Mathematical model of a radial sliding bearing with a porous layer on its operating surface with a low-melting metal coating on shaft surface

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Abstract. The paper considers the steady flow of a liquid lubricant and a melt of a low-melting metal coating of the shaft surface, which have viscous rheological properties in the operating clearance of an endless radial sliding bearing, as well as in the body of the porous coating of the surface of bearing bush. In order to solve this problem, the authors used the equation of the movement of viscous lubricant for a thin layer, the equation of continuity and also the equation describing the radius of the molten contour of the fusible coating of the shaft surface, taking into account the rate of dissipation of mechanical energy. Having averaged the acceleration of the movement of the liquid lubricant in the operating clearance, the authors obtained the dependence for the contour of the molten coating of the shaft surface. As a result of the integration of the obtained dependence, the expression was found that allowed solving the function characterizing the contour of the molten surface of the coating by the method of successive approximations. Using the well-known method of finding an exact self-modal solution, the velocity field and pressure in the lubricating and porous layer, as well as the load capacity and friction force, were found. On the basis of the performed theoretical studies, the experimental study was carried out taking into account the parameters characterizing the melt of a low-melting metal coating, as well as a porous coating on the surface of the bearing bush and the rheological properties of a viscous lubricant. As a result, the original expression was obtained for the calculation of the load capacity of a radial bearing and the magnitude of the friction force in the range under the following loading conditions $v=0.5\div3 \text{ m/s}$, $\sigma=2\div7 \text{ MPa}$. The research results can be used in mechanical engineering, aircraft construction, instrument engineering, where the supply of lubricant is associated with great difficulties.

1. Introduction
Many works are devoted to the hydrodynamic calculation of sliding bearings [1-8] operating in conditions of the melt on one of their operating surfaces. One of the drawbacks of the existing design models [9-15] is that the lubrication process in the considered bearing designs is not self-sustaining.

In order to ensure damping properties and a self-sustaining lubrication process, it is necessary to have a porous layer on the operating surfaces of these bearings [16-20]. In our design model, the presence of a porous layer on the operating surface of the bearing and the presence of melt on the surface of the shaft are taken into account.
2. Results

The authors consider the steady movement of a viscous incompressible fluid between an eccentrically located pin and a bearing. A bearing with a porous layer on the operating surface is stationary, and the pin with the melt on its surface rotates with an angular velocity $\Omega$ (Fig. 1). In the polar coordinate system in the center of the bearing, the equations of the contours $C_2$ and $C_3$, $C_0$ and $C_1$ take the following form: $r' = b$, $r' = b + h$, $O_iM = a - \beta f(\theta)$ - radius equal to the molten shaft contour, $a$ - shaft radius to the melt.

![Figure 1. Scheme of the determination of the contours of shaft and bearing](image)

For contour points of pin $C_0$, we have:

$$OM = O_O \cos \theta + O_iM \cos \varphi$$

where $O_i$ - center of pin, $O$ - center of bearing, $OO_i = e$ - eccentricity, $\theta$ - polar angle of point M of the contour $C_i$, $\varphi$ - angle between $OM$ and $O_iM$. From the triangle $OO_iM$, by the sine theorem, we find $\sin \varphi$, and then $\cos \varphi$:

$$\sin \varphi = \frac{OO_i}{OM} \sin \theta = \frac{e}{a - \beta f(\theta)} \sin \theta = e \left[1 + \frac{\beta}{a} f(\theta)\right] \sin \theta$$

$$\cos \varphi = \sqrt{1 - \xi^2 \left(1 + \frac{\beta}{a} f(\theta)\right)} \sin^2 \theta = 1 + O(\varepsilon^2)$$

(1)

where $\varepsilon = \frac{e}{a}$ - eccentricity ratio.
Taking into account (1) for the \( C_0 \) contour in the adopted approximation, we obtain the equation:

\[
OM = a \left( 1 + \varepsilon \cos \theta - \frac{\beta}{a} f(\theta) \right)
\]  

(2)

The initial equations are flow equations of a viscous incompressible fluid for a thin layer, the continuity equation, the equation describing the lubricant flow in the body of the porous bush and the equation describing the radius of the molten contour of the shaft surface.

