Risk Assessment of Multi-timescale Cascading Outages based on Markovian Tree Search

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Abstract— In the risk assessment of cascading outages, the rationality of simulation and efficiency of computation are both of great significance. To overcome the drawback of sampling-based methods that huge computation resources are required and the shortcoming of conventional enumeration-based practices that the correlations in sequences of outages are omitted, this paper proposes a novel risk assessment approach by searching on Markovian Tree (MT). The MT model is reformulated from the quasi-dynamic multi-timescale simulation model proposed recently to ensure reasonable modeling and simulation of cascading outages. Then a tree search scheme is established to avoid duplicated simulations on cascade paths, which saves computation time. To accelerate convergence of risk assessment, a risk estimation index (REI) is proposed to guide the search for states with major contributions to the risk, and the risk assessment is realized based on REI with a forward-search-backward-update algorithm. The effectiveness of the proposed method is illustrated on a 4-node power system, and its efficiency and potentials for online application are demonstrated on the RTS-96 test system.

Index Terms— Cascading outages, risk assessment, Markovian Tree, tree search

I. INTRODUCTION

Risk assessment of cascading outages in power systems is an important topic since cascading outages are big threats to electricity supply as well as to the society[1-3]. One of the most challenging problems of risk assessment is the limitation of calculation speed brought by the huge number of possible cascade paths and the lack of computation resources. One commonly-utilized approach of risk assessment is based on sampling, i.e. Monte-Carlo method, which repeatedly creates samples of events based on their real probabilities until risk index converges. However, Monte-Carlo method requires large numbers of samples, especially for rare events [4]. Since the convergence of sampling-based methods relies on the variance of sampling, several variance reduction methods are proposed, e.g. importance sampling [5], cross-entropy [6], stratification [7], etc. These methods can accelerate computation by several times compared to Monte-Carlo method. However the improvements are limited and risk assessment still requires huge computation.

Enumeration or selection of cascading outage paths is another approach for risk assessment. Similarly, various contingency or state combination selection techniques have been proposed [8-10]. However these methods treat outages as independent of each other, which neglects the fundamental “cascading” nature of cascading outages. Reasonable risk assessment requires updating the probability of element outages during simulation, which makes the techniques utilized in state combination selection methods ineffective.

Correctly capturing the correlation within sequences of outages is a prerequisite of reasonable and practical risk assessment, so the simulation of cascading outages is inevitable in risk assessment and cannot be simplified. An effective way of efficiency enhancement is to reduce simulation invocation. In sampling-based methods, lots of time is wasted in duplicated sampling of same cascade paths. Actually upon knowing the conditional probabilities on each level of cascading outages, efficiency of risk assessment can be significantly improved by enumerating outage paths and calculate risk indices since enumeration can avoid duplicated simulation of same paths. All the possible paths can be formulated as a tree-like structure, and as cascading outages can be regarded as Markovian processes [12][27], the risk assessment is processed as enumeration on a Markovian Tree (MT)[11].

Ref. [12] proposes a Markovian Tree model of cascading line outages. The model considers the effect of random load fluctuation and incorporates time into simulation, which enhances practicality. However, the mechanisms of line outage directly caused by triggering protection and the slow outage process [13] caused by tree contact are not distinguished. And processes as slow change of load level and generation re-dispatch are not considered.

Risk assessment should be based on reasonable modeling and simulation. Various models have been proposed, including topology-based models [14], high-level statistical models [15], power-flow based models [16-18], models treating cascading outages in different stages [19], and models mainly considering severe initial outage combinations [20], etc. To reflect the essence of cascading outages as a multi-timescale complex process which is essential for reasonable simulation and risk assessment, our previous work [21] proposes a multi-timescale quasi-dynamic (QD) simulation model. In the model, the events
in cascading outages are categorized into three timescales, and the interactions among timescales as well as representation of time elapse are realized within the quasi-dynamic framework. Based on the model in [21], this paper proposes a risk assessment method of multi-timescale cascading outages based on Markovian tree search. The multi-timescale modeling and quasi-dynamic simulation of cascading outage paths are reformulated as a Markovian tree, and then the risk assessment is realized by searching on MT. Since duplicated simulation of same paths is avoided, the efficiency is expected to be significantly enhanced. According to the analytical expression of the risk index, a searching strategy based on a risk estimation index (REI) is proposed to speed up convergence of risk index, which further accelerates risk assessment.

