Anisotropy in Dijet Production in Exclusive and Inclusive Processes

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We investigate the effect of soft gluon radiations on the azimuthal angle correlation between the total and relative momenta of two jets in inclusive and exclusive dijet processes. We show that the final state effect induces a sizable \( \cos(2\phi) \) anisotropy due to gluon emissions near the jet cones. The phenomenological consequences of this observation are discussed for various collider experiments, including diffractive processes in ultraperipheral \( pA \) and \( AA \) collisions, inclusive and diffractive dijet production at the EIC, and inclusive dijet in \( pp \) and \( AA \) collisions at the LHC.

I. INTRODUCTION

Dijet productions are the most abundant events in hadronic collisions and have been under intensive investigations from both experiment and theory sides. Typically the two final state jets are produced in the so-called correlation configuration, namely, back-to-back in the transverse plane with nearly balanced transverse momenta \([1-8]\).

Deviations from the perfect back-to-back configuration inform important strong interaction physics, and, in particular, the non-perturbative structure of the nucleon and nucleus \([9-22]\). It is expected to reveal the medium property by studying de-correlation of dijet in heavy ion collisions as well \([23-30]\).

Let \( \vec{q}_\perp = \vec{k}_1\perp + \vec{k}_2\perp \) be the total transverse momentum of the two jets. We also define \( \vec{P}_\perp = (\vec{k}_1\perp - \vec{k}_2\perp)/2 \) as the leading jet transverse momentum, see Fig. 1. The correlation limit is defined by the condition \( q_\perp \ll P_\perp \sim k_1\perp \sim k_2\perp \). In general, the azimuthal angular correlation between \( \vec{q}_\perp \) and \( \vec{P}_\perp \) is isotropic. However, anisotropic distribution can be generated from nontrivial correlations in the transverse momentum dependent parton distribution (TMD) associated with the incoming hadrons. This provides a unique probe to the novel tomography imagining of the nucleon/nucleus in dijet production.

Among the proposed observables, of particular interest is the \( \cos(2\phi) \) anisotropic correlation in exclusive dijet production \([20]\) where \( \phi \) is the angle between \( \vec{P}_\perp \) and the target recoil momentum \( \vec{\Delta}_\perp \). A simple interpretation of this anisotropic correlation is provided by the so-called elliptic gluon Wigner distribution of the target \([20, 31, 32]\). Partly motivated by this observation (see, also, \([33]\)), very recently, the CMS collaboration has reported a preliminary measurement of this anisotropy in ultraperipheral collisions (UPC) at the LHC \([34]\). Using \( q_\perp \) as a proxy for \( \Delta_\perp \), CMS reported a significant \( \cos(2\phi) \) asymmetry. Intrigued by this result, in this paper, we will perform a detailed analysis of this anisotropic distribution, focusing mainly on the soft gluon radiation contributions.

Soft gluon radiation contributions to the azimuthal angular anisotropy of \( \cos(2\phi) \) and their resummation for various processes have been investigated in Refs. \([35-42]\). For the jet-related processes, they were first studied in Refs. \([41, 42]\) and it was found that their contributions are not power suppressed and can be resummed to all orders in perturbation theory. In this paper, we will follow these developments and apply to dijet production processes. For azimuthally symmetric distributions, the soft gluon resummation has been derived in Refs. \([43-51]\).

The physical picture is as follows. Soft gluons emitted from the final state jets tend to be aligned with the jet directions. Since gluons emitted too close to the jet axis become part of the jet, what matters is the emissions slightly outside the jet cones, and \( \vec{q}_\perp \) is essentially the recoil momentum against these gluons. This means that \( \vec{q}_\perp \) also tends to point to jet directions on average, resulting in a positive \( \langle \cos(2\phi) \rangle \). When \( q_\perp \ll P_\perp \), one needs to perform the resummation of logarithms \( \alpha_s^n \ln(P_\perp/q_\perp)^n \) which can be done in the TMD framework. We shall demonstrate that this simple picture can explain, at least partly, the observation by the CMS collaboration.

