Research on finite element analysis and modelling of bolted joint

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Abstract. The bolted joint is one of the most widely used fixed joints in mechanical structures, and has considerable impact on the dynamic characteristics of mechanical structures. Therefore, the correctness of bolt joint modeling is the key to the dynamic performance analysis and behavior prediction of the entire mechanical system. In this paper, the stiffness of the lapped beam structures was calculated according to the empirical formula for the dynamic stiffness of the joint surface, and an equivalent model of the bolt joint was established using the spring-damping element in ANSYS. Meanwhile, another equivalent model of the bolt joint was established by using the virtual material layer method. Finally, the modal test of the lapped beam structure with the bolted joint was carried out, and the simulation results obtained from the two theoretical modeling methods were compared with the modal test results. The results showed that the modeling method using the spring-damping element has higher modeling accuracy compared with the virtual material layer method, and the errors in the first six natural frequencies were within 2.2%, which verified the correctness and effectiveness of the spring-damping element modeling method, and provided a basis for the modeling of bolted joints in practical structures.

1. Introduction

The dynamic characteristics of the joint surface have a considerable influence on the dynamic characteristics of the mechanical structure[1], and so, research on the dynamic characteristics of the entire mechanical system is relevant for the study of its joint surface as well. At present, research on the joint interface can be broadly classified as micro-research and macro-research.

In micro-research, based on the elastic-plastic theory, a real rough contour was used to replace the sinusoidal surface, and the linear contact process of the asperities was studied by the elastic-plastic finite element method [2]. The influence of the moment on the equivalent plastic strain and stress of a rigid sphere was studied by the numerical method. The stress and equivalent plastic strain in the contact process of asperities were calculated, and an empirical formula was proposed to predict the contact area of the asperity[3]. A new multi-scale computational method was proposed to calculate the nonlinear dynamic response of bolt joints under different values of roughness by linking the surface roughness with contact pressure and contact stiffness [4]. The normal contact stiffness of elastic solids was approximated to the power function of the normal force. The normal contact stiffness and contact damping of two elastomers with fractal rough surfaces and different dimensions are calculated[5].
Macro-research mainly focuses on the study of joint modeling methods. At present, the two main methods are the virtual material layer method and spring-damping element method. An equivalent spring element was used to establish the numerical model of the joint surface under different bonding conditions, and the correctness of the numerical model was verified by a modal experiment [6]. Based on mode identification and considering contact stiffness as the design variable, a dynamic parameter identification model for iterative optimization was established, and the validity of the method was verified by experiment[7]. Considering the coupling relationship of the relative motion and degree of freedom among the substructures of the fixed joint, a dynamic model of the bolt fixed joint was established, and the correctness and validity of the dynamic model were verified by experiment [8]. Modeling techniques of the bolt joint that take into consideration the pre-tightening force and contact characteristics of bolts were studied. A solid bolt model, coupled bolt model, spider bolt model, and no-bolt model were examined. By comparing the theoretical and experimental results, the coupling modeling method, when compared with the other three modeling methods, had better validity and practicability [9]. The bolt joint was replaced by a layer of virtual material and the parameters of the virtual material layer were identified using fractal theory and experimental method, respectively. A dynamic model of the bolt joint was established, which provided a basis for more accurate dynamic modeling of the fixed joint in the numerical control machine tool [10, 11]. According to the principle of material strain energy equivalence, the parameters of the virtual material layer were computed, which provided a theoretical basis for more accurate modeling of the joint surface [12].

In summary, few scholars have made a systematic comparison between the two bolt joint modeling methods. Therefore, this paper mainly established the dynamic models of a bolted joint using spring-damping element and virtual material layer method from macro aspect, respectively, and carried out free modal analysis. Finally, the overlapped beam structure, which is analogous to the simulation specimen, was used for the modal test, and the simulation results of the two theoretical modeling methods are compared with the modal test results to determine the optimal modeling method for the bolt joint.

2. Dynamic modeling of bolt joint

The spring-damping element and virtual material layer method were used to model the dynamics of the overlapped beam structure with bolted joints. The stiffness of the joint affects the natural frequencies and modes of the entire structure. Meanwhile, damping influences only the vibration amplitude of the structure, and not the natural frequencies and modes. In this paper, only the stiffness of the bolt joint is considered. A dynamic finite element model of the overlapped beam structure with a bolted joint is established by two theoretical modeling methods, and modal analysis is carried out.

