SOME HADAMARD-TYPE INEQUALITIES FOR COORDINATED P–CONVEX FUNCTIONS AND GODUNOVA-LEVIN FUNCTIONS

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Abstract. In this paper we established new Hadamard-type inequalities for functions that co-ordinated Godunova-Levin functions and co-ordinated P–convex functions, therefore we proved a new inequality involving product of convex functions and P–functions on the co-ordinates.

1. INTRODUCTION

Let \( f : I \subseteq \mathbb{R} \to \mathbb{R} \) be a convex function and let \( a, b \in I \), with \( a < b \). The following inequality;

\[
\frac{f(a) + f(b)}{2} \leq \frac{1}{b-a} \int_a^b f(x) \, dx \leq \frac{f(a) + f(b)}{2}
\]

is known in the literature as Hadamard’s inequality. Both inequalities hold in the reversed direction if \( f \) is concave.

In [1], E.K. Godunova and V.I. Levin introduced the following class of functions.

Definition 1. A function \( f : I \subseteq \mathbb{R} \to \mathbb{R} \) is said to belong to the class of \( Q(I) \) if it is nonnegative and, for all \( x, y \in I \) and \( \lambda \in (0, 1) \) satisfies the inequality;

\[
f(\lambda x + (1-\lambda)y) \leq \frac{f(x)}{\lambda} + \frac{f(y)}{1-\lambda}
\]

In [2], S.S. Dragomir et.al., defined following new class of functions.

Definition 2. A function \( f : I \subseteq \mathbb{R} \to \mathbb{R} \) is \( P \) function or that \( f \) belongs to the class of \( P(I) \), if it is nonnegative and for all \( x, y \in I \) and \( \lambda \in [0, 1] \), satisfies the following inequality;

\[
f(\lambda x + (1-\lambda)y) \leq f(x) + f(y)
\]

In [2], S.S. Dragomir et.al., proved two inequalities of Hadamard’s type for class of Godunova-Levin functions and \( P \)– functions.

Theorem 1. Let \( f \in Q(I), a, b \in I \), with \( a < b \) and \( f \in L_1[a, b] \). Then the following inequality holds.

\[
\frac{f(a) + f(b)}{2} \leq \frac{1}{b-a} \int_a^b f(x) \, dx \leq \frac{f(a) + f(b)}{2}
\]
\( f(\frac{a+b}{2}) \leq \frac{4}{b-a} \int_a^b f(x)dx \)

**Theorem 2.** Let \( f \in P(I), a, b \in I \), with \( a < b \) and \( f \in L_1[a,b] \). Then the following inequality holds.

\( f(\frac{a+b}{2}) \leq \frac{2}{b-a} \int_a^b f(x)dx \leq 2|f(a)+f(b)| \)

In [10], Tunc proved following theorem which containing product of convex functions and P–functions.

**Theorem 3.** Let \( a, b \in [0, \infty), a < b \), \( I = [a,b] \) with \( f, g : [a,b] \rightarrow \mathbb{R} \) be functions \( f, g \) and \( f, g \) are in \( L_1 ([a,b]) \). If \( f \) is convex and \( g \) belongs to the class of \( P(I) \) then,

\[
\frac{1}{b-a} \int_a^b f(x)g(x)dx \leq \frac{M(a,b) + N(a,b)}{2}
\]

where \( M(a,b) = f(a)g(a) + f(b)g(b) \) and \( N(a,b) = f(a)g(b) + f(b)g(a) \).

In [3], S.S. Dragomir defined convexity on the co-ordinates, as following:

**Definition 3.** Let us consider the bidimensional interval \( \Delta = [a,b] \times [c,d] \) in \( \mathbb{R}^2 \) with \( a < b, c < d \). A function \( f : \Delta \rightarrow \mathbb{R} \) will be called convex on the co-ordinates if the partial mappings \( f_y : [a,b] \rightarrow \mathbb{R}, f_y(u) = f(u, y) \) and \( f_x : [c,d] \rightarrow \mathbb{R}, f_x(v) = f(x, v) \) are convex where defined for all \( y \in [c,d] \) and \( x \in [a,b] \). Recall that the mapping \( f : \Delta \rightarrow \mathbb{R} \) is convex on \( \Delta \) if the following inequality holds,

\[
f(\lambda x + (1-\lambda)z, \lambda y + (1-\lambda)w) \leq \lambda f(x, y) + (1-\lambda)f(z, w)
\]

for all \( (x, y), (z, w) \in \Delta \) and \( \lambda \in [0,1] \).

