Cannonballs in the context of Gamma Ray Bursts

Formation sites?

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Abstract. We investigate possible formation sites of the cannonballs (in the gamma ray bursts context) by calculating their physical parameters, such as density, magnetic field and temperature close to the origin. Our results favor scenarios where the cannonballs form as instabilities (knots) within magnetized jets from hyperaccreting disks. These instabilities would most likely set in beyond the light cylinder where flow velocity with Lorentz factors as high as 2000 can be achieved. Our findings challenge the cannonball model of gamma ray bursts if these indeed form inside core-collapse supernovae (SNe) as suggested in the literature; unless hyperaccreting disks and the corresponding jets are common occurrences in core-collapse SNe.

Key words. Gamma-ray bursts: cannon balls

1. Introduction

It has been argued in the literature that, as an alternative to the fireball scenario (e.g. Piran [1999] and references therein), the so-called cannonball (CB) model provides a good fit to the observed GRB flux and temporal variations (Dar & De Rújula 2004). For example, to explain GRBs, CBs must be created in supernova explosions and accelerated to high Lorentz factors, $\Gamma_{CB} \sim 1000$. However, the origin of these highly relativistic “balls” of matter has not yet been investigated and is the subject of much debate and controversy. In order to shed some light on the still open questions of their formation and early evolution we investigate, in this paper, the CB physical conditions at the origin given their features at the distance when they become transparent to their enclosed radiation as required to explain GRBs. Our proposal is that the conditions within the CB as we scale the distance down along its path to the origin should be an indication of their formation site. This, despite the simplicity of our approach, we hope might help elucidate some questions related to the origin/existence of these CBs. We start in Sect. 2 by a brief introduction to the CB model as described in Dar & De Rújula (2004). In particular CBs conditions at infinity which best fits GRB lightcurves are isolated. In Sect. 3 we present the methods we adopted to extrapolate back to the CB source. In Sect. 4, given the conditions at the origin, we study possible formation sites and explore formation mechanisms. Sect. 5 is devoted to the study of mechanisms capable of acceleration CBs to Lorentz factors as high as $\sim 1000$. We summarize our results and conclude in Sect. 6.

2. The CB model for GRBs

In the CB model for GRBs, the prompt gamma ray emission is assumed to be produced when ambient light from the supernova is Compton up scattered by the electrons in the CB. These CBs move with $\Gamma_{CB} \sim 1000$ with respect to the supernova remnant and as such the emitted radiation is highly beamed in the observer frame (Dar & De Rújula 2004). For the first $10^3$ s in the CB rest frame, the CB in a fast cooling phase emits via thermal bremsstrahlung (Dado et al. 2002), but eventually, it is argued, its emissivity is dominated by synchrotron emission from ISM electrons that penetrate it. A CB will become transparent to the bulk of its enclosed radiation in a time of $O(1)$ s in observer frame after it exits the transparent outskirts of the shell of the associated SN. The internal radiation pressure drops abruptly and its transverse expansion rate is quenched by collisionless, magnetic-field mediated interactions with the ISM (Dado et al. 2002).

Typical values for CB parameters as derived by Dado et al. (2002) and Dar & De Rújula (2004) are given in Table [Ill] where we denote the radius of the CB by $R_{CB}$, the distance travelled by the CB from its origin by $D$, the time passed in the CB rest frame by $t$. The expansion velocity of the CB is denoted

¹ The wind from the SN progenitor star is ionized and is semi-transparent to photons in the visible and UV frequencies.
Table 1. Cannon ball parameters as given by Dado et al. (2002) and Dar & De Rújula (2004)

| parameter      | value         |
|----------------|---------------|
| $R_{CB,max}$   | $2.2 \times 10^{14}$ cm |
| $R_{CB,trans}$ | $10^{13}$ cm  |
| $D_{trans}$    | $1.7 \times 10^{16}$ cm |
| $\Gamma_{CB}$  | $1.0 \times 10^{3}$ |
| $\delta$       | $1.0 \times 10^{3}$ |
| $z$             | 1             |
| $\beta_{i}$    | $1/\sqrt{3}$  |
| $N_{CB}$        | $10^{50}$     |
| $M_{CB}$        | $10^{26}$ g   |

by $\beta_{i} = v_{i}/c$, the number of baryons and the mass of the CB by $N_{CB}$ and $M_{CB}$, respectively. The subscript "trans" refers to the point where the CB becomes transparent ("transparency radius").

By fitting the observed GRB afterglow the CB Lorentz factor is estimated to vary between $\Gamma_{CB} = 250$ and $\Gamma_{CB} = 1600$, while the number of baryons is of the order of $10^{50}$. With this information at hand, our goal is to derive the conditions in the CB as we integrate back to a plausible source. As an indication of the close proximity to a compact source (e.g. black-holes and neutron stars) when applicable we will make use of the notion of light cylinder which we take to be about $R_{LC} \sim 10^{7}$ cm.

