1. Introduction

In a hot strip mill, it is crucial to acquire a capability for rigorously predicting the thermo-mechanical behavior of the strip and roll, for successful on-line process set-up and control. However, the thermo-mechanical behavior occurring in the roll–strip system is complex, due to the coupled nature of the strip temperature and roll temperature, and also due to the coupled nature of plastic deformation and heat transfer in the strip. In addition, there are so many input process parameters to consider, making it difficult to develop such a capability.

It is demonstrated during the last two decades that the thermo-mechanical behavior can be made far more accurately predicted on the basis of the finite element (FE) process models than on the basis of the elementary models which inherently involve many simplifying assumptions. However, a precision process model, such as a FE process model, tends to require a large central processing unit (CPU) time, rendering itself inadequate for on-line calculation.

In this paper, we present general, dimensionless expressions for the parameters describing the thermo-mechanical behavior of the roll–strip system, on the basis of the boundary value problem associated with hot strip rolling. It is shown that, by conducting process simulation with an integrated finite element process model, the dimensionless expressions may be transformed into various on-line models which may be applied to precision process set-up and control. The validity of the proposed approach is examined through comparison with predictions from finite element process simulation.

KEY WORDS: finite element method; thermo-mechanical behavior; effective strain; non-dimensional analysis; hot strip rolling.

2. A Hypothetical Mode of Rolling

Suppose that, in actual rolling, the thickness of a strip (inlet temperature \( T_1 \) and flow stress \( \sigma(\tilde{E}, \tilde{T}) \)) is reduced from \( H_1 \) to \( H_2 \), with its exit speed and exit temperature \( V_2 \) and \( T_2 \), respectively. Now, for the same rolling geometry and the same strip, consider an hypothetical mode of rolling in which each segment of the strip is uniaxially compressed from \( H_1 \) to \( H_2 \) while passing through the roll bite, and no friction is present at roll/strip interface, as shown in Fig. 1. Further, the strip temperatures in the roll bite are uniform along the thickness direction and vary linearly along the rolling direction such that the inlet and exit temperature coincide with \( T_1 \) and \( T_2 \), respectively. Also, assume that the exit speed is equal to \( V_2 \).

\[
P' = V_2 H_2 E_1' \quad \text{..................................(1)}
\]
\[
E_1' = \frac{2}{\sqrt{3}} \int_{H_1}^{H_2} \frac{\tilde{\sigma} (\tilde{E}, \tilde{T})}{h} dh \quad \text{............................(2)}
\]
\[
F' = \frac{2}{\sqrt{3}} \int \sigma (\tilde{E}, \tilde{T}) \cos \phi d\phi \quad \text{..........................(3)}
\]
\[
E_2' = \int_0^\phi \tilde{\sigma} (\tilde{E}, \tilde{T}) \cos \phi d\phi \quad \text{........................(4)}
\]
Neglecting the front and back tension, it is clear from Eqs. (17) and (24) that distribution of the roll force and roll power, respectively, in the context of the assumed distribution of strip temperatures in the bite zone.

3. Dimensionless Expressions for the Process Parameters

Let us consider a 2-D boundary value problem for the analysis of the rigid-plastic deformation of the strip, with the process geometry given in Fig. 2.

- equilibrium equation:
  \[ \sigma_{ij} = 0 \quad \text{in } \Omega \] ..............................(8)
- constitutive equation:
  \[ \sigma_i = \frac{\zeta}{\omega} V_{i,k} \delta_{ij} + \frac{2\sigma}{3\bar{\varepsilon}} \frac{\dot{\varepsilon}_i}{\omega} \] ..............................(9)
where \( \zeta \) is a very large constant.
- boundary conditions:
  \[ \sigma_{ij} n_j = -\frac{\zeta}{\omega} (V_i - \bar{V}_i) \quad \text{on } \Gamma_v \] ..............................(10)
  \[ \sigma_{ij} n_j = \mu \frac{\zeta}{\omega} (V_i - \bar{V}_i) \quad \text{on } \Gamma_v \] ..............................(11)
where \( \bar{V}_i \) is the normal component of the roll velocity vector \( (V_i = 0) \) and

\[ \bar{r} = \frac{(V - \bar{V})}{\omega} \] ..............................(12)

\[ \sigma_{11} = \sigma_1, \quad \sigma_{22} = \sigma_2 = 0 \quad \text{on } \Gamma_{e_1} \] ..............................(13)
\[ \sigma_{11} = \sigma_1, \quad \sigma_{22} = \sigma_2 = 0 \quad \text{on } \Gamma_{e_2} \] ..............................(14)
\[ V_0 = 0, \quad \sigma_{11} = \sigma_{22} = 0 \quad \text{on } \Gamma_{e_0} \] ..............................(15)

Neglecting the front and back tension, it is clear from Eqs. (8)–(15) that distribution of \( \dot{\varepsilon} / \omega \) is dependent only on \( \xi, \mu \), shape of the strip domain, and distribution of \( \bar{\varepsilon} \) or \( \bar{\sigma} \).

