Minimal Modification To The Tri-bimaximal Neutrino Mixing

Xiao-Gang He\(^1\) and A. Zee\(^2\)

\(^1\)Center for Theoretical Sciences, Department of Physics, National Taiwan University, Taipei, Taiwan

and

\(^2\)Kavli Institute for Theoretical Physics, UCSB, Santa Barbara, CA 93106, USA

Abstract

Current experimental data on neutrino oscillations are consistent with the tri-bimaximal mixing. If future experimental data will determine a non-zero $V_{e3}$ and/or find CP violations in neutrino oscillations, there is the need to modify the mixing pattern. We find that a simple neutrino mass matrix, resulting from $A_4$ family symmetry breaking with residual $Z_3$ and $Z_2$ discrete symmetries respectively for the Higgs sectors generating the charged lepton and neutrino mass matrices, can satisfy the required modifications. The neutrino mass matrix is minimally modified with just one additional complex parameter compared with the one producing the tri-bimaximal mixing. In this case, the CP violating Jarlskog factor $J$ has a simple form ($|J| = |V_{e1}V_{e3}|/2\sqrt{3}$ for real neutrino mass matrix), and also $V_{\mu i} = 1/\sqrt{3}$. We also discuss how this mixing matrix can be tested experimentally.
**Introduction**

The current neutrino mixing matrix from various experimental data \[1, 2\] can be described by three neutrino mixing \[3, 4\]. The mixing matrix $V$ can be parameterized, using the Particle Data Group convention \[2\], by three mixing angles $\theta_{12}$, $\theta_{23}$ and $\theta_{13}$, and one intrinsic CP violating phase $\delta$ for Dirac neutrinos. For Majorana neutrinos there are additional two independent Majorana CP violating phases. Present constraints on the mixing angles, at the 99% confidence level, are as the following \[4\]:

$$30^\circ < \theta_{12} < 38^\circ, \quad 36^\circ < \theta_{23} < 54^\circ, \quad \theta_{13} < 10^\circ.$$

At present there is no experimental information about CP violating phases.

The above data can be well fitted by the tri-bimaximal mixing of the form

$$V_{\text{tri-bi}} = \begin{pmatrix} -\frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}.$$  \hspace{1cm} (2)

With a suitable normalization of the signs for the matrix elements, the above tri-bimaximal mixing has $\theta_{12} = \sin^{-1}(1/\sqrt{3}) = 35.2^\circ$, $\theta_{23} = 45^\circ$ and $\theta_{13} = 0$. Here we have omitted a possible diagonal Majorana phase matrix $P = \text{Diag}(e^{i\alpha_1}, e^{i\alpha_2}, e^{i\alpha_3})$ on the right. Since an overall phase does not play a role in any physical process, only two of the $\alpha_{1,2,3}$ are physically independent.

The tri-bimaximal form for the mixing matrix was first proposed by Harrison, Perkins and Scott \[5\] and further studied by authors in Ref. \[6\]. Later, we independently arrived at the same Ansatz \[7\]. Many theoretical efforts have been made to produce such a mixing pattern \[8, 9, 10, 11\]. Among them theories based on $A_4$ symmetry provide some interesting examples \[9, 10, 11\].

If future experimental data will find a non-zero value for $V_{e3}$, it is necessary to modify the mixing pattern. Another class of experimental data which may also lead to the requirement of modifying the tri-bimaximal mixing is the observation of CP violation in neutrino oscillations. CP violation in neutrino oscillations is proportional to the CP violating Jarlskog factor \[12\]:

$$J = \text{Im}[V_{\mu1}V_{\mu2}^* V_{\mu1}^* V_{\mu2}].$$

A non-zero $J$ is related to the non-removable phase in $V$ ("intrinsic" CP violation). This is different than the source of CP violation due to Majorana phases which do not show up in neutrino oscillations. The tri-bimaximal mixing leads to
$J = 0$ and therefore has no intrinsic CP violation. In this note we analyze a simple mixing matrix\cite{10, 11}, resulted from theories based on $A_4$ family symmetry breaking, satisfying the required modifications.

