Abstract

We apply a recently proposed Kerr/CFT correspondence to extremal supersymmetric five-dimensional charged spinning black holes, constructed by Breckenridge, Myers, Peet and Vafa. By computing the central charge of the dual CFT and Frolov-Thorne temperature, Cardy’s formula succeeds in reproducing Bekenstein-Hawking area law.
1 Introduction

Considerable progress in deriving black hole entropy statistically has been made by resorting to state counting approaches. Among them, while Cardy’s formula in conformal field theory (CFT) plays an indispensable role, this can be better understood in the context of AdS/CFT correspondence via string compactification and wrapped branes [1]. For example, a 2D $\mathcal{N} = (0, 4)$ CFT living on an M5-brane wrapping spatially $S^1 \times P_4$ ($P_4 \subset CY_3$) was shown to be dual to a 4D BPS black hole formed by a Type IIA D0-D2-D4 system which has an attractor geometry near its horizon [2]. The entropy in terms of Cardy’s formula

$$2\pi \sqrt{\frac{c_L L_0}{6}}$$

agrees with Bekenstein-Hawking area law. Here $c_L$ denotes the central charge and $L_0$ is the eigenvalue of left-moving Virasoro zero mode.

On the other hand, an alternative pioneered much earlier by Brown and Henneaux [3] is to take into account the asymptotic symmetry preserved at the boundary under suitable boundary conditions. They dealt with $AdS_3$ with $SL(2, R)_L \otimes SL(2, R)_R$ isometry which a 3D BTZ black hole asymptotically approaches. There, two copies of Virasoro algebra emerge as a result of infinitely many Fourier modes of the boundary diffeomorphism $\xi^\mu(x) \partial_\mu$. The central term arising from commutators of Virasoro generators was later used to reproduce the macroscopic entropy $S_{BTZ} = 2\pi \sqrt{\frac{c_L L_0}{6}} + 2\pi \sqrt{\frac{c_R L_0}{6}}$ by Strominger [7].

In much the same spirit of Brown-Henneaux, chiral auxiliary 2D CFTs dual to 4D extremal Kerr black holes have recently been proposed by Strominger et al. [8]. In their paper and a series of related works [9, 10, 11, 12, 13, 14], on the black hole near-horizon geometry certain crucial boundary conditions are imposed such that the asymptotic symmetry group (ASG) preserving it includes ultimately two kinds of generators, i.e.

$$K^t = \partial_t,$$

$$K^\phi = \epsilon(\phi) \partial_\phi - r \epsilon'(\phi) \partial_r,$$

where $\phi$ denotes some angular coordinate and $r$ stands for the radial direction. Decomposing an arbitrary periodic $\epsilon(\phi)$ into infinitely many Fourier modes labeled by $n$, one

\footnote{See also [4, 5, 6] for the appearance of Virasoro algebra in generic black holes.}
may identify $K_n^\phi$ with the generator $L_n$ of Virasoro algebra. Consequently, the central charge $c$ can be determined completely from the near-horizon metric and (1.2) owing to techniques developed in literatures [15, 16]. Quite remarkably, by further introducing Frolov-Thorne temperature $T_{FT}$ associated with $\phi$ [17], Cardy’s formula

$$S = \frac{\pi^2}{3} c T_{FT}$$

(1.3)

reproduces the macroscopic entropy perfectly. (1.3) can be regarded as a Legendre transformed version of (1.1) with an effective $T_{FT}$.

In this article, we apply the above procedure as well as (1.3) to a well-known 5D extremal supersymmetric charged spinning black hole constructed by Breckenridge, Myers, Peet and Vafa (BMPV) [18]. Unlike 4D Kerr-Newman black holes, this solution still exhibits unbroken supersymmetry even extremality is satisfied. As BTZ black holes mentioned above, the microscopic origin of BMPV entropy first roots in its D-brane realization. Nevertheless, the degeneracy counting that we derive below will rely thoroughly on Virasoro algebra stemming from ASG and the corresponding Frolov-Thorne temperature.

In section 2, we introduce briefly the BMPV black hole, a solution to the equation of motion of 5D Einstein-Maxwell-Chern-Simons gravity and present its conserved charges. In section 3, we carry out the computation of its dual chiral CFT central charge and Frolov-Thorne temperature. By making use of Cardy’s formula, perfect agreement with Bekenstein-Hawking area law is found. Finally, we conclude with some comments in section 4.

