Anomaly-Mediated Supersymmetry Breaking with Axion

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Abstract

We construct hadronic axion models in the framework of the anomaly-mediated supersymmetry breaking scenario. If the Peccei-Quinn symmetry breaking is related to the supersymmetry breaking, mass spectrum of the minimal anomaly-mediated scenario is modified, which may solve the negative slepton mass problem in the minimal anomaly-mediated model. We find several classes of phenomenologically viable models of axion within the framework of the anomaly mediation and, in particular, we point out a new mechanism of stabilizing the axion potential. In this class of models, the Peccei-Quinn scale is related to the messenger scale. We also study phenomenological aspects of this class of models. We will see that, in some case, the lightest particle among the superpartners of the standard-model particles is stau while the lightest superparticle becomes the axino, the superpartner of the axion. With such a unique mass spectrum, conventional studies of the collider physics and cosmology for supersymmetric models should be altered.
1 Introduction

In modern particle physics, symmetries play extremely significant roles in solving problems related to various fine tunings. Among them, in this paper, we consider two serious problems, that is, the strong CP problem and the gauge hierarchy problem.

Taking account of the instanton effect, the Θ-parameter in QCD, which is given by

\[ \mathcal{L} = \frac{g_3^2}{64\pi^2} \Theta \epsilon^{\mu\nu\rho\sigma} G^{a}_{\mu\nu} G^{a}_{\rho\sigma}, \]  

(1.1)

with \( g_3 \) being the gauge coupling constant for \( SU(3)_{C} \) and \( G^{a}_{\mu\nu} \) the field strength of the gluon, is constrained to be smaller than \( 10^{-9} \) [1]; otherwise, the electric dipole moment of the neutron becomes too large to be consistent with the experimental constraint. Such a smallness is, however, unnatural in the standard model, since \( \Theta \) is just a parameter in the Lagrangian. One smart solution to this problem is to introduce the Peccei-Quinn (PQ) symmetry [2, 3, 4, 5], which is a spontaneously broken global abelian symmetry which is anomalous under QCD. With such a symmetry, which we call \( U(1)_{PQ} \), Nambu-Goldstone boson (called “axion” \( a \)) shows up and couples to the gluon as

\[ \mathcal{L} = \frac{g_3^2}{64\pi^2} \left( \Theta + \frac{a}{f_a} \right) \epsilon^{\mu\nu\rho\sigma} G^{a}_{\mu\nu} G^{a}_{\rho\sigma}, \]  

(1.2)

where \( f_a \) is the decay constant of the axion. In this case, the \( \Theta \) parameter is promoted to a dynamical variable. Minimizing the potential of the axion, \( \Theta_{\text{eff}} \equiv \Theta + a/f_a \to 0 \), and hence there is no strong CP problem in this case. Since the PQ symmetry solves the strong CP problem very beautifully, it is desirable to implement the PQ symmetry in models of new physics beyond the standard model.

Another serious fine-tuning in the standard model is related to the stability of the electroweak scale against radiative corrections; in the standard model, radiative corrections to the Higgs mass parameter quadratically diverge and hence the electroweak scale is expected to be as large as the cutoff scale. Taking the cutoff at the gravitational scale, we need more than 30 orders of magnitude of fine tuning.

Once the supersymmetry (SUSY) is introduced, quadratic divergences cancel between bosonic and fermionic diagrams. Thus, SUSY is currently regarded as one of the most promising candidate of the solution to the naturalness problem, and we consider axion model in supersymmetric framework [6]. Among various models, in this paper, we consider the anomaly-mediated SUSY breaking model [7]. Interestingly, the anomaly-mediated model may solve the SUSY CP and FCNC problems without fine tuning, since in this model, all SUSY breaking parameters are well controlled by the super-Weyl (SW) anomaly. In addition, this model may provide solutions to cosmological difficulties such as gravitino problem and cosmological moduli problem [8, 9]. Thus, we will study how axion models...
Table 1: Particle content of the model. The index $i$ is the flavor index which runs from 1 to $N_5$.

|             | $SU(3)_C$ | $SU(2)_L$ | $U(1)_Y$ | $U(1)_{PQ}$ |
|-------------|-----------|-----------|-----------|-------------|
| $X$         | 1         | 1         | 0         | 1           |
| $Q_i$       | 3         | 1         | $-1/3$    | $-1/2$      |
| $L_i$       | 1         | 2         | $1/2$     | $-1/2$      |
| $\bar{Q}_i$| 3         | 1         | $1/3$     | $-1/2$      |
| $\bar{L}_i$| 1         | 2         | $-1/2$    | $-1/2$      |

can be constructed in the framework of the anomaly-mediated models. In particular, in some class of the axion model, we will emphasize that the axion multiplet changes the prediction of the minimal anomaly-mediated model and that the negative sneutrino mass problem may be solved. We will show that, in this case, the resulting superparticle mass spectrum is similar to that of the deflected anomaly mediation [10].

Organization of this paper is as follows. In Section 2, we introduce several models of axion in the framework of the anomaly-mediated SUSY breaking. Then, in Section 3, we study phenomenology of models introduced in Section 2. We summarize our results in Section 4.

2 Model

Let us discuss the framework of our model. Although our mechanism works in a large class of models with various particle content, we consider a model with $N_5$-pairs of 5 and $\bar{5}$ representations in the $SU(5)$ as an example. The particle content of our model is given in Table 1, where the representations for the standard-model gauge groups as well as the charge for the $U(1)_{PQ}$ are also shown. In our model, the lowest component of $X$ acquires a vacuum expectation value (VEV) and it breaks the $U(1)_{PQ}$ symmetry. Thus, we call $X$ as the axion multiplet.

In addition to the fields in the observable sector, we also introduce hidden-sector field which is responsible for the SUSY breaking. We denote the hidden-sector field as $z$. In our model, we adopt the sequestered structure

$$K = -3 \log \left[ \zeta(z^\dagger, z) + \xi(X^\dagger, X) \right], \tag{2.1}$$

which may arise, for example, when the hidden and observable sectors are geometrically separated [12].

Once the sequestered structure is assumed, phenomenology in the observable sector is insensitive to how the SUSY is broken in the hidden sector since the effect of the SUSY breaking is mediated by the axion multiplet.

