Frictional drag between quantum wells mediated by fluctuating electromagnetic field

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March 23, 2022

Abstract

We use the theory of the fluctuating electromagnetic field to calculate the frictional drag between nearby two- and three dimensional electron systems. The frictional drag results from coupling via a fluctuating electromagnetic field, and can be considered as the dissipative part of the van der Waals interaction. In comparison with other similar calculations for semiconductor two-dimensional system we include retardation effects. We consider the dependence of the frictional drag force on the temperature $T$, electron density and separation $d$. We find, that retardation effects become dominating factor for high electron densities, corresponding thing metallic film, and suggest a new experiment to test the theory. The relation between friction and heat transfer is also briefly commented on.

1 Introduction

A great deal of attention has been recently devoted to double layer systems in which two parallel quasi-two-dimensional (2D) subsystems (electron or hole gases) are separated by a potential barrier thick enough to prevent particles
from tunneling across it but allowing for the interaction between particles on both its sides. Some time ago, Pogrebinskii and later Price [1] predicted that the Coulomb interaction between two 2D electron systems will induce a frictional drag force between the layers: a current in one film will induce a current in the adjacent film. The first frictional drag experiment was performed by Gramila et al. for two electron layers [2] and by Sivan, Solomon and Shtrikman for an electron-hole system [4]. In these experiments a current is drawn in the first layer, while the second layer is an open circuit. Thus no dc-current can flow in the second layer, but an induced electric field occur that opposes the “drag force” from the first layer. These experiments spurred a large body of theoretical work both on electron-hole systems [5] and on electron-electron systems [6]-[15]. Most of this work focused on interlayer Coulomb interaction, the most obvious coupling mechanism and the one considered in the original theoretical papers [1], though the contributions due to an exchange of phonons between the layers have also been considered [3, 8, 9, 16]. The origin of Coulomb drag is due to quantum and thermal fluctuations of the charge and current densities and can be considered as the dissipative part of van der Waals interaction. The static aspects of van der Waals interaction is well understood, and from the theory of Lifshitz [17] it is known that one must distinguish between two distance regime: (a) The nonretarded limit, when the separation between bodies \(d\) is small compared to wavelengths \(\lambda \sim c/\omega_0\), where \(\omega_0\) is a characteristic frequency of the charge fluctuation, and \(c\) the light velocity, the interaction is determined by the fluctuations in an instantaneous Coulomb field. For metal \(\omega_0 \sim \omega_p\), where \(\omega_p\) is the plasma frequency. (b) Retardation effects become important, when \(d > \lambda\). As we have shown in Ref. [18, 19], when calculating the dissipative part of the van der Waals interaction for two semi-infinite bodies in relative motion, retardation effects become important for \(d > c/\omega_0\), where \(\omega_0 \sim \omega_p (\omega_p \tau)\) and \(\tau\) is the relaxation time. For \(\omega_p \sim 10^{16}s^{-1}\) and \(\tau \sim 10^{-14}s\) retardation effects become important for very short distances \(d > 1\) Å. However for 2D systems there is no investigation of the role of retardation effects in the frictional drag experiments. For large distances the retarded contribution to the frictional drag becomes important, and it is interesting to compare this contribution to the non-retarded contribution. To evaluate the retarded contribution from photon exchange we use the general theory of the fluctuating electromagnetic field developed by Rytov [20] and applied by Lifshitz [17] for studying the conservative part, and by us [18] for studying dissipative part of the van der Waals interaction. In this
approach the interaction between the bodies is mediated by the fluctuating electromagnetic field which is always present in the vicinity of any collection of atoms. Beyond the boundaries of a solid this field consist partly of traveling waves and partly of evanescent waves which are damped exponentially with the distance away from the surface of the body. The method we use for calculating the frictional drag between two nearby 2D systems is quite general, and is applicable to any body at arbitrary temperature. It takes into account retardation effects, which become important for large enough separation between the bodies.

We shall calculate frictional stress \( \sigma = \gamma v \) acting on the electrons in layer 1 due to the current density \( J_2 = n_2 e v \) in the layer 2, where \( n_2 \) is the carrier concentration (per unit area). If no current is allowed to flow in layer 1 (open circuit) an electric field \( E_1 \) develops whose influence cancels the frictional stress \( \sigma \) between the layers. The frictional stress \( \sigma = \gamma v \) must equal the induced stress \( n_1 e E_1 \) so that

\[
\gamma = n_1 e E_1 / v = n_1 n_2 e^2 E_1 / J_2 = n_1 n_2 e^2 \rho_{12},
\]

where the transresistivity \( \rho_{12} = E_1 / J_2 \) is defined as the ratio of the induced electric field in the first layer to the driving current density in the second layer. The transresistivity is often interpreted in terms of a drag rate which, in analogy with a Drude model, is defined by \( \tau_D^{-1} = \rho_{12} n_2 e^2 / m^* = \sigma / n_1 m^* v. \)

We find that for modulation-doped semiconductor quantum wells retardation effects are not important under typical experimental conditions, supporting earlier calculations where retardation effects always have been neglected \[5, 6\]. However, although previous calculation for friction drag between two-dimensional semiconductor systems are equivalent to ours, other approaches were very different. The present derivation offers an alternative insight and is more general. A striking new result we find, that for systems with high 2D-electron density, e.g., thin metallic films, retardation effects becomes crucial and in fact dominates the frictional shear stress \( \sigma \).

To test the theoretical predictions presented below we therefore suggest performing experiments on thin metallic layers grown on insulating substrates and separated by thin insulating layers. For example, for two thin (of order monolayer) silver films separated by \( d \approx 100 \text{ Å} \), we estimate that the induced voltage \( U_1 \) in metal film 1, due to a current \( J_2 \) in layer 2, will be of order \( U_1 \approx 10^{-8} U_2 \), where \( U_2 \) is the driving voltage applied to metal film 2. Thus if \( U_2 \approx 1 \text{ V} \) the induced voltage will be of order 10 pV which should be possible
to detect experimentally. We note that the study of this problem is also of
direct interest in the context of sliding friction, since the electronic friction
probed when two metallic bodies slide relative to each other, should be the
same as the electronic friction probed by the transresistivity measurement,
see Fig. 1. The electronic sliding friction (usually called vacuum friction)
has recently been invoked to explain experimental results for the damping of
a small metal particle vibrating in the vicinity of a flat metal surface [21],
but this explanation is controversial [19], and it is clear that independent
studies of the electronic friction would be of great interest.

