Getting chirality right: top-philic scalar leptoquark solution to the \((g - 2)_{e,\mu}\) puzzle

Innes Bigaran\(^1\) and Raymond R. Volkas\(^1\)

ARC Centre of Excellence for Particle Physics at the Terascale, School of Physics, The University of Melbourne, Victoria 3010, Australia

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We identify the two scalar leptoquarks capable of generating sign-dependent contributions to leptonic magnetic moments, \(R_2 \sim (3, 2, 7/6)\) and \(S_1 \sim (3, 1, -1/3)\), as favoured by current measurements. We consider the case in which the dominant contributions originate from top-containing loops, within so-called \textit{top-philic} coupling textures. The strongest constraints on these models arise from flavour-violating leptonic decays, and contributions to the \(Z\) leptonic decay widths. To be a comprehensive solution to the \((g - 2)_{e,\mu}\) puzzle, we find that the mass of the \(S_1\) leptoquark must be less than 5.96 TeV. We find no allowable mass regions for \(R_2\) that pass all constraints. This analysis has the potential to be embedded within broader flavour anomaly studies, including those of hierarchical leptoquark coupling structures. It can also be straightforwardly adapted to accommodate future measurements of leptonic magnetic moments, such as those expected from the Muon \(g - 2\) collaboration in the near future.

I. INTRODUCTION

The remarkable agreement between measurements and predictions of the muon and electron magnetic dipole moments has long been testament to the success of quantum field theory. Precise measurements of the deviation of this observable from the classical, tree-level value, \(g_t = 2\), give a sensitive probe of higher-order effects – within the Standard Model (SM) and beyond. SM corrections are precisely known, and therefore these specify the quantity \(a_t^{\text{SM}}\), where

\[
a_t = \frac{1}{2} (g - 2) t. \tag{1}
\]

This makes anomalies in \((g - 2)_t\), particularly if these differ between lepton flavour, a very strong indication of new physics (NP) effects at loop-level\(^1\).\(^2\).

For the muon, there is persistent deviation between the SM prediction and the measured value\(^3\)\(^4\),

\[
\Delta a_\mu = a_\mu^{\text{exp}} - a_\mu^{\text{SM}}, \tag{2}
\]

corresponding to a 3.6\(\sigma\) anomaly\(^5\)

\[
\Delta a_\mu = (286 \pm 63 \pm 43) \times 10^{-11}. \tag{3}
\]

Similarly, recent experimental results have indicated a deviation for the electron magnetic moment, of 2.5 \(\sigma\) significance\(^6\):

\[
\Delta a_e = -(0.88 \pm 0.36) \times 10^{-12}. \tag{4}
\]

It is important to note that the sensitivity of \(a_t\) to NP at energy scale \(\Lambda\) scales generally as \(m_t^n/\Lambda^m\), for some integer \(n, m\). This indicates that a heavier lepton generally provides a more sensitive probe of NP. However, due to the short lifetime of the tau, a precise measurement of its magnetic moment (and, consequentially, any deviation from the SM) is beyond the reach of current experiments. The important issue to be addressed in this paper is that the discrepancies, \(\Delta a_\mu\) and \(\Delta a_e\), are of \textit{opposite sign}, which is a difficulty to be overcome when searching for a common explanation\(^7\).

The leading candidates to explain these deviations involve flavour-dependent, loop-level, NP effects. It has long been established that exotic scalar-only extensions to the SM are capable of generating sizeable corrections to \((g - 2)_{e,\mu}\). Of particular interest are scalar leptoquark (LQ) models, which have proven to be useful for reconciling other well-known flavour-dependent anomalies (\textit{e.g.} see references\(^{16, 20}\)), provide a portal to generating radiative neutrino mass\(^{21}\), and are embedded

\[\text{TABLE I. Scalar LQs and their transformation properties, under the hypercharge convention } Q = I_3 + Y. \text{ The second-last column indicates whether the model is able to generate one-loop (1L) corrections to the muon and electron magnetic moments with opposite sign – \textit{i.e.} the LQ has mixed-chiral couplings.}\]

| Symbol   | \(SU(3)_C \otimes SU(2)_L \otimes U(1)_Y\) | \((g - 2)_t\) at 1L | \(|F|\) |
|----------|------------------------------------------|----------------------|------|
| \(\bar{S}_1\) | (3, 1, -4/3) | \(\times\) | 2 |
| \(S_1\)   | (3, 1, -1/3) | \(\checkmark\) | 2 |
| \(S_3\)   | (3, 3, -1/3) | \(\times\) | 2 |
| \(S_7\)   | (3, 1, 2/3) | \(\times\) | 2 |
| \(R_2\)   | (3, 2, 7/6) | \(\checkmark\) | 0 |
| \(\bar{R}_2\) | (3, 2, 1/6) | \(\times\) | 0 |

\(^1\)The uncertainty values refer to the experimental and theoretical prediction uncertainties, respectively. See also the ‘note added’ at the end of the conclusion.

