Factors Influencing on Flux Distribution on Focal Region of Parabolic Concentrators

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Abstract. The random surface error, tracking error, position error and shadow of receiver of parabolic dish concentrators are analysed by using the ray tracing and the Monte-Carlo method. Several examples are given. The analysis is help to optimizing the system efficiency and measuring the flux density distribution.

1. Introduction

Analyzing the flux density distribution on the focal region of concentrators when considering errors is very important for optimize systems efficiency. Many scholars have done a lot of researches about it. In 1985, K.S.Jefferies of NASALewis research centre considered the random surface error, the pointing error, the rim angle and solar limb darkening [1]. In 1995, Peter D analyzed the optical characteristic of column receiver by using geometry optics method. He considered the finite sunshape, concentrator surface errors, pointing system zero-mean and constant offset errors [2]. In this paper the direct influence on flux distribution of main errors are analyzed by using the ray tracing and the Monte-Carlo method.

2. Calculation of flux density

The incidence energy is seen $N_s$ rays. Suppose the receiver is a disk that radius is $RA$, and $N$ equidistant annuli are defined in the disk. The number of reflection rays in each annulus is $N_j$. So the flux density $E_j$ of each annulus is:

$$ E_j = \begin{cases} \frac{\phi(\theta_j)N_jS_c\rho R^2 N^2}{N_s \cdot RA^2 \cdot j^2} & j = 1 \\ \frac{\phi(\theta_j)N_jS_c\rho R^2 N^2}{N_s \cdot RA^2 \cdot (2j-1)} & 1 < j \leq N \end{cases} $$

(1)

Where $S_c$ is the beam normal irradiance, $\rho$ is the reflectivity of concentrator surface, $R$ is the radius of the concentrator and $\phi(\theta)$ is the sunshape model which expression is:
Where \( k = 0.9 \ln (13.5 \chi^{0.3}) \), \( \gamma = 2.2 \ln (0.52 \chi^{0.43}) - 0.1 \), \( \chi \) is the circumsolar ratio and \( i = 1: N_i \).

The flux density of each annulus can be obtained if \( N_j \) has been known when \( \Phi \left( \theta_i \right) = 1 \) that is ideal sunshape. \( N_j \) lies on the ubiety between the intersection of reflection rays and the center of the receiver surface. Suppose that \( M \) is the incidence ray vector, \( P \) is the reflection ray vector and \( N \) is the normal vector. Then the unit vector of \( P \) is:

\[
P' = 2(N' \cdot M') \cdot N' - M'
\]

Where \( N' \) and \( M' \) are the unit vectors of \( N \) and \( M \). So the parametric equation of reflection ray is:

\[
\begin{align*}
x &= x_0 + m_x \cdot t \\
y &= y_0 + m_y \cdot t \\
z &= z_0 + m_z \cdot t
\end{align*}
\]

Where \( (m_x, m_y, m_z) \) is the vector of \( P' \), \( (x_0, y_0, z_0) \) is a point on the reflection ray and \( t \) is a parameter.

The incidence rays are parallel and the normal vector at each point is the gradient of the surface at the point when ideal situation. However when the system has tracking error, there is an angle between the incidence rays and the axis of concentrators; when has random surface error, actual surface normal can take an infinite number of positions around this direction; when the receiver has position errors, the expression of receiver plane will be changed. So the main task of analyzing these errors is to derive the incidence vector, the normal vector and the expression of the receiver plane. By expression (3), (4) and the equation of receiver plane, the equation of the reflection ray and the intersection of the ray and the receiver plane can be obtained. Thereby the distance between the intersection and the center of the receiver is also obtained. Then the annulus where the reflected ray lies in can be judged.

3. Analysis of errors

The system coordinate system \( OXYZ \) is built which origin is the peak of the concentrator. Looking from the concentrator onto the target plane, the \( X \)-axis is the horizontal axis, the \( Y \)-axis is the vertical axis and the \( Z \)-axis is the optical axis (figure 1). \( F \) is the focus of paraboloid. Suppose \( M_0O_i \) is the ideal incidence ray and \( O_i \) is the intersection of \( M_0O_i \) and the paraboloid. \( O_iN_0 \) is the ideal normal.

