How one can estimate relativistic contribution into nucleon observables in the relativistic constituent quark model

T.P. Ilichova

Francisk Skaryna Gomel State University, Laboratory of Particle Physics, JINR

Talk at the VIth International School-Seminar “Actual Problems of High Energy Physics” August 7-16, 2001, Gomel, Belarus

Abstract

A nonrelativistic decomposition for the quark energy by the ratio of the dispersion of quark momentum squared and the effective quark mass is investigated in the framework of the relativistic oscillator constituent quark model as bound systems of three valence quarks. It is shown that relativistic corrections are defined by dispersion of the squared absolute value of the quark momentum. The variations of the quark mass and oscillator parameter are studied in detail both in the spectrum and in the nucleon magnetic moments.

The nonrelativistic constituent quark model gives good results in the study of the static properties of the nucleon, however, the dynamical characteristics require calculation of the pionic cloud or/and including quark structure parameters.

This might be an indication that a static parameters are weakly sensitive to the quark structure, and the quark anomalous magnetic moments are small and the quarks show no structure in the observables of the baryons magnetic moments. In this case a spectrum and magnetic moments of the nucleon can be used for the study a relativistic contribution separately from the contribution of quark structure. The static parameters also weakly depend on a potential type [1, 2]. Therefore, choosing particular type of the potential one can in general conclude about the relativistic contribution into model observables and spread these conclusions into dynamical properties in our model.

We will consider a generalization [3] of the nucleon nonrelativistic oscillator model [1] based on a relativistic quasipotential equation for wave
function of the relative motion $\varphi$:  
\[
\delta \left( \sum_{k=1}^{3} \vec{p}_k \right) \left( \sum_{k=1}^{3} E_{\vec{p}_k} - M \right) \varphi(\vec{p}) = \int V(\vec{p} | \vec{p}') \ast \delta \left( \sum_{k=1}^{3} \vec{p}_k' - \sum_{k=1}^{3} \vec{p}_k \right) \delta \left( \sum_{k=1}^{3} \vec{p}_k' \right) \varphi(\vec{p}') \prod_{k=1}^{3} d\Omega_{\vec{p}_k}.
\]

Where $M$ - nucleon mass, $V$ - quasipotential, $\vec{p}_k$ - quark momentum, $\vec{p}_i$ - quark momentum in the nucleon rest frame, $\vec{p}_1, \vec{p}_2, \vec{p}_3, \vec{p}_1', \vec{p}_2', \vec{p}_3'$, $d\Omega_{\vec{p}_k} = d\vec{p}_k / E_{\vec{p}_k}$ - element of volume, $m$ - quark mass.

Let us suppose that the nonrelativistic decomposition of the quark energy $E_{\vec{p}_i} = \sqrt{m^2 + \vec{p}_i^2}$, one should perform without using the ratio $\frac{\vec{p}_i}{m}$, but using a deviation momentum from average momentum, i.e. the dispersion of the quark momentum squared. For this aim we introduce the definition of the effective quark mass $m_{\text{eff}} = \sqrt{m^2 + \langle \vec{p}_k^2 \rangle}$, here $\langle f \rangle = \int d\Omega_{\vec{p}_i} d\Omega_{\vec{p}_j} |\varphi|^2 f$. The quark energy is:  
\[
E_{\vec{p}_k} = \sqrt{m^2 + \langle \vec{p}_k^2 \rangle + \frac{\Delta_k}{2m_{\text{eff}}} - \frac{\Delta_k^2}{8m_{\text{eff}}^3} + \ldots}.
\]

Let us consider the nonrelativistic decomposition of the quark energy via ratio $\Delta_k / m_{\text{eff}}: E_{\vec{p}_k} = m_{\text{eff}} + \frac{\Delta_k}{2m_{\text{eff}}} - \frac{\Delta_k^2}{8m_{\text{eff}}^3} + \ldots$. Our aim is to investigate the lowest-order relativistic corrections in the quark energy:

\[
\delta E_k \equiv \langle | E_{\vec{p}_k} - E_{\text{nonrel}}^{\vec{p}_k} | \rangle = \langle | E_{\vec{p}_k} - m_{\text{eff}} - \frac{\Delta_k}{2m_{\text{eff}}} | \rangle \approx \frac{\langle \Delta_k^2 \rangle}{8m_{\text{eff}}^3} = \\
= \frac{1}{8m_{\text{eff}}^3} \left[ \langle \vec{p}_k^4 \rangle + \langle \vec{p}_k^2 \rangle^2 - 2 \langle \vec{p}_k^2 \rangle^2 \right] = \frac{1}{8m_{\text{eff}}^3} \left[ \langle \vec{p}_k^4 \rangle - \langle \vec{p}_k^2 \rangle^2 \right] = \frac{\sigma_{\vec{p}_k}^2}{8m_{\text{eff}}^3} \tag{2}
\]

where, $\sigma_{\vec{p}_k} = \sqrt{\langle \vec{p}_k^4 \rangle - \langle \vec{p}_k^2 \rangle^2}$ - dispersion of momentum squared.

