PHYSICS PROSPECTS AT FUTURE $e^+e^-$ COLLIDERS*

Abdelhak Djouadi

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1 Introduction

1.1 The Physics Motivations

The $e^+e^-$ colliders which have operated in the past twenty years, from SPEAR to LEP1 and SLC, have been very successful in searching for the fundamental constituents of matter and in exploring their interactions. In particular, the recent high–precision studies of the properties of the $Z$ boson at LEP1 and SLC, have laid a very solid base for the Standard Model (SM) of the electroweak and strong interactions. However, within the SM, many problems remain to be solved and despite of the spectacular agreement with the present experimental data, extensions of the model are anticipated.

- The top quark, recently discovered at the Tevatron, has a mass of $\sim 180$ GeV. This high value renders a thorough analysis of the properties of this particle mandatory: since the mass is close to the Fermi scale, the top quark might hold important clues to the electroweak symmetry breaking mechanism.

- The strength and the form of the electroweak $W$ and $Z$ boson self–couplings are determined by the non–abelian gauge symmetry of the SM. However, deviations from these values are expected in more general scenarios. These couplings can be measured at existing colliders only with large errors, and the precision must be improved by one or two orders of magnitude to unambiguously establish the nature of these particles.

- The Higgs particle, and the electroweak symmetry breaking mechanism, are the most important ingredients of the SM. The search for the Higgs particle will be a major task and it is very important that no way of escape is left for this particle. Once discovered, its fundamental properties, spin–parity quantum numbers and couplings to the other particles, must be precisely determined.

- Supersymmetry (SUSY) is the most attractive extension of the SM. It not only stabilizes the huge hierarchy between the weak and the GUT scales against radiative corrections, but also if SUSY is broken at a sufficiently high scale, it allows us to understand the origin of the hierarchy in terms of radiative gauge symmetry breaking. Moreover, SUSY models offer a natural solution to the dark matter problem and allow for a consistent unification of the all known gauge couplings. Many new particles are predicted in these theories; the search for these states and the study of their properties will be one of the major goals of present and future colliders.

- Many other possibilities for extensions of the SM can be anticipated. For instance, the SM group $\text{SU}(3)\times\text{SU}(2)\times\text{U}(1)$ is expected to be embedded in a unifying group at a
high–energy scale. In this case, extra gauge bosons with masses not too far from the Fermi scale might exist. In these unifying groups, many new matter particles are also predicted and some of them might be relatively light. These new gauge and matter particles have also to be searched for at present and future colliders.

While strongly interacting particles, as well as new gauge bosons, can be easily searched for at hadron colliders, $e^+e^-$ colliders provide unique opportunities to search for non–colored particles such as Higgs bosons, SUSY partners of leptons and electroweak/Higgs bosons and, due to a clean experimental environment, to make detailed studies of their properties. Even for strongly interacting (s)particles and new gauge bosons, the $e^+e^-$ colliders can play an important role in pinning down their properties.

In this talk, I will discuss the potential of future high–energy $e^+e^-$ linear colliders to address these issues, after summarizing some aspects of the $e^+e^-$ reaction and the future machines. The talk is based on the work done at the various workshops on $e^+e^-$ colliders [1–6] which took place in the last years to which I refer for original references.

1.2 The $e^+e^-$ Collision

The $e^+e^-$ collision is a very simple reaction, with a well defined initial state and rather simple topologies in the final state. It has a favorable signal to background ratio, leading to a very clean experimental environment which allows to easily search for new phenomena and to perform very high–precision studies as has been shown at PEP/PETRA and more recently at LEP/SLC.

The physical processes are mainly mediated by $s$–channel photon and $Z$ boson exchanges with cross sections which scale like $1/s$, and $t$–channel gauge boson or electron/neutrino exchange, with cross sections which rise like $\log(s)$. The $s$–channel exchange is the most interesting process: it is democratic in the sense that it gives approximately the same rates for weakly and strongly interacting (matter) particles and for the production of known and new particles.

However the rates are low at high energies, and one needs to increase the luminosity to compensate for the $1/s$ drop of the interesting cross sections. At $\sqrt{s} \sim 1$ TeV, one needs a luminosity of $\mathcal{L} \sim 10^{34}$ cm$^{-2}$s$^{-1}$, or equivalently an integrated luminosity of $\int \mathcal{L} \sim 100$ fb$^{-1}$ per year, to produce 10.000 muon pairs as at PEP and PETRA. Such a luminosity is necessary to allow for thorough data analyses, including cuts on the event samples and allowing for acceptance losses in the detectors. At higher energies, the luminosity should be scaled up as $s$ to generate the same number of events.
1.3 Aspects of the Colliders

Various reference designs of future high–energy $e^+e^−$ colliders are being studied at DESY, SLAC, KEK, Novosibirsk and CERN. In the following, let me briefly list a few points about these future colliders [1–3]:

- Because of synchrotron radiation which rises as the fourth power of the c.m. energy in circular machines, $e^+e^−$ colliders beyond LEP2 must be linear machines, a type of accelerator which has been pioneered by the SLC.

- These machines will be probably realized in two steps. In a first phase, they will operate in the energy range between 300 and 500 GeV with a yearly luminosity of $\mathcal{L} \sim 50$ fb$^{-1}$, and in a second phase with a center of mass energy up to $\sqrt{s} \sim 2$ TeV and an integrated luminosity of $\mathcal{L} \sim 200$ fb$^{-1}$. The c.m. energy of the colliders can be adjusted in order to make detailed studies: for instance to scan the $t\bar{t}$ threshold or to maximize the cross section for Higgs boson production.

