Implications from $B \to K^*\ell^+\ell^-$ observables using 3 fb$^{-1}$ of LHCb data.

Rusa Mandal* and Rahul Sinha†
The Institute of Mathematical Sciences,
Taramani, Chennai 600113, India
and
Homi Bhabha National Institute Training School Complex,
Anushakti Nagar, Mumbai 400085, India
(Dated: January 30, 2017)

The decay mode $B \to K^*\ell^+\ell^-$ results in the measurement of a large number of related observables by studying the angular distribution of the decay products and is regarded as a sensitive probe of physics beyond the standard model (SM). Recently, LHCb has measured several of these observables using 3 fb$^{-1}$ data, as a binned function of $q^2$, the dilepton invariant mass squared. We show how data can be used without any approximations to extract theoretical parameters describing the decay and to obtain a relation amongst observables within the SM. We find three kinds of significant disagreement between theoretical expectations and values obtained by fits. The values of the form factors obtained from experimental data show significant discrepancies when compared with theoretical expectations in several $q^2$ bins. We emphasize that this discrepancy cannot arise completely due to resonances and non-factorizable contributions from charm loops. Further, a relation between form factors expected to hold at large $q^2$ is very significantly violated. Finally, the relation between observables also indicates some deviations in the forward-backward asymmetry in the same $q^2$ regions. These discrepancies are possible evidence of physics beyond the SM.

PACS numbers: 11.30.Er, 13.25.Hw, 12.60.-i

I. INTRODUCTION

The rare decay $B \to K^*\ell^+\ell^-$ involves a $b \to s$ flavor changing loop induced transition at the quark level making it attractive as a mode to search for physics beyond the standard model (SM). Indirect searches for new physics (NP) involving loop processes require a comparison of theoretical estimates with experimental observations. The theoretical estimates thus need to be extremely reliable in order to make a conclusive claim on the existence or non-existence of NP. Fortunately, significant progress has been made in understanding the hadronic effects involved in the decay $B \to K^*\ell^+\ell^-$. The mode $B \to K^*\ell^+\ell^-$ is also of special interest as it allows for the measurement of several observables using the angular distribution [1]. The large number of observables depend on theoretical parameters that describe this decay. In this paper we show how some of the parameters can be extracted directly from LHCb measurements allowing us to verify our theoretical understanding. Any discrepancy observed must be attributed either to a failure of our understanding of hadronic effects or to the existence of NP. We also test the relation between observables that provides another clean test for NP.

The differential decay distribution [1–3] of $B \to K^*\ell^+\ell^-$ results in the measurement of at least nine observables using the angular distribution, as a function of $q^2$ the dilepton invariant mass squared. These observables are commonly chosen to be the differential decay rate with respect to $q^2$, two independent helicity fractions that describe the decay, the three asymmetries that describe the real part of the interference between different helicity amplitudes and three asymmetries that describe the imaginary part of the interference.

Recently LHCb [4] has reported measurements of all these observables that have been averaged in eight $q^2$ bins. A lot of studies on this decay mode are widely discussed in literature [5, 6]. We use the LHCb data to obtain estimates of hadronic form factors that describe the decay. Previously some of the form factors have been determined [7] using 1 fb$^{-1}$ of LHCb data. We emphasize that our approach does not involve evaluating the decay amplitude in terms of theoretically estimated parameters. Instead we start with the most general parametric form of the amplitude without any hadronic approximations within the SM (see Eq. (5) below). Experimental data alone is used to fit the theoretical parameters introduced in the parametric amplitude. These experimentally fitted theoretical parameters are simply compared to the estimates by other authors [10, 14] which are widely regarded as the state of the art. The values of form factors obtained from experimental data show significant discrepancy when compared with theoretical expectations in several $q^2$ bins.

In addition to extracting form factors from data, this mode also allows a relation among observables that can provide a clean signal [2, 3] of NP. We find that the measurements do not satisfy the expected relation between the observables in the same $q^2$ domains where the fitted form factors also show a large discrepancy with the theoretical estimates. The simultaneous observation of these discrepancies points to possible evidence of NP.

The paper is organized as follows. In Sec. II we de-
scribe the theoretical framework developed to write the most general parametric form of the amplitude and cast the observables in a form where hadronic parameters can be obtained from data. The relation among observables are also derived here. A numerical analysis is presented in Sec. III which contains two subsections. The Sub-sec. III A gives elaborate description of extraction of form factors using LHCb measurements, whereas, the validity of the relations derived assuming SM are examined in Sub-sec. III B with experimental data. In Sec. IV we summarize the important results obtained in this paper. Appendix. A and B estimate the complex part the amplitude and the systematic uncertainty arising mainly due to bin average effect of the observables, respectively.

II. OBSERVABLES AND THEORETICAL FRAMEWORK

In this section we briefly discuss the theoretical framework derived to take into account all possible contributions within SM for the decay $B \to K^{\ast} \ell^+ \ell^-$. We start with the observables as defined in Ref. [2] to be the $F_L$, $F_\perp$, $A_4$, $A_5$, $A_{FB}$, $A_7$, $A_8$, $A_9$ and $d\Gamma/dq^2 \equiv \Gamma_f$. The observables $F_L$, $A_4$, $A_5$, $A_{FB}$, $A_7$, $A_8$ and $A_9$ are related to the CP averaged observables $S_1$, $S_4$, $S_5$, $A_{FB}^{LHCb}$, $S_7$, $S_8$ and $S_9$ measured by LHCb [4, 8] as follows:

$$ F_\perp = \frac{1}{2} F_L + 2 S_3, \quad A_4 = -\frac{2}{\pi} S_4, \quad A_5 = \frac{3}{4} S_5, $$

$$ A_{FB} = -A_{FB}^{LHCb}, \quad A_7 = \frac{3}{4} S_7, \quad A_8 = -\frac{2}{\pi} S_8, \quad A_9 = \frac{3}{2\pi} S_9. \quad (1) $$

It may be remarked that LHCb collaboration observes a local tension with some observables based on the hadronic estimates of Refs. [10, 11].

