Toward Ultimate Control of Terahertz Wave Absorption in Graphene

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It is commonly believed that weak light-matter interactions in low-mobility graphene dramatically limits tunability of graphene-based optoelectronic devices, such as tunable absorbers or switches. In this paper, we develop and use a simple circuit model to understand absorption in graphene sheets. In particular, we show that light interacts weakly also with very high-mobility graphene sheets and propose systematic design means to overcome these problems. The results have allowed us to demonstrate in the terahertz band that perfect absorption with excellent electrical tunability can be achieved within a wide span of mobility values (from 200 to 20000 cm²V⁻¹s⁻¹) which almost covers the whole range of ever reported room-temperature mobilities. Remarkably, concentrating on the most practical low-mobility graphene devices, we exemplify our theory with two cases: frequency-tunable and switchable absorbers with near 100% modulation efficiencies. Our work provides systematic and instructive insights into the design of highly tunable absorbers, without restrictions on graphene mobility. The design strategy and the developed analytical model can, in principle, be generalized to other wavelength regions from microwave to mid-infrared range, and other two-dimensional materials such as transition metal dichalcogenides (TMDs) and black phosphorus.

I. INTRODUCTION

Graphene is an excellent platform for optoelectronics, enabling better photodetectors, wave modulators, and tunable plasmonic devices. The performance of graphene-based devices is largely determined by the intensity of light-matter interactions in graphene. It is well known that a pristine graphene layer naturally absorbs only a small portion of incident power in the full electromagnetic spectrum [1] (in the visible and near-infrared ranges it is <2.3%, in mid-infrared, <3% and in far-infrared it is <10%). In recent years, a lot of efforts have been made to enhance absorption in graphene, targeting at different wavelength regions. In the visible and near-infrared ranges, absorption can be improved using dielectric [2,3] or metallic resonators [4]. In this range, electrical tuning the device is not possible since graphene exhibits universal conductance (σₛ = e²/4ℏ) dominated by interband transitions. Probably, the most attractive spectrum for tunable graphene applications is from mid-infrared to microwave band, where the conductivity of graphene can be dynamically controlled by electrostatic gating. According to the Kubo formula, the sheet impedance of graphene is inversely proportional to the carrier mobility and the Fermi level. For a homogeneous graphene sheet, if the sheet impedance is engineered to the level of the free-space impedance η₀ the sheet is positioned in an appropriate resonant structure, the absorption in graphene will be strong. Based on this fact, most of the theoretical studies purposefully assume a large mobility or high Fermi level to make the sheet impedance of graphene close to η₀ [0,8]. For example, under this assumption on mobility, graphene is patterned into different shapes to enable plasmon-induced absorption and realize various functionalities. In particular, full absorption [9,10] as well as broadband [11,12] and multiband [13,14] absorption spectra have been reported.

Despite the appeal of these theoretical predictions, it has been very difficult to realize this performance in the laboratory. Many researchers fabricate such nanopatterned graphene structures, but the observed absorptivity is much smaller than unity [15,10]. The main reason of poor absorption is that the carrier mobility of processed graphene is much lower than those assumed in corresponding theoretical works. For example, in [9], graphene mobility is assumed to be μₘ = 10000 cm²V⁻¹s⁻¹. Such high mobility value is possible to obtain from mechanically exfoliated graphene [17], but extremely hard from processed CVD graphene. The typically reported mobility of CVD graphene is only around 1000 cm²V⁻¹s⁻¹ [18,20], corresponding to sheet resistivity of 9000 Ω/sq (at a low Fermi level, E_F = 0.1 eV) which is severely mismatched with the free-space wave impedance. The low carrier mobility of graphene is attributed to charged impurities and defects introduced during graphene growth, transfer and nanopatterning, which is unavoidable and difficult to control. Due to the hardship of obtaining high quality (high mobility) graphene, many experimental works resort to improvements of the doping level of graphene, either with chemical or electrical methods, to compensate the poor carrier mobility [21,23]. For example, in [22], full absorption of a graphene Salisbury screen occurs only when the Fermi level is raised up to E_F = 1 eV. As an alternative, in some works, graphene is subjected to chemical dopings [24,25]. Although in this way the graphene impedance can be easily matched to the free-space impedance, strong chemical doping may disrupt the band structure of graphene and thus compromise its tunability. It should be noticed that there are some experimental verifications of near-perfect absorp-
tion with weakly-doped graphene and metallic patterns, but in these structures the power is absorbed mostly in lossy metals and dielectrics \[26, 27\], as also pointed out in \[18, 22\]. Therefore, the tuning ability of such structures are quite limited.

