Bosonic String and String Field Theory: a solution using the holomorphic representation *

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Abstract

In this paper we show that the holomorphic representation is appropriate for description in a consistent way string and string field

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theories, when the considered number of component fields of the string field is finite. A new Lagrangian for the closed string is obtained and shown to be equivalent to Nambu-Goto’s Lagrangian. We give the notion of anti-string, evaluate the propagator for the string field, and calculate the convolution of two of them.

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1 Introduction

In a series of papers [1, 2, 3, 4, 5] we have shown that Ultradistribution theory of Sebastiao e Silva [6, 7, 8] permits a significant advance in the treatment of quantum field theory. In particular, with the use of the convolution of Ultradistributions we have shown that it is possible to define a general product of distributions (a product in a ring with divisors of zero) that sheds new light on the question of the divergences in Quantum Field Theory. Furthermore, Ultradistributions of Exponential Type (UET) are adequate to describe Gamow States and exponentially increasing fields in Quantum Field Theory [9, 10, 11].

In four recent papers ([12, 13, 14, 15]) we have demonstrated that Ultradistributions of Exponential type provide an adequate framework for a consistent treatment of string and string field theories. In particular, a general state of the closed string is represented by UET of compact support, and as a consequence the string field is a linear combination of UET of compact support (CUET). Thus a sting field theory result be a superposition of infinitely many of fields. The corresponding development is convergent due to that the superposition of infinitely many of complex Dirac’s deltas and its derivatives is convergent.
However for experimental purposes is suitable consider the string field theory as a superposition of a finite but sufficient great number \( n \) of fields.

The resultant theory can be described in a simplified way with the use of the holomorphic representation \([20]\). In this case we can not consider a superposition of infinitely many of fields due to that we can not assure the convergence of the corresponding development in powers of the variable \( z \).

In this paper we show that holomorphic representation provides an adequate method for a consistent simplified treatment of closed bosonic string. In particular, a general state of the closed bosonic string is represented by a polynomial function of a given number of complex variables.

This paper is organized as follows: In section 2 we give a new Lagrangian for bosonic string and solve the corresponding Euler-Lagrange’s equations for closed bosonic string. In section 3 we give a new representation for the states of the string using the holomorphic representation. In section 4 we give expressions for the field of the string, the string field propagator and the creation and annihilation operators of a string and a anti-string. In section 5, we give expressions for the non-local action of a free string and a non-local interaction lagrangian for the string field similar to \( \lambda \phi^4 \) in Quantum Field Theory. Also we show how to evaluate the convolution of two string field
propagators. In section 6 we realize a discussion of the principal results.

2 The Constraints for a Bosonic String

As is known the Nambu-Goto Lagrangian for the bosonic string is given by

\[ \mathcal{L}_{NG} = T \sqrt{ (\dot{X} \cdot X')^2 - \dot{X}^2 X'^2 } \]  

(2.1)

where

\[
\begin{align*}
X_\mu &= X_\mu(\tau, \sigma) ; \quad \dot{X}_\mu = \partial_\tau X_\mu ; \quad X'_\mu = \partial_\sigma X_\mu \\
X_\mu(\tau, 0) &= X_\mu(\tau, \pi) \\
-\infty &< \tau < \infty ; \quad 0 \leq \sigma \leq \pi
\end{align*}
\]

(2.2)

If we use the constraint

\[ (\dot{X} - X')^2 = 0 \]  

(2.3)

we obtain:

\[ \dot{X}^4 + X'^4 = 4(\dot{X} \cdot X')^2 - 2\dot{X}^2 X'^2 \geq 0 \]  

(2.4)

On the other hand

\[ (\dot{X}^2 - X'^2)^2 = \dot{X}^4 + X'^4 - 2\dot{X}^2 X'^2 \]  

(2.5)

and from (2.4) we have

\[ 4\mathcal{L}_{BS}^2 = T^2 (\dot{X}^2 - X'^2)^2 = 4T^2 [(\dot{X} \cdot X')^2 - \dot{X}^2 X'^2] = 4\mathcal{L}_{NG}^2 \geq 0 \]  

(2.6)
As a consequence of (2.6):

\[ \mathcal{L}_{\text{NG}} = T \sqrt{(\dot{X} \cdot X')^2 - \dot{X}^2 X'^2} = \frac{T}{2} |\dot{X}^2 - X'^2| = \mathcal{L}_{\text{BS}} \quad (2.7) \]

