A Theoretical Study on the Perception of Weight in a Moving Elevator

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Abstract:

The present study shows the time dependence of weight, as perceived by a man inside an elevator in motion. The sensation of this weight is caused by the upward force exerted by the elevator upon the man. A function has been empirically chosen to express the time dependence of velocity. Using this function, the upward force acting on the man has been calculated to determine thereby his apparent weight. Fractional change of weight, perceived by the man, has been estimated. The ratio of the apparent weight to the actual weight has been determined. For the upward and downward motions of the elevator, the present article graphically shows the time dependence of its speed, acceleration, ratio of the apparent to actual weight and the fractional change of weight, for different values of a parameter that controls the sharpness of the velocity-versus-time curve. To quantify the degree of comfort that one gets, while riding in an elevator, a parameter called comfort factor has been defined. It has been defined in two different ways, to show clearly the aspects that reduce or enhance the level of comfort in such a journey. A major purpose of this study is to show the science students, the time evolution of what we perceive as our weight inside an elevator, using very simple mathematics, going far beyond the qualitative or semi-quantitative discussions of the school-level textbooks for senior students.

Keywords: Weight, Apparent Weight, Weightlessness, Elevator, Physics Education

1. Introduction:

The weight of an object is the force of earth’s gravitational attraction acting on it. The magnitude of this force is actually the product of its mass and the free-fall acceleration (also known as the acceleration due to gravity), denoted by $g$. The direction of this force is always downward, i.e., in a direction towards the centre of the earth. The value of $g$ depends upon the location of the object with respect to the earth, in accordance with Newton’s law of gravitation. On the surface of the earth, we have $g = 9.78 \, m/s^2$ near the equator and we have $= 9.83 \, m/s^2$ near the poles. At the latitude of $45^0$ we have $g = 9.80 \, m/s^2$ [1-5].

An object, lying at rest on a table, is acted by an upward force that is equal and opposite to its weight. If it has to be accelerated upward, an upward force, having a magnitude greater than its weight, should be applied upon it. The weight of an object is proportional to its mass for locations having the same values of $g$. The bodyweight of a man, as perceived by himself, can be different from his actual weight ($mg$). Inside an elevator that is at rest, a man experiences an upward force exerted by the floor, which is equal and opposite to his weight.
The apparent weight, in this case, is equal to the actual weight. In an elevator that is accelerating upward (or decelerating downward), a man feels heavier than usual. He feels lighter in an elevator that is accelerating downward (or decelerating upward). All these perceptions depend upon the force exerted by the floor of the elevator upon the man. If an elevator is allowed to fall with an acceleration which is equal to \( g \), no force will be acting on the man and he will perceive perfect weightlessness. A rocket, carrying a spacecraft to be put into its orbit round the earth, moves with an upward acceleration which is typically of the order of \( 15g \). An astronaut, strapped to a couch in that spacecraft, would experience a net force of \( 15mg \) in the upward direction. This net upward force is actually the vector sum of the upward force exerted by the spacecraft and the downward pull (\( mg \)) exerted by the earth on the astronaut. Therefore, the upward force exerted by the spacecraft on the astronaut is \( 16mg \), which is 16 times the weight of the astronaut. Thus, the apparent weight, perceived by the astronaut, is extremely large in this case. An astronaut, in a spacecraft orbiting round the earth, experiences weightlessness. In that case, the spacecraft and the astronaut have the same acceleration \( g \), directed towards the centre of the earth, at each point on the orbit. It is not that the astronaut is weightless. Our sensation or perception of weight depends upon the upward force exerted by the floor on us. All these discussions are based on the valuable materials obtained from some standard textbooks of physics [1-5]. There are some excellent textbooks and research articles that have detailed information, demonstration and interpretation of the matters connected to weight and weightlessness [6, 7].

In the present article, an attempt has been made to find theoretically the time dependence of the apparent weight of a man inside an elevator. As the elevator moves from one floor to the next, the sensation or perception of weight changes with time, depending upon the change in the upward force exerted by the floor. For this purpose, we have empirically chosen a half cycle of a sinusoidal function of time to represent the time dependence of elevator speed. This function has been used to obtain the upward force exerted by the elevator upon the man, which is his apparent weight. Using its expression, fractional change of weight (\( FCW \)) has been calculated. The time dependence of the velocity, acceleration, ratio of apparent to actual weight and \( FCW \) has been shown graphically, for the upward and downward motions of the elevator. To determine theoretically, the degree of comfort that one gets, while travelling in an elevator, we have proposed to calculate a parameter, called comfort factor (\( CF \)), in two different ways. We have thereby identified the parameters that determine the level of comfort in such a journey, quantified by \( CF \).

