Performance evaluation method of closed-loop system considering sensor degradation

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Abstract—To improve the closed loop control system in the accuracy of the parts when degraded performance evaluation, this paper proposes a new performance evaluation method based on improved relative entropy index, first of all, the original asymmetric relative entropy distance measure improvement for symmetric distance measure, and then through the unbounded function mapping index improved to (0, 1) range of bounded indicators. In the Gaussian case, the index value can be calculated from the mean and variance of the data. Finally, the influence of sensor degradation in the control system on the performance of the control system is considered, and a simulation model including sensor degradation is established by Matlab/Simulink. The improved relative entropy index is used to evaluate the performance. Compared with the minimum variance index, this method can evaluate the performance of the control system flexibly and effectively when the sensor degradation.

1. Introduction
At the beginning of the thermal process, the controller can achieve the desired control performance. With the passage of time, the performance degradation of system components or disturbances inside and outside the system will degrade the controller performance and the loop control quality. This requires the use of control system performance evaluation technology to evaluate the performance information of the controller in operation, and then optimize the control loop to maintain the performance of the control system at a normal level [1]. In general, deterministic index and stochastic index are used to evaluate the performance of the control system. Deterministic index refers to the tracking ability of the control system to the change of input signal under the perturbation of a given value, but it is difficult to obtain the exact model parameters, so this kind of index is rarely used alone. Random index refers to the control performance of the control system for the controlled quantity under the interference of random noise or deterioration of components. It mainly includes minimum variance index [2], minimum entropy index [3] and other customized indexes [4-5]. The sensor is one of the important components of the control system. When the sensor is abnormal, faulty or degraded, it will affect the operation and control of the system, and even cause a catastrophic accident [6]. So how to accurately evaluate the performance of the control system is one of the current research focuses when the performance of the sensors in the control system degrades.

At present, the randomness index based on minimum variance has been developed very mature. Harris was the first to propose a performance evaluation method of control system based on minimum variance [7]. Subsequently, scholars at home and abroad conducted extended studies in this aspect [8-9]. However, the minimum variance index needs to know the precise delay time information of the loop, which is difficult to be applied in practice [10]. In order to overcome the limitation of minimum variance index, scholars at home and abroad have done a lot of research in this aspect. Srinivasan et al. [11]
proposed to use de-trended fluctuation analysis method to calculate Hurst index and use this index for performance evaluation, but this index does not require prior knowledge and is greatly affected by noise, resulting in low accuracy. In recent years, an entropy-based control system performance evaluation method has gradually developed. In literature [12], the least square method is used to estimate the minimum entropy, and the effectiveness of the algorithm is verified by simulation. Literature [13] uses minimum information entropy to evaluate the performance of the control system. However, most performance evaluation methods based on entropy take a long time to calculate, and can not objectively represent the actual performance of the control system, which has certain evaluation defects.

To solve the above problems, this paper proposes a stochastic performance evaluation index of control system based on improved relative entropy. The new index has symmetry and boundedness and is suitable for distance measure. This index does not need to obtain the precise model of the control system in the process of operation, but can only use process data to represent the actual performance of the system. The effectiveness of the new performance index is verified by simulation experiments of a control system with sensor degradation model.

2. Process Description

2.1. Improved symmetric relative entropy index (ISRE index)

Relative entropy, also known as KL divergence, can be used to measure the distance or similarity between two random distributions [14]. When two random distributions are the same, their relative entropy is zero. Anyway, as the difference between two random distributions increases, so does the relative entropy. There are two probability distribution functions \( p(x) \) and \( q(x) \), initial relative entropy can be defined as:

\[
D(p||q) = \int p(x) \log \frac{p(x)}{q(x)} \, dx
\]  

(1)

Relative entropy is a statistical distance measurement method, which satisfies the following three properties:

1) Self-similarity: \( D(p||p) = 0 \);
2) Non-negative: \( \forall p, q \quad D(p||q) \geq 0 \);
3) Asymmetry: \( D(p||q) \neq D(q||p) \).

