Abstract

We study the process $B^- \to K^- \chi_{c0}$ considering intermediate charmed meson rescattering effects. For this decay mode the naive factorization ansatz would predict a vanishing amplitude. We estimate contributions from the $D_s^*(\ast) D^{(*)} \to K^- \chi_{c0}$ rescattering amplitudes, and compare the result with recent experimental measurements. We find that rescattering effects are able to produce a large branching ratio consistent with measurements by Belle Collaboration. We also consider rescattering effects in $B^- \to K^- J/\psi$, arguing that they play a similar role in producing a large branching fraction for this colour-suppressed decay mode.
1 Introduction

Understanding strong interaction effects in weak exclusive heavy hadron decays is of great importance to gain information on fundamental aspects of strong and weak interaction phenomenology, both in the Standard Model and beyond. In this respect the factorization ansatz, that allows a treatment of nonleptonic decay amplitudes by factorizing hadronic matrix elements of four-quark operators as products of two current matrix elements, has been a widely used working tool for the analysis of $B$ decays to charmed and charmless hadrons in the final state. In those decays in which the effective Wilson coefficients are not colour suppressed and the tree-level $(V-A) \times (V-A)$ current matrix elements do not vanish, it is found that factorization provides a reasonable description of data [1]. However, the recent observation of the decay mode $B^- \to K^-\chi_{c0}$, reported by the Belle Collaboration together with the measurement of the branching fraction [2]:

$$B(B^- \to K^-\chi_{c0}) = (6.0^{+2.1}_{-1.8} \pm 1.1) \times 10^{-4},$$  

(1)

demonstrates the inadequacy of the factorization model in the calculation of nonleptonic $B$ decay amplitudes for colour suppressed $B$ to charmonium transitions. A large non-factorizable term is needed to account for the observed branching ratio. As a matter of fact, the result [1] implies that the rate of $B$ decays into a kaon and the $0^{++}$ state of the charmonium system, $\chi_{c0}$, is comparable with the $B$ decay rate into a kaon and $J/\psi$, and indeed the measurement of the ratio of the two branching fractions, reported by the same Collaboration, is:

$$\frac{B(B^- \to K^-\chi_{c0})}{B(B^- \to K^- J/\psi)} = (0.60^{+0.21}_{-0.18} \pm 0.05 \pm 0.08).$$  

(2)

The experimental results [1] and [2] are in conflict with the vanishing of the amplitude of $B \to K\chi_{c0}$ computed by the factorization ansatz, while the amplitude governing $B \to KJ/\psi$ is different from zero in the same approximation. This can be easily shown: the effective Hamiltonian governing both the transitions $^1$:

$$H_W = \frac{G_F}{\sqrt{2}} \left\{ V_{cb}V_{cs}^* \left( c_1(\mu)O_1(\mu) + c_2(\mu)O_2(\mu) \right) - V_{tb}V_{ts}^* \sum_i c_i(\mu)O_i(\mu) \right\} + h.c.$$  

(3)

involves only vector and axial-vector $\bar{c}c$ operators:

$$O_1 = (\bar{c}b)_{V-A}(\bar{s}c)_{V-A}$$

$^1$We neglect the $B^-$ annihilation transition, which is governed by the CKM matrix element $V_{ub}$. 

2
\[ O_2 = (sb)_{V-A}(\bar{c}c)_{V-A} \]
\[ O_{3(5)} = (sb)_{V-A} \sum_q (\bar{q}q)_{V-A}[V+A] \]
\[ O_{4(6)} = (\bar{s}b_j)_{V-A} \sum_q (\bar{q}j q_l)_{V-A}[V+A] \]  
\[ (4) \]
\[ O_{7(9)} = \frac{3}{2} (sb)_{V-A} \sum_q e_q (\bar{q}q)_{V+A}[V-A] \]
\[ O_{8(10)} = \frac{3}{2} (\bar{s}b_j)_{V-A} \sum_q e_q (\bar{q}j q_l)_{V+A}[V-A] \]

