Magnetic properties of a long-lived sunspot

Vertical magnetic field at the umbral boundary *

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ABSTRACT

Context. In a recent statistical study of sunspots in 79 active regions, the vertical magnetic field component \( B_{\text{ver}} \) averaged along the umbral boundary is found to be independent of sunspot size. The authors of that study conclude that the absolute value of \( B_{\text{ver}} \) at the umbral boundary is the same for all spots.

Aims. We investigate the temporal evolution of \( B_{\text{ver}} \) averaged along the umbral boundary of one long-lived sunspot during its stable phase.

Methods. We analysed data from the HMI instrument on-board SDO. Contours of continuum intensity at \( I_{\text{c}} = 0.5I_{\text{qs}} \), whereby \( I_{\text{qs}} \) refers to the average over the quiet sun areas, are used to extract the magnetic field along the umbral boundary. Projection effects due to different formation heights of the Fe i 617.3 nm line and continuum are taken into account. To avoid limb artefacts, the spot is only analysed for heliocentric angles smaller than 60°.

Results. During the first disc passage, NOAA AR 11591, \( B_{\text{ver}} \) remains constant at 1693 G with a root-mean-square deviation of 15 G, whereas the magnetic field strength varies substantially (mean 2171 G, rms of 48 G) and shows a long term variation. Compensating for formation height has little influence on the mean value along each contour, but reduces the variations along the contour when away from disc centre, yielding a better match between the contours of \( B_{\text{ver}} = 1693 \text{ G and } I_{\text{c}} = 0.5I_{\text{qs}} \).

Conclusions. During the disc passage of a stable sunspot, its umbral boundary can equivalently be defined by using the continuum intensity \( I_{\text{c}} \) or the vertical magnetic field component \( B_{\text{ver}} \). Contours of fixed magnetic field strength fail to outline the umbral boundary.

Key words. sunspots – Sun: photosphere – Sun: magnetic fields – Sun: activity

1. Introduction

The boundary between umbra and penumbra of sunspots has long been defined in terms of the continuum intensity \( I_{\text{c}} \). This brightness difference is the consequence of the different magneto-convective processes running in umbrae and penumbrae. We have evaluated magnetic quantities to identify which of them may cause the different behaviour on the two sides of the umbral boundary.

Jurčák (2011) investigated the properties of the magnetic field at umbral boundaries and noted that the vertical magnetic field component \( |B_{\text{ver}}| \) changes little along the boundaries of the ten sunspots he analysed and could neither verify nor falsify a dependence of the median value along the boundary on the area of the umbral boundary.

Jurčák et al. (2015) extended the analysis by investigating a 4.5h time series of a forming sunspot using GFPI/VTT data and noting an increase of \( |B_{\text{ver}}| \) at the migrating umbral boundary during penumbra formation and stabilization of this value after completion of the formation. Shortly thereafter, that part of the umbral boundary was observed with Hinode/SP and a \( |B_{\text{ver}}| \) value of 1810 G measured. They propose that the umbral mode of magneto-convective prevails in areas with \( |B_{\text{ver}}| > B_{\text{ver}}^{\text{stable}} \), whereas outside, the penumbral mode takes over.

Following this line of investigation, Jurčák et al. (2017) studied a pore whose \( |B_{\text{ver}}| \) remained below this critical value. They found that a developing penumbra completely cannibalized the pore, thus supporting the assertion that in umbral areas with \( |B_{\text{ver}}| < B_{\text{ver}}^{\text{stable}} \), the penumbral mode of magneto-convective takes over the umbral mode.

Jurčák et al. (2018) extends the analysis of 2011 to 88 scans of 79 different active regions again using Hinode/SP and showed that the \( I_{\text{c}} = 0.5I_{\text{qs}} \) contours match mostly the \( |B_{\text{ver}}| = 1867 \text{ G} \) contours. A Bayesian linear regression showed that a model with constant \( B_{\text{ver}} \) is more likely to explain the data than a first or second order polynomial with log area as independent variable. Furthermore the most likely \( |B_{\text{ver}}| = 1867 \text{ G}, \text{ with a } 99\% \text{ probability for } 1849 \text{ G} \leq |B_{\text{ver}}| \leq 1885 \text{ G}. \) A dependence on the solar cycle could not be verified.

