Control of self-oscillations in a diode laser with external cavity

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Abstract. We study possibilities of controlling instabilities in a diode laser with external cavity by adding a second external cavity and adjusting its length and feedback strength. This method is approved numerically with a model of Lang-Kobayashi equations for the laser with two external cavities. We find that chaotic behaviour of the laser output can be completely stabilized to periodic orbits of different periods.

1. Introduction
The use of an external cavity for optical feedback in a semiconductor laser has been studied by many researchers since it has many applications in optical communications, interferometric sensors, etc. The induced feedback tends to destabilize the laser output; this effect is known as coherence collapse [1]. Variations on the optical power are generated when a small amount of the light emitted by the semiconductor laser reflects back into the laser cavity causing self-mixing. At a moderate feedback strength, intermittent drops of the laser intensity are observed, that gives rise to low-frequency fluctuations. This class of dynamical systems are described by delayed differential equations which are frequently used to model control systems [2]. The dynamics of semiconductor lasers with a single feedback was studied extensively and the basic mechanism for the low-frequency fluctuations is well understood [3,4].

In this paper we study the possibility of controlling instabilities and chaos in a semiconductor laser with an external cavity by adding a second external cavity. Early, this scheme was investigated by Rogister et al. [5,6]. They demonstrated the stabilizing effects with particular attention to the bifurcation diagram of the steady-state solutions. In this work we concentrate on phase-locking regimes which can be achieved when two identical external cavities are used. We show that both the length and feedback strength of the second external cavity can be used as control parameters and the variation of these two parameters allows us to realize the phase-locking and obtain different dynamical regimes.

The paper is organized as follow. First, we consider the model of the laser with a single external cavity, and then we study the laser with two external cavities and compare the results of the two models. Finally, the main conclusions are given.
2. Lasers with a single external cavity

A schematic arrangement of a semiconductor laser with a single external cavity is shown in figure 1.

![Figure 1. Scheme of laser diode with external cavity. Ld and Lex are the lengths of the internal and external cavities, r1 and r2 are the laser facets and r3 the external mirror.](image)

The dynamics of a feedback laser is described by the Lang-Kobayashi equations [2]:

\[
\frac{d}{dt} E_0(t) = \frac{1}{2} \left[ G_N (N(t) - N_0) - \frac{1}{\tau_p} \right] E_0(t) + \frac{\kappa}{\tau_L} E_0(t - \tau) \cos[\omega_0 \tau + \varphi(t) - \varphi(t - \tau)] ,
\]

\[
\frac{d}{dt} \varphi(t) = \frac{1}{2} \alpha G_N [N(t) - N_T] - \frac{\kappa}{\tau_L} \frac{E_0(t - \tau)}{E_0(t)} \sin[\omega_0 \tau + \varphi(t) - \varphi(t - \tau)],
\]

\[
\frac{d}{dt} N(t) = R_p - \frac{N(t)}{\tau_S} - G_N [N(t) - N_0] E_0^2(t),
\]

where \( E_0(t) \) is the complex amplitude of the electric field, \( \omega_0 \) is the angular frequency of the unperturbed laser, \( G_N \) is the modal gain coefficient, \( N(t) \) is the average density of carriers, \( N_0 \) and \( N_T \) are the carrier densities at transparency and at threshold, \( \varphi(t) \) is the phase, \( \tau_p \) is the photon lifetime, \( \tau_L \) and \( \tau \) are the diode cavity and external cavity round trip times, \( \tau_S \) is the carrier lifetime, \( R_p \) is the pumping term, \( \alpha \) is the linewidth enhancement factor, and \( \kappa \) is the feedback parameter of the external cavity. For an ideal system, the photon lifetime term can be neglected. The parameter values used in calculations are presented in table 1. All parameters have been rescaled [7] by ns\(^{-1}\) since most of the parameters are multiples of it.

| Parameter | Value |
|-----------|-------|
| \( \alpha \) | 3.5 |
| \( G_N \) | 50 |
| \( \tau_L \) | 1 |
| \( \tau \) | 0.22 |
| \( \tau_S \) | 1 |
| \( R_p \) | 8 |
| \( \kappa \) | 40 |
| \( N_T \) | 3 |
| \( N_0 \) | 5 |
The numerical solution of equations (1)-(3) with the parameters presented in table 1 are shown in figure 2. A formation of pulse packages in the laser intensity is observed. The pulse intensities are modulated by a low frequency envelope that forms the individual pulse packages.