(i, j are color indices and \((\bar{q}q)_{V=0} = \bar{q}\gamma^\mu(1 \mp \gamma_5)q\), and therefore the factorized amplitude
\[ A_F(B^- \to K^-\chi_{c0}) = \frac{G_F}{\sqrt{2}} V_{cb} V_{c}\bar{c}^* \left[ a_2 + \sum_{i=3,5,7,9} a_i \langle K^-|(\bar{c}c)_{V=A}|B^-\rangle \langle \chi_{c0}|(\bar{c}c)_{V=0}|0 \rangle \right] \]  
(5)

vanishes since the current matrix elements \(\langle \chi_{c0}|(\bar{c}c)_{V,A}|0 \rangle\) are zero. Instead, the experimental result \([1]\) corresponds to \(A_{\exp}(B^- \to K^-\chi_{c0}) = (3.39 \pm 0.68) \times 10^{-7}\) GeV. On the other hand in the case of \(B \to KJ/\psi\), since \(\langle J/\psi|(\bar{c}c)_{V}|0 \rangle \neq 0\), a nonvanishing factorized amplitude, analogous to \([\mathbb{E}]\), can be obtained once the combinations of Wilson coefficients \(a_2 = c_2 + c_1/N_c\) and \(a_i = c_i + c_{i+1}/N_c\) \([\mathbb{F}]\) and the matrix element \(\langle K^-|(\bar{c}c)_{V=A}|B^-\rangle\) are provided.

Corrections to naive factorization involve gluon exchanges between the charmonium system and the quarks in \(B\) and \(K\) mesons. For a class of nonleptonic \(B \to M_1 M_2\) decays it has been argued that, in the large \(m_b\) limit, non factorizable corrections are dominated by hard (perturbatively calculable) gluon exchanges, while soft effects are confined to the \((B, M_1)\) system, where \(M_1\) is the meson picking up the spectator quark in \(B\) decay. This is the case of several processes where \(M_1\) and \(M_2\) are light mesons. However, when the meson which does not pick up the spectator quark is heavy, such a result no longer holds \([\mathbb{G}]\). In order to apply the QCD-improved factorization model to \(B\) decays to charmonium plus a kaon, either the \(c\bar{c}\) state should be considered light with respect to the \(B\) meson, or one has to invoke the small transverse size of the \(c\bar{c}\) system in order to assume a tiny overlap of the quarkonium wave function with the kaon wave function. However, an analysis of non factorizable corrections due to hard gluon exchanges in \(B \to K\chi_{c0}\) has revealed the presence of infrared singularities, showing a difficulty of the method when applied to this decay mode \([\mathbb{H}]\). In the present note we investigate another effect, namely the \(K\chi_{c0}\) production by rescattering of open charm mesons \(D_s^{(*)}D^{(*)}\) etc. primarily produced in \(B^-\) decays. The corresponding amplitude is mainly obtained by the operators \(O_1\) and \(O_2\) in

\(^2\)The definition of \(a_7\) and \(a_9\) includes a factor \(e\).
and therefore this process could produce a sizeable contribution to \( B \to K \chi_{c0} \) owing to the relatively large values of the corresponding Wilson coefficients \( c_1 \) and \( c_2 \). Analogous effects were investigated in \([3]\), and have recently received new attention \([3, 4]\).

Rescattering of intermediate \( D_s^{(*)} D^{(*)} \) mesons can also contribute to the transition \( B \to K J/\psi \) and therefore we also consider this decay mode. Our conclusion is that, although the calculation presents uncertainties the size of which we shall try to assess, rescattering effects represent a non-negligible contribution to the decay channels \( B^- \to K^- \chi_{c0} \) and \( B^- \to K^- J/\psi \).

2 Process \( B^- \to D_s^{(*)} D_s^{(*)0} \to K^- \chi_{c0} \)

In the charm sector, rescattering effects have been recognized as a source of sizeable contributions in hadronic \( D \) meson decays. As the mass of the decaying \( B \) meson is larger than the \( D \) meson mass, one could suppose a minor role of such processes in \( B \) transitions, since one naively expects that high momentum final state particles move fast away from the interaction region without having the possibility to rescatter \([3]\). However, in a number of analyses it has been shown that rescattering effects can play an important role even in \( B \) decays \([1, 10, 11, 12]\).

It is worth attempting an estimate of the size of rescattering effects in color-suppressed \( B \) decays to final states containing heavy particles. We concentrate on two-body charmed meson contributions \([10]\), which can be included through a number of dynamical assumptions. We consider a set of amplitudes corresponding to the diagrams in fig.4, that represent \( t \)-channel contributions to the final state interaction. The charmed intermediate states \( D_s^{(*)} \) and \( D^{(*)} \) rescatter to \( \chi_{c0} \) and \( K \) by the exchange of one resonance states, \( D \) and \( D^* \). We treat the exchanged resonances as virtual particles, with their propagators taken as Breit-Wigner forms.