![FIG. 1. Dijet in transverse plane perpendicular to the beam direction at hadron colliders. Their total transverse momentum \( \vec{q}_\perp = \vec{k}_1\perp + \vec{k}_2\perp \) is much smaller than the individual jet momentum \( \vec{P}_\perp = (\vec{k}_1\perp - \vec{k}_2\perp)/2 \). Angular distribution between \( \vec{q}_\perp \) and \( \vec{P}_\perp \) has an anisotropy due to soft gluon radiation associated with the final state jet with a non-zero \( \langle \cos(2\phi) \rangle \).](image-url)
Our result in this paper will have a broad impact on the tomographic study of nucleons/nuclei at the future electron-ion collider (EIC) \cite{52, 53}. Various anisotropy observables have been proposed to study the novel structure of nucleon/nucleus \cite{13-22}. The results in these previous works have to be re-examined. In addition, since the induced \( \cos(2\phi) \) asymmetry is dominated by final state radiation, we can apply this approach to explore the soft gluon radiation from a fast-moving jet in the hot QCD medium in heavy-ion collisions. This provides a unique perspective to investigate the medium interaction with the jet and the associated energy loss mechanism.

### II. DIFFRACTIVE PHOTOPRODUCTION OF DIJET

We first study the diffractive photoproduction of dijets, \( \gamma A \to q\bar{q} + A \). The photon fluctuates into a quark-antiquark pair which then scatters off the nucleon/nucleus target and forms a final state dijet with momentum \( k_1 \) and \( k_2 \). This process can be studied, for example, in ultra-peripheral \( pA \) and \( AA \) collisions and the planned EIC in the future.

Let us define the soft factor \( S_J(q_{\perp}) \) as the probability for the dijet to emit total transverse momentum \( q_{\perp} \) outside the jet cones. Implicitly, it should also depend on the dijet relative transverse momentum \( P_{\perp} = \frac{1}{2}(k_{1\perp} - k_{2\perp}) \). The dijet cross section then reads

\[
\frac{d\sigma}{d^2P_{\perp}d^2q_{\perp}} = \int d^2q'_{\perp}\frac{d\sigma_0}{d^2P_{\perp}d^2q'_{\perp}}S_J(q_{\perp} - q'_{\perp}),
\]

where \( \sigma_0 \) is the cross section without soft radiations. We assume that the two jets are back-to-back in azimuth, and consider the soft regime \( |q_{\perp}| \ll |P_{\perp}| \). To lowest nontrivial order, \( S_J \) is given by the standard eikonal formula

\[
S_J(q_{\perp}) = \delta(q_{\perp}) + \frac{\alpha_s}{2\pi^2} \int dy_g \left( \frac{k_1 \cdot k_2}{k_1 \cdot k_2 k_2 \cdot k_g} \right) \eta_{\perp}^{-1} \eta_{\perp}^{-1},
\]

where \( k_g \) is the soft gluon momentum emitted from the dijet and \( y_g \) is its rapidity. A power-counting argument shows that \( S_J(q_{\perp}) \sim 1/q_{\perp}^2 \) at small \( q_{\perp} \), and we neglect power corrections \( (q_{\perp}/P_{\perp})^n \) to this leading behavior such as coming from gluon emissions from the \( t \)-channel Pomeron (see, e.g., \cite{54}). We shall be particularly interested in the \( \phi \)-distribution where \( \phi \) is the azimuthal angle between \( q_{\perp} \) and \( P_{\perp} \). In general one can expand

\[
S_J(q_{\perp}) = S_{J0}(q_{\perp}) + 2\cos(2\phi)S_{J2}(q_{\perp}) + \cdots,
\]

where for simplicity higher harmonics are neglected in this work. Ref. \cite{49} only considered the symmetric part \( S_{J0} \) and studied its impact on the ‘primordial’ \( \langle \cos(2\phi) \rangle \) correlation from the elliptic Wigner distribution. We shall see that the \( S_{J2} \) term leads to significant consequences for dijet angular correlations. To \( O(\alpha_s) \), \( S_{J0, J2} \) can be obtained by calculating the Fourier coefficients

\[
\int_0^{2\pi} d\phi \int_{-\infty}^{\infty} dy_g \frac{k_1 \cdot k_2}{k_1 \cdot k_2 k_2 \cdot k_g} (1, \cos(2\phi)).
\]