2 Calculation of the fluctuating electromagnetic field

Let us firstly calculate the fluctuating electromagnetic field from one 2D
system, surrounded by a dispersionless dielectric medium. We introduce a
coordinate system with the \(xy\)-plane in the 2D layer. Following Lifshitz
[17], to calculate the fluctuating field we shall use the general theory due
to Rytov, which is described in his book [20]. This method is based on
the introduction of a “random” field in the Maxwell equations (just as, for
example, one introduce a “random” force in the theory of Brownian motion
of a particle). For a monochromatic field (time factor \(\exp(-i\omega t)\)) in a dielectric,
nonmagnetic medium, these equations are:

\[
\nabla \times \mathbf{E} = i \frac{\omega}{c} \mathbf{B} \\
\nabla \times \mathbf{H} = -i \frac{\omega}{c} \mathbf{D} + \frac{4\pi}{c} (\mathbf{j} + \mathbf{j}_f) \delta(z)
\]

where, following to Rytov, we divided the total current density \(\mathbf{j}_{tot}\) into two
parts, \(\mathbf{j}_{tot} = \mathbf{j} + \mathbf{j}_f\), the fluctuating current density \(\mathbf{j}_f\) associated with thermal
and quantum fluctuations, and the current density \(\mathbf{j}\) induced by the electric
field \(\mathbf{E}\). \(\mathbf{D}, \mathbf{H}\) and \(\mathbf{B}\) are the electric displacement field, the magnetic and
the magnetic induction fields, respectively. For nonmagnetic medium \(\mathbf{B} = \mathbf{H}\)
and \(\mathbf{D} = \varepsilon \mathbf{E}\), where \(\varepsilon\) is the dielectric constant of the surrounded media.
Eliminating \(\mathbf{B}\) and \(\mathbf{H}\) from (1) one get

\[
\nabla^2 \mathbf{E} + \left(\frac{\omega}{c}\right)^2 \varepsilon \mathbf{E} - \frac{4\pi}{\varepsilon} \nabla (\rho + \rho_f) \delta(z) + \frac{4\pi i \omega}{c^2} (\mathbf{j} + \mathbf{j}_f) \delta(z) = 0
\]
where the total charge density $\rho + \rho_f$ is the sum of the induced and fluctuating electron densities. We represent the current and electron densities in the form of a Fourier series

$$j(r) = \frac{1}{\sqrt{A}} \sum_q j(q) e^{i q \cdot r}$$

$$\rho(r) = \frac{1}{\sqrt{A}} \sum_q \rho(q) e^{i q \cdot r}$$

where $q$ and $r$ are 2D vectors in the $xy$-plane and $A$ is the surface area. From the equation of continuity

$$\frac{\partial \rho}{\partial t} + \nabla \cdot j = 0$$

one get $\omega \rho = q \cdot j$. Then (2) takes the form

$$\frac{d^2 E(q)}{d^2 z} + p^2 E(q) - \frac{4\pi i q}{\varepsilon \omega} q \cdot (j + j_f) \delta(z) - \frac{4\pi \hat{z}}{\varepsilon \omega} q \cdot (j + j_f) \delta'(z)$$

$$+ \frac{4\pi i \omega}{c^2} (j + j_f) \delta(z) = 0$$

where

$$p = \sqrt{\left(\frac{\omega}{c}\right)^2 \varepsilon - q^2}$$

and $\hat{z}$ is a unit vector along the $z$-axis. We shall consider separately the two cases where the electric field $E$ is in the plane determined by the vectors $\hat{q} = q/q$ and $\hat{z} (\rho-\text{polarized waves})$ and perpendicular to this plane along the vector $n = \hat{z} \times \hat{q} (s-\text{polarized waves}).$

Let us firstly suppose that $E$ is parallel the vector $n$. In this case (3) gives

$$\frac{d^2 E_n}{d^2 z} + p^2 E_n + \frac{4\pi i \omega}{c^2} \left(\sigma_t(q, \omega) E_n + j_{fn}\right) \delta(z) = 0$$

where we have used Ohm’s law

$$j_n = \sigma_t(q, \omega) E_n$$

where $\sigma_t$ is the transverse conductivity of the layer. The solution of (7) can be written in the form

$$E_n = u_n e^{ip|z|}$$

5
From (7) one can get the following boundary conditions at \( z = 0 \)

\[
\frac{dE_n}{dz} \bigg|_{z=+0} - \frac{dE_n}{dz} \bigg|_{z=-0} = -\frac{4\pi i \omega}{c^2} \left( \sigma_t(q, \omega)E_n + j_{fn} \right)
\] 

Substituting (9) into boundary conditions (10) we get the following relation between the fluctuating current density \( j_{fn} \) and electric field \( E_n \)

\[
j_{fn} = - \left( \sigma_t + \frac{pc^2}{2\pi \omega} \right) E_n(z = 0)
\] 

On the other hand the electric field can be calculated using linear response theory [22]. The Hamiltonian of the system has the form \( \hat{H} + \hat{H}_{\text{int}} \), where \( \hat{H} \) is the Hamiltonian of the body and radiation field, while

\[
\hat{H}_{\text{int}} = -\frac{1}{c} \int \hat{A}_n(r) j_{fn}(r)e^{-i\omega t} d^2r = -\frac{1}{c} \sum_q \hat{A}_n(-q) j_{fn}(q, \omega)e^{-i\omega t}
\] 

Accordingly to the linear response theory the average value of the vector potential is determined by

\[
\langle \hat{A}_n(q, \omega) \rangle = \alpha_t(q, \omega) j_{fn}(q, \omega)/c
\] 

Taking into account that \( \mathbf{E} = (i\omega/c) \mathbf{A} \), from comparison (11) and (13) we obtain

\[
\alpha_t^{-1} = -\frac{i\omega}{c^2} \left( \sigma_t + \frac{pc^2}{2\pi \omega} \right)
\] 

Using fluctuation-dissipation theorem [23] we get

\[
\langle j_{fn}(q, \omega) j_{fn'}(q', \omega') \rangle = \frac{\hbar c^2}{2\pi i} \left( \frac{1}{2} + n(\omega) \right) \left( \alpha_t^{*-1} - \alpha_t^{-1} \right) \delta_{qq'} \delta(\omega - \omega')
\]

\[
= \frac{\hbar \omega}{\pi} \left( \frac{1}{2} + n(\omega) \right) \text{Re} \sigma_t \delta_{qq'} \delta(\omega - \omega'),
\] 

where the Bose-Einstein factor

\[
n(\omega) = \frac{1}{e^{\hbar \omega/k_B T} - 1},
\]