While the previous works rely on carefully tailoring the mobility value or the Fermi level for perfect absorption, in this paper, we demonstrate that positioning a graphene sheet on a specific metasurface substrate, perfect and strongly tunable absorption is always possible even in very weakly-doped graphene, without strict requirements of the carrier mobility. Understanding and design of such tunable graphene devices needs a clear physical model and systematic design strategy. We use a circuit model as an analytical tool and show what factors restrict absorption and tunability and how to optimize performance for graphene with arbitrary mobility values. We demonstrate that, in the terahertz range, both low and high carrier mobility weaken the absorption in graphene. For graphene with low mobility, the large sheet resistance restricts absorption, while for high quality graphene the kinetic inductance becomes a dominant decoupling factor.

We show that both of these two decoupling problems can be solved by positioning graphene sheets on patch-type metasurfaces. We show that properly choosing the structural parameters enables perfect and strongly tunable absorption in graphene within a wide range of mobility values (from 200 to 20000 cm/s, even if the graphene sheet is very weakly doped. Particularly, in the low mobility regime, we design two tunable absorbers with near 100% modulation efficiencies, one has the tunable frequency of absorption and the other operates as a switch between high absorption and high reflection regimes.

**II. FUNDAMENTAL LIMITS OF ABSORPTION IN CONTINUOUS GRAPHENE SHEET**

The graphene layer is modeled as an infinitesimally thin surface characterized by surface conductivity \(\sigma_g(\omega, T, E_F, \gamma)\), where \(\omega\) is the angular frequency, \(T\) is the temperature, \(E_F\) is the Fermi level with respect to the charge neutrality point (CNP), and \(\gamma\) is the scattering rate. The surface conductivity of graphene is given by the well-known Kubo formula

\[
\sigma_g = -\frac{e^2 K_B T}{\pi \hbar^2 (\omega - j \gamma)} \left[ \frac{E_F}{2 \pi} + 2 \ln \left(1 + e^{-\pi B \gamma} \right) \right] - j \frac{e^2}{4 \pi \hbar} \ln \left[ \frac{2|E_F| - (\omega - j \gamma) \hbar}{2|E_F| + (\omega - j \gamma) \hbar} \right],
\]

where \(e\) is the electron charge, \(K_B\) is the Boltzmann constant, and \(\hbar\) is the Plank constant. The first term represents the intraband transitions contribution, and the second term refers to the interband contribution. In the terahertz frequency band and at lower frequencies the intraband transitions dominate. Throughout this paper, we discuss applications in the terahertz band, approximating the surface conductivity of graphene with the first term in [1], but we note that the proposed design methodology is applicable in the general case.

The Fermi energy is a function of the carrier density \(n\), \(E_F = \hbar v_F \sqrt{n}\), which can be dynamically adjusted by applying a vertical electrostatic field upon graphene layer \((v_F = 10^8 \text{ cm/s is the Fermi velocity})\). The scattering rate is inversely proportional to the Fermi energy, \(\gamma = ev_F^2/(\mu m E_F)\). \(\mu m\) is the carrier mobility of graphene. In this work, we assume that the mobility is independent of the carrier concentration. The surface conductivity of graphene is a complex-valued parameter, and the graphene sheet can be considered as a complex-impedance sheet, with the sheet impedance \(Z_g = 1/\sigma_g = R_g + j X_g\).

