Early GRB Afterglows from Reverse Shocks in Ultra-relativistic, Long-lasting Winds

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Abstract

We develop a model of early Gamma-Ray Burst (GRB) afterglows with dominant X-ray contribution from the reverse shock (RS) propagating in highly relativistic (Lorentz factor $\gamma_w \sim 10^3$) magnetized wind of a long-lasting central engine. The model reproduces, in a fairly natural way, the overall trends and yet allows for variations in the temporal and spectral evolution of early optical and X-ray afterglows. The high energy and the optical synchrotron emission from the RS particles occurs in the fast cooling regime; the resulting synchrotron power $L_{\gamma} \propto L_w / (1 + \sigma_w)$ ($L_w$ and $\sigma_w$ are wind power and magnetization). Thus, plateaus—parts of afterglow light curves that show slowly decreasing spectral power—are a natural consequence of the RS emission. Contribution from the forward shock (FS) is negligible in the X-rays, but in the optical both FS and RS contribute similarly: FS optical emission is in the slow cooling regime, producing smooth components, while RS optical emission is in the fast cooling regime, and thus can both produce optical plateaus and account for fast optical variability correlated with the X-rays, e.g., due to changes in the wind properties. We discuss how the RS emission in the X-rays and combined FS and RS emission in the optical can explain many puzzling properties of early GRB afterglows.

Key words: gamma-ray burst: general

1. Introduction

Gamma-Ray Bursts (GRBs) are produced in relativistic explosions (Paczynski 1986; Piran 2004) that generate a two shock structure—a forward blast wave and the termination shock in the prompt ejecta. The standard fireball model (Rees & Meszaros 1992; Sari & Piran 1995; Piran 1999; Mészáros 2006; Kumar & Zhang 2015) postulates that (i) the prompt emission is produced by internal collision of matter-dominated “shells” within the ejecta; (ii) afterglows are generated in the relativistic blast wave after the ejecta deposited much of its bulk energy into circumburst medium.

One of the most surprising results of Swift observations of early afterglows, at times $\lesssim 1$ day, is the presence of unexpected features—flares and plateaus (e.g., Nousek et al. 2006; Gehrels & Razzaque 2013; Lien et al. 2016). This challenges the standard fireball model, which states that the early X-rays are produced in the forward shock (FS; Lyutikov 2009; Kann et al. 2010).

Though the overall hydrodynamic structure of the outflow consisting of the FS, contact discontinuity (CD), and (possibly) a reverse shock (RS) is hardly questionable, the observational signatures of shock components are not clear at the moment. In the Swift era, the early afterglow light curves, at times $\lesssim 10^5$ s, show numerous variability phenomena, like fast decays, plateaus, flashes, and various light curve breaks. This variability is hard to explain within the FS model, which is expected to produce smooth light curves. Also, the conventional RS signature in matter-dominated models, a bright flash followed by a smooth power law decay in the optical band, is rarely observed.

Flares and plateaus in early X-ray afterglows are interpreted in terms of a long-lived central source, which could remain active (keeps producing relativistic wind) for long times after the explosion, $\sim 10^2 - 10^4$ s, and perhaps even longer (e.g., Troja et al. 2007); see Lyutikov (2009) for a critique. (We use the term “afterglow” in a relation to late, post-prompt, emission, not necessarily coming from the FS.)

In this paper we consider a possibility that most of the X-ray afterglow emission comes from the RS of a long-living central engine, not the FS. The FS shock is there, but it might not radiate. Previously, Katz et al. (1998), Kumar & Piran (2000), Ulm & Beloborodov (2007), Genet et al. (2007), and Ghisellini et al. (2007) advocated that X-ray afterglows may come from the RS emission. (Laskar et al. 2013 argued that radio and millimeter afterglows of GRB 130427A come from the RS.) All these works assumed a “shell paradigm” for the ejecta—a collection of colliding (presumably matter-dominated) blobs with mild Lorentz factors. In contrast, in this work we consider long-lived central engine that produces very fast wind, with Lorentz factors much larger than that of the FS. Building on the previous ideas outlined in Lyutikov (2009) we consider temporal and spectral evolution of the RS emission and discuss how various problems with early X-ray afterglows can be resolved with the RS paradigm.

2. Extended Source Activity: Highly Relativistic, Highly Magnetized Wind

The primary GRB explosion may be driven by a highly magnetized central source (Lyutikov 2006). The long-lasting wind from the central source is even more likely to be magnetized and highly relativistic, with Lorentz factors much larger than the Lorentz factor of the primary shock. In the present work we do not specify the nature of the long-lasting central engine. All three types of proposed central engine may do: (i) magnetar (Usos 1992; Metzger et al. 2011); (ii) black hole jet powered by late accretion (Komissarov & Barkov 2009; Barkov & Komissarov 2010; Barkov & Poznенко 2011); (iii) BH keeping its magnetic field for a long period of time (Lyutikov & McKinney 2011; Lyutikov 2013). In all of these cases it is expected that late activity produces
The interaction of the wind with the shocked circumburst medium creates a second FS (2ndFS below) in the already shocked blast wave, and the RS in the ejecta. The wind pushes the CD with time toward the primary FS. After the external medium absorbs an amount of energy from the wind comparable to the initial explosion, the overall dynamics follows a self-similar blast wave with energy injection (Blandford & McKee 1976, B&Mc afterwards). But before this time the dynamics of the RS and the primary FS are very different: the primary FS decelerates as it sweeps more material, while the RS, CD and the second FS propagate through a very tenuous tail of the blast wave, pushed by the wind. As a result, the Lorentz factor of the RS/CD/2ndFS combination decreases more slowly, see Section 3. This naturally leads to flat emission profiles—the plateaus.

At the same time, the wind produced by the central source can be highly relativistic, so that the RS is always relativistically strong and accelerates particles to high Lorentz factors, of the order of the relative Lorentz factor between the RS and the wind. In addition, the wind can be highly magnetized, leading to efficient production of synchrotron radiation. All these factors: high bulk Lorentz factor, high Lorentz factor of the accelerated particles (in the flow frame) and high magnetization are very beneficial for the production of high energy radiation. Also, in the fast cooling regime a large fraction of the wind luminosity is radiated, while short cooling times are encouraging for explaining short duration flares.

Finally, we know that RSs in relativistic outflows do accelerate particles. Lyutikov et al. (2016) and Yuan & Blandford (2015) clearly demonstrated that the Inner Knot of the Crab Nebula is the surface of the relativistic termination shock.

3. Point Explosion Followed by Fast Wind: Approximately Self-Similar Dynamics at Early Times

3.1. Reverse Shock in Fast Wind

The initial explosion with isotropic equivalent energy \( E_1 \) creates an ultra-relativistic blast wave propagating with Lorentz factor \( \Gamma_{FS} \). Figure 1.

\[
\Gamma_{FS} = \frac{1}{2} \sqrt{\frac{17}{2\pi}} \frac{E_1}{c^2 \rho t^{-3/2}} = \left( \frac{17E_1}{8\pi c^5 \rho_{ob}^2} \right)^{1/8} \tag{1}
\]

where \( t_{ob} = t/\Gamma_{FS}(t)^2 \) is the observer time associated with the FS.

The post-primary shock structure is self-similar (B&Mc). Suppose that the initial explosion is followed by a wind with constant luminosity \( L_w \) (generalization to other temporal behaviors of \( L_w \) are straightforward, but, perhaps are not interesting: we do not expect that the wind power increases with time, while the decreasing wind luminosity will make the effects considered less important). The wind launches a second FS in the medium already shocked by the primary blast and the RS in the wind; they are separated by the CD. We expect the dynamics of this triple-shock structure (triple: primary FS, second FS and the RS, Figure 1) to have two stages. At the early stage, the energy deposited by the wind into the shocked media is much smaller than the initial explosion. In this case the CD is located far downstream the first shock; moving with time in the self-similar coordinate \( \chi \) toward the first shock. The
motion of the first shock is unaffected by the wind at this stage. After the wind has deposited energy comparable to the initial explosion at time $t_{eq}$, the whole flow approaches the Blandford–McKee self-similar solution with energy supply.