The rest of this paper is organized as follows. Section II reformulates the quasi-dynamic multi-timescale simulation as a Markovian Tree. Section III proposes the idea of risk assessment with Markovian Tree search. Section IV proposes a risk estimation index (REI) for guiding of the search. And then a forward-search-backward-update scheme of risk assessment based on REI is established. Section V presents test case studies of the risk assessment method on a 4-node system and RTS-96 test system. Section VI draws the conclusion.

II. CASCADING OUTAGE SIMULATION WITH MARKOVIAN TREE

A. Brief Retrospect of Multi-Timescale Simulation Model

To represent the time elapse in simulation, ref. [21] categorizes events in cascading outages by timescales, as shown in Fig. 1. Then a quasi-dynamic (QD) simulation framework is utilized, which inserts the simulation of shorter timescales between adjacent longer-timescale events (Fig. 2). Thus multi-timescale simulation is realized and approximate time elapse is provided. Reasonable modeling and simulation of cascading outages is a prerequisite of practical risk assessment. So next, we show how to reformulate the quasi-dynamic multi-timescale model as Markovian tree (MT) and then propose the mechanism of risk assessment on MT.

If we merge same states of all the possible cascading outage sequences from the beginning state, a tree structure will be formulated, as shown in Fig. 3. Assume that cascading outages are Markovian [12] [22], the tree is then a Markovian Tree (MT). Each node on MT represents a state, and each branch on MT represents an MTRO. Similar to the QD model, every mid-term transition corresponds to a time elapse \( \tau_D \), and the reasonable values of \( \tau_D \) will be discussed afterwards with test cases.

The labelling of states on an MT is determined as follows. The beginning state is level-0 node, and subsequent states are labelled sequentially as level-1 nodes, level-2 nodes, etc. Note that not every state transition corresponds to an outage, it is possible that no outage occurs during \( \tau_D \). If we label elements with positive integers, then a state is coded as the sequence of labels of failed elements in each level (no-outage is labelled as 0), e.g. \((i_k, i_k, \ldots i_k)\). Any state \((i_k, i_k, \ldots i_k)\) may correspond to costs (loss of load or economic loss, etc.) caused by re-dispatch or emergency control, denoted as \(C(i_k, i_k, \ldots i_k)\). The probability of any state depends on previous states, so the conditional probability of event \(i_{k_{out}}\) is represented with \(Pr(i_{k_{out}} | i_k, i_k, \ldots i_k)\). With these terms, take the risk assessment of expected load loss as example, the risk index is expressed as

\[
R = C_0 + \sum_{k_1} Pr(i_{k_1})C(i_{k_1}) + \sum_{k_1} \sum_{k_2} Pr(i_{k_1} | i_{k_2})C(i_{k_2}) + \sum_{k_1} \sum_{k_2} \sum_{k_3} Pr(i_{k_1} | i_{k_2} | i_{k_3})C(i_{k_3}) + \ldots
\]

Different from the QD model which is able to sample more than one outage in each interval, here similar to [12, 22, 23], each interval \(\tau_D\) allows at most one element outage. To ensure the equivalency between the QD model and this model, the mid-term interval \(\tau_D\) in the MT is set as \(1/N_q\) of that in the QD model, thus the MT model is equivalent to the case of sampling up to \(N_q\) outages during the same period. It is found through tests that generally \(N_q = 3 \sim 5\) can satisfy practical needs. Moreover, the MT model considers the sequence of outages, which is closer to system reality than sampling-based methods.

The requirement of single outages on the MT renders different definitions of probability. In the QD model, outage events are sampled independently, while on MT, the outage probability is defined as “the probability that the outage is the first to occur”. Assume that the occurrence of outages follow
Poisson process, where the outage rate of element $i$ is $\lambda_i$. Then in the QD model [21], the outages are sampled independently with probabilities in interval $\tau_D$:  

$$ \Pr_{r,i}^{MC} = 1 - e^{-\lambda_i \tau_D} \quad (2) $$

while on the MT, the outage probability of each element is  

$$ \Pr_{r,i}^{MT} = \frac{\lambda_i}{\sum_j \lambda_j} \left( 1 - e^{-\lambda_j \tau_D} \right) \quad (3) $$

In (3) the outage probability of element $i$ is not only dependent on $\lambda_i$, but also on outage rates of other elements.

