Dependence of the Star Formation Efficiency on the Parameters of Molecular Cloud Formation Simulations

Yetli Rosas-Guevara\textsuperscript{1} *, Enrique Vázquez-Semadeni\textsuperscript{1} †, Gilberto C. Gómez\textsuperscript{1} ‡, and A.-Katharina Jappsen\textsuperscript{2} §

\textsuperscript{1}Centro de Radioastronomía y Astrofísica, Universidad Nacional Autónoma de México, Apdo. Postal 3-72, Morelia, 58089, México
\textsuperscript{2}School of Physics & Astronomy, Cardiff University, Queens Buildings, The Parade, Cardiff CF24 3AA, UK

9 November 2009

\textbf{ABSTRACT}

We investigate the response of the star formation efficiency (SFE) to the main parameters of simulations of molecular cloud formation by the collision of warm diffuse medium (WNM) cylindrical streams, neglecting stellar feedback and magnetic fields. The parameters we vary are the Mach number of the inflow velocity of the streams, $M_{s,\text{inf}}$, the rms Mach number, $M_{s,\text{bgd}}$ of the initial background turbulence in the WNM, and the total mass contained in the colliding gas streams, $M_{\text{inf}}$. Because the SFE is a function of time, we define two estimators for it, the “absolute” SFE, measured at $t = 25$ Myr into the simulation’s evolution ($\text{SFE}_{\text{abs,25}}$), and the “relative” SFE, measured 5 Myr after the onset of star formation in each simulation ($\text{SFE}_{\text{rel,5}}$). The latter is close to the “star formation rate per free-fall time” for gas at $n = 100$ cm$^{-3}$. We find that both estimators decrease with increasing $M_{\text{inf}}$, although by no more than a factor of 2 as $M_{s,\text{inf}}$ increases from 1.25 to 3.5. Increasing levels of background turbulence (injected at scales comparable to the streams’ transverse radius) similarly reduce the SFE, because the turbulence disrupts the coherence of the colliding streams, fragmenting the cloud, and producing small-scale clumps scattered through the numerical box, which have low SFEs. Finally, the SFE is very sensitive to the mass of the inflows (at roughly constant density and temperature), with $\text{SFE}_{\text{rel,5}}$ decreasing from $\sim 0.4$ to $\sim 0.04$ as the mass in the colliding streams decreases from $\sim 2.3 \times 10^4 M_\odot$ to $\sim 600 M_\odot$ or, equivalently, the virial parameter $\alpha$ increases from $\sim 0.15$ to $\sim 1.5$. This trend is in partial agreement with the prediction
by Krumholz & McKee (2005), since the latter lies within the same range as the observed efficiencies, but with a significantly shallower slope. We conclude that the observed variability of the SFE is a highly sensitive function of the parameters of the cloud formation process, and may be the cause of significant scatter in observational determinations.

Key words: interstellar matter – stars: formation – turbulence

1 INTRODUCTION

The control of the star formation efficiency (SFE) by turbulence is a central issue in our present understanding of star formation, and currently a topic of intense study (see, e.g., the reviews by Mac Low & Klessen 2004; McKee & Ostriker 2007). In recent years, several groups have studied the SFE of molecular clouds (MCs) using numerical simulations of isothermal turbulence, in which the entire numerical box represents the interior of a molecular cloud (see, e.g., the reviews by Mac Low & Klessen 2004; Ballesteros-Paredes et al. 2007; Vázquez-Semadeni 2007). One confusing issue is that simulations of driven turbulence seem to indicate that the SFE decreases as the turbulent rms Mach number $M_s$ increases (e.g., Klessen, Heitsch & Mac Low 2000; Vázquez-Semadeni, Ballesteros-Paredes & Klessen 2003; Vázquez-Semadeni, Kim & Ballesteros-Paredes 2005), while simulations of decaying turbulence suggest that the SFE increases with increasing $M_s$ (Nakamura & Li 2005). This has prompted the question of how does turbulence actually originate and behave in real MCs. To answer this question, it has become necessary to investigate the entire evolutionary process of MCs.

The formation of MCs by collisions of warm neutral medium (WNM) streams has been intensely studied in recent years. A vast body of numerical simulations has shown that moderate, transonic compressions in the WNM can nonlinearly trigger a phase transition to the cold neutral medium (CNM) (e.g., Hennebelle & Pérault 1999; Koyama & Inutsuka 2000, 2002; Walder & Folini 2000), and that the dense gas produced by this mechanism is over-pressured with respect to the mean WNM thermal pressure (Vázquez-Semadeni et al. 2006) and turbulent, due to the combined action of Kelvin-Helmholz, thermal (Field 1965) and...
nonlinear thin-shell \cite{Vishniac1994} instabilities \cite{Heitsch2003,Heitsch2006}. The turbulence produced by this mechanism is continually driven for as long as the compression lasts.

The physical scenario of MC evolution was outlined by \cite{Hartmann2001}, who estimated the column densities necessary for the cloud to become self-gravitating, molecular, and magnetically supercritical, finding them to be comparable. The one-dimensional physical conditions in the dense gas were calculated analytically by \cite{Hennebelle1999} and \cite{Vazquez-Semadeni2006}. A recent review of the subject has been presented by \cite{Hennebelle2008}.

