Probabilistic temperature forecasting: a summary of our recent research results

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Abstract

We summarise the main results from a number of our recent articles on the subject of probabilistic temperature forecasting.

1 Introduction

Industries such as finance, insurance, reinsurance and energy have been using quantitative probabilistic methods for many years. But when these industries incorporate weather forecasts into their calculations they tend to only use forecasts of the expectation of future outcomes. Why don’t they use probabilistic forecasts? There are a number of reasons, including the following:

• Adequate probabilistic meteorological forecasts are not available commercially. There are two parts to this problem. First, that very few forecast vendors produce probabilistic forecasts at all and second that those that are produced are not correctly calibrated.

• There is considerable confusion in the academic meteorological literature about what a probabilistic forecast even is, with the words ‘ensemble forecast’ and ‘probabilistic forecast’ often being used as if they are the same thing.

• There is considerable confusion about how ensemble forecasts, and in particular the ensemble spread, should be interpreted and used.

• The methods suggested in the academic literature for making probabilistic forecasts are complicated, ad-hoc, poorly understood and mostly contain obvious flaws. Complex new methods have been introduced but have not been compared with simpler methods.

• Meteorologists have generally evaluated probabilistic forecasts using methods they have invented themselves rather than using well known and well understood methods from statistics. This is not to say that the methods used by meteorologists are not intrinsically good ones (see further comments on this below), but just that they are not well understood or accepted by anyone outside the rather narrow field of meteorological forecast verification. There are methods, however, that are used throughout statistics to solve essentially the same problem and that are well understood by much of the statistical and scientific community.

• Past forecasts are generally very hard to get hold of. Users of forecasts are generally rather suspicious of the claims of forecast vendors, and the fact that neither forecast vendors nor modellers make their past forecasts available compounds this problem. It also makes it hard for users to evaluate whether forecasts are any good. We ourselves have extreme difficulties in obtaining past forecasts, even for research purposes.

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1Our rather straightforward view is that a probabilistic forecast is a forecast that gives probabilities, while an ensemble forecast is a forecast that consists of many members. They are not the same thing: an ensemble forecast is not a probabilistic forecast because it doesn’t give probabilities, and a probabilistic forecast is not an ensemble forecast because it doesn’t contain members. The two are, of course, related: probabilistic forecasts can be made from single or ensemble forecasts using statistical methods, and probabilistic forecasts can be turned into ensemble forecasts by sampling. From the point of view of temperature forecasting, the main reason for running ensemble forecasts is not to generate probabilities, but to improve the forecast of the expectation: see Jewson (2003a).
We would like to use probabilistic forecasts of site-specific temperatures, and, given the problems listed above, have decided to investigate ourselves how such forecasts might be produced. Our hope is that the methods we have developed might be taken up by forecast vendors from whom we could then obtain the resulting forecasts. We have published a series of articles describing the results of our research, and the purpose of this current article is to summarise the results thus far. The articles on which this summary is based are Jewson et al. (2003), Jewson (2003d), Jewson (2003a), Jewson (2003c), Jewson (2003b), Jewson (2004). There articles are all freely available on the internet, and comments are welcome.

2 Modelling philosophy

We believe that it is crucially important to use an appropriately scientific modelling philosophy, and that such a philosophy is generally lacking in the academic literature on this subject. Our philosophy is as follows:

- To start with extremely simple models and build up, step by step, to more complex models. At each stage we compare the more complex models with the simpler models, and attempt to understand what each new effect brings to the forecast. As we will see below, this approach has led to significant insight into what is important and what not in the generation of probabilistic forecasts. We don’t have any particular axe to grind with respect to different techniques such as classical statistics, Bayesian statistics or analogue methods. However we do believe that classical statistics offers the simplest models and so we start there. If other methods can be shown to be materially better than the classical statistical methods that we propose we would be happy to adopt them.

- To test everything empirically. For instance, it is often claimed that the ensemble spread is a measure of the uncertainty in a forecast, and this is assumed to be true by a number of the calibration methods that have been suggested in the academic literature and are used to make commercial probabilistic forecasts. Whether spread really is a useful measure of the uncertainty in a forecast may or may not be true and will likely depend on the forecast model, the variable being predicted, and other factors. But this question should always be tested by including and excluding the spread and observing the effect on the final forecast skill. Considering the spread-skill correlation is not relevant to this question since it does not consider the affects of spread on the skill of the final forecast produced.