In the modified mixing matrix, $V_{\mu i} = 1/\sqrt{3}$ which are the same as those in the tri-bimaximal mixing. However, the matrix element $V_{e3}$ is no longer zero, and is given by $V_{e3} = i(ce^\rho - s)$. Here $c = \cos \theta$, $s = \sin \theta$ with $\theta$ being a new mixing angle. The phase $\rho$ is related to phases in the neutrino mass matrix. The detailed meaning will be given later.

The modified mixing matrix in various limits reduces to some of the forms considered in Refs\cite{13, 14, 15, 16}. In the case $c = s = 1/\sqrt{2}$ and $\rho = 0$, the mixing matrix reduces to the tri-bimaximal form. This mixing matrix has intrinsic CP violation with the Jarlskog factor $J$ given by $-(c^2 - s^2)/6\sqrt{3}$. The two parameters $\theta$ and $\rho$ can be determined by measuring $|V_{e3}|$, $|V_{\mu 3}|$ and $J$. Therefore this mixing pattern can be tested experimentally in details.

The key point in obtaining the tri-bimaximal mixing pattern in theories based on $A_4$ family symmetry is to first get the matrices $U_l$ and $U_\nu$, which diagonalize the charged lepton mass matrix $M_l$ and neutrino mass matrix $M_\nu$, $U_l^T M_l U_l = D_l$ and $U_\nu^T M_\nu U_\nu = D_\nu$ (assuming Majorana neutrinos), to have the following forms\cite{9, 10, 11}\cite{3}\cite{10, 11}\cite{10, 11}

$$U_l^T = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ \omega & 1 & \omega^2 \\ \omega^2 & 1 & \omega \end{pmatrix}, \quad U_\nu = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 & -1 \\ 0 & \sqrt{2} & 0 \\ 1 & 0 & 1 \end{pmatrix},$$

(3)

where $\omega^3 = 1$ and $1 + \omega + \omega^2 = 0$. Then using the definition for the mixing matrix $V = U_l^T U_\nu$ to obtain

$$V = V_{tri-bi} V_\phi,$$

(4)

where $V_\phi = Diag(-1, 1, -i)$.

For $U_\nu$, we recognize that it is just a rotation through $45^\circ$ in the $(1 - 3)$ plane. Recalling that $U_\nu$ is determined by requiring $U_\nu^T M_\nu U_\nu = D_\nu$ be diagonal, the form of $M_\nu$ is

$$M_\nu = \begin{pmatrix} \alpha & 0 & \beta \\ 0 & \gamma & 0 \\ \beta & 0 & \alpha \end{pmatrix}.$$

(5)

Tri-bimaximal mass matrix and modifications
Let us now briefly discuss how tri-bimaximal mixing can arise and how it is minimally modified in $A_4$ models following Ref.[10]. The basic issue is that $A_4$ symmetry is broken down to two different discrete subgroups upon charged leptons and upon the neutrinos acquiring masses, namely $Z_3$ and $Z_2$ respectively. The clash between these two different subgroups was called the “sequestering problem”[11]. To explain the clash, let us be more specific with Higgs mechanism supplying the lepton masses. We emphasize, however, that the results of this paper are not dependent on any specific model.