#3 The function $\xi$ also depends on the observable sector fields such as quark, lepton, and Higgs multiplets. Such fields are irrelevant in studying the axion potential, and we omit these fields in the expressions.
breaking is mediated to the observable sector only by the SW anomaly. Thus, we only assume that the SUSY is somehow broken in the hidden sector. In this framework, SUSY breakings in the observable sector are only from couplings to the compensator field Φ, whose VEV is given by

$$\Phi = 1 + F_\Phi \theta^2. \quad (2.2)$$

Here, $F_\Phi$ is an auxiliary field in the gravitational multiplet, and its VEV is given by, assuming vanishing cosmological constant,

$$F_\Phi = \frac{1}{M_*^2} \left( W + \frac{1}{3} \frac{\partial K}{\partial z} F_z \right), \quad (2.3)$$

where $M_* \simeq 2.4 \times 10^{18}$ GeV is the reduced Planck scale. With this compensator field, relevant part of the supergravity Lagrangian has the following form [13]:

$$\mathcal{L} \simeq \int d^4 \theta \Phi^\dagger \Phi e^{-K/3} + \left[ \int d^2 \theta \Phi^3 W + \text{h.c.} \right]$$

$$\simeq \int d^4 \theta \Phi^\dagger \Phi \left[ \zeta(z^\dagger, z) + \xi(X^\dagger, X) \right] + \left[ \int d^2 \theta \Phi^3 W + \text{h.c.} \right]. \quad (2.4)$$

Soft SUSY breaking parameters in the observable sector are obtained by expanding the above Lagrangian in the background with non-vanishing $F_\Phi$.

Now, let us consider the potential of $X$ and study how the PQ symmetry can be broken. As we will see below, there are a couple of different approaches, each of which have different consequences on the mass spectrum of the squarks and sleptons. Thus, we will study each of them separately.

Importantly, because of the $U(1)_{\text{PQ}}$ symmetry, the function $\xi$ in the Kähler potential depends on the combination $X^\dagger X$. At the tree level, expanding $\xi$ around $|X| = 0$, we express

$$\xi_{\text{tree}}(X^\dagger, X) = M_*^2 \sum_{p=1}^{\infty} c_p \left( \frac{X^\dagger X}{M_*^2} \right)^p = c_1 X^\dagger X + \frac{c_2}{M_*^2} (X^\dagger X)^2 + \cdots, \quad (2.5)$$

where $c_p$ are constants. Notice that, as we will see below, radiative correction to the function $\xi$ becomes significant in some case. In addition, the axion multiplet $X$ is coupled to other fields in the superpotential as

$$W_{X^{55}} = \lambda_Q X \bar{Q} Q + \lambda_L X \bar{L} L. \quad (2.6)$$

Due to this superpotential, the axion multiplet $X$ couples to the field strength of the gluon multiplet, and the imaginary part of the lowest component of $X$ becomes the axion.

#4 We use the same expression for the chiral superfield and its bosonic (lowest) component.
At the tree level, and with the simplest Kähler potential of \( \xi = c_1 X^\dagger X \), VEV of \( X \) is undetermined. Indeed, after the rescaling
\[
\hat{X} \equiv \Phi X, \quad \hat{Q} \equiv \Phi Q, \quad \cdots
\]
all the compensators in Eq. (2.4) are absorbed in the “hatted” superfields. (Hereafter, “hatted” superfields denote superfields after the rescaling with the compensator \( \Phi \), like Eq. (2.7).) In this case, no potential is generated for \( \hat{X} \). Taking account of the higher dimensional operators and/or radiative corrections, however, non-trivial potential for \( \hat{X} \) is generated. In the following subsections, we study several cases.

### 2.1 Model 1

The first approach is to use the radiative correction to the wave function renormalization factor of \( X \) and the higher dimensional terms in the Kähler potential. Taking account of the wave function renormalization, at the loop level, we expect the function \( \xi \) to become
\[
\xi(X^\dagger, X) = c_1 Z_X(X^\dagger, X) X^\dagger X + \frac{c_2}{M_s^2} (X^\dagger X)^2 + \cdots
\]
(2.8)
The \( X \)-dependence of the wave function renormalization factor \( Z_X \) is from the scale dependence; below the scale of the VEV of \( X \), all the particles coupled to \( X \) decouple and \( Z_X \) does not run. Thus, we obtain
\[
Z_X(X^\dagger, X) = \sum_n \frac{1}{\Gamma(n+1)} \left[ \frac{d^n Z_X}{d(\log \mu)^n} \right]_{\mu = \Lambda} \left( \frac{1}{2} \log \frac{X^\dagger X}{\Lambda^2} \right)^n
\]
\[
= Z_X(\Lambda) + Z'_X(\Lambda) \left( \frac{1}{2} \log \frac{X^\dagger X}{\Lambda^2} \right) + \frac{1}{2} Z''_X(\Lambda) \left( \frac{1}{2} \log \frac{X^\dagger X}{\Lambda^2} \right)^2 + \cdots
\]
(2.9)
where \( \Lambda \) is an arbitrary scale, and the “prime” represents derivative with respect to \( \log \mu \).

Taking account of these effects, the supergravity Lagrangian given in (2.4) becomes, after the rescaling (2.7),
\[
\mathcal{L} \simeq \int d^4 \theta Z_X \left\{ \left[ 1 + \frac{1}{2} \frac{Z'_X}{Z_X} \log \frac{\hat{X}^\dagger \hat{X}}{\Phi^\dagger \Phi} + \frac{1}{8} \frac{Z''_X}{Z_X} \left( \log \frac{\hat{X}^\dagger \hat{X}}{\Phi^\dagger \Phi} \right)^2 + \cdots \right] \hat{X}^\dagger \hat{X}
\right.
\]
\[
+ \frac{c_2}{M_s^2} \frac{(\hat{X}^\dagger \hat{X})^2}{\Phi^\dagger \Phi} + \cdots \right\}.
\]
(2.10)
where we performed an \( X \)-independent rescaling of \( X \), and the potential for \( \hat{X} \) is given by
\[
V(X^\dagger, X) \simeq -\frac{1}{4} \left[ \frac{d^2 Z_X}{d(\log \mu)^2} \right]_{\mu = \Lambda_X} |F_\Phi|^2 \hat{X}^\dagger \hat{X} - \frac{c_2}{M_s^2} |F_\Phi|^2 (\hat{X}^\dagger \hat{X})^2 + \cdots
\]
\[
\simeq m_X^2 \hat{X}^\dagger \hat{X} - \frac{c_2}{M_s^2} |F_\Phi|^2 (\hat{X}^\dagger \hat{X})^2 + \cdots
\]
(2.11)
With the superpotential given in Eq. (2.3), the mass for the $X$ field is given by, at the one-loop level,

