Intriguing microstructures of five-dimensional neutral Gauss-Bonnet AdS black hole

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In this paper, we analytically study the phase structure and construct the Ruppeiner geometry in the extended phase space for the five-dimensional neutral Gauss-Bonnet AdS black hole. Through calculating the scalar curvature of the Ruppeiner geometry and combining the phase transition, we show that the attractive interaction is dominant in the microstructure of the black hole system. More significantly, there is an intriguing property that the normalized scalar curvature has the same expression for the saturated small and large black hole curves. This implies that although the microstructure is different before and after the small-large black hole phase transition, the interaction between the microscopic constituents keeps unchanged. These results are quite valuable on further understanding the microstructure of the AdS black hole in modified gravity.

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I. INTRODUCTION

The study of black hole thermodynamics and phase transition has been one of the increasingly active areas among the last couple of decades. In particular, black hole chemistry attracts much more attention. The cosmological constant was treated as a thermodynamic pressure and its conjugate quantity as volume in the extended phase space [1]. Rich phase transitions and phase structures were observed, for examples, the small-large black hole phase transition, reentrant phase transition, triple point, λ-line phase transition [2–16], for a recent review, see Refs. [17, 18].

It is extensively shown that the small-large black hole phase transition is extremely similar to the liquid-gas phase transition of Van der Waals (VdW) fluids. As we know, VdW fluids consist of micromolecules with finite size and interaction between them. Therefore, a natural question is what is the microstructure of the black holes. In order to examine it, we proposed that an AdS black hole is also constructed by some unknown micromolecules [19]. Combining with the black hole phase transition, we studied the properties of the black hole microstructures. It was also shown that the interaction between these micromolecules can be tested by the Ruppeiner geometry. This meaningful approach has been generalized to other black holes in AdS space [20–31].

Recently, we further investigated this issue [32, 33]. When taking the temperature and volume as the fluctuation coordinates, a universal Ruppeiner metric was worked out and the corresponding scalar curvature was calculated. For a VdW fluid, the scalar curvature indicates that the attractive interactions dominate amongst the fluid microstructures. While for the charged AdS black hole, we found that the Ruppeiner geometry will become problematic due to the vanishing heat capacity. In order to cure this problem, we introduced a new quantity, the normalized scalar curvature, for the charged AdS black hole [32]. Employing this new scalar curvature, we observed that the repulsive interaction can exist for the small black hole of high temperature. This novel property shows that, even they share the similar phase transition and critical phenomena, there still exists a large difference between the microstructures of a black hole and VdW fluid. This property was also observed for the higher-dimensional charged AdS black holes [33].

Furthermore, we also examined the critical phenomena of the normalized scalar curvature [33]. At the critical point, the normalized scalar curvature $R_N$ goes to negative infinity, which indicates that the correlation length tends to infinity. It was also found that the normalized scalar curvature has a universal exponent 1/2. Another universal dimensionless parameter is $R_N(1 - \tilde{T})^2$, where $\tilde{T}$ is the reduced temperature. For the VdW fluid, numerical result shows that it is $-1/8$ near the critical point. For the four-dimensional charged AdS black hole, the analytical result confirms this constant. While for higher-dimensional charged AdS black holes, the values are more negative than $-1/8$. These results cast new light on understanding the black hole microstructures.

Since the analytical results can give us the exact information of the black hole microstructures, we would like to

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further study them. As we know, the coexistence curve of the five-dimensional neutral Gauss-Bonnet (GB) AdS has an analytical form [34]. This gives us a good chance to exactly investigate it. The result will provide us a possible way to understand the microstructures of the black hole in this modified gravity.

This work is organized as follows. In Sec. II, we briefly introduce the Ruppeiner geometry. Thermodynamics and phase diagrams of the GB-AdS black hole are discussed in Sec. III. Then we apply the geometrical method to the black hole in Sec. IV. The black hole microstructures are studied in detail. Furthermore, the critical phenomena of the normalized scalar curvature are investigated. Finally, the conclusions and discussions are given in Sec. V.

