Comparison of clustering properties of observed objects and dark matter halos on different mass and spatial scales

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ABSTRACT

We investigate the large-scale distribution of galaxy clusters taken from several X-ray catalogs. Different statistics of clustering like the conditional correlation function (CCF) and the minimal spanning tree (MST) as well as void statistics were used. Clusters show two distinct regimes of clustering: 1) on scales of superclusters (∼ 40h⁻¹ Mpc) the CCF is represented by a power law; 2) on larger scales a gradual transition to homogeneity (∼ 100h⁻¹ Mpc) is observed. We also present the correlation analysis of the galaxy distribution taken from DR6 SDSS main galaxy database. In case of galaxies the limiting scales of the different clustering regimes are 1) 10-15 h⁻¹ Mpc; 2) 40 – 50h⁻¹ Mpc. The differences in the characteristic scales and scaling exponents of the cluster and galaxy distribution can be naturally explained within the theory of biased structure formation. We compared the density contrasts of inhomogeneities in the cluster
and galaxy distributions in the SDSS region. The value of the density contrast should be taken into account to reconcile the observed gradual transition to homogeneity with the apparent presence of structures on the corresponding scales. The estimation of the relative cluster-galaxy bias (comparing number of clusters in different SDSS regions with corresponding number of galaxies) gives the value $b = 5 \pm 2$. The distribution of real clusters is compared to that of simulated (model) clusters (the MareNostrum Universe simulations). We selected a cluster sample from $500 \, h^{-1} \text{Mpc}$ simulation box with WMAP3 cosmological parameters and $\sigma_8 = 0.8$. We found a general agreement between the distribution of observed and simulated clusters. The differences are mainly due to the presence of the Shapley supercluster in the observed sample. On the basis of SDSS galaxy sample we study properties of the power law behavior showed by the CCF on small scales. We show that this phenomenon is quite complex, with significant scatter in scaling properties, and characterized by a non-trivial dependence on galaxy properties and environment.

Keywords: galaxies, galaxy clusters, large-scale structure of the Universe, dark matter simulations

1. Introduction

Clusters of galaxies are the most massive virialised structures in the Universe. Within the framework of the modern "picture of the world" with dark matter and dark energy (cosmological constant) the structures evolve via gravitational instability starting from small seeds which are described by the spectrum of initial inhomogeneities. Clusters of galaxies are perfect probes of the matter distribution on large scales. Studying their spatial distribution one can constrain the parameters of the $\Lambda$CDM model ($\Omega_m, \sigma_8$). Baryonic gas falls into cluster potential wells and heats up to temperatures of the order of $10^7$ K so that it emits X-rays. Clusters of galaxies were historically identified first as overdensities in the galaxy distribution (ACO [1] and APM [2] optical galaxy cluster catalogs). This kind of selection leads to some spurious objects due to projection effects. When clusters are detected according to their X-ray flux we undoubtedly deal with deep potential wells. In this work we investigate the distribution of galaxy clusters selected by X-ray flux from several catalogs of X-ray clusters and compare it with simulated clusters taken from the "MareNostrum Universe" [3].
Fig. 1.— Distribution in equatorial coordinates of 400 X-ray clusters with redshifts $z \leq 0.1$ and $L_X(0.1-2.4keV) > 1.25 \cdot 10^{43}h^{-2}$ erg/s. The edges of minimal spanning tree shorter than 45 $h^{-1}$ Mpc are shown by solid lines. Circles of constant galactic latitude ($b = -20^\circ$, $0^\circ$, $+20^\circ$) are plotted by dotted lines. Gray circles represent the spectral plates of the SDSS-DR6.

