High-energy spectra of the atmospheric neutrinos: predictions and measurements

A. A. Kochanov, A. D. Morozova, T. S. Sinegovskaya, and S. I. Sinegovsky

1 Institute of Solar-Terrestrial Physics, Siberian Branch, Russian Academy of Sciences, RU-664033, Irkutsk, Russia
2 Irkutsk State University, RU-664003 Irkutsk, Russia
3 Dzelepev Laboratory of Nuclear Problems, Joint Institute for Nuclear Research, RU-141980 Dubna, Russia
4 Irkutsk State Transport University, RU-664074, Irkutsk, Russia
5 Dzelepev Laboratory of Nuclear Problems, Joint Institute for Nuclear Research, RU-141980 Dubna, Russia

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Statistical analysis is performed of the atmospheric neutrino flux models as compared with the data of Frejus, AMANDA-II, IceCube, ANTARES, and Super-Kamiokande experiments. The main objective is to characterize models of hadron-nucleus interactions from the point of view of the statistical significance of the atmospheric neutrino flux predictions compared to the measurements.

The flux calculations were performed in the framework of the single computational scheme involving a set of hadronic models combined with parameterizations of the primary cosmic rays spectrum by Hillas & Gaisser and Zatsepin & Sokolskaya. The analysis showed satisfactory agreement of the conventional \( \nu_{\mu} \) flux calculations with the measurements. The prompt neutrinos contribution obtained with a set of charm production models (QGSM, SIBYLL 2.3c, PROSA, GRRST, BEJKRSS, and GM-VFNS) is statistically negligible in the energy range covered by the neutrino telescopes.

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I. INTRODUCTION

Decays of pions, kaons, and charm particles produced in cosmic rays interactions with the Earth atmosphere generate high-energy neutrinos which form an unavoidable background for detecting astrophysical neutrinos. To date, the atmospheric muon and the electron neutrino spectra are measured in Frejus [1], AMANDA-II [2, 3], ANTARES [4, 5], IceCube [6–11], and Super-Kamiokande [12] experiments within the energy range \( \sim 10 \) GeV through 600 TeV.

By the time, when detector AMANDA at the South Pole was constructed, the Monte Carlo calculations of the atmospheric neutrinos (AN) spectra had been performed for the neutrino energies of no more than 10 TeV [13–15]. Later on, these results were used to reconstruct the events in the IceCube [8] and Super-Kamiokande experiments.

The reference model for the AN spectra in IceCube is based on the Monte Carlo calculation (for \( E \leq 10 \) TeV) [14]. This model was extrapolated to energies beyond 10 TeV, taking into consideration the knee of the primary cosmic ray spectrum with normalizing corrections depending on the energy [9]. Thus, there is a necessity of a consistent scheme for AN spectra calculations over a wide neutrino energy range. The scheme validation should be done via a thorough comparison with the experimental data.

Here we apply the Z(\( E, h \)) method [16–19], developed to solve the high-energy atmospheric hadron cascade equations and to compute the atmospheric muon and neutrino fluxes. The Z(\( E, h \)) method enables us to compute atmospheric fluxes of hadrons, muons, and neutrinos for a non-power-law spectrum of cosmic rays, the non-scaling behavior of inclusive cross sections, and rising cross sections for inelastic hadron-nucleus collisions. The method have been tested [18, 20] by comparing the calculated fluxes of high energy atmospheric hadrons and muons with the data of the past decade experiments. The atmospheric muon spectra at various zenith angles have been thoroughly examined for wide energy range [18, 21, 22]. The method enables one to estimate directly the effect of the primary cosmic ray spectrum and the hadronic interactions models on the absolute values of muon and neutrino fluxes without recourse to any normalizing factors.

In this study, we analyze statistically the predicted atmospheric neutrinos spectra as compared with measured ones, by using standard \( \chi^2 \) criterion. This work continues and extends the topic touched upon in the conference talk [23].

II. SPECTRA OF ATMOSPHERIC NEUTRINOS

The atmospheric neutrinos comprise two components, “soft” and “hard”, clearly distinguishable in zenith-angle distributions and energy spectra. The anisotropic component originated from decays of pions and kaons has
A. Conventional neutrinos

Conventional atmospheric neutrinos within ~100 GeV to 10 PeV, produced in decays of $\pi^\pm$, $K^\pm$, $\Lambda$'s, and $K^0$ mesons were discussed in references [13, 23, 24]. The neutrino spectra were computed for a set of hadron-nucleus interaction models GQGSM-III-03, SIBYLL 2.1, ZS, and model by Kimel & Mokhov (KM) [34, 35], which are used also in Monte Carlo simulations of extensive air showers produced by cosmic rays. We apply two cosmic ray spectrum models by Zatsepin and Sokolskaya (ZS) [36] and by Hillas and Gaisser [57] (chosen was the H3a version for mixed composition of the extragalactic component).

The ZS comprises contributions from three classes of Galaxy cosmic rays sources: isolated SNe exploding into a random interstellar medium (ISM), high mass SNe exploding into a dense ISM (OB associations), and weak sources associated with novae explosions. The ZS spectrum supported by direct measurements of ATIC-2 experiment [38, 39] within 10 GeV–50 TeV serves, indeed, as an extrapolation of the CR spectrum beyond PeV (up to 100 PeV), and therefore the calculated neutrino flux in case of ZS spectrum should be restricted to $E_\nu < 10$ PeV.

The Hillas and Gaisser model [55] includes three classes of sources: supernovae remnants in the Galaxy, Galaxy high-energy sources of unknown origin (that contribute to the cosmic ray flux between the knee (3 PeV) and the ankle (4 EeV)), and extragalactic astrophysical objects (Active Galactic Nuclei, sources of the gamma-ray bursts, and others). The composite spectrum is formed of five nuclei groups (p, He, CNO, Mg-Si, and Mn-Fe). Each of the three populations accelerates five nuclei groups, whose spectrum cuts off at a characteristic rigidity.

Figures 1, 2 present the conventional neutrinos fluxes, calculated with the hadronic models (QGSJET-II-03, SIBYLL 2.1, and KM), together with the experimental data. The main content of these figures is the comparison of calculated CN flux with experiments. However, the curves of the prompt neutrino spectra also shown in figures to exhibit distinctions between CN and PN fluxes. Hereinafter, $\nu_\mu$ and $\bar{\nu}_e$ designate the sums of neutrinos and antineutrinos, $\nu_\mu + \bar{\nu}_\mu$ and $\nu_e + \bar{\nu}_e$, respectively.

