The High-Temperature Two-Loop Effective Potential of the Electroweak Theory in a General ’t Hooft Background Gauge

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Abstract

We calculate the high-temperature two-loop effective potential using a general ’t Hooft background gauge. The dependence on the gauge-fixing parameter $\xi$ is investigated. The effective coupling constant at the critical temperature $g_3(T_c)^2$ is decreased considerably compared to the one-loop result, independent of $\xi$.

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There are strong indications that the electroweak standard theory predicts a first order phase transition at the electroweak scale \[1\]-\[9\]. Generating the baryon asymmetry of the universe at the electroweak phase transition is an exciting possibility. A better understanding of this phase transition is required, however, in order to clarify whether this is indeed the case. The electroweak phase transition cannot be treated completely by perturbative techniques. Problems are caused by infrared singularities of the symmetric phase, requiring the summation of infinite sets of diagrams. Lattice simulations take care of this automatically, and therefore are indispensable tools for the investigation of the electroweak phase transition \[7\]-\[9\]. However, they are not well suited for the study of important physical quantities like the sphaleron transition rate, or the rate of critical bubble formation. These quantities have been studied in quasiclassical approximation to one-loop order. Two-loop calculations should help to control to what extend the corresponding results are reliable, though of course genuine nonperturbative contributions to the potential in the infrared have to be taken into account differently.

Both sphalerons and critical bubbles are static field configurations, i.e. they do not depend on the imaginary time variable \(\tau\). Therefore, it becomes useful to integrate out the non-static Matsubara frequencies first, to some order in the loop expansion. This perturbative expansion should be reliable because the nonstatic Matsubara frequencies become heavy at high temperature. The longitudinal component of the gauge field \(A_0\) develops a Debye-mass proportional to \(gT\) and may be integrated out as well \[10\].

The resulting three-dimensional effective theory is of course non-local. Usually higher derivative terms are neglected in the spirit of the high temperature expansion. One also neglects the Weinberg mixing and considers the action of the three-dimensional SU(2)-Higgs model:

\[
S_{\text{ht}} = \frac{1}{g_3(T)^2} \int d^3 \bar{x} \left[ \frac{1}{4} F^a_{ij} F^a_{ij} + (D_i \Phi)^\dagger (D_i \Phi) + V_{\text{ht}}(\Phi^\dagger \Phi) \right] ,
\]

where we have introduced dimensionless coordinates and fields

\[
\bar{x} \rightarrow \frac{\bar{x}}{g v}, \quad \Phi \rightarrow v \Phi, \quad A \rightarrow v A .
\]

The scale \(v\) is left open for the moment.

The effective 3-dimensional gauge coupling is defined as

\[
g_3(T)^2 = \frac{gT}{v} .
\]

The gauge coupling \(g\) has been scaled out of the covariant derivative and the field strength tensor. The high temperature effective potential is

\[
V_{\text{ht}}(\Phi^\dagger \Phi) = \lambda \left( (\Phi^\dagger \Phi)^2 - v_0^2 \Phi^\dagger \Phi \right) .
\]

For compactness of notation we use a rescaled “\(\lambda\)” and “\(v_0^2\)”. They are temperature dependent constants which correspond as \(\lambda \approx \lambda_T / g^2\) and \(v_0^2 \approx (v_0(T)/v)^2\) to the one-loop quantities used in reference \[11\], respectively as \(\lambda \approx \bar{\lambda}_3 / g_3^2\) and \(\lambda v_0^2 \approx -\bar{m}_3^2 / v^2\) to the two-loop quantities used in \[6\].

We divide the fields into a background and fluctuations

\[
\Phi \rightarrow \hat{\Phi} + g_3 \tilde{\Phi}, \quad \hat{\Phi} = \frac{1}{\sqrt{2}} \phi \bar{e}, \quad \tilde{\Phi} = \frac{1}{\sqrt{2}} (\sigma 1 + i \pi^a \tau^a)e
\]

\[
A \rightarrow \hat{A}_i^a + g_3 \tilde{A}_i^a, \quad \hat{A} = 0, \quad \tilde{A}_i^a = a_i^a .
\]
(The $\tau^a$ are the Pauli matrices.) In order to describe critical bubbles responsible for the onset of the electroweak phase transition it is sufficient to work with only one nonvanishing background component $\varphi$ of $\Phi$ in an arbitrary but constant direction $\vec{e}$. We consider this type of background only.