- The requirement of a high–luminosity is achieved by squeezing the electrons and positrons into bunches of very small transverse size, leading to beamstrahlung which results into beam energy loss and the smearing of the initially sharp $e^+e^−$ c.m. energy. Since the precise knowledge of the initial state is very important for precision studies, beamstrahlung should be reduced to a very low level, as it is already the case in narrow beam designs.

- The longitudinal polarization of the electron (and possibly positron) beam should be easy to obtain as it has already been shown at the SLC. The longitudinal polarization might be very important when it comes to make very precise measurements of the properties of the top quark and the $W/Z$ bosons for instance.

- Last but not least, the $e^+e^−$ collider can run in three additional modes. First, one just needs to replace the positron bunches by electron bunches to have an $e^−e^−$ collider. Then, by illuminating the electron and positron bunches by laser photons, one can convert the $e^+e^−$ collider into an $e\gamma$ or $\gamma\gamma$ collider with a comparable total c.m. energy and luminosity as the initial $e^+e^−$ collider. These modes will be very useful to address problems such as the Higgs–photons coupling and the properties of the $W$ bosons.

I the rest of this talk, I will nevertheless mainly focus on the $e^+e^−$ option of the collider and discuss only the physics potential at the first phase with a center of mass energy around $\sqrt{s} \sim 500$ GeV. Occasionally, I will comment on the benefits of raising the energy or changing the option.
2 SM Physics

2.1 The Top Quark and QCD

The first—and most obvious—thing to do at a 500 GeV $e^+e^-$ collider is the study of the properties of the top quark. Since the top quark is very massive, $m_t \sim 180$ GeV, it might play a very important role in the understanding of the mechanism of electroweak symmetry breaking and the properties of this particle should be studied in detail.

In $e^+e^-$ collisions, top quarks are mainly produced through $s$–channel $\gamma$ and $Z$ boson exchange, $e^+e^- \rightarrow \gamma, Z \rightarrow t\bar{t}$. The maximum cross section is reached at about 50 GeV above threshold, i.e. at $\sqrt{s} \sim 400$ GeV, Fig.1. A rate of 40,000 top quark pairs can be produced with a luminosity of $\mathcal{L} \simeq 50$ fb$^{-1}$, a sample comparable to the number of top quark which can be used for physics at the LHC, after filtering out the background. The top quarks will mainly decay into $b$ quarks and $W$ bosons, $t \rightarrow bW^+$, and in the clean environment of $e^+e^-$ colliders, the top quarks can be easily isolated.

The very sharp rise of the $t\bar{t}$ production cross near threshold, due to the well–known Coulomb singularity, would allow a very high–precision measurement of the top quark mass. Conservative estimates of the theoretical and experimental uncertainties lead to an error of about 200 MeV on $m_t$, more than an order of magnitude smaller than what is expected to be the case at the LHC. This precision will allow the first stringent constraint on the mass of the Higgs boson if this particle has not been found.

Because of its large mass, the top quark will decay before it forms $t\bar{t}$ bound states, and the $t$ and $\bar{t}$ will separate at distances less than $10^{-2}$ fm. However, the $e^+e^- \rightarrow t\bar{t}$ cross section near the production threshold is still sensitive to the quark–antiquark binding potential. Therefore, the threshold region will be the best place to study QCD: the binding is strong, but it is completely determined by perturbation theory. From the QCD potential, the strong coupling constant $\alpha_s$ could be determined with an error of less than 0.4%, and this would be the most precise individual measurement of this coupling.

The measurement of $m_t$ and $\alpha_s$ from the excitation curve will be however correlated. To decorrelate the two measurements, one can determine the momentum distribution of the top quark, the average of which is proportional to the product of the two quantities, $<p> \sim m_t\alpha_s$. The top quark width [as well as $m_t$ and $\alpha_s$] can be determined from the forward–backward asymmetry. Indeed, the interference between S and P–wave contributions near threshold is rather strong, leading to a FB asymmetry of about 10%. For fixed $m_t$ and $\alpha_s$, this allows the determination of $\Gamma_t$ with a precision of less than 20%. Note
that the QCD coupling constant can also be determined independently by measuring the
annihilation cross section into jets, allowing for a stringent test of asymptotic freedom
down to distances of $\mathcal{O}(10^{-3})$ fm.

Because of the short lifetime of the top quark, its polarization is not lost during the
decay making possible a general helicity analysis of the angular distribution of the
process $e^+e^- \rightarrow tt \rightarrow bbWW \rightarrow bb + 4$–fermions. This will allow the determination of the
electroweak $tt$ production and the weak ($tb$) decay currents. The anomalous magnetic
moment, the electric dipole moment [a sign of CP violation in the $t$ sector] and the chirality of the ($tb$) current [left–handed in the SM] can be determined at a high level. This allows
to constrain many extensions of the SM, such as multi–Higgs doublet models, models of
dynamical symmetry breaking and compositeness, which might manifest themselves first
in the top sector due the large value of $m_t$.

