Probing the charged Higgs quantum numbers through the decay \( H_\alpha^+ \to W^+ h_\beta^0 \)

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Abstract

The vertex \( H_\alpha^+ W^- h_\beta^0 \), involving the gauge boson \( W^\pm \) and the charged (\( H_\alpha^+ \)) and neutral Higgs bosons (\( h_\beta^0 \)), arises within the context of many extensions of the SM, and it can be used to probe the quantum numbers of the Higgs multiplet. After presenting a general discussion for the expected form of this vertex with arbitrary Higgs representations, we discuss its strength for several specific models, which include: i) the Two-Higgs Doublet Model (THDM), both the generic and the SUSY case, and ii) models with additional Higgs triplets, including both SUSY and non-SUSY cases. We find that in these models, there are regions of parameters where the decay \( H_\alpha^+ \to W^+ h_\beta^0 \), is kinematically allowed, and reaches Branching Ratios (BR) that may be detectable, thus allowing to test the properties of the Higgs sector.

1 Introduction

The Higgs spectrum of many well motivated extensions of the Standard Model (SM) often includes a charged Higgs, whose detection at future colliders would constitute a clear evidence of a Higgs sector beyond the minimal SM [1]. In particular, the two-Higgs doublet model (THDM) has been extensively studied as a prototype of a Higgs sector that includes a charged Higgs boson (\( H^+ \))[1]. However, a definitive test of the mechanism of electroweak symmetry breaking will require further studies of the complete Higgs spectrum. In particular, probing the properties of charged Higgs could help to find out whether it is indeed associated with a weakly-interacting theory, as in the case of the...
popular minimal SUSY extension of the SM (MSSM) [2], or to a strongly-interacting scenario [3]. Furthermore, these tests should also allow to probe the symmetries of the Higgs potential, and to determine whether the charged Higgs belongs to a weak-doublet or to some larger multiplet.

Decays of a charged Higgs boson have been studied in the literature, including the radiative modes \( W^+\gamma, W^+Z \) [4], mostly within the context of the THDM, or its MSSM incarnation, and more recently for the effective Lagrangian extension of the THDM [5]. Charged Higgs production at hadron colliders was studied long ago [6], and recently more systematic calculations of production processes at LHC have been presented [7]. Current bounds on charged Higgs mass can be obtained at Tevatron, by studying the top decay \( t \to bH^+ \), which already eliminates some region of parameter space [8], whereas LEP bounds give approximately \( m_{H^+} > 100 \text{ GeV} \) [9].

On the other hand, the vertex \( H_\alpha^+W^-h_\beta^0 \), deserves special attention because it can give valuable information about the underlying structure of the gauge and scalar sectors. In the first place, the decay mode \( H_\alpha^+ \to W^+h_\beta^0 \) could be detected at the future Large Hadron Collider (LHC) as it was claimed in reference [10] within the context of the MSSM. Furthermore, the vertex \( H_\alpha^+W^-h_\beta^0 \), can also induce the associated production of \( H_\alpha^+ + h_\beta^0 \) at hadron colliders, through a virtual \( W^{*+} \) in the s-channel, which could become a relevant production mechanism for heavy charged Higgs bosons.

In this paper we are interested in studying thoroughly the physics behind this important vertex. Its organization goes as follows: in section 2, we present a general analysis of the case when the Higgs sector includes arbitrary Higgs representations; we derive a general formula for the vertex \( H_\alpha^+W^-h_\beta^0 \) in terms of the isospin components and the hypercharge of the Higgs multiplet. Then, in section 3, we apply this general discussion to study the charged Higgs vertex within the THDM and the minimal SUSY extension of the SM (MSSM). Results for the BR of the charged Higgs decay in the MSSM are presented including the leading radiative corrections. In section 4, we discuss the strength of the vertex for an extended supersymmetric model that includes a complex Higgs triplet; we perform a numerical analysis to search for values of Higgs masses that allow the decays \( H_\alpha^+ \to W^+h_\beta^0 \) to proceed. Finally, in section 5 we present our conclusions.

### 2 General Analysis

Let us consider a Higgs sector that includes several Higgs multiplets \( \Phi_\alpha \), which transforms under \( SU(2) \times U(1) \) with isospin \( (T_\alpha) \) and hypercharge \( (Y_\alpha) \). The components of the isospin \( T_\alpha, T_{i\alpha} (i = 1, 2, 3) \) are \( n \)-dimensional representa-
tions of \(SU(2)\), and satisfy the algebra \([T_{i\alpha}, T_{j\alpha}] = i\epsilon_{ijk}T_{k\alpha}\). From the operators \(T_{i\alpha}\) one can define the raising and the lowering operators \(T_{a}^{\pm} = T_{1\alpha} \pm iT_{2\alpha}\), which will be used next.

The kinetic terms of the Higgs multiplets are given by

\[
\mathcal{L}_K = \sum_{\alpha} (D^\mu\Phi_\alpha)^\dagger (D_\mu\Phi_\alpha),
\]

(1)

where \(D_\mu\) denotes the covariant derivative, and it takes the following form for a general multiplet,

\[
D_\mu = \partial_\mu - \frac{ig}{\sqrt{2}} (T^{+}W_\mu^{+} + T^{-}W_\mu^{-}) - \frac{ig}{c_w} (T_{3} - s_{w}Q)Z_\mu - ieQA_{\mu},
\]

(2)

where \(Q\) is the charge operator and the hypercharge \(Y\) is normalized to satisfy the relation \(Q = T_{3} + Y/2\). Equation (1) will induce the gauge boson masses and the Higgs-gauge vertices after spontaneous symmetry breaking (SSB).

