Fine structure of the topological defect cores studied for disclinations in lyotropic chromonic liquid crystals

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The detailed structure of singularities of ordered field represents a fundamental problem in diverse areas of physics. At the defect cores, the deformations are so strong that the system explores states with symmetry different from that of an undistorted material. These regions are difficult to explore experimentally as their spatial extension is very small, a few molecular lengths in the condensed matter. Here we explore the cores of disclinations in the so-called chromonic nematics that extend over macroscopic length scales accessible for optical characterization. We demonstrate that the amplitude $S$ and the phase $\mathbf{n}$ (the director) of the order parameter vary along both the radial and azimuthal directions, in contrast to the classic models in which $S$ varies only with the distance from the centre and $\mathbf{n}$ depends only on the azimuthal coordinate. This unexpected core structure is explained by a strong coupling of the phase and amplitude of the order parameter in the free energy.
Topological defects represent an important concept in many branches of modern physics ranging from cosmology and optics to hard and soft matter. One of the most difficult problems is the fine structure of the so-called core region of defects, where the deformations of the order parameter are so strong that the phenomenological description valid in the far-field fails. The difficulty is not only that the theory should account for the strong spatial gradients of the order but also in limited accessibility of the core region to experimental exploration. For example, in the case of topological defects in hard and soft matter, such as dislocations and disclinations, the gradient energy becomes comparable to the condensation energy at the atomic/molecular length scales. Narrow extension of the core makes it practically impossible to use standard optical and even electron microscopy\(^1\)\(^-\)\(^3\) in establishing how the properties of the medium are modified as one approaches the core. Because of the very limited experimental data\(^1\)\(^-\)\(^3\), modern theories\(^4\)\^-\(^6\) of the topological defects treat the core problem with a number of strong simplifying assumptions, such as angular and radial independence of the order parameter inside and outside the core region. In the case of linear defects, the core is often treated simply as a cylinder of a more symmetric phase with zero-order parameter (an isotropic melt), embedded into an outside region with a constant amplitude of the order parameter.

In this work, we take advantage of the peculiar nature of the so-called lyotropic chromonic liquid crystals (LCLC) of a nematic type with a constant amplitude of the order parameter.\(^7\)\(^-\)\(^9\) The LCLC is disodium cromoglycate (DSCG) dissolved in water and saturates at optical birefringence at wavelength \(\lambda = 546\) nm of the probing beam. The quantity \(\Lambda = [\Delta n] / d\) yields the in-plane birefringence, which is the measure of the tensorial orientational order parameter, as a function of coordinates (Fig. 1c,d). To facilitate the discussion, we introduce polar coordinates \((r, \theta)\); the azimuthal angle \(\theta\) is measured from the symmetry axis \(x\). The experimentally measured azimuthal orientation of the director is represented as a combination of a linear term \(m\theta\) and a periodic anisotropic function \(\tilde{\phi}^{(m)}(r, \theta)\), that is, \(\tilde{\phi}^{(m)}(r, \theta) = m\theta + \tilde{\phi}^{(m)}(r, \theta)\). We further examine \(\tilde{\phi}^{(m)}(r, \theta)\) by its Fourier expansion: \(\tilde{\phi}^{(m)}(r, \theta) = \tilde{\phi}^{(m)}(r) \sin \theta + \tilde{\phi}^{(m)}(r) \sin 2\theta\). We find that, different from the expectation of classical theories (see next paragraph), the Fourier harmonics \(\tilde{\phi}^{(m)}(r)\) and \(\tilde{\phi}^{(m)}(r)\) are functions of the distance to the core: in the far field, \(r > 20\) \(\mu\)m, both \(\tilde{\phi}^{(1)}(r)\) and \(\tilde{\phi}^{(3)}(r)\) approach their limiting \(r\)-independent values \(\tilde{\phi}^{(1)}(\infty)\) and \(\tilde{\phi}^{(3)}(\infty)\). At \(r < r_s \approx 20\) \(\mu\)m, however, \(\tilde{\phi}^{(1)}(r)\) and \(\tilde{\phi}^{(3)}(r)\) change with \(r\). For \(m = 1\) disclinations, the experimental data on \(\tilde{\phi}^{(m)}(r, \theta) = \tilde{\phi}^{(m)}(r, \theta) - m\theta\) show that the first harmonic \(\tilde{\phi}^{(1)}(r)\) increases as \(r\) increases and saturates at \(\tilde{\phi}^{(1)}(\infty) \approx 14\). The third harmonic \(\tilde{\phi}^{(3)}(r)\), although distinguishable, does not contribute much to the main effect (Fig. 1e). For \(m = 2\) disclinations, the first harmonic is zero within the experimental error, \(\tilde{\phi}^{(2)}(r) = 0\), while the third harmonic \(\tilde{\phi}^{(3)}(r)\) increases its absolute value as \(r\) increases and saturates at \(\tilde{\phi}^{(3)}(\infty) \approx -3.5\). Far-field director and anisotropy of Frank–Oseen elasticity. The radial dependence of the anisotropy function \(\tilde{\phi}^{(m)}(r, \theta)\) represents a major departure from the assumed behaviour of disclinations in classic models\(^6\). In the widely popular one-constant approximation to the elasticity of liquid crystals, the director field is assumed to be of the form \(\tilde{\phi}^{(m)}(r, \theta) = m\theta\), so that the anisotropic function vanishes, \(\tilde{\phi}^{(m)}(r, \theta) = 0\). In real materials, the elasticity is anisotropic, with different elastic constants attached to deformations of splay \(K_1\), twist \(K_2\), and bend \(K_3\). The Frank–Oseen elastic energy density \(f_{\text{FO}}\) is then written in terms of the bulk director gradient as:

\[
f_{\text{FO}} = \frac{1}{2} K_1 (\text{div} \mathbf{n})^2 + \frac{1}{2} K_2 (\mathbf{n} \cdot \text{curl} \mathbf{n})^2 + \frac{1}{2} K_3 (\mathbf{n} \times \text{curl} \mathbf{n})^2
\]
differential equation\(^4,16\):
\[
\alpha \frac{\partial^2 \varphi}{\partial \theta^2} \left[ 1 - \varepsilon \cos 2(\varphi - \theta) \right] - \left[ 2 \frac{\partial \varphi}{\partial \theta} - \left( \frac{\partial \varphi}{\partial \theta} \right)^2 \right] \varepsilon \sin 2(\varphi - \theta) = 0
\]

where \(\varepsilon = (K_s - K_b)/(K_s + K_b)\) is the splay–bend anisotropy. Therefore, the elastic anisotropy \(\varepsilon\) modifies the linear dependency, \(\varphi^{(m)}(r, \theta) = m \theta + \varphi^{(\text{D})}(\theta)\), where the anisotropy-induced function \(\varphi^{(m)}\) is \(\varphi^{(m)}(\theta) = \varphi_1^{(m)} \sin \theta + \varphi_3^{(m)} \sin 3\theta\) depending on
the polar angle $\theta$ but not on the radial distance $r$. The implicit dependence of the director on $r$ (equation (2)) can be used to measure the latter, as was demonstrated by Hudson and Thomas.\textsuperscript{16} We further develop the analytical solution of equation (2) in the Fourier space of $\theta$, and establish the following relationship between $\varepsilon$ and the main harmonics $\phi_1^{(1/2)}$ and $\phi_3^{(-1/2)}$ of the anisotropy function $\phi^{(m)}$:

$$
\varepsilon = 4 \frac{\phi_k^{(m)}}{4k^2 - 1},
$$

where $k = 2(1 - m)$. As shown in Fig. 1e, the far-field director ($r > 20\mu m$) exhibits a significant contribution from the anisotropy functions, with $\phi_1^{(1/2)}(\infty) \approx 14^\circ$ for the $m = 1/2$ disclination (Fig. 1e) and $\phi_3^{(-1/2)}(\infty) \approx -3.5^\circ$ for the $m = -1/2$ line (Fig. 1f). Using these asymptotic values and equation (3), we determine the elastic anisotropy of DSCG at 15 wt% to be $\varepsilon = 0.34$, when extracted from the analysis of $m = 1/2$ defects, and $\varepsilon = 0.44$, when $m = -1/2$ lines are analysed. These values match well the measurements of $K_1$ and $K_3$ by dynamic light scattering\textsuperscript{17} for DSCG solutions of concentration 14 and 16 wt%, which yield the range $\varepsilon = (0.33 - 0.52)$ for the elastic anisotropy values.