The analysis of the diagrams in fig.4 involves the weak matrix elements governing the transitions \( B^- \to D_s^{(*)} D_s^{(*)0} \), and the strong couplings between the charmed states \( D_s^{(*)} D_s^{(*)0} \) and the kaon and \( \chi_{c0} \). There is experimental evidence that the calculation of the amplitude by factorization reproduces the main features of the \( B^- \to D_s^{(*)} D_s^{(*)0} \) decay modes \([14]\). Therefore, neglecting the contribution of the operators \( O_{3–10} \) in \([3]\), we can write:

\[
\langle D_s^{(*)} D_s^{(*)0} | H_W | B^- \rangle = \frac{G_F}{\sqrt{2}} V_{cb} V^*_{cs} a_1 \langle D_s^{(*)0} | (V - A)^\mu | B^- \rangle \langle D_s^{(*)} | (V - A)_\mu | 0 \rangle \]  

\(3\)The role of inelastic effects in \( B \) decays has been emphasized in \([13]\).
Figure 1: Diagram contributing to the decay $B^- \rightarrow K^- \chi_{c0}$. The boxes represent weak vertices, the dots strong couplings

with $a_1 = c_1 + c_2/N_c$. Using the heavy quark effective theory, the above matrix elements can be expressed in terms of a single form factor, the Isgur-Wise function $\xi$, and a single leptonic constant $\hat{F}$. This can be shown expressing the fields $H_a$ describing the negative parity $J^P = (0^-,1^-)$ $\bar{q}Q$ meson doublet in terms of operators $P^*_a\mu$ and $P^a_{\mu}$ respectively annihilating the $1^-$ and $0^-$ mesons of four-velocity $v$ ($a = u,d,s$ is a light flavour index), and writing the $B^- \rightarrow D^{(*)0}$ matrix elements as follows:

$$<D^0(v')|V^\mu|B^-(v)> = \sqrt{m_Bm_D} \xi(v \cdot v')(v + v')^\mu$$
$$<D^{*0}(v',\epsilon)|V^\mu|B^-(v)> = -i\sqrt{m_Bm_{D^*}} \xi(v \cdot v') \epsilon^\beta_\alpha \epsilon^{\alpha\beta\gamma\mu} v_\alpha v'_\gamma$$
$$<D^{*0}(v',\epsilon)|A^\mu|B^-(v)> = \sqrt{m_Bm_{D^*}} \xi(v \cdot v') \epsilon^\beta_\alpha [(1 + v \cdot v')g^{\beta\mu} - v^\beta v'^\mu] .$$ (8)

In (8) $\epsilon$ is the $D^*$ polarization vector and $\xi(v \cdot v')$ represents the Isgur-Wise form factor.

The weak current for the transition from a heavy to a light quark $Q \rightarrow q_a$, given at the quark level by $\bar{q}_a \gamma^\mu (1 - \gamma_5)Q$, can be written in terms of a heavy meson and light pseudoscalars. The octet of the light pseudoscalar mesons is represented by $\xi = e^{i\mathcal{M}}$, with

$$\mathcal{M} = \begin{pmatrix} \sqrt{\frac{1}{2}}\pi^0 + \sqrt{\frac{1}{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\sqrt{\frac{1}{2}}\pi^0 + \sqrt{\frac{1}{6}}\eta & K^0 \\ K^- & K^0 & -\sqrt{\frac{2}{3}}\eta \end{pmatrix}$$ (9)

and $f \simeq f_\pi = 131$ MeV, and the effective heavy-to-light current, written at the lowest
order in the light meson derivatives, reads:

\[ L_a' = \frac{\hat{F}}{2} Tr [\gamma^\mu (1 - \gamma_5) H_b \xi^d_{ba}] \]  

(10)

In this way, all the matrix elements \(< 0 | \bar{q}_a \gamma^\mu (1 - \gamma_5) c | D_a^\nu (v) >\) are related to the single constant \(\hat{F}\):

\[ < 0 | \bar{q}_a \gamma^\mu \gamma_5 c | D_a (v) > = f_{D_a} m_{D_a} v^\mu \]

\[ < 0 | \bar{q}_a \gamma^\mu c | D_a^* (v, \epsilon) > = f_{D_a} m_{D_a} \epsilon^\mu \]  

(11)

with \(f_{D_s} = f_{D_s} = \frac{F}{\sqrt{m_{D_a}}}\).

