Mapping a photonic random walk by tuning the coupling coefficient

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We present a method to map the evolution of photonic random walks that is compatible with nonclassical input light. Our approach is based on the insight that tuning the jumping rate between spatial modes can be equivalent to varying the evolution time. In a proof-of-principle demonstration we reconstruct the evolution of photons through an array of coupled waveguides by tuning the coupling coefficient and monitoring the end-face. This approach enables direct observation of mode occupancy at arbitrary resolution, extending the utility of photonic random walks for quantum simulations and related applications.

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Quantum random walks have been studied extensively in the context of quantum computing and simulation [1–4]. Systems built around a quantum random walk have been proposed as physical simulators for a wide variety of quantum [5] and classical [6] phenomena. When random walks are leveraged for physical simulation the dynamics and evolution of the random walker as it traverses the graph are often of primary interest.

In the experimental domain, quantum random walks have been demonstrated across a range of physical systems, employing trapped particles [7–9] and photons [10–14]. The field of photonic random walks is particularly developed, owing to the ease of access to long coherence times and the ability to perform high fidelity manipulation of single quanta using relatively low cost devices. Within this class, devices comprising waveguide arrays have proved popular due to their favourable scaling properties [15].

When simulating a physical system, access to the evolution of the random walker is usually provided by excitation of fluorescence in the host device [11, 12]. Unfortunately, this signal is unable to capture non-classical properties of light, and due to its inherently low efficiency such experiments utilize strong laser illumination. Systems which leverage non-classical states of light such as squeezed states or multi-photon entangled states are only able to obtain a snapshot of the random walk at a fixed propagation length, i.e. the output face [19]. Even where the evolution of the random walker is not of primary interest, access to this information may be desired as a diagnostic or calibration tool.

We have designed and implemented an optical circuit in which the evolution of a photonic random walk can be observed in a chip of constant length. In our system, a sequence of observations at the end face combined with appropriate tuning of the device parameters exposes the evolution of the input state in a manner that is compatible with single photon intensities, and extensible to multiphoton quantum states. We demonstrate this technique by implementing the well-studied one-dimensional, continuous-time random walk.

A continuous-time, discrete-space random walk can be described with a graph containing a set of vertices connected with edges (as depicted in Fig. 1 middle row). A spatial step is represented by the transition between two vertices connected by an edge. Given the probability of a transition occurring per unit time as \( \gamma \), the probability of a random walker occupying vertex \( a \) in time \( t \) follows

\[
\frac{dp_a(t)}{dt} = -\sum_b M_{ab} p_b(t). \tag{1}
\]

The matrix \( M \) encodes nearest-neighbour coupling with elements \( a, b \) defined as

\[
M_{ab} = \begin{cases} 
-\gamma & \text{if } a \neq b, \text{ and } a \text{ and } b \text{ are connected}, \\
0 & \text{if } a \neq b, \text{ and } a \text{ and } b \text{ are not connected}, \\
\beta \gamma & \text{if } a = b.
\end{cases} \tag{2}
\]

Here \( \beta \) is the number of vertices that are connected to site \( a \), and \( \gamma \) is the jumping rate, which may be identified as the coupling coefficient. For a quantum random walk, a state introduced to this system will evolve along the propagation direction \( z \) following a unitary operator \( \hat{U} \) given by \([20, 21]\)

\[
\hat{U} = \exp (-i M z) \tag{3}
\]

The evolution of the system depends on both \( z \) and \( \gamma \), which appear as a product in the elements of \( \hat{U} \). As the units of \( \gamma \) are inverse distance, \( \gamma z \) is a dimensionless variable identifying a unique location in the random walk. Physically this implies that two different systems,
FIG. 1. The operational concept for a random walk in a waveguide array mapped via modification of the coupling coefficient (denoted $\gamma$).

