ALTITUDE LIMITS FOR ROTATING VECTOR MODEL FITTING OF PULSAR POLARIZATION

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Received 2012 March 16; accepted 2012 June 13; published 2012 August 3

ABSTRACT

Traditional pulsar polarization sweep analysis starts from the point dipole rotating vector model (RVM) approximation. If augmented by a measurement of the sweep phase shift, one obtains an estimate of the emission altitude (Blasikiewicz et al.). However, a more realistic treatment of field line sweepback and finite altitude effects shows that this estimate breaks down at modest altitude \(\sim 0.1 R_{\text{LC}}\). Such radio emission altitudes turn out to be relevant to the young energetic and millisecond pulsars that dominate the \(\gamma\)-ray population. We quantify the breakdown height as a function of viewing geometry and provide simple fitting formulae that allow observers to correct RVM-based height estimates, preserving reasonable accuracy to \(R \sim 0.3 R_{\text{LC}}\). We discuss briefly other observables that can check and improve height estimates.

Key words: methods: numerical – polarization – pulsars: general

Online-only material: color figures

1. INTRODUCTION

After nearly a half-century of pulsar observations, we still do not know the detailed location of the emission zones in the neutron star magnetosphere. However, the general consensus is that the radio emission arises from the “open” field line zone above the magnetic poles at modest altitudes, from a few to a few tens of neutron star radii. In contrast, the \(\gamma\)-ray emission, as measured by Fermi (Abdo et al. 2010), is dominated by high altitudes \(> 0.1 R_{\text{LC}}\), where the light cylinder radius is \(R_{\text{LC}} = c P/2\pi\). Thus, the emission zones and light curves for these two bands generally differ. However, recently Fermi has detected \(\gamma\)-ray emission from a number of millisecond pulsars where the entire magnetosphere is outside of \(R_{\text{NS}} \approx 0.2 / P_{\text{ms}} R_{\text{LC}}\), so that radio emission must be from “high altitude” (Kerr et al. 2012). Further, Karastergiou
& Johnston (2007) and Johnston & Weisberg (2006) have found evidence that for young energetic pulsars, the radio emission is dominated by an altitude of \(\sim 1000\) km \(\sim 100 R_{\text{NS}}\). This is \(\sim 0.2 R_{\text{LC}}\) for \(P = 100\) ms, and it is precisely such young, energetic pulsars which are \(\gamma\)-bright. Thus, if one is interested in \(\gamma\)-emitting pulsars, one must also consider radio emission from an appreciable fraction of the light cylinder radius.

Since the first radio observations, the high linear polarization, and rapid position angle sweep of many pulsars at centimeter wavelength have been used as a clue to the geometry of the emission zone. The foundation for such study is the Radhakrishnan & Cooke (1969) “rotating vector model” (RVM), which follows the sweep of the magnetic field line tangent of a point dipole as projected on the sky. Of course, finite altitude radio emission violates the point source RVM assumption and (Blasikiewicz et al. 1991, hereafter BCW) gave simple approximations for the effects of relativistic aberration at small altitude. In this approximation, the polarization position angle is

\[
\psi = \arctan \left[ \frac{3 r \sin(\zeta) - \sin(\alpha) \sin(\phi + \rho)}{\sin(\zeta) \cos(\alpha) - \cos(\zeta) \sin(\alpha) \cos(\phi + \rho)} \right],
\]

(1)

where the inclination angle between the rotation axis and magnetic axis is \(\alpha\), the viewing angle is \(\zeta\), and the pulse phase is \(\phi\). The RVM formula is recovered in the limit as the scaled emission height, \(r \equiv r_{\text{em}} / R_{\text{LC}}\), goes to zero. Here, the principal effect is a lag in the phase of the maximum rate of the polarization sweep \(d\psi / d\phi|_{\text{max}}\) of \(\Delta \phi \approx 2r\) from the phase of the magnetic axis.

