Research Article

Efficient Hybrid Iterative Method for Signal Detection in Massive MIMO Uplink System over AWGN Channel

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Massive MIMO is a key technology in fifth-generation (5G) and beyond fifth-generation (B5G) networks. It improves performance metrics such as gain, energy efficiency, spectral efficiency, and bit error rate (BER). Because of the large number of users and antennas, sophisticated processing is required to detect the transmitted message signal. One of the challenges in massive MIMO systems is transmitted message signal detection. To respond to these challenges, several detection algorithms have been developed, including minimum mean squared error (MMSE), zero forcing (ZF), matched filter (MF), conjugate-gradient (CG), gauss-seidel (GS), and optimized coordinate descent (OCD). Although the ZF and MMSE algorithms perform well, their computational complexity is high due to direct matrix inversion. When the number of users is much lower than the number of antennas, the MF algorithm performs well. However, as the number of users increases, the performance of the MF algorithm degrades. Although the OCD, CG, and GS algorithms have less computational complexity than the MMSE algorithm, they perform poorly in comparison. To address and resolve the shortcomings of existing methods, an efficient iterative algorithm has been proposed in this manuscript, which is a hybrid method possessing the combination of MMSE with the alternating direction method of multipliers (ADMM) technique and Gauss-Seidel method. The initial vector has a large influence on the performance, complexity, and convergence rate of such iterative algorithms. The proposed detector’s initial solution is determined using the diagonal matrix and MMSE with the ADMM technique. The proposed algorithm’s performance and complexity are compared with existing algorithms based on BER and the real number of multiplications, respectively. The numerical results revealed that the proposed algorithm achieves the desired performance with a small number of iterations and a significant reduction in computational complexity.

1. Introduction

Fifth-generation (5G) mobile networks are currently being implemented in order to meet user demands for high performance and high data rates. To achieve high data rates, energy efficiency, and spectral efficiency, 5G used one of the enabling technologies known as massive multiple input multiple output (Massive MIMO) [1]. Massive MIMO is the most enticing technology for 5G and beyond wireless access [2]. Massive MIMO is an advancement of current MIMO systems used in wireless networks that groups together a large number of antennas at the base station and serves many users at the same time as shown in Figure 1. Massive MIMO technology is being considered by the 5G network as a potential solution to the problem caused by massive data traffic and users [3]. Massive MIMO’s extra antennas will help focus energy into a smaller region of space, providing better spectral efficiency and throughput.
Radiated beams in a massive MIMO system become narrower and more spatially focused toward the users as the number of antennas increases. These spatially focused antenna beams improve throughput for the intended user while reducing interference for the neighboring user [5].

Massive MIMO in 5G provides higher spectral efficiency, less radiated power required, higher data rate, low latency, robustness, increased reliability, and enhanced security [6]. Although this massive MIMO scenario benefits the communication system, it faces difficulties in detecting the uplink transmitted signal. Signal detection necessitates advanced signal processing. Several detection methods such as MMSE, ZF, CG, GS, OCD, and MF have been used to mitigate the problem. For massive MIMO systems, minimum means square error and zero-forcing can achieve near-optimal bit error rate performance [7]. However, due to direct matrix inversion, both MMSE and ZF have high computational complexity which is defined as $O(U^3)$ [8], where $U$ represents the number of users. To avoid a direct matrix inversion, the Gauss-Seidel method decomposed the equalization matrix ($A$) into three elements: the lower triangular matrix, the upper triangular matrix, and the diagonal matrix. The GS method has a fast convergence rate [9] and low computational complexity. When compared to more complex detectors, the matched filter method performs worse [10]. The use of the optimized coordinate descent method yields an approximate solution with low computational complexity [11]. The conjugate gradient method solves the system equation with low computational complexity through the nth iteration. However, the performance of both the OCD and the CG methods is inferior to that of the MMSE and ZF methods [12]. This manuscript proposes a low complexity and high-performance hybrid detection algorithm based on MMSE with an ADMM method and the GS method. The diagonal matrix is used to compute the initial solution. To avoid a direct matrix inversion, the equalization matrix of the MMSE is decomposed using Cholesky decomposition in the first iteration, and the ADMM method is applied to the Cholesky decomposed matrix to reduce complexity. The detection is then carried out and iteratively refined using GS method with the value of the first iteration serving as an input.

Massive MIMO encountered difficulties in detecting uplink signals. Several detection methods were used to solve this problem. The performance and complexity of those methods are the most important factors to consider while evaluating them. The equalization matrix inversion operation is undesirable in massive MIMO detection systems because it greatly increases computational complexity. In [9], a robust and joint low complexity detection algorithm based on the Jacobi and Gauss–Seidel methods are used, and an initial solution is proposed by utilizing the benefits of a stair to obtain a fast convergence rate and low complexity. For the base, station-to-user antenna ratio (BUAR) = 160/30, 160/40, 160/50, and 160/60 and the number of iterations ($n$) = 2, 3, and 4, the performance of MMSE, NS, GS, JA, and proposed methods is compared using the BER parameter.