Finally, it might well be that the top quark does not decay only into $bW$ final states. Due
to the large value of $m_t - m_b$, there could be enough phase space for exotic decay
modes to occur. For instance, in supersymmetric extensions of the SM, the top quark
could decay into a $b$ quark and a charged Higgs boson, $t \rightarrow bH^+$, or into the lightest
neutralino $\chi^0_1$ [which is the lightest supersymmetric particle] and the scalar top squark $\tilde{t}$
[which in many scenarios is lighter than the $t$ and all the other squarks], $t \rightarrow \tilde{t}\chi^0_1$. Even
if these exotic decay modes are allowed by phase space, they are expected to be rare,
with branching ratios of the order of a few percent. This rates would be frequent enough,
however, to be analysed thoroughly at $e^+e^-$ colliders.

2.2 $W$ and $Z$ Bosons

The gauge symmetries of the SM completely determine the form and the strength of
the self-interactions of the electroweak bosons. Deviations of the triple $WW\gamma$, $WWZ$
and quartic $W^4, W^2Z^2, W^2\gamma^2$ couplings, as well as new couplings $Z^3, Z^2\gamma^2$ in addition
to the usual SM couplings, could be expected in more general scenarios. For instance
in composite models and models where the $W/Z$ bosons are generated dynamically or
interact strongly with each other, these couplings can be altered.

The couplings $WW\gamma$ and $WWZ$ are in general described by seven parameters each.
Assuming C, P and T invariance in the bosonic sector, the number of parameters is reduced
to three: $g_{\gamma,Z}, \kappa_{\gamma,Z}$ and $\lambda_{\gamma,Z}$. $g_{\gamma,Z}$ are the electric and $Z$ charges of the $W$ bosons and the
parameters $\kappa_{\gamma,Z}$ and $\lambda_{\gamma,Z}$ are related to the corresponding magnetic dipole moments and
electric quadripole moments of the $W$ bosons. Symmetries of the underlying dynamical
interactions may further constrain these couplings. The gauge symmetries of the SM, demand that $g_\gamma = 1, g_Z = -\tan^2 \theta_W, \kappa_{\gamma,Z} = 1$ and $\lambda_{\gamma,Z} = 0$ at tree–level.

The magnetic dipole and electric quadripole moments can be measured directly in the production of electroweak bosons at hadron and $e^+e^-$ colliders. Modifications of the self–couplings vertices destroy the SM unitarity cancellations between interfering boson and fermion exchange amplitudes for longitudinal vector boson production. As a result, small deviations of the moments from their SM values are magnified by a coefficient $\beta^2 W s/M^2_W$ where $\beta_W$ is the velocity of the $W$ bosons. Because the constraints also scale with the accumulated luminosity, a 500 GeV $e^+e^-$ collider with a luminosity of $\mathcal{L} \sim 50 \text{ fb}^{-1}$ will have a sensitivity which is two orders of magnitude larger than LEP2.

The main reaction to be exploited at high–energy $e^+e^-$ colliders is $W$ pair production $e^+e^- \rightarrow W^+W^-$. A large number, $\mathcal{O}(10^5)$, of $W$ pairs will be produced in a clean environment for the energy and luminosity specified above, so that a very high reconstruction efficiency can be achieved. Additional and complementary information can be obtained from $WW$ fusion to $Z$ bosons, $e^+e^- \rightarrow \nu\bar{\nu}WW \rightarrow \nu\bar{\nu}Z$, and in particular if laser induced $\gamma$ beams are available, from $\gamma e^- \rightarrow \nu W^-$ and $\gamma\gamma \rightarrow W^+W^-$. These processes are separately sensitive to $\gamma$ and $Z$ couplings, and can help to disentangle $\kappa_{\gamma}, \lambda_{\gamma}$ from $\kappa_{Z}, \lambda_{Z}$.

From the combined analysis of the $W$ pair production cross section and the angular distribution of the $W$ decay products [which allow to separate the various $W$ helicity components], very tight constraints on the anomalous couplings can be obtained. If all $\kappa$’s and $\lambda$’s are allowed to vary freely, they can be probed to an accuracy of $|\kappa_{\gamma} - 1|, |\kappa_{Z} - 1|, |\lambda_{\gamma}|, |\lambda_{Z}| \lesssim 0.01$. More stringent bounds can be obtained if the anomalous couplings are constrained by additional symmetry requirements. To disentangle $\gamma$ from $Z$ couplings in this reaction, longitudinal polarization is very important.

The analysis of $W\gamma$ final states at the Tevatron leads to bounds of $\mathcal{O}(1)$ on the anomalous photonic couplings. At the LHC, bounds of order $|\lambda_{\gamma}| \lesssim 0.02$ and $|\kappa_{\gamma} - 1| \lesssim 0.1$ are expected in this process, while information on $\kappa_{Z}$ and $\lambda_{Z}$ is more difficult to extract. The bounds on $|\kappa_{\gamma} - 1|$ and $|\lambda_{\gamma}|$ that can be reached at an $e^+e^-$ collider operating at $\sqrt{s} = 500$ GeV with a luminosity of 10 fb$^{-1}$ are compared with the expected values at LEP2 [with a very high luminosity $\int \mathcal{L} = 1.3 \text{ fb}^{-1}$] and LHC [which has the same reach as the late SSC in the high–luminosity option $\int \mathcal{L} = 100 \text{ fb}^{-1}$]; Fig. 2. $e^+e^-$ colliders are more powerful by not only probing $\kappa_{\gamma}$ and $\lambda_{\gamma}$ to a higher accuracy, but also in providing stringent constraints on $\kappa_{Z}$ and $\gamma_{Z}$. Note that the constraints become more stringent at higher energies or higher luminosities.

