A Minimal Superstring Standard Model II:
A Phenomenological Study

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Abstract

Recently, we demonstrated the existence of heterotic–string solutions in which the observable sector effective field theory just below the string scale reduces to that of the MSSM, with the standard observable gauge group being just $SU(3)_C \times SU(2)_L \times U(1)_Y$ and the $SU(3)_C \times SU(2)_L \times U(1)_Y$–charged spectrum of the observable sector consisting solely of the MSSM spectrum. Associated with this model is a set of distinct flat directions of vacuum expectation values (VEVs) of non–Abelian singlet fields that all produce solely the MSSM spectrum. In this paper, we study the effective superpotential induced by these choices of flat directions. We investigate whether sufficient degrees of freedom exist in these singlet flat directions to satisfy various phenomenological constraints imposed by the observed Standard Model data. For each flat direction, the effective superpotential is given to sixth order. The variations in the singlet and hidden sector low energy spectrums are analyzed. We then determine the mass matrices (to all finite orders) for the three generations of MSSM quarks and leptons. Possible Higgs $\mu$–terms are investigated. We conclude by considering generalizations of our flat directions involving VEVs of non–Abelian fields.

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1 Minimal Superstring Standard Models

Recently \[1, 2\] we demonstrated that it is indeed possible for a string model \[3, 4\] to have exactly the minimal supersymmetric standard model (MSSM) fields as the SU\(^{(3)}_C \times SU(2)_L \times U(1)_Y\)–charged matter content of its low energy effective field theory. We propose that string models with this property be classified as “Minimal Superstring Standard Models” (MS\(_{str}\)SM ) \[2\]. In our MS\(_{str}\)SM, decoupling of all MSSM–charged exotics from the low energy effective field theory was accomplished by sets of vacuum expectation values (VEVs) that eliminate the anomalous U\(^{(1)}_A\) endemic to several classes of string models (in particular, those of bosonic lattice, orbifold, or free fermionic construction). Besides restoring spacetime supersymmetry through cancellation of the Fayet–Iliopoulos (FI) D–term, these sets of VEVs also give FI–scale (\(\approx 4 \text{ to } 7 \times 10^{16} \text{ GEV}\)) mass to the MSSM exotics.

If the underlying “initial” state of the universe truly was an anomalous U\(^{(1)}_A\) string model, then determination of the specific flat VEV direction chosen to cancel the FI D–term was a result of non–perturbative dynamics. That is, the physically preferred flat direction cannot be identified perturbatively. However, through perturbative means we can locate and classify the possible flat directions most consistent with observed data. Classification of non–Abelian (NA) singlet flat directions that produce the MSSM gauge group and matter fields, while simultaneously decoupling all MSSM exotics, was performed in \[2\]. Based on our “stringent” F–flatness constraints, we found three directions flat to all order, one direction flat to 12\(^{th}\) order, and around 100 remaining directions only flat to seventh order or less.

The existence of free fermionic models with solely the MSSM spectrum below the string scale reinforces the motivation to improve our understanding of this particular class of string models. Both from the point of view of understanding the non–perturbative dynamics, as well as improving the techniques that are needed in order to confront the perturbative string models with the low energy experimental data. In this paper we perform studies of the phenomenological features of these first four flat directions of the “FNY” model of \[3, 4\]. We explore the phenomenology of our MS\(_{str}\)SM flat directions and investigate which (if any) of our four singlet directions appear most consistent with observed phenomenological criteria.

We remark that phenomenological studies, similar to the one performed in this paper, were done in the past for other three generation free fermionic models. The new features in this paper are as follows. First, the FNY model is the first known example of a semi–realistic string model, which produces solely the MSSM–charged spectrum just below the string scale. Thus, for the first time such a phenomenological analysis is carried out in a Minimal Superstring Standard Model. Second, and more importantly, in the phenomenological analysis performed in this paper, we implement the systematic techniques for the analysis of F and D flat directions that were developed over the last few years \[1, 2, 5, 6, 7\]. Relative to the more primitive studies performed in the past, our study here has the advantage that it incorporates
in much of the analysis the non–renormalizable terms to all finite orders. In the analysis we systematically decouple from the effective low energy field theory the fields that become superheavy and their superpotential couplings. Furthermore, the solutions that we study in this paper are flat to all orders. We emphasize that such an exhaustive analysis is performed for the first time in a semi–realistic string model. Thus, our paper further advances the methodology needed to confront potentially viable superstring models with experimental data.

Our paper is organized as follows. In Section 2 we present a brief review of $Z_2 \times Z_2$ free fermionic models, the class from which the “FNY” model originates. In Section 3 we review $D$– and $F$–flatness constraints. Our study and discussion of the phenomenology of our four flat directions appears in Section 4. For each of the flat directions, we analyze the three generation mass matrices for up, down, electron, and neutrino states, and study the effective Higgs $\mu$ terms. We also investigate coupling constant strength for high order superpotential terms. We conclude Section 4 with study of additional $F$–flatness constraints imposed when both fields in a vector–like pair (with opposite $U(1)$ charges) acquire VEVs. Lastly, in Section 5 we include some general discussion and overview of our singlet flat directions. We briefly consider generalizations of them in which NA fields are also allowed to take on VEVs.

2 $Z_2 \times Z_2$ free fermionic models

Constructing minimal superstring models, i.e. models with solely the MSSM spectrum below the string scale, is clearly the coveted goal of superstring phenomenology. However, it should be emphasized that the success of the FNY model in achieving this goal should not be viewed as indicating that the model of ref. [3] is the correct string vacuum. In this respect it is important to understand that the FNY model belongs to a large class of three generation free fermionic models, which possess an underlying $Z_2 \times Z_2$ orbifold structure. Many of the issues pertaining to the phenomenology of the Standard Model and supersymmetric unification have been addressed in the past in the framework of the quasi–realistic free fermionic models. An important property of these models is the fact that they produce three generation models with the standard $SO(10)$ embedding of the Standard Model spectrum. No other orbifold heterotic–string compactification has yielded a similar structure. The FNY model should be viewed as a prototype example of a semi–realistic free fermionic model. The success of the FNY model in producing solely the MSSM spectrum below the string scale should then be regarded as providing further evidence for the assertion that the true string vacuum is connected to the $Z_2 \times Z_2$ orbifold in the vicinity of the free fermionic point in the Narain moduli space.

For completeness we recall the basic structure of the free fermionic superstring models. The purpose is to highlight the fact that the free fermionic models correspond to a large set of viable three generation models, which differ in their detailed phenomenological characteristics. In this respect, the FNY model should be regarded
as a representative example.

A model in the free fermionic formulation is defined by a set of boundary condition basis vectors, and one–loop GSO phases, which are constrained by the string consistency requirements, and which completely determine the vacuum structure of the models. The physical spectrum is obtained by applying the generalized GSO projections.

The first five basis vectors of the \( Z_2 \times Z_2 \) free fermionic models consist of the NAHE set. The gauge group after the NAHE set is \( SO(10) \times E_8 \times SO(6)^3 \) with \( N = 1 \) space–time supersymmetry, and 48 spinorial 16’s of \( SO(10) \), sixteen from each sector \( b_1 \), \( b_2 \) and \( b_3 \). The three sectors \( b_1 \), \( b_2 \) and \( b_3 \) are the three twisted sectors of the corresponding \( Z_2 \times Z_2 \) orbifold compactification. The \( Z_2 \times Z_2 \) orbifold is special precisely because of the existence of three twisted sectors, with a permutation symmetry with respect to the horizontal \( SO(6)^3 \) symmetries. The NAHE set is depicted in the table below which highlights its cyclic permutation symmetry. The NAHE set is common to a large class of three generation free fermionic models. The construction proceeds by adding to the NAHE set three additional boundary condition basis vectors which break \( SO(10) \) to one of its subgroups, \( SU(5) \times U(1) \), \( SO(6) \times SO(4) \) or \( SU(3) \times SU(2) \times U(1)^2 \), and at the same time reduces the number of generations to three, one from each of the sectors \( b_1 \), \( b_2 \) and \( b_3 \). The various three generation models differ in their detailed phenomenological properties. These detailed properties depend on the specific assignment of boundary condition basis vector for the internal world–sheet fermions \( \{ y, \omega | \bar{y}, \bar{\omega}, 1, \ldots, 6 \} \). However, many of the characteristics of the three generation models can be traced back to the underlying NAHE set structure. One such important property to note is the fact that, as the three generations are obtained from the three twisted sectors \( b_1 \), \( b_2 \) and \( b_3 \), they automatically possess the Standard \( SO(10) \) embedding. Consequently, the weak hypercharge, which arises as the usual combination \( U(1)_Y = U(1)_T + \frac{1}{2} U(1)_{B-L} \), has the standard \( SO(10) \) embedding. To date, of the three generation heterotic–orbifold models that have been constructed, only the free fermionic models have yielded such a structure.

**THE NAHE SET**

| \( \psi^\mu \) | \( \chi^{12} \) | \( \chi^{34} \) | \( \chi^{56} \) | \( \bar{\psi}^{1,...,5} \) | \( \tilde{\eta}^{1} \) | \( \tilde{\eta}^{2} \) | \( \tilde{\eta}^{3} \) | \( \bar{\phi}^{1,...,8} \) |
|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 1,...,1 | 1 | 1 | 1,...,1 |
| S | 1 | 1 | 1 | 1 | 0,...,0 | 0 | 0 | 0,...,0 |
| \( b_1 \) | 1 | 1 | 0 | 0 | 1,...,1 | 1 | 0 | 0,...,0 |
| \( b_2 \) | 1 | 0 | 1 | 0 | 1,...,1 | 0 | 1 | 0,...,0 |
| \( b_3 \) | 1 | 0 | 0 | 1 | 1,...,1 | 0 | 0 | 1 | 0,...,0 |

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It should be emphasized that the success of free fermionic models in providing a viable framework for reproducing the low energy phenomenology makes evident the need for better understanding of this class of models, both from the phenomenological point of view, as well as trying to understand the nonperturbative mechanism which fixes the string vacuum. It should be further emphasized that because the free fermionic construction is formulated at an enhanced symmetry point in the string moduli space, it is very natural to expect that the true string vacuum should indeed be found in the vicinity of this point. The structure of the $Z_2 \times Z_2$ orbifold, which underlies the free fermionic models, then seems particularly suited for constructing three generation models. It is a very intriguing fact that precisely where one would have expected to find the true string vacuum, indeed the most realistic string models have been found. The further success of the free fermionic models in producing models with solely the MSSM charged spectrum in the observable sector then provides further evidence for the assertion that the true string vacuum is indeed located in this vicinity. Elaborate exploration of the realistic free fermionic models is therefore vital.

The detailed massless spectrum of the FNY model and quantum charges are given in refs. [3, 2]. Here we briefly recap the notation used in this paper. The massless spectrum includes three generations from the sectors $b_1$, $b_2$ and $b_3$. The Neveu–Schwarz (NS) sector produces the gravity and gauge multiplets, three pairs of electroweak doublets $\{h_1, h_2, h_3, \bar{h}_1, \bar{h}_2, \bar{h}_3\}$, seven pairs of $SO(10)$ singlets with observable $U(1)$ charges, $\{\Phi_{12}, \Phi_{12}, \Phi_{23}, \Phi_{23}, \Phi_{13}, \Phi_{13}, \Phi_{56}, \Phi_{56}, \Phi_{56}, \Phi_4, \Phi_4, \Phi_4, \Phi_4\}$, and three scalars that are singlets of the entire four dimensional gauge group, $\{\Phi_1, \Phi_2, \Phi_3\}$. The states from the NS sector carry vector–like charges with respect to all unbroken $U(1)$ symmetries. The states from the sectors which are combinations of $\{1, b_{1,2,3,4}, \alpha\} + 2\beta$ are generically denoted by $V_n$, $n = 1, 2, \cdots$. The states from the sectors with some combination of $\{1, b_{1,2,3,4}, \alpha\} + \beta$ are generically denoted by $H_n$, $n = 1, 2, \cdots$. A superscript “$s$” denotes when a respective $H$ or $V$ field carries only $U(1)$ charges and is a singlet for each of the non-Abelian gauge groups. The $V_n$ and $H_n$ states are vector–like with respect to some $U(1)$ currents but can be chiral with respect to others.
3 Flat MSSM directions of the FNY model

3.1 Spacetime supersymmetry and $D$– & $F$–constraints

In supersymmetric models, each chiral spin–$\frac{1}{2}$ fermion $\psi_m$ is paired with a scalar field $\phi_m$ to form a superfield $\Phi_m$. The potential $V(\varphi)$ for the scalar fields receives contributions from $D$–terms

$$D^\alpha_a \equiv \sum_m \varphi_m^\dagger T^\alpha_a \varphi_m,$$  \hspace{1cm} (3.1)

where $T^\alpha_a$ is a matrix generator of the gauge group $g_\alpha$ for the representation $\varphi_m$, and from $F$–terms,

$$F_{\Phi_m} \equiv \frac{\partial W}{\partial \Phi_m}.$$  \hspace{1cm} (3.2)

The scalar potential has the form

$$V(\varphi) = \frac{1}{2} \sum_{\alpha,a} g_\alpha D^\alpha_a D^\alpha_a + \sum_m |F_{\Phi_m}|^2.$$  \hspace{1cm} (3.3)

For an Abelian group, $U(1)_i$, (3.1) reduces to

$$D^i \equiv \sum_m Q_m^{(i)} |\varphi_m|^2,$$  \hspace{1cm} (3.4)

where $Q_m^{(i)}$ is the $U(1)_i$ charge of $\varphi_m$. When one Abelian group $U(1)_A$ is anomalous\footnote{If initially the anomaly is contained in two or more $U(1)_{A,i}$, then the anomaly can always be rotated into a single $U(1)_A$ by a unique rotation.}, i.e., when the trace over the massless fields of its charge is non–zero,

$$\text{Tr} Q^{(A)} \neq 0,$$  \hspace{1cm} (3.5)

then (3.4) is modified by the appearance of an additional term $\epsilon$ on the right–hand side, where

$$\epsilon \equiv \frac{g_s^2 M_P^2}{192\pi^2} \text{Tr} Q^{(A)}.$$  \hspace{1cm} (3.6)

Here $g_s$ is the string coupling and $M_P$ is the reduced Planck mass, $M_P \equiv M_{\text{Planck}}/\sqrt{8\pi} \approx 2.4 \times 10^{18}$. The FI $D$–term $\epsilon$ results from the standard string theory anomaly cancellation mechanism [12]. The universal Green–Schwarz relations, which result from modular invariance constraints, remove all Abelian triangle anomalies except those involving either one or three $U_A$ gauge bosons. The string anomaly cancellation mechanism breaks $U_A$ and, in the process, generates $\epsilon$.\footnote{The form of the FI $D$–term was determined from string theory assumptions. Therefore, a more encompassing $M$–theory [13] might suggest modifications to this FI $D$–term. However, recently it was argued that $M$–theory does not appear to alter the form of the FI $D$–term [14]. Instead an $M$–theory FI–term should remain identical to the FI–term obtained for a weakly–coupled $E_8 \times E_8$ heterotic string, independent of the size of $M$–theory’s 11th dimension.}
Spacetime supersymmetry is broken when the scalar potential acquires a positive-definite VEV. Thus, the FI $D_A$–term $\epsilon$ breaks supersymmetry near the string scale, with

$$V \sim g_s^2 \epsilon^2 ,$$

(3.7)

unless a set of scalar VEVs, $\{\langle \varphi_{m'} \rangle\}$, carrying anomalous charges $Q^{(A)}_{m'}$ can cancel $\epsilon$ by making a contribution to $D^{(A)}$ of equal magnitude but opposite sign:

$$< D^A > = \sum_{m'} Q^{(A)}_{m'} |\langle \varphi_{m'} \rangle|^2 + \epsilon = 0 .$$

(3.8)

Further, maintaining supersymmetry also requires any set of scalar VEVs satisfying eq. (3.8) to also be $D$–flat for all of the non–anomalous Abelian and non-Abelian gauge groups as well,

$$< D^{(i,\alpha)} >= 0 .$$

(3.9)

The appearance of a given superfield $\Phi_m$ in the superpotential $W$ imposes additional constraints on flat directions via the associated $F$–term, (3.12). $F$–flatness (and thereby supersymmetry) can be broken through an $n^{th}$ order $W$ term containing $\Phi_m$ when all of the additional fields in the term acquire VEVs,

$$< F_{\Phi_m} > \sim \frac{\partial W}{\partial \Phi_m} > \sim \lambda_n < \varphi >^2 \left( \frac{< \varphi >}{M_{Pl}} \right)^{n-3} .$$

(3.10)

(3.11)

where $< \varphi >$ denotes a generic scalar VEV. If $\Phi_m$ also takes on a VEV, then supersymmetry can be broken simply by $< W > \neq 0$. Therefore we also demand that $< W > = 0$ for each individual term.