Fig. 2. Strip and roll, a definition sketch.

where \( \Delta T = T_2 - T_1 \). Note that \( V_2 \) and \( T_2 \) may be precisely predicted from the FE process model described previously. It is to be noted that \( F^* \) and \( P^* \) represent the theoretical minimum (or very close to the theoretical minimum) of roll force and roll power, respectively, in the context of the assumed distribution of strip temperatures in the bite zone.

The flow stress of the strip, which is rate dependent as well as temperature dependent, may be expressed by a dimensionless form

\[ \frac{\bar{V}_j}{\bar{R} \bar{o}} = \frac{\hat{\bar{\sigma}}}{\hat{\bar{\sigma}}_j} \left( \begin{array}{c} \hat{V}_j \\ \hat{T}_j \\ \hat{\omega} \\ \hat{T}_1 \\ \hat{T}_2 \\ \hat{f}_s \\
\end{array} \right) \] ..............................(16)

where a variable with \( \sim \) denotes its distribution in the strip domain, and \( \hat{\bar{\sigma}} \) denotes an operator which relates the quantities in the parenthesis with the quantity on the left-hand side of the equation.

According to Buckingham pi theorem, Eq. (16) may be replaced by a dimensionless form with five independent dimensionless parameters, since seven variables in Eq. (16) involve two independent units (force and length). Among many possible dimensionless forms, we consider

\[ \frac{\bar{V}_j}{\bar{R} \bar{o}} = \frac{\hat{\bar{\sigma}}}{\hat{\bar{\sigma}}_j} \left( \begin{array}{c} \hat{V}_j \\ \hat{T}_j \\ \hat{\omega} \\ \hat{T}_1 \\ \hat{T}_2 \\ \hat{f}_s \\
\end{array} \right) \] ..............................(17)

The flow stress of the strip, which is rate dependent as well as temperature dependent, may be expressed by a dimensionless form

\[ \frac{\bar{\sigma}}{K} = \phi \left( \bar{\xi}, \bar{\varepsilon}, \bar{T}, C_2 \right) \] ..............................(18)

where \( K, C_1, C_2 \), are constants that possess the same unit as \( \bar{\sigma}, \bar{T}, \bar{\varepsilon} \), respectively. Note that \( C_1 \) and \( C_2 \) are introduced since \( \bar{\sigma} \) is governed by non-dimensional \( \bar{T} \) and \( \bar{\varepsilon} \) and therefore, their values may be chosen arbitrarily. In the present investigation, \( C_1 = 1°C \) and \( C_2 = 1 \text{ rad/s} \) are assumed.

Let us define the average values of the flow stress for the hypothetical mode of rolling, as follows:

\[ \bar{\sigma}_{01} = \frac{2 \ln H_1}{\sqrt{3}} \] ..............................(19)
\[ \bar{\sigma}_{02} = \frac{E_2^*}{\sin \phi_1} \] ..............................(20)

Then, we may construct, from Eqs. (18), (19), and (20), a new dimensionless form for the expression of the flow stress

\[ \frac{\bar{\sigma}}{\bar{\sigma}_{0j}} = \phi \left( \bar{\xi}, \bar{\varepsilon}, \bar{T}, C_2 \right) \] ..............................(21)

where \( f_s \) denotes the forward slip. In the present investigation, Eq. (21) is used for the flow stress expression instead of Eq. (18), since many parameters describing the thermo-mechanical behavior of the strip are found to be linearly dependent on \( \bar{\sigma}_{0j} \), as will be illustrated later.

