Modelling of dynamics of the suspension of tandem axles of a multi-axle vehicle provided with a novel load-equalising system

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Abstract: The paper presents the problem of using multi-axle suspension systems in vehicles, concerning unbalanced distribution of axle loads while moving in off-road conditions. An innovative load balancing system for tandem axles, which limits the occurring overloads, is presented. A mathematical model is proposed to represent the suspension of a front “quarter” of a four-axle vehicle provided with a novel system to equalise the loads on two closely spaced (tandem) axles. The “quarter” of the vehicle is described as a set of rigid bodies linked with each other by spring and damping elements. The model has three degrees of freedom and defines vehicle’s responses to excitations (inputs) generated by road irregularities. The mathematical models known from the literature describe typical suspension systems and specific types of load balancing systems for two- and three-axle vehicles. This makes it impossible to apply them directly to simulation tests of a vehicle with two front axles. A new model dedicated to the proposed load equalizing system, which allows simulation tests for the design and verification of the new system’s operation, has been prepared.

Keywords – vehicle, vehicle dynamics, suspension

1. Introduction

One of the crucial problems that arise from using in vehicles multi-axle suspension systems is the necessity to equalise the loads on all the vehicle axles when moving on uneven surfaces typical for unpaved roads or roadless tracts [1,2]. The loads on vehicle axles must be kept uniform to prevent axle overloads, which might result in damage to components of vehicle suspension or power transmission systems. Moreover, the correct distribution of loads on vehicle axles is important for the performance of vehicle braking and steering systems, which is critical for driving safety. The multi-axle vehicles’ suspension systems, where each of a pair of closely spaced axles, referred to as “tandem axles” or “twin axles”, is suspended separately from each other, do not ensure uniform distribution of axle loads, especially when the vehicle is negotiating separate and repeating ground bumps or pits. As an example, when the wheels of one of the axles run onto a high bump, the other axle may be suspended in mid-air. In such a situation, one of the tandem axles carries the entire static load normally carried by both of them [2, 3]. To overcome this problem, engineering solutions are used whose common feature is the
fact that the suspensions of each of the tandem axles are coupled together. Examples of the solutions of this kind, having the form of mechanical, hydraulic, or pneumatic load equalizing systems, can be found in the literature (Fig. 1).

Based on results of analyses of the good and bad points of the solutions used, a new concept of the system to equalise the loads on tandem axles has been proposed. A schematic of such a system has been shown in Fig. 2. The conventional arrangement of the front suspension system of a four-axle vehicle with rigid drive axles and leaf springs on each wheel has been developed by adding four hydraulic cylinders fixed to the vehicle frame over the axles next to the leaf springs and four other hydraulic cylinders pin-jointed in place of spring shackles. All the cylinders are of the single-acting type. The cylinders situated on one vehicle side are interconnected in such a way that the cylinder situated over the first axle is hydraulically linked with the one pin-connected to the end of the leaf spring of the second axle and the cylinder situated over the second axle is hydraulically linked with the one pin-connected to the end of the leaf spring of the first axle. There is some clearance between each axle and the piston rod of the cylinder situated over it. In the rest position, there is no overpressure in the hydraulic system, the pistons in the cylinders situated over the axles are fully extended, and the pistons in the cylinders situated at the leaf spring ends are fully retracted. In the schematic in Fig. 2, the system has been presented in its working position, without showing the damping elements.
Fig. 2. A concept of the multi-axle vehicle’s suspension with a hydraulic load equalising system [7]

The most important good points of the solution proposed include:

- extended range of system operation in comparison with the designs known hitherto;
- applicability to all the vehicles of this kind, whether newly manufactured or already being in service (after minor modifications in the design of the latter);
- ensured correct operation of the suspension in case of a failure of the equalising system;
- simple construction;
- elimination of permanent presence of high pressure in the hydraulic system.

The essence of the problem addressed in the scientific work undertaken lies in the modelling of the system proposed. During the tests of vehicle dynamics, simplified “quarter-vehicle” or “half-vehicle” models, described in numerous publications [8, 9, 10, 11, 12, 13, 14] are often used. Ready-to-use equations of the dynamics are used, substituting only parameters of spring and damping elements of the suspension and values of sprung and unsprung masses. In spite of the fact that the first load balancing systems for axles suspended on leaf springs appeared at the beginning of the 20th century, the use of these systems has not become widespread until now and no universal mathematical models for this type of systems have been developed. The simulation models described in the literature refer to two-axle vehicles in which the front and rear axle shock absorbers were replaced by double-acting hydraulic cylinders [15] and three-axle vehicles in which the suspension of all axles was connected hydraulically [16]. These models refer to suspension systems that use two cylinders per axle (one per wheel). Fig. 3 shows examples of models related to specific design solutions.

