Abstract. The construction of effective Hamiltonians arising from Loop Quantum Gravity and incorporating Planck scale corrections to the dynamics of photons and spin 1/2 particles is summarized. The imposition of strict bounds upon some parameters of the model using already existing experimental data is also reviewed.

1. INTRODUCTION

The possibility of bringing quantum gravity induced effects to the observational realm has sparkled a lot of attention recently. It is expected that the observation of high energy cosmological particles, such as photons \(^1\) and neutrinos \(^2\) arising from gamma ray bursts, will provide the appropriate arena to test such predictions. Also, very precise experiments already performed in atomic and nuclear physics to search for minute Lorentz covariance violations have been used to place strict bounds upon such effects \(^3\).

One of the leading theories providing a consistent description of quantum gravity is Loop Quantum Gravity (LQG) \(^4\). This theory predicts the quantization of space in units of \(\ell_P\), with \(\ell_P\) being the Planck length \(^5\). An intuitive way of thinking about this is to imagine space being described by discrete cells at very small distances \(d \sim \ell_P\), with the standard continuous description being recovered for large distances \(d \geq \ell_P\). From the point of view of a particle immersed in such a space, this granular structure will act as an effective media modifying the particle propagation properties with respect to those usually assumed in the standard vacuum. Such granularity will induce also minute violations of Lorentz covariance, which have been the subject of very precise experimental investigations \(^6\), as well as theoretical descriptions leading to a standard model extension which can account for the diversity of observations that have been made \(^7\). Modifications arising from LQG constitute a specific realization of such general scheme, providing a physical interpretation of the parameters involved.

\(^1\) Dedicated to A. García and A. Zepeda on their sixtieth birthday
To obtain such modifications starting from the full LQG requires a semiclassical approximation where the particles (photons and spin 1/2 particles, for example) are treated as classical fields, while an appropriate integration is performed upon the gravitational degrees of freedom. In this sense, we are interested in the regime where the matter fields are slowly varying while the gravitational variables are rapidly varying. The full Hamiltonian is known in LQG, being a well defined regularized operator acting upon cylindrical functions. These are functions of generalized connections defined upon graphs \( \Gamma \), characterized by a set of vertices \( \{ v \} = \{ v_1, v_2, \ldots \} \) and edges \( \{ e \} = \{ e', e'', e''', \ldots \} \) joining those vertices. What is missing is the strict construction of the semiclassical state describing the matter field of interest, together with the corresponding large scale continuous space-time metric (flat space in our case). With these two ingredients one would define and calculate the semiclassical effective Hamiltonian as the expectation value of the full LQG Hamiltonian in the corresponding semiclassical state. A rigorous formulation of this problem has turned out to be complicated and is presently in the process of development [8]. Here we take an heuristical point of view, starting from the exact operator version of LQG and defining its action upon the semiclassical state through some plausible requirements.

Central to our approach is Thiemann’s regularization of the LQG Hamiltonian [9]. This is based upon a triangulation of space, adapted to the corresponding graphs which define a given state. The regularization is provided by the volume operator, with discrete eigenvalues arising only from the vertices of the graph.

The paper is organized as follows: section 2 contains a very compact summary of Thiemann’s regularization, exemplified in the context of the magnetic sector of QED, together with the heuristical scheme employed in our estimations. Section 3 summarizes the results for the case of photons and spin 1/2 particles and contains a brief discussion regarding the choice of some relevant parameters in the model. Section 4 contains the description of the phenomena from the point of view of the laboratory frame attached to earth, moving at a speed \( v/c \approx 10^{-3} \) with respect to the Cosmic Microwave Background frame (CMB), yielding modifications which can be tested with already existing experimental data.