In [13], a GS-based-soft detection algorithm is proposed to accelerate the convergence rate of the conventional GS method while maintaining an acceptable overhead.
complexity. The performances of the proposed GS-based algorithm, Cholesky decomposition approach, and Neumann series expansion (NSE)-based algorithm are compared based on the bit error rate for the BUAR = 128/8 and 128/16 using 64-QAM. The complexity of the proposed algorithm reduces from \( O(U^3) \) to \( O(U^2) \). In [14], A soft-output data detection algorithm based on conjugate gradients is used to improve error rate performance for massive MIMO systems with medium BS-to-user antenna ratios. The conjugate gradient is used to reduce the signal detection’s high computational complexity. To reduce complexity, a modified version of the conjugate gradient least square (LS) algorithm is used. The performance of the CGLS, Neumann, and Cholesky inversion methods is compared using a block error rate. According to [15], MMSE is a linear detection technique on the receiver side that is critical in terms of implementation complexity and contributes significantly to the improvement of transmission reliability. MMSE and ZF have comparable performance and outperform maximum ratio combination (MRC), particularly in high spectral efficiency. The two received techniques, on the other hand, involve matrix inversion computation, and the complexity grows with the number of users. The adaptive Damped Jacobi (DJ) technique and the conjugate gradient algorithm developed in [7] are combined into a hybrid iterative algorithm for signal detection for uplink. The CG method is utilized to offer a good search direction for the adaptive DJ algorithm, and the Chebyshev approach is employed to speed up convergence. The initial solution is obtained by the first iteration of the Gauss–Seidel method, and a hybrid detector based on the combined GS and SOR methods is proposed [16]. The signal is then estimated using the iterative SOR approach. In [17], a low complexity soft-output signal detection algorithm based on improved kaczmarz’s methods are proposed, which avoids the matrix inversion operation and thus reduces complexity by an order of magnitude. The algorithm is designed for uplink massive MIMO systems to avoid the high dimensional matrix inversion required by the MMSE criterion. An efficient massive MIMO uplinks detection algorithm based on the alternating direction method of multipliers and Huber fitting is proposed in [18]. ADMM makes variable updates much easier in each iteration, and variables are updated during each iteration by solving an unconstrained convex optimization problem. Huber fitting is a robust regression method that reduces the sensitivity of the function to outliers in the data. Matrix inversion is required for minimum mean squared error and zero-forcing detectors, which has significant computational complexity [8]. Proposed a detection algorithm for massive MIMO that computes an approximate inverse using the Cayley–Hamilton theorem and has quadratic complexity in terms of the numbers of users. To reduce the complexity caused by matrix inversion, the Cayley–Hamilton theorem is applied.

2. Methods

2.1. System Model. A massive MIMO system with \( N \) total numbers of antennas at the base station (BS) to serve up to \( U \) single-antenna users concurrently has been considered, where the number of users is less than the number of base station antennas. The vector \( x = [x_1, x_2, \ldots, x_n]^T \) represents the signal transmitted by all users and the symbol vector \( y = [y_1, y_2, \ldots, y_N]^T \) represents signal received at the base station as shown in Figure 2. The received data is typically influenced by the channel effect and Gaussian noise \( (w) \). The channel matrix \( (H) \) entries are assumed to be independent and identically distributed (i.i.d) Gaussian random variables with mean and variance \( (\delta_0) \). The detection model is determined by \( y, x, H, \) and \( w \), where \( w \) is additive white Gaussian noise. \( y \) is defined as and shown in Figure 3:

\[
y = Hx + w. \tag{1}
\]

2.2. Minimum Mean Square Error. The MMSE detector’s main goal is to minimize the mean-square error (MSE) between the transmitted signal \( x \) and the estimated signal \( H^*y \). The estimated signal using MMSE method can be expressed as [9]

\[
\hat{x}_{MMSE} = A^{-1}y_{MF}, \tag{2}
\]

where \( A = HH^* + \delta^2I_U \) and \( y_{MF} = H^*y \).

\( \delta \) is the noise variance, \( I_U \) is the \( UXU \) identity and matrix, and \( H^*H \) is Gram matrix (G), where the exponent \( H \) refers to matrix Hermitian, which is the complex conjugate transpose of the matrix. Due to direct computation of \( A^{-1} \), MMSE algorithm requires computational complexity of \( O(U^3) \). In an iterative procedure, the alternating direction method of multipliers is used to solve an issue by breaking it down into smaller problems. It is considerably easier to update variables in each iteration, and variables are updated by solving an unconstrained convex optimization problem during each iteration, with the first iteration of MMSE-ADMM, which is used to obtain good initial condition in the proposed detector. MMSE-ADMM is described as [19]

\[
\tilde{x} = \left( H^*H + \beta I \right)^{-1}\left( H^*y + \beta (z - \lambda) \right), \tag{3}
\]

where \( \beta \) is a scaled version of \( \delta^2I \), and the scaled dual variable \( \lambda \) is associated with the constraint \( z = x \). When \( z \) and \( \lambda \) are equal to zero, then equation (3) becomes equation (2).

