Propagating Higgs Boundstates in SUSY

RICHARD DAWID\textsuperscript{a} and SERGUEI REZNOV\textsuperscript{b}

Institut für Theoretische Physik, Technische Universität München, James–Franck–Straße, D–85747 Garching, Germany

Abstract

The possibilities for an underlying theory behind an effective supersymmetric Nambu–Jona-Lasinio like model of electroweak symmetry breaking with propagating Higgs boundstates (e.g. supersymmetric topcondensation) are investigated. The concept of a renormalizable underlying theory turns out not to be appropriate. We argue that the new theory should come from the Planck scale and be connected to supergravity. Such a model can be constructed but necessarily implies a nonlinear definition of the full Lagrangian.

\textsuperscript{a}Email: Richard.Dawid@Physik.TU-Muenchen.DE
address after August 1996:
Institut für Theoretische Physik, Universität Wien, Boltzmanng. 5, 1090 Wien, Austria
Email: Richard.Dawid@ariel.pap.univie.ac.at
\textsuperscript{b}Email: Serguei.Reznov@Physik.TU-Muenchen.DE
Whether the electroweak symmetry is spontaneously broken by fundamental scalars or by a dynamical mechanism still remains an open question. In case of dynamical symmetry breaking two alternatives can be distinguished: Models with no propagating scalars at all like technicolour and models which replace the fundamental Higgs by a propagating bound state, like e.g. top condensation. The latter approach which involves propagating scalar bound states is based on the separation of the bound state mass scale and the scale where this bound state breaks up. While this separation can be achieved by an albeit undesirable fine tuning of the four–fermion coupling in standard model(SM)–like top condensation, it turns out to become a major complication in SUSY extensions due a basically very attractive SUSY property: The cancellation of quadratic divergences. The question of our paper is whether there exists an underlying theory of supersymmetric top condensation which is able to accomplish the required separation of scales. We will first give a very short introduction to top condensation and its supersymmetric extension. Then we will investigate the approach of introducing heavy particle exchange in an underlying theory to explain the 4–fermion terms and we will see that it fails for several reasons. Next we present a new approach in connection with supergravity which, as far as we can see, is the only possible solution in the framework of established theoretical concepts.

Before dealing with the supersymmetric case let us shortly reconsider the mechanism of isospin breaking in non–supersymmetric top condensation. A new strong four–fermion interaction is introduced which induces spontaneous electroweak symmetry breaking by a self consistent Nambu–Jona-Lasinio gap equation and generates a composite Higgs sector. The Lagrangian contains the usual covariant kinetic terms for all gauge fields, quarks and leptons and the new interaction term involving left–handed quark doublets and right–handed singlets:

\[ \mathcal{L} = \mathcal{L}_{\text{kin,cov}} + G \bar{t}_L t_R \]

For \( G > G_{\text{cr}} = 8\pi^2/N_c \Lambda^2 \) the gap equation has an energetically favoured non–trivial solution for a top–mass \( m_t > 0 \) and a top condensate emerges. The high energy cutoff \( \Lambda \) corresponds to new physics which is resolved at this scale, e.g. a heavy gauge boson exchange. In other words the coupling becomes non–local at this scale and the scalar bound state breaks up. As there is one free parameter less than in the SM, this theory predicts a certain ratio between the top quark mass \( m_t \) and the W–mass \( m_W \) in slight dependence of \( \Lambda \). The resulting top–mass, becoming slightly smaller with increasing \( \Lambda \) still has an unrealistic value of about 220 GeV for \( \Lambda \) at the Planck scale. Moreover to achieve the large scale separation between \( \Lambda \) and \( m_t \) one has to fine–tune the four fermion coupling \( G \) towards the critical coupling \( G_{\text{cr}} \).
A supersymmetric version of a Nambu–Jona-Lasinio Model [3], formulated in the framework of top condensation in [4], appears to be able to solve both problems mentioned above: A lower top mass is natural first due to a lower value of the quasi fixed point of $g_t$ in the minimal supersymmetric standard model (MSSM) and second because of the additional Higgs doublet which allows to lower the top mass by the factor $\sin(\beta)$, the ratio between the values of the vacuum expectation values (VEVs) for the two Higgs fields. The hierarchy problem which appears as the fine–tuning problem of $G$ in top condensation is alleviated in supersymmetry because of the cancellation of quadratic divergences.

