Design and Analysis Of an Optimization Model for Allocating the upcoming trains to the Platforms

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Abstract. In railways, day by day assigning trains to the platforms is becoming a task by using the available resources. In this paper, an optimization model is proposed to minimize the waiting time of the upcoming trains to get into the platform on time. The waiting time of a train is minimized by reducing the overtime of the platform and service completion time of the existing train in the platform. The objective of the problem is minimize the overtime of the platform and service completion time for each trains. The solution of the model is to obtain the optimum scheduling decision of trains which leads to the result of Mixed Integer Programming problem.

1. Introduction.
Railway networks are directed every minutes in different cities to take away more number of passengers and goods. For such transportations it is highly essential to improve the utility and efficacy of platforms and services. So, setting up a schedule or planning can be carried out to increase the capability of platforms and services and fulfilling the satisfaction of passengers by minimizing the waiting time. Scheduling determines, direction for dispatching a set of trains traveling on a railway path and service start time and allocate the required resources to each service over a scheduled period. A substantial scheduling intention of this problem is the overall delays and ways to minimize the overtime. And it’s necessary to keep a gap between services and departures of two trains. Or else certain trains becomes congested and it leads to train delay issues. On such issues, compatible adjustment should be decided shortly and performed for track down the feasibility condition, a frequent task that involves service timetable and allocation. In this project, mathematical model has been constructed and optimum scheduling decision of trains which leads to the result of Mixed Integer Programming problem.

2. Literature survey
Maryam Youngespour et.al (2019) has proposed “Using Mixed integer programming and Constraint programming for operating rooms scheduling with modified block strategy”. In this research, they found the optimal sequence and minimized the cost of overtime, make span and completion time of surgeon, using MIP and constraint programming and compared the results.

Matteo Fischetti et.al (2017) has presented “Using a general-purpose Mixed-Integer Linear Programming solver for the practical solution of real-time train rescheduling”. In this paper, the mainly considered the train delays as the issue of the problem.
Keisuke Sato et.al (2013) has proposed “A MIP-based timetable rescheduling formulation and algorithm minimizing further inconvenience to passengers”. They used mixed integer programming and the results of this paper has been derived mainly on satisfaction of passengers taken into account.

Andrea Lodi et.al (2013) has discussed “Performance Variability in Mixed-integer programming”. On considering the platforms scheduling they found the variability that occurs unexpectedly for a mixed integer programming solver.

Leonardo Lamargese et.al (2017) has developed and described a model “Integer programming techniques for Train Dispatching in Mass transit and Main line”. In this paper, they produced IP that can be used to get optimal solutions to large dispatching problems and real-life implementation in addition to it, they used Main-line applications.

Mohd Arshad et.al (2019) has presented a model on the title, “Train Delay Estimation in Indian Railways by Including weather factors through Machine Learning Techniques”. In this paper, the delay of trains under few constraints has been predicted and the well-defined ideas for preventing such issues using 4 different type of machine learning methods and found the appropriate results.

Marin Marinov et.al (2012) has modelled “Railway operations, time tabling and control”. In this research, they concentrated on planning and managing the train movement and evaluated the behaviour of railway networks.

Jiajian Liang et.al (2017) has proposed “Increasing performance of railway system by exploitation of the relationship between capacity and operation quality”. In this article, they presented an optimization of dispatching algorithm to evaluate the impact on capacity and operation quality.

Shoichiro Watanabe et.al (2015) has given “Energy-saving train scheduling diagram for automatically operated electric railway”. In this paper, they produced a method for developing an automatic train operation, which leads to the result of increased level of energy-saving efficiency.

Norman Weik et.al (2016) has discussed “Capacity analysis of railway lines in Germany- A rigorous discussion of the queueing based approach”. In this paper, the capacity analysis of railway lines and limitation are investigated and results is produced.