II. RUPPEINER GEOMETRY

In this section, we would like to briefly review the Ruppeiner geometry [35].

Let us first consider a thermodynamic system, which contains two parts, one is a small subsystem S, and the other is its environment E. Thus the total entropy of the system with two independent thermodynamic variables \( x^0 \) and \( x^1 \) can be written as

\[
S(x^0, x^1) = S_S(x^0, x^1) + S_E(x^0, x^1),
\]

where we require \( S_S \ll S_E \sim S \). Near the local maximum of the entropy at \( x^\mu = x_0^\mu \), the total entropy can be expanded in the following form

\[
S = S_0 + \left. \frac{\partial S_S}{\partial x^\mu} \right|_{x_0^\mu} \Delta x^\mu_S + \left. \frac{\partial S_E}{\partial x^\mu} \right|_{x_0^\mu} \Delta x^\mu_E \\
+ \frac{1}{2} \left. \frac{\partial^2 S_S}{\partial x^\mu \partial x^\nu} \right|_{x_0^\mu} \Delta x^\mu_S \Delta x^\nu_S + \frac{1}{2} \left. \frac{\partial^2 S_E}{\partial x^\mu \partial x^\nu} \right|_{x_0^\mu} \Delta x^\mu_E \Delta x^\nu_E + \cdots , \quad (\mu, \nu = 0, 1),
\]

(2)

where \( S_0 \) is the local maximum of the entropy \( S(x_0^\mu) \). By using the conservation law, we can get \( \left. \frac{\partial S_S}{\partial x^\mu} \right|_0 \Delta x^\mu_S + \left. \frac{\partial S_E}{\partial x^\mu} \right|_0 \Delta x^\mu_E = 0 \), and then, we arrive

\[
\Delta S = \frac{1}{2} \left. \frac{\partial^2 S_S}{\partial x^\mu \partial x^\nu} \right|_0 \Delta x^\mu_S \Delta x^\nu_S + \frac{1}{2} \left. \frac{\partial^2 S_E}{\partial x^\mu \partial x^\nu} \right|_0 \Delta x^\mu_E \Delta x^\nu_E + \cdots .
\]

(3)

Because \( S_E \) is of the same order as that of the total system, the second term is quite smaller than the first one, and thus can be ignored. Then, the probability of finding the system in the internals \( x_0 + \Delta x_0 \) and \( x_1 + \Delta x_1 \) will be of the following form

\[
P(x^0, x^1) \propto e^{-\frac{1}{2} \Delta l^2},
\]

(4)

where

\[
\Delta l^2 = -1 \frac{1}{k_B} g_{\mu\nu} \Delta x^\mu \Delta x^\nu,
\]

(5)

\[
g_{\mu\nu} = \left. \frac{\partial^2 S_S}{\partial x^\mu \partial x^\nu} \right|_0.
\]

(6)

According to the thermodynamic information geometry, \( \Delta l^2 \) measures the distance between two neighboring fluctuation states. Different from the Weinhold geometry [36], the thermodynamic potential of the Ruppeiner geometry is the entropy rather the internal energy of the system. Given the metric (6), we can calculate the scalar curvature for the geometry following GR approach. Then according to the interpretation that positive (negative) Ruppeiner scalar curvature indicates the repulsive (attractive) interaction, we are allowed to test the property of the system microstructures.