2) Simulation of Re-dispatch

Re-dispatch is categorized as a mid-term process. When overload occurs, dispatchers adjust generators or dump loads to relieve the overload. In conventional models, re-dispatch is modeled as an optimization problem and the optimal solution is instantly applied as the new state. However, re-dispatch takes time [24]; when overloading occurs, the system needs time to acquire data, analyze system conditions and reflect the data to operators; the operators also need time to judge and make decisions before taking actions. Due to the generation ramping speed constraints, it also takes time from the beginning of actions till the fulfillment of re-dispatch objectives. Therefore, the re-dispatch is a process with time delay $\Delta_{\text{delay}}$ and ramping.

![Illustration of Re-dispatch Simulation](image)

As shown in Fig. 4, when an overloading event occurs at $t_0$, the re-dispatch action for the event is not immediately started. During interval $t_0 < t < t_0 + \Delta_{\text{delay}}$, the system may be taking actions dealing with previous events or there is no action at all. Considering the time-delay nature of re-dispatch, a queue of re-dispatch commands is prepared in simulation. As an event occurs, add the corresponding command to the queue and wait until the execution is due. The latest command meeting the beginning time is offered from the queue and starts executing. The command in action is kept until re-dispatch is finished or it is replaced by a new command.

3) Simulation of Short-term Processes

Ref. [21] mentions that short-term processes mainly refer to outages directly triggered by protection relays and actions of emergency control. These processes usually finish in several seconds and are much shorter than processes in other timescales, and these processes follow strict preset logics.

In cascading outage simulation, when system states change, check if short-term processes occur. If so then first simulate them. As illustrated in Fig. 5, event $i_{k+1}$ triggers short-term event $i_{k+1}$, and consequently triggers $i_{k+2}$, then afterwards the short-term process ends. Since this process is very short compared with the MT structure of mid-term processes, the short-term process can be modeled as an equivalent node.

![Fig. 5. Illustration of Short-term Processes in MT](image)

In the simulation of short-term processes, there may be load losses caused by island balancing when system separates or by emergency load shedding. It should be noted that since these losses are not caused by market-based measures, the unit cost of these losses are usually much higher than electricity market prices. Therefore when calculating the expected economic loss, the unit cost is $\mu_k$ (e.g. 100[25]) times of dispatch operations.

4) Simulation of long-term processes

A long-term process corresponds to the variation of load level. Since searching in depth on the MT also means elapse of time, the simulation of load variation can be realized by updating system loads according to the load curve.

III. Enhancing Efficiency Using Tree Search

Following the details proposed in Section II, simulation of cascading outages can be realized on the MT as equivalent to the model in [21], and risk assessment can be carried out by sampling on the MT. Yet sampling duplicates simulation of same cascade paths. To enhance efficiency, the simulated cascade paths can be recorded and avoided in further simulation. Thus risk assessment becomes searching on the MT.

The risk index (1) can be regarded as the sum of risk index terms of all the single states on the MT. Therefore the risk assessment based on the MT can be regarded as simulating new cascade paths on the MT and adding new terms to (1). Since the terms in (1) are non-negative, the risk index is expected to keep increasing until stopping at a value $R$, which is the cascading outage risk of the system.

Moreover, to accelerate the convergence of the risk index, a strategy of searching can be proposed which guides searching to the paths with major contribution to the risk index. Observing the risk index (1), since risk assessment is to add terms onto (1), the strategy should be first searching for states with larger risk terms. Next we will establish strategies that guide searching to such states.

