Background Independent Matrix Models

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Abstract

A class of background independent matrix models is made for which the structure of both local gauge symmetries and classical solutions is clarified. These matrix models do not involve a space-time metric and provide the matrix analogs of topological Chern-Simons and BF theories. It is explicitly shown that the BF type of matrix model can be formulated in any space-time dimension and include 3+1 dimensional gravity as a special case. Moreover, we discuss some generalization of the model to include a fermionic BRST-like symmetry whose partition function is related to the Casson invariant.
1 Introduction

One of the most important observations in the recent developments of string theory is that D-instanton and D-particle \cite{1} may be the microscopic degrees of freedom of IIB superstring theory \cite{2} and M-theory \cite{3}, respectively. In both the theories, the space-time coordinates are expressed in terms of $N \times N$ hermitian matrices describing coordinates of D-instanton or D-particle. For instance, M-theory in the infinite momentum frame is expected to be equivalent to a quantum mechanics of $U(N)$ matrices in the $N \to \infty$, with the Hamiltonian that comes from the one dimensional reduction of ten dimensional super Yang-Mills theory \cite{3}. This feature of the space-time coordinates as matrices yields a totally new interpretation about the space-time structure. Namely, the conventional description of the space-time as a continuous manifold is in itself meaningful only in the long distance region where the space-time coordinates are commutable and diagonal while the space-time is quantized and has a discretized structure in the short distance regime.

Despite such impressive developments of matrix models, it is fair to say that we are still far from having a complete understanding of non-perturbative formulation of string theory. In particular, a big mystery is the problem of background independence. In the matrix models constructed so far \cite{2, 3}, it is always assumed space-time background to be flat. Since the matrix theories must involve a theory of quantum gravity, they are not allowed to have their most fundamental formulation in a fixed classical space-time manifold and the space-time geometry should emerge from a more fundamental theory that is independent of background metric.

Closely related to the background independence is that we have no clear understanding of how the matrix theories are connected with Einstein’s general relativity. Even if there is circumstantial evidence that the low energy theory of the matrix models contains general relativity, it is quite obscure how general relativity is derived from the matrix models in a comprehensive manner \cite{4}. These two big questions also have a deep connection with the crucial question of what the underlying gauge symmetry and the fundamental principle behind the matrix theories are.

In the present paper, we would like to address the question of whether a background independent formulation of the matrix model is possible. The main idea of this article is to construct the matrix models of topological Chern-Simons \cite{5} and BF theories \cite{6, 7, 8, 9}. As a consequence, the matrix models obtained in this way do not depend on background metric and contain the local translation invariance in a manifest way. Incidentally, a different type of the matrix models has been already made on a basis of the topological quantum field theory \cite{10, 11}.

The paper is organized as follows. In section 2 we study Chern-Simons type of background independent matrix model that was originally introduced by Smolin \cite{12} and examine some intriguing problems such as its classical solutions and canonical formalism. In section 3, we construct a new background independent matrix model based on topological BF theory. In contrast with the Chern-Simons type, this new matrix model is not only formulated in any space-time dimension but also yields general relativity reduced to a point in 2, 3 and 4
dimensions by selecting appropriate classical solutions. In section 4, we incorporate the spinors in the above theory and construct a new matrix model with BRST-like supersymmetry whose partition function yields the Casson invariant \[13\]. The final section is devoted to discussions.

2 The Chern-Simons matrix model

In this section, we shall not only review the Chern-Simons type of matrix model which was originally introduced in \[12\], but also examine its classical solutions and canonical formalism. Let us consider a simple game constructing the action which consists of only the hermitian matrices \(X_\mu(\mu = 0, 1, \ldots, D - 1)\) and is independent of the background metric. Almost a unique answer is to just line up all the \(X_\mu\)'s, take the trace of them and then contract the \(D\) indices by the Levi-Civita tensor density \(\varepsilon^{\mu_1\mu_2\cdots\mu_D}\) to make a c-number scalar. As a consequence, we obtain the topological matrix model

\[
S_{CS}^D = \varepsilon^{\mu_1\mu_2\cdots\mu_D} \text{Tr} X_{\mu_1} X_{\mu_2} \cdots X_{\mu_D}. \tag{1}
\]