These findings have led to the Jurčák criterion, an empirical law stating that the umbral boundary of stable sunspots can be equivalently defined by a continuum intensity \( I_{\text{c}} \) or a vertical magnetic field component \( |B_{\text{ver}}| \). In other words, in areas with \( |B_{\text{ver}}| > B_{\text{ver}}^{\text{stable}} \), only the umbral mode of convection exists, hindering other modes of magneto-convection. A conjecture can also be stated from these findings: umbral areas with \( |B_{\text{ver}}| < B_{\text{ver}}^{\text{stable}} \) are unstable against more vigorous modes of convection, that is, they are prone to vanish.

In this work we have investigated the behaviour of the magnetic field along the umbral boundary in a time series of a single stable sunspot. We used the spot of NOAA AR 11591 during its

* videos associated with Fig. 3 are available at [http://www.aanda.org](http://www.aanda.org)
first disc passage. This allows us to verify whether \( \langle B_{\text{ret}} \rangle_0(t) \) remains constant over \( \approx 10 \) days, which would provide support to the Jurčák criterion. Hereby \( \langle \cdot \rangle_0 \) stands for average along the \( I_x \) contour.

## 2. Data and analysis

The used data are retrieved after processing by the Solar Dynamics Observatory’s (SDO) Helioseismic and Magnetic Imager (HMI) Vector Magnetic Field Pipeline \((\text{Hoeksema et al. 2014})\) cutout service for NOAA AR 11591. Using this NOAA AR number on \http://jsoc.stanford.edu/ajax/exportdata.html\, the \texttt{fitshead2wcs}, \texttt{wcs_convert_from_coord}, and \texttt{exportdata} routines were modified from and tested against Xudong Sun’s \texttt{hmi_b2ptr}, \texttt{im_patch}, and those they call \(\text{Gary \\& Hagyard 1990; Thompson 2006; Sun 2013}).\)

The heliographic Stonyhurst coordinates are calculated using procedures modified from and tested against sswidl’s \texttt{wcs} routines \texttt{fitshead2wcs}, \texttt{wcs_get_coord}, \texttt{wcs_convert_from_coord} and those they call \(\text{Thompson 2006}).\) The canonical value for HMI of \( R_0 = 696 \text{Mm} \) is used. The transformation of the magnetic field vector into the local reference frame was performed with a code modified from and tested against Xudong Sun’s \texttt{sswidl} routine \texttt{hmi_b2ptr}, \texttt{fitshead2wcs}, \texttt{wcs_convert_from_coord} and those they call \(\text{Gary \\& Hagyard 1990; Thompson 2006; Sun 2013})).\)

### Quiet sun intensity.

The limb darkening correction in the HMI pipeline was based on \(\text{Pierce \\& Slaughter 1977, Eq. 9}\) which does not consider all orbital artefacts introduced into the continuum intensity \( I_c \) of SDO/HMI data. Even after limb darkening removal and normalization there is a change over the day in \( I_c \) of the order of 1% towards the limb with opposite signs on the western and eastern hemisphere. To compensate for this, the quiet sun intensity \( I_{qs} \) for each time step was chosen such that \( I_{qs} \) is the mean of all the quiet sun pixels within the \( 500 \times 500 \) cutout, where quiet sun is defined as having quiet sun intensity. To compensate for this, the direction of the surface normal does not consider all orbital artefacts introduced into the coordinate system.

The limits of the time series we analyse are given as \( t_{\text{start}} \) and \( t_{\text{end}} \) in Table 1. A total of 1063 time steps in this time range are available. This time range was chosen to select data sets, for which the heliocentric angle \( \psi \) of the centroid of the umbra was smaller than 60°.