We assume that the wind is magnetized with luminosity

$$L_w = 4\pi r^2 \left( n_w m_e c^2 + \frac{b_w^2}{4\pi} \right) c,$$

where $n_w$ and $b_w$ are density and magnetic field measured in the wind rest frame. Thus

$$b_w = \sqrt{\sigma_w} \sqrt{\frac{L_w}{c \gamma_w}},$$

where

$$\sigma_w = \frac{b_w^2}{4\pi n_w m_e c^2},$$

is the wind magnetization parameter. We assume that the wind is very fast, $\gamma_w \gg \Gamma_{FS}, \Gamma_{CD}$.

Interaction of the wind with the CD produces the RS. The post-RS wind has

$$\gamma_{RS} \approx \Gamma_{CD}. \quad \text{(5)}$$

(We use capital letters to denote the Lorentz factors of the special surfaces, e.g., FS, CD, and the RS, while lower cases to denote the flow velocity; primes denote quantities measured in the post-RS flow frame.)

The RS propagates with respect to the CD with velocity $\beta_\sigma$, given by the post-shock velocity of magnetized relativistic shock (Kennel & Coroniti 1984a, Equation (4.11)).

$$\beta_\sigma = \sqrt{1 - \frac{16(\sigma_w + 1)}{8\sigma_w^2 + \sqrt{(2\sigma_w + 1)^2(16\sigma_w^2 + 16\sigma_w + 1) + 26\sigma_w + 17}}} \quad \text{(6)}$$

in the observer frame

$$\Gamma_{RS} = \Gamma_{CD} \sqrt{\frac{1 - \beta_\sigma}{1 + \beta_\sigma}} \quad \text{(7)}$$

Neglecting numerical factors of the order of unity we can approximate

$$\Gamma_{RS} \approx \frac{\Gamma_{CD}}{\sqrt{2(1 + \sigma_w)}} \quad \text{(8)}$$

(for $\sigma_w = 0$ and fully self-similar structure this is the exact result; a more correct approximation would be $\sqrt{1 + \sigma_w} \to \sqrt{1 + 2\sigma_w}$—we neglect this small difference for the sake of simplicity).

In physical coordinate $r$, the FS and the RS remain very close, $r_{FS} \approx r_{RS} \approx ct$,

$$\frac{\Delta r}{r_{FS}} = \frac{r_{FS} - r_{RS}}{r_{FS}} \approx 1 + \sigma_w \Gamma_{FS}^2. \quad \text{(9)}$$

Only for $\sigma_w \sim 1^2$ (sub-Alfvénic wind) this becomes of the order of unity.

For sufficiently strong wind, with time, the second FS approaches the primary FS. We can estimate the catch-up time by noticing that the power deposited by the wind in the shocked medium scales as $L_w/\Gamma_{CD}^2$. Thus, in coordinate time the wind deposits energy similar to the initial explosion at time at time when $\Gamma_{CD} \sim \Gamma_{FS}$,

$$t_{eq} = \Gamma_{FS}^2 \frac{E_1}{L_w} \approx \left( \frac{E_1^2}{c^5 \rho L_w} \right)^{1/4}. \quad \text{(10)}$$

This corresponds to the observer time

$$t_{eq,ob} = t_{eq}/\Gamma_{FS}(t_{eq}) = \frac{E_1}{L_w}. \quad \text{(11)}$$

After time (11) the two FSs merge, see Section 6.

### 3.2. Self-similar Structure of Second Shocks

The fast wind propagating with $\gamma_w$ will launch a second FS (2ndFS) in the already shocked medium. The wind is separated from the double-shocked external plasma by a CD; there is also the RS in the wind. With time, the system of RS/CD/2ndFS may catch up with the first FS. The motion of the CD is approximately self-similar at early times, well before the catch-up time (11), and well after. There are several self-similar regimes for the motion of the RS/CD/2ndFS, depending on the initial set-up (e.g., importance of time delay) and the dynamical approximations, see Appendices A and B. Though the details of dynamics are somewhat different in each case they all follow similar patterns.

Let us first assume that there is no time delay between the first explosion and the onset of the wind (for the applicability of the no time delay condition (see Appendix B)). The motion of the CD is expected to be self-similar before it catches up with the primary FS. Let us assume it scales as $\Gamma_{CD} \propto r^{-m/2}$; then the second shock/CD are located at a self-similar coordinate associated with the primary shock (B&Mc).

$$\chi_{CD} = \frac{\Gamma_{FS}^2}{\Gamma_{CD}^2} \propto t^{m-3}. \quad \text{(12)}$$

In contrast to fully self-similar solution of B&Mc, in which case $\chi_{CD}$ is constant, now it depends on time, $\chi_{CD}(t)$. To estimate the motion of the CD, let us assume $\Gamma_{CD} \propto r^{-m/2}$, then the second shock/CD are located at a self-similar coordinate associated with the primary shock (B&Mc).

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Let us first assume that there is no time delay between the first explosion and the onset of the wind (for the applicability of the no time delay condition (see Appendix B)). The motion of the
total accumulated enthalpy \( w_2 \),
\[
w_2 = 16\pi \int r^2 p dr = \frac{4\pi}{3} m_p c^5 n_{ext} t^3 \int_{\chi_{CD}}^{\infty} \frac{d\chi}{\chi^{3/2}}
\]
\[= \frac{16\pi p c^3}{5} \chi_{CD}^{5/12},\]  
\tag{13}
\]
times \( \Gamma_{CD}^2 \),
\[
E_2 \approx w_2 \Gamma_{CD}^2.
\tag{14}
\]
This energy should equal the momentum flux through the RS in the wind, integrated over time
\[
E_2 = \int L_w \frac{4\pi r^2 c^2}{4\pi \Gamma_{CD}^2} dt = \frac{L_w t}{(m + 1) \Gamma_{CD}^2}
\tag{15}
\]
thus,
\[
\Gamma_{CD} = \left( \frac{5}{2} \frac{\frac{\chi_{CD}}{c^5}}{\chi_{CD}^{5/4}} \right)^{1/4} \frac{L_w^{1/4} \chi_{CD}^{5/48}}{c^{5/4} \rho^{3/4} \sqrt{t}}.
\tag{16}
\]
Solving (12) and (16) for \( \Gamma_{CD} \) and \( \chi_{CD} \), we find
\[
\Gamma_{CD} = 0.50 \left( \frac{E_1}{c^{5/8} L_w^{6/29}} \right)^{1/4} \frac{L_w^{6/29}}{\rho^{29/68} \chi_{CD}^{29/68}}
\tag{17}
\]
thus, \( m = 39/29 \). The observer time for the CD is
\[
t_{ob} = \frac{t}{2(m + 1) \Gamma_{CD}^2} = \frac{29 t}{1361 \Gamma_{CD}^2}.
\tag{18}
\]
Thus,
\[
t = 1.08 \frac{E_1}{c^{5/8} L_w^{3/34}} \frac{L_w^{3/34}}{\chi_{CD}^{29/68}}
\tag{19}
\]
In terms of the observer time,
\[
\Gamma_{CD} = 0.48 \left( \frac{E_1}{c^{5/8} L_w^{3/34}} \right) \frac{L_w^{3/34}}{\rho^{29/68} \chi_{CD}^{29/68}}
\tag{20}
\]
With time, \( \chi_{CD} \) decreases—the CD is getting closer to the FS. When \( t_{ob} \sim t_{eq} = E_1/L_w \), the second shock catches up with the primary shock.