\(^2\)See, for example, references\(^{16, 20}\) for alternative methods to explain these anomalies in a single model.
in a number of theories of unification (e.g. see reference [22]).

A. Chirality of scalar LQ models

To begin characterising scalar LQ models, we first need to introduce some terminology. Motivated by the introduction of direct lepton-quark couplings, rather than separately considering lepton (L) or baryon (B) number conservation, these are absorbed into the definition of a new conserved quantity [23] fermion number, \( F \);

\[
F = 3B + L. \tag{5}
\]

\( F \) is well-defined for each of the finite number of LQ models, and characterises the types of interactions mediated. The \( |F| = 2 \) LQs couple to multiplets of the form \( \ell q \) and \( |F| = 0 \) couple to \( \ell q \) [24]. Table I gives an overview of the scalar LQs and their gauge-group transformations, adopting the symbol notation from reference [16].

The important characteristic of models that can generate contributions to \((g-2)_c/\mu\) that are consistent with experiment is the chirality of their Yukawa couplings. To generate one-loop corrections whose sign can vary between lepton flavours, the LQ must have mixed-chiral couplings, i.e. both left- and right-handed couplings to charged leptons are present. To see why this is, we begin with a brief overview of the established calculations for \( a_\ell \) corrections from scalar LQ states.

B. Scalar LQs for \((g-2)_\ell\)

In this section, we follow the calculation procedure for \( \ell \to \ell' \gamma \) from reference [16], but adapt it specifically for \((g-2)_\ell\). The generic effective Lagrangian corresponding to contributions to \( a_\ell \) is given by:

\[
\mathcal{L}_{a_\ell} = e \ell \left( \gamma_\mu A^\mu + \frac{a_\ell}{4m_\ell} \sigma_{\mu\nu} F^{\mu\nu} \right) \ell, \tag{6}
\]

where \( F^{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \), and \( \sigma^\ell_{L/R} \) parameterise the effective left- and right-chiral interactions. Equation (6) reveals, via coefficient matching, that

\[
\Delta a_\ell = im_\ell (\sigma^\ell_L + \sigma^\ell_R). \tag{7}
\]

For the purpose of this discussion, for general scalar LQ \((\phi)\) models, the couplings to charged-leptons \((\ell)\) and quarks \((q)\) can be expressed as:

\[
\mathcal{L}_\ell = \bar{\ell}^{(c)} (y^R_\ell P_R + y^L_\ell P_L) \; q \; \phi^1 + h.c. \tag{8}
\]

where \( \ell^{(c)} = \ell \) for \( |F| = 0 \) and \( \ell^{(c)} = \ell' \) for \( |F| = 2 \).

We consider the two leading-order topologies for these corrections illustrated in Figure 1. Their contributions to \( \sigma^\ell_{L/R} \) are well established in the literature [16]. For \( F = 0 \) scalar LQs:

\[
\sigma^\ell_L = \frac{iN_c}{16\pi^2 m_\phi^2} \sum_q \left[ m_\ell (|y^R_\ell|^2 + |y^L_\ell|^2) \kappa + y^R_\ell y^L_\ell^* m_q \kappa' \right]. \tag{9}
\]

\[
\sigma^\ell_R = \frac{iN_c}{16\pi^2 m_\phi^2} \sum_q \left[ m_\ell (|y^R_\ell|^2 + |y^L_\ell|^2) \kappa + y^R_\ell y^L_\ell^* m_q \kappa' \right]. \tag{10}
\]

In equations (9) and (10),

\[
\kappa = Q_\phi f_S(x_q) - f_F(x_q),
\]

\[
\kappa' = Q_\phi g_S(x_q) - g_F(x_q), \tag{11}
\]

where \( x_q = m_q^2/m_\phi^2 \). These contributions are proportional to the number of colours, \( N_c = 3 \), and are summed over quark flavours \( q \) running in the loop. The electric charge of the field \( \phi \) is given by \( Q_\phi \), and the loop functions in (11) are [16] 25:

\[
f_S(x) = \frac{x + 1}{4(x-1)^2} - \frac{x \log(x)}{2(x-1)^3},
\]

\[
f_F(x) = \frac{x^2 - 5x - 2}{12(x-1)^3} + \frac{x \log(x)}{2(x-1)^4},
\]

\[
g_S(x) = \frac{1}{x - 1} - \frac{\log(x)}{(x-1)^2},
\]

\[
g_F(x) = \frac{x - 3}{2(x-1)^2} + \frac{\log(x)}{(x-1)^3}. \tag{12}
\]

Therefore, we conclude via equation (7) that

\[
\Delta a_\ell = \frac{-3m_\ell}{8\pi^2 m_\phi^2} \sum_q \left[ m_\ell (|y^R_\ell|^2 + |y^L_\ell|^2) \kappa + m_q \text{Re}(y^L_\ell y^R_\ell^*) \kappa' \right]. \tag{13}
\]