Since the original relative entropy formula does not meet the symmetry of distance, it cannot be used as a distance measurement method, the following symmetric relative entropy formula is considered as a distance measure:

\[
D_S(p || q) = \frac{1}{2} \left[ \int p(x) \log \frac{p(x)}{q(x)} \, dx + \int p(x) \log \frac{p(x)}{q(x)} \, dx \right] = \frac{1}{2} \{ D(p||q) + D(q||p) \}
\]  

(2)

For Equation (1), this paper uses natural logarithm function to calculate relative entropy. The improved symmetric relative entropy formula (2) is also non-negative, and the value range of the result is \((0, +\infty)\), which is unbounded. It has certain defects when applied to the performance evaluation of closed-loop system. Therefore, a new performance evaluation index ISRE based on symmetric relative entropy is proposed, and the definition is as follows:

\[
ISRE = \frac{1}{kD_S + 1} = \frac{1}{k \left[ D(p||q) + D(q||p) \right] + 1}
\]  

(3)

\( k \) is the sensitivity coefficient, which can be taken as \( k > 0 \). The larger the coefficient is, the stronger the perception ability of \( ISRE \) index to system changes is. Under this definition, the newly defined \( ISRE \) indicator can be considered as a distance measure. It satisfies two properties,

1) Boundedness: \( 0 < ISRE < 1 \);
2) Symmetry: \( ISRE(p \parallel q) = ISRE(q \parallel p) \).
When the probability distribution function \( p(x) \) is closer to \( q(x) \), the \textit{ISRE} index tends to 1. When it is applied to performance evaluation, the evaluation result is better. On the contrary, the greater the difference between \( p(x) \) and \( q(x) \), the more \textit{ISRE} index tends to 0, the worse the performance evaluation results of the control system;

2.2. \textit{ISRE} index calculation method

When two distributions obey the Gaussian distribution, the improved symmetric relative entropy formula can be used as the metric method between the Gaussian distributions. That is, \( p \sim N(\mu_1, \sigma_1^2) \), \( q \sim N(\mu_2, \sigma_2^2) \). \( \mu_1, \mu_2 \) and \( \sigma_1, \sigma_2 \) respectively represent the mean value and variance of the data, so the original relative entropy calculation formula is:

\[
D(p \| q) = \frac{1}{2} \left( \frac{(\mu_1 - \mu_2)^2}{\sigma_2^2} + \ln \frac{\sigma_2^2}{\sigma_1^2} + 1 \right)
\]

(4)

The symmetric relative entropy can be calculated as follows:

\[
D_s(p \| q) = \frac{1}{2} \left( D(p \| q) + D(q \| p) \right) = \frac{1}{2} \left( \frac{(\mu_1 - \mu_2)^2}{\sigma_2^2} + \frac{1}{\sigma_1^2} + \frac{1}{\sigma_2^2} + \frac{\sigma_1^2}{\sigma_2^2} + \frac{\sigma_2^2}{\sigma_1^2} - 1 \right)
\]

(5)

When to take \( k = 1 \). In this case, the calculation formula of the newly defined \textit{ISRE} indicator is:

\[
\text{ISRE} = \frac{2}{D_s + 1} \left( \frac{(\mu_1 - \mu_2)^2}{\sigma_2^2} + \frac{1}{\sigma_1^2} + \frac{1}{\sigma_2^2} + \frac{\sigma_1^2}{\sigma_2^2} + \frac{\sigma_2^2}{\sigma_1^2} \right)
\]

(6)

Equation (6) shows that in the Gaussian case, the calculation of the new indicator is only related to the mean and variance of data.

2.3. Degradation model of sensor

When the sensor works normally, its output should be able to accurately reproduce the change of the input, that is, in a stable state, the input and output of the sensor can maintain a linear relationship. When the sensor degrades, it can be divided into two types according to its external input and output characteristics: additive degradation and multiplicative degradation [15].

In the closed-loop control system, it is assumed that the actual output of the controlled object is \( y_a \), while the measured output of the sensor is \( y_m \). According to the operating status of the sensor, its input and output characteristics can be described by the following formula:

\[
y_m = D_n \times y_a + D_a
\]

(7)

In Formula (7), \( D_n \) represents the gain coefficient, and its magnitude represents the degree of sensor multiplicative degradation. \( D_a \) represents the deviation coefficient, and its size represents the additive degradation degree of the sensor.

