Leptonic Decay Constants of $D_s$ and $B_s$ Mesons at Finite Temperature

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Abstract
In the present work, $D_s$ and $B_s$ meson parameters are investigated in the framework of thermal QCD sum rules. The temperature dependences of the mass and the leptonic decay constants are investigated by using Borel transform sum rules and Hilbert moment sum rules. To increase sensitivity, the vacuum contributions are subtracted from thermal expressions and the temperature dependences of the leptonic decay constants and meson masses are studied.

1 Introduction
In order to explain the heavy ion collision results, some information about hadrons parameters at finite temperature and density is required. Some of the characteristic parameters at finite temperature and density are the masses and leptonic decay constants of hadrons. The investigation of these parameters requires non-perturbative approaches. One of these non-perturbative methods is the QCD sum rules [1], formulated by Shifman, Vainshtein and Zakharov.

The extension of the QCD sum rules method to finite temperatures has been made by Bochkarev and Shaposhnikov [2]. This extension is based on two basic assumptions that the OPE and notion of quark-hadron duality remain valid, but the vacuum condensates are replaced by their thermal expectation values. The thermal QCD sum rules method has been extensively used for studying thermal properties of both light and heavy hadrons as a reliable and well-establish method [3]-[8].

The investigation of heavy meson decay constants at zero temperature has been widely discussed in the literature [9]. The knowledge of these constants is needed in order to predict numerous heavy flavor electroweak transitions and to determine Standard Model parameters.
from the experimental data. Also leptonic decay constants play essential role in the analysis of CKM matrix, CP violation and the mixings \( B_d B_d, B_s B_s \). The first determination of these constants were made twenty years ago \[10\]-\[12\] and due to further theoretical and experimental progress, this problem was reconsidered taking into account the running quark masses and perturbative three-loop \( \alpha_s^2 \) corrections to the correlation function \[13\], \[14\]. At finite temperatures the nonperturbative nature of QCD vacuum induces temperature dependences of the leptonic decay constants and masses. Recently, first attempts have been made in order to calculate the leptonic decay constants of heavy mesons at finite temperature in the framework of thermal QCD sum rules \[15\].

In the present paper, we investigate the temperature behavior of the masses and leptonic decay constants of \( D_s \) and \( B_s \) mesons using QCD sum rules. Taking into account perturbative two-loop order \( \alpha_s \) corrections to the correlation function and nonperturbative corrections up to the dimension six condensates \[16\] we investigated the temperature dependences of masses and leptonic decay constants using Borel transform sum rules and Hilbert moment sum rules. For increased sensitivity, we subtract the vacuum contributions from thermal expressions and study the temperature dependences of the leptonic decay constants and meson masses.

## 2 Pseudoscalar thermal correlator at finite temperature

We start with pseudoscalar two-point thermal correlator

\[
\psi_5(q^2) = i \int d^4x e^{iq \cdot x} \langle T(J(x)J^+(0))\rangle,
\]

where \( J(x) = (m_Q + m_s) : \bar{s}(x) i \gamma_5 Q(x) : \) is the heavy-light quark current and has the quantum numbers of the \( D_s \) and \( B_s \) mesons, \( m_Q \) and \( m_s \) are heavy and strange quark masses respectively. We shall not neglect \( s \) quark mass throughout this work. Thermal average of any operator \( O \) is defined in the following way

\[
\langle O \rangle = Tre^{-\beta H} O / Tr e^{-\beta H},
\]

where \( H \) is the QCD Hamiltonian, \( \beta = 1/T \) stands for the inverse of the temperature \( T \) and traces are over any complete set of states. Up to a subtraction polynomial, which depends on the large \( q^2 \) behavior, \( \psi_5(q^2) \) satisfies the following dispersion relation \[11\], \[9\]

\[
\psi_5(Q^2) = \int ds \frac{\rho(s)}{s + Q^2} + \text{subtractions},
\]
where $Q^2 = -q^2$ is Euclidean momentum, $\rho(s) = \frac{1}{\pi}Im\psi_5(s)$ is spectral density and in perturbation theory at zero temperature in the leading order has the following form [16]:

$$\rho(s) = \frac{3(m_s + m_Q)^2}{8\pi^2s}v(s)q^2(s)\left[1 + \frac{4\alpha_s}{3\pi}f(x)\right],$$