We then see that is sufficient to use only one constraint to obtain the Lagrangian for a bosonic string theory from the Nambú-Goto Lagrangian. Another constraint from which (2.6) follows is

\[ (\dot{X} + X')^2 = 0 \quad (2.8) \]

Thus, the problem for the bosonic string reduces to:

\[ \begin{cases} 
\mathcal{L} = \frac{T}{2} |\dot{X}^2 - X'^2| \\
(\dot{X} + X')^2 = 0 \\
X_\mu(\tau, 0) = X_\mu(\tau, \pi) 
\end{cases} \quad (2.9) \]

or

\[ \begin{cases} 
\mathcal{L} = \frac{T}{2} |\dot{X}^2 - X'^2| \\
(\dot{X} - X')^2 = 0 \\
X_\mu(\tau, 0) = X_\mu(\tau, \pi) 
\end{cases} \quad (2.10) \]

The Euler-Lagrange equations for (2.9) and (2.10) are respectively:

\[ 4\delta(\dot{X}^2 - X'^2)[(\dot{X} \cdot \ddot{X} - X' \cdot \ddot{X}')X_\mu - (X' \cdot \ddot{X}' - X' \cdot X'')X'_\mu] + \\
\text{Sgn}(\dot{X}^2 - X'^2)(\ddot{X} - X'') + \lambda(\ddot{X} + 2\dddot{X}' + X'') = 0 \quad (2.11) \]
\[ 4\delta(\dot{X}^2 - X'^2)[(\dot{X} \cdot \ddot{X} - X' \cdot \dot{X}')\dot{X}_\mu - (X' \cdot \ddot{X}' - X' \cdot X'\ '')X'_\mu] + \]
\[ \text{Sgn}(\dot{X}^2 - X'^2)(\dddot{X} - X'') + \lambda(\dddot{X} - 2\dot{X}' + X'') = 0 \quad (2.12) \]

where \( \lambda \) is a Lagrange multiplier.

Let \( X_\mu \) be given by:
\[ X_\mu = \text{Sgn}(\dot{Y}^2 - Y'^2)Y_\mu \quad (2.13) \]

where
\[
\begin{cases}
Y_\mu(\tau, \sigma) = y_\mu + l^2 p_\mu \tau + \frac{i}{2} \sum_{n=-\infty \; ; \; n \neq 0}^{\infty} \frac{a_n}{n} e^{-2i(n-\sigma)} \\
p^2 = 0
\end{cases}
(2.14)
\]
or
\[
\begin{cases}
Y_\mu(\tau, \sigma) = y_\mu + l^2 p_\mu \tau + \frac{i}{2} \sum_{n=-\infty \; ; \; n \neq 0}^{\infty} \frac{\tilde{a}_n}{n} e^{-2i(n+\sigma)} \\
p^2 = 0
\end{cases}
(2.15)
\]

(2.14) satisfy
\[ \dot{Y}_\mu + Y'_\mu = p_\mu \quad (2.16) \]

and (2.15)
\[ \dot{Y}_\mu - Y'_\mu = p_\mu \quad (2.17) \]

For both we have:
\[ \dot{X}^2 - X'^2 = \dot{Y}^2 - Y'^2 \neq 0 \quad (2.18) \]
and then

\[(\dot{X}^2 - X'')^2 = (\dot{Y}^2 - Y'')^2 \neq 0 \quad (2.19)\]

((2.13), (2.14)) and ((2.13), (2.15)) are solutions of (2.11) and (2.12) respectively. To prove this we take into account that for ((2.13), (2.14)) we have

\[\ddot{X}_\mu = -\dot{X}' = X'' \quad (2.20)\]

and for ((2.13), (2.15))

\[\ddot{X}_\mu = \dot{X}' = X'' \quad (2.21)\]

To quantum level we have respectively for (2.14) and (2.15):

\[
\begin{aligned}
Y_\mu(\tau, \sigma) &= y_\mu + l^2 p_\mu \tau + \frac{i}{2} \sum_{n=-\infty ; n \neq 0}^{\infty} \frac{a_{n\mu}}{n} e^{-2i n (\tau - \sigma)} \\
p^2 |\phi > &= 0
\end{aligned} \quad (2.22)
\]

and

\[
\begin{aligned}
Y_\mu(\tau, \sigma) &= y_\mu + l^2 p_\mu \tau + \frac{i}{2} \sum_{n=-\infty ; n \neq 0}^{\infty} \frac{\tilde{a}_{n\mu}}{n} e^{-2i n (\tau + \sigma)} \\
p^2 |\phi > &= 0
\end{aligned} \quad (2.23)
\]

where |\Phi > is the physical state of string.

