THE MACHO PROJECT LARGE MAGELLANIC CLOUD VARIABLE-STAR INVENTORY. XIII. FOURIER PARAMETERS FOR THE FIRST-OVERTONE RR LYRAE VARIABLES AND THE LMC DISTANCE

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ABSTRACT

Shapes of RR Lyrae light curves can be described in terms of Fourier coefficients that past research has linked with physical characteristics such as luminosity, mass, and temperature. Fourier coefficients have been derived for the \( V \) and \( R \) light curves of 785 overtone RR Lyrae variables in 16 MACHO fields near the bar of the LMC. In general, the Fourier phase differences \( \phi_{21}, \phi_{31}, \) and \( \phi_{41} \) increase and the amplitude ratio \( R_{21} \) decreases with increasing period. The coefficients for both the \( V \) and \( R \) magnitudes follow these patterns, but the phase differences for the \( R \) curves are on average slightly greater, and their amplitudes are about 20% smaller, than the ones for the \( V \) curves. The \( \phi_{31} \) and \( R_{31} \) coefficients have been compared with those of the first-overtone RR Lyrae variables in the Milky Way and in the LMC. The results indicate that many of the LMC variables have properties similar to the ones in M2, M3, M5, and the Oosterhoff type I variables in \( \omega \) Cen, but they are different from the Oosterhoff type II variables in \( \omega \) Cen. Equations derived from hydrodynamic pulsation models have been used to calculate the luminosity and temperature for the 330 bona fide first-overtone variables. The results indicate that they have log \( L \) in the range 1.6–1.8 \( L_\odot \) and log \( T_{\text{eff}} \) between 3.85 and 3.87. Based on these temperatures, a mean color excess \( E(B-V) = 0.14 \) mag, equivalent to \( E(B-V) = 18.99 \pm 0.02 \) (statistical) \( \pm 0.16 \) (systematic) has been estimated for these 330 stars. The 80 M5-like variables (selected according to their location in the \( \phi_{31} - \log P \) plot) are used to determine an LMC distance. After correcting for the effects of extinction and crowding, a mean apparent magnitude \( \langle V_0 \rangle = 18.99 \pm 0.02 \) (statistical) \( \pm 0.16 \) (systematic) has been estimated for these 80 stars. Combining this with a mean absolute magnitude \( M_V = 0.56 \pm 0.06 \) for M5-like stars derived from Baade-Wesselink analyses, main-sequence fitting, Fourier parameters, and the trigonometric parallax of RR Lyrae, we derive an LMC distance modulus \( \mu = 18.43 \pm 0.06 \) (statistical) \( \pm 0.16 \) (systematic) mag. The large systematic error arises from the difficulties of correcting for interstellar extinction and for crowding.

Key words: galaxies: distances and redshifts — Magellanic Clouds — RR Lyrae variable

On-line material: machine-readable tables

1. INTRODUCTION

The MACHO Project database is a valuable resource for studying the characteristics of variable stars in the LMC. In Paper II of this series, Alcock et al. (1996, hereafter A96) identified 7900 RR Lyrae variables in 22 fields in the region of the LMC bar. The period-frequency distribution that they plotted for these variables showed that the mode was 0.583 days, indicative of an Oosterhoff (1939, 1944) type I population. In addition, there were two other peaks in the distribution, at 0.342 and 0.281 days, which they attributed to variables pulsating in the first- and second-overtone modes, respectively. The purpose of the present investigation is to perform a Fourier analysis of the first-overtone (RR1)18 RR Lyrae variables in order to determine an LMC distance. The LMC is a well-known benchmark in the extragalactic distance scale, and thus new measurements of its distance are important in order to test the accuracy of standard cosmological models.

The distance to the LMC has a controversial history, and yet in recent years a standard distance modulus has emerged.
This is due in part to the completion of the *Hubble Space Telescope’s (HST)* key project to measure the Hubble constant with variable stars and standard candles, which employs \( \mu_{MC} = 18.5 \text{ mag} \) (Freedman et al. 2001). The Freedman et al. (2001) result of \( H_0 = 71 \pm 10 \text{ km s}^{-1} \text{ Mpc}^{-1} \) (statistical and systematic error total) is in strikingly good agreement with that derived from *Wilkinson Microwave Anisotropy Probe* data \( (H_0 = 72 \pm 5 \text{ km s}^{-1} \text{ Mpc}^{-1}; \text{Spergel et al. 2003})*. These measurements of \( H_0 \) are based on entirely different physics, and thus their agreement lends support to the accuracy of the standard LMC distance modulus adopted by Freedman et al. (2001). It is, in fact, a recent trend in the literature that most new LMC distance measurements are in excellent agreement with the standard model, and in many cases systematic errors in prior measurements are being found and corrected (e.g., Alves et al. 2002; Mitchell et al. 2002).

In this investigation, we employ the Fourier decomposition technique, a method for quantifying the structural characteristics of the observed light curves of variable stars. It was first applied to RR Lyrae variables by Simon & Teays (1982) who analyzed the light curves of 70 field RR Lyrae stars. Later, Clement, Jankulak, & Simon (1992) and Simon & Clement (1993, hereafter SC93) used the technique to compare the RR1 variables in six Galactic globular clusters (GGCs) with metal abundances ranging from [Fe/H] = \(-0.99\) to \(-2.17\) on the Zinn & West (1984, hereafter ZW84) scale. In particular, they studied the Fourier phase parameter \( \phi_{31}\). By plotting \( \phi_{31} \) versus log \( P \), they discovered that the clusters were segregated according to metallicity and that, within each cluster, \( \phi_{31} \) increases with period. To understand the physical significance of this result, SC93 analyzed hydrodynamic pulsation models for first-overtone variables and found that they could derive equations for expressing both the mass and the luminosity in terms of \( \phi_{31} \) and the pulsation period. An application of these equations to the six GGCs indicated that there was a strong correlation between mean RR1 luminosity and metal abundance of the cluster. This provided independent evidence for the existence of an RR Lyrae luminosity-metallicity relation. It also demonstrated that Fourier decomposition is a useful technique for estimating the luminosity of an RR1 variable.

Most LMC distance determinations based on RR Lyrae variables depend on the luminosity-metallicity relation. In these studies, a mean metal abundance must be adopted because spectroscopic studies by A96, Bragaglia et al. (2001), and Clementini et al. (2003, hereafter C03) have all shown that there is a range of metal abundance among the field RR Lyraes in the LMC. However, since we do not have [Fe/H] values for the individual stars in our sample, we take a different approach. In this investigation, our modus operandi will be to compare the Fourier parameters of the LMC RR1 variables with the ones in some well-studied GGCs. We will look for a subset of LMC RR1 variables that are similar to those in one of these clusters. Then we will assume that their RR Lyrae variables have the same mean absolute magnitude and use independent studies to determine the RR Lyrae absolute magnitudes.

2. THE OBSERVATIONAL DATA

Our investigation is based on the RR Lyrae data from 16 LMC\(^{19}\) fields observed for the MACHO project. The program stars were selected from a preliminary sample that included all of the (approximately 1200) RR Lyrae variables deemed to be RR1 stars according to their periods and light-curve shapes. These preliminary data were instrumental magnitudes derived from the observations acquired between 1992 July and 1995 December through the MACHO \( B_M \) and \( R_M \) filters. The first step of our analysis was to derive the periods using Stellingwerf’s (1978) phase dispersion minimization (PDM) technique and then to fit both the \( B_M \) and \( R_M \) magnitudes to a Fourier series of the form

\[
\text{mag} = A_0 + \sum_{j=1}^{n} A_j \cos \left( j \omega t + \phi_j \right),
\]

where \( \omega = 2\pi/\text{period} \), \( t \) is the time of the observation, and \( n \) is the order of the fit. In each case, the order of the fit was six, and all magnitudes for which the assessed error was greater than 0.1 mag were excluded. The phase differences, \( \phi_j = (\phi_j - \phi_1) \), and amplitude ratios, \( R_j = (A_j/A_1) \), were calculated, and their standard errors were evaluated using the formula of Petersen (1984). Since the Fourier decomposition technique is not useful for studying stars with large uncertainties in their coefficients, we included for further study only the stars with an error less than 0.3 in \( \phi_{21} \) or an error less than 0.4 in \( \phi_{31} \) in at least one of \( B_M \) or \( R_M \). There were 785 stars that met these criteria. The instrumental magnitudes of these stars for the observations obtained between 1992 July and 1999 December were transformed to the Kron-Cousins \( V \) and \( R \) system using the equations derived by Alcock et al. (1999, hereafter A99). This calibration has been designated version 9903018. Calibration version numbers may also appear in some MACHO database documentation and released light-curve data (e.g., Allsmann & Axelrod 2001; Alcock et al. 2003).

3. THE FOURIER ANALYSIS OF THE PROGRAM STARS

3.1. The Fourier Coefficients

The calibrated \( V \) and \( R \) magnitudes for the 785 program stars extended over a longer time base than the preliminary data, and so we used the PDM technique to revise the periods before performing the Fourier analysis. Only observations obtained under good transparency conditions were included. In addition, all magnitudes for which the assessed photometric error was greater than 0.1 mag were excluded. The magnitudes were then fitted to equation (1) using a sixth-order fit. It turned out that the sample included 105 stars that were found to be multiperiodic\(^{20}\) by Kovács and the MACHO collaboration (Kovács et al. 2000; A00), so that only 680 stars in the sample were monoperiodic. We present the data for these stars in Table 1. For each star, we report the results of the Fourier analysis for both the \( V \) and \( R \) magnitudes. \( N \) denotes the number of observations. The quantities \( A_0, A_1, R_1, \phi_1 \) and their standard errors (\( \sigma \)), the amplitude, and \( \sigma_{\text{fit}} \) were all obtained from the fit of equation (1) to the data. The coefficients for the 105 multiperiodic stars are listed in Tables 2–5,

\(^{19}\) The LMC fields included are Nos. 2, 3, 5, 6, 10, 11, 12, 13, 14, 15, 18, 19, 47, 80, 81, and 82, all of which are close to the bar. The field of view for each field is 0.52 deg\(^2\). An identification chart and a list of the right ascension and declination of the field centers are available at http://www.macho.mcmaster.ca.