**Landau-de Gennes model of the order parameter at the core.**

The consideration above is based on the Frank-Oseen functional is naturally limited by the condition of a constant scalar order parameter. Near the defect core, such an assumption is not valid, as demonstrated in theoretical models by Lyubskiy and Schopohl and Sluckin.\textsuperscript{3} The enormous size of the disclination cores in LCLCs offers an opportunity to explore the spatial variations of both the phase and the amplitude of the order parameter experimentally, by mapping the optical axis and optical retardation of the material (Fig. 1a-d).

The spatial distribution of optical retardation $\Gamma(x,y)$ clearly exhibits two regions. Far from the disclination core, $r > 20\mu m$, $\Gamma$ = const. As one approaches the centre of the disclination, $r < 20\mu m$, $\Gamma$ decreases to 0 as $r \to 0$. Also in this region, $\Gamma$ shows a strong azimuthal asymmetry, with the core shape deviating from the circular one. In the case of $m = 1/2$ disclination, there is a cusp-like region, located in the sector where the director is radial; along the cusp, the retardation changes over an extended distance (Fig. 1g). In the $m = -1/2$ case, there are three such cusp regions, all expended along the regions with a radial director (Fig. 1h).

The experimentally measured dependence of retardance on the polar coordinate can be presented in a Fourier form:

$$
\Gamma^{(m)}(r, \theta) = \Gamma_0^{(m)}(r) + \Gamma_1^{(m)}(r) \cos \theta + \Gamma_3^{(m)}(r) \sin \theta.
$$

By averaging images of about 25 different individual disclinations to eliminate spurious factors in the textures, we determine the radial dependences of the harmonic terms $\Gamma_{0}^{(1/2)}(r)$ and $\Gamma_{1}^{(1/2)}(r)$ for $m = 1/2$, and $\Gamma_{0}^{(-1/2)}(r)$ and $\Gamma_{3}^{(-1/2)}(r)$ for $m = -1/2$ lines (Fig. 1g,h). The $\Gamma_{1}^{(1/2)}(r)$ term for $m = 1/2$ disclination can be up to $20\%$ of the zero harmonic $\Gamma_{0}^{(0)}(r)$. The $m = -1/2$ disclination shows a similar influence of the $\Gamma_{3}^{(-1/2)}(r)$ term, at the level of about $7\%$. Other harmonics are of the amplitudes comparable to experimental errors, and can be neglected.

To understand qualitatively the observed experimental features, we describe the core structure using the standard traceless symmetric tensor order parameter of the nematice\textsuperscript{18}

$$
Q_{ij} = S(n_i \hat{n}_j - \delta_{ij}/3) + P \left(\hat{l}_i \hat{l}_j - \hat{m}_i \hat{m}_j\right),
$$

which is diagonal $Q = \text{diag}(P - S/3, -P - S/3, 2S/3)$ in the frame of three mutually orthogonal directors $\hat{1} \equiv -\hat{l}$, $\hat{m} \equiv -\hat{m}$, and $\hat{n} \equiv -\hat{n}$; the quantities $S$ and $P$ are the uniaxial and biaxial order parameters, respectively, that depend on temperature $T$ and composition.

As one approaches the core of disclinations, the assumption of constant $K_1$ and $K_3$ values in Dzyaloshinsky’s model, which implies $S = S_N, P = 0$, no longer holds, as discussed earlier (Fig. 1c,d). Since $\Gamma \propto S - P$ changes markedly in space as the function of $(r, \theta)$, we use the Landau-de Gennes free energy density $f_{\text{DG}} = f_{A} + f_{B}$, where $f_{A}$ and $f_{B}$ are associated with the uniform and gradient terms of the $Q$ tensor, respectively. The first contribution writes\textsuperscript{18,19}

$$
f_{A} = \frac{1}{2} A Q_{00} Q_{00} - \frac{1}{3} B Q_{00} Q_{00} Q_{00} + \frac{1}{4} C (Q_{00} Q_{00})^2,
$$

where $A = a(T \to T^*)$, and $a, b, c, d$ are the material parameters. The equilibrium scalar order parameters correspond to the minimum of $f_A$ and represent the uniaxial state, $S = S_N = \frac{B}{4C} + \sqrt{\frac{B^2}{16C^2} - \frac{3A^2}{4C^2}}$ and $P = 0$.