More recently, simulations including self-gravity and “sink particles”, which represent gravitationally collapsed objects (stars or stellar clusters), and using finite-duration compressions in the WNM, although lacking stellar feedback and magnetic fields, have been used to study the evolution of the turbulent motions and of the SFE in a self-consistent manner \cite{Vazquez-Semadeni2007}. Concerning the velocity dispersion in the clouds, these authors found that the turbulence is intermediate between driven and decaying, since what decays is the driving rate of the turbulence as the inflows weaken with time. However, they also found that the random motions are gradually replaced by global infall motions, as the cloud begins to contract gravitationally. Concerning the SFE, Paper I measured the masses of dense gas $M_{\text{dense}}$ and of the collapsed stellar objects $M_{\text{stars}}$ in the simulations, allowing a measurement of the SFE, defined as

$$\text{SFE} = \frac{M_{\text{stars}}}{M_{\text{dense}} + M_{\text{stars}}}. \quad (1)$$

The resulting SFE was however too high, reaching $\sim 50\%$ roughly 6 Myr after the time at which star formation (SF) had begun (denoted $t_{\text{SF}}$), although this excessive SFE can possibly be attributed to the neglect of stellar feedback in that simulation. Indeed, Paper I estimated, using a prescription by \cite{Franco1994} and a standard IMF \cite{Kroupa2001}, that by 3 Myr after $t_{\text{SF}}$, enough massive stars would have formed as to be able to destroy the cloud by ionization. At that point, the SFE was $\sim 15\%$, closer to the typical values $\lesssim 5\%$ reported observationally for full MC complexes \cite{Myers1986}.

One obvious possible reason for the relatively high values of the SFE obtained in this type of simulations is the neglect of stellar feedback and magnetic fields. However, it is also of interest to perform a more systematic investigation of the degree of variability that the SFE could exhibit within the scenario of Paper I, simply by varying the parameters of the WNM compressions triggering the formation of the cloud complex. In this paper we
undertake a first approach to such task, by varying three parameters of the WNM stream collisions modeled in the simulations. First, we consider the inflow speed \( v_{\text{inf}} \) and the velocity dispersion of the background turbulence initially present in the medium, both measured by their respective Mach numbers, \( M_{s,\text{inf}} \) and \( M_{s,\text{bgd}} \), with respect to the unperturbed WNM. Subsequently, we consider the mass in the colliding streams \( M_{\text{inf}} \), as determined by their radius \( R_{\text{inf}} \) and length \( l_{\text{inf}} \). Since the parameter space covered by these three parameters is already quite large, in this work we do not consider variations in the collision angle of the streams, instead having them collide head-on in all cases.

The paper is organized as follows. In §2 we describe the numerical model and experiments. In §3 we present our results, and in §4 we present a summary and our conclusions.

## 2 NUMERICAL MODEL AND EXPERIMENTS

For the numerical simulations, we use the same numerical setup as that used in Paper I, except that we now use the smoothed particle hydrodynamics (SPH) + N-body code Gadget-2 [Springel 2005] (in Paper I we used the previous version of the code, Gadget), modified to include random turbulence driving and sink particles according to the prescription of Jappsen et al. [2005], and including parameterized heating and cooling, as applied in Paper I, using the fit of Kovama & Inutsuka [2002] to a variety of atomic and molecular cooling processes. This cooling function causes the gas to be thermally unstable, under the isobaric mode, in the density range \( 1 \lesssim n \lesssim 10 \, \text{cm}^{-3} \). We assume that the gas is all atomic, with a mean atomic weight of 1.27. The numerical box is periodic, with size \( L_{\text{box}} \). In all cases, we use \( 118^3 = 1.64 \times 10^6 \) particles, and set the mean number of particles within a smoothing volume to 40. According to the criterion of Bate & Burkert [1997], the mass resolution is twice the number of particles within a smoothing volume, or \( \sim 80 \) times the mass per particle. The critical density for sink formation is set at \( 3.2 \times 10^7 \, \text{cm}^{-3} \), and the outer sink accretion radius is set at 0.04 pc.

An initial turbulent velocity field of one-dimensional velocity dispersion, characterized by its rms Mach number \( M_{s,\text{bgd}} \) and applied at scales between 1/4 and 1/8 of the box size, is added to the inflow velocity field, in order to trigger the instabilities that render the cloud turbulent. Note that this added turbulent velocity field is applied by turning on the random driver for the first few timesteps of the simulation’s evolution, and that what we actually

---

1 Our “molecular clouds” are thus only so in the sense of density and temperature, but not of chemical composition.
control is the energy injection rate parameter. Thus, simulations intended to have the same
turbulence strength do so only approximately, as the flow’s response is slightly different in
every realization.