### 2.1 The Goldstone bosons

The Higgs multiplet \(\Phi_\alpha\) can be expanded in terms of the spinors \(\chi^{n(+)}\), eigenstates of \(T_{3a}^2\) and \(T_{3a}\), as follows:

\[
\Phi_\alpha = \sum_n \phi^{n(+)}_\alpha \chi^{n(+)}_\alpha, \quad (\Phi_\alpha)^\dagger = \sum_n (\phi^{n(+)}_\alpha)^* (\chi^{n(+)}_\alpha)^\dagger,
\]

(3)

where the components \(\phi^{n(+)}_\alpha\) denote the scalar state with \(n\) units of the electric charge, and include: \(\phi^{0}_\alpha, \phi^{\pm}_\alpha, \phi^{\pm\pm}_\alpha, \ldots\), for \(n = 0, \pm 1, \pm 2, \ldots\). The spinors \(\chi^{n(+)}_\alpha\), being eigenstates of \(T_{3a}^2\) and \(T_{3a}\), satisfy the following relations,

\[
(\chi^{m(+)}_\alpha)^\dagger (\chi^{n(+)}_\beta) = \delta_{\alpha,\beta} \delta^{m,n},
\]

\[
T_{3a} \chi^{m(+)}_\alpha = T_{3a} \chi^{m(+)}_\alpha, \quad T_{3a}^m = T_{\alpha}, T_{\alpha} - 1, \ldots, -T_{\alpha},
\]

\[
T^{\pm, m} \chi^{m(+)}_\alpha = T^{\pm, m} \chi^{m(+)}_\alpha, \quad T^{\pm, m} = \left[ (T_{\alpha} \mp T_{3a}) (T_{\alpha} \pm T_{3a} + 1) \right]^{1/2},
\]

(4)

where \(T_{3a}^m\) and \(T^{\pm, m}_{\alpha}\) are the eigenvalues of the operators \(T_{3a}\) and \(T^{\pm}_{\alpha}\) respectively.

After SSB, the neutral Higgs components acquire vacuum expectation values (v.e.v.'s), and one can write:

\[
< \Phi_\alpha > = v_\alpha \chi^{0}_\alpha, \quad < \Phi_\alpha >^\dagger = v_\alpha^* (\chi^{0}_\alpha)^\dagger.
\]

(5)

While in some particular model it is possible that some of the \(< \Phi_\alpha >\) could be absent, in the following we shall assume that \(v_\alpha^* = v_\alpha\), which corresponds
to a CP-invariant vacuum. Thus, in order to obtain the Higgs mass matrices and interactions we need to make the substitution $\Phi_\alpha \to \Phi_\alpha + <\Phi_\alpha>$ into the lagrangian (1).

Expanding equation (1), gives the following term for the linear term involving the charged gauge boson,

$$\left( L_K \right)_{W^\pm, \phi^\pm, \phi^0} = \frac{ig}{\sqrt{2}} W^-_\mu \sum_\alpha \left[ \Phi^\dagger_\alpha T^- (\partial_\mu \Phi_\alpha) - (\partial_\mu \Phi_\alpha)^\dagger T^- \Phi_\alpha \right] - \frac{ig}{\sqrt{2}} W^+_\mu \sum_\alpha \left[ (\partial_\mu \Phi_\alpha)^\dagger T^+ \Phi_\alpha - \Phi^\dagger_\alpha T^+ (\partial_\mu \Phi_\alpha) \right].$$ (6)

From this equation one can identify the combination of fields that correspond to the charged Goldstone boson $G^\pm_W$, by separating terms of the form $im_W(W^-_\mu \partial^\mu G^+_W - W^+_\mu \partial^\mu G^-_W)$, which leads to

$$G^+_W = \frac{g}{\sqrt{2} m_W} \sum_\alpha \left[ (T^+ <\Phi_\alpha>)^\dagger \Phi_\alpha - \Phi^\dagger_\alpha T^- <\Phi_\alpha> \right].$$ (7)

The expression for the charged Goldstone bosons can be written then in terms of the components $\phi^\pm_\alpha$, as follows:

$$G^+_W = \frac{g}{\sqrt{2} m_W} \sum_\alpha v_\alpha \left[ T^{+,0}_\alpha \phi^+_\alpha - T^{-,0}_\alpha (\phi^-_\alpha)^* \right] C_{Y_\alpha}. $$ (8)

For the cases when $T_\alpha$ is integer and $Y = 0$ (i.e. in real representation), one has: $T^{+,0}_{\alpha} = T^{-,0}_{\alpha} = \sqrt{T_\alpha(T_\alpha + 1)}$. On the other hand, when $2T_\alpha = Y_\alpha$, which corresponds to a complex representation, and $T_\alpha$ could be either integer or half-integer (e.g., Higgs doublets with $Y = 1$ or triplets with $Y = 2$), one has $T^{+,0}_{\alpha} = 0$ for $Y_\alpha < 0$ and $T^{-,0}_{\alpha} = 0$ when $Y_\alpha > 0$. Furthermore, one can fix the following phase convention: $\phi^{+,T}_{\alpha} = - (\phi^-_{\alpha,T})^*$. We also notice that when $T_\alpha$ is integer and $Y_\alpha = 0$, $<\Phi_\alpha(T,Y)>$ can contribute to $m_W$.

Thus, the most general expression for the charged Goldstone bosons for a Higgs sector that includes an arbitrary number of multiplets $\Phi_\alpha(T,Y)$ (either with $Y_\alpha = 0$ or $Y_\alpha = 2T_\alpha$) is given by:

$$G^+_W = \frac{g}{\sqrt{2} m_W} \sum_\alpha v_\alpha T^{+,0}_\alpha \phi^+_\alpha.$$ (9)

### 2.2 The vertex $H^+_\alpha W^- h^0$

To derive the form of this vertex, we have to determine the physical charged Higgs states $H^+_\alpha$, which must be orthogonal of the Goldstone boson $G^+_W$. For this, one needs to construct and diagonalize the Higgs mass matrix, which
requires to study the Higgs potential. However, in this section we shall proceed as general as possible, and we will only indicate the mixing matrix for the charged and neutral Higgs states.

Using the previous expansion for $\Phi_\alpha$ (equation (3)), as well as the properties of the spinors $\chi_\alpha$ (equation (4)), and substituting them in the equation (6), we obtain the general expression for the vertices of the type $W^+H^{n(+)}H^{(n+1)(-)}$ as follows

$$
(L_K)_{W^+\phi^+\phi^0} = \frac{ig}{\sqrt{2}} W^- \sum_\alpha \sum_n T^{-,-}_\alpha \left[ (\phi^{(+n-1)}_\alpha)^* \partial_\mu \phi^{(+n)}_\alpha \right]

- \frac{ig}{\sqrt{2}} W^+ \sum_\alpha \sum_n T^{+,n-1}_\alpha \left[ (\phi^{(+n-1)}_\alpha)^* \partial_\mu \phi^{(+n)}_\alpha \right],
$$

(10)

where $a \leftrightarrow \partial_\mu b = a \partial_\mu b - b \partial_\mu a$. We pick now the terms with $n = 0, 1$, to obtain the following expression for the vertex $W^-\phi^+\phi^0$,

$$
(L_K)_{W^-\phi^+\phi^0} = \frac{ig}{\sqrt{2}} W^- \sum_\alpha \left[ T^{-,-}_\alpha (\phi^{-}_\alpha)^* \partial_\mu \phi^0_\alpha + T^{-,-}_\alpha (\phi^{-}_\alpha)^* \partial_\mu \phi^{-}_\alpha \right]

