Synthesized micro-theoretical analysis of performance of magnetorheological fluids subjected to shear mode operation: transmission, slip and sedimentation characteristics

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Keywords: magnetorheological fluids, shear mode, shear yield stress, slip, sedimentation

Abstract
This paper aims to formulate a synthesized micro-theoretical analysis that consists of transmission, slip and sedimentation characteristics for the evaluation of the performance of magnetorheological fluids (MRFs) subjected to the shear mode operation. To begin, based on the magnetic dipole theory and the typical micro-structure of MRFs, the shear yield stress of MRFs is analyzed considering the contribution of the magnetic force, excluded-volume force and viscous force. Then, an improve micro-structure of MRFs that is in correspondence with the actual observation is built, and the slip power of MRFs per unit area that takes into account the friction of the particles, walls and carrier liquid is investigated. Moreover, the sedimentation characteristic of MRFs described by the sedimentation velocity of the particles with the coating is analyzed based on the Stokes’ rule. The main influencing factors, such as the magnetic field intensity, the volume fraction of particles, the radius of particles, the coating thickness, the shear strain and the shear strain rate, etc, are taken into account in the micro-theoretical analysis, and the effects of these factors on the transmission, slip and sedimentation characteristics of MRFs are investigated respectively. It shows that the micro-theoretical analysis can describe these characteristics of MRFs accurately and reasonably, and can effectively be utilized for the initial design and optimization of the performance of MRFs and the useful guidance on the external control of magnetorheological transmission devices.

1. Introduction

Magnetorheological fluids (MRFs) are a kind of advanced smart materials comprised of ferromagnetic particles with the micron-size and the non-ferromagnetic carrier liquid. When exposed to an external magnetic field, they can translate instantly and reversibly from a Newtonian fluid into a semi-solid with an adjustable shear yield stress, and demonstrate tremendous changes in macroscopic rheological characteristics. There are three main working modes of MRFs: shear mode, squeeze mode and flow mode [1]. When MRFs work in the shear mode operation, they can be utilized for the application in magnetorheological transmission devices, such as brakes and clutches [2, 3]. In generally acknowledged, there are three key characteristics that affect the performance of MRFs subjected to the shear mode operation, which are transmission, slip and sedimentation characteristics. Firstly, the transmission characteristic of MRFs consists of the shear yield stress of particles and the viscous shear stress of the carrier liquid under a magnetic field and shear deformation, and strongly affects the output power of magnetorheological transmission devices. Secondly, the slip characteristic of MRFs is that the high temperature of MRFs caused by the friction of the particles, walls and carrier liquid under the slip difference reduces the viscosity of the carrier liquid. Thirdly, the sedimentation characteristic of MRFs is that the huge density mismatch between the particles and carrier liquid results in the agglomeration and sedimentation of particles, which reduces the stability of the particle suspension.
Great efforts were respectively made to investigate on the transmission, slip and sedimentation characteristics of MRFs in recent years. For the transmission characteristic of MRFs, Ginder [4] presented a simple model and obtained a set of explicit expressions of the yield stress at different magnetic fields, but this model may overestimate the shear yield stress due to neglecting the nonlinear magnetization. Peng et al [5] formulated a micro-macro model to calculate the shear yield stress of MRFs based on micromechanics and a statistical approach, which can take into account the effect of each of the main governing factors. Zhao et al [6] presented an enhanced dipole model that can improve the calculation accuracy of the shear yield stress of MRFs. Güth et al [7] investigated the behavior of MRFs at high shear rates and formulated the corresponding model of the shear yield stress. Zhou et al [8] investigated the quasi static shear deformation of magnetic particles in a Couette flow of MRFs through Stokes and dynamic simulations, but the effects of the radius of particles and the coating thickness are ignored. Li et al [9] compared and analyzed the influence of the particle size, particle spacing and magnetic field intensity on the model error of the single-chain dipole based on the finite element to use the shear yield stress model of MRFs more accurately. However, these studies did not take into account the effects of the excluded-volume force and viscous force of particles.

Unlike so many analyses of the shear yield stress on MRFs, it is regretted that there are few researches on the analysis of the slip characteristic of MRFs, especially on a micro scale. Many researchers measured the temperature rise of MRFs caused by the slip difference through experimental methods [10–15]. Besides, for the sedimentation characteristic of MRFs, one of the most available approaches to prolong the sedimentation time of particles is having ferromagnetic particles coated with non-ferromagnetic and lightweight materials. Many studies were devoted to searching suitable coating materials to improve the sedimentation stability of MRFs [16–18]. Currently, very few studies paid attention to the essential link of the transmission, slip and sedimentation characteristics of MRFs and carried out the theoretical analyses of them on the micro scale.