3. Cannonball propagation and evolution

In this section, we will explore the evolution of the CB by extrapolating backward from the location where the GRB occurs to where the CB reaches nuclear saturation density (applying CB parameters as given in Table 1). For simplicity, we assume that the CB is expanding with a constant velocity and is moving with a constant Lorentz factor. The natural assumption for the expansion velocity is the sound speed of the hot blob of matter, $v_{exp} \simeq c_{s} \simeq c/\sqrt{3}$. Six different cases of CB Lorentz factor and mass are investigated (see Table 2).

We will first apply a simple model of the CB’s internal energy obeying a simple equation of state. Using this we calculate the evolution of the density, magnetic field and temperature as the CB moves away from the origin. In a second step we extend our model approach applying an energy equation where pressure degeneracy and neutrino effects are included. As we will see, spatial back integration from the transparency radius will give strong indication that the CB may be launched close to a black hole. The spatial integration back to the source is carried out until the CB temperature reaches extreme values, $T_{i} \sim 100$ MeV, unless the CB density reaches nuclear saturation density before $T_{i}$.

With the CB expanding at a constant expansion velocity equal to the sound speed of the matter $v_{exp} = c/\sqrt{3}$, we can use certain estimates about the CB at the distance of transparency to derive an interrelation between radius and distance from origin the CB has traveled. As radius of the CB at the distance of transparency we apply the estimate by Dado et al. (2002),

$$R_{trans} \simeq 10^{13} \left( \frac{N_{CB}}{6 \times 10^{50}} \right)^{1/2} \text{cm.} \quad (1)$$

To reach this radius, the CB has traveled a period of time

$$t_{trans,CB} = \frac{R_{trans}}{v_{exp}} \simeq 577 \left( \frac{N_{CB}}{10^{50}} \right)^{1/2} \text{sec.} \quad (2)$$

in the CB rest frame. As it travels essentially with the speed of light, at the time when it becomes transparent, the CB has traveled a distance

$$D_{trans} = \Gamma_{CB} c t_{trans} \quad (3)$$

from its origin where it was ejected. This gives a linear scaling factor

$$l = R_{trans}/D_{trans} = \frac{1}{\Gamma_{CB} \sqrt{3}} \quad (4)$$

The radius of a CB is then simply expressed as $R_{CB} = D/\Gamma_{CB} \sqrt{3}$. The radius at nuclear saturation density and at the light cylinder ($D_{LC} \sim 1.5 \times 10^{7}$ cm) are listed in Table 2, as well as the radius at which the CB becomes transparent to the enclosed radiation for the different cases of CB masses and Lorentz factors.

Table 2. The different cases of CBs explored in this work. Note that cases 6, 8 and 9 ($\Gamma_{CB} = 1000$ and $N_{CB} = 10^{51}$, $\Gamma_{CB} = 2000$ and $N_{CB} = 10^{50}$, $\Gamma_{CB} = 2000$ and $N_{CB} = 10^{51}$ respectively) are not consistent with our assumptions as they reaches nuclear saturation density at a distance beyond the light cylinder, and have therefore been left out in this paper.

| $\Gamma_{CB}$ | $N_{CB}$ |
|---------------|----------|
| Case 1        | $1.0 \times 10^{5}$ $10^{29}$ |
| Case 2        | $1.0 \times 10^{5}$ $10^{50}$ |
| Case 3        | $1.0 \times 10^{5}$ $10^{51}$ |
| Case 4        | $1.0 \times 10^{5}$ $10^{69}$ |
| Case 5        | $1.0 \times 10^{5}$ $10^{50}$ |
| Case 7        | $2.0 \times 10^{3}$ $10^{59}$ |

Table 3. The CB radius at the point where it approaches density reaches nuclear saturation density (left), at the light cylinder $R_{LC} = 1.5 \times 10^{7}$ cm (middle) and at the distance where the CB becomes transparent to radiation (right).

| $\Gamma_{CB}$ | $N_{CB}$ |
|---------------|----------|
| Case 1        | $2.5 \times 10^{3}$ $1.2 \times 10^{5}$ $1.0 \times 10^{12}$ |
| Case 2        | $5.6 \times 10^{3}$ $1.2 \times 10^{5}$ $1.0 \times 10^{13}$ |
| Case 3        | $12.0 \times 10^{3}$ $1.2 \times 10^{5}$ $1.0 \times 10^{14}$ |
| Case 4        | $2.5 \times 10^{3}$ $1.2 \times 10^{4}$ $1.0 \times 10^{12}$ |
| Case 5        | $5.6 \times 10^{3}$ $1.2 \times 10^{4}$ $1.0 \times 10^{13}$ |
| Case 7        | $2.5 \times 10^{3}$ $5.8 \times 10^{3}$ $1.0 \times 10^{12}$ |
the field strength is between

\[ B \] (G) from origin (Fig. 2). Close to the hypothetical CB origin where we may compute the CB magnetic field versus the distance we reach nuclear saturation density, the magnetic field strength by the CB. Depending on the CB parameters, the densities at reaches nuclear saturation density.