\(^{20}\) A00 introduced a new system of subclasses to describe the frequency spectra of the multiperiodic variables: RR01 for double-mode stars (fundamental and first-overtone), RR12 for double-mode stars (first- and second-overtone), RR1-PC for stars with period changes, RR1-9L for Buzhkov variables, and RR1-1v, RR1-2v, and RR1-1vM for other multifrequency stars, where 1, 2, and M indicate that there are one, two, or more than two additional frequencies.
**TABLE 1**

FOURIER PARAMETERS OF LMC RR1 VARIABLES

| Star ID          | Nobs | Color | Amplitude | σₘₚ | Period | A₀    | A₁    | R₃₁  | R₃₃  | R₄₁  | φ₂₁  | φ₃₁  | φ₄₁  |
|------------------|------|-------|-----------|------|--------|-------|-------|------|------|------|------|------|------|
| 80.6589.1879,... | 751  | V     | 0.304     | 0.066 | 0.249987 | 19.24 | 0.149 | 0.1129 ± 0.0236 | 0.0324 ± 0.0238 | 0.0318 ± 0.0244 | 4.81 ± 0.22 | 3.36 ± 0.74 | 1.87 ± 0.75 |
| 80.6589.1879,... | 736  | R     | 0.213     | 0.071 | 0.249987 | 18.94 | 0.104 | 0.1198 ± 0.0368 | 0.0359 ± 0.0371 | 0.0546 ± 0.0378 | 4.56 ± 0.32 | 2.39 ± 1.05 | 2.16 ± 0.69 |
| 2.5389.1478,... | 469  | V     | 0.258     | 0.082 | 0.250557 | 19.52 | 0.178 | 0.0679 ± 0.0303 | 0.0611 ± 0.0310 | 0.117 ± 0.0299 | 4.99 ± 0.46 | 3.04 ± 0.51 | 4.03 ± 2.69 |
| 2.5389.1478,... | 455  | R     | 0.286     | 0.074 | 0.250557 | 19.30 | 0.136 | 0.1228 ± 0.0353 | 0.0481 ± 0.0366 | 0.0561 ± 0.0354 | 4.57 ± 0.32 | 3.54 ± 0.75 | 4.59 ± 0.67 |
| 6.5849.1114,... | 363  | V     | 0.391     | 0.060 | 0.255347 | 19.55 | 0.189 | 0.1462 ± 0.0247 | 0.0593 ± 0.0235 | 0.0161 ± 0.0239 | 4.39 ± 0.17 | 3.43 ± 0.41 | 3.23 ± 1.48 |

Note.—Table 1 is presented in its entirety in the electronic edition of the Astronomical Journal. A portion is shown here for guidance regarding its form and content.

**TABLE 2**

FOURIER PARAMETERS OF LMC RR01 VARIABLES

| Star ID          | Nobs | Color | Amplitude | σₘₚ | Period | A₀    | A₁    | R₃₁  | R₃₃  | R₄₁  | φ₂₁  | φ₃₁  | φ₄₁  |
|------------------|------|-------|-----------|------|--------|-------|-------|------|------|------|------|------|------|
| 80.7193.1485,... | 514  | V     | 0.433     | 0.084 | 0.328817 | 19.50 | 0.215 | 0.1431 ± 0.0244 | 0.0766 ± 0.0248 | 0.0157 ± 0.0249 | 5.14 ± 0.18 | 4.98 ± 0.34 | 1.23 ± 1.62 |
| 80.7193.1485,... | 394  | R     | 0.379     | 0.099 | 0.328817 | 19.24 | 0.168 | 0.1701 ± 0.0409 | 0.1139 ± 0.0430 | 0.0528 ± 0.0428 | 4.99 ± 0.28 | 5.33 ± 0.40 | 5.55 ± 0.85 |
| 81.8639.1450,... | 435  | V     | 0.256     | 0.069 | 0.335260 | 19.14 | 0.124 | 0.1152 ± 0.0380 | 0.1500 ± 0.0389 | 0.0356 ± 0.0386 | 4.35 ± 0.34 | 2.77 ± 0.28 | 1.21 ± 1.07 |
| 81.8639.1450,... | 466  | R     | 0.203     | 0.068 | 0.335260 | 18.92 | 0.101 | 0.1169 ± 0.0449 | 0.0766 ± 0.0455 | 0.0296 ± 0.0447 | 4.21 ± 0.39 | 3.10 ± 0.59 | 4.66 ± 1.56 |
| 80.7439.1836,... | 244  | V     | 0.483     | 0.187 | 0.434457 | 19.80 | 0.228 | 0.2093 ± 0.0752 | 0.1234 ± 0.0750 | 0.0476 ± 0.0745 | 4.96 ± 0.40 | 3.36 ± 0.65 | 2.35 ± 1.64 |

Note.—Table 2 is presented in its entirety in the electronic edition of the Astronomical Journal. A portion is shown here for guidance regarding its form and content.

**TABLE 3**

FOURIER PARAMETERS OF LMC RR12 VARIABLES

| Star ID          | Nobs | Color | Amplitude | σₘₚ | Period | A₀    | A₁    | R₃₁  | R₃₃  | R₄₁  | φ₂₁  | φ₃₁  | φ₄₁  |
|------------------|------|-------|-----------|------|--------|-------|-------|------|------|------|------|------|------|
| 12.10443.367,... | 674  | V     | 0.215     | 0.056 | 0.336557 | 18.91 | 0.099 | 0.1408 ± 0.0320 | 0.1096 ± 0.0319 | 0.0261 ± 0.0311 | 1.30 ± 0.23 | 6.16 ± 0.30 | 3.10 ± 1.22 |
| 12.10443.367,... | 725  | R     | 0.174     | 0.057 | 0.336557 | 18.60 | 0.076 | 0.1588 ± 0.0400 | 0.1035 ± 0.0406 | 0.0541 ± 0.0399 | 1.73 ± 0.26 | 0.29 ± 0.39 | 3.80 ± 0.75 |
| 12.10202.285,... | 565  | V     | 0.262     | 0.056 | 0.398113 | 18.76 | 0.111 | 0.2345 ± 0.0313 | 0.2008 ± 0.0314 | 0.0918 ± 0.0316 | 2.11 ± 0.15 | 0.09 ± 0.17 | 4.14 ± 0.34 |
| 12.10202.285,... | 601  | R     | 0.216     | 0.052 | 0.398113 | 18.43 | 0.092 | 0.2535 ± 0.0341 | 0.2196 ± 0.0343 | 0.0843 ± 0.0344 | 2.19 ± 0.15 | 0.32 ± 0.17 | 4.42 ± 0.41 |
### Table 4
Fourier Parameters of LMC RR1 Variables

| Star ID         | N<br><sub>obs</sub> | Color | Amplitude | σ<sub>fit</sub> | Period | A<sub>0</sub> | A<sub>1</sub> | R<sub>21</sub> | R<sub>31</sub> | R<sub>41</sub> | φ<sub>21</sub> | φ<sub>31</sub> | φ<sub>41</sub> |
|-----------------|---------------------|-------|------------|----------------|--------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|
| 14.9702.401.....| 532                 | V     | 0.455      | 0.067         | 0.275403| 19.39       | 0.224       | 0.1919 ± 0.0187 | 0.0823 ± 0.0181 | 0.0562 ± 0.0182 | 4.50 ± 0.10 | 2.62 ± 0.24 | 1.42 ± 0.34 |
| 14.9702.401.....| 539                 | R     | 0.389      | 0.063         | 0.275403| 19.23       | 0.183       | 0.2491 ± 0.0216 | 0.0787 ± 0.0207 | 0.0460 ± 0.0209 | 4.51 ± 0.09 | 2.52 ± 0.28 | 1.18 ± 0.47 |
| 5.730.4057..... | 519                 | V     | 0.262      | 0.075         | 0.276320| 19.41       | 0.117       | 0.2582 ± 0.0416 | 0.0262 ± 0.0415 | 0.0478 ± 0.0402 | 4.18 ± 0.19 | 2.61 ± 1.53 | 2.10 ± 0.88 |
| 5.730.4057..... | 483                 | R     | 0.218      | 0.068         | 0.276320| 19.30       | 0.100       | 0.2406 ± 0.0450 | 0.0803 ± 0.0451 | 0.0455 ± 0.0446 | 4.25 ± 0.22 | 2.14 ± 0.57 | 1.76 ± 1.01 |
| 3.6603795.........| 406                 | V     | 0.508      | 0.059         | 0.276507| 19.48       | 0.256       | 0.2259 ± 0.0169 | 0.0732 ± 0.0168 | 0.0715 ± 0.0167 | 4.60 ± 0.08 | 2.85 ± 0.23 | 1.68 ± 0.24 |

Note.—Table 4 is presented in its entirety in the electronic edition of the Astronomical Journal. A portion is shown here for guidance regarding its form and content.

