ON THE GENUS OF A CYCLIC PLANE CURVE OVER A FINITE FIELD

FABIO PASTICCI

ABSTRACT. Cyclic curves, i.e. curves fixed by a cyclic collineation group, play a central role in the investigation of cyclic arcs in Desarguesian projective planes. In this paper, the genus of a cyclic curve arising from a cyclic $k$-arc of Singer type is computed.

Key words: Projective Plane, Cyclic Arc, Singer Group, Algebraic Curve.
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1. Introduction

Let $PG(2,q)$ be the projective plane over the finite field $\mathbb{F}_q$, $q = p^h$ for some prime $p$ and some $h \in \mathbb{N}$. A $k$-arc in $PG(2,q)$, $k \geq 3$ is a set of $k$ points, every 3 of which are not collinear. A $k$-arc in $PG(2,q)$ is said to be complete if and only if it cannot be extended to a $(k+1)$-arc by a point of $PG(2,q)$. A $k$-arc is called cyclic [7] if it consists of the points of a point orbit under a cyclic collineation group $G$ of $PG(2,q)$. A cyclic $k$-arc is said to be of Singer type if it consists of a point orbit under a subgroup of a cyclic Singer group of $PG(2,q)$.

An essential tool in the investigation of $k$-arcs is the following result due to B. Segre (see [12]).

If $q$ is odd, then there exists a plane curve $\Gamma'$ in the dual plane of $PG(2,q)$ such that:

1. $\Gamma'$ is defined over $\mathbb{F}_q$.
2. The degree of $\Gamma'$ is $2t$, with $t = q - k + 2$ being the number of 1-secants through a point of $K$.
3. The $kt$ 1-secants of $K$ belong to $\Gamma'$.
4. Each 1-secant $\ell$ of $K$ through a point $P \in K$ is counted twice in the intersection of $\Gamma'$ with $\ell_P$, where $\ell_P$ denotes the line corresponding to $P$ in the dual plane.
5. The curve $\Gamma'$ contains no 2-secant of $K$.
6. The irreducible components of $\Gamma'$ have multiplicity at most 2, and $\Gamma'$ has at least one component of multiplicity 1.
7. If $k > \frac{2}{3}(q + 2)$, then there exists a unique curve in the dual plane of $PG(2,\mathbb{F}_q)$ satisfying properties (2), (3), (4), (5).

[12]

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The investigation of the algebraic envelope of a cyclic $k$–arc of Singer type was initiated by Cossidente and Korchmáros in 1998 [4], and continued by Giulietti in [7]. In [7] the terminology of cyclic curve is introduced to denote the algebraic plane curve defined over $\mathbb{F}_q$ corresponding to the algebraic envelope of a cyclic $k$–arc of Singer type.

In this paper we deal with the problem of computing the genus of a cyclic curve. Our main result is the following:

**Theorem 1.1.** Let $C$ be a cyclic curve of order $n = 2(q - k + 2) = 2t$. If $k \geq q - \sqrt{2q + \frac{1}{4} + \frac{a}{2}}, k^2 - k + 1 \neq 0 \pmod{p}$, and $C$ is irreducible of genus $g$, then $C$ is singular and either $g = 2(t - 1)(t - 2)$ or $g = \frac{(t-1)(t-2)}{2}$ holds.

2. Preliminaries on Singer cycles

Let $\mathbb{F}_{q^3}$ be a cubic extension of $\mathbb{F}_q$. Following Singer [16], we identify the projective plane $PG(2, q)$ with $\mathbb{F}_{q^3} \mod \mathbb{F}_q$. This means that points of $PG(2, q)$ are non-zero elements of $\mathbb{F}_{q^3}$ and two elements $x, y \in \mathbb{F}_{q^3}$ represent the same point of $PG(2, q)$ if and only if $x/y \in \mathbb{F}_q$. Let $\omega$ be a primitive element of $\mathbb{F}_{q^3}$ and its minimal polynomial over $\mathbb{F}_q$.

