Conditional Production of Superpositions of Coherent States with Inefficient Photon Detection

A.P. Lund,1,* H. Jeong,1,* T.C. Ralph,1 and M.S. Kim2
1Center for Quantum Computer Technology, Department of Physics, University of Queensland, St Lucia, Qld 4072, Australia
2School of Mathematics and Physics, Queen’s University, Belfast BT7 1NN, United Kingdom

(Dated: October 9, 2018)

It is shown that a linear superposition of two macroscopically distinguishable optical coherent states can be generated using a single photon source and simple all-optical operations. Weak squeezing on a single photon, beam mixing with an auxiliary coherent state, and photon detecting with imperfect threshold detectors are enough to generate a coherent state superposition in a free propagating optical field with a large coherent amplitude (\(\alpha > 2\)) and high fidelity (\(F > 0.99\)). In contrast to all previous schemes to generate such a state, our scheme does not need photon number resolving measurements nor Kerr-type nonlinear interactions. Furthermore, it is robust to detection inefficiency and exhibits some resilience to photon production inefficiency.

Since Schrödinger suggested his famous cat paradox \(\dagger\), there has been great interest in generating and observing a quantum superposition of a macroscopic system. The component states composing such a superposition state should be macroscopically distinguishable, i.e., they should give macroscopically distinct measurement outcomes \(\ddagger\). Two coherent states can be discriminated by a homodyne measurement, which can be considered a macroscopic measurement, when they are well separated in the phase space. Therefore, a superposition of two optical coherent states with sufficiently large amplitudes and with a \(\pi\) phase difference between these amplitudes is considered a realization of such a macroscopic superposition.

Recently, the coherent state superposition (CSS) in a free propagating optical field has been studied for application to quantum information processing including quantum teleportation \(\ddagger\), quantum computation \(\ddagger\), entanglement purification \(\ddagger\) and error correction \(\ddagger\). In particular, it was found that quantum computation can be realized using only linear optics and photon counting, given pre-arranged CSS's \(\ddagger\). In this framework, initial CSS's of amplitude \(\alpha \geq 2\) are required for efficient quantum computation with simple optical networks \(\ddagger\).

Unfortunately, it is extremely demanding to generate a free propagating CSS using current technology. It is well known that the CSS can be generated from a coherent state by a nonlinear interaction in a Kerr medium \(\ddagger\). However, Kerr nonlinearity of currently available nonlinear media is extremely small and attenuation is not negligible compared with the required level to generate a CSS \(\ddagger\).

Some alternative methods have been studied to generate a superposition of macroscopically distinguishable states based upon conditional measurements \(\ddagger\). A crucial drawback of these schemes is that a highly efficient detector which can discriminate photon numbers is necessary. Cavity quantum electrodynamics has been studied to enhance nonlinearity \(\ddagger\). Some success has been reported in creating such superposition states within high Q cavities in the microwave \(\ddagger\) and optical \(\ddagger\) domains. However, all the suggested schemes for quantum information processing with coherent states \(\ddagger\) require a free propagating CSS.

In this Letter, we show that a free propagating optical CSS can be generated with a single photon source and simple optical operations. A CSS with a small coherent amplitude (\(\alpha \leq 1.2\)) and high fidelity (\(F > 0.99\)) can be deterministically generated by squeezing a single photon. A large CSS (\(\alpha > 2\)) with high fidelity (\(F > 0.99\)) can be obtained in a non-deterministic way from small CSS's. Weak squeezing, beam mixing with an auxiliary coherent field and photon detecting with threshold detectors are enough to generate such large CSS's given a single photon source. Remarkably, neither discrimination of photon numbers nor \(\chi^{(3)}\) nonlinear interactions are required in our scheme. Furthermore, our scheme is robust to detection inefficiency and somewhat resilient to photon production inefficiency. In a more general sense, our examples reveal some previously unrealized relations between the quantum states of harmonic oscillators: we learn that the first excited energy eigenstates can be converted to superpositions of large coherent states by linear operations and projections.