The initial conditions consist of a uniform medium at \( n = 1 \text{ cm}^{-3} \) and \( T = 5000 \text{ K} \),
in which two cylinders of length \( l_{\text{inf}} \) along the \( x \) direction and radius \( R_{\text{inf}} \) are set to collide
head-on at the \( x = L_{\text{box}}/2 \) plane of the simulation (refer to Fig. 1 of Paper I). Note that the
cylindrical inflows are entirely contained within the numerical box, since the boundaries are
periodic. The length \( l_{\text{inf}} \) is measured from the central collision plane, and is always shorter
than the half-length of the box, implying that a small region between the edge of the inflows
and the box boundaries is not given any velocity. This region is partially evacuated during
the subsequent evolution of the simulations, as the gas within it tends to fill the void left by
the inflows.

Table 1 shows the various runs we performed for the present study, indicating the relevant
parameters for each one. In addition to the inflow length, radius, and velocity, defined above,
Table 1 gives the Mach number \( M_{s,\text{inf}} \) corresponding to \( v_{\text{inf}} \) at the initial temperature of the
gas, for which the adiabatic sound speed is \( 7.54 \text{ km s}^{-1} \), the one-dimensional rms Mach
number of the initial turbulent motions, \( M_{s,\text{bgd}} \), the total mass contained in the simulation
box, \( M_{\text{box}} \), the mass contained in the two inflows, \( M_{\text{inf}} \), and the mass per SPH particle,
\( M_{\text{part}} \). The runs are labeled mnemonically, with their names giving, in that order, the inflow
Mach number \( M_{s,\text{inf}} \), the background Mach number \( M_{s,\text{bgd}} \), and the inflow mass, \( M_{\text{inf}} \).

Note that run Mi1-Mb.02-Ma2e4 has the same parameters as run L256\( \Delta v0.17 \) from
Paper I, and we take these as the “fiducial” set of parameters. Note, however, that these two
runs are not identical because they were performed with different codes. Gadget-2 differs
in many ways from Gadget, and in particular it takes longer timesteps, implying that the
initial forcing (used to trigger the instabilities in the dense layer) is applied at different
time intervals, and with different random seeds. Thus, the two runs are similar only in a
statistical sense, but are not identical. For reference, in Fig. 11 we show a face-on view of
Mi1-Mb.02-Ma2e4 at the time when it is beginning to form stars, showing that the general
morphology it develops is similar to that of run L256\( \Delta v0.17 \) from Paper I (compare to Fig.
4 of that paper, noting that the linear scales shown are different in each figure).

We report the SFE as defined by eq. (1), with the “dense” gas defined as that with
number density \( n \geq 100 \text{ cm}^{-3} \). However, because the SFE is actually a function of time, and
sink formation begins at different times in different runs, in order to report a number for the
SFE, we estimate it in two different ways. One is to measure the “absolute” SFE 25 Myr after the start of the simulation, which we denote as \( \text{SFE}_{\text{abs,25}} \). The other is to measure the “relative” SFE, which we define as the SFE 5 Myr after the onset of star formation in the simulation, and which we denote as \( \text{SFE}_{\text{rel,5}} \). This is actually close to the “star formation rate per free-fall time”, \( \text{SFR}_{\text{ff}} \) (after SF has begun), as defined by Krumholz & McKee (2005), since the free-fall time for gas at \( n \sim 100 \text{ cm}^{-3} \) is \( \sim 4.6 \) Myr.

3 RESULTS

3.1 SFE vs. inflow velocity

In this section, we consider the dependence of the SFE on the inflow velocity of the colliding streams, \( v_{\text{inf}} \). Figure 2 shows the evolution of the dense gas mass (top panel) and the sink mass (bottom panel) for runs Mi1-Mb.02-Ma2e4, Mi2-Mb.02-Ma2e4, and Mi3-Mb.02-Ma2e4.
Star Formation Efficiency in Simulations of Molecular Cloud Formation

Figure 2. Evolution of the dense gas mass (left panel) and sink mass (right panel) for runs Mi1-Mb.02-Ma2e4, Mi2-Mb.02-Ma2e4, and Mi3-Mb.02-Ma2e4, which differ only by the Mach number of the inflows (indicated by the “Mi#” entry in the run’s name), having $M_{s,\text{inf}} = 1.25, 2.5$ and $3.5$ respectively.

These runs have all parameters equal, except for the speed of the inflows (cf. Table I), which are varied from $M_{s,\text{inf}} = 1.25$ in Mi1-Mb.02-Ma2e4 to $M_{s,\text{inf}} = 3.5$ in Mi3-Mb.02-Ma2e4. Note that the time at which the runs begin to form sinks, $t_{\text{SF}}$, is different in each case. The top panels of Fig. 3 then show the SFE, defined as in eq. (1). The top left panel shows $\text{SFE}_{\text{abs,25}}$, i.e., starting from the beginning of the simulation, and up to a total time of 25 Myr. The top right panel shows the SFE starting from the time at which sink formation begins in each run, allowing one to read off $\text{SFE}_{\text{rel,5}}$.

From the top right panel of Fig. 3 we see that the mean slope of the curve $\text{SFE}(t)$, denoted $\langle \dot{\text{SFE}} \rangle$, decreases, although moderately, with increasing $M_{s,\text{inf}}$. It is interesting, however, that all three runs form stars at roughly the same rate (i.e., the slopes of the curves in the right panel of Fig. 2, which give the mass accretion rate onto the sinks, and which we identify with the star formation rate, SFR, are all similar). The decrease in $\langle \dot{\text{SFE}} \rangle$ is thus due to the larger mass growth rate of the cloud induced by the larger inflow velocities (Fig. 2, top), not to a smaller SFR.