The two forms for $U_l$ and $U_\nu$ in eq.(3) are very different. In $A_4$ theories, this requires at least two separate Higgs sectors. We consider a case with three Higgs fields, \(\Phi, \phi\) (standard model doublet) and \(\chi\) (standard model singlet). Under the $A_4$, \(\Phi\) and \(\chi\) both transform as 3, and \(\phi\) as 1. The standard left-handed leptons \(l_L\), right-handed charged leptons \((l_{1R}, l_{2R}, l_{3R})\), and right-handed neutrinos \(\nu_R\) transform as 3, \((1', 1, 1')\) and 3, respectively. We refer the readers for more details on $A_4$ group properties to Refs.[9, 10, 11, 17]. The Lagrangian responsible for the lepton mass matrix is

\[
L = \lambda_e \bar{l}_L \Phi l_R^1 + \lambda_\mu \bar{l}_L \Phi l_R^2 + \lambda_\tau \bar{l}_L \Phi l_R^3 + H.C.
+ \lambda_l \bar{l}_L \nu_R \phi + m_\nu R \nu_R^C + \lambda_{\chi} \bar{\nu}_R \nu_R^C \chi.
\] (6)

If the vev structure is of the form \(<\Phi_{1,2,3}>=v_\Phi, <\chi_{1,3}>=0, <\chi_2>=v_\chi\), and \(<\phi>=v_\phi\), one would obtain the charged lepton mass term as

\[
\begin{pmatrix}
1 & \omega^2 & \omega \\
\omega & 1 & 1 \\
1 & \omega & \omega^2
\end{pmatrix}
\begin{pmatrix}
\lambda_e v_\Phi & 0 & 0 \\
0 & \lambda_\mu v_\Phi & 0 \\
0 & 0 & \lambda_\tau v_\Phi
\end{pmatrix}
\begin{pmatrix}
l_{1R}^1 \\
l_{2R}^2 \\
l_{3R}^3
\end{pmatrix}.
\] (7)

From the above, we can identify the charged lepton mass to be \(\sqrt{3}\lambda_i v_\Phi\), and \(U_l\) to have the “magic” form in eq.(3). \(U_r\) is a unit matrix.

The neutrino mass matrix has the see-saw form with

\[
M = \begin{pmatrix} 0 & M_D \\ M_D^T & M_R \end{pmatrix}, \quad M_R = \begin{pmatrix} m & 0 & m_\chi \\ 0 & m & 0 \\ m_\chi & 0 & m \end{pmatrix},
\] (8)

where \(M_D = \text{Diag}(1, 1, 1)\lambda_\nu v_\phi\), and \(m_\chi = \lambda_\chi v_\chi\). From this one obtains the light neutrino mass matrix \(M_\nu\) of the form given in eq.(5). One therefore has a model for the tri-bimaximal mixing.
The vev structure of the Higgs fields breaks $A_4$, but left some residual symmetries. The Higgs doublet $\Phi_i$ with equal vacuum expectation values breaks $A_4$ down to a $Z_3$ generated by $\{I, c, a\}$, and the vev of only the $\chi_2$ component to be non-zero in $\chi$ breaks $A_4$ down to a $Z_2$ generated by $\{1, r_2\}$. Here $a, c, r_2$ are $A_4$ group elements defined in Ref. 10. We note that the charged lepton mass matrix and the neutrino mass matrix are related to two separate Higgs sectors, $\Phi$, and, $\chi$ and $\phi$, respectively. If there is no communication between the two Higgs sectors, the residual $Z_3$ and $Z_2$ symmetries will be maintained. In general $\Phi$ and $\chi$ mix in the Higgs potential, it is not possible to keep the vev structure for $\Phi$ and $\chi$ discussed above. One needs to separate them from communicating in the Higgs potential and therefore the sequestering problem. This sequestering problem will complicate the situation. However, models realizing such separation have been constructed with additional symmetries. For our purpose here, we will assume that the sequestering problem is solved and study the consequences.

