Abstract

We present a model where neutrino masses are generated by a combination of spontaneous R-parity violation and Type III seesaw. In addition to the usual MSSM particle content, our model consists of one extra triplet matter chiral superfield containing heavy SU(2) triplet fermions and its superpartners. R-parity is broken spontaneously when the sneutrinos associated with the one heavy neutrino as well as the three light neutrinos get vacuum expectation values, giving rise to the mixed $8 \times 8$ neutralino-neutrino mass matrix. We show that our model can comfortably explain all the existing neutrino oscillation data. Due to the presence of the triplet fermion, we have a pair of additional heavy charged leptons which mix with the standard model charged leptons and the charginos. This gives rise to a $6 \times 6$ chargino-charged lepton mass matrix, with 6 massive eigenstates. Finally we discuss about the different R-parity violating possible decay modes and the distinctive collider signatures which our model offers.
1 Introduction

Despite its inimitable success, it is now universally accepted that the standard model of particle physics is only the low energy limit of a more complete theory of elementary particles. Various experimental evidences on the existence of neutrino flavor oscillations have proved beyond doubt that the standard model needs to extended in order to explain neutrino masses and flavor mixing. Experiments like SNO, KamLAND, K2K and MINOS [1–4] provide information on the two mass square differences $\Delta m^2_{21}$ and $\Delta m^2_{31}$ and on the two mixing angles $\theta_{12}$ and $\theta_{23}$. The third mixing angle $\theta_{13}$ is not yet determined, but from the null result of the CHOOZ [5] experiment it is known to be certainly small. The current $3\sigma$ allowed intervals of the oscillation parameters are given as [6]

$$7.1 \times 10^{-5} \text{eV}^2 < \Delta m^2_{21} < 8.3 \times 10^{-5} \text{eV}^2, \ 2.0 \times 10^{-3} \text{eV}^2 < \Delta m^2_{31} < 2.8 \times 10^{-3} \text{eV}^2$$

(1)

$$0.26 < \sin^2 \theta_{12} < 0.42, \ 0.34 < \sin^2 \theta_{23} < 0.67, \ \sin^2 \theta_{13} < 0.05.$$  \hspace{1cm} (2)

Any theory of physics beyond the standard model must offer a natural explanation of the neutrino masses. The two main challenges involved at this frontier are, (i) explaining the smallness of neutrinos masses, which are at least 12 orders of magnitude smaller compared to the top mass, and (ii) explaining the mixing pattern which is very distinct from the mixing observed in the quark sector. Very small Majorana neutrino masses can be generated by the dimension 5 operator $\frac{1}{\Lambda}LLHH$ [7], where the masses are suppressed naturally by the scale of new physics $\Lambda$. Note that this term breaks lepton number which is mandatory for the generation of Majorana masses. In the seesaw mechanism [8], $\Lambda$ is associated with the mass of new heavy particle(s). In the so-called Type I seesaw [8], the new particles are heavy right-handed neutrinos which are singlets under the standard model gauge group. In Type III seesaw [9–16], the new particles are heavy triplet fermions with hypercharge $Y = 0$. The fermions therefore are self-conjugate and belong to the adjoint representation of $SU(2)$. Small Majorana masses can also be realized in the Type II seesaw models [17], where the model is extended by including one heavy complex scalar with hypercharge $Y = 2$, which transforms as a triplet of $SU(2)$. Observed neutrino mixing can be obtained very naturally by imposing flavor symmetry. Among the numerous viable flavor symmetry models [18], the models based on the group $A_4$ are the most popular ones [19].

Among the main drawbacks of the standard model is the problem of explaining the stability of the Higgs mass. Supersymmetry in particle physics [20] has been one of the most widely accepted way of alleviating this problem. Supersymmetry demands that for every fermion/boson in the model, there is a superpartner which is a boson/fermion. In the limit of exact supersymmetry, this results in the cancellation of all quadratic divergences of the Higgs mass, which is hence naturally stabilized against quantum corrections. The most general superpotential allows for terms which break lepton number and baryon number.

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1 We define $\Delta m^2_{ij} = m^2_i - m^2_j$.
2 All possible terms in the Minimal Supersymmetric Standard Model (MSSM) superpotential are discussed in section 2 and given in Eqs. (3) and (4).
However these lepton and baryon number violating couplings are severely constrained by non-observation of proton decay and data on heavy flavor physics from Belle and Babar [21]. In order to avoid all such terms in the superpotential, one imposes a \( Z_2 \) symmetry called R-parity, which is defined as \( R_p = (-1)^{3(B-L)+2S} \) [22, 24], where \( B \) and \( L \) are the baryon and lepton number of the particle and \( S \) is the spin. In the limit where R-parity is conserved both lepton and baryon number are conserved, whereas the breaking of R-parity ensures the breaking of lepton and/or baryon number. On the other hand, allowing for breaking of R-parity opens up the possibility of generating Majorana mass terms for the neutrinos [25–34]. This can be done through one loop [28, 29] and two loop [30] diagrams generated via the lepton number breaking trilinear couplings \( \lambda \) and \( \lambda' \) (see Eq. (4)). Small neutrino masses can also be generated by the R-parity violating bilinear coupling \( \hat{H}_u \hat{L} \) [25, 26] where \( \hat{H}_u \) is the Higgs superfield and \( \hat{L} \) is the leptonic superfield. This term gives rise to higgsino-neutrino mixing, whereas the gaugino-neutrino mixing will be generated from the kinetic terms (which we give in Eq. (61)), once the sneutrino fields get vacuum expectation values. Hence this \( \hat{H}_u \hat{L} \) bilinear coupling alongwith the gaugino-neutrino mixing terms bring about a neutralino-neutrino mixing and one can easily obtain a seesaw like formula for the light neutrinos, where the neutrino mass is suppressed by the neutralino mass. However, unlike the higher loop neutrino mass models with trilinear R-parity violating couplings, the bilinear R-parity breaking gaugino seesaw model can only generate one massive neutrino. Neutrino oscillation experiments have confirmed that we have at least two massive neutrinos. Therefore, one needs something extra in the gaugino seesaw models for generating the correct neutrino mass matrix which is viable with experiments. Models with explicit R-parity violation have been put forth where one introduces additional chiral superfields containing heavy right-handed neutrinos [31, 32], in addition to usual MSSM field content. However, in all models with R-parity violation, proton decay remains a severe constraint, and one has to explain why the lepton number violating terms in the R-parity violating part of the superpotential should be so much more larger compared to the one which involves only quarks (the \( \lambda'' \) coupling in Eq. (4)) and which should be heavily suppressed in order to suppress proton decay. One would generally have to impose additional symmetry arguments in order to explain this.

A possible way of circumventing this problem is by invoking spontaneous breaking of R-parity [23, 33–38]. In this scenario, R-parity is conserved initially in the superpotential, and only once the sneutrino fields acquire vacuum expectation value, R-parity breaking terms are generated spontaneously. In presence of additional singlet or triplet matter chiral fields, this provides a natural explanation for the origin of the R-parity and lepton number violating bilinear term \( \epsilon \), without generating the baryon number violating \( \lambda'' \) term in the superpotential. Therefore, one can generate neutrino masses without running into problems with proton decay in this class of supersymmetric models.

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3 Proton decay obviously requires simultaneous presence of baryon and lepton number violation. Lepton number is necessarily violated to generate Majorana neutrino masses. However, proton decay can be forbidden by forbidding the \( \lambda'' \) coupling.

4 It is possible to realize the other terms \( \lambda \) and \( \lambda' \) from the R-parity conserving MSSM superpotential only after redefinition of basis [25].
There have been earlier attempts to construct spontaneous R-parity violating models within the MSSM gauge group. In all these models the R-parity is broken spontaneously when the sneutrino acquires a VEV. This automatically breaks lepton number spontaneously. Since lepton number is a global symmetry, this might give rise to a massless Goldstone called the Majoron \[23\,33\,35\,39\]. All models which predict presence of Majoron are severely constrained, unless the Majoron is mostly singlet. The phenomenology of these latter types of models has been studied in detail in \[34\,35\]. A possible way of avoiding the Majoron is by gauging the U(1) symmetry associated with lepton number such that the spontaneous R-parity and lepton number violation comes with the new gauge symmetry breaking. This gives rise to an additional neutral gauge boson and the phenomenology of these models have also been studied extensively in the literature \[36\,37\]. This idea has been used in a series of recent papers \[38\].

In this work we study spontaneous R-parity violation in the presence of a SU(2) triplet \(Y = 0\) matter chiral superfield, where we stick to the gauge group of the minimal supersymmetric standard model. Lepton number is broken explicitly in our model by the Majorana mass term of the heavy fermionic triplet, thereby circumventing the problem of the Majoron. We break R-parity spontaneously by giving vacuum expectation values to the 3 MSSM sneutrinos and the one additional sneutrino associated with the triplet which leads to the lepton-higgsino and lepton-gaugino mixing in addition to the conventional Yukawa driven neutrino-triplet neutrino mixing. This opens up the possibility of generating neutrino mass from a combination of the conventional Type-III seesaw and the gaugino seesaw. We restrict ourself to just one additional SU(2) triplet \(Y = 0\) matter chiral superfield, and explore the possibility of getting viable neutrino mass splitting and the mixing angles at tree level. With one generation of heavy triplet we get two massive neutrinos while the third state remains massless. Like the neutralino sector we also have R-parity conserving mixing between the standard model charged leptons and the heavy triplet charged lepton states. The spontaneous R-parity breaking brings about mixing between the charginos and the charged leptons, and hence modifies the chargino mass spectrum. In addition to the usual charged leptons, our model contains a pair of heavy charged fermions coming from the fermionic component of the triplet superfield. Therefore, the charginos have contributions from these heavy exotic charged particles as well and we expect distinctive collider signatures due to this. Another novel feature of our model comes from the fact that the additional triplet fermions and sfermions have direct gauge interactions. Hence they offer a much richer collider phenomenology. We discuss in brief about the possibility of detecting our model at colliders and the predicted R-parity violating signatures.

The main aspects of our spontaneous R-parity violating model compared to the ones already discussed in the literature are the following:

- We introduce one chiral superfield containing the triplets of SU(2) and with \(Y = 0\). The earlier viable models have considered one or more singlet chiral superfields containing the right-handed neutrino.

- We have an explicit breaking of the lepton number due to the presence of the mass term of this chiral superfield in the superpotential. Therefore unlike as in \[23\,33\,35\],
the spontaneous breaking of R-parity does not create any Majoron in our model. Since we do not have any additional gauge symmetry, we also do not have any additional neutral gauge boson as in [36–38].

• Since we have only one additional triplet chiral superfield we have two massive neutrinos, with the lightest one remaining massless. One of the neutrinos get mass due to type III seesaw and another due to gaugino seesaw. Combination of both gives rise to a neutrino mass matrix which is consistent with the current data.

• The triplet chiral superfield in our model modifies not only the neutrino-neutralino mass matrix, but also the charged lepton – chargino mass matrix. Being a triplet, it contains one neutral Majorana fermion \( \Sigma^0 \), two charged fermions \( \Sigma^\pm \), one sneutrino \( \tilde{\Sigma}^0 \), and two charged sfermions \( \tilde{\Sigma}^{\pm} \). Therefore, our neutral fermion mass matrix is a \( 8 \times 8 \) matrix, giving mixing between the gauginos, higgsinos as well as the new TeV-scale neutral fermion \( \Sigma^0 \). Likewise, the new charged fermions \( \Sigma^\pm \) will mix with the charginos and the charged leptons.