IV. Markovian Tree Search Using REI

A. Risk Estimation Index (REI)

Take the partial MT in Fig. 6 to study the searching strategy. Assume that searching has reached “*$*$” state (labelled as $i_{k+1}$) and is about to select a subordinate state $i_k$ (hollow nodes pointed by solid line arrows) to simulate. The strategy should let the increment of risk index of the selected path be as large as possible, so the first task is to estimate the risks of all the subsequent states with acceptable computation complexity.
Here a risk estimation index (REI) $\rho_{\bar{b}_i}$ is established, and probabilities for searching are determined using $\rho_{\bar{b}_i}$. The REI consists of the following three parts:

1) Risk of System Separation

If the outage of a branch causes the grid to separate, then the branch is called a cut branch of the grid. According to [26], cut branches can be identified with complexity of $O(\mathcal{E})$, where $|\mathcal{E}|$ is the number of connected branches. Denote the admittance matrix of the grid as $Y$, then its Penrose-Moore pseudo-inverse uniquely exists, denoted as $Z$.

$$Z = Y^+$$  \hspace{1cm} (4)
A branch $i_k = \{u, v\}$ is a cut branch if and only if

$$Y_{uv}^{-1} - 2Z_{uv} + Z_{uu} + Z_{vv} = 0$$  \hspace{1cm} (5)

Considering numerical errors, set a sufficiently small threshold $\varepsilon$ (e.g. $10^{-10}$), if

$$\left| Y_{uv}^{-1} - 2Z_{uv} + Z_{uu} + Z_{vv} \right| < \varepsilon$$  \hspace{1cm} (6)
then the branch is identified as a cut branch. If $i_k = \{u, v\}$ is a cut branch and it fails, the separated two parts of the system will have unbalanced power $\pm F_{uv}$, which needs power balancing and generates cost. Therefore, the cost of system separation caused by cut branch $i_k$ outage is estimated as

$$\alpha_{\bar{b}_i} = \left\{ \begin{array}{ll} 2|F_{uv}| & Y_{uv}^{-1} - 2Z_{uv} + Z_{uu} + Z_{vv} < \varepsilon \\ 0 & Y_{uv}^{-1} - 2Z_{uv} + Z_{uu} + Z_{vv} \geq \varepsilon \end{array} \right.$$  \hspace{1cm} (7)

And the risk of system separation is estimated as

$$\rho_{\bar{b}_i} = \Pr(i_{k_1} \cdots i_{k_r}i_{\bar{b}_i})\alpha_{\bar{b}_i}$$  \hspace{1cm} (8)

2) Risk of Overloading

After the outage of non-cut branches, the power flow will re-distribute throughout the system and may cause overloading on other elements, leading to costs generated by re-dispatch or emergency control actions. The influence of a branch outage on other branches can be quantified by the power transfer distribution factor (PTDF). The PTDF of a non-cut branch $\{u, v\}$ to any other branch $\{p, q\}$ is

$$\delta_{pq} = \frac{Z_{up} + Z_{vq} - Z_{uv} - Z_{pq}}{1 + Y_{uv}(Z_{uu} + Z_{vv} - 2Z_{uv})} F_{pq}$$  \hspace{1cm} (9)

The power flow on $\{p, q\}$ after the outage of $\{u, v\}$ is

$$F_{pq}^{\text{new}} = F_{pq} + \delta_{pq} F_{uv}$$  \hspace{1cm} (10)

And the extent of overloading on branch $\{p, q\}$ is

$$\pi_{pq}^{*\text{new}} = \max\{F_{pq}^{\text{new}} - F_{pq}^{\text{old}}, 0\}$$  \hspace{1cm} (11)
Define the overloading index of branch $\{u, v\}$ outage as

$$\sigma_{i_k} = \sum_{\{p, q\} \in \mathcal{E}} \pi_{pq}^{*\text{new}}$$  \hspace{1cm} (12)
and define the estimation of overloading risk as

$$\rho_{i_k} = \Pr(i_{k_1} \cdots i_{k_r}i_{\bar{b}_i})\sigma_{i_k}$$  \hspace{1cm} (13)

Observing the denominator of (9), if $\{u, v\}$ is a cut branch, then the denominator is 0. So the PTDF of a cut branch has no definition. Therefore if (6) is satisfied, then $\rho_{i_k} = 0$.

3) Secondary Risk

Considering Fig. 6, when selecting the next-level states of “*” state, the risk of subsequent states of next-level states should also be accounted for. This risk is called a secondary risk in this paper. Since the secondary risk is hard to analytically quantify, a rough estimation is given in this paper.