For a scalar LQ with maximally chiral Yukawa couplings, the second term will not be present and contributions from each propagator will be of definite relative sign. However, for mixed-chiral scalar LQs, we can access terms proportional to \( \kappa' \), allowing us to vary the
sign of the NP contribution. As these contributions scale proportional to \(m_q\), we expect the dominant contributions to be those with the third-generation quarks entering the loop. Taking \(m_t \ll m_q\), the like-handed terms are subdominant to the mixed-handed contributions – exactly the terms required for generating contributions with relative-sign. This leaves the following:

\[
\Delta a_t \sim -\frac{3m_t}{8\pi^2 m_\phi^2} \sum_{q=t,b} m_q \text{Re}(y_{t,q}^L y_{t,q}^R \kappa').
\]  

(14)

For \(|F| = 2\) LQs, the above applies but with \(Q_\phi \rightarrow -Q_\phi\), \(y^R_\ell \rightarrow y^L_\ell\), and \(y^L_\ell \rightarrow y^R_\ell\).

For mixed-chiral scalar LQ models, this provides a portal to flavour-dependent sign allocation for the correction. We have the clear prospect of meeting current experimental measurements with simple, single-scalar field extensions\(^3\), such as those identified in Table I.

**II. MODELS OF INTEREST**

As summarised in Table I, the \(S_1\) and \(R_2\) leptoquarks are able to induce opposite-sign \((g - 2)_\mu\) contributions. These LQs have also garnered recent attention in other flavour anomaly studies\(^27\)\(^\rightarrow\)\(^22\). The relevant LQ couplings for each extension, represented here as \(3 \times 3\) Yukawa coupling matrices, are given by\(^2\)

\[
\mathcal{L}_{int}^{S_1} = (\mathcal{L}_L^s \lambda_{LQ} Q_L + \mathcal{L}_R^a \lambda_{eQ} u_R) S_1^L + h.c.,
\]  

(15)

\[
\mathcal{L}_{int}^{R_2} = (\mathcal{L}^R \lambda_{Lu} u_R + \mathcal{L}^R \lambda_{eQ} Q_L) R_2^L + h.c.
\]  

(16)

For completeness, the SM fermion field transformations used are as defined in Section 1.1 of reference\(^16\).

The doublet, \(R_2\), can be expressed in terms of its electric charge-definite components:

\[
R_2 \sim \begin{pmatrix} R_2^{5/3} \\ R_2^{2/3} \end{pmatrix},
\]  

(17)

with charges as indicated by the superscripts. We assume negligible mass-splitting between the components of the multiplet, i.e.,

\[
m_{R_2} \approx m_{R_2^{5/3}} \approx m_{R_2^{2/3}}.
\]

When rotated into the flavour eigenbasis, we redefine the couplings in accordance with the mappings

\[
\mathcal{R}_e \lambda_{eQ} \mathcal{R}_u \rightarrow y^{Seu},
\]  

(18)

\[
\mathcal{L}_c \lambda_{LQ} \mathcal{L}_u \rightarrow y^{SLQ},
\]  

(19)

\[
\mathcal{L}_c^1 \lambda_{Lu} \mathcal{R}_u \rightarrow y^{RLu},
\]  

(20)

\[
\mathcal{L}_c^1 \lambda_{eQ} \mathcal{L}_u \rightarrow y^{ReQ}.
\]  

(21)

Here, \(L\) and \(R\) represent the basis mapping between the gauge and flavour eigenstates, and \(V = \Sigma^2 \mathcal{L}_d\) is the standard CKM matrix. There are two independent coupling matrices for each model, and the interaction Lagrangians may be re-expressed as:

\[
\mathcal{L}^{S_1} \supset y_{ij}^{SLQ} \left[ \overline{c_{L,i}} u_{L,j} - V_{jk} \overline{c_{L,i}} d_{L,k} \right] S_1^L + y_{ij}^{Seu} \overline{c_{R,i}} u_{R,j} S_1^L + h.c.,
\]  

(22)

\[
\mathcal{L}^{R_2} \supset y_{ij}^{RLu} \left[ \overline{y_{L,i}} u_{R,j} R_2^{5/3} - \overline{y_{L,i}} u_{R,j} R_2^{2/3} \right] + y_{ij}^{ReQ} \overline{y_{R,i}} R_2^{5/3} + V_{jk} d_{L,k} R_2^{5/3} + h.c.
\]  

(23)

Recalling the discussion in Section I, we note that \(R_2^{5/3}\) and \(S_1\) both have left- and right-handed couplings to charged leptons and SM up-type quarks. The parameters \(\kappa'\) for each model (equation\(^11\)) are given by the following, assuming \(m_q = m_t\) and \(m_\phi \ll m_\phi^2\):

\[
\kappa'(S_1) = \frac{7}{6} + \frac{2}{3} \log \left( \frac{m_t^2}{m_{S_1}} \right),
\]  

