Unsteady Laminar Flow Analysis of ER Valve Systems: Modeling and Simulation

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Abstract. This paper proposes a new method to model electrorheological valve systems based on the unsteady flow analysis. After describing the mechanism and operational principle of the ER valve, the unsteady flow in the valve is identified. A method to model this unsteady flow is developed based on the assumptions that the ER fluid follows Bingham behavior and the pressure drop of the unsteady flow results from the independent pressure drops due to the yield stress, inertia and fluid viscosity. The dynamic model of the valve is then derived based on the unsteady flow analysis. The time response of ER effect is also considered in the dynamic model. Using the proposed model, the performance of the ER valve is predicted and validated by comparing with experimental results. In addition, the flow rate predicted by the unsteady flow model is compared with that by the steady flow model.

1. Introduction

A hydraulic system is very important in industrial applications where large inertia and torque loads have to be handled [1]. Various types of valves have been developed as the key element of the hydraulic system. Among them, the electrorheological (ER) valve has been successfully employed in many engineering systems such as cylinder for position control, pressure modulator for pressure control, damper for suspension control, etc. When the ER fluid is employed in valve systems, its rheological change causes a controllable pressure drop which can be continuously tuned by the intensity of the imposed electric field. This feature has triggered a large amount of research to develop accurate control model and design mechanism of the ER valve [2,3]. However, in previous studies, the behaviour of the ER valve was predicted based on the assumption of the steady flow of ER fluid through the valve duct although the ER fluid unsteadily flows at real situations. Consequently, the main contribution of the present work is to propose a new modeling methodology for the ER valve based on unsteady laminar flow analysis of the ER fluid. After manufacturing a cylindrical type of ER valve, its pressure drop is analyzed under unsteady flow conditions with randomly varying electric field in which the ER fluid follows Bingham behaviour. The dynamic model of the valve is then derived by considering the pressure drop under unsteady flow conditions and time response of the ER fluid. In order to validate the proposed model, the predicted valve performance is compared with experimental result.
2. Modeling of the ER Valve

The geometric configuration and photograph of a cylindrical ER valve devised in this work is shown in Figure 1. As shown in the Figure 1(a), it is composed of cylindrical inner and outer electrodes which are fixed by two fixtures at both ends. The ER fluid flows through the gap between the inner and outer cylinders. The pressure drop is then continuously controlled by the intensity of the input electric field. Figure 1(b) shows the manufactured ER valve. The materials for the outer electrode and inner electrode are aluminum and stainless steel, respectively. A nonconductive fixture is used to maintain the electrode gap, and the ‘o’ ring is adopted to prevent the leakage of the ER fluid.

Figure 2 shows the flow through an annular duct. By neglecting the gravity and assuming the linear axial distribution of pressure, the axial momentum equation of motion for the steady-state laminar flow of an incompressible fluid can be expressed in cylindrical coordinate system as:

\[
\frac{1}{r} \frac{\partial}{\partial r} (r \tau) = \frac{dp}{dz} = -\frac{\Delta P}{L} ; \quad R_i \leq r \leq R_o
\]

where \( \tau \) is the shear stress across the flow at radius \( r \), \( L \) is the duct length, \( R_i \) and \( R_o \) are respectively the inner and outer radius of the duct, respectively, and \( \Delta P \) is the pressure drop of the flow through the duct. Because the radius of the duct is much larger than its gap it is assumed that the flow through the annular duct is equivalent to the flow through the duct between two parallel plates shown in Figure 3. Therefore, equation (1) becomes

\[
\frac{\partial \tau}{\partial y} = -\frac{\Delta P}{L} ; \quad (0 \leq y \leq d)
\]

where \( d \) is the duct gap. In this work, the ER fluid is assumed to follow Bingham rheological laws, thus the following rheological equation of state can be obtained.

\[
\tau = \tau_y \text{sgn}(u) + \mu \frac{\partial u}{\partial y}
\]

where \( u \) is the velocity of the flow at coordinate \( y \), \( \tau_y \) is the yield stress of the ER fluid induced by the applied voltage, and \( \mu \) is the field-independent plastic viscosity.

