APPLICATION OF THE HIERARCHICAL BOOTSTRAP TO MULTI-LEVEL DATA IN NEUROSCIENCE

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ABSTRACT

A common feature in several types of neuroscience datasets is the presence of hierarchical data structures, such as recording the activity of multiple neurons in multiple animals across multiple trials. Accordingly, the measurements constituting the dataset are not independent, even though the traditional statistical analyses often applied in such cases (e.g. Student’s t-test) treat them as such. The hierarchical bootstrap has been shown to be an effective tool to accurately analyze such data and while it has been used extensively in the statistical literature, its use is not widespread in neuroscience, despite the ubiquity of hierarchical datasets. In this paper, we illustrate the intuitiveness and utility of the hierarchical bootstrap to analyze hierarchically nested datasets. We use simulated neural data to show that traditional statistical tests can result in a false positive rate of over 45%, even if the Type-I error rate is set at 5%. While summarizing data across the non-independent points (or lower levels) can potentially fix this problem, this methodology greatly reduces the statistical power of the dataset. The hierarchical bootstrap, when applied sequentially over the levels of the hierarchical structure, keeps the Type-I error rate within the intended bound and retains more statistical power than summarizing methods. We conclude by demonstrating the effectiveness of the method in two real-world examples, first analyzing singing data in male Bengalese finches (Lonchura striata var. domestica) and second quantifying changes in behavior under optogenetic control in flies (Drosophila melanogaster).

Keywords Hierarchical Bootstrap · Multi-level datasets

1 Introduction

It is commonplace for studies in neuroscience to collect multiple samples from within a category (e.g. multiple neurons from one animal) to boost the sample size. A recent survey found that of 314 papers published in prominent journals covering neuroscience research over an 18 month period in 2013-14, roughly 53% of those studies had nested datasets featuring hierarchical data [1]. When data are collected in this manner, the resulting data are not independent. Commonly deployed statistical tests like the Student’s t-test and ANOVA, however, treat all data points as independent. This assumption results in an underestimation of uncertainty in the dataset and a corresponding underestimation of the p-value [2,3,4]. Intracluster correlation (ICC) [5,6] and pseudoreplication [7,8], in which variance within a cluster and variance between clusters are not propagated appropriately, often are the cause of such biases. For example, consider a hypothetical example in which one measures changes in dendritic spine size during learning. Since one can typically only measure from a few animals each in different treatment conditions, researchers usually increase sample sizes by measuring multiple spines from each neuron and by measuring multiple neurons within an animal. The hierarchical nature of such datasets can result in different samples being statistically dependent on each other: spines measured from the same neuron may be more similar than spines measured across different neurons, even more so than spines measured from different animals within the same treatment condition.
Linear Mixed Models (LMMs) can be used to account for the variance across different levels [1,9] and have recently been used to do so in several studies [10,11,12,13]. However, LMMs assume that all hierarchical structure present is linear, which is often not true for typical datasets. Additionally, the parameters returned by LMM fits may exhibit bias and be unreliable when the number of clusters is small, as is also often the case in neuroscience datasets [14,15,16].

The hierarchical bootstrap [17,18,19,20] is a statistical method that has been applied successfully to a wide variety of clustered datasets, including census and polling data, education and psychology, and phylogenetic tree data [21,22,16]. Unlike LMMs, the hierarchical bootstrap is relatively agnostic to the underlying structure present in the data and has consistently performed better at quantifying uncertainty and identifying signal than traditional statistics [23,22,24], though some concerns have been raised that the bootstrap may be excessively conservative in a limited subset of cases [25,26]. However, the use of the hierarchical bootstrap in neuroscience is limited, even though its application is increasingly warranted.

This paper is divided into two parts. In the first, we simulate a typical dataset studied in neuroscience and use it to illustrate how the Type-I error is inflated in hierarchical datasets when applying traditional statistical methods but can be averted using the hierarchical bootstrap. In the second, we demonstrate the use of the hierarchical bootstrap in two real-world examples using singing data from songbirds [27] and optogenetic control of behavior in flies [28]. In both cases, the data have a strong hierarchical structure and our analyses highlight the need to use appropriate statistical tests when analyzing hierarchical datasets in neuroscience.

2 Materials and Methods

The simulations for this paper were run in the Jupyter Notebooks environment using Python (version 3.7.2) and importing the following libraries: NumPy (version 1.15.4), SciPy (version 1.1.0), Matplotlib (version 3.0.2) and Pandas (version 0.23.4). Reanalysis of data from Hoffmann and Sober (2014) was performed using MATLAB (version 2017a). The codes for both simulation and the data analysis are available on Github.