Fig. 3. „Half-vehicle” models with hydraulically interconnected suspension of:
  a) a two-axle vehicle, b) a three-axle vehicle [15, 16]
2. **Model of the suspension with a load equalising system**

To obtain effective operation of the system installed in a vehicle, many tests must be carried out both in laboratory conditions and in real driving tests. First, however, engineering computations and simulation tests of the system functioning are necessary for the prototype to be built. With this goal in view, a model of dynamics of the suspension of tandem axles using a load-equalising system was prepared. The model was developed to represent the suspension system of front tandem axles of a four-axle motor truck (Fig. 4), taken as an example.

![Fig. 4. Visualisation of the suspension system of tandem axles of an off-road truck [17]](image)

When building the model, the following simplifying assumptions were adopted:

- the sprung mass is a rigid solid;
- the vehicle frame remains in the horizontal position;
- the system has three degrees of freedom;
- only vertical movements are considered;
- the vehicle body has a longitudinal symmetry plane;
- the damping and stiffness characteristics are linear;
- the ground surface is undeformable;
- the impact of antiroll bars and rubber-metal elements is ignored;
- the tangent tyre-ground interactions are ignored;
- the gravitational components of the forces acting in the suspension system are ignored;
- the vehicle is driven along a rectilinear path with a constant velocity.

The physical model of the equalising system proposed has been presented in Fig. 5. Its components with definite physical properties include:

- unsprung masses $m_1$ [kg] and $m_2$ [kg] as the parts of the vehicle axle mass taken per a single wheel of the first and second axle, respectively;
- sprung mass $m_3$ [kg] as the part of the vehicle body mass acting upon the two wheels of the tandem axle that are situated on the same vehicle side;
- displacements $z_1$ [m] and $z_2$ [m] as the vertical displacements of the first and second axle, respectively, relative to the axles’ positions of static equilibrium;
- displacement $z_3$ [m] as the vertical displacement of the vehicle body relative to its position of static equilibrium;
clearance values $L_1$ [m] and $L_2$ [m] as the distances between the top surface of the first and second axle, respectively, and the end surface of the piston rod of the hydraulic cylinder situated over the corresponding axle;

damping values $c_1$ [N·s/m] and $c_2$ [N·s/m] as the values of the viscous damping of the first and second axle’s shock absorbers, respectively;

stiffness values $k_{11}$ [N/m] and $k_{12}$ [N/m] as the component stiffness values of the first axle’s leaf spring;

stiffness values $k_{21}$ [N/m] and $k_{22}$ [N/m] as the component stiffness values of the second axle’s leaf spring;

dry friction $T_1$ [N] and $T_2$ [N] between leaves of the springs of the first and second axle, respectively;

displacements $h_{11}$ [m] and $h_{21}$ [m] as the vertical displacements of pistons of the hydraulic cylinders situated over the first and second axle, respectively;

displacements $h_{12}$ [m] and $h_{22}$ [m] as the vertical displacements of pistons of the hydraulic cylinders situated at leaf springs’ ends;

areas $s_1$ [m²] and $s_2$ [m²] of hydraulic cylinders’ pistons;

leaf spring length $l$ [m] as the distance between the points of fastening the spring to the vehicle frame;

dimensions $a$ [m] and $b$ [m] as the distances between the axle and individual points of fastening the spring to the vehicle frame;

damping value $c_k$ [N·s/m] as the value of tyre damping;

stiffness value $k_k$ [N/m] as the value of tyre stiffness;

coordinate $q(t)$ [m] representing the kinematic input generated by the road profile.