### Time series fit.

For every time step and magnetic quantity, an average was computed along the contours, thereby creating time series of the form \( X(t) \in \langle B_{\text{ret}} \rangle_0(t), \langle B \rangle_0(t), \langle \gamma_{\text{art}} \rangle_0(t) \). Similarly, standard deviations along the contours \( \sigma_{\gamma_0}(t) \) were calculated. These time series \(\text{cf. Sect. 3 and Figs. 1 and 2}\) show a daily variation of an approximately sinusoidal shape. We believe them to be an artefact of SDO’s geosynchronous orbital motion. For the ranges from \( t_{\text{start}} \) to \( t_{\text{end}} \) given in Table 1 these time series are least square fitted against functions of the form

\[
X_{\text{fit}}(t) = X_0 + X_1 \sin(2\pi t) + X_2 \cos(2\pi t)
\]

whereby \( t \) is in days and \( t \in \mathbb{N} \) is at noon. \( X_0 \) is the value we are interested in and will be henceforth called offset. It is used instead of a time average \( \langle X(t) \rangle \) because it correctly accounts for missing data (most importantly the gap in the afternoon of Oct 17) and that \( t_{\text{start}} \) & \( t_{\text{end}} \) have a different time of day. Here we have \( |\langle X(t) \rangle - X_0| < 0.5 \text{G} \) for all \( X(t) \) in \( G \) and \( < 0.01 \text{G} \) for \( X(t) = \langle \gamma_{\text{art}} \rangle_0(t) \). While \( X_1 \) & \( X_2 \) are used internally during the fitting process to guarantee numeric stability, the results are presented with parameters \( X_0, X_1, X_2 \) and \( X_0 \) in Table 2. \( X_1 \) and \( X_2 \) are the amplitude and phase of the orbital artefacts. Also listed are the standard deviations of the residuals \( \sigma = \sigma(\langle X(t) \rangle - X_{\text{fit}}(t)) \) and the means of the standard deviations along the contours over the range in time \( \langle \sigma_\gamma(t) \rangle \).

### Levels of magnetic contours.

The offsets \( X_0 \) from the fits to \( \langle B_{\text{ret}} \rangle_0(t), \langle B \rangle_0(t) \), and \( \langle \gamma_{\text{art}} \rangle_0(t) \) for the 0.5 (0.4) \( I_{qs} \) contours are then used as contour level on the \( B_{\text{ret}}, B \) and \( \gamma_{\text{art}} \) maps, respectively. They are discussed in Sect. 3.2 and plotted in Figs. 3 and 5 and the videos.

### Distance between contours.

To quantify how well two contours match we calculated the average distance between them \( \langle d \rangle_0 \), which we define as the area of symmetric difference divided by the length of the intensity contour, \( \ell(t) \). The area of

\[
\langle x', y' \rangle = \left( 1 + \frac{\Delta h}{R_0} \right) (x, y)
\]
Fig. 1. Mean magnetic field strength $\langle |B| \rangle_\psi(t)$ in black (it’s vertical component $\langle B_{\text{ver}} \rangle_\psi(t)$ in blue) along the $I_c = 0.5I_{qs}$ contour, with $\Delta h = 465$ km accounted for, Sinusodial fits and the residuals for NOAA AR 11591.

NOAA AR 11591, mean $|B|$ & $-B_{\text{ver}}$ along contour $I_c=0.5I_{qs}$ with formation height difference of 465km
Stonyhurst longitude of umbral centroid

Fig. 2. As Fig. 1 but from contours at $I_c = 0.4I_{qs}$. 

NOAA AR 11591, mean $|B|$ & $-B_{\text{ver}}$ along contour $I_c=0.4I_{qs}$ with formation height difference of 465km
Stonyhurst longitude of umbral centroid
symmetric difference, $\Delta t(t)$, is the area surrounded by either of the contours but not both. When averaging in time we weighed by the contour length, giving

$$
(d)_{\phi,t} = \frac{\sum_t \Delta t(t)}{\sum_t t(t)}.
$$

These average distances between contours are listed in Table 2 in km pixel. For $(d)_{\phi,t} \ll 1$ pixel only the total ordering should be relied upon due to gridding and other computational effects.

Fit along each contour. For every point along a contour, a reference angle $\psi = \angle(\text{PCD})$ is calculated, whereby $P$ is the point on the contour, $C$ is the centroid of the $l_c = 0.5 I_{ls}$ contour in the CCD frame and $D$ is the centre of the solar disc as observed by SDO. The angles are calculated on the sphere. For every time step and every contour, $Y(\psi) \in \{B(\psi), |B(\psi), \gamma_{\text{ref}}(\psi)\}$ is least square fitted against functions of the form

$$
Y_{\text{fit}}(\psi) = Y_0 + Y_1 \sin(\psi) + Y_2 \cos(\psi)
$$

Those fits are plotted in the right panels of the videos (cf. Sect. 3.2 and bottom panels of Figs. 3 and 5). Furthermore the time averages of the fit amplitudes $\langle Y(\psi) \rangle_t$ are listed in Table 2.