Various quartic vector boson couplings can also be probed at $e^+e^-$ colliders: the
$W^4, W^2 Z^2$ and $Z^4$ couplings in the reaction $e^+e^- \to WWZ$ and $ZZZ$, and the $W^2 \gamma^2$ and $\gamma^2 Z^2$ couplings in the reaction $e^+e^- \to WW\gamma$ and $e^+e^- \to ZZ\gamma$. The new physics effective energy scale which can be probed at a 500 GeV collider is of the order of a TeV.

Finally, $WW$ scattering cannot be studied at energies around 500 GeV. This is a very important process to study if light Higgs particles do not exist and $W$ bosons become strongly interacting at high energies. Raising the c.m. energy of the collider to $\sqrt{s} \sim 1.5$ to 2 TeV, allows to study in detail this strongly interacting scenario.

### 2.3 The Higgs Particle

a) The Higgs in the SM

The most important mission of a future high–energy collider will be the search for scalar Higgs particles and the exploration of the electroweak symmetry breaking mechanism. In the SM, one doublet of complex scalar fields is needed to spontaneously break the SU(2)×U(1) symmetry. Among the four initial degrees of freedom, there Goldstones will be absorbed by the $W^\pm$ and $Z$ bosons to get their masses, and the remaining degree of freedom will correspond to a physical scalar particle, the Higgs boson.

Since the couplings of the Higgs boson to fermions and gauge bosons are proportional to the masses of these particles, the only unknown parameter in the SM is the Higgs boson mass, $M_H$. It is a free parameter and the only thing we know about it is that it should be larger than $\sim 65$ GeV, from the negative searches at LEP1, and that it is probably smaller than 1 TeV, from fits of the high–precision LEP1 data. [For $M_H \gtrsim 1$ TeV, the gauge bosons would interact strongly to insure unitarity in the scattering of the electroweak gauge bosons; the residual final state interaction can be studied at c.m. energies beyond 1 TeV].

However, there is a theoretical argument which indicates that the Higgs boson might be light, $M_H \lesssim 200$ GeV. Indeed, the quartic Higgs coupling is proportional to $M_H^2$ and since the scalar sector of the SM is not an asymptotically free theory, the coupling will grow with the energy until it reaches the Landau pole, where the theory does not make sense anymore. If the cut–off $\Lambda$ where new phenomena should occur is of $O(1 \text{ TeV})$, the Higgs mass should be smaller than $\sim 700$ GeV [as verified by simulations on the lattice]; Fig.3. But if one wants to extend the SM up the GUT scale $\Lambda \sim 10^{16}$ GeV [a prerequisite for the perturbative renormalization of the electroweak mixing angle from the GUT symmetry value 3/8 down to the experimentally observed value at low energies],
$M_H$ is restricted to values smaller than $\sim 200$ GeV. In addition, radiative corrections due to top quark loops could drive the Higgs self-coupling to negative values, therefore destabilizing the vacuum. The stability and the triviality bounds, constrain the Higgs boson mass to lie in the range $100$ GeV $\lesssim M_H \lesssim 200$ GeV; Fig. 3.

A Higgs bosons with $M_H \gtrsim 100$ GeV, cannot be produced at LEP2 at a c.m. energy of $\sqrt{s} = 192$ GeV and the discovery of the Higgs boson will have to wait for the LHC. At hadron colliders, Higgs particles will be copiously produced; however, because of the huge QCD background, one has to rely on very rare Higgs decay channels to see the signal. In the low mass range, $M_H \lesssim 140$ GeV, the Higgs will mainly decay into $b\bar{b}$ final states [and to a lesser extent to $\tau^+\tau^-$, $c\bar{c}$ and gluon pairs] and one has to use the clean $\gamma\gamma$ decay channel which has a branching ratio of $\mathcal{O}(10^{-3})$. For larger masses, where the Higgs decays mainly into $W$ pairs, the $ZZ$ decays [where one of the $Z$ bosons can be virtual] become important, but one needs to make the $Z$ bosons decays into charged leptons which brings again the branching to the level of a few times $10^{-3}$.

This would nevertheless be sufficient to discover the Higgs particles at LHC after a few years of running with a rather high luminosity and with dedicated detectors [to resolve for instance the narrow $\gamma\gamma$ peak for $M_H \lesssim 140$ GeV]. But it will be very difficult to make a detailed study of the properties of the Higgs bosons and to test unambiguously the mechanism of electroweak symmetry breaking. In contrast, an $e^+e^-$ collider with a c.m. energy of 500 GeV will be the ideal machine to produce Higgs particles with masses between 100 and 200 GeV and to study in detail their fundamental properties.

b) Higgs Production in $e^+e^-$ Collisions

The main production mechanism of Higgs particles in $e^+e^-$ collisions are the Higgs–strahlung process, $e^+e^- \rightarrow (Z^*) \rightarrow HZ$ [with a cross section which scales as $1/s$ and therefore dominates at low energies] and the $WW$ fusion mechanism, $e^+e^- \rightarrow \nu\bar{\nu}(W^*W^*) \rightarrow \nu\bar{\nu}H$ [with a cross section rising like $\log(s/M_H^2)$ and which dominates at high energies]. At $\sqrt{s} \sim 500$ GeV, the two processes have approximately the same cross sections for the interesting range $100$ GeV $\lesssim M_H \lesssim 200$ GeV; Fig. 4. With an integrated luminosity $\int \mathcal{L} \sim 50$ fb$^{-1}$, approximately 2000 events per year can be collected in each channel; a sample which is more than enough to discover the Higgs boson and to study it in detail.