The supersymmetric extension of (1) is

$$\mathcal{L} = \mathcal{L}_{YM} + \int d^2 \theta d^2 \bar{\theta} (\overline{Q} e^{2V_Q} Q + T^c e^{-2V_T} T^c + B^c e^{-2V_B} B^c) (1 - \Delta^2 \theta^2 \bar{\theta}^2)$$

$$+ G \int d^2 \theta d^2 \bar{\theta} (\overline{Q} T^c) e^{2V_Q - 2V_T}(QT^c) (1 - 2\Delta^2 \theta^2 \bar{\theta}^2 + \delta \theta^2 + \delta \bar{\theta}^2) \, , \quad (2)$$

where $\mathcal{L}_{YM}$ contains the usual SUSY kinetic terms for gauge fields, $Q \ (T^c, B^c)$ are SU(2) doublet (singlet) chiral quark superfields. $\Delta$ and $\delta$ are SUSY soft breaking parameters. Throughout this paper superfields will be denoted by capital letters and component fields by small letters except for the vectorfield which is identifiable by its Dirac index. Reformulated in auxiliary fields eq.(2) corresponds to:

$$\mathcal{L} = \mathcal{L}_{YM} + \int d^2 \theta d^2 \bar{\theta} (\overline{Q} e^{2V_Q} Q + T^c e^{-2V_T} T^c + B^c e^{-2V_B} B^c) (1 - \Delta^2 \theta^2 \bar{\theta}^2)$$

$$+ \int d^2 \theta d^2 \bar{\theta} \overline{H}_1 e^{2V_{H_1}} H_1 (1 - M_H^2 \theta^2 \bar{\theta}^2)$$

$$- \int d^2 \theta \epsilon_{ij} (\mu_0 H^i_1 H^j_2 (1 + B_0 \theta^2) - g_{T_0} H_2^i T^c (1 + A_0 \theta^2))$$

$$- \int d^2 \theta \epsilon_{ij} (\mu_0 \overline{H}_1^i \overline{H}_2^j (1 + B_0 \theta^2) - g_{T_0} \overline{T}^c \overline{Q} H^j_2 (1 + A_0 \theta^2)) \, , \quad (3)$$

with $M_H^2 = 2\Delta^2 + \delta^2$, $A_0 - B_0 = \delta$, $V_{H_1} = V_Q - V_T$ and $G = \frac{g^2}{\mu_0}$. It is an essential feature of propagating boundstates that they correspond to the auxiliary fields of the binding effective coupling which receive kinetic contributions at lower energies. Thus the auxiliary fields in eq.(3) represent the two MSSM Higgs superfields.

A top mass is produced as a nontrivial solution of the following self consistent gap equation [4]:

$$\mathcal{L} = \mathcal{L}_{YM} + \int d^2 \theta d^2 \bar{\theta} (\overline{Q} e^{2V_Q} Q + T^c e^{-2V_T} T^c + B^c e^{-2V_B} B^c) (1 - \Delta^2 \theta^2 \bar{\theta}^2)$$

$$+ \int d^2 \theta d^2 \bar{\theta} \overline{H}_1 e^{2V_{H_1}} H_1 (1 - M_H^2 \theta^2 \bar{\theta}^2)$$

$$- \int d^2 \theta \epsilon_{ij} (\mu_0 H^i_1 H^j_2 (1 + B_0 \theta^2) - g_{T_0} H_2^i T^c (1 + A_0 \theta^2))$$

$$- \int d^2 \theta \epsilon_{ij} (\mu_0 \overline{H}_1^i \overline{H}_2^j (1 + B_0 \theta^2) - g_{T_0} \overline{T}^c \overline{Q} H^j_2 (1 + A_0 \theta^2)) \, , \quad (3)$$
\[
G^{-1} = \frac{N_c \Delta^2}{16 \pi^2} \left[ (1 + \frac{2m_t^2 + \delta^2 \alpha}{2 \Delta^2}) \ln \left( \frac{\Lambda^4}{(m_t^2 + \Delta^2)^2 - \frac{m_{QQ}^4}{m_t^2}} \right) - \frac{2m_t^2}{\Delta^2} \ln \left( \frac{\Lambda^4}{m_t^2} \right) \right],
\]

where \(m_{QQ}^2\) is the self-consistent squark mass and \(\alpha = \frac{m_{QQ}^2}{(\delta m_t)}\) is given by

\[
\alpha^{-1} = 1 + \frac{GM_H^2 N_c}{32 \pi^2} \ln \left( \frac{\Lambda^4}{(m_t^2 + \Delta^2)^2 - \frac{m_{QQ}^4}{m_t^2}} \right).
\]

One can see that, as consequence of the SUSY cancellation of quadratic divergences, the cutoff appears only logarithmically in this gap equation. But the quadratic contributions of the right hand side of (4) have to cancel the suppression factor of \(G\) to provide a nontrivial solution. Due to this fact, a nontrivial solution of the gap equation can be achieved only if the coupling \(G\) is of the order \(\frac{1}{\Delta^2}\) which is much larger than \(\frac{1}{\Lambda^2}\). One can of course introduce such couplings by hand but now the theory lacks any explanation for the high scale of the boundstate breakup (the cutoff scale). It seems to be advisable to investigate whether an underlying theory is able to provide an explanation.

