Effects of anomalous couplings of the tau lepton

T. Huang, Z.-H. Lin and X. Zhang

Institute of High Energy Physics, Academia Sinica, Beijing, 100039, P. R. China

Abstract

The dimension-six CP-conserving $SU_L(2) \times U_Y(1)$ invariant operators involving the tau lepton are studied. It leads to the new physics effects to the lepton flavor violation (LFV) and lepton flavor-changing neutral current (FCNC). With the available experimental data on the decays $Z \to \tau^+ + \tau^-$, $Z \to \mu^+ + \mu^-$ and $\mu^- \to e^- + \gamma$, the constraints on parameters of the effective lagrangian can be given. According to such constraints, the branching ratio of the decay $Z \to \mu^\pm + \tau^\mp$ could reach to $10^{-6} - 10^{-7}$. The size of the LFV effects depends crucially on the dynamics of the lepton mass generation. Assuming the lepton mass matrices in the form of a Fritzsch ansatz, we point out that the experiment on $\mu \to e\gamma$ will put stronger limits on the anomalous magnetic and electric dipole moments of the tau lepton than obtained by Escribano and Massó.
1 Introduction

Although the Standard Model (SM) has been successful in describing the physics of the electroweak interaction [1], it is quite possible that the SM is only an effective theory which breaks down at higher energies as the deeper structure of the underlying physics emerges.

There are reasons to believe that the deviation from the SM might first appear in the interactions involving the third-generation fermions [2, 3, 4]. The tau lepton is possibly a special probe of new physics in the lepton sector due to the fact that it is the only lepton which is heavy enough to have hadronic decays and that the heavier fermions are more sensitive to the new physics related to mass generation.

There has been extensive studies on the tau physics and related possible new physics of various extension of the SM models. We take a model-independent approach and formulate new physics effects in terms of an effective lagrangian.

Without specifying the detail of the underlying new physics, the effective lagrangian to dimension 6 can be written as,

$$L_{\text{eff}} = L_0 + \frac{1}{\Lambda^2} \sum_i C_i O_i,$$  \hspace{1cm} (1)

where $L_0$ is the SM Lagrangian, $\Lambda$ is the new physics scale and $O_i$ involve only ($\nu_\tau$, $\tau_L$, $\tau_R$), the gauge and the scalar bosons and are $SU_c(3) \times SU_L(2) \times U_Y(1)$ invariant. $C_i$ are constants which represent the coupling strengths of $O_i$ [5]. A complete list of CP-violating operators of leptons has been given in Ref. [6].

To restrict ourselves to the lowest order, we consider only tree diagrams and to the order of $1/\Lambda^2$, so only one vertex in a given diagram can contain anomalous couplings. Under these conditions, operators connected by the field equations are not independent. The fermion and the Higgs boson equations of motion can be used to list the operators but the equations of motion of the gauge bosons cannot. Furthermore, all the operators $O_i$ are Hermitian and the coefficients $C_i$ are real, since we assume that the available energies are below the unitarity cuts of new-physics particles and no imaginary part can be generated by the new physics effect. The higher dimension operators modify the couplings of the tau lepton and gauge (or Higgs) bosons, and generate also lepton flavor violation interaction. However, the size of the lepton flavor violating effect depends crucially on the lepton mass mixing matrices. We take the lepton mass ansatz of Fritzsch, which was discussed twenty years ago [7] and developed in lepton sector in recent years [8, 9]. This ansatz is based on "democratic symmetry" of fermion mass matrix, which has the form

$$M_{0i} = c_{0i} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix},$$ \hspace{1cm} (2)

Here $i$ stands for $u, d$ in case of quarks and $l$ in case of the charged leptons. For the charged leptons,
this matrix could be diagonalized as

\[
M'_H = c_l \begin{pmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 3
\end{pmatrix}
\]

(3)

implying that only the third family lepton has nonzero mass 3c_l under the democratic limit.

We study the correlated effects of anomalous couplings of the tau lepton, such as

- Anomalous magnetic and electric dipole moments of the tau lepton by allowing the mixing of three generations in the lepton sector;
- The decay \( Z \to \mu^\pm + \tau^\mp \) with the experimental data at LEP on the decays \( Z \to \tau^+ + \tau^- \), \( Z \to \mu^+ + \mu^- \);
- Others.

2 Anomalous couplings of the tau lepton

The expressions of the CP-conserving operators involving the third family leptons are parallel to their corresponding ones involving the third family quarks, but the number of independent operators is much less due to the absence of right-handed neutrino and the strong interactions. The possible CP-conserving \( SU_L(2) \times U_Y(1) \) invariant operators are given by

