Amplified propagating plasmon in asymmetrical graphene periodic structure

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Abstract

The excitation and amplification of the propagating plasmons in a periodic active (with population inversion of charge carriers) graphene-based structure with dual grating-gate with an asymmetric unit cell is studied theoretically. Such structure transforms the normally incident THz electromagnetic wave into sustained or amplified unidirectionally propagating plasmon. This effect is caused by simultaneous excitation of radiative and ‘non-radiative’ plasmon modes at the same frequency.

1. Introduction

Graphene, representing a true two-dimensional material with a thickness of one atom, is a suitable platform for the excitation and propagation of plasmons [1–3]. Due to high mobility of charge carriers [4] and tunable dynamic mass [5], plasmons in graphene can be efficiently excited in the terahertz (THz) frequency range at room temperature. Linear carrier dispersion as well as zero band gap of charge carriers in graphene [6] allow for creating the population inversion [7, 8], which can promote the amplification of THz radiation and THz plasmons [9]. The population inversion in graphene is observed in picoseconds time-scale, which might be utilized for amplification of THz radiation [10, 11]. The optical pumping of graphene leads to the optical phonon emission and following strong Auger recombination [12], which creates hot two-dimensional electron-hole gas. However, improved methods of graphene pumping like carrier injection [11, 13–15], diffusion pumping [16] (including diffusion pumping by cold electrons [17]) and Landau-Zener pumping [18] can solve the problem. Radiative recombination of charge carriers in graphene leads to its negative dynamic conductivity [19]. Experimental evidences of population inversion in graphene were found in different papers [20–22].

Plasmons in graphene structures can be used to generate [23], detect [24], and transform [25] THz radiation. Since the plasmons are slow surface waves, their excitation in graphene are only possible using different scattering elements or slow-wave structures [26, 27]. The efficient energy transfer from electromagnetic wave to plasmons can be achieved with using the periodical metallic gratings, [28, 29] which provides the coupling of THz radiation with plasmons. The gratings without the center of inversion are used for excitation of plasmons in graphene with noncentrosymmetric spatial electric field distribution [30]. Such noncentrosymmetric electric fields trigger the quadratic nonlinear effects in graphene and can induce the unidirectional propagation of excited plasmon in graphene structure [31]. However, in spite of effective energy transfer from electromagnetic wave into propagating plasmon in graphene structure, the estimated plasmon mean free path reaches only several wavelength for reasonable value of carrier relaxation time (1 ps for 300 K) [32].

In this work, we investigate the excitation and amplification of unidirectionally propagating plasmons in periodic graphene structure with population inversion. We theoretically study the plasmon lasing conditions and conditions of excitation of propagating plasmon in order to obtain a sustaining and amplifying unidirectionally propagating plasmons.
2. Model and theoretical approach

The dual-grating-gate graphene (DGGG) structure under study is shown in figure 1. The active graphene layer is encapsulated between two dielectric slabs and located at a distance \(d\) from the grating–gate and at a distance \(h\) from homogeneous metallic screen, which serves as a back gate. The grating gate and back gate form Fabry–Perot resonator in the THz range. The conductivity of the back gate metal and grating gate metal corresponds to gold in the THz frequency range. The dual grating gate consists of two periodical interdigitated subgratings. The lateral shift of subgratings in respect to each other creates the spatial asymmetry of DGGG structure. A unit cell of the grating-gate with a spatial period \(L\) contains two metal electrodes of zero thickness with widths \(w_1\) and \(w_2\), and width of the gaps between them \(l_1\) and \(l_2\). We assume that the plane THz electromagnetic wave with the electric field polarized along the \(x\)-direction (across the grating-gate fingers) is incident from vacuum normally onto the structure plane.