As long as the $Z_3$ symmetry is not broken, i.e. equal vev for $\Phi_i$, the form of $U_l$ obtained in the above is stable against higher order corrections. Also if the $Z_2$ symmetry is not broken, the “12”, “21”, “23” and “32” entries in $M_D$ and $M_R$ and therefore $M_\nu$ are prevented from getting non-zero values. However it does not protect the “11” and “22” entries be equal. Therefore after symmetry breaking a more general form of the light neutrino mass matrix $M_\nu$ will emerge with

$$M_\nu = \begin{pmatrix} \alpha - \varepsilon & 0 & \beta \\ 0 & \gamma & 0 \\ \beta & 0 & \alpha + \varepsilon \end{pmatrix}, \tag{9}$$

rather than the $M_\nu$ in (5). The above neutrino mass matrix has been obtained previously in Refs. 10, 11. The above form of neutrino mass matrix is a minimal modification to the one which generates the tri-bimaximal mixing in the sense that there is just one additional complex parameter $\varepsilon$ introduced in the mass matrix. With the new $M_\nu$ the most general form for $U_\nu$ is given by

$$U_\nu = V_\phi' \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{i\delta} \end{pmatrix} \begin{pmatrix} c & 0 & -s \\ 0 & 1 & 0 \\ s & 0 & c \end{pmatrix} V_{\phi''}^*, \tag{10}$$
where $c = \cos \theta$, $s = \sin \theta$, and
\[
\tan^2 2\theta = \frac{4|\beta|^2}{(|\alpha - \varepsilon| - |\alpha + \varepsilon|)^2} \left( 1 - \frac{4|\alpha - \varepsilon||\alpha + \varepsilon|}{(|\alpha - \varepsilon| + |\alpha + \varepsilon|)^2} \sin^2 \sigma \right),
\]
\[
\tan \delta = -\frac{|\alpha - \varepsilon| - |\alpha + \varepsilon|}{|\alpha - \varepsilon| + |\alpha + \varepsilon|} \tan \sigma,
\]
\[
\sigma = \text{Arg} (\beta) - \frac{1}{2} (\delta_{11} + \delta_{33}),
\]
\[
V_{\nu'} = \text{Diag}(e^{-i\delta_{11}/2}, 1, e^{-i\delta_{33}/2}),
\]
\[
\delta_{11} = \text{Arg} (\alpha - \varepsilon), \delta_{33} = \text{Arg} (\alpha + \varepsilon),
\]
and $(V_{\nu'}^\dagger)^2$ is related to the neutrino Majorana phases which are functions of $\alpha$, $\beta$, $\gamma$ and $\varepsilon$.

Finally after reorganizing the Majorana phases, in the basis when taking the limit that $\varepsilon$ goes to zero $V$ reduces to $V_{\text{tri-bi}}$, we obtain a neutrino mixing matrix in the following form
\[
V = \frac{1}{\sqrt{3}} \begin{pmatrix}
1 & 1 & 1 \\
\omega & 1 & \omega^2 \\
\omega^2 & 1 & \omega
\end{pmatrix} \begin{pmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & e^{i\rho}
\end{pmatrix} \begin{pmatrix}
c & -s \\
c & s \\
0 & 0
\end{pmatrix} \begin{pmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & i
\end{pmatrix}
\]
\[
= \frac{1}{\sqrt{3}} \begin{pmatrix}
-(c + se^{i\rho}) & 1 & i(c e^{i\rho} - s) \\
-(\omega c + \omega^2 se^{i\rho}) & 1 & i(\omega^2 c e^{i\rho} - \omega s) \\
-(\omega^2 c + \omega se^{i\rho}) & 1 & i(\omega c e^{i\rho} - \omega^2 s)
\end{pmatrix} V_{\text{tri-bi}} \begin{pmatrix}
\cos \tau & 0 & i \sin \tau e^{i\eta} \\
0 & 1 & 0 \\
i \sin \tau e^{-i\eta} & 0 & \cos \tau
\end{pmatrix} V_p,
\]
where $\rho = \delta - (\delta_{33} - \delta_{11})/2$. $V_p = \text{Diag}(e^{i(\xi + \rho/2)}, 1, e^{i(-\xi + \rho/2)})$ with $\tan \xi = (s - c) \tan(\rho/2)/(s + c)$, $\tan \eta = 2 \tan(2\theta) \tan(\rho/2)/(1 + \tan^2(\rho/2))$, and $\sin^2 \tau = (1 - 2sc \cos \rho)/2$.