The backward integration is stopped when the CB density

Fig. 2. Magnetic field vs distance from the origin for the CB. The backward integration is stopped when the CB density reaches nuclear saturation density.

\[ \rho \] [g/cm³]

\[ v \] (s)

\[ M \] [g]

\[ R \] [cm]

\[ N \] (8)

\[ T \] (K)

\[ a \] is the number density of the CB.

\[ k \] is Boltzmann’s constant.

\[ T \] is the CB temperature.

\[ \epsilon_m \] is a parameter that allow us to write the magnetic energy in terms of the CB rest mass energy.

This parameter is fixed by imposing energy equipartition at the specified CB origin. Note that the magnetic energy is constant, as we assume that the magnetic field is not dissipated in reconnection events and it is not expelled from the CB. The gravitational energy is always negligible compared to the other energy channels. The total energy is then given as:

\[ E_{\text{tot}} = 3 \times E_{\text{mag}} = 3 \epsilon_m M_{\text{CB}} c^2. \]  (10)

We can now write the energy equation as:

\[ 4/3 \pi R^3 a T^4 + 3 N k T = 2 \epsilon_m M c^2 \]  (11)

This equation is solved to find \( T \) as a function of \( D \).

In what follows, we explore two scenarios: (i) the first one is indicative of the close proximity of a compact source and as such it corresponds to the case where the energy equation is integrated assuming equipartition at nuclear saturation densities; (ii) the second reflect scenarios where the CB originates from the coronal region of compact stars of their associated accretion disks. Specifics below.

3.1. Simple energy equation

We continue our simple estimates by assuming energy conservation in the CB

\[ E_{\text{rad}} + E_{\text{th}} + E_{\text{mag}} = E_{\text{tot}}. \]  (6)

The radiation energy is written as,

\[ E_{\text{rad}} = a T^4 \frac{4}{3} \pi R^3_{\text{CB}}, \]  (7)

the magnetic energy is,

\[ E_{\text{mag}} = \epsilon_m M_{\text{CB}} c^2 \]  (8)

and the gas thermal energy is,

\[ E_{\text{th}} = 3 N_{\text{CB}} k T, \]  (9)

where \( a = 7.5657 \times 10^{-15} \text{ erg cm}^{-3} \text{ K}^{-4} \) is the radiation constant, \( k = 1.3807 \times 10^{-16} \text{ erg K}^{-1} \) is Boltzmann’s constant, \( T \) is the CB temperature, \( \epsilon_m \) is a parameter that allow us to write the magnetic energy in terms of the CB rest mass energy.

This parameter is fixed by imposing energy equipartition at the specified CB origin. Note that the magnetic energy is constant, as we assume that the magnetic field is not dissipated in reconnection events and it is not expelled from the CB. The gravitational energy is always negligible compared to the other energy channels. The total energy is then given as:

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3.1.1. Equipartition at nuclear densities: source origin

To set equipartition at nuclear saturation density, \( \epsilon_m \) has to be 0.5 for all cases. By rearranging Eq. (11) it can be seen that it becomes a function of \( T \) and \( \rho \):

\[ a T^4 + 3 p k T / m_H = 2 p c^2 \epsilon_m \]  (12)

we may compute the CB magnetic field versus the distance from origin (Fig. 2). Close to the hypothetical CB origin where we reach nuclear saturation density, the magnetic field strength approaches values up to \( B \sim 10^{15} G \) (see also [4]). At a distance from the origin of the order of the light cylinder radius, the field strength is between \( 4.8 \times 10^{13} \) and \( 1.4 \times 10^{18} \) Gauss depending on the choice of Lorentz factor and mass of the CB. The densities ranges from \( 10^9 \) to \( 10^{14} \) g/cm³.

The temperature therefore depends on the density only and implies a temperature of about \( 10^{12} \) K at nuclear saturation density for all cases. The temperature as a function of distance travelled is shown in Fig. 3 while Fig. 4 shows the energy components for case 4. The radiation energy is dominant everywhere except at nuclear saturation density where there is equipartition. We note that nuclear saturation density would be reach at distances larger than \( 10^9 \) cm for most cases. It is unrealistic to
find object with such high densities much larger than $10^6$ cm. Also, CBs with such densities need a magnetic field $B = 10^{18}$ G, which is unrealistically high. We can therefore rule out CBs formed with nuclear saturation density.