The Lorentz factor of the CD decreases much slower than that of the FS: \( \Gamma_{CD} \propto t^{-39/58} \) compared to \( \Gamma_{FS} \propto t^{-3/2} \). (The difference in observer time is not that large though, \( \Gamma_{CD} \propto t_{ob}^{39/136} \) versus \( \Gamma_{FS} \propto t^{-3/8} \).

We can also verify that the Lorentz factor of the CD (20) is larger than the local Lorentz factor of the post-primary shock flow in front of the CD. For \( \chi_{CD} \propto \Gamma_{FS}/(2\chi_{CD}) \) we find
\[
\frac{\Gamma_{CD}}{\Gamma_{FS}} = \left( \frac{17}{5} \frac{E_1}{L_w t_{ob}} \right)^{6/17} \gg 1 \text{ for } t_{ob} \ll t_{eq}.
\tag{21}
\]
Thus, the wind will drive a second shock into the already shocked external medium, see Figure 2.

This approach, to treat the second shock/CD in a self-similar way, has limitations. We should require then that the CD advances with time to smaller \( \chi \). For a source luminosity scaling as \( L \propto t_{em}^q \), we find that this requires \( q > -1 \).

4. Synchrotron Emission from Self-Similar RS

4.1. Estimate of Efficiency

Let us first point out the most important result: high efficiency of the RS emission in the fast wind. Following the treatment of the RS emission in pulsar winds by Kennel & Coroniti (1984b) we assume that the RS accelerates all the incoming particles to a typical Lorentz factor \( \gamma_f \sim \gamma^f \)—the Lorentz factor of the wind in the RS frame (plus an energetically subdominant power-law tail \( f(\gamma^f) \propto \gamma^{-p} \); the standard choice of \( p \) is 2.5). The total synchrotron power \( L_s \) is the number of emitting leptons times the power of each electron:
\[
L_s \approx N P_e.
\tag{22}
\]
In the fast cooling regime the number of emitting electrons is the injection rate \( \dot{N} \) times the cooling time \( \tau_e \). The cooling time is the energy of an electron divided by the synchrotron power, \( \tau_e = \gamma_e m_e c^2 / P_e \), where \( \gamma_e \approx \gamma_f \Gamma_{CD} \) is the electron Lorentz factor in the observer frame. Thus
\[
L_s \approx \dot{N} \tau_e P_e = N \gamma_e m_e c^2.
\tag{23}
\]
Estimating the injection rate as
\[
\dot{N} = \frac{L_w}{(1 + \sigma_w) \gamma_w m_e c^2},
\tag{24}
\]
gives
\[
L_s \approx \frac{\gamma_e L_w}{(1 + \sigma_w) \gamma_w}.
\tag{25}
\]
Since the minimum lepton Lorentz factor in observer frame is
\[ \gamma_e = \gamma'_w \Gamma_{CD} = \left( \frac{\gamma_w}{2\Gamma_{RS}} \right) \Gamma_{CD} = \frac{\gamma_w}{\sqrt{2(1 + \sigma_w)}}. \quad (26) \]

The total luminosity is then
\[ L_s = \frac{L_w}{\sqrt{2(1 + \sigma_w)}}. \quad (27) \]

We arrived at an important result: the RS emission is just the wind power down, so that for constant Lorentz factor of the wind \( \gamma_w \), the relative Lorentz factor of wind in the frame of the RS \( \gamma'_w \propto \gamma_w/\Gamma_{CD} \) increases.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig3.png}
\caption{Evolution of the distribution function of the RS-accelerated leptons in the fast cooling regime. The RS injects leptons with a power-law distribution \( f_{\text{inj}} \propto \gamma_w^{-p} \) and the minimal energy of the order of the wind’s Lorentz factor in the frame of the RS, \( \gamma'_w \sim \gamma'_w \). Quickly, in the observer time \( \tau_{c,m} \), all leptons cool to the lowest energy, producing the cooled spectrum \( \propto \gamma^{-p/2} \) above \( \gamma_w \), and then at observer time \( \tau_c \sim t_{\text{ob}} \), cool down to the cooling break energy, producing the spectrum \( \propto \gamma^{-1/2} \), after Sari et al. (1998).}
\end{figure}

4.2. Typical Frequencies

Next we estimate the corresponding synchrotron peak frequency. The Lorentz factor of the wind in the frame of the RS is
\[ \gamma'_w = \frac{\gamma_w}{2 \Gamma_{RS}} = \frac{\gamma_w}{\epsilon_{\text{rs}}^{1/2} \Gamma_{CD}}. \quad (28) \]

Using (3) for magnetic field in the wind, in the post-RS region the magnetic field is
\[ B_{RS} = \frac{\gamma_w}{\Gamma_{CD}} B_w = \frac{\sqrt{t_w}}{\sqrt{\epsilon_{\text{rs}} t_{\text{cd}} \sqrt{\sigma_w + 1}}}. \quad (29) \]

Synchrotron photon energy at the break (assuming \( \gamma'_e \sim \gamma'_w \)) is
\[ \epsilon_m = \frac{n_e B_{RS}^2}{2 m_e c} \frac{e \hbar}{2\sqrt{5/2}} \frac{L_{w/2}^{1/2}}{t_{\text{cd}}^{1/2}} \sqrt{\sigma_w \left( \sigma_w + 1 \right)} \gamma_w^2 \]
\[ = 2 \sqrt{17} \frac{145}{4} \frac{n_{\text{ex}}}{m_e} \frac{e^2}{m_e} \sqrt{\sigma_w \left( \sigma_w + 1 \right)} \gamma_w^2 \frac{5/34}{\Gamma_{CD}^{5/34}} \frac{5/34}{\Gamma_{CD}^{5/34}} \frac{\gamma_w^6}{E_{1.54}^{5/34}} \text{eV}, \quad (30) \]

where \( n_{\text{ex}} \) is the external medium number density, and numerical estimates assume that the quantities are measured in cgs; we use the notation \( X_\text{e} = X/10^9 \). The energy \( (31) \) is the minimum energy—the break energy. Acceleration of particles at the RS will produce a cooled power-law emission above \( (31) \), stretching, presumably, to hundreds of keV. Note that \( \epsilon_m \) increases with time—this is due to the fact that the CD slows down, so that for constant Lorentz factor of the wind \( \gamma_w \), the relative Lorentz factor of wind in the frame of the RS \( \gamma'_w \propto \gamma_w/\Gamma_{CD} \) increases.

\[ P_s \approx \gamma'_{\text{RS}} \gamma_r^2 \frac{e^4 h^2}{m_e^2 c^3} \approx \frac{e^4 \sigma_w L_w \gamma_w^2}{2 \epsilon_{\text{rs}}^{1/2} \Gamma_{CD}^{5/34} m_e^2 c^3}. \quad (32) \]

The cooling time for the electrons emitting at \( \epsilon_m \) is
\[ \tau_{c,m} \approx \frac{\gamma'_w \gamma_w m_e c^2}{P_s} = \frac{m_e^2 c^8}{e^4} \frac{\Gamma_{CD}^{5/34}}{\Gamma_{CD}^{5/34}} \gamma_w \sigma_w. \quad (33) \]

We find
\[ \tau_{c,m} \approx 3 \times 10^{-2} \frac{E_{1.54}^{15/68}}{n_{\text{ex}}^{3/4} L_{w/50}^{5/34} \rho_{0,3}^{40/68} \gamma_w^{6/6}} \text{cm} \text{s} \quad (34) \]

placing X-ray emission, and higher energy \( \gamma \) ray emission, well in the fast cooling regime (after \( t_{\text{ob}} \sim \) few seconds).

