The Linear Multiplet and Quantum String Effective Actions

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Abstract
Quantum symmetries of four-dimensional superstrings are frequently re-
alized in an anomaly-cancellation mode in the effective low-energy super-
gravity. The massless antisymmetric tensor plays an important rôle in
this mechanism and the choice of its supersymmetric description, using ei-
ther a chiral or a linear multiplet, appears to introduce significant concep-
tual and practical differences at the string loop level. This paper reviews
the construction of loop-corrected string effective supergravities with the
dilaton and antisymmetric tensor embedded in a linear multiplet. Using
anomaly cancellation and the linear multiplet allows to obtain an all-order
renormalization-group invariant effective lagrangian for a pure gauge sector
with field-dependent gauge coupling constant.

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1 The linear multiplet

A massless antisymmetric tensor $b_{\mu\nu}$ plays an important rôle in the low-energy effective supergravity of superstrings. Its existence is a universal property: the gravity sector of four-dimensional superstrings contains, together with the graviton, $b_{\mu\nu}$ and a real scalar, the dilaton. It is then natural to take advantage of the particular properties of the supermultiplet generalizing the antisymmetric tensor when discussing the structure of the effective supergravity theory.

This section will review some aspects of the supersymmetric description of the antisymmetric tensor. For simplicity, the discussion will first concentrate on global supersymmetry. The case of supergravity which will be of importance for the superstring case will be introduced at the end of the section.

In global supersymmetry, the description of the antisymmetric tensor uses two superfields\[1\], the prepotential $\psi^\alpha$ and the linear multiplet $L$. The prepotential is a chiral, spinor (the index $\alpha$) superfield with the gauge transformation

$$\delta \psi^\alpha = i \overline{D} \overline{D}^\alpha K,$$

where $K$ is a real vector superfield. This transformation can be used to choose a gauge in which some of the components of $\psi^\alpha$ are eliminated. In this gauge, $\psi^\alpha$ contains the antisymmetric tensor, a real scalar $C$ and a spinor $\chi$. And (1) reduces to the residual (bosonic) gauge symmetry

$$\delta b_{\mu\nu} = \partial_\mu k_\nu - \partial_\nu k_\mu,$$

which preserves the gauge choice.

The real linear multiplet\[1\] is the supersymmetrization of the field strength

$$v_\mu = \frac{1}{\sqrt{2}} \epsilon_{\mu\rho\sigma} \partial^{\rho\sigma} b,$$

of $b_{\mu\nu}$. In terms of the prepotential, it is defined by

$$L = D^\alpha \psi_\alpha + \overline{D}_\dot{\alpha} \overline{\psi}^{\dot{\alpha}},$$

which is invariant under the transformation (1). Alternatively, it can be obtained by imposing the constraints

$$\overline{D} D L = \overline{D} D L = 0,$$

on a real vector superfield. The component expansion of $L$ is

$$L = C + i \theta \chi - i \overline{\theta} \overline{\chi} + \theta \sigma^\mu \overline{\theta} v_\mu - \frac{1}{2} \theta \theta \theta (\partial_\mu \chi \sigma^\mu) - \frac{1}{2} \overline{\theta} \theta \theta (\sigma^\mu \partial_\mu \overline{\chi}) - \frac{1}{4} \theta \theta \theta \theta \Box C.$$

It is interesting to remark that $L$ does not contain any auxiliary field. A lagrangian involving the linear multiplet $L$ only does not include a scalar potential and supersymmetry cannot spontaneously break.

A generic lagrangian for $L$ and chiral matter denoted collectively by $\Sigma$ is of the form

$$\mathcal{L}_{\text{global}} = 2 \int d^2 \theta d^2 \overline{\theta} \Phi(L, \Sigma, \overline{\Sigma}) + \int d^2 \theta w(\Sigma) + \int d^2 \overline{\theta} w(\overline{\Sigma}),$$

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with an arbitrary real function $\Phi$ and a superpotential $w$ independent from $L$. Coupling this theory to gauge fields leads to

$$\mathcal{L}_{\text{global}} = 2 \int d^2 \theta d^2 \bar{\theta} \Phi(L, \Sigma, \Sigma e^V) + \int d^2 \theta w(\Sigma) + \int d^2 \bar{\theta} \bar{w}(\Sigma),$$

(6)

where $V = V^a T^a$ is the Lie-algebra-valued vector superfield of gauge potentials, the superpotential is a gauge-invariant analytic function,

$$\hat{L} = L - 2\Omega,$$

(7)

and $\Omega$ is the Chern-Simons superfield. This last object can be defined by its relation with the chiral superfield of gauge curvatures,

$$W^a = -\frac{1}{4} \overline{DD} e^{-V} D^a e^V = W^{aT}_a,$$

which reads

$$\overline{DD} \Omega = W^a W^a, \quad D D \Omega = W^a \overline{W}^a.$$

(8)

Since theory (6) only depends on $b_{\mu\nu}$ through its curl $v_\mu$ [eq. (3)], a duality transformation can be performed to replace $b_{\mu\nu}$ by a (pseudo)scalar field. With supersymmetry, the duality transformation replaces $L$ by a chiral superfield, an operation which will be discussed in detail in the next section.

