Neutrino Oscillations, Lorentz/CPT Violation, and Dark Energy

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(Dated: October 22, 2009)

If dark energy (DE) couples to neutrinos, then there may be apparent violations of Lorentz/CPT invariance in neutrino oscillations. The DE-induced Lorentz/CPT violation takes a specific form that introduces neutrino oscillations that are energy independent, differ for particles and antiparticles, and can lead to novel effects for neutrinos propagating through matter. We show that ultra-high-energy neutrinos may provide one avenue to seek this type of Lorentz/CPT violation in $\nu_{\mu}-\nu_{\tau}$ oscillations, improving the current sensitivity to such effects by seven orders of magnitude. Lorentz/CPT violation in electron-neutrino oscillations may be probed with the zenith-angle dependence for high-energy atmospheric neutrinos. The “smoking gun,” for DE-neutrino coupling would, however, be a dependence of neutrino oscillations on the direction of the neutrino momentum relative to our peculiar velocity with respect to the CMB rest frame. While the amplitude of this directional dependence is expected to be small, it may nevertheless be worth seeking in current data and may be a target for future neutrino experiments.

PACS numbers: 98.80.-k, 95.36.+x, 95.85.Ry, 14.60.Pq, 11.30.Er

I. INTRODUCTION

The accelerated cosmic expansion\cite{1} poses difficult questions for theoretical physics\cite{2,3,4}. Is it simply due to a cosmological constant? Is some new negative-pressure dark energy (DE) required? Is general relativity modified at large distance scales? The major thrust of the empirical assault on these questions has been to determine whether the expansion history and growth of large-scale structure are consistent with a cosmological constant or require something more exotic\cite{5}.

However, it may be profitable to explore whether there are other experimental consequences of the new physics—which we collectively refer to as DE, although it may involve a modification of gravity rather than the introduction of new substance—responsible for accelerated expansion. If cosmic acceleration is due to a cosmological constant (i.e., if general relativity is valid and the equation-of-state parameter is $w = -1$), then the vacuum is Lorentz invariant. If, however, something else is going on, then the “vacuum” has a preferred frame: the rest frame of the cosmic microwave background (CMB). If, moreover, dark energy couples somehow to standard-model particles, then there may be testable (apparent) violations of Lorentz invariance. For example, if DE is coupled to the pseudoscalar $\tilde{F}F$ of electromagnetism\cite{6}, there may be a “cosmological birefringence” that rotates the linear polarization of cosmological photons; CMB searches for such a rotation\cite{7} constrain this rotation to be less than a few degrees\cite{8}.

Here we explore DE-induced Lorentz/CPT-violating effects in the neutrino sector. We show that the form of a Lorentz-violating coupling between neutrinos and dark energy is highly restricted under fairly general assumptions.\footnote{The coupling of neutrinos to dark energy has also been considered in the context of “mass-varying neutrinos”\cite{9}, but that implementation of the DE-neutrino coupling does not lead to the type of Lorentz/CPT-violating effects we discuss here.} The coupling engenders an additional source for neutrino mixing (e.g., Ref.\cite{10}), resulting in neutrino oscillations with a different energy dependence than vacuum oscillations and different oscillation probabilities for neutrinos and antineutrinos. While similar Lorentz/CPT-violating oscillations have been considered before\cite{11,12,13}, we emphasize here that cosmic acceleration dictates a specific form for such effects.

Data from Super-Kamiokande and K2K\cite{14} and AMANDA/IceCube\cite{15} already tightly constrain CPT-violating parameters for $\nu_{\mu}-\nu_{\tau}$ mixing, and those from solar-neutrino experiments and KamLAND\cite{16} do so for $\nu_{\mu}-\nu_{\mu}$ mixing. However, the effects of DE-induced CPT violation become more significant at higher energies\cite{17}. Here we show that next-generation measurements of ultra-high-energy neutrinos produced by spallation of ultra-high-energy cosmic rays will increase the sensitivity to CPT-violating $\nu_{\mu}-\nu_{\tau}$ oscillations by seven orders of magnitude. We also show that these CPT-violating couplings may lead to novel effects in the zenith-angle dependence for atmospheric neutrinos in the $\sim 100$ GeV range.

