That which we call *private*

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**Abstract**

The guarantees of security and privacy defenses are often strengthened by relaxing the assumptions made about attackers or the context in which defenses are deployed. Such relaxations can be a highly worthwhile topic of exploration—even though they typically entail assuming a weaker, less powerful adversary—because there may indeed be great variability in both attackers’ powers and their context.

However, no weakening or contextual discounting of attackers’ power is assumed for what some have called “relaxed definitions” in the analysis of differential-privacy guarantees. Instead, the definitions so named are the basis of refinements and more advanced analyses of the worst-case implications of attackers—without any change assumed in attackers’ powers.

Because they more precisely bound the worst-case privacy loss, these improved analyses can greatly strengthen the differential-privacy upper-bound guarantees—sometimes lowering the differential-privacy epsilon by orders-of-magnitude. As such, to the casual eye, these analyses may appear to imply a reduced privacy loss. This is a false perception: the privacy loss of any concrete mechanism cannot change with the choice of a worst-case-loss upper-bound analysis technique. Practitioners must be careful not to equate real-world privacy with differential-privacy epsilon values, at least not without full consideration of the context.

**1 Introduction**

“*What’s in a name? That which we call a rose
By any other name would smell as sweet.*”

Romeo and Juliet (II, ii, 1–2)

Alongside greatly increased adoption of machine learning (ML), its privacy aspects have seen increased attention, both offensively [17, 18, 4, 20] and defensively [1, 13, 14, 15]. In this, the gold-standard definitions of differential privacy (DP) have rightly played a key role [7]. In particular, in industry, DP techniques are being used to enable training of high-utility models that preserve the privacy of training data [9, 2, 5, 15].

While DP provides rigorous privacy guarantees, they take the form of analytic upper bounds that hold equally true for worst-case, artificial adversarially-crafted scenarios as they do for real-world ML applications. As a result, these upper-bound DP guarantees can be very loose (i.e., overly pessimistic) and the actual privacy loss in real-world applications may be many orders-of-magnitude lower than what is indicated by DP guarantees; this is especially true in the analysis of ML models trained using DP stochastic gradient descent (DP-SGD) [1]. In addition, the same model may be subjected—without any change or retraining—to different DP analyses that give different upper bounds, making DP guarantees even harder to understand.

In particular, a casual reader of the study by Jayaraman and Evans in USENIX Security 2019 might conclude that “relaxed definitions of differential privacy” should be avoided, because they “increase the measured privacy leakage” in the empirical study of DP machine-learning models [11].

In this note, we demonstrate that this study is consistent with a different interpretation. Namely, the “relaxed definitions” are strict improvements and provide orders-of-magnitude tighter guarantees without changing the real-world privacy loss. We also reproduce the results of Jayaraman and Evans [11], extend the bounds on privacy leakage defined by Yeom et al. [20], and demonstrate empirically a tighter connection between privacy leakage and differential privacy bounds.

**2 An Apparent Paradox?**

The privacy of ML models trained by iterative DP-SGD with clipping and Gaussian noise [1] can be analyzed via naive [6] or advanced [8] composition theorems. They may also be analyzed via more sophisticated and refined definitions such as zero-Concentrated DP (zCDP) [3] and Rényi DP (RDP) [12].

For a fixed $\varepsilon$ upper-bound DP guarantee, Jayaraman and Evans train models that achieve the target $\varepsilon$ (irrespective of utility) under each of the above definitions; subsequently, they measure the models’ empirical privacy loss as the success rate of a variant of the membership inference attack in [20]. They find that the empirical attacks have higher success probability for models associated with refined definitions (zCDP and RDP). This result feels like a paradox, as zCDP...
and RDP were developed to provide \textit{stronger} privacy guarantees incorporating tighter and more advanced techniques than prior works.

This apparent paradox is resolved as follows. A model trained with DP-SGD has some (unknown) fixed actual privacy guarantee as well as (known) fixed utility. Such an existing model’s empirical privacy cannot be changed by its re-analysis under refined definitions (we can only aim to get tighter upper bounds) and this is consistent with the study’s results \cite{11}. Conversely, simpler DP definitions have (orders-of-magnitude) greater gaps between the upper-bound \(\varepsilon\) and actual privacy loss whereas empirical measurements are tied only to the actual privacy loss. By training models to a fixed upper-bound \(\varepsilon\) under different DP definitions, the models’ actual privacy loss will vary wildly (as will their utility). We should expect the lowest empirical attack success rate (and the lowest utility) for the simplest DP definitions’ models which is borne out by the study \cite{11}.

### 3 A New Interpretation

| \(\varepsilon\) | (cross-entropy loss, count of attack success) |
|---------------|-----------------------------------------------|
|               | Naïve | Advanced | zCDP | RDP               |
| 1             | (.93, 0) | (.93, 0) | (.88, 0) | (.51, 122)         |
| 10            | (.90, 0) | (.87, 0) | (.47, 157) | (.09, 329)         |
| 100           | (.48, 152) | (.53, 138) | (.08, 362) | (.00, 456)         |

Consider this subset of Table 5 in Jayaraman and Evans \cite{11} where pairs denote cross-entropy loss and count of attack successes at 5% FPR respectively.