First calculate the power flow $F_{pq}$ of all the connected lines $\{p, q\}$ after outage of branch $i_k = \{u, v\}$, and calculate the corresponding probabilities of outage $\Pr_{i_k}$ during the next interval using (3). According to the overloading extent of $\pi_{pq}$, give an estimation of the subsequent cost $C_{pq}$ (it is difficult to accurately analyze, so in this paper $C_{pq}$ is set as 1% of system load), then the secondary risk of $i_k = \{u, v\}$ is

$$\rho_{i_k} = \Pr(i_{k_1} \cdots i_{k_r}i_{\bar{b}_i}) \sum_{\{p, q\} \in \mathcal{E}} \pi_{pq}^{*\text{new}} \bigg| E_{i_{k_1} \cdots i_{k_r}} \bigg|$$  \hspace{1cm} (14)

where $E_{i_{k_1} \cdots i_{k_r}}$ is the set of connected branches at state $i_{k_1} \cdots i_{k_r}$ and $|E_{i_{k_1} \cdots i_{k_r}}|$ is the number of connected branches.

If a next-level state has no outage, i.e. $i_k = 0$, then system separation risk and overloading risk are both considered as 0, but the secondary risk may be non-zero. In this case, if approximately regarding the system state at $i_k = 0$ the same as that of $i_{k_1} \cdots i_{k_r}$, then the secondary events can be seen as shifting the next-level events of $i_{k_1} \cdots i_{k_r}$ to $i_k$. If the probability of $i_k = 0$ is $\Pr_{i_k}$, then the corresponding secondary risk can be defined as

$$\rho_{i_k}^\mu = \mu \Pr_{i_k} \sum_{i_k} \rho_{i_k}$$  \hspace{1cm} (15)

$\mu \leq 1$ is a discount factor considering that risk will be reduced by control schemes in the system.

4) Establishing REI

Till now, at arbitrary newly-searched state $i_{k_1} \cdots i_{k_r}$, the REI of next-level branch $i_k = \{u, v\}$ outage is

$$\rho_{i_k} = \alpha \rho_{i_k}^\alpha + \beta \rho_{i_k}^\beta + \gamma \rho_{i_k}^\gamma$$  \hspace{1cm} (16)

where $\alpha, \beta, \gamma$ are weights of risk terms. In this paper we select $\alpha = \beta = \gamma = 1$. The REI of $i_k = 0$ is

$$\rho_0 = \gamma \rho_0$$  \hspace{1cm} (17)

B. Forward-Backward Scheme of Tree Search Using REI

1) Forward Searching using REI

As shown in Fig. 6, if a new state (labeled with asterisk) is reached on the MT, then all the subsequent states and paths are unknown. The REIs of next-level states are calculated and
probabilities for selecting these states can be obtained using REIs. If REI is thought as accurate reflection of risks, then the optimal search strategy is to guide to the path with the highest REI, which is a deterministic strategy

$$\Pr_{i_k}^{calc} = \begin{cases} 1, & i_k = \arg \max \{\rho_{k_{i_{k-1}}\cdots q_{i_{1}}} \} \\ 0, & \text{otherwise} \end{cases}$$

(18)

Another strategy is random search with equal probability

$$\Pr_{i_k}^{calc} = 1/(|E_{q_{i_{k-1}}\cdots q_{i_{1}}}| + 1)$$

(19)

The searching strategies in (18) and (19) represent two extremes: deterministic search vs. pure random search. An actual searching strategy can be selected in between. Introduce a parameter $\lambda \geq 0$ and set probability

$$\Pr_{i_k}^{calc} = \frac{(\rho_x)^{\lambda}}{\sum_{\xi = E_{q_{i_{k-1}}\cdots q_{i_{1}}}} (\rho_{\xi})^{\lambda}}$$

(20)

For $\lambda = 0$, equation (20) is equivalent to (19), and for $\lambda \to +\infty$, equation (20) is approximately (18).