2.1 Traditional vs Summarized vs Bootstrap

Throughout this paper, we compare 3 statistical methods that we refer to by shorthand as “Traditional”, “Summarized” and “Bootstrap” respectively. Throughout this paper, when we refer to the “Bootstrap” method, we mean a hierarchical bootstrap procedure. We will detail what each of those terms mean here (see Fig. 1 for schematics of each). For the sake of clarity, let us consider a fictitious example. Suppose our dataset involves recording the neural activity of neurons in the amygdala when an individual was exposed to an aversive auditory cue either in the presence or absence of a drug of interest believed to reduce anxiety. Each neuron was recorded for around one hundred trials of exposure to the auditory cue and the process was repeated for several hundreds of neurons in both the presence and absence of the drug (see Fig. 1). We could add a layer of complexity by considering that the experiment was repeated across several individuals but for the sake of simplicity, let us assume that all the data were collected from a single individual. In the “Traditional” method, every data point (i.e. the firing rate of every neuron to every instance of the auditory cue) is treated as independent, regardless of the hierarchical structure present in the dataset (see Fig. 1). All the data points are used to calculate the mean and the uncertainty in the estimate of the mean, namely the standard error of the mean (SEM) and a Student’s t-test is used to ascertain statistically significant differences between the mean firing rate of the neurons in the presence versus absence of the drug of interest. The “Summarized” method, on the other hand, acknowledges the possibility that repeated trials within the same neuron may be more similar to each other than trials across neurons. As a result, the mean firing rate for each neuron is calculated first and the mean of the group is calculated as the mean of the population of mean firing rates for each neuron in the group and the SEM is computed from this population of means (see Fig. 1). Note that the mean for each group in this case is equal to that in the “Traditional” case if and only if the number of trials recorded for every neuron within a group is represented equally. A Student’s t-test is thus applied to the population of mean firing rates between the two groups. An additional complication that we circumvent in our toy example by considering all the data to be obtained from a single subject is the decision as to which level one must summarize the data. In the case of multiple subjects, one may summarize either at the level of individual neurons or individual subjects. While summarizing at the level of subjects is the most appropriate way to avoid non-independence between data points, it can seriously reduce sample size and therefore power. Finally in the “Bootstrap” method, we perform the hierarchical bootstrap on the two groups to compute posterior distributions of the range of means possible from each group (see Fig. 1), as follows. First, we sample with replacement (i.e., we sample from the current distribution in such a way that replications of previously drawn samples are allowed) from the neurons in the group. Then, for the neurons selected, we then sample with replacement from the individual trials for the number of times each neuron was recorded. We then compute the mean firing rate across the group for that resampled population and repeat the entire process N_bootstrap times (N_bootstrap=10^4 for all instances in this paper unless...
An overview of Hierarchical Datasets

Level 1: Experimental Condition
Level 2: Neurons
Level 3: Trials

b) “Traditional” Statistics

Drug
Mean + SEM
Control
Mean + SEM

Compare with Two sample t-test

Calculate Mean for each Neuron

Drug
Mean + SEM
Control
Mean + SEM

Compare with Two sample t-test

c) “Summarized” Statistics

Calculate Mean for each Neuron

Drug
Mean + SEM
Control
Mean + SEM

Compare with Two sample t-test

d) The “Hierarchical Bootstrap” Method

Resample each level with replacement

Repeat \( N_{\text{bootstrap}} \) times for each condition

Mean across all trials

Population of \( N_{\text{bootstrap}} \) means

Compare with Statistical tests

Population of \( N_{\text{bootstrap}} \) means

Compare with Statistical tests

Figure 1: a) An example of a hierarchical dataset. Here the dataset is divided into 3 levels with the first level containing the experimental groups to be compared, the second containing the individual neurons and the third containing the neuronal firing rate during each trial. Each neuron is color coded and the trials per neuron are distinguished by the position of the colored diamond. b) In “Traditional” statistics, the means for each group is computed across all the trials and are then compared using a two sample t-test. c) In “Summarized” statistics, the mean for each neuron is computed first. These means are then used to compute an overall mean for each group and the groups are compared using a two-sample t-test. d) In the “Hierarchical Bootstrap” method, we create new datasets \( N_{\text{bootstrap}} \) times by resampling with replacement first at the level of neurons followed by trials within a neuron. We then compute the mean across all trials every time we perform resampling. The final statistic is computed on this population of resampled means (see Methods for details).

otherwise noted). The mean for each group in this case is identical to that computed in the “Traditional” method. The 67% confidence interval (or equivalently, the standard deviation) of the population of means so obtained gives an accurate estimate of the uncertainty in the mean. Note that the mean is a special case where this uncertainty is more commonly referred to as the Standard Error of the Mean or SEM. We can then compute the probability of one group being different from the other using the population of resampled means obtained above for each group (see Hypothesis testing with Bootstrap below for complete details).
2.2 Hypothesis testing using Bootstrap

We described above how bootstrap samples can be used to compute the uncertainty in measuring the mean of a population. However, the bootstrap can be used more broadly to measure the uncertainty in any metric of interest as long as it obeys the law of large numbers and scales linearly in the probability space. In addition, the bootstrap is used to compute posterior distributions of the range of values possible for the metric of interest from the data of limited sample size. As a result, the distribution of bootstrap samples can be used to compute probabilities that the data supports particular hypotheses of interest directly. We will describe below how this can be done with an example. Note that while this is not the only way of hypothesis testing using bootstrapping, we found this to be a particularly simple and effective way of doing so.