In this basis, the neutrino masses $m_{1,2,3}$ have Majorana phases $-2\alpha_{1,2,3}$ with
\[
\alpha_1 = -[\text{Arg}(c^2|\alpha - \varepsilon| + 2sc|\beta|e^{i(\delta + \gamma)} + s^2|\alpha + \varepsilon|e^{i2\delta}) + \delta_{11}] / 2 + \pi,
\]
\[
\alpha_2 = -\text{Arg}(\gamma)/2,
\]
\[
\alpha_3 = -[\text{Arg}(s^2|\alpha - \varepsilon| - 2sc|\beta|e^{i(\delta + \gamma)} + c^2|\alpha + \varepsilon|e^{i2\delta}) + \delta_{11}] / 2 - \pi / 2.
\]

The masses are given by
\[
|m_1|^2 = |c^2|\alpha - \varepsilon| + 2sc|\beta|e^{i(\delta + \gamma)} + s^2|\alpha + \varepsilon|e^{i2\delta}|^2
\]
\[
= \left| \frac{1}{2} |\alpha - \varepsilon|(1 + \frac{1}{\cos(2\theta)}) + \frac{1}{2} |\alpha + \varepsilon|(1 - \frac{1}{\cos(2\theta)})e^{i2\delta} \right|^2
\]
\[
= \left| \frac{1}{2} |\alpha - \varepsilon|^2 + 2|\beta|^2 + |\alpha + \varepsilon|^2 + \frac{1}{\cos(2\theta)}(|\alpha - \varepsilon| - |\alpha + \varepsilon|^2),
\]
\[
|m_2|^2 = |\gamma|^2,
\]
\[ |m_{3}|^{2} = \left| s^{2}|\alpha - \varepsilon| - 2sc|\beta|e^{i(\delta + \sigma)} + c^{2}|\alpha + \varepsilon|e^{i2\delta} \right|^{2}. \]
\[ = \frac{1}{2}|\alpha - \varepsilon|\left(1 - \frac{1}{\cos(2\theta)}\right) + \frac{1}{2}|\alpha + \varepsilon|\left(1 + \frac{1}{\cos(2\theta)}\right)e^{i2\delta} \right|^{2} \]
\[ = \frac{1}{2}\left[|\alpha - \varepsilon|^{2} + 2|\beta|^{2} + |\alpha + \varepsilon|^{2} - \frac{1}{\cos(2\theta)}(|\alpha - \varepsilon|^{2} - |\alpha + \varepsilon|^{2}) \right]. \quad (13) \]

**Properties of the modified mixing matrix**

One clearly sees that the new mixing matrix can be very different from the tri-bimaximal, but the entries \( V_{\mu i} = 1/\sqrt{3} \) are the same as those of the tri-bimaximal mixing. This can be tested in the near future by experiments. There are of course many new features. Two important qualitative differences are:

(a). \( V_{e3} \) is not zero any more. We have \( |V_{e3}| = |(ce^{i\rho} - s)|/\sqrt{3} \). In the real neutrino mass matrix case, for small \( \varepsilon \) [10],
\[ |V_{e3}| \approx \frac{|\varepsilon|}{\sqrt{6}|\beta|}. \quad (14) \]

(b). There are intrinsic CP violation. This can be easily checked by evaluating the Jarlskog factor, we obtain [11]
\[ J = -\frac{1}{9}(c^{2} - s^{2}) \sin \frac{2\pi}{3}. \quad (15) \]

It is surprising to note that the CP violating Jarlskog factor \( J \) does not contain the phase \( \rho \) implying that even if the parameters \( \alpha, \beta \) and \( \varepsilon \) are real (or \( \rho = 0 \)) there is intrinsic CP violation. In this case the value of \( J \) is equal to \(-iV_{e1}V_{e3}/2\sqrt{3} \) whose size can be as large as 0.04. This is sizeable enough to be measured in future experiments.