It should be noted that the solutions are not very sensitive to the choice of total internal energy. As an example, increasing the total internal energy by an order of magnitude we find a temperature at the light cylinder, $T_{\text{lc}}$, for case 3 to be $9 \times 10^{11}$ K, compared to $5 \times 10^{11}$ K in our initial calculation.

3.1.2. Equipartition at the light cylinder: coronal origin

Figure 5 show the evolution of the CB conditions when equipartition at a distance of about a light cylinder radius is assumed, and in Fig. 4 the energy components for case 4. Table 4 shows the temperature found at the light cylinder and the value for the energy equipartition parameter $\epsilon_m$ for different kinematic parameters of the CB (see Table 1). Note that $\epsilon_m$ is now determined by the condition that we have equipartition at the CB origin (i.e. at a light cylinder distance).

**Table 4.** Coronal CB origin. The values for the temperature and $\epsilon_m$ found for the simple energy equation assuming equipartition between the magnetic, gas thermal and radiation energy at the light cylinder.

| Case | $T$ [K] | $\epsilon_m$ |
|------|--------|-------------|
| 1    | $5.9 \times 10^{10}$ | 0.0165 |
| 2    | $1.3 \times 10^{11}$ | 0.035 |
| 3    | $2.7 \times 10^{11}$ | 0.075 |
| 4    | $6.1 \times 10^{11}$ | 0.175 |
| 5    | $1.2 \times 10^{12}$ | 0.325 |
| 7    | $1.2 \times 10^{12}$ | 0.325 |

By assuming an equal number of electrons $n_e$ and baryons in the CB the Fermi temperature can be computed. If we take the light cylinder distance as a typical length unit at the CB origin and compute the Fermi temperature at this location, we see that the CB temperature is much smaller than the electron Fermi temperature (Table 5). We therefore have to improve our approach considering also electron degeneracy and neutrino effects. In the next section we will explore a more appropriate energy equation where neutrino effects are added. However, we will still apply a kinematic approach assuming a linear expansion of the CB.

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2 The expansion energy of the CB is of the same order as the magnetic energy. For simplicity we have included it in the expression for the total energy.
density is found at the light cylinder for the simple energy equation and assuming equipartition at the light cylinder. The temperature and magnetic field at nuclear saturation is computed from Eq. 3.8 and Eq. 3.9 in Popham et al. (1999): 

\[ \frac{3}{2}RTM_{CB} \left( \frac{1 + 3X_{\text{nuc}}}{4} \right) + \frac{11}{4}aT^4 \left( \frac{4\pi D^3}{\Gamma_{\text{CB}}^{3.5/2}} \right) \]

Note that we apply the same total internal energy \( (E_{\text{tot}}) \) of the CB as in Sec. 3.3, thus the same energy parameter \( \epsilon_{\text{m}} \). Close to the origin, however, we add an energy component due to neutrino effects (emissivity and cooling) and is denoted by \( E_{\nu}(t) \) in Eq. (15).

Two types of neutrino losses may occur, i.e. neutrino emission due to pair annihilation and neutrino losses due to the capture of pairs on nuclei. Their contribution to the energy budget is computed from Eq. 3.8 and Eq. 3.9 in Popham et al. (1999):

\[ q_{\nu,T} = 5.0 \times 10^{33} \left( \frac{T}{10^{11}K} \right)^9 \text{ ergs cm}^{-3} \text{s}^{-1} \]

\[ q_{\nu,N} = 9.0 \times 10^{33} \left( \frac{\rho}{10^{10}\text{g/cm}^3} \right) \left( \frac{T}{10^{11}K} \right)^6 \text{ ergs cm}^{-3} \text{s}^{-1} \]

These expressions are integrated over the time it takes the CB to reach conditions for which neutrino cooling is not significant. We find the latter cooling method (Eq. 20) to be dominant, so we limit ourselves to using that.

To compute the effects due to neutrinos, we must know the temperature. However, in turn, we want to use the neutrino effects to find the temperature. We therefore first solve the energy equation (13) without adding neutrino effects. Then we use the temperature found to calculate the neutrino emissivity which is then added to the total energy in equation (15) and this equation is solved to find the temperature as a function of distance travelled. The neutrinos are released in small successive bursts, mimicking a continuous emission.

We emphasize again that because of the neutrinos effects and the different energy equation used in sec 3.3 there is no assumption on energy equipartition applied in this section.

The temperature is shown in Fig. 7 as a function of distance travelled. In general, the release of neutrinos are seen as a small jump in the temperature curve. We note that cases 1 to 4 reach the light cylinder with reasonable temperatures \( (T < 10^{13} \text{ K}) \) and densities \( (\rho < 10^{14}\text{g/cm}^3) \). Cases 5 and 7 are ruled out as they reach even more extreme conditions before reaching the light cylinder. For illustrative purposes in Fig. 8 we show the energy components for case 3. Because of the neutrino effects, the radiation energy is now the dominant energy, even close to the light cylinder.