We begin by assuming the massless lepton limit but generalize to include the lepton mass. The corrections due to the mass of the leptons are easily taken into account [2]. In the massless lepton limit the decay is described in terms of six transversity amplitudes which can be written in the most general form as,

$$ A_{\lambda}^{L,R} = C_{L,R}^\lambda F_\lambda - \tilde{G}_{\lambda} = (\tilde{C}_{\lambda}^9 + C_{10}) F_\lambda - \tilde{G}_{\lambda}. \quad (2) $$

This form of the amplitude [2] is the most general parametric form of SM amplitude for $B \to K^{\ast} \ell^+ \ell^-$ decay that comprehensively takes into account all contributions up to $O(G_F)$ within it. The form includes all short-distance and long-distance effects, factorizable and nonfactorizable contributions and resonance contributions. In Eq. (2) $C_{10}$ and $C_{10}'$ are Wilson coefficients with $\tilde{C}_{\lambda}^9$ being the redefined “effective” Wilson coefficient defined [2, 12] such that

$$ \tilde{C}_{\lambda}^9 = C_9 + \Delta C_{9}^{(fac)}(q^2) + \Delta C_{9}^{(non-fac)}(q^2) \quad (3) $$

where $\Delta C_{9}^{(fac)}(q^2)$, $\Delta C_{9}^{(non-fac)}(q^2)$ correspond to factorizable and soft gluon non-factorizable contributions.

Strong interaction effects coming from electromagnetic corrections to hadronic operators do not affect $C_{10}$.

The form factors $F_\lambda$ and $\tilde{G}_\lambda$ introduced in Eq. (2) can be related to the conventional form factors describing the decay as shown in the appendix of Ref. [2]. The form-factors $F_\lambda$ are of particular interest here as we show that they can be extracted directly from data. The $F_\lambda$ can be related to the well known form-factors $V$, $A_1$ and $A_{12}$ by comparing with [10]:

$$ F_{\perp} = N \sqrt{2} \frac{\lambda (m_B^2, m_{K^*}^2, q^2)}{m_B + m_{K^*}} \frac{V(q^2)}{m_B + m_{K^*}}, \quad (4a) $$

$$ F_\parallel = -N \sqrt{2} (m_B + m_{K^*}) A_1(q^2), \quad (4b) $$

$$ F_0 = \frac{N}{\sqrt{q^2}} (m_B m_{K^*} A_{12}(q^2)). \quad (4c) $$

It should be noted that $F_\lambda$’s and $C_{10}$ are completely real in the SM, with all imaginary contributions to the amplitude arising only from the imaginary part of complex $\tilde{C}_{\lambda}$ and $\tilde{G}_{\lambda}$ terms. Thus with the introduction of two variables $r_\lambda$ and $\varepsilon_\lambda$ the amplitude $A_{\lambda}^{L,R}$ in Eq. (2) can be rewritten as,

$$ A_{\lambda}^{L,R} = (\mp C_{10} - r_\lambda) F_\lambda + i\varepsilon_\lambda, \quad (5) $$

where,

$$ r_\lambda = \frac{\text{Re} (\tilde{G}_{\lambda})}{F_\lambda} - \frac{\text{Re} (\tilde{C}_{\lambda}^9)}{F_{\perp}}, \quad (6) $$

$$ \varepsilon_\lambda = \frac{\text{Im} (\tilde{C}_{\lambda}^9)}{F_\lambda} - \frac{\text{Im} (\tilde{G}_{\lambda})}{F_{\perp}}. \quad (7) $$

The imaginary contributions arise mostly from resonant long-distance contributions, which can be removed by studying only those $q^2$ regions where no resonances can contribute. In practice this means the removal of charmonium resonance regions from the whole $q^2$ range. LHCb 3 fb$^{-1}$ measurements [4] conservatively exclude the resonance region. Moreover, the contributions from imaginary parts are bounded directly from the LHCb measurements and the bin average values of the $\varepsilon_\lambda$’s are found to be very small as shown in Appendix. A. Hence for now we are neglecting the $\varepsilon_\lambda$’s and will address it’s contribution in the numerical analysis.

It is convenient to define $P_1$ and $P_2$ as,

$$ P_1 = \frac{F_\lambda}{F_{\perp}}, \quad P_2 = \frac{F_{\parallel}}{F_0}. \quad (8) $$

The observables $F_\parallel$, $F_L$, $A_{FB}$, $A_5$ and $A_4$ can be written [2] as

$$ F_\parallel = u_1^2 \pm 2 \zeta \quad (9) $$

$$ F_L P_2^2 = u_3^2 \pm 2 \zeta \quad (10) $$

$$ A_{FB}^{LHCb} = \frac{9 \zeta}{2 P_1^2} (u_0 \pm u_\perp)^2 \quad (11) $$

$$ A_5^2 = \frac{9 \zeta}{4 P_2^2} (u_0 \pm u_\perp)^2 \quad (12) $$

$$ A_4 = \sqrt{\frac{3}{P_1 P_2}} (2 \zeta \pm u_0 u_\parallel) \quad (13) $$

The form-factors $F_\lambda$ and $\tilde{G}_\lambda$ introduced in Eq. (2) can be related to the conventional form factors describing the decay as shown in the appendix of Ref. [2].
where,

$$\zeta = \frac{F_{\perp}C_0^2}{I_f}$$

$$u_{\lambda}^2 = \frac{2F_{\perp}^2}{I_f} = \frac{2F_{\perp}^2}{I_f}(\text{Re}(\tilde{G}_{\lambda}) - \text{Re}(\tilde{C}_{0}\lambda)^2).$$

$u_{\lambda}$ is always taken to be positive and the sign ambiguities introduced in Eqs. (11)-(13) ensure that we can make this assumption. The five observables $F_{\perp}, F_L, A_{FB}, A_5$ and $A_4$ have been expressed above in terms of five parameters $P_1, P_2, \zeta, u_0$ and $u_\perp$. The other three observables $A_7, A_8$ and $A_9$ have already been used to solve for the three $\varepsilon_{\lambda}$ values which are presented in Ref. [2]. It may be noted that since $F_{\parallel} = F_{L} - F_{\perp}, u_{\parallel}$ is not independent and is related to the other parameters by, $u_{\parallel}^2 = P_1^2(1 - P_2^{-2}(u_0^2 + 2\zeta) - (u_0^2 + 2\zeta)) - 2\zeta$.