For reasons to be explained below, conformal supergravity\cite{2} offers the appropriate formalism to extend the global lagrangian (6) to local supersymmetry. One needs to introduce a chiral compensating supermultiplet $S_0$ with conformal and chiral weights equal to one\cite{2}. Since the conformal weight of $L$ is two, the supergravity lagrangian generalizing (6) is

$$\mathcal{L}_{\text{local}} = \left[ S_0 \overline{S}_0 \Phi\left(\frac{\hat{L}}{S_0 \overline{S}_0}, \Sigma, \Sigma e^V\right) \right]_D + \left[ S_0^3 w(\Sigma) \right]_F.$$

(9)

The real and chiral density formula are denoted respectively by $[\ldots]_D$ and $[\ldots]_F$. The Poincaré supergravity theory is obtained by imposing in (9) gauge-fixing conditions for the superconformal symmetries not contained in the super-Poincaré algebra. These gauge choices use in particular the scalar and fermion components $z_0$ and $\psi_0$ of $S_0$, and leave the auxiliary component $f_0$ and the gauge field $A_\mu$ of the chiral, internal $U(1)$ of the superconformal algebra as supergravity auxiliary fields.

It is worth recalling that the linear multiplet has made at least two appearances in past developments of supergravity theories. Firstly, ‘new minimal supergravity’ is obtained in the conformal framework using a linear compensating multiplet\cite{4, 3}, with lagrangian $[L \log L]_D$. And secondly, early investigations\cite{5} of the tree-level effective supergravity of superstrings suggested the relevance of ‘16+16 supergravity’\cite{6}, which corresponds in the conformal approach to a theory with a linear and a chiral multiplet, with some particular gauge fixing\cite{6}.

\footnote{This choice of compensator leads to the old minimal set of supergravity auxiliary fields, which allows the most general matter couplings\cite{3}.}
2 Gauge kinetic lagrangian and gauge coupling constants

Since field-dependent gauge coupling constants in superstring effective supergravities will be at the center of the discussion, we briefly consider\[8\] the sector of gauge kinetic terms in the global lagrangian (6) and in its supergravity extension (9).

In global supersymmetry, the expansion of the Chern-Simons superfield in the Wess-Zumino gauge contains the following terms:

\[\Omega = -\frac{1}{2}\theta\bar{\theta}\theta \left( \frac{i}{2}D^a D^a - \frac{1}{2} i \lambda^a \sigma^\mu \partial_\mu \bar{\lambda}^a + \frac{1}{4} i \partial_\mu \lambda^a \sigma^\mu \bar{\lambda}^a - \frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} \right) + \frac{1}{2} (\theta \sigma^\mu \bar{\theta}) \epsilon^{\mu\nu\rho\sigma} \omega_{\nu\rho\sigma} + \frac{1}{4} \theta \bar{\theta} \lambda^a \bar{\lambda}^a + \ldots,\]

where \(\omega_{\nu\rho\sigma}\) is the gauge Chern-Simons form, normalised by

\[\partial_\mu \epsilon^{\mu\nu\rho\sigma} \omega_{\nu\rho\sigma} = \frac{1}{4} \epsilon^{\mu\nu\rho\sigma} F^a_{\mu\nu} F^a_{\rho\sigma} = \frac{1}{2} \tilde{F}^a_{\mu\nu} F^a_{\mu\nu}.\]

Its highest component is the super-Yang-Mills lagrangian (multiplied by \(-\frac{1}{2}\)) and it does not contain any \(\tilde{F}^a_{\mu\nu} F^a_{\mu\nu}\) contribution. Gauge kinetic terms in lagrangian (6) will then clearly be of the form

\[2\Phi_C \left( \frac{1}{2} D^a D^a - \frac{1}{2} i \lambda^a \sigma^\mu \partial_\mu \bar{\lambda}^a + \frac{1}{4} i \partial_\mu \lambda^a \sigma^\mu \bar{\lambda}^a - \frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} \right),\]

with \(\Phi_C = \frac{\partial}{\partial C} \Phi(C, z, \bar{z}) = \left[ \frac{\partial}{\partial C} \Phi(L, \Sigma, \Sigma e^V) \right]_{\theta = \bar{\theta} = 0}.\) The field-dependent gauge coupling constant is then\[9, 8\]

\[\frac{1}{g^2} = 2\Phi_C.\] (10)

With the linear multiplet, the gauge coupling constant is a real function which is not constrained to be harmonic. It is obtained in a real D-term, contrary to the chiral case where it is related to a holomorphic function. This illustrates the fact that holomorphicity properties of gauge couplings do not follow from supersymmetry only but require also a selection of supersymmetric multiplets\[4].