In $f_B$, we include the bulk second-order gradients terms, and to remove the degeneracy $K_1 = K_3$, we add cubic terms with the coefficients $L_2^{(3)}$ and $L_3^{(3)20-23}$,

$$
f_{B_{\text{DG}}} = f_{B_{\text{O}}} + L_1 Q_{00} Q_{00} + L_2 Q_{00} Q_{00} Q_{00} + L_3^{(3)} Q_{00} Q_{00} Q_{00} Q_{00} + L_{3}^{(3)2} Q_{00} Q_{00} Q_{00} Q_{00}.
$$

where $L_m$ are the elastic constants and $k = \partial / \partial S_N$ are the derivatives. Comparison of $f_{B_{\text{O}}}$ and $f_{B_{\text{DG}}}$ with $Q_{00}^{(N)} = S_N (n_i n_j - \delta_{ij}/3)$ results in the following relations between the elastic constants: $K_1 = 2 S_N^2 (2L_1 + L_3) + 2 S_N^3 \left(2L_3^2 - L_1^2\right)/3$, $K_3 = 4 S_N^2 L_1$, and $K_3 = 2 S_N^2 (2L_1 + L_3) + 2 S_N^3 \left(2L_3^2 - L_1^2\right)/3$. As seen from the latter expressions, the difference between the splay $K_1$ and bend $K_3$ constants is reflected in the cubic terms, $K_1 - K_3 = 2 S_N^2 \left(L_3^2 - L_1^2\right)$.

The Q-tensor field of a disclination is determined by the minimization of the functional $F = \int f_{\text{DG}} dV$. We consider planar disclinations with a two-dimensional (2D) director field $\mathbf{n}$ confined to the $(x,y)$ plane. As prompted by the experimental data, we describe the Q-tensor (equation (4)) in the polar coordinates $(r, \theta)$ with the disclination centre at $r = 0$ by the order parameters $S(r, \theta), P(r, \theta)$ and the angle $\phi(r, \theta) = m \theta + \phi_0(r, \theta)$ between $\mathbf{n}$ and $x$-axis. The experiment demonstrates strong axial asymmetry of the cores, with the main harmonic $k = 2(1 - m)$, that is, $k = 1$ for $m = 1/2$ (Fig. 1e) and $k = 3$ for $m = -1/2$ (Fig. 1f). Note that $k$ counts the number of cusps, or how many times the director field around the core assumes the orientation parallel to the radius-vector $r$. Using the relevant main harmonics, we find the radial distributions of the order parameters around the disclinations, by minimizing the functional $F$, as detailed in Methods:

$$
S(r, \theta) = S_0(r) + S_k(r) \cos k \theta,
$$

$$
P(r, \theta) = P_0(r) + P_k(r) \cos k \theta,
$$

$$
\phi(r, \theta) = \phi_0(r) \sin k \theta.
$$

The 3D and contour line representation of spatially varying $\Gamma \propto S - P$ in Fig. 1c,d indicate that the scalar order parameters $S$ and $P$ are function of $(r, \theta)$. The theoretically deduced anisotropy functions and the harmonics of the optical retardation, $\Gamma \propto S - P$, are shown in Fig. 1g,h.

**Experimental determination of nanoscale core structure.**

We performed an independent experiment by exploring the structure of the $m = 1/2$ core at the scale of tens of nanometres, using cryo-transmission electron microscopy (cryo-TEM) (Fig. 2a). The data on $\phi_1^{(1/2)}$ versus $\theta$ (Fig. 2b) clearly show that within the circular band $60 nm < r < 250 nm$, the anisotropy function $\phi_1^{(1/2)}$ becomes
The physical origin zero, as the dependence is of the type \(j = C_0/\sqrt{r} \approx 1/2\)) made by the director with respect to the chosen fixed axis (dashed white line) measured in the region \(60 \, \text{nm} < r < 250 \, \text{nm}\). It shows a linear relationship \(\phi^{(1/2)}(r, \theta) \approx \theta/2\). At \(r < 60 \, \text{nm}\), the image is too blurred to extract meaningful \(\phi^{(1/2)}\) values.