• There are thus new TeV-mass neutral and charged leptons and charged scalars in our model, which will have mixing with other MSSM particles. These could be probed at future colliders and could lead to a rich phenomenology. We give a very brief outline of the collider signatures in this work. We plan to make a detailed study of the collider aspects of our model in a future work.

The paper is organized as follows. In section 2 we describe the model and in section 3 we present the symmetry breaking analysis. In section 4, we discuss the neutralino-neutralino mass matrix and in section 5 we discuss the chargino-charged lepton mass matrix. We concentrate on the neutrino phenomenology in section 6 and discuss the possibility of getting correct mass splittings and mixings even with one generation of SU(2) triplet matter chiral supermultiplet. In section 7 we discuss about detecting our model in colliders and finally in section 8 we present our conclusion. Discussion on soft-supersymmetry breaking terms, gaugino-lepton-slepton mixing and the exact analytic expression of the low energy neutrino mass matrix have been presented in Appendix A, Appendix B and in Appendix C, respectively. In Appendix C we also analyze the constraints on the different sneutrino vacuum expectation values coming from the neutrino mass scale.

2 The Model

The discrete \( Z_2 \) symmetry R-parity is defined as \( R_p = (-1)^{3(B-L)+2S} \) where \( B, L \) represents the baryon number and lepton number of the particles and \( S \) is the spin. The matter chiral superfields have R-parity \(-1\) whereas the Higgs chiral superfields have R-parity \(+1\). The R-parity conserving superpotential of the MSSM is

\[
W_{\text{MSSM}} = Y_e \hat{H}_d \hat{L} \hat{E}^c + Y_d \hat{H}_d \hat{Q} \hat{D}^c - Y_u \hat{H}_u \hat{Q} \hat{U}^c + \mu \hat{H}_u \hat{H}_d.
\] (3)
The other terms which are allowed by supersymmetry invariance and gauge invariance but violate R-parity are

\[ W_{R_p} = -\epsilon_i \hat{H}_u \hat{L}_i + \lambda_{ijk} \hat{L}_i \hat{L}_j \hat{E}^c_{k} + \lambda'_{ijk} \hat{Q}_j \hat{D}^c_k + \lambda''_{ijk} \hat{U}_i \hat{D}^c_j \hat{D}^c_k. \]  

(4)

As already mentioned in the introduction, in this work we will explore spontaneous R-parity violation in the presence of SU(2) triplet \( Y = 0 \) matter chiral superfield. The matter chiral supermultiplets of the model are:

\[ \hat{Q} = (\hat{\bar{U}} \hat{D}), \hat{\bar{L}} = (\hat{\bar{\nu}} \hat{E}), \hat{\bar{U}}_c, \hat{\bar{D}}_c \] and

\[ \hat{E}^c. \]

In addition to the standard supermultiplet contents of the MSSM we introduce one SU(2) triplet matter chiral supermultiplet \( \hat{\Sigma}^c_R \) with \( U(1)_Y \) hypercharge \( Y = 0 \). We represent \( \hat{\Sigma}^c_R \) as

\[ \hat{\Sigma}^c_R = \frac{1}{\sqrt{2}} \left( \hat{\Sigma}^{0c}_R + \sqrt{2} \hat{\Sigma}^{-c}_R \right). \]  

(5)

The three different chiral superfields in this multiplet are

\[ \hat{\Sigma}^{+c}_R = \hat{\Sigma}^c_R + \sqrt{2} \theta \hat{\Sigma}^{+c}_R + \theta \theta F_{\Sigma_R^{+c}}, \]

(6)

\[ \hat{\Sigma}^{-c}_R = \hat{\Sigma}^c_R + \sqrt{2} \theta \hat{\Sigma}^{-c}_R + \theta \theta F_{\Sigma_R^{-c}}, \]

(7)

\[ \hat{\Sigma}^{0c}_R = \hat{\Sigma}^c_R + \sqrt{2} \theta \hat{\Sigma}^{0c}_R + \theta \theta F_{\Sigma_R^{0c}}. \]

(8)

The SU(2) triplet fermions are \( \hat{\Sigma}^{+c}_R, \hat{\Sigma}^{-c}_R \) and \( \hat{\Sigma}^{0c}_R \) and their scalar superpartners are represented as \( \hat{\Sigma}^{+c}_R, \hat{\Sigma}^{-c}_R \) and \( \hat{\Sigma}^{0c}_R \) respectively. \( F_{\Sigma_R^{+c}}, F_{\Sigma_R^{-c}}, F_{\Sigma_R^{0c}} \) represent the different auxiliary fields of the above mentioned multiplet. R-parity of \( \hat{\Sigma}^c_R \) is \(-1\) where componentwise the fermions \( \hat{\Sigma}^{+c}_R, \hat{\Sigma}^{-c}_R \) and \( \hat{\Sigma}^{0c}_R \) have \( R_p = +1 \) and their scalar superpartners have \( R_p = -1 \). With these field contents, the R-parity conserving superpotential \( W \) will be

\[ W = W_{MSSM} + W_{\Sigma}, \]

(9)

where \( W_{MSSM} \) has already been written in Eq. (3) and \( W_{\Sigma} \) is given by

\[ W_{\Sigma} = -Y_{\Sigma} \hat{H}_u^T (i \sigma_2) \hat{\Sigma}^{c}_R \hat{L}_i + \frac{M}{2} Tr[\hat{\Sigma}^{c}_R \hat{\Sigma}^{c}_R]. \]  

(10)
$W_\Sigma$ is clearly R-parity conserving. The scalar fields $\tilde{\Sigma}_0^c$ and $\tilde{\nu}_{Li}$ are odd under R-parity. Hence in this model, R-parity would be spontaneously broken by the vacuum expectation values of these sneutrino fields. We will analyze the potential and spontaneous R-parity violation in the next section. On writing explicitly, one will get these following few terms from the above superpotential

\[ W_\Sigma = \frac{Y_{\Sigma}}{\sqrt{2}} \hat{H}_u^0 \Sigma_{R}^0 \hat{l}_i + \frac{Y_{\Sigma}}{\sqrt{2}} \hat{H}_u^+ \Sigma_{R}^+ \hat{i}_i - \frac{Y_{\Sigma}}{\sqrt{2}} \hat{H}_u^0 \Sigma_{R}^{-} \hat{l}_i - \frac{Y_{\Sigma}}{\sqrt{2}} \hat{H}_u^+ \Sigma_{R}^{+} \hat{i}_i + M \Sigma_{R}^c \Sigma_{R}^c. \] (11)

The kinetic terms of the $\hat{\Sigma}_R$ field is

\[ L_k^\Sigma = \int d^4 \theta Tr[\Sigma_R^c \Sigma_R^c], \] (12)

The soft supersymmetry breaking Lagrangian of this model is

\[ L_{soft} = L_{soft}^{MSSM} + L_{soft}. \] (13)

For completeness we write the $L_{soft}^{MSSM}$ Lagrangian in the Appendix A. $L_{soft}$ contains the supersymmetry breaking terms corresponding to scalar $\tilde{\Sigma}_R^c$ fields and is given by

\[ L_{\Sigma}^{soft} = -m_{\Sigma}^2 Tr[\Sigma_R^c \Sigma_R^c] - (\tilde{m}_{\Sigma}^2 Tr[\tilde{\Sigma}_R^c \tilde{\Sigma}_R^c] + h.c) - (A_{\Sigma} \Sigma_R^c \Sigma_R^c + h.c), \] (14)

where

\[ \tilde{\Sigma}_R^c = \frac{1}{\sqrt{2}} \left( \begin{array}{c} \tilde{\Sigma}_R^0 \\ \sqrt{2} \tilde{\Sigma}_R^+ \\ -\tilde{\Sigma}_R^0 \\ \sqrt{2} \tilde{\Sigma}_R^- \end{array} \right). \] (15)

We explicitly show in the Appendix A all the possible trilinear terms which will be generated from Eq. (11) and the interaction terms between gauginos and SU(2) triplet fermion and sfermion coming from Eq. (12). In the next section we analyze the neutral component of the potential and discuss spontaneous R-parity violation through $\tilde{\Sigma}_R^0$ and $\tilde{\nu}_{Li}$ vacuum expectation values.

### 3 Symmetry Breaking

In this section we write down the scalar potential which will be relevant to analyze the symmetry breaking of the Lagrangian. The potential is

\[ V = V_F + V_D + V_{soft}, \] (16)

where $V_F$ and $V_D$, the contributions from different auxiliary components of the chiral superfield and different vector supermultiplets are given by

\[ V_F = \sum_k F_k^a F_k, \] (17)

\[ V_D = \frac{1}{2} \sum_a D^a D^a \] (18)
where the index \( k \) denotes all possible auxiliary components of the matter chiral superfields whereas the index \( \alpha \) is the gauge index. The contribution from the soft supersymmetry breaking Lagrangian is given by \( V_{soft} \). Below we write down the neutral component of the potential which would be relevant for our symmetry breaking analysis. The neutral component of the potential is given by

\[
V_{neutral} = V^n_F + V^n_D + V^n_{soft},
\]

where

\[
V^n_F = |F_{H_d^0}|^2 + |F_{H_u^0}|^2 + |F_{\tilde{\nu}_{L_i}}|^2 + |F_{\tilde{\Sigma}^0_R}|^2.
\]

The different \( F_k \) are given by

\[
F^*_k = \mu H^0_k + \ldots,
\]

\[
F^*_k = -\sum_i \frac{Y_{\Sigma_i}}{\sqrt{2}} H^0_u \tilde{\nu}_{L_i} - M \tilde{\Sigma}^0_R + \ldots,
\]

\[
F^*_k = -\sum_i \frac{Y_{\Sigma_i}}{\sqrt{2}} H^0_u \tilde{\nu}_{L_i} + \ldots.
\]

In the above equations ... represents other terms which will not contribute to the neutral component of the potential. With all these \( F_k \)'s, \( V^n_F \) is given by

\[
V^n_F = |\mu H^0_d|^2 + |\mu H^0_u|^2 + \frac{1}{2} \sum_i Y_{\Sigma_i} \tilde{\Sigma}_{R} \tilde{\nu}_{L_i} |^2 + \frac{1}{2} \sum_i |Y_{\Sigma_i} H^0_u \tilde{\Sigma}_{R}^0|^2 + \frac{1}{2} \sum_i Y_{\Sigma_i} H^0_u \tilde{\nu}_{L_i} |^2
\]

\[
-|\mu H^0_d (\sum_i \frac{Y_{\Sigma_i}}{\sqrt{2}} \tilde{\Sigma}_{R} \tilde{\nu}_{L_i})|^2 + c.c + |M|^2 \tilde{\Sigma}_{R}^0 \tilde{\Sigma}_{R}^0 + \left( \sum_i \frac{Y_{\Sigma_i}}{\sqrt{2}} H^0_u \tilde{\nu}_{L_i} (M \tilde{\Sigma}^0_R) c.c. \right).
\]

