SUSY QCD CORRECTIONS TO THE $t \to H^+ b$ DECAY*

HEINZ KÖNIG
Département de Physique
Université du Québec à Montréal
C.P. 8888, Succ. Centre Ville, Montréal
Québec, Canada H3C 3P8

ABSTRACT

I present the contribution of gluinos and scalar quarks to the decay rate of the top quark into a charged Higgs boson and a bottom quark within the minimal supersymmetric standard model, including the mixing of the scalar partners of the left- and right-handed top quark. I show that for certain values of the supersymmetric parameters the standard QCD loop corrections to this decay mode are diminished or enhanced by several tens of per cent. I show that not only the small value of 3 GeV for the gluino mass (small mass window) but also much larger values of several hundreds of GeV's have a non-negligible effect on this decay rate, against general belief. Last but not least, if the ratio of the vacuum expectation values of the Higgs bosons are taken in the limit of $v_1 \ll v_2$ I obtain a drastic enhancement due to a $\tan \beta$ dependence in the couplings.

I. INTRODUCTION

Recently there has been a lot of interest in the electroweak loop corrections\(^1,2\) as well as in the QCD loop corrections\(^3,4,5,6\) to the top quark decay into a charged Higgs boson and a bottom quark.

In the standard model we have no charged Higgs particle and therefore this decay can be used as a test for models beyond the standard model; such as a two Higgs doublet model\(^7\) and the minimal supersymmetric extensions of the standard model (MSSM)\(^8,9\), which is the favorite model beyond the standard model.

In this talk I take the last one as the underlying model to consider the QCD corrections to the $t \to H^+ b$ decay mode. Czarnecki and Davidson\(^6\) showed that the effect of the mass of the bottom quark to this decay rate is negligible. They also showed that the ratio of the first order to the zeroth order is constant at about $-9\%$ for a wide range of the Higgs mass ($0 \leq m_{H^\pm} \leq 90$ GeV); the top quark mass was taken to be 150 GeV.

In this talk I present the SUSY QCD loop corrections to the $t \to H^+ b$ decay if gluinos and scalar quarks are taken within the loop. Throughout the calculations I neglect the mass of the bottom quark, but I do not neglect the mixing of the scalar partners of the left- and right-handed top quark, which is proportional to the top quark mass.

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quark mass.

In the next section I only present the results and refer the interested reader for detailed calculation to ref.10. I end with final remarks and the conclusions.

II. SUSY QCD CORRECTIONS TO THE TOP QUARK DECAY INTO A CHARGED HIGGS BOSON AND BOTTOM QUARK

In the MSSM the interaction Lagrangian relevant to the decay $t \rightarrow H^+ b$ leads to the following decay rate for $m_b \ll m_{top}$:

$$\Gamma^0(t \rightarrow H^+ b) = \frac{G_F}{\sqrt 2} |V_{tb}|^2 \frac{\cot^2 \beta}{8\pi} m_{top}^3 (1 - \frac{m_H^2}{m_{top}^2})^2$$  \hspace{1cm} (1)

$V_{tb} \approx 1$ is the Kobayashi-Maskawa matrix value, $\cot \beta = v_1/v_2$ is the ratio of the vacuum expectation values (vev) of the two Higgs doublets.

To calculate the 1 loop diagram with gluinos and scalar quarks within the loop we need the couplings of the scalar quarks to the charged Higgs boson and the scalar-quark-gluino-quark coupling. The first coupling is given in Fig.115* in ref.7 and the latter one in Eq.(C89) in ref.9.

When neglecting the bottom quark mass only the scalar partner of the left handed bottom quark $\tilde b_L$ occurs within the loop whereas for the top quark we have to take both left- and right-handed superpartner $\tilde t_L$ and $\tilde t_R$ into account. Furthermore since the mixing of $\tilde t_L$ and $\tilde t_R$ is proportional to the top quark mass we have to include the full scalar top quark matrix, which is given by:

$$M^2_{\tilde t} = \left( \begin{array}{cc} m_{\tilde t_L}^2 + m_{\tilde t_{top}}^2 + 0.35 D_Z^2 & -m_{\tilde t_{top}} (A_{\tilde t_{top}} + \mu \cot \beta) \\ -m_{\tilde t_{top}} (A_{\tilde t_{top}} + \mu \cot \beta) & m_{\tilde t_R}^2 + m_{\tilde t_{top}}^2 + 0.16 D_Z^2 \end{array} \right)$$ \hspace{1cm} (2)