There are collinear divergences when \( k_g \) is parallel to either \( k_1 \) or \( k_2 \). They are factorized into the jet functions associated with the two final state jets. In practice, we introduce the theta-function constraints

\[
\Theta(2\cos(y_1 - y_2) - 2\cos(\phi - R^2)) \approx 1 - \Theta(R^2 - (y_1 - y_2)^2 - \phi^2),
\]

where \( R \) is the jet radius, and similarly for the second jet. In the narrow jet approximation \( R \ll 1 \), we obtain

\[
S_{J0}(q_{\perp}) = \delta(q_{\perp}) + \frac{\alpha_0}{\pi^2} \frac{1}{q_{\perp}^2}, \quad S_{J2}(q_{\perp}) = \frac{\alpha_2}{\pi^2} \frac{1}{q_{\perp}^2},
\]

where

\[
\alpha_0 = \frac{\alpha_s C_F}{2\pi} \ln \frac{a_0}{R^2}, \quad \alpha_2 = \frac{\alpha_s C_F}{4\pi} \ln \frac{a_2}{R^2}.
\]

\( \alpha_0 \) depends on the rapidity difference \( \Delta y_{12} = |y_1 - y_2| \) as \( \alpha_0 = 2 + 2\cos(\Delta y_{12}) \). A closed analytic expression is not available for \( \alpha_2 \) except in the two limiting cases \( \Delta y_{12} = 0, \infty \), but it has very mild dependence on \( \Delta y_{12} \). It increases slightly from \( \alpha_2 = 1/4 \) for \( \Delta y_{12} = 0 \) to \( \alpha_2 = 1/e \) for \( \Delta y_{12} \to \infty \).

We observe that the \( \cos(2\phi) \) term is not power suppressed, which is consistent with the finding in Ref. \cite{42}. In the small-\( R \) limit, the ratio \( \alpha_2/\alpha_0 \) is essentially unity, and the \( \phi \) distribution reduces to two delta functions around 0 and \( \pi \). However, for realistic values of \( R \), typically \( \alpha_2/\alpha_0 \lesssim 0.2 \) because \( \alpha_0 \approx 2a_0 \).

To have a reliable prediction, we need to perform the resummation of higher order contributions to \( S_J \) \cite{49}. The constraint \( q_{\perp} = -k_{1g\perp} - k_{2g\perp} - \cdots \) from multiple gluon emissions in momentum space can be conveniently deconvoluted by Fourier-transforming to \( b_{\perp} \)-space,

\[
\tilde{S}_J(b_{\perp}) = \int d^2q_{\perp} e^{i\vec{q}_{\perp} \cdot \vec{b}_{\perp}} S_J(q_{\perp}),
\]

where \( \phi_b \) represents the angle between \( \vec{b}_{\perp} \) and \( \vec{P}_{\perp} \). Again we have only kept the leading two harmonics. To one-loop, we find

\[
\tilde{S}_{J0}(b_{\perp}) = 1 + a_0 \ln(\mu_b^2/P_{\perp}^2), \quad \tilde{S}_{J2}(b_{\perp}) = a_2, \quad a_2 = \frac{2e^{-\gamma_E} b_{\perp}}{b_{\perp}^2} \gamma_E \Gamma_{\perp},
\]

where \( \mu_b = e^{-\gamma_E} b_{\perp} \) and \( \gamma_E \) being the Euler constant. In the above equation, we have taken into account the virtual diagram contribution to \( S_{J0} \) whose natural scale is the jet momentum \( P_{\perp} \). Note that in the \( S_{J2} \) term, the Fourier transformation does not introduce IR divergence. We extend \( (9) \) to all orders by exponentiating to the Sudakov form factor,