Other hadronic quantities appearing in the diagrams in fig.4 are the strong couplings \(D^s_s D^s_s K\) and \(D^s_s D^s_s \chi_{c0}\). The \(D^s_s D^s_s K\) couplings, in the soft \(\vec{p}_K \rightarrow 0\) limit, can be related to a single effective constant \(g\), as it turns out considering the effective QCD Lagrangian describing the strong interactions between the heavy \(D^s_s D^s_s\) mesons and the octet of the light pseudoscalar mesons [16]:

\[ L_I = ig Tr [H_b \gamma_\mu \gamma_5 A_{ba}^{\mu} H_a] \]  

(12)

with the operator \(A\) in (12) given by

\[ A_{\mu ba} = \frac{1}{2} \left( \xi^d_\mu \partial \xi - \xi \partial_\mu \xi^d \right)_{ba} . \]  

(13)

This allows to relate the \(D^s_s D^s_s K\) couplings, defined through the matrix elements

\[ < D^0 (p) K^- (q) | D^s_s^- (p + q, \epsilon) > = g_{D_s^-D_s^- K^-} (\epsilon \cdot q) \]

\[ < D^a (p, \eta) K^- (q) | D^s_s^- (p + q, \epsilon) > = i \epsilon^{\alpha \beta \mu \gamma} p_\alpha \epsilon_\beta q_\mu \eta^\gamma g_{D^-s_s-D^-s_s K^-} \]  

(14)

to the effective coupling \(g\):

\[ g_{D_s^-D_s^- K^-} = -2 \sqrt{m_{D_s} m_{D_s}} \frac{g}{f_K} \]

\[ g_{D^-s_s-D^-s_s K^-} = 2 \sqrt{m_{D_s} m_{D_s}} \frac{g}{f_K} \]  

(15)

As for the coupling of the \(\chi_{c0}\) state to a pair of \(D\) mesons, defined by the matrix element:

\[ \langle D^0 (p_1) D^0 (p_2) | \chi_{c0} (p) \rangle = g_{DD\chi_{c0}} \]  

(16)

an estimate can be obtained considering the \(D\) matrix element of the scalar \(\bar{c}c\) current:

\[ \langle D (v') | \bar{c}c | D (v) \rangle \), assuming the dominance of the nearest resonance, i.e. the scalar \(\bar{c}c\) state,
in the $(v-v')^2$-channel and using the normalization of the Isgur-Wise form factor at the
zero-recoil point $v = v'$. This allows us to express $g_{DD\chi_{c0}}$ in terms of the constant $f_{\chi_{c0}}$
that parameterizes the matrix element

$$
\langle 0|\bar{c}c|\chi_{c0}(q)\rangle = f_{\chi_{c0}} m_{\chi_{c0}}.
$$

(17)

The method can also be applied to $g_{D^*D^*\chi_{c0}}$. One obtains:

$$
g_{DD\chi_{c0}} = -2\frac{m_D m_{\chi_{c0}}}{f_{\chi_{c0}}} \quad g_{D^*D^*\chi_{c0}} = 2\frac{m_{D^*} m_{\chi_{c0}}}{f_{\chi_{c0}}}.
$$

(18)

It is worth noticing, however, that the determinations of the couplings described above
do not account for the off-shell effect of the exchanged $D$ and $D^*$ particles, the virtuality
of which can be large. As discussed in the literature, a method to account for such effect
relies on the introduction of form factors:

$$
g_i(t) = g_{i0} F_i(t),
$$

(19)

with $g_{i0}$ the corresponding on-shell couplings \[14], \[16]. A simple pole representation for
$F_i(t)$ is: $F_i(t) = \frac{\Lambda_i^2 - m_{D(i)}^2}{\Lambda_i^2 - t}$, consistent with QCD counting rules \[17]. The parameters
in the form factors represent a source of uncertainty in our analysis.

We have the elements for computing the diagrams in fig.1. The absorptive part of the
amplitude (1) simply reads:

$$
\text{Im} A_1 = \sqrt{\frac{\lambda(m_B^2, m_{D_s}^2, m_D^2)}{32\pi m_B^2 m_D}} \int_{-1}^{+1} dz A(B^- \rightarrow D_s^- D^0) A(D_s^- D^0 \rightarrow K^- \chi_{c0})
$$