Probably the simplest arrangement which allows total absorption in graphene is the Salisbury screen \[22, 23, 24\], where the transmission is totally blocked. The classical Salisbury screen is a resistive sheet placed on a grounded dielectric substrate of the quarter wavelength thickness. Total absorption is realizable if the input impedance of the structure can be made equal to the free-space impedance \(\eta_0\). Here, a graphene layer is used as a resistive sheet, as shown in Fig. 1(a). It is noteworthy that the thickness of the substrate is not necessary equal to one quarter of the wavelength since the graphene is not a purely resistive layer and possesses some reactive properties. We can analyse this structure using a simple transmission-line model shown in Fig. 1(b). Instead of writing \(Z_g\) as a series connection of a resistor and a reactive element, as many other authors do \[29\], we express the graphene sheet impedance as two parallel components, the shunt resistance \(R_g\) and the shunt reactance \(j X'_g\) [see Fig. 1(b)]. Obviously, \(R'_g\) and \(j X'_g\) can be expressed in terms of the series-circuit parameters \(R_g\) and \(X_g\):

\[
R'_g = R_g + \frac{X_g^2}{R_g}
\]

and

\[
j X'_g = j \left( X_g + \frac{R_g^2}{X_g} \right).\]

The grounded substrate is represented by a shorted transmission line with the characteristic impedance \(\eta_d = \sqrt{\mu_0/\epsilon_0}\). For normal incidence of plane waves, the shorted transmission line can be viewed as a reactance whose impedance is \(j X_d = j \eta_d \tan(k_d d)\), where \(k_d = \omega/\sqrt{\epsilon_\infty \mu_0}\) is the wave number in the substrate. The total input admittance of the structure can be written as \(Y_{in} = R_g^{-1} + (j X'_g)^{-1} + (j X_d)^{-1}\). Perfect absorption takes place when \(Y_{in} = Y_0 = 1/\eta_0\), which satisfies the impedance matching condition. Thus, the real and imaginary parts of \(Y_{in}\) should satisfy

\[
\frac{1}{R_g} = \frac{1}{\eta_0}
\]
Condition (4) means that the shunt resistance of the graphene sheet should be equal to the free-space impedance at the frequency of perfect absorption. The difference between \( R'_g \) and \( \eta_0 \) actually determines the absorption level. Equation (5) tells that the reactances of the graphene sheet and the input reactance of the grounded substrate should cancel out (forming a high-impedance surface). This condition is usually easier to satisfy (compared to (4)) because it can be easily met by adjusting the substrate thickness or its dielectric constant, for an arbitrary value of \( X'_g \). Assuming that the condition (5) is satisfied, the reflection coefficient is written as

\[
r = \left( \frac{R'_g - \eta_0}{R'_g + \eta_0} \right) \frac{1}{jX'_g + \frac{1}{jX_d}} = 0.
\]

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Next, in the terahertz region, we study the decoupling mechanisms for graphene with arbitrary mobility and Fermi levels. Figure 2 shows the calculated \( D_e \) as a function of the carrier mobility and Fermi level at 4 THz. We study the doping range from 0.1 eV to 1 eV, which can be realized with high capacitance ion gel films [15, 22, 30]. For low quality graphene (\( \mu_m < 2000 \text{ cm}^2\cdot\text{V}^{-1}\cdot\text{s}^{-1} \)), we can see that decoupling is strong and absorption is very weak at low Fermi levels. This is caused by the huge sheet resistance of weakly-doped graphene (\( R'_g \) is dominated by \( R_g \)). Therefore, improving the doping level (decreasing \( R_g \)) will somewhat enhance the absorption. This can clearly explain the recent experimental observation of perfect terahertz absorption in graphene [22], where graphene is subjected to a high electrical doping \( (E_F = 1 \text{ eV}) \). For high quality graphene, the impedance mismatch becomes very severe and it cannot be alleviated by the increase of Fermi level. The decoupling is dominated by the kinetic inductance of graphene (\( R'_g \) is mostly affected by \( X_g \)). This theory can successfully explain the absorption enhancement in graphene plasmonic patterns. In fact, the capacitive interactions between the neighboring graphene cells cancel the natural kinetic inductance of graphene, thus increasing the coupling. We see that at low doping levels, both low and high quality graphene sheets are difficult to couple with incident waves, although decoupling is caused by different mechanisms (large sheet resistance or reactance). Hereafter, we aim at perfect absorption in weakly-doped graphene.