As we have three generation of leptons hence the generation index \( i \) runs as \( i=1,2,3 \). The D term contribution of \( V_{neutral} \) is given as

\[
V^n_D = \frac{1}{S} (g^2 + g'^2) (|H^0_d|^2 - |H^0_u|^2 + \sum_i |\tilde{\nu}_{L_i}|^2)^2.
\]

Note that \( \tilde{\Sigma}^0_R \) which has \( Y = 0 \) and the third component of the isospin \( T_3 = 0 \) does not contributes to \( V^n_D \). The soft supersymmetry breaking contributions to the neutral part of the potential is given by \( V^n_{soft} \) where

\[
V^n_{soft} = - (b H^0_d H^0_u + c.c) + m^2_{H_u} |H^0_u|^2 + m^2_{H_d} |H^0_d|^2
\]

\[
+ m^2_{\tilde{\Sigma}^0_R} \tilde{\Sigma}^0_R + [m^2_{\tilde{\Sigma}^0_R} \tilde{\Sigma}^0_R + c.c]
\]

\[
+ \sum_i \frac{m^2_{\tilde{\nu}_{L_i}} \tilde{\nu}_{L_i} + [\sum_i \frac{\tilde{A}_{\Sigma_i}}{\sqrt{2}} H^0_u \tilde{\Sigma}^0_R \tilde{\nu}_{L_i} + c.c].
\]
We represent the vacuum expectation values of $H_u^0$, $H_d^0$, $\bar{\nu}_L$, and $\Sigma_R^0$ as $\langle H_u^0 \rangle = v_2$, $\langle H_d^0 \rangle = v_1$, $\langle \bar{\nu}_L \rangle = u_i$ and $\langle \Sigma_R^0 \rangle = \tilde{u}$. We have considered a diagonal $m_\Sigma^2$ matrix. In terms of these vacuum expectation values the neutral component of the potential is

$$
\langle V_{\text{neutral}} \rangle = (|\mu|^2 + m_{H_u}^2)|v_2|^2 + (|\mu|^2 + m_{H_d}^2)|v_1|^2 - (bv_1 v_2 + c.c)
+ \frac{1}{8}(g^2 + g'^2)(|v_1|^2 - |v_2|^2 + \sum_i |u_i|^2)^2 + (|M|^2 + m_{\Sigma}^2)|\tilde{u}|^2
+ \sum_i m_{i}\bar{u}_i u_i + \frac{1}{2}\sum_i Y_{\Sigma_i} \tilde{u} u_i + \frac{1}{2}\sum_i |Y_{\Sigma_i}|^2 |\tilde{u}|^2 v_2^2
+ \frac{1}{2}\sum_i Y_{\Sigma_i} u_i v_2^2 + \left(\sum_i \frac{A_{\Sigma_i}^2}{\sqrt{2}} v_2 u_i \tilde{u} + c.c\right) - (\mu v_1 \sum_i \frac{Y_{\Sigma_i}}{\sqrt{2}} \tilde{u} u_i) + c.c
+ \sum_i \frac{Y_{\Sigma_i}}{\sqrt{2}} v_2 u_i (M \tilde{u})^* + c.c).

(29)

For simplicity we assume all the vacuum expectation values and all the parameters are real. Hence $\langle V_{\text{neutral}} \rangle$ simplifies to

$$
\langle V_{\text{neutral}} \rangle = (\mu^2 + m_{H_u}^2)|v_2|^2 + (\mu^2 + m_{H_d}^2)v_1^2 - 2bv_1 v_2 + \frac{1}{8}(g^2 + g'^2)(v_1^2 - v_2^2 + \sum_i u_i^2)^2
+ (M^2 + m_{\Sigma}^2)\tilde{u}^2 + \sum_i m_{i}\bar{u}_i u_i + 2m_{\Sigma}^2 \tilde{u}^2 + \frac{1}{2}\left(\sum_i Y_{\Sigma_i} u_i \right)^2 \tilde{u}^2 + \frac{1}{2}\sum_i |Y_{\Sigma_i}|^2 \tilde{u}^2 v_2^2
+ \frac{1}{2}\sum_i Y_{\Sigma_i} u_i v_2^2 + \sqrt{2} \sum_i (A_{\Sigma_i} v_2 u_i \tilde{u} - \mu v_1 Y_{\Sigma_i} \tilde{u} u_i + Y_{\Sigma_i} M v_2 u_i \tilde{u}).

(30)

Minimizing $\langle V_{\text{neutral}} \rangle$ with respect to $v_1$, $v_2$, $\tilde{u}$ and $u_i$ we get the following four equations,

$$
2(\mu^2 + m_{H_u}^2)v_1 - 2bv_2 + \frac{v_1}{2}(g^2 + g'^2)(v_1^2 - v_2^2 + \Sigma_i u_i^2) - \sqrt{2} \mu \bar{u} \sum_i Y_{\Sigma_i} u_i = 0,

(31)

2(\mu^2 + m_{H_u}^2)v_2 - 2bv_1 - \frac{v_2}{2}(g^2 + g'^2)(v_1^2 - v_2^2 + \sum_i u_i^2) + v_2 ((\sum_i Y_{\Sigma_i} u_i)^2 + \sum_i (Y_{\Sigma_i})^2 \tilde{u}^2)
+ \sqrt{2} \sum_i (A_{\Sigma_i} + Y_{\Sigma_i} M) u_i \tilde{u} = 0

(32)

2(m_{\Sigma}^2 + 2M^2 + 2m_{\Sigma}^2)\tilde{u} + (\sum_i Y_{\Sigma_i} u_i)^2 \tilde{u} + \sqrt{2} \sum_i (A_{\Sigma_i} v_2 u_i - \mu Y_{\Sigma_i} v_1 u_i + Y_{\Sigma_i} M v_2 u_i)
+ \sum_i (Y_{\Sigma_i})^2 \tilde{u} v_2^2 = 0,

(33)

\frac{u_i}{2} (g^2 + g'^2)(v_1^2 - v_2^2 + \sum_j u_j^2) + (v_2^2 + \tilde{u}^2)[Y_{\Sigma_i}^2 u_i + Y_{\Sigma_i} \sum_{j \neq i} Y_{\Sigma_j} u_j] + \sqrt{2} A_{\Sigma_i} v_2 \tilde{u}
+ \sqrt{2} [Y_{\Sigma_i} M v_2 \tilde{u} - \mu Y_{\Sigma_i} v_1 \tilde{u}] + 2m_{\Sigma}^2 u_i = 0.

(34)
respectively. In the last equation note that the index $i$ is *not summed over*. As mentioned before, since $\tilde{\nu}_L$ and $\Sigma^c_R$ have nontrivial R-parity, hence R-parity is spontaneously broken when $\tilde{\nu}_L$ and $\Sigma^c_R$ take vacuum expectation values. As a result of this spontaneous R-parity violation, the bilinear term $LH_u$ which will contribute to the neutrino mass matrix is generated. We will discuss in detail about the neutralino-neutrino and chargino-charged lepton mass matrix in the next section.

The minimization conditions given in Eqs. (31)-(34) can be used to give constraints on the vacuum expectation values $u_i$ and $\tilde{u}_i$. In order to get such relations we drop the generation indices for the moment and consider $u_i = u$ for simplicity. In this case $Y_{\Sigma}$, $A_{\Sigma}$ and $m_{\tilde{\nu}}^2$ contain no generation index and would be just three numbers. From the simplified version of the last two equations Eq. (33) and Eq. (34) it can then be shown that in the limit that $u$ is small the two $R_p$ breaking vacuum expectation values $u$ and $\tilde{u}$ are proportional to each other. Combining these two equations one gets

$$\tilde{u}^2 = \frac{u^2}{Y_{\Sigma}v_2^2 + 2(m_{\tilde{\nu}}^2 + 2M^2)} \left[ \frac{1}{2} (g^2 + g'^2)(v_1^2 - v_2^2 + u^2) + 2m_{\tilde{\nu}}^2 + Y_{\Sigma}^2 v_2^2 \right].$$  (35)

Hence for small $u$ which is demanded from the smallness of neutrino mass (see the next section and Appendix C), $\tilde{u}$ will also be of the same order as $u$ unless there is a cancellation between the terms in the denominator. In this work we will stick to the possibility of small $u$ and $\tilde{u}$. We will show in the next section that one needs $u \sim 10^{-3}$ GeV to explain the neutrino data. Hence $\tilde{u}$ will also have to be $10^{-3}$ GeV. In the $u = 0$ limit, $\tilde{u}$ would also be 0 and this is our usual R-parity conserving scenario.

### 4 Neutralino-Neutrino Mass Matrix

In this section we discuss the consequence of R-parity violation through the neutralino-neutrino mixing. It is well known [40] that R-parity violation results in mixing between the neutrino-neutralino states. In our model the neutrino sector is enlarged and includes both the standard model neutrino $\nu_L$, as well as the heavier neutrino state $\Sigma^c_R$, which is a component of SU(2) triplet superfield. Since R-parity is violated we get higgsino-neutrino mixing terms $\frac{Y_{\Sigma}}{\sqrt{2}} u \tilde{H}_u^0 \nu_L$ and $\frac{Y_{\Sigma}}{\sqrt{2}} u \tilde{H}_u^0 \Sigma^c_R$, in addition to the conventional R-parity conserving Dirac mass term $\frac{Y_{\Sigma}}{\sqrt{2}} v u \Sigma^0_R \nu_L$. The R-parity breaking former two terms originated from the term $\frac{Y_{\Sigma}}{\sqrt{2}} \tilde{H}_u^0 \Sigma^c_R \nu_L$ in Eq. (11), once the sneutrino fields $\tilde{\nu}_L$ and $\Sigma^c_R$ get vacuum expectation values. The third term also has the same origin and it is the conventional Dirac mass term in Type-I or Type-III seesaw. In addition to the higgsino-neutrino mixing terms generated from the superpotential $W_{\Sigma}$, there would also be gaugino-neutrino mixing terms generated from the kinetic term of the $\tilde{L}$ and $\tilde{\Sigma}^c_R$. In the Appendix B we show explicitly the contributions coming from $W_{\Sigma}$ and the neutrino-sneutrino-gaugino terms originating from the kinetic term of the triplet superfield $\Sigma^c_R$ written down in Eq. (12).