Replacing \( \bar{\xi}, \bar{\varepsilon}, \bar{T} \) and \( \bar{R} \bar{o} \) in Eq. (21) by \( \bar{\varepsilon}, \bar{\sigma}, \bar{T} \), and \( \bar{R} \bar{o} \), and noting that

\[ \bar{\varepsilon} = \bar{\eta} (\bar{V}_i, R, H_1, H_2) \] ..............................(22)
\[ \frac{\bar{\varepsilon}}{\bar{\sigma}} = \bar{\eta} (\bar{V}_i, \bar{T}, \bar{H}_1, \bar{H}_2, \bar{\omega}) \] ..............................(23)

a dimensionless expression is obtained for the flow stress distributions in the strip, as follows:

\[ \frac{\hat{\bar{\sigma}}}{\hat{\bar{\sigma}}_j} = \bar{\eta} \left( \bar{\xi}, \bar{\varepsilon}, \bar{T}, \bar{\omega}, C_2 \right) \] ..............................(24)

Selecting \( \xi = C \bar{\sigma}_{0j} \), where \( C \) is a prescribed constant, it follows from Eqs. (17) and (24) that


\[ \tilde{N}_s = \tilde{n} \left( \frac{\tilde{N}_s}{C_2}, \frac{\tilde{N}_s}{C_1}, \mu, s, r, \beta_1, \beta_2, \tilde{\beta}_1 \right) \] ..........................(26)

where \( \tilde{N}_s \) represents \( \tilde{N}, \tilde{E}, \tilde{E}/\tilde{\omega}, \tilde{\sigma}, \tilde{\sigma}/\tilde{\omega} \), as well as \( V_R/\tilde{\omega} \).

Now, let us consider the boundary value problem for the analysis of heat transfer in the strip, with the process geometry given in Fig. 2.

- **energy balance equation:**
  \[ \rho c_p V T_n = (k T_n)_{,1} \] ..........................(27)

- **boundary conditions:**
  \[ k \frac{\partial T}{\partial n} = q_s \] on \( \Gamma_s \) .................(28)
  \[ T = T_i \] on \( \Gamma_{in} \) .....................(29)
  \[ k \frac{\partial T}{\partial n} = 0 \] on \( \Gamma_{ex} \) ....................(30)
  \[ k \frac{\partial T}{\partial n} = 0 \] on \( \Gamma_{sym} \) .................(31)

It is clear from the boundary value problem that the strip temperature distributions are given by

\[ \tilde{T} = \tilde{n}(\tilde{V}_R, \tilde{E}, \tilde{q}_s, k, \rho c_p, T_s, R, H_s, H_2) \] ..........................(32)

Replacing \( \tilde{V}_R, \tilde{E}, \tilde{q}_s, k, \rho c_p, T_s, R, H_s, H_2 \) by Eq. (26), and removing all the dependent variables, we obtain

\[ \tilde{T} = \tilde{n} \left( \frac{\tilde{E}}{C_2}, \frac{T}{C_1}, \mu, s, r, \beta_1, \beta_2, \tilde{\beta}_1 \right) \] ..........................(33)

Equation (33), which involves thirteen variables, may be reduced to a non-dimensional form with nine independent dimensionless variables, since four independent units (temperature, force, length, and time) are identified in the equation. Among many possible dimensionless forms, we consider

\[ \frac{\tilde{T}}{T_1} = \tilde{n} \left( \frac{\rho c_p V R H_1}{k}, \frac{\beta_1}{k T_1}, \frac{2 l q_s}{k T_1}, \frac{P^*}{k T_1} \right) \] ..........................(34)

where \( V_R = R \tilde{\omega} \), and

\[ P^* = E^* V R H_2 = P'(1 + f_2) \] ..........................(35)

Note that \( E^* \) is selected to derive Eq. (34) from Eq. (33). However, it may be shown that selecting \( E^* \) may also lead to Eq. (34). Replacing \( \tilde{T}/C_1 \) in Eq. (26) by \( \tilde{T}/C_1 \) (noting that \( \tilde{T}/C_1 = \tilde{T}/C_1 \)), we obtain

\[ \tilde{N}_s = \tilde{n} \left( \frac{\tilde{E}}{C_2}, \frac{T}{C_1}, \mu, s, r, \beta_1, \beta_2, \tilde{\beta}_1 \right) \] ..........................(39)

where \( \tilde{N}_s \) represents all the basic non-dimensional fields, which are, \( \tilde{E}, \tilde{E}/\tilde{\omega}, \tilde{\sigma}, \tilde{\sigma}/\tilde{\omega}, \tilde{\omega}/\tilde{\omega} \), and \( \tilde{T}/C_1 \).