3.1. The normal vector of considering random surface error

A coordinate system \( O_iX_iY_iZ_i \) is built which origin is Point \( O_i \), the \( Y_i \)-axis and the \( X_i \)-axis are built on the tangent plane at point \( O_i \) and the \( Z_i \)-axis is the normal of the tangent plane. \( O_iN \) is the deviation normal vector, which radial angle and tangential angle are \( \theta \) and \( \phi \) [3]. \( O_iP \) is reflection vector; point \( P \) is the intersection with the focal plane. Thus the directional number of \( O_iN \) at coordinate system \( O_iX_iY_iZ_i \) is:

\[
O_iN = \left\{ \sin \theta \cos \phi, \sin \phi \sin \theta, \cos \theta \right\}
\]

The probability models of \( \theta \) and \( \phi \) are:

\[
\theta = \sqrt{-2\sigma^2 \ln(1 - r_\theta)}
\]

\[
\phi = 2\pi \cdot r_\phi
\]

Where \( r_\theta \) and \( r_\phi \) are random numbers between 0 and 1, \( \sigma \) is the standard deviation of \( \theta \).

3.2. The incidence ray vector of considering tracking error

A coordinate system \( O_iX_iY_iZ_i \) is built which origin is \( O_i \) and the direction of \( Z_i \)-axis is the ones of \( O_iM_0 \) (figure 1.). \( M_0O_i \) is the central ray of an incidence cone when tracking error is considered, which
radial angle and tangential angle are $\theta_1$ and $\phi_1$. So the directional number of $O_1M_1$ at coordinate system $O_1X_2Y_2Z_2$ is:

$$O_1M_1 = \{\sin \theta_1 \cos \phi_1 , \sin \theta_1 \sin \phi_1 , \cos \theta_1 \}$$

The probability model of $\phi_1$ can be determined by a random number $r_\phi_1$ (between 0 and 1):

$$\phi_1 = 2\pi \cdot r_\phi_1$$

3.3. The receiver plane of considering the position errors of receiver

A coordinate system $FX_fY_fZ_f$ is built which origin is $F$ and $Z_f$-axis is $Z$-axis. The position errors are the position orientation errors $\Delta x$, $\Delta y$, $\Delta z$ and the angle errors $\Delta \omega_x$, $\Delta \omega_y$, $\Delta \omega_z$ (figure 2).

![Figure 1. The incident and reflected ray path.](image1.png)

![Figure 2. The six position errors of receiver.](image2.png)

Suppose the center position of the receiver is $F'$, and a coordinate system $F'X_f'Y_f'Z_f'$ is built which origin is $F'$ and the three axes parallel with $X$-axis, $Y$-axis and $Z$-axis respectively (figure 1.). Suppose $\alpha_1$, $\beta_1$ and $\gamma_1$ are the angles between receiver plane and $X_f$-axis, $Y_f$-axis and $Z_f$-axis. So the equation of receiver plane at $F'X_f'Y_f'Z_f'$ is:

$$x_f' \sin \alpha_1 + y_f' \sin \beta_1 + z_f' \sin \gamma_1 = 0$$

3.4. The shadow of receiver

Here we consider the sunshape and diffuse reflection [4]. $MO_1$ is the real incident ray which radial angle and tangential angle at $O_1X_2Y_2Z_2$ is $\theta_1$ and $\phi_1$. (figure 1.). So the directional number $O_1M$ at coordinate system $O_1X_2Y_2Z_2$ is:

$$O_1M = \{\sin \theta_1 \cos \phi_1 , \sin \theta_1 \sin \phi_1 , \cos \theta_1 \}$$

The probability models of $\theta_1$ and $\phi_1$ can be determined by random numbers $r_\theta_1$ and $r_\phi_1$:

$$\theta_1 = \arcsin \sqrt{r_\theta_1 \cdot \sin^2 \alpha_{\max}}$$

$$\phi_1 = 2\pi \cdot r_\phi_1$$

The equation of receiver plane at $O_1X_2Y_2Z_2$ is:

$$z_2 = f - z_0$$

According to (11) and (14) the intersection of $O_1M$ and the receiver plane and the distance defined $d$ between this intersection and focus can be obtained. If $d > RA$, the incident ray can fell on the surface of concentrator.

