THREE-DIMENSIONAL SIMULATIONS OF GYROSYNCHROTRON EMISSION FROM MILDLY ANISOTROPIC NONUNIFORM ELECTRON DISTRIBUTIONS IN SYMMETRIC MAGNETIC LOOPS

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ABSTRACT

Microwave emission of solar flares is formed primarily by incoherent gyrosynchrotron radiation generated by accelerated electrons in coronal magnetic loops. The resulting emission depends on many factors, including pitch-angle distribution of the emitting electrons and the source geometry. In this work, we perform systematic simulations of solar microwave emission using recently developed tools (GS Simulator and fast gyrosynchrotron codes) capable of simulating maps of radio brightness and polarization as well as spatially resolved emission spectra. A three-dimensional model of a symmetric dipole magnetic loop is used. We compare the emission from isotropic and anisotropic (of loss-cone type) electron distributions. We also investigate effects caused by inhomogeneous distribution of the emitting particles along the loop. It is found that the effect of the adopted moderate electron anisotropy is the most pronounced near the footpoints and it also depends strongly on the loop orientation. Concentration of the emitting particles at the looptop results in a corresponding spatial shift of the radio brightness peak, thus reducing effects of the anisotropy. The high-frequency ($\gtrsim 50$ GHz) emission spectral index is specified mainly by the energy spectrum of the emitting electrons; however, at intermediate frequencies (around 10–20 GHz), the spectrum shape is strongly dependent on the electron anisotropy, spatial distribution, and magnetic field nonuniformity. The implications of the obtained results for the diagnostics of the energetic electrons in solar flares are discussed.

Key words: radiation mechanisms: non-thermal – Sun: corona – Sun: flares – Sun: radio radiation

Online-only material: color figures, supplemental data (tar.gz)

1. INTRODUCTION

Microwave emission produced during solar flares is known to contain highly important information about fast electron acceleration and transport, coronal magnetic field, and thermal plasma (Bastian et al. 1998; Fleishman et al. 2009). However, this potential of the microwave emission has not yet been converted to routine diagnostics for two main reasons. The first is the absence of well-calibrated radio imaging spectroscopy data with needed spatial, temporal, and spectral resolutions (Gary 2003; Gary & Keller 2004). Currently, the situation has started to change as a number of solar radio instruments (e.g., OVSA and SSRT) experience significant upgrade (Upgraded SSRT, USSRRT) and expansion (Expanded OVSA, EOVSA), while even more powerful solar radio facilities are planned to be built in the near future, and a general-purpose radio facility, the Expanded Very Large Array, will soon be operational in solar observing mode. This implies that the required radio data will soon become available. However, even with these there is a second reason why microwave emission potential has not been converted to routine diagnostics, which is an apparent lack of realistic three-dimensional modeling of the microwave emission from flares. This modeling is highly important because the gyrosynchrotron (GS) emission depends in a complicated nonlinear way on many involved parameters and source geometry including spatial inhomogeneity and angular anisotropy. The realistic modeling has to establish a clear quantitative picture and solid detailed understanding of how the involved physics (i.e., source properties and parameter regimes) affects the emission produced, e.g., what changes in the emission can be expected from the variation of a given parameter.

Available three-dimensional models of the GS emission (Preka-Papadema & Alissandrakis 1992; Kucera et al. 1993; Bastian et al. 1998; Tzatzakis et al. 2008; Simões & Costa 2006; Fleishman et al. 2009; Simões & Costa 2010) have established valuable examples of the flaring microwave emission; however, they are neither numerous nor comprehensive and rely on isotropic pitch-angle distribution and uniform spatial distribution in most of the cases. The observations, however, suggest that fast electrons in flares have often anisotropic and/or inhomogeneous distributions (e.g., Melnikov et al. 2002; Fleishman et al. 2003; Fleishman 2006; Alyntsev et al. 2008).

Analysis of the pitch-angle anisotropy effect has yielded controversial conclusions: although the anisotropy was found to have a huge effect on the GS emission from a uniform (or spatially resolved) source (Fleishman & Melnikov 2003a; Fleishman et al. 2003), this effect can become much weaker when averaging over a significant volume with a nonuniform loop magnetic field comes into play (Simões & Costa 2010). This calls for a more systematic study of the GS emission from anisotropic electron distributions in three-dimensional magnetic loops. The situation is additionally complicated by the fact that the mentioned pitch-angle anisotropy implies, as a side effect, a spatial inhomogeneity of the electron distribution due to the fast electron accumulation at the looptop (Lee et al. 1994; Melnikov et al. 2002).

As a first step toward addressing the whole problem of GS modeling in realistic three-dimensional coronal magnetic loops, this paper presents a convenient modeling tool, the “GS Simulator,” which gives any user an ability to build an analytical dipole magnetic loop, select a desired viewing angle, populate the loop with thermal plasma and nonthermal electrons, and compute microwave images and spatially resolved spectra. The
vacuum-like propagation from the source to the observer is adopted in the tool, so any propagation effect on the radiation in the coronal plasma including polarization modification, which might be present in the reality, is ignored. A reasonably quick computation of a given model is made possible by the use of recently developed fast GS codes (Fleishman & Kuznetsov 2010) that proved to be highly accurate for both isotropic and anisotropic electron distributions. We intentionally consider here analytical models for the magnetic field and electron distribution to fully control the input and reliably interpret the outcome; numerical input in the form of corresponding datacubes is under development and will be presented soon elsewhere. This paper considers the microwave (GS + free–free) emission produced by mildly anisotropic electron distributions from a symmetric dipole magnetic loop viewed at different angles and for different parameter combinations. We find that even in such nonextreme conditions the anisotropy makes noticeable fingerprints on the emission properties, which are discussed in detail below.