In this paper, a synthesized micro-theoretical analysis that consists of transmission, slip and sedimentation characteristics is formulated to evaluate the performance of MRFs subjected to the shear mode operation. To begin, for the preparation of the micro-theoretical analysis, some assumptions of the typical micro-structure of MRFs are made and the working micro-mechanism of MRFs is introduced. Based on the magnetic dipole theory, the shear yield stress of MRFs is analyzed considering the contribution of the magnetic force, excluded-volume force and viscous force. In order to study the slip characteristic of MRFs described by the slip power, an improved micro-structure of MRFs that is in correspondence with the actual observation is built, and the radial components of the magnetic force, excluded-volume force and viscous force that lead to the slip heat are derived. Considering the friction of the particles, walls and carrier liquid, the slip power of MRFs per unit area is determined. Besides, based on the Stokes’ rule, we define the ratio of the sedimentation velocities of the particles with the coating and without the coating to describe the sedimentation characteristic of MRFs. The main influencing factors, such as the magnetic field intensity, the volume fraction of particles, the radius of particles, the coating thickness, the shear strain and the shear strain rate, etc, are taken into account in the micro-theoretical analysis, and the effects of them on the transmission, slip and sedimentation characteristics of MRFs are investigated respectively. The proposed micro-theoretical analysis in this study may serve to provide a theoretical basis for the initial design and optimization of the performance of MRFs and the useful guidance on the external control of magnetorheological transmission devices.

2. Micro-theories of the transmission, slip and sedimentation characteristics of MRFs

2.1. Transmission characteristic
Magnetorheological transmission technology depends on the shear yield stress of MRFs. When exposed to an external magnetic field, ferromagnetic particles are magnetized and each of them has a magnetic dipole moment parallel to the direction of the magnetic field based on the magnetic dipole theory. Meanwhile, each of the magnetized particles yields an induced magnetic field, which makes it subjected to the action of the surrounding particles. Therefore, the particles attract each other and aggregate to form chains along the direction of the magnetic field. When the chains keep stationary, there is no shear yield stress between the particles. However, if the chains are subjected to a shear deformation, there is a shear yield stress between the neighbouring particles along the shear direction. In order to avoid complications, the following assumptions of the typical micro-structure of MRFs are made:

(a) All the particles are of spheres with the identical size and are made of the same material [5].
(b) The distance between the chains is sufficiently large so that the interaction between the particles in different chains can be negligible [5].
The particles are mainly subjected to the magnetic force, excluded-volume force and viscous force, since other forces, such as the van der Waals force, brown force, gravity and buoyancy, etc, are much smaller than them [19].

The magnetic field is assumed as uniform.

When the particles are subjected to the magnetic field and shear deformation, the magnetic force, excluded-volume force and viscous force of the particle $i$, see in figure 1, are respectively expressed as [20]

\[
\begin{align*}
F_{m}^{i} &= \sum_{i \neq j} \frac{4\pi\mu_{0}\chi H^{2}R^{6}}{3r_{ij}^{4}}[(1 - 5 \cos^2 \theta_{ij})r_{ij} + 2 \cos \theta_{ij}] \\
F_{ev}^{i} &= \sum_{i \neq j} \frac{8\pi\mu_{0}\chi H^{2}R^{6}}{3r_{ij}^{4}} \exp \left[-\beta \left(\frac{r_{ij}}{2R} - 1\right)\right]r_{ij} \\
F_{v}^{i} &= -6\pi R\eta \dot{x}
\end{align*}
\]

where $\mu_{0} = 4\pi \times 10^{-7}$ H m$^{-1}$ is the permeability in free space, $\chi$ is the susceptibility of particles, $H$ is the magnetic field intensity, $R$ is the radius of particles, $r_{ij}$ is the distance between the cores of the particle $i$ and the particle $j$, $\theta_{ij}$ is the included angle between the inclined chains and applied magnetic field, $\vec{r}_{ij}$ is the unit vector pointing from the particle $i$ to the particle $j$, $\vec{y}$ is the unit vector parallel to the direction of the magnetic field, $\beta$ is a material-dependent constant, $\eta$ is the viscosity of the carrier liquid, $\dot{u}$ is the velocity of the particles relative to the carrier liquid in the same layer, and $\vec{x}$ is the unit vector vertical to the direction of the magnetic field.

It is worth noting in equation (3) that $\dot{u}$ is the velocity difference between the particles and carrier liquid in the same layer. When the shear strain rate is stable, the velocity difference between the particles and carrier liquid in the same layer is very small so that the viscous force of particles can be negligible.

Thus, the resultant force of particle $i$ in the direction $x$ is

\[
F_{ix} = \frac{8\pi\mu_{0}\chi H^{2}R^{6}}{3r_{ij}^{4}}(5 \cos^2 \theta_{ij} - 1) \sin \theta_{ij} - \frac{16\pi\mu_{0}\chi H^{2}R^{6}}{3r_{ij}^{4}} \sin \theta_{ij} \exp \left[-\beta \left(\frac{R + t}{R} - 1\right)\right]
\]  

(4)

Considering a non-ferromagnetic coating of thickness $t$ of each particle and a uniform stretching of the chain in the typical micro-structure (figure 2), the distance between any two particles is given by

\[
r_{ij} = \frac{2k(R + t)}{\cos \theta_{ij}} - k = 1, 2, 3, \ldots N - 1
\]  

(5)

where $N$ is the number of particles in a single chain.

Besides, it is known that the magnetic induction intensity of ferromagnetic particles increases with the magnetic field intensity, but it has a saturation value. This phenomenon can be described by the susceptibility $\chi$ according to the Frohlich-Kennelly Equation [21].

\[
\chi = \frac{\chi_{0}M_{s}}{\chi_{0}H + M_{s}}
\]  

(6)

where $\chi_{0}$ is the initial susceptibility when the magnetic field intensity $H$ tends to zero, and $M_{s}$ is the saturation intensity of the magnetization.