- To try and find the optimum blend between information from forecast models and from past forecast error statistics. There is a general principle that should be followed when designing calibration models: forecast models tell us about variability, while only past forecast error statistics can tell us constants. This is because the processes that determine constants are not included in the forecast models. This principle applies equally well to the mean temperature, the forecast uncertainty and the forecast correlation. For instance the mean level of uncertainty and the amplitude of variability of the uncertainty must both come from past forecast error statistics and should not be taken directly from model output.

3 Statement of the problem

Weather forecast information is produced from numerical models of the atmosphere. The information from these models, however, should not be thought of as consisting of a prediction, but rather as a set of predictors. These predictors consist of ensemble members, ensemble means and single integrations, possibly from several models. As mentioned above, it is dangerous to make assumptions about what information there may be in any of these predictors: everything should be tested empirically. An additional source of information is contained in the performance of forecasts in the past. The challenge is to take these sources of information and combine them to produce an optimal probabilistic prediction of the weather.

4 Forecast scoring

Scoring probabilistic forecasts is all about putting samples from the distribution of weather variability up against the predicted distribution and comparing them. The generic statistical question of comparing observations with a distribution was addressed in Fisher (1912) and Fisher (1922). In these papers
Fisher suggested the likelihood, defined as the probability of the observations given the model, as a useful measure. The likelihood rapidly gained wide acceptance in statistics as the most appropriate way to measure the goodness of fit of a distribution and has been used ever since. There are many variations of the likelihood such as the various information criteria (AIC, BIC, SIC). We use the likelihood (or versions thereof) for all of our forecast scoring.

Academic meteorologists have tended to use other measures, however, and in particular one measure known as the Brier score [Brier, 1950]. The Brier score is popular principally because it can be decomposed in a way that is similar to a well known decomposition of the mean square error. We believe, however, that the Brier score is not a sensible generic measure for probabilistic forecasts. Consider the following example: forecast 1 predicts a probability of 10% for a certain event, while forecast 2 predicts a probability of 0%. The real probability is 4%. The Brier score chooses forecast 2 as best, while we believe that the correct choice would be forecast 1. This problem is also shared by derivatives of the Brier score such as the CRPS, and is discussed in more detail in Jewson (2003).

5 Forecast comparison

Forecasts can be compared in 3 ways: in sample, out of sample or in real forecast mode. The latter is the ideal, but is seldom practical. In sample testing can be used for the comparison of parametric calibration models. If the models being compared have the same number of parameters then likelihood scores can be used directly. If they do not then one has to use one of the various information criteria to correct for over-fitting. Out of sample testing should be used for all non-parametric models. The results we describe below were based on a combination of in and out of sample testing.

6 Data

All of our calibration experiments have been based on a single year of daily ECMWF forecasts for London Heathrow. These have been compared with the appropriate climate observations. This data is deseasonalised, and calibration is applied to the anomalies. We are well aware that using only a single year of data is not ideal. However forecast models are changed rather frequently, and if much more than a year of data is used it is unlikely to be stationary, thus invalidating the whole idea of building a statistical model between model output and observations.

7 Calibration methods and results

The calibration methods we have tested are listed below, along with a brief description of the main results. The details are given in the relevant articles. We fit all the models described below by maximising the likelihood: in some cases this requires numerical methods.

7.1 Linear regression

Our first model is simply linear regression between temperature on day $i$ ($T_i$) and the ensemble mean on day $i$ ($m_i$), which we write as

$$T_i \sim N(\alpha + \beta m_i, \gamma)$$

This model corrects biases using $\alpha$, optimally "damps" the variability of the ensemble mean and merges optimally with climatology using $\beta$, and predicts flow-independent uncertainty using $\gamma$. The bias and the uncertainty produced by this model vary seasonally because of the deseasonalisation and reseasonalisation steps.

Linear regression has been used for calibration of forecasts of the expected temperature since at least the early 1970s [Leith, 1974] but it has not always been realised that it also yields a probabilistic forecast. We believe that all probabilistic calibration experiments should start with linear regression as a baseline for comparison. It is very simple, very easy to use, very well understood and gives good results.
7.2 Linear regression with the uncertainty predicted using the ensemble spread

It is often claimed that the ensemble spread contains information about the uncertainty in a forecast and so it makes sense to adapt linear regression so that the uncertainty is given by the ensemble spread rather than being fitted using past forecast error statistics. This gives

$$T_i \sim N(\alpha + \beta m_i, s_i)$$

(2)

where $s_i$ is the ensemble spread on day $i$. When we tested this model in Jewson (2003d) we found that the resulting forecasts performed significantly worse than linear regression, and even worse than climatology at short lead times. The explanation for this is that the uncalibrated ensemble spread underestimates the uncertainty in the forecast, which has been well known for a number of years.