(4)

where $x = m_Q^2/s$, $\alpha_s = \alpha_s(m_Q^2)$ and

$$q(s) = s - (m_Q - m_s)^2, \quad v(s) = (1 - 4m_s m_Q/q(s))^{1/2},$$

(5)

$$f(x) = \frac{9}{4} + 2Li_2(x) + \ln x \ln(1 - x) - \frac{3}{2} \ln \left(\frac{1}{x} - 1\right) - \ln(1 - x) + x \ln \frac{1}{x} - \frac{x}{1 - x} \ln x. \quad (6)$$

The subtraction terms are removed by using the Borel transformation or Hilbert moment methods. Therefore we will omit these terms. The thermal propagator contains on-shell soft quarks which do not exist in the confined phase. Therefore, in obtaining the OPE of the thermal correlator (1), vacuum propagators must be used [4]. The non-perturbative contributions at zero temperature to the correlator has the following form

$$\psi_{5, np}(Q^2) = -m_Q \lambda \langle 0 | \bar{s}s|0 \rangle \left[1 + \frac{1}{2} \varepsilon (3 - \lambda) - \lambda \varepsilon^2 (1 - \lambda) - \frac{1}{2} \varepsilon^3 (1 + \lambda - 4\lambda^2 + 2\lambda^3)\right]$$

$$+ \frac{1}{12\pi} \lambda \langle 0 | \alpha_s G^2 |0 \rangle \left[1 + 3 \varepsilon \left(1 - \frac{8}{3} \lambda + 2\lambda^2 - 2\lambda(1 - \lambda) \ln(\varepsilon \lambda)\right)\right]$$

$$- \frac{m_Q^2}{2m_Q} \langle 0 | \bar{s}s|0 \rangle \lambda^2 (1 - \lambda) (1 + \varepsilon (2 - \lambda)) - \frac{8\pi \rho}{27m_Q^2} \alpha_s \langle 0 | \bar{s}s|0 \rangle^2 \lambda^2 (2 - \lambda - \lambda^2),$$

(7)

where $\lambda = m_Q^2/(Q^2 + m_Q^2)$ and $\varepsilon = m_s/m_Q$. Also, for the mixed condensate the parameterization:

$$g \langle 0 | \bar{q} \sigma_{\mu\nu} \frac{\lambda_a}{2} G^{\mu\nu} q|0 \rangle = M_0^2 \langle 0 | \bar{q}q|0 \rangle$$

(8)

is used. It is assumed, that the expansion (7) also remains valid at finite temperatures, but the vacuum condensates must be replaced by their thermal expectation values [2]. For the light quark condensate at finite temperature we use the results of [17], [18] obtained in chiral perturbation theory. Temperature dependence of quark condensate in a good approximation can be written as

$$\langle \bar{q}q \rangle = \langle 0 | \bar{q}q|0 \rangle \left[1 - 0.4 \left(\frac{T}{T_c}\right)^4 - 0.6 \left(\frac{T}{T_c}\right)^8\right],$$

(9)

where $T_c = 160 \text{ MeV}$ is the critical temperature. The low temperature expansion of the gluon condensate is proportional to the trace of the energy momentum tensor [19] and can be approximated by [15]

$$\langle \alpha_s G^2 \rangle = \langle 0 | \alpha_s G^2 |0 \rangle \left[1 - \left(\frac{T}{T_c}\right)^8\right].$$

(10)
The value of the QCD scale $\Lambda$ is extracted from the value of $\alpha_s(M_Z) = 0.1176$ [20]. Equating OPE and hadron representations of the correlation function and using quark-hadron duality the sum rules is obtained as

$$\frac{f_H^2 m_H^4}{Q^2 + m_H^2} = \int_{(m_Q + m_s)^2}^{s_0} ds \frac{\rho(s)}{s + Q^2} + \psi_{5,np}(Q^2),$$

where $f_H$ is the leptonic decay constant and is defined by the matrix element of the axial-vector current between the corresponding meson and the vacuum as:

$$\langle 0 | \bar{s} \gamma_\mu \gamma_5 Q | H(q) \rangle = if_H q_\mu,$$

where $Q = c, b$ and $H = D_s, B_s$ in the same normalization as $f_\pi = 130.56$ MeV. In thermal field theories the parameters $m_H$ and $f_H$ must be replaced by their temperature dependent values. The continuum threshold $s_0$ also depends on temperature; to a very good approximation it scales universally as the quark condensate [15]

$$s_0(T) = s_0 \frac{\langle \bar{q} q \rangle}{\langle 0 | \bar{q} q | 0 \rangle} \left[ 1 - \frac{(m_Q + m_s)^2}{s_0} \right] + (m_Q + m_s)^2,$$

where in the right hand side $s_0$ is hadronic threshold at zero temperature: $s_0 \equiv s_0(0)$. Analysis shows that thermal non-perturbative correlator is basically driven by the quark condensates.

