Transport through short quantum wires

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Abstract

At temperatures $T < \hbar v_F/K_B d \equiv T_{\text{wire}}$ the collective excitations are negligible and the spectrum of the short wire is dominated by the ”zero modes” particle excitations. At temperature $T > T_{\text{wire}}$ a spin polarized state controlled by the electron-electron interaction and electron density is identified at short times. As a result the anomaly in the conductance $G > 2 \times 0.5e^2/h$ appears. At $T \to 0$ by varying the gate voltage we find that our problem is equivalent to a resonant impurity level. As a result perfect transmission with a conductance $G \simeq \frac{2e^2}{h}$ is obtained. The model presented here can be used as a spin filter which operate by varying the temperature. Transport through a short quantum wire with electron-electron interaction and length “$d$” coupled to Luttinger leads is considered. The short wire model might explain the “0.7 anomaly” observed in quantum point contacts.

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Recent experiments in Quantum Wires show that the spin degrees of freedom play a crucial role in the "Quantum Ballistic transport". Few years ago the Cavendish group [1] reported a new quasiplateau with the conductance $G \approx 2 \times 0.7\frac{e^2}{h}$. With increase in the external magnetic field, the new quasiplateau shifts to lower values approaching $G = 2 \times 0.5\frac{e^2}{h}$ which corresponds to the conductance of a single polarized channel. An attempt to explain these results have been presented at the Ban-Ilan Conference (1997) [2]. The explanation was based on the fact that for short quantum wires a spin polarized state can occur causing the appearance of the new quasiplateau. Additional explanation based on spin-polarization have been given in references [3-5]. On the experimental side the additional quasiplateau have been observed by a number of groups [6-8]. The additional quasiplateau has been also observed in a Quantum point contacts (Q.P.C.) experiment [9] suggestive of the Kondo effect [10].

Recently a density spin polarization in ultra-low disorder quantum wires has been observed [11]. In particular it has been reported that the feature in the range $0.5 - 0.7 \times 2\frac{e^2}{h}$ conductance depends on the electron density, length of the quantum wire and temperature. In order to explain these results we will introduce the following model. We consider a short wire with electron-electron (e-e) interaction and length $2d$ described by the hamiltonian $H^{\text{wire}}$. The short wire is coupled to the left and right leads of length $L$, $H^{\text{leads}} = H^L + H^R$ ($H^L$ and $H^R$ represent the left and right Luttinger liquid). The coupling between the leads and the wire is described by the tunneling hamiltonian $H_T$. The conductance is determined by the properties of $H^{\text{wire}}$. We find that when the wire is short the e-e interaction causes the appearance of a spin polarized state for temperatures $T^{\text{wire}} < T$. At $T < T^{\text{wire}}$ we obtain a problem which is equivalent to a resonant impurity problem. Varying the gate voltage we reach perfect transmission. Therefore at $T \to 0$ one obtains a perfect conductance $G = 2e^2/h$. At $T > T^{\text{wire}}$ we find that at short time the tunneling electrons see the short wire polarized causing a conductance $G > 2 \times 0.5\frac{e^2}{h}$.

These results are obtained within the zero mode bosonization [12]. In particular, we find that for short wire the backward scattering amplitude for the sine-Gordon spin liquid does not normalize to zero as is the case for long wire. As a result a spin polarization occurs. This new feature is obtained within a new non-linear zero mode calculation.
The conductance is determined by the zero mode correlation function of the short wire
\[ K_\sigma(t - t_1) = \langle e^{i q_\sigma(t)} e^{-i q_\sigma(t_1)} \rangle \]
where \( q_\sigma, \sigma = \uparrow, \downarrow \) is the zero mode coordinate \(-\pi \leq q_\sigma \leq \pi\) conjugated to the number of fermion \( N_\sigma, \sigma = \uparrow, \downarrow \) in the wire. At \( T \to 0 \), the two correlation functions \( K_\uparrow(t - t_1) = K_\downarrow(t - t_1) \) are equal giving rise to a conductance \( G \approx 2e^2/h \), contrary to finite temperatures where the spin polarized wire gives \( K_\uparrow(t - t_1) \neq K_\downarrow(t - t_1) \) causing the new feature in the conductance \( G > 2 \times 0.5e^2/h \). The range of the temperatures where this is obtained is determined by the length of the wire and electron density.

