Backtesting Trading Strategies with GAN To Avoid Overfitting

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Abstract—Many works have shown the overfitting hazard of selecting a trading strategy based only on good IS (in-sample) performance. But most of them have merely shown such phenomena exist without offering ways to avoid them. We propose an approach to avoid overfitting: A good (meaning non-overfitting) trading strategy should still work well on paths generated in accordance with the distribution of the historical data. We use GAN with LSTM to learn or fit the distribution of the historical time series. Then trading strategies are backtested by the paths generated by GAN to avoid overfitting. (This paper is an translated English version of [1] which was originally written in Chinese in 2018, where some statements and claims are outdated in 2022.)

Index Terms—Backtest, Backtest Overfitting, GAN, LSTM, Algorithm trading.

1 INTRODUCTION

In algorithm trading, strategy and backtesting are important elements. The so-called strategy refers to the rules that determine when to buy, when to sell, and the amount of the transaction [2]. When we have a strategy, the most common practice is to do backtesting on historical data, meaning that we would apply this strategy to past data to see how it performs, as a way of evaluation for this strategy.

This sounds good: we use some methods to find strategies, then apply them to historical stock prices and get an indicator such as Sharpe ratio to assess the performance of strategies. We pick the strategies with good indicator values, and finally adopt these strategies in the real world, and then we can enjoy a good life.

However this barely happens in real life. The gap between the dream and the reality is due to a phenomenon called backtest overfitting.

The purpose of this study is to propose a backtesting framework, a data-generation-based one where the data generating process is learned use the generative adversarial network (GAN). This may extend the usage of GAN.

2 RELATED WORK

There have been many studies on backtest overfitting. In [3], the authors severely criticized some of the problems that were overlooked in the backtesting, including excessive data mining without considering the increasing probability of false positive. They proposed a concept, the minimum Backtest length (MinBTL), to help to control this risk. The authors’ idea is as following: Suppose we chose N numbers i.i.d. from the standard normal distribution, and they calculate $E_{\max}^n$, which is the expected value of the maximum of the N chosen numbers.

Then with a experiment which conducts N-number-choose (corresponding to the number of times we try different configurations on the same data) and a null hypothesis that the mean of the distribution is greater than zero, if the value $E_{\max}^n$ appears, we should treat it as not significant. From the perspective of the annual Sharpe Ratio, $E_{\text{annual}}[\max_n] \approx y^{-1/2}E_{\max}^n$, where y is the length of the sample. If we increase y we can increase $E_{\text{annual}}[\max_n]$ so that the Sharpe Ratio of the experiment becomes more convincing (for example, $E_{\text{annual}}[\max_n]$ is close to zero, then the Sharpe Ratio greater than zero is considered to be significant result rather than an false positive result). More discussion on this topic on be found in [4].

The concept of probability of backtest overfitting was proposed in [3].

$$\sum_{n=1}^{N} E[\bar{r}_n | r \in \Omega_n^*] \cdot \text{Prob}[r \in \Omega_n^*] < N/2$$

where $E[\bar{r}_n | r \in \Omega_n^*]$ refers to the ranking of the best strategy r in the sample outside the sample. The intuition of the formula is: Overfitting is happening when the OOS(Out Of Sample) performance of the strategy who owns the best IS(In Sample) performance is lower than the average performance. More specially, the best-ranked strategy within the sample has a lower rank outside the sample than the average. The author’s method is to observe the strategy-representation matrix, where $M_{st}$ represents the performance of strategy s at time t (such as how much profit or loss). Given a constant $K_t$, then $M_{st}$ is divided into K equal time blocks to form a set, from which the method will to chose any two of them to form a sample pair (in the sample, outside the sample), and then use these sample pairs to do the backtesting and the selection. But note that this method only applies to the situation that there are multiple strategies and you need to choose one of these. The goal is to avoid the overfitting of selection process.