In order to calculate the scalar curvature easily and connect it with the phase transition, the geometry can be transformed to a new expression. Here we take the temperature \( T \) and thermodynamic volume \( V \) as the fluctuation coordinates. Actually, one can change the volume \( V \) to the number density of the system. However in order to easily extend it to the black hole system, we adopt the volume \( V \). Then this two-dimensional Ruppeiner geometry has the following line element

\[
\Delta l^2 = -\frac{1}{T} \left( \frac{\partial^2 F}{\partial T^2} \right) \Delta T^2 + \frac{1}{V} \left( \frac{\partial^2 F}{\partial V^2} \right) \Delta V^2.
\]

(7)
Here $F = U - TS$ is the free energy, and it satisfies the differential law $dF = -SdT - PdV$. With the help of the heat capacity at constant volume $C_V = T(\partial_T S)_V$, the line element can be further expressed as

$$\Delta t^2 = \frac{C_V}{T^2} \Delta T^2 - \frac{(\partial_T P)_T}{T} \Delta V^2.$$  

(8)

This diagonalized metric has the following scalar curvature [33]

$$R = \frac{1}{2C_V^2(\partial_V P)^2} \left\{ T(\partial_V P) \left[ (\partial_T C_V)(\partial_V P - T\partial_{T,V} P) - (\partial_V C_V)^2 \right] \right. $$

$$+ \left. C_V \left[ (\partial_V P)^2 - T \left( (\partial_T C_V)(\partial_V, V P) + T(\partial_{T,V} P)^2 \right) + 2T(\partial_V P)((\partial_V, V C_V) + T(\partial_{T,V}, V P)) \right] \right\}.$$

(9)

Adopting this formula, one can obtain the scalar curvature for the Ruppeiner geometry. Then the microscopic property could be revealed from the scalar curvature.

### III. THERMODYNAMICS AND PHASE TRANSITION

The action describing a $d$-dimensional charged GB-AdS black hole is

$$S = \int d^d x \sqrt{-g} \left( \frac{1}{16\pi G_d} (R - 2\Lambda + \alpha_{GB}L_{GB}) - L_{\text{matter}} \right),$$

(10)

where

$$L_{\text{GB}} = R_{\mu\nu\gamma\delta}R^{\mu\nu\gamma\delta} - 4R_{\mu\nu}R^{\mu\nu} + R^2,$$

(11)

$$L_{\text{matter}} = 4\pi F_{\mu\nu}F^{\mu\nu},$$

(12)

and $\alpha_{GB}$ is the GB coupling constant. The Maxwell field strength is defined as $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ with $A_\mu$ the vector potential. The metric of the black hole is

$$ds^2 = -f(r)dt^2 + f^{-1}(r)dr^2 + r^2(d\theta^2 + \sin^2 \theta d\phi^2 + \cos^2 \theta d\Omega_{d-4}^2),$$

(13)

with the metric function given by [37][10]

$$f(r) = 1 + \frac{r^2}{2\alpha} \left( 1 - \sqrt{1 + \frac{2\alpha}{d-2} \left( \frac{32\pi M}{\Sigma_{d-2} r^{d-1}} - \frac{4Q^2}{(d-3)r^{2d-4}} + \frac{4\Lambda}{d-1} \right) } \right),$$

(14)

where $\alpha = (d-3)(d-4)\alpha_{GB}$ and $\Sigma_{d-2}$ is the area of a unit $(d-2)$-dimensional sphere. The parameters $M$ and $Q$ are the black hole mass and charge, respectively. In the extended phase space, the cosmological constant $\Lambda$ was interpreted as the thermodynamic pressure [11]

$$P = -\frac{1}{8\pi} \Lambda.$$  

(15)

It was shown that the five-dimensional neutral GB-AdS black hole demonstrates a small-large black hole phase transition. Especially, the coexistence curve of the small and large black holes has an analytical expression, which allows us to exactly study the thermodynamics and phase transition. Moreover, it also provides us an opportunity to determine the effect of the GB coupling constant on the black hole thermodynamics. Therefore, in the following, we will take $d=5$ and $Q=0$. Then the metric function [11] will be of the following form

$$f(r) = 1 + \frac{r^2}{2\alpha} \left( 1 - \sqrt{1 + \frac{32M\alpha}{3\pi r^4} - \frac{16P\alpha}{3} } \right).$$