Interestingly enough, we can construct such an action only in the case that \(D\) is odd numbers since the action with even numbers of \(X_\mu\) is identically zero by the following identity:

\[
S_{CS}^D = \varepsilon^{\mu_1\mu_2\cdots\mu_D} \text{Tr} X_{\mu_1} X_{\mu_2} \cdots X_{\mu_D} = (-1)^{D-1} \varepsilon^{\mu_1\mu_2\cdots\mu_D} \text{Tr} X_{\mu_D} X_{\mu_1} \cdots X_{\mu_{D-1}} = (-1)^{D-1} S_{CS}^D, \tag{2}
\]

where we have used the cyclic property of trace and the totally antisymmetric property of the Levi-Civita tensor density. Thus we will set \(D\) to be \(2d + 1\) with \(d \in \mathbb{Z}_+ \cup \{0\}\) in this section. Incidentally, the topological matrix model with any number of \(X_\mu\) will be built in the next section.

The equations of motion derived from the action (1) read

\[
\varepsilon^{\mu_1\mu_2\cdots\mu_{2d}} X_{\mu_1} X_{\mu_2} \cdots X_{\mu_{2d}} = 0. \tag{3}
\]

Note that (3) does not include the metric tensor in comparison with the equations of motion derived from IIB and M matrix models \[3, 4\] whose formal expression is provided by

\[
\eta^{\mu\nu} [X_\mu, [X_\nu, X_\rho]] = 0 \tag{4}
\]

with the flat Minkowskian metric \(\eta^{\mu\nu} = \text{diag}(-1, +1, \cdots, +1)\). At this stage, it is useful to find the classical solutions satisfying the equations of motion (3). One obvious solution is the one satisfying the equation \([X_\mu, X_\nu] = 0\), that is, this solution has the form of the diagonal \(N \times N\) matrix

\[
X_\mu = \begin{pmatrix} X_\mu^{(1)} \\ \vdots \\ X_\mu^{(N)} \end{pmatrix}, \tag{5}
\]
which we call "classical space-time" in this paper. Next nontrivial solution is "string" solution given by

$$X_\mu = (X_0, X_1, 0, \cdots, 0), \quad (6)$$

where we have considered the string along 1st axis without losing generality. Similarly, "membrane" solution stretched out in the direction of 1st and 2nd axes reads

$$X_\mu = (X_0, X_1, X_2, 0, \cdots, 0). \quad (7)$$

It is obvious that this kind of solutions continues to exist until "(2d − 1)-brane"

$$X_\mu = (X_0, X_1, X_2, \cdots, X_{2d-1}, 0). \quad (8)$$

Moreover, a solution associated with several "k-branes" (1 ≤ k ≤ 2d − 1) can be built out of the above solution for single "k-brane" in a perfectly similar way to the case of IIB matrix model [2]. For instance, the solution for two "strings" separated by the distance b along 2nd axis is given by

$$X_0 = \begin{pmatrix} x_0 & 0 \\ 0 & x_0 \end{pmatrix}, \quad X_1 = \begin{pmatrix} x_1 & 0 \\ 0 & x_1 \end{pmatrix},$$

$$X_2 = \begin{pmatrix} \frac{b}{2} & 0 \\ 0 & -\frac{b}{2} \end{pmatrix}, \quad X_3 = \cdots = X_{2d} = 0, \quad (9)$$

where $x_0$ and $x_1$ are certain nonzero elements. Of course, the specific choice of a form of the matrix $X_\mu$ leads to other classical solutions of the equations of motion (3), but I could not find any physical importance on them as solutions of Matrix Theory.

Now let us turn our attention to the symmetries in the action (1). It is remarkable that as well as the conventional gauge symmetry

$$X_\mu \rightarrow X'_\mu = U X_\mu U^{-1} \quad (10)$$

with $U \in U(N)$, the action (1) is invariant under the local translation of the diagonal element

$$X_\mu \rightarrow X'_\mu = X_\mu + V_\mu(X) \quad 1$$

with $V_\mu(X)$ being not a matrix but a c-number function of $X_\mu$. This symmetry is in sharp contrast with the matrix models [4, 5] where $V_\mu$ is a global parameter of c-number. In other words, the global translation in [4, 5] is now promoted to the local translation. In this respect, it is of interest to recall the following things. Firstly, in the matrix models [4, 5] the diagonal matrix like (3) corresponds to the classical space-time coordinates while the non-diagonal matrix describes the interactions. Hence the local symmetry (11) coincides with the local space-time translation at the classical level. Secondly, it is well known that general relativity is the gauge theory with the local translation as the gauge symmetry, so the existence of this
symmetry might be a signal of the existence of general relativity in this matrix model though we need more studies to confirm this conjecture in future.