\( T \) is the temperature and \( \text{Re} \sigma \) is the real part of conductivity. For \( q < (\omega/c) \sqrt{\varepsilon} \) (15) includes a term which does not depend on the conductivity,
and which result from the second term in the expression (14). This term is associated with the black-body radiation which exist in the dielectric even without the 2D layer. Thus this term is irrelevant to the problem under consideration, and must be omitted. For \( p \)- polarized waves we get a similar expression as (15), except we must replace \( \sigma_t \rightarrow \sigma_l \), where \( \sigma_l \) is the longitudinal conductivity of the layer. Since the \( s \)- and \( p \)- polarized waves are not coupled, the average value of the product \( \langle j_{1q}(\mathbf{q}, \omega)j_{1q'}^{*}(\mathbf{q}', \omega') \rangle = 0 \).

Let us now consider two parallel 2D electron layers separated by a distance \( d \). We introduce two reference systems \( K \) and \( K' \), with coordinate axes \( xyz \) and \( x'y'z' \). The \( xy \)- and \( x'y' \)- planes coincide with layer 1, with the \( x \)- and \( z \)- axes pointing in the same direction, and the \( z' \)- axes pointing toward layer 2. In the \( K \) system both layers are at rest. Assume now that in the layer 2 the conduction electrons move with the drift velocity \( v \), corresponding to the current density \( j_2 = n_2 e v \), while no current flow in layer 1. The \( K' \) reference system moves with velocity \( v \hat{x} \) along to the \( x \)- axis relative to frame \( K \). In the \( K' \) frame there is no current density in layer 2, while the surrounding dielectric moves with velocity \( -v \hat{x} \).

In the \( K \) frame for \( z < d \) the Maxwell equations have the form (1) with \( j \) and \( j_f \) replaced by \( j_1 \) and \( j_f^1 \), respectively. After decomposition of the components of the electromagnetic field into a Fourier series the general solution of the Maxwell equation for \( z < d \) can be written in the form

\[
E = \begin{cases} 
  v e^{ipz} + w e^{-ipz}, & 0 < z < d \\
  u_1 e^{-ipz}, & z < 0
\end{cases}
\]

\[
B = \begin{cases} 
  ( [q \times v] + p [\hat{z} \times v]) e^{ipz} + ([q \times w] - p [\hat{z} \times w]) e^{-ipz}, & 0 < z < d \\
  ([q \times u_1] - p [\hat{z} \times u_1]) e^{-ipz}, & z < 0
\end{cases}
\]

where \( v, w \) and \( u_1 \) satisfy the transversality conditions

\[
v \cdot q + pv_z = 0, \quad w \cdot q - pw_z = 0, \quad u_1 \cdot q + pu_{1z} = 0
\]

(18)

We now decompose the electromagnetic field into \( s \)- and \( p \)- polarized waves. The boundary conditions at \( z = 0 \) for \( s \)- polarized waves is determined by (10). For \( p \)-polarized waves, from (7) one obtain the boundary conditions

\[
\begin{align*}
  E_q(z) &= +0) = E_q(z = -0) \\
  \frac{dE_q}{dz}|_{z=+0} - \frac{dE_q}{dz}|_{z=-0} &= -\frac{4\pi i p^2}{\varepsilon\omega} (\sigma_{ll}(q, \omega)E_q + j_{f1q})
\end{align*}
\]

(19)
From (10) and (19) we can obtain the following equations:

\[ v_q + R_{1p} w_q = -\frac{4\pi pj_{f1q}}{\varepsilon \omega (\varepsilon_{1p} + 1)} \]  

\[ v_n + R_{1s} w_n = -\frac{4\pi \omega j_{f1n}}{pc^2(\varepsilon_{1s} + 1)} \]  

where \( v_q = \hat{q} \cdot \mathbf{v} \) and so on, and

\[ R_{1s(p)} = \frac{\varepsilon_{1s(p)} - 1}{\varepsilon_{1s(p)} + 1}, \quad \varepsilon_{1s} = \frac{4\pi \omega \sigma_t}{pc^2} + 1, \quad \varepsilon_{1p} = \frac{4\pi \omega \sigma_t}{\omega \varepsilon} + 1 \]

The Maxwell equations in the \( K' \)-system for \( z > 0 \) have the same form as (1) with \( j \to j_2 \) and \( j_f \to j_{f2} \). However, to first order in \( v/c \) the relations between \( D, E, \) and \( B, H \) are

\[ D = \varepsilon E - (\varepsilon - 1)\frac{v}{c} \hat{x} \times B \]  

\[ H = B - (\varepsilon - 1)\frac{v}{c} \hat{x} \times E \]  

After eliminating \( D, B \) and \( H \) from Maxwell equations, and writing \( E, j \) and \( j_f \) in Fourier series we get

\[ \frac{d^2 \mathbf{E}'(\mathbf{q}')}{d^2 z} + p'^2 \mathbf{E}'(\mathbf{q}') - \frac{4\pi ip'^2}{\varepsilon \omega} \hat{q}' \cdot (\mathbf{j}_2 + \mathbf{j}_{f2}) \mathbf{e}_q \delta(z - d) - \frac{4\pi i}{\varepsilon \omega} q' \cdot (\mathbf{j}_2 + \mathbf{j}_{f2}) \delta'(z - d) \]

\[ + \frac{4\pi i \omega}{c^2} \mathbf{n}' \cdot (\mathbf{j}_2 + \mathbf{j}_{f2}) \mathbf{n}' \delta(z - d) - \frac{4\pi i \beta q_y}{\varepsilon c} (j_{n'q'} + j_{q'n'}) \delta(z - d) = 0 \]  

where

\[ p' = \sqrt{\left(\frac{\omega'}{c}\right)^2 \varepsilon' - q'^2}, \quad \varepsilon' = \varepsilon + \frac{2\beta q_x c}{\omega}, \quad \beta = (\varepsilon - 1)v/c \]