Marcella Samà et.al (2015) had designed “A multi-criteria decision support methodology for real-time train scheduling”. In this research, they worked on the real-time optimization of train scheduling decisions during congested traffic situation and solved using mixed integer linear programming method.

Ismail sahin (2017) had modelled “Markov chain model for delay distribution in train schedules: Assessing the effectiveness of time allowances”. In this paper, they considered the train departure, arrival delays and analysed the successive state change in single track railway and they used markov chain model and developed retrieval transition matrix and deterioration matrix.

Akilesh kumar et.al (2018) has discussed “A bi-level programming model for operative decisions on special trains: An Indian railways perspective”. In this paper, they maximized the expected revenue, rolling-stock grouping plan and few more criteria using mixed integer bi-level programming model.
Jose Luis Espinosa - Aranda et.al (2013) has given “A demand-based weighted train delay approach for rescheduling railway networks in real time”. In this model, they formulated make span minimization using mixed integer linear programming and AMDAA, AMCC algorithms.

Emma V. Andersson et.al (2015) has proposed “Reduced railway traffic delays using mixed integer linear programming approach to increase Robustness in critical points”. In this article, they presented timetable to reduce delay propagation and to promote robustness in critical points (RCP), using mixed integer linear programming approach and new timetables are evaluated.

Many researchers have shown their attention and interest towards finding an optimization of railway operations in the past researches. Many number of approaches were found and used to get the optimized solution of the problems. These methods are categorized as mathematical programming, expert solving tools and solutions or results. As similar to the above papers, in this project, for finding the minimized time of the waiting trains and service completion, a mathematical model has been constructed and formulation has been done mathematically.

3. Mathematical Formulation

The mathematical programming model is proposed to find the optimal solution and the notations used for this model are presented as follows:

**PARAMETERS:**

\( p, p' \)  
No. of platforms,  \( p, p' \in P \)

\( P \)  
Total no. of Platforms.

\( t_r \)  
No. of Trains,  \( t_r, t_r' \in T_r \)

\( T_r \)  
Total no. of Trains.

\( t \)  
Time.

\( i, i' \)  
Index of service,  \( i, i' \in I \)

\( J \)  
Stages of service,  \( j \in \{1,2,3\} \)

\( K_t \)  
Kind of tools.

\( L \)  
Normal service session length of the platform.

\( w, w' \)  
Index of Station master and track laborers,  \( w, w' \in W \)

\( MSS \)  
Master Service Scheduling.

\( AT \)  
Set of available tools belonging to each tools kinds.

\( SSE_{t_r} \)  
Turn over time of entering train  \((t_r)\) between two consecutive services.
Max $A_{tp}$ Maximum available time in each platforms.

$Stt_{tr}$ The start time of leaving train ($t_r$)

$d_{i,j}$ Duration of service of a train in $j^{th}$ stage.

**DECISION VARIABLES:**

$Need_{tr,p}$ Binary variable; 1 if stationary train($t_r$) needs service, 0 otherwise.

$MSS_{w,p}$ Binary variable; 1 if the track laborer is allocated to platforms(p) according to MSS, 0 otherwise.

$y_{i,p}$ Binary variable ; 1 if service(i) is allocated to platforms(p), 0 otherwise.

$Z_{i,i',p}$ Binary variable ; 1 if service(i) is precedes service(i´) in platforms(p), 0 otherwise.

$Q_{i,AT}$ Binary variable ; 1 if the tools(AT) is assigned to service(i), 0 otherwise.

$U_{i,i',w}$ Binary variable ; 1 if service(i) is precedes service(i´) with track laborers(w), 0 otherwise.

$W_{i,i',AT}$ Binary variable ; 1 if service(i) is precedes service(i´) on using the available tools(AT), 0 otherwise.

$Ct_{w}$ Binary variable ; 1 if completion time of track laborers(w´) service in assigned platform (based on MSS) is less than the normal session length of platform, 0 otherwise.

$C_{i,p}$ Completion time of service(i) in the platforms (p).