In [6], this problem is discussed from a statistical point of view: when searching the best configuration (hypperparameters) for a model on a single data set, and evaluated by the Sharpe Ratio, the multiple-testing needs to replace the original single test, which means an additional haircut is required for the Sharpe Ratio obtained from the test. Although the adjustment of the Sharpe Ratio has been used in practical
3 PREREQUISITE

3.1 Strategy

There seems to be no standard in the definition of a strategy. The trading rules defined in [2] represent the actual buying and selling. For the convenience of later discussion, we also made a definition of the strategy: we can think of it as a function that maps a time series (such as stock price) to -1, 0, 1, where -1 is to short, 0 is to not operate, 1 is to long as shown in eq. (1)

\[ S_\theta : \{p_t\}^T \times \{-1, 0, 1\}^T \rightarrow R \]  

And there’s two strategies we will use to discuss next:

3.1.1 MAC strategy

The MAC strategy is moving average cross [2]. This model accepts two parameters \( p_1 \) and \( p_2 \). The strategy calculates the mean \( MA_1 \) of the last \( p_1 \) periods stock prices and \( MA_2 \) of the last \( p_2 \) periods. If \( p_1 > p_2 \), let the position become 1 and vice versa. For more specific information, refer to table 1.

| Parameters | \( p_1 \) | \( p_2 \) |
|------------|-----------|-----------|
| Range      | 1~50      | 1~50      |
| Exp.       | average price | average price |

| TABLE 1: MAC strategy
Parameters space for Moving Average Cross Strategy |

3.1.2 BH strategy

The buy and hold strategy (buy and hold, referred to as BH) will accept four parameters entry, hold, stop-loss and side. The logic of the strategy is to divide the trading day into months, and the transaction is carried out in each month: enter on the entry (e.g. 1-31) day, hold for hold days and then sell [3]. And if the loss during the holding period exceeds the stop-loss, sell it in advance. For more specific information, refer to table 2.

| Parameters | entry day | holding days | stop loss | side |
|------------|-----------|--------------|-----------|------|
| Range      | 1~30      | 1~30         | 0~20      | -1,1 |
| Exp.       | entry day | holding days | stop loss | long/short |

| TABLE 2: BH strategy
Parameters space for Buy and Hold Strategy |

3.2 Backtesting

The so-called backtesting refers to simulating a trading strategy in history, and then calculating a certain indicator to help judge the pros and cons of the trading strategy. Define a backtesting function \( B \) as eq. (2)

\[ B : \{p_t\}^T \times \{-1, 0, 1\}^T \rightarrow R \]

The outcomes of a strategy \( S \) are mapped to a real value as a measure of the quality of strategy \( S \). The specific \( B \) will vary according to different trading goals, but we stipulate that the larger the value of \( B \) is, the better the strategy is (otherwise, it is multiplied by -1), and we call the value of \( B \) the performance of the strategy.

The common Sharpe Ratio used in our study:

\[ SR : \{p_t\}^T \times \{-1, 0, 1\}^T \rightarrow \frac{E[X] - R_f}{\sigma_X} \]

where \( X \) is defined as gain or loss calculated by each pair if \( \{p_t\} \) and \( \{-1, 0, 1\}^T \)

3.3 Objective function of backtesting

Under the definition of eq. (1) and eq. (2), we can define the object of a trader to eq. (4)

\[ \theta^* = \arg \max_{\theta} E_\mu \left[ B[S(p_\mu; \theta)] \right] \]

where \( p_\mu \) are the paths sampled from the distribution \( \mu \).

That is we want to maximize the expected performance of the strategy with sampled paths. And here is the reason: we may easily move the which is similar to the bagging in the machine learning. That is we let the paths vote whether this strategy is good or bad. In the experiment we use the mean to

\[ \theta^* = \arg \max_{\theta} \frac{1}{N} \sum_{\mu \in A_N} B[S(p_\mu; \theta)] \]

where \( A_N \) refers to the set of sampled paths.

3.4 Backtesting Overfitting

The process of solving eq. (5) is likely to cause the most troublesome problem in quantitative trading: overfitting. For a detailed discussion, please refer to the following chapters.

Here is an example to illustrate. Suppose we have a buy-and-hold strategy, call it \( S' \). We assume that the generation of stock prices is a random walk:

\[ P(t+1) = P(t) + \epsilon(t) \]

where \( \epsilon(t) \sim N(0,1) \)

Let us generate a 600-day data with eq. (6) and take the first 300 days as historical data and the last 300 days as future data. So our goal is to find the best \( \theta^* \) on historical data to get a strategy \( S'' \) then apply \( S'' \) to future data and examine its performance.