Integrating out the fluctuating fields in the loop expansion generates an effective action to be used to find the saddle point solutions corresponding to sphalerons and critical bubbles. To higher loop order this expansion will break down for small values of the Higgs field $\varphi$. Near the broken minimum, however, the loop expansion is expected to work quite well. Whether the saddle point actions can be estimated reliably depends on how important different regions in field space are for the corresponding solutions. This question deserves further studies.

In praxi it is not possible to calculate the full effective action, but one has to expand it in some way, cut off the expansion and calculate the coefficient in powers of derivatives of the field. This expansion must of course break down at small values of the field $\varphi$, because the derivative operator $\partial$ has the mass dimension 1, which must be compensated by powers of $\varphi^{-1}$. Indeed calculating the contribution of a higher term in the derivative expansion to the effective action of a quasiclassical configuration one finds a divergent result, except for the potential and the $\partial_i \varphi \partial_i \varphi$ term. However for the latter kinetic term one finds a Z-factor [11] which in one-loop order is very strongly gauge dependent and therefore even this term, if considered separately, is rather unphysical.

What seems to be needed is some less gauge dependent (nonlocal) combination of kinetic terms. As we will demonstrate, the effective potential is less gauge dependent. In a strict expansion in $g^2$ its extrema are completely gauge-independent. It will play an essential role in the case of inhomogeneous field configurations as well.

In order to integrate out the fluctuating fields one has to fix the gauge. We choose as gauge-fixing condition the ‘t Hooft background gauge

$$ F^a = D(A)_{i} A^a_i + \frac{i}{2} \xi (\hat{\Phi} \tau^a \hat{\Phi} - \hat{\Phi} \tau^a \hat{\Phi}) = \partial_i a^a_i - \frac{1}{2} \xi \varphi \pi^a . $$

The resulting effective action $\Gamma[\varphi]$ depends in general on the gauge-fixing. Physical quantities should, on the other hand, be independent of it. This is due to the fact that they are described by extrema of $\Gamma[\varphi]$. Kobes et.al. [12] showed, that the gauge-fixing dependence of the effective action can be written as

$$ \delta \Gamma[\varphi] = \frac{\delta \Gamma[\varphi]}{\delta \varphi} \delta X[\varphi] , $$

where $\varphi$ represents all kinds of fields and $\delta X[\varphi]$ is a functional of the fields which can be calculated from the gauge-fixing condition and from the generators of the gauge transformation. One reads of immediately that the value of the effective action is gauge-fixing independent for solutions of the equations of motion, as it should.

An often raised objection against the ‘t Hooft background gauges is (see e.g. [13]) that the field $\hat{\varphi}$ used in the gauge-fixing should be of another type than the background field and should therefore not be varied in calculating the equation of motion

$$ \left( \frac{\delta \Gamma[\varphi, \hat{\varphi}]}{\delta \varphi} \right)_{\varphi = \hat{\varphi}} \neq \frac{\delta \Gamma[\varphi, \varphi]}{\delta \varphi} . $$

Nevertheless equation (8) holds even for these class of gauges with $\Gamma[\varphi] = \Gamma[\varphi, \hat{\varphi} = \varphi]$ as is explicitly shown in reference [12]. Similar statements, although less general, have been verified long time ago [14, 13].
As a consistency check, we shall explicitly demonstrate the gauge-fixing independence (i.e. the \( \xi \)-independence) of the value of the effective potential at its extrema. In order to achieve this one has to be careful to work consistently to a given order of \( g_3^2 \).

Expanding in terms of the fluctuating fields we obtain the action

\[
S_{\text{ht}} + \int d^3 x \left( \frac{1}{2 \xi} F^a F^a + \mathcal{L}_{FP} \right) \rightarrow \int d^3 x \left\{ \right.
\]

\[
\frac{1}{g_3^2} \left( \frac{1}{2} \partial_i \varphi \partial_i \varphi + V \left( \frac{1}{2} \varphi^2 \right) \right) + \frac{1}{g_3} \left( \partial_i \varphi \partial_i \sigma + \lambda \left( \varphi^2 - v_0^2 \right) \sigma \varphi \right) + \left( \frac{1}{2} \left( \partial_i a_j^a \right)^2 + \frac{1}{2 \xi} \left( 1 - \xi \right) \left( \partial_i a_j^a \right)^2 + \frac{1}{8} \varphi^2 a_i^a a_i^a + \frac{1}{2} \partial_i \sigma \partial_i \sigma + \frac{1}{2} \lambda \left( 3 \varphi^2 - v_0^2 \right) \sigma^2 \right.
\]