In the polar coordinate system, the above mentioned system of equations will be written in the following form:

\[
\frac{\partial}{\partial \theta} \left[ a \left( 1 + \varepsilon \cos \theta - \frac{\beta}{a} f(\theta) \right) \right] = \frac{2\mu}{\Omega L} \int_{\Omega(\theta)} \left( \frac{\partial v_\theta}{\partial \theta'} \right)^2 d\theta',
\]

\[
H(\theta) = a(1 + \varepsilon \cos \theta - \frac{\beta}{a} f(\theta))
\]  

(3)

For a certain flow of lubricant in the body of the porous bush, we will assume that the bush is homogeneous and isotropic and that the flow in the body follows Darcy's law.

\[
\vec{V}' = -k \frac{\mu}{\mu} \text{grad} P'
\]  

(4)

Here \( v_r, v_\theta \) - velocity vector coordinates, \( P' \) - hydrodynamic pressure in the lubricating layer, \( k \) - permeability coefficient, \( P' \) - pore pressure in the bush, \( \mu \) - dynamic viscosity of the lubricant, \( a \) - shaft radius to melt, \( b \) - radius of bush, \( L^* \) - specific heat of fusion per the unit of volume, \( \tilde{H} \) - porous layer thickness.

From the equation (4) and the equation of continuity it follows that the pressure in the pores of the bush is equal to:

\[
\frac{\partial ^2 P'}{\partial \theta'^2} + \frac{1}{r'} \frac{\partial P'}{\partial r'} + \frac{1}{r} \frac{\partial P'}{\partial \theta} = 0
\]  

(5)

The boundary conditions for the velocity and pressure in the lubricating and porous layer will be as follows:

\[
\vec{v} = \frac{\Omega \chi O_M}{C_0} \text{ on } C_0;
\]

\[
v_r = -k \frac{\partial P'}{\mu \partial r'}, \ v_\theta = 0 \text{ on } C_2, \ \frac{\partial P'}{\partial r'} = 0 \text{ on } C_3, \ P' = P' \text{ on } C_2, \ P'(0) = P'(2\pi) = P_s, \ a - \beta f(\theta) = h_0 \text{ at } \theta = 0, \ \theta = 2\pi
\]  

(6)

If we design the velocity vector \( \vec{v} = \frac{\Omega \chi O_M}{C_0} \text{ of shaft spindle on the coordinate axis, then we get} \)

\[
v_r = -\Omega (a - \beta f(\theta)) \sin \phi, \ v_\theta = \Omega (a - \beta f(\theta)) \cos \phi, \text{ on the contour } C_0 \text{ taking into account (1)}
\]
\[ v_r = -\varepsilon \Omega a \sin \theta + O \left( \frac{\beta^2}{a^2} \varepsilon \right), \quad v_\theta = \Omega a \left( 1 - \frac{\beta}{a} f(\theta) \right) \] on the contour \( C_0 \)

Before the formulation of an exact self-similar solution to the system of equations (3) and (5) satisfying the boundary conditions (6), we average integrally in the interval \([b, H(\theta)]\), we get:

\[ \frac{\partial v_\theta}{\partial r'} = \frac{\Omega a \left( 1 - \frac{\beta}{a} f(\theta) \right)}{a \left( 1 + \varepsilon \cos \theta - \frac{\beta}{a} f(\theta) \right) - b}, \quad \text{then} \]

\[ \int_b^H \left( \frac{\partial v_\theta}{\partial r'} \right)^2 dr' = \frac{\Omega^2 a^2 \left( 1 - \frac{\beta}{a} f(\theta) \right)^2}{a \left( 1 + \varepsilon \cos \theta - \frac{\beta}{a} f(\theta) \right) - b} \]