While such CPT-violating effects, if detected, could be attributed simply to intrinsic CPT violation in fundamental physics, not related to DE, a DE-neutrino coupling further predicts a directional effect: the neutrino-mixing parameters depend on the neutrino propagation direction relative to our peculiar velocity with respect to the CMB rest frame. While this signature will likely remain elusive even to next-generation experiments, it would, if detected, be a “smoking gun” for DE beyond a
cosmological constant. It is therefore worth considering as a long-range target for future neutrino experiments. It may also be worthwhile to search current data in case an implementation of DE-neutrino coupling different from that we consider here leads to a different energy dependence for these directional effects. We therefore work out explicitly the directional dependence to aid experimentalists who may wish to look for such correlations in current data.

Below, we first derive in Sec. [III] the form of the Lorentz/CPT violation allowed by a DE-neutrino coupling and discuss the resulting neutrino-oscillation physics. In Sec. [IV] we apply the formalism to cosmogenic ultra-high-energy neutrinos, and obtain projected sensitivities of future detectors to these effects in oscillations. In Sec. IV we discuss matter-induced effects for neutrino-oscillations in high-energy atmospheric neutrinos, and obtain projected sensitivities of future detectors to these effects in oscillations. In Sec. II, we comment briefly on possible consequences below.

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The smallness of the CMB quadrupole demands that the 3-dimensional hypersurfaces of constant DE density must be closely aligned with those of constant CMB temperature [18]. The preferred frame associated with the cosmic expansion is then parametrized by a unit four-vector \( l^\mu \) which is orthogonal to surfaces of constant CMB temperature; i.e., in the CMB rest frame, it is \( l^\mu = (1,0,0,0) \). The symmetry of the problem thus dictates that \( a^\mu \propto l^\mu \) and \( b^\mu \propto l^\mu \). The tensor \( H^{\mu\nu} \) is antisymmetric, and there is no way to construct an antisymmetric tensor \( H^{\mu\nu} \) from a single four-vector; we thus expect \( H^{\mu\nu} = 0 \) for DE-neutrino coupling.

Furthermore, since neutrinos are produced and interact in weak eigenstates, it is only the combination \( (a_L)^{\mu}_{ab} \equiv (a + b)^{\mu}_{ab} \) that is relevant for neutrino phenomenology. Thus, the Lorentz/CPT violation induced in neutrino physics can be parametrized entirely by a single four-vector-valued \( (a_L)^{\mu}_{ab} \propto l^\mu \) matrix in the flavor space.

### B. Dark-energy–induced Lorentz violation

Lorentz violation in Eq. (2) is parametrized by the four-vectors \( a^\mu \), \( b^\mu \), and the antisymmetric tensor \( H^{\mu\nu} \). The parameters \( a^\mu \) and \( b^\mu \) are both CPT and Lorentz violating, while \( H^{\mu\nu} \) is Lorentz violating but CPT conserving. While these parameters are non-zero for the most general Lorentz/CPT-violating Dirac equation [12], the allowable forms for \( a^\mu \), \( b^\mu \), and \( H^{\mu\nu} \) are highly restricted if the Lorentz/CPT violation is induced by coupling to dark energy.

The propagation of the flavor eigenstates is then described by an effective Hamiltonian

\[ P_R = (1 + \gamma_5)/2. \]

The 2N×2N mass matrix \( m_R \) is written in terms of N×N matrices \( L \), \( R \), and \( D \), through

\[ m_R = \begin{pmatrix} L & D \\ D^T & R \end{pmatrix}. \]  \( \text{(3)} \)

Here, \( R \) and \( L \) are the right- and left-handed Majorana neutrino masses (\( L = 0 \) if required if electroweak gauge invariance is preserved), and \( D \) is the Dirac-mass matrix. The \( R \) and \( L \) matrices are required to be symmetric, and \( R \), \( L \), and \( D \) can most generally be complex.

### C. Neutrino Oscillations

The propagation of the flavor eigenstates is then described by an effective Hamiltonian

\[ (h_{\text{eff}})_{ab} = \begin{pmatrix} p_{\delta ab} + (\bar{m}^2)_{ab}/2p + (a_L)_{ab} \mu_{\mu}/p & 0 \\ 0 & p_{\delta ab} + (\bar{m}^2)_{ab}/2p - (a_L)_{ab} \mu_{\mu}/p \end{pmatrix}. \]  \( \text{(4)} \)
where the flavor indices \( a \) and \( b \) run over the flavor eigenstates \( e, \mu, \tau \) and \( e^c, \mu^c, \tau^c \). Here, \( p \equiv |\mathbf{p}| \), with \( \mathbf{p} \) the neutrino momentum, and \( m_i^2 \equiv m_i m_i^\dagger \) is the usual mass matrix, with \( m_i = L - D R^{-1}D^T \).

