Two photon decay of neutral scalars below 1.5 GeV in a chiral model for $\bar{q}q$ and $\bar{q}qqq$ states

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We study the two photon decay of neutral scalars below 1.5 GeV in the context of a recently proposed chiral model for $\bar{q}q$ and $\bar{q}qqq$ states. We find good agreement with experimental results for the $a_0(980) \rightarrow \gamma \gamma$. Our calculations for $f_0(980) \rightarrow \gamma \gamma$ shows that further work is necessary in order to understand the structure of this meson. The model predicts $\Gamma(a_0(1450) \rightarrow \gamma \gamma) = 0.16 \pm 0.10$ keV, $\Gamma(\sigma \rightarrow \gamma \gamma) = 0.47 \pm 0.66$ KeV, $\Gamma(f(1370) \rightarrow \gamma \gamma) = 0.07 \pm 0.15$ KeV, $\Gamma(f(1500) \rightarrow \gamma \gamma) = 0.74 \pm 0.78$ KeV.

I. INTRODUCTION

The understanding of the lowest lying scalar mesons remains as one of the most challenging problems in low energy QCD. Over the past years experimental evidence has accumulated for the existence of a light scalar nonet $\bar{q}q$ but there is still an intense debate about the structure of these mesons. There are essentially two proposals for the structure of light scalar mesons: a $\bar{q}q$ structure and a $\bar{q}qqq$ one. In the latter case the are at least three possible dynamical configurations: a meson-meson molecule, a diquark-diquark state and a compact genuine $\bar{q}qqq$ state. In addition there exist the possibility that isosinglet scalars contain a certain amount of glueball. Most of recent work has been dedicated to the understanding the properties of light scalars in the light of these pictures, explored in different formalisms.

Since photons couple to charge, electromagnetic decays of mesons have been intensively used in the past to obtain information on their structure. As for scalar mesons, there exist calculations for the $a_0(980), f_0(980) \rightarrow 2\gamma$ decays using a variety of approaches $\bar{q}q$, $\bar{q}qqq$, in particular, in different versions of the quark model $\bar{q}q$, with very different results depending on the details of the model. The generally accepted conclusion, is that the measured $a_0(980), f_0(980) \rightarrow \gamma \gamma$ decay widths $\bar{q}q$ are not consistent with a $\bar{q}q$ structure. In the light of these results, other possibilities for the structure of light scalars such as a molecule picture $\bar{q}q$ and a $\bar{q}qqq$ structure were explored.

More recently, some tools to determine the glueball content of mesons from their branching fractions in radiative quarkonium decays and production cross sections in $\gamma \gamma$ collisions, were developed in $\bar{q}q$. Also, the two photon decay of the lightest scalar, the $f_0(600)$ or $\sigma$, has been studied using different formalisms $\bar{q}q$, yielding very disparate predictions. The possibility that this meson has a large glueball content was analyzed in $\bar{q}q$ using the two photon decay channel. As shown in this work, the extraction of the two photon coupling of light isoscalars from data on $\gamma \gamma \rightarrow \pi \pi$ is not straightforward and requires a careful analysis in order to get reliable results.

The two photon decay of the $a_0(980)$ and $f_0(980)$ mesons were also studied in $\bar{q}q$ in the framework of a chiral model involving scalars and a linear realization of chiral symmetry, a linear sigma model ($\bar{q}q$), where the key interaction is the one violating $U_A(1)$ symmetry. This interaction is assumed to be the manifestation at the hadron level of the effective six-quark interaction ($\bar{q}q$, $\bar{q}qqq$) induced by instantons. In this concern this interaction identifies the fields entering the model as $\bar{q}q$ states. In this formalism, the two photon decay of neutral scalars is induced at the one loop level. The scalar decay firstly into two (real or virtual) charged mesons which annihilate into two photons. As to the information on the internal structure of scalars which can be inferred in this calculation, we must stress that a naive estimation of the distances explored by the photons in this decay are of the order of $d \approx 1/k = 2/m_3 = 0.4$ fm which is of the same order as the kaon charge radius, thus the effective degrees of freedom seen by the photons are actually mesons rather than quarks. In this sense, we can infer that the decaying mesons are $q\bar{q}$ states but this bare state has large quantum fluctuations into meson-meson states whose exact amount is difficult to quantify.

