Rotation of the Universe and the angular momenta of celestial bodies

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Abstract

We discuss the equation of motion of the rotating homogenous and isotropic model of the Universe. We show that the model predicts the presence of a minimum in the relation between the mass of an astronomical object and its angular momentum. We show that this relation appears to be universal, and we predict the masses of structures with minimal angular momenta in agreement with observations. In such a manner we suggest the possibility at
acquirement of angular momenta of celestial bodies during their formation from the global rotation of the Universe.

**keywords** angular momenta, Universe rotation

# 1 Introduction

The pioneering idea of the rotation of the Universe should be attributed to G. Gamow [1], who expressed the opinion that the rotation of galaxies is due to the turbulent motion of masses in the Universe, and “we can ask ourselves whether it is not possible to assume that all matter in the visible universe is in a state of general rotation around some centre located far beyond the reach of our telescopes?”. The idea of turbulence as a source of the rotation of galaxies was afterwards developed by C.F. von Weizsäcker [2], Ozernoy and Chernin [3], Ozernoy [4], but presently is only of historical value. [If the angular momenta of galaxies had originated in such a way, their spins should be perpendicular to the main protostructure plane [5], which is not observed.] The exact solution of the Einstein equation for the model of a homogeneous universe with rotation and spatial expansion was proposed by Goedel [6, 7]. The observational evidence of global rotation of the Universe was presented by Birch [8]. He investigated the position angles and polarisation of classical high-luminosity double radio sources, finding that the difference between the position angle of the source elongation and of the polarisation are correlated with the source position in the sky. Immediately there appeared a paper by Phinney and Webster [9] concluding that “the data are unsufficient to substantiate the claim” and the statistics are applied incorrectly. Answering, Birch [10] pointed out the difference in the quantity investigated by him and that by Phinney and Webster, showing that their data exhibit the such effect. At the request
of Birch, Phinney and Webster [11] reanalysed the data, introducing new “indirectional statistics” and taking into account possible observational uncertainties. They concluded “that the reported effect (whatever may be its origin) is strongly supported by the observations”. Bietenholz and Kronberg [12] repeated the analysis for a larger sample of objects, finding no effect of the Birch type. New statistical tests were later applied to the data [13].

Nodland and Ralston [14] studied the correlation between the direction and distance to a galaxy and the angle $\beta$ between the polarisation direction and the major galaxy axis. They found an effect of a systematic rotation of the plane of polarisation of electromagnetic radiation, which depends on redshifts. As usually, the result was attacked for the point of incorrectly applied statistics [15, 16, 17] see the reply [18] with a claim that the new, better data do not support the existence of the effect [19].

The problem of the rotation of the whole Universe has attracted the attention of several scientists. It was shown that the reported rotation values are too big when compared with the CMB anisotropy. Silk [20] pointed out that the dynamical effects of a general rotation of the Universe are presently unimportant, contrary to the the early Universe, when angular velocity $\Omega \geq 10^{-13}\text{rad/yr}$. He stressed that now the period of rotation must be greater than the Hubble time, which is a simple consequence of the CMB isotropy. Barrow, Juszkiewicz and Sonoda [21] also addressed this question. They showed that cosmic vorticity depends strongly on the cosmological models and assumptions connected with linearisation of homogeneous, anistropic cosmological models over the isotropic Friedmann Universe. For the flat universe, the value is $\frac{\omega}{H_0} \sim 2 \cdot 10^{-5}$.

Another interesting problem was the discussions on the empirical rela-
tion between the angular momentum and mass of celestial bodies \( J \sim M^{5/3} \) [22]. Li-Xin Li [23] explained this relation for galaxies as a result of the influence of the global rotation of the Universe on galaxy formation.

2 Universe and its angular momentum.

Homogeneous and isotropic models of the Universe with matter may not only expand, but also rotate relative to the local gyroscope. The motion of the matter can be described by Raychaudhuri equation. This is a relation between the scalar expansion \( \Theta \), the rotation tensor \( \omega_{ab} \) and the shear tensor \( \sigma_{ab} \) [24] [25]. The perfect fluid has the stress-energy tensor: \( T_{ab} = (\rho + p)u_au_b + pg_{ab} \), where \( \rho \) is mass density and \( p \) is pressure. The Raychaudhuri equation can be written as:

\[
-\nabla_a A^a + \dot{\Theta} + \frac{1}{3} \Theta^2 + 2(\sigma^2 - \omega^2) = -4\pi G(\rho + 3p),
\]

where \( A^a = u^b \nabla_b u^a \) is the acceleration vector (vanishing in our case), while \( \omega^2 \equiv \omega_{ab}\omega^{ab}/2 \) and \( \sigma^2 \equiv \sigma_{ab}\sigma^{ab}/2 \) are rotation and sheer scalar respectively, \( \Theta \) is scalar expansion.

It has been shown that the spatial homogeneous, rotating and expanding universe filled with perfect fluid must have non-vanishing shear [26].

