Dynamic Parameter Design Method and Application under a General Linear Relationship between Response Variables and Signal Factors

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Abstract. In dynamic parameter design, there is often a different functional relationship between response variables and signal factors. SNR is an important tool for the parameter design of dynamic characteristics, and its estimation results change with the change of function relationships. This paper describes the dynamic parameter design method in a linear relationship between response variables and signal factors, and provides a case for verification. The results of this study are applicable to engineers and academics who need to conduct product design, process development and process optimization through dynamic parameter design.

1. Introduction

Product quality is first designed, then manufactured and tested [1, 2, 3]. The design of a product can be carried out in three stages, namely: system design, parameter design and tolerance design [4, 5]. The three-stage design method is of great significance to improve the design quality of products. Among them, parameter design is the core. The so-called parameter design is to adjust the design parameters so that the noise factors such as environmental interference have little and stable influence on the characteristics of products or processes [6]. According to whether the target value of the property is fixed or not, the parameter design can be divided into static and dynamic [7]. For the parameter design of static systems, the characteristic value has fixed target, so the function of signal factor is very weak. In the dynamic parameter design, the expected output of the system depends on the level of the signal factor and the signal factor is manipulated to attain the varying target values of the dynamic system output [3, 8], which means the system output (i.e. the response variable) is a function of the signal factors [9].

As the most critical step of the three-stage design method, parameter design has always been the focus of scholars’ research and discussion, and its application scope covers numerous fields, such as optimization of process parameter [10, 11, 12, 13, 14], improvement of product performance [15, 16], analysis of factors affecting quality characteristics [17, 18, 19]. However, it is not difficult to find that the existing research on the application of parameter design for quality improvement mostly focuses on the static characteristics of products or processes. Although many scholars have studied the parameter design of dynamic systems, few literature has analyzed the differences of parameter design between response variables and signal factors in different functional relationships. Thus, in this paper, we study the dynamic parameter design method under the general linear relationship between response variables and signal factors.
This paper is organized as follows. In Section 2, we provide different functional relationships between response variables and signal factors. In Section 3, we describe the general steps for dynamic parameter design in general linear relationships. Section 4 provides an example with the general linear relationship to verify the accuracy and effectiveness of the proposed method. And some concluding remarks are given in Section 5.

2. Multiple functional relations in dynamic systems

In dynamic systems, the response variable and signal factor will present several different functional relationships, which can be divided into three types: zero-point proportional equation, reference-point proportional equation, and general linear equation [20].

Both the origin-point proportional equation and reference-point proportional equation are proportional equations and are suitable for cases where the response variable exhibits a clear proportional relationship with the signal factor. For example, in the calibration of measuring equipment, if changes in the scale intervals of the measuring equipment are proportional to the measured size, all scale marks will be calibrated according to a proportional equation. For a reference sample, the reference-point proportional equation is specifically used to perform SNR estimation and error calibration. When no specific constraint conditions exist between the response variable and signal factor, a general linear relationship is recommended for use. For example, in investigations of a hematomanometer, the situation where the blood pressure is zero is not crucial, and whether the regression line passes through the origin is also not crucial. In this situation, a general linear relationship is employed to examine the optimal design of this system.

The parameter design of a dynamic system is designed to achieve the minimum fluctuation of the desired output within the range of signal factor under various noise conditions, that is, robustness. In addition, the dynamic parameter design also requires the results to have a higher sensitivity to the signal factor, and the ideal relationship between the response variable and signal factor should also have linearity, which makes the output easy to be adjusted or corrected to the required target value.

The key to the success of parameter design is the use of the SNR [1, 2, 4, 21]. Compared with static characteristics, the SNR estimation of dynamic characteristics is more complicated for that its selection is usually based on the research objective or the target function of the product or process. In a parameter design of dynamic characteristics, different research objects, or in other words, different relationships between response variables and signal factors, will lead to different estimation results of SNR. Three types of mathematical models are given below.

2.1. Origin-point proportional equation

Assuming that the truth value of the signal factor is known, the relationship between the response variable and the signal factor is origin-point proportional formula, i.e.

\[ Y = \beta M + \varepsilon \]  

(1)

Where the effective divisor \( r = \frac{1}{n} \sum_{i=1}^{k} (M_i - \overline{M})^2 \).