Lagrangian (6) also contains a term

\[- [\Phi_{CC} v_\mu - i \Phi_{Cz} \partial_\mu z + i \Phi_{C\bar{z}} \partial_\mu \bar{z}] \epsilon^{\mu\nu\rho\sigma} \omega_{\nu\rho\sigma}.\] (11)

This contribution will only give rise, by partial integration, to an \(\tilde{F}^a_{\mu\nu} F^a_{\mu\nu}\) with the very special choice

\[\Phi = \tilde{L}[\phi(\Sigma) + \bar{\phi}(\Sigma)].\] (12)

In this case, \(\Phi_{CC} = 0, \Phi_{Cz} \partial_\mu z = \partial_\mu \phi(z), \Phi_{C\bar{z}} \partial_\mu \bar{z} = \partial_\mu \bar{\phi}(\bar{z}),\) and (11) becomes

\[i \partial_\mu [\phi(z) - \bar{\phi}(\bar{z})] \epsilon^{\mu\nu\rho\sigma} \omega_{\nu\rho\sigma} = 2\text{Im} \phi(z) \partial_\mu \epsilon^{\mu\nu\rho\sigma} \omega_{\nu\rho\sigma} + \text{total derivative}\]

\[= \text{Im} \phi(z) \tilde{F}^a_{\mu\nu} F^a_{\mu\nu} + \text{total derivative}.\]
In this case, the gauge coupling constant is proportional to the real part of $\phi(z)$:

$$\frac{1}{g^2} = 4\text{Re} \phi(z).$$

Notice however that since $\int d^2\theta d^2\bar{\theta} L[\phi(\Sigma) + \bar{\phi}(\Sigma)] = \text{total derivative}$, the choice (12) completely eliminates the linear multiplet. Then,

$$\mathcal{L} = -4 \int d^2\theta d^2\bar{\theta} \Omega[\phi(\Sigma) + \bar{\phi}(\Sigma)] = \int d^2\theta \phi(\Sigma) W^a W^a + \text{h.c.},$$

which is the usual super-Yang-Mills theory with gauge coupling depending on chiral matter. The fact that the gauge coupling is the real part of a holomorphic function appears then to depend on the description of matter with chiral multiplets only.

### 3 Supersymmetric duality

Any theory with a linear multiplet can be in principle transformed into a theory with chiral multiplets only, using a supersymmetric duality transformation. Suppose we want to transform (6) into a theory with a chiral superfield $S$ replacing $L$. The starting point is to write a classically equivalent theory, of the form:

$$\mathcal{L}_1 = 2 \int d^2\theta d^2\bar{\theta} \Phi(\hat{L} - U, \Sigma, \Sigma e^V) + \left\{ \int d^2\theta \left[ -\frac{1}{2} S \tilde{D}D U + w(\Sigma) \right] + \text{h.c.} \right\}. \quad (13)$$

This lagrangian possesses a new gauge invariance under

$$U \rightarrow U + U, \quad L \rightarrow L + U, \quad (14)$$

where the gauge parameter is a linear superfield $U$: $\tilde{D}DU = \tilde{D}D\bar{U} = 0$. The equation of motion for $S$ implies that $U$ is a linear superfield and (6) is then simply the gauge choice $U = 0$ in (13), after solving for $S$. Notice that a superfield $U$ submitted to the gauge transformation (14) can be regarded as the supersymmetric extension of an antisymmetric tensor $a_{\mu\nu\rho}$ with gauge variation $\delta a_{\mu\nu\rho} = \partial_{[\mu} U_{\nu\rho]}$. The expansion of the superfield $L - U$ contains

$$\frac{1}{\sqrt{2}} (\theta_{\sigma} \theta_{\nu} \sigma) \epsilon_{\mu\rho\sigma} D[\nu \rho \sigma],$$

with a derivative

$$D[\nu \rho \sigma] = \partial[\nu \rho \sigma] - a_{\nu \rho \sigma}$$

which is invariant under $\delta b_{\mu\nu} = U_{\mu\nu}$. Theory (13) can be rewritten

$$\mathcal{L}_2 = 2 \int d^2\theta d^2\bar{\theta} \left[ \Phi(\hat{L} - U, \Sigma, \Sigma e^V) - (S + \bar{S})(\hat{L} - U) - 2(S + \bar{S})\Omega \right]$$

$$+ \left\{ \int d^2\theta w(\Sigma) + \text{h.c.} \right\}$$

$$= 2 \int d^2\theta d^2\bar{\theta} \left[ \Phi(\hat{L} - U, \Sigma, \Sigma e^V) - (S + \bar{S})(\hat{L} - U) \right]$$

$$+ \left\{ \int d^2\theta \left[ SW^a W^a + w(\Sigma) \right] + \text{h.c.} \right\}. \quad (15)$$
The equation of motion for $U$ (or $\hat{L} - U$) can in principle be used to express $\hat{L} - U$ as a function of $S + \overline{S}$, to finally obtain the dual theory

$$\mathcal{L}_S = \int d^2\theta d^2\overline{\theta} K(S + \overline{S}, \Sigma, \Sigma e^V) + \left\{ \int d^2\theta \ [SW^aW^a + w(\Sigma)] + \text{h.c.} \right\}. \quad (16)$$