One of the objectives of the present study is to explore the concept of apparent weight, in an accelerating elevator, with a greater mathematical rigour than the qualitative or semi-quantitative discussions that one finds in several textbooks read by senior school students.

2. Theoretical Analysis:

Consider an elevator that is moving upward with an acceleration of magnitude \( a \). A man of mass \( m \) is standing inside the elevator. The magnitude of the downward force, acting on the man due to earth’s gravitational field, is \( mg \) where \( g \) stands for the acceleration due to gravity (free-fall acceleration). Taking \( F \) to be the magnitude of the upward force exerted by the floor of the elevator, the magnitude of the net upward force acting on the man would be \( F - mg \). Due to this force, if the magnitude of the upward acceleration of the man is \( a \), which is the same as the acceleration of the elevator itself, we can write,

\[
ma = F - mg
\]
According to equation (1), we have \( F = m(a + g) \), which is actually the magnitude of the upward force applied by the elevator upon the man, when its upward acceleration magnitude is \( a \). This force is perceived by the man as his weight during his stay inside the elevator. This apparent weight depends on \( a \), which can have both positive and negative values depending upon the direction of acceleration. When the elevator is stationary, we have \( F = mg \), implying that the apparent weight is equal to the actual weight of the man. As the elevator just starts moving upward, it gets suddenly accelerated, causing the apparent weight to increase. Just before coming to a halt, the elevator gets decelerated, causing \( a \) to have a negative value, leading to a magnitude of the apparent weight \( F \) that is smaller than the actual weight \( (mg) \). During its downward motion, as the elevator starts moving from rest, its acceleration is downward, implying a negative value for \( a \). It causes the apparent weight to be smaller than the actual weight. Just before coming to a halt, it has a downward deceleration, leading to a positive value of \( a \), causing the apparent weight to be greater than the actual weight. If the elevator is allowed to fall freely under gravity, we have \( a = -g \), causing \( F \) to be zero which means complete weightlessness for the man.

The vector, representing apparent weight (i.e. the force exerted by the elevator upon the man), can be expressed as,

\[
\vec{F} = m \frac{d\vec{v}}{dt} + mg \hat{k} \tag{2}
\]

Here \( \vec{v} \) is the velocity of the elevator and \( \hat{k} \) is the unit vector in the vertically upward direction. The acceleration \( \frac{d\vec{v}}{dt} \) can be both parallel and anti-parallel to \( \hat{k} \). In equation (1), \( a \) and \( F \) are respectively the magnitudes of the vectors \( \frac{d\vec{v}}{dt} \) and \( \vec{F} \).

For the present study we choose the following expression for the velocity \( (\vec{v}) \) of the elevator.

\[
\vec{v} = \epsilon f(t) \hat{k} \tag{3}
\]

Here \( \epsilon \) is a constant, whose values are \(+1\) and \(-1\) respectively, for the vertically upward and downward motions of the elevator. The time dependence of the magnitude of \( \vec{v} \) is governed by the function \( f(t) \) where \( f(t) > 0 \) for all values of \( t \). Substituting for \( \vec{v} \) in equation (2), one obtains,

\[
\vec{F} = m \left( \epsilon \frac{df(t)}{dt} + g \right) \hat{k} \tag{4}
\]

Here \( \vec{F} = |\vec{F}| \hat{k} = F\hat{k} \), where \( F \) is the magnitude of the upward force exerted by the elevator upon the man. From equation (4), the apparent weight is therefore given by,

\[
F = m \left( \epsilon \frac{df(t)}{dt} + g \right) \tag{5}
\]

The fractional change of weight \( (FCW) \) can be expressed as,

\[
FCW = \frac{\text{apparent weight} - \text{actual weight}}{\text{actual weight}} = \frac{F - mg}{mg} \tag{6}
\]

Using equation (5) in equation (6), one gets,
The sign of this \( FCW \) depends upon the signs of \( \varepsilon \) and \( \frac{df(t)}{dt} \). When both of them have the same sign, \( FCW \) is positive, indicating an increase of weight, as perceived by the man inside the elevator. When \( \varepsilon \) and \( \frac{df(t)}{dt} \) have opposite signs, \( FCW \) becomes negative, indicating a fall in weight, as perceived by the man in the elevator.