\[
+ \left( \partial_i \pi^a \right)^2 + \frac{1}{2} \left( \frac{1}{4} \xi \varphi^2 + \lambda \left( \varphi^2 - v_0^2 \right) \right) \pi^a \pi^a - c^a \delta^2 \sigma^a + \frac{1}{4} \xi \varphi^2 c^a c^a + a_i^a \pi^a \partial_i \varphi \right) + \left( \frac{1}{2} \varepsilon^{abc} a_j^a a_j^b \left( \partial_i a_j^c - \partial_j a_i^c \right) + \frac{1}{2} \varepsilon^{abc} a_j^a \pi^b \partial_j \pi^c - \varepsilon^{abc} \partial_i c^a \partial_j c^b a_i^c - \frac{1}{4} \varepsilon^{abc} \varphi a^a c^b c^c \right.
\]

\[
\left. + \lambda \left( 3 \varphi^2 - v_0^2 \right) \sigma + \frac{1}{2} a_i^a \left( \partial_i \sigma \pi^a - \sigma \partial_i \pi^a \right) + \frac{1}{4} \varphi \sigma a_i^a a_i^a + \frac{1}{4} \xi \varphi c^a c^a \right) \right. + \left( \frac{1}{4} \varepsilon^{abc} \varepsilon^{ade} a_j^a a_j^d a_j^e + \frac{1}{8} a_i^a \pi^b \pi^b + \frac{1}{4} \lambda \left( \sigma^2 + \pi^a \pi^a \right)^2 + \frac{1}{8} \left( \sigma^2 + \pi^a \pi^a \right) a_i^a a_i^a \right) \}
\]

One reads off the propagators and vertices. The gauge boson \((a_i^a)\) propagator may be written as

\[
D_W(k)_{ij}^{ab} = \delta^{ab} \left( \frac{1}{k^2 + m_W^2} \left( \delta_{ij} + \frac{k_i k_j}{m_W^2} \right) - \frac{k_i k_j}{m_W^2} \right) .
\]  

The Higgs (\(\sigma\)), Goldstone (\(\pi^a\)) and ghost (\(c^a\)) propagators are as usual with the masses

\[
m_H^2 = \lambda (3 \varphi^2 - v_0^2) , \quad m_{Gs}^2 = \lambda (\varphi^2 - v_0^2) + \frac{1}{4} \xi \varphi^2 , \quad m_W^2 = \frac{1}{4} \xi \varphi^2 , \quad m_{gh}^2 = \frac{1}{4} \xi \varphi^2 .
\]  

Note that there are no IR-divergences due to massless Goldstone-bosons at the broken minimum, for \(\xi \neq 0\). In the background field formalism the fluctuation appear only in the inner lines while the external lines consist of background fields [16]. Only the 1PI-graphs contribute to the effective action.

In the following we study the effective potential. Up to now it has been calculated to two-loop order in Landau gauge from the four-dimensional theory [17, 18] and from the high-temperature theory [6]. In a recent work M. Laine [19] presented a calculation using a general covariant gauge (\(\tilde{F}^a = \partial_i A_i^a\)).

The aim of our work is to calculate the potential in two-loop order using the 't Hooft background gauge with an arbitrary gauge-fixing parameter \(\xi\), and to investigate the \(\xi\)-dependence.

At one-loop one obtains the effective potential

\[
V_{\text{eff}}^{(1)} = \lambda \left( \frac{1}{4} \varphi^4 - \frac{1}{2} \varphi^2 \right) - \frac{g_3^2}{12 \pi} \left( m_H^2 + m_{Gs}^2 + (6 + 3 \xi^{3/2}) m_W^2 - 6 m_{gh}^2 \right) .
\]  

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The lines are:

- Higgs ($\sigma$);
- Goldstone ($\pi^a$);
- gauge boson ($a^i$);
- ghost ($c^a$);
- background field ($\varphi$)

Figure 1: The Graphs contributing to the two-loop effective potential.

