The Quantum Stress Tensor in the Three Dimensional Black Hole

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ABSTRACT: The quantum stress tensor $< T_{\mu\nu} >$ is calculated in the 2+1 dimensional black hole found by Banados, Teitelboim, and Zanelli. The Greens function, from which $< T_{\mu\nu} >$ is derived, is obtained by the method of images. For the non-rotating black hole, it is shown that $< T_{\mu\nu} >$ is finite on the event horizon, but diverges at the singularity. For the rotating solution, the stress tensor is finite at the outer horizon, but diverges near the inner horizon. This suggests that the inner horizon is quantum mechanically unstable against the formation of a singularity.
Recently, Banados, Teitelboim, and Zanelli [1] found a black hole solution in 2 + 1 dimensions which shares many of the features of its 3 + 1 dimensional counterpart [2]. In particular, the static solution has a singularity and event horizon, while the rotating black hole like Kerr possesses outer and inner horizons and an ergosphere. Asymptotically, however, the 2 + 1 solution is not flat, but approaches anti-de Sitter space [3]. 2 + 1 dimensions provides a simpler setting than 3+1 and possibly a more realistic one than 1+1 [4] in which to study the quantum properties of black holes, and specifically, the endpoint of black hole evaporation. Such an investigation should begin with the quantum stress tensor $\langle T_{\mu \nu} \rangle$ which describes the quantum effects of the black hole on a propagating field in a way that allows one to analyze the back reaction. Provided it can be properly renormalized, $\langle T_{\mu \nu} \rangle$ is a well defined local quantity in contrast to particle number which is not, in general, a meaningful concept in curved spacetime. Another motivation for studying $\langle T_{\mu \nu} \rangle$ in the rotating black hole is to investigate the quantum stability of the inner horizon. The maximally extended Reissner-Nordstrom and Kerr solutions include an infinite number of asymptotic regions which in principle could be accessed. However, it has been shown that since the inner horizon is an infinite blueshift surface, classical perturbations will diverge there [5], and the associated back reaction will produce a singularity [6]. Quantum effects for the 1+1 dimensional analog of the Reissner-Nordstrom solution were investigated in [7] where it was shown that $\langle T_{\mu \nu} \rangle$ diverges near the inner horizon. Attempts to include quantum corrections in 3 + 1 dimensions [8] are somewhat inconclusive suggesting that the classical instability either is enhanced or is dampened resulting in a regular spacetime. In this paper, the exact expression for the quantum stress tensor is found for the rotating 2 + 1 dimensional black hole and is shown to diverge near the inner horizon. An estimation of the back reaction suggests that the inner horizon will be replaced by a curvature singularity. We use units in which $\hbar = c = G = 1$.

The 2+1 dimensional black hole solution found by Banados, Teitelboim, and Zanelli [1] is most easily described as three dimensional anti-de Sitter space (ADS$_3$) identified under a discrete subgroup of its isometry group. Recall that ADS$_3$ is the three dimensional hypersurface

$$-T_1^2 + X_1^2 - T_2^2 + X_2^2 = -l^2$$

imbedded in four dimensional flat space with metric $\eta_{ab}$

$$ds^2 = -dT_1^2 + dX_1^2 - dT_2^2 + dX_2^2$$
where \( l = (-\Lambda)^{-1/2} \). The hypersurface (1) is a pseudohyperbolic analog of a three sphere with radius vector \( x^a \equiv (T_1, X_1, T_2, X_2) \), radius \( \sqrt{-x^ax_a} = l \), and constant curvature \( R = -6/l^2 \). We will use lowercase Latin indices for the four dimensional imbedding space and lowercase Greek indices for \( \text{ADS}_3 \). The isometry group of \( \text{ADS}_3 \) is \( SO(2, 2) \) and corresponds to the subgroup of the isometry group of the imbedding space which leaves (1) invariant. Since boosts and rotations in two dimensional planes generate the isometry group, the simplest coordinate systems for \( \text{ADS}_3 \) parameterize these symmetries. As we will see, the black hole solution is constructed by identifying the parameters describing boosts in the \( (T_1, X_1) \) and \( (T_2, X_2) \) planes. Thus, it is in terms of these boost parameters that we wish to express the metric for \( \text{ADS}_3 \). We view it in terms of two copies of \( 1 + 1 \) Minkowski space, \( M_1 \) with coordinates \( (T_1, X_1) \) and \( M_2 \) with coordinates \( (T_2, X_2) \), with the constraint (1) \( \rho_1 + \rho_2 = l^2 \) where \( \rho_i = T_i^2 - X_i^2 \). In each space \( M_i \), one can define Rindler coordinates

