Large non-Gaussianity from non-local inflation

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Abstract. We study the possibility of obtaining large non-Gaussian signatures in the cosmic microwave background in a general class of single-field non-local hilltop inflation models. We estimate the non-linearity parameter $f_{NL}$ which characterizes non-Gaussianity in such models and show that large non-Gaussianity is possible. For the recently proposed $p$-adic inflation model we find that $f_{NL} \sim 120$ when the string coupling is order unity. We show that large non-Gaussianity is also possible in a toy model with an action similar to those which arise in string field theory.

Keywords: string theory and cosmology, inflation

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1. Introduction

Though the simplest models of inflation yield a negligible degree of non-Gaussianity in the cosmic microwave background (CMB) there has been considerable interest recently in constructing models which can give rise to large non-Gaussian signatures [1]–[8] (see [9] for a review). Non-Gaussianity is typically characterized by the dimensionless non-linearity parameter $f_{\text{NL}}$ which is of the order of $|n_s - 1| \ll 1$ (where $n_s$ is the spectral index) in conventional models [1]–[4]. The current observational limit is $|f_{\text{NL}}| \lesssim 300$ [11] (for the WMAP 3 Year data [12]) and future missions are expected to be able to probe $f_{\text{NL}}$ as small as order unity [13]. Though it has been difficult to find models which can yield large non-Gaussianity (indeed, observation of $|f_{\text{NL}}| \gtrsim 5$ would be strongly indicative of some novelty in the dynamics driving inflation) there exist several examples in the literature:

Non-Gaussianity can further be characterized using the trispectrum, which is also small in the simplest models [10].
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1. Single-field inflationary models in which the inflaton has a small sound speed [14], as in [15].
2. Single-field models in which the inflaton potential has a sharp feature [16].
3. Hybrid inflation [17] can generate significant non-Gaussianity during the preheating phase for certain values of the model parameters [18,19]. Non-Gaussianity from preheating has also been considered in [20].
4. The curvaton model [21].
5. Ghost inflation [22].

The simplest multi-field models do not yield large non-Gaussianity [23], however, this need not be true also of more complicated models. In this paper we show that large non-Gaussianity may also be generated in certain inflationary models based on non-local field theory. In particular, we will show that large non-Gaussianity is possible in the recently proposed $p$-adic inflation model [24].

Recently, cosmological applications of field theories containing an infinite number of derivatives have attracted considerable interest in the literature [24]–[31]. In [26] higher derivative modifications of gravity were considered and it was shown that such theories can have novel features such as improved ultraviolet (UV) behaviour and the existence of non-singular bouncing solutions. In [27] it was shown that similar non-local gravity theories can lead to self-inflation.

In this paper we will be interested in the cosmology of scalar field theories containing an infinite number of derivatives (we assume a standard gravitational sector) as in [25] and [28]–[31]. Non-local scalar field theories of this type can be derived from string theory, the most popular examples being the $p$-adic string theory [32]–[34] and cubic string field theory [35]–[37]. In both cases the scalar field is a tachyon describing the instability of some unstable D-brane configuration. Our interest in these types of non-local field theories is motivated by their novel cosmological behaviour. Non-local field theories can exhibit a variety of interesting cosmological properties including the possibility of slow roll inflation with steep potentials [24,25], bouncing cosmologies [29] and an equation of state $w < -1$ for dark energy within a sensible microscopic theory [28,29,31].

In this paper we will focus on non-local field theories in which inflation is realized near the top of an unstable maximum of the potential (we refer to such constructions as non-local hilltop [38] inflation models). In [24] a particular model based on $p$-adic string theory, $p$-adic inflation, was proposed. A novel feature was the possibility of slow roll dynamics for the inflaton, despite the presence of a steep potential, which suggests an intriguing possible resolution for the fine tuning problems which typically plague inflationary model building. In [25] a more general analysis of non-local hilltop inflation was performed and it was shown that this behaviour is possible also in other models. In this paper we study the possibility of obtaining non-Gaussian perturbations in general models of the form considered in [25], specializing at the end to the case of $p$-adic inflation. We find that in $p$-adic inflation $f_{NL} \sim 120$ is possible for $g_s \sim 1$. We show that large non-Gaussianity is possible also in other models.

The plan of the paper is as follows. In section 2 we consider the inflationary dynamics of non-local hilltop inflation, generalizing the results of [25] to include the possibility of a $\phi^3$ term in the scalar field potential. In section 3 we show how these results apply to the case of $p$-adic inflation. In section 4 we provide an estimate for the non-linearity
parameter in non-local hilltop inflation. In section 5 we use this result to estimate the non-linearity parameter in \( p \)-adic inflation, showing that \( |f_{NL}| \gg 1 \) is possible. Finally, we consider a toy model with an exponential kinetic function (which is typical of string field theory Lagrangians) showing that large non-Gaussianity is possible.

2. Non-local hilltop inflation

We consider general non-local theories of the form [25] (see also [28, 29])

\[
\mathcal{L} = \gamma^4 \left[ \frac{1}{2} \phi F \left( \frac{\Box}{m_s^2} \right) \phi - U(\phi) \right],
\]

where we assume a hilltop potential of the form

\[
U(\phi) = U_0 - \frac{\mu^2}{2} \phi^2 + \frac{g}{3} \phi^3 + \cdots,
\]

where the \( \cdots \) denotes terms of order \( \mathcal{O}(\phi^4) \) and higher. Such terms will generically be present, but since we are interested in the dynamics close to the false vacuum \( \phi = 0 \), they will play a subleading role in our calculation. Indeed, the potential (2.2) should only be thought of as an effective description close to \( \phi = 0 \) since it must be supplemented by additional terms to ensure that \( U(\phi) \) is bounded from below—at least on one side. We use metric signature \( \eta_{\mu \nu} = \text{diag}(-1, +1, +1, +1) \) so that \( \Box = -\partial^2 + \nabla^2 \) in flat space. In the above we take \( \phi, U_0, \mu \) and \( g \) to be dimensionless while \( \gamma, m_s \) have mass dimension one. With no loss of generality we can set \( F(0) = 0 \), since \( F(0) \) can always be absorbed into the definition of \( \mu \). The differential operator \( F(\Box/m_s^2) \) should be understood as a series expansion

\[
F \left( \frac{\Box}{m_s^2} \right) = \sum_{n=0}^{\infty} c_n \left( \frac{\Box}{m_s^2} \right)^n,
\]

where

\[
c_n = \frac{1}{n!} \left. \frac{d^n F(z)}{dz^n} \right|_{z=0},
\]

and \( c_0 = 0 \). Lagrangians of the form (2.1) arise frequently in string field theory, in which case \( m_s \) coincides with the string mass scale. In the following we keep \( m_s \) unspecified.

Examples of theories of the form (2.1) include the \( p \)-adic string theory [32] (see also [33, 34])

\[
\mathcal{L} = \frac{m_s^4}{g_p^2} \left[ -\frac{1}{2} \bar{\psi} p^{-\Box/m_s^2} \psi + \frac{1}{p+1} \bar{\psi}^p + \mathcal{O}(\psi^{p+1}) \right],
\]

with

\[
\frac{1}{g_p^2} = \frac{1}{g_s^2} \frac{p^2}{p-1}
\]

and, in order to put (2.4) in the form (2.1), we take \( \psi = \phi + 1 \). In (2.4) \( m_s \) and \( g_p \) are the string mass and coupling respectively. The action (2.4) was derived assuming \( p \) to be...
a prime number, though it appears to make sense for any integer value of $p$. A second popular example is the tachyon action [35] (see also [28, 36, 37])

$$L = \frac{m_4^4}{g_s^2} \left[ \frac{1}{2} \psi \left( 1 + \lambda^2 \frac{\Box}{m_s^2} \right) e^{-\Box/4m_s^2} \psi - \frac{1}{4} \psi^4 \right],$$  \hspace{1cm} (2.5)

where $\lambda^2 \simeq 0.9556$. This tachyon action is derived in cubic superstring field theory (SFT) in a flat background, incorporating only massless fields.