In Refs. [2, 3] a relation depending on observables including all possible effects within SM which was derived as,

$$\sqrt{4(F_{L} + F_{\parallel} + \sqrt{2}\pi A_1)F_{\perp} - \frac{16}{9} (A_{FB} + \sqrt{2}A_5)^2} = \sqrt{4F_{L}F_{\perp} - \frac{16}{9} A_{FB}^2} + \sqrt{4F_{L}F_{\perp} - \frac{32}{9} A_5^2}.\quad (16)$$

This equation can be used to express any of the observables in terms of the others. A solution for $A_4$ [3] is

$$A_4 = \frac{8A_5 A_{FB}}{9\pi F_{\perp}} + \sqrt{\frac{4F_{L}F_{\perp} - \frac{16}{9} A_{FB}^2}{\frac{2\sqrt{2}\pi F_{\perp}}{2\sqrt{2}\pi F_{\perp}}} \frac{4F_{L}F_{\perp} - \frac{32}{9} A_5^2}{4\sqrt{2}F_{\perp}}}.\quad (17)$$

Whereas, the solution for $A_5$ and $A_{FB}$ are given by,

$$A_5 = \frac{\pi A_4 A_{FB}}{2F_{\parallel}} \pm \frac{3\sqrt{4F_{L}F_{\perp} - \frac{16}{9} A_{FB}^2} \sqrt{2F_{L}F_{\perp} - \pi^2 A_4^2}}{8F_{\parallel}}.\quad (18)$$

$$A_{FB} = \frac{\pi A_4 A_5}{F_{L}} \pm \frac{3\sqrt{4F_{L}F_{\perp} - \frac{32}{9} A_5^2} \sqrt{2F_{L}F_{\perp} - \pi^2 A_4^2}}{4\sqrt{2}F_{L}}.\quad (19)$$

It may be noted that Eqs. (17), (18) and (19) depend only on observables and not on any theoretical parameters and thus provides an exact test of the gauge structure of SM with experimental measurements.
III. NEW PHYSICS ANALYSIS

In this section we discuss the detailed numerical analysis using 3 fb$^{-1}$ of LHCb data [4]. It contains two different parts, at first we show how the experimental data can be used to extract out the form factors which are involved in this decay mode. Secondly we present the consistency of data to test the relation among observables derived relying only on the gauge structure of SM.

A. Form Factor Extraction

We demonstrate the technique to extract out the hadronic parameters by including complex contributions of the amplitudes and considering systematic uncertainty arising mainly due to bin average effect. It is shown in Ref. [2] that $\varepsilon_A$’s contribute to the helicity fractions $F_A$ and asymmetry $A_F$. We refer to Appendix. A for thorough details of evaluation of the complex part of the amplitudes. Using the bin average central values of $\varepsilon_A/\sqrt{I}$, with ±1σ errors from Table. III we can numerically separate out the complex contributions from experimental measured values of the observables. We calculate the central value with ±1σ error of the modified observables $F^\text{ex}_\pm$ and $A^\text{ex}_4$ given by,

\begin{equation}
F^\text{ex}_\pm = \frac{F_{\pm}}{\varepsilon_\lambda} - \frac{2\varepsilon_\perp^2}{\sqrt{I}};
\end{equation}
\begin{equation}
A^\text{ex}_4 = A_4 - \frac{2\varepsilon_{0}^2}{\pi I},
\end{equation}

which enter in the $\chi^2$ definition Eq. (23) below. It enables us to take into account the complex corrections in our analysis and extract out the variables $P_1$, $P_2$, $\zeta$, $u_\perp$ and $u_\perp$ (which only deal with the real part of amplitude) from experimental measurements of the observables accurately.

It should be noted that Eqs. (9) – (13) are valid for each $q^2$ point. However, experiments can provide bin integrated values of observables over a certain $q^2$ intervals. Thus a $\chi^2$ fit with bin average values of the observables may lead to a biased conclusion. To avoid this issue we have added systematic uncertainties for each observables due to bin average effect with the introduction of new parameter $\beta$, where the change in each observable $\mathcal{O}$ is given by,

\begin{equation}
\mathcal{O} \rightarrow \mathcal{O} + \beta \mathcal{O}^*.
\end{equation}

$\mathcal{O}^*$ is the maximum shift for each observables with a best fitted $q^2$ function to 14 bin LHCb data [4] within the concerned bin interval. The precise determination of $\mathcal{O}^*$ is described in Appendix. B. Therefore the $\chi^2$ definition is

\begin{equation}
\chi^2 = \left[ \frac{F_\pm^\text{ex} - F_\pm - \beta F_\pm^*}{\Delta F_\pm^\text{ex}/\Delta F_\pm} \right]^2 + \left[ \frac{F_L^\text{ex} - F_L - \beta F_L^*}{\Delta F_L^\text{ex}/\Delta F} \right]^2
+ \left[ \frac{A_4^\text{ex} - A_4 - \beta A_4^*}{\Delta A_4^\text{ex}/\Delta A_4} \right]^2 + \left[ \frac{A_2^\text{ex} - A_2 - \beta A_2^*}{2A_2^\text{ex}/\Delta A_2} \right]^2
+ \left[ \frac{A_0^\text{ex} - A_0 - \beta A_0^*}{2A_0^\text{ex}/\Delta A_0} \right]^2 + \beta^2,
\end{equation}

