An Intuitionistic Fuzzy Pseudo Enlarged Ideal of a BH-Algebra

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Abstract. In this work, the concepts of an intuitionistic fuzzy pseudo ideal of a pseudo BH-algebra are introduced. Several propositions and examples are scrupulous to study properties of this idea.

Keywords. BH-Algebra, Pseudo BH-Algebra, intuitionistic fuzzy pseudo ideal in pseudo BH-algebra, intuitionistic enlarged ideal in pseudo BH-algebra.

1. Introduction:
The algebraic design named BCK-algebra & BCI-algebra a generality of BCK-algebra are come in by K. ISEKI and Y. IMAI in 1966[2]. In 1998 Y. B. Jun, et al show the idea of a BH-algebra [8]. Furthermore, Y.B Jun, et al introduce the idea of a pseudo BH-algebra in 2015[8]. In 2017, A.H. Nouri and H.H. Abbass thought about some kinds of ideals of pseudo BH-algebra [9]. The most writer deem the year 1965 is the starting of a fuzzy logic when L. A. Zadeh knew a subset in fuzzy sets [1]. In 1991 Xi. O. thought in BCK-algebra a fuzzy sense [10]. Ever after then, the researchers have on a comprehensive scale, fuzzy ideals about an element of pseudo BH-algebra defined by A. A. Mutesher & H. H. Abbass[11]. H.H. Abbass & H.A. Dahham offer a fuzzy completely closed ideal of BH-algebra in 2012[5]. A fuzzy closed ideal relies on an element in BH-algebra thought by H. M. A. Saeed & H. H. Abbass in 2011 [7], we intuitionistic fuzzy if pseudo ideal and pseudo enlarged ideal in a pseudo BH-algebra.

2. Preliminaries.
In this work, several basic connotations about a BH-algebra, ideal in BH-algebra, intuitionistic enlarged ideal in BH-algebra, pseudo ideal pseudo BH-algebra, intuitionistic fuzzy ideal in BH-algebra are given.

2.1. Definition
A set X is not equal to ∅ with a dual operation (*) and a constant 0 is named a BH-algebra if achieved:
∀ μ, λ ∈ X
• μ * μ = 0
• μ * λ = 0 and λ * μ = 0 ⇒ μ = λ
• μ * 0 = μ

2.2. Definition
Assume that S ≠ ∅ is a subset of a BH-X is named a BH-subalgebra of X signify by BH-S if μ * λ ∈ S, ∀ μ, λ ∈ S.
2.3. Definition
Assume that $I \neq \emptyset$ and a subset of a BH-$X$. Therefore $I$ is named an ideal of $X$ if that was achieved: $\forall \mu, \lambda \in X$
- $0 \in I$.
- $\mu \cdot \lambda \in I$ and $\lambda \in I \Rightarrow \mu \in I$.

2.4. Definition
Assume that $I \neq \emptyset$ and a subset of a P.BH-algebra $X$ and there is no need an ideal of $X$, a subset $J$ of $X$ is named an Enlarged ideal of $X$ related to $I$, and signify by $E$. $I$ if that was achieved: for every $\mu, \lambda \in X$
- $I$ is a subset of $J$
- $0 \in J$
- $\mu \cdot \lambda \in I$ and $\lambda \in I \Rightarrow \mu \in J$.

2.5. Definition
A pseudo BH indicates $P.BH$ is a set $X$ is not equ $\emptyset$ with a fixed 0 and dual operations $\ast, \#$ check the next conditions:
- $\mu \ast \mu = \mu \# \mu = 0, \forall \mu \in X$
- $\mu \ast \lambda = 0 \& \lambda \# \mu = 0 \Rightarrow \mu = \lambda, \forall \mu, \lambda \in X$
- $\mu \ast 0 = \mu \# 0 = \mu, \forall \mu \in X$.

2.6. Definition
Assume that $S \neq \emptyset$ is a subset of a P.BH-$X$ is named a P.BH-subalgebra of $X$ signify by P.BH-$S$ if that was achieved: $\mu \ast \lambda$ and $\mu \# \lambda \in S, \forall \mu, \lambda \in S$.