A CSS can be defined as \(|\text{CSS}_\varphi(\alpha)\rangle = N_\varphi(\alpha)(|\alpha\rangle + e^{i\varphi}| - \alpha\rangle)\), where \(N_\varphi(\alpha)\) is a normalization factor, \(|\alpha\rangle\) is a coherent state of amplitude \(\alpha\), and \(\varphi\) is a real local phase factor. The amplitude \(\alpha\) is assumed to be real for simplicity without loss of generality. In this paper we refer to the magnitude of \(\alpha\) as the size of the CSS. Note that CSS's such as \(|\text{CSS}_{\pm}(\alpha)\rangle = N_\pm(\alpha)(|\alpha\rangle \pm | - \alpha\rangle)\) are called even and odd CSS's respectively because the even (odd) CSS, \(|\text{CSS}_{+}(\alpha)\rangle\) \(|\text{CSS}_{-}(\alpha)\rangle\), always contains an

*The first two authors contributed equally to this work.
even (odd) number of photons.

An arbitrarily large CSS can be produced out of arbitrarily small CSS’s using the simple experimental set-up depicted in Fig. 1. Let us first illustrate this procedure with a simple example. Suppose that one has a collection of identical small odd CSS’s with known amplitude \( \alpha_i \). Two of the small CSS’s are selected and incident onto a 50:50 beam splitter BS1 as

\[
|CSS_-(\alpha_i)a|CSS_-(\alpha_i)b \xrightarrow{BS1} |0\rangle_f (|\sqrt{2}\alpha_i\rangle_g + | - \sqrt{2}\alpha_i\rangle_g) - (|\sqrt{2}\alpha_i\rangle_f + | - \sqrt{2}\alpha_i\rangle_f) |0\rangle_g
\]  

(1)

where the normalization factor is omitted on the right hand side. One can then say that if one could condition on detecting \( |0\rangle_g \), a larger CSS with amplitude \( \sqrt{2}\alpha_i \) would be obtained at mode \( f \). An additional step is therefore needed to unambiguously discriminate between the vacuum and coherent states \( |\pm \sqrt{2}\alpha_i\rangle_g \) with inefficient detectors. Another 50:50 beam splitter, BS2, mixes the field at mode \( g \) and an auxiliary coherent state \( |\sqrt{2}\alpha_i\rangle_c \) as

\[
|BS1\rangle_{f,g} |\sqrt{2}\alpha_i\rangle_c \xrightarrow{BS2} |0\rangle_f (|2\alpha_i\rangle_{t1}|0\rangle_{t2} + |0\rangle_{t1}|2\alpha_i\rangle_{t2}) - (|\sqrt{2}\alpha_i\rangle_f + | - \sqrt{2}\alpha_i\rangle_f) |\alpha_i\rangle_{t1} - \alpha_i\rangle_{t2}
\]  

(2)

where \( |BS1\rangle_{f,g} \) represents the right hand side of Eq. (1) and the normalization factor is omitted. Finally, photodetectors A and B are set to detect photons at modes \( t1 \) and \( t2 \). The remaining state at mode \( f \) is selected only when both the detectors detect any photon(s) at the same time. In this case, it is obvious that the right hand side of Eq. (2) is reduced to a larger CSS’s. If either of the detectors fails to click, the resulting state is discarded. This process can be recursively applied until a sufficiently large CSS is obtained. Suppose that an even CSS with amplitude \( \alpha \geq 2 \) is required while the initial amplitude of small odd CSS’s is \( \alpha_i = 1/\sqrt{2} \). After a sufficient number of CSS’s of the amplitude \( \sqrt{2}\alpha_i \) are obtained, the second step will be taken with the same experimental set-up and another auxiliary coherent state \( |2\alpha_i\rangle_c \). In this second stage, larger even CSS’s of amplitude \( 2\alpha_i \) will be gained from pairs of even CSS’s of \( \sqrt{2}\alpha_i \). Eventually, the amplitude will reach the required value by four recursive applications of the process, i.e., \( \alpha = 4\alpha_i \approx 2.83 \).

The process described above can be generalized for arbitrarily small CSS’s with known amplitudes as shown in Fig. 1. Suppose two small CSS’s, \( |CSS_\varphi(\alpha)\rangle \) and \( |CSS_\phi(\beta)\rangle \), with amplitudes \( \alpha \) and \( \beta \). The reflectivity \( r \) and transmittivity \( t \) of BS1 are set to \( r = \beta/\sqrt{\alpha^2 + \beta^2} \) and \( t = \alpha/\sqrt{\alpha^2 + \beta^2} \), where the action of the beam splitter is represented by \( \tilde{B}_{a,b}(r,t)|a\rangle_b|b\rangle_b = |ta + rt\rangle_f | - ra + t\beta\rangle_g \). The other beam splitter BS2 is a 50:50 beam splitter \( (r = t = 1/\sqrt{2}) \) regardless of the conditions and the amplitude \( \gamma \) of the auxiliary coherent field is determined as \( \gamma = 2\alpha\beta/\sqrt{\alpha^2 + \beta^2} \). The resulting state for mode \( f \) then becomes \( |CSS_{\varphi + \phi}(\mathcal{A})\rangle \propto |\mathcal{A}\rangle + e^{i(\varphi + \phi)} | - \mathcal{A}\rangle \), whose coherent amplitude \( \mathcal{A} = \sqrt{\alpha^2 + \beta^2} \) is larger than both \( \alpha \) and \( \beta \). The relative phase of the resulting CSS is the sum of the relative phases of the input CSS’s. The success probability \( P_{\phi,\phi}(\alpha,\beta) \) for a single iteration is