- \frac{ig}{\sqrt{2}} W^+ \sum_\alpha \left[ T^{+,n-1}_\alpha \phi^+_\alpha \partial_\mu (\phi^0_\alpha)^* + T^{+,n-1}_\alpha \phi^+_\alpha \partial_\mu (\phi^+_\alpha)^* \right].
$$

(11)

If we focus our attention on Higgs multiplets with integer $T_\alpha$ and $Y_\alpha = 0$ (or $Y_\alpha = 2T_\alpha$), then we have $T^{-,-}_\alpha = T^{-,-}_\alpha = T^{+,n-1} = T^{+,n-1} = 0$ and $T^{+,n-1}_\alpha = T^{+,n-1}_\alpha$). Furthermore, for the case $Y_\alpha = 0$ we use the phase conventions: $\phi^+_{\alpha T} = -(\phi^{-}_{\alpha T})^*$. Thus, for these cases the coupling $W^-\phi^+\phi^0$, has the following form:

$$
(L_K)_{W^-\phi^+\phi^0} = \frac{ig}{\sqrt{2}} W^- \sum_\alpha T^{+,0}_\alpha \phi^+_\alpha \partial_\mu \phi^+_\alpha + \text{h.c.}.
$$

(12)

Since $T^{+,0}_\alpha = \text{const}$, the strength of the vertex will depend on the mixing factors. Namely, in order to obtain the coupling $W^-H^+h^0$ it is necessary to determine the physical Higgs states of the charged and neutral sector (CP-even). Since we have assumed that the Higgs potential is CP-invariant, the imaginary and real parts of the neutral scalar fields do not mix. Thus, the physical neutral Higgs bosons (CP-even) are determined from $\text{Re} \phi^0_\alpha = \varphi^0_\alpha$.

For the charged Higgs, we define a unitary rotation that gives the physical mass eigenstates $H^+_{\alpha}$ as:

$$
H^+_\alpha = \sum_\beta U_{\alpha\beta} \phi^+_\beta.
$$

(13)

1 A phenomenological study of the double-charged Higgs vertex is underway [11].
Then, we choose the first charged field $H_1^+$ to be the Goldstone boson $G_W^+$, while the physical charged fields $H_\alpha^+$ (for $\alpha \geq 2$) are orthogonal states to the Goldstone boson. Then, we can fix the first row of the matrix $U$, through the following expression

$$U_{1\beta} = \frac{g}{\sqrt{2m_W}} v_\beta T^{+,0}_\beta. \quad (14)$$

For the physical neutral Higgs eigenstates $H_\beta^0$, one only needs to consider $Re\phi_\alpha^0$, and we introduce a similar unitary rotation ($V_{\beta\gamma}$) that gives the mass-eigenstates, namely:

$$H_\beta^0 = \sum_\beta V_{\beta\gamma} \phi_\beta^0. \quad (15)$$

We can also choose the first physical neutral field $H_1^0$ to be the lightest Higgs boson, which can be identified as the light SM-like state preferred by EW radiative corrections.

Thus, using these rotations $U$ and $V$, as well as the previous conventions, we find that the vertex $W^+H_\alpha^-H_\beta^0$ is given by

$$(\mathcal{L}_K)_{W^+H_\alpha^-H_\beta^0} = \frac{ig}{2} \left[ \sum_{\alpha \geq 2, \beta} \eta_{\alpha,\beta} H_\beta^0 \partial_\mu H_\alpha^+ \right] W^-_{\mu}. \quad (16)$$

where:

$$\eta_{\alpha,\beta} = \sqrt{2} \sum_\gamma T^{+,0}_\gamma V_{\beta\gamma}^* U_{\alpha\gamma}; \quad (17)$$

it depends on the quantum number $T_\alpha$ and the mixing matrices $U$&$V$, but not on the vev's.

2.3 Application: a model with one Higgs doublet and one real triplet

To illustrate the above formulae, we can write the corresponding expressions for a model that includes a complex Higgs doublet, i.e. $T = 1/2, T_3 = \pm 1/2$, and a real triplet, with $T = 1, Y = 0$. In matrix representations, the eigenstates of $T$ for the doublet are given by

$$|\frac{1}{2}, \frac{1}{2} > = \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \chi^+, \quad |\frac{1}{2}, -\frac{1}{2} > = \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \chi^0, \quad \quad (18)$$

$$T_3 = \frac{1}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$
Then, we can expand the Higgs doublet $\Phi_D$ in terms of the spinors $\chi^{+,0}$ as follows:

$$\Phi_D = \begin{pmatrix} \phi_D^+ \\ \phi_D^0 \end{pmatrix} = \phi_D^+ \chi^+ + \phi_D^0 \chi^0 = \sum_{n=0}^{1} \phi_D^{n(+)} \chi^{n(+)}. \quad (19)$$

For this representation the operator $T^\pm$ are

$$T^+ = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \quad T^- = (T^+)^\dagger = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}. \quad (20)$$

And their action on the spinors $\chi^{+,0}$ are:

$$T^+ \chi^+ = 0, \quad T^+ \chi^0 = \chi^+, \quad T^- \chi^+ = \chi^0, \quad T^- \chi^0 = 0.$$

On the other hand, for a Higgs triplet with $T = 1$, one has $T_3 = 1, 0, -1$ and the spinors associated with the isospin eigenstates are given by:

$$|1, 1>= \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \chi^+, \quad |1, 0>= \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \chi^0, \quad |1, -1>= \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \chi^- . \quad (21)$$

$T_3$ has the matrix representation

$$T_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}. \quad (22)$$

Thus, the Higgs triplet $\Phi_T$ with hypercharge $Y = 0$, can be expanded in terms of the spinors $\chi^{+,0,-}$ as follows

$$\Phi_\alpha = \begin{pmatrix} \phi_\alpha^+ \\ \phi_\alpha^0 \\ \phi_\alpha^- \end{pmatrix} = \phi_\alpha^+ \chi^+ + \phi_\alpha^0 \chi^0 + \phi_\alpha^- \chi^- = \sum_{n=-1}^{1} \phi_\alpha^{n(+)} \chi^{n(+)}. \quad (23)$$