$C_{S,w,p}$ Service Completion time of track laborers(w) in the platforms (p).

$O_{p}$ Overtime of platforms (p).

$C_{serve,w}$ Service Completion time of Track laborers (w).

It must be considered that the precedence relation in variables, $Z_{i,i',p}$, $U_{i,i',w}$ and $W_{i,i',AT}$ does not notify instant precedence relation between services i and i´ but indicates general precedence. On using the above notations, a mixed integer programming model (MIP) is constructed.
The objective function (eq.1) is to minimize the platform’s overtime and completion time of service.

\[ \text{Min } ( \sum_{p} O_{p} + \sum_{w} C_{\text{ser}_{w}} ) \] \hspace{1cm} \text{(1)}

The objective function (eq.1) is to minimize the platform’s overtime and completion time of service.

\[ \sum_{p} Y_{i,p} = 1 \hspace{1cm} \forall \ i \] \hspace{1cm} \text{(2)}

Constraint (2) denotes that each service is performed in one platform.

\[ L - CS_{w,p} \leq Ct_{w} \forall w, p / MSS_{w,p} = 1 \] \hspace{1cm} \text{(3)}

Constraint (3) denotes that each service is performed in one platform.

\[ (Ct_{w} - 1) \leq L - CS_{w,p} \forall w, p / MSS_{w,p} = 1 \] \hspace{1cm} \text{(4)}

if the laborer’s completion time in platform p is less than the normal service length(L), the binary variable \( Ct_{w} \) takes one.

\[ Y_{i,p} \leq 1 - Ct_{w} \forall i, w, p / k = w_i MSS_{w,p} = 0 \] \hspace{1cm} \text{(5)}

Constraint (5), prevent track laborers from performing service in platform that have not assigned to them. \( MSS_{w,p} \) is a binary matrix parameter whose rows and columns indicate the labourer’s code & the platform number, if the labourer belongs to the platform, its value is one. Or it is zero.

\[ Z_{i,i',p} + Z_{i,i',p} \leq Y_{i,p} \forall i/l < i' \] \hspace{1cm} \text{(6)}

\[ Z_{i,i',p} + Z_{i,i',p} \geq Y_{i,p} \forall i/l < i' \] \hspace{1cm} \text{(7)}

\[ Z_{i,i',p} + Z_{i,i',p} \geq Y_{i,p} + Y_{i',p} - 1 \forall i/l < i' \] \hspace{1cm} \text{(8)}

Constraint (6) to (8) assures that precedence relation takes place between two services if and only if they are both assigned to the same platform.

\[ U_{i,i',w} + U_{i,i',w} = 1 \forall l, i', w / l < i' \text{, w=t l' } \] \hspace{1cm} \text{(9)}

Equation (9) indicates another relation occurs between two services if and only if they are both related to the same laborer.
Constraint (10) assures that the completion time of a service in every individual platform is zero if it is not assigned to it.

\[ C_{i,p} \leq Y_{i,p} b_i, p \quad \ldots \ldots \ldots \ldots (10) \]

Equation (11) defines that the completion time of two services in single platform by considering their precedence relation.

\[ C_{i,p} \geq C_{i',p} + \sum_{j} d_{i,j} - (1 - Z_{i,i',p}) \quad \forall \ i, i', p/i \neq i' \quad \ldots \ldots (11) \]

Equation (12) followed by the previous one, regulates the completion times of two service which are done by one set of laborers considering their precedence relation.

\[ \sum_{p=1}^{np} C_{i,p} \geq \sum_{p=1}^{np} C_{i',p} - d_{3i} + SSE_{ir} + d_{2i'} + d_{3i'} - (1 - U_{i,i',w}) \]

\[ \forall \ i, i', w/i \neq i', w = ser i, w = ser i' \quad \ldots \ldots (12) \]

Equation (13) are relevant to tools. These constraints defines the relation between two services that need identical tools.