$$m_X^2(X^\dagger, X) = -\frac{1}{4} \left[ \frac{d^2Z_X}{d(\log \mu)^2} \right]_{\mu=\lambda_X} |F^c|^2$$

$$= -\left(\frac{1}{16\pi^2}\right)^2 N_5 \left[ (16g_3^2 + \frac{4}{3}g_1^2) \lambda_Q^2 + (6g_2^2 + 2g_1^2) \lambda_L^2 \right.$$

$$\left. -N_5(3\lambda_Q^2 + 2\lambda_L^2)^2 - 2(3\lambda_Q^4 + 2\lambda_L^4) \right] |F^c|^2$$

$$\approx -\left(\frac{1}{16\pi^2}\right)^2 N_5 \left[ 16g_3^2 - 5(5N_5 + 2)\lambda^2 \right] \lambda^2 |F^c|^2,$$

where, in the last equality, we used the approximations $g_3 \gg g_2$ and $g_1$, and $\lambda \equiv \lambda_Q \sim \lambda_L$. Hereafter, we adopt these approximations for simplicity. Our results are qualitatively unchanged even if we use the exact formula.

For the spontaneous breaking of the PQ symmetry, $m_X^2$ should become negative. This happens when the gauge coupling constants are larger than the Yukawa couplings $\lambda_Q$ and $\lambda_L$. Such a situation can be naturally realized by assuming relatively small values of the coupling constants $\lambda_Q$ and $\lambda_L$. Of course, for a realistic model of SUSY and PQ symmetry breakings, potential of $\hat{X}$ should be somehow stabilized. Pomarol and Rattazzi [10] proposed to use the inverted hierarchy mechanism [11] to stabilize the potential; if $m_X^2$ changes its sign at some scale with $d m_X^2 / d(\log \mu) > 0$, the potential of $\hat{X}$ has stable minimum at the scale $m_X^2 = 0$. Such a scenario is possible when there exists asymptotically-free gauge interaction with relatively large gauge coupling constant. We found, however, that the $SU(3)_C$ gauge interaction cannot play this role since its gauge coupling constant is not large enough to realize the inverted hierarchy mechanism. We checked that this is always the case, by solving renormalization group equations numerically. Pomarol and Rattazzi suggested to gauge (some part of) the flavor symmetry to introduce a new gauge interaction for the stabilization. Unfortunately, it does not work for the axion model since, once $Q$ and $\bar{Q}$ (and/or $L$ and $\bar{L}$) have non-trivial gauge quantum numbers under extra gauge interaction, the axion potential is modified and the strong CP problem cannot be solved.

In our model, we do not rely on the inverted hierarchy mechanism, since there is another simple way to stabilize the potential. As suggested in Eq. (2.11), the higher dimensional terms in the Kähler potential naturally exist which result in quartic and higher order terms in the scalar potential. Assuming the negativity of $c_2$, the potential given in Eq. (2.11) has a minimum, and the VEV of $\hat{X}$ is given by

$$f_a^2 \equiv \langle |\hat{X}|^2 \rangle = \frac{m_X^2}{2c_2|F_0|^2N_5^2} M_*^2 \simeq \left[ \frac{1}{4\pi^2} \sqrt{\frac{1}{2c_2|N_5|}} M_* \right]^2,$$

where we assumed $g_3 \gg \lambda$. As one can see, the PQ scale is suppressed by the loop factor, and it decreases as the coupling constant $\lambda$ becomes smaller. Even with $\lambda \sim O(10^{-1})$, $f_a$
is $O(10^{15} \text{ GeV})$, and $f_a$ becomes $O(10^{12} \text{ GeV})$ when $\lambda \sim O(10^{-4})$. Phenomenological and cosmological implications of this fact will be discussed later.

Next, let us discuss the mass spectrum of the superparticles in the MSSM sector. Since the $X$ field couples to the chiral multiplets which have standard-model quantum numbers, it affects the soft SUSY breaking parameters in the observable sector at the loop level. Indeed, at the scale of the VEV of $X$, the chiral superfields $Q$, $\bar{Q}$, $L$ and $\bar{L}$ decouple. Then, the running effects from the cutoff scale $\Lambda$ to the scale of the VEV of $X$ is via the combination $\log(X^\dagger X/\Lambda^2) = \log(\hat{X}^\dagger \hat{X}/\Lambda^2 \Phi^\dagger \Phi)$. Since the VEV of the highest component of $\hat{X}$ (i.e., the axion multiplet after the rescaling), which is denoted as $F_{\hat{X}}$, vanishes, we obtain $F_{\Phi}$ dependence from $\log(\hat{X}^\dagger \hat{X}/\Lambda^2 \Phi^\dagger \Phi)$. Expanding this logarithm, we obtain the SUSY breaking parameter at the messenger scale

$$M_\lambda(M_{\text{mess}}) = -\frac{b_i - N_5}{4\pi} \alpha_i(M_{\text{mess}}) F_{\Phi}, \quad (2.14)$$

$$m_f^2(M_{\text{mess}}) = \frac{1}{(4\pi)^2} \left[ 2C_f^i (b_i - N_5) \alpha_i^2(M_{\text{mess}}) \right. \left. - N_u \alpha_t(M_{\text{mess}}) \left\{ \frac{13}{15} \alpha_1(M_{\text{mess}}) + 3\alpha_2(M_{\text{mess}}) + \frac{16}{3} \alpha_3(M_{\text{mess}}) \right\} \right] |F_{\Phi}|^2, \quad (2.15)$$

$$A_f(M_{\text{mess}}) = -\frac{y_f(M_{\text{mess}})}{4\pi} \sum_{\text{fields} \in f} \left[ 2C_f^i \alpha_i(M_{\text{mess}}) - N_u \alpha_t(M_{\text{mess}}) \right] F_{\Phi}, \quad (2.16)$$

where the messenger scale is given by

$$M_{\text{mess}} = \lambda \langle X \rangle, \quad (2.17)$$

and $\alpha_i$ is top-quark Yukawa coupling, $\alpha_i$ are gauge coupling constants ($i$ runs over the MSSM gauge groups.), $b = (-\frac{33}{5}, -1, 3)$, and $C_f^i$ is the second-order Casimir. (For the fundamental representations of $SU(3)_C$ and $SU(2)_L$, $C_3^f = \frac{4}{3}$ and $C_2^f = \frac{3}{4}$, and for $U(1)_Y$, $C_1^f = \frac{3}{5}Y^2$ with $Y$ being the hypercharge quantum number.) In addition, $N_u = (1, 2, 3)$ for $\tilde{q}_{3\text{rd}}^L$, $t_R$, and $h_u$ and $N_u = 0$ for other particles.