First, one consequence is that the expected value of strategy \( S'' \) should theoretically not be greater than 0 on the paths generated by eq. (6). This is because if strategy
Fig. 1: IS vs OOS performance
Corresponding to the Y-axis on the right, the green part is the equity curve in the sample, and the red part is the equity curve of the same strategy outside the sample. Corresponding to the left Y-axis, the solid black line is the in-sample price, and the black dashed line is the out-of-sample price.

$S'^\ast$ leaves the market early, the profit and loss must be negative. However, if there is no early exit, the profit and loss is only related to the price of entry and exit. Suppose that the market is entered at time $t$ and held for $L$ days, and because

$$
E[P(t + L) - P(t)] = E \left[ \sum_{k=t}^{t+L} \epsilon(k) \right] = 0
$$

(7)

so the expected value should be zero.

But if we go through all the parameter combinations in table 2 and pick the configuration that has the best performance on historical data, fig. 1 will be the result. The green part is the equity curve (equity curve, which records the change of funds during the investment process) obtained on the historical data, and the red part is the equity curve obtained on the future data. The green equity curve clearly has a non-zero return. In addition, if we do not consider the above conclusion that the expected value is 0, we can also see an obvious result from the graph: although the historical performance is very good, the future performance is far from the historical performance.

The reason for this result is very simple: the configuration of this strategy performs so well in the sample because the configuration of this strategy fits the noise in the sample, for example, the backtest data happens to be on the 12th of each month On the 17th, it is all rising, and the parameters of the strategy happen to be buying and holding on the 12th for 5 days, but eq. (5) indicates that the probability of rising and falling from the next 12th to 17th is the same, so the strategy will be on the 12th to 17th. The 17th doesn’t actually make money.

3.5 Anti Backtesting Overfitting
To illustrate further, since we know the stock price generation formula eq. (5), we use Monte Carlo simulation to do another experiment: we apply $S'^\ast$ to 2000 stock-price-time-series generated with eq. (5) and get backtest results (for each generated path).

The results are shown in fig. 2. The blue frequency plot represents the distribution of the Sharpe Ratio of the strategy in fig. 1 on the generated 2000 paths, the green dashed line represents the distribution of the Sharpe Ratio of the IS in fig. 1 and the red dashed line represents the OOS.

The distribution of the Sharpe Ratio of the strategy in fig. 1 on the generated 2000 paths, the green dashed line represents the distribution of the Sharpe Ratio of the IS in fig. 1 and the red dashed line represents the OOS.

From this small experiment we can feel that perhaps referencing the results of backtesting on multiple paths as a distribution is a better goal for backtesting, see below for a further discussion.

Another angle to examine over-fitting can be examined from the smoothness of the parameter configuration corresponding to the performance, we use MAC and eq. (6) and white noise:

$$
P(t) = \epsilon(t)
$$

(8)

where

$$
\epsilon(t) \sim N(0,1)
$$

It should be noted that MAC has an expected value of 0 on the random walk procedure of eq. (6), while there is a strategy with an expected value greater than 0 on eq. (8), the results refer to fig. 3b and fig. 3a.

The figures show the performance of traversing all the configurations of the two parameters: the more blue it is, the higher the Sharpe Ratio is, the better the backtesting effect is; and the more red it is, the opposite is true.

On random walk, the dark blue part (that is, the of the corresponding configuration is significantly greater than 0) is not presented smoothly but jumps, while the result on white noise is much smoother.

This example shows that if a configuration of a strategy is found to perform well in the backtest, but the backtest of
its surrounding configurations is poor as shown in fig. 3b, then we should be concerned that this may be an overfitting result. Conversely, if the backtest function is smoother, it indicates that the probability of configuration overfitting will be smaller as shown in fig. 3a.

3.6 LSTM with GAN

Long-and-short term memory model neural network (LSTM) was first proposed by [9]. Different from the traditional neural network, LSTM not only uses the current input and output, but also uses the results calculated by the previous time steps, as shown in Figure 2.9, where $\sigma$ represents the sigmoid function.

