Circular dichorism-like effect in interaction of a Bose-Einstein condensate with a Laguerre-Gaussian beam

Pradip Kumar Mondal,1 Bimalendu Deb,2 and Sonjoy Majumder1
1Department of Physics, Indian Institute of Technology Kharagpur, Kharagpur-721302, India.
2Department of Materials Science, Indian Association for the Cultivation of Science, Jadavpur, Kolkata 700032, India.

We predict circular dichorism-like effect in interaction of vortices of an atomic Bose-Einstein condensate with a Laguerre-Gaussian beam. This is done by demonstrating the sensitivity of the electric dipole and quadrupole interactions of the vortex state of condensate to the handedness of the orbital angular momentum of the beam. We show that the quantum mechanical motion of the external (center-of-mass) degrees of freedom of atoms are important for the effect. We demonstrate that the dipole and the quadrupole Rabi frequencies critically depend on the ratio of the width of center-of-mass wavefunction of atoms to the waist of the beam. The predicted effect can be useful in detection of the vorticity and the handedness of a matter-wave vortex of atomic superfluids.

I. INTRODUCTION

The interaction of chiral molecules with light is sensitive to the circular polarization or helicity of the photons. One manifestation of this optical activity is circular dichorism (CD), i.e., light absorption in such material is sensitive to the handedness of the circular polarization. Such interaction is enantiomerically specific and depends on the structure of the chiral matter. The work of Allen et al. [2] have shown that in addition to spin angular momentum (SAM) associated with its polarization, Laguerre-Gaussian (LG) beams carry well defined orbital angular momentum (OAM) associated with its spatial mode. This has triggered new research on interaction of matter with laser beams having certain spatial profiles, like, LG or Bessel profile. From extensive theoretical [3–7] and experimental works [8, 9] it has been believed that the OAM of LG beam does not play a role in CD. Here we prove that the vortex state of a Bose-Einstein condensate (BEC) interacting with an LG beam can lead to CD like effects, i.e., the light absorption becomes sensitive to the handedness of the field OAM.

Numerous theoretical and experimental studies related to creation of matter-wave vortex states, persistent current in BEC and coherent control of the OAM of atoms using interaction of ultra-cold atoms with optical vortex have been reported over past two decades. Quantized vortex states play an essential role in macroscopic quantum phenomena like superfluidity and superconductivity. There have been studies to detect and measure the angular momentum of a vortex state using different techniques, like, imaging the density distribution after free expansion, interference between vortex states, and exciting the quadrupole mode of a BEC using an auxiliary laser beam as stirrer. The matter-wave interference technique has been proposed by Bolda and Walls to determine the handedness of angular momentum of a vortex state. Here we show how the CD-like effect that arises in interaction of a BEC with an LG beam can be useful to detect the vorticity and its handedness of a matter-wave vortex of the BEC.

Here we study interaction of an LG beam with matter-wave vortices of BEC. We explicitly consider quantized c.m. motion of condensate atoms as discussed in our previous work. Atoms are assumed to be cooled down to their recoil limit so that de Broglie wavelength of an atom has become comparable to the wavelength of the light. Generally, the interaction of atoms and molecules with laser beams are dominated by electric dipole-active transitions. Electric quadrupole transition rates are smaller than the dipole transition rates by a factor . It has been theoretically predicted that quadrupole Rabi frequency will scale up with the square of the winding number of the light beam. Recently, Lembessis and Babiker have theoretically predicted that quadrupole Rabi frequency will scale up with the square of the winding number of the vortex beam in interaction of an atom with an LG beam. So, while exploring CD like effects we calculate both dipole and quadrupole Rabi frequencies of a BEC interacting with an LG beam and make a comparison between them. With the advent of sophisticated optical measurement techniques, many recent studies are aimed to observe, utilize and enhance quadrupole effects.

The paper is organized as follows. In Sec. II, we give a brief theory of OAM exchange in interaction of LG beam with the BEC following our previous work and discuss analytically the dipole and quadrupole Rabi frequencies. Sec. III presents numerical calculations and results. Finally, in Sec. IV, we have made some concluding remarks.