As will be done several times in this paper, suppose we wished to evaluate the support for the hypothesis that the mean of particular sample was significantly different from a fixed constant - zero for our example. In order to do so, we would compute the proportion of means in our population of bootstrapped means of the sample that were greater than or equal to zero. If we set the acceptable false positive (Type-I error) rate to $\alpha = 0.05$ throughout this paper, then if the computed proportion was greater than $1 - \alpha/2$ (or $p>0.975$ for $\alpha = 0.05$) we would conclude that the sample of interest had a mean significantly greater than zero. Alternatively, if the computed proportion was less than $\alpha/2$ (or $p<0.025$ for $\alpha = 0.05$) then we would conclude that the sample of interest had a mean significantly less than zero. Any proportion $\alpha/2 \leq p \leq (1 - \alpha/2)$ would indicate a relative lack of support for the hypothesis that the mean of the sample of interest is different from zero. In the case of multiple comparisons, we use the Bonferroni correction to adjust the threshold for significance accordingly. We would also like to make a distinction between the probabilities we referred to above and the p-values typically associated with statistical tests. p-values refer to the probability of obtaining a result as extreme or more extreme than those obtained under the assumption that the null hypothesis is true. As such, they do not provide a direct measure of support for the hypothesis one truly wishes to test. The ‘p’ referred to in the bootstrapping procedure above, however, directly provides a probability of the tested hypothesis being true. For the rest of this paper, in order to distinguish the direct probabilities obtained using the bootstrapping procedure from p-values reported from traditional statistical tests, we will use ‘$p_{\text{boot}}$’ to refer to bootstrap probabilities and ‘p’ to refer to p-values from other tests.

While it is not performed in this paper, the procedure described above can also be used to compare the means of two different groups using their respective samples. In this case, we would compute a joint probability distribution of the two samples with each sample forming the two axes of a 2-D plot. In this case, the null hypothesis would be a circle centered on the line $y = x$. Therefore, to test if the two groups are different, one would compute the total density of the joint probability distribution on one side of the unity line. If the volume computed is greater than $1 - \alpha/2$ then the first group is significantly greater than or equal to the second while if the volume computed is less than $\alpha/2$, the second group is significantly greater than or equal to the first with all other volumes indicating no significant differences between the groups. We can also extend this formulation to comparisons between multiple groups by performing pairwise-comparisons between the groups and adjusting the threshold for significance accordingly (by Bonferroni correction for example).

2.3 Design Effect (DEFF)

When one analyzes data from hierarchical datasets, the unique information provided by each additional data point at the lowest level of the hierarchy depends on the average number of samples in the cluster and the relative variance within and between clusters. This relationship was mathematically quantified using the Intra-cluster correlation (a.k.a. intra-class correlation) or ICC. ICC is a useful metric that provides a quantitative measure of how similar data points are to each other within an individual cluster in a hierarchical dataset [5][6]. While there are some differences in how it is calculated, in general it is defined as the following ratio:

$$ICC \quad or \quad \rho = \frac{s^2_{\text{between}}}{s^2_{\text{between}} + s^2_{\text{within}}}$$  \hspace{1cm} (1)$$

Where $s^2_{\text{between}}$ represents the variance across cluster means while $s^2_{\text{within}}$ represents the variance within clusters. Hence, the ICC is a metric that varies from zero to one, where a measure of zero represents no clustering of data and every data point being independent and a measure of one represents a perfect reproduction of samples within clusters (i.e., all points within a cluster are exactly the same). Kish further formalized the relationship between ICC and the adjusted effect size that was termed the “Design Effect” or DEFF with a corresponding correction to be applied to the standard error of the mean computed from the dataset termed DEFT, defined as the square root of DEFF [6][29]. Formally, DEFF was defined as:

$$DEFF = \frac{\sigma^2(\text{data})}{\sigma^2(\text{data if independent})} = 1 + \rho \ast (\bar{n}_j - 1)$$  \hspace{1cm} (2)$$
Where $\bar{n}_j$ represents the average sample size within each cluster and $\rho$ is the ICC. Hence, as the number of samples within a cluster increases, the DEFF increases, resulting in a need for a larger correction (increase) to the standard errors. Conversely, as the number of samples within clusters increase, the standard error of the mean is underestimated potentially resulting in underestimation of the p-values and inflation of the Type-I error rate.

3 Results

Our results section has been organized into two sub-sections: Simulations and Examples. In the Simulations sub-section, we show results from simulations that illustrate the utility of the hierarchical bootstrap and in the Examples sub-section, we highlight the differences in results when analyzing data in two examples with and without the hierarchical bootstrap. Throughout the results section, we will compare statistical tests we refer to by shorthand as “Traditional”, “Summarized” and “Bootstrap” respectively. See Traditional vs Summarized vs Bootstrap in Materials and Methods and Figure 1 for a detailed description of the differences between the three conditions. Also note that whenever we refer to the “Bootstrap” in this paper, we mean the hierarchical bootstrap unless otherwise specified.

3.1 Simulations

We used simulations of neuronal firing in order to highlight the key characteristics of the hierarchical bootstrap as applied to nested data in neuroscience and the differences between the bootstrap and other, more commonly used statistical tests. Specifically, we were interested in whether the bootstrap displayed a conservative bias for independent and non-independent datasets, as well as in quantifying the bootstrap’s statistical power compared to other techniques. While these results may be derived from other mathematical results previously published \[30, 20]\, we found them instructive to depict explicitly.

