Supplemental Material for “Dilation of subglacial sediment governs incipient surge motion in glaciers with deformable beds”

Brent M. Minchew\textsuperscript{1} and Colin R. Meyer\textsuperscript{2}

\textsuperscript{1}Department of Earth, Atmospheric and Planetary Sciences, Massachusetts Institute of Technology, Cambridge, MA, USA
\textsuperscript{2}Thayer School of Engineering, Dartmouth College, Hanover, NH, USA

Corresponding author: B. M. Minchew, minchew@mit.edu

\section*{Notation}

\begin{tabular}{lll}
\textbf{Variables} & \textbf{Descriptions} & \textbf{Units} \\
\hline
\(a\) & direct (velocity) effect on coefficient of internal friction & - \\
\(b\) & evolution effect on coefficient of internal friction & - \\
\(A\) & rate factor in constitutive relation for ice & \(\text{Pa}^{-n} \text{s}^{-1}\) \\
\(d_c\) & characteristic slip displacement & \(m\) \\
\(\mu\) & coefficient of internal friction of till & - \\
\(\mu_n\) & nominal coefficient of internal friction & - \\
\(g\) & gravitational acceleration & \(\text{m s}^{-2}\) \\
\(h\) & ice thickness & \(m\) \\
\(h_s\) & thickness of deformable till layer & \(m\) \\
\(\ell\) & glacier length & \(m\) \\
\(m_{wi}\) & water mass per unit volume of till & \(\text{kg m}^{-3}\) \\
\(M\) & glacier surface mass balance & \(\text{m s}^{-1}\) \\
\(N\) & effective pressure at the glacier bed \((N = p_i - p_w)\) & \(\text{Pa}\) \\
\(n\) & stress exponent in constitutive relation for ice & - \\
\(p_i\) & ice overburden pressure \((p_i = \rho_i gh)\) & \(\text{Pa}\) \\
\(p_w\) & pore water pressure in deformable till layer & \(\text{Pa}\) \\
\(p_{w,\infty}\) & pore water pressure in non-deforming substrate & \(\text{Pa}\) \\
\(p_{w,v}\) & water pressure in subglacial hydrological system & \(\text{Pa}\) \\
\(q_{wi}\) & water flux in deformable till layer & \(\text{kg m}^{-2} \text{s}^{-1}\) \\
\(t_h\) & hydraulic diffusion timescale of deformable till layer & \(s\) \\
\(\bar{u}\) & depth-averaged speed of glacier & \(\text{m s}^{-1}\) \\
\(u_b\) & basal slip rate & \(\text{m s}^{-1}\) \\
\(u_{bn}\) & nominal basal slip rate & \(\text{m s}^{-1}\) \\
\(u_s\) & surface speed of glacier & \(\text{m s}^{-1}\) \\
\(u_{sw}\) & balance surface speed & \(\text{m s}^{-1}\) \\
\(w\) & glacier half-width & \(m\) \\
\(\alpha\) & ice surface slope & - \\
\(\beta\) & till compressibility & \(\text{Pa}^{-1}\) \\
\(\gamma_h\) & till permeability & \(\text{m}^2\) \\
\(\epsilon_e\) & elastic compressibility coefficient & - \\
\(\epsilon_p\) & plastic dilatancy coefficient & - \\
\end{tabular}
| Symbol | Description                                                                 | Unit  |
|--------|-----------------------------------------------------------------------------|-------|
| $\dot{\varepsilon}_{ij}$ | strain rate tensor                                                        | $s^{-1}$ |
| $\dot{\varepsilon}_e$ | effective strain rate ($\dot{\varepsilon}_e = \sqrt{\dot{\varepsilon}_{ij}\dot{\varepsilon}_{ij}/2}$) | $s^{-1}$ |
| $\zeta$ | ratio of depth-averaged velocity to surface velocity                        | $-$   |
| $\kappa_h$ | hydraulic diffusivity of till                                               | $m^2 s^{-1}$ |
| $\theta$ | state of deformable till                                                   | $s$   |
| $\eta_w$ | dynamic viscosity of water                                                  | $Pa \cdot s$ |
| $\rho_i$ | mass density of ice                                                        | $kg m^{-3}$ |
| $\rho_w$ | mass density of water                                                      | $kg m^{-3}$ |
| $\tau_{ij}$ | deviatoric stress tensor                                                   | $Pa$  |
| $\tau_b$ | basal drag                                                                | $Pa$  |
| $\tau_d$ | gravitational driving stress                                               | $Pa$  |
| $\tau_{ds}$ | balance driving stress                                                    | $Pa$  |
| $\tau_e$ | effective deviatoric stress ($\tau_e = \sqrt{\tau_{ij}\tau_{ij}/2}$)     | $Pa$  |
| $\tau_t$ | till shear strength                                                       | $Pa$  |
| $\phi$ | till porosity                                                             | $-$   |
| $\phi_p$ | plastic component of till porosity                                         | $-$   |
| $\psi$ | hydraulic transmittance                                                    | $-$   |