Every convex function is co-ordinated convex but the converse is not generally true.

In [3], S.S. Dragomir established the following inequalities of Hadamard’s type for co-ordinated convex functions on a rectangle from the plane \( \mathbb{R}^2 \).

**Theorem 4.** Suppose that \( f : \Delta = [a,b] \times [c,d] \rightarrow \mathbb{R} \) is convex on the co-ordinates on \( \Delta \). Then one has the inequalities;

\[
f(\frac{a+b}{2}, \frac{c+d}{2}) \leq \frac{1}{2} \left[ \frac{1}{b-a} \int_a^b f(x, \frac{c+d}{2})dx + \frac{1}{d-c} \int_c^d f(\frac{a+b}{2}, y)dy \right]
\]

\[
\leq \frac{1}{(b-a)(d-c)} \int_a^b \int_c^d f(x, y)dxdy
\]

\[
\leq \frac{1}{4} \left[ \frac{1}{b-a} \int_a^b f(x, c)dx + \frac{1}{b-a} \int_a^b f(x, d)dx + \frac{1}{d-c} \int_c^d f(a, y)dy + \frac{1}{d-c} \int_c^d f(b, y)dy \right]
\]

\[
\leq \frac{f(a,c) + f(a,d) + f(b,c) + f(b,d)}{4}
\]
For recent results which similar to above inequalities see [5], [6], [7], [8] and [9].

In [4], M.E. Ozdemir et.al., established the following Hadamard’s type inequalities as above for co-ordinated $m$-convex and $(\alpha, m)$-convex functions.

**Theorem 5.** Suppose that $f : \Delta = [0, b] \times [0, d] \to \mathbb{R}$ is $m$-convex on the co-ordinates on $\Delta$. If $0 \leq a < b < \infty$ and $0 \leq c < d < \infty$ with $m \in (0, 1]$, then one has the inequality:

\[
\frac{1}{(b-a)(d-c)} \int_a^b \int_c^d f(x, y)\,dx\,dy \\
\leq \frac{1}{4(b-a)} \min \{v_1, v_2\} + \frac{1}{4(d-c)} \min \{v_3, v_4\}
\]

where

\[
v_1 = \int_a^b f(x, c)\,dx + m \int_a^b f(x, \frac{d}{m})\,dx \\
v_2 = \int_a^b f(x, d)\,dx + m \int_a^b f(x, \frac{c}{m})\,dx \\
v_3 = \int_c^d f(a, y)\,dy + m \int_c^d f(\frac{b}{m}, y)\,dy \\
v_4 = \int_c^d f(b, y)\,dy + m \int_c^d f(\frac{a}{m}, y)\,dy.
\]

**Theorem 6.** Suppose that $f : \Delta = [0, b] \times [0, d] \to \mathbb{R}$ is $m$-convex on the co-ordinates on $\Delta$. If $0 \leq a < b < \infty$ and $0 \leq c < d < \infty$, $m \in (0, 1]$ with $f_x \in L^1[0, d]$ and $f_y \in L^1[0, b]$, then one has the inequalities:

\[
\frac{1}{b-a} \int_a^b f(x, \frac{c+d}{2})\,dx + \frac{1}{d-c} \int_c^d f(\frac{a+b}{2}, y)\,dy \\
\leq \frac{1}{(b-a)(d-c)} \left[ \int_a^b \int_c^d f(x, y) + m f(x, \frac{c}{m}) \right] \frac{dy\,dx}{2} \\
+ \int_c^d \int_a^b f(x, y) + m f(\frac{c}{m}, y) \frac{dx\,dy}{2}
\]

Similar results can be found for $(\alpha, m)$-convex functions in [4]. In this paper we established new Hadamard-type inequalities for Godunova-Levin functions and $P$-functions on the co-ordinates on a rectangle from the plane $\mathbb{R}^2$ and we proved a new inequality involving product of co-ordinated convex functions and co-ordinated $P$-functions.