where $A_2^\text{ex}/\Delta A_2^\text{ex}$ and $A_0^\text{ex}/\Delta A_0^\text{ex}$ indicate experimental central values of the observables $A_2^\text{ex}$ and $A_0^\text{ex}$ with ±1σ errors as $\Delta A_2^\text{ex}$ and $\Delta A_0^\text{ex}$, respectively. Similarly $F_\pm^\text{ex}$, $F_L^\text{ex}$ and
$A_2^3$ are the central values of the modified observables defined in Eqs. (20) and (21) and $\Delta F_{1,1,1}^V$, $\Delta F_{1,1,2}^V$, $\Delta A_1^V$ are ±1σ uncertainties in it. The systematic uncertainties added for each observables are denoted as $F_{1,1}^V$, $F_{1,2}^V$, $A_1^F$, $A_2^F$ and $A_3^F$ and these values are quoted in Table IV of Appendix B. The observables $F_{1,1}$, $F_{1,2}$, $A_1$, $A_2^F$ and $A_3^F$ are evaluated in terms of the five parameters $P_1$, $P_2$, $\zeta$, $u_0$ and $u_\perp$ using Eqs. (9) – (13). Considering the inverse of the covariance matrix the error ellipsoids are constructed for all the eight bins corresponding to the $q^2$ values in the range (0.1 – 0.98) GeV$^2$, (1.1 – 2.5) GeV$^2$, (2.5 – 4.0) GeV$^2$, (4.6 – 6.8) GeV$^2$, (11.0 – 12.5) GeV$^2$, (15 – 17) GeV$^2$ and (17 – 19) GeV$^2$. It can be seen that $\beta$ is treated as a nuisance parameter with values $0 \pm 1$. The $\chi^2$ function is minimized with r.t six parameters $P_1$, $P_2$, $\zeta$, $u_0$, $u_\perp$ and $\beta$ and the contours shown in Fig. 1 and Fig. 2 are the allowed regions in the corresponding planes. The minimum values of the $\chi^2$ function for first to eighth bins are $6.9 \times 10^{-9}$, $3.4 \times 10^{-10}$, 0.055, 8.6 $\times 10^{-30}$, 1.094, 0.538, 0.221 and 0.044, respectively. The best fitted values with ±1σ errors of the parameter $\beta$ for all eight bins are $7.4 \times 10^{-5}$ ± 0.015, $1.6 \times 10^{-5}$ ± 0.020, 0.153 ± 0.011, $1.0 \times 10^{-17}$ ± 0.005, 0.736 ± 0.020, 0.251 ± 0.003, 0.261 ± 0.001 and 0.161 ± 0.012, respectively.

The contours corresponding to 1σ, 3σ and 5σ permitted regions for $P_1$ versus $P_2$ plane are presented in Fig. 1. These contours are compared with the estimated values of $P_1$ and $P_2$ using Ref. [10] for $q^2 \leq 8$ GeV$^2$ and Ref. [14] for $q^2 \geq 11$ GeV$^2$. The center black point denotes the best fit point by minimizing the chi-square function defined in Eq. (23). In most cases reasonable agreement is found between theoretical values of $P_1$ and $P_2$ and their values obtained from data. However, there are some significant disagreements. The values of form factor ratio $P_2$ differ by 9σ in the $0.1 \leq q^2 \leq 0.98$ GeV$^2$ bin. It may be noted that this region in $q^2$ is highly affected by finite lepton mass and hence the large discrepancy may not accurately reflect the significance due to the unaccustomed lepton mass correction systematics. Significant deviations are also found for the three bins 6 $\leq q^2 \leq 8$ GeV$^2$, 11.0 $\leq q^2 \leq 12.5$ GeV$^2$ and 15 $\leq q^2 \leq 17$ GeV$^2$ where $P_1$ ($P_2$) differ by 4.2σ (0.8σ), 5.2σ (4.8σ) and 5.5σ (5.3σ), respectively. The light blue bands denote exact solutions for the SM observables including charmonium resonances from Ref. [17] parametrization and are shown only for the relevant $q^2$ bins. The detailed analysis of resonance effect will be discussed later in this section.

Finally the form factor $F$ can be evaluated using Eq. (4a) and the value of $F_{1,1}$ obtained. Since the recent $3 fb^{-1}$ of LHCb data [4], Round brackets indicate the standard deviation between fitted values and theoretical estimates [10, 14]. Significant discrepancies are found for $F_1$ and $A_1$ in several $q^2$ region.

In Fig. 2 contours similar to Fig. 1, but corresponding to $P_1$ versus $\zeta$ permitted regions are presented for 1σ, 3σ and 5σ confidence level regions. These contours are similarly compared with the estimated values of $P_1$ and $\zeta$ using Refs. [10, 14] and assuming the theoretical estimate of $C_{10}^*$ [15]. Data shows consistency with theoretical values of $P_1$ and $\zeta$ in most cases except for the two bins 11.0 $\leq q^2 \leq 12.5$ GeV$^2$ and 15 $\leq q^2 \leq 17$ GeV$^2$ where $\zeta$ disagrees by 2.8σ and 1.7σ respectively. The best fit value of $\zeta$ with ±1σ error obtained from the fit can be used to calculate the form factor $F_{1,1}$ using Eq. (14).