B. Charm production models

In this study, we use the prompt neutrino (PN) contributions obtained with the charm production models: QGSM [24], SIBYLL 2.3c [40], PROSA Collaboration [41, 42], GRRST [13], BEJKRSS [44], and GM-VFNS [45].

The non-perturbative quark-gluon string model (QGSM) was developed [46, 47] to describe the soft and semihard hadronic processes at high energies. It has been applied to successfully describe the meson and baryon production in hadron-nucleon collisions. The QGSM was one of the first models to estimate the atmospheric prompt muon and neutrino fluxes [48–50]. Here, we use the results of the prompt muon neutrino flux calculations at 1 TeV – 100 PeV performed with the updated QGSM [24]. Parameters of the updated QGSM were examined by comparing the calculated cross sections for the charm hadron production with the measurements in the LHCb and ALICE experiments. Although the LHCb does not enable an unique choice of the QGSM parameters, the intercept of the Regge trajectory $\alpha(0) = -2.2$ appears a more preferable value versus $\alpha(0) = 0$. Another QGSM free parameter is the coefficient $a_1$ providing an unified description for the $x \to 0$ and $x \to 1$ kinematic regions in the case of the leading fragmentation. There are no clear arguments for choosing $a_1$, and we calculate the PN flux for this parameter, varying from $a_1 = 2$ and $a_1 = 30$ (shaded band in Figure 3).

The SIBYLL 2.3c [40] was used to calculate the PN spectra based on numerical solver for the system of the coupled cascade equations (Matrix Cascade Equations, MCE method) [51, 52]. The model for charm quark production is based on the LO QCD computations and the probability for the replacing $s$ quarks by $c$ ones in the fragmentation process.

The PROSA collaboration [41, 42] has presented the thorough study of the atmospheric prompt neutrino problem with usage of the PROSA Monte Carlo event generator. The PROSA computations were based on the LHCb and ALICE measurements of the charmed hadron production and improved constraints on the parton distribution functions (PDFs) in NLO QCD analysis using DIS and $pp$ collision data. The prompt $\nu_\mu$ spectrum was obtained through the atmospheric cascade equations that describe the production and decay of secondary particles arising from cosmic ray interactions which produce finally the atmospheric muons and neutrinos. The cascade equations admit approximate solutions in the $z$-moment approach with use of the superposition model for $pA$ and $AA$ interactions. The $z$-moments were calculated with the PROSA PDFs at the next-to-leading order (NLO) perturbative QCD (pQCD) in the fixed/variable flavour number schemes (FFNS/VFNS) consistent with the LHCb, ALICE and HERA measurements of charmed and beauty-flavoured hadrons. It was shown that PDF
uncertainties lead to the smaller flux uncertainty with respect to those arising from a choice of the QCD renormalization and factorization scales. The variations of phenomenological parameters of the charm fragmentation functions, as well as the choice of the cosmic ray model (the CR composition and spectrum) have also considerable impact on the flux uncertainty.

The PN spectra predicted with GRRST model \cite{43} are based on the Monte Carlo event generator using the z-moment approach to simulate of particles propagation and decays in the framework of NLO pQCD calculations through the same set of high energy charged hadrons as in the PROSA approach. Cross sections of charged particles production are obtained with the PDFs integrated the measurements of charm production at the LHCb experiment. Uncertainties of calculations are defined by the “scale uncertainties” of the NLO perturbative QCD and are reduced by NNLO calculations. Also the uncertainties of gluon PDF at small $x$ variable make a sizable contribution to the total errors of predicted PN, especially at $E_\nu > 1$ PeV.

The PN flux predicted with BEJKRSS model \cite{44} was evaluated using the scheme for calculation of the charm production cross section, which comprises the NLO pQCD computations, the $k_T$ factorization approach with the low $x$ resummation, and the color dipole model comprising the gluon saturation. The QCD parameters were chosen to provide the best fit of the heavy quark production cross sections measured in RHIC and LHCb experiments. The latest version of BEJKRSS model incorporates nuclear effects in the target PDFs, which usually are neglected in the perturbative approach, although the nuclear shadowing effects may be considerable at very low $x$ region. It was found that the reduction of the neutrino flux due to nuclear effects varies from 10% to 50% at the highest energies, depending on the used scheme.

The general-mass variable flavour number scheme (GM-VFNS) model \cite{45} bases on the NLO pQCD approach in which the matrix elements for the charm hadroproduction of light and heavy partons are combined with a set of fragmentation functions to describe the hadronization process (transition from partons to charged hadrons). The scheme involves only one massive heavy quark $m_Q$, other quarks are massless. The GM-VFNS appropriates different approaches for calculation of cross sections: the FFNS scheme uses in low and intermediate $p_T$, and zero mass VFNS framework does in high regions of $p_T$. The validity of the approach has been cross-checked by comparisons with data measured with the LHCb experiment. Uncertainty of predicted PN spectra arises from renormalization scale around central value (with a permanent level of half of order of PN magnitude) and from the PDF uncertainty, which rapidly grows to two and a half orders of magnitude for very high energies.

### 2. PN flux predictions

This work analysis concerns only atmospheric muon and electron neutrinos, because tau neutrinos substantially are the prompt ones (originate from $D^0$ and $\tau$ decays) and they are suppressed by one order of magnitude \cite{40}.

The PN spectra are plotted in Figs. 3 the calculations are performed for the H3a spectrum of primary cosmic rays. Prompt electron neutrinos dominate over the conventional ones at energies beyond 50 TeV, while the prompt muon neutrinos become the dominant component at the PeV scale. The PN energy spectra are calculated for the vertical direction (no averaging over zenith angles). The isotropic approximation provides a reasonable estimate at energies below 3 PeV where the PN flux weakly depends on the zenith angle. The directions around the vertical are most suitable to reveal PN neutrinos because of the best PN/CN flux ratio. The green band shows the QGSM calculation uncertainty relating to varying free parameter $a_1$, that provides a unified description for the kinematic regions $x \rightarrow 0$ and $x \rightarrow 1$ in the case, when the valence quarks participate in fragmentation \cite{24}. We do not show uncertainties of the spectra computed by the PROSA \cite{11} which absorb those of the rest models. All $\chi^2$ values are obtained for the central values of PN predictions.

These models predictions display the spread of the prompt neutrino flux obtained for the H3a spectrum of cosmic rays, i.e. these models mark the approximate range of the PN contribution calculated with H3a spectrum (the use of the ZS spectrum weakly affects the range). The PN fluxes obtained with QGSM \cite{24}, PROSA \cite{11} are very close to each other within a wide energy range (Fig. 3). The SIBYLL 2.3c \cite{40} also predicts the PN flux rather close to that of PROSA and QGSM. Shaded area in Figure 3 shows the spread of model predictions for the median “crossing energy”, i.e. the energy, above which the atmospheric neutrino flux is dominated by the PN component (see also Table IV). All the fluxes are calculated for the H3a model of primary cosmic rays spectrum.