(16)

Solving the above equation, we obtain the black hole mass

$$M = \frac{\pi}{8}(4P\alpha r_h^4 + 3r_h^2 + 3\alpha).$$

(17)
According to Ref. [10], the black hole mass should be treated as the enthalpy \( H \equiv M \). Other thermodynamic quantities read

\[
T = \frac{8\pi Pr_h^3 + 3r_h}{6\pi r_h^2 + 12\pi \alpha}, \quad S = \frac{\pi^2 r_h(r_h^2 + 6\alpha)}{2},
\]

\[
A = -\frac{\pi}{8} \frac{32\pi Pr_h^4 + 9r_h^2 - 6\alpha}{r_h^2 + 2\alpha}, \quad V = \frac{\pi^2 r_h^4}{2}, \quad v = \frac{4}{3} r_h.
\]

Here \( A \) is a conjugate quantity to the GB coefficient \( \alpha \). The parameters \( V \) and \( v \) are, respectively, the thermodynamic volume and specific volume of the black hole system. It is easy to check that the following first law and Smarr relation hold

\[
dH = TdS + VdP + A d\alpha,
\]

\[
2H = 3TS - 2PV + 2A\alpha.
\]

The state equation for this black hole system is

\[
P = \frac{T}{v} - \frac{2}{3\pi v^2} + \frac{32\alpha}{9v^3}.
\]

Obviously, the state equation closely depends on the GB coupling \( \alpha \). For non-vanishing \( \alpha \), there exists a small-large black hole phase transition of VdW type. The critical point can be obtained by solving \((\partial_v P)_T = (\partial_v v)_T = 0\), which gives

\[
P_c = \frac{1}{48\pi \alpha}, \quad T_c = \frac{1}{2\pi \sqrt{6\alpha}}, \quad V_c = 18\pi^2 \alpha^2, \quad v_c = \sqrt{\frac{32\alpha}{3}}.
\]

In the reduced parameter space, the state equation can be expressed as

\[
\hat{P} = \frac{3\hat{T}}{\hat{v}} - \frac{3}{\hat{v}^2} + \frac{\hat{T}}{\hat{v}^3},
\]

where the reduced quantities are defined by

\[
\hat{P} = \frac{P}{P_c}, \quad \hat{T} = \frac{T}{T_c}, \quad \hat{v} = \frac{v}{v_c}.
\]

It is quite clear that the reduced state equation (24) is independent of the parameter \( \alpha \), which is mainly because that the neutral GB-AdS black hole is a single-characteristic-parameter system [41]. Moreover, we can express the state function with the thermodynamic volume

\[
P = \frac{3\pi^{3/2} \alpha T}{2 \times (2V)^{3/4}} + \frac{3\sqrt{\pi} T}{4 \times (2V)^{1/4}} - \frac{3}{8\sqrt{2V}},
\]

or in the reduced parameter space

\[
\hat{P} = \frac{\hat{T}}{V^{3/4}} - \frac{3}{\sqrt{V}} + \frac{3\hat{T}}{\sqrt{V}^{1/4}}.
\]

Now, let focus on the spinodal curve first, which satisfies

\[
(\partial_v \hat{P})_T = 0.
\]

This curve divides the metastable phase from the unstable phase, and the heat capacity diverges along this curve. Solving the equation, one can obtain the spinodal curve. Here we show a compact form of the spinodal curve

\[
\hat{T}_{sp} = \frac{2V^{1/4}}{1 + \sqrt{V}}.
\]

In the reduced parameter space, the analytical coexistence curve is [34]

\[
\hat{P} = \frac{1}{2} (3 - \sqrt{9 - 8\hat{T}^2}),
\]
FIG. 1: Phase structures of the five-dimensional neutral GBA-dS black hole. The red solid curves are the coexistence curve of small and large black holes. The blue dashed curves are the spinodal curves. (a) $\tilde{P}$-$\tilde{T}$ diagram. (b) $\tilde{T}$-$\tilde{V}$ diagram.