2.2. Reference-point proportional equation

Assuming that the truth value of the signal factor is known and \( M_s \) is the reference-point, the relationship between the response variable and the signal factor is origin-point proportional formula, i.e.

\[ Y - \overline{Y}_s = \beta (M - M_s) + \varepsilon \]  

(2)

Where \( \overline{Y}_s \), the average of the output characteristics of the reference-point, \( \overline{Y}_s = \frac{1}{n} \sum_{i=1}^{n} Y_{si} \).  

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2.3. General linear equation

Generally, the relationship between response variables and signal factors is assumed to be linear, i.e.

\[ Y = a + \beta (M - \bar{M}) + \varepsilon \]  

(3)

3. Steps for dynamic parameter design under a general linear relationship

3.1. Design orthogonal experiment

Orthogonal experiment design is one of the basic tools of Taguchi robust parameter design [22]. In this method, orthogonal table is adopted to obtain the optimal combination of controllable factors by arranging the levels of experimental factors. In the design of orthogonal experiment, Taguchi divided factors affecting product or process quality into controllable factors and noise factors. By making full use of the interaction between controllable factors and noise factors, the optimal level combination of controllable factors was changed, so as to achieve the goal of reducing the fluctuation of noise factor to output response.

3.1.1. Define factor and level

Dynamic parameter design involves signal factors, controllable factors, and noise factors. A signal factor is a factor that guides changes in dynamic characteristics according to the operator input. Controllable factors and noise factors influence the final quality of products; the level of controllable factors can be artificially controlled, whereas that of noise factors cannot. Noise factors include factors such as environmental state and equipment depreciation.

The levels of controllable factors can be arranged in an arithmetic or geometric sequence and can also be determined according to the series values of components; the signal factor is similar. However, regardless of the method of setting levels, one principle should be followed. That is, the interval among levels should be maximized within the range of the discipline to discover the optimal combination.

3.1.2. Design inner-outer array and arrange experiment

In addition to noise factors, the parameter design of dynamic characteristics requires the introduction of signal factors into the outer design. The type of outer array is determined according to the number of levels, and the size is selected according to the number of factors and the number of interactions.

After completing the design of inner and outer arrays, an orthogonal array is employed to arrange the experiment plan. In the inner array, the number of plans determines the number of noise factor level tables. Correspondingly, the number of times the outer design is performed for the signal factor indicates the number of sets of outer array data.

3.2. Calculate SNR

To obtain the best input level combination of controllable factors, Taguchi proposed using SNR as an index of two steps for the first time[1, 22, 23]: the first step is building regression model with position effect and dispersion effect and the corresponding SNR formula according to the different characteristics of the output response, and then selecting the optimal level of controllable factors combination to maximize the SNR; the second step is to select the appropriate regulator to adjust the output response to the appropriate target value. This method is simple in principle and widely used in practice.

The different function relationships between the response variable \( Y \) and signal factor \( M \) are first reflected in the least-squares estimation results of the first-order coefficient in the regression and these differences affect calculations of the sum of squares of variation and the degree of freedom. Moreover, different functional relationships between response variables and signal factors also result in different meanings represented by the sum of squares of variation. In a proportional relationship, \( S_\beta \) represents the sum of squares of variation caused by the proportional relationship. In a general linear relationship,
$S_\beta$ reflects the variation caused by the linear effect of the signal factor $M$. The specific estimation process and calculation formula of SNR under the linear relationship is as follows:

1. Calculate $\hat{a}$, $\hat{b}$
   Using the principle of least squares, obtain
   
   $$a = \hat{a} = -b\bar{M} \tag{4}$$
   
   $$b = \hat{b} = \frac{1}{r} \sum_{i=1}^{k} (M_i - \bar{M}) T_i \tag{5}$$
   
   Where the effective divisor $r = \sum_{i=1}^{k} (M_i - \bar{M})^2$.

2. Calculate the sum of squares of variation $S_r$, $S_\beta$, $S_e$
   
   $$S_r = \sum_{i=1}^{k} \sum_{j=1}^{n} (y_{ij} - \bar{y})^2 = \sum_{i=1}^{k} \sum_{j=1}^{n} y_{ij} - n \bar{y}^2$$
   
   $$S_\beta = \frac{1}{r} \left( \sum_{i=1}^{k} (M_i - \bar{M}) T_i \right)^2 \tag{6}$$
   
   $$S_e = S_T - S_\beta \tag{7}$$
   
   Where the correction term $CT = \frac{r^2}{k_n}$.

