Abstract

We consider waveguides formed by single or multiple two-dimensional chaotic cavities connected to leads. The cavities are chaotic in the sense that the ray (or equivalently, classical particle) dynamics within them is chaotic. Geometrical parameters are chosen to produce a mixed phase space (chaotic regions surrounding islands of stability where motion is regular). Incoming rays (or particles) cannot penetrate into these islands but incoming plane waves dynamically tunnel into them at a certain discrete set of frequencies (energies). The support of the corresponding quasi-bound states is along the trajectories of periodic orbits trapped within the cavity. We take advantage of this difference in the ray/wave behavior to demonstrate how chaotic waveguides can be used to design beam splitters and microlasers. We also present some preliminary experimental results in a microwave realization of such chaotic waveguide.

Key words: Beam splitter, Microlaser, Quantum chaos/Wave chaos
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1 Introduction

During the last years there has been a growing interest in the use of chaotic cavities as resonators for microlasers, mainly due to the possibility of quality factor tuning and its highly directional emission, see for example [1]. The key feature behind these characteristics is the cavity geometry, which is designed in most cases to produce mixed chaotic dynamics.

Here we propose a two-dimensional (2D) locally deformed waveguide that can be used as beam splitter or as (single or multi-cavity) resonator for microlasers. The novel and important feature of our model is that the cavities are open, unlike the well known model of Ref. 1. This difference allows for a wider versatility and range of applications. The splitting and lasing mechanism requires that the deformation yields particle (ray) motion in a mixed phase space. We remark that due to the equivalence between the problem of a TM wave inside a 2D waveguide with Dirichlet boundary conditions (Helmholtz equation) to that of a quantum wave in a 2D billiard (Schrödinger equation) [3], our results are applicable to electromagnetic as well as electronic setups.

2 The Model

The 2D waveguide we shall use consists of a cavity connected to two collinear semi-infinite leads of width $d$ extended along the $x$-axis. The prototype cavity has the geometry of the so-called cosine billiard extensively studied in [4,5]: it has a flat wall at $y = 0$ and a deformed wall given by $y(x) = d + a[1 - \cos(2\pi x/L)]$, where $a$ is the amplitude of the deformation and $L$ is the length of the cavity. In Fig. 1(a) we show the geometry of the cavity.

For any $a > 0$ the cavity develops a ternary horseshoe [2] that can be either incomplete (proper of mixed chaotic dynamics) or complete (prescribing global chaos), depending on the particular values of $(d, a, L)$.[6] When a waveguide is constructed with a cavity characterized by an incomplete horseshoe, its conductance fluctuates strongly with sharp resonances [4b,5]. The wave functions corresponding to the sharpest conductance resonances can be identified with energy eigenstates living in phase space stability islands, whence they are quasi bound states (QBS) [5]. Note that incoming trajectories cannot penetrate into the resonance (stability) island as motion in the islands correspond to trajectories trapped within the cavity. However, the wave function dynamically tunnels into the islands at the resonance values, allowed by Heisenberg’s uncertainty principle. All QBS have support on stability islands surrounding low period periodic orbits. In particular, for $(d, a, L) = (1.0, 0.305, 5.55)$ the QBS reveal I- and M-shaped patterns while for $(d, a, L) = (1.0, 0.5, 5.55)$ V- and
W-shaped patterns are observed. In Fig. 2 examples of I-, M-, V-, and W-type QBS are shown.

3 Discussion

3.1 Microlasers

If the waveguide described above is constructed with a semiconductor material having a refraction index $n$ and a wave is introduced into one of the leads with an energy corresponding to a sharp conductance resonance, the cavity acts as a laser resonator since light will bounce in the cavity region having the chance to escape only for certain incidence angles, which depend on the type of the QBS excited and the index of refraction of the waveguide. In Fig. 3 ray dynamics was used to predict the direction and intensity of the lasing produced by M- and I-type QBS for typical values of refraction index, showing both high quality and high directionality [7]. The great advantage of a waveguide-based resonator may be in that no pedestals or couplers (pumpers) close to the cavity are needed, as is the case with the currently investigated 3D and 2D micro lasers.
Fig. 2. Wave function density plots for the waveguide of Fig. 1(a), showing (a) M-, (b) I-, (c) W-, (d) Π-, and (e) V-type quasi-bound states. See [5,6,7] for the calculation details.