If the absolute position angle of the magnetic axis on the plane of the sky is known (e.g., from the position angle of the spin axis), Hibschman & Arons (2001) show that the observed polarization gives a second height estimate, \(\Delta \psi \approx 10/3r \cos(\alpha)\), where

\[
\psi = \arctan \left[ \frac{-\sin(\alpha) \sin(\phi - 2r)}{\sin(\xi) \cos(\alpha) - \cos(\xi) \sin(\alpha) \cos(\phi - 2r)} \right] + \Delta \psi
\]

(2)

(Dyks 2008). In practice it is generally unclear how to measure the magnetic axis polarization angles; most authors treat \(\Delta \psi\) as a nuisance parameter.

Of course, both formulae presume knowledge of the phase of closest approach of the magnetic axis \(\phi = 0\). The phase of the radio pulse peak is often used, but these pulses can have complex, multi-component morphology. Further, the special relativistic effects shift the intensity peak forward, giving a net observable lag of the polarization sweep from the intensity peak of \(\Delta \phi \approx 4r\). The shifts have been clarified and extended to include the effects of field line sweepback by Dyks & Harding (2004) and Dyks (2008). Nevertheless, observers generally fit to the zero altitude (RVM) limits of the formula to constrain \(\alpha\) and \(\zeta\) and, when possible, estimate the shift of \(d\psi / d\phi|_{\text{max}}\) to constrain the altitude, using the linear (BCW) scaling. While this works adequately for many non-recycled pulsars, relatively high altitude emission is inferred for young energetic objects. For millisecond pulsars the basic RVM model often does not fit well.

Thus, recent strong interest in \(\gamma\)-ray emitting pulsars draws our attention to objects where the radio emission may extend to 0.1 \(R_{\text{LC}}\) or higher, where the standard RVM treatment is suspect. We seek here to quantify this breakdown: if one applies an RVM/BCW fit and obtains estimates of the magnetic inclination angle \(\alpha_f\), viewing angle \(\zeta_f\), and emission height \(r_f\), for what ranges of these parameters are these fits “valid,” i.e., when do the fit values and uncertainty ranges include (at some prescribed
probability) the real value \( r_e \). We develop this analysis as a guide to observers wishing to interpret pulsar polarization data and as an indication to situations where detailed fits to numerical models (e.g., Parent et al. 2011) are required. In addition, we suggest analytic corrections to allow useful \( r_e \) estimates from simple RVM fits to extend to somewhat higher altitude.

2. SIMULATION MODEL ASSUMPTIONS

Our approach is to use a specific three-dimensional magnetosphere model with plausible radio emission zones to “fit” the resulting light curve and polarization sweep with the point dipole RVM formula and to parameterize the errors. For simplicity the field lines are given by the basic swept-back (retarded) dipole popular in models of high altitude \( \gamma \)-ray emission (Romani & Watters 2010) and we assume that the radiating particle bunches follow the magnetic field lines. In the spirit of the RVM model, we make a simple geometric construction, projecting the field line tangent at the emission point in the lab frame onto the plane of the sky and assume that the radio emission is polarized parallel to (or perpendicular to) this vector. We do not attempt here to superpose multiple emission heights or to compute intrinsic polarization fractions. Nor do we include other physical effects such as possible cross-field drift of the emitting charge bunches, current-induced departure from the vacuum structure for energetic pulsars (Spitkovsky 2006), or higher-order multipole/offset dipole effects that may be important in the small magnetospheres of millisecond pulsars (Harding & Muslimov 2011). While our simple construction ignores these possible effects, we do capture the dominant effect of dipole sweepback and our computed polarization sweeps pass smoothly to the RVM model curves at low altitude; the other physical effects likely only dominate very close to the light cylinder.

We assume here that the radio emission comes from a single altitude, within the open zone. We then must define the open zone shape and the illumination across it. Of course, there is a formal cap shape for the vacuum retarded dipole solution, where the locus of field line tangents to the light cylinder traces to a cap on the surface with opening angle \( \theta_{\text{cap}}(\phi_{\text{cap}}) \) varying with azimuth \( \phi_{\text{cap}} \) around the magnetic axis. Alternatively, it is common to assume a simple circular cap, with surface angle \( \theta_{\text{c}}(\phi_{\text{cap}}) = \text{constant} \). To roughly match the open zone beam sizes at an emission height of 0.1 \( R_{\text{LC}} \) we chose a surface cap angle of \( \theta_{\text{c}} = 2^\circ \) for a neutron star of \( R_{\text{NS}} = 10^{-3} R_{\text{LC}} \), i.e., a \( \sim 0.2 \) s pulsar.

