A NATURAL SOLUTION TO THE $\mu$ PROBLEM

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Abstract

We propose a simple mechanism for solving the $\mu$ problem in the context of minimal low–energy supergravity models. This is based on the appearance of non–renormalizable couplings in the superpotential. In particular, if $H_1H_2$ is an allowed operator by all the symmetries of the theory, it is natural to promote the usual renormalizable superpotential $W_o$ to $W_o + \lambda W_o H_1 H_2$, yielding an effective $\mu$ parameter whose size is directly related to the gravitino mass once supersymmetry is broken (this result is maintained if $H_1 H_2$ couples with different strengths to the various terms present in $W_o$). On the other hand, the $\mu$ term must be absent from $W_o$, otherwise the natural scale for $\mu$ would be $M_P$. Remarkably enough, this is entirely justified in the supergravity theories coming from superstrings, where mass terms for light fields are forbidden in the superpotential. We also analyse the $SU(2) \times U(1)$ breaking, finding that it takes place satisfactorily. Finally, we give a realistic example in which supersymmetry is broken by gaugino condensation, where the mechanism proposed for solving the $\mu$ problem can be gracefully implemented.
1 Introduction

One of the interesting features of low–energy supergravity (SUGRA) models is that the electroweak symmetry breaking can be a direct consequence of supersymmetry (SUSY) breaking [1]. In the ordinary SUGRA models, SUSY breaking takes place in a hidden sector of the theory, so that the gravitino mass $m_{3/2}$ becomes of the electroweak scale order. Below the Planck mass, $M_P$, one is left with a global SUSY Lagrangian plus some terms (characterized by the $m_{3/2}$ scale) breaking explicitly, but softly, global SUSY. As we will briefly review below, the breakdown of $SU(2) \times U(1)_Y$ appears as an automatic consequence of the radiative corrections to these terms. The so–called $\mu$ problem [2] arises in this context.

Let us consider a SUGRA theory with superpotential $W(\phi_i)$ and canonical kinetic terms for the $\phi_i$ fields\footnote{We will consider this case throughout the paper for simplicity. Our general conclusions will not be modified by taking a more general case.}. Then, the scalar potential takes the form [3]

$$ V = e^K \left[ \sum_i \left| \frac{\partial W}{\partial \phi_i} + \bar{\phi}_i W \right|^2 - 3|W|^2 \right] + \text{D terms} , \quad (1) $$

where $K = \sum_i |\phi_i|^2$ is the Kähler potential. It is customary to consider $W$ as a sum of two terms corresponding to the observable sector $W^{\text{obs}}(\phi^{\text{obs}}_i)$ and a hidden sector $W^{\text{hid}}(\phi^{\text{hid}}_i)$

$$ W(\phi^{\text{obs}}_i, \phi^{\text{hid}}_i) = W^{\text{obs}}(\phi^{\text{obs}}_i) + W^{\text{hid}}(\phi^{\text{hid}}_i) . \quad (2) $$

$W^{\text{hid}}(\phi^{\text{hid}}_i)$ is assumed to be responsible for the SUSY breaking, which implies that some of the $\phi^{\text{hid}}_i$ fields acquire non–vanishing vacuum expectation values (VEVs) in the process. Then, the form of the effective observable scalar potential obtained from eq. (1), assuming vanishing cosmological constant, is [4]

$$ V^{\text{obs}}_{\text{eff}} = \sum_i \left| \frac{\partial W^{\text{obs}}}{\partial \phi^{\text{obs}}_i} \right|^2 + m_{3/2}^2 \sum_i |\phi^{\text{obs}}_i|^2 + \left( A m_{3/2} W^{\text{obs}}_t + B m_{3/2} W^{\text{obs}}_b + \text{h.c.} \right) + \text{D terms} \quad (3) $$