The most obvious idea for an underlying theory would be to consider a renormalizable theory with heavy degrees of freedom. We will investigate whether it is possible to regain the four-scalar coupling of (2) effectively by integrating out such heavy degrees of freedom of an underlying theory. To get a reasonable structure for the effective theory it is necessary to assume a supersymmetric structure of the heavy mass terms in the underlying theory. All physical components of the heavy superfields acquire the same mass and have to be integrated out. Recently a way of integrating out heavy degrees of freedom in superfield formalism has been used in SUSY GUTs [11]. But due to some subtleties concerning the supersymmetric structure of our effective theory which will be discussed in an upcoming paper we integrate out in component fields.

There are two different kinds of superfields that can be exchanged in an underlying theory: Real (vector) superfields involving vectorfields plus their fermionic superpartners and chiral superfields which, to allow Yukawa couplings to quarks in agreement with R-parity, should consist of scalars and their fermionic superpartners.

Following the usual approach of top-condensation we first have a look at heavy gauge boson exchange. We introduce a new gauge group \(G_S\), a supersymmetric gauge coupling to the heavy gauge bosons \(V_S\), the corresponding fieldstrength contribution \(W^a_S W^a_S\) and a coupling of \(V_S\) to a heavy Higgs sector to produce the mass of \(V_S\). This Higgs sector should break the symmetry \(G_S\) in a supersymmetric way and should not couple to the light sector. We assume a Higgs potential \(P_{HS}\) which is able to accomplish this without looking into the problematic
details. Additionally we will need soft breaking terms (gaugino and heavy Higgs mass terms) denoted by $\mathcal{L}_{\text{soft}}$. Written in superfields the new interaction sector looks like:

$$
\mathcal{L}_{NI} = \int d^2\theta d^2\overline{\theta} \left( Q e^{gS} V S + T^c e^{gS} V S + H S e^{gS} H S ight) + \left( \frac{1}{4} W_{sa} W_s^a + P_{H S} \right) \delta^2(\overline{\theta}) + h.c. + \mathcal{L}_{\text{soft}} .
$$

(6)

In component fields this corresponds to the following Lagrangian:

$$
L_{NI} = L_{\text{kin pure}}(\overline{q}, \overline{t}, q, t, \lambda, \overline{H}) + g_s \overline{q} D_s q + F_q^+ F_q + \frac{1}{2} D_s D_s + M_s^2 V_{\mu} V^\mu + (g_s \overline{q} \overline{\sigma}^\mu V_{\mu} q + i g_s (\partial^\mu q^+) V_{\mu} \overline{q} + i \sqrt{2} g_s q^+ \lambda_s q) + M_s \lambda_s \overline{H}_s + (q \leftrightarrow t) + h.c.) + \ldots
$$

(7)

where $L_{\text{kin pure}}$ denotes the pure, non–covariant kinetic terms and the dots contain the terms which will not contribute to order $1/M_s^2$ in the effective theory. In order to integrate out the heavy degrees of freedom it is important to notice that it will not be correct to just neglect the kinetic terms of the heavy fields before extracting the equations of motion. We have to deal with linear fermionic and quadratic bosonic mass terms at the same time. Additionally trilinear mass valued coupling terms can appear in the Lagrangian (not now but in case of heavy scalar exchange), which decrease the order of suppression of effective coupling terms. Thus derivative couplings coming from heavy particle kinetic terms are not necessarily suppressed by orders higher than $1/M_s^2$ and we have to take them into consideration. So we have to do the following: We extract the equations of motion from the full Lagrangian to get “constituent relations” for the heavy fields. These relations also include suppressed derivative terms of heavy fields coming from their kinetic terms. We insert these relations to eliminate the heavy fields in lowest order. Then we reinsert the same relations again to eliminate the suppressed derivative terms of heavy fields. This gives us the correct effective theory up to order $1/M_s^2$. We do the calculation in the Wess–Zumino gauge which can be done because all arguments are tree level arguments.