2It can also be realized via an M-theory lift of a D0-D2-D6 system wrapped on CY3, see [19] and references therein.
2 BMPV black hole and near-horizon geometry

2.1 BMPV black hole

As shown in [20], the BMPV solution can be embedded in 5D $\mathcal{N} = 2$ supergravity and is charged under a graviphoton. The metric reads (Planck length $l_5 = 1$)

$$ds^2 = -\left(1 - \frac{\mu}{r^2}\right)^2 dt^2 + \frac{dr^2}{\left(1 - \frac{\mu}{r^2}\right)^2} - \frac{\mu a}{r^2} \left(1 - \frac{\mu}{r^2}\right) \sigma_3 dt - \frac{\mu^2 a^2}{4r^4} \sigma_3^2 + \frac{r^2}{4} d\Omega_3^2,$$

$$\sigma_3 = d\phi + \cos \theta d\psi,$$

$$d\Omega_3^2 = d\theta^2 + \sin^2 \theta d\psi^2 + \sigma_3^2,$$

$$\theta \in [0, \pi], \quad \phi \in [0, 2\pi], \quad \psi \in [0, 4\pi]$$

(2.1)

with gauge potentials

$$A = B(r) dt + C(r) \sigma_3,$$

$$B(r) = \sqrt{3\mu} \frac{2r^2}{\lambda}, \quad C(r) = -\frac{\sqrt{3\mu a}}{4r^2}$$

(2.2)

and a constant dilaton field. $d\Omega_3^2$ is the line element on $S^3$.

The conserved energy, angular momentum and graviphoton charge for the BMPV black hole are as follows:

$$M = \frac{3}{4} \pi \mu, \quad J = \frac{1}{4} \pi a \mu, \quad Q = \frac{\sqrt{3}}{2} \pi \mu,$$

(2.3)

which satisfy the first law of black hole thermodynamics ($\Omega_\psi = 0$)

$$dM = T_H dS + \Omega_\phi dJ + \Phi dQ.$$  

(2.4)

Due to extremality, Hawking temperature $T_H$ and angular velocity $\Omega_\phi$ are zero, while the chemical potential $\Phi$ is equal to $B(\sqrt{\mu})$. A tricky point is that the ratio $\frac{T_H}{\Omega_\psi}$ is definitely finite at $r = \sqrt{\mu}$. We will use this fact later in section 3.3.

2.2 Near-horizon geometry

By taking near-horizon limit: $r = \sqrt{\mu}(1 + \frac{\lambda}{2} \hat{t})$ and $t = \frac{\sqrt{\mu}}{2\lambda} \hat{t}$ with $\lambda \to 0$, the BMPV metric (2.1) becomes

$$ds^2 = \frac{\mu}{4} (-\hat{r}^2 dt^2 + \frac{d\hat{r}^2}{\hat{r}^2}) - \frac{a\sqrt{\mu}}{2} \hat{r}(d\phi + \cos \theta d\psi) d\hat{t}$$

$$+ \frac{\mu - a^2}{4} (d\phi + \cos \theta d\psi)^2 + \frac{\mu}{4} (d\theta^2 + \sin^2 \theta d\psi^2).$$

(2.5)
It is seen that (2.5) possesses a structure of $AdS_2$ (in Poincaré patch) fibered over $S^3$. This strongly suggests at this limit the existence of a dual 2D chiral CFT whose central charge will be obtained thereof. Before proceeding to the dual CFT computation, we note that the horizon area is equal to

$$A_{\text{horizon}} = 2\pi^2 \mu \sqrt{\mu - a^2}$$  \hspace{1cm} (2.6)

and thus the macroscopic Bekenstein-Hawking entropy is given by ($G_5 = 1$)

$$S_{\text{macro}} = \frac{A_{\text{horizon}}}{4} = \frac{\pi^2}{2} \mu \sqrt{\mu - a^2}.$$  \hspace{1cm} (2.7)

### 3 Entropy from chiral CFT

#### 3.1 Boundary condition and asymptotic symmetry group

Following the work [3], we have to impose carefully boundary conditions on the asymptotic variation of the metric (2.5) and single out the desired ASG.