2. THE CALCULATION

2.1. The regularized Hamiltonian operator

The curved space magnetic contribution to the QED Hamiltonian is

\[
H^B = \frac{1}{Q^2} \int_{\Sigma} d^3 x \, q_{ab} \, \frac{1}{\sqrt{q}} \, 2 \, B^a B^b ,
\]

(1)

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2 Limitations of space prevent us to provide a more complete list of references. We apologize to the corresponding authors.
where \( B^a = \epsilon_{abc} F_{bc} \), \( F_{bc} = \partial_b A_c - \partial_c A_b \), in standard notation. The underline identifies the electromagnetic variables. Here \( q_{ab} \) is the three-metric, \( q = \det (q_{ab}) \) with \( a, b, c, \ldots \) being spatial indices and \( Q \) is the electromagnetic coupling constant. Thiemann’s regularized expression for the above contribution is \([9]\)

\[
\hat{H}^B = \frac{1}{2\ell_p Q^2} \sum_{v \in V(\gamma)} G(v) \sum_{v(\Delta) = v(\Delta')} \epsilon^{JKL} \epsilon^{MNP} \times \\
\times \hat{w}_{i\Delta} \left( h_{\alpha JK(\Delta)} - 1 \right) \hat{w}_{iP\Delta'} \left( h_{\epsilon MN(\Delta')} - 1 \right),
\]

(2)

where

\[
\hat{w}_{P\Delta} = Tr \left( v_{h_{ij}(\Delta)} \left[ h_{ij(\Delta)}^{-1}, \sqrt{V_i} \right] \right),
\]

(3)

with \( \sqrt{V_i} \) being the volume operator. The above equations are written in terms of the holonomies \( h_\gamma \) of the corresponding connections: \( A^i_a \) (gravitational) and \( A^i_a \) (electromagnetic) along the curves \( \gamma : s_I(\Delta), a_{IJ}(\Delta) \), to be defined below. The triangulation involved in \([2]\) is adapted to the graph \( \Gamma \) corresponding to the state acted upon, in such a way that at each vertex \( v \) of \( \Gamma \) and triplet of edges \( e, e', e'' \) joining the vertex, a tetrahedron is defined with basepoint at the vertex \( v(\Delta) = v \) and segments \( s_I(\Delta) \), \( I = 1, 2, 3 \), in the directions of \( e, e', e'' \) respectively. The arcs connecting the end points of \( s_I(\Delta) \) and \( s_J(\Delta) \) are denoted \( a_{IJ}(\Delta) \), so that a loop \( a_{IJ} := s_I \circ a_{IJ} \circ s_J^{-1} \) can be formed.

### 2.2. The semiclassical approximation

We think of the semiclassical configuration describing the particular matter field \((\tilde{E}, \tilde{B})\) plus flat-space at large distances, as given by an ensemble of graphs \( \Gamma \), each occurring with probability \( P(\Gamma) \). To each of such graphs we associate a wave function \( |\Gamma, \mathcal{L}, \tilde{E}, \tilde{B} \rangle \equiv |\Gamma, S \rangle \) which is peaked with respect to the classical electromagnetic field configuration together with a flat gravitational metric and a zero value for the gravitational connection. In other words, the contribution for each operator inside the expectation value are estimated as \([10, 11]\)

\[
\langle \Gamma, \mathcal{L}, \tilde{E}, \tilde{B} | \tilde{Q}_a \rangle = \delta_{ab} + \mathcal{O} \left( \frac{\ell_p}{\mathcal{L}} \right)
\]

\[
\langle \Gamma, \mathcal{L}, \tilde{E}, \tilde{B} | \tilde{A}_{ia} \rangle = 0 + \frac{1}{\mathcal{L}} \left( \frac{\ell_p}{\mathcal{L}} \right)^\gamma,
\]

