Electroproduction of $\psi'$
and polarized gluon distribution in a proton

Kazutaka SUDOH

Graduate School of Science and Technology,
Kobe University, Nada, Kobe 657–8501, Japan
E-mail: sudou@radix.h.kobe-u.ac.jp

Han–Wen HUANG$^a$ and Toshiyuki MORII$^b$

Faculty of Human Development, Kobe University, Nada, Kobe 657–8501, Japan
$^a$E-mail: huanghw@radix.h.kobe-u.ac.jp
$^b$E-mail: morii@kobe-u.ac.jp

Abstract

In order to get information about the polarized gluon distribution in a proton, we studied the electroproduction of $\psi'$ in polarized electron and polarized proton collisions in the framework of the NRQCD factorization approach. The value of the cross section $d\Delta\sigma/dp_T^2$ for color–octet mechanism is about 5 times larger than that for color–singlet one, and it might be another test of the color–octet model if the polarized gluon distribution $\Delta g(x)$ is well known. Furthermore, we found that this reaction is quite effective for testing the model of gluon polarization by measuring the double spin asymmetry $A_{LL}$ for the initial electron and proton. Though the shape of $p_T^2$ distribution of $A_{LL}$ is quite similar for the production mechanism with color–octet and color–singlet, we can see a big difference in $A_{LL}$ among various models of the polarized gluon distribution function $\Delta g(x)$.

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The proton spin puzzle [4] has been one of the most challenging topics in high energy hadron physics. As is well known, the proton spin is composed of the spin and angular momenta of quarks and gluons which constitute a proton. To understand the spin structure of a proton, it is necessary to study the behavior of the parton distribution functions which plays an important role in describing the spin structure of a proton. So far, a number of good parameterization models of the spin–independent parton distributions have been already obtained. In these years, we have also a large number of experimental data on the spin–dependent structure functions $g_1(x, Q^2)$ [2] which lead to extensive study on the spin–dependent parton distributions. However, knowledge of the polarized gluon distribution is still poor, though many processes have been suggested so far for extracting information about it.

On the other hand, since the advent of surprisingly big cross sections of prompt $J/\psi$ and $\psi'$ hadroproduction at large $p_T$ regions observed by the CDF collaboration [3], production mechanism of heavy quarkonium has been one of the most challenging topics in current particle physics with QCD. The CDF results were larger than the prediction by the conventional color–singlet model by more than one order of magnitude. $J/\psi$ photoproduction data at $ep$ collider HERA experiment are also at variance with the prediction by the color–singlet model [4]. In order to solve these serious discrepancies between the experimental data and the theoretical prediction by the color–singlet model [5], the NRQCD factorization formalism has been recently proposed [6] as one of the most promising candidates. The NRQCD factorization approach separates the effects of short distance scales that are comparable to or smaller than the inverse of heavy quark mass from the ones of longer distance scales that cause the hadronization of heavy quark pair. A heavy quark and its antiquark pair is allowed to be produced not only in color–singlet but also in color–octet intermediate state at a short distance of the process, and subsequently the color–octet pair hadronizes into a final color–singlet quarkonium via emission or absorption of dynamical gluons, which is called the color–octet mechanism. A short distance coefficient can be computed using perturbative QCD, and a long distance parameter is described by nonperturbative NRQCD matrix elements whose values should be determined from experiments or lattice gauge theory. Unfortunately, the present uncertainties of the NRQCD matrix elements are not so small and thus the discussion on whether the NRQCD works well or not is quite controversial. There have been several discussions that to test the color–octet mechanism, it is important to study the heavy quarkonium production in various polarized reactions [7].

In this work, to extract information about the polarized gluon density in a proton, we study the polarized electroproduction of $\psi'$ at large–$p_T$ regions by taking account of color–octet contribution,

$$\vec{e} + \vec{p} \rightarrow \psi' + X,$$

which could be observed in the forthcoming $e$–RHIC or TESLA–$\vec{N}$ experiments, where incident electron and proton are longitudinally polarized. Typical Feynman diagram of this process is illustrated in Fig. [4]. Since the process is dominated by photon–gluon fusion, the cross section must be sensitive to the gluon density in a proton and thus one can obtain good information about the polarized gluon distribution function.
Figure 1: Feynman diagram for large–$p_T$ $\psi'$ lepton production at the leading order. Incident electron and proton are polarized.