2. SIMULATION TOOL AND METHOD

As has been said, the diversity of the microwave flaring emission is huge, so neither a paper nor a series of papers can, perhaps, offer a truly comprehensive table of options fully covering all relevant parameter combinations. Thus, an entirely different approach to address the whole problem of GS modeling is called for: we need a simulation tool capable of smoothly changing all the involved source parameters and quickly computing and returning the datacubes describing the microwave emission produced. Here, we present such a tool and give an example of using it for the microwave emission simulation.

2.1. Dipole Magnetic Flux Tube Model

The simulations were performed using the interactive IDL tool GS Simulator. This tool allows one to change the shape and orientation of the flaring loop, choose the parameters of the magnetic field, thermal plasma, and energetic electrons, and calculate the brightness maps of GS emission. In the model used, the magnetic field is produced by a dipole located below the solar surface. The loop is formed by a set of field lines such that at the looptop it has a circular cross-section with a given radius; near the footpoints, the loop becomes narrower and non-circular due to conservation of the magnetic flux. As shown in Figure 1, which pictures an actual implementation of such geometry in the GS Simulator tool, the user is allowed to freely rotate the dipole loop model in any direction, so that to obtain an arbitrary line of sight relative to the dipole’s central plane.

The magnetic model geometry and adjustable parameters are described in detail in the Appendix. The source code and documentation of the GS Simulator tool are provided in the online version of the journal.

2.2. Flaring Loop Model and Simulation Method

To keep the range of options manageable within one paper, in the present study we restrict the flexibility of the model parameters in several ways. We consider only a simple case of symmetric dipole magnetic loops, i.e., with the dipole perpendicular to a local vertical. The adopted geometry (visualized via a few reference magnetic field lines demarcating the surface of the magnetic loop) for two different loop orientations is shown in Figure 1. Dependence of the magnetic field strength at the loop axis on the distance from the looptop along the field line is shown in Figure 2(a).

We assume that the density and temperature of the thermal plasma component within the loop are constant (since the plasma in flaring loops is often heated up to the temperatures of \( \gtrsim 10^7 \) K, the corresponding barometric scale heights far exceed the loop heights, so the density variations with height can be neglected). Parameters of the energetic electrons can either be constant or vary with the distance from the looptop (see below). By using the above assumptions, we construct a three-dimensional model of the flaring loop (which is observed at a given direction) with all the source parameters depending on the Cartesian coordinates \((x, y, z)\), where \(x\) and \(y\) are the coordinates in the image plane, and \(z\) is the coordinate along the line of sight. The source volume is divided into a number of volume elements (voxels); each voxel is considered to be quasi-homogeneous. The radio brightness map (or the observed emission intensity as a function

![Figure 1. Magnetic field model used in the simulations, as implemented in the GS Simulator tool. The dashed box inscribes the portion of the magnetic loop (visualized by a few bold field lines) situated above the solar surface, while the solid rectangle represents the top view of an inscribing box that is perpendicular to the observer’s line of sight. The two panels show two different orientations of the same model corresponding to (a) a loop located near the center of the solar disk and (b) a loop located near the solar limb.](image1)

![Figure 2. (a) Magnetic field strength, (b) loss-cone boundary, and (c) relative density of the energetic electrons vs. coordinates along the loop. All the values correspond to the loop axis.](image2)
of two-dimensional coordinates x and y) at a given frequency f is calculated by numerical integration of the radiation transfer equation

$$\frac{dI}{dr} = I_{L,R}(f,x,y,z) - \rho_{L,R}(f,x,y,z)I_{L,R}(f,x,y,z)$$

(1)

along all selected lines of sight. In Equation (1), $I_{L}$ and $I_{R}$ are the spectral intensities of the left- and right-polarized emission components, respectively, $\rho_{L}$ and $\rho_{R}$ are the corresponding GS emissivities, and $\rho_{L}$ and $\rho_{R}$ are the absorption coefficients. We use the “strong coupling” model, i.e., the left- and right-polarized emission components propagate independently. Left-polarized emission corresponds to either an ordinary or extraordinary magnetoeicnic mode, depending on the magnetic field direction, and right-polarized emission corresponds to the opposite mode. The plasma emissivities $\rho_{O,X}$ and absorption coefficients $\rho_{E,X}$ for the ordinary and extraordinary modes accounting for both GS and free–free contributions at each voxel are calculated using fast GS codes developed by Fleishman & Kuznetsov (2010). Outside the flaring loop, the emission propagates as it does in a vacuum.

The loop orientation is described in general by three Euler angles; however, since rotation of the loop around the z-axis results simply in the same rotation of the brightness maps, variation of only two angles is considered. We adopt the loop to be located at the solar equator; in this case, the loop orientation is characterized by the angle $\psi$ between the magnetic dipole and the equatorial plane and by the longitude $\lambda$.

The energetic electrons are described by the distribution function $F$ in a factorized form: $F(E, \mu) = u(E)g(\mu)$, where $E$ is the electron kinetic energy, $\mu = \cos \alpha$, and $\alpha$ is the electron pitch angle (the angle between the particle velocity and the local magnetic field vectors). The tool allows using a number of different model electron distribution functions, including thermal, power law, broken power law, a power law matched to a Maxwellian core (so-called thermal/nonthermal, TNT, distribution), kappa distribution, etc. In particular, the use of TNT distribution allows one to consistently take into account both GS (nonthermal) and gyrososonat (GR, thermal) contributions to the emission and absorption. For our modeling, however, it has a disadvantage that two distributions with different anisotropies must be matched, which eventually increases the number of model parameters needing to be varied in the modeling. For this reason, we assume that the electrons have a power-law energy spectrum $u(E) \sim E^{-5}$ in the energy range $E_{\text{min}} < E < E_{\text{max}}$ and so neglect the GR contribution entirely. However, we make an assessment of the GR contribution by considering the TNT distribution in the supplemental material available in the online version of the journal. Although the effect of GR absorption is modest for our adopted source model and parameters, it does have a noticeable imprint on the spectrum and polarization (see Preka-Papadema & Alissandrakis 1992) from the lower parts of the loop legs; see Figures 29 and 30 in the online supplemental material.

