Merging Locally Correct Knowledge Bases: A Preliminary Report

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Abstract

Belief integration methods are often aimed at deriving a single and consistent knowledge base that retains as much as possible of the knowledge bases to integrate. The rationale behind this approach is the minimal change principle: the result of the integration process should differ as less as possible from the knowledge bases to integrate. We show that this principle can be reformulated in terms of a more general model of belief revision, based on the assumption that inconsistency is due to the mistakes the knowledge bases contain. Current belief revision strategies are based on a specific kind of mistakes, which however does not include all possible ones. Some alternative possibilities are discussed.

1 Introduction

Most of the existing belief revision semantics are based—in some way—on a principle that has been formulated at the very beginning of the investigation on this topic: the minimal change principle [1, 6]. According to this principle, the result of integrating two or more knowledge bases should be as similar as possible to them. Semantics proposed for merging agree on this principle, and only differ in the way it is applied, i.e., in how to combine the several possibilities arising, in how to measure the difference between knowledge bases, in how the knowledge bases are represented, in what is the relative reliability of sources, etc. Nevertheless, very few arguments against the basic principle exist [16].

This paper does not contain arguments against the minimal change principle, but only as it being a first principle. Taking a different perspective, we show that
it is indeed a particular consequence of a more general assumption. Namely, we present a model of how the knowledge bases to integrate are obtained that justifies the minimal change principle, as it is currently applied, only in particular cases. This model explains inconsistencies between knowledge bases by assuming that mistakes have been done in the process of knowledge acquisition.

This model is not completely new, as existing merging semantics actually rely on its particularization to the case in which mistakes are changes of value of literals. For example, Dalal’s revision semantics [3] can be reformulated as the result of assuming that one knowledge base is free of mistakes, and the other one results from introducing mistakes in the value of literals in an otherwise correct knowledge base. In formulae, while revising $K$ with $P$, we assume that the process of acquiring $P$ is error-free, while $K$ contains some mistakes, each changing the value of a single literal in a model. Therefore, Dalal’s revision can be reformulated as the correction of a minimal number of mistakes. Other belief revision semantics are based on the same principle, but have different rules for combining the different possibilities that arise [17, 5, 2]. Iterated belief revision semantics [18, 12], updates [11, 8, 7], and merging/arbitration operators [13, 9, 14, 15], are based on similar principles.

The model proposed in this paper, however, does not only formalize existing semantics; being more general, it is applicable to other scenarios, leading to different revision techniques. While cases like the example of the stock market experts [9] are perfectly modeled in the “mistake of value” model, other ones are not. Some examples, like the following one, comes from everyday life.

Example 1 Yesterday, I met an old friend I have not been seeing in years. While talking about the high school days, we shared information about other friends we knew at that time. In particular, he told me that George earned a lot of money by creating a startup company he then sold, and now he lives in the Nukunonu island. On the other hand, I knew that George become incredibly rich with some illegal business, and he is currently in jail (but I do not know whether he still has some of the money.)

The union of our knowledge bases is inconsistent, as there are no jails in the Nukunonu island. On the other hand, both of us are completely certain of our current knowledge. We then had to conclude that we were talking about two different Georges. The correct conclusion of merging information should then be that “George A is rich”, “George A lives in the Nukunonu island”, and that “George B is in jail”.

Merging based on the minimal change principle, combined with the “mistake of value” assumption as it is usually done, would have led to a completely different result. Namely, since we assumed that we are talking about the same George, and since both of us have the same confidence on our knowledge, we could only conclude that either “George is in jail” or that “George lives in the Nukunonu island”, but not both (since no jail is in the Nukunonu island.) This is already a problem, as this information is not complete about George’s current location, while in fact we both
know exactly where the Georges are. Still worst, since I do not know whether George is still rich while my friend is sure he is, I will incorrectly conclude that the George I am talking about is still rich, a fact that is not backed up by any evidence.

This scenario is about a common life incident, but similar problems are common in computer science: putting together two \LaTeX source files creates the problem of the same name for two different macros; similar problems arise in compiling C code fragments, etc. In the rest of the paper, we make the simplifying assumption that each knowledge base is the knowledge of a different agent involved in the process of merging.

One of the characteristics of the example above is the “local” correctness of the involved knowledge base: both me and my friend had correct information about the George we were thinking about. The fact that each agent regards its knowledge base as correct, and then has to correct it during the merging process, is true in current belief semantics as well. However, the “mistake of value” model implies that the conclusions drawn by each single knowledge base were in fact incorrect. On the contrary, if the only mistakes are like the same name for two different objects, then the conclusions drawn from each knowledge base separately (before the merging) are correct, e.g., the conclusion that George cannot travel any more was correctly entailed by my knowledge base, and this is a correct conclusion, as I am referring to the George who is in jail. The correction to the knowledge bases is therefore only necessary to avoid inconsistency while merging the knowledge bases.

While inconsistency is undoubtedly the most serious problem that may arise during merging, it is not the only one. There are mistakes that cannot be be discovered just by checking for inconsistency the union of the knowledge bases. Indeed, a mistake does not necessarily create an inconsistency. On the contrary, some mistakes make the union of the knowledge bases weaker than it should be. An example of this case is when two knowledge bases give different names to the same object, which forbids drawing conclusions based on two facts contained in the two knowledge bases.

\textbf{Example 2} Still talking with my high school friend, I mentioned Teddy, who entered the Law school; I though that if he ever had graduated, he would have ended up in jail. The friend I was talking with, however, does not remember this Teddy, and the only guy he knows entered Law was Bobby, who actually graduated. In fact, Teddy was a nickname for Bobby, but we did not remember this fact.

No inconsistency arises in this case. However, the conclusion that Bobby is (likely) in jail could not be drawn by simply putting together the knowledge we had. Contrary to common merging scenarios, the conjunction of the knowledge bases is weaker than it should be. Such problems are clearly difficult to diagnose, as they do not create an inconsistency. The only way to find them out is from the fact that the resulting knowledge base is weaker than it should be. For example, knowing that only
one person from our class entered the Law school would have allow us to find out that Bobby and Teddy must be the same person.