\[
\begin{align*}
O_{LW} &= \left[ \bar{L}\gamma^\mu \tau^I D^\nu L + D^\nu L \gamma^\mu \tau^I L \right] W^{I\mu}, \\
O_{LB} &= \left[ \bar{L}\gamma^\mu D^\nu L + D^\nu L \gamma^\mu L \right] B_{\mu\nu}, \\
O_{\tau B} &= \left[ \bar{\tau}_R \gamma^\mu D^\nu \tau_R + D^\nu \tau_R \gamma^\mu \tau_R \right] B_{\mu\nu}, \\
O^{(1)}_{\Phi L} &= i \left[ \Phi^\dagger D_\mu \Phi - (D_\mu \Phi)^\dagger \Phi \right] \bar{L}\gamma^\mu L, \\
O^{(3)}_{\Phi L} &= i \left[ \Phi^\dagger \tau^I D_\mu \Phi - (D_\mu \Phi)^\dagger \tau^I \Phi \right] \bar{L}\gamma^\mu \tau^I L, \\
O_{\Phi \tau} &= i \left[ \Phi^\dagger D_\mu \Phi - (D_\mu \Phi)^\dagger \Phi \right] \bar{\tau}_R \gamma^\mu \tau_R, \\
O_{\tau 1} &= (\Phi^\dagger \Phi - \frac{v^2}{2}) \left[ \bar{L}_T \Phi + \Phi^\dagger \bar{\tau}_R L \right], \\
O_{\Phi r} &= \left[ (LD_\mu \tau_R) D^\mu \Phi + (D^\mu \Phi)^\dagger (D_\mu \tau_R L) \right], \\
O_{\tau \Phi \Phi} &= \left[ (\bar{L}\sigma^{\mu\nu} \tau^I \tau_R) \Phi + \Phi^\dagger (\bar{\tau}_R \sigma^{\mu\nu} \tau^I L) \right] W_{\mu\nu}^{I}, \\
O_{\tau B \Phi} &= \left[ (\bar{L}\sigma^{\mu\nu} \tau_R) \Phi + \Phi^\dagger (\bar{\tau}_R \sigma^{\mu\nu} L) \right] B_{\mu\nu}.
\end{align*}
\]

(4)
The standard notation is:

- $L$ the third family left-handed doublet leptons,
- $\Phi$ the Higgs doublet,
- $W_{\mu \nu}$ the SU(2) gauge boson field tensors,
- $B_{\mu \nu}$ the U(1) gauge boson field tensors,
- $D_\mu$ the appropriate covariant derivatives.

The expressions of these operators after electroweak symmetry breaking in the unitary gauge are given by

\[
O_{L_W} = \frac{1}{2} W^3_{\mu \nu} \left[ \bar{\nu}_\tau \gamma^\mu P_L \partial^\nu \nu_\tau + \partial^\nu \bar{\nu}_\tau \gamma^\mu P_L \nu_\tau - \bar{\tau} \gamma^\mu P_L \partial^\nu \tau - \partial^\nu \bar{\tau} \gamma^\mu P_L \tau \right]
\]
\[
+ \frac{1}{\sqrt{2}} \left[ W^+_{\mu \nu} (\bar{\nu}_\tau \gamma^\mu P_L \partial^\nu \tau + \partial^\nu \bar{\nu}_\tau \gamma^\mu P_L \tau) + W^-_{\mu \nu} (\bar{\tau} \gamma^\mu P_L \partial^\nu \tau + \partial^\nu \bar{\tau} \gamma^\mu P_L \tau) \right]
\]
\[-i g_2 \bar{L}_\gamma^\mu \tau \partial^\nu \tau - \partial^\nu \bar{\tau} \gamma^\mu P_L \tau \],

\[
O_{L_B} = B_{\mu \nu} \left[ \bar{L} \gamma^\mu \partial^\nu \tau + \partial^\nu \bar{L} \gamma^\mu \tau \right] B_{\mu \nu},
\]

\[
O^{(1)}_{\Phi L} = \frac{m_Z}{v} (H + v)^2 Z \left[ \bar{\nu}_\tau \gamma^\mu P_L \nu_\tau + \tau \gamma^\mu P_L \tau \right],
\]

\[
O^{(3)}_{\Phi L} = -\frac{m_Z}{v} (H + v)^2 Z \left[ \bar{\nu}_\tau \gamma^\mu P_L \nu_\tau - \bar{\tau} \gamma^\mu P_L \tau \right]
\]
\[
+ \frac{1}{\sqrt{2}} g_2 (H + v)^3 \left[ W^+_{\mu \nu} \bar{\nu}_\tau \gamma^\mu P_L \tau + W^-_{\mu \nu} \bar{\tau} \gamma^\mu P_L \tau \right],
\]

\[
O_{\Phi \tau} = \frac{m_Z}{v} (H + v)^2 \bar{Z} \bar{\tau} \gamma^\mu P_R \tau,
\]

\[
O_{\tau 1} = \frac{1}{2 \sqrt{2}} \left[ H (H + v) (H + 2v) \bar{\tau} \tau \right],
\]