2.1. Dynamic modeling of spring-damping element

2.1.1. Dynamic stiffness of bolt joint. The dynamic parameters of the bolt joint include contact stiffness and contact damping. To establish the dynamic model of the bolt joint, the relevant dynamic parameters must first be identified. For the first time, a complete empirical formula for the dynamic stiffness of the joint surface of unit area can be expressed as [13]

\[ k_n = \alpha_n P_n^{\beta_n} \omega^{\gamma_n} X_n^{\eta_n} \]  
\[ k_t = \alpha_t P_n^{\beta_t} \omega^{\gamma_t} X_n^{\eta_t} \]  

where \( k_n, k_t \) are the normal and tangential dynamic stiffness of the joint surface in unit area, respectively, in MPa/µm; \( P_n \) is the normal surface pressure of the joint surface, the unit is MPa; \( \omega \) is the excitation frequency, the unit is Hz; \( \alpha_n, \beta_n, \gamma_n \) and \( \eta_n \) are the normal characteristic parameters of the joint surface, and \( \alpha_t, \beta_t, \gamma_t \) and \( \eta_t \) are the tangential characteristic parameters of the joint surface. The parameter values are all related to the processing method of the joint, material of the joint, bonding medium and surface roughness.
In 2000 and 2010, experiments on relevant dynamic characteristics were carried out for the joint surfaces of different bonding conditions [14, 15]. The relationship between the normal dynamic stiffness and the tangential dynamic stiffness on the joint surface of unit area was determined by data fitting. In this study, the two bonding materials are carbon steel (1045), the bonding surface is no-oil, surface roughness is 0.8, and normal stiffness and tangential stiffness of unit area was calculated by

\[ k_n = 3.262540P_n^{0.604} \quad (3) \]
\[ k_r = 0.268894P_n^{0.48} \quad (4) \]

2.1.2. Dynamic modelling. Because the joint has the complex characteristics of elasticity and damping, energy storage and energy consumption, it is equivalent to a dynamic model composed of multiple springs and dampers [16]. The combin14 element in the ANSYS cell library is selected to represent the bolt joint, and its structure diagram is shown in Figure 1. The combin14 element has two nodes with each node having three degrees of freedom, which has the functions of axial tension, compression, and torsion. The input parameters include the two parameters of spring stiffness and damping. Since the combin14 cannot have a zero-valued solution, the distance between the two contact surfaces is set to 0.001 mm by simulation and comprehensive consideration.

Figure 2 shows the dynamic model of the bolt joint. That is, four groups of combin14 elements are uniformly arranged around the bolt hole. Each group has three combin14 elements including one normal element and two tangential elements.

![Figure 1. Structure diagram of combin14.](image1)

![Figure 2. Dynamic model of bolt joint.](image2)

The contact surface pressure between bolt joints is not uniformly distributed over the entire joint surface, but unevenly around the bolt hole, as shown in Figure 3. The relationship between the equivalent pressure diameter \( D_A \) between the joint surfaces of bolts and the diameter \( d_W \) of the bolt head is \( D_A = (1.5-2) d_W \), taken as 1.7. The diameter \( d_W \) of the M12 bolt head is taken as 18 mm, and the equivalent spring elements are distributed along the circumference of the diameter \( D_A \).

The three dimensional model and structural sizes of the overlapped beam structure with the bolted joint are shown in Figure 4. The two plates are connected by M12 outer six angle bolt and the sizes are 380mm × 50mm × 8 mm. The material property parameters of the two plates are shown in Table 1.

| Materials  | Elastic Modulus | Poisson’s ratio | Density       |
|------------|-----------------|----------------|---------------|
| Steel–steel| 200 GPa         | 0.25           | 7800 kg/m³    |

Table 1. Material property parameter table.
There is an air medium between the two contact surfaces, surface roughness of both mating surfaces is 0.8, while that of the remaining surfaces are all 3.2, and the pre-tightening torque applied to the bolt is $T = 30 \text{ N•m}$. The relationship between the bolt pre-tightening force and the preload moment is obtained as

$$F = \frac{T}{0.2d}$$  \hspace{1cm} (5)

where $T$ is the pre-tightening moment of the bolt, in N•m; $F$ is the pre-tightening force of the bolt, in N; $d$ is the nominal diameter of the bolt in m.