In terms of creation and annihilation operators we have for (2.22) and (2.23):

\[
\begin{aligned}
Y_\mu(\tau, \sigma) &= y_\mu + l^2 p_\mu \tau + \frac{i}{2} \sum_{n>0} b_{n\mu} \sqrt{n} e^{-2i n (\tau - \sigma)} - \frac{b_{n\mu}}{\sqrt{n}} e^{-2i n (\tau - \sigma)} \\
p^2 |\phi > &= 0
\end{aligned} \quad (2.24)
\]
\[
\begin{align*}
Y_\mu(\tau, \sigma) &= y_\mu + l^2 p_\mu \tau + \frac{il}{2} \sum_{n>0} \frac{\tilde{b}_n}{\sqrt{n}} e^{-2\text{Im}(\tau+\sigma)} - \frac{\tilde{b}_n}{\sqrt{n}} e^{-2\text{Im}(\tau+\sigma)} \\
p^2|\phi| &\geq 0
\end{align*}
\] (2.25)

where:

\[
[b_{\mu m}, b^+_{\nu n}] = \eta_{\mu \nu} \delta_{mn} \tag{2.26}
\]

\[
[\tilde{b}_{\mu m}, \tilde{b}^+_{\nu n}] = \eta_{\mu \nu} \delta_{mn} \tag{2.27}
\]

A general state of the string can be written as:

\[
|\phi| = [a_0(p) + a_{i_1}^{i_1}(p) b^{+\mu_1} + a_{i_1 i_2}^{i_1 i_2}(p) b^{+\mu_1} b^{+\mu_2} + ... + ... + a_{\mu_1 \mu_2 ... \mu_n}^{i_1 i_2 ... i_n}(p) b^{+\mu_1} b^{+\mu_2} ... b^{+\mu_n} + ... + ...]0 >
\] (2.28)

or

\[
|\phi| = [a_0(p) + a_{i_1}^{i_1}(p) \tilde{b}^{+\mu_1} + a_{i_1 i_2}^{i_1 i_2}(p) \tilde{b}^{+\mu_1} \tilde{b}^{+\mu_2} + ... + ... + a_{\mu_1 \mu_2 ... \mu_n}^{i_1 i_2 ... i_n}(p) \tilde{b}^{+\mu_1} \tilde{b}^{+\mu_2} ... \tilde{b}^{+\mu_n} + ... + ...]0 >
\] (2.29)

where:

\[
p^2 a_{\mu_1 \mu_2 ... \mu_n}^{i_1 i_2 ... i_n}(p) = 0 \tag{2.30}
\]

It is immediate to prove that ((2.13), (2.14)) and ((2.13), (2.15)) are solutions of Nambu-Goto equations on physical states. (Nambu-Goto equations arise from Euler-Lagrange equations corresponding to the Lagrangian (2.1), and it is easy to prove that the currently used solution for the closed string...
movement is not solution of Nambu-Goto equations due to the fact that Virasoro operators $L_n$ and $\tilde{L}_n$ does not annihilate the physical states for $n < 0$ and moreover, does not form a set of commuting operators).

3 A representation of the states of the closed string

Let $A(\mathbb{R}^2)$ be the complex Euclidean space defined as (see ref. [19] for a definition of complex Euclidean space)

$$A(\mathbb{R}^2) = \left\{ f(z)/f(z) \text{ is analytic } \wedge \frac{i}{2} \int f(z)\overline{f(z)}e^{-z\bar{z}}dz \, d\bar{z} < \infty \right\} \quad (3.1)$$

supplied with the complex scalar product (see ref. [20]):

$$< f(z), g(z) > = \frac{i}{2} \int f(z)\overline{g(z)}e^{-z\bar{z}}dz \, d\bar{z} = \quad (3.2)$$