### 3.2. Coronal CB origin: neutrino effects

In this section we improve our approach by taking into account degeneracy pressure and neutrino cooling (e.g. Popham et al. 1999). The energy equation becomes,

\[ E_{\text{tot}} + E_{\nu}(t) = E_{\text{th}} + E_{\text{rad}} + E_{\text{deg}} + E_{\text{mag}} \]

where

\[ E_{\text{deg}} = 3K M_{\text{CB}} \left( \frac{\rho}{\mu_{\text{e}}} \right)^{1/3} \]

is the degeneracy energy,

\[ E_{\text{rad}} = \frac{11}{4}aT^4 \left( \frac{\rho}{M_{\text{CB}}} \right) \]

is the radiation energy and

\[ E_{\text{th}} = \frac{3}{2}RTM_{\text{CB}} \left( 1 + 3X_{\text{nuc}} \right) \]

the gas thermal energy where

\[ X_{\text{nuc}} = 30.97 \left( \frac{\rho}{10^{10}\text{g/cm}^3} \right)^{-3/4} \left( \frac{T}{10^{10} \text{K}} \right)^{9/8} \times \exp \left( -6.096 \times \frac{10^{10} \text{K}}{T} \right) \]

\[ \rho < 10^{14} \text{ g/cm}^3 \]

\[ T < 10^{13} \text{ K} \]

\[ X_{\text{nuc}} < 1, \text{ and } X_{\text{nuc}} = 1 \text{ elsewhere.} \]

In the equation above, \( K = (2\pi h c/3)(3/8\pi m_{\text{nuc}})^{1/3} = 1.24 \times 10^{15} \), \( m_{\text{nuc}} \) is the nucleon mass, \( R \) is the gas constant, \( a \) is the radiation constant and \( \mu_{\text{e}} = 2 \) is the mass per electron. Inserting Eqs. (8), (14), (15) and (16) into (15) gives in terms of \( D \)

\[ E_{\text{tot}} + E_{\nu}(t) = 3K M_{\text{CB}} \left( \frac{\rho}{\mu_{\text{e}}} \right)^{1/3} \Gamma_{\text{CB}} \left( \frac{D}{\pi \mu_{\text{e}} \rho} \right)^{3.5/2} \]

\[ + \frac{3}{2}RTM_{\text{CB}} \left( 1 + 3X_{\text{nuc}} \right) \left( \frac{T}{10^{11} \text{K}} \right)^6 \]

\[ \text{ergs cm}^{-3} \text{s}^{-1} \]

This section we discuss sites that are best suited to account for the CBs conditions at the source derived in our previous section. We also explore possible formation scenarios.
find that the “flows are gravitationally stable under almost all conditions of interest”. Exceptions exist for strong accretion rates and in the outer part of the disk (see also Narayan et al. 2001). However as can be seen for Eq. [A.3] in the appendix these extreme cases favor lower densities and temperatures than those expected for CBs.

The magneto-rotational instability (MRI, Balbus & Hawley 1991 and Hawley & Balbus 1991) works only for low magnetic field strengths and cannot account for the strong magnetic fields required at the origin for CBs. Let us also mention the accretion-ejection instability (Tagger et al. 1992) as a possible formation mechanism. This instability works for intermediate magnetic field strengths and will transfer angular momentum to Alfvén waves toward the corona of the disk. At extreme magnetization the accretion-ejection instability is reminiscent of the interchange instability (Spruit et al. 1995) but it seems unlikely that these can lead to CBs formation since most of the perturbations are carried by Alfvén waves.

It is thus not clear how a CB can form within a hyperaccretion disk. There is also the issue of accelerating the CB to \( \Gamma > 100 \) which is also a major challenge. We will return to this in \[\square\] after we discuss other possible formation sites.

4.2. Neutron tori

The thick, self-gravitating, neutron tori around 2-3 \( M_\odot \) black holes are known to be affected by a runaway instability on time scales below the evolutionary time scale of GRBs (Nishida & Eriguchi 1996) and we therefore exclude them as source for CBs. Simulations of neutron star mergers have also shown that about 0.01 \( M_\odot \) of the thick disk of 0.2 \( M_\odot \) around a 1.5 to 3.1 \( M_\odot \) final central mass distribution becomes gravitationally unbound (Rosswog et al. 1999). However, in difference to the hyperaccreting disk model, this unbound mass stays rather cold (10\(^8\) \( K \)) and do not constitute a formation site for CBs.