The concept of Generative Adversarial Networks, proposed by Ian Goodfellow in [10] in 2014, has sparked a wave of research in recent years. There are countless papers appearing, all of which are various variants.

Almost all versions of GAN have the same idea: there is a generator $G$, which is responsible for mapping from the distribution $Z$ on a latent space to the distribution $T$ we want. The trick is to extract a sample from the potential space and throw it to the generator, which will generate a sample of the distribution $T$. In the process, a discriminator $D$ is also needed, which is responsible for distinguishing whether the data is generated from the $T$ distribution or the generator is generated from the $Z$ distribution.

RGAN was proposed in [11] earlier this year. The purpose of RGAN was to solve the privacy problem when real patient data was used as research data.

The R in RGAN corresponds to the meaning that both the generator and the discriminator are recurrent neural networks. RGAN was proposed in [11] earlier this year. The purpose of RGAN is to solve the privacy problem when real patient data was used as research data.

In [11], the target data is a time series with four dimensions: pulse oximetry, heart rate, respiratory rate, and mean arterial pressure. It can be seen that if we don’t care about the specific physical meaning of the value, we can completely replace the data with the opening price, the highest price, the lowest price and the closing price and bring it into the framework of RGAN. In addition, in this study, we changed the discriminator part of RGAN from LSTM to bidirectional LSTM, the purpose of which is to enhance the discriminator’s ability.

3.7 Model

First, let’s take a look at the performance of GAN on the common data generation model: whether it can learn this model, the requirements for the amount of data, and so on. From a verification point of view, we use the stock price model GBM, which is often assumed in the financial field.

fig. 3 is an overview of our model: data sampled from the latent space is thrown to a generator. The output of the generator has the same shape as the real data, so they will be thrown to the discriminator respectively. Since the produced data and the real data are time series, the discriminator at each moment will also have a corresponding output. When all the time data is input, we summarize the output of the discriminator for the generated data and the output of the discriminator for the real data separately, and finally put them together to calculate the cross entropy as loss.

fig. 3 is the overall architecture of the model. From a separate point of view, in the latent space part, we choose 5 dimensions, and each dimension is an independent standard normal distribution. In the generator part, we use the LSTM model, in which the number of hidden layer nodes, we have tested different numbers in the experimental part for comparison. The activation function passed by the hidden layer uses sigmoid, and the activation function of the output layer uses tanh. In the discriminator part, we tried a single RGAN and a two-way RGAN. The activation function passed by the hidden layer uses sigmoid, and the activation function of the output layer uses tanh. In the discriminator part, we tried a single RGAN and a two-way RGAN. The activation function passed by the hidden layer uses sigmoid, and the activation function of the output layer uses tanh, and the number of hidden layer nodes is that we did another experiment to compare.

The part that has the greatest impact on the model is the number of hidden layer nodes of LSTM, and its impact on parameters is shown in table 3.

| # hidden units | # parameters generator | discriminator | total |
|----------------|------------------------|---------------|-------|
| 100            | 42,901                 | 82,800        | 125,701 |
| 50             | 11,451                 | 21,400        | 32,851 |
| 10             | 691                    | 1,080         | 1,771  |

TABLE 3: Number of parameters and that of hidden units
follow the normal distribution where the mean value is
\[(\mu - \sigma^2/2) t\]
and the variance value is
\[\sigma^2 t\]

4.2 Preprocessing

In addition, since the final activation function of the generator is tanh, which means that the value range is (-1, 1), we need to normalize the samples to the interval of (-1, 1), here we use:

\[sp_{\text{new}} = 2 \times \left( \frac{sp_{\text{raw}} - sp_{\text{min}}}{sp_{\text{max}} - sp_{\text{min}}} \right)\]

(12)

Among which, \(sp_{\text{new}}\) generally refers to the samples generated by the model, \(sp_{\text{min}}\) and \(sp_{\text{max}}\) represent the minimum and maximum values of the samples generated by the model, and \(sp_{\text{new}}\) represents the samples we really use for RGAN.

