This paper reports the spatial mathematical modeling of the process of dressing the working surface of grinding wheels for implementing the double-sided grinding of the ends of cylindrical components. Parts with high-precision end surfaces that are commonly used include bearing rollers, piston fingers, crosspieces of cardan shafts, and others. The geometric accuracy of surfaces is ensured by simultaneously grinding the ends at two-sided end-grinding machines with crossed axes of the part and wheels that operate under a self-blunting mode. Before starting the machining, the wheels are dressed in a working position. Moreover, the total orientation angle of the tools is selected subject to the condition of uniform distribution of allowance along the rough sections of wheels. Dressing involves a single-crystal diamond tool with a variable feed. That ensures different development of the surface of abrasive tools, which prolongs their operating time between dressings and improves overall stability. The constant size of micro irregularities at the calibration site enhances the quality of machining. The calibration site is made in the form of a straight line belonging to the plane that passes through the axis of rotation of the wheel and is perpendicular to the plane of the machined part. Based on the spatial mathematical models of the processes of removal of allowance and shape formation when dressing the wheel, the surface of the grinding wheel was investigated. Mathematical models for shaping the ends of parts when grinding with wheels with conical calibration sites have been proposed; it is shown that when applying the proposed machining scheme, there is no geometric error in the size of the part. In addition, due to the uniform distribution of the allowance along the rough area of the wheel, the quality of the surface layer of the ends of parts increases. The devised method for dressing the working surface of wheels could be used to grind the ends of non-circular components.

Keywords: double-sided grinding, crossed axes, wheel dressing, diamond pencil

1. Introduction

Trends in the development of the machine-building industry require constant improvement of technological and operational characteristics of various articles, as well as an increase in production performance while reducing its cost; this ensures the competitiveness of individual enterprises and the industry of the state as a whole [1].

To improve the working parameters of machine parts, special materials and advanced methods of manufacturing parts are used. Despite the rapid development of new technologies in the design and construction of parts, for example, by 3D printing [2] etc., the methods of cutting parts are still relevant. Methods of high-quality sharpening are actively developing [3], which significantly reduces energy consumption for machining but the resulting accuracy and quality of the surface are traditionally ensured by finishing operations at grinding machines [4].

One of the important tasks for fabricating a finished article is high-quality machining of the ends of round and non-round parts of small sizes, such as bearing rollers, piston fingers, crosspieces of cardan shafts, and others. In order to improve machining productivity, the surfaces are simultaneously polished at double-sided end-grinding machines, which ensures the high accuracy of the mutual location of the ends and their roughness. The parameters of the quality of grinding are significantly influenced by the condition of the surfaces of abrasive wheels. When machining with
parallel axes of tools and parts, there is rapid wear of grinding wheels since the main part of the allowance is removed when the part enters the machining area. And when grinding with crossed axes of the wheels and part, the allowance is removed gradually, which reduces the wear of tools but introduces the geometric error of the part, caused by the machining scheme.

Therefore, devising more progressive schemes for dressing the wheel at machines with numerical control could improve the accuracy and quality of machining the ends of round parts of small diameters.

In modern machines, the process of control over machining in general and, in particular, the dressing of grinding wheels, is executed using CNC systems. Their estimation and geometric basis are the mathematical and spatial models of all elements of the machined system and the model of the process of removal of allowance and shape formation.

Thus, it is a relevant task to construct and investigate spatial mathematical models for dressing the grinding wheel, the process of removing the allowance, and the shape formation of a part’s ends at the simultaneous double-sided grinding with wheels with cone calibration sites.

2. Literature review and problem statement

Junker Group [5] has been presenting its machine tools on the world market since 1962. For simultaneous machining of two ends of parts, the Saturn machine is offered. This line of equipment makes it possible to grind a significant range of parts, including components of couplings, bearings, stamped parts of various types, etc. Machining is proposed to be carried out either under the mode of mortise grinding when the working surfaces of the wheels are parallel, or to grind in a run while the wheels are arranged at some angle relative to each other and the drum of workpiece feed. In the case of mortise grinding, all allowance is removed when the part enters the machining area, which causes a rapid increase in temperature in the cutting zone and significant wear of grinding wheels. When grinding in a run, the allowance is removed gradually, so the heat intensity in the contact area of the part and tool is much lower. The resulting surface of the part is formed when it leaves the machining area but the machining scheme itself reduces the accuracy of the part, due to the presence of turning wheels.

