Photonic crystal waveguide for second harmonic generation: exact solution

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Abstract. We provide the exact solution of the dispersion equation for a waveguided semiconductor structure with a photonic crystal core that provides the efficient second harmonic generation in the visible range. The comparison of the results with an approximate solution shows good agreement.

1. Introduction
Recently, much attention has been paid to the development of optical waveguides with controlled dispersion. These we designed by introduction of spatial inhomogeneities into such structures [1-2]. The second harmonic generation of optical radiation in the visible range is one of the most promising applications of structures with controlled light dispersion being of high demand in biomedicine, laser television etc. [3,4]. However, the efficiency of such converters is limited due to material dispersion of the refractive index. Material dispersion leads to a phase mismatch between the fundamental mode and second harmonic. So, the most crucial criterion for the efficiency of nonlinear light conversion is the coherence length L – the distance on which the phase difference between the fundamental mode and second harmonic approaches the value of π. On the distance over L, the second harmonic intensity begins to decrease. In modern frequency doublers based on periodical polarization of ferroelectric crystals, the coherence length is of the order of tens of microns [5].

We propose a device that converts pump radiation to the second harmonic and provides a coherence length at a centimeter scale. A similar structure was described by us earlier in [5-6]. However, in those works, the dispersion was calculated by approximate methods. In this paper, the dispersion equation's exact solution is given, showing a good agreement with the previously obtained results.

In this work we consider a structure that consists of a photonic crystal core located between two semiconductor cladding layers with refractive indices \(n_i\). The considered photonic crystal is a periodic structure, containing \(p\) pairs of alternating semiconducting layers with refractive indices \(n_a\) and \(n_b\) and with thicknesses \(a\) and \(b\) correspondingly.

2. Calculation of eigenmodes dispersion
An approximate approach to the dispersion characteristics calculation is described in [6-8]. The exact calculation of the dispersion characteristics is based on the analysis of the Helmholtz equation:

\[
\Delta E + \frac{\omega^2}{c^2} \epsilon(\omega, z) E = 0
\]
with standard boundary conditions - continuity of the field amplitude and its derivative at the interfaces. The solution of this equation has the form \( E(x) = C_1 \cdot \exp(kx) \) for the area 1; 
\[ E(z) = C_1 \cdot \exp(ik_1z) + C_2 \cdot \exp(-ik_1z) \] for areas 2, 4, 6 etc.; 
\[ E(z) = C_3 \cdot \exp(ik_3z) + C_4 \cdot \exp(-ik_3z) \] for areas 3,5,7 etc.; 
\[ E(z) = C_n \cdot \exp(-k_n(z-h)) \] for area \( n+1 \). Here \( h = p(a+b) \) is the thickness of the structure. The areas 1 and \( (n+1) \) are waveguide claddings, the areas with even numbers are semiconductor layers of thickness \( a \), the odd numbered areas are layers with thickness \( b \).

The dispersion equation for such a system is

\[ J_s(\lambda, \beta) = 0 \quad (1) \]

where \( J_s(\lambda, \beta) = \det(K) \) is the determinant of the matrix \( K \) of coefficients in a system of \( s = 2(p-1) \) equations for the field amplitude and its derivate, \( \beta \) is an effective refractive index. Thus, the dispersion calculation is reduced to a calculation of the determinant of the order \( s \). It is a strip determinant containing nonzero elements along the main diagonal. Expanding the determinant by the elements of its rows and columns, one can obtain a recurrence system that allows to calculate \( J_s \).

The solution of equation (1) determines the dependence of the waveguide effective refractive index \( \beta \) on the radiation wavelength \( \lambda \). Then, the coherence length can be determined as follows

\[ L = \frac{c}{\omega} \left| \frac{\pi}{\beta_1^2 - 2\beta} \right| \geq 10 \text{mm} . \]

The other important parameter determining the efficiency of nonlinear conversion is group velocity dispersion GVD. It can be calculated as:

\[ \text{GVD}(s^2/m) = -\frac{\lambda^2}{2\pi c} \cdot \frac{(\beta_2 - \beta_1)}{c \cdot \lambda^2} \cdot 10^9, \]

where \( \lambda^* \) is the pump radiation wavelength in nanometers.

3. Discussion of results

Figure 1 shows the coherence length as a function of the thickness of the intrinsic conduction layer. The considered structure contains a waveguide with AlGaN cladding and a core of 9 pairs of alternating layers of AlN intrinsic conductivity and AlN, doped to the level of surface concentration \( N_s = 3.79 \cdot 10^{19} \text{cm}^{-2} \). The pumping wavelength is 1100 nm. It can be seen from Figure 1 that while the structure has an acceptable sensitivity to changes in the thickness of the dielectric layers (the width of the peaks at the level \( L = 1 \text{ cm} \) is just several nanometers), the coherence length can exceed 1 cm. The peak positions calculated by exact and approximate methods differ by no more than 10 nm.

Figure 2 shows the calculated dependence of the coherence length and the group velocity dispersion on the thickness of the intrinsic conductivity layer, calculated by the exact method. It follows from the calculations that the group velocity dispersion does not exceed 0.0005 ps\(^2\)/m and crosses zero at \( a = 1.254 \mu\text{m} \), where the coherence length becomes infinity. Thus, the proposed waveguide structure suppresses the dispersion of the refractive index and provides an efficient second harmonic-generation.
Figure 1. The dependence of the coherence length $L$ on the thickness $a$ of layers of intrinsic conductivity in a photonic crystal calculated by the exact (red squares) and approximate (green squares) approaches.

Figure 2. The dependence of the coherence length $L$ and the group velocity dispersion GVD on the thickness $a$ of the intrinsic conductivity layer.
4. Conclusions
The exact approach for the calculation of the light dispersion in a waveguide with a photonic-crystal core region is presented. The calculation based on the Maxwell's equations solution in a medium with periodically varying optical density. It is shown that a proper choice of waveguide parameters can achieve zero group velocity dispersion between the fundamental wave with a length of 1100 nm and the second harmonic with a wavelength of 550 nm. In this case, the coherence length exceeds 1 cm. Thus, the considered photonic crystal waveguide is an efficient device for frequency doubling, capable of generating coherent radiation in the optical range from near-infrared pump radiation.

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