In order to produce a knowledge base in which as many mistakes as possible are corrected, we use two formulae that act as integrity constraints. Formally, we are given a multiset of knowledge bases $\mathcal{K}$ and two formulae $A$ and $B$; the result of the integration process is a formula $K = \mathcal{I}_B^A\mathcal{K}$ such that $K \models A$ and $K \land B \not\models \bot$. This way, we constraint the resulting knowledge base to have (at least) a specific set of consequences $A$, and not to have some undesired other consequences $B$. The formula $A$ formalizes the usual integrity constraints (facts that should remain true after integration), while $B$ extends the usual consistency requirement: $B = \top$ only enforces the result of integration to be consistent. Since $\bar{K} = \mathcal{I}_B^A\mathcal{K}$ is a formula whose set of models is contained in $\text{Mod}(A)$, and is not contained in $\text{Mod}(\neg B)$, we call $A$ and $B$ the upper and lower bound of the merging operator, respectively.

If the union of the knowledge bases of $\mathcal{K}$ implies $A$ and is consistent with $B$, we assume that there is no problem, i.e., the knowledge bases do not contain any mistake. This assumption may be wrong anyway, but we have no way to realize it. The interesting case is when either constraint is not satisfied. In this case, we assume that some mistakes have been made while acquiring the knowledge bases. Some possible mistakes are listed below. The three last mistakes of the list are the only ones leading to a locally incorrect knowledge base.

**homonymy:** two agents use the same variable while they should use two different ones;

**synonimies:** two agents use different variables while they should use the same one;

**subject misunderstanding:** a formula is stated using one variable, while it should use a different one;

**extension:** a formula $F$ is extended to another variable or set of variables: formally, the agent assumes $F[X/Y]$ in addition of $F$;

**generalization:** a formula is assumed to hold in general, while it holds only under some assumptions;

**particularization:** a formula is assumed to hold only in a specific scenario, while it is more general;

**ambiguity:** a formula containing $a \lor b$ is assumed to be $a$ alone (or $b$ alone, or both $a$ and $b$);

**exclusion:** a formula containing an inclusive or is taken to refer to the exclusive or, or vice versa;
value: the formula is correct because it contains a model with a wrong value.

Besides the mistake of value, these mistakes can be grouped in three categories: mistakes due to a wrong interpretation of variables (homonyms, synonimies, and subject misunderstanding); mistakes due to a wrong interpretation of context (generalization, particularization, and extension); mistakes due to a wrong of the logic (ambiguity and exclusion).

Some other mistakes are particular cases of the above ones. For example, an agent may incorrectly assume that a previously true fact continues to hold while it does not: this is a subcase of incorrect generalization. Another similar mistake is the incorrect simplification of a definition, like “the water boils at 100°C” instead of “the water boils at 100°C at sea level”.

In the domain we consider, each agent introduces some mistakes into a truly correct knowledge base. This is modeled by assuming that each agent modified its original knowledge base in some way. Clearly, this is only a theoretical model: if the agent ever had a correct knowledge base, it had not modify it. However, this way we can say that “the agent modified the knowledge base”, that simplifies the more correct sentence “the agent incorrectly considered the information x to be y”.

Merging is the process of first correcting mistakes in the knowledge bases, and then conjoining them. Correcting mistakes, in turns, is a two-phase process: first, we have to find out which mistakes have been made, and then correcting them. We initially assume that an ordering of likeliness of mistakes is known, and then consider the problem of how to derive it from the knowledge bases. In this second case, however, we cannot expect the merging process to do much, given the high number of possible mistakes: for example, the multiset \{a, \neg a\} may be inconsistent because the second a should be b, or because a is only true when b is true (that is, the first formula should be \(b \rightarrow a\) instead of a alone), or because the ambiguity \(a \lor b\) of the first formula has been interpreted as a choice, and the agent has incorrectly assumed a, etc. The number of possibilities increases with the number of variables and with the size and complexity of the knowledge bases. The process of correcting the mistakes can also be problematic: knowing that a formula has been obtained by changing a name is not enough if we do not know the original name.

We make some simplifications. The first one is to neglect the mistakes of logic (ambiguity and exclusion). The second one is to restrict our study to propositional knowledge bases. While first-order logic (even without function symbols) is uncommon in belief revision studies, this assumption makes us disregard the very relevant case of epistemic bases [10], which contain not only the agent’s belief, but also what it considers more or less plausible.
2 A Model of the Sources

In this section, we give a formal definition of our framework. The general belief merging process can be visualized as in Figure 1: there are a number of agents (sources) each sending a knowledge base to a centralized “knowledge merger”. We do not consider the more sophisticated models that are sometimes used (e.g., an agent supplies more than one knowledge base.)

![Diagram of basic model of belief merge.](image)

Figure 1: The basic model of belief merge.

We improve over this simple schema by providing a model of how the sources get the knowledge bases $K_i$'s they pass to the merger: each $K_i$ is obtained by applying one or more transformations to a knowledge base $S_i$, which is assumed to be correct. Figure 2 is a graphical representation of this model.

![Diagram of model of belief merge with mistakes.](image)

Figure 2: The model of belief merge, with mistakes.
The “mistake of value” revision semantics fit in this model: each $K_i$ is obtained from $S_i$ by applying the transformation that changes the value of a variable in a model. Namely, let $\tau_{M,x}^v$ be the transformation that takes a formula, and gives another formula in which the model $M$ is replaced by the model with the opposite value of $x$. Each $K_i$ is obtained from $S_i$ by applying a suitable number of such transformations. Specific revision/arbitration/merging operators can be then formalized by assuming a form of minimality of the mistakes, and then combining in some way the possible results of this assumption.

For example, Dalal’s revision assumes that a. one of the knowledge base is correct (no transformation has been applied to it); b. the other knowledge base results from the application of a number of transformations $\tau_{M,x}^v$ to a correct one; and c. a minimal number of transformations have been applied. If more than one knowledge base result from inverting these transformations, they are disjoined. This semantics fits into the proposed model: the $K_i$’s are obtained by applying transformations to the $S_i$’s, and the process of integration attempts to invert them.