### Table 5
Fourier Parameters of Other Multifrequency LMC RR1 Variables

| Star ID         | N<br><sub>obs</sub> | Color | Amplitude | σ<sub>fit</sub> | Period | A<sub>0</sub> | A<sub>1</sub> | R<sub>21</sub> | R<sub>31</sub> | R<sub>41</sub> | φ<sub>21</sub> | φ<sub>31</sub> | φ<sub>41</sub> |
|-----------------|---------------------|-------|------------|----------------|--------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|
| 3.6243.404..... | 128                 | V     | 0.467      | 0.101         | 0.270850| 19.41       | 0.196       | 0.2331 ± 0.0826 | 0.0761 ± 0.0812 | 0.2712 ± 0.0738 | 4.64 ± 0.33 | 3.29 ± 1.07 | 2.23 ± 0.47 |
| 3.6243.404..... | 189                 | V     | 0.362      | 0.085         | 0.270850| 19.19       | 0.168       | 0.1906 ± 0.0548 | 0.1450 ± 0.0563 | 0.1716 ± 0.0562 | 4.93 ± 0.29 | 2.38 ± 0.42 | 1.88 ± 0.37 |
| 8.7441.933..... | 369                 | V     | 0.349      | 0.098         | 0.273310| 19.28       | 0.165       | 0.1864 ± 0.0464 | 0.0331 ± 0.0446 | 0.0491 ± 0.0441 | 4.61 ± 0.25 | 3.37 ± 1.33 | 2.49 ± 0.92 |
| 8.7441.933..... | 359                 | R     | 0.292      | 0.068         | 0.273310| 19.04       | 0.141       | 0.1653 ± 0.0376 | 0.1261 ± 0.0363 | 0.0518 ± 0.0359 | 4.62 ± 0.23 | 3.34 ± 0.31 | 2.80 ± 0.72 |
| 8.6353.1458.....| 474                 | V     | 0.423      | 0.067         | 0.275327| 19.40       | 0.203       | 0.1951 ± 0.0226 | 0.0294 ± 0.0220 | 0.0405 ± 0.0217 | 4.75 ± 0.12 | 2.98 ± 0.78 | 0.40 ± 0.57 |

Note.—Table 5 is presented in its entirety in the electronic edition of the Astronomical Journal. A portion is shown here for guidance regarding its form and content.
with the double modes (RR01 and RR12) in Tables 2 and 3, respectively, the RR1-w1 stars in Table 4, and other multifrequency variables in Table 5. For the rest of this investigation, we consider only the stars listed in Table 1.

Because there is overlap between some of the LMC fields in our sample (i.e., numbers 2 and 19, 3 and 80, 5 and 10, 6 and 13, and 11 and 14), there were 29 stars that were included twice in Tables 1–5. These stars are listed, in order of increasing right ascension, in Table 6. The table also includes the period, the mean $V$ and $R$ magnitudes, denoted $\langle V \rangle_F$ and $\langle R \rangle_F$, because they are the $A_B$ values derived from equation (1), and $\phi_{31} \pm \sigma$ for the $V$ data from each field so that the two sets of observations can be compared. In the last column, we list the number of the table where all of the data for the particular star can be found. It turns out that 24 of them are in Table 1. Thus, although there are 680 entries in Table 1, they represent 656 different stars. Our subsequent analysis is based on all 680 entries, since the duplicate entries do not sensibly alter the results.

Figure 1 is a plot of $\langle V \rangle_F$ versus $\langle V \rangle_F - \langle R \rangle_F$ for the data listed in Table 1. For the bulk of the points, there is a general increase in $\langle V \rangle$ with increasing color, an expected consequence of reddening. The line shown in the diagram is the reddening vector, which has a slope of 5.35, the relative extinction $A_V/E(V-R)$, for the Cerro Tololo $V$ and $R$ bandpasses (Schlegel, Finkbeiner, & Davis 1998). The bright stars that appear in the top right of the diagram are either foreground or blended stars. Their $V$ amplitudes, mean $V$ and $R$ magnitudes, and colors are listed in Table 7. Since blending of stars causes the amplitude of light variation to be reduced, we assume that the stars with $V$ amplitudes less than 0.35 are probably blended, but the ones with larger amplitudes may be foreground stars. The 17 stars of Table 7 have been excluded from the rest of our investigation.

In Table 8, we list the mean magnitudes and colors for the remaining program stars in each field. The angular coordinates of the field centers ($\rho$ and $\Phi$) are tabulated in columns (2) and (3). Following van der Marel & Cioni (2001, hereafter vMC01), we define $\rho$ as the angular distance between the field center and the LMC center and $\Phi$ as the position angle of the field center measured eastward from north. The origin of our adopted coordinate system is the location given by van der Marel (2001):

![Diagram](image-url)

**Fig. 1.—**Plot of $\langle V \rangle_F$ vs. $\langle V \rangle_F - \langle R \rangle_F$ for the stars listed in Table 1. The reddening vector [$A_V/E(V-R) = 5.35$] is marked with an arrow.
The detected (in brightness as a function of position angle). The peak-to-peak variation that
the plane of the LMC disk is inclined to terms of $D_0$, the distance from the
to the LMC center. These
clination angle of the plane of the LMC disk is 34.8° with the near side at position angle
and the line of nodes has a position angle $\Theta = -90^\circ$ ($\Phi = 30^\circ$) and the
far side at $\Theta = +90^\circ$ ($\Phi = 210^\circ$). Using these values along with
equation (8) of vMC01, we calculated for each field the distance $D$ from the observer to the point
where the field center intersects the plane of the LMC disk in terms of $D_0$, the distance from the
observer to the LMC center. These $D/D_0$ values are tabulated in column (4). In column (5), we list the
average densities of the fields (number of objects per square arcminute) that were
estimated by Alcock et al. (2001). In columns (6)–(8), we list the
mean $(V)_F$ and $(R)_F$ magnitudes and the mean colors $(V)_F - (R)_F$ along with their standard deviations. The number of
program stars in each field is in column (9). The mean $V$
magnitudes range from 19.24 for field 19 to 19.53 for field 15.
These variations may be due to a combination of differences in
distance, reddening, or the intrinsic properties of the stars
among the different fields. Any differences due to calibration
are expected to be small. A99 found an internal precision of
$\sigma_V = 0.021$, $\sigma_R = 0.019$, and $\sigma_{V-R} = 0.028$ for stars (with $V < 18$) in overlapping fields, and it appears that this precision extends to fainter magnitudes. Among the 29 RR Lyrae variables
listed in Table 6, 17 are included in both fields 6 and 13.
The mean $\Delta(V)_F$ for these stars is 0.012, and the mean $\Delta(R)_F$ is
0.022. Since the LMC is inclined to the plane of the sky, some of
our fields must be closer than others. According to the vMC01
model, we would expect the mean magnitudes of the RR Lyrae
stars in the MACHO fields that are closest to us, fields 3 and 82,

\[ \alpha_0 = 3^h29^m \text{ and } \delta_0 = -69^\circ.5. \] According to vMC01, the
inclination angle of the plane of the LMC disk is $34.8^\circ \pm 6.7^\circ$, and the line of
nodes has a position angle $\Theta = 122.5^\circ \pm 8.3^\circ$, with the near side at position angle $\Theta = -90^\circ$ ($\Phi = 30^\circ$) and the
far side at $\Theta = +90^\circ$ ($\Phi = 210^\circ$). Using these values along with

\[ \Delta(V)_F \] and \( \Delta(R)_F \) along with their standard deviations. The number of
program stars in each field is in column (9). The mean $V$
magnitudes range from 19.24 for field 19 to 19.53 for field 15.

\[ \sigma_V = 0.021, \sigma_R = 0.019, \text{ and } \sigma_{V-R} = 0.028 \text{ for stars (with } V < 18) \text{ in overlapping fields, and it appears that this precision}
\]
to be approximately 0.06 mag brighter than the ones in the most
distant fields, 10 and 13. However, this is not indicated by the
data of Table 8. Differences in extinction seem to be more
important. The fields with the faintest mean \( \langle V \rangle \) magnitudes
(3 and 15) have mean colors that are redder than most of the
other fields. If the distribution of unreddened colors of the RR
Lyrae stars in these two fields is similar to the other fields,
higher extinction can account for their faint mean magnitudes.
We will discuss the effect of extinction in \( \S \) 4. Another source of
the variations may be inhomogeneities in the properties of the
stars themselves. This is a problem we will address using
Fourier decomposition.

In Figure 2, we plot the Fourier phase differences \( \phi_{21}, \phi_{31}, \)
and \( \phi_{41} \) versus \( \log P \) for the \( V \) data. The points are plotted as
open circles with three different sizes to denote different error
levels: the larger the size, the smaller the error. In general,
the phase differences increase with increasing period. Some of
the outliers, the stars with \( \phi_{31} \sim 3.5, \phi_{41} \sim 2.0, \) and \( \log P <
-0.55, \) are probably second-overtone pulsators (RR2 vari-
bles). A96 have already pointed out that there may be second-
overtone pulsators among the LMC RR Lyrae population.

Histograms of Figure 3 illustrate the range of errors in the
Fourier phase differences. As expected, the errors in \( \phi_{41} \) are
larger than those for \( \phi_{31}, \) which in turn are larger than the
errors in \( \phi_{21}. \) This occurs because the amplitudes for the
higher orders are smaller and thus it becomes increasingly
difficult to derive their phases with sufficient precision.

Figure 4 illustrates the relationship between the \( \phi_{21}, \phi_{31}, \) and
\( \phi_{41} \) values for \( R \) and \( V. \) Horizontal lines are drawn at \( \Delta \phi_{j} = 0
\) on each plot. Although there is a great deal of scatter, it can be
noted that, in each case, the majority of the points lie above
the line. The mean differences \( \langle [\phi_{j}(R) - \phi_{j}(V)] \rangle \) are 0.03,
0.07, and 0.10 for \( j = 2, 3, \) and 4, respectively, indicating that,
in general, the Fourier phase differences for \( R \) magnitudes are
greater than the ones for \( V. \) In a comparison of \( \phi_{21} \) and \( \phi_{31} \) for
classical Cepheids, Simon & Moffett (1985) obtained a sim-
ilar result. They found that \( \phi_{j}(R) > \phi_{j}(V) > \phi_{j}(B) \) for \( j = 2 \)
and 3. They also found that the differences were greater for
\( \phi_{31} \) than for \( \phi_{21}. \)

In Figure 5, we plot the \( V \) amplitude, the Fourier amplitude,
\( A_{1}, \) and the amplitude ratios \( R_{21}, R_{31}, \) and \( R_{41} \) versus \( \log P \) for
the \( V \) data. The distributions of the estimated errors for \( R_{21}, \)
\( R_{31}, \) and \( R_{41} \) are shown in Figure 6. The error distribution is
similar for all three, but since the \( R_{31} \) and \( R_{41} \) ratios are sig-
ificantly lower than \( R_{21}, \) their errors are relatively large.
Figure 5 illustrates that, in general, \( R_{31} \) decreases with in-
creasing period, but there is not such a clear trend for \( R_{31} \) or
\( R_{41}, \) possibly because of their larger uncertainties. Some of the
short-period variables with low values for \( A_{1}, A_{2}, \) and \( R_{31} \) are
probably RR2 variables. Figure 7 shows the amplitude ratios

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**Fig. 2.**—Fourier phase differences \( \phi_{21}, \phi_{31}, \) and \( \phi_{41} \) as a function of \( \log P \) for
the \( V \) data of the 663 program stars summarized in Table 8. The plotted points
are open circles that have three sizes: the larger the size, the lower the error. For
\( \phi_{21}, \) the largest size denotes standard errors less than 0.2, the smallest size
denotes errors greater than 0.4, and the intermediate size denotes errors be-
tween 0.2 and 0.4. For the \( \phi_{31}, \) and \( \phi_{41} \) plots, the three sizes denote standard
errors less than 0.4, greater than 0.8 and between 0.4 and 0.8, respectively.