\[
T_i = \sqrt{\rho_i} \cosh \chi_i, \quad X_i = \sqrt{\rho_i} \sinh \chi_i, \quad \rho_i > 0, \quad -\infty < \chi_i < \infty
\]

\[
T_i = \sqrt{-\rho_i} \sinh \chi_i, \quad X_i = \sqrt{-\rho_i} \cosh \chi_i, \quad \rho_i < 0, \quad -\infty < \chi_i < \infty
\]

valid in the lightcone interior (\( \rho_i > 0 \)) and exterior (\( \rho_i < 0 \)) respectively. Defining \( \chi_1 \equiv \phi \) and \( \chi_2 \equiv t \), we see that there are three qualitatively distinct regions: (I) \( \rho_1 > l^2 \) \( (\rho_2 < 0) \), (II) \( 0 < \rho_1, \rho_2 < l^2 \), and (III) \( \rho_1 < 0 \) \( (\rho_2 > l^2) \), in which the vectors \( \frac{\partial}{\partial \phi} \) and \( \frac{\partial}{\partial t} \) are spacelike and timelike, spacelike and spacelike, and timelike and spacelike, respectively. It is natural to view \( I \) as the asymptotic region of the spacetime. Substituting (3) in (2) with \( r^2 \equiv \rho_1 = l^2 - \rho_2 \), one obtains the metric for \( \text{ADS}_3 \)

\[
ds^2 = -(\frac{r^2}{l^2} - 1)dt^2 + (\frac{r^2}{l^2} - 1)^{-1}dr^2 + r^2d\phi^2, \quad t, \phi \in (-\infty, \infty)
\]

valid in regions \( I \) and \( II \). Since \( t \) and \( \phi \) parameterize boosts, they take on all real values.

The black hole solution is now constructed by making some combination of \( \phi \) and \( t \) periodic. For the static black hole with mass \( M \), one identifies \( \phi \) with period \( 2\pi \sqrt{M} \). This is somewhat analogous to the identification which leads to the static cone solution in \( 2 + 1 \) gravity without a cosmological constant [3]. A salient difference, however, is that the cone reduces to flat space as \( M \to 0 \), while \( \text{ADS}_3 \), the covering space of the black hole, is recovered as \( M \to \infty \). One would expect the event horizon and singularity of the black hole to have a natural geometric interpretation in terms of \( \text{ADS}_3 \). Indeed, the event horizon is located at \( (r = l) \) and coincides with the boundary between regions \( I \) and \( II \).
in $\text{ADS}_3$ as well as with the light cone in the $1+1$ space $M_2$. The black hole singularity is located at $r = 0$ corresponding to the boundary between regions $II$ and $III$ and to the light cone in $M_1$. $r = 0$ is not a curvature singularity since the curvature is bounded and in fact, constant in $\text{ADS}_3$. It is however a singularity because there are inextendible incomplete geodesics. $r = 0$ is directly analogous to the Misner space light cone $[10]$ on which incomplete null geodesics pile up. Asymptotically, the black hole solution approaches anti-deSitter space.

The black hole with non-zero angular momentum $J$ is obtained from (4) by making a linear combination of $\phi$ and $t$ periodic: $(t, r, \phi) \sim (t - n \alpha_-, r, \phi + n \alpha_+)$ where

$$\alpha_\pm = \pi(\sqrt{M + J/l} \pm \sqrt{M - J/l}).$$

It is possible to transform to coordinates $(\tilde{t}, \tilde{r}, \tilde{\phi})$ :

$$t = \frac{1}{2\pi}(\alpha_+ \tilde{t} - \alpha_- \tilde{\phi})$$
$$\phi = \frac{1}{2\pi}(\alpha_+ \tilde{\phi} - \alpha_- \tilde{t}/l)$$
$$r^2 = \frac{(2\pi \tilde{r})^2 - \alpha_-^2 l^2}{\alpha_+^2 - \alpha_-^2}$$