Taking into account (8) for the determination \( a - \beta f(\theta) \) we have the following equation:

\[ \frac{d \left( a - \beta f(\theta) \right)}{d\theta} = \varepsilon \sin \theta + \frac{2\mu \Omega}{L} \frac{(a - \beta f(\theta))^2}{(a + \varepsilon \cos \theta - \beta f(\theta) - b)} \]

Integrating this equation, we get

\[ \Phi = 1 - \frac{\beta}{a} f(\theta) = \frac{h_0^*}{a} + \varepsilon \cos \theta + K \int_0^\theta \frac{\Phi^2(\theta) d\theta}{\Phi(\theta) + \xi \cos \theta - \frac{b^*}{a}}, \quad K = \frac{2\mu \Omega a}{L} \]

Solving this equation by the method of successive approximations, we get:

\[ \Phi_0 = \frac{h_0^*}{a}, \quad \Phi_1 = \frac{h_0^*}{a} \varepsilon \cos \theta + K \int_0^\theta \frac{\left( \frac{h_0^*}{a} + \varepsilon \right)^2 d\theta}{\frac{h_0^*}{a} + \varepsilon - \frac{b}{a} + \varepsilon \cos \theta} \]

We integrally average \( \Phi_1(\theta) \) in the interval \([0, 2\pi]\). For the averaged radius of a shaft with a molten surface, we get:

\[ \alpha^* = \frac{1}{2\pi} \int_0^{2\pi} a \Phi_1(\theta) d\theta \]

Taking into account (12) for \( OM \), we finally get the expression in the following form:

\[ OM = \alpha^* + \varepsilon \cos \theta = \alpha^* \left( 1 + \xi \cos \theta \right), \quad \text{where} \quad \xi = \frac{\varepsilon}{\alpha^*} \]

In the lubricating layer, we pass to dimensionless variables by the formulas:

\[ v_r = \delta \Omega u, \quad v_\theta = \alpha^* \Omega v, \quad p' = p^*, \quad r' = b - \delta r, \quad \delta = b - \alpha^*, \quad p^* = \frac{\mu \Omega a^* b}{\delta^2} \]
\[ P' = \rho' P', \quad r' = br' \]  

We substitute (14) and (15) into (3) and (6), respectively, we arrive to the following system of equations and boundary conditions to them (with an accuracy up to the terms \( O\left( \frac{\delta}{b} \right) \))

\[
\begin{align*}
\frac{\partial^2 v}{\partial r^2} &= \frac{\partial p}{\partial \theta}, \quad \frac{\partial u}{\partial r} + \frac{\partial v}{\partial \theta} = 0, \quad \frac{\partial^2 P}{\partial r'^2} + \frac{1}{r'} \frac{\partial P}{\partial r'} + \frac{1}{r'^2} \frac{\partial P}{\partial \theta} = 0, \\
v = 1, \quad u = -\tilde{\eta} \sin \alpha \theta & \text{ at } r = 1 - \tilde{\eta} \cos \theta, \quad \tilde{\eta} = \frac{e}{\delta}, \\
v = 0, \quad u \bigg|_{r=0} = -\tilde{K} \frac{\partial P}{\partial r} \bigg|_{r=1}, \quad \tilde{K} = -k \alpha' b \frac{\partial P}{\partial r}, \quad \frac{\partial P}{\partial r} \bigg|_{r=\frac{b}{b+1}} = 0
\end{align*}
\]  

(16)

Exact self-similar solution of problem (16) - (17) up to the terms \( O\left( \eta^2 \right) \) we will find using following form:

\[
\begin{align*}
v &= \frac{\partial \psi}{\partial r} + V(r, \theta), \quad u = -\frac{\partial \psi}{\partial \theta} + U(r, \theta), \quad \psi = \psi(\xi), \quad V(r, \theta) = \tilde{v}(\xi), \quad U(r, \theta) = \tilde{u}(\xi) \tilde{\eta} \sin \theta \\
\xi &= \frac{r}{1 - \tilde{\eta} \cos \theta}, \quad \frac{dP}{d\theta} = \frac{\tilde{C}_1}{(1 - \tilde{\eta} \cos \theta)^2} + \frac{\tilde{C}_2}{(1 - \tilde{\eta} \cos \theta)^3}
\end{align*}
\]  