The higher the order, $n$, of an $F$–breaking term, the stronger the suppression of the supersymmetry breaking scale below the string scale. This scale suppression normally results from a product of two effects: (i) a factor $\sim \left( \frac{< \varphi >}{M_{Pl}} \right)^n < 1$ (where typically $< \varphi > \sim \frac{1}{10} M_{Pl}$ in weakly coupled models) and (ii) less than “factorizable” growth, with increasing $n$, of the world-sheet correlation function integral $I_{n-3}$, which is contained within the non–normalizable ($n > 3$) coupling constants, $\lambda_n$. By this we mean that $I_{n-3} \ll (I_1)^{n-3}$ for $n > 4$. In the weakly coupled case, $F$–breaking terms with orders as high as $n \sim 17$ can generate supersymmetry breaking at an energy scale too far above the electroweak scale. Note that for our flat directions, each $< \varphi > / M_{Pl}$ contributes a suppression factor of approximately $1/30$ to (3.11).

As the string coupling increases (and the physics becomes less perturbative in nature) the string scale can be significantly lowered. For strong coupling, the mass scale in the denominator of (3.11) should be replaced by the string scale $M_{str}$. The diminished suppression from each $< \varphi > / M_{str}$ factor then requires that $F$–flatness be maintained to an even higher order.
$F$–flatness can be broken by two classes of superpotential terms, those composed of: (i) only the VEV’ed fields and (ii) the VEV’ed fields, and a single field without a VEV. Obviously, $F$–flatness is guaranteed to a specific order $n$ in $W$ when neither class of terms appears at order $n$ or below. In [2], the three all–order flat directions, and the one 12th–order flat direction satisfying our “exotic decoupling” requirements were found by this technique. While lack of the appearance of either class of term for a given $D$–flat direction is sufficient to guarantee $F$–flatness, this requirement is not necessary. The non–presence of such terms can be relaxed. Several terms can appear without breaking $F$–flatness, provided that the sum over all the terms in each $\langle F_{\Phi_m} \rangle$, and in $\langle W \rangle$, vanishes.

3.2 MSSM flat directions

Tables I and II in Appendix A form a review of the the four basic classes of VEV directions (denoted FD1, FD2, FD3, and FD4) presented in [3]. These directions sustain the MSSM gauge group and produce, in the low energy effective field theory, exactly the three standard generations of MSSM matter fields and a single set of Higgs $h$ and $\bar{h}$ fields as the only MSSM–charged fields. All other MSSM exotics are decoupled, acquiring FI–scale masses. FD1 is the “root” direction contained within the three others. That is, all of the fields, $\Phi_{12}, \Phi_{23}, \Phi_4, \Phi'_4, \Phi'_{56}, H'_{15}, H_{30}, H'_{31},$ and $H_{38},$ that take on VEVs in FD1, do so likewise in FD2, FD3, and FD4, FD2, FD3, and FD4 each contain one additional field with a VEV: $\Phi'_{56}, H'_{15},$ and $H_{30},$ respectively. FD1, FD2, and FD3 are flat to all order, while $F$–flatness in FD4 is broken at twelfth order. The Fayet–Iliopoulos scales (that is, the overall scales of the VEVs) for these four flat directions are approximately $6.7 \times 10^{16}$ GeV, $3.9 \times 10^{16}$ GeV, $4.8 \times 10^{16}$ GeV, and $6.7 \times 10^{16}$ GeV, respectively.

The above charged $\Phi$ fields are all vector–like for all Abelian groups, while the corresponding $H$ and $V$ fields are not. Thus, a possible variation of this class of flat directions is to allow both a $\Phi$ field and its vector partner $\overline{\Phi}$ to each take on a VEV. For example, in FD1 both $\Phi_{12}$ and $\overline{\Phi}_{12}$ can acquire VEVs, so long as

$$|\langle \Phi_{12} \rangle|^2 - |\langle \overline{\Phi}_{12} \rangle|^2 = 3 < \alpha >^2,$$

(3.12)

where $\alpha$ is an overall scale factor for a given flat direction and is specified in Table II of Appendix A. Indeed, we examine flat directions varieties wherein $\overline{\Phi}_{12}$ picks up a VEV in each of FD1 through FD4; we refer to these respective modified directions as FD1V through FD4V. For FD2 we also consider the case where $\Phi'_{56}$ receives a VEV alongside $\overline{\Phi}'_{56}$. This particular variation is denoted FD2V. Thus, FD2V signifies the presence of VEVs for both $\overline{\Phi}_{12}$ and $\Phi'_{56}$. Generally, the appearance of VEVs for both fields in a vector–pair results in new flatness constraints with often important phenomenological implications. We discuss vector–pair constraints in Subsection 4.3.

$^\dagger$The first class of terms was referred to as “type A” and the second as “type B” in [1].

$^\ddagger$FD1V is therefore embedded in FD2V, FD3V, and FD4V.

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4 Phenomenology of flat directions

Phenomenology of a model is generally vastly altered by the turning on of a flat direction [1, 2]. Further, the specific phenomenological modifications from that of the “unVEVed” model can vary substantially for different directions. We survey the respective phenomenological features of our three all–order and our one twelfth order singlet flat directions (the latter considered primarily for comparison) presented in [2] for the FNY model. We investigate, in particular, the MSSM three generation mass hierarchies and the effective $\mu$ term(s) for each direction.

4.1 Field content and decoupling of FI–scale massive fields

The functions of our singlet flat directions were twofold. In addition to cancelling the FI–term, we required each direction to decouple all exotic fields carrying fractional electric charge (that is, one $SU(3)_C$ triplet/antitriplet pair, four $SU(2)_L$ doublets, 16 NA singlets, two $SU(2)_H$ doublets, and two $SU(2)_{H'}$ doublets) from the effective low energy field theory. The fractional fields, denoted in (4.1) as $E'_i$, are decoupled from the low energy effective field theory of each flat direction through a generalized Higgs effect. (See Appendix B.) That is, all of the fractional fields appear in various (relatively low) $n$th–order superpotential terms containing $n−2$ flat direction field VEVs $<X_j>$,

$$E'_iE'_{i_2}<X_{j_1}> <X_{j_2}> <X_{j_3}> \cdots <X_{j_{n−2}}> (M_{Pl})^{n−3}. \tag{4.1}$$

The flat direction VEVs $<X_i>$ are generally constrained to be around the FI scale, $<\alpha>$. Thus, masses for the fields that are decoupled through renormalizable (i.e., $n=3$) terms are also expected to be around the FI scale, while masses of fields decoupled through non–renormalizable ($n>3$) order terms are suppressed below the FI scale by factors of $\sim (<\alpha>/M_{Pl})^{n−3}$. All of the fractionally charged exotics decouple through renormalizable terms, except for the $SU(3)_C$ vector–like triplet pair. The triplets appear in a fifth order mass term [1]. Thus, we expect the vector triplet to receive a mass smaller, perhaps, by a factor of around 1/10 to 1/100.

Each of our four flat directions decouples not only the fractionally–charged fields, but also the remaining MSSM–charged exotics with integer electric charge. Like their fractionally charged counterparts, the six integer charged $SU(2)_L$ doublets receive mass through third order terms. Various sets of additional NA singlets and NA hidden sector fields are also decoupled by the flat directions.

The additional fields decoupled by each direction, along with the superpotential terms responsible for the decoupling, are also specified in Appendix B. The massless and the massive (decoupled) superfield content and related superpotential terms are

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*An exception to this is the overall scale of the four $\Phi_4$–related VEVs, which is unconstrained, provided the difference of the norms of these fields is around the FI scale. The overall scale for VEVs of vector–like pairs of fields provides a similar exception.
first presented for FD1 in this appendix. Following this, the alterations to mass eigenstates resulting from replacement of FD1 by each of the other VEV directions are listed. In our “root direction” FD1, 15 of the 44 electrically uncharged $U(1)$–charged NA singlets and one of the totally uncharged fields, $\Phi_1$, receive masses from third order terms; four more singlets receive mass at fifth order; eight of the 30 NA hidden sector states with no electric charge become massive from third order terms, four such fields from fourth order terms, and four from fifth order terms. Several fields remain massless through at least seventh order; 21 of the $U(1)$–charged NA singlets, (including the right–handed neutrino singlets, $N^c_{1,2,3}$), two uncharged singlets, $\Phi_{2,3}$, one $SU(3)_H$ triplet/anti–triplet pair, six $SU(2)_H$ doublets, and six $SU(2)_{H'}$ doublets.

The renormalizable term $\Phi_5^5 H_s^{19} H_s^{20}$ prevents more than one of its three associated fields from taking on a VEV simultaneously. Hence the distinctions in VEVs between FD2, FD3, and FD4. In FD2 (and all of its variations), $H_s^{19}$ and $H_s^{20}$ become FI–scale massive, while in FD3(V) the corresponding massive states are $\Phi_5^5$ and $H_s^8$, and in FD4(V) they are $\Phi_5^5$ and $H_s^8$. FD3 and FD3V also give FI–scale mass both to another $SU(3)_H$ doublet and to another $SU(2)_{H'}$ doublet. A novel aspect of the FD4 and FD4V classes, perhaps identifying them from the rest, is their rendering of a near FI–scale Majorana mass to a right–handed neutrino singlet, $N_1^c$, and to the associated singlet field $V_{s1}^s$ through a seventh order term,

$$< \Phi_4^4 H_s^{15} H_s^{20} H_s^{30} H_s^{31} > N_1^c V_{s1}^s . \tag{4.2}$$

A few other eigenstate differences between FD1 and others are also especially significant. For example, in all “non–V” directions (that is FD1, FD2 (′), FD3, and FD4) wherein $< \Phi_{12} > = 0$, $\Phi_{12}$ is a massive eigenstate and $\Phi_{23}$ is a massless eigenstate. However, in the “V” directions, FD1V, FD2(′)V, FD3V, and FD4V, $\Phi_{12}$ receives a VEV. This triggers the rotation of the the $\Phi_{12}$ and $\Phi_{23}$ eigenstates into the massless state

$$\Phi_{12}^{(1)} \equiv \frac{1}{\sqrt{|< \Phi_{12} > |^2 + |< \Phi_{23} > |^2}} (< \Phi_{12} > \Phi_{12} - < \Phi_{23} > \Phi_{23}) , \tag{4.3}$$

and into the orthogonal FI–scale massive eigenstate,

$$\Phi_{12}^M \equiv \frac{1}{\sqrt{|< \Phi_{12} > |^2 + |< \Phi_{23} > |^2}} (< \Phi_{23} > \Phi_{12} + < \Phi_{12} > \Phi_{23}) . \tag{4.4}$$

Acquisition of a VEV by $\Phi_{12}$ also transforms the massless Higgs eigenstate from simply

$$\bar{h} \equiv \bar{h}_1 \tag{4.5}$$

into

$$\bar{h} \equiv \frac{1}{\sqrt{|< \Phi_{12} > |^2 + |< H_{s1}^8 > |^2}} (< H_{s1}^8 > \bar{h}_1 - < \Phi_{12} > \bar{h}_4) . \tag{4.6}$$
In Section 4.3 we will see that the scale of $<\Phi_{12}>$, undetermined perturbatively, is extremely relevant to both the inter- and intra-generational mass hierarchy.

Another important phenomenological effect appears in the FD2' and FD2'V directions. In all but these two directions, $\Phi'_5_6$ is massive while $H^s_{15}$ is massless. However, the VEV of $\Phi'_5_6$ in FD2' and FD2'V, rotates these fields into the massless eigenstate,

$$\Phi'^{(1)}_{5_6} \equiv \frac{1}{\sqrt{|<\Phi'_5_6>|^2 + |<H^s_{15}>|^2}} (<\Phi'_5_6> - <\Phi^s_{15}>) . \quad (4.7)$$

and into the orthogonal massive eigenstate

$$\Phi'^M_{5_6} \equiv \frac{1}{\sqrt{|<\Phi'_5_6>|^2 + |<H^s_{15}>|^2}} (<\Phi^s_{15} > <\Phi'_5_6> + <\Phi'_5_6> H^s_{15}) . \quad (4.8)$$

The VEV of $\Phi'_5_6$ also carries important hierarchical implications.

### 4.2 Superpotentials

We have computed the effective low energy superpotentials, denoted herein as $W^{FD1}$, $W^{FD2}$, $W^{FD3}$, $W^{FD4}$, $W^{FD1V}$, $W^{FD2V}$, $W^{FD3V}$, $W^{FD4V}$, $W^{FD2'}$, and $W^{FD2'V}$, for each of the four singlet directions FD1, FD2, FD3, FD4, and six of their vector pair variations, respectively. We present each superpotential up through sixth order in Appendix C. The superpotential for FD1 is listed in total. For the remaining flat directions, only the additional terms not present in $W^{FD1}$ are given. The terms in each superpotential are categorized according to whether they contain, in addition to singlet fields: (i) nothing else, (ii) only standard MSSM fields, and/or right-handed neutrinos, (iii) both MSSM fields and hidden sector non-Abelian fields, or (iv) only hidden sector non-Abelian fields.

The terms in each of our 10 low energy effective superpotentials are those originating from seventh or lower order terms in the original “un-VEV’ed” superpotential, $W^o$. A specific term of order $n_o$ in $W^o$ generates for a given flat direction, a term of effective order $n_e = n_o - n_v$, when $n_v$ of the fields in the $W^o$ term acquire VEVs. Since we expect the effective coupling constant coming from an $n_o$-order term in $W^o$ to be smaller than that of a comparable $n^{th}_e$ order $W^o$ term when $n_e > 0$ (except perhaps when $n_v = 1$ and $n_e = 3$), we refer to such a term as a “suppressed” effective $n^{th}_e$ order term in the flat direction’s effective superpotential.