It is known that a pristine graphene layer always becomes doped by some residual carriers during the growth and processing of graphene. This initial doping is unintentional, and varies from sample to sample. We take a typical value of this unintentional doping as \( n_0 = 7.3 \times 10^{11} \text{ cm}^{-2} \), corresponding to the Fermi level of \( E_{F0} = 0.1 \text{ eV} \) [31, 32]. Realizing perfect absorp-
tion with such low Fermi levels is more difficult than in heavily-doped graphene, but beneficially, it may enable the use of graphene in perfect absorbers without external assists (electrical or chemical doping). More importantly, if the matching condition is satisfied in a weakly-doped graphene, it is possible to maximize the tunability because one can drive the Fermi level from a very low state to the practical upper limit, which provides a maximum force to break the established matching. Figure 6 shows the decoupling factor for $E_D = 0.1$ eV at different frequencies and for different mobility values. In the high mobility regime, the inductance-induced decoupling becomes evident as the frequency increases, while for low quality graphene it is almost frequency-independent. We see that strong decoupling is a universal problem in the whole terahertz range. In the next section, we will demonstrate that strong but differently induced decoupling can be eliminated by using one type of metasurface substrates, and perfect absorption is always achievable in graphene irrespective of the carrier mobility.

III. PERFECT ABSORPTION IN GRAPHENE USING METALLIC METASURFACE SUBSTRATE

A. Analytical modeling

The schematic of the metasurface-based graphene absorber is displayed in Fig. 4(a). Compared to the Salisbury structure, we use one auxiliary layer formed by thin metal patches periodically positioned on the substrate. The patch array period is small compared to the wavelength, so that there are no propagating Floquet modes. The graphene sheet is directly transferred on the patterned metal layer. The metasurface (formed by the periodic square patch array and the grounded substrate) plays two important roles: creation of a resonant cavity which improves the absorption in graphene layer, and reducing the effective shunt resistance of graphene.

With the presence of metallic patches, the graphene layer is equivalent to be "patterned" into a square mesh (a complementary structure to the square patches), as shown in Fig. 5(left). This is because in the graphene-metal contact areas, graphene is actually shorted by highly conductive metal. Tangential electric field on graphene surface where it is covered by metal is zero due to the boundary condition on metal. Thus, the graphene-metal hybrid still behaves as metal. Assuming that the electric field is polarized along the $x$-direction, strong capacitive coupling between the patches occurs via $y$-directed gaps. Therefore, in the effectively "patterned" graphene only $y$-oriented strips are excited. The effective impedance of graphene in one cell is $Z_{\text{eff}} = Z_{g}/p$, where $p = (D - g)/g$ will be called the scaling factor, which is determined by the width/length ratio of the effective graphene area (see the red dashed box in Fig. 5). $D$ and $g$ are the cell period and the gap width, respec-

\begin{equation}
C_m = \frac{2\epsilon_{\text{eff}}\epsilon_0 D\ln \left(\frac{\csc \frac{\pi}{2(p+1)}}{\pi} \right)}{\pi}, \quad (6)
\end{equation}

where $\epsilon_{\text{eff}} = (\epsilon_r + 1)/2$.

FIG. 4. (a) Schematics of the proposed graphene absorber based on metasurface substrate (b) equivalent circuit model of the proposed structure.

FIG. 5. Top view of the “patterned” graphene (left) and metallic patches (right). The red dashed box in the left figure shows the effective graphene in one unit. The arrows (orange) represents for the current in graphene.