2. X-ray galaxy cluster sample

The X-ray cluster sample consists of the all sky ROSAT clusters with X-ray flux $F_X \geq 3 \cdot 10^{-12}$ erg/cm$^2$/s in the (0.1-2.4 keV) energy band. Clusters were selected from the following catalogs: 1) REFLEX (N=186 clusters) [4]; eBCS (N=108) [5,6]; NORAS (N = 36) [7]; CIZA (N = 70, galactic latitude $|b_{gal}| < 20^\circ$) [8,9]. The final all-sky sample consists of 400 X-ray clusters up to redshift $z = 0.1$ with luminosities $L_X > 1.25 \cdot h^{-2}10^{43}$ erg/s (assuming the current rate of universal expansion - Hubble constant $H_0 = 100$ km/s/Mpc, $h=H/H_0$, where $H$ is the true value of Hubble constant). The volume-limited sample (VL) extracted from this compilation contains 233 X-clusters with redshifts limited by $Z_{VL} = 0.09$ (which corresponds to the radial distance of 265.3 $h^{-1}$ Mpc ($\Omega_m = 0.24$, $\Omega_\Lambda = 0.76$) with $L_X > 2.5 \cdot h^{-2}10^{43}$ erg/s.
3. Model cluster sample

Model clusters The MareNostrum clusters (MN-clusters) were extracted from the 500 $h^{-1}$ Mpc simulation box MUWHS [10] with cosmological parameters $\Omega_m = 0.24$, $\Omega_\Lambda = 0.76$, $h=0.73$, $\sigma_8 = 0.8$ ($\sigma_8$, the present-day rms mass fluctuations on spheres of radius $8h^{-1}$ Mpc is slightly higher than predicted by WMAP3 and in better agreement with WMAP5). Within a box of 500$h^{-1}$ Mpc size the linear power spectrum at redshift $z = 40$ has been represented by $512^3$ DM particles of mass $m_{DM} = 8.3 \cdot 10^9 h^{-1} M_{\odot}$ (In the following we assume $h = 1$). The nonlinear evolution of structures has been followed by the GADGET II code of V. Springel [11]. Clusters were identified in the simulation by the FOF (friend-of-friend) algorithm. For comparison with observations we extracted the 233 most massive clusters in a sphere of radius 265.3 Mpc (we slightly expanded the simulation box using the periodic boundary conditions). We have chosen this simulated sample by keeping its number density equal to the observed cluster density (independent of an $L_X$ - mass relation) so that the most massive simulated
clusters correspond to the most luminous observed clusters. We use the 3D velocities of the simulated clusters and place an observer to the center of the sphere extracted from the simulation box. Then the cluster positions were converted to redshift space. The mass of the lightest dark matter cluster in the simulated sample is $2.46 \cdot 10^{14} M_{\odot}$.

4. Statistics of clustering

4.1. Conditional correlation function

In order to compare the X-ray and simulated cluster distributions we use different statistical methods. The conditional correlation function (CCF) [12,13] measures the number density of objects in spheres of radius $R$ averaged over the spheres around all objects of the sample that are away from sample boundaries by more then $R$ Mpc (integral CCF). The clusters show three distinct regimes of clustering (Fig.3a): 1) on scales of superclusters up to a scale of 35-40 Mpc the CCF is represented by a power law; 2) on larger scales a gradual transition to homogeneity is observed; 3) starting from about 100 Mpc the CCF becomes a constant, i.e. the number density does not anymore decline with increasing radius of spheres. Fluctuations on scales $>100$ Mpc exist but evidently they doesn’t contribute to cluster number density contrasts on such scales. Here we reach a mean number density of clusters (which can not be obtained on smaller scales) in our VL-sample.