4.3. Accretion disk corona

Another possibility is CB formation in the disk corona, for example as a huge magnetic flare which ejects a large part of the accretion disk corona into a bullet of high velocity. A CB of such a size would have a density of about \( 2.4 \times 10^5 \) g cm\(^{-3} \) which is, for comparison, in the range of white dwarf densities. The maximum initial size of the CB we expect not to exceed \( R_{L,C} \approx 10^7 \) cm. Comparing the CB asymptotic kinetic energy to the magnetic energy contained in a volume of that size, provides an estimate for the mean magnetic field strength of about 10\(^15\) G. This corresponds to a magnetic flux of \( \sim 10^{29} - 10^{30} \) G cm\(^2\) and is unrealistically high for such coronae.

4.4. Disk-jets and funnel-jets

Figure \[\text{\textcopyright}\] is an illustration of the type of jets that could emanate from the vicinity of a compact star. The disk-jet material is ejected from the accretion disk while the funnel-jet is ejected
Fig. 9. Illustration of Funnel-jet and Disk-jet. The funnel-jet is launched from a region close to the compact star. The disk-jet is launched from the accretion disk.

from the innermost parts of the disk at the interface with the compact star.

Recent general relativistic magneto-hydrodynamic simulations by De Villiers et al. (2005) of a black hole and an initial torus seeded with a weak poloidal magnetic field show that a funnel jet with \( \Gamma_{\text{CB}} \gtrsim 50 \) is formed. Instabilities do occur in funnel-jets, however, the induced instabilities have densities much lower than the CB values found in the previous section. Funnel-jets can therefore be ruled out as a possible formation site for CBs.

A disk-jet becomes cylindrically collimated on a length scale of the order of 1-2 light cylinder distances (Fendt & Memola 2001). Knot generating instabilities reminiscent of CBs are known to occur as jets collimate (Ouyed et al. 1997). This is a possible formation mechanism for CBs. What remains is to show how they can be accelerated to high \( \Gamma \). Specifics below.

5. CB acceleration to ultra-relativistic velocity

Having isolated, or more precisely favored, jets from hyper-accretion disks as plausible formation sites for CBs we now discuss acceleration mechanisms with which CBs can reach Lorentz factors in the thousands.

Assuming that the CB is accelerated by converting the internal magnetic energy to kinetic energy, we can find an estimate for the magnetic field needed to explain such Lorentz factors. Using typical CB radii close to the source and parameters from Table I, we find that the magnetic field must be of the order of \( 10^{18} - 10^{19} \) G. This is unrealistically high. Either our approach is too simple, or a different acceleration mechanism must be at work.

5.1. MHD Acceleration: CB speed

The ability of the magnetic field to accelerate particles to high Lorentz factors is given by the magnetization parameter (Michel 1969)

\[
\sigma = \frac{\Phi^2 \Omega_F^2}{4M_{\text{jet}} c^3},
\]

where \( \Phi = B r^2 \) is the magnetic flux, \( \Omega_F = c/D_{\text{lc}} \) is the angular frequency of the magnetic field and \( M_{\text{jet}} = \pi \rho v_p r^2 \) is the mass flow rate within the flux surface. For spherical outflow Michel (1969) found that the Lorentz factor at infinity scales as

\[
\Gamma_{\infty} = \sigma^{1/3}.
\]

Fendt & Ouyed (2004) finds a modified Michel scaling in the case of a non-spherical magnetic field distribution. In this case they find a linear relation between \( \sigma \) and \( \Gamma_{\infty} \). If the field distribution is \( \Phi(r; \Psi) \sim r^{-0.1} \), they find that \( \Gamma_{\infty} = 10^{-1/3} \sigma \), and if \( \Phi(r; \Psi) \sim r^{-0.2} \), they find

\[
\Gamma_{\infty} = 10^{-1/5} \sigma,
\]

in which case hyperaccreting disks with ejection rates of the order \( 10^{-5} M_\odot/s \) and magnetic field of the order \( 10^4 G \) can lead to jets with a Lorentz factor \( \Gamma_{\infty} \sim 1875 \).

5.2. MHD instability: CB mass

To a first order, instabilities related to Alfvén crossing time can develop on timescales

\[
t_{\text{ins}} = t_A = \frac{2R_{\text{jet}}}{v_A},
\]

where \( R_{\text{jet}} \) is the radius of the disk-jet. For \( 1R_{\text{lc}} < R_{\text{jet}} < 10R_{\text{lc}} \), we arrive at \( t_{\text{ins}} \sim 1 - 10 \) ms which would imply the plausible formation of blob of matter as massive as \( M_{\text{ins}} = t_{\text{ins}} M_{\text{jet}} \sim 10^{-8} - 10^{-7} M_\odot \). This can be compared to the typical CB mass of the order \( M_{\text{CB}} = 10^{-7} M_\odot \).