Similarly to the Salisbury screen discussed in the previous section, perfect absorption takes place when the
following two conditions are satisfied:

\[ \frac{R'_g}{\rho} = \eta_0 \]  

and

\[ \frac{1}{jX'_g/\rho} + \frac{1}{jX_m} + \frac{1}{jX_d} = 0. \]  

In what follows, the design procedures for unity absorption at a specified frequency \( f_0 \) are introduced. For a given graphene sample with arbitrary mobility \( \mu_m \) and initial Fermi-level \( E_{F0} \), we can obtain the sheet impedance in its shunt form using Equation (1)-(3). Then \( p \) is determined from condition (7). For a fixed graphene sample and substrate, \( jX_d \) and \( jX'_g \) are already known. Thus, \( D \) can be uniquely solved to satisfy condition (8). However, the obtained \( D \) may be much larger than the substrate thickness \( d \). In this case the analytical formula for the grid capacitance (6) becomes not accurate due to ignorance of evanescent-mode coupling with the ground plane [31]. In this case, one should use more accurate expressions of \( C_m \) [31] where the influence of evanescent modes is accounted for. In the following validation examples we set up a relation between the thickness of each graphene sample and substrate, \( d \), \( G \) and \( s \) (see Figure 7). The proposed method is also suitable for designing perfect absorbers at different frequencies while using graphene of the same quality. Here we use a low-mobility graphene with \( \mu_m = 2000 \text{ cm}^2\text{V}^{-1}\text{s}^{-1} \) and the target absorption frequencies are \( 2 \text{ THz}, 5 \text{ THz} \) and \( 8 \text{ THz} \) with required scaling factors \( p = 14.4, 17.2 \) and \( 22.9 \). The an-

\[ \gamma_{Au} = 3.33 \times 10^{13} \text{ rad/s} \]  

and \( \omega_p = 1.36 \times 10^{16} \text{ rad/s} \) [19]. Figure 6 displays the simulated absorption coefficients for each assumed mobility, using the analytically solved structural parameters of metasurface. All of the samples have nearly perfect absorption at the designated frequency of \( 4 \text{ THz} \).

\[ \text{Unit: cm}^2\text{V}^{-1}\text{s}^{-1} \]

\[ \text{FIG. 6. Simulated absorption coefficient in terms of frequency for different qualities of graphene. The structure dimensions are analytical solutions.} \]

\[ D \]  

\[ \text{FIG. 7. Analytical and numerical results for total absorption designed at different frequencies.} \]  

\[ D \]  

\[ \text{FIG. 8. Displayed the simulated absorption coefficients for each assumed mobility, using the analytically solved structural parameters of metasurface. All of the samples have nearly perfect absorption at the designated frequency of 4 THz.} \]  

B. Perfect absorption in weakly-doped graphene

Following the above design procedure we can achieve perfect absorption in graphene of different quality (within a wide span of carrier mobilities). Here, we investigate graphene mobilities from \( \mu_m = 200 \text{ cm}^2\text{V}^{-1}\text{s}^{-1} \) to \( \mu_m = 20000 \text{ cm}^2\text{V}^{-1}\text{s}^{-1} \). The initial doping of graphene is assumed to be \( E_{F0} = 0.1 \text{ eV} \). The example target frequency for unity absorption is \( 4 \text{ THz} \). The required \( D \) and \( p \) for each graphene mobility value are solved from conditions (7) and (8). Numerical tool HFSS (based on the finite elements method) is employed to simulate the proposed structure. Graphene is modeled as a zero-thickness sheet, with the frequency-dependent sheet impedance given by (6). We use a 75 nm-thick gold layer for the metallic ground plane and the patches. Gold conductivity is modeled by the Drude formula with the parameters

\[ \begin{align*}
\gamma_{Au} = 3.33 \times 10^{13} \text{ rad/s} \\
\omega_p = 1.36 \times 10^{16} \text{ rad/s}
\end{align*} \]

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alytical absorption spectra are calculated using (9) and (10), as shown in Fig. 7 (lines). With the same structural parameters, the simulated absorbances (symbols) are compared with the analytical ones in Fig. 7. Good agreement between analytical and numerical absorbance in the frequency range confirms the accuracy of the developed analytical model.