---

$^5$We will show later that small $u$ is demanded from the smallness of neutrino mass.
Here we write the color singlet neutral-fermion mass matrix of this model in the basis
\[ \psi = (\lambda^0, \lambda^3, H_u^0, H_d^0, \Sigma_R^c, \nu_{L_1}, \nu_{L_2}, \nu_{L_3})^T \] where with one generation of \( \Sigma_R \), the neutral fermion mass matrix is a \( 8 \times 8 \) matrix. The mass term is given by
\[ L_n = -\frac{1}{2} \psi^T M_n \psi + h.c. \] (36)
where
\[ M_n = \frac{1}{\sqrt{2}} \left( \begin{array}{cccccc}
\sqrt{2} M^1 & 0 & -g' v_1 & g' v_2 & 0 & -g' u_1 & -g' u_2 & -g' u_3 \\
0 & \sqrt{2} M^2 & g v_1 & -g v_2 & 0 & g u_1 & g u_2 & g u_3 \\
g' v_1 & g v_1 & 0 & -\sqrt{2} \mu & 0 & 0 & 0 & 0 \\
g' v_2 & -g v_2 & -\sqrt{2} \mu & 0 & \Sigma_i Y_{\Sigma_i} u_i & Y_{\Sigma_1} u \tilde{u} & Y_{\Sigma_2} u \tilde{u} & Y_{\Sigma_3} u \tilde{u} \\
0 & 0 & 0 & \Sigma_i Y_{\Sigma_i} u_i & \sqrt{2} M & Y_{\Sigma_1} v_2 & Y_{\Sigma_2} v_2 & Y_{\Sigma_3} v_2 \\
-g' u_1 & g u_1 & 0 & Y_{\Sigma_1} \tilde{u} & Y_{\Sigma_1} v_2 & 0 & 0 & 0 \\
-g' u_2 & g u_2 & 0 & Y_{\Sigma_2} \tilde{u} & Y_{\Sigma_2} v_2 & 0 & 0 & 0 \\
-g' u_3 & g u_3 & 0 & Y_{\Sigma_3} \tilde{u} & Y_{\Sigma_3} v_2 & 0 & 0 & 0 \\
\end{array} \right). \] (37)

Here \( M^{1,2} \) are the soft supersymmetry breaking gaugino mass parameters (see Appendix B), whereas \( M \) corresponds to the triplet-fermion bilinear term. We define the \( 3 \times 5 \) matrix \( m_D \) as
\[ m_D^T = \frac{1}{\sqrt{2}} \left( \begin{array}{ccc}
-g' u_1 & g u_1 & 0 \\
-g' u_2 & g u_2 & 0 \\
-g' u_3 & g u_3 & 0 \\
\end{array} \right) \left( \begin{array}{c}
Y_{\Sigma_1} u \tilde{u} \\
Y_{\Sigma_1} v_2 \\
Y_{\Sigma_2} u \tilde{u} \\
Y_{\Sigma_2} v_2 \\
Y_{\Sigma_3} u \tilde{u} \\
Y_{\Sigma_3} v_2 \\
\end{array} \right), \] (38)
Defined in this way, the \( 8 \times 8 \) neutral fermion mass matrix can be written as
\[ M_n = \left( \begin{array}{cc}
M' \\
m_D^T \\
0 \\
\end{array} \right), \] (39)
where \( M' \) represents the \( 5 \times 5 \) matrix
\[ M' = \frac{1}{\sqrt{2}} \left( \begin{array}{ccccccc}
\sqrt{2} M^1 & 0 & -g' v_1 & g' v_2 & 0 \\
0 & \sqrt{2} M^2 & g v_1 & -g v_2 & 0 \\
g' v_1 & g v_1 & 0 & -\sqrt{2} \mu & 0 \\
g' v_2 & -g v_2 & -\sqrt{2} \mu & 0 & \Sigma_i Y_{\Sigma_i} u_i \\
0 & 0 & 0 & \Sigma_i Y_{\Sigma_i} u_i & \sqrt{2} M \\
\end{array} \right). \] (40)

The low energy neutrino mass would be generated once the neutralino and exotic triplet fermions get integrated out. Hence the low energy neutrino mass matrix \( M_\nu \) is
\[ M_\nu \sim -m_D^T M'^{-1} m_D. \] (41)

For \( M' \) in the TeV range, \( M_\nu \sim 1 \) eV demands that \( m_D \) should be \( 10^{-3} \) GeV. If one takes \( v_2 \sim 100 \) GeV then this sets \( Y_{\Sigma_i} \sim 10^{-5} \) and the scale of \( u \) to be \( 10^{-3} \) GeV. Since in our model for small value of \( u \), the \( \tilde{u} \) and \( u \) are proportional to each other, hence we naturally get \( \tilde{u} \sim u \sim 10^{-3} \) GeV. We have discussed in more detail in Appendix C how the smallness of neutrino mass can restrict the vacuum expectation values \( u_i, \tilde{u} \) and the Yukawas \( Y_{\Sigma_i} \).

One can clearly see from Eq. (37) that in the \( u = 0 \) and \( \tilde{u} = 0 \) limit, the gaugino-higgsino sector completely decouples from the standard model neutrino-exotic neutrino sector and the low energy neutrino mass would be governed via the usual Type-III seesaw only.
5 Chargino-Charged Lepton Mass Matrix

Like the neutralino-neutrino mixing as discussed in the previous section, R-parity violation will also result in chargino-charged lepton mixing, which in our model is significantly different compared to the other existing models of spontaneous and explicit R-parity violation, because of the presence of extra heavy triplet charged fermionic states in our model. Like the enlarged neutrino sector ($\nu_L, \Sigma^0_{\tau}$) we have also an extended charged lepton sector. With one generation of $\Sigma^c_R$ we have two additional heavier triplet charged leptons $\Sigma^+_{cR}$ and $\Sigma^-_{cR}$ in our model, in addition to the standard model charged leptons. Hence we get mixing between the charginos and the standard model charged leptons as well as the heavier triplet charged leptons. The possible contributions to the different mixing terms would come from the superpotential as well as from the kinetic terms of the different superfields. Since we have written down explicitly the charginos-charged lepton-sneutrino interaction terms in Appendix, it is straightforward to see the contribution to the mass matrix coming from Eq. (58), Eq. (59) and Eq. (61) once the $\tilde{\Sigma}^0_{cR}$ and $\tilde{\nu}_L$ states get vacuum expectation values. The chargino-charged lepton mass matrix in the basis

$$L_c = -\psi_1^T M_c \psi_2 + h.c,$$

where

$$M_c = \frac{1}{\sqrt{2}} \begin{pmatrix}
\sqrt{2}M^2_{v_1} & \sqrt{2}gv_1 & \sqrt{2}gu_1 & \sqrt{2}gu_2 & \sqrt{2}gu_3 & g\tilde{u} \\
\sqrt{2}gv_2 & \sqrt{2}\mu & Y_{\Sigma_1} \tilde{u} & Y_{\Sigma_2} \tilde{u} & Y_{\Sigma_3} \tilde{u} & -\Sigma_1 \sqrt{2}Y_{\Sigma_1} u_i \\
0 & -\sqrt{2}Y_{e_1} u_1 & \sqrt{2}Y_{e_1} v_1 & 0 & 0 & 0 \\
0 & -\sqrt{2}Y_{e_2} u_2 & 0 & \sqrt{2}Y_{e_2} v_1 & 0 & 0 \\
0 & -\sqrt{2}Y_{e_3} u_3 & 0 & 0 & \sqrt{2}Y_{e_3} v_1 & 0 \\
-g\tilde{u} & 0 & \sqrt{2}Y_{\Sigma_1} v_2 & \sqrt{2}Y_{\Sigma_2} v_2 & \sqrt{2}Y_{\Sigma_3} v_2 & \sqrt{2}M
\end{pmatrix}.$$

The chargino-charged lepton mass matrix would be diagonalized by bi-unitary transformation $M_c = T M_c^d S^T$.

6 Neutrino Mass and Mixing

R-parity violation can contribute significantly to the $3 \times 3$ standard model light neutrino mass matrix. In this section we concentrate on determining the neutrino mass square differences and the appropriate mixings. It is well known that with only one generation of singlet/triplet heavy Majorana neutrino it is not possible to get viable neutrino mass square differences and mixings in the R-parity conserving Type-I or Type-III seesaw scenario. Since R-parity is violated, we have neutrino-neutralino mixing apart from the conventional standard model neutrino-heavy neutrino mixing, which has significant effect in determining the low energy neutrino mass square differences and mixing angles.
of PMNS mixing matrix, through the gaugino and higgsino mass parameters $M^{1,2}$, $\mu$ and the R-parity violating sneutrino vacuum expectation values $u_i$ and $\tilde{u}$. Below we write the approximate $3 \times 3$ standard model neutrino mass matrix. Since $Y_{\Sigma_i}$, $u_i$ and $\tilde{u}$ are very small, all the terms which are proportional to $Y^2_{\Sigma_i} u_i^2$ and the terms $Y^3_{\Sigma_i} u_i \tilde{u}$ are neglected and we write down only the leading order terms. The exact analytical expression of the low energy neutrino mass matrix for our model has been given in the Appendix C. The approximate light neutrino mass matrix has the following form,

$$M_\nu \sim \frac{v_2^2}{2M} A + \frac{\alpha \mu}{2} B + \frac{\alpha \tilde{u} v_1}{2\sqrt{2}} C,$$  \hspace{1cm} (44)$$

where the matrix $A$, $B$ and $C$ respectively are,

$$A = \begin{pmatrix} Y^2_{\Sigma_1} & Y_{\Sigma_1} Y_{\Sigma_2} & Y_{\Sigma_1} Y_{\Sigma_3} \\ Y_{\Sigma_1} Y_{\Sigma_2} & Y^2_{\Sigma_2} & Y_{\Sigma_2} Y_{\Sigma_3} \\ Y_{\Sigma_1} Y_{\Sigma_3} & Y_{\Sigma_2} Y_{\Sigma_4} & Y^2_{\Sigma_3} \end{pmatrix}, \hspace{1cm} (45)$$

$$B = \begin{pmatrix} u_1^2 & u_1 u_2 & u_1 u_3 \\ u_1 u_2 & u_2^2 & u_2 u_3 \\ u_1 u_3 & u_2 u_3 & u_3^2 \end{pmatrix}, \hspace{1cm} (46)$$

$$C = \begin{pmatrix} 2 u_1 Y_{\Sigma_1} & u_1 Y_{\Sigma_2} + u_2 Y_{\Sigma_1} & u_1 Y_{\Sigma_3} + u_3 Y_{\Sigma_1} \\ u_1 Y_{\Sigma_2} + u_2 Y_{\Sigma_1} & 2u_2 Y_{\Sigma_2} & u_2 Y_{\Sigma_3} + u_3 Y_{\Sigma_2} \\ u_1 Y_{\Sigma_3} + u_3 Y_{\Sigma_1} & u_2 Y_{\Sigma_3} + u_3 Y_{\Sigma_2} & 2u_3 Y_{\Sigma_3} \end{pmatrix}. \hspace{1cm} (47)$$

The parameter $\alpha$ depends on gaugino masses $M^{1,2}$, the higgsino mass parameter $\mu$ and two vacuum expectation values $v_1, v_2$ as follows

$$\alpha = \frac{(M_1 g^2 + M_2 g'^2)}{M_1 M_2 \mu - (M_1 g^2 + M_2 g'^2)v_1 v_2}. \hspace{1cm} (48)$$

Note that the 1st term in Eq. (44) which depends only on the Yukawa couplings $Y_{\Sigma_i}$, triplet fermion mass parameter $M$ and the vacuum expectation value $v_2$, is the conventional R-parity conserving Type-I or Type-III seesaw term. The 2nd and 3rd terms involve the gaugino mass parameters $M^{1,2}$, the higgsino mass parameter $\mu$ and R-parity violating vacuum expectation values $u_i$ and $\tilde{u}$. Hence the appearance of these two terms are undoubtedly the artifact of R-parity violation.