The procedure outlined above leads to the following structure: There are 3 phenomenons corresponding to vector superfield exchange, which are gauge boson exchange, gaugino exchange, and “exchange” of the auxiliary $D_{V_s}$–field (see Figure 1). Each of them can roughly be connected to one four–superfield term:
(1): The vector boson exchange (Fig. 1a) produces component terms that fit into the $1/M_S^2$-suppressed supersymmetric four-fermion term

$$\mathcal{L}_{\text{eff}1} = \frac{g_2^2}{M_S^2} \int d^2\theta d^2\bar{\theta} [(\bar{Q} \Gamma T^c)(QT^c)] .$$

However not all components of (8) are produced this way. The remaining ones come from second order kinetic term contributions of gaugino exchange (b). Altogether all component terms corresponding to (8) are correctly produced up to order $1/M_S^2$.

(2): The gaugino exchange (Fig. 1b) would not give any contribution to first order in an exactly supersymmetric theory because the heavy VEV connects the two component gaugino spinor to the Higgsino as its opposite chirality and the Higgsino does not couple to the low energy sector. Only the soft breaking gaugino mass term gives soft breaking contributions of the order $(1/M_S)(\delta/M_S)$ which can be written in superfields as

$$\mathcal{L}_{\text{eff}2} = \frac{g_2^2}{M_S^2} \int d^2\theta d^2\bar{\theta} [(\bar{Q} \Gamma T^c)(QT^c)](\delta^2 + \delta\bar{\theta}^2) .$$

with the soft breaking parameter $\delta$. This happens the same way in SUSY GUTs [12].

---

**Figure 1:** Vector superfield exchange; (a) vectorfield, (b) gaugino, (c) auxiliary field

**Figure 2:** Diagram cancelling Fig. 1c up to soft breaking terms
(3): Also similar to SUSY GUTs is the situation of auxiliary field “exchange” (Fig. 1c). The contribution of Fig. 1c is exactly cancelled by the contribution of Fig. 2 in the supersymmetric case, only the soft breaking Higgs mass $\Delta^2$ leads to a soft breaking contribution of the order $(\Delta/M_S)^2$:

$$L_{eff} = -\frac{g_S^2}{M_S^2} \int d^2\theta d^2\bar{\theta}[\bar{Q} \tau Q^c]2\Delta^2\theta^2\bar{\theta}^2.$$  \hfill (10)

Finally we end up with an effective coupling term of the form:

$$L_{eff} = \frac{g^2}{M^2} \int d^2\theta d^2\bar{\theta}[\bar{Q} \tau Q^c](1 - 2\Delta^2\theta^2\bar{\theta}^2 + \delta\theta^2 + \delta\bar{\theta}^2).$$  \hfill (11)

The structure and the size of the soft breaking contributions of the four superfield coupling in equation (2) are correctly regained but, not surprisingly, the suppression factor of the four–superfield coupling is of the order of the heavy mass scale while we want it to be of the soft breaking scale.

Still this is not the whole story: For a realistic model it is necessary to take into consideration also the electroweak gauge couplings, which will lead to even more fundamental problems of heavy vector superfield exchange. We write the kinetic terms of the full theory as:

$$L_{kin} = \int d^2\theta d^2\bar{\theta}[(\partial^\mu + ig^2V^\mu_2 + igQV^\mu_1)Q + T^c e^{(gsVs+gtV_1)}Q + \frac{1}{4}(W_{sa}W^a_s + W_{ewa}W^a_{ew})\delta^2(\theta) + h.c.]$$  \hfill (12)

Now we have a look at the gauge structure: In the effective theory we are confronted with different covariant derivations $D^\mu_Q = \partial^\mu + ig_2V^\mu_2 + igQV^\mu_1$ and $D^\mu_T = \partial^\mu + igTV^\mu_1$ for $q, \tilde{q}$ and $t, \tilde{t}$ respectively. One of the dimension 6 effective coupling terms is the four–scalar coupling term which includes two covariant derivatives. As described in above in the framework of auxiliary fields, this term has to be identified with the kinetic term of the scalar boundstate $h_1 = \tilde{q}\tilde{t}$ at the cutoff scale. Therefore it should have the structure $[D^\mu_Q(q\tilde{q}\tilde{t})]D^\mu_T(q\tilde{q}\tilde{t})$ with $D^\mu_Q = \partial^\mu + ig_2V^\mu_2 + ig(Q - gT)V^\mu_1$. But the term generated by integrating out a heavy vector field is the $D_QD_T$ term $[\tilde{q}\tilde{t}D_Q\tilde{q}\tilde{t}]D_T\tilde{q}\tilde{t}$. (One covariant derivative of the $\tilde{q}$ and $\tilde{t}$ scalar kinetic term is used up by the intermediating heavy gauge Boson.) This is, of course, a gauge invariant coupling term as well, but it is simply the wrong one. It does not fit into the structure of eq.(2) and it is not consistent with low energy propagating boundstates whose structure should resemble the structure of the leading order binding coupling terms.
Heavy vector superfield exchange is therefore not able to give the correct gauge structure for finetuned supersymmetric dynamical electroweak breaking.

