LQG based control design of electric vehicle driveline

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**Abstract.** In this paper LQG (Linear-Quadratic-Gaussian) based control design for drive line of Electric vehicle is presented. Driveline oscillations is a phenomenon of fluctuating vehicle speed caused by torsional vibrations in the vehicle drive line which results as jerks in the vehicle motion. These oscillations result in uncomfortable vehicle ride which is contemporary research problem. The control objective is to achieve steady state driveline in lesser time for this purpose LQG based control is pursued with certain modification in vehicle dynamic model. This paper includes driveline modelling, control design and verification by simulations. Results of simulation show the considerable mitigation of oscillations and LQG based control for driveline of electric vehicle works very well.

1. Introduction

Instead of using precise but expensive components in drive line to remove jerks in vehicle motion we will apply control such that these jerks in vehicle motion remove. In [1] torsional oscillations are due to of fast response from the electric motor, poor damping in the driveshaft material, elasticity of various components in drive shaft, perturbation coming from road, non linearity in drive line and variable load applied to wheel. In [2] torsional oscillations lead to many problems such as driver comfort can be affected, oscillations in the torque can cause wheel blockage or slip and oscillations in the drive shaft torque will result in accelerated fatigue of mechanical components. Another problem in drive line is backlash [3]. Basically backlash is defined as play between adjacent move able parts. Sources of backlash are gear and joints of shaft. Normal load on each axle can be determined from free body diagram of static vehicle inclined to road [1,4].Where \( l_f, l_r \) represent longitudinal distance of centre of gravity from front and rear wheel

\[
F_{nf} = \frac{mg(l_f \cos(\beta) + h\sin(\beta))}{l_f + l_r} \cdot \frac{mah}{l_f + l_r} \cdot \frac{F_{pDhD}}{l_f + l_r}
\]

\[
F_{nr} = \frac{mg(l_r \cos(\beta) - h\sin(\beta))}{l_f + l_r} \cdot \frac{mah}{l_f + l_r} + \frac{F_{pDhD}}{l_f + l_r}
\]

2. Free Body Diagram of accelerating electric vehicle
Vehicle longitudinal dynamics is affected by driving resistance. These driving resistances consist of rolling resistance, air resistance, gradient resistance [5,6,7]. Rolling resistance is roughly proportional to normal force on each axle. The rolling resistance coefficient is:

\[ F_{r} = f F_{n} \tag{3} \]

\[ F_{r}' = f' F_{n}' \tag{4} \]

where \( F_{r} \) and \( F_{r}' \) are rolling resistances of front and rear wheel. Air resistance is product of dynamic pressure, frontal area of vehicle \( A \) and drag coefficient \( c_{d} \).

\[ F_{D} = \frac{1}{2} \rho c_{d} A v^{2} \tag{5} \]

Gradient resistance is force due to of gravitation acting on vehicle body moving up or down hill.

\[ F_{g} = mg \sin(\beta) \tag{6} \]

where \( \beta \) is angle of inclination of road on which vehicle is moving. In [5,6,7] traction forces, \((F_{f}, F_{r})\) for front and rear wheel are forces acting on wheels due to contact with ground:

\[ F_{f} = (F_{f}' + F_{r}) = m\ddot{v} + F_{\text{drain}} + F_{\text{rolling}} + F_{\text{gradient}} \tag{7} \]

In [5,8] drive line control of internal combustion engine vehicle is simulated whose system was consist of engine, drive shaft, fly wheel, clutch, wheel and transmission but we will control drive line for electric vehicle which is consist of motor, gear, Power train housing (PTH), drive shaft, backlash and wheel. PTH and drive shaft can be expressed by spring and damper in parallel. In [2] PID control and in [9] output feedback control used to compensate oscillations but we will use optimal control. In this paper simulink model is generated for drive line modelling and all forces acting on vehicle body. The effect of nonlinearity due to of backlash on drive line is analyzed. Kalman filter is used for state estimation instead of expensive sensors to measure system states. At the end LQG controller is used to remove jerks in vehicle motion for smooth drive.

The paper is organized as follow: Section 2 describes the drive line model. Section 3 describes mathematical modelling. Section 4 describes estimation of states. Section 5 describes control design and at the end section 6 and 7 describes results and conclusion.

2. Drive Line Model

Drive line model consists of electric motor, gear box, PTH, backlash, drive shaft, and wheel.