(24)

\[
\kappa'(R_2) = \frac{1}{6} - \frac{2}{3} \log \left( \frac{m_t^2}{m_{R_2}} \right).
\]  

(25)

Their dominant \(\Delta a_t\) contributions are therefore, via equation\(^{14}\):

\[
\Delta a_t^{S_1} \sim -\frac{m_t m_\phi}{4\pi^2 m_{S_1}^2} \frac{7}{4} - 2 \log \left( \frac{m_{S_1}}{m_t} \right) \text{Re}(y^{L_s}_{L} R_{L}^{S_1}),
\]  

(26)

\[
\Delta a_t^{R_2} \sim -\frac{m_t m_\phi}{4\pi^2 m_{R_2}^2} \frac{1}{4} - 2 \log \left( \frac{m_{R_2}}{m_t} \right) \text{Re}(y^{L_s}_{L} R_{L}^{R_2}).
\]  

(27)

where for \(S_1\),

\[
y^R = y^{Seu} \text{ and } y^L = y^{SLQ},
\]

and for \(R_2\),

\[
y^R = -y^{RLu} \text{ and } y^L = y^{ReQ}.
\]

For consistency between \(|F| = 0\) and \(|F| = 2\) models, and with equation\(^0\), we have labelled the chirality to be that of the quark field in the associated interaction term.
Using equations (26) and (27), assuming the dominant contributions in equations (22) and (23) of reference [41]. To a good approximation, for $m_\nu \gtrsim 900$ GeV, LQ contributions to the effective couplings of the Z have little dependence on $|F|$ [41]. We find the effective axial-vector coupling, $g_A^e$, to be the most constraining of the two; at leading order, this is given by:

$$g_A^e \sim -0.5 + 9.5 \times 10^{-3} \left( |y_{23}^L|^2 - |y_{23}^R|^2 \right) x_t \log(x_t), \quad (31)$$

where $x_t = m_t^2/m_\phi^2$. Combining (31) and (30), with the central values for the effective couplings in Table II, we obtain the following expression, up to order $O(x_t^2)$:

$$(y_{13}^L)^2 \approx 0.0338 x_t (y_{23}^L)^4 \log(x_t) + 0.00429 (y_{23}^L)^2. \quad (32)$$

From this we expect a finite set of solutions in the $y_{13}^L - y_{23}^L$ plane, representing points of preference for these $(g - 2)_{\mu/\ell}$ models to also satisfy electroweak constraints. However, it is non-trivial to algebraically derive a simple analytic consideration of parameter space

To avoid CP-violation constraints, we restrict our input couplings to real values\(^5\) and so refine equation (29) to read:

$$y_{13}^L y_{13}^R \sim -0.063 y_{23}^L y_{23}^R. \quad (30)$$

We also implement the minimal nonzero Yukawa coupling textures given by equation (28), and so restrict our discussion of constraints to those relevant to these couplings.

We follow the calculation of effective $Z$ couplings to charged-leptons $\ell$, and associated observables, from reference [41]. The couplings $g_A^{\ell} (V)$ are the effective axial-vector (vector) couplings, as calculated in equations (22) and (23) of reference [41]. To a good approximation, for $m_\nu \gtrsim 900$ GeV, LQ contributions to the effective couplings of the Z have little dependence on $|F|$ [41]. We find the effective axial-vector coupling, $g_A^e$, to be the most constraining of the two; at leading order, this is given by:

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From this we expect a finite set of solutions in the $y_{13}^L - y_{23}^L$ plane, representing points of preference for these $(g - 2)_{\mu/\ell}$ models to also satisfy electroweak constraints. However, it is non-trivial to algebraically derive a simple

\(^5\) We switch off charm contributions so as to determine a conservative parameter space. If more freedom is required in model-building, these additional contributions could also be considered.

\(^6\) Note that the constraints from leptonic decays with three-body final states, such as $\ell \to 3\ell'$, give Yukawa coupling constraints that are orders-of-magnitude weaker than their $\ell \to \ell'\gamma$ counterparts. Constraints from nuclear $\mu - e$ conversion and neutrinoless $2\beta$ decay are negligible within this framework; switching-off the first-generation quark Yukawa couplings ensure they are suppressed on the relevant scales [37].

