Linewidth of power spectrum originated from thermal noise in spin torque oscillator

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A theoretical formula for the linewidth caused by thermal activation in a spin torque oscillator with a perpendicularly magnetized free layer and an in-plane magnetized pinned layer is derived on the basis of the LLG equation in the energy-phase representation. It is shown that the linewidth can be suppressed to 0.1 MHz by applying a large current (10 mA for typical material parameters). A quality factor larger than 10⁴ is predicted in the large current limit, which is two orders of magnitude larger than a recently observed experimental value.

The spin torque oscillator (STO) is a promising candidate for a future nanocommunication device because of its small size, high emission power, and frequency tunability. Recently, it was found that an STO with a perpendicularly magnetized free layer and an in-plane magnetized pinned layer can achieve a large emission power of close to 1 µW. Therefore, this type of STO will be the model structure for practical STO applications.

Another important quantity characterizing the STO’s properties is the linewidth of the power spectrum. For example, a narrow linewidth is necessary to obtain a high quality factor (Q-factor) Q = f₀/Δf, where f₀ is the peak frequency of the power spectrum. The physical origin of the linewidth is the nonuniform magnetization dynamics due to thermal activation. Therefore, theoretical evaluation of the linewidth due to thermal activation and of the Q-factor is desirable to clarify the theoretically possible values of these parameters.

In the self-oscillation state of an STO, the energy supplied by the spin torque balances the energy dissipation due to damping; therefore, the magnetization steadily precesses almost on a constant energy curve. This situation is very similar to magnetization switching in the thermally activated region, where the magnetization precesses on a constant energy curve many times during switching. Recent studies have shown that the energy-phase representation of the Landau–Lifshitz–Gilbert (LLG) equation is useful for theoretical investigations of the magnetization switching properties, such as the switching probability, in the thermally activated region. Accordingly, we were motivated to develop a theory of the linewidth of an STO based on the LLG equation in the energy-phase representation.

In this letter, the theoretical formula for the linewidth of an STO is derived on the basis of the LLG equation in the energy-phase representation. It is shown that the linewidth can be suppressed to 0.1 MHz by applying a large current (~10 mA). The Q-factor can reach more than 10⁴ in this type of STO, which is two orders of magnitude larger than the previously reported value.

Figure 1 shows a schematic view of the STO under consideration, where the unit vectors pointing in the magnetization direction of the free and pinned layers are denoted as m and p, respectively. The z-axis is normal to the film plane, and the x-axis is parallel to p. The external field Happl is applied along the z-axis. The current I flows uniformly along the z-axis, where positive current corresponds to electron flow from the free layer to the pinned layer. The LLG equation is given by

\[
\frac{d\mathbf{m}}{dt} = -\gamma \mathbf{m} \times \mathbf{H} - \gamma \mathbf{m} \times (\mathbf{p} \times \mathbf{m}) - \gamma \mathbf{m} \times \mathbf{h} + \alpha \mathbf{m} \times \frac{d\mathbf{m}}{dt},
\]

where γ and α are the gyromagnetic ratio and Gilbert damping constant, respectively. The magnetic field is defined as \( \mathbf{H} = -\partial E/\partial (M\mathbf{m}) \), where M is the saturation magnetization. The energy density is given by

\[
E = -MH\text{appl}m_z - \frac{M(H_k - 4\pi M)}{2} m_z^2,
\]

where \( H_k \) and \( 4\pi M \) represent the crystalline uniaxial anisotropy and shape anisotropy (demagnetization) fields along the z-axis, respectively. Because the LLG equation conserves the magnetization magnitude, the magnetization dynamics can be regarded as the motion of a point particle on a unit sphere, as schematically shown in Fig. 1. The constant energy curves of Eq. (2) correspond to latitude lines on this sphere. The spin torque strength \( H_s \) is given by \( H_s = \hbar v/[2\epsilon(1+\lambda \mathbf{m} \cdot \mathbf{p})MV] \), where V is the volume of the free layer. The parameters \( \eta \) and \( \lambda \) characterize the spin torque strength. The third term on the right-hand side of Eq. (1) represents the random torque due to thermal activation, where the components of the random field \( \mathbf{h} \) satisfy the fluctuation–dissipation theorem. The diffusion coefficient is \( (\hbar^2/\eta\gamma^2)\delta(t-t') \), where \( D = \alpha\gamma k_B T/(MV) \) is the diffusion coefficient.