In this case, the action of matrix $T^\pm$ on the spinors $\chi^{+,0,-}$ are given by $T^+ \chi^+ = 0, \quad T^+ \chi^0 = \sqrt{2} \chi^+, \quad T^+ \chi^- = \sqrt{2} \chi^0, \quad T^- \chi^+ = \sqrt{2} \chi^0, \quad T^- \chi^- = \sqrt{2} \chi^-, \quad T^- \chi^0 = 0$. The model includes one charged Higgs and two neutral CP-even states, which are obtained from the weak eigenstates by orthogonal $2 \times 2$ rotations, which are parameterized by mixing angles $\alpha$ and $\delta$, respectively. Therefore, one can write the factor $\eta$ in terms of these mixing angles; for the light state it goes
as follows:

$$\eta_{h^0} = \cos \gamma \sin \delta + \sqrt{2} \sin \gamma \cos \delta. \quad (24)$$

Similarly, for the heavier neutral Higgs we obtain:

$$\eta_{H^0} = \sin \gamma \sin \delta + \sqrt{2} \cos \gamma \cos \delta, \quad (25)$$

where $\tan 2\gamma$ depends of parameters of the Higgs potential.

Thus, the coupling $H^+ W^- h^0$ is quite sensitive to the structure of the covariant derivative, and could be one place where to look for deviations from the minimal THDM (or SUSY) prediction at tree-level.

3 The vertex $H^+ W^- h^0$ in the THDM and MSSM

One of the simplest models that predicts a charged Higgs is the THDM, which includes two scalar doublets of equal hypercharge, namely $\Phi_1 = (\phi_1^+, \phi_1^0)$ and $\Phi_2 = (\phi_2^+, \phi_2^0)$, this is indeed the Higgs content used in the minimal SUSY extension of the SM (MSSM). Besides the charged Higgs ($H^\pm$), the spectrum of the THDM includes two neutral CP-even states ($h^0, H^0$, with $m_{h^0} < m_{H^0}$), as well as a neutral CP-odd state ($A^0$).

Diagonalization of the charged mass matrices gives the expression for the charged Higgs mass-eigenstate: $H^+ = \cos \beta \phi_1^+ + \sin \beta \phi_2^+$, where $\tan \beta = v_2/v_1$ denotes the ratio of v.e.v.'s from each doublet. In this case the factor $\eta$ that appears in the vertex $H^+ W^- h^0$ is given by:

$$\eta_{h^0} = \sin \beta \sin \alpha + \cos \beta \cos \alpha$$

$$= \cos(\beta - \alpha). \quad (26)$$

Similarly, for the heavier neutral Higgs we obtain: $\eta_{H^0} = \sin(\beta - \alpha)$. In these cases it is possible for the vertices $H^+ W^- h^0(H^0)$ to vanish, but only thanks to ad hoc combination of mixing angles.

3.1 The decay $H^+ W^- h^0$ in the THDM

The vertex $H^+ W^- h^0$ could induce the decay $H^+ \rightarrow W^+ h^0(H^0)$, whenever it is kinematically allowed. Since the charged and neutral Higgs masses are given
by:

\[
\begin{align*}
    m_{H^\pm}^2 &= m_A^2 + \frac{2m_W^2}{g^2}(\lambda_5 - \lambda_4),
    \\
    m_{h^0}^2 &= m_A^2c_\beta^2 - a + v^2 \left[ \lambda_1c_\beta^2s_\alpha^2 + \lambda_2s_\beta^2c_\alpha^2 - 2\lambda_Tc_\alpha c_\beta s_\alpha s_\beta + \lambda_5c_\beta^2 \right], \\
    m_{H^0}^2 &= m_A^2s_\beta^2 - a + v^2 \left[ \lambda_1c_\beta^2s_\alpha^2 + \lambda_2s_\beta^2c_\alpha^2 + 2\lambda_Tc_\alpha c_\beta s_\alpha s_\beta + \lambda_5s_\beta^2 \right],
\end{align*}
\]

where \( \lambda_i \) are the parameters of the quartic terms that appear in the Higgs potential, \( \lambda_T = \lambda_3 + \lambda_4 + \lambda_5 \) [12,13]. Therefore, one can write the following conditions on the Higgs parameters for the decay \( H^+ \rightarrow W^+ h^0 \) to proceed:

\[
\cos^2 \beta \lambda_1 + \sin^2 \beta \lambda_2 \leq -\frac{2}{v^2} \left[ m_W^2 - \frac{\mu_{12}^2}{\cos \beta \sin \beta} \right] - \lambda_4 - \lambda_5
\]

In the decoupling limit approximation, where \( \alpha \approx \beta - \frac{\pi}{2} \) and \( \mu_{12}^2 = 0 \) (with \( \lambda_6 = \lambda_7 = 0 \)) this condition becomes

\[
\cos^2 \beta \lambda_1 + \sin^2 \beta \lambda_2 \leq -\frac{2m_W^2}{v^2} - \lambda_4 - \lambda_5
\]

which also corresponds to the case when the scalar potential respects a discrete symmetry under which one doublet changes sign. Thus we see that there are regions of parameters where the decay \( H^+ \rightarrow W^+ h^0 \) can proceed. The corresponding decay width is given by:

\[
\Gamma(H^+ \rightarrow W^+ h^0) = \frac{g^2\lambda^2}{64\pi m_H^3} \left[ m_H^2 - 2(m_H^2 + m_{h^0}^2) + \frac{(m_H^2 - m_{h^0}^2)^2}{m_W^2} \right]
\]

where \( \lambda \) is the usual kinematic factor, \( \lambda(a,b,c) = (a - b - c)^2 - 4bc \). This decay mode has been studied in the literature [10], and it is concluded that the coming large hadron collider (LHC) can detect it. For the light SM-like Higgs, this decay is proportional to the factor \( \eta_{h^0}^2 = \cos^2(\beta - \alpha) \), which will determine its strength.

Other relevant decays of the charged Higgs boson are the modes into fermion pairs, which include the decays \( H^+ \rightarrow \tau \nu_\tau, c\bar{b} \), and possibly into \( t\bar{b} \). If the charged Higgs is indeed associated with the Higgs mechanism, its couplings to fermions should come from the Yukawa sector, and the corresponding decays should have a larger BR for the modes involving the heavier fermions. A very simple test of this could be done through a comparison of the modes \( H^+ \rightarrow \tau \nu_\tau \) and \( H^+ \rightarrow \mu \nu_\mu \), which should have very different BR’s.

