Entanglement between two Rydberg atoms induced by a thermal field

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Abstract. We investigated two Rydberg atoms successively passing a vacuum or a thermal cavity taking into account the detuning. The atoms was assumed to be initially prepared in the Bell types entangled atomic states. Calculating the negativity we investigated the dynamics of atom-atom entanglement both for the vacuum and the thermal field. The special features of negativity behavior have been studied comprehensively foe small and large values of detunings. For thermal field and small detunings we established the effect of sudden death and birth of entanglement.

1. Introduction
Entanglement is not only one of the most surprising features of quantum theory, but also provides an important resource for various quantum information processes such as quantum information, quantum communication, and quantum cryptography. Therefore, great efforts have been made to investigate entanglement characterization, entanglement control, and entanglement production in different systems. Much attention is given to a system containing two two-level atoms, since they can represent two qubits, the building blocks of the quantum gates that are essential to implement quantum protocols in quantum information processing. Two-atom entangled states have been demonstrated experimentally using Rydberg atoms, cold trap ions, impurity spins in solids, superconducting circuits and cavity quantum electrodynamics schemes (cavity QED) [1]. Cavity QED with Rydberg atoms has been a favorite tool to checking the foundations of quantum mechanics including entanglement. Rydberg atoms was proposed a two decade ago to implement quantum gates between neutral atoms. The theoretical and experimental aspects of Rydberg-mediated quantum information processing received for the first decade are reviewed by M. Saffman and T. Walker [2]. The newest review of quantum computation with neutral atom qubits are presented by M. Saffman an coauthors [3, 4]. A brief overview of the current state of the experimental research on the development of the element base of quantum computers with qubits based on single neutral atoms trapped in optical traps are made by I. Ryabtsev and coauthors [5].

The entanglement between two initially independent Rydberg atoms successively passing a vacuum cavity have been demonstrated by S. Haroche et all. [6]. The entanglement procedure involves the resonant coupling, one by one, of the atoms to a high Q microwave superconducting cavity. The atoms, prepared in circular Rydberg states, exchange a single photon in the cavity and become entangled by this indirect interaction. Later S. Haroche and coauthors demonstrated
the multiparticle entanglement for two atoms and a single-photon cavity mode [7]. A number of theoretical works have been devoted to investigations of entanglement properties of two atoms passing through cavities one after another, thereafter interacting with a various state radiation fields inside the cavities [8]-[13]. In our paper [14] we examined the dynamics of entanglement of two initially entangled Rydberg atoms successively passing a thermal cavity with different velocities. In this paper we continued this investigations taking into account the detuning between the cavity mode and the atom frequency. The dynamics and transfer of entanglement of two atoms interacting with quantum field of a cavity in the presence of detuning has been investigated in a number of papers [8, 15-19]. Some of these papers deal with the atoms simultaneously interacting with common cavity field. In other papers most attention has been concentrated on the entanglement dynamics of atoms passing consecutively through a cavity [8, 18, 19]. B. Gosh et al. [8] considered the model with effective two-photon interaction between atom and cavity field taking into account detuning and Stark shift. They found that though the two-atom entanglement generally diminishes with the increase of the two-photon detuning and the Stark shift, it is possible to sustain the entanglement over a range of interaction times by making the detuning and the Stark shift compensate each other. Consideration is being given to atoms prepared in their upper states before they enter the cavity. D. Gonta and S. Fritzsche [18] proposed a scheme to generate an entangled state between two four-level atoms that interact effectively by means of a detuned optical cavity and a laser beam that acts perpendicularly to the cavity axis. It is shown how the degree of entanglement for two atoms passing through the cavity can be controlled by manipulating their velocity and the (initial) distance between the atoms. Caldererón and coauthors [19] considered the sequential passage of two Rubidium atoms with effective two-photon transitions, prepared in specific Rydberg states, through a high-Q cavity, initially prepared in its vacuum state. They shown that, choosing a particular interaction time, the two-photon detuning can be used to control the atomic entanglement. We concerned our attention to investigation of the entanglement properties of two initially entangled Rydberg atoms passing through a single-mode cavity with a thermal field. The main goal of the present paper is to investigate the influence of detuning and thermal photons on entanglement between Rydberg atoms.

2. Model and negativity calculations

The physical system under consideration consists of two separate identical two-level Rydberg atoms passing through a cavity one after another and interacting with the cavity field. With the exception of the initial atomic state and cavity temperature, the parameters of the considered model are assumed to be the same as in experiment [6]. In [6] the author used two circular Rydberg states of Rb with principal quantum number 51 or 50 before crossing the cavity. The relevant cavity mode has is slightly off resonant with the transition at \( \omega_0 = 51.1 \) GHz between exited and ground states (detuning \( \delta = 170 \) kHz). Exact resonance between the atoms and the cavity is obtained by switching on a small electric field across the cavity mirrors while each atom is crossing a cavity. The Rabi frequency \( \Omega/2\pi \) of the atom transition resonant with the cavity mode was 48 kHz. One can easily calculate that \( \delta \approx 1.2g \), where \( g \) is the atom-field coupling constant. The cavity was cooled to 0.6 K, i.e. the mean thermal photon number was negligible. The cavity photon damping time was 112 ms, much shorter than the interval between two experimental sequences. In our theoretical consideration we took into account the detuning and thermal photons. We also considered the entangled initial atomic states.