It may be deduced from Eq. (39) that any, reasonably selected, dimensionless parameters that describe the thermo-mechanical behavior of the strip should, in general, be influenced by eight independent dimensionless parameters appearing in the right hand side of Eq. (39). Note that all of them represent design variables (variables to be prescribed by an engineer), except \( \tilde{\beta}_1 \), since \( \tilde{\beta}_1 \) is unknown.

For the work roll, the boundary value problem associated with the steady-state thermal behavior of the roll may be given, with the definition sketch shown in Fig. 2, by

- **energy balance equation:**
  \[ \rho c_p V R T_n = (k T_n)_{,1} \] ..........................(40)

- **boundary conditions:**
  \[ k \frac{\partial T}{\partial n} = q_s \] on \( \Gamma_s \) .................(41)
  \[ k \frac{\partial T}{\partial n} = -h_{rv} (T_r - T_w) \] on \( \Gamma_{rv} \) .................(42)

Assuming a uniform roll cooling system (water is uniformly sprayed on the entire roll surface, except the roll–strip interface), it may be deduced from the boundary value problem that

\[ \tilde{T}_r = \tilde{n}(\tilde{q}_r, l_p, \omega, R, h_{rv}, T_{rv}, \rho c_p, k_r) \] ..........................(43)

Equation (43), which involves nine variables, may be reduced to a dimensionless form with five independent dimensionless variables, since four independent units (temperature, force, length, and time) are identified in the equation. Among many possible dimensionless forms, we consider

\[ \frac{\tilde{T}_r}{T_w} = \tilde{n}(\xi_1, \xi_2, \xi_3, \xi_4) \] ..........................(44)

where

\[ \xi_1 = \frac{l_p}{2 \pi R} \] ..........................(45)
\[ \xi_2 = \frac{2 l q_r}{k_r} \] ..........................(46)
\[ \xi_3 = \frac{h_{rv}}{\rho c_p V R} \] ..........................(47)
\[ \xi_4 = \frac{\rho c_p V R l_p}{k_r} \] ..........................(48)

Note that \( \xi_1, \xi_3, \) and \( \xi_4 \) are design variables, but \( \xi_2 \) is not, since \( \tilde{q}_r \) is unknown.

### 4. An Integrated FE Process Model

An integrated finite element (FE) process model applied for the present investigation consists of three basic FE models: a model for the analysis of steady-state thermoviscoplastic deformation of the strip, a model for the analysis of steady-state heat transfer in the strip, and a model for the analysis of steady-state heat transfer in the work roll. As shown in Fig. 3, interaction between the thermal behavior of the work roll and that of strip caused by roll–strip contact, as well as interaction between the thermal behavior of the strip and the mechanical behavior of the strip, are taken into account by iterative solution schemes. Details regarding the process model and its solution accuracy are given in Ref. 11.

It is to be noted that heat transfer in the direction of the roll axis as well as in the direction of strip width are neglected. Also, plane-strain deformation of the strip is assumed. Consequently, all the basic FE models applied for the present investigation are 2-D models.

As illustrated in Fig. 4, a sufficiently large number of elements, along with the mesh refinement near the contact zones, are used to construct the roll and strip meshes in order to remove the mesh dependency of the solution accu-
An integrated FE process model for the analysis of the thermo-mechanical behavior of the roll-strip system. The element type used is a linear quadrilateral element. 10,965 elements are used for the roll, and 1,975 elements are used for the strip. The predicted temperature distributions in the strip as well as in the roll are illustrated in Fig. 5. Clearly seen is the effect of heat transfer from the strip to the roll at the roll–strip interface, as well as the effect of heat generation in the strip due to plastic deformation. The process simulation required approximately 20 min of CPU time of a modern engineering workstation (such as COMPAQ XP 1000 (Hewlett Packard, Palo Alto, CA)), which clearly indicates that a FE process model cannot be directly employed as an on-line model, at least not in the near future, considering that calculations should be carried out in a tiny fraction of a second for on-line process control.

5. On-line Models for the Prediction of $F$, $P$, $P_r$, $P_s$, $f_s$, and $\bar{e}_c$

Some dimensionless parameters describing the thermo-mechanical behavior of the strip are found to be dependent mainly on $\mu$, $s$, and $r$, while the effect of the rest of the dimensionless process parameters is negligible. Examples are, $F/F^*$, $P/P^*$, $P_r/P^*$, and $P_s/P^*$, where $F$ is the roll force per unit strip width and $P$, $P_r$, and $P_s$ are roll power, frictional energy, and deformation energy per unit strip width defined by

$$P = P_e + 2P_f$$

$$P_f = \int_{\Gamma_p} [\mu \sigma_{\epsilon} - \bar{V} - \vec{V}^r] d\Gamma$$

$$P_d = \int_{\Omega} \overline{\sigma} \bar{e}_d d\Omega$$

Other examples are $f_s$, forward slip, and $\bar{e}_c$, effective strain of the rolled strip at the plane of symmetry. That the effect of the design variables other than $\mu$, $s$, and $r$ are negligible is found from process simulation with the integrated FE process model, as illustrated in Refs. 13, 14.