In addition, since eq. (12) will make the maximum (small) value in the sample 1 (-1), when RGAN wants to generate a time series longer than 7, it will encounter a problem: the generated maximum value is limited. So our data preprocessing changed from eq. (12) to eq. (13)

\[sp_{\text{new}} = \frac{1}{\text{scaling}} \times \left( 2 \times \left( \frac{sp_{\text{raw}} - sp_{\text{min}}}{sp_{\text{max}} - sp_{\text{min}}} \right) \right)\]

(13)

where scaling is a constant of our choosing.

4.3 Evaluation

The evaluation for different models should be different, because it is quite difficult to test the distribution of time series [11], [13], but because of our choice of GBM, we can take the natural logarithm of its value in each period, then The distribution of each period will be a normal distribution, and its expected value and variance on the time scale will be linear with time.

In training, the criteria for judging whether a GAN is good at learning is vague [11], but since the characteristics of the time series we choose are obvious (expected value and variance increase linearly in time),

thus we use the theoretical The linear equation is used as our goal to calculate \(R^2\) with the data generated by GAN, as shown in fig. 5a and fig. 5b and the discussion of the entire training process, including the amount of data, the amount of parameters, etc., will be discussed in the experimental results and analysis.

4.4 Result

4.4.1 Statistical

fig. 5a shows what the samples generated by the GBM model look like. fig. 5b shows the generated frequency plot of the distribution of these samples at each moment. Just like eq. (11), the distribution of each period is close to the normal distribution.

From fig. 5c it seems that the distribution is moving and the standard deviation of the distribution is increasing over time, which can be seen more clearly in From fig. 5d and From fig. 5e and From fig. 5f show the expected...
value and variance at each time of the GBM time series, which, as in theory, have a linear relationship with time. As mentioned earlier, this linear relationship is very helpful in our training, one of the goals of our training is to observe whether the expected value of the time series generated by RGAN at each time point is linear with time.

Fig. 6a shows what the path generated by the (trained) RGAN looks like. As we said, it is difficult to compare it with the naked eye to Figure 4.1. It is easier to refer to fig. 6b, the distribution of fig. 6b, and the property of fig. 5b, compared to fig. 5b, symmetry, increasing expected value over time, increasing variance over time in Increase. For the latter two, it would be more helpful to compare fig. 5c and fig. 5d, fig. 5c and fig. 5d. We can see that both fig. 5c and fig. 5d indeed exhibit the property of linear inertia with time. The red line in fig. 5c and fig. 5d represents the theoretical line: reference (3.3). It can be seen that the blue one has a fairly high fit, where the $R^2$ of the expected value is 0.994935 and the $R^2$ of the variance is 0.9946425.

4.4.2 Data size and Number of Parameters

In the previous section, we realized that RGAN can indeed learn the results of GBM quite well from some interpretations, but as mentioned earlier, the training of RGAN is not easy. The choice of hyperparameters (learning rate, etc.) has a relatively strong relationship [14]. In this section, we further experiment the influence of the following parameters on the results of RGAN, including the angle of the model and the angle of the data. The data perspective has been discussed in Research Methods.

First, we compare the impact of the number of hidden layer nodes and the amount of data on training. fig. 7a and fig. 7b show the performance of different configurations over time.

The horizontal axis represents the number of batches fed to the model, the fixed batch size is 50, and the vertical axis is the expected value and theory of the model trained at that time. The $R^2$ of the expected value straight line corresponds to the variance, and the meaning of $R^2$ can be referred to fig. 4a and fig. 4b.

As can be seen from the figure, the learning effect of the model with too few hidden layer nodes is not ideal, and the $R^2$ performance of the model with 10 hidden layer nodes is poor. This is actually very reasonable. The huge number of parameters is a characteristic of deep learning. As for the further discussion of the number of parameters, model complexity and model generalization ability, please refer to [15].

Surprisingly, RGAN does not require a high number of samples in this experiment! When the number of samples is only 100, the ideal training state can still be achieved. However, this may be caused by overfitting. Overfitting in GAN means that GAN completely memorizes the sample instead of generating the distribution of the sample.