To remove some trivial redundancies, the terms we list in the various effective superpotentials are only those for which the maximal number of possible VEVs are realized. For each such term, the existence in the same superpotential of related higher order terms containing fewer VEVs is implied. Consider a specific effective third order MSSM superpotential term for FD1: $<\Phi^s_{4}H^s_{15}H^s_{30}H^s_{31}> N^c_{1}H^s_{20}V^s_{31}$. This term implies the presence of a quartet of related effective fourth order terms, including
for example, \(< H_{15}^s H_{30}^s H_{31}^s > N_1^c \Phi_4 H_{20}^s V_{31}^s \), several effective fifth and sixth order terms, and the seventh order one, \( N_1^c \Phi_4 H_{15}^s H_{20}^s H_{30}^s H_{31}^s V_{31}^s \) \(\square\).

For each flat direction, the decoupling of the associated FI–scale massive fields drastically simplifies the corresponding superpotential in comparison to \( W^o \). Only a handful of third through sixth order \( W^o \) terms survive field decoupling for any direction. Large numbers of \( W^o \) terms do not appear until the seventh order. Relatedly, seventh order \( W^o \) terms are the primary source of the variations among our ten low energy effective superpotentials. It is for these reasons that we include up to seventh order \( W^o \) contributions to those ten superpotentials.

The small number of surviving third through six order \( W^o \) terms results in a division between singlet, MSSM, and NA hidden sector terms at low order. For example, the only mixed MSSM–hidden sector terms (henceforth, simply referred to as “mixed terms”) in \( W^{FD1} \), besides the many with seventh order \( W^o \) origin, are four effective fifth order terms appearing at sixth order in \( W^o \). Similarly, besides gaining four additional mixed terms from seventh order in \( W^o \), FD1V only acquires two new effective fourth order terms. The two latter terms have their origin at fifth and sixth order in \( W^o \). FD2V merely results in an additional mixed effective fifth order term with seventh order origin, while FD3 leads to three similar terms. FD2, FD2′, and FD2V, FD3V, FD4, and FD4V generate no additional mixed terms.

Effective low order, especially renormalizable, terms with unsuppressed couplings are few in number. For example, in the FD1 superpotential, \( W^{FD1} \), the singlet sector content of \( W^{FD1} \) is just two unsuppressed renormalizable terms. FD1V and FD2V offer a sole additional singlet term, while the remaining flat directions provide for no others at all.

A general property of the flat direction superpotentials is, indeed, that their more distinguishing terms owe their origin to seventh (or higher) order in \( W^o \). This implies a type of “strong stability” for an MSSM field theory realized from the FNY model. That is, the MSSM low energy effective field theory is quite robust against variation of the MS\(_{str}\)-SM generating VEVs that are flat to at least 12\(^{th}\) order.

### 4.3 MSSM three generation mass matrices

Viable three generation mass hierarchies present very strong constraints that a realistic model should satisfy. Whether or not the FNY model in particular can face this challenge remains to be seen. Even models in the close neighborhood of a candidate solution may likely not be able to satisfy all mass criteria. Here we investigate the structure of the three generation up, down, electron, and neutrino mass matrices for each of our singlet flat solutions. First we will consider the up, down, and electron matrices and then separately discuss the Dirac and Majorana matrices for neutrinos.

\(^{11}\)A term containing a VEV of an FI–scale massive field does not imply higher order terms wherein the VEV is replaced by the field.
4.3.1 Quark and electron masses

The root of all of our flat directions, FD1, presents a good start to viable quark and electron mass matrices. When respective components $\bar{h}_1$ and $h_3$ of the Higgs pair $\bar{h}$ and $h$ acquire VEVs, FD1 gives an unsuppressed renormalizable mass to exactly one generation, via the terms

$$gh_1[Q_1 u_c^c + L_1 N_1^c] + gh_3[Q_3 d_3^c + L_3 e_3^c], \quad (4.9)$$

where $g \equiv g_s \sqrt{2}$ is the physical four–dimensional gauge coupling constant. These four terms are, in fact, the only surviving renormalizable $W$–MSSM–class terms. An interesting property of this model is that the top and bottom quarks (or at least their primary components) do not come from the same $(3,2)$ representation of $SU(3)_C \times SU(2)_L$. Rather the top is a component of $Q_1$ and the bottom quark is a component of $Q_3$. From eq. (4.9) we see that the phenomenologically successful relation $[15]$, $m_b = m_\tau$ at the unification scale, is maintained. Note also that in this model all the heavy generation Yukawa couplings are obtained at the cubic level of the superpotential, which differs from the case in some other free fermionic models in which the bottom quark and tau lepton Yukawa couplings necessarily arise from nonrenormalizable terms $[10]$.

The additional, non–renormalizable mass terms for the ups, downs, and electrons resulting from each of the other flat directions appear in the mass matrices $M_{u_c, Q}$, $M_{d_c, Q}$, $M_{e_c, L}$. The components $m_{ij}$ of these matrices are defined by the convention

$$\sum_{i,j} m_{ij} f_i^c F_j = (f_1^c, f_2^c, f_3^c) M_{u_c, Q}(F_1, F_2, F_3)^T, \quad (4.10)$$

where $(f^c, F)$ ranges over $(u^c, Q), (d^c, Q), (e^c, L)$, and $(N^c, L)$. In these matrices $h_1$ and $h_3$ are two components of the physical Higgs mass eigenstate $h$. $\tilde{h}_1$ and $\tilde{h}_4 \equiv H_{34}^s$ are the parallel for $\bar{h}$.

The zero in these matrices are valid to all finite orders. The terms that are generated by a particular flat direction are marked by the superscripts $(r)$ of the coupling constants $\lambda^{(r)}$. The given mass term is also generated by all flat directions that contain the one specified. For instance, an $r = 1V$ superscript indicates the associated mass term appears for FD1V and for all flat direction in which FD1V is embedded. Likewise, a term with superscript $r = 2'$ appears for both FD2' and FD2V. The coupling constant subscript $n$ indicates that the given mass term appears at $n^{th}$–order in $W^o$.

**Our index on an MSSM field does not correspond to generation number, but rather to the string boundary sector $b_1$, $b_2$, or $b_3$ from which the field originates.

††The component indices, $i$ and $j$, of our mass matrices $[11]$, $[16]$, carry the same boundary sector interpretation as the fields themselves. Thus, the top quark mass term appears in position $(1,1)$ in the up–class matrix, rather than in position $(3,3)$.**
\[
M_{u_i, Q_j} = \begin{pmatrix}
\bar{h}_{11} g & \bar{h}_1 \lambda_8 \langle \Phi_{23} \Phi_4 H^*_L H^*_R \rangle M^2_{Pl} & 0 \\
\bar{h}_1 \lambda_7 \langle \Phi_{23} H^*_L H^*_R \rangle M^2_{Pl} & \bar{h}_{12} \langle \Phi_{23} \Phi_3 H^*_L H^*_R \rangle M^2_{Pl} & 0 \\
0 & 0 & \{ \bar{h}_4 \lambda_6 \langle \Phi_{23} H^*_L H^*_R \rangle M^2_{Pl} + \bar{h}_1 \lambda_7 \langle 2^* \Phi_{23} \Phi_3 \Phi_4 H^*_L H^*_R \rangle M^2_{Pl} \}
\end{pmatrix}
\]

(4.11)

Notice that several terms in our mass matrices contain doublets denoted as either \(h'\) and \(\bar{h}\). These are linear combinations of \(SU(2)_L\) doublets generically defined as,

\[
h' \equiv \frac{1}{c_{h'}} (\langle \Phi_{12} > h_1 + l_h < \Phi_{23} > h_3 )\) and \(\bar{h} \equiv \frac{1}{c_{\bar{h}'}} (\langle \Phi_{12} > \bar{h}_1 + l_{\bar{h}} < \Phi_{23} > \bar{h}_3 )\) \hspace{1cm} (4.12)

\[
where (c_{h'})^{-1} \equiv \frac{1}{\sqrt{|\langle \Phi_{12} >|^2 + |l_h < \Phi_{23} >|^2}}\) and \((c_{\bar{h}'})^{-1} \equiv \frac{1}{\sqrt{|\langle \Phi_{12} >|^2 + |l_{\bar{h}} < \Phi_{23} >|^2}}\). The coefficients \(l_h\) and \(l_{\bar{h}}\) are defined below.

For generic values of \(l_h\) \((l_{\bar{h}}\) = 1 results in both massless and FI–scale massive components for \(h'\) \((\bar{h}')\). Only in the \(l_h \to 1\) \((l_{\bar{h}} \to 1)\) limit does \(h'\) \((\bar{h}')\) become an eigenstate, with an FI–scale mass. In the respective limit for each,

\[
h' \to h^M \equiv \frac{1}{\sqrt{|\langle \Phi_{12} >|^2 + |\Phi_{23} >|^2}} (\langle \Phi_{12} > h_1 + \Phi_{23} > h_3 )\) \hspace{1cm} (4.14)
\[
\bar{h}' \to \bar{h}^M \equiv \frac{1}{\sqrt{|\langle \Phi_{12} >|^2 + |\Phi_{23} >|^2}} (\langle \Phi_{12} > \bar{h}_1 + \Phi_{23} > \bar{h}_3 )\) \hspace{1cm} (4.15)
\]

\(h^M\) and \(\bar{h}^M\) are orthogonal to the massless eigenstates \(h\) and \(\bar{h}\) given in (4.25, 4.20).

For a given mass term, \(l_h\) \((l_{\bar{h}})\) is determined by the ratio of the coupling constants for the contributions from \(h_1\) and \(h_3\) \((\bar{h}_1\) and \(\bar{h}_4\). Because the terms in which \(h'\) and \(\bar{h}'\) appear are all seventh order or higher, computation of the related \(l_h\) and \(l_{\bar{h}}\) is extremely non–trivial. (See Subsection 4.3.5.) However, the symmetries of the world-sheet charges of the respective fields in both the \(h'\) down and electron mass terms and in the \(\bar{h}'\) up and neutrino mass terms strongly suggest \(l_h = 1\) and \(l_{\bar{h}} = 1\) in all cases. If this is true (as we suspect), then all \(h'\) and \(\bar{h}'\) terms become decoupled from the mass matrices.
By giving a VEV to $\Phi_{12}$, FD1V provides for several additional up mass terms, but no further terms for downs or electrons. To the up matrix (4.11) FD1V contributes: (i) a fourth order diagonal term in $m_{3,3}$ for the second generation, involving the $\tilde{h}$ component of $\tilde{h}$; (ii) a twelfth order diagonal term, $m_{2,2}$, for the first generation, involving the $\tilde{h}_1$ component of $\tilde{h}$; and (iii) seventh and eighth order off-diagonal terms in $m_{1,2}$ and $m_{2,1}$. The seventh and eighth order off-diagonal terms involve $\tilde{h}'$. Only if the associated $l_h \neq 1$ will the respective terms contain the massless $\tilde{h}$ field and contribute to the matrix. Should this be the case, then (as we will argue below) physical constraints would most likely require the $\tilde{h}_1$ component of $\tilde{h}$ to strongly dominate over the $\tilde{h}_4$ component. Additionally, even if these off-diagonal terms are non-zero, we believe they would offer only minimal perturbations to the first generation mass. Most likely, $m_{1,2}m_{2,1}/m_{1,1} \ll m_{2,2}$. That is, relatively speaking, we estimate that even twelfth is too “low” of an order for the first generation mass term $m_{2,2}$ to allow for a see-saw mechanism. Nevertheless, the fact that the dominant mixing would be between the first and third generations is interesting and may have testable phenomenological implications. FD2' contributes the only additional up mass term: a twelfth order second generation term that, based on (i) above can be ignored. Thus, the generational up mass ratios are identical for FD1V, FD2V, FD2'V, FD3V, FD4V. For the remaining directions, only the top quark receives mass.

$$M_{d^c, Q_j} = \begin{pmatrix} 0 & 0 & h_1 \lambda_{10}(2') \frac{<\Phi_{56}^\prime H_{15}^+ H_{31}^+>}{M_{pl}} & 0 \\ h_1 \lambda_{10}(2') \frac{<\Phi_{56}^\prime H_{15}^+ H_{31}^+>}{M_{pl}} & 0 & 0 \\ 0 & 0 & 0 & h_3 g \end{pmatrix}$$

(4.16)

$$M_{e^c, L_j} = \begin{pmatrix} 0 & h_1 \lambda_{10}(2') \frac{<\Phi_{56}^\prime H_{15}^+ H_{31}^+>}{M_{pl}} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & h_3 g + h_3 \lambda_{12}(2'V) \frac{<\Phi_{56}^\prime H_{15}^+ H_{31}^+>}{M_{pl}} \\ 0 & 0 & 0 & h_3 g \end{pmatrix}$$

(4.17)

With the exception of the trivial order FD2'V electron mass (for which the tenth order FD2' term always dominates over) all first and second generation down and electron mass terms are derived from FD2'. Thus, the $d$ and $e$ mass ratios are identical.
for FD2’ and FD2’V. In the other directions only the bottom and tau fields receive mass. If \( h' \) contains a massless \( h \) component, then FD2’ produces a seventh order mass term for a second down generation via \( m_{2,1} \) in \( M_{d_i,Q_j} \) and an eighth order mass term for a second electron generation via \( m_{1,2} \) in \( M_{e_i,L_j} \). In this case then FD2’ also “begins” to provide for both types of first generation masses through tenth order terms \( m_{2,2} \) in \( M_{d_i,Q_j} \) and \( M_{e_i,L_j} \). However, while these mass terms might suggest a first generation mass, they are insufficient because (if the seventh order terms exist) they simply rotate the second generation \( d^c \) and \( e \) eigenstates, respectively. On the other hand, if \( h' \) is a massive eigenstate, then it is the \( m_{2,2} \) term that is responsible for the charm and muon masses.

Our up, down, and electron mass matrices indicate that the “better” phenomenology is clearly found along the flat direction FD2’V. Since FD2’V contains both FD1V and FD2’, all of the terms in the (4.11), (4.16), and (4.17) appear in this direction. However, as we shall see in Section 4.4, there is a cost for this superior phenomenology: allowing both components of the two vector pairs \((\Phi_{12, \Phi_{12}})\) and \((\Phi_{56, \Phi_{56}}')\) to acquire VEVs offers new dangers to low order \( F \)-breaking. However, phenomenologically consistent solutions to this do appear possible. As we discuss in Section 4.3.4, part of the solution is suggested independently by our mass matrices and the Higgs fields components. However, before analysis of the Higgs fields, we investigate the neutrino mass matrices.

### 4.3.2 Neutrino masses

Let us now consider the three types of neutrino mass terms: Majorana doublet terms \( m_{L_iL_j} \), Dirac terms \( m_{L_iN_j^c} \), and Majorana singlet terms \( m_{N_i^cN_j^c} \). Simply by conservation of gauged Abelian charges, we can show that no Majorana doublet terms appear for any of our flat directions. This is favorable for a good see–saw mechanism.