### 3 Numerical analysis of masses and leptonic decay constants

In this section we present our results for the temperature dependence of $D_s$ and $B_s$ meson masses and leptonic decay constants. Performing Borel transformation with respect to $Q^2_0$ on both sides of equation (11) and differentiating with respect to $1/M^2$, we obtain:

$$m_H^2(T) = \frac{f_H^2 m_H^6 \exp(-m_H^2/M^2) + \overline{B}(T)}{f_H^2 \exp(-m_H^2/M^2) + \overline{A}(T)},$$

$$f_H^2(T) = \frac{1}{m_H^2(T)} \left[ \overline{A}(T) + f_H^2 m_H^4 \exp(-m_H^2/M^2) \right] \exp \left[ \frac{m_H^2(T)}{M^2} \right],$$

where the bar on the operators means subtractions of their vacuum expectation values from thermal expectation values; for example $\overline{\psi_{5,np}}(M^2, T) = \psi_{5,np}(M^2, T) - \psi_{5,np}(M^2, T = 0)$. Here

$$\overline{A}(T) = \int_{s_0(T)}^{s_0} ds \rho(s) \exp \left( -\frac{s}{M^2} \right) + \overline{\psi_{5,np}}(M^2, T),$$

(16)
\[
\overline{\psi}_{5, np}(M^2, T) = -m_Q^2 \langle 0|\bar{s}s|0 \rangle e^{-\beta} [1 + \frac{3}{2} \beta - 3 \beta \varepsilon - \beta \varepsilon^2 (1 - \frac{1}{2} \beta) - \frac{1}{2} \varepsilon^3 (1 + \beta - 2 \beta^2 + \frac{3}{2} \beta^3)] \\
+ \frac{1}{12 \pi} m_Q \langle 0|\alpha_s G^2|0 \rangle m_Q^2 e^{-\beta} [1 + 3 \beta (1 - \frac{8}{3} \beta + \beta^2 - 2 \beta (\ln(\beta \varepsilon) + \gamma - 1) + \beta^2 (\ln(\beta \varepsilon) + \gamma - \frac{3}{2}))] \\
- \frac{1}{2} m_Q^2 \beta m_Q \langle 0|\bar{s}s|0 \rangle e^{-\beta} [1 - \frac{1}{2} \beta + 2 \varepsilon (1 - \frac{3}{4} \beta (1 - \frac{1}{9} \beta))] - \frac{4}{81} \pi \rho \alpha_s \langle 0|\bar{s}s|0 \rangle^2 \beta e^{-\beta} \\
\times (12 - 3 \beta - \beta^2),
\]

where \(\gamma\) is the Euler constant, \(\beta = m_Q^2/M^2\) and \(\overline{T}(T) = -m_Q^2 \frac{d\overline{A}(T)}{dT}\). To investigate the meson parameters at finite temperature we also use Hilbert moments methods, which eliminate the subtraction terms. Calculating Hilbert moments at \(Q^2 = -q^2 = 0\) and using first two moments we obtain

\[
m_H^2(T) = \frac{F(T) - \int_{s_0(T)}^{s_0} ds \rho(s) s^{-3} + \frac{f_H^2}{m_H^2}}{G(T) - \int_{s_0(T)}^{s_0} ds \rho(s) s^{-4} + \frac{f_H^2}{m_H^4}},
\]

\[
f_H^2(T) = m_H^2(T)[F(T) - \int_{s_0(T)}^{s_0} ds \rho(s) s^{-3} + \frac{f_H^2}{m_H^2}],
\]

where \(F(T)\) and \(G(T)\) functions are expressed by thermal expectation values of condensates

\[
F(T) = -\frac{1}{m_Q^2} \langle 0|\bar{s}s|0 \rangle (1 + 3 \varepsilon^2) + \frac{1}{12 \pi m_Q^4} \langle 0|\alpha_s G^2|0 \rangle [1 + \varepsilon (21 + 18 \ln \varepsilon)] \\
+ \frac{1}{2 m_Q^2} M_6^2 (0|\bar{s}s|0) (3 + 2 \varepsilon) + \frac{80}{27 m_Q^4} \pi \rho \alpha_s \langle 0|\bar{s}s|0 \rangle^2,
\]

\[
G(T) = -\frac{1}{m_Q^2} \langle 0|\bar{s}s|0 \rangle (1 - \frac{1}{2} \varepsilon + 6 \varepsilon^2 - \frac{5}{2} \varepsilon^3) + \frac{1}{12 \pi m_Q^4} \langle 0|\alpha_s G^2|0 \rangle [1 + \varepsilon (52 + 36 \ln \varepsilon)] \\
+ \frac{1}{m_Q^2} M_6^2 (0|\bar{s}s|0) (3 + \varepsilon) + \frac{176}{27 m_Q^4} \pi \rho \alpha_s \langle 0|\bar{s}s|0 \rangle^2.
\]