At a given observer time \( t_{\text{ob}} \), we can estimate the photon energy \( \epsilon_c \) below which emission would be in a slow cooling region, \( \tau_c \approx t_{\text{ob}} \). The regime of fast cooling starts at \( \gamma_c \), which satisfies the condition
\[ \gamma_c m_e c^2 \approx \gamma_{\text{RS}} \gamma_c m_e c^2 = P_{\text{inj}} t_{\text{ob}} \]
\[ P_c = \gamma_{\text{RS}}^2 \frac{e^4 h^2}{m_e^2 c^3} = \frac{0.1}{E_{1.54}^{7/8}} \frac{L_w^{1/2}}{\rho_{0,3}^{5/8}} \frac{1 + \sigma_w}{\gamma_w^{5/8}} \frac{m_e^{39/8}}{E_{1.54}^{7/8}}. \quad (35) \]
frequencies and values are given for law index assumed that both the RS and the FS accelerate particles with the same power estimated at the observer time $\sim$ the break energy ($D$ where $D$ is the distance to the source). Evolution of the electron distribution function of the regime. Evolution of the electron distribution function of the RS models of early afterglows assume that fraction of $\frac{F_0}{F_X} = \left(\frac{\epsilon_0}{\epsilon_X}\right)^{-1/2} \left(\frac{\epsilon_m}{\epsilon_X}\right)^{-p/2} \approx 10^3$ (Equation 40) for $\left(\frac{\epsilon_m}{\epsilon_x}\right)^{1/2} \sim 30$ and $\left(\frac{\epsilon_0}{\epsilon_x}\right)^{1/2} \sim 30$ ($\epsilon_0 \sim 1$ eV, $\epsilon_m \sim$ few keV, $\epsilon_X \sim$ tens of keV). The total energetics in the optical will be a factor $\epsilon_0/\epsilon_X \sim 10^{-4}$ smaller than the estimate (40).

The estimate (40) compares well with observations. For example, for GRB 130603B (Fong et al. 2014, Figure 2), at 0.47 days the ratio of optical to hard X-ray spectral fluxes from the RS is $\sim 5 \times 10^{-7} \mu J$ is $\sim 10^{10}$.

5.2. Forward Shock: Optical, no X-Ray

The FS models of early afterglows assume that fraction of energy $\epsilon_F$ and $\epsilon_B$ are converted into accelerated particles and magnetic field. The emission from the FS in our model follows the conventional prescription for afterglow emission (Sari et al. 1998), but with smaller $\epsilon_B$, $\epsilon_F \sim 10^{-3}$. These values, especially much smaller $\epsilon_B$, are better justified than the usually used ones of $\epsilon_B, \epsilon_F \sim 10^{-1}-10^{-2}$ (e.g., Silva et al. 2003; Spitkovsky 2008). The lower values of $\epsilon_B, \epsilon_F$ shift the spectrum down in intensity, the peak to lower frequencies, and makes the slow cooled range of energies wider.

The magnetic field and the Lorentz factor of accelerated particles in the FS region are (Sari et al. 1998)

$$b_{FS} = \sqrt{32\pi p c^3 m_e} \Gamma_{FS} \gamma_c = \epsilon_c m_p / m_e$$

We estimate using our fiducial numbers

$$\epsilon_{FS} = h \Gamma_{FS} \gamma_c^2 \frac{e b_{FS}}{m_e c} = \frac{17}{2} \frac{e c h m_e^2}{m_e^2 c^5/2} \sqrt{\epsilon_B} \epsilon_F^{1/2} \frac{\sqrt{E_1}}{t_{ob}}$$

$$\tau_{FS} = \frac{m_e^4 c^3}{32 \pi e^4 m_p \Gamma_{FS}^4 \rho \epsilon_c \epsilon_B}$$

$$\tau_{FS} = \frac{e^{11/2} m_e^4 \Gamma_{FS}^{1/2}}{4 \sqrt{17 \pi e^4 m_p \sqrt{E_1} \rho \epsilon_c \epsilon_B}} = 100 \frac{E_1^{1/2} t_{ob}^{1/2} \epsilon_B^{-3} \epsilon_c^{-3}}{m_e^{1/2} \Gamma_{FS}^{1/2}}.$$

Thus the peak emission is in IR, uncooled.

5. Spectra and Time-evolution

5.1. Reverse Shock

Let us calculate the expected spectra and light curves. Below, we restrict our calculations to the case of a constant luminosity source and constant external density. The external wind environment is discussed in Appendix C.
The energy of the particles with $\tau_c \sim t_{ob}$ is

$$\gamma_c = \frac{m_e^3 c^3}{32\pi e^4 \rho_{ob} \Gamma_{FS}^2 \epsilon B} = 27 \frac{E_{1,54}^{1/2} n_{ex}^{1/2} \epsilon_{e,-3}^{-2} \epsilon_{B,-3}^{1/2}}{\epsilon_{c,-3}^{1/2}}.$$  \hspace{1cm} (43)

Emission at the cooling break falls into UV

$$\epsilon_{c,FS} = h \Gamma_{FS} \gamma_c^2 \left( \frac{e B_{FS}}{m_e c} \right) = 16 \text{ eV} E_{1,54}^{-1/2} n_{ex}^{1/2} \epsilon_{e,-3}^{1/2} \epsilon_{B,-3}^{1/2}.$$  \hspace{1cm} (44)

Thus FS produces optical into the uncooled regime. Synchrotron emission in the slowly cooling regime is expected to produce smoother light curves. The ratio of optical fluxes from the FS and RS are compared in Figure 5.

Figure 5. Evolution of typical frequencies (cooling and break) for the RS shock ($\epsilon_m$ and $\epsilon_c$) and the FS ($\epsilon_{m,FS}$ and $\epsilon_{c,FS}$). (Parameters are $E_1 = 10^{54}$ erg, $L_{ox} = 10^{38}$ erg s$^{-1}$, $\gamma_m = 10^5$, $\epsilon_c = \epsilon_B = 10^{-3}$, $p = 2.5$, $n_{ex} = 1$ cc, $\sigma_e = 1$; distance to the source is $3 \times 10^9$ pc. These parameters are used in the following figures as well. For the plots we also assume constant luminosity wind and constant external density.)

### 5.3. Comparing Optical and X-Ray Fluxes of FS and RS

The particles from the high-energy tail in the FS can still emit X-rays. The required Lorentz factor is

$$\gamma_X \approx 3 \times 10^5 \frac{E_{1,54}^{3/8} \epsilon_{e,-3}^{1/8} \epsilon_{B,-3}^{1/4}}{n_{ex}^{1/4} \epsilon_{m,FS}} \approx 10^2 \gamma_c.$$  \hspace{1cm} (47)

The ratio of X-fluxes from the RS (30) and the FS (42) are (assuming the same $p$ for both)

$$\frac{F_{X,FS}}{F_{X,RS}} = \frac{\frac{E_{m,FS}}{E_c} \epsilon_{c,FS}^{p/2} \epsilon_{m,FS}^{-p/2}}{\epsilon_m} = 4 \times 10^{-3} \frac{E_{1,54}^{1/16} \epsilon_{e,-3}^{0.125} \epsilon_{B,-3}^{1.5}}{n_{ex}^{1/4} \epsilon_{e,50}^{1/4}}.$$  \hspace{1cm} (48)

Thus, the X-ray contribution from the FS is negligible. On the other hand, the ratio of optical fluxes from the RS (30) and the FS (42) is

$$\frac{F_{O,FS}}{F_{O,RS}} = \frac{\frac{E_{m,FS}}{E_c} \epsilon_{e,FS} \epsilon_{m,FS}^{(p-1)/2} \epsilon_{m}^{-1/2}}{\epsilon_m} = 0.16 \frac{E_{1,54}^{1.3} n_{ex}^{3/4} \epsilon_{e,-3}^{0.875} \epsilon_{B,-3}^{1.5}}{L_{w,50}^{1.04} \epsilon_{c,50}^{1.11}}.$$  \hspace{1cm} (49)

where $\epsilon_O$ is the energy of the optical photons.