\[
P_{\phi,\phi}(\alpha,\beta) = \frac{(1 - e^{-\frac{2\alpha^2 + 2\beta^2}{\alpha^2 + \beta^2}})^2 [1 + \cos(\varphi + \phi) e^{-2(\alpha^2 + \beta^2)}]}{2(1 + \cos \varphi e^{-2\alpha^2})(1 + \cos \phi e^{-2\beta^2})},
\]

which is plotted for a number of different combinations in Fig. 2. The success probability approaches 1/2 as the amplitudes of initial CSS’s becomes large. Note that the probabilities depend on the type of CSS’s (odd or even).
used. The effect of detector inefficiency is just to decrease this success probability.

We now show that a small odd CSS with $\alpha \leq 1.2$ is surprisingly well approximated by a squeezed single photon. The single mode squeezing operator is $\hat{S}(r) = e^{-\frac{r}{2} (\hat{a}^2 - \hat{a}^2)}$, where $r$ is the squeezing parameter and $\hat{a}$ is the annihilation operator. This operator reduces quantum noise of a vacuum state in the phase quadrature by a factor of $e^{-2r}$. When the squeezing operator is applied to a single photon the resultant state can be expanded in terms of photon number states as

$$\hat{S}(r) |1\rangle = \sum_{n=0}^{\infty} \frac{(\tanh r)^n \sqrt{(2n+1)!}}{(\cosh r)^{\frac{n}{2}}} \frac{1}{2^n n!} (2n+1). \quad (3)$$

The state contains only odd photon numbers and has coefficients decaying exponentially as $n$ increases in a similar fashion to an odd CSS. The fidelity of this state with an odd CSS is

$$F(r, \alpha) = |\langle CSS_-(\alpha)|\hat{S}(r)|1\rangle|^2 = \frac{2\alpha^2 \exp[\alpha^2 (\tanh r - 1)]}{(\cosh r)^3 (1 - \exp[-2\alpha^2])}.$$  

Fig. 3 shows the maximized fidelity on the y-axis plotted against iteration $t$ (a) and the number of steps in (b) against the desired $\alpha$ in the CSS’s. For example, four iterations starting from the initial amplitude $\alpha_i = 1/2$ is required to gain the maximum fidelity $F = 0.995$ for $\alpha = 2$. It is evident from Fig. 4 that high fidelity, $F > 0.99$, can be obtained up to $\alpha = 2.5$. The error rate for discrimination between coherent states with $\alpha = \pm 2.5$ via a classical measurement (homodyne detection) is only $3 \times 10^{-7}$.

Current technology does not produce pure single photon states; the single photon is always in a mixture with the vacuum as $p \langle 0 | \langle 0 | + (1-p) | 1 \rangle \langle 1 |$, where $p$ is the inefficiency of the photon production. Hence the squeezed single photon state will also be a mixture with a squeezed vacuum. However, an interesting aspect of our scheme is that it may be somewhat resilient to the photon production inefficiency because its first iteration purifies the mixed CSS’s while amplifying them. The initial input states for the CSS amplification process from the imperfect single photon source are
\[
\rho_{a,b,c} = \left[ (1-p)^2 |S_1\rangle \langle S_1| \otimes |S_1\rangle \langle S_1| + p^2 |S_0\rangle \langle S_0| \otimes |S_0\rangle \langle S_0| \right.
+ p(1-p) \left( |S_0\rangle \langle S_0| \otimes |S_1\rangle \langle S_1| + |S_1\rangle \langle S_1| \otimes |S_0\rangle \langle S_0| \right) \right]_{a,b} \otimes (|\gamma\rangle \langle \gamma|)_c.
\]

In the CSS amplification process, the zero amplitude coherent states that occur in the detection modes in Eq. (2) may be slightly different from zero because of imperfect mode matching at beam splitters. This will lead to a small probability of accepting the wrong state. Good mode matching is a requirement in any linear optical network where one wishes to measure manifestly quantum mechanical effects. Highly efficient mode matching of a single photon from parametric down conversion and a weak coherent state from an attenuated laser beam at a beam splitter has been experimentally demonstrated using optical fibers 14. Such techniques could be employed for the implementation of our scheme.