\[
\tilde{S}_{J0}(b_{\perp}) = e^{-\Gamma_0(b_{\perp})}, \quad \tilde{S}_{J2}(b_{\perp}) = a_2 e^{-\Gamma_0(b_{\perp})},
\]
The anisotropy in dijet production $\gamma + A \rightarrow q\bar{q} + A$ in ultra peripheral heavy ion collisions at the LHC. The kinematics correspond to the CMS measurements \cite{34} with the leading jet $P_\perp = 35\text{GeV}$ and the two jets are at the same rapidity $\Delta y_{12} = 0$. The figure shows $\langle \cos(2\phi) \rangle$ as a function of $q_\perp$, where $\phi$ is the azimuthal angle between $q_\perp$ and $P_\perp$. Where $\Gamma_0(b_\perp) = \int_{0}^{P_{T,\perp}^2} \frac{d\mu_{b\perp}^2}{P_{T,\perp}^2}\alpha_0$. Here and in what follows, we neglect the non-global logarithms (NGLs) \cite{44, 57} which are relevant to the observable at hand, since $q_\perp$ is sensitive to soft gluons emitted outside the jet cones. While NGLs are parameterically of the same order as the single-logarithmic terms that we keep, in practice they are numerically not very significant at least for large, but not too large values of $P_\perp/q_\perp$ ($q_\perp \gg \Lambda_{QCD}$) considered in this paper. We leave the resummation of NGLs for the present observable for future work. Note that Ref. \cite{49} did include NGLs, but did not consider angular dependence.

In the fixed coupling case, the Fourier transform of \eqref{10} can be obtained analytically, and we find that $\langle \cos(2\phi) \rangle = S_{J2}(q_{\perp})/S_{J0}(q_{\perp})$ remains a constant independent of $q_{\perp}$, but its value is reduced by a factor of $(1 - \alpha_0)$ compared to the leading order result $\alpha_2/\alpha_0$. In the running coupling case, we proceed numerically and adopt the $b_\perp$-prescription \cite{58}: $\Gamma_0(b_\perp) \Rightarrow \Gamma_0(b_\perp) + g_a b_\perp^2$, where $b_\perp = b_\perp/\sqrt{1 + b_\perp^2/b_{\max}^2}$ with $b_{\max} = 1.5$ GeV$^{-1}$. The parameter $g_a$ is applied to include non-perturbative effects at large $b_\perp$, which should be order $\Lambda_{QCD}^2$. In the numerical calculations, we take three different values $g_\Lambda = 0.05, 0.1, 0.2$ GeV$^2$. In Fig. 2, we show our prediction for $\langle \cos(2\phi) \rangle$ for dijet production $\gamma + A \rightarrow q\bar{q} + A$, with $P_\perp = 35\text{GeV}$, $R = 0.4$ and $\Delta y_{12} = 0$. We have also plotted the leading order result $\alpha_2/\alpha_0 \approx 0.14$ for comparison. From this plot, we can see that the anisotropy does not change dramatically with $q_\perp$ in most kinematics except in the very small $q_\perp$ region, where the prediction is sensitive to the non-perturbative contributions as expected.

In the above result, we have neglected the possible contribution from the elliptic gluon Wigner distribution \cite{20}. The anisotropy $\langle \cos(2\phi) \rangle$ due to the elliptic Wigner is at most a few percent \cite{21, 31}, which is further suppressed after convoluted with the symmetric part $S_{J0}$ \cite{49}. It is thus negligible compared to the result in Fig. 2. As originally discussed in \cite{20}, in order to see this effect one has to measure the $\cos(2\phi)$ correlation between $P_\perp$ and the nucleon recoil momentum $\Delta_\perp$. The dijet total momentum $q_\perp$ is not suitable for this purpose because it is very sensitive to radiations in the final state.

In Ref. \cite{22}, a significant $\cos(2\phi)$ anisotropy was also found from a leading order color-glass-condensate calculation in the kinematics of $q_\perp \gg P_\perp$, which is an opposite to the correlation limit considered in this paper. Comparison of these $\cos(2\phi)$ anisotropies will help us to understand the underlying physics in dijet processes.