The pitch-angle distribution can be either isotropic or a loss cone described by the model function

$$g(\mu) \sim \begin{cases} 1, & \text{for } |\mu| < \mu_c, \\ \exp\left[-\frac{(|\mu| - \mu_c)^2}{\Delta \mu^2}\right], & \text{for } |\mu| \geq \mu_c, \end{cases}$$

(2)

where $\mu_c = \cos \alpha_c$, $\alpha_c$ is the loss-cone boundary, and the parameter $\Delta \mu$ determines the sharpness of this boundary. The loss-cone boundary is adopted to exactly follow the transverse adiabatic invariant

$$\sin^2 \alpha_c = \frac{B}{B_I},$$

(3)

where $B$ and $B_I$ are the magnetic fields at a given point and at the loop footpoint, respectively. Dependence of the loss-cone boundary on the coordinate along the loop is shown in Figure 2(b); this parameter equals 90° at the footpoint and decreases with height, so that the distribution becomes closer to the isotropic one. Since within the adopted model the anisotropy is only strong at and around the footpoints, while it becomes much weaker across most of the magnetic loop, we call this “a moderate anisotropy.”

We consider both the homogeneous spatial distribution of the energetic electrons (when their number density $n_e$ is constant) and the case when energetic electrons are accumulated at the footpoint (Melnikov et al. 2002). The inhomogeneous distribution of the particles along the loop is described by the following model function:

$$n_e \sim \exp[-c^2(\psi - \pi/2)^2],$$

(4)

where $\psi$ is the magnetic latitude (or the angle between the dipole axis and the vector drawn from the dipole center to a given point) and the parameter $c$ determines the inhomogeneity degree ($c = 0$ corresponds to the homogeneous case). Density profiles of the energetic electrons along the loop for different values of $c$ are shown in Figure 2(c).

In the simulations, we use the following parameters of the flaring loop: height $H = 10^6 \approx 7270$ km from the base of the corona, dipole depth below the base of the corona $D = 6^\circ \approx 4360$ km (so that the distance between the footpoints $\Delta \approx 11.5 \approx 8400$ km), radius at the top $R_t = 2^\circ \approx 1450$ km, and magnetic field at the top $B_t = 75$ G (that results in the footpoint magnetic field of $B_f \approx 500$ G). Figures 1 and 2 correspond to these parameters. Two loop orientations are considered in the paper in some detail: $\psi = 60^\circ$, $\lambda = 20^\circ$ (a loop near the center of the solar disk, Figure 1(a)) and $\psi = 60^\circ$, $\lambda = 80^\circ$ (a loop at the limb, Figure 1(b)); many more examples are given in the online supplemental material. The thermal plasma density and temperature are $n_0 = 10^{10}$ cm$^{-3}$ and $T_0 = 2 \times 10^7$ K, respectively. The energetic electrons have a power-law index of $\delta = 4$, cutoff energies of $E_{\text{min}} = 100$ keV and $E_{\text{max}} = 10$ MeV, and a loss-cone boundary width of $\Delta \mu = 0.2$. Thus, in each loop orientation, the variable parameters are a type of the pitch-angle distribution (isotropic or loss-cone), the number density of the energetic electrons $n_e$, and the inhomogeneity parameter $c$.

3. SIMULATION RESULTS

3.1. Effect of the Electron Anisotropy

First, we consider the effect of electron anisotropy on the GS emission. Figure 3 shows the brightness maps at four frequencies for the loop located near the center of the solar disk. The displayed Stokes parameters are $I = I_R + I_L$ and $V = I_R - I_L$. The isotropic (top row) and loss-cone (bottom row) pitch-angle distributions are considered. In both cases, the number density of the energetic electrons $n_e$ is constant and equals $3 \times 10^6$ cm$^{-3}$. Figure 4 shows the spatially resolved emission spectra for the footpoint and looptop sources (obtained by summation over the circled pixels) as well as the total emission (obtained by summation over all pixels). The figure also shows the degree of
Figure 3. Radio brightness maps for a loop located near the center of the solar disk for the isotropic distribution (top row) and loss-cone distribution (bottom row); north is up and west is to the right. Concentration of the accelerated electrons is assumed to be constant along the loop. Solid lines are the intensity contours which are evenly distributed between zero and the maximum brightness temperature $T_m$ (the corresponding temperatures are given in each panel). Color shades (see the online version of the journal) represent the circular polarization (Stokes $V$ normalized by the absolute value of $V_{\text{peak}}$); red and blue correspond to the right and left circular polarizations, respectively.

(A color version of this figure is available in the online journal.)

Figure 4. Emission intensity, degree of polarization, and spectral index vs. frequency for the loop shown in Figure 3. The columns (from left to right) correspond to northern footpoint source, southern footpoint source, loop top source, and the emission from the entire loop (spatially unresolved). Solid lines: isotropic distribution; dashed lines: loss-cone distribution. The regions taken to calculate the spatially resolved spectra are indicated in Figure 3 by thick dashed circles.
polarization $\eta$ and spectral index $\delta_r$ which are defined as

$$\eta = \frac{V}{I}, \quad \delta_r = \frac{-f}{I \frac{dI}{df}}. \quad (5)$$

From the spatially resolved spectra, one can see that the influence of the electron anisotropy is most important at the footpoints. In the optically thin frequency range, the emission from the loss-cone distribution is lower than that from the isotropic electrons by a factor of about 2–6. This reflects the fact that the GS radiation is emitted mainly in the direction of the electron velocity. Near the footpoints, the electrons with the loss-cone distribution are concentrated around the pitch angle of $90^\circ$ whereas the magnetic field (in the adopted geometry) is nearly parallel to the line of sight. This is why the electrons with loss-cone distribution produce only a relatively weak radiation flux toward the observer. In the optically thick frequency range, the emission intensities from the isotropic and anisotropic distributions are almost the same, although the differences in the degree of polarization and spectral index can be visible. Near the looptop, the loss-cone boundary $\alpha_c$ falls to about $20^\circ$ and thus the loss-cone distribution does not noticeably differ from the isotropic one; as a result, these distributions produce almost identical emission.