3.1.1 The hierarchical bootstrap does not have a conservative bias in a hierarchical dataset

If the bootstrap had a strong conservative bias regardless of the nature of the data (hierarchical or independent), it may not be the right metric with which to address the problem of statistical analysis in hierarchical datasets. For our first simulation, we wished to evaluate whether the bootstrap displayed a conservative bias when processing a typical hierarchical dataset one might encounter in neuroscience research. Specifically, we simulated a condition in which we recorded the activity of 1000 neurons for 100 trials each. The neurons would then be split randomly into two groups of 500 neurons each and the mean firing rate across groups would be compared. Each neuron had a mean firing rate of 5Hz, simulated using a Poisson process. However, in order to introduce hierarchical structure into the dataset, each neuron’s firing rate was offset by a constant drawn from Gaussian noise of width 3Hz. This constant was the same for all 100 trials simulated for each neuron but varied between neurons. Since neurons were split randomly into two groups, there should be no statistical difference between the groups and we would expect to see a false positive rate of $\alpha$ (here set to 0.05).

We also varied the number of trials per neuron to study its effect on the false positive rate. We simulated the experiment 1000 times for each value of number of trials and computed the false positive rate from each. We used bootstrapping on the obtained results to estimate error bars and to test for significant differences away from 0.05. Given the relationship between the number of points within a cluster to the Design Effect (DEFF; see Design Effect in Materials and Methods), we would expect the false positive rate to increase with the number of trials per neuron \[31,32,31\].

As shown in Figure 2a, the false positive rate for the traditional method does increase with the number of trials per neuron rising from around 46% for 10 trials to almost 96% in the case of 3000 trials per neuron (probability of resampled proportions being greater than or equal to 0.05 was $p_{\text{boot}} > 0.9999$ in all cases; limit due to resampling $10^4$ times). On the other hand, both the summarized and bootstrap methods stayed remarkably similar in value and were not significantly different from 0.05 in all cases (adjusting for threshold of significance with Bonferroni corrections for 3 comparisons).

We also computed the estimate for the SEM using all 3 cases for each number of trials simulated, and the result is shown in Figure 2b. As shown, the SEM estimate remains fairly constant for both the summarized and bootstrap methods but decreases with an increase in the number of trials per neuron in the traditional case. Furthermore, the SEM estimate for the traditional case starts out much lower than either the summarized or bootstrap case suggesting that the increased false positive rate is at least partially due to the underestimation of error in the traditional case.
Figure 2: False positive rate and size of error bars quantified for the simulation in which there was no difference between the groups but the points were not independent. a) The proportion of significant results using each statistical method as a function of the number of trials per neuron. As expected, the false positive rate for the traditional method rises with increasing number of samples within each neuron. The Summarized and Bootstrap methods on the other hand have almost identical false positive rates close to the theoretical 5% in all cases. b) The size of SEMs computed for all 3 methods as a function of the number of trials per neuron. While those for both summarized and bootstrap stay roughly the same, those for the traditional method reduces with increasing number of trials. Note that for both traces, since the Bootstrap and Summarized almost perfectly overlap, the green trace has been thickened for visualization.

3.1.2 The hierarchical bootstrap is more conservative than Traditional and Summarized methods for independent data points

In the previous experiment, we reported that the hierarchical bootstrap does not have a conservative bias for hierarchical datasets. However, it has been reported earlier that the bootstrap has a conservative bias [25, 26], resulting in larger error bars than strictly necessary for the chosen threshold of Type-I error $\alpha$ (here set to 0.05). It has also been argued that this is not a bug or bias in the algorithm, but rather a more generic property of hypothesis testing by resampling [33, 21] and newer algorithms have claimed to reduce bias further [34, 35]. Here we tested the conservative bias of the hierarchical bootstrap in a similar situation as the first experiment above but where all the data points were independent. Given that we set $\alpha$ to 0.05, we would expect a 5% false positive rate if there were no bias in the algorithm.

As before we simulated a situation in which we recorded the activity of 1000 neurons over 100 trials each. In order to make each trial independent though, we removed the Gaussian noise term for each neuron. The neurons were thus simulated using only a Poisson random number generator with an average firing rate of 5Hz (each trial was thought to be 1 second of activity). Since the data points are independent, we would not expect differences between the Traditional and Summarized methods. We then split these 1000 neurons into two groups of 500 each randomly and computed the mean firing rate for each group. We then tested whether the means were significantly different from each other using the Traditional, Summarized and Bootstrap methods. We repeated this analysis 10000 times and plotted the
Figure 3: Results from the simulation in which there was no difference between the groups of two neurons. a) Proportion of significant results when comparing the 2 groups with each statistical method at \( \alpha \) of 0.05. As expected, both the Traditional and Summarized methods give roughly 5% false positive results. However, the bootstrap gives a much smaller proportion of significant results suggesting a conservative bias. b) The size of SEMs computed using each of the methods. The bootstrap does give an error bar roughly 1.4 times that of the other two metrics.

