Computational research on the formation mechanism of double humps in pump–turbines

Yong Liu, Dezhong Wang and Hongjuan Ran

School of Mechanical Engineering, Shanghai Jiaotong University, Shanghai, People’s Republic of China

ABSTRACT

The hump formation mechanism has not been fully understood so far, mainly because of two difficulties in analysis. The first one is how to find the most important external characteristic factor inducing humps, and the other is how to avoid too many assumptions when analysing the change in the runner’s capacity for work (the direct use of Euler’s formula makes too many assumptions). This paper proposes a new method based on the second derivative of the external characteristics to identify the most critical factor inducing humps quantitatively. The runner blades’ capacity for work is calculated and shown as a cloud diagram using as few assumptions as possible to help understand the actual relationship between flow fields and humps. Additionally, the importance of stall in affecting the runner’s capacity for work is analysed and explained. Thus, the double hump formation mechanism in pump–turbines has finally been understood.

ARTICLE HISTORY

Received 8 June 2021
Accepted 3 September 2021

KEYWORDS

Flow simulation; pump–turbine; hump; flow instability; stall; flow separation

1. Introduction

As the key energy conversion component of pumped-storage power stations, pump–turbines have the functions of both pump and turbine. Owing to their special structure, pump–turbines have hump problems under pump mode. During the process of pump starting, stopping, and working conditions changing, severe vibration and noise appear when passing through the hump region, and the loop flowrate oscillates hugely, even leading to pump surge. Humps not only threaten the safety, reliability, and service life of pumps, but also lead to operational failure (Lu, 2018; Yin, 2012). With the continuous large-scale production of renewable energy by means such as nuclear power, solar power, and wind power, the power grid system increasingly relies on pumped storage technology to cope with load changes, and pump–turbines need to be started and stopped more frequently. Thus, hump problems are even more critical (Capelo et al., 2017; Li, 2017; Li et al., 2020; Sampredo et al., 2021). Pump–turbines have always developed rapidly in the direction of higher heads, and the maximum head of a single-stage pump–turbine has reached about 800 m. How to deal with humps is becoming more and more important (Li, 2017; Li et al., 2017). Hump problems are key to restricting the development of pumped storage technology, and it has always been a research hotspot, but the hump formation mechanism has not been fully understood.

It needs to be emphasized here that there is a positive slope in the hump region, which is the source of various problems, while the sudden head drop under large flowrate conditions does not destroy the monotone decreasing curve so that it does not destroy the stable operation of the pump–turbine. They are absolutely two different situations. After long-term research, many scholars have found that hump formation is closely related to the flow field (Braun, 2009; Eisele et al., 1998; Güllich, 2008; Zhou, 2015). Güllich (2008) found that flow regime change caused a sudden increase in hydraulic loss and caused hump. Lu (2018) and Ye et al. (2020) indicated that flow regime change was mainly related to change in flow separation intensity. Fisher and Webb (1978) concluded that backflow at the runner inlet was an important cause of hump, while Guedes et al. (2002) observed the main unstable flow at the runner outlet through the particle image velocimeter (PIV) and the Laser Doppler Velocimetry (LDV) tests. Braun et al. (2005) and Gentner et al. (2012) also had similar findings. Li et al. (2016), Lino and Tanaka (2004), Ran and Luo (2018), and Ran et al. (2020) validated that flow separation in the guide vane and runner was at the root of hump, while other scholars concluded that stall in the guide vane was the more important cause (Guedes et al., 2002; Shibata et al., 2016;
Wang et al. (2011). Zhou (2015) conducted simulations and found that stall in the runner did not cause hump. Sun (2016) declared that the main reason for hump was the excessive hydraulic loss caused by stall in the guide vane. The same conclusion was obtained by Xiao et al. (2016), Yang (2015), and Yang et al. (2018). With developing research, scholars have gradually discovered that the change in runner’s capacity for work is also an important reason for hump. Lu (2018) found that overall hydraulic loss did not increase much before or after hump and concluded that hump was caused by both increasing hydraulic loss and a change in runner’s capacity for work. This opinion was confirmed by Li (2017), Tao et al. (2014), and Xiao et al. (2014). Lu et al. (2019) indicated that stall in the guide vane increased hydraulic loss, enhanced the runner’s capacity for work, and caused hump.

It is still unclear whether an increase in hydraulic loss or a change in runner’s capacity for work plays the key role in hump formation, because the investigation methods do not yet have full confidence. Simply comparing hydraulic loss under hump conditions with that under other conditions is not enough to quantify its significance in hump formation. The method of evaluation comparing the first derivative of the head coefficient with the discharge coefficient was adopted by Lu et al. (2019). It is not ideal yet, because the change of the slope is closely related to the hump, not the magnitude of the slope. Regarding how to trace the hydraulic loss to the flow field, both entropy generation analysis and energy-equation-based loss analysis can achieve accurate results (Li et al., 2017; Lu, 2018; Wilhelm et al., 2016). However, there is a lack of an accurate and effective method to trace the runner’s capacity for work to the flow field. Li et al. (2015, 2018) applied Euler’s pump formula to establish a relation between the runner’s capacity for work and the flow velocities at the inlet and outlet of the runner. This method works well, but it cannot display the capacity for work at any position of the blade. Additionally, there are many assumptions in the derivation of Euler’s formula. In particular, it is assumed that the flow velocity and flow angle at the inlet and outlet of the runner are exactly the same in the circumferential direction, and that the flow is always in the plane perpendicular to the pump axis (Luo, 2007). The farther the operating condition is from the design point, the greater the error produced by Euler’s formula. As the hump is often far from the design point, Euler’s formula inevitably brings more errors to the hump analysis.

The hump formation mechanism has not been fully understood so far mainly because of two difficulties in analysis. The first one is how to find the most important external characteristic factor inducing hump, and the other is how to avoid too many assumptions when analysing the change in the runner’s capacity for work (the direct use of Euler’s formula makes too many assumptions). This paper proposes a new method based on the second derivative of the external characteristics to identify the most critical factor inducing humps quantitatively. The runner blades’ capacity for work is calculated and shown as a cloud diagram using as few assumptions as possible to help understand the actual relationship between flow fields and humps. Additionally, the importance of stall in affecting the runner’s capacity for work is analysed and explained. Thus, the double hump formation mechanism in pump–turbines has finally been understood.

2. Pump–turbine model and numerical schemes

2.1. Model and simulation settings

A pump–turbine with medium specific speed consisting of draft tube, runner, guide vanes, stay vanes, and volute is researched in this paper, as shown in Figure 1. The main design and performance parameters under the design point of the pump mode are listed in Table 1. Hexahedral grids generated by ANSYS®-Turbogrid and ANSYS-Icem are adopted in the fluid domain of the draft tube, runner and guide vanes, and unstructured grids generated by ANSYS-Icem are used in fluid domain of stay vanes and volute, as shown in Figure 1 (Ez Abadi et al., 2020; Ghalandari et al., 2019). Grids near walls are refined to

![Figure 1. Pump–turbine geometry and mesh.](image)

| Parameter | Value | Parameter | Value |
|-----------|-------|-----------|-------|
| $Z_r$     | 9     | $Z_g/Z_s$ | 20    |
| $D_1$     | 79.48 mm | $\alpha$ | 26°   |
| $D_2$     | 119.77 mm | $D_0$ | 134.28 mm |
| $b_2$     | 17.13 mm | $\alpha_5$ | 17.18° |
| $\beta_2$ | 27°   | $n_0$     | 49    |
| $\delta$  | 10.41° | $Q_{BEP}$ | 7.02 kg/s |
| $b_3/b_4$ | 16.84 mm | $H_{BEP}$ | 2.91 m |
Table 2. $Y^+$ of key components.

| Component    | Maximum | Minimum | Mean |
|--------------|---------|---------|------|
| Runner       | 2.200   | 0.004   | 1.200|
| Guide vanes  | 3.169   | 0.006   | 2.830|
| Stay vanes   | 9.620   | 3.100   | 7.760|

Figure 2. Grid schemes.

ensure that the $Y^+$ level is one in key components, as shown in Table 2.