![Figure 2. Chaotic time series in a laser with external cavity.](image)

3. Lasers with two external cavities
In figure 3 we show the scheme when a second cavity is added.

![Figure 3. Scheme of laser diode with two external cavities.](image)

The theoretical model is expanded for this case. For the complex amplitude of the electric field $E_0(t)$ and phase $\phi(t)$ parts of the equations, an extra term is introduced to include the external round trip and the feedback parameter of the second cavity. The new set of equations takes the form of:

\[
\frac{d}{dt} E_0(t) = \frac{1}{2} \left[ G_N (N(t) - N_0) - \frac{1}{\tau_p} \right] E_0(t) + \frac{\kappa_1}{\tau_L} E_0(t - \tau_1) \cos[\omega_0 \tau + \phi(t) - \phi(t - \tau_1)] + \frac{\kappa_2}{\tau_L} E_0(t - \tau_2) \cos[\omega_0 + \phi(t) - \phi(t - \tau_2)] \tag{4}
\]

\[
\frac{d}{dt} \phi(t) = \frac{1}{2} \alpha G_N \left[ N(t) - N_\gamma \right] - \frac{\kappa_1}{\tau_L} \frac{E_0(t - \tau_1)}{E_0(t)} \sin[\omega_0 \tau + \phi(t) - \phi(t - \tau_1)] - \frac{\kappa_2}{\tau_L} \frac{E_0(t - \tau_2)}{E_0(\tau \tau)} \sin[\omega_0 + \phi(t) - \phi(t - \tau_2)] \tag{5}
\]

\[
\frac{d}{dt} N(t) = R_p - \frac{N(t)}{\tau_s} - G_N [N(t) - N_0] E_0^2(t), \tag{6}
\]
where the new terms $\kappa_1$ and $\kappa_2$ and $\tau_1$ and $\tau_2$ are the feedback strengths and the round trip times, respectively, for the first and second external cavities.

The numerical solutions were obtained by using the parameters from table 1, only the values for $\kappa_1$, $\kappa_2$, $\tau_1$ and $\tau_2$ are varied. The second external cavity can stabilize chaos to a periodic orbit when it has the same length as the first one, $L_{ext1}=L_{ext2}$, or the ratio of their lengths is a rational number. The time series for the former case is shown in figure 4.

![Figure 4. Period 1 in laser with two external cavities with equal feedback strengths $\kappa_1 = \kappa_2 = 40$ and round trip times $\tau_1 = \tau_2 = 0.22.$](image)

Figures 5 and 6 illustrate the time series of the laser intensity for two external cavities with equal lengths but with different feedback strengths. The envelope of the pulse packages in figure 5 encloses a chaotic regime.

![Figure 5. Two external cavities were $\kappa_1 = 40$, $\kappa_2 = 15$, and $\tau_1 = \tau_2 = 0.22$.](image)

![Figure 6. Two external cavities were $\kappa_1 = 40$, $\kappa_2 = 5$, and $\tau_1 = \tau_2 = 0.22$.](image)

Finally in figure 7 we show a chaotic regime which is obtained when both the lengths and feedback parameters of the two cavities are aliquant.
The periodic regimes displayed in figures 4-6 are just examples of a variety of locking states obtained by numerical simulations of equations (4)-(6) for different control parameters. The bifurcation structure can be analyzed in the two parameter space of the cavity length and feedback strength. A complex behavior, like Arnold’s tongues and Devil’s stairs, is observed, and different routes to chaos can be realized by varying these two control parameters.

4. Conclusions
We have demonstrated that the chaotic output of a semiconductor laser with an external cavity can be controlled by another external cavity using its length and feedback strength as control parameters. Different periodic, quasi-periodic or chaotic regimes can be obtained depending on the ratios between the lengths and feedback strengths of the two external cavities. When these ratios are rational numbers the phase locking occurs.

We believe that the results of this research can be of interest for some technological applications, especially in optical communications.

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