In the remaining part we are going to present our model and results,

\[ H = H^L + H^R + H^{\text{wire}} + H_T \quad (1) \]

\( H^L \) and \( H^R \) represent the left and right Luttinger leads characterized by the tunneling exponent \([12, 13]\), \( r = \frac{1}{2} \left( \frac{1}{K_c} + \frac{1}{K_s} \right) \geq 1 \) where \( K_c \) and \( K_s \) are the charge and spin interaction parameters.

The leads are confined to the region \(-L \leq x \leq -d \) and \( d \leq x \leq L \), where \( L \gg d > a \), "a" being the lattice constant and "2d" the length of the short wire described by the hamiltonian \( H^{\text{wire}} \). The "wire" is coupled to the leads through the tunneling matrix elements \( t_L \) at \( x = -d \) and \( t_R \) at \( x = d \).

We will use open boundary condition for the two leads and wire \([14, 15]\). As a result the fermions in the leads will be given by one chiral fermion, \( L_\sigma(x) \) (left leads) \( R_\sigma(x) \) (right leads) and \( \chi_\sigma(x) \) (for the wire). The tunneling hamiltonian \( H_T \) takes the form,

\[ H_T = \sum_{\sigma = \uparrow, \downarrow} \left\{ \lambda_L \left[ L_\sigma(-d) \chi_\sigma(-d) + h.c. \right] + \lambda_R \left[ R_\sigma^{\dagger}(d) \chi_\sigma(d) + h.c. \right] \right\} \quad (2) \]

where \( \lambda_L = 4t_L \sin(k_F^{(L)} a) \sin(k_F^{(d)} a) \), \( \lambda_R = 4t_R \sin(k_F^{(R)} a) \sin(k_F^{(d)} a) \), \( t_L \sim t_R \), \( k_F^{(R)} \sim k_F^{(L)} \sim k_F^{(d)} \) represent the Fermi momentum in the two leads and in the short wire.

The current will depend on the properties of the short wire of length "2d" with e-e interaction. Contrary to the long wires, for short wires we are not allowed to neglect the "Backward" scattering amplitude for spin excitations and "Umklapp" scattering for the
where \( b \) \( (H_\delta \) and mode coordinate). 4

Within the Bosonization method the "Backward" and "Umklapp" term will be described by two sine-Gordon models \[16\] \( H_{\text{wire}} = H_{\text{cwire}} + H_{\text{swire}} \), where \( H_{\text{cwire}} \) and \( H_{\text{swire}} \) are the charge and spin hamiltonian. Since we are working with a short wire, we will use the zero modes \[12\].

\[
H_{\text{cwire}} = \int_{-d}^{d} dx [v_c(\partial_x \Phi_c(x))^2 + \frac{U}{2(\pi a)^2} \cos((4k_F - G) x + \sqrt{8\pi}(\Phi_c(x) - \Phi_c(-x)) + \frac{2\pi}{d} P_c x)] + \frac{\hbar \pi}{4d} v_c P_c^2
\]

where \( P_c = N_\uparrow + N_\downarrow, q_c = \frac{1}{2}(q_\uparrow + q_\downarrow) \) and \([P_c, q_c] = i \) ("\( P_c \)" is the charge and \( q_c \) is the zero mode coordinate). \( 4k_F - G = \delta \) with \( G = \frac{2\pi}{a}, \delta = 0 \) corresponds to half filling (when \( d \rightarrow \infty \) and \( \delta \neq 0 \) the sine-Gordon term can be ignored.)