with

$$ m_{3/2}^2 = e^{K^{\text{hid}}} |W^{\text{hid}}|^2 \quad (4) $$

$$ B = A - 1 = \sum_i \left( |\phi^{\text{hid}}_i|^2 + \frac{\bar{\phi}^{\text{hid}}_i}{W^{\text{hid}}} \frac{\partial W^{\text{hid}}}{\partial \phi^{\text{hid}}_i} \right) - 1 \quad (5) $$
where $K^{hid} = \sum_i |\phi_i^{hid}|^2$, $\hat{W}^{obs}$ is the rescaled observable superpotential $\hat{W}^{obs} = e^{K^{hid}/2} W^{obs}$, the subindex $t(b)$ denotes the trilinear (bilinear) part of the superpotential, and $A, B$ are dimensionless numbers of $O(1)$, which depend on the VEVs of the hidden fields. Since we are assuming that SUSY breaking takes place at a right scale, the gravitino mass given by eq.(4) is hierarchically smaller than the Planck mass (i.e. of order the electroweak scale).

In the minimal supersymmetric standard model (MSSM) the matter content consists of three generations of quark and lepton superfields plus two Higgs doublets, $H_1$ and $H_2$, of opposite hypercharge. Under these conditions the most general effective observable superpotential has the form

$$W^{obs} = \sum_{generations} \left( h_u Q_L H_2 u_R + h_d Q_L H_1 d_R + h_e L_L H_1 e_R \right) + \mu H_1 H_2 .$$

(6)

This includes the usual Yukawa couplings (in a self–explanatory notation) plus a possible mass term for the Higgses, where $\mu$ is a free parameter. From eq.(3) the relevant Higgs scalar potential along the neutral direction for the electroweak breaking is readily obtained

$$V(H_1, H_2) = \frac{1}{8} \left( g^2 + g'^2 \right) \left( |H_1|^2 - |H_2|^2 \right)^2 + \mu_1^2 |H_1|^2 + \mu_2^2 |H_2|^2 - \mu_3^2 (H_1 H_2 + h.c.) ,$$

(7)

where

$$\mu_{1,2}^2 = m_{3/2}^2 + \hat{\mu}^2$$
$$\mu_3^2 = -B m_{3/2} \hat{\mu}$$
$$\hat{\mu} \equiv e^{K^{hid}/2} \mu .$$

(8)

This is the SUSY version of the usual Higgs potential in the standard model. In order for the potential to be bounded from below, the condition

$$\mu_1^2 + \mu_2^2 - 2 |\mu_3^2| > 0$$

(9)

must be imposed all over the energy range $[M_Z, M_P]$. This implies in particular $\langle H_{1,2} \rangle = 0$ at the Planck scale. Below the Planck scale, one has to consider the radiative corrections to the scalar potential. Then the boundary conditions of eq.(8) are substantially modified in such a way that the determinant of the Higgs mass–squared matrix becomes negative, triggering $\langle H_{1,2} \rangle \neq 0$ and $SU(2) \times U(1)_Y$ symmetry breaking [1].

For this scheme to work, the presence of the last term in eq.(3) is crucial. If $\mu = 0$, then the form of the renormalization group equations (RGEs) implies that such a term is not generated at any $Q$ scale since $\mu(Q) \propto \mu$. The same occurs for $\mu_3$, i.e. $\mu_3(Q) \propto \mu$. 

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Then, the minimum of the potential of eq. (7) occurs for $H_1 = 0$ and, therefore, $d$-type quarks and $e$-type leptons remain massless. Besides, the superpotential of eq. (8) with $\mu = 0$ possesses a spontaneously broken Peccei–Quinn symmetry [5] leading to the appearance of an unacceptable Weinberg–Wilczek axion [6].

Once it is accepted that the presence of the $\mu$ term in the superpotential is essential, there arises an immediate question: Is there any dynamical reason why $\mu$ should be small, of the order of the electroweak scale? Note that, to this respect, the $\mu$ term is different from the SUSY soft–breaking terms, which are characterized by the small scale $m_{3/2}$ once we assume correct SUSY breaking. In principle the natural scale of $\mu$ would be $M_P$, but this would reintroduce the hierarchy problem since the Higgs scalars get a contribution $\mu^2$ to their squared mass [see eq.(8)]. Thus, any complete explanation of the electroweak breaking scale must justify the origin of $\mu$. This is the so–called $\mu$ problem [2]. This problem has been considered by several authors and different possible solutions have been proposed [2,7,8]. In this letter we suggest a scenario in which $\mu$ is generated by non–renormalizable terms and its size is directly related to the gravitino mass. A comparison with the scenarios of refs.[2,7,8] is also made.