As we have already shown in (4.9), FD1 provides for a renormalizable third generation Dirac mass term for \( m_{1,1} \) from \( g\bar{h}_1L_1N_{c}^{\ast} \). To this, FD1V would add three diagonal terms: a fifth order minor perturbation to \( m_{1,1} \) (containing \( \bar{h}_4 \)), a sixth order \( \bar{h}_4 \) contribution for \( m_{2,2} \), and a possible twelfth order \( \bar{h'} \) to \( m_{3,3} \) (See the Dirac mass matrix (4.18) below). Thus, FD1V would imply significant mass difference between first and second generation neutrinos. FD2’V would, however, alters this through its addition of a sixth order \( \bar{h}_4 \) contribution to \( m_{3,3} \), thereby balancing the first and second generation mass scales. This flat direction also provides a (trivial) 14\(^{th}\) order perturbation to \( m_{2,2} \). FD4V keeps the first and second generation mass scale distinction by only providing trivial 16\(^{th}\) and 22\(^{nd}\) order perturbations to FD1V’s \( m_{2,2} \) and \( m_{3,3} \) terms, respectively. However, should \( h' \) have a \( h \) component, then FD4 yields mixing between the third generation and that associated with \( m_{3,3} \) via (i) a twelfth order contribution to \( m_{1,3} \) and (ii) a thirteenth order contribution \( m_{3,1} \).

\[\text{‡‡} \text{Each of these FD2’V terms requires a VEV for the vector–partner of } \Phi_{56}.\]
where,
\[
X_{1,1} \equiv \Phi_{12} H_{31}^s, \\
X_{1,3} \equiv \Phi_{12} \Phi_{23} (\Phi_4^2 + \Phi_4'^2) H_{15}^s H_{20}^s H_{30}^s H_{31}^s H_{38}^s, \\
X_{1,3}^a \equiv \Phi_{12}^2 \Phi_{23}^2 \Phi_4^2 \Phi_4'^2 \Phi_{56} H_{15}^s H_{30}^s, \\
X_{1,6}^b \equiv \Phi_{12}^2 \Phi_{23}^2 \Phi_4 H_{15}^s H_{20}^s H_{30}^s H_{31}^s H_{38}^s, \\
X_{2,2} \equiv \Phi_{23} H_{30}^s, \\
X_{3,1} \equiv \Phi_{12} \Phi_{23} \Phi_4 H_{15}^s H_{20}^s H_{30}^s H_{31}^s H_{38}^s, \\
X_{3,3} \equiv \Phi_{12}^2 \Phi_{23} \Phi_4 \Phi_{56} H_{15}^s H_{30}^s H_{31}^s, \\
X_{3,3}^b \equiv \Phi_{12}^2 \Phi_{23}^2 (\Phi_4 + \Phi_4')^3 H_{15}^s H_{20}^s H_{30}^s H_{31}^s H_{38}^s, 	ext{ and} \\
X_{3,3}^c \equiv \Phi_{56} H_{30}^s.
\]

While some Dirac terms appear for our flat directions, Majorana singlet terms do not. Local $U(1)$ charge conservation forbids neutrino singlet Majorana mass terms of the form
\[
N_i^c N_j^c \prod_{\alpha, \beta} < \Phi_\alpha > < H_\beta^s >, \tag{4.19}
\]
for any of our flat directions. Alternative Majorana singlet masses arising via terms of the form,
\[
N_i^c S \prod_{\alpha, \beta} < \Phi_\alpha > < H_\beta^s >, \tag{4.20}
\]
where $S$ is a generic singlet, are likewise forbidden, with one important exception. FD4 converts the effective trilinear term, $< \Phi_4' H_{15}^s H_{20}^s H_{30}^s H_{31}^s > N_i^c H_{20}^s V_{31}^s$, into the FI-scale mass term,
\[
< \Phi_4' H_{15}^s H_{20}^s H_{30}^s H_{31}^s > N_i^c V_{31}^s. \tag{4.21}
\]
Thus, a complete and viable see–saw mechanism appears for the third generation under FD4. If they are not decoupled, the off–diagonal FD4V Dirac terms could then propagate this see–saw mechanism to another generation.

The shortage of two generations of Majorana singlet masses for all but FD4 and FD4V, combined with the previously discussed missing first generation down and electron masses, present evidence that VEVs of NA fields may be necessary if viable mass matrices are to be obtained in this model. However, FD1V offers a possible way around this for neutrinos: another field, $V_{32}$, might play the neutrino singlet role if $\tilde{h}'$ should contain a massless $\tilde{h}$ component. FD1V produces the seventh order Dirac mass term,

$$<\Phi_4 H^s_{15} H^s_{38} > \tilde{h}' L_3 V^s_{32}. \quad (4.22)$$

This term, along with the FD4 term, $<\Phi_4' H^s_{15} H^s_{30} H^s_{20} H^s_{31} > N^c_1 V^s_{31}$, also appears in the FD4V superpotential. Furthermore, FD1V contains an interaction term for $V^s_{31}$ and $V^s_{32}$:

$$<\Phi_{12} H^s_{30} > H^s_{29} V^s_{31} V^s_{32}, \quad (4.23)$$

(which also appears in FD4V). The combination of these terms offers some interesting neutrino dynamics for FD4V.

### 4.3.3 Proton decay

In the MSSM, the dangerous proton decay operators arise from baryon and lepton number violating superpotential terms of the form

$$W = [\eta_1 u^c d^c \bar{d}^c + \eta_2 Q d^c L + \eta_3 L L e^c]$$

$$+ [\lambda_1 Q Q Q L + \lambda_2 u^c d^c \bar{d}^c e^c] / M_{Pl}$$

(4.24)

where generational indices are suppressed [10]. $\eta_i$ and $\lambda_j$ represent terms of generic order and can contain built-in suppression factors of $(N^c / M_{Pl})$ and/or $(<\phi> / M_{Pl})^n$, where $<\phi>$ represents either an NA singlet state VEV or a singlet product of NA fields, I such as a condensate of two hidden sector vector-like fields. Proton decay limits imply $\eta_1 \eta_2 \lesssim 10^{-24}$ and $\lambda_j / M(\text{GeV}) \lesssim 10^{-25}$ for $\Delta(B - L) = 0$ decays.

In the FNY model, the VEVs of FD1 and FD1V produce several sets of the dangerous operators in (4.24). While these operators originate from terms of at least sixth or seventh order in $W^o$, the associated suppression factors in these terms do not appear strong enough to slow proton decay sufficiently. That is, the proton lifetime would be significantly shorter than known limits. We remark that local discrete

---

*FD3V (which also contains (1.22)) generates a similar $\tilde{h}_4$ term $< H^s_{15} H^s_{19} H^s_{31} > \tilde{h}_4 L_1 V^s_{32}$ at sixth order, while not producing a $\tilde{h}_1$ counterpart.

†For products of NA fields, the appropriate extra number of $1/M_{Pl}$ factors is implied.
symmetries which do forbid proton decay mediating operators to all orders of non-renormalizable terms do appear in some three generation free fermionic models [16]. However, such a symmetry does not seem to operate in the case of the flat directions of the FNY model. Therefore, there would still appear the need to find a model that incorporates the attractive features of the FNY model, while at the same time, incorporates such local discrete symmetries.

4.3.4 Effective $\mu$ term for the Higgs

In all of our flat directions the massless MSSM Higgs doublets have the general composition,

$$h \equiv \frac{1}{\sqrt{|<\Phi_{12}>|^2 + |<\Phi_{23}>|^2}}(<\Phi_{23} > h_1 - <\Phi_{12} > h_3)$$ and \hspace{1cm} (4.25)

$$\bar{h} \equiv \frac{1}{\sqrt{|<\Phi_{12}>|^2 + |H_{31}^s|^2}}(<H_{31}^s > \bar{h}_1 - <\Phi_{12} > \bar{h}_4).$$ \hspace{1cm} (4.26)

Thus, when $\Phi_{12}$, but not $\Phi_{12}^\prime$, receives a VEV (as in FD1, FD2, FD2', FD3, and FD4), then $\bar{h}$ is simply $\bar{h}_1$. However, when $<\Phi_{12}> \neq 0$ (as in FD1V, FD2V, FD2'V, FD3V, and FD4V) $\bar{h}$ becomes a linear combination of $\bar{h}_1$ and $\bar{h}_4 \equiv H_{34}$. $<\Phi_{12}> \neq 0$ appears a problematic issue. As we have commented, the four possible non-renormalizable up mass terms containing $\bar{h}_1$ in (4.11) (including the two $\bar{h}_1^\prime$ terms) all require $<\Phi_{12}> \neq 0$. Thus, these terms actually imply that $\bar{h}$ has a non-zero $\bar{h}_4$ component. Furthermore, all of the remaining non-renormalizable up mass terms involve $\bar{h}_4$, thereby also requiring $<\Phi_{12}> \neq 0$. Hence, either all or none of the eight non-renormalizable up terms appear.

Also note that, while all of the down mass terms in (4.10) and electron mass terms in (4.17) are seventh order or higher, the $<\bar{h}_4>$ up term in $m_{3,3}$ is fourth order. Therefore, if we assume that seventh (or tenth) order terms can produce viable down and electron second and first generations mass scales, then we would expect a fourth order $<\bar{h}_4>$-related mass term to upset the generational mass hierarchy. However, this problem can be eliminated if the fourth order up mass can be sufficiently suppressed to be of similar magnitude to a seventh (tenth) order down mass. This is indeed possible: since $\bar{h}_4$ is only a component of the eigenstate $\bar{h}$, the fourth order mass eigenvalue contains a suppression factor,

$$\frac{<\Phi_{12}>}{\sqrt{|<\Phi_{12}>|^2 + |H_{31}^s|^2}}.$$ \hspace{1cm} (4.27)

Since $<H_{31}^s>$ is an FI-scale VEV, when $<\Phi_{12}>$ is several orders of magnitude below the FI-scale, it would indeed seem possible for (4.27) to suppress the fourth
order up mass, making it comparable to a seventh order down mass.

Investigation of the set of possible effective Higgs \( \mu \) terms leads to similar conclusions regarding \( < \Phi_{12} > \). An effective \( \mu \) term for the Higgs will appear at twelfth order in the superpotential,

\[
< \Phi_{12} \Phi_{23} \Phi_{4} \Phi'_{56} \Phi'_{56} H_{15}^s H_{30}^s H_{31}^s H_{38}^s > H_{36}^s h_{1} \bar{h}_1 / (M_{Pl})^9 ,
\]

(4.28)

if the massive field \( H_{36}^s \) were to acquire an appropriate intermediate scale VEV. However, preceding this term at ninth order is,

\[
< \Phi_{12} \Phi_{23} \Phi_{4} \Phi'_{56}' H_{15}^s H_{30}^s H_{31}^s > h_1 \bar{h}_1 / (M_{Pl})^6.
\]

(4.29)

(4.29) would clearly produce too large of an effective \( \mu \)–term unless \( \Phi_{12} \) and/or \( \Phi'_{56} \) is far below the FI–scale. The appearance of \( \Phi'_{56} \) in all of the down and electron mass terms suggests that \( < \Phi'_{56} > \) should be around the FI–scale. Therefore, these effective \( m \mu \) terms also imply that \( < \Phi_{12} > \) cannot receive an FI–scale value in a good flat direction. Rather, the VEV of \( \Phi_{12} \) should be at a low to intermediate scale. Specifically, production of a phenomenologically viable effective \( \mu \) term,

\[
\mu \equiv< \Phi_{12} \Phi_{23} \Phi_{4} \Phi'_{56} \Phi'_{56} H_{15}^s H_{30}^s H_{31}^s > / (M_{Pl})^6 ,
\]

(4.30)

in the 100 GeV to 10 TeV range requires,

\[
< \Phi_{12} > 256 \left( \frac{< \alpha >}{M_{Pl}} \right)^6 \approx 100 \text{ GeV to 10 TeV},
\]

(4.31)

where \( < \alpha > \approx 3.9 \times 10^{16} \text{ GeV} \) and \( M_{Pl} \approx 2.4 \times 10^{18} \). This predicts a range for \( < \Phi_{12} > \) of around \( 10^{10} \) to \( 10^{12} \) GeV.

Additional evidence against a large \( < \Phi_{12} > \) similarly appears in a tenth order \( \mu \) term for \( h_1 \bar{h}_4 \),

\[
< \Phi_{23} \Phi'_{56} \Phi'_{56} H_{30}^s H_{15}^s H_{31}^s > h_1 \bar{h}_4 / (M_{Pl})^7.
\]

(4.32)

Here the set of VEVs generating an effective \( \mu \) does not contain \( \Phi_{12} \), but \( \bar{h}_4 \) is replaced by \( \bar{h}_1 \). Thus, we should again expect \( \mu \) to be significantly above the EW scale unless the \( \bar{h}_4 \) contribution to \( \bar{h} \) is either zero or extremely small, which again implies that \( < \Phi_{12} > \) is also far below the FI–scale. The same conclusions are suggested by 16\(^{th} \) order terms from FD4:

\[
( < \Phi_{12} > h_1 + < \Phi_{23} > h_3 ) \bar{h}_4 < \Phi_{23} \Phi'_{56} [ \Phi_4^2 + \Phi'_{4}^2 ] H_{15}^s H_{20}^s H_{31}^s H_{38}^s / (M_{Pl})^{13} \]

(4.33)

\[ \text{Note that in this case, the leading component of the seventh order down mass term should not contain a } < \Phi_{12} > \text{ factor.} \]
4.3.5 Non–renormalizable coupling constants

An important stringy aspect to the generational mass hierarchy is the numeric scale of the the coupling constants $\lambda_n$ in $n^{th}$–order non–renormalizable mass terms. In free fermionic models, these coupling constant can be expressed in terms of an $n$–point string amplitude $A_n$. This amplitude is proportional to a world-sheet integral $I_{n-3}$ of the correlators of the $n$ vertex operators $V_i$ for the fields in the superpotential terms. [17, 18, 19],

$$A_n = \frac{g}{\sqrt{2}}(\sqrt{8/\pi})^{n-3} C_{n-3} I_{n-3}/(M_{Pl})^{n-3} .$$

The integral has the form,

$$I_{n-3} = \int d^2 z_3 \cdots d^2 z_{n-1} <V_1^f(\infty)V_2^f(1)V_3^b(z_3)\cdots V_{n-1}^b(z_{n-1})V_n^b(0)>$$

where $z_i$ is the world–sheet coordinate of the fermion (boson) vertex operator $V_i^f$ ($V_i^b$) of the $i^{th}$ string state. $C_{n-3}$ is an $O(1)$ coefficient that includes renormalization factors in the operator product expansion of the string vertex operators and target space gauge group Clebsch–Gordon coefficients. $SL(2,C)$ invariance is used to fix the location of three of the vertex operators at $z = z_\infty, 1, 0$. When $n_v$ of the fields $\Pi_{i=1}^l X_i$ take on VEVs, $<\Pi_{i=1}^l X_i>$, then the coupling constant for the effective $n_e = (n - n_v)$–th order term becomes $A'_{n_e} \equiv A_n <\Pi_{i=1}^l X_i>$.