Equation 4 has several implications: (i) Since the matrix is block-diagonal, there is no mixing between neutrinos and antineutrinos (as may arise in more general Lorentz-violating scenarios [13]). (ii) Since \( a_L \) appears with opposite sign in the neutrino and antineutrino entries in the Hamiltonian, a nonzero \( a_L \) implies (apparent) CPT violation—i.e., the propagation of neutrinos and antineutrinos is not the same. Thus, for example, if the anomalous LSND results had stood, the CPT-violating explanations (e.g., Ref. [19]) for them [20] may have implied DE-neutrino coupling. (iii) The mixing induced by DE-neutrino coupling is energy independent (like in the explanations (e.g., Ref. [19]) for them [20]) as opposed to vacuum mixing, which declines as \( E \). Thus, these effects will become increasingly visible at higher energies. The detailed form of CPT violation implied by this effect is also thus different than that obtained with different \( \Delta m^2 \) for neutrinos and antineutrinos. (iv) There may also be novel effects for neutrinos propagating through matter, an effect we discuss further in Sec. [V] below.

Finally, (v) the neutrino oscillations induced by DE-neutrino coupling are frame dependent. If the observer is in the rest frame of the CMB, then \( (a_L)^\mu p_\mu \propto E \), and neutrino oscillations are independent of the neutrino direction. However, the Solar System moves with respect to the CMB rest frame with a velocity \( v \approx 370 \text{ km s}^{-1} \). DE-induced neutrino oscillations will therefore depend on \( (a_L)^\mu p_\mu \propto (1 - \mathbf{v} \cdot \mathbf{p}) \), where \( \mathbf{p} \) is the neutrino-propagation direction and \( \mathbf{v} \) is our peculiar velocity with respect to the CMB rest frame. There will thus be an annual modulation in solar-neutrino oscillations, a diurnal modulation in laboratory neutrino-mixing experiments, and a direction dependence in oscillations of cosmogenic neutrinos.

Since neutrino mixing arises only as a consequence of the traceless part of the propagation Hamiltonian, the DE-neutrino coupling must (like the vacuum mass matrix) be flavor-violating if neutrino oscillations are to be affected.

D. Two-flavor oscillations

The evolution equation for DE-induced two-flavor mixing is of the form,

\[
\frac{d}{dt} \begin{pmatrix} \nu_a \\ \nu_b \end{pmatrix} = \frac{1}{2} \begin{pmatrix} -\Delta m^2_{2E} \cos 2\theta_e - m_{\text{eff}}(1 - \mathbf{v} \cdot \hat{\mathbf{p}}) \cos 2\theta_d & m_{\text{eff}}(1 - \mathbf{v} \cdot \hat{\mathbf{p}}) \sin 2\theta_d e^{-i\eta} \\ \Delta m^2_{2E} \sin 2\theta_e + m_{\text{eff}}(1 - \mathbf{v} \cdot \hat{\mathbf{p}}) \sin 2\theta_d e^{i\eta} & m_{\text{eff}}(1 - \mathbf{v} \cdot \hat{\mathbf{p}}) \cos 2\theta_d \end{pmatrix} \begin{pmatrix} \nu_a \\ \nu_b \end{pmatrix},
\]

then the probability for one species of neutrino to convert to a different neutrino after a distance \( L \) is

\[
P(\nu_a \rightarrow \nu_b) = \sin^2 2\theta \sin^2 (ML).
\]

Here we have neglected the \( CP \)-violating phase in Eq. [6] because it does not affect the oscillation probability. The propagation Hamiltonian in Eq. [5] can be written in the form of Eq. [6] with the following relations [22]:

\[
\sin^2 2\theta = \frac{1}{M^2} \left[ \left( \frac{\Delta m^2}{4E} \right)^2 \sin^2 2\theta_e + \frac{m_{\text{eff}}^2 (1 - \mathbf{v} \cdot \hat{\mathbf{p}})^2}{4} \sin^2 2\theta_v \right] + \frac{\Delta m^2}{4E} m_{\text{eff}}(1 - \mathbf{v} \cdot \hat{\mathbf{p}}) \cos 2\theta_v \cos 2\theta_d + \sin 2\theta_v \cos 2\theta_d \cos \eta,
\]