In this expression, which possesses the symmetry $S \rightarrow S + i$(real constant), the gauge coupling constant is always

$$\frac{1}{g^2} = 4\text{Re} \ s, \quad (17)$$

$s$ being the scalar lowest component of $S$. And there is a term of the form

$$\text{Im} \ s F^a_{\mu\nu} F^{a\mu\nu} = -2(\partial_\mu \text{Im} \ s) \epsilon^{\mu\nu\rho\sigma} \omega_{\nu\rho\sigma} + \text{total derivative.} \quad (18)$$

The duality transformation is performed by solving the equations of motion for the components ($c, \tilde{c}, m, n, \tilde{v}_\mu, \tilde{\lambda}, d$) of $U$, for instance in the gauge $L = 0$. In particular, these equations indicate that

$$\text{Re} \ s = \frac{1}{2} \Phi_c, \quad \partial_\mu \text{Im} \ s = \frac{1}{2} \Phi_{cc} \tilde{v}_\mu - \frac{i}{2} \Phi_{cz} \partial_\mu z + \frac{i}{2} \Phi_{c\overline{z}} \partial_\mu \overline{z} + \text{fermionic terms.} \quad (19)$$

These relations show the equivalences of (17) and (18) with respectively (10) and (11).

These results have a straightforward extension to conformal and Poincaré supergravities. Equation (17), in particular, remains true in the local case. It provides an important information on the description of superstring effective supergravities. The two dual theories have a different physical interpretation. Working with the chiral theory, with $S$, implies that the gauge coupling constant appearing in the lagrangian is the expectation value of $\text{Re} \ s$. This field is then directly related to a bare, unphysical quantity. The physical (renormalized) gauge coupling constant, which appears in the effective action (the generating functional of one-particle irreducible Green’s functions), is a function of $\text{Re} \ s$ (as well as other fields, in general). It is certainly not given by the expectation value of $\text{Re} \ s$. Then, since $\text{Re} \ s$ is related to a bare quantity of the effective field theory, it is not necessarily in simple and natural relation with string parameters at the (string) quantum level. And there is no reason to expect that quantum symmetries of the superstring have a natural action on this field.

### 4 Superstring effective lagrangians

The effective supergravity of superstrings describes the low-energy physics of string massless modes. It then contains an ultraviolet physical cutoff, above which ‘string physics has been integrated’. The effective supergravity can be defined using a Wilson lagrangian $\mathcal{L}_W$. At the level of superstring perturbation theory, in which supersymmetry does not break, the Wilson effective lagrangian has a string loop expansion

$$\mathcal{L}_W = \sum_{N \geq 0} \mathcal{L}_W^{(N)}. \quad (20)$$

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2With respect to the inverse string length and compactification radius.
The Wilson lagrangian and the corresponding action $S_W = \int d^4x L_W$ should not be confused with the more familiar effective action of quantum field theory,

$$S_\Gamma = \int d^4x L_\Gamma,$$

which is the generating functional of one-particle irreducible Green’s functions. While $L_W$ is a quantized lagrangian, $L_\Gamma$ is a functional of classical fields.

For large classes of superstrings, the tree-level lagrangian $L_0^W$ is well understood. Since the present discussion focusses on the rôle of the antisymmetric tensor described by a linear multiplet $L$, or on the dual theory with a chiral $S$, it will be sufficient to use

$$L_0^W = -\frac{1}{\sqrt{2}} \left[ \hat{L} \left( \frac{\hat{K}/3}{S_0 S_0} \right)^{-3/2} \right]_D + [S_0^3 w]_F, \quad (21)$$

as tree-level Wilson lagrangian\[10\]. The Kähler function $\hat{K}$ and the superpotential $w$ depend in a gauge invariant way on the chiral multiplets describing moduli and charged matter. Using (16), theory (21) is, by duality, equivalent to the chiral lagrangian

$$L_0^S = -\frac{3}{2} \left[ S_0 \overline{S}_0 e^{-\kappa^2/3} \right]_D + [S W^a W^a + S_0^3 w]_F, \quad \mathcal{K} = -\log(S + \overline{S}) + \hat{K}. \quad (22)$$

The tree-level Wilson lagrangian possesses the (formal) Kähler symmetry

$$S_0 \rightarrow e^{\varphi/3} S_0, \quad w \rightarrow e^{-\varphi} w, \quad \hat{K} \rightarrow \hat{K} + \varphi + \overline{\varphi}, \quad \hat{L} \rightarrow \hat{L}, \quad (23)$$

where $\varphi$ is an arbitrary holomorphic function of chiral multiplets.