![Drive Line model](image)

Figure 2: Drive Line model

Driveshaft will deliver torque to wheels, according to the driver’s torque request. It is described as spring and damper in parallel. Basically spring and damper are used to represent flexible nature of drive shaft and bushing. In [4] driveshaft can be described as

\[ T_{s} = k_{s}(\theta_{s} - \dot{\theta}_{s}) + c_{s} (\dot{\theta}_{s} - \ddot{\theta}_{s}) \tag{8} \]

where \( k_{s} \) is driveshaft stiffness and \( c_{s} \) is driveshaft damping coefficient. \( \theta_{s} \) is shaft position connected to gear end. \( \dot{\theta}_{s} \) is shaft position connected to wheel end. Power train housing (PTH) consists of non rotating parts of electric motor and gear box mounted together [4]. Where \( k_{p} \) is stiffness coefficient, \( c_{p} \) is damping coefficient and \( J_{p} \) rotational inertia of PTH. Rotational acceleration of lumped PTH is

\[ w_{p} = \frac{T_{p} - K_{p} \theta_{p} - C_{p} \omega_{p}}{J_{p}} \tag{9} \]

Effective rotational acceleration\( = \dot{\dot{\theta}}_{\text{eff}} = \dot{\dot{\theta}}_{s} - \dot{\dot{\theta}}_{p} \).
3. Mathematical Modelling

3.1. Wheel acceleration
In [4,5] according to Newton’s second law of rotational motion,

\[ J_w \ddot{\omega}_w = T_g - T_r \]

Where \( T_r \) is traction torque which is related to traction force \( F_r \). \( T_g \) is drive shaft torque. \( J_w \) is wheel inertia. \( \dot{\omega}_w \) is wheel acceleration. \( F_{res} \) is sum of rolling, air and gradient resistances.

\[ J_w \ddot{\omega}_w = T_g - (ma + F_{res})r \]

\[ \dot{\omega}_w = \frac{T_g - T_{res}}{J_w + mr^2} \]

\[ \dot{\omega}_w = \frac{k_w(\theta_g - \theta_w)}{(J_w + mr^2)} + \frac{c_{w1}w_w}{(J_w + mr^2)i} - \frac{c_{w2}w_w}{(J_w + mr^2)i} - \frac{T_{res}}{(J_w + mr^2)} \]

3.2. Torsion
Torsion in drive shaft is difference between shaft position connected to gear end \( \theta_g \) and shaft position connected to wheel end \( \theta_w \).

3.3. Motor acceleration
In [1,4] motor and transmission dynamics are given as

\[ J_d \ddot{\omega}_m = T_m - \frac{T_f}{i} \]

Where \( J_m \) is motor inertia, \( \ddot{\omega}_m \) is motor rotational acceleration, \( T_m \) is input torque of motor, \( i \) is final gear ratio, \( i_{g1} \) is primary gear ratio, \( i_{g2} \) is secondary gear ratio, \( J_1, J_2, J_3 \) are primary secondary and final gear ratio. Total speed conversion from motor to gear box out coming shaft is \( \omega_j = \frac{\omega_m}{i} \).

\[ J_d \ddot{\omega}_m = T_m - \frac{T_f}{i} \]

\[ \ddot{\omega}_m = \frac{T_m}{J_d} - \frac{T_f}{J_d i} \]

\[ \dot{\omega}_m = \frac{-k_w(\theta_g - \theta_w)}{J_d i^2} + \frac{c_{w1}w_w}{J_d i^2} + \frac{c_{w2}w_w}{J_d i^2} + \frac{T_m}{J_d i} \]

3.4 State Space Model
State space model of drive line can be represent as

\[ \dot{x} = Ax + Bu + E\ddot{u} = \begin{bmatrix} 0 & \frac{1}{i} & \frac{-k_w}{J_d i} & \frac{-c_{w1}}{J_d i^2} & \frac{-c_{w2}}{J_d i^2} & -1 & 0 & 0 & 0 \end{bmatrix} \]

\[ x = \begin{bmatrix} \theta_g - \theta_w & \omega_m & \omega_w \end{bmatrix}^T \]

\[ \ddot{u} = \begin{bmatrix} \frac{1}{i} & \frac{1}{J_d} & \frac{1}{J_d} & r \end{bmatrix} \]

\[ F_{res} = \begin{bmatrix} 0 & 0 \end{bmatrix} \]

\[ E = \begin{bmatrix} 0 & 0 \end{bmatrix} \]

\[ A = \begin{bmatrix} 0 & \frac{1}{i} & \frac{-k_w}{J_d i} & \frac{-c_{w1}}{J_d i^2} & \frac{-c_{w2}}{J_d i^2} & -1 & 0 & 0 & 0 \end{bmatrix} \]

\[ B = \begin{bmatrix} \frac{1}{i} & \frac{1}{J_d} & \frac{1}{J_d} & r \end{bmatrix} \]

\[ E = \begin{bmatrix} 0 & 0 \end{bmatrix} \]

\[ F_{res} = \begin{bmatrix} 0 & 0 \end{bmatrix} \]

\[ x = \begin{bmatrix} \theta_g - \theta_w & \omega_m & \omega_w \end{bmatrix}^T \]

\[ \ddot{u} = \begin{bmatrix} \frac{1}{i} & \frac{1}{J_d} & \frac{1}{J_d} & r \end{bmatrix} \]