2.1. Background dynamics

We first consider the homogeneous dynamics of the theory (2.1). The Klein–Gordon equation for the scalar field $\phi$ is

$$F \left( \frac{\Box}{m_s^2} \right) \phi(t) = -\mu^2 \phi(t) + g\phi^2(t).$$ \hspace{1cm} (2.6)

The Friedmann equation is

$$3H^2 = \frac{1}{M_p^2} T_{00},$$ \hspace{1cm} (2.7)

where the stress tensor is given by [25] (see also [40])

$$\frac{1}{\gamma^4} T_{\mu\nu} = \sum_{l=1}^{\infty} c_l m_s^{-2l} \sum_{j=0}^{l-1} \left[ (\partial_\mu \Box^l \phi) (\partial_\nu \Box^{l-1-j} \phi) - \frac{1}{2} g_{\mu\nu} \left( \partial_\alpha \Box^l \phi \right) \left( \partial_\alpha \Box^{l-1-j} \phi \right) 
- \frac{1}{2} g_{\mu\nu} \left( \Box^j \phi \right) \left( \Box^{l-j} \phi \right) \right] \left[ \phi \sum_{l=0}^{\infty} c_l \left( \frac{\Box}{m_s^2} \right)^l \phi - 2V \right].$$ \hspace{1cm} (2.8)

In order to solve the system (2.6), (2.7) we make the ansatz of a series expansion

$$\phi(t) = \sum_{r=1}^{\infty} \phi_r e^{r\lambda t},$$ \hspace{1cm} (2.9)

$$H(t) = H_0 - \sum_{r=1}^{\infty} \phi_r e^{r\lambda t}$$

and solve for $\lambda$ and $\{\phi_r, H_r\}$ order-by-order in powers of $u = e^{\lambda t}$. We have parametrized the solution so that as $t \to -\infty$ the field $\phi$ sits at the unstable maximum $\phi = 0$ and the universe undergoes de Sitter expansion with Hubble constant $H_0$. We use the freedom to choose the origin of time to set $\phi_1 \equiv 1$. We also set $H_1 \equiv 0$ which is consistent because $T_{\mu\nu}$ does not contain any terms linear in $\phi$. Thus, to solve the equations of motion up to order $\mathcal{O}(u^2)$ the truncated expansion

$$\phi(t) = u + \phi_2 u^2,$$ \hspace{1cm} (2.10)

$$H(t) = H_0 - H_2 u^2$$ \hspace{1cm} (2.11)

suffices.
2.1.1. Scalar field evolution. We now proceed to solve (2.6) using (2.10). It was shown in [24] that
\[
(-\Box)^n \phi = + \left( \lambda^2 + 3H_0 \lambda \right)^n u + \left( 4\lambda^2 + 6H_0 \lambda \right)^n \phi_2 u^2 + \mathcal{O}(u^3),
\]
so that the left-hand side of (2.6) is
\[
F \left( \frac{\Box}{m_s^2} \right) \phi = F \left( \frac{-\lambda^2 + 3H_0 \lambda}{m_s^2} \right) u + F \left( \frac{-4\lambda^2 + 6H_0 \lambda}{m_s^2} \right) \phi_2 u^2 + \mathcal{O}(u^3),
\]
while the right-hand side of (2.6) gives
\[
-\mu^2 \phi + g\phi^2 = -\mu^2 u + (g - \mu^2 \phi_2) u^2 + \mathcal{O}(u^3).
\]
Matching at linear order in \(u\) implies
\[
F \left( \frac{-\lambda^2 + 3H_0 \lambda}{m_s^2} \right) = -\mu^2.
\]
It is convenient to define
\[
\omega^2 = -m_s^2 F^{-1} (-\mu^2),
\]
since consistency of this approach requires that \(F^{-1}\) exists and is single valued, so that (2.15) takes the form
\[
\lambda^2 + 3H_0 \lambda - \omega^2 = 0.
\]
It is straightforward to solve (2.17) for \(\lambda\)
\[
\lambda = \frac{-3H_0}{2} \pm \frac{3H_0}{2} \sqrt{1 + \frac{4\omega^2}{9H_0^2}}.
\]
The slow roll solution corresponds to taking the positive root and assuming that \(\omega^2 \ll H_0^2\) so that
\[
\lambda \approx \frac{\omega^2}{3H_0}.
\]
We also define a dimensionless parameter
\[
\eta = -\frac{\omega^2}{3H_0^2},
\]
so that slow roll corresponds to \(|\eta| < 1\) (notice that \(\eta < 0\) in this model). For the slowly rolling solution we have \(\lambda/H_0 = |\eta| < 1\) and \(\lambda^2 \ll 3H_0 \lambda\) so that the evolution is friction dominated in the usual sense, namely \(\ddot{\phi} \ll H \dot{\phi}\) when \(|\eta| < 1\). In passing, notice that to leading order in the small-\(u\) expansion the field obeys the eigenvalue equation
\[
\Box \phi = -\omega^2 \phi.
\]
However, the correspondence between the solutions of (2.20) and the solutions of (2.6) breaks down beyond leading order in the small-\(u\) expansion.

We now solve (2.6) up to second order in the small-\(u\) expansion. Notice that the arguments of \(F\) in (2.13) are approximately \(-\omega^2/m_s^2\) and \(-2\omega^2/m_s^2\) for the coefficients of the \(\mathcal{O}(u)\) and \(\mathcal{O}(u^2)\) terms respectively. Matching (2.13) and (2.14) at order \(u^2\) gives
\[
\phi_2 \approx \frac{g}{\mu^2 + F (-2(\omega^2/m_s^2))}
\]
for \(|\eta| < 1\). Notice that in the case \(g = 0\) we have \(\phi_2 = 0\) and the correspondence between the solutions of (2.20) and the solutions of (2.6) holds quite generally. As we have pointed out previously, however, for \(g \neq 0\) this equivalence breaks down at order \(u^2\).
2.1.2. Friedmann equation. It now remains to determine the coefficients $H_0$, $H_2$ in (2.11). In order to determine $H(t)$ up to order $u^2$ we need only consider $\phi(t)$ up to first order in the small-$u$ expansion. This is so because $T_{\mu\nu}$ does not contain any term linear in $\phi$ (see equation (2.8)). Working strictly with $\phi(t) = u$ the scalar field obeys (2.20) so that it is straightforward to resum the infinite series in (2.8). The result is

$$\gamma^{-4} T_{00} = U_0 - \frac{\omega^2}{2m_s^2} F' \left( - \frac{\omega^2}{m_s^2} \right) \left[ 1 - \frac{\eta}{3} \right] u^2 + \mathcal{O}(u^3).$$

(2.22)

Then we have

$$\sqrt{T_{00}} \approx \frac{\gamma^2 U_0^{1/2}}{\sqrt{3} M_p} \left[ 1 - \frac{\omega^2}{4m_s^2 U_0} F' \left( - \frac{\omega^2}{m_s^2} \right) u^2 + \mathcal{O}(u^3) \right]$$

(2.23)

at leading order in the $\eta$ slow roll parameter. The fact that there is no $\mathcal{O}(u)$ term on the right-hand side of (2.22) demonstrates that it was consistent to set $H_1 = 0$ in (2.11). From the Friedmann equation

$$H(t) = \frac{\sqrt{T_{00}}}{\sqrt{3} M_p},$$

it is straightforward to identify $H_0$, $H_2$ by equating (2.11) to (2.23). We find that

$$H_0 = \frac{\gamma^2 U_0^{1/2}}{\sqrt{3} M_p}$$

(2.24)

and

$$\frac{H_2}{H_0} = \frac{\omega^2}{4m_s^2 U_0} F' \left( - \frac{\omega^2}{m_s^2} \right).$$

(2.25)

Or, using (2.16) to eliminate the derived parameter $\omega^2$, we have

$$\frac{H_2}{H_0} = -\frac{1}{4U_0} F^{-1}(-\mu^2) F' \left[ F^{-1}(-\mu^2) \right].$$

(2.26)

It is useful to define a second dimensionless ‘slow roll’ parameter by

$$\epsilon(t) = \frac{H_2}{H_0} e^{2\lambda t}.$$  

(2.27)