or

$$\tilde{T} = \sqrt{\frac{\tilde{P}(3 - \tilde{P})}{2}}. \tag{31}$$

Interestingly, we obtain the formula of the coexistence curve in the $\tilde{T}$-$\tilde{V}$ diagram, which reads

$$\tilde{T} = \frac{3\tilde{V}^{1/4}(1 + \sqrt{\tilde{V}})}{1 + 4\sqrt{\tilde{V} + \tilde{V}}} \tag{32}.$$

With these results, we show the phase structures in Fig. 1(a). From Fig. 1(a), we show three black hole phases, the small black hole, large black hole, and the supercritical black hole. The red solid curve is the coexistence curve of the small and large black holes. It starts from the origin, then increases with the temperature, and ends at the critical point marked with a black dot. Top and bottom blue dashed curves are the spinodal curves for the large and small black holes, respectively. Note that the small black hole spinodal curve starts at $\tilde{T} = \sqrt{3}/2$. The phase structure is also shown in the $\tilde{T}$-$\tilde{V}$ diagram in Fig. 1(b). Two more metastable phases, the superheated small black hole phase and the supercooled large black hole phase, are clearly presented.

Next, we would like to examine the change of the thermodynamic volume among the black hole phase transition. After a simple calculation, we obtain the thermodynamic volumes $\tilde{V}_s$ and $\tilde{V}_l$ for the small and large black holes in terms of the phase transition temperature

$$\tilde{V}_s = \left(\frac{2\tilde{T} - \sqrt{6}\sqrt{2\tilde{T}^2 - 3 + \sqrt{9 - 8\tilde{T}^2}}}{3 - \sqrt{9 - 8\tilde{T}^2}}\right)^4, \tag{33}$$

$$\tilde{V}_l = \left(\frac{2\tilde{T} + \sqrt{6}\sqrt{2\tilde{T}^2 - 3 + \sqrt{9 - 8\tilde{T}^2}}}{3 - \sqrt{9 - 8\tilde{T}^2}}\right)^4. \tag{34}$$

In terms of the phase transition pressure, these volumes can also be expressed as

$$\tilde{V}_s = \frac{(\sqrt{3 - 3\tilde{P}} - \sqrt{3 - \tilde{P}})^4}{4\tilde{P}^2}, \tag{35}$$

$$\tilde{V}_l = \frac{(\sqrt{3 - 3\tilde{P}} + \sqrt{3 - \tilde{P}})^4}{4\tilde{P}^2}. \tag{36}$$

Moreover, we find the follow interesting relations

$$\tilde{V}_l + \tilde{V}_s = 14 + \frac{36 - 48\tilde{P}}{\tilde{P}^2}, \quad \tilde{V}_l \ast \tilde{V}_s = 1. \tag{37}$$
We describe the change of the thermodynamic volume $\Delta \tilde{V} = \tilde{V}_l - \tilde{V}_s$ in Fig. 2 as a function of the phase transition temperature and pressure, respectively. From the figures, we find that with the increase of the temperature or the pressure, $\Delta \tilde{V}$ decreases, and when the critical point is approached, $\Delta \tilde{V}$ vanishes. Note that at the critical point, $\tilde{V}_l = \tilde{V}_s$, and thus $\log_{10} \Delta \tilde{V} = -\infty$. We have made a cut-off in the figures at the critical point.

