Spectral fluctuations effects on conductance peak height statistics in quantum dots

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Within random matrix theory for quantum dots, both the dot’s one-particle eigenlevels and the dot-lead couplings are statistically distributed. While the effect of the latter on the conductance is obvious and has been taken into account in the literature, the statistical distribution of the one-particle eigenlevels is generally replaced by a picket-fence spectrum. Here we take the random matrix theory eigenlevel distribution explicitly into account and observe significant deviations in the conductance distribution and magnetoconductance of closed quantum dots at experimentally relevant temperatures.

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The universal statistical fluctuations observed at the low-energy part of the spectrum of quantum systems whose associated classical dynamics are chaotic can be described by random matrix theory (RMT). This type of description can be justified for diffusive quantum dots and quantum dots with irregular shapes [1] which makes quantum dots a particular example for the study of RMT fluctuations. While in open quantum dots (which have a strong dot-lead coupling) the electron-electron interaction effects are mostly neglected, in closed quantum dots this interaction leads to the Coulomb blockade (see [2] for a review): The low-temperature conductance is heavily suppressed due to the large charging energy for adding an electron to the quantum dot, except for the Coulomb blockade peaks at which the potential of the quantum dot is adjusted such that \( N \) and \( N + 1 \) electrons in the dot have the same energy. The RMT approach [1, 4] successfully described the mesoscopic fluctuations of these Coulomb blockade peaks, i.e., the statistical distribution of their height \( \langle G^{\max} \rangle \) and its dependence upon magnetic field [1]. On the other hand, recent improved experiments show significant deviations from the RMT prediction, suggesting that interaction effects beyond charging should be considered as well. In particular, dephasing of the single-particle states due to interactions modifies the conductance peak height statistics (see [1, 2] and references therein). In a recent experiment, Patel et al. [3] found that the statistical distribution has a smaller ratio of standard deviation to mean peak height \( \sigma(G^{\max})/\langle G^{\max} \rangle \) than predicted by RMT [4] and attributed this to dephasing effects. In another experiment, Folk et al. [5] measured the change of the conductance in a magnetic field \( B \)

\[
\alpha = \frac{\langle G^{\max} \rangle_{B \neq 0} - \langle G^{\max} \rangle_{B = 0}}{\langle G^{\max} \rangle_{B = 0}},
\]

as a probe of dephasing times. This is the closed dot analog of the weak localization magnetoconductance which had proven to be an effective measure for open dots dephasing times [6]. It was pointed out that \( \alpha = 1/4 \) long as the transport is dominated by elastic scattering [7, 8]. Therefore, any deviation of the measured \( \alpha \) from 1/4 was considered an indication for dephasing.

In this paper we discuss the effects of spectral fluctuations of the RMT one-particle eigenlevels on the statistical distribution \( P(G^{\max}) \) and the weak localization correction \( \alpha \). Previous works [1, 4, 7, 10, 11] have generally considered a picket fence spectrum, i.e., a rigid level spacing between successive eigenlevels in the quantum dot, for the calculation of the conductance. This ignores the effect of spectral eigenlevel fluctuations. The picket fence spectrum is a good approximation for both very high temperatures and very low temperatures [4], and a comparison of \( P(G^{\max}) \) with full RMT statistics and a picket fence spectrum without spin-degeneracy at three temperatures showed only minor deviations [12].

In the present paper, we study the full RMT statistics in detail with and without spin-degeneracy, and find significant differences compared to the picket fence spectrum, in particular in an experimentally relevant regime \( k_B T < \Delta \). The spectral fluctuations lead to lower values of \( \alpha \) than 1/4 such that this value is not universal, even in the absence of any dephasing mechanism. One therefore has to be careful while using \( \alpha \) as a probe for dephasing in this temperature regime.