An $n$–point string function trivially vanishes when the correlator $<\Pi_{i=1}^l X_i>$ itself vanishes, resulting from non–conservation of at least one or more gauged or global (including “Ising”) world-sheet charges. When all charges are conserved, one must compute $I_{n-3}$ to determine the numeric value of $A_n$. It might actually be possible for an $n$–point function to vanish upon integration of $<\Pi_{i=1}^l V_i>$, even when $<\Pi_{i=1}^l V_i>$ is non–zero (i.e., when all gauge, picture–changed global world-she t, and Ising charges are conserved). Typical non–zero values of $I_1$ and $I_2$ integral for 4– and 5–point string amplitudes are around 100 and 340 [17, 18] for free fermionic models.

As (4.16) and (4.17) indicate, when the relevant $h_1$– and $h_3$–term coupling constants are non–equal, our second generation down and electron mass terms appear at seventh and eight order, respectively, in the superpotential.\textsuperscript{§} Thus, in the case of coupling constant inequality, an order of magnitude estimate of the mass ratio between the second and third generations requires knowing the coupling strength of the seventh order term. This necessitates numeric computation of the associated correlation function integral. To our knowledge, this has not been done to date for sixth order or beyond. Thus, for an order of magnitude comparison of 7$^{th}$ order couplings to 3$^{rd}$, (i.e. between third and second generations), we have analyzed the integral

\textsuperscript{§}Alternatively, when the couplings are equal, then actual mass terms are tenth order.
$I_4$ associated with the superpotential term, $h_3Q_1d_2^c < \Phi_23\Phi_5^iH_1^sH_3^s >$. In the zero external momentum limit we find,

$$
I_4 = \int d^2z_3d^2z_4d^2z_5d^2z_6 \left\{ |z_3|^{5/4}|z_5| \right\}
\frac{\left|z_\infty - 1\right| |z_\infty - z_3| |z_\infty - z_5|^2 |z_\infty - z_6|^2 |z_3 - z_4| |z_3 - z_5| |z_4|}{\left|z_\infty - z_3| |z_\infty - z_5|^2 |z_\infty - z_6|^2 |z_3 - z_4| |z_5 - z_4| \right\}
\times \frac{|z_5|^2|z_6|^2/\zeta_\infty + 2|z_4|^2\zeta_\infty}{(\left|1 - z_3| |z_6\right|)^{3/2}(\left|1 - z_6| |z_3 - z_6\right|)^{3/4}|z_4 - z_5|^4|z_4 - z_6|^2} \right. \right)
(4.37)

The integrand in $I_4$, computed directly from the product of correlations functions, initially contained many more terms than shown in \(4.37\). However, (relating to our comments about the possibility of an $I_{n-3} = 0$ integral above), the vast majority of terms were in fact not holomorphic with regard to coordinates, but instead contained extra $(z_i - z_j)$ or $(\bar{z}_i - \bar{z}_j)$ factors that could not be paired to form $|z_i - z_j|$ factors. Since integration of such non–holomorphic terms results in a zero contribution to $I_4$, only the holomorphic (equivalently, “non–zero”) terms are included in \(4.37\).

The second term in the integrand of $I_4$ clearly dominates over the first term, except in the $z_6 \to \infty$ limit. Thus, for a first order of magnitude approximation, $I_4$ reduces to,

$$
I_4 = 2 \int d^2z_3d^2z_4d^2z_5d^2z_6 \left\{ |z_3|^{5/4}|z_4| \right\}
\frac{\left|z_\infty - z_3| |z_\infty - z_5|^2 |z_\infty - z_6|^2 |z_3 - z_4| |z_3 - z_5| |z_4|}{\left|z_\infty - z_3| |z_\infty - z_5|^2 |z_\infty - z_6|^2 |z_3 - z_4| |z_5 - z_4| \right\}
\times \frac{1}{(\left|1 - z_3| |z_6\right|)^{3/2}(\left|1 - z_6| |z_3 - z_6\right|)^{3/4}|z_4 - z_5|^4|z_4 - z_6|^2} \right. \right)
(4.38)

Several poles and fixed factors of infinity (i.e. of $z_\infty$) are found in the integrand, making evaluation difficult at best. This could indicate that the spacetime momentum should not be set to zero before performing the integral.

General consideration of \(4.38\) might appear to imply something contrary to that expected of a seventh order coupling constant. For a weakly coupled model, when $n - 3$ fields acquire VEVs in an $n$th order term, we expect the magnitude of the effective third order coupling constant to be far below unity, i.e., to produce strongly suppressed terms. However, integration over a space with infinite limits of a positive–semidefinite integrand involving several poles makes a finite result difficult to imagine. In this case, the resolution to obtaining a finite value seems to rest on the infinities in the denominator. The approach to computing the integral is, perhaps, not to actually integrate, but to examine the divergent areas and expand about them. Clearly the most highly divergent region is near $z_4 = z_5$, but the zeros from the $1/z_\infty$ factors might be enough to damp out the four resulting infinities.

What value of $I_4$ is required of the seventh order down term to satisfy the physical $m_s : m_t \approx 10^{-3}$? The coupling constants of the third order mass terms are all $A_3 = g,$
Thus, a correct order of magnitude mass ratio requires,

\[
\frac{A_7}{A_3} \left< \Phi_{23} \Phi_{56}^* H_{15}^s H_{30}^s \right> = \frac{1}{\sqrt{2}} \left(\sqrt{8/\pi}\right)^4 C_4 I_4 \frac{\left< \Phi_{23} \Phi_{56}^* H_{15}^s H_{30}^s \right>}{(M_{Pl})^4} \\
\approx 10^{-3}.
\] (4.39)

Insertion of the FD2 VEVs in (4.39) results in

\[
C_4 I_4 \approx 43.
\] (4.40)

In NAHE class $Z_2 \times Z_2$ models, $I_1$ and $I_2$ take on respective values typically around 70 and 400. Furthermore, $I_n$ generally increases with $n$. Thus, (4.33) is in disagreement with the probable magnitude of $I_4 >> 400$, given that $C_4$ is 0(1). However, if the seventh order down mass term contains only the massive eigenstate $h^m$ (i.e. equal couplings) then (4.16) suggests the second generation results from a tenth order term. The constraint on the associated $I_7$ becomes,

\[
\frac{A_{10}}{A_3} \left< \Phi_{23} \Phi_{56}^* H_{15}^s H_{30}^s H_{31}^s \right> = \frac{1}{\sqrt{2}} \left(\sqrt{8/\pi}\right)^7 C_7 I_7 \frac{\left< \Phi_{23} \Phi_{56}^* H_{15}^s H_{30}^s H_{31}^s \right>}{(M_{Pl})^7} \\
\approx 10^{-3}.
\] (4.41)

This implies $I_7 \approx 200,000$. While extrapolating a pattern for values of $I_n$ based only on the known ranges of $I_4$ and $I_5$ in FNY is untenable, a value of $I_7$ in this range appears possible. Thus, it is phenomenologically preferable that for the seventh order down term $h'$ reduces to the the massive $h^M$ eigenstate.

4.4 $F$–constraints from vector pairs

The FD2' and FD2'V variations of FD2, in which vector partners of FD2 fields $\Phi_{56}$ and of both $\Phi_{56}$ and $\Phi_{12}$, respectively, also take on VEVs, result in the more phenomenologically viable mass matrices. Note especially that if $\Phi_{56}'$ does not acquire a VEV, then no new up, down, or electron mass terms are produced by the FD2 class models. The appearance of these vector partner VEVs does generate some new and dangerous (low order) non–zero $F$–terms,

\[
< \partial W / \partial \Phi_{13} > = g < \Phi_{12} \Phi_{23} > \tag{4.42}
\]
\[
< \partial W / \partial \Phi_{13}^* > = \lambda_7 < \Phi_{56}' \Phi_{56}^* H_{30}^s H_{15}^s H_{31}^s > / (M_{Pl})^4 \tag{4.43}
\]
\[
< \partial W / \partial H_{16}^s > = g < \Phi_{56}^* H_{15}^s > \tag{4.44}
\]
along with a non–zero $< W >$ term,

\[
\lambda_8 < \Phi_{12} \Phi_{56}' \Phi_{23} \Phi_{4}^* H_{15}^s H_{30}^s H_{31}^s > / (M_{Pl})^5 . \tag{4.45}
\]

\*In strongly coupled models, for which substitution of $M_{string} \ll M_{Pl}$ in (4.39) might be justified, an even smaller $C_4 I_4$ value is found.
Examination of the FNY superpotential indicates that, when $\Phi'_{56}$ and $\Phi_{12}$ acquire VEVs, survival of spacetime supersymmetry down to the EW scale requires VEVs for some NA fields as well. No other singlet terms can lead to cancellation of $F$–term components. Particularly worrisome is (4.45) when $<\Phi_{12}> \neq 0$. Cancellation of all of the first derivatives of (4.45) may necessitate that several NA fields acquire VEVs. However, other phenomenological issues, such as the Higgs mass scale, also imply that $\Phi_{12}$ should not acquire an FI–scale VEV. In the case where $<\Phi_{12}> = 0$ or is extremely small, the dangerous terms reduce to (4.43), (4.44), and

$$<\partial W/\partial \Phi_{12} > = \lambda_8 <\Phi'_{56} \Phi_{23} \Phi'_{4} H_{15}^s H_{30}^s H_{31}^s > / (M_{Pl})^5 + \ldots (4.46)$$

Consider now (4.43). Inclusion of singlet and hidden sector terms in (4.43) through sixth order results in

$$\partial W/\partial H_{16}^s = g <\Phi'_{56} H_{15}^s > + \lambda_4 N_3 V_{37} H_{28} / (M_{Pl}) + \lambda_5 \Phi_{2} N_3 V_{37} H_{28} / (M_{Pl})^2 + \lambda_5 <\Phi'_{4} > N_3 V_{40} H_{28} / (M_{Pl})^2 + \lambda_6 <H_{30} > N_3 H_{29} V_{31} H_{20} / (M_{Pl})^3 (4.47)$$

A non–zero contribution to $<\partial W/\partial H_{16}^s >$ from $\Phi_{2} N_3 V_{37} H_{28}$ implies a larger contribution from $N_3 V_{37} H_{28}$. We should also expect the suppression factors for the $\lambda_5 <\Phi'_{4} > N_3 V_{40} H_{28}$ and $\lambda_6 <H_{30} > N_3 H_{29} V_{31} H_{20}$ terms to be too small to enable these terms to cancel $<\Phi'_{56} H_{15}^s >$. Thus, $N_3$, $V_{37}$, and $H_{28}$ must all receive FI–scale VEVs when $<\Phi'_{56} > \neq 0$. This is a viable solution because fourth order string couplings can actually be as large as third order couplings [13].

5 Conclusions

In this paper we investigated phenomenological issues of the four flat directions (and some of their variations) derived in [2] for the “FNY” model [3, 4]. Using our “stringent” $F$–flat requirements, these were the only directions found to be flat beyond seventh order that decoupled all fractionally charged MSSM exotics from the low energy superpotential. (It was as a serendipitous bonus that these four directions decoupled all MSSM–charged exotics.) Approximately 100 other VEV directions were found that removed the fractionally charged fields, but $F$–flatness for these was broken at only sixth or seventh order.

The first three of our flat directions are actually flat to all finite order, while $F$–flatness of the fourth is broken at twelfth order. Variations on these flat directions, in which both components of vector pairs of fields were allowed to acquire VEVs, were also investigated. The variations of $FD2$ were found to be the most phenomenologically viable with regard to mass matrices.

We examined, for the different flat directions, the inter–generational mass hierarchy for the MSSM quarks and leptons and possible effective $\mu$ terms for the MSSM.
Higgs. Reasonable, but not perfect, up, down, electron, and neutrino hierarchy–producing mass matrices were found for particular variations of our flat directions. Possible additional $F$–flatness constraints on the vector–pair flat directions were also examined.

An interesting aspect of the Higgs fields was mutually suggested by the mass matrices, the possible $\mu$ terms, and the additional $F$–flatness constraints. In the FNY field basis given in [3], the MSSM Higgs eigenstates $h$ and $\tilde{h}$ each have two components. For several phenomenological reasons, the $SU(2)_L$ doublet $\tilde{h}_1$ contribution to to $\tilde{h}$ must greatly outweigh $\tilde{h}_4$'s. Relatedly, it is only the $\tilde{h}_4$ component of $\tilde{h}$, and not the $\tilde{h}_1$ contribution, that couples to the second generation up field, ultimately giving mass to it. This results in an additional cosine suppression factor for the charm mass term. This is a favorable feature for this model, since the charm mass term originates at fourth order, while those for the strange and muon masses appear at seventh and eight order, respectively. Thus, without the extra cosine factor, this model would predict a several orders of magnitude mass difference between the charm and strange fields.

Our investigation of singlet flat directions led to several pieces of evidence suggesting that some non–Abelian fields must also acquire VEVs for the FNY model to produce viable phenomenology. The additional $F$–terms arising from FD1V and FD2$'$ especially implied this. In particular, for FD2$'$ it appears necessary for two $SU(2)_H$ doublets (along with a neutrino singlet) to also acquire FI–scale VEVs.

In forthcoming [20], we will generalize our current flat directions by permitting non–Abelian fields to acquire VEVs. The simplest non–Abelian extension to each of our current direction is the addition of the only completely chargeless product of hypercharge–singlet non–Abelian fields: $V_5V_{40}V_{10}V_{37}$. We shall also examine the additional flat directions available when our basis set of 24 nontrivial $D$–flat singlet directions increases to a basis set of 53 non–trivial directions involving the 30 hypercharge–singlet hidden sector non–Abelian fields in the FNY model.

We conclude by reemphasizing that the success of the FNY model in producing Minimal Superstring Standard Models should not be regarded as suggesting that the FNY model is the true string vacuum. Rather, it is our firm opinion that the success of the FNY model in this regard, together with the other properties of the free fermionic NAHE–based models, such as the natural emergence of three generations, can be regarded as providing evidence to the assertion that the true string vacuum will possess some of the characteristics shared by this class of models. Furthermore, the existence of highly realistic string models near a maximally symmetric point in the moduli space, provides further indication for the relevance of string theory in nature. The continued development of the techniques needed to seriously confront string models with experimental data is therefore imperative. The main focus of the present paper being the phenomenological analysis of exact flat directions.
6 Acknowledgments

This work is supported in part by DOE Grants No. DE-FG-0294ER40823 (AF) and DE-FG-0395ER40917 (GC,DVN,JW).
### A  $D$– and $F$–flat Directions

| FD# | \# VEVs | $O(W)$ breaking | $\{VEV_1\}$ | $\Phi_{56}$ | $\Phi'_{56}$ | $H_{19}^s$ | $H_{20}^s$ |
|-----|--------|-----------------|--------------|------------|-------------|----------|----------|
| 1   | $8+3$  | $\infty$       | *            | *          | 0           | 0        | 0        |
| 2   | $9+3$  | $\infty$       | *            | *          | 0           | 0        | 0        |
| 3   | $9+3$  | $\infty$       | *            | *          | *           | 0        | 0        |
| 4   | $9+3$  | 12              | *            | *          | *           | 0        | 0        |

Table I: Classes of $D$ & $F$–flat NA singlet directions that yield an MSSM low energy effective field theory.

