Temperature considerations in non-spherical bubble collapse near a rigid wall

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Abstract. The inertial collapse of cavitation bubbles is known to be capable of damaging its surroundings. While significant attention has been dedicated to investigating the pressures produced by this process, less is known about heating of the surrounding medium, which may be important when collapse occurs near objects whose mechanical properties strongly depend on temperature (e.g., polymers). Using a newly developed computational approach that prevents pressure and temperature errors generated by naively implemented shock- and interface-capturing schemes, we investigate the dynamics of shock-induced collapse of gas bubbles near rigid surfaces. We characterize the temperature fields based on the relevant non-dimensional parameters entering the problem. In particular, we show that bubble collapse causes temperature rises in neighboring solid objects via two mechanisms: the shock produced at collapse and heat diffusion from the hot bubble close to the object.

1. Introduction
Cavitation bubble dynamics appear in a wide range of hydraulic applications, such as turbomachinery and naval structures. One of the important consequences of cavitation is the structural damage to neighboring surfaces due to bubble collapse [7, 8]. The modern description of cavitation erosion to metallic surfaces is based on a sequence of four steps [6]: production of small-scale vapor structures, impact loads due to bubble collapse, pitting, and failure. Based on this model, the origin of cavitation erosion lies in the mechanical loading produced on the material. However, most recent experiments show that high temperatures produced by bubble collapse near soft materials (e.g., polymer-based) may influence the damage mechanism(s).

Due to the complexity of the physics and the wide range of spatial and temporal scales in cavitation, numerical simulations have emerged as a reliable tool to complement analytical and experimental studies. However, direct simulations are challenging due to algorithmic difficulties. To overcome these difficulties, we employ a shock- and interface-capturing approach. However, a naive implementation of this approach has been known to give rise to spurious pressure and temperature oscillations across the interfaces [1, 2, 3]. Therefore, special care must be taken to prevent these errors [2, 5].

The objective of the present work is to better understand the potential damage caused by the shock-induced collapse of a single bubble near a rigid wall. We provide three-dimensional numerical simulations of this problem for different geometrical configurations to investigate the basic flow physics, with a particular emphasis on temperature effects. We further demonstrated that significant maximum wall temperatures can be achieved, depending on the heat diffusivity of the material.
2. Numerical method
The compressible Navier-Stokes equations with no mass transfer or surface tension effects are solved using a shock- and interface-capturing approach that prevents spurious numerical errors at material interfaces. The time marching is handled by a third-order accurate explicit SSP Runge-Kutta scheme. For the spatial discretization, a three-dimensional solution-adaptive high-order accurate central difference/discontinuity-capturing method is used. Smooth regions are computed using central differences, a finite difference weighted essentially non-oscillatory (WENO) scheme with Lax-Friedrichs flux splitting handles shock waves, and the approach of Johnsen & Colonius [3] is used for material interfaces.

3. Results
The governing equations are non-dimensionalized by the density and sound speed of air at atmospheric pressure, initial bubble radius $R_0 = 0.2$ mm, and $T = 300$ K. The schematic of the problem setup is shown in Fig. 1. An initially spherical bubble located a distance $H/R_o$ from a rigid wall is impacted by a Mach 1.018 shock (500:1 pressure ratio).

The simulations are done for different initial standoff distance $H$ to study its effects on pressure and temperature along the wall. Fig. 2 shows the pressure and temperature contours during the collapse of a gas bubble for $H/R_o = 1.2$. The right-moving shock interacts with the bubble, thus producing a reflected rarefaction wave. As the bubble starts its collapse, the incoming shock hits the rigid wall and reflects back onto the bubble. This doubles the surrounding pressure and leads to a stronger collapse. During the collapse, a re-entrant jet directed toward the wall is produced, which, upon impact with the distal side, generates an outward propagating shock. Thereafter, the bubble takes the form of a vortex ring, which is hot due to the rapid compression and is convected toward the wall. This process produces regions of high pressure and temperature along the wall.