\(^7\) The parameter space could be extended to complex couplings. One would then need to carefully consider the contribution of the diagrams in Figure 1 to electric dipole moments [32].

| PROCESS | OBSERVABLE | LIMITS |
|---------|------------|--------|
| $\mu \to e\gamma$ | $5\ell$ | $< 4.2 \times 10^{-13}$ [33] |
| $Z \to \ell\ell$ | $y_{\ell}^L$ | $-0.03817 \pm 0.00047$ [31] |
| $y_{\ell}^R$ | $-0.0367 \pm 0.0023$ [31] |
| $y_{\ell}^d$ | $-0.50111 \pm 0.00035$ [31] |
| $y_{\ell}^u$ | $-0.50120 \pm 0.00054$ [31] |
| $\text{Br}(Z \to \ell\ell)$ | $< 7.5 \times 10^{-7}$ [35] |
| $\text{Br}(Z \to e\gamma)$ | $(3.6332 \pm 0.0042) \times 10^{-2}$ [34] |
| $\text{Br}(Z \to \mu\mu)$ | $(3.3662 \pm 0.0066) \times 10^{-2}$ [34] |

TABLE II. Processes most constraining on this model.

As the dominant contributions in equations (26) and (27) are proportional to the mass of the quark in the loop, we restrict LQ couplings to represent a top-phic coupling texture: the only nonzero NP quark-couplings are those to the top quark. In this framework, the necessary non-zero couplings to generate $\Delta a_\ell \neq 0$, $\ell \in \{e, \mu\}$ are shaded in grey:

$$y^R \sim \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}, \quad y^L \sim \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}; \quad (28)$$

Here, the coupling values themselves are a priori completely arbitrary. For the remainder of this paper we will assume a top-phic model\(^7\). The mixed-chiral contributions in the $R_2$ model arise from interactions of the $R^{5/3}$ component of the LQ doublet, which has couplings of both chiralities to charged leptons and up-type quarks.

A. Constraints from $Z$ and LFV leptonic processes

Using equations (26) and (27), assuming the dominant contribution from the mixed-chiral component, we find that satisfying the current central values of both $(g - 2)_{\ell}$ measurements requires an allocation of couplings such that:

$$\frac{\text{Re}(y_{13}^L y_{13}^R)}{\text{Re}(y_{23}^L y_{23}^R)} = \frac{m_\mu}{m_\ell} \frac{\Delta a_\ell^\text{exp}}{\Delta a_\mu^\text{exp}} \sim -0.063, \quad (29)$$

independent of which model is chosen. We also require a sign difference between the contributions such that $\Delta a_\ell < 0$. Equation (29) suggests an order-of-magnitude difference between these products of couplings.

By the crossing-symmetry between the topologies in Figure 1 and contributions of the form $\ell \to \ell'\gamma / Z$, we expect the strongest constraint\(^6\) to be from $\ell \to \ell'\gamma$ and $Z \to \ell\ell'$ processes [38,39,41]. By virtue of the structure of equations (22) and (23), there are unavoidable couplings to neutrinos, so contributions to the invisible width of the Z must also be considered — particularly for $R_2$, where neutrino LQ couplings are not CKM suppressed.

1. Analytic consideration of parameter space

To avoid CP-violation constraints, we restrict our input couplings to real values\(^5\) and so refine equation (29) to read:

$$y_{13}^L y_{13}^R \sim -0.063 y_{23}^L y_{23}^R. \quad (30)$$

We also implement the minimal nonzero Yukawa coupling textures given by equation (28), and so restrict our discussion of constraints to those relevant to these couplings.

We follow the calculation of effective $Z$ couplings to charged-leptons $\ell$, and associated observables, from reference [41]. The couplings $g_A^{\ell} (V)$ are the effective axial-vector (vector) couplings, as calculated in equations (22) and (23) of reference [41]. To a good approximation, for $m_\nu \gtrsim 900$ GeV, LQ contributions to the effective couplings of the Z have little dependence on $|F|$ [41]. We find the effective axial-vector coupling, $g_A^e$, to be the most constraining of the two; at leading order, this is given by:

$$g_A^e \sim -0.5 + 9.5 \times 10^{-3} \left( |y_{23}^L|^2 - |y_{23}^R|^2 \right) x_t \log(x_t), \quad (31)$$

where $x_t = m_t^2/m_\phi^2$. Combining (31) and (30), with the central values for the effective couplings in Table II, we obtain the following expression, up to order $O(x_t^2)$:

$$(y_{13}^L)^2 \approx 0.0338 x_t (y_{23}^L)^4 \log(x_t) + 0.00429 (y_{23}^L)^2. \quad (32)$$

From this we expect a finite set of solutions in the $y_{13}^L - y_{23}^L$ plane, representing points of preference for these $(g - 2)_{\mu/\ell}$ models to also satisfy electroweak constraints. However, it is non-trivial to algebraically derive a simple
relationship for the couplings to ensure that these constraints are met. To match with experiment, the mixed-chiral term needs to dominate $\Delta a_t$, but it alone will not completely represent the NP contributions; also, due to the interplay between couplings for various constraints, simplifying expression (32) under particular limits will not capture the full scope of these models. This necessitates numerical studies to determine their viability. For numerical evaluation in Section III we use the full analytic expressions for constraints, and for the calculation of $\Delta a_t$ (13).