A complementary way of thinking of the emission is via the radio images at various frequencies (Figure 3). At low frequencies (3.75 GHz), the whole loop is seen on the map (although the footpoints are brighter than the top); the images for the isotropic and anisotropic distributions are very similar. At higher frequencies, the emission is strongly concentrated at the footpoints. However, since the footpoint emission from the electrons with loss-cone distribution is weaker than that from the isotropic distribution, the difference between the footpoints and the looptop is smaller as well. Therefore, the emission from the anisotropic electrons is more evenly distributed along the loop.

Figures 5 and 6 show the brightness maps and emission spectra for the loop located near the limb (all other parameters are the same as in Figures 3 and 4). One can see that now the electrons with loss-cone distribution produce stronger footpoint emission in the optically thin frequency range than the isotropic distribution does (due to the same reasons as discussed above). In the optically thick frequency range, the considered characteristics of the emission are very similar for both isotropic and anisotropic cases except the effects of the individual cyclotron harmonics that are much more pronounced for the anisotropic distribution, which is discussed in Section 3.4 in greater detail. The looptop emissions from the isotropic and loss-cone electron distributions are almost the same.

In the brightness maps (Figure 5), the maximum of the emission at 3.75 GHz is located between the looptop and the footpoints; the images for the isotropic and anisotropic distributions are very similar. At the higher frequencies, the emission is concentrated at the footpoints. In contrast to the previous case (Figure 3), the footpoint sources are now more compact for the loss-cone electron distribution; this effect becomes more pronounced with the frequency increase.

Changing the pitch-angle distribution from the isotropic one to the loss cone results in a shift of the spectral peak of the footpoint emission toward lower frequencies for the loop located near the disk center, and toward higher frequencies for the loop near the limb (which could be observationally addressed via center-to-limb variation analysis of the spatially resolved footpoint spectra). This spectral peak variation affects the emission spectral index around the peak. With an increasing frequency (in the optically thin range), the spectral indices of the emission from the isotropic and anisotropic electrons approach each other and gradually become the same. We have confirmed that at $f \to \infty$, the emission spectral indices asymptotically approach the ultrarelativistic limit $\delta_{\text{rel}} = (\delta - 1)/2$ (provided that the high-energy cutoff $E_{\text{max}} \to \infty$, Ginzburg & Syrovatskii
and neglecting the free–free contribution) for both the isotropic and anisotropic electron distributions. However, in the frequency range of \(\sim 10–100 \text{ GHz}\), the emission spectral index is strongly affected by the magnetic field inhomogeneity in the source; as a result, the spectral index varies with frequency and can be noticeably different at the different footpoints and the looptop. At frequencies \(\gtrsim 100 \text{ GHz}\), the spectrum is affected by the high-energy cutoff of the electron distribution (note that Figures 3–6 are calculated for \(E_{\text{max}} = 10 \text{ MeV}\) which makes the spectrum steeper (the spectral index increases with frequency); this effect is clearly visible for the looptop emission where the magnetic field inhomogeneity is the lowest. On the other hand, at these and higher frequencies the free–free contribution can make the spectrum flatter (see below examples of such flattening).

The loss-cone anisotropy of the fast electrons has the opposite effect on the GS emission for the loops located near the solar disk center and the limb. Thus, for some intermediate longitude, the anisotropy effect should be minimal. We have found that this occurs at a longitude of \(\lambda \simeq 60^\circ\).

### 3.2. Effect of the Inhomogeneous Electron Distribution

Figures 7–10 show the brightness maps and spatially resolved emission spectra for the models with the inhomogeneous distribution of energetic electrons along the flaring loop (the particles are accumulated at the looptop). The number density of the energetic electrons at the looptop is taken to be \(n_t = 2.8 \times 10^6 \text{ cm}^{-3}\). The density profiles of the energetic electrons are described by Equation (4) with \(\epsilon = 4, 6, \text{and } 8\), which corresponds to the ratios of the footpoint and looptop number densities of \(n_f/n_t = 1.1 \times 10^{-2}, 3.6 \times 10^{-5}, \text{and } 1.3 \times 10^{-8}\), respectively (for \(\epsilon = 4\), we obtain the footpoint number density of \(n_f = 3 \times 10^6 \text{ cm}^{-3}\), as in the previous section). In all cases, the loss-cone distribution of the energetic electrons over the pitch angle is used.

Figures 7 and 8 correspond to the loop located near the center of the solar disk. Since the number density of the energetic electrons at the looptop is assumed to be constant, the looptop emission is almost independent of the parameter \(\epsilon\). In the footpoints, the increase of \(\epsilon\) results in a decrease of the number density of the energetic electrons and, consequently, in a decrease of the emission intensity in the optically thin frequency range; the spectral peak shifts toward lower frequencies. In the brightness maps, the intensity maximum is always located near the looptop. For \(\epsilon = 4\), at a frequency of 3.75 GHz, the whole loop is seen with relatively weak brightness variations along it. At higher frequencies and/or with increasing \(\epsilon\), the visible emission source becomes more compact.