We next discuss the neutrino oscillation parameters, the three angles in the $U_{PMNS}$ mixing matrix and two mass square differences $\Delta m^2_{21}$ and $\Delta m^2_{31}$. Note that with the mass matrix $M_\nu$ given in Eq. (45), i.e taking only the effect of triplet Yukawa contribution into

$$\text{Note: The standard charged lepton mass matrix which is obtained from Eq. (43) turns out to be almost diagonal and therefore the PMNS mixing comes almost entirely from } M_\nu.$$
account one would get the three following mass eigenvalues for the three light standard model neutrinos,

\[ m_1 = 0, \quad m_2 = 0, \quad m_3 = \frac{v^2}{2M}(Y^2_{\Sigma_1} + Y^2_{\Sigma_2} + Y^2_{\Sigma_3}) \]

and the eigenvectors

\[
\frac{1}{\sqrt{Y^2_{\Sigma_1} + Y^2_{\Sigma_3}}} \begin{pmatrix} -Y_{\Sigma_3} \\ Y_{\Sigma_1} \end{pmatrix}, \quad \frac{1}{\sqrt{Y^2_{\Sigma_1} + Y^2_{\Sigma_2}}} \begin{pmatrix} -Y_{\Sigma_2} \\ Y_{\Sigma_1} \end{pmatrix}, \quad \frac{1}{\sqrt{Y^2_{\Sigma_1} + Y^2_{\Sigma_2}}} \begin{pmatrix} Y_{\Sigma_1} \\ Y_{\Sigma_2} \end{pmatrix}
\]

respectively. Clearly, two of the light neutrinos emerge as massless while the third one gets mass, which is in conflict with the low energy neutrino data. Similarly if one has only matrix \( B \) which comes as a consequence of R parity violation then one also would obtain only one light neutrino to be massive, in general determining only the largest atmospheric mass scale \([41, 42]\). However the simultaneous presence of the matrix \( A, B \) and \( C \) in Eq. \((44)\) make a second eigenvalue non-zero, while the third one remains zero. With the choice \( \tilde{u} \) as of the same order of \( u \), the third term in Eq. \((44)\) would be suppressed compared to the first two terms. Hence the simultaneous presence of the matrix \( A \) and \( B \) are very crucial to get both the solar and atmospheric mass splitting and the allowed oscillation parameters. Eigenvalues of the full \( M_\nu \) given in Eq. \((44)\) are

\[ m_1 = 0, \quad m_{2,3} \sim \frac{1}{2}[W \mp \sqrt{W^2 - V}], \quad (49)\]

where

\[ W = \frac{v^2}{2M} \sum_i Y^2_{\Sigma_i} + \frac{\mu\alpha}{2} \sum_i u_i^2 + \frac{\tilde{u}v_1\alpha}{\sqrt{2}} \sum_i u_i Y_{\Sigma_i}, \quad (50)\]

and

\[ V = 4\left(\frac{v^2 \mu \alpha}{4M} - \frac{\tilde{u}^2 v_1^2 \alpha^2}{8}\right)[Y^2_{\Sigma_1}(u_2^2 + u_3^2) + Y^2_{\Sigma_2}(u_3^2 + u_1^2) + Y^2_{\Sigma_3}(u_1^2 + u_2^2) - 2(u_1 u_2 Y_{\Sigma_1} Y_{\Sigma_2} + u_1 u_3 Y_{\Sigma_1} Y_{\Sigma_3} + u_2 u_3 Y_{\Sigma_2} Y_{\Sigma_3})]. \quad (51)\]

Similarly we have obtained approximate analytic expressions for the mixing matrix, however the expressions obtained are too complicated and hence we do not present them here. Instead we show in Tables 1 and 2 an example set of model parameters \((M^{1,2}, M, \mu, Y_{\Sigma_i}, v_{1,2}, \tilde{u}, u_i)\) which give the experimentally allowed mass-square differences \( \Delta m^2_{21} \) and \( \Delta m^2_{31} \) and mixing angles \( \sin^2 \theta_{12} \), \( \sin^2 \theta_{23} \) and \( \sin^2 \theta_{13} \), as well as the total neutrino mass \( m_t \). The values obtained for these neutrino parameters for the model points given in Tables 1 and 2 is shown in Table 3. We have presented these results assuming \( \Delta m^2_{31} > 0 \) (normal hierarchy).
\begin{center}
\begin{tabular}{|c|c|c|c|c|c|c|}
\hline
$M^1$ (GeV) & $M^2$ (GeV) & $M$ (GeV) & $\mu$ (GeV) & $Y_{\Sigma_1}$ & $Y_{\Sigma_2}$ & $Y_{\Sigma_3}$ \\
\hline
300 & 600 & 353.24 & 88.31 & $5.62 \times 10^{-7}$ & $8.72 \times 10^{-7}$ & $3.84 \times 10^{-8}$ \\
\hline
\end{tabular}
\end{center}

Table 1: Sample point in the parameter space for the case of normal hierarchy. $M^{1,2}$ is the gaugino mass parameter, $\mu$ is higgsino mass parameter, $M$ is triplet fermion mass parameter, $Y_{\Sigma_1}$, $Y_{\Sigma_2}$ and $Y_{\Sigma_3}$ correspond to the superpotential coupling between the standard model Lepton superfields $\tilde{L}_i$, SU(2) triplet superfield $\Sigma^0_R$ and Higgs superfield $H_u$.

\begin{center}
\begin{tabular}{|c|c|c|c|c|c|}
\hline
$v_1$ (GeV) & $v_2$ (GeV) & $\tilde{u}$ (GeV) & $u_1$ (GeV) & $u_2$ (GeV) & $u_3$ (GeV) \\
\hline
10.0 & 100.0 & $5.74 \times 10^{-3}$ & $1.69 \times 10^{-5}$ & $9.55 \times 10^{-5}$ & $1.26 \times 10^{-4}$ \\
\hline
\end{tabular}
\end{center}

Table 2: Sample point in the parameter space for the case of normal hierarchy. $v_{1,2}$ are the vacuum expectation values of $H^0_u$ fields respectively, $\langle \tilde{\nu}_{Li} \rangle = u_i$ for i=1,2,3 and $\tilde{u}$ is the vacuum expectation value of triplet sneutrino state $\tilde{\Sigma}^0_R$.

At this point we would like to comment on the possibility of radiatively-induced neutrino mass generation in our model. In a generic R-parity violating MSSM, both the lepton number and baryon number violating operators are present. Apart from the tree level bilinear term $\hat{L}\hat{H}_u$ which mixes neutrino with higgsino, the trilinear lepton number violating operators $\lambda \hat{L}\hat{L}\hat{E}^c$, $\lambda' \hat{L}\hat{Q}\hat{D}^c$ and also the bilinear operator $\hat{L}\hat{H}_u$ contribute to the radiatively-induced neutrino mass generation [28,30]. In general there could be several loops governed by the $\lambda\lambda'$, $BB$, $\epsilon B$ couplings. However in the spontaneous R-parity violating model, working in the weak basis we would only have $\hat{L}\hat{H}_u$ operator coming from $H_u\Sigma^0_R\hat{L}$ term in the superpotential. Similarly in the scalar potential we would obtain $H_u\tilde{L}$ coupling coming out from $H_u\tilde{\Sigma}^0_R\tilde{L}$ term in Eq. (14). Hence we can have loops governed by $A_{\Sigma}A_{\Sigma}$ couplings, like the $BB$ loop in Fig. 3 of [28]. For the sake of completeness we have presented the diagram in Fig. 1. The one loop corrected neutrino mass coming out from this diagram would be $m_{\nu} \sim \frac{g^2u^2}{64\pi^2\cos^2\beta} \frac{A_{\Sigma}A_{\Sigma}}{m^2}$. In general for moderate values of $\cos\beta$, if one chooses the average slepton mass $\bar{m}$ to be in the TeV order and the soft supersymmetry breaking coupling $A_{\Sigma} \sim 10^2$ GeV, then because $\tilde{u} \sim 10^{-3}$ GeV, the contribution coming from this diagram would be roughly suppressed by a factor of $10^{-2}$ compared to the tree level neutrino mass. Similar conclusion can be drawn for the $\epsilon A_{\Sigma}$ loop induced neutrino mass, shown in Fig. 2. In our model the R parity violating $\lambda$ and $\lambda'$ couplings do not get generated in the weak basis. However, from the R-parity conserving superpotential given in Eq. (3) it would be possible to generate $\lambda$ and $\lambda'$ couplings via mixings and only after transforming to a mass basis. Hence, in general for our model we would expect the contributions coming from the $\lambda\lambda$ and $\lambda'\lambda'$ loops to be suppressed compared to the tree level neutrino masses. Apart from these different bilinear and trilinear radiatively induced neutrino mass generation, there could be another source of radiative neutrino mass, namely the non-universality in the slepton mass matrices. In the R parity conserving limit the analysis would be same as of [43], only triplet fermions in our model are generating the Majorana sneutrino masses.
\[ \Delta m^2_{21} (eV^2) \quad \Delta m^2_{31} (eV^2) \quad \sin^2 \theta_{12} \quad \sin^2 \theta_{23} \quad \sin^2 \theta_{13} \quad m_t (eV) \\
7.44 \times 10^{-3} eV^2 \quad 2.60 \times 10^{-3} eV^2 \quad 0.33 \quad 0.507 \quad 4.34 \times 10^{-2} \quad 10^{-2} \]

Table 3: Values for neutrino oscillation parameters for the input parameters specified in Table 1 and in Table 2.

\( \tilde{\nu}_i \tilde{\nu}_j \). However, in this work we stick to the universality of the slepton masses, hence this kind of radiative neutrino mass generation will not play any non trivial role. Due to the RG running from the high scale to the low scale the universal soft supersymmetry breaking slepton masses could possibly get some off-diagonal contribution \[44\], which we do not address in this present work.

7 Collider Signature

In this section we discuss very briefly about the possibility of testing our model in collider experiments. Because R-parity is violated in our model there will be extra channels compared to the R-parity conserving minimal supersymmetric standard model, which carry the information about R-parity violation. As R-parity gets broken, the triplet neutral heavy lepton \( \Sigma^0_{3R} \) and standard model neutrino \( \nu_{Li} \) mix with the neutral higgsino \( \tilde{H}^0_u \) and \( \tilde{H}^0_d \), with bino \( \tilde{\chi}^0 \) and wino \( \tilde{\lambda}^3 \), in addition to the usual R-parity conserving Dirac mixing between them. Hence in the mass basis with one generation of heavy triplet fermion, there will be 5 neutralinos in our model. We adopt the convention where the neutralino state \( \tilde{\chi}^0_i \) are arranged according to the descending order of their mass and \( \tilde{\chi}^0_5 \) is the lightest neutralino. If one adopts gravity mediated supersymmetry breaking as the origin of soft supersymmetry breaking Lagrangian, then the lightest neutralino is in general the lightest supersymmetric particle (LSP). But as R-parity is broken, in any case LSP will not be stable \[41, 45\]. For MSSM, the production mechanism of neutralinos and sneutrinos in colliders have been extensively discussed in \[24, 46\]. Depending on the parameters, the lightest neutralino can be gaugino dominated or higgsino dominated. In our model in addition to the standard model neutrinos we also have a heavy triplet neutral fermion \( \Sigma^0_{3R} \). Hence in this kind of model where heavy triplet fermions are present, the lightest neutralino can also be \( \Sigma^0_{3R} \) dominated. Detail analysis of the collider signatures of model and implications for LHC will be presented in a forthcoming paper. Here we present a qualitative discussion on the possible neutralino, sneutrino, slepton and chargino decay modes.