Taking into account that:

$$\frac{i}{2} dz \wedge d\bar{z} = \frac{i}{2} (dx + idy) \wedge (dx - idy) =$$

$$\frac{1}{2} dx \wedge dy - dy \wedge dx = dx \wedge dy$$

(3.3)
where $\wedge$ denotes the outer product of two 1-forms, we have for the scalar product (3.2)

$$< f(z), g(z) > = \int_{-\infty}^{\infty} f(z)g(z)e^{-\pi z^2} \, dx \, dy = \int_{-\infty}^{\infty} \int_{0}^{2\pi} f(z)g(z)e^{-\rho^2} \, d\theta \, d\rho \quad (3.4)$$

Let $Z(\mathbb{R}^2)$ be the complex Euclidean space defined as:

$$Z(\mathbb{R}^2) = \left\{ f(z)/z^p \frac{d^q f(z)}{dz^q} \in \Lambda(\mathbb{R}^2) \right\} \quad (3.5)$$

supplied with the scalar product (3.2), where $p$ and $q$ are natural numbers.

For $f \in Z(\mathbb{R}^2)$ we have:

$$< \frac{df(z)}{dz}, g(z) > = \frac{i}{2} \int \frac{df(z)}{dz} \overline{g(z)} e^{-\pi z^2} \, dz \, d\overline{z} =$$

$$\frac{i}{2} \int f(z)zg(z) e^{-\pi z^2} \, dz \, d\overline{z} = < f(z), zg(z) > \quad (3.6)$$

If we define:

$$a = \frac{d}{dz} ; \quad a^+ = z \quad (3.7)$$

we obtain:

$$[a, a^+] = 1 \quad (3.8)$$

Representation (3.7) is called the holomorphic representation (see ref. [20]) for annihilation and creation operators.
The vacuum state annihilated by $\frac{d}{dz}$ is the number $1/\sqrt{\pi}$ and the orthonormalized states obtained by successive application $z$ to $1/\sqrt{\pi}$ are:

$$F_n(z) = \frac{z^n}{\sqrt{\pi} \ n!} \quad (3.9)$$

Using this representation a general state of the string can be written as:

$$\phi(x, \{z\}) = a_0(x) + a_{\mu_1}^i(x)z_{t_1}^{\mu_1} + a_{\mu_1 \mu_2}^{i_1 i_2}(x)z_{t_1}^{\mu_1}z_{t_2}^{\mu_2} + \ldots + a_{\mu_1 \mu_2 \ldots \mu_n}^{i_1 i_2 \ldots i_n}(x)z_{t_1}^{\mu_1}z_{t_2}^{\mu_2} \ldots z_{t_n}^{\mu_n} \quad (3.10)$$

where $\{z\}$ denotes $(z_{1\mu}, z_{2\mu}, \ldots, z_{n\mu})$.

The functions $a_{\mu_1 \mu_2 \ldots \mu_n}^{i_1 i_2 \ldots i_n}(x)$ are solutions of

$$\Box a_{\mu_1 \mu_2 \ldots \mu_n}^{i_1 i_2 \ldots i_n}(x) = 0 \quad (3.11)$$

4 The String Field

According to (2.25), (2.25) and section 3 the equation for the string field is given by:

$$\Box \phi(x, \{z\}) = (\partial_0^2 - \partial_1^2 - \partial_2^2 - \partial_3^2)\phi(x, \{z\}) = 0 \quad (4.1)$$

where $\{z\}$ denotes $(z_{1\mu}, z_{2\mu}, \ldots, z_{n\mu}, \ldots, \ldots)$, and $\phi$ is a analytic function in the set of variables $\{z\}$. Thus we have:

$$\phi(x, \{z\}) = [A_0(x) + A_{\mu_1}^i(x)z_{t_1}^{\mu_1} + A_{\mu_1 \mu_2}^{i_1 i_2}(x)z_{t_1}^{\mu_1}z_{t_2}^{\mu_2} + \ldots + \ldots$$
where the quantum fields $A_{\mu_1 \mu_2 \ldots \mu_n}^{i_1 i_2 \ldots i_n}(x)$ are solutions of

$$\square A_{\mu_1 \mu_2 \ldots \mu_n}^{i_1 i_2 \ldots i_n}(x) = 0$$  \hspace{1cm} (4.3)$$