The lagrangian (21) is superconformal invariant. Two different choices of compensator fixings are of interest. Imposing canonical normalisation of the Einstein term leads to

$$e^{-1} L_0^W = -\frac{1}{2\kappa^2} R - \frac{1}{4}(\kappa^2 C)^{-1} F_{\mu\nu}^a F^{a\mu\nu} + \frac{1}{4}\kappa^2 (\kappa^2 C)^{-2} v_\mu v^\mu + \ldots,$$

and the gauge coupling constant is

$$g_{\text{tree}}^{-2} = (\kappa^2 C)^{-1} = 4\text{Re } s. \quad (24)$$

The second choice is the ‘string frame’ where the same bosonic terms become

$$e^{-1} L_0^W = \frac{\phi^{1/2}}{\kappa^2} \left[ -\frac{1}{2} R - \frac{1}{4} M_s^{-2} F_{\mu\nu}^a F^{a\mu\nu} + \frac{1}{4} M_s^{-4} v_\mu v^\mu \right] + \ldots,$$

where $\varphi = \kappa^2 C$ is the string ‘loop-counting parameter’ and $M_s^2 = \langle C \rangle$ is the string scale. With this choice,

$$g_{\text{tree}}^{-2} = (\kappa^2 C)^{-1} = (4\text{Re } s)^{2/3},$$

since the relation between Re $s$ and $C$ depends on the superconformal gauge fixing.
5 One loop: a string symmetry, anomaly cancellation

Some contributions to $\mathcal{L}^{(1)}_W$, in eq. (21), have been revealed by the effective field-theory description of string one-loop calculations of gauge kinetic terms in $(2,2)$ symmetric orbifolds \cite{11, 12, 13}. The moduli dependence of these corrections is in large part dictated by target-space duality, a perturbative string symmetry. In the effective supergravity, the crucial observation \cite{9} is that since target-space duality is an anomalous symmetry of $\mathcal{L}^{(0)}$, invariance is restored by an anomaly-cancellation mechanism due to specific contributions in $\mathcal{L}^{(1)}$. This mechanism is similar to the cancellation of gauge and gravitational anomalies in ten-dimensional superstrings \cite{14}. Target-space duality acts on the tree-level lagrangian (21) like a Kähler transformation (23). The cancellation of target-space duality anomaly is then a mechanism of Kähler anomaly cancellation in supergravity \cite{15}.

This mechanism will be reviewed here in its simplest version only. Specifically, we will consider the case of vanishing threshold corrections \cite{3}. We will also for simplicity restrict the discussion to a simple gauge group, which could be the hidden $E_8$ group present in symmetric $(2,2)$ orbifolds. The discussion of the general case can be found in the original literature \cite{9, 16}.

The nature of Kähler symmetry is easily elucidated. Clearly, it is a sigma-model symmetry: it acts on $\hat{K}$ and $\hat{K}$ is the Kähler potential of the supersymmetric sigma-model defining the couplings of chiral multiplets. But in supergravity, it also acts as an R-symmetry which, in particular, rotates all fermions in the theory. This is due to the presence of the supergravity auxiliary field $A_\mu$, which in the superconformal theory is the gauge field of the internal chiral $U(1)$. This chiral group does not commute with supersymmetry transformations and hence acts like an R-symmetry. The equation of motion of its gauge field is

$$A_\mu = \frac{i}{3} \left[ \frac{\partial \hat{K}}{\partial z} (\partial_\mu z) - \frac{\partial \hat{K}}{\partial \bar{z}} (\partial_\mu \bar{z}) \right] + \ldots,$$

for an arbitrary chiral scalar component $z$. The Kähler transformation (23) is then a chiral $U(1)$ gauge transformation of $A_\mu$, once the equation of motion for $A_\mu$ has been solved, and $\hat{K}$ can be assimilated to the (superfield) Kähler connection.

The existence of chiral couplings of the Kähler connection to fermions implies that a non-zero anomalous triangle graph with, for instance, two external gauge fields and one external Kähler connection will in general arise at one-loop. This diagram corresponds to a non-local contribution to the effective lagrangian $\mathcal{L}_T$ (not to $\mathcal{L}_W$), which, in the absence of chiral charged matter as assumed here, is of the form

$$\Delta_{\text{triangle}} = -\frac{A}{4} \left[ \frac{1}{3} W^a W^a \mathcal{P}_C \hat{K} \right],$$

(25)

This means that anomaly cancellation also cancels the one-loop modulus-dependent correction to physical gauge coupling constants.
where $\mathcal{P}_C$ is the non-local chiral projector. Gauginos only contribute to the anomaly and its coefficient is $A = 3C(G)/(8\pi^2)$, $C(G)$ being the quadratic Casimir of the gauge group $G$. The transformation of $\Delta_{\text{triangle}}$ under (23) is

$$\delta(\Delta_{\text{triangle}}) = -\frac{A}{4} \left[ \frac{1}{3} W^a W^a \varphi \right]_F,$$

a local expression.