**Figure 2 | Experimentally determined nanostructure of a disclination core.** (a) Cryo-TEM texture of the 1/2 disclination in DSCG thin film. Dark short lines in the texture represent the aggregates whose electron density is higher as compared to background (water) environment. Scale bar, 50 nm. (b) Azimuthal dependence of the polar angle \(\phi^{(1/2)}\) made by the director with respect to the chosen fixed axis (dashed white line) measured in the region \(60 \, \text{nm} < r < 250 \, \text{nm}\) shows a linear relationship \(\phi^{(1/2)}(r, \theta) = \theta/2\). At \(r < 60 \, \text{nm}\) from the core, the image is too blurred to extract meaningful \(\phi^{(1/2)}\) values.

**Landau-de Gennes model versus experimental core structure.** The qualitative features of the experimental and model data are in a reasonable agreement. First of all, the experimental and numerical radial dependences of the main Fourier harmonics \(\phi_1^{(1/2)}\) for \(m = 1/2\) (Fig. 1e) and \(\phi_1^{(1/2)}\) for \(m = -1/2\) (Fig. 1f) are qualitatively similar and vanish as \(r \to 0\). The radial dependence of the leading harmonics of the optical retardance is rather well reproduced by the numerical simulations (Fig. 1g,h). The simulations also reproduce the qualitative character of the non-monotonous radial dependence of the functions describing asymmetry of the cores (Fig. 1g,h), but fail to describe these variations quantitatively. It is hard to expect a better agreement between the experimental behaviour and the Landau-de Gennes model, since the chromonic nematics depart rather strongly from an idealized Landau-de Gennes system. Namely, the building units of the nematic phase, the aggregates, are not fixed by the covalent bonds and their length and length distribution change markedly as a function of temperature, concentration \(\text{la} \to \text{lc}\) and, presumably, order parameter gradients. One can thus expect a spatially varying concentration and length distribution of the aggregates within the core; the Landau-de Gennes model does not capture these degrees of freedom, thus limiting our description.

**Asymmetric melted core and Landau-de Gennes elasticity.** An independent verification of the strong anisotropy of elastic properties of the chromonic nematics that is at the basis of the complex core structure is supplied by the behaviour of the defect cores at elevated temperatures, when the centre of the core is melted and expands towards the nematic periphery (Fig. 3a). The disclinations show asymmetric non-circular interfaces between the nematic and the isotropic phases with strongly developed cusps, one in the case of \(m = 1/2\) and three in the case of \(m = -1/2\). The physical origin of the axial asymmetry is that the energy associated with the gradients of order parameters depends on the director orientation. Consider an interface between the nematic and isotropic phases for two local orientations of the director; first, with \(\hat{n}\) in the plane of the interface and second, with \(\hat{n}\) being perpendicular to the interface. The theory above allows us to estimate the width over which the order parameter changes (see Methods). This width is proportional to \(w_1 \propto \sqrt{6L_1 + \epsilon_n}\) for the tangential alignment and to \(w_p \propto \sqrt{6L_1 + 4\epsilon_n}\) for the perpendicular alignment, respectively. The two are different because of the ‘anisotropy’ constant \(\epsilon_n\). As discussed above, \(L_1 \approx 40\, \text{nm}\), thus the width of the interfacial regions with a perpendicular director should be about two times wider than in the tangential case, \(w_p/w_1 \approx 2\), which is exactly the feature
observed experimentally (Fig. 3b). Note that the large value of \( L_4 \) as compared to \( L_2 \) is essentially the main factor that makes the disclination core in the chromonic liquid crystals so much larger than in the thermotropic materials where it is on the order of 10–100 nm, see, for example, ref. 3.

Discussion

To summarize, we demonstrate a complex structure of the cores of topological defects in the uniaxial nematic liquid crystals, the so-called disclinations. As an experimental system, we used the nematic phase of LCLCs based on water dispersions of organic molecules. The reason is that in these materials, the defect cores, that is, the regions over which the order parameter changes, are large, extending over tens of micrometres which allows one to use optical microscopy to characterize the fine structure of the cores\(^9\). The studied disclinations are of strength 1/2 and \(-1/2\), defined as a number of rotations by \(2\pi\) when one circumnavigates the centre of the defect once. Strong spatial variations of the order parameter near the defect cores are characterized in terms of the local preferred orientation (the director) and the degree of orientational order (the scalar order parameter). An unexpected feature of the defect cores is the appearance of ‘cusps’ along which the radial gradients of the scalar order parameters are ‘healing’ more slowly than along other directions. The observed radial and azimuthal dependences demonstrate a strong coupling between the gradients of the director and the scalar order parameter in the free energy. At the very core of the defects, \(<250\) nm from its geometrical centre, the core structure becomes azimuthally symmetric. The findings are in sharp contrast to the available theoretical models of the defect cores in which the phase of the order parameter is considered to be independent of the radial coordinate and the amplitude of the order parameter is considered to be independent of azimuthal coordinate. Since the azimuthal and radial dependencies of the phase and amplitude of the order parameter are caused by the elastic anisotropy, similar features are expected for other materials, including the thermotropic liquid crystals, for which the experimental observations are currently impossible because of the small spatial scale of distortions.