Contrary to the conventional scenario, there is another important particle in our model, that is, the axino which is the superpartner of the axion. The axino may become the LSP in our model which has significant implications for collider physics and cosmology. Axino mass arises from the Lagrangian (2.10); expanding the Lagrangian, the axino mass is given by

$$m_{\tilde{a}} = -\frac{1}{4} \frac{d^2 Z_X}{d \log \mu^2} \left\langle X^\dagger \right\rangle \left\langle X \right\rangle F_{\Phi}^1 \simeq \frac{g_5^2 \lambda^2}{16\pi^4} N_5 F_{\Phi}, \quad (2.18)$$

where we neglected unimportant phase in the second equality. As one can see, the axino mass arises at the two-loop level and is much smaller than the masses of the superparticles in the MSSM sector. Thus, in the model 1, the axino becomes the LSP.
Interactions of the axino with the observable sector fields depend on the PQ charges of the Higgs (and other) fields. The charge assignment is model-dependent, and in particular, the charge assignment is related to how the $\mu_H$- and $B_\mu$-parameters are generated. This issue will be discussed later.

2.2 Model 2

It is also possible to break the PQ symmetry by higher dimensional operators in the superpotential. For this purpose, we introduce extra chiral multiplet $Y$. With this superfield, we write down the following superpotential

$$W = W_{\chi \bar{55}} + \frac{1}{\Lambda_{pX+pY-3}} X^{pX} Y^{pY}, \quad (2.19)$$

where $p_X$ and $p_Y$ are positive integers, and $\Lambda$ is some mass parameter which is assumed to be of order the Planck scale. Obviously, this superpotential is $U(1)_{PQ}$ invariant assigning the PQ charge $-p_X/p_Y$ to $Y$.

In the background with non-vanishing $F_\Phi$, the scalar potential is given by

$$V = \left| \frac{1}{\Lambda_{pX+pY-3}} \right|^2 \left[ p_X^2 \left| \hat{X}^{pX-1} \hat{Y}^{pY} \right|^2 + p_Y^2 \left| \hat{X}^{pX} \hat{Y}^{pY-1} \right|^2 \right]$$

$$+ (p_X + p_Y - 3) \left[ \frac{F_\Phi}{\Lambda_{pX+pY-3}} \hat{X}^{pX} \hat{Y}^{pY} + h.c. \right]. \quad (2.20)$$

Minimizing this potential, we obtain

$$p_Y |\langle \hat{X} \rangle|^2 = p_X |\langle \hat{Y} \rangle|^2 = C \left| F_\Phi \Lambda_{pX+pY-3} \right|^{2/(p_X+pY-2)}, \quad (2.21)$$

where

$$C = \left( \frac{p_X + p_Y - 3}{p_X + p_Y - 1} \right)^{2/(p_X+pY-2)} \frac{p_X^{(p_Y-2)/(p_X+pY-2)} p_Y^{(p_X-2)/(p_X+pY-2)}}{p^{(p_Y-2)/(p_X+pY-2)}}, \quad (2.22)$$

which is a constant of $O(1)$. Thus, we obtain

$$f_a \sim \frac{|\langle \hat{X} \rangle|}{N_5} \sim \frac{|\langle \hat{Y} \rangle|}{N_5} \sim \frac{|F_\Phi \Lambda_{p-3}|^{1/(p-2)}}{N_5}, \quad (2.23)$$

and

$$\frac{F_\hat{X}}{X} = -\frac{p - 3}{p - 1} F_\Phi, \quad (2.24)$$

where $p = p_X + p_Y$. In this case, the PQ scale is determined by the relative size of $F_\Phi$ and $\Lambda$. Taking $\Lambda \sim M_*$, for example, and $F_\Phi \sim 100$ TeV, we obtain $f_a \sim O(10^{10}$ GeV),
$O(10^{13}$ GeV), and $O(10^{14}$ GeV), for $p = 4$, 5, and 6, respectively. Especially, in the case of $p = 4$, $N_5 \sim 10$.

With the VEV given in Eq. (2.24), we obtain

$$M_\lambda(M_{\text{mess}}) = -\frac{b_i - N_5}{4\pi} \alpha_i(M_{\text{mess}}) F_\Phi,$$

(2.25)

$$m^2_f(M_{\text{mess}}) = \frac{1}{(4\pi)^2} \left[ 2C^f_i \left( b_i - \frac{4(p - 2)}{(p - 1)^2} N_5 \right) \alpha_i^2(M_{\text{mess}}) - N_u \alpha_t(M_{\text{mess}}) \right] |F_\Phi|^2,$$

(2.26)

$$A_f(M_{\text{mess}}) = -\frac{y_f(M_{\text{mess}})}{4\pi} \sum_{\text{fields} \in f} \left[ 2C^f_i \alpha_i(M_{\text{mess}}) - N_u \alpha_t(M_{\text{mess}}) \right] F_\Phi.$$  

(2.27)

One important difference between this model and the model 1 is the axino mass. In this model, axino is a linear combination of the fermionic component in the chiral superfields $X$ and $Y$. The mass matrix for these fermions are, after relevant phase rotation to remove unwanted phases,

$$M_{\text{axino}} = C^{(p+2p')} p_X^{(2-p')/2} p_Y^{(2-p)/2} \left( \frac{p_X - 1}{\sqrt{p_X p_Y}} - \frac{1}{p_Y - 1} \right) |F_\Phi|.$$  

(2.28)

As one can easily see, both fermions acquire masses as large as $|F_\Phi|$, which is of order $10 - 100$ TeV. Thus, in this model, axino mass is much heavier than the masses of the superparticles in the MSSM sector and hence the axino cannot be the LSP.