The two-loop potential receives contributions from the graphs shown in figure 1. We use the $\overline{\text{MS}}$-renormalization scheme. The sunset graphs consist of integrals with three denominators; the figure-eight graphs have two denominators. The latter can be calculated easily while the momenta in the numerators of the sunsets cause some trouble. We removed them by a procedure which is similar to the one used in reference [6]. The remaining integrals are of the following form ($d = 3 - 2\epsilon$)

$$G(m_1, m_2) = \mu^{4\epsilon} \int \frac{d^d k}{(2\pi)^d} \int \frac{d^d p}{(2\pi)^d} \frac{1}{k^2 + m_1^2} \frac{1}{p^2 + m_2^2} = \frac{1}{16\pi^2} m_1 m_2$$ (15)

$$H(m_1, m_2, m_3) = \mu^{4\epsilon} \int \frac{d^d k}{(2\pi)^d} \int \frac{d^d p}{(2\pi)^d} \frac{1}{k^2 + m_1^2} \frac{1}{p^2 + m_2^2} \frac{1}{(k + p)^2 + m_3^2}$$ (16)

$$= \frac{1}{32\pi^2} \left( \frac{1}{2\epsilon} - \gamma + \ln(4\pi) \right) + \frac{1}{16\pi^2} \left( \frac{1}{2} - \ln \left( \frac{m_1 + m_2 + m_3}{\mu} \right) \right)$$ (17)

The two-loop potential is a rather long expression and therefore not displayed here. All the ingredients are given in the appendix. While single graphs have $\xi$-dependent
divergences the overall divergence is $\xi$-independent

$$\frac{9}{2} \lambda \varphi^2 - 6\lambda^2 \varphi^2 - 3\lambda v_0^2 + \frac{51}{32} \varphi^2 .$$  \hspace{1cm} (18)

From equation (17) one sees that the $\mu$-dependent part of the potential is proportional to the divergence and therefore $\xi$-independent. Hence it is possible to treat the $\mu$-dependence for one value of $\xi$ and the $\xi$-dependence for one value of $\mu$. The former is usually discussed by means of the renormalization group. This has been done for the Landau gauge elsewhere and will not be repeated here. Instead we are setting $\mu$ to be the value of the field in the broken minimum later on.

As mentioned above the value of the action must be independent of the gauge-fixing on its extrema. In our case the value of the potential at its extrema should be $\xi$-independent. In order to expand it systematically in orders of $g_3^2$ one has to expand the value of the field at the minimum first

$$\varphi_{\text{min}} = \varphi_{\text{min}}^{(0)} + g_3^2 \varphi_{\text{min}}^{(1)} + g_3^4 \varphi_{\text{min}}^{(2)} + \mathcal{O}(g_3^6) .$$  \hspace{1cm} (19)

Plugging this into the effective potential one gets

$$V_{\text{min}} = V_{\text{eff}}(\varphi_{\text{min}}) = V_{\text{min}}^{(0)} + g_3^2 V_{\text{min}}^{(1)} + g_3^4 V_{\text{min}}^{(2)} + \mathcal{O}(g_3^6) .$$  \hspace{1cm} (20)

We have to distinguish two cases

(i) One tree-level extremum is at $\varphi_{\text{min}}^{(0)} = 0$ and will stay there ($\varphi_{\text{min}}^{(1)} = \varphi_{\text{min}}^{(2)} = 0$).

From equation (20) on gets:

$$V_{\text{min}}^{(0)} = 0 , \hspace{1cm} (21)$$

$$V_{\text{min}}^{(1)} = -\frac{1}{3\pi} (-\lambda v_0^2)^{3/2} , \hspace{1cm} (22)$$

$$V_{\text{min}}^{(2)} = \frac{3}{64\pi^2} \lambda v_0^2 \left( -3 - 8\lambda + 4 \ln \left( 2\sqrt{-\lambda v_0^2} \right) \right) . \hspace{1cm} (23)$$

$V_{\text{min}}^{(1)}$ and $V_{\text{min}}^{(2)}$ are real for $v_0^2 < 0$, i.e. above the tree-level roll-over temperature, and complex below. This is due to the fact that $\varphi_{\text{min}}^{(0)} = 0$ is a maximum of $V_{\text{ht}}(\varphi)$ for $v_0^2 > 0$.

(ii) At this temperatures one should expand around the broken minimum $\varphi_{\text{min}}^{(0)} = v_0$

$$\varphi_{\text{min}}^{(1)} = \frac{3}{32\lambda^2} \left( 1 + 2\sqrt{\xi} \lambda + 25/2 \lambda^{3/2} \right) , \hspace{1cm} (24)$$

$$V_{\text{min}}^{(0)} = -\frac{\lambda v_0^4}{4} , \hspace{1cm} (25)$$

$$V_{\text{min}}^{(1)} = -\frac{1}{48\pi} \left( 3 + 27/2 \lambda^{3/2} \right) v_0^3 , \hspace{1cm} (26)$$

$$V_{\text{min}}^{(2)} = \frac{3}{1024\pi^2} \lambda \left( -3 + (11 + 42 \ln(2/3)) \lambda - 29/2 \lambda^{3/2} + 24\lambda^2 - 211/2 \lambda^{5/2} - 128\lambda^3 ight.$$

$$+ 8\lambda \left( 1 - 4\lambda + 8\lambda^2 \right) \ln \left( 1 + \sqrt{2} \lambda \right) + 64\lambda^3 \ln \left( 3\sqrt{2} \lambda \right)$$

$$+ 2\lambda(-17 - 16\lambda + 64\lambda^2) \ln \left( \frac{v_0}{\mu} \right) \right) . \hspace{1cm} (27)$$