**Fig. 3.**—Histograms illustrating the distribution of the standard errors in
\( \phi_{21}, \phi_{31}, \) and \( \phi_{41} \) for the \( V \) data that are plotted in Fig. 2.
\( R / V \) for the light-curve amplitude and \( A_1 \) through \( A_4 \), \( R \) amplitudes are generally lower than \( V \) amplitudes because pulsating stars like RR Lyrae variables have lower amplitudes when observed at longer wavelengths. In each panel, horizontal lines indicating the median are shown, and in each case, the median \( R / V \) ratio is approximately 0.8. The scatter is greater for the higher orders because their amplitudes are small and have large uncertainties. Another factor that may contribute to the scatter in the amplitude ratios is contamination. The presence of a nearby unresolved companion will reduce the observed amplitude. This is an effect we need to consider when we select a sample of stars for deriving the LMC distance.

3.2. Comparison with the RR1 Variables in GGCs

For this comparison, we examine the plots of \( \phi_{31} \) versus \( \log P \) and \( R_{21} \) versus \( \log P \). We prefer \( \phi_{31} \) to \( \phi_{21} \), even though \( \sigma(\phi_{31}) > \sigma(\phi_{21}) \), because the range in values of \( \phi_{21} \) is very small, making it difficult to detect the differences among the clusters. The \( \phi_{31} \) versus \( \log P \) plots are shown in Figure 8. In the top panel, we plot \( \phi_{31}(V) \) versus \( \log P \) for the RR1 variables in five well-studied GGCs. The data for these clusters are taken from the following sources: NGC 6441 (Pritzl et al. 2001), M107 (Clement & Shelton 1997), M5 (kahuzny et al. 2000, hereafter K00), M2 (Lee & Carney 1999, hereafter LC99), and M68 (Walker 1994). For NGC 6441, we have included all the variables that Pritzl et al. (2001) listed as “RRc” but not the questionable ones (indicated as “RRc?”). V79 was also excluded because it had a large amount of scatter on its light curve. For M5, we...
have included all of the variables in Table 1 of the K00 paper, with the exception of V78 (considered to be an RR2 variable), V76 (classification uncertain), and V130 (large error in $\phi_{31}$).

Since the M5 Fourier decomposition was based on a sine series, we subtracted 3.14 from all of the published $\phi_{31}$ values before plotting them in Figure 8. The LC99 study of M2 did not include Fourier analysis of the variables, so we analyzed their published observations. Our results for the three M2 stars that we consider to be bona fide22 RR1 variables are summarized in Table 9. For M68, we excluded V5, which is probably an RR2 variable. The straight lines in the top panel are least-squares fits to the data for each cluster. In the middle panel, these lines are plotted again, along with the $\phi_{31}(V)$ values for the LMC stars with $\phi_{31} < 0.4$. The bottom panel is a repeat of the middle panel, but it also includes points plotted for the RR1 variables in M3 from Kahužny et al. (1998) and $\omega$ Centauri from the Clement & Rowe (2000) study based on the observations of Kahužny et al. (1997). The $\phi_{31}$–log $P$ plots for the GGCs indicate that, in general, the higher the metallicity,23 the higher the line lies in the diagram. Most of the LMC RR1 variables are distributed between the M107 and M68 lines. Many have $\phi_{31}$–log $P$ values similar to the ones in M2, M3, M5, and the Oosterhoff type I (OoI) variables in $\omega$ Centauri, i.e., the ones with log $P < -0.44$. However, not many are similar to the OoII variables in $\omega$ Cen, the ones with

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Footnotes:

22 LC99 presented photometry for 30 RR Lyrae variables, 12 of which they classified as type c. We include only three of these “type c” stars in Table 9 because three of them (LC 651, 715, and 733) appear to be pulsating in the second-overtone mode. Four others (V15, V18, V20, and LC 939) were excluded because their light curves have night-to-night variations similar to those of the variables in M55 that Olech et al. (1999) classified as nonradial pulsators. The stars LC 608 and 1047 were also excluded because of a large amount of scatter on their light curves. In the case of the former, it may be caused by large period changes, and for the latter it may be due to light contamination from a nearby star.

23 The $[\text{Fe/H}]$ values that Harris (1996) lists in his 2003 catalog update for NGC 6441, M107, M5, M3, M2, and M68 are $-0.53$, $-1.04$, $-1.27$, $-1.57$, $-1.62$, and $-2.06$, respectively. The ZW84 values for the same six clusters are $-0.59$, $-0.99$, $-1.40$, $-1.66$, $-1.62$, and $-2.09$. 

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Fig. 6.—Histograms illustrating the distribution of the standard errors in $R_{21}$, $R_{31}$, and $R_{41}$ for the $V$ data plotted in Fig. 5.

Fig. 7.—Plots of the ratios $A_2/A_1$, $A_3(A_1(V), A_2(R)/A_3(V))$, $A_3(R)/A_2(V)$, and $A_4(R)/A_3(V)$ to show the effect of wavelength band. The symbols are the same as in Fig. 5.
longer periods. There are also a significant number (with \( \phi_{31} \) between 2.0 and 3.0) that do not have counterparts among the RR1 variables in the well-studied GGCs. These objects are worthy of further investigation.

The \( R_{21} - \log P \) plots are shown in Figure 9. In the top panel, we plot the data for the RR1 variables in M107, M5, M2, and M68. Lines based on least-squares fits to the points are plotted for M5, M2, and M68. However, we do not plot a line for M107 because, although the five stars with the shortest periods show a steady decrease in \( R_{21} \) with increasing period, the other two (with \( \log P \sim -0.5 \)) do not follow the sequence. Instead, they lie among the M5 points. In the middle panel, these lines are plotted again, along with the LMC variables. The bottom panel is the same as the middle panel, but includes points for M3 and \( \omega \) Centauri. Like the previous figure, Figure 9 illustrates that some of the LMC RR1 variables have \( R_{21} - \log P \) values similar to the ones in M2, M3, M5, and the OoI variables in \( \omega \) Cen, but they are different from the majority of the \( \omega \) Cen variables, which have OoII characteristics.

Since one of the aims of our investigation is to determine the LMC distance, we want to select a group of LMC variables that have properties similar to the variables in a well-studied GGC. We will then make the assumption that these stars have a similar distribution of luminosities so that their absolute magnitudes can be derived by independent methods. We conclude that M5 is more suitable for this purpose than M2 or M3 because the M5 study by K00 includes a more complete sample (13) of RR1 variables with well-determined Fourier coefficients. In addition, there have been independent studies of the absolute magnitude of the horizontal-branch (HB) stars in M5. Storm, Carney, & Latham (1994) performed a Baade-Wesselink analysis on two of its RR Lyrae variables, and Carretta et al. (2000) derived \( M_V \) (HB) from main-sequence fitting. These studies will be discussed further in \( \S \) 5.3.

3.3. Luminosity, Mass, and Temperature Derived from Fourier Coefficients

SC93 derived an equation, based on hydrodynamic pulsation models, for calculating the luminosity of an RR1 variable from \( \phi_{31} \) and period:

\[
\log L/L_\odot = 1.04 \log P - 0.058 \phi_{31} + 2.41. \tag{2}
\]

In an independent investigation, based on observations of 93 RR1 variables in eight different stellar systems, Kovács (1998, hereafter K98) derived an equation relating the absolute magnitude \( M_V \) of an RR1 variable to its period and Fourier coefficients \( \phi_{21} \) and \( A_4 \):

\[
M_V = 1.261 - 0.961 P - 0.044 \phi_{21} - 4.447 A_4. \tag{3}
\]

The phase difference \( \phi_{21} \) in the K98 equation is based on a sine series fit, so we subtracted 1.57 from our \( \phi_{21} \) values, which are based on a cosine series. SC93 also used their pulsation models to derive an equation relating mass to \( \phi_{31} \) and period:

\[
\log M/M_\odot = 0.52 \log P - 0.11 \phi_{31} + 0.39. \tag{4}
\]

Combining this with the period/mean density law (eq. [2] in their paper), we can derive an equation for calculating the temperature:

\[
\log T_{\text{eff}} = 3.775 - 0.1452 \log P + 0.0056 \phi_{31}. \tag{5}
\]