in terms of which the metric (4) becomes

$$ds^2 = -\left(\frac{\tilde{r}^2}{l^2} - M\right)d\tilde{t}^2 - Jd\tilde{t}d\tilde{\phi} + \left(\frac{\tilde{r}^2}{l^2} - M + \frac{J^2}{4\tilde{r}^2}\right)^{-1}d\tilde{r}^2 + \tilde{r}^2 d\tilde{\phi}^2$$

and $\tilde{\phi}$ is periodic in $2\pi$. The rotating solution possesses both an outer and inner horizon at $\tilde{r} = \alpha_+ l/2\pi$ ($r = l$) and $\tilde{r} = \alpha_- l/2\pi$ ($r = 0$) corresponding respectively to the boundaries between regions $I$ and $II$ and between $II$ from $III$ in $\text{ADS}_3$. In addition, the region $\alpha_+ l/2\pi < \tilde{r} < \sqrt{Ml}$ defines an ergosphere, in which the asymptotic Killing field $\partial/\partial \tilde{t}$ is spacelike. Finally, one should note that in contrast to the static $J = 0$ black hole, the rotating solution is geodesically complete.

The points identified in the rotating black hole are related by an element of $SO(2,2)$ which as a matrix acting on the imbedding space coordinates $x^a \equiv (T_1, X_1, T_2, X_2)$ takes the form

$$\Lambda^a_b \equiv \begin{pmatrix}
\cosh \alpha_+ & \sinh \alpha_+ & 0 & 0 \\
\sinh \alpha_+ & \cosh \alpha_+ & 0 & 0 \\
0 & 0 & \cos \alpha_- & \sin \alpha_- \\
0 & 0 & -\sin \alpha_- & \cosh \alpha_- 
\end{pmatrix}.$$

(8)
For $J = 0$ ($\alpha_- = 0$), $\Lambda$ reduces to a boost in the $M_1$ space, or equivalently a translation in $\phi$, and has fixed points coinciding with the singular surface $r = 0$. For $J \neq 0$, $\Lambda$ has no fixed points accounting for the non-singular nature of the rotating solution.

We now introduce a propagating quantum field in the black hole background and calculate its Greens function. Consider a conformally coupled massless scalar field $\phi$ governed by the action

$$S = -\int \left( \frac{1}{2} (\nabla \phi)^2 + \frac{1}{16} R \phi^2 \right) \sqrt{g} d^3x \quad (9)$$

with $R$ the scalar curvature. We first review the construction of the Greens function in $\text{ADS}_3$, the covering space of the black hole [11]. $\text{ADS}_3$ is a static spacetime with a globally defined timelike Killing field corresponding to the generator of rotations in the $(T_1, T_2)$ plane in the imbedding space. There is therefore a natural vacuum state defined by modes which are positive frequency with respect to this time parameter. Since anti-deSitter space is not globally hyperbolic, it is important to address the issue of boundary conditions at infinity. $\text{ADS}_3$ can be conformally mapped to half of the Einstein static universe with infinity mapped to the equator [10]. Therefore, solutions to the equations of motion in one space can be mapped to solutions in the other, and similarly, boundary conditions at infinity correspond to conditions on the fields at the equator. As discussed in [11], there are three natural choices of boundary conditions. The first which is known as “transparent” simply corresponds to quantizing the field using modes which are smooth on the entire Einstein static universe. The other two boundary conditions are obtained by imposing Dirichlet or Neumann conditions on the field at the equator in the Einstein static universe. The Greens function is given by

$$\tilde{G}_\lambda(x, x') = \frac{1}{4\pi} \frac{1}{|x - x'|} + \frac{\lambda}{4\pi} \frac{1}{|x + x'|} \quad (10)$$

with $\lambda = 0, 1, -1$ for transparent, Neumann, and Dirichlet boundary conditions respectively. Observe that $|x - x'| \equiv ((x - x')^a (x - x')_a)^{1/2}$ is the chordal distance between $x$ and $x'$ in the four dimensional imbedding space and not the distance in $\text{ADS}_3$. The second term in (10) is obtained from the first by the antipodal transformation $x' \rightarrow -x'$, a discrete isometry of $\text{ADS}_3$. In this paper, we will be considering only the $\lambda = 0$ Greens function corresponding to transparent boundary conditions

$$\tilde{G}(x, x') = \frac{1}{4\pi} \frac{1}{|x - x'|} \quad (11)$$
Note that the Greens function coincides with its form in three-dimensional Minkowski space. This is expected as φ is conformally coupled and ADS$_3$ is conformally flat. We now verify that the Greens function satisfies the φ equation of motion as derived from (9)

$$\left(\nabla^\mu \nabla_\mu + \frac{3/4}{l^2}\right) \tilde{G}(x, x') = 0, \quad x \neq x'.$$