Under a Lorentz transformation, with accuracy to the term linear in \( v/c \), we have \( \omega' = \omega - q_x v \) and \( q' = q - \hat{x} \omega v/c^2 \). Note also that \( p \) is invariant under the Lorentz transformation, i.e. \( p = p' \). The last term in (24) gives rise to a coupling between \( s- \) and \( p- \) polarized waves. However, it can be shown
that this coupling gives a corrections $\sim (v/c)^2$ to the frictional drag force between the layers, so this term can be omitted. The solution of the Maxwell equations in the $K'$ reference frame can be written as

$$
E' = \begin{cases} 
v'e^{ipz} + w'e^{-ipz}, & 0 < z < d \\
u_2e^{ipz}, & z > d 
\end{cases}
$$

(25)

From the boundary conditions for the $s-$ and $p-$ polarized waves, which follow from (24), we get the equations

$$
w'_q + R_{2p}(q', \omega')e^{2ipd}v'_q = -\frac{4\pi pj_{2q}e^{ipd}}{\varepsilon\omega' (\varepsilon_2 + 1)}
$$

(26)

$$
w'_n + R_{2s}(q', \omega')e^{2ipd}v'_n = -\frac{4\pi \omega' j_{2n}e^{ipd}}{p\varepsilon (\varepsilon_2 + 1)}
$$

(27)

The relations between the fields in the $K$ and $K'$ reference frames are determined by the Lorentz transformation. As it was shown in Ref. [18], such a Lorentz transformation gives terms of the order $(v/c)^2$. Thus we can take this transformation in zero order in $v/c$ so that $v'_q(\omega') = v_q(\omega)$, $v'_n(\omega') = (\omega'/\omega) v_n(\omega)$ and similar equations for $w$. After the transformation the solution of the system of the equations (20, 21, 26, 27) take the form

$$
v_q = \frac{4\pi p}{\Delta_p} \left[ \frac{j_{2q}(q', \omega') e^{ipd} R_{1p}(q, \omega)}{(\varepsilon_2 + 1)(\varepsilon_1)(q, \omega)} - \frac{j_{1q}(q, \omega)}{(\varepsilon_1)(q, \omega)} \right]
$$

(28)

$$
w_q = \frac{4\pi p}{\Delta_p} \left[ \frac{j_{1q}(q, \omega) e^{ipd} R_{2p}(q', \omega')}{(\varepsilon_2)(q', \omega') + 1} - \frac{j_{2q}(q', \omega') e^{ipd}}{(\varepsilon_2)(q', \omega')} \right]
$$

(29)

$$
v_n = \frac{4\pi \omega}{\Delta_s p^2} \left[ \frac{j_{2n}(q', \omega') e^{ipd} R_{1s}(q, \omega)}{(\varepsilon_1)(q, \omega)} - \frac{j_{1n}(q, \omega)}{(\varepsilon_1)(q, \omega)} \right]
$$

(30)

$$
w_n = \frac{4\pi \omega}{\Delta_s p^2} \left[ \frac{j_{1n}(q, \omega) e^{ipd} R_{2s}(q', \omega')}{(\varepsilon_1)(q', \omega') + 1} - \frac{j_{2n}(q', \omega') e^{ipd}}{(\varepsilon_1)(q', \omega')} \right]
$$

(31)

$$
v_z = -\frac{qv_q}{p}, \quad w_z = \frac{qw_q}{p}
$$

(32)

where we have introduce the notation

$$
\Delta_p = 1 - e^{2ipd} R_{2p}(q', \omega') R_{1p}(q, \omega)
$$

$$
\Delta_s = 1 - e^{2ipd} R_{2s}(q', \omega') R_{1s}(q, \omega)
$$

9
3 Calculation of the frictional drag force between 2D systems

The frictional drag stress $\sigma$ which acts on the conduction electrons in layer 1 can be obtained from the $xz-$ component of the Maxwell stress tensor $\sigma_{ij}$, evaluated at $z = \pm 0$

$$\sigma = \frac{1}{8\pi} \int_{-\infty}^{+\infty} d\omega \left\{ \varepsilon \langle E_z E_x^* \rangle + \langle B_z B_x^* \rangle + c.c \right\}_{z=+0} - \left\{ \cdots \right\}_{z=-0}$$  (33)

Here the $\langle \cdots \rangle$ denote statistical averaging over the fluctuating current densities. The averaging is carrying out with the aid of (15) for $s-$polarized waves and the similar equation for $p-$ polarized waves. Note that the components of the fluctuating current density $j_1$ and $j_2$ refer to different layers, and are statistically independent, so that the average of their product is zero. Expanding the electric field and magnetic induction in Fourier series we obtain

$$\sigma = \frac{1}{8\pi} \int \frac{d\omega d^2q}{(2\pi)^2} q \left\{ \varepsilon \langle E_z (q, \omega) E_x^* (q, \omega) \rangle + \langle B_z (q, \omega) B_x^* (q, \omega) \rangle + c.c \right\}_{z=+0} - \left\{ \cdots \right\}_{z=-0}$$  (34)

For a given value of $q$ it is convenient to express the component $E_x$ and $B_x$ in terms of the components along the vectors $\hat{q}$ and $n$

$$E_x = \frac{q_x}{q} E_q - \frac{q_y}{q} E_n$$  (35)
$$B_x = \frac{q_x}{q} B_q - \frac{q_y}{q} B_n$$  (36)

After substitution of expressions (35,36) into (34) and taking into account that the term which is proportional to $q_y$ is equal to zero [18], we obtain

$$\sigma = \frac{1}{8\pi} \int \frac{d\omega d^2q}{(2\pi)^2} q \left\{ \varepsilon \langle E_z (q, \omega) E_x^* (q, \omega) \rangle + \langle B_z (q, \omega) B_x^* (q, \omega) \rangle + c.c \right\}_{z=+0} - \left\{ \cdots \right\}_{z=-0}$$  (37)

where

$$E_z(z = +0) = (v_z + w_z) = (q/p)(w_q - v_q) = (qp^*/|p|^2)(w_q - v_q)$$  (38)
$$E_z(z = -0) = u_{1z} = (q/p)u_q = (q/p)(w_q + v_q)$$  (39)
After substituting these expressions into formula (37) we obtain

\[
\sigma = \frac{1}{16\pi^2} \int_0^{\infty} d\omega \int \frac{d^2 q}{q^2} \left( \frac{\varepsilon}{p^2} \left[ (|p + p^*|)^2 - |\langle |v_q| \rangle - |\langle |v_n| \rangle \right] 
- (|v_q + w_q|^2) + (p - p^*)\langle |v_q w^*_q - v_q w^*_q| \rangle 
+ \left( \frac{c}{\omega} \right)^2 \left[ (p + p^*)\langle |w_n|^2 \rangle - |\langle |v_n| \rangle - |\langle |v_n + w_n|^2 \rangle 
- (p - p^*)\langle |v_n w^*_n - v_n w^*_n| \rangle \right] \right)
\]

(44)

where we integrate only over positive values of \( \omega \), which gives an extra factor of two.