To evaluate the branching ratios within the THDM we have used the expressions for the decay widths of the tree-level modes, as appearing in ref. [1]. We
take $m_t = 175$ GeV, and the values for the electroweak parameters of the Table of Particle Properties [14]. We shall only comment on the resulting BR for the charged Higgs for the following scenarios, and assume here $m_h = 115$ GeV. A more detailed discussion of the THDM case was presented in ref. [5].

a) Non-decoupling scenario-A. We consider here a large mass difference between $A^0$ and $H^+$, i.e. $m_{H^+} - m_A = 300$ GeV, with $m_H \simeq m_{H^+}$, and also $\alpha \simeq \beta - \pi/2$. In this case we find that the mode $W^+h^0$ has a BR about $10^{-3}$ (2 $\times$ $10^{-5}$), for $\tan \beta = 7$ (30) and $H^+ = 300$ GeV. On the other hand, the BR for the radiative modes $H^+ \rightarrow W^+Z$ and $H^+ \rightarrow W^+\gamma$ is about $4 \times 10^{-2}$ (4 $\times$ $10^{-3}$), for is about $2 \times 10^{-6}$ (2 $\times$ $10^{-7}$), respectively. Thus, for this case, $H^+ \rightarrow W^+Z$ dominates.

b) Non-decoupling scenario-B. Here we also assume a large mass difference between $A^0$ and $H^+$, i.e. $m_A - m_{H^+} = 300$ GeV, with $m_H \simeq m_{H^+}$, but now with $\alpha \simeq \beta - \pi/4$. In this case we find that the BR for the mode $W^+h^0$ has a BR about 1 (0.2) for $\tan \beta = 7$ (30) and $H^+ = 300$ GeV. Similarly, the BR for the mode $H^+ \rightarrow W^+Z$ is about $10^{-2}$ (4 $\times$ $10^{-3}$), whereas the BR for $H^+ \rightarrow W^+\gamma$ is about $4 \times 10^{-7}$ (2 $\times$ $10^{-7}$). In this scenario $H^+ \rightarrow W^+h^0$ clearly dominates, even above the $t\bar{b}$ mode.

3.2 The decay $H^+W^-h^0$ in the MSSM with radiative corrections

As it was mentioned before, one of the most popular motivations for the THDM, is that such Higgs sector is in fact the one of the minimal SUSY extension of the SM (MSSM). The masses of the two CP-even neutral Higgses ($h^0, H^0$) and the charged pair ($H^\pm$), are conveniently determined in terms of the mass of the CP-odd state ($A^0$) and $\tan \beta = v_2/v_1$. The Higgs potential of the MSSM has less free parameters than the THDM; in particular the quartic couplings are given in terms of the gauge couplings, which then implies that the light Higgs boson must satisfy the (tree-level) bound $m_{h^0} \leq \cos 2\beta m_Z$. However this relation receives important corrections from top/stop loops, which give an approximate bound $m_{h^0} \leq 130$ GeV [15].

In the decoupling limit ($m_A >> m_Z$) the parameters of the potential give the approximate relation: $c_{\beta-\alpha}^2 \sim m_Z^2/m_{A^0}^2$, which tends to be small for large values of $m_{A^0}$. One also obtains an approximately degenerated spectrum of heavy Higgs bosons, i.e. $m_{H^+} \simeq m_{H^0} \simeq m_{A^0}$, while the mixing angles satisfy the approximate relation: $\alpha \simeq \beta - \pi/2$. Therefore, only the decay mode $W^+h^0$ is allowed for most regions of parameter space of the MSSM. One obtains a typical BR of the order $2 \times 10^{-2}$ (7 $\times$ $10^{-5}$) for $m_{H^+} = 300$ GeV and $\tan \beta = 7$ (30).

We have performed a detailed parametric search for contour regions for the
branching ratio of $H^\pm \rightarrow W^\pm + h^0$, using the program HDECAY [16], and the results are shown in fig. 1. We thus see that the BR is larger for small values of $\tan \beta$ and almost independent of $m_{A^0}$.

4 The vertex $H^+W^-h^0$ in a SUSY models with a Higgs triplet

The supersymmetric model with two doublets and a complex triplet is one of the simplest extension of the minimal supersymmetric model that allows to study phenomenological consequences of an explicit breaking of the custodial symmetry SU(2) [17].

4.1 The Higgs sector of the model

The model includes two Higgs doublets and a (complex) Higgs triplet given by

$$
\Phi_1 = \begin{pmatrix}
\phi_1^0 \\
\phi_1^-
\end{pmatrix},
\Phi_2 = \begin{pmatrix}
\phi_2^+
\phi_2^0
\end{pmatrix},
\sum = \begin{pmatrix}
\sqrt{\frac{1}{2}}\xi^0 & -\xi_2^+ \\
\xi_1^- & -\sqrt{\frac{1}{2}}\xi^0
\end{pmatrix}.
$$

(30)

The Higgs triplet is described in terms of the $2 \times 2$ matrix representation; $\xi^0$ is the complex neutral field, and $(\xi_1^-)^*, (\xi_2^+)$ denote the charged scalars. The most general gauge invariant and renormalizable superpotential, that can be written for the Higgs superfields $\Phi_{1,2}$ and $\Sigma$ is given by:

$$
W = \lambda \Phi_1 \cdot \Sigma \Phi_2 + \mu_D \Phi_1 \cdot \Phi_2 + \mu_T \text{Tr}(\Sigma^2),
$$

(31)

where we have used the notation $\Phi_1 \cdot \Phi_2 \equiv \epsilon_{ab} \Phi_1^a \Phi_2^b$. The resulting scalar potential involving only the Higgs fields is then written as

$$
V = V_{SB} + V_F + V_D,
$$

where $V_{SB}$ denotes the most general soft-supersymmetry breaking potential [18]. In turn, the full scalar potential can be splitted into its neutral and charged parts, i.e. $V = V_{\text{charged}} + V_{\text{neutral}}$.

From the Higgs potential one derives the minimization conditions and the scalar mass matrices. For the neutral scalars we have that the resulting mass matrix splits into two blocks, one of them (the imaginary components) is associated with the pseudoscalar Higgs states, while the other one (real components) describes the masses of the scalar Higgs bosons. The mass matrix for the imaginary parts contains a massless state, which is the Goldstone boson $G^0$ that gives mass to the $Z$ boson. Whereas the mass matrix for charged states include also a massless state $G^+$, which give mass to $W^+$ boson ($G^{+*} \equiv G^-$).
Besides the supersymmetry-breaking mass terms, $m_i^2$ ($i = 1, 2, 3$), the potential depends on the parameters $\lambda$, $\mu_D$, $\mu_T$, $A$, $B$. For simplicity we shall assume that there is no CP violation in the Higgs sector, and thus all the parameters and the v.e.v.’s are assumed to be real. The explicit expressions of the Higgs potential are given in ref [18].

