Violations of lepton-flavour universality in $P \rightarrow \ell\nu$ decays: a model-independent analysis

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Abstract

We analyse the violations of lepton-flavour universality in the ratios $\mathcal{B}(P \rightarrow \ell\nu)/\mathcal{B}(P \rightarrow \ell'\nu)$ using a general effective theory approach, discussing various flavour-symmetry breaking patterns of physics beyond the SM. We find that in models with Minimal Lepton Flavour Violation the effects are too small to be observed in the next generations of experiments in all relevant meson systems ($P = \pi, K, B$). In a Grand Unified framework with a minimal breaking of the flavour symmetry, the effects remain small in $\pi$ and $K$ decays while large violations of lepton-flavour universality are possible in $B \rightarrow \ell\nu$ decays.

1 Introduction

In the last few years there has been a great experimental progress in quark and lepton flavour physics. On the quark side, several stringent tests of neutral-current flavour-changing (FCNC) processes have been performed finding no significant deviations from the Standard Model (SM) predictions. This fact naturally points towards new physics models with highly constrained flavour structures. In models with flavoured degrees of freedom around the TeV scale, a natural option is the so-called hypothesis of Minimal Flavour Violation (MFV). Within a general Effective Field Theory (EFT) approach, this hypothesis can be formulated in general terms as the assumption that the Yukawa couplings are the only relevant sources of flavour symmetry breaking both within and beyond the SM, at least in the quark sector $^1$. As discussed in the recent literature, this hypothesis can naturally be implemented in supersymmetric extensions of the SM $^2$ or, in slightly modified forms, also in models with new strongly interacting dynamics at the TeV scale $^3$ $^4$.

The situation of the lepton sector is more uncertain but also more exciting. The discovery of neutrino oscillations provide an unambiguous indication that the SM is not
a complete theory and a clear evidence about the existence of new flavour structures in addition to the three SM Yukawa couplings. In various frameworks these new flavour structures can have non-trivial implications in other sectors of the model. In particular, deviations from SM predictions are expected in FCNC decays of charged leptons and, possibly, in a few meson decays with leptons pairs in the final state (see e.g. Ref. [5] for a recent review).

As far as meson decays are concerned, an interesting role is played by the helicity suppressed $P \to \ell\nu$ decays and particularly by the lepton-universality ratios in these processes. As pointed out in Ref. [6] (see also [7]), lepton-flavour violating effects in supersymmetric extensions of the SM could induce up to $\mathcal{O}(1\%)$ deviations from SM in the $R_{K}^{\mu e} = \mathcal{B}(K \to e\nu)/\mathcal{B}(K \to \mu\nu)$ ratio. Such level of precision is well within the reach of present [8, 9] and near-future experiments in kaon physics [10]. A very significant test of lepton-flavour universality (LFU) will soon be performed also in $\pi \to \ell\nu$ decays, measuring the $R_{\pi}^{\mu e}$ ratio at the $\mathcal{O}(0.1\%)$ level [11]. Finally, the purely leptonic decays of the $B^{\pm}$ mesons have just been observed at the $B$ factories [12, 13, 14, 15] opening the possibility of interesting LFU tests also in these modes [16].

A general theoretical tool for analyzing the results of all these future experiments is provided by low-energy EFT approaches based on specific flavour-symmetry assumptions (such as the MFV hypothesis). This approach allows us to test the implications of flavour symmetries which are independent from specific dynamical details of the new-physics model. As far as lepton-flavour mixing is concerned, two general constructions of this type are particularly interesting: the EFT based on the Minimal Lepton Flavour Violation (MLFV) hypothesis [17] (see also [18, 19]) and the implementation of the MFV hypothesis in a Grand Unified Theory framework (MFV-GUT) [20]. The purpose of this paper is to analyze the role of the $P \to \ell\nu$ decays in both these frameworks.

The paper is organized as follows: in Section 2 we analyze the dimension-six effective operators contributing to $P \to \ell\nu$ decays and discuss their flavour structure according to the MLFV hypothesis. In Section 3 we present the results for the $P \to \ell\nu$ decay rates and discuss their natural size in the MLFV framework. Section 4 is devoted the MFV-GUT framework and its peculiar features. The phenomenology is discussed in Section 5 starting from the model-independent bounds derived from FCNC processes within the EFT approach. The final results concerning the maximal deviations from SM predictions on $R_{K}^{\mu e}$ ratios are presented in Section 6 and summarized in the Conclusions.

2 Operator basis and flavour structures

In order to construct the effective theory relevant to our analysis we introduce two energy scales: i) the scale $\Lambda_{\text{LFV}}$, close to the electroweak scale, where new degrees of freedom carrying lepton-flavour quantum numbers appear; ii) a very high energy scale $\Lambda_{\text{LN}}$ (or $M_{\nu}$), well above $\Lambda_{\text{LFV}}$, associated to the breaking of total lepton number. At low-energies we integrate out the new degrees of freedom and describe their effects by a series of non-renormalizable operators suppressed by inverse powers of $\Lambda_{\text{LFV}}$.

In order to determine the relevant effective operators contributing to $P \to \ell\nu$ decays we need to specify the structure of the low-energy EFT. This is characterized by: i) the
low-energy field content; ii) the flavour symmetry; iii) the flavour-symmetry breaking terms. In all cases we assume that the low-energy field content of the EFT is the SM one, with the exception of the Higgs sector, where we assume two Higgs doublets coupled separately to up-type quarks \((H_u)\), and down-type quarks and charged leptons \((H_d)\). This way we avoid large contributions to FCNC, but we explore possible \(\tan \beta = \langle H_u \rangle / \langle H_d \rangle\) enhancements of flavour-violating effects.

As far as the flavour symmetry and symmetry-breaking terms are concerned, we consider first the two MLFV frameworks introduced in Ref. [17], and briefly summarised below. Since in these two cases the deviations from the SM turn out to be very small, in Section 4 we analyse the GUT framework of Ref. [20] with the aim of identifying if, and under which conditions, larger deviations from the SM are possible.

