Generating functionals of correlation functions of $p$-form currents in AdS/CFT correspondence

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Abstract

The generating functional of correlation functions of the currents corresponding to general massless $p$-form potential is calculated in $AdS/CFT$ correspondence of Maldacena. For this we construct the boundary-to-bulk Green’s functions of $p$-form potentials. The proportional constant of the current-current correlation function, which is related to the central charge of the operator product expansion, is shown to be $c = \frac{d-p}{2\pi^2} \Gamma(\frac{d-p}{2}) \Gamma(\frac{d}{2}-p)$. The result agrees with the known cases such as $p = 1$ or $2$. 

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1 Introduction

Ever since the introduction of Maldacena’s proposal\cite{1} that the classical supergravity action on Anti-de Sitter space ($AdS$) can be used to determine the current-current correlation function of the dual conformal field theory ($CFT$) on the boundary of $AdS$\cite{2,3}, various successful applications are reported in the direction of confirming it\cite{4,5,6}.

Two point correlators for the case of scalar fields\cite{2,3,7,8,9}, three\cite{10} and four-point correlators\cite{12,13} are investigated. For massless vectors it is rather simple when one benefits from the gauge transformations to eliminate the $A_0$ component of $AdS$ vector potential\cite{3}. But for massive vector fields there is no such transformations. In fact the determination of $A_0$ component of a vector potential is an essential element of holographic projection of massive vector fields\cite{14,15}. Spinors\cite{16,17,14}, gravitinos\cite{18,19}, and gravitons\cite{11,12} are also investigated. The next natural step is the consideration of $p$-form potential. Since supergravity actions and brane theories contain various $p$-form potentials, it is rather one of the imperative for the full understanding of $AdS/CFT$ correspondence.

In this paper the generating functional of current-current correlation functions of general $p$-form potentials are constructed using the $AdS/CFT$ correspondence. We follow Witten’s intuitive methodology\cite{3} to construct the boundary-to-bulk Green’s function. But rather than delving into the $p$-form potential we begin with the 2-form case. Gauge transformation of 2-form potential is used to elemenate unnessesary components of the potential. The boundary-to-bulk Green’s function which is in fact a 2-form is constructed using the differential form language. The classical action which is served as the generating functional of current-current correlation functions is determined from this 2-form Green’s function. This is presented in the next section. The generization to the general $p$-form case is rather straight forward. This is done in section 3. Appendix is devoted to prove that the Witten’s method causes no Ward identity problem for massless $p$-form cases which we consider.
2 General consideration of boundary-to-bulk Green’s function

According to the Maldacena’s proposal the supergravity partition function $\mathcal{Z}_{AdS_{d+1}}[\phi_a]$ of general fields $\phi_a$ on $AdS_{d+1}$ can be used as the generating functional $\langle \exp \int d^d x \varphi_i(x) \mathcal{O}_i(x) \rangle_{CFT}$ of the operator product expansions of sources $\mathcal{O}_i(x)$ on the boundary of $AdS$. The relation between $\phi_a$ and $\varphi_i$ is that when one goes to the boundary of $AdS$, $\phi_a$ reduces to $\varphi_i$. When the classical approximation is employed to the supergravity partition function, Maldacena’s proposal implies that the classical action $I[\phi_a]$ of the bulk field $\phi_a$, when written in terms of boundary field $\varphi_i$, serves as the generating functional of the OPE. This means that it is essential for the calculation of OPE of the sources $\mathcal{O}_i(x)$ to determine the boundary-to-bulk Green’s function.

For our purpose it is sufficient to employ the Euclidean $AdS$ which is characterized by the Lobachevsky space $\mathbb{R}^{1+d}_+ = \{(x_0, \mathbf{x}) \in \mathbb{R}^{1+d} \mid x_0 > 0\}$ with the Poincare metric of the following form,

$$ds^2 = \frac{1}{x_0^2} \left[(dx^0)^2 + (d\mathbf{x})^2\right].$$

(1)

The $x_0 \to \infty$ point and the $x_0 \to 0$ region consist the boundary of $AdS$. To simplify the notation we sometimes confuse $x^0$ with $x_0$ unless it is stated explicitly. Roman characters such as $i$ and $j$ are used to denote the indices $1, \ldots, d$, and Greek characters such as $\mu$ and $\nu$ are preserved to denote whole indices $0, 1, \ldots, d$.

