Light quark masses using domain wall fermions

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We compute the one–loop self–energy correction to the massive domain wall quark propagator. Combining this calculation with simulations at several gauge couplings, we estimate the strange quark mass in the continuum limit. The perturbative one–loop mass renormalization is comparable to that for Wilson quarks and considerably smaller than that for Kogut–Susskind quarks. Also, scaling violations appear mild in comparison to other errors at present. Given their good chiral behavior and these features, domain wall quarks are attractive for evaluating the light quark masses. Our preliminary quenched result is

$$m_s(2 \text{ GeV}) = 82(15) \text{ MeV}$$

in the $\overline{\text{MS}}$ scheme.

1. INTRODUCTION

Computing light quark masses is a high priority. Present lattice predictions of $m_l \equiv (m_u + m_d)/2$ and $m_s$ give differing results depending on the particular method. Domain wall (DW) fermions respect the chiral symmetries of the continuum exactly in the limit $N_s \to \infty$, $N_s$ being the number of sites in a fictitious extra dimension. This is an especially attractive feature for simulating light quark physics where chiral symmetry is crucial. Furthermore, simulations have demonstrated that $N_s \sim 10$ for $\beta \geq 6.0$ suffices for very good chiral behavior rendering DW fermions quite practical. We report on first results for the strange quark mass using DW fermions.

2. ACTION

We use the boundary fermion variant of the DW formulation. The fifth dimension has a finite number of sites $N_s$; a light right–handed mode is bound to the 4–d surface at one end and a light left–handed mode at the other end. A 5–d mass $M$ determines the strength of the binding, and a parameter $m$ controls the coupling between the two 5–d boundaries.

The chiral symmetry breaking is due to the explicit term proportional to $m$ and to implicit mixing between the two modes which should be suppressed $\sim e^{-\alpha N_s}$, where $\alpha > 0$. Therefore the latter can be made arbitrarily small compared to the former by increasing $N_s$. For example, at tree level the quark mass is

$$am_q = M(2 - M) \left( m + |1 - M|^{-\alpha N_s} \right).$$

(1)

If, in the free theory, $M$ is in the range $0 < M < 2$, there is a single light mode fixed to either boundary. More properties of DW fermions are discussed in and references therein.

3. RENORMALIZATION

Interactions renormalize the five–dimensional mass $M$ additively; thus the range $0 < M < 2$ is shifted by an amount $M_c$. In perturbation theory $M_c$ can be computed from terms proportional to $a^{-1}$. Numerically, the tadpole graph gives larger contributions to the self–energy than the half–circle graph; so one way to estimate $M_c$ is to use the tadpole term alone, giving $M_t^{\text{rad}} = 12.6a_s$.

Noting the importance of $M_c$ and the limitations of perturbation theory, a nonperturbative determination of $M_c$ is more desirable. Through the overlap formulation the Wilson–Dirac operator defines a transfer matrix which governs propagation in the 5th dimension. Therefore $M_c$ can be determined directly from the critical Wilson hopping parameter $\kappa_c^W$:

$$M_c^W = 4 - (2\kappa_c^W)^{-1}. $$

(2)

Using data from we give $M_c^W$ relevant to this work in Table 1. The superscript is to distinguish this calculation of $M_c$ from other estimates. For
reasonable definitions of the coupling constant $\alpha_s$ (see later), we find $M_{c}^{\text{tad}} \approx M_{c}^{W}$.

In order to compute the quark mass renormalization, we have extended the explicit one–loop calculation [7] of the DW fermion self–energy to nonzero fermion mass [11]. The renormalization factor $Z_m$ can be defined by equating the one–loop fermion propagators on the lattice and the continuum [12]. The result is

$$Z_m = 1 - \left( 2\alpha_s/\pi \right) \left[ \ln(\mu a) - C_m \right],$$

(3)

where the DW quark masses are computed with a lattice spacing $a$ and the $\overline{\text{MS}}$ quark masses at a scale $\mu$. $C_m$ depends only on $M - M_c$ and is plotted in Fig. 1. The $C_m$ for the $M - M_c$ used in our simulations are shown in Table 1 and should be compared to 2.16, 3.22, and 6.54 for Wilson, Sheikholeslami–Wohlert (SW), and Kogut–Susskind (KS) fermions.