On the diagonal, in yellow, the empirical privacy loss can be seen to be approximately the same for models with the same accuracy, as predicted by the above discussion.

Training loss is a proxy for the effective noise added during DP-SGD training; therefore, we can “rotate the table” to fix the noise (or loss) for each row and reformat the table to show \(\varepsilon\) upper bound, count of attack successes at 5% FPR).

\[ \approx \]

| Loss | Naïve | Advanced | zCDP | RDP |
|------|-------|----------|------|-----|
| .93  | (1, 0) | (1, 0)   | (.1, 0) | (.05, 0) |
| .65  | (50, 16) | (50, 73) | (5, 45) | (.5, 27) |
| .50  | (100, 152) | (100, 138) | (10, 157) | (1, 122) |

From this reformatted table, it is obvious that—as expected—the more advanced DP analysis provides orders-of-magnitude tighter upper-bound guarantees, without changing the empirical privacy loss.

### 4 Reproducing Results

We reproduce a subset of the results of Jayaraman and Evans \cite{11}. In our first set of results, we picked \(\varepsilon\)-value targets for training a Logistic Regression classifier for the Purchase-100 dataset and evaluated two aspects: (1) the resulting training accuracy relative to the baseline non-private training, and (2) the success probability of the membership inference attack from Yeom et al. \cite{20} comparing cross-entropy loss of the input to the average loss over the training set to classify the input as train or test. We present the results of (2) at a 5% false positive rate, as in \cite{11}.

| \(\varepsilon\) | (training accuracy rel. to baseline, attack TPR) |
|---------------|-----------------------------------|
|               | Naïve | Advanced | RDP               |
| 1             | (2%, 5.1%) | (4%, 5.2%) | (14%, 5.6%)       |
| 10            | (3%, 5.1%) | (12%, 5.6%) | (71%, 7.8%)       |
| 100           | (11%, 5.9%) | (34%, 6.1%) | (92%, 8.1%)       |

We note a couple of changes. First, we do not report values where \(\varepsilon\) was targeted under a zCDP analysis. In the stochastic gradient descent setting of training models with Gaussian noise, zCDP and RDP definitions are identical and the results reported in \cite{11} are simply weaker bounds that ignore how sampling affects the privacy bounds.

Second, we present much higher success rates for counts of attack success because we report this in a slightly different manner. Jayaraman and Evans report the number of consistently recovered members across five trials at 5% false positive rate for each trial. We simply report the average number of recovered members across these trials. While the former gives an intuitive look into the efficacy of these attacks, the latter enables us to translate the true TPR and FPR rates into lower-bound estimates for \(\varepsilon\)-DP values of the trained model (see Section 5 for more information).

Similarly, in Table \cite{11} we reproduce the results that go into the reformatted table from Section 3 by keeping the noise that goes into training as the constant across which both analysis upper bounds and attacks are compared. Once again, we conclude that advanced DP analyses provide significant improvements in the upper-bound guarantees while the empirical privacy loss, as measured by attacks, remains the same.

### 5 Epsilon Bounds from Attacks

A model trained with \((\varepsilon, \delta)\)-DP guarantee mitigates membership inference attacks. Yeom et al. \cite{20} showed that the advantage in a membership inference game (defined equivalently as the difference between the TPR and FPR of the attack) can be bounded above as a function of \(\varepsilon\). We get a tighter estimate (as a function of \(\varepsilon\) and \(\delta\)) below.

Equipped with this tighter bound, we can translate a membership inference attack with some success advantage to a lower bound on the \emph{actual} \(\varepsilon\) of the trained model. In other words, \textit{if} the model had an \(\varepsilon\) guarantee lower than this lower bound, the membership inference adversary cannot have such
a high success probability. This allows us to consider a uniform picture of such an evaluation on differentially privately trained models—the best analysis provide the tightest upper bounds on the actual $\epsilon$, while the best attacks translate to the tight lower bounds on the actual $\epsilon$. We can then conclude that the actual privacy guarantee of the trained model lies somewhere in between.

5.1 A tighter upper bound

We strengthen the bound in [20, Theorem 1] by starting with the following proposition from Hall et al. [10, Proposition 4] paraphrased.

**Proposition 1.** Let $Y \sim \mathcal{M}(D)$ be the output of a mechanism over a dataset satisfying $\epsilon, \delta$-$\text{DP}$. Any test of the hypothesis that a particular $x \in D$ on input $Y$ with a false positive rate $\alpha$ has a true positive rate bounded above by $e^\epsilon \cdot \alpha + \delta$.

Applying this, we can derive the following proposition in a straightforward manner.

**Proposition 2.** The membership inference attack advantage of an adversary against a model trained with $\epsilon, \delta$-$\text{DP}$ is bounded above by $1 - e^{-\epsilon} + \delta \cdot e^{-\epsilon}$.