(20)

with $\lambda$ the triangular function. Analogous expressions correspond to the diagrams (2)
and (3). Explicitly, the imaginary parts are given by

$$
\text{Im} A_1 = \frac{K f_{D_s} m_{D_s} \sqrt{m_B m_D}(m_B + m_D)}{32\pi m_B^2 m_D} \lambda^{1/2}(m_B^2, m_{D_s}^2, m_D^2) \int_{-1}^{1} dz g_{D_s D K}(t) g_{D D \chi}(t)
$$

$$
\times \left( \frac{m_B^2 - m_{D_s}^2 + m_D^2}{2m_B m_D} \right) - q^0 + \frac{k^0 q \cdot k}{m_D^2}
$$

$$
\frac{t - m_{D_s}^2}{t - m_D^2}
$$

$$
\text{Im} A_2 = \frac{K f_{D_s} \sqrt{m_B m_D}}{32\pi m_B^2} \lambda^{1/2}(m_B^2, m_{D_s}^2, m_{D^*}^2) \int_{-1}^{1} dz g_{D_s D^* K}(t) g_{D^* D^* \chi}(t)
$$

$$
\times \left( \frac{m_B^2 - m_{D_s}^2 + m_{D^*}^2}{2m_B m_{D^*}} \right) \left\{ \left[ \frac{m_K^2 - q \cdot k}{m_{D_s}^2} - 1 \right] \left( 1 + v \cdot v_D \right) q \cdot k + v_D \cdot k (q^0 + v_D \cdot q) \right\}
$$
\[-\frac{m_K^2 - q \cdot k}{m^2_{D_s}} \{ -(1 + v_D)m^2_{D_s} + v_D \cdot k(k^0 + v_D \cdot k) \} \]

\[\text{Im } A_3 = \frac{-Kf_D^* m_D}{16\pi m_B^3} \sqrt{m_B m^2} \lambda^{1/2}(m_B^2, m_{D_s}^2, m_{D^*}^2) \int_{-1}^1 dz g_D(t)g_D^*(t) \xi \left( \frac{m_B^2 - m_{D_s}^2 + m_{D^*}^2}{2 m_B m_D^*} \right) q^0 v_D \cdot q - k^0 v_D \cdot q \]

\[
\xi \left( \frac{m_B^2 - m_{D_s}^2 + m_{D^*}^2}{2 m_B m_D^*} \right) q^0 v_D \cdot q - k^0 v_D \cdot q, \tag{21}
\]

where: \( K = \frac{G_F}{\sqrt{2}} V_{cb} V^*_{cs} a_1, q^0 = m_B^2 + m_K^2 - m_\chi^2, |q| = \frac{\lambda^{1/2}(m_B^2, m_K^2, m_\chi^2)}{2 m_B}, \) together with:
\[v \cdot v_D = \frac{m_B - k^0}{m_{D_s}}, v_D \cdot k = \frac{m_B k^0 - m_{D_s}^2}{m_{D_s}}, v_D \cdot q = \frac{m_B q^0 - q \cdot k}{m_{D_s}}, k^0 = \frac{m_B^2 + m_{D_s}^2 - m_{D_s}^2}{2 m_B}, \]
\[|k| = \frac{\lambda^{1/2}(m_B^2, m_{D_s}^2, m_{D^*}^2)}{2 m_B} \] and \( m_{D_{s,1}} = m_{D_{s,3}} = m_{D^*_s}, m_{D_{s,2}} = m_{D_s}; m_{D_s} = m_D, m_{D_2} = m_{D^*}. \)

The dispersive parts of the amplitudes in fig.1 can be estimated using
\[
\text{Re} A_i(m_B^2) = \frac{1}{\pi} PV \int_{s_{th}^{(i)}}^{+\infty} ds' \frac{\text{Im} A_i(s')}{s' - m_B^2}, \tag{22}
\]

with the thresholds \( s_{th}^{(i)} \) given by: \( s_{th}^{(1)} = (m_{D_s} + m_D)^2, s_{th}^{(2)} = (m_{D_s} + m_{D^*})^2 \) and \( s_{th}^{(3)} = (m_{D^*_s} + m_{D^*})^2 \), respectively. It can be assumed that such expressions are dominated by the region close to the pole \( m_B^2 \). Therefore, we compute the integrals, that in general depend on the asymptotic behavior of the spectral functions \( \text{Im} A_i(s') \), by using a cutoff not far from the \( B \) meson mass, chosen in the range \( 35 - 40 \text{ GeV}^2 \).

A comment on other contributions to \( K^- \chi_{c0} \) via final state rescattering is in order, since also the \( D^{(*)} \) and \( D_s^{(*)} \) excitations could be considered as intermediate states. These terms are suppressed by smaller values of the universal form factors and of the leptonic decay constants; therefore, the amplitudes in fig.1 represent the main contributions that need to be analyzed.