### III. INCLUSIVE DIJET IN $\gamma p$ COLLISIONS

In experiments at the EIC, the photoproduction contains both direct photon and resolved photon contributions. In this section, we focus on direct photon contribution, where the leading order dijet production comes from the partonic processes $\gamma g \rightarrow q\bar{q}$ and $\gamma q \rightarrow qg$. We take the example of $\gamma g \rightarrow q\bar{q}$ channel and similar derivation of $\gamma q \rightarrow qg$ can follow. In this process, we will have contributions from the soft gluon radiation associated with incoming gluon. Two important features emerge from the initial state soft gluon radiations. First, they contribute to the leading power with double logarithms at low $q_\perp$. Second, they are symmetric in the azimuthal angle distribution between $q_\perp$ and $P_\perp$. The differential cross section from the soft gluon radiation can be written as,

$$
\frac{d^4\sigma}{d\Omega} = \sigma_0 x g f_g(x_g) \frac{1}{q_\perp} \left[ \frac{\alpha_s^2}{\pi} + \frac{\alpha_s^2}{\pi} \cos(2\phi) \right], \tag{11}
$$

where $\sigma_0$ represents the leading order cross section, $d\Omega = dx_1 dx_2 dP_{T,\perp}^2 d^2 q_\perp$ for the phase space, and $f_g(x_g)$ is the gluon distribution with $x_g = P_\perp (e^{y_1} + e^{y_2})/\sqrt{2} S_{\gamma p}$ is momentum fraction of the nucleon carried by the gluon. At this order, the coefficient $\alpha_{0,2}$ can be derived from the soft gluon radiation amplitude of \cite{46},

$$
\alpha_{0}^{\gamma g} = \frac{\alpha_s}{2\pi} \left[ C_A \ln \frac{P_{T,\perp}^2}{q_\perp^2} + 2C_F \ln \frac{a_0}{R^2} \right],
\alpha_{2}^{\gamma g} = \frac{\alpha_s}{2\pi} \left[ C_A \ln \frac{a_1}{R^2} - \frac{N_c}{3} \ln \frac{a_2}{R^2} \right], \tag{12}
$$

where $a_1 = 1/e$ and $a_2$ is the same as defined in previous section. Clearly, in $\gamma g \rightarrow q\bar{q}$ process, there exist leading double logarithmic contribution, the first term in $\alpha_0$, which comes from the soft gluon radiation from the incoming gluon. Again, it is convenient to introduce the Fourier transform with respect to $q_\perp$,

$$
\frac{d^4\sigma}{d\Omega} = \sum_{ab} \sigma_0 \int \frac{d^2 b_{\perp}^2}{(2\pi)^2} e^{-i q_\perp b_{\perp}} \left[ \tilde{W}_0^{\gamma p}(|b_{\perp}|) \right. \\
\left. - 2 \cos(2\phi) \tilde{W}_2^{\gamma p}(|b_{\perp}|) \right], \tag{13}
$$
where the azimuthal symmetric term can be written as

$$\tilde{W}_{0}^{\gamma p}(b_{\perp}) = x_{g} f_{g}(x_{g}, \mu_{b}) e^{-S_{\gamma p}^{p}(P_{\perp}^{2}, b_{\perp})}.$$  \hfill (14)

we separate the Sudakov form factor $S(P_{\perp}, b_{\perp})$ into perturbative and non-perturbative parts in $\gamma p$ collisions:

$$S(P_{\perp}, b_{\perp}) = S_{\text{pert}}(P_{\perp}, b_{\perp}) + S_{\text{NP}}(P_{\perp}, b_{\perp})$$

with the perturbative part at one-loop order is defined,

$$S_{\text{pert}}^{\gamma p} = \int \frac{d \mu^{2}}{\mu^{2}} \frac{\alpha_{s} C_{A}}{2\pi} \left[ \ln \frac{P_{\perp}^{2}}{\mu^{2}} - 2\beta_{0} + \frac{2C_{F}}{C_{A}} \ln \frac{a_{0}}{R^{2}} \right]$$

where $\beta_{0} = 11/12 - N_{f}/18$. For the non-perturbative part, we follow those for the TMD quark distributions in Refs. [59, 60] with an appropriate color factor,

$$S_{\text{NP}}^{\gamma p} = \frac{C_{A}}{2C_{F}} \left[ 0.106 b_{\perp}^{2} + 0.42 \ln \frac{P_{\perp}}{Q_{0}} \ln \frac{b_{\perp}}{b_{s}} \right]$$

with $Q_{0}^{2} = 2.4$ GeV$^{2}$. For the $\cos(2\phi)$ term, we have

$$\tilde{W}_{2}^{\gamma p}(b_{\perp}) = \alpha_{g}^{g g} \tilde{W}_{0}^{\gamma p}(b_{\perp}).$$  \hfill (17)