2.7. Definition
Assume that $I \neq \emptyset$ and subset of a P.BH-$X$. Therefore $I$ is named a pseudo ideal & signify by $P. I$ of $X$ if achieved: $\forall \mu, \lambda \in X$
- $0 \in I$.
- $\mu \ast \lambda, \mu \# \lambda \in I$ and $\lambda \in I \Rightarrow \mu \in I$.

2.8. Definition
Assume that $I \neq \emptyset$ and subset of a P.BH-algebra $X$, a subset $J$ of $X$ is named a pseudo Enlarged ideal of $X$ related to $I$, and signify by $P. E. I$ if that was achieved: $\forall \mu, \lambda \in X$
- $I$ is a subset of $J$
- $0 \in J$
- $\mu \ast \lambda \in I, \mu \# \lambda \in I$ and $\lambda \in I \Rightarrow \mu \in J$.

2.9. Definition
Assume $X$ that is a non-empty set, fuzzy subset $\omega, \sigma$ in $X$ are a formula from $X$ into $[0, 1]$ of the real number.

2.10. Definition
Assume that $A$ is an intuitionistic fuzzy set in $X$, shortened by $I. F. S$ and the set $U(\omega, \alpha) = \{ \mu \in X : \omega_A(\mu) \geq \alpha \}$ is named upper $\alpha$ -level cut of $A$ and $L(\sigma, \alpha) = \{ \mu \in X : \sigma_A(\mu) \leq \alpha \}$ is named lower $\alpha$ - level cut of $A$.

2.11. Definition
Assume $A = (\omega_A(\mu), \sigma_A(\mu)) \& B = (\omega_B(\mu), \sigma_B(\mu))$ are $I. F. S$ in $X : \forall \mu \in X$
- $(A \cup B)(\mu) = \{< \mu, \max (\omega_A(\mu), \omega_B(\mu)), \min (\sigma_A(\mu), \sigma_B(\mu)) > | \mu \in X \}$
- $(A \cap B)(\mu) = \{< \mu, \min (\omega_A(\mu), \omega_B(\mu)), \max (\sigma_A(\mu), \sigma_B(\mu)) > | \mu \in X \}$
A U B & A \cap B are I. F. S in X, \forall \mu \in X in broadly, if \( \{ A_i, i \in \Omega \} \) be a chain of intuitionistic sets in X

\[(\cap A_i)(\mu) = (\inf \omega_{A_i}(\mu), \sup \sigma_{A_i}(\mu))\]

\[(\cup A_i)(\mu) = (\sup \omega_{A_i}(\mu), \inf \sigma_{A_i}(\mu))\]

Which are too I. F. S in X.

3. The Main Results
In the work, is defined the concepts of intuitionistic fuzzy pseudo enlarged ideal in P.BH-algebra. for our conversation, we will study the advantages of these concepts.

3.1. Definition
Assume A & B are two I. F. S of a BH-algebra X, so that A \subseteq B then B is named intuitionistic fuzzy enlarged ideal of X related to A & signify by I. F. E. I if that was achieved :

- \( \omega_B(0) \geq \omega_B(\mu) \) & \( \sigma_B(0) \leq \sigma_B(\mu) \), \forall \mu \in X.
- \( \omega_B(\mu) \geq \min \{ \omega_A(\mu \ast \lambda), \omega_A(\lambda) \} \), \forall \mu, \lambda \in X.
- \( \sigma_B(\mu) \leq \max \{ \sigma_A(\mu \ast \lambda), \sigma_A(\lambda) \} \), \forall \mu, \lambda \in X.

3.2. Example
Assume that X = \{0, k, v, h\} is a BH-algebra with the next cayley tables :

\[
\begin{array}{cccc}
* & 0 & k & v & h \\
0 & 0 & 0 & 0 & 0 \\
k & k & 0 & 0 & k \\
v & v & v & 0 & v \\
h & h & h & h & 0 \\
\end{array}
\]

Define A = (\( \omega_A(\mu), \sigma_A(\mu) \)), B = (\( \omega_B(\mu), \sigma_B(\mu) \)) are two I. F. S of X by

\[
\omega_A(\mu) = \begin{cases} 0.5 & \text{if } \mu = 0, h \\ 0.4 & \text{if } \mu = k, v \end{cases}
\]

\[
\sigma_A(\mu) = \begin{cases} 0.2 & \text{if } \mu = 0 \\ 0.4 & \text{if } \mu = k, v, h \end{cases}
\]

\[
\omega_B(\mu) = \begin{cases} 0.6 & \text{if } \mu = 0, k \\ 0.5 & \text{if } \mu = v, h \end{cases}
\]

\[
\sigma_B(\mu) = \begin{cases} 0.2 & \text{if } \mu = 0, v \\ 0.3 & \text{if } \mu = k, h \end{cases}
\]

Then B is an I. F. E. I of X related to A.