Work [6] reports a study of double-sided grinding of bearing rollers, which showed that the accuracy of their ends significantly affects the resource of the bearing. In [7], the indicators of roughness of the ends of parts during double-sided grinding were investigated but it is not defined how the geometry of the grinding wheel affects it. Study [8] considers the methodology for measuring machining parameters. A significant number of schemes for dressing wheels were devised that have certain advantages and disadvantages, for example, according to the technique reported in [9], the dressing tool is placed in a special staple and moves perpendicular to the axis of rotation of the wheel that forms the straight profile of the wheel. At the same time, the presence of an elastic element in the set-up prevents the destruction of the surface of the abrasive tool.

Several studies, in particular [10], demonstrate that the accuracy indicators for bilateral end grinding largely depend on the shape of the working surface of wheels. Thus, when using a tool with a direct profile during the machining of the first parts of the batch, significant wear and tear of the surface of the wheel occurs. At the same time, there is a worse quality of the first parts in a batch. To solve this problem, [11] proposes a preliminary dressing of the grinding wheel, obtaining a parabolic profile of the work surface. This scheme reduces the time for the tool to adjust but the resulting shape of the part is formed by a curvilinear surface, which introduces a flatness error.

Spatial modeling of various finishing surfaces with double-sided end grinding is described in [12]. The authors consider cylindrical parts and articles with different diameters of the machined ends, using an example of an internal combustion engine valve. The accuracy of grinding parts with ends of different diameters was analyzed. Spatial mathematical modeling of tool and machining surfaces was also carried out in [13] when grinding the ends of round and non-round parts. It is proposed to dress the wheels with a diamond tool with a round work surface. However, the proposed mathematical models do not take into consideration the rotational movements of the grinding wheel and the dressing tool.

Work [14] proposes a method for dressing the periphery of the grinding wheel with a variable feed, which ensures different development of the working surface of the wheel and more favorable conditions in the machining area during the longitudinal grinding of cylindrical parts. A mathematical model of the dressing tool in the form of a spatial cutting wedge was proposed; the shape formation process when dressing a wheel was mathematically modeled. However, the features of the formation of the working end surface of the grinding wheel for the scheme of double-sided cutting of parts were not considered. In addition, dressing the wheel with an octahedra-shaped tool, as proposed in the cited work, causes different parameters of the roughness of the tool depending on the location of the dressed area, which adversely affects the quality of the calibration site.

The issue related to improving the accuracy of simultaneous grinding of two ends of round parts can be addressed by conducting spatial mathematical modeling of the processes of removal of allowance and shape formation when dressing grinding wheels while ensuring the straightness of the cone calibration site. Analysis of the developed models would make it possible to devise high-performance schemes of double-sided grinding and improve the performance characteristics of the end surface of grinding wheels by obtaining special shapes of the tool profile.

3. The aim and objectives of the study

The purpose of this study is to spatially model the process of shaping the working surfaces of wheels at the two-sided grinding of round ends at the machine with crossed axes of the tool and part. This would make it possible to more thoroughly choose the scheme of grinding and dressing the tool surface, to analyze them. That could improve the accuracy of the machining of parts at the double-sided end grinding, as well as increase the resource of the tool.

To accomplish the aim, the following tasks have been set:
- to design a scheme for dressing grinding wheels with a diamond tool at a CNC machine;
- to build a reference model of the tool, on the basis of which it would be possible to develop a control program for a CNC machine;
- to construct the spatial models of surfaces of the dressing tool and grinding wheel;
– to build a general model of the process of simultaneous two-way grinding of the ends of round parts with abrasive wheels with conical calibration sites, to investigate the accuracy of the shaping process.

4. The study materials and methods

The object of this study is the process of removing allowance and shape formation when dressing an abrasive tool at the double-sided end grinding. To define the main parameters of dressing and to analyze the accuracy of shape formation, we perform modular spatial modeling. Spatial modular models of the process of double-sided end grinding of cylindrical parts are constructed using a matrix apparatus for converting coordinate systems. According to this procedure, a unit vector of the origin of the coordinate system \( \mathbf{e}_4 \) and the transformation matrix are distinguished. A unit vector is a column in the matrix of 1x4. Coordinate conversion matrices \( M_1, M_2, M_3 \) are responsible for moving the starting point along the coordinate axes \( X, Y, Z \), respectively. Matrices \( M_4, M_5, M_6 \) – for the turn of the point relative to the coordinate axes. Thus, by sequentially multiplying a unit vector by transformation matrices, one can describe any three-dimensional shape. To simplify the notation and visibility of mathematical models, rectangular, cylindrical, and spherical modules are introduced. Each of these modules, depending on its type and purpose, contains a series of matrices.