The $ZZ$ fusion mechanism, $e^+e^- \rightarrow e^+e^-(Z^*Z^*) \rightarrow e^+e^-H$, and the associated production with top quarks, $e^+e^- \rightarrow t\bar{t}H$ have much smaller cross sections. But these processes will be very useful when it comes to study the Higgs properties as will be discussed later.

In the Higgs–strahlung process, $e^+e^- \rightarrow HZ$, the recoiling $Z$ boson [which can be
tagged through its clean $\mu^+\mu^-$ decay mode e.g. is mono–energetic and the Higgs mass can be derived from the energy of the $Z$ boson if the initial $e^+$ and $e^-$ beam energies are sharp [beamstrahlung, which smears out the c.m. energy should be thus suppressed as strongly as possible, and this is already the case for machine designs such as TESLA]. Therefore, it will be easy to separate the signal from the backgrounds. For low Higgs masses, $M_H \lesssim 140$ GeV, the main background will be $e^+e^- \to ZZ$. The cross section is large, but it can be reduced by cutting out the forward and backward directions [the process is mediated by $t$–channel electron exchange] and by selecting $b\bar{b}$ final states by means of $\mu$–vertex detectors [while the Higgs decays almost exclusively into $b\bar{b}$ jets in this mass range, BR$(Z \to b\bar{b})$ is small, $\sim 15\%$]. The background from single $Z$ production, $e^+e^- \to Zq\bar{q}$, is small and can be further reduced by flavor tagging. In the high mass range where the decay $H \to WW^*$ is dominant, the main background is triple gauge boson production and is suppressed by two powers of the electroweak coupling.

The $WW$ fusion mechanism, $e^+e^- \to \nu\bar{\nu}H$, offers a complementary production channel. For small $M_H$, the main backgrounds are single $W$ production, $e^+e^- \to e^+\nu W^+$ [$W \to q\bar{q}$ and the $e^\pm$ escape detection] and $WW$ fusion into a $Z$ boson, $e^+e^- \to \nu\bar{\nu}Z$, which have cross sections 60 and 3 times larger than the signal, respectively. Cuts on the rapidity spread, the energy and momentum distribution of the two jets in the final state [as well as flavor tagging for small $M_H$] will suppress these background events.

It has been shown in detailed simulations, that just a few fb$^{-1}$ of integrated luminosity are needed to obtain a 5$\sigma$ signal for a Higgs boson with a mass $M_H \sim 140$ GeV at a 500 GeV collider [in fact, in this case, it is better to go to lower energies where the cross section is larger], even if it decays invisibly [as it could happen in SUSY models for instance]; Fig.5a. Higgs bosons with masses up to $M_H \sim 350$ GeV can be discovered at the 5$\sigma$ level, in both the strahlung and fusion processes at an energy of 500 GeV and with a luminosity of 50 fb$^{-1}$; Fig.5b. For even higher masses, one needs to increase the c.m. energy of the collider, and as a rule of thumb, Higgs masses up to $\sim 70\%$ of the total energy of the collider can be probed. This means than a $\sim 1$ TeV collider will be needed to probe the entire Higgs mass range in the SM.

c) Determination of Higgs Properties

Once the Higgs boson is found it will be of great importance to explore all its fundamental properties. This can be done at great details in the clean environment of $e^+e^-$ linear colliders: the Higgs mass, the spin and parity quantum numbers and the couplings to fermions and gauge bosons can measured, as discussed below.
In the Higgs–strahlung process with the $Z$ decaying into visible particles, the mass resolution achieved with kinematical constraints is close to $5$ GeV, and a precision of about $\pm 200$ MeV can be obtained on the Higgs mass with $\int L = 10$ fb$^{-1}$ if the effects of beamstrahlung are small. For masses below $250$ GeV, the Higgs boson is extremely narrow and its width cannot be resolved experimentally; only for higher masses [or possibly in extended models], $\Gamma_H$ can be measured.

The angular distribution of the $Z/H$ in the Higgs–strahlung process is sensitive to the spin–zero of the Higgs particle: at high–energies the $Z$ is longitudinally polarized and the distribution follows the $\sim \sin^2 \theta$ law which unambiguously characterizes the production of a $J^P = 0^+$ particle. The spin–parity quantum numbers of the Higgs bosons can also be checked experimentally by looking at correlations in the production $e^+e^- \rightarrow HZ \rightarrow 4$–fermions or decay $H \rightarrow WW^* \rightarrow 4$–fermion processes, as well as in the more difficult channel $H \rightarrow \tau^+\tau^-$ for $M_H \lesssim 140$ GeV. An unambiguous test of the CP nature of the Higgs bosons can be made in the process $e^+e^- \rightarrow ttH$ or at laser photon colliders in the loop induced process $\gamma\gamma \rightarrow H$.

The masses of the fermions are generated through the Higgs mechanism and the Higgs couplings to these particles are proportional to their masses. This fundamental prediction has to be verified experimentally. The Higgs couplings to $ZZ/WW$ bosons can be directly determined by measuring the production cross sections in the bremsstrahlung and the fusion processes. In the $e^+e^- \rightarrow H\mu^+\mu^-$ process, the total cross section can be measured with a precision of less than $10\%$ with $50$ fb$^{-1}$.