The proportion of trials that resulted in significant differences for each of the methods in Figure 3a. The error bars were computed by bootstrapping the results obtained from the simulation runs. As shown in the figure, both the Traditional and Summarized methods resulted in a proportion of significant results close to and not significantly different from 5% as expected (Traditional – 4.86 ± 0.21%; probability of proportion of significant results being greater than or equal to 0.05 was \( p_{\text{boot}} = 0.26 \); Summarized – 4.95 ± 0.22%; probability of proportion of significant results being greater than or equal to 0.05 was \( p_{\text{boot}} = 0.42 \)). By contrast, when using the bootstrap method, the proportion of significant results was significantly lower than the expected 5% at 0.66 ± 0.08%. Even when we increased the value of \( \alpha \) to 0.1, the proportion of significant results was still only 2.19 ± 0.15% (probability of proportion of significant results being greater than or equal to 0.05 was \( p_{\text{boot}} < 10^{-4} \) in both cases; limit due to resampling 10^4 times). This was a marked departure from Figure 3a where we saw that the bootstrap had no significant conservative bias for hierarchical datasets.

We also computed the standard error of the mean (SEM) in each case and reported the results in Figure 3b. As shown, the error bars for both the Traditional and Summarized methods are almost identical at 1.002 ± 0.002 * 10^{-2} for Traditional and 0.994 ± 0.023 * 10^{-2} for Summarized respectively. The error bars computed using the Bootstrap method are roughly 1.4 times larger at 1.407 ± 0.035 * 10^{-2}. Since the effect size is inversely proportional to the uncertainty in the dataset [36], which is captured here by the error bars, we conclude that the larger error bars do partially account for the drop in proportion of significant results observed and that the bootstrap seems to have a conservative bias for independent datasets.
3.1.3 The Bootstrap balances intended Type-I error rate and statistical power better than the Traditional or Summarized methods at low sample sizes

As we saw in the previous section, both the summarized and bootstrap methods bind the Type-I error rate at the intended 5% and the estimate of the SEM is roughly the same for both methods (Fig. 2). What then is the advantage of the bootstrap method over simply using the summarized method? The answer lies in the fact that the summarized methods result in a loss of statistical power (the ability to detect a statistical difference when one truly exists), particularly for low sample sizes of the upper hierarchical levels and for small effect sizes (a situation commonly found in neuroscience datasets). We used simulations to calculate the power for each of the three methods and the results are shown in Figure 4.

Since power depends on the effect size and the number of samples, we chose to examine the change in power with respect to the number of neurons per group (N) and the effect size for these simulations (Δ(mean)). In order to do so, we varied N between 2 and 16, keeping the number of trials per neuron constant at 100 each. We kept the mean firing rates of one group of neurons at 5Hz as before and varied the mean firing rate of the other group by Δ(mean), adding an additional 3Hz random Gaussian noise term to each neuron in both groups. As before, this term is constant for all trials within a neuron and varies between neurons. Since the previous simulations did not estimate the false positive rate when the number of neurons was as low, we first kept the mean firing rate for both groups of neurons equal at
We now present two real-world examples of the utility of the hierarchical bootstrap as applied to behavioral data. We then computed the power for the three methods as a function of the number of neurons per group and the difference in mean firing rate between the groups. Accordingly, we repeated the simulations described above changing the mean firing rate of one of the groups to 6.5Hz, 8Hz and 9.5Hz (light, medium and dark traces in Fig. 4) respectively. Since there is an actual difference between the groups in this case, the ideal plot will have a very high proportion of significant results barring adjustments for extremely low sample sizes. As shown, the traditional method has the most power (blue traces in Fig. 4), but as was seen in Figure 4, it also has an unacceptably high false positive rate for this type of data. The summarized method has the lowest power among the three methods, but does catch up for large effect sizes and with increasing group sizes (red traces in Fig. 4). The bootstrap is between the two extremes and has more power than the summarized metric particularly for small effect sizes and small group sizes (green traces in Fig. 4). As a result, we see that the bootstrap helps retain statistical power while also being sensitive to the Type-I error rate. However, as was mentioned when discussing Figure 4, the bootstrap can weight trials within levels more heavily than one would expect if the number of samples in the upper levels is very low and one must therefore be mindful when dealing with very low sample sizes that their data collected may not represent the true distribution in the population.

3.2 Examples

We now present two real-world examples of the utility of the hierarchical bootstrap as applied to behavioral data collected from experiments in songbirds [27] and flies [28]. These examples provide concrete instances of why one should use the appropriate statistical tests depending on the nature of their data and how the popular tests can result in more false positives or less statistical power than one desires.

3.2.1 The bootstrap highlights the risk of false positives when analyzing hierarchical behavioral data (vocal generalization in songbirds) using traditional statistical methods

As described above, although the bootstrap provides a better compromise between statistical power and false-positive rate than the Traditional or Summarized methods, its use is not widespread in the neuroscience literature, including in some of our own prior published work. To illustrate the practical importance of these issues, and to encourage other authors to critically re-evaluate their prior analyses, we here present a case in which we have used the bootstrap to reexamine one of our prior results – which used both Traditional and Summarized methods – and found the choices made can significantly affect the outcome of our analyses. As a reminder, when discussing Traditional or Summarized statistical tests, we will report a p-value denoted by ‘p’ which yields a significant result if p < 0.05. When talking about the Bootstrap tests however, we will report a p\textsuperscript{boot} which in turn yields a significant result if p\textsuperscript{boot} < 0.025 or p\textsuperscript{boot} > 0.975. In addition, p\textsuperscript{boot} provides a direct probability of the hypothesis being true.