Substituting equations (3) and (6) into equation (4) and considering the contribution of all the particles in a single chain, the resistance against the shear deformation is given by

\[
\text{Figure 1. Forces between the neighbouring particles in a single chain.}
\]
Figure 2. Stretching of the chain with the shear deformation in the typical micro-structure.

Figure 3. The metabolic process of MRFs subjected to the shear deformation. (a) Chaining; (b) Stretching; (c) Breaking; (d) Re-formation.

\[ F_x = \sum_{k=1}^{N-1} F_{x_k} = \sum_{k=1}^{N-1} \frac{1}{k^4} \frac{\pi \mu_0 \chi_0^2 M_s^2 H^2 R^6}{3 (R + t)^4 (\chi_0 H + M_s)^2} \cos^2 \theta_0 \sin \theta_0 \left\{ \frac{5 \cos^2 \theta_0 - 1}{2} - \exp \left[ -\beta \left( \frac{R + t}{R} - 1 \right) \right] \right\} \]

where let \( A = \sum_{k=1}^{N-1} \frac{1}{k^4} \approx 1.0823 \).

For MRFs with the volume fraction of particles \( \phi \), the number of chains per unit area can be estimated with

\[ G = \frac{\phi Sh}{SN (4\pi R^3 / 3)} = \frac{3\phi (R + t)}{2\pi R^3} \]

where \( S \) is the area of MRFs perpendicular to the direction of the magnetic field, and \( h = 2N (R + t) \) is the spacing between the upper and lower walls.

Noticing that the macroscopic shear deformation makes the distance between the neighbouring particles larger and larger, and the interaction between them is correspondingly smaller and smaller. At this time, the magnetic chains are tilted and stretched (figure 3(b)). When the interaction is unable to maintain the connection of the particles, the chains break between any two particles (figure 3(c)). However, the broken chain instantly connects with the next broken chain into a new integrated straight chain through the action of the coupled field (figure 3(d)). Then, the re-formation chains will be tilted and stretched again subjected to the shear deformation.
This process of the deformation and re-formation of the chain is continuously metabolic and can be described by the angle of \( \theta_{ij} \).

Thus, the macroscopic shear yield stress of MRFs contributed by all the chains can be expressed as

\[
\tau_m = F_x G = \frac{A\mu_0 \chi_0^2 M_i^2 H^2 R^3 f}{2(R + t)^3(x_0 H + M_i)^2} \cos^8 \theta_{ij} \sin \theta_{ij} \left\{ \frac{5 \cos^2 \theta_{ij} - 1}{2} - \exp \left[ -\beta \left( \frac{R + t}{R} - 1 \right) \right] \right\}
\]

(9)

Utilizing the relation between the angle \( \theta_{ij} \) and the shear strain \( \gamma \) defined by

\[
\gamma = \tan \theta_{ij}
\]

(10)

The shear yield stress \( \tau_m \) can be expressed as a function of the shear strain \( \gamma \) by

\[
\tau_m = F_x G = \frac{A\mu_0 \chi_0^2 M_i^2 H^2 R^3 f}{2(R + t)^3(x_0 H + M_i)^2} \gamma \left\{ \frac{5 \gamma^2 + 4}{2} - \exp \left[ -\beta \left( \frac{R + t}{R} - 1 \right) \right] \right\}
\]

(11)

If the viscous shear stress of the carrier liquid is \( \tau_0 \) without applying the magnetic field, the overall macroscopic shear stress of MRFs in the magnetic field based on the Bingham model is given by

\[
\tau = \tau_m + \tau_0 = \frac{A\mu_0 \chi_0^2 M_i^2 H^2 R^3 f}{2(R + t)^3(x_0 H + M_i)^2} \gamma \left\{ \frac{5 \gamma^2 + 4}{2} - \exp \left[ -\beta \left( \frac{R + t}{R} - 1 \right) \right] \right\} + \eta \dot{\gamma}
\]

(12)

where \( \dot{\gamma} \) is the shear strain rate of MRFs.

### 2.2. Slip characteristic

In generally acknowledged, when particle chains are subjected to the shear deformation under the magnetic field, there is a relative velocity difference between the upper and lower walls. Since the space between the upper and lower walls consists of two main parts: (1) the relative velocity difference between the upper and lower ends of particles and between the walls and the upper and lower ends of particles, and (2) the relative velocity difference inside the carrier liquid and between the walls and carrier liquid. Then, the slip difference of the particles, walls and carrier liquid result in the enormous heat within MRFs, which reduces the viscosity of the temperature-dependent carrier liquid. Correspondingly, the slip heat of MRFs consists of two main parts: (1) the slip heat between the particles and between the walls and the upper and lower ends of particles, and (2) the viscous heat inside the carrier liquid and between the walls and carrier liquid. Within this section, these two heat sources of MRFs described by the slip power are demonstrated in the following content.

#### 2.2.1. Slip power of particles

According to section 2.1, the particles are mainly subjected to the magnetic force, excluded-volume force and viscous force under the magnetic field and shear deformation. The shear components of these forces along the shear direction are used for the resistance to the shear deformation, and the radial components of these forces along the direction of the chain result in the continuous friction of the particles and walls [22]. Considering the negligible viscous force through the analysis in section 2.1, the mutual forces \( F_{ij} \) and \( F_{ji} \) between the particle \( i \) and the particle \( j = i + 1 \) are derived as (figure 1)

\[
F_{ij} = F_{ij}^m - F_{ij}^{ev}
\]

(13)

\[
F_{ji} = F_{ji}^m - F_{ji}^{ev} = -F_{ij}^m + F_{ij}^{ev}
\]

(14)

where \( F_{ij}^m \) and \( F_{ij}^{ev} \) are respectively the radial components of the magnetic force and excluded-volume force.