Formalizing Dalal’s revision in this way shows how integration can be done in general: inverting the transformation applied to $S_i$, and merging what results. Ideally, we should be able to obtain the knowledge bases $S_i$, which are assumed correct. Unfortunately, inverting the transformations cannot be done uniquely, as the merger only knows the $K_i$’s, but has no direct knowledge of the transformations used or the original $S_i$’s. For example, $K_i = a$ may be correct, or may be the result of changing a variable name to $S_i = b$, or may be a wrong generalization of $S_i = c \rightarrow a$, and so on.

The mistakes listed in Section 1 can be formalized by the following transformations.

**Variable substitution**: $\tau_{x,y}^h(F) = F[x/y];$

**Generalization**: $\tau_x^g(F) = F[x/true];$

**Particularization**: $\tau_x^p(F) = x \rightarrow F;$

Variable substitution models all mistakes due to mistakes relative to variable names: homonymies, renaming, and subject misunderstanding. Wrong generalization is the mistake of neglecting some assumptions of an (otherwise true) fact. This can be formalized by taking the original (correct) formula $F$, and replacing the assumption $x$ with $true$. Note that the resulting formula $\tau_x^g(F)$ does not contain $x$ at all, but has exactly the models $F$ would have if $x$ is true. The simplest case of generalization is when $x \rightarrow F$ is taken to be $F$: if $F$ does not contain $x$, then $F = \tau_x^g(x \rightarrow F)$. However, $\tau_x^g$ also models more complex cases of generalization. Particularization is easy to formalize: some assumptions are believed to be required for some fact to hold, while they are not. Generalization and particularization can
be, to some extent, been considered the opposite of each other, since $\tau^g_x(\tau^g_y(F)) \equiv F$. However, the converse does not hold, as it may be $\tau^g_x(\tau^g_y(F)) \not\equiv F$; this is the case, for example, if $F$ does not mention $x$ at all. We neglect mistakes of logic, that is, ambiguity and exclusion, as they are too hard to detect and invert. Especially ambiguity is difficult to detect without a lot of additional information: given a formula, it may be that each of its subformulae was originally disjoined with another formula (that may be an arbitrary formula of the domain). Even restricting to literals, the number of possibilities makes the problem quite difficult.

3 The Merging Process

The merging process consists in inverting the transformations, and then putting together the resulting knowledge bases. Since we only have the knowledge bases $K_i$’s after the changes, we do not know for sure which transformations are the ones to invert. Extending the principles used for revision and arbitration, we make some hypotheses about the kind of mistakes that have been made. Considering only the most likely possibilities, we are still left with a number of possible scenarios. For each of them, however, we know how to invert the transformations and obtain the original knowledge bases $S_i$, which can be then conjoined to get the maximum possible information. What result is the merged knowledge base in one of the possible scenarios we assumed. Therefore, we have one knowledge base for each scenario: since these are alternative possibilities, the right way of combining them is by disjunction.

Formally, we begin with the knowledge bases $K_1, K_2, \ldots, K_n$, and make an assumption about the transformations that have been used to obtain them. Inverting these transformations, we obtain $K'_1, K'_2, \ldots, K'_n$. If the assumption about the transformations is correct, the best way of merging them is simply by putting them together, thus obtaining $K = K'_1 \land K'_2 \land \cdots \land K'_n$.

On the other hand, this is only a possible scenario. In another scenario, we may get a different result of merging $K^1$, in another one we may have yet another result $K^2$, etc. Since these are the results of considering different alternatives we consider equally likely, the final result of merging should be the disjunction (logical or) of them.

Figure 3 shows this process. Finding and inverting the transformations are central steps of this process: on the one hand, we should select as few possible scenarios as possible to avoid a too weak result; on the other hand, including too few possibilities may lead us to neglect the one that really represents the state of the world.

For simplicity, we replace these first two steps of the process by the one of finding one (or more) $n$-tuples of inverse transformations, one for each knowledge base. Indeed, finding the transformations that have been applied and inverting them can be formalized by the single step of finding the transformations that lead from the
knowledge bases we have to the original ones; we call them “inverse transformations” simply because they invert the transformations that have been previously applied, but they are still the transformations previously considered, like variable substitution, etc.

In order to select one (or more) \( n \)-tuple of inverse transformations, we define an ordering over all possible \( n \)-tuples of sets of transformations. This way, we can compare a possible scenario with another one, and tell which one is the most likely. A different and simpler model is that in which there is one ordering for each knowledge base. We do not adopt this model because mistakes in one knowledge base should be ranked not only according to that source, but also as a result of comparing it with the other knowledge bases. This is why we consider an ordering ranking \( n \)-tuples rather than comparing transformations locally, i.e., source by source.

This ordering may originate in different ways: it can be part of the knowledge of each source (that is, each agent has its own idea of the mistakes it likely makes), or it can be an information the merger has (possibly based on the meaning of the literals and other related knowledge), or it is derived from the knowledge bases \( K_i \)'s using some heuristics. In the first two cases, we can simply assume that the ordering is given; the problem of obtaining it from the knowledge bases is discussed in the next session. Either way, in the rest of this section we assume that this ordering is given. In particular, we assume that \( \mathcal{R} \) is a function that associates an integer to each \( n \)-tuple of sets of transformations, giving the likeliness they correct the mistakes in the knowledge bases \( K_1, \ldots, K_n \). As is common in belief revision, we interpret a lower rank as an higher degrees of likeliness, and therefore prefer \( n \)-tuples with the lowest rank.
The set of possible transformation that may have been used for generating \( K_i \) from \( S_i \) is defined as follows:

\[
T(K_i) = \{ \tau^h_{x,y} \mid y \in \text{Var}(K_i), \ x \notin \text{Var}(K_i) \} \cup \{ \tau^g_x \mid x \notin \text{Var}(K_i) \} \cup \{ \tau^p_x \mid \neg x \models K_i \}
\]