Figure 2 shows five schemes with different grid densities, which are adopted under hump operating conditions to verify grid independence, as shown in Figure 3. Energy coefficient $E_{nD}$ and torque coefficient $T_{nD}$ in Figure 3 are obtained according to the provisions of the International Standards Committee (IEC).

\[ Q_{nD} = \frac{Q}{nD^3} \]  
\[ E_{nD} = \frac{gH}{n^2D^2} \]  
\[ T_{nD} = \frac{T}{\rho n^2D^2} \]

where $E_{nD}$ and $T_{nD}$ and the discharge coefficient $Q_{nD}$ are all dimensionless numbers. $Q$, $n$, $D$, $H$, $g$, $\rho$, and $T$ represent discharge, rotation speed, nominal runner diameter, head, gravitational acceleration, fluid density, and shaft torque, respectively. Figure 3 indicates that, when the grid density becomes larger than that of Scheme 3, $E_{nD}$, efficiency $\eta$, and $T_{nD}$ tend to be stable with a small change of less than 3%. Considering calculation accuracy and cost, Scheme 4 with 13.499 million grid cells is finally selected for simulation.

ANSYS CFX 19.0 is employed to conduct the flow simulation in this paper. Mass flow rate and static pressure are set as the inlet and outlet conditions, and the turbulence intensity of flow at the inlet is 5%. All walls are defined to be smooth and non-slip. High Reynolds number flow in the pump–turbine is predicted by the SST $k$–$\omega$ model, which uses the low Reynolds number $k$–$\omega$ model in the boundary layer region, and applies the high Reynolds number $k$–$\epsilon$ model in the free shear layer to capture the flow separation under reverse pressure gradient. The SST $k$–$\omega$ model has been verified to predict humps in pump–turbine accurately (Li, 2017).

The interfaces between the rotor and stators are coupled by a frozen rotor method that fully considers the uneven distribution of the flow parameters in the circumferential direction of the interface. The interface setting method of None mode is used for the interfaces between stators. The high-resolution scheme is set for the advection scheme and turbulence numeric. For convergence control, the RMS (root mean square) residual for each time step is set below $10^{-5}$ with a maximum iteration of 1000 steps in the steady simulation and $10^{-5}$ with a maximum iteration of 10 steps for each time step in unsteady simulation. A time step of $2.564 \times 10^{-4}$ s, corresponding to a runner-rotating angle of $2^\circ$ per time step is used in unsteady simulation. The duration of unsteady

Figure 3. Grid independence test.
Table 3. Calculation results selected for flow field analysis.

|                  | 1st hump valley | 1st hump peak | 2nd hump valley | 2nd hump peak |
|------------------|-----------------|---------------|-----------------|---------------|
| Initial condition|                 |               |                 |               |
| Timestep selected|                 |               |                 | The 1800th unsteady calculation timestep |
| Steady calculation results |                 |               |                 |               |

Figure 4. Residual convergence curve in simulation.

calculation is 1 s, corresponding to about 20 rotations of the runner. The external characteristic curve in pump mode is predicted to be in the INC direction (discharge increasing) and the DEC direction (discharge decreasing), respectively, by the method of continuously using the previous calculation result as the initial file of the next calculation (Li, 2017). In flow field analysis, unsteady calculation results are adopted, which are initialized by steady calculation results. In order to ensure that the position relationship between runner and guide vane is consistent in the analysis of different working conditions, the timestep corresponding the end of the 10th runner revolution is selected, namely the 1800th unsteady calculation timestep as shown in Table 3. Until this timestep, the unsteady calculation is sufficiently stable and credible, and the residual convergence curve is shown in Figure 4.

2.2. Experimental validation

The external characteristic in pump mode is tested on the test bench as shown in Figure 5. Runner speed $n$, pump shaft torque $M$, discharge $Q$, draft tube inlet pressure $P_{inlet}$, and volute outlet pressure $P_{outlet}$ are collected by torsiograph, vortex flowmeter, and pressure sensor. The signals collected by all sensor are sent to an NI USB-6003 acquisition card connected to the PC. The uncertainty of key measurement equipment is listed in Table 4. According to the equipment uncertainty and the random uncertainty calculated by multiple test records, comprehensive uncertainty can be obtained. The comprehensive uncertainty of efficiency in the experiment is 0.65%, which is the comprehensive value of the comprehensive uncertainty of flowrate, head, rotational speed, and torque, and can represent the test error. The test was carried out in the discharge range 0.4–1.2$Q_{BEP}$ in the INC and DEC directions, respectively. For each operating condition, data of 10 s duration is collected after the operating condition has been stable for at least 1 min, and the average value is calculated to reduce the random error.

Comparison between the simulation and test results is shown in Figure 6. The test results show double humps in the energy coefficient curve: the 1st hump is close to the design point, and the 2nd hump is located under a small discharge condition far from the design point. The test and calculation results are generally in good agreement, and the highest error of $E_{nD}$ occurs at small flowrate, not exceeding 9.4%. The reason for the $E_{nD}$ error is that the pressure sensor range greatly exceeds the actual pressure, resulting in a slight lack of linearity near the measured pressure. The linearity problem mainly affects the overall height of the $E_{nD}$ curve, but has little effect on its changing trend. Therefore, the measured hump position is close to the simulation result. Overall, the simulation results are credible.

3. Numerical analysis

3.1. Analysis of the most critical factor causing hump

If the external characteristic parameters are regarded as functions of the discharge coefficient, the following relationship exists:

$$E_{nD}(Q_{nD}) = E_{nD}^t(Q_{nD}) + E_{nD}^{\text{loss}}(Q_{nD})$$

$$= E_{nD}^t(Q_{nD}) + \sum_{k=1}^{5} E_{nDk}^{\text{loss}}(Q_{nD})$$

(4)

where $E_{nD}^{\text{loss}}(Q_{nD})$ represents the dimensionless hydraulic loss and is agreed to be a negative value, $\sum_{k=1}^{5} E_{nDk}^{\text{loss}}(Q_{nD})$ represents the dimensionless hydraulic loss of each flow component, and $k = 1, 2, \ldots, 5$ in turn represents the draft tube, runner, guide vane, stay vane, and volute. $E_{nD}^t(Q_{nD})$ calculated from the theoretical head $H_t$ according to Equation (2) represents the runner’s capacity for work excluding hydraulic loss, and $H_t$ is solved as
Figure 5. The test bench: (a) schematic of the test bench; (b) the test bench picture; (c) the vortex located between the draft and the runner (in turbine mode).