We would like to comment on two important points before we present the numeric results. First, for inclusive dijet production, the leading double logarithms dominate the $q_{\perp}$ distribution. Second, The non-perturbative part associated with the final state jet is negligible compared to the TMD parton distribution which dominates the non-perturbative part.

In Fig. 3, we show the $\langle \cos(2\phi) \rangle$ as function of $q_{\perp}$. In the numerical calculations, we take the typical kinematics for inclusive dijet production at the EIC with $\sqrt{s_{\gamma p}} = 100$ GeV, $P_{\perp} \sim 15$ GeV and the two jets are at the same rapidity. Compared to the results in the diffractive case, the asymmetry increases with $q_{\perp}$ more rapidly. The stronger $q_{\perp}$ dependence comes from the fact that the incoming gluon distribution leads to a broader distribution. For the diffractive case with a similar kinematics at the EIC, the prediction follows that of Fig. 2 with $q_{\perp}$ scaled down by a factor of 2.5.

The $\cos(2\phi)$ asymmetry in inclusive dijet production in DIS ($Q^{2} \neq 0$) has been proposed to study the so-called linearly polarized gluon distribution in nucleon [13–15, 17]. As we have seen, the final state radiation from the dijet system contributes to exactly the same $\cos(2\phi)$ asymmetry. It is important to quantitatively compare these two contributions to unambiguously extract the linearly polarized gluon distribution. We will publish the relevant results in a separate paper.

IV. INCLUSIVE DIJET IN $pp$ COLLISIONS

We can also extend the above studies to inclusive dijet production in $pp$ collisions. Again we have to take into account the soft gluon radiation associated with incoming partons. In the following, we take the example of $gg \rightarrow gg$ channel, which is also the dominant contribution for dijet production in the typical kinematics at the LHC. The differential cross section and the resummation formula follow the similar expressions as Eqs. (11,13,14). For the soft gluon radiation contribution, we find the following results,

$$a_{s}^{gg} = \alpha_{s} C_{A} \frac{2 \ln a_{1}^{2}}{R^{2}} + \frac{t^{2} + u^{2}}{2(s^{2} - t^{2})} \ln \frac{a_{2}}{a_{1}},$$

and the associated $a_{s}^{gg}$ has been derived in Ref. [48], where $s$, $t$ and $u$ are usual Mandelstam variables in the partonic process. For simplicity, we take the leading logarithmic approximation, for example, for the azimuthal symmetric one,

$$\tilde{W}_{0}^{gg}(b_{\perp}) = x_{1} f_{g}(x_{1}, \mu_{b}) x_{2} f_{g}(x_{2}, \mu_{b}) e^{-S_{gg}^{g g}(P_{\perp}^{2}, b_{\perp})}.$$  \hfill (19)

where $f_{a,b}(x, \mu_{b})$ are parton distributions for the incident partons $a$ and $b$, $x_{1,2} = P_{\perp} (e^{y_{1}} + e^{y_{2}})/\sqrt{s}$ are momentum fractions of the incoming hadrons carried by the partons. The perturbative part is defined as one-loop order,

$$S_{\text{pert}}^{gg} = \int \frac{d \mu^{2}}{\mu^{2}} \frac{\alpha_{s} C_{A}}{\pi} \left[ \ln \frac{P_{\perp}^{2}}{\mu^{2}} - 2\beta_{0} + \ln \frac{a_{0}}{R^{2}} \right].$$  \hfill (20)

For the $\cos(2\phi)$ term, we have

$$\tilde{W}_{2}^{gg}(b_{\perp}) = \alpha_{g}^{gg} \tilde{W}_{0}^{gg}(b_{\perp}).$$  \hfill (21)

There is no contribution from the linearly polarized gluon distributions, similar to that of the photo-production process discussed in previous section.