**MLFV minimal field content**

In this case the lepton flavour symmetry group is

\[ G_{LF} = SU(3)_{LL} \times SU(3)_{eR} , \]

and the lepton sector is invariant under two \(U(1)\) symmetries which can be identified with the total lepton number, \(U(1)_{LN}\), and the weak hypercharge. In order to describe charged-lepton and neutrino masses we introduce the symmetry-breaking Lagrangian

\[
L_{\text{Sym.Br.}}^\text{min} = -\lambda_e^{ij} \bar{\psi}_R (H_u^T L_L^i) - \frac{1}{2\Lambda_{LN}} g_\nu^{ij} (\bar{L}_L^i \tau_2 H_u) (H_u^T \tau_2 L_L^j) + \text{h.c.} \tag{2}
\]

where \(\lambda_e\) and \(g_\nu\) are the two irreducible sources of \(G_{LF}\) breaking.\(^1\) As anticipated, \(\Lambda_{LN}\) denotes the scale of the flavour-independent breaking of the \(U(1)_{LN}\) symmetry. The smallness of neutrino masses, \(m_\nu \equiv g_\nu v_u^2 / \Lambda_{LN}\), is attributed to the smallness of \(v_u / \Lambda_{LN}\).

It is convenient to treat the matrices \(\lambda_e\) and \(g_\nu\) as spurions of \(G_{LF}\), such that the Lagrangian (2) and the complete low-energy EFT is formally invariant under \(G_{LF}\). The transformation properties of \(\lambda_e\) and \(g_\nu\) are

\[
\lambda_e \to V_R \lambda_e V_L^\dagger, \quad g_\nu \to V_L^* g_\nu V_L^\dagger, \tag{3}
\]

where \(V_L \in SU(3)_{LL}\) and \(V_R \in SU(3)_{eR}\). The simplest spurion combination controlling LFV transitions in the charged-lepton sector is \(g_\nu^\dagger g_\nu\). Working in the basis where \(\lambda_e\) is diagonal we can write

\[
g_\nu^\dagger g_\nu = \frac{\Lambda_{LN}^2}{v_u^4} U \bar{m}_\nu^2 U^\dagger, \tag{4}
\]

where \(\bar{m}_\nu = \text{diag}(m_{\nu_1}, m_{\nu_2}, m_{\nu_3})\) and \(U\) is the usual PMNS mixing matrix (see Ref. [17] for notations). Up to the overall normalization, the LFV spurion in (4) is completely determined in terms of physical observables of the neutrino sector. Its explicit form in the

\(^1\)In Eq. (2) the indices \(i, j\) are lepton-flavour indices (that in the following will often be omitted, or equivalently indicated up or down), and \(\psi^c \equiv -i \gamma^2 \psi^*\). \(L_L\) and \(e_R\) denote the lepton doublet and the right-handed lepton singlet, respectively \((L_L^T \equiv (\nu_L, e_L), \bar{L}_L^c \equiv (\bar{\nu}_L^c, \bar{e}_L^c))\). We also denote by \(v_u\) and \(v_d\) the vacuum expectation values (vevs) of the Higgs doublets: \(v_u \equiv v \sin \beta = \langle H_u \rangle\) and \(v_d \equiv v \cos \beta = \langle H_d \rangle\), where \(v \approx 174\text{ GeV}\).
case of normal hierarchy \((m_{\nu_1} < m_{\nu_2} \ll m_{\nu_3})\) or inverted hierarchy \((m_{\nu_1} \ll m_{\nu_1} < m_{\nu_2})\) is
\[
\begin{align*}
[g^\dagger_{\nu \nu}]^{(\text{norm.})}_{ij} & = \frac{\Lambda_{\nu \nu}^2}{v^4} \left[ m_{\nu_1}^2 \delta_{ij} + U_{i2} U_{j2}^* \Delta m^2_{\text{sol}} + U_{i3} U_{j3}^* \Delta m^2_{\text{atm}} \right], \quad (5) \\
[g^\dagger_{\nu \nu}]^{(\text{inv.})}_{ij} & = \frac{\Lambda_{\nu \nu}^2}{v^4} \left[ m_{\nu_1}^2 \delta_{ij} + U_{i1} U_{j1}^* (\Delta m^2_{\text{atm}} - \Delta m^2_{\text{sol}}) + U_{i2} U_{j2}^* \Delta m^2_{\text{atm}} \right]. \quad (6)
\end{align*}
\]

For simplicity, in the numerical analysis of the following sections we assume that the mass of the lightest neutrino vanishes \((m_{\nu_1} = 0)\) in the normal hierarchy and \(m_{\nu_3} = 0\) in the inverted one). Furthermore, we adopt the convention where \(s_{13} \geq 0\) and \(0 \leq \delta < 2\pi\) \([21]\).

**MLFV extended field content**

In this case we assume the existence of three right-handed neutrino singlets under the SM gauge group, beside the SM degrees of freedom. The Majorana mass matrix of these neutrinos is flavour-blind \([M_R]_{ij} = M_\nu \delta_{ij}\), it is the only source of \(U(1)_{\text{LN}}\) breaking and it is assumed to be much heavier than the electroweak scale \(|M_\nu| \gg v\). The lepton flavour symmetry group is

\[
SU(3)_{LL} \times SU(3)_{e_R} \times O(3)_{\nu_R} = G_{\text{LF}} \times O(3)_{\nu_R}.
\]

The irreducible sources of flavour symmetry breaking are \(\lambda_\nu\) and \(\lambda_e\), defined by

\[
L^\text{ext \ Sym.Br.} = -\lambda_{ij}^e \bar{e}_R (H^d_R L^L_i) + i \lambda_{ij}^\nu \bar{\nu}_R (H^T_\nu \tau_2 L^L_i) + \text{h.c.}, \quad (8)
\]

which have the following spurion transformation properties

\[
\lambda_e \to V_R \lambda_e V^\dagger_L, \quad \lambda_\nu \to O_\nu \lambda_\nu V^\dagger_L, \quad (9)
\]

with \(V_L \in SU(3)_{LL}, V_R \in SU(3)_{e_R}\) and \(O_\nu \in O(3)_{\nu_R}\). Integrating out the heavy right-handed neutrinos the effective left-handed Majorana mass matrix is \(m_\nu = \left( v^2_u / M_\nu \right) \lambda_\nu^T \lambda_\nu\).