The one-loop Wilson lagrangian $\mathcal{L}^{(1)}_W$ should then contain a Kähler-variant term able to cancel (26) by its variation. This contribution is

$$\mathcal{L}^{(1)}_W = \frac{A}{3} \left[ \hat{L} \hat{K} \right]_D,$$

which includes a coupling of the antisymmetric tensor to the Kähler connection. With this one-loop correction, the Wilson lagrangian becomes

$$\mathcal{L}^{(0)}_W + \mathcal{L}^{(1)}_W = \left[-\frac{1}{\sqrt{2}} \hat{L} \left( \frac{\hat{L} \hat{K} / 3}{S_0 S_0} \right)^{-3/2} + \frac{A}{12} \hat{L} \hat{K} \right]_D + [S_0^3 w]_F.$$

Repeating the steps (15) and (16) to perform the duality transformation leads to the substitution

$$S + \overline{S} \longrightarrow S + \overline{S} - \frac{A}{3} \hat{K}$$

in the tree-level Kähler function $K$ given in eq. (22). This result is reminiscent of the mechanism cancelling gauge anomalies for $U(1)$ factors of the gauge group, where the substitution is

$$S + \overline{S} \longrightarrow S + \overline{S} - \alpha V,$$

$V$ being the vector superfield of the $U(1)$ gauge potential and $\alpha$ the coefficient of the anomaly.

### 6 An effective all-order gauge sector

In the previous section, the information that target-space duality is a quantum string symmetry has been used to derive a one-loop contribution to the Wilson effective lagrangian, using anomaly cancellation as a basic principle. The one-loop lagrangian (28) has been written in the superconformal formalism. It is then also scale invariant as long as a superPoincaré invariant gauge choice has not been taken. It is tempting to extend the method applied to restore the anomalous target-space duality also to the dilatation symmetry contained in the conformal algebra. Since the gauge beta-function corresponds to the dilatation anomaly obtained when varying the compensator field $S_0$, the superconformal formalism used here is certainly appropriate.

The discussion given in this section only applies to a super-Yang-Mills theory without chiral charged matter. This situation corresponds, for instance, to the hidden $E_8$..
sector of \((2,2)\) superstrings. The reason for this limitation is not only simplicity. The generalization to a string gauge sector with chiral charged matter and a non simple gauge group is not known at present. Furthermore, the all-order field theory results on beta-functions, which are necessary to check the results of the construction, are not available with arbitrary chiral matter.

To proceed, consider the effect of a scale change on the gauge coupling constant. Under

\[ M \rightarrow \lambda M, \]

the variation of the one-loop gauge coupling constant is

\[ \delta(g^{-2}) = A \log \lambda, \]

where \(A = \frac{3C(G)}{8\pi^2}\) is the coefficient of the one-loop beta function which already appeared in the Kähler/target-space duality anomaly \((23)\). In an effective lagrangian formalism, this anomalous variation reads

\[
\delta \left( -\frac{1}{4} g^2 F^a_{\mu\nu} F^{a\mu\nu} \right) = -\frac{1}{4} A \log \lambda F^a_{\mu\nu} F^{a\mu\nu}. \tag{30}
\]

The next step is to construct a supersymmetric expression such that its variation compensates in particular \((30)\). This should be done without introducing any new Kähler anomaly. Using the same notation as in the preceding section, there exists a unique supersymmetric expression with a variation including \((30)\):

\[
\frac{A}{4} \left[ \hat{L} \log(\hat{L}/\mu^2) \right]_D. \tag{31}
\]

Its anomaly behaviour follows from the conformal weight two of \(\hat{L}\). The quantity \(\mu\) is an arbitrary number. It is not a scale since a constant supermultiplet must have zero conformal dimension. It is only after conformal gauge fixing that \(\mu\) will acquire the rôle of a reference scale, as will be seen below.

Collecting its tree, one-loop and running contributions, the complete Wilson lagrangian is

\[
\mathcal{L}_W = -\frac{1}{\sqrt{2}} \left[ \hat{L} \left( \frac{\hat{L}e^{\hat{K}/3}}{S_0\overline{S}_0} \right)^{-3/2} + \frac{A}{4} \hat{L} \log \left( \frac{\hat{L}e^{\hat{K}/3}}{\mu^2} \right) \right]_D + \left[ S_0^3 \omega \right]_F. \tag{32}
\]

Notice that the anomaly term can also be written:

\[
\frac{A}{4} \left[ \hat{L} \log \left( \frac{\hat{L}e^{\hat{K}/3}}{S_0\overline{S}_0} \right) + \hat{L} \log \left( \frac{S_0\overline{S}_0}{\mu^2} \right) \right]_D.
\]

This last expression shows that both Kähler and dilatation anomalies are conveniently carried by the chiral compensator \(S_0\), a natural observation since \(S_0\) has chiral \textit{and} Weyl weights equal to one and Kähler symmetry finds its superconformal origin in the chiral internal \(U(1)\).
The effective lagrangian $\mathcal{L}_\Gamma$ can be easily obtained from the Wilson theory (32). It differs from $\mathcal{L}_W$ (viewed as a functional of classical fields instead of an operator-valued lagrangian) by contributions arising from loop diagrams, which are partially non-local:

$$\mathcal{L}_\Gamma = \mathcal{L}_W - \frac{A}{4} \left[ W^a W^a \mathcal{P}_C \log \left( \frac{\hat{L} e^{\hat{K}/3}}{\mu^2} \right) \right] + \ldots,$$  \hspace{1cm} (33)

the dots indicating contributions unrelated to gauge kinetic terms which are not of interest here. As it should, it is invariant under both Kähler and scale (conformal) transformations.