III. PHENOMENOLOGY

In both models, the free parameters are summarised by
\[ \{y_{13}^L, y_{23}^L, y_{13}^R, y_{23}^R, m_{\phi}\}. \] (33)

Presently the scalar leptoquark mass, $m_{\phi}$, is most strongly constrained using LHC searches for couplings predominantly to first-generation leptons (43):
\[ m_{\phi} > 1435 \text{ GeV at 95\%CL}. \] (34)

This represents the worst-case coupling scenario with respect to mass constraints, and so provides a conservative lower bound.

We aim to fix the values of $\Delta a_t$ to the central values from (3) and (4), reducing the parameter degrees of freedom to $5-2 = 3$. To achieve this, we scan logarithmically over positive perturbative left-handed couplings, and fix the right-handed couplings according to the full calculation of $\Delta a_t$ (13); the relative sign is absorbed into the allocation of right-handed couplings. Since this equation is a polynomial in $y_{13}^R$, we restrict the couplings to be real-valued, we solve for the value of $y_{13}^R$ for which Re$(y_{13}^R)$ is perturbative, and Im$(y_{13}^R)$ is minimal. We truncate the input value for $y_{13}^R$ to be the real-component of the root. This is done algorithmically such that the input value satisfies both requirements, listed in order of preference. We then check the calculated value for $\Delta a_t$ and identify points which remain within one-sigma of the central values.

The conditions for this parameter study are as follows, where experimental values implemented are as given in Table II. Unless otherwise stated, the below represent the requirements for values to pass constraints for each observable:

1. Perturbativity bound: $|y_{13}^{L/R}| \leq \sqrt{4\pi}$;
2. Br$(Z \rightarrow e\mu)$ and Br$(\mu \rightarrow e\gamma)$ below the experimental bounds (Table II);
3. Effective $N_e$, within 2σ;
4. $g_\lambda^2$ and $g_{\lambda}^\mu$ within 2σ, (Section III-B);
5. Br$(Z \rightarrow ee)$ and Br$(Z \rightarrow \mu\mu)$ within 2σ, (Section III-C), called ‘$Z \rightarrow \ell\ell$ preferred’;
6. Most importantly, $\Delta a_e$ and $\Delta a_\mu$ within 1σ.

We do not consider correlation between effective couplings for the indicative studies in Section III-A and Section III-B. However, we consider the partial decay widths rather than effective couplings in Section III-C to capture these effects. We employ calculations from (10) and (11) to evaluate contributions to the leptonic decays and Z-decay parameters, respectively.

To begin, we first explore the relationship between the generational lepton couplings by performing a scan over couplings using a fixed $m_{\phi}$ (Section III-A). Then, leaving this ratio between couplings fixed, we perform a scan over masses and couplings to establish an approximate upper-bound on the LQ masses capable of rectifying the $(g-2)_{\ell}$ anomalies, within the notable experimental constraints (Section III-B). We perform these checks independently for both the $S_1$ and $R_2$ models. We then perform a full parameter scan for each model, with the aim of achieving an upper-bound on the LQ masses capable of tackling the $(g-2)_{\mu/e}$ puzzle, and explore the coupling textures of these solutions (Section III-C).

A. Ratio of generational LQ couplings

In this section, we fix the LQ mass to a benchmark value of $m_{\phi} = 1.5$ TeV. We emphasise that the points of preference illustrated in Figure 2 represent a projection onto a subset of parameter space, and as such they are indicative points. With varying the mass $m_{\phi}$, other solutions $(y_{13}^L - y_{23}^L$ relationships) may also arise.

1. $S_1$, the |$F$| = 2 LQ solution

Beginning with $S_1$, contributions to $N_e^{\text{eff}}$, via $Z \rightarrow \nu\nu$, are either inverse-top-mass or CKM suppressed – with significant enough suppression to render these negligible for the considered parameter space. This numerical study illustrated in Figure 2 indicates a preference for Yukawa couplings such that
\[ y_{13}^L \sim y_{23}^L \quad \text{or} \quad y_{13}^L \sim 10^3 y_{23}^L, \] (35)
for a LQ mass $m_{\phi} = 1.5$ TeV.

2. $R_2$, the |$F$| = 0 LQ solution

For the benchmark LQ mass $m_{R_2} = 1.5$ TeV, the $R_2$ leptoquark is unable to reconcile the $\Delta a_\mu$ anomaly within
one sigma, in any regions permitted by the other constraints. For $R_2$, there is no CKM suppression of neutrino couplings, and $N_{\nu_{\ell}}$ does contribute a bound—although here it is not competitive. For the set of constraints excluding that from $\Delta a_\mu$, the numerical study illustrated in Figure 2 indicates a preference for Yukawa couplings related as per

$$y_{13} \sim y_{23}^{L}. \quad (36)$$

We repeated this exercise with different benchmark $R_2$ masses and achieved similar results indicating this preferred coupling relationship; nevertheless, these too were unable to ameliorate the $\Delta a_\mu$ anomaly.