The Hamiltonian of the joint "one atom+field" system with the dipole and rotating wave approximation can be written as

\[
H = (1/2)\hbar \omega_0 \sigma^z + \hbar \omega \ a^+ a + \hbar g (a^+ \sigma^- + a \sigma^+),
\]

where \( (1/2)\sigma^z \) is the inversion operator, \( \sigma^+ = |+\rangle \langle -| \), and \( \sigma^- = |--\rangle \langle +| \) are the transition
operators between the excited $|+\rangle$ and the ground $|−\rangle$ states, $a^+$ and $a$ are the creation and the annihilation operators of photons of the cavity mode and $\omega$ is the frequency of the cavity mode. We introduce the detuning as $\delta = \omega_0 - \omega$.

The evolution operator corresponding to the Hamiltonian (1) is

$$U_{Ai}(t) = e^{-\frac{\text{i}}{\hbar} \int_0^t \{ A_n |n + 1, -i\rangle \langle n + 1, -i| + B_n |n, +i\rangle \langle n, +i| + C_n (|n + 1, -i\rangle \langle n, +i| + |n, +i\rangle \langle n + 1, -i|) \}}.$$ 

Here

$$A_n = \cos(\Delta_n t/2) + \frac{\text{i} \delta}{\Delta_n} \sin(\Delta_n t/2), \quad B_n = \cos(\Delta_n t/2) - \frac{\text{i} \delta}{\Delta_n} \sin(\Delta_n t/2), \quad C_n = \frac{\Omega_n}{\Delta_n} \sin(\Delta_n t/2),$$

where $\Delta_n = \sqrt{\delta^2 + \Omega_n^2}$, $\Omega_n = 2g\sqrt{n + 1}$ and $|n\rangle$ is the Fock state for the cavity mode.

Suppose that before the first atom enters the cavity the two atoms have been prepared in Bell-type entangled states of the form

$$|\Psi(0)\rangle = \cos(\Theta)|+,-\rangle + \sin(\Theta)|-,+\rangle \quad (2)$$

or

$$|\Psi(0)\rangle = \cos(\Theta)|+,+\rangle + \sin(\Theta)|-,-\rangle, \quad (3)$$

where $\Theta$ defines the degree of initial atomic entanglement ($0 \leq \Theta \leq \pi/2$), and one-mode cavity field is in a thermal state $\rho_F(0) = \sum_p p_n |n\rangle \langle n|$, where the probabilities $p_n = \bar{n}^n/(1 + \bar{n})^{n+1}$. Here $\bar{n}$ is the mean photon number in the cavity mode $\bar{n} = (\exp[h\omega/k_B T] - 1)^{-1}$, $k_B$ is the Boltzmann constant and $T$ is the equilibrium temperature of the cavity mirrors.

Solving the evolution equation we derived the density matrix of considered system for time moment $\tau$ when the first atom leaves the cavity. The considered density matrix on the other hand is the initial state of the system prior to entering into the cavity of the second atom. We deduced the density matrix of the whole system at the moment $t$ when the second atom leaves the cavity. Taking a partial trace over the field variables we obtained from the density matrix of the whole system the reduced atomic density operator $\rho_{at}(t, \tau)$ in the two-atom basis $|+,+\rangle$, $|+,-\rangle$, $|-,+\rangle$, $|-,-\rangle$ both for entangled state (2) and (3).

For two-qubit system described by the density operator $\rho_{at}(t, \tau)$, a measure of entanglement or negativity can be defined in terms of the negative eigenvalues $\mu_i$ of partial transpose of the reduced atomic density matrix $\rho_{at}^{(T_i)}$ [20, 21]. The negativity is $\varepsilon = -2 \sum_i \mu_i$. When $\varepsilon = 0$ two qubits are separable and $\varepsilon > 0$ means the atom-atom entanglement. The case $\varepsilon = 1$ indicates maximum entanglement.

Using the exact expressions for reduced atomic density matrix $\rho_{at}(t, \tau)$ we derived the partial transpositions and calculated the exact formulae for negativity both for state (2) and (3). We do not give the explicit form of negativities, as they are too cumbersome. The results of numerical calculations of negativity for entangled initial atomic state (2) and (3) are shown in Figs. 1-5. The curves were obtained under the assumption that $\tau = t/2$ as in experiment [6].

3. Results and discussion

In numerical calculations we have turned our attention to the effects of the detuning both for the vacuum and the thermal field. Without loss of generality, we derived that the detuning is necessary and meaningful. Fig. 1 illustrates the dynamics of negativity as a function of a scaled time $gt$ for separable atomic state $|+,-\rangle$. The detuning sharply reduces the maximum degree of
Figure 1. The negativity as a function of a scaled time $gt$ for separable atomic state $|+,-\rangle$ with $\delta = 0$ (solid) and $\delta = 5$ (dashed). The mean photon number $\bar{n} = 0$ (a) and $\bar{n} = 0.5$ (b).