\[ W_{i,i',AT} \leq Q_{i,AT} \quad \forall \ i, i', AT, i < i', Need_{i,kt} = Need_{i',kt} = 1 \]

\[ W_{i,i',AT} + W_{i',i,AT} \leq Q_{i,AT} \quad \forall \ i, i', AT, i < i', Need_{i,kt} = Need_{i',kt} = 1 \]

\[ W_{i,i',AT} + W_{i',i,AT} = Q_{i,AT} + Q_{i,AT} - 1 \quad \forall \ i, i', AT, i < i', Need_{i,kt} = Need_{i',kt} = 1 \]

Equation (14)-(16) are relevant to tools. These constraints defines the relation between two services that need identical tools.

\[ \sum_{p=1}^{np} C_{i,p} \geq \sum_{p=1}^{np} C_{i',p} - (1 - W_{i,AT}) \quad \forall \ i, i', AT/i \neq i' \quad \ldots \ldots (17) \]

Constraint (17) defines the completion time of two services that have similar tools.

\[ Y_{i,p} \leq MSS_{w,p} \quad \forall \ i, p, w = \text{ser} i. \quad \ldots \ldots (18) \]

In constraint (18), the allocation of services for any platform can only be based on MSS.
Constraints (19) ensures that if labourer(w) works in other labourer’s platform, the start time of labourer(w)’s service in a particular platform should be after the service completion time of the labourer who is the proprietor of that platform according to MSS.

\[ C_{ip} - \sum_j d_{ij} \geq CS_{w,p} - (1 - Y_{i,p}) \forall i, p, w, \text{MSS}_{w,p}=0 , w=\text{ser } i, w \neq w' \]

\[ \text{MSS}_{w,p} = 1 \quad \ldots \ldots (19) \]

The completion time of labourer(w) service is defined in constraint (20).

\[ CS_{w,p} \leq CS_{w,p} \forall p, w \quad \ldots \ldots (20) \]

Constraint (20) determines the completion time of labourer(w) in platform(p).

\[ CS_{w,p} \geq C_{ip} - d_{3,i} \forall i, p, w/w=\text{ser } i \quad \ldots \ldots (21) \]

Constraint (21) particularize the maximum available time for doing service in every platform.

\[ C_{ip} \geq Stlt_r + \sum_j d_{ij} - (1 - Y_{i,p}) \forall i, p, w/w=\text{ser } i \quad \ldots \ldots (22) \]

\[ C_{ip} \leq \text{Max } A t_r \forall i, p \quad \ldots \ldots (23) \]

4. Methodology

The Mixed integer programming problems can be solved by any methods under integer programming, few methods are mentioned below:

4.1. Branch-and-bound(B&B) Algorithm

This method is a typical example, used for solving combinatorial optimization problems and mathematical optimization. This algorithm is examined against upper and lower estimated bounds on the optimal solution.

4.2 Cutting Plane Algorithm
This method was proposed by Ralph E. Gomory for solving integer and mixed integer programming problems. It is a different variety of optimization method which produces a feasible set or objective function for a linear inequality cuts. This algorithm is mostly common used for solving MIP problems and convex optimization problems.

In general, the methodology for solving a MIP problem is as follow:

Step 1: In the beginning identifying and analyzing the reference model.

Step 2: The model’s assumptions.

Step 3: Fixing the decision variables and parameters

Step 4: The model’s mathematical formulation.

Step 5: If the problems is a real time assignment or scheduling then proceeding with the next step.

Step 6: Preliminary Computation and producing Numerical results.

And more importantly, this problem can also be solved by any algorithmic approach using software such as, matlab, Mathematica, etc.

5. Conclusion

In this paper, the model has been designed using mixed integer programming, subject to various constraints which minimizes the overtime of the platform and completion time of service which leads to reduce the waiting time of the trains, outside the platform. This process leads to the result if arriving the minimized and computational time for the platform overtime and service completion time.

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