The Higgs couplings to light fermions are harder to measure, except if $M_H \lesssim 140$ GeV. The Higgs branching ratios to $b\bar{b}$, $\tau^+\tau^-$ and $c\bar{c} + gg$ can be measured with a precision of $\sim 5, 10$ and $40\%$ respectively for $M_H \sim 110$ GeV. For $M_H \sim 140$ GeV, BR($H \rightarrow WW^*$) becomes sizeable and can be experimentally determined; in this case the absolute magnitude of the $b$ coupling can be derived since the $HWW$ coupling is fixed by the production cross section. The Higgs coupling to top quarks, which is the largest coupling in the electroweak theory, is directly accessible in the process $e^+e^- \rightarrow t\bar{t}H$. For $M_H \lesssim 130$ GeV, $\lambda_t$ can be measured with a precision of about $10$ to $20\%$ at $\sqrt{s} \sim 500$ GeV with $\int L \sim 50$ fb$^{-1}$. For $M_H \gtrsim 350$ GeV, the $Ht \bar{t}$ coupling can be derived by measuring the $H \rightarrow t\bar{t}$ branching ratio at higher energies.

Finally, the measurement of the trilinear Higgs self–coupling, which is the first non–trivial test of the Higgs potential, is possible in the double Higgs production processes $e^+e^- \rightarrow ZHH$ and $e^+e^- \rightarrow \nu\bar{\nu}HH$. However, the cross sections are rather small and very high luminosities [and very high energies in the second process] are needed.
3 Extensions of the SM

3.1 Supersymmetry

An even stronger case for $e^+e^-$ colliders in the 300–500 GeV energy range is made by supersymmetric theories. The minimal supersymmetric extension of the Standard Model (MSSM) requires the existence of two isodoublets of Higgs fields, leading to three neutral, $h/H$(CP=+), $A$(CP=−) and a pair of charged scalar particles. Besides the four masses, two additional parameters define the properties of these particles: a mixing angle $\alpha$ in the neutral CP–even sector and the ratio of the two vacuum expectation values $\tan \beta$, which from GUT restrictions is assumed in the range $1 < \tan \beta < m_t/m_b$. Supersymmetry leads to several relations among these parameters and only two of them [in general $\tan \beta$ and $M_A$] are in fact independent. In the MSSM, the upper bound on the mass of the lightest Higgs boson $h$ is shifted from the tree level value $M_Z$ to $\sim 140$ GeV. The masses of the heavy neutral and charged Higgs particles can be expected, with a high probability, in the range of the electroweak symmetry breaking scale. Some of these features are not specific to the minimal extension and are expected to be realized also in more general SUSY models. For instance, a light Higgs boson with a mass below $\mathcal{O}(200 \text{ GeV})$ is quite generally predicted by SUSY theories.

At hadron colliders, the search for the Higgs bosons of the MSSM will be even more difficult than for the SM Higgs boson. This is mainly due to the fact that the important decays of the neutral Higgs particles into $\gamma\gamma$ and $ZZ$ final states have in general smaller branching ratios, especially if decays into SUSY particles are kinematically allowed. If $M_h$ is close to its maximum value, $h$ has SM like couplings and the situation is similar to the SM case with $M_H \sim 100–140$ GeV.

In $e^+e^-$ collisions, besides the usual Higgs–strahlung and fusion processes for the production of the CP–even Higgs bosons $h$ and $H$, the neutral Higgs particles can also be produced pairwise: $e^+e^- \rightarrow A + h/H$. The cross sections [Fig. 6a] for the Higgs–strahlung and the pair production as well as the cross sections for the production of $h$ and $H$ are mutually complementary, coming either with a coefficient $\sin^2(\beta - \alpha)$ or $\cos^2(\beta - \alpha)$. The cross section for $hZ$ production is large for large values of $M_h$, being of $\mathcal{O}(50 \text{ fb})$; by contrast, the cross section for $HZ$ is large for light $h$ [implying small $M_H$]. In major parts of the parameter space, the signals consist of a $Z$ boson and a $b\bar{b}$ or a $\tau^+\tau^-$ pair, which is easy to separate from the main background, $e^+e^- \rightarrow ZZ$ [for $M_h \simeq M_Z$, efficient $b$ detection is needed]. For the associated production, the situation is opposite: the cross section for $Ah$ is large for light $h$ whereas $AH$ production is preferred in the complementary
region. The signals consist mostly of four $b$ quarks in the final state, requiring efficient $b\bar{b}$ quark tagging; mass constraints help to eliminate the QCD jets and $ZZ$ backgrounds. The CP–even Higgs particles can also be searched for in the $WW$ and $ZZ$ fusion mechanisms. Charged Higgs bosons can be produced pairwise, $e^+e^- \rightarrow H^+H^-$ through $\gamma, Z$ exchange. The cross section depends only on $M_{H^\pm}$ and is large up to $M_{H^\pm} \sim 230$ GeV. Charged Higgs bosons can also be produced in top decays as discussed previously.