The values of $V_{\text{min}}^{(i)}$ ($i = 1, 2, 3$) are independent of $\xi$ as they should. Note that we do not have any IR-divergences showing up in covariant non-background gauges [19].
Equations (25) - (27) show that the effective expansion parameter at small $\lambda$ is $\frac{g_3^2}{4\pi\lambda}$. Consequently it becomes of order 1 if one approaches the critical temperature from below. Going to the limit $\lambda \to \text{small}$ therefore does not help in improving the convergence of the loop expansion close to the critical temperature. The overall $\frac{1}{\lambda}$ in Eq. (27) arises entirely from inserting $\varphi_{\text{min}}^{(1)}$ into the tree- and one-loop potential.

Although the expansion around $\varphi_{\text{min}}^{(0)} = v_0$ is manifestly gauge-fixing independent, it is not useful close to the critical temperature. As soon as $\varphi_{\text{min}}^{(0)}$ tends towards zero the true position of the minimum may no longer be considered as being obtained as a small perturbation around $v_0(T)$. In the following, we therefore do no longer insist on the $g_3^2$ expansion of $\varphi_{\text{min}}$. Consequently, we do no longer work consistently to a given order in $g_3^2$, and some $\xi$ dependence must be expected. A small $\xi$ dependence would be an indication of a reasonable convergence of the approximation.

Both the one loop and the two-loop potential predict a first order phase transition. They have two local minima, which are degenerated at the respective critical temperature. Up to now the value of $v$ used to rescale the field in equation (3), is arbitrary. It is an appropriate choice to take $v$ and $\mu$ to be the value of the scalar field at the broken minimum. The asymmetric minimum of the rescaled field $\varphi$ is then at $\varphi_a = 1$

\[ V'_{\text{eff}}(\varphi = 1) = 0 \quad . \tag{28} \]

In addition, we have the condition

\[ V_{\text{eff}}(\varphi = 0) = V_{\text{eff}}(\varphi = 1) \quad . \tag{29} \]

at the critical temperature. $g_3^2$ and $v_0^2$, the two parameters of the high temperature effective action (1), can be calculated from equations (28) and (29). From this it follows that the two coupling constants $\lambda$ and $g_3^2$ are not independent at the critical temperature. Since $g_3^2$
determines the sphaleron rate this relation is important to determine cosmological bounds on the Higgs mass \([20]\).

In Figure 2, \(g_3^2\) is taken at the corresponding one and two-loop critical temperature, respectively, and is plotted versus \(\lambda\) for \(\xi = 0, 1, 2\). The gauge-fixing dependence is weak, both at one and two-loop. The inclusion of the two-loop contributions changes the magnitude of \(g_3(T_c)^2\) by about 40\%. This would reduce the sphaleron rate by many orders of magnitude. Therefore bounds on the Higgs mass from the wash-out of the baryon asymmetry may be less reliably than thought so far.

The large corrections to \(g_3(T_c)^2\) are not caused by large corrections to \(\varphi_{\text{min}}(T)/T\) at fixed temperature, but by the shift of \(T_c\). This is demonstrated in figure 3. (The \(\xi\)-dependence is again not significant here.) Crosses denote the one (×) and two (+) loop critical temperatures. Going from one to two-loop one essentially moves along an almost universal curve.

The determination of the critical temperature from the perturbative potential is of course very questionable because the latter is unreliable for small \(\varphi\) values. Still, it is remarkable that the two-loop potential leads to a lowering of \(T_c\) and an increase of \(g_3^2\) almost independently of the gauge-fixing parameter \(\xi\).