It is well known that to have correct correlation functions which preserve the Ward identity one should be careful to have all fields to approach the boundary of AdS uniformly. For this an imaginative $x_0 = \epsilon$ boundary is first used to compute the classical action, and then one take the $\epsilon \to 0$ limit. In [15] this procedure in coordinate-space is analysed. Using this algorism one may prove that for massless $p$-form field we do not need to worry about the $\epsilon$-boundary procedure. For more details, see appendix.
3 The generating function of OPE of 2-form currents

Consider a 2-form potential $\mathcal{A}$ in $AdS$. The free action is given by

$$I = \frac{1}{2} \int_{AdS_{d+1}} \mathcal{F} \wedge \ast \mathcal{F},$$

(2)

where $\mathcal{F} = d\mathcal{A}$ is the field strength 3-form. The classical equation of motion of $\mathcal{A}$ obtained from this action is

$$d^\ast d\mathcal{A} = 0.$$  

(3)

To calculate the current-current correlation function in $AdS/CFT$ correspondence we need to know the boundary-to-bulk 2-form Green’s function,

$$K(x_{bk}, x_{bd}) = \frac{1}{2} K_{\mu\nu}(x_{bk}, x_{bd}) dx_{bk}^\mu \wedge dx_{bk}^\nu.$$  

(4)

Here $x_{bk}$ is a point in $AdS$, and $x_{bd}$ is a point on the boundary of it. $K$ satisfies the same equation which $\mathcal{A}$ satisfies, and its component becomes the boundary-space delta function as $x_{bk}$ approaches $x_{bd}$. Using the gauge transformation,

$$\mathcal{A} \rightarrow \mathcal{A} + d\Lambda,$$  

(5)

of a 2-form potential we may assume that all $K_{0i}$, $i = 1, ..., d$, vanish. For simplicity we take as $x_{bd}$ the boundary point at $x_0 \rightarrow \infty$. The boundary of $AdS$ and the Poincare metric are both invariant under the following translation,

$$\mathbf{x} \rightarrow \mathbf{x} - \mathbf{x}',$$  

(6)

where $\mathbf{x}'$ is a constant vector. This allows us to assume that $x_{bk}$ is $(x_0, 0, ..., 0)^3$. The general form of $K$ under these restrictions is

$$K_\infty = K_\infty(x_0) dx^i \wedge dx^j,$$  

(7)

where $i < j$. The infinity symbol in this equation denotes the fact that the boundary point of $K$ is chosen to be $x_0 \rightarrow \infty$. 

3
The equation which is satisfied by $K_\infty$ is

$$d^*dK_\infty = 0. \quad (8)$$

To solve this we compute $dK_\infty$,

$$dK_\infty = \partial_0 K_\infty(x_0) dx^0 dx^i dx^j, \quad (9)$$

and use the relation,

$$*(dx^0 dx^i dx^j) = (-)^{i+j+1} x_0^{-(d-5)} \tilde{dx}^0 \tilde{dx}^1 ... \tilde{dx}^i ... \tilde{dx}^j ... dx^d, \quad (10)$$

where $\tilde{dx}^0$, etc., means that $dx^0$ should be omitted in the wedge product. The result is

$$\partial_0 \left( x_0^{-(d-5)} \partial_0 K_\infty(x_0) \right) = 0, \quad (11)$$

whose solution is

$$K_\infty = \alpha x_0^{d-4} dx^i dx^j. \quad (12)$$

Here $\alpha$ is an unknown constant.

For the case $d > 4$, it is clear that as $x_0$ goes to 0, $K_\infty(x_0)$ vanishes. When $x_0$ goes to $\infty$, $K_\infty(x_0)$ diverges. This means that $K_\infty$ looks like a boundary-to-bulk Green’s function. Before proving this fact explicitly we benefit from the following gauge transformation,

$$K_\infty \rightarrow K_\infty + d(\beta x_0^{d-4} x^i dx^j - \gamma x_0^{d-4} x^j dx^i). \quad (13)$$

The transformed $K_\infty$ is given by

$$K_\infty = (\alpha + \beta + \gamma) x_0^{d-4} dx^i dx^j + (d-4) \beta x_0^{d-5} x^i dx^0 dx^j - (d-4) \gamma x_0^{d-5} x^j dx^0 dx^i. \quad (14)$$