### 3 Mode \( B^- \rightarrow K^- J/\psi \)

Before attempting a numerical estimate of the amplitudes in fig.1, let us consider the decay mode \( B^- \rightarrow K^- J/\psi \). In this case the amplitude obtained in the naive factorization approach, keeping only the contribution of the operators \( \mathcal{O}_1 \) and \( \mathcal{O}_2 \) in (3), is given by
\[
\mathcal{A}_F(B^- \rightarrow K^- J/\psi) = \frac{2 G_F}{\sqrt{2}} V_{cb} V^*_{cs} a_2 f_{J/\psi} m_{J/\psi} F_{1B}(m_{J/\psi}^2) (\epsilon^* \cdot q) = \tilde{\mathcal{A}}_F(\epsilon^* \cdot q), \tag{23}
\]
with the constant $f_{J/\psi}$ defined by

$$\langle 0|\bar{c}\gamma^\mu c|J/\psi(p', \epsilon)\rangle = f_{J/\psi} m_{J/\psi} \epsilon^\mu,$$  \hspace{1cm} (24)

$\epsilon$ the $J/\psi$ polarization vector, $q$ the kaon momentum and $F^{BK}_1$ one of the two form factors parameterizing the matrix element $\langle K^-|\bar{s}\gamma^\mu b|B^-\rangle$.

Identifying eq.(23) with the experimental amplitude obtained from the measurement $B(B^- \to K^- J/\psi) = (1.00 \pm 0.10) \times 10^{-3}$ \cite{18}: $\tilde{A}_{\text{exp}} = (1.41 \pm 0.07) \times 10^{-7}$, and using the value $f_{J/\psi} = 405 \pm 14$ MeV, one obtains a result for the product $|a_2 F^{BK}_1(m^2_{J/\psi})|$. This determination of $a_2$ is mainly affected by the uncertainty on $F^{BK}_1$; scanning several form factor models, as done in \cite{19}, one gets $|a_2| = 0.2 - 0.4$, while considering the calculation in \cite{20} one obtains $|a_2| = 0.38 \pm 0.05$. Such results are obtained using $V_{cb} = 0.040$ and $V_{cs} = 0.9735$ that correspond to the central values reported by the Particle Data Group \cite{18}.

As discussed at length in the literature, the above values of $a_2$ are different from the combination $a_2 = c_2 + c_1/N_c$ of the Wilson coefficients in (3). As a matter of fact, from the values $c_1 = 1.085(1.109)$ and $c_2 = -0.198(-0.243)$ computed for $\overline{m}_b(m_b) = 4.4$ GeV and $\Lambda^{(5)}_{MS} = 290$ MeV in the naive dimensional regularization (or ’t Hooft-Veltman) scheme \cite{21}, one would get: $a_2 = 0.163(0.126)$. \footnote{Similar values are obtained varying $\overline{m}_b(m_b)$ and $\Lambda^{(5)}_{MS}$.}

Therefore, nonfactorizable effects are sizeable in $B^- \to K^- J/\psi$, and indeed in a generalized factorization ansatz $a_2$ is treated as an effective parameter used to fit the data. The calculation in the framework of QCD factorization does not allow to reproduce the fitted value, although an improvement towards the experimental datum is obtained \cite{22}. It is worth considering rescattering contributions of intermediate charm mesons, described by diagrams as depicted in fig.2. The hadronic information for determining such amplitudes are the same as in Section 2, with the only difference in the strong $D^{(*)} D^{(*)} J/\psi$ couplings that can be expressed in terms of the parameter $f_{J/\psi}$, using the same vector meson dominance method applied to derive eq. (18).

4 Numerical calculation and discussion

In order to evaluate the amplitudes in figs.1,2 we have to fix the values of the various hadronic parameters. The Wilson coefficient $a_1$, common to all the amplitudes, can be put to $a_1 = 1.0$ as obtained by the analysis of exclusive $B \to D_s^{(*)} D^{(*)}$ transitions.
Figure 2: Rescattering diagrams contributing to $B^- \rightarrow K^- J/\psi$.

Moreover, we use $f_{D_s} = 240$ MeV, in the range quoted by the Particle Data Group \cite{18}, and $f_{D_s^*} = f_{D_s}$ consistently with our approach that exploits the large $m_Q$ limit. For the Isgur-Wise universal form factor $\xi$, the expression $\xi(y) = \left(\frac{2}{y+1}\right)^2$ is compatible with the current results from the semileptonic $B \rightarrow D^{(*)}$ decays.