As we have shown above, first forming the CB in the disk and then accelerating it will require unrealistic magnetic fields of the order \( 10^{19} \) G. However, first accelerating the wind to the light cylinder and then forming the CB through an instability beyond the light cylinder requires much smaller magnetic field strength \( (< 10^{14} \) G). This is a possible mechanism for forming and accelerating CBs.

6. Conclusion

Assuming that CBs move and expand with a constant velocity we have estimated the CB conditions as close as possible to their origin. CBs require extremely high internal magnetic fields when they are formed with field strength exceeding \( \sim 10^{15} \) G. The temperature was found to be of the order of \( 10^{11} - 10^{12} \) K. The physical parameters of the CBs at the origin are, within an order of magnitude estimates, indicative of hyperaccreting disks. However, if formed in the accretion disk we find it challenging to accelerate the CBs to the high Lorentz factors. The coronal origin is ruled out because of the unrealistically high corona magnetic flux necessary to form the CBs. Our results instead hint to a jet origin of CBs. The radius \( (< D_{\text{lc}}) \) and mass flow \( (10^{-5} M_\odot/s) \) in a jet from a hyperaccreting can account for the CB mass and density. Furthermore,
this outflow can be accelerated to $\Gamma \sim 2000$ by MHD processes (Fendt & Ouyed 2004). Any instability in this outflow beyond the light cylinder could lead to CB formation. We thus suggest that CBs form as instabilities in ultra-relativistic jets emanating from the surface of hyperaccretion disks. The tight link between SNe and the CB model for GRB requires that all (or almost all) core collapse SNe will produce CBs. Our work, within its limitations, implies that hyperaccretion disks must be a common occurrence in core collapse SNe to accommodate the CB model - a notion which remains to be confirmed.

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Appendix A: Accretion disks

A.1. “Standard” accretion disks

A standard Shakura-Sunyaev disk (Shakura & Sunyaev 1973) will have a density

$$\rho [\text{g cm}^{-3}] = 7.2 \times 10^{-4} \left( \frac{\alpha_v}{0.001} \right)^{-1} \left( \frac{M}{M_{\text{Edd}}} \right)^{-2} \left( \frac{r}{3r_S} \right)^{3/2} \times \left( \frac{M}{M_{\odot}} \right)^{-1} \left( 1 - \left( \frac{r}{3r_S} \right)^{-1/2} \right)^{-2}. \quad (A.1)$$

where $\alpha_v$ is a viscosity parameter, $r_S$ is Schwarzschild radius and $M_{\text{Edd}}$ is the Eddington mass. With this density, the radius of a CB with mass $M = 10^{50}$ baryons becomes $3.8 \times 10^9$ cm, assuming the default parameters. For the equipartition magnetic field one gets

$$B [\text{G}] = 10^8 \left( \frac{M}{M_{\odot}} \right)^{-1/2} \left( \frac{r}{3r_S} \right)^{-3/4}. \quad (A.2)$$

The temperature is

$$T [\text{K}] = 1.3 \times 10^8 \left( \frac{\alpha_v}{0.001} \right)^{-1/4} \left( \frac{M}{M_{\odot}} \right)^{-1/4} \left( \frac{r}{3r_S} \right)^{-3/4}. \quad (A.3)$$

Advection Dominated Accretion Flow (ADAF) disks have density (Narayan & Yi 1995):

$$\rho [\text{g cm}^{-3}] = 6.5 \times 10^{-3} \left( \frac{\alpha_v}{0.001} \right)^{-1} c_1 c_3^{-1/2} \left( \frac{M}{M_{\text{Edd}}} \right)^{11} \times \left( \frac{M}{M_{\odot}} \right)^{-1} \left( \frac{r}{3r_S} \right)^{-3/2}, \quad (A.4)$$

where $c_1$ and $c_3$ are defined in Eq. (2.1) in Narayan & Yi (1995).

The corresponding magnetic field is

$$B [\text{G}] = 5.5 \times 10^9 \left( \frac{\alpha_v}{0.001} \right)^{-1/2} c_1 c_3^{-1/2} c_3^{-1/4} (1 - \beta)^{1/2} \times \left( \frac{M}{M_{\text{Edd}}} \right)^{1/2} \left( \frac{M}{M_{\odot}} \right)^{-1/2} \left( \frac{r}{3r_S} \right)^{-5/4}, \quad (A.5)$$

while the ion temperature of such disks are (Narayan et al. 1998)

$$T_i [\text{K}] = 2 \times 10^{12} \beta \left( \frac{r}{2r_S} \right)^{-1}, \quad (A.6)$$