IV. ELECTRICAL TUNABILITY

In this section, we discuss tunability of perfect absorption at THz frequencies as the Fermi level of graphene changes. To better explain the tunability property, first we simplify the complex Kubo formula of graphene conductivity in (1) into Drude model (valid in low THz band), \( \sigma_g = e^2 \gamma \omega / \pi \hbar (\omega + \gamma) \) which corresponds to the following expression for the sheet impedance of graphene:

\[
Z_g = R_g + j \omega L_g = \frac{\pi \hbar^2 \sigma_g^2}{e \mu_m \hbar^2} + j \omega \left( \frac{\pi \hbar^2}{e^2 \gamma \omega} \right).
\] (11)

The effective shunt resistance and inductance of graphene [see Fig. 4(b)] can then be expressed in terms of \( R_g \) and \( L_g \): \( R'_{\text{eff}}(\omega, E_F) = R_g + \omega^2 L_g^2 / R_g \) and \( L'_{\text{eff}}(\omega, E_F) = L_g / p + R_g^2 / \omega^2 L_g p \). Note that for a fixed value of graphene mobility, both \( R'_{\text{eff}} \) and \( L'_{\text{eff}} \) depend on the frequency and the Fermi level in graphene. The \( E_F \)-dependent effective inductance indicates that the resonant frequency of the \( RCL \)-circuit in Fig. 4(b) is tunable with the chemical potential. The resonant frequency \( \omega_0 \) can be solved from condition (8):

\[
\frac{\omega_0^2 L_g p}{\omega_0^2 L_g^2 + R_g^2} - \omega_0 C_m - \frac{1}{\eta_d \tan(\omega_0 \sqrt{\pi e \gamma_0 \mu_0 d})} = 0.
\] (12)

The curves in Fig. 8 shows the resonant frequency of four different structures which all realize perfect absorption at \( f_0 = 4 \text{ THz} \) and \( E_{V0} = 0.1 \text{ eV} \) using graphene sheets of different mobilities. The structural parameters are given in the caption of Fig. 8. We can see that the resonant frequency always climbs up when the graphene Fermi level increases. However, the modulation of \( \omega_0 \) does not necessarily mean frequency-tunable absorption. Another necessary condition is impedance matching with free space at the resonant frequency. In order to realize frequency-tunable perfect absorption, the best scenario is to ensure that the condition \( R'_{\text{eff}}(\omega_0, E_F) = \eta_0 \) is always satisfied under the change of the Fermi level. Although the effective resistance of graphene is easy to tailor to \( \eta_0 \) at a fixed frequency and a given graphene state, it is hard to sustain this established impedance matching when the Fermi level of graphene is changing. The impedance matching condition is almost determined by the intrinsic properties of graphene.

Figure 8 shows the dependence of \( R'_{\text{eff}}(\omega, E_F) \) on the frequency and the Fermi level for different qualities of graphene. From Fig. 8(a) we can see that for low-mobility graphene \( R'_{\text{eff}} \) is quite susceptible to the Fermi level while it is not sensitive to the frequency. The value of \( R'_{\text{eff}} / \eta_0 \) corresponding to the resonant frequencies [the curve of \( \omega_0(E_F) \) in Fig. 8(a)] decreases from unity to almost zero. Apparently, very poor quality graphene is not suitable to achieve frequency-tunable perfect absorption. On the other hand, quickly decreasing shunt resistance at a fixed frequency makes it a good candidate to realize a switch, which will be discussed in Section IVB. For the graphene sheet with \( \mu_m = 1500 \text{ cm}^2 \text{V}^{-1} \text{s}^{-1} \), as the chemical potential increases, the frequency point where \( R'_{\text{eff}}(\omega, E_F) = \eta_0 \) also shifts to higher frequencies. This is in harmony with increased resonant frequency. Also, the values of \( R'_{\text{eff}} \) at the resonant frequencies are very close to \( \eta_0 \), which means that the peak absorption level remains close to unity even when the resonant frequency is tuned to higher values. For high-quality graphene (\( \mu_m = 10000 \text{ cm}^2 \text{V}^{-1} \text{s}^{-1} \)), the effective resistance of graphene appears to be insensitive to the change of the graphene Fermi level, and increases gradually with the frequency. Therefore, it can be expected that the peak absorbance decreases with the increase of the chemical potential.