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\[ E = \begin{bmatrix} 0 & 0 \end{bmatrix} \]

\[ F_{res} = \begin{bmatrix} 0 & 0 \end{bmatrix} \]

\[ x = \begin{bmatrix} \theta_g - \theta_w & \omega_m & \omega_w \end{bmatrix}^T \]
4. Estimator

Generally it is not possible to measure all states of system because of sensors used to measure states of system are very expensive and measured states of system using these sensors are too noisy. To solve these problems we estimate unknown states of system with the help of estimator using mathematical model of system and state measurement.

Plant with process noise \( w \) and measurement noise \( v \) is

\[
\dot{x} = Ax + Bu + w, \quad y = Cx + v
\]

Covariance for noises

\[
Q = E(ww^T), \quad R = E(vv^T)
\]

Kalman filter equation

\[
\dot{x}_k = A\dot{x}_{k-1} + Bu_k + K_1(y_k - C(A\dot{x}_{k-1} + Bu_k))
\]

For drive line of electric vehicle wheel speed and motor speed both can measure but wheel speed measurement is far lower than motor speed measurement so we will use motor speed as measurement state for estimator so \( y = C\dot{x} = [0 \ 1 \ 0]^T\dot{x} \)

In [10] Kalman filter uses system model and state measurement to find kalman gain \( K_1 \) minimizing error covariance \( P_1^- \). Kalman estimator is a two step process

**Prediction:** \[ \dot{x}_k^- = A\dot{x}_{k-1} + Bu_k \]

**Update:** \[ \begin{align*}
K_1 & = (P_1^-C^T)(CP_1^-C^T + R)^{-1} \\
\dot{x}_k & = \dot{x}_{k^-} + K_1(y_k - C\dot{x}_{k^-}) \\
P_2 & = (I - K_1C)P_1^-
\end{align*} \]

5. Control Design

To bring the drive line to steady state quickly for smoother drive, controller regulates torque demand to electric motor. LQG is combination of kalman filter and linear quadratic regulator (LQR). Kalman estimates unknown states of system and LQR regulates torque demand to electric motor. In [11] the state control will try to minimize cost function \( J \)

\[
J = \frac{1}{2} \int_0^T (x^TQ_{lqr}x + u^TR_{lqr}u)dt
\]

\( Q_{lqr} \) and \( R_{lqr} \) are weighted squares of deviation of states from target and weighted squares of control activity. Square are used because these lead to easier analysis and well behaved solutions. \( Q_{lqr} \) and \( R_{lqr} \) allow the trade of between input activity and rate of convergence. Term \( x^TQ_{lqr}x \) is related to convergence rate such as Rise time, Settling time. Term \( u^TR_{lqr}u \) is related to aggressive use of input. \( P_{lqr} \) is a positive definite matrix.

\[
u = -K_{lqr}x
\]

\[
K_{lqr} = R_{lqr}^{-1}B^TP_{lqr}
\]

\[
A^TP_{lqr} + P_{lqr}A - P_{lqr}BR_{lqr}^{-1}B^TP_{lqr} + Q_{lqr} = 0
\]

\[
\dot{x} = (A - BK_{lqr})x \quad \text{where} \quad (A - BR_{lqr}^{-1}B^TP_{lqr})x
\]

6. Results

We can see oscillations in drive shaft and electric vehicle acceleration for step input of 200 Nm motor torque in figures [4-6]. Torque oscillations in drive shaft relate to oscillations in acceleration of electric vehicle. Controlled response of drive shaft and electric vehicle is shown in figures [5-9].

Figure 4. Shaft torque for 200 Nm step input

Figure 5. Shaft torque with 30 degree backlash
7. Conclusion
In this paper we have presented the concept of drive line control and how it can be applied to automotive application. Due to torsional vibrations vehicle motion results as jerks in start so an anti-jerk controller has been designed for an electric vehicle using LQG to meet multiple objectives of velocity tracking performance and enhance drivability. We use Kaman filter for estimation of states using motor speed as measurement state. LQG control is a feasible and successful in reducing vehicle oscillations which regulates torque demand to electric motor. We have also implemented dynamics of wheel, tyre, resistive forces and vehicle body in this paper. The control algorithm is implemented in existing motor control without any additional cost.

8. References
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