Note that the mixing matrix, apart from the Majorana phases \( \alpha_{i} \), is a two-parameter, \( \rho \) and \( \theta \), matrix. They can be completely determined experimentally.

The sign of \( J = -(c^{2} - s^{2})/6\sqrt{3} \) will decide whether \( c \) is larger or smaller than \( s \). We have
\[ \cos 2\theta = -6\sqrt{3}J. \quad (16) \]

One can always choose a convention with both \( s \) and \( c \) be positive. We then have \( \sin 2\theta = \sqrt{1 - (\cos 2\theta)^{2}} \).

The phase factor \( \rho \) can be determined by additional measurements of \( V_{e3} \) and \( V_{\mu 3} \). We have
\[ \cos \rho = \frac{1 - 3|V_{e3}|^{2}}{\sqrt{1 - (6\sqrt{3}J)^{2}}}. \quad (17) \]
Combining
\[
\tan \rho = -\frac{2}{\sqrt{3}} \left( \frac{1}{2} + \frac{1 - 3|V_{\mu 3}|^2}{1 - 3|V_{e3}|^2} \right),
\]
the sign of \( \sin \rho \) can also be determined. The consistency of the above two equations can provide tests for the mixing matrix proposed.

We comment that the mixing matrix contains some of the cases studied by Xing [13], Bjorken, Harrison and Scott [14], Friedberg and Lee [15], and Xing, Zhang and Zhou [16] in various limiting cases. We find the following two limiting cases interesting.

(1). \( c = s = 1/\sqrt{2} \). In this case there is no intrinsic CP violation (no CP violation can be observed in neutrino oscillations). We have from eq. (12),
\[
V = V_{\text{tri}} - V_{\text{bi}} \begin{pmatrix}
\cos(\rho/2) & 0 & \sin(\rho/2) \\
0 & 1 & 0 \\
-\sin(\rho/2) & 0 & \cos(\rho/2)
\end{pmatrix} \begin{pmatrix}
\cos(\rho/2) & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & e^{i\rho/2}
\end{pmatrix}. \quad (19)
\]
This is the same mixing matrix, up to some Majorana phases, in eq. (1.18) obtained in Ref. [15]. From this we also see that the phase \( \rho \) indeed does not play the role of a Dirac phase which causes intrinsic CP violation.

(2). \( \rho = 0 \), in this case there is intrinsic CP violation. We have
\[
V = V_{\text{tri}} - V_{\text{bi}} \begin{pmatrix}
\cos(\theta - \pi/4) & 0 & i\sin(\theta - \pi/4) \\
0 & 1 & 0 \\
i\sin(\theta - \pi/4) & 0 & \cos(\theta - \pi/4)
\end{pmatrix}, \quad (20)
\]
and \( J = -iV_{e1}V_{e3}/2\sqrt{3} \). This mixing matrix looks similar to that in eq. (1.18) of Ref. [15], but the appearance of “\( i \)” makes it CP violating.

It would be interesting to see how these two limiting cases can be experimentally distinguished. An obvious way to tell the difference is to determine whether \( J \) is zero or not by measuring CP violation in neutrino oscillations. If \( J \) turns out to be non-zero, case (1) has to be abandoned. Before \( J \) can be measured, precise measurements of \( |V_{\mu 3}| \) and \( |V_{e3}| \) can also tell the difference since for case (1), one has
\[
|V_{\mu 3}|^2 = \frac{1}{2}(1 - |V_{e3}|^2) \pm \frac{1}{\sqrt{2}}|V_{e3}|\sqrt{1 - 3|V_{e3}|^2}/2, \quad (21)
\]
where “+” should be taken if \( \cos(\rho/2) \) and \( \sin(\rho/2) \) have the same sign. Otherwise “−” should be taken. While for case (2), one has
\[
|V_{\mu 3}|^2 = \frac{1}{2}(1 - |V_{e3}|^2). \quad (22)
\]
Since $|V_{e3}|$ is small, $|V_{\mu 3}|$ in case (2) has a weaker dependence on $|V_{e3}|$ compared with case (1).