In the self-oscillation state, the magnetization precesses almost on a constant energy curve because the energy dissipation due to damping balances the energy supplied by the spin torque. Thus, it is convenient to divide the magnetization dynamics into the directions orthogonal to and along the constant energy curve. From Eq. (1), the former dynamics is described by

\[
\frac{d\mathbf{m}}{dt} = -\gamma \mathbf{m} \times \mathbf{H} - \gamma \mathbf{m} \times (\mathbf{p} \times \mathbf{m}) - \gamma \mathbf{m} \times \mathbf{h} + \alpha \mathbf{m} \times \frac{d\mathbf{m}}{dt},
\]
\[
\frac{\dot{E}}{\gamma M} = -\alpha[H^2 - (m \cdot H)^2] \\
+ H_s[p \cdot H - (m \cdot p)(m \cdot H)] + (H \times m) \cdot \mathbf{h},
\]
where the first and second terms on the right-hand side represent the energy dissipation due to damping and the energy change due to the spin torque, respectively. On the other hand, to describe the dynamics along a constant energy curve, we introduce the phase \(\psi\)^{(17)} whose dynamics is described by
\[
\dot{\psi} = \left(\frac{2\pi}{\tau}\right) - \left(\frac{2\pi}{\tau}\right) \frac{[m \times (m \times H)] \cdot \mathbf{h}}{|m \times H|^2}.
\]
Here, \(\tau(E) = \int dt\) is the precession period on the constant energy curve. The terms \((H \times m) \cdot \mathbf{h}\) and \((2\pi/\tau)[m \times (m \times H)] \cdot \mathbf{h}/|m \times H|^2\) are the projections of the random torque to the \(\dot{E}\) and \(\dot{\psi}\) directions, and can be replaced by
\[
\sqrt{2D[H^2 - (m \cdot H)^2]}\xi_E/\gamma \quad \text{and} \quad \sqrt{2D(2\pi/\tau)^2/|m \times H|^2}\xi_\psi/\gamma,
\]
with \(\xi_E(t)\) and \(\xi_\psi(t)\) being the random functions of time, respectively. The random torque makes the probability function of the magnetization direction, which satisfies the Fokker–Planck equation,\(^{(17)}\) independent of \(\psi\)\(^{(5,16)}\) after a sufficiently long time compared with \(\tau\). Thus, we assume that the deterministic torques and diffusion coefficients of \(\dot{E}\) and \(\dot{\psi}\) can be replaced by their averages on the constant energy curve.\(^{(16,17)}\) Accordingly, the LLG equation in the energy-phase representation is
\[
\frac{dE}{dt} = -\frac{M}{\gamma\tau} (\alpha \cdot \mathbf{H}_a - \mathbf{M}_a) + \frac{D}{\gamma} \frac{1}{\tau} \frac{d}{dE} \frac{M}{\gamma} \left(\frac{1}{\tau}\right) \frac{d\mathbf{H}_a}{dE} \\
+ \frac{M}{\gamma} \sqrt{2D \cdot \mathbf{H}_a/\tau} \xi_E(t),
\]
\[
\frac{d\psi}{dt} = \frac{2\pi}{\tau} - \frac{2D \cdot \mathbf{N}_\psi}{\sqrt{\gamma^2}} \xi_\psi(t),
\]
where\(^{(15–20,24)}\)
\[
\mathbf{H}_a(E) = \gamma^2 \int dt[H^2 - (m \cdot H)^2],
\]
\[
\mathbf{M}_a(E) = \gamma^2 \int dtH_s[p \cdot H - (m \cdot p)(m \cdot H)],
\]
\[
\mathbf{N}_\psi(E) = \frac{2\pi}{(\gamma^2)} \int \frac{dt}{|m \times H|^2}.
\]
In the first term on the right-hand side of Eq. (3), the energy dissipation due to damping \((\propto \alpha \cdot \mathbf{H}_a)\) is always negative, whereas the sign of the energy change due to the spin torque \((\propto \mathbf{M}_a)\) depends on the current direction. The second term on the right-hand side of Eq. (3) is the thermally generated drift term pointed out by Ref.\(^{16}\) and represents the energy supplied by the heat bath. The last terms of Eqs. (3) and (4) describe the fluctuations of the energy and phase, where \(D_E = D(M/\gamma)M/\gamma\) \(\mathbf{H}_a/\tau\) and \(D_{\psi} = D \cdot \mathbf{N}_\psi/\tau\) are the averaged diffusion coefficients of the random torque along the \(\dot{E}\) and \(\dot{\psi}\) directions, respectively. The constant energy curve on which the magnetization steadily precesses is determined by the condition
\[
-\frac{M}{\gamma\tau} (\alpha \cdot \mathbf{H}_a - \mathbf{M}_a) + \frac{1}{\tau} \frac{d}{dE} \frac{M}{\gamma} \left(\frac{1}{\tau}\right) \frac{d\mathbf{H}_a}{dE} = 0.
\]
Let us calculate the fluctuations of the energy density \(E\) and the phase \(\psi\). By expanding Eq. (3) around the steady-state energy \(E\) up to the first order of the fluctuation \(\delta E\), the solution of \(\delta E\) is obtained as
\[
\delta E(t) = c_1 e^{-\Omega_a t} + \Xi \int_{-\infty}^{t} dt' \xi_E(t') e^{-\Omega_a(t-t')},
\]
where the term proportional to the integral constant \(c_1\) is determined by the initial condition and is negligible in the following calculation. The quantity \(\Omega_a\), which characterizes the relaxation of \(\delta E\), is defined as
\[
\Omega_a = \frac{M}{\gamma\tau} d(\alpha \cdot \mathbf{H}_a - \mathbf{M}_a)/dE - D \left(\frac{M}{\gamma}\right)^2 \frac{1}{\tau} \frac{d^2 \mathbf{M}_a}{dE^2},
\]
whereas \(\Xi = (M/\gamma \sqrt{2D \cdot \mathbf{H}_a/\tau})\). Similarly, the solution of the phase is obtained from Eq. (4). Note that \(\tau\) in Eq. (4) depends on the energy density \(E\); therefore, the fluctuation of \(E\) should be taken into account to determine the phase. By expanding \(1/\tau\) up to the first order of \(\delta E\), the solution of the phase is obtained as
\[
\psi(t) = \psi(0) + \frac{2\pi\tau}{\tau} \frac{d}{dE} \int_{0}^{t} dt' \delta E(t') \quad - \frac{2D \cdot \mathbf{H}_a}{\tau} \int_{0}^{t} dt' \xi_\psi(t'),
\]
where \(\psi(0)\) is the initial phase. Then, the phase variance \(\Delta \psi^2(t) = (\psi(t) - \langle \psi(t) \rangle)^2\)
\[
= \Delta \psi^2(t) = 2 \Delta \psi_0 \left[(1 + \nu^2)|t| - \nu^2 \left(\frac{1 - e^{-\Omega_a |t|}}{\Omega_a}\right)\right],
\]
where \(\Delta \psi_0 = D \cdot \mathbf{N}_\psi/\tau\), and \(\nu^2\) is defined as
\[
\nu^2 = \frac{\mathbf{M}_a}{\gamma \mathbf{N}_\psi} \left(\frac{2\pi M}{\gamma^2} \frac{d}{dE} \frac{M}{\gamma} \left(\frac{1}{\tau}\right) \frac{d\mathbf{H}_a}{dE}\right)^2.
\]
The power spectrum \(\mathcal{P}(f)\) of an STO is the Fourier transformation of the correlation function \(\langle \exp[i(\psi(t) - \psi(0))] \rangle \approx \exp[-(i(f - f_0))^2/(2\Delta f_L^2)]\). Then, the linewidth of the power spectrum is determined by the phase variance. The power spectrum becomes the Lorentz function \(\mathcal{P}(f) \propto \Delta f/L(f - f_0)^2 + (\Delta f_L)^2\) with a linewidth \(\Delta f_L\) of
\[
\Delta f_L = \frac{\Delta \psi_0}{2\pi} \left(1 + \nu^2\right)
\]
when \(\Delta f_L \ll \Omega_a\).\(^{(25)}\) In the opposite limit, \(\mathcal{P}(f)\) is the Gaussian \(\mathcal{P}(f) \propto \exp\left[-(f - f_0)^2/(2\Delta f_G^2)\right]\), with
\[
\Delta f_G = \frac{\sqrt{\Delta \psi_0^2 \Omega_a}}{2\pi}.
\]
Equation (9) or (10) is applicable to an arbitrary type of STO. The term \(\Delta \psi_0/(2\pi)\) in Eq. (9) arises from the phase fluctuation and is called the phase noise or linear linewidth.\(^{(25)}\) On the other hand, the term \(\Delta \psi_0^2/(2\pi)\) arises from the energy fluctuation and is called the amplitude noise or nonlinear linewidth.\(^{(25)}\)