The four classes are defined by their respective set of non–Abelian singlet field VEV components. The class identification number appears in column one. Column two specifies the number of singlet VEVs, with its first entry specifying the number of VEVs resulting from FI cancelation and its second entry indicating the number of $\Phi_4$–related fields to which additionally VEVs are assigned. Column three indicates the superpotential order at which flatness is broken. The *’s in the remaining columns identify the field VEVs. An * implies a given field takes on a VEV, while a $\bar{*}$ implies the vector partner does instead. A * below $\{VEV_1\}$ indicates that all fields in the set $\{\Phi_{12}, \Phi_{23}, (\Phi_4, \Phi'_4, \overline{\Phi}_4, \overline{\Phi'}_4), H_{30}^s, H_{38}^s, H_{15}^s, H_{31}^s\}$ acquire VEVs.

| FD# | $Q^{(A)}_{112}$ | $<\alpha>$ | $\{\Phi_{12}, \Phi_{23}, (\Phi_4, \Phi'_4), H_{30}^s, H_{38}^s, H_{15}^s, H_{31}^s\}$ | $\Phi_{56}$ | $\Phi'_{56}$ | $H_{19}^s$ | $H_{20}^s$ |
|-----|--------------|----------|---------------------------------|------------|-------------|----------|----------|
| 1   | -2           | $6.7 \times 10^{16}$ GeV | 3, 1, 1, 3, 2, 2, 1 1 0 0 0 |            |            |          |          |
| 2   | -6           | $3.9 \times 10^{16}$ GeV | 10, 2, 2, 8, 6, 4, 2 3 -1 0 0 |            |            |          |          |
| 3   | -4           | $4.8 \times 10^{16}$ GeV | 4, 3, 2, 8, 2, 6, 2 1 0 2 0 |            |            |          |          |
| 4   | -2           | $6.7 \times 10^{16}$ GeV | 3, 2, 3, 3, 4, 4, 3 2 0 0 2 |            |            |          |          |

Table II: Examples of flat directions from the four classes presented in Table I. Column one indicates the class to which an example direction belongs. Column two contains the anomalous charges $Q^{(A)}/112$ of the example flat directions. Column three specifies the overall common scale of the associated VEVs. The remaining column entries specify the ratios of the norms of the VEVs. $\{\Phi_{12}, \Phi_{23}, (\Phi_4, \overline{\Phi}_4, \overline{\Phi'}_4), H_{30}^s, H_{38}^s, H_{15}^s, H_{31}^s\} \equiv \{VEV_1\}$ form the set of fields possessing VEVs in all four directions. The third component VEV involves all of the $\Phi_4$–related states and is the net value of $|<\Phi_4>|^2 + |<\Phi'_4>|^2 - |<\overline{\Phi}_4>|^2 - |<\overline{\Phi'}_4>|^2$. E.g., a “1” in the $\Phi_4$ column for FD1 specifies that $|<\Phi_4>|^2 + |<\Phi'_4>|^2 - |<\overline{\Phi}_4>|^2 - |<\overline{\Phi'}_4>|^2 = 1 \times <\alpha>^2$. 

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B Field content for flat directions

Massless Fields for flat direction 1:

- 3 generations of MSSM fields: \((Q, u^c, d^c, L, e^c, N^c)_{i=1,2,3}\)
- 2 MSSM Higgs:
  \[
  h \equiv \frac{1}{\sqrt{\langle \Phi_{12} \rangle^2 + \langle \Phi_{23} \rangle^2}} (\Phi_{23} > h_1 - \Phi_{12} > h_3),
  \]
  \[
  \bar{h} \equiv \frac{1}{\sqrt{\langle \Phi_{12} \rangle^2 + \langle H^{s}_{31} \rangle^2}} (H^{s}_{31} > \bar{h}_1 - \Phi_{12} > H^{s}_{34}),
  \]
- 21 Non-Abelian Singlets: \(\Phi_2, \Phi_3, \Phi_{12}, \Phi_{23}, \Phi^{(1)}_{23}, \Phi^{(2)}_{23}, \Phi^{(3)}_{23}\)
- 2 Hidden Sector \(SU(3)_H\) (Anti)-Triplets: \(V_{4, 13}\)
- 6 Hidden Sector \(SU(2)_H\) Doublets: \(H_{23}, H_{26}, V_{5, 15}, V_{39}, V_{40}\)
- 6 Hidden Sector \(SU(2)_{H'}\) Doublets: \(H_{25}, H_{28}, V_{10, 19}, V_{35}, V_{37}\)
Fields with (near) FI–scale masses for flat direction 1:

(i) Exotic MSSM-Charged States

- \( SU(3)_C \) Triplets: \( H_{33}, H_{40} \)
- \( SU(2)_L \) Doublets: \( h_2, \tilde{h}_2, h_3 \)

\[
h^M \equiv \frac{1}{\sqrt{|<\Phi_{12}>|^2 + |<\Phi_{23}>|^2}} <\Phi_{12} > h_1 + <\Phi_{23} > h_3 ,
\]

\( \tilde{h}_4 \equiv H_{34}, h_4 \equiv H_{41}, V_{45}, V_{46}, V_{51}, V_{52} \)

- Hidden Sector \( SU(2)_H \) Doublets (with fractional electric charge): \( H_1, H_2 \)
- Hidden Sector \( SU(2)_{H'} \) Doublets (with fractional electric charge): \( H_{11}, H_{13} \)

(ii) Non-Abelian Singlets

- \( \Phi_1, \bar{\Phi}_{12}, \Phi_{13} \)

\[
\bar{\Phi}_{13}^M \equiv \frac{1}{\sqrt{|<H_{30}>|^2 + |<X>|^2}} (<H_{30} > \Phi_{13} + <X> > H_{36}^s)
\]

\[
H_{36}^{s*} M \equiv \frac{1}{\sqrt{|X| X}} (<X > \bar{\Phi}_{13} - <H_{30} > H_{36}^s)
\]

\[
\bar{\Phi}_{23}^M \equiv \frac{1}{\sqrt{|<H_{30}>|^2 + |<X>|^2}} (<X > \bar{\Phi}_{23} + (<H_{30} > + <X>)H_{30} + (<X > <\Phi_{12} >/ <H_{30} >)H_{32} >)
\]

\[
H_{37}^{s*} M \equiv \frac{1}{\sqrt{|X| X}} (<X > \bar{\Phi}_{23} - <H_{30} > H_{37}^s)
\]

\[
\Phi_4^M \equiv \frac{1}{\sqrt{|<\Phi_4>|^2 + |<\Phi_4'>|^2 + |\bar{\Phi}_4|^2 + |\bar{\Phi}_4'>|^2}} <\Phi_4 > \Phi_4 + <\Phi_4' > \bar{\Phi}_4 + <\bar{\Phi}_4 > \Phi_4 + <\bar{\Phi}_4' > \bar{\Phi}_4
\]

\( \Phi_{56}, H_{5}^s, H_{4}^s, H_{5}^s, H_{6}^s, H_{7}^s, H_{8}^s, H_{9}^s, H_{10}^s, H_{16}^s, H_{17}^s, H_{18}^s, H_{21}^s, H_{22}^s, H_{32}^s, H_{37}^s, H_{39}^s, V_{2}^s, V_{11}^s, V_{21}^s, V_{22}^s, V_{41}^s, V_{42}^s, V_{43}^s, V_{44}^s, V_{47}^s, V_{48}^s, V_{49}^s, V_{50}^s \)

(iii) Hidden Sector Non-Abelian Fields:

- \( SU(3)_H \) (Anti)-Triplets: \( H_{32}, V_{14}, V_{24}, V_{34}, H_{35}, V_{3}, V_{23}, V_{33} \)
- \( SU(2)_H \) Doublets: \( V_7, V_{17}, V_{25}, V_{27} \)
- \( SU(2)_{H'} \) Doublets: \( V_9, V_{20}, V_{29}, V_{30} \)
Effective FI-scale tadpole in FD1 superpotential:

\[ \Phi^1[\Phi_4 \Phi_4^\dagger] + \Phi_4^\dagger \Phi_4 \] (B.1)

Effective FI–scale mass terms in FD1 superpotential:

**W_3** terms:

\[ \Phi^1[\Phi_4 \Phi_4^\dagger] + \Phi_4^\dagger \Phi_4 + \Phi_4^\dagger \Phi_4^\dagger + \Phi_4 \Phi_4^\dagger + \Phi_4 \Phi_4^\dagger \] +
\[ \Phi_{12} < h_1 \tilde{h}_2 + H_{36}^s H_{37}^s + V_{21}^s V_{22}^s + V_{29} V_{30} + V_{25} V_{27} + V_{23} V_{24} > +
\[ \Phi_{23} [\Phi_{13}^2 + h_3 \tilde{h}_2 + V_{33} V_{34}] +
\[ \Phi_4 [V_{45} V_{46} + H_1 H_2] + \Phi_4^\dagger [V_{51} V_{52} + H_8^s H_8^s + H_{10}^s H_{10}^s] +
\[ \Phi_4^\dagger [H_3^s H_4^s + H_5^s H_6^s + V_{41} V_{42}^s + V_{43} V_{44}^s] + \Phi_4^\dagger [V_{47}^s V_{48}^s + V_{49}^s V_{50}^s + H_{11} H_{13}] +
\[ \Phi_5^\dagger [H_{17}^s H_{18}^s + H_{21} H_{22}^s] +
\[ H_{15}^s \Phi_5 H_{16}^s + H_{30}^s \Phi_{13} H_{29}^s + H_{31}^s h_2 \tilde{h}_4 + H_{38}^s H_{41} \tilde{h}_3 ] (B.2)

**W_4** terms:

\[ < H_{15}^s H_{30}^s > [V_3 V_{14} + V_7 V_{17}] \] (B.3)

**W_5** terms:

\[ < \Phi_{23} H_{31}^s H_{38}^s > [h_4 H_{41} + H_{33} H_{40}] + < \Phi_4^\dagger H_{15}^s H_{30}^s > [V_9 V_{11}^s + V_9 V_{20}] +
\[ < \Phi_{56} H_{31}^s H_{38}^s > [H_{32}^s H_{39}^s + H_{35} H_{42}] \] (B.4)

**W_6** terms:

\[ < \Phi_{23} \Phi_{56} H_{31}^s H_{38}^s > H_{29}^s H_{36}^s \] (B.5)
Additional Fields receiving (near) FI–scale masses for other flat directions:

| fd # | non–FD1 VEVs | New mass and tadpole terms |
|------|--------------|----------------------------|
| 1V   | \( \Phi_{12} \) | \(< \Phi_{12} > [\Phi_{13}\Phi_{23} + h_2 h_1], < \Phi_{12}\Phi_{23} > \Phi_{13} \) |
| 2    | \( \Phi'_{56} \) | \(< \Phi'_{56} > H^s_{19}H^s_{20} \) |
| 2V   | \( \Phi'_{56}, \Phi_{12} \) | - |
| 2'   | \( \Phi'_{56}, \Phi'_{56} \) | \(< \Phi'_{56} > H^s_{15}H^s_{16}, < \Phi'_{56}H^s_{15} > H^s_{16} \) |
| 2'V  | \( \Phi'_{56}, \Phi'_{56}, \Phi_{12} \) | - |
| 3    | \( H^s_{19} \) | \(< H^s_{19} > \Phi'_{56}H^s_{20}, < H^s_{19}H^s_{21} > V_{19}V_{37}, < \Phi'_{4}H^s_{19}H^s_{15} > V_{15}V_{40}, < \Phi'_{56}H^s_{15}H^s_{19} > H^s_{15}H^s_{22} \) |
| 3V   | \( H^s_{19}, \Phi_{12} \) | - |
| 4    | \( H^s_{20} \) | \(< H^s_{20} > \Phi'_{56}H^s_{19}, < H^s_{15}H^s_{20}H^s_{31}H^s_{38} > V_{41}V_{50}, < \Phi'_{4}H^s_{15}H^s_{20}H^s_{30}H^s_{31} > N_1V_{31}^s \) |
| 4V   | \( H^s_{20}, \Phi_{12} \) | - |

The first column denotes the flat direction. A “V” in this column indicates a vector–like partner to one of the standard fields associated with a given flat direction is also allowed a VEV. The listings in column two indicate a flat direction’s VEVs that are not present in flat direction 1. Column three lists the VEV–induced mass terms new to the respective flat directions and not present in flat direction 1. If a mass term for a given field is already present in a standard flat direction, respective higher order mass terms (should they exist) are not necessarily listed.

Rotated massless (with “(1)” superscript) and FI–scale massive (with “M” superscript) eigenstates for non–FD1 directions:

- **FD1V (and all embeddings):**

  \[ \Phi^{(1)}_1 \equiv \frac{1}{\sqrt{|< \Phi_{12}|^2 + |< \Phi_{23}|^2|}} (< \Phi_{12} > \Phi_{12} - < \Phi_{23} > \Phi_{23}) \]

  \[ \Phi^{M}_1 \equiv \frac{1}{\sqrt{|< \Phi_{12}|^2 + |< \Phi_{23}|^2|}} (< \Phi_{23} > \Phi_{12} + < \Phi_{12} > \Phi_{23}) \]

- **FD2’, FD2’V:**

  \[ \Phi^{(1)}_56 \equiv \frac{1}{\sqrt{|< \Phi'_{56}|^2 + |< H^s_{15}|^2|}} (< \Phi'_{56} > \Phi'_{56} - < H^s_{15} > H^s_{15}) \]

  \[ \Phi^{M}_56 \equiv \frac{1}{\sqrt{|< \Phi'_{56}|^2 + |< H^s_{15}|^2|}} (< H^s_{15} > \Phi'_{56} + < \Phi'_{56} > H^s_{15}) \]
C Superpotentials for Low–Energy Effective Field Theories for Flat Directions

Flat direction 1 Low Energy Effective Superpotential, \( W^{FD1} \):

**Singlet Terms**

\( W_3: \)

\[ g[\bar{\Phi}^6_{56}H_{19}^sH_{20}^s + \bar{\Phi}_{23}V_{31}^sV_{32}^s] \] (C.1)

**MSSM Terms**

\( W_3: \)

\[ g\bar{h}_1[Q_1u_1^c + L_1N_1^c] + gh_3[Q_3d_3^c + L_3e_3^c] + \]
\[ <\Phi_1^sH_{15}^sH_{30}^sH_{31}^s > N_1^cH_{20}^sV_{31}^s \] (C.2)

\( W_4: \)

\[ <H_{30}^s + (\Phi_4\Phi_4 + \Phi_4^s\Phi_4)H_{30}^s > [h_1Q_3d_3^c + h_1L_3e_3^c]H_{29}^s + \]
\[ <\Phi_{12} + (\Phi_4\Phi_4 + \Phi_4^s\Phi_4)\Phi_{12} > [Q_1Q_2u_1^c d_2^c + Q_1u_1^c L_2e_1^c + Q_1u_1^c L_2e_1^c + Q_2d_1^c L_1N_1^c + \]
\[ Q_2d_1^c L_1N_1^c + L_1L_1e_1^cN_1^c + \]
\[ <\Phi_{12}\Phi_{4}^s > [Q_1Q_1Q_2L_2 + Q_1Q_1u_2^c d_2^c + \]
\[ <H_{15}^sH_{30}^s > [Q_1Q_3Q_3L_3 + Q_1Q_2u_2^c d_2^c + Q_1Q_3Q_3L_3e_3^c + Q_1u_2^c L_2e_3^c + Q_1u_2^c L_2e_3^c] + \]
\[ <\Phi_4^sH_{15}^sH_{38}^s > [Q_2d_3^c L_3 + L_2L_3e_2^c]V_{32}^s + <\Phi_4^sH_{15}^sH_{38}^s > [Q_2d_3^c L_2 + u_2^c d_2^c]V_{32}^s + \]
\[ <\Phi_4^sH_{15}^sH_{38}^s > [Q_3u_1^c L_3e_3^c + Q_3u_2^c L_3e_3^c + u_1^c u_2^c d_2^c + u_1^c u_2^c d_2^c + u_2^c u_3^c d_3^c] + \]
\[ <\Phi_4^sH_{15}^sH_{38}^s > [Q_2Q_3u_3^c d_3^c + Q_2u_3^c L_3e_3^c + Q_2u_3^c L_3e_3^c] + \]
\[ <\Phi_{23}^sH_{15}^sH_{30}^s > [Q_1Q_2Q_2L_2 + Q_1Q_2u_2^c d_2^c + Q_1u_2^c L_2e_2^c] \] (C.3)