2 A natural solution to the $\mu$ problem

Let us start with a simple scenario with superpotential

$$W = W_o + \lambda W_o H_1 H_2, \quad (10)$$

where $W_o$ is the usual superpotential (including both observable and hidden sectors) without a $\mu H_1 H_2$ term. We have allowed in (10) a non–renormalizable term, characterized by the coupling $\lambda = O(1)$ (in Planck units), which mixes the observable sector with the hidden sector (other higher–order terms of this kind could also be included, but they are not relevant for the present analysis). The $\mu H_1 H_2$ term must be absent from $W_o$ since, as was mentioned above, the natural scale for $\mu$ would otherwise be $M_P$. Certainly, this is technically possible in a supersymmetric theory, since the non–renormalization theorems assure that this term cannot be generated radiatively if initially $\mu = 0$. One may wonder, however, whether there is a theoretical reason for the absence of the $\mu H_1 H_2$ term from $W_o$ in eq. (10), since it is not forbidden by any symmetry of the theory[4]. It is quite remarkable here that this is provided in the low–energy SUSY theory obtained from superstrings. In

\footnote{The $\mu H_1 H_2$ term can be forbidden by invoking a Peccei–Quinn (PQ) symmetry [2,8]. This is not possible here since (10) does not possess any PQ symmetry.}
this case mass terms (like $\mu H_1 H_2$) are forbidden in the superpotential. We will see in section 4 an explicit example in this context. Finally, non-renormalizable terms (like $\lambda W_o H_1 H_2$) are in principle allowed in a generic SUGRA theory. Next, we show that the $\lambda W_o H_1 H_2$ term yields dynamically a $\mu$ parameter.

Using the general expression of eq.(1), the scalar potential $V$ generated by $W$ has the form

$$V = e^K \left\{ \sum_i \left| \frac{\partial [W_o (1 + \lambda H_1 H_2)]}{\partial \phi_i} \right|^2 + \bar{\phi}_i W_o (1 + \lambda H_1 H_2) \right\} - 3|W_o (1 + \lambda H_1 H_2)|^2 \right\}$$

$$+ \quad \text{D terms}, \tag{11}$$

which can be written as

$$V = V^{(1)} |1 + \lambda H_1 H_2|^2 + e^K \left\{ \left| \frac{\partial [W_o (1 + \lambda H_1 H_2)]}{\partial H_1} \right|^2 + H_1 W_o (1 + \lambda H_1 H_2) \right\} + (H_1 \leftrightarrow H_2)$$

$$+ \quad \text{D terms}, \tag{12}$$

where

$$V^{(1)} \equiv e^K \left( \sum_i \left| \frac{\partial W_o}{\partial \phi_i} + \bar{\phi}_i W_o \right|^2 - 3|W_o|^2 \right); \quad \phi_i \neq H_{1,2}. \tag{13}$$

Since $H_{1,2}$ enter in $W_o$ only through the ordinary Yukawa couplings and we are assuming vanishing VEVs for the observable scalar fields, it is clear (recall that $W_o$ does not contain a $\mu H_1 H_2$ coupling) that $\left. \frac{\partial W_o}{\partial H_{1,2}} \right|_{\min} = 0$. Besides, the vanishing of the cosmological constant implies $V^{(1)} = 0$ at the minimum of the potential. So, we can extract from the second term in eq.(12) the soft terms associated with $H_{1,2}$:

$$V(H_1, H_2) = \frac{1}{8} \left( g^2 + g'^2 \right) \left( |H_1|^2 - |H_2|^2 \right)^2 + m_{3/2}^2 (1 + \lambda^2) |H_1|^2 + m_{3/2}^2 (1 + \lambda^2) |H_2|^2$$

$$+ 2 m_{3/2}^2 \lambda (H_1 H_2 + \text{h.c.}) \tag{14}.$$}

Comparing eqs.(3–8) with eqs.(10,14) it is clear that $\lambda W_o H_1 H_2$ behaves like a $\mu$ term when $W_o$ acquires a non–vanishing VEV dynamically. Defining $\lambda \langle W_o \rangle \equiv \mu$ we can write eq.(14) as eqs.(7,8) where now the value of $B$ is

$$B = 2. \tag{15}$$

The value of $A$ is still given by eq.(3), but the relation $B = A - 1$ is no longer true. The fact that the new "$\mu$ parameter" is of the electroweak–scale order is a consequence of our
assumption of a correct SUSY–breaking scale \( m_{3/2} = e^{K/2}W = O(M_Z) \). Finally, note that the usual condition for the potential to be bounded from below (14) is automatically satisfied by (14) for any value of \( \lambda \).

One may wonder how general is the simple scenario of eq.(10). First of all, let us note that the fact that \( H_1 H_2 \) is not forbidden by any symmetry of the theory is a key ingredient for this scenario to work. An obvious generalization of (10) arises when \( W_o \) consists of several terms \( W_o = W_o^{(1)} + W_o^{(2)} + ... \) and \( H_1 H_2 \) couples with a different strength to each term, i.e. \( (\lambda_1 W_o^{(1)} + \lambda_2 W_o^{(2)} + ... )H_1 H_2 \). However, provided that the hierarchical small value for \( \langle W_o \rangle \) is not achieved by a fine–tuning between the VEVs of the various terms \( W_o^{(1)}, W_o^{(2)}, ... \), it is clear that the order of magnitude of \( \mu \) continues being \( m_{3/2} \).

Apart from this, it should be noticed that \( \lambda_i = O(1) \) (in Planck units) is only natural if \( W_o^{(i)} \) is not an operator with a extremely small coupling constant. However, this would be a naturalness problem by itself. This would happen, for instance, for \( W_o^{(i)} = m \Phi^2 \) with \( m << M_P \). (These terms are forbidden in string theories.)

To conclude this section, it is worth noticing that in the context of supergravity theories there is another possible solution to the \( \mu \) problem. Since the Kähler potential \( K \) is an arbitrary real–analytic function of the scalar fields, we can study for example a theory with the following \( K \)

\[
K = \sum_i |\phi_i|^2 + f(g(\phi_j, \overline{\phi}_j))H_1 H_2 + \text{h.c.},
\]

where \( \phi_j \neq H_{1,2} \) and \( f \) and \( g \) are generic functions (\( \langle g(\phi_j, \overline{\phi}_j) \rangle = O(1) \)). Then, although \( W_o \) does not contain a \( \mu \) term, this is generated in the scalar potential. This is trivial to see for the simplest case (i.e. \( f(x) = x \), \( g = \text{const.} \equiv \lambda \)). Then the theory is equivalent to one with Kähler potential \( \sum_i |\phi_i|^2 \) and superpotential \( W_o e^{\lambda H_1 H_2} \), since the function \( G = K + \log |W|^2 \) that defines the SUGRA theory is the same for both. Expanding the exponential, the first two terms coincide with eq.(10) and hence we obtain the same \( \mu \) term as in eq.(14). The possibility (16) was examined in ref.[7] for \( f(x) = x \) and when \( g \) is a non–trivial function of the hidden fields, in particular for the simplest case \( g(\phi_j, \overline{\phi}_j) = \xi \), where \( \xi \) is a hidden field. It remains to be explored whether a Kähler potential similar to that of eq.(16) can arise in the context of superstring theories.