3.3. Definition
Assume A & B are an I. F. S of a BH-algebra X so that A \subseteq B then B is named intuitionistic fuzzy pseudo enlarged ideal of X related to A, signify by I. F. P. E. I if that was achieved : \forall \mu, \lambda \in X

- \( \omega_B(0) \geq \omega_B(\mu) \) & \( \sigma_B(0) \leq \sigma_B(\mu) \)
- \( \omega_B(\mu) \geq \inf \{ \omega_A(\mu \ast \lambda), \omega_A(\mu \# \lambda), \omega_A(\lambda) \} \)
- \( \sigma_B(\mu) \leq \sup \{ \sigma_A(\mu \ast \lambda), \sigma_A(\mu \# \lambda), \sigma_A(\lambda) \} \)
3.4. Example
Assume that $X = \{0, k, v, h\}$ is a P.BH with the following Cayley tables:

| * | 0 | k | v | h |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| k | k | 0 | 0 | k |
| v | v | v | 0 | v |
| h | h | h | h | 0 |

Define $A = (\omega_A(\mu), \sigma_A(\mu))$, $B = (\omega_B(\mu), \sigma_B(\mu))$ are two I. F. S of $X$ by

$\omega_A(\mu) = \begin{cases} 0.5 & \mu = 0, k \\ 0.4 & \mu = v, h \end{cases}$

$\sigma_A(\mu) = \begin{cases} 0.2 & \mu = 0 \\ 0.3 & \mu = k, v, h \end{cases}$

$\omega_B(\mu) = \begin{cases} 0.6 & \mu = 0 \\ 0.5 & \mu = k, v, h \end{cases}$

$\sigma_B(\mu) = \begin{cases} 0.1 & \mu = 0, h \\ 0.3 & \mu = k, v \end{cases}$

Then $B$ is an I. F. P. E. I of $X$ related to $A$.

3.5. Theorem
Assume that $\{B_i \mid i \in \Omega\}$ is a family of I. F. P. E. I of a P.BH-algebra $X$ related to $A$. Then $\cap_{i \in \Omega} B_i$ is an I. F. P. E. I of $X$ related to $A$.

Proof:

- Assume that $\mu \in X, i \in \Omega, \omega_{B_i}(0) \geq \omega_{B_i}(\mu) \Rightarrow \inf_{i \in \Omega} \omega_{B_i}(0) \geq \inf_{i \in \Omega} \omega_{B_i}(\mu) \Rightarrow \omega_{\Lambda B_i}(0) \geq \omega_{\Lambda B_i}(\mu)$.
- Let $\mu \in X, i \in \Omega, \sigma_{B_i}(0) \leq \sigma_{B_i}(\mu) \Rightarrow \sup_{i \in \Omega} \sigma_{B_i}(0) \leq \sup_{i \in \Omega} \sigma_{B_i}(\mu) \Rightarrow \sigma_{\Lambda B_i}(0) \leq \sigma_{\Lambda B_i}(\mu)$.

3.6. Theorem
Assume that $X$ is a P.BH-algebra. $A = (\omega_A, \sigma_A)$ & $B = (\omega_B, \sigma_B)$ are two I. F. S of $X$, such that $A \subseteq B$ then $B$ is an I. F. P. E. I of $X$ related to $A \leftrightarrow$ the set upper level $U(\omega_B, \sigma_B)$ is P. E. I of $X$ related to $U(\omega_A, \sigma_A)$ or empty of $X$, $\forall \alpha_1 \in [0,1]$ and the set lower level $L(\sigma_B, \alpha_2)$ is P. E. I of $X$ related to $L(\sigma_A, \alpha_2)$ or empty of $X$, $\forall \alpha_2 \in [0,1]$. 