Fig. 3.— a) Comparison of CCFs for X-clusters and MN-clusters; b) CCFs for VL cluster sample in 2 hemispheres: GAC ($123^0 < \ell_{gal} < 303^0$) and GC ($303^0 < \ell_{gal}$ and $\ell_{gal} < 123^0$).
The CCFs for observed and simulated clusters look quite similar (the second regime of clustering from $\sim 40$ to $\sim 100$ Mpc is perfectly reproduced by the simulated clusters) however the value of the slopes on scales below 40Mpc are different: $\gamma_X \sim 1.6$ for the observed and $\gamma_{MN} \sim 1.25$ for the simulated clusters. This difference reflects the lack of close pairs in the simulated cluster distribution with respect to the observed ones. The comparison with a model cluster distribution obtained from the same realization but using a smaller linking length of the FOF algorithm (in order to identify substructure and possible close pairs which could be linked by the original linking length) showed that the FOF parameters have negligible influence on the value of the slope. In order to understand the difference in the slopes we have gathered observed cluster pairs with separations smaller than 5 Mpc into single objects. This reduces the total number of objects in the observed sample by 10. Mostly the clusters that belong to the Shapley supercluster (located at $l_{gal} \sim 311^o$, $b_{gal} \sim +30^o$ and redshift $z \sim 0.05$) (Fig.6) were linked. The CCF of the reduced sample (Fig.3a) looks nearly identical to the simulated one ($\gamma_{MN}$ is very close to the $\gamma_X$ of the reduced observational sample). When we calculate CCF separately in two hemispheres we obtain the value of slope $\gamma_{GAC}$ very close to the model $\gamma_{MN}$ in the GAC hemisphere that doesn’t contain the Shapley supercluster (Fig.3b). This means that the difference is mainly produced by the Shapley supercluster – we don’t have such a structure in the simulated cluster sample. It is an open question how often such outstanding structures appear in the Universe and whether they can be reproduced by ΛCDM simulations of larger volumes which should contain larger wavelength perturbations that could be responsible for formation of more massive objects. Therefore, still, it is a question if we really see in our VL-sample with CCF the universal mean X-ray cluster number density.

4.2. Void statistics

We have also performed a void analysis of the same observed and simulated samples. Starting from the largest empty spheres non-spherical voids have been constructed by extending the original spherical void with empty spheres of smaller radii the center of which was inside the original void. The radius of the smaller spheres is limited to be larger than an ad hoc parameter 0.9 of the radius of initial sphere. The process is repeated a few times. It produces voids which are slightly non-spherical. The mean distance between the observed (and simulated) clusters is $\sim 28$ Mpc. Therefore, we have limited our voids to a minimal radius of 20 Mpc. The cumulative void volume functions (CVF) $\Delta V/V_{sample}$ show that the observed and simulated voids fill the sample volumes in a similar way though in Fig.4a we see a difference at $R_{void} \sim 80$ Mpc: the largest simulated voids are bigger than the observed ones. Here the differential void function (DVF) is presented as $R_{void}$ versus rank [14] (largest void
has rank 1, \( R_{\text{void}} = (3 \cdot V_{\text{void}} / 4\pi)^{1/3} \). It shows rather good agreement between observation and simulation. There are two breaks in the DVF (Fig. 4b). The one at \( R_{\text{void}} \sim 45 \) Mpc (less prominent) is associated with the scale where the power law regime of clustering vanishes. The break at \( R_{\text{void}} \sim 100 \) Mpc can be directly associated with the scale of homogeneity. The slope of the \( R_{\text{void}} - \text{Rank} \) relation after the break is \( z_v = 1/(3 - \gamma_{\text{voids}}) \) which gives \( \gamma_{\text{voids}} \sim 1.2 \) close to the slope of the CCF on small scales.

Fig. 4.— a) CVF for X-ray and MareNostrum-clusters; b) DVF for X-ray and MareNostrum-clusters.