It is clear that once $\epsilon(t) = 1$ then the perturbative expansion (2.11) breaks down and our solution is no longer valid. At this point one may assume that inflation has ended and hence the condition $\epsilon(t_{\text{end}}) = 1$ implicitly defines the time $t_{\text{end}}$ at which inflation ends. It is worth pointing out that the definition (2.27) differs from the usual $\epsilon$ slow roll parameter since

$$-\frac{\dot{H}}{H^2} \approx 2|\eta|\epsilon$$

with our definitions (2.19), (2.27). A quantity in which we will be interested in later on is the value of $u = e^{\lambda t}$ at the end of inflation. Using (2.26) we see that

$$\frac{1}{u_{\text{end}}^2} = -\frac{1}{4U_0} F^{-1}(-\mu^2) F' \left[ F^{-1}(-\mu^2) \right].$$

(2.28)
2.2. Fluctuations

We now study the scalar field fluctuations about a homogeneous background,
\[ \phi(t, x) = \phi_0(t) + \delta\phi(t, x). \] (2.29)

As we have shown the background solution during inflation is described by \( \phi_0(t) \approx e^{\lambda t}, \) \( H(t) \approx H_0 - H_2 e^{2\lambda t}. \) In order to solve for the fluctuation \( \delta\phi \) we treat the background as pure de Sitter space (which is equivalent to working to zeroth order in the small-\( u \) expansion so that \( \phi_0 = 0 \) and \( H = H_0 \)). In this limit the fluctuation \( \delta\phi \) obeys the equation
\[ F\left(\frac{\Box}{m_s^2}\right) \delta\phi = -\mu^2 \delta\phi. \] (2.30)

We can obtain solutions of (2.30) by choosing \( \delta\phi \) to satisfy the eigenvalue equation
\[ \Box \delta\phi = -\omega^2 \delta\phi \] (2.31)
with \( \omega^2 \) given by (2.16) as above. The solutions of (2.31) are well known. However, to make contact with the usual treatment of cosmological perturbations we need to define a field in terms of which the action looks canonical. This presents a difficulty because, in general, there is no local field redefinition which will bring the kinetic term \( \phi F\left(\frac{\Box}{m_s^2}\right) \phi \) into canonical form. However, as in [24,25], we can circumvent this difficulty by noticing that (2.30) can be derived from the perturbed Lagrangian
\[ \mathcal{L}^{(2)} = \frac{\gamma 4}{2} \left[ \delta\phi F\left(\frac{\Box}{m_s^2}\right) \delta\phi + \mu^2 (\delta\phi)^2 \right]. \] (2.32)

Then, for on-shell fields (that is, when (2.31) is satisfied),
\[ \mathcal{L}_{\text{on-shell}}^{(2)} = \frac{\gamma 4}{2} \left[ \delta\phi F\left(-\frac{\omega^2}{m_s^2}\right) \delta\phi + \mu^2 (\delta\phi)^2 \right] \]
\[ = \frac{\gamma 4}{2} \left[ \delta\phi F\left(-\frac{\omega^2}{m_s^2}\right) \right] \]
\[ = \frac{\gamma 4}{2} \left[ \delta\phi F\left(-\frac{\omega^2}{m_s^2}\right) \delta\phi + \mu^2 (\delta\phi)^2 \right] \]
\[ = \frac{1}{2} \delta\varphi \Box \delta\varphi + \frac{\omega^2}{2} (\delta\varphi)^2, \]
where we have defined
\[ \delta\varphi = A \delta\phi \] (2.33)
with
\[ A^2 = \gamma 4 \left(-\frac{\omega^2}{m_s^2}\right) F\left(-\frac{\omega^2}{m_s^2}\right) = \frac{\gamma 4}{m_s^2} \frac{-\mu^2}{F^{-1}(\mu^2)}. \] (2.34)

In the second equality in (2.34) we have used (2.16) to eliminate the derived parameter \( \omega. \) Thus \( \delta\varphi \) is the variable which has canonical kinetic term in the action\(^2\).

\(^2\) Our choice of normalization coincides with the definition of a ‘canonical’ inflaton advocated in [24]. However, this choice differs from the normalization employed in [25], where the inflaton was normalized in such a way that the stress tensor \( T_{\mu\nu} \) takes canonical form, though the kinetic term in the action does not. In the case of \( p \)-adic inflation, our main interest, the discrepancy is only a factor of \( \sqrt{m_p} \) which is of order unity for the values of \( p \) which we consider.
Notice that the definition of the canonical field, equation (2.33), has been derived by studying the linearized theory. However, we will continue to adopt this definition even up to second order in cosmological perturbation theory when we compute the non-Gaussianity in section 4. This is justified since in the ADM formalism [39] it suffices to use the free-field solution to compute the interaction Hamiltonian. This is so because the terms in the Lagrangian which would provide non-linear corrections to the free-field dynamics are always multiplied by terms proportional to first order equations of motion, and hence these corrections vanish [14]. Similar comments apply to our construction of the curvature perturbation in subsection 2.3. In any case, contributions to \( f_{\text{NL}} \) coming from the free-field dynamics must always be present and, barring anomalous cancellation, the contribution of such terms provides a lower bound on the actual non-Gaussianities produced.

We now proceed to solve (2.31), bearing in mind that \( \delta \varphi \) is the appropriate canonically normalized field. We write the Fourier transform of the inflaton fluctuation as

\[
\delta \varphi(t, \vec{x}) = \int \frac{d^3k}{(2\pi)^{3/2}} e^{i\vec{k} \cdot \vec{x}} \xi_k(t),
\]  

where the operator-valued Fourier coefficients \( \xi_k(t) \) can be decomposed into annihilation/creation operators \( a_k, a_k^\dagger \) and \( c \)-number-valued mode functions \( \varphi_k \) as

\[
\xi_k(t) = a_k \varphi_k(t) + a_k^\dagger \varphi_k^*(t).
\]

The mode functions \( \varphi_k(t) \) are given by

\[
\varphi_k(t) = \frac{1}{2} \sqrt{\frac{\pi}{a^3 H_0}} e^{i(\pi/2)(\nu+1/2)} H^{(1)}_{\nu} \left( \frac{k}{aH_0} \right),
\]

where the order of the Hankel functions is

\[
\nu = \sqrt{\frac{9}{4} + \frac{\omega^2}{H_0^2}}
\]

and of course \( a = e^{H_0 t} \). In writing (2.37) we have used the usual Bunch–Davies vacuum normalization so that on small scales, \( k \gg aH_0 \), one has

\[
|\varphi_k| \approx \frac{a^{-1}}{\sqrt{2k}},
\]

which reproduces the standard Minkowski space fluctuations. This is the usual procedure in cosmological perturbation theory. However, we note that the quantization of the theory (2.1) is not transparent and it might turn out that the usual prescription is incorrect in the present context. We defer this and other subtleties to future investigation. On large scales, \( k \ll aH_0 \), the solutions (2.37) behave as

\[
|\varphi_k| \approx \frac{H}{\sqrt{2k^3}} \left( \frac{k}{aH_0} \right)^{3/2-\nu},
\]

which gives a large scale power spectrum for the fluctuations

\[
P_{\delta\varphi} = \left( \frac{H_0}{2\pi} \right)^2 \left( \frac{k}{aH_0} \right)^{n_s-1}
\]
with spectral index
\[ n_s - 1 = 3 - 2\nu \cong 2\eta, \]  
(2.39)

where \( \eta \) is given by (2.19). Since \( \eta < 0 \) this model always gives a red-tilted spectrum, in agreement with the latest WMAP data [12].

Using (2.19) and (2.24) we can rewrite (2.39) as
\[ \frac{1}{\gamma^4} = \frac{|n_s - 1|}{2} \frac{U_0}{M_p^2 m_s^2 |F^{-1}(-\mu^2)|}, \]  
(2.40)

which can be used to eliminate \( \gamma \) in favour of other parameters.

It is worth noting that the equivalence between the solutions of (2.31) and (2.30), which implies the equivalence between local and non-local theories described in [25], appears only at linear order in perturbation theory. Beyond linear order (as long as \( g \neq 0 \)) this equivalence breaks down, as we illustrate for the case of \( p \)-adic inflation in appendix A. The breakdown of this equivalence, once interactions are included, is crucial for understanding why non-local theories of the type (2.1) can give rise to significant non-Gaussianity. Indeed, if it were true that the equivalence persisted at all orders in perturbation theory then the calculation of \( f_{NL} \) in the theory (2.1) would be exactly equivalent to a calculation of \( f_{NL} \) in some local field theory where \( f_{NL} \ll 1 \) is generic.