5 Experiment: Backtesting on Generated Paths

Next, look at the results of GAN from the perspective of trading strategies. In practice, the training of GAN is very difficult [16], and sometimes the results learned by GAN are not completely ideal. In this case, is it really not helpful for backtesting at all? Not so. To give a simple example, suppose our real model is a random walk model, that is, the price of the next period is equal to the price of this period plus a value derived from a standard normal distribution, such as the mean and standard deviation of the normal distribution of each period. The difference will not affect, even if it does not pass the test of the normal distribution, it does not matter, the important thing is that it is symmetrical and the mean is 0 (this means that the expected value of the next period’s profit can be calculated and guaranteed by the confidence interval), it can be seen that whether to learn the complete generation distribution of stock prices may not be so important for the strategy. The specific approach of this part is to use the generation process of hypothetical stock prices, and we study the strategies that can theoretically have positive expected values on these stock prices and
Fig. 5: Characteristic of Monte Carlo Simulation (as benchmark)

Fig. 6: Characteristic of GAN generated data
strategies with zero expected values (on OOS), so that we have an OOS on A reference value for the performance of the strategy. Based on this reference value, we can compare the methods of centrally generating strategies and compare whose results are closer to the reference value.

5.1 Model selection
On the basis of GBM, a new process AR(2) is added to the experiment here:

\[
y_t = a + by_{t-1} + cy_{t-2} + \epsilon_t
\]

where \( \epsilon_t \sim N(0, 1) \)

In order to compare GBM, we selected \( a = 0, b = 1.1, c = -0.5 \). The reason for this selection is that the absolute value of the root of the characteristic polynomial of eq. (14) can be less than 1, which ensures that eq. (14) is stationary. In addition, we have listed the theoretical results of each theoretical strategy in each process at the bottom.

**BH and GBM** Since the logarithmic expected value of GBM increases linearly in time (see eq. (11)), the expected value held by BH should indeed be positive.

**BH and AR(2)** Since the expected value of our AR(2) is constant over time (because our AR(2) is a stationary process), the expected value for how long it is held is 0.

**MAC and GBM** Since the past of GBM has no effect on the future, and under our model and parameters, the expected value of the GBM logarithm increases linearly in time, so the expected value held by the MAC should indeed be positive.

**MAC and AR(2)** Due to the stationary nature of our AR(2), the past mean can be used as an estimate of the expected value. Therefore, when the strategy finds that the current price is lower than this average value (that is, \( p_1 \) is selected long enough and \( p_2 = 1 \), that is, the current period), choosing to buy instead of selling can ensure that it is profitable online.

| Strategy | Model | GBM | AR(2) |
|----------|-------|-----|-------|
| BH       | YES   | NO  |       |
| MAC      | YES   | YES |       |

TABLE 4: Theoretical strategies have positive return on model

5.2 Evaluation
As mentioned in [11], how to evaluate the quality of the data generated by GAN is challenging. Unlike traditional machine learning models, the loss of the generator and discriminator has little significance for the evaluation of model performance. In the image part, assessments that usually judge the quality of images generated by GANs mostly rely on human ratings. TSTR (Train on Synthetic, Test on Real) and TRTS (Train on Real, Test on Synthetic) proposed in [20]. They consist in training a traditional classifier on a generated (real) dataset, then testing the results on real (generated) data, and if the two are close, the GAN is considered to have learned.

We made some adjustments to the algorithm, as indicated in algorithm 1. We now use the training set to train a generator, and then we find a best-performing strategy from the data generated on the training set, and get a strategy score indicator (such as Sharpe Ratio), preferably we apply this strategy to the test. The score index on the test set is obtained on the set, and we compare the two score indicators to score it.

**Algorithm 1:** Algorithm for backtesting strategies

1. train data, test data \( \leftarrow \) split(data)
2. discriminator, generator \( \leftarrow \) train_GAN (train data)
3. for config in configs do
4. strategy = \text{argmax} (backtest (synthetic data, config))
5. synthetic_score = backtest (test data, strategy)
6. test_score = backtest (test data, strategy)
7. score = score (synthetic_score, test_score)
8. end

Since there is a model, we first find the strategy, then use our model to generate paths, and apply this strategy to these generated paths, so that we get a distribution of backtest performance, which we become the target backtest result distribution.

Secondly, we then apply the strategy found above to what we learned with the GAN, so that we can also get a distribution.
Finally, we compare the distribution of target backtest results with the distribution of experimental backtest results, including setting a confidence interval for the distribution, comparing the results of Monte Carlo with the results of GAN’s rejection, which becomes a classification problem.