The larger-mass clouds appear to be incapable of forming stars at a proportionally larger rate because the larger inflow velocity produces larger turbulent velocity dispersions $v_{\text{rms}}$ in the clouds, which have been shown to reduce the SFR in simulations of driven turbulence (e.g., Klessen, Heitsch & Mac Low 2000; Vázquez-Semadeni, Ballesteros-Paredes & Klessen 2003; Mac Low & Klessen 2004; Vázquez-Semadeni, Kim & Ballesteros-Paredes 2005). This is illustrated in the bottom panel of Fig. 3 which shows the evolution of the one-dimensional

---

$^2$ Note that, because the cloud’s mass is in general not constant in time, eq. (1) implies that $M_{\text{dense}} \times \langle \dot{\text{SFE}} \rangle \neq \text{SFR}$.

$^3$ Note that this velocity dispersion, which is the response of the flow in its dense regions to the collision of the inflows, is different from the initial background turbulent velocity dispersion applied to the runs, measured by the parameter $M_{s,\text{bgd}}$. 
Figure 3. Top panels: Evolution of the “absolute” SFE ($\text{SFE}_{\text{abs},25}$, left), shown out to 25 Myr after the start of the runs, and the “relative” SFE (right), shown from the onset of sink formation, for runs Mi1-Mb.02-Ma2e4, Mi2-Mb.02-Ma2e4, and Mi3-Mb.02-Ma2e4. $\text{SFE}_{\text{rel},5}$ is the value of this curve at a relative time of 5 Myr. Bottom panel: Evolution of the one-dimensional velocity dispersion perpendicular to the direction of the inflows for the same three runs.

$v_{\text{rms}}$ perpendicular to the direction of the inflows for the same three runs\(^4\). The trends of $\text{SFE}_{\text{abs},25}$, $\text{SFE}_{\text{rel},5}$, and $v_{\text{rms}}$ with the inflow speed are summarized in Fig. 4. In this figure, $v_{\text{rms}}$ is measured for Run Mi1-Mb.02-Ma2e4 at the time when it exhibits a minimum in the bottom panel of Fig. 3, i.e., $t = 8$ Myr. This is equivalent to $\sim 2/3 \, t_{\text{cros}}$, where $t_{\text{cros}} \equiv l_{\text{inf}} / v_{\text{inf}}$ is the inflow crossing time, the time taken by the tail of the inflowing cylinders to reach the collision plane. We do this in an attempt to capture the true velocity dispersion in the cloud after the initial transients have ended, but before global collapse sets in. The latter is indicated by the smooth rise in $v_{\text{rms}}$ for this run during the time interval $9 \lesssim t \lesssim 23$ Myr. Then, for consistency, we measure $v_{\text{rms}}$ for the other two runs also at $2/3$ of their own $t_{\text{cros}}$.

\(^4\) Note that the velocity dispersions in this figure are significantly smaller than those reported in Figs. 5 (bottom panel) and 9 of Paper I. In that paper, those figures suffered from a typographical error, having the velocities erroneously multiplied by one too many factors of the velocity unit, $v_0 = 7.362$ km s\(^{-1}\). Thus, the correct velocities in those figures are obtained by dividing by this factor.
3.2 SFE vs. background turbulence strength

We now consider the response of the SFE to the amplitude of the initial turbulent velocity field. Note that this field was not originally intended to produce density condensations on its own, but just to sufficiently disorganize the inflow velocity field as to trigger the instabilities that render the cloud turbulent. However, in the cases of stronger turbulence, we do observe clump formation everywhere in the box as a result of the initial background turbulence, and not just at the collision site of the inflows.

Figure 5 shows the evolution of the SFE for runs Mi1-Mb.11-Ma2e4, Mi1-Mb.27-Ma2e4, Mi1-Mb.02-Ma2e4 and Mi0-Mb.10-Ma2e4. The first three runs differ only in the strength of the initial turbulence, measured by \(M_{s,\text{bgd}}\) (cf. Table I). The last run has nearly the same value of \(M_{s,\text{bgd}}\) as Mi1-Mb.11-Ma2e4, but with no inflow velocity, in order to assess the amount of star formation induced solely by the turbulent field, in the absence of colliding streams.

From these figures, we see that \(t_{\text{SF}}\) becomes longer and \(\langle \text{SFE} \rangle\) becomes smaller as larger
Figure 5. Evolution of the “absolute” SFE (SFE$_{abs,25}$, left panel) and the “relative” SFE (right panel) for four runs, characterized by various values of the initial background turbulent Mach number $M_{s,bgd}$, indicated by the entry “Mb.##” in the runs’ names. Three of the runs have the same inflow Mach number, $M_{inf} = 1.25$, while the fourth run has no inflows ($M_{inf} = 0$), in order to assess the SFE due exclusively to the initial background turbulence.

values of $M_{s,bgd}$ are considered. This appears to be due to the stronger fragmentation induced by the turbulent velocity field, which in the extreme case of Mi1-Mb.27-Ma2e4 almost obliterates the inflows, and produces scattered clumps throughout the simulation box, with very little remaining of the cloud formed by the inflows, as illustrated in Fig. 6, left panel.