7.3 Linear regression with the uncertainty predicted as a multiple of the ensemble spread

An obvious way to overcome the problem with the previous model is to scale the ensemble spread. This is then similar to the methods suggested by Roulston and Smith (2003) and Mylne et al. (2002). This gives:

$$T_i \sim N(\alpha + \beta m_i, \delta s_i)$$

(3)

When we tested this model we found that the resulting forecasts were worse than those from linear regression. The reason for this is that a single scaling factor transformation of the ensemble spread sets both the mean level of the uncertainty and the amplitude of the variability of the uncertainty with only one free parameter. The mean level of uncertainty is underestimated in the ensemble, and $\delta$ is forced to be greater than one to correct for this. However, a $\delta$ greater than one also inflates the variability in the uncertainty. In the forecast data we looked at the variability of the uncertainty actually needs to be deflated (see below).

7.4 Linear regression extended to include non-normality using kernel densities

It has been suggested that the non-linear dynamics of the atmosphere might lead to non-normality in the distribution of future temperatures. For this reason it seems sensible to relax the assumption of normality in the linear regression model, and we used a flexible kernel density model for this purpose. We call this model kernel regression (Jewson, 2003c):

$$T_i \sim K(\alpha + \beta m_i, \gamma, \lambda)$$

(4)

where $\lambda$ is the bandwidth of the kernel density. The results from the kernel regression model showed no improvement over the normal model, however, indicating that any non-normality present in the ensemble does not contain useful information. We don’t know whether this is because the ensemble doesn’t contain much normality, or because it does contain non-normality but that that non-normality is unrealistic.

We do not conclude that such kernel methods should not be used. They perform as well as the normal distribution, and are a reasonable alternative for those who dislike imposing highly parametric models.

7.5 Linear regression with the uncertainty predicted using a linear function of ensemble spread

We have seen that a simple scaling of the ensemble spread did not improve on linear regression because it tries to calibrate the mean and the amplitude of the variability of the ensemble spread at the same time. In this model we calibrate them separately. We call this model spread regression (see Jewson et al. (2003) and Jewson (2003d)).

$$T_i \sim N(\alpha + \beta m_i, \gamma + \delta s_i)$$

(5)
The parameters in this model were all significantly different from their default values out to day 10, justifying their inclusion in the model. That $\delta$ is significantly different from zero indicates that there is some kind of spread-skill relationship.

We find that while the mean of the uncertainty is best predicted by increasing the ensemble spread (as we saw in section 7.3) the variability of the uncertainty is best predicted by decreasing the amplitude of the variability of the ensemble spread (i.e. $\delta$ is less than 1). In fact we find that in the calibrated forecast the variability of the uncertainty is rather small (between 5% and 20% of the mean level): the calibrated forecasts show more or less the same level of uncertainty every day with only small variations. That singular-vector forecasting systems such as that used at ECMWF would tend overestimate the amplitude of the variability in the uncertainty in this way is to be expected (Jewson et al., 2004).

We have learnt an important lesson from the spread regression model: that calibration schemes should treat the mean and the amplitude of the variability of the uncertainty separately. A simple thought experiment is useful to test whether a calibration scheme satisfies this requirement. Imagine that the ensemble spread contains no information whatsoever: does the calibration scheme ignore it and estimate uncertainty based on past forecast error statistics alone? If not, the scheme is unlikely to be as good as the spread regression model, and may not even be as good as linear regression. Interestingly, most of the published calibration schemes (such as Roulston and Smith (2003), Mylne et al. (2002) and Raftery et al. (2003)) and all the the commercially calibrated ensemble or probabilistic forecasts we have seen fail this test. We therefore recommend that these schemes and forecasts should not be used until they have been adapted to calibrate the spread in a more appropriate way.

The results from the spread regression model were the first of our results to beat the linear regression model, although the difference was very small and could not be detected as significant in out of sample tests. Why is the benefit from using the spread so small? Presumably because the variations in the calibrated uncertainty forecast are not large relative to the mean uncertainty and relative to the variability in the mean temperature. Why, then, are the fluctuations in the predicted uncertainty so small? This could be either because the ensemble does a poor job in predicting the fluctuations in the uncertainty, or because the fluctuations in uncertainty are inherently small. We don’t know how to distinguish between these two cases.

Another lesson we have learnt is that a statistically significant spread skill relationship does not guarantee that the ensemble spread is a useful predictor: we found a significant relationship but saw only a tiny benefit from actually using the spread as a predictor.

