Localization of electrons in two-dimensional spatially-correlated random magnetic fields

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Abstract. The localization properties of electrons moving in a plane perpendicular to a spatially-correlated static magnetic field of random amplitude and vanishing mean are investigated. We apply the method of level statistics to the eigenvalues and perform a multifractal analysis for the eigenstates. From the size and disorder dependence of the variance of the nearest neighbor energy spacing distribution, $P_{W,L}(s)$, a single branch scaling curve is obtained. Contrary to a recent claim, we find no metal-insulator-transition in the presence of diagonal disorder. Instead, as in the uncorrelated random magnetic field case, conventional unitary behavior (all states are localized) is observed.

The eigenstates at the band center, which in the absence of diagonal disorder are believed to belong to the chiral unitary symmetry class, are shown to exhibit a $f(\alpha)$-distribution for not too weak random fields. The corresponding generalized multifractal dimensions are calculated and found to be different from the results known for a QHE-system.

Keywords: localization in 2d systems, random flux model, chiral symmetry, multifractality

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1 Introduction

The localization properties of non-relativistic electrons moving in a two-dimensional disordered system in the presence of static random magnetic fields (RMF) with zero mean have been discussed controversially for more than a decade. Investigations of Anderson localization of 2d electrons with random complex hopping matrix elements date back to the early eighties [1]. The actual study of this model was particularly inspired by physical situations encountered in high-$T_c$ superconductors and for composite-fermions in the quantum Hall effect at half filling factor.

Now, for lattice models of width $M$ and length $L$ with uncorrelated random magnetic fields the results gradually seem to converge to the notion that all states are localized (see, e.g., [2] and papers quoted therein). In the absence of diagonal disorder, the localization length for systems with $M$ odd is believed to diverge at $E = 0$ [3], where the eigenstates are suggested to become critical [4]. This special state at $E = 0$ is a consequence of the chiral symmetry [4, 5] which can be broken by diagonal disorder or, as reported in the case of quasi-1d systems, by application of periodic boundary conditions [6]. The random-flux model was shown to exhibit the same symmetry [7, 3] as the Gade-Wegner model [8]. The critical behavior is supposed to be similar to that of Dirac fermions with random gauge fields [9, 10, 11].
Recently, a metal-insulator transition has been asserted for the lattice random-flux model incorporating *spatially-correlated* random magnetic fields and additional diagonal disorder [11]. This conclusion was based on a finite-size scaling study of the longitudinal conductance which was calculated numerically.

The purpose of the present paper is twofold. First, we check the alleged metal-insulator transition by investigating the scaling behavior of the eigenvalue statistics. Second, the spatial fluctuations of the eigenstates near energy $E = 0$ are calculated and the scaling of the corresponding moments are analyzed. In the presence of diagonal disorder, we find only localized states for the spatially-correlated random flux model. Due to the huge localization length the wave functions near the band center show multifractal behavior which, however, differs from the quantum Hall case.

### 2 Model with correlated random flux

A model of non-interacting electrons on a square lattice is considered in a one-band tight-binding approximation. The spatially-correlated random fluxes are generated as in Ref. [11],

\[ \phi(m) = 4h_0/l_c^2 \sum_n R_n \exp(-|r_m - r_n|^2/l_c^2), \]  

where $h_0$ and $l_c$ are the strength and correlation length of the random magnetic fields. The uncorrelated random numbers $R_n$ are evenly distributed between $(-1, +1)$ and the $r_m, r_n$ denote the spatial positions of the flux plaquettes considered. The magnetic flux per plaquette, $\phi(m)$, enters the Hamiltonian via the Peierls phase factors in the transfer terms from site $k$ to the neighboring sites $k'$,

\[ H = \sum_k \varepsilon_k c_k^\dagger c_k + \sum_{\langle k \neq k' \rangle} V \exp(i\theta_{kk'}) c_k^\dagger c_{k'} \ldots \]  

The total flux through the plaquette labeled by $m$ is given by the sum of the phases $\theta_{kk'}$ associated with its four links. $V = 1$ is taken as the unit of energy and the lattice constant $a = 1$ as the unit of length. The diagonal disorder potentials $\varepsilon_k$ are realized by a set of uncorrelated random numbers taken from the interval $(-W, +W)$. The eigenvalues have been calculated for square lattices of size $L = 64, 128, 192$. In addition, some selected eigenstates were obtained for systems of size up to $L = 305$.

### 3 Results and discussion

The results of the level statistics are shown in Fig. [4] where the second moment, $I_0(W, L) = 1/2 \int_0^\infty P_{W,L}(s) ds$, of the level spacing distribution, $P_{W,L}(s)$, is plotted as a function of the system size $L$ and disorder strength $W$. We take the same parameters as in [11] so that the amplitude of the correlated random fields $h_0 = 1.0$, the correlation length $l_c = 5.0$, and the Fermi energy $E_F = -1.0$. The eigenvalues were calculated within the interval $(E_F - 0.25, E_F + 0.25)$. Many realizations have been considered such that the number of accumulated eigenvalues for each pair $(L, W)$ is $\approx 1 \cdot 10^5$. 
Fig. 1  Second moment of the level statistics, $I_0(W, L)$, versus disorder strength $W$. The system sizes are $L = 64$ ($\bigcirc$), $L = 128$ ($+$), and $L = 192$ ($\bigtriangledown$). The inset shows the corresponding single branch scaling curve.