2.3. The curvature perturbation

A full calculation of the power spectrum should include metric perturbations and also deviation of the background expansion from pure de Sitter. Such a computation is beyond the scope of the present paper. Here we neglect metric fluctuations and take \( H \cong H_0 \) to compute the field perturbation, as we have done in the last subsection. (These approximations reproduce the full calculation up to acceptable accuracy in the standard theory.) We assume that the curvature perturbation is given by
\[ \zeta = -\frac{H}{\dot{\varphi}_0} \delta \varphi, \]  
(2.41)

where \( \varphi_0 = A\phi_0 = Au \). To evaluate the prefactor \( H/\dot{\varphi}_0 \) we must work beyond zeroth order in the small-\( u \) expansion. We take \( \phi_0 = u \) to compute \( H/\dot{\varphi}_0 \) even though the perturbation \( \delta \varphi \) is computed in the limit that \( \phi_0 = 0 \). This should reproduce the full answer up to \( \mathcal{O}(u) \) corrections. It will come in handy later on to define \( \zeta = c_\zeta \delta \varphi \) so that the prefactor \( c_\zeta \) is
\[ c_\zeta = -\frac{H}{\dot{\varphi}_0} = -\frac{H_0}{A\lambda} \frac{1}{u}, \]

To compute the spectrum this must be evaluated at horizon crossing, \( N_e \) e-foldings before the end of inflation. Noting that
\[ u(t) = a(t)^{\eta}, \]

and the scale factor at horizon crossing is related to that at the end of inflation by \( a_\star = e^{-N_e a_{\text{end}}} \), it follows that
\[ u_\star = e^{-N_e u_{\text{end}}} = e^{-(N_e/2)|n_s-1|} u_{\text{end}}. \]  
(2.42)
while \( u_{\text{end}} \) is given by (2.28). In the above \( u_\ast = u(t_\ast) = e^{\lambda t_\ast} \) is the value of \( u(t) \) at the time of horizon crossing, \( t_\ast \). Further, since \( \lambda/H_0 = |\eta| = |n_s - 1|/2 \) and using (2.34), (2.28), (2.42) we can write the prefactor \( c_\zeta = -H/\dot{\phi}_0 \) at the time of horizon crossing as

\[
c^2_\zeta = \left( -\frac{H}{\dot{\phi}_0} \right)^2 \frac{4}{|n_s - 1|^2} \frac{m_s^2 F^{-1}(-\mu^2) e^{N_0|n_s-1|}}{u_{\text{end}}} \frac{\gamma^4}{(-\mu^2)} \frac{u_{\text{end}}^2}{|n_s - 1|^2} = \frac{e^{N_0|n_s-1|} m_s^2}{|n_s - 1|^2} \frac{1}{\gamma^4 U_0 \mu^2} \left[ F^{-1}(-\mu^2)^2 F' \right]^2 F' \left[ F^{-1}(-\mu^2) \right]. \tag{2.43}
\]

It follows that the power spectrum of the curvature perturbation is

\[
P_\zeta = A^2_\zeta \left( \frac{k}{aH_0} \right)^{n_s-1}, \tag{2.44}
\]

where

\[
A^2_\zeta = \left( -\frac{H}{\dot{\phi}_0} \right)^2 \left( \frac{H_0}{2\pi} \right)^2 = c^2 \left( \frac{H_0}{(2\pi)^2} \right)^2 \approx 25 \times 10^{-10}. \tag{2.45}
\]

Using (2.43) and (2.24) we arrive at the result

\[
A^2_\zeta = \frac{1}{12\pi^2} \left( \frac{m_s}{M_p} \right)^2 e^{N_0|n_s-1|} \frac{1}{|n_s - 1|^2} G(\mu^2), \tag{2.46}
\]

where we have defined the function

\[
G(\mu^2) = \frac{1}{\mu^2} \left[ F^{-1}(-\mu^2)^2 F' \right] F' \left[ F^{-1}(-\mu^2) \right]. \tag{2.47}
\]

3. Specialization to \( p \)-adic inflation

We now consider how our previous results apply to the case of \( p \)-adic inflation (2.4). For the dimensionful parameters we have

\[
\gamma^4 = \frac{m_s^4}{g_p^2} = \frac{m_s^4}{g_s^2} \frac{p^2}{p-1} \tag{3.1}
\]

and \( m_s \) is identified with the string mass scale. Notice that \( \gamma > m_s \) for typical values of \( g_s \) and \( p \). The dimensionless coefficients appearing in the potential (2.2) are

\[
U_0 = \frac{1}{2} \frac{p-1}{p+1}, \tag{3.2}
\]

\[
\mu^2 = p - 1, \tag{3.3}
\]

\[
g = \frac{p}{2} (p - 1). \tag{3.4}
\]

The function \( F \) associated with the kinetic operator is

\[
F(z) = 1 - p^{-z/2}, \tag{3.5}
\]

so that

\[
F^{-1}(-\mu^2) = -2, \tag{3.6}
\]
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\[ F' \left[ F^{-1}(-\mu^2) \right] = \frac{p}{2} \ln p, \]  
\[ G(\mu^2) = \frac{2p \ln p}{p - 1}. \]  

Notice also that (2.40) can be used to eliminate the ratio \( m_s/M_p \) in favour of \( p, g_s \) as

\[ \left( \frac{m_s}{M_p} \right)^2 = \frac{8(p + 1)}{p^2} \frac{g_s^2}{|n_s - 1|}. \]  

Using equations (3.6)–(3.8) it is easy to show that we exactly reproduce the expressions for \( \lambda, \phi_2, H_0, H_2 \) and \( n_s - 1 \) given in [24]. However, our results for \( u_{\text{end}} \) and \( A^2_\zeta \) (equations (2.28) and (2.46)) differ somewhat from the results of [24]. To see this we first compute \( u_{\text{end}} \) for \( p \)-adic inflation using (2.28)

\[ \frac{1}{u^2_{\text{end}}} = p \left[ \frac{1}{2} \frac{p + 1}{p - 1} \ln p \right]. \]  

Now, using (2.46) to compute the COBE normalization for \( p \)-adic inflation we find

\[ A^2_\zeta = \frac{8}{3\pi^2} \frac{g_s^2}{|n_s - 1|^3} \left[ \frac{1}{2} \frac{p + 1}{p - 1} \ln p \right]. \]  

Equations (3.9) and (3.10) should be compared to equations (5.18) and (5.20) in [24]. In both cases the equations differ by a factor of \( (p + 1) \ln p/[2(p - 1)] \). This discrepancy arises because the end of inflation was defined here as the end of the validity of the perturbative expansion is powers of \( u \), whereas in [24] the end of inflation was defined using the friction-dominated approximation, which has a longer range of validity. (This discrepancy was noted also in [25].) Thus, in the particular case of \( p \)-adic inflation, the slow roll dynamics actually persist even after the perturbative description (2.9) has broken down. Thus, choosing (2.28) to define the end of inflation is more stringent.

Notice that in the case of \( p \)-adic inflation this discrepancy does not lead to any significant quantitative change in the results since the factor \( (p + 1) \ln p/[2(p - 1)] \) is order unity for the values of \( p \) which were considered in [24], and which will also interest us here.