\[
O_{D \tau} = \frac{1}{2 \sqrt{2}} \partial^\mu H \left[ (\partial_\mu \bar{\tau}) \tau + \bar{\tau} \gamma^5 \partial_\mu \tau - (\partial_\mu \bar{\tau} \gamma^5 \tau) + 2g_1 B_\mu \bar{\tau} \gamma^5 \tau \right]
\]
\[
+ \frac{i}{2 \sqrt{2}} \frac{m_Z}{v} (H + v) Z^\mu \left[ (\partial_\mu \bar{\tau}) \tau - \bar{\tau} \partial_\mu \tau - \partial_\mu \bar{\tau} \gamma^5 \tau - i2g_1 B_\mu \bar{\tau} \gamma^5 \tau \right]
\]
\[-\frac{i}{2} g_2 (H + v) \left[ W^+_{\mu \nu} (\bar{\nu}_\tau \partial_\mu P_R \tau + \partial_\tau P_R \partial_\mu \tau) + W^-_{\mu \nu} (\bar{\tau} \partial_\mu \nu_\tau + \partial_\tau \nu_\tau \partial_\mu \tau) - W^+_{\mu \nu} (\bar{\tau} \gamma^5 \partial_\mu \nu_\tau - ig_1 B_\mu \gamma^5 \partial_\mu \nu_\tau) \right],
\]

\[
O_{\tau W \Phi} = \frac{1}{2} (H + v) \left[ W^+_{\mu \nu} (\bar{\nu}_\tau \sigma^{\mu \nu} P_R \tau) + W^-_{\mu \nu} (\bar{\tau} \sigma^{\mu \nu} P_L \nu_\tau) - \frac{1}{\sqrt{2}} W^3_{\mu \nu} (\bar{\tau} \sigma^{\mu \nu} \tau) \right]
\]
\[
+i g_2 (W^+_{\mu W_\nu} - W^3_{\mu W_\nu}) (\bar{\nu}_\tau \sigma^{\mu \nu} P_R \tau) - ig_2 (W^-_{\mu W_\nu} - W^3_{\mu W_\nu}) (\bar{\tau} \sigma^{\mu \nu} P_L \nu_\tau)
\]
\[-i \frac{g_2}{\sqrt{2}} (W^+_{\mu} W^-_{\nu} - W^-_{\mu} W^+_{\nu}) (\bar{\tau} \sigma^{\mu \nu} \tau) \],
\[ O_{\tau B\Phi} = \frac{1}{\sqrt{2}} (H + v) B_{\mu\nu} (\bar{\tau} \sigma^{\mu\nu} \tau). \]  

(5)

The possibilities of contributions of the dimension-six CP-conserving operators to some three-particle couplings are shown in Table 1.

|     | \( W_{\nu\tau} \) | \( Z_{\tau\tau} \) | \( \gamma_{\tau\tau} \) | \( H_{\tau\tau} \) |
|-----|-------------------|-------------------|-------------------|-------------------|
| \( O_{\Phi L}^{(3)} \) | \( x \) | \( x \) | \( x \) | \( x \) |
| \( O_{LW} \) | \( x \) | \( x \) | \( x \) | \( x \) |
| \( O_{D\tau} \) | \( x \) | \( x \) | \( x \) | \( x \) |
| \( O_{\tau W\Phi} \) | \( x \) | \( x \) | \( x \) | \( x \) |
| \( O_{LB} \) | \( x \) | \( x \) | \( x \) | \( x \) |
| \( O_{\tau B} \) | \( x \) | \( x \) | \( x \) | \( x \) |
| \( O_{\tau B\Phi} \) | \( x \) | \( x \) | \( x \) | \( x \) |
| \( O_{\Phi L}^{(1)} \) | \( x \) | \( x \) | \( x \) | \( x \) |
| \( O_{\Phi \tau} \) | \( x \) | \( x \) | \( x \) | \( x \) |
| \( O_{\tau 1} \) | \( x \) | \( x \) | \( x \) | \( x \) |

Table 1: The contribution status of dimension-six CP-conserving operators to the tau couplings. The contribution of a CP-conserving operator to a particular vertex is marked by \( \times \).

According to their contribution to the three-particle vertices of charged and neutral current, we classify the operators as:

Class A: \( O_{\Phi L}^{(3)}, O_{LW}, O_{D\tau}, O_{\tau W\Phi}, \) contributing to both charged and neutral currents.

Class B: \( O_{\Phi L}^{(1)}, O_{\Phi \tau}, O_{LB}, O_{\tau B}, O_{\tau B\Phi}, \) contributing to neutral currents.

Class C: \( O_{\tau 1}, \) no contribution to charged and neutral currents.

Since Class C operators contribute only to the \( H_{\tau\tau} \) coupling, they may not be probed at future colliders. We do not consider these operators here. Both Class A and Class B operators affect neutral currents of the tau.