**B. LQ mass constraints: fixed ratio**

The results of Section III-A support the anzatz that left-handed couplings will follow a simple proportionality relationship between lepton generations, indicated by the points of interest on the $y_{13}^{L} - y_{23}^{L}$ plane. To begin exploring the constraints on LQ mass in these models, we first fix the $y_{13}^{L} : y_{23}^{L}$ ratio.

For both models we select the relationship $y_{13}^{L} \sim y_{23}^{L}$ from equations (35) and (36). Figure 3 illustrates that an approximate upper bound for $m_\phi$ can reasonably be derived for these couplings structures, in each model, via a thorough consideration of constraints. We have also included a vertical dashed line to show the collider lower-bound on $m_\phi$ given by equation (34).

1. $S_1$, the $|F| = 2$ LQ solution

Note that we selected only one of the relationships suggested by equation (35) simply to demonstrate a viable parameter space. In the upper plot of Figure 3 contributions to $\text{Br}(Z \rightarrow e\mu)$ and coupling perturbativity border the allowed region—a subset of that labelled as ‘$Z \rightarrow \ell\ell$ preferred’. The boundary of this region suggests an upper-bound on $m_{S_1} \sim O(5)\text{TeV}$.

2. $R_2$, the $|F| = 0$ LQ solution

As per the earlier scan with the benchmark mass, in Figure 3 we find that the $R_2$ model is incapable of satisfying $\Delta a_\mu$ within one sigma together with other constraints. Notably, the couplings needed to satisfy $\text{Br}(\mu \rightarrow e\gamma)$ appear to be completely incompatible for this benchmark. Although many constraints appear to lie on top of one another in this figure, we reiterate that it is presented on a double-logarithmic scale.

**C. LQ mass constraints: parameter scan**

Using Figure 3 as a rough guide for allowed mass-scales, we perform a random full-parameter scan for each of these models. These more general scans do not incorporate the assumption of a fixed proportionality between
coupings, so parameter space additional to that explored in the previous sections is probed, which is especially relevant for $R_2$ given the strong constraints derived above. We logarithmically vary the parameters within the following ranges:

$$m_\phi \in [1, 100] \text{ TeV}, \quad y_L^{13}, y_L^{23} \in [0, \sqrt{4\pi}] \subset \mathbb{R}^+. \quad (37)$$

The right-handed couplings are fixed, as per earlier discussion, and we require that points satisfy the constraints outlined at the beginning of Section III.

1. $R_2$, the $|F| = 0 \ LQ$ solution

We are unable to identify points for $R_2$ within this top-philic parameter space that pass all constraints. This is complementary to the investigation in reference [45], where $R_2$ is considered in a charmphilic $(g - 2)_\mu$ model, and find that it is unsuccessful within experimental bounds.

We emphasise that this conclusion is based on the current central values and error ranges for the anomalous magnetic moments, and should be revisited when new experimental results are obtained.

2. $S_1$, the $|F| = 2 \ LQ$ solution

For generating both $(g - 2)_\mu$ and $(g - 2)_\mu$ within one-sigma in top-philic parameter space, constraints imply a mass upper-bound of

$$m_{S_1} \leq 5.96 \text{ TeV} . \quad (38)$$

This is consistent with the benchmark in Section III-B.

The coupling textures for points which pass all constraints are illustrated in Figure 3. These show a clear substructure in the assignment of both chiralities of couplings. This is consistent with the expectations from our discussion in Section II-A, and observations in Section III-A.

In the leftmost subfigure, we see two distinct coupling structures appearing in the data:

$$y_{13}^{L} \sim y_{23}^{L}, \quad \text{and} \quad y_{13}^{L} \sim 10^3 \times y_{23}^{L}, \quad (39)$$

which are consistent with those found in equation (35). From equation (30), a dominant mixed-chiral $\Delta a_\mu$ contribution, together with proportionality in the $y_{13}^{L} - y_{23}^{L}$ plane, translates to a simple proportionality relation for the right-handed couplings. This can be seen in the rightmost subfigure in Figure 3.

The patterns emergent in these datasets motivate a simple algebraic relationship between couplings of the top-quark to charged-lepton generations. Known theoretical approaches can be explored to generate such hierarchical coupling structures in UV-complete NP models; for example, Froggatt-Nielson mechanisms [46]. We refrain from discussing this concept further here, but rather identify it as an avenue for future exploration.