The dark count rate of photodetectors will affect the fidelity of the CSS’s. Currently, highly efficient detectors have relatively high dark count rates while less efficient detectors have very low dark count rates 15. We emphasize again that our scheme does not require highly efficient detectors because the inefficiency of the detectors does not affect the quality of CSS’s even though it decreases the success probability. Silicon avalanche photodiodes operating at the visible wavelength have relatively high efficiency and a small dark count rate, which is preferred in our proposal.

The single photons required for our scheme could be generated conditionally from a down-converter 17. This is a \(\chi^{(2)}\) process (like squeezing) and does not require photon number resolving detection. Once free propagating CSS’s are generated, they can be detected by homodyne measurements, which can be highly efficient in quantum optics experiments.

Our scheme non-deterministically generates large CSS’s. However, a non-deterministic CSS source is useful enough for quantum information processing 16, 20. Efficient gate operations for coherent-state quantum computation 20 are based on teleportation via an entangled coherent state 20. Entangled coherent states can be simply generated from CSS’s using a beam splitter and can be used as off-line resources for quantum computation. We note that such entanglement of macroscopically distinguishable states is perhaps more closely aligned with Schrödinger’s original concept 1.

In conclusion, we have proposed a simple all-optical scheme to generate a linear superposition of macroscopically distinguishable coherent states in a propagating optical field. We have found a previously unrealized connection between squeezed number states and superpositions of coherent states as well as the interesting additive
properties of the latter. In stark contrast to all previous schemes our scheme requires neither $\chi^{(3)}$ nonlinearity nor photon number resolving detection to generate a macroscopic superposition state.

We would like to thank A. Gilchrist for useful comments. This work was supported by the Australian Research Council and the University of Queensland Excellence Foundation. M.S.K. acknowledges the UK Engineering and Physical Science Research Councils and the Korea Research Foundation (2003-070-C00024) for financial support.

[1] E. Schrödinger, Naturwissenschaften. 23, 807-812; 823-828; 844-849 (1935).
[2] M.D. Reid, quant-ph/0101062 and references therein.
[3] S.J. van Enk and O. Hirota, Phys. Rev. A 64, 022313 (2001).
[4] H. Jeong, M.S. Kim, and J. Lee, Phys. Rev. A 64, 052308 (2001).
[5] H. Jeong and M.S. Kim, Phys. Rev. A 65, 042305 (2002).
[6] T.C. Ralph, W.J. Munro, and G. J. Milburn, Proceedings of SPIE 4917, 1 (2002); quant-ph/0110115
[7] T.C. Ralph et al., Phys. Rev. A 68, 042319 (2003).
[8] H. Jeong and M.S. Kim, Quantum Information and Computation 2, 208 (2002); J. Clausen, L. Knöll, and D.-G. Welsch, Phys. Rev. A 66, 062303 (2002).
[9] P. T. Cochrane, G. J. Milburn, and W. J. Munro, Phys. Rev. A 59, 2631 (1999); S. Glancy, H. Vasconcelos, and T.C. Ralph, quant-ph/0311093
[10] B. Yurke and D. Stoler, Phys. Rev. Lett. 57, 13 (1986).
[11] R.W. Boyd, J. Mod. Opt 46, 367 (1999).
[12] S. Song, C.M. Caves and B. Yurke, Phys. Rev. A. 41, 5261 (1990).
[13] M. Dakna et al., Phys. Rev. A. 55, 3184 (1997).
[14] Q.A. Turchette et al., Phys. Rev. Lett. 75, 4710 (1995).
[15] M. Brune et al., Phys. Rev. Lett. 77, 4887 (1996).
[16] C. Monroe et al., Science 272, 1131 (1996).
[17] A.I. Lvovsky et al., Phys. Rev. Lett. 87, 050402 (2001).
[18] S. Takeuchi et al., Appl. Phys. Lett. 74, 1063 (1999).
[19] T.B. Pittman and J.D. Franson, Phys. Rev. Lett. 90, 240401 (2003).