Numerical Study on Emission Characteristics of a Point Cathode Electron Gun: Determination of Space Charge using Random Emitting Conditions

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The emission characteristics of a point cathode thermionic electron gun have been studied with numerical method. The method is based on the integral form of Poisson equation. The field including the space charge is obtained by the iterative procedure, consisting of the field calculation, the electron ray tracing in 3D, and the determination of the space charge distribution. The space charge distribution was determined from the rays traced with random emitting conditions considering energy and angular distributions and treated as many coaxial ring charges. The use of random emitting conditions leads to a self-consistent result after several iterations. The emission currents were calculated for different bias voltages and different cathode temperatures. The method provides the emission currents close to the measured values. [DOI: 10.1380/ejssnt.2006.339]

Keywords: Electron emission; Thermionic emission; Space charge effects; Computer simulations; Field computation

I. INTRODUCTION

Electron emission is influenced by space charge, which is caused by emitted electrons themselves. The negative charge causes a decrease of the electric potential near the emitter and limits the emission current density. It also varies the field distribution acting as the immersion lens and the energy distribution of the emitted beam. Space charge is an important factor limiting the performance of electron guns and other electron emission systems. The study of the space charge effects in practical electrode geometry requires numerical calculation, because the space charge distribution is generally unknown. The space charge is estimated tracing the electron trajectories. Then, Poisson equation is solved with the iterative method to obtain the field [1]. The finite difference method (FDM) [2] and the finite element method (FEM) [3] are commonly utilized for the field calculation, where the space charge is treated as a negative potential on each mesh point or each element node.

We have studied the space charge effects in a tungsten point cathode gun to obtain a better understanding of the emission characteristics at high cathode temperatures or high emission current densities. The cathode has a sharply etched tip of a submicron radius of curvature, which can enhance the electric field on the tip and reduce the space charge effects in the gun. Experiments [4, 5] showed that the point cathode provides the beam brightness near the theoretical value even at a cathode temperature as high as 3100 K. The brightness is maximized at a bias shift of several volts from the cutoff. Such a small bias shift means that the emission is restricted to a very narrow area near the tip apex. The other area is surrounded with negative potential, which repels the electrons having low energies and results in an increase of the space charge density. The charge may affect the emission area and current as well as the field distribution acting as the immersion lens.

The numerical study of the point cathode gun was described by some authors [6–9], where the field was analyzed with the surface charge method based on the integral form of Laplace equation. The space charge was not taken into account. The reason that the method was used is that it can treat accurately the cathode geometry having a large difference in dimensions compared to the other electrodes. We have developed a numerical method based on the integral form of Poisson equation. In this method, the electrode geometry is treated in the same way as the surface charge method, and the space charge distribution is approximated with many coaxial charged rings with different radii and different axial positions. In the previous papers [10, 11], we described some details of the method and the results obtained with the space charge estimation from the rays traced for some fixed emitting conditions. The rays were started with the most probable emitting energy and some emitting angles. Because the emitting conditions were restricted, two additional calculations were needed in the estimation of the space charge, such as the replacement of each ring charge with a mean value and the multiplication of the weight factor to each ring charge. The former is needed to obtain convergence of the field calculations, and the latter to obtain the emission current near the measured value [12].

This paper describes the numerical analysis of the emission characteristics of the gun using random emitting conditions. It can be expected that the accuracy of the calculation is improved when the space charge distribution is estimated from a large number of the trajectories started considering the energy and angular distributions of thermally emitted electrons. The emitting energies and angles were given by random numbers and the trajectories over 9000 were traced for the determination of the space charge. It will be shown that the iterations combined with the determination of the space charge using random emitting conditions lead to a self-consistent result and give the emission currents close to the measured values with no additional calculations.

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FIG. 1: Electrode geometry of the electron gun. The gun consists of the point cathode, the negatively biased Wehnelt electrode and the anode. The cathode is a tungsten wire of 0.1 mm in diameter. The tip radius is 0.4 μm. The tip is placed at the height 0.25 mm from the Wehnelt aperture. The aperture hole is 1.2 mm in diameter. Distance between the cathode tip and the anode is 9.2 mm.