4. The typical calculated results

The flux density distribution is calculated by applying the above probability models and ray racing. Where $R=2.5$ m, $f=3$ m, $\varepsilon=0.1$, $\chi=0.05$, $S_c=1000W/\text{m}^2$ and $N_s =10^7$. Figure 3, 4 and 5 are the flux.
density distribution considering the random surface error, tracking error and shadow of receiver respectively. Table 1 and figure 6 are the flux density distribution considering the position error of receiver. Then the conclusions can be drawn:

(1) The peak value of the flux density decrease with higher $\sigma$, and the characteristic width of the focal spot increase. When $\sigma$ changed from 1 mrad to 1.5 mrad, The peak value decrease 55%, \(R90(90\% \text{ of the focal spot radius})\) change from 12 mm to 17.5; when $\sigma=2$ mrad, The peak value decrease 75% and $R90=24mm$; when $\sigma=3$ mrad, The peak value decrease 89% and $R90=36mm$; 

(2) The peak value decrease and the characteristic width increase with bigger $\theta_1$, and the flux density distribution is far away from the center. When $\theta_1$ change from 1 mrad to 2, 3 and 4 mrad, The peak value decrease 64%, 83% and 89%, respectively. When $\theta_1=1, 2, 3$ and 4 mrad, $R_s$ (the focal spot radius)$= 2.6mm, 5.2mm, 6.5mm$ and $9.1mm$. The distances of the focal spot far away from the focus center are $3.9mm, 7.8mm, 10.4mm$ and $13mm$;

(3) The bigger the size of the receiver is, the smaller of The peak value and the characteristic width are. As can be seen from figure 5, when $RA=0.25$ m, The peak value is 122.39MW/m², $R90=15mm$; When $RA$ increased 0.25 m, the peak value and $R90$ decrease 6.063% and 50%; when $RA$ increase 0.5 m, The peak value and $R90$ decrease 12.86% and 66.7%. So the size of receiver mainly influence on the characteristic width;

(4) $\Delta \omega_y$ had no influence on the flux distribution because the receiver was a disk. The sloped angle of the center axis of the focal spot increases when $\Delta \omega_y$ increase and the shape of focal spot changed from ellipse to irregular shape. The peak value decrease, the characteristic width increase and the center of focal spot have some offset when each errors increase (figure 6. and table 1.).
Table 1. The flux distribution at differential position receiver.

| Errors | Peak values (MW/m²) | X-axis | Y-axis | The offset of focal spot (mm) |
|--------|---------------------|--------|--------|-----------------------------|
| Δx=5mm | 4.2719              | 43.3   | 51.4   | Along X-axis: -5             |
| Δx=10mm| 4.2264              | 48.8   | 57.7   | Along X-axis: -10            |
| Δx=50mm| 4.0759              | 71.8   | 76.3   | Along X-axis: -50            |
| Δy=5mm | 5.6503              | 43.2   | 51.2   | Along Y-axis: -5             |
| Δy=10mm| 4.6939              | 46.9   | 55.8   | Along Y-axis: -10            |
| Δy=50mm| 4.0735              | 64.6   | 72.6   | Along Y-axis: -50            |
| Δz=5mm | 5.6039              | 43.7   | 52     | Along Y-axis: -5             |
| Δz=10mm| 5.4157              | 44.3   | 53.7   | Along Y-axis: -13.5          |
| Δz=50mm| 3.2329              | 62.6   | 91.1   | Along Y-axis: -65.3          |
| Δαx=0.02 mrad| 5.7216 | 42.9 | 51.4 | =0 |
| Δαx=0.2 mrad | 4.7506 | 44.1 | 61.2 | =0 |
| Δαx=0.5 mrad | 3.8870 | 47.1 | 105.6 | =0 |

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