The undimension parameter can be written as $\delta E = \frac{\sum_{k=1}^{3} \delta E_k}{\sum_{k=1}^{3} E_{\vec{p}_k}} \ast 100\%$.

Now we can decompose the quark energy in the Eq. (2):  
\[
E_{\vec{p}_i} = \sqrt{m_{\text{eff}}^2 + \Delta} = m_{\text{eff}} + \frac{\Delta_k}{2m_{\text{eff}}} + W_{\vec{p}_k}^{\text{rel}} = m_{\text{eff}} + \frac{\vec{p}_k^2}{2m_{\text{eff}}} - \frac{\langle \vec{p}_k^2 \rangle}{2m_{\text{eff}}} + W_{\vec{p}_k}^{\text{rel}},
\]

where
$W^\text{rel}_k$ is the relativistic contribution. Thus,

$$\delta \left( \sum_{k=1}^{3} \frac{\hat{P}_k}{2m_{\text{eff}}} \right) \left( \frac{\hat{P}_1}{2m_{\text{eff}}} + \frac{\hat{P}_2}{2m_{\text{eff}}} + \frac{\hat{P}_3}{2m_{\text{eff}}} + \sum_k W^\text{rel}_k + 3m_{\text{eff}} - M + C \right) \varphi(\hat{P}) =$$

$$= \delta \left( \sum_{k=1}^{3} \frac{\hat{P}_k}{2m_{\text{eff}}} \right) \int V(\hat{P} | \hat{P}') \varphi(\hat{P}') |_{\sum_{\hat{p}_k=0}} d\Omega_{\hat{p}_1} d\Omega_{\hat{p}_2}$$

(3)

Here, we may perform the overdefinition of the potential and involve constant $C = -\sum_k \frac{\hat{p}_k^2}{2m_{\text{eff}}}$ into the potential $V$.

Choosing the potential $V$ in the Eq. (3) as a generalization of the 3-particle nonrelativistic oscillator, Eq. (3) can be represented as ( $k$ - harmonic oscillator parameter ) :

$$\delta \left( \sum_{k=1}^{3} \frac{\hat{P}_k}{2m_{\text{eff}}} \right) \left[ \frac{\hat{P}_1}{2m_{\text{eff}}} + \frac{\hat{P}_2}{2m_{\text{eff}}} + \frac{\hat{P}_3}{2m_{\text{eff}}} + \sum_k W^\text{rel}_k + 3m_{\text{eff}} - M - k(\nabla^2_{\hat{p}_1} + \nabla^2_{\hat{p}_2} + \nabla_{\hat{p}_1} \nabla_{\hat{p}_2}) \right] \varphi(\hat{P}) = 0$$

(4)

In order to estimate the relativistic contribution into spectrum and nucleon magnetic moments we will use wave function in the zero-order approximation. After neglecting $\sum_k W^\text{rel}_k$ in the Eq. (4) one can solve exactly the zero-order approximation equation (4) in analogous as in the nonrelativistic quark model: $\varphi^{\text{osc}} = N \exp \left( -\frac{k^2}{2\lambda^2} - \frac{k'^2}{2\lambda'^2} \right)$, here $\lambda^2 = \gamma^2/2, \lambda'^2 = 2\gamma^2/3, \gamma^2 = m_{\text{eff}} \omega, \omega = \sqrt{3k/m_{\text{eff}}}, k = \frac{1}{2}(\hat{p}_1 - \hat{p}_2), k' = (\hat{p}_1 + \hat{p}_2)$.

The calculation of nucleon current matrix element is described in Ref. [6]. The extraction of the magnetic moment has been performed using magnetic form factor: $\mu = G_M(0) = \lim_{K \to 0} G_M(K)$

$$\mu = \lim_{K \to 0} \frac{i(2\pi)^3 J_2^{\frac{3}{2}}(K, 0) \sqrt{2E_K(M + E_K)}}{e |K|} = \lim_{K \to 0} \frac{i(2\pi)^3 J_2^{\frac{3}{2}}(K, 0) 2M}{e |K|}$$

(5)

here $J_2^{\lambda\lambda'}(K, 0) \equiv \langle \lambda | K | J_\mu(0) | \lambda \rangle$

The current matrix element is represented as :

$$\langle K\lambda\tau' | J_\mu | 0\lambda\tau \rangle = 3 \int d\Omega_{p_1} d\Omega_{p_2} \varphi(\hat{P}_1, \hat{P}_2, \hat{P}_3) \lambda_{\hat{p}_1} \lambda_{\hat{p}_2} \lambda_{\hat{p}_3} \lambda_{\hat{p}_1} \lambda_{\hat{p}_2} \lambda_{\hat{p}_3}^*$$