For simplicity and to follow the BCW picture, we illuminate the open zone with a simple Gaussian profile

\[
I \propto e^{-(\theta_{\text{cap}}/\theta_0)^2}, \quad \text{with} \quad \theta_0 = 2^\circ/\sqrt{\ln 5} \quad (3)
\]

so that the intensity falls by 5× at the “edge” of the simple circular cap. The angles are measured at the star surface, although the corresponding radio flux may be emitted at high altitude. We note that there is some evidence that a conal intensity distribution with a patchy illumination may be more typical of many pulsars (Lyne & Manchester 1988; Karastergiou & Johnston 2007).

To generate a model polarization sweep, we select a magnetic inclination, \( \alpha_r \), and emission height, \( r_e \). We then project the swept-back field lines at this altitude to the plane of sky and record the results on a two-dimensional sky map. Horizontal cuts across this map at a given viewing angle, \( \zeta_r \), give the polarization angle sweep, \( \psi(\phi) \). We assign “measurement” errors to each value inversely proportional to the pulse flux at its phase. We assume that the observer’s integration achieves a uniform signal to noise (S/N) at pulse maximum, so that the polarization measurement error there is 1°. For pulsars observed far from the magnetic axis at large \( |\beta| \approx |\zeta - \alpha| \) this implies longer integration. As the pulse flux falls toward the edge of the open zone the polarization angle uncertainties increase.

2.1. Estimating \( \phi = 0 \)

Use of the simple Gaussian illumination with the pulse phase at the intensity peak (the projected phase of closest approach to the magnetic axis) corresponds to the BCW assumptions. Except for very high altitude emission, where field lines overlap in the sky map and pulse caustics can occur, this gives a simple prescription from which \( \phi = 0 \) may be estimated via the BCW shift. However, conal emission concentrated to the cap edge significantly complicates the determination of pulse phase. One effect is the variable sweepback at the leading and trailing edge of the cap. Another is the particular shape of the open zone boundary. We illustrate these effects by marking a “peak phase,” the midpoint of the projected open zone boundary, both for a simple circular cap and for the more detailed retarded dipole cap. Figure 1 displays the peak phase shifts for these different definitions.

Not unexpectedly, Figure 1 shows that the pulse phase is more sensitive to the details of the open zone geometry for a conal emission zone. The offsets shown there illustrate the effect of the retarded potential field line flaring at high altitude. To this should be added the uncertainties associated with identifying the magnetic axis phase in the presence of patchy conal emission and non-dipole field structure (for millisecond pulsars). Nevertheless, as we shall show, a substantial fraction of the phase offset is insensitive to the choice of cap center and can be corrected.

2.2. “Fitting” an RVM Curve

The retarded dipole field structure increasingly departs from the point dipole as the scaled emission height \( r = r_e/R_{\text{LC}} \) approaches unity. Thus, if one fits polarization data for a

![Figure 1](image_url)
Our principal goal here is to test the utility of standard RVM fits and to provide a prescription to allow these fits wider applicability for pulsars with high altitude emission. To do this we compare the RVM fit estimate $r_f$ with the simulated value $r_r$. Since the mapping is not simple, statements about ranges of validity are perforce statistical. This makes our answers mildly sensitive to the distribution in the underlying pulsar population. Here, we assume that our parent pulsar population has isotropically distributed inclination and viewing angles, i.e., $\text{Prob}(\alpha) \propto \sin(\alpha)$, $\text{Prob}(\zeta) \propto \sin(\zeta)$ while the altitude is distributed uniformly on $0 \leq r_f \leq 0.3$. Note that we only observe a usable polarization sweep if a pulsar produces a minimum number of phase bins (here $\Delta \phi_{\text{obs}} > 0.1$). In turn, this means that our observable pulsar population is biased toward modest $|\beta| = |\alpha - \zeta|$.