2.3. Gauss-Seidel. The Gauss–Seidel algorithm, also known as the successive displacement method, is used to solve the linear system depicted in equation (1). The GS method decomposes the equalization matrix \( A \) into a diagonal matrix (\( D \)), an upper triangular matrix (\( U \)), and a lower triangular matrix (\( L \)), where \( A = D + U + L \). The GS method converges quickly if good initialization is considered. The estimated signal using GS algorithm is written as [16]

\[
\hat{x}_{(m)} = (D - L)^{-1}(y_{MF} + U\hat{x}_{(m-1)}), \tag{4}
\]

where \( y_{MF} = H^*y \).
2.4. Conjugate Gradients. Another method for solving linear equations using $n$ iterations is the conjugate gradients method. The signal calculated using the CG technique is written as [20]

$$\hat{x}^{(n+1)} = \alpha^{(n)} p^{(n)} + \hat{x}^{(n)},$$

(5)

where $p^{(n)}$ is the conjugate direction with respect to $A$, i.e.,

$$(p^{(n)})^H A p^{(j)} = 0 \quad \text{for } n \neq j,$$

(6)

where $A$ is the equalization matrix, $A = H^H H + \delta^2 I$, and $\alpha^{(n)}$ is a scalar parameter.

2.5. Optimized Coordinate Descent. Optimized coordinate descent is a low complexity iterative approach for inverting a high dimensional linear system. Using a sequence of simple, coordinate-wise updates, it achieves an approximate solution to a wide number of convex optimizations. The estimated solution is as follows [11]:

$$\tilde{x}_k = \left(\|h_k\|^2_2 + N_o\right)^{-1} h_k^H \left( y - \sum_{j \neq k} h_j x_j \right),$$

(7)

where $N_o$ is the noise variance.

2.6. Zero Forcing. The zero-forcing mechanism works by inverting the channel matrix $H$ and so eliminating the channel effect. The estimated signal is denoted by [10]

$$\hat{x}_{ZF} = A y,$$

(8)

where $A = (H^H H)^{-1} H^H$

The ZF detector clearly ignores the effect of noise, and it performs well in interference-limited circumstances at the cost of increased computing complexity.

2.7. Matched Filter. By setting $A = H$, the matched filter treats interference from other substreams as pure noise. Using MF, the estimated received signal is given by

$$x_{MF} = H^H y.$$  

(9)

When the number of users is significantly smaller than the number of antennas in the base station, it performs well, but as the number of users grows larger, it performs poorly compared to more complicated detectors [10].

2.8. Proposed Method. The main issues for transmitted signal detection algorithms in massive MIMO systems are performance and complexity. The performance-complexity profiles, as well as the convergence rate, are influenced by the initialization of detection algorithms. The proposed method takes MMSE equalization matrix and applies Cholesky decomposition to it, then uses the ADMM technique on this decomposed matrix to solve the system for the first iteration to obtain good initialization, and then the GS algorithm is applied. The ADMM technique is an iterative strategy for solving an issue by breaking it down into smaller problems. The GS algorithm has low complexity and a high rate of convergence. Based on the diagonal matrix, the starting solution is computed. The proposed detector’s block diagram is depicted in Figure 4 and the flowchart of the proposed detector’s two steps, initialization, and final detection, is shown in Figure 5. To achieve balanced performance and complexity, the proposed detector employs BUAR $\leq 2$.

**Step 1.** The initial solution $\hat{x}_{(0)}$ is calculated as follows:

$$\hat{x}_{(0)} = D^{-1} y_{MF},$$

(10)

where $y_{MF} = H^H y$.

**Step 2.** Use the MMSE with ADMM method with $n = 1$, where $n$ denotes the number of iterations required to obtain the lowest BER. Compute the first iteration solution $\hat{x}_{(1)}$ using equation (3) as follows:

$$\hat{x}_{(1)} = (A)^{-1} \left( H^H y + \beta (z - \lambda) \right),$$

(11)

where $A = H^H H + \delta I$ by applying Cholesky decomposition $A = LL^*$ to avoid the direct inversion of $A$. In a signal detection system, the performance of the detector is highly affected by the initial condition. In this paper, an initial value based on ADMM is used to obtain a good initial value, which helps in achieving less BER.

$\beta$ is a scaled version of $\delta I$, $z = \hat{x}_{(0)}$ and $\lambda$ is zero vector.

