Hybrid methods for composite scattering from conductive target above a rough surface

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Abstract. A hybrid method combining the method of moment (MoM) and physical optical (PO) is used to compute the scattering from 3D conductive target above a rough sea surface in this paper. Firstly, the composite model of 3D conductive target above the sea surface is constructed. The rough sea surface is simulated by the Pierson-Moscowitz spectrum and Monte-Carlo method. The target is divided into MoM region and the rough sea surface is divided into PO region. Then, the current on the facet of target is obtained by MoM method and the current on the facet of rough surface is obtained by PO approximation. Compared to the method of moment, the hybrid MoM-PO method proves to be high accurate and efficient. Finally, composite scattering under different condition are studied in detailed.

1 Introduction

The composite scattering from the target above the rough surface has been widely used in radar detection, ground remote sensing, et al.¹² The fast multiple method (FMM)³ and the forward and backward iterative algorithm⁴ is of high accuracy for computing the composite scattering. However, they are hard to meet the need of computing the composite scattering with the electrically large size rough surface. The reciprocity theorem method combining the Kirchhoff approximation is of high efficiency for computing the composite scattering.⁵ However, they are not accurate enough. The method of moment combining the Kirchhoff approximation (MoM-KA) is used to calculate the composite scattering from a 2D conductive target above a 1D rough surface by R Wang⁶, which is of high accuracy and efficiency. Nevertheless, more complex 3D problems for composite scattering are rarely studied in previews paper.

In this paper, a MoM-PO hybrid method is used to calculate the scattering from the 3D conductive target above a rough sea surface. The composite model is divided into MoM region for the method of moment and PO region for PO approximation. Compared to the traditional MoM, this hybrid method is accurate and efficient.

2 Composite Model

As we see, the geometry of conductive target above rough sea surface is shown in figure 1. The altitude of the target is $H$; $\theta_i$, $\theta_r$ is the incident angle and the scattering angle, the position vector of field point and source point is $r = x\hat{x} + y\hat{y} + z\hat{z}$ and $r' = x'\hat{x} + y'\hat{y} + z'\hat{z}$, respectively.

Both the target and the rough surface is discrete as facets of triangulation. The surface of the target in MoM region is divided into N triangular patches, and the surface of the rough surface in PO region is divided into M triangular patches. Supposing that the induced electric current on the facets of target and rough sea surface is $\mathbf{J}_t$ and $\mathbf{J}_r$, respectively. It can be expressed by the RWG base function $f_m$ and $\mathbf{f}_m$ as⁷:

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\[ J_s = \sum_{n=1}^{N} \alpha_n f_n \]  
\[ J_s = \sum_{n=1}^{M} \beta_n f_m \]  
(1)

Where, \( \alpha_n \) and \( \beta_n \) is the electric current expansion coefficient. The scattering electric field and the scattering magnetic field is:

\[ E^i = L^i(J) \]
\[ H^i = L^h(J) \]  
(2)

The definition of \( L^i \) and \( L^h \) is:

\[ L^i(J) = ik \int (I + \frac{\sqrt{V}}{k}) J(r') g(r,r') dr' \]
\[ L^h(J) = \int J \times e^i g(r,r') dr' \]  
(3)

Where, \( \vec{I} \) is unite vector, \( k = \omega \sqrt{\mu_0 \varepsilon_0} \) is the wavenumbers in free space, \( g(r,r') = e^{-jkr-r} / 4\pi |r-r'| \) is the green function in free space, \( J(r') \) is the induced electric current. The scattering magnetic field on the surface of target \( S_{mom} \) is:

\[ H^i = L^i(\sum_{n=1}^{N} (\alpha_n f_n)) \]  
(4)

The electric increment on rough sea surface that excited by the scattering field from the target is:

\[ J_{sm}(r_m) = 2\hat{n} \times H^i + 2\hat{n} \times L^i(\sum_{n=1}^{N} (\alpha_n f_n)) \]  
(5)

Where, \( \hat{n} \) is the normal unit vector of the facet \( m \) on the rough sea surface. The electric current on rough surface excited by the incident waves and the scattering wave from the target is expressed as:

\[ J_{sm}(r_m) = 2\hat{n} \times H^m + 2\hat{n} \times L^i(\sum_{n=1}^{N} (\alpha_n f_n)) \]  
(6)

Where, \( H^m(r_m) \) is the incident magnetic field. By plugging the equation (1) into equation (6), the electric current coefficient on the rough sea surface is:

\[ \beta_m = (t^+ + t^-) \cdot (n^+ \times H^m(r_m)) \]
\[ + (t_m^+ + t_m^-) \cdot (n^+ \times L^i(\sum_{n=1}^{N} (\alpha_n f_n))) \]  
(7)

Where, \( t_m^+ \) and \( t_m^- \) is the tangential unit vector that perpendicular to the edge on the left facet and right facet, respectively. \( n^+ \) and \( n^- \) is the unite normal vector on the left facet and right facet of the edge \( m \).