II. THEORY

We consider an LG beam without any off-axis node propagating along axis of the laboratory frame inter-
acting with $^{23}$Na atoms whose c.m. wavefunction has extension as large as comparable to the wavelength of the light but smaller than the waist of the beam. Under this condition, the atom experiences a local field of the type $^{28, 38}$

$$E(r', t) = \frac{E_0}{\sqrt{|I|!}} \left( \frac{r'_i}{w_0} \right)^{|l|} \exp(ikz') \exp[i(kz' - \omega t)], \quad (1)$$

where $r'_i$ is the projection of $r'$ on $xy$ plane, $l$, $w_0$, $\phi'$ are the winding number, the waist, and the azimuth of the beam, respectively. We consider very simple model of the atomic system composed of a core (including the nucleus and the correlated core electrons) of mass $m_c$ and a correlated valance electron of mass $m_e$. For simplicity, the spin of the particles is ignored. The center-of-mass of the atomic system is $R_{c.m.} = (m_c r_c + m_e r_e)/m_t$ with $m_t = m_c + m_e$ being the total mass, $r_e$ and $r_c$ the coordinates of the valance electron and the center of the core, respectively. The Hamiltonian of the atom field system is $H = H_0 + H_1$ where $H_0$ is unperturbed atomic Hamiltonian and the interaction Hamiltonian $H_1$ is derived in Power-Zineau-Wooley (PZW) scheme $^{28, 38, 39}$ as given by

$$H_1 = - \int dr' \mathcal{P}(r') \cdot E(r', t) + \text{H.c.} \quad (2)$$

$\mathcal{P}(r')$ is the electric polarization given by

$$\mathcal{P}(r') = -e \frac{m_c}{m_t} \int_0^1 d\lambda (r' - R_{c.m.} - \lambda \frac{m_c}{m_t} r), \quad (3)$$

where relative coordinate (internal) $r = r_e - r_c$. Considering $m_c \approx m_t$ and following the way described in Ref. $^{28}$, we can write the dipole and quadrupole parts of the interaction Hamiltonian as

$$H_1^d = \sqrt{\frac{4\pi}{3}} \left( \frac{1}{w_0} \right)^{|l|} r \sum_{\sigma = 0, \pm 1} \epsilon_\sigma Y_1^\sigma(\hat{r}) R_{c.m., \pm} e^{i\theta \Phi_{c.m.}} e^{ikZ_{c.m.}} + \text{H.c.} \quad (4)$$

$$H_1^q = \frac{1}{2} \sqrt{\frac{4\pi}{3}} \left( \frac{1}{w_0} \right)^{|l|^2} r^2 \sum_{\sigma = 0, \pm 1} \epsilon_\sigma Y_1^\sigma(\hat{r}) \left\{ |l| R_{c.m., \pm}^{l-1} e^{sgn(l) i |l| - 1} \Phi_{c.m.} e^{sgn(l) i \phi} \sin \theta \right. \left. + (ik) R_{c.m.} e^{i\theta \Phi_{c.m.}} e^{ikZ_{c.m.}} \cos \theta \right\} + \text{H.c.} \quad (5)$$

The dot products of the type $r' \cdot E_0$ are replaced by $r' \sqrt{4\pi/3} \sum_{\sigma = 0, \pm 1} \epsilon_\sigma Y_1^\sigma(\theta', \phi')$ with $\epsilon_{\pm 1} = (E_\pi \pm iE_y)/\sqrt{2}$ and $\epsilon_0 = E_z$. In paraxial approximation, the $E_z$ component makes negligible contribution. Operators, described in Eq. (4) and (5), clearly operate on atomic states whose rotation of c.m. motion is quantized. The dipole and quadrupole transition matrix elements are given by $M_{\ell'^{-}\ell}^{d,q} = \langle \Psi_{\ell'} | H_{\ell'}^{d,q} | \Psi_{\ell} \rangle$ and $M_{\ell'^{-}\ell}^{q,q} = \langle \Psi_{\ell'} | H_{\ell'}^{q} | \Psi_{\ell} \rangle$, respectively, where $\Psi$ denotes an unperturbed atomic state, i.e., eigenstate of $H_0$. We assume $\Psi(R_{c.m., r}) = \Psi_{c.m.}(R_{c.m.}) \psi(r)$, where $\Psi_{c.m.}(R_{c.m.})$ is the c.m. wavefunction and depends on the external potential that traps the atom. $\psi(r)$ is the internal electronic wavefunction which can be considered to be a correlated orbital obtained from many-body theory $^{40}$. Therefore,