Proof.- Let $B = (\omega_B, \sigma_B)$ be an I. F. P. E. I of $X$ related to $A$ & $U (\omega_B, \alpha_1) \neq L (\sigma_B, \alpha_2)$, for every $\alpha_1, \alpha_2 \in [0,1]$. Obviously $0 \in U (\omega_B, \alpha_1) \land L (\sigma_B, \alpha_2)$ since $\omega_B(0) \geq \alpha_1$ & $\sigma_B(0) \leq \alpha_2$. Assume $\mu, \lambda \in X$ such that $\mu \# \lambda \in U (\omega_B, \alpha_1)$ & $\lambda \in U (\omega_A, \alpha_1)$ Then $\omega_A(\mu \# \lambda) \geq \alpha_1$, $\omega_A(\mu \# \lambda) \geq \alpha_1$, and $\omega_A(\lambda) \geq \alpha_1$.

Therefore, $\inf \{ \omega_A(\mu \# \lambda), \omega_A(\mu \# \lambda), \omega_A(\lambda) \} \geq \alpha_1$, but $\omega_B(\mu) \geq \alpha_1 \Rightarrow \omega_B(\mu) \geq \alpha_1 \Rightarrow \omega_B(\mu) \geq \alpha_1$ & $\omega_B(\mu \# \lambda) \geq \alpha_1$ & $\omega_B(\lambda) \geq \alpha_1$.

Let $\omega_B(\mu) = \alpha_1$ & $\omega_B(\mu \# \lambda) = \alpha_2$ then $\mu \# \lambda \in L (\sigma_B, \alpha_2) \land \lambda \in L (\sigma_B, \alpha_2)$ then $\sigma_B(\mu \# \lambda) \leq \alpha_2$, $\sigma_B(\mu \# \lambda) \leq \alpha_2$ and $\sigma_B(\lambda) \leq \alpha_2$, therefore, $\sup \{ \sigma_B(\mu \# \lambda), \sigma_B(\mu \# \lambda), \sigma_B(\lambda) \} \leq \alpha_2$ but $\sigma_B(\mu) \leq \sup \{ \sigma_B(\mu \# \lambda), \sigma_B(\mu \# \lambda), \sigma_B(\lambda) \}$ since $B$ is an I. F. P. E. I of $X$ related to $A$ previously, $\sigma_B(\mu) \leq \alpha_2 \Rightarrow \mu \in U (\omega_B, \alpha_2)$

Then $L (\sigma_B, \alpha_2)$ is an P. E. I of $X$. Conversely, assume that $\alpha_1, \alpha_2 \in [0,1]$ and $U (\omega_B, \alpha_1) \land L (\sigma_B, \alpha_2)$ are P. E. I of $X$ related to $U (\omega_B, \alpha_1) \land L (\sigma_B, \alpha_2)$ respectively, $\forall \mu \in X$.

Let $\omega_B(\mu) = \alpha_1$ & $\omega_B(\mu) = \alpha_2$ then $\mu \in U (\omega_B, \alpha_1) \land L (\sigma_B, \alpha_2) \land U (\omega_B, \alpha_1) \neq L (\sigma_B, \alpha_2)$ & $\omega_B(\mu) \neq \emptyset$ since $U (\omega_B, \alpha_1) \land L (\sigma_B, \alpha_2)$ Hence $\omega_B(0) \geq \alpha_0 = \omega_B(\mu)$ & $\omega_B(0) \geq \alpha_0 = \omega_B(\mu)$, $\forall \mu \in X$, we take the opposite. Let $u, v \in X$ such that $\omega_B(\mu) \leq \inf \{ \omega_A(u \# v), \omega_A(u \# v), \omega_A(v) \}$, now let $\alpha_3 = \frac{1}{2}(\omega_B(u) + \inf \{ \omega_A(u \# v), \omega_A(u \# v), \omega_A(v) \}$, then $\omega_B(u) < \alpha_3 < \inf \{ \omega_A(u \# v), \omega_A(u \# v), \omega_A(v) \}$.