**Proof.** Following the lines of the proof in [20, Theorem 1], the membership advantage is equal to TPR $-$ FPR. Let $\alpha$ denote the FPR and $(1 - \beta)$ denote the TPR. Proposition 1 implies that $1 - \beta \leq e^{-\epsilon} \cdot \alpha + \delta \implies \alpha \geq e^{-\epsilon} (1 - \beta) - \delta \cdot e^{-\epsilon}$.

Therefore, the membership advantage, $(1 - \beta) - \alpha$ is less than or equal to $(1 - \beta) - e^{-\epsilon} (1 - \beta) + \delta \cdot e^{-\epsilon}$, which is equal to $(1 - e^{-\epsilon}) (1 - \beta) + \delta \cdot e^{-\epsilon}$. The proposition follows by noting that $\beta \geq 0$ and $e^{-\epsilon} \leq 1$.

5.2 Translating bounds

We translate the stronger upper bound on attack advantage to a lower bound on the actual $\epsilon$ of the trained model in a straightforward manner by rearranging terms of Proposition 2.

**Proposition 3.** For a given value of $\delta$, a noisily trained model admitting a membership inference attack, as defined by Yeom et al. [20] with advantage $\gamma$ can only be $(\epsilon, \delta)$-$\text{DP}$ for

$$\epsilon \geq \log \left( \frac{1 - \delta}{1 - \gamma} \right).$$

We can reproduce a subset of Table I with attack-derived lower bounds noting that the actual privacy of the model lies somewhere between the best upper-bound estimate and this lower bound. (We also note that the lower bounds are loose as they are computed at FPR of 5%, which does not necessarily translate to the best possible membership inference advantage.)

| Noise mult. | Accuracy | Attack TPR | $\epsilon$ at $\delta = 10^{-5}$ |
|-------------|-----------|------------|---------------------------------|
| Naive       | Advanced  | RDP        |
| 136.67x     | 1.9%      | 5%         | 4.07                            |
| 13.76x      | 7.0%      | 5.3%       | 51.2                            |
| 6.99x       | 14%       | 5.6%       | 125                             |
| 1.06x       | 71%       | 7.8%       | 8421                            |
| 0.44x       | 92%       | 8.9%       | 44770                           |

Table 1: Reproduced and re-oriented results on training models with noise.

| Noise mult. | $\epsilon$-lower bound | $\epsilon$-upper bound |
|-------------|-------------------------|------------------------|
| RDP         | Naive                   | Advanced               |
| 136.67x     | 0                       | 0.05                   |
| 13.76x      | 0.003                   | 0.5                    |
| 6.99x       | 0.006                   | 1                      |
| 1.06x       | 0.04                    | 10                     |
| 0.44x       | 0.05                    | 100                    |

Table 2: Translated $\epsilon$-lower bounds and their corresponding upper bounds for various noise scales.

5.3 Region of privacy

We combine the lower bound from Proposition 3 on the best possible membership inference advantage (by choosing a threshold that maximizes TPR $-$ FPR) and the upper bound from the best possible analysis (RDP) to produce the graph below. The graph plots both bounds across differentially-privately trained models with noise varying over several orders of magnitude. The region between the two curves constitute the “region of privacy” where the actual privacy of the model lies. By improving the privacy analysis of models and getting stronger attacks which imply better lower bounds, we can narrow this region and get a more accurate estimate of how private the model is.
5.4 Discussion

We conclude by noting important subtleties related to deriving epsilon bounds from attacks. Throughout this discussion, we make an implicit assumption about the training and test data. An attacker’s advantage as defined in Yeom et al. [20] being zero in an ideal scenario requires that the training and test distributions themselves be indistinguishable. Trivially, even a model trained with very good privacy properties might still yield a large attack advantage if the distributions are sufficiently different. In fact, one might be able to artificially construct pathological examples of good machine learning models trained with good accuracy/privacy tradeoffs that trivially fail these tests because, say, all train images start with a 0 pixel, and all test images start with a 1 pixel.

These tests shed a more accurate light on the privacy of the model if we simultaneously endeavor to make sure that the train and test distributions are as close as possible. This is closely related to similar work in the broader ML community exploring the connection between model generalization and artefacts in the train/test sets such as in the case of MNIST [19] and CIFAR [16].

A second subtlety we do not address in this work is the choice of threshold when applying the attacks from Yeom et al. [20]. One straightforward way to choose this would be to consider average loss, as suggested in the original paper, but we could strengthen the attacks by considering the best possible threshold that yields the largest attack advantage. While this sort of arbitrary auxiliary information (albeit minimal) would break the rigorous nature of the analysis of $\varepsilon$-lower bounds, we still deem the resulting lower bound useful in practice for two reasons—the present upper bound analyses are loose, and the attacks mounted are fairly simplistic and therefore fairly loose lower bounds themselves. Strengthening lower bounds a little bit with arbitrary auxiliary information will not lead to inaccurate results.

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