4. Calculation of the non-Gaussianity

We now proceed to estimate the level of non-Gaussianity in non-local models, based on the bispectrum. As explained above, a full calculation including metric contributions appears to be prohibitively difficult; we do not know how to quantize the metric perturbations due to the complexity of the stress–energy tensor. However using local scalar field theories as a guide, we hypothesize that the scalar contribution to the curvature perturbation is of the same order of magnitude as that of the metric contribution, so that a valid estimate can be obtained from the scalar perturbation alone. Its contribution to the bispectrum comes from the cubic term in the action for small fluctuations.
4.1. Perturbing the Lagrangian

It is straightforward to perturb the Lagrangian (2.1) up to cubic order in the field \( \delta \varphi = A \delta \phi \). The quadratic Lagrangian is

\[
L^{(2)} = \frac{\omega^2}{2 \mu^2} \left[ \delta \varphi F \left( \frac{\Box}{m_s^2} \right) \delta \varphi + \mu^2 (\delta \varphi)^2 \right].
\]  (4.1)

The cubic Lagrangian is

\[
L^{(3)} = c_H (\delta \varphi)^3,
\]  (4.2)

where we have defined

\[
c_H = -\frac{g m_s^3}{3 \gamma^2} \left[ \frac{F^{-1}(-\mu^2)}{(-\mu^2)} \right]^{3/2}.
\]  (4.3)

4.2. Calculation of the bispectrum

Variation of the quadratic Lagrangian (4.1) yields the free perturbation equation of motion (2.30) whose solutions are given by (2.35)–(2.37), as we have already discussed. To leading order in slow roll parameters the order of the Hankel functions in (2.37) is \( \nu \sim 3/2 \) so that

\[
\varphi_k(\tau) \simeq \exp \left( \frac{i \delta H_0}{\sqrt{2k^3}} \right) e^{-ik\tau} (1 + i k \tau),
\]  (4.4)

where we have employed conformal time \( \tau \), which is related to cosmic time \( t \) by \( d\tau = dt \). It follows that \( a \simeq e^{H_0 t} \simeq -1/(H_0 \tau) \). In (4.4) \( k \) is the comoving wavenumber of the perturbation and \( \delta \) is a constant real phase whose value is irrelevant for our calculation.

The quantity of interest for our calculation is the three-point correlation function of the curvature perturbation \( \langle \zeta_k(\tau_1) \zeta_k(\tau_2) \zeta_k(\tau_3) \rangle \) where the expectation value is computed in the interaction vacuum \( \exp(-i \int_0^t dt' H_{\text{int}}(t')) |0\rangle \). The leading contribution is

\[
\langle \zeta_k(\tau_1) \zeta_k(\tau_2) \zeta_k(\tau_3) \rangle = -i \int_{\tau_i}^{\tau_f} d\tau' \langle 0 | \zeta_k(\tau_1) \zeta_k(\tau_2) \zeta_k(\tau_3) , H_{\text{int}}(\tau') | 0 \rangle
\]  (4.5)

\[
= -i c_\zeta^3 \int_{\tau_i}^{\tau_f} d\tau' \langle 0 | \zeta_k(\tau_1) \zeta_k(\tau_2) \zeta_k(\tau_3) , H_{\text{int}}(\tau') | 0 \rangle,
\]  (4.6)

where we have used the fact that

\[
\zeta_k(\tau) = c_\zeta \zeta_k(\tau) = c_\zeta \left[ a_k \varphi_k(\tau) + a_k^\dagger \varphi_k^*(\tau) \right].
\]

The interaction Hamiltonian takes the form

\[
H_{\text{int}} = -\int d^3x a^3 L^{(3)}
\]  (4.7)

\[
= -c_H \int d^3x a^3 \delta \varphi^3,
\]  (4.8)

where \( c_H \) is defined in (4.3).
In order to compute (4.5) we first consider the commutator
\[
[\xi_k(\tau), \delta \varphi(\tau', x')] = \frac{e^{-ik \cdot x'}}{(2\pi)^{3/2}} (\varphi_k(\tau) \varphi^*_k(\tau') - \varphi_k(\tau') \varphi^*_k(\tau))
\]
\[
\cong -i \frac{H^2}{3} (\tau - \tau')^3 \frac{e^{-ik \cdot x'}}{(2\pi)^{3/2}},
\]
where on the last line we have taken the large scale limit \(-k\tau, -k\tau' \ll 1\).

Noting that the commutator (4.9) is a c-number we have
\[
[\xi_k(\tau), \delta \varphi^3(\tau', x')] = 3 [\xi_k(\tau), \delta \varphi(\tau', x')] \delta \varphi^2(\tau', x').
\]

The expectation value in (4.5) can be written as
\[
\langle 0 | [\xi_{k_1}\xi_{k_2}\xi_{k_3}(\tau), \delta \varphi^3(\tau', x')] | 0 \rangle = 3 \langle 0 | \xi_{k_1}\xi_{k_2}\xi_{k_3}(\tau) \delta \varphi^2(\tau', x') | 0 \rangle
+ 3 \langle 0 | \xi_{k_1}\delta \varphi(\tau', x') \xi_{k_2}\xi_{k_3}(\tau) | 0 \rangle
+ 3 \langle 0 | \xi_{k_1}(\tau) \delta \varphi(\tau', x') \xi_{k_2}\xi_{k_3}(\tau) | 0 \rangle.
\]

Consider, for example, the first term. Carrying out the Wick contractions we have

\[
3 \langle 0 | \xi_{k_1}\xi_{k_2}\xi_{k_3}(\tau) \delta \varphi^2(\tau', x') | 0 \rangle
= 6 \langle 0 | \xi_{k_1}(\tau) \delta \varphi(\tau', x') \rangle e^{-i(k_1+k_2) \cdot x'} D_{k_1}(\tau - \tau') D_{k_2}(\tau - \tau'),
\]

where the momentum space propagator is
\[
D_{k}(\tau - \tau') = \frac{1}{(2\pi)^{3/2}} \varphi_k(\tau) \varphi^*_k(\tau')
= \frac{1}{(2\pi)^{3/2}} \frac{H^2}{2k^3} e^{-ik(\tau - \tau')}(1 + i k \tau)(1 - ik \tau')
\cong \frac{1}{(2\pi)^{3/2}} \frac{H^2}{2k^3},
\]
and on the last line we have restricted to large scales. The remaining terms in (4.10) are similar.

We are now in a position to calculate (4.5). On large scales \(-k_i \tau, -k_i \tau' \ll 1\) we have
\[
\langle \xi_{k_1}\xi_{k_2}\xi_{k_3}(\tau) \rangle \cong -c_{H}^{3} \frac{H^2}{2} \int d\tau' \frac{(\tau - \tau')^3}{(\tau')} k_1^3 + k_2^3 + k_3^3 \delta^{(3)}(k_1 + k_2 + k_3).
\]

The time integral is divergent and needs to be regulated. Taking \(\tau\) to correspond to the end of inflation and \(\tau_i\) to correspond to the beginning of inflation we have
\[
\int_{\tau_i}^{\tau} d\tau' \frac{(\tau - \tau')^3}{(\tau')} \cong \ln \left( \frac{\tau_i}{\tau} \right) = \ln e^{N_e} = N_e,
\]
where \(N_e\) is the total number of e-foldings of inflation.

It is worth commenting on the infrared (IR) divergence in the \(d\tau\) integral in (4.12) which does not appear in the standard calculation of \(f_{\text{NL}}\) [3, 14]. Naively, this divergence would seem to suggest that the curvature perturbation in \(\zeta\) is not freezing out on super-horizon scales, as it does in the standard theory. However, we suspect that the IR divergence in (4.13) is in fact an artifact of the approximations which we have made.
Indeed, a calculation of $f_{NL}$ in a local field theory which makes precisely the same approximations as we have made (neglecting the metric perturbations and the departure of the background from pure de Sitter space) yields exactly the same IR divergence [41]. It is clear from the calculation of [41] that this IR divergence will generically arise in the three-point function of any light scalar field in de Sitter space. In appendix B we show that, despite the presence of the IR divergence, the calculation of [41] can be used to estimate $f_{NL}$ in a local field theory and their result does reproduce the result of the complete calculation [3] up to factors of order unity. We assume that the same holds true in the non-local theories which we consider.

Though we believe that in a more complete calculation one would find that $\zeta$ is conserved outside the horizon in the non-local theory, we were not able to demonstrate this conclusively. A proper resolution of this issue would require us to perform a complete second order cosmological perturbation calculation, including metric perturbations and departures of the background from pure de Sitter space. Such a calculation is complicated by the non-local structure of the theory and is beyond the scope of the present work. Though it is a logical possibility that $\zeta$ is actually not conserved in the non-local theory, it would be surprising.