Using equations (2), (3), (4), and (5) with the \( V \) data, we calculated \( \log L, M_V, \log M, \) and \( \log T_{\text{eff}} \) for the “Table 1” stars with \( \sigma(\phi_{31}) < 0.4 \). Since these equations are valid only for RR1 variables, we have included only stars with periods in the range \(-0.56 < \log P < -0.4 \) and amplitudes \( A_V > 0.3 \).
### TABLE 9
**FOURIER PARAMETERS OF M2 RR1 VARIABLES**

| Star                  | Period (2) | $N_{\text{obs}}$ (3) | $A_0$ (4) | Amplitude $A_1$ (5) | $\sigma_{\text{fit}}$ (7) | $R_{21}$ (8) | $R_{31}$ (9) | $R_{41}$ (10) | $\phi_{21}$ (11) | $\phi_{31}$ (12) | $\phi_{41}$ (13) |
|-----------------------|------------|-----------------------|-----------|---------------------|---------------------------|-------------|-------------|---------------|-------------------|-------------------|-------------------|
| V19 ................... | 0.319416   | 114                   | 16.06     | 0.441               | 0.226                     | 0.025       | 0.1624 ± 0.0153 | 0.0772 ± 0.0164 | 0.0507 ± 0.0168 | 5.21 ± 0.10       | 3.74 ± 0.21       | 2.40 ± 0.31       |
| V24 (LC450)........... | 0.358162   | 125                   | 16.00     | 0.421               | 0.211                     | 0.031       | 0.1010 ± 0.0200 | 0.0573 ± 0.0193 | 0.0245 ± 0.0195 | 4.58 ± 0.19       | 4.06 ± 0.34       | 3.09 ± 0.77       |
| V32 (LC864)........... | 0.361938   | 145                   | 16.05     | 0.429               | 0.212                     | 0.050       | 0.0714 ± 0.0285 | 0.0845 ± 0.0290 | 0.0403 ± 0.0296 | 4.77 ± 0.41       | 4.41 ± 0.36       | 1.93 ± 0.70       |

**Note.**—LC99 discovered 13 new RR Lyrae variables in M2. In the catalog of Variable Stars in Globular Clusters at [http://www.astro.utoronto.ca/people.html](http://www.astro.utoronto.ca/people.html) (Clement et al. 2001), these have been numbered V22 through V34.
Stars with shorter periods and lower amplitudes are probably RR2 variables and stars with longer periods have anomalous light curves, indicating that they are probably not bona fide RR1 variables. In addition, we restricted the sample to stars with amplitude ratios in the range $0.75 < \frac{A_R}{A_V} < 0.85$. Amplitude ratios outside this range may occur if the phase coverage on the light curve is incomplete or if the star has a faint unresolved companion, in which case the mean magnitude and amplitude are not reliable. In addition, if a variable is an eclipsing binary, its amplitude ratio should be close to unity and therefore greater than 0.85. A total of 330 stars met the above criteria. We will refer to these as the “bona fide” RR1 stars. In the electronic edition of Table 1, these stars are denoted “bf.”

Figure 10 is a plot of $\log \frac{L}{L_\odot}$ versus $\log T_{\text{eff}}$ for these 330 bona fide RR1 variables. It demonstrates that the stars in our sample have $\log L$ ranging from $\sim 1.6$ to $\sim 1.8 L_\odot$ and $\log T_{\text{eff}}$ between $\sim 3.85$ and $\sim 3.87$, values appropriate for first-overtone pulsators on the blue side of the instability strip according to the models of Bono et al. (1997) and Yoon & Lee (2002). In Figures 11 and 12, we plot $\log L$ and $M_V$ versus $\langle V \rangle_F$ with the results for each of the 16 fields shown in different panels. The lines in Figure 11 have a slope of $-0.4$. They are plotted at an arbitrary position, but are set at the same position in each panel so that any variations among the different fields can be readily recognized. The standard deviation of the fit of the hydrodynamic models to equation (2) was $\Delta \log L = 0.035$, but the scatter for the individual fields is greater than that. The situation is similar in Figure 12 where the lines are plotted with a slope of unity. The standard deviation that K98 derived for equation (3) was 0.042. Clement & Rowe (2000) made similar plots for the RR1 variables in $\omega$ Centauri, and the fit was much better. If the variations in apparent magnitude among the stars in the individual LMC fields are due primarily to differences in luminosity, we would not expect to see correlations in these figures. However, a difference of 0.5 mag would require a difference of about 25% in distance, which is certainly not expected a priori. We conclude that differential reddening and crowding within the individual fields must also contribute to the scatter.

In Figure 13, we plot $\log L$ versus $M_V$, and it is clear that these two quantities are correlated even though equations (2) and (3) were derived by independent methods. Since C03 established that there exists a luminosity-metallicity relation...
with slope equal to $\Delta M_V/\Delta [Fe/H] = 0.214 \pm 0.047$ among the LMC RR Lyrae variables, we assume that the brighter stars are more metal-poor than the faint ones. This means that their bolometric corrections will be different, and since $\log L$ refers to the bolometric luminosity, we should take this into account when comparing $\log L$ with $MV$. Bessell & Germany (1999, hereafter BG99) showed the relationship between bolometric correction (BC) and $V-R$ color for four different values of $[Fe/H]$ (Fig. 8 in their paper). From a comparison of BC for $[Fe/H] = -1.0$ and $-2.0$ at $(V-R)_0 = 0.14$ mag, a typical intrinsic color for an RR1 variable, we see that $BC = 0.03$. Combining this with C03’s slope for the luminosity-metallicity relation (0.21), we derive a slope of $-0.46$ for the $\log L/L_\odot$ versus $M_V$ plot. The envelope lines in the diagram are plotted with this slope and are separated by $\Delta \log L = 0.07$, twice the standard deviation in the fit of equation (2) to the models. The actual slope of the plotted points ($-0.53$) is steeper than $-0.46$. Nevertheless, $72\%$ of the points lie between the envelope lines. In $\S$ 5.3, we will derive a mean absolute magnitude for the M5-like variables based on Fourier coefficients.

4. THE INTERSTELLAR EXTINCTION

In their discussion of the LMC distance, Benedict et al. (2002) pointed out that the average extinction-corrected magnitude of RR Lyrae variables in the LMC remains a significant uncertainty. Establishing the effect of interstellar extinction on the observed magnitudes of LMC stars is a difficult problem because the amount of extinction is not constant. Schwering & Israel (1991) found that the foreground reddening ranges from $E(B-V) = 0.07$ to $0.17$ mag over the LMC surface. Among the 16 fields in our study, their $E(B-V)$ values range from approximately $0.07$ to $0.14$ mag. Thus it is not appropriate to make one reddening correction for all of the stars in our sample. It is more accurate to consider each star separately. Therefore, our approach is to calculate the effective temperatures from equation (5) and then use a color-temperature calibration to derive the unreddened colors. Equations relating $\log T_{\text{eff}}$ to $(V-R)_0$ have been derived by BG99 and by Kovács & Walker (1999, hereafter KW99). BG99’s equations, which are based on model atmospheres of Castelli (1999), apply only for $\log g = 2.5$ and four different values of $[Fe/H]$. However, the temperature-color relation of KW99 is more general. They derived a linear expression (eq. [10] in their paper) relating the temperature to color, $\log g$, and $[M/H]$ based on models of Castelli, Gratton, & Kurucz (1997):

$$\log T_{\text{eff}} = 3.8997 - 0.4892(V + R)_0 + 0.0113 \log g + 0.0013[M/H], \quad (6)$$

and they also derived an expression (eq. [12] in their paper) for estimating the gravity from mass, temperature, and the fundamental-mode pulsation period:

$$\log g = 2.9383 + 0.2297 \log (M/M_\odot) - 0.1098 \log T_{\text{eff}} - 1.2185 \log P_0. \quad (7)$$

We have used equations (4), (5), (6), and (7) to derive the unreddened color for each star. In each case, we assumed
[M/H] = −1.5, and the fundamental-mode pulsation period was computed from the overtone period (P1) using a period ratio, P1/P0 = 0.7445. A plot of log Teff versus ((V − R) − (V − R)0) for the bona fide RR1 variables in each of the 16 fields is shown in Figure 14. The unreddened line in each panel is derived from equation (6) assuming log g = 2.924 and [M/H] = −1.5. The diagram indicates that the color excess varies from field to field and also within the individual fields. Thus, differential reddening may be responsible for at least some of the scatter in Figures 11 and 12. For each of the stars, we calculated the color excess and the corrected mean V magnitude:

\[ E(V-R) = \langle V \rangle_F - \langle R \rangle_F - (V-R)_{0} \]  
\[ V_0(F) = \langle V \rangle_F - 5.35E(V-R). \]  

In Table 10, we summarize the mean \( \langle V \rangle_F \), the mean extinction \( E(V-R) \), and the mean corrected magnitude \( V_0(F) \) for the bona fide RR1 stars in each field. N is the number of RR1 stars in the field, and in each case, the errors represent the standard error of the mean (i.e., the standard deviation divided by \( \sqrt{N} \)). In addition to these errors, there are systematic errors in \( E(V-R) \) and \( V_0 \) because of the uncertainties in the derivation of log Teff and \( (V-R)_{0} \). These will be discussed in §5.2.

The temperature-color relations that BG99 derived apply only for log g = 2.5, which is lower than the 2.9 that we have assumed. According to equation (6), the lower log g would decrease \( (V-R)_{0} \) by \( \sim 0.01 \) and thus increase the derived extinction. If we compare the \( (V-R)_{0} \) colors derived from the BG99 relations with those predicted by our equation (6) for the same log g (i.e., 2.5), we find that the BG99 colors

---

Fig. 12.—Plots of \( M_V \) calculated from eq. (3) vs. the mean V magnitude for the 330 RR1 variables plotted in Fig. 10. The stars in each field are plotted separately, and the line drawn through the points in each panel has a slope \( \Delta M_V/\Delta V = 1 \).

Fig. 13.—Plot of log \( L/L_{\odot} \) vs. \( M_V \) calculated from eqs. (2) and (3) using the V data for the 330 RR1 variables plotted in Fig. 10. The envelope lines have a slope of −0.46, the predicted slope for \( \Delta \log L/\Delta M_V \), and are separated by \( \Delta \log L = 0.07 \).