We conclude that the FS and the RS can produce similarly to optical fluxes (given the uncertainties of the parameters). This is important: it allows us to explain smooth optical light curves (as emission of the FS and constant RS) and possible fast variations at the RS due to changing wind conditions.

### 5.4. Light Curves and Spectra

In the present model the RS is highly relativistic, magnetized, and produces emission in the fast cooling regime. In contrast, the FS is low-magnetized and produces emission in the slow cooling regime. Evolution of typical frequencies with time for the FS and RS are compared in Figure 5.

Next, we combine our calculations of the spectra produced by RS and the FS (Figure 6). The X-ray afterglows are dominated by the RS, while in the optical, contribution from the FS may also be important.

In Figure 7 we plot observed fluxes in X-rays and in the optical. In the X-rays the RS produces a very flat light curve, see Equation (38), while the FS emission is fast decaying, see Equation (46). In the optical, both FS and RS emission experience a break near $t_{ob} \sim 100$ s. But the nature of the breaks is different: for the RS it is the passing of the cooling break, while for the FS it is the effect of minimal Lorentz factor.
To be in the fast cooling regime it is required that \( \epsilon_c < \epsilon_m \).

This puts a lower limit on \( \gamma_w \):

\[
\gamma_w \gg 0.4 \frac{e^{17/4} m_e^3}{e^4} \frac{E_{\text{15/68}}}{L_{w,7/4}^{8/7} / t_{\text{ob}}^{49/68}} \frac{(1 + \sigma_w)}{\sigma_w} \approx \gamma_w \gg 3 \times 10^4.
\] (50)

5.5. Variable Wind Properties

All the calculations performed above are for a constant Lorentz factor and magnetization of the wind. Variations in these quantities are expected to produce variations in the light curves. These variations can be taken into account, approximately, if we notice that the time when a particular element of wind left the central source is approximately the emission observer time. Thus, changing wind properties can be approximately taken into account by introducing \( L_{w}(t_{\text{ob}}), \sigma_w(t_{\text{ob}}), \gamma_w(t_{\text{ob}}) \).

To illustrate the overall behavior, in Figure 8 we plot the spectra and light curves for assumed wind luminosity \( L_w = L_{w,0}/(1 + (t_{\text{ob}}/t_0)^2) \) with \( t_0 = 10^4 \) s. Such power dependence is expected for magnetar-powered wind with spin-down time \( t_0 \). For times \( t_{\text{ob}} \gg t_0 \) the time behavior of the X-ray spectral flux is expected to follow

\[
F_{\gamma} \propto \epsilon_{m}^{p-1} t_{\text{ob}}^{-2} \propto \gamma_{w,73}^{(p-73)/34} \approx t_{\text{ob}}^{-1.78}.
\] (51)

This compares favorably with the post-plateau flux decays.

Variations in the wind Lorentz factor \( \gamma_w \) can also produce features in the light curves. For example, if \( \gamma_w \) increases with time, the peak energy moves to higher values—passing through the detector range it will produce a light curve break (associated with the spectral change from \(-p/2\) to \(-1/2\), see Figure 9).

6. Later Afterglows, \( t_{\text{ob}} \geq E_{\gamma}/L_w \)

At times \( t_{\text{ob}} \geq E_{\gamma}/L_w \) the wind deposits an amount of energy into the FS that exceeds the initial explosion (provided the source produces sufficiently high luminosity for sufficiently long time). After this time the FS is affected by the long lasting engine. (This case also applies if the initial explosion was produced by the wind itself.) At this stage the dynamics of the RS changes: now the wind is working against all the swept-up material, so that the CD/RS follow the FS. There is no simple analytical solution for \( f(\chi) \), hence below we just estimate the typical frequencies, not their temporal evolution/light curves.

The FS flow follows the self-similar solution of B&Mc for blast in constant density with constant energy supply. The FS moves with

\[
\Gamma_{FS} \approx \left( \frac{3}{\pi} \right)^{1/4} \frac{L_w}{c^2 \rho t_{\text{ob}}^2} \approx \left( L_w / c^2 \rho \right)^{1/8} t_{\text{ob}}^{1/4} \] (52)

(Figure 10). The Lorentz factor of the RS is then

\[
\Gamma_{RS} \approx \Gamma_{FS} / (2 \sqrt{1 + \sigma_w}).
\] (53)

The wind Lorentz factor in the frame of the RS is

\[
\gamma_{w,RS} \approx \gamma_w / 2 \Gamma_{RS} = \gamma_w \sqrt{1 + \sigma_w} / \Gamma_{FS} \propto t_{\text{ob}}^{1/2}. \] (54)

Thus the relative Lorentz factor of the wind with respect to the RS (and thus, by assumption, the energy of accelerated particles) increases with time (at the catch-up stage it was nearly constant (Equation (28))). As a result, the typical synchrotron energy remains constant

\[
\epsilon_m \approx \frac{3}{\pi} \frac{1}{4} \frac{e^4 h}{m_e^2} \gamma_{w,6}^2 \sqrt{\sigma_w (1 + \sigma_w)} \approx 2 \text{ keV} \gamma_{w,6}^2 \sqrt{\sigma_w (1 + \sigma_w)} n_{\text{ex}}^{1/2}. \] (55)

The cooling time,

\[
\tau_c = \frac{c^{17/4} m_e^3}{2 e^4} \frac{1}{\sigma_w} \frac{L_{w,7/4}^{8/7} / t_{\text{ob}}^{49/68}}{\gamma_{w,6}^{3/4}},
\] (56)

is much shorter than the observer time,

\[
\tau_c / t_{\text{ob}} \approx 10^{-2} \frac{\sqrt{1 + \sigma_w}}{\sigma_w} L_{w,30}^{1/4} n_{\text{ex}}^{1/4} / t_{\text{ob}}^{1/4} \gamma_{w,6}. \] (57)

Since \( \tau_c / t_{\text{ob}} \ll t_{\text{ob}}^{1/2} \), the electrons stay in the fast cooling regime (this is because their energy increases with time, \( \gamma_{e,RS} \sim \gamma_{w,RS} \) (54)). Since \( \epsilon_m \) is constant, the spectral power just follows the wind luminosity \( L_w \).
Let us next hypothesize how emission from the long-lasting RS in fast wind can resolve a number of contradicting GRB observations, contrasting with the FS model, extending the discussion of Lyutikov (2009). In short, we propose that X-rays are coming from the RS, and optical emission is a mixture of RS and FS emission.

Plateaus. In the present model the synchrotron emission is just the wind power (Equation (27)). Thus, to produce a plateau all is needed is an approximately temporally constant wind power. The total energy produced by the long-lasting sources can be subdominant to the primary explosion. In the present model the plateaus appear simultaneously in the optical and in the X-ray; explaining the results of de Pasquale et al. (2007) (both optical and X-ray light curves exhibit a broad plateau).

In contrast, if associated with FS, this requires that the FS energy, \( E_{\text{FS}} \), increases with time, \( E_{\text{FS}} \propto t^{1/2} \). Thus, a central engine should inject \( \sim 10 \) times more energy at the plateau phase than was available during the prompt phase (e.g., Zhang & Mészáros 2002). Since the total energy in the outflow is constrained by late radio observations, this puts unreasonable demands on efficiency of prompt emission.

Flares. In the present model the afterglow flares correspond to changing parameters of the wind. These changes affect the emission instantaneously (when a perturbation arrives at the RS).