The above analysis suggests possibilities to realize broad tunability of graphene perfect absorber with both low and high quality graphene. In the following, we will show two different modulation scenarios in detail: frequency-tunable perfect absorbers and intensity-tunable perfect absorbers (switches) using low mobility graphene.

A. Frequency-tunable absorber

Here we present tuning characteristics of a frequency-reconfigurable absorber designed based on the developed theory. The mobility of graphene is chosen as \( \mu_m = 1500 \text{ cm}^2 \text{V}^{-1} \text{s}^{-1} \), and the primary doping is assumed as \( E_{V0} = 0.1 \text{ eV} \). The structural parameters are given in the caption of Fig. 6. Figure 9 shows the analytically calculated absorption coefficient from 1 THz to 15 THz when the Fermi level is raised from the original 0.1 eV to 1 eV. Apparently, the absorption frequency is blue-shifted from 4 THz to 12.57 THz, realizing a large modulation bandwidth (about 100%). The peak absorbance at each Fermi level is very close to unity, only with a small degradation around \( E_F = 0.3 \text{ eV} \) (92% absorption). Numerical results for several discrete Fermi levels are shown in Fig. 10. Very similar absorption behavior with the analytical results reported in Fig. 9 further confirms the accuracy of the developed circuit model. The near perfect tunability in absorption frequency indicates that the critical coupling between light and graphene is sustainable during the Fermi level modulation, even if graphene’s property dramatically changes. It should be pointed out that the mobility of graphene is not strictly limited at \( \mu_m = 1500 \text{ cm}^2 \text{V}^{-1} \text{s}^{-1} \). The most effective frequency tuning is achieved with the mobility between
FIG. 8. Normalized effective shunt resistance of graphene as a function of the frequency and Fermi level, for graphene mobility (a) $\mu_m = 200 \text{ cm}^2\text{V}^{-1}\text{s}^{-1}$, (b) $\mu_m = 1500 \text{ cm}^2\text{V}^{-1}\text{s}^{-1}$ and (c) $\mu_m = 10000 \text{ cm}^2\text{V}^{-1}\text{s}^{-1}$.

$\mu_m = 1000 \text{ cm}^2\text{V}^{-1}\text{s}^{-1}$ and $\mu_m = 2000 \text{ cm}^2\text{V}^{-1}\text{s}^{-1}$. This mobility range is also quite reasonable and can be obtained using CVD-grown graphene [18–20, 36].

FIG. 9. Analytically calculated absorbance in terms of the frequency and doping level. The mobility of graphene is assumed to be $\mu_m = 1500 \text{ cm}^2\text{V}^{-1}\text{s}^{-1}$.

FIG. 10. Simulated absorption intensity in terms of the frequency and doping level when varying the Fermi level from 0.1 eV to 1 eV. The graphene mobility is set to $\mu_m = 1500 \text{ cm}^2\text{V}^{-1}\text{s}^{-1}$.

### B. Intensity-tunable absorber

As explained above, the rapidly decreasing $R'_\text{eff}$ of biased low-quality graphene makes it possible to be an efficient switchable absorber. Here we assume a graphene sample with a very poor mobility ($\mu_m = 200 \text{ cm}^2\text{V}^{-1}\text{s}^{-1}$) and study the tunability of the absorption level at the resonant frequency. Following the design procedures, the structural parameters are determined as $D = 7 \mu m$, $p = 137$ and $a = 0.5$ if the perfect absorption is expected at 4 THz for $E_{F0} = 0.1 \text{ eV}$. The analytically calculated absorbance is shown in Fig. 11. In striking contrast with the case of higher mobility $\mu_m = 1500 \text{ cm}^2\text{V}^{-1}\text{s}^{-1}$, the absorption is quickly suppressed by increasing the chemical potential, while the resonant frequency practically does not shift. At 4 THz, the absorption level drops below 10% when the Fermi level increases to 0.6 eV.