3. Calculate estimations of $\beta^2$, $\sigma^2$
   
   $$\hat{\beta}^2 = \left( \frac{S_e}{r} \right) \cdot \frac{\hat{b}^2}{\hat{a}} = \frac{S_e \hat{b}^2}{r \hat{a}^2} \tag{8}$$
   
4. Calculate estimation of SNR
   
   $$\eta = \frac{\hat{\beta}^2}{\hat{\sigma}^2} = \frac{1}{r} \frac{S_e \hat{b}^2}{V_r} = 10 \log_{10} \frac{1}{r} \frac{S_e \hat{b}^2}{V_r} (dB) \tag{9}$$

3.3. Statistical analysis

The SNR is used as an indicator to evaluate plans and perform statistical analysis of the inner array. The steps are as follows:

1. Calculate the auxiliary table of the analysis of variance (ANOVA) for the SNR;
2. Find the total sum of squares of variation $S_T$ for $\eta$ and the sum of squares of variation $S_a$ and $S_\beta$ caused by the factors, as well as the sum of squares of variation $S_e$ caused by error;
3. Organize the aforementioned results into an ANOVA table;
4. A larger $S$ value for a factor indicates that the factor has a greater effect on the SNR and can be called a robust factor. Select the optimal level for each factor using the auxiliary table of ANOVA and complete the parameter design.

4. Case application

An application to a practical case is the final step of the dynamic parameter design method. The purpose of introducing the case is to verify the effectiveness of the aforementioned parameter design
method. The parameter optimization process of a sound pressure meter measurement system is as follows.

4.1. Factors and their levels

Three levels were obtained for signal factor $M$: $M_1 = 8$, $M_2 = 12$, and $M_3 = 16$. The controllable factors and levels of this measurement system are listed in Table 1.

Table 1. Controllable factors and levels.

| Level | Controllable factor |
|-------|---------------------|
| 1     | Large              |
| 2     | Small              |
| 3     | -                  |
|       | A 15 B 100 C 50 D 10 E |
|       | A 30 B 200 C 100 D 20 E |
|       | A 45 B 300 C 200 D 30 E |

An analysis revealed that six factors may cause errors in this measurement system: G, H, I, J, K, and L. To minimize the number of tests, the aforementioned six error factors were integrated and the worst, normal, and best case conditions of each factor were obtained, from which it was determined that the levels of noise factors comprised $N'_1$, $N'_2$, and $N'_3$.

4.2. Design and data

Taguchi’s orthogonal array $L_{18} (2^1 \times 3^7)$ was used to arrange the controllable factors for the inner design. The two-way layout of the noise factors and signal factor $M$ were used for the outer design. The experimental plan and data are listed in Table 2.

After completing the design of the inner and outer arrays, an orthogonal array is employed to arrange the experiment plan. In the inner array, the number of plans determines the number of noise factor level tables. Correspondingly, the number of times the outer design is performed for the signal factor indicates the number of sets of outer array data.

Table 2. Experimental Plan and Data.

| No. | A | B | C | D | E | M_1 | M_2 | M_3 | η |
|-----|---|---|---|---|---|-----|-----|-----|---|
| 1   | 1 | 1 | 1 | 1 | 1 | 6.62 | 10.87 | 8.06 | 5.08 |
| 2   | 1 | 2 | 2 | 1 | 2 | 6.92 | 11.22 | 11.39 | 6.99 |
| 3   | 1 | 3 | 3 | 3 | 3 | 6.09 | 11.12 | 12.79 | 5.75 |
| 4   | 2 | 1 | 1 | 2 | 2 | 6.88 | 11.25 | 11.45 | 9.02 |
| 5   | 1 | 2 | 2 | 3 | 3 | 7.38 | 12.53 | 12.79 | 16.66 |
| 6   | 1 | 2 | 3 | 1 | 1 | 7.21 | 12.12 | 11.45 | 16.73 |
| 7   | 1 | 3 | 1 | 2 | 3 | 8.21 | 12.27 | 11.83 | 17.03 |
| 8   | 1 | 3 | 2 | 1 | 3 | 7.41 | 12.13 | 11.46 | 16.73 |
| 9   | 1 | 3 | 1 | 3 | 2 | 7.11 | 11.66 | 11.37 | 17.47 |
| 10  | 2 | 1 | 3 | 2 | 2 | 7.02 | 12.76 | 11.99 | 16.4 |
| 11  | 2 | 1 | 1 | 3 | 3 | 7.44 | 12.15 | 11.63 | 16.44 |
| 12  | 2 | 1 | 3 | 2 | 1 | 7.32 | 11.02 | 11.38 | 16.87 |
| 13  | 2 | 2 | 1 | 3 | 1 | 6.93 | 12.01 | 11.55 | 17.27 |
| 14  | 2 | 2 | 3 | 1 | 2 | 8.03 | 12.04 | 11.58 | 16.22 |
| 15  | 2 | 2 | 3 | 1 | 2 | 6.97 | 12.19 | 11.97 | 17.02 |
| 16  | 2 | 2 | 3 | 2 | 1 | 6.58 | 13.13 | 11.97 | 16.12 |
| 17  | 2 | 3 | 2 | 3 | 1 | 8.08 | 11.99 | 12.28 | 17.54 |
| 18  | 2 | 3 | 1 | 3 | 1 | 7.33 | 11.55 | 11.58 | 16.46 |