(4)

while the expectation values including the electric and magnetic operators are estimated through their corresponding classical values \( \tilde{E} \) and \( \tilde{B} \). The parameter \( \gamma \geq 0 \) is a real number. Not surprisingly, the semiclassical state specifies both the classical coordinate and the classical momentum for each pair of canonical variables. The scale \( \mathcal{L} >> \ell_p \) of the wave function is such that the continuous flat metric approximation is appropriate for distances much larger that \( \mathcal{L} \), while the granular structure of spacetime becomes relevant when probing distances smaller that \( \mathcal{L} \). Such scale will have a natural realization according to each particular physical situation.
We summarize now the method of calculation [2, 10, 11, 12]. For each graph \( \Gamma \) the effective Hamiltonian is defined as \( H_\Gamma = \langle \Gamma, S | H_T | \Gamma, S \rangle \). For a given vertex, inside the expectation value, one expands each operator in powers of the segments \( s_1(\Delta) \) plus derivatives of the matter fields operators. Schematically, in the case of (2) this produces

\[
H^B_\Gamma = \sum_{v \in V(\Gamma)} \sum_{v(\Delta) = v} \langle \Gamma, S | \hat{T}_{p_1 q_1} (v) \ldots \hat{T}_{pq} (v) \hat{T}_{a_1} \ldots p q p_1 q_1 \ldots (v, s(\Delta)) | \Gamma, S \rangle. \tag{5}
\]

where \( \hat{T} \) contains gravitational operators together with contributions depending on the segments of the adapted triangulation in the particular graph. Next, space is considered to be divided into boxes, each centered at a fixed point \( \vec{x} \) and with volume \( \mathcal{L}^3 \approx d^3 x \). The choice of boxes is the same for all the graphs considered. Each box contains a large number of vertices of the semiclassical state (\( \mathcal{L} \gg \ell_p \)), but it is considered as infinitesimal in the scale where the space can be regarded as continuous. The sum over the vertices in (2) is subsequently split as the sum over the vertices in each box, plus the sum over boxes. Also, one assumes that the electromagnetic operators are slowly varying within a box (\( \mathcal{L} \ll \lambda \), with \( \lambda \) been the particle wavelength), in such a way that for all the vertices inside a given box one can write \( \langle \Gamma, S | \ldots \hat{E}_{ab} (v) \ldots | \Gamma, S \rangle = \mu \hat{F}_{ab} (\vec{x}) \). Here \( \hat{F}_{ab} \) is the classical electromagnetic field at the center of the box and \( \mu \) is a dimensionless constant which is determined in such a way that the standard classical result in the zeroth order approximation is recovered. Applying the procedure just described to (2) leads to

\[
H^B_\Gamma = \sum_{\text{Box}} F_{p_1 q_1} (\vec{x}) \ldots \left( \partial^{a_1} \ldots F_{pq} (\vec{x}) \right) \sum_{v \in \text{Box}} \ell_p^3 \sum_{v(\Delta) = v} \mu^{n+1} \times
\]

\[
\times \langle \Gamma, S | \frac{1}{\ell_p^3} \hat{T}_{a_1} \ldots p q p_1 q_1 \ldots (v, s(\Delta)) | \Gamma, S \rangle, \tag{6}
\]

where \( n + 1 \) is the total number of factors \( F_{pq}(\vec{x}) \). The expectation value of the gravitational contribution is supposed to be a rapidly varying function inside each box. Finally, the effective Hamiltonian is defined as an average over the graphs \( \Gamma \), i.e. over the adapted triangulations : \( H^B = \sum_{\Gamma} P(\Gamma) H^B_\Gamma \). This effectively amounts to average the expectation values remaining in each box of the sum (5). We call this average \( T_{a_1} \ldots p q p_1 q_1 \ldots (\vec{x}) \) and estimate it by demanding \( T \) to be constructed from the flat space tensors \( \delta_{ab} \) and \( \epsilon_{abc} \). In this way we are imposing isotropy and rotational invariance on our final Hamiltonian, which consequently describes the modified dynamics in a specific reference frame which we take to be the CMB frame. Also, the scalings given in (4) together with the additional assumptions: \( \langle \Gamma, S | \ldots \hat{V} \ldots | \Gamma, S \rangle \rightarrow \ell_p^3, \ s_i^2 \rightarrow \ell_p \) are used in this estimation. After replacing the summation over boxes by the integral over space, the resulting Hamiltonian has the final form

\[
H^B = \int d^3 x \ F_{p_1 q_1} (\vec{x}) \ldots \left( \partial^{a_1} \ldots F_{pq} (\vec{x}) \right) T_{a_1} \ldots p q p_1 q_1 \ldots (\vec{x}). \tag{7}
\]