The problem of excitation of the plasmon modes in DGGG structure is solved in the framework of a self-consistent electromagnetic approach developed by the authors of this paper, which is based on the integral equation method using the full system of the Maxwell equations (similar approach see in [31]). The induced electric field is written as follows:

\[
E_x(x, y, t) = \exp(-i\omega t) \sum_{p=-\infty}^{\infty} e_{xp}(y) \exp(iq_px),
\]

(1)

where \(e_{xp}(y)\) is the Fourier amplitude of the \(x\)-component of the electric field, \(q_p = 2\pi p/L\) is the Fourier harmonic wave vector, where \(L\) is the spatial period of the structure, and \(\omega\) is the frequency of the incident THz wave. The THz response of graphene is described by the complex dynamic THz conductivity [19]:

\[
\sigma(\omega) = \frac{e^2}{4\hbar} \left\{ \frac{8k_B T\tau}{\pi \hbar (1 - i\omega\tau)} \ln \left[ 1 + \exp \left( \frac{E_F}{k_B T} \right) \right] + \tanh \left( \frac{\hbar\omega - 2E_F}{4k_B T} \right) \frac{4\hbar\omega}{i\pi} \int_0^{\infty} \frac{G(E - G(\hbar\omega)/2)}{(\hbar\omega)^2 - 4E^2} dE \right\},
\]

(2)

where \(e\) is the electron charge, \(h\) is the reduced Planck constant, \(k_B\) is the Boltzmann constant, \(\tau\) and \(T\) are the momentum relaxation time and temperature of the charge carriers in graphene, respectively, \(E_F\) is the quasi-Fermi energy in graphene determining the inversion of charge carriers (+\(E_F\) and –\(E_F\) for electrons and holes, respectively), and \(G(\xi) = \sinh(\xi/(k_B T))/[\cosh(\xi/(k_B T)) + \cosh(E_F/(k_B T))]\). Inversion of charge carriers triggers their radiative recombination which corresponds to the negative real part of the graphene dynamic conductivity [19]. Consequently, the Joule heating in graphene becomes negative, which leads to the negative values of the absorbance coefficient. Graphene amplifies each Fourier harmonic of the oscillating electric field (1), which falls into the frequency range of the negative graphene conductivity (2). However, only few of those Fourier harmonics are important at a particular plasmon mode resonance.

Figure 1. The schematic view of DGGG structure.
The energy flux density in the structure is described by the time-averaged Poynting vector \( \mathbf{S} = 0.5 \text{Re}[\mathbf{E} \mathbf{H}^*] \), where \( \mathbf{E} \) is the total electric field and \( \mathbf{H} \) is the total magnetic field. The energy flux density of induced TM electromagnetic wave has two components: normal to graphene \( S_y = 0.5 \text{ Re}[E_z H_y^*] \) and lateral plasmon flux \( S_x = -0.5 \text{ Re}[E_x H_y^*] \). The lateral energy flux of the plasmon in the \( x \)-direction is given by averaged over the spatial period of the structure and integrated over the \( y \)-coordinate the \( x \)-component of the Poynting vector

\[
S_x^P = \frac{1}{L} \int_{-\infty}^{\infty} \int_0^L S_x \, dx dy.
\]

The Poynting flux of the plasmon in the DGGG structure can be divided into two oppositely propagating fluxes as \( S_x^P = S_x^{P+} + S_x^{P-} \), where \( S_x^{P+} \) is the Poynting flux in the positive \( x \)-direction (to the right in figure 1), and \( S_x^{P-} \) is the Poynting flux in the negative \( x \)-direction. Accordingly, we introduce two coefficients of transformations of the incident wave into left and right propagating plasmons as

\[
T^+_p = \frac{S_{x,y}^{P+}}{S_{x,y}^{\text{inc}}}, \quad T^-_p = \frac{S_{x,y}^{P-}}{S_{x,y}^{\text{inc}}},
\]

where \( S_{x,y}^{\text{inc}} = LS_{x,y}^{\text{inc}} \) is the Poynting flux of THz wave incident onto the unit cell of the periodic DGGG structure. The total transformation coefficient is \( T_p = T^+_p + T^-_p \). The energy flux densities of incident, reflected, and transmitted waves can be written as

\[
S_y^{\text{inc}} = \frac{1}{2} \sqrt{\frac{\varepsilon \varepsilon_0}{\mu_0}} |\mathbf{e}_x^{\text{inc}}|^2, \quad S_y^R = \frac{1}{2} \sqrt{\frac{\varepsilon \varepsilon_0}{\mu_0}} |\mathbf{e}_x^{(1)}|^2,
\]

\[
S_y^T = \frac{1}{2} \sqrt{\frac{\varepsilon \varepsilon_0}{\mu_0}} |\mathbf{e}_x^{(3)}|^2.
\]