In this framework the basic spurion combination controlling LFV transitions in the charged-lepton sector is \(\lambda_\nu^T \lambda_\nu\). This can be unambiguously connected to the low-energy neutrino mass matrix only if we impose a further hypothesis, namely if we neglect CP violation in the neutrino mass matrix \([17]\):

\[
\lambda_\nu^T \lambda_\nu \xrightarrow{\text{CP limit}} \frac{M_\nu}{v^2_u} U \bar{m}_\nu U^\dagger. \quad (10)
\]

In the CP limit and neglecting the mass of the lightest neutrino, we can write

\[
\begin{align*}
[\lambda^T_\nu \lambda_\nu]^{(\text{norm.})}_{ij} & = \frac{M_\nu}{v^2_u} \left[ U_{i2} U_{j2} \sqrt{\Delta m^2_{\text{sol}}} + U_{i3} U_{j3} \sqrt{\Delta m^2_{\text{atm}}} \right], \quad (11) \\
[\lambda^T_\nu \lambda_\nu]^{(\text{inv.})}_{ij} & = \frac{M_\nu}{v^2_u} \left[ U_{i1} U_{j1} \sqrt{\Delta m^2_{\text{atm}} - \Delta m^2_{\text{sol}}} + U_{i2} U_{j2} \sqrt{\Delta m^2_{\text{atm}}} \right]. \quad (12)
\end{align*}
\]
2.1 Relevant effective operators and mixing matrices

We are now ready to analyse the effective operator basis relevant for $P \rightarrow \ell \nu$ decays. We are interested in operators of dimension up to six, invariant under the SM gauge group, that can contribute to LFV processes with a single charged-lepton-neutrino pair and a single meson. The presence of a single meson implies we can neglect all operators with a tensor Lorentz structure, because of their vanishing hadronic matrix element. Moreover, we can safely neglect all the dimension-five operators, which necessarily break the total lepton number and are suppressed by inverse powers of $\Lambda_{\text{LN}}$ or $M_\nu$.

The basic building blocks for the relevant dimension-six operators are the bilinears

$$\bar{L}_L \Delta_{LL} L_L \quad \text{and} \quad \bar{e}_R \Delta_{RL} L_L,$$

where $\Delta_{LL}$ and $\Delta_{RL}$ are spurions transforming under $G_{\text{LF}}$ as $(8, 1)$ and $\bar{(3,3)}$ respectively.

The specific structure of the two spurions depends on the considered scenario. The lepton bilinears in (13) must be combined with corresponding quark bilinears, that we construct following the MFV rules [1]. Restricting the attention to terms with at most one power of the quark Yukawa couplings, the basis of relevant dimension-six operators is:

$$O^{(1)}_{RL} = \bar{\epsilon}_R \Delta_{RL} L_L \bar{Q}_L \lambda d_R \quad O^{(1)}_{LL} = \bar{L}_L \gamma_\mu \tau_a \Delta_{LL} L_L \left( i D_\mu H_u \right) \tau_a H_u$$
$$O^{(2)}_{RL} = \left( D_\mu H_d \right) \bar{\epsilon}_R \Delta_{RL} D_\mu L_L \quad O^{(2)}_{LL} = \frac{1}{2} \bar{L}_L \gamma_\mu \tau_a \Delta_{LL} L_L \bar{Q}_L \gamma_\mu \tau_a Q_L$$
$$O^{(3)}_{RL} = \bar{\epsilon}_R \Delta_{RL} L_L^T \bar{u}_R \lambda_i \tau_2 Q_L$$

In principle an additional independent operator is obtained replacing $H_u$ with $H_d$ in $O^{(1)}_{LL}$. However, this operator has the same flavour and Lorentz structure as $O^{(1)}_{LL}$ and it is suppressed in the large $\tan \beta$ limit, therefore we ignore it.

Expanding the lepton spurions in powers of $\lambda_e, g_\nu$ and $\lambda_\nu$ and retaining only the leading terms in the expansion (we assume all these couplings have perturbative elements), the explicit form of $\Delta_{LL}$ and $\Delta_{RL}$ in the charged-lepton mass basis is:

- **minimal case**

$$\Delta_{LL} = \frac{\Lambda^2_{\text{LN}}}{v^4_\nu} U \bar{m}_\nu U^\dagger, \quad \Delta_{RL} = \bar{\lambda}_e \left[ \frac{\Lambda^2_{\text{LN}}}{v^4_\nu} U \bar{m}_\nu U^\dagger \right]$$

- **extended case**

$$\Delta_{LL} = \frac{M_\nu}{v^4_\nu} U \bar{m}_\nu U^\dagger, \quad \Delta_{RL} = \bar{\lambda}_e \left[ \frac{M_\nu}{v^4_\nu} U \bar{m}_\nu U^\dagger \right]$$

3 $P \rightarrow \ell \nu$ matrix elements and decay rates

Having defined the operator basis, we write the effective Lagrangian encoding new physics (NP) contributions as

$$L_{\text{eff}}^{\text{LFV}} = \frac{1}{\Lambda^2_{\text{LFV}}} \sum_{n=1}^2 c^{(n)}_{LL} O^{(n)}_{LL} + \frac{1}{\Lambda^2_{\text{LFV}}} \sum_{n=1}^3 c^{(n)}_{RL} O^{(n)}_{RL} + \text{h.c.},$$