Quite generally, the time derivative of the large scale curvature perturbation is proportional to the non-adiabatic pressure, $\dot{\zeta} \propto P_{\text{nad}}$ [42]. In a model with only one degree of freedom there are no entropy perturbations and $P_{\text{nad}} = 0$ so that $\dot{\zeta} = 0$ on large scales. Thus we expect that as long as (2.1) describes only a single degree of freedom then the large scale curvature perturbation should be conserved (in a complete calculation). It is well known that higher derivative theories with finitely many derivatives can be rewritten in terms of multiple scalar fields (at least one of which will generically be ghost-like). However, for infinite derivative theories of the type which we consider this need not be the case [26]. For infinite derivative theories one should consider the pole structure of the propagator $G_{k} \sim [F(-k^2/m_s^2) + \mu^2]^{-1}$. Our restriction to models where $F^{-1}(-\mu^2)$ is single valued (below equation (2.16)) ensures that the propagator has only one pole (at least near the false vacuum $\phi = 0$). It follows that our analysis is restricted to infinite derivative theories which describe only a single scalar degree of freedom and hence one expects $P_{\text{nad}} = 0$ (and therefore $\dot{\zeta} \simeq 0$) for such theories.

4.3. The non-linearity parameter

We now estimate the non-linearity parameter by comparing (4.12) to the WMAP ansatz

$$\langle \zeta_{k_1} \zeta_{k_2} \zeta_{k_3} (\tau) \rangle = (2\pi)^7 \left( -\frac{3}{10} f_{NL} \right) A_{\zeta}^4 \frac{k_1^3 + k_2^3 + k_3^3}{k_1^3 k_2^3 k_3^3} \delta^{(3)} (k_1 + k_2 + k_3),$$

(4.14)

where, as we have shown previously, $A_{\zeta}^2 = H^4/(2\pi \dot{\phi}_0)^2 = c_{\zeta}^2 H_0^2/(4\pi^2)$. Comparing (4.14) to (4.12) we have

$$f_{NL} = \frac{5 N_{e}}{3 (2\pi)^5 c_{H} c_{\zeta}} \frac{1}{A_{\zeta}^2}. \quad (4.15)$$

\footnote{For example, in the case of $p$-adic inflation the propagator near the unstable maximum has only one pole corresponding to a scalar field with mass-squared $-2m_{s}^2$: the bosonic open string tachyon. In contrast there are no poles about the true minimum, which is the $p$-adic version of the statement that there are no open strings in the tachyon vacuum.}
It is straightforward to compute the product $c_H c_\zeta$ using (4.3) and (2.43). The result is

$$c_H c_\zeta = -\frac{g}{3U_0^{1/2}} \left( \frac{m_s}{\gamma} \right)^4 e^{(N_s/2)|n_s-1|} \frac{(F^{-1}(-\mu^2))^{3/2}}{|\mu|^3} G(\mu^2),$$

(4.16)

where $G(\mu^2)$ is defined in (2.47). It is possible to further simplify this expression by using (2.40) to eliminate $\gamma^{-4}$ with the result

$$c_H c_\zeta = -\frac{gU_0^{1/2}}{6} \left( \frac{m_s}{M_p} \right)^2 e^{(N_s/2)|n_s-1|} \frac{|F^{-1}(-\mu^2)|^{1/2}}{|\mu|^3} \sqrt{G(\mu^2)}.$$

(4.17)

We can eliminate $(m_s/M_p)^2$ from (4.17) using the COBE normalization (2.46). The final result is

$$f_{NL} \cong -\frac{5N_s}{48\pi^3 gU_0^{1/2} e^{-(N_s/2)|n_s-1|}|n_s-1|^2} K(\mu^2),$$

(4.18)

where we have defined

$$K(\mu^2) = \frac{1}{\mu^2} \frac{1}{|F^{-1}(-\mu^2)|^{1/2}} \frac{1}{|\mu|^3}.$$

(4.19)

Equation (4.18) is the main result of this section.

### 4.4. The perturbative regime

Since $f_{NL} \propto g$ and we are interested in models with $|f_{NL}| \gg 1$ it is important to understand what values of $g$ are acceptable. In conventional (local) inflationary models one would require $|g| \ll 1$ in order not to spoil slow roll; however, this need not be the case for the non-local hilltop models which we consider. Indeed, in the case of $p$-adic inflation one has $|g| \sim p^2 \gg 1$ (see section 3). In terms of the canonical field the Lagrangian is of the form

$$\mathcal{L} = \frac{\omega^2}{2\mu^2} \varphi F \left( \frac{\Box}{m_s^2} \right) \varphi - V(\varphi),$$

(4.20)

where

$$V(\varphi) = V_0 - \frac{\omega^2}{2} \varphi^2 + c_H \varphi^3$$

(4.21)

and $V_0 = \gamma^4 U_0$. In a local field theory the one-loop correction to the cubic term of the potential (4.21) would be of the order

$$\delta c_H \sim \frac{1}{16\pi^2} \frac{c_H^3}{\omega^3},$$

so that the dimensionless quantity $c_H^2/\omega^2$ controls the perturbative expansion. We therefore expect that the calculation is under control as long as $c_H^2 < \omega^2$. Using equations (4.3) and (2.16) the condition $c_H^2 < \omega^2$ implies the following upper bound on $g$:

$$|g| < \frac{3\gamma^3}{|F^{-1}(-\mu^2)|} \left( \frac{\gamma}{m_s} \right)^2.$$

(4.22)

In a given model the largest reliable value of $f_{NL}$ is achieved by choosing $g$ to saturate the bound (4.22).
5. Examples

5.1. p-adic inflation

With the results of section 3 we can compute $f_{NL}$ for the p-adic inflation model, using (4.18), (4.19). The result is

$$f_{NL} \approx \frac{5N_e}{192\pi^3} e^{-(N_e/2)|n_s-1|} |n_s - 1|^2 \sqrt{p} \frac{2(p-1)}{\ln p + 1}^{1/2},$$

so that $f_{NL} \sim \sqrt{p}$ for $p \gg 1$. This result may slightly underestimate the non-Gaussianity produced in p-adic inflation because of the extra factor $2(p-1)/(\ln p + 1)$, which appears because the perturbative approach breaks down before the apparent end of the slow roll phase. This is precisely the discrepancy discussed below (3.10). Taking into account the fact that in p-adic inflation slow roll ends when $u \sim p^{-1/2}$, rather than when $u$ is given by (3.9), we obtain

$$f_{NL} \approx \frac{5N_e}{192\pi^3} e^{-(N_e/2)|n_s-1|} |n_s - 1|^2 \sqrt{p}.$$  \hspace{1cm} (5.1)

For $n_s \approx 0.95$ and $N_e \approx 60$ this implies

$$f_{NL} \approx 2.8 \times 10^{-5} \sqrt{p}.$$  \hspace{1cm} (5.2)

so that large $f_{NL}$ requires large values of $p$.

How large can (5.2) be made? The COBE normalization relates $g_s$ and $p$ as [24]

$$g_s = \frac{5\pi^3}{2\sqrt{2}} 10^{-5} e^{-(N_e/2)|n_s-1|} |n_s - 1|^{3/2} \sqrt{p}.$$  \hspace{1cm}

For $N_e \approx 60$ and $n_s \approx 0.05$ the condition $g_s < 1$ bounds $p$ from above as

$$p \lesssim 1.7 \times 10^{13}$$

so that, using (5.2), the non-linearity parameter is bounded as

$$f_{NL} \lesssim 120$$

with the upper limit corresponding to $g_s = 1$. The maximum possible non-Gaussianity is within the observational limit $f_{NL} < 300$ though it should be observable in future missions. We see that large non-Gaussianity is possible in p-adic inflation. It is interesting that the largest values of $f_{NL}$ correspond to $g_s$ close to unity, which is considered natural from the string theory perspective.