### 2.3 Model 3

In the previous models, the PQ symmetry is unbroken in the supersymmetric limit. In some case, however, the PQ symmetry can be spontaneously broken even if the supersymmetry is preserved. Let us finally consider such a case.

The simplest superpotential realizing such a situation is

$$W = W_{X55} + \kappa Y'(XX - N_5^2 f_a^2),$$

(2.29)

where $f_a$ here is some constant, and $Y'$, whose $U(1)_{\text{PQ}}$ charge is 0, is a chiral superfield which is singlet under $SU(5)$. Contrary to the previous models where the PQ scale $f_a$ is somehow related to the SUSY breaking parameter $F_\Phi$, $f_a$ is an input parameter in this case. As one can see, the PQ symmetry is broken solving the condition for the vanishing $F$-component of $Y'$: $\partial W/\partial Y' = 0$.

In this model, in fact, the PQ symmetry is broken but the masses of the squarks and sleptons are not modified. This can be easily seen by solving the equations of motion.
After the rescalings $\hat{X} = \Phi X$ and so on, we obtain $\langle \hat{X} \hat{X} \rangle = f_a^2 \Phi^2$, by solving $\partial W/\partial \hat{Y}' = 0$. At this stage, the relative size of $\hat{X}$ and $\hat{X}$ is undetermined since it corresponds to a flat direction in the supersymmetric limit. The relative size is determined taking account the effect of the SUSY breaking. With non-vanishing $F_\Phi$, $\hat{Y}'$ acquires a VEV as

$$\kappa \langle \hat{Y}' \rangle \simeq F_\Phi,$$

which gives equal masses to $\hat{X}$ and $\hat{X}$. Then, the VEVs of these chiral superfields become the same and

$$\langle \hat{X} \rangle = \langle \hat{X} \rangle = N_5 f_a \Phi,$$

up to an irrelevant phase. Thus, substituting this VEV into $\log(\hat{X}^\dagger \hat{X}/\Lambda^2 \Phi^\dagger \Phi)$, effect of the SUSY breaking disappears, and hence the axion multiplet does not modify the masses of the superparticles. In this case, extra mechanism should be introduced to solve the negative slepton mass problem, and we do not pursue this direction anymore.

### 2.4 The $\mu_H$- and $B_\mu$-Parameters

Before closing this section, let us comment on the $\mu_H$- and $B_\mu$-parameters. The $\mu_H$- and $B_\mu$-parameters can be generated by slightly modifying the model by Pomarol and Rattazzi [10]. The new superpotential we introduce is

$$W = \lambda_{ST} S T X + \lambda_{SST} S^2 T + \lambda_{THH} T H_u H_d,$$

where $S$ and $T$ are chiral superfield which are singlet under the $SU(5)$, whose PQ charges are +1 and $-2$, respectively. Notice that we also assigned the PQ charge $-1$ for the up- and down-type Higgses.

As in the case of the model by Pomarol and Rattazzi, kinetic mixing between $X$ and $S$ appears at the one-loop level:

$$\mathcal{L}_{\text{mix}} = \int d^4 \Phi \Phi Z_{ST}(\mu = |\lambda X|) S X^\dagger + \text{h.c.}$$

In addition, once $X$ acquires a VEV, $S$ and $T$ becomes massive and these fields can be integrated out. In particular, by solving the condition $\partial W/\partial T = 0$, we obtain

$$S = -\frac{\lambda_{THH} H_u H_d}{\lambda_{STX} X},$$

and substituting Eq. (2.34) into Eq. (2.33), we obtain

$$\mathcal{L}_{\text{mix}} = -\frac{\lambda_{THH}}{\lambda_{STX}} \int d^4 \Phi Z_{ST}(\mu = |\lambda X/\Phi|) \frac{\hat{X}^\dagger}{X} H_u H_d + \text{h.c.}$$
The model 1

| \tilde{e}_1 | 144 | \tilde{e}_2 | 265 |
| \tilde{\tau}_1 | 104 | \tilde{\tau}_2 | 270 |
| \tilde{\nu}_e | 253 | \tilde{\nu}_\tau | 248 |
| d_1 | 686 | d_2 | 732 |
| b_1 | 648 | b_2 | 695 |
| \tilde{u}_1 | 690 | \tilde{u}_2 | 728 |
| \tilde{t}_1 | 579 | \tilde{t}_2 | 737 |
| \tilde{\chi}_1^{\pm} | 306 | \tilde{\chi}_2^{\pm} | 615 |
| \tilde{\chi}_1^0 | 299 | \tilde{\chi}_2^0 | 319 |
| \tilde{\chi}_3^0 | 528 | \tilde{\chi}_4^0 | 616 |
| h^0 | 117 | H^0 | 332 |
| A^0 | 333 | H^\pm | 345 |
| \tilde{g} | 941 | - | - |

The model 2 (p = 4 case)

| \tilde{e}_1 | 167 | \tilde{e}_2 | 460 |
| \tilde{\tau}_1 | 104 | \tilde{\tau}_2 | 462 |
| \tilde{\nu}_e | 453 | \tilde{\nu}_\tau | 447 |
| d_1 | 1675 | d_2 | 1735 |
| b_1 | 1623 | b_2 | 1664 |
| \tilde{u}_1 | 1677 | \tilde{u}_2 | 1733 |
| \tilde{t}_1 | 1512 | \tilde{t}_2 | 1691 |
| \tilde{\chi}_1^{\pm} | 706 | \tilde{\chi}_2^{\pm} | 1458 |
| \tilde{\chi}_1^0 | 702 | \tilde{\chi}_2^0 | 711 |
| \tilde{\chi}_3^0 | 1192 | \tilde{\chi}_4^0 | 1458 |
| h^0 | 130 | H^0 | 712 |
| A^0 | 716 | H^\pm | 721 |
| \tilde{g} | 2538 | - | - |

Table 2: Mass spectra of the model 1 and 2 in units of GeV. We take \tan \beta = 30, M_{mess} = 10^{12}\text{GeV} and 10^{11}\text{GeV}, N_5 = 7 and 13 for the model 1 and 2, respectively. The Wino mass \tilde{M}_2 is taken to be 600 GeV for the model 1 and 1450 GeV for the model 2.