One of the curious properties of afterglow X-ray flares is their short duration, shorter than the expected \( \Delta t/t_{\text{ob}} \sim 1 \) (this is because an instantaneous emission pulse from radius \( r \sim ct \) will be spread in terms of observational time over \( \Delta t \sim t/\Gamma^2 \approx t_{\text{ob}} \) due to time-of-flight effects). If a flare is produced by a luminosity pulse of relative strength \( L_{r} = \eta_{L} L_{w} \), with \( \eta_{L} \sim 10 \), then when it arrives at the RS the Lorentz factor of the RS will change instantaneously by \( \eta_{L}^{-1/2} \) (Equation (20)). The corresponding duration will be shorter by \( \eta_{L}^{-1/2} \sim 3 \). (Also, at the end of the “early stage” the Lorentz factor of the RS exceeds the Lorentz factor of the FS.)

In contrast, in the framework of FS afterglows, flares correspond to additional injection of energy into the FS. Each flare with the flux increase of the order of unity requires energy injection of the order of total energy already in the outflow. Thus, in the case of GRBs with many flares, the total energy in the FS grows in geometrical progression. Also, the models that advocate X-flare as late collisions of shells with low Lorentz factor (e.g., Kumar & Piran 2000) suffer from the efficiency problem. Only the energy of relative motion of shells can be converted into radiation. For collision of low-\( \Gamma \) shells this is a very inefficient process. Lazazzati & Perna (2007) concluded that it is problematic to produce flares in the FS.

Abrupt endings of the plateau phases. One of the most surprising observations of early afterglow is the sudden

---

**Figure 8.** Evolution of X-ray (left panel) and optical (right panel) light curves for time-dependent wind luminosity, \( L_w = L_{w,0}/(1 + t_{ob}/t_0)^2 \) with \( t_0 = 10^3 \) s. Solid lines are for RS; dashed lines are for FS.

**Figure 9.** X-ray light curve for varying wind Lorentz factor, \( \Gamma_w = 10^9(1 + t_{ob}/100 \text{ s}) \). The break is associated with the increasing peak energy that passes through detector’s energy range.

**Figure 10.** Temporal evolution of the Lorentz factors of the FS and the CD for constant wind power and constant external density. At time \( t_{ob} \approx L_w/E_1 \) the second FS merges with the first; at later times the energy deposited by the wind dominates over the initial explosion energy. This changes the FS from point explosion to constant energy supply.

**7. Early Afterglows from RS Emission**

Let us next hypothesize how emission from the long-lasting RS in fast wind can resolve a number of contradicting GRB observations, contrasting with the FS model, extending the discussion of Lyutikov (2009). In short, we propose that X-rays are coming from the RS, and optical emission is a mixture of RS and FS emission.
disappearance of the X emission at the end of the plateau phase (e.g., Troja et al. 2007). First, these observations exclude, in our view, the FS origin of the plateaus—the FS cannot just disappear. Thus, if a central source switches off at coordinate time $t_{\text{off}}$, after time $t_{\text{off}} \sim \Gamma_{\text{RS}}^2$ the information will reach the RS—it will disappear at the observer time $t_{\text{obs}} \sim t_{\text{off}}$. Since the particles are in a fast cooling regime, effective emissivity from the RS will become zero instantaneously. The observed light curve will be modified by the time-of-flight and shock curvature effects.

In contrast, the RS emissivity can become zero instantaneously. The observed light curve will decay on a timescale $\sim (r/c)/\Gamma_{\text{RS}}^2$. Since $\Gamma_{\text{RS}}$ can exceed $\Gamma_{\text{FS}}$, the drop can be sharper than the observer time $t_{\text{obs}} \sim (r/c)/\Gamma_{\text{FS}}^2$.

Fast optical variations. There are a number of GRBs that show fast optical variability (e.g., GRB021004 and most notoriously GRB 080916C). This is hard to explain within the standard matter-dominated FS–RS paradigm, since optical electrons should be in a slow cooling regime (due to mild Lorentz factors of matter-dominated “shells” and small magnetization).

In the present model the optical emission from the RS is in the fast cooling regime. Variations of the wind parameters then can produce fast optical variations of the observed flux.

Naked GRBs (GRBs without an appreciable afterglow emission). If the explosion does not produce a long-lasting engine, there is no X-ray afterglow.

In the FS paradigm, naked GRBs, e.g., GRB 050421, are hard to produce in the FS model (Kumar & Paniaescu 2000).

Similarity of short and long GRBs’ afterglows. In the FS paradigm, both short and long GRBs’ afterglows are driven by the highly magnetized/highly relativistic wind, which might have similar character in both cases: unipolar inductor-driven light ultra-relativistic pair wind.

This similarity is surprising in the FS model: the properties of the FS do depend on the external density, while the prompt emission is independent of it. (Granot & Sari 2002 give a detailed discussion of evolution of FS spectra; in some cases the dependence on the external density is indeed weak.)

Missing jet breaks. The FS model of afterglows predicts that when the bulk Lorentz factor becomes $\Gamma \sim 1/\theta_ j$ ($\theta_ j$ is jet opening angle), an achromatic break should be observed in the light curve (Rhoads 1997). In some cases there is no jet break (most exciting being 130427A, De Pasquale et al. 2016, with no jet break between few hundred seconds and 80 Ms). In other cases breaks are achromatic.

In the present model, the RS/CD propagate with slowly changing Lorentz factor before the catch-up time (which can be very long for a powerful initial explosion). No jet break is expected then. Alternatively, for an early jet break, the “later stage,” Section 6, can be applicable: a very long lasting central engine drives a FS–RS configuration, producing emission at the RS in the fast cooling regime.

Chromatic jet breaks. Temporal changes observed in the light curves—jet breaks—are often interpreted in terms of the morphology of the FS (Rhoads 1997). The prediction is then that such changes should be achromatic—occurring simultaneously at different frequencies. In fact, often the breaks are achromatic with no break in optical contemporaneous with the X-ray break or with different jumps in the optical and X-ray, and sometimes breaks are not observed at all, e.g., GRB 061007 (Paniaescu 2007; Racusin et al. 2009).

In the present model, the X-ray afterglows are coming from the RS, while optical ones are a mixture of RS and FS. This allows one to explain achromatic breaks (in cases when the optical is dominated by RS and thus is tied to X-rays) and chromatic breaks (in cases when the optical is dominated by FS and thus is unrelated to X-rays).

Missing orphan afterglows. Decelerating FS emits into larger angles at later times. Thus, we should have seen many orphan afterglows in the X-ray and optical without the prompt emission (e.g., Granot et al. 2002). None have been solidly detected, though the rate estimate is not straightforward (van Eerten et al. 2010; Ghirlanda et al. 2015).

In the present model, the afterglow’s optical and X-emission depend on the details of the RS, e.g., on the collimation properties of the fast wind. If the later wind has similar collimation to that of the prompt emission, and since the RS’s Lorentz factor remains nearly constant in time, fewer orphan afterglows are expected.

Missing RSs problem. The standard model had a clear prediction of a bright optical flare with definite decay properties (Sari et al. 1996; Meszaros & Rees 1997; Sari & Piran 1999). Though a flare closely resembling the predictions was indeed observed (GRB 990123, Mészáros & Rees 1999), this was an exception. In the Swift era, optical flashes are rare (Gomboc et al. 2009); even when they are seen, they rarely correspond to RS predictions (often optical emission correlates with X-rays, too variable, wrong decay laws).

In the present model the early X-ray afterglows are the RS emission.

