Anomalous scaling dimensions and critical points in type-II superconductors

Asle Sudbø\textsuperscript{a,1}, Anh Kiet Nguyen\textsuperscript{a} and Joakim Hove\textsuperscript{a}

\textsuperscript{a}Department of Physics, Norwegian University of Science and Technology, N-7491 Trondheim, Norway

Abstract

The existence of a stable critical point, separate from the Gaussian and XY critical points, of the Ginzburg-Landau theory for superconductors, is demonstrated by direct extraction via Monte-Carlo simulations, of a negative anomalous dimension $\eta_{\phi}$ of a complex scalar field $\phi$ forming a dual description of a neutral superfluid. The dual of the neutral superfluid is isomorphic to a charged superfluid coupled to a massless gauge-field. The anomalous scaling dimension of the superfluid order-field is positive, while we find that the anomalous dimension of the dual field is negative. The dual gauge-field does not decouple from the dual complex matter-field at the critical point. These two critical theories represent separate fixed points. The physical meaning of a negative $\eta_{\phi}$ is that the vortex-loop tangle of the superfluid at the critical point fills space more efficiently than random walkers, without collapsing. This is due to the presence of the massless dual gauge-field, and the resulting long-ranged vectorial Biot-Savart interaction between vortex-loop segments, which is a relevant perturbation to the steric $|\psi|^4$ repulsion term. Hence, the critical dual theory is not in the universality class of the $|\psi|^4$-theory.

A charged superfluid (spin-singlet superconductor) is described by the Ginzburg-Landau theory of a complex matter field $\psi$ coupled to a massless gauge-field $A$, with coupling constant $2e$ and a local gauge-symmetry; here $e$ is the electron charge. In 3D, the dual of this theory is given by a dual complex matter-field $\phi$ coupled to a massive dual gauge-field $h$ with mass $e$, exhibiting a global $U(1)$-symmetry due to the mass of $h$ \cite{1}. It renders the critical point of the theory in the universality class of the $|\phi|^4$-theory. In renormalization group sense, the short-range interaction mediated by the massive $h$, is an irrelevant perturbation to the steric repulsion stemming from the $|\psi|^4$-term \cite{2}. However, in the limit $e \to 0$, the dual gauge-field becomes massless, and the symmetry of the problem changes to a local gauge-symmetry. This profoundly changes the physics.

In this case, the $|\psi|^4$-theory of the 3D superfluid, $H_{\psi} = |\partial_i \psi|^2 + m_{\psi}^2 |\psi|^2 + (u_{\psi}/2)|\psi|^4$ has a dual theory given by $H_{\phi, h} = |(\partial_i - ig h_i)\phi|^2 + m_h^2 |\phi|^2 + (u_{\phi}/2)|\phi|^4 + (1/2)(\nabla \times h)^2$ \cite{1}. The coupling constant $g$ is given by $g = 2\pi m_h/e$, where $m_h = e\omega$, where $\omega$ is the amplitude of the order-field $\psi$. Thus, the dual of the neutral superfluid is isomorphic to the Ginzburg-Landau theory of a charged superfluid coupled to a massless (dual) gauge-field $h$. In Ref. [3] it is argued that as $T \to T_c^-$, $\omega \to 0$. This would imply that at and above $T_c$, $h$ decouples from $\phi$. Thus, the critical dual theory would
be that of a $|\phi|^4$-theory, i.e. the superfluid and its dual theory would be self-dual.

This is a mean-field argument. At the true $T_c$, $\omega > 0$, and hence the dual gauge-field does not decouple from $\phi$ as $T \to T_c^-$. At the true critical point, the dual of the $|\psi|^2$-theory is not a $|\phi|^4$-theory, in particular the symmetries of $H_\phi$ and $H_{\phi,h}$ are different. We thus expect the critical exponents of these two theories to be different, describing two separate critical points. This has been checked by large-scale Monte-Carlo simulations [4].

The main point of this short communication is the following. Direct evaluation of the anomalous dimension of the matter field $\psi$, yields \( \eta_\psi = 0.038 \) [5]. In contrast, the anomalous dimension of the dual field $\phi$, is given by $\eta_\phi = -0.18 \pm 0.07$ [4]. The latter result is obtained in our Monte-Carlo simulations by analysing the statistics of vortex-loops in the 3DXY-model, using a mapping to the dual theory. Therefore, we demonstrate that the critical behavior of the superfluid, which is a $|\psi|^2$-theory, is in a different universality class than the critical behavior of the dual theory. The results of Ref. [6] thus rest on a firm theoretical footing, in that the existence of a novel charged fixed point need not be assumed, it is demonstrated.

The results for the vortex-loop distribution function $D(p)$ as a function of vortex-loop perimeter $p$ are shown in Fig. 1 [4]. We fit $D(p)$ to the form $D(p) = Ap^{-\alpha} \exp(-\epsilon(T)/k_BT)$ where $T$ is temperature, $\alpha \approx 2.5$, and $\epsilon(T)$ is the line tension of the thermally generated vortex-loops, the topological defects destroying superfluid order [4].

We fit $T$-dependence of $\epsilon(T)$ to the form $\epsilon(T) \sim |T - T_c|^\gamma$ [4]; $\gamma$ is identified as the susceptibility exponent $\gamma_\phi$ of the dual field $\phi$ [4]. By the scaling law $\gamma_\psi = \nu_\phi(2 - \eta_\phi)$ and the observation that $\nu_\psi = 0.672$ [5], we extract $\eta_\phi$ from the line-tension of the vortex-loops of the 3DXY-model. We find $\gamma_\phi = 1.45 \pm 0.05$, and hence $\eta_\phi = -0.18 \pm 0.07$. This should be compared to $\eta_\psi = 0.038$ [5]. Thus, the interaction mediated by the massless gauge-field $h$ is a relevant perturbation to the non-trivial fixed-point of the $|\phi|^4$-theory, in renormalization group sense.

We have established a difference in sign of $\eta_\psi$ and $\eta_\phi$ via Monte-Carlo simulations of the vortex-loop gas of the neutral superfluid at the critical point. Hence, $h$ does not decouple from $\phi$ at $T_c$. The dual theory is isomorphic to a superconductor coupled to an electromagnetic gauge-field. Thus, we establish the existence of a new stable charged fixed point of the Ginzburg-Landau theory, of the type proposed in Ref.[6].

This dual description is also useful in investigating broken symmetries in the vortex-liquid phase of type-II superconductors in magnetic fields [7].

References

[1] H. Kleinert, in Gauge fields in condensed matter, (World Scientific Press, Singapore, 1989), Vol. 1.
[2] C. Dasgupta and B. I. Halperin, Phys. Rev. Lett., 47, 1556 (1981).
[3] M. Kiometzis and A. M. J. Schakel, Int. J. Mod. Phys. B 7, 4271 (1993).
[4] A. K. Nguyen and A. Sudbø, submitted to Phys. Rev. B. J. Hove, A. K. Nguyen, and A. Sudbø, in preparation.
[5] See M. Hasenbusch and T. Török, condmat/9904408, and references therein.
[6] I. F. Herbut and Z. Tesanović, Phys. Rev. Lett., 76, 4588 (1996); ibid, 78, 980 (1997).
[7] Z. Tesanovic, Phys. Rev. B 59, 6449, (1999). A. K. Nguyen and A. Sudbø, Europhys. Lett., 46, 780 (1999).