Note that this time \( \sin 2\theta \) does indeed depend on \( \eta \), as it is not the overall phase, but the relative one, that cannot be rotated away by redefinition of wave functions.
The oscillation length is then \( L_{\mathrm{osc}} = \pi M^{-1} \). In the absence of DE-neutrino coupling, we recover the standard oscillation length \( L_{\mathrm{osc}} = 4\pi E/\Delta m^2 \) and mixing angle \( \theta = \theta_e \). If \( m_{\text{eff}} \gg \Delta m^2/2E \), then the oscillation length is \( L_{\mathrm{osc}} = 2\pi m_{\text{eff}}^{-1}(1 - \nu \cdot \vec{p})^{-1} \).

In general, \( m_{\text{eff}} \) can be either positive or negative. However, from the symmetry of the Hamiltonian, the relevant parameter space can be limited to \( m_{\text{eff}} \geq 0 \), \( 0 \leq \theta_d \leq \pi/4 \), and \( 0 \leq \eta \leq \pi/4 \).

Thus far, no deviations from standard three-flavor neutrino oscillations have been discovered in experimental data (except LSND \( \cite{20} \)), and this yields constraints on CPT-violating parameters, especially for \( m_{\text{eff}} \). By analyzing solar-neutrino and KamLAND data, Ref. \( \cite{16} \) obtained an upper limit of \( 3.1 \times 10^{-20} \text{ GeV} \) for \( \nu_e-\nu_\mu \) mixing. Atmospheric and accelerator data provide an upper limit for \( \nu_\mu-\nu_\tau \) mixing of \( m_{\text{eff}} < 5 \times 10^{-23} \text{ GeV} \) \( \cite{14} \).

III. ULTRA-HIGH-ENERGY NEUTRINOS

A. Prediction

Given that DE-induced neutrino mixing becomes increasingly important, relative to vacuum mixing, at high energies, the DE-neutrino coupling can be probed with ultra-high-energy cosmicogenic neutrinos. These neutrinos are produced by the interaction of ultra-high-energy cosmic-ray protons with CMB photons \( \cite{23} \):

\[
p_{\gamma} \rightarrow n_{\pi^+} \rightarrow n_{\mu^+} \nu_\mu \rightarrow n_{e^+} \nu_\mu \bar{\nu}_\mu.
\]

The fact that the Greisen-Zatsepin-Kuzmin cutoff \( \cite{24} \) has now been observed by the HiRes \( \cite{25} \) and Auger \( \cite{26} \) collaborations implies that this interaction must be occurring. And if so, there must be a population of cosmicogenic neutrinos with energies \( 10^{17} - 10^{20} \text{ eV} \) \( \cite{23, 24, 28, 29} \).

The characteristic distance between the source of these neutrinos and the Earth is the Hubble distance \( cH_0^{-1} \), which is much longer than the oscillation length—i.e., \( cH_0^{-1} \gg M^{-1} \)—as long as \( m_{\text{eff}} \gg H_0 = 10^{-42} \text{ GeV} \), as is always the case here. Therefore, any oscillatory features in neutrino mixing will be washed out; the probability for conversion of a cosmicogenic neutrino from its production flavor to another flavor en route from the source is then simply \( \sin^2 2\theta/2 \). Cosmicogenic neutrinos mostly originate from pion decays, with the characteristic flavor ratio \( \nu_e: \nu_\mu: \nu_\tau = 1:2:0 \). The result of standard vacuum mixing would be a flavor ratio at the Earth of \( \nu_e: \nu_\mu: \nu_\tau = 1:1:1 \). While possible corrections to this flavor ratio can be induced by small three-flavour oscillation effects or other new physics, here we concentrate on exploring the consequences of the DE-induced mixing.

For the sake of simplicity, we focus on \( \nu_{\mu}-\nu_\tau \) mixing (and their antiparticles). In the absence of a DE-neutrino interaction, these two flavors are maximally mixed; i.e., \( \theta_\tau = \pi/4 \), and thus even if only \( \nu_\mu \) are produced at the source, an equal number of \( \nu_\mu \) and \( \nu_\tau \) is generated by mixing. However, this can be altered if there is a DE-neutrino interaction. The flux of \( \nu_\mu \) and \( \nu_\tau \) at the detector is related to the \( \nu_\mu \) flux at the source through,

\[
\phi_{\nu_\mu} = \left( 1 - \frac{1}{2} \sin^2 2\theta \right) \phi_{\nu_\mu}^0,
\]

\[
\phi_{\nu_\tau} = \frac{1}{2} \sin^2 2\theta \phi_{\nu_\mu}^0,
\]

and \( \theta \) will in general differ from \( \theta_e \) if \( m_{\text{eff}} \neq 0 \).

