Simulation of the effect of secondary Doppler modulation for air targets with a turbojet engine

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Abstract. The paper analyzes various approaches to modeling the effect of secondary Doppler modulation for turbojet engines. The issue of studying the conditions for the appearance of the effect of secondary Doppler modulation of the sounding signal of onboard radar stations for the further formation of the base of standards for the detection and recognition of air targets with reduced radar signature is considered. The results of modeling this effect based on ray approximation in the MATLAB environment are presented using the example of the situation of reflection of an electromagnetic wave from the blades of the first stage of a low-pressure compressor of a hypothetical turbojet engine. The stages of modeling the process of propagation of an electromagnetic wave from a radar station to a target and back, as well as "collecting" rays in the receiver antenna and forming a reflected signal with subsequent spectral analysis are described.

Effective scattering area (area) (ESA) the air intake and nozzle of the turbojet engine makes a significant contribution to the total aircraft in the front and rear hemisphere, accordingly, which largely determines the probability of its detection [1].

When the aircraft is irradiated with turbojet engine (Turbojet engine) into the front (rear) hemisphere, the effect of secondary Doppler modulation (SDM) occurs, which consists in transforming the time-frequency structure of the sounding signal due to reflection from the rotating blades of the first stages of the low-pressure compressor (the last stages of the turbine) [2].

The use of this effect makes it possible to carry out the procedure for recognizing various types of air targets at a distance of no more than 65% of the maximum by taking into account the features of the spectrum of the reflected signal for their aircraft power plants [3].

The recognition method based on this effect is more applicable in tracking and guidance radars, since it requires an increase in the duration of the observation interval (up to 50-100 milliseconds at a repetition rate of sounding pulses of more than 5 kHz).

It should be noted that the operation the same power plants on different types of targets, as well as several modes of operation (rotational speeds), inevitably leads to a decrease in the quality of recognition [4].
The absence of real SDM spectra for various types of targets leads to the need for simulation of the indicated effect to obtain their calculated values and subsequent use as a data bank.

Each stage of the low-pressure compressor (LPC) of the turbojet engine has an impeller rotating on the rotor axis at a given frequency and consisting of a certain number of blades.

There is an approximate formula for calculating the SDM harmonics based on the known impeller rotation frequency and the number of blades [2]:

\[ f_{sd} = kF_{rot}N_b, \]  

(1)

where

- \( k = 0, \pm 1, \pm 2 \ldots \) – harmonic number;
- \( F_{rot} \) – impeller speed;
- \( N_b \) – number of blade.

Having set the rotor speed at 7468 rpm and the number of rotor blades 18, we obtain the following form of the SDM spectrum. The amplitudes of the harmonics are set arbitrarily.

In the case of a two-stage rotor, the SDM spectrum, in addition to the harmonics caused by each of the impellers, contains their combinations [2]:

\[ f_{sd} = (nN_{b1} + mN_{b2})F_{rot}, \]  

(2)

where

- \( n, m = 0, \pm 1, \pm 2, \ldots \);
- \( F_{rot} \) – impeller speed;
- \( N_{b1}, N_{b2} \) – the number of blades of the first and second stages, respectively.

In view of the rapid attenuation of electromagnetic waves (EMW) in the rotor stages, taking into account the subsequent stages impractical.

The indicated formulas (1–2) are suitable only for evaluating the calculated values of the SDM spectrum, since they take into account only the number of blades and their rotation frequency.

Chen’s formula for synthesizing a complex reflected signal in the case of a single-stage rotor is known [5]:

\[ \hat{s}(t) = L \exp \left(-j \frac{4\pi R_0}{\lambda} \right) \sum_{k=1}^{N_b} \text{sinc}(\Phi_k(t)) \exp[-j\Phi_k(t)], \]  

(3)

where

- \( L \) – blade length;
- \( \lambda \) – radiation wavelength;
- \( R_0 \) – initial range from the radar to the center of the impeller;
- \( \beta \) – impeller elevation angle;
- \( k = 1, N_b \) – blade number;
- \( N_b \) – number of blade;
- \( \text{sinc}(x) \) – sign function;
- \( \Phi_k(t) = \frac{4\pi L}{\lambda} \cos\beta \cos \left(2\pi F_{rot}t + \varphi_0 + k \frac{2\pi}{N} \right) \) – signal phase;
- \( F_{rot} \) – vane rotation frequency;
- \( \varphi_0 = \frac{4\pi R_0}{\lambda} \) – initial phase.