According to the Hertzian contact theory, the radius and maximum contact stress of the contact surface between the neighbouring particles are given by

\[
d_{pp} = \left[ \frac{3R(1 - \mu^2)(F_{ij} - F_{ji})}{4E} \right]^{1/2}
\]

(15)

\[
d_{pp} = \left[ \frac{6E^2(F_{ij} - F_{ji})}{\pi R^2(1 - \mu^2)^2} \right]^{1/2}
\]

(16)

where \( E \) is the elastic modulus of particles, and \( \mu \) is the Poisson ratio of particles.

It is known that the typical micro-structure in figure 2 is very effective in the analysis of the interaction of particles. However, what we should pay attention to is that there is a spacing between the neighbouring particles during the shear deformation. Due to the comprehensive impact of multiple complex factors, the actual chain is not same as the typical micro-structure and exists in the relatively stable form. Throughout the observation of the micro-structure of MRFs, it is demonstrated that the single chain stretches and breaks between any two
neighbouring particles during the shear deformation [23]. Thus, an improved micro-structure shown in figure 4 is built, and it can be seen that the other neighbouring particles except two stretched and broken particles contact and rub against each other. It can be understood that the interaction of particles corresponding to the typical micro-structure and the contact between the neighbouring particles corresponding to the improved micro-structure belong to two different systems, and they do not affect each other. Thus, they can be collaboratively taken into account in the analysis of the heat generation of MRFs. Thus, the slip heat flow of the particles in a single chain during the shear deformation is given by

\[
J_{pp} = (N - 2) \sum_{k=1}^{N-1} \left( \frac{2 \mu_{pp} v_{ij}}{3} \right) \pi a_{pp}^2 v_{ij} = \frac{2 \mu_{pp} A \pi \mu_0 \chi_0^2 M_i^2 H^2 R^6 v_{pp} (N - 2)}{3 (R + t)^4 (\chi_0 H + M_i)^2 (N - 1)} \cos^4 \theta_{ij}
\]

where \( \mu_{pp} \) is the frictional coefficient between the neighbouring particles, \( v_{ij} \) is the relative velocity between the neighbouring particles, and \( v_{pp} \) is the relative shear velocity between the upper and lower ends of particles.

Then, the slip power of the particles per unit area is given by

\[
P_{pp} = J_{pp} G = \frac{\mu_{pp} A \mu_0 \chi_0^2 M_i^2 H^2 R^6 v_{pp} (N - 2)}{(R + t)^4 (\chi_0 H + M_i)^2 (N - 1)} \cos^4 \theta_{ij} \left\{ \frac{3 \cos^2 \theta_{ij} - 1}{2} \right\} - \exp \left[ -\beta \left( \frac{R + t}{R} - 1 \right) \right]
\]

Substituting equation (10) into equation (18) and considering \( N \gg 1 \) give

\[
P_{pp} = \frac{\mu_{pp} A \mu_0 \chi_0^2 M_i^2 H^2 R^6 v_{pp} (N - 2)}{(R + t)^4 (\chi_0 H + M_i)^2 (N - 1)} \left\{ \frac{3 \cos^2 \theta_{ij} - 1}{2} \right\} - \exp \left[ -\beta \left( \frac{R + t}{R} - 1 \right) \right]
\]

Since the material of the walls is usually taken as the low carbon steel, the walls are hard to be magnetized under the magnetic field. However, there is the attracting interaction between the walls and the upper and lower ends of particles. Similarly, based on the magnetic dipole theory, the interaction between the walls and the upper and lower ends of particles is given by

\[
F_{pw} = F_{pw}^m - F_{pw}^v
\]

The radius and maximum contact stress of the contact surface between the walls and the upper and lower ends of particles are given by

\[
\text{Figure 4. Typical and improved micro-structures of MRFs.}
\]
\[ a_{pw} = \left[ \frac{3R(1 - \mu^2)F_{pw}}{4E} \right]^{\frac{1}{2}} \]  

(21)

\[ q_{pw} = \left[ \frac{6E^2F_{pw}}{\pi^2R^4(1 - \mu^2)^2} \right]^{\frac{1}{2}} \]  

(22)

Then, the slip heat flow between the walls and the upper and lower ends of particles during the shear deformation is given by

\[ J_{pw} = \frac{2m_{pw}}{3} q_{pw} \pi a_{pw}^2 v_{pw} = \frac{16\mu_0 \chi_0^2 M^2 H^2 R^2 v_{pw}}{3(R + t)^4(\chi_0 H + M_r)^2} \left\{ \frac{3 \cos^2 \theta_{ij} - 1}{2} - \exp\left[-\beta \left( \frac{R + t}{R} - 1 \right) \right] \right\} \]  

(23)

where \( \mu_{pw} \) is the frictional coefficient between the walls and the upper and lower ends of particles, and \( v_{pw} \) is the relative shear velocity between the walls and the upper and lower ends of particles.