Other miscellaneous points include: (i) the present model requires fast wind, $\gamma_ w \geq 10^4$ (Equation (50)). It is expected that as the surroundings of the central engine become cleaner with time, the Lorentz factor of the wind will increase. If initially $\gamma_ w$ is not sufficiently high, the RS emission will be in the slow cooling regime, with smaller luminosity. As the wind becomes cleaner, the Lorentz factor $\gamma_ w$ increases and its luminosity increases as well. (ii) We associate smooth optical emission with the FS component. We expect that at the onset of a “later phase,” Section 6, the dynamics of the FS changes from point explosion to constant wind luminosity. This may be reflected in flattening of optical light curves at later times. (iii) The emission from the second FS can also contribute to the observed fluxes. During the early stage its Lorentz factor is smaller than that of the the primary FS (Equation (20)). Hence its emission is generally weaker (but may still be more pronounced at lower energies). On the other hand, the second FS propagates through particles accelerated in the primary shock. The primary shock may work as an injector, making acceleration at the second FS more effective. (iii) Very early X-ray afterglows, with steep temporal decay, are probably the “high latitude” tails of prompt emission. (iv) Post-plateau steepening could be due to the declining wind power, e.g., magnetar spin-down (Figure 8). (iv) Infra-red and radio afterglows also could be coming form the RS, but our experience with radio-emitting electrons in pulsar wind nebulae indicates that they form a different population from the high-energy electrons (Kennel & Coroniti 1984b; Atoyan 1999); hence, extending simple models of high-energy emission to lower frequencies may not be justified. (v) Extended emission in short GRBs (Norris & Bonnell 2006; Troja et al. 2008) may also originate in the RS of long-lasting central engine.
8. Discussion

In this paper we argue the observed temporal and spectral properties of early X-ray afterglows are generally consistent with the synchrotron emission coming from particles accelerated behind a RS propagating in a long-lasting ultra-high relativistic wind. Since the late wind is expected to be fast and highly magnetized, the RS emission is very efficient, converting a large fraction of the wind power into radiation. This high efficiency of conversion of wind power into radiation does not place tough constraints on the wind luminosity and total power. In the simplest model, the plateaus are very flat in time (Equation (38)).

The main points of the work are as follows.

1. Since the circumburst cavity is cleared by the preceding forward blast, and since the RS is powered by the central engine, the Lorentz factor of the RS behaves very differently from the FS, typically decreasing with a slower rate than that of the FS.

2. In the fast cooling regime the RS synchrotron emission is, approximately, the wind power. Thus, in order to power afterglows a mildly luminous central source is needed.

3. The X-ray spectral flux is nearly constant (Equation (38)). This provides a natural explanation for the early X-ray plateaus.

4. In the optical, contributions from FS and RS are approximately comparable. The FS is expected to produce smooth light curves (in the slow cooling regime), while the optical emission of RS (in the fast cooling regime, Equation (36)) may show fast variability correlated with X-rays. The optical emission from the RS can also produce plateaus.

Importantly, RS luminosity depends on the instantaneous wind power at the location of the RS, and thus can be highly variable (e.g., due to development of instabilities in the jet Lazzati et al. 2011). This is in contrast to the FS emission, which depends on the total integrated energy absorbed by the media; thus in order to produce, e.g., flares in the FS model, one must change the total energy of the FS shock. Also, the fast cooling regime ensures that at any moment the observed emission corresponds to the instantaneous shock power (modulo time-of-flight effects). Thus, flares, as well as sudden dimming of the afterglows (akin to the one discussed by Troja et al. 2007), are more naturally explained in the RS scenario. For example, if the wind from the central engine terminates, the RS disappears the moment that the back part of the CD with respect to the upstream fluid. This may change the particular time scalings, but as long as the RS emission remains in the fast cooling regime, our key estimates will remain true. To illustrate this point (that the RS emission properties are roughly independent of the assumed approximations for strong RS, in Appendix A we calculate the RS dynamics using Kompaneets approximation—balancing wind and the post-shock pressures—and in Appendix B we calculate the RS with a delayed start; though the particularities are different, the main conclusions stand).

In conclusion, the emission from the RS of the long-lived engine that produces highly relativistic wind is (i) highly efficient (due to the high Lorentz factor of the wind and high magnetic field supplied by the central source); (ii) stable for constant wind parameters (its Lorentz factor is nearly constant as a function of time); (iii) can react quickly to the changes of the wind properties (and thus can explain rapid variability). All these properties point to the RS emission as the origin of early X-ray afterglows. RS also contributes to optical emission—this explains correlated X-optical features often seen in afterglows. FS emission occurs in the optical range, and, at later times, in the radio range.

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Appendix A

RS Dynamics in the Kompaneets Approximation

The Kompaneets approximation (Kompaneets 1960) balances post-shock and ram pressure (for non-relativistic application in astrophysics see, e.g., Icke 1988); the relativistic case was discussed by Lyutikov (2011). Since the post-first shock plasma is relativistically hot and moving away from the center of explosion, this pressure depends on the local enthalpy and the relative Lorentz factor between the local plasma flow and the second shock/CD. We can estimate both those values using the self-similar solution for a relativistic point explosion (B&Mc).

The pressure balance on the CD between the wind and the immediate pre-shock ram pressure is

\[ \frac{L_w}{4\pi r^2 c \Gamma_{CD}^2} = \frac{w_{CD}}{2 \gamma_{CD}} \left( \frac{\Gamma_{CD}}{2} \right)^2 \approx \frac{8}{3} \Gamma_{CD}^3 \rho c^2 \chi_{CD}^{7/12} \]

where \( w_{CD} \) is the enthalpy and \( \gamma_{CD} \) is the post-first shock fluid right in front of the second shock/CD. Here on the left is the ram pressure created by the freely expanding wind; it is smaller by \( \Gamma_{CD}^{-2} \) in the frame of the CD. On the right is the ram pressure of the second FS, which is approximately the pressure in the post second shock fluid. The ram pressure is the enthalpy (since the primary shocked fluid has \( p \gg \rho \)) times the Lorentz factor\(^2\) of the motion of the CD with respect to the upstream fluid. This Lorentz factor is, approximately, the Lorentz factor
of the CD, \( \Gamma_{\text{CD}} \), divided by the Lorentz factor of the fluid in front of it, \( \chi_{\text{CD}} \). (One of the tests of this scaling is the Blandford–McKee scaling for the FS with energy supply; in that case the enthalpy equals the outside density, the Lorentz factor of the fluid in front of it is 1, which gives \( \Gamma_{\text{CD}} \propto t^{-1/2} \).

Importantly, the Lorentz factor of the primary shock (and of the pre-second FS flow at the location of the CD, \( \chi_{\text{CD}} \)) cancels out: pre-second shock enthalpy \( \propto \gamma_{\text{CD}}^2 \propto \Gamma_{\text{FS}}^2 \), but the ram pressure that the second shock creates is \( \propto \gamma_{\text{CD}}^2 \propto \Gamma_{\text{FS}}^2 \). (We remind the reader that the lower-case \( \gamma_{\text{CD}} \) refers to the Lorentz factor of the fluid in front of the CD, while the capital \( \Gamma_{\text{CD}} \) refers to the Lorentz factor of the CD.)

Solving (58) for \( \Gamma_{\text{CD}} \), we find

\[
\Gamma_{\text{CD}} \approx \frac{G^1/4}{c^{3/4} \gamma_{\text{CD}}^{1/2} \rho_{\text{CD}}^{1/4} \chi_{\text{CD}}^{3/4}}.
\]

This gives the Lorentz factor of the CD between the doubly shocked external medium and the constant power wind with luminosity \( L_w \); the CD is located at the self-similar coordinate \( \chi_{\text{CD}} \) of the primary shock.