Since the approach presented here has made use only of the main features that semiclassical states should have, all dimensionless coefficients in the expectation values that contribute to \( T_{a_1} \ldots p q p_1 q_1 \ldots (\vec{x}) \) in (7) remain undetermined. They are subsequently denoted by \( \theta \)’s and \( \kappa \)’s.
3. THE RESULTS

Here we summarize the corresponding effective Hamiltonians and modified dispersion relations for the cases of photons and two-component spin 1/2 particles.

3.1. Photons

A detailed discussion can be found in Refs. [10, 12]. The effective Hamiltonian is

\[ H^{EM} = \frac{1}{Q^2} \int d^3 \vec{x} \left[ \left( 1 + \theta_7 \left( \frac{\ell_P}{\mathcal{L}} \right)^{2+2\Upsilon} \right) \frac{1}{2} \left( \vec{B}^2 + \vec{E}^2 \right) + \theta_3 \ell_p^2 \left( \mathcal{B}^a \nabla^2 B_a + \mathcal{E}^a \nabla^2 E_a \right) \right. \]

\[ + \theta_2 \ell_p \mathcal{E}^a \partial_a \partial_b \mathcal{E}_b + \theta_8 \ell_p \left( \vec{B} \cdot (\nabla \times \vec{B}) + \vec{E} \cdot (\nabla \times \vec{E}) \right) + \theta_5 \mathcal{L}^2 \ell_p \left( \frac{\mathcal{L}}{\ell_p} \right)^{2\Upsilon} \left( \vec{B}^2 \right)^2 + \ldots \right], \tag{8} \]

up to order \( \ell_p^2 \). The corresponding dispersion relation is

\[ \omega_{\pm} = k \left( 1 + \theta_7 \left( \frac{\ell_P}{\mathcal{L}} \right)^{2+2\Upsilon} \right) - 2 \theta_3 (k \ell_P)^2 \pm 2 \theta_8 (k \ell_P) \right). \tag{9} \]

The \( \pm \) signs correspond to the different polarizations of the photon. From the above we obtain the speed of the photon ( \( v_{\pm}(k, \mathcal{L}) = \partial \omega_{\pm}(k, \mathcal{L}) / \partial k \) )

\[ v_{\pm} = 1 \pm 4 \theta_8 (k \ell_P) - 6 \theta_3 (k \ell_P)^2 + \theta_7 (k \ell_P)^{2+2\Upsilon} + \ldots \tag{10} \]

The last expression gives \( v \) expanded to leading order in \( \ell_P \), with the estimation \( \mathcal{L} = 1/k \). To first order in \( (k \ell_P) \) we recover the helicity dependent correction found already in the seminal work of Gambini and Pullin [13]. As far as the \( \Upsilon \) dependent terms we have either a quadratic (\( \Upsilon = 0 \)) or a quartic (\( \Upsilon = 1 \)) correction. The only possibility to have a first order helicity independent correction amounts to set \( \Upsilon = -1/2 \) which corresponds to that of Ellis et. al. [14]. However, we do not have an interpretation for such a value of \( \Upsilon \).