Collecting all the relevant terms we get the effective CP-conserving couplings,

\[
\mathcal{L}_{W_{\nu\tau}} = \frac{C_{\Phi L}^{(3)}}{\Lambda^2} \frac{g_2}{\sqrt{2}} v^2 W_{\mu}^{\nu}(i\bar{\nu}_{\tau}\gamma^\mu P_L \tau) - \frac{C_{D\tau}}{\Lambda^2} \frac{g_2}{\sqrt{2}} \sqrt{2} W_{\mu}^{+}(i\bar{\nu}_{\tau} P_R \partial^\mu \tau)
\]
\[ \mathcal{L}_{Z\tau\tau} = \frac{C_{tW\Phi}}{\Lambda^2} \left[ \frac{1}{2} \left( W^+_{\mu\nu}(\tilde{\nu}_\tau \sigma^{\mu\nu} P_R \tau) + \frac{C_{\Phi L}}{\Lambda^2} \right) \right] + \frac{C_{lW}}{\Lambda^2} \left[ \frac{1}{2} \left( W^+_{\mu\nu}(\tilde{\nu}_\tau \gamma^\mu P_L \partial^\nu \tau + \partial^\nu \tilde{\nu}_\tau \gamma^\mu P_L \tau) \right) \right], \]

\[ \mathcal{L}_{\gamma\tau\tau} = \frac{C_{tW\Phi}}{\Lambda^2} \left[ \frac{1}{2} \left( W^+_{\mu\nu}(\tilde{\nu}_\tau \sigma^{\mu\nu} P_R \tau) + \frac{C_{\Phi L}}{\Lambda^2} \right) \right] + \frac{C_{tW}}{\Lambda^2} \left[ \frac{1}{2} \left( W^+_{\mu\nu}(\tilde{\nu}_\tau \gamma^\mu P_L \partial^\nu \tau + \partial^\nu \tilde{\nu}_\tau \gamma^\mu P_L \tau) \right) \right], \]

where \( P_{L,R} \equiv (1 \mp \gamma_5)/2, Z_\mu = -\cos \theta_W W^3_\mu + \sin \theta_W B_\mu, A_\mu = \sin \theta_W W^3_\mu + \cos \theta_W B_\mu, s_W \equiv \sin \theta_W \) and \( c_W \equiv \cos \theta_W \). Now we can write down the \( Z\tau\tau \) and \( \gamma\tau\tau \) vertices including both the SM couplings and new physics effects as

\[ \Gamma^{Z,\gamma}_\mu = -ie g^{Z,\gamma}_\mu \left[ \gamma_\mu V^{Z,\gamma} - \gamma_\mu \gamma_5 A^{Z,\gamma} - \frac{1}{2m_\tau} (ik_\nu \sigma^{\mu\nu}) S^{Z,\gamma} \right], \]

where \( g^Z = 1/(4s_W c_W), g^\gamma = 1, \) and \( k = p_\tau + p_\tau \) (\( p_\tau \) and \( p_\tau \) are the momenta of outgoing tau and anti-tau respectively). We neglect the scalar and pseudoscalar couplings, \( k_\mu \) and \( k_\mu \gamma_5 \), since these terms give contributions proportional to the electron mass in \( e^+e^- \) collisions. The vector and axial-vector couplings \( V^{Z,\gamma} \) and \( A^{Z,\gamma} \) contain both the SM and new physics contributions, while \( A^{\gamma} \) and \( S^{Z,\gamma} \) contain only new physics contributions. One can write the vector and axial-vector couplings as

\[ V^{Z,\gamma} = (V^{Z,\gamma})^0 + \delta V^{Z,\gamma}, \quad A^{Z,\gamma} = (A^{Z,\gamma})^0 + \delta A^{Z,\gamma}, \]

where \((V^{Z,\gamma})^0\) and \((A^{Z,\gamma})^0\) represent the SM couplings and \( \delta V^{Z,\gamma}, \delta A^{Z,\gamma} \) the anomalous new physics contributions. At the tree level the SM couplings are \((V^{Z,\gamma})^0 = 1 - 4s_W^2, 1 \) and \((A^{Z,\gamma})^0 = 1, 0 \), while the new physics contributions are given by

\[ \delta V^Z = \frac{2s_W c_W v m_\tau}{\Lambda^2} \left[ C_{lW} \frac{c_W k^2}{2v m_\tau} + (C_{tB} + C_{\Phi L}) s_W k^2 \right] - C_{\Phi L}^0 - C_{\Phi L}^0 - C_{\Phi L}^0, \]

\[ \delta A^Z = \frac{2s_W c_W v m_\tau}{\Lambda^2} \left[ C_{lW} \frac{c_W k^2}{2v m_\tau} + (C_{tB} - C_{\Phi L}) s_W k^2 \right] - C_{\Phi L}^0 - C_{\Phi L}^0 + C_{\Phi L}^0, \]
\[ S^Z = -\frac{8s_W c_W m_\tau v}{e} \frac{v}{\Lambda^2} \sqrt{2} \left[ C_{D\tau} \frac{m_Z}{2v} - C_{\tau W} \phi c_W - 2C_{\tau B} \phi s_W \right], \]
\[ \delta V^\gamma = \frac{1}{e} \frac{k^2}{2\Lambda^2} \left[ C_{LW} \frac{s_W}{2} - (C_{LB} + C_{\tau B}) c_W \right], \]
\[ \delta A^\gamma = \frac{1}{e} \frac{k^2}{2\Lambda^2} \left[ C_{LW} \frac{s_W}{2} - (C_{LB} - C_{\tau B}) c_W \right], \]
\[ S^\gamma = \frac{2m_\tau \sqrt{2v}}{e} \frac{1}{\Lambda^2} \left[ C_{\tau W} \phi \frac{s_W}{2} - C_{\tau B} \phi c_W \right]. \] (9)