B. Proposed Measurement

Here we investigate the possibility of measuring \( \theta \) using current or future ultra-high-energy neutrino experiments such as Auger \( \cite{30} \) and ANITA \( \cite{31} \). It is in principle possible to discriminate \( \nu_e \) from \( \nu_\mu \) by separately measuring the Earth-skimming events (\( \nu_\tau \)) and almost horizontal events originating in air (\( \nu_\mu \)).

We assume that a given experiment detects \( N_{\nu}^{\mathrm{tot}} \) neutrino events. This quantity is for the total neutrino and antineutrino flux; i.e., \( N_{\nu}^{\mathrm{tot}} = N_{\nu_e} + N_{\nu_\mu} \) (here \( \nu \) represents both neutrinos and antineutrinos). If the flavor democracy expected from vacuum mixing is realized, then one expects \( N_{\nu_\mu} = N_{\nu_\tau} = N_{\nu}^{\mathrm{tot}}/2 \). The number of neutrino events at the detector is related to the flux through,

\[
N_{\nu} = \int_{E_{\min}}^{E_{\max}} dE \phi_{\nu}(E) \Xi(E),
\]

where \( \Xi(E) \) is the detector exposure to neutrinos in units of cm\(^2\) sr, and it generally depends on neutrino energy. Here we assume \( \phi_{\nu}^{\mathrm{tot}} = \phi_{\nu} = KE^{-2} \) with a normalization constant \( K \), \( E_{\min} = 2 \times 10^{17} \text{ eV} \), and \( E_{\max} = 2 \times 10^{19} \text{ eV} \). This provides a good approximation for the spectrum of cosmicogenic neutrinos (e.g., Ref. \( \cite{29} \)). For simplicity we further assume that the detector exposure is independent of energy; the Auger exposure indeed depends on neutrino energy only weakly \( \cite{30} \). Therefore, the total number of neutrino events is given by

\[
N_{\nu}^{\mathrm{tot}} = \frac{K \Xi}{E_{\min}^2},
\]

and the number of \( \nu_\tau \) events is given by

\[
N_{\nu_\tau} = \int_{E_{\min}}^{E_{\max}} dE \frac{1}{2} \sin^2 2\theta \phi_{\nu_\mu}(E) \Xi
= \frac{K \Xi}{2} \int_{E_{\min}}^{E_{\max}} dE E^{-2} \sin^2 2\theta,
= \frac{N_{\nu}^{\mathrm{tot}} E_{\min}}{2} \int_{E_{\min}}^{E_{\max}} dE E^{-2} \sin^2 2\theta,
\]

where we used Eq. (14) in the last equality.
To investigate the sensitivity of a given experiment, we assume a null detection of new physics; i.e., the result of events. We also note that a weaker sensitivity, albeit still much better than the current sensitivity, may be achieved with neutrinos of slightly lower energies.

FIG. 1: Sensitivity on \((m_{\text{eff}}, \sin^2 2\theta_d)\) plane of future experiments that would yield \(N^\text{tot}_\nu = 12\) and 100 total neutrino events.

We now turn our attention to Lorentz/CPT-violating effects in electron-neutrino oscillations, showing here that novel effects may arise with DE-neutrino coupling as neutrinos propagate through the Earth. These effects may also be used to access atmospheric neutrinos regions of the DE-neutrino–coupling parameter space significantly below those currently probed. In this section, we consider two-flavor and three-flavor oscillations.

As neutrinos travel through matter, there is an additional contribution to oscillations from the matter potential \(\sqrt{2}G_F N_e\) (where \(G_F\) and \(N_e\) are, respectively, the Fermi constant and electron density) relevant if electron neutrinos are involved. Recalling that the matter potential is \(\gtrsim 10^{-22}\) GeV, the vacuum-mixing term \(\Delta m^2/2E\) is small for neutrino energies \(\gtrsim 10\) GeV. The mixing matrix Eq. (5) then becomes for \(\nu_e-\nu_\mu\) mixing (neglecting the overall factor of 1/2, the directional dependence, and the phase \(\eta\)),

\[
\begin{pmatrix}
-m_{\text{eff}} \cos 2\theta_d + \sqrt{2}G_F N_e & m_{\text{eff}} \sin 2\theta_d \\
m_{\text{eff}} \sin 2\theta_d & m_{\text{eff}} \cos 2\theta_d - \sqrt{2}G_F N_e
\end{pmatrix},
\]

(17)

Note that here, both the DE term and the matter potential change sign for antineutrinos, unlike the usual MSW effect, in which the vacuum term does not change sign. Unlike MSW mixing, there is essentially no energy dependence, at sufficiently high energies, in this mixing matrix.