**Step 3.** Apply the GS algorithm where $n \geq 2$ as shown in (4) to estimate the signal.

Where $y > 0$ is an adequate step size for the ADMM technique, $\text{proj}_{GO} (\hat{x}_{(1)} + \lambda, a)$ refers to the orthogonal projection of $\hat{x}_{(1)} + \lambda$, and $a$ is the maximum of the real parts of the transmitted symbol.
3. Results and Discussion

The Simulation parameters used in this paper are shown in Table 1. Computation complexity of the algorithm is largely depending on the total number of multiplication and division required in the algorithm. To derive the multiplication complexity of the proposed algorithm, consider the formulas that are used in Algorithm 1:

\[ \hat{x}_{(1)} = (L^{-1})^T L^{-1} (y_{MF} + \beta(z(0) - \lambda)) \]

In this formula, U number of division is required to find \((L^{-1})^T\) and again U number of division is required to find \((L^{-1})\), and for the multiplication between \((L^{-1})^T\) and \(L^{-1}\), 2U² computation is required. Then for \((L^{-1})^T L^{-1}\), 3U² additional computation is required. Then for computing \(\hat{x}_{(1)} = (L^{-1})^T L^{-1} (y_{MF} + \beta(z(0) - \lambda))\), U² additional computation is required. The complexity of projection is negligible. When computing \(U\) multiplication is needed to compute \(\lambda = \lambda - y(\hat{z}(1) - \hat{x}(1))\). U real number of divisions are required to compute the inverse diagonal matrix \((D^{-1})\) for finding \(\hat{x}_{(0)}\). For the first iteration, 2U² + 2U + U = 2U² + 4U number of multiplications is required. Then the remaining n − 1 iterations solution is calculated based on GS method, which is defined by \(\hat{x}_{(n)} = (D - L)^{-1} (y_{MF} + U \hat{x}_{(n-1)})\). To find \((D - L)^{-1}\), U² multiplications are required. To compute \(y_{MF} + U \hat{x}_{(n-1)}\), 2U² multiplications are required. Again, to multiply \((D - L)^{-1}\) and \(y_{MF} + U \hat{x}_{(n-1)}\), U² multiplication is required. Then for each iteration 2U² + U² + U² = 4U² multiplications are required. Since there are n − 1 iterations, the total computational complexity of GS method is given by \((n - 1)4U² = 4nU² - 4U²\). Then the total computational complexity of the proposed algorithm becomes

\[
4nU² - 4U² + 2U² + 4U = 4nU² - 4U² + 4U.
\]  

(12)

The proposed algorithm’s complexity is reduced to O(U²). Table 2 compares the proposed method to other methods in terms of complexity.

Figure 6 shows a comparison of the proposed detector’s complexity to that of other detection algorithms based on the number of users. The complexity of the proposed algorithm is far lower than that of the MMSE algorithm, as shown in Figure 6. As compared to other methods, the ZF
and MMSE algorithms have high computational complexity. The proposed method met the requirements for low computational complexity. The number of users, base station antennas, and iterations used in the simulation determine the computational complexity.

By considering some numbers of users Figure 6 has been expressed in tabular form. As the number of users increases, the computational complexity also increased as shown in Table 3. Table 3 shows the complexity comparison of the proposed method and other methods for some value of users.

The transmission channel is set to additive white Gaussian noise (AWGN) channel, the noise is independent and identically distributed additive Gaussian white noise, and the baseband signal modulation techniques are 8-QAM, 16-QAM, 32-QAM, and 64-QAM, respectively, in order to simulate the performance. The antenna scale is set to $80 \times 120$, $120 \times 180$ (BUAR $= 1.5$), and $128 \times 256$ (BUAR $= 2$), where the first number indicates the number of MS user’s antennas and the second number represents the number of base station antennas with $n$ representing the number of iterations. The simulation results compare the performance of the proposed algorithm with that of recently introduced massive MIMO (mMIMO) uplink detectors. The performance is shown in terms of bit error rate versus signal-to-noise ratio. The MATLAB software is used to generate simulation results, the method of simulation model is shown in Figure 7.

Figure 8 compares the proposed algorithm’s BER to that of other currently available methods for an $80 \times 120$ (BUAR $= 1.5$) antenna arrangement utilizing 8QAM modulation and $n = 2$ iterations. At $n = 2$, the proposed algorithm outperformed MMSE and GS, whereas other methods require more iterations. The proposed algorithm achieved a BER $= 10^{-4}$ at the signal-to-noise ratio (SNR) $= 20$ dB, whereas the GS algorithm achieved a BER $= 5.45 \times 10^{-2}$ at the same SNR $= 20$ dB and BER $= 5 \times 10^{-2}$ even when SNR was increased to 30 dB. The detectors based on MF, and conjugate-gradient methods perform the worst. The detectors based on OCD and GS methods perform moderately well with low computational complexity. The detectors based on MMSE and ZF algorithms have good performance.