In Model 1 where \hat{F}_X = 0, expanding the above Lagrangian, we obtain the \mu_H- and B_\mu- parameters to be

\[
\mu_H = -\frac{\lambda_{THH}}{2\lambda_{STX}^2} \frac{dZ_{SX}}{d \log \mu} \frac{F_\Phi^\dagger}{\mu = \lambda_X}, \quad (2.36)
\]

\[
B_\mu = -\frac{\lambda_{THH}}{4\lambda_{STX}} \left[ \frac{d^2 Z_{SX}}{(d \log \mu)^2} \right]_{\mu = \lambda_X} |F_\Phi|^2. \quad (2.37)
\]

3 Phenomenology

Here we would like to discuss phenomenological issues of the models we presented. As we mentioned earlier, our models have superparticle masses identical to those in the deflected anomaly mediation. Some of our results were already discussed in Refs. [10, 14].

3.1 Mass Spectrum

Let us first consider the model 1. In this model, the soft masses are given in Eqs. (2.14), (2.15) and (2.16), which are parameterized by the scale of the SUSY breaking \hat{F}_\Phi, the messenger scale \tilde{M}_{mess} and the number of the messenger fields \tilde{N}_5. In addition, we have the \mu_H and B_\mu parameters. Here we do not take any particular mechanism to generate them. \mu_H is solely determined so as to reproduce the correct electroweak scale, and B_\mu is related
Figure 1: Lower bound on the messenger scale $M_{\text{mess}}$ as a function of the number of messengers $N_5$. The light-shaded region is excluded by tachyonic slepton for $\tan \beta = 5$. The lower limit for $\tan \beta = 30$ is also shown as a solid line. The dark-shaded region is excluded because the gauge coupling constants blow up below the GUT scale.

To $\tan \beta$, the ratio of the vacuum expectation values of the two Higgs in the MSSM. Thus the superparticle mass spectrum is specified by $F_\Phi, M_{\text{mess}}, N_5$ and $\tan \beta$.

To obtain a realistic mass spectrum (positive slepton masses etc.), the number of the messenger should be large enough. In fact for $N_5 = 3$, the gluino mass vanishes, resulting in an unrealistic mass spectrum. For $N_5 = 4$, we found that the sleptons are too light to survive the experimental mass bounds, unless the SUSY breaking scale is very high. This is disfavored, because the other superparticle masses will exceed 1 TeV, causing the fine-tuning problem. Thus we will consider the case $N_5 \geq 5$. In Fig. 1, we show the lower bound on the messenger scale $M_{\text{mess}}$ as a function of $N_5$. The bound comes from the two requirements: (i) the positive slepton mass squared (ii) the perturbativity of the gauge coupling constants up to the GUT scale. The former requirement is severer for smaller $N_5$, while the latter is severer for larger $N_5$. One finds that the messenger scale $M_{\text{mess}}$ must be larger than $\sim 10^9$ GeV.\textsuperscript{#5}

In Figs. 2 and 3, the masses of some superparticles are shown for $M_{\text{mess}} = 10^{12}$ and $10^{14}$ GeV, respectively. One finds that the lightest superparticle among the MSSM particles

\textsuperscript{#5}In Fig. 1, we require that all the soft SUSY breaking mass squared parameters for sleptons be positive to make the figure being independent of $F_\Phi$. Notice that such a constraint is slightly different from the actual experimental constraint based on the mass eigenvalues. (See Figs. 2 and 3.) Constraint on $M_{\text{mess}}$ is, however, almost unchanged.
Figure 2: Mass spectrum of $\tilde{\tau}_1$, $\tilde{\chi}^0_1$, $\tilde{\chi}^\pm_1$ and $\tilde{t}_1$, with $\tan \beta = 30$, $M_{\text{mess}} = 10^{12}$ GeV and $M_2 = 600$ GeV.

Figure 3: Same as Fig. 2, except $M_{\text{mess}} = 10^{14}$ GeV and $M_2 = 500$ GeV.
Figure 4: The SUSY contribution of the muon magnetic dipole moment $a_\mu = \frac{1}{2}(g_\mu - 2)$, for $M_{\text{mess}} = 10^{12}$ GeV and $M_2 = 600$ GeV.

Figure 5: Same as Fig. 4, except $M_{\text{mess}} = 10^{14}$ GeV and $M_2 = 500$ GeV.
is almost always a stau, the superpartner of the tau lepton. Note that in the model 1, the stau is not stable since the axino is the lightest superparticle and the stau decays to the axino. We will discuss phenomenological and cosmological implications to the stau decay. In addition, it is also notable that the slepton masses are more enhanced relative to the gauginos if we adopt larger value of $M_{mess}$.

In Table 2, we show the superparticle mass spectrum for some typical cases. As we mentioned, the lightest superparticle in the MSSM sector is the stau. The spectrum is rather compact, and squarks and gluinos are much lighter for the same stau mass than in the case of gauge mediation. Another interesting point is that the lightest neutralino is higgsino-like, which is not achieved in many scenarios of supersymmetry breaking, including the minimal supergravity model with all the SUSY particles being lighter than $\sim 1$ TeV. In fact, one sees that the lightest and the second lightest neutralinos and the lighter chargino are quite degenerate in mass, indicating that they are higgsino-like.

In the minimal anomaly mediation, the sign of the gluino mass is opposite to that of the Wino mass. This causes a potential conflict between the muon anomalous magnetic moment $a_{\mu}$ and $Br(b \to s\gamma)$ [15]. The result reported by the Brookhaven E821 experiment [18] suggests a deviation from the standard model prediction, indicating a new contribution from new physics. If this is real, the new physics should give a positive contribution to $a_{\mu}$, which constrains the sign of $\mu_H M_2 (> 0)$ in the context of SUSY. This choice also fixes the sign of $\mu_H M_3 (< 0$ in the minimal anomaly mediation). This sign plays an important role in the SUSY contributions to $b \to s\gamma$. With this choice, the charged Higgs loop and the chargino/stop loop give the contributions with the same sign, conflicting with the experimental constraints. One of the advantages of our scenario is that the gluino mass has the same sign as the Wino mass as far as $N_5 > 3$, and thus the two constraints can be simultaneously satisfied.