Measurements on quantities related to neutrino masses can also determine the parameters in the neutrino mass matrix. We list a few of them in the below.

i) Neutrinoless double beta decay measurement determines the 11-element $m_{ee}$ of the neutrino mass matrix in the basis where the charged lepton mass matrix has been diagonalized. In this model $|m_{ee}| = |2\alpha + 2\beta + \gamma|/3$.

ii) Tritium beta effective electron neutrino mass $m_{\nu e} = \sqrt{\sum_i |V_{ei}m_i|^2}$ determines

$$m_{\nu e}^2 = |\alpha - \varepsilon|^2 + 2|\beta|^2 + |\varepsilon + \alpha|^2 + |\gamma|^2 + 4Re(\beta\alpha^*).$$

(23)

iii) Measurements of $\Delta m^2_{12}$, $\Delta m^2_{23}$ from neutrino oscillation data and $m_{sum} = |m_1| + |m_2| + |m_3|$ from cosmology data can also provide information about the mass matrix parameters using eq.(13). The modified mass matrix allows both normal and inverted neutrino mass hierarchies. If $\cos(2\theta)$ and $(|\alpha - \varepsilon|^2 - |\alpha + \varepsilon|^2)/\cos(2\theta) > 0$ the mass hierarchy is inverted and otherwise the mass hierarchy is a normal one.

We finally comment that to rule out the mixing proposed here it is necessary to have precise measurement of $|V_{e2}|$. If future experiments will determine a $|V_{e2}|$ significantly deviate from $1/\sqrt{3}$, one has to further modify the model. In the $A_4$ based model discussed earlier, this implies that a further break down of the residual $Z_3$ and $Z_2$ must happen. If just $Z_3$ is broken, the vev of the components $<\Phi_i>$ will not be equal, this will affect the “magic” form $U_l^\dagger$ in eq.(3), whereas if $Z_2$ is broken, the zero entries in eq.(9) will become non-zero. In general both $Z_3$ and $Z_2$ may be broken at the same time. The form of the mixing will become the most general one with corrections to all elements[18]. We have to wait more precise data to tell us if the simple mixing proposed here need to be further modified.

Acknowledgments

AZ thanks the hospitality of the Department of Physics at the National Taiwan University where this work was initiated. This work was supported in part by NCTS. XGH was supported by Taiwan National Science Council. AZ was supported in part by US National Science Foundation under grants PHY 99-07949 and PHY00-98395.
References

[1] SNO Collaboration, B. Aharmim et al., nucl-ex/0502021; SK Collaboration, Y. Ashie et al., hep-ex/0501064; GNO Collaboration, M. Altmann et al., Phys. Lett. B616, 174(2005); KamLAND Collaboration, K. Eguchi et al., Phys. Rev. Lett. 94, 081801(2005); K2K Collaboration, E. Aliu et al., Phys. Rev. Lett. 94, 081802(2005); MARCO Collaboration, M. Ambrosio et al., Eur. Phys. J. C36, 323(2004); Soudan 2 Collaboration, M. Sanchez et al., Phys. Rev. D68, 113004(2003); CHOOZ Collaboration, M. Apollonio et al., Eur. Phys. J. C27, 331(2003).

[2] S. Eidelman, et al., Particle Data Group, Phys. Lett. B592, 1(2004).