Table 4. Equipment specifications.

| Equipment         | Manufacturer         | Type | Range          | Precision       |
|-------------------|----------------------|------|----------------|-----------------|
| Torsigraph        | Szechwan Chengbang   | YB1  | 0–50 Nm        | Revolution speed ≤ 0.2%FS |
|                   |                      |      | 0–6000 rpm     | Torque ≤ 0.5%FS  |
| Pressure sensor   | Kunshan Danrui       | TS110| −100–100 kPa   | ≤ 0.25%FS       |
| Vortex flowmeter  | Shanghai Kente       | KVFN-23| 0–51.43 m³/h  | ≤ 0.55%FS       |

follows:

$$H_t = \frac{T \omega}{\rho g Q}$$  \hspace{1cm} (5)

where $\omega$ is the rotational angular velocity of the runner.

Hump refers to the interval where the slope of the $E_{nD}$-$Q_{nD}$ curve changes from negative to positive, as shown in Figure 7(a). Actually, there are differences in hump position along the INC and DEC directions, and Figure 7 ignores the difference for simplicity. The slope change can be expressed by the second derivative of $E_{nD}$, and Equation (6) can be derived after the second derivative of $Q_{nD}$ has been calculated on both sides of Equation (5). Then, the ‘Deflection Contribution Factor’ $C_f$ and the ‘Deflection Degree’ $D_f$ can be defined as shown in Equations (7) and (8):

$$d \left( \frac{dE_{nD}}{dQ_{nD}} \right) / dQ_{nD} + d \left( \sum_{k=1}^{5} \frac{dE_{loss,nD}}{dQ_{nD}} \right) / dQ_{nD} = d \left( \frac{dE_{nD}^t}{dQ_{nD}} \right) / dQ_{nD} \quad (6)$$
for the INC and DEC directions, the discretization formulas are given as follows:

\[
C_f(\epsilon) = \left( \frac{d}{dQ_{nD}} \frac{d\epsilon}{dQ_{nD}} \right) / \left( \frac{d}{dQ_{nD}} \frac{dE_{nD}}{dQ_{nD}} \right) \quad (7)
\]

\[
D_f(\epsilon) = \frac{d}{dQ_{nD}} \frac{d\epsilon}{dQ_{nD}} \quad (8)
\]

where \( \epsilon \) represents \( E_{nD}^t \), \( E_{nD}^{loss} \), and \( E_{nD}^{loss} \). The actual \( E_{nD}^{loss} \) curve is composed of a series of discrete values as shown in Figure 7(b), so it is necessary to use the discretization method to solve Equations (7) and (8). In the INC and DEC directions, the discretization formulas for \( C_f \) and \( D_f \) are given as follows:

\[
C_{f,INC} = \left( \frac{\epsilon_{i+1} - \epsilon_i}{(Q_{nD})_{i+1} - (Q_{nD})_i} \right) / \left( \frac{E_{nD}^{loss}_{i+1} - E_{nD}^{loss}_i}{(Q_{nD})_{i+1} - (Q_{nD})_i} \right) \quad (9)
\]

\[
D_{f,INC} = \frac{\epsilon_{i+1} - \epsilon_i}{(Q_{nD})_{i+1} - (Q_{nD})_i} \quad (10)
\]

\[
C_{f,DEC} = \left( \frac{\epsilon_{i+1} - \epsilon_i}{(Q_{nD})_{i+1} - (Q_{nD})_i} \right) / \left( \frac{E_{nD}^{loss}_{i+1} - E_{nD}^{loss}_i}{(Q_{nD})_{i+1} - (Q_{nD})_i} \right) \quad (11)
\]

\[
D_{f,DEC} = \frac{\epsilon_{i+1} - \epsilon_i}{(Q_{nD})_{i+1} - (Q_{nD})_i} \quad (12)
\]

where \( C_f \) and \( D_f \) have different physical meanings and functions. \( C_{f,INC} \) represents the ratio of the changing speed of the \( E_{nD}^{loss} \) curve slope to the changing speed of the \( E_{nD}^{loss} \) curve slope in the INC direction. A positive \( C_f(\epsilon) \) value means that the change in the direction of the \( E_{nD}^{loss} \) curve is consistent with that of the \( E_{nD}^{loss} \) curve. The larger the value of \( C_f(\epsilon) \), the greater the promotion of the factor \( e \) in hump formation. Negative \( C_f(\epsilon) \) signifies that \( e \) does not promote hump formation. The concept of \( C_f \) can be used to compare quantitatively the effects of factors such as \( E_{nD}^t \) and \( E_{nD}^{loss} \) on hump formation. \( D_{f,INC} \) indicates the degree of \( E_{nD}^{loss} \) curve deflection in the INC direction. The larger the value of \( D_f \) \( (\epsilon) \), the greater the degree of \( E_{nD}^{loss} \) curve deflection under this operating condition. The concept of \( D_f \) can be used to compare quantitatively the degree of \( E_{nD}^{loss} \) curve deflection under different operating conditions.

For ease of analysis, Equation (10) can be resolved into the sum of two terms, the first and second of which are called \( D_{f,INC,1} \) and \( D_{f,INC,2} \):

\[
D_{f,INC} = D_{f,INC,1} + D_{f,INC,2} \quad (13)
\]

\[
D_{f,INC,1} = (\epsilon_{i+1} - \epsilon_i) \left[ \frac{1}{(Q_{nD})_{i+1} - (Q_{nD})_i} \right] ^2 \quad (14)
\]

\[
D_{f,INC,2} = (\epsilon_{i+1} - \epsilon_i) \left[ \frac{1}{(Q_{nD})_{i+1} - (Q_{nD})_i} \right] \quad (15)
\]
The \( D_f \) in the DEC direction can also be resolved in the same way, and will not be repeated here.

The \( E_{nD} - Q_{nD} \) curve with double humps in the INC direction is shown in Figure 8(a), and Figure 8(b) shows the \( C_f \) distribution over the entire flow interval in the INC direction. In the 1st and 2nd hump regions, the \( C_f \) distribution reveals that the \( C_f(E_{nD}^t) \) value is mostly positive and much larger than \( C_f(E_{nD}^{loss}) \) and \( C_f(E_{nD}^{loss}) \), indicating that \( E_{nD}^t \) is the critical factor causing both humps, according to the physical meaning of \( C_f \) explained above. At the double humps valley, \( C_f \) analysis is made further as shown in Figures 8(c) and 8(d). At the 1st hump valley, \( C_f(E_{nD}^t) \) is close to one, while \( C_f(E_{nD}^{loss}) \) is approximately zero as well as \( C_f(E_{nD}^{loss}) \), which shows that \( E_{nD}^t \) plays the most critical role in the 1st hump formation. At the 2nd hump valley, \( C_f(E_{nD}^t) \) is close to 2, while \( C_f(E_{nD}^{loss}) \) is close to \(-1\), and \( C_f(E_{nD}^{loss}) \) is a smaller positive value or even negative. In conclusion, \( E_{nD}^t \) plays the most critical role in double humps formation, and hydraulic loss plays a much smaller role.