Hence $u \notin U (\omega_B, \alpha_3)$, $u \neq v$, $u \# v \in U (\omega_B, \alpha_3)$ and $v \in U (\omega_A, \alpha_3)$ then $U (\omega_B, \alpha_3)$ is not P. E. I. And let $k, h \in X$ such that $\sigma_B(k) \sup \{ \sigma_A(k \# h), \sigma_A(k \# h), \sigma_A(h) \}$, now let $\alpha_3 = \frac{1}{2}(\sigma_k(h) + \sup \{ \sigma_A(k \# h), \sigma_A(k \# h), \sigma_A(h) \}$ then $\sup \{ \sigma_A(k \# h), \sigma_A(k \# h), \sigma_A(h) \} < \alpha_3 < \alpha_3$.

Hence $k \neq h, k \# h \in L (\sigma_A, \alpha_4)$, and $h \in L (\sigma_A, \alpha_4)$, but $k \notin L (\sigma_A, \alpha_4)$, then $L (\sigma_A, \alpha_4)$ is not P. E. I. This is impossible from the assumption, therefore, $B = (\omega_B, \sigma_B)$ is an I. F. P. E. I of $X$ related to $A$.

3.7. Remark

Assume that $A = (\omega_A, \sigma_A)$ is an I. F. S of $X$ then the mappings $\hat{A} = (\hat{\omega}_A, \hat{\sigma}_A)$ is define as follows $\hat{\omega}_A(\mu) = \omega_A(\mu) + 1 - \omega_A(0)$ and $\hat{\sigma}_A(\mu) = \sigma_A(\mu) - \sigma_A(0)$.

3.8. Theorem

Assume that $X$ is a P.BH such that $\hat{B}$ is an I. F. S of $X$ so that $\omega_B(0) = \omega_A(0) & \sigma_B(0) = \sigma_A(0)$, then $B$ is an I. F. P. E. I of $X$ related to $\hat{A}$. Suppose $B$ is an I. F. P. E. I of $X$ related to $A$ & $\mu \in X \Rightarrow \omega_B(\mu) \geq \omega_A(\mu)$, $\omega_B(0) \geq \omega_A(0)$ & $\sigma_B(\mu) \leq \sigma_A(\mu)$, $\sigma_B(0) \leq \sigma_A(0)$

Hence $\omega_B(\mu) = \omega_A(\mu) + 1 - \omega_A(0)$ & $\omega_B(\mu) = \omega_A(\mu) + 1 - \omega_A(0)$.

Then $\omega_B(\mu) \geq \omega_A(\mu) + 1 - \omega_A(0)$ & $\omega_B(\mu) \geq \omega_A(\mu) + 1 - \omega_A(0)$.

\[ \inf \{ \omega_A(\mu \# \lambda), \omega_A(\mu \# \lambda), \omega_A(\lambda) \} + 1 - \omega_A(0) \]

\[ \inf \{ \omega_A(\mu \# \lambda) + 1 - \omega_A(0), \omega_A(\mu \# \lambda) + 1 - \omega_A(0), \omega_A(\mu \# \lambda) + 1 - \omega_A(0) \} \]

\[ \omega_B(\mu) \geq \inf \{ \omega_A(\mu \# \lambda), \omega_A(\mu \# \lambda), \omega_A(\lambda) \} \]

\[ \omega_B(\mu) \geq \inf \{ \omega_A(\mu \# \lambda), \omega_A(\mu \# \lambda), \omega_A(\lambda) \} \]
\[ \leq \sup \{ \sigma_A(\mu \ast \lambda), \sigma_A(\mu \# \lambda), \sigma_A(\lambda) \} - \sigma_A(0) \]
\[ \leq \sup \{ \delta_A(\mu \ast \lambda) - \sigma_A(0), \sigma_A(\mu \# \lambda) - \sigma_A(0), \sigma_A(\lambda) - \sigma_A(0) \} \]
\[ \leq \sup \{ \sigma_A(\mu \ast \lambda), \sigma_A(\mu \# \lambda), \sigma_A(\lambda) \} \Rightarrow \]
\[ \sigma_B(\mu) \leq \sup \{ \sigma_A(\mu \ast \lambda), \sigma_A(\mu \# \lambda), \sigma_A(\lambda) \} \]