$$M_{\ell'^{-}\ell}^{d} = \sqrt{\frac{4\pi}{3}} \left( \frac{w_{c.m.}}{w_0} \right)^{|l|} \sum_{\sigma = 0, \pm 1} \epsilon_\sigma (\psi_{\ell'} | r Y_1^\sigma(\hat{f}) | \psi_{\ell}) M_{c.m.}^{\ell'} \quad (6)$$

$$M_{\ell'^{-}\ell}^{q} = \frac{1}{2} \sqrt{\frac{4\pi}{3}} \left( \frac{w_{c.m.}}{w_0} \right)^{|l|^2} \sum_{\sigma = 0, \pm 1} \epsilon_\sigma \left\{ |l| \left( \frac{w_{c.m.}}{w_0} \right)^{|l| - 1} \left( \frac{w_c}{w_0} \right) \langle \psi_{\ell'} | r^2 Y_1^\sigma(\hat{f}) \sin \theta e^{sgn(l) i \phi} | \psi_{\ell} \rangle M_{c.m.}^{sgn(l) |l| - 1} \right. \left. + (ik) \left( \frac{w_{c.m.}}{w_0} \right)^{|l|} \langle \psi_{\ell'} | r^2 Y_1^\sigma(\hat{f}) \cos \theta | \psi_{\ell} \rangle M_{c.m.}^{l'} \right\} \quad (7)$$

where $w_{c.m.}$ and $w_c$ stand for the average spatial width of $\Psi_{c.m.}(R_{c.m.})$ and $\psi(r)$, respectively and $M_{c.m.}^{\ell'} = \langle R_{c.m., r} | \psi_{c.m.}^{l'} \rangle e^{il' \Phi_{c.m.}} e^{ikZ_{c.m.}} | \Psi_{c.m.} \rangle. \quad \text{The first term in R.H.S. of Eq. (7) shows exchange of one unit of angular momentum between the quantized c.m. motion and the electronic motion. The second term corresponds to usual electric quadrupole transition. The Rabi frequency corresponding to resonant transition is defined as } \Omega_e = \frac{M_{\ell'^{-}\ell}^{q}}{\hbar}, \quad a = d, q \text{ corresponds to dipole and quadrupole transition, respectively.}
We consider the c.m. state of cold atoms in a trap as the state of a trapped BEC \cite{10, 41, 43}. The system is dilute enough so that the Gross-Pitaevskii (GP) theory \cite{44} for trapped bosons is applicable to the c.m. motion. Vortex states in BEC will be created due to transfer of OAM from the LG beam to the c.m. motion of the atoms. In addition to the OAM, linear momentum (LM) of light will also be transferred to the c.m. motion of atoms. Hence, we write the initial and final stationary state of c.m. motion of a single atom as \cite{42}

$$
\Psi_{c.m.}(R_{c.m.}) = \psi_{c.m.}(R_{c.m.}, Z_{c.m.}) e^{i\kappa_c R_{c.m.}}
$$

and

$$
\Psi_{c.m.}(R_{c.m.}) = \psi_{c.m.}(R_{c.m.}, Z_{c.m.}) e^{i\kappa_f R_{c.m.}}
$$

where \(\kappa_c\) and \(\kappa_f\) are the quantum of circulation about z axis in the initial and final states of the atom. \(\kappa \neq 0\) represents vortex states of the BEC. The expression of \(\mathcal{M}_{c.m.}'\) gives the selection rule for c.m. motion as \(\Delta\kappa = \kappa'\). According to time-independent GP theory, the nonlinear Schrödinger equation obeyed by the c.m. motion in an anisotropic simple harmonic potential trap is given by \cite{42, 44}

$$
\left[ \frac{\hbar^2}{2m} \left( -\nabla^2 + \kappa^2 R_{c.m.}^{-2} \right) + \frac{m\omega_z^2}{2} \left( \omega_\perp^2 R_{c.m.}^{-2} + \omega_Z^2 Z_{c.m.}^2 \right) + \frac{4\pi\hbar^2 a}{m} |\psi_{c.m.}|^2 \right] \psi_{c.m.} = \mu \psi_{c.m.}
$$

where \(\omega_\perp\) and \(\omega_Z\) are the transverse and axial angular frequencies associated with the external potential of the anisotropic harmonic trap, \(a\) is the s-wave scattering length, and \(\mu\) is the chemical potential. In Eq. (6) and (7), \(w_{c.m.}\) is of the order of the trap characteristic length \(a_\perp = \left( \frac{\hbar}{m \omega_\perp} \right)^{1/2}\) and \(w_z\) is of the order of Bohr radius \(a_0\). Hence, if we consider the first term of Eq. (7) then the ratio of quadrupole to dipole Rabi frequency will be proportional to the factor \(\left( \frac{a_0}{a_\perp} \right)^2\). We evaluate the c.m. wavefunction at temperature \(T = 0\) using the steepest descent method for functional minimization as prescribed in Ref. \cite{42}. The electronic portion of the transition matrix element is calculated using Coupled-Cluster theory \cite{40}.