3.2. Two-component spin 1/2 particles

The details can be found in Refs. [2, 11]. The effective Hamiltonian is

\[ H_{1/2} = \int d^3 x \left[ i \pi(\vec{x}) \tau^d \partial_d \hat{A} \xi(\vec{x}) + c.c. + \frac{i}{4\hbar} \frac{1}{\mathcal{L}} \pi(\vec{x}) \hat{C} \xi(\vec{x}) \right. \]

\[ + \frac{m}{2\hbar} \gamma^T(\vec{x}) (i \sigma^2) (\alpha + 2\hbar \beta \tau^a \partial_a) \xi(\vec{x}) + \frac{m}{2\hbar} \pi^T(\vec{x}) (\alpha + 2\hbar \beta \tau^a \partial_a) (i \sigma^2) \pi(\vec{x}), \tag{11} \]
non perturbative states of the spin-network can be approximated by the classical flat probe to distances of order \( L \). The propagating particle (photon or neutrino) is characterized by energies which approach can be found in Ref. [15].

Alternative results based on a string theory inspired with the former. Bounds for \( \nu \) with neutrino oscillations [17] produces a universal scale estimation which is consistent with the former. Bounds for \( \Upsilon \) have been estimated in Ref. [11] based on the observation.

The corresponding dispersion relation is

\[
E_\pm(p, \mathcal{L}) = \left[ p + m^2/2p \pm \ell_p \left( \frac{1}{2} m^2 \kappa_9 \right) + \ell_p^2 \left( -\frac{1}{2} \kappa_3 p^2 + \frac{1}{8} (2 \kappa_3 + \kappa_7^2) m^2 p \right) \right]
\]

\[
+ \left( \frac{\ell_p}{\mathcal{L}} \right)^{\Upsilon+1} \left[ \left( \kappa_1 p - \Theta_{11} m^2/4p \right) \pm \ell_p \left( -\kappa_2 p^2/4 + \Theta_{12} m^2/16 \right) \right]
\]

\[
+ \left( \frac{\ell_p}{\mathcal{L}} \right)^{\Upsilon+1} \left[ \left( \kappa_1 + \Theta_{11} m^2/4p^2 \right) \pm \frac{\kappa_7}{2} (\ell_p p) \right]
\]

\[
+ \left( \frac{\ell_p}{\mathcal{L}} \right)^{\Upsilon+2} \left( \kappa_2 + \frac{m^2}{64 p^2} \Theta_{22} \right),
\]

within the same approximation. Alternative results based on a string theory inspired approach can be found in Ref. [15].

3.3. The parameters \( \mathcal{L} \) and \( \Upsilon \)

In order to produce numerical estimations of some of the effects arising from the modifications to the dynamics previously obtained, we must further fix the value of the scales \( \mathcal{L} \) and \( \Upsilon \). Recall that \( \mathcal{L} \) is a scale indicating the onset distance from where the non perturbative states of the spin-network can be approximated by the classical flat metric. The propagating particle (photon or neutrino) is characterized by energies which probe to distances of order \( \lambda \). In order to preserve the description in terms of a classical continuous equation it is necessary that \( \mathcal{L} < \lambda \). Two distinguished cases arise: (i) the mobile scale, where we take the marginal choice \( \mathcal{L} = \lambda \) and (ii) the universal scale, which has been considered in Ref. [16] in the context of the GZK anomaly. The study of the different reactions involved produces a preferred bound on \( \mathcal{L} : 4.6 \times 10^{-8} GeV^{-1} \geq \mathcal{L} \geq 8.3 \times 10^{-9} GeV^{-1} \). A recent study of the gravitational Cerenkov effect together with neutrino oscillations [17] produces a universal scale estimation which is consistent with the former. Bounds for \( \Upsilon \) have been estimated in Ref. [11] based on the observation.
that atmospheric neutrino oscillations at average energies of the order $10^{-2} \sim 10^{2}$ GeV are dominated by the corresponding mass differences via the oscillation length $L_m$. This means that additional contributions to the oscillation length, in particular the quantum gravity correction $L_{QG}$, should satisfy $L_{QG} > L_m$. This is used to set a lower bound upon $\Upsilon$. Within the proposed two different ways of estimating the scale $\mathcal{L}$ of the process we obtain: (i) $\Upsilon > 0.15$ when $\mathcal{L}$ is considered as a mobile scale and (ii) $1.2 < \Upsilon$ when the scale $\mathcal{L}$ takes the universal value $\mathcal{L} \approx 10^{-8}$ GeV$^{-1}$.