\[
H_{\text{swire}} = \int_{-d}^{d} dx [v_s(\partial_x \Phi_s(x))^2 + \frac{U}{2(\pi a)^2} \cos(\sqrt{\delta \pi}(\Phi_s(x) - \Phi_s(-x)) + \frac{\pi}{d} (P_s + \mu_s)x)] + \frac{\hbar \pi v_s}{4d} (P_s + \mu_s)^2
\]

In equation 3b, \( N_\uparrow - N_\downarrow \) and \( q_s = \frac{1}{2}(q_\uparrow - q_\downarrow) \) are the spin zero mode variables. If a spin polarized solution exist one has to find the difference between spin up and spin down is nonzero \( \langle N_\uparrow - N_\downarrow \rangle \equiv \mu_s \neq 0 \). Therefore the conjugate variables to \( q_s \) will be \( P_s \) defined by \( P_s = : N_\uparrow - N_\downarrow := N_\uparrow - N_\downarrow - \langle N_\uparrow - N_\downarrow \rangle \). \( \Phi_{c(s)}(x) \) represent the renormalized bosonic fields, \( \Phi_{c(s)}(x) = \frac{K_{c(s)}^{1/2}}{2} (\Phi_{c(s)}(x) - \Phi_{c(s)}(-x)) + \frac{K_{c(s)}^{1/2}}{2} (\Phi_{c(s)}(x) + \Phi_{c(s)}(-x)) \) where \( \Phi_{c(s)}(x) \) are the bare bosonic fields, \( \Phi_{c(s)}(x) = (\Phi_\uparrow(x) \pm \Phi_\downarrow(x))/\sqrt{2} \). \( K_c < 1, K_s \geq 1 \) are the charge and spin interaction parameters. We will use the Renormalization Group (R.G.) in order to investigate the long wave behavior of eqs 3a - 3b. The hamiltonian in eqs 3a-3b has two different behaviors a "high temperature" crossover behavior (a) for temperature such that the thermal length \( L_T = \frac{\hbar v_F}{k_B T} \) is shorter than the length of the wire \( 2d, T > T_{\text{wire}} = \frac{\hbar v_F}{k_B d} \) and (b) the "low" temperature when \( T_{\text{wire}} < T \). In order to investigate this situation we will use a two cut-off renormalization group for the hamiltonian in eqs 3a-3b. We introduce a bandwidth cutoff \( v_F \Lambda \equiv K_B T_F \). In order to compute the tunneling current a drain-source voltage \( \mu_L - \mu_R = eV_{DS} \) and the gate source voltage \( eV_G = (\mu_R + \mu_L)/2 \equiv \mu \) will be applied. \( \mu_L \) and \( \mu_R \) are the chemical potential for the left and right leads).

Using the Renormalization group (R.G.) we scale down the problem from \( \Lambda \) to \( \Lambda/b_0 \equiv \Lambda_0 \) where \( b_0 = d/a \). At the scale \( b_0 \) our problem is equivalent to an effective single impurity
problem governed by the wire Hamiltonian. The integration of the Fermion in the leads gives rise to a line width of the “impurity level” (single particle state) given by: $\Gamma \sim 2\lambda^2$.

Next we bosonize the leads fermions $R_\sigma(d)$ and $L_\sigma(-d)$; $R_\sigma(x) = \frac{1}{\sqrt{2\pi a}} e^{i\hat{\varphi}_\sigma(x)}$ with $\hat{\varphi}_\sigma = \sqrt{4\pi \varphi_\sigma + \beta_\sigma}$ and $L_\sigma(x) = \frac{1}{\sqrt{2\pi a}} e^{i\hat{\theta}_\sigma(x)}$ with $\hat{\theta}_\sigma(x) = \sqrt{4\pi \theta_\sigma + \alpha_\sigma}$. $\alpha_\sigma$ and $\beta_\sigma$ are the zero mode variables of the leads with the conjugate number of particles $\hat{N}_\sigma$ and $n_\sigma$, $[\alpha_\sigma, \hat{N}_\sigma] = [\beta_\sigma, n_\sigma] = -i$.