A three-variable polynomial expression for each dimensionless parameter, which is obtained by performing least square regression of the data predicted from the integrated FE process model, and valid for the reference sets of process parameters shown in Table 1, is given in Tables 2 and 3. Note that the polynomials are incomplete (some terms are missing), because of iterative application of least square regression in which insignificant terms are eliminated and the coefficients are recalculated.

The ranges of $s$ and $r$ for which the present on-line models are valid, $1 < s < 7$ and $0.02 < s < 0.6$, are taken from the hot strip mills in POSCO, indicating that actual rolling is rarely conducted with either $s$ or $r$ out of the aforementioned range. Also, note that all the non-dimensional parameters describing the thermo-mechanical behavior of the roll–strip system, including $F/F^*$ and $P/P^*$, are dependent on $s$ and $r$, but not on the actual values of $R$, $H_1$, and $H_2$. Therefore, any combination of $R$, $H_1$, and $H_2$ may be used to generate the values of $s$ and $r$. They may be produced by fixing $R$ and varying $H_1$ and $H_2$, as is done in the present investigation, or they may be produced by selecting randomly $R$, $H_1$, and $H_2$. While the actual values of $R$, $H_1$, and $H_2$ do
not affect the expressions given in Tables 2 and 3 and other tables, they do affect $F'$ and $P'$, and consequently, affect $F$, $P$, $F'$, and $P'$. The prediction accuracy of each on-line model thus obtained may be examined through comparison with the predictions from the integrated FE process model. For all the reference and perturbed sets of process conditions listed in Table 1, a good agreement is noted for roll force, roll power, $\bar{e}_c$, forward slip, frictional energy, and deformation energy, with differences being smaller than 10% in most cases, as shown in Figs. 6–11. Regarding the prediction of roll force, Fig. 12 shows that predictions from the present on-line model are in excellent agreement with predictions from the integrated FE process model for various $R$, $H_1$, and $H_2$ with less than 3% deviation, while Orowan's model15) underestimates, and Sims model16) overestimates the roll force, with 20% deviation in maximum for the combinations of the process variables selected for the investigation.

Note that the present on-line models are valid only in the context of two dimensional deformation and heat transfer, with a rigid roll. In order to take into account the effect of roll elastic deformation, or roll flattening, Eq. (39) may be extended to cover the effect of roll elastic deformation, which would lead to the inclusion of two more dimensionless parameters (one related to Young's modulus of the roll, and Poisson's ratio of the roll), in addition to the eight parameters. Then, least square regression may be performed with the data predicted from a new integrated FE process model, in which the coupled aspect of roll elastic deformation and strip deformation is reflected. An alternative, simplified approach would be to develop an on-line model for the prediction of $s$ as a function of a group of independent dimensionless parameters extracted from $p$, $R$, $H_1$, $H_2$, $E$, and $v$, where $p$ denotes roll pressure along the contact surface which is assumed to be constant, from regression of data predicted from FE simulation of the elastic deformation of the roll. Hitchcock's formula17) may also be considered as an approximate model for the prediction of $s$. Then the model for the prediction of $s$ and the present on-line model for the prediction of $F$ may be solved together by an iterative procedure. Prediction of other quantities, such as $P$, $f_s$, etc., may then be performed, with the value of $s$ thus found.