Figure 1 shows the SDM spectrum calculated by formula (3) with the following initial data:

- the number of blades is 18;
- blade length 0.15 m;
- radiation carrier frequency 10 GHz (wavelength 3 cm);
- the elevation angle of the impeller is 0 degrees;
- impeller rotation frequency 7468 rpm;
- the initial range from the radar to the center of the impeller is 100 km.
This approach does not take into account the reflective properties of the materials from which the blades are made and other elements of the aircraft power plant.

For a pulsed sounding signal with a sufficiently high repetition rate, so that the spectrum of the reflected signal is free from frequency ambiguities, (or for continuous unmodulated radiation), the signal reflected from the compressor impeller is a superposition of reflections from the axis of the rotation rotor and the blades of the first stage of the LPC. view [6]:

\[
\hat{u}(t) = \left[ \hat{U}_k + \sum_{n=0}^{N-1} U_b \text{sinc}\left\{ m \cos\left( 2\pi F_{\text{rot}} (t - nT_p) + \beta_0 \right) \right\} \right] e^{j(2\pi f_0 t + 2\pi f_d t + \phi_0)},
\]

where \( \hat{U}_k \) – the complex amplitude of the signal reflected from the motor housing; \( f_0 \) – carrier frequency of radiation; \( r \) – rotor radius; \( n=0,1\ldots N-1 \) – blade number; \( F_{\text{rot}} \) – blade rotation frequency; \( T_p = 1/NF_{\text{rot}} \) – period of "pulsations"; \( L \) – blade length; \( \beta_0 \) – the starting angle of the zero blade; \( \alpha \) – the angle of incidence of the electromagnetic wave relative to the normal to the plane of rotation; \( U_b = B L d \) – the amplitude of the signal reflected from the blades; \( B \) – the amplitude of the distribution of the electromagnetic field over the surface of the blades; \( d \) - blade width; \( m=2\pi L/\lambda \) – the reciprocal of the "ripple" width; \( \lambda \) – radiation wavelength; \( \phi_0 = 4\pi R_0 / \lambda \) - initial phase; \( f_d = 2V_r / \lambda \) – Doppler frequency shift; \( R_0 \) – initial range from the radar to the center of the impeller; \( V_r \) – radial approach speed; \( \text{sinc}(x) \) – sign function.

When deriving the expression, an assumption was made about the uniform distribution of electromagnetic field over the surface of the blades. Figure 2 shows the SDM spectrum for a single-stage rotor. The following values were used in the calculation:
carrier frequency 10 GHz (wavelength 3 cm);
angle of irradiation 70 degrees;
the initial angle of the zero blade is 45 degrees;
blade length 0.39 m;
rotor radius 0.59 m;
the number of blades is 18;
blade rotation speed 7468 rpm.

The prevalence of the components of the lower lateral spectrum over the components of the upper lateral spectrum is due to the observation angle from the side of the "outgoing" blades, that is, having a radial component of velocity relative to the airframe in the direction opposite to the radar.

The reflected signal passing through the two compressor stages is represented as follows:

\[
\dot{u}(t) = \dot{u}_1(t) + \dot{u}_2(t) + \dot{u}_1(t)\dot{u}_2(t),
\]

(5)

Where \(\dot{u}_1(t)\) and \(\dot{u}_2(t)\) – the reflected signal from the first and second stages of the LPC, respectively, calculated by the formula (5).

The presented mathematical model of the reflected signal makes it possible to take into account the influence of the second stage of the LPC, but does not take into account, like the above models, reflections and the shape of the blades.

To improve the adequacy of the above models of the formation of the reflected signal, it is proposed to use a facet model with a hypothetical shape of swirling blades, taking into account their orientation relative to the incident beam.

The main advantages of the method based on the approximation of the surface of an object with facets (elementary areas such as a triangle, square, etc.) are:

- no restrictions on the geometry of the object;
- the possibility of detailed accounting of phase relationships when calculating radar characteristics.