The slip power between the walls and the upper and lower ends of particles per unit area is given by

\[ P_{pw} = J_{pw} G = \frac{8\mu_0 \chi_0^2 M^2 H^2 R v_{pw} \phi}{(R + t)^4(\chi_0 H + M_r)^2} \left\{ \frac{3\gamma^2 + 2}{2} - \exp\left[-\beta \left( \frac{R + t}{R} - 1 \right) \right] \right\} \]  

(24)

Therefore, the slip power of the particles and walls per unit area is given by

\[ P_p = P_{pp} + P_{pw} = \frac{\mu_0 \chi_0^2 M^2 H^2 R \gamma \phi}{(R + t)^4(\chi_0 H + M_r)^2} \left\{ \frac{3\gamma_{pp}^2 + 2}{2} - \exp\left[-\beta \left( \frac{R + t}{R} - 1 \right) \right] \right\} \]  

(25)

where \( v_{pp} + v_{pw} = v_r \) is the relative shear velocity between the upper and lower walls.

Utilizing the relation between the relative shear velocity \( v_r \) and the shear strain rate \( \gamma \) defined by

\[ \gamma = \frac{v_r}{2N(R + t)} \]  

(26)

The slip power of the particles and walls per unit area \( P_p \) can be expressed as a function of the shear strain rate \( \gamma \) by

\[ P_p = \frac{\mu_0 \chi_0^2 M^2 H^2 R \gamma \phi}{(R + t)^4(\chi_0 H + M_r)^2} \left\{ \frac{3\gamma_{pp}^2 + 2}{2} - \exp\left[-\beta \left( \frac{R + t}{R} - 1 \right) \right] \right\} \]  

(27)

where \( \gamma_{pp} \) is the shear strain rate between the upper and lower ends of particles in a single chain, and \( \gamma_{pw} \) is the shear strain rate between the walls and the upper and lower ends of particles.

2.2.2. Slip power of the carrier liquid

In generally acknowledged, the viscous heat of the fluid is caused by its viscous resistance based on the law of the internal friction. It is assumed that the carrier liquid flow is laminar, the viscous forces between the neighbouring layers of the carrier liquid and between the walls and carrier liquid cause the heat generation. However, the study [24] shows that the viscous heat inside the carrier liquid is a much more significant heat source so that the viscous heat between the walls and carrier liquid can be negligible.

The viscous resistance of the carrier liquid per unit area based on the law of internal friction can be expressed as

\[ F_f = \eta \frac{v_r}{2N(R + t)} \]  

(28)

The slip power of the carrier liquid per unit area is given by

\[ P_f = (1 - \phi) \int_0^{v_r} F_f \cdot v_f dv = \frac{\eta v_r^3(1 - \phi)}{4N(R + t)} = 2\eta \gamma^3 N^2(R + t)^2(1 - \phi) \]  

(29)

where \( v_r \) is the velocity of the carrier liquid.
Therefore, the slip power of MRFs per unit area is given by

\[
P = P_p + P_f = \frac{\mu_0 \chi_0 M^2 H^2 R^3 / \phi}{(R + t)^3 (\chi_0 H + M_s)^2} 2N (R + t) \left[ \mu_p \chi_p (\gamma^2 + 1) + 8 \mu_{pw} \gamma_{pw} \right]
\times \left\{ \frac{3 \gamma^2 + 2}{2} - \exp \left[ -\beta \left( \frac{R + t}{R} - 1 \right) \right] \right\} + 2 \eta \gamma^3 N^2 (R + t)^3 (1 - \phi)
\]

(30)

2.3. Sedimentation characteristic

Due to the huge density mismatch between the particles and carrier liquid, the particles in the carrier liquid will sink. In order to prolong the sedimentation time and improve the sedimentation stability of MRFs, one of the most available approaches is to have the particles coated with non-ferromagnetic and lightweight materials. Within this section, the sedimentation characteristic of MRFs described by the sedimentation velocity of particles is demonstrated.

According to the Stokes’ rule, the sedimentation velocity of the particles in the carrier liquid can be expressed as

\[
v_p = \frac{8 \pi R^2 (\rho_p - \rho_f) g}{\eta}
\]

(31)

where \( \kappa \) is a constant related to the dimension system adopted, \( \rho_p \) and \( \rho_f \) are respectively apparent densities of the particles and carrier liquid, and \( g \) is the gravitational acceleration.

When the particles are coated with the non-ferromagnetic and lightweight materials, the radius and apparent density of the particles with the coating are respectively given by

\[R_{pc} = R + t\]

(32)

\[\rho_{pc} = \frac{m_{pc}}{V_{pc}} = \frac{\rho_p R^3 + \rho_c [(R + t)^3 - R^3]}{(R + t)^3}\]

(33)

where \( \rho_c \) is the apparent density of the coating.