Now we discuss the neutral current processes involving three-family charged leptons. With assumption that the new physics correction only resides in the third-family lepton, the neutral currents effective lagrangian has a non-universal form in three-family lepton sector, like

\[ \mathcal{L}_{\text{eff}} = \frac{2m_Z}{v} Z^\mu Z^\nu \left[ \begin{array}{l} \tau \\ \mu \\ \tau \end{array} \right]^T U_L^{(l)} \left\{ g_L \left( \begin{array}{cc} 1 & 1 \\ 0 & 0 \end{array} \right) + \left( \begin{array}{l} \delta_L \\ \delta_R \end{array} \right) \right\} U_L^{(l)} \gamma_\mu \rho L \left( \begin{array}{l} e \\ \mu \\ \tau \end{array} \right) \]
\[ + (L \leftrightarrow R), \] (10)

where L (R) denotes the left (right) hand, the SM tree level couplings are

\[ g_L = -\frac{1}{2} + \sin^2 \theta_W, \]
\[ g_R = \sin^2 \theta_W, \] (11)

the new physics couplings are

\[ \delta_L = \frac{v^2}{2\Lambda^2} \left( C_{\Phi L}^{(1)} + C_{\Phi L}^{(3)} - \frac{C_{LB} s_W k^2}{vm_Z} - \frac{C_{LW} c_W k^2}{2vm_Z} \right), \]
\[ \delta_R = \frac{v^2}{2\Lambda^2} \left( C_{\Phi \tau} - \frac{C_{\tau B} s_W k^2}{vm_Z} \right), \] (12)

and \( U_{L,R}^{(l)} \) are unitary flavor-mixing matrices diagonalizing the left-handed and right-handed charged leptons respectively.

Here, we classify the effective lagrangian not as vector part and axial-vector part but as left hand part and right hand part by the reason that \( \delta_L \) and \( \delta_R \) are independent by containing different parameters. Noticing that the SM corresponds to \( \mathcal{L}_{\text{eff}} \) in the limit \( \Lambda \to \infty \), the corrections of new physics vanish and \( U_{L,R}^{(l)} \) are not measurable in neutral current processes in the SM.

6
3 Fritzsch Ansatz on the lepton mass

The flavor-mixing matrix $U (=U_L^{(l)} = U_R^{(l)})$ which diagonalizes the ”democratic matrix” $M_{0l}$ by the transformation $UM_{0l}U^\dagger = M_H^l$, is given by [10]

$$
U = \begin{pmatrix}
\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\
\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & -\frac{2}{\sqrt{6}} \\
\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}}
\end{pmatrix}.
$$

Then we have

$$
U \begin{pmatrix} 0 \\ 0 \\ \delta_L \end{pmatrix} U^\dagger = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{2\delta_L}{3} & -\frac{\sqrt{2}}{3}\delta_L \\ 0 & -\frac{\sqrt{2}}{3}\delta_L & \frac{1}{3}\delta_L \end{pmatrix}. \tag{13}
$$

In this approximation, there is no extra correction to the vertices of $Zee$, $Z\mu\mu$ and $Z\tau\tau$. However, some small corrections proportional to $\delta_L$ are generated naturally in the second and third families.

By taking into account the existence of the mass of electron and muon, the democratic symmetry would be broken and there is

$$
M_l = c_l \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + \Delta M_l, \tag{14}
$$

where $\Delta M_l$ are the symmetry breaking terms for the charged leptons. $\Delta M_l$ would be a diagonal mass shift, i.e.

$$
\Delta M_l = \begin{pmatrix} \delta_l & 0 & 0 \\ 0 & \varrho_l & 0 \\ 0 & 0 & \varepsilon_l \end{pmatrix}, \tag{15}
$$

and the neutrino mass matrix $M_\nu$ has the diagonal form with three different eigenvalues. Following Ref. [8], we take $\delta_l = -\varrho_l$ to simplify the problem, and, the mixing angles can be completely expressed in terms of the charged lepton mass. The flavor-mixing matrix $V (VM_lV^\dagger = M_{\text{diag}})$ can be parametrized in terms of three Euler angles as follow

$$
V = \begin{pmatrix}
c_{12}c_{13} & s_{12}c_{13} & s_{13} \\
-s_{12}c_{23} - c_{12}s_{23}s_{13} & c_{12}c_{23} - s_{12}s_{23}s_{13} & s_{23}c_{13} \\
s_{12}s_{23} - c_{12}c_{23}s_{13} & -c_{12}s_{23} - s_{12}c_{23}s_{13} & c_{23}c_{13}
\end{pmatrix}. \tag{16}
$$
where \( s_{ij} \equiv \sin \theta_{ij} \), \( c_{ij} \equiv \cos \theta_{ij} \) and the three mixing angles are determined as follows

\[
\begin{align*}
\tan \theta_{12} &= -1 + \frac{2}{\sqrt{3}} \frac{m_e}{m_\mu}, \\
\tan \theta_{23} &= -\sqrt{2} - \frac{3 m_\mu}{\sqrt{2} m_\tau}, \\
\tan \theta_{13} &= -\frac{2}{\sqrt{6}} \frac{m_e}{m_\mu}.
\end{align*}
\]