1) Normal operation: \( D_n = 1 \), \( D_a = 0 \); In this case, \( y_m = y_a \), the formula indicates that the sensor output under ideal conditions is equal to the input;

2) Additive degradation: \( D_n = 1 \), \( D_a \) is a constant that is not 0; In this case, \( y_m = y_a + D_a(D_a \neq 0) \);

3) Multiplicative degradation: \( D_a = 0 \), \( D_n \) is a constant that is not 1; In this case, \( y_m = D_n \times y_a(D_n \neq 1) \).

3. Test Results and Discussions

Consider the level control system shown in Figure 1.
In Figure 1, \( Q(s) \) represents the controller transfer function; \( T(s) \) represents the controlled object transfer function, \( N(s) \) represents the disturbance transfer function; The loop is controlled by proportional integral controller and the parameters are pre-set. Since this paper mainly studies the performance evaluation of the control system when the sensor degrades, the parameters of the controller will not change after setting.

Matlab/Simulink was used to simulate the performance of the control system under the condition of sensor degradation, and the least variance index and the ISRE index were used to evaluate the performance of the loop under different degrees of degradation. Under the normal operation of the system, the control level is set at 80cm. When the simulation time is 500s, sensor degradation models of different types and degrees are applied to the system. Fig. 2 shows the output liquid level curve of the closed-loop system when the sensor works normally and there is additive degradation (Deviation coefficient is \( D_a = 0.8 \)) and multiplicative degradation (Gain coefficient is \( D_m = 1.2 \))[16].

![Fig.2 Output signal of liquid level control system](image)

The output without sensor degradation is used as the baseline data. The minimum variance index and the improved symmetric relative entropy (ISRE) index values are calculated for different degradation types and degrees. The calculation results of specific indicators are shown in Table 1. (Note: ISRE index is defined in section 2.1 and calculated by the calculation method provided in section 2.2. The new index has no unit.)

| Type                  | The degree of degradation | Minimum variance index | ISRE index (defined in section 2.1) |
|-----------------------|---------------------------|------------------------|------------------------------------|
| Additive degradation  | \( D_a = 0.2 \)          | 0.943                  | 0.981                              |
|                       | \( D_a = 0.8 \)          | 0.917                  | 0.922                              |
|                       | \( D_a = 1.5 \)          | 0.884                  | 0.898                              |
|                       | \( D_m = 1.1 \)          | 0.936                  | 0.935                              |
| Multiplicative        | \( D_m = 1.2 \)          | 0.852                  | 0.749                              |
| degradation           | \( D_m = 1.5 \)          | 0.785                  | 0.325                              |
As can be seen from Table 1, with the deepening of sensor degradation, the oscillation amplitude of system liquid level output intensifies. Compared with the minimum variance indicator, the change of ISRE indicator is more obvious and indicative. So the new index is more sensitive to the performance change of the closed-loop control system. Take the multiplier degradation \( D_m = 1.5 \) of the sensor as an example. At this time, the loop oscillation is severe and the performance degradation is serious. The minimum variance index is 0.785, which does not clearly reflect the impact of the degradation degree on the closed-loop loop. However, the ISRE index shows 0.325, which is obviously different from \( D_m = 1.2 \), so this index shows certain effectiveness and superiority.

4. Conclusion
Aiming at the problem that the performance evaluation index of the current control system is not accurate enough, the improved relative entropy index is introduced into the control system, and a symmetric bounded relative entropy index ISRE is proposed to evaluate the performance. In the Gaussian case, only the mean and variance of the data are used to calculate the index value. Matlab/Simulink was used for simulation modeling, the degradation model of sensors was added into the liquid level control system, and the ISRE index values of the control system under different degradation degrees were calculated, and compared with the traditional random performance index (minimum variance index). The experimental results show that ISRE can evaluate the performance of the closed-loop control system correctly. The new index proposed in this paper does not need to estimate the time delay of the data and identify the system model, but only needs the closed-loop operation data to obtain performance evaluation of the current data, which has the advantages of flexibility and effectiveness.

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