An interesting observable is the double spin asymmetry $A_{LL}$ which is defined by

$$A_{LL}(\beta) = \frac{d \Delta \sigma / d \beta}{d \sigma / d \beta}, \tag{2}$$

where $\beta$ is some kinematical variables. $d \Delta \sigma$ represents spin–dependent cross section and is defined by

$$d \Delta \sigma = \frac{1}{4} [d \sigma_{++} - d \sigma_{+-} + d \sigma_{-+} - d \sigma_{--}], \tag{3}$$

where $d \sigma_{+-}$, for instance, denotes the cross section with positive electron helicity and negative proton helicity. The asymmetry $A_{LL}$ is generally more sensitive to the mechanism of hard processes than the cross section itself. This is because, since the asymmetry is normalized, the input parameters such as the quark mass, strong coupling constant, etc. which are not related to dynamics, are dropped out from the numerator and the denominator.

The cross sections of large–$p_T$ $J/\psi$ production in $\gamma p$ collisions were calculated for the unpolarized case \cite{8,10} and the polarized case \cite{9,11}. Here we use the Weizsäcker–Williams approximation \cite{12} to evaluate the cross sections for the process (1). In this scheme we can write the electroproduction cross section in terms of the photoproduction cross section convoluted by the photon flux factor in the electron,

$$d \Delta \sigma(ep \rightarrow \psi' + X) = \int dy d\Delta f_{\gamma/e}(y) d\Delta \sigma(\gamma p \rightarrow \psi' + X), \tag{4}$$

where $y$ is the energy fraction of electron carried by photon, defined by $y = p_e \cdot q_\gamma / p_p \cdot p_e$ with $p_p$, $p_e$, and $q_\gamma$ being the momentum of proton, electron, and photon, respectively. In this calculation, we have neglected the contribution from the resolved photon process, which can be safely eliminated by introducing appropriate kinematical cuts \cite{3}. $\Delta f_{\gamma/e}(y)$ is the polarized photon distribution function in the electron, which is taken as \cite{3}

$$\Delta f_{\gamma/e}(y) = \frac{\alpha_{em}}{2\pi} \left[ \frac{1 - (1 - y)^2}{y} \log \frac{Q^2_{\max} (1 - y)}{m_e^2 y^2} + 2m_e^2 y^2 \left( \frac{1}{Q^2_{\max}} - \frac{1 - y}{m_e^2 y^2} \right) \right], \tag{5}$$
where $m_e$ is the electron mass. $Q^2_{\text{max}}$ is the maximum value of $Q^2$ for photoproduction processes and we adopt $Q^2_{\text{max}} = 1 \text{ GeV}$ as in Ref [13].

The process is dominated by photon–gluon fusion. We neglected here the contribution from the photon–quark fusion process, since its contribution to the cross sections is only a few percent [9]. Photoproduction cross section in the integrand of Eq. (4) is calculated by using the subprocess cross section as

$$d\Delta\sigma(\gamma p \to \psi' + X) = \int dx \Delta g(x, Q^2) d\Delta\hat{\sigma}(\gamma g \to \psi' + X),$$

where $\Delta g(x, Q^2)$ is the polarized gluon distribution function in a proton.

In the NRQCD factorization formalism, the subprocess cross section can be factorized into the short and long distance factors as

$$d\Delta\hat{\sigma}(\gamma g \to \psi' + X) = \sum_n \Delta C_n \langle O_{1,8}^{\psi'}(2S+1L_J) \rangle.$$

The label $n$ represents color and angular momentum configuration of intermediate $c\bar{c}$ pair. $\Delta C_n$ is a short distance coefficient for the polarized process and can be calculated as perturbation series with a QCD coupling constant $\alpha_s$. $\langle O_{1,8}^{\psi'}(2S+1L_J) \rangle$ is the vacuum expectation value of the NRQCD matrix element, whose relative importance is determined by the NRQCD velocity scaling rule. This long distance nonperturbative parameter represents the probability for $c\bar{c}$ pair being in the $n$ configuration which evolves into physical state $\psi'$, and can be extracted from experiments at present.

At large–$p_T$ regions, the process is dominated by the subprocess $\gamma^* + g \to (c\bar{c}) + g$ as illustrated in Fig. 2. At the leading order, we can expect the contributions from the following 4 processes:

$$\gamma + g \to c\bar{c}^{(3S_1, \frac{1}{2})} + g, \quad (8)$$
$$\gamma + g \to c\bar{c}^{(1S_0, \frac{3}{2})} + g, \quad (9)$$
$$\gamma + g \to c\bar{c}^{(3S_1, \frac{5}{2})} + g, \quad (10)$$
$$\gamma + g \to c\bar{c}^{(3P_J, \frac{3}{2})} + g, \quad (11)$$
each of which has an intermediate $c\bar{c}$ pair with definite angular momentum and color. The first process is the conventional color–singlet process. The other three processes are color–octet processes induced by the NRQCD factorization formalism.