Circular dichroism-like behaviour

In experiments of Refs. \cite{10, 12, 14}, the authors have used two-photon transitions to transfer the OAM of an LG beam to the atomic BEC. The two-photon pulse consists of an LG beam and a Gaussian beam. Here we study how \(\Omega^d\) and \(\Omega^s\) depend on the handedness of the OAM of the LG beams. The density distribution of matter-vortex states with vorticity \(\pm\kappa\) are identical. Let us suppose that the BEC is initially in a vortex state with vorticity \(+\kappa\). Let us consider two LG beams \(LG^0\) and \(LG^0\) \(+\) having winding numbers \(+l\) and \(-l\), respectively. The LG beams are identical otherwise. Considering electric dipole transition, after interacting with \(LG_0\) the BEC will have vorticity \(\kappa + l\). On the other hand, BEC will have vorticity \(\kappa - l\) when it interacts with \(LG_0^{-l}\) beam. The radial portions of c.m. wavefunction of BEC corresponding to vorticities \(\kappa + l\) and \(\kappa - l\) are different. This makes the c.m. matrix elements \(\mathcal{M}_{c.m.}'\) corresponding to the two transitions different. Thus, in principle, a BEC vortex state is expected to show CD like behavior in interaction with LG beams having OAM of opposite handedness. If initially the BEC was in a non-vortex state then after interaction with LG beams the final states of BEC will have vorticities \(+l\) and \(-l\), respectively. The radial portion of c.m. wavefunctions of these two states are identical. This makes the c.m. matrix elements and Rabi frequencies corresponding to these transitions identical. Hence, non-vortex state in BEC does not show CD like behavior in interaction with LG beam.

III. NUMERICAL RESULTS

We now proceed to evaluate the Rabi frequencies numerically and explore their dependency on the handedness and OAM of light, the atom number density and the nature of mean-field interaction of BEC. For numerical illustration we consider sodium BEC \cite{10} in an anisotropic harmonic trap interacting with LG beam. Let us consider a left circularly polarized LG beam (i.e., \(\sigma = +1\) in Eqs. (4-7)) interacting with a BEC of \(10^4\) number of \(^{23}\)Na atoms in an anisotropic harmonic trap. We choose the characteristics of the experimental trap as given in Ref. \cite{10}. The asymmetry parameter of the trap is \(\lambda_{tr} = \omega_Z/\omega_\perp = 2\). The axial frequency is \(\omega_Z/2\pi = 40\) Hz. The corresponding characteristic length is \(a_\perp = 4.673\) \(\mu\)m. s- wave scattering length is \(a = 2.75\) nm \cite{11, 13}. The waist of the LG beam is set to be \(w_0 = 10^{-4}\) m and the intensity \(I = 10^2\) Wcm\(^{-2}\). The amplitude of the LG beam in Eq. (6) and (7) is related to the intensity by \(I = c_0\epsilon_0c^2/2\) where \(\epsilon_0\) is vacuum permittivity and \(c\) is velocity of light. For external motion of the gas, we assume here that both the initial and final electronic states of the atoms are confined by the same
TABLE I. This illustrates circular dichorism-like effect in BEC vortex states interacting with LG beams. The condensate initially has vorticity $\kappa_i$. After interacting with LG beam having winding number $l$, the vorticity of the condensate becomes $\kappa_f$. $\Omega^d$ is corresponding dipole Rabi frequency (s$^{-1}$).

| $\kappa_i$ | $l$ | $\kappa_f$ | $\Omega^d$ | $l$ | $\kappa_f$ | $\Omega^d$ |
|-----------|-----|------------|------------|-----|------------|------------|
| $+$1      | 2   | $1.20 \times 10^6$ | $-$1   | 0   | $1.05 \times 10^6$ |
| $+$2      | 3   | $8.33 \times 10^6$ | $-$2   | $-$1 | $6.99 \times 10^6$ |
| $-$1      | 0   | $1.05 \times 10^6$ | $-$1   | 2   | $1.20 \times 10^6$ |
| $+$2      | 1   | $6.99 \times 10^6$ | $-$2   | $-$3 | $8.33 \times 10^6$ |