Calculations and necessary transformations, in order to obtain results based on the built mathematical models, are carried out in the mathematical software package Mathcad. At the same time, standard functions of working with matrices and graphic objects are used. Special logic units are also built using Mathcad software tools.

5. Results of studying the dressing of the working surfaces of wheels at the two-sided grinding of the ends of round parts

5.1. Building a scheme for dressing grinding wheels with a diamond tool

The scheme of dressing grinding wheels 1, 2 at the two-sided grinding of the ends is shown in Fig. 1. Dressing is carried out by a diamond in holder 3, which is installed in the drum of article feed 4 and moves in a circle with a radius \( R_d \) at a variable angular velocity \( \omega_0 \). The coordinate system of the dressing tool \( O_3X_3Y_3Z_3 \) is parallel to the coordinate system of the drum, and its position differs by the radius of installation \( R_0 \) and by the length of the pencil reach relative to the central plane of the drum. At the initial time point, the \( O_3X_3Y_3 \) axis is strictly vertical, and the \( O_3dY_3 \) and \( O_3dZ_3 \) axes are in a horizontal plane. The origin of the coordinate system is selected in the center of the radius of rounding the top of the diamond. Wheels with radius \( R_{w1} = R_{w2} \) are set to working position, that is, they are rotated at angles \( \alpha_{w1} \) and \( \alpha_{w2} \) relative to the \( O_{w1}X_{w1} \) and \( O_{w2}X_{w2} \) axes, respectively, and at angles \( \beta_{w1} \) and \( \beta_{w2} \) relative to the \( O_{w1}X_{w1} \) and \( O_{w2}X_{w2} \) axes.

At the same time, the total angle of rotation \( \Delta_\alpha \) is determined from the following equality:

\[
\delta_\alpha = \sqrt{\alpha_{w1}^2 + \beta_{w1}^2} = \sqrt{\alpha_{w2}^2 + \beta_{w2}^2}.
\]  

In the process of dressing, grinding wheels rotate at a working frequency, and \( \omega_{w1} = \omega_{w2} \).

In order to improve the accuracy of simultaneous machining of round parts, cone calibration sections are proposed at the end working surfaces of abrasive wheels. To ensure their shape formation, the tool is additionally set into a relative movement \( S \) in the axial direction, the value of which is controlled by the CNC system.

![Fig. 1. Schematic of dressing the grinding wheel with a diamond pencil](image)

After dressing the first grinding wheel, diamond tool 5 is turned 180° to adjust the working surface of the second wheel. The speed of rotation of the article feed drum is chosen variable in time depending on its angle of rotation. This allows for greater roughness along the radius on the rough and finishing areas of the cutting surface, and ensures constant roughness on the calibration cone site.

5.2. Building a reference model of the grinding tool

The resulting shape of a cylindrical part is set at its exit from the machining area. To exclude the error in the machined ends, the grinding wheel provides for a straight cone area. In this case, the generatrix of a cone should lie in the plane perpendicular to the end plane of the machined article, and each point at the end of the part should contact a certain point on the calibration site. In order to optimize the machining process, we take the length of the calibration site \( l \) on the surface of a grinding wheel equal to the diameter of the part that is to be machined \( d_p \).