\( W_5: \)

\[ [1 + <\Phi_4^s\Phi_4 + \Phi_4^s\Phi_4>]h_3Q_2d_2^c + h_3L_2e_2^c]V_{31}^sV_{32}^s + \]
\[ <\Phi_{12} > [h_3Q_2d_2^c V_{12}^sV_{12}^s + Q_1Q_3u_1^c d_3^c \bar{\Phi}_{23} + Q_1u_3^c L_3e_1^c \bar{\Phi}_{23} + Q_3d_3^c L_1N_1^c \bar{\Phi}_{23} + \]
\[ Q_3d_3^c L_1N_1^c \bar{\Phi}_{23} + L_1L_3e_3^cN_1^c \bar{\Phi}_{23}] + \]
\[ <H_{15}^s > [Q_2d_2^c L_1 + L_1L_2e_3^c]V_{19}^s + \]
\[ <H_{30}^s > [Q_1Q_3u_1^c d_3^c + Q_1u_1^c L_3e_3^c + Q_1u_3^c L_3e_3^c + Q_3d_1^c L_1N_1^c + \]
\[ Q_3d_1^c L_1N_1^c + L_1L_3e_3^cN_1^c]H_{29}^s + \]
\[ <H_{38}^s > [\bar{h}_1N_3^c \bar{\Phi}_{56}^sH_{29}^s + (Q_1d_2^c L_3 + Q_1d_2^c L_3)H_{29}^sV_{32}^s + \]
\[ <\Phi_{12} \Phi_{23} > [Q_1Q_2u_1^c d_2^c + Q_1u_1^c L_2e_2^c + Q_1u_2^c L_2e_2^c + Q_2d_1^c L_1N_1^c + Q_2d_2^c L_1N_1^c + L_1L_2N_1^c e_2^c \bar{\Phi}_{23} + \]
\[ <\Phi_{12} \Phi_4 > [Q_3u_1^c L_3e_3^c + Q_3d_1^c L_3N_1^c + u_1^c u_1^c d_3^c e_3^c + u_1^c u_3^c d_3^c e_3^c + u_3^c d_1^c d_3^c N_1^c + u_3^c d_1^c d_3^c N_1^c] \bar{\Phi}_{23} + \]

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\[ \Phi_{12} \Phi_1 > [Q_1 Q_1 Q_3 L_3 + Q_1 Q_1 d_3 u_3^c + L_1 L_1 e_3^c N_3^c] \Phi_{23} + \]
\[ \Phi_{12} \Phi_4 > h_3 L_1 e_2^c V_{12}^s + \]
\[ \Phi_{12} \Phi_{56} > [Q_3 Q_2 u_3^c d_3 + Q_1 u_1^c L_2 e_2^c + Q_2 d_1^c L_1 N_2^c + Q_2 d_2^c L_1 N_1^c + L_1 L_2 e_3^c e_2^c] \Phi_{56} + \]
\[ \Phi_{12} H_{38}^s > [Q_3 d_2^c L_1 N_2^c + L_1 L_2 e_3^c N_2^c] H_{20}^s + \]
\[ \Phi_{23} H_{30}^s > [h_1 Q_3 d_3^c \Phi_{23} + h_3 L_1 e_2^c \Phi_{23} + Q_1 Q_2 u_1^c d_2^c + Q_1 u_1^c L_2 e_2^c + Q_1 u_2^c L_2 e_1^c + Q_2 d_1^c L_1 N_2^c + Q_2 d_2^c L_1 N_1^c + L_1 L_2 e_3^c e_2^c] H_{20}^s + \]
\[ \Phi_{4} H_{30}^s > [Q_3 u_3^c L_3 e_1^c + Q_3 d_2^c L_3 N_1^c + L_1 L_1 e_3^c e_1^c + u_1^c u_2^c d_3 e_1^c + u_1^c u_2^c d_3 e_2^c + u_1^c u_2^c e_3 d_3 N_1^c + u_1^c u_2^c e_3 d_3 N_2^c] H_{20}^s + \]
\[ \Phi_{4} H_{38}^s > [h_1 L_1 N_1^c \Phi_{56} V_{31}^s + L_2 L_2 e_1^c H_{20}^s V_{32}^s] + \]
\[ \Phi_{4} H_{15}^s > [Q_2 d_1^c L_2 + u_2^c d_1^c d_2^c] H_{19}^s V_{32}^s + \]
\[ \Phi_{4} H_{38}^s > u_1^c u_2^c d_3 H_{29}^s V_{32}^s + \]
\[ \Phi_{56} H_{30}^s > h_1 [Q_3 d_3^c + L_3 e_3^c] \Phi_{56} H_{20}^s + \]
\[ H_{15} H_{30}^s > [Q_1 Q_3 Q_3 L_3 + Q_1 Q_1 d_3^c u_3^c + L_1 L_1 e_3^c N_3^c] H_{20}^s + \]
\[ H_{15} H_{30}^s > [Q_1 Q_2 Q_4 L_3 + Q_1 Q_2 d_3^c u_3^c + Q_1 Q_2 u_2^c d_3^c + Q_1 u_2^c L_3 e_3^c + L_2 L_2 e_3^c] \Phi_3 + \]
\[ H_{15} H_{30}^s > [Q_1 d_1^c L_3 N_3^c + Q_1 d_3^c L_2 N_3^c + Q_1 d_3^c L_3 N_2^c] \Phi_{56} \] (C.4)

\[ W_6: \]
\[ h_3 h_1 [Q_1 Q_1 Q_3 L_3 + Q_1 Q_1 u_3^c d_3 + L_1 L_1 e_3^c N_3^c] + \]
\[ [Q_1 d_1^c L_2 + L_1 L_2 e_1^c] H_{19}^s H_{20}^s V_{32}^s + \]
\[ < H_{30}^s > h_1 h_1 [Q_3 d_3^c + L_3 e_3^c] H_{29}^s + \]
\[ c_4 h_1 [Q_2 Q_3 d_3^c + Q_2 d_2^c L_3 e_3^c + Q_3 d_2^c L_2 e_3^c + Q_3 d_2^c L_2 e_3^c + Q_3 d_2^c L_2 e_3^c + Q_3 d_2^c L_2 e_3^c + L_2 L_2 e_3^c] + \]
\[ c_4 h_1 [Q_1 Q_2 u_1^c d_2^c + Q_1 u_1^c L_2 e_2^c + Q_1 u_2^c L_2 e_1^c + Q_2 d_1^c L_1 N_2^c + \]
\[ Q_2 d_1^c L_1 N_1^c + L_1 L_2 e_3^c N_1^c] + \]
\[ < \Phi_4 > h_3 h_1 [Q_1 Q_1 Q_3 L_3 + Q_1 Q_1 u_3^c d_3 + L_1 L_1 e_3^c N_3^c] + \]
\[ < \Phi_4 > h_3 h_1 [Q_3 u_3^c L_3 e_1^c + Q_3 d_2^c L_3 N_1^c + u_3^c d_1^c d_3^c N_3^c + u_3^c d_1^c d_3^c N_2^c + u_3^c d_1^c d_3^c N_3^c + u_3^c d_1^c d_3^c N_2^c] + \]
\[ c_4 h_1 [Q_2 d_2^c + L_2 e_2^c] \Phi_{23} V_{31}^s V_{32}^s + \Phi_{56} H_{19}^s H_{20}^s + \]
\[ < \Phi_{56} > [h_3 Q_2 d_2^c + h_3 L_2 e_2^c] \Phi_{56} V_{31}^s V_{32}^s + \]
\[ < H_{30}^s > [(h_1 Q_2 d_2^c + h_1 L_2 e_2^c) V_{31}^s V_{32}^s + (h_1 Q_3 d_3^c + h_1 L_3 e_3^c) (\Phi_2 \Phi_2 + \Phi_3 \Phi_3)] H_{20}^s + \]
\[ < H_{30}^s > [h_1 L_1 N_1^c \Phi_2 \Phi_{56} H_{20}^s] + \]
\[ < \Phi_{12} > [Q_1 Q_2 u_1^c d_2^c + Q_1 u_1^c L_2 e_2^c + Q_1 u_2^c L_2 e_1^c + \]
\[ Q_2 d_1^c L_1 N_2^c + Q_2 d_1^c L_1 N_1^c + L_1 L_2 e_3^c N_1^c] (\Phi_2 \Phi_2 + \Phi_3 \Phi_3) + \]
\[ [< \Phi'_4 > u_1^c d_1^c d_2^c + < \Phi'_4 > Q_1 d_1^c L_1] H_{19}^s H_{20}^s V_{32}^s + \]
\[ < H_{15} H_{30}^s > [(Q_2 d_2^c L_1 + L_1 L_2 e_2^c) \Phi_2 + (Q_3 d_3^c L_1 + L_1 L_3 e_3^c) \Phi_{23}] H_{19}^s V_{32}^s + \]
\[ < H_{38} > [Q_1 d_2^c L_3 + Q_1 d_3^c L_2] \Phi_3 H_{20}^s V_{32}^s \] (C.5)
Mixed MSSM–Hidden Terms

\(W_5:\)

\[
< \Phi_{12} > [h_3 Q_2 d_1^c V_{10} V_{19} + h_3 L_1 e_2^c (V_4 V_{13} + V_5 V_{15})] + \\
< H^s_{31} > h_1 \tilde{h}_1 H^s_{19} V_{19} V_{37} + \\
< \Phi^*_{4} H^s_{30} > h_1 h_1 V_{31}^s H_{23} V_{39} + < \Phi^*_{4} H^s_{31} > h_1 \tilde{h}_1 H^s_{19} V_{15} V_{40} + \\
< \Phi_{12} \Phi^*_{4} > h_3 Q_2 d_1^c [V_4 V_{13} + V_5 V_{15}] + < \Phi_{12} \Phi^*_{4} > h_3 L_1 e_2^c V_{10} V_{19} + \\
< \Phi_{12} H^s_{38} > [(Q_1 d_3^r L_3 + Q_1 d_3^r L_2) H_{25} V_{35} + (u_1^c d_2^c d_3^c + L_2 L_3 e_1^r) H_{24} V_{39}] + \\
< \Phi_{23} H^s_{30} > N_2^c [H_{26} H_{26} H_{25} V_{19} + H_{26} V_{15} H_{25} H_{28} + H_{26} H_{26} H_{28} V_{19} + H_{25} V_{15} H_{28} H_{28}] + \\
< H^s_{30} H^s_{30} > N_2^c \Phi_{56} V_{31}^s V_{19} V_{35} + \\
< H^s_{30} H^s_{38} > [h_1 Q_1 u_3^c V_{19} V_{35} + \bar{h}_1 Q_3 u_1^c V_{15} V_{39}] + \\
< H^s_{31} H^s_{38} > [\bar{h}_1 L_1 N_3^c V_{19} V_{37} + \bar{h}_1 L_3 N_1^c V_{15} V_{40} + \\
< H^s_{38} H^s_{38} > \bar{h}_1 L_2 V_{32}^s V_{15} V_{39} \quad \text{(C.6)}
\]

\(W_6:\)

\[
[Q_3 d_2^c L_3 + Q_3 d_3^c L_2 + u_3^c d_2^c d_3^c + L_2 L_3 e_3^c] H^s_{19} H_{25} V_{37} + \\
< \Phi_{12} > [(Q_1 d_1^r L_2 + L_1 L_2 e_1^r) H^s_{19} H_{25} V_{35} + u_1^c d_1^c d_2^c H^s_{19} H_{23} V_{39} + \\
h_3 Q_2 d_1^c \Phi_{3} V_{10} V_{19} + h_3 L_1 e_2^c \Phi_{3} V_{5} V_{15}] + \\
< \Phi_{23} > [Q_2 d_2^c L_3 + u_2^c d_2^c d_3^c + L_2 L_2 e_2^c] H^s_{19} H_{25} V_{37} + \\
[< \Phi^*_{4} > (Q_3 d_2^c L_3 + u_3^c d_2^c d_3^c)] H^s_{19} H_{23} V_{40} + \\
< H^s_{15} > h_3 Q_1 d_3^c V_{32}^s H_{28} V_{10} + \\
< H^s_{38} > \bar{h}_1 L_3 [V_4 V_{13} H_{23} V_{40} + H_{23} V_{5} V_{15} V_{40} + H_{25} V_{10} V_{19} V_{37} + V_1^s V_{12} H_{25} V_{37}] \quad \text{(C.7)}
\]
Hidden Terms

$W_3$:

\[
\begin{align*}
\langle H_{30}^s \rangle + \langle \Phi_4 \Phi_4 H_{30}^s \rangle + \langle \Phi_4 \Phi_4 H_{30}^s \rangle \rangle V_{31}^s H_{25} V_{35} + \\
\langle H_{31}^s \rangle + \langle \Phi_4 \Phi_4 H_{31}^s \rangle + \langle \Phi_4 \Phi_4 H_{31}^s \rangle \rangle H_{19} V_{19} V_{37} + \\
\langle \Phi_4 H_{31}^s \rangle + \langle \Phi_4 \Phi_4 \Phi_4 H_{31}^s \rangle + \langle \Phi_4 \Phi_4 \Phi_4 H_{31}^s \rangle + \langle \Phi_4 \Phi_4 \Phi_4 H_{31}^s \rangle \rangle H_{19} V_{15} V_{40} + \\
\langle \Phi_4 H_{30}^s \rangle + \langle \Phi_4 \Phi_4 \Phi_4 H_{30}^s \rangle + \langle \Phi_4 \Phi_4 \Phi_4 H_{30}^s \rangle + \langle \Phi_4 \Phi_4 \Phi_4 H_{30}^s \rangle \rangle V_{31}^s H_{23} V_{39}
\end{align*}
\]

\( (C.8) \)