We note that the disclinations of strength 1/2 and \(-1/2\) considered in our work are the main ingredients of the topological turbulence regime in active matter\(^11\)–\(^13\). Furthermore, disclinations considered in our work are the main ingredients of the topological defects in active matter\(^11\)–\(^13\). The parameters are indicated in the text.

latter. We expect that the main findings of the present work, namely, radial and azimuthal dependences of the orientational order would also be present in out-of-equilibrium systems with a potential impact on the dynamic behaviour of these systems.

Methods

Lyotropic chromonic liquid crystals. DSCG was purchased from Spectrum Chemicals, 98% purity, and dissolved in ultrapure water (resistivity 18.2 MΩ cm) at 15 wt%.

Glass cell preparation. Glass substrates were soaked in Piranha solution (98% sulfuric acid: 30% H₂O₂ = 3:1) at 80 °C for \(>1\) h. The substrates were then rinsed with ultrapure water and dried with dry nitrogen. Two substrates were bound at four corners with a NOA65 glue mixed with 5 μm glass spacers to keep a uniform separation between the two substrates. The cell gap \(d\) is measured by spectroscopy when the cell is empty. After filling the cell with DSCG, open edges were well sealed with epoxy glue. The transition temperature from nematic to nematic/isotropic coexistence phase is checked before and after the experiments and found to be different by \(<0.2°\) C, indicating no change of the DSCG concentration during the experiments. During the LC-PolScope measurements, the sample temperature is controlled (±0.1 °C) by Linkman hot stage LTS120 and controller PE94.

LC-PolScope microscopy. The DSCG textures were examined by a polarizing microscope (Nikon E600) equipped with LC-PolScope (Cambridge Research). Unlike a conventional microscope which maps the intensity of a transmitted light, LC-PolScope maps the optical phase retardance and the projection of the optical axis (director in our case) onto the plane of the sample. The sample is probed by a monochromatic light of wavelength 546 nm at different settings of a liquid crystal retarder. At each pixel of the image, LC-PolScope maps orientation of the director (Fig. 1a,b) and the optical retardance \(\Gamma(x,y)\) in the range \((0–273)\) nm (Fig. 1c,d, ref. 14). To clarify the structure of the disclination cores, in Fig. 1c,d, we present the radial and azimuthal dependences of the orientational order would also be present in out-of-equilibrium systems with a potential impact on the dynamic behaviour of these systems.

Cryo-TEM. Thin film of 15 wt% DSCG supported on a carbon-coated copper grid (Ted Pella) was prepared by plunge-freezing technique in a controlled environment vitrification chamber (Vitrobot, FEI) and observed using low-dose cryo-TEM (Technai TF20, FEI) operated at 200 kV (ref. 30). The pixel intensity of the image reveals the local electron density of the sample, and dark short lines represent the linear aggregates of DSCG molecules. To determine the orientation of director at each point, first we calculate the spatial autocorrelation function of an 18 by 18 nm sub-image around that point. Then a discrete Radon transform is performed on the autocorrelation function to generate its sinogram. The local orientation of the aggregates \(\phi\) is then determined to be the angle that corresponds to the Radon cross-section with the highest intensity variation. This procedure is repeated at each pixel of the image that is larger than 9 nm away from the boundaries. From the 2D mapping of azimuthal angle of the director, we extract values of \(\phi^{(1/2)}(r,\theta)\) around the 1/2 disclination.