For numerical values of the NRQCD matrix elements, we used the results of recent analysis by [14] and [15]:

$$\langle O_{1}^{\psi'}(3S_{1}) \rangle = 6.70 \times 10^{-1} \text{ GeV}^{3}, \quad (12)$$

$$\langle O_{8}^{\psi'}(1S_{0}) \rangle = 0.75 \times 10^{-2} \text{ GeV}^{3}, \quad (13)$$

$$\langle O_{8}^{\psi'}(3S_{1}) \rangle = 0.37 \times 10^{-2} \text{ GeV}^{3}, \quad (14)$$

$$\langle O_{8}^{\psi'}(3P_{0}) \rangle / m_{c}^{2} = 0.01 \times 10^{-2} \text{ GeV}^{3}, \quad (15)$$

$$\langle O_{8}^{\psi'}(3P_{J}) \rangle = (2J + 1) \langle O_{8}^{\psi'}(3P_{0}) \rangle. \quad (16)$$

The color–singlet matrix element given in Eq. (12) is related to the radial wave function at the origin, whose value is extracted from potential model calculations or directly from experiments. In fact, its value can be determined from the leptonic decay width of $\psi'$ with good accuracy. The values of the color–octet matrix elements were taken from the analysis on charmonium hadroproduction data. However, their values have not well been determined at present and various set of parameter values have been proposed [15]. The main source of ambiguity of those matrix elements comes from the higher order correction and uncertainties of unpolarized parton distribution functions. Especially, the value of $P$–wave color–octet matrix element $\langle O_{8}^{\psi'}(3P_{0}) \rangle$ for the $\psi'$ production becomes negative in some regions, which is unphysical. For example, combining the data of Ref [14] with the Type I set of Ref [15] and the data of Ref [14] with the Type II set of Ref [15], we obtain the value of $\langle O_{8}^{\psi'}(3P_{0}) \rangle$ as

$$\langle O_{8}^{\psi'}(3P_{0}) \rangle / m_{c}^{2} = -0.22 \pm 0.13 \times 10^{-2} \text{ GeV}^{3} \quad \text{for Type I}, \quad (17)$$

$$\langle O_{8}^{\psi'}(3P_{0}) \rangle / m_{c}^{2} = -0.08 \pm 0.11 \times 10^{-2} \text{ GeV}^{3} \quad \text{for Type II}. \quad (18)$$

As shown above, though the value of $\langle O_{8}^{\psi'}(3P_{0}) \rangle$ for Type I set is negative and unphysical, there can be a possibility of positive value of $\langle O_{8}^{\psi'}(3P_{0}) \rangle$ for Type II set by taking its observed error into consideration. Here we have adopted Eq. (13) as a possible parameter for Type II set.

Setting $Q^{2} = 4m_{c}^{2}$ with a charm quark mass $m_{c} = 1.5$ GeV and the relevant center–of–mass energy $\sqrt{s} = 300$ GeV, we calculated the spin–independent and spin–dependent cross sections and double spin asymmetry. The spin–dependent cross sections are shown in Fig. 3 as a function of $p_{T}^{2}$ (left panel), and as a function of $z$ (right panel), where $z$ is defined as $z = p_{T} / p_{c} \cdot q_{T}$ with $p_{c}$ being the momentum of outgoing $\psi'$. In this calculation, we have required the kinematical cut on $p_{T}$ as $p_{T}^{2} > 1$ GeV for $z$ distribution in order to suppress the contributions from the diffractive process and higher twist contribution. We used the GS set A [16] and GRSV “standard scenario” [17] parameterizations for the polarized gluon distribution function, and the GRV parameterization [18] for the unpolarized one. In these figures, the solid and dashed lines show the case of set A of GS and the “standard scenario” of GRSV, respectively. Bold
Figure 3: The spin–dependent differential cross section as a function of \( p_T^2 \) (left) and \( z \) (right). The bold lines represent the sum of the color–singlet and octet contributions, while the normal lines represent the color–singlet contribution only. The solid and dashed lines show the case of set A of GS [16] parameterization and the “standard scenario” of GRSV [17] parameterization, respectively.