In the remainder of this section, we would like to consider the possibility of deriving the gauge symmetries (10) and (11) in the canonical formalism. In order to tame the action (1) in the canonical framework, it is necessary to introduce a fictitious time \( \tau \) into the theory and assume that \( X_\mu \)'s are function of \( \tau \). The idea is then to consider a similar (but different) action to (1), from which to gain useful information about constraints of the action (1) through the canonical formalism of the similar action. As such a deformed action, let us consider

\[
I_{CS}^{d+1} = \int d\tau \varepsilon^{\mu_1\mu_2\cdots\mu_{d+1}} T\tau (D_\tau X_{\mu_1}) X_{\mu_2} \cdots X_{\mu_{d+1}},
\]

where the covariant derivative \( D_\tau X_\mu = \partial_\tau X_\mu + [A_\tau, X_\mu] \) with a fictitious gauge field \( A_\tau \) is introduced. Note that this action (12) reduces simply to the original action (1) at the boundary in a gauge with \( A_\tau = 0 \). This is the reason why we have chosen the action (12) in order to implement the canonical analysis of the action (1).

The canonical conjugate momenta corresponding to \( X_\mu \) are given by

\[
P_\mu = \varepsilon^{\mu_1\mu_2\cdots\mu_{d}} X_{\mu_1} X_{\mu_2} \cdots X_{\mu_d},
\]

from which \( Tr P_\mu = 0 \) holds identically. On the other hand, the canonical conjugate momentum \( \pi \) corresponding to \( A_\tau \) vanishes trivially since the action (12) does not involve the kinetic term for \( A_\tau \). The Hamiltonian \( H \) can be easily calculated to

\[
H = -\varepsilon^{\mu_1\mu_2\cdots\mu_{d+1}} Tr A_\tau [X_{\mu_1}, X_{\mu_2} \cdots X_{\mu_{d+1}}].
\]

Thus, the condition that the time evolution of the primary constraint \( \pi \approx 0 \) also vanishes weakly under this Hamiltonian gives rise to the secondary constraint

\[
[X_\mu, P_\mu] \approx 0,
\]

which is nothing but Gauss's law constraint. In terms of this Gauss's law constraint, the Hamiltonian (14) becomes weakly zero so that no more constraint occurs. Therefore all the constraints are summarized to be in the form

\[
\phi^\mu \equiv P_\mu - \varepsilon^{\mu_1\mu_2\cdots\mu_{d}} X_{\mu_1} X_{\mu_2} \cdots X_{\mu_d} \approx 0,
\]

\[
\chi \equiv [X_\mu, P_\mu] \approx 0,
\]

where the constraint \( \pi \approx 0 \) is excluded from this constraint system by picking a gauge with \( A_\tau = 0 \). Then it is straightforward to check that these constraints (16) are the first-class constraints so that they generate the infinitesimal gauge transformations. Actually, we can easily see that \( \phi^\mu \approx 0 \) generates the topological symmetry \( X_\mu \rightarrow X_\mu + \varepsilon_\mu (X) \) and \( \chi \approx 0 \) does the usual gauge transformation. The reason why we have the topological symmetry is quite simple. This is because as suggested above in the gauge with \( A_\tau = 0 \) the action (12) precisely...
reduces to a surface term. The role of the fictitious gauge field $A_\tau$ is just to isolate the usual gauge symmetry from the topological symmetry.

So far we have developed the canonical formalism for the deformed action (12). We are now in a position to extract the information about the first-class constraints describing the gauge symmetries of the action (1) from the constraints (16) of the action (12). All we have to do is to take the constraints with the forms of $\text{Tr} \phi^\mu = T \tau \phi^\mu \approx 0$ plus $\chi \approx 0$ from (16). It is then obvious that the algebra closes among these constraints and these constraints generate the gauge symmetries (10) and (11).