Radial distribution of the order parameters. To reproduce the radial dependence of the main harmonics \(j = 0, k\) in equations (7)–(9) around the disclinations, we represent them as sums of a constant and decaying exponents:

\[
S_j(r) = S_0 + \sum_{n=-\infty}^{\infty} S_{j(n)}(r) \exp(-q_n \cdot r)
\]

\[
P_j(r) = P_0 + \sum_{n=1}^{\infty} P_{j(n)} \exp(-q_n \cdot r)
\]

\[
\phi_j(r) = \phi_0 + b_j \exp(-\eta \cdot r)
\]

Figure 4 | The radial dependence of the main harmonics of order parameters of disclinations. (a) \(m = 1/2\) disclination and (b) \(m = -1/2\) disclinations. The parameters are indicated in the text.
Then, minimization of the integral $F$ leads to the general conditions far away from the disclination, $r \to \infty$, and at the core centre, $r = 0$, as well as determines the amplitudes in equation (10) that depend on the values of the parameters in equations (5) and (6). As expected, minimization results in an equilibrium core structure that is uniaxial far away from the disclination centre, with the constant order parameter $S_0$ and the director azimuthal angle obeying the Dzyaloshinsky model (equations (2), (3), (10)):

$$\bar{s}_0 = S_0, \quad \bar{s}_1 = \bar{p}_0 = \bar{p}_1 = 0, \quad \bar{b}_k = \phi_k^{(m)}$$

where $\phi_k^{(m)} = (1 - 4k^2)/2L = (2k^2 + 4k^2L^2)/2L(2 + 4k^2L^2)$. At the core centre, $r = 0$, the equilibrium structure is also uniaxial, but the symmetry axis is along the disclination line with $S_0(0) = P_0(0), \bar{S}_0(0) = \bar{P}_0(0) = 0$ and $\phi_0(0) = 0$. These conditions yield the following relations between the coefficients:

$$S_0 + \phi_1^{(2)} + \phi_2^{(2)} = \phi_1^{(0)} + \phi_2^{(0)}$$

$$L_1^{(1)} + S_1^{(1)} + \phi_1^{(2)} = \phi_1^{(0)} + \phi_2^{(0)} = 0$$

(11)

Minimization of the integral $F = \int f_{3D} dV$ using equations (4)–(12) results in radial dependencies of the scalar order parameters $S$ and $P$, shown in Fig. 4. To plot the dependencies on Fig. 4, we have adopted the following approach to select the values of the thermodynamic parameters $A, B, C$, and elastic constants $L_0, L_1, L_2^{(0)}$ and $L_2^{(1)}$. First, we define a dimensionless radius of the core as $r = r\sqrt{C/E_0}$, where $r$ is the real radius of dimension (m). The free energy is made dimensionless by presenting it in the units of $C$. The energy is minimized for a set of chosen parameters $A/C, B/C$ and the dimensionless elastic constants $L_0, L_2^{(1)} / L_1$ and $L_1^{(1)} / L_1$, to find the amplitudes and exponents in equation (10). The results are then presented in the form that can be compared to the experimentally measured parameters such as optical retardance, $\Gamma = (S - P)$ and the anistropy function for the azimuthal angle. The value of coefficient $\gamma$ is determined from the saturated birefringence far away from the defect cores and the corresponding theoretical value, $S = S_0, P = 0$. The characteristic length $\xi = L_1/C$ of the core, defined through the relationship $r = r\sqrt{C/E_0}$, is determined by scaling the theoretical dependencies ($S_0 - P_0$) versus $r$ to the experimental dependencies of $\Gamma(0)$. The procedure is repeated until one finds the combination of $A/C, B/C, L_0, L_2^{(1)} / L_1$ and $L_1^{(1)} / L_1$ that most closely matches the experimental results for both negative and positive disclination cores. Figure 4 shows a result with the following parameters: $A/C = -0.12, B/C = 0.6, L_0/L_1 = 40, L_2^{(1)} / L_1 = 170$ and $L_1^{(1)} / L_1 = 85$. These values and $\xi = 143$ nm were used to calculate the theoretical curves in Fig. 1e–h.

The characteristic length $\xi = L_1/C$ for chromonic nematics that emerges in numerical simulations is on the order of 100 nm, which is one order of magnitude larger than the core size $\sim 10$ nm expected for thermotropic disclinations. Moreover, in the chromonic materials, the defect core is much larger than $\xi = L_1/C \sim 10^4$ nm, as a result of a large value of the elastic constant $L_0$ and a large width of the nematic-isotropic interface as discussed in the main text.

Data availability. All relevant data are available from the authors upon request.

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