To determine the trajectory of the relative movement of the dressing tool and grinding wheel, we record the reference surface of the tool, which must be obtained as a result of dressing. According to the proposed dressing technique, the grinding wheel contains two main areas – rough and calibration. Each section has two independent parameters – the current radius of the wheel and the current angle of rotation, that is, the surface of the grinding tool can be described using...
the cylindrical tool module $\mathbf{C}_{\mathbf{w}_{\mathbf{n}, \mathbf{b}_{\mathbf{n}}}}$. We separate a cone calibration site in the general mathematical model of the reference surface of the wheel using the Heaviside function, that is,

$$
\bar{r}_e(\theta_{w}, R_{e}) = C_{\mathbf{w}_{\mathbf{n}, \mathbf{b}_{\mathbf{n}}}} = 
M_6(\theta_{w}) \cdot M_1(R_{w}) \cdot \bar{F} \cdot \Phi(\theta_{w}) - 
m_6(\theta_{w}) \cdot M_1(R_{w}) \cdot \bar{F} \cdot \Phi(\theta_{w} - R_{w}) + 
+M_6(\theta_{w}) \cdot M_3((R_{w} - R_{e}) \tan \delta_{w}) \times 
\times M_1(R_{w}) \cdot \bar{F} \cdot \Phi(\theta_{w} - R_{w}),
$$

where $M_1, M_2...M_6$ are the matrices of the transformation of coordinate systems responsible for moving and rotating a unit radius vector relative to the $M_1, M_2...M_6$ coordinate axes; $\bar{F}$ is the unit radius vector; $R_{w}$ – the current linear coordinate of the tool along its radius and, in the initial position, the $O_{w}X_{w}$ axis (Fig. 1), mm; $\theta_{w}$ – the current angle of rotation of a single point around the axis of rotation of the grinding wheel $O_{w}Z_{w}$, rad; $\Delta \theta_{w}$ is the total angle of orientation of the dressing tool, rad; $R_{e}$ is the radius that determines the transition to the calibration site at the end surface of the wheel, with $R_{e} = R_{w} - l_{w}$, mm.

The graphical representation of the mathematical spatial model of the reference surface of a grinding wheel is shown in Fig. 2.

Fig. 2. Mathematical model of the reference surface of a grinding wheel with conical calibration sites

The orientation of the tool surface in the process of operation is determined by the spherical module $S_{\mathbf{w}_{\mathbf{n}, \mathbf{b}_{\mathbf{n}}}}$, accordingly:

$$
\bar{r}_e(\theta_{w}, R_{e}) = S_{\mathbf{w}_{\mathbf{n}, \mathbf{b}_{\mathbf{n}}}} \cdot C_{\mathbf{w}_{\mathbf{n}, \mathbf{b}_{\mathbf{n}}}} \cdot \bar{F} = 
M_4(\beta_{w}) \cdot M_5(\alpha_{w}) \cdot \bar{F}(\theta_{w}, R_{e}).
$$

where $\alpha_{w}, \beta_{w}$ are the angles of orientation of the grinding wheel relative to the $O_{w}X_{w}$ and $O_{w}Y_{w}$ axes, respectively, rad.

The constructed spatial mathematical model of the wheel (3) is used to derive the equation of the line of intersection of the tool with the axial plane, and the trajectory of the diamond vertex is equidistant to this line and is at a distance from it by the radius value at the top of the diamond.

5.3. Construction of spatial models of the surfaces of the dressing tool and grinding wheel

We analyze dressing parameters and calculate the required trajectory and relative speed of movement of the diamond tool based on spatial mathematical modeling.

The selected tool for machining is the diamond in a holder. The structure of the tool is described in specialized reference books. Conventionally, the shape of the working surface is considered conical with an angle at the top $\psi = 90^\circ$. The radius of diamond rounding can vary in a wide range of $\rho = 0.001...0.01$ mm. Let us record a mathematical model of the working part of the diamond. In this case, the radius vector of the tool surface can be represented by the product of three single-coordinate matrices with two angular parameters. The transition from the spherical section of the top of the diamond to its rectilinear part is described using the Heaviside function.

Thus, the radius vector of the dressing instrument is:

$$
\bar{r}_e(\varphi, \theta) = M_6(\theta) \cdot M_5(\psi) \cdot M_3(\rho) \cdot \bar{F} \cdot \Phi(\varphi) - 
-M_6(\theta) \cdot M_5(\psi) \cdot M_3(\rho) \cdot \bar{F} \cdot \Phi(\varphi - \varphi_{s}) + 
+M_6(\theta) \cdot M_1(h(\psi)) \cdot M_3(-h(\psi)) \cdot M_5(\psi) \times 
\times M_1(R_{w}) \cdot \bar{F} \cdot \Phi(\varphi - \varphi_{s}),
$$