(17)
and compute the decay rates using $\mathcal{L}_{\text{tot}} = \mathcal{L}_{\text{SM}} + \mathcal{L}_{\text{LFV}}^{\text{eff}}$. At the level of accuracy we are working, the SM amplitude for the $P^- \to \ell \bar{\nu}_\ell$ decay is

$$A_{\text{SM}} = \frac{4G_F V_{ab}}{\sqrt{2}} \langle \ell \bar{\nu}_\ell | e_L^\alpha \gamma^\mu \nu_\mu | 0 \rangle \langle 0 | \bar{u}^a_L \gamma_\mu d^b_L | P^- \rangle.$$  \hfill (18)

where $a, b$ are the quark-flavour indices of the corresponding meson and $V$ is the CKM matrix. Defining the meson decay constant, $\langle 0 | \bar{u}^a \gamma_\mu \gamma_5 d^b | P^-(p) \rangle = i \sqrt{2} F_P p^\mu$, the corresponding rate is

$$\Gamma(P^- \to \ell \bar{\nu}_\ell)_{\text{SM}} = \frac{G_F^2}{4\pi} |V_{ab}|^2 F_P^2 m_\ell^2 M_P \left(1 - \frac{m_\ell^2}{M_P^2}\right)^2.$$  \hfill (19)

The LL operators of $\mathcal{L}_{\text{LFV}}^{\text{eff}}$ have SM-like matrix elements, while the RL operators have a different structure:

$$A_{\text{RL}} \sim \langle \ell \bar{\nu}_k | e_R^\alpha \nu_\mu | 0 \rangle \times \begin{cases} \langle 0 | \bar{u}^a \gamma_5 d^b | P \rangle & O^{(1)}_{RL}, O^{(3)}_{RL}, \\ (p_\ell)_\mu \langle 0 | \bar{u}^a \gamma_\mu d^b | P \rangle & O^{(2)}_{RL}. \end{cases}$$  \hfill (20)

For light mesons ($\pi$ and $K$), the hadronic matrix element of the pseudoscalar current

$$\langle 0 | \bar{u}^a \gamma_5 d^b | P^-(p) \rangle = -i \sqrt{2} F_P B_0,$$  \hfill (21)

leads to a substantial enhancement of the first two terms in Eq. (20). Looking at the complete structure of the RL terms, it is easy to realise that $O^{(1)}_{RL}$ is the potentially dominant one: $O^{(2)}_{RL}$ is suppressed by the extra charged-current interaction needed to mediate $P \to \ell \nu$ decays, while $O^{(3)}_{RL}$ is suppressed by the small value of the up-quark mass (appearing because of $\lambda_u$). On the other hand, there is no clear difference among the two LL operators. Since we are interested in evaluating the maximal deviations from the SM, we concentrate the following phenomenological analysis on the three potentially dominant terms: $O^{(1)}_{RL}, O^{(1)}_{LL}$ and $O^{(2)}_{LL}$.

In order to analyse the relative strength of SM and new-physics contributions, for each of these operators we define the ratio

$$R_{P\ell\nu}[O^{(n)}_{\chi\chi}] = \frac{\Gamma(P^- \to \ell \bar{\nu}_\ell)_{\text{SM}} + \delta \Gamma(P^- \to \ell \bar{\nu}_\ell)_{\text{int}} + \sum_{k \neq \ell} \Gamma(P^- \to \ell \bar{\nu}_k)_{\text{LFV}}}{\Gamma(P^- \to \ell \bar{\nu}_\ell)_{\text{SM}}} ,$$  \hfill (22)

where $\delta \Gamma_{\text{int}}$ takes into account the lepton-flavour-conserving contributions generated by $\mathcal{L}_{\text{LFV}}^{\text{eff}}$ (including the interference with SM amplitude). The explicit expressions of this ratio for the three dominant operators are:

$$R_{P\ell\nu}[O^{(1)}_{LL}] = \begin{array}{c} 1 + \frac{2v_u^2 c_{LL}^{(1)}}{\Lambda_{\text{LFV}}^2} \Delta_{\ell \ell}^L |^2 + \sum_{k \neq \ell} \frac{4v_u^4 |c_{LL}^{(1)}|^2}{\Lambda_{\text{LFV}}^4} |\Delta_{\ell \ell}^L|^2, \\
1 - \frac{c_{LL}^{(2)}}{\sqrt{2}G_F \Lambda_{\text{LFV}}^2} \Delta_{\ell \ell}^L |^2 + \sum_{k \neq \ell} \frac{|c_{LL}^{(2)}|^2}{2G_F^2 \Lambda_{\text{LFV}}^4} |\Delta_{\ell \ell}^L|^2, \\
1 - \frac{c_{RL}^{(1)}}{2\sqrt{2}G_F \Lambda_{\text{LFV}}^2} \Delta_{\ell \ell}^R \frac{m_d B_0}{v_d m_\ell} |^2 + \sum_{k \neq \ell} \frac{|c_{RL}^{(1)}|^2}{8G_F^2 \Lambda_{\text{LFV}}^4} |\Delta_{\ell \ell}^R|^2 \left(\frac{m_d B_0}{v_d m_\ell}\right)^2 \end{array}.$$  \hfill (23)
where \( m_d \) is the mass of the down-type quark inside the hadron (the apparent dependence from light-quark masses is canceled by the corresponding dependence of \( B_0 \)).

A closer look to these expressions allows us to identify the potentially dominant terms and the maximal size of the NP effects. The basic hypothesis of our approach is that \( \Lambda_{\text{LFV}} \) is around the TeV scale and that all the Wilson coefficients are at most of \( \mathcal{O}(1) \). This implies that the dimensionless coefficients of the \( \Delta_{LL} \) terms in Eq. (23) are at most of \( \mathcal{O}(10^{-1}) \). The size of \( \Delta_{LL} \) is controlled by the scale of lepton-number violation (\( \Lambda_{\text{LN}} \) or \( M_\nu \)): as shown in [17], within this general EFT approach the scale of lepton-number violation cannot be too large because of the bounds from \( \mu \to e\gamma \). We shall come back on the precise bounds from LFV processes in Section 5.1, here we simply note that an order of magnitude estimate give \( \Delta_{LL} \sim \Lambda_{\text{LN}}^2 \Delta m^2_{\text{atm}}/v_u^4 \approx \mathcal{O}(10^{-4}) \). Thus for \( R_{P\ell\nu}[O^{(1)}_{LL}] \) and \( R_{P\ell\nu}[O^{(1)}_{RL}] \) the non-standard effect is necessarily small and the interference terms dominate.

