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Damage Modeling of Metallic Fiber-Reinforced Concrete

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Abstract

This contribution presents a practical 3D numerical model to predict the mechanical behavior of concrete matrix reinforced with sliding metallic fibers. Considering fiber-reinforced concrete (FRC) as two phase-composite, constitutive behavior laws of plain concrete and fibers were described first and then they were combined according to anisotropic damage theory to predict the mechanical behavior of metallic fiber-reinforced concrete. The behavior law used for the plain concrete is based on damage and plasticity theories. The constitutive law of the action of fibers in the matrix is based on the effective stress carried by the fibers. This effective stress depends on a damage parameter related to on one hand, on the content and the mechanical properties of the fiber and on the other hand, on the fiber-matrix bond. The proposed model for FRC is easy to implement in most of the finite element codes based on displacement formulation; it uses only measurable parameters like elasticity coefficient, tensile and compressive strengths, fracture energies and strains at peak stress in tension and compression. A comparison between the experimental data and model results has been also provided in this paper.

Keywords: concrete; fiber reinforcement; damage modeling;

1. Introduction

Despite a wealth of empirical knowledge of properties and behavior of fiber-reinforced concrete (FRC), relatively few attempts have been made so far to rationally explain the basic mechanical behavior and to develop mathematical models capable of simulating the behavior of FRC subjected to complex loading histories. An overview of different methods (such as shear lag-based, analytical, fiber bundle model, fracture mechanics, continuum damage mechanics and numerical continuum mechanical) of the
mathematical modeling of deformation, damage and fracture in FRC composites has been reported by Leon and Povl [1]. Recent developments in the modeling of FRC composites include work of Pyo and Lee [2], Radtke et al [3], George and Peter [4], and Dutra et al [5].

When FRC composite fails due to fiber pull-out the fibers are qualified as sliding fibers. Sliding of fibers occurs due to low bond strength with the concrete matrix. Results of an experimental program carried out to study mechanical behavior of concrete reinforced with sliding metallic fibers have been reported in [6].

In the present contribution, a three dimensional (3D) finite element model to predict the mechanical behavior of concrete matrix reinforced with sliding metallic fibers is proposed. Considering fiber reinforced concrete as two-phase composite, constitutive behavior laws of plain concrete and sliding metallic fibers are described first and then they are combined according to anisotropic damage theory to predict the mechanical behavior of fiber-reinforced concrete.

2. Model Presentation

Since fibers act only when the matrix is cracked, the behavior of FRC is decomposed into two phases as shown in Fig.1 obtained by performing direct tension tests:

- Pre-cracking phase: this corresponds to the plain concrete behavior
- Post-cracking phase: this corresponds to the behavior of composite after the initiation of cracking. This phase is highly depended on the fiber effects which depend on the content and type of the fibers used.

![Figure 1: Pre and post cracking behavior of FRC in direct tension](image)

Modeling of the behavior of FRC composite implies combining the behavior laws for plain concrete with the effect of fibers in the concrete. For matrix reinforced with sliding metallic fibers, direct tension test results show that after when the matrix is cracked, a sudden drop in the stress is followed by an important value of residual stress (compared to control composite) by bridging localized macro-crack. Moreover, this residual stress is maintained over a wide range of crack opening until the fibers are fully pulled-out from the matrix. This shows that fibers act at macro-cracking level and their action evolves along with the increase of crack opening as shown in Fig.2.
The stress carried by the sliding metallic fibers, when they bridge the macro-crack, is added to the stress carrying capacity of the matrix in the contiguous remaining un-cracked concrete; as a result, value of post-peak stress of the brittle matrix is increased importantly (refer to Fig.2). Since the sliding fibers act after the localization of macro-crack and their action vary along with the increase of opening of macro-crack, to model the behavior of FRC containing this type of fibers, value of the crack width is required.

To calculate the crack width, following equation is used:

\[ w = \varepsilon(\vec{n}) \times l(\vec{n}) \]  

(1)

Where \( \varepsilon \) and \( l \) are strain and length of the finite element in \( \vec{n} \) direction as shown in Fig.3. The direction \( \vec{n} \) is always normal to the crack. One can imagine that with the change of orientation of the crack, length of the finite element required to calculate crack width will be different. For this reason, an anisotropic damage model for the concrete matrix is required which must be capable of determining the orientation and width of the crack. An orthotropic damage model of plain concrete proposed by Sellier [7] is used here to simulate the behavior of concrete matrix.