- (A) Neutralino two body decay: As R-parity is violated, the lightest neutralino can decay through R-parity violating decay modes. It can decay via the two body decay mode \( \tilde{\chi}^0_i \rightarrow l^\pm W^{\mp} \) or \( \tilde{\chi}^0_i \rightarrow \nu Z \). Other heavier neutralinos can decay to the lighter neutralino state \( \tilde{\chi}^0_i \rightarrow \tilde{\chi}^0_j Z \), or through the decay modes \( \tilde{\chi}^0_i \rightarrow l^\pm W^{\mp}/\nu Z \). The gauge boson can decay leptonically or hadronically producing

\[ \tilde{\chi}^0_i \rightarrow l^\pm W^{\mp} \rightarrow l^\pm l ^\mp + E_T, \]
\[\tilde{\chi}_i^0 \rightarrow l^\pm W^\mp \rightarrow l^\pm + 2j,\]
\[\tilde{\chi}_i^0 \rightarrow \nu Z \rightarrow l^\pm \bar{l}^\mp + E_T,\]
\[\tilde{\chi}_i^0 \rightarrow \nu Z \rightarrow 2b+ E_T.\]

- **(B) Sneutrino and slepton decay:** Because of R-parity violation the slepton can decay to a charged lepton and a neutrino \[\bar{l} \rightarrow \nu l.\] The sneutrino can also have the possible decay \[\tilde{\nu} \rightarrow l^+ l^- .\] Note that in the explicit \[R_p\] violating scenario, this interaction term between \[\tilde{\nu} l^\pm l^\mp\] and \[\bar{l} l \nu\] would have significant contribution from \[\lambda L L E^c.\] Here \[l^\pm, \nu\] and \[\bar{l}\] all are in mass basis. In the spontaneous R-parity violating scenario these interactions are possible only after basis redefinition. The different contributions to the above mentioned interaction terms will come from the MSSM R-parity conserving term \[\tilde{H}_d \tilde{L} \tilde{E}^c\] as well as from the kinetic terms of the different superfields after one goes to the mass basis.

- **(C) Three body neutralino decay modes:** The other possible decay modes for the neutralino are \[\tilde{\chi}_i^0 \rightarrow \nu \bar{\nu} \rightarrow l^\pm l^\mp, \nu h.\] If the lightest neutralino \[\tilde{\chi}_i^0\] is the lightest supersymmetric particle, then the slepton or sneutrino would be virtual. The sneutrino or slepton can decay through the R-parity violating decay modes. Hence neutralino can have three body final states such as \[\tilde{\chi}_i^0 \rightarrow \nu \bar{\nu} \rightarrow l^\pm l^\mp, \tilde{\chi}_i^0 \rightarrow l^\pm \bar{l}^\mp \rightarrow l^\pm \nu l^\mp.\]

Our special interest is in the case where the lightest neutralino state is significantly triplet fermion \[\Sigma^0\] dominated. Besides the Yukawa interaction, the triplet fermion \[\Sigma^\pm, \Sigma^0\] have gauge interactions. Hence they could be produced at a significant rate in a proton proton collider such as LHC through gauge interactions. The triplet fermions \[\Sigma^\pm\] and \[\Sigma^0\] could be produced via \[pp \rightarrow W^\pm/Z \rightarrow \Sigma^\pm \Sigma^0/\Sigma^\mp\] channels apart from the Higgs mediated channels. In fact the production cross section of these triplet fermions should be more compared to the production cross section of the singlet neutrino dominated neutralino states [31]. as for the former case the triplet fermions have direct interactions with the gauge bosons. The decay channels for these triplet neutral fermions are as \[\Sigma^0 \rightarrow \nu Z, \Sigma^0 \rightarrow l^\pm W^\mp, \Sigma^0 \rightarrow \nu h\] and also \[\Sigma^0 \rightarrow \nu \bar{\nu}^*, l^* l^*\] followed by R-parity violating subsequent decays of the slepton/sneutrino.

Apart from the neutralino sector in our model, the charged higgsino and charged winos mix with standard model leptons and heavy charged leptons \[\Sigma^\pm.\] Hence, just like in the neutralino sector, there will also be R-parity violating chargino decay. The chargino \[\tilde{\chi}_i^\pm\] can decay into the following states, \[\tilde{\chi}_i^\pm \rightarrow l^\pm Z, \nu W, \nu h^\pm\] and also to \[\tilde{\chi}_i^\pm \rightarrow l \bar{\nu} \rightarrow l l^\pm l^\mp, \tilde{\chi}_i^\pm \rightarrow l \bar{l} \rightarrow l l.\] Just like as for the neutralinos, depending on the parameters, in this kind of model with heavy extra triplet fermions the lightest chargino could be as well \[\Sigma^\pm\] dominated. Moreover, because of R-parity violation, there will be mixing between the the slepton and charged Higgs and sneutrino and neutral Higgs. The sneutrino state \[\tilde{\Sigma}\] can have the decays \[\tilde{\Sigma}^\pm \rightarrow \nu l^\pm, \nu W^\mp, l^\pm Z.\] Similarly, \[\tilde{\Sigma}^0\] can decay into \[\tilde{\Sigma}^0 \rightarrow l^+ l^- .\]

\footnote{By \[\Sigma^\pm, \Sigma^0\] we mean chargino state \[\tilde{\chi}^\pm\] or neutralino state \[\tilde{\chi}^0\] significantly dominated by \[\Sigma^\pm\] and \[\Sigma^0\] respectively.}
Apart from these above mentioned decay channels, some other possible decay channels are, $\Sigma^+ \rightarrow d\bar{u}$, $\Sigma^0 \rightarrow u\bar{u}$, $\Sigma^+ \rightarrow u\bar{d}$. We reiterate that by $\Sigma^{\pm,0}$ and $\tilde{\Sigma}^{\pm,0}$, we mean triplet dominated chargino/neutralino or sfermionic states. Note that for the R-parity conserving seesaw scenario, these decay modes will be totally absent, as there is no mixing between the triplet/singlet fermion and the higgsino/gauginos and mixing between sfermions and Higgs bosons.

We would also like to comment about the possibility of lepton flavor violation in our model. For the non-supersymmetric Type III seesaw, the reader can find detailed study on lepton flavor violation in [14]. Embedding the triplet fermions in a supersymmetric framework opens up many new diagrams which can contribute to lepton flavor violation, for example $\mu \rightarrow e\gamma$. In general the sneutrino, triplet sneutrino, different sleptons and the charginos or neutralinos can flow within the loop [44, 48–50]. In our model the R-parity violating effect comes very selectively. The main contribution comes via the bilinear R-parity violating terms which get only generated spontaneously. Hence as discussed before, we ignore $\lambda$ and $\lambda'$ dominated diagrams [49] and we expect that the lepton flavor violating contribution would be mainly governed by the soft supersymmetry breaking off-diagonal slepton mass contribution. Since we stick to the universal soft supersymmetry breaking slepton masses, only RG running might generate the off diagonal supersymmetry breaking slepton masses [44]. In the R parity conserving limit the soft supersymmetry breaking slepton masses get a contribution $m_L^2 \sim (3m_0^2 + A_0^2)(Y_{\Sigma}^T Y_{\Sigma}/M_{\Sigma}) \log(M_{\Sigma}/M_X)^8$. For $m_0, A_0 \sim \text{TeV}$ and $Y_{\Sigma} \sim 10^{-6}$, $m_L^2 \sim 10^{-6} \log(M_{\Sigma}/M_X)\text{GeV}^2$. The branching ratio would be roughly $\frac{m_0^2 |m_L^2|^2}{\alpha_F^2 m^2 \tan^2 \beta}$. However, because of extremely small Yukawa $Y_{\Sigma} \sim 10^{-6}-10^{-8}$ our model would not violate the bound coming from $\mu \rightarrow e\gamma$.

Before concluding this section we present a qualitative comparison between our model and other models having $SU(2)$ triplet. There have been several studies on another class of models containing $SU(2)$ triplet and $Y = 2$ chiral superfields [17]. The triplet fermions in our model share a very distinguishing feature compared to the Higgs triplet models because of hypercharge $Y = 0$. While in a model with Higgs triplet and in its supersymmetrized version one would inevitably have a doubly charged Higgs and higgsino, the model with triplet fermion and its supersymmetrized version offer only singly charged and charge neutral fermions and their scalar superpartners. Moreover though in our model we consider R parity violation, but the R parity violation comes in a very specific way. For example in [51] the authors have extended the superfield contents by adding $SU(2)$ triplet $Y = 2$ chiral superfield and in addition to this they have incorporated the explicit bilinear R parity violation. In our model we have extended the particle contents by adding one $SU(2)$ triplets $Y = 0$ chiral superfield and in our work we address the issue of spontaneous R parity violation. In fact these two different class of models offer very different collider signatures. There have been several studies on the collider signatures of Higgs triplets. The readers can look at [12, 52]. The different channels are $H^{--} \rightarrow l_i l_j$, $H^{++} \rightarrow W^- W^+$, $H^{++} \rightarrow H^+ W^+$ and so on. However, for Higgs mass in the range of $M_{H^{++}} \sim 300$ GeV the dileptonic channel

\footnote{For simplicity we have taken $Y_{\Sigma}$ to be real.}
is the most dominant one [52]. In [51] the authors have related the neutrino mixing angle with the dileptonic decay branching ratio. On the other hand in the fermionic triplet models the important decay modes are $\Sigma^\pm \rightarrow l^\pm h$, $\Sigma^\pm \rightarrow W^\pm \nu$, $\Sigma^0 \rightarrow \nu h$, $\Sigma^0 \rightarrow Z\nu$ and so on. Again, for non supersymmetric version, detailed analysis on the collider signatures could be found in [11,12]. In [16] the authors have addressed the Type-III seesaw in the context of an additional Higgs which offers very drastic difference in the triplet fermion and Higgs phenomenology. In this present work we have addressed what could be interesting collider signatures if one embeds the triplet fermion into a supersymmetric framework while incorporating R parity violation very selectively. The hypercharge $Y = 0$ and $Y = 2$ SU(2) triplet models would also differ in the different lepton flavor violating processes. Above we have presented a qualitative discussion on the lepton flavor violation for $Y = 0$ triplet. Very detailed study on the lepton flavor violating processes, mediated by the Higgs triplet fields have been done in [13,53]. In general the doubly charged Higgs can mediate the $\mu \rightarrow 3e$ processes at the tree level, while for the process $\mu \rightarrow e\gamma$ the Higgs triplet fields would contribute via loops. Some detailed analysis have been presented in [13,53].

8 Conclusion

In this work we have explored the possibility of spontaneous R-parity violation in the context of basic MSSM gauge group. Since the R-parity violating terms also break lepton number, spontaneous R-parity violation could potentially generate the massless mode Majoron. We avoid the problem of the Majoron by introducing explicit breaking of lepton number in the R-parity conserved part of the superpotential. We do this by adding a SU(2) triplet $Y = 0$ matter chiral superfield $\Sigma^c_R$ in our model. The gauge invariant bilinear term in these triplet superfields provides explicit lepton number violation in our model. The superpartners of the standard model neutrino and the triplet heavy neutrino states acquire vacuum expectation values, thereby breaking R-parity spontaneously.