The matrix

$$
C = \begin{pmatrix}
0 & 1 & 0 \\
0 & 0 & 1 \\
c & b & a
\end{pmatrix}
$$

induces a linear collineation $\phi$ of $PG(2, q)$ of order $q^2 + q + 1$ called a Singer cycle of $PGL(3, \mathbb{F}_q)$. All Singer cycles of $PGL(3, \mathbb{F}_q)$ form a single conjugacy class and the matrix $C$ is conjugate in $GL(3, \mathbb{F}_q)$ to the diagonal matrix

$$
D = \begin{pmatrix}
\omega & 0 & 0 \\
\omega^2 & \omega^q & 0 \\
0 & 0 & \omega^{q^2}
\end{pmatrix}
$$

by the matrix

$$
E = \begin{pmatrix}
1 & 1 & 1 \\
\omega & \omega^q & \omega^{q^2} \\
\omega^2 & \omega^{2q} & \omega^{2q^2}
\end{pmatrix}
$$

Let $\sigma$ denote the linear collineation of $PG(2, q^3)$ induced by $D$. It fixes the points $E_0 = (1, 0, 0)$, $E_1 = (0, 1, 0)$ and $E_2 = (0, 0, 1)$. The linear collineation $T$ of $PG(2, q^3)$ defined by
has order 3 and acts on the points \( E_0, E_1, E_2 \) as the cycle \((E_0 \ E_1 \ E_2)\).

**Proposition 2.1.** Cyclic Singer groups of \( PG(2, q) \) are equivalent under conjugation by the elements of \( PGL(3, q) \).

**Proof.** See [19] and [12], Corollary 4 to Theorem. 4.2.1. \( \square \)

**Definition 2.2.** A \( k \)-arc of Singer type in \( PG(2, q) \) is a \( k \)-arc which consists of a point orbit under a subgroup of a cyclic Singer group of \( PG(2, q) \).

From now on \( K \) will be a \( k \)-arc of Singer type in \( PG(2, q) \); obviously, \( k \) divides \( q^2 + q + 1 \).

Since by Proposition 2.1 two orbits of the same size under subgroups of cyclic Singer groups are projectively equivalent, we will assume without loss of generality that

\[
K = \{ 1, \omega^{\frac{q^2+q+1}{k}}, \ldots, \omega^{(k-1)\frac{q^2+q+1}{k}} \}.
\]

Let \( K \) be a \( k \)-arc of \( PG(2, q) \), we consider the unique envelope \( \Gamma' \) of \( K \) as a plane algebraic curve in \( PG(2, \overline{F}_q) \) defined over \( F_q \) and we will denote it by \( \Gamma'_{2t} \).

Following [7], we will study a curve \( C_{2t} \) projectively equivalent to \( \Gamma'_{2t} \) in \( PG(2, q^3) \). Let \( \phi = [L] \) be the element of \( PGL(3, q^3) \) such that

\[
LBL^{-1} = \begin{pmatrix}
\omega^q & 0 & 0 \\
0 & \omega^q & 0 \\
0 & 0 & \omega
\end{pmatrix}
\]

and \( \phi(1, 0, 0) = (1, 1, 1) \). Let \( \Pi \) be the image of \( PG(2, q) \) by \( \phi \); \( \Pi \) is a subplane of \( PG(2, q^3) \) isomorphic to \( PG(2, q) \). We can consider \( \alpha := \phi \sigma \varphi^{-1} \), collineation of \( \Pi \); we have that if \( a := \omega^{q-1} \), then

\[
\alpha : \begin{cases}
\rho x'_1 = ax_1 \\
\rho x'_2 = a^{q+1} x_2 \\
\rho x'_3 = x_3
\end{cases}, \quad \rho \in \overline{F}_q^*.
\]

So the points of \( \Pi \) are those in the orbit of \((1, 1, 1)\) under the action of \( < \alpha > \):

\[
\Pi = \{(a^i, a^{i(q+1)}, 1) \mid i = 0, 1, \ldots, q^2 + q \}.
\]