| $q^2$ range in GeV$^2$ | $V(q^2)$ | $A_1(q^2)$ | $A_{12}(q^2)$ |
|------------------------|----------|------------|--------------|
| 0.1 $\leq q^2 \leq 0.98$ | 0.677 ± 0.092 (3.05σ) | 0.570 ± 0.077 (3.40σ) | 0.246 ± 0.034 (0.88σ) |
| 1.1 $\leq q^2 \leq 2.5$ | 0.625 ± 0.071 (2.78σ) | 0.409 ± 0.046 (2.00σ) | 0.326 ± 0.047 (0.69σ) |
| 2.5 $\leq q^2 \leq 4.0$ | 0.230 ± 0.150 (1.36σ) | 0.180 ± 0.118 (1.09σ) | 0.214 ± 0.109 (0.81σ) |
| 4.0 $\leq q^2 \leq 6.0$ | 0.552 ± 0.043 (1.07σ) | 0.400 ± 0.032 (1.69σ) | 0.359 ± 0.041 (1.09σ) |
| 6.0 $\leq q^2 \leq 8.0$ | 0.485 ± 0.045 (1.27σ) | 0.598 ± 0.073 (3.18σ) | 0.252 ± 0.025 (1.78σ) |
| 11.0 $\leq q^2 \leq 12.5$ | 0.166 ± 0.018 (5.64σ) | 0.560 ± 0.065 (1.76σ) | 0.450 ± 0.054 (1.81σ) |
| 15.0 $\leq q^2 \leq 17.0$ | 0.828 ± 0.120 (2.79σ) | 0.649 ± 0.098 (1.38σ) | 0.496 ± 0.074 (1.51σ) |
| 17.0 $\leq q^2 \leq 19.0$ | 1.813 ± 0.436 (0.78σ) | 0.698 ± 0.171 (0.80σ) | 0.461 ± 0.112 (0.91σ) |

In Table I. The form factor values obtained from fit to 3 fb$^{-1}$ of LHCb data [4]. Round brackets indicate the standard deviation between fitted values and theoretical estimates [10, 14]. Significant discrepancies are found for $V$ and $A_1$ in several $q^2$ region.
affected by resonances. This is easily seen if we consider a situation where NP is absent and all the parameters for resonances (strength, phase etc.) are known, the observables calculated using Eqs. (9)–(13) should agree with the experimental measured observables. Thus the consistent set of Eqs. (9)–(13) must provide the same set of parameters that we would have started with, as best fit solutions. In the absence of NP the measured observables should result in the solutions for parameters matching with SM values. Since $P_1$ and $P_2$ are unaffected by resonances their best fit solutions also remain unaffected by it. Our best fit values of $P_1$ and $P_2$ differ from the SM estimates and this discrepancy cannot be accounted for by resonances.

To establish the above arguments we further undertake an extensive study illustrated in Fig. 3. We choose the region $q^2 > 11 \text{GeV}^2$ as resonance effects can be dominant here and assume SM form factor values of the observables from lattice calculations [14]. The solutions for $P_1$ and $P_2$ are obtained using Eqs. (9)–(13) for seven different $q^2$ points; 11 GeV$^2$, 12 GeV$^2$, 15 GeV$^2$, 16 GeV$^2$, 17 GeV$^2$, 18 GeV$^2$ and 19 GeV$^2$. The observables $F_L$, $F_\perp$, $A_2$, $A_{FB}$ and $A_1^\perp$ are SM estimates calculated using lattice form factors. These seven solutions of $P_1$ and $P_2$ are denoted by ‘star’ symbols in the corresponding plots. The black central line with gray band is the form factor estimate (mean with $\pm 1\sigma$ error) of $P_1$ and $P_2$. It can be seen that the set of Eqs. (9)–(13) are completely consistent with SM structure and produces expected solutions. In case the solutions were completely analytically obtained, the ‘stars’ should sit on the black curves. However the solutions for hadronic parameters are very complicated and has been evaluated numerically, resulting in small shifts that are visible. The blue error bars are the solutions for $P_1$ and $P_2$ using the bin average values of SM observables. It can be seen that as the Eqs. (9)–(13) are valid at each $q^2$ point, bin averaging has induced some shifts in the solutions. However the results are in agreement within $\pm 1\sigma$ confidence level region. To illustrate the effect of resonances we have considered the parametrization from Ref. [17]. The charmonium bound states $J/\psi(1S)$, $\psi(3770)$, $\psi(4040)$, $\psi(4160)$ and $\psi(4415)$ are included in the mentioned five observables. Interestingly, the change in the value of observables including the resonances affected the solutions for $\zeta$, $u_1$ and $u_0$. However, solutions to $P_1$ and $P_2$ remained unaltered (upto second decimal place), hence, the solutions completely superimpose with the ‘stars’ obtained without resonance contributions. We have also investigated the effect of resonances in the bin average where the observables are evaluated with lattice form factors including the above mentioned resonances and the solutions to $P_1$ and $P_2$ are shown in light blue bands for the three $q^2$ bins $11.0 \leq q^2 \leq 12.5$ GeV$^2$, $15 \leq q^2 \leq 17$ GeV$^2$ and $17 \leq q^2 \leq 19$ GeV$^2$. In this case the results with and without resonances do not completely superimpose however are quite consistent within $\pm 1\sigma$ error bars. These solutions are also shown in Fig. 1 and 2, in same light blue bands, for the relevant bins where resonance effect may in principle be significant. The red error bars are the solutions for $P_1$ and $P_2$ obtained from data (as discussed and highlighted in contours is Fig. 1) that have been shown here again for convenience. We reiterate that effect of resonances in $P_1$, $P_2$ solutions is independent of the parametrization choice as the solutions do not depend on Wilson coefficient $C_9$ and our conclusions derived for $P_1$ and $P_2$ parameters are unaffected by resonance effect. It is justified that bin average can induce some errors in the solutions. However, we have allowed a shift in the
observable values (in Eq. 23 and Table. IV) of more than the 1σ error for each observable which hopefully is sufficient to compensate such effects.

It is important to note that in our analysis no hadronic estimates are used to solve for the five parameters from exactly five measurements. Whereas, in other approaches, when considering the same $B \rightarrow K^* \ell^+ \ell^-$ model all six form factors, Wilson coefficients and non factorisable corrections based on conservative estimations are needed. We compare $P_1$ and $P_2$ obtained from experimental data alone, with the three form factor $V$, $A_1$ and $A_{12}$ to which they are related as theoretical inputs. The form-factors $T_1, T_2$ and $T_{23}$ are not used in this comparison. Thus, our comparisons are different in nature and have reduced uncertainties, in terms of number of theoretical estimates. This may result in higher significance level of deviation observed here.