From the minimization condition of the scalar potential, we showed that in our model, the vacuum expectation values of the superpartner of the triplet heavy neutrino and the standard model neutrinos turn out to be proportional to each other. For the supersymmetry breaking soft masses in the TeV range, smallness of neutrino mass ($\sim eV$) constrains the standard model sneutrino vacuum expectation value $u \sim 10^{-3}$ GeV. Since for small $u$ the sneutrino vacuum expectation values $u$ and $\tilde{u}$ are proportional, hence in the absence of any fine tuned cancellation between the different soft parameters, one can expect that $\tilde{u}$ will be of the same order as $u$, i.e $10^{-3}$ GeV. We have analyzed the neutralino-neutrino mass matrix and have shown that the R-parity violation can have significant effect in determining the correct neutrino mass and mixing. Since neutrino experiments demand at least two massive neutrinos, we restrict ourselves to only one generation of $\Sigma^c_R$ superfield. While in R-parity conserving Type-III susy seesaw with only one generation of triplet neutrino state $\Sigma^0_{R}$, two of the standard model neutrinos turn out to be massless. If one invokes R-Parity violation in addition then one among the massless states can be made massive. We showed that the main impact on the neutrino oscillation parameters comes from the gaugino mass
dominated seesaw term in the neutrino mass matrix. The correct mass splitting and mixing angles for the low energy neutrino could be achieved. Alongwith the neutralino-neutrino mixing, we have chargino-charged lepton mixing in our model. Because of the presence of the triplet fermionic states, we have heavy charged fermions in our model. The heavy fermionic states modify the chargino-charged lepton mass matrix. In particular, we can have charginos which are heavy triplet fermion dominated. We also have qualitatively discussed on the radiatively induced neutrino mass for our model.

Finally we discussed the neutralino, chargino, slepton and sneutrino decay decays and the possible collider signature of this model. Because of R-parity violation the neutralino and chargino can decay through a number of R-parity violating decay channels alongwith the possible R-parity violating decays of sneutrinos and sleptons. In the context of our model, we have listed few of these channels. Depending on the parameters of the neutralino-neutrino mass matrix, the lightest neutralino could be a gaugino dominated or could be higgsino dominated. Since heavy triplet fermions are present, hence the lightest neutralino state in this kind of models could even be triplet neutrino dominated. Similar kind of conclusion would hold for the chargino states as well. Unlike the $SU(2)_L \times U(1)_Y$ singlet neutrino states, the triplet neutrino/sneutrino states have direct interaction with the standard model gauge bosons. Hence the production cross section of triplet neutrino dominated neutralino should be more than the singlet neutrino dominated neutralino states. The production cross section of triplet charged lepton dominated chargino should also be significantly higher. We expect multileptonic final states associated with jets or missing energy as a collider signature of our model. Detailed collider signatures and effective cross-sections of model warrants a separate study, which is underway and will be reported elsewhere. In addition to this we also have presented a qualitative discussion on the lepton flavor violation in our model and the relative comparison between different models having $SU(2)$ triplets with hypercharge $Y = 0$ and $Y = 2$.

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Appendix
A   Soft supersymmetry breaking lagrangian of MSSM

The soft supersymmetry breaking Lagrangian of this model is given by,

\[ L^{\text{soft}} = L^{\text{soft}}_{\text{MSSM}} + L^{\text{soft}}_{\Sigma}, \]

where \( L^{\text{soft}}_{\Sigma} \) has been written in Eq. (14) and the MSSM soft supersymmetry breaking lagrangian has the following form,

\[
- \mathcal{L}^{\text{soft}}_{\text{MSSM}} &= (m_Q^2\bar{Q}_i^j Q^j_i + (m_{\tilde{e}}^2)^{ij}\bar{\tilde{e}}_i^j \tilde{e}^c_j + (m_{\tilde{d}}^2)^{ij}\bar{\tilde{d}}_i^j \tilde{d}^c_j + (m_{\tilde{L}}^2)^{ij}\bar{\tilde{L}}_i^j \tilde{L}_j \\
&+ (m_{\tilde{e}}^2)^{ij}\bar{\tilde{e}}_i^j \tilde{e}^c_j + m^2_{\tilde{H}_u} H_u^0 H_u + m^2_{\tilde{H}_d} H_d^0 H_d + (b H_u H_d + \text{H.c.}) \\
&+ \left[-A_u^i H_u \tilde{Q}_i \tilde{u}^c_i + A_d^i H_d \tilde{Q}_i \tilde{d}^c_i + A_{\tilde{e}}^i H_d \tilde{L}_i \tilde{e}^c_i + \text{H.c.}\right] \\
&+ \frac{1}{2} \left( M^3 \tilde{g} \tilde{g}^* + M^2 \tilde{\lambda} \tilde{\lambda}^* + M^1 \tilde{\lambda} \tilde{\lambda}^0 + \text{H.c.} \right). \tag{53}
\]

where \( i \) and \( j \) are generation indices, \( m_Q^2, m_{\tilde{e}}^2 \) and other terms in the first two lines of the above equation represent the mass-square of different squarks, sleptons, sneutrino\(^9\) and Higgs fields. In the third line the trilinear interaction terms have been written down and in the fourth line \( M^3, M^2 \) and \( M^1 \) are respectively the masses of the gluinos \( \tilde{g} \), winos \( \tilde{\lambda}^{1,2,3} \) and bino \( \tilde{\lambda}^0 \).

B  Gaugino-lepton-slepton mixing

In this section we write down explicitly all the interaction terms generated from \( W_{\Sigma} \), as well as the gaugino-triplet leptons-triplet sleptons interaction terms originating from \( L^k_{\Sigma} \). As has already been discussed in section 2, \( W_{\Sigma} \) is given by

\[
W_{\Sigma} = -Y_{\Sigma_i} \hat{H}^T_u (i \sigma_2) \hat{\Sigma}^c_R \hat{L}_i + \frac{M}{2} Tr[\hat{\Sigma}_R^c \hat{\Sigma}_R]. \tag{54}
\]

\[
W_{\Sigma} = Y_{\Sigma_0} \hat{H}^0_u \tilde{\Sigma}^0_R \tilde{c} \tilde{l}_i + Y_{\Sigma_1} \hat{H}^0_u \tilde{\Sigma}^0_R \tilde{c} \tilde{l}_i - \frac{Y_{\Sigma_2}}{\sqrt{2}} \hat{H}^0_u \tilde{\Sigma}^0_R \tilde{l}_i + \frac{M}{2} \Sigma^0_R \tilde{\Sigma}^0_R + M \Sigma^0_R \tilde{\Sigma}^0_R. \tag{55}
\]

In Table. 4 we show all the trilinear interaction terms generated from \( Y_{\Sigma_i} \hat{H}^0_u \tilde{\Sigma}^0_R \tilde{l}_i \).

The kinetic terms of the \( \hat{\Sigma}^c_R \) field is given by

\[
L^k_{\Sigma} = \int d^4 \theta Tr[\hat{\Sigma}^c_R \hat{\Sigma}^c_R]. \tag{56}
\]

\(^9m_{\tilde{e}}^2 \) represents the mass square of the superpartner of the standard model neutrino and \( m_{\tilde{e}}^2 = m_{\tilde{\nu}}^2 \).
Table 4: Trilinear interaction terms between standard model leptons/sleptons, Higgs/higgsinos and the SU(2) triplet fermions/sfermions. These interaction terms originate from the superpotential $W_{\Sigma}$.

where $V$ represents the SU(2) vector supermultiplets. From the above kinetic term one will get the following gaugino-triplet fermion-triplet sfermion interactions

$$- L_{\Sigma^- \Sigma^{-} \lambda^{-}} = \frac{g}{\sqrt{2}} (\tilde{\Sigma}_R^{c} \Sigma_{R}^{-c} \lambda^{-} - (\tilde{\Sigma}_R^{+} \Sigma_{R}^{+c} \lambda^{+}) + h.c, \quad (57)$$

$$- L_{\Sigma^- \Sigma^{+} \lambda^{+}} = \frac{g}{\sqrt{2}} ((\tilde{\Sigma}_R^{c} \Sigma_{R}^{+} \lambda^{+} - (\tilde{\Sigma}_R^{0} \Sigma_{R}^{0c} \lambda^{0}) + h.c, \quad (58)$$

$$- L_{\Sigma^{-} \Sigma^{0} \lambda^{-}} = \frac{g}{\sqrt{2}} (\tilde{\Sigma}_R^{0c} \Sigma_{R}^{0} \lambda^{-} - (\tilde{\Sigma}_R^{0} \Sigma_{R}^{-c} \lambda^{-}) + h.c. \quad (59)$$

Note that the mixing terms between the gauginos and triplet fermions $\tilde{\lambda}^{+} \Sigma_{R}^{0c}$ and $\tilde{\lambda}^{-} \Sigma_{R}^{-c}$ contribute to the color singlet charged fermion mass matrix Eq. (13) and these mixing terms are generated from Eq. (58) and Eq. (59) respectively, once $\tilde{\Sigma}_R^{0c}$ gets vacuum expectation value. In addition to these interaction terms between gauginos, triplet leptons and triplet sleptons, we also explicitly write down the gaugino-standard model lepton-slepton interaction terms which will be generated from the kinetic term of the $\hat{L}_i$ superfields

$$L_L^k = \int d^4\theta \hat{L}_i \hat{L}_i^\dagger e^{2gV + 2g'\hat{V}' \hat{L}_i}. \quad (60)$$

$$L_L^k = - \frac{g}{\sqrt{2}} \tilde{\nu}_L^i \tilde{\lambda}^3 \nu_{Li} - \frac{g}{\sqrt{2}} \tilde{l}_i^+ \tilde{\lambda}^3 l_i^- - g \tilde{\nu}_L^i \tilde{\lambda}^+ l_i^- - g \tilde{l}_i^+ \tilde{\lambda}^- \nu_{Li}$$

$$+ \frac{g'}{\sqrt{2}} \tilde{\nu}_L^i \tilde{\lambda}^0 \nu_{Li} + \frac{g'}{\sqrt{2}} \tilde{l}_i^+ \tilde{\lambda}^0 l_i^- + h.c. \quad (61)$$

Similar interaction terms would be generated from $\hat{E}^c$ kinetic term and kinetic terms of other superfields like the Higgs $\tilde{H}_{u,q}$ and other quarks. Looking at Eq. (61) it is clear that once the sneutrino fields $\tilde{\nu}_L$ get vacuum expectation values the first and fifth term of the above equation would contribute to gaugino-neutrino mixing while the third term would contribute in the chargino-charged lepton mixing, as have already been shown in Eq. (37) and in Eq. (13).
The Neutrino Mass, $Y_\Sigma$ and $u$