Let $h_{\mu\nu}$ be the perturbation around the near-horizon metric (2.5). We choose the following boundary condition:

$$\begin{array}{l}
h_{tt} = O(r^2) \quad h_{tr} = O\left(\frac{1}{r}\right) \quad h_{t\theta} = O\left(\frac{1}{r}\right) \quad h_{t\phi} = O(1) \quad h_{t\psi} = O(r) \\
h_{rt} = h_{tr} \quad h_{rr} = O\left(\frac{1}{r^2}\right) \quad h_{r\theta} = O\left(\frac{1}{r}\right) \quad h_{r\phi} = O\left(\frac{1}{r}\right) \quad h_{r\psi} = O\left(\frac{1}{r}\right) \\
h_{\theta t} = h_{t\theta} \quad h_{\theta r} = h_{r\theta} \quad h_{\theta\theta} = O\left(\frac{1}{r}\right) \quad h_{\theta\phi} = O\left(\frac{1}{r}\right) \quad h_{\theta\psi} = O\left(\frac{1}{r}\right) \\
h_{\phi t} = h_{t\phi} \quad h_{\phi r} = h_{r\phi} \quad h_{\phi\theta} = h_{\theta\phi} \quad h_{\phi\phi} = O(1) \quad h_{\phi\psi} = O(1) \\
h_{\psi t} = h_{t\psi} \quad h_{\psi r} = h_{r\psi} \quad h_{\psi\theta} = h_{\theta\psi} \quad h_{\psi\phi} = h_{\phi\psi} \quad h_{\psi\psi} = O\left(\frac{1}{r}\right)
\end{array}$$  \hspace{1cm} (3.1)

The most general diffeomorphism which respects the above boundary condition reads

$$\zeta = \left[C + O\left(\frac{1}{r^3}\right)\right] \partial_t + [-r e'(\phi) + O(1)] \partial_r + O\left(\frac{1}{r}\right) \partial_\theta + O\left(\frac{1}{r^2}\right) \partial_\psi + \left[\epsilon(\phi) + O\left(\frac{1}{r^2}\right)\right] \partial_\phi,$$  \hspace{1cm} (3.2)

where $C$ is an arbitrary constant and $\epsilon(\phi)$ is an arbitrary periodic function of $\phi$. We have dropped the hat over $t$ and $r$ for brevity. As a result, ASG here is simply generated by

$$\zeta^t = \partial_t,$$
$$\zeta^{[1]} = \epsilon(\phi) \partial_\phi - r e'(\phi) \partial_r.$$  \hspace{1cm} (3.3)
3.2 Central charge

We use the method developed in [15, 16] to compute the central charge of the dual chiral CFT. Let us start with

\[
\zeta^{[1]}_{(n)} = -e^{-in\phi} \partial_\phi - in re^{-in\phi} \partial_r,
\]

\[
\zeta^{[2]}_{(n)} = -e^{-in\psi} \partial_\psi - in re^{-in\psi} \partial_r,
\]

which correspond to Fourier modes of the periodic function $\epsilon$. Naively, commutators of $\zeta^{[i]}_{(n)}$'s constitute two copies of chiral Virasoro algebra without central terms. Nevertheless, the central extension $c^{(j)}$ ($j = 1, 2$) can be given as follows:

\[
\frac{1}{8\pi} \int_{\partial\Sigma} k_\zeta[h, g, g] = -\frac{i}{12} (m^2 + \xi m) c^{(j)} \delta_{m+n,0},
\]

where $\partial\Sigma$ is a spatial slice and $L_\zeta$ denotes Lie derivative with respect to $\zeta$. The 3-form $k_\zeta$ is defined by

\[
k_\zeta[h, g] = \frac{1}{2} \left[ \zeta_\nu D_\mu h - \zeta_\nu D_\sigma h_\mu^\sigma + \zeta_\sigma D_\nu h_\mu^\sigma + \frac{1}{2} h D_\nu \zeta_\mu - h_\nu^\sigma D_\sigma \zeta_\mu + \frac{1}{2} h_\nu^\sigma (D_\mu \zeta_\sigma + D_\sigma \zeta_\mu) \right] * (dx^\mu \wedge dx^\nu),
\]

where covariant derivatives and Einstein summation are performed with respect to $g_{\mu\nu}$. The coefficient $\xi$ in (3.5) is irrelevant because it can be absorbed by a shift of Virasoro zero mode. Equipped with generators of ASG (3.4) and the near-horizon metric (2.5), we have

\[
(L_{\zeta^{[1]}_{(n)}} g)_{tt} = \frac{i}{2} \mu r^2 ne^{-in\phi},
\]