Figure 2. The negativity as a function of a scaled time $gt$ for entangled initial state (2) with $\Theta = \pi/4$ and a vacuum cavity field. The detuning $\delta = 0$ (solid), $\delta = 2g$ (dashed) and $\delta = 5g$ (dotted) (a) and $\delta = 10g$ (solid), $\delta = 20g$ (dashed) and $\delta = 50g$ (dotted) (b).

Figure 3. The negativity as a function of a scaled time $gt$ for entangled initial state (2) with $\Theta = \pi/4$ and a thermal cavity field with $\bar{n} = 0.5$. The detuning $\delta = 0$ (solid), $\delta = 2g$ (dashed) and $\delta = 5g$ (dotted) (a) and $\delta = 10g$ (solid), $\delta = 20g$ (dashed) and $\delta = 50g$ (dotted) (b).

entanglement both for the vacuum and the thermal field. The above effects were noted firstly for initial separable atomic state $|+,-\rangle$ [8]. For vacuum state results agree closely with experiment.
Figure 4. The negativity as a function of a scaled time $gt$ for entangled initial state (2) with $\Theta = \pi/4$ and detuning $\delta = 5$ (a) and $\delta = 50$ (b). The mean photon number $\bar{n} = 0.5$ (solid), $\bar{n} = 0.7$ (dashed) and $\bar{n} = 1.5$ (dotted).

S. Haroche et al. [6] noted that atom-cavity coupling was negligible when detuning ($\delta \approx 1.2g$) takes place. When the atom-field detuning is large enough, there is no energy exchange between the cavity and atom. The field, which acts as a medium, is virtually excited during the atom-atom coupling process. In the considered case the interaction between atoms through virtual media doesn’t lead to entanglement. Note that for atoms simultaneously interact with common field the virtually excited cavities may produce the maximally entangled two-atom states [22].

In Fig. 2 we plot the negativity as a function of a scaled time $gt$ for entangled state (2) and a vacuum field. Fig. 2 shows that small detunings leads to decreasing of the negativity oscillations and stabilization of the entanglement. For large detunings the virtually excited medium doesn’t destroy the initial atomic quantum correlations or entanglement. Fig. 3 shows the same for thermal field. Fig 3(a) shows that for slight detunings the entanglement behavior is very similar to that for a vacuum field, but the time periodicity destroys because system has a set of Rabi frequencies. One can see from Fig. 3(a) that for a thermal cavity field and small detunings the sudden death and birth of entanglement takes place. Figs. 3(b) demonstrates that for large detunings the amplitude of entanglement oscillations are scarcely affected by detuning. This result is more or less to be expected from the fact that the quantum correlations of atoms are affected by the medium involving not only the virtual but the real photons of thermal field. The frequencies of oscillations increase with increasing the Rabi frequencies $\Omega_n \approx \delta$ ($\delta \gg g$ and $\bar{n} \sim 1$). In Fig. 4 we plot the negativity as a function of a scaled time $gt$ for entangled initial state (2) and different values of mean photon numbers with detuning $\delta = 5g$ (a) and $\delta = 30g$ (b). For small detunings the negativity decreases with increasing of the mean photon number. For large detunings and relatively large mean photon numbers ($\bar{n} > 1$) the negativity behavior is scarcely affected by $\bar{n}$.

The negativity behavior for entangled initial state (3) is qualitatively similar to that for entangled initial state (2). In Fig. 4 we show the negativity as a function of a scaled time $gt$ for thermal field and atomic state (3) for small and large values of detunings.

4. Conclusions

We used the negativity to study the entanglement of the system of two initially entangled Rydberg atoms successively passing a thermal cavity taking into account the detuning. We derived the exact expressions for the reduced atomic density matrix and calculated the negativity formulae for different Bell’s initial atomic state and thermal cavity field. We investigated the entanglement turning our attention to the role of detuning in entanglement behavior. Our
The negativity as a function of a scaled time $gt$ for entangled initial state (3) with $\Theta = \pi/4$ and a thermal cavity field with $n = 0.5$. The detuning $\delta = 0$ (solid), $\delta = 2g$ (dashed) and $\delta = 5g$ (dotted) (a) and $\delta = 10g$ (solid), $\delta = 20g$ (dashed) and $\delta = 50g$ (dotted) (b).

numerical results reveal that for small detunings the presence of detuning leads to decreasing of the entanglement amplitude oscillations and stabilization of the degree of entanglement both for the vacuum and for the thermal field. For vacuum field and large detunings the initial entanglement ceases to vary in time. For thermal field and large detunings the entanglement amplitude oscillations not decrease with detuning increasing. For thermal field and small detunings the effect of sudden death and birth of entanglement takes place. For large detunings such effect vanishes. For separable initial atomic states the detuning reduces the maximum degree of entanglement both for the vacuum and the thermal field. These results show that the atom-atom entanglement can be controlled by changing the system parameters, such as the mean photon numbers and detuning.

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