In evaluating our estimate for $f_{NL}$, equation (5.2), we have used the total number of e-foldings between horizon crossing and the end of inflation $N_e \approx 60$, rather than the number of e-foldings which can be observed. This is justified because the factor $N_e$ has its origins in an IR divergence (4.13). This divergence should be regulated by the largest scale to which we have experimental access, namely, our current horizon size. The need for such IR regulators in cosmological perturbation theory was discussed in [6] where it was argued that these divergences simply reflect our ignorance about scales beyond the horizon. Notice that typically the total number of e-foldings between horizon crossing and the end of inflation is $N_e \sim |n_s - 1|^{-1}$ so that one of the factors of $n_s - 1$ in (4.18) is cancelled by the factor $N_e$. An identical cancellation occurs in local field theory models, see appendix B.
Finally, we note that even taking $N_e \approx 6$ we would still obtain $f_{\text{NL}} \sim 12 g_s$ which, for $g_s \sim 1$, is still several orders of magnitude larger than the prediction of the simplest inflationary models. As long as $f_{\text{NL}} > O(1)$ then the primordial non-Gaussianity should be detectable in future missions [13]. For the primordial $f_{\text{NL}} = O(1)$ the situation is more complicated since post-inflationary super-horizon evolution will also generate an order unity contribution to the non-linearity parameter [7] (see also [8]) and it may be difficult to disentangle these two distinct sources of non-Gaussianity.

In principle one could make $f_{\text{NL}}$ even larger than $f_{\text{NL}} \sim 120$ in this model by relaxing the requirement that $g_s < 1$, however, at large $g_s$ we can no longer rely upon perturbation theory to compute the bispectrum or other quantities from the tree-level $p$-adic action. String loop effects will invalidate this action when $g_s \gg 1$. This is also consistent with our discussion in section 4.4 since the bound (4.22), for the case of $p$-adic inflation, corresponds to $g_s < 3$.

5.2. A toy model with exponential kinetic function

As an example of further possibilities within the framework of non-local field theories, we next investigate a toy model which can give rise to significant non-Gaussianity. We choose a kinetic function

$$F(z) = \frac{1}{\alpha} (1 - e^{-\beta z}),$$

(5.3)

where it is assumed that $\alpha, \beta > 0$. Equation (5.3) is typical of the kinds of kinetic functions which arise in string field theory. It is straightforward to compute

$$F^{-1}(-\mu^2) = -\frac{1}{\beta} \ln (1 + \alpha \mu^2),$$

(5.4)

$$F' [F^{-1}(-\mu^2)] = \frac{\beta}{\alpha} (1 + \alpha \mu^2)$$

(5.5)

so that

$$G(\mu^2) = \frac{1}{\beta} \left( \frac{1 + \alpha \mu^2}{\alpha \mu^2} \right) \ln^2 (1 + \alpha \mu^2),$$

(5.6)

$$K(\mu^2) = \frac{1}{\mu^3} \sqrt{\frac{\alpha \mu^2}{1 + \alpha \mu^2}} \sqrt{\frac{1}{\ln (1 + \alpha \mu^2)}}.$$  

(5.7)

In passing, it is interesting to compute the effective mass of the field fluctuation for this model using (5.4) and (2.16)

$$\omega^2 = \frac{m_s^2}{\beta} \ln (1 + \alpha \mu^2).$$

The fluctuations $\delta \varphi$ behave as though the field had mass-squared $-\omega^2$. However, for an exponential kinetic function (5.3) this effective mass is only logarithmically sensitive to the actual mass $\mu$. This explains the novel behaviour first noted in [24] that the cosmology of the $p$-adic tachyon is virtually insensitive to the naive mass of the field.

We set

$$\gamma^4 = \frac{m_s^4}{g_s^2}.$$  

(5.8)
by analogy with the $D$-brane tension. Taking $U_0 = 1$ the equation (2.40) fixes the string scale as

$$\left(\frac{m_s}{M_p}\right)^2 = \frac{2g_s^2}{|n_s - 1|} \frac{\ln(1 + \alpha \mu^2)}{\beta}, \quad (5.9)$$

while the COBE normalization requires

$$\frac{g_s^2}{\beta^2} = 6\pi^2 |n_s - 1|^3 e^{-N_e |n_s - 1|} \frac{\alpha \mu^2}{1 + \alpha \mu^2 \ln^3 (1 + \alpha \mu^2)} \cdot 25 \times 10^{-10}. \quad (5.10)$$

If we take $\alpha = \mu^2 = 1$ then (5.10) requires

$$\beta \frac{g_s}{g_s} = 10^6 \quad (5.11)$$

and (5.9) gives $(m_s/M_p)^2 = 3 \times 10^{-5} g_s$ so that $m_s \ll M_p$ unless $g_s$ is very large.

To obtain an upper bound on $f_{NL}$ we note that $g$ is bounded from above by (4.22) so that

$$g < \frac{3\mu^3}{\ln(1 + \alpha \mu^2) g_s} = \frac{3}{\ln 2} g_s = 0.4 \times 10^7 \quad (5.12)$$

(we are taking $g > 0$). Using (5.11), (5.12), $n_s \approx 0.95$ and $N_e \approx 60$ we obtain

$$|f_{NL}| \approx 9.5 \times 10^{-5} g$$

so that $f_{NL} \lesssim 300$ bounds $g$ from above as $g \lesssim 0.31 \times 10^7$. Since this upper bound is stronger than the bound (5.12) it follows that it is possible to saturate the observational limit on $f_{NL}$.

6. Conclusions

In this paper we have studied the non-Gaussianity produced during non-local hilltop inflation. We were particularly interested in $p$-adic inflation, in which case large non-Gaussianity is possible, and indeed is natural since the upper bound on $f_{NL}$ corresponds to $g_s \sim 1$. We also considered a toy model with an exponential kinetic function, typical of string field theory Lagrangians, and showed that for certain values of the parameters large non-Gaussianity is possible in this model. Thus, non-local hilltop inflation models are among the few inflationary scenarios which can give rise to $f_{NL} \gg 1$.

There are several caveats to our work. In computing the curvature perturbation $\zeta$ we have neglected both metric perturbations and also departures of the background from pure de Sitter space. Although these approximations reproduce the correct answer up to acceptable accuracy in standard (local) theories, it is not clear if this is also true in the non-local theories which we have considered. A cause for concern is the IR divergence (4.13) which we have regulated by the number of e-foldings of inflation. This divergence does not occur in the standard theory and seems to indicate that our definition of $\zeta$ does not freeze out on large scales, as it should. We speculated that this divergence is an artifact of the approximations which we made, and this claim is supported by the fact that an identical divergence occurs also in the local theory when one makes exactly the same approximations which we have made [41]. That being said, we have not proven that $\zeta$...
will really freeze out in a more comprehensive treatment. It is possible that the relation \( \zeta \sim -H\delta \phi/\dot{\phi}_0 \) does not hold for non-local theories. In order to answer this question in a satisfactory manner it will be necessary to perform a complete and rigorous analysis of the cosmological perturbations in this theory, incorporating also metric perturbations. Such an analysis is complicated by the fact that the kinetic operator \( F(\Box) \) will be of the form \( F(\Box_0 + \delta \Box) \) where \( \Box_0 \) is the covariant d'Alembertian associated with the homogeneous background and \( \delta \Box \) is the perturbation which contains both metric perturbations \( \delta g_{\mu\nu} \) and also derivatives. In general \( \Box_0 \) and \( \delta \Box \) do not commute, making it extremely difficult to solve the non-local evolution equations. We leave a rigorous treatment of the cosmological perturbations to future analysis.

In a local field theory the gravitational sector is known to give a small contribution to the non-Gaussianity and hence the neglect of the metric perturbations is justified when \( f_{\text{NL}} \gg 1 \) [14]. It is not clear if this is also the case in non-local theories. In a complete calculation one would find terms in the perturbed Lagrangian which involve the operator \( F(\Box_0/m_s^2) \) acting on the scalar metric perturbations and hence it is not clear that the contribution of such terms to \( f_{\text{NL}} \) will be suppressed. Barring anomalous cancellations we would expect such terms to be of the same order of magnitude as those which we consider and hence our estimate should be reliable up to factors of order unity.

Another caveat is the complication in setting up the initial value problem for differential equations with infinitely many derivatives. It is known that the initial value problem for equations with infinitely many derivatives is fundamentally different from the initial value problem for an equation with \( N \) derivatives where \( N \gg 1 \) [33]. Infinite derivative equations have been studied from a mathematical physics perspective in [43]. For further discussions about the initial value problem see [24, 27, 44].