Two mechanisms raise the temperature of the wall: the shock produced at collapse and heat diffusion from the hot bubble if sufficiently close to the solid wall. Fig. 3 shows the maximum temperature along the wall (from the simulations) and the maximum temperature caused by the shock wave (calculated based on the corresponding shock pressure and the stiffened equation of state for water) for different values of $H/R_o$. This figure shows that for the cases where the
bubble is initially close to the wall ($H/R_o \lesssim 1.3$), the maximum temperature is higher than the shock temperature, indicating that its source is the hot vortex ring touching the wall. For larger $H/R_o$, the maximum temperature is caused by the shock produced at collapse.

This mechanism can be discussed further by considering the time evolution of the wall pressure and temperature at “point A” for three different $H/R_o$. Johnsen & Colonius [4] showed that proximity to the wall leads to a faster and stronger collapse. This phenomenon can be observed through the pressure plot in Fig. (4). For the largest $H/R_o$, the maximum temperature happens right at the time of maximum pressure. However, the story is different for bubbles initially close to the wall. The time lag between the maximum pressure and maximum temperature indicates that the source of high temperature is not the shock wave. A large rise in temperature is observed, which can be explained by noticing that the vortex ring is in contact with the wall. This can be seen better in the other temperature plots in Fig. (4) where increasing $H/R_o$ exhibits a larger time lag (e.g. for $H/R_o = 1.25$ the shock wave from the collapse results in the first peak, while the second peak is caused by the hot vortex ring).

The maximum temperature in the whole domain happens inside the air bubble at the collapse time where we have the maximum pressure. After the collapse, the pressure decreases rapidly due to the bubble expansion, which corresponds to an immediate reduction in bubble temperature. This implies that if the bubble is initially close to the wall, the collapse is stronger, the air has a higher temperature, and the hot vortex ring touches the wall closer to the collapse time. The combination of all of these effects leads to a higher temperature measured in the liquid along the wall. However, this temperature is not the actual temperature of the wall. This is schematically illustrated in Fig. (5), which shows the thermal boundary layer between the fluid touching the wall and the wall surface. To estimate the wall temperature, we solve a simplified model problem based on the data from our simulations.
We start with the heat equation \( \partial T/\partial t = \alpha \partial^2 T/\partial x^2 \) where \( \alpha \) is the thermal diffusivity. We solve this equation in the wall with appropriate initial condition \( T(x, 0) = T_{Amb} \), and boundary conditions \( T(\infty, t) = T_{Amb} \) and \( q'' = k_f (T_{Hot} - T_{Amb})/\delta \), where \( T_{Amb} \) is the ambient temperature, \( T_{Hot} \) is the flow temperature, \( k_f \) is the flow temperature, \( q'' \) is the heat flux, and \( \delta \) is an estimate of the thermal boundary layer thickness. The wall temperature is highly dependent on the heat diffusivity of the solid. Fig. (6) shows the maximum temperature of the surface of the wall for different materials using this diffusion model. The results show that if the wall material has a low heat diffusivity (e.g., polymers) the surface temperature of the wall may increase significantly, while materials with high heat diffusivity dissipate the incoming heat flux through the wall and the surface temperature does not change significantly. However, this simple heat transfer model has some limitations: adiabatic and rigid wall assumptions are made in the simulations, as well as finite resolution. Heat transfer coupling between the wall and the flow, and also a compliant wall, may affect on the final results. In addition, because of the finite mesh size, one can never fully resolve the thermal boundary layer. Despite these limitations, this analysis provides us with a useful estimate of the wall temperature.

4. Conclusion
To provide a better understanding of cavitation erosion, we conducted numerical simulations of shock-induced collapse of a gas bubble near a rigid wall for different geometrical configurations. We introduced a simple heat transfer model to estimate the temperature of the wall, and showed that the initial location of the bubble plays a critical role in achieving a particular temperature.

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