where $D_Z^2 = m_Z^2 \cos 2\beta$. $m_{\tilde t_{top}}^2$ are soft breaking masses, $A_{\tilde t_{top}}$ is the trilinear scalar interaction parameter and $\mu$ is the supersymmetric mass mixing term of the Higgs bosons. The mass eigenstates $\tilde t_1$ and $\tilde t_2$ then are related to the current eigenstates $\tilde t_L$ and $\tilde t_R$ by $\tilde t_1 = \cos \Theta \tilde t_L + \sin \Theta \tilde t_R$ and $\tilde t_2 = -\sin \Theta \tilde t_L + \cos \Theta \tilde t_R$. In the following we take $m_{\tilde t_L} = m_{\tilde t_R} = m_S = A_{\tilde t_{top}}$ (global SUSY), $m_{\tilde b_L}^2 = m_S^2 - 0.42 D_Z^2$ and $m_{\tilde b_R}^2 = m_S^2 - 0.08 D_Z^2$. With negelecting bottom quark mass the scalar partners of the left and right handed bottom quarks do not mix and therefore $m_{\tilde b_L} = m_{\tilde b_R}$. The gluino mass $m_{\tilde g}$ is a free parameter, which in general is supposed to be larger than 100 GeV, although there is still the possibility of a small gluino mass window in the order of 1 GeV$^{11,12}$.

In Eq.(10) in ref.6 the authors present the results of the standard QCD 1 loop corrections within the two Higgs doublet model and the MSSM, which I will include in my calculation. The results of the calculation of the loop diagram with scalar quarks and gluinos are finite, there are no dimensional divergencies. As a result I get for the first order in $\alpha_s$:

$$\Gamma^1(t \rightarrow H^+ b) = \Gamma^0(t \rightarrow H^+ b) \left[ 1 + \frac{4\alpha_s}{3\pi} \tilde G_+ - \frac{2\alpha_s}{3\pi} (S + A) \right]$$ \hspace{1cm} (3)

* $\mu$ has to be replaced by $-\mu$
\[ \hat{G}_+ = 2 \text{Li}_2(1 - \chi^2) - \frac{\chi^2}{1 - \chi^2} \log \chi^2 + \log \chi^2 \log (1 - \chi^2) + \frac{1}{\chi^2} \left( 1 - \frac{5}{2} \chi^2 \right) \]

\[ \log (1 - \chi^2) - \frac{2\pi^2}{3} + \frac{9}{4} \]

\[ \chi^2 = \frac{m_{H^+}^2}{m_{\text{top}}^2} \]

\[ S = S_t + \frac{m_{\tilde{g}}}{m_{\text{top}}} S_{\tilde{g}} \]

\[ A = A_t + \frac{m_{\tilde{g}}}{m_{\text{top}}} A_{\tilde{g}} \]

\[ S_t = K_{11} \left[ \frac{\alpha}{\beta} C^\beta_{1i} + s^2_\Theta C^\beta_{2i} \right] + K_{21} \left[ s_\Theta \alpha (C^\beta_{1i} - C^\beta_{1i}) \right] \]

\[ A_t = S_1 \]

\[ S_{\tilde{g}} = K_{11} \left[ \alpha s_\Theta (C^\beta_{0i} - C^\beta_{0i}) \right] - K_{21} \left[ \alpha s_\Theta C^\beta_{0i} + s^2_\Theta C^\beta_{0i} \right] \]

\[ A_{\tilde{g}} = S_{\tilde{g}} \]

\[ K_{11} = 1 - \frac{m_{W}^2}{m_{H^0}^2} \tan \beta \sin 2\beta \]

\[ K_{21} = \frac{1}{m_{\text{top}}^2} (A_{\text{top}} + \mu \tan \beta) \]

\[ C^\beta_{0i} = - \int_0^1 \int_0^{1-\alpha_1} \frac{m_{\text{top}}^2}{f^\beta_{1i}} \, d\alpha_1 \, d\alpha_2 \]

\[ C^\beta_{1i} = - \int_0^1 \int_0^{1-\alpha_1} \frac{m_{\text{top}}^2 \alpha_1}{f^\beta_{1i}} \, d\alpha_1 \, d\alpha_2 \]

\[ f^\beta_{1i} = m_{\tilde{g}}^2 - (m_{\tilde{g}}^2 - m_{\tilde{g}}^2) \alpha_1 - (m_{\tilde{g}}^2 - m_{\tilde{g}}^2) \alpha_2 - m_{\text{top}}^2 \alpha_1 (1 - \alpha_1 - \alpha_2) - m_{H^+}^2 \alpha_1 \alpha_2 \]

where \( c_\Theta = \cos \theta \) and \( s_\Theta = \sin \theta \). \( S \) and \( A \) indicate that the contribution comes from the scalar and axial scalar coupling of the matrix element. For no mixing of the scalar top quark masses the contribution of \( S_{\tilde{g}} \) and \( A_{\tilde{g}} \) are zero. The Feynman integration can be done numerically.