There seems to occur another problem connected to the phenomenon that the effective dimension 6 operators do not show the correct supersymmetric structure. An investigation of this point is in progress.

The alternative to vector superfield exchange would be the exchange of massive chiral superfields which do not break any gauge symmetry. In this case the exchanged chiral superfields have to resemble exactly the gauge structure of the auxiliary fields describing the effective couplings. Therefore problems with a wrong gauge structure like in vector superfield exchange are safely excluded. A point we made in the beginning of this discussion can be seen quite impressively now: The part of the underlying theory responsible for the effective four–coupling includes only F–terms (Yukawa coupling term and the $\mu$–term) and the kinetic terms of the heavy fields. The F–terms which do not contain any derivatives in component fields will never be able to lead to the D–term in our effective theory (simply because the derivatives that occur there can not emerge out of nothing.) So it is a crucial point to take into consideration the kinetic terms of the heavy fields which produce the effective D–term in a perfectly supersymmetric way. If we neglect the Yukawa term for the second Higgs field the effective F–term is zero and the structure of the effective theory is in agreement with (2) except for the notorious scale problem for $G$. Once again the big suppression factor of effective coupling $G$ does not allow dynamical symmetry breaking.

From this discussion we conclude that a renormalizable underlying gauge field theory is not able to produce propagating effective Higgs fields for several reasons. One has to look therefore for alternative concepts.

An interesting approach to gain a low suppression scale for $G$ has been suggested by Ellwanger \cite{5} in the framework of supersymmetric nonlinear sigma models. However this remains an effective theory approach. It is not clear to us how it could be connected to a fundamental theory behind.

A good way to understand the problem of separating the scales $\Delta$ and $\Lambda$ is to have a careful look at the effective Lagrangians. It is a fundamental principle of top condensation that the SM serves as a good effective theory below the cutoff scale $\Lambda$ which means that the SM Lagrangian can be identified with the top condensation Lagrangian at the cutoff scale. This requirement is best formulated by rewriting the top condensation Lagrangian

\begin{enumerate}
\item[1] However we are inclined to doubt the physical meaning of a concept of producing effective scalar boundstates by heavy scalar exchange in general. A discussion of this point is in preparation.
\end{enumerate}
in auxiliary field formalism and corresponds to conditions for the SM parameters at \( \Lambda \), the 3 so called constituent conditions \[2\]: At the cutoff scale the top Yukawa coupling \( g_t \) and the four Higgs coupling \( \lambda \) of the SM have to run into a pole and, which is important for our discussion, the SM Higgs mass parameter \( m^2 \) has to be identified with the inverse four–fermion coupling \(-G^{-1}\) of top condensation. While the first two conditions have to be fulfilled by renormalization group running of \( g_t \) and \( \lambda \) and predict certain low energy values for these parameters, the situation for \( m^2 \) is different. The running of \( m^2 \) from the electroweak scale up to the cutoff has to change its sign and shift its scale from electroweak to cutoff size. This running behaviour cannot stem from mere renormalization group running. The SM renormalization group running of the parameter \( m^2 \) cannot change its sign, i. e. the SM does not allow radiative breaking of the electroweak symmetry. In contrast with renormalization group running, the kind of running that is responsible for the identification of \( m^2 \) with \(-G^{-1}\) at the cutoff scale in top condensation has to include quadratic contributions. This phenomenon is connected to the fact, that the whole concept of Nambu–Jona-Lasinio like dynamical symmetry breaking is based on the relevance of quadratic divergences in the self consistent gap equation. The quadratic running behaviour of \( m^2 \) can be more or less understood as the effective temperature running of \( m^2 \) which comes up to a mass contribution of the order temperature.