For example, \( K_i \) may result from replacing \( x \) with \( y \), and this is why the renaming of \( x \) with \( y \) is in the first part of \( T(K_i) \) only if \( y \) is mentioned in \( K_i \) while \( x \) is not. The other parts of \( T(K_i) \) are motivated in a similar way. The set \( T(K_i) \) is potentially infinite, as there are potentially infinite possible variables \( x \notin \text{Var}(K_i) \). For example, if \( S_i = x \lor y \), and the source renamed \( x \) with \( z \), it ends up with \( K_i = z \lor y \). Inverting this transformation amounts to deciding which name \( z \) originally had, and this is impossible by looking at \( K_i \) only. When a variable disappears from a knowledge base, like in this case, we either use a variable that appears in another knowledge base, or introduce a new one. This limits the set of possible transformations: when we write \( x \notin \text{Var}(K_i) \) we assume that either \( x \) is a variable occurring in some other knowledge base, or \( x \) is a new variable created on purpose.

In order to invert the transformations, we define an inverse relation \( \text{Inverse}_i(\tau_1, \tau_2) \), which relates two transformations \( \tau_1 \) and \( \tau_2 \) in such a way \( \tau_2 \) undoes the changes made by \( \tau_1 \) on the knowledge base \( K_i \). Note that \( \text{Inverse}_i \) is indexed by \( i \), thus making this relation dependent on the considered knowledge base. However, only the names of the variables in \( K_i \) are really needed. Also note that \( \text{Inverse}_i \) is not a function, as renaming and generalization cannot be uniquely inverted. This relation is formally defined as follows.

\[
\text{Inverse}_i = \{ (\tau^h_{x,y}, \tau^h_{y,z}) \mid \tau^h_{x,y} \in T(K_i), \ x \in \text{Var}(K_i), \ z \notin \text{Var}(K_i) \} \cup \{ (\tau^g_x, \tau^p_y) \mid \tau^g_x \in T(K_i), \ y \notin \text{Var}(K_i) \} \cup \{ (\tau^p_x, \tau^g_y) \mid \tau^p_x \in T(K_i), \ y \notin \text{Var}(K_i) \}
\]

The relation \( \text{Inverse}_i \) defines the set of all possible inverse transformations on the knowledge base \( K_i \). Since there are too many such transformations, we also consider the ordering that tells their degree of likeliness. This ordering is formalized as a functions from \( n \)-tuples of sets of transformations to integers. Formally, an integer is associated to each subset of \( \text{Inverse}_1 \times \cdots \times \text{Inverse}_n \). The idea is that each subset of this set contains a set of transformations for each knowledge base; implicitly, it tells the mistakes that have been done. The ordering simply tells the degree of likeliness of these mistakes. We denote this function as \( R \).

This ranking makes the process of merging possible. As it is common in belief revision, we consider all possible changes to the knowledge bases, select only the ones that lead to the expected result (\( A \) should be derivable but \( \neg B \) should not), and then use the ranking to further reduce the set of possibilities.

In order to define the first step (selection of transformations), we have to specify, for each \( n \)-tuple of sets of transformations, what is the resulting knowledge base. Let
therefore \((\mathcal{L}_1, \ldots, \mathcal{L}_n)\) be this n-tuple, where \(\mathcal{L}_i \subseteq \{\tau \mid \exists \tau'. \langle \tau', \tau \rangle \in \text{Inverse}_i\}\). The result of applying the transformations in \(\mathcal{L}_i\) to \(K_i\) is as follows:

\[
\mathcal{I}_{\mathcal{L}_i}(K_i) = \tau_1(\ldots(\tau_n(K_i))) \quad \text{where} \quad \mathcal{L}_i = \{\tau_1, \ldots, \tau_n\}
\]

We extend this operator to tuples of set of transformations and to tuples of knowledge bases as follows.

\[
\mathcal{I}_{(\mathcal{L}_1, \ldots, \mathcal{L}_n)}(\mathcal{K}) = \bigwedge \mathcal{I}_{\mathcal{L}_i}(K_i)
\]

This is the result of merging only if \(\mathcal{L}_1, \ldots, \mathcal{L}_n\) is known to be the way in which the transformations have to be inverted, or it is the only way in which both the constraint on \(A\) and the constraint on \(B\) can be satisfied. Usually, this is not the case, so we have to use the ranking \(\mathcal{R}\) to make a selection.

The transformations we consider are the minimal ones among those making the result of merging to imply \(A\) but not to imply \(\neg B\). Minimality is defined using the ranking.

\[
\mathcal{M}_\tau = \min_{\mathcal{R}}(\{\{\mathcal{L}_1, \ldots, \mathcal{L}_n\} \mid \bigwedge_{i=1}^{n} \mathcal{I}_{(\mathcal{L}_1, \ldots, \mathcal{L}_n)}(\mathcal{K}) \models A \quad \text{and} \quad \bigwedge_{i=1}^{n} (\mathcal{I}_{(\mathcal{L}_1, \ldots, \mathcal{L}_n)}(\mathcal{K}) \land B \models \bot)\})
\]

This formula defines a set of transformations for each source. Clearly, there is no warranty that such a minimum is unique. The merger applies each set of possible transformations, and disjoins the results:

\[
\mathcal{I}_A^A(\mathcal{K}) = \bigvee_{\mathcal{L} \in \mathcal{M}_\tau} \mathcal{I}_{\mathcal{L}}(\mathcal{K})
\]

By construction, \(\mathcal{I}_A^A(\mathcal{K})\) implies \(A\) simply because it is a disjunction of terms, each implying \(A\). For the same reason, since each term is consistent with \(B\), so is the result of merging.

### 4 Selection Heuristics

The merging process outlined in the last section depends on which the most likely transformations are. So far, we simply assumed the knowledge of the ordering \(\mathcal{R}_i\), either because it is an additional information the agents have, or because it is known to the centralized merger. However, the case in which no additional information, besides the knowledge bases, is known is also important. In this section we consider the case in which no information about the likeliness of mistakes is given, and the ordering must be drawn from the knowledge bases \(K_i\).