where $\rho$ is the radius of rounding the top of the diamond tool, mm; $\varphi$ is the current angle of rotation of a single point around the $O_{w}Y_{w}$ axis, $\varphi = 0...\varphi_{s}$, rad; $\varphi_{s}$ is the angle that determines the transition from the spherical section of the top of the diamond to its conical part, $\varphi_{s} = 0.5 \psi$, rad; $\theta$ is the current angle of rotation of a single point around the $O_{w}Z_{w}$ axis, $\theta = 0...2 \pi$, rad; $h(\psi) = \sin(\pi/4) \cdot \rho \cdot \tan(\psi - \varphi_{s})$ is the function that specifies the current coordinates of the point of the rectilinear section of the diamond tool along the $O_{w}X_{w}$ and $O_{w}Z_{w}$ axes, depending on a change in the current angle $\varphi$, mm; $\Phi(\varphi)$ is the Heaviside function that determines the scope of each equation.

In this case, the first component of the equation of the radius-vector of the dressing tool (4) is responsible for the formation of the radius of rounding, the latter – for the formation of the conical part of the diamond, the second component stops the action of the first term during the transition from the spherical area to the rectilinear. The graphical representation of the dressing tool surface is shown in Fig. 3.

Fig. 3. Mathematical model of the tool for dressing a grinding wheel

To describe the dressing process and ultimately the surface of the grinding wheel, it is necessary to transfer the radius vector of the diamond dressing tool to the drum coordinate system, set the relative position and orientation of the grinding wheel, and define working movements. Using specialized modules and a matrix apparatus for converting coordinate systems, we derive a spatial model of the shape formation process at dressing:

$$
\bar{r}_e(\varphi, \theta) = C_{\mathbf{w}_{\mathbf{n}, \mathbf{b}_{\mathbf{n}}}} \cdot S_{\mathbf{w}_{\mathbf{n}, \mathbf{b}_{\mathbf{n}}}} \cdot P_{\mathbf{w}_{\mathbf{n}, \mathbf{b}_{\mathbf{n}}}} \cdot C_{\mathbf{w}_{\mathbf{n}, \mathbf{b}_{\mathbf{n}}}} \cdot \bar{F}(\varphi, \theta).
$$
where $C_{s,R}^\beta$ is the cylindrical module of transferring the coordinate system of the diamond tool to the coordinate system of the feed drum; $P_{s,R}^\gamma$ is the rectangular module of transition to the coordinate system of the machine; $S_{s,R}^\delta$ is the spherical orientation module; $C_s^\gamma$ is the cylindrical shape formation module at dressing.

Moreover:

$$C_{s,R}^\beta = M6(\theta_s) \cdot M2(R_s),$$  \hspace{1cm} (6)

$$P_{s,R}^\gamma = M3(z_s) \cdot M2(y_s) \cdot M1(x_s),$$  \hspace{1cm} (7)

$$S_{s,R}^\delta = M1(x_s) \cdot M4(\beta_s) \cdot M5(\alpha_s),$$  \hspace{1cm} (8)

$$C_s^\gamma = M6(\theta_s),$$  \hspace{1cm} (9)

where $R_s$ is the radius of movement of the center of the blanks in the feed drum, mm; $\theta_s$ is the current angle of rotation of the feed drum, rad; $x_s$, $y_s$, $z_s$ is the distance from the center of rotation of wheels, that is, the coordinate system of the machine, to the origin of the coordinate system of the article feed drum, mm; $\alpha_s$, $\beta_s$ are the angles of orientation of the grinding wheel relative to the $O_6X_w$ and $O_6X_m$ axes, respectively, rad; $x_m$ is the displacement of the center of the grinding wheel along the $O_6X_m$ axis, mm; $\theta_w$ is the current angle of rotation of the grinding wheel, rad.

Taking into consideration equations (6) to (9), the radius vector of the surface of the grinding wheel at dressing (5) can be rewritten as:

$$\begin{align*}
\mathbf{r}_{gs}(\theta_s, \theta_b, \varphi, \theta) &= M6(\theta_s) \cdot M4(\beta_s) \times
\times M5(\alpha_s) \cdot M3(z_s) \cdot M2(y_s) \times
\times M1(x_s) \cdot M6(\theta_s) \cdot M2(R_s) \cdot \mathbf{r}_{gs}(\varphi, \theta).
\end{align*}$$  \hspace{1cm} (10)

In the mathematical model (10), there are currently four variable parameters, with $\varphi$ and $\theta$ responsible for the shape of the diamond’s tool surface, and $\theta_s$ and $\theta_w$ set the relative movement of the article feed drum and the grinding wheel. All other values are constant and depend on the selected machining conditions and the specific configuration of the double-sided end-grinding machine.

