. Through cutting magnetic line, the kinetic energy of the moving mass converts to the electric energy of the linear generator. After simulation, the result of indicate power fulfills the design value. Some design targets with their respective simulation results are compared in Table 1. Through comparison, the simulation results generally meet the requirement of the design. The error between the design expectation and simulation result is limited in 10%. It turns out, at least, the structure design of the OPFPLG is feasible in the zero-dimensional simulation. Due to the ideal hypotheses of the simulation, the result needs further test in the prototype experiment. Combining with the experiment data, the simulation model can be revised.

![Indicate power of the linear generator](image)

**Figure 7.** Indicate power of the linear generator

| Design index       | Target value | Simulation result |
|--------------------|--------------|-------------------|
| CR                 | 10           | 10                |
| Frequency          | 30Hz         | 33Hz              |
| Peak temperature   | Less than 3000K | 2900K           |
| Mechanical efficiency | 35%       | 38.2%             |
| Indicate power     | 6kW          | ≈6kW              |

**Table 2.** Expectation and simulation results of the design indexes

4. The influence of variable controllable parameters

4.1. The mass of moving mass

Figure 8 illustrates the pressure of the bounce chamber versus time for various masses of moving mass. The peak pressure of the combustion chamber increases with the heavier moving mass, but the velocity of the moving mass reduces so the working frequency of the linear generator declines. Excessive burst pressure will cause problems to the strength of the bounce chamber and the dynamic piston.
Figure 8. Pressure of the bounce chamber for various moving mass

Figure 9 shows the indicate power of various masses. Because the velocity of the moving mass declines with the heavier mass, omitting electromagnetic effects of the linear generator, the indicate power also decreases, which means the extra energy from the higher burst pressure converts to the bigger CR of the bounce chamber, so the mechanical efficiency is declined.

Figure 9. Indicate power of various masses

4.2. The ignition advance angle

Figure 10 is a plot of piston velocity versus time for various ignition advance angles. Generally, the optimum ignition advance angle is 15-25°CA, if exceeding this scope, the negative work of the compression stroke increases or the positive work of the expansion stroke losses. Thus, the simulation samples are chosen from this scope. The increase of ignition advance angle leads to the increase of the velocity of the moving mass, and the working frequency increases a little as well, but not notable. Since the velocity increases, the indicate power will increase correspondingly.
Figure 10. Piston velocity for various ignition advance angles

Figure 11 is a plot of pressure of combustion chamber versus time for various ignition advance angles. The less the ignition advance angle, the higher the burst pressure, and the combustion is improved, which contributes to the overall performance of the engine. Therefore, thinking of both the performance of the free-piston engine and the electric indicate power, adopting variable ignition advance angle with respect to different working conditions is necessary.

Figure 11. Pressure of combustion chamber for various ignition advance angles

5. Conclusions

The structure design and the working principle of an OPFPLG are introduced. The dynamic and thermodynamic model analyses of the OPFPLG has been presented. Through simulation, the dynamics and thermodynamics performance of OPFPLG are investigated, and the validity of the PI controller is illustrated. The characteristics of the OPFPLG are analyzed. According to the simulation, the structure design is proved and the expected performance is guaranteed, and the errors between design and simulation are controlled in 10%. Then, the influence of mass and ignition advance angle are discussed. The increase of mass reduces the indicate power but the increase of ignition advance power, in the optimum scope, raises the indicate power.
References
[1] Mohamed N A N, Ariffin A K and Fonna S 2006 Simulation of a two-stroke spark ignition free piston linear engine motion J. Univ. Tek. Mas. 44(A) 27–40
[2] Wang H, Zhao C and Zhang F 2018 Design of synchronous drive mechanism of opposed-piston hydraulic-output engine J. B. Inst. Techno, 27(2) 250-6
[3] Callahan T J and Ingram S K 1995 Free-piston engine linear generator for hybrid vehicles modeling study Southwest Res. Inst. Report
[4] Huang L 2012 An opposed-piston free-piston linear generator development for HEV SAE Technical Paper doi:10.4271/2012-01-1021
[5] Nguyen B H and Ock T L 2014 A study of a two-stroke free piston linear engine using numerical analysis J. Mech. Sci. Technol. 28 (4) 1545-57
[6] Lee C 2014 Turbine-compound free-piston linear alternator engine (Ann Arbor: University of Michigan) pp 63-76
[7] Terry A J and Michael T L 2016 Experimental evaluation of a prototype free piston engine – linear alternator (FPLA) system SAE Technical Paper doi:10.4271/2016-01-0677
[8] Hohenberg G F 1979 Advanced approaches for heat transfer calculations SAE Spec. Publ. SP-449 61-72
[9] Zhang Y 2010 Study on sealing performance of new composite piston ring Trans. CSICE 28(03) 281-7
[10] Mohammad A 2016 Modeling and Simulation of a Free-Piston Engine with Electrical Generator Using HCCI Combustion (Morgantown: West Virginia University) pp 47-9