We can also combine the v.e.v.’s of the Higgs doublet as $v_D^2 \equiv v_1^2 + v_2^2$ and define $\tan\beta \equiv v_2/v_1$. Further, the relations between $(v_D, v_T)$ and $(m_W^2, m_Z^2)$ are

$$m_W^2 = \frac{1}{2} g^2 (v_D^2 + 4v_T^2),$$
$$m_Z^2 = \frac{1}{2} g^2 v_D^2 \cos^2\theta_W.$$

which imply that the tree-level $\rho$-parameter is different from one, namely,

$$\rho \equiv \frac{M_W^2}{M_Z^2 \cos^2\theta_W} = 1 + 4R^2, \quad R \equiv \frac{v_T}{v_D}.$$

The bound on $R$ is obtained from the $\rho$ parameter, which lays in the range 0.9799-1.0066 at 95 % c.l., thus, $R \leq 0.04$ (95 % c.l.) and then $v_T \leq 9$ GeV, at 95 % c.l. [19]. This bound must be respected in our numerical analysis.

4.2 Mass spectrum

The diagonalization of the mass matrices, and the resulting mass eigenvalues and mixing matrix, will allow us to analyze the coupling $H^\pm W^- h^0_\beta$ ($\alpha = 1, 2, 3$ and $\beta = 1, 2, 3$). The CP-even mass eigenstates are denoted by $h^0$, $H^0_1$, $H^0_2$, ordered according to their masses, $m_{h^0} < m_{H^0_1} < m_{H^0_2}$. The charged Higgs states are denoted by $H^\pm_\alpha$ with $m_{H^+_1} < m_{H^+_2} < m_{H^+_3}$. Because of the large number of parameters appearing in the model, which include $\tan\beta$, $R$, $\lambda$, $\mu_D$, $\mu_T$, $A$, $B_D$, $B_T$, one has to consider some simplified cases, for which we shall try to identify useful relations or trends for the behaviour of Higgs masses and couplings. For our numerical analysis of the allowed region and Higgs masses, we shall consider: a) $\tan\beta$ as an independent-variable; b) $R$ will take the representative value 0.01; c) the parameter $\lambda$ will take the values 0.5, 1.0; and d) the remaining parameters will cover the ranges allowed by SUSY, namely masses in the range between 0 and 1000 GeV. Furthermore, we shall analyze the following specific scenarios (which were defined and studied in ref. [18]):

**Scenario I:** $B_D = \mu_D = 0$, which represent the scenario when SSB is dominated by the effects of the Higgs triplets, here we shall also consider the following cases

A) $B_T = \mu_T = A$
B) $B_T = \mu_T = -A$
C) $B_T = -\mu_T = A$
D) $-B_T = \mu_T = A$

**Scenario II:** $B_T = \mu_T = 0$, which represent the scenario when SSB is dominated by the effects of the Higgs doublets; now the following cases will be considered:

A) $B_D = \mu_D = A$
B) $B_D = -\mu_D = -A$
C) $B_D = -\mu_D = A$
D) $-B_D = \mu_D = A$

**Scenario III:** $|B_D| = |B_T| = |\mu_D| = |\mu_T| = |A|$, both doublets and the triplet contribute to SSB. Within this scenario we shall consider several cases; for instance A) $B_D = B_T = \mu_D = \mu_T = A$, as well as 15 others combinations with positive and negative signs.

Then, for each point in parameter space, within the above scenarios, we shall determine the allowed regions, by requiring the scalar squared mass eigenvalues to be positive, and the Higgs potential laying in a global minimum. In these allowed regions the masses of the physical Higgs states of the model are computed numerically.

### 4.3 The vertex $W^+ H_{\alpha}^- h^0$ and $W^+ H_{\alpha}^- Z$

We shall apply now the general expression for the vertex $W^+ H_{\alpha}^- h^0$ derived in sect. 2, for the present SUSY model with a Higgs triplet; we shall only consider the case of the lightest neutral CP-even scalar. To present a complete study of the branching ratios of the charged Higgs, we shall also discuss the vertex $W^+ H_{\alpha}^- Z$, which could dominate in some specific scenarios.

Substituting the expression for the rotation matrices of the charged and neutral Higgs sectors, $U$ and $V$, in the expression for $\eta_{\alpha,\beta}$ (equation (16)) allows us to derive the following expression for the coefficient of the vertex $W^+ H_{\alpha}^+ h^0$, namely,

$$\eta_{h^0} = \left( \frac{1}{\sqrt{2}} V_{11}(U_{2i} - U_{1i}) + \frac{1}{4} V_{31}(U_{4i} - U_{3i}) \right),$$

where $H_{\alpha}^+$ denote the charged Higgs bosons of the model, and $h^0$ corresponds to the lightest scalar Higgs boson of the model. The terms $U_{ji}$ denote the coefficients in the expansion for $H_{\alpha}^+$, which is given by:

$$H_{\alpha}^\pm = U_{1i} H_2^\pm + U_{2i} H_1^- + U_{3i} \xi_2^\pm + U_{4i} \xi_1^- \ast$$

(35)
while $V_{ij}$ denote the elements of the rotation matrix for the CP-even neutral sector.

On the other hand, in this model the vertex $H_\alpha^+ W^- Z$ is also induced at tree level because of the violation of the custodial symmetry. The expression for the vertex $H_\alpha^+ W^- Z$ is given by

$$H_\alpha^+ W^- Z : i e^2 v_T (U_{3i} - U_{4i}) \frac{\cos \theta_W}{\sin^2 \theta_W}. \quad (36)$$

One can see than only the triplet components contribute to this vertex, while the dependence on $v_T$ gives a suppression effect. In what follows, the coefficients $U, V$, are calculated at tree level.