The discussion on the MSSM Higgs sector in $e^+e^-$ linear colliders can be summarized in the following points: i) The Higgs boson $h$ can be detected in the entire range of the MSSM parameter space, either through the bremsstrahlung process or through pair production; Fig. 6b. In fact, this conclusion holds true even at a c.m. energy of 300 GeV and with a luminosity of a few fb$^{-1}$. ii) There is a substantial area of the $(M_h, \tan \beta)$ parameter space where all SUSY Higgs bosons can be discovered at a 500 GeV collider; Fig. 6b. This is possible if the $H, A$ and $H^\pm$ masses are less than $\sim 230$ GeV. For Higher masses, one simply has to increase the c.m. energy. iii) In some parts of the MSSM parameter space, the lightest Higgs $h$ can be detected, but it cannot be distinguished from the SM Higgs boson. In this case, Higgs production in $\gamma\gamma$ fusion [which receives extra contributions from SUSY loops] can be helpful. iv) The properties of the Higgs bosons can also be accurately determined; for instance, in the case of $h$, the same tests as for the SM Higgs can be made.

The lightest neutralinos and charginos, which are mixtures of the supersymmetric partners of the electroweak gauge bosons and Higgs bosons, are expected to be the lightest supersymmetric particles [especially the lightest neutralino, which in the MSSM with conserved R–parity, is the lightest SUSY particle and is stable]. To avoid unnatural fine tuning in radiative corrections, these particles should have masses below the Fermi scale.

Neutralinos and charginos are difficult to find at hadron colliders, but they are easy to detect at $e^+e^-$ colliders. They are produced in pairs, $e^+e^- \rightarrow \chi_i^+\chi_j^- (i, j = 1, 2)$ and $e^+e^- \rightarrow \chi_i^0\chi_j^0 (i, j = 1-4)$, through $s$–channel $\gamma, Z$ exchange, and $t$–channel sneutrino or selectron exchange. The cross sections can be rather large, $O(100$ fb) at $\sqrt{s} \sim 500$ GeV, and enough events will be produced to discover these particles and study their properties. Detailed experimental simulations have shown that these particles can be found with masses up to the beam energy, if the mass difference with the lightest neutralino is not less than 20 GeV, substantially improving on the discovery reach of LEP2.

Left or right–handed scalar particles correspond to each chiral SM fermion. Starting with a universal scalar mass $m_0$ at the GUT scale, squark and slepton masses evolve differently down to low energies, the later being significantly smaller than the former [with
the possible exception of the stop squark]. The SUSY partners of leptons may have masses below the Fermi scale. In $e^+e^-$ collisions, sleptons are produced pairwise $e^+e^- \rightarrow \tilde{l}\tilde{l}$, through $s$–channel $\gamma, Z$ exchange. For sneutrinos only $Z$ exchange is present, for selectrons and electron sneutrinos additional chargino/neutralino $t$–channel exchanges are present. The cross sections are large, $\sigma \gtrsim 50$ fb for $\sqrt{s} \sim 500$ GeV, and these states can be discovered up to the kinematical limit if the slepton masses are larger than the lightest neutralino mass by more than a few ten GeV.

While colored squarks [except for the stop squarks] and gluinos can be detected up to masses of $\mathcal{O}(1$ TeV) at LHC, the sleptons are very hard to detect due to the low production rates and the large backgrounds, so that $e^+e^-$ colliders are unique in this sector. In $e^+e^-$ collisions, one can also measure the masses of the scalar particles [which is difficult at hadron colliders because of the escaping LSP’s] and their couplings. This is very important in order to constrain the various SUSY scenarios.

Finally, due to the large value of $m_t$, the lightest stop quark can be lighter than the top quark and all the other squarks. Because of the large backgrounds, stop squarks are also difficult to find at the LHC while they can be easily detected in $e^+e^-$ collisions with masses up to the beam energy.

### 3.2 Grand Unified Theories

The SM does not unify the electroweak and strong forces in a satisfactory way since the couplings of these interactions are different. Therefore one would expect that a more fundamental theory exists which describes the three forces within the context of a single gauge group [which will contain the SM as a subgroup and will reduce to it at low energies] and hence, with only one coupling constant. The LEP data show that this can be indeed achieved in Supersymmetric Grand Unified Theories.

Two predictions of GUT can have dramatic phenomenological consequences in the $\mathcal{O}$(TeV) energy range: (i) The unifying group must be spontaneously broken at the unification scale for the proton to be stable; however, it is possible that the breaking to the SM group occurs in several steps and that some subgroups remain unbroken down to a scale of order 1 TeV allowing for new neutral gauge bosons ($Z'$) with masses not far from the Fermi scale. (ii) The GUT groups incorporate fermion representations in which a complete generation of SM quarks and leptons can be naturally embedded and in most cases these representations are large enough to accommodate additional new fermions. These fermions, which are needed to have anomaly–free theories, can have masses not much larger than
the Fermi scale. In addition, particles with exotic quantum numbers such as difermions [leptoquarks, diquarks and dileptons], could occur.

The direct search for these new matter and gauge particles and tests of their indirect effects will be a major goal of the next generation of accelerators and high-energy $e^+e^-$ linear colliders have a rather rich potential for these searches.