Substituting (28) and (32) into (44) and taking into account that \( p = p^* \) for \( q < \omega/c \) and \( p = -p^* \) for \( q > \omega/c \), we obtain

\[
\sigma = \frac{\hbar}{8\pi^3} \int_0^{\infty} d\omega \int_{q<\omega/c} d^2 q d\omega \left[ T_{1p}(\omega)T_{2p}(\omega - q_x v)(n(\omega - q_x v) - n(\omega)) 
- \left( |1 - e^{2ipd} R_{1p}(\omega) R_{2p}(\omega - q_x v) |^2 
- T_{1p}(\omega)(|1 - R_{1p}(\omega)|^2) + (|1 - e^{ipd} R_{1p}(\omega)|^2)(n(\omega) + 1/2) \right) 
\right]
\]

\[
\times \frac{\hbar}{2\pi^3} \int_0^{\infty} d\omega \int_{q>\omega/c} d^2 q d\omega e^{-2|p|d} 
\times \frac{\text{Im} R_{1p}(\omega)\text{Im} R_{2p}(\omega - q_x v)}{1 - e^{-2|p|d} R_{1p}(\omega) R_{2p}(\omega - q_x v) |^2} (n(\omega - q_x v) - n(\omega)) 
+ [p \to s]
\]

(45)

where

\[
T_{1p}(\omega) = 1 - |R_{1p}|^2 - |1 - R_{1p}|^2 = \frac{16\pi \text{Re} \sigma (\omega)p}{\omega \varepsilon |\epsilon_{it} + 1|^2}
\]

\[
T_{is}(\omega) = 1 - |R_{is}|^2 - |1 - R_{is}|^2 = \frac{16\pi \text{Re} \sigma (\omega)\omega}{pc^2 |\epsilon_{it} + 1|^2}
\]
The first integral in (66) is the contribution to the frictional drag force from propagating electromagnetic waves. This integral contains terms which formally diverge upon integration over \( \omega \). These terms are proportion to \( \omega^{-1} \) at large frequencies and appear as a result of the expansion of the reflection factor \( R_{2p(s)}(\omega - q_x v) \) in power series in \( q_x v \), and upon performing the \( q \) integration. A similar divergence also occur in the derivation of the static van der Waals interaction \[17\], and result from zero point vacuum fluctuations of electromagnetic field. The solution of this problem consists of subtraction from the integrand the terms which do not depend from separation between layers \( d \) in the limit \( d \to \infty \). In our case this procedure consists of subtraction from the first integrand in (66) the same expression taken at \( T = 0 \) K and with denominator equal to unity. The second term in (66) is derived from the evanescent field.

4 Some limiting cases

Consider distances \( d << d_W \sim c h/k_B T \) (at \( T = 3 K \) we have \( d_W \sim 10^6 \text{Å} \)). In this case we can neglect by the first integral in (66), put \( p \approx i q \) and extend the integral over \( q \) to the whole \( q \)− plane. Using these approximations, the second integral in (66) can be written as

\[
\sigma = \frac{\hbar}{2 \pi^3} \int_{-\infty}^{+\infty} dq_y \int_0^{\infty} dq_x q_x e^{-2qd} \\
\times \left\{ \int_0^{\infty} d\omega [n(\omega) - n(\omega + q_x v)] \\
\times \left[ \left( \frac{\text{Im} R_{1p}(\omega) \text{Im} R_{2p}(\omega + q_x v)}{1 - e^{-2qd} R_{1p}(\omega) R_{2p}(\omega + q_x v)} \right)^2 + (1 \leftrightarrow 2) \right] \right. \\
- \int_0^{q_x v} d\omega \left[ n(\omega) + 1/2 \right] \left[ \left( \frac{\text{Im} R_{1p}(\omega - q_x v) \text{Im} R_{2p}(\omega)}{1 - e^{-2qd} R_{1p}(\omega - q_x v) R_{2p}(\omega)} \right)^2 + (1 \leftrightarrow 2) \right] \\
\left. \right\} + (s \to p) \right\} \quad (46)
\]

The second term in this expression is proportional to \( v^2 \) as \( v \to 0 \) and can be neglected in the limit of small \( v \). In the first term we can use approximation

\[
n(\omega) - n(\omega + q_x v) \approx -q_x v \frac{dn}{d\omega} = \frac{e^{\hbar \omega/k_B T}}{(e^{\hbar \omega/k_B T} - 1)^2} \frac{\hbar q_x v}{k_B T}
\]
Thus

\[
\sigma = \frac{\hbar v}{2\pi^2} \int_0^\infty dq q^2 e^{-2qd} \int_0^\infty d\omega \left( -\frac{dn}{d\omega} \right) \left\{ \frac{\text{Im}R_{1p}(\omega)\text{Im}R_{2p}(\omega)}{|1 - e^{-2qd}R_{1p}(\omega)R_{2p}(\omega)|^2} + [p \to s] \right\}
\]

(47)

Note that \(\sigma\) is linear in the velocity \(v\). Let us describe the 2D layers in RPA approximation. Thus for \(q < k_F\) (corresponding to separations \(d > k_F^{-1}\), where \(k_F\) is the Fermi wave vector of the degenerate electron gas system; for 2D electron layer with electron density \(n_s \approx 1.5 \times 10^{11} \text{cm}^{-2}\), \(k_F = (2\pi n_s)^{1/2} \sim 10^6 \text{cm}^{-1}\)) the transverse and longitudinal parts of the conductivity for 2D electron layer can be written in the form [25, 26]

\[
\sigma_l = \frac{i\omega e^2 n_s}{q^2 \epsilon_F} \left\{ \frac{\omega \overline{u}}{(\omega + i\gamma)\sqrt{\overline{u}^2 - 1} - i\gamma \overline{u}} - 1 \right\}
\]

(48)

\[
\sigma_t = -\frac{2ie^2 n_s \overline{u}(\sqrt{\overline{u}^2 - 1} - \overline{u})}{m^*(\omega + i\gamma)}
\]