4.4 Branching ratios for the modes $H_\alpha^+ \rightarrow W^+ Z$ and $H_\alpha^+ \rightarrow W^+ h^0$

We now discuss the BR for the charged Higgs, including the decay widths of the dominant modes of $H_\alpha^+$, which turn out to be the following modes: 1) $H_\alpha^+ \rightarrow W^+ Z$; 2) $H_\alpha^+ \rightarrow W^+ h^0$; 3) $H_\alpha^+ \rightarrow \tau \nu_\tau$; and 4) $H_\alpha^+ \rightarrow t \bar{b}$. The decay width for each of the above modes is:

(1) The decay $H_\alpha^+ \rightarrow W^+ h^0$:

$$\Gamma \left( H_\alpha^+ \rightarrow W^+ h^0 \right) = \frac{g^2 \lambda^{1/2}(m_{H_\alpha^+}^2, m_W^2, m_{h^0}^2)}{64 \pi m_{H_\alpha^+}^3} \left| \eta_{h^0} \right|^2 \times \left[ m_W^2 - 2(m_{H_\alpha^+}^2 + m_{h^0}^2) + \frac{(m_{H_\alpha^+}^2 + m_{h^0}^2)^2}{m_W^2} \right]. \quad (37)$$

where $\lambda^{1/2}$ is the usual kinematic factor $\lambda^{1/2}(a, b, c) = (a - b - c)^2 - 4bc$; this decay is proportional to the factor $\eta_{h^0}^2$.

(2) The decay $H_\alpha^+ \rightarrow W^+ Z$:

$$\Gamma \left( H_\alpha^+ \rightarrow W^+ Z \right) = \frac{m_{H_\alpha^+}}{16 \pi} \lambda^{1/2}(1, w, z) \left[ |M_{LL}|^2 + |M_{TT}|^2 \right]. \quad (38)$$

Here $w = (\frac{m_W}{m_{H_\alpha^+}})^2$ and $z = (\frac{m_Z}{m_{H_\alpha^+}})^2$; $|M_{LL}|^2 = \frac{g^2}{4z} (1 - w - z) F_Z |^2$ and $|M_{TT}|^2 = 2g^2 w |F_Z|^2$ are the final polarization contributions of the gauge bosons.
The decay $H^+_a \to t\bar{b}$:

$$\Gamma(H^+_a \to t\bar{b}) = \frac{3g^2}{32\pi m^2_W m^3_{H^+_a}} \lambda^{1/2}(m^3_{H^+_a}, m^2_T, m^2_\tau)$$

$$\times \left[ (m^3_{H^+_a} - m^2_T - m^2_\tau)(m^2_\tau\tan^2\beta T_2 + m^2_\tau\cot^2\beta T_1) - 4m^2_\tau m^2_T T_1 T_2 \right],$$

(39)

where $T_1$ and $T_2$ depend on the mixing angles that diagonalize the charged Higgs mass matrix, namely: $T_1 = \cot \beta \left( \frac{U_{22}}{\cos \beta} \right)$ and $T_2 = \tan \beta \left( \frac{U_{12}}{\sin \beta} \right)$.

(4) The decay $H^+_a \to \tau\nu_\tau$:

$$\Gamma(H^+_a \to \tau\nu_\tau) = \frac{g^2}{32\pi m^2_W m^3_{H^+_a}} \lambda^{1/2}(m^2_{H^+_a}, 0, m^2_\tau)$$

$$\times m^2_\tau, \tan^2\beta T_2 (m^3_{H^+_a} - m^2_\tau),$$

(40)

We have then evaluated numerically the BR for these four modes, using the previous expressions. For the numerical analyses, we considered the scenarios listed above, which have fixed values of $\lambda$ and $\tan \beta$.

In scenario I, the calculation are performed for $\lambda = 0.5, 1$ and $\tan \beta = 5, 10, 15$. We considered the cases A and D within this scenario, for each of the charged Higgs bosons $H^+_a$. In Figs. 2 we present the BR for case A, with $\lambda = 0.5$. In this scenario, the decay $H^+_a \to W^+h^0$ is not allowed for the lightest charged Higgs boson, thus and we only show the results for $H^+_2$ and $H^+_3$. For $H^+_2$, the second lighter charged Higgs boson, we can see that $WZ$ is the dominant mode, which would be a clear signature of the Higgs triplet. The mode $W^+h^0$ reaches an important BR, although it is smaller than the BR for $t\bar{b}$. For the state $H^+_3$, $W^+Z$ has a BR of the order $10^{-1}$; here the mode $W^+h^0$ is dominant, while the mode $t\bar{b}$ becomes dominant when $\tan \beta$ increases. In turn, the mode $\tau\nu_\tau$ is the most supressed one, and it reaches a maximum BR of order $10^{-3}$ when $\lambda = 0.5$ and large $\tan \beta$.

For $\lambda = 1$ in scenario I (see Fig. 3), the mode $W^+h^0$ is dominant for $H^+_2$, whereas for $H^+_3$ it becomes dominant when $\tan \beta$ increases. The $W^+Z$ mode gets a BR of the order $10^{-2}$, this is dominating for $H^+_3$ boson when $\tan \beta$ is small, while the BR for the mode $t\bar{b}$ increases when $\tan \beta$ increase. For case D, we notice a different behavior, as it is show in Figs. 4 and Fig. 5, which show the BR for $\lambda = 0.5$ y $\lambda = 1$. Now the mode $W^+h^0$ mode gets a BR of order $10^{-1}$ for $H^+_2$; the same occurs for $H^+_3$ boson. When $\lambda$ increase, $W^+h^0$ mode is dominant for $H^+_3$, while $W^+Z$ is dominant for $H^+_2$.

In scenario II, which mimics the MSSM, Fig. 5 and 6 show the BR’s for $\lambda = 0.5$ and $\lambda = 1$. Now, the mode $t\bar{b}$ becomes dominant for $H^+_1$, the lightest charged Higgs boson, for any combination of of parameters. We also notice that the mode $W^+h^0$ has a BR of order of $10^{-2}$. On the other hand, the mode $W^+Z$ is
dominant for $H_2^+$ and $H_3^+$, for any combination of the parameters. The mode $W^+h^0$ mode gets suppressed when $\tan \beta$ increase.

Finally, for scenario III we considere the case F. Here both doublets and tripet contribute equally to SSB. The results for the BR’s are show in Fig. 8 and Fig. 9 for $\lambda = 0.5$ and $\lambda = 1$, respectively. Again, we find that for $H_1^+$ the dominant mode is $\bar{b}b$. The behavior of $W^+Z$ and $+h^0$ modes is similar to the ones from scenario II. We also find that the mode $W^+h^0$ reaches larger BR’s when $\tan \beta$ is large.