(12)

This is most easily checked by expressing the wave operator $\nabla^\mu \nabla_\mu$ in ADS$_3$ in terms of derivatives $\partial_a$ in the imbedding space. $P^{ab} = \eta^{ab} + x^a x^b/l^2$ satisfies $P^{ab} x_b = 0$ and is a projection operator for ADS$_3$. Applying it to the wave operator $\partial^a \partial_a$, one obtains

$$\nabla^\mu \nabla_\mu = P^{ab} \partial_a (P^c_b \partial_c) = P^{ab} \partial_a \partial_b + 3 x^a l^2 \partial_a.$$  

(13)

Using this one verifies that (11) satisfies (12). Since the black hole solution corresponds to ADS$_3$ with discrete identifications, the Greens function $G(x, x')$ for the black hole can be obtained from the Greens function (11) for its covering space by the method of images [12]. Since the images of $x'$ are $\Lambda^n x'$ with $\Lambda$ given in (8), the Greens function is

$$G(x, x') = \sum_{n=-\infty}^{\infty} G(x, \Lambda^n x') = \frac{1}{4\pi} \sum_{n=-\infty}^{\infty} \frac{1}{|x - \Lambda^n x'|}.$$  

(14)

The contributions from the $n$th and $-n$th terms insure that (14) is symmetric in $x$ and $x'$. The quantum stress tensor can now be obtained from $G(x, x')$. Varying the action (3) with respect to $g_{\mu\nu}$ yields

$$T_{\mu\nu} = \frac{3}{4} \nabla_\mu \phi \nabla_\nu \phi - \frac{1}{4} g_{\mu\nu} (\nabla \phi)^2 - \frac{1}{4} \phi \nabla_\mu \nabla_\nu \phi + \frac{1}{4} g_{\mu\nu} \phi \nabla^3 \phi + \frac{1}{8} G_{\mu\nu} \phi^2$$  

(15)

with $G_{\mu\nu}$, the Einstein tensor for the background spacetime. It follows from the equation of motion for $\phi$ that $T_{\mu\nu}$ is traceless and conserved. The quantum stress-tensor $\langle T_{\mu\nu} \rangle$ is obtained by point splitting (13) and then taking its expectation value. Using the $\phi$ equation of motion in the fourth term, and substituting in $G_{\mu\nu} = l^{-2} g_{\mu\nu}$ for ADS$_3$, one obtains

$$\langle T_{\mu\nu} \rangle = \lim_{x' \rightarrow x} \left(\frac{3}{4} \nabla_\mu x' \nabla_\nu x' G - \frac{1}{4} g_{\mu\nu} g^{\alpha\beta} \nabla_\alpha x' \nabla_\beta x' G - \frac{1}{4} \nabla_\mu x' \nabla_\nu x' G - \frac{1}{16l^2} g_{\mu\nu} G \right)$$  

(16)

in terms of the Greens function (14). The renormalization of the stress tensor, ordinarily a difficult procedure in 3 + 1 dimensions [13], is achieved here by simply subtracting off the
coincident $n = 0$ term in the image sum \[14\]. Substituting \[14\] into \[16\] and using

$$\nabla_\mu \nabla_\nu x^a = g_{\mu\nu} x^a / l^2,$$

one eventually finds

$$\langle T^\mu_\nu \rangle = \frac{3}{16\pi} \sum_{n \neq 0} \left( S^a_{\mu\nu} - \frac{1}{3} g_{\mu\nu} g^{\lambda\rho} S^a_{\lambda\rho} \right),$$

$$S^a_{\mu\nu} = \partial_\mu x^a \partial_\nu x^b S^{ab}_{n}, \quad S^{ab}_{n} = \frac{(\Lambda^a)^{bc} (\Lambda^b)^{cd} (\Lambda^c)^{de} (\Lambda^d)^{ef} (\Lambda^e)^{fg} (\Lambda^f)^{gh} (\Lambda^g)^{hi} (\Lambda^h)^{ij}}{|x - \Lambda^n x|^3}.$$  \(17\)

$S^a_{\mu\nu}$ is the pull back to ADS$_3$ of $S^{ab}_{n}$.