Top, a stylized profile of the waveguide array cut orthogonal to the propagation direction. Centre, the associated graph representation, a 1D array of vertices (circles) connected by edges (lines). Bottom, simulated evolution of a random walk on the structure depicted above, over an interval \( \{z : 0, 200\} \). The left column illustrates the initial array, for which $\gamma = 0.02$, while the right column illustrates a tuned configuration, with $\gamma = 0.01$. The associated range of the product $\gamma z$ is also shown, with the tuned configuration (right) exhibiting evolution over precisely half the range of its undistorted counterpart (left). All units are arbitrary.

observed over the same interval of $\gamma z$, will evolve identically. In other words, the same quantum random walk is observed either by modification of the propagation distance or the jumping rate. Hence jumping rate and propagation distance determine the system dynamics on an equal footing, allowing them to be studied interchangeably in experiment.

A visual representation of this scheme is shown in Fig. 1. The evolution of optical modes in a one dimensional random walk is obtained following Equations 2 and 3 and plotted for two different conditions. Following our insights, we plot the random walk as a function of $\gamma z$ for each step of the evolution, enabling representation of the random walk on a standardized scale. In the example depicted, the second (tuned) state exhibits a 50% reduction in the coupling coefficient, and consequently the evolution of its random walk is halted at half the range of its unmodified counterpart.

We propose that a quantum random walk can be mapped through the continuous tuning of the coupling coefficient of a single waveguide array. This contrasts with previous studies where the spatial evolution of photons was sampled at the output of multiple devices of different length [15, 22]. Experimentally, our approach requires a waveguide array fabricated on a suitable platform to enabled controlled modification of the coupling coefficient. In our earlier work [23], we developed a waveguide platform in the soft polymer polydimethylsiloxane (PDMS), and demonstrated continuous tuning of a single beamsplitter by stretching of the host chip. The ability to control the separation between neighbouring waveguides of an array through elastic stretching allows us to vary the coupling coefficient, without significantly affecting the device length.

As a proof of principle experiment, we fabricate a 1D array of waveguides on a PDMS chip using the casting technique previously reported [23]. The device consists of an array of 51 single mode rib waveguides with a pitch of 17.5 $\mu$m. On each side of the array, an isolated pair of waveguides act as “reference structures”, with simple coupling behaviour [24] enabling independent measurement of the system-wide coupling coefficient. A simplified schematic of the chip is shown in Fig. 2, together with a microscope image of the device end-face.

FIG. 2. A simplified representation of the array device, comprising an evanescently coupled array of rib waveguides with a 17.5 $\mu$m pitch defined in a dual-layer polydimethylsiloxane chip. Also shown are the reference structures: pairs of waveguides flanking the main array. The optical microscope image shows the end-face of a typical device.

For controlled stretching along the direction transverse to mode propagation, we mount our chip on a jig driven by a miniature translation stage. Light is input into the array through edge-coupling a single mode optical fiber to the desired waveguide channel. The output of the
device is then imaged onto a CMOS camera via a 5× microscope objective lens. The output corresponding to each deformation of the device is recorded, then stacked and aligned to visualise the evolution of the random walk. The values of $\gamma z$ corresponding to each step in the walk are determined from the reference structures.

Intensity distributions at the end-face with no stretching and with maximum stretching are shown in Fig. 3. The spatial mode profiles are similar and relatively unaffected by the global distortion, supporting the assertion that to first order, device tuning only results in a change of separation between neighbouring modes. A larger number of excited modes is observed for larger values of $\gamma z$. As the device is stretched, coupling between neighbouring waveguides decreases, resulting in a reduced power transfer from the source waveguide and hence fewer excited modes. The decrease in coupling due to stretching allows us to probe the random walk at different stages of the evolution.

![Intensity distributions observed from the end-face of the waveguide array after a propagation distance of ~7 mm, with the central waveguide coupled to a laser source of 532 nm. The excited optical modes at the output are shown. Top: Optical modes observed from the initial device state. Bottom: Optical modes observed when the device is maximally stretched, with a chip-scale distortion of approximately +10%. The decrease in the number of excited modes is a consequence of the decreased coupling coefficient between waveguides in the array, and enables us to probe the field structure at an earlier stage of the random walk. Measured values of $\gamma z$ are 0.2 and 0.6 for the stretched and unstretched states respectively.