TABLE II. Listed are dipole ($\Omega^d$) and quadrupole ($\Omega^q$) Rabi frequencies (s$^{-1}$) for different winding numbers $l$ of the LG beam while the vorticity of initial BEC state $\kappa_i = 0$.

| $l$ | $\Omega^d$ | $\Omega^q$ | ($\Omega^q/\Omega^d$) |
|-----|------------|------------|---------------------|
| 1   | $1.05 \times 10^6$ | $4.24 \times 10^6$ | $4.05 \times 10^{-6}$ |
| 2   | $6.76 \times 10^6$ | $4.19 \times 10^6$ | $6.20 \times 10^{-6}$ |
| 3   | $4.18 \times 10^6$ | $3.31 \times 10^6$ | $7.93 \times 10^{-6}$ |
| 4   | $2.50 \times 10^6$ | $2.37 \times 10^6$ | $9.48 \times 10^{-6}$ |
| 5   | $1.45 \times 10^6$ | $1.58 \times 10^6$ | $1.09 \times 10^{-5}$ |

TABLE III. This illustrates variation of dipole ($\Omega^d$) and quadrupole ($\Omega^q$) Rabi frequencies (s$^{-1}$) with vorticity of initial state of c.m. wavefunction. The condensate having initial vorticity $\kappa_i$ is interacting with LG beam having winding number $l$.

| $l$ | $\kappa_i$ | $\Omega^d$ | $\Omega^q$ | ($\Omega^q/\Omega^d$) |
|-----|------------|------------|------------|---------------------|
| 1   | $1$        | $1.02 \times 10^6$ | $4.24 \times 10^6$ | $3.53 \times 10^{-6}$ |
| 2   | $2$        | $1.37 \times 10^6$ | $4.24 \times 10^6$ | $3.11 \times 10^{-6}$ |
| 3   | $3$        | $1.52 \times 10^6$ | $4.24 \times 10^6$ | $2.79 \times 10^{-6}$ |
| 4   | $4$        | $1.67 \times 10^6$ | $4.24 \times 10^6$ | $2.54 \times 10^{-6}$ |
| 5   | $5$        | $1.80 \times 10^6$ | $4.24 \times 10^6$ | $2.36 \times 10^{-6}$ |

 harmonic potential. This is possible because the excited atomic states are tuned far off the resonance $|46\rangle$. We consider the state $|3S_{1/2}\rangle$ as initial electronic state, $|\psi_i\rangle$, and $l'$ be the winding number of the LG beam. The electronic portion of Eq. (6), $\langle \psi_f | r Y_l^a (\hat{f}) | \psi_i \rangle$, ensures the final electronic state $|\psi_f\rangle$ to be $|3P_{l' \pm \frac{1}{2}}\rangle$ in dipole transition. Simultaneously, vorticity of the c.m. state will be changed by $l'$ due to OAM of the LG beam. For electric quadrupole transition, the electronic portion of the first term in R.H.S of Eq. (7), $\langle \psi_f | r^2 Y_l^a (\hat{f}) \sin \theta e^{ig \frac{l}{2} \phi} | \psi_i \rangle$, ensures $|\psi_f\rangle$ to be $|3D_{l' \pm \frac{1}{2}}\rangle$. Here one unit of field OAM is transferred to the electronic motion via c.m. motion and hence, the rest $(l' - 1)$ unit of field OAM will change the vorticity of the c.m. state.

Table I gives numerical results showing CD like effects depending on dipole Rabi frequencies using LG beams of opposite handedness. It shows that if $\kappa_i > 0$ then Rabi frequency is larger in interaction with LG$_{0}^{l}$ beam and if $\kappa_i < 0$ then it is larger in interaction with LG$_{0}^{-l}$ beam. If we consider electric quadrupole transition then also we get similar effects. For $\kappa_i > 0$ the quadrupole Rabi frequency is larger in interaction with LG$_{0}^{l}$ beam and for $\kappa_i < 0$ it is larger in interaction with LG$_{0}^{-l}$ beam.