The relationship between the angle of rotation of the feed drum $\theta_s$ and the grinding wheel $\theta_w$ can be determined by knowing their angular rotation speeds:

$$\theta_w = \theta_s - \frac{\omega_w}{\omega_s},$$  \hspace{1cm} (11)

where $\omega_w$, $\omega_s$ is the angular speed of rotation of the feed drum and grinding wheel, respectively, rad/min.

The exclusion of another parameter from the mathematical model is ensured by the use of the bending relationship as the main condition for shape formation:

$$\frac{\partial \mathbf{r}_{gs}(\theta_s, \varphi, \theta)}{\partial \theta} \cdot \frac{\partial \mathbf{r}_{gs}(\theta_s, \varphi, \theta)}{\partial \varphi} \cdot \frac{\partial \mathbf{r}_{gs}(\theta_s, \varphi, \theta)}{\partial \theta_w} = 0.$$  \hspace{1cm} (12)

Taking into consideration dependences (11), (12), equations (10) describes the specified surface of the grinding wheel with conical calibration sites, which is obtained at dressing. Based on this model, further analysis of the resulting tool surface is carried out, as well as modeling of the grinding process of the ends of cylindrical parts.

However, the constant speed of the feed drum with the installed dressing tool should be used only when dressing the boat and finishing sections of the tool. Under such conditions, a variable roughness along the radius line would form on the end surface of the grinding wheel, with its increase as the radius of the wheel increases. At the calibration site, it is more advisable to obtain a constant development of the profile of the wheel. To this end, when switching the diamond dressing tool to the formation of conical calibration sites, it is necessary to gradually reduce the angular speed of rotation of the feed drum (Fig. 4).

Reducing the angular speed of the feed drum must begin when reaching the top of the diamond point $F_0$ – the intersection of the arc of the minimum radius of the calibration site $R_{w_0} = R_w - L_w$ with the arc of the motion of the dressing tool $R_0$:

$$\theta_b = \arccos \left( \frac{L_w^2 + R_{w_0}^2 - R_b^2}{2 \cdot L_w \cdot R_b} \right).$$  \hspace{1cm} (13)

where $L_w$ is the distance of the mutual arrangement of the feed drum and grinding wheel along the horizontal $O_6X_w$ axis, mm.

The diagram (Fig. 4) shows that in order to ensure the constant step $x_p$ of micro irregularities on the profile of the grinding wheel, it must meet with the diamond at points $F_0, F_1, \ldots, F_n$. So, when forming the second groove, the wheel should make one revolution and additionally turn at some angle $\Delta \theta_{w_1}$.

The current angle of rotation of the feed drum will be $\theta_s$. It is necessary to align the angles of rotation of the feed drum $\theta_{s_1}, \theta_{s_2}, \ldots$ with the angle of rotation of the grinding wheel while adding the value of additional displacement $\Delta \theta_{w_1}, \Delta \theta_{w_2}$.

The function of changing the additional angle of displacement is as follows:

$$\theta_{w}(R_{w_1}) = \arccos \left( \frac{L_w^2 + R_{w_1}^2 - R_{b_1}^2}{2 \cdot L_w \cdot R_{b_1}} \right).$$  \hspace{1cm} (14)

where $R_{w_1}$ is the current radius on the calibration site of the grinding wheel $R_{w_1} = R_i + i \cdot x_p$, mm; $i$ is the rotation number of the grinding wheel in the formation of the calibra-
tion site; $s_p$ is the micro irregularity step at the calibration site of the wheel.

An equality similar to (13) assigns functional dependence to the change in the angle of rotation of the feed drum on the angle of rotation of the grinding wheel. And, accordingly, the dependence of the angular speed of the diamond feed on the speed of rotation of the drum.

When using this dependence in the mathematical model of dressing grinding wheels (10), a constant step of micro irregularities at the calibration conical part of the working end surface of the wheel would be ensured.