The one and two-loop potential at the corresponding critical temperatures is plotted for \(\xi = 0, 1, 2\) in figure 4. One first notices that the bulge between the symmetric and the asymmetric minimum grows from one to two loop, which is in agreement with previous calculations using Landau gauge \([17, 18]\). Accordingly, the phase transition is stronger first order, as predicted by lattice calculations \([7, 9]\). While \(V_{\text{eff}}^{(1)}(\varphi)\) depends strongly on \(\xi\), the shape of \(V_{\text{eff}}^{(2)}(\varphi)\) is only slightly \(\xi\)-dependent. This indicates that the loop-expansion might asymptotically converge towards an \(\xi\)-independent effective potential if one uses ’t Hooft background gauges.
It has a strong $\xi$-dependence for small $\varphi$-values and becomes even negative at some range as already pointed out in reference [11]. Near the broken minimum however it behaves well but is still $\xi$-dependent. One can discuss if this $\xi$-dependence cancels the one of $v(T)$ calculating the renormalized field value $\sqrt{Z(v(T))v(T)}$. We found that there is indeed some 45% reduction if the one-loop $Z$-factor with the two-loop $g^2$ and the two-loop $v(T)$ is used. Calculating the temperature dependent $W$-mass would also require the $Z_W$-factor.

In conclusion, we have demonstrated the applicability of the class of 't Hooft background gauges (eq. 7) for studies of the electroweak phase transition. The main advantage of this class of gauge-fixings is the absence of IR-divergences in the broken phase, which are caused by massless Goldstone bosons in the class of covariant gauges ($F^a = \partial_i A_i^a$) which is prominent in literature. The latter class has been used by M. Laine in a recent publication [19] to calculate the two loop potential. He found that the loop expansion converges even in the broken phase only for small values of $\xi$. This is essentially due to a
ξ-dependent infrared divergence which does not show up in ’t Hooft background gauges. In our opinion this shows the superiority of these gauges. The severe problems of perturbation theory in the symmetric phase caused by nonperturbative condensates can of course not be cured either.

Besides the improved IR-behavior in the broken phase there are some technical advantages due to the absence of a mixed Goldstone-gauge boson propagator. If one restricts oneself to the ’t Hooft-Feynman gauge (ξ = 1), the gauge boson propagator turns out to be quite simple as well (cf. eq. (11)). Note that the background gauges are also used in the computation of high energy cross sections[21].

We showed explicitly that the value of the effective potential at its minima is independent of the gauge-fixing parameter order by order if it is expanded consistently in \( g_3^2 \).

Close to the critical temperature this expansion breaks down. Here we worked with the full two-loop potential and used a rescaling procedure which is especially suited to the treatment of quasiclassical solutions like critical bubbles [11] and sphalerons [20]. This procedure is not gauge-fixing independent but the ξ-dependence becomes substantially weaker from one to two-loop order in the case of the effective potential (figure 4).
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Appendix

The propagators and vertices are read off equation (10). We used the form of the gauge boson propagator given in equation (11). It is straightforward to write down the two-loop graphs shown in figure 1. While the “figure 8” graphs can be evaluated easily the momenta in the numerators of the “sunset” graphs have to be removed first.

We did this in two steps:
(i) after use of momentum conservation two loop-momenta $k$ and $p$ are left. The mixed scalar products $kp$ are removed using the following identities:

\[
\frac{2kp}{(k+p)^2 + m^2} = 1 - \frac{k^2 + p^2 + m^2}{(k+p)^2 + m^2} \tag{31}
\]

\[
\int \frac{d^d p}{(2\pi)^d} kp F(p^2, k^2) = 0 \tag{32}
\]

\[
\int \frac{d^d p}{(2\pi)^d} (kp)^2 F(p^2, k^2) = \frac{1}{d} \int \frac{d^d p}{(2\pi)^d} k^2 p^2 F(p^2, k^2) \tag{33}
\]

(ii) The remaining momenta in the numerators are removed using the identities:

\[
\frac{k^2}{k^2 + m^2} = 1 - \frac{m^2}{k^2 + m^2} \tag{34}
\]

\[
\int \frac{d^d k}{(2\pi)^d} \int \frac{d^d p}{(2\pi)^d} \frac{k^m p^n}{k^2 + m^2} = 0 \tag{35}
\]

Formula (35) holds only due to the cancelation of IR-singularities. This reduction procedure has been performed with FORM. The integrals left are of one of the types given in equations (15, 16).