We find that for $W > 2$, $I_0(W, L)$ increases with increasing size $L$. This behavior is expected for localized eigenstates which in the limit $L \to \infty$ obey the Poisson statistics ($I_P = 1.0$) of uncorrelated energy levels. For $W < 2$ no scale dependence is observed for the sizes investigated but $I_0(W, L)$ equals the random matrix result $I_{RMT} = 0.59$. Therefore, the localization length considerably exceeds the system size $L$. If there were a critical point below $W = 2$ as suggested in [11], $I_0(W_c, L)$ should be scale independent at the critical disorder $W_c$ and larger than $I_{RMT}$. Hence, we have to conclude that for the suggested parameters [11] there is no localization-delocalization-transition in square systems. The corresponding single branch scaling curve is shown in the inset of Fig 1. This result is similar to the situation found previously for the uncorrelated magnetic field case [12].

The increase of the conductance for small $W$ in quasi-1d systems reported in [11] is presumably a finite size effect because for narrow systems the increase in width $M$ may raise the conductance due to the additional transport channels. In the limit of large $M$, however, quantum interference will eventually localize the conductance.

Turning now to the analysis of the eigenstates, we remind that the spatial fluctuations of critical eigenstates $\psi_{E,L}(r)$ exhibit multifractal properties, i.e., the scaling of the appropriate moments, $P_q(l, E, L) = \left( \sum_{r \in \Omega(l)} |\psi_{E,L}(r)|^2 \right)^q$, follow power-laws with an infinite set of unrelated exponents $\tau(q) = (q - 1)D(q)$. Alternatively, one considers the corresponding $f(\alpha)$-distribution which for $|q| < 1$ can often be approximated by $f(\alpha) = D(0) - (\alpha - \alpha_0)^2/[4(\alpha_0 - D(0))]$, where in 2d for our lattice system $D(0) = 2$. Hence, $\alpha_0$ effectively characterizes the whole distribution.

In Fig. 2 the $f(\alpha)$-distribution of an eigenstate near $E = 0$ of the correlated random magnetic flux model is shown. Similar results are obtained for even $L$ or different $h_0$ and $l_c$ parameters as long as the localization length considerably exceeds the system size. In most cases, a parabola can be fitted with $\alpha_0 = 2.14 \pm 0.03$. This number is quite distinct from the value $\alpha_0 = 2.29$ of a QHE-system [13]. From the scaling of
the second moment we find a fractal dimension \( D(2) = 1.87 \pm 0.03 \) for the correlated random magnetic field case which also differs from the QHE value \( 1.6 \) \cite{14}.

Our values for spatially-correlated RMF are similar to those reported for the uncorrelated case. The authors of Ref. \cite{15} determined \( D(2) = 1.8 \pm 0.06 \) from the exponent \( \delta \) of the temporal decay of the autocorrelation function using the relation \( D(2) = 2\delta \) \cite{14}, and \( D(2) = 1.79 \) was extrapolated in Ref. \cite{16} from a multifractal analysis similar to ours. Multifractal wave functions have also been reported \cite{14} for massless Dirac fermions near \( E = 0 \) with a \( f(\alpha) \)-distribution very close to the one shown here.

We find two exceptions to the multifractal behavior at \( E \approx 0 \) discussed above. First, in the presence of diagonal disorder \( W > 1 \) a tendency to strong localization can be observed which is fully developed for \( W > 4 \). Second, if the amplitudes of the spatially-correlated \( (l_c = 5.0) \) random magnetic fields are weak, \( h_0 \lesssim 0.2 \), the eigenstates for odd system size \( L \leq 305 \) and \( W = 0 \) appear to be extended \( (\alpha_0 \approx 2.04) \) which may be due to the small \( L \) or an indication of the chiral unitary symmetry class.

References

[1] P. A. Lee and D. S. Fisher, Phys. Rev. Lett. 47 (1981) 882
[2] A. Furusaki, Phys. Rev. Lett. 82 (1999) 604
[3] J. Miller and J. Wang, Phys. Rev. Lett. 76 (1996) 1461
[4] M. R. Zirnbauer, J. Math. Phys. 37 (1996) 4986
[5] A. Altland and B. D. Simons, preprint cond-mat/9811134 (1999)
[6] C. Mudry, P. W. Brouwer, and A. Furusaki, preprint cond-mat/9903026 (1999)
[7] A. W. W. Ludwig et al., Phys. Rev. B 50 (1994) 7526
[8] R. Gade, Nucl. Phys. B 398 (1993) 499
[9] C. de C. Chamon, C. Mudry, and X.-G. Wen, Phys. Rev. Lett. 77 (1996) 4194
[10] Y. Hatsugai, X.-G. Wen, and M. Kohmoto, Phys. Rev. B 56 (1997) 1061
[11] D. N. Sheng and Z. Y. Weng, preprint cond-mat/9901019 (1999)
[12] M. Batsch, L. Schweitzer, and B. Kramer, Physica B 249-251 (1998) 792
[13] B. Huckestein, B. Kramer, and L. Schweitzer, Surf. Science 263 (1992) 125
[14] B. Huckestein and L. Schweitzer, Phys. Rev. Lett. 72 (1994) 713
[15] T. Kawarabayashi and T. Ohtsuki, Phys. Rev. B 51 (1995) 10897
[16] K. Yakubo and Y. Goto, Phys. Rev. B 54 (1996) 13432