In case of SUSY top condensation the situation is different: The parameter that has to be identified with \( G \) at the cutoff scale is \( \frac{g^2}{\mu^2} \) involving the SUSY invariant \( \mu \)–term. This \( \mu \)–term necessarily produces a positive scalar mass term \( \mu^2 \) after integrating out F–terms. Therefore no change of sign has to take place to identify \( \frac{g^2}{\mu^2} \) with \( G \). Now as the contribution of temperature breaks SUSY \[7\] it must have the structure of softbreaking scalar mass terms and cannot show the structure of the supersymmetric \( \mu \)–term. Thus the \( \mu \)–term is not touched by temperature running, contrary to standard top condensation the identification with \( G \) has to be achieved by mere renormalization group running. Notice that this fact is essential for making a small \( G \)–coupling suppression factor possible. Otherwise the suppression factor would necessarily stay at the cutoff scale due to temperature arguments.

Now one should have a look at the renormalization group running of \( \frac{g^2}{\mu^2} \). Both \( g_t^2 \) and \( \mu^2 \) run into a pole driven by the same contributions. These pole making contributions cancel\[4\], the rest is logarithmic running that does not change the order of the scale drastically. Noticing that low energy \( g_t^2 \) is approximately 1, we see that the low energy value of \( \mu^2 \) indeed gives

---

\[2\]One cannot see this cancellation by expanding in the coupling constant, as this expansion obviously breaks down in the pole region. However the cancellation is correctly produced by the first order \( 1/N_c \) \( \beta \)–functions.
the order of the $G$–coupling suppression factor. This makes quite an interesting difference compared to standard top condensation.

The conclusion of this discussion is that the problem how to separate softbreaking and cutoff scale is directly related to the SUSY $\mu$–problem \cite{3}. This $\mu$–problem can be stated as follows: The $\mu$–term in a MSSM has to be of the order softbreaking scale to allow electroweak symmetry breaking. But being a supersymmetric term it is not restricted by a SUSY breaking scale and should therefore be expected to be naturally of a high (e. g. the Planck) scale. Here we find again our problem of separating the scales. As we assume the MSSM to be an effective theory of SUSY top condensation, the two problems are identical for us, just formulated once in the framework of an effective theory (MSSM) and the other time in the framework of the underlying theory (top condensation). Now there exist solutions of the $\mu$–problem in connection with supergravity. The next step should be to see whether these solutions can be maintained in the framework of a constituent Higgs model.

The basic idea of these solutions of the $\mu$–problem is to forbid the direct $\mu$–term in the Lagrangian (something which could be natural in a superstring scenario) and to regain a low energy $\mu$–term from other sources within supergravity \cite{10}. A quite general approach is to couple the Higgses to some singlet from the hidden sector like:

$$\int d^2\theta d^2\theta' \frac{1}{M_P} Z^+ H_1 H_2 + h.c.$$  

In global SUSY this term can be written as an F–term:

$$\int d^2\theta d^2\theta' \frac{1}{M_P} Z^+ H_1 H_2 = -\frac{1}{4} \int d^2\theta \frac{D^2 Z^+}{M_P} H_1 H_2$$  

(13)

If SUSY is broken in the hidden sector by a vacuum expectation value of the auxiliary field of $Z$ at a scale $< F_Z > \sim (10^{10} - 10^{11})^2$, an effective $\mu$–term of the desired scale $\mu = \frac{< F_Z >}{M_P} \sim 10^4 GeV$ is produced.

Now, to apply this procedure to a constituent Higgs model, it is necessary to be able to interpret the Higgs fields of (13) as auxiliary fields. As above we use the condition that the kinetic term of $H_2$ vanishes at the cutoff scale $\Lambda$ and write the Lagrangian

$$\mathcal{L} = \mathcal{L}_{YM} + \int d^2\theta d^2\theta' (Q e^{2V_Q} Q + T e^{-2V_T} T^c + B e^{-2V_B} B^c)(1 - \Delta^2 \theta^2 \theta')$$

$$+ \int d^2\theta d^2\theta' \overline{H}_1 \exp(2V_{H_1}) H_1 (1 - \frac{\theta^2 \theta'}{M_P})$$

$$- \int d^2\theta \epsilon_{ij} \frac{D^2 Z^+}{M_P} H_1^i H_2^j (1 + B_0 \theta^2) - g r_0 H_2^i Q^i T^c (1 + A_0 \theta^2))$$
\[- \int d\theta^2 \epsilon_{ij} \left( \frac{D^2 Z}{M_P} \right)_{ij} H_1 \bar{H}_2 (1 + B_0 \theta^2) - g_{\tau_0} T \bar{Q} \bar{H}_2 (1 + A_0 \bar{\theta}^2) \right). \quad (14)\]