However, in order to reduce the high resistance of such low-quality graphene, a narrow gap ($p = 136$ and $g = 50 \text{ nm}$) between the metallic patches is required in this case. This significantly increases the fabrication difficulties. The situation will become even worse if the operating frequency is higher ($D$ becomes smaller). On the other hand, the tiny width of metallic slots is not desirable, since the Fermi level pinning effect in graphene is not negligible. This practical issue is caused by the transfer of electrons in metal to the graphene sheet via the metal-graphene contact. The transferred charges result in an inhomogeneous distribution of the Fermi level in graphene [19, 37].

In order to avoid these problems, we propose a novel metallic pattern with meandered metal channels instead of straight ones (formed by the patch array), as shown in...
The mobility of graphene is assumed to be \( \mu = 200 \text{ cm}^2\text{V}^{-1}\text{s}^{-1} \). Using graphene of the same quality (\( \mu_{\text{m}} = 200 \text{ cm}^2\text{V}^{-1}\text{s}^{-1} \)), perfect absorption is achieved when \( D = 5 \mu\text{m} \), \( l_1 = 3.6 \mu\text{m} \), \( g = 250 \text{ nm} \) and \( N = 8 \). Perfect absorbance is shown in Fig. 13 (red curve). When the Fermi level is tuned from 0.1 eV to 1 eV, the absorption level fastly decreases from unity to 5%. The device acts as a perfect absorber in its natural state (\( E_{\text{F}0} = 0.1 \text{ eV} \)) while in the biased state it is a nearly perfect reflector.

Similarly with graphene, many other two-dimensional (2D) materials, such as semiconducting transition metal dichalcogenides (TMDs) [38] and black phosphorus (BP) [39], also suffer from weak light-matter interactions. According to the explanations in Section II, weak absorption stems from the impedance mismatch between the shunt sheet resistance of the material layers and the free-space impedance. In the case of larger shunt resistance than the free-space impedance, it is possible to use our proposed metasurface as a substrate for realizing perfect absorption.

Here, we consider an example of monolayer black phosphorus which exhibits anisotropic Drude-type conductivity in the crystal plane, \( \sigma_{\text{bp}}^{\text{i}} = -jn_{\text{c}}e^2/m_i(\omega - j\gamma) \). In this expression, \( i \) labels the direction along \( x- \) (armchair) or \( y- \) (zigzag) axes, \( n_{\text{c}} \) is the carrier density; and \( m_i \) is the effective electron mass which is different along two orthogonal axes (the \( x- \) and \( y- \) directions) [40–42]. According to [40], the material parameters can be typically assumed as \( \gamma = 10 \text{ meV} \), \( n_{\text{c}} = 2 \times 10^{13} \text{ cm}^{-2} \), \( m_x = 0.15m_0 \) and \( m_y = 0.7m_0 \) (\( m_0 \) is the static mass of electron). From 4 THz to 15 THz, the shunt resistance along the \( x- \) direction varies from 1500 to 16000 \( \Omega \) which is much larger than \( \eta_0 \). This means that in this range we can use a patch-type metasurface to fully absorb \( x- \) polarized waves. All of the analytical formulas developed for graphene are valid for other 2D materials only by replacing the graphene impedance \( Z_g \) with the sheet impedance of thin layers of other materials. The conductivity of other 2D materials is also tunable either by electrostatic or optical means. Thus, the absorption level can be dynamically modulated. The tunability analysis can follow the example of graphene discussed in Section IV.
VI. CONCLUSION

To summarize, we have developed an analytical model of graphene-metasurface structures which allows us to identify physical mechanisms behind tunable absorption in graphene and design structures possessing required electromagnetic properties with the use of different graphene samples. Our systematic analysis demonstrated that, in the terahertz frequency range, perfect absorption as well as excellent tunability can be realized with graphene exempt from mobility requirements. In particular, using low quality graphene, we show that the absorption can be dynamically tuned either in the absorption frequency or absorption levels, with nearly 100% modulation efficiencies. The examples are provided for the terahertz frequency range, but the developed graphene-metasurface analytical model is not limited to the terahertz band. It can be scaled from microwave to mid-infrared ranges by tailoring the metasurface dimensions. This theory is applicable also to other two-dimensional materials such as black phosphorus and transition metal dichalcogenides.

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