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24 In order to plot the unreddened line in Figure 14, we calculated a mean value of log g for the 330 stars in our sample based on the mean log Teff(3.863), the mean log M(−0.2085), and log P0 = −0.3617, which corresponds to the mean log P1(−0.4898).
Fig. 14.—Plots of log $T_{\text{eff}}$ vs. $(V)_F - (R)_F$ for the 330 RR1 variables plotted in Fig. 10. The stars in each field are plotted separately to show the differences in color excess. The ridge lines represent the log $T_{\text{eff}}$ vs. $(V-R)_0$ relations derived from eq. (6) assuming log $g = 2.9$ and [M/H] = −1.5.

### TABLE 10
CORRECTED MAGNITUDES AND REDEENINGs BY FIELD

| FIELD | N | $(V)_F$ | $E(V-R)$ | $(V)_0$ | $N$ | $(V)_F$ | $E(V-R)$ | $(V)_0$ |
|-------|---|---------|---------|--------|---|---------|---------|--------|
| 2     | 32 | 19.25 ± 0.03 | 0.06 ± 0.01 | 18.92 ± 0.04 | 8 | 19.23 ± 0.07 | 0.06 ± 0.02 | 18.91 ± 0.08 |
| 3     | 22 | 19.50 ± 0.05 | 0.13 ± 0.01 | 18.84 ± 0.05 | 4 | 19.48 ± 0.05 | 0.13 ± 0.02 | 18.80 ± 0.05 |
| 5     | 34 | 19.25 ± 0.03 | 0.06 ± 0.005 | 18.92 ± 0.03 | 8 | 19.23 ± 0.04 | 0.08 ± 0.01 | 18.81 ± 0.05 |
| 6     | 36 | 19.36 ± 0.02 | 0.09 ± 0.01 | 18.87 ± 0.03 | 9 | 19.45 ± 0.05 | 0.11 ± 0.01 | 18.86 ± 0.07 |
| 10    | 14 | 19.22 ± 0.04 | 0.06 ± 0.01 | 18.93 ± 0.03 | 6 | 19.23 ± 0.07 | 0.05 ± 0.01 | 18.94 ± 0.06 |
| 11    | 25 | 19.36 ± 0.05 | 0.05 ± 0.01 | 19.10 ± 0.06 | 2 | 19.27 ± 0.08 | 0.06 ± 0.05 | 18.94 ± 0.20 |
| 12    | 13 | 19.41 ± 0.04 | 0.09 ± 0.01 | 18.94 ± 0.04 | 2 | 19.42 ± 0.15 | 0.10 ± 0.03 | 18.90 ± 0.02 |
| 13    | 32 | 19.31 ± 0.02 | 0.08 ± 0.01 | 18.91 ± 0.03 | 8 | 19.34 ± 0.05 | 0.08 ± 0.01 | 18.92 ± 0.06 |
| 14    | 13 | 19.30 ± 0.04 | 0.03 ± 0.01 | 19.15 ± 0.05 | 5 | 19.26 ± 0.05 | 0.04 ± 0.01 | 19.05 ± 0.07 |
| 15    | 13 | 19.56 ± 0.04 | 0.11 ± 0.01 | 19.00 ± 0.03 | 3 | 19.62 ± 0.09 | 0.11 ± 0.01 | 19.02 ± 0.02 |
| 18    | 11 | 19.27 ± 0.04 | 0.06 ± 0.01 | 18.97 ± 0.09 | 2 | 19.35 ± 0.05 | 0.05 ± 0.001 | 19.07 ± 0.05 |
| 19    | 13 | 19.22 ± 0.04 | 0.03 ± 0.01 | 19.05 ± 0.05 | 4 | 19.33 ± 0.03 | 0.03 ± 0.01 | 19.14 ± 0.07 |
| 47    | 12 | 19.25 ± 0.05 | 0.05 ± 0.01 | 18.99 ± 0.04 | 2 | 19.28 ± 0.15 | 0.03 ± 0.03 | 19.11 ± 0.04 |
| 80    | 33 | 19.39 ± 0.04 | 0.13 ± 0.01 | 18.72 ± 0.04 | 10 | 19.43 ± 0.06 | 0.12 ± 0.02 | 18.79 ± 0.05 |
| 81    | 14 | 19.45 ± 0.05 | 0.07 ± 0.01 | 19.05 ± 0.07 | 3 | 19.50 ± 0.13 | 0.09 ± 0.02 | 19.00 ± 0.23 |
| 82    | 13 | 19.37 ± 0.03 | 0.11 ± 0.01 | 18.76 ± 0.06 | 4 | 19.43 ± 0.06 | 0.13 ± 0.01 | 18.73 ± 0.11 |
| All   | 330 | 19.34 ± 0.01 | 0.08 ± 0.003 | 18.92 ± 0.01 | 80 | 19.35 ± 0.02 | 0.09 ± 0.005 | 18.91 ± 0.02 |

**Note.**—In each case, the errors listed represent the standard error of the mean. However, for both the extinction and $V_0$, there are systematic errors as well: 0.022 mag in $E(V-R)$, which propagates to an error of 0.12 mag in $V_0$. 

\[ \frac{0.005}{0.01} \]
would be about 0.01 less for $T_{\text{eff}} \sim 7300$, a typical temperature for RR1 variables. Thus the extinction we derive for the same log $g$ is lower than that predicted by BG99’s equations.

On the other hand, the mean extinction we derive for our bona fide RR1 stars is larger than the value that C03 adopted for the same region. C03 observed RR Lyrae variables in two fields that overlap with our fields 6 and 13 for which we derived mean $E(V-R) = 0.09$ and 0.08 mag, respectively. This is equivalent to $E(B-V) \sim 0.15$ mag. They used two methods to determine the color excess for the stars in their sample. First, they used Sturch’s (1966) method. For this, they compared the observed $B-V$ colors of 62 RR0 (RRab) variables at minimum light with unreddened colors that were calculated from Walker’s (1990) equation that relates color to period and metal abundance. The $E(B-V)$ values that they derived for their fields A and B were 0.133 and 0.115 mag, respectively. In their second method, they derived the mean $B-V$ colors for five RR Lyrae at the blue and five at the red edge of the instability strip and compared these with the colors Corwin & Carney (2001) observed for the instability strip boundaries in the globular cluster M3. To account for any color differences due to different metallicity, they applied a metallicity-color shift relation derived by Walker (1998). Using this method, they derived $E(B-V) = 0.116$ mag for their field A and 0.086 mag for field B, values that are lower than the ones they derived from the Sturch method. They adopted the latter values because independent studies of globular clusters have indicated that Sturch’s method gives $E(B-V)$ values that are about 0.02 mag larger than those determined by other techniques. The mean extinction we derived for fields 6 and 13 is comparable to what C03 found from the Sturch method, but considerably larger than those determined by other techniques. The mean extinction we derive for the reddening for these fields from observations of red clump stars, they found that the extinction had been overestimated in the earlier study. Their revision indicates the reddening for these fields from observations of red clump stars (Udalski et al. 1998). It was based on a photometric ($UBV$) and spectroscopic study by Oestreich & Schmidt-Kaler (1996). However, the revision was not actually observed by these latter authors. It lies about 2° south of a region where they found high extinction. When Udalski et al. (1999) reestimated the reddening for these fields from observations of red clump stars, they found that the extinction had been overestimated in the earlier study. Their revision indicates $E(V-I) \sim 0.23$ mag, which corresponds to $E(V-R) \sim 0.11$ mag, the mean extinction that we list for field 15 in Table 10.

Another benchmark for our extinction values is the recent study of LMC bump Cepheids by Keller & Wood (2002), based on MACHO data. Using pulsation theory, they derived the extinction for 20 stars, two of which were in field 19. The mean $E(V-R)$ that they derived for these two stars, 0.03 mag, is the same mean that we have derived for this field. We cannot consider this to be a conclusive result, however, because their sample is so small.

5. THE M5-LIKE VARIABLES

5.1. The Selection of the M5-like Variables

Only the M5-like stars will be considered for our derivation of the LMC distance, and we select these stars according to their location in the $\phi_{31}$--log $P$ plots. Points that lie less than 2 $\sigma$ ($\Delta \phi_{31} = 0.214$) from the M5 line in Figure 8 are considered to be M5-like ($\sigma$ is the standard deviation of the fit of the K00 $\phi_{31}$ values to the line). In Figure 15, we show $\phi_{31}$--log $P$ plots for the individual fields with the lines for M5 and M68 superposed. The stars that are plotted as crosses are the ones we classify as “M5-like.” In the electronic version of Table 1, these stars are denoted “M.” The mean values of $(V)_F$, extinction $E(V-R)$, and $V_0(F)$ for the M5-like stars in each field are listed in the last three columns of Table 10. In each case, the quoted error is the standard error of the mean. These errors have been derived in the same manner as those listed in columns (3)–(5) of the table.

5.2. The Mean Apparent Magnitude of the M5-like Variables

In order to determine an LMC distance, we must derive the mean apparent magnitudes of the M5-like stars in our sample, correct these magnitudes for the effects of interstellar extinction, and assess the errors. The mean $(V)_F$, $(R)_F$ magnitudes listed in Table 10 are the mean $A_0$ values calculated from equation (1). The first step is to determine how well the $A_0$ values represent the data. To do this, we selected a subset of the M5-like stars, the 10 in field 80, and applied Efron’s bootstrap method (Diaconis & Efron 1983). The results indicated that the mean error in $A_0(V)$ was 0.0026 mag with a standard deviation of 0.0010. The equivalent numbers for $A_0(R)$ were 0.0034 and 0.0014, respectively. Thus, the errors in $A_0$ are very small. The next step is to compare $A_0$ with the “intensity” mean magnitude $<(V)_{int}>$ for most studies of pulsating variable stars, the intensity means are used. To make this comparison, we calculated $(V)_{int}$ and $(R)_{int}$ for the same 10 stars, and the results are presented in Table 11. It turns out that the $A_0$ values are generally about 0.01 mag fainter than the intensity means. We will take this into account when we adopt our final value for the mean-corrected apparent magnitude.