We have found that, as in vortex state of BEC, both electric dipole and quadrupole Rabi frequencies for non-vortex state of BEC decrease monotonically with increasing winding number $l$ of the LG beam. This happens because the dipole and quadrupole frequencies depend on the factors $\left(\frac{w_{c.m.}}{w_0}\right)^{|l|}$ and $\left(\frac{w_{c.m.}}{w_0}\right)^{|l| - 1}$, respectively and $w_{c.m.} < w_0$ (see Eqs. (6, 7)). Table II further shows that the ratio of quadrupole to dipole Rabi frequencies increases with the increasing value of $l$. As $w_{c.m.} < w_0$, with increasing $l$ the Rabi frequencies themselves decrease so rapidly that one can not find a value of $l$ for which $\Omega^q$ can be comparable to $\Omega^d$. That is why we have considered only dipole transitions in Table I to illustrate the CD effect.

Let the initial c.m. wavefunction be a vortex state of BEC in a simple harmonic potentialtrap. The c.m. portions $\mathcal{M}_{c.m.}^l$ of Eq. (6) and (7) change due to the change of initial and final vortex states. As is demonstrated in Figure 1 of ref. [28], the spread of the c.m. wavefunction increases with the increase in vorticity and this in turn increases the value of $\mathcal{M}_{c.m.}^l$. Table III presents the variation of dipole and quadrupole Rabi frequencies for different initial vortex states. It clearly shows that with increase in vorticity ($\kappa_i$) of the initial c.m. wavefunction both the dipole and quadrupole Rabi frequencies increase. Only exception is the case $l = 1$ when the total OAM is transferred to the electronic motion in quadrupole transition and vorticity of the c.m. motion remains unchanged. This makes the quadrupole Rabi frequency becomes independent of $\kappa_i$. But the ratio ($\Omega^q/\Omega^d$) always decreases gradually with increasing $\kappa_i$.

Next, for the sake of completeness, we study the effect of mean-field interaction expressed in terms of the s-wave scattering length, $a$, on the Rabi frequencies. If $a$ is positive, the atoms repel each other and the c.m. wavefunction becomes more spread with increase in number of atoms $N$ in the condensate. This increases the value of $\mathcal{M}_{c.m.}^l$, which in turn increases both the Rabi frequencies. Figure 1 shows the variation of dipole and quadrupole Rabi frequencies with number of condensate atoms. The situation is totally different for the BEC with negative s-wave scattering length (7Li) where the atoms experience attractive interactions with each other. The spread of the c.m. wavefunction decreases with increase in $N$.
and above a critical value of $N$ the condensate collapses. Therefore, in case of BEC having negative $s$-wave scattering length, the values of $M^l_{m}$ and Rabi frequencies decrease with increase in number of atoms. From these results we conclude that the dipole Rabi frequencies are always far too larger than the quadrupole Rabi frequencies.

We discuss how CD like effect can be useful to detect vortex state in BEC and determine the handedness of vorticity using two LG beams having OAM of opposite handedness. Let the sodium BEC is initially prepared in electronic state $|3S_{1/2}, F = 1, m_{F} = -1\rangle$. The condensate atoms are allowed to undergo two-photon transitions as given in Refs. [10, 13, 14]. But now two sets of pulses are applied simultaneously. One pulse consists of an LG$_{0}^{l}$ beam and a Gaussian beam $G_{1}$ (LG$_{0}^{l}$/G$_{1}$ pulse). The other pulse consists of an LG$_{0}^{l}$ beam and a Gaussian beam $G_{2}$ (LG$_{0}^{l}$/G$_{2}$ pulse). Figure 2 shows a schematic diagram of the relevant levels and transitions. If a condensate atom undergoes two-photon transition due to interaction with the LG$_{0}^{l}$/G$_{1}$ pulse then the final electronic state is $|3S_{1/2}, F = 1\rangle$ and let us call it as type-I two-photon transition. But if the atom undergoes two-photon transition due to interaction with the LG$_{0}^{l}$/G$_{2}$ pulse then the final electronic state will be $|3S_{1/2}, F = 2\rangle$ and let us call it as type-II two-photon transition. The final electronic states are decided by the polarization of the light fields. The intensities of the beams are selected such that electronic portions of Rabi frequencies corresponding to the type-I and type-II transitions are equal. The difference in Rabi frequencies will come from the c.m. portion of transition matrix element given in Eq. (6). The direction of the applied beams are such that after free expansion, the atoms undergoing type-I two-photon transition should be spatially separable from the atoms undergoing type-II two photon transition. Next, in a similar detection process as given in Ref. [10], one can image how much fraction of the initial number of atoms has undergone type-I transition and how much fraction has type-II transition. If almost same number of atoms have taken part in both types of transitions then the initial BEC state is a non-vortex state. If a larger fraction of atoms takes part in type-I (type-II) transition then the initial BEC state has vorticity $\kappa > 0$ ($\kappa < 0$). The exact number of atoms that have undergone a particular type of transition is not essential. We only need to know the relative number of atoms undergone the two transitions.