While it is always theoretically possible to select all possible transformations that satisfy the constraints \(A\) and \(B\), these transformations may be too many to give useful
information. Indeed, the more the possible considered scenarios are, the weaker the 
resulting knowledge base is, and the number of possible scenarios may be very large 
even for very simple knowledge bases. For example, the knowledge base \(K_1 = x \rightarrow y\) 
may result from the renaming of \(z\) to \(x\), or from the particularization of \(y\) (i.e. we 
iccorrectly assumed that \(y\) holds only when \(x\) is true), or from the generalization of 
\(x \land z \rightarrow y\), etc. A first selection criteria is that we only accepts sets of transformations 
that produce a knowledge base that satisfies the upper and the lower bounds. This, 
however, may be still too weak a constraint to limit the number of transformations.

For this reason, we also assume some minimality criteria; namely, we assume 
that as few mistakes as possible have been made while producing \(K_i\) from \(S_i\). In a 
sense, this is the minimal change principle in disguise: assuming a minimal number 
of mistakes, we still consider a minimal number of (inverse) transformations to be 
applied to the knowledge bases. On the other hand, the minimal change principle in 
this form is not a first principle any longer, but only a consequence of a more general 
assumption.

The principle of minimizing the number of mistakes/transformations, however, 
may still be not enough, that is, the number of possible scenarios may still be too 
high. Therefore, we use the knowledge bases to further limit the number of possible 
alternatives. In this section, we present a selection heuristics that is based only on 
the knowledge bases. We assume that no further information is given about the 
meaning of literals, the likeliness of mistakes, etc. and that we cannot perform any 
information-gathering actions (a common assumption in belief revision, less in the 
real world.)

Another problem of the merging process is that some transformations cannot be 
inverted uniquely. In particular, knowing that \(K_i = \tau^g_x(S_i)\) does not allow to derive 
\(S_i\). In such cases, we simply assume that \(S_i = \tau^p_x(K_i)\). This is equivalent to assuming 
that \(\tau^g_x\) is only applied to formulae like \(x \rightarrow F\), i.e., having \(x\) as a precondition.

In order to define this ranking \(R\), we observe that it only needs to rank the 
transformations according to their plausibility, regardless of whether they lead to 
satisfy the lower and upper bounds of merging: it is the merging process that enforces 
these constraints to be satisfied.

The ranking \(R\) is based on (besides assuming a minimal number of mistakes,) 
assuming that the initial knowledge bases \(S_i\)’s are similar to each other. Therefore, 
the best inverse transformations are those making the resulting knowledge bases \(K_i'\) 
as similar to each other as possible.

Examples justifying this way of operating are easy to find: if a knowledge base is 
identical to another one except for a different variable name, the change of the name 
is intuitively the most reasonable action to do before integrating the two knowledge 
bases.

This example can be generalized to the case in which applying a transformation
to \( K_i \) makes it equal to \( K_j \); this transformation is likely to be the inverse of the one that changed \( S_i \) to \( K_i \). In this case, \( S_i = S_j \), but is not always the case. To make this criteria to have general applicability, we need a way for applying it even when the two knowledge bases cannot be made identical. To this aim, we measure the similarity between knowledge bases, and trade off between the number of inverse transformations and the degree of similarity of the resulting knowledge bases. We therefore need a way for measuring the similarity between two knowledge bases, and then a way for combining this measure with the number of changes needed to make the knowledge bases similar.

The measure of similarity can be defined either syntactically or semantically; we define a semantical measure. There are two reasons for this choice: first, it is possible to express the same knowledge in different ways (so that \( S_i \) and \( S_j \), while identical in their sets of models, are syntactically different); second, each source may have further changed the syntactic form of its knowledge base to suit its purposes.

Let \( K_1 \) and \( K_2 \) be two knowledge bases, and let \( \text{Mod}(K_1) \) and \( \text{Mod}(K_2) \) be their sets of models. The measure of similarity should grow as the size of the intersection \( \text{Mod}(K_1) \cap \text{Mod}(K_2) \), and as the intersection of their complements \( \text{Mod}(\neg K_1) \cap \text{Mod}(\neg K_2) \). The total size of these two sets is in fact equal to \( |\text{Mod}(K_1 \equiv K_2)| \).

The degree of similarity should also decrease with the number of models that satisfy only one formula, that is, the size of \( \text{Mod}(K_1 \not\equiv K_2) \). A possible choice is the linear combination of these two measures:

\[
\delta(K_1, K_2) = |\text{Mod}(K_1 \equiv K_2)| - |\text{Mod}(K_1 \not\equiv K_2)| = 2 * |\text{Mod}(K_1 \equiv K_2)| - |\text{Mod}(\text{true})|
\]

This function is in practice the same as \( |\text{Mod}(K_1 \equiv K_2)| \). Another possibility is that of using a quotient: \( \delta(K_1, K_2) = |\text{Mod}(K_1 \equiv K_2)| / |\text{Mod}(K_1 \not\equiv K_2)| \).

Having defined the measure of similarity \( \delta \) of two knowledge bases, we can now combine it with the number of transformations to define the ranking. Let us therefore consider a specific \( n \)-tuple \( \langle \mathcal{L}_1, \ldots, \mathcal{L}_n \rangle \). The knowledge bases generated by the transformations are \( \mathcal{I}_{\mathcal{L}_i}(K_i) \). We compare them using \( \delta \) and the number of transformations in each set \( \mathcal{L}_i \). We use a simple linear combination of these two measures:

\[
\mathcal{R}(\langle \mathcal{L}_1, \ldots, \mathcal{L}_n \rangle) = \sum_{K_i, K_j} \log(\delta(\mathcal{I}_{\mathcal{L}_i}(K_i), \mathcal{I}_{\mathcal{L}_j}(K_j)) + 1) + \sum |\mathcal{L}_i|
\]

The logarithm is used to make the measure of similarity and the number of transformations to be on the same scale: without it, the measure of similarity can be exponentially large, thus making the contribution of the number of transformations irrelevant. We used a logarithm (instead of a multiplying factor) because a difference of distances should be less important when the number of different models is high: the difference between one model and two is more important than the difference between 1000 and 1001.
This ranking $\mathcal{R}$ defines a measure of goodness of transformations, and therefore completes the merging process outlined in the previous section: given the knowledge bases $K_i$’s, we can now tell exactly what the result of merging is.