Then the absorption coefficient of the DGGG structure is

\[
\alpha = 1 - \left( S_y^T + S_y^R \right) / S_y^{\text{inc}}.
\]

3. Results and discussions

The electric field of each excited plasmon mode has a number of important Fourier–harmonics with wave vectors \( k_x = \frac{\pi n}{d} \). The most effective excitation of propagating plasmons in a lossy DGGG structure corresponds to excitation of the plasmon modes with wave vectors \( k_x = 2\pi p/L \) (\( p \geq 2 \) is a positive integer) quantized by the length of the DGGG structure period \( L \) [31]. This is because, in spatially asymmetric structure, the frequencies of the radiative (bright) and ‘non-radiative’ (weak) plasmon modes with wave vectors \( k_x = 2\pi p/L \) (\( p \geq 2 \)) can be tuned to the same frequency [33]. Appropriate interaction of strongly excited bright plasmon and weakly excited ‘non-radiative’ but essentially asymmetric plasmon yields the propagating plasmon. It is worth noting that the frequencies of radiative and ‘non-radiative’ plasmons for \( p = 1 \) cannot coincide due to different screening of oscillating electric fields by the gate electrodes, and, in this case, the propagating plasmon is suppressed.

We made calculations for the DGGG structure with parameters: \( \omega_1 = 500 \text{ nm}, l_z = 80 \text{ nm}, d = 120 \text{ nm}, h = 7 \mu \text{m}, E_p = 50 \text{ meV}, \tau = 0.5 \text{ ps} \). At such parameters of \( E_p \) and \( \tau \), the dynamic conductivity of graphene has negligible real part at frequencies greater than 3.95 THz (in accordance with (2)). By choosing the thickness \( h \), the frequency of the Fabry–Perot resonance was tuned to 4.3 THz in order to increase the energy transfer from the electromagnetic wave to the plasmon. We introduce the grating–gate filling factor \( K = (w_1 + w_2)/L \), which describes the metallic filling of each unit cell. Keeping the sum \( w_1 + l_z = 970 \text{ nm} \) and length of the period \( L = w_1 + l_z + w_2 + l_z = 1550 \text{ nm} \) we calculated the absorbance and transformation coefficients spectra (figure 2) depending on gate width \( w_1 \) described by corresponding filling factor \( K(w_1) \). The negative value of absorbance corresponds to the amplification regime that occurs due to radiative recombination of charge carriers in graphene when plasmon gain prevails over the total damping in DGGG structure, which corresponds to the negative part of the graphene dynamic conductivity. The values of the absorbance coefficient \( \alpha \) exceeding 100 correspond to the plasmon lasing mode, which can be excited when accurate balance of the Drude plus radiative losses and the gain caused by radiative electron–hole recombinations in graphene is reached [34].

Figure 2 shows the excitation of bright and weak plasmon modes with wave vector \( k_x = 2\pi p/L \) depending on the filling factor \( K \). The maximum negative absorbance values for the bright and weak plasmons are achieved at different frequencies mostly due to the difference in radiative losses of these plasmon modes (figure 2(a)).

Depending on the grating–gate filling factor, the frequencies of plasmon modes lay between square root and linear dependence on the wave vector. The resonant frequencies of the bright and weak plasmons can be shifted from each other due to different screening efficiency of bright and weak plasmons. This effect of different screening efficiency is caused by the fact of which part of the oscillation charge density of plasmon mode are located under the gate strips. The variation of the filling factor \( K \) changes the screening efficiency of each plasmon mode and, consequently, changes their resonant frequencies. Approaching the plasmon resonances of...
bright and weak plasmon modes changes the radiative damping of the structure, which governs the conditions of plasmon lasing in the DGGG structure (when gain balances total losses in the structure). Tuning the coupling between plasmon resonances we found the maxima of the transformation coefficient $T_p$ (figure 2(b)). All these maxima of the transformation coefficient $T_p$ correspond to the total lateral energy flux along the graphene caused by the excitation of amplifying plasmon.