3 Expectation values for the Higgses

In the above analysed solution to the \( \mu \) problem it is assumed that the observable scalar fields have vanishing VEVs at the Planck scale. Since the non–renormalizable term
\( \lambda W_o H_1 H_2 \) mixes observable and hidden fields, one may wonder whether that assumption is still true for the Higgses. We will show now that this is in fact the case.

We assume here that the initial superpotential \( W_o \) gives a correct SUSY breaking, i.e. small gravitino mass and vanishing cosmological constant. This means that \( V_o \), i.e. the scalar potential derived from \( W_o \), is vanishing at the minimum \( V_o|_{\text{min}} = 0 \) and thus positive–definite. Using the general expression of eq.(1), \( V_o \) can be decomposed in three pieces

\[
V_o = V^{(1)} + e^K \left\{ \left| \frac{\partial W_o}{\partial H_1} + \bar{H}_1 W_o \right|^2 + (H_1 \rightarrow H_2) \right\} + \text{D terms} , \tag{17}
\]

where \( V^{(1)} \) is defined in eq.(13). Recalling that we are assuming that \( W_o \) does not contain a \( \mu H_1 H_2 \) term and that \( \frac{\partial W_o}{\partial H_{1,2}} \big|_{\text{min}} = 0 \) (since squarks and sleptons are supposed to have vanishing VEVs), it is clear that \( V^{(1)} \) is flat in \( H_{1,2} \). So, the minimum of the second piece of \( V^{(1)} \) is zero and occurs at \( H_{1,2} = 0 \) (for any value of \( W_o \)). Therefore, necessarily \( V^{(1)} \big|_{\text{min}} = 0 \), i.e. \( V^{(1)} \) is also positive–definite. All this is very ordinary: it simply means that the hidden sector is entirely responsible for the breaking. (Note that the \( H_{1,2} \) F–terms are vanishing, while some of the hidden fields F–terms must be different from zero.)

Notice also that from (17) one obtains

\[
e^K |W_o|^2 (|H_1|^2 + |H_2|^2) = m_{3/2}^2 |H_1|^2 + m_{3/2}^2 |H_2|^2
\]

but, because of the absence of a \( \mu H_1 H_2 \) term in \( W_o \), there is no \( B m_{3/2} \bar{\mu} H_1 H_2 \) term in the scalar potential.

Let us now study the impact of doing, according to our approach, \( W_o \rightarrow W = W_o + \lambda W_o H_1 H_2 \). The corresponding scalar potential, \( V \), has already been written in eq.(12). Now, since \( V^{(1)} \) is positive–definite, so is \( V \). In fact, the minimum of \( V \) is for \( V = 0 \) and occurs when the three pieces of (12) are vanishing. Clearly, the minimum of the first and third pieces of (12) coincides with that of eq.(17) above, implying \( V^{(1)} \big|_{\text{min}} = 0 \) and thus the VEV of \( W_o \) is the same as when we started with just \( W_o \). Finally, recalling that \( \frac{\partial W_o}{\partial H_{1,2}} \big|_{\text{min}} = 0 \), it is clear that the second piece of \( V \) in eq.(12) has two possible minima

\[
H_1, H_2 = 0 , \tag{18}
\]

\[
\lambda H_2 + (1 + \lambda H_1 H_2) \bar{H}_1 = 0
\]

\[
(\bar{H}_1 \leftrightarrow H_2) = 0 \tag{19}
\]

\(^3\)The only exception occurs if \( \lambda H_1 H_2 = -1 \), but then the second piece of (12), which is also positive–definite, is different from zero, so this is not a solution for the minimization of the whole potential.
As was explained in section 1, the solution (18) is the phenomenologically interesting one, whereas the solution (19) leads to $H_{1,2} \sim M_P$, so it is not phenomenologically viable. We can ignore this solution since if $H_{1,2}$ are initially located at $H_{1,2} = 0$ (e.g. by thermal effects) they will remain there as long as (18) continues to be a minimum solution. Of course, radiative corrections will trigger non–zero VEVs of the correct size for $H_1, H_2$.