Recently we pointed out the existence of a quasi-degenerate chiral nonet in the energy region around 1.4 GeV and studied the possibility that mesons below 1.5 GeV arise as admixtures of normal $q\bar{q}$ and $\bar{q}qqq$ states, with the latter lying at its natural scale as dictated by the linear rising of meson masses with the number of constituent quarks $\bar{q}q$. This model has the nice feature of reducing to the one explored in $\bar{q}q$ in the case when we decouple the $\bar{q}qqq$ states.

In this work we explore the predictions of the model presented in $\bar{q}q$ for the two photon decay widths of all neutral scalars below 1.5 GeV.
II. MESON LOOP CONTRIBUTIONS TO $S \to \gamma \gamma$.

The most general form for the $S(p) \to \gamma(k, \epsilon) \gamma(q, \eta)$ transition amplitude is dictated by Lorentz covariance and gauge invariance as

$$M(S \to \gamma(k, \epsilon) \gamma(q, \eta)) = \frac{i\alpha}{\pi f_K} V^S(g^{\mu\nu} k \cdot q - k^\mu q^\nu) \eta_\mu \epsilon_\nu,$$

where $f_K$ denotes the kaon weak decay constant and $\alpha$ denotes the electromagnetic fine constant. The factors $\alpha, \pi, f_K$ in Eq. (1) are introduced just for convenience in future manipulations. With this normalization the form factor $V^S(p^2)$ is dimensionless.

In the effective theory formulated in [15] the two photon decays of neutral scalar mesons below 1.5 GeV are induced by loops of charged mesons as depicted in Fig. 1. A straightforward calculation yields

$$V^S_M = \frac{2f_K g_{SM}}{m_S^2} \left[ -\frac{1}{2} + \xi_M^S I(\xi_M^S) \right],$$

where $\xi_M^S = \frac{m_M^2}{m_S^2}$, $g_{SM}$ denotes the coupling constant of the decaying scalar $S$ to the meson pair $(M^+M^-)$ in the loops and $I(x)$ denotes the loop integral

$$I(x) = \begin{cases} 
2 \left( \arcsin \sqrt{\frac{1}{4x}} \right)^2 & x > \frac{1}{4} \\
\frac{2}{x^2} + i \ln \left( \sqrt{\frac{1}{4x}} + \sqrt{\frac{1}{4x} - 1} \right) \right)^2 & x < \frac{1}{4}
\end{cases}.$$

The decay width is given by

$$\Gamma(S \to \gamma \gamma) = \frac{\alpha^2 m_S^2}{64\pi^3 f_K^2} |V^S|^2.$$

The experimental data for $a_0(980)$, $f_0(980)$ decay into two photons is [6]

$$\Gamma(a_0(980) \to \gamma \gamma) \times BR(a_0(980) \to \pi^0 \eta) = 0.24^{+0.08}_{-0.07} \text{ KeV},$$

$$\Gamma(f_0(980) \to \gamma \gamma) = 0.39^{+0.10}_{-0.13} \text{ KeV},$$

and assuming that the decay $a_0(980) \to \pi \eta$ is the dominant mode ($BR(a_0(980) \to \pi^0 \eta) \simeq 1$) we obtain the form factors $|V^{a(980)}|$ and $|V^{f(980)}|$ at $p^2 = m_S^2$ as

$$|V^{a(980)}|_{Exp} = 0.34 \pm 0.05, \quad |V^{f(980)}|_{Exp} = 0.44 \pm 0.07.$$

There exists no confident experimental information for the two photon decays of the $a_0(1450)$, $f_0(600)$, $f_0(1370)$ and $f_0(1500)$ mesons [6].
III. TWO PHOTON DECAY OF $a_0(980)$ AND $a_0(1450)$.

For the $a_0(980) \rightarrow \gamma \gamma$ and $a_0(1450) \rightarrow \gamma \gamma$ decays the model in [13] yields contributions coming from are $K$, $\kappa(900)$ and their respective heavy “partners” $K(1469)$ and $K^*_0(1430)$ in the loops. For the sake of simplicity we denote these mesons as $K$, $\kappa$, $\hat{K}$ and $\hat{\kappa}$ respectively. The couplings entering in the loops were calculated in [16] and are listed in Table I, we refer the reader to [16] for details of the notation.