Substituting equations (32) and (33) into equation (31) gives the sedimentation velocity of the particles with the coating.

\[
v_{pc} = \kappa \frac{8 \pi R^2 (\rho_p - \rho_f) g}{\eta} = \kappa \frac{8 (R + t)^2 g}{\eta} \left\{ \frac{\rho_p R^3 + \rho_c [(R + t)^3 - R^3]}{(R + t)^3} - \rho_f \right\}
\]

(34)

Defining the ratio of the sedimentation velocities of the particles with the coating and without the coating to describe the sedimentation characteristic of MRFs yields

\[
\frac{v_{pc}}{v_p} = \frac{R_p^2 (\rho_p - \rho_f)}{R^2 (\rho_p - \rho_f)} = \frac{(R + t)^2}{R^2 (\rho_p - \rho_f)} \left\{ \frac{\rho_p R^3 + \rho_c [(R + t)^3 - R^3]}{(R + t)^3} - \rho_f \right\}
\]

(35)

3. Results and discussion

Before the results and discussion some parameter values of the micro-theoretical analysis can be obtained, when the materials of the particles and carrier liquid are determined. For the material of the particles, it is usually designated as the carbonyl iron powder, and it also can be replaced by the pure iron in some ferromagnetic properties. For the material of the carrier liquid, it is usually designated as the silicone oil. These exact parameter values are shown in table 1 [5, 20, 25, 26].

| Parameter | \( \mu_0 \) (H m\(^{-1}\)) | \( \chi_0 \) | \( M_s \) (A m\(^{-1}\)) | \( \phi \) | \( \eta \) (Pa s) |
|-----------|-----------------|--------|-----------------|------|----------|
| Value     | \( 4 \pi \times 10^{-7} \) | 1000   | \( 1.671 \times 10^6 \) | 10   | 0.09     |
| Value     | \( \mu_{pp} \) | \( \mu_{pw} \) | \( \rho_p \) (kg m\(^{-3}\)) | \( \rho_f \) (kg m\(^{-3}\)) | \( \rho_c \) (kg m\(^{-3}\)) |

| Value     | 0.3 | 0.2 | 8000 | 1000 | 500 |

---

Table 1. Parameter values.
3.1. Effect of the magnetic field intensity $H$

It is known that MRFs are a kind of controllable materials whose output power can be adjusted by the external governing current [25]. More precisely, the adjustment of the external current is mainly to change the magnetic field intensity. Given $R = 5 \mu m$, $t = 1 \mu m$, $\gamma = 0.2$, $\dot{\gamma} = 500 \text{ s}^{-1}$, $N = 100$, and $\dot{\gamma}_{pp}/\dot{\gamma}_{pw} = 9$, the variations of $\tau$ and $P$ versus $H$ at different $f$ are shown in figure 5.

It can be seen from the figure that, as $H$ increases, $\tau$ and $P$ increase, and the speeds of their increases are getting slower and slower. It is known that the particles subjected to the magnetic force and the excluded-volume force that are determined by $H$ align in the direction of the magnetic field and aggregate into chains. Based on the magnetic dipole theory, the magnetic force and the excluded-volume force increase with $H$. On the one hand, the shear components of the magnetic force and the excluded-volume force are used for the resistance against the shear deformation. On the other hand, the radial components of the magnetic force and the excluded-volume force result in the extrusion of particles and thereby cause the slip heat during the shear deformation. However, due to the magnetization saturation of particles, the magnetic force and the excluded-volume force will not continue to increase when they reach certain values. It also can be seen that $\tau$ and $P$ are proportional to $f$. That is because that the number of particle chains per unit area increases with $f$.

For the better application of MRFs in magnetorheological transmission devices, we hope that MRFs are capable of supplying a sufficient output torque when the maximum temperature caused by the slip heat does not invalidate MRFs. When the macro-structures and dimensions of magnetorheological transmission devices are determined, it is better for MRFs to provide a sufficient shear stress and work within an allowable slip power of MRFs per unit area at the certain operating hours. Thus, we can calculate the slip power of the devices through the micro-theoretical analysis of the present paper and thereby carry out the thermal field simulation to decide the maximum of $H$. Moreover, $f$ is usually taken from 30% to 45% in magnetorheological transmission devices. On the one hand, when $f$ is less than 30%, there are many broken chains in MRFs. On the other hand, when $f$ is more than 45%, the number of the particle chains will increase, which reduces the average distance between the neighbouring particle chains and thereby yields the non-negligible interaction between the neighbouring particle chains. Thus, $f$ can be taken as about 40%, and it can be adjusted to meet the requirement of the actual output torque.

3.2. Effect of the radius of the particles $R$

It is known that $R$ is usually from 0.1 $\mu m$ to 100 $\mu m$, and its typical value is taken from 3 $\mu m$ to 5 $\mu m$ [27]. When $H = 5 \text{ kA} \text{ m}^{-1}$, $t = 1 \mu m$, $\phi = 0.4$, $\gamma = 0.2$, $\dot{\gamma} = 500 \text{ s}^{-1}$, $N = 100$, and $\dot{\gamma}_{pp}/\dot{\gamma}_{pw} = 9$, the variations of $\tau$, $P$ and $v_{pc}/v_{s}$ versus $R$ are shown in figure 6.

It can be seen from figure 6(a) that, as $R$ increases, $\tau$ increases, and the speed of its increase is getting slower and slower. That is because the magnetic force and the excluded-volume force are directly proportional to the volume of the particle but are inversely proportional to the distance between any two particles. However, it can be seen from figure 6(b) that $P$ increases approximately linearly with $R$. That is because, in addition to the effect...
of $R$ on the magnetic force and the excluded-volume force, the shear velocity and the slip power of the carrier liquid all increase with $R$ at a certain $\gamma$. Meanwhile, it can be seen from figure 6(c) $v_{pc}/v_p$ increases with $R$, and the speed of its increase is getting slower and slower. That means the sedimentation velocity of the particles with the coating increases with $R$. It can be imagined that the average density of the particles with the coating decreases with the increase of $R$.