4 A realistic example

As we saw in section 2, the assumption of correct SUSY breaking was crucial for obtaining the $\mu$ parameter of the electroweak–scale order. As a matter of fact, gaugino condensation effects in the hidden sector [9] are the most satisfactory mechanism so far explored, able to break SUSY at a scale hierarchically smaller than $M_P$ [10]. The reason is that the scale of gaugino condensation corresponds to the scale at which the gauge coupling becomes large, and this is governed by the running of the coupling constant. Since the running is only logarithmically dependent on the scale, the gaugino condensation scale is suppressed relative to the initial one by an exponentially small factor $\sim e^{-1/2\beta g^2}$ ($\beta$ is the one–loop coefficient of the beta function of the hidden sector gauge group $G$). This mechanism has been intensively studied in the context of SUGRA theories coming from superstrings [11,12], where the gauge coupling is related to the VEV of the dilaton field $S$ (more specifically $\text{Re}S = g^{-2}$). Recall that we have argued in section 2 that superstring theories are precisely a natural context where the solution of the $\mu$ problem presented here can be implemented, since mass terms, such as $\mu H_1 H_2$, appearing in the superpotential are automatically forbidden in superstrings. Besides, non–renormalizable terms like $\lambda W_o H_1 H_2$ in eq.(10) are in principle allowed and, in fact, they are usually present [13].

In the absence of hidden matter, the condensation process is correctly described by a non–perturbative effective superpotential

$$W_o \propto e^{-3S/2\beta_o},$$

with $\beta_o = 3C(G)/16\pi^2$, where $C(G)$ is the Casimir operator in the adjoint representation of $G$. It is difficult to imagine, however, how the mechanism expounded in section 2 could be implemented here. More precisely, it is not clear that we could have something like $W = W_o + \lambda W_o H_1 H_2$, due to the effective character of (20).

Fortunately, things are different in the presence of hidden matter, which is precisely the most frequent case in string constructions [13]. There is not at present a generally accepted formalism describing the condensation in the presence of massless matter, but
the case of massive matter is well understood [14]. For example, in the case of $G = SU(N)$ with $M(N + \bar{N})$ “quark” representations $Q_\alpha$, $\bar{Q}_\alpha$, $\alpha = 1, \ldots, M$, with a mass term given by

$$W^{\text{pert}}_o = - \sum_{\alpha, \beta} M_{\alpha, \beta} Q_\alpha \bar{Q}_\beta ,$$

(21)

the complete condensation superpotential can be written as [12]

$$W_o \propto [\det \mathcal{M}]^{1\over N} e^{-3S/2\beta_o} .$$

(22)

It should be noticed here that, strictly speaking, there are no mass terms like (21) in the context of string theories. However the matter fields usually have trilinear couplings which play the role of mass terms with a dynamical mass given by the VEV of another matter field. The simplest case occurs when there is an $SU(N)$ singlet field $A$ giving mass to all the quark representations. Then (21) takes the form

$$W^{\text{pert}}_o = - \sum_{\alpha = 1}^M A Q_\alpha \bar{Q}_\alpha ,$$

(23)

and $\det \mathcal{M} = A^M$. Now, if $H_1 H_2$ is an allowed coupling from all the symmetries of the theory, it is natural to promote $W^{\text{pert}}_o$ to

$$W^{pert} = - \sum_{\alpha} A(1 + \lambda' H_1 H_2) Q_\alpha \bar{Q}_\alpha ,$$

(24)

so that $\det \mathcal{M} = [A(1 + \lambda' H_1 H_2)]^M$, and (22) takes the form

$$W_o \to W \propto [A(1 + \lambda' H_1 H_2)]^{1\over N} e^{-3S/2\beta_o} \simeq A^{1\over N}(1 + M \lambda' H_1 H_2) e^{-3S/2\beta_o} .$$

(25)

Thus

$$W = W_o + \lambda W_o H_1 H_2 ,$$

(26)

where we have defined $\lambda \equiv {M \over N} \lambda'$. This is precisely the kind of superpotential we wanted (see eq.(10)) in order to generate the $\mu$ term dynamically.