For the model cleared of its constraints, a system of forces acting on individual masses of the model (Fig. 6) was identified, which included:

- elasticity forces $F_{S11}$ [N] and $F_{S21}$ [N] as the elasticity force components acting on the points of fastening the front ends of the first and second axle’s springs, respectively;
- elasticity forces $F_{S_{12}}$ [N] and $F_{S_{22}}$ [N] as the elasticity force components acting on the piston rods of the hydraulic cylinders at the rear ends of the first and second axle’s springs, respectively;
- damping forces $F_{t_{1}}$ [N] and $F_{t_{2}}$ [N] as the forces of viscous damping of the first and second axle’s shock absorbers, respectively;
- forces $F_{p_{1}}$ [N] and $F_{p_{2}}$ [N], exerted by the hydraulic cylinders situated over the first and second axle, respectively;
- forces $F_{T_{1}}$ [N] and $F_{T_{2}}$ [N], generated by dry friction between leaves of the springs of the first and second axle, respectively;
- elasticity forces $F_{sk_{1}}$ [N] and $F_{sk_{2}}$ [N] generated in tyres of the first and second axle, respectively;
- damping forces $F_{tk_{1}}$ [N] and $F_{tk_{2}}$ [N] generated in tyres of the first and second axle, respectively.

Fig. 6. System of forces acting in the equivalent model of suspension dynamics

Based on the system of forces acting in the model of suspension dynamics, equations of the dynamics of individual masses in the model have been formulated as follows, using the d’Alembert’s method:

$$m_{1} \cdot \ddot{z}_{1} = F_{sk_{1}} + F_{t_{k_{1}}} - F_{s_{11}} - F_{t_{1}} - F_{p_{1}} - F_{T_{1}} - F_{s_{12}}$$

$$m_{2} \cdot \ddot{z}_{2} = F_{sk_{2}} + F_{t_{k_{2}}} - F_{s_{21}} - F_{t_{2}} - F_{p_{2}} - F_{T_{2}} - F_{s_{22}}$$

$$m_{3} \cdot \ddot{z}_{3} = F_{s_{11}} + F_{t_{1}} + F_{p_{1}} + F_{T_{1}} + F_{s_{12}} + F_{s_{21}} + F_{t_{2}} + F_{p_{2}} + F_{T_{2}} + F_{s_{22}}$$

2.1. Leaf spring elasticity forces

In the model, the leaf spring has been represented as a system of two coil springs combined with a rigid horizontal beam, which can move in vertical direction only. For the first axle, the equations have the form as shown below:

$$F_{S_{11}} = u_{11} \cdot k_{11}$$

$$F_{S_{12}} = u_{12} \cdot k_{12}$$

$$k_{11} = k_{1} \cdot \frac{b}{l}$$
\[ k_{12} = k_1 \cdot \frac{a}{l} \]  \hspace{1cm} (7)

\[ F_{S1} = F_{S11} + F_{S12} \]  \hspace{1cm} (8)

where: \( k_i \) is stiffness of the leaf spring of the first axle and \( u_{i1} \) and \( u_{i2} \) are equivalent dynamic deflections [m] of the first and second coil spring of the first axle, determined from equations:

\[ u_{i1} = u_i \]  \hspace{1cm} (9)

\[ u_{i2} = z_i - z_3 + h_{i2} \]  \hspace{1cm} (10)

where: \( u_i \) is deflection [m] of the first axle’s suspension, determined from an equation:

\[ u_i = z_i - z_3 \]  \hspace{1cm} (11)

and, simultaneously:

\[ h_{i2} = h_{2i1} \cdot i_h \]  \hspace{1cm} (12)

where: \( i_h \) is hydraulic ratio:

\[ i_h = \frac{s_i}{s_2} \]  \hspace{1cm} (13)

\[ for \quad u_2 \leq L_2 \quad h_{2i1} = 0 \]  \hspace{1cm} (14)

\[ for \quad u_2 > L_2 \quad h_{2i1} = u_2 - L_2 \]  \hspace{1cm} (15)

and: \( u_2 \) is deflection [m] of the second axle’s suspension, determined from an equation:

\[ u_2 = z_2 - z_3 \]  \hspace{1cm} (16)