The graphical representation of the fitted surface of a wheel with conical calibration sites, constructed from (10), taking into consideration (11) to (13), is shown in Fig. 5, a. Fig. 5, b demonstrates the formation of micro irregularities at the calibration site of the wheel.

The model (Fig. 5, a) shows the general configuration of the end surface of the grinding tool, with dashes left by the diamond dressing tool. For clarity, the radius of rounding the diamond and its size are enlarged. The intersection of the calibration section of the wheel with an axial plane (Fig. 5, b) illustrates the continuity of the micro irregularity steps.

5.4. Building a general model of the process of two-sided grinding of the ends of round parts and studying the accuracy of machining

The scheme of double-sided end grinding of cylindrical parts is similar to the scheme of dressing wheels (Fig. 1) with the difference that round blanks are installed in the feed drum and the function of removing the allowance is carried out by grinding wheels. In accordance with this, a modular mathematical model of the end surface of the machined part can be written in the following form:

$$\tau_{\theta_p}(\theta_p, \phi, \theta) = C_{\theta_p}^{f_r} S_{\theta_p}^{O_p} \rho_{\theta_p}^{r} C_{\theta_p}^{f_t} \tau_{\theta_p}(\theta_p, \phi, \theta),$$

(15)

where $C_{\theta_p}^{f_r}$ is the cylindrical module of transferring the coordinate system of a part to the coordinate system of the feed drum; $C_{\theta_p}^{f_t}$ is the cylindrical module of shape formation when grinding the end of a part.

Moreover:

$$C_{\theta_p}^{f_r} = 6Q(\theta_p) M3(h_p) M2(R_p),$$

(16)

$$C_{\theta_p}^{f_r} = 6Q(\theta_p),$$

(17)

where $h_p$ is the displacement value of the machined end relative to the coordinate system of the article feed drum along the axis, which coincides with the axis of rotation of the drum, mm; $\theta_{p_p}$ is the current angle of rotation of the grinding wheel when grinding the ends of parts, rad.

To define the condition of shape formation, to construct and analyze the mathematical model of the end of the machined part, we use the relationship of bending similar to (12).

We shall analyze the accuracy in the shape formation of the ends of cylindrical parts. We shall build plots of deviations from the flatness by the ends of cylindrical parts for the proposed method of dressing and for known techniques (Fig. 6). The procedure for constructing the plots of end’s accuracy from the mathematical spatial model of the shape formation process is similar to that described in [12, 13].

![Fig. 5. Mathematical model of the surface of a grinding wheel: a — spatial model; b — the surface profile of the wheel along the radius](image)

![Fig. 6. Plots of deviation from the flatness by the ends of cylindrical parts: 1 — according to the traditional machining scheme; 2 — when machining wheels with conical calibration sites](image)

Our analysis of the plot (Fig 6) reveals that machining according to traditional schemes, the error of deviation from flatness increases at a distance from the center of the part (curve 1). And when using two-sided machining with oriented wheels with conical calibration sites, the error is absent over the entire plane of the cylindrical part (curve 2).

6. Discussion of results of studying the process of dressing wheels at the two-side end grinding of cylindrical parts

The suggested procedure for dressing grinding wheels (Fig. 1) ensures the presence of rectilinear conical areas on their end working surfaces. Due to the orientation of the wheels in two planes, a uniform distribution of the removable allowance along working surface of the cutting tool is ensured. This reduces heat intensity in the machining area, which improves the quality of the surface layer of parts. The amount of wear of the working surface of the wheel is decreased, too.

In contrast to existing methods of dressing, a given method ensures uniform development of the cutting surface of grinding wheels at calibration sites (Fig. 5). This is achieved by controlling the angular speed of rotation of the article feed drum with the installed dressing tool (13), (14). This differs from the classic case of machining with a constant feed of diamond where there is an increase in geometric roughness as it moves away from the center of rotation of the wheel.

The constructed mathematical models (10), (15) and additional dependences (13), (14) form the estimation base for writing control programs for CNC machines. They are also used to determine and control the parameters of the process of dressing wheels at the double-sided end grinding of cylindrical parts.

For the devised method of dressing grinding wheels, we also studied the accuracy in the shape formation of torus of cylindrical parts. It has been established that when machining
with wheels with conical calibration sites, the geometric error of the ends is zero (Fig. 6, curve 2). That ensures better performance of the part during operation and prolongs its service life in general. In addition, the division of the working part of the end of the wheel into rough, finishing, and calibration sites increases its resource by 10 %, by prolonging the period of work between dressing operations [15].