A problem of this ranking, however, is that it is based on assuming that all knowledge bases $K_i$ derives from similar knowledge bases $S_i$ by applying some, equally likely, transformations. While the equal likeliness is the natural result of assuming no information about the likeliness of transformations, the assumption that the $S_i$’s are similar is questionable. In particular, it may be more reasonable to assume that each knowledge base is “targeted” to a different subject. Indeed, it is likely that each source uses the knowledge base for a specific purpose; as a result, the knowledge bases may contain only information about some specific subjects.

To take this consideration into account, we do not measure how similar the knowledge bases are, but how similar they are when restricted to a subset of variables. Namely, let $K^Y$ be the restriction of the formula $K$ to the variables in $Y$. The difference between two formulae $K_1$ and $K_2$ is:

$$\delta(K_1, K_2) = \sum_{\emptyset \neq Y \subseteq X} \frac{\delta(K^Y_1, K^Y_2)}{|X| - |Y| + 1}$$

This is how we formalize the assumption that the result of the inverse transformations may be a formula that is similar to the other one only for a subset of its variables. The quotient is defined in such a way to avoid a difference in the case $|Y| = 1$ to count the same as in the case $Y = X$.

### 5 The Renaming Merging Operator

The ranking defined in the previous section allows for determining the result of merging from the knowledge bases alone, without any additional information. In the belief revision terminology, this is a merging operator, as opposed to merging schemas, which require some additional information such as ranking, preferences, etc. (they are called schemas because they are the backbones of a merging process, but something has to be added to make them complete merging operators.)

The operator defined in the last section allows for checking the validity of properties that should hold for the merging process. However, the number of possible transformations make the operator quite complicated. We therefore make the simplifying assumption that the only mistakes are those involving renamings. The set of possible transformations is therefore defined as follows.

**Definition 1** Given a set of variables $X$, a permitted inverse transformation is a substitution $X/Y$ in which each $x_i$ is either substituted with another variable in $X$, or it is renamed as the new variable $x'_i$. 
This definition forbids the proliferation of new variables: if we have to replace $x_i$ with a new variable, we are forced to name it $x'_i$. This rule limits the number of choices while renaming variables. Intuitively, if we have to change the name of a variable, this rule allows not to care about the name of the new variable.

The merging operator is based on a particularization of the general model of the sources; namely, the only considered transformations are renamings. Therefore, in order to define a specific merging operator, we only need an ordering over the renamings. Assuming all transformations equally likely we get a merging operator we call Renaming Merging with Equal Likeliness Operator, or RMEL for short. For the sake of clarity, we only consider two knowledge bases, as is common in the merging/arbitration literature.

**Definition 2** The Renaming Merging with Equal Likeliness Operator $^{A,B}_{RMEL}$ associates any two knowledge bases $K_1$ and $K_2$ to another knowledge base $K_1 *^{A,B}_{RMEL} K_2$ defined as follows:

$$K_1 *^{A,B}_{RMEL} K_2 = \bigvee_{(Y,Z) \in PIT} K_1[X/Y] \land K_2[X/Z]$$

where $(Y,Z) \in PIT$ if and only if $x/Y$ and $x/Z$ are permitted inverse transformations that satisfy $K_1[X/Y] \land K_2[X/Z] \models A$ and $K_1[X/Y] \land K_2[X/Z] \land B \neq \bot$ and are of minimal combined size (that is, the size of $Y$ plus that of $Z$ is minimal.) $X$ is a subset of the variables in $K_1$, $K_2$, $A$, and $B$.

In this definition, we consider renamings to the variables in $K_1$ and in $K_2$ that satisfy the bounds $A$ and $B$. Using only permitted inverse transformations reduces the number of disjuncts in the definition. Indeed, for each variable in each of the two knowledge bases, we can either substitute it with another variable in $X$, or with a new variable not appearing anywhere else. The use of new variables is necessary as the two knowledge bases may use the same variables for different facts, so that either one or both of them have to be renamed. Using only permitted inverse transformations we avoid the problem of having to consider transformations that differ only for the name of the new variables, since the name of new variables is defined uniquely. On the other hand, permitted transformations are liberal enough to allow for making the alphabets of $K_1[X/Y]$ and $K_2[X/Z]$ disjoint (just substitute each $x_i$ with $x'_i$ in $K_1$, and make no changes to $K_2$.) This may be necessary when the two knowledge bases use exactly the same variables to represent completely different facts.

The rule of minimality excludes transformations that introduce renamings that are not justified. This particular ordering is the one that reduces the number of renamings the most, but other rules can be used instead: minimality w.r.t. set containment, minimal size of $Y$ and $Z$ considered separately, user supplied ranking of transformations, etc.
Let us consider some properties of this operator. We assume that both $A$ and $B$ are consistent, and do not contradict each other. This is important: for example, if $A = a$ and $B = \neg a$, there is no way to make $A$ implied and $B$ consistent at the same time. We therefore assume that $A$, $B$, and $A \land B$ are all consistent. A good property this merging operator should have is that of success, that is, it should produce a meaningful result. In our case, since merging is defined as a disjunction of possible hypotheses, this amounts to checking whether the resulting knowledge base is consistent. Unfortunately, this may not be the case, as the following example shows:

$$
K_1 = \neg x_1 \\
K_2 = \neg x_2 \\
A = x_1 \\
B = \top
$$

The problem here is that the two knowledge bases both tell that something is false, while we wanted a variable to be true after the merging, as $A = x_1$. The problem could be overcome by considering transformations involving negative literals, but this is quite unintuitive in this case: if we assume that the only problem is that we are giving the wrong name to a fact, we cannot infer that a fact is true from a statement saying that a fact is false.