### III. EXPERIMENTAL DATA

The reconstructed neutrino energy spectra are derived from the neutrino telescopes with large statistical and systematical errors due to the restricted set of data and necessity to resort to sophisticated technique of handling the neutrino events. The total errors of the $\nu_\mu$ and $\nu_e$ spectra measured in the Fréjus experiment \cite{11} at zenith angles $90^\circ < \theta < 180^\circ$, vary from $\sim 26\%$ to $\sim 55\%$ in low and high neutrino energies within the range of $0.25 < E_\nu < 10^3$ GeV.

Preliminary results of the AMANDA-II experiment obtained at 90% CL without zenith angles cuts were reported in 2009 \cite{2} (grey band in the Figure 4(a)).
Figure 1: Calculated atmospheric neutrino spectra (scaled by $E_\nu^3$) averaged over zenith angles, $\nu_\mu$ (a), (c), (d), and $\nu_e$ (b) compared to the data of experiments: Fréjus [1], AMANDA-II [2, 3], ANTARES [4, 5], IceCube-40 [6], IceCube-59 [7], IceCube-79 [7], IceCube-86 [11], and Super-Kamiokande I–IV [12]. Error bars correspond to the uncertainties, including all statistical and systematic errors. Grey band in panel (a) indicates the 90% CL from the forward-folding analysis of AMANDA-II 2009 [2]. The total errors (yellow rectangles) and statistical ones (dark rectangles) for the IceCube-40 data and AMANDA-II 2010 [8] are shown in panels (c) and (d). The spectra are calculated with H3a parameterization of the cosmic rays spectrum [37] for hadronic models KM (solid lines), SIBYLL 2.1 (dashed), and QGSJET-II-03 (short dashed). Also shown are the PN spectra calculated with QGSM [24], SIBYLL 2.3c [40], and BEJKRSS [44].

The recent data published by the ANTARES Collaboration in 2021 [5] were collected in the period 2007–2017.
Figure 2: Calculated energy spectra of the atmospheric muon neutrinos averaged over 90° < θ < 120° (a), and 120° < θ < 180° (b) in comparison with the IceCube-59 data [9]. The total and statistical measurement errors are shown as light and dark shaded rectangles. Notations of experimental errors and theoretical predictions of the CN and PN fluxes are the same as in Figure 1.

Figure 3: Prompt $\nu_\mu$ spectra (scaled by factor $E_\nu^3$) at $\theta = 0^\circ$ calculated with charm production models QGSM [24], SIBYLL 2.3c [40], GRRST [43], BEJKRSS [44], GM-VFNS [45], and PROSA [41]. The CN spectra averaged over zenith angles are also shown. The shaded rectangle displays the “crossing energy” range predicted with different PN flux models. All the fluxes are calculated for the H3a model of primary cosmic rays spectrum.

in the energy range between $\sim 100$ GeV and $\sim 50$ TeV (5 bins for $\nu_\mu$ and 3 bins for $\nu_e$); the zenith angles interval is 90° < θ < 180°. The statistical uncertainties of the ANTARES 2021 reconstruction are rather large: 10% − 100% ($\nu_\mu$) and 30% − 200% ($\nu_e$). During of 3012 days of the livetime, about 130 $\nu_e$ and 850 $\nu_\mu$ events were reconstructed in the instrumented volume of ANTARES detector.

The density of ANTARES optical modules is insufficient to reconstruct a considerable number of events induced by neutrinos at energies below 100 GeV. The low statistics of the $\nu_e$ events prevents from testing charm production models above tens of TeV, i.e. in the energy range, where one could expect an appreciable contribution of the prompt electron neutrinos. The energy estimate for the semi-contained events relative to the through-going ones reduces the overall uncertainty of the measured flux as compared with ANTARES 2013 measurements. The ANTARES 2021 $\nu_\mu$ flux is close to that of IceCube-40 (2011) and IceCube-59 (2015), and 20% − 25% below the flux reported in the ANTARES 2013 measurement.

The total experimental errors of the neutrino spectra reconstructed in the Super-Kamiokande [12] (operated intermittently since 1996, but the data sets used in this paper include the data until 2015) vary from 15% to 19% ($\nu_e$) and 15% − 21% ($\nu_\mu$) in the energy range $0.25 < E_\nu < 10^4$ GeV.

In the period 2008 − 2009, the IceCube detector was operated in the 40-strings configuration [6]. The results of the measurements were presented for three zenith an-
gles intervals: $97^\circ < \theta < 124^\circ$, $124^\circ < \theta < 180^\circ$, and $97^\circ < \theta < 180^\circ$ (joint interval). The authors of the experiment have published the errors only for the joint interval. The total errors for the joint analysis were estimated from $21\%$ to $158\%$ for low and high energies in the range $10^2 < E_\nu \lesssim 10^6$ GeV.

The first data on the atmospheric $\nu_e$ flux in TeV energy range were obtained in 2010–2011 using the DeepCore infill array in the IceCube-79 (the 79-strings configuration) [4]. DeepCore included the six specialized strings and the seven adjacent standard IceCube strings (DeepCore-13) and allowed reducing the energy threshold to $\sim 10$ GeV. Four data points of the reconstructed atmospheric $\nu_e$ spectrum were obtained from a selected data sample of $496 \pm 66$ (stat) $\pm 88$ (syst) cascade events observed in 281 days of data. They included $\nu_e$ charge current interactions and neutral current interactions of neutrinos of all flavors. The experimental errors of measured $\nu_e$ spectra are from $54\%$ to $100\%$ in the energy range $80$ GeV $– 6$ TeV.

In 2011–2012, a new IceCube analysis of the $\nu_e$ spectrum was based on the data taken for $97^\circ < \theta < 180^\circ$ with the full 86-string configuration during 332.3 days of livetime [11]. The total errors of data are estimated as $25\%$–$94\%$ for energies $10^2 < E_\nu \lesssim 10^5$ GeV. Whereas the information on zenith angles cuts is not entirely clear, we do not apply any cuts in our analysis. The IceCube-59 experimental data for $\nu_\mu$ were taken in 2009–2010 with the 59-string configuration of the detector [8]. Like in the IceCube-40, the analysis of events addresses three intervals of zenith angles, $90^\circ < \theta < 120^\circ$, $120^\circ < \theta < 180^\circ$, and $90^\circ < \theta < 180^\circ$. The total errors are estimated from $25\%$ to $\sim 250\%$ for energies $10^2 < E_\nu \lesssim 10^6$ GeV. We do not use the IceCube-79 data [11] in our analysis, because they contain uncertain admixture of astrophysical neutrinos.