In Fig. 4, we show the $\langle \cos(2\phi) \rangle$ as function of $q_{\perp}$ for dijet production through $gg \rightarrow gg$ channel. In the numerical calculations, we take the typical kinematics at the

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1 For the real production process, the linearly polarized gluon distribution does not contribute.
and jets are at mid-rapidity. Here we plot $\langle \cos(2\phi) \rangle$ as function and $q_{\perp}$, where $\phi$ is the azimuthal angle between $q_{\perp}$ and $P_{\perp}$.

LHC with $\sqrt{S_{pp}} = 7$ TeV, leading jet $P_{\perp} \sim 100$ GeV and the two jets are both at mid-rapidity.

In heavy ion collisions, the final state jet suffers multiple interactions with the hot QCD matter when it traverses through the medium. It also generates medium induced soft gluon radiations. Taking into account of the two effects, we have the following modifications on the Fourier transform $\tilde{W}$,

$$\tilde{W}_0 \rightarrow \tilde{W}_0^{med} = \tilde{W}_0 + Q_s^2 p_{\perp}^2 / 4,$$

$$\tilde{W}_2 \rightarrow (\alpha_2 + \alpha_2^m) \tilde{W}_0^{med},$$

where $Q_s^2 = \langle qL \rangle$ represents the medium $p_T$-broadening effects with the medium transport coefficient $\tilde{q}$ and jet traverse length $L$ and $\alpha_2^m$ for the medium induced soft gluon radiation contribution to the $\cos(2\phi)$ anisotropy. As shown in Ref. [23], the $p_T$-broadening effects does not change much the total transverse momentum distribution for the dijet production because it is dominated by the Sudakov effects, i.e., $\tilde{W}_0^{med} \approx \tilde{W}_0$ for the LHC kinematics. As a result, the $\cos(2\phi)$ anisotropy will be additive effects from the medium contribution. Because the medium induced radiation also has a collimation associated with the jet [61], this contribution will be similar to what was discussed in previous sections. Therefore, we may utilize the modification of the $\cos(2\phi)$ anisotropy as a measure of the medium induced soft gluon radiation in quark-gluon plasma in heavy-ion collisions.

V. CONCLUSIONS

In summary, we have studied the soft gluon radiation contribution to the dijet production in various hadronic collisions, focusing on the anisotropy of the total transverse momentum with respect to the leading jet. Our results have shown that these soft gluon radiation contributions lead to characteristic behaviors of $\langle \cos(2\phi) \rangle$ as functions of the total transverse momentum $q_{\perp}$. Experimental studies, as shown in a preliminary analysis of the diffractive dijet production in ultraperipheral $AA$ collisions at the LHC [34], will provide a unique opportunity to investigate the final state radiation associated with the jet. Our finding has far-reaching consequences in extracting the linearly polarized gluon distribution and the elliptic Wigner distribution from the measurement of angular anisotropies in inclusive and exclusive dijet processes. We also argued that comparing the inclusive dijet production in heavy-ion collisions to that in proton-proton collisions may lead to a novel probe to the medium interaction with the energetic jets. We expect more experimental activities in this direction in the near future.

A number of further developments shall follow. In this paper we focused on the $\cos(2\phi)$ anisotropy. Following the examples of Ref. [42], other harmonics such as the $\cos(4\phi)$ anisotropy can be derived as well. Second, we have made the narrow jet approximation in the derivations. The finite-$R$ corrections, together with the sub-kineal corrections which might be relevant at large $q_{\perp}$, should be taken into account when we compare to the precise experimental data from the LHC and future EIC. Third, we only applied the leading logarithmic resummation for all-order soft gluon radiations. A more rigorous derivation beyond this approximation needs to be carried out along the line of Refs. [41, 42]. Nontrivial features may emerge when we study higher-order corrections, e.g., following the example of semi-inclusive jet function formalism [62].

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