The Poincaré invariant theory is obtained by a superconformal gauge fixing which uses in particular the scalar $z_0$ and fermionic $\psi_0$ components of the compensating multiplet $S_0$. The gravitational lagrangian will take the canonical form $-\frac{1}{2\kappa^2} \kappa R$ if one chooses

$$U = \frac{1}{\sqrt{2}} \left( \frac{C e^{\hat{K}/3}}{z_0 \bar{z}_0} \right)^{-3/2} = \frac{1}{\kappa^2 C} - \frac{A}{6},$$  \hspace{1cm} (34)

which defines $|z_0|$ and introduces the Planck scale via $\kappa = \sqrt{8\pi G_N} = (M_P/8\pi)^{-1}$. Notice that the quantity $U$ is Kähler and scale invariant.

The field-dependent Wilson gauge coupling constant is defined as the coefficient of gauge kinetic terms in $\mathcal{L}_W$:

$$\mathcal{L}_W = -\frac{1}{4} g^2 W^a F^a_{\mu\nu} F^{a\mu\nu} + \ldots$$  \hspace{1cm} (35)

It is a bare, unphysical quantity, like all parameters appearing in $\mathcal{L}_W$. In theory (32), one finds,

$$\frac{1}{g^2 W(z_0 \bar{z}_0)} = U - \frac{A}{3} \log U + \frac{A}{2} \log \left( \frac{z_0 \bar{z}_0}{\mu^2} \right) + c,$$  \hspace{1cm} (36)

where the constant $c = \frac{A}{2} (1 - \frac{1}{3} \log 2)$ can be absorbed in $\mu$. The Wilson coupling is neither Kähler nor scale invariant. Notice also that the Poincaré gauge fixing promotes the quantity $\mu$ to a scale, once $|z_0|$ is measured in units of the Planck scale.

The field-dependent effective gauge coupling constant is defined as the coefficient of gauge kinetic terms in the effective lagrangian:

$$\mathcal{L}_\Gamma = -\frac{1}{4} g^2 F^a_{\mu\nu} F^{a\mu\nu} + \ldots$$  \hspace{1cm} (37)

The non-local contributions to $\mathcal{L}_\Gamma$ in eq. (33) include local gauge kinetic terms, and one simply obtains

$$\frac{1}{g^2 \Gamma(\mu)} = U,$$  \hspace{1cm} (38)

a Kähler and target-space duality invariant result. According to (34), the physical effective gauge coupling constant in the loop-corrected theory is specified by the expectation value of the real, dimensionless field $\kappa^2 C$. And, as discussed in section 2, the Wilson lagrangian (32) does not contain any $F^a_{\mu\nu} \tilde{F}^{a\mu\nu}$ term.

\* The field $\kappa^2 C$ is the lowest component of $\kappa^2 \hat{L}$ in the Wess-Zumino gauge.
The dependence on the arguments \(z_0\) and \(\mu\) of \(g_W\) and \(g_\Gamma\) follows from the following remark. Using eqs. (36) and (38), one finds:

\[
\partial_\mu g_\Gamma = -\frac{3C(G)}{16\pi^2}g_\Gamma^3\left[1 - \frac{C(G)}{8\pi^2}g_\Gamma^2\right]^{-1},
\]

\[
|z_0|\partial_{|z_0|}g_W = -\frac{3C(G)}{16\pi^2}g_W^3.
\]

The scale dependence of the physical coupling \(g_\Gamma\) coincides with the all-order renormalisation-group equation in pure super-Yang-Mills theory\([20]\). And the Wilson gauge coupling is only affected by one-loop running. Relation (40) leads to

\[
\frac{1}{g_W^2(\mu^2)} = \frac{1}{g_\Gamma^2(\mu)} + \frac{A}{3} \log g_\Gamma^2(\mu).
\]

Equations (40) and (41) have been established in pure super-Yang-Mills theories by Shifman and Vainshtein\([21]\). They have been obtained here from anomaly cancellation in an effective lagrangian construction.

Finally, the dual Wilson lagrangian with \(L\) replaced by the chiral multiplet \(S\) has a gauge coupling constant given by

\[
\frac{1}{g_W^2(z_0\overline{z}_0)} = 4\text{Re}\,s,
\]

as in (14). The duality transformation is performed by expressing \(U\) as a function of \(\text{Re}\,s\), using eqs. (36) and (42). The solution can however only be found in a perturbative expansion: while the linear multiplet theory (32) is known exactly, with all-order gauge couplings (36) and (38), the dual chiral theory can only be expressed as an infinite series.