In ref.[8] an interesting solution to the $\mu$ problem was proposed in a similar context with a PQ symmetry, using the presence of a term $H_1 H_2 Q\bar{Q}$ in the superpotential and assuming that the scalar components of $Q$ and $\bar{Q}$ condense at a scale $\Lambda \simeq 10^{11}$ GeV.

\footnote{We neglect here higher-order non-renormalizable couplings since they do not contribute to the $\mu$ term.}
As mentioned above, the only accepted formalism describing the condensation is in the presence of massive matter. Thus the previous term behaves as a dynamical mass term for the squarks and the complete superpotential (22) becomes \( W \propto \left( H_1 H_2 \right)^* e^{-3S/2\beta_o} \). This is phenomenologically unviable since the Higgses must have vanishing VEVs at \( M_P \) for a correct phenomenology, which would imply \( \langle W \rangle = 0 \) and thus no SUSY breaking.

We can improve this model by including a mass term for \( Q \bar{Q} \). However, a genuine mass term for \( Q \bar{Q} \) would break the PQ symmetry, so one should consider something similar to (23). Then the perturbative superpotential is

\[
W_{\text{pert}} \sim A Q \bar{Q} + H_1 H_2 Q \bar{Q},
\]

and the scenario becomes much more similar to that given by eq.(24). However, there still is an important difference. In eq.(24) \( H_1 H_2 \) couples to \( AQ \bar{Q} \) (which is the natural thing if \( H_1 H_2 \) is invariant under all the symmetries of the theory) instead of \( Q \bar{Q} \); thus there is no PQ symmetry. Moreover, (24) leads to (26) in which the \( \mu \) scale is directly given by the \( m_{3/2} \) scale \( (\mu = O(m_{3/2})) \). However from (27) the \( \mu \) scale is given by the squark condensation scale [12] \( \langle Q \bar{Q} \rangle / M_P \simeq m_{3/2} M_P / N \langle A \rangle \), so that the value of \( \mu \) in this case tends to be a bit too large.

5 Summary and conclusions

We have proposed a simple mechanism for solving the \( \mu \) problem in the context of minimal low–energy SUGRA models. This is based on the appearance of non–renormalizable couplings in the superpotential. In particular, if \( H_1 H_2 \) is an allowed operator by all the symmetries of the theory, it is natural to promote the usual renormalizable superpotential \( W_o \) to \( W_o + \lambda W_o H_1 H_2 \), yielding an effective \( \mu \) parameter whose size is directly related to the gravitino mass once SUSY is broken (this result is essentially maintained if \( H_1 H_2 \) couples with different strengths to the various terms present in \( W_o \)).

On the other hand, the \( \mu \) term must be absent in \( W_o \), otherwise the natural scale for \( \mu \) would be \( M_P \). Certainly this is technically possible in a supersymmetric theory since the non–renormalization theorems assure that this term cannot be generated radiatively if initially \( \mu = 0 \). Remarkably enough, however, a theoretical reason for the absence of the \( \mu H_1 H_2 \) term from \( W_o \) is provided in the low–energy SUSY theory obtained from superstrings. In this case mass terms (such as \( \mu H_1 H_2 \)) are forbidden in the superpotential (however, non–renormalizable terms like \( \lambda W_o H_1 H_2 \) are in principle allowed and, in fact, they are usually present).
We have also addressed other alternative solutions, comparing them with the one proposed here. On the other hand, we have analysed the $SU(2) \times U(1)$ breaking, finding that it takes place satisfactorily.

Finally, we have given a realistic example in which SUSY is broken by gaugino condensation in the presence of hidden matter (which is the usual situation in strings), and where the mechanism proposed for solving the $\mu$ problem can be gracefully implemented.

ACKNOWLEDGEMENTS

We gratefully acknowledge J. Louis for extremely useful discussions.