Then we have

\[
V \begin{pmatrix}
0 \\
0 \\
\delta_L
\end{pmatrix} V^\dagger = \begin{pmatrix}
s_{13}^2 & s_{13}s_{23} c_{13} & s_{13} c_{23} c_{13} \\
s_{13}s_{23} c_{13} & s_{23}^2 c_{13} & s_{23}^2 c_{23} c_{13} \\
s_{13} c_{23} c_{13} & s_{23} c_{13} c_{23} & c_{23}^2 c_{13}
\end{pmatrix} \delta_L
\]

\[
= \begin{pmatrix}
0.0035 & -0.049 & 0.032 \\
-0.049 & 0.701 & -0.455 \\
0.032 & -0.455 & 0.296
\end{pmatrix} \delta_L, \quad (17)
\]

where \( \sqrt{m_e/m_\mu} \approx 0.0696 \) and \( m_\mu/m_\tau \approx 0.0594 \) are used. Note that the corrections to the vertices of \( Zee, Ze\mu \) and \( Ze\tau \) are suppressed much more than the second and third family charged leptons, since the angle \( \theta_{13} \) is approximated to zero compared with \( \theta_{12} \) and \( \theta_{23} \). The right hand part can be calculated in the same way.

\section{4 Bounds on anomalous magnetic and electric moments \[11,12\]}

In general, the anomalous magnetic and the electric dipole moments of the tau lepton are defined by (which we follow the notation of Ref. \[11\])

\[
a_\tau = \frac{g_\tau - 2}{2} = F_2(q^2 = 0), \quad \text{and} \quad d_\tau = eF_2(q^2 = 0), \quad (18)
\]

where \( F_2 \) and \( \tilde{F}_2 \) are form factors in the electromagnetic matrix element

\[
< p'|J_{em}^\mu(0)|p> = e\bar{u}(p')(F_1 \gamma^\mu + \frac{i}{2m_\tau} F_2 + \gamma_5 \tilde{F}_2) \sigma^{\mu\nu} q_\nu u(p), \quad (19)
\]

where \( q = p' - p \) and \( F_1(q^2 = 0) = 1 \).

Theoretically the standard model predicts \( a_\tau = 1.1769(4) \times 10^{-3} \) and a very tiny \( d_\tau \) from CP violation in the quark sector \[\[13\].
The most stringent bounds are that inferred from the width \( \Gamma(Z \rightarrow \tau^+\tau^-) \), which are \(-0.004 \leq a_\tau \leq 0.006 \) and \( |d_\tau| \leq 1.1 \times 10^{-17} \text{e cm} \) \[1\]. To obtain these limits, Escribano and Massó took two effective operators \( O_{\tau B} \) and \( O_{\tau W} \) in Eq. (4).

When \( \Phi \) gets vacuum expectation value, operators \( O_{\tau B} \) and \( O_{\tau W} \) give rise to anomalous magnetic moment of the tau lepton and also corrections to the decay width of \( Z \) into \( \bar{\tau}\tau \). Given that the experimental data on \( Z \) width is quite consistent with the prediction of the SM, Escribano and Massó put a strong bound on anomalous magnetic moment of the tau lepton listed above.

Considering the dimension-six operators, \( O_{\tau B} \) and \( O_{\tau W} \), the full effective lagrangian now can be written as:

\[
L_{\text{eff}} = L_0 + \frac{1}{\Lambda^2} (c_{\tau B} O_{\tau B} + c_{\tau W} O_{\tau W} + \text{h.c.}).
\]

After the electroweak symmetry is broken and the mass matrices of the fermions and the gauge bosons are diagonalized, the effective neutral current couplings of the leptons to gauge boson \( Z \) and the photon \( \gamma \) are

\[
L_{\text{eff}}^{Z,\gamma} = e g^{Z,\gamma} \left( \begin{array}{c} \tau \\ \mu \\ \tau \end{array} \right)^T U_l \left\{ \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} (\gamma_\mu V^{Z,\gamma} - \gamma_\mu \gamma_5 A^{Z,\gamma}) \\ \frac{1}{2m_\tau} (ik_\nu \sigma^{\mu\nu}) S^{Z,\gamma} \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \right\} U_l^\dagger \begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix} (Z^\mu, \gamma^\mu),
\]

where \( g^Z = 1/(4s_W c_W) \), \( g^\gamma = 1 \), and \( V^{Z,\gamma} = 1 - 4s_W^2, 1, A^{Z,\gamma} = 1, 0 \) for \( Z \) and photon respectively, and

\[
S^Z = -\frac{8s_W c_W m_\tau v}{e \Lambda^2 \sqrt{2}} [C_{\tau W} c_W - 2C_{\tau B} s_W], \\
S^\gamma = \frac{2m_\tau \sqrt{2} v}{e \Lambda^2} \left[ C_{\tau W} s_W - C_{\tau B} c_W \right].
\]

The decay width of \( l \rightarrow l' + \gamma \) is given by

\[
\Gamma_{l \rightarrow l' \gamma} = \frac{m_l}{32\pi} \left( V_{l' l} e S^\gamma \frac{m_l}{m_\tau} \right)^2,
\]

where \( V_{l' l} \ (l \neq l') \) are the nondiagonal elements of matrix \( V \).

\[
V = U_l \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} U_l^\dagger
\]
We have neglected the mass of the light lepton $l'$.