4. OBSERVATIONAL BOUNDS USING EXISTING DATA

The previously found Hamiltonians were obtained under the assumption of flat space isotropy so that they account for the dynamics in a preferred reference frame. We have identified it as the frame in which the Cosmic Microwave Background looks isotropic. Our velocity $w$ with respect to that frame has already been determined to be $w/c \approx 1.23 \times 10^{-3}$ by COBE. Thus, in the earth reference frame one expects the appearance of signals indicating minute violations of space isotropy encoded in $w$-dependent terms appearing in the transformed Hamiltonian or Lagrangian [3]. On the other hand, many high precision experimental test of rotational symmetry, using atomic and nuclear system, have been already reported in the literature. Amazingly such precision is already enough to set very stringent bounds on some of the parameters arising from the quantum gravity corrections. In Ref. [3] we have considered the case of non-relativistic Dirac particles obtaining corrections which involve the coupling of the spin to the CMB velocity together with a quadrupolar anisotropy of the inertial mass. The calculation was made with the choices $\Upsilon = 0$ and $\mathcal{L} = 1/M$, where $M$ is the rest mass of the fermion. Keeping only terms linear in $\ell_P$, the equation of motion arising from the two-component Hamiltonian (11) can be readily extended to the Dirac case as

$$\left( i \gamma^\mu \partial_\mu + \Theta_1 m \ell_P i \gamma^\nu \nabla_\nu - \frac{K}{2} \gamma_5 \gamma^0 - m (\alpha - i \Theta_2 \ell_P \Sigma \cdot \nabla) \right) \Psi = 0,$$

where we have used the representation in which $\gamma_5$ is diagonal, the spin operator is $\Sigma^k = (i/2) \epsilon_{klm} \gamma^l \gamma^m$, $K = \Theta_4 m^2 \ell_P$ and $\alpha = 1 + \Theta_3 m \ell_P$. The normalization has been chosen so that in the limit $(m \ell_P) \to 0$ we recover the standard massive Dirac equation. The term $m (1 + \Theta_3 m \ell_P)$ can be interpreted as a renormalization of the mass whose physical value is taken to be $M = m (1 + \Theta_3 m \ell_P)$. After this modification the corresponding effective Lagrangian is

$$L_D = \frac{1}{2} i \bar{\Psi} \gamma^0 (\partial_0 \Psi) + \frac{1}{2} i \bar{\Psi} \left( (1 + \Theta_1 M \ell_P) \gamma^k - \Theta_2 \ell_P M \Sigma^k \right) \partial_k \Psi - \frac{1}{2} M \bar{\Psi} \Psi - \frac{K}{4} \bar{\Psi} \gamma_5 \gamma^0 \Psi + \text{h.c.},$$

which describes the time evolution as seen in the CMB frame. In order to obtain the dynamics in the laboratory frame we implement an observer Lorentz transformation. To this end we rewrite (16) in a covariant looking form, by introducing explicitly the CMB
frame’s four velocity $W^\mu = \gamma(1, w/c)$. In the metric with signature $-2$ the result is

$$L_D = \frac{1}{2} \bar{\Psi} \gamma^\mu \partial_\mu \Psi - \frac{1}{2} M \bar{\Psi} \gamma^\mu \gamma_\mu \Psi + \frac{1}{2} i(\Theta_1 M \ell_P) \bar{\Psi} \gamma^\mu (g^\mu\nu - W^\mu W^\nu) \partial_\nu \Psi$$

$$+ \frac{1}{4} (\Theta_2 M \ell_P) \bar{\Psi} \epsilon_{\mu\nu\alpha\beta} W^\mu \gamma^\nu \gamma^\alpha \gamma^\beta \Psi - \frac{1}{4} (\Theta_4 M \ell_P) M W^\mu \bar{\Psi} \gamma^\mu \Psi + \text{h.c.} .$$ \hspace{1cm} (17)