We generate a set of pulsar models and apply the RVM fits. This delivers a set of observables $\alpha_f, \zeta_f, r_f; \sigma_{\alpha}, \sigma_{\zeta}, \sigma_r$, where the fit values are determined by $\chi^2$ minimum and the error ranges are estimated from the curvature of the $\chi^2$ surface. An observer presented with this set of measurements must infer the original pulsar properties.

Focusing here on the height measurement, we test the systematic bias in the RVM estimate. For best comparison with the BCW assumptions, we work with the height determined from the phase lag measured from the peak of a Gaussian pulse centered on the magnetic axis. In Figure 2, the color scale represents the number of pulsars in the simulated population at a given altitude derived from fitting RVM versus the difference between fitted and real altitude. Figure 2 shows that $r_f$ increasingly underestimates $r_r$ at increasing altitude. A simple formula to provide improved height estimates $r_f'$ from RVM fits is then

$$r_f' = r_f + 0.2(r_f/0.5)^2,$$

as plotted in Figure 2. The line fits best to the darkest ridge (the ridge that contains a majority of simulated pulsars) for the
models using the maximum of a simple Gaussian intensity peak ($G_C$) and the center of the cap edges for a circular cap ($C_C$). For the case using the center of the cap edge for a retarded dipole cap ($C_R$), the line slightly underpredicts the darkest ridge and does not capture the behavior of the second ridge which is caused by the shift of the central line from the cap notch (see Figure 1). We can (unrealistically) assume that we know where in phase the magnetic axis is located and calculate the altitude from the shift in polarization directly. Inaccuracies in altitude are then from Equation (2) alone. With the assumption of perfect knowledge of the magnetic axis ($E_B$), we see that the departure from the BCW formulation occurs at lower altitudes. Apparently, the estimate $\Delta \phi = -2r_f \phi$ for the peak intensity shift preserves good accuracy to higher altitude than the $\Delta \phi = +2r_f \phi$ shift of the polarization sweep, especially when the intensity arises from a circular cap.

In practice, the height offset depends on the geometrical angles ($\alpha, \zeta$). In addition, the height estimate is affected by uncertainty in estimating the polarization sweep lag, i.e., in determining the phase of the pulse (or equivalently the phase of the magnetic axis). These effects are shown in Figure 3. For each panel we show, as a function of the estimated angles ($\alpha_f, \zeta_f$), the maximum height (color bar) at which the estimated altitude is accurate. For the estimate to be useful, we require that $r_f$ lies in the range $r_f \pm \sigma_{rf}$ for a large fraction (99%) of the observable model pulsars. At small altitude this is always true. At large altitude the distortion due to the retarded field structure causes increasing departure from the BCW estimate. Once too small a fraction of models produce useful fits, the BCW approximation “breaks down.” Lowering the required fraction does not drastically change the results seen in Figure 3, since the fraction of failing models increases very rapidly with fit altitude. Also shown is a green contour that marks the area where the bins contain at least 50 simulated pulsars. Uncolored bins are where the BCW approximation is inaccurate at the lowest altitude. The contours are independent of the intensity model (the contours are the same for each model) because the $\alpha_f, \zeta_f$ bin depends only on the polarization sweep which is calculated independently of the intensity model.

A strong dependence between the breakdown altitude and $\alpha_f$ and $\zeta_f$ exists as can be seen in Figure 3. This is not due to any difficulty in finding the phase center but arises from the nontrivial relation between the shift in the maximum sweep of the polarization and the geometry angles. In Figure 3, we can see that for $\alpha_f$ and $\zeta_f$ further from 90°, BCW tends to break down at a lower altitude. The shift in the maximum sweep of the polarization angles for these values is smaller than predicted by the BCW model. Since the BCW model has no dependence on $\alpha$ and $\zeta$, it is not surprising that the breakdown altitude has a dependence on these angles.