4.3. Cross-correlation of clusters and galaxies

From SDSS DR6 main galaxy database we selected a region in galactic coordinates \( (48^\circ < l_{\text{gal}} < 210^\circ, 50^\circ < b_{\text{gal}} < 86^\circ) \) and built a VL sample with \( z_{\text{max}} = 0.1 \) and \( M_{\text{lim}} = -19.67 \) (see section 5). There are 23 X-ray clusters from our compilation in this region. In Fig. 5b we present the cluster-galaxy cross-CCF (clusters were used as centers of spheres in which the number of galaxies inside was calculated) and compare with the CCF of SDSS galaxies in the selected region (star symbols in Fig. 5b). We defined a local number galaxy contrast \( \Delta = 11/(4\pi R_{10}^3/3)/\rho_{\text{mean}} - 1 \), where \( R_{10} \) is the distance to the 10th neighbour \( (R_{10} \sim 4 \) Mpc is the mean value for entire sample) and \( \rho_{\text{mean}} \) is the mean galaxy number density in the sample. In this regard, the clusters located in the sample have a median galaxy contrast of \( \Delta \sim 40 \). We have randomly selected 23 galaxies separated by more than 10 Mpc and with \( \Delta < 0 \) (10 realizations) located in void regions. The filled triangles in Fig. 5b show
the mean cross CCF for these low density regions. We see the same scale of plateau on CCF at about 40 Mpc independently of the way of the calculation. The cluster-galaxy cross-CCF shows a stronger correlation than observed in the entire galaxy population (CCF for SDSS galaxies) and inherits from the cluster-cluster correlations the length of the scaling regime. Note that all three curves in Fig.5b, that were calculated in rather different ways converge to the homogeneity regime at the same scale of $\sim 40$ Mpc.

![Graph](image_url)

Fig. 5.— a) MST analysis for X-ray and model MN-clusters; b) CrossCCF clusters-galaxies (open circles), CCF of SDSS galaxies (stars; error bars are smaller than symbol size), low density region galaxies crossCCF (triangles, 10 realizations).

### 4.4. Minimal Spanning Tree

We built the minimal spanning trees (MST) of the cluster samples. The MST consists of knots and edges and is constructed by appending new knots satisfying the condition for the distance to the already constructed part of the tree being at a minimum [15]. The MST and void analyses give us a clue to outline the ”skeleton” of structures represented by clusters. The full length of the truncated MST when only knots having more then 1 edge left, normalized to the number of such knots is $L_X^{tr} = 37$ Mpc for the X-ray cluster sample and $L_{MN}^{tr} = 38$ Mpc for the simulated sample (for comparison, random samples with the same number density give after averaging $L_R^{tr} = 47$ Mpc). Using the MST linking lengths $LL = 20, LL = 30$ and $LL = 40$ Mpc we have constructed the cumulative functions of structure abundances (Fig.5a) for the observed, simulated and randomly generated samples.
Clusters in the observed sample are slightly more structured than the simulated ones (largest differences are for $LL = 20$ Mpc which reflects the same effect we discussed already before (section 4.1): a lack of close pairs in the simulated sample). But, the largest structures in both samples (for $LL = 30$ and 40 Mpc) have nearly the same number of clusters: we see an overall agreement of abundances of observed and simulated structures detected in the levels of connectivity chosen. Again we see large deviation with respect to the Poisson sample for both values of LL which is another signature of clustering in our samples.

Fig. 6.— X-ray luminous ($L_X(0.1 - 2.4keV) > 1.25 \cdot 10^{43}h^{-2}$ erg/s) member clusters of Shapley supercluster ($0.039 < z < 0.059$) connected by the edges of minimal spanning tree shorter than $40h^{-1}$ Mpc.

5. CCF of galaxy samples from the SDSS DR6 main galaxy database

When analyzing the SDSS DR6 data, we have selected 3 rectangular regions from the region of spectroscopic sky coverage for the convenience of allowance for the boundary con-
ditions for CCF and for ensuring sample completeness. In the \((\lambda, \eta)\) coordinate system of the survey, the selected regions are S1: \(48^\circ < \lambda < 30^\circ, 6^\circ < \eta < 35^\circ\); S2: \(25^\circ < \lambda < 48^\circ, 6^\circ < \eta < 35^\circ\); S3: \(-54^\circ < \lambda < -16^\circ, 33^\circ < \eta < -17^\circ\).