Figures 9 and 10 show the brightness maps and emission spectra for the loop located near the limb. As in the previous case, the looptop emission is independent of the parameter \(\epsilon\) while the footpoint emission intensity (in the optically thin frequency range) decreases with increasing \(\epsilon\). Distributions of the radio brightness along the loop can be qualitatively different for the different frequencies and inhomogeneity models: for \(\epsilon = 4\), the emission maximum at 3.75 GHz is located at the looptop and gradually shifts toward the footpoints with increasing frequency. For \(\epsilon = 6 \text{ and } 8\), the intensity maximum is always located at the looptop, but the visible emission source becomes more compact with increasing frequency. Thus, at high frequencies (\(\gtrsim 15 \text{ GHz}\)), a brightness map will be dominated by either two footpoint sources (for the loops with a moderate accumulation of the particles at the top, up to \(n_f/n_t \simeq 100\)) or one looptop source (for the loops with higher inhomogeneity). We note an interesting
behavior of the polarization map: for the loop-like optically thin sources (at 17 or 34 GHz) in Figure 9 most of the image area is dominated by the right-hand polarization, which is indicative of the same direction of the line-of-sight magnetic field component throughout most of the loop area. Similar unipolar polarization patterns are highly typical for the Nobeyama Radioheliograph data on the limb flares. If the concentration of the electrons at the looptop develops at the course of flare due to electron transport (Lee et al. 1994; Melnikov et al. 2002; Fleishman 2006), this process of the electron inhomogeneity buildup can be roughly described in terms of the increase, which implies the apparent brightness peak motion (at the optically thin frequencies) from footpoints to the looptop in agreement with observations of some limb flares (Tzatzakis et al. 2008; Reznikova et al. 2009).

In all the models, the emission spectral index at the looptop behaves like that for a homogeneous source: the index is nearly constant ($\delta_r \simeq 1.7–1.8$) in the frequency range of 20–50 GHz (for both loop orientations) and increases at higher frequencies due to the high-energy cutoff of the electron spectrum. At the footpoints, an increase of the parameter $\epsilon$ results in a decrease of the spectral peak frequency. With an increasing frequency (in the optically thin range), the spectral indices for all values of $\epsilon$ become the same (if the free–free emission is negligible, see below). However, in contrast to the looptop, the spectral index of the footpoint emission is also affected by the source inhomogeneity. For the loop located near the disk center, we can note an asymmetry of the emission spectra produced at the different footpoints, which is caused by a difference of the viewing angles.

Changing the pitch-angle distribution of the energetic electrons affects the spatially resolved emission spectra in the same way as discussed in the previous section: in comparison to the isotropic distribution, the loss-cone distribution provides stronger footpoint emission for a loop near the limb and weaker footpoint emission for the loop near the disk center, while the looptop emission remains almost unchanged. Since in the considered inhomogeneous models the looptop emission source often dominates the images, the effect of the electron anisotropy on the brightness maps is only moderate.
In this section, we consider the total (spatially integrated) emission from the flaring loop. Although in light of the currently operational and soon to be available radio instruments, considering the total radiation may look old-fashioned, we feel that this still makes sense for the following reasons. Most of the historically accumulated databases and corresponding statistical studies are done based on the total power observations (e.g., Guidice & Castelli 1975; Nita et al. 2004). The total power data are more manageable as they can easily be visualized by dynamic spectra and characterized by only a few simple numbers, such as spectral indices, rise and decay times, peak flux, and frequency (Nita et al. 2004). In particular, the corresponding high-frequency spectral index is widely used to evaluate the fast electron energy spectral index (Dulk & Marsh 1982). From this perspective, a statement made by Simões & Costa (2010) that such a spectral index is a good measure of the electron spectral index even for anisotropic electron distributions, if confirmed, could be of a great practical value.

The parameters characterizing the total emission are shown in the right columns in Figures 4, 6, 8, and 10. Considering the right top panel in Figure 4 as a vivid example, one can easily isolate three distinct regions of the spectra—a low-frequency part (region I), a middle-frequency part (region II), and a high-frequency part (region III). Visual comparison of these total power spectra with the spatially resolved spectra from the footpoints and looptop in Figure 4 suggests that the low-frequency part is formed primarily at the looptop region with low magnetic field, the high-frequency part in the footpoints where the magnetic field is large, and the middle-frequency part by the entire loop and so related to the magnetic field nonuniformity.

The low-frequency part is known to be formed by the effect of the GS optical thickness and/or the Razin effect (suppression of the GS emission in a dense background plasma, e.g., Bastian et al. 1998) possibly accompanied by the free–free absorption in the dense plasma (Bastian et al. 2007); the slope of the spectrum can here be quantified by an index of $-2$ or less; see the right bottom panel in Figure 4. The high-frequency part is mainly determined by the distribution of fast electrons including the energy spectrum (Bastian et al. 1998) and pitch-angle anisotropy (Fleishman & Melnikov 2003a); note the anisotropy-related difference between the solid (isotropic) and dashed (loss-cone) curves in this panel.

The middle-frequency part is clearly seen in Figure 4 as it is almost flat (the spectral index is around zero). Solar microwave bursts with flat spectra have been observed for decades. For example, Ramaty & Petrosian (1972) proposed that the free–free absorption of GS emission can form such spectra, while Lee et al. (1994) recognized that the electrons trapped in a large dipole magnetic loop can produce the flat radio spectra due to the source nonuniformity in certain parameter regimes. Our modeling confirms the finding made by Lee et al. (1994). In addition, we have found that this middle-frequency part is not always flat but can have either negative or positive spectral index, see, e.g., Figure 6, which can be misinterpreted as either region I or III in observations with a limited spectral coverage. In fact, the observations (see, e.g., Figure 11 in Nita et al. 2004) reveal that the histograms of both low- and high-frequency spectral indices extend to zero implying that both low- and high-frequency spectra can be much flatter than those determined by the optical thickness effect or electron energy index, respectively. For practical application, this means that having the spectrum falling with the frequency does not guarantee that its slope is formed by
either energetic or angular properties of the electron distribution function but can instead be related to the source nonuniformity.

One of the parameters controlling the shape of the middle-frequency part is the viewing angle: Figure 6 presents the case when this part grows with frequency, which overall broadens the spectrum peak. Not surprisingly, nonuniform spatial distribution also significantly affects this part of the spectrum. In fact, with an inhomogeneous nonthermal electron spatial distribution (with their concentration at the looptop), the radio spectrum begins to resemble emission from a roughly uniform source; see Figures 8 and 10. The reason for this to happen for an inhomogeneous source is very simple: with the adopted inhomogeneous electron distribution most of the electrons reside at the looptop, where the spatial variation of the magnetic field and the viewing angle are minor, so we have a situation similar to a uniform source.