IV. CONCLUSION

In conclusion, we have predicted CD like effects in interaction of a matter-wave vortex with an LG beam. We have compared the electric dipole and quadrupole effects on cold atoms in interaction with LG beams in order to find which one would dominantly influence CD effect. We have shown that both dipole and quadrupole Rabi frequencies decrease with increase in value of the winding number $l$ of the LG beam. Since, the OAM of the field is due to the spatial inhomogeneity, OAM transfer from the beam to the c.m. motion cannot take place unless the spread of the c.m. wavefunction is large enough to feel the spatial variation of the field. This makes the use of dipole effects preferable in application of interaction of LG beam with BEC. Recently, Lembessis and Babiker have theoretically studied [32] quadrupole effects in the interaction of atoms with LG beams. They have...
predicted that the weak electric quadrupole interaction in atoms can be enhanced when the atoms interact resonantly with an optical vortex and the quadrupole Rabi frequency scales up with the square of the winding number of the vortex beam. We can not draw direct comparison of our results with theirs as the physical conditions considered in that paper are completely different from ours. They have considered a freely moving atom as a point mass and they treated the c.m. motion classically. While, our work holds good for cold atoms whose spread of the c.m. wavefunction is comparable to the wavelength of the laser beam but much smaller than the waist of the beam. We have further shown that the c.m. portion of the transition matrix elements depends on other system parameters such as the vorticities of the c.m. states, scattering length and number of atoms present in the condensate. The CD like effect predicted in this paper will have wide applications including detection of the handedness of the vortex. We have proposed a possible method to detect BEC vortex states and its handedness.

We are thankful to Mr. Narendra Nath Dutta of Indian Institute of Technology, Kharagpur for useful discussion. Pradip Kumar Mondal acknowledges financial support from the Council of Scientific and Industrial Research (CSIR), India.