Since the thermodynamic volume has analytical form, we can expand the change of the volume $\Delta \tilde{V}$ near the critical point. The results are

$$\Delta \tilde{V} = 8\sqrt{6}(1 - \tilde{T})^{3/2} + 118\sqrt{6}(1 - \tilde{T})^{3/2} + O(1 - \tilde{T})^{5/2}, \quad (38)$$

$$\Delta \tilde{V} = 4\sqrt{6}(1 - \tilde{P})^{3/2} + 17\sqrt{6}(1 - \tilde{P})^{3/2} + O(1 - \tilde{P})^{5/2}. \quad (39)$$

Therefore, at the critical point, $\Delta \tilde{V}$ has a universal exponent $1/2$. Considering the behavior of the $\Delta \tilde{V}$, it can serve as an order parameter to describe the small-large black hole phase transition.

IV. RUPPEINER GEOMETRY AND SCALAR CURVATURE

In this section, we would like to study the Ruppeiner geometry for the five-dimensional neutral GB-AdS black hole. Employing the scalar curvature of the geometry, we can examine the microstructures of the black hole.

By using Eqs. (8) and (26), the line element of the Ruppeiner geometry reads

$$ds^2 = \frac{C_V}{T^2} dT^2 + \frac{3}{16} \frac{(6\pi^{3/2}T\alpha + \sqrt{2\pi V}T - (2V)^{1/4})^2}{2^{3/4}V^{7/4}T} dV^2. \quad (40)$$

For this neutral GB-AdS black hole, the heat capacity at fixed volume and GB coefficient vanishes. Therefore, we would like to adopt the normalized scalar curvature $R_N = C_V \ast R$ defined in [32]. After a simple calculation, the normalized scalar curvature is

$$R_N = \frac{2^{1/4}\sqrt{V} - 2\sqrt{2\pi TV^{3/4} - 12\pi^{3/2}\alpha TV^{1/4}}}{2^{3/4} \left(6\pi^{3/2}\alpha T + \sqrt{2\pi V}T - (2V)^{1/4}\right)}. \quad (41)$$

Obviously, $R$ closely depends on the coupling constant $\alpha$. In the reduced parameter space, it is

$$R_N = \frac{2\sqrt{V}^{1/4} \left(\tilde{V}^{1/4} - \tilde{T} \left(1 + \sqrt{\tilde{V}}\right)\right)}{\left(\tilde{T} - 2\tilde{V}^{1/4} + \tilde{T}\sqrt{\tilde{V}}\right)^2}. \quad (42)$$

Similar to the charged AdS black hole [33], this normalized scalar curvature $R_N$ is independent of $\alpha$. We show the behavior of $R_N$ in Fig. 3 for $\tilde{T}=0.4, 0.8, 0.9, 1.0$. When $\tilde{T}=0.4, 0.8$ and $0.9$, we observe two negative divergent
FIG. 3: Behavior of the normalized scalar curvature $R_N$ as a function of the reduced volume $\tilde{V}$. (a) $\tilde{T}=0.4$. (b) $\tilde{T}=0.8$. (c) $\tilde{T}=0.9$. (d) $\tilde{T}=1.0$.

points. And these two points get closer with the increase of $\tilde{T}$. At the critical temperature $\tilde{T}=1.0$, these two divergent points coincide with each other at $\tilde{V}=1$. More importantly, for $\tilde{T}=0.4$, we observe a positive $R_N$ near the region $\tilde{V} \in (0.06, 0.16)$, which, according to the interpretation of the Ruppeiner geometry, implies a repulsive interaction among the microstructure of the black hole system.

From the expression of normalized scalar curvature (42), we find that $R_N$ exactly diverges at the spinodal curve. Moreover, solving $R_N = 0$, we can obtain the sign-changing curve, which is

$$\tilde{T}_0 = \frac{\tilde{T}_{sp}}{2} = \frac{\tilde{V}^{1/4}}{1 + \sqrt{\tilde{V}}}.$$  \hspace{1cm} (43)

Therefore, the temperature $T_0$ is half of that of the spinodal curve, which is similar to the charged AdS black hole [33].