Finally, let us consider to what extent we could rely on the high-frequency spectral index in evaluating the fast electron energy spectrum or pitch-angle anisotropy. The obtained spectral behavior of the local spectral index can easily be tracked in the figures, so we do not describe it here in any detail. Instead, in order to quantify it with measures relevant to available observations, we form a few distinct spectral indices, namely, an index \( \delta_{15} \) at 15 GHz relevant to the OVSA spectral range (Nita et al. 2004), the “Nobeyama” index \( \delta_{17-34} \) calculated with two well-separated frequencies, 17 and 34 GHz (e.g., Reznikova et al. 2009), and higher-frequency spectral index \( \delta_{50} \) taken at 50 GHz relevant to a number of earlier separated frequency observations (e.g., Melnikov & Magun 1998); the results are gathered in Table 1.

One can see that for the same energy electron distribution, \( \delta = 4 \), the measured spectral indices are highly different depending on the pitch-angle anisotropy, the viewing angle, spatial inhomogeneity of the fast electron distribution, and the instrument used to measure the spectral index. In fact, the numbers given in Table 1 are bounded between 0.13 and 2.98, although as has already been said, the ultrarelativistic approximation predicts the emission spectral index of \( \delta_{\text{rel}} = (\delta - 1)/2 \), or \( \delta_{\text{rel}} = 1.5 \) for the adopted value of \( \delta = 4 \), while a widely used approximation proposed by Dulk & Marsh (1982) predicts the value of \( \delta_{\text{D-M}} \approx 0.90\delta - 1.22 \), or \( \delta_{\text{D-M}} \approx 2.38 \) for \( \delta = 4 \); none of these approximate values is favored by Table 1. Stated another way, the radio indices from 0.13 to 2.98 would imply the electron energy index range from 1.26 to 6.96 if the relativistic asymptote is used and from 1.50 to 4.67 if the Dulk–Marsh approximation is applied. Again, none of the values is truly informative in terms of recovering the original electron energy index as there is no unique way of linking the measured radio spectral index with the original electron energy.
Figure 10. Same as in Figure 8, for the loop models shown in Figure 9 (the loop is located near the solar limb). The regions taken to calculate the spatially resolved spectra are indicated in Figure 5 by thick dashed circles.

Figure 11. Radio brightness maps (emission intensity) for a loop located near the center of the solar disk. Brighter areas correspond to higher intensity. The number density of the accelerated electrons is constant and their pitch-angle distribution is of the loss-cone type.

spectral index. This conclusion holds (with narrower scatter of the recovered values, 4.34–5.92 and 3.21–4.09, respectively) if we consider only higher-frequency indices $\delta_{17-34}$ and $\delta_{50}$. On the other hand, the OVSA index $\delta_{15}$ is more sensitive to the particle anisotropy, which, thus, can be studied through a forward fit of the radio spectra as in Fleishman et al. (2009).

3.4. Harmonic Structure

The GS emission from a homogeneous source can demonstrate an oscillatory spectral structure in the low-frequency range ($f/f_B \lesssim 10$), when the emission intensity increases at a narrow spectral region near the harmonics (small integer multiples) of the electron cyclotron frequency (see, e.g., the figures in the articles of Ramaty 1969; Benka & Holman 1992; Fleishman & Melnikov 2003a, 2003b; Fleishman & Kuznetsov 2010). In an inhomogeneous source, however, this harmonic structure can be hidden because of natural smoothing: the resonance giving rise to a gyroharmonics at a given location will vary with frequency due to the spatially dependent resonant condition in the spatially nonuniform magnetic field. Thus, even if a spectrum from a single pixel contains harmonics they often disappear after integration over even a relatively small part of the source. This is why no harmonic structure is present in either the footpoint or looptop spectra in Figures 4, 6, 8, and 10 other than a number of extremely large peaks in Figures 6 and 10 for the loss-cone case provided that the conditions for the electron cyclotron maser (ECM) instability (see, e.g., Stepanov 1978; Wu & Lee 1979; Holman et al. 1980; Hewitt et al. 1982; Dulk & Marsh 1982; Sharma & Vlahos 1984; Wu 1985; Aschwanden 1990; Fleishman & Melnikov 1998; Fleishman & Arzner 2000; LaBelle & Treumann 2002; Treumann 2006; Kuznetsov 2011) are locally fulfilled. We do not consider the coherent ECM
emission here, concentrating instead on the harmonic structure of the incoherent GS emission.

In the simplest case of a uniform GS source, the harmonic structure is more prominently pronounced for the source viewed at a quasi-transverse direction relative to the source magnetic field. Note that for the magnetic model adopted here, the harmonic structure is only expected from the biggest flaring loops and at low frequencies (where scattering of radio waves by coronal density inhomogeneities can additionally fuse the images, however; Bastian 1994), which is comparable with the anticipated capability of the Frequency Agile Solar Radiotelescope (Gary 2003); although for the instruments currently under development (e.g., USSRT and EOVSA) detecting the harmonic stripes can only be expected from the biggest flaring loops and/or for the cases with a more uniform magnetic field. Such favorable cases, although atypical, are not unlikely: Staehler et al. (2008; see also Benka & Holman 1992), which might indicate gyroharmonic contribution; however, such an interpretation remains ambiguous unless confirmed by direct imaging data.