[1] L. D. Barron, *Molecular Light Scattering and Optical Activity* (Cambridge University Press, Cambridge, 2004).
[2] L. Allen, M. W. Beijersbergen, R. J. C. Spreeuw, and J. P. Woerdman, Phys. Rev. A **45**, 8185 (1992).
[3] L. Allen, S. M. Barnett, and M. J. Padgett, *Optical Angular Momentum* (Institute of Physics Publishing, London, 2003).
[4] D. L. Andrews, M. Babiker, *The Angular Momentum of Light* (Cambridge University Press, New York, 2013).
[5] D. L. Andrews, L. C. Dvila Romero, and M. Babiker, Opt. Commun. **237**, 133 (2004).
[6] D. L. Andrews and M. M. Coles, Opt. Lett. **37**, 3009 (2012).
[7] M. M. Coles and D. L. Andrews, Phys. Rev. A **85**, 063810 (2012).
[8] F. Araoka, T. Verbiest, K. Clays, and A. Persoons, Phys. Rev. A **71**, 055401 (2005).
[9] W. Löffler, D. J. Broer, and J. P. Woerdman, Phys. Rev. A **83**, 065801 (2011).
[10] M. F. Andersen, C. Ryu, P. Cladé, V. Natarajan, A. Vaziri, K. Helmerson, and W. D. Phillips, Phys. Rev. Lett. **97**, 170406 (2006) and references therein.
[11] T. P. Simula, N. Nygaard, S. X. Hu, L. A. Collins, B. I. Schneider, and K. Molmer, Phys. Rev. A **77**, 015401 (2008).
[12] J. -J. Song and B. A. Foreman, Phys. Rev. A **80**, 033602 (2009).
[13] K. C. Wright, L. S. Leslie, and N. P. Bigelow, Phys. Rev. A **77**, 041601(R) (2008).
[14] K. C. Wright, L. S. Leslie, A. Hansen, and N. P. Bigelow, Phys. Rev. Lett. **102**, 030405 (2009).
[15] A. Jaouadi, N. Gaaloul, B. Viaris de Lesegno, M. Telmini, L. Pruvost, and E. Charron, Phys. Rev. A **82**, 023613 (2010).
[16] N. Lo Gullo, S. McEndoo, T. Busch, and M. Paternostro, Phys. Rev. A **81**, 053625 (2010).
[17] M. E. Taggin, Ö. E. Müstecaplıoğlu, and L. You, Phys. Rev. A **84**, 063628 (2011) and references therein.
[18] V. E. Lembessis, D. Ellinas, and M. Babiker, Phys. Rev. A **84**, 043422 (2011) and references therein.
[19] J. F. S. Brachmann, W. S. Bakr, J. Gillen, A. Peng, and M. Greiner, Optics Express **19**, 12984 (2011).
[20] R. Kanamoto and E. M. Wright, Journal of Optics **13**, 064011 (2011).
[21] A. Ramanathan, K. C. Wright, S. R. Muniz, M. Zelan, W. T. Hill, C. J. Lobb, K. Helmerson, W. D. Phillips, and G. K. Campbell, Phys. Rev. Lett. **106**, 130401 (2011).
[22] G. R. M. Robb, Phys. Rev. A **85**, 023426 (2012).
[23] A. Yu. Okulov, Physics Letters A **376**, 650 (2012).
[24] S. Beattie, S. Moulder, R. J. Fletcher, and Z. Hadzibabic, Phys. Rev. Lett. **110**, 025301 (2013) and references therein.
[25] M. Cozzini, B. Jackson, S. Stringari, Phys. Rev. A **73**, 013603 (2006).
[26] E. L. Bolda and D. F. Walls, Phys. Rev. Lett. **81**, 5477 (1998).
[27] F. Chevy, K. W. Madison, J. Dalibard, Phys. Rev. Lett. **85**, 2223 (2000).
[28] P. K. Mondal, B. Deb, S. Majumder, Phys. Rev. A **89**, 063418 (2014).
[29] C. Cohen-Tannoudji, *Atoms in Electromagnetic Fields* (World Scientific Publishing, Singapore, 1994).
[30] D. P. Craig and T. Thirunamachandran, *Molecular Quantum Electrodynamics* (Academic Press, London, 1984).
[31] B. H. Bransden, and C. J. Joachain, *Physics of Atoms and Molecules 2nd edition* (Dorling Kindersley, India, 2008).
[32] V. E. Lembessis and M. Babiker, Phys. Rev. Lett. **110**, 083002 (2013).
[33] S. -M. Hu, H. Pan, C. -F. Cheng, Y. R. Sun, X. -F. Li, J. Wang, A. Campargue, and A. -W. Liu, Astrophys. J. **749**, 76 (2012).
[34] S. Tojo, M. Hasuo, and T. Fujimoto, Phys. Rev. Lett. **92**, 053001 (2004).
[35] A.M. Kern and O.J.F. Martin, Phys. Rev. A **85**, 022501 (2012).
[36] A.M. Kern and O.J.F. Martin, Nano Lett. **11**, 482 (2011).
[37] V.V. Klimov and V.S. Letokhov, Phys. Rev. A **54**, 130401 (1996).
[38] A. Alexandrescu, E. Di Fabrizio, and D. Cojoc, J. Opt. B: Quantum Semiclass Opt. **7**, 87 (2005).
[39] C. Cohen-Tannoudji, J. Dupont-Roc, and G. Gryenberg, *Photons and Atoms An Introduction to Quantum Electrodynamics* (Wiley, New York, 1989).
[40] P. K. Mondal, N. N. Dutta, G. Dixit, S. Majumder, Phys. Rev. A **87**, 062502 (2013) and references therein.
[41] J. Tempere, J. T. Devreese, E. R. I. Abraham, Phys. Rev. A **64**, 023603 (2001).
[42] F. Dalfovo and S. Stringari, Phys. Rev. A **53**, 2477 (1996).
[43] M. Edwards, R. J. Dodd, C. W. Clark, P. A. Ruprecht, and K. Burnett, Phys. Rev. A 53, R1950 (1996).
[44] L. Pitaevskii, S. Stringari, Bose-Einstein Condensation (Clarendon Press, Oxford, 2003).
[45] S. Inouye, M. R. Andrews, J. Stenger, H.-J. Miesner, D. M. Stamper-Kurn, W. Ketterle, Nature 392, 151 (1998).
[46] G. Nandi, R. Walser, W. P. Schleich, Phys. Rev. A 69, 063606 (2004).