The large $q^2$ region where the $K^*$ has low-recoil energy has also been studied [18, 19] in a modified heavy quark effective theory framework which is a model independent approach. In this limit the number of independent hadronic form factors reduces to only three and one finds [3] that $r_0 = r_{kl} = r_\perp$ or equivalently $u_0 = u_{ll} = u_{\perp}$ must hold as long as non-factorizable charm loop contributions are negligible. We find that this relation does not hold for either of the bins $15 \leq q^2 \leq 17$ GeV$^2$ and $17 \leq q^2 \leq 19$ GeV$^2$. The values of $u_0$, $u_{ll}$ and $u_{\perp}$ obtained from the fit with ±1σ errors are listed in Table II. We note that $u_{ll}$’s receive problematic resonance contribution coming from $C_3$. To address this issue we have introduced more systematics in measured observable than the one arising only from bin average effect. We have checked our analysis by doubling the systematics of the observables given in Table. IV of Appendix. B for the $q^2$ range $11 \leq q^2 \leq 12.5$ GeV$^2$ and $15 \leq q^2 \leq 17$ GeV$^2$ and our results are stable with it. The actual significance of the deviations observed here can be obtained with the detailed study of resonance systematics which is a subject of an independent paper. However the significance level is evaluated by conservatively adding systematics varying between 10% – 100% in the observables. The large discrepancies observed are equally hard to explain solely due to non-factorizable charm loop contributions and may be additional evidence of physics beyond the SM.

| $q^2$ range in GeV$^2$ | $u_0$          | $u_{ll}$       | $u_{\perp}$       |
|------------------------|---------------|---------------|-------------------|
| $15 \leq q^2 \leq 17$ | $0.000 \pm 0.016$ | $0.013 \pm 0.153$ | $0.367 \pm 0.025$ |
| $17 \leq q^2 \leq 19$ | $0.166 \pm 0.014$ | $0.000 \pm 4.579$ | $0.260 \pm 0.048$ |
| $15 \leq q^2 \leq 19$ | $0.120 \pm 0.007$ | $0.004 \pm 0.441$ | $0.244 \pm 0.026$ |

TABLE II. The values of $u_0$, $u_{ll}$ and $u_{\perp}$ obtained from fit to 3 fb$^{-1}$ of LHCb data [4]. In large $q^2$ region [18, 19] the equality $u_0 = u_{ll} = u_{\perp}$ is expected to hold if non-factorizable charm loop contributions are negligible. The errors in the value of $u_{ll}$ for the larger $q^2$ bin is unexpectedly large to draw any conclusions. Significant discrepancies which are too large to be solely due to non-factorizable charm loop corrections are observed between the values of $u_{\perp}$ and $u_0$ in both bins.

B. Testing relation between observables

The relation between the observables for asymmetries $A_4$, $A_5$ and $A_{BF}$ given in Eqs. (17) – (19) can also be tested using LHCb data [4]. In Fig. 4, top left panel, we compare theoretically calculated $A_{BF}$ mean values and ±1σ errors (in yellow bands) with experimental measurements (red error bars) for the respective $q^2$ bins. All observables in the r.h.s of Eq. (19) (‘relation’) are assumed to be Gaussian distributions in data and the predictions for $A_{BF}$ in yellow bands are obtained using the expression of the ‘relation’. A very good agreement is evident for most $q^2$ regions, however, for the ranges $11.0 \leq q^2 \leq 12.5$ GeV$^2$ and $15 \leq q^2 \leq 17$ GeV$^2$ a deviation of 2.1σ and 1.8σ is observed. Similarly ‘relation’ for $A_4$ in Eq. (17) results in a very good agreement except for showing a discrepancy of $2.3\sigma$ only in the $0.1 \leq q^2 \leq 0.98$ GeV$^2$ bin, in right top panel of Fig. 4. The disagreement in the value of $A_{BF}$ and $A_4$ in some $q^2$ bins indicates that there is no set of form factors and Wilson coefficients which can explain $A_{BF}$ and $A_4$ completely. Observables $A_5$ or equivalently $P_5^\prime$ [20] are found to be in complete agreement i.e. within about ±1σ deviation for all $q^2$ bins as shown in the two lower panels of Fig. 4. The solutions for $A_5$ and $A_{BF}$ have ambiguities. We chose the ambiguity for which the chi-squared deviations are the least. Our conclusions have no bearing on and do not rule out the observation made by LHCb in observable $P_5^\prime$ in Refs. [4, 9]. The predictions of observable $P_5^\prime$ derived from the relation is a signal of consistency of LHCb results. We note that the relation remains valid except in the presence of NP operators that result in modified new angular distribution. Hence we do not expect to see the discrepancy observed by LHCb [9] if right-handed currents or extra vector current such as $Z'$ contributes to the decay. The discrepancy observed by LHCb depends on the comparison with model based calculation of form factors. Whereas, the predictions of these asymmetries made in this paper, are independent of any form factor values and depend purely on the gauge structure of SM. If the model dependent calculations of form factors are correct, signal of new physics may well be indicated in the bins suggested by Ref. [9]. We find that LHCb data indicates yet another independent discrepancy.

IV. CONCLUSION

In conclusion, we have used the 3 fb$^{-1}$ of LHCb data to determine some hadronic parameters governing the decay $B \rightarrow K^* \ell^+ \ell^-$ assuming contributions from SM alone. We obtain the values of the form factors $V(q^2)$, $A_1(q^2)$ and $A_{12}(q^2)$ that are used to describe the matrix element $\langle K^* | \bar{s} \gamma_{\mu} P_\mu b | B \rangle$ directly from data. Very significant deviations are seen for the form factors $V$ and $A_1$ especially in the regions $q^2 < 2.5$ GeV$^2$ and $q^2 > 6$ GeV$^2$. We point out that the presence of resonances in data can induce more systematic uncertainties in the fits. How-
ever in the view of absence of such a existence of resonances in $B \to K^{*}\ell^{+}\ell^{-}$ data, we emphasize that the significant deviations observed in the form factor values can not be completely explained by resonances and non-factorizable contributions. We would like to point out that there exist major differences between the global fit approaches [6] to study the anomalies in $b \to s$ transitions and the approach adopted in our work. Our work relies only on $B \to K^{*}\ell^{+}\ell^{-}$ decay mode, whereas, global fit techniques incorporate various decay modes and hence either use LCSR, Lattice based estimates of form factors or treat form-factors as parameters in the fit procedure. The number of inputs and fitted parameters differ making a number by number comparison of the different approaches difficult. Furthermore due to the absence of accurate estimates of non factorisable corrections, the global fit techniques rely on some conservative estimations of these corrections. However, the formalism we have developed parametrizes such corrections and the conclusions drawn here are independent of non factorisable estimates. These are perhaps the reasons why we find larger significance. However, qualitatively we don’t see a significant disagreement with the other approaches as we do observe $\sim 3\sigma$ discrepancy in $P_{1} - P_{2}$ plane in $q^{2}$ region [6 - 8]GeV² where observable $P'_{5}$ also deviates by $2.7\sigma$ from its SM prediction.