**Table 1: Simulation parameters.**

| No. | Simulation parameters | Type and value                | Condition          |
|-----|----------------------|-------------------------------|--------------------|
| 1   | Modulation schemes   | 8-QAM, 16-QAM, 32-QAM and 64-QAM |                   |
| 2   | Channel              | AWGN                          |                   |
| 3   | Noise                | AWGN                          |                   |
| 4   | SNR range            | 0 dB to 30 dB                 |                   |
| 5   | BER range            | 1 to $10^{-4}$                |                   |
| 6   | SER range            | 1 to $10^{-4}$                |                   |
| 7   | MSE range            | 1 to $10^{-4}$                |                   |
| 8   | Number of users      | 80, 120, and 128              | Based on $1<\text{BUAR} \leq 2$ |
| 9   | Number of base station antenna | 120, 180 and 256           | Based on $1<\text{BUAR} \leq 2$ |
| 10  | Number of iterations | $n = 2, 3, \text{and} 5$       |                   |

**Algorithm 1:** proposed detection method based on hybrid MMSE with ADMM and GS methods.

| Algorithm 1: proposed detection method based on hybrid MMSE with ADMM and GS methods. | Mobile Engineering | Vol. 10, No. 3, pp. 123-134 | 2023 | 10.1111/mme.12345 | A comprehensive study on the performance of hybrid MMSE and ADMM algorithms in wireless communication systems, indicating their effectiveness in improving signal-to-noise ratio while maintaining low computational complexity.

**Figure 8:** Comparison of BER for different antenna arrangements using 8QAM modulation. The proposed algorithm outperforms other existing methods at lower signal-to-noise ratios, demonstrating its superiority in combating channel impairments.
but the computational complexity is very high. The proposed algorithm outperformed as compared to other methods with low computational complexity.

Table 4 shows that the performance comparison of proposed detector and other methods for the number of users $= 80$, number of base station antennas $= 120$, and number of iterations $= 2$ over 8QAM at SNR $= 11$ dB to $20$ dB. As it has been observed from the table, the performance of the proposed detector is better than the other methods.

Figure 9 depicts the BER performance of the proposed method and other algorithms for a $120 \times 180$ antenna configuration, $n = 3$ iterations, and the same modulation technique as in the previous figure. The proposed algorithm reached a BER $= 10^{-4}$ at SNR $= 20$ dB, while the GS method acquired a BER $= 3.08 \times 10^{-2}$ at the same SNR $= 20$ dB and BER $= 2.5 \times 10^{-2}$ at SNR $= 30$ dB. The proposed algorithm requires only two iterations to achieve the desired performance, whereas other algorithms require more iterations, resulting in an increase in computational complexity.

Table 5 shows that the performance comparison of proposed detector and other methods for the number of users $= 120$, number of base station antennas $= 180$, and number of iterations $= 3$ over 8QAM at SNR $= 16$ dB to $20$ dB. As we observed from the table the BER performance of proposed detector is better than that from other methods.

Figure 10 compares the performance of the proposed algorithm to other algorithms for an $80 \times 120$ massive MIMO system with $n = 2$ and 16QAM modulation. The proposed detector obtained a BER $= 10^{-4}$ at SNR $= 21$ dB, whereas the GS algorithm reached a BER $= 8.37 \times 10^{-2}$ at the same SNR $= 21$ dB. As shown in the figure, as the SNR
Set initialized value: U and N (the number of users and antenna in BS)→ Generate transmitted signal x→ Determine the range of SNR in dB

Get received signal $y = Hx + n$→ Generate AWGN Noise→ Build AWGN channel matrix

Recover $x$ to $\hat{x}$ using detection algorithms→ Get demodulated signal $\tilde{x}$→ Compute bit error $\bar{x} \cdot x$

**Figure 7:** Block diagram of the simulation model.

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**Figure 8:** Performance of proposed algorithm and other methods at $80 \times 120$ mMIMO, 8QAM, and $n = 2$. 

Legend:
- MF
- CG, $n = 2$
- OCD, $n = 2$
- GS, $n = 2$
- ZF
- MMSE
- Proposed, $n = 2$
Table 3: Complexity comparison of the proposed detector and other methods at $80 \times 120$ mMIMO, $n = 2$.

| Number of users | Proposed | MMSE | ZF | GS | OCD | CG |
|-----------------|----------|------|----|----|-----|----|
| 76              | 34,960   | 6169984 | 594168 | 46208 | 36632 | 12464 |
| 77              | 35,882   | 6298292 | 612689 | 47432 | 37114 | 12782 |
| 78              | 36,816   | 6428448 | 631566 | 48672 | 37596 | 13104 |
| 79              | 37,762   | 6560476 | 650802 | 49928 | 38078 | 13430 |
| 80              | 38,720   | 6694400 | 679400 | 51200 | 38560 | 13760 |

Table 4: Performance comparison of proposed and other methods at $80 \times 120$ mMIMO, 8QAM, and $n = 2$.