Given the current experimental upper limits on $\mu^- \rightarrow e^- \gamma$, $4.9 \times 10^{-11}$ [13], we have

$$|S^\gamma| < 1.3 \times 10^{-10}. \quad (25)$$

The new physics contribution to the anomalous magnetic moments of the tau lepton is given by

$$|\delta \alpha_\tau| = |V_{\tau\tau} S^\gamma|. \quad (26)$$

With the bounds on $S^\gamma$ and $V_{\tau\tau}$, we obtain that

$$|\delta \alpha_\tau| \leq 3.9 \times 10^{-11}. \quad (27)$$

This limit is much stronger than that obtained by Escribano and Massó. The limits from other LFV processes, such as $\tau^- \rightarrow e^- \gamma$, are weaker than that given from $\mu^- \rightarrow e^- \gamma$.

Similarly, considering operators below which are introduced by Escribano and Massó,

$$\hat{O}_{\tau B\phi} = \bar{L}_\sigma \sigma^\mu i \gamma_5 \tau_R \Phi_B \mu + h.c.,$$
$$\hat{O}_{\tau W\phi} = \bar{L}_\sigma \sigma^\mu i \gamma_5 \bar{\sigma} \tau_R \Phi_{\bar{W}} \mu + h.c., \quad (28)$$

and following the procedure above in obtaining the bounds on tau lepton magnetic moment, we put

$$|d_\tau| \leq 2.2 \times 10^{-25} \text{ e cm}. \quad (29)$$

Again this is stronger than that obtained by Escribano and Massó.

Note the following relation:

$$|\delta \alpha_\tau|^2 \Gamma_{(\mu \rightarrow e\gamma)} = \frac{8 m_\mu^3}{\alpha m_\tau^3} \Gamma_{(\tau \rightarrow e\gamma)} \Gamma_{(\tau \rightarrow \mu\gamma)}, \quad (30)$$

which is independent of the lepton mass mixing matrix [12].

5 Effects on the decay $Z \rightarrow \mu^\pm + \tau^\mp$

The decays $Z \rightarrow \tau^+ + \tau^-$ and $Z \rightarrow \mu^+ + \mu^-$ can be discussed. For convenience, we define

$$M \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} M^\dagger \equiv \begin{pmatrix} U_{ee} & U_{e\mu} & U_{e\tau} \\ U_{\mu e} & U_{\mu\mu} & U_{\mu\tau} \\ U_{\tau e} & U_{\tau\mu} & U_{\tau\tau} \end{pmatrix}, \quad (31)$$

where $M$ is $U$ or $V$. Defining $\delta \Gamma_{l^l}$ ($l = e, \mu, \tau$) to be the pure correction of the new physics beyond the SM to the $Z \rightarrow l^l l^l$ width, $\Gamma_{l^l}$, we have

$$\frac{\delta \Gamma_{l^l}}{\Gamma_{l^l}} \simeq \frac{2U_{ll}(g_L^* \delta L + g_R \delta R)}{g_L^2 + g_R^2} = (-4.28 \delta L + 3.68 \delta R)U_{ll}, \quad (32)$$
where \( U_{ll} (l = e, \mu, \tau) \) are diagonal elements of matrix, and we have used \( \sin^2 \theta_W = 0.231 \). It is clear that different corrections of width would be given by using different approximations.

The constraints of the parameters \( \delta_L \) and \( \delta_R \) can be given in the following two cases in term of the experimental data of the decays,

\[
\frac{\Gamma_{\mu\bar{\mu}}}{\Gamma_{\text{full}}} = (3.367 \pm 0.013)\%,
\]

\[
\frac{\Gamma_{\tau\bar{\tau}}}{\Gamma_{\text{full}}} = (3.360 \pm 0.015)\%.
\]

The width of the FCNC decays can be given by

\[
\Gamma_{ll'} = \frac{|U_{ll'}|^2}{6\pi v^2} m_Z^3 (\delta_L^2 + \delta_R^2),
\]

where \( U_{ll'} (l \neq l') \) are the nondiagonal elements, and the mass of charge lepton is approximated to zero compared with the heavy mass of \( Z \).

Under the constraints derived from \( \frac{\delta_{\mu\mu}}{\Gamma_{\mu\mu}} \), we can get the limit of the FCNC effects.

According to our calculation, it is found that the limit of FCNC is directly related to the experimental data of the decays \( Z \to \tau^+ + \tau^- \) and \( Z \to \mu^+ + \mu^- \). Under assumption that the experimental error of the width \( Z \to \tau^+ + \tau^- \) and \( Z \to \mu^+ + \mu^- \) is contributed mainly by the effects of new physics, the branching ratio of the decay \( Z \to \mu^\pm + \tau^\mp \) could reach to \( 10^{-5} \) to \( 10^{-7} \).