5.3 Result

5.3.1 Performance
As mentioned earlier, GANs can approximate common stock processes, but is such an approximation enough for us to backtest? We did an experiment here.

fig. 8a and fig. 8b show the experimental results of BH-GBM. The choice of our strategy is to pick the best within the sample. It can be seen that the red and blue PDFs are quite close: this means that under the BH-GBM combination, the reference value of the model learned by GAN in backtesting is similar to that of the real model. fig. 8c and fig. 8d show the experimental results of MAC-GBM. At this time, the P.D.Fs of red and blue are not so close, and the mean of blue is obviously larger than that of red, but from the CDF, the difference between the two is not large, so when we choose a reasonable confidence interval to reject false positives.

fig. 9a and fig. 9b and Figures fig. 9a and fig. 9a performed well, with similar conclusions to fig. 8a, fig. 8a.

5.3.2 Confusion matrix
In addition, we also do tests, sample 100 for each (process, strategy) combination and compare with Monte Carlo results. We define how effective the strategy is: the frequency of Sharpe > 0 exceeds 0.75 (while will be applied for Monte Carlo and RGAN. We can also adjust this value to control our recall and precision).

Table 5 presents the results for strategy BH and process GBM. It can be seen that the overall accuracy rate is good, only the part that RGAN thinks is invalid and Monte Carlo thinks it is effective may be adjusted by adjusting the threshold of RGAN.

Table 6 shows that the results of strategy MAC and process GBM are not very satisfactory, which may be related to the performance of fig. 8c.

Table 7 shows the results of the strategy BH and the process AR(2). It can be seen that the correct rate is 100%. As mentioned earlier, the expected value of BH and AR(2) cannot be positive.

Table 8 shows the results of strategy BH and process AR(2). It can be seen that the correct rate is 100%. As mentioned earlier, the expected value of BH and AR(2) is impossible to be positive.

Through the above results, we find that under these stock price models and strategies, our method is quite close to the expected correct answer, that is, in the backtesting part, the distribution learned by GAN can replace the real distributed.

| GBM-BH          | Monte Carlo |
|-----------------|-------------|
| RGAN positive   | 72          |
| RGAN negative   | 8           |

*TABLE 5: BH strategy and GBM*

| GBM-MAC         | Monte Carlo |
|-----------------|-------------|
| RGAN positive   | 22          |
| RGAN negative   | 18          |

*TABLE 6: MAC strategy and GBM*

| AR(2)-BH        | Monte Carlo |
|-----------------|-------------|
| RGAN positive   | 0           |
| RGAN negative   | 100         |

*TABLE 7: BH strategy and AR(2)*

| AR(2)-MAC       | Monte Carlo |
|-----------------|-------------|
| RGAN positive   | 39          |
| RGAN negative   | 1           |

*TABLE 8: MAC strategy and AR(2)*

6 Conclusions
This paper starts from the backtesting overfitting problem that is easily encountered in quantitative trading, and hopes to use GAN to learn the distribution of stock prices to alleviate this problem. From a machine learning perspective, we study the nature of GAN learning, including the generator and discriminator architectures and the number of parameters, as well as the number of samples required by GANs for the task of mitigating overfitting. From an application perspective, we investigate whether the distributions learned by GANs help to mitigate backtest overfitting.

We obtained the following conclusions through experiments. Under our hypothetical model and selection strategy: (1) GAN can learn some properties of commonly used stock price models. (2) The importance of the sample size in this experiment is not that great. (3) Scaling can alleviate some related problems of its value range in the generation stage, but at the cost of sacrificing some statistical properties of the variance. (4) The distribution learned by GAN is enough to avoid the problem of overfitting to some extent under some strategies we choose, which can help us to better screen strategies.

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Fig. 8: Simulation results for the stationary GBM process.

(a) GBM-BAH-PDF
(b) GBM-BAH-CDF
(c) GBM-MAC-PDF
(d) GBM-MAC-CDF

Fig. 9: Simulation results for the stationary AR2 process.

(a) AR2-BAH-CDF
(b) AR2-BAH-PDF
(c) AR2-MAC-CDF
(d) AR2-MAC-PDF
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