The Euler–Lagrange equations give the following constituent relations for the fields $H_1, H_2$:

\[
\frac{D^2 Z}{M_P} H_1 (1 + B_0 \theta^2) = g_{\tau_0} QT \bar{c} (1 + A_0 \theta^2), \quad (15)
\]

\[
\frac{Z}{M_P} H_2 (1 + B_0 \theta^2) = -\frac{1}{4} T_1 e^{2V_{H_1}} (1 - M^2_{H_1} \theta^2 \bar{\theta}^2). \quad (16)
\]

At low energies the couplings to the hidden sector field $Z$ are suppressed by the Planck scale and therefore neglectable with one exception: the coupling to the VEV of the auxiliary field $F_Z$ is important. Like in the fundamental Higgs case the VEV produces a mass term $\mu_0 = \langle F_Z \rangle / M_P$ of the order $10^4$ GeV. Inserting $\langle F_Z \rangle$ into (15) and (16) and neglecting all terms suppressed by $M_P$ we get the following constituent relations in the broken phase:

\[
H_1 (1 + B_0 \theta^2) = g_{\tau_0} QT \bar{c} (1 + A_0 \theta^2), \quad (17)
\]

\[
H_2 (1 + B_0 \theta^2) = -\frac{D^2}{4\mu_0} T_1 e^{2V_{H_1}} (1 - M^2_{H_1} \theta^2 \bar{\theta}^2), \quad (18)
\]

which are exactly the constituent relations of Bardeen et al. Thus we get a reasonable four–fermion theory in the SUSY broken phase which becomes nonlocal at the cutoff scale $\Lambda$ not due to a propagating heavy gauge boson but due to some underlying finite theory at the Planck scale, e. g. a string theory. Consequently the scale $\Lambda$ has to be identified with the Planck scale.

The situation in the unbroken phase is much less attractive. The constituent relations (15), (16) are highly nonlinear, it seems to be impossible to eliminate the auxiliary fields. Still the existence of this non–dynamical relation means that there is no independent Higgs degree of freedom in the theory. We claim that this nonlinear realization is the only possibility to achieve a critical Nambu–Jona-Lasinio like gap equation in a supersymmetric framework. These are once more the summarized arguments leading to this statement:

We want to produce a Higgs bound state that propagates up to a scale $\Lambda$ higher than the scale where SUSY becomes relevant, i.e. the SUSY soft breaking scale. Because of the cancellation of quadratic divergences a critical Nambu–Jona-Lasinio gap equation in a SUSY theory requires a four–fermion coupling $G$ suppressed only by the soft breaking scale. A renormalizable underlying gauge field theory, i.e. an exchange of heavy particles with mass
of the cutoff scale $\Lambda$ would not be able to separate the scales of $\Lambda^2$ and $G^{-1}$. The alternative is to maintain the nonrenormalizable structure up to the Planck scale. As the constituent conditions for RG running parameters in a SUSY theory of propagating boundstates are formulated without involving temperature running, the SUSY $\mu$–term gives the scale for the suppression factor of $G$. Since the SUSY theory with Higgses has to be an effective theory of the Nambu–Jona-Lasinio model, the structure that produces the low suppression of $G$ has to appear as a solution to the $\mu$–problem in this effective SUSY theory. But the known solutions to the $\mu$–problem correspond to a nonlinear constituent relation in a constituent Higgs model.

While the nonlinear constituent relation in the unbroken phase is not a highly attractive feature of this model, the model shows apart from this obstacle a number of nice features:

The model gives a natural solution to the $\mu$–problem. There is no need of a symmetry like R–symmetry or Peccei–Quinn symmetry that forbids the $\mu$–term in the superpotential like in a theory with fundamental Higgses. An effective $\mu$–term in the superpotential would be produced by a 4–superfield D–term

$$\int d^2\theta d^2\overline{\theta} \frac{1}{M_P^2} \overline{T} Q T^c,$$

which is suppressed by the Planck–scale. But adding this additional interaction term to the Lagrangian \[\text{Lag}\] only gives highly suppressed additional contributions to the low energy theory. The low energy four fermion interaction becomes:

$$\int d^2\theta d^2\overline{\theta} \frac{1}{M_P^2} \overline{T} Q T^c + \int d^2\theta d^2\overline{\theta} \frac{1}{\Delta^2} \overline{T} Q T^c \simeq \int d^2\theta d^2\overline{\theta} \frac{1}{\Delta^2(1 - \frac{\Delta^2}{M_P^2})} \overline{T} Q T^c.$$

By constructing the $\mu$–term from four–superfield interactions the low scale becomes the dominating one. There is no necessity to forbid any type of possible interaction.