Much of our prior work involves hierarchical datasets whether that involves vocal behavior in songbirds (Drs. Saravanan and Sober) or motor behavior in flies (Dr. Berman). In songbirds, each bird sings a variety of syllables and each syllable is repeated a different number of times. In many of our studies, we examine changes in the pitch of these syllables in response to manipulations. In a prior study, we examined the “generalization” (see below) of vocal learning in songbirds in response to an induced auditory pitch shift on a particular (target) syllable [27]. In these studies, the auditory feedback of one syllable was shifted in pitch and relayed to the bird through custom-built headphones with very short (10 ms) latency, effectively replacing the bird’s national auditory feedback with the manipulated version [37, 38, 27]. Note that while the headphones provided auditory feedback throughout the song, only the feedback for the single syllable targeted for pitch shift was shifted in pitch. We reported that in addition to birds changing the pitch of the target syllable in response to the pitch shift, the birds also “generalized” by changing the pitch of other syllables that had not been shifted in pitch. Specifically, we reported that syllables of the same-type (acoustic structure) as the target syllable changed pitch in the same direction as the target syllable (“generalization”) while syllables of a different-type than the target syllable did not change pitch in the same direction as the target syllable. A summary of these results is shown in Figure 4. As described above, although the bootstrap provides a better compromise between statistical power and false-positive rate than the Traditional or Summarized methods, its use is not widespread in the neuroscience literature, including in some of our own prior published work. To illustrate the practical importance of these issues, and to encourage other authors to critically re-evaluate their prior analyses, we here present a case in which we have used the bootstrap to reexamine one of our prior results – which used both Traditional and Summarized methods – and found the choices made can significantly affect the outcome of our analyses. As a reminder, when discussing Traditional or Summarized statistical tests, we will report a p-value denoted by ‘p’ which yields a significant result if p < 0.05. When talking about the Bootstrap tests however, we will report a p\textsuperscript{boot} which in turn yields a significant result if p\textsuperscript{boot} < 0.025 or p\textsuperscript{boot} > 0.975. In addition, p\textsuperscript{boot} provides a direct probability of the hypothesis being true.

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syllable changed pitch in the direction opposite to that of the target syllable (‘anti-adaptive generalization’; see Fig. 5b). Since in Hoffmann and Sober (2014) we employed traditional and summarized (at a syllable level) statistics when analyzing generalization, we decided to reanalyze the data from that study to ask if the generalization observed was still statistically significant when statistical tests were computed using the hierarchical bootstrapping procedure. In order to do so, we first recapitulated the results reported by computing statistics on the last 3 days of the shift period using the traditional and summarized methods as was reported earlier [27]. We focus our reporting on changes in the target syllable and anti-adaptive generalization in different-type syllables for the purpose of this example.

Figure 5: Reanalysis of generalization in the headphones learning paradigm. a) The results when quantified using traditional statistics. As shown, both target and different type syllables differ significantly from zero over the last 3 days of the shift with target syllable moving adaptively and the different type syllables showing anti-adaptive generalization respectively. b) The results when quantified using summarized statistics when summarized over the syllables. In this case, the target syllable is significantly different from zero while the different type syllables are just over the threshold for significance. c) The results when bootstrap is applied over the hierarchical levels. The target syllable is significantly different from zero but the different type syllables are not.
When we computed the change in pitch over the last 3 days of the shift period for the syllable targeted with auditory pitch shifts, we found that the birds compensated for the pitch shift of 1 semitone by 0.341 ± 0.007 (mean ± SEM in all cases) semitones with traditional statistics (one sampled t-test comparing to zero; t = 47.3; p < 2×10^{-38}; limit due to smallest number in MATLAB; red trace in Fig. 5a) and by 0.34 ± 0.08 semitones with summarized statistics (one sampled t-test comparing to zero; t = 4.25; p = 0.004; red trace in Fig. 5b). We did see anti-adaptive generalization in different-type syllables of -0.087 ± 0.003 semitones with traditional statistics (one sampled t-test; t = 23.9; p = 4×10^{-125}; blue trace in Fig. 5b). With summarized statistics, the different-type syllables changed by -0.12 ± 0.06 semitones (one sampled t-test; t = 2.00; p = 0.053; blue trace in Fig. 5b) and was (just) not statistically significant. We note a minor confound in our reanalysis: due to a discrepancy in our data archiving, our recapitulation of the old analysis for the paper yielded a slightly different p-value (p = 0.053) for summarized analysis of different type syllables than was originally reported in the original paper (p = 0.048). The point of this analysis is therefore not to replicate the exact findings but to highlight how choices made for statistical analyses can define the interpretation of one’s results as we detail below.