![Figure 2: FRC composite containing sliding fibers](image)

2.1. Constitutive Law of Plain Concrete

The constitutive law of plain concrete is based on plasticity and damage theories. The particularity of this model lies in considering residual stresses in opened cracks which will allow easy modeling of FRC composites. After a brief description of the model for plain concrete, its extension to FRC is presented in this part of the paper.

In the model for plain concrete, like in most of the finite element codes, a six terms vectorial representation of symmetric stress and strain matrix has been adopted. The classical stress tensor of the continuum mechanic becomes a vector \( \vec{\sigma} \), and the symmetric strain tensor becomes also a six components vector \( \varepsilon \). Similarly, the stiffness matrix of the healthy material \( \mathbf{S}^0 \) is represented by a second order tensor. In the model, positive stress stands for tension and negative stress stands for compression.
2.1.1. Constitutive Equations

Behavior Law - For a comprehensive equivalent idealized one-dimensional problem, let $\mathbf{D}'$ be the tensile damage tensor which represents the effect of the tensile cracks on the behavior law. The undamaged zone in tension is represented by the tensor $(\mathbf{1} - \mathbf{D}')$. The effective stress, called $\tilde{\sigma}^t$, represents the loading effects concentrated in the undamaged zone. According to the equivalent strain principle of the damage mechanics [8], it is assessed directly from the mechanical elastic strain using the elasticity stiffness matrix of the healthy material (material without damage) $\mathbf{S}^0$ (Equation 2).

$$
\tilde{\sigma}^t = \mathbf{S}^0 \tilde{\varepsilon} \tag{2}
$$

Concerning the stress state in a tensile crack, it is represented by $\tilde{\sigma}^f$. It is a function of the inelastic strain $\tilde{\varepsilon}^f$ associated to the crack opening, and of $\tilde{\varepsilon}$ as expressed in Equation 3. Adopting $\mathbf{S}^0$ as stiffness to assess $\tilde{\sigma}^f$ allows recovering the stiffness of the healthy material in case of strong compressive loading.

$$
\tilde{\sigma}^f = \mathbf{S}^0 (\tilde{\varepsilon} - \tilde{\varepsilon}^f) \tag{3}
$$

The inelastic strain will be chosen such as that the most of crack re-closure effects described by Jefferson [9] could be considered. The combination of $\tilde{\sigma}^t$ and $\tilde{\sigma}^f$ is done in accordance with their relative zone of application: $\mathbf{D}'$ for $\tilde{\sigma}^f$ and $\mathbf{1} - \mathbf{D}'$ for $\tilde{\sigma}^t$. The resulting stress $\tilde{\sigma}^c$ is given by Equation 4.

$$
\tilde{\sigma}^c = (\mathbf{1} - \mathbf{D}') \tilde{\sigma}^t + \mathbf{D}' \tilde{\sigma}^f \tag{4}
$$

For the undamaged zone in compression, $\tilde{\sigma}^c$ can lead to a compressive damage represented by a second order tensor called $\mathbf{D}^c$. This damage corresponds physically to crushing of the material. That is why it affects as well the zone undamaged in tension and causes damage. After this damage, the resulting stress becomes the apparent stress $\tilde{\sigma}$; it is given by Equation 5.
\[
\tilde{\sigma} = (1 - D^t)(1 - D^t)^0_\bar{\sigma} + D^t \bar{\sigma}^f \\
\tilde{\sigma} = (1 - D^t)(1 - D^t)_0^S \bar{\varepsilon} + (1 - D^t)_D^t S^0_0 \bar{\varepsilon} - \bar{\varepsilon}^f \]

(5)

(6)

Internal variables - The damages \((D^t)\) and the inelastic strain in cracks \((\bar{\varepsilon}^f)\) are the internal
variables of the presented model for the plain concrete. Their evolution laws were chosen according to the
thermodynamic principles. Considering \(\bar{S}\) as the stiffness matrix of a material damaged by an oriented
micro-cracking, the behavior law of the cracked material can be expressed by Equation 7. At this stage, it
is assumed that there is no stress in the cracks during their propagation.