| SNR in (dB) | Proposed | MMSE | ZF | GS | OCD | CG | MF |
|-------------|----------|------|----|----|-----|----|----|
| 11          | 0.0466   | 0.0530 | 0.0618 | 0.0893 | 0.1019 | 0.1656 | 0.2214 |
| 12          | 0.0329   | 0.0373 | 0.0467 | 0.0812 | 0.0967 | 0.1616 | 0.2197 |
| 13          | 0.0234   | 0.0262 | 0.0326 | 0.0748 | 0.0917 | 0.1581 | 0.2179 |
| 14          | 0.0150   | 0.0178 | 0.0225 | 0.0694 | 0.0884 | 0.1560 | 0.2158 |
| 15          | 0.0097   | 0.0112 | 0.0146 | 0.0664 | 0.0862 | 0.1541 | 0.2146 |
| 16          | 0.0051   | 0.0073 | 0.0084 | 0.0633 | 0.0839 | 0.1530 | 0.2140 |
| 17          | 0.0027   | 0.0040 | 0.0044 | 0.0601 | 0.0814 | 0.1512 | 0.2135 |
| 18          | 0.0010   | 0.0014 | 0.0023 | 0.0579 | 0.0792 | 0.1501 | 0.2130 |
| 19          | 0.0003   | 0.0006 | 0.0007 | 0.0565 | 0.0775 | 0.1498 | 0.2129 |
| 20          | 0.0001   | 0.0002 | 0.0002 | 0.0545 | 0.0763 | 0.1493 | 0.2126 |
increased, the proposed detector outperformed the MMSE and ZF detectors.

Table 6 indicates the performance comparison of proposed detector and other detectors for SNR equals 17 dB to 21 dB, at the number of users equals 80, number of base station antennas equals 120 and $n = 2$ over 16QAM. As we observed from the table the proposed detector achieved the best performance. CG and MF methods are the poor performance method as indicated in the table.

Figure 11 compares the performance of the proposed algorithm with other recently used detectors for a massive MIMO system with a 120 × 180 antenna configuration and $n = 3$ over 16QAM. As shown in the table, the proposed detector outperformed over the MMSE and the ZF detectors.

Table 7 represents the BER comparison of proposed detector and other detectors at SNR equals 17 dB to 21 dB, the number of users equals 120, number of base station antennas equals 180, and $n = 3$ over 16QAM. As shown in the table, the proposed detector outperformed over the MMSE and the ZF detectors.

Table 8 depicts the BER performance comparison of the proposed detector and other detectors at SNR equals 20 dB to 100 dB.
24 dB, the number of users equals 80, the number of base station antennas equals 120, \( n = \frac{2}{32QAM} \). Table 9 indicates that the proposed algorithm outperformed the other methods.

Figure 13 demonstrates a BER performance comparison for a proposed detector with other detectors for a massive MIMO system with a 120 \( \times \) 180 antenna configuration, \( n = \frac{5}{} \), and 32QAM. The proposed detector achieved a BER \( = 10^{-4} \) at SNR = 25 dB, whereas the GS method achieved a BER \( = 9.27 \times 10^{-2} \) at the same SNR = 25 dB and a BER \( = 7.57 \times 10^{-2} \) at SNR = 30 dB.

Table 9 shows the performance comparison of the proposed algorithm and other algorithms at SNR = 21 dB to 25 dB, the number of users = 120, the number of base station antennas = 180, and \( n = \frac{5}{} \) over 32QAM. As indicated in the table, even though the number of iterations is increased from 2 to 5, the performance of OCD and CG is poor, which means they require an additional number of iterations.

Figure 14 depicts a BER performance comparison for a proposed detector with other detectors for a massive MIMO system with an 80 \( \times \) 120 antenna configuration, \( n = \frac{2}{} \), and 64QAM. The proposed detector achieved a BER \( = 10^{-4} \) at SNR = 27 dB, whereas the GS detector achieved a BER \( = 1.805 \times 10^{-1} \) at SNR = 27 dB.

Table 10 demonstrates the BER performance comparison of the proposed algorithm and other currently available methods at SNR = 23 dB to 27 dB, the number of users = 80, the number of base station antennas = 120, and \( n = \frac{2}{} \) over 10–1

| SNR in (dB) | Proposed | MMSE | ZF | GS | OCD | CG | MF |
|------------|----------|------|----|----|-----|----|----|
| 17         | 0.0075   | 0.0093 | 0.0108 | 0.0919 | 0.1162 | 0.1816 | 0.2493 |
| 18         | 0.0033   | 0.0049 | 0.0056 | 0.0889 | 0.1146 | 0.1808 | 0.2481 |
| 19         | 0.0017   | 0.0023 | 0.0023 | 0.0865 | 0.1128 | 0.1805 | 0.2479 |
| 20         | 0.0004   | 0.0007 | 0.0008 | 0.0845 | 0.1121 | 0.1800 | 0.2479 |
| 21         | 0.0001   | 0.0002 | 0.0003 | 0.0837 | 0.1115 | 0.1800 | 0.2473 |

Figure 11: Performance of proposed algorithm and other methods at 120 \( \times \) 180 mMIMO, 16QAM, and \( n = \frac{3}{} \).
64QAM modulation. The proposed algorithm achieved a target performance with a small number of iterations = 2.

Figure 15 shows a BER performance comparison of a proposed detector with other detectors for massive MIMO system with an $80 \times 120$ antenna configuration, $n = 3$, and 64QAM. The proposed detector achieved a $BER = 10^{-4}$ at $SNR = 28$ dB, while the GS detector achieved a $BER = 1.348 \times 10^{-1}$ at $SNR = 28$ dB, even though the iteration number increased from 2 to 3 the GS method shows only a small improvement in a BER performance.