The motion of an elevator, from one floor to another, is such that the function \( f(t) \) must be zero at the beginning and the end of this journey. Let \( T \) be the time that elapses during this travel from one point of zero speed to another. Keeping these facts under consideration, we propose the following functional form for \( f(t) \).

\[
f(t) = A \left\{ \sin \left( \frac{\pi}{T} t \right) \right\}^n
\]  

(8)

Here, \( A \) and \( n \) are constants. To satisfy the requirements pertaining to equation (3) and also regarding the zeroes of \( f(t) \), we must have \( A > 0 \) and \( n > 0 \). This function \( f(t) \) is defined over the domain of \( 0 \leq t \leq T \), causing \( f(t) \) to vary over the range of \( 0 \leq f(t) \leq A \).

Using equation (8) in equation (3), one obtains,

\[
\vec{v}' = \varepsilon A \left\{ \sin \left( \frac{\pi}{T} t \right) \right\}^n \hat{k}
\]  

(9)

In equation (9), the constant \( A \) is a measure of the maximum velocity of the elevator since \( |\varepsilon| = 1 \) and \( \sin \left( \frac{\pi}{T} t \right) \leq 1 \) over the domain of \( 0 \leq t \leq T \). For plotting the results graphically, in the later part of this theoretical study, we have taken \( A = 1 \text{m/sec} \). The parameter \( n \) determines the nature of change of \( \vec{v}' \) with time. Generally, an elevator is found to have an accelerated motion only at the beginning and at the end of its journey from one floor to another. The smaller the value of \( n \), closer would be the time variation of the above function (\( \vec{v}' \)) to this observed behaviour. For the present study we have used three fractional values of \( n \), which are 0.05, 0.1 and 0.5, for the graphical depiction of the theoretical findings.

The acceleration of the elevator, obtained from equation (9), is given by,

\[
\vec{a} = \frac{d\vec{v}}{dt} = \varepsilon \frac{nA\pi}{T} \cos \left( \frac{\pi}{T} t \right) \left\{ \sin \left( \frac{\pi}{T} t \right) \right\}^{n-1} \hat{k}
\]  

(10)

From equation (10), an approximate estimate of the magnitude of the acceleration would be a fraction of \( \frac{nA\pi}{T} \). As per equation (9), \( A \) is a measure of the maximum velocity. The elevator has to attain this velocity in an interval of time \( T/2 \), since the velocity varies symmetrically about the instant of \( t = T/2 \), according to equation (9). Thus \( A/T \), having the dimension of acceleration, can be regarded as an estimate of the acceleration of the elevator.

Using equations (4) and (8), the force exerted by the elevator upon the man is given by,

\[
\vec{F} = m \left[ \varepsilon \frac{nA\pi}{T} \cos \left( \frac{\pi}{T} t \right) \left\{ \sin \left( \frac{\pi}{T} t \right) \right\}^{n-1} + g \right] \hat{k}
\]  

(11)
Using equation (8) in equation (7), one obtains the following expression for the fractional change of weight (FCW).

\[
FCW = \frac{e^{\frac{n \pi \epsilon}{g} \cdot c \left( t \right) \left( \sin \left( \frac{\pi}{T} t \right) \right)^n}}{g \cdot c \left( t \right)}^{n-1}
\]  

(12)

We propose to define a ratio (R) of the apparent weight (F) to the actual weight (mg) of the man inside the elevator. Using equation (5), it can be expressed as,

\[
R = \frac{F}{mg} = \frac{e^{\frac{d \left( \epsilon \right)}{dt}}}{g \cdot c \left( t \right)} + 1
\]  

(13)

Thus, R depends on the ratio of the acceleration of elevator \(e^{\frac{d \left( \epsilon \right)}{dt}}\) to the acceleration due to gravity or the free-fall acceleration (g).