Within the constant interaction model, the conductance of a quantum dot is given by the formula [3]

\[
G = \frac{e^2}{kT} \sum_{p=1}^{\infty} \frac{\Gamma_i^L \Gamma_i^R}{\Gamma_i^L + \Gamma_i^R} P_{eq}(N) P(E_i[N][1 - f(E_i - \mu)])
\]

where \( \Gamma_i^{L(R)} \) is the tunneling rate between the ith one-particle eigenlevel of the dot and the left (right) lead, \( P_{eq}(N) \) is the equilibrium probability to find \( N \) electrons in the dot with the Coulomb blockade allowing for \( N \) and \( N + 1 \) electrons, \( P(E_i[N]) \) is the canonical probability to have the level \( i \) occupied given the presence of \( N \) electrons in the dot, and \( f(E) \) is the Fermi function at the effective chemical potential \( \mu \), which includes the charging energy. In a typical experimental situation, the charging
energy is much large than temperature, and thus only one term contributes to the sum over $N$. In Eq. (3), $\Gamma_i^{L(R)}$ is Porter-Thomas distributed in the Gaussian orthogonal ensemble (GOE) and Gaussian unitary ensemble (GUE) without and with a magnetic field, respectively, and the eigenlevel energies $E_i$ obey the respective RMT distribution [4]. In contrast, the picket fence spectrum has $E_{2i} = E_{2i-1} = i\Delta$ in the case of spin-degeneracy and $E_i = i\Delta/2$ without spin-degeneracy. The first term in the sum $\Gamma_i^L\Gamma_i^R/(\Gamma_i^L + \Gamma_i^R)$ depends only on the eigenfunctions of the dot, and thus is uncorrelated with the spectrum within the RMT approach. The ensemble average of this term in the absence (GOE) or presence (GUE) of a magnetic field is

$$\left\langle \left( \frac{\Gamma_i^L\Gamma_i^R}{\Gamma_i^L + \Gamma_i^R} \right) \right\rangle = \begin{cases} 1/4 & \text{GOE} \\ 1/3 & \text{GUE} \end{cases}.$$ (3)

This yields the value $\alpha = 1/4$ if the weights $P(E_i|N)$ are the same for both ensembles. This should be the case in the low temperature regime $k_B T \ll \Delta$ since only one level $E_0$ contributes with maximal weight, $P(E_i|N) \approx \delta_{i0}$. In general, the main contribution to the sum comes from $O(k_B T/\Delta)$ levels around the Fermi energy which gives the same contribution at large temperatures $k_B T \gg \Delta$ for the GOE and GUE, $\alpha = 1/4$ in this regime as well.

However, for $k_B T \lesssim \Delta$, the probability to have more than one level in an energy window $k_B T$ around the Fermi energy is increased for the RMT eigenlevel distribution compared to the picket-fence spectrum. These additional levels enhance the conductance. Since there are more close-by levels for the GOE case, due to the weaker level repulsion, the GOE conductance is enhanced more, and $\alpha$ is suppressed.

A second important effect is the optimization of the chemical potential for the Coulomb blockade peak. This effect was generally ignored, as it is technically cumbersome to consider, and is not significant for both very low and very high temperatures. Disregarding this effect means that a theorist optimized the chemical potential w.r.t. the averaged conductance, instead of optimizing for every realization as in the experiment. Whenever there is a close-by level, the position of the peak is shifted to optimize the contribution from both levels. Typically, a level with very low tunneling rates (and, thus, suppressed conductance peak) would get enhanced significantly by contributions from its neighbors. If the tunneling rate of a neighboring level is much higher, the peak position $\mu_{\text{max}}$ is shifted towards it. As the distribution of level spacings is different depending on the existence of magnetic field, this enhancement mechanism is again more effective in the absence of magnetic field (GOE), where probabilities of small spacing and of small conductances are higher. Thus, this effect which was neglected in [2] suppresses $\alpha$ even further.

We evaluated the sum (3) numerically by drawing $\Gamma_i^{L(R)}$ from the Porter-Thomas distribution and $E_i$ according to the Wigner-Dyson distribution. Levels within a window of $\pm 4k_B T$ around the Fermi energy have been taken into account and the Fermi energy $\mu$ in Eq. (3) has been adjusted to yield $G_{\text{max}}$ for every realization.