3 The BF matrix model

In the previous section, we have considered the Chern-Simons matrix model, but this model has some problems. In particular, it is quite unsatisfactory that we cannot construct the matrix model in even space-time dimensions. Furthermore, it seems to be difficult to make a supersymmetric extension of the Chern-Simons matrix model without introducing the background metric. Finally, it is at present unclear that the Chern-Simons matrix model has a relationship with general gravity. Luckily, we have already met a similar situation to this in topological quantum field theories where the Chern-Simons theory is replaced with the BF theory in order to overcome these impasse. In the case of the matrix model at hand we also proceed with the same line of argument as the topological quantum field theories.

Now we would like to present BF matrix model which has the form

$$S^D_n = \varepsilon^{\mu_1 \mu_2 \cdots \mu_D} \text{Tr} X_{\mu_1} X_{\mu_2} \cdots X_{\mu_n} B_{\mu_{n+1} \cdots \mu_D},$$

where a totally antisymmetric tensor matrix $B$ is introduced. In this respect let us recall that the original form of topological BF theory is

$$S_{BF} = \int \varepsilon^{\mu_1 \mu_2 \cdots \mu_D} \text{Tr} F_{\mu_1 \mu_2} B_{\mu_3 \cdots \mu_D},$$

where the 2-form field strength $F$ is defined as $F = dA + A^2$. Thus, precisely speaking, the straightforward generalization of the topological BF theory to the matrix model corresponds to the case of $n = 2$ in (17). Of course, owing to the introduction of the matrix $B$ the action makes sense in arbitrary space-time dimension.

The classical equations of motion derived from the BF matrix model (17) read

$$\varepsilon^{\mu_1 \mu_2 \cdots \mu_D} X_{\mu_1} X_{\mu_2} \cdots X_{\mu_n} = 0,$$

$$\sum_{i=1}^n (-1)^i \varepsilon^{\mu_1 \cdots \hat{\mu}_i \cdots \mu_D} X_{\mu_{i+1}} \cdots X_{\mu_n} B_{\mu_{n+1} \cdots \mu_D} X_{\mu_1} \cdots X_{\mu_{i-1}} = 0,$$

where $\hat{\mu}_i$ denotes that the index $\mu_i$ is excluded. Note that apart from the number of $X_\mu$, Eq. (19) accords with (3) in the Chern-Simons matrix theory. Thus the structure of the
solutions with respect to $X_\mu$ is almost the same as that case. On the other hand, it is Eq. (20) that appears for the first time in the BF matrix model. In fact, this equation would have an important implication in relating the model at hand to general relativity later.

As for the gauge symmetries, besides the usual U(N) gauge symmetry, at first glance the action (17) looks like it might be invariant under the following natural generalization of the local translation symmetry (11)

$$X_\mu \rightarrow X'_\mu = X_\mu + V_\mu(X) \mathbf{1},$$
$$B_{\mu_{n+1} \cdots \mu_D} \rightarrow B'_{\mu_{n+1} \cdots \mu_D} = B_{\mu_{n+1} \cdots \mu_D} + W_{\mu_{n+1} \cdots \mu_D}(X) \mathbf{1}. \tag{21}$$

However, it is interesting to notice that only the action (17) with $n$ being even integers has such a local translation symmetry while the action (17) with odd $n$ has neither the local nor the global translation symmetry. Concerning the canonical formalism of the BF matrix model, although some formulae become more complicated than in the Chern-Simons matrix model, the canonical formalism explained in the previous section applies equally well to this case. The key point here is to start with the following matrix model consisting of an almost surface term:

$$I^D_n = \int d\tau \varepsilon^{\mu_1 \mu_2 \cdots \mu_D} T_F \left[ (D_\tau X_{\mu_1})X_{\mu_2} \cdots X_{\mu_n} B_{\mu_{n+1} \cdots \mu_D}ight.
+ X_{\mu_1}(D_\tau X_{\mu_2})X_{\mu_3} \cdots X_{\mu_n} B_{\mu_{n+1} \cdots \mu_D} + \cdots
+ X_{\mu_1} \cdots X_{\mu_{n-1}}(D_\tau X_{\mu_n}) B_{\mu_{n+1} \cdots \mu_D}
+ X_{\mu_1} \cdots X_{\mu_n} \bar{D}_\tau B_{\mu_{n+1} \cdots \mu_D} \right], \tag{22}$$