The average contact surface pressure of the joint surface can be written as

$$P = \frac{F}{S}$$  \hspace{1cm} (6)

In the formula, $P$ is the joint surface pressure in MPa; $S$ is the joint area in mm$^2$.

The normal dynamic stiffness and tangential dynamic stiffness of the single bolt joint obtained from (3), (4), (5), and (6) are shown in Table 2.

| Equivalent stiffness          | Bolt joint | Single spring element |
|------------------------------|------------|-----------------------|
| Normal stiffness (N/m)       | 2.11e10    | 5.28e9                |
| Tangential stiffness (N/m)   | 1.42e9     | 3.55e8                |

2.1.3. Finite element model

For finite element modeling, the 20-noded element 186 is selected as the element type and the element length is set as 0.5. Since the spring element is to be established at the corresponding node, it is necessary to define a hard point before free meshing. The dynamic finite element model is established as shown in Figure 5.
2.2. Virtual material layer dynamics modeling
In the virtual material layer method, the asperities between the two surfaces are assumed equivalent to a layer of virtual material with a certain thickness. The virtual material layer and the component surface were coupled by a fixed connection [17]. The equivalent schematic diagram is shown in Figure 6.

![Figure 6. Equivalent schematic diagram of virtual material layer.](image)

2.2.1. Parameter calculation
The property parameters of the virtual material layer include three material parameters and one structural parameter, namely the elastic modulus, Poisson's ratio, density, and thickness, and four additional parameters are calculated as follows.

The relationships between the elastic modulus and normal dynamic stiffness, and between the shear modulus and tangential dynamic stiffness of the virtual material layer are given by

\[ E = k_n h \]
\[ G = k_\tau h \]  

(7)

(8)

where \( E \) and \( G \) are the elastic modulus and shear modulus of the virtual material, respectively, and the units are Pa; \( k_n, k_\tau \) are the normal and tangential dynamic stiffness per unit area of the surface of the bolt joint, Pa/m; \( h \) is the thickness of the virtual material layer in m, approximately 0.001 m [11]; in addition, the relationship between \( E, G \), and \( \mu \) is defined as

\[ G = \frac{E}{2(1+\mu)} \]  

(9)

From equations (3), (4), (5), (6), (7), and (8), \( E = 8.87 \) GPa, \( G = 0.595 \) GPa, the Poisson's ratio \( \mu = 6.45 \) of the virtual material obtained by the formula (9), while the Poisson's ratio of commonly used isotropic materials is generally in the range of -0.5. Therefore, the isotropic virtual material is remodelled as an orthotropic virtual material, the Z direction of the virtual material corresponds to the direction of the normal stiffness \( K_n \) of the joint surface, the tangential direction corresponds to the direction of the tangential stiffness \( K_\tau \) of the interface, and Poisson's ratio of the virtual material is taken to be the Poisson's ratio of the actual material. The other parameters used are the equivalent elastic modulus and equivalent shear modulus of the joint surface [18], while the equivalent elastic modulus and equivalent shear modulus of the interface are calculated by

\[ \frac{1}{E'} = \frac{1-\mu_1^2}{E_1} + \frac{1-\mu_2^2}{E_2} \]  

(10)
\[
\frac{1}{G'} = \frac{2 - \mu_1}{G_1} + \frac{2 - \mu_2}{G_2}
\]  

(11)

where \(E', G'\) are the equivalent elastic modulus and shear modulus of the joint surface, respectively; \(E_1, G_1, \mu_1\) are the elastic modulus, shear modulus and Poisson's ratio of structure 1, respectively; \(E_2, G_2, \mu_2\) is the elastic modulus, shear modulus and Poisson's ratio of the structure 2, respectively.