When we reanalyzed the data using bootstrapping over the hierarchal levels, we found that we did not have enough statistical power to claim that the anti-adaptive generalization was statistically significant. As expected, the targeted syllable shifted significantly away from zero to a final value of 0.34 ± 0.12 semitones (probability of resampled mean being greater than or equal to zero was p_{boot} = 0.995; red trace in Fig. 5). As a reminder, p_{boot} gives the probability of the hypothesis tested being true. Therefore, a value of 0.5 indicates minimal support for either the hypothesis (or its opposite) while values close to 1 (or 0) represent strong support for (or for the opposite of) the hypothesis. Different-type syllables however shifted to a final value of -0.09 ± 0.09 (probability of resampled mean being greater than or equal to zero was p_{boot} = 0.25; blue trace in Fig. 5). Hence, this result shows that the anti-adaptive generalization was too small an effect to detect with the sample size in the original study, suggesting that the generalization effects observed were driven largely by a small number of individual birds rather than a population wide effect. This result was reaffirmed by an independent study that also did not find evidence of anti-adaptive generalization in songbirds [39].

We also reanalyzed generalization in same-type syllables and similarly did not find a significant effect (probability of resampled mean being greater than or equal to zero was p_{boot} = 0.85) again indicating that we did not perform the generalization experiment on sufficient number of birds to adequately power the study. However it is worth noting that while the results did not meet the threshold for statistical significance, reporting probabilities in support of the hypotheses (p_{boot}) provides more information than simply determining whether or not a statistical threshold was met. In this case, a p_{boot} of 0.85 means that if we measured data from more birds drawn from the same distribution, we will see adaptive generalization in 85% of cases which is much higher than chance (50%) and is still useful information. Furthermore, we will note that an independent study [39] did find evidence of adaptive generalization for same type syllables in songbirds.

### 3.2.2 The hierarchical bootstrap captures the signal better than traditional or summarized methods in optogenetic control of behavior in flies

We wanted to test the utility of the hierarchical bootstrap in an independent example, and so we chose to analyze the data from an experiment studying the role of descending neurons in the control of behavior in flies [28]. Studies involving optogenetics are another area where hierarchical datasets are the norm. Each fly, since it can be tracked over extended periods of time, will exhibit each behavior multiple times within the period of observation. Additionally, multiple flies can be tracked simultaneously and the behavior is typically averaged across flies across trials for each experimental group. In this study, we used optogenetics to activate descending neurons in flies and studied the corresponding changes in behavior displayed. In order to do so, we first created a two-dimensional representation of the behavior of the flies in the absence of any manipulations, as has been described in detail previously [40, 28]. We then mapped the behavior of experimental animals both in the presence and absence of light stimulation as well as control animals that were not fed retinol, a binding co-factor needed for functionality of the light-activated channels, both in the presence and absence of light stimulation onto the behavioral representation. The resulting behavioral map for one class of descending neurons is shown in Figure 6. In order to assess whether the light stimulation caused a statistically significant change in the frequency of behavior observed, we argued that the frequency of behavior had to be significantly greater during optical stimulation than during periods of no stimulation within experimental animals. In addition, the frequency of behavior during optical stimulation had to be greater in experimental animals than in control animals. We used Wilcoxon rank summed tests coupled with Sidak corrections [41] for multiple comparisons in order to test for statistically significant differences. This would fall under the category of traditional methods as we have described previously. We compared the regions obtained from the original analysis with regions we obtained when using summarized or bootstrap statistics on this dataset and the result is shown in Figure 6. As shown, the traditional method seems to overestimate the region of significant differences and includes a false positive area that is separate from the main region where signal is present. The summarized method, on the other hand, does not identify any regions as being statistically significant despite
Discussion

The hierarchical bootstrap is a powerful statistical tool that was developed to quantify the relationship between school class sizes and achievement [20] and has since been used to quantify effect sizes in a wide variety of fields. The use of the hierarchical bootstrap in neuroscience however is still limited in spite of the need for it being increasingly clear. Through our simulations, we have shown the utility of the hierarchical bootstrap in neuroscience by examining the shortcomings of more common statistical techniques in a typical example one might encounter. While the results of our simulations may be inferred from other mathematical work on the subject [60, 20, 23, 42], to our knowledge, our results have not been shown explicitly in previous work. We first illustrated that the bootstrap does not have a conservative bias for hierarchical datasets in which the assumption of independence between data points is violated (Fig. 2). We then showed that the bootstrap performs better than summarized statistical measures by not sacrificing as much statistical power especially at low sample sizes and small effect sizes (Fig. 4). Finally, we showed real world applications of applying the hierarchical bootstrap to two datasets from songbirds and flies to demonstrate the advantages of the hierarchical bootstrap over other more commonly used statistical analysis techniques in neuroscience (Figs. 5 and 6).

“Pseudoreplication” refers to studies in which samples were either not replicated or the replicated samples were not independent, yet statistical tests performed treated the data points as independent replicates [27]. While pseudoreplication was first extensively reported on in ecological studies [7, 43], it has since been identified as a common problem in other fields, including neuroscience [8]. While resampling methods including bootstrapping were originally suggested as tools by which one could overcome the pseudoreplication problem [44], the bootstrap was argued to have a conservative bias resulting in larger error bars than necessary [25, 20]. Since then, however, several versions of the bootstrap algorithm...
have been developed to apply to hierarchical data and have been found to be unbiased and more robust for calculation of uncertainty in clustered data than other statistical methods [30][20][23][42][22][16]. In order to test the bootstrap for any potential bias in a typical example we might encounter in neuroscience, we produced simulations to quantify differences in mean firing rates between two groups of neurons when there was no difference between the groups. We illustrated that the bootstrap produced a false positive rate significantly below the expected 5% (Fig. [3]) and had larger error bars (Fig. [3]) than other statistical methods when the data were independent. However, when the independence between data points was abolished by introducing a hierarchical structure, the bootstrap was not statistically different from the expected 5% false positive rate (green bars in Fig. [2a]) and the error bars computed were similar to those computed using summarized statistics (red and green bars in Fig. [2b]) suggesting that the hierarchical bootstrap is robust to bias for applications in neuroscience.