5 Conclusions

We have studied the charged Higgs vertex $H_\alpha^+ W^+ h_\beta^0$, within the context of several extensions of the SM that predict this vertex. For a Higgs sector that includes arbitrary Higgs representations, we were able to derive the general form of this vertex, i.e. its dependence on the isospin and hypercharge of the Higgs multiplet. Then, we evaluate the strength of this vertex for several specific models, which include: i) the THDM, both generic and the MSSM version, and ii) models with additional Higgs triplets, for both SUSY and non-SUSY cases. When the decay $H_\alpha^+ \rightarrow W^+ h^0$ is allowed, it can reach a BR that could be detected at LHC, and would permit to test the charged Higgs quantum numbers. We can summarize our results in terms of the following classification for the BR, namely:

- Dominant: Large BR (when $H_\alpha^+ \rightarrow W^+ h^0$ is the dominant mode)
- Moderate: $\text{BR} \sim 10^{-1} - 10^{-2}$
- Small: $\text{BR} \sim 10^{-2} - 10^{-4}$
- Negligible: $\text{BR} < 10^{-4}$

We can appreciate that for each model there area regions or values of parameters that correspond to one of those scenarios. Therefore, the observation of the decay $H^+ \rightarrow W^+ h^0$, as the dominant mode, would correspond to the THDM or scenario I of the SUSY tripet case, while the moderate case could arise either of THDM or MSSM (observation of more Higgs bosons with the predicted properties would then be needed to discriminate among them), while the detection of several charged and neutral Higgs bosons would correspond to a model with more elaborated Higgs sector (such as Higgs triplets). Then, some results for typical BR’s within each model are shown in Table 1.

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| BR($H^+ \rightarrow W^+h^0$) | THDM$^{(1)}$ | MSSM$^{(2)}$ | Triplets$^{(3)}$ |
|-------------------------------|-------------|-------------|------------------|
| a) Dominant                   | $\alpha = \beta - \frac{\pi}{4}$ | not possible | Scenario I:  
$m_{H^+} = 100 - 550$ GeV  
tan $\beta = 5$ |
|                              | tan $\beta = 7$ |             |                  |
| b) Moderate                   | $\alpha = \beta - \frac{\pi}{4}$ | $m_{H^+} = 300$ GeV  
tan $\beta \cong 7$ | Scenario I:  
$m_{H^+} = 200 - 300$ GeV  
tan $\beta = 5, 10$  
Scenario II:  
$m_{H^+} = 200$ GeV  
tan $\beta = 5, 15$  
Scenario III:  
$m_{H^+} = 150 - 300$ GeV  
tan $\beta = 30$ |
|                              | tan $\beta = 30$ |             |                  |
| c) Small                      | $\alpha = \beta - \frac{\pi}{2}$ | $m_{H^+} = 300$ GeV  
$10 < \tan \beta \leq 25$ | Scenario I:  
$m_{H^+} \simeq 250 - 400$ GeV  
tan $\beta = 10$  
Scenario III:  
$m_{H^+} \simeq 200$ GeV  
tan $\beta = 5, 15, 30$ |
|                              | tan $\beta = 7$ |             |                  |
| d) Negligible                 | $\alpha = \beta - \frac{\pi}{2}$ | $m_{H^+} = 300$ GeV  
tan $\beta \geq 25$ | Scenario I:  
$m_{H^+} \simeq 280$ GeV  
tan $\beta = 15$  
Scenario II:  
$m_{H^+} \simeq 200$ GeV  
tan $\beta = 5, 15, 30$  
Scenario III:  
$m_{H^+} \simeq 200$ GeV  
tan $\beta = 5, 15, 30$ |
|                              | tan $\beta = 30$ |             |                  |

Table 1  
Classification of BR ($H^+ \rightarrow W^+h^0$) according to the scheme discussed in the text.  
(1) For the THDM we take $m_{H^+} - m_A = 300$ GeV, $m_{h^0} = 115$ GeV.  
(2) We have $m_{H^+} \approx m_{A^0}$ for most regions of parameters.  
(3) We consider the cases with $\lambda = 1$,  
and the scenarios I, II, III defined in the text.
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(1) BR ($H^+ \to W^+ h^0$) in the MSSM, with radiative corrections to the Higgs mass as included in HDECAY, with $m_{\tilde{q}} = 500$ GeV, $\mu = 100$ and $A_0 = 1500$.

(2) Branching Ratios of the charged Higgs bosons $H_\alpha^+$ in the principal modes for the scenario I, case A, considering $\lambda = 0.5$. The kind of lines correspond to the different modes as: (a) dashed: $H_\alpha^+ \to t\bar{b}$; (b) dotted: $H_\alpha^+ \to W^+ Z$; (c) solid: $H_\alpha^+ \to \tau\bar{\nu}_\tau$; and dot-dashed: $H_\alpha^+ \to W^+ h^0$. The figure show this modes for each charged Higgs boson, the first row correspond to the $H_2^+$ and the second row to $H_3^+$. In the columns is shown the different results to tan $\beta = 5, 10, 15$.

(3) The same of Fig. 2, with $\lambda = 1.0$.

(4) Scenario I, case D, with $\lambda = 0.5$.

(5) The same of Fig. 4, with $\lambda = 1.0$.

(6) Scenario II, case D, with $\lambda = 0.5$. In this scenario, we considered tan $\beta = 5, 15, 30$.

(7) The same of Fig. 6, with $\lambda = 1.0$.

(8) Scenario III, case F, with $\lambda = 0.5$.

(9) The same of Fig. 8, with $\lambda = 1.0$. 
This figure "Fig1.JPG" is available in "JPG" format from:

http://arxiv.org/ps/hep-ph/0309097v1
$\tan \beta = 5$

$\tan \beta = 10$

$\tan \beta = 15$

Fig. (2)
Fig. (3)

\[ \tan \beta = 5 \quad \tan \beta = 10 \quad \tan \beta = 15 \]

\begin{align*}
\text{BR} & \quad m_{H_2} \\
\text{BR} & \quad m_{H_3}
\end{align*}
\[ \text{Fig. (4)} \]
\[ \tan \beta = 5 \]  
\[ \tan \beta = 10 \]  
\[ \tan \beta = 15 \]  

\[ \text{BR} \]  
\[ m_{H^-} \]  
\[ m_{H^+} \]  

Fig (5)
Fig. (6)
\[ \tan \beta = 5 \]  
\[ \tan \beta = 15 \]  
\[ \tan \beta = 30 \]  

Fig. (7)
$\tan \beta = 5$

$\tan \beta = 15$

$\tan \beta = 30$

$\tan \beta = 5$

$\tan \beta = 15$

$\tan \beta = 30$

$\tan \beta = 5$

$\tan \beta = 15$

$\tan \beta = 30$

Fig. (8)
\[ \tan \beta = 5 \]
\[ \tan \beta = 15 \]
\[ \tan \beta = 30 \]

Fig. (9)