However, this work has not calculated the minimum permissible length of the calibration site. Its size would depend on the diameter of the parts being machined, as well as the angles of orientation of the tool. At two-sided grinding, it is desirable to make a calibration site as small as possible; this could make it possible to more fully use the working surface layer of the wheel to remove the main allowance. In addition, we did not define clear functional dependencies between the speed of movement of the feed drum and the wheel. In the future, it is desirable to conduct research in order to determine the relationship between the angular speeds of the grinding wheel and the diamond tool. As well as determine the dependence of the axial movement of the tool on the angle of rotation of the feed drum.

### 7. Conclusions

1. We have proposed a scheme for dressing grinding wheels in order to form a straight-line conical calibration site located in a plane that passes through the axis of rotation of the wheel and belongs to the machined workpiece. The presence of such a site eliminates the geometric error in shape formation at the two-side end grinding of round parts. The variable feed of a diamond pencil provides different development of the working rough area of the grinding wheel, which makes it possible to improve productivity. As well as the possibility of ensuring constant roughness at the calibration site, which increases the grinding accuracy and the quality of the surface layer of the ends of cylindrical parts.

2. In order to determine the current coordinates of the position of the top of the diamond pencil in the process of dressing, we have built a mathematical model of the reference surface of a grinding wheel with a conical calibration site. This forms the mathematical basis for developing a control program for a CNC machine.

3. General modular spatial mathematical models of diamond dressing tool, grinding wheel, and the processes of removal of allowance and shape formation at dressing have been constructed. The process of shape formation and a condition of the surface of the grinding wheel at dressing have been analyzed. It is proposed to carry out dressing at different angular speeds, which could ensure different development of the working end surface of the grinding wheel. Increasing roughness in the rough area would prolong the resource of the tool. And its constant value at the conical calibration site could improve the quality of the surface layer and the accuracy of the machined blanks. We have established the dependence of the angular speed of the feed drum when dressing with a diamond pencil, which ensures constant roughness at the rectilinear calibration site.

4. A general modular mathematical spatial model of the process of simultaneous double-sided grinding of the ends of round parts with conical calibration sites has been developed. Based on which, the accuracy in the shape formation of ends was investigated. It has been proven that when using the proposed method for dressing wheels, there is no geometric error of the end shape.

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Ensuring the reliable operation of the dust fuel preparation system at thermal power plants (TPP) is a topical issue since it determines the energy strategy of any country that fires coals for thermal energy production. This unit is one of the most energy-intensive units in TPP. Those systems are outdated, poorly automated and high energy-intensive. Furthermore, they must ensure efficient and safe operation of the facility while being environmentally friendly. The current work focuses on the process of grinding coals in ball drum mills for further pulverized combustion. An experimental study was performed in order to determine the main factors (rotational speed of the drum mill, the degree of loading with the grinding balls, and the velocity of the supplied air) that affect the efficiency of the fuel preparation system. The obtained experimental data and performed mathematical modeling resulted in regression equations describing the energy performance of the mill. Three regression equations for mill productivity, power consumed, and specific surface area of the final product were obtained and validated. The study reveals that the lowest specific energy consumption is achieved when the relative rotational speed of the mill is between 0.81 and 0.87; the weighted average diameter of the balls ranges from 33.5 up to 34.5 mm; the load factor of the grinding media ranges from 0.325 up to 0.335; the supplied air velocity is between 0.2 and 0.3 m/s. The proposed methodology allows adjustment of the operating parameters of the grinding process to achieve the lowest energy consumption. The power consumption for the preparation can be reduced up to 5 % for the selected operation mode of the grinding facility.

Keywords: drum ball mill, coal grinding, efficiency improvement, thermal power plant, regression analysis

1. Introduction

Worldwide, TPPs are facilities that are used for the simultaneous production of thermal and electric energy. Different types of primary energy carriers such as natural gas, coals, biomass are used to run facilities. Some of them are directly used but the others must be initially processed before combustion. The fuel preparation process is energy-intensive, which results in an increased final energy price. Following this, to reduce final costs for energy production, mechanisms are being sought to improve the energy efficiency of each unit of the plant.