Similarly, for the better application of MRFs in magnetorheological transmission devices, in addition to a sufficient shear stress and an allowable slip power of MRFs per unit area at the certain operating hours, it is also necessary for MRFs to possess a low sedimentation velocity of the particles with the coating. Thus, when the slip power of MRFs per unit area is determined at the certain operating hours through the micro-theoretical analysis of present paper, we need to calculate the maximum sedimentation velocity to ensure the superior sedimentation characteristic of particles within the operating hours.

3.3. Effect of the coating thickness $t$

Having the particles coated with non-ferromagnetic and lightweight materials can improve the sedimentation stability of MRFs, but it also may partly reduce the magnetorheological property of MRFs. In order to achieve a relative balance between the magnetorheological property and the sedimentation stability as much as possible, it is necessary to investigate the effect of $t$ on $\tau$, $P$, and $v_{pc}/v_p$. Given $H = 5$ kA m$^{-1}$, $R = 5$ $\mu$m, $\phi = 0.4$, $\gamma = 0.2$, $\dot{\gamma} = 500$ $s^{-1}$, $N = 100$, and $\dot{\gamma}_{pp}/\dot{\gamma}_{pw} = 9$, the variations of $\tau$, $P$, and $v_{pc}/v_p$ versus $t$ are shown in figure 6. It can be seen from figure 6(d) that, as $t$ increases, $\tau$ increases, and reaches a maximum at $t \approx 0.4$ $\mu$m, and then decreases. That is because, as $t$ increases, the magnetic force decreases, but the excluded-volume force increases. When $t < 0.4$ $\mu$m, the decrement of the magnetic force with $t$ is less than the increment of the excluded-volume force with $t$. However, when $t > 0.4$ $\mu$m, the decrement of the magnetic force with $t$ is more than the increment.

![Figure 6. Curves of the shear stress $\tau$, the slip power of MRFs per unit area $P$ and the ratio of the sedimentation velocities of the particles with the coating and without the coating $v_{pc}/v_p$ in terms of the radius of particles $R$ and the coating thickness $t$. (a) Variation of $\tau$ versus $R$; (b) Variation of $P$ versus $R$; (c) Variation of $v_{pc}/v_p$ versus $R$; (d) Variation of $\tau$ versus $t$; (e) Variation of $P$ versus $t$; (f) Variation of $v_{pc}/v_p$ versus $t$.](image-url)
of the excluded-volume force with \( t \). It can be seen from figure 6(e) that \( P \) increases with \( t \), and reaches a maximum at \( t \approx 0.9 \, \mu m \), and then decreases with \( t \). That is because, in addition to the effect of \( t \) on the magnetic force and excluded-volume force, the shear velocity and the slip power of the carrier liquid all increase with \( t \) at a certain. It also can be seen from figure 6(f) that \( \gamma_{pv}/\gamma_p \) decreases with \( t \), which means the sedimentation velocity of the particles with the coating decreases with \( t \). That is because the average density of the particles with the coating decreases with the increase of \( t \).

Since having the particles coated with non-ferromagnetic and lightweight materials is mainly to improve the sedimentation stability of MRFs, the effect of \( t \) on \( \gamma_{pv}/\gamma_p \) should have an priority. In order to possess the superior transmission, slip and sedimentation characteristics of MRFs, it is necessary to be taken as a suitable value. On the one hand, according to figure 5(f), \( t \) can be taken from 0.4 \( \mu m \) to 1.5 \( \mu m \). On the other hand, figures 6(d) and (e) are shown that \( t \) is better to be taken from 0.4 \( \mu m \) to 0.9 \( \mu m \). Thus, \( t \) is taken from 0.4 \( \mu m \) to 0.9 \( \mu m \), and is as small as possible.

### 3.4. Effect of the shear strain \( \gamma \)

It is known that the process of the deformation and re-formation of chains is continuously metabolic, and it can be described by \( \gamma \) and \( \dot{\gamma} \) in one cycle. When \( H = 5 \, kA \, m^{-1}, R = 5 \, \mu m, t = 1 \, \mu m, \phi = 0.4, \dot{\gamma} = 500 \, s^{-1}, \) and \( \gamma_{pp}/\gamma_{pu} = 9 \), the variations of \( \tau \) and \( P \) versus \( \gamma \) at different \( N \) are shown in figure 7.

It can be seen from figure 7(a) that \( \tau \) increases with \( \gamma \), and reaches a maximum at \( \gamma \approx 0.8 \), and then decreases with \( \gamma \). That is because when \( \gamma \) is small the shear components of the magnetic force and excluded-volume force are small. With the increase of \( \gamma \), the shear components of the magnetic force and excluded-volume force increase, and reach a maximum at \( \gamma \approx 0.8 \), and then decrease. It can be seen from figure 7(b) that \( \dot{\gamma}_{pp} \) and the slip power of the carrier liquid all increase with \( \gamma \). It also can be seen that \( \dot{\gamma} \) increases with \( N \). That is because the shear velocity and the slip power of the carrier liquid all increase with \( N \) at a certain \( \gamma \).

Since \( \gamma \) is bound to change from 0 to 1 in the process of the deformation and re-formation of chains, it is hard to be adjusted to meet the requirement of the application. However, the average \( P \) can be calculated in one cycle to prepare for the continuously metabolic shear deformation according to \( \gamma \). Moreover, the gap thickness of MRFs increases with \( N \) at the certain \( R \) and \( t \), which means the large gap thickness is not advantageous for the shear characteristic of MRFs. That agrees with the actual gap thickness of MRFs which is usually taken as 1 mm [28].