(49)

where \(\overline{u} = (\omega + i\gamma)/qv_F\), \(\gamma = 1/\tau\), \(v_F = \hbar k_F/m^*\) is the Fermi velocity, \(\tau\) is a relaxation time, \(\epsilon_F = \hbar^2 k_F^2/2m^*\) is the Fermi energy. In the experiment [2, 3] \(m^* = 0.067 m_e\), \(v_F = 1.6 \times 10^7 \text{cm/s}, \epsilon_F \sim 60 \text{K}\) and the mobility \(\mu \sim 2 \times 10^6 \text{cm}^2/\text{Vs}\), so that \(\tau \sim 7.6 \times 10^{-11} \text{s}\). Let us divide the integration over \(0 < q < \infty\) into the two parts \(0 < q < \omega/v_F\) and \(\omega/v_F < q < \infty\). In the first part of integration \(\overline{u} > 1\), and taking the limit \(\overline{u} \gg 1\) we obtain in this limit the Drude formula for conductivity

\[
\sigma_l = \sigma_t = \frac{ie^2 n_s}{m^*(\omega + i\gamma)}
\]

(50)

In the second part of integration \(\overline{u} < 1\) and taking the limit \(\overline{u} \ll 1\) we obtain

\[
\sigma_l = \frac{\omega e^2 n_s}{q^2 \epsilon_F} (u - i)
\]

(51)

\[
\sigma_t = \frac{e^2 n_s v_F}{\epsilon_F q} ;
\]

where we put \(\gamma\) equal to zero because it gives only a small contribution in this limit.
Let us consider the case of small separation $d$ when $a = (2k_B T d / \hbar v_F) < 1$. Introducing the dimensionless variables $q = x/2d$ and $\omega = (k_B T / \hbar) y$ we obtain in this limit for $ay < x < \infty$

$$R_p = \frac{\lambda_p(x + iay)}{x^2 + \lambda_p(x + iay)} \approx 1 - \frac{x^2}{\lambda_p(x + iay)}$$

$$R_s = \frac{i \lambda_s y}{i \lambda_s y - x^2}$$

and for $0 < x < ay$

$$R_p = \frac{\lambda'_p x}{\lambda'_p x - 2y^2 - 2iy \delta}$$

$$R_s = \frac{\lambda'_s y}{2xy + \lambda'_s y + i2x \delta}$$

where

$$\lambda_p = \frac{8\pi e^2 n_s d}{\varepsilon m^* \varepsilon_F^2}, \quad \lambda_s = 8\pi a d \left( \frac{e^2 n_s}{m^* c^2} \right), \quad \lambda'_p = \frac{2\pi n_s e^2}{\varepsilon m^* d} \left( \frac{\hbar}{k_B T} \right)^2, \quad \lambda'_s = \frac{8\pi n_s e^2 d}{m^* c^2}$$

We note that the expression (54) has pole at

$$\omega^2 = \frac{2\pi n_s e^2}{\varepsilon m^*} q$$

what corresponds to the plasmon excitations [28]. After substituting (52) (55) in (57) we obtain for the frictional drag rate

$$\tau^{-1}_{Dp} \approx 0.2360 \left( \frac{kT}{\epsilon_F} \right)^2 \left( \frac{q_T F d}{k_F d} \right)^2 + 10 \left( \frac{k_B T}{\epsilon_F} \right)^5 \left( \frac{k_B T}{\epsilon_{TF}} \right)^2 \gamma$$

$$\tau^{-1}_{Ds} \approx 3.3 \cdot 10^{-5} \left( \frac{k_B T}{m^* c^2} \right) \left( 4 \frac{k_B T}{\hbar} + \gamma \right)$$

where $\tau^{-1}_{Dp}$ and $\tau^{-1}_{Ds}$ are the contributions from $s-$ and $p-$ polarized waves, respectively, $q_{TF} = 2e^2 m^*/\hbar^2 \varepsilon$ is the single-layer Thomas-Fermi screening wavevector, $\epsilon_{TF} = \hbar^2 q_{TF}^2 / 2m^*$. The first term in (57) agrees with the result of Gramila et al [3] and Persson and Zhang [27]. From comparison (57) and (58) it follows that for

$$n_s < n_c \sim 10^2 \left( \frac{mk_BT}{\pi \hbar^2} \right) \left( \frac{\varepsilon a^2 k_B T}{m^* c^4} \right)^{1/5}$$

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the contribution from $p$-polarized waves exceeds the contribution from $s$-polarized waves for all distances $d < \hbar v_F/k_B T$. However for $n_s > n_c$ the contribution from $s$-polarized waves will dominate for $d > 10(\varepsilon/n_s)^{1/2}$. For example, for $T = 3 K$ and for the conditions of experiment of Ref.[2,3] $n_c \sim 10^{12}$ cm$^{-2}$ and we find that in this case the retardation effects are small. However retardation effects are important for high electron densities. For example, assuming that $\varepsilon = 1$ and $n_s \approx 10^{15}$ cm$^{-2}$, which correspond to about 1 monolayer of silver, we find that in this case the retardation effects are small. However retardation effects are important for high electron densities. For example, for $T = 3 K$ and for the conditions of experiment of Ref.[2,3] $n_c \sim 10^{11}$ cm$^{-2}$ and we find that in this case the retardation effects are small.

Let us estimate the voltage $U_1$ induced in a thin silver film (layer 1) (open circuit) when a current flow in another parallel silver film (layer 2). A voltage difference of order 1 pV can be measured with standard equipment so that if $U_1$ is of order pV or larger, it is possible to probe retardation effects with this experimental setup. If $L$ denote the length of the metallic films (assumed identical) in the direction of the driving current, then $U_1 = LE_1$ and $U_2 = LE_2 = LJ_2/\sigma_2$ where $\sigma_2 = n_2e^2\tau_2/m^*$ is the conductivity ($\tau_2$ is a Drude relaxation time and $m^*$ the electron effective mass). Thus, using the equation (see introduction) $\gamma = n_1n_2e^2E_1/J_2$ with $E_1/J_2 = U_1/(\sigma_2U_2) = (U_1/U_2)m^*/(n_2e^2\tau_2)$ gives $U_1 = (\gamma\tau/m^*n_1)U_2$. In a typical case $\tau = 4 \times 10^{-14}$ s and $n_1 \approx 10^{15}$ cm$^{-2}$, and from Fig. 2a, $\gamma \approx 10^{-6}$ Ns/m$^2$, giving $U_1 \approx 10^{-8} U_2$. Thus if the applied voltage $U_2 \approx 1$ V, the induced voltage would be of order 10 pV, which should be possible to measure.