Now let us calculate the linewidth of an STO with a perpendicularly magnetized free layer and an in-plane magnetized pinned layer. Using Eq. (2), the energy density \(E\) can be directly related to the cone angle of the magnetization \(\theta = \cos^{-1} m_z\). The explicit forms of \(\tau, \mathbf{M}_a, \mathbf{M}_s, \mathbf{N}_\psi\) are given by
\[
\tau = \frac{2\pi}{\gamma[H_{\text{app}} + (H_K - 4\pi M) \cos \theta]},
\]
\[
\mathbf{M}_a = \frac{(2\pi \sin \theta)^2}{\tau},
\]
balances the energy supply from the heat bath at the angle 
\( \frac{\pi}{4} \) which can be obtained from Eq. (11) in the limit of 
I = equilibrium direction for the damping constant. The deviation angle 
\( \theta_0 \) from the z-axis in the absence of the current can be estimated by the condition 
\( 1 - \epsilon = 0 \). In the zero-temperature limit, Eq. (11) is identical to 
I(\theta) derived in Ref. 13. The threshold current at zero 
temperature is 
\[ I_c = \left( \frac{4\alpha MV}{\hbar} \right) (H_{\text{appl}} + H_K - 4\pi M), \]
which can be obtained from Eq. (11) in the limit of \( T \to 0 \)
and \( \theta \to 0 \). Because the energy dissipation due to damping balances the energy supply from the heat bath at the angle 
\( \theta_0 \), an infinitesimal current can excite the magnetization 
dynamics with the cone angle \( \theta > \theta_0 \). Thus, our theory gives the 
linewidth for the current region of \( |I| > 0 \) without a separation of the current region at \( I_c \). This differs from the previous theory,25) in which the magnetization is fixed in the 
equilibrium direction for \( I < I_c \), so the linewidth due to thermal activation becomes finite only for \( I > I_c \). The explicit forms of Eqs. (9) and (10) are 
\[ \Delta f_L = \frac{\alpha \gamma k_B T}{2\pi MV} \sin^2 \theta \left[ 1 + \left( \frac{\omega_K}{\Omega_a} \right)^2 \sin^4 \theta \right], \]
\[ \Delta f_G = \frac{\omega_K}{2\pi} \sqrt{\frac{\alpha \gamma k_B T}{MV\Omega_a}} \sin \theta, \]
where \( \omega_K = \gamma (H_K - 4\pi M) \). In \( \Delta f_G, 1 \ll \nu^2 = \left( \frac{\omega_K}{\Omega_a} \right)^2 \sin^4 \theta \) is again assumed. The explicit form of \( \Omega_a \) is 
\[ \Omega_a = \alpha \gamma (2H_{\text{appl}} \cos \theta - (H_K - 4\pi M)(1 - 3 \cos^2 \theta)) \]
\[ + \frac{\gamma \hbar I}{2\alpha MV} \left[ 1 - \frac{\lambda^2}{2} \left( \frac{1 - \lambda^2 \sin^2 \theta}{\lambda^2 \sin^2 \theta} \right)^{3/2} \right] \]
\[ + D \left[ \frac{H_{\text{appl}}^2}{[H_{\text{appl}} + (H_K - 4\pi M) \cos \theta]} \right]. \]
These equations give the relation between the current, 
cone angle of the oscillation, and linewidth.