The reason of why we cannot get success in this example is that the operator is based on assuming that the knowledge bases are obtained by renamings of correct ones, but the knowledge bases $K_1$ and $K_2$ of this example contradict this assumption. Indeed, $K_1 = \neg x_1$ cannot be the result of changing a name to a knowledge base that implies $x_1$. Obtaining a consistent result from the knowledge bases above would therefore be counterintuitive, as the merging operator would be saying that the assumption on the transformations (only name changes are possible) is consistent with the available data, while in fact it is not.

This example shows that we cannot expect the merging operator to work correctly even when the assumptions it is based on do not hold. On the contrary, the properties of this operator have to be checked with respect to two knowledge bases $K_1$ and $K_2$ that actually result from renaming some variables in two knowledge bases $S_1$ and $S_2$, both consistent with $B$ and both implying $A$.

If this is the case, the transformations can be inverted, and therefore the bounds $A$ and $B$ can be satisfied. It does not matter that the inverse transformation is not unique: to achieve derivability of $A$ and consistency with $B$, all that is needed is that there is at least a pair $\langle Y, Z \rangle$ such that $K_1[X/Y] \land K_2[X/Z] \models A$ and $K_1[X/Y] \land K_2[X/Z] \land B \not\models B$. All other disjuncts involved in the definition (if any) are consistent with $B$, and therefore their disjunction is consistent with $B$ as well. The upper bound
A is satisfied for the same reason: since each element of the disjunction implies \( A \), all of its models are models of \( A \).

A second property that we wish to obtain is that the original knowledge is correctly, even if not completely, recovered. This is to say that, if \( S_1 \land S_2 \not\models C \), then the result of merging \( K_1 \) with \( K_2 \) should not imply \( C \) either. However, this is not always the case, as the following example shows.

\[
\begin{align*}
K_1 &= x_1 \quad (S_1 = x_2) \\
K_2 &= \top \quad (S_2 = \top) \\
A &= \top \\
B &= \top
\end{align*}
\]

This example clearly shows a problem that has been already mentioned in the introduction: if we have no way to realize that a mistake has been made, then there is no way to recover from it. In this case, assuming that both knowledge bases are free of mistakes is not inconsistent with the bounds \( A \) and \( B \). Therefore, \( K_1 \land K_2 = x_1 \) is the result of merging simply because we have no reason to assume that a name change is necessary. This conclusion is incorrect, as \( x_1 \) is not a consequence of \( S_1 \land S_2 = x_2 \). This example also shows the obvious fact that we cannot enforce completeness either: \( x_2 \) is a consequence of the original knowledge base, but is not a consequence of the result of merging.

The fact that we cannot always recover the original knowledge bases, however, it is not unique to this operator. Even in the “mistake of value” assumption (that is, in “traditional” belief revision operators), the way the result of merging is related to the real world is conditioned to the validity of the minimal change principle. To make a concrete example, if our real world is \( a \land b \), and we have to revise \( K = \neg a \) to \( P = b \), we will always get the incorrect conclusion \( \neg a \). This is simply because:

1. there is no evidence we need to make any change;
2. we commit to the principle of making as few changes as possible.

In our scenario, we do not have any evidence that makes us thinking that a mistake has been made, and we therefore assume that the knowledge bases are correct. Making any other choice without any additional justifying information would be unmotivated.

We now consider the operator obtained by adding the ranking over transformations defined in the previous section. The definition of ranking specializes to the case of renamings only as follows.
Definition 3 The degree of a permitted inverse transformation \(X/Y, X/Z\) w.r.t. \(K_1, K_2, A,\) and \(B\) is given by the following formula:

\[
R(\langle X/Y, X/Z \rangle) = |X/Y| + |X/Z| + \log(\delta(K_1[X/Y], K_2[X/Z]))
\]

In words, this ranking combines the number of name changes with the similarity of the knowledge bases after the changes (the similarity measure \(\delta\) can be defined as shown in the previous section.) In this case, we have used a simple linear combination, but other combinations are possible (for example, we can first consider the number of changes, and then the similarity only in case of ties.)

Definition 4 The Renaming Merging Operator \(\ast_{RM}^{A,B}\) associates with any two knowledge bases \(K_1\) and \(K_2\) another knowledge base \(K_1 \ast_{RM}^{A,B} K_2\) defined as follows:

\[
K_1 \ast_{RM}^{A,B} K_2 = \bigvee_{\langle Y, Z \rangle \in MPIT} K_1[X/Y] \land K_2[X/Z]
\]

where \(\langle Y, Z \rangle \in MPIT\) if and only if \(X/Y\) and \(X/Z\) are minimal permitted inverse transformations w.r.t. \(K_1, K_2, A,\) and \(B\).

This operator differs from the previous one only in that the similarity between the two knowledge bases is taken into account, and it is in the same degree as the number of substitutions.

The same drawbacks of the operator with equal likeliness appear here. The difference is that, using an ordering, we select less transformations. Thus, we have less terms in the disjunction, and therefore the result of merging can be logically stronger.

6 Complexity Results

In this section, we consider the complexity of inference for the renaming merging with equal likeliness operator. Formally, given \(K_1, K_2, A, B,\) and \(Q,\) we want to check whether \(Q\) is implied by the merge of \(K_1\) with \(K_2,\) where \(A\) and \(B\) are the upper and lower bound, respectively.

Theorem 1 The problem of checking whether \(K_1 \ast_{RM}^{A,B} K_2 \models Q\) is \(\Pi_2^p\)-hard, and is in \(\Delta_3^p[\log n]\).

Proof. Membership: finding the size of the minimal renamings that make \(K_1\) and \(K_2\) consistent with \(B\) and not with \(\neg A\) can be done with a logarithmic number of queries to an oracle that checks the existence of a substitution that satisfies both
constraints (this oracle must be in the second level of the polynomial hierarchy due to the upper bound: it has to check the existence of a substitution such that the resulting knowledge bases imply \( A \)).