FIG. 3. Intensity distributions observed from the end-face of the waveguide array after a propagation distance of ~7 mm, with the central waveguide coupled to a laser source of 532 nm. The excited optical modes at the output are shown. Top: Optical modes observed from the initial device state. Bottom: Optical modes observed when the device is maximally stretched, with a chip-scale distortion of approximately +10%. The decrease in the number of excited modes is a consequence of the decreased coupling coefficient between waveguides in the array, and enables us to probe the field structure at an earlier stage of the random walk. Measured values of $\gamma z$ are 0.2 and 0.6 for the stretched and unstretched states respectively.

The coupling coefficient between neighbouring modes is also expected to vary with wavelength, due to the wavelength dependence of the spatial mode profile. Several wavelengths of light can be used to map out distinct ranges of $\gamma z$. By varying the wavelengths of the input light, we can explore an extended random walk by combining the random walks from each wavelength.

Experimental and simulated quantum random walks for wavelengths of 450 nm, 532 nm and 630 nm are shown in Fig. 4. Reconstructed evolutions qualitatively match their simulated counterparts for all three wavelengths. For 630 nm, we observe an offset in the values of $\gamma z$ compared to the simulated distribution. At this wavelength, the reference structures did not provide sufficient resolution to capture the magnitude of $\gamma z$ with high precision. We are exploring alternative designs for reference structures in future experiments.

The results of our experiment demonstrate the validity of this method for observing the evolution of optical modes in quantum random walks. By using a tunable photonic device to control the coupling coefficient between copropagating optical modes, we are able to reconstruct the evolution of random walks with extended range. Random walks are observed over a range of $\gamma z = 0$ to 1.3, with wavelengths from 450 nm to 630 nm. These measurements are performed on a low cost device, fabricated following polymer casting techniques that are broadly accessible to the research community. Combined with a straightforward and intuitive mechanism to tune their optical properties, integrated chips of this type represent an attractive platform for research in the field of photonic simulators.

This approach to observing a random walk provides a specific advantage compared to other chips used in quantum photonic experiments [11, 13, 16, 17] in that it does not require the use of fluorescence for visualisation of field evolution. This is an important consideration in experiments employing light of extremely low intensity, such as the single-photon regime. In particular, it should be possible using these devices to reconstruct the evolution of a wide variety of non-classical states in photonic random walks. For example multi-photon states can be used to simulate random walks in higher dimensional graphs [12], or of particles obeying non-Bosonic statistics [19, 25].

The approach demonstrated here (utilizing chip-scale stretching) is directly applicable to any graph structure in which the jumping rate $\gamma$ may be factorized from the evolution operator. It can be adapted for more complex graphs with a corresponding increase in complexity of mechanical tuning. For example, out-of-plane distortions could enable non-uniform, localized changes, and with suitable actuation, one can even consider dynamical changes to the underlying array. The technique need not be restricted to continuous-time random walks, as localized tuning would also enable the effective removal of successive generations of splitters in a discrete-time, discrete-space system [25] to similar effect. Combined, we believe that these techniques have the potential to greatly expand the capabilities of the photonic random walk as a platform for quantum simulations.

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FIG. 4. Photonic random walks observed for a range of wavelengths: (a) and (d) 450 nm, (b) and (e) 532 nm, (c) and (f) 630 nm. The experimentally observed random walks (d)-(f) were constructed from the output intensity distribution of the device (see Fig. 3) observed over a large tuning range (up to ∼630 nm). They are in good agreement with their simulated random walks, (a)-(c) respectively. The range of random walk changes with the wavelength of the input source, with the longest wavelength resulting in a larger range probed.