Figure 2(a) shows the dependence of the cone angle \( \theta \) and 
linewidth \( \Delta f_L \) on the current, where the values of the parameters are 
\( M = 1448 \text{ emu/cm}^3, H_K = 18.6 \text{ kOe}, \gamma = 1.732 \times 10^7 \text{ }\text{ }\text{ }\text{ }\text{ }\text{ }\text{ }\text{ }\text{ }\text{ }\text{ }\text{ }1/(\text{Oe-s}), V = \pi \times 60 \times 60 \times 2 \text{ mm}^3, \alpha = 0.005, \eta = 0.54, \lambda = \eta^2, H_{\text{appl}} = 3.0 \text{ kOe}, \) and \( T = 300 \text{ K}. \) Using 
these values, \( \Omega_a \) is typically on the order of \( 0.1-1 \text{ GHz}, \)
satisfying the condition that \( \beta \) must be the Lorentz function, 
\( \Delta f_L \ll \Omega_a \). Therefore, the linewidth of the Lorentz function is shown in Fig. 2(a). The deviation angle of the magnetization 
from the z-axis, which satisfies \( 1 - \epsilon = 0 \), is \( \theta_0 = 1.6^\circ \). 
The threshold current at zero temperature is \( I_c = 2.2 \text{ mA}. \) From \( \theta_0 \), the cone angle \( \theta \) monotonically increases as the 
current increases. On the other hand, the linewidth above \( I_c \) 
monotonically decreases as the current increases, whereas 
that below \( I_c \) depends nonmonotonically on the current. Such 
complex current dependence of the linewidth was found in 
previous experimental studies.7,20) Above \( I_c \), the length of the 
precession trajectory becomes long as the current, as well as 
the cone angle, increases. Then, the phase fluctuation by 
one random torque becomes relatively small as the current increases, 
resulting in linewidth decreases. On the other hand, 
below \( I_c \), not only the fluctuation but also the effective 
reduction of the damping constant, \( \alpha(1 - \epsilon) \), by the energy 
supplied by the heat bath affects the linewidth. Whereas 
the linewidth due to the former contribution increases as the 
current decreases, the linewidth due to the latter contribution 
decreases. Because of this competition, the linewidth shows a 
nonmonotonic dependence on the current below \( I_c \). 
The current dependence of \( \nu^2 \) is shown in the inset of Fig. 2, 
indicating that the amplitude noise dominates the linewidth in the 
current region of \( I > I_c \).