\[
(L_{\zeta^{[1]}_{(n)}} g)_{t\psi} = \frac{i}{4} anr \sqrt{\mu} \cos \theta e^{-in\phi},
\]

\[
(L_{\zeta^{[1]}_{(n)}} g)_{r\phi} = -\frac{1}{4} \mu n^2 r e^{-in\phi},
\]

\[
(L_{\zeta^{[1]}_{(n)}} g)_{\phi\phi} = -\frac{i}{2} (a^2 - \mu) n e^{-in\phi},
\]

\[
(L_{\zeta^{[1]}_{(n)}} g)_{\phi\psi} = -\frac{i}{4} (a^2 - \mu) n \cos \theta e^{-in\phi}
\]
and

\[
\begin{align*}
(\mathcal{L}_{\zeta^{[2]}}g)_{tt} &= \frac{i}{2} \mu r^2 n e^{-i\psi} \\
(\mathcal{L}_{\zeta^{[2]}}g)_{t\phi} &= \frac{i}{4} a n r \sqrt{\mu} \cos \theta e^{-i\psi} \\
(\mathcal{L}_{\zeta^{[2]}}g)_{r\psi} &= -\frac{1}{4} \frac{\mu n^2}{r} e^{-i\psi} \\
(\mathcal{L}_{\zeta^{[2]}}g)_{\phi\psi} &= -\frac{i}{4} (a^2 - \mu) n \cos \theta e^{-i\psi} \\
(\mathcal{L}_{\zeta^{[2]}}g)_{\psi\psi} &= -\frac{i}{2} (a^2 \cos^2 \theta - \mu) n e^{-i\phi}
\end{align*}
\]

(3.8)

Substituting these back to (3.6), we obtain

\[c^{(1)} = 3\pi a \mu, \quad c^{(2)} = 0.\]  

(3.9)

This result can be reasoned as below. Due to a different coordinate choice, in [20] the black hole has two equal but opposite spins \(\pm J\). Here, by using Hopf fiber description of \(S^3\), one of them turns into a spin \(2J\) (associated with \(\phi\) coordinate), while the other (associated with \(\psi\) coordinate) vanishes.

### 3.3 Frolov-Thorne temperature

Let us determine the so-called Frolov-Thorne temperature. First, through equating eigen-modes near the horizon and elsewhere (hat is restored)

\[e^{-i\omega t + im_\phi \phi + im_\psi \psi} = e^{-im_R t + im_{L\phi} \hat{\phi} + im_{L\psi} \hat{\psi}},\]  

(3.10)

one has the relation between quantum numbers like

\[m_R = \frac{\omega \sqrt{\mu}}{2\lambda}, \quad m_{L\phi} = m_\phi, \quad m_{L\psi} = m_\psi\]  

(3.11)

for \(t = \frac{\sqrt{\mu}}{2\lambda} \hat{t}\), \(\phi = \hat{\phi}\) and \(\psi = \hat{\psi}\).

Next, we rewrite Boltzmann factor as \((\Omega_\psi = 0)\)

\[\exp \left( -\frac{\omega - \Omega_\phi m_\phi}{T_H} \right) = \exp \left( -\frac{m_R}{T_R} - \frac{m_{L\phi}}{T_\phi} \right).\]  

(3.12)

\(T_R\) and \(T_\phi\) are Frolov-Thorne temperatures. From (3.11), we get

\[T_R = \frac{T_H \sqrt{\mu}}{2\lambda}, \quad T_\phi = -\frac{T_H}{\Omega_\phi}.\]  

(3.13)
Automatically, $T_R = 0$ due to extremality and
\[
T_\phi = -\lim_{r \to \sqrt{\mu}} \frac{T_H(r)}{\Omega_\phi(r)} = \frac{\sqrt{\mu - a^2}}{2\pi a}.
\] (3.14)

As advertised, the ratio $\frac{T_H}{\Omega_\phi}$ remains non-vanishing at the horizon by carefully examining (3.14). We present this procedure in Appendix 3.

3.4 Microscopic entropy

Substituting (3.9) and (3.14) into Cardy’s formula (1.3), we obtain the microscopic entropy
\[
S_{\text{micro}} = \frac{1}{2} \pi^2 \mu \sqrt{\mu - a^2}.
\] (3.15)

This agrees precisely with Bekenstein-Hawking entropy (2.7).

