Quantum teleportation of a three-qubit entangled states via W-class states

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Abstract. We propose a novel and efficient scheme for quantum teleportation a three-qubit entanglement state via two sets W-class states. The local measurement on Alice’s particle using two three von-Neumann projective and Bob reconstruct using unitary transformation. This quantum teleportation scheme is perfect, i.e. the success of probability and fidelity can reach one.

1. Introduction
Quantum teleportation is prime example of a quantum formation processing task, where an unknown state can be perfectly teleported from one place to another by using previously shared entanglement and classical communication between the sender and the receiver. Bennett et al [1] proposed quantum teleportation of one-qubit via maximally entangled two qubit states. Thereafter, entangled states of the W-class are considered as a quantum channel for teleportation of an entangled state [2,3]. The W states channels was used in quantum communication [5,6,7,8,9]. The quantum teleportation for an unknown two-qubit states [4], three-qubit states [8] and N-qubit states [10] via W states.

In this article, we propose a quantum teleportation protocol to transmit a three-qubit entangled states via twofold of three-qubit W-class states. This teleportation protocol is supported by Bob’s additional particles [8] and the SWAPP operator [11] to assist Alice’s projective measurements and Bob’s unitary transformations.

Furthermore, the structure of this article in Sect. 2, we present quantum teleportation of a three-qubit via two W-class states. Finally, in Sect.3, we present some conclusions.

2. Quantum teleportation of a three-qubit entangled states via W-class states

The quantum information of a three-qubit entangled states is given by,

\[ |\psi\rangle_{ABC} = a|001\rangle + b|010\rangle + c|100\rangle + d|111\rangle, \]

with \(|a|^2 + |b|^2 + |c|^2 + |d|^2 = 1\).

Generally in quantum teleportation protocol scheme, Alice as sender wants to send to distant Bob’s receiver. The quantum channel Alice and Bob is two W-class states,

\[ |\psi\rangle_{123} = \frac{1}{2} (|100\rangle + |010\rangle)_{123} + \frac{\sqrt{2}}{2} |001\rangle_{123}, \]

\[ |\psi\rangle_{456} = \frac{1}{2} (|100\rangle + |010\rangle)_{456} + \frac{\sqrt{2}}{2} |001\rangle_{456}. \]
Particles A, B, C, 1, 2, 4 and 5 belong to Alice, particle 3 and 6 belong to Bob. Then Bob introduce an additional particle \( |0\rangle \). Initially, the states of the joint system can be written as,
\[
|\Psi\rangle_{ABC123456} = |\psi\rangle_{ABC} \otimes |\psi\rangle_{123} \otimes |\psi\rangle_{456} \otimes |0\rangle_7
\]
To teleporting an unknown three-qubit state \( |\psi\rangle_{ABC} \) to Bob, Alice first perform two three-qubit von Neumann projective measurement on her qubit pairs (A,1,2) and (B,4,5) in the following set orthogonal bases in the W-class states category given by,

\[
|\eta\rangle^{(\pm)}_{ijk} = \frac{1}{2} (|001\rangle + |010\rangle \pm \sqrt{2}|100\rangle)_{ijk},
\]

\[
|\xi\rangle^{(\pm)}_{ijk} = \frac{1}{2} (|101\rangle + |110\rangle \pm \sqrt{2}|000\rangle)_{ijk}.
\]

The following result Alice’s measurement are,

\[
|\Gamma\rangle_{ABC123456} = |\psi\rangle_{ABC} \otimes |\psi\rangle_{123} \otimes |\psi\rangle_{456} \otimes |0\rangle_7
\]

Without loss generality, if Alice’s measurement result are \( |\eta^{+}\rangle_{A12} |\eta^{+}\rangle_{B45} \) respectively, then the particle pair (3,6,7,C) are collapsed into the state,

\[
\frac{1}{4} \{ |0001\rangle_{367} + b|0010\rangle_{367} + c|1000\rangle_{367} + d|1101\rangle_{367} \}
\]
and suppose SWAPP operator,

\[
\text{SWAPP}_{7C} = |00\rangle_7 \langle 01| + |01\rangle_7 \langle 10| + |10\rangle_7 \langle 01| + |11\rangle_7 \langle 11|.
\]

The SWAPP operator applied on the particle pairs (7,C), therefore equation (8) can be expressed in,

\[
\frac{1}{4} \{ |0001\rangle_{367} + b|0010\rangle_{367} + c|1000\rangle_{367} + d|1101\rangle_{367} \}
\]

Then Alice measures particle C by basis \{0,1\}. When the measurement result is \( |0\rangle \), particles (3,6,7) therefore equation (10) can be expressed in,

\[
\frac{1}{4} \{ |000\rangle_{367} + b|010\rangle_{367} + c|100\rangle_{367} + d|111\rangle_{367} \}.
\]
After doing that, Alice transforms the measurement result to Bob by classical channel. Bob applied appropriate unitary operation to recover the input state. Alice’s measurement results and Bob’s unitary operations are shown in Table 1.