Thus, the total data set for the statistical analysis for $\nu_\mu$ spectra contains 54 data points measured with Fréjus (4 points), AMANDA-II (9 points), IceCube-40 (12 points), ANTARES 2013 (10 points), ANTARES 2021 (5 points), Super-Kamiokande (4 points), and IceCube-59 (10 points) at zenith angles in the interval $90^\circ < \theta < 180^\circ$. The set for the analysis of $\nu_e$ spectra includes 11 data points obtained in Super-Kamiokande [12], IceCube-79 [4], IceCube-86, and ANTARES 2021 [13]. Figures 1(a) and 2 show the measured and predicted spectra calculated with the models tested in our analysis. Figure 1(b) shows the experimental data for $\nu_e$ spectra measured in Fréjus 1995 [3], Super-Kamiokande [12] for zenith angles $90^\circ < \theta < 180^\circ$ along with the predicted spectra averaged over the narrowed angle interval, $97^\circ < \theta < 180^\circ$.

V. RESULTS AND DISCUSSION

A set of $n$ independent measurements $\Phi_i$ (energy spectrum) at points $E_i$ (energy) is considered as Gaussian distributed with the mean $\mu(E_i; \vec{\alpha}, \vec{\beta})$ and known variance $\sigma_i^2$. The goal of the statistical analysis is to construct estimators for the unknown parameters $\vec{\alpha}, \vec{\beta}$. In our case, $\vec{\alpha}$ stands for hadronic models $(\alpha_j, j = 1, 2, 3)$ and $\vec{\beta}$ labels models of cosmic ray spectrum $(\beta_k, k = 1, 2)$. That is, index $j$ implies a hadronic interaction model KM, QGSJET-II-03 and SIBYLL 2.1, and $j$ marks CR models H3a and ZS.

For the statistical analysis loads [53, 54] we use $\chi^2$ values

$$
\chi^2(\vec{\alpha}, \vec{\beta}) = \sum_{i=1}^{n \text{dof}} \frac{\left(\Phi_i^{\text{exp}} - \phi(\vec{\alpha}, \vec{\beta})(E_i)\right)^2}{(\delta \Phi_i^{\text{exp}})^2},
$$

where $\Phi_i^{\text{exp}}$ is the detected neutrino flux for $i$-th energy bin; $\phi(\vec{\alpha}, \vec{\beta})$ is the calculated one for the chosen flux model $(j, k)$; ndf is the number of the data bins (data points) index $i$ enumerates the measured mean values $E_i$; $\delta \Phi_i^{\text{exp}}$ designates experimental errors (considering the systematic and statistical uncertainties). For each bin $\Phi(E_{\nu}) = \langle dN_{\nu}/dE_{\nu} \rangle_\theta$ denotes the differential neutrino flux (the energy spectrum) averaged over zenith angles. The predicted neutrino flux was averaged over energy in the $i$-th bin:

$$
\phi(\vec{\alpha}, \vec{\beta})(E_i) = \frac{1}{\Delta E_i} \int_{E_i}^{E_{i+1}} \phi(\vec{\alpha}, \vec{\beta})(E) dE.
$$

The quality of the overall fit can be judged from the global $\chi^2$ divided by ndf. For each data set included in the analysis, a partial $\chi^2$/ndf relating to single experiment is provided. The second column in Tables I and II presents $\chi^2$/ndf obtained for the CN flux calculated with hadronic models, KM, QGSJET-II-03 and SIBYLL 2.1 (the flux model indices are dropped). Columns 3–5 are $\chi^2$/ndf for the total neutrino flux, the conventional and the prompt one (CN+PN). Here, we show also results for the three charm production models QGSM, SIBYLL 2.3c, and BEJKRS (see Section II).

The partial and global $\chi^2$/ndf values for the conventional muon neutrinos illustrate a satisfactory agreement among all the data sets (Tables I and II) except for AMANDA-II data. We may state, that the prompt muon neutrinos predicted with charm production models under study are statistically insignificant (Tables I and II). More optimistic picture is seen for the contribution of the prompt electron neutrinos with the SIBYLL 2.3c (4th column in Table III): for IceCube-86 experiment $\chi^2$/ndf value is reduced by $\sim 8$–$10\%$. Unfortunately, total statistical significance of the $\nu_e$ data is not so high in our analysis due to restricted energy range.

IV. STATISTICAL ANALYSIS

The obtained $\chi^2$ values for each flux model and each experiment are shown in Tables I and II.
Table I: $\chi^2$/ndf values for the predicted $\nu_\mu$ spectra versus the experimental data. Calculations are made for the H3a and ZS (in brackets) cosmic ray spectrum.