FIG. 3. Constraining mass for $S_1$ (upper plot) and $R_2$ (lower plot) with a fixed coupling relationship: Grey andhashed shaded regions indicate that they are ruled out by the labelled constraint. The green shaded region labelled ‘$Z \rightarrow \ell\ell$ preferred’ indicates that the central values for both $g_\mu^a$ and $g_\mu^c$ are satisfied within $2\sigma$. The grey shaded areas are ruled-out by perturbativity of the generated right-handed couplings (upper and lower plots), and/or the input couplings (upper plot). The lower mass bound from colliders is shown as a vertical dashed line, and note the difference in scales on both plots to clearly illustrate regions of interest. For this benchmark coupling relationship, the $R_2$ model is unable to reconcile the $(g - 2)_\mu$ anomaly (excluded region labelled as $\Delta a_\mu$) within the bounds from other constraints. Conditions for each constraint are outlined at the beginning of Section III.
D. Future of predicting $(g - 2)_\tau$

There are proposed measurement techniques for the quantity $(g - 2)_\tau$, however up to the present there are only rough bounds on its experimental value \[ -0.052 < a_\tau < 0.013 \quad (95\% C.L) \] The short lifetime of the tau makes measurement of this quantity notoriously difficult.

By observation of the other two lepton flavours, we would expect that a measurement of this observable could provide further evidence for lepton non-universality in NP models. Furthermore, exploring the generation hierarchical coupling structures could be further motivated with future measurement of $(g - 2)_\tau$, particularly if such a measurement implies related substructure to that observed in Figure 4.

To generate a correction to the tauon magnetic moment in a similar manner to the above, we would extend the coupling textures in \[ y_{13} \] to include top-quark couplings to the tau, with an important constraint coming from the observed rate for $Z \to \tau\tau$. We expect that these parameters will also be heavily constrained by kinematically-allowed rare tauonic decays to semileptonic final states. As the framework above does not consider tauonic couplings, we refrain from discussing explicit bounds. However, the future of tau physics could have very interesting things to say about the potential for simple scalar-only extensions to the SM as avenues for generating lepton-flavour violation and higher-order corrections to precision observables.

IV. CONCLUSIONS

We have argued that the apparent lepton-flavour universality violation in measurements of the anomalous magnetic moments of the electron and muon can be explained using the mixed-chiral subset of scalar leptoquark models. This mixed-chiral nature is necessary for the sign dependence of one-loop BSM corrections, consistent with the observed anomalies. There are precisely two scalar LQs which have the required coupling structure: the $SU(2)_L$ singlet $S_1 \sim (3,1,-1/3)$ and the doublet $S_2 \sim (3,2,7/6)$. We demonstrated the allowed parameter space, assuming real-values for Yukawa couplings, showing that only one of these models ($S_1$) is capable of generating $\Delta a_\mu$ and $\Delta a_e$ within $1\sigma$ of the current observed values, whilst also satisfying constraints from precision electroweak observables and flavour-violating leptonic decays. An upper limit on the LQ mass was found to be $m_{S_1} \lesssim 5.96$ TeV. Combining this with the collider bound (equation 34) gives the following solution for the $(g - 2)_{e/\mu}$ puzzle: a mixed chiral scalar leptoquark, $S_1$, with mass in the range

\[ 1.44 \leq \frac{m_{S_1}}{\text{TeV}} \leq 5.96. \] (41)

Hierarchical Yukawa coupling relationships emerge in the allowed points for $S_1$, which motivates further study into possible texture generation mechanisms and their extension to models including effects in $(g - 2)_\tau$.

We were unable to numerically identify a region of parameter space for a top-philic model of $R_2$ that could simultaneously satisfy these constraints, given the present experimental values. This was due, in particular, to an interplay between the couplings required to rectify the $(g - 2)_\mu$ anomaly together with the constraints from $\mu \to e\gamma$, $(g - 2)_e$ and $Z \to \mu\mu$.

The framework outlined in this paper can be easily adapted when new experimental results for $(g - 2)_\ell$ are obtained, as expected in the near-future from experiments such as Muon $g - 2$ \[ 4\]. We motivate the ongoing consideration of mixed-chiral LQs for flavour phenomenol-
ology in new physics models, particularly those permitting lepton-flavor violation at one-loop level.

**Note added:** After this work was completed, a result from the BMWc collaboration [49] was released giving an improvement on the theoretical prediction for hadronic vacuum polarization contribution to \((g-2)_\mu\) from lattice QCD. Their result indicates that within the current experimental measurements, NP explanations in this quantity may be entirely unnecessary. In the \(\Delta \alpha_\mu\) result quoted in equation (3), the hadronic contribution to the corresponding theoretical calculation originates from the averaged \(R\)-ratio procedural results for \((g-2)_\mu\) [50–52]. Prior to this BMWc result, the \(R\)-ratio procedure was highly favoured over lattice methods – due to lattice methods having very large associated theoretical uncertainties. Reference [49] is the first to achieve competitive precision for the lattice method; however, as the authors themselves acknowledge, this result is yet to be independently verified. For this reason, we use the \(R\)-ratio theoretical values for our analysis.

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