Using equation (8) in equation (13), one gets,

\[
R = \frac{e^{\frac{n \pi \epsilon}{g} \cdot c \left( t \right) \left( \sin \left( \frac{\pi}{T} t \right) \right)^n}}{g \cdot c \left( t \right)}^{n-1} + 1
\]  

(14)

The vertical distance (y) travelled by the elevator in time t can be obtained from the solution of the following differential equation, which is based on equation (9).

\[
\frac{dy}{dt} = \epsilon \cdot A \cdot \left( \sin \left( \frac{\pi}{T} t \right) \right)^n
\]  

(15)

Here T is a fixed interval of time that elapses during a continuous journey from one floor to another. Therefore, the vertical distance travelled by the elevator, during this interval, must be equal to the vertical separation between two such floors of the building. Taking this distance to be L we can write,

\[
\left| \int_0^T \epsilon \cdot A \left( \sin \left( \frac{\pi}{T} t \right) \right)^n dt \right| = L
\]  

(16)

Equation (16) is obtained from equation (15) by integrating \(\frac{dy}{dt}\) and equating its absolute value or modulus to the distance (L) between two positions of zero speed. Since \(|\epsilon| = 1\), we have,

\[
A = L \left[ \int_0^T \left( \sin \left( \frac{\pi}{T} t \right) \right)^n dt \right]^{-1}
\]  

(17)

Evaluating the integral of equation (17) we write,

\[
A = \frac{L \sqrt{\pi} \cdot \Gamma \left( \frac{\epsilon + n}{2} \right)}{\Gamma \left( \frac{\epsilon + n}{2} \right)}
\]  

(18)

In equation (18), \(L \cdot \frac{1}{T}\) can be regarded as a measure of the average velocity magnitude of the elevator where A is its maximum speed according to equation (9). This equation serves as a relation between the parameters A, L and T. If any two of them are regarded as independent, the third one can be calculated from them, for a certain value of the parameter n.

Using equation (18) in equation (9), one gets the following expression for the velocity.
Using equation (18) in equation (12), one gets the following expression for $FCW$.

\[ FCW = \frac{\varepsilon \pi L}{T^2} \frac{\sqrt{T}}{L} \frac{\Gamma(\frac{2+n}{2})}{\Gamma(\frac{1+n}{2})} \cos \left( \frac{\pi}{T} t \right) \left\{ \sin \left( \frac{\pi}{T} t \right) \right\}^{n-1} \]  

(20)

Equation (20) shows that a small decrease in $T$ (for a fixed $L$) causes a large increase in the value of $FCW$. Taking $\frac{\varepsilon}{L}$ to be a measure of the average velocity magnitude, one may write,

\[ FCW = \frac{\varepsilon \pi L}{g} v_{av}^2 \frac{\sqrt{T}}{L} \frac{\Gamma(\frac{2+n}{2})}{\Gamma(\frac{1+n}{2})} \cos \left( \frac{\pi}{T} t \right) \left\{ \sin \left( \frac{\pi}{T} t \right) \right\}^{n-1} \]  

(21)

It is clearly evident from equation (21) that the fractional change of weight ($FCW$) is proportional to the square of the average velocity magnitude ($v_{av}$) of the elevator. A slight rise in $v_{av}$ (for a fixed $L$) would cause a large increase in the value of $FCW$.

When the elevator goes up more than one floor at a stretch, the time variation of $FCW$ can be obtained from equation (21), by adjusting the value of $L$, which is actually the distance between the initial and the final positions of the elevator, the velocity being zero only at these two positions. For a fixed value of $v_{av}$, larger values of $L$ causes a reduction in $FCW$ at any time $t$. It implies that our perception of feeling lighter or heavier will be less pronounced for longer vertical paths, traversed with the same average velocity.

The ratio of two gamma functions, i.e. $\frac{\Gamma(\frac{2+n}{2})}{\Gamma(\frac{1+n}{2})}$, in equation (21), is found to increase with the parameter $n$, resulting in a larger value of $FCW$. According to equation (9), larger values of $n$ ensure longer durations of accelerated motion of the elevator, which is also evident from the plots of the time variation of velocity in Figure 1. For $n = 0.05, 0.10, 0.50, 1.00$, this ratio of gamma functions becomes $0.5835, 0.6024, 0.7397$ and $\sqrt{\pi}$ respectively. In most elevators, the accelerated motion takes place over a very short duration, close to the beginning and the end of its journey from one floor to the next. Change of weight is perceived only during these short periods.