In this section we discuss in detail how the smallness of neutrino mass restricts the order of magnitude of the Yukawa $Y_\Sigma$ and the sneutrino vacuum expectation value $u$. Below we write the analytical expression of the low energy neutrino mass of our model. The $3 \times 3$ neutrino mass matrix is

$$M_\nu \sim -m_D^T M'^{-1} m_D,$$

where $m_D$ and $M'$ have already been given in Eq. (38) and in Eq. (40) respectively. With the specified $m_D$ and $M'$, the $3 \times 3$ standard model neutrino mass matrix $M_\nu$ has the following form,

$$-M_\nu = 2v_2^2\alpha_1 A + 2M_\mu^2\alpha_t B + \sqrt{2}\alpha_t\bar{u}v_1 M_\mu C + M_\alpha v_1^2 \bar{u}^2 A$$

$$-\sqrt{2}\alpha_t\bar{u}v_1^2 v_2 \sum_{i=1,2,3} u_i Y_\Sigma_i A - \alpha_t v_1 v_2 \mu F.$$

(63)

The matrix $A$, $B$ and $C$ have already been presented in Eq. (45), Eq. (46) and Eq. (47). The matrix $F$ has the following form,

$$F = \begin{pmatrix} F_{11} & F_{12} & F_{13} \\ F_{12} & F_{22} & F_{23} \\ F_{13} & F_{23} & F_{33} \end{pmatrix},$$

(64)

where the elements of $F$ are

$$F_{ij}(i \neq j) = Y_{\Sigma_i} Y_{\Sigma_j} (u_i^2 + u_j^2) + u_i u_j (Y_{\Sigma_i}^2 + Y_{\Sigma_j}^2) + u_k Y_{\Sigma_k} (u_i Y_{\Sigma_j} + u_j Y_{\Sigma_i}),$$

(65)

and

$$F_{ii} = 2Y_{\Sigma_i}^2 u_i^2 + 2u_i Y_{\Sigma_i} \sum_{k \neq i} Y_{\Sigma_k} u_k.$$

(66)

The indices $i$, $j$, $k$ in Eq. (65) and the index $i$ in Eq. (66) are not summed over. $\alpha_t$ has this following expression,

$$\alpha_t = \frac{(M_1 g^2 + M_2 g'^2)}{4M_1 M_2 \mu^2 - 4M_\mu (M_1 g^2 + M_2 g'^2) v_1 v_2 - (M_1 g^2 + M_2 g'^2) v_1^2 (\Sigma_i u_i Y_{\Sigma_i})^2},$$

(67)

while the parameter $\alpha_1$ is

$$\alpha_1 = \frac{M_1 M_2 \mu^2 - (M_1 g^2 + M_2 g'^2) v_1 v_2 \mu}{4M_1 M_2 \mu^2 - 4M_\mu (M_1 g^2 + M_2 g'^2) v_1 v_2 - (M_1 g^2 + M_2 g'^2) v_1^2 (\Sigma_i u_i Y_{\Sigma_i})^2}.$$

(68)

Below we show how the choice of TeV scale gaugino masses and the triplet fermion mass $M$ dictate the sneutrino vacuum expectation value $u$ to be smaller than $10^{-3}$ GeV and the
Yukawa $Y_\Sigma \lesssim 10^{-5}$ to have consistent small ( $\lesssim$ eV) standard model neutrino mass. We consider the following three cases and show that only Case (A) is viable.

Case (A): If one assumes $MM_1M_2\mu^2$ and $M\mu(M_1g^2+M_2g'^2)v_1v_2 \gg (M_1g^2+M_2g'^2)u^2Y_\Sigma^2v_1^2$, the first and second term dominate over the third one in the denominator of Eq. (67) and Eq. (68). Then $\alpha_t$ and $\alpha_1$ simplify to

$$\alpha_t \sim \frac{(M_1g^2 + M_2g'^2)}{4M_1M_2\mu^2} + \ldots,$$

and

$$\alpha_1 \sim \frac{1}{4M} + \ldots \quad (70)$$

The neutrino mass matrix Eq. (68) will have the following form,

$$M_\nu \sim \frac{v_2^2Y_\Sigma^2}{2M} + \frac{u^2(M_1g^2 + M_2g'^2)}{2M_1M_2} + \sqrt{2\bar{u}v_1uY_\Sigma}(M_1g^2 + M_2g'^2) + \frac{v_2^2\bar{u}Y_\Sigma^2(M_1g^2 + M_2g'^2)}{4M_1M_2\mu^2} - \sqrt{2\bar{u}v_1uY_\Sigma^2}(M_1g^2 + M_2g'^2) + \frac{v_1v_2u^2\bar{u}Y_\Sigma^2(M_1g^2 + M_2g'^2)}{4M_1M_2\mu^2}. \quad (71)$$

The order of $Y_\Sigma$ and $u$ would be determined from the first two terms of the above equation respectively. The fourth, fifth and sixth terms of Eq. (71) cannot determine the order of $Y_\Sigma$ and $u$. To show this with an example let us consider the contribution coming from the fourth term $\frac{(M_1g^2 + M_2g'^2)v_1^2\bar{u}Y_\Sigma^2}{M_1M_2\mu^2}$. Contribution of the order of 1 eV from this term demands $Y_\Sigma^2\bar{u}^2 \sim 10^{-4}$ GeV$^2$ for $M^{1.2}$ in the TeV range, $\mu \sim 10^2$ GeV and $v_1$ in the GeV range$^{11}$. But if one assumes that $Y_\Sigma \sim 1$ and $\bar{u}^2 \sim 10^{-4}$ GeV$^2$ then we get the contribution from the first term of Eq. (71) $\frac{v_2^2Y_\Sigma^2}{M} \gg$ 1 eV, for $M \sim$ TeV and $v_2 \sim$ 100 GeV. The choice $Y_\Sigma^2 \sim 10^{-4}$ and $\bar{u} \sim 1$ GeV also leads to $\frac{v_2^2Y_\Sigma^2}{M} \gg$ 1 eV for $v_2 \sim$ 10$^2$ GeV. The other option is to have $Y_\Sigma^2 \sim 10^{-12}$ and $\bar{u}^2 \sim 10^{8}$ GeV$^2$ so that $Y_\Sigma^2\bar{u}^2 \sim 10^{-4}$ GeV$^2$. But to satisfy this choice one needs $10^7$ order of magnitude hierarchy between the two vacuum expectation values $\bar{u}$ and $u$.$^{12}$ This in turn demands acute hierarchy between the different soft supersymmetry breaking mass terms in Eq. (65). Similarly, $Y_\Sigma$ and $u$ could also not be fixed from fifth and sixth terms of Eq. (71). Hence, we fix $Y_\Sigma$ and $u$ from the first two terms only. The third term is larger than the fourth, fifth and sixth terms, but will still be subdominant compared to the first two terms.$^{13}$ The contribution from the first three terms to the low energy neutrino mass matrix is

$$M_\nu \sim \frac{v_2^2Y_\Sigma^2}{2M} + \frac{u^2(M_1g^2 + M_2g'^2)}{2M_1M_2} + \sqrt{2\bar{u}v_1uY_\Sigma}(M_1g^2 + M_2g'^2) + \frac{v_1v_2u^2\bar{u}Y_\Sigma^2(M_1g^2 + M_2g'^2)}{4M_1M_2\mu^2}. \quad (72)$$

$^{10}$Most likely between the first and second terms, the first term would have larger value for the choice of TeV scale gaugino mass.

$^{11}$As an example $v_1 \sim 10$ GeV.

$^{12}$As the scale of $u$ is fixed from the second term of Eq. (71) and $u < 10^{-3}$ GeV.

$^{13}$As the third term $\propto \bar{u}uY_\Sigma$ and from the 1st two terms $Y_\Sigma$ and $u$ will turn out to be small.
It is straightforward to see from the first term of this approximate expression, that $M_{\nu} \lesssim 1eV$ demands $Y_{\Sigma}^2 \lesssim 10^{-10}$ for $M$ in the TeV range and $v_2 \sim 100$ GeV. On the other hand for $M_{1,2}$ also in the TeV range, the bound on sneutrino vacuum expectation value $\bar{u}$ comes from the second term as $u^2 \lesssim 10^{-6}$ GeV$^2$. For the choice of $\bar{u} \sim u$, which is a natural consequence of the scalar potential minimization conditions, one can check that the third term in Eq. (72) would be much smaller compared to the second term because of the presence of an additional suppression factor $Y_{\Sigma}$. Hence, neutrino masses demand that $Y_{\Sigma} \leq 10^{-5}$ and $u$ and $\bar{u} \leq 10^{-3}$ GeV. One can explicitly check that for this above mentioned $Y_{\Sigma}$, $u$ and $\bar{u}$ the contributions coming from 4th, 5th and 6th terms of Eq. (72) are much smaller compared to the 1st three terms.

Case (B): If one assumes $MM_1M_2\mu^2 \ll (M_1g^2 + M_2g'^2)u_1^2Y_{\Sigma}^2 v_1^2$, which is possible to achieve only for large value of the vacuum expectation value $u$ and Yukawa $Y_{\Sigma}$ so that in the denominator of Eq. (68) the third term dominates over the first two. Then the last term in Eq. (68) contributes as $\sim \frac{Y_{\Sigma}^2 v_1^2}{u_1^2 \mu Y_{\Sigma}^2} \sim \frac{\mu v_2}{v_1} \gg 1eV$ for $\mu$, $v_2$ and $v_1$ in the TeV range and GeV range. Therefore, this limit is not a viable option.

Case (C): We consider the last possibility $MM_1M_2\mu^2 \sim (M_1g^2 + M_2g'^2)u_1^2Y_{\Sigma}^2 v_1^2$, which also demands large Yukawas and large vacuum expectation value $u$. If one considers $M_{1,2}$ and $M$ in the TeV range and $\mu$, $v_2$ in the TeV range and $v_1$ in the GeV range, then demanding that the first term in Eq. (63) should be smaller than eV results in the bound $Y_{\Sigma}^2 \lesssim 10^{-10}$. Hence, in order to satisfy $MM_1M_2\mu^2 \sim (M_1g^2 + M_2g'^2)u_1^2Y_{\Sigma}^2 v_1^2$, one needs $u \geq 10^9$ GeV for $v_1$ in the GeV range. For such large values of $u$, the second term in Eq. (63) will give a very large contribution to the neutrino mass. This case is therefore also ruled out by the neutrino data.

Hence Case (B) and Case (C) which demand large $u$ and $Y_{\Sigma}$ are clearly inconsistent with the neutrino mass scale, and the only allowed possibility is Case (A). This case requires $u \lesssim 10^{-3}$ GeV and $Y_{\Sigma} \lesssim 10^{-5}$, for the gaugino and triplet fermion masses $M_{1,2}$ and $M$ in the TeV range. As for small $u$, $\bar{u}$ and $u$ are proportional to each other, one obtains $\bar{u} \sim 10^{-3}$ GeV as well.

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\textsuperscript{14}To satisfy this condition, the combination of $Y_{\Sigma}$ and $u$ has to be such that $u_1^2Y_{\Sigma}^2 \gg 10^6$ GeV$^2$ for $M_{1,2}, M \sim TeV$, $\mu$ in hundred GeV and $v_1$ in GeV range.
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Figure 1: The $A_A^\Sigma A_A^\Sigma$ loop-generated neutrino mass. The cross on the internal solid line represents the majorana mass for the neutralino and the blob on the dashed line represents the mixing between sneutrino and the neutral Higgs.

Figure 2: Neutrino Majorana mass generated by $\epsilon A_A^\Sigma$ loop.