### 3.5. Effect of the shear strain rate \( \dot{\gamma} \)

Moreover, given \( H = 5 \, kA \, m^{-1}, R = 5 \, \mu m, t = 1 \, \mu m, \phi = 0.4, \gamma = 0.2, \) and \( N = 100, \tau \) and \( P \) versus \( \dot{\gamma} \) at different \( \gamma_{pp}/\gamma_{pu} \) are shown in figure 8.

It can be seen from figure 8(a) that \( \dot{\gamma} \) has little effect on \( \tau \). That is because the viscous shear stress of the carrier liquid related to \( \dot{\gamma} \) is much smaller than the shear yield stress of particles. It can be seen from figure 8(b)
that $P$ is proportional to $\gamma$, and decreases linearly with $\gamma_{pp}/\gamma_{pw}$. That is because $P$ is proportional to the shear velocity, and the effect of $\gamma_{pp}/\gamma_{pw}$ is nearly 8 times greater than that of $\gamma_{pp}$ by equation (30).

Therefore, a small shear strain rate is beneficial for the slip characteristic of MRFs, and does not reduce the transmission capability of MRFs. Meanwhile, a small $\gamma_{pw}$ can reduce $P$, which means increasing the roughness of walls is also beneficial for the slip characteristic of MRFs. However, a large roughness of walls leads to a great wear damage on the walls and the upper and lower ends of particles. Thus, a relative balance between the slip characteristic and the wear damage need to be achieved.

3.6. Comparison with the experimental results

In order to demonstrate the validity of the micro-theoretical analysis, some typical governing parameters are investigated and compared with the experimental results [29]. Meanwhile, other parameters are taken precisely based on the condition of the experiments.
Making use of the magnetic induction intensity $B = \mu_0(1 + \chi)H$, figure 9(a) shows the variation of $\tau$ versus $B$ at $\phi = 0.25$ and $\dot{\gamma} = 105 \text{ s}^{-1}$ and the comparison with the experimental results. It can be seen from figure 9(a) that $\tau$ calculated by the micro-theoretical analysis of the present paper agrees with the testing results, and the minor difference is due to the interaction between the particles and carrier liquid at an unstable shear velocity and the neglect of the the van der Waals force, brown force, gravity and buoyancy. Figure 9(b) shows the variation of $\tau$ versus $\dot{\gamma}$ at $f = 0.25$ and $g = 4k \text{ G s}^{-1}$. It can be seen from figure 9(b) that $\tau$ obtained by the micro-theoretical analysis is in reasonable agreement with the experimental results, and the minor difference can be attributed to not considering the effect of the temperature on the viscosity of the carrier liquid.

4. Numerical example

It is desired to quantify the performance of MRFs comprising of transmission, slip and sedimentation characteristics as they work in magnetorheological transmission devices. For data representative of MRFs it is assumed that $\rho_p = 8000 \text{ kg m}^{-3}$, $\rho_f = 1000 \text{ kg m}^{-3}$, $\rho_c = 500 \text{ kg m}^{-3}$ and $\eta = 0.09$ as previously assumed. When $H = 8 \text{ kA m}^{-1}$, $\gamma = 0.5$, and $\dot{\gamma} = 500 \text{ s}^{-1}$, the values of $\tau$, $P$ and $v_{pc}/v_p$ are shown in tables 2–4 from equations (12), (30) and (35) by Matlab software.

5. Conclusion

This paper formulates a synthesized micro-theoretical analysis that consists of transmission, slip and sedimentation characteristics to evaluate the performance of MRFs subjected to the shear mode operation. The main influencing factors, such as the magnetic field intensity, the volume fraction of particles, the radius of particles, the coating thickness, the shear strain and the shear strain rate, etc, are taken into account in the micro-theoretical analysis, and the effects of these factors on the transmission, slip and sedimentation characteristics of MRFs are investigated respectively. Throughout comparison with the typical experimental results of the shear stress, the micro-theoretical analysis is effective in description of the performance of MRFs. It is shown that these designable and controllable factors have great effects on the transmission, slip and sedimentation characteristics of MRFs, and the effects are quantitatively studied. Furthermore, based on the micro-theoretical analysis, a relative balance of the performance of transmission, slip and sedimentation characteristics can be achieved more
reasonably and precisely to improve the efficiency of MRFs for the application in magnetorheological transmission devices. The micro-theoretical analysis is significant for the analysis and optimization of the properties of MRFs, and for the design of the high-performance MRFs and the external control of magnetorheological transmission devices. However, the micro-theoretical analysis is only suitable for the evaluation of the performance of MRFs subjected to the shear mode operation instead of the squeeze mode due to huge differences of the micro-structures and working micro-mechanism between them, and it is unknown how to recycle the lost slip heat. These aspects will be significant topics in future studies.

Acknowledgments

The authors gratefully acknowledge the financial support of this work from the National Natural Science Foundation of China under Grant No. 51875068, Youth Project of Science and Technology Research Program of Chongqing Education Commission of China under Grant No. KJQN201901120, and Research Startup Funds for Chongqing University of Technology under Grant No. 2015ZD03.

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