We constructed volume limited samples to eliminate the incompleteness in the radial coordinate. We have set the limit on the r-band absolute magnitude \(M_r\) for the sample of galaxies to \(M_{\text{lim}} = r_{\text{lim}} - 25 - 5\log(R_{\text{max}}(1+z_{\text{max}})) - K(z)\), where \(r_{\text{lim}} = 17.77\) was taken as the limiting r-band magnitude, \(K(z)\) is the K-correction, and \(R_{\text{max}}\) is the maximum value of the radial coordinate corresponding to \(z_{\text{max}}\), (assuming \(\Omega_\Lambda = 0.7, \Omega_m = 0.3\)). Therefore, we have in the VL-sample all galaxies with \(M_r < M_{\text{lim}}\). The \(r\) magnitudes used here were corrected for extinction. To estimate the absolute magnitudes of the galaxies, we used an approximation for K-correction for SDSS galaxies of the form \(K(z) = 2.3537z^2 + 0.5735z + 0.18437\) [16,17].

Here, we present results for two cuts on redshift \(z_{\text{max}}\): VL1 \((z_{\text{max}} = 0.12, M_{\text{lim}} = -20.11)\) and VL2 \((z_{\text{max}} = 0.15, M_{\text{lim}} = -20.68)\) (Fig7a,b).

![CCFs for different VL samples from DR6 SDSS main galaxy database. Ngal - number of galaxies in a sample.](image)

The CCF method deals with spheres that are fully included in the sample. For spheres with centers located at large radii they tend to be located close to each other. We have limited our analysis to the scale defined by the condition that spheres of large radii do not overlap by more than half of their volumes. As can be seen in the Figs, the power law regime is limited to scales \(\sim 10 – 15\) Mpc and the CCFs of different samples show rather concerted convergence to the homogeneity regime. We should note a small but distinct differences in the amplitudes of the homogeneous regime in the CCFs for three different regions. This
means that, at such scales, we can measure the mean density with some scatter that is caused by cosmic variance, i.e., the presence of different structures in different samples. The characteristic scales of galaxy correlations are significantly smaller than the ones produced by clusters in proportions that are similar to those obtained by early application of the more traditional two-point correlation function $\xi(r)$ (see e.g. [18]). Such differences have a natural explanation in the theory of biased galaxy formation.

6. Estimation of the relative cluster-galaxy bias on 200 Mpc scale

The theory of structure formation predicts that the clustering of the most massive dark matter halos (clusters) is enhanced relative to that of the general mass distribution (galaxies) [19, 20]. Fig.1 shows large inhomogeneities in the distribution of clusters on scales of $100 - 300$ Mpc. In the northern region of the SDSS (Fig.2) we can estimate a relative clustering bias ($b$) for the volume-limited samples of clusters with $L_X > 2.5 \cdot 10^{43}$ erg/s and galaxies with $M_r < -19.67$. In three equal volumes of $5.2 \cdot 10^6$ Mpc$^3$ defined in the redshift intervals $0.020 - 0.069$, $0.069 - 0.087$ and $0.087 - 0.100$ for the area covered by the SDSS-DR6 spectral survey ($\sim 6100$ square degrees) there are 12, 32 and 13 clusters and 36612, 46785 and 38566 galaxies, respectively. These counts give a rough estimate of cluster-galaxies relative bias $b = 5 \pm 2$ on the scale of 200 Mpc. This estimate is consistent with the bias ($b = 3$) measured for massive halos ($\sim 3 \cdot 10^{14} M_{\odot}$) in N-body simulations, but on scales of 15-30 Mpc [21].