II. NUMERICAL METHOD

A. Electrode geometry and operating conditions

The numerical model of the electron gun is given in Fig. 1. The electrode geometry is that for the point cathode gun we have developed [9]. The electrodes are axial symmetry. The cathode is a tungsten filament of 0.1 mm in diameter and the tip is 0.4 μm in radius. The tip is set at a height of around 0.25 mm from the aperture of the Wehnelt electrode. The aperture size is 1.2 mm in diameter. The anode is placed at 9.2 mm from the cathode tip. The gun is operated at the anode voltage 50 kV and the cathode temperatures above 2800 K. The electron emission from the cathode tip is controlled by adjusting the negative bias on the Wehnelt electrode. Numerical analysis was carried out for such operating conditions.

B. Field computation

1. Basic expressions

The integral form of Poisson equation is expressed as

$$
\Phi(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \int_S \frac{\sigma(\vec{r}')}{|\vec{r} - \vec{r}'|} dS' + \frac{1}{4\pi\varepsilon_0} \int_V \frac{\rho(\vec{r}')}{|\vec{r} - \vec{r}'|} dV',
$$

where Φ(\vec{r}) is the potential at a position of \vec{r}, \varepsilon_0 is the dielectric constant of vacuum, \sigma(\vec{r}') is the charge density on the electrode surface S' and \rho(\vec{r}') is the space charge density surrounding the cathode. The electric field is given by the following equation.

$$
\vec{E}(\vec{r}) = -\frac{1}{4\pi\varepsilon_0} \int_S \sigma(\vec{r}') \nabla \left( \frac{1}{|\vec{r} - \vec{r}'|} \right) dS' - \frac{1}{4\pi\varepsilon_0} \int_V \rho(\vec{r}') \nabla \left( \frac{1}{|\vec{r} - \vec{r}'|} \right) dV'.
$$

2. Numerical expressions and iterative procedure

For the field calculation, the term for the surface charge density in Eqs. (1) and (2) was discretized, dissecting the electrode surfaces into small circular sections and putting the charge density on each section constant, in the same way as the surface charge method. The second term for the space charge density was discretized, dividing the space around the cathode into small ring-shape volume elements and approximating the space charge within each individual element with a coaxial charged ring located in the centre of it. When the position vector \vec{r}' puts on the i-th section of the electrode surface, the above discretiza-
tion gives the following form for Eq. (1),

\[
\Phi(\vec{r}_i) = \frac{1}{\pi \varepsilon_0} \sum_j \sigma_j F_{ij} + \frac{1}{4\pi \varepsilon_0} \sum_n q_n G_{in},
\]

(3)

where \(\Phi(\vec{r}_i)\) is the electrode potential on the \(i\)-th surface section, \(\sigma_j\) is the charge density of the \(j\)-th surface section, \(F_{ij}\) is a potential factor derived from the geometrical relation between the \(i\)-th and \(j\)-th surface sections, \(q_n\) is the \(n\)-th ring charge, \(G_{in}\) is the function including the radius and the \(z\)-position of the \(i\)-th surface section and those of the \(n\)-th ring charge [10].

Applying Eq. (3) to all surface sections, we derive a set of linear equations for the surface charge density \(\sigma\). Since the space charge is initially unknown, firstly the surface charge density in the absence of the space charge \((q_n = 0)\) is calculated by solving the linear equations at given electrode potentials. Then the electron trajectories are traced, where the electric field is calculated using the corresponding discretized form of Eq. (2). The space charge in each volume element is determined from the rays. The space charge causes a negative potential. So, the surface charge density is recalculated taking into account the space charge potential on each surface section. The recalculated charge density provides the potential and field distributions including the space charge, but the space charge is an estimated one in the Laplace field. To obtain a self-consistent result for Poisson equation, these calculations are repeated until the potential and field distributions become unchanged. The main portions of the iterative procedure consist of the following three calculations;

( a ) Determination of the charge density \(\sigma\) on the electrode surfaces using the electrode potential and the space charge potential,

( b ) Ray tracing in 3D,

( c ) Estimation of the space charge density \(\rho\) from the rays.