The coefficients of the \( \Delta_{RL} \) terms in \( R_{P\ell\nu}[O^{(1)}_{RL}] \) are apparently enhanced by an inverse dependence from the lepton mass. However, as shown in Eqs. (15)–(16), in the MLFV framework the \( \Delta_{RL} \) spurion contains a charged lepton Yukawa that cancels this dependence:

\[
\Delta_{RL}^{\ell_k} \approx (\tan \beta)^2 \frac{m_{\bar{\nu}}}{v_u^2} \Delta_{LL}^{\ell_k}.
\]

The above result implies that the non-standard effect in \( R_{P\ell\nu}[O^{(1)}_{RL}] \) are: i) negligible for \( \pi \) and \( K \) decays, even for large \( \tan \beta \); ii) of similar size of those in \( R_{P\ell\nu}[O^{(1,2)}_{LL}] \) in the \( B \)-meson case at large \( \tan \beta \). We thus conclude that in the MLFV framework, both within the minimal and the extended field content, there is no way to generate large deviations from the SM in \( P \to \ell\nu \) decays.

The key difference with respect to the case discussed in Ref. [6] is that the MLFV hypothesis imply the same helicity suppression for SM and non-standard amplitudes. A general EFT framework where such condition is not necessarily enforced is the MFV-GUT framework that we analyse below.

### 4 Beyond the minimal case: MFV-GUT framework

Following Ref. [20], we consider the implementation of the MFV principle in a Grand Unified Theory (GUT) based on the \( SU(5) \) gauge group: the three generations of SM fermions fall into a \( \bar{5} \) \( [\psi_i \equiv (d^c_{iR}, L_{iL})] \) and a \( 10 \) \( [\chi_i \equiv (Q_{iL}, u^c_{iR}, e^c_{iR})] \) of \( SU(5) \), and we add three singlets for the right-handed neutrinos \( [N_i \equiv \nu_{iR}] \). The maximal flavour group is then reduced to \( SU(3)_\bar{5} \times SU(3)_{10} \times SU(3)_1 \).

Introducing three Higgs fields, \( H_5, H_\bar{5} \) and \( \Sigma_{24} \), with appropriate \( U(1) \) charges to avoid tree-level FCNCs, the Yukawa Lagrangian defining the irreducible sources of flavour-symmetry breaking is

\[
\mathcal{L}_{Y-\text{GUT}} = \lambda^{ij}_5 \psi_i^T \chi_j H_5 + \lambda^{ij}_{10} \chi_i^T \chi_j H_\bar{5} + \frac{1}{M}(\lambda'_5)^{ij} \psi_i^T \Sigma_{24} \chi_j H_5
+ \lambda^{ij}_1 N_i^T \psi_j H_5 + M^{ij}_RN_i^TN_j + \text{h.c.} \tag{25}
\]
Imposing the invariance of $\mathcal{L}_{\text{Y-GUT}}$ under the $SU(3)_5 \times SU(3)_{10} \times SU(3)_1$ group implies

\begin{align}
\lambda_5^{(i)} &\rightarrow V_5^* \lambda_5^{(i)} V_{10 \dagger}^\dagger, & \lambda_{10} &\rightarrow V_{10\dagger}^* \lambda_{10} V_{10\dagger}^\dagger, \\
\lambda_1 &\rightarrow V_1^* \lambda_1 V_1^\dagger, & M_R &\rightarrow V_1^* M_R V_1^\dagger.
\end{align}

(26) (27)

with $V_5 \in SU(3)_5$, $V_{10} \in SU(3)_{10}$ and $V_1 \in SU(3)_1$.

The non-renormalizable term in (25) has been introduced to break the exact GUT relations between down-type quark and charged-lepton masses, which are known to be violated in the case of the first two generations.\footnote{The high-scale vev of $\Sigma_{24}$ breaks $SU(5)$ preserving $SU(2)_L \times U(1)_Y$, $\langle \Sigma_{24} \rangle = M_{\text{GUT}} \text{diag}(1,1,1,-3/2,-3/2)$. Moreover, we assume $M \gg M_{\text{GUT}}$, such that this non-renormalizable term is negligible but for the first two generations.}

Expressing the low-energy Yukawa couplings in terms of the high-energy ones we have

$$
\lambda_u = a_u \lambda_{10}, \quad \lambda_d = a_d (\lambda_5 + \lambda_5'), \quad \lambda_e = a_e \left( \lambda_5 - \frac{3}{2} \lambda_5' \right), \quad \lambda_\nu = a_\nu \lambda_1,
$$

(28)

where $a_i = \mathcal{O}(1)$ are appropriate renormalization-group factors and we have redefined the spurion $\lambda_5'$ incorporating a suppression factor $\sim \langle \Sigma_{24} \rangle / M$. In the basis where the down-type quark Yukawa coupling is diagonal, the complete set of low-energies Yukawa couplings assume the form:

$$
\lambda_d = \lambda_d, \quad \lambda_u = V^T \lambda_u V, \quad \lambda_e = C^T \lambda_e G^*, \quad \left[\lambda_\nu^\dagger \lambda_\nu\right]_{\text{CP-limit}} = \frac{M_\nu^2}{v_u^2} G^T U \bar{m}_\nu U^\dagger G^*,
$$

(29)

where, in analogy with the MLFV case with extended field content, we have assumed that $M_R$ is flavour blind $[(M_R)_{ij} = M_\nu \delta_{ij}]$.