As shown in Fig. 3, the dominant contribution comes from the color–octet \(^1S_0\) state. We can see that the sum of the color–singlet and –octet contribution is larger than the color–singlet one alone by a factor of about 5 in the whole \( p_T^2 \) regions. It is remarkable that the difference of the cross section due to these two production mechanisms is larger than the one of the parameterization models for the polarized gluon distribution functions. Therefore, this polarized reaction could be another independent test of the NRQCD factorization approach. In addition, as shown in Fig. 3 (left panel), it might be effective even for testing the model of polarized gluon distributions, if we have precise values of the NRQCD matrix elements and can observe \( p_T^2 \) distribution of the cross sections precisely.

The \( z \) distribution of the spin–dependent cross section is also shown in Fig. 3 (right panel). Here we do not see big difference between the color–singlet and –octet contribution in lower \( z \) regions. Taking account of the ambiguity of the polarized gluon density, we cannot distinguish between the color–singlet and –octet contribution in these regions. In the larger \( z \) regions the color–octet contribution rapidly increases and we could see a rather big difference for two production mechanisms in these regions. However, as is well known, the inelastic photoproduction in the NRQCD framework has an important issue on kinematical enhancement in the region \( z \to 1 \) arising from higher order terms in the NRQCD velocity expansion [18]. It is considered that the soft gluon resummation becomes important in the larger \( z \) regions. The prediction for the cross sections by the NRQCD factorization approach in this regions should be modified with higher order corrections. The prediction for the leading order calculation might be not reliable for this region. Based on these considerations, the \( z \) distribution of
the spin–dependent cross section is not so effective for testing the color–octet model. As shown in Fig. 3 (right panel), it is also not good for testing the polarized gluon distribution in the proton.

Next we move to the analysis on the spin correlation for inelastic $\psi'$ production in polarized $ep$ collisions. The double spin asymmetries $A_{LL}$ at $\sqrt{s} = 300$ GeV are calculated and presented in Fig. 4 as a function of $p_T^2$ (left panel) and $z$ (right panel). As seen in the $p_T^2$ distribution of $A_{LL}$, upper two solid lines show the calculations for the case of GS set A parameterization. The lower two dashed lines represent the ones for the case of GRSV “standard scenario” parameterization. We found that the dependence of the production mechanisms on the $p_T^2$ distribution of $A_{LL}$ is quite small in the whole $p_T^2$ regions. Instead, the difference due to the models of the polarized gluon density is large. It becomes larger and larger with $p_T^2$. Therefore, we can rather clearly test the models of the polarized gluon distribution functions $\Delta g(x)$ by measuring the double spin asymmetry for large $p_T$ regions. In the $z$ distribution, the difference due to the polarized gluon density is again larger than the one due to the production mechanisms, though the color–octet contribution dominates over the color–singlet one in the whole region of $z$.

A similar analysis was done by Yuan et al. for the case of $J/\psi$ productions. They have discussed the color–octet contribution to $J/\psi$ production and insisted that the process is effective not only for the test of the color–octet mechanism but also for the measurement of the gluon polarization in a proton. However, only direct $J/\psi$ production is considered there, and they do not take account of the feedback processes like $\psi' \rightarrow J/\psi + X$ and $\chi \rightarrow J/\psi + X$, which have been shown to play an important role in total $J/\psi$ production from experiment. For $\psi'$ case, there is no such complication, almost all contribution come from direct production. The advantage of our analysis is that the analysis for $\psi'$ productions is much clearer than the one for $J/\psi$.

In summary, in order to obtain information about the polarized gluon distribution in a proton, the electroproduction of $\psi'$ was studied in polarized electron and polarized
proton collisions by taking account of color–octet contribution. As shown in the left panel in Fig. 3, precise measurement of $d\Delta\sigma/dp_T^2$ for this reaction might give another test for the NRQCD factorization approach, if the polarized gluon distribution function is sufficiently established. Furthermore, if we have precise values of the NRQCD matrix elements, we could even test the model of the polarized gluon distribution $\Delta g(x)$ in a proton from it. On the other hand, $d\Delta\sigma/dz$ is not so effective for testing the color–octet model and also $\Delta g(x)$. As shown in Fig. 4 (left panel), the $p_T^2$ distribution of double spin asymmetry $A_{LL}(p_T^2)$ does not show a large difference of the production mechanism with color–octet and color–singlet. Rather, we see a big difference among the models of the polarized gluon distribution function $\Delta g(x)$. The $z$ distribution of $A_{LL}$ can be also a good test of $\Delta g(x)$, if values of the NRQCD matrix elements are well determined. Therefore, the process is quite effective for extracting information about $\Delta g(x)$.

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