$W_4$:

\[
\begin{align*}
H_{37}^s V_{32}^s H_{26} V_{40} + \\
\langle \Phi_4 > H_{37}^s V_{32}^s H_{26} V_{37} + \langle H_{30}^s > \Psi_3 V_{31}^s H_{25} V_{35} + \\
\langle \Phi_2 \Phi_5 \Phi_6 \rangle [H_{23} H_{23} H_{28} H_{28} + H_{23} H_{26} H_{25} H_{28} + H_{26} H_{26} H_{25} H_{25}] + \\
\langle H_{31}^s \rangle + \langle \Phi_4 H_{31}^s \rangle + \langle \Phi_5 \Phi_6 \rangle H_{19} V_{19} V_{37} + \\
\langle \Phi_4 H_{30}^s \rangle + \langle \Phi_4 \Phi_4 H_{30}^s \rangle \rangle V_{31}^s H_{23} V_{39} + \\
\langle H_{15}^s H_{30}^s \rangle + \langle H_{20}^s V_{31}^s H_{25} V_{40} + \\
\langle \Phi_4 \Phi_4 H_{30}^s \rangle + \langle \Phi_4 \Phi_4 H_{30}^s \rangle \rangle \Psi_3 V_{31}^s H_{25} V_{35} + \\
\langle \Phi_4 \Phi_4 \Phi_4 \Phi_4 H_{31}^s \rangle + \langle \Phi_4 \Phi_4 \Phi_4 \Phi_4 H_{31}^s \rangle \rangle H_{19} V_{15} V_{40} + \\
\langle \Phi_4 H_{15}^s H_{30}^s \rangle + \langle H_{20}^s V_{31}^s H_{23} V_{5} + \langle \Phi_4 H_{15}^s H_{30}^s \rangle + \langle H_{20}^s V_{31}^s H_{23} V_{5}
\end{align*}
\]

\( (C.9) \)

$W_5$:

\[
\begin{align*}
\Phi_2 H_{37}^s V_{32}^s H_{26} V_{40} + \\
\langle H_{30}^s > (\Phi_2 \Phi_3 + \Psi_3 \Phi_3) V_{31}^s H_{25} V_{35} + \langle H_{31}^s > (\Phi_2 \Phi_2 + \Phi_3 \Phi_3) H_{19} V_{19} V_{37} + \\
\langle \Phi_4 H_{30}^s \rangle (\Phi_2 \Phi_2 + \Phi_3 \Phi_3) V_{31}^s H_{25} V_{35} + \langle \Phi_4 H_{30}^s \rangle (\Phi_2 \Phi_2 + \Phi_3 \Phi_3) H_{19} V_{15} V_{40} + \\
\langle H_{15}^s H_{30}^s \rangle \Phi_2 H_{20}^s V_{31}^s H_{25} V_{40} + \langle H_{20}^s V_{31}^s H_{23} V_{35} + \langle \Phi_5 \Phi_6 H_{30}^s \rangle (\Phi_2 \Phi_3) V_{31}^s H_{25} V_{35}
\end{align*}
\]

\( (C.10) \)

$W_6$:

\[
\begin{align*}
\langle \Phi_4 V_{32}^s V_{32}^s [H_{26} H_{26} H_{25} H_{25} + H_{23} H_{26} H_{25} H_{28} + H_{23} H_{23} H_{28} H_{28}] + \\
\langle H_{30}^s > (\Phi_2 \Phi_2 \Phi_3 + \Phi_3 \Phi_3 \Phi_3) V_{31}^s H_{25} V_{35}
\end{align*}
\]

\( (C.11) \)
Flat direction 1V modifications to superpotential, $W^{FDIV}$:

**Singlet Terms**

$W_3$:

$$[<\bar{\Phi}_{12}H^s_{30}> + <\Phi_4\bar{\Phi}_{12}\Phi_{12}H^s_{30}> + <\bar{\Phi}_{12}\Phi_4\Phi_{12}H^s_{30}> + <\bar{\Phi}_{12}\Phi_4\Phi_{12}H^s_{30}>]H^s_{29}V^s_{31}V^s_{32}$$

(C.12)

$W_4$:

$$<\bar{\Phi}_{12}\Phi_{23}H^s_{30}> [\bar{\Phi}_{23}H^s_{29}V^s_{31}V^s_{32} + \bar{\Phi}_{56}H^s_{19}H^s_{29}H^s_{30}] + <\bar{\Phi}_{12}\Phi_56H^s_{30}> \Phi_56H^s_{29}V^s_{31}V^s_{32}$$

$W_5$:

$$<\bar{\Phi}_{12}H^s_{30}> [\Phi_2\Phi_2 + \Phi_3\Phi_3]H^s_{29}V^s_{31}V^s_{32}$$

(C.14)

**MSSM Terms**

$W_3$:

$$[<H^s_{30}> + <\Phi_4\bar{\Phi}_{12}H^s_{30}>]\bar{h}_4Q_3u^c + [\Phi_2\Phi_{23}H^s_{31}> + <\Phi_1\Phi_{12}\Phi_{12}H^s_{31}> + <\Phi_1\Phi_{12}\Phi_{12}H^s_{31}>][\bar{h}_4Q_1u_1^c + \bar{h}_4L_1N_1^c] + <\bar{\Phi}_{15}\Phi_{15}H^s_{30}> c_{\kappa}\bar{h}_1\bar{Q}_1u_1^c + <\Phi_4\Phi_{15}H^s_{30}> c_{\kappa}\bar{h}_1\bar{L}_1V^s_{32}$$

(C.15)

$W_4$:

$$[<H^s_{31}> + <\Phi_4\Phi_{12}H^s_{31}> + <\Phi_4\Phi_{12}H^s_{31}> + <\Phi_4\Phi_{12}H^s_{31}>][\bar{Q}_1Q_2u^c_1d^c_2 + Q_1u^c_1L_2e^c_2 + Q_1u^c_1L_2e^c_1 + Q_2d^c_1L_1N_2^c + \bar{Q}_2d^c_2L_1N_1^c + \bar{L}_1L_2e^c_2N_1^c] +$$

$$<\bar{\Phi}_{12}H^s_{30}> c_{\kappa}\bar{h}_3^c[Q_3d^c_3 + L_3e^c_3]H^s_{29} +$$

$$<\Phi_{23}H^s_{30}> c_{\kappa}\bar{h}_3[Q_1u^c_1 + L_1N_1^c]H^s_{29}H^s_{30} +$$

$$<\bar{\Phi}_{12}\Phi_{23}H^s_{31}> [\bar{h}_4Q_1u_1^c + \bar{h}_4L_1N_1^c]\bar{\Phi}_{23} +$$

$$<\bar{\Phi}_{12}\Phi_{56}H^s_{31}> [\bar{h}_4Q_1u_1^c + \bar{h}_4L_1N_1^c]\Phi_{56}$$

(C.16)

$W_5$:

$$<H^s_{30}> [\bar{h}_1\bar{h}_1\bar{h}_4Q_3u^c_3 + \bar{h}_4Q_3u^c_3(\Phi_2\Phi_2 + \Phi_3\Phi_3) + \bar{h}_4L_1N_1^c\Phi_{56}] +$$

$$<\Phi_{56}H^s_{31}> \bar{h}_3\bar{h}_4\Phi_{56}V^s_{31}V^s_{32} + <H^s_{30}H^s_{31}h_1\bar{h}_4H^s_{29}V^s_{31}V^s_{32} +$$

$$<\Phi_{12}\Phi_{12} > [h_3Q_2e^c_2 + h_3L_2e^c_2]V^s_{31}V^s_{32} +$$

$$<\Phi_{12}H^s_{31} > [h_1\bar{h}_4\Phi_{56}H^s_{15}H^s_{29} + (\bar{h}_4Q_1u_1^c + \bar{h}_4L_1N_1^c)(\Phi_2\Phi_2 + \Phi_3\Phi_3)] +$$

$$<H^s_{31} > [c_{\kappa}\bar{h}_4\Phi_{23}V^s_{31}V^s_{32} + c_{\kappa}\bar{h}_4\bar{h}_1\bar{h}_4Q_1u_1^c + c_{\kappa}\bar{h}_4\bar{h}_1\bar{h}_4L_1N_1^c + c_{\kappa}\bar{h}_3\bar{h}_4Q_3d^c_3 + c_{\kappa}\bar{h}_3\bar{h}_4L_3e^c_3]$$

(C.17)

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\( W_6: \)
\[
< H^{s}_{30} > [\hat{h}_4 Q_2 w_2^c \Phi_{56} + \hat{h}_4 L_2 N_2^c \Phi_{56}] V^s_{31} V^s_{32} \\
< H^{s}_{31} > [h_1 h_3 \hat{h}_4 h_3 \hat{h}_4 (\Phi_2 \Phi_2 + \Phi_3 \Phi_3)] V^s_{31} V^s_{32} + \\
< H^{s}_{38} > \hat{h}_1 \hat{h}_4 Q_1 w_3^c L_2 V^s_{32}
\] (C.18)

**Mixed MSSM–Hidden Terms**

\( W_4: \)
\[
[< \Phi_{12} > + < \Phi_{12} \Phi_{12} \Phi_{12} > + < \Phi_{12} \Phi_{4} \Phi_{4} > + < \Phi_{12} \Phi_{4} \Phi_{4} >] \hat{h}_4 L_1 H_{23} V_5 + \\
< \Phi_{12} \Phi_{4} > \hat{h}_4 L_1 H_{23} V_{10}
\] (C.19)

\( W_5: \)
\[
[< \Phi_{12} \Phi_{23} > \hat{h}_1 + < \Phi_{23} H^{s}_{31} > \hat{h}_4] L_2 H^s_{19} H_{25} V_{37} + \\
< \Phi_{12} \Phi_{23} > \hat{2} + < \Phi_{12} \Phi_{56} > \Phi_{56} + < \Phi_{23} H^{s}_{30} > H^s_{29} \hat{h}_4 L_1 H_{23} V_{5}
\] (C.20)

\( W_6: \)
\[
[< \Phi_{12} > h_1 \hat{h}_1 + < \Phi_{23} > h_3 \hat{h}_1 + < \Phi_{12} > (\Phi_2 \Phi_2 + \Phi_3 \Phi_3)] \hat{h}_4 L_1 H_{23} V_{5}
\] (C.21)

**Hidden Terms**

\( W_3: \)
\[
< \Phi_{12} \Phi_{12} H^{s}_{30} > V^s_{31} H_{25} V_{35} + < \Phi_{12} \Phi_{12} H^s_{31} > H^s_{19} V_{19} V_{37} + \\
< \Phi_{12} \Phi_{12} \Phi_{4} H^{s}_{30} > V^s_{31} H_{25} V_{39} + < \Phi_{12} \Phi_{12} \Phi_{4} H^s_{31} > H^s_{19} V_{15} V_{40}
\] (C.22)

\( W_4: \)
\[
< \Phi_{12} \Phi_{12} H^{s}_{30} > \Phi_3 V^s_{31} H_{25} V_{35}
\] (C.23)
Flat direction 2' modifications to superpotential, $W_{FD2'}$:

**MSSM Terms**

**$W_3$:**

\[ < H_{15}^s H_{31}^s \Phi_{56}' > h_1 q^c_1 d_2^c > q^c_1 d_2^c > \]

\[ (C.24) \]

**$W_4$:**

\[ < \Phi_{12} \Phi_{56}^I \Phi_{56}^I > [Q_1 Q_2 u^c_2 d_2^c + Q_1 u^c_1 L_2 e^c_2 + Q_1 u^c_1 L_2 e^c_2 + Q_2 d_1^c L_1 N_2^c + Q_2 d_1^c L_1 N_2^c + L_1 L_2 e^c_2 N_1^c] \]

\[ C.25 \]

**$W_5$:**

\[ < \Phi_{56}^I H_{15}^s H_{30}^s > [Q_1 Q_3 Q_3 L_2 + Q_1 Q_3 u^c_3 d_2^c + Q_1 u^c_3 L_2 e^c_3] + \]

\[ C.25 \]

**Hidden Terms**

**$W_3$:**

\[ < \Phi_{56}^I \Phi_{56}^I > [h_3 Q_2 d_2^c + h_3 L_2 e^c_2] V_{31}^s V_{32}^s \]

\[ C.26 \]

Flat direction 2V modifications to superpotential, $W_{FD2V}$:

**MSSM Terms**

**$W_4$:**

\[ < H_{30}^s \Phi_{56}^I > \bar{h}_1 L_3 N_3^c \]

\[ C.28 \]

**$W_5$:**

\[ < H_{30}^s \Phi_{56}^I > \bar{h}_4 Q_2 u^c_2 V_{31}^s V_{32}^s \]

\[ C.29 \]

Flat Direction 2'V modifications to superpotential:

**Singlet Terms**

**$W_3$:**

\[ < \Phi_{12} \Phi_{56}^I \Phi_{56}^I > H_{30}^s V_{31}^s V_{32}^s \]

\[ C.30 \]

**MSSM Terms**

**$W_3$:**

\[ < H_{30}^s \Phi_{56}^I \Phi_{56}^I > \bar{h}_4 Q_3 u^c_3 + \]

\[ C.31 \]
\[ W_4: \]
\[
< H_{30}^s \Phi_{56}' \overline{\Phi}_{56} > \bar{h}_4 Q_3 u_c^c 
\]

**Mixed MSSM–Hidden Terms**

\[ W_5: \]
\[
< \Phi_{12} \overline{\Phi}_{56} > \bar{h}_4 L_1 H_{23} V_5 \Phi_{56}' 
\]

Flat direction 3 modifications to superpotential, \( W^{FD3} \):

**MSSM Terms**

\[ W_4: \]
\[
< H_{15}^s H_{19}^s > [Q_2 d_2^c L_1 + L_1 L_2 e_2^c] V_{32}^s + \\
< \Phi_{15}' H_{19}^s > [Q_2 d_1 L_2 + u_2^c d_1^c d_2^c] V_{32}^s 
\]

\[ W_5: \]
\[
< H_{19}^s > [Q_1 d_1^c L_2 + L_1 L_2 e_1^c] H_{29}^s V_{32}^s + \\
< \Phi_{19}' H_{19}^s > u_1^c d_1^c d_2^c H_{29}^s V_{32}^s + < \overline{\Phi}_{19} H_{19}^s > Q_1 d_2^c L_1 H_{29}^s V_{32}^s + \\
< H_{15}^s H_{19}^s > [Q_2 d_2^c L_1 \Phi_2 + Q_3 d_3^c L_1 \Phi_{23} + L_1 L_2 e_2^c \Phi_2 + L_1 L_2 e_3^c \Phi_{23}] V_{32}^s 
\]

\[ W_6: \]
\[
< H_{19}^s > [Q_1 d_1^c L_2 + L_1 L_2 e_1^c] (\Phi_2 + \Phi_3^s) H_{29}^s V_{32}^s 
\]

**Mixed MSSM–Hidden Terms**

\[ W_5: \]
\[
< \Phi_{12} H_{19}^s > [Q_1 d_1^c L_2 + u_2^c d_1^c d_2^c + L_1 L_2 e_1^c] H_{25} V_{35} 
\]

Flat Direction 3V modifications to superpotential, \( W^{FD3V} \):

**MSSM Terms**

\[ W_3: \]
\[
< H_{15}^s H_{19}^s H_{31}^s > \bar{h}_4 L_1 V_{32}^s 
\]

\[ W_4: \]
\[
< H_{15}^s H_{19}^s H_{31}^s > \bar{h}_4 L_1 V_{32}^s \Phi_2 
\]

Flat Direction 4 modifications to superpotential \( W^{FD4} \):

**MSSM Terms**

\[ W_4: \]
\[
< \Phi_{12} H_{38}^s H_{20}^s > [Q_3 d_2^c L_1 N_2^c + L_1 L_2 e_3^c N_2^c] 
\]

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