Figs. 4 and 5 depict the SUSY contribution to $a_{\mu}$ in the model 1 for $M_{mess} = 10^{12}$ and $10^{14}$ GeV. (For the formula for the SUSY contribution to $a_{\mu}$, see [19], and for recent works see [20].) In general, for a fixed value of of the gaugino mass, the stau mass decreases as $\tan \beta$ increases. Furthermore, as mentioned before, slepton mass relative to the gaugino mass becomes smaller as we adopt smaller value of the messenger scale. As a result, for relatively small $M_{mess}$, we obtain an upper bound on $\tan \beta$ to evade the experimental upper bound on the stau mass, unless the overall scale of the SUSY breaking is large enough. (End points of the lines in Fig. 4 correspond to such points.) One consequence of this fact is that, when the messenger scale is low, the SUSY contribution to the muon anomalous magnetic moment is constrained to be relatively small, in particular, compared to the deviation observed by the E821 experiment. Once the messenger scale is pushed up, however, the stau mass can be large enough and one finds that the requisite contribution of $O(10^{-9})$ is easily realized. Notice that, for $M_{mess} = 10^{14}$ GeV, $f_a = (10^{14} - 10^{15})$ GeV

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#6 The Higgs masses are computed by using the effective potential at one loop level. Higher order corrections will somewhat reduce the $h^0$ mass.

#7 In the focus point scenario of the supergravity model [13, 14], however, the lightest neutralino becomes higgsino-like [7].
is required since the parameter $\lambda$ has to be smaller than $O(0.1)$ in our model. (See Eq. (2.17).) For such a large value of $f_a$, energy density of the axion field becomes larger than the critical density if the standard evolution of the Universe is assumed [21]. For a possible cosmological scenario in this case, see the next subsection.

We also studied the superparticle mass spectrum for the model 2. A typical mass spectrum in this case is given in Table 2 as well. The lightest superparticle in the MSSM sector is again the stau. Because the axino is heavy in the model 2, the stau becomes the LSP in the whole theory. Since such a charged LSP is ruled out cosmologically [22], it must decay through $R$-parity violation. Furthermore the superparticle masses are somewhat spread compared to the previous case, which may cause the naturalness problem. Thus we conclude that the model 2 is less attractive than the model 1.

### 3.2 Other Issues

Let us return to the model 1, and consider the effects of the stau decay into the axino. The axion-multiplet coupling to the matter is suppressed by the decay constant $f_a$. Since we are considering the hadronic axion, the axion coupling to leptons vanishes at tree level. In fact, it arises at two-loop order, which originates from the anomalous coupling of the axion to two photons. Equivalently one can compute this coupling by using the superfield techniques. The axion coupling is then contained in the field dependent wave function renormalization of the lepton multiplet. The same technique enables us to compute the axino-lepton-slepton coupling in a simple matter as in Ref. [14]. Their result is given for the right-handed slepton

$$
\mathcal{L}_{\tilde{a} \tilde{l}} = C_l \frac{F_{\Phi}}{\langle X \rangle} \tilde{a} \tilde{l} \tilde{l}^\dagger + \text{h.c.},
$$

where

$$
C_l = \frac{1}{8\pi^2} \frac{2N_5(N_5 + 33/5)}{11} \left[ \alpha_2^2(M_{\text{mess}}) - \alpha_1^2(m_Z) \right].
$$

It follows from this that the lifetime of the stau is approximately

$$
\tau_{\tilde{\tau}} = N_5^2 \left( \langle X \rangle / 10^{13}\text{GeV} \right)^2 (200\text{GeV}/m_{\tilde{\tau}})^3 \text{sec}.
$$

Thus in collider experiments, the staus do not decay inside detectors, and they are practically regarded as stable particles. Thus highly ionized tracks produced by the heavy charged particles will be a signature of this scenario [23].

Cosmological implications of our model are also interesting since there exist various exotic particles in our model, like axion, axino, and saxino, which may affect evolution of the Universe. In particular, in the model 1, saxino may significantly affect the cosmology, contrary to the conventional scenario. Therefore, let us comment on this point.

In the model 1, there are two relatively light scalar fields, which are axion and saxino. The initial amplitude of these fields are in general displaced from the minimum of the
potential and they start to oscillate as the Universe expands. As a result, we should worry about cosmological difficulties possibly caused by these coherent oscillations. The effects of the axion field have been extensively studied.

The effects of the saxino is quite different from those of the axion, since in our model the saxino is much heavier than the axion. Consequently, the lifetime of the saxino becomes much shorter than that of axion, and hence the saxino may decay before the present epoch. The saxino \( \sigma \) dominantly decays into the axion pair, and the lifetime is calculated as

\[
\tau_{\sigma} = \left[ \frac{1}{64\pi} \frac{m_{\chi}^3}{\langle X \rangle^2} \right]^{-1} \simeq 10 \text{ sec} \times c_2^{-1} \lambda^{-1} \left( \frac{F_\phi}{100 \text{ TeV}} \right)^{-3} \left( \frac{N_5}{8} \right)^{-1/2},
\]

where we approximated that the mass of the saxino is comparable to \( m_X \). The lifetime is much longer than 1 sec as far as \( \lambda \ll 1 \) and \( F_\phi \lesssim 100 \text{ TeV} \). As a result, if the coherent oscillation exists in the early Universe, it decays after the big-bang nucleosynthesis (BBN) starts. Therefore, if the energy density of \( \sigma \) is too large at the time of the BBN, it spoils the great success of it. Assuming that the initial amplitude of the saxino field as \( \sim f_a \), the saxino energy density normalized by the entropy density is estimated as

\[
\frac{\rho_\sigma}{s} \sim \frac{m_{\chi}^{1/2} f_a^2}{M_*^{3/2}} \sim 1 \times 10^6 \text{ GeV} \times \lambda^{5/2} \left( \frac{F_\phi}{100 \text{ TeV}} \right)^{1/2} \left( \frac{N_5}{8} \right)^{-3/2}.
\]

Notice that this ratio is independent of time.