Table I

| $g_{a(980)K^+K^-}$ | $-\frac{1}{\sqrt{2(a+b)}} \left( m_{K^+}^2 + m_{K^-}^2 - m_a^2 - m_A^2 + \mu_1^2 - \mu_{1/2}^2 \right) \cos \phi_1 \cos^2 \theta_{1/2}$, |
| $g_{a(980)\kappa^+\kappa^-}$ | $-\frac{1}{\sqrt{2(b-a)}} \left( m_{\kappa^+}^2 + m_{\kappa^-}^2 - m_a^2 - m_A^2 + \mu_1^2 - \mu_{1/2}^2 \right) \cos \phi_1 \cos^2 \theta_{1/2}$, |
| $g_{a(1450)K^+K^-}$ | $-\frac{1}{\sqrt{2(a+b)}} \left( m_{K^+}^2 + m_{K^-}^2 - m_a^2 - m_A^2 + \mu_1^2 - \mu_{1/2}^2 \right) \sin \phi_1 \cos^2 \theta_{1/2}$, |
| $g_{a(1450)\kappa^+\kappa^-}$ | $-\frac{1}{\sqrt{2(b-a)}} \left( m_{\kappa^+}^2 + m_{\kappa^-}^2 - m_a^2 - m_A^2 + \mu_1^2 - \mu_{1/2}^2 \right) \sin \phi_1 \cos^2 \theta_{1/2}$, |
| $g_{a(980)\hat{K}^+\hat{K}^-}$ | $-\frac{1}{\sqrt{2(a+b)}} \left( m_{\hat{K}^+}^2 + m_{\hat{K}^-}^2 - m_a^2 - m_A^2 + \mu_1^2 - \mu_{1/2}^2 \right) \cos \phi_1 \sin^2 \theta_{1/2}$, |
| $g_{a(980)\hat{\kappa}^+\hat{\kappa}^-}$ | $-\frac{1}{\sqrt{2(b-a)}} \left( m_{\hat{\kappa}^+}^2 + m_{\hat{\kappa}^-}^2 - m_a^2 - m_A^2 + \mu_1^2 - \mu_{1/2}^2 \right) \cos \phi_1 \sin^2 \theta_{1/2}$, |

It is worth noticing that the coupling of light scalars to light pseudoscalars quoted above reduce to the ones obtained in the linear sigma model [12] when we decouple the heavy fields. The two photon decays are of course dominated by light meson in the loops, heavy meson contributions being suppressed by the corresponding inverse mass powers. The values extracted in [13] for the mixing angles entering these expressions are

| Angle | Prediction |
|-------|------------|
| $\theta_1$ | $18.16^\circ \pm 4.3^\circ$ |
| $\theta_{1/2}$ | $22.96^\circ \pm 4.8^\circ$ |
| $\phi_1$ | $39.8^\circ \pm 4.5^\circ$ |
| $\phi_{1/2}$ | $46.7^\circ \pm 9.5^\circ$ |
| $\gamma$ | $-9.11^\circ \pm 0.5^\circ$ |
| $\delta$ | $21.45^\circ \pm 6.5^\circ$ |
| $\delta'$ | $51.36^\circ \pm 8.3^\circ$ |

We use also the relations

$$a = \frac{f_\pi}{\sqrt{2} \cos(\theta_1)}, \quad a + b = \frac{\sqrt{2} f_K}{\cos(\theta_{1/2})}. \quad (5)$$

Using these values and the masses quoted in [13] we obtain the results listed in Table II for the form factors. We include the contributions of different mesons step by step in order to have an idea on the effects of different mesons in the loops.