For the second axle, the equations have the form:

\[ F_{S21} = u_{21} \cdot k_{21} \]  \hspace{1cm} (17)

\[ F_{S22} = u_{22} \cdot k_{22} \]  \hspace{1cm} (18)

\[ k_{21} = k_2 \cdot \frac{b}{l} \]  \hspace{1cm} (19)

\[ k_{22} = k_2 \cdot \frac{a}{l} \]  \hspace{1cm} (20)

\[ F_{S2} = F_{S21} + F_{S22} \]  \hspace{1cm} (21)

where: \( k_2 \) is stiffness of the leaf spring of the second axle and \( u_{21} \) and \( u_{22} \) are equivalent dynamic deflections [m] of the first and second coil spring of the second axle, determined from equations:

\[ u_{21} = u_2 \]  \hspace{1cm} (22)

\[ u_{22} = z_2 - z_3 + h_{22} \]  \hspace{1cm} (23)

and, simultaneously:

\[ h_{22} = h_{11} \cdot i_h \]  \hspace{1cm} (24)

\[ for \quad u_1 \leq L_1 \quad h_{11} = 0 \]  \hspace{1cm} (25)

\[ for \quad u_1 > L_1 \quad h_{11} = u_1 - L_1 \]  \hspace{1cm} (26)
2.2. Damping forces in the viscous damper

\[
F_{t1} = \dot{u}_1 \cdot c_1 \\
\dot{u}_1 = z_1 - \dot{z}_3 \\
F_{t2} = \dot{u}_2 \cdot c_2 \\
\dot{u}_2 = \dot{z}_2 - \dot{z}_3
\]

(27)  
(28)  
(29)  
(30)

where: \( \dot{u}_1 \) and \( \dot{u}_2 \) are suspension deflection velocities [m/s] of the first and second axle, respectively; \( \dot{z}_1 \) and \( \dot{z}_2 \) are displacement velocities [m/s] of the first and second axle, respectively; and \( \dot{z}_3 \) is vehicle body displacement velocity [m/s].

2.3. Forces exerted by the hydraulic cylinders situated over the axles

The interaction between the leaf spring and the hydraulic cylinder is only possible when the clearance between the axle and the piston rod end is reduced to zero, i.e. when these parts come into contact with each other, which is described as follows:

\[
for \ u_1 \leq L_1 \quad F_{p1} = 0 \\
for \ u_1 > L_1 \quad F_{p1} = F_{s22} \cdot i_h
\]

(31)  
(32)

and

\[
for \ u_2 \leq L_2 \quad F_{p2} = 0 \\
for \ u_2 > L_2 \quad F_{p2} = F_{s12} \cdot i_h
\]

(33)  
(34)

2.4. Forces generated by dry friction between spring leaves

\[
F_{d1} = \text{sig}(\dot{u}_1) \cdot T \\
F_{d2} = \text{sig}(\dot{u}_2) \cdot T
\]

(35)  
(36)

2.5. Forces coming from vehicle tyres

\[
F_{sk1} = (q_1 - z_1) \cdot k_k \\
F_{sk2} = (q_2 - z_2) \cdot k_k \\
F_{tk1} = (q_1 - \dot{z}_1) \cdot c_k \\
F_{tk2} = (q_2 - \dot{z}_2) \cdot c_k
\]

(37)  
(38)  
(39)  
(40)

Unlike the well-known models, in which the leaf spring is replaced by a single spring element, the prepared model uses two spring elements located at the ends of a horizontal beam, the combined stiffness of which corresponds to the leaf spring stiffness. The value of the force, exerted by the hydraulic cylinder situated over the axle, depends on the deflection of the spring and the load value of the second axle. The mathematical model includes a variable for the clearance between the hydraulic cylinder and the axle to ensure that the system only acts when it is needed, i.e. when a specified spring deflection limit is exceeded. Once the model has been implemented into a computational environment such as MATLAB Simulink, it will be possible to carry out a number of simulation tests for different loading scenarios.
The results of the simulation tests will be used to select the parameters of the load equalizing system. In the next stage, experimental studies are planned to verify the model.

3. Conclusion

A load equalising system is required to ensure a balanced load distribution on tandem axles of multi-axle vehicles operating in off-road conditions. These systems are not commonly used, and the available mathematical models are not universal and their application is limited to specific construction solutions. The development of a new concept of the load equalising system required the preparation of a mathematical model, taking into account the characteristic features of the system. The equivalent model described herein and representing a “quarter” of a four-axle vehicle provided with the innovative system to equalise the loads on tandem vehicle axles may be a tool for the carrying out of simulation tests aimed at the preparation of data for prototype designing and construction purposes. It may also be used to check whether the system functioning ensures the obtaining of the effects expected, i.e. an adequate reduction in the dynamic loads on vehicle axles during drives in off-road conditions. At the next stage of the work, the model developed must be verified by comparing the simulation results with results of experimental testing of the vehicle when driven in real off-road conditions.

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