To compare the standard QCD correction given in ref.6 with the gluino and scalar quarks contribution I present in Fig.1 and Fig.2 the results for different masses of the gluino and \( \tan \beta \). I take \( \mu = 500 \text{ GeV} \) and \( A_{\text{top}} = m_S \). In the MSSM we have \( m_{H^+}^2 = m_W^2 + m_{H^0}^2 \) where \( H^0_3 \) is the pseudo Higgs particle. That is the mass of the charged Higgs particle has to be larger than the mass of the W boson. I put the charged Higgs mass to be equal \( m_W \) and the top quark mass to be the recently released CDF value of 174 GeV\(^{13}\).

In Fig.1 and Fig.2 the solid straight line is the standard contribution given in ref.6 and lies at \(-9.5\%\). I present the results for three different values of the gluino mass that is 3 GeV (solid line), 100 GeV (dotted line) and 500 GeV (dash-dotted line).
In Fig.1 I consider the case \( v_1 = v_2 \). The lighter scalar top quark mass is about 250 GeV for \( m_S \) smaller than 100 GeV, decreases constantly to about 70 GeV for \( m_S = 350 \) GeV and increases again to 260 GeV in the range considered here. The heavier one varies from 358 GeV to 631 GeV and \( m_{b_1,2} = m_S \). Here \( \cos \Theta = 1/\sqrt{2} \) and the influence of the scalar and axial scalar coupling of the gluino is larger than the one of the top quark. As a result we have in this case that the standard QCD corrections are diminished for small gluino masses whereas we get an enhancement up to \(-18\%\) for a gluino mass of 500 GeV. Changing the \( \mu \)-parameter hardly affects the results. If the Higgs mass is enhanced all curves are pushed up closer to 0, but the shape of the curves remains the same. For \( m_{H^+} = 120 \) GeV the standard QCD correction is about \(-8.1\%\).

In Fig.2 I consider the same case, but with \( \tan \beta = 10 \). Here the contribution of the gluino are much larger than the top quark contribution due to a \( \mu \tan \beta \) dependence in the couplings. The gluino mass also becomes more important, whereas for very large gluino masses \( (m_\tilde{g} \gg 100 \) GeV) the 1 loop contribution \( \Gamma^1(t \rightarrow H^+b) \) is decreasing again. Here \( \cos \Theta \approx 1/\sqrt{2} \), the lighter scalar top quark mass is about 115 to 110 GeV for \( m_S \) smaller than 100 GeV and increases constantly to 379 GeV for \( m_S = 450 \) GeV. The heavier one varies from 219 GeV to 564 GeV. The heavy scalar bottom quark mass varies from 78 GeV to 454 GeV and the lighter one from 56 GeV to 451 GeV. As a result we see that the gluino mass contribution enhances the standard QCD correction drastically. This decay mode therefore can be used to put constraints on the ratio of the vevs \( v_1 \) and \( v_2 \). Smaller values for \( \mu \) diminishes the results whereas higher values for \( \mu \) enlarges them. For \( m_S \leq 50 \) GeV the ratio \( \Gamma^1/\Gamma^0 \) is decreasing again.

III. FINAL REMARKS AND CONCLUSIONS

\( v_1 \ll v_2 \) has to be taken with care, because we neglected the mass of the bottom quark. In a full analysis I also have to include the scalar partner of the right
handed bottom quark, whose coupling are proportional to the bottom quark mass. Although if I take \(\tan \beta = 2\), that is \(m_b \tan \beta \ll m_{\text{top}} \cot \beta\), the shape of the curves remains the same as in Fig.2, but with values closer to the standard model.

Furthermore the most competitive decay mode to \(t \rightarrow H^+ b\) is the equivalent decay mode of \(t \rightarrow W^+ b\), which was considered in ref.14–16. There it was shown that the corrections are in the same order as for the \(t \rightarrow H^+ b\) decay, that is \(-10\%\) according to the equivalence theorem. In ref.6 it was also shown that the \(t \rightarrow W^+ b\) decay is becoming more important than the \(t \rightarrow H^+ b\) for increasing \(\tan \beta\).

In ref.2 and ref.3 it was shown that the electroweak corrections to \(t \rightarrow H^+ b\) are in the order of \(-5\%\) for a heavy top quark and decreases rapidly for higher values of \(\tan \beta\).

Last but not least within the MSSM the electroweak corrections to the \(t \rightarrow W^+ b\) decay mode was recently considered in ref.17. They considered the contribution of neutralinos and charginos to this decay mode. They showed that the contributions are of the order of \(-5\%\) to \(-10\%\) and increasing for higher values of \(\tan \beta\).

In this talk I have shown that in a full analysis of the \(t \rightarrow H^+ b\) decay one cannot neglect the gluino and scalar quark masses. As a result, because of the equivalence theorem, the same must be true for the \(t \rightarrow W^+ b\) decay. A full analysis of the gluino and scalar quarks contribution to this decay rate has not been done yet and will be presented elsewhere\(^{18}\).

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