Light-matter interaction at the single-quantum level is of fundamental importance to modern quantum technologies. Strong interaction of a qubit with a single photon of an electromagnetic field mode is described by the cavity/circuit electrodynamics (QED) regime which is one of the most advanced platforms for quantum computing. The opposite regime of the waveguide QED, where qubits interact with a continuum of modes in an infinite one-dimensional space, is also at the focus of recent research revealing novel quantum phenomena. Despite the demonstration of several key features of waveguide QED, the transition from an experimentally realizable finite-size system to the theoretically assumed infinite device size is neither rigorously justified nor fully understood [1].

In this paper, we formulate a unifying theory which under a minimal set of standard approximations accounts for physical boundaries of a system in all parameter domains. Considering two qubits in a rectangular waveguide which naturally exhibits a low frequency cutoff we are able to account for infinite number of modes and obtain an accurate description of the waveguide transmission, a life-time of a qubit-photon bound state and the exchange interaction between two qubit-photon bounds states. For verification, we compare our theory to experimental data obtained for two superconducting qubits in a rectangular waveguide demonstrating how the infinite size limit of waveguide QED emerges in a finite-size system. Our theory can be straightforwardly extended to other waveguides such as the photonic crystal and coupled cavity arrays [2].

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