During the simulation of cascading outages, the matrices $Y$, $Z$ need to be updated. If a set of branches $\{i_k\}$ are removed from the grid, then the admittance matrix can be updated with

$$Y' = Y - M_{i_k} \text{Diag}(y_{i_k}) M_{i_k}^T$$

(21)

where $M_{i_k}$ is a $[L \times |\{i_k\}|]$ matrix. Each of its column $M_{i_k}$ corresponding to a branch $i_k \in \{u,v\}$ satisfies $M_{i_k,u} = 1$, $M_{i_k,v} = -1$ and all other entries are 0. $\text{Diag}(y_{i_k})$ is a matrix with branch admittances of $\{i_k\}$'s as its diagonal elements. The update of matrix $Y$ (21) can be finished with a very small amount of calculation, with complexity of $O(|\{i_k\}|)$.

The update of $Z$ is realized with

$$Z' = Z + Z M_{i_k} z_{i_k}^{-1} M_{i_k}^T Z$$

(22)

where

$$z_{i_k} = \text{Diag}(y_{i_k})^{-1} - M_{i_k}^T Z M_{i_k}$$

(23)

Since outages usually occur to very few branches at a time, $z_{i_k}$ is small and (22) has complexity of $O(|Y|^2)$. However the inverse of (23) requires that $\{i_k\}$ is not a cut set. If $\{i_k\}$ is a cut set, then the update of $Z$ has to be realized with SVD of $Y$, which has complexity of $O(|Y|^3)$ [26].

2) Backward Updating REI

After states on cascading outage paths are visited and recorded, reaching these states again in the future will not contribute to the increment of the risk index. Therefore after a cascading outage path is found, the REIs should also be updated. On the contrary to the searching direction from the root to terminals of the MT, the updating of REIs should go backwards from terminals to the root. As shown in Fig. 7, assume that solid nodes are visited states and the node labelled as 3 in the bottom is the terminal of the path. Since searching to a visited terminal again will not make any contribution to the risk, then for a terminal state $i_k$ on a path $(i_k, i_{k-1}, \ldots i_1)$, assign its REI a sufficiently small value $\rho_{i_k} = \varepsilon_k$ to avoid visiting it again.

For a non-terminal state $i_k$, since it has been visited and the risk term on the state $i_k$ itself will not contribute to the risk index again, then its REI will only reflect risks of its subsequent states. Since REIs of all its next-level states $\rho_{i_{k+1}, i_{k+2}, \cdots i_{k'}}$ must have been calculated or even updated, and the probabilities for searching are $\Pr_{i_{k+1}}^{calc}$ according to (20), then the REI of $i_k$ is

$$\rho_{i_k} = \sum_{i_k} \rho_{i_{k+1}} \rho_{i_{k+2}} \cdots \rho_{i_{k'}} \Pr_{i_{k+1}}^{calc}$$

(24)

Equation (24) represents recursive backward updating of REIs. In the risk assessment on MT, first do forward search along a path, and then reversely update REIs using (24).

V. CASE STUDIES

A. Illustrative 4-Node System

First verify the accuracy of the method in this paper with a small scale 4-node system. The system has 5 branches, 2 generator nodes and 2 load nodes, as shown in Fig. 8.

1) Verification of Risk Assessment Performance

Set outage of branch 2-3 as the initial failure, and use the MT method to assess the post-failure risk of the system. Set $T_{max} = 60$ min and $\tau_D = 15$ min, then the number of intervals is $K_D = \lceil T_{max} / \tau_D \rceil = 4$, where $\lceil \cdot \rceil$ is “ceil” operation. By analysis, if there are $N$ elements at the initial state, then the number of all possible cascade paths on MT is

$$N_T = \sum_{i=0}^{K} C_N^{K, K-i} A_{K-i}$$

(25)

With $N = 4$ and $K_D = 4$ we can get $N_T = 73$, it is not a big number and we can calculate through enumeration the theoretical value of the risk index as $R = 0.14208$ MW.

Fig. 9. Variation of Risk Index ($\lambda = 5$)
Then use the method proposed in this paper. Set $\lambda = 5$ and $\lambda = 0.01$ respectively, and search on the MT for up to 100 paths, the variation of the risk index as searching continues are shown in Figs. 9 and 10 separately. In both cases, the risk reaches theoretical value $R = 0.14208\text{MW}$, but the speeds of risk index convergence are distinct. The case with the larger $\lambda$ achieves a faster convergence profile. In the case with $\lambda = 5$, the convergence of the risk index only takes 35 search attempts, while the case of $\lambda = 0.01$ takes 87 attempts.