By choosing the coefficients which satisfy

$$\alpha + \beta + \gamma = 1, \quad \beta = \gamma = - \frac{1}{d-4}, \quad (15)$$

$K_\infty$ reduces to

$$K_\infty = x_0^{d-4} dx^i dx^j - x_0^{d-5} x^i dx^0 dx^j + x_0^{d-5} x^j dx^0 dx^i. \quad (16)$$
It is still possible to rescale \( K_\infty \) by a constant \( c \) without losing the desired property.

To prove that \( K_\infty \) in fact generates the correct boundary-to-bulk Green’s function we use the following isometry of (1) which is discussed in detail in [10],

\[
x^\mu \to \frac{x^\mu}{x_0^2 + |x'|^2}.
\] (17)

It is important to note that under this isometry the boundary point corresponding to \( x_0 \to \infty \) transforms to the boundary point \((x_0, x) \to 0\). This isometry transforms \( K_\infty \) to

\[
K_0 = \frac{x_0^{d-4}}{(x_0^2 + |x|^2)^{d-2}} dx^i dx^j - \frac{x_0^{d-5} x^i}{(x_0^2 + |x|^2)^{d-2}} dx^0 dx^j + \frac{x_0^{d-5} x^j}{(x_0^2 + |x|^2)^{d-2}} dx^0 dx^i,
\] (18)

where the subscript “0” in \( K_0 \) means that the boundary point of \( K_0 \) corresponds to \((x_0, x) \to 0\).

To get the Green’s function corresponding to a general boundary point \((0, x')\) we use the translational symmetry (6). This gives the final Green’s function,

\[
c K = \frac{x_0^{d-4}}{(x_0^2 + |x - x'|^2)^{d-2}} dx^i dx^j - \frac{x_0^{d-5} x^i}{(x_0^2 + |x - x'|^2)^{d-2}} dx^0 dx^j + \frac{x_0^{d-5} x^j}{(x_0^2 + |x - x'|^2)^{d-2}} dx^0 dx^i,
\] (19)

where \( c \) is a normalization constant. On the other hand, it is not difficult to show that

\[
\lim_{x_0 \to 0} \frac{x_0^{d-2
u}}{(x_0^2 + |x - x'|^2)^{d-\nu}} = c_{d,\nu} \delta^{(d)}(x - x'),
\] (20)

where,

\[
c_{d,\nu} = \pi^\frac{d}{2} \Gamma(\frac{d}{2} - \nu) \Gamma(d - \nu).
\] (21)

This means that by choosing the constant \( c \) of (19) which is equal to \( c_{d,2} \), \( K \) of (19) is in fact the desired boundary-to-bulk Green’s function.

Now let \( A = \frac{1}{2} A_{ij}(x)dx^i dx^j \) be a 2-form potential on the boundary of \( AdS \). Using (19) we have the following 2-form bulk potential,

\[
c_{d,2} A(x_0, x) = \left[ x_0^{d-4} \int d^d x' \frac{A_{ij}(x')}{(x_0^2 + |x - x'|^2)^{d-2}} \right] \frac{1}{2} dx^i dx^j - 2 \left[ x_0^{d-5} \int d^d x' \frac{(x^i - x'^i) A_{ij}(x')}{(x_0^2 + |x - x'|^2)^{d-2}} \right] \frac{1}{2} dx^0 dx^j,
\] (22)
where we used the anti-symmetric property of $A_{ij}$. The field strength 3-form $F$ is given by

$$c_{d,2} F(x_0, \mathbf{x}) = (d - 2)x_0^{d-5} \int d^d x' \frac{A_{ij}(\mathbf{x'})}{(x_0^2 + |\mathbf{x} - \mathbf{x'}|^2)^{d-2}} \frac{1}{2} dx^0 dx^i dx^j$$

$$-2(d - 2)x_0^{d-3} \int d^d x' \frac{A_{ij}(\mathbf{x'})}{(x_0^2 + |\mathbf{x} - \mathbf{x'}|^2)^{d-1}} \frac{1}{2} dx^0 dx^i dx^j$$

$$-4(d - 2)x_0^{d-5} \int d^d x' \frac{(x^j - x'^j)(x^k - x'^k) A_{ij}(\mathbf{x'})}{(x_0^2 + |\mathbf{x} - \mathbf{x'}|^2)^{d-1}} \frac{1}{2} dx^0 dx^i dx^k + ...$$

where the abbreviated terms are those which do not contain $dx^0$.