Optimal height difference. $\Delta h = 465 \text{ km}$ was chosen because it minimizes the average distance $(d)_{\phi,t}$ between the $l_c = 0.5 I_{ls}$ contours after transformation with Eq. 1 and the $B_{\text{ref}}$ contours, whereby the contour level $X_0$ on the $B_{\text{ref}}$ map has been derived with the fit to $(B_{\text{ref}})(t)$ as described above (Eq. 2). An optimal height difference of $\Delta h = 465 \text{ km}$ means that the intensity contour at the limb is shifted outwards by $465 \text{ km} \cdot r_c \approx 0.65'' \approx 1.3$ pixel. The difference of the formation heights for continuum and Fe i 617.3 nm line core amounts to $\approx 250 \text{ km}$ for a typical umbral model atmosphere (see e.g. Norton et al. 2006, Table 1).

The fact that the value for $\Delta h$ is larger may be explained with the Wilson depression of the umbra, which typically amounts to $800 \text{ km}$. The latter causes the $\tau = 1$ surface to be strongly inclined relative to horizontal. Minimizing the standard deviation of $B_{\text{ref}}$ along the $l_c = 0.5 I_{ls}$ contour $(\sigma_{\phi})_c$ column in Table 2 instead would give an optimal $\Delta h = 520 \text{ km}$.

3. Results

Based on the time series of approximately ten days, in which the spot of NOAA AR 11591 has heliocentric angles smaller than $60^\circ$, we determine the magnetic properties for two distinct contour levels of the continuum intensity. As intensity levels we use $l_c = 0.5 \text{ (0.4) } I_{ls}$. Along each contour, the azimuthal average of $B_{\text{ref}}$, $|B|$ and $\gamma_{\text{ref}}$ are calculated. The respective values of those averages for $B_{\text{ref}}$ (in blue) and $|B|$ (in black) as well as sinusoidal fit of the orbital variation are displayed in the upper panels of Fig. 1 for $l_c = 0.5 I_{ls}$ and of Fig. 2 for $l_c = 0.4 I_{ls}$. The lower panels show the residuals after subtracting the fit.

3.1. Temporal evolution

The parameters of the sinusoidal fits, offset $X_0$, amplitude $X_3$, and the rms of the corresponding residuals, $\sigma_r$, are given in Table 2 for all considered cases. In addition, they are printed into the plots of Figs. 1 and 2. For the contours at $l_c = 0.5 I_{ls}$, we find for $(B_{\text{ref}})(t)$ that $\sigma_r \approx 15 \text{ G}$ is smaller than the orbital amplitude $X_3 = 18 \text{ G}$, with an offset of $X_0 = 1693 \text{ G}$. For the contours of $l_c = 0.4 I_{ls}$, $\sigma_r = 19 \text{ G}$ is also smaller than $X_3 = 20 \text{ G}$ with an $X_0 = 1850 \text{ G}$. For the residuals of $B_{\text{ref}}$ no long-term trend is noticeable.

In contrast, the residuals of $(|B|)(t)$ amount to $\sigma_r = 48 \text{ G}$ which is larger than the amplitudes of the sinusoidal fit $(16 \text{ G})$, and it shows a long-term variation. Since $\gamma_{\text{ref}}$ is dependent on $B_{\text{ref}}$ and $|B|$, it has a long-term variation which compensates for that of $|B|$ (not shown). The offsets $X_0$ for $(|B|)(t)$ and $(\gamma_{\text{ref}})(t)$ are 2171 G and 141.4$^\circ$ respectively at the contours with $l_c = 0.5 I_{ls}$.