4. QUARK MASS

We have computed the pion mass and decay constant on a few dozen quenched configurations at $\beta = 5.85$, 6.0, and 6.3. We give other simulation parameters in Table 1. In addition we studied the pion mass as a function of both $m$ and $M$ at $\beta = 6.0$. For each $M$, the pion mass squared extrapolates linearly to zero at $m = 0$ within errors as in [4]. Fig. 2 shows the dependence of the pion mass squared on $M - M_c$. In lowest order chiral perturbation theory $M_\pi^2 \sim m_q$. Therefore, from Eqn. (1) with $M \rightarrow M - M_c^W$ the pion mass squared should obey

$$\left( aM_\pi \right)^2 = (\text{const.}) (M - M_c^W)(2 - M + M_c^W).$$

(4)

The dotted lines are fits to (4) and have good $\chi^2$'s. At present the data do not permit a more general fit.

To numerically compute the renormalization constant (3), one must choose a definition for the coupling constant $\alpha_s$ and the scale $\mu$ at which it is evaluated. We follow [13] and use $\alpha_{\overline{\text{MS}}}$. Specifically, we compute $\alpha_V(3.41/a)$ from the plaquette and convert to $\alpha_{\overline{\text{MS}}}$ perturbatively [14]. We run that coupling constant to $\mu$ to apply (3) and finally run $m_s^{\overline{\text{MS}}}$ perturbatively to 2 GeV. We choose $\mu = 2/a$ for the matching; the final result varies by 2 MeV for $0.5 < \mu a < \pi$. We use the physical pion decay constant to set the lattice spacing and the physical kaon mass to fix the parameter $m$ to the strange quark system.

In Fig. 3 we plot our final results for the strange quark mass along with recent results using other actions. The results for DW fermions appear to be in rough agreement with other lattice cal-

| Table 1 | Summary of simulation parameters and results. $m_s^{\text{LAT}}$ and $m_s^{\overline{\text{MS}}}(2 \text{ GeV})$ are in MeV. |
|---------|---------------------------------------------------------------|
| $\beta$ = 6.3 | $\beta$ = 6.0 | $\beta$ = 5.85 |
| # configs. | 18 | 33 | 18 |
| volume | $24^3 \times 60$ | $16^3 \times 32$ | $16^3 \times 32$ |
| $N_s$ | 10 | 10 | 14 |
| $M$ | 1.5 | 1.7 | 1.7 |
| $M_c^W$ | 0.708 | 0.819 | 0.908 |
| $C_m$ | 2.35 | 2.47 | 2.42 |
| $a^{-1}$ (GeV) | 3.4(3) | 2.39(12) | 1.69(10) |
| $m_{\text{strange}}$ | 0.020(3) | 0.027(3) | 0.041(6) |
| $m_s^{\text{LAT}}$ | 64(11) | 62(8) | 66(10) |
| $\langle \text{Tr } U_{\text{plaquet}}/3 \rangle$ | 0.6224 | 0.5937 | 0.5751 |
| $\alpha_{\overline{\text{MS}}}(2/a)$ | 0.131 | 0.146 | 0.157 |
| $m_s^{\overline{\text{MS}}}(2 \text{ GeV})$ | 85(15) | 81(10) | 83(13) |
calculations of the strange quark mass using the kaon mass and the perturbative $Z_m$. Within our rather large errors, the DW results appear scale independent, so for now we take a weighted average. Our statistical errors are large due to our small data set. We attribute a 2 MeV error to the ambiguity in $q^*$ and tentatively assign a 15% error due to determining $a^{-1}$ from $f_\pi$. Quenching errors are not included at present. We add the systematic and statistical errors in quadrature to obtain the preliminary result for the $\overline{MS}$ quark mass: $m_s(2 \text{ GeV}) = 82(15)$ MeV.

We find that DW fermions are a viable method of studying the light quark masses. We have shown that perturbative corrections to the mass renormalization are under control, the lattice quark mass depends on $m$ and $M$ as expected, and scaling violations appear mild at this stage.

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\footnote{For other recent results see [19].}