It is also worth noting that run Mi0-Mb.10-Ma2e4, which has no inflows, has a much larger $t_{SF}$ and a lower $\langle SFE \rangle$ than run Mi1-Mb.11-Ma2e4, which differs from the former only in the presence of the inflows. As shown in Fig. 6, this run also produces scattered clumps throughout the numerical box, although not as profusely as Mi1-Mb.27-Ma2e4. So, we conclude that sink formation is still dominated by the colliding streams in Mi1-Mb.11-Ma2e4, although a small fraction of the sinks is contributed by the global turbulence.

Figure 7 summarises the results of this section. A clear trend of a decreasing SFE (seen in both SFE$_{abs,25}$ and SFE$_{rel,5}$) with increasing $M_{s,bgd}$ is seen, which we interpret as a result of the reduction of the fragment mass with increasing turbulence strength (at constant total mass; Ballesteros-Paredes et al. 2006), and of the fact that smaller-mass fragments tend to have smaller SFEs (3.3).

3.3 SFE vs. inflow mass

The last dependence of the SFE we analyse is on the mass content of the colliding inflows. So, we consider inflows of various radii. However, since a very narrow inflow is necessarily more poorly resolved, we consider smaller simulation boxes in two of the cases, in order to better resolve the resulting clouds. Specifically, as shown in Table 1, $R_{inf}$ in run Mi1-Mb.02-
Ma5e3 is half that in Mi1-Mb.02-Ma2e4. Run Mi1-Mb.06-Ma2e3 has $R_{\text{inf}}$ equal to that of Mi1-Mb.02-Ma5e3, but half the length, since the numerical box size of the former is half that of the latter. Finally, Mi1-Mb.06-Ma6e2 has $R_{\text{inf}}$ equal to half that of Mi1-Mb.06-Ma2e3. So, the total mass contained in the inflows of runs Mi1-Mb.02-Ma2e4, Mi1-Mb.02-Ma5e3, Mi1-Mb.06-Ma2e3 and Mi1-Mb.06-Ma6e2 is, respectively, $2.26 \times 10^4$, $5.64 \times 10^3$, $2.42 \times 10^3$, and $6.04 \times 10^2 M_\odot$.

Figure 8 shows the evolution of $\text{SFE}_{\text{abs},25}$ and $\text{SFE}_{\text{rel},5}$ for these runs. We see that there is a general trend for the SFE, in both its forms, to increase with the total mass involved in the stream collision. There is only a reversal to this trend in the relative SFE between runs Mi1-Mb.02-Ma2e4 and Mi1-Mb.02-Ma5e3 because the latter has a large early maximum of $\text{SFE}_{\text{rel},5}$, although later it decreases, in a period of mass accumulation in the cloud at low SFR. Other than that, the trend is general, as shown in Fig. 9.

It is important to note that for all runs in this series we used the same energy injection rate of the turbulence driver. However, runs Mi1-Mb.06-Ma6e2 and Mi1-Mb.06-Ma2e3, performed in a smaller computational box, have an initial, background rms turbulent Mach number that is roughly 2.5 times larger than that of runs Mi1-Mb.02-Ma2e4 and Mi1-Mb.02-Ma5e3. This may additionally reduce the SFE because of the additional fragmentation it produces, but we see that the trend of the SFE to decrease with decreasing inflow mass holds generally even at the same physical box size, so the result appears robust.

The trend discussed above can be put in the context of the theory of Krumholz & McKee.
Figure 7. Dependence of $\text{SFE}_{\text{abs}, 25}$ and $\text{SFE}_{\text{rel}, 5}$ on the rms Mach number, $M_{\text{s,bgd}}$, of the initial (background) turbulent velocity perturbations. Both indicators are seen to decrease with increasing $M_{\text{s,bgd}}$ as a consequence of the progressively stronger fragmentation induced by the turbulence.

Figure 8. Evolution of the “absolute” SFE ($\text{SFE}_{\text{abs}, 25}$, left panel) and the “relative” SFE (right panel) for runs Mi1-Mb.02-Ma2e4, Mi1-Mb.02-Ma5e3, Mi1-Mb.06-Ma2e3, and Mi1-Mb.06-Ma6e2. The radius of the cylindrical inflows for these runs is respectively 32, 16, 16, and 8 pc. Runs Mi1-Mb.02-Ma2e4 and Mi1-Mb.02-Ma5e3 are performed in a 256-pc box, with inflow length 112 pc, while runs Mi1-Mb.06-Ma6e2 and Mi1-Mb.06-Ma2e3 are performed in a 128-pc box with the same number of SPH particles (thus being better resolved) and a 48-pc inflow length.
Figure 9. Dependence of SFE_{abs,25} and SFE_{rel,5} on the inflows’ mass. A general trend for the SFE to increase with inflow mass is observed.