To see when DE-induced mixing may be significant, recall that the value of the matter potential is \(\sqrt{2}G_F N_e = 7.6 \times 10^{-14} Y_e (\rho / g \text{ cm}^{-3})\) eV. The Earth core has average density \(\rho_{\text{core}} = 11.83\) g cm\(^{-3}\) and electron fraction \(Y_e = 0.466\), while the mantle has average density \(\rho_{\text{mantle}} = 4.66\) g cm\(^{-3}\) and \(Y_e = 0.494\), with the surface layer of the Earth having density as low as 2.6 g cm\(^{-3}\). The matter potential is thus about \(10^{-13}\) eV, so the effects of DE-induced mixing may be manifest for \(m_{\text{eff}}\) around \(10^{-22}\) GeV, well below current upper limits. In the absence of matter, as discussed in Ref. [11], it is possible to obtain a resonance when all mixing angles involved are maximal

\[
\frac{\Delta m^2}{2E} \cos 2\theta_\nu + m_{\text{eff}} \cos 2\theta_d = 0.
\]

Here, in the presence of matter and at high energies, a resonance can occur for a small mixing angle \(\theta_d\) when

\[
m_{\text{eff}} \cos 2\theta_d = \sqrt{2}G_F N_e.
\]

The presence of a resonance is thus entirely determined by the densities encountered along the path and the DE

\[\text{IV. MATTER EFFECTS IN ATMOSPHERIC NEUTRINO OSCILLATIONS IN THE PRESENCE OF A DARK-ENERGY COUPLING}\]

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decreases to values comparable to the oscillation probability as a function of zenith angle. As there are regular large-amplitude variations of the oscillation probability decreases, and the oscillation length is determined almost entirely by the DE term; to calculate the neutrino-propagation equation (including the small vacuum-mixing term) to calculate the oscillation probability as a function of zenith angle below those coming from the Lorentz-invariance-violating terms.

V. DIRECTIONAL DEPENDENCE

While detection of CPT/Lorentz-violating effects would be spectacular—it would imply new physics regardless of whether it is DE-related or not—the real “smoking gun” for a DE effect would be the directional dependence, \(\propto (1 - v \cdot \hat{p})\), of neutrino-oscillation parameters. Given that our peculiar velocity with respect to the CMB rest frame is \(10^{-3}\) times the speed of light, the magnitude of this effect is going to be suppressed relative to the other effects, discussed above, of a DE-neutrino interaction. Statistics well beyond the reach of current and forthcoming neutrino experiments will be required to detect this effect. Still, it is worth keeping in mind for future generations of experiments.

It may also be worth searching for such a directional dependence in current data, just in case there is a DE-neutrino coupling that is manifest in ways different than what we have seen before. For example, if DE somehow produces Lorentz violation through a modification of the kinetic term in the Dirac equation, the energy dependence of the mixing induced by Lorentz/CPT-violation could be different. We therefore work out in this Section expressions for the factor \(v \cdot \hat{p}\) to aid experimentalists who may wish to look for direction-dependent effects in their neutrino (or other) data.

To proceed, we first set our coordinate system. We set the origin at the center of Earth and align the \(z\) axis along the rotational axis of Earth, so that the north pole has positive \(z\) coordinate. We set the \(x\) axis along the direction to the Sun at vernal equinox. Since the Sun moves eastbound, its position at summer solstice aligns with the \(y\)-axis. We can thus represent the seasonal shift by an azimuthal angle \(\varphi\), where \(\varphi = 0, \pi/2, \pi,\) and \(3\pi/4\) for vernal equinox, summer solstice, autumn equinox, and winter solstice, respectively. Note also that the orbital plane of the Sun is inclined from the \(x\)-\(y\) plane by \(\theta_{\text{sun}} = 23.5^\circ\).