| Experiment, CN | $\chi^2$ (CN) | QGSJET-II-03 | SIBYLL 2.1 |
|---------------|---------------|---------------|-------------|
| CN | PN models | KM | QGSM | (CN) | SIBYLL 2.3c | BEJKRSS |
| Frejus 1995 | $E_\nu \lesssim 10^3$ GeV, $90^\circ < \theta < 180^\circ$ | | | | |
| KM | 2.30/ 4 = 0.57 | 0.23/ 2 = 0.11 | 6.78/ 2 = 3.39 |
| QGSJET-II-03 | 2.77/ 4 = 0.69 | 0.36/ 2 = 0.18 | 6.77/ 2 = 3.39 |
| SIBYLL 2.1 | | | |
| AMANDA-II 2010 | $1.0^3 \lesssim E_\nu \lesssim 10^6$ GeV, $100^\circ < \theta < 180^\circ$ | | | | |
| KM | 21.4/ 9 = 2.38 | 20.5/ 9 = 2.28 | 20.1/ 9 = 2.23 | 21.0/ 9 = 2.33 |
| QGSJET-II-03 | (29.1/ 9 = 3.23) | (28.1/ 9 = 3.12) | (27.6/ 9 = 3.07) | (28.6/ 9 = 3.18) |
| SIBYLL 2.1 | 31.4/ 9 = 3.49 | 30.3/ 9 = 3.36 | 29.7/ 9 = 3.30 | 30.9/ 9 = 3.43 |
| IceCube-40 2011 | $1.0^2 < E_\nu < 10^6$ GeV, $90^\circ < \theta < 180^\circ$ | | | | |
| KM | 0.78/12 = 0.06 | 0.87/12 = 0.07 | 0.92/12 = 0.08 | 0.82/12 = 0.07 |
| QGSJET-II-03 | (0.70/12 = 0.06) | (0.72/12 = 0.06) | (0.75/12 = 0.06) | (0.71/12 = 0.06) |
| SIBYLL 2.1 | 13.3/12 = 1.10 | 13.5/12 = 1.13 | 13.7/12 = 1.14 | 13.4/12 = 1.12 |
| ANTA Res 2013 | $1.0^2 < E_\nu < 10^6$ GeV, $100^\circ < \theta < 180^\circ$ | | | | |
| KM | 4.46/10 = 0.45 | 4.23/10 = 0.42 | 4.12/10 = 0.41 | 4.35/10 = 0.44 |
| QGSJET-II-03 | (5.35/10 = 0.53) | (5.10/10 = 0.51) | (4.98/10 = 0.50) | (5.24/10 = 0.52) |
| SIBYLL 2.1 | 7.17/10 = 0.72 | 6.90/10 = 0.69 | 6.77/10 = 0.68 | 7.05/10 = 0.70 |
| IceCube-IV 2016 | $1.0^2 < E_\nu < 10^4$ GeV, $90^\circ < \theta < 180^\circ$ | | | | |
| KM | 3.65/ 4 = 0.91 | 4.29/ 4 = 1.07 |
| QGSJET-II-03 | 4.01/ 2 = 2.01 | 4.00/ 2 = 2.00 |
| SIBYLL 2.1 | 1.38/ 2 = 0.69 | 1.26/ 2 = 0.63 |
| Combined data | $1.0^2 < E_\nu < 10^6$ GeV | | | | |
| KM | 32.6/39 = 0.84 | 31.5/39 = 0.81 | 31.1/39 = 0.80 | 32.1/39 = 0.82 |
| QGSJET-II-03 | (42.2/39 = 1.08) | (41.0/39 = 1.05) | (40.4/39 = 1.04) | (41.6/39 = 1.07) |
| SIBYLL 2.1 | 43.4/35 = 1.24 | 42.1/35 = 1.20 | 41.3/35 = 1.18 | 42.8/35 = 1.22 |

Present the values of $\chi^2$/ndf calculated for $\nu_\mu$, Table III does the same for $\nu_e$ spectra. The energy ranges and cuts for zenith angles are indicated for each experiment. Namely, Tables I and II show $\chi^2$/ndf for $\nu_\mu$ spectra predicted by Fréjus [1], IceCube-40 2011 [6], IceCube-59 2015 [3], ANTA Res 2013 [4], ANTA Res 2021 [5], AMANDA-II 2010 [3], and Super-Kamiokande I–IV 2016 [12], compared with the CN and CN+PN neutrino spectra predicted by QGSJET-II-03 [30–32], SIBYLL 2.1 [33], and KM [34, 35], for H3a and ZS (in brackets) cosmic ray spectrum. The CN spectra were averaged over the zenith angles according to the cuts provided by experimentalists.

The analysis is performed for five combinations: 1) $\nu_\mu$ and $\nu_e$ separately for each experiment (ndf = 4 ÷ 12); 2) the combined $\nu_\mu$ data except for IceCube-59 and ANTA Res 2021 (ndf = 35, 39) (Table II); 3) combined all $\nu_\mu$ data (ndf = 50, 54) (Table II); 4) the combined ANTA Res 2013 and ANTA Res 2021 $\nu_\mu$ data (ndf = 15) (Table III); 5) the combined $\nu_e$ data of IceCube-79, IceCube-86, and ANTA Res 2021 (Table III).
Table II: \( \chi^2/\text{ndf} \) values obtained for predicted 3\( \nu_\mu \) spectra and measured ones in the IceCube and ANTARES experiments.

| Experiment, CN | \( \chi^2 \) (CN) | QGSJET II-03 | SIBYLL 2.1 | BEJKRSS |
|----------------|-------------------|--------------|------------|---------|
| ICECUBE-59 2015 | \( 10^2 < E_\nu \leq 10^6 \text{ GeV}, 90^\circ < \theta < 120^\circ \) | | | |
| KM | 11.0/ 9 = 1.22 | 12.7/ 9 = 1.41 | 11.0/ 9 = 1.22 | 10.9/ 9 = 1.21 |
| QGSJET-II-03 | 4.60/ 9 = 0.51 | 4.70/ 9 = 0.52 | 4.51/ 9 = 0.50 | 4.56/ 9 = 0.51 |
| SIBYLL 2.1 | 35.1/ 9 = 3.90 | 35.1/ 9 = 3.90 | 35.2/ 9 = 3.91 | 35.1/ 9 = 3.90 |
| ICECUBE-59 2015 | \( 10^2 < E_\nu \leq 10^6 \text{ GeV}, 120^\circ < \theta < 180^\circ \) | | | |
| KM | 0.97/ 8 = 0.12 | 1.26/ 8 = 0.16 | 1.43/ 10 = 0.44 | 1.10/ 8 = 0.14 |
| QGSJET-II-03 | 0.41/ 8 = 0.05 | 0.55/ 8 = 0.07 | 0.64/ 8 = 0.08 | 0.47/ 8 = 0.06 |
| SIBYLL 2.1 | 10.6/ 8 = 1.33 | 11.1/ 8 = 1.39 | 11.4/ 8 = 1.42 | 10.8/ 8 = 1.35 |
| ICECUBE-59 2015 | \( 10^2 < E_\nu \leq 10^6 \text{ GeV}, 90^\circ < \theta < 180^\circ \) | | | |
| KM | 4.79/ 10 = 0.48 | 4.49/ 10 = 0.45 | 4.40/ 10 = 0.44 | 4.64/ 10 = 0.46 |
| QGSJET-II-03 | 3.58/ 10 = 0.36 | 3.15/ 10 = 0.32 | 3.00/ 10 = 0.30 | 3.38/ 10 = 0.34 |
| SIBYLL 2.1 | 18.0/ 10 = 1.80 | 17.8/ 10 = 1.78 | 17.8/ 10 = 1.78 | 17.9/ 10 = 1.79 |
| ANTARES 2021 | \( 10^2 < E_\nu \leq 10^4 \text{ GeV}, 90^\circ < \theta < 180^\circ \) | | | |
| KM | 1.74/ 5 = 0.35 | 1.80/ 5 = 0.36 | 1.84/ 5 = 0.37 | 1.76/ 5 = 0.35 |
| QGSJET-II-03 | 0.19/ 5 = 0.04 | 0.20/ 5 = 0.04 | 0.20/ 5 = 0.04 | 0.19/ 5 = 0.04 |
| SIBYLL 2.1 | 20.0/ 5 = 4.00 | 20.2/ 5 = 4.04 | 20.4/ 5 = 4.08 | 20.1/ 5 = 4.02 |
| ANTARES 2013 & ANTARES 2021 | \( 10^2 < E_\nu \leq 10^6 \text{ GeV} \) | | | |
| KM | 6.20/ 15 = 0.41 | 6.03/ 15 = 0.40 | 5.96/ 15 = 0.40 | 6.11/ 15 = 0.41 |
| QGSJET-II-03 | 7.37/ 15 = 0.49 | 6.86/ 15 = 0.46 | 6.77/ 15 = 0.45 | 6.98/ 15 = 0.47 |
| SIBYLL 2.1 | 21.6/ 15 = 1.44 | 21.6/ 15 = 1.44 | 21.8/ 15 = 1.45 | 21.6/ 15 = 1.44 |

(ndf = 10, 11).