where the second covariant derivative $\bar{D}_\tau B_{\mu_{n+1} \cdots \mu_D} = \partial_\tau B_{\mu_{n+1} \cdots \mu_D} + [\bar{A}_\tau, B_{\mu_{n+1} \cdots \mu_D}]$ with another fictitious gauge field $\bar{A}_\tau$ is also introduced. Following the similar line of arguments to the case of the Chern-Simons matrix model, we can also reach the similar results whose details are skipped over now. Only the difference lies in the fact that the trace of both canonical conjugate momenta to $X_\mu$ and $B_{\mu_{n+1} \cdots \mu_D}$ identically vanishes only for even $n$ which would be related to the existence of the local translation invariance for even $n$.

Before closing this section, we turn to the problem of relating the present model to general relativity. Indeed, there are some methods for it in lower dimensions. This follows from the fact that the topological BF theory includes the content of general relativity in two, three and four dimensions. In contrast, we have no clear understanding of how to formulate general relativity in terms of the topological BF theory in the dimensions more than four. In this paper, we shall confine our consideration to general relativity in four space-time dimensions since the treatment in both two and three dimensions is also similar to or easier than in four dimensions.

To this aim, one has to consider the specific case $n = 2$, which exactly corresponds to the matrix model of the original, topological BF theory. First of all, let us consider the first possibility of deriving the action of general relativity in four dimensions by starting with the BF matrix model in the dimensions more than four. Then, for a notational convenience, let us decompose the index $\mu = 0, 1, \cdots, D - 1$ into the four-dimensional part $A = 0, 1, 2, 3$
and the remaining part \( a = 4, \cdots, D - 1 \). Moreover, we introduce the definition \( \tilde{B}^{\mu_1 \mu_2} = \varepsilon^{\mu_1 \mu_2 \cdots \mu_D} B_{\mu_3 \cdots \mu_D} \). Consequently, the starting action is of the form

\[
S_{n=2}^D = Tr X_\mu X_\nu \tilde{B}^{\mu_1 \mu_2} = Tr (X_{A_1} X_{A_2} \tilde{B}^{A_1 A_2} + [X_A, X_B] \tilde{B}^{A_B} + X_{a_1} X_{a_2} \tilde{B}^{a_1 a_2}).
\]

(23)

Then, the key idea is to find a special solution satisfying the equations of motion for \( \tilde{B} \) without affecting the space-time coordinates \( X_\mu \) in order to yield the first-order Palatini action of general relativity in four dimensions. We can easily find the desirable solution given by

\[
\begin{align*}
\tilde{B}^{A_1 A_2} &= \varepsilon^{A_1 A_2 A_3 A_4} e_{A_3} e_{A_4}, \\
\tilde{B}^{A a} &= \tilde{b}^{A a}, \tilde{B}^{a_1 a_2} = \tilde{b}^{a_1 a_2} 1,
\end{align*}
\]

(24)

where \( e_A \) is the one-form vierbein and \( \tilde{b} \) is not a matrix but a \( c \)-number with the same symmetric property as the corresponding matrix. If we substitute (24) into (23), we can obtain

\[
S_{n=2}^D = \varepsilon^{A_1 A_2 A_3 A_4} Tr X_{A_1} X_{A_2} e_{A_3} e_{A_4}.
\]

(25)

This is exactly the same form of the first-order Palatini action reduced to a point. In this way, we can derive the action of general relativity from the BF matrix model in a simple manner.