where $\beta$ is given by

$$p_m = \frac{B^2}{24\pi} = (1 - \beta) \rho c_s^2, \quad (A.7)$$

A.2. Hyperaccreting disks

Hyperaccreting disk (Popham et al. 1999) density is:

$$\rho [\text{g cm}^{-3}] = 1.3 \times 10^{12} \left( \frac{\alpha_v}{1.0} \right)^{-1.3} \left( \frac{M}{M_{\odot} 8.7^{+1}} \right)^{1} \times \left( \frac{M}{M_{\odot}} \right)^{-1.7} \left( \frac{r}{3r_S} \right)^{-2.55}, \quad (A.8)$$

their disk scale height is

$$H [\text{cm}] = 1.9 \times 10^5 \left( \frac{\alpha_v}{1.0} \right)^{0.1} \left( \frac{M}{M_{\odot}} \right)^{0.9} \left( \frac{r}{3r_S} \right)^{1.35}, \quad (A.9)$$

while the temperature is

$$T [\text{K}] = 7.6 \times 10^{10} \left( \frac{\alpha_v}{1.0} \right)^{0.2} \left( \frac{M}{M_{\odot}} \right)^{-0.2} \left( \frac{r}{3r_S} \right)^{-0.3}, \quad (A.10)$$

The corresponding equipartition magnetic field is of the order of

$$B [\text{G}] \sim 10^{14} - 10^{15}. \quad (A.11)$$
Appendix B: Forward integration

For completeness and for self-consistency check, here we consider CBs with hyperaccreting disk conditions at the origin and perform a forward integration until the CBs reach the distances where they become transparent.

Assuming that the CB radius evolves as before, \( R = c/(\Gamma_{CB} \sqrt{3}) \), then the density at a distance corresponding to the surface of the hyperaccreting disk (\( D = 10^5 \) cm) will be too high. We will thus make a slight adjustment by rewriting the radius as \( R = D/(\Gamma_{CB} \sqrt{3}) + x \), where \( x \) is a number that ensures that the density at the origin does not exceed \( \rho = 10^{12} \text{g/cm}^3 \). Therefore, \( x \) is found by solving the following equation:

\[
10^{12} \text{g/cm}^3 = \frac{M_{CB}}{\frac{4}{3} \pi \left( \frac{10^5 \text{cm}}{\Gamma_{CB} \sqrt{3}} + x \right)^3},
\]

which implies:

\[
x[\text{cm}] = \frac{3M_{CB}^{1/3}(6/\pi)^{1/3}\Gamma_{CB} - \sqrt{12} \times 10^9}{60000 \Gamma_{CB}}.
\]

This also ensures the correct expansion velocity \( v_{\text{exp}} = c/\sqrt{3} \). Table B.1 shows the corresponding parameter values for \( x \) and \( \epsilon_m \). We should also note that in this case \( \epsilon_m \) will be chosen as to insure energy equipartition at the disk surface. The temperature thus found is used to calculate the neutrino emissivity, which is then added to the total energy in Eq. 13 to find the new temperature. As before, the neutrinos are released in small successive bursts mimicking a continuous release of neutrinos.

Figs. B.2 and B.3 show the temperature, magnetic field and density as a function of distance. All cases starts with \( T \sim 2 \times 10^{11} \) K, \( \rho = 10^{12} \text{g/cm}^3 \) and \( B = 6 \times 10^{16} \) G at \( D = 10^5 \) cm. The neutrino effects can be seen as small jumps in the temperature curves, but in general the neutrinos do not change the overall picture a lot. The neutrino contribution were of the same order or smaller than the total energy, and as discussed before the temperature is not very sensitive to changes in the total energy.

Table B.1. The parameter \( x \) used in the relation between \( R_{CB} \) and \( D \), and \( \epsilon_m \) for the different CB cases when starting with disk conditions and integrating forward.

| Case | \( x[\text{cm}] \) | \( \epsilon_m \) |
|------|-----------------|----------------|
| 1    | 15279.2         | 0.045          |
| 2    | 33584.5         | 0.045          |
| 3    | 73022.3         | 0.045          |
| 4    | 15798.9         | 0.045          |
| 5    | 34104.2         | 0.045          |
| 7    | 15827.7         | 0.045          |
The temperature at $D_{\text{trans}}$ is of the same order as for the backward integration ($T = 10^4 \text{ K}$ to $T = 10^5 \text{ K}$), and also close to the value given by Dado et al. (2002) of $T_{\text{trans}} \simeq 4 \text{ eV}$. The difference between the backward and forward integration at large distances is due to the different $\epsilon_m$ parameter. For large distances, the $x$-parameter does not play any role.

To summarize, the results of the forward integration indicate that CBs formed within hyperaccretion disks could in principle provide the necessary conditions at $D_{\text{trans}}$ to account for GRB features as claimed in Dado et al. (2002).