By partial integration the bulk action becomes

$$I = \lim_{\epsilon \to 0} \frac{1}{2} \int_{x_0 = \epsilon} A \wedge * F.$$

The following relation

$$dx^i dx^j \wedge *(dx^0 dx^l dx^m) = x_0^{-(d-5)} \delta_{im} dx^1 ... dx^d, \quad \delta_{im} = \delta_i^l \delta_j^m - \delta_i^m \delta_j^l,$$

is useful for the computation of the action. The final result is given by

$$I[A] = c \sum_{i < j} \int d^d x d^d x' \frac{A_{ij} (\mathbf{x}) A_{ij} (\mathbf{x'})}{|\mathbf{x} - \mathbf{x'}|^{2(d-2)}}$$

$$-2c \sum_{ijk} \int d^d x d^d x' \frac{A_{ij} (\mathbf{x}) A_{ik} (\mathbf{x'}) (x^j - x'^j)(x^k - x'^k)}{|\mathbf{x} - \mathbf{x'}|^{2(d-1)}},$$

where

$$c = \frac{d - 2}{2 \pi \frac{d}{2} \Gamma(d - 2)}.$$

It agrees with the result of \[20\].

4 The generating function of OPE of $p$-form currents

A general $p$-form potential $A$ on the boundary of $AdS$ can be lifted to a bulk one $A$ by using the boundary-to-bulk Green’s function $K$, the determination of which is our next step. Similarly to the 2-form case we assume that

$$K_\infty = K_\infty (x_0) dx^{i_1} ... dx^{i_p},$$

(28)
where $i_1 < \ldots < i_p$. $K_{\infty}$ satisfies the free-field equation $d^*dK_{\infty} = 0$ which reduces to
\[
\partial_0 \left[ x_0^{-d-2p-1} \partial_0 K_{\infty}(x_0) \right] = 0.
\] (29)
The solution of this equation which vanishes at $x_0 \to 0$ is $K_{\infty}(x_0) = c_0 x_0^{d-2p}$, where $c_0$ is a constant. In fact it is true only for $d > 2p$. But for $d \leq 2p$ one may use dual $p^*$-form such as $p^* = d - p - 1$. Using this $p^*$ we have the relation $d - 2 \geq 2p^*$, or the relation $d > 2p^*$ is satisfied.

To simplify the final calculation we apply the following gauge transformation,
\[
K_{\infty} \to c_0 x_0^{d-2p} dx^{i_1} \ldots dx^{i_p} + d \left[ x_0^{d-2p} \left( c_1 x^{i_1} dx^{i_2} \ldots dx^{i_p} - c_2 dx^{i_1} \cdot x^{i_2} \cdot dx^{i_3} \ldots dx^{i_p} \ldots \right) + (-)^{p+1} c_p dx^{i_1} \ldots dx^{i_{p-1}} \cdot x^{i_p} \right].
\] (30)
The transformed $K_{\infty}$ is given by
\[
K_{\infty} = (c_0 + c_1 + \ldots + c_p) x_0^{d-2p} dx^{i_1} \ldots dx^{i_p} + c_1 (d - 2p) x_0^{d-2p-1} x^{i_1} dx^0 \cdot \widehat{dx^{i_2}} \ldots dx^{i_p} - c_2 (d - 2p) x_0^{d-2p-1} x^{i_2} dx^0 dx^{i_1} \cdot \widehat{dx^{i_3}} \ldots dx^{i_p} \ldots + (-)^{p+1} c_p (d - 2p) x_0^{d-2p-1} x^{i_p} dx^0 dx^{i_1} \ldots dx^{i_{p-1}} \cdot \widehat{dx^{i_p}},
\] (31)
where $\widehat{dx^{i_k}}$, $k = 1, \ldots, p$, means that $dx^{i_k}$ should be omitted in the wedge product. When one choose constants such as
\[
c_1 = c_2 = \ldots = c_p = -\frac{1}{d - 2p}, \quad c_0 = \frac{d - p}{d - 2p},
\] (32)
$K_{\infty}$ simplifies to
\[
K_{\infty} = x_0^{d-2p-1} \sum_{\mu=0}^{p} (-)^{\mu} x^{i_0} dx^{i_1} \ldots \widehat{dx^{i_\mu}} \ldots dx^{i_p}.
\] (33)
Here we use the notation $x^{i_0} = x^0$. Under the isomorphism (17) $K_{\infty}$ transforms to
\[
c_{d,p} K_0 = x_0^{d-2p-1} \sum_{\mu=0}^{p} (-)^{\mu} \frac{x^{i_\mu}}{(x_0^2 + x^2)^{d-p}} dx^{i_0} \ldots \widehat{dx^{i_\mu}} \ldots dx^{i_p}
\] (34)
\[
= \frac{x_0^{d-2p}}{(x_0^2 + x^2)^{d-p}} dx^{i_1} \ldots dx^{i_p} + x_0^{d-2p-1} \sum_{k=1}^{p} (-)^k \frac{x^{i_k}}{(x_0^2 + x^2)^{d-p}} dx^0 dx^{i_1} \ldots \widehat{dx^{i_k}} \ldots dx^{i_p},
\]
where the normalization constant $c_{d,p}$ is multiplied to get the correct $K$. The final step for the construction of the boundary-to-bulk Green’s function is to use the translation (3).