Even if the above derivation is itself of interest, some people may complain that we have just selected a special solution by hand among many classical solutions. Here, to make the process of selecting the special solution (24) more convincing, we can make use of the strategy adopted in the references [14], which amount to adding an additional term to the starting action such that the above solution becomes the general solution. To elucidate our strategy, let us just confine ourselves to four space-time dimensions from the outset. Then relevant action equals

\[
S_{n=2}^{D=4} = Tr (X_{A_1} X_{A_2} \tilde{B}^{A_1 A_2} - \frac{1}{2} \Psi \tilde{B}^{A_1 A_2} B_{A_1 A_2}).
\]

(26)

The variational equation with respect to \( \Psi \) produces the equation \( \tilde{B}^{A_1 A_2} B_{A_1 A_2} = 0 \). According to the proposition in [15], the general solution of this equation is given by \( \tilde{B}^{A_1 A_2} = \varepsilon^{A_1 A_2 A_3 A_4} e_{A_3} e_{A_4} \). Thus, the substitution of this solution into (26) leads to the first-order Palatini action (23) like before. It is quite interesting to examine whether the above-mentioned strategy can also be applied to the case of the higher space-time dimensions.

4 Generalization with fermionic symmetry

Now we will discuss some generalizations of the BF model to include fermionic symmetry. Indeed, in the matrix models [2, 3] the fermionic symmetry, in particular, the supersymmetry, was needed to guarantee the cluster and BPS properties of instantons.
One possibility is to add fermions of integer spin to achieve a BRST-like symmetry. It is known that the partition function of the BF theory is related to the Ray-Singer torsion \[16\] while that of the BF theory with such a BRST-like symmetry corresponds to the Casson invariant \[13\]. We think that this statement is valid even in the BF matrix model treated in this paper. Let us start by the following BRST-like fermionic symmetry:

\[
\delta X_\mu = \eta \psi_\mu, \quad \delta \psi_\mu = 0, \\
\delta \chi_{\mu_{n+1} \cdots \mu_D} = -\eta B_{\mu_{n+1} \cdots \mu_D}, \quad \delta B_{\mu_{n+1} \cdots \mu_D} = 0.
\] (27)

We can check explicitly the following action to be invariant under the fermionic symmetry (27):

\[
S^D_n = \varepsilon^{\mu_1 \mu_2 \cdots \mu_D} \text{Tr}(X_{\mu_1} X_{\mu_2} \cdots X_{\mu_n} B_{\mu_{n+1} \cdots \mu_D} \\
- \sum_{i=1}^n X_{\mu_1} X_{\mu_2} \cdots X_{\mu_{i-1}} \psi_{\mu_i} X_{\mu_{i+1}} \cdots X_{\mu_n} \chi_{\mu_{n+1} \cdots \mu_D}).
\] (28)

For even integers \(n\), this action is still invariant under the enlarged local translation which constitutes of Eq.(21) and

\[
\psi_\mu \rightarrow \psi'_\mu = \psi_\mu + v_\mu(X) \textbf{1}, \\
\chi_{\mu_{n+1} \cdots \mu_D} \rightarrow \chi'_{\mu_{n+1} \cdots \mu_D} = \chi_{\mu_{n+1} \cdots \mu_D} + w_{\mu_{n+1} \cdots \mu_D}(X) \textbf{1}.
\] (29)

A more interesting possibility of incorporating fermions of half integer spin would be to twist the action (28) like the topological quantum field theory \[17\]. Here note that even if the bosonic action (17) is nontrivial its BRST-like generalization (28) is BRST-exact form so that we can use the twisting technique developed in the reference \[17\]. This problem will be reported in a separate publication in future.

### 5 Discussions

In this paper, we have proposed two candidates for the background independent formulation of the matrix model. One is based on the Chern-Simons theory \[5\] in odd dimensions \[12\], and the other is on the BF theory \[6, 7, 8, 9\] in any space-time dimension. Both of the models share some common features, for instance, the existence of similar gauge symmetries and classical solutions. However, it seems that the latter matrix model is currently more interesting than the former one in that the BF matrix model not only can be formulated in an arbitrary dimension, in particular, in four dimensions, but also has a close connection with gravity. Note that these advantageous features of the BF theory are already seen at the level of topological field theory.

As mentioned in the introduction, the main purpose of constructing the background independent matrix models is to understand the non-perturbative aspects of string theory without reference to the specific background metric. To this end, we have to clarify the relation in
detail between the present matrix models and the ones in \cite{2,3} in future. At any rate, it seems to be essential to twist the model at hand for the purpose of getting $N = 2$ supersymmetric matrix model.

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