Fig. 3. Ray prediction of lasing emission for M- and I-type quasi-bound states.
3.2 Multi-cavity resonators

Our waveguide model can also be used to construct multi-cavity resonators formed, for example, by a co-linear array of coupled 2D chaotic cavities, c.f. Fig. 1(b). Here, as the number of cavities is increased most of the ray trajectories take longer to be transmitted or reflected; they oscillate irregularly around the widest parts of the coupled cavities. The dwelling of the rays in the outskirts of the resonance islands induces a higher probability of dynamical tunneling into the classical inaccessible regions. Thus, the trapping of scattering wave functions along bounded ray trajectories is enhanced. See in Fig. 4 an I-type QBS excited in the multi-cavity resonator of Fig. 1(b). Even though a multi-cavity resonators is of general interest in the field of optics, one may also consider the construction of a micro laser using a resonant multi-cavity. A multi-cavity micro laser would have two main advantages: (i) the multi-cavity array would enhance considerably the emission intensity compared to a micro laser constructed with one cavity only, and (ii) the micro laser would emit several parallel beams in one or more directions since I-, M-, V-, W-, or Π-type QBS can be excited by choosing the appropriate cavity geometry [9].

3.3 Beam splitters

Once one knows the type of QBS that can be excited for a given set of cavity parameters \((d, a, L)\), it is possible to construct electromagnetic or electronic beam splitters by attaching transversal leads to the waveguide. The transversal leads are prescribed as follows [9]: (i) they have to be located on the stability islands supporting the QBS, and (ii) their width must be small enough to preserve the global phase space structure. For example, Figs. 1(c1-c2), (c3), and (c5-c6) show beam splitter setups suitable for I-, M-, and V-type QBS, respectively. We still consider plane waves coming from the horizontal leads. Then, the scattering wave functions which tunnel into stability islands will be guided out of the cavity through the transversal leads. Fig. 1(c4) is an example of a beam splitter using both I- and M-type QBS, where the beam will be guided up or down depending on the excited QBS, as shown in Fig. 5. Obviously, one can also anticipate the construction of multi beam splitters, as suggested in Fig. 1(c7).
3.4 Experimental

We have recently performed measurements on the experimental realization of the cosine billiard in the microwave regime. Figure 6 shows the experimental setup. Not shown in the figure is a top metallic plate. The distance between the top and bottom plates, the resonator height $h$, is 0.8 cm. As long as the frequency $\nu$ is less than $c/2h = 18.75$ GHz ($c$ is the speed of light) the cavity is a 2D system and there is a one to one correspondence between the electric field strength and the quantum mechanical wave function (see [3]). The present setup includes three electric dipole antennas. The first one is fixed on the left lead, near the top left corner, at $(x, y) = (-18.0, 7.725)$ cm, the second one is fixed on the opposite side, at $(x, y) = (18.0, 7.725)$ cm. The third antenna is inside the cavity, scanning in small steps the whole cavity region (normally in steps of 2.5 mm) for each value of frequency in the range 1-18.75 GHz. A vector network analyzer is used to measure reflection in each of the antennas as well as transmission between them. Using the methods developed in [8] the modulus and sign of the wave function can be obtained from the transmission measurements, while the reflection measurement provides only the modulus of the wave function. In Fig. 6 we present a wave function obtained from the reflection measurements from the scanning antenna at $\nu = 13.7708$ GHz, a resonance frequency. Experimental results like this confirm the existence of
I-type QBS, corresponding to the theoretical predictions of Fig. 2(b), and forecast the realization of the proposed waveguide-based beam splitters and (multi-cavity) resonators.

Finally, as an additional note we want to mention that in the case of a waveguide with a cavity characterized by a complete horseshoe the conductance is in general a smooth function of the energy (or frequency). However, some non-generic complete horseshoes anticipate the appearance of scars [10], which in turn produce conductance resonances and II-type QBS [9]. See an example of a II-type QBS in Fig. 2(d) where \((d, a, L) = (1.0, 1.0, 5.55)\) was used. Then the construction of microlasers and beam splitters based on scared QBS could be also accomplished.

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