Along the same discharge changing direction, repeated external characteristic tests show the same humps position all the time, and the reason can be understood using \( D_f \) analysis. In the INC direction, the \( E_{nD} - Q_{nD} \) curve, the \( E_{nD}^t - Q_{nD} \) curve, and the distribution of \( D_f \) are shown in Figure 8(c) and 8(d). At the 1st hump valley, \( C_f(E_{nD}^t) \) is close to one, and \( C_f(E_{nD}^{loss}) \) is approximately zero as well as \( C_f(E_{nD}^{loss}) \), which shows that \( E_{nD}^t \) plays the most critical role in the 1st hump formation. At the 2nd hump valley, \( C_f(E_{nD}^t) \) is close to 2, while \( C_f(E_{nD}^{loss}) \) is close to \(-1\), and \( C_f(E_{nD}^{loss}) \) is a smaller positive value or even negative. In conclusion, \( E_{nD}^t \) plays the most critical role in double humps formation, and hydraulic loss plays a much smaller role.

\[
E_{nD}^t = \sum_{m=1}^{9} (E_{nD})_m \tag{17}
\]

\[
d_f,INC,1(E_{nD}^t) = \sum_{m=1}^{9} d_f,INC,1,m(E_{nD}^t) \tag{18}
\]

The \( D_f,INC(E_{nD}^t) \) fluctuates up and down near the zero axis in the entire discharge range and there is no obvious regularity, but it reaches a maximum under the humps valley operating condition. The \( D_f,INC,1(E_{nD}^t) \) value fluctuates on the negative semi-axis of the vertical axis, and rises close to zero at the two hump valleys, reaching a maximum in the entire discharge interval. The \( D_f,INC,2(E_{nD}^t) \) value fluctuates on the positive semi-axis of the vertical axis, and drops close to zero at both hump peaks, reaching a minimum in the entire discharge interval.

It can be seen that, if there is a hump, its valley occurs under the operating condition with the maximum \( D_f,INC(E_{nD}^t) \) and the maximum \( D_f,INC,1(E_{nD}^t) \). Because all the operating conditions are set at equal flow intervals in the simulation, \([1/(Q_{nD})_{i+1} - (Q_{nD})_i]^2\) in formula (14) is the same under every operating condition. Therefore, reaching the maximum of \( D_f,INC,1(E_{nD}^t) \) is equivalent to reaching the maximum of \( (E_{nD}^t)_{i+1} - E_{nD,i}^t \), and the latter is defined as Equation (16). Since the \( E_{nD}^t \) value is determined by the flow field in the pump–turbine, it is fixed under every operating condition. Thus, the operating condition with the maximum \( d_f,INC,1(E_{nD}^t) \) is fixed as well, indicating that, if a hump exists, its position is fixed.

\[
d_f,INC,1(E_{nD}^t) = E_{nD,i+1}^t - E_{nD,i}^t \tag{16}
\]

In conclusion, the runner’s capacity for work, namely \( E_{nD}^t \), is the most critical factor inducing humps, and a hump’s position is determined by \( E_{nD}^t \) as well. In the INC direction, if there is a hump, its valley occurs under the operating condition where the runner’s capacity for work increases fastest. In other words, the hump valley appears where \( d_f,INC,1(E_{nD}^t) \) reaches a maximum.

### 3.2. Formation mechanism of the 1st hump

In the INC direction, the 1st hump valley, peak and adjacent operating point on the left side of valley are called \( i, (i + 1) \), and \( (i - 1) \), respectively, as shown in Figure 8(a). Since the crown and band of the runner do little work on the fluid and can be ignored, the following relationship exists:

\[
E_{nD}^t = \sum_{m=1}^{9} (E_{nD})_m \tag{17}
\]

\[
d_f,INC,1(E_{nD}^t) = \sum_{m=1}^{9} d_f,INC,1,m(E_{nD}^t) \tag{18}
\]

where \( m \) represents the blade number, and 1–9 represent the 9 blades in order.

Generally, the \( E_{nD}^t - Q_{nD} \) curve decreases monotonously along the INC direction, and \( d_f,INC,1(E_{nD}^t) \) is always negative. But not all \( d_f,INC,1,m \) values are negative because of asymmetry and non-uniformity in runner flow field, and blades with non-negative \( d_f,INC,1,m \) value are called ‘aberrant blades’ in the paper. Figure 9(a) shows the \( d_f,INC,1,m \) distribution of the whole blade, pressure side and suction side under the design point, and two aberrant blades can be found, namely blades No. 4 and No. 5, marked with orange boxes on the abscissa. Since the asymmetry and non-uniformity in the flow field under the pump condition intensifies greatly, the difference in each blade \( d_f,INC,1,m \) increases as shown in Figure 9(b). There are five aberrant blades under the 1st hump valley condition, shown by red lines, much more than the other two conditions. Figure 9(c) shows the \( d_f,INC,1,m \) distribution on the whole blade, pressure side and suction side under the valley condition, and blade No. 7 is called a ‘typical aberrant blade’ because of it has the largest \( d_f,INC,1,m \) value.
As discussed above, a hump valley occurs under the operating condition with the maximum $d_{f,\text{INC},1}$, here the $d_{f,\text{INC},1,m}$ distribution verifies that the maximum $d_{f,\text{INC},1}$ is caused by the excessive number of aberrant blades. The hump formation mechanism can be understood by analysing how the excessive aberrant blades are formed.

### 3.2.1. The flow field source of the 1st hump

Under the peak and valley conditions of the 1st hump, the flow field in the runner and guide vane is shown in cascade form in Figure 10. The SPAN ranging from 0.1, 0.3, 0.5, 0.7, to 0.9 represents different blade heights from crown to band, and the numbers of aberrant blades are marked with boxes. Under the valley condition, Figure 10(a) shows that multiple vanes of the guide vane suffer from serious stall, and the flow inside runner is smooth except for severe flow separation at the No. 7 blade inlet. In Figure 11(a), the upper and lower red curves represent the circumferential pressure distribution at the guide vane outlet and runner outlet, respectively, and the dashed lines represent the average

---

**Figure 8.** Analysis of $D_f$ and $C_f$ in the INC direction: (a) $D_f$ distribution in the INC direction; (b) $C_f$ distribution in the INC direction; (c) $C_f$ at the 1st hump valley; (d) $C_f$ at the 2nd hump valley.
pressure. The three cascade diagrams from top to bottom are the flow fields in the guide vane, the pressure contour in the guide vane, and the pressure contour of the runner. The stall cells cause pressure rise both upstream and downstream as shown in Figure 11(a). The pressure at the outlet of the guide vane flow passage where stall exists is significantly higher than the average pressure, and the pressure at the runner outlet near the guide vane stall cells is also higher than the average pressure. This is caused by the effect of stall choke pressure. This phenomenon is caused by the blocking and pressurization effect of stall.

Under the 1st hump peak condition, Figure 10(b) shows weaker guide vane stall than that at the valley, and the severe flow separation at the No. 7 blade inlet has disappeared. Thus, the pressure rise area at the guide vane outlet is much smaller than that under the valley condition, and there no longer exists obvious pressure rise at the runner outlet, as shown in Figure 11(b).

Table 5 shows the distribution of surface shear stress τ, static pressure P, and \( (E_{nD})_7 \) of the No. 7 blade. The τ distribution displays a severe flow separation zone TSV 1 on the pressure side at the blade inlet under the valley condition, while TSV 1 disappears under the peak condition. Comparatively, the flow separation zone TSV 2 exists under both valley and peak conditions. The P distribution demonstrates that the flow separation TSV 1 causes significant low pressure locally, which disappears under the peak condition. In addition, the stall intensity on the No. 1 vane of the guide vane significantly decreases from the valley to the peak condition, resulting in large drop in P at the No. 7 blade outlet section from valley to peak.