(2005, hereafter KM05) for the SFR_{ff}. This theory predicts a dependence of the SFR_{ff} on the virial parameter \( \alpha \) and the rms Mach number \( M_s \). Here,

\[
\alpha \equiv 2E_{\text{kin}}/|E_{\text{grav}}|,
\]

(2)

where \( E_{\text{kin}} = M \Delta v^2/2 \) is the cloud’s kinetic energy, \( M \) is the cloud’s mass, \( \Delta v \) is its rms turbulent velocity dispersion, and \( E_{\text{grav}} \) is the cloud’s gravitational energy.

For our flattened clouds, we compute the gravitational energy assuming they can be approximated as infinitely thin, uniform disks of radius \( R \), and write

\[
E_{\text{grav}} = \int_A \Sigma \phi \, d^2x = 2\pi \Sigma \int_0^R r\phi(r)\,dr,
\]

(3)

where \( A \) is the area of the disk, \( \Sigma \) is the (uniform) surface density, and \( \phi \) is the gravitational potential. In our case, the latter is given by \cite{Wyse1942, Burkert2004}

\[
\phi(r) = -4G\Sigma RE(r/R),
\]

(4)
where $E$ is the second complete elliptic integral. Thus, the gravitational energy is

$$E_{\text{grav}} = -8\pi G\Sigma^2 R \int_0^R rE(r/R)dr = -8\pi G\Sigma^2 R^3 \int_0^1 xE(x)dx$$

$$= -8\pi \left(\frac{28}{45}\right) G\Sigma^2 R^3. \quad (5)$$

To compute the kinetic energy of the clouds, we note that the relevant velocity dispersion is the one produced in the clouds as a consequence of the inflow collision (Heitsch et al. 2005; Vázquez-Semadeni et al. 2006), rather than the initial background turbulent velocity, which is much smaller. Since all four simulations analysed in this section have the same $v_{\text{inf}}$, we use the value of $v_{\text{rms}}$ measured for Run Mi1-Mb.02-Ma2e4 in § 3.1 (Fig. 4) for all of them, namely $v_{\text{rms}} = 0.5$ km s$^{-1}$. Noting that this value is a one-dimensional velocity dispersion, we take $\Delta v = \sqrt{3} v_{\text{rms}}$.

We finally obtain, from equations (2) and (5),

$$\alpha = \frac{224\pi R\Delta v^2}{45GM}. \quad (6)$$

Figure 10 shows the results of this exercise. The solid line shows the simulation data, while the straight dotted line shows a least squares fit to them. The dashed line shows the result from Krumholz & McKee (2005), given by

$$\text{SFR}_{\text{ff}} \approx 0.014 \left(\frac{\alpha}{1.3}\right)^{-0.68} \left(\frac{M_s}{100}\right)^{-0.32}, \quad (7)$$

where we have taken $M_s = \Delta v/c_s$. We see that the prediction by KM05, although being numerically within the same range as the data, exhibits a significantly shallower slope. Specifically, the fit to our data has a slope $-1.12 \pm 0.37$, where the uncertainty is the 1σ error of the fit, while the slope of the KM05 prediction, $-0.68$, lies beyond this error. We discuss this result further in § 4.

4 SUMMARY AND DISCUSSION

In this paper, we have considered the scenario of molecular cloud formation by WNM stream collisions, and investigated the dependence of the SFE on three parameters of this scenario, namely the inflow speed, the rms Mach number of the background medium, and the total mass contained in the inflows. Since the SFE, defined as in eq. (1), is a time-dependent function because the cloud continues to accrete mass from the WNM while it forms stars, we have considered two estimators of its time integral, namely the absolute SFE after 25
Figure 10. Dependence of SFE$_{rel,5}$ on the virial parameter $\alpha$. The straight dotted line shows a least squares fit to the simulation data, with slope $-1.12$, while the dashed line shows the result from Krumholz & McKee (2005), with slope $-0.68$.

Myr from the start of the simulation, SFE$_{abs,25}$, and the “relative” SFE, 5 Myr after the onset of SF in the cloud, denoted SFE$_{rel,5}$.

We have found a wide range of values of these estimators as we vary the parameters of the simulations. In general, the SFE decreases, although moderately, with increasing inflow velocity. In particular, SFE$_{rel,5}$ decreases from $\sim 0.4$ to $\sim 0.2$ as $M_{s,inf}$ increases from 1.25 to 3.5. Note that the runs in this case have similar SFRs, and the decrease in the SFE and $\langle SFE \rangle$ is due to the faster increase in cloud mass rather than to a decrease in the SFR. That the SFR is similar in all three runs, in spite of the larger gas mass is probably due to the larger turbulent velocity dispersion in the dense gas caused by the larger inflow speed, which tends to inhibit the SFR, thus compensating the tendency to have a larger SFR due to the larger cloud masses.

Similarly, the SFE decreases with increasing background turbulence strength, as the latter progressively takes a dominant role in the production of the dense gas but, due to the relatively small scales at which the turbulence is excited, the clouds and clumps formed by
it are significantly smaller than the cloud formed by the coherent stream collision. In this case, $\text{SFE}_{\text{rel,5}}$ decreases from $\sim 0.4$ to $\sim 0.03$ as the rms Mach number of the background turbulence increases from $\sim 0.02$ to $\sim 0.3$.