For the numerical evolution of the above sum rule, the values of the QCD parameters used are shown in Table 1. The criterion we adopt here is to fix \(s_0\) in such a way as to reproduce the zero temperature values of meson masses and leptonic decay constants. For \(D_s\) meson \(s_0\) is 6 GeV\(^2\) and 8 GeV\(^2\), for \(B_s\) meson \(s_0\) is 34 GeV\(^2\) and 35 GeV\(^2\) in Borel and Hilbert moment sum rules methods, respectively. The temperature dependences of the \(D_s\) and \(B_s\) meson masses and leptonic decay constants obtained using the Borel and Hilbert moment methods are shown in Fig. 1 and Fig. 2, respectively. The results for leptonic decay constants are shown in Fig. 3 and Fig. 4. As seen in figures, \(f_{D_s}\) and \(f_{B_s}\) decrease with increasing temperature and vanish approximately at critical temperature \(T_c = 160\) MeV. This may be interpreted as a signal for deconfinement and agrees with light and heavy-light mesons investigations [15, 21]. Numerical analysis shows that the temperature dependence of \(f_{D_s}\) is independent of \(M^2\), when \(M^2\) changes between 3 GeV\(^2\) and 4 GeV\(^2\) and \(f_{B_s}\) is independent of the Borel parameter, when \(M^2\) changes.
Table 1: QCD input parameters used in the analysis.

| Parameters | References |
|------------|------------|
| $m_{D_s} = 1968$ MeV | 20 |
| $m_{B_s} = 5366$ MeV | 20 |
| $m_s = 120$ MeV | 20 |
| $m_c = 1.47$ GeV | 13, 20 |
| $m_b = 4.4$ GeV | 13, 20 |
| $f_{D_s} = 235$ MeV | 13, 20 |
| $f_{B_s} = 240$ MeV | 13, 20 |
| $\rho = 4$ | 15, 16 |
| $\langle 0|\bar{q}q|0\rangle = -0.014$ GeV$^3$ | 1 |
| $\langle 0|\frac{1}{x}\alpha_s G^2|0\rangle = 0.012$ GeV$^4$ | 1 |
| $\alpha_s \langle 0|\bar{q}q|0\rangle^2 = 5.8 \times 10^{-4}$ GeV$^6$ | 13 |
| $M_0^2 = 0.8$ GeV$^2$ | 13 |
| $\langle 0|\bar{s}s|0\rangle = 0.8\langle 0|\bar{q}q|0\rangle$ | 13 |

between 16 GeV$^2$ and 24 GeV$^2$. Obtained results can be used for the interpretation of heavy ion collision experiments. It is also essential to compare these results with other model calculations. We believe these studies to be of great importance for understanding phenomenological and theoretical aspects of thermal QCD.

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Figure 1: Temperature dependence of $D_s$ meson mass in Hilbert and Borel sum rules methods. Here Borel parameter is $M^2 = 3 \, GeV^2$, hadronic threshold $s_0 = 6 \, GeV^2$ for Borel and $s_0 = 8 \, GeV^2$ for Hilbert moment sum rules methods.

Figure 2: Temperature dependence of $B_s$ meson mass in Hilbert and Borel sum rules methods. Here Borel parameter is $M^2 = 20 \, GeV^2$, hadronic threshold $s_0 = 34 \, GeV^2$ for Borel and $s_0 = 35 \, GeV^2$ for Hilbert moment sum rules methods.
Figure 3: Temperature dependence of $f_{D_s}$ in Hilbert and Borel sum rules methods. Here Borel parameter is $M^2 = 3 \text{ GeV}^2$, hadronic threshold $s_0 = 6 \text{ GeV}^2$ for Borel and $s_0 = 8 \text{ GeV}^2$ for Hilbert moment sum rules methods.

Figure 4: Temperature dependence of $f_{B_s}$ in Hilbert and Borel sum rules methods. Here Borel parameter is $M^2 = 20 \text{ GeV}^2$, hadronic threshold $s_0 = 34 \text{ GeV}^2$ for Borel and $s_0 = 35 \text{ GeV}^2$ for Hilbert moment sum rules methods.