This conclusion is larger than that from the calculation of loop level in the SM, which could be \( 10^{-7} \) to \( 10^{-8} \).

Since the recent experimental data of the branching ratio of the rare decay \( Z \to \mu^\pm + \tau^\mp \) is less than \( 1.2 \times 10^{-5} \), our prediction can be experimentally accessible and hopeful for observation in the future colliders.

6 Summary

1. The phenomena of the lepton FCNC in \( Z \) decays by means of the effective lagrangian and the lepton flavor-mixing matrices are studied. The dimension-six CP-conserving \( SU_L(2) \times U_Y(1) \) invariant operators involving the tau lepton, generated by new physics at a higher energy scale, are listed. The phenomena of the lepton FCNC and the lepton number violation are subject to the existing experimental limits and are hopeful for observation in the future collider.

2. We extend the work by Escribano and Massó to bound the anomalous magnetic and electric dipole moments of the tau lepton in the effective lagrangian by allowing the mixing of three generations in the lepton sector. In the standard model, these mixing effects are not measurable because of vanishing neutrino masses and the universal gauge interactions. With non-universal interaction, the lepton flavor violation happens even with zero neutrino masses. By taking the lepton mass matrix of Fritzsch ansatz, we have demonstrated that the experimental limit on \( \mu \to e\gamma \) puts stronger limits on the anomalous magnetic and electric dipole moments of the tau lepton than obtained by Escribano and Massó.
\[ |\delta \alpha_\tau| \leq 3.9 \times 10^{-11}, \quad |d_\tau| \leq 2.2 \times 10^{-25} \text{ e cm.} \]  

(34)

3. Our results depend on the lepton mass matrix. So qualitatively our results also indicates the equal importance of probing the anomalous magnetic and electric dipole moments of tau lepton as well as the lepton flavor violation. Furthermore, the future experimental data on anomalous magnetic and electric dipole moments of the tau lepton together with lepton flavor violation will provide an experimental test on various lepton mass ansatz.

4. Among all of 10 such operators, 6 operators \((O_{\Phi L}^{(1)}, O_{\Phi L}^{(3)}, O_{LB}, O_{LW}, O_{\Phi \tau} \text{ and } O_{\tau B})\) contribute mainly to the \(Z\tau\tau\) coupling. Their corresponding constants \(C_{\Phi L}^{(1)}, C_{\Phi L}^{(3)}, C_{LB}, C_{LW}, C_{\Phi \tau} \text{ and } C_{\tau B}\) are contained by two parameters \(\delta_L\) and \(\delta_R\), which could be constrained by the experimental data of the decays \(Z \rightarrow \tau^+ + \tau^-\) and \(Z \rightarrow \mu^+ + \mu^-\). According to these constraints, the branching ratio of the decay \(Z \rightarrow \mu^+ + \tau^-\) could reach to \(10^{-6} - 10^{-7}\), which is 100 times larger than that of \(Z \rightarrow e^\pm + \tau^\mp\) and \(Z \rightarrow e^\pm + \mu^\mp\).

References

[1] R.D. Peccei, hep-ph/9811309; J. L. Hewett, hep-ph/9810316

[2] K. Whisnant, J. M. Yang, B.-L. Young and X. Zhang, Phys. Rev. D 56, 467 (1997).

[3] J. M. Yang and B.-L. Young, Phys. Rev. D 56, 5907 (1997).

[4] X. Zhang and B.-L Young, Phys. Rev. D 51, 467 (1995)

[5] C. J. C. Burgess and H. J. Schnitzer, Nucl. Phys. B 228, 454 (1983); C. N. Leung, S. T. Love and S. Rao, Z. Phys. C 31, 433 (1986); W. Buchmueller and D. Wyler, Nucl. Phys. B 268, 621 (1986).

[6] T. Huang, J. M. Yang, B.-L. Young and X. Zhang, Phys. Rev. D 58, (1998) 073007.

[7] H. Fritzsch, Nucl. Phys. B 155 (1979) 189.

[8] H. Fritzsch and Z. Z. Xing, Phys. Lett. B 372 (1996) 265.

[9] H. Fritzsch and J. Plankl, Phys. Lett. B 237 (1990) 451; H. Fritzsch and D. Holtmannspötter, Phys. Lett. B 338 (1994) 290; H. Fritzsch and Z. Z. Xing, hep-ph/9808272, to be published in Phys. Lett. B; H. Fritzsch and Z. Z. Xing, hep-ph/9807234; M. Fukugita, M. Tanimoto and T. Yanagida, Phys. Rev. D 57, (1998) 4429; M. Tanimoto, hep-ph/9807283

[10] For example, Z. Xing, hep-ph/9804433.

[11] R. Escribano and E. Masso, Phys. Lett. B395, 369 (1997).
[12] T. Huang, Z.-H. Lin and X. Zhang, Phys. Lett. B450, 257 (1999).

[13] A. Czarnecki and W. J. Marciano, hep-ph/9810512.

[14] C. Caso et al., Particle Data Group, Eur. Phys. J. C3 (1998) 1.