The density of the virtual material layer can be obtained by

\[
\rho = \frac{\rho_1 V_1 + \rho_2 V_2}{V_1 + V_2} = \frac{\rho_1 + \rho_2}{2}
\]

(12)

where \(\rho\) is the density of the virtual material layer, \(\rho_1, V_1\) are the density and volume of structure 1, respectively. \(\rho_2, V_2\) are the density and volume of structure 2, respectively.

The material property and structural parameters of the virtual material layer are obtained as shown in Table 3.

**Table 3. Parameters of orthotropic virtual material.**

| Elastic modulus (GPa) | Shear modulus (GPa) | Poisson's ratio | Density (kg/m³) | Thickness (mm) |
|-----------------------|---------------------|-----------------|-----------------|----------------|
| \(E_1\) | \(E_2\) | \(E_3\) | \(G_1\) | \(G_2\) | \(G_3\) | \(\mu_1\) | \(\mu_2\) | \(\mu_3\) | \(\rho\) | \(h\) |
| 10 | 7 | 107 | 8.87 | 0.595 | 0.595 | 23 | 0.25 | 0.25 | 0.25 | 7800 | 1 |

2.2.2. Finite element model

The material property parameters in Table 3 are in-built in ANSYS. The virtual material layer and the component surfaces are operated by GLUE. The dynamic model finite element is established, the local finite element model is shown in Figure 7, and modal analysis is carried out.

![Figure 7. Dynamic finite element model.](image)

3. Modal test

Depending on the plate size of the lapped beam structure in the previous section, two plates which are consistent with the simulation parameters are machined for the modal test. In this study, the hammering modal test of the overlap beam structure with bolt joint is carried out using the LMS Test Lab 14A vibration test and analysis system. The experimental device is composed of a hammer, three-way acceleration sensor, LMS Test Lab vibration test and analysis software, data acquisition system, structural specimen of overlapped beam, signal line, elastic rope, suspension, and the PC. The schematic diagram of the experimental setup is shown in Figure 8.
Before the modal test, the geometric model of the lapped beam structure is first created. Surface elements are used to model the overlapped beam structure. The two-dimensional model of the experimental test specimen of 24 nodes in total, along with the bolt joint is shown in Figure 9. The experimental vibration testing system is shown in Figure 10. A digital torque wrench controls the pre-tightening torque of the bolt. The piezoelectric hammer is used to exert exciting force to the lapped beam structure. The hammer head is made of nylon and equipped with a force sensor with a sensitivity of 2.25 mV/N. The first channel in the data acquisition system is set as the reference channel, that is, the force hammer channel, and the other channels are those of the three-way acceleration sensor.

**Figure 8.** Schematic diagram of experimental.

**Figure 9.** Experimental model with 24 nodes.

The experimental specimen is suspended with two elastic ropes to simulate its free boundary as shown in Figure 11 to avoid the influence of external objects on the results. The main methods of
vibration test include single-input/single-output, single-input/multi-output, multi-input/single-output, and multi-input/multi-output methods. This paper uses the single-input/multi-output method in which the hammer is used to knock at positions corresponding to the 24 nodes of the two-dimensional experimental model along the Z direction in turn, and the average value of three knocks per point is used to reduce random error, and improve the signal-to-noise ratio and reliability of experimental results. The vibration is picked up by two three-directional acceleration sensors arranged at 8 and 14 nodes respectively. The sensitivity is X: 105.30 mV/g, Y: 108.51 mV/g, Z: 100.92 mV/g, X: 99.67 mV/g, Y: 103.81 mV/g and Z: 99.7 mV/g, respectively.

Taking the simulation data as a reference, the first sixth order modal parameters of the overlapped beam structure are extracted in the frequency range of 60-1200Hz, and the modal parameters are abstracted as shown in Figure 12.

![Figure 12. Extraction of modal parameters](image)

The modal guarantee criterion is applied to verify the reliability of the experimental results. It is a mathematical criterion for checking the independence and consistency between the two-order modes, and its assurance criterion is shown as [19]

$$MAC_j = \frac{(\phi_i^T \phi_j)^2}{(\phi_i^T \phi_i)(\phi_j^T \phi_j)} \in [0,1]$$  \hspace{1cm} (13)

where $\phi_j$ is a reduced numerical eigenvector containing only the $j$th order mode of the measured degree of freedom, and $\phi_i$ is the eigenvectors of the experimentally derived $i$th order mode.