Thence \( \bar{B} \) is an I. F. P. E. I of \( X \) related to \( \bar{A} \). Conversely, assume that \( \bar{B} \) is an I. F. P. E. I of \( X \) related to \( \bar{A} \) & \( \mu \in X \)
\[ \omega_B(\mu) \geq \omega_A(\mu) \Rightarrow \omega_B(0) \geq \omega_A(0) \]
\[ \omega_B(\mu) + 1 - \omega_B(0) \geq \omega_A(\mu) + 1 - \omega_A(0) \]
\[ \omega_B(\mu) + 1 - \omega_B(0) \geq \omega_A(\mu) (\mu) + 1 - \omega_A(0) \]

[since \( \bar{B} \) is an I. F. P. E. I of \( X \) related to \( \bar{A} \)] therefore, \( [\omega_B(\mu) \geq \omega_A(\mu)] \) & \( \sigma_B(\mu) \leq \sigma_A(\mu) \Rightarrow \sigma_B(0) \leq \sigma_A(0) \)
\[ \sigma_B(\mu) = \sigma_B(0) \& \sigma_A(\mu) = \sigma_A(0) \]
\[ \sigma_B(\mu) \leq \sigma_A(\mu) \Rightarrow \sigma_B(0) \leq \sigma_A(0) \]

[since \( \bar{B} \) is an I. F. P. E. I of \( X \) related to \( \bar{A} \)] therefore, \( [\sigma_B(\mu) \leq \sigma_A(\mu)] \). Now
i. \( \omega_B(0) = \omega_B(0) - 1 + \omega_B(0) \geq \omega_B(\mu) - 1 + \omega_A(0) = \omega_B(\mu) \Rightarrow \)
\[ [\omega_B(0) \geq \omega_B(\mu)] \] for every \( \mu \in X \& \)
\[ \sigma_B(0) = \sigma_B(0) + \sigma_B(0) \leq \delta_B(\mu) + \sigma_B(0) = \sigma_B(\mu) \Rightarrow [\sigma_B(0) \leq \sigma_B(\mu)] \]

ii. \( \omega_B(\mu) = \omega_B(\mu) - 1 + \omega_B(0) \geq \inf \{ \omega_A(\mu \ast \lambda), \omega_A(\mu \# \lambda), \omega_A(\lambda) \} - 1 + \omega_A(0) \)
\[ \geq \inf \{ \omega_A(\mu \ast \lambda) - 1 + \omega_A(0), \omega_A(\mu \# \lambda) - 1 + \omega_A(0), \omega_A(\lambda) - 1 + \omega_A(0) \} \]
\[ \geq \inf \{ \omega_A(\mu \ast \lambda), \omega_A(\mu \# \lambda), \omega_A(\lambda) \} \]
\[ \omega_B(\mu) \geq \inf \{ \omega_A(\mu \ast \lambda), \omega_A(\mu \# \lambda), \omega_A(\lambda) \} \]

iii. \( \sigma_B(\mu) = \sigma_B(0) \leq \sup \{ \delta_B(\mu \ast \lambda), \delta_A(\mu \# \lambda), \delta_A(\lambda) \} + \sigma_A(0) \}
\[ \leq \sup \{ \delta_B(\mu \ast \lambda) + \sigma_A(0), \delta_A(\mu \# \lambda) + \sigma_A(0), \delta_A(\lambda) + \sigma_A(0) \} \]
\[ \leq \sup \{ \sigma_B(\mu \ast \lambda), \sigma_A(\mu \# \lambda), \sigma_A(\lambda) \} \Rightarrow \]
\[ \sigma_B(\mu) \leq \sup \{ \sigma_B(\mu \ast \lambda), \sigma_A(\mu \# \lambda), \sigma_A(\lambda) \} \]

Then \( \overline{B} \) is an I. F. P. E. I of \( X \) related to \( \overline{A} \).

4. Conclusion
In this work, the ideas (I.P.I & I.P.E.I & I.F.P.I) of a P.BH-algebra are offered. Moreover, the consequences are studied in idiom of the relationship with an I.P.E.I, I.F.P.I & I.F.P.E.I of a P,BH-algebra.

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