Because \( \sigma \) falls off more rapidly than the rotation \( \omega \) as the universes expand it is reasonable to consider such generalization of Friedmann equation in which only the “centrifugal” term is present i.e.

\[
\frac{\dot{a}^2}{2} + \frac{\omega^2 a^2}{2} - \frac{4\pi Ga^2}{3c^2} \epsilon = -\frac{kc^2}{2},
\]

where \( \epsilon = \rho c^2 \) is energy density, \( k \) is curvature constant, \( a \) is scalar factor and \( \dot{a} \equiv \frac{d}{dt} a \) (or \( \dot{a} \equiv \frac{d}{da} \)). Equation (2) should be completed with the principle of conservation energy momentum (tensor) and that of angular
momentum:

\[
\dot{\epsilon} = -(\epsilon + p) \Theta, \quad \Theta \equiv 3 \frac{\dot{a}}{a}
\]  

(3)

\[
\frac{p + \epsilon}{c^2} a^5 \omega = J.
\]

(4)

From that we can observe that if \( p = 0 \) (dust) then \( \rho \propto a^{-3} \) and \( \omega \propto a^{-2} \), while in general \( \sigma \) falls as \( a^{-3} \) [24]. The momentum conservation law should be satisfied for each kind of matter, and consequently the angular velocity of the universe will evolve according to different laws in different epochs.

Before decoupling \( (z = 1000) \), matter and radiation interact but after decoupling dust and radiation evolve separately with their own angular velocities \( \omega_d \) and \( \omega_r \). Quantities \( \omega \) and \( \rho \) can be written as \( \omega = \omega_0 (1 + z)^2 \), \( \rho = \rho_{d0} (1 + z)^3 + \rho_{r0} (1 + z)^4 \) the total mass density of matter and radiation.

The conservation of the angular momentum of a galaxy relative to the gyroscopic frames in dust epochs gives [23]:

\[
J = k M^{5/3} - l M,
\]

(5)

where \( k = \frac{2}{5} \left( \frac{3}{4 \pi \rho_0} \right)^{2/3} \omega_0 \), \( \rho_0 \) is the density of matter in the present epoch, \( l = \beta r_f^2 (1 + z_f)^2 \omega_0 \), \( r_f \) is galaxy radius, and \( \beta \) is a parameter determined by the distribution of mass in the galaxy.

In [23] the (present) value of the angular velocity of the Universe is estimated. A suitable value for \( k \) is 0.4 (in CGS Units). Taking \( \rho_{d0} = 1.88 \cdot 10^{-29} \Omega h^2 g cm^{-3} \) and \( h = 0.75 \), \( \Omega = 0.01 \) (Peebles [27] for rich clusters of galaxies, see also [28, 29]), we obtain \( \omega_0 \simeq 6 \cdot 10^{-21} rad s^{-1} \simeq 2 \cdot 10^{-13} rad yr^{-1} \)

It is interesting to note that there are the minimal values of \( J_{min} \), corresponding to same \( M_{min} \).

From the analysis of relation \( J(M) \) [eq(5)], we obtain the presence of
the global minimum at

\[ M_{\text{min}} = \left( \frac{3l}{5k} \right)^{3/2} = 1.95r_f^3(1 + z_f)^3 \rho_d, \quad (6a) \]

\[ J_{\text{min}} = -\frac{6\sqrt{3}}{25\sqrt{5}k^{5/2}}, \quad (6b) \]

For us it is important that \( J \) grows as a function of \( M \) after the minimal value of \( M \). It should be stressed that the value of \( M_{\text{min}} \) does not depend on the value of \( \omega \), i.e. the value of the rotation of the Universe.

Li-Xin Li [23] considered the way an object of the size of our Galaxy is gaining angular momentum. It is an interesting approach to the cosmogonical problem. Following the considerations of Li-Xin Li [23] by accepting \( \Omega_m = 0.01, \ z_f \) between 1 and 3, \( r_f = 30Kpc \approx 10^{23}cm \), and assuming the value of \( \beta \) 0.5 or 0.4 as the coefficient of the inertia momenta in the equation for a celestial object (i.e assuming disk like spherical shapes) we obtain the value of \( M_{\text{min}} \sim 5 \cdot 10^{39}g \sim 2.5 \cdot 10^{3}M_\odot \). Fig. 1 shows dependence of \( J(M) \) in that case.

From the observational point of view, only absolute values of \( J \) in relation (5) are important. Due to this fact, the minimum value of \( |J| \) is easily observed. From Equation (5) and (6a) it is seen that this value equals 0 for:

\[ M_0 = \left( \frac{l}{k} \right)^{3/2} \approx 2.15M_{\text{min}}. \quad (7) \]

In the considered case \( M_0 \approx 5 \cdot 10^{3} M_\odot \). This is sub-globular cluster mass. It seems to be accepted that such structures are not rotating.

Because \( M_{\text{min}} \) as well as \( M_0 \) do not depend on \( \omega \), it is possible to consider relation (6a) as a universal one for any collapsing-dust proto-structure.