As a result we find that the tunneling Hamiltonian is given by:

$$h_T = -\frac{i\lambda^2}{\hbar} \sum_{\sigma=\uparrow, \downarrow} \eta_{R,\sigma} \eta_{L,\sigma} \int_0^\ell dt \{ K_\sigma(t - t_1) [e^{-i\hat{\varphi}_\sigma(t)} e^{i\hat{\theta}_\sigma(t_1)} - e^{-i\hat{\varphi}_\sigma(t)} e^{i\hat{\varphi}_\sigma(t_1)}] - h.c. \}$$ (4a)

$\eta_{R,\sigma}, \eta_{L,\sigma}$ are real Majorana fermions. $K_\sigma(t - t_1)$ is given by the short wire expectation value:

$$K_\sigma(t - t_1) = e^{-\Gamma(t-t_1)} \hat{K}_\sigma(t - t_1), \quad t > t_1$$

$$\hat{K}_\sigma(t - t_1) = \langle \chi_\sigma(d,t) \chi_\sigma^\dagger(-d,t) \rangle_{wire}, \quad t > t_1$$ (4b)

Using the bosonic representation of the short wire Hamiltonian we find that $\hat{K}_\sigma(t - t_1)$, is given by:

$$\hat{K}_\sigma(t - t_1) = \frac{1}{2\pi a} \langle e^{iq_\sigma(t)} e^{-iq\sigma(t_1)} e^{i\sqrt{4\pi} \Phi_\sigma(d,t)} e^{-i\sqrt{4\pi} \Phi_\sigma(-d,t)} \rangle_{wire}$$ (4c)

In eq. 4c $q_\sigma(t)$ is the zero mode coordinate. The correlation function will be computed for the two cases:

(a) The high temperature case, $T > T^{wire}$.

Using the R.G. method we compute the form of the Hamiltonian at the scale $b = b_T = T_T/T = \exp \ell_T$. At this scale we find that the two body potentials $\hat{U}_c(\ell_T)$ and $\hat{U}_s(\ell_T)$ are given by, $\hat{U}_c(\ell_T) = \frac{\hat{U}_c(0)}{1 + \hat{U}_c(0)\ell_T}$, $\hat{U}_s(\ell_T) = \frac{\hat{U}_s(0)}{1 + \hat{U}_s(0)\ell_T}$ where $\hat{U}_c(0) = \frac{\mu}{\pi v_c}$ (at half filling $\delta = 0$), $\hat{U}_c \approx 0$ (for $\delta \neq 0$) and $\hat{U}_s(0) = \frac{\mu}{\pi v_s}$.

For $L_T < d$ the spectrum can be replaced by a continuum variable, $P_c \rightarrow \hat{\mu}_c$, $P_s + \hat{\mu}_s \rightarrow \hat{\mu}_s$.

For the charge part we have:

$$H_c \approx \frac{\epsilon_c}{2} \hat{\mu}_c^2, \quad \delta \neq 0$$ (5a)

for $H_s$ we have

$$H_s = \epsilon_s \frac{\hat{\mu}_s^2}{2} + \hat{\mu}_s(T) \cos(\pi \frac{L_T}{d} \hat{\mu}_s)$$ (5b)
where $\hat{g}_s(T) = (\frac{\pi}{\hbar})(\frac{\nu}{v_s})(\frac{d}{L_T})\frac{1}{\pi} \frac{\nu}{v_s \bar{T}}$. In obtaining this expression we have used $\frac{\nu}{v_s} \simeq \frac{\pi^2}{\hbar} \bar{T}$ with $\bar{T} \simeq 1$, $\frac{\pi^2}{\hbar} \simeq \frac{1}{137}$, $\frac{\pi^2}{\hbar} \simeq 10^2$, we find for $K_\sigma(t-t_1)$,