A discussion is needed about the $D^{(*)} D^{(*)} K$ vertices. For the effective coupling $g$ in (15) one can use the CLEO result $g = 0.59 \pm 0.01 \pm 0.07$ obtained by the measurement of the $D^*$ width \cite{23}. This value is in the upper side of several theoretical calculations \cite{24,25}. We choose to be conservative, and vary this parameter in the range: $0.35 < g < 0.65$ that encompasses the largest part of the predictions.

We use the expression (18) for the $D^{(*)} D^{(*)} \chi_{c0}$ vertices, with $f_{\chi_{c0}} = 510 \pm 40$ MeV obtained by a standard two-point QCD sum rule analysis. As for the couplings $D^{(*)} D^{(*)} J/\psi$, expressions analogous to (18) involve $f_{J/\psi}$, for which we use the experimental measurement. To account for the off-shell effects of the $D^{(*)}$ exchanged particles, we use eq.(19) with two choices for the parameters: $\Lambda_i = 2.5$ GeV and $\Lambda_i = 2.8$ GeV, corresponding to typical values of the mass of the radial excitations of $D^{(*)}$ mesons.

The results are reported in Table 1. One has to notice that the rescattering amplitudes contribute with different signs to the final result, with significant cancellations among the various terms.

A few observations are in order. First, in the chosen range of values for the vari-
Table 1: Numerical results for the rescattering amplitudes

| $B^- \rightarrow K^-\chi_{c0}$ | Re$\tilde{A}$ (GeV) | Im$\tilde{A}$ (GeV) | $\Lambda_i$ (GeV) |
|---------------------------------|---------------------|---------------------|------------------|
| $(0.9 - 1.7) \times 10^{-7}$    | $(0.5 - 1.0) \times 10^{-7}$ | 2.5               |
| $(1.4 - 2.7) \times 10^{-7}$    | $(0.6 - 1.2) \times 10^{-7}$ | 2.8               |

| $B^- \rightarrow K^-J/\psi$    | Re$\tilde{A}$ (GeV) | Im$\tilde{A}$ (GeV) | $\Lambda_i$ (GeV) |
|---------------------------------|---------------------|---------------------|------------------|
| $(0.1 - 0.2) \times 10^{-7}$    | $(0.5 - 0.9) \times 10^{-7}$ | 2.5               |
| $(0 - 0.3) \times 10^{-7}$      | $(0.9 - 1.7) \times 10^{-7}$ | 2.8               |

Oous parameters, the rescattering amplitudes are sizeable, and become comparable to the experimental ones. This observation can be made more quantitative. Assuming that the amplitude relative to $B^- \rightarrow K^-J/\psi$ deviates from the factorized result because of the contribution of the rescattering term: $\tilde{A}_{exp} = \tilde{A}_{fact} + \tilde{A}_{resc}$, one can constrain the values of $\Lambda_i$ for the calculation of $\mathcal{B}(B^- \rightarrow K^-\chi_{c0})$. One obtains: $\Lambda_i \approx 2.7$ GeV and $\mathcal{B}(B^- \rightarrow K^-\chi_{c0}) = (1.1 - 3.5) \times 10^{-4}$, to be compared to [1]. The result seems noticeable, considering the rather schematic description of the rescattering process.

The second observation is that a correspondence similar to that between $B^- \rightarrow K^-\chi_{c0}$ and $B^- \rightarrow K^-J/\psi$ is expected with analogous decay modes, namely $B^- \rightarrow K^-\chi_{c1}$ and $B^- \rightarrow K^-\chi_{c2}$, the mesons $\chi_{c1,2}$ being the axial vector and the tensor states of the charmonium system. It is worth observing that $B^- \rightarrow K^-\chi_{c2}$ is another process the amplitude of which vanishes in the naive factorization model; in our approach, we expect a branching fraction analogous to that of $B^- \rightarrow K^-\chi_{c0}$.