In order to derive the interstellar extinction for each star, we need to determine the color. The mean colors listed in Tables 10 and 11 were computed from $(V)_F$--$(R)_F$. However, Fernie (1990) found that a straight average of the color over the pulsation cycle (e.g., $(V-R)_{mag}$) is a better indicator of the temperature. We computed $(V-R)_{mag}$ for the M5-like variables in field 80, and these are also listed in Table 11. The colors derived by the two methods differ by less than 0.01 mag. Thus, we conclude that using $(V)_F$--$(R)_F$ to derive the extinction does not introduce any systematic errors. The main sources of error in $(V-R)$ are errors in the derived temperatures and in the color-temperature relation. Any errors in the observed colors are assumed to be minor and random, so that over the sample of 80 M5-like stars, they cancel out. On the other hand, the other two effects are not random because the stars in our sample have similar properties. Based on the errors in the fits of SC93’s equations relating $log L$ and $log M$ of the pulsation models to the period and $\phi_{31}$ ($\sigma_{log L} = 0.035$ and $\sigma_{log M} = 0.025$), we estimate that the error in $log T_{\text{eff}}$ calculated from equation (5) is 0.01. This translates to an error of 0.02 mag in $(V-R)_0$. Since the dependence of $log T_{\text{eff}}$ on $\phi_{31}$ is very weak, the errors in $\phi_{31}$ do not make a significant contribution to the errors in our derived temperatures. The discussion of the different $log T_{\text{eff}}$--color relations in § 4 indicates there is an additional uncertainty of $\sim 0.01$ mag in $(V-R)_0$ colors. Adding in quadrature, we estimate that the systematic error in the color excess $E(V-R)$ is 0.022 mag. Since $A_V = 5.35E(V-R)$, this propagates to an error of 0.12 mag in $V_0$. 
The distribution of \( V_0(F) \) for the RR1 variables is shown in Figure 16. All of the 330 bona fide RR1 variables have been plotted, with the M5-like stars represented as solid areas. This diagram and the data in Table 10 both illustrate that the mean \( V_0 \) of the M5-like stars does not differ significantly from that of all of the RR1 variables. In Figure 16, each field is plotted separately in case there are systematic field-to-field variations. Within some of the individual fields, the range in \( V_0 \) for the M5-like stars is 0.5 mag or more. Based on the K00 study of M5, one would expect a scatter of at least 0.1 mag in \( V_0 \) among the M5-like variables. Some of the additional scatter must be due to variations in distance from the observer, but some is probably caused by crowding. Artificial star tests in the MACHO fields support this conclusion. The results from these tests will be discussed further in § 5.4, where we estimate the LMC distance using M5-like stars and include a correction for crowding bias. Figure 17, a plot of \( \langle V_0 \rangle_F \) versus the density for each field, also illustrates that crowding affects our derived \( V_0 \) values. One would not expect a perfect correlation in Figure 17 because it is possible for stars in low-density fields to have unresolved companions. Nevertheless, the faintest mean \( V_0 ( \sim 19.1 ) \) occurs in low-density fields, and the mean \( V_0 \) is \( \sim 0.3 \) mag brighter in the highest density fields where the crowding is expected to be the most severe. Although we have tried to overcome the crowding problem by restricting our sample to stars with an amplitude ratio \( A_R/A_F \)....

### Table 10

| Star     | \( \langle V \rangle_F \) | \( \langle V \rangle_{int} \) | \( \langle R \rangle_F \) | \( \langle R \rangle_{int} \) | \( \langle V \rangle_F - \langle R \rangle_F \) | \( \langle V-R \rangle_{mag} \) |
|----------|--------------------------|--------------------------|--------------------------|--------------------------|-----------------------------|-----------------------------|
| 80.6351.2358       | 19.32                     | 19.31                     | 19.10                     | 19.09                     | 0.22                        | 0.22                        |
| 80.6354.3658       | 19.81                     | 19.79                     | 19.48                     | 19.48                     | 0.33                        | 0.32                        |
| 80.6475.3588       | 19.46                     | 19.45                     | 19.20                     | 19.20                     | 0.26                        | 0.25                        |
| 80.6589.2425       | 19.56                     | 19.54                     | 19.30                     | 19.29                     | 0.26                        | 0.26                        |
| 80.6596.3127       | 19.51                     | 19.50                     | 19.24                     | 19.23                     | 0.27                        | 0.27                        |
| 80.6710.2075       | 19.41                     | 19.40                     | 19.18                     | 19.17                     | 0.23                        | 0.23                        |
| 80.6832.2030       | 19.59                     | 19.58                     | 19.31                     | 19.30                     | 0.28                        | 0.27                        |
| 80.7192.4927       | 19.49                     | 19.48                     | 19.19                     | 19.18                     | 0.30                        | 0.30                        |
| 80.7320.1224       | 19.26                     | 19.25                     | 19.03                     | 19.02                     | 0.23                        | 0.23                        |
| 80.7437.1665       | 19.39                     | 19.38                     | 19.11                     | 19.10                     | 0.29                        | 0.29                        |
between 0.75 and 0.85, it appears that it has not been completely eliminated.

Another method for checking our adopted mean magnitudes is to compare with other investigations. Seven of the bona fide RR1 stars from Table 1 were included in C03's study. Their mean magnitudes will be published in a forthcoming paper by Di Fabrizio et al. (2004). In Table 12, we list the mean $h V_i$ magnitudes derived for these stars from the two studies; for both groups, the means that we list are arithmetic means. The average of the C03 mean magnitudes is 0.07 mag fainter than ours. It is more difficult to compare our results with those of Udalski et al. (1999) because most of the OGLE observations were in the $I$ band and the mean magnitudes that they published were corrected for extinction. In addition, the mean that they quoted included RR Lyrae variables in four different LMC fields. In Udalski's (1998) study, a mean $V_0 = 18.86$ mag was adopted for 110 RR0 variables in these fields, two of which overlap with our field 15. This mean was derived from ~65 $I$ and six $V$ magnitudes for each star. Later, when more observations were available for these stars (~140 in the $I$ band and 20 in the $V$ band) and the extinction was revised, Udalski et al. (1999) changed the mean $V_0$ to 18.94 ± 0.04. The mean $(V_0)$ that we list in Table 10 for the 13 stars in field 15 is 19.00 ± 0.13.

5.3. The Absolute Magnitudes of the M5-like Variables

In our derivation of the absolute magnitude, we consider four independent methods: Baade-Wesselink (B-W) analysis, main-sequence fitting, trigonometric parallax of the star RR Lyrae, and Fourier analysis.

As we noted at the end of § 3.2, Storm et al. (1994) performed a B-W analysis of two RR Lyrae variables (V8 and V28) in M5. They derived $M_F = 0.65$ and 0.67 mag, using a value of 1.30 for $p$, the conversion factor between observed and true pulsation velocity. Later, Clementini et al. (1995) revised these values to $M_F = 0.52 ± 0.26$ and 0.54 ± 0.26 mag, by assuming $[\text{Fe/H}] = -1.17$ and $p = 1.38$. However, if the assumed metal abundance is less, e.g., the ZW84 value of $-1.40$ instead of $-1.17$, these stars would be 0.02 mag
fainter. The adopted $p$-factor (1.38) is the value that gives the brightest possible value of the luminosity, and Fernley (1994) considered it to be a more appropriate value than 1.30. More recently, Cacciari et al. (2000) reexamined the calibration of B-W results and suggested that $p$ might be a few percent smaller than 1.38. Based on all of this, we adopt $p$-factor (1.38) is the value that gives the brightest possible value of the luminosity, and Fernley (1994) considered it to be a more appropriate value than 1.30. More recently, Cacciari et al. (2000) reexamined the calibration of B-W results and suggested that $p$ might be a few percent smaller than 1.38. Based on all of this, we adopt $M_V(BW) = 0.55 \pm 0.26$ mag for the M5 RR Lyrae variables.

Carretta et al. (2000) derived $M_V = 0.54 \pm 0.09$ mag for the HB of M5 by fitting the main sequence to subdwarfs with parallaxes determined from Hipparcos. However, Gratton et al. (2003a, 2003b) have subsequently concluded that this is too bright. Using the ESO VLT telescopes, they made new spectroscopic observations of subdwarfs in the Galactic field and in the three globular clusters NGC 6397, NGC 6752, and 47 Tuc and have revised the metal abundances. They also derived new reddening values. The new data indicate that the absolute $V$ magnitude for the HB of M5 may be as bright as 0.58 or as faint as 0.65, depending on the assumed metal abundance. We adopt $M_V(HB) = 0.61 \pm 0.12$ mag.

Benedict et al. (2002) used HST astrometry to derive a trigonometric parallax for RR Lyrae and then calculated its absolute magnitude: $M_V = 0.61_{-0.12}^{+0.11}$. Because of the location of RR Lyrae in a period-amplitude plot, we assume that its absolute magnitude is comparable to that of an M5 RR Lyrae variable. It is well known that the light curve of the star RR Lyrae ($P = 0.567$ days) is modulated with a longer secondary period ($\sim$40 days); i.e., it exhibits the Blazhko effect (Smith 1995). However, according to Szeidl (1988), the maximum light amplitude of a Blazhko variable always fits the period-amplitude relation for singly periodic variables. According to some recently published observations of RR Lyrae (Smith et al. 2003), its maximum $V$ amplitude is $\sim$0.9 mag, which places it on the period-amplitude relation that K00 plotted for the RR0 variables in M5. If there is a period-luminosity-amplitude relation for RR0 variables as demonstrated by Sandage (1981), then we may assume that the absolute magnitude of RR Lyrae is comparable to that of an M5-like variable. We adopt $M_V(\{\pi\}) = 0.61_{-0.12}^{+0.11}$ mag.