$$K_\sigma(t-t_1) \approx \frac{1}{2\pi L_T} \langle e^{i\sigma q(t)} e^{-i\sigma q(t_1)} \rangle_{H_c} \langle e^{i\sigma q(t)} e^{-i\sigma q(t_1)} \rangle_{H_s}$$

$$= \frac{1}{2\pi L_T} K_c(t-t_1) K_s^{(\sigma)}(t-t_1) \quad (6a)$$

where $K_c(t-t_1)$ in the presence of $(\mu_R + \mu_L)/2 = eV_G$ is given by $K_c(t-t_1) = \langle e^{-i\pi^2 \hat{\mu}}(t-t_1) \rangle_{H_c} e^{-i\pi^2 \hat{\mu}_s(t-t_1)}$, $\omega_c \equiv \frac{eV}{\hbar}$. Using the eq. of motion for $q_s$, $\dot{q}_s = \frac{i}{\hbar}[q_s, H^{wire}]$ we find from eq. 5b that $K_s^{(\sigma)}(t-t_1) \approx \langle \exp[-i\sigma \pi^2 \hat{\mu}_s(t-t_1)] \rangle_{H_s}$ represents the expectation value with respect to $\hat{\mu}_s$ controlled by the Hamiltonian $H_s$ in eq. 5b. We define the parameters, $\omega_s \equiv \frac{eV}{\hbar}$, $y = \sqrt{\beta\epsilon_s}$, $\tilde{\beta} = \beta\epsilon_s = \frac{\pi\hbar}{\nu_s L_T}$ and introduce the Euclidean time, $\tau - \tau_1 = i(t-t_1)$. We compute the spin correlation function $K_s^{(\sigma)}(\tau - \tau')$:

$$K_s^{(\sigma)}(\tau - \tau_1) = Z^{-1} \int_{-\infty}^{\infty} dy e^{y^2} \langle \exp[-\frac{1}{2} y^2 + \kappa_s(T) \cos(2\sqrt{\pi}(\frac{L_T}{d})^{1/2} y) + \sigma \frac{\omega_s}{\sqrt{\pi}} (2\pi L_T)^{1/2} y(\tau - \tau_1)] \rangle$$

$$= \frac{1}{2\pi L_T} K_c(t-t_1) K_s^{(\sigma)}(t-t_1) \quad (6b)$$

where $Z = \int dy e^{-\beta H_s}$ and $\kappa_s(T) = \hat{g}_s(T) \pi L_T/2d$. In order to compute the integral in eq. 6b we use the saddle point method. The saddle point solution of eq. 6b is given by:

$$y \equiv \bar{y} = -\sigma \frac{\omega_s}{\sqrt{\pi}} (2\pi L_T)^{1/2} (\tau - \tau_1) + \bar{x}_0 \quad (6c)$$

A polarized solution exists if $\bar{x}_0 \neq 0$. $\bar{x}_0$ is the solution of the eq. 6d

$$\bar{x}_0 - \kappa_s(T) 2\sqrt{\pi}(\frac{L_T}{d})^{1/2} \sin[2\sqrt{\pi}(\frac{L_T}{d})^{1/2} \bar{x}_0 - \sigma 2\omega_s(\tau - \tau_1)] = 0 \quad (6d)$$

From eq. 6d we see that a non zero solution for $x_0$ exists if $2\omega_s(\tau - \tau_1) \ll 1$ and is given by:

$$\text{sinc}(2\sqrt{\pi}(\frac{L_T}{d})^{1/2} \bar{x}_0) \equiv \hat{g}_s(T) \pi (\frac{L_T}{d})^{1/2} \bar{x}_0. \quad (6d)$$