6. On-line Models for the Prediction of $T_{mtr}$, $T_{mtr}$, $P_{t}r$, $q_s$, and $\Delta T$

When an on-line model is derived for a parameter de-
scribing the thermo-mechanical behavior of the strip from Eq. (39), a prerequisite for its application, in general, is to provide the value of $\tilde{q}_s$. The same is true for an on-line model derived from Eq. (44), where it is necessary to provide the value of $\tilde{q}_r$. It is then evident that an essential requirement is to be capable of predicting $\tilde{q}_s$ and $\tilde{q}_r$. In the following, we approximate $\tilde{q}_s$ and $\tilde{q}_r$ by their average values, $q_s$ and $q_r$, and attempt to develop a strategy for their on-line prediction, considering their mutual dependence. The total heat loss from the strip to the roll is defined by

$$P_s = \int_{T_s} h_{\text{lub}} (T - T_s) \, d\Gamma$$

Assuming that $h_{\text{lub}}$ is uniform along the roll–strip interface, we have

$$\frac{P}{l_d} = h_{\text{lub}}(T_{ms} - T_{mr}) \quad (53)$$

where $T_{ms}$ and $T_{mr}$ are the average temperatures defined by

$$T_{ms} = \frac{\int T \, d\Gamma}{l_d} \quad (54)$$

$$T_{mr} = \frac{\int T \, d\Gamma}{l_d} \quad (55)$$

Assuming that one half of the frictional heat generated at the roll–strip interface goes into the strip, and the other half into the roll, the average incoming heat flux to the strip and that to the roll are given by

$$q_s = \frac{P_f - 2P_r}{2l_d} \quad (56)$$

$$q_r = \frac{P_f + 2P_r}{2l_d} \quad (57)$$

Replacing $\tilde{q}_s$ and $\tilde{q}_r$ by $q_s$ and $q_r$, respectively, we have

$$\tilde{\beta}_3 = \beta_3 = \frac{P_f - 2P_r}{P_f + 2P_r} \quad (58)$$

$$\frac{\bar{q}_s}{\bar{q}_r} = \frac{P_f - 2P_r}{P_f + 2P_r} \quad (59)$$

It then follows from Eqs. (39) and (44) that

$$\frac{T_{ms}}{T_s} = g_1 \left( \frac{\omega}{C_2} \frac{T_1}{C_1}, \mu, s, r, \beta_1, \beta_2, \beta_3 \right) \quad (60)$$

$$\frac{T_{mr}}{T_r} = g_4 (\bar{q}_s, \bar{q}_r, \bar{q}_s, \bar{q}_r) \quad (61)$$

Multi-variable polynomial expressions for $T_{ms}/T_s$, which are obtained by performing least square regression of the data predicted from FE process simulation of deformation and heat transfer in the strip, and valid for the reference sets of process parameters shown in Table 4, are given in Tables 6. That the expressions do not include $T_1/C_1$ and $\omega/C_2$ is due to their negligible effect on $T_{ms}/T_s$. Also, from FE process simulation of heat transfer in the roll, obtained are multi-variable polynomial expressions for $T_{mr}/T_r$, which are valid for the reference sets of process parameters shown in Table
The result is given in Table 7.

Substituting Eqs. (60)–(61) into Eq. (53), and noting that $P_{f}$ can be calculated from the mathematical expression shown in Table 2, we have

$$\frac{P_{r}}{H_{1}} = \frac{1}{H_{1}} \left( \frac{P_{r}}{H_{1}} \right)_{\text{gr}} \left( \frac{T_{w}}{H_{1}} \right)_{\text{ld}}$$

which may be solved for $P_{r}$ by an iterative solution technique such as the Newton–Raphson method.

As shown in Fig. 13, the values of $P_{r}$ predicted by the present method for various cases are in good agreement with the predictions from the integrated FE process model, with the differences being smaller then 10% in most cases, indicating that the present method of predicting $P_{r}$ may be successfully applied to the precise evaluation of $q_{s}$ and $q_{r}$, from Eqs. (56) and (57).

In addition, considering the energy balance in the bite zone, it may be shown that the temperature increase in the bite zone is given by

$$\Delta T = \frac{P_{r} + P_{p} - 2P_{f}}{\rho c_{p}H_{2}}$$

The values of $\Delta T$ predicted by Eq. (64) are in good agreement with the predictions from the integrated FE process model, as shown in Fig. 14.

### 7. Application to On-line Prediction of Effective Strain Distribution at the Roll Exit

Evolution of effective strain during rolling affects the recrystallization behavior, and therefore, on-line process control to achieve desired microstructures may require online calculation of effective strain. However, due to surface chilling as well as severe shear deformation at the strip surface in the bite zone, effective strain may become severely non-uniform along the thickness direction. Therefore, it is desired to predict variation of effective strain along the thickness direction, in addition to $\bar{e}_{\text{avg}}$, average effective strain of a rolled strip.