The stress tensor \[17\] can be evaluated in a particular set of coordinates $y^\mu$ in ADS$_3$ by substituting in the corresponding imbedding $x^a = x^a(y^\mu)$. For the static $J = 0$ ($\alpha_- = 0$) black hole in coordinates $(t, r, \phi)$ \[4\], \[17\] takes the form

$$\langle T^\nu_\mu \rangle = \frac{A(M)}{r^3} \text{diag}(1, 1, -2), \quad A(M) \equiv \frac{\sqrt{2}}{32\pi} \sum_{n=1}^{\infty} \frac{\cosh n\pi \sqrt{M + 3}}{(\cosh n\pi \sqrt{M - 1})^{3/2}}.$$  \(18\)

where $M$ is the black hole mass. Since the series converges exponentially for all real $M$, the stress tensor is finite everywhere except near the singularity where it diverges as $r^{-3}$. The divergence there arises from the fact that since $r = 0$ remains invariant under the action of $\Lambda$, the denominator in the Greens function \[14\] vanishes. Even though the coordinates $(t, r, \phi)$ breakdown near the event horizon, it is clear that the scalar $\langle T^\mu_\nu \rangle < T^\mu_\nu >$ is smooth there. For $M >> 1$, the first term in the series gives the leading order behavior $A(M) \sim e^{-\pi \sqrt{M}}$. Recall that as $M \to \infty$, $\phi$ becomes unidentified and ADS$_3$ is recovered. Since $\langle T^\mu_\nu \rangle$ was renormalized with respect to ADS$_3$, it vanishes in this limit. For small $M$, the series can be approximated by an integral yielding $A(M) \sim M^{-3/2}$. From the invariance of the vacuum under the anti-deSitter group, one would expect $\langle T^\nu_\mu \rangle \sim \delta^\nu_\mu$. However, the identification in $\phi$ breaks the underlying symmetry picking out $\phi$ as a preferred direction. $\langle T^\nu_\mu \rangle$ is traceless and conserved. One should note, however, that in analogy to the Casimir effect the energy density is negative.
For the rotating black hole, the stress tensor (17) becomes

\[
< T_{t t} > = \frac{1}{4\pi} \sum_{n=1}^{\infty} \left( (\cosh n\alpha_+ + 2\cosh n\alpha_- - 3)r^2 - 2(\cosh n\alpha_- - 1)l^2 \right) \frac{c_n}{|d_n|^{5/2}}
\]

\[
< T_{r r} > = \frac{1}{4\pi} \sum_{n=1}^{\infty} \left( (\cosh n\alpha_+ - \cosh n\alpha_-)r^2 + (\cosh n\alpha_- - 1)l^2 \right) \frac{c_n}{|d_n|^{5/2}}
\]

\[
< T_{\phi \phi} > = \frac{1}{4\pi} \sum_{n=1}^{\infty} \left( -2(\cosh n\alpha_+ + \cosh n\alpha_- - 3)r^2 + (\cosh n\alpha_- - 1)l^2 \right) \frac{c_n}{|d_n|^{5/2}}
\]

\[
< T_{t \phi} > = \frac{3}{4\pi} \sum_{n=1}^{\infty} \cosh n\alpha_+ \cosh n\alpha_- \frac{r^2/l^2 - 1}{|d_n|^{5/2}}
\]

\[
< T_{t r} > = < T_{r t} > = 0
\]

\[
c_n \equiv \cosh n\alpha_+ + \cosh n\alpha_- + 2
\]

\[
d_n \equiv |x - \Lambda^nx|^2 = 2(\cosh n\alpha_+ - \cosh n\alpha_-)r^2 + 2(\cosh n\alpha_- - 1)l^2
\]

with \(\alpha_\pm\) given in (5). In the \(J = 0\) \((\alpha_- = 0)\) limit, (19) reduces to (18). Recall that in \((t, r, \phi)\) coordinates, the outer and inner horizons are located at \(r = l\) and \(r = 0\).