In the analysis of Super-Kamiokande data, we use only 4 (\( \nu_\mu \)) and 2 (\( \nu_e \)) data points measured at high energies. We consider high energy models QGSJET-II-03 and SIBYLL 2.1 as reasonable ones at energies \( E_\nu \geq 100 \text{ GeV} \), while KM is valid in the wider energy range, \( E_\nu \geq 10 \text{ GeV} \). Thus, we compare the KM-predicted \( \nu_\mu \) spectrum (ndf = 4) and that of other theoretical models (ndf = 2) with measured ones in Fréjus 1995 and Super-Kamiokande at neutrino energies above 10 GeV. The same relates to four and three points of the \( \nu_e \) flux measured in the IceCube.
The $\chi^2$ values obtained with KM are QGSJET-II are closely related for all data, differing from those for SIBYLL 2.1. QGSJET-II and KM give the best fit for SIBYLL 2.1 appears as the preferred model (Table III). As may be seen from these tables, the prompt neutrinos contribution is practically negligible for all measurements.

The PN fluxes were calculated using all listed charm production models, but only three of them, QGSM [24], SIBYLL 2.3c [40], and BEJKRSS [44], are presented separately. The GRRST [43] and BEJKRSS [44] predict highest “crossing energies”.

The bulk analysis of conventional muon neutrino spectra shows that all flux models are consistent with measurements. However, SIBYLL 2.1 is in a tension with the IceCube-59 data for zenith angles $90^\circ < \theta < 180^\circ$.

Table III: $\chi^2$/ndf values calculated for the measured and predicted $\nu_e$ spectra.
in comparison with KM ($p \approx 0.01$) and QGSJET-II ($p \approx 2 \times 10^{-4}$).

Notice also that close agreement of the SIBYLL 2.1 prediction ($p = 0.999$) with ANTARES 2013 data is ruined by ANTARES 2021 ($p = 1.2 \times 10^{-3}$), while KM and QGSJET-II-03 keep the accordance with the latter data ($p \approx 0.88-0.99$). Similar results are obtained also for the combined data ANTARES 2013+2021: $p = 0.98$ (KM) and $p = 0.95$ (QGSJET-II) against $p = 0.12$ for SIBYLL 2.1 (Tables III and IV).

The IceCube-59 2015 data within the interval $90^\circ < \theta < 120^\circ$ are described by models KM, QGSJET-II, and SIBYLL 2.1 with a lower confidence level ($p = 0.28, 0.87, 6 \times 10^{-4}$) as compared with that for angles $120^\circ < \theta < 180^\circ$ ($p = 0.998, 0.999,$ and 0.225). The discrepancy may result from an inaccuracy in analysis of events induced by neutrinos passing the detector near the horizon.

Our calculations showed that the zenith-angle cut influences moderately on the angle-averaged conventional neutrino flux: reducing the angle interval by $\sim 1^\circ$ near the horizon leads to decrease in the spectra by $\sim 3\%$ for neutrino energies above 100 TeV.

Table III presents the comparative statistical significance of hadronic interactions models used in the analysis of $\nu_\mu$ spectra derived in IceCube-59 (10 data points) and in ANTARES experiments [4, 5] (15 points). This table demonstrates the proximity of the QGSJET-II-03 and Kimel & Mokhov predictions ($\sim 1\sigma$) relative to these experiments, while the SIBYLL 2.1 proves a certain tension with the data ($>3.5\sigma$).

As regards to $\nu_e$ spectra, SIBYLL 2.1 and QGSJET-II-03 give a good description of IceCube-79 (with 3 data points involved in the analysis) [2]. The KM model gives similar result for the same data (3 points) but fails with $\chi^2/\nu df \approx 5.6$ if all 4 points are included. Nevertheless KM and QGSJET-II-03 give fairly good fit for the IceCube-86 $\nu_e$ spectrum (4 points) [10]: $p \approx 0.21$ (KM) and 0.31 (QGSJET-II).

Although, for reasons mentioned above, KM seems suitable for describing two of four $\nu_e$ data points (beyond 10 GeV) measured with Super-Kamiokande [12], actually the model hardly fits them ($p \approx 1.5 \times 10^{-2}$).

The $\chi^2$ analysis of the spectra measured in the IceCube-59 [3], IceCube-79 [2], and IceCube-86 [10] shows a slight preference of the H3a model for the cosmic ray spectrum as compared to the ZS parameterization.

### VI. CONCLUSIONS

Predicted differential spectra of atmospheric muon neutrino successfully describe the experimental data within the experimental uncertainties. Both parameterizations of the cosmic rays spectrum, by Zatsepin & Sokolovskaya and Hillas & Gaisser, produce close $\chi^2$ values for the datasets under analysis, i.e. they are statistically indistinguishable.

The calculated spectra of atmospheric muon neutrinos agree well with data obtained in Frejus, AMANDA, IceCube, and ANTARES experiments. QGSJET-II-03 and KM lead to the best description of IceCube-59 data and ANTARES 2021 measurements of the $\nu_\mu$ spectrum, SIBYLL 2.1 is a good model to describe the AMANDA-II and the ANTARES 2013 muon neutrino data. The Kimel & Mokhov model also provides suitable predictions for the IceCube-59 and ANTARES 2021, and the best one for combined data ANTARES 2013 + ANTARES 2021 ($p = 0.98$). The minimal $\chi^2$ value for the total data on the $\nu_\mu$ spectrum is also derived with Kimel & Mokhov model ($p = 0.94$).

As concerns atmospheric electron neutrinos,low event statistics in the measurements of the $\nu_e$ flux beyond 100 GeV impedes the unique choice of the preferred hadronic interactions model.