We are aware of the various sources of theoretical uncertainty. The uncertainty related to the $B^- \rightarrow D_s^{(*)}D^{(*)}$ vertices can be minimized gaining experimental information on these decay modes. The uncertainties in the strong vertices can be reduced by dedicated analyses using nonperturbative QCD methods (QCD sum rules or lattice QCD). However, all such uncertainties only affect the precise numerical predictions and not our main conclusion. We have found that rescattering amplitudes, describing a rearrangement of the quarks in the final state after the production of pairs of charmed mesons, not only cannot be neglected both in $B^- \rightarrow K^-\chi_{c0}$, both in $B^- \rightarrow K^-J/\psi$, but can provide a large part of the decay amplitudes. Analogous effects are expected to be important in similar colour suppressed decay modes, namely $B^- \rightarrow K^-\chi_{c1}$ and $B^- \rightarrow K^-\chi_{c2}$.
References

[1] For a review see: M. Neubert and B. Stech, Adv. Ser. Direct. High Energy Phys. 15 (1998) 294.

[2] K. Abe et al. [Belle Collab.], Phys. Rev. Lett. 88 (2002) 031802.

[3] M. Beneke, G. Buchalla, M. Neubert and C. T. Sachrajda, Phys. Rev. Lett. 83 (1999) 1914; Nucl. Phys. B 591 (2000) 313.

[4] Z. Song and K. T. Chao, arXiv:hep-ph/0206253.

[5] P. Colangelo, G. Nardulli, N. Paver and Riazuddin, Z. Phys. C 45 (1990) 575.

[6] M. Ciuchini, E. Franco, G. Martinelli, M. Pierini and L. Silvestrini, Phys. Lett. B 515 (2001) 33.

[7] C. Isola, M. Ladisa, G. Nardulli, T. N. Pham and P. Santorelli, Phys. Rev. D 64 (2001) 014029; ibidem 65 (2002) 094005.

[8] J. D. Bjorken, Nucl. Phys. Proc. Suppl. 11 (1989) 325; M. J. Dugan and B. Grinstein, Phys. Lett. B 255 (1991) 583; H. D. Politzer and M. B. Wise, Phys. Lett. B 257 (1991) 399.

[9] M. Gronau and J. L. Rosner, Phys. Rev. D 57 (1998) 6843.

[10] M. Neubert, Phys. Lett. B 424 (1998) 152.

[11] M. Gronau and J. L. Rosner, Phys. Rev. D 58 (1998) 113005.

[12] A. F. Falk, A. L. Kagan, Y. Nir and A. A. Petrov, Phys. Rev. D 57 (1998) 4290.

[13] J. F. Donoghue, E. Golowich, A. A. Petrov and J. M. Soares, Phys. Rev. Lett. 77 (1996) 2178.

[14] Z. Luo and J. L. Rosner, Phys. Rev. D 64 (2001) 094001.

[15] For reviews see: M. Neubert, Phys. Rept. 245 (1994) 259; A. V. Manohar and M. B. Wise, Cambridge Monogr. Part. Phys. Nucl. Phys. Cosmol. 10 (2000) 1; F. De Fazio, in "At the Frontier of Particle Physics - Handbook of QCD", edited by M. A. Shifman, (World Scientific, 2001), page 1671.
[16] M. B. Wise, Phys. Rev. D 45 (1992) 2188; G. Burdman and J. F. Donoghue, Phys. Lett. B 280 (1992) 287; T. M. Yan et al., Phys. Rev. D 46 (1992) 1148 [Erratum-ibid. D 55 (1992) 5851].

[17] O. Gortchakov, M. P. Locher, V. E. Markushin and S. von Rotz, Z. Phys. A 353 (1996) 447.

[18] D. E. Groom et al. [Particle Data Group Collab.], Eur. Phys. J. C 15 (2000) 1.

[19] H. Y. Cheng and K. C. Yang, Phys. Rev. D 59 (1999) 092004.

[20] P. Colangelo, F. De Fazio, P. Santorelli and E. Scrimieri, Phys. Rev. D 53 (1996) 3672 [Erratum-ibid. D 57 (1998) 3186].

[21] A. J. Buras, arXiv:hep-ph/9806471.

[22] H. Y. Cheng and K. C. Yang, Phys. Rev. D 63 (2001) 074011.

[23] A. Anastassov et al. [CLEO Collab.], Phys. Rev. D 65 (2002) 032003.

[24] For reviews see: P. Colangelo and A. Khodjamirian, in At the Frontier of Particle Physics / Handbook of QCD, ed. by M. Shifman (World Scientific, Singapore, 2001) 1495; D. Becirevic and A. L. Yaouanc, JHEP 9903 (1999) 021.

[25] For new analyses see: P. Colangelo and F. De Fazio, Phys. Lett. B 532 (2002) 193; F. S. Navarra, M. Nielsen and M. E. Bracco, Phys. Rev. D 65 (2002) 037502; A. Abada et al., arXiv:hep-ph/0206237.