Finally, the SFE in general decreases with decreasing mass of the inflows (at constant $v_{\text{inf}}$). This may be a consequence of the fact that clouds formed by the collision of our inflows always have roughly the same density, temperature, and velocity dispersion, so smaller clouds are more weakly gravitationally bound, a condition known to decrease the SFE (Clark et al. 2005). The end result is that $\text{SFE}_{\text{rel,5}}$ decreases from $\sim 0.4$ to $\sim 0.03$ as the mass in the colliding streams decreases from $\sim 2.3 \times 10^4 M_\odot$ to $\sim 600 M_\odot$. It is important to stress that this result is not at odds with the well known fact that the SFE increases as the object mass decreases from the mass scale of a giant molecular cloud ($M \sim 10^4 - 10^6 M_\odot$, $\text{SFE} \sim 0.02$; Myers et al. 1986) to that of a cluster-forming core ($M \sim 10^3 M_\odot$; SFE $\sim 0.3 - 0.5$, Lada & Lada 2003), because in this case the cores’ mean densities are much larger than those of the GMCs, while in our case the mean densities of the various clouds are always comparable. Thus, our clouds do not conform to Larson’s (1981) density-size scaling.

The latter results, expressed in terms of the virial parameter $\alpha$, exhibit partial agreement with the prediction by KM05 for the dependence of the SFE after a free-fall time (what those authors called the star formation rate per free-fall time, or SFR$_{ff}$), which is directly comparable to our $\text{SFE}_{\text{rel,5}}$. Although their prediction, without any rescaling, lies in the same range of values as our observed efficiencies, it contains a much shallower dependence on $\alpha$ than we observe. This may be due to the fact that those authors assumed that the clouds were supported by turbulent pressure, possibly provided by stellar feedback, while our simulations lack such support. However, the notion of turbulent support has been questioned recently by Vázquez-Semadeni et al. (2008), and the low observed efficiency may be the result of the dispersal (rather than support) of the parent cloud by its stellar products (Hartmann, Ballesteros-Paredes & Bergin 2001). More work is needed to determine the causes of the discrepancy. However, it is noteworthy that Run Mi1-Mb.06-Ma2e3, which has a value of $\alpha$ closest to unity, has a reasonably realistic value of $\text{SFE}_{\text{rel,5}} \sim 4\%$.

We conclude that the SFE, even in the absence of further agents such as magnetic fields and stellar feedback, is a highly sensitive function of the parameters of the cloud formation process, and may be responsible for significant intrinsic scatter in observational determinations of the SFE.
ACKNOWLEDGMENTS

The numerical simulations were performed in the cluster at CRyA-UNAM acquired with CONACYT grants 36571-E and 47366-F to E.V.-S. G.C.G. acknowledges financial support from grants IN106809 (UNAM-PAPIIT) and J50402-F (CONACYT). A.-K.J. acknowledges support by the Human Resources and Mobility Programme of the European Community under the contract MEIF-CT-2006-039569.