## 2. MAIN RESULTS

We define Godunova-Levin functions and $P$-functions on the co-ordinates as the following:

**Definition 4.** Let us consider the bidimensional interval $\Delta = [a, b] \times [c, d]$ in $\mathbb{R}^2$ with $a < b$, $c < d$. A function $f : \Delta \to \mathbb{R}$ is said to belong to the class of $Q(I)$ if it is nonnegative and for all $(x, y), (z, w) \in \Delta$ and $\lambda \in (0, 1)$ satisfies the following
Theorem 7. Levin functions on the co-ordinates.

Let \( \nu \) be a convex function; which shows convexity of \( f \) \( \nu \) coordinated Godunova-Levin function if the partial mappings \( f_y : [a, b] \to \mathbb{R}, \), \( f_y(u) = f(u, y) \) and \( f_x : [c, d] \to \mathbb{R}, f_x(v) = f(x, v) \) are belong to the class of \( Q(I) \) where defined for all \( y \in [c, d] \) and \( x \in [a, b] \).

We denote this class of functions by \( Q(V \nu) \). If the inequality reversed then \( f \) is said to be concave on \( \Delta \) and we denote this class of functions by \( QV(f, \Delta) \).

Definition 5. Let \( f : \Delta = [a, b] \times [c, d] \to \mathbb{R} \) be a \( P \)-function with \( a \leq b, c \leq d \). If it is nonnegative and for all \( (x, y), (z, w) \in \Delta \) and \( \lambda \in (0, 1) \) the following inequality holds:

\[
f(\lambda x + (1-\lambda)z) + (1-\lambda)w) \leq f(x, y) + f(z, w)
\]

A function \( f : \Delta \to \mathbb{R} \) is said to belong to the class \( Q(X \nu) \) if the partial mappings \( f_y : [a, b] \to \mathbb{R}, f_y(u) = f(u, y) \) and \( f_x : [c, d] \to \mathbb{R}, f_x(v) = f(x, v) \) are \( P \)-functions where defined for all \( y \in [c, d] \) and \( x \in [a, b] \).

We denote this class of functions by \( PX(f, \Delta) \). We need following lemma for our main theorem.

Lemma 1. Every \( f \) function that belongs to the class \( Q(I) \) is said to belongs to class \( Q(X \nu) \).

Proof. Suppose that \( f : \Delta = [a, b] \times [c, d] \to \mathbb{R} \) is said to belong to the class \( Q(I) \) on \( \Delta \). Consider the function \( f_x : [c, d] \to [0, \infty), f_x(v) = f(x, v) \). Then \( \lambda \in (0, 1) \) and \( v_1, v_2 \in [c, d] \), one has:

\[
f_x(\lambda v_1 + (1-\lambda)v_2) = f(x, \lambda v_1 + (1-\lambda)v_2) = f(\lambda x + (1-\lambda)x, \lambda v_1 + (1-\lambda)v_2) \leq \frac{f(x, v_1)}{\lambda} + \frac{f(x, v_2)}{1-\lambda} = f_x(v_1) + f_x(v_2) \frac{\lambda}{1-\lambda}
\]

which shows convexity of \( f_x \). The fact that \( f_y : [a, b] \to \mathbb{R}, f_y(u) = f(u, y) \) is also convex on \([a, b]\) for all \( y \in [c, d] \) goes likewise and we shall omit the details. \( \square \)

The following inequalities is considered the Hadamard-type inequalities for Godunova-Levin functions on the co-ordinates.