Another interesting feature is the fact that the argument of Hasenfratz et al. \[\text{Ref.}\] lanced against four–fermion top condensation is not valid in our case. This argument states that higher dimensional operators can change the predictions of a four fermion top condensation model arbitrarily so that this model would turn out to be just a different parametrization of the SM. In our case higher dimensional operators are suppressed by higher orders of the Planck scale while the suppression factor of the effective four–fermion coupling is just the SUSY softbreaking scale $\Delta$. Thus these higher order operators are irrelevant in low energy physics.

In summary we have analyzed the possibilities for theories underlying supersymmetric Nambu–Jona-Lasinio like models which produce a supersymmetric propagating Higgs bound state.
(e.g. supersymmetric top condensation). The general problem of such a concept is to find some mechanism which is able to separate the suppression scale of the effective four–superfield coupling from the effective cutoff scale, the scale of new physics in the underlying theory. It turns out that a renormalizable gauge field theory, which is a theory of heavy particle exchange, is not able to achieve this. The approach of underlying vector superfield exchange additionally is not able to produce the gauge structure of the four–superfield coupling that would be needed to build Higgs boundstates. We argued that the only reasonable alternative in the framework of established physical concepts is to couple dynamical electroweak breaking to supergravity. In this scenario the correct effective structure of a Nambu–Jona-Lasinio like model is produced as a result of SUSY breaking in the hidden sector. Such a concept automatically implies the desired vanishing of the $\mu$–term in the SUGRA superpotential and suppresses unwanted higher dimensional operators that could change the predictions of top condensation. A drawback of this approach is the fact that, in spite of being a model without independent Higgs degrees of freedom, it does not allow to integrate out the auxiliary fields linearly.

**Acknowledgments:** We would like to thank E. Dudas who called our attention at the relevance of the $\mu$–problem in this context and A. Blumhofer, M. Lindner, U. Nierste and M. Yamaguchi for useful discussions and helpful comments on the draft version of this paper. This work is in part supported by DFG (contract Li 519/2-1).

**References**

[1] S. Weinberg, Phys. Rev. D13 (1976) 974; D19 (1979) 1277;
L. Susskind, Phys. Rev. D20 (1979) 2619;
E. Fahri and L. Susskind, Phys. Rep. 74 No.3 (1981) 277 .

[2] W.A. Bardeen, C.T. Hill and M. Lindner, Phys. Rev. D41 (1990) 1647 ;
V. A. Miranski, M. Tanabashi and K. Yamawaki, Mod. Phys. Lett. A4 (1989) 1043,
Phys. Lett. B221 (1989) 177 .

[3] W. Buchmüller and U. Ellwanger, Nucl. Phys. B245, (1984) 237 .
[4] W.A. Bardeen, T.E. Clark and S.T. Love, Phys. Lett. B237 (1990) 235;  
   W.A. Bardeen, M. Carena, T.E. Clark, K. Sasaki and C.E.M. Wagner, Phys. Lett. B369 (1992) 33 .

[5] U. Ellwanger, Nucl. Phys. B356 (1991) 46 .

[6] E. Dudas, private discussion .

[7] L. Girardello, M.T. Grisaru and P. Salomonson, Nucl. Phys. B178 (1981) 331 ;  
   Z. Lalak, J. Paweczyk and S. Pokorski, MPI-Ph/93-42 hep-th/9307162

[8] A. Hasenfratz, P. Hasenfratz, K. Jansen, J. Kuti, Y. Shen, Nucl. Phys. B365 (1991) 79 .

[9] G.F. Giudice and G. Ridolfi, Z. Phys. C41 (1988) 447

[10] J.E. Kim and H.P. Nilles, Phys. Lett. B138 (1984) 150, B263 (1991) 79 ;  
    G.F. Giudice and A. Masiero, Phys. Lett. B206 (1988) 480 ;  
    J.A. Casas and C. Munoz, Phys. Lett B306 (1993) 288 .

[11] S. Dimopoulos and A. Pomarol, Nucl. Phys. B453 (1995) 83 .

[12] W. Lang, Nucl. Phys. B203 (1982) 277 .

[13] J. P. Derendinger and C. A. Savoy, Phys. Lett. 118B (1982) 374 .