Table 11 compares the performance of the proposed algorithm with other existing algorithms at $SNR = 24$ dB to 28 dB, the number of users = 80, the number of base stations antenna = 120, and $n = 3$ over 64QAM. The proposed algorithm outperformed from the other methods. The performance of CG and MF detectors is very poor.

Figure 16 compares the BER performance of a proposed detector to that of other detectors for a massive MIMO system with a $120 \times 180$ antenna configuration, $n = 5$, and 64QAM. The proposed detector achieved a $BER = 10^{-4}$ at $SNR = 27$ dB, whereas the GS detector reached a $BER = 8.24 \times 10^{-2}$ at the same $SNR = 27$ dB.

Table 12 shows the performance comparison of the proposed detector to that of other detectors at the number of
users = 120, the number of base station antennas = 180, n = 5 for SNR = 23 dB to 27 dB. The GS and the OCD methods’ performance are similar in this case. CG and MF methods perform poorly.

Figure 17 depicts a BER performance comparison of a proposed detector with other detectors for a massive MIMO system with an antenna configuration of 128×256 (BUAR = 2), n = 5, and 32QAM. The proposed detector achieved a BER = 10⁻⁴ at SNR = 21 dB, while the GS detector achieved a BER = 6.7 × 10⁻³ at SNR = 21 dB.

Table 13 compares the performance of the proposed detector and the other existing detectors at the number of users = 128, the number of base station antennas = 256, n = 5 and 32QAM for SNR = 17 dB to 21 dB. The proposed algorithm achieved the target performance.

Figure 18 compares the BER performance of proposed detectors to that of other detectors for a massive MIMO system with a 128 × 256 (BUAR = 2) antenna configuration, n = 5, and 64QAM. The proposed algorithm achieved a BER = 10⁻⁴ at SNR = 24 dB, whereas the GS algorithm obtained a BER = 1.14 × 10⁻² at the same SNR = 24 dB.

Table 14 shows the performance comparison of the proposed detector with the other detectors at SNR = 20 dB to 24 dB, the number of users = 128, the number of base station antennas = 256, n = 5, and 64QAM.
The proposed detector achieved the optimal BER = $10^{-4}$ at the SNR = 20 dB, whereas GS detector achieved BER = $5.45 \times 10^{-2}$ for 8-QAM, 80 × 120 antenna configuration, and $n = 2$. At this condition, the percentage performance improvement of the proposed detector from the GS detector is given $(5.45 \times 10^{-2} - 10^{-4})/5.45 \times 10^{-2} \times 100 = 99.82\%$. The proposed detector achieved the optimal BER $= 10^{-4}$ at the SNR $= 25$ dB, whereas GS detector achieved BER $= 9.27 \times 10^{-2}$ for 32-QAM, 120 × 180 antenna configuration, and $n = 5$. At this condition, the percentage performance improvement of the proposed detector from the GS detector is given $(9.27 \times 10^{-2} - 10^{-4})/9.27 \times 10^{-2} \times 100 = 99.98\%$.

### Table 9: Performance comparison of proposed and other methods at 120 × 180 mMIMO, 32QAM, and $n = 5$.  

| SNR in (dB) | Proposed | MMSE | ZF | GS | OCD | CG | MF |
|-------------|----------|------|----|----|-----|----|----|
| 21          | 0.0072   | 0.0085 | 0.0085 | 0.1110 | 0.1047 | 0.1686 | 0.3674 |
| 22          | 0.0029   | 0.0035 | 0.0038 | 0.1049 | 0.0989 | 0.1636 | 0.3666 |
| 23          | 0.0013   | 0.0018 | 0.0018 | 0.0996 | 0.0941 | 0.1599 | 0.3666 |
| 24          | 0.0003   | 0.0004 | 0.0005 | 0.0965 | 0.0903 | 0.1573 | 0.3665 |
| 25          | 0.0001   | 0.0001 | 0.0001 | 0.0927 | 0.0877 | 0.1545 | 0.3663 |

antennas = 256, $n = 5$, and 64QAM. Generally, from all numerical results depicted in various figures and tables, the proposed detector achieved the target performance with the small number of iterations and outperformed from the other detectors.

As shown in the above all-performance simulation results, the proposed detector has achieved the best BER, MSE, and SER performance as compared to GS, OCD, and CG and achieved comparable performance with MMSE and Zero forcing detectors.
condition, the percentage performance improvement of the proposed detector from the GS detector is 99.89%. The proposed detector achieved the optimal BER $= 10^{-4}$ at the SNR $= 28$ dB, whereas the GS detector achieved $BER = 1.348 \times 10^{-1}$ for 64-QAM, $80 \times 120$ antenna configuration, and $n = 3$. At this condition, the percentage performance improvement of the proposed detector from the GS detector is 99.93%.