Theorem 7. Suppose that \( f : \Delta = [a, b] \times [c, d] \to \mathbb{R} \) is said to belong to the class \( Q(X \nu) \) on the co-ordinates on \( \Delta \) with \( f_x \in L_1[c, d] \) and \( f_y \in L_1[a, b] \), then one has the inequalities:

\[
(2.1) \quad \frac{1}{16} \left[ \int_{\frac{a+b}{2}} f \frac{c+d}{2} \right] \leq \frac{1}{8} \left[ \frac{1}{b-a} \int_a^b f \left( \frac{c+d}{2} \right) dx + \frac{1}{d-c} \int_c^d f \left( \frac{a+b}{2} , y \right) dy \right] \leq \frac{1}{(b-a)(d-c)} \int_a^b \int_c^d f(x, y) dy dx
\]
Proof. Since $f : \Delta = [a, b] \times [c, d] \to \mathbb{R}$ is said to belong to the class $QX(f, \Delta)$ on the co-ordinates it follows that the mapping $g_x : [c, d] \to \mathbb{R}$, $g_x(y) = f(x, y)$ is Godunova-Levin function on $[c, d]$ for all $x \in [a, b]$. Then by Hadamard’s inequality (1.1) one has:
\[
g_x \left( \frac{c + d}{2} \right) \leq \frac{4}{d - c} \int_c^d g_x(y) dy, \forall x \in [a, b].
\]
That is,
\[
f(x, \frac{c + d}{2}) \leq \frac{4}{d - c} \int_c^d f(x, y) dy, \forall x \in [a, b].
\]
Integrating this inequality on $[a, b]$, we have:
\[
\frac{1}{b - a} \int_a^b f(x, \frac{c + d}{2}) dx \leq \frac{4}{(b - a)(d - c)} \int_c^d \int_c^b f(x, y) dy dx.
\]
A similar argument applied for the mapping $g_y : [a, b] \to \mathbb{R}$, $g_y(x) = f(x, y)$, we get:
\[
\frac{1}{d - c} \int_c^d f \left( \frac{a + b}{2}, y \right) dy \leq \frac{4}{(b - a)(d - c)} \int_c^d \int_a^b f(x, y) dx dy.
\]
Summing the inequalities (2.2) and (2.3), we get the last inequality in (2.1).

Therefore, by Hadamard’s inequality (1.1) we also have:
\[
f(\frac{a + b}{2}, \frac{c + d}{2}) \leq \frac{4}{d - c} \int_c^d f(\frac{a + b}{2}, y) dy
\]
and
\[
f(\frac{a + b}{2}, \frac{c + d}{2}) \leq \frac{4}{b - a} \int_a^b f(x, \frac{c + d}{2}) dx
\]
which give, by addition the first inequality in (2.1).

This completes the proof. $\square$

Corollary 1. Suppose that $f : \Delta = [a, b] \times [a, b] \to \mathbb{R}$ is said to belong to the class $QX(f, \Delta)$ on the co-ordinates, then one has the inequalities:
\[
(2.4) \frac{1}{16} \left[ f(\frac{a + b}{2}, \frac{a + b}{2}) \right] \leq \frac{1}{8} \left[ \frac{1}{b - a} \int_a^b \left\{ f(x, \frac{a + b}{2}) + f(\frac{a + b}{2}, x) \right\} dx \right]
\]
\[
\leq \frac{1}{(b - a)^2} \int_a^b \int_a^b f(x, y) dy dx.
\]

Corollary 2. In (2.7), under the assumptions Theorem 4 with $f(x, y) = f(y, x)$ for all $x \in [a, b] \times [a, b]$, we have:
\[
f(\frac{a + b}{2}, \frac{a + b}{2}) \leq \frac{1}{4} \left[ \frac{1}{b - a} \int_a^b f(x, \frac{a + b}{2}) dx \right]
\]
\[
\leq \frac{1}{(b - a)^2} \int_a^b \int_a^b f(x, y) dy dx.
\]

Lemma 2. Every $P-$functions are coordinated on $\Delta$ or belong to the class of $PX(f, \Delta)$. 
Proof. Let $f$ be a $P$-function and defined by $f_x : [a, b] \to \mathbb{R}$, $f_y(u) = f(u, y)$ and $f_z : [c, d] \to \mathbb{R}$, $f_z(v) = f(x, v)$ where $y \in [c, d]$, $x \in [a, b]$ and $\lambda \in [0, 1]$, $v_1, v_2 \in [a, b]$, then

\[ f_z(\lambda v_1 + (1 - \lambda)v_2) = f(x, \lambda v_1 + (1 - \lambda)v_2) = f(\lambda x + (1 - \lambda)x, \lambda v_1 + (1 - \lambda)v_2) \leq f(x, v_1) + f(x, v_2) = f_z(v_1) + f_z(v_2) \]

which shows convexity of $f_z$. The fact that $f_y : [a, b] \to \mathbb{R}$, $f_y(u) = f(u, y)$ is also convex on $[a, b]$ for all $y \in [c, d]$ goes likewise and we shall omit the details. \hfill \Box