For unidirectional plasmon propagation, it is necessary to realize the appropriate interaction of radiative and ‘nonradiative’ plasmon modes. The excitation of unidirectional propagating plasmon resonance demands for inherent asymmetry of the nonradiative plasmon mode and for radiative damping of the radiative plasmon mode. Such interaction can be achieved by changing the filing factor $K$. We found three values of $K$ when the incident electromagnetic wave is transformed into unidirectionally propagating plasmon (horizontal lines in figures 2(a) and (b)). At such values of $K$, the transformation coefficients $T_p^+$ exceed the coefficients $T_p^-$ (or vice versa) by an order of magnitude, which means the excitation of the unidirectionally propagating plasmon (figures 3(a), (c), (e)). Absorbance coefficients are negative for all points of excitation of unidirectional plasmons: $A = -1.3$ for $K = 0.92$ (figures 3(a), (b)), $A = -0.65$ for $K = 0.77$ (figure 3(c), (d)), and $A = -1.1$ for $K = 0.57$ (figures 3(e), (f)). Corresponding spatial distributions of the in-plane electric field shown in figures 3(b), (d), and (f) reveal the excitation of unidirectionally propagating plasmons as a weak plasmon modes. The electric field distribution of such weak plasmon modes is shifted by a quarter of plasmon wavelength from that of the radiative plasmon mode.

Now we choose the length $l_1 = 270$ and calculate the dependence of absorbance spectra on width $w_1$, keeping all other parameters constant. For finding the best excitation of the propagating plasmon, we tune the losses by changing the momentum relaxation time $\tau$ in order to move the lasing points of the bright plasmon mode and the point of best amplification of the weak plasmon mode close to the frequency of the best excitation of propagating plasmon (figure 4). The laser points of the bright mode are shown by yellow circles and the points

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Figure 2. The dependences of (a) the absorbance spectrum $A$ and (b) the transformation coefficient spectrum $T_p$ on the grating-gate filing factor $K = (w_1 + w_2)/L$. Horizontal lines show the strongest transformation of normally incident electromagnetic wave into propagating plasmon with wave vector $k_x = 2\pi 2/L$. The plasmon lasing point is shown by yellow circle in panel (a).
The best amplification of weak modes are shown by purple circles in figure 4. Absorbance values exceed $-100$ for all laser points (figure 4). The best values of absorbance for weak modes reach $-8$ for $\tau = 0.5$ ps (figure 4(c)), and are lower then $-0.5$ for $\tau = 0.4$ ps and $\tau = 0.6$ ps (figure 4(b) and (d)). The efficiency of transformation of the incident THz wave into the unidirectionally propagating plasmon grows with increasing amplification. However lasing regime cancels the unidirectional plasmon propagation and leads to two oppositely propagating plasmons instead. With increasing the relaxation time, the resonances become too narrow and, hence, cannot effectively interact with each other, which detune the coupling of bright and weak (‘non-radiative’) modes, and consequently, deteriorates the unidirectional plasmon propagation. (figure 4).

*Figure 3.* (a), (c), (e) The coefficients of transformation of the incident THz wave into the left ($T_L$) and right ($T_R$) propagating plasmons and the total transformation coefficient ($T_p$). (b), (d), and (f) The spatial distributions of the in-plane component of the plasmon electric field as functions of frequency.

*Figure 4.* The absorbance spectra of the DGGG structure as a function of width $w_1$ for different values of $\tau$ in graphene. The laser points of the bright mode are shown by yellow circles and the points of the best amplification of weak mode are shown by purple circles. The red dots show the position of the best excitation of the unidirectional plasmon.
4. Conclusion

In conclusion, the excitation and amplification of the propagating plasmons in a periodic active (with population inversion of charge carriers) graphene-based structure with dual grating-gate having an asymmetric unit cell was studied theoretically. It was found that the maximum transformation efficiency of the incident THz wave into a propagating plasmon occurs when the radiative and 'non-radiative' plasmon modes interact with each other in the amplification regime. For unidirectional plasmon propagation, the energy flux of the plasmons propagating in a certain direction exceeds the energy flux of the plasmons propagating in the opposite direction by more than an order of magnitude. Thus, the structure under study can be used as a unidirectional source and amplifier of propagating THz plasmons in graphene.

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