The propagator of the string field can be expressed in terms of the propagators of the component fields:

$$\Delta(x - x', \{z\}, \{\bar{z}'\}) = \Delta_0(x - x') + \Delta_{\mu_1 \mu_2}^{i_1 j_1}(x - x') z_{i_1}^{\mu_1} \bar{z}_{j_1}^{\nu_1} + \ldots + \ldots +$$

$$\Delta_{\mu_1 \ldots \mu_n \nu_1 \ldots \nu_n}^{i_1 \ldots i_n j_1 \ldots j_n}(x - x') z_{i_1}^{\mu_1} \ldots z_{i_n}^{\mu_n} \bar{z}_{j_1}^{\nu_1} \ldots \bar{z}_{j_n}^{\nu_n}$$  \hspace{1cm} (4.4)$$

For the fields $A_{\mu_1 \mu_2 \ldots \mu_n}^{i_1 i_2 \ldots i_n}(x)$ we have:

$$A_{\mu_1 \mu_2 \ldots \mu_n}^{i_1 i_2 \ldots i_n}(x) = \int_{-\infty}^{\infty} a_{\mu_1 \mu_2 \ldots \mu_n}^{i_1 i_2 \ldots i_n}(k) e^{-ik_{\mu} x^\mu} + b_{\mu_1 \mu_2 \ldots \mu_n}^{i_1 i_2 \ldots i_n}(k) e^{ik_{\mu} x^\mu} \, d^3k$$  \hspace{1cm} (4.5)$$

We define the operators of annihilation and creation of a string as:

$$a(k, \{z\}) = a_0(k) + a_{\mu_1}^{i_1}(k) z_{i_1}^{\mu_1} + \ldots + \ldots +$$

$$a_{\mu_1 \ldots \mu_n}^{i_1 \ldots i_n}(k) z_{i_1}^{\mu_1} \ldots z_{i_n}^{\mu_n}$$  \hspace{1cm} (4.6)$$

$$a^+(k', \{\bar{z}'\}) = a_0^+(k') + a_{\nu_1}^{i_1}(k') \bar{z}_{j_1}^{\nu_1} + \ldots + \ldots +$$

$$a_{\nu_1 \ldots \nu_n}^{i_1 \ldots i_n}(k') \bar{z}_{j_1}^{\nu_1} \ldots \bar{z}_{j_n}^{\nu_n}$$  \hspace{1cm} (4.7)$$

and the annihilation and creation operators for the anti-string

$$b(k, \{\bar{z}\}) = b_0(k) + b_{\mu_1}^{i_1}(k) \bar{z}_{i_1}^{\mu_1} + \ldots + \ldots +$$
\[ b_{\mu_1 \ldots \mu_n}^{i_1 \ldots i_n} (k) \bar{z}_{i_1}^{\mu_1} \ldots \bar{z}_{i_n}^{\mu_n} \quad (4.8) \]

\[ b^+ (k', \{ z' \} ) = b_0^+ (k') + b_{\nu_1}^{+ j_1} (k') z_{j_1}^{\nu_1} + \ldots + \ldots + \]

\[ b_{\nu_1 \ldots \nu_n}^{+ j_1 \ldots j_n} (k') z_{j_1}^{\nu_1} \ldots z_{j_n}^{\nu_n} \quad (4.9) \]

If we define

\[ [a_{\mu_1 \ldots \mu_n}^{i_1 \ldots i_n} (k), a_{\nu_1 \ldots \nu_n}^{+ j_1 \ldots j_n} (k')] = f_{\mu_1 \ldots \mu_n}^{i_1 \ldots i_n} j_1 \ldots j_n (k) \delta (k - k') \quad (4.10) \]

the commutations relations are

\[ [a(k, \{ z \}), a^+(k', \{ z' \})] = [f_0 (k) + f_{\mu_1 \nu_1}^{i_1 \ldots i_n} j_1 \ldots j_n (k) z_{i_1}^{\mu_1} \bar{z}_{j_1}^{\nu_1} + \ldots + \ldots + \]