Such a solution is possible since the conductance is determined by the cutoff $\tau - \tau_1 \leq \frac{L_T}{v_s}$. This gives $\omega_s(\tau - \tau_1) \leq \frac{L_T}{v_s} < 1$. Therefore, in spite of the fact that no real breaking of symmetry occurs, for $\tau - \tau_1 \leq \frac{L_T}{v_s}$ we have $K_s^{(\sigma)}(t-t_1) \approx e^{\frac{-\omega_s^2}{\sqrt{\pi}} (\frac{L_T}{d})^{1/2}(t-t_1)^2} e^{-i\sigma \frac{\omega_s}{\sqrt{\pi}} (\frac{L_T}{d})^{1/2}(t-t_1)}$. As a result we find that $K_\sigma(t-t_1)$ in eq. 6a is equivalent to impurity in a magnetic field:

$$K_\sigma(t-t_1) \approx e^{-i\frac{E_\sigma}{\hbar} (t-t_1)}; \quad (t-t_1) \leq \frac{L_T}{v_s} \quad (6e)$$

$$E_\sigma = E_c + \sigma \Delta_s; \quad \sigma = \pm \quad (6f)$$

where $E_c = \hbar \omega_c$, $\Delta_s \equiv \frac{\omega_s}{\sqrt{\pi}} (\frac{L_T}{d})^{1/2} \bar{x}_0$. Using eqs. 6e and 4a we compute the tunneling current.
The tunneling current is given by the difference of charge between the left and right leads, 
\[ I_\sigma = -e \frac{d n_\sigma}{dt} = -e \frac{\dot{N}_\sigma}{dt} = \frac{1}{2} e \frac{d L_\sigma}{dt}, \quad J_\sigma = n_\sigma - \dot{N}_\sigma = i \left( \frac{d}{d\alpha_\sigma} - \frac{d}{d\beta_\sigma} \right) \equiv 2i \frac{d}{d\gamma_\sigma} \] where \( \gamma_\sigma = \alpha_\sigma - \beta_\sigma \). Using the Keldysh [17] formalism we obtain the current:

\[ I_\sigma = \frac{e}{2} \left( \frac{i}{R} \right)^2 i \left\langle \int_0^t dt_1 \{ h_T(t_1 - i\varepsilon) \frac{d}{d\gamma_\sigma} h_T(t) - \frac{d}{d\gamma_\sigma} h_T(t_1 + i\varepsilon) \} \right\rangle \] (7)

where \( \left\langle \right\rangle \) stands for the thermodynamic average at temperature \( T \) with respect to the leads: \( H_L + H_R + \mu_L(N_\uparrow + N_\downarrow) + \mu_R(n_\uparrow + n_\downarrow) \). We introduce the drain source voltage \( V_{DS} = \frac{\mu_L - \mu_R}{2e} \) and the gate voltage \( V_G = \frac{\mu_L + \mu_R}{2e} \). The chemical potentials \( \mu_L \) and \( \mu_R \) are used to perform the thermodynamic average with respect to the ”zero modes” in the leads. The current is controlled by the short wire correlation function \( K_\sigma(t - t_1) \) (see eq. 6e).

Using eq. 7 we can compute the tunneling conductance. Changing the gate voltage \( V_G \) we can reach a situation that \( |E_\uparrow - V_G| \ll \lambda^2 \) and \( |E_\downarrow - V_G| > \lambda^2 \).

As a result \( E_\downarrow \) is off-resonance and \( E_\uparrow \) is at resonance. Since the conductance in eq. 7 depends on \( K_\sigma(t - t_1) \) given by eq. 6e we have two different situations for the two conductances \( G_\uparrow \) and \( G_\downarrow \). For \( \sigma = \downarrow \), the off-resonance condition for \( E_\downarrow \) allows to replace \( K_\downarrow(t - t_1) \) by \( \delta(t - t_1) \). This gives rise in eq. 7 to “weak link” problem in a Luttinger liquid. As a result we find that \( G_\downarrow \) is given by \( G_\downarrow \sim \frac{\sigma^2}{h} \left( \frac{T}{T_F} \right)^2(r-1), \ r > 1 \). Therefore lowering the temperature causes \( G_\downarrow \) to decrease towards zero.