4 Conclusion and comments

We have succeeded in reproducing the BMPV black hole entropy using Kerr/CFT correspondence. This non-trivial check suggests that counting entropy semi-classically by evaluating the dual CFT central charge and effective temperature is also applicable to 5D extremal supersymmetric charged spinning black holes.

Since the central charge in BMPV cases is proportional to $J$ (spin), we expect that a lifted solution in 6D similar to [12] can be constructed in order to reproduce the entropy in the degenerate limit $a \to 0$. Also, in $\mathcal{N} = 2$ Type IIA string compactified on $CY_3$, BMPV black holes can be realized via an M-theory lift of D0-D2-D6 systems wrapped on $CY_3$ with brane charges $(q_0, q_A, 1)$. Due to one single D6-brane, the 5D black hole is located at the center of a Taub-NUT space and $q_0 \propto J$ is associated with its spin over the $S^1$ bundle of Taub-NUT. In addition, $q_A$ is related to $Q$ by $q_A = \frac{3Q}{Y_A}$ where scalar fields $Y$’s in vector multiplets take their horizon values with normalization $1 = D_{ABC} Y^A Y^B Y^C (D_{ABC}$: triple intersection number of $CY_3$). It will be interesting to check whether Kerr/CFT prescription works as well for generic configurations of IIA brane charges $(q_0, q_A, p^A, p^0)$.

\textsuperscript{3}We are grateful to Chiang-Mei Chen who pointed out that a missing factor 2 in version one may be attributable to our previous Frolov-Thorne temperature.

\textsuperscript{4}A is the index for the 2-cycle basis of $CY_3$. 

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As another remark, the central charge in (3.9) differs from what is microscopically derived in [18] where \( c \propto \mu^2 \sim Q^2 \) for large \( \mu \). This feature is not encountered in 3D BTZ cases because the central charge of Brown-Henneaux is exactly equal to that in the dual 2D \( \mathcal{N} = (4,4) \) CFT [7]. It remains interesting to understand the exotic Kerr/CFT correspondence by pursuing this discrepancy further.

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A Appendix

According to [21], one is able to have a general black hole embedded in Gödel universe parameterized by \((m, q, j, a)\), namely,

\[
\begin{align*}
 ds^2 &= -f(r)dt^2 - 2g(r)\sigma_3 dt + h(r)\sigma_3^2 + \frac{dr^2}{V(r)} + \frac{r^2}{4}d\Omega_3^2, \\
 f(r) &= 1 - \frac{2m}{r^2} + \frac{q^2}{r^4}, \\
 V(r) &= 1 - \frac{2m - 8j(m + q)(a + 2j(m + 2q))}{r^2} + \frac{2(m - q)a^2 + q^2(1 - 16ja - 8j^2(m + 3q))}{r^4}, \\
 h(r) &= -j^2r^2(r^2 + 2m + 6q) + 3jqa + \frac{(m - q)a^2}{2r^2} - \frac{q^2a^2}{4r^4}, \\
 g(r) &= jr^2 + 3jq + \frac{(2m - q)a}{2r^2} - \frac{q^2a}{2r^4}.
\end{align*}
\]
Further, Hawking temperature $T_H(r_+)$ and the angular velocity $\Omega_\phi(r_+)$ ($\Omega_\psi(r_+)=0$) at the outer horizon are defined via

$$
T_H(r) = \frac{rV'(r)}{4\pi\sqrt{4h(r)+r^2}},
$$
$$
\Omega_\phi(r) = \frac{g(r)}{h(r)+\frac{r^2}{4}}.
$$

(A.6)

The BMPV solution saturates $m=q$ with $j=\sqrt{\frac{2(m-q)}{4(m+q)}}$, and $r_+=r_-=\sqrt{m}$. Since both Hawking temperature and the angular velocity go to zero as $r_+\to\sqrt{m}$, Frolov-Thorne temperature of BMPV

$$
T_\phi = -\lim_{r\to\sqrt{m}} \frac{T_H(r)}{\Omega_\phi(r)}
$$

(A.7)

should be calculated instead by $\frac{T_H(r)}{\Omega_\phi(r)}$ according to l’Hopital’s theorem.

Adhering to the three steps in order: first setting $j=\sqrt{\frac{2(m-q)}{4(m+q)}}$, then taking $m=q$ followed by substituting $r=\sqrt{m}$ as the final step, one can show that

$$
T_\phi = \frac{\sqrt{m-a^2}}{2\pi a}.
$$

(A.8)

This is nothing but (3.14) by putting $m=\mu$.

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