7. Scaling properties of galaxies

For samples containing galaxies with absolute magnitudes less than $M^*$ the galaxy integral CCF behaves pretty much like a power law with an exponent $\gamma_{gal} \sim 1.0$ (see section 5) up to scales $\sim 10 - 15$ Mpc. Galaxies more luminous than $M^*$ tend to be more clustered [13, 22, 23]. To investigate scaling properties of individual galaxies on such scales, we chose all galaxies from the sample (section 5) VL1-S1 ($z_{max} = 0.12$) that are located more than 10 Mpc away from the sample boundaries. For each selected galaxy we calculated number $N$ of neighbours in the spheres of radii $R_{sp}$ (changing $R_{sp}$ from 2 to 10 Mpc with step 1Mpc) and used linear approximation with slope $s_{NR}$ of $\log(N) - \log(R_{sp})$ dependencies. We excluded from the analysis very isolated galaxies and galaxies that have an error in the slope, $\sigma_S$, larger than 0.3 (taking into account the dispersion of slope values and excluding "bad" approximations). After completing this procedure, about 37000 galaxies were left. Surprisingly the mean slope $s_{NR}^m = 1.7$ (which also corresponds to the maximum in the
histogram of slope values) does not correspond to an exponent $\gamma_{S1} \sim 1.0$ of the CCF of the sample on scales $\sim 10\,\text{Mpc}$. (For instance, in a homogeneous scale-invariant distribution, it should be $s_{NR}^{m} = 3 - \gamma_{CCF}$). The dispersion of the distribution of slopes is significant: $\sigma_{NR} = 0.6$. By defining galaxy structures according to their density contrast and connectivity (using the abovementioned MST technique) we came to the conclusion that it is hard to associate galaxies with slopes in a certain narrow range with identified structures - there is a complex mixture of slopes with significant dispersion, but the possibility to trace structures by their scaling properties is still an open question. We found some amount of slopes $s_{NR} > 3$: all of them are associated with galaxies in relatively low density regions with $\delta < 10$ (see section 4.3). Following the approach of [24] we performed a multi-scaling analysis that weights high and low density regions in different way according to the positive or negative counting-weighting exponent $q-1$. The generalized dimensions $D_q$ differs significantly for different values of $q$ ($D_{-4}$ increases from $D_{-4} = 1.1$ to $D_{2} = 1.8$) which is another signature of the complexity of the scaling properties on small scales. Usually this effect is interpreted as a manifestation of multifractality. It is evident that galaxy clustering on small scales can not be described by simple models like the ones with a unique scaling exponent.

8. Conclusions

The application of different complementary statistics to samples of observed and simulated clusters of galaxies, chosen in a way to fit the observed number density of the volume limited X-ray cluster sample, show general agreement in the distribution of most luminous virialised objects in the Universe and most massive halos in cosmological simulations. Based on the CCF we found the same scale ($\sim 100\,\text{Mpc}$) of statistical homogeneity (in a sense in which we have a definite, but fluctuating, mean number density of objects on such scales) for observed and simulated clusters. This scale can be related to the comoving scale of the largest wavelength of the acoustic oscillation of the photon-baryon plasma before recombination [25]. It is very interesting to note that the second (transitional) clustering regime (beyond the characteristic scale of superclusters) shown both by the observed and simulated CCF happen to be at the same scales of $\sim 40 - 100\,\text{Mpc}$. The Shapley supercluster strongly affects the value of the CCF slope of clusters on small scales and it is responsible for the differences in the distribution characteristics of observed and simulated clusters: we see a lack of close massive cluster pairs in simulations. Larger computational boxes are necessary to find out Shapley-like structures in simulations. The MST analysis shows that the observed clusters are slightly more structured than the simulated ones. The observed and simulated void functions agree except for the largest voids. In summary, the distribution of most massive $\Lambda$CDM dark matter halos show a reasonable agreement with the distribution
of most luminous X-ray clusters of galaxies. The significant differences in the characteristic scales of the distribution of X-ray clusters and SDSS galaxies (power law scales at 40 Mpc and 10-15 Mpc and homogeneity scales at approximately 100 and 40-50 Mpc, respectively) are similar to differences obtained in earlier works using the two-point correlation functions. They can be explained by the theory of biased structure formation. Our estimation of the relative cluster-galaxy bias value ($b \sim 5$) is in general agreement with the theoretical prediction. The power law behaviour of the decline of the galaxy density with distance indicated by the CCF (correlation exponent $\gamma$) on small scales has a very complex nature: a dependence of $\gamma$ on colors and luminosities of galaxies, a significant scatter of individual exponents of the number-radius relation for different galaxies in the sample, and the evidences of multifractality - differences in the scaling properties depending on environment (high and low density regions).

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