For the calculation of the surface charge density \(\sigma\), the electrode surface (Fig. 1) was divided into 200 circular sections. Each section was subdivided into 24 for the Gaussian quadrature in the numerical integration of the potential and the field. For the calculation of the space charge, the space around the cathode, in the range between \(z = -40 \mu m\) and \(z = 10 \mu m\) (the tip position \(z = 0\)), was divided into the width \(\Delta r = \Delta z = 0.2 \mu m\). The number of the ring charges required for the approximation of the space charge depends on the size of the volume element as well as the broadening of the traced rays. The broadening varies with the Wehnelt bias. At shallow bias conditions, the
emission area and the broadening of the traced rays are increased. In such a case, the number of the charged rings was over 50,000.

3. Random emitting conditions and ray tracing

The partial current density distribution for thermionic emission [13] gives the following expressions for the normalized energy and angular distributions:

\[ f(E)dE = \frac{E}{(kT)^2} \exp\left(-\frac{E}{kT}\right) dE, \]  
\[ g(\alpha)d\alpha = 2\sin \alpha \cdot \cos \alpha \cdot d\alpha, \]  

where \( E \) is the emitting energy, \( k \) Boltzmann constant, \( T \) the cathode temperature, and \( \alpha \) the emitting angle to the normal direction. In the azimuth direction \( \beta \), the elections are emitted uniformly. Considering these distributions, random numbers for the emitting energy \( E \) and the angle \( \alpha \) were generated using the rejection method [14]. The azimuths \( \beta \) from 0 to \( 2\pi \) were given by uniform random numbers. The seeds were varied at each starting position using the cpu-time.

From each starting position, fifty rays were started with \( (E, \alpha, \beta) \). The rays were traced by solving directly the equation of motion in 3D. The starting positions were put as follows. The cathode area of 35 \( \mu m \) in the axial-length from the tip apex was divided into 184 circular sections with different widths \( \Delta z \). The starting position was placed in the middle of each width. At the shallow bias conditions, the starting positions were increased to the cathode area of 50 \( \mu m \), and the area was divided into 259 circular sections. The equation of motion was normalized with time \( (\tau = \sqrt{\epsilon/mt}) \) and a set of differential equations [15] was solved using the Adams-Moulton (predictor-corrector) method, putting an initial integrating time-step \( (\Delta \tau) \) to \( 1 \times 10^{-7} \) and a tolerance for error control to \( 1 \times 10^{-4} \). These values were chosen to save the computing time. No difference was found in the trajectories, when the rays were traced with a time-step and a tolerance smaller than the above values.

4. Estimation of the space charge

The space charge in each individual volume element was estimated from all the rays passing through it using the relation of \( \Delta Q = i_c \Delta t \), where \( i_c \) is the current assigned to each ray and \( \Delta t \) is its transit time through the element. The current \( i_c \) was assigned as follows. The current \( \Delta I \) of the starting area was calculated using the Richardson-Dushman equation, where the Richardson constant and the work function were put to 80 A/cm\(^2\)K and 4.5 eV, respectively. These values are typical ones for polycrystalline tungsten cathode. The calculated current \( \Delta I \) was equally assigned to the fifty rays from each starting area \( (i_c = \Delta I/50) \).

III. RESULTS AND DISCUSSION

The iterations were done as monitoring the variations of the potential distribution and the emission current and area. The emission current was obtained by summing up the currents of the rays passing through the Wehnelt aperture. The emission area was estimated from the starting positions of these rays. Figure 2 shows an example of the variations of the emission current and area, observed in the iterations at the Wehnelt bias −203 V and the cathode temperature 2950 K. The emission current and area are decreased after the space charge is taken into account and the variations become small after 6 iterations, where the potential distribution around the cathode tip becomes unchanged. The iterations provide the emission current of 39 \( \mu A \). The determination of the space charge using the random emitting conditions gives the self-consistent result of Poisson equation with no additional calculations. The calculations showed that the number of iterations required for convergence depends on the bias voltage (the emission
FIG. 5: (a) Potential distribution without the space charge. 0.2 V step. Wehnelt bias $-214$ V and the cathode temperature 2950 K. (b) some samples of the trajectories traced for the space charge estimation. 920 rays are plotted. 50 $\mu$m $\times$ 50 $\mu$m area around the cathode tip.