\[
\tilde{\sigma}^c = S_0 \bar{\varepsilon} = (1 - D^t)_0 S \bar{\varepsilon}
\]

(7)

The tensor \((1 - D^t)\) is then obtained from Equation 7.

\[
(1 - D^t)^0 = S^0_0 - S
\]

(8)

\[
(1 - D^t)^-1_0^S = S^0_0 S^{-1}
\]

(9)

From Equation 8, \(D^t\) is obtained as (Equation 10):

\[
D^t = 1 - S^{-1} S^0_0
\]

(10)

\((1 - D^t)^-1\) tensor deduced from the homogenization theory is a function of the micro crack density \(d_i^t\).
\(d_i^t\) ranges from zero for the healthy material (without damage) to one for a full damage in direction 1.

Crack mouth opening displacement (CMOD) - It is well known that the crack localizes from the peak
of the uni-axial tensile behavior law, and in the case of uni-axial tensile loading the opening of the
localized crack becomes equal to the displacement imposed to the edge of the finite element as soon as
the crack crosses it; simultaneously the crack density tends to one.

If \(d_i^t \rightarrow 1\) then \(w_i \rightarrow \varepsilon_{11} I_{(e_i)}\)

(11)

In Equation 11, \(\varepsilon_{11}\) is the normal strain in the main direction of cracking \(e_i\), \(l_{(e_i)}\) the finite element
size in direction \(e_i\), and \(d_i^t\) the cracking density in direction \(e_i\). Before this extreme configuration, a
localized crack can exist but doesn’t cross the finite element.

Since the crack is localized only from the peak, the cracking density lesser than that one corresponding
to the peak of the tensile behavior law \(d_i^{peak}^t\) concerns only the diffuse cracking. Consequently only the
part \((d_i^t - d_i^{peak})\) corresponds to the localized part of the crack, considering this, a CMOD
approximation for the softening phase is proposed. Currently, an approximated value of the crack width is
used. It leads to Equation 12. This expression is interpreted as the average value between a part
\( (d_i^t - d_i^{t,\text{peak}}) \) including a crack with an opening equal to \( w_i \) and a complementary part without localized crack. The proposed form verifies \( w_i = 0 \) while \( d_i^t \leq d_i^{t,\text{peak}} \), and \( w_i \to \varepsilon_{11} \cdot I(\dot{\phi}) \) when \( d_i^t \to 1 \).

\[
w_i = \frac{\tilde{\sigma}_f^i I \left( v - 1 + 2v^2 d_i^t \right)}{E(v - 1)(1 - d_i^t)} \cdot \frac{d_i^t - d_i^{t,\text{peak}}}{1 - d_i^{t,\text{peak}}} H(\tilde{\sigma}_f^i I) H \left( \frac{d_i^t - d_i^{t,\text{peak}}}{1 - d_i^{t,\text{peak}}} \right) \tag{12}
\]

In Equation 12, \( H \) stands for the Heaviside function defined as \( H(X) = 1 \) if \( X > 0 \), 0 otherwise. The Heaviside function is applied to the effective stress to consider that a crack can be opened only if the finite element is in tension in the corresponding direction. It is applied also to \( (d_i^t - d_i^{t,\text{peak}}) \) to consider that only a crack density greater than \( d_i^{t,\text{peak}} \) leads to a localized crack.

2.2. Constitutive Law of Sliding Metallic Fibers in Concrete

Let \( \tilde{\sigma}_f^i \) be the effective stress transferred through the crack by the sliding fibers, which can be expressed as follows:

\[
\tilde{\sigma}_f^i = K_f \cdot w_i \text{, with } I = [1,2,3] \text{ (three main directions)}
\tag{13}
\]

Where \( K_f \) is the rigidity of fibers; \( w_i \) is crack mouth opening displacement (Equation 12). Equation 13 implies that the fibers work only after the crack localization. To consider the fiber-matrix bond, a new damage variable \( d_i \) is introduced such as when \( d_i \) is the damage caused to fiber-matrix bond by the relative sliding of the fibers, the residual bearing capacity of fibers can be expressed as:

\[
\sigma_{f,1} = \tilde{\sigma}_f^i \cdot (1 - d_i) \tag{14}
\]