[3] G. Fogli et al, hep-ph/0506083; M. Maltoni, T. Schwetz, M. A. Tortotla and J. W. F. Valle, Phys. Rev. D68, 113010(2003); G. Goswami and A. Y. Smirnov, hep-ph/0411359; S. Goswami, A. Bandyopadhyay and S. Choubey, Nucl. Phys. 143(Proc. Suppl.), 121(2005); M.C. Gonzalez-Garcia, hep-ph/0410030; H. Back et al., hep-ex/0412016.

[4] A. Strumia and F. Vissani, hep-ph/0606054.

[5] P. F. Harrison, D.H. Perkins and W.G. Scott, Phys. Lett. B530, 167(2002) hep-ph/0202074.

[6] P. F. Harrison and W.G. Scott, Phys. Lett. B535, 163(2002) hep-ph/0203209; P.F. Harrison and W.G. Scott, Phys. Lett. B557, 76(2003) hep-ph/0302025; Z.-Z. Xing, Phys. Lett. B533, 85(2002) hep-ph/0204049.

[7] X. G. He and A. Zee, Phys. Lett. B560, 87(2003) hep-ph/0301092.

[8] C. I. Low and R. R. Volkas, Phys. Rev. D68, 033007 (2003) hep-ph/0305243; J.E. Kim and J.-C. Park, JHEP 0605:017(2006) hep-ph/0512130; W. Grimus and L. Lavoura, JHEP, 0601:018(2006) hep-ph/0509239; I. Varizelas, S.-F. King and G.G. Ross, hep-ph/0512313 hep-ph/0607045; N. Singh, M. Rajkhowa and A. Borach, hep-ph/0603189; P. Kovtun and A. Zee, hep-ph/0604169; R. Mohapatra, S. Naris and Y.-H. Yu, hep-ph/0605020; N. Haba, A. Watanabe and K. Yoshioke, hep-ph/-603116.

[9] G. Altarelli and F. Feruglio, Nucl. Phys. B720, 64(2005) hep-ph/0504165; Nucl. Phys. B741, 215(2006) hep-ph/0512103; E. Ma, Phys. Rev. D72, 037301 (2005); Phys. Rev. D 73, 057304 (2006); Mod. Phys. Lett. A 20, 2601 (2005) hep-ph/0508099; K. S. Babu and X-G. He, hep-ph/0507217.

[10] A. Zee, Phys. Lett. B630, 58(2005) hep-ph/0508278.
[11] Xiao-Gang He, Yong-Yeon Keum and Ray Volkas, JHEP, 0604:039(2006)\textsuperscript{[hep-ph/0601001].}

[12] C. Jarlskog, Phys. Rev. Lett. 55, 1039(1985); Z. Phys. C\textbf{29}, 491(1985).

[13] Z.-Z. Xing, in Ref.\textsuperscript{[6].}

[14] J. Bjorken, P. Harrison and W. Scott,\textsuperscript{hep-ph/0511201}

[15] R. Friedberg and T.D. Lee,\textsuperscript{hep-ph/0606071}

[16] Z.-Z. Xing, H. Zhang and S. Zhou,\textsuperscript{hep-ph/0607091}

[17] E. Ma and G. Rajasekaran, Phys. Rev. D\textbf{64}, 113012 (2001); E. Ma, Mod. Phys. Lett. A\textbf{17}, 289 (2002); ibid. A\textbf{17}, 627 (2002), K. S. Babu, E. Ma, and J. W. F. Valle, Phys. Lett. B\textbf{552}, 207 (2003; B. Adhilary et al., Phys. Lett. B638, 345(2006)\textsuperscript{[hep-ph/0603059].}

[18] N. Li and B.-Q. Ma, Phys. Rev. D\textbf{71}, 017302(2005)\textsuperscript{[hep-ph/0412126]; F. Plentinger and W. Rodejohann, Phys. Lett. B\textbf{625}, 264(2005)\textsuperscript{[hep-ph/0507143].}