Figure 2(b) shows the current dependence of the Q-factor 
\( f_0/\Delta f_L \), where \( f_0 = 1/\tau \). The Q-factor reaches \( 10^4 \) for 
the current \( I \approx 10 \text{ mA}, \) which is two orders of magnitude larger 
than the experimentally observed value (135 in Ref. 7). This 
result should motivate future experimental research for 
practical STO applications. The current magnitude \( I \approx 10 \text{ mA} \).
mA is larger than the experimentally available applied current (typically, 3 mA\(^7\)) for an MgO-based magnetic tunnel junction. A reduction in the resistance due to the tunnel barrier will be necessary to apply a large current. A giant magnetoresistive system would also be an interesting candidate for this purpose.

The linewidth shown in Fig. 2(a) is narrower than the experimental value. For example, the calculated linewidth in Fig. 2(a) is, at maximum, about 100 MHz. On the other hand, the experimental value is, at minimum, 50 MHz, and is typically much wider than 100 MHz.\(^7\) This fact indicates that other physical mechanisms contribute to the linewidth in the experiments. One possible contribution is the shot noise effect, which arises from the discrete transmission of electrons through the tunnel barrier and produces fluctuations of the current as well as the spin torque. This fluctuating spin torque can be effectively included in the random torque by renormalizing the damping constant \(\alpha\) and leads to an increase in the linewidth.\(^17\) Another possibility is a reduction in the perpendicular anisotropy observed in the experiment,\(^7\) whose origin was assumed to be Joule heating in Ref. 7. The reduction in \(H_K\) make the fluctuations of \(E\) and \(\varphi\) relatively large, which leads to an increase in the linewidth.

Finally, let us discuss the relation between the present work and previous theoretical work. Slavin and Tiberkevich\(^22\) calculated the linewidth by a similar approach, but using the oscillation amplitude of the magnetization (\(\rho\) in their notation) instead of the energy density \(E\). In Ref. 25, the magnetization direction of the pinned layer is assumed to be parallel to the easy axis of the free layer, and the spin torque parameter \(\lambda\) is neglected. However, a finite \(\lambda\) should be taken into account to define physical quantities such as the threshold current \(I_c\)\(^13\) and linewidth when the pinned layer magnetization is orthogonal to the easy axis of the free layer, as is the case for the STO studied in this letter. More generally, when the constant energy curve is symmetric with respect to an axis perpendicular to the pinned layer magnetization, \(\lambda\) should be finite. In fact, if \(\lambda\) is set to zero, \(I_c\rightarrow \infty\), and the linewidth becomes independent of the current, which clearly contradicts the experiments. Mathematically, Ref. 25 assumed that \(\mathbf{p}\) is parallel to the easy axis of the free layer, and \(H_F\) is independent of \(\mathbf{m}\) and \(\mathbf{p}\) in the calculation of \(\mathcal{M}\). On the other hand, we did not make this assumption when calculating \(\mathcal{M}\). The energy supplied by the heat bath,\(^16\) which is expressed by the second term of Eq. (3), is also neglected in Ref. 25. In light of these differences, our formula can be regarded as a generalization of the formulae in Ref. 25.

In conclusion, we developed a general formula for the linewidth of an STO by solving the LLG equation in the energy-phase representation. We applied the formula to an STO with a perpendicularly magnetized free layer and an in-plane magnetized pinned layer to estimate the linewidth and quality factor. It was shown that the linewidth is suppressed to 0.1 MHz when a large current is applied. A quality factor larger than 10\(^4\) is predicted in the large current limit, which is two orders of magnitude larger than the value recently observed in experiments.

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