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Table 1. Timestamps of our spot, year=2012

| NOAA AR | $t_{\text{ref}}$ | $t_{\text{start}}$ | $t_{\text{end}}$ | $t_{\text{W}}$ | Stonyhurst Lon | Lat |
|---------|-----------------|-----------------|----------------|--------------|----------------|-----|
| 11591   | 10.11.17:24     | 10.13.19:24     | 10.22.22:24     | 10.25.08:00  | 10.17.23:59-59Z | -7  |

Table 2. Results: fit parameters and time averages

| $l_c/l_{qs}$ | $\Delta h [\text{km}]$ | variable | $X_0$ | $X_1$ | $X_2[\text{rad}]$ | $\sigma_r$ | $\langle Y(\psi) \rangle_t$ | $(d)_{\phi,t} [\text{px}]$ |
|--------------|------------------------|----------|-------|-------|-----------------|-----------|--------------------------|--------------------------|
| 0.50         | 465 $-B_{\text{ref}}[G]$ | 1693     | 18    | 3.067 | 15              | 81        | 41                       | 0.45                     |
| 0.50         | 0 $-B_{\text{ref}}[G]$  | 1695     | 17    | 2.981 | 16              | 113       | 97                       | 0.59                     |
| 0.40         | 465 $-B_{\text{ref}}[G]$ | 1850     | 20    | 3.109 | 19              | 83        | 58                       | 0.50                     |
| 0.40         | 0 $-B_{\text{ref}}[G]$  | 1849     | 18    | 3.107 | 21              | 114       | 103                      | 0.66                     |
| 0.50         | 465 $|B| [G]$           | 2171     | 16    | -3.120| 48              | 111       | 102                      | 0.97                     |
| 0.50         | 0 $|B| [G]$           | 2175     | 14    | 3.116 | 47              | 124       | 112                      | 1.09                     |
| 0.40         | 465 $\gamma_{\text{ref}}[^\circ]$ | 2265     | 17    | -3.118| 54              | 117       | 116                      | 1.18                     |
| 0.40         | 0 $\gamma_{\text{ref}}[^\circ]$ | 2267     | 16    | -3.082| 55              | 131       | 126                      | 1.31                     |
| 0.50         | 465 $\gamma_{\text{ref}}[^\circ]$ | 141.4    | 0.2    | 2.835 | 1.6             | 2.6       | 2.5                      | $a$ 0.82                 |
| 0.50         | 0 $\gamma_{\text{ref}}[^\circ]$ | 141.4    | 0.2    | 2.728 | 1.5             | 2.8       | 2.6                      | $a$ 0.84                 |
| 0.40         | 465 $\gamma_{\text{ref}}[^\circ]$ | 145.0    | 0.2    | 2.968 | 1.6             | 2.5       | 2.3                      | 0.77                     |
| 0.40         | 0 $\gamma_{\text{ref}}[^\circ]$ | 144.8    | 0.2    | 2.943 | 1.4             | 2.5       | 2.3                      | 0.77                     |

(a) Excluding five snapshots due to faulty 180°-disambiguation: t=10.22. [12:14,12:36,13:36,14:00,14:12]
Fig. 3. NOAA AR 11591, the top panels show continuum intensity maps for longitudes $-60^\circ$, $0^\circ$ & $30^\circ$. The legend in the lower right corner of the top left panel defines the contour levels. Different formation heights are accounted for (Eq. 1, $\Delta h = 465$ km). The cyan arrow originates in the centroid of the umbra and points towards disc centre. The bottom 3 rows show the magnetic field parameters retrieved along the $I_c = 0.5I_q$ contour. The temporal evolution is available online at http://www.aanda.org
The fact that the residuals $\sigma_\psi(B_{\text{vert}}; t)$ are smaller than $\sigma_\psi(|B|; t)$ is remarkable, but is even more remarkable if one considers that the gradient of $B_{\text{vert}}$ perpendicular to the contour is larger than that of $|B|$. This can be inferred from Table 2. The difference of the $\langle B_{\text{vert}}; t \rangle$ offset, $X_0$, between the two different intensities amounts to 157 G while that of $\langle |B|; t \rangle$ is only 94 G. Hence, a small shift of the contour implies a larger deviation in $B_{\text{vert}}$ than in $|B|$. Therefore, our result of a smaller deviation in $B_{\text{vert}}$ relative to $|B|$ gives further evidence that $\langle B_{\text{vert}}; t \rangle$ can be considered constant in time.