The statistical analysis shows that none of the discussed neutrino flux models leads to the statistically significant contribution of the prompt atmospheric neutrino.
nos in the energy range covered by the neutrino telescopes. Thus we can infer from the analysis that the high-energy atmospheric neutrino spectra calculated with the consistent scheme [13] [24] [29] are sufficiently reliable and might be suitable for numerical simulation of the atmospheric neutrino events in the operating neutrino telescopes, as well as in the future experiments, Baikal-GVD [55] [57], IceCube-Gen2 [58, 59], and KM3NeT/ORCA [60]. We expect that increased statistics on the muon, electron and tau neutrino events due to functional capabilities of the next generation of neutrino telescopes will enable one to solve the prompt neutrino problem.

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[1] K. Daum et al. (Fréjus Collaboration), Determination of the atmospheric neutrino spectra with the Fréjus detector, Z. Phys. C 66, 417 (1995).
[2] R. Abbasi et al. (IceCube Collaboration), Determination of the atmospheric neutrino flux and searches for new physics with AMANDA-II, Phys. Rev. D 79, 102005 (2009) [arXiv:0902.0675].
[3] R. Abbasi et al. (IceCube Collaboration), The energy spectrum of atmospheric neutrinos between 2 and 200 TeV with the AMANDA-II detector, Astropart. Phys. 34, 48 (2010) [arXiv:1004.2357].
[4] S. Adrian-Martinez et al. (ANTARES Collaboration), Measurement of the atmospheric $\nu_\mu$ energy spectrum from 100 GeV to 200 TeV with the ANTARES telescope, Eur. Phys. J. C 73, 2606 (2013) [arXiv:1308.1599].
[5] A. Albert et al. (ANTARES Collaboration), Measurement of the atmospheric $\nu_e$ and $\nu_\mu$ energy spectra with the ANTARES neutrino telescope, Phys. Lett. B 816, 136228 (2021) [arXiv:2101.12170].
[6] R. Abbasi et al. (IceCube Collaboration), Measurement of the atmospheric neutrino energy spectrum from 100 GeV to 400 TeV with IceCube, Phys. Rev. D 83, 012001 (2011) [arXiv:1010.3980].
[7] M. G. Aartsen et al. (IceCube Collaboration), Measurement of the atmospheric $\nu_\tau$ flux in IceCube, Phys. Rev. Lett. 110, 151105 (2013) [arXiv:1212.4700].
[8] M. G. Aartsen et al. (IceCube Collaboration), Search for a diffuse flux of astrophysical muon neutrinos with the IceCube 59-string configuration, Phys. Rev. D 89, 062007 (2014) [arXiv:1311.7048].
[9] M. G. Aartsen et al. (IceCube Collaboration), Development of a general analysis and unfolding scheme and its application to measure the energy spectrum of atmospheric neutrinos with IceCube, Eur. Phys. J. C 75, 116 (2015) [arXiv:1409.4535].
[10] M. G. Aartsen et al. (IceCube Collaboration), Measurement of the atmospheric $\nu_e$ spectrum with IceCube, Phys. Rev. D 91, 122004 (2015) [arXiv:1504.03753].
[11] M. G. Aartsen et al. (IceCube Collaboration), Measurement of the $\nu_\mu$ energy spectrum with IceCube-79, Eur. Phys. J. C 77, 692 (2017) [arXiv:1705.07780].
[12] E. Richard et al. (Super-Kamiokande Collaboration), Measurements of the atmospheric neutrino flux by Super-Kamiokande: energy spectra, geomagnetic effects, and solar modulation, Phys. Rev. D 94, 052001 (2016) [arXiv:1510.08127].
[13] G. D. Barr et al., A three–dimensional calculation of atmospheric neutrinos, Phys. Rev. D 70, 023006 (2004) [arXiv:astro-ph/0403630].
[14] M. Honda et al., Calculation of atmospheric neutrino flux using the interaction model calibrated with atmospheric muon data, Phys. Rev. D 75, 043006 (2007) [arXiv:astro-ph/0611418].
[15] M. Honda et al., Improvement of low energy atmospheric neutrino flux calculation using the JAM nuclear interaction model, Phys. Rev. D 83, 123001 (2011) [arXiv:1102.2688].
[16] V. A. Naumov, T. S. Singegovskaya, Simple method for solving transport equations describing the propagation of cosmic ray nucleons in the atmosphere, Phys. Atom. Nucl. 63, 1927 (2000).
[17] V. A. Naumov, T. S. Singegovskaya, Atmospheric proton and neutron spectra at energies above 1-GeV, in Proceedings of the 27th International Cosmic Ray Conference (Hamburg, 2001), Vol. 1, p. 4173 [arXiv:hep-ph/0106015].
[18] A. A. Kochanov, T. S. Singegovskaya, and S. I. Singegovsky, High-energy cosmic ray fluxes in the Earth atmosphere: calculations vs experiments, Astropart. Phys. 30, 219 (2008) [arXiv:0803.2943].
[19] T. S. Singegovskaya, A. D. Morozova, and S. I. Singegovsky, High-energy neutrino fluxes and flavor ratio in the Earth’s atmosphere, Phys. Rev. D 91, 063011 (2015) [arXiv:1407.3501].
[20] S. I. Singegovsky et al., Atmospheric muon flux at PeV energies, Int. J. Mod. Phys. A 25, 3733 (2010).
[21] A. A. Kochanov, T. S. Singegovskaya, and S. I. Singegovsky, High-energy cosmic ray muons in the Earth’s atmosphere, J. Exp. Theor. Phys. 116, 395 (2013).
[22] A. A. Kochanov et al., High-energy atmospheric muon flux calculations in comparison with recent measurements, J. Phys. Conf. Ser. 1181, 012054 (2019) [arXiv:1907.00640].
[23] A. A. Kochanov et al., Atmospheric neutrino spectra: a statistical analysis of calculations in comparison with experiment, Bull. Russ. Acad. Sci. Phys. 85, 433 (2021).
[24] S. I. Singegovsky and M. N. Sorokovikov, Prompt atmospheric neutrinos in the quark–gluon string model, Eur. Phys. J. C 80, 34 (2020) [arXiv:1812.11341].
[25] A. D. Morozova et al., Calculation of atmospheric high-
energy neutrino spectra and the measurement data of IceCube and ANTARES experiments, Bull. Russ. Acad. Sci. Phys. 81, 516 (2017).

[26] A. A. Kochanov et al., Examination of calculations of the atmospheric muon and neutrino spectra using new measurements, Bull. Russ. Acad. Sci. Phys. 83, 933 (2019).