Important constraint on the saxino abundance is from the overproduction of \(^4\text{He}\). If the saxino energy density is too large at the time of the neutron decoupling, it boosts up the expansion rate of the Universe and increases the freeze-out temperature of the neutron. If this happens, the neutron number density after the freeze out becomes larger, resulting in an overproduction of \(^4\text{He}\). To avoid this problem, the ratio \( \rho_\sigma/s \) should be much smaller than the ratio \( \rho_{\text{rad}}/s \) at the time of the neutron decoupling, and we obtain an upper bound on \( \lambda \) of \( \sim O(10^{-4}) \), corresponding to \( M_{\text{mess}} \lesssim 10^9 \text{ GeV} \). As seen in Fig. 1, with the messenger scale lower than \( \sim 10^9 \) GeV, a large number of the messenger multiplet should be introduced in order to avoid the tachyonic sfermion mass, which makes the gauge coupling constants non-perturbative below the GUT scale. One might think that we may have a consistent scenario of cosmology if \( M_{\text{mess}} \sim 10^9 \) GeV. In this case, however, the lifetime of the saxino is as long as \( \sim 10^5 \) sec and hence the saxino decays after dominating the Universe. Since the saxino dominantly decays into the axions, energy density of the axion becomes 2 – 3 orders of magnitude larger than that of the radiation and hence there exists large extra energy density in the form of the relativistic matter (i.e., the axion). First, this fact changes the epoch of the radiation-matter equality to \( z \sim 10 \). In addition, the axion is a very weakly interacting particle and hence the cosmic density fluctuation for the scale which enters the horizon before the equality is washed out by the effect of the free-streaming. These effects cause a serious difficulty in the galaxy formation. Furthermore, some part of the saxino will decay into the pion pair through the QCD anomaly. Such pions dissociate the light elements produced by the BBN and spoil the great success of the
standard BBN. Thus, we should conclude that the scenario with $M_{\text{mess}} \sim 10^9$ GeV does not work. Therefore, if we assume the conventional scenario of cosmology, our model has a serious cosmological disaster.

This problem is, however, easily solved if a large amount of entropy is produced after the onset of the saxino oscillation. With a large entropy production, the energy density of the saxino field is diluted and the ratio $\rho_{\sigma}/s$ becomes much smaller than the value given in Eq. (3.5). In the anomaly-mediated models, the moduli fields may be heavy and they can become a natural source of the late-time entropy production. It is also important to point out that, if a large late-time entropy production occurs, axinos are also diluted. Thus, the axino relic density becomes much smaller than the present critical density even if the axinos are thermally produced in the early Universe. In addition, if the entropy production occurs after the QCD phase transition, coherent oscillation of the axion field is also diluted and the cosmological bound on the axion scale $f_a$ can be relaxed [24]. In this case, $f_a$ much larger than $10^{13}$ GeV becomes possible. This fact may be of some help if a relatively large value of the SUSY contribution to the muon magnetic moment is required, as mentioned in the previous subsection. It should be also noted that the relic staus are also diluted by the entropy production. Thus, the relic staus do not significantly affect the BBN even if their lifetime is longer than $\sim 1$ sec. The scenario with the late-time entropy production by the modulus decay may be tested by precisely measuring the cosmic microwave anisotropy since the decay of the modulus field may generate correlated mixture of the adiabatic and isocurvature fluctuations [25].

In the model 2, the axino and saxino are as heavy as $\sim O(F_\Phi)$. As a result, they are short-lived, and irrelevant in cosmology.

4 Summary

In this paper, we have considered hadronic axion models in the supersymmetric theory where SUSY breaking is mediated by the super-Weyl anomaly. SUSY breaking plays an important role in determining the VEV of the axion multiplet $X$ which spontaneously breaks $U(1)_{\text{PQ}}$ symmetry. We constructed three models in each of which the role of the SUSY breaking is different. The first model has no superpotential for $X$ and thus the scalar potential solely comes from SUSY breaking. In fact, the VEV of $X$ is determined by the balance between the two terms: one is the negative soft SUSY breaking mass generated by the super-Weyl anomaly, and the other is from the higher order term in the Kähler potential. In the second model, the PQ symmetry would not be broken in the absence of the SUSY breaking. Once the SUSY breaking is switched on, the potential minimum shift to the vacuum with the symmetry breaking. In the third model, the PQ symmetry is already broken in the exact SUSY limit, but the vacuum is degenerate along a non-compact flat direction. It is then the effect of the SUSY breaking that lifts the flat direction, thus fixing the PQ scale. In all three models, the PQ scale can be adjusted to the allowed range by appropriate choice of the coupling constants.
In the first two models, the PQ multiplet possesses non-trivial SUSY breaking, and thus it mediates SUSY breaking to the MSSM sector via a mechanism similar to the gauge mediation. Then soft SUSY breaking masses are deflected from those from the pure anomaly mediation. Our models provide a realization of the deflected anomaly mediation advocated in Ref. [10]. On the other hand, in the model 3 the $X$ field has trivial SUSY breaking, and thus it does not affect the mass spectrum of the superparticles in the MSSM. Therefore the sleptons remain tachyonic unless some other mechanism lifts up the slepton masses.

Phenomenological issues of the models 1 and 2 are briefly discussed. In these models, the stau becomes the lightest superparticle in the MSSM sector. In collider experiments, charged tracks due to the stau will be an important signature of the models. We also computed the SUSY contribution to the muon anomalous magnetic moment. We found that, despite relatively heavy Winos, the contribution from the SUSY sector can be comparable to or even larger than that from the weak interaction sector. Thus it can easily explain the possible discrepancy from the standard model prediction reported by the E821 experiment.

One of the crucial differences between the two models is the mass of the axino. In the model 1, the axino is light and becomes the lightest superparticle of the whole theory. In this case, the stau is no longer stable, but decays to the axino, with lifetime typically larger than 1 sec. In the model 2, the axino acquires mass at the tree level and thus it is as heavy as the gravitino. In this case, the stau will be the LSP of the whole sector and so we need $R$-parity violation to avoid the charged stable particle.

We also discussed cosmology of the models. In model 1, the saxino is light and long-lived. We argued that its coherent oscillation would spoil the success of the BBN. To avoid it, we invoke the late time entropy production, and thus cosmological evolution of the model 1 should differ from the standard thermal history of the Universe. On the other hand, in model 2, the saxino has a mass comparable to $F_\Phi$, and thus is heavy. Thus it is short-lived, and cosmologically harmless.

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