3.2. Contours

Using the offsets $X_0$ from the fits in Table 2 with $I_c = 0.5I_{qs}$ and $\Delta h = 465$ km, the upper panels of Fig. 3 (overplot the contours of intensity $I_c = 0.5I_{qs}$ (red), $|B| = 2171$ G (green), $-B_{\text{vert}} = 1693$ G (blue), and $\gamma_{qs} = 141.4^\circ$ (yellow). The background images consist of 100x100 pixel cutouts of grey-scale intensity maps with a minimum (maximum) of $I_c = 0.1 (1.2) I_{qs}$. A close inspection of the figure shows that the $B_{\text{vert}}$ contour matches best with the intensity contour. The cyan arrow originates in the centroid of the umbra and points towards disc centre. The centroid is determined by the $I_c = 0.5I_{qs}$ contour and is derived using CCD coordinates.

The three bottom rows of panels of Fig. 3 show the magnetic field quantities along the $I_c = 0.5I_{qs}$ contour as well as their sinuosoidal fits in black. The azimuth is determined relative to the centroid and the direction towards disc centre, which corresponds to $\psi = 0^\circ$ and runs counter-clockwise.

To quantify the azimuthal variation of the magnetic parameters, Table 2 gives the time average of the standard deviations along the contours, $\langle \sigma_\psi \rangle_t$. Again $\langle \sigma_\psi \rangle_t$ is smaller for $B_{\text{vert}}$ (81 G) than for $|B|$ (111 G). As before, the small value for $B_{\text{vert}}$ is remarkable, since its gradient perpendicular to the contour is larger than for $|B|$. The lower panels demonstrate that the azimuthal variations are smallest for $B_{\text{vert}}$. Again, we note that this is remarkable considering the fact that the gradient of $B_{\text{vert}}$ perpendicular to the contour is larger than the gradient of $|B|$.

A video of the temporal evolution of those contours during the disc passage of the spot is available at http://www.aanda.org. This animation demonstrates that an iso-contour of $B_{\text{vert}} = -1693$ G coincides nicely with the intensity contour at $0.5I_{qs}$. This animation also demonstrates that contours of $|B|$ and $\gamma_{qs}$ do not coincide.

To quantify the match or mismatch of two contours, we have introduced the average distance between two sets of contours, $\langle d \rangle_{\psi,t}$ (cf. Eq. 3). It is given in the last column of Table 2. $\langle d \rangle_{\psi,t}$ is smallest for the $B_{\text{vert}}$ contours with $\Delta h = 465$ km (see the first and final row of Table 2).

In Fig. 4 the average distance is plotted for intensities changing from 0.30 to 0.65. The corresponding contour levels for $B_{\text{vert}}$ are calculated as described in Sect. 2 (fit to Eq. 2). The best match, $\langle d \rangle_{\psi,t} = 0.44$, is found for $I = 0.53I_{qs}$ with $-B_{\text{vert}} = 1639$ G ($X_3 = 17$ G, $\sigma_\psi = 15$ G, and $\langle \sigma_\psi \rangle_t = 82$ G). Distances for $|B|$ and $\gamma_{qs}$ are in all cases larger and not plotted. Hence, by minimizing the distance, $-B_{\text{vert}} = 1639$ G results as the value that defines the umbral boundary at $I = 0.53I_{qs}$. This is additional proof that our chosen value of $I = 0.5I_{qs}$ is very close to the optimum value.

3.3. Effect of neglecting formation heights compensations

For the results presented so far, we corrected for the projection effects due to different formation heights of continuum and line. As discussed in the end Sect. 2 we assume a height difference of $\Delta h = 465$ km. Table 2 also gives the results for the case in which these projection effects are not considered, i.e. $\Delta h = 0$ km. As a general trend, it is seen that the values for $X_0$, $X_3$, and $\sigma_\psi$ change only marginally. A plot like in Fig. 1 with $\Delta h = 0$ km looks almost identical (not shown).

However, $\langle \sigma_\psi \rangle_t$ and $\langle d \rangle_{\psi,t}$ increase significantly. For example, for $B_{\text{vert}}$ at $I = 0.5I_{qs}$, $\langle \sigma_\psi \rangle_t$ and $\langle d \rangle_{\psi,t}$ increase by more than 30% from 81 to 113 G, and from 0.45 to 0.59 pixel, respectively. This is illustrated in Fig. 5 which shows the same snapshot as in the left column of Fig. 3 with the only difference that $\Delta h = 0$ km. In this case, the heliocentric angle is 60°. It is seen that the magnetic contours are shifted relative to the intensity, which results in an increase of $\langle d \rangle_{\psi,t}$, and the variation of $B_{\text{vert}}$ along the contour (bottom panels) are larger for $\Delta h = 0$ km. This can also be seen in the corresponding video of the disc passage of the spot, which is available at http://www.aanda.org.