The level of comfort that one gets in the entire journey, from one floor to the next, may be expected to depend upon the time average of the change of weight that one experiences during the motion of the elevator. The smaller the value of this quantity, greater will be the comfort of the passenger. Based on this consideration, we propose to define a parameter, called Comfort Factor ($CF$), in the following way.

\[ CF = \left[ \frac{1}{T} \int_0^T |F - mg| dt \right]^{-1} \]  

(22)

Instead of using $|F - mg|$, if we choose $(F - mg)$ as the integrand here, the result of integration will be zero, since, $\int_0^{T/2} (F - mg) dt = -\int_{T/2}^T (F - mg) dt$. The reason is, during the upward (downward) journey, we have accelerated (decelerated) motion in the first half and decelerated (accelerated) motion in the second half, in the upward direction. By definition, this parameter $CF$ is the reciprocal of the time average of the change of weight $(F - mg)$ perceived by the man inside the elevator.
Using equation (13), i.e. \( R = F/mg \), in equation (22), one gets,

\[
\frac{CF}{2m} = \left[ \frac{1}{T} \int_0^T mg |R - 1| \, dt \right]^{-1} \tag{23}
\]

Based on the property of \( R \), as evident from the Figures 5 and 6, equation (23) can be expressed as,

\[
\frac{CF}{2m} = \left[ \frac{1}{T/2} \int_0^{T/2} mg (R - 1) \, dt \right]^{-1} \tag{24}
\]

Using equation (14) in equation (24), one gets,

\[
\frac{CF}{2m} = \left[ \frac{2m \Delta n}{T^2} \int_0^{T/2} \cos \left( \frac{\pi}{T} t \right) \left( \sin \left( \frac{\pi}{T} t \right) \right)^{n-1} \, dt \right]^{-1} \tag{25}
\]

In obtaining equation (25), we have taken \( \epsilon \) to be unity, since we are interested in the absolute value of \( CF \), which would be same for upward and downward journeys, as evident from the figures showing the time dependence of \( R \).

The result of the integration, in equation (25), is \( T/n\pi \). Using this result in equation (25), one gets,

\[
\frac{CF}{2m} = \frac{T}{2mA} = \frac{1}{m} \frac{1}{A/(T/2)} = \frac{1}{m} \frac{1}{v_{max}/(T/2)} = \frac{1}{m} \frac{1}{a_{av}} \tag{26}
\]

Here we have, \( a_{av} = \frac{1}{T/2} \int_0^{T/2} |a| \, dt = \frac{A}{T/2} \), from equation (10), with \( A = v_{max} \) from equation (9). It shows that, the comfort in the journey, inside an elevator, is proportional to \( T/2mA \). The value of Comfort Factor (\( CF \)) increases as the product of \( m \) and \( a_{av} \) decreases.

For the formulation of equation (26), we have used \( A \) and \( T \) as independent parameters, where \( L \) would be dependent upon them, according to equation (18). By using \( L \) and \( T \) as independent parameters, one would get \( CF \) as a function of them. Combining equation (18) with (26), for this purpose, we get the following expression for \( CF \).

\[
\frac{CF}{2mL/\sqrt{\pi}} = \frac{\Gamma \left( \frac{1+n}{2} \right)}{\Gamma \left( \frac{2+n}{2} \right)} \tag{27}
\]

The ratio of gamma functions, in equation (27), increases as \( n \) decreases. Thus we see that, for an elevator, which is travelling a vertical distance \( L \) in an interval of time \( T \), the comfort of the ride is greater for smaller values of the parameter \( n \). As evident from Figures 1 and 2, smaller values of \( n \) mean longer duration of a nearly uniform velocity for the elevator.

Using equation (9), the time average of velocity magnitude comes out to be,

\[
\frac{1}{T} \int_0^T |v| \, dt = \frac{A}{\sqrt{\pi}} \frac{\Gamma \left( \frac{1+n}{2} \right)}{\Gamma \left( \frac{2+n}{2} \right)} \tag{28}
\]

Thus, average velocity increases as \( n \) decreases, for a fixed value of \( A \) (maximum velocity). Substituting for \( A \) from equation (18), in equation (28), one gets,
We have already used this expression of \( v_{av} \) in equation (21). Combining equation (29) with (27), we get the following expression for \( CF \).

\[
CF = \frac{L}{2m v_{av}^2 \sqrt{\pi} \left( \frac{1+n}{2} \right)}
\]

(30)

According to equation (30), \( CF \) is proportional to the ratio of \( L/v_{av}^2 \).