As mentioned above, although P on both sides of the No. 7 blade decreases, the \( (E_{nD})_7 \) distribution shows that \( (E_{nD})_7 \) on the suction side is affected by the decrease of stall intensity much more than on the pressure side.
The reasons will be analysed in detail in the next section. Therefore, the effect of the stall change on $d_{f, INC, 1.7}$ of the whole blade is consistent with its effect on $d_{f, INC, 1.7}$ of the suction side. Under the valley condition, the stall intensity on the No. 1 vane of the guide vane is large, with strong blocking and pressurization effect, and then $(E_{nD})_{7}$ on the suction side of the No. 7 blade is large. Until the peak condition, the stall strength is reduced with weak blocking and pressurization effect, and $|E_{nD})_{7}$ on the suction side decreases. Since $E_{nD}$ on the blade suction side is always negative, $(E_{nD})_{7}$ on the suction side increases much in the process from the valley to the peak condition, that is, $d_{f, INC, 1.7}$ on the suction side is large, which also indicates that $d_{f, INC, 1.7}$ on the whole No. 7 blade is large. In addition, under the peak condition, the low pressure zone generated by TSV 1 disappears together with flow separation TSV 1, resulting in a further $d_{f, INC, 1.7}$ increase on the whole No. 7 blade. It can be seen that, from the valley to the peak condition, owing to the significant reduction of stall intensity and the disappearance of flow separation at the No. 7 blade inlet, $d_{f, INC, 1.7}$ on the No. 7 blade is the largest among all blades.

After analysing the No. 1, No. 3, No. 4, and No. 9 blade in the same way, it is found that these aberrant blades are mainly caused by the significant reduction in intensity of stall in the guide vane during the process from valley to peak operating conditions.

**Figure 10.** The flow field in the 1st hump: (a) the 1st hump valley; (b) the 1st hump peak.
In conclusion, the runner $d_{f_{INC}}(E_{nD})$ forms the maximum in the entire discharge range because of too many aberrant blades, and then the 1st hump occurs here. The reason for the excessive number of abnormal blades is that the flow separation at the blade inlet disappears and the intensity of stall in the guide vane greatly decreases in the process from the valley to the peak operating conditions.

3.2.2. The important basis for the assertion that stall induces hump

Based on the above analysis, stall in the guide vane affects the runner’s capacity for work by its blocking and pressurization effect, and then induces hump. The pressure rises caused by stall on the outlet sections of both sides of the runner blade are equivalent, and it is not difficult to imagine that, if the pressure rise has the same degree of influence on the capacity for work of both sides, the influence on both blade sides will completely cancel each other out so that the capacity for work of the whole blade would not be influenced by stall. Thus, stall may not be able to induce hump. However, the actual situation is that the influence of pressure rise on the capacity for work of the suction side is far greater than that of the pressure side. Therefore, the pressure rise can affect the capacity for work of the whole blade, which is an important basis for the assertion that stall in the guide vane induces hump. The reason is analysed in detail below.

The relationship between capacity for work on a unit area of the runner blade $d(E_{nD})_m$ and the torque generated by the unit area $dT$ is as follows:

$$d(E_{nD})_m = dT \cdot \frac{\pi}{30 \rho QnD^2}$$

(19)
Table 5. Analysis of the No. 7 blade under the 1st hump.

|            | Pressure side | Suction side | Pressure side | Suction side | Pressure side | Suction side |
|------------|---------------|--------------|---------------|--------------|---------------|--------------|
| $\tau$     |               |              |               |              |               |              |
|            |               |              |               |              |               |              |
| $P$        |               |              |               |              |               |              |
|            |               |              |               |              |               |              |
| $(E_{nD}^i)^2$ |               |              |               |              |               |              |

Valley

Peak

Legend

where $\rho$, $Q$, $n$, and $D$ represent the fluid density, the volume rate of flow, the rotation speed and the nominal diameter of the runner, respectively. The relationship between $dT$ and blade angle $\alpha$ is shown in Figure 12(a), and $d(E_{nD}^i)_m$ can be further expressed as

$$d(E_{nD}^i)_m = (P \cdot dS \cdot R \cdot \sin \alpha) \cdot \frac{\pi}{30 \rho Q n D^2} \quad (20)$$

where $dS$ represents unit area of the blade, $R$ represents the distance from $dS$ to the runner shaft, and $P$ represents the static pressure on $dS$.

Affected by the stall in the guide vane, the pressure on unit area of the outlet section of the runner blade increases by $\Delta P$, and the change in capacity for work of unit area $\Delta [d(E_{nD}^i)_m]$ can be expressed as

$$\Delta [d(E_{nD}^i)_m] = (\Delta P \cdot dS \cdot R \cdot \sin \alpha) \cdot \frac{\pi}{30 \rho Q n D^2} \quad (21)$$

Equation (21) reveals that, when pressure rise $\Delta P$ occurs at the blade outlet section, the change in capacity for work $\Delta [d(E_{nD}^i)_m]$ is larger on the blade side with larger blade angle $\alpha$. The blade angles $\alpha$ of both blade sides are compared in Figure 12(b): $\alpha$ on both sides of the inlet section Region A differs little, while there is a big difference in $\alpha$ on both blade sides in the outlet section Region B, where $\alpha$ of the suction side is much larger than that of the pressure side. Thus, when stall induces a pressure rise $\Delta P$ at the blade outlet section, the capacity for work of the suction side must be affected much more than the pressure side. As shown in Figure 12(c), the $E_{nD}^i$ distribution on the runner blade can also confirm this point. The part with the largest capacity for work of the blade suction side is much closer to the blade outlet than that of the pressure side, and therefore the pressure rise near the blade outlet inevitably has greater impact on the suction side.

Stall weakening in the process from the hump valley to the peak produces the effect of enhancing the capacity for work of the suction side and the effect of reducing the capacity for work of the pressure side. According to the above analysis, stall has greater impact on the suction side, resulting in the stall weakening producing the same effect on the whole blade as on the suction side, namely the effect of enhancing the capacity for work. This is an important basis for the assertion that stall induces hump.
3.2.3. Analysis of flow separation at the runner blade inlet

The 1st hump is formed under the dual influence of stall in the guide vane and backflow at the entrance of the runner blade. The mechanism of the stall effect has been analysed above, and the flow separation is analysed below in detail.

The draft tube, which has a great influence on the runner inflow, is now selected for analysis. As shown in Figure 13(a), planes A, B, C, D, E, F, and G are representative cross sections of the draft tube and runner. The flow pattern evolution process is summarized in Figure 13(b), where the horizontal axis represents different positions in the flow direction, the coordinates are representative cross sections, and ordinates represent the valley and peak conditions. Under the valley and peak conditions, the flow patterns in cross sections A and B are the same, and both them have gone through ‘Stage 1 Uniform Flow’ in the straight pipe section and ‘Stage 2 Dean Vortex Forming’ at the elbow. However, these two operating conditions have obvious differences in flow pattern downstream of the elbow.

Under the peak condition, the dean vortex in the draft tube has undergone a long-time ‘Stage 31 Dean Vortex Fusion’ downstream of the elbow and upstream of the runner blade inlet. Since vortices I and II have opposite
spiral directions, they gradually show a state of being connected end to end and weakening each other, resulting in continuous weakening in dean vortex strength. In addition, the fluid viscosity continues to weaken the dean vortex strength as well. When the dean vortex reaches the runner blade inlet, its intensity is already very low and it quickly dissipates under the action of blade cutting. Therefore, the dean vortex does not have significant impact on the blade inlet attack angle, and this stage is called 'Stage 41 Vortex Breakage'.