The single graphs are given below where the combinatorical factors are chosen in a way that the contribution to the two-loop potential is given by:

\[
g_3^4 \sum \text{figure 8's} - \frac{1}{2} g_3^4 \sum \text{sunsets} \tag{36}
\]

The dimension is $d = 3 - 2\epsilon$.

\[
\text{WWW} =
\]

\[
H(m_W, m_{gh}, m_{gh}) \left( -3\xi m_W^2 + \frac{3}{4} m_W^2 \right) + H(m_W, m_W, m_W) \left( -9 m_W^2 + \frac{45}{4} m_W^2 \right)
\]

\[
+ G(m_{gh}, m_{gh}) \left( -\frac{3}{4} + \frac{3}{2} \xi - 3\xi^2 \frac{1}{d} + 3\xi^2 \right) + G(m_W, m_{gh}) \left( \frac{3}{2} + 6\xi \frac{1}{d} + 6\xi d - 12\xi \right)
\]

\[
+ G(m_W, m_W) \left( -\frac{9}{4} - \frac{3}{d} + 3d \right)
\]

\[
\text{WGHGh} =
\]

\[
H(m_{gh}, m_{gh}, m_{gh}) \left( -\frac{3}{2} \xi^2 m_W^2 \right) + H(m_W, m_{gh}, m_{gh}) \left( +6\xi m_W^2 - \frac{3}{2} m_W^2 \right)
\]

