A Naturally Narrow Positive Parity $\Theta^+$

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We present a consistent color-flavor-spin-orbital wave function for a positive parity $\Theta^+$ that naturally explains the observed narrowness of the state. The wave function is totally symmetric in its flavor-spin part and totally antisymmetric in its color-orbital part. If flavor-spin interactions dominate, this wave function renders the positive parity $\Theta^+$ lighter than its negative parity counterpart.

We consider decays of the $\Theta^+$ and compute the overlap of this state with the kinematically allowed final states. Our results are numerically small. We note that dynamical correlations between quarks are not necessary to obtain narrow pentaquark widths.

Introduction. The recent discovery of pentaquark states has stimulated a significant body of theoretical [8–37] and experimental research. Pentaquarks are baryons whose minimal Fock components consist of four quarks and an antiquark. The first observed pentaquark was the $\Theta^+ (1540)$ with strangeness $S = +1$, and with quark content $ududs$. More recently, the NA49 Collaboration has reported a narrow $\Xi^- (1860)$ baryon with $S = -2$ and quark content $dsds$, together with evidence for its isosquartet partner $\Xi_0$ at the same mass.

The existence of the $\Theta^+$, as well as its flavor quantum numbers, seems to be well established (for a different view, see Ref. [38]). If the $\Theta^+$ were a member of an isovector or isoscalar multiplet, then one would expect to observe its doubly charged partner experimentally. The SAPHIR Collaboration searched for a $\Theta^{++}$ in $\gamma p \rightarrow \Theta^{++} K^- \rightarrow p K^+ K^-$, with negative results. They concluded that the $\Theta^+$ is an isosinglet and hence a member of a pentaquark antidecuplet. All but one theoretical paper treat the $\Theta^+$ as an isosinglet.

The spin and parity quantum numbers of the $\Theta^+$ have yet to be determined experimentally. The spin of $\Theta^+$ is taken to be $1/2$ by all theory papers to our knowledge and various estimates show that spin-$3/2$ pentaquarks must be heavier. A more controversial point among theorists is the parity of the state. For example, QCD sum rule calculations [10], quenched lattice QCD [11], and a minimal constituent quark treatment by the present authors [12], predict that the lightest $\Theta^+$ is a negative parity isosinglet. All chiral soliton papers [13, 14], some correlated quark models [15, 16], and some works within the constituent quark model [17, 18, 19], including a second work by the present authors [20], predict the lightest $\Theta^+$ pentaquark as a positive parity isosinglet.

The photoproduction and the pion-induced production cross sections of the $\Theta^+$ were studied in [21]. It was shown in both cases that the production cross sections for a negative parity $\Theta^+$ are much smaller than those for the positive parity state (for a given $\Theta^+$ width). In Ref. [21], results for the $\Theta^+$ production cross section in photon-proton reactions were compared with estimates of the cross section based on data obtained by the SAPHIR Collaboration, and odd-parity pentaquark states were argued to be disfavored.

Here, we present new results following from a consistent treatment of the color-flavor-spin-orbital wave function for a positive parity $\Theta^+$. In [21] (inspired by [17]), we showed that dominant flavor-spin interactions render the positive parity $\Theta^+$ lighter than its negative parity counterpart. Here we will present decompositions of the quark model wave function of the $\Theta^+$, explicitly including the orbital part. We will see that the narrowness of the $\Theta^+$ follows naturally from the group theoretic structure of the state.

Wave Function. If flavor-spin interactions dominate [39], the lightest positive parity $\Theta^+$ will have a flavor-spin (FS) wave function that is totally symmetric. Fermi-Dirac statistics dictates that the color-orbital (CO) wave function must be fully antisymmetric. We present two decompositions of the wave function, one in terms of quark pairs and the antiquark, and another in terms of the quantum numbers of $q^4$ and $q\bar{q}$ subsystems.

In the first decomposition, the overall $q^4$ flavor state must be a $6$. This is the only representation that one can combine with a flavor 3 (the antiquark) to form an antidecuplet. This further implies that the overall $q^4$ spin is 0, since the only possible fully symmetric $q^4$ ($F, S$) wave functions are $(6, 0)$ or $(15_M, 1)$. A flavor 6 can be obtained if both quarks pairs are in either a $6$ or 3, while a spin-0 state can be obtained if both are either spin-0 or 1. Since we want a fully symmetric FS wave function, we must combine these possibilities as follows:

$$|FS\rangle_{(6, 0)} = a |(3, 0)(3, 0)\rangle_{(6, 0)} + b |(6, 1)(6, 1)\rangle_{(6, 0)}. \quad (1)$$

The parentheses on the right hand side delimit the flavor and spin quantum numbers of the first and second pair of quarks, each of which is combined into an overall $(6, 0)$. For the $\Theta^+$, the $q^4$ states on the right-hand-side are:

$$| (3, 0)(3, 0)\rangle_{(6, 0)} =$$
which is a mixed symmetry state, whose Young tableaux symmetry given by the conjugate tableaux in order to obtain combined to yield a totally antisymmetric CO state.