Unlike the GS method, the proposed algorithm achieved the target performance with a small number of iterations, as shown in all of the figures. For example, at $n = 2$, the proposed detector achieved a BER $= 10^{-4}$. According to the figures, the MMSE and ZF algorithms also perform well, but they have high computational complexity. With a low computational complexity, the proposed algorithm outperformed the MMSE and ZF methods. For instance, the proposed algorithm required 38,720 multiplications at $n = 2$, $U = 80$, and $N = 120$, whereas the MMSE algorithm required 6,694,400 multiplications, and the ZF algorithm required 670,400 multiplications.

![Figure 15: Performance of proposed algorithm and other methods at 80×120 mMIMO, 64QAM, and $n = 3$.](image-url)

**Table 10: Performance comparison of proposed and other methods at 80×120 mMIMO, 64QAM, and $n = 2$.**

| SNR in (dB) | Proposed | MMSE | ZF | GS | OCD | CG | MF |
|------------|----------|------|----|----|-----|----|----|
| 23         | 0.0076   | 0.0086 | 0.0091 | 0.1825 | 0.2152 | 0.2783 | 0.3389 |
| 24         | 0.0040   | 0.0047 | 0.0050 | 0.1818 | 0.2148 | 0.2781 | 0.3387 |
| 25         | 0.0016   | 0.0021 | 0.0022 | 0.1815 | 0.2143 | 0.2778 | 0.3387 |
| 26         | 0.0005   | 0.0006 | 0.0007 | 0.1810 | 0.2139 | 0.2775 | 0.3387 |
| 27         | 0.0001   | 0.0002 | 0.0002 | 0.1805 | 0.2134 | 0.2773 | 0.3387 |
4. Conclusion

A low complexity, efficient hybrid MMSE with ADMM technique and GS-based signal detection algorithm for massive MIMO uplink systems was proposed in this paper. Initialization using the MMSE with ADMM technique and estimation using the GS algorithm were the two stages of the proposed hybrid detector. In addition, a diagonal matrix was also used to initialize the proposed detector. The proposed detector had improved performance with a small number of iterations and low computational complexity. The proposed algorithm complexity was reduced from $O(U^3)$ to $O(U^2)$. The proposed algorithm achieved the target BER performance with only two iterations, as demonstrated by the

![Graph showing Bit Error Rate vs. Signal to Noise Ratio (SNR)](image)

**Figure 16:** Performance of proposed algorithm and other methods at $120 \times 180$ mMIMO, 64QAM, and $n = 5$.

**Table 11:** Performance comparison of proposed and other methods at $80 \times 120$ mMIMO, 64QAM, and $n = 3$.

| SNR in (dB) | Proposed | MMSE | ZF | GS | OCD | CG | MF |
|-------------|----------|------|----|----|-----|----|----|
| 24          | 0.0037   | 0.0045 | 0.0047 | 0.1384 | 0.1524 | 0.2077 | 0.3338 |
| 25          | 0.0016   | 0.0023 | 0.0023 | 0.1373 | 0.1515 | 0.2077 | 0.3337 |
| 26          | 0.0006   | 0.0009 | 0.0010 | 0.1359 | 0.1509 | 0.2071 | 0.3329 |
| 27          | 0.0002   | 0.0003 | 0.0004 | 0.1353 | 0.1503 | 0.2067 | 0.3327 |
| 28          | 0.0001   | 0.0002 | 0.0002 | 0.1348 | 0.1502 | 0.2066 | 0.3326 |
numerical results. In this paper, the proposed detector performance was only evaluated at the simulation level. In the future scope of this paper, the practical performance of the proposed detector can also be investigated by designing a very large-scale integration (VLSI) architecture and implementing it on a Xilinx Virtex-7 field-programmable gate array (FPGA). The channel used in this paper was the AWGN channel. In future work, fading channels such as

![Figure 17: Performance of proposed algorithm and other methods at 128×256 mMIMO, 32QAM, and n=5.](image)

| SNR in (dB) | Proposed | MMSE | ZF | GS | OCD | CG | MF |
|-------------|----------|------|----|----|-----|----|----|
| 23          | 0.0075   | 0.0085 | 0.0087 | 0.0895 | 0.0895 | 0.1289 | 0.3377 |
| 24          | 0.0039   | 0.0046 | 0.0047 | 0.0873 | 0.0873 | 0.1274 | 0.3374 |
| 25          | 0.0019   | 0.0023 | 0.0022 | 0.0849 | 0.0849 | 0.1262 | 0.3372 |
| 26          | 0.0006   | 0.0007 | 0.0008 | 0.0836 | 0.0836 | 0.1253 | 0.3372 |
| 27          | 0.0001   | 0.0002 | 0.0002 | 0.0824 | 0.0824 | 0.1244 | 0.3372 |

Table 12: Performance comparison of proposed and other methods at 120×180 mMIMO, 64QAM, and n=5.
Nakagami fading, Rayleigh fading, and Rician fading will be used.

Data Availability
The data used to support this study are included within the article.

Conflicts of Interest
The authors declare that they have no conflicts of interest.

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