For spin up, we have a resonance condition. \( E_\uparrow \) is at the Fermi level, therefore \( K_\uparrow(t - t_1) \) is replaced by \( K_\uparrow(t - t_1) \sim 1 \). Substituting \( K_\uparrow(t - t_1) \sim 1 \) in eq. 7 introduces a long time correlation expressed mathematically by a shift in the exponent \( r \to r - 1 \). As a result, we find in this case that the conductance \( G_\uparrow \sim \frac{\sigma^2}{h} \left( \frac{T}{T_F} \right)^2(r-2) \). Contrary to the previous case here we observed that by lowering the temperature \( G_\uparrow \) increases towards the maximal value \( G_\uparrow \sim \frac{\sigma^2}{h} \). Since \( G_\downarrow \sim 0 \) we find that \( G_\uparrow + G_\downarrow \geq \frac{\sigma^2}{h} \) in agreement with the experimental situation.

(b) Next we consider the situation in the limit \( T < T^{wire} \). In this case we stop scaling at the length scale \( b = d/a \). At this scale we have a ”zero dimension” quantum problem.

At the length scale \( b = \frac{d}{a}, \ell = \log(d/a) \) the bosonic fields \( \Phi_{\ell}(x) \) are integrated out and eqs. 3a and 3b are replaced by the zero mode hamiltonians \( H_c^{(n=0)} \) and \( H_s^{(n=0)} \). (The zero mode representation is valid at low temperature \( T \) such that the thermal length \( L_T \equiv \frac{\hbar v_F}{k_B T} > d \equiv \frac{\hbar v_F}{k_B T^{wire}} \) (For wires of the length \( d \sim \mu m \) we obtain temperatures, \( T \sim 1 - 2K^0 \)). Therefore for temperatures \( T < T^{wire} \) we replace eqs 3a and 3b only by the zero mode
hamiltonian $H_c^{(n=0)} = \varepsilon_c h_c$ and $H_s^{(n=0)} = \varepsilon_s h_s$ where $\varepsilon_c = \frac{h v}{2a} v_c$ and $\varepsilon_s = \frac{h v}{2a} v_s$.

In the quantum region $P_c$ and $P_s$ are discrete and if a non-zero value of $\mu_s$ occurs it must be an integer. But in this case the partition function is invariant if we shift $P_s \rightarrow P_s \pm 1$ we conclude that no broken symmetry takes place, $Z(\mu_s) = Z(0)$. The charge part is given by:

$$h_c = \frac{1}{2} P_c^2 + g_c \cos(\pi P_c)$$

(8a)

$$g_c = g_c(\delta, d), \text{ at half filling } \delta = 0, \ g_c \approx U/\pi v_c; \ g_c \neq 0 \text{ only for } \delta d < \pi.$$

The spin part is given by:

$$h_s = \frac{1}{2} (P_s)^2 + g_s \cos(\pi P_s)$$

(8b)

$$g_s \approx \frac{U}{1 + U \ell}, \ U = U/\pi v_s, \ \ell = \ln(d/a). \text{ Therefore away from half filling we will have } g_s \gg g_c.$$

We want to compute the spectrum of the hamiltonian in eqs 8a-8b. The spectrum of the free part of the hamiltonian in eqs 8a-8b is given by $|: \delta N \uparrow ; : \delta N \downarrow : \rangle = |P_c\rangle \otimes |P_s\rangle$ where $P_c$ is the charge sector and $P_s/2$ is the spin sector (: $\delta N \uparrow \downarrow :$ $\equiv \ N \uparrow \downarrow - \langle N \uparrow \downarrow \rangle$) The low energy particles excitation (which do not include the bosonic particle hole excitation) obey the condition: $P_c = P_s$ (modulo 2), where $P_s = 0$ corresponds to a singlet and $P_s/2 = \pm 1/2$ to the spin half doublet. Therefore the non-interacting spectrum will be given by $|P_c = 2n\rangle \otimes |P_s = 0\rangle$ and $|P_c = 2n + 1\rangle \otimes |P_s = \pm 1\rangle$ where $n = 0, \pm 1, \pm 2, \ldots$.