## 7 Conclusions

Formally, a supergravity theory with a linear multiplet can always be transformed into a theory with chiral multiplets only, using duality. String effective supergravities indicate however that this formal equivalence does not imply conceptual equivalence, besides the fact that duality can be hard or impossible to perform analytically. The duality transformation is applied to the Wilson lagrangian, with the loop expansion (20). Since the scalar component \(C\) of the linear multiplet appears to be the string loop-counting parameter, it seems natural to formulate the loop expansion with \(L\). The relation between \(S\) and \(L\) changes order by order. Duality will then induce a rearrangement of perturbation theory which can obscure the identification of string parameters and amplitudes with the fields of the effective supergravity. This is exemplified by the effective description of threshold corrections in orbifolds\([11, 9]\), and also by substitution (29), at one loop.
The construction of the effective gauge sector provides a number of interesting results. The all-order renormalization-group behaviour is obtained from anomaly cancellation. In some sense, there is a non-renormalization property similar to an Adler-Bardeen theorem. The field-dependent gauge coupling, a physical parameter, is the expectation value of the real field $C$. The chiral version of the theory uses a complex scalar $s$, which gives the bare unphysical Wilson coupling. And the linear theory does not include any $F \bar{F}$ term.

Finally, the strength of the gauge interaction and the scale at which this force becomes confining are characterized by the renormalization-group invariant scale $\Lambda$, given by the real superfield

$$\Lambda^3 = \mu^3 U e^{-3U/\Lambda},$$

which is also target-space duality invariant. This is also the scale at which dynamical supersymmetry breaking is expected to be induced by gaugino condensates. The gauge sector defined by the Wilson lagrangian (32) can be used for an effective lagrangian treatment of this phenomenon[22].

8 References

References

[1] S. Ferrara, J. Wess and B. Zumino, *Phys. Lett.* B51 (1974) 239; W. Siegel, *Phys. Lett.* B85 (1979) 333.

[2] M. Kaku, P.K. Townsend and P. van Nieuwenhuizen, *Phys. Rev.* D17 (1978) 3179; T. Kugo and S. Uehara, *Nucl. Phys.* B222 (1983) 125; B226 (1983) 49.

[3] S. Ferrara, L. Girardello, T. Kugo and A. Van Proeyen, *Nucl. Phys.* B223 (1983) 191.

[4] B. de Wit and M. Roček, *Phys. Lett.* B109 (1982) 439.

[5] W. Lang, J. Louis and B. A. Ovrut, *Phys. Lett.* B158 (1985) 40.

[6] G. Girardi, R. Grimm, M. Müller and J. Wess, *Phys. Lett.* B147 (1984) 81.

[7] W. Siegel, *Class. Quantum Grav.* 3 (1986) L47; C. S. Aulakh, J.-P. Derendinger and S. Ouvry, *Phys. Lett.* B169 (1986) 201.

[8] J.-P. Derendinger, F. Quevedo and M. Quirós, *Nucl. Phys.* B428 (1994) 282.

[9] J.-P. Derendinger, S. Ferrara, C. Kounnas and F. Zwirner, *Nucl. Phys.* B372 (1992) 145.

[10] S. Cecotti, S. Ferrara and M. Villasante, *Int. J. Mod. Phys.* A2 (1987) 1839.

[11] L. J. Dixon, V. S. Kaplunovsky and J. Louis, *Nucl. Phys.* B355 (1991) 649.
[12] I. Antoniadis, K. S. Narain and T. R. Taylor, Phys. Lett. B267 (1991) 37;  
I. Antoniadis, E. Gava and K. S. Narain, Phys. Lett. B283 (1992) 209; Nucl. Phys. B383 (1992) 93.

[13] P. Mayr and S. Stieberger, Nucl. Phys. B407 (1993) 725; B412 (1994) 502.

[14] M. B. Green and J. H. Schwarz, Phys. Lett. B149 (1984) 117.

[15] G. L. Cardoso and B. A. Ovrut, Nucl. Phys. B369 (1992) 351; B392 (1993) 315.

[16] J.-P. Derendinger, in Particles, Strings and Cosmology, edited by P. Nath and S. Reucroft (World Scientific, Singapore, 1992), p. 766;  
J. Louis, in Particles, Strings and Cosmology, edited by P. Nath and S. Reucroft (World Scientific, Singapore, 1992), p. 751;  
D. Lüst and L. E. Ibáñez, Nucl. Phys. B382 (1992) 305.

[17] M. Dine, N. Seiberg and E. Witten, Nucl. Phys. B289 (1987) 589.

[18] J.-P. Derendinger, S. Ferrara, C. Kounnas and F. Zwirner, Phys. Lett. B271 (1991) 307.

[19] See for instance: M. T. Grisaru and P. C. West, Nucl. Phys. B254 (1985) 249,  
or: S. J. Gates Jr., M. T. Grisaru, M. Roček and W. Siegel, Superspace (Benjamin/Cummings, Reading, 1983).

[20] D. R. T. Jones, Phys. Lett. B123 (1983) 45;  
V. A. Novikov, M. A. Shifman, A. I. Vainshtein and V. I. Zakharov, Nucl. Phys. B229 (1983) 381; Phys. Lett. B166 (1986) 329.

[21] M.A. Shifman and A.I. Vainshtein, Nucl. Phys. B277 (1986) 456; Nucl. Phys. B359 (1991) 571.

[22] C. P. Burgess, J.-P. Derendinger, F. Quevedo and M. Quirós, in preparation.