By 2.1 \( K \) is fixed by \( \sigma^{\frac{q^2+q+1}{k}} \) and \( \tau \), where \( \tau \) is defined by \( \tau(\omega^i) := \omega^{qi} \). We are interested in \( \beta := \varphi \sigma^{\frac{q^2+q+1}{k}} \varphi^{-1} \) and \( \delta := \varphi \tau \varphi^{-1} \), collineations of \( \Pi \). We have that if \( b := a^{\frac{q^2+q+1}{k}} \) then

\[
\beta : \begin{cases}
\rho x'_1 = bx_1 \\
\rho x'_2 = b^{q+1} x_2 \\
\rho x'_3 = x_3
\end{cases}, \quad \rho \in \overline{F}_q^*
\]

and

\[
\delta : \begin{cases}
\rho x'_1 = x_1^q \\
\rho x'_2 = x_2^q \\
\rho x'_3 = x_3^q
\end{cases}, \quad \rho \in \overline{F}_q^*.
\]
δ is a non-linear collineation of $PG(2, q^3)$, but it acts on $\Pi$ as $\mu^{-1}$, where $\mu$ is defined by

$$
\mu : \begin{cases}
\rho x'_1 = x_3 \\
\rho x'_2 = x_1 \\
\rho x'_3 = x_2
\end{cases}, \quad \rho \in \mathbb{F}_{q^3}^*.
$$

In [7] the following proposition is proved.

**Proposition 2.3.** Let $\Gamma_{2t}$ be the algebraic curve associated to the envelope of $K$ and let $C_{2t}$ be its image by $\varphi$. Then $C_{2t}$ has the following properties:

1. it is preserved by $\beta$;
2. it is preserved by $\mu$;
3. it has degree $2(q - k + 2)$;
4. it has no fundamental line as a component;
5. it is defined over $\mathbb{F}_q$.

**Definition 2.4.** Let $\omega$ be a primitive element of $\mathbb{F}_{q^3}$, $k$ be a divisor of $q^2 + q + 1$ such that $k > \frac{2}{3}(q + 2)$, and suppose $\beta$ and $\mu$ defined as above. An algebraic plane curve defined over $\mathbb{F}_q$ is called cyclic if it satisfies (1), (2), (3), (4) of Proposition 2.3.

For the rest of the section $C$ will denote a cyclic curve of degree $n = 2(q - k + 2)$.

**Proposition 2.5.** Each vertex of the fundamental triangle is a 2-fold cuspidal point of $C$ such that one of the fundamental line through the vertex is the tangent and has intersection multiplicity $n - 2$ with $C$ at the vertex, namely, letting $A_1 := (1, 0, 0)$, $A_2 := (0, 1, 0)$, $A_3 := (0, 0, 1)$,

$$
I(A_2; C \cap \{x_1 = 0\}) = I(A_3; C \cap \{x_2 = 0\}) = I(A_1; C \cap \{x_3 = 0\}) = n - 2.
$$

Besides, the only two possibilities for branches of $C$ centered at $A_i$ ($i = 0, 1, 2$) are the following:

1. there exist two linear branches (not necessarily distinct) centered at $A_i$ and the tangent meets each of them with multiplicity $\frac{n}{2} - 1$;
2. there exists a unique quadratic branch centered at $A_i$.

**Proof.** See [4], Prop. 5. □

Note that $< \mu >$ preserves $C$ and acts transitively on the vertices of the fundamental triangle; so the number of branches through $A_i$ and their characteristics do not depend on $i$. According to [7], if (1) of the above Proposition holds, then $C$ is said to be cyclic of the first type, otherwise of the second type.

### 3. ON THE GENUS OF A CYCLIC CURVE

In this section we will prove that if $k \geq q - \sqrt{\frac{2}{3}q + \frac{1}{4} + \frac{9}{4}}$ and if the envelope of the $k$-arc of Singer type $K$ in $PG(2, q)$ is irreducible, than we can establish its genus. More precisely the following theorem holds (the notation will be as in Section 2).
Theorem 3.1. Let $\mathcal{C}$ be a cyclic curve of order $n = 2(q - k + 2) = 2t$, as defined in Section 2. If $k \geq q - \sqrt{\frac{2}{3}q + \frac{3}{4} + \frac{9}{4}}$, $k^2 - k + 1 \not\equiv 0 \mod p$, and $\mathcal{C}$ is irreducible of genus $g$, then

- if $\mathcal{C}$ is of the first type then $2g - 2 = 4t^2 - 12t + 6$;
- if $\mathcal{C}$ is of the second type then $2g - 2 = t^2 - 3t$.