Under the valley condition, the flow velocity is slower than that under the peak condition, hence it takes a long time for the dean vortex to flow from the elbow to the blade inlet, such that the dean vortex has enough time to evolve into a more complicated pattern. Firstly, the dean vortex formed at the elbow undergoes 'Stage 32 Fast Fusion' upstream of the runner inlet. Vortices I and II, which have opposite spiral directions, quickly connect to each other end to end and cancel each other, and soon the dean vortex evolves into a single vortex II. Then the flow pattern goes through 'Stage 33 Wall Vortex Forming' downstream of the runner inlet and upstream of the runner blade inlet. In this stage, the wall surface of cross section is the runner band rotating quickly as shown by the red dashed line and arrow. Because the region farther from vortex II on the cross section is less constrained by the vortex core, and the flow direction of this region is opposite to the wall boundary layer, these two fluids gradually merge to form an obvious wall flow separation vortex III. Since the swirling direction of vortex II
is opposite to the wall boundary layer, it is constantly weakened by the wall boundary layer, while vortex III is continuously strengthened. When the vortices II and III touch the runner blade inlet, 'Stage 42 Vortex Disturbance' starts. Vortex II is weak here because of being weakened continuously, and it quickly dissipates under blade cutting. Vortex III is stronger because of enhancement in 'Stage 33' so that it maintains a complete shape and disturbs the attack angle at the blade inlet in the initial stage of contact with the runner blades. As vortex III contacts the blades more and more, the cutting intensity of the blades is greater, and vortex III quickly dissipates as well. Therefore, vortex III only exists at the blade inlet close to the band, and the blade attack angle here is significantly changed by it.

The following is a detailed analysis of how wall vortex III changes the attack angle at the runner blade inlet and causes flow separation. Different blade heights near the blade leading edge are chosen to analyse the circumferential flow angle $\phi$ as shown in Figure 14(a). The SPAN from 0.1 to 0.9 represents the direction from crown to band, and the flow angle $\phi$ is defined in Figure 14(b). The $\phi$ distribution near the blade inlet under the valley condition is shown in Figure 14(c), the horizontal axis of which represents the circumferential position, and the vertical axis represents $\phi$. Corresponding to Figure 14(a), different curve colours in Figure 14(c) represent five different blade heights. The range where the No. 7 blade of the runner is located and the coverage of wall vortex III are marked with shading. Figure 14(c) reveals that $\phi$ in the range covered by wall vortex III decreases significantly relative to other positions in the circumferential direction, and the No. 7 blade is just within this area. $\phi$ is more affected near the band because wall vortex III is mainly concentrated near the band at the No. 7 blade inlet. Under the peak condition, there is no wall vortex III, and the sudden drop in $\phi$ no longer exists, as shown in Figure 14(d). Comparing Figures 14(c) and 15(d), it can be seen that the $\phi$ distribution pattern under the valley condition is the superposition of the distribution pattern.
under the peak condition and the $\varphi$ distortion pattern caused by wall vortex III.

The velocity triangle as shown in Figure 15 can be used to analyse the flow around blade No. 7, where $V$, $W$, $U$, $\varphi$ and $\alpha$ represent the flow velocity in the stationary coordinate system, the flow velocity in the rotational coordinate system, the runner rotational linear velocity, the flow angle, and the blade angle, and subscripts P, V, 1, and 2 represent peak condition, valley condition, inlet and outlet, respectively. Figure 15(a) shows an equivalence between $\alpha_1$ and $\varphi_{P1}$ (both of them are about 15°) under the peak condition at blade No. 7 inlet. Until the valley condition, affected by vortex III and with flowrate decreasing, $V_{V1}$ decreases and deviates from $V_{P1}$ so that $\varphi_{V1}$ drops to 10°, much smaller than $\alpha_1$, causing flow separation on the blade suction side, while at the outlet as shown in Figure 15(b), $V_{P2}$ decreases for flowrate reduction from peak to valley condition, but its direction deviates just a bit from $V_{P1}$ because of a slight influence from vortex III. Thus, $\varphi_{V2}$ is somewhat less than $\varphi_{P2}$, and both of them are less than $\alpha_1$, of about 3°, indicating that flow slip at the outlet under the valley condition intensifies a little.

Under the valley condition, wall vortex III greatly reduces the flow angle at the No. 7 blade inlet, resulting in significant flow separation TSV 1 on the pressure side, originating from point C1 near the blade inlet and the band, as shown in Figure 16(a). TSV 1 is strong, bypasses the blade leading edge and extends to the suction side. The flow separation TSV 2 forming on the No. 7 blade suction side is shown in Figure 16(b), and originates from point C2 near the blade inlet and band. Under the peak condition, there is no wall vortex so that the flow separation TSV 1 disappears as shown in Figure 16(c). Additionally, TSV 2 on the suction side is also weaker than that under the peak condition and its source point C2 moves towards the blade outlet direction slightly, as shown in Figure 16(d).

It can be concluded that the change in the flow pattern at the blade inlet plays an important role in 1st hump formation, and the change results from the difference in flow pattern of the draft tube.

### 3.3. Formation mechanism of the 2nd hump

The 2nd hump formation mechanism is analysed below in the same way as the 1st hump analysis. The 2nd hump peak, valley, and the normal operating condition on the left side of the valley are called the $(j + 1)$, $(j)$, and $(j - 1)$ conditions, respectively, as shown in Figure 8(a). As shown in Figure 17(a), among these three conditions, the valley condition has the largest number of aberrant blades, namely the No. 1, No. 4, No. 5, No. 6, and No. 8 blades, of which the No. 1 blade is a typical aberrant blade. The aberrant blades are mainly caused by the intensity change of the guide vane stall. According to the previous discussion, the influence of stall intensity change on the capacity for work of the suction side is more significant than that of the pressure side, so the $df_{INC,1,m}$ distribution law of the whole blade is consistent with that of the suction side, as shown in Figure 17(b).

The flow field inside the runner and guide vane in the 2nd hump is expanded in the form of a cascade, as shown in Figure 18. Under the valley condition, there is obvious flow separation at nearly all the runner blade inlets near the band, especially the No. 4 and No. 5 blade inlets, and there is no obvious difference in the runner flow field between the peak and valley conditions. Under the valley condition, almost all guide vanes suffer from stall, covering most of the flow passages, and stall has the greatest strength from the top cover to middle blade height. Until the peak condition, there is almost no change in stall cell number, but the stall coverage in the flow passage decreases, especially in the range from the middle blade span to the bottom ring. Among all the vanes, stall in the No. 7, No. 16, and No. 3 vanes declines the most, resulting in the largest influence on the No. 1, No. 5, and No. 8 runner blades which are the closest to the above vanes. Thus, the change in capacity for work, namely the $df_{INC,1,m}$ values of the No. 1, No. 5, and No. 8 runner blades, are the largest among all runner blades.

The No. 1 runner blade is taken as an example to analyse the influence of the stall change in the guide vane on the blade from the 2nd hump valley to the peak condition, as shown in Table 6. The $\tau$ distribution on the No. 1 blade displays the fact that flow separation mainly occurs...
Figure 16. Flow separation on blade No. 7 in the 1st hump: (a) pressure side under valley condition; (b) suction side under valley condition; (c) pressure side under peak condition; (d) suction side under peak condition.