Combining (12) and (60) we find the self-similar dynamics of the second shock: \( m = 3/17 \). The Lorentz factor and the location of the CD are then given by

\[
\Gamma_{\text{CD}} = 0.25 \left( \frac{L_w}{E_1^{1.25}} \right)^{1/34} t^{-3/34},
\]

\[
\chi_{\text{CD}} \approx 10.9 \left( \frac{L_w t}{E_1 \Gamma_{\text{FS}}} \right)^{-12/17}.
\]

In terms of the observer time,

\[
\Gamma_{\text{CD}} = 0.4 \left( \frac{L_w}{E_1^{1.40} \rho^{1.5} \rho_{\text{FS}}^{1.5} \chi_{\text{FS}}^{1.5}} \right)^{3/10} t^{3/10}.
\]

The corresponding peak energy is

\[
\epsilon_m \approx 1.7 \times 10^3 \text{eV} \frac{E_1^{7/10} \rho_{\text{ex}}^{2/5} \rho_{\text{FS}}^{2/5} \chi_{\text{FS}}^{2/5}}{L_w^{7/10} \rho_{\text{FS}}^{12/18} \chi_{\text{FS}}^{12/18}}.
\]

The peak energy (63) has, qualitatively—and given the uncertainties of the parameters—the same order-of-magnitude value as the estimate (31). The cooling energy evaluates to

\[
\epsilon_c \approx 10^{-3} \text{eV} \frac{E_1^{7/10} \rho_{\text{ex}}^{2/5} \rho_{\text{FS}}^{2/5} \chi_{\text{FS}}^{2/5}}{E_1^{7/10} \rho_{\text{FS}}^{12/18} \chi_{\text{FS}}^{12/18}} \ll \epsilon_m.
\]

Thus, though the dynamics of the CD in the Kompaneets approximation is somewhat different, the key conclusions remain valid: the X-ray emission is in the fast cooling regime.

Appendix B

**RS Dynamics with Delayed Start**

Another scaling for the dynamics of the second shock is possible if the onset of the wind is delayed with respect to the initial explosion. Suppose that the second explosion/secondary wind occurs at time \( t_d \) after the initial one and the second shock/CD is moving with the Lorentz factor \( \Gamma_{\text{CD}} \propto (t - t_d)^{-m} \gg \Gamma_1^2 \). Then, the location of the second shock at time \( t \) is

\[
R_{\text{CD}} = (t - t_d) \left( 1 - \frac{1}{2\Gamma_{\text{CD}}^2 (m + 1)} \right).
\]

The corresponding self-similar coordinate of the second shock in terms of the primary shock self-similar parameter \( \chi \)

\[
\chi_{\text{CD}} = (1 + 8\Gamma_1^2 \chi) \left( 1 - \frac{R}{t} \right) \approx \left( \frac{8\Gamma_{\text{FS}}^2}{t} + \frac{4}{(m + 1)\Gamma_{\text{CD}}^2} \right)^{1/2}.
\]

For \( t_d \gg t/(2(m + 1)\Gamma_{\text{CD}}^2) \) the location of the CD in the self-similar coordinate associated with the first shock is

\[
\chi_{\text{CD}} \approx \frac{8\Gamma_1^2 t_d}{t} \propto t^{-4}.
\]

In this case

\[
\Gamma_{\text{CD}} = 0.52 \frac{E_1^{5/18} \rho_{\text{FS}}^{5/18} L_w^{1/4}}{\chi_{\text{FS}}^{5/18} \rho_{\text{FS}}^{12/18} \chi_{\text{FS}}^{12/18}} \approx 0.46 \frac{E_1^{5/18} \rho_{\text{FS}}^{5/18} L_w^{1/4}}{\chi_{\text{FS}}^{5/18} \rho_{\text{FS}}^{12/18} \chi_{\text{FS}}^{12/18}}.
\]

The peak energy

\[
\epsilon_m \approx 2.6 \times 10^3 \text{eV} \frac{E_1^{5/18} \rho_{\text{FS}}^{5/18} L_w^{1/4}}{\chi_{\text{FS}}^{5/18} \rho_{\text{FS}}^{12/18} \chi_{\text{FS}}^{12/18}}
\]

is much larger than the cooling frequency

\[
\epsilon_c \approx 0.2 \text{eV} \frac{E_1^{5/18} \rho_{\text{FS}}^{5/18} L_w^{1/4}}{\chi_{\text{FS}}^{5/18} \rho_{\text{FS}}^{12/18} \chi_{\text{FS}}^{12/18}}.
\]

We conclude that various approximations for the second shock dynamics do not affect our main conclusion (that the RS in the fast wind is a very efficient in converting the wind energy into the high energy radiation).

Appendix C

**Wind Environment**

For the wind environment \( \rho = \rho_0 (r/r_0)^{-2} \), the initial blast wave propagates according to

\[
\Gamma_{\text{FS}} = \frac{3}{2\sqrt{2\pi} c^3/2} \left( \frac{E_1}{\rho_0 r_0^2 \rho} \right)^{1/3},
\]

and creates a profile (B&Mc) \( f \propto \chi^{3/2} \), \( g \propto 1/\chi \). Accumulated enthalpy,

\[
w_2 = \frac{16\pi}{3} \frac{\rho_0^2 c^3 \rho_0 t}{\chi_{\text{FS}}^{3/2}},
\]

gives the Lorentz factor of the CD

\[
\Gamma_{\text{CD}} = 0.41 \frac{L_w^{1/4} \chi_{\text{CD}}^{1/8}}{c^{3/4} \sqrt{\rho_0 r_0^2}}.
\]

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Together with (12) and (71) we find
\[
\Gamma_{CD} = 0.49 \frac{E_1^{1/12} L_w^{1/6}}{c^{9/10} \rho_0^{3/10} \eta_0^{3/5} t} = 0.55 \frac{E_1^{1/12} L_w^{1/6}}{c^{3/4} \rho_0^{1/4} \eta_0^{1/2}}
\]
\[
\chi_{CD} = 1.46 \frac{E_1^{1/5}}{c^{6/5} \rho_0^{2/5} \eta_0^{4/5} L_w^{1/5}} = 3.7 \left( \frac{E_1}{L_w t_{ob}} \right)^{2/3}
\]
\[
t = 0.31 \frac{E_1^{1/6} L_w^{1/3} L_t^{1/6}}{\rho_0^{1/2}}.
\] (74)

The peak and the cooling energies evolve according to
\[
\epsilon_m = 5.2 \frac{\sqrt{c} \epsilon_0 \rho_0^{2} \sqrt{\sigma_w (1 + \sigma_w) \gamma_w^2}}{E_1^{1/3} m_e L_w^{1/6} t_{ob}^{2/3}}
\]
\[
\epsilon_c = 10^{-2} \frac{E_1^{2/3} m_e^2 \sigma_w^2 (1 + \sigma_w) \gamma_w^{1/3}}{\epsilon_0^{2} \rho_0^{2} \sigma_w^{2} L_w^{1/6}}.
\] (75)

At later times, when the CD catches the FS, we find
\[
\Gamma_{CD} \approx \Gamma_{FS} \approx \frac{L_w^{1/4}}{c^{3/4} \rho_0^{1/4} \eta_0^{1/2}}
\]
\[
\epsilon_m = \frac{\epsilon h}{m_e c} \frac{\sqrt{\sigma_w (1 + \sigma_w)} \sqrt{\rho_0 \rho_0 \gamma_w^2}}{t}
\]
\[
= \frac{\sqrt{c} \epsilon_0 \rho_0^{2} \sqrt{\sigma_w (1 + \sigma_w) \gamma_w^2}}{m_e \sqrt{L_w t_{ob}}}
\]
\[
\epsilon_c = \frac{E_1^{1/5} m_e^5 h \sqrt{L_w t_{ob}} (\sigma_w + 1)}{\epsilon_0^{2} \rho_0^{2} \sigma_w^{2} L_w^{1/6}}.
\] (76)