Figure 15 illustrates a typical effective strain distribution in the strip. The curve may be characterized by $\bar{e}_{s}$, the maximum effective strain occurring at $y' = \delta$, $\bar{e}$, the effective strain at the plane of symmetry, or at $y' = 0$, and the effective strain distributions in $0 < y' < \delta$ and in $\delta < y' < 1$. By examining the actual effective strain distributions predicted from FE process simulation, it may be deduced that the distribution in $\delta < y' < 1$ may be approximated by a linear function, while the distribution in $0 < y' < \delta$ may be approximated by

$$E(y') = E(y')_{\delta} \left( \frac{E(y')_{\delta} - E(y')_{0}}{E(y')_{\delta} - E(y')_{\delta}} \right) \tan^{-1}(a(d - y'))$$

In summary, $\delta$, $\bar{E}$, $\bar{E}$, as well as $E_{\text{max}}$, are the dimensionless parameters to be predicted for the prediction of the effective strain distribution.
It is to be noted that when the surface chilling effect is not severe, the maximum effective strain may occur at the surface. In such a case, $\bar{e}_s / H_{11005} \approx 0$ and $d / H_{11005} \approx 1$. We may classify such a case as type 1, while classifying the former case as type 2. However, due to the discrete nature of the finite element method, $d$ value predicted from FE process simulation may not vary continuously but reveal sudden jumps with a slight change in the process condition, unless an excessively fine mesh is used. To remedy such a problem, an interpolation scheme may be employed to predict $d$, as follows:

Suppose that $\bar{e}_{\text{max}}$ occurs at element $m$, and its position is given by $d / H_{11005} = d_m$. Define $\bar{e}_u$ and $\bar{e}_b$, the strain occurring at one element layer above $m$ and below $m$, respectively. Then $d$ may be calculated from

$$d = \frac{\bar{e}_u - \bar{e}_b}{\bar{e}_{\text{max}} - \bar{e}_u} \quad \text{(65)}$$

where $l$ denotes the thickness of an element layer. Also, if $\bar{e}_{\text{max}}$ occurs at an element layer at the strip surface, we may assume

$$d = \frac{\bar{e}_u - \bar{e}_b}{\bar{e}_{\text{max}} - \bar{e}_u} \quad \text{(66)}$$

where $\bar{e}_f$ and $\bar{e}_f$ denote the strain occurring at the first element layer below the element layer at the strip surface and at the second element layer, respectively.

It is found from FE process simulation of deformation and heat transfer in the strip, which are performed with the reference sets of process parameters shown in Table 4, that the effect of some of the design variables is negligible. The result may be summarized as

$$a = f(T, C_i, \mu, s, r, \beta) \quad \text{(68)}$$

$$\delta = f(T, C_i, \mu, s, r, \beta) \quad \text{(69)}$$

$$\bar{e}_u = f(u, s, r, \beta) \quad \text{(for type 2)} \quad \text{(70)}$$

$$\bar{e}_{\text{avg}} = f(u, s, r, \beta) \quad \text{(71)}$$

Multi-variable polynomial forms of $a$, $\delta$, $\bar{e}_u$, $\bar{e}_{\text{avg}}$, and obtained from FE process simulation and valid for the reference sets of process parameters shown in Table 4, are shown in Tables 8–11.

It may be shown, from Eq. (64) and noting that

$$\bar{e}_{\text{avg}} = \int_0^1 \bar{e}(y')dy' \quad \text{(72)}$$

$\bar{e}_{\text{max}}$ can be calculated from

$$\bar{e}_{\text{max}} = f_{\text{avg}} - \frac{1 - \delta}{1 - B + \frac{1 - \delta}{2}} \quad \text{(73)}$$

where

$$B = \frac{\ln[1 + (\alpha \delta)^2]}{\tan^{-1}(\alpha \delta)} \quad \text{(74)}$$

The effective strain distribution may then be predicted, as follows:

1) Calculate $\delta$ and $a$ from the mathematical expressions given in Tables 8–9. Also, calculate $\bar{e}_{\text{avg}}$ and $\bar{e}_u$ from the mathematical expressions given in Table 11, and Table 3, respectively.

2) If $\delta < 1$, calculate $\bar{e}_s$ from the mathematical expression given in Table 10.
3) If $\delta < 1$ calculate $\bar{e}_{\text{avg}}$ from Eqs. (73) and (74). If $\delta > 1$, assume $\delta = 1$ and calculate $\bar{e}_{\text{avg}} = (\bar{e}_{\text{max}})$ from Eqs. (73) and (74).