Comparing the benefit of using the ensemble mean with the benefit of using the ensemble spread (Jewson, 2003) we see that the ensemble mean is dramatically more useful. The spread is most useful at the shortest leads, but even then the ensemble mean is still much more useful.

### 7.6 Linear regression with the uncertainty predicted using a linear function of ensemble spread and non-normality using kernel densities

Our next model is a combination of two of the previous models: we extend the spread regression model to include non-normality using the kernel density, to give the kernel spread regression model.

$$T_i \sim K(\alpha + \beta_m, \gamma + \delta_s, \lambda)$$  \hspace{1cm} (6)

As with the kernel regression model we find no benefit from including non-normality (Jewson, 2003c).

### 7.7 Linear regression with seasonally varying parameters

Everything in meteorology varies seasonally, and there is no particular reason to think that the optimum parameters in our calibration models should be any different. We thus tested a number of the preceding models but with seasonally varying parameters (Jewson, 2004).

The most complex of the models we tested was:

$$T_i \sim N(\alpha_i + \beta_i m_i, \gamma_i + \delta_i s_i)$$  \hspace{1cm} (7)

where

$$\alpha_i = \alpha_0 + \alpha_s \sin \theta_i + \alpha_c \cos \theta_i$$

$$\beta_i = \beta_0 + \beta_s \sin \theta_i + \beta_c \cos \theta_i$$

$$\gamma_i = \gamma_0 + \gamma_s \sin \theta_i + \gamma_c \cos \theta_i$$

$$\delta_i = \delta_0 + \delta_s \sin \theta_i + \delta_c \cos \theta_i$$
We also tested a number of models with seasonality in only some of the parameters. Adding seasonally varying parameters gave a huge improvement in our forecasts, including a big improvement in the linear correlation between the forecast and observed anomalies. This was mainly due to the inclusion of seasonally varying bias correlation, although making the other parameters vary seasonally helped too. Interestingly there was a synergistic effect whereby making all the parameters vary seasonally at once gave a greater benefit than the sum of the benefits from making them vary seasonally individually. The importance of seasonally varying bias correction can be interpreted as being due to discrepancies between the seasonal cycles in the observations and the model.

7.8 Correlation calibration

In Jewson (2003b) we considered how best to predict the correlation between different days of a forecast. Our results showed that correlations predicted from past forecast error statistics were more accurate than those predicted directly from the ensemble, but that an 80-20 mix of past forecast errors statistics and correlations based on the ensemble gave better predictions than either alone.

8 Summary of results

We summarise our main results as follows:

- The linear regression model is a very good starting point for the comparison of different calibration methods.
- We have found it very hard to see any more than a tiny benefit from using the ensemble spread.
- We found it essential to calibrate the spread using at least two degrees of freedom: one for calibrating the mean level of the spread and the other for calibrating the amplitude of the variability of the spread. We consider all other calibration methods that we have seen in the literature or in the commercial sector to be flawed because they confuse the calibration of the mean and the variability of the spread.
- We did not find any benefit from using non-normal rather than normal distributions.
- Extending the linear regression model to include seasonally varying parameters was extremely beneficial.

Putting all this together, we currently recommend the seasonal parameter linear regression model as both the best method for producing probabilistic forecasts and an appropriate baseline for judging new calibration methods.

8.1 Recommendations for forecast users

Forecast users should be wary of using the forecast calibration methods described in the academic literature, and wary of the probabilistic and ensemble forecast products currently available from forecast vendors. We are not convinced that any of these methods have been well tested or do as well as the seasonal-parameter linear regression model described above, and they are certainly much more complex and liable to model error.

8.2 Recommendations for forecast vendors

We believe that forecast vendors need to take a simple, transparent and pragmatic approach to producing probabilistic forecasts. They should start by using the seasonal-parameter linear regression model given above (or something similar) and progress beyond that model only when more complex models have been empirically proven to be better.

8.3 Recommendations for numerical modellers

It is slightly disappointing that we haven’t been able to find more use for the ensemble spread. One of the reasons for this is that the information contained in the ensemble spread is subtle and hard to calibrate. A great help in this respect would be longer series of past forecasts from stationary models.
8.4 Recommendations for calibration research

The production of probabilistic meteorological forecasts is still in its infancy, as this article has showed. There are many areas for future research. This includes:

- Trying to beat the models described above, perhaps using different transformations of the ensemble spread, perhaps using Bayesian methods, perhaps by calibrating full fields rather than anomalies, perhaps using high resolution forecasts, etc.
- Testing the models described above on different locations.
- Testing the models described above on longer forecast records.
- Applying the same straightforward calibration philosophy to wind and precipitation forecasts.

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