Table II

| Contr. | $K$ | $K, \kappa$ | $K, \kappa, \hat{K}$ | $K, \kappa, \hat{K}, \hat{\kappa}$ |
|--------|-----|-------------|---------------------|----------------------------------|
| $|V_{a(980)}|$ | $0.331 \pm 0.078$ | $0.321 \pm 0.080$ | $0.323 \pm 0.080$ | $0.320 \pm 0.084$ |
| $|V_{a(1450)}|$ | $0.149 \pm 0.045$ | $0.154 \pm 0.051$ | $0.153 \pm 0.051$ | $0.154 \pm 0.053$ |

As expected, the main contribution comes from kaon loops due to its relatively light mass. These results are in good agreement with the world average in the case of the two photon decay of the $a_0(980)$. The modifications introduced by the mixing between $q\bar{q}$ and $Qq\bar{q}\bar{q}$ fields to the picture in the conventional (updated) linear sigma model [12] are also exhibited in Table III.
Table III

| Form Factor | $\Lambda$M | This model | Exp. |
|-------------|----------|------------|------|
| $|\gamma|^2(980)$ | $0.348 \pm 0.038$ | $0.320 \pm 0.084$ | $0.34 \pm 0.05$ |
| $|\gamma|^2(1450)$ | — | $0.154 \pm 0.053$ | — |

The form factor for to the $a_0(1450)$ decay as predicted by the chiral model for $\pi\pi$ and $\eta\eta$ states corresponds to a width

$$\Gamma(a_0(1450) \rightarrow \gamma\gamma) = 0.16 \pm 0.10 \text{ KeV}. \quad (6)$$

IV. TWO PHOTON DECAY OF ISOSINGLET SCALARS.

Next, we work out the predictions of the model for $f_0(600)$ (or $\sigma$), $f_0(980)$, $f_0(1370)$ and $f_0(1370)$ decay into two photons. In this case we expect the main contribution to come from $\pi$, $K$ and $\kappa$ meson loops. In Table IV we list the couplings involved in these processes as arising from the chiral model for $\pi\pi$ and $\eta\eta$ states.

Table IV

| $g_{\pi^+\pi^-}$ | $\frac{1}{\sqrt{2}a}(m_\pi^2 + m_\pi^2 - m_\pi^2 - m_\pi^2 + \mu_\pi^2 - \mu_\pi^2) \cos \gamma \cos \delta \sin^2 \theta_1$, |
| $g_{\pi^+K^-}$ | $\frac{1}{\sqrt{2}(a+b)} (m_\pi^2 + m_\pi^2 - m_K^2 - m_K^2 + \mu_\pi^2 - \mu_\pi^2) \cos \gamma \cos \delta \sin^2 \theta_1$, |
| $g_{\pi^K+\pi^-}$ | $\frac{1}{\sqrt{2}(a-b)} (m_\pi^2 + m_\pi^2 - m_K^2 - m_K^2 + \mu_\pi^2 - \mu_\pi^2) \cos \gamma \cos \delta \sin^2 \theta_1$, |
| $g_{\pi^+\pi^-}(980)$ | $\frac{1}{\sqrt{2}(a+b)} (m_\pi^2 + m_\pi^2 - m_\pi^2 - m_\pi^2 + \mu_\pi^2 - \mu_\pi^2) \sin \gamma \cos \delta \sin^2 \theta_1$, |
| $g_{\pi^+K^-}(980)$ | $\frac{1}{\sqrt{2}(a-b)} (m_\pi^2 + m_\pi^2 - m_K^2 - m_K^2 + \mu_\pi^2 - \mu_\pi^2) \sin \gamma \cos \delta \sin^2 \theta_1$, |
| $g_{f_0(980)\pi^+\pi^-}$ | $\frac{1}{\sqrt{2}(a+b)} (m_f^2 + m_f^2 - m_\pi^2 - m_\pi^2 + \mu_f^2 - \mu_f^2) \sin \gamma \cos \delta \sin^2 \theta_1$, |
| $g_{f_0(980)\pi^+K^-}$ | $\frac{1}{\sqrt{2}(a-b)} (m_f^2 + m_f^2 - m_K^2 - m_K^2 + \mu_f^2 - \mu_f^2) \sin \gamma \cos \delta \sin^2 \theta_1$, |
Again, couplings for light scalars to light pseudoscalars listed in Table IV reduce to the linear sigma model ones when we decouple heavy fields. In this sense, we would like to notice that the naive mixing factor used in [12] for \( f_{(980)\kappa^+\kappa^-} \) contains an incorrect minus sign \([13]\). Even though contributions coming from charged \( \kappa \) loops are suppressed, they are crucial in the understanding of two photon decay of the \( f(980) \). Indeed, the sign assumed in [12] takes the result arising from kaon and pion loops in the right direction to match experiment. We recalculated this decay in the \( \Delta \rho \)M correcting for this sign finding a \( \kappa \) contribution in the opposite direction. The different contributions to this form factor as obtained in the \( \Delta \rho \)M are quoted in the Table VI. Predictions of the chiral model for \( \bar{q}q \) and \( \bar{q}q\bar{q}q \) states for the different contributions to the form factor describing \( f_0(980) \rightarrow \gamma\gamma \) decay are also shown in the table V.