The panels show the maximum useful height for four different estimates of the phase lag: (top-to-bottom) perfect knowledge of the magnetic axis, a Gaussian pulse peaked on the magnetic axis field line, a “conal” pulse from a field line with a circular cap on the star, and a “conal” pulse with a cap determined by the detailed open zone of the retarded vacuum solution. Note that most observed pulsars have modest $|\beta| = |\zeta - \alpha|$ and are close to the diagonal. The right panels show the equivalent maximum useful height when the estimate has been corrected according to Equation (4). While the uncorrected estimates for the Gaussian pulse peak model are useful only to an average (over $\alpha_f$ and $\zeta_f$) height of $R_f = 0.11R_{LC}$, the corrected estimates are usable to higher altitudes (reaching $r'_f \sim 0.3$ for the commonly observed case of near-orthogonal rotators) with an average of $R_f = 0.22R_{LC}$. Again, corrected RVM estimates from a model radio pulse do better than estimates assuming perfect knowledge of the magnetic axis, since the retarded potential phase shifts are a fractionally larger contribution to the phase offset in this case.

We can improve the heuristic correction function by including the viewing geometry. The bulk of the sensitivity is evidently due to $\zeta_f$, as illustrated by the relatively small dispersion of the $r_f$ error for individual $\zeta_f$ slices (see Figure 4 for a Gaussian central pulse). Accordingly, we have made an alternate corrected height estimate

$$r'_f = r_f + [0.3 + 0.7(|\cos(\zeta)|)](r_f/0.5)^3,$$

where $r_f = \Delta \phi/4$, as usual. This greatly extends the range for which a simple RVM height estimate can be used (Figure 5). This estimate, based on a Gaussian radio pulse emitted along the swept-back magnetic axis, is in general the best function...
for an observer to use with no other information. It provides significant improvement in the emission height accuracy for the circular cone pulse profiles.

Of course if one has reason to believe that a particular pulse profile shape is more accurate, a different correction function may be preferred. For example if one had a double pulse arising from the open zone edges (\(C_R\)) and had high confidence that this pulse filled the retarded vacuum dipole open zone, one would correct by

\[
r_f' = r_f + [0.3 + 2 \cos(\zeta)]^2 r_f / 0.5^3.
\]

This formulation raises the average over \(\alpha_f\) and \(\zeta_f\) of the maximum useful height from \(\bar{r}_f = 0.05 R_{LC}\) with no correction to \(\bar{r}_f = 0.15 R_{LC}\).

In general, we recommend that when an observer fits an RVM model to pulsar data, obtaining viewing angle and polarization sweep lag measurements, they correct their height estimate using Equation (5). This is particularly useful whenever the RVM fit appears statistically adequate, but the resulting phase lag suggests a significant emission height. The change to the estimated height will be small for \(r_f < 0.2\), but the accuracy of the resulting estimate will be greatly increased.

Of course, whenever \(\chi^2 / \text{DoF} \gg 1\) at the fit minimum, it is a sign that the model is inadequate. In many cases, this will be due to unmodeled orthogonal mode jumps and intervening scattering (Karastergiou 2009), higher order multipoles, etc. However, for large altitudes and multi-altitude emission the effects of sweepback and the formation of caustics (which dominate \(\gamma\)-ray light curves) become dominant. The observer should be aware that large \(\chi^2\) at the fit minimum can signal such effects and, when the inferred altitude is large, consider fitting the data to numerical models of three-dimensional pulsar magnetospheres.