Consider a $p$-form potential on the boundary of $AdS$,

$$A = \frac{1}{p!} \sum A_{i_1...i_p} dx^{i_1}...dx^{i_p}. \quad (35)$$

The bulk potential $A$ which is lifted from this is

$$c_{d,p} A(x, x) = \sum_{i_1...i_p} x_0^{d-2p} \int d^d x' \frac{A_{i_1...i_p}(x')}{(x_0^2 + |x - x'|^2)^{d-p}} \frac{1}{p!} dx^{i_1}...dx^{i_p}$$

$$+ \sum_{k=1}^p \sum_{i_1...i_p} (-)^k x_0^{d-2p-1} \int d^d x' (x_{i_k} - x'_{i_k}) A_{i_1...i_p}(x') \frac{1}{(x_0^2 + |x - x'|^2)^{d-p}} \frac{1}{p!} dx^{0} dx^{i_1}...dx^{i_p}. \quad (36)$$

We confuse notation $x_{i_k}$ with $x'^{i_k}$ for convenience. Using the anti-symmetric property of $A_{i_1...i_p}$ it can be further simplified to

$$c_{d,p} A(x, x) = \sum_{i_1...i_p} x_0^{d-2p} \int d^d x' \frac{A_{i_1...i_p}(x')}{(x_0^2 + |x - x'|^2)^{d-p}} \frac{1}{p!} dx^{i_1}...dx^{i_p} \quad (37)$$

$$- p \sum_{i_1...i_p} x_0^{d-2p-1} \int d^d x' (x_{i_1} - x'_{i_1}) A_{i_1...i_p}(x') \frac{1}{(x_0^2 + |x - x'|^2)^{d-p}} \frac{1}{p!} dx^{0} dx^{i_2}...dx^{i_p}.$$ 

The field strength $(p + 1)$-form $F = dA$ is given by

$$c_{d,p} F = (d-p)x_0^{d-2p-1} \sum_{i_1...i_p} \int d^d x' \frac{A_{i_1...i_p}(x')}{(x_0^2 + |x - x'|^2)^{d-p}} \frac{1}{p!} dx^{0} dx^{i_1}...dx^{i_p} \quad (38)$$

$$- 2(d-p)x_0^{d-2p+1} \sum_{i_1...i_p} \int d^d x' \frac{A_{i_1...i_p}(x')}{(x_0^2 + |x - x'|^2)^{d-p+1}} \frac{1}{p!} dx^{0} dx^{i_1}...dx^{i_p}$$

$$- 2p(d-p)x_0^{d-2p-1} \sum_{i_1...i_p} \int d^d x' (x_{i_1} - x'_{i_1})(x_{i_2} - x'_{i_2}) A_{i_1...i_p}(x') \frac{1}{(x_0^2 + |x - x'|^2)^{d-p+1}} \frac{1}{p!} dx^{0} dx^{i_3}...dx^{i_p}$$

$$+ F' ,$$

where $F'$ are the terms which do not have $dx^0$.