Figure 1 compares the probability distribution $P(G_{\text{max}})$ for a picket fence spectrum vs. the full RMT level statistics. As explained above, RMT spectral fluctuations enhance the conductance. In particular, the probability to have a very low $G_{\text{max}}$ is reduced and the probability to have an intermediate $G_{\text{max}}$ is enhanced. The rea-

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**FIG. 1:** Probability distribution $P(g)$ of the dimensionless closed dot conductance $g$ defined by $G_{\text{max}}^g = 2 \sum_{k_B T} g$ at $k_B T = 0.2\Delta$ in the presence of spin-degeneracy (left: GOE; right: GUE; solid line: RMT spectral fluctuations; dashed line: picket fence)

**FIG. 2:** Magnetoconductance $\alpha$ vs. $k_B T/\Delta$ for the spin-degenerate case (dashed line) and without spin-degeneracy (solid line). Taking into account the RMT spectral fluctuations, $\alpha$ is reduced from its “universal” value $\alpha = 1/4$, in particular in the experimental relevant regime $0.1\Delta < k_B T < 0.8\Delta$. 

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1. There is a close-by level, the position of the peak is shifted to optimize the contribution from both levels. Typically, a level with very low tunneling rates (and, thus, suppressed conductance peak) would get enhanced significantly by contributions from its neighbors. If the tunneling rate of a neighboring level is much higher, the peak position $\mu_{\text{max}}$ is shifted towards it. As the distribution of level spacings is different depending on the existence of magnetic field, this enhancement mechanism is again more effective in the absence of magnetic field (GOE), where probabilities of small spacing and of small conductances are higher. Thus, this effect which was neglected in [2] suppresses $\alpha$ even further.

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2. We evaluated the sum (3) numerically by drawing $\Gamma_i^{L(R)}$ from the Porter-Thomas distribution and $E_i$ according to the Wigner-Dyson distribution. Levels within a window of $\pm 4k_B T$ around the Fermi energy have been taken into account and the Fermi energy $\mu$ in Eq. (3) has been adjusted to yield $G_{\text{max}}$ for every realization.
son for the reduction is that a very low $G_{max}$ requires $\Gamma_L$ or $\Gamma_R$ in Eq. (3) to be low. RMT spectral fluctuations enhance the contributions from close-by levels, which typically do not have a low value of $\Gamma_L^{L(R)}$ at the same time. Thus, the peak position of $\mu$ is shifted towards a close-by level and the conductance occurs through both levels. Notably, the effect of phase-breaking inelastic scattering processes leads to similar changes [1].

Deviations of $\alpha$ from the “universal” value 1/4 have been interpreted as being a result of dephasing. While dephasing would certainly suppress $\alpha$, we note here that in the regime $k_B T \lesssim \Delta$, the spectral fluctuation effects discussed above, lead to a similar effect. In Figure 2 we present the results for $\alpha$ as a function of the scaled temperature $k_B T/\Delta$, for both spin-degenerate spectrum and the case of broken symmetry. While the effect seems to be small, one should keep in mind that in the low temperature regime, even very strong dephasing does not suppress $\alpha$ to zero [10], and thus the correction due to spectral fluctuations is comparable with or even larger than the effect of dephasing [10, 11]. One should therefore cautiously use $\alpha$ as a probe of dephasing in this regime.

In conclusion, we have shown that RMT spectral fluctuation effect the probability distribution function $P(G_{max})$, leading to non-negligible deviations in measurable quantities in the regime $0.1 \Delta < k_B T < 0.8 \Delta$. In particular, the weak localization correction $\alpha$ which was recently used as a probe of dephasing in closed quantum dots is affected. $\alpha$ is different from 1/4 and, moreover, turns out to be temperature dependent, even in the absence of dephasing. At low temperatures, $\alpha$ is reduced down to $\alpha \approx 0.2$, which can be below the lower limit of a picket fence model with dephasing. This should also be taken into account while analyzing the ongoing experiments aimed at measuring dephasing times in closed dots in the low temperature regime $k_B T \lesssim \Delta$.

Finally, we would like to note that during the completion of the present paper some of the results have been independently arrived at in [14].

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