Let us consider a proto-solar cloud with a diameter of about 1 ps. Because the formation time of the solar system is certainly shorter than
that of the galaxy formation, equation (7) gives $M_0 \approx 10^{24}\, g$. Such are the masses of giant satellites in the Solar System. Disregarding the Moon, their angular momenta are smaller than those of planets and asteroids [30]. Thus, the mass corresponding to the minimal momentum of a celestial body shows correctly those structures which in reality have the minimal value of angular momenta.

Numerical simulations with dark matter taken into account show that primordial picture of large scale structure formation consists of a network of filaments. During gravitational collapse, clusters of galaxies are formed at the intersections of filaments. The question arises: how great $M_0$ (for dust) should be. Assuming the radius of the proto-structure to be of the order of 30 Mpc, which is consistent with the Perseus-Pisces and Hydra-Centaurus superclusters [31] and $z_f = 6$ then we obtain $M_0 \approx 5\cdot10^{13}\, M_\odot$. Taking into account that this is the mass of dust, it corresponds to the total mass of
galaxy cluster of the order of $10^{14}$ to $10^{15} \, M_\odot$. These contributions are consistent with observations under the assumption that the evolution of dark matter density follows that of dust density. We point out that presently there is no evidence of rotation of cluster of galaxies. In other words, our considerations show that the predicted masses of structures having minimal angular momenta are in agreement with observations. Assuming that the density, in which the proto-structure is formed is equal to the dust density of the Universe, the radius of the proto-structure together with the redshift formation univocally determines the mass $M_0$ for which the absolute value of the angular momentum of the structure is minimal. This relation is schematically presented in the Fig. 2. In such a manner it is possible to consider a universal mechanism of structure rotation.

References

[1] Gamow, G. 1946 Nature 158, No 4016, 549 (1946)

[2] von Weizsaeker, C.F., ApJ 114, 165, (1951)
[3] Ozernoy, L.M., Chernin, A.D., Astr. Zh., 45, 1137 (1968)

[4] Ozernoy, L.M., 1978 in : Origin and Evolution of Galaxies and Stars (ed Pikelner, S.B.,) 105, Nauka, Moscow (1978)

[5] Shandarin, S.F., Sov. Astr. 18, 392 (1974)

[6] Goedel K., Rev. Mod.Phys 21, 447 (1949); GRG 32,1409 (2000),

[7] Goedel K., In: Int. Cong. Math. ed L.M. Graves et al (1952); GRG 32, 1419 (2000).

[8] Birch P., Nature 298, 451 (1982)

[9] Phinney E.S., Webster R.L., Nature 301, 735 (1983)

[10] Birch P., Nature 301, 736 (1983)

[11] Phinney E.S., Webster R.L, Kendall D.G., Young G.A.,(1984) MN-RAS 207, 637, (1984)

[12] Bietenholz M.F., Kronberg, ApJ 287, L1 (1984)

[13] Bietenholz, M.F., AJ 91, 1249 (1986)

[14] Nodland, B., Ralston, J.P., Phys Rev Lett. 78, 3043 (1997)

[15] Carrol, S.M., Field, G.B., Phys.Rev.Lett 79,2394, (1997)

[16] Loredo, T.J., Flanagan, Wasserman, I.M., Phys.Rev.Lett D56, 7507 (1997)

[17] Eisenstein, D.J., Bunn, E.F., Phys.Rev.Lett. 79, 1957 (1997)

[18] Nodlan, B., Ralston, J.P., Phys. Rev.Lett. 79, 1958 (1997)

[19] Wardle J.F.C., Perley R.A., Cohen M.H., Phys. Rev. Lett. 79, 1801 (1997)
[20] Silk J., MNRAS 147, 13 (1970)

[21] Barrow, J.D., Juszkiewicz, R., Sonoda, D.H., MNRAS 213, 917, (1985)

[22] Brosche, P., Comm.Astroph. 11, 213 (1986).

[23] Li-Xin Li., GRG 30, 497 (1998)

[24] Hawking, S.W., MNRAS 142, 129 (1969)

[25] Ellis, G.F.R., in Cargese Lecture in Physics Vol 6 (ed. Schatzman E.)
    1, New York Gordon and Brach Science Publishers (1973)

[26] King, A.R., Ellis G.F.R., 1973 Commun Math Phys 31 209 (1973)

[27] Peebles, P.J.E., Principles of Physical Cosmology, Clarendon Press,
    Oxford (1993)

[28] Peebles, P.J.E., Ratra, B., astro-ph/0207347 (2002)

[29] Lahav, O., astro-ph/0208297 (1997)

[30] Wesson, P.S., Astr. Astroph. 80, 296, (1980)

[31] Giovanelli, R. Haynes M.P. in Large Scale Motions in the Universe
    Princeton University Press, (eds V.C.Rubin G.V. Coyne), 31, Princeton
    (1988)

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