Using the minimal size of substitutions, all is needed is to check whether the knowledge bases imply \( Q \) using all substitutions of minimal size that satisfy both constraints.

Hardness is proved by reduction from \( \forall \exists \text{QBF} \). We prove that \( \forall X \exists Y.F \) is valid if and only if \( K_1 \stackrel{A,B}{\star} \text{RM EL} K_2 \models Q \), where \( Q = a \) and

\[
K_1 = a \land x_1 \land \cdots \land x_n \\
K_2 = \neg a \land \neg x_1 \land \cdots \land \neg x_n \\
A = a \lor (\neg F[Y/Y_1] \land \cdots \land \neg F[Y/Y_{n+1}]) \\
B = \top
\]

In order to satisfy the lower bound, the substitutions must make \( K_1 \) and \( K_2 \) consistent. This is only possible by changing the name of each variable in \( \{a, x_1, \ldots, x_n\} \) either in \( K_1 \) or in \( K_2 \). This way, putting together \( K_1 \) and \( K_2 \), we obtain a formula that contains exactly one literal between \( x_i \) and \( \neg x_i \) and one literal between \( a \) and \( \neg a \), that is, a formula having exactly one model over variables \( X \cup \{a\} \).

Changing the names this way is necessary to satisfy the lower bound \( B \). We can also prove that \( n + 1 \) name changes are sufficient to satisfy the upper bound \( A \). The substitutions that rename \( a \) in \( K_2 \) are such that \( a \) is implied by \( K_1 \) and \( K_2 \) after the renaming; therefore, \( A \) is implied as well. As a result, exactly \( n + 1 \) variable name changes are needed to make both constraints satisfied. In particular, each variable in \( X \cup \{a\} \) has to be renamed in either \( K_1 \) or \( K_2 \): if the knowledge base that results satisfies \( A \), then this substitution is considered.

Let us first consider the case in which all variables in \( K_1 \) and \( K_2 \) are replaced with new ones. In order to make \( K_1 \) and \( K_2 \) consistent, we have to rename any variable in \( \{a, x_1, \ldots, x_n\} \) either in \( K_1 \) or in \( K_2 \). After the change, \( K_1 \) and \( K_2 \) is a knowledge base with exactly one model. The substitution is considered only if \( A \) is implied by this model. By construction, \( A \) is implied only if either \( a \) is true, or the value of the variables \( \{x_1, \ldots, x_n\} \) satisfy \( \neg F \) for all possible values of \( Y \). As a result, a substitution that makes \( a \) false satisfies the upper bound if and only if the corresponding evaluation of the variables \( X \) falsifies \( F \) for any possible assignment of the variables \( Y \). As a result, \( Q = a \) is implied if and only if such assignments do not exist, that is, for all values of \( X \), there is a value of \( Y \) that satisfy \( F \).

The reduction is proved only if we restrict to substitution changing the name of a variable with a new name. Let us now consider the other substitutions. If a variable
\(x_i\) is renamed to \(x_j\), all is said above still holds (as the variable \(x_i\), in a way or another “disappears” from the knowledge base, and therefore it is set to the value it has in the other one.) The only substitutions that cause problems are those changing the value of \(x_i\) (or \(a\)) into a variable in \(Y\). Let for example consider the case in which the substitution \(x_1/y_1\) is applied to \(K_1\). Then, \(x_1\) is set to false in the resulting merging. At the same time, however, \(y_1\) is set to true as well. This is a problem if \(\neg F\) is not satisfied by the values of \(X\) alone, but it is if \(y_1\) is true: if \(a\) is set to false, we obtain that \(Q = a\) is not implied any more, while we know that the partial evaluation of \(X\) does not satisfy \(F\). This is why \(A\) contains \(n + 1\) copies of \(F\): however we change the names of variables in \(X\) to variables in \(Y\), the upper bound \(A\) always contain a copy of \(F\) whose variables in \(Y\) are not mentioned in \(K_1\) and \(K_2\) after renaming. This ensures that \(A\) can only be derived if the partial evaluation of \(X\) falsifies \(F\).

7 Conclusions

The contribution of this paper is in the approach taken, rather than the proposed specific belief revision method. Starting from a very general model of the integration domain, we have shown that the existing semantics for knowledge integration correspond to a specific assumption. In this model, the sources get the knowledge bases they have by a process of acquisition that is prone to errors; previous integration semantics correspond to the assumption that mistakes are of a specific kind (which we called “mistakes of value”). Other mistakes are considered in this paper, leading to completely new integration semantics. The work reported here is still preliminary, as the properties of merging in the new models have not yet been fully investigated (comments and suggestions are welcome.)

New issues come from further generalizing this model. For example, we have only considered the case in which all knowledge bases are propositional. For first order logic, new interesting cases arise: a form of generalization is to transform \(P(a)\) into \(\forall x.P(x)\); the opposite of particularization is also interesting; subject misunderstanding is in this context different (it is the change of a constant, not the change of a literal), etc.

Other issues arise from comparing the approach taken here with “classical” belief revision. In the usual formalization of belief revision, the knowledge expressed by each source is actually a set of preferences, rather than simply a knowledge base. This is because each agent involved in the merging process not only has some beliefs, but also acknowledges the possibility that they may be indeed false. As a result, it also has a measure of preference (degree of belief) over all facts it considers to be false, generating an ordering over the possible worlds.

Modeling merging with the assumption of mistakes, such ordering cannot be
used. As it is clear from the heuristics presented, it is impossible to express merging as a merging of ranking, as the most likely transformations of each source depend on the other knowledge bases. It is also true that we could consider a more sophisticated model accounting both rankings (expressing the measure of likeliness of worlds according to each agent) and mistakes (that each agent did while getting its knowledge).

Finally, let us briefly discuss the computation issues. The result of Section 5 shows that the proposed semantics is computationally harder than the propositional calculus, as expected. Nevertheless, it is not much harder than most of the revision operators, that are $\Pi_2^p$ complete [4]. A simplified definition has been used, but it seems unlikely it did reduce complexity much.

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