Figure 2. ER fluid flow in the annular valve duct

Figure 3. Equivalent ER fluid flow.
From equations (2) and (3), the flow rate of steady state ER fluid flow through the duct is derived as follows [4]:

\[
Q = \frac{W \Delta P}{12 \mu L} (d - \delta)^2 (d + \delta / 2)
\]  

(4)

In the above, \( W \) is the width of the equivalent duct which is calculated by \( W = 2\pi R \); \( R \) is the average radius of the annular duct, and \( \delta \) is plug thickness of the ER flow given by

\[
\delta = \frac{2L \tau_y}{|\Delta P|}
\]  

(5)

Equation (4) is usually used to calculate steady state flow rate of ER fluid flow due to the given pressure at the inlet and the outlet of the duct. On the other hand, when the flow rate is given, the pressure drop of the steady state ER fluid flow can be calculated by

\[
\Delta P = \frac{12 \mu L}{Wd^3} Q + c \frac{L}{d} \tau_y
\]  

(6)

where the coefficient \( c \) can be approximately estimated as follows:

\[
c = 2.07 + \frac{12Q\mu}{12Q\mu + 0.8\pi Rd^2}\tau_y
\]  

(7)

In the case of unsteady state flow, the momentum equation of the ER fluid in the duct is expressed as follows:

\[
\rho \frac{\partial u}{\partial t} - \frac{\partial}{\partial y} \left( \frac{\partial \tau}{\partial y} \right) = \frac{\Delta P}{L}, \quad (0 \leq y \leq d)
\]  

(8)

where \( \rho \) is the density of the fluid. By multiplying \( W \) in both sides of Eq. (8) and then integrating with \( y \), equation (8) can be rewritten by

\[
\Delta P_u = \rho L \frac{dQ}{dt} \frac{d}{L} \int_0^d \frac{\partial \tau}{\partial y} dy = \Delta P_1 + \Delta P_2; \quad (0 \leq y \leq d)
\]  

(9)

where \( \Delta P_1 \) is pressure drop due to the flow inertia and \( \Delta P_2 \) is pressure drop due to fluid rheological behavior. The pressure drop due to fluid rheological behavior of the unsteady state flow can be approximately calculated from that of the steady state flow as follows:

\[
\Delta P_2 = - \frac{L}{d} \int_0^d \frac{\partial \tau}{\partial y} dy = k\Delta P
\]  

(10)

The coefficient \( k \) depends on the variation of unsteady flow rate. Substitution of \( \Delta P \) obtained from equations (4) and (6) into equation (10) for \( \Delta P_2 \), then plug \( \Delta P_2 \) into equation (9) yields

\[
\Delta P_u = \rho L \frac{dQ}{dt} + \frac{12k\mu LQ}{W(d - \delta)^2(d + \delta / 2)}
\]  

(11)

\[
\Delta P_u = \rho L \frac{dQ}{dt} + \frac{6k\mu L}{\pi d^3 R} Q + k \frac{cL}{d} \tau_y
\]  

(12)

Noteworthily, equation (11) is suitable for calculating flow rate of the unsteady flow due to the given variation of pressure drop while equation (12) is used for calculating pressure drop of unsteady flow due to the given variation of flow rate. By neglecting time response of electronic devices of power supply unit, the time response of ER fluid to the applied electric field is equal to the switching time of the ER fluid itself. Thus, the effect of electric field change on the ER yield stress can be described by
\[ \tau_y + \lambda \dot{\tau}_y = \alpha E^\beta \]  

(13)

where \( \lambda \) is the switching time of ER fluid that is same as the response time of the ER fluid itself. In this study it is assumed that \( \lambda = 5 \text{ms} \).

3. Results and Discussion

In this study, the predicted results obtained by the proposed model are presented and compared with experimental results to validate the proposed model. The significant geometric dimensions of the ER valve are as follows: \( L=265\text{mm}; d=0.8\text{mm}; R_i=12\text{mm} \). For the ER fluid, chemically-treated starch and silicone oil are chosen as particles and liquid, respectively. The viscosity of the base oil is 0.003 Pa \( \cdot \) s. The size of the particles ranges from 10\( \mu \text{m} \) to 50\( \mu \text{m} \). The weight ratio of the particles to the ER fluid is 40\%. At testing temperature of 45\( ^\circ \text{C} \), the coefficients \( \alpha \) and \( \beta \) are respectively evaluated by 405.7 and 1.45 using least square curve fitting method [2]. Figure 4 shows the pressure drop of the ER valve calculated from equation (11) due to step input electric fields. In this case, the flow rate through the valve is kept constant at 0.015 l/s. As clearly observed from the results, the predicted results by the proposed model are well matched with experimental results.

Figure 4. Pressure drop of the valve.

Figure 5. Flow rate of the valve.

4. Conclusion

In this work, a new modeling method based on unsteady flow analysis has been proposed to provide accurate control model for ER valve systems. The pressure drop of the cylindrical ER valve was analytically expressed with unsteady flow condition of the ER fluid and varying electric field. After designing and manufacturing of the ER valve, the predicted pressure drop for the proposed dynamic model has been compared with experimental results in order to validate the proposed model. The difference of the flow rate between steady and unsteady analysis cases has been also demonstrated.

Reference

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