\section{4. HEIGHT CALCULATION FROM SHIFT IN \(\psi\)}

We can alternatively estimate \(r_f\) and errors using the shift in \(\psi\) (Hibschman & Arons 2001),

\[
\Delta \psi \approx \frac{10}{3} r_f \cos(\alpha) \left[ \frac{3}{8} + \frac{5}{8} \cos(\alpha - \zeta) \right] - \frac{47}{18} r_f \sin(\alpha) \sin(\xi - \alpha)
\]

(7)

or, in the small \(|\beta| = |\zeta - \alpha|\) limit, \(\Delta \psi \approx 10/3 r_f \cos(\alpha)\). As before, we compute the residual, \(r_f - r_f'\), as a function of \(\alpha_f, \zeta_f\), and \(r_f\). To estimate an emission height from the polarization shift in \(\phi\), one needs an estimate for \(\phi = 0\), e.g., from a pulse peak intensity model; no such intensity model is needed if we have a measurable shift in \(\psi\). The increase with \(r_f\) is shown in Figure 6, where the left panel uses the small \(\beta\) limit while the right uses the full formula. As for the \(\Delta \psi\) estimate, the errors increase with \(r_f\). However here, even when the full equation (7) is used, the corrections show a substantial spread. In fact the uncorrected formula proves accurate (\(|r_f - r_f'|\) within \(\sigma_{r_f}\) 99% of the time) only for \(\bar{r}_f < 0.08\) (where \(\bar{r}_f\) is again the average over \(\alpha_f\) and \(\zeta_f\)) and for \(\xi_f < 60^\circ\) or \(\zeta_f > 120^\circ\). For near-orthogonal rotators the estimate is unreliable at the lowest altitudes.

A heuristic correction to the \(\Delta \psi\) estimate for Equation (7) can be made for \(\xi_f < 60^\circ\) or \(\xi_f > 120^\circ\)

\[
r_f' = r_f + 0.4 (r_f / 0.5)^2
\]

(8)

which allows accurate estimates to \(\bar{r}_f = 0.12 R_{LC}\). Including the \(\zeta\) dependence,

\[
r_f' = r_f + [0.2 + 0.1 \cos(\zeta)] (r_f / 0.5)^2
\]

(9)

raises the useful range to \(\bar{r}_f = 0.18 R_{LC}\). Considering that the correction for the common orthogonal rotator case is especially poor, and that it is often difficult to infer the intrinsic \(\psi_0\), height estimates from the phase shift remain much more useful.
5. PULSE WIDTH DEPENDENCE ON EMISSION HEIGHT

Since the field lines flare in the open zone, the full phase width $W$ of the observed radio pulse can also be checked against the expected radio emission altitude. The standard prescription assumes a circular cap and static dipole field lines to infer a minimum height

$$r_W = \frac{4}{9} \arccos^2 \left[ \cos(\alpha) \cos(\xi) + \sin(\alpha) \sin(\xi) \cos \left( \frac{W}{2} \right) \right].$$

(Equation (10))

In Figure 7, we show that the retarded dipole field flares more than predicted by this simple formula and hence the minimum height in Equation (10) is an overestimate. Thus, in general, lower altitudes are consistent with a given observed pulse width than suggested by this formula. Moreover, we expect that the general effect of currents in the magnetosphere will be to increase the footprint angles of the open zone. This further increases the allowed $W$ at a given height.

In general, larger widths are still most easily accommodated at large $r$ or small $\alpha$, but sweepback and magnetospheric currents substantially weaken the minimum altitude constraints from the commonly used Equation (10). Given the large sensitivity to the details of the open zone volume and the presently unknown effect of magnetospheric currents, it is not worth developing corrections to this formula.

6. CONCLUSIONS; EXAMPLES FROM LITERATURE

We conclude by examining a few RVM/BCW estimates of emission height present in the literature.

In Romani et al. (2011), $\Delta \psi$ estimates were used to suggest large emission heights for two young energetic pulsars. For J0538+2817 the shift gives $r_f = 0.15 R_{LC}$, but RVM fitting only weakly constrains $\zeta$. Applying Equation (4), we would infer $r_f' = 0.17 R_{LC}$, a small, but significant, increase which makes it easier to accommodate the large observed pulse width. Similarly PSR J1740+1000 gives $r_f = 0.12 R_{LC}$. Here, we constrain $\zeta = 80^\circ$ to $130^\circ$, so that the corrected fit altitude (Equation (4) or Equation (5)) is $r_f' = 0.13 R_{LC}$, again a small but statistically significant increase.