The damage to the fiber-matrix bond by the relative sliding of the fibers varies between 0 and 1. When \( d_i = 1 \), this shows that fiber is fully pulled-out from the matrix. The evolution of the \( d_i \) was modeled using Weibull equation. Finally, the behavior law of concrete reinforced with sliding fibers (Equation 15) is obtained by adding (according to the damage theory) the plain concrete contribution and the fiber stress carrying capacity beyond the macro-crack localization:

\[
\bar{\sigma} = \left(1 - D^c\right)\left(1 - D^s\right)S^0 \varepsilon + \left(1 - D^c\right)D^s\left(S^0 \left(\varepsilon - \varepsilon^f\right) + \tilde{\sigma}_f\right) \tag{15}
\]

2.3. Fitting and Validation of Model

This section is intended to fit various model parameters and validation of the proposed model. An experimental investigation was carried out to study the mechanical behavior of FRC composites containing sliding fibers at content of 20 kg/m³. Displacement controlled direct tensile tests were performed on notched prismatic specimen of size 100x100x160 mm. For the loading rate, RILEM recommendations for this type of test were followed.

Since the specimen was symmetrical in all direction \( (x,y,z) \), in finite element code CASTEM [10], to reduce computation time, one fourth of the specimen was modeled as three dimensional problem.
Material parameters for the uni-axial direct tensile tests simulation for the control and FRC composites are given in Table 1. In this table, SF-20 stands for concrete containing sliding fibers at content of 20 kg/m². Comparison of the results of model fitting and experiments is shown in Fig.4 for control and FRC composites.

Table 1: Material parameters for the uni-axial direct tension tests

| Model Parameters          | Unit | Composite         |
|---------------------------|------|-------------------|
| Name                      |      | Control           |
| Young modulus, \( E \)    | GPa  | SF-20             |
| Poisson coefficient, \( \nu \) |      | 0.27              |
| Tensile strength, \( R_t \) | MPa  | 3.5               |
| Fracture energy in tension, \( G_f \) | MN/m | 1.37 x10⁻⁴       |
| Compressive strength, \( R_c \) | MPa  | 50                |
| Fracture energy in compression, \( G_f' \) | MN/m | 1.37 x10⁻²       |
| \( K_f \)                  | MPa  | 11000             |
| \( (\nu_1)_{peak} \)      | m    | 0.5x10⁻²          |
| \( (R_{t_f})_1 \)         | MPa  | 1.2               |

Figure 4: Model fitting – Direct tension test (control and SF-20 composites)

This part of the paper is intended to validate the presented model for FRC through three point bending tests. The model predictions have been compared to experimental data. To obtain experimental data, three point bending tests were performed on notched prismatic specimens of size 100 x 100 x 500 mm...
constructed using FRC containing 20 kg/m³ of sliding metallic fibers. The values of the model parameters used in the simulation were the same as listed in Table 1.

Comparison of the experimentally obtained load-deflection & load-CMOD response for the three samples and model results is presented in Fig.5, where it can be noticed that the model results agree quite well with the experimental results in both cases (load-deflection and load-CMOD curves). Crack localization in the finite element mesh of test specimen is shown in Fig.6.

![SF-20 - Load vs Deflection Curves](image1)

![SF-20 - Load vs CMOD Curves](image2)

**Figure 5:** Comparison of Load-Deflection and Load-CMOD curves of SF-20 in three point bending test (Model and experimental results)

**Figure 6:** Crack localization and propagation (SF-20 in 3PBT)

### 3. Conclusions and future work

From the damage behavior laws for plain concrete on one hand and fiber-reinforced concrete on the other, taking into account fiber–matrix bond damage for sliding fibers, a 3D numerical model to predict the behavior of fiber reinforced concrete has been proposed. The proposed numerical model involves such parameters which have a definite physical meaning and can, therefore, be readily determined from conventional tests on concrete specimens in tension and compression. Model predictions in bending test simulation showed good agreement when compared to experimental data.

An ongoing program focuses on modeling of flexural behavior of concrete reinforced with conventional steel bars and metallic fibers simultaneously (reinforced fibrous concrete). In this case, proposition of an effective and more practical approach to simulate the steel bar-concrete interface is envisaged. The findings are expected to be finally integrated into a finite element code to predict the flexural behavior of reinforced fibrous concrete beam using the present model for FRC composites containing sliding metallic fibers.
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