| Alice’s results | Bob’s results | Bob’s operations |
|-----------------|--------------|-----------------|
| $|\eta^+\rangle_{A12}|\eta^+\rangle_{B45}|0\rangle_c$ | $a|001\rangle_{367} + b|010\rangle_{367} + c|100\rangle_{367} + d|111\rangle_{367}$ | $I_3 \otimes I_6 \otimes I_7$ |
| $|\eta^-\rangle_{A12}|\eta^-\rangle_{B45}|0\rangle_c$ | $a|001\rangle_{367} - b|010\rangle_{367} + c|100\rangle_{367} - d|111\rangle_{367}$ | $I_3 \otimes (\sigma_x)_{6} \otimes I_7$ |
| $|\eta^+\rangle_{A12}|\eta^-\rangle_{B45}|0\rangle_c$ | $a|001\rangle_{367} + b|010\rangle_{367} - c|100\rangle_{367} - d|111\rangle_{367}$ | $(\sigma_x)_{3} \otimes (\sigma_x)_{6} \otimes I_7$ |
| $|\eta^-\rangle_{A12}|\eta^-\rangle_{B45}|0\rangle_c$ | $a|001\rangle_{367} - b|010\rangle_{367} - c|100\rangle_{367} + d|111\rangle_{367}$ | $(\sigma_x)_{3} \otimes (\sigma_x)_{6} \otimes I_7$ |
| $|\xi^+\rangle_{A12}|\xi^+\rangle_{B45}|0\rangle_c$ | $a|111\rangle_{367} + b|000\rangle_{367} + c|110\rangle_{367} + d|101\rangle_{367}$ | $I_3 \otimes (\sigma_x)_{6} \otimes I_7$ |
| $|\xi^-\rangle_{A12}|\xi^-\rangle_{B45}|0\rangle_c$ | $a|111\rangle_{367} - b|000\rangle_{367} - c|110\rangle_{367} + d|101\rangle_{367}$ | $(\sigma_x)_{3} \otimes (\sigma_x)_{6} \otimes I_7$ |
| $|\xi^+\rangle_{A12}|\xi^-\rangle_{B45}|0\rangle_c$ | $a|111\rangle_{367} + b|000\rangle_{367} - c|110\rangle_{367} - d|101\rangle_{367}$ | $(\sigma_x)_{3} \otimes (\sigma_x)_{6} \otimes I_7$ |
| $|\xi^-\rangle_{A12}|\xi^-\rangle_{B45}|0\rangle_c$ | $a|111\rangle_{367} - b|000\rangle_{367} + c|110\rangle_{367} - d|101\rangle_{367}$ | $(\sigma_x)_{3} \otimes (\sigma_x)_{6} \otimes I_7$ |

2.1 The success probability and fidelity calculations of an information

a. The success probability

Suppose the success probability of an information is given by

$$P_i = |\langle \chi_{in}|\chi_{(out)i}\rangle|^2, \quad i = 1, \ldots, 16,$$

(12)

where $|\chi_{in}\rangle$ is the information sent by Alice and $|\chi_{(out)i}\rangle$ is the information recovered by Bob for i-term on joint states equation. For example, suppose the first term in equation (7). $|\chi_{(out)i}\rangle = 1/4 \{a|001\rangle + b|010\rangle + c|100\rangle + d|111\rangle\}_{367}$.

$$P_1 = |\langle 1/4 (a|001\rangle + b|010\rangle + c|100\rangle + d|111\rangle \rangle_{ABC}\times 1/4 (a|001\rangle + b|010\rangle + c|100\rangle + d|111\rangle \rangle_{367}|^2,$$

$$= 1/16 [2(\alpha)^2 + |b|^2 + |c|^2 + |d|^2]^2 = 1/16 \left(\frac{1}{2}\right)^2,$$

$$P_1 = 1/16.$$

(13)

Similar as the above calculation for fifteen other terms, we obtain the total success probability for this information type, $P = 16 \times (1/16) = 1$.

b. Fidelity

Suppose fidelity for an information [11],

$$F_i(\rho_{in}, \rho_{(out)i}) = \left| \text{tr} (\sqrt{\rho_{in}\rho_{(out)i}})\sqrt{\rho_{in}} \right|^2,$$

(14)

where, $\rho_{in} = |\chi_{in}\rangle \langle \chi_{in}|$ is information density which is sent by Alice and $\rho_{(out)i} = |\chi_{(out)i}\rangle \langle \chi_{(out)i}|$ is information density which is recovered by Bob for the i-th term. Equation (14) can be simplified by using trace properties,
\[
\mathcal{F}_i(\rho_{in}, \rho_{(out)i}) = \left[ tr \left( \sqrt{\rho_{in}} \rho_{(out)i} \sqrt{\rho_{in}} \right) \right]^2,
\]

\[
= \left[ tr(\sqrt{\rho_{in}} \rho_{(out)i}) \right]^2 = \left[ tr \left( \sqrt{\langle x_{in} \rangle \langle x_{in} \rangle} (|x_{(out)i} \rangle \langle x_{(out)i}|) \right) \right]^2,
\]

\[
= \left[ tr \left( \langle x_{in} \rangle (|x_{(out)i} \rangle \langle x_{(out)i}|) \right) \right]^2 = \left[ tr \left( \sqrt{|x_{in} \rangle \langle x_{in}|} \right) \right]^2,
\]

\[
\mathcal{F}_i(\rho_{in}, \rho_{(out)i}) = |\langle x_{in} \rangle |^2.
\] (15)

The fidelity in equation (15) is similar as equation (12) and we obtain the total fidelity for this information type, \( \mathcal{F}(\rho_{in}, \rho_{(out)}) = 16 \times (1/16) = 1. \) The total success probability and fidelity of the teleportation each are one.

3. Conclusion

In this paper, we have demonstrated that two sets of W-class states can be used as the quantum channel to realize the deterministics teleportation of a three-qubit entangled states based on the three-qubit von Neumann measurement and the local unitary operations. The main results of these protocols are both the probability of success and fidelity can attain the values of one and it can be considered as a perfect quantum teleportation.

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