Figure 17. The $d_f^{INC,1,m}$ distribution in the 2nd hump: (a) operating conditions in the 2nd hump region; (b) the 2nd hump valley condition.

on the suction side and extends from the inlet near the band to the outlet with little difference between the valley and peak conditions. Since the stall on the No. 7 vane decreases significantly from valley to peak condition, its blocking and pressurization effect is greatly reduced so that $P$ on both sides of the runner No. 1 blade outlet section is obviously reduced as well. As the blade angle $\alpha$ on the suction side outlet section is much larger than that on the pressure side, the capacity for work, namely $E^t_{nD}$, of the suction side is more affected. The $(E^t_{nD})_1$ distribution in Table 6 shows that $(E^t_{nD})_1$ on the suction side outlet section changes much more than the pressure side.
After all the other aberrant blades, namely the No. 4, No. 5, No. 6, and No. 8 blades, are analysed by the same method, the same conclusion as for the No. 1 blade can be drawn. From the 2nd hump valley to the peak condition, the stall intensity in the guide vane is significantly reduced, leading to a decrease in the blocking and pressurization effect of stall. Therefore, $P$ at these blade outlet sections close to stall drops significantly, thus inducing aberrant blades with non-negative $d_f,INC,1,m$ values. Under the comprehensive influence of all aberrant blades, the runner $d_f,INC,1,m$ under this operating condition becomes maximum in the entire discharge range, and here the 2nd hump is formed.

During the process from the 2nd hump valley to peak, the flow separation intensity at the runner blade inlet changes little, so it causes little change in the runner’s capacity for work. Therefore, the flow separation at the runner blade inlet plays a negligible role in 2nd hump formation. In conclusion, the 2nd hump is mainly induced by the stall change in the guide vane and has a smaller relationship with the flow separation at the runner blade inlet.
4. Conclusions

The double hump formation mechanism in the INC direction (the direction of increasing discharge) in a pump–turbine of medium specific speed has been researched, and the conclusions are as follows.

(1) The key reason for hump formation is not the large change in hydraulic loss, but the difference in the theoretical energy coefficient between a certain working condition \((i)\) to its adjacent working condition \((i+1)\) in the INC direction, namely the \((E_{t,nD,i+1} - E_{t,nD,i})\) value is too large; when selecting operating conditions at equal discharge intervals in the entire discharge range, if there is a hump, then the hump occurs under the operating condition with the largest \((E_{t,nD,i+1} - E_{t,nD,i})\) value; since the \(E_{t,nD}\) value under each operating condition is determined by the flow field, so that its value is fixed, the operating condition with the maximum \((E_{t,nD,i+1} - E_{t,nD,i})\) value is determined, and thus the hump position is determined.

(2) Stall in the guide vane is a necessary but not sufficient condition for hump formation. In the INC direction, humps occur near the operating conditions where the stall intensity weakens the fastest with increasing discharge. Stall affects the runner’s capacity for work through its blocking and pressurization effect, i.e. it affects the runner’s theoretical energy coefficient \(E_{t,nD}\) to induce humps.

(3) The important basis for the assertion that the ability of stall in the guide vane to affect \(E_{t,nD}\) and induce humps is that the blade angle \(\alpha\) on the suction side of the runner blade outlet section is much larger than that on the pressure side, leading to a larger impact of stall blocking and pressurization on the capacity for work on the suction side than the pressure side.

(4) The formation of the 1st hump near the design point is due to the fact that the intensity of the stall in the guide vane is greatly reduced, and the flow separation at the inlet of the runner blade pressure side disappears rapidly. The latter is caused by vortex structure changes in the draft tube. The formation of the 2nd hump at the small discharge operating point is due to the sudden decrease in the intensity of the stall in the guide vane, but flow separation at the runner inlet also plays a very small role in its formation.

Humps in the DEC direction (the direction of decreasing discharge) can also be analysed by the method in this paper, and similar conclusions can be drawn as for the INC direction. The applicability of the method proposed in this paper to pump–turbine characteristic analysis and humps under cavitation conditions needs to be tested, and this is also an important direction for further development of the method.

Author contribution

Yong Liu: experiment, simulation and analysis; Dezhong Wang: supervision; Hongjuan Ran: data curation.

Disclosure statement

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Funding

This work was supported by Youth Science Foundation Project (China), project number 51406170.

Nomenclature

| Parameter | Symbol |
|-----------|--------|
| Number of runner blades | \(Z_r\) |
| Runner inlet diameter | \(D_1\) |
| Runner outlet diameter | \(D_2\) |
| Runner blade outlet width | \(b_2\) |
| Blade angle at runner outlet | \(\beta_2\) |
| Runner blade wrap angle | \(\varphi\) |
| Vaneless zone gap | \(\delta\) |
| Blade height of guide/stay vane | \(b_3/b_4\) |
| Number of guide / stay vanes | \(Z_g/Z_s\) |
| Degree of guide vane opening | \(\alpha\) |
| Guide vane distribution diameter | \(D_0\) |
| Degree of stay vane opening | \(\alpha_s\) |
| Specific speed | \(n_q\) |
| Rated speed | \(n_r\) |
| Flux under maximum efficiency point | \(Q_{\text{BEP}}\) |
| Head under maximum efficiency point | \(H_{\text{BEP}}\) |
| Shear stress | \(\tau\) |
| Static pressure | \(P\) |

References

Braun, O. (2009). Part load flow in radial centrifugal pumps [No. thesis]. EPFL.
Braun, O., Kueny, J. L., & Avellan, F. (2005, January). Numerical analysis of flow phenomena related to the unstable energy-discharge characteristic of a pump–turbine in pump mode. Fluids Engineering Division Summer Meeting, 41987, 1075–1080. https://doi.org/10.1115/FEDSM2005-77015
Capelo, B., Pérez-Sánchez, M., Fernandes, J. F., Ramos, H. M., López-Jiménez, P. A., & Branco, P. C. (2017). Electrical behaviour of the pump working as turbine.
in off grid operation. Applied Energy, 208, 302–311. https://doi.org/10.1016/j.apenergy.2017.10.039

Eisele, K., Muggli, F., Zhang, Z., Casey, M., Sallaberger, M., & Sebestyen, A. (1998, September). Experimental and numerical studies of flow instabilities in pump–turbine stages. 18th IAHR Symp. on Hydr. Machinery and Systems, Valencia, Spain.

Ez Abadi, A. M., Sadi, M., Farzaneh-Gord, M., Ahmadi, M. H., Kumar, R., & Chau, K. W. (2020). A numerical and experimental study on the energy efficiency of a regenerative Heat and Mass Exchanger utilizing the counter-flow Maisotsenko cycle. Engineering Applications of Computational Fluid Mechanics, 14(1), 1–12. https://doi.org/10.1080/19492060.2019.1617193

Fisher, R. K., & Webb, D. R. (1978). Effect of cavitation on the discontinuity point and on alternating pressures and gate torques on a pump/turbine model in the pump cycle. Allis-Chalmers Corporation, Hydro-Turbine Division.

Gentner, C., Sallaberger, M., Widmer, C., Braun, O., & Staubli, T. (2012, November). Numerical and experimental analysis of instability phenomena in pump turbines. In IOP conference series: Earth and environmental science (Vol. 15, No. 3, pp. 032–042). IOP Publishing.

Ghalandari, M., Mirzadeh Kooshashi, E., Mohamadian, F., Shamshirband, S., & Chau, K. W. (2019). Numerical simulation of nanofluid flow inside a root canal. Engineering Applications of Computational Fluid Mechanics, 13(1), 254–264. https://doi.org/10.1080/19492060.2019.1578696

Guedes, A., Kueny, J. L., Ciocan, G. D., & Avellan, F. (2002). Unsteady rotor-stator analysis of hydraulic pump–turbine: CFD and experimental approach. In Proceedings of the 21st IAHR symposium on hydraulic machinery and systems, Lausanne, Switzerland (Vol. 1, No. CONF, pp. 767–780). International Association For Hydraulic Research.