From Eq. (26) of [18] we obtain the non-relativistic limit of the Hamiltonian corresponding to (17), up to first order in $\ell_P$ and up to order $(w)/c^2$, which is

$$\tilde{H} = \left[ M c^2 (1 + \Theta_1 M \ell_P (w/c)^2) + \left( 1 + 2 \Theta_1 M \ell_P \left( 1 + \frac{5}{6} (w/c)^2 \right) \right) \left( \frac{p^2}{2M} + g \mu s \cdot B \right) \right]$$

$$+ \left( \Theta_2 + \frac{1}{2} \Theta_4 \right) M \ell_P \left[ \left( 2 M c^2 - \frac{2 p^2}{3 M} \right) s \cdot \frac{w}{c} + \frac{1}{M} s \cdot Q_P \cdot \frac{w}{c} \right] + \Theta_1 M \ell_P \left[ \frac{w \cdot Q_P \cdot w}{M c^2} \right] .$$ \hspace{1cm} (18)

where $s = \sigma / 2$. Here we have not written the terms linear in the momentum since they average to zero. In (18) $g$ is the standard gyromagnetic factor, and $Q_P$ is the momentum quadrupole tensor with components $Q_{Pij} = p_i p_j - 1/3 p^2 \delta_{ij}$. The terms in the second square bracket represent a coupling of the spin to the velocity with respect to the “rest” (privileged) frame. The first one has been measured with high accuracy in references [6] where an upper bound for the coefficient has been found. The second term is a small anisotropy contribution and can be neglected. Thus we find the correction

$$\delta H_S = \left( \Theta_2 + \frac{1}{2} \Theta_4 \right) M \ell_P (2 M c^2) \left[ 1 + O \left( \frac{p^2}{2 M c^2} \right) \right] s \cdot \frac{w}{c} .$$ \hspace{1cm} (19)

The last term of (18), which represents an anisotropy of the inertial mass, has been bounded in Hughes-Drever like experiments. With the approximation $Q_P = -5/3 < p^2 > Q/R^2$ for the momentum quadrupole moment, with $Q$ being the electric quadrupole moment and $R$ the nuclear radius, we obtain

$$\delta H_Q = -\Theta_1 M \ell_P \frac{5}{3} \left( \frac{p^2}{2 M} \right) \left( \frac{Q}{R^2} \right) \left( \frac{w}{c} \right)^2 P_2(\cos \theta) ,$$ \hspace{1cm} (20)

for the quadrupole mass perturbation, where $\theta$ is the angle between the quantization axis and $w$. Using $< p^2 / 2 M > \sim 40$ MeV for the energy of a nucleon in the last shell of a typical heavy nucleus, together with the experimental bounds of references [6] we find

$$| \Theta_2 + \frac{1}{2} \Theta_4 | < 2 \times 10^{-9} , \hspace{1cm} | \Theta_1 | < 3 \times 10^{-5} .$$ \hspace{1cm} (21)

The above bounds on terms that were formerly expected to be of order unity, already call into question the scenarios inspired on the various approaches to quantum gravity, suggesting the existence of Lorentz violating Lagrangian corrections which are linear in Planck’s length. To this respect it is interesting to notice that a very reasonable fit to the gamma ray spectrum beyond de GZK cutoff has been recently made by using dispersion relations of higher order than linear in $\ell_P$ [19]. Observational bounds upon parameters of related theories are obtained in the works of Ref. [20]
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