Through the rest of the section we will prove Theorem 3.1. Let $\mathcal{C}$ be a cyclic curve and let $g(x, y) = 0$ its minimal equation. Let $g(x, y) = \varphi_j(x, y) + \ldots + \varphi_n(x, y)$, $\varphi_u$ homogeneous of degree $u$; we will denote the generic term of $\varphi_u(x, y)$ by $a_{s, u}x^s y^{u-1}$. In [4] the following four lemmas are proved.

Lemma 3.2. For every $m$, $(0 \leq m \leq n)$, $g(x, y)$ has at most one term of degree $m$.

Lemma 3.3. For every $l$, $(0 \leq l \leq n)$, $g(x, y)$ has at most one term of degree $l$ in $x$ and at most one term of degree $n - l$ in $y$.

Lemma 3.4. For any two integers $l$, $m$ $(0 \leq l \leq m \leq n)$, we have

$$a_{l, m-l} = \epsilon a_{n-m, l} = \epsilon^2 a_{m-l, n-m},$$

with $\epsilon^3 = 1$.

Lemma 3.5. Let $a_{l, m-l}x^l y^{m-l}$ be a term of $g(x, y)$ different from zero; then

$$m \equiv (s - 1)l + 2 \mod k.$$ 

Now we can prove the following proposition.

Proposition 3.6. Let $\mathcal{C}$ be a cyclic curve of order $n = 2(q - k + 2) = 2t$. If $k \geq q - \sqrt{\frac{2}{3}q + \frac{3}{4} + \frac{9}{4}}$ then $\mathcal{C}$ has equation

$$g(x, y) = y^2 + \epsilon_1 x^2 y^{2t-2} + \epsilon_2 x^{2t-2} + c(x^t y^{l-1} + \epsilon_2 x^{t-1} y + \epsilon_2 xy^l) = 0,$$

with $\epsilon_1^3 = \epsilon_2^3 = 1$.

Proof. For any term $a_{l, m-l}x^l y^{m-l}$ of $g(x, y)$ there exist by Lemma 3.4 two other terms of $g(x, y)$ of type $a_{n-m, l}x^{n-m} y^l$ and $a_{m-l, n-m}x^{m-l} y^{n-m}$. By permutating indexes we may assume that $0 \leq l \leq \frac{1}{3}n$; so

$$(t-1)l + 2 \leq \frac{2}{3}(t-1)t + 2;$$

the hypothesis concerning $k$ yields $\frac{2}{3}(t-1)t + 2 \leq k$. By Lemma 3.5 we have $m = (t-1)l + 2$ and since $m \leq 2t$ we have $l \leq 2$. So the couple $(l, m-l)$ is equal to $(0, 2)$, $(1, t)$ or $(2, 2t-2)$ and the proposition is proved (use Lemma 3.4 again).

We will compute the genus of $\mathcal{C}$ by applying the famous Hurwitz Theorem. Let $\mathcal{L} := \overline{\mathbb{F}}_q$ and let $\Sigma = \mathcal{L}(\mathcal{C})$ the field of rational functions of $\mathcal{C}$. Let $x$ and $y$ be elements of $\Sigma$ such that $\Sigma = \mathcal{L}(x, y)$. Following Seidenberg’s book approach to algebraic curves (see [15]) we
define a rational transformation \( \Phi \) of \( C \) in \( PG(2, \mathcal{L}) \) by choosing two elements \( x' \) and \( y' \) in \( \Sigma \):

\[
x' := \frac{y}{x^t-1}, \quad y' := \frac{y'^{-1}}{x^{t-2}}.
\]

The image \( C' \) of \( C \) by \( \Phi \) is a plane algebraic curve of genus 0; for \( x' \) and \( y' \) satisfy

\[
x'^2 + \epsilon_1 y'^2 + \epsilon_2 y' + c \epsilon_2 x' y' + \epsilon c^2 x^t y' = 0,
\]

so \( C' \) is a conic or a line. Let \( \Sigma' = \mathcal{L}(x', y') \).

**Lemma 3.7.** The degree \( [\Sigma : \Sigma'] \) of the extension \( \Sigma : \Sigma' \) is \( t^2 - 3t + 3 \) or \( 2(t^2 - 3t + 3) \).