4.3. Calculation of SNR

The experimental data in Plan No.1 of the inner array was used as an example to perform the calculation of SNR. The rearranged data is shown in Table 3.

Table 3. Experimental Data of Plan No.1.

$$\begin{array}{|c|c|c|c|c|} \hline M & j & T_i \\
\hline M_1=8 & 6.62 & 7.08 & 7.23 & 20.93 \\
M_2=12 & 10.87 & 11.24 & 12.31 & 34.42 \\
M_3=16 & 18.06 & 18.37 & 18.68 & 55.11 \\
\hline \end{array}$$

110.46
Similarly, the SNR under other conditions can be obtained. The results are displayed in the last column of Table 4.

4.4. Analysis of variance

The auxiliary table of the ANOVA for SNR is shown in Table 4.

| $T_i$ | $\Sigma$ |
|-------|---------|
|       | A       | B       | C       | D       | E       |
| 1     | 70.46   | 47.33   | 53.9    | 58.14   | 58.52   |
| 2     | 98.28   | 59.29   | 59.26   | 56.53   | 53.77   |
| 3     | 62.12   | 55.58   | 54.07   | 56.45   | 56.45   |

The ANOVA demonstrates the importance of factors. The analysis results of Table 5 reveal that factors A and B exerted relatively large effects on the SNR, where A was the most influential parameter for this measurement system.

| Source | $S$  | $f$ | $V$  | $F$  | $S'$ | $\rho$    |
|--------|------|-----|------|------|------|-----------|
| A      | 42.00| 1   | 42.00| 6.89 | 35.9 | 81.13%    |
| B      | 20.55| 2   | 10.28| 1.69 | 8.35 | 18.87%    |
| C      | 2.51 | 2   | 1.26 | 0.21 | 0.16 |           |
| D      | 1.40 | 2   | 0.7  | 0.11 |      |           |
| E      | 1.89 | 2   | 0.95 | 0.16 |      |           |
| e      | 12.19| 8   | 6.10 |      |      |           |
| T      | 80.54| 17  |      |      |      |           |
4.5. Optimal conditions

According to the ANOVA results, the characteristics of larger and more favourable SNR combinations can be determined from Table 4. The optimal levels of controllable factors A and B are A₁ and B₃, respectively, whereas the levels of other factors can be selected according to considerations of changes in needs such as the costs and ease of operation in practical applications.

5. Conclusions

This study examined a dynamic parameter design method under a general linear relationship between response variables and signal factors. First, mathematical models of different general linear relationships between response variables and signal factors were established and steps of dynamic parameter design in the general linear relationship were proposed. The case of parameter optimization for a sound pressure meter measurement system was used for verification, which revealed that when the response variables and the signal factor exhibited a general linear relationship, the parameter design had the following characteristics:

(1) Different linear relationships between the response variable and signal factor cause differences in least-squares estimation results, thereby affecting the calculations of the sum of squares of variation and the degree of freedom. When the response variables and the signal factors exhibit a general linear relationship, the \( f_i \) and \( f_e \) differ from the degree of freedom under a proportional relationship by 1, and the total sum of squares of variation differs by one correction term;

(2) Under different relationships, differences in the SNR calculation results are reflected in the ANOVA, and the optimal conditions are confirmed by the ANOVA table. That is, differences in the SNR calculation results affect the selection of optimal parameters.

The aforementioned results indicate that SNR estimations based on the different functional relationships between response variables and signal factors generate different calculation results, affect the ANOVA, and even change the selection and combination of optimal parameters. Therefore, it is necessary to distinguish the type of functional relationship between response variables and the signal factor prior to performing the statistical analysis and parameter selection of the SNR. A targeted and scientific parameter design can be achieved by establishing a mathematical model between response variables and signal factors, clarifying the SNR estimation under the specific relationship, and accordingly performing statistical analysis and parameter selection. This also leads to highly favourable minimization of variation.

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