Fig. 11. Relationship between Required Attempts to Convergence and $\lambda$

Fig. 11 demonstrates the average search attempts to convergence under different values of $\lambda$. It is shown that with the increase of $\lambda$, the required number of search attempts decreases and finally stays at 35 times. To further assess the convergence of the risk index with $N_S$ search attempts, construct the following convergence metric

$$
\phi_{N_S} = \sum_{j=1}^{N_s} j \cdot (R_{N_S} - R_j)
$$

(26)

where $R_j$ is the risk index after $j$ th search attempt and $R_{N_S}$ is the converging risk index after $N_s$ search attempts. A smaller $\phi_{N_s}$ metric means a better convergence profile.

### Table I. Convergence Metric under Different Trust Factors

| $\lambda$ | Average Search Attempts | Average value of $\phi_{N_s}$ | Standard deviation of $\phi_{N_s}$ |
|-----------|-------------------------|-------------------------------|----------------------------------|
| 0.01      | 101.4                   | 145.98                        | 59.34                            |
| 0.05      | 62                      | 53.76                         | 16.80                            |
| 0.1       | 50.6                    | 29.33                         | 7.647                            |
| 0.5       | 37.6                    | 10.58                         | 2.261                            |
| 1         | 35                      | 8.144                         | 1.276                            |
| 5         | 35                      | 6.176                         | 0.315                            |
| 10        | 35                      | 5.994                         | 0.214                            |
| 500       | 35                      | 6.045                         | 0.235                            |
| 2000      | 35                      | 5.834                         | 0.145                            |
| 10000     | 35                      | 5.871                         | 0.256                            |

Table I and Fig. 12 demonstrate convergence metrics under different values of $\lambda$. It shows that a larger $\lambda$ gets a smaller $\phi_{N_s}$ so as to achieve a better convergence profile. Moreover, by rearranging the sequence of all cascade paths, a theoretically optimal sequence with minimum $\phi_{N_s}$ can be obtained for evaluating convergence of the risk assessment. In this case the theoretically minimum is $\phi_{N_s} = 3.01$. Compared with the $\phi_{N_s}$ under a nearly random search ($\lambda = 0.01$), the $\phi_{N_s}$ of search in which REI plays more important role under $\lambda \geq 1$ is much closer to $\phi_{N_s}$, and the deviation of $\phi_{N_s}$ is much smaller, which means more stable performance.

Fig. 12. Convergence metric under values of different $\lambda$

Analyzing the mechanism of MT search, a larger $\lambda$ tends to select cascading outage paths with higher REIs. The effectiveness of REI-guided search also verifies that the REIs can well reflect the distribution of actual risks. Since REI has a low computation complexity, the practicality of REI is verified.

However, the selection of $\lambda$ is not simply “the larger the better” since a too large $\lambda$ annihilates the randomness in path selection, and thus might miss some “hidden” risky states. Moreover, a too large $\lambda$ may cause overflow of floating point numbers in (20). Therefore in this case selecting $\lambda$ within 1~100 is suggested.

2) Influence of Mid-term Interval Length

The mid-term interval length $\tau_D$ is an important parameter in MT search. The determination of $\tau_D$ was discussed in [21], but since there are some difference between [21] and this paper in detail, the influence of $\tau_D$ should also be studied. Assign $\tau_D$ from 3min to 20min, and do risk assessment respectively.

Fig. 13. Influence of different $\tau_D$ on risk assessment results.

From Fig. 13, although the value of $\tau_D$ are quite different, the calculated risks are fairly close, showing insensitivity of results to the $\tau_D$, which is reasonable since $\tau_D$ is a simulation parameter. As $\tau_D$ increases, risk slightly decreases because a larger $\tau_D$ allows fewer outages in the same period of time, so that some cases of multiple outages cannot be covered and the calculated risk is lower. But from the results in Fig. 13, the contribution of successive multiple outages to risk is rather limited since the probability of such a case is usually small. So a larger $\tau_D$ can satisfy the requirement of accuracy and has the advantage of better efficiency. If accuracy is preferred, then a
smaller $\tau_D$ is desirable, while if efficiency is the priority, a larger $\tau_D$ is more suitable with satisfactory accuracy.