Further, a relation between form factors expected to hold in the large $q^{2}$ region as long as non-factorizable charm loop contributions are negligible, seems to fail. Finally, the relation between observables also indicates some deviations in the same regions where the form factors were found to disagree. The forward-backward asymmetry $A_{FB}$ deviates in the $q^{2} > 11$ GeV² region, where as $A_{4}$ differs in the region $q^{2} \leq 0.98$ GeV². As the systematic error arises from the experimental measurements of observables in terms of binned dilepton invariant mass are accounted, the magnitude of discrepancies observed would be hard to accommodate either as systematics from long distance resonance contributions or possible corrections to theoretical estimates. All these features can be understood if there are other unaccounted for operators contributing to the decay mode. In view of this, we speculate that these deviations are likely to be a signature of physics beyond SM.
ACKNOWLEDGMENTS

We are indebted to Tom Browder and thank him for several suggestions and discussions. We also thank J. Martin Camalich, Hai-Yang Cheng, N. G Deshpande, Jim Libby and Arjun Menon for discussions. R.S. thanks Hai-Yang Cheng and Institute of Physics, Academia Sinica, Taipei, Taiwan for hospitality during final stages of manuscript preparation.

Appendix A: Complex contribution \( \varepsilon_\lambda \) estimates from data

In Ref. [2] it was shown that the complex contributions \( \varepsilon_\lambda \) to the amplitude of the decay mode \( B \to K^* \ell^+ \ell^- \), can be taken into consideration. \( \varepsilon_\lambda \) can be solved in terms of iterative solutions proportional to the observables \( A_7 \), \( A_8 \), \( A_9 \) and a form factor ratio \( P_1 \). The expressions for all the three \( \varepsilon_\lambda \)'s are shown in Eqs. (76)–(78) of Ref. [2]. They are reproduced here for convenience.

\[
\varepsilon_\perp = \frac{\sqrt{2\pi} \Gamma_f}{(r_0 - r_1) \mathcal{F}_\perp} \left[ \frac{A_0 P_1}{3\sqrt{2}} + \frac{A_3 P_2}{4} - \frac{A_7 P_1 P_2 r_\perp}{3\pi C_{10}} \right], \tag{A1}
\]

\[
\varepsilon_\parallel = \frac{\sqrt{2\pi} \Gamma_f}{(r_0 - r_1) \mathcal{F}_\perp} \left[ \frac{A_0 r_0}{3\sqrt{2} r_\perp} + \frac{A_3 P_2 r_\parallel}{4 P_1 r_\perp} - \frac{A_7 P_2 r_\parallel}{3\pi C_{10}} \right], \tag{A2}
\]

\[
\varepsilon_0 = \frac{\sqrt{2\pi} \Gamma_f}{(r_0 - r_1) \mathcal{F}_\perp} \left[ \frac{A_0 P_1 r_0}{3\sqrt{2} P_2 r_\perp} + \frac{A_3 r_\parallel}{4 r_\perp} - \frac{A_7 P_1 r_0}{3\pi C_{10}} \right]. \tag{A3}
\]

A point to be noted as explained in detail in Ref. [2], is that the \( \langle \varepsilon_\lambda / \Gamma_f \rangle \)'s are completely expressed in terms of observables and the form factor ratio \( P_1 \). However, these solutions are essentially iterative, since the \( r_\lambda \)'s and \( C_{10} \) are derived in terms of the primed observables that depend on \( \varepsilon_\lambda \). If \( \langle \varepsilon_\lambda / \Gamma_f \rangle \)'s are small as should be expected, accurate solutions for them can be found with a few iterations. In Ref. [2] the variation of \( \varepsilon_\lambda \) with \( P_1 \) was studied for 1 fb\(^{-1}\) LHC data and it was found that the solutions are not sensitive to the value of \( P_1 \).

We generate a set of events for every bin, with each event consisting of randomly chosen values drawn from Gaussian distributions generated for each of the observables \( F_L \), \( F_\perp \), \( A_4 \), \( A_5 \), \( A_7 \), \( A_8 \) and \( A_9 \). The distributions are generated using experimental results from Ref. [4], with the experimentally measured value as mean and the uncertainty as standard deviation.

\( \varepsilon_\lambda \) are solved iteratively for every set of observables. We find converged iterative solutions for \( \varepsilon_\lambda / \sqrt{\Gamma_f} \) for each set of observables with the histograms shown in Fig. 5. The red (dark), light brown (lightest) and green histograms denote the solutions for \( \varepsilon_\perp / \sqrt{\Gamma_f} \), \( \varepsilon_\parallel / \sqrt{\Gamma_f} \) and \( \varepsilon_0 / \sqrt{\Gamma_f} \) respectively for all the eight bins with \( q^2 \) range \((0.1 - 0.98)\) GeV\(^2\), \((1.1 - 2.5)\) GeV\(^2\), \((2.5 - 4.0)\) GeV\(^2\), \((4 - 6)\) GeV\(^2\), \((6 - 8)\) GeV\(^2\), \((11.0 - 12.5)\) GeV\(^2\), \((15 - 17)\) GeV\(^2\) and \((17 - 19)\) GeV\(^2\). We have also quoted the mean and \( \pm 1\sigma \) errors for each \( \varepsilon_\lambda / \sqrt{\Gamma_f} \) in Table. III calculated from the distributions shown in Fig. 5. It can be easily seen that all the mean values of \( \varepsilon_\lambda / \sqrt{\Gamma_f} \) are consistent with zero. From Eqs. (37)–(40) of Ref. [2], the contributions from imaginary part of the amplitude to the observables \( F_L \), \( F_\parallel \), \( F_\perp \) and \( A_4 \) are quadratic in the corresponding \( \varepsilon_\lambda / \sqrt{\Gamma_f} \) and thus are negligible.