5 Frictional drag between 3D systems

For high electron densities, when the thickness of the layers $h >> n^{-1/3}$, where $n$ is a volume electron density, the electrons behave as in 3D systems. It was shown in Ref.[18] that for 3D systems the frictional drag stress is also given by formula (47), where the electromagnetic reflection coefficients

$$R_{ip} = \frac{\varepsilon_ip - \varepsilon_is}{\varepsilon_ip + \varepsilon_is}, \quad R_{is} = \frac{p - s_i}{p + s_i},$$  

(60)
where \( \varepsilon_i \) is the complex dielectric constant for layer \( i \),

\[
s_i = \sqrt{\frac{\omega^2}{c^2} \varepsilon_i - q^2}
\]

Consider two identical 3D layers described by the dielectric function

\[
\varepsilon(\omega) = 1 - \frac{\omega_p^2}{\omega(\omega + i/\tau)}
\]

where \( \tau \) is the Drude relaxation time and \( \omega_p \) the plasma frequency. For \( k_B T > \hbar \gamma \), for small frequencies and \( d < c/\omega_p (\hbar \gamma/k_B T)^{1/2} \)

\[
\text{Im}R_p \approx \frac{2\varepsilon \omega}{\omega_p^2 \tau}, \quad \text{Re}R_p \approx 1,
\]

\[
\text{Re}R_s \approx 0, \quad \text{Im}R_s \approx 4 \left( \frac{\omega_p}{c} \right)^2 \frac{\omega}{\gamma q^2}, \quad \text{for} \quad q^2 > \left( \frac{\omega_p}{c} \right)^2 \frac{\omega}{\gamma},
\]

then, taking into account that for 3D systems \( n_s = nh \), (61) gives

\[
\tau_D^{-1} = \frac{13.32 \varepsilon k_B T^2}{\hbar \epsilon_F (k_F d)^2 (k_T d)^2 (k_F h) (\omega_p \tau)^2}
\]

\[
\tau_D^{-1} = \frac{\varepsilon k_B T^2 (\omega_p \tau)^2}{8\pi \hbar (m^* c^2)^2},
\]

where \( k_F = \frac{6\pi n e^2}{\epsilon_F} \) is the 3D Thomas-Fermi screening wavevector, and \( k_F = (3\pi^2 n)^{1/3} \) is the 3D Fermi wavevector. From comparison (63) and (64) it follows that, for \( d > \varepsilon^{1/2} (c/\omega_p)(\omega_p \tau)^{-1} \), the \( s \)-wave contribution exceeds the \( p \)-wave contribution. Thus, in the case of the dissipative van der Waals interaction between 3D bodies, retardation effects become important for much shorter distances in comparison with conservative one, when the retardation effects become important for \( d > c/\omega_p \).  

Fig. 3 shows the calculated shear stress for two semi-infinite silver bodies moving with the relative velocity \( v = 1 \text{m/s} \) parallel to the flat surfaces. Results are shown for the \( s \)- and \( p \)-wave contribution, where in the latter case we have taken into account non-local effects [the dashed lines show the result when the local (long-wavelength) dielectric function is used]. Results are shown for two different temperatures, \( T = 70 \text{K} \) and \( 300 \text{K} \), and the observed temperature dependence reflect both that of the temperature prefactor \( T^2 \) in the expression for the shear stress, as well as the temperature dependence of the
6 Relation between friction and heat transfer

The frictional shear stress studied above is closely related to the heat transfer from one solid to another when the solids have different temperatures. For large separation the heat transfer is given by the Stefan’s law

\[ J_z = \frac{\pi^2 k_B^4}{60 h^4 c_0^3} \left( T_1^4 - T_2^4 \right) \] (65)

where \( T_1 \) and \( T_2 \) are the temperatures of solid 1 and 2, respectively. This formula corresponds to emission of real photons. However, for short separation \( d \) it is possible for the evanescent near field to transfer energy from one solid to the other. This corresponds to photon-tunneling. In general, the heat flux (energy flow per unit area and unit time) is given by a formula very similar to that for the frictional stress \[29, 30\] :

\[
J_z = \frac{\hbar}{8\pi^3} \int_{0}^{\infty} d\omega \omega \int_{q<\omega/c} d^2 q \times \left[ \frac{(1 - | R_{1p}(\omega) |^2)(1 - | R_{2p}(\omega) |^2)(n_1(\omega) - n_2(\omega))}{1 - e^{2ipd R_{1p}(\omega) R_{2p}(\omega)} |^2} \right]
+ \frac{\hbar}{2\pi^3} \int_{0}^{\infty} d\omega \omega \int_{q>\omega/c} d^2 q e^{-2|p|d} \times \frac{\text{Im}R_{1p}(\omega)\text{Im}R_{2p}(\omega)}{1 - e^{-2|p|d R_{1p}(\omega) R_{2p}(\omega)} |^2} (n_1(\omega) - n_2(\omega))
+ [p \rightarrow s] \]

(66)

where

\[ n_1(\omega) = \left( e^{\hbar \omega/k_B T_1} - 1 \right)^{-1} \] (67)

is the Bose-Einstein factor of solid 1 and similar for \( n_2 \). Fig. 4a shows the heat transfer between two semi-infinite silver bodies separated by the distance \( d \) and at the temperatures \( T_1 = 273 \text{K} \) and \( T_2 = 0 \text{K} \). The \( s \) and \( p \)-wave contribution are shown separately, and the \( p \)-wave contribution has been calculated using non-local optics (the dashed line shows the result using local optics). It is remarkable how important the \( s \)-contribution is even for short distances. The detailed distance dependence of \( J_z \) has been studied by Ploder and Van Hove within the local optics approximation, and will not be repeated here. The nonlocal optics contribution to \((J_z)_p\), which is important
only for \( d < l \) (where \( l \) is the electron mean free path in the bulk), is easy to calculate for free electron like metals. The non-local contribution to \( \text{Im} R_p \) is given by \[ 27 \]

\[
(\text{Im}g)_{\text{surf}} = 2\xi \frac{\omega}{\omega_p} \frac{q}{k_F}.
\]