Table 1
Spectral Parameters of the Total (Spatially Unresolved) Emission

| $g(\mu)$ | $\epsilon$ | $f_{\text{peak}}$ (GHz) | $\delta_{15}$ | $\delta_{17-34}$ | $\delta_{50}$ |
|----------|-----------|-------------------------|------------|----------------|-------------|
| Loop location: near the disk center |
| Isotropic 0 | 6.03 | 2.40 | 2.46 | 2.05 |
| Loss cone 4 | 4.37 | 2.98 | 2.21 | 1.86 |
| Isotropic 4 | 7.59 | 1.75 | 1.82 | 1.79 |
| Loss cone 6 | 7.59 | 1.76 | 1.77 | 1.77 |
| Loss cone 8 | 6.92 | 1.66 | 1.72 | 1.76 |
| Loss cone 10 | 6.92 | 1.64 | 1.71 | 1.75 |
| Loop location: near the limb |
| Isotropic 0 | 12.02 | 0.57 | 2.10 | 2.14 |
| Loss cone 4 | 13.80 | 0.13 | 1.67 | 2.16 |
| Isotropic 4 | 10.47 | 0.94 | 1.67 | 1.92 |
| Loss cone 6 | 10.47 | 0.79 | 1.72 | 1.95 |
| Loss cone 8 | 7.24 | 1.77 | 1.78 | 1.78 |
| Loss cone 10 | 6.61 | 1.66 | 1.71 | 1.76 |

Notes. The values $\delta_{15}$ and $\delta_{50}$ are the exact spectral indices at 15 and 50 GHz, respectively; $\delta_{17-34}$ is an approximate spectral index calculated using two points at 17 and 34 GHz; for comparison $\delta_{\text{rel}} = 1.5$ while $\delta_{\text{irr-M}} = 2.38$. For the models with $\epsilon = 0$ and $\epsilon \neq 0$, the number densities of the energetic electrons at the looptop are different (see the text).

4. DISCUSSION

We have considered microwave emission produced by moderately anisotropic electron distributions populating (uniformly or nonuniformly) a nonuniform symmetric dipole magnetic loop. We emphasize that for the adopted loop geometry, the mirror ratio is more than 10, so in most of the loop volume (except the close vicinity of the footpoints) the loss-cone angle defined by the adiabatic invariant (Figure 2(b)) is small and, accordingly, the angular distribution of the fast electrons is close to the isotropic distribution, which we here conventionally call a “moderate anisotropy.” But even this moderate anisotropy noticeably affects both images and spectral characteristics of the emission.

In particular, for the uniform electron distribution along the loop the pitch-angle anisotropy enhances the optically thin...
emission from the footpoint vicinity for the loop located at the limb, while it suppresses it for the loop located at the disk center. In the case of nonuniform electron distributions due to accumulation at the looptop, the relative contribution of the electrons in the vicinity of the footpoints decreases and, so, the effect of the anisotropy becomes less pronounced. Nevertheless, in most of the cases (except looptop emission, where the electron angular distribution is almost isotropic) the effect of anisotropy on the images, spectrum, polarization, and spectral index can easily be recognized. We note that for a larger thermal density than that adopted for our restricted modeling, which is indeed often the case (Bastian et al. 2007), the looptop emission will be strongly suppressed by the Razin effect (which is specified by the \( n_0/B \) ratio and becomes especially strong at \( f < 20n_0/B \)), so the emission from the footpoints and legs of the loop, where the electron anisotropy is stronger than at the looptop, will dominate the emission; thus, the anisotropy effect will be even stronger than for the cases discussed above; see the supplemental material in the online version of the journal.

An interesting feature of the low-frequency images is the presence of the “gyrostripes,” which are indicative of distinct gyroharmonics produced at certain heights changing with frequency (since a few small integer multiples of the local gyrofrequency produce strong enhancement of the flux density). Having radio imaging spectroscopy data with the spatial resolution sufficient to resolve such gyrostripes would offer a nice model-independent diagnostics of the coronal magnetic field during flares. In the near future the EOVSA will be capable of such measurements at least for relatively large flaring sources.

An important issue is the anisotropy/nonuniformity effect on the total power radio spectrum. Since the total power spectrum is the result of emission integration over the entire source area, the result of this integration depends on how the distinct spatially resolved contributions are weighted, which, in turn, depends on the anisotropy, inhomogeneity, and the viewing angle. We found that all the spectrum peak, peak flux, polarization, and the spectral index depend on both anisotropy and electron distribution inhomogeneity, although the effects are counterdirected. In particular, the radio spectral index changes noticeably with frequency and its behavior depends noticeably on the pitch-angle anisotropy, e.g., one can see that the maximal value of the total power spectral index (Figure 4, right bottom) for the anisotropic case exceeds that for the isotropic case by roughly 1.

We compared the spectral indices with asymptotic relativistic (synchrotron) and approximate Dulk–Marsh values as the observed radio spectral indices are often used to evaluate the fast electron energy spectral indices. We found that neither the synchrotron nor the Dulk–Marsh value is a good approximation of the true spectral index: although the synchrotron index does represent the true index at high frequencies (higher than considered in our plots), the Dulk–Marsh index is less meaningful even though it sometimes coincides with the true one at a given single frequency. Therefore, we do not confirm the conclusion made by Simões & Costa (2010) that the Dulk–Marsh index is a quantitatively good approximation of the spectral index of the spatially integrated spectrum, which might be an artifact specific to their adopted source model.

5. CONCLUSION

We have introduced a flexible simulation tool capable of fast computing three-dimensional models of the microwave emission utilizing recently developed fast GS codes and presented an example of its use for the microwave emission modeling. Considering a highly restricted parameter space (symmetric dipole magnetic loop, moderate anisotropy, weak or no Razin effect), we have analyzed images, spectra, and polarization of the model radio emission in the view of the available and future (more complete) radio data. In particular, we note that the high-frequency spectral index does not have a unique value for a given energy spectral index of radiating electrons; instead, it noticeably varies with the frequency, the viewing angle, the anisotropy, and the inhomogeneity and is also different for various parts of the source and the entire source. This implies that the use of the radio spectral index for constraining the electron energy spectral index is not straightforward; instead, the electron spectrum recovery must rely on the forward fitting of the entire radio datacube (ideally, including polarization) as in the example presented by Fleishman et al. (2009).