If the energy can be raised high enough, the $e^+e^-$ collider will operate as a $Z'$ factory; the event rates will be very high and the properties of the $Z'$ can be studied in great details. A heavy $Z'$ boson, even if its mass is substantially larger than the available center of mass energy, will manifest itself through its propagator effects in the process $e^+e^- \rightarrow \gamma, Z, Z' \rightarrow$ fermions, producing potentially sizeable effects on the leptonic cross section $\sigma_{\text{lept}}$, the ratio $R = \sigma_{\text{had}}/\sigma_{\text{lept}}$, the forward–backward asymmetry $A_{\text{FB}}^{\text{lept}}$ and if longitudinal polarization is available, the left–right asymmetries $A_{\text{LR}}^{\text{lept}}$ and $A_{\text{LR}}^{\text{had}}$. Masses up to 6 times the c.m. energy of the collider can be probed for the expected luminosities. If a $Z'$ with mass below 3 TeV is discovered at LHC, even a 500 GeV $e^+e^-$ collider would give valuable contributions to its detailed investigation by allowing the distinction between different classes of models and the determination of the model parameters. The two types of colliders would then provide complementary information.

$e^+e^-$ colliders are well suited machines for the search of new leptons. These particles can be pair produced, $e^+e^- \rightarrow L\bar{L}$, with large rates if their masses are smaller than the beam energy. They can also be singly produced in association with their standard light partners, $e^+e^- \rightarrow L + \bar{L}$ if the mixing between the light and heavy states is not prohibitively small; one can then reach masses close to the total energy of the collider. The signatures, with final states involving charged leptons and gauge bosons, have clear characteristics so that the detection of these particles should not be difficult in the clean environment of $e^+e^-$ colliders. Since they are strongly interacting particles, quarks, leptoquarks and diquarks will be produced at the LHC with very large rates. However, because of the difficult hadronic jet background, the signals would be hard to analyze in detail. $e^+e^-$ colliders would provide the ideal framework for highly precise analyses of the properties of these new exotic particles if they are found at the hadron colliders.

4 Conclusions

$e^+e^-$ linear colliders operating in the energy range of $\sim 500$ GeV [and which can be extended to energies up to 2 TeV] have a very rich physics potential, which in many aspects is complementary to that of the LHC. High–precision measurements in the top quark and
W/Z bosons sectors can be performed, allowing further tests of the Standard Model and eventually opening a window to new physics. The exploration of the electroweak symmetry breaking mechanism can be made at great details: Higgs particles can be easily searched for, and the clean environment of the collider allows detailed studies of their basic properties. The $e^+e^-$ colliders provide a unique opportunity to explore in a deep manner central aspects of supersymmetric theories: the Higgs spectrum and its properties, the higgsino/gaugino and slepton sectors. Finally, gauge extensions of the Standard Model can be explored by searching for new gauge bosons and new matter particles.

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**References**

[1] Proceedings of the Workshop “Physics and Experiments with Linear Colliders”, Saariselkä (Finland), 1991; R. Orava et al. (eds).

[2] Proceedings of the Workshop “Physics and Experiments with Linear Colliders”, Waikoloa (Hawaii), 1993; F. Harris et. al (eds).

[3] Proceedings of the Workshop “Physics and Experiments with Linear Colliders”, Morioka (Japan), 1995.

[4] Proceedings of the Workshop “$e^+e^-$ Collisions at 500 GeV: the Physics Potential”, Reports DESY 92–123A+B and DESY–93–123C, P. Zerwas (ed.).

[5] Proceedings of the Workshop “Physics with $e^+e^-$ Linear Colliders”, Annecy–Gran Sasso–Hamburg, Feb–Sept. 1995, to appear.

[6] Proceedings of the DPF Workshop “Electroweak Symmetry Breaking and Beyond the Standard Model”, 1995, T. Barlow et al., eds.
Fig. 1: The top quark production cross sections in $e^+e^-$ collisions as a function of the c.m. energy.

Fig. 2: 95% CL constraints on $(\alpha_2, \lambda_2)$ for LEP2 ($\sqrt{s} = 1.3$ TeV), LHC ($\sqrt{s} = 100$ TeV) and a 500 GeV $e^+e^-$ collider ($\sqrt{s} = 500$ GeV). SM values are assumed for $\alpha_2$ and $\lambda_2$. From T. Barklow (in ref. [1]).
Fig. 3: Allowed values of the top and Higgs masses from triviality and stability, if the SM can be extended up to a scale $\Lambda$.

Fig. 4: Higgs boson production cross sections in $e^+e^-$ collisions at $\sqrt{s} = 500$ GeV.
Fig. 5: (a) Distribution of the missing mass $M^2 = (p_e + p_\nu - p_\mu - p_\mu')^2$ for the $e^+e^- \rightarrow H\mu^+\mu^-$ signal (assuming $M_H = 130$ GeV) and the $e^+e^- \rightarrow \mu^+\mu^-\gamma$ background (with an angular cut $\theta_{\nu\mu} < 90^\circ$). Distributions $d\sigma/dM_{WW}$ for heavy Higgs signals and backgrounds in the Higgs-strahlung (b) and the $W^+W^-Z$ fusion (c) processes. Acceptance cuts are applied, and beamstrahlung, beamstrahlung and the energy spread are taken into account everywhere; $\sqrt{s} = 500$ GeV and the DESY-Darmstadt design (results for TESLA are similar) are assumed.
Fig. 6a: Cross sections for the SUSY Higgs production at $\sqrt{s} = 500$ GeV and $\tan \beta = 20$.

Fig. 6b: Regions of the $(M_h, \tan \beta)$ parameter space where the four cross sections $e^+e^- \rightarrow h/H/A(\tilde{Z})$ are larger than 2.5 fb. The dashed area is the theoretically forbidden region, and the thin lines correspond to the region which can be probed at LEP2.