Among the reasons Linear Mixed Models (LMMs) gained in popularity for statistical testing was the fact that they could accommodate hierarchical datasets by controlling for various levels as “random” effects while still using all available data thereby minimizing loss in statistical power [32][45][31][1][46]. Though we did not directly compare the loss in power between bootstrapping and LMMs, we showed that the bootstrap also does not lose power to the degree that using summarized statistics does (see green traces versus red traces in Fig. [4]) while also keeping the false positive rate within the intended bound (see green trace in Fig. [4]). Additionally, unlike LMMs, which assume linearity, the hierarchical bootstrap as applied in this paper does not make assumptions about the relationships underlying the latent variables that define the hierarchical structure. However, as we saw in Figure [4], where we sampled multiple trials from a very small number of neurons, the bootstrap assumes that the data collected captures essential characteristics of the population distribution. In this case, the bootstrap initially considered the trial level data as independent before switching to neuron level data as the number of neurons increased. Hence, one may have to adjust the resampling procedure to ensure that the distribution of resampled data points most accurately matches the population distribution one wishes to study.

We then used the hierarchical bootstrap on two independent examples to showcase its utility in analyzing hierarchical datasets in neuroscience. First, we reanalyzed data from Hoffmann and Sober, 2014 in which we used both Traditional and Summarized statistical analysis to conclude that songbirds generalize changes in pitch targeted on a single syllable anti-adaptively to syllables of a different type. When reanalyzed with the bootstrap however, we found that the anti-adaptive generalization of different type syllables (blue trace in Fig. [5]) did not meet the threshold for statistical significance. This was a striking result, as the original study did report statistically significant changes from zero even while using summarized statistics [27]. A probable reason for the differences between the summarized and bootstrap methods for this dataset stems from a decision point regarding the level to which one must summarize the data when using summarized statistics (we avoided this decision in the simulations by assuming all data came from a single subject). The summary statistics reported were summarized at the level of syllables for this dataset. However, in order to truly make sure all points are independent, one must summarize at the highest level, i.e., at the level of individual birds in the dataset. The differences in results between the summarized and bootstrap methods here suggest that the generalization effects were driven largely by strong effects in a subset of birds as opposed to a population-wide effect and that, by failing to take the hierarchical nature of the dataset into account, we overestimated our statistical power and chose too low an N. Further evidence for this interpretation comes reanalysis of data from a separate study looking at learning birds display in response to pitch shift of their entire song through custom-built headphones [37] using the hierarchical bootstrap. Since the changes in pitch were far more systemic across birds in this experiment, we did not see any changes in statistically significant results [47].

Second, we used the hierarchical bootstrap on an independent experiment studying the role of descending neurons in controlling behavior in flies using optogenetics [28]. As shown in Figure [6] the hierarchical bootstrap performs better than the traditional and summarized statistical methods in isolating the true signal in the experiment. The traditional method includes areas that are likely false positives and the summarized method does not return any statistically significant areas.

We would also like to reiterate another advantage of the direct probabilities returned by the bootstrap ($p_{\text{boot}}$) over traditionally reported p-values. p-values represent the probability of obtaining results at least as extreme as the ones obtained under the assumption that the null hypothesis is true. It is a cumbersome definition that has led to numerous misconceptions regarding what it actually means [48][49]. The value returned by the bootstrap however, $p_{\text{boot}}$, provides a direct probability in support of a particular hypothesis. As we reported in the songbirds example, we found that the probability of same-type syllables generalizing was 0.85. This means that if we measured data from more birds drawn from the same distribution, we will see adaptive generalization in 85% of cases which is much higher than chance (50%). Hence, the hierarchical bootstrap method can provide a measure of the relative support for the hypothesis which is both easier to understand and can be useful information for both positive and negative results in research.
To conclude, neuroscience research is at a crossroads wherein, on the one hand, exciting new technologies are being built promising bigger and more complex datasets to help understand brain function \[50, 51, 52\], and on the other, we have rising concerns over the incorrect use of statistical tests \[53, 54, 55\] and the lack of reproducibility of a number of past findings \[56, 57, 58\]. We propose the hierarchical bootstrap as a powerful but easy-to-implement method that can be scaled to large and complicated datasets, that returns a direct probability in support of a tested hypothesis reducing the potential for misinterpretation of p-values and that can be checked for correct implementation through sharing of analysis code. As we have shown through this paper, we believe that widespread use of the bootstrap will reduce the rate of false positive results and improve the use of appropriate statistical tests for a given type of dataset.

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6 Conflicts of Interest

The authors declare no competing financial interests.

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