A related difficulty is the issue of quantizing a non-local theory. In our analysis we have assumed the usual Bunch–Davies vacuum for the perturbation \( \delta \phi \), though it is not clear that this is correct in the present context. Quantization of non-local theories with finitely many derivatives generically leads to ghost excitations (and also to a classical pathology called the Ostrogradski instability) though non-local theories with infinitely many derivatives (such as we consider) can evade these difficulties. For further discussion see [24, 26, 27].

Despite these difficulties, we believe that our calculation does provide a rough (order of magnitude) estimate of the actual non-Gaussianity produced. It is easy to intuitively understand why \( f_{\text{NL}} \) can be made large in these kinds of models. Our result (4.18) is equivalent to computing the three-point function for a light scalar field in de Sitter space. This correlator is proportional to coefficient of the cubic term in the potential, \( g \), and hence will be large when \( g \) is large. In conventional inflationary models this is impossible since a large coupling \( g \) would spoil slow roll. However, in non-local theories this is not necessarily true. (Similarly a large mass term \( \mu^2 \) would spoil inflation in a local theory, though there is no problem with exponentially large \( \mu^2 \) in our models.) Indeed, in the examples where we have found large non-Gaussianity this is simply because \( g \gg 1 \), and the novelty is that such theories can support inflation.

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Appendix A. \(p\)-adic scalar field evolution beyond linear order

In this appendix we demonstrate that the equivalence between local and non-local theories breaks down beyond linear order in perturbation theory by studying the \(p\)-adic scalar field equation of motion

\[
p^{-\Box/2}\psi = \psi^p
\]

(in units \(m_s \equiv 1\), which we employ throughout this appendix) up to second order in perturbation theory. For simplicity we neglect metric perturbations and assume pure de Sitter background. Expanding the \(p\)-adic scalar in perturbation theory as

\[
\psi = 1 + \phi = 1 + \delta^{(1)} \phi + \frac{1}{2} \delta^{(2)} \phi.
\]

It is straightforward to perturb the field equation (A.1) up to second order with the result

\[
[p^{-\Box/2} - p] \delta^{(1)} \phi = 0,
\]

\[
[p^{-\Box/2} - p] \delta^{(2)} \phi = p(p-1) \left( \delta^{(1)} \phi \right)^2.
\]

At linear order one may construct solutions of (A.2) by taking \(\Box \delta^{(1)} \phi = -\omega^2 \delta^{(1)} \phi\), however, this prescription fails for the second order equation (A.3). It is interesting to notice that in terms of the canonical field

\[
\varphi = \frac{p}{\sqrt{2} g_s} \phi
\]

the second order equation (A.3) becomes

\[
[p^{-\Box/2} - p] \delta^{(2)} \varphi = c \left( \delta^{(1)} \varphi \right)^2,
\]

where

\[
c = \frac{p(p-1)}{A} = \sqrt{2} g_s (p-1).
\]

The COBE normalization gives \(g_s \sim 10^{-7} \sqrt{p}\) (for \(n_s \cong 0.95\) and \(N_e \cong 60\)) so that \(c\) is of the order

\[
c \sim 10^{-7} p^{3/2}
\]

for \(p \gg 1\). We see that the second order effects can be made large by taking \(p \gg 1\) which is precisely the regime in which \(f_{NL} > 1\) occurs. As one might expect, this also coincides with the regime in which the non-local structure of the theory is playing an important role in the dynamics, as emphasized in [24].
Appendix B. Comparison to local theory

In [41] Falk et al studied the three-point function of the inflaton perturbation making the same approximations as we have made in our calculation. Namely [41], neglect metric perturbations and departures of the background expansion from pure de Sitter. For the potential

\[ V(\varphi) = V_0 - \frac{g}{6}\varphi^3 \]  

(B.1)

Falk et al find

\[ \langle \delta \varphi_{k_1}(\tau) \delta \varphi_{k_2}(\tau) \delta \varphi_{k_3}(\tau) \rangle \approx + \frac{2\pi^3}{3} g H^2 N_e \sum_{i=1}^3 \frac{k_i^3}{\Pi_i k_i^3} \delta^{(3)} \left( \sum_{i=1}^3 k_i \right). \]  

(B.2)

The factor of \( N_e \) in (B.2) arises from regulating a time dependence of the form \( \ln \tau \), exactly as in (4.13). We now use this result to estimate \( f_{NL} \) for the potential (B.1), showing that the answer agrees with a more careful calculation [3] up to factors of order unity. For simplicity we assume that \( g > 0 \) throughout this appendix and restrict ourselves to the case where \( \varphi \) rolls from the unstable point \( \varphi = 0 \) towards some positive value \( \varphi > 0 \).

We assume that \( V_0 \gg g\varphi^3 \) throughout inflation so that \( H \cong H_0 = \sqrt{V_0/\sqrt{3}M_p} \). The slow roll parameters, \( \epsilon = 2^{-1}M_p^2(V'/V)^2 \) and \( \eta = M_p^2V''/V \), evaluated at the time of horizon crossing \( t = t_* \), are

\[ \eta \approx -M_p^2 \frac{g\varphi^*}{V_0} \approx -\frac{g\varphi^*}{3H_0^2}, \]
\[ \epsilon \approx |\eta| \frac{g\varphi^3}{V_0} \ll |\eta|, \]

where \( \varphi^* = \varphi(t = t_*) \) is the value of the inflaton at horizon crossing. The spectral tilt is

\[ n_s - 1 = 2\eta - 6\epsilon \approx 2\eta. \]  

(B.3)

The slow roll Klein–Gordon equation

\[ 3H \dot{\varphi} \approx -V' \]  

(B.4)

has solution

\[ \varphi(N) \approx \varphi^* \left[ 1 - \frac{|\eta|}{2} N \right]^{-1}, \]  

(B.5)

where \( N = H_0(t - t_*) \) so that \( \varphi(N = 0) = \varphi^* \) is the inflaton value at horizon crossing. It is straightforward to see that

\[ \frac{1}{2M_p^2 H^2} \approx \eta^2 \frac{\varphi^2}{8M_p^2} \left( 1 - \frac{|\eta|}{2} N \right)^{-4} \]

so that inflation ends \( N_e \) e-foldings after horizon crossing when \( |\eta| N_e/2 \approx 1 \). It follows that \( |n_s - 1| N_e = \mathcal{O}(1) \).

We assume that

\[ \zeta = c_\zeta \delta \varphi = -\frac{H}{\varphi} \delta \varphi. \]  

(B.6)
The value of \( c_\zeta \) at horizon crossing is

\[
\left. c_\zeta \right|_{N=0} \simeq - \frac{1}{\eta} \frac{d\varphi}{dN} \bigg|_{N=0} \simeq - \frac{2}{\eta \varphi_*},
\]

where in the last equality we have used (B.5). The COBE normalization is

\[
A_\zeta^2 = c_\zeta^2 \left( \frac{H_0}{2\pi} \right)^2 = 25 \times 10^{-10}
\]

so that, using (B.7), we have

\[
A_\zeta^2 \simeq \frac{H_0^2}{\pi^2 \eta^2 \varphi_*^2}.
\]

Using (B.6) and (B.7) we can convert (B.2) into an estimate for the bispectrum of the curvature perturbation

\[
\langle \zeta_{k_1}(\tau) \zeta_{k_2}(\tau) \zeta_{k_3}(\tau) \rangle \simeq \frac{2(2\pi)^3}{3} \frac{gH_0^2}{\eta^3 \varphi_*^3} \sum_{i=1}^3 k_i^3 \Pi_{i=1}^3 k_i^3 \delta(3) \left( \sum_{i=1}^3 k_i \right).
\]

Comparing (B.10) to the WMAP ansatz (4.14) we obtain the following estimate for the non-linearity parameter

\[
|f_{NL}| \sim \frac{5}{24} (n_s - 1)^2 N_e,
\]

whereas the complete calculation [3] gives \( |f_{NL}| \sim (5/12)(n_s - 1) \). The estimate (B.11) reproduces the result of the complete calculation up to an order of magnitude since \( |n_s - 1| N_e = \mathcal{O}(1) \).

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