(18)

\[ P = R(r^*) \tilde{\eta} \tilde{C}_1 \sin \theta \]

Substituting (18) into (16) and (17), we get:

\[
\begin{align*}
\frac{d^3 \tilde{v}}{d\xi^3} &= \tilde{C}_2, \quad \frac{d^2 \tilde{v}}{d\xi^2} = \tilde{C}_1, \quad \frac{d\tilde{u}}{d\xi} + \xi \frac{d\tilde{v}}{d\xi} = 0, \quad \frac{d^2 R}{dr^*} + \frac{dR}{dr^*} - \frac{R}{r^*} = 0 \\
\tilde{v} &= 0 \text{ at } \xi = 0; \quad \tilde{u} \bigg|_{\xi=0} = -\tilde{K} \frac{dR}{dr^*} \bigg|_{r^*=1}, \quad R(1) = 1, \quad \tilde{v} = 1, \quad \tilde{u} = 1 \text{ at } \xi = 1, \\
\frac{dR}{dr^*} = 0 & \text{ at } r^* = 1 + \frac{b}{h}, \quad \tilde{u}' = 0 \text{ at } \xi = 0, \quad \xi = 1, \quad \int_0^1 \tilde{v} d\xi = \tilde{K} \frac{dR}{dr^*} \bigg|_{r^*=1}
\end{align*}
\]  

(19)

(20)

The solution of the task (19) - (20) in the adopted approximation is found by direct integration. As a result, we have:

\[
\psi' = \frac{\tilde{C}_2}{2} (\xi^2 - \xi), \quad \tilde{v} = \tilde{C}_1 \frac{\xi^2}{2} + \left( 1 - \frac{\tilde{C}_2}{2} \right) \xi - 1, \quad R(r^*) = \frac{r^*}{1 + \left( 1 \frac{h}{b} \right)^2} + \frac{\left( 1 + \frac{h}{b} \right)^2}{1 + \left( 1 + \frac{h}{b} \right)^2} r^*
\]

\[ p = \tilde{C}_2 \tilde{\eta} \sin \theta + \frac{p_s}{p}, \quad \tilde{C}_2 = -\tilde{C}_1, \quad \tilde{C}_1 = \frac{6}{1 + 12KR'(1)}
\]  

(21)

Using formula (19) for the bearing capacity and friction force, we have:
Based on the dependences given in the results (20), according to the numerical analysis data, it was proved that a radial bearing with a low-melting metal coating on the shaft surface and a porous coating on the bearing bush surface has a bearing capacity that exceeds standard radial plain bearings by 9 - 11%. In this case, the friction coefficient decreases by 10 - 12%.

In addition, experimental studies confirmed the conclusions of the theoretical research (Table 1).

Table 1. Experimental studies

| Coefficient of friction | Radial bearing |  |
|-------------------------|----------------|---|
|                         | Standard       | Low-melting metal coating | Low-melting metal and porous coating |
| Parallel experiments    | 0.0038         | 0.0031                      | 0.0021                     |
|                         | 0.0042         | 0.0032                      | 0.0022                     |
|                         | 0.0043         | 0.0034                      | 0.0024                     |
|                         | 0.0045         | 0.0038                      | 0.0028                     |
|                         | 0.0048         | 0.0041                      | 0.0029                     |
| Average                 | 0.0043         | 0.0035                      | 0.0024                     |

3. Conclusion

According to the complex of theoretical and experimental studies of a mathematical model of a radial sliding bearing with a porous layer on its operating surface with a low-melting metal coating on the shaft surface and taking into account all additional factors, the authors obtained the original expression for the calculation of the load capacity of a radial sliding bearing and the magnitude of the friction force in the range under the following loading conditions $v=0.5÷3$ m/s, $\sigma=2÷7$ MPa.

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