The two new mixing matrices $C$ and $G$ appearing in (29) control the diagonalization of $\lambda_e$ in the basis where $\lambda_d$ is diagonal. In the spirit of minimising the unknown sources of flavour symmetry breaking, in the following we work in the limit $C = G = I$. This assumption, which is justified in the limit where we can neglect the breaking of the flavour symmetry induced by $\lambda_5'$ ($C, G \rightarrow I$ in the limit $\lambda_5' \rightarrow 0$ [20]), allows us to express all mixing effects in terms of the CKM and the PMNS matrices.

In the GUT framework the number of independent spurion combinations contributing to LFV processes is much larger than in the MLFV case; however, only few of them can give rise to a substantial parametrical difference. The potentially most interesting effect is obtained replacing the $\Delta_{RL}$ spurion of the MLFV case with $\Delta_{RL}^{\text{GUT}} = \lambda_u \lambda_u^\dagger \lambda_d'$. In the basis where the charged-lepton Yukawa is diagonal, this take the form

$$
\left[\Delta_{RL}^{\text{GUT}}\right]_{ik} = \left[\left( C \Delta^{(q)} \lambda_\nu^* G^\dagger \right)_{ik} \right]_{\text{C=G=I}} ^* \left[\Delta^{(q)} \lambda_d^* \lambda_d \right]_{ik} ^*.
$$

(30)

where

$$
\Delta^{(q)}_{ij} \equiv (V^\dagger \lambda_\nu^* V)_{ij} \approx \frac{m_\nu^2}{v_u^2} (V)_{3i}^* (V)_{3j}.
$$

(31)

The key feature of $\Delta_{RL}^{\text{GUT}}$ in (30) is the presence of the suppressed down-type Yukawa coupling on the right, and not on the left, as in Eqs. (15) - (16). This could allow to overcome the SM helicity suppression in LFV processes with light charged leptons and neutrinos of the third generation.
| Coefficients | MLFV minimal $\Lambda_{LN} = 10^{13}$ GeV | MLFV extended $M_\mu = 10^{12}$ GeV | MFV-GUT |
|-------------|-------------------------------|---------------------------|--------|
| $c_{LL}^{(1)}$ $(1 \text{ TeV}/\Lambda_{LFV})^2$ | $< 1.9 \times 10^{-1}$ | $< 2.7 \times 10^{-2}$ | $< 3.5 \times 10^{-2}$ | $-$ |
| $c_{LL}^{(2)}$ $(1 \text{ TeV}/\Lambda_{LFV})^2$ | $< 5.3 \times 10^{-1}$ | $< 3.8 \times 10^{-2}$ | $< 5.0 \times 10^{-2}$ | $-$ |
| $c_{LR}^{(1)} (\tan \beta/10)^2 (1 \text{ TeV}/\Lambda_{LFV})^2$ | $< 36$ | $< 4.9$ | $< 6.6$ | $< 6.3$ |

Table 1: Bounds on the Wilson coefficients of the LFV effective Lagrangian, from $\mu \to e$ conversion, in different flavour symmetry breaking frameworks. In the MFV-GUT case we report the values in the $C,G \to I$ limit (see text), and only for the potentially dominant operator $(O_{RL}^{(1)})$.

## 5 Phenomenology

### 5.1 Bounds from FCNC processes

One of the advantages of the EFT approach is that we can derive model-independent bounds on the coefficients of the effective Lagrangian in (17) from experiments. In particular, we can extract some interesting bounds from FCNC transitions of charged leptons which receive tree-level contributions from the operators in (17).

At present the most stringent bound is obtained by the bounds on the $\mu-e$ conversion in nuclei and in particular by this result:

$$B_{\mu\to e} = \sigma(\mu^- \text{Au} \to e^- \text{Au})/\sigma(\mu^- \text{Au} \to \text{capture}) < 7 \times 10^{-13}$$ \[32\]

Starting from the Lagrangian (17), assuming $\tan \beta \gg 1$ and using the notation of [22], we get:

$$B_{\mu\to e} = \frac{m_\mu^5}{\Gamma_{\text{capt}} \Lambda_{LFV}^4} \left| \left[ (1 - 4s_{2\nu}^2) V(p) - V(n) \right] c_{LL}^{\mu \mu} + (-V(p) + V(n)) c_{LL}^{\mu \mu} (\Delta_{RL}^\mu + (\Delta_{RL}^\mu)^\dagger) \right|^2,$$ \[33\]

where $g^{(p)}_{RS} = 4.3\lambda_d + 2.5\lambda_s$, $g^{(n)}_{RS} = 5.1\lambda_d + 2.5\lambda_s$ and $V^{(p,n)}$, $S^{(p,n)}$ are dimensionless nucleus-dependent overlap integrals, whose numerical value can be found in [22].

Barring accidental cancellations among the contributions of different operators, expressing the $\Delta$‘s in terms of neutrino masses and mixing angles, according to Eqs. [15]–[16], we extract the bounds on the ratios of Wilson coefficients and effective scales reported in Table 1. For simplicity, in the case of the two MLFV frameworks we show

\[3\] The numerical values of neutrino masses and mixing angles used in the analysis are those reported in Ref. [17].
Operators | MLFV minimal | MLFV extended | MFV-GUT
---|---|---|---
| Norm. hier. | Inv. hier. | |

| $O^{(1)}_{LL}$ | $O^{(2)}_{LL}$ | $O^{(1)}_{RL}$ | $O^{(1)}_{LL}$ | $O^{(2)}_{LL}$ | $O^{(1)}_{RL}$ |
|---|---|---|---|---|---|
| $< 2.7 \times 10^{-6}$ | $< 2.3 \times 10^{-6}$ | $< 3.3 \times 10^{-6}$ | $< 2.7 \times 10^{-6}$ | $< 2.3 \times 10^{-6}$ | $< 3.3 \times 10^{-6}$ |
| $< 2.1 \times 10^{-7}$ | $< 1.7 \times 10^{-7}$ | $< 2.5 \times 10^{-7}$ | $< 6.3 \times 10^{-5}$ | $< 2.1 \times 10^{-7}$ | $< 1.7 \times 10^{-7}$ | $< 2.5 \times 10^{-7}$ | $< 6.3 \times 10^{-5}$ |

Bounds on $|\Delta r^e_\pi|$:
$O^{(1)}_{RL}$

Bounds on $|\Delta r^e_K|$:

Table 2: Bounds on $|\Delta r^e_\pi|$ and $|\Delta r^e_K|$ and in the various symmetry-breaking frameworks.