3) The Impact of Re-dispatch Delay

This risk assessment method can be utilized to study the impact of some system parameter on the cascading outage risk, and gives a clue on how to lower the risk.

![Fig. 14. The impact of re-dispatch delay on risk](image)

The operation delay is an important performance factor related to system security but is seldom studied in existing research on cascading outages. Here, we change $\Delta \tau_{\text{Delay}}$ to study the impact of delay on risk as shown in Fig. 14. It is shown that the risk rises as $\Delta \tau_{\text{Delay}}$ increases, and the impact of delay is more significant when $\Delta \tau_{\text{Delay}}$ is small. Because the re-dispatch is usually activated in 5-10 minutes[24], the risk can be more effectively reduced if delay is shortened. In reality the decrease in delay usually requires a control system upgrade, and the results in Fig. 14 can be helpful in the cost/effect analysis for system upgrade.

B. RTS-96 Test System

In this part the method is tested on a larger RTS-96 3-area system[27] having 73 buses, 120 branches, 33 generator nodes and 51 load nodes.

Select outages of branches 22, 23 and 24 as initial outages and assess risk. In simulation, set $\tau_D = 15 \text{ min} \, , \, T_{\text{max}} = 150 \text{ min} \, , \, \Delta \tau_{\text{Delay}} = 30 \text{ min} \, \cdot \, $ According to (25), the number of possible cascading paths is about $3.549\times 10^{30}$, so it is unrealistic to enumerate them all and calculate theoretical risk index. Here use a relatively large number of search attempts $N_S$ and regard the risk index at $N_S$ attempts $R_{N_S}$ as the theoretical risk index $R$. The test program is developed and tested in MATLAB on a workstation with a 2.6 GHz processor and 32GB RAM.

Set $N_S = 300000$ and get risk $R_{N_S} = 252.76\text{MW}$ through risk assessment. From Fig. 15, the risk index rises sharply at the beginning and approaches $R_{N_S}$ quickly in the first several thousands of search attempts, and then its rising speed becomes much slower. As Table II shows, after 19 search attempts the risk index has reached $0.5R_{N_S}$, and then after 2709 attempts the index reaches $0.9R_{N_S}$, with calculation time less than 10 min. But reaching $0.99R_{N_S}$ takes a much larger amount of computation, consuming several hours. From the perspective of application, it is of practical sense to apply risk assessment with limited computation time. In this case, no more than 5000 attempts and 1000 seconds of computation time can account for more than 90% of the cascading risk.

![Fig. 15. Risk index on RTS-96 test system](image)

![Fig. 16. Coverage of probability in risk assessment](image)

With a high-performance computer and software-level optimization, this method is expected to meet the need for online applications. Moreover, using risk assessment results, all the simulated cascading outage paths and risks can be analyzed and measures for lowering risk can be established[28].

VI. CONCLUSIONS

In this paper, a risk assessment method of multi-timescale cascading outages based on Markovian Tree (MT) search is proposed. The method first reformulates our previous work of quasi-dynamic multi-timescale simulation of cascading outages as an MT, thus the method maintains the advantage of reasonable modeling and simulation. Then the idea of risk assessment by searching on MT is proposed, which enhances efficiency by avoiding duplicated searches on cascade paths and effectively exploiting computation resources.

To accelerate the convergence of the risk index, this paper proposes a search strategy with a risk estimation index (REI) and a “forward search \(-\) backward REI update” scheme. The strategy is capable of guiding the search to paths with major contributions to the risk index, further enhancing efficiency of risk assessment.
The method is first tested on a 4-bus system to verify the accuracy and effectiveness of risk assessment by a comparison with theoretical results. The selection of search strategy is also tested and analyzed from the perspective of the balancing between the deterministic maximum-REI searching and random searching, and the effectiveness of REI is verified. The selection of the mid-term process interval length is also analyzed, which verifies the insensitivity of risk to interval length, and thus demonstrates the practicality of modeling, simulation and risk assessment proposed in this paper. The risk assessment method is also tested on the RTS-96 system, showing that the method is able to effectively search out most risky cascade paths and states accounting for more than 90% risk in less than 10 minutes, which has potential for online assessment and mitigation.

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