| Contr. | \( |V_f(980)| \) | \( |V_f(1500)| \) | \( |V_f(600)| \) | \( |V_f(1370)| \) |
|--------|----------------|----------------|----------------|----------------|
| \( K \) | 0.409 ± 0.127 | 0.283 ± 0.094 | 0.014 ± 0.020 | 0.011 ± 0.016 |
| \( K, \pi \) | 0.601 ± 0.133 | 0.254 ± 0.085 | 1.487 ± 0.928 | 0.060 ± 0.069 |
| \( K, \pi, \kappa \) | 0.662 ± 0.145 | 0.306 ± 0.134 | 1.445 ± 0.937 | 0.102 ± 0.103 |
| \( K, \kappa, \pi, K \) | 0.664 ± 0.144 | 0.308 ± 0.135 | 1.444 ± 0.937 | 0.102 ± 0.104 |
| \( K, \kappa, \pi, K, \pi \) | 0.664 ± 0.144 | 0.308 ± 0.135 | 1.444 ± 0.937 | 0.102 ± 0.104 |
| Total | 0.682 ± 0.149 | 0.323 ± 0.152 | 1.431 ± 0.939 | 0.116 ± 0.107 |

whereas in the linear sigma model \([12]\) we obtain results listed in Table VI

| Contr. | \( |V_f(980)|_{\Delta \rho M} \) | \( K \) | \( K, \pi \) | \( K, \pi, \kappa \) |
|--------|----------------|--------|---------|---------|
| \( \bar{q}q \) states | 0.446 ± 0.115 | 0.779 ± 0.200 | 0.898 ± 0.135 |

The form factors in Table V yield the following decay widths

\[
\Gamma(\sigma \rightarrow \gamma\gamma) = 0.470 \pm 0.665 \text{ KeV}
\]
\[
\Gamma(f(1370) \rightarrow \gamma\gamma) = 0.071 \pm 0.155 \text{ KeV}
\]
\[
\Gamma(f(1500) \rightarrow \gamma\gamma) = 0.741 \pm 0.778 \text{ KeV}.
\]

V. CONCLUSIONS

In this paper we work out the predictions of the chiral model for \( \bar{q}q \) and \( \bar{q}q\bar{q}q \) states \([13,14]\) for the two photon decay of all neutral scalars below 1.5 GeV. Except for the \( a_0(980) \) and the \( f_0(980) \), there is no experimental information on these decays. The predictions of the model for the \( a_0(980) \rightarrow \gamma\gamma \) decay are in very good agreement with experiment. As for the \( f_0(980) \rightarrow \gamma\gamma \) we recalculate and update the linear sigma model predictions for this decay finding a discrepancy with the experimental results. The situation is improved by the mixing inherent to the chiral model for \( \bar{q}q \) and \( \bar{q}q\bar{q}q \) states \([13,14]\). Nevertheless, on the one side the extraction of the width from experimental data is not an easy task as shown in \([6]\), thus the world average quoted in \([6]\) should be taken with some care; on the other side we expect the \( f_0(980) \) to arise actually as a mixing of \( \bar{q}q \), \( \bar{q}q\bar{q}q \) and glueball. The latter has not been included in the model under consideration and according to recent analysis based on chiral symmetry the \( f_0(980) \) can acquire some component along the glueball direction \([17,18]\), thus we expect modifications to the present picture upon the inclusion of glueball degrees of freedom. Finally the model gives definite predictions for the two photon decays of the \( f_0(600) \), \( f_0(1370) \) and \( f_0(1500) \). It is particularly interesting the small decay width of the latter two mesons into two photons, even if they are composed of quarks in this model. The small width of these mesons has been usually argued as the signal for a large glueball component.

VI. ACKNOWLEDGMENTS

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