The Sun is moving with respect to the CMB rest frame with a speed of \(v_{\odot} = 369\) km \(s^{-1}\) towards the direction \(\alpha = 168^\circ\), \(\delta = -7.22^\circ\), where \(\alpha\) is right ascension and \(\delta\) is declination of the celes-
tial coordinates \([34]\). In our coordinates, the velocity of the Sun is \(v_\odot = v_\odot (\cos \delta \cos \alpha, \cos \delta \sin \alpha, \sin \delta) = (-358, 76.1, -46.4) \text{ km s}^{-1}\). The Earth is moving around the Sun with average orbital speed of \(V = 29.8 \text{ km s}^{-1}\). Thus, the velocity of the Earth with respect to the CMB rest frame is

\[
v_\odot = v_\odot + V \begin{pmatrix}
\sin \varphi \\
- \cos \varphi \cos \theta_{\text{inc}} \\
- \cos \varphi \sin \theta_{\text{inc}}
\end{pmatrix} = \begin{pmatrix}
-358 \\
76.1 \\
-46.4
\end{pmatrix} \text{ km s}^{-1}. \quad (20)
\]

We neglect the contribution from the rotation of Earth \((\lesssim 0.5 \text{ km sec}^{-1})\) to our velocity with respect to the CMB rest frame.

Now we evaluate the direction of the neutrino beam \(\hat{p}\). We suppose that the beam runs from some point A on the Earth’s surface to another point B on its surface (or vice versa). It should be straightforward to generalize the arguments below so that extraterrestrial neutrino production can be taken into account. We set the origin of time coordinate \(T\) to “noon” (i.e., when the Sun reaches highest) at the point A. Therefore, the positions of A and B in our coordinate are

\[
x_A = R_B \left( \cos \phi_A \cos (\omega T_A + \varphi) \over \sin \phi_A \right), \quad (21)
\]

\[
x_B = R_B \left( \cos \phi_B \cos (\omega T_A + \Delta \lambda + \varphi) \over \sin \phi_B \right), \quad (22)
\]

where \(R_B\) is the radius of the Earth, \(\omega\) is the rotational frequency \((2\pi/\text{day})\), \(T_A\) is the time at the position A relative to noon, \(\phi_{A,B}\) is the geometric latitude of the points A and B, and \(\Delta \lambda = \lambda_A - \lambda_B\) is the difference of the geometric longitude. The quantity \(\Delta \lambda\) appears in \(x_B\) because we measure the time (for both A and B) with respect to noon of the point A, so the time difference is given by the latitude difference (note also that the longitude increases to the west). The direction of the neutrino beam \(\hat{p}\) is then proportional to \(x_B - x_A\) with proper normalization as

\[
\hat{p} = \frac{1}{\sqrt{2(1 - \cos \phi_A \cos \phi_B \cos \Delta \lambda - \sin \phi_A \sin \phi_B)}} \begin{pmatrix}
\cos \phi_B \cos (\omega T_A + \Delta \lambda + \varphi) - \cos \phi_A \cos (\omega T_A + \varphi) \\
\cos \phi_B \sin (\omega T_A + \Delta \lambda + \varphi) - \cos \phi_A \sin (\omega T_A + \varphi)
\end{pmatrix}. \quad (23)
\]

Therefore, by combining Eqs. (20) and (23), we obtain the directional factor \(1 - v_\odot \cdot \hat{p}\). Since it is a scalar quantity, the final result does not depend on the choice of the coordinate system.

VI. THEORETICAL IMPLICATIONS

Before closing, we discuss, for illustration, the implications of a measurement of a particular value of the Lorentz-invariance–violating effective mass parameter \(m_{\text{eff}}\) in terms of a specific model of DE-neutrino coupling.

Perhaps the simplest interaction of this kind has the form

\[
\mathcal{L}_{\text{int}} = -\lambda_{\alpha\beta} \phi \over \sqrt{M_\star} \gamma^\mu (1 - \gamma_5) \nu_{\beta}, \quad (24)
\]

where \(\phi\) is a quintessence field, \(\lambda_{\alpha\beta}\) is a coupling-constant matrix, and \(M_\star\) is some mass scale. Thus, \(a_L^\mu \sim \lambda \phi(t)^\mu M_\star\), and \(m_{\text{eff}} \sim \Delta \lambda \phi(t)/M_\star\), where \(\Delta \lambda\) is the difference between eigenvalues of the \(\lambda\) matrix. For quintessence, one expects \(\phi \sim M_{\text{Pl}} H_0 (1 + w)^{1/2}\) (e.g., Ref. [3]), where \(M_{\text{Pl}}\) is the Planck energy scale. In this case, the mass scale \(M_\star\) corresponding to a given \(m_{\text{eff}}\) is