The action, using the equation of motion, is given by

$$I = \frac{1}{2} \int_{AdS_{d+1}} F \wedge *F = \frac{1}{2} \lim_{\epsilon \to 0} \int_{x^0 = \epsilon} A \wedge *A. \quad (39)$$

To compute this we make use of

$$dx^{i_1}...dx^{i_p} \wedge (*dx^0 dx^{i_1}...dx^{i_p}) = x_0^{-(d-2p-1)} \delta_{j_1...j_p} dx^{i_1}...dx^{i_p}. \quad (40)$$
where $\delta_{i_1...i_p}^{j_1...j_p}$ denotes the sign of the corresponding permutation.

Using (38) and (39) this can be simplified to give

$$I = c \sum_{i_1<...<i_p} \int d^d x d^d x' \frac{A_{i_1...i_p}(x) A_{i_1...i_p}(x')}{|x - x'|^{2(d - p)}}$$

$$- 2c \sum_{ij} \sum_{i_2<...<i_p} \int d^d x d^d x' \frac{(x_i - x'_i) (x_j - x'_j) A_{i_2...i_p}(x) A_{i_2...i_p}(x')}{|x - x'|^{2(d - p + 1)}},$$

where

$$c = \frac{d - p}{2\pi^{d/2}} \frac{\Gamma(d - p)}{\Gamma(d/2 - p)}.$$ (42)

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**Appendix**

In this appendix we follow the notation of [15]. Suppose that $\phi(x_0, x)$ is a field in $AdS$ which has following asymptotic behaviour

$$\phi \to x_0^{-\lambda} \text{ as } x_0 \to 0.$$ (A.1)

Then one defines a holographically projected field $\phi_h(x)$ by

$$\phi_h(x) = \lim_{\epsilon \to 0} \epsilon^{-\lambda} \phi(\epsilon, x).$$ (A.2)

This is exactly the definition which Witten employed in his paper [3]. This boundary field is related to the bulk field $\phi(x_0, x)$ by

$$\phi(x_0, x) = \int d^d x' K(x_0, x - x') \phi_h(x').$$ (A.3)
Here $K(x_0, x - x')$ is a boundary-to-bulk Green’s function which obeys the same equation which $\phi(x_0, x)$ satisfies, and has following boundary condition

$$\lim_{x_0 \to 0} K(x_0, x - x') = \delta^{(d)}(x - x'). \quad (A.4)$$

This Green’s function is clearly different from the Green’s function $G_\epsilon(x_0, x - x')$ defined to relate the original $AdS$ field $\phi(x_0, x)$ and $\phi_\epsilon(x)$, where

$$\phi_\epsilon(x) = \phi(\epsilon, x). \quad (A.5)$$

Even though Witten’s procedure is quite practical it may give wrong results when one considers massive cases. In fact one should use $G_\epsilon$ rather than $K$ in order to preserve the Ward identity. But for massless case there is no such problem. It comes from the fact that for massless $p$-form field, $\lambda = 0$. In this case $AdS$ field at the boundary $\phi(0, x)$ is exactly equal to $\phi_h(x)$, and two Green’s functions $G_\epsilon(x_0, x - x')$ and $K(x_0 + \epsilon, x - x')$ are equal. It thus causes no problem at all.

Now we prove that for $p$-form, $\lambda = 0$. Suppose $\phi$ is a $p$-form with the equation of motion $d^* d\phi = 0$. The asymptotic form of this equation, which is exactly the same as (29) which the Green’s function satisfies, is given by

$$\partial_0 \left[ x_0^{-(d-2p-1)} \partial_0 \phi_\infty(x_0) \right] = 0, \quad (A.6)$$

where $\phi_\infty(x_0)$ is the asymptotic form of independent component of $\phi$. The general solution of this equation is

$$\phi_\infty(x_0) = c_1 + c_2 x_0^{d-2p}. \quad (A.7)$$

It is already proven that we may assume that $d > 2p$ without loss of any generality. This shows that for $p$-form we do not need to introduce a rescaling factor such as $x_0^\lambda$. In other word, $\lambda = 0$. 

10
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