Outside the inner horizon, where \(d_n\) is positive and the infinite sums converge exponentially, \(< T_{\mu \nu} >\) is smooth. The inner horizon, in terms of the imbedding coordinates, is the surface \(r^2 = T_1^2 - X_1^2 = 0\) corresponding to the lightcone in the 1+1 space \(M_1\). Inside the horizon, \(\rho = r^2 = T_1^2 - X_1^2\) becomes negative, and the denominators \(d_n\) in (19) vanish on a sequence of timelike surfaces

\[
\rho = \rho_n, \quad \rho_n \equiv -\frac{\cosh n\alpha_- - 1}{\cosh n\alpha_+ - \cosh n\alpha_-}l^2, \quad M > J/l.
\]

As we now demonstrate, the \(n^{th}\) surface in (20) consists of points \(x^a\) connected to their image \(\Lambda^nx\) by a null geodesic and is known as a polarized hypersurface [15]. Since \(x\) and \(\Lambda^n x\) are identified in the black hole solution, the connecting null geodesic is self-intersecting. In \(\text{ADS}_3\), geodesics are the analogs of great circles on ordinary spheres. In other words, they are curves which also lie on a two dimensional plane passing through the origin in the four dimensional imbedding space. Two points \(x\) and \(y\) are connected by a spacelike, lightlike, or timelike geodesic depending on whether \(x^a y_a < -l^2\), \(x^a y_a = -l^2\), or \(-l^2 < x^a y_a < l^2\) respectively [13]. [Points with \(x^a y_a > l^2\) lie on different branches of a hyperboloid and, therefore, are not connected by any geodesic.] Since a point \(x\) on the \(n^{th}\) polarized hypersurface satisfies \(d_n = |x - \Lambda^n x|^2 = 0\) implying \(x^a (\Lambda^n)_{ab} x^b = -l^2\), \(x\) and \(\Lambda^n x\) are connected by a null geodesic. As one approaches a polarized hypersurface (20) from
a geodesic distance \( s \), \(<T_{\mu\nu}>\) diverges as \( s^{-5/2} \). Since these surfaces in the \( n \to \infty \) limit approach the inner horizon, \( r = 0 \), the stress tensor will diverge there. [It should be noted that \(<T_{\mu\nu}>\) is in fact finite at the inner horizon as it is approached from the outside. This is due to the fact that though each of the polarized hypersurfaces contains null geodesics, the inner horizon itself does not and is said to be non-compactly generated.] One can estimate the back reaction due to the diverging stress tensor by substituting \(<T_{\mu\nu}>\) into the field equation. Integrating twice, one finds that the metric perturbation diverges as \( \delta g_{\mu\nu} \sim s^{-1/2} \) on each of the polarized hypersurfaces. This suggests that the inner horizon is quantum mechanically unstable against formation of a curvature singularity.

For the extremal case \((M = J/l)\), the stress tensor \((19)\) becomes

\[
\begin{align*}
<T^t_t> &= K(3r^2 - 2l^2) \\
<T^r_r> &= Kl^2 \\
<T^\phi_\phi> &= -K(3r^2 - l^2) \\
<T^t_\phi> &= \frac{3}{2}K(\frac{r^2}{l^2} - 1)l \\
<T^r_\phi> &= <T^\phi_r> = 0 \\
K \equiv \frac{\sqrt{2}}{16\pi l^5} \sum_{n=1}^{\infty} \frac{\cosh n\pi \sqrt{2M} + 1}{(\cosh n\pi \sqrt{2M} - 1)^{3/2}}.
\end{align*}
\]

For \(M = J/l >> 1\), one has \(K \approx l^{-5}e^{-\pi \sqrt{M/2}}\). Note that in contrast to the non-extremal case, \((21)\) is smooth everywhere but diverges asymptotically.

In this paper, we studied the stress tensor for a propagating quantum field in the \(2+1\) black hole. Considering the relatively simple geometric structure of the black hole solution, one would hope that further investigation would lead to a greater understanding of its quantum properties.

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