4) Calculate the effective strain distribution, from Eq. (64) for the range $0 < y < \delta$, and from the assumption of a linear pattern for the range $\delta < y < 1$.

As shown in Fig. 16, the average effective strain values predicted from the mathematical expression given in Table 11 are in excellent agreement with predictions from the integrated FE process model. Also, as shown in Fig. 17, the effective strain distributions predicted from the proposed model are in good agreement with the predictions from the integrated FE process model, demonstrating the capability of the present approach for on-line prediction of a field as well as a parameter characterizing the thermo-mechanical behavior occurring in hot strip rolling.

8. Concluding Remarks

Derived from the present investigation are general dimensionless forms that relate the parameters describing the thermo-mechanical behavior of the strip and roll with the design variables. It is shown that there are eight independent dimensionless design variables influencing a 2-D thermo-mechanical behavior of the strip (two more if front and back tension are included), and four independent dimensionless design variables influencing a 2-D, steady-state thermal behavior of the work roll. Then, it is demonstrated
that, in conjunction with an integrated FE process model, the dimensionless forms may be transformed into precision on-line models for the prediction of the parameters describing the thermo-mechanical behavior. It is to be noted that precision process control would eventually require on-line prediction of a field, such as temperature field, in a three-dimensional domain. The present investigation demonstrates that such a task is also possible, in conjunction with a sound 3-D FE process modeling capability.

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Nomenclature

\[ T_i \] = Strip inlet temperature  
\[ T_e \] = Strip exit temperature  
\[ T_w \] = Water temperature  
\[ T_r \] = Roll temperature  
\[ T_{\text{ave}} \] = Average strip temperature at the roll/strip interface  
\[ T_{\text{ave}} \] = Average roll temperature at the roll/strip interface  
\[ H_1 \] = Strip inlet thickness  
\[ H_z \] = Strip exit thickness  
\[ R \] = Roll radius  
\[ s \] = Shape factor, \( s = [2/(2-r)]^{1/2} R / H_1 \)  
\[ r \] = Reduction ratio, \( r = (H_1 - H_z) / H_1 \)  
\[ \mu \] = Coefficient of friction  
\[ l_c \] = Contact length at the roll/strip interface  
\[ V_{f_r}, \omega \] = Roll tangential velocity, roll angular velocity  
\[ V_r \] = Strip inlet velocity  
\[ V_{f_r} \] = Strip exit velocity  
\[ V_s \] = Strip velocity vector  
\[ V_{f_r}, V_s \] = Roll velocity vector  
\[ V_n \] = Normal component of the velocity vector  
\[ \sigma_y \] = Stress tensor  
\[ \dot{e}_{ij} \] = Strain rate tensor  
\[ \dot{\epsilon}_{ij} \] = Deviatoric strain rate tensor  
\[ q_r \] = Heat flux to the strip  
\[ k_r \] = Thermal conductivity of the strip  
\[ \rho_c \] = Density multiplied by heat capacity of the strip  
\[ k \] = Thermal conductivity of the roll  
\[ \rho_c \omega \] = Density multiplied by heat capacity of the roll  
\[ h_{rs} \] = Heat transfer coefficient due to water cooling, at the roll surface  
\[ h_{slip} \] = Heat transfer coefficient at the roll/strip interface  
\[ \sigma_h \] = Back tension, in a stress unit  
\[ \sigma_f \] = Front tension, in a stress unit  
\[ \sigma_n \] = Normal stress  
\[ \sigma_t \] = Tangential stress  
\[ F_r \] = Roll force in a hypothetical mode of rolling  
\[ P^* \] = Deformation energy in a hypothetical mode of rolling  
\[ P \] = Roll power per unit strip width (roll power is \( P/2 \) for each roll)  
\[ f_s \] = Forward slip, \( f_s = -R \omega / R \omega \)  
\[ \bar{e} \] = Effective strain  
\[ \bar{e} \] = Effective strain rate  
\[ \bar{\sigma} \] = Flow stress  
\[ \varepsilon \] = Stefan–Boltzmann constant multiplied by emissivity  
\[ \xi \] = Penalty constant  
\[ \Omega \] = Plastic deformation zone (bite zone) in the strip  
\[ \Gamma_{\text{c}} \] = Roll/strip interface  
\[ n_i \] = Components of a unit normal vector  
\[ t_i \] = Components of a unit tangent vector  
\[ \phi_i \] = Contact angle  
\[ E \] = Young’s modulus  
\[ \nu \] = Poisson’s ratio

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