\[
\begin{align*}
\frac{1}{4} (ud - du)(ud - du) \otimes (\uparrow\downarrow - \downarrow\uparrow)(\uparrow\downarrow - \downarrow\uparrow), \\
\equiv |(6, 1)(6, 1)(6, 0)\rangle = \big(\frac{1}{12} (2udd + 2ddu - udd - udu - duu - dd) \otimes (2 \uparrow\uparrow\downarrow\downarrow + 2 \downarrow\downarrow\uparrow\uparrow - \uparrow\uparrow\downarrow\downarrow - \uparrow\downarrow\uparrow\uparrow - \downarrow\uparrow\downarrow\downarrow - \downarrow\uparrow\uparrow\uparrow)\big).
\end{align*}
\]

Total symmetry of the wave function demands \(a = b\). To properly normalize the state, we choose \(a = b = 1/\sqrt{2}\).

The next step is to construct the totally antisymmetric CO wave function. The \(q^3\) color state must be a \(3\), which is a mixed symmetry state, whose Young tableaux is shown in Fig. 1. The orbital state, containing three \(S\)-states and one \(P\)-state, must have a permutation symmetry given by the conjugate tableaux in order to obtain overall antisymmetry. Hence the structure of our wave functions implies that the strange antiquark is not orbitally excited; simple estimates suggest that a state with the \(s\) excited would be considerably heavier [20]. The possible color and orbital representations for two pairs of quarks are shown in Fig. 1.

From Fig. 1 a totally antisymmetric CO wave function must have the form:

\[
|CO\rangle = a' |(3, S)(3, S)\rangle + b' \{|(6, A)(3, S)\rangle + |(3, S)(6, A)\rangle\}\),
\]

The coefficients \(a'\) and \(b'\) are fixed by the constraint that the wave function must be antisymmetric under interchange of the first and third quarks. When the \(q^1\) color state is red, the explicit expressions for the wave functions on the right-hand-side are:

\[
|3, S\rangle = \frac{1}{\sqrt{8}} \{ (RG - GR)(BR - RB) \\
- (BR - RB)(RG - GR) \},
\]

\[
\otimes \frac{1}{2} \{ SS(SP + PS) - (SP + PS)SS \},
\]

\[
|6, A\rangle = \frac{1}{4} \{ (2RR(GB - BG) \\
+ (RG + GR)(BR - RB) + (RB + BR)(RG - GR) \}
\]

\[
\otimes \frac{1}{\sqrt{2}} (SP - PS) SS.
\]

The wave function is properly normalized with the choice \(a'' = b' = 1/\sqrt{3}\). In our construction, the total spin of the \(q^3\bar{s}\) can only be 1/2. Appropriate Clebsch-Gordan coefficients may be chosen to combine the orbital angular momentum of the excited \(q\) so that the total \(\Theta^+\) spin is 1/2. We leave this implicit.

For the second decomposition, we note that the \(q^3\) and \(q\bar{q}\) flavor wave functions must both be \(8\)'s if one is to form a flavor \(16\). Since the \(q^3\) FS wave function is fully symmetric, the \(q^3\) FS wave function must be fully symmetric also. The mixed symmetry of the \(q^3\) flavor wave function implies that the \(q^3\) spin wave function must have mixed symmetry also and hence is spin-1/2. Total symmetrization of the \(q^3\) FS wave function is obtained as follows:

\[
|\langle 8, 1/2\rangle\rangle_q = \frac{1}{\sqrt{2}} \left( |\langle 8, 1/2S\rangle\rangle + |\langle 8A, 1/2A\rangle\rangle \right),
\]

where symmetry of the first two quarks. The \(q\bar{q}\) spin can be 0 or 1. The fully symmetric FS wave function is of the form

\[
|\langle FS\rangle(10, 1/2)\rangle = a''|\langle 8, 1/2\rangle(8, 0)\rangle(10, 1/2)\rangle + b'' |\langle 8, 1/2\rangle(8, 1)\rangle(10, 1/2)\rangle,
\]

where the coefficients \(a''\) and \(b''\) are fixed by requiring that the wave function is symmetric under the interchange of the first and fourth quarks. For the \(\Theta^+\), the part of the states on the right-hand-side that have \(z\)-component spin projection 1/2 are:

\[
|\langle 8, 1/2\rangle(8, 0)\rangle(10, 1/2)\rangle = \frac{1}{\sqrt{2}} \left( \frac{1}{4} (ud - du)(ud - du)\bar{s} \otimes (\uparrow\downarrow - \downarrow\uparrow)(\uparrow\downarrow - \downarrow\uparrow) \\
+ \frac{1}{12} (2udd + 2ddu - udd - udu - duu - dd)\bar{s} \\
\otimes (2 \uparrow\uparrow\downarrow\downarrow + 2 \downarrow\downarrow\uparrow\uparrow - \uparrow\uparrow\downarrow\downarrow - \uparrow\downarrow\uparrow\uparrow - \downarrow\uparrow\downarrow\downarrow - \downarrow\uparrow\uparrow\uparrow)\right),
\]