FIG. 5. (color online) The solutions for \( \varepsilon_\perp / \sqrt{\Gamma_f} \), \( \varepsilon_\parallel / \sqrt{\Gamma_f} \) and \( \varepsilon_0 / \sqrt{\Gamma_f} \) using distributions for first through eighth \( q^2 \) bins are depicted in red (dark), light brown (lightest) and green respectively. All the \( \varepsilon_\lambda / \sqrt{\Gamma_f} \)'s are consistent with zero.
TABLE III. The $\epsilon_{9}/\sqrt{\Gamma_{f}}$ mean values with $\pm1\sigma$ errors from Fig. 5

Appendix B: Systematic uncertainty evaluation

TABLE IV. The systematic uncertainties for each observables $F_{1}$, $F_{2}$, $A_{1}$, $A_{2}$, $A_{4}$ and $A_{L}$ are shown. The values denote magnitude of maximum deviation of the bin average central value with the fitted $q^{2}$ polynomial within every $q^{2}$ bin.

We discuss the evaluation of systematic uncertainties arising mainly due to bin average effect of observables.

As written in Eq. (22), the shift $O^{2}$ in each observable is calculated for each $q^{2}$ bin, by considering the maximum deviation of the bin average value of the observable $O$ from a fitted $q^{2}$ polynomial of entire range. It is highlighted in Fig. 6 where red error bars are LHCb measurements and gray curves represent best fitted polynomial in $q^{2}$ for 14 bin LHCb data. We use 14 bin measurement (based on the method of moments [13]) from LHCb to fit the polynomial in $q^{2}$, rather than the 8 bin data set as it provides more information to determine the shape of the polynomial for entire $q^{2}$ region. The black dashed line denotes the maximum deviation of bin average central value of the observable with the $q^{2}$ function for the region $6 \leq q^{2} \leq 8$ GeV and $O^{*}$ is the length of the line for observable $O$. Similar technique is applied for other $q^{2}$ bins also and the values of systematic errors are given in Table IV for all observables.

It should be noted that as discussed in Sec. III finite lepton mass can affect the analysis in the first two $q^{2}$ region namely $q^{2} \leq 2.5$ GeV$^{2}$ and in the absence of a measurement of asymmetries $A_{10}$ and $A_{11}$ [2] we have to rely on some hadronic estimates. This in principle may cause more uncertainties and we took a conservative approach by considering two times the $O^{*}$ values for all observables given in Table IV for the two bins $0.1 \leq q^{2} \leq 0.98$ GeV$^{2}$ and $1.1 \leq q^{2} \leq 2.5$ GeV$^{2}$.

We emphasize that resonances in our analysis will only affect the fitted function in $q^{2}$, which in turn will induce more systematic uncertainties to the observables. We have checked the $\chi^{2}$ fit (in Sub-sec. IIIA) by increasing the systematic uncertainties two times of the values given in Table IV for the regions $11 \leq q^{2} \leq 12.5$ GeV$^{2}$ and $15 \leq q^{2} \leq 17$ GeV$^{2}$ and our results are stable with it. However a detailed study of resonance systematics on this decay mode is currently going on and will be a subject of an independent paper itself.
FIG. 6. (color online). The procedure to calculate systematic errors are shown for observables $F_\perp$, $F_L$, $A_4$, $A_{FB}^2$ and $A_5^2$, respectively. The red error bars are LHCb measurements and gray curves represent best fitted polynomial in $q^2$ for 14 bin LHCb data [4]. The black dashed lines denote the maximum deviation of bin average central value of the observables with the $q^2$ function for bin $6 \leq q^2 \leq 8$ GeV$^2$. The length of these black lines are denoted by $F^s_\perp$, $F^s_L$, $A^s_4$, $A^s_{FB}$ and $A^s_5$, respectively. Similar lines can be drawn for other $q^2$ bins also and the values of systematic errors are given in Table. IV for all observables.

[13] F. Beaujean, M. Chrzaszcz, N. Serra and D. van Dyk, Phys. Rev. D 91, 114012 (2015) [arXiv:1503.04100 [hep-ex]].
[14] R. R. Horgan, Z. Liu, S. Meinel and M. Wingate, Phys. Rev. Lett. 112, 212003 (2014); [arXiv:1310.3887 [hep-ph]]; R. R. Horgan, Z. Liu, S. Meinel and M. Wingate, arXiv:1501.00367 [hep-lat].
[15] W. Altmannshofer, P. Ball, A. Bharucha et al., JHEP 0901, 019 (2009). [arXiv:0811.1214 [hep-ph]].
[16] N. G. Deshpande, J. Trampetic and K. Panose, Phys. Rev. D 39, 1461 (1989).
[17] F. Kruger and L. M. Sehgal, Phys. Lett. B 380, 199 (1996) [hep-ph/9603237].
[18] B. Grinstein, D. Prijol, Phys. Rev. D 70, 114005 (2004). [arXiv:hep-ph/0404250v3].
[19] C. Bobeth, G. Hiller and D. van Dyk, JHEP 1007, 098 (2010). [arXiv:1006.5013 [hep-ph]].
[20] S. Descotes-Genon, J. Matias, M. Ramon and J. Virto, JHEP 1301, 048 (2013) [arXiv:1207.2753 [hep-ph]].