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APPENDIX

MAGNETIC MODEL

Similar to other models used in the past (e.g., Antiochos & Sturrock 1976), in our simulation tool the magnetic field is produced by a magnetic dipole with a moment \( \mu \) that makes an adjustable angle \( \pi/2 - \varphi_0 \) with its corresponding solar radius, and is located below the solar surface at a depth \( D \). The flaring loop is constructed around a central field line that is chosen to lie in the plane defined by the magnetic dipole vector and the local vertical, henceforth referred to as the central plane of the loop. As pictured in Figure 13, in the dipole’s Cartesian system of coordinates \( (\xi, \eta, \zeta) \), which is defined as having the axis \( \vec{O}_{\xi} \) oriented along the magnetic dipole \( \mu \), and the axis \( \vec{O}_{\eta} \) in the central plane, the central magnetic field line may be parameterized as

\[
\begin{align*}
\xi_c &= H \sin^2 \varphi \cos \varphi \\
\eta_c &= H \sin^3 \varphi, \\
\zeta_c &= 0
\end{align*}
\] (A1)

where \( \varphi \in [0, 180] \) represents the angle between the position vector \( \vec{r}_c = (\xi, \eta, 0) \) and the dipole’s axis, i.e., \( \cos \varphi = \xi/r_c \), and \( H \) is the height of the central line, i.e., the maximum distance between the central line and the dipole’s axis, which corresponds to \( \varphi = \pi/2 \). In a solar coordinate system defined as having the axis \( \vec{O}_{\xi} \) along the solar radius and the axis \( \vec{O}_{\zeta} \) identical with the axis \( \vec{O}_{\xi} \), the same central line is expressed as

\[
\begin{align*}
x_c &= H \sin^2 (\varphi) \cos (\varphi + \varphi_0) \\
y_c &= H \sin^2 (\varphi) \sin (\varphi + \varphi_0) \\
z_c &= 0
\end{align*}
\] (A2)

Equations (A1) and (A2) are a direct consequence of the fact that any magnetic dipole field line is described by the general polar equation \( r = H \sin^2 \varphi \) and that the dipole’s system of coordinates is rotated relative to the solar system of coordinates by the angle \( \varphi_0 \) around the \( \vec{O}_{\zeta} \) axis, i.e.,

\[
\begin{align*}
x &= \xi \cos \varphi_0 - \eta \sin \varphi_0 \\
y &= \xi \sin \varphi_0 + \eta \cos \varphi_0 \\
z &= \zeta
\end{align*}
\] (A3)
The magnetic dipole moment model implemented in the simulation tool.

The magnetic flux tube centered on the central field line is defined in terms of a circular cross-section of radius $\rho_0$ which is normal to the central field at $\varphi = \pi/2$, i.e., the magnetic looptop. Hence, the Cartesian coordinates of an arbitrary magnetic field line, $r = \{\xi, \eta, \zeta\}$, that intersects the looptop cross-section at distance $\rho$ from the central field line, may be expressed in terms of three convenient free parameters $\{\rho, \alpha, \varphi\}$ as

$$\begin{align*}
\xi &= \sqrt{H^2 + 2\rho H \cos \alpha + \rho^2 \sin^2 \varphi \cos \varphi} \\
\eta &= (H + \rho \cos \alpha) \sin^3 \varphi \\
\zeta &= \rho \sin \alpha \sin^3 \varphi,
\end{align*}$$

where, as shown in Figure 13(b), $\alpha$ represents the fixed angle between the vector $\rho$ and the dipole’s vertical axis $\rho \eta$, and $\varphi$ represents the variable angle between the vector $r$ and the dipole moment $\mu$ that is measured in the plane containing the field line, which makes the angle $\theta = \arctan (\zeta/\eta) = \rho \sin \alpha / (H + \rho \cos \alpha)$ with the central plane of the magnetic flux tube.

For $\rho = \rho_0$, the set of Equation (A4) defines the envelope of the magnetic flux tube, which has a circular cross-section shape only for $\varphi = \pi/2$, i.e., at the looptop, while the shape of its cross-section is continuously changing along the central field line.

The condition for an arbitrary point $r$ to belong to a magnetic flux tube of height $H$ and top circular cross-section of radius $\rho_0$, i.e., the condition for the magnetic field line passing through the point $\{\xi, \eta, \zeta\}$ to intersect the looptop circular cross-section, may be expressed as

$$
\left[ H - \eta \left(1 + \frac{\xi^2}{\xi^2 + \eta^2}\right)^{3/2}\right]^2 + \xi^2 \left(1 + \frac{\xi^2}{\xi^2 + \eta^2}\right)^3 \leq \rho_0^2,
$$

which has to be combined with the condition

$$x^2 + (y - D + R_{\text{Sun}})^2 + z^2 \geq R_{\text{Sun}}^2$$

in order to determine if such flux tube point is also located above the solar surface.

Considering a normal cross-section of the flux tube corresponding to an angle $\varphi$ measured in the central plane of the flux tube, the parameter

$$s = \frac{1}{2} H \left\{\cos \varphi \sqrt{1 + 3 \cos^2 \varphi} + \frac{1}{\sqrt{3}} \ln \left[3 \cos \varphi + \sqrt{1 + 3 \cos^2 \varphi}\right]\right\}$$

may be used, instead of the angular coordinate $\varphi$, as a convenient flux tube longitudinal coordinate corresponding to the normal cross-section of interest.

The strength of the magnetic field is controlled by a unique adjustable parameter $B_0$ that defines the absolute value of the looptop magnetic field vector $B_0$, which is perpendicular to the looptop cross-section. Since the vector $B_0$ uniquely determines the dipole moment $\mu = -B_0 H^3$, the magnetic field vector in any point characterized by the position vector $r \equiv \{x, y, z\}$ relative to the dipole origin is given by

$$B = \frac{3(\mu r) r - r^2 \mu}{r^5}.$$