The following inequalities is considered the Hadamard-type inequalities for $P$-functions on the co-ordinates.

**Theorem 8.** Suppose that $f : \Delta = [a, b] \times [c, d] \to \mathbb{R}$ is said to belong to the class $PX(f, \Delta)$ on the co-ordinates on $\Delta$ with $f_x \in L_1[a, b]$ and $f_y \in L_1[a, b]$, then one has the inequalities:

\[ f\left(\frac{a + b}{2}, \frac{c + d}{2}\right) \leq \frac{1}{b - a} \int_a^b f(x, \frac{c + d}{2})dx + \frac{1}{d - c} \int_c^d f\left(\frac{a + b}{2}, y\right)dy \leq \frac{4}{(b - a)(d - c)} \int_a^b \int_c^d f(x, y)dydx \leq \frac{2}{b - a} \left[ \int_a^b f(x, c)dx + \int_a^b f(x, d)dx \right] \]

\[ + \frac{2}{d - c} \left[ \int_c^d f(a, y)dy + \int_c^d f(b, y)dy \right] \]

Proof. Since $f : \Delta = [a, b] \times [c, d] \to \mathbb{R}$ is said to belong to the class $PX(f, \Delta)$ on the co-ordinates it follows that the mapping $g_x : [c, d] \to \mathbb{R}$, $g_x(x) = f(x, y)$ is $P$-function on $[c, d]$ for all $x \in [a, b]$. Then by Hadamard’s inequality (1.2) one has:

\[ f\left(\frac{a + b}{2}, \frac{c + d}{2}\right) \leq \frac{2}{d - c} \int_c^d f(x, y)dy \leq 2 \left[ f(x, c) + f(x, d) \right] \]

Integrating this inequality on $[a, b]$, we have:

\[ \frac{1}{b - a} \int_a^b f(x, \frac{c + d}{2})dx \leq \frac{2}{(b - a)(d - c)} \int_a^b \int_c^d f(x, y)dydx \leq \frac{2}{b - a} \left[ \int_a^b f(x, c)dx + \int_a^b f(x, d)dx \right] \]

A similar argument applied for the mapping $g_y : [a, b] \to \mathbb{R}$, $g_y(x) = f(x, y)$, we get:

\[ \frac{1}{d - c} \int_c^d f\left(\frac{a + b}{2}, y\right)dy \leq \frac{2}{(b - a)(d - c)} \int_c^d \int_a^b f(x, y)dydx \leq \frac{2}{d - c} \left[ \int_c^d f(a, y)dy + \int_c^d f(b, y)dy \right] \]
Theorem 9. Let □ Which gives the first inequality in (2.5). This completes the proof.

\[ \frac{1}{(b-a)(d-c)} \int_a^b \int_c^d f(x,y)dydx \leq \frac{1}{2(b-a)} \left[ \int_a^b f(x,c)dx + \int_a^b f(x,d)dx \right] + \frac{1}{2(d-c)} \left[ \int_c^d f(a,y)dy + \int_c^d f(b,y)dy \right] \]

Which gives the last inequality in (2.5). We also have:

\[ \frac{1}{b-a} \int_a^b f(x, c + \frac{d}{2})dx + \frac{1}{d-c} \int_c^d f(\frac{a+b}{2}, y)dy \]

\[ \leq \frac{4}{(b-a)(d-c)} \int_a^b \int_c^d f(x,y)dydx \]