From a practical point of view one can say that, the faster it attains the uniform velocity, the greater would be the comfort level in the elevator. Figures 1, 2 suggest that, by decreasing \( n \), we can increase the span of time in the middle of the journey, where the acceleration is much smaller than that near the ends. One can determine the time required to attain a velocity that is half its maximum velocity. Let us denote this interval by \( T_{1/2} \). The maximum velocity magnitude, as per equation (9), is \( A \). Thus, from equation (9), one gets,

\[
\frac{T_{1/2}}{T} = \frac{1}{\pi} \left[ \sin^{-1} \left( \left( \frac{1}{2} \right)^{1/n} \right) \right]
\]

(31)

As the velocity gets closer to its maximum value, the acceleration would be smaller. Therefore, the smaller the value of \( \frac{T_{1/2}}{T} \), greater would be the degree of comfort of a passenger in an elevator. We propose to define a new comfort factor \( (CF_n) \), which would be the reciprocal of \( T_{1/2}/T \). Its expression, based on equation (31), is given by,

\[
CF_n = \pi \left[ \sin^{-1} \left( \left( \frac{1}{2} \right)^{1/n} \right) \right]^{-1}
\]

(32)

In equation (32), the value of \( CF_n \) is found to increase as \( n \) decreases. For \( n = 0.05, 0.10, 0.50, 1.00 \), the values of \( CF_n \) are found to be 3294198.66, 3216.99, 12.43 and 6.00 respectively.

3. Graphical Depiction of Theoretical Findings:

Figure 1 shows the time variation of the velocity of the elevator during its upward motion, for three values of the parameter \( n \). As \( n \) decreases, the sharpness of the curve decreases. Greater flatness means greater resemblance to the actual situation where an accelerated motion is observed only near the start and the end of the journey of an elevator.

Figure 2 shows the time variation of the velocity of the elevator during its downward motion, for three values of the parameter \( n \). The values are negative due to downward motion. As \( n \) decreases, the flatness of the curve increases, making it closer the real situation where an elevator has an almost uniform motion except at the beginning and the end of its journey.

Figure 3 shows the variation of acceleration as a function of time, during the upward motion of the elevator for three values of the parameter \( n \). Starting from a positive value it decreases gradually to a negative one. Its values are closer to zero for smaller values of \( n \), indicating the
velocity to be (nearly) uniform for a longer duration of time. The acceleration is positive during the first half of the journey and it is negative in the second half. This is due to the fact that the elevator has an accelerated motion in the first half and a decelerated motion in the second half, in the upward direction.

Figure 4 shows the variation of acceleration as a function of time, during the downward motion of the elevator for three values of the parameter $n$. Starting from a negative value it increases gradually to a positive one. Its values are closer to zero for smaller values of $n$, implying a nearly uniform velocity for a longer duration. The acceleration is negative during the first half of the journey and it is positive in the second half. This is due to the fact that the elevator has an accelerated motion in the first half and a decelerated motion in the second half, in the downward direction.

Figure 5 shows the time variation of the ratio ($R$) of apparent weight to actual weight of the man during the upward motion of the elevator for three values of the parameter $n$. Its value is found to decrease with time gradually. For smaller values of $n$, its curve remains closer to the value of $R = 1$. For the first half of the journey, we have $R > 1$ and, for the second half we have $R < 1$, with $R = 1$ exactly at $t = T/2$. The passenger perceives greater weight than actual in the first half and he feels smaller weight than actual in the second half of the trip. In the first and the second halves, the elevator has accelerated and decelerated motions, respectively, in the upward direction. It causes the apparent weight to be respectively greater and smaller than the actual weight in these two halves.

Figure 6 shows the time variation of the ratio ($R$) of apparent weight to actual weight of the man during the downward motion of the elevator for three values of the parameter $n$. Its value is found to increase with time gradually. For smaller values of $n$, the values of $R$ remain closer to unity. For the first half of the journey, we have $R < 1$ and, for the second half we have $R > 1$, with $R = 1$ exactly at $t = T/2$. The man inside the elevator feels lighter than usual in the first half and he feels heavier than usual in the second half of the trip. In the first and the second halves, the elevator has accelerated and decelerated motions, respectively, in the downward direction. It causes the apparent weight to be respectively smaller and larger than the actual weight in these two halves.