Gürnberg, C., Sallaberger, M., Widmer, C., Braun, O., & Staubli, T. (2012, November). Numerical and experimental analysis of instability phenomena in pump turbines. In IOP conference series: Earth and environmental science (Vol. 15, No. 3, pp. 032–042). IOP Publishing.

Ghalandari, M., Mirzadeh Kooshashi, E., Mohamadian, F., Shamshirband, S., & Chau, K. W. (2019). Numerical simulation of nanofluid flow inside a root canal. Engineering Applications of Computational Fluid Mechanics, 13(1), 254–264. https://doi.org/10.1080/19492060.2019.1578696

Guedes, A., Kueny, J. L., Ciocan, G. D., & Avellan, F. (2002). Unsteady rotor-stator analysis of hydraulic pump–turbine: CFD and experimental approach. In Proceedings of the 21st IAHR symposium on hydraulic machinery and systems, Lausanne, Switzerland (Vol. 1, No. CONF, pp. 767–780). International Association For Hydraulic Research.

Güllich, J. F. (2008). Centrifugal pumps (Vol. 2). Springer.

Li, D., Chang, H., Zuo, Z., Wang, H., Li, Z., & Wei, X. (2020). Experimental investigation of hysteresis on pump performance characteristics of a model pump–turbine with different guide vane openings. Renewable Energy, 149, 652–663. https://doi.org/10.1016/j.renene.2019.12.065

Li, D., Wang, H., Qin, Y., Han, L., Wei, X., & Qin, D. (2017). Entropy production analysis of hysteresis characteristic of a pump–turbine model. Energy Conversion and Management, 149, 175–191. https://doi.org/10.1016/j.enconman.2017.07.024

Li, D., Wang, H., Qin, Y., Wei, X., & Qin, D. (2018). Numerical simulation of hysteresis characteristic in the hump region of a pump–turbine model. Renewable Energy, 115, 433–447. https://doi.org/10.1016/j.renene.2017.08.081

Li, D., Wang, H., Xiang, G., Gong, R., Wei, X., & Liu, Z. (2015). Unsteady simulation and analysis for hump characteristics of a pump turbine model. Renewable Energy, 77, 32–42. https://doi.org/10.1016/j.renene.2014.12.004

Li, D. Y. (2017). Investigation on flow mechanism and transient characteristics in hump region of a pump–turbine [Doctoral dissertation, Ph. D. thesis]. Harbin Institute of Technology, Harbin, People’s Republic of China.

Li, X., Zhu, Z., Li, Y., & Chen, X. (2016). Experimental and numerical investigations of head-flow curve instability of a single-stage centrifugal pump with volute casing. Proceedings of the Institution of Mechanical Engineers, Part A: Journal of Power and Energy, 230(7), 633–647. https://doi.org/10.1177/0957650916663326

Lino, M., & Tanaka, K. (2004). Numerical analysis of unstable phenomena and stabilizing modification of an impeller in a centrifugal pump. Proceedings of 22nd IAHR symposium on hydraulic machinery and systems, Stockholm, Sweden.

Lu, G., Zuo, Z., Liu, D., & Liu, S. (2019). Energy balance and local unsteady loss analysis of flows in a low specific speed model pump–turbine in the positive slope region on the pump performance curve. Energies, 12(10), 1829. https://doi.org/10.3390/en12101829

Lu, G. C. (2018). Investigations on the influence of the flow separation in guide vane channels on the positive slope on the pump performance curve in a pump–turbine [Doctoral dissertation, Ph. D. thesis]. Tsinghua University, Beijing, People’s Republic of China.

Luo, T. Q. (2007). Hydromechanics. Chinese Mechanical Industry Press.

Ran, H., Liu, Y., Luo, X., Shi, T., Xu, Y., Chen, Y., & Wang, D. (2020). Experimental comparison of two different positive slopes in one single pump turbine. Renewable Energy, 154, 1218–1228. https://doi.org/10.1016/j.renene.2020.01.023

Ran, H., & Luo, X. (2018). Experimental study of instability characteristics in pump turbines. Journal of Hydraulic Research, 56(6), 871–876. https://doi.org/10.1080/00221686.2017.1422193

Sampedro, E. O., Colas, F., Rousselte, O., Coutier-Delgosha, O., & Caignaert, G. (2021). Multistage radial flow pump–turbine for compressed air energy storage: Experimental analysis and modeling. Applied Energy, 289, 116705. https://doi.org/10.1016/j.apenergy.2021.116705

Shibata, A., Hiramatsu, H., Komaki, S., Miyagawa, K., Maeda, M., Kamei, S., Hazama, R., Sano, T., & lino, M. (2016). Study of flow instability in off design operation of a multistage centrifugal pump. Journal of Mechanical Science and Technology, 30(2), 493–498. https://doi.org/10.1007/s12206-016-0101-1

Sun, Y. K. (2016). Instability characteristics and influencing factors of positive slope on pump performance curves of a low-speed pump–turbine [Doctoral dissertation, Ph. D. thesis]. Tsinghua University, Beijing, People’s Republic of China.

Tao, R., Xiao, R. F., Yang, W., & Liu, W. C. (2014). Hump characteristic of reversible pump–turbine in pump mode. Journal of Drainage and Irrigation Machinery Engineering, 32(11), 927–930. https://doi.org/10.3969/j.issn.1674-8530.13.0668

Wang, L. Q., Liu, J. T., Zhang, L. F., Qin, D. Q., & Jiao, L. (2011). Low flow fluctuation characteristics in pump–turbine’s pump mode. Journal of Zhejiang University (Engineering Science), 45(7), 1239–1243. https://kns.cnki.net/kcms/detail/detail.aspx?FileName=ZDZC2011070717&DbName=CJFQ2011

Wilhelm, S., Balarac, G., Métais, O., & Séguinou, C. (2016). Analysis of head losses in a turbine draft tube by means of 3D unsteady simulations. Flow, Turbulence and Combustion, 97(4), 1255–1280. https://doi.org/10.1007/s10494-016-9767-9

Xiao, R. F., Tao, R., & Liu, W. C. (2014). Numerical study on unstable head-discharge performance in pump-turbine considering the transient rotor-stator interactions. Chinese
Xiao, Y., Yao, Y., Wang, Z., Luo, Y., Zeng, C., & Zhu, W. (2016). Hydrodynamic mechanism analysis of the pump hump district for a pump–turbine. *Engineering Computations, 33*(3), 957–976. https://doi.org/10.1108/EC-02-2015-0038

Yang, J. (2015). Flow patterns causing saddle instability in the performance curve of a centrifugal pump with vaned diffuser.

Yang, J., Pavesi, G., Liu, X., Xie, T., & Liu, J. (2018). Unsteady flow characteristics regarding hump instability in the first stage of a multistage pump–turbine in pump mode. *Renewable Energy, 127*, 377–385. https://doi.org/10.1016/j.renene.2018.04.069

Ye, W., Ikuta, A., Chen, Y., Miyagawa, K., & Luo, X. (2020). Numerical simulation on role of the rotating stall on the hump characteristic in a mixed flow pump using modified partially averaged Navier-Stokes model. *Renewable Energy, 166*, 91–107. https://doi.org/10.1016/j.renene.2020.11.066

Yin, J. L. (2012). *Study on the internal flow and optimum design of pump turbine in the 'S' zone* [Doctoral dissertation, Ph.D. thesis]. Zhejiang University, Hangzhou, People’s Republic of China.

Zhou, P. J. (2015). *Investigation of stall characteristics in centrifugal pumps*. China Agricultural University.