The bounds obtained for reference values of $\Lambda_{LN}$ and $M_\nu$: for different values of these energy scales, the bounds can easily be rescaled according to Eqs. (15)–(16). This overall normalization problem does not appear in the GUT case: here we report the bound in the $C, G \rightarrow I$ limit and we analyse only the case of the potentially dominant operator $O^{(1)}_{RL}$.

5.2 Predictions for the lepton universality ratios

Using the general expressions for the ratios in Eq. (23) and the bounds on the Wilson coefficients reported in Table 1, we are ready to derive predictions for the possible deviations from the SM in $P \rightarrow \ell \nu$ decays. The most interesting observables are the lepton universality ratios, $R^{\ell \ell'}_P = B(P \rightarrow \ell \nu)/B(P \rightarrow \ell' \nu)$, whose values can be computed with high accuracy within the SM [23]. We parametrise possible deviations from the SM in the $R^{\ell \ell'}_P$ as follows:

$$\frac{R^{\ell \ell'}_P |_{\text{exp}}}{R^{\ell \ell'}_P |_{\text{SM}}} = 1 + \Delta r^{\ell \ell'}_P.$$ (34)

The maximal allowed values for $\Delta r^e_\pi$ and $\Delta r^e_K$ are shown in Table 2. As can be seen, in the two MLFV frameworks these values are far too small compared to the experimental precision expected in future experiments: few×0.01% in $R_\pi$ and few×0.1% in $R_K$. Undetectable effects are found also in the $B \rightarrow \ell \nu$ system, within the two MLFV frameworks.

In the MFV-GUT case the situation is definitely more interesting. According to the last column of Table 2 in such case $\Delta r^e_K$ might be within the reach of future experiments. However, this result must be taken with some care. The bounds in Table 2 are obtained saturating the constraints on the Wilson coefficients from $\mu \rightarrow e$ conversion: in the MFV-GUT framework these are not very stringent and are saturated only for unnatural large values of the Wilson coefficients or unnatural low values of the effective scale $\Lambda_{LFV}$. Imposing natural constraints on the parameters of the EFT, such as $|c^{(1)}_{LR}| < 1$,
Table 3: Bounds on the universality ratios $R_{B}^{\ell\ell}$ in the MFV-GUT framework. The model-independent bounds are obtained saturating the constraint on $c_{LR}^{(1)}$ reported in Table 1 and imposing $\mathcal{B}(B \to \tau\nu) = \mathcal{B}(B \to \tau\nu)_{SM}$ (see text). The bounds in the last line are obtained evaluating all the decays with the natural conditions $|c_{LR}^{(1)}| < 1$, $\tan \beta < 50$ and $\Lambda_{\text{LFV}} > 1$ TeV.

| Model       | $R_{B}^{e\tau}$ | $R_{B}^{e\mu}$ | $R_{B}^{\tau\tau}$ |
|-------------|-----------------|----------------|-------------------|
| SM          | $\approx 4.4 \times 10^{-3}$ | $\approx 2.4 \times 10^{-5}$ | $\approx 1.1 \times 10^{-7}$ |
| MFV - GUT (mod. ind.) | $< 7.0 \times 10^{-3}$ | $< 1.6 \times 10^{-2}$ | $< 7.4 \times 10^{-5}$ |
| MFV - GUT ($|c_{LR}^{(1)}| < 1$) | $< 6.5 \times 10^{-3}$ | $< 4.2 \times 10^{-4}$ | $< 2.8 \times 10^{-6}$ |

The reason why our bounds on $\Delta r_{e\mu}$ are stronger with respect to the maximal effects discussed in Ref. [6], even in the MFV-GUT framework, is the assumption of minimal breaking of the flavour symmetry. We have assumed that both $M_R$ and the new mixing matrices $C$ and $G$ are flavour blind. In this limit the mixing structure of $[\Delta_{GUT}^{RL}]_{i3}$ is controlled by the CKM matrix. This implies that the enhancement of $[\Delta_{GUT}^{RL}]_{i3}$ due to the large Yukawa coupling of the third generation is partially compensated by the suppression of $|V_{i3}| \ll 1$ ($i = 1, 2$). This suppression could be removed only with new flavour-mixing structures. We thus conclude that if some violation of LFU will be observed in $\pi$ decays at the 0.1% level, or in $K$ decays at the 1% level, this would unambiguously signal the presence of non-minimal sources of lepton-flavour symmetry breaking.

The only system where sizable violations of LFU universality can be produced in the minimal set-up we are considering is the case of $B \to \ell\nu$ decays (assuming the MFV-GUT framework). Here the large meson mass provides a substantial enhancement of the contribution of the $\Delta_{RL}$ terms (see Eq. (24)). As a result, in the helicity suppressed modes $B \to \mu\nu$ and, especially, $B \to e\nu$, the contribution of the $\Delta_{RL}$ terms is large enough to compete with the SM.

The enhancement of $\mathcal{B}(B \to e\nu)$ allowed by the model-independent bound on $c_{LR}^{(1)}$ (Table 1) is huge ($\sim 10^3 \times$ SM), and even imposing $|c_{LR}^{(1)}| < 1$ large non-standard effects are possible. The maximal deviations from the SM for the three LFU ratios are shown in Table 3. Given the suppression of the $B \to \ell\nu$ rates, the experimental sensitivity needed to go below these bounds is still far from the presently available one. However, it could possibly be reached in future with high-statistics dedicated experiments.