At present, it is the facet method of representing the geometry of an object that is most widely used. The main disadvantages include the computational complexity of algorithms for determining
illuminated (shaded) facets, as well as a large number of elementary areas required for surface modeling [7]. The latter is solved by reducing their number on relatively flat surfaces (wing, plumage, etc.).

Simulation of the processes of propagation of an electromagnetic wave (EMW) from the radar to the target and back was carried out by the method of geometric optics (ray approximation) [8].

The essence of the ray approximation is to represent the front of a plane EMW (in the far zone) as a grid of rays with a given step and dimensions corresponding to the maximum linear size of the target (figure 3).

In the process of EMW propagation, the propagation delay, power, taking into account all multiple reflections, and direction (in two planes) are determined for each beam. In addition, a feature has been introduced that indicates the reflection of rays from the facets belonging to the blades at some stage of propagation.

To determine the point of intersection of the ray with the facet plane, the Moller-Trumbor algorithm is used [9].

The formation of the envelope of the reflected signal consists in the accumulation of the power of the rays reflected from the target towards the radar, depending on the time of arrival and the presence of a sign of reflection from the blade.

\[ s(t) = \sum_{n=1}^{N} \sum_k P_{rk} \text{rect}(t - nT_s), \]  

where

- \( N \) – the number of counts on the duration of the probe pulse;
- \( k = 1, 2 \ldots \) – the number of the beam reflected from the blade;
- \( P_{rk} \) – beam power \( k \);
- \( T_s \) – sampling interval of the probe pulse.

Next, the spectrum is estimated by the Welch method (averaging the modified periodograms) [10] with the Hamming window function (the length of the window corresponds to the length of the weighted sequence) with 0\% overlap of the signal sample vector segments. After that, the value and relative level of the first harmonic in the SDM spectrum is determined.

Modeling was carried out with the following initial data:

- radiation carrier frequency: 10 GHz (wavelength: 3 cm);
- type of probing signal: rectangular pulse without intra-pulse modulation;
- pulse duration (observation interval): 5 ms;
- slant target range: 100 km;
- target radial speed: 0 m / s;
- target azimuth (positive direction is counted counterclockwise relative to the plane of the blades from the side of the coca, figure 7): 10 degrees;
- target elevation angle: 0 degrees;
- distance from the front of the flat EMV to the target: 50 m;
- Beam grid step: 2.4 cm;
- the maximum number of beam re-reflections: 10;
- coefficient of losses in case of re-reflection: 0.028;
- number of blades: 18;
- blade rotation frequency: 7468 rpm.

According to formula (1), when reflected from one compressor stage, the spectrum of the reflected signal should contain components that are multiples of the product of the number of blades and their rotation frequency.

The simulation results presented in figures 4 testify to the adequacy of the algorithm for the formation of the SDM spectrum: the envelope of the reflected pulse is modulated in amplitude the first harmonic of the SDM frequency corresponds to the theoretical value and is 2.3 kHz.

![Figure 4. SDM spectrum for a hypothetical engine (figure 5) - formula (6).](image)

When the impeller with the spinner is placed into the pipe cavity (figure 10), the sector of azimuthal angles, at which the effect of secondary Doppler modulation is quite well observed, is reduced, while the maximum value of the first harmonic of the SDM is achieved at angles close to 35 (145) degrees (figure 11).

![Figure 5. Geometry of modeling the EMW propagation process for a hypothetical turbojet engine.](image)
When an impeller with IGV blades and a fairing is placed into the pipe cavity and an air intake is added (figure 14), the sector of azimuthal angles, when the effect of secondary Doppler modulation is quite well observed, is further reduced, while the maximum value of the first harmonic of the SDM is achieved at angles close to 85 (95) degrees (figure 15).

In addition, in order to form a reflected signal in this case, it is necessary to increase the maximum number of beam reflections to 30.

Thus, taking into account multiple re-reflections of the beam in the cavity of the air intake (pipe) made it possible to increase the adequacy of the above models. Further work is aimed at studying the influence of the target elevation angles on the SDM spectrum, using a burst of pulses as a probing signal and increasing the adequacy of the proposed model of the formation of the reflected signal by taking into account the ESA of facets in accordance with formula (4).

![Figure 6. VDM spectrum for a hypothetical engine (figure 5) - formula (6).](image)

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