[27] A. D. Morozova et al., Influence of cosmic-ray spectrum and hadron-nucleus interaction model on the properties of high-energy atmospheric-neutrino fluxes, Phys. Atom. Nucl. 82, 491 (2019).

[28] A. D. Morozova et al., The comparison of the calculated atmospheric neutrino spectra with the measurements of IceCube and ANTARES experiments, J. Phys. Conf. Ser. 798, 012101 (2017).

[29] A. D. Morozova et al., Influence of hadronic interaction models on characteristics of the high-energy atmospheric neutrino flux, J. Phys. Conf. Ser. 934, 012008 (2017).

[30] N. N. Kalmykov, S. S. Ostapchenko, and A. I. Pavlov, Quark-gluon string model and EAS simulation problems at ultra-high energies, Nucl. Phys. B (Proc. Suppl.) 52, 17 (1997).

[31] S. Ostapchenko, QGSJET-II: Towards reliable description of very high energy hadronic interactions, Nucl. Phys. B (Proc. Suppl.) 151, 143 (2006) [arXiv:hep-ph/0412332].

[32] S. Ostapchenko, Hadronic interactions at cosmic ray energies, Nucl. Phys. B (Proc. Suppl.) 175–176, 73 (2008) [arXiv:hep-ph/0612068].

[33] E. J. Ahn et al., Cosmic ray interaction event generator SIBYLL 2.1, Phys. Rev. D 80, 094003 (2009) [arXiv:0906.4113].

[34] A. N. Kalinovsky, N. V. Mokhov, and Y. P. Nikitin, Passage of high-energy particles through matter (American Institute of Physics, New York, 1989).

[35] L. R. Kimel and N. V. Mokhov, Particle distributions in $10^{-2} - 10^{12}$ eV energy range initiated by high-energy hadrons in dense media, Izv. Vuz. Fiz. 10, 17 (1974).

[36] V. I. Zatsepin and N. V. Sokolskaya, Three-component model of cosmic ray spectra from 100 GeV up to 100 PeV, Astron. Astrophys. 458, 1 (2006) [arXiv:astro-ph/0601475].

[37] T. K. Gaisser, Spectrum of cosmic-ray nucleons, kaon production, and the atmospheric muon charge ratio, Astropart. Phys. 35, 801 (2012) [arXiv:1111.6675].

[38] A. D. Panov et al., Elemental energy spectra of cosmic rays from the data of the ATIC-2 experiment, Bull. Russ. Acad. Sci. Phys. 71, 494 (2007) [arXiv:astro-ph/0612377].

[39] A. D. Panov et al., Energy spectra of abundant nuclei of primary cosmic rays from the data of ATIC-2 experiment: final results, Bull. Russ. Acad. Sci. Phys. 73, 564 (2009) [arXiv:1101.3216].

[40] A. Fedynitch et al., Hadronic interaction model SIBYLL 2.3c and inclusive lepton fluxes, Phys. Rev. D 100, 103018 (2019) [arXiv:1806.04140].

[41] O. Zenaiev et al. (PROSA Collaboration), Improved constraints on parton distributions using LHCb, ALICE and HERA heavy-flavour measurements and implications for the predictions for prompt atmospheric-neutrino fluxes, JHEP 04, 118 (2020) [arXiv:1911.13164].

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[42] M. V. Garzelli et al. (PROSA Collaboration), Prompt neutrino fluxes in the atmosphere with PROSA parton distribution functions, JHEP 05, 004 (2017) [arXiv:1611.03815].

[43] R. Gauld et al., The prompt atmospheric neutrino flux in the light of LHCb, JHEP 02, 130 (2016) [arXiv:1511.06346].

[44] A. Bhattacharya et al., Prompt atmospheric neutrino fluxes: perturbative QCD models and nuclear effects, JHEP 11, 167 (2016) [arXiv:1607.00193].

[45] M. Benze et al., Prompt neutrinos from atmospheric charm in the general-mass variable-flavor-number scheme, JHEP 12, 021 (2017) [arXiv:1705.10386].

[46] A. B. Kaidalov and O. I. Piskunova, Production of charmed particles in the quark - gluon string model, Sov. J. Nucl. Phys. 43, 994 (1986).

[47] A. B. Kaidalov, High-energy hadronic interactions (20 years of the quark gluon strings model), Phys. Atom. Nucl. 66, 1994 (2003).

[48] E. V. Bugaev et al., Prompt leptons in cosmic rays, Nuovo Cim. C 12, (1989).

[49] E. V. Bugaev et al., Atmospheric muon flux at sea level, underground and underwater, Phys.Rev. D 58, 054001 (1998).

[50] V. A. Naumov, T. S. Sinegorskaya, and S. I. Sinegovsky, The $K_{3\ell}$ form factors and atmospheric neutrino ratio at high energies, Nuovo Cim. A 111, 129 (1998).

[51] A. Fedynitch et al., MCEQ2 – numerical code for inclusive lepton flux calculations, PoS (ICRC2015) 1129.

[52] A. Fedynitch, Phenomenology of atmospheric neutrinos, EPJ Web Conf. 116, 11010 (2016).

[53] G. Cowan et al. Asymptotic formulae for likelihood-based tests of new physics, Eur. Phys. J. C 71, 1554 (2011).

[54] M. Tanabashi et al. (Particle Data Group), Review of Particle Physics, Phys. Rev. D 98, 030001 (2018).

[55] I. Belolaptikov and Zh.-A. Dzhilkibaev on behalf of the Baikal-GVD Collaboration, Neutrino telescope in Lake Baikal: Present and nearest future, PoS (ICRC2021) 002.

[56] A. V. Avrorin et al. High-energy neutrino follow-up at the Baikal–GVD neutrino telescope, Astronomy Lett. 47, 94 (2021).

[57] J. Stasielak, P. Malecki, D. Naumov et al. (IceCube Collaboration), The IceCube–Gen2 Collaboration, High-energy neutrino astronomy – Baikal-GVD neutrino telescope in Lake Baikal, Symmetry 13, 377 (2021).

[58] M. G. Aartsen et al. (IceCube-Gen2 Collaboration), IceCube-Gen2: the window to the extreme Universe, J. Phys. G: Nucl. Part. Phys. 48, 060501 (2021) [arXiv:2008.04323].

[59] A. Ishihara et al. (IceCube Collaboration), The IceCube upgrade - design and science goals, PoS (ICRC2019) 1031 [arXiv:1908.09441].

[60] S. Adrián-Martínez et al. (KM3NeT Collaboration), KM3NeT 2.0 – Letter of intent for ARCA and ORCA, J. Phys. G: Nucl. Part. Phys. 43, 084001 (2016) [arXiv:1601.07459].