**Proof.** Note that the only branches of \( C \) whose image by \( \Phi \) is centered at a point of the line \( y' = 0 \) are those centered at \( A_3 \). If \( C \) is of the first type, then at \( A_3 \) are centered two branches \( \gamma_1, \gamma_2 \) of \( C \) with \( \gamma_1 = (\tau, a_0 \tau^{\frac{3}{t}} - 1 + \ldots), \gamma_2 = (\tau, a'_0 \tau^{\frac{3}{t}} - 1 + \ldots) \) and \( a_0, a'_0 \neq 0 \); \( \gamma_1 \) and \( \gamma_2 \) are transformed by \( \Phi \) in \( \gamma'_1 \) and \( \gamma'_2 \), branches with (imprimitive) representations \((a_0 + \ldots), \tau^{t^2-3t+3} (a_0 + \ldots)^{-1}\) and \((a'_0 + \ldots), \tau^{t^2-3t+3} (a'_0 + \ldots)^{-1}\) respectively. Since \( \gamma'_1 \) and \( \gamma'_2 \) are branches of a conic or a line, their intersection multiplicity \( I \) with the line \( y' = 0 \) has to be 1 or 2; but \( t^2 - 3t + 3 \) is odd so \( I = 1 \) and the ramification index of \( \gamma_1 \) and \( \gamma_2 \) with respect to \( \Phi \) is \( t^2 - 3t + 3 \). Therefore \( [\Sigma : \Sigma'] = t^2 - 3t + 3 \) or \( [\Sigma : \Sigma'] = 2(t^2 - 3t + 3) \) according to whether \( \gamma'_1 \) and \( \gamma'_2 \) are distinct or not. If \( C \) is of the second type, then at \( A_3 \) is centered exactly one branch \( \gamma \) of \( C \) with \( \gamma = (\tau^2, b_0 \tau^{n-2} + \ldots), b_0 \neq 0 \). \( \gamma \) is transformed by \( \Phi \) in \( \gamma' \) whose (imprimitive) representation is of type \((a_0 + \ldots), \tau^{2(t^2-3t+3)} (a_0 + \ldots)^{-1}\); the ramification index of \( \gamma \) is equal to the degree of \( \Sigma : \Sigma' \) since \( \gamma \) is the only branch over \( \gamma' \); it can be \( 2(t^2 - 3t + 3) \) or \( t^2 - 3t + 3 \) as before and so we are done. \( \square \)

We will calculate the order of the different \( D \) of \( \Sigma : \Sigma' \). We recall that \( D \) is the divisor of \( \Sigma \)

\[
D := \sum \gamma D_{\gamma}
\]

where \( \gamma \) runs in the set of all branches of \( C \) and \( D_{\gamma} \) is defined as follows: if \((\gamma_1(\tau), \gamma_2(\tau)) \) is a primitive representation of \( \gamma \) and if \((\psi_1(z), \psi_2(z)) \) is a primitive representation of \( \gamma' \), the image of \( \gamma \) by \( \Phi \), with \( z = z(\tau) = \tau^2 + \ldots \) and \((\psi_1(z(\tau)), \psi_2(z(\tau))) = (\Phi(\gamma_1(\tau), \gamma_2(\tau))) \), then

\[
D_{\gamma} = \text{ord}_\tau \frac{dz}{d\tau}.
\]

**Lemma 3.8.** Let \( u \) be an element of \( \mathcal{L} \) such that \( u^{t^2-3t+3} = 1 \). Then the linear collineation of \( PG(2, \mathcal{L}) \) \( \eta \) defined by

\[
\eta : \begin{cases}
\rho x_1' = u x_1 \\
\rho x_2' = u^{-1} x_2, & \rho \in \mathcal{L}^*
\end{cases}
\]

preserves \( C \).