Equations (2) and (3), which were derived by SC93 and K98, respectively, show relationships between Fourier coefficients and the luminosity of RR1 variables. K98 based the zero point for his equation on absolute magnitudes derived from B-W analyses of field and cluster RR Lyrae variables. We have already taken into account the B-W analyses of two RR Lyrae variables in M5, but we cannot assume that the other stars for which B-W analyses have been carried out, and on which K98 based his zero point, have properties similar to the RR Lyrae in M5. Therefore, we will not derive an absolute magnitude from equation (3); we will consider only equation (2). The mean log $L/L_\odot$ for the 80 M5-like stars in our sample is 1.688. Assuming that the Sun’s $M_{bol} = 4.74$ (BG99), we derive a mean $(M_{bol}) = 0.52$ for these M5-like stars. From the BC-$(V-R)_0$ plot of BG99, we read BC = 0.02 at [Fe/H] $\sim$ $-1.5$ and $(V-R)_0 = 0.14$ (a typical color for an RR1 variable). Since the standard deviation of the fit of equation (2) to SC93’s models was $\sigma_{log L} = 0.035$, the mean $M_V$ derived from $\phi_{31} = 0.50 \pm 0.09$ mag.

Using a weighted average of these four values, our final adopted mean $M_V$ for M5-like RR Lyrae variables is 0.56 $\pm$ 0.06 mag.

### 5.4. The LMC Distance

The average $V_0(F)$ for the 80 M5-like stars is 18.91 $\pm$ 0.02, where the quoted error is the standard error of the mean. If the $(V_0)$ values for the individual fields are corrected for $D/D_0$, this is revised to 18.89 $\pm$ 0.02. We emphasize that making this correction assumes that the RR Lyraes lie in the main disk of the LMC. Fortunately, since our stars lie near the center of the LMC, the difference between the two values is small. Because of the systematic difference between $A_0(V_0(F))$ and $(V_0)_int$, we subtract 0.01 from $(V_0(F))$ and adopt a mean $V_0$ of 18.88 $\pm$ 0.02 for the 80 M5-like stars in our 16 fields. In addition, there is a systematic error of 0.12 mag due to the uncertainty in $E(V-R)$. We must also account for the crowding bias, which is known to exist in the MACHO data because of the artificial star tests.

As the final step in calculating the LMC distance, we estimate a correction for the apparent brightness of the RR1 stars using artificial star tests that were part of the MACHO microlensing detection efficiency calculation (Alcock et al. 2001). This calculation is fraught with difficulty because many of the steps in the process of selecting bona fide RR1 variables, for example, the removal of severely blended stars, cannot be modeled accurately. Therefore, our approach is to interpret the artificial star tests in two different ways, and thus derive 2 mag corrections for the RR1 stars. The difference

### Table 12

| ID (1) | $\langle V \rangle$ (2) | $N_{obs}$ (3) | ID (4) | $\langle V \rangle$ (5) | $N_{obs}$ (6) | $\Delta(V)$ (7) |
|-------|----------------|--------------|-------|----------------|--------------|---------------|
| 6.6689.563 | 19.22 | 443 | 2249 | 19.39 | 70 | 0.17$^a$ |
| 13.6689.305 | 19.24 | 393 | ... | ... | ... | 0.15$^b$ |
| 6.6812.1063 | 19.61 | 240 | 8837 | 19.58 | 64 | -0.03 |
| 6.7054.310 | 19.43 | 318 | 7864 | 19.48 | 67 | 0.05 |
| 13.7054.300 | 19.36 | 171 | ... | ... | ... | 0.12 |
| 13.5838.667 | 19.39 | 253 | 7648 | 19.40 | 69 | 0.01 |
| 13.5959.584 | 19.20 | 423 | 7783 | 19.30 | 70 | 0.10 |
| 13.6079.604 | 19.25 | 373 | 4749 | 19.32 | 63 | 0.07 |
| 13.6201.670 | 19.10 | 462 | 7490 | 19.18 | 70 | 0.08 |
| Mean | ... | ... | ... | ... | ... | 0.07 |

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$^a$ MACHO 6.6689.563 is the same star as 13.6689.305.

$^b$ C03 found excessive scatter in the light curve for their star No. 2249.

$^c$ MACHO 6.7054.310 is the same star as 13.7054.300.
between the two estimates obtained is taken to indicate the level of systematic error associated with the final adopted magnitude correction and hence also with our LMC distance result.

The MACHO artificial star tests were made in 10 regions of the LMC that span the range of different crowding conditions found in the MACHO database. The parameter that we adopt as a measure of the crowding is the number of objects detected per square arcminute, or what we call the “Density” in Table 8 (see also Tables 1 and 2 of Alcock et al. 2001). In the most crowded MACHO fields, this “Density” may have been underestimated because it is more difficult to detect the faint objects. The correction that we seek, \( \Delta V \), is the difference between the input and recovered magnitudes of the artificial stars. For our first method, we calculated the average \( \Delta V \) for stars in four (1 mag wide) bins centered at \( V = 17.5, 18.5, 19.5, \) and 20.5 mag. A high-order polynomial function was fitted to these binned data points in order to predict \( \Delta V \) in terms of recovered magnitude and field density for any value of these two parameters. Since the frequency-weighted mean density for the M5-like RR1 stars in our sample is 200 objects arcmin\(^{-2}\), we evaluated the function for a density of 200 objects arcmin\(^{-2}\) and derived \( \Delta V \) (recovered – input) = –0.21 mag at \( V = 19.3 \). However, because we used an (uncropped) straight average for \( \Delta V \) in each bin, heavily blended artificial stars may tend to inflate the correction. In fact, this correction is probably an upper limit for the RR1 stars in our sample because we have already removed the most severely blended stars, but these are given full weight when interpreting the artificial stars. For our second estimate, we constructed one bin of artificial stars with \( V = 19.1 \) to 19.5 mag and calculated the median \( \Delta V \) value at each of the 10 different field densities. A second-order polynomial that was forced to pass through zero in an empty field was fitted to these data.\(^{25}\) In this case, \( \Delta V = -0.11 \pm 0.10 \) mag at a density of 200 objects arcmin\(^{-2}\). Since the median value is not significantly affected by outlying points like those due to severe blends, we believe that this approach is more appropriate for our data. When this correction is applied, our \( V_0 \) is revised to 18.99 mag. In § 5.2 and Table 12, we showed that our mean \( V \) magnitudes for seven RR1 stars in fields 6 and 13 appear to be \(-0.07\) brighter than those of C03. When the crowding correction for these two fields is applied to the MACHO magnitudes for these stars, the mean \( V \) magnitude for the MACHO data is 0.03 mag fainter than the C03 values, well within the estimated errors.

There are two sources of systematic error in our estimate of \( V_0 \). The first is the error in our derived \((V-R)\) color (0.022 mag), which propagates to an error of 0.12 mag in \( V_0 \). The second is the error in our crowding correction (0.10 mag). Combining these in quadrature, we estimate that the systematic error in \( V_0 \) is 0.16 mag. Another factor that should be considered is the effect of crowding on the derived Fourier coefficients. To address this problem, SC93 performed simulations in which they added constant light to the observed magnitudes at all phases on the light curves. They found that, although \( A_0 \) brightened, \( \phi_{31} \) remained largely unaltered. We therefore conclude that crowding has not seriously affected our selection of the “M5-like” stars.

The final mean LMC distance modulus that we obtain for the 80 M5-like RR1 variables, based on \( M_V = 0.56 \) and \( V_0 = 18.99 \), is \( \mu_{LMC} = 18.43 \pm 0.06 \) (statistical) \( \pm 0.16 \) (systematic). Our analysis has illustrated that the two major impediments to the derivation of a precise LMC distance from RR Lyrae variables are the uncertainty in the extinction and the uncertainty in establishing the effects of light contamination due to crowding. Near-infrared photometry (see, for example, the recent study of the Reticulum cluster by Dall’Ora et al. 2003) is an effective way to deal with the first problem. To address the second problem, investigations like the SuperMACHO project, which is based on observations obtained with the 4 m telescope at Cerro Tololo, will provide images with better resolution. Preliminary results indicate that the median seeing of the CTIO images is a factor of 3 smaller than the FWHM of the images that were obtained with the Mount Stromlo 50 inch (1.3 m) telescope for the MACHO project.

6. SUMMARY

We have determined Fourier coefficients for 785 stars deemed to be first-overtone RR Lyrae (RR1) variables, according to their periods, magnitudes, and colors. We established that 330 of these stars are bona fide RR1 stars. By using a \( \phi_{31} - \log P \) plot, we compared the LMC stars with the RR1 variables in some well-studied GGCs and found that they have properties similar to the ones in M2, M3, M5, and the OoII variables in \( \omega \) Centauri. However, they are different from the OoI variables in \( \omega \) Cen. In addition, there are a significant number that do not have counterparts in the well-studied GGCs.

There are several problems that must be addressed in deriving the LMC distance from RR Lyrae variables. Perhaps the most important is the correction for differential extinction. We have shown that there is a large range in the color excess from field to field [0.03 < \( E(V-R) < 0.13 \)], in good agreement with the results of other studies. We also found a significant variation among the stars within individual fields. These variations indicate that it is advantageous to correct for the extinction on a star-by-star basis. We did this by computing the temperature of each star from \( \phi_{31} \) and \( \log P \) and then using the temperature to derive the unreddened color.

Another consideration is that the star fields in the LMC, particularly those in the region of the bar, are extremely crowded. By comparing the \( V \) and \( R \) pulsation amplitudes, we removed the most severely blended stars from our sample. Further, a statistical correction based on artificial star tests was made to rectify the photometric errors caused by more moderate blending.

Finally, it is important to select a homogenous group of stars for which the absolute magnitude is well-determined. To do this, we identified the M5-like RR1 stars in our sample and applied absolute magnitudes determined from four independent methods to derive a distance modulus of \( 18.43 \pm 0.06 \) (statistical) \( 0.16 \) (systematic) mag. This method has the advantage that the result does not depend on the still somewhat uncertain \( M_V -[Fe/H] \) relation for RR Lyrae variables.

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\(^{25}\) For \( V = 19.1-19.5 \) mag, \( \Delta V = (-6.06 \times 10^{-6})O - (2.34 \times 10^{-6})O^2 \), where \( O \) is objects arcmin\(^{-2}\).
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