(6)
where $D$ differs in the isospin third projection function of the nucleon 1/2$^+$ with 1% the statistic accuracy in despite of the simple random sampling. VEGAS and MISER give result faster on 20% the following methods: VEGAS \cite{8, 10}, MISER \cite{9, 10}, method of the situation results were weakly sensitive to $x$ suppose quark mass as following:

$$m_{q} = 0.65, 0.22 \div 0.26, 0.38 \div 0.39, 0.07, 0.22 \div 0.3, 0.40 \div 0.42, 0.08, 0.24 \div 0.3, 0.42 \div 0.44, 0.095, 0.22 \div 0.32, 0.46 \div 0.48, 0.1, 0.24 \div 0.32, 0.47 \div 0.49.$$

It should be noticed, that the region with $m_{q} \sim 0.2 GeV$ corresponds the good agreement with the experimental data both for nucleon magnetic moments with 2% accuracy and for relativistic contribution into quark energy $\delta E$ with 6% accuracy. That has been considered as

\begin{equation}
D^{(1/2)}(L_{-1}^{-1}, P_{3}') \langle P_{3} \lambda_{3}^{\prime} \tau_{3} | j_{\mu}^{(3)} | P_{3} \lambda_{3} \tau_{3} \rangle \chi^{1}_{2 \lambda_{1}} \lambda_{2} \lambda_{3} \tau_{1} \tau_{2} \tau_{3} \varphi(P_{1}, P_{2}, P_{3}), \end{equation}

Conclusions.

- The value of the relativistic contribution in the energy $\delta E$ is less than 20% in the whole investigated region; $\delta E$ is less than 6% for $m_{q} \sim 0.2 GeV$.

- The magnetic moments of the proton and neutron are obtained with 2% accuracy from experimental data for following parameters:

$[\gamma^{2}(GeV^{2}), m_{q}(GeV), m_{eff}(GeV)]$: [0.065, 0.22 $\div$ 0.26, 0.38 $\div$ 0.39], [0.07, 0.22 $\div$ 0.3, 0.40 $\div$ 0.42], [0.08, 0.24 $\div$ 0.3, 0.42 $\div$ 0.44], [0.095, 0.22 $\div$ 0.32, 0.46 $\div$ 0.48], [0.1, 0.24 $\div$ 0.32, 0.47 $\div$ 0.49].
a proof of the applicability of the nonrelativistic expansion via ratio $\Delta_k/m_{eff}$.

- In ref. [6] was obtained, that best simultaneous reproduction of the proton electric and magnetic form factors $G_E$ and $G_M$ in oscillator model can be obtained for $m_q = 0.162 \text{ GeV}$, $\gamma^2 = 0.35 \text{ GeV}^2$. For this parameters the proton magnetic moments differ from experimental data up to 20%. Therefore one can suppose that agreement data for static and dynamical characteristics can be obtained if we allow quarks to have structure: radius or magnetic moments.

- Effective quark mass is not fixed in region where nucleon magnetic moments are reproduced. The value of $m_q^{eff}$ depends on $m_q$, $\gamma^2$ and varied up 0.48 GeV to 1.9 GeV for $m_q \in [0.1 \div 0.32]$, $\gamma^2 \in [0.06 \div 0.6]$.

References

[1] N.Isgur, G.Karl, Phys. Rev. D18 (1978) 4187-4205.

[2] N.V.Maksimenko, S.G.Shulga, Yad.Fiz. (Sov. J. Nucl. Phys.) 52 (1990) 524-534.

[3] A.A.Logunov, A.N.Tavkhelidze, Nuovo Cimento 29 (1963) 380.

[4] R.N.Faustov, Teor. Mat. Fiz. (Sov. J. Theor. Math. Phys.) 3 (1970) 240

[5] T.P. Ilichova, S.G.Shulga, In: II Opened Scientific Conference of Young Scientist and Specialists JINR. Proceedings Conference. Dubna.(1998) 55-57.

[6] T.P. Ilichova, S.G.Shulga, In Proceed. of International School-Seminar ”Actual problems of particle physics”, E-1,2-2000-208, Dubna, 2000, vol.1, p.190-194.

[7] F.E. Close, An Introduction to Quarks and Partons, Acad. Press, London, 1979.

[8] G.P. Lepage, J. Comp. Phys. 27 (1978) 192-203.

[9] W.H. Press, G.R. Farrar, Computers in Physics 4 (1990) 190-195.
[10] Numerical Recipes in C: The art of scientific computing (ISBN 0-521-43108-5), Cambridge University Press. 1992