The spectrum in the quantum case for the charg part is given by $E_c(2n) \approx \frac{1}{2} \varepsilon_c(2n)^2$ and $E_c(2n+1) \approx \frac{1}{2} \varepsilon_c(2n+1)^2$ and for the spin part we have $E_s(0) = \varepsilon_s g_s$ and $E_s(\pm 1) = (\frac{1}{2} - g_s) \varepsilon_s$.

At length scale $\ell = \ln(d/a)$ the creation and annihilation fermion operators for the wire are replaced by the zero mode part, $\chi_\sigma(x) \rightarrow \hat{\chi}_\sigma \equiv \frac{e^{iqx}}{\sqrt{2\pi a}}$. In order to compute the tunneling current we need the short wire correlation functions, $\langle \hat{\chi}_\sigma(t) \hat{\chi}_\sigma^\dagger(t_1) \rangle \equiv \frac{1}{2\pi a} K_\sigma(t - t_1)$. This can be done with the help of the spectrum of the short wire.

$$e^{iq_\uparrow} = \sum_{P_c = \{even\}} \langle [P_c] (P_c + 1) \otimes [P_s = 0] (P_s = 1) + [P_c - 1] (P_c) \otimes [P_s = -1] (P_s = 0) \rangle$$

(9)

$$e^{iq_\downarrow} = \sum_{P_c = \{even\}} \langle [P_c] (P_c + 1) \otimes [P_s = 0] (P_s = -1) + [P_c - 1] (P_c) \otimes [P_s = 1] (P_s = 0) \rangle$$

(10)
Using the spectrum of $h_c$ and $h_s$ we compute $K_\sigma(t-t_1)$ using eqs 9, 10. At low temperature case (b) $T < T_{\text{wire}}$ we find:

$$K_\sigma(t-t_1) = (1 + 2e^{-\beta\Delta})^{-1} \left[ e^{i\Delta(t-t_1)} + e^{-\beta\Delta}e^{-i\Delta(t-t_1)} \right]$$

(11)

where $\Delta$ is given by

$$\Delta = E_c(\pm 1) - E_c(0) + E_s(\pm 1) - E_s(0) = \frac{1}{2}(\varepsilon_c + \varepsilon_s) - \frac{1}{2}g_s\varepsilon_s$$

(12)

In the limit of $\beta \to \infty$ we have a resonant impurity problem of energy $\Delta$ above the Fermi energy.

Comparing eq. 11 with eq 6f we observe that eq 11 is spin independent. Therefore by tuning the gate voltage we obtain a resonant impurity problem which obeys $G_\uparrow \sim G_\downarrow \sim \frac{e^2}{h}(T/T_F)^{2(r-2)}$. At low temperatures $G_\sigma$ grows reaching the maximum value $G_\uparrow + G_\downarrow \simeq \frac{2e^2}{h}$.

In conclusion we have shown that a system consisting of two Luttinger leads coupled through a short wire with e-e interaction has the following behavior: at temperature $T > T_{\text{wire}} = \frac{\hbar v_F}{k_B d}$ the short wire acts as a spin polarizer with a conductance $G \geq \frac{e^2}{h}$. From the other hand at $T < T_{\text{wire}}$ the system is equivalent to a resonant impurity level with a conductance $G \simeq \frac{2e^2}{h}$ at $T \to 0$. Therefore we suggest that our system can work as a spin filter by varying the temperature. This avoids the difficult practical task of applying a magnetic field on the short wire.

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