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4 The model-independent bound on $c_{LR}^{(1)}$ allow a lepton-flavour conserving contribution to $B \to \tau\nu$, from $O_{RL}^{(1)}$, of the same order of the SM amplitude. Since the experimental measurement of $\mathcal{B}(B \to \tau\nu)$ is consistent with the SM expectation and we have not systematically analysed all the lepton-flavour conserving dimension-six operators, in Table 3 we report the model-independent bounds on the $R_{B}^{e\mu}$ assuming $\mathcal{B}(B \to \tau\nu) = \mathcal{B}(B \to \tau\nu)_{SM}$. 
6 Conclusions

Within the SM $P \to \ell \nu$ decays are mediated only by the Yukawa interaction (the decay amplitudes can indeed be computed to an excellent accuracy in the gauge-less limit of the model). This implies a strong helicity suppression and a corresponding enhanced sensitivity to possible physics beyond the SM. In particular, the lepton-flavour universality ratios $R_{P}^{\ell\ell'} = \mathcal{B}(P \to \ell\nu)/\mathcal{B}(P \to \ell'\nu)$, which can be predicted within high accuracy, are interesting probes of the underlying lepton-flavour symmetry-breaking structure.

In this work we have analysed the deviations from the SM in the $R_{P}^{\ell\ell'}$ ratios using a general effective theory approach, employing different ansatz about the flavour-symmetry breaking structures of physics beyond the SM. The main results can be summarised as follows:

- In models with Minimal Lepton Flavour Violation, both in the minimal and in the extended version (as defined in [17]), we find that the effects are too small to be observed in the next generations of experiments in all relevant meson systems ($P = \pi, K, B$). This is because the MLFV hypothesis, by construction, implies the same helicity suppression of $P \to \ell \nu$ amplitudes as in the SM.

- In a Grand Unified framework with a minimal breaking of the flavour symmetry (as defined in Section 4) the effects remain small in $\pi$ and $K$ decays, while large violations of lepton-flavour universality are possible in the $B$ system (see Table 3). These are possible mainly because of the enhancement of the flavour-violating $B \to e\nu$ rate.

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References

[1] G. D’Ambrosio, G. F. Giudice, G. Isidori and A. Strumia, Nucl. Phys. B 645 (2002) 155 [arXiv:hep-ph/0207036].

[2] P. Paradisi, M. Ratz, R. Schieren and C. Simonetto, Phys. Lett. B 668 (2008) 202 [arXiv:0805.3989 [hep-ph]]; G. Colangelo, E. Nikolidakis and C. Smith, Eur. Phys. J. C 59 (2009) 75 [arXiv:0807.0801 [hep-ph]].

[3] G. Cacciapaglia, et al. JHEP 0804 (2008) 006 [arXiv:0709.1714 [hep-ph]]; A. L. Fitzpatrick, G. Perez and L. Randall, [arXiv:0710.1869 [hep-ph]]; C. Csaki, A. Falkowski and A. Weiler, [arXiv:0806.3757 [hep-ph]].

[4] A. L. Kagan, G. Perez, T. Volansky and J. Zupan, [arXiv:0903.1794 [hep-ph]].

[5] M. Raidal et al., Eur. Phys. J. C 57 (2008) 13 [arXiv:0801.1826 [hep-ph]].

[6] A. Masiero, P. Paradisi and R. Petronzio, Phys. Rev. D 74 (2006) 011701 [arXiv:hep-ph/0511289]; JHEP 0811 (2008) 042 [arXiv:0807.4721 [hep-ph]].
[7] J. Ellis, S. Lola and M. Raidal, Nucl. Phys. B 812 (2009) 128 [arXiv:0809.5211 [hep-ph]].

[8] M. Antonelli et al. [FlaviaNet Working Group on Kaon Decays], [arXiv:0801.1817 [hep-ph]].

[9] B. Sciascia [KLOE Collaboration], [arXiv:0810.3436 [hep-ex]].

[10] R. Fantechi [NA48 Collaboration], J. Phys. Conf. Ser. 110 (2008) 072009.

[11] D. Bryman, PoS KAON (2008) 052.

[12] K. Ikado et al., Phys. Rev. Lett. 97 (2006) 251802 [arXiv:hep-ex/0604018].

[13] B. Aubert et al. [BABAR Collaboration], Phys. Rev. D 77 (2008) 011107 [arXiv:0708.2260 [hep-ex]].

[14] N. Satoyama et al. [Belle Collaboration], Phys. Lett. B 647 (2007) 67 [arXiv:hep-ex/0611045].

[15] B. Aubert et al. [BABAR Collaboration], [arXiv:0903.1220 [hep-ex]].

[16] G. Isidori and P. Paradisi, Phys. Lett. B 639 (2006) 499 [arXiv:hep-ph/0605012].

[17] V. Cirigliano, B. Grinstein, G. Isidori and M. B. Wise, Nucl. Phys. B 728 (2005) 121 [arXiv:hep-ph/0507001].

[18] V. Cirigliano and B. Grinstein, Nucl. Phys. B 752 (2006) 18 [arXiv:hep-ph/0601111].

[19] M. B. Gavela, T. Hambye, D. Hernandez and P. Hernandez, [arXiv:0906.1461 [hep-ph]].

[20] B. Grinstein, V. Cirigliano, G. Isidori and M. B. Wise, Nucl. Phys. B 763 (2007) 35 [arXiv:hep-ph/0608123].

[21] C. Amsler et al. [Particle Data Group], Phys. Lett. B 667 (2008) 1.

[22] R. Kitano, M. Koike and Y. Okada, Phys. Rev. D 66 (2002) 096002 [Erratum-ibid. D 76 (2007) 059902] [arXiv:hep-ph/0203110].

[23] V. Cirigliano and I. Rosell, Phys. Rev. Lett. 99 (2007) 231801 [arXiv:0707.3439 [hep-ph]].