The electric current on the facet of rough surface can be obtained by having the coefficient \( \beta_m \) into equation (1):

\[ J_s = \sum_{m=1}^{M} f_m \{(t_m^+ + t_m^-) \cdot (n^+ \times H^m(r_m)) \]
\[ + (t_m^+ + t_m^-) \cdot (n^+ \times L^i(\sum_{n=1}^{N} (\alpha_n f_n))) \]  
(8)

The scattering field on the surface of target \( S_{mom} \) can be expressed as:

\[ E^i(r) = L^i(J_s) + L^i(J) \]  
(9)

Based on the Dirichlet boundary condition on the conduct surface, the electric integral equation on the surface of target \( S_{mom} \) is:

\[ E(r) = E^m(r) + E^i(r) = 0 \]  
(10)

Where, \( E^m(r) \) is incident field, \( E^i(r) \) is the scattering electric field that excited by the electric current on the facet of target and rough surface. By having the equation (9) into equation (10), we can get that:

\[ E^m(r) + L^i[J_s(r)] + L^i[J] = 0 \]  
(11)

By having the equation (1) and equation (8) into equation (11), we can get that:

\[ L^i \left[ \sum_{m=1}^{M} f_m \left\{ (t_m^+ + t_m^-) \cdot (n^+ \times H^m(r_m)) \right\} \right] 
\[ + \sum_{m=1}^{M} \beta_m f_m \right\} \]  
(12)

Equation (12) can be simplified as:

\[ \alpha_l \left[ L^i(f_m)_{\mid m=i} \right] + \sum_{m=1}^{M} \beta_m f_m \right\]  
(13)

Where, \( t_m^i \) and \( t_m^f \) can be expressed as:

\[ t_m^i = (t_m^+ + t_m^-) \cdot (n^+ \times H^m(r_m)) \]
\[ + (t_m^+ + t_m^-) \cdot (n^+ \times L^i(\sum_{n=1}^{N} (\alpha_n f_n))) \]  
(14)

The RWG function \( f_m(r') \) is applied to test equation (11), and the matrix equation is obtained:

\[ (Z^l_{S_{mom}} + [Z^l_{S_{mom}}] \cdot [P^{PO}_{S_{mom}}]) \alpha \]
\[ = [V_{S_{mom}}] - [Z^l_{S_{mom}}] \cdot [I_{S_{mom}}] \]  
(15)

Where, \( Z^l_{S_{mom}} \) denotes the self-impedance matrix. \( [Z^l_{S_{mom}}] \cdot [P^{PO}_{S_{mom}}] \) denotes the matrix that contains the interaction between the target and the rough surface, \( \alpha \) denotes the matrix of unknown electric current coefficient; \( V_{S_{mom}} \) denotes the excitation matrix on the
surface of target $S_{mm}^m$, $[Z_{SS}^{\text{MOM-PO}}]-[T_{SS}^{\text{PO}}]$ denotes the excitation matrix on target $S_{mm}^m$, which contains the interaction between the incident waves and the rough sea surface. By solving the equation (15), the electric current can be obtained, and then the electric current on the facet of target and rough surface are obtained furthermore. The remote composite scattering field for target above rough surface is obtained by Stratton-Chu equation[8].

### 3 Numerical Results

In this paper, the parameter are set as: working frequency is $f=0.3\text{GHz}$, $g=\frac{L}{4}$, the polarization of the incident waves is VV polarization, the size of the rough surface is $L_x \times L_y = 100\lambda \times 100\lambda$.

#### 3.1 Validation Of Hybrid Method

As we see, the comparison results of the MoM method and the MoM-PO hybrid method for scattering from a cubic above a rough surface is shown in figure 2. In the example, the length of the cubic is $l=2\lambda$, the polarization of the target from underlying rough surface is $H=6\lambda$, the polarization model is Vertical-to-vertical (VV) polarization, the altitude of the target is $H=6\lambda$, $\delta=0.1\lambda$, the correlation of rough surface is $l_x \times l_y = 1.0\lambda$. Compared to the method of moment, the MoM-PO hybrid method proves to be accurate and efficient. what’s more, the MoM-PO hybrid method have great advantages in decreasing the computing time and storage requirement.

#### 3.2 Numerical Results With Different Parameter Of Target

As we see, composite scattering from a cubic above a rough surface with different root mean square is analyzed in detail. The altitude of the target above rough surface is $H=6\lambda$, the size of the cubic is $l \times l \times l = 2\lambda \times 2\lambda \times 2\lambda$, the root mean square of the rough surface is $\delta=0.1\lambda$, $\delta=0.2\lambda$, $\delta=0.3\lambda$, the other parameter is as same as figure 2. As we see, with the root mean square of rough surface increases, the roughness of the rough surface increases, which contributes to specular scattering decreasing and backward scattering getting larger.

### 4 Conclusions

In this paper, a hybrid method combining the method of moment (MoM) and physical optic (PO) is proposed to solve the composite scattering from a 3D conductive target above a sea surface. Firstly, a composite model of target above the rough surface is constructed. The target is divided into MoM region, the rough surface is divided into PO region. Then, the electric current on the facets of target is obtained by MoM method, and the electric current on the facet of rough surface is obtained by PO method. Compared to MoM method, the hybrid method proves to be accurate and efficient. Finally, composite scattering from target above rough surface with different root mean square is analyzed in detail.

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