Using this expression for \( \text{Im} R_p \) in (66) gives the (surface) contribution:

\[
J_{\text{surf}} \approx \frac{\hbar \xi^2}{\omega_p^2 k_F^2 d^4} \left( \frac{k_B T_1}{\hbar} \right)^4 f(T_1/T_2)
\]

where

\[
f(T_1/T_2) = \frac{1}{4\pi^2} \int_0^\infty dx \frac{x^3 e^{-x}}{(1 - e^{-x})^2} \int_0^\infty dy y^3 \left( \frac{1}{1 - e^{-y}} - \frac{1}{1 - e^{-(T_1/T_2)y}} \right)
\]

\[= 0.1827 \int_0^\infty dy y^3 \left( \frac{1}{1 - e^{-y}} - \frac{1}{1 - e^{-(T_1/T_2)y}} \right) \to 1.186\]

as \( T_2/T_1 \to 0 \). Note from Fig. 4a that the local optics contribution to \( (J_z)_p \) depends nearly linearly on \( 1/d \) in the studied distance interval, and that this contribution is much smaller than the \( s \)-wave contribution. Both these observations differ from Ref. \[31\], where it is stated that the \( s \) contribution can be neglected for small distances and that the \( p \)-wave contribution (within local optics) is proportional to \( 1/d^2 \) for small distances. However, for the very high-resistivity materials, the \( p \)-wave contribution becomes much more important, and a crossover to a \( 1/d^2 \)-dependence of \( (J_z)_p \) is observed at very short separations \( d \). This is illustrated in Fig. 4b and 4c, which have been calculated with the same parameters as in Fig. 4a, except that the electron mean free path has been reduced from \( l = 560 \) Å (the electron mean free path for silver at room temperature) to 20 Å (roughly the electron mean free path in lead at room temperature) (Fig. 4b) and 3.4 Å (of order the lattice constant, representing the minimal possible mean free path) (Fig. 4c). Note that when \( l \) decreases, the \( p \) contribution to the heat transfer increases while the \( s \) contribution decreases. Since the mean free path cannot be much smaller than the lattice constant, the result in Fig. 4c represent the largest possible \( p \)-wave contribution for normal metals. However, the \( p \)-wave contribution may be even larger for other materials, e.g., semimetals, with lower carrier concentration than in normal metals. This fact has already
been pointed out by Pendry: the p-wave contribution for short distances is expected to be maximal when the function

$$\text{Im} R_p \approx \text{Im} \frac{\varepsilon - 1}{\varepsilon + 1} = \text{Im} \left[ 1 - 2 \frac{\omega}{\omega_p} \left( \frac{\omega}{\omega_p} + \frac{i}{\omega_p \tau} \right) \right]^{-1}$$

is maximal with respect to variations in $1/\tau$. This gives:

$$\omega_p \tau = \frac{2kB T}{\hbar \omega_p}$$

where we have used that typical frequencies $\omega \sim k_B T/\hbar$. Since the DC resistivity $\rho = 4\pi/(\omega_p^2 \tau)$ we get (at room temperature) $\rho \approx 2\pi \hbar/k_B T \approx 0.14 \ \Omega \text{cm}$.

7 Summary and conclusion

We have used a general theory of a fluctuating electromagnetic field to calculate the frictional drag force between 2D and 3D electron systems. The separation $d$ between the parallel electron layers is assumed to be so large that the only interaction between the layers is via the electromagnetic field associated with thermal and quantum fluctuations in the layers; the resulting friction force can be considered as the dissipative part of the van der Waals interaction. A general formula has been obtained, in which the frictional drag force is expressed through the electromagnetic reflection coefficients for $s$ and $p-$waves. We have found that the non-retarded Coulomb interaction, connected with evanescent $p-$polarized waves, is the predominant process for small layer separations and small electron densities. For high electron densities retardation effects (connected with evanescent $s-$polarized waves) become very important, and we have suggested a new experiment, involving thin metallic films, where the theory can be tested. We have shown that retardation effects are even more important for interaction between 3D electron systems. For very large separations the interaction is dominated by the traveling electromagnetic waves, which results from black-body radiation. However, the latter interaction appears negligible in comparison with phonon mediated process. Finally, we have pointed out the close relation between heat transfer and friction.

Acknowledgment
FIGURE CAPTIONS

Fig. 1 Left: a metallic block sliding relative to the metallic substrate with the velocity $v$. An electronic frictional shear stress $\sigma$ will act on the block (and on the substrate). Right: The shear stress $\sigma$ can be measured if instead of sliding the upper block, a voltage $U_2$ is applied to the block resulting in a drift motion of the conduction electrons (velocity $v$). The resulting frictional stress on the substrate electrons will generate a voltage difference $U_1$ (proportional to $\sigma$) as indicated in the figure, which can be measured experimentally.

Fig. 2 The shear stress as a function of the distance $d$ between the surfaces (the log-function is with basis 10). (a) For $\sim$monolayer-films of silver for two different temperatures. The s and p-wave contributions are shown separate. In the calculation $\tau = 4 \times 10^{-14}$ s and $20 \times 10^{-14}$ s for $T = 273$ K and 77 K, respectively. We have assumed $n_s = 1.05 \times 10^{19} \text{m}^{-2}$, $m^* = m_e$, and $v_F = 1.4 \times 10^6 \text{m/s}$. (b) For quantum wells at $T = 3$ K. In the calculation $\tau = 7.6 \times 10^{-11}$ s, $n_s = 1.5 \times 10^{15} \text{m}^{-2}$, $m^* = 0.067 m_e$, and $v_F = 1.6 \times 10^6 \text{m/s}$.

Fig. 3 The shear stress as a function of the distance $d$ between the surfaces of two semi-infinite silver bodies. The s and p wave contributions are shown separately and for two different temperatures, $T = 70$ K and 300 K. The p-wave contribution has been calculated both using a local dielectric function (dashed lines) and using a theory which takes into account nonlocality within the jellium model.

Fig. 4 (a) The heat transfer flux between two semi-infinite silver bodies, one at temperature $T_1 = 273$ K and another at $T_2 = 0$ K. (b) The same as (a) except that we have reduced the electron relaxation time $\tau$ for solid 1 from a value corresponding to a mean free path $v_F \tau = l = 560 \text{Å}$ to 20 Å. (c) The same as (a) except that we have reduced $l$ to 3.4 Å.

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\[ J_2 = n_2 ev \]