\[
+ G(m_{gh}, m_{gh}) \left( +\frac{3}{2} - \frac{3}{2} \xi \right) + G(m_W, m_{gh}) (3)
\]
\[
\begin{align*}
\text{WG}_{\text{G}S\text{G}_{\text{S}}} &= \\
&= H(m_W, m_{\text{G}_{\text{S}}}, m_{\text{G}_{\text{S}}}) \left( \frac{3}{4} m_W^2 - 3 m_{\text{G}_{\text{S}}}^2 \right) + G(m_W, m_{\text{G}_{\text{S}}}) \left( \frac{3}{2} \right) \\
&+ G(m_{\text{G}_{\text{S}}}, m_{\text{gh}}) \left( \frac{3}{2} \xi \right) + G(m_{\text{G}_{\text{S}}}, m_{\text{G}_{\text{S}}}) \left( -\frac{3}{4} \right) \\
\text{GhGh}_{\text{G}_{\text{S}}} &= \\
&= + H(m_{\text{G}_{\text{S}}}, m_{\text{gh}}, m_{\text{gh}}) \left( \frac{3}{8} \xi^2 \varphi^2 \right) \\
\text{WW} &= \\
&= G(m_{\text{gh}}, m_{\text{gh}}) \left( -\frac{3}{2} \xi^2 \frac{1}{d} + \frac{3}{2} \xi^2 \right) + G(m_W, m_{\text{gh}}) \left( +3 \xi \frac{1}{d} + 3 \xi d - 6 \xi \right) \\
&+ G(m_W, m_W) \left( +\frac{9}{2} - \frac{3}{2} \frac{1}{d} - \frac{9}{2} d + \frac{3}{2} d^2 \right) \\
\text{WG}_{\text{G}_{\text{S}}} &= \\
&= G(m_W, m_{\text{G}_{\text{S}}}) \left( -\frac{9}{8} + \frac{9}{8} d \right) + G(m_{\text{G}_{\text{S}}}, m_{\text{gh}}) \left( +\frac{9}{8} \xi \right) \\
\text{GsG}_{\text{G}_{\text{S}}} &= \\
&= + G(m_{\text{G}_{\text{S}}}, m_{\text{G}_{\text{S}}}) \left( +\frac{15}{4} \lambda \right) \\
\text{WG}_{\text{G}_{\text{S}}H} &= \\
&= H(m_W, m_{\text{G}_{\text{S}}}, m_H) \left( -\frac{3}{2} \frac{m_{\text{G}_{\text{S}}}^2 m_H^2}{m_W^2} + \frac{3}{4} \frac{m_{\text{G}_{\text{S}}}^4}{m_W^2} + \frac{3}{4} \frac{m_H^4}{m_W^2} + \frac{3}{4} m_W^2 - \frac{3}{2} m_{\text{G}_{\text{S}}}^2 - \frac{3}{2} m_H^2 \right) \\
&+ H(m_{\text{G}_{\text{S}}}, m_H, m_{\text{gh}}) \left( +\frac{3}{2} \frac{1}{m_W} m_{\text{G}_{\text{S}}}^2 m_H^2 - \frac{3}{4} \frac{1}{m_W} m_{\text{G}_{\text{S}}}^4 - \frac{3}{4} \frac{1}{m_W} m_H^4 \right) \\
&+ G(m_W, m_{\text{G}_{\text{S}}}) \left( +\frac{3}{4} + \frac{3}{4} \frac{1}{m_W} m_{\text{G}_{\text{S}}}^2 - \frac{3}{4} \frac{1}{m_W} m_H^2 \right) \\
&+ G(m_W, m_H) \left( +\frac{3}{4} + \frac{3}{4} \frac{1}{m_W} m_{\text{G}_{\text{S}}}^2 + \frac{3}{4} \frac{1}{m_W} m_H^2 \right) \\
&+ G(m_{\text{G}_{\text{S}}}, m_{\text{gh}}) \left( +\frac{3}{4} \xi - \frac{3}{4} \frac{1}{m_W} m_{\text{G}_{\text{S}}}^2 + \frac{3}{4} \frac{1}{m_W} m_H^2 \right) + G(m_{\text{G}_{\text{S}}}, m_H) \left( -\frac{3}{4} \right) \\
&+ G(m_H, m_{\text{gh}}) \left( +\frac{3}{4} \xi + \frac{3}{4} \frac{1}{m_W} m_{\text{G}_{\text{S}}}^2 - \frac{3}{4} \frac{1}{m_W} m_H^2 \right) \\
\text{GsG}_{\text{S}}H &= \\
&= H(m_{\text{G}_{\text{S}}}, m_{\text{G}_{\text{S}}}, m_H) \left( +6 \lambda^2 \varphi^2 \right) \\
\text{GhG}_{\text{h}}H &= \\
&= H(m_H, m_{\text{gh}}, m_{\text{gh}}) \left( -\frac{3}{16} \xi^2 \varphi^2 \right)
\end{align*}
\]
\[ \text{WWH} = H(m_W, m_W, m_H) \left( + \frac{3}{32} \varphi^2 \frac{1}{m_W^4} m_H^4 - \frac{3}{8} \varphi^2 \frac{1}{m_W^2} m_H^2 + \frac{3}{8} \varphi^2 d - \frac{3}{8} \varphi^2 \right) 
+ H(m_W, m_H, m_{gh}) \left( \frac{3}{8} \xi \varphi^2 \frac{m_H^2}{m_W^2} + \frac{3}{8} \xi \varphi^2 - \frac{3}{8} \xi \varphi^2 - \frac{3}{16} \varphi^2 \frac{m_H^4}{m_W^4} + \frac{3}{16} \varphi^2 \frac{m_H^2}{m_W^2} - \frac{3}{16} \varphi^2 \right) 
+ H(m_H, m_{gh}, m_{gh}) \left( - \frac{3}{8} \xi \varphi^2 \frac{1}{m_W^2} m_H^2 + \frac{3}{8} \xi \varphi^2 + \frac{3}{32} \varphi^2 \frac{1}{m_W^4} m_H^4 \right) 
+ G(m_{gh}, m_{gh}) \left( + \frac{3}{16} \xi \varphi^2 \frac{1}{m_W^2} - \frac{3}{32} \varphi^2 \frac{1}{m_H^2} m_{gh}^2 \right) 
+ G(m_W, m_{gh}) \left( - \frac{3}{16} \xi \varphi^2 \frac{1}{m_W^2} + \frac{3}{16} \varphi^2 \frac{1}{m_W^4} m_H^2 - \frac{3}{16} \varphi^2 \frac{1}{m_W^2} \right) 
+ G(m_W, m_W) \left( - \frac{3}{32} \varphi^2 \frac{1}{m_W^4} m_H^2 + \frac{3}{16} \varphi^2 \frac{1}{m_W^2} \right) 
+ G(m_W, m_H) \left( + \frac{3}{16} \xi \varphi^2 \frac{1}{m_W^2} - \frac{3}{16} \varphi^2 \frac{1}{m_W^4} \right) 
+ G(m_H, m_{gh}) \left( - \frac{3}{16} \xi \varphi^2 \frac{1}{m_W^2} + \frac{3}{16} \varphi^2 \frac{1}{m_W^2} \right) \]

\[ \text{HHH} = \quad H(m_H, m_H, m_H) \left( + 6 \lambda^2 \varphi^2 \right) \]

\[ \text{WH} = \quad G(m_W, m_H) \left( - \frac{3}{8} + \frac{3}{8} d \right) + G(m_H, m_{gh}) \left( + \frac{3}{8} \xi \right) \]

\[ \text{GsH} = \quad G(m_{gs}, m_H) \left( + \frac{3}{2} \lambda \right) \]

\[ \text{HH} = \quad + G(m_H, m_H) \left( + \frac{3}{4} \lambda \right) \]

\[ \text{GhGs} = \quad G(m_{gs}, m_{gh}) \left( + \frac{3}{4} \xi \right) \]