4. Conclusion

Investigating the physical properties along the umbra-penumbra boundary of a stable sunspot for a time span of approximately ten, we find three main results:

1. $B_{\text{vert}}$ averaged along the $I = 0.5I_{qs}$ contour is nearly constant in time.
2. Contours of intensity and of $B_{\text{vert}}$ match at the umbral boundary. The best match is obtained for $I = 0.53I_{qs}$ and $|B_{\text{vert}}| = 1639$ G.
3. Projection effects due to different formations height of the spectral line and continuum need to be considered. If not, variation of $B_{\text{vert}}$ along the contour increases significantly.

These results are obtained by analysing 1063 consecutive SDO/HMI data sets (with a time step of 12 min) of the first disc passage of NOAA AR 11591.

Using $I_c = 0.5I_{qs}$ to define the umbral boundary, we obtain $|B_{\text{vert}}| = 1693$ G $\pm 15$ (1 $\sigma$-error). Jurčák et al. (2018) used Hinode/SP data to find $|B_{\text{vert}}| = 1867^{+106}_{-96}$ G (99%-error) at $I_c = 0.5I_{qs}$.

The values for $|B_{\text{vert}}|$ differ by some 175 G. In general, a difference is expected due to differences in the experimental setup.
The temporal evolution is available online at http://www.aanda.org.

and analysis methods. Sainz Dalda (2017) investigates the differences between HMI and SP vector magnetograms and obtained comparable differences. He concludes that the filling factor followed by spatial and spectral resolution are the main source. At the umbral boundary the filling factor is 1, and causes therefore no differences. The other effects are particularly strong at the sharp boundary between umbra and penumbra, where the intensity gradient is large.

Hence, these investigations provide evidence that $|B_{\text{ver}}|$ is constant for a statistical sample of sunspots as well as during the evolution of one stable spot, thereby supporting the Jurčák criterion.

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References

Borrero, J. M., Tomczyk, S., Kubo, M., et al. 2011, Sol. Phys., 273, 267
Gary, G. A. & Hagyard, M. J. 1990, Sol. Phys., 126, 21
Hoeksema, J. T., Liu, Y., Hayashi, K., et al. 2014, Sol. Phys., 289, 3483
Jurčák, J. 2011, A&A, 531, A118
Jurčák, J., Bello González, N., Schlichenmaier, R., & Rezaei, R. 2015, A&A, 580, L1
Jurčák, J., Bello González, N., Schlichenmaier, R., & Rezaei, R. 2017, A&A, 597, A60
Jurčák, J., Rezaei, R., Bello González, N., Schlichenmaier, R., & Vonmlel, J. 2018, A&A, 611, L4
Leka, K. D., Barnes, G., Crouch, A. D., et al. 2009, Sol. Phys., 260, 83
Metcalf, T. R. 1994, Sol. Phys., 155, 235
Norton, A. A., Graham, J. P., Ulrich, R. K., et al. 2006, Sol. Phys., 239, 69
Pesnell, W. D., Thompson, B. J., & Chamberlin, P. C. 2012, Sol. Phys., 275, 3
Pierce, A. K. & Slaughter, C. D. 1977, Sol. Phys., 51, 25
Rimmele, T. R. 1995, A&A, 298, 260
Sainz Dalda, A. 2017, ApJ, 851, 111
Schou, J., Scherrer, P. H., Bush, R. I., et al. 2012, Sol. Phys., 275, 229
Sun, X. 2013, ArXiv e-prints [arXiv:1309.2392]
Thompson, W. T. 2006, A&A, 449, 791
Westendorp Plaza, C., del Toro Iniesta, J. C., Ruiz Cobo, B., et al. 2001a, ApJ, 547, 1130
Westendorp Plaza, C., del Toro Iniesta, J. C., Ruiz Cobo, B., & Martínez Pillet, V. 2001b, ApJ, 547, 1148
Wilson, A. 1774, Phil. Trans. R. Soc. London, Series I, 64, 1