**Proof.** The proof is a simple computation. \( \square \)

**Lemma 3.9.** If \( k^2 - k + 1 \neq 0 \mod p \) then there exist exactly \( t^2 - 3t + 3 \) distinct elements \( u \) in \( \mathcal{L} \) such that \( u^{t^2-3t+3} = 1 \).
Proof. The polynomial $X^{t^2-3t+3} - 1$ is inseparable in $\mathcal{L}[X]$ since the characteristic $p$ of $\mathcal{L}$ does not divide $t^2 - 3t + 3$; for $t^2 - 3t + 3 = k^2 - k + 1 \mod p$. □

Proposition 3.10. If $k^2 - k + 1 \neq 0 \mod p$ then $[\Sigma : \Sigma'] = t^2 - 3t + 3$ and the order of the different $D$ of $\Sigma : \Sigma'$ is $6(t^2 - 3t + 2)$ or $3(t^2 - 3t + 2)$ according to whether $\mathcal{C}$ is of the first type or not.

Proof. Let $\gamma$ be a branch of $\mathcal{C}$ centered at a point $(x_0, y_0)$ not belonging to any fundamental line and let $\gamma'$ be its image by $\Phi$; $\gamma'$ is centered at $(x_1, y_1) := \left(\frac{y}{x^{t-1}}, \frac{y^{t-1}}{x^t}\right)$. For every $u$ in $\mathcal{L}$ such that $u^{t^2-3t+3} = 1$ the point $Q := (ux_0, u^{t-1}y_0)$ is a point of $\mathcal{C}$ and every branch centered at $Q$ has as image $\gamma'$, the only branch of $\mathcal{C}'$ centered at $(x_1, y_1)$. Moreover, it is easy to see that all the branches of $\mathcal{C}$ that lie over $\gamma'$ are centered at points of type $(ux_0, u^{t-1}y_0)$ with $u^{t^2-3t+3} = 1$. So the set of branches $\gamma'$ of $\mathcal{C}'$ such that there exist exactly $t^2 - 3t + 3$ distinct branches of $\mathcal{C}$ that lie over $\gamma'$ is infinite; for otherwise there would be an infinite set of singular points of $\mathcal{C}$. Suppose $[\Sigma : \Sigma'] = 2(t^2 - 3t + 3)$; then there exist an infinite set of branches $\gamma$ of $\mathcal{C}$ such that $D_\gamma$ is greater than 0, a contradiction. So by Lemma 3.7 the first part of the statement is proved. Now it is clear that if $\gamma$ is a branch of $\mathcal{C}$ centered at a point not belonging to any fundamental line then $D_\gamma = 0$; on the other hand, if $\gamma$ is centered at a vertex of the fundamental triangle, we have that (with notation as before Lemma 3.8) $z(\tau) = \tau^{t^2-3t+3} + \ldots$ so

$$\frac{dz}{d\tau} = (t^2 - 3t + 3)\tau^{t^2-3t+2} + \ldots;$$

since $p$ does not divide $t^2 - 3t + 3$ we have $\text{ord} \frac{dz}{d\tau} = t^2 - 3t + 2$ and we are done. □

We recall the statement of the Hurwitz Theorem.

Theorem 3.11. Let $\mathcal{G}$ be an irreducible algebraic curve over the algebraically closed field $\mathcal{L}$ and let $\mathcal{G}'$ be the image of $\mathcal{G}$ by a rational transformation. Let $\Sigma := \mathcal{L}(\mathcal{G})$ and $\Sigma' := \mathcal{L}(\mathcal{G}')$, $\Sigma' \subseteq \Sigma$. Let $D$ be the different of $\Sigma : \Sigma'$, $g$ be the genus of $\mathcal{G}$ and $g'$ be the genus of $\mathcal{G}'$. Then

$$(3.1) \quad 2g - 2 = [\Sigma : \Sigma'](2g' - 2) + \text{ord}(D).$$

Finally we prove the main result of the section.

Proof. (Theorem 3.1) We apply (3.1) and Proposition 3.10 so we have that if $\mathcal{C}$ is of the first type then

$$2g - 2 = (t^2 - 3t + 3)(-2) + 6(t^2 - 3t + 2) = 4t^2 - 12t + 6,$$

otherwise

$$2g - 2 = (t^2 - 3t + 3)(-2) + 3(t^2 - 3t + 2) = t^2 - 3t.$$
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Dipartimento di Matematica e Informatica Università degli Studi di Perugia, 06123 Perugia, Italy

E-mail address: pasticci@dipmat.unipg.it