FIG. 6: (a) Potential distribution including the space charge (after 10 iterations). 0.2 V step. Wehnelt bias $-214$ V and the cathode temperature of 2950 K. (b) some samples of the electron trajectories. 920 rays are plotted. 50 $\mu$m $\times$ 50 $\mu$m area around the cathode tip.

area) and the cathode temperature (the initial energy). A large number of iterations were needed at shallow bias conditions and at high cathode temperatures.

Figure 3 represents the histograms of the initial energies and angles of the rays used for estimating the space charge in the last iteration and the theoretical curves: (a) the distributions of 250 rays (5 starting positions) and (b) the distributions of 9200 rays (all rays). The generated random numbers give the distributions near the theoretical curves. The most probable energy $kT$ varies with the cathode temperature, which is 0.241 eV at 2800 K, 0.254 eV at 2950 K, and 0.267 eV at 3100 K.

To examine the emission characteristics of the gun, calculations were done for different bias voltages and different cathode temperatures. Figure 4(a) shows the calculated currents as a function of the Wehnelt bias. The cathode temperature was varied from 2800 K to 3100 K. The cut-off bias is about $-220$ V. The bias was varied from $-222$ V (the cutoff region) to $-198$ V. The range is enough for the analysis of the emission characteristics of the gun, because the beam brightness is maximized at the bias shift of around 5-6 V from the cutoff [9]. At the cathode temperature 3100 K, the calculations including the space charge provide the emission current of about 11 $\mu$A at the bias $-214$ V. When the bias is reduced to $-198$ V, the current is limited to 120 $\mu$A. These values are close to the measured ones. In the experiments [16], we observed that at a cathode temperature of 3100 K the
emission current remains a value of around 100 μA, when the bias is shifted by 20 V from the cutoff. At cathode temperatures of less than 3000 K, the current was limited to 40 μA at the same bias shift from the cutoff.

For a comparison, the emission currents in the absence of the space charge (before iteration) are given in Fig. 4(b). In this case, the emission current at 3100 K is over 250 μA at the bias −198 V. The discrepancy between the calculated and measured value is increased with the bias reduction and the temperature rise of the cathode. As can be expected, the currents in the case of no space charge are directly proportional to the current density of emission, 5.0 A/cm² at 2800 K, 14.3 A/cm² at 2950 K and 37.1 A/cm² at 3100 K. The comparison of Figs. 4(a) and (b) indicates that the space charge plays an important role in keeping the emission current small.

The emission current in Fig. 4(a) results from a decrease in the emission area due to the space charge. Figures 5 and 6 represent the potential distribution and some samples of the traced rays before and after iterations at the Wehnelt bias −214 V and the cathode temperature 2950 K. The equi-potential lines are 0.2 V steps, where the line intersecting with the cathode is the zero-volt line. In Fig. 5, the cathode area fronting the positive potential is about 6 μm in length from the tip apex. The cone part of the cathode is surrounded with negative potential, where most electrons are repelled within a short range. These electrons produce a dense space charge cloud. The ring charges are distributed along the traced rays. The potential distribution after 10 iterations is varied by the space charge as shown in Fig. 6(a). Due to the space charge, the zero-volt line is moved downward. The emission area becomes nearly perpendicular. The field strength at the tip apex is reduced by the space charge. It can be seen from Figs. 5(b) and 6(b) that the space charge also has an effect keeping the radial broadening of the rays small.

IV. CONCLUSIONS

The emission characteristics of the point cathode thermionic emission gun have been studied with the numerical method. The field distribution including space charge was obtained solving Poisson equation. The space charge distribution was determined from the electron trajectories traced with random emitting conditions considering energy and angular distributions of thermionic emission.

Calculations showed that the determination of the space charge using random emitting conditions leads to the self-consistent result for Poisson equation. The emission current was calculated for different bias voltages and different cathode temperatures, and the dependence of the emission current on the bias was examined. The method provides the emission currents close to the measured values. The method gives useful information about the space charge effects on the emission characteristics of the gun at high brightness operations.

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