REFERENCES

Ballesteros-Paredes, J., Gazol, A., Kim, J., Klessen, R. S., Jappsen, A.-K., & Tejero, E. 2006, ApJ, 637, 384
Ballesteros-Paredes, J., Klessen, R. S., Mac Low, M.-M., & Vazquez-Semadeni, E. 2007, Protostars and Planets V, 63
Bate, M. R., & Burkert, A. 1997, MNRAS, 288, 1060
Burkert, A., & Hartmann, L. 2004, ApJ, 616, 288
Clark, P. C., Bonnell, I. A., Zinnecker, H., & Bate, M. R. 2005, MNRAS, 359, 809
Field, G. B., 1965, ApJ, 142, 531
Franco, J. Shore, S. N., & Tenorio-Tagle, G. 1994, ApJ, 436, 795
Hartmann, L., Ballesteros-Paredes, J., & Bergin, E. A. 2001, ApJ, 562, 852
Heitsch, F., Burkert, A., Hartmann, L., Slyz, A. D. & Devriendt, J. E. G. 2005, ApJ, 633, L113
Heitsch, F., Slyz, A., Devriendt, J., Hartmann, L., & Burkert, A. 2006, ApJ, 648, 1052
Heitsch F., Hartmann L. W., Slyz A. D., Devriendt J. E. G., Burkert A., 2008, ApJ, 674, 316
Hennebelle, P., Pérault, M., 1999, A&A, 351, 309
Hennebelle P., Banerjee R., Vázquez-Semadeni E., Klessen R., Audit E., 2008, A&A, 486, L43
Hennebelle, P., Mac Low, M.-M., & Vázquez-Semadeni 2007, in Structure Formation in the Universe: Galaxies, Stars, Planets, ed. G. Chabrier (Cambridge: Cambridge University Press), in press (arXiv:0711.2417)
Jappsen, A.-K., Klessen, R. S., Larson, R. B., Li, Y., and Mac Low, M.-M. 2005, A&A, 435, 611
Klessen, R. S., Heitsch, F., & MacLow, M. M. 2000, ApJ, 535, 887
Koyama, H. & Inutsuka, S.-I. 2000, ApJ, 532, 980
Koyama, H. & Inutsuka, S.-I. 2002, ApJ, 564, L97
Kroupa, P. 2001, MNRAS, 322, 231
Krumholz, M. R., & McKee, C. F. 2005, ApJ, 630, 250 (KM05)
Heitsch, F., & Hartmann, L. 2008, ApJ, 689, 290
Lada, C. J., & Lada, E. A. 2003, ARAA, 41, 57
Larson, R. B. 1981, MNRAS, 194, 809
Mac Low, M.-M., & Klessen, R. S. 2004, Rev. Mod. Phys., 76, 125
McKee, C. F., & Ostriker, E. C. 2007, ARAA, 45, 565
Myers, P. C., Dame, T. M., Thaddeus, P., Cohen, R. S., Silverberg, R. F., Dwek, E. & Hauser, M. G. 1986, ApJ, 301, 398
Nakamura, F., & Li, Z.-Y. 2005, ApJ, 631, 411
Springel, V. 2005, MNRAS, 364, 1105
Vázquez-Semadeni, E. 2007, in Triggered Star Formation in a Turbulent ISM, eds. B. G. Elmegreen & J. Palous (Cambridge: Cambridge Univ. Press), 292
Vázquez-Semadeni, E., Ballesteros-Paredes, J. & Klessen, R. 2003, ApJ, 585, L131
Vázquez-Semadeni, E., Kim, J. & Ballesteros-Paredes, J. 2005, ApJ, 630, L49
Vázquez-Semadeni, E., Ryu, D., Passot, T., González, R. F., & Gazol, A., 2006, ApJ, 643, 245
Vázquez-Semadeni, E., Gómez, G. C., Jappsen, A. K., Ballesteros-Paredes, J., González, R. F., & Klessen, R. S. 2007, ApJ, 657, 870 (Paper I)
Vázquez-Semadeni, E., González, R. F., Ballesteros-Paredes, J., Gazol, A., & Kim, J. 2008, MNRAS, 390, 769
Vishniac, E. T. 1994, ApJ, 428, 186
Walder, R. & Folini, D. 2000, ApSS, 274, 343
Wyse, A. B., & Mayall, N. U. 1942, ApJ, 95, 24
| Run number | Run name       | $L_{\text{box}}$ [pc] | $l_{\text{inf}}$ [pc] | $v_{\text{inf}}$ [km s$^{-1}$] | $M_{\text{s,inf}}$ | $M_{\text{s,bgd}}$ | $R_{\text{inf}}$ [pc] | $M_{\text{box}}$ [$M_{\odot}$] | $M_{\text{inf}}$ [$M_{\odot}$] | $M_{\text{part}}$ [$M_{\odot}$] |
|------------|---------------|------------------------|------------------------|-------------------------------|-------------------|-------------------|------------------------|--------------------------|------------------------|-------------------------|
| 1          | Mi1-Mb.11-Ma2e4 | 256                    | 112                    | 9.20                          | 1.25              | 0.11              | 32                     | $5.25 \times 10^5$     | $2.26 \times 10^4$     | 0.32                    |
| 2          | Mi2-Mb.11-Ma2e4 | 256                    | 112                    | 18.41                         | 2.50              | 0.11              | 32                     | $5.25 \times 10^5$     | $2.26 \times 10^4$     | 0.32                    |
| 3          | Mi1-Mb.27-Ma2e4 | 256                    | 112                    | 9.20                          | 1.25              | 0.27              | 32                     | $5.25 \times 10^5$     | $2.26 \times 10^4$     | 0.32                    |
| 4          | Mi1-Mb.02-Ma2e4 | 256                    | 112                    | 9.20                          | 1.25              | 0.021             | 32                     | $5.25 \times 10^5$     | $2.26 \times 10^4$     | 0.32                    |
| 5          | Mi2-Mb.02-Ma2e4 | 256                    | 112                    | 18.41                         | 2.50              | 0.024             | 32                     | $5.25 \times 10^5$     | $2.26 \times 10^4$     | 0.32                    |
| 6          | Mi3-Mb.02-Ma2e4 | 256                    | 112                    | 25.77                         | 3.50              | 0.025             | 32                     | $5.25 \times 10^5$     | $2.26 \times 10^4$     | 0.32                    |
| 7          | Mi1-Mb.02-Ma5e3 | 256                    | 112                    | 9.20                          | 1.25              | 0.020             | 16                     | $5.25 \times 10^5$     | $5.64 \times 10^3$     | 0.32                    |
| 10         | Mi1-Mb.06-Ma6e2 | 128                    | 48                     | 9.20                          | 1.25              | 0.057             | 8                      | $6.57 \times 10^4$     | $6.04 \times 10^2$     | 0.04                    |
| 11         | Mi0-Mb.10-Ma2e4 | 256                    | 112                    | 0.                            | 0.                | 0.10              | 32                     | $5.25 \times 10^5$     | $2.26 \times 10^4$     | 0.32                    |
| 12         | Mi1-Mb.06-Ma2e3 | 128                    | 48                     | 9.20                          | 1.25              | 0.058             | 16                     | $6.57 \times 10^4$     | $2.42 \times 10^3$     | 0.04                    |