and

\[
|\langle 8, 1/2\rangle(8, 1)\rangle(10, 1/2)\rangle = \frac{1}{\sqrt{3}} \left( \frac{1}{2} (ud - du)(ud - du)\bar{s} \otimes \left\{ (\uparrow\downarrow - \downarrow\uparrow)(\uparrow\downarrow - \downarrow\uparrow) \\
+ \frac{1}{\sqrt{12}} (2udd + 2ddu - udd - udu - duu - dd)\bar{s} \\
- duu\bar{s} \otimes \left\{ \frac{1}{3} (\uparrow\downarrow - \uparrow\downarrow - \uparrow\downarrow)(\uparrow\downarrow - \uparrow\downarrow) \right\} \\
- \frac{1}{6} (2 \uparrow\uparrow\downarrow - \uparrow\uparrow\downarrow - \uparrow\uparrow\downarrow)(\uparrow\downarrow + \uparrow\downarrow) \right\} \right).
\]

These are sufficient to show \(a'' = -1/2\) and \(b'' = -\sqrt{3}/2\), using a sign convention consistent with our previous decomposition.

The CO wave function includes two possibilities. Either the orbital wave function is totally symmetric,
The totally symmetric orbital wave functions with a $P$-state included correspond to a ground state baryon or meson with center-of-mass motion. From the previous section we know $a'' = 1/2$ and $b'' = -1/\sqrt{2}$. Hence the total probability of the $\Theta^+$ overlap with $NK$ is:

$$c_+ = \left(\frac{a'' a''/\sqrt{2}}{(a'' b'')^2}\right)^2 + (a'' b'')^2 = \frac{5}{96},$$

which implicitly includes a sum over $z$-component spin projections. This is interestingly small. The $\Theta^+$ width for a positive ($\Gamma_+$) or negative ($\Gamma_-$) parity state is

$$\Gamma_\pm = c_\pm g_\pm^2 \cdot \frac{M}{16\pi} \left[1 - \left(\frac{m + \mu}{M}\right)^2\right]^{1/2} \times \left[1 \pm \frac{m^2 - \mu^2}{M^2}\right],$$

where $M, m, \mu$ are the masses of the $\Theta^+$, the final state baryon and the meson, respectively, $c_\pm$ is the dimensionless spin-flavor-color-orbital overlap factor ($c_+ = 5/96$, or $c_- = 1/4$ from Ref. 12), and $g_\pm$ is an effective meson-baryon coupling constant, $\mathcal{E}_{\text{eff}}(c_\pm = 1) = g_- N K \Theta^+$ or $g_+ N^3 K \Theta^+$. Applying the rules of naive dimensional analysis (NDA) 10, one estimates that $g_\pm \sim 4\pi$, up to order one factors. One then finds

$$\Gamma_+ \approx 4.4 \text{ MeV while } \Gamma_- \approx 1.1 \text{ GeV.}$$

In the effective theory approach, effects associated with long-distance dynamics are subsumed in the values of the couplings $g_\pm$. For example, an explicit computation of quark wave function overlaps in baryons with both $S$- and $P$-wave constituents could lead to a smaller estimate for $g_+$. However, the precise outcome is strongly model dependent and we do not pursue this issue further. Our result implies that a positive parity $\Theta^+$ is narrow, independent of these uncertainties.

It has been noted 18 that the correlated diquark state advocated in Ref. 14 has a small overlap with the $NK$ state, even if one just considers the color-flavor-spin wave function. However, the $q^4$ part of the correlated diquark state presented in 14 is not perfectly antisymmetric. The state is a good approximation to a Fermi-Dirac allowed state only to the extent that the diquarks are significantly more compact than the overall state. The significant likelihood that the diquarks are comparable in size to the entire pentaquark is reason for concentrating on a consistent, antisymmetrized wave function. (We can nonetheless report for the correlated diquark model that inclusion of the orbital wave function reduces the $\Theta^+$ overlap with $NK$ from the Jennings-Maltman color-flavor-spin result of 1/24 to a remarkably small 5/576.)

**Conclusions.** We have presented an explicit framework in which the width of a positive parity $\Theta^+$ is narrow. We find that the spin-flavor-color-orbital overlap probability for decays to kinematically allowed final states is $5/96$. By comparison, the same overlap probability for the negative parity case is $1/4$, as was shown in Ref. 12.
Without any incalculable dynamical suppression (that could render \( g_* \) substantially less than \( g_* \) above), one may infer that a negative parity pentaquark state, if it exists, is significantly broader than its positive parity cousin. Aside from its \( N \bar{K} \) component, the even parity \( \Theta^+ \) wave function overlaps with other color-singlet-color-singlet baryon-meson states that are together heavier than the \( \Theta^+ \), and with color-octet-color-octet baryon-meson states. Hence, even though the decay proceeds via a fall-apart mode, the amplitude to kinematically allowed baryon-meson states is small.

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