For millisecond pulsars the effects can be larger. For example, Keith et al. (2012) find that RVM fitting can be usefully applied to several recycled pulsars. PSR J1502−6752 ($P = 26.7$ ms) is a mildly recycled pulsar for which the phase lag implies $r_f = 0.2 R_{LC}$. With no significant $\zeta$ constraints, we apply Equation (4) to infer a 16% altitude increase to $r_f' = 0.23 R_{LC}$.

Similarly PSR J1708−3506 ($P = 4.5$ ms) has a phase shift implying 0.19 $R_{LC}$, which we correct to 0.21−0.22 $R_{LC}$. For this pulsar, a naive application of the pulse width formula (10) gives altitudes of $r_{w,0} \simeq 0.65 R_{LC}$ (10% peak width). However, the increased $r_f'$ and decreased pulse width height from sweepback effects (Figure 7), along with additional current-induced open zone growth, make it likely that the pulse width can be accommodated at the corrected height.

Keith et al. (2012) also report an RVM/BCW height $r_f = 0.44 R_{LC}$ for the $P = 2.7$ ms pulsar J1811−2404, along with well-constrained viewing angles of $\alpha = 89.7^\circ$ and $\beta = 21^\circ$. While our full analysis does not cover this altitude, as Figure 5 shows the corrections of Equation (5) give a very high accuracy for orthogonal rotators viewed near $90^\circ$. Note in Figure 4, bottom panel, that the correction function is nearly linear, thus extrapolation to somewhat higher values may be justified. Naively applying this correction we get $r_f' = 0.81 R_{LC}$. We certainly cannot trust this value in detail since plasma effects and other perturbations may be relevant at such altitudes. However, the correction is certainly large and it brings the expected height up to an altitude where the very wide observed radio pulse, and the likely detection of emission from both open zones, can be easily accommodated. Certainly simple RVM/BCW fitting is inadequate for this pulsar and one should use a detailed model for the high altitude field geometry.

Our exercise extends the range of utility of RVM-fit polarization sweeps for inferring the altitudes of radio pulsar emission. For fit altitudes less than $r_f = 0.25 R_{LC}$ the corrections are not large, but they are systematic and, for high S/N data localizing the phase of maximum polarization sweep, they can be highly significant. We thus believe it is worth applying our recommended correction. For larger altitudes the corrections grow rapidly, but we caution that as one approaches the light cylinder, current-induced distortions should increase and, except for near-orthogonal rotators, one would expect the RVM formulae to provide a poor fit in any case. Fitting to detailed numerical models is then preferred. In all cases, the dominant residual uncertainty is likely in locating the phase of the radio pulse. We also checked the use of absolute polarization axis position angles and pulse width to constrain the emission height. Here, the difficulties in establishing the unperturbed $\psi_0$ and the
expected distortions of the open zone boundaries by currents, etc., make the estimates much less useful. Nevertheless, we have shown that the effects of sweepback do go in the direction of reconciling observed pulsar properties to a consistent emission height: larger heights are inferred by a given $\Delta \psi$ shift and larger pulse widths can be accommodated at a given height. We feel, however, that the corrections are less quantitative than for $\Delta \phi$.

In sum, since observers will continue to apply analytic RVM fits to pulsar polarization data, by applying our recommended correction (Equation (5)), these results can continue to give accurate height estimates to $\leq 0.3 \, R_L$. At higher heights which will be common for millisecond pulsars, a fit to more detailed numerical models is likely warranted.

This work was supported in part by NASA grants NNX10AP65G and NAS5-00147.

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Figure 7. Altitude limits for effective pulse width. Each panel shows the distribution of simulated model fits (color bar) in offset from the true altitude as a function of fit altitude $r_f/R_L$. $W_C$: circular cap. $W_R$: retarded dipole cap. The simple static dipole formula overestimates the altitude needed to accommodate a given pulse with $W$ in the open zone. The error depends on the viewing geometry $\alpha$ and $\zeta$, and bifurcates for the “notched” cap of the formal retarded potential open zone.

(A color version of this figure is available in the online journal.)