Figure 7 shows the time evolution of the fractional change of weight ($FCW$) during the upward motion of the elevator for three values of the parameter $n$. Its values are closer to zero for smaller values of $n$. For each value of $n$, it decreases with time, remaining positive in the first half and negative in the second half of the journey. This is due to the fact that the elevator has an accelerated motion in the first half and a decelerated motion in the second half, in the vertically upward direction. Multiplying the vertical scale of this plot by hundred, one can get the percentage of change of the weight perceived by the passenger. Its positive value indicates greater apparent weight than the actual weight and its negative value indicates smaller apparent weight than the actual weight.

Figure 8 shows the time evolution of the fractional change of weight ($FCW$) for the downward motion, for three values of the parameter $n$. Its values are closer to zero value for smaller values of $n$. For each value of $n$, it increases with time, remaining negative and positive respectively in the first and the second halves of the journey. Multiplying the vertical scale of this plot by hundred, one can get the percentage of change of the weight perceived by the man. The positive and negative values of $FCW$ correspond to the situations where one perceives a bodyweight, greater and smaller, respectively, than its actual value ($mg$).
FIGURES

Figure 1: Time dependence of velocity during the upward motion of the elevator for three values of the parameter $n$. Unit of $A$ is m/sec.

Figure 2: Time dependence of velocity during the downward motion of the elevator for three values of the parameter $n$. Unit of $A$ is m/sec.

Figure 3: Time dependence of acceleration during the upward motion of the elevator for three values of the parameter $n$. Units of $A$ and $T$ are m/sec and sec, respectively.

Figure 4: Time dependence of acceleration during the downward motion of the elevator for three values of the parameter $n$. Units of $A$ and $T$ are m/sec and sec, respectively.
FIGURES

Figure 5: Time dependence of the ratio of apparent weight to actual weight during the upward motion of the elevator for three values of the parameter n. Units of A and T are m/sec and sec, respectively.

Figure 6: Time dependence of the ratio of apparent weight to actual weight during the downward motion of the elevator for three values of the parameter n. Units of A and T are m/sec and sec, respectively.

Figure 7: Time dependence of the fractional change of weight (FCW) during the upward motion of the elevator for three values of the parameter n. Units of A and T are m/sec and sec, respectively.

Figure 8: Time dependence of the fractional change of weight (FCW) during the downward motion of the elevator for three values of the parameter n. Units of A and T are m/sec and sec, respectively.
4. Concluding Remarks:

One of the purposes of the present study is to demonstrate theoretically the fact that one perceives a greater or smaller bodyweight while travelling in an elevator, during its non-uniform motion. In the present article, the time variation of our sensation of weight, during a ride in an elevator, has been explored theoretically and we have tried to quantify the ease or comfort with which one makes this journey. To achieve the maximum comfort, one must ideally minimize the duration of its non-uniform motion, irrespective of the vertical distance traversed by the elevator. It is undoubtedly a difficult thing to accomplish in reality. In the present study, we have not explored the technological challenge involved in it.

The entire study depends on the functional form of \( f(t) \), given by equation (8). It governs the time dependence of the velocity of the elevator, as given by equation (9). The parameter \( n \) controls the sharpness of its time variation, as shown by Figures 1 and 2. Experience of riding in a modern elevator tells us that the velocity versus time curve must be sufficiently flat, except at the beginning and the end of the journey, where it is likely to have a sharp rise and fall respectively. This flatness can be increased here by decreasing the value of \( n \), which is clearly evident from the plots of Figures 1 and 2. A limitation of the present model is that the velocity is never actually uniform, due to the sinusoidal nature of time dependence, as shown by equation (9). An alternative functional form of \( f(t) \) can be expressed as, \( f = A/4(1 + \tanh \gamma t)[1 + \tanh \gamma(T - t)] \) which gives an almost rectangular pulse of height \( A \) and width \( T \) (duration of the pulse) with \( f = 0 \) at \( t \leq 0 \) and \( t \geq T \). The length of its flat region increases by increasing the value of the parameter \( \gamma \). Time dependence of the apparent weight perceived by a man inside an elevator can be determined using this function, which would perhaps be a part of a future project to explore this field.

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