\[ f_{\mu_1 \ldots \mu_n \nu_1 \ldots \nu_n}^{i_1 \ldots i_n} j_1 \ldots j_n (k) z_{i_1}^{\mu_1} \ldots z_{i_n}^{\mu_n} \bar{z}_{j_1}^{\nu_1} \ldots \bar{z}_{j_n}^{\nu_n} ] \delta (k - k') \quad (4.11) \]

and for the anti-string:

\[ [b_{\mu_1 \ldots \mu_n}^{i_1 \ldots i_n} (k), b_{\nu_1 \ldots \nu_n}^{+ j_1 \ldots j_n} (k')] = g_{\mu_1 \ldots \mu_n \nu_1 \ldots \nu_n}^{i_1 \ldots i_n} j_1 \ldots j_n (k) \delta (k - k') \quad (4.12) \]

the commutations relations are

\[ [b(k, \{ \bar{z} \}), b^+(k', \{ \bar{z}' \})] = [g_0 (k) + g_{\mu_1 \nu_1}^{i_1 \ldots i_n} j_1 \ldots j_n (k) \bar{z}_{i_1}^{\mu_1} z_{j_1}^{\nu_1} + \ldots + \ldots + \]

\[ g_{\mu_1 \ldots \mu_n \nu_1 \ldots \nu_n}^{i_1 \ldots i_n} j_1 \ldots j_n (k) \bar{z}_{i_1}^{\mu_1} \ldots \bar{z}_{i_n}^{\mu_n} z_{j_1}^{\nu_1} \ldots z_{j_n}^{\nu_n} ] \delta (k - k') \quad (4.13) \]

With this anihilation and creation operators we can write:

\[ \Phi (x, \{ z \}) = \int_{-\infty}^{\infty} a(k, \{ z \}) e^{-ik_\mu x^\mu} + b^+(k(z)) e^{ik_\mu x^\mu} \, d^3 k \quad (4.14) \]
5 The Action for the String Field

The action for the free bosonic bradyonic closed string field is:

\[ S_{\text{free}} = \frac{i^n}{2^n} \int_{-\infty}^{\infty} \partial_{\mu} \Phi(x, \{z\}) e^{-\{z\} \cdot \partial_{\mu} \Phi^+(x, \{z\})} \, d^3x \, \{dz\} \, \{d\bar{z}\} \]  

(5.1)

A possible interaction is given by:

\[ S_{\text{int}} = \lambda \frac{i^n}{2^n} \int_{-\infty}^{\infty} \Phi(x, \{z\}) e^{-\{z\} \cdot \Phi^+(x, \{z\})} e^{-\{\bar{z}\} \cdot \Phi(x, \{z\})} \times \]

\[ e^{-\{z\} \cdot \Phi^+(x, \{z\})} \, d^3x \, \{dz\} \, \{d\bar{z}\} \]  

(5.2)

Both, \( S_{\text{free}} \) and \( S_{\text{int}} \) are non-local as expected.

The convolution of two propagators of the string field is:

\[ \check{\Delta}(k, \{z_1\}, \{z_2\}) \ast \check{\Delta}(k, \{z_3\}, \{z_4\}) \]  

(5.3)

where \( \ast \) denotes the convolution of Ultradistributions of Exponential Type on the \( k \) variable only. With the use of the result

\[ \frac{1}{\rho} \ast \frac{1}{\rho} = -\pi^2 \ln \rho \]  

(5.4)

(\( \rho = x_0^2 + x_1^2 + x_2^2 + x_3^2 \) in euclidean space)

and

\[ \frac{1}{\rho \pm i0} \ast \frac{1}{\rho \pm i0} = \mp i\pi^2 \ln(\rho \pm i0) \]  

(5.5)

(\( \rho = x_0^2 - x_1^2 - x_2^2 - x_3^2 \) in minkowskian space)

the convolution of two string field propagators is finite.
6 Discussion

We have shown that holomorphic representation is appropriate for the description in a consistent way string and string field theories. By means of a new Lagrangian for the closed string strictly equivalent to Nambu-Goto Lagrangian we have obtained a movement equation for the field of the string and solve it. We shown that this string field is a polynomial in the variables $z$. We evaluate the propagator for the string field, and calculate the convolution of two of them, taking into account that string field theory is a non-local theory. For practical calculations and experimental results we have given expressions that involve only a finite number of variables.

As a final remark we would like to point out that our formulas for convolutions follow from general definitions. They are not regularized expressions
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