Which gives the mid inequality in (2.5). By Hadamard’s inequality we also have:

\[ f(\frac{a+b}{2}, c + \frac{d}{2}) \leq \frac{2}{b-a} \int_a^b f(x, \frac{c+d}{2})dx \]

and

\[ f(\frac{a+b}{2}, \frac{c+d}{2}) \leq \frac{2}{d-c} \int_c^d f(\frac{a+b}{2}, y)dy \]

Adding these inequalities we get,

\[ f(\frac{a+b}{2}, \frac{c+d}{2}) \leq \frac{1}{b-a} \int_a^b f(x, c + \frac{d}{2})dx + \frac{1}{d-c} \int_c^d f(\frac{a+b}{2}, y)dy \]

Which gives the first inequality in (2.5). This completes the proof. \( \square \)

Theorem 9. Let a, b, c, d \( \in [0, \infty) \), a < b and c < d, \( \Delta = [a, b] \times [c, d] \) with \( f, g : \Delta \rightarrow \mathbb{R} \) be functions \( f, g \) and \( fg \) are in \( L_1 ([a, b] \times [c, d]) \). If \( f \) is co-ordinated convex and \( g \) belongs to the class of \( PX(f, \Delta) \), then one has the inequality;

\[ \frac{1}{(d-c)(b-a)} \int_a^b \int_c^d f(x,y)g(x,y)dydx \leq L(a,b,c,d) + M(a,b,c,d) + N(a,b,c,d) \]

where

\[ L(a,b,c,d) = f(a,c)g(a,c) + f(b,c)g(b,c) + f(a,d)g(a,d) + f(b,d)g(b,d) \]

\[ M(a,b,c,d) = f(a,c)g(a,d) + f(a,d)g(a,c) + f(b,c)g(b,d) + f(b,d)g(b,c) \]

\[ + f(b,c)g(a,c) + f(b,d)g(a,d) + f(a,c)g(b,c) + f(a,d)g(b,d) \]

\[ N(a,b,c,d) = f(b,c)g(a,c) + f(b,d)g(a,c) + f(a,c)g(b,d) + f(a,d)g(b,c) \]

Proof. Since \( f \) is co-ordinated convex and \( g \) belongs to the class of \( PX(f, \Delta) \), by using partial mappings and from inequality (2.5), we can write

\[ \frac{1}{d-c} \int_c^d f_x(y)g_x(y)dy \leq \frac{f_x(c)g_x(c) + f_x(d)g_x(d) + f_x(c)g_x(d) + f_x(d)g_x(c)}{2} \]

That is

\[ \frac{1}{d-c} \int_c^d f(x,y)g(x,y)dy \leq \frac{f(x,c)g(x,c) + f(x,d)g(x,d) + f(x,c)g(x,d) + f(x,d)g(x,c)}{2} \]
Dividing both sides of this inequality \((b - a)\) and integrating over \([a, b]\) respect to \(x\), we have

\[
\frac{1}{(d - c) (b - a)} \int_a^b \int_c^d f(x, y) g(x, y) \, dy \, dx \leq
\]

\[
\frac{1}{2 (b - a)} \int_a^b f(x, c) g(x, c) + \frac{1}{2 (b - a)} \int_a^b f(x, d) g(x, d)
\]

\[
+ \frac{1}{2 (b - a)} \int_a^b f(x, c) g(x, d) + \frac{1}{2 (b - a)} \int_a^b f(x, d) g(x, c)
\]

By applying (1.3) to each integral on right hand side of (2.10) and using these inequalities in (2.10), we get the required result as following

\[
\frac{1}{(d - c) (b - a)} \int_a^b \int_c^d f(x, y) g(x, y) \, dy \, dx
\]

\[
\leq \frac{4}{4} f(a, c) g(a, c) + \frac{4}{4} f(b, c) g(b, c) + \frac{4}{4} f(a, c) g(b, c) + \frac{4}{4} f(b, c) g(a, c)
\]

\[
+ \frac{4}{4} f(a, d) g(a, d) + \frac{4}{4} f(b, d) g(b, d) + \frac{4}{4} f(a, d) g(b, d) + \frac{4}{4} f(b, d) g(a, d)
\]

By a similar argument, if we apply (1.3) for \(f(y)g(x)\) on \([a, b]\), we get the same result.

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