\[
M_\star \approx 10^6 (\Delta \lambda) \left(\frac{1 + w}{0.01}\right)^{1/2} \left(\frac{m_{\text{eff}}}{10^{-30} \text{ GeV}}\right) \text{ GeV}, \quad (25)
\]

to the mass scale that controls the DE-neutrino interaction. The ultra-high-energy \(\nu_\mu - \nu_\tau\) oscillation effects we have discussed thus probe up to mass scales \(M_\star \sim 10^6\) GeV. The \(\nu_\tau - \nu_\mu\) oscillations induced by the matter effects we discussed probe up to mass scales \(M_\star \sim 100\) MeV.

VII. CONCLUSIONS

We studied the implications of an interaction between dark energy and neutrinos for neutrino oscillations. The most general Lorentz/CPT-violating term induced by dark energy (DE) takes the form \((a_L)^\mu \nu_\mu (1 - \gamma_5) \nu\), where \((a_L)^\mu\) is a four-vector normal to the CMB rest frame. This introduces a new source for neutrino oscillations that are energy independent and different for neutrinos and antineutrinos. Furthermore, the motion of the Earth with respect to the cosmic rest frame induces a directional dependence in the oscillation probabilities.
The current best limits to the DE-neutrino coupling we considered are obtained from atmospheric- and accelerator-neutrino experiments for $\nu_\mu$-$\nu_\tau$ mixing, and from solar and reactor experiments for $\nu_e$-$\nu_\mu$ mixing. However, the higher the neutrino energy, the more prominent the effect of the DE-neutrino interaction. We therefore considered in this paper cosmogenic ultra-high-energy (energies of $10^{17}$-10$^{18}$ eV) neutrinos produced by the interaction of ultra-high-energy cosmic rays with CMB photons. We showed that future experiments targeting these neutrinos will improve the sensitivity to a DE-neutrino interaction by seven orders of magnitude, down to $m_{\text{eff}} \sim 10^{-30}$ GeV compared with the current upper bound $m_{\text{eff}} \lesssim 5 \times 10^{-23}$ GeV (Fig. 1). This corresponds to a sensitivity to an energy scale as large as $\sim 10^6$ GeV for the DE-neutrino interaction. We then showed that the interplay of DE- and matter-induced neutrino mixing could induce a novel zenith-angle dependence for $\nu_e$ oscillations in atmospheric neutrinos. This effect may extend the sensitivity to Lorentz/CPT-violating parameters in the $\nu_e$ by roughly three orders of magnitude.

The real smoking gun of a DE-neutrino interaction (as opposed to some other origin for Lorentz/CPT violation) would be a directional dependence of the oscillation probabilities. The notion that Lorentz violation may give rise to a directional dependence is not new (e.g., Ref. [39]) and searches for directional dependence in neutrino experiments have already been carried out (e.g., Ref. [50]), but prior work has considered Lorentz-violating parameters introduced in an ad hoc manner and/or tested for direction-dependent effects in a Sun-centered inertial frame. We emphasize here that cosmic acceleration suggests that we seek a specific form of Lorentz violation, that where the preferred frame is aligned with the CMB rest frame. Even though such a signal is expected to be small, it is still worth seeking in existing and future experimental data.

We have not discussed specific models for a DE-neutrino interaction, beyond an illustrative toy model, but it may be interesting to do so (see also Ref. [38]). The theoretical motivation to expect such a coupling may admittedly be slim. However, we are at square one in our understanding of DE, and such a coupling is no less likely to be expected, perhaps, than any of the many other manifestations of new cosmic-acceleration physics that have been considered. Discovery of Lorentz/CPT-violating effects would be extremely important, even if not attributable directly to dark energy. A directional dependence, if discovered, would be absolutely remarkable, as it would provide moreover clear evidence that there is more to cosmic acceleration than simply a cosmological constant.

Acknowledgments

We thank Alexander Friedland and Stephon Alexander for useful discussions, and we acknowledge the hospitality of the Aspen Center for Physics. This work was supported by the Sherman Fairchild Foundation (SA), DoE DE-FG03-92-ER40701 (MK), and NSF grant PHY-0555368 (IM).

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