References

[1] For a recent review, see: L.E. Ibañez and G.G. Ross, CERN–TH.6412/92 (1992), to appear in Perspectives in Higgs Physics, ed. G. Kane, and references therein

[2] J.E. Kim and H.P. Nilles, Phys. Lett. B138 (1984) 150

[3] E. Cremmer, S. Ferrara, L. Girardello and A. Van Proeyen, Nucl. Phys. B212 (1983) 413

[4] R. Barbieri, S. Ferrara and C.A. Savoy, Phys. Lett. 119B (1982) 343; L. Hall, J. Lykken and S. Weinberg, Phys. Rev. D27 (1983) 2359

[5] R. Peccei and H. Quinn, Phys. Rev. Lett. 38 (1977) 1440

[6] S. Weinberg, Phys. Rev. Lett. 40 (1978) 223; F. Wilczek, Phys. Rev. Lett. 40 (1978) 229

[7] G.F. Giudice and A. Masiero, Phys. Lett. B206 (1988) 480

[8] J.E. Kim and H.P. Nilles, Phys. Lett. B263 (1991) 79; E.J. Chun, J.E. Kim and H.P. Nilles, Nucl. Phys. B370 (1992) 105
[9] H.P. Nilles, Phys. Lett. B115 (1982) 193, Nucl. Phys. B217 (1983) 366; S. Ferrara, L. Girardello and H.P. Nilles, Phys. Lett. B125 (1983) 457

[10] For a review, see: H.P. Nilles, Int. J. Mod. Phys. A5 (1990) 4199

[11] J.P. Derendinger, L.E. Ibáñez and H.P. Nilles, Phys. Lett. B155 (1985) 65; M. Dine, R. Rohm, N. Seiberg and E. Witten, Phys. Lett. B156 (1985) 55; N.V. Krasnikov, Phys. Lett. B193 (1987) 37; L. Dixon, talk presented at the A.P.S. D.P.F. Meeting at Houston (1990); V. Kaplunovsky, talk presented at the "Strings 90" Workshop at College Station (1990); J.A. Casas, Z. Lalak, C. Muñoz and G.G. Ross, Nucl. Phys. B347 (1990) 243; A. Font, L. Ibáñez, D. Lüst and F. Quevedo, Phys. Lett. B245 (1990) 401; M. Cvetič, A. Font, L. Ibáñez, D. Lüst and F. Quevedo, Nucl. Phys. B361 (1991) 194; S. Ferrara, N. Magnoli, T.R. Taylor and G. Veneziano, Phys. Lett. B245 (1990) 409; H.P. Nilles and M. Olechowsky, Phys. Lett. B248 (1990) 268; P. Binétruy and M.K. Gaillard, Phys. Lett. B253 (1991) 119; J. Louis, SLAC–PUB–5645 (1991); B. de Carlos, J.A. Casas and C. Muñoz, CERN–TH.6436/92 (1992), to appear in Nucl. Phys. B

[12] D. Lüst and T.R. Taylor, Phys. Lett. B253 (1991) 335; B. de Carlos, J.A. Casas and C. Muñoz, Phys. Lett. B263 (1991) 248; D. Lüst and C. Muñoz, Phys. Lett. B279 (1992) 272

[13] J.A. Casas, E.K. Katehou and C. Muñoz, Nucl. Phys. B317 (1989) 171; J.A. Casas and C. Muñoz, Phys. Lett. B214 (1988) 63; A. Font, L. Ibáñez, H.P. Nilles and F. Quevedo, Phys. Lett. B210 (1988) 101; I. Antoniadis, J. Ellis, J.S. Hagelin and D.V. Nanopoulos, Phys. Lett. B205 (1988) 459, B213 (1988) 56; A. H. Chamseddine and M. Quirós, Nucl. Phys. B316 (1989) 101

[14] T.R. Taylor, G. Veneziano and S. Yankielowicz, Nucl. Phys. B218 (1983) 493; I. Affleck, M. Dine and N. Seiberg, Nucl. Phys. B241 (1984) 493; D. Amati, K. Konishi, Y. Meurice, G.C. Rossi and G. Veneziano, Phys. Rep. 162 (1988) 169