The histogram of the modal assurance criterion of the experimental results is shown in Figure 13. The result of the orthogonality of the mode vectors of each order with themselves are all 100%, and are represented in red. The other colors indicate the result of the orthogonality of the mode vectors of each order and the vectors representing the other modes, for which the theoretical value should be zero. The error arises mainly due to the nonlinearity of the structure, external noise interference of the measurement results, etc. However, the error is limited to less than 10%. Therefore, the experimental test results can be considered to satisfy the orthogonality, which also indicate the accuracy of the experimental results.
4. Comparison of results

Experimental verification includes validity verification and accuracy verification. Validity verification means that the theoretical mode shape is consistent with the experimental mode shape. Accuracy verification means that the relative error between the theoretical natural frequency and experimental natural frequency is as small as possible. Therefore, the experimental results are used to verify the two theoretical modes, and the modes are qualitatively compared, while the corresponding natural frequencies are compared quantitatively as shown in Table 4 and Table 5.

It can be seen from Table 5 that the experimental mode shape and the mode shape obtained by the two theoretical modeling methods are consistent with each other. From Table 4, it can be seen that the errors of the natural frequencies of the first six mode orders obtained by the virtual material layer modeling method and the experimental natural frequencies are within 3.9%. Only the single bolt joint is discussed in this work. The error in a multi-bolt joint may be greater, but the errors in the first six natural frequencies of the spring-damping element modeling method are less than 2.2% compared with the virtual material modeling method. Therefore, the modeling method that uses the spring-damping element is ideally suited and more accurate for bolted joints. Accordingly, when studying the dynamic characteristics of complex mechanical structures with bolted fixed joints such as machine tools, this method can be used to approximate or represent the bolted joints.

### Table 4. Comparison of natural frequencies.

| Mode order | 1         | 2         | 3         | 4         | 5         | 6         |
|------------|-----------|-----------|-----------|-----------|-----------|-----------|
| Test natural frequencies (Hz) | 81.871    | 231.692   | 435.260   | 699.377   | 736.605   | 1077.819  |
| Spring element natural frequencies (Hz) | 80.050    | 226.75    | 426.92    | 688.96    | 727.47    | 1059.2    |
| Relative errors (%) | 2.2       | 2.1       | 1.9       | 1.5       | 1.2       | 1.7       |
| Virtual material frequencies (Hz) | 84.326    | 226.81    | 448.00    | 720.35    | 728.59    | 1119.7    |
| Relative errors (%) | 3.0       | 2.1       | 2.9       | 3.0       | 1.1       | 3.9       |
Table 5. Comparison of mode shapes for experiment and simulations.

| Order | Test mode shape | Damping element mode shape | Virtual material mode shape |
|-------|----------------|---------------------------|-----------------------------|
| 1     | ![Test mode shape](image1) | ![Damping element mode shape](image2) | ![Virtual material mode shape](image3) |
| 2     | ![Test mode shape](image4) | ![Damping element mode shape](image5) | ![Virtual material mode shape](image6) |
| 3     | ![Test mode shape](image7) | ![Damping element mode shape](image8) | ![Virtual material mode shape](image9) |
| 4     | ![Test mode shape](image10) | ![Damping element mode shape](image11) | ![Virtual material mode shape](image12) |
| 5     | ![Test mode shape](image13) | ![Damping element mode shape](image14) | ![Virtual material mode shape](image15) |
| 6     | ![Test mode shape](image16) | ![Damping element mode shape](image17) | ![Virtual material mode shape](image18) |

5. Conclusions
The dynamic finite element model of overlapped beam structure with the bolted joint was established by using spring-damping element and virtual material layer respectively, and the finite element modal analysis was carried out. The theoretical results based on the two modeling methods are compared with the modal test results. The results show that the theoretical and experimental mode shapes obtained by the two modeling methods are similar, but the relative errors associated with the spring-damping element method are smaller than those associated with virtual material method. Therefore, the modeling method using the spring-damping element has higher modeling accuracy, which provides a basis for more accurate modeling of bolt joints in actual structures.

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