In Fig. 4, we list the coexistence curve (red solid line), spinodal curve (blue dashed line), and the sign-changing curve (black dot dashed line) in the $\tilde{T}$-$\tilde{V}$ diagram. The region marked in light purple color below the sign-changing curve has positive $R_N$, while the region above it has negative $R_N$. It is clear that the spinodal curve and the sign-changing curve are both under the coexistence curve. Consider that the state equation (27) does not hold in the coexistence region of small and large black holes, the region of positive $R_N$ will be excluded, and thus for the five-dimensional neutral GB-AdS black hole system, $R_N$ is always negative, which implies that only attractive interaction exists among the microstructures. This property is quite different from that of the charged AdS black hole shown in Refs. [32, 33], where there can exist a repulsive interaction for the small black holes. So not all the black hole systems allow the existence of the repulsive interaction.

Next, we would like to examine the behavior of $R_N$ along the coexistence curve and near the critical point.

With the help of (33) and (34), we express $R_N$ along the saturated small and large black hole curves. The result strongly suggests that the normalized scalar curvature $R_N$ along these two curves shares the same expression, which is

$$R_N = \frac{6\tilde{T}^2 - (3 - 2\tilde{T}^2)\sqrt{9 - 8\tilde{T}^2} - 7}{4 \left(1 - \tilde{T}^2\right)^{\frac{1}{2}}}.$$  \hspace{1cm} (44)
This result is novel, and indicates that the interaction among the microstructures is the same before and after the phase transition. However the microstructure does change. Therefore, it gives us the first example that the black hole microscopic interaction does not change during the phase transition. Moreover, the numerator of $R_N$ is negative for $\tilde{T} \in (0,1)$, which leads to a negative $R_N$. We also plot it in Fig. 5. From it, we observe a negative $R_N$. With the increase of $\tilde{T}$, $R_N$ decreases and diverges at the critical temperature.

Near the critical point, we expand $R_N$ as

$$R_N = -\frac{1}{8}(1 - \tilde{T})^{-2} - \frac{13}{8}(1 - \tilde{T})^{-1} - \frac{27}{32} + \mathcal{O}(1 - \tilde{T}).$$ \hspace{1cm} (45)

This implies that $R_N$ has a universal exponent 2 near the critical point. Moreover, ignoring the high orders, we obtain the following relation

$$R_N(1 - \tilde{T})^2 = -\frac{1}{8},$$ \hspace{1cm} (46)

which is the same constant as the VdW fluid and four-dimensional charged AdS black hole, while it is different from that of the higher-dimensional charged AdS black holes [33].

V. CONCLUSIONS AND DISCUSSIONS

In this paper, we analytically studied the thermodynamic phase transition and Ruppeiner geometry for the five-dimensional neutral GB-AdS black hole. Combining with the phase transition, we understood the microscopic properties of the black hole by analyzing the behavior of the scalar curvature of the Ruppeiner geometry along the coexistence curve and near the critical point.
By using analytical form of the coexistence curve of the small and large black holes, we showed the phase structure of the black hole in the $P-T$ and $T-V$ diagrams, respectively. Especially, in the $T-V$ diagram, two metastable black hole phases, the superheated small black hole phase and supercooled large black hole phase, were displayed. The spinodal curves of the small and large black holes were also shown in the phase diagrams, along which the heat capacity diverges. We also obtained the thermodynamic volumes along the saturated small and large black holes. Their change $\Delta V$ decreases with the phase transition temperature or pressure, and vanishes at the critical point. The result also shows that $\Delta V$ has a universal exponent $\frac{1}{2}$ at the critical point. Therefore, $\Delta V$ can serve as an order parameter to describe the small-large black hole phase transition.

|                | saturated SBH/liquid | saturated LBH/gas | critical point | critical exponent $R_N(1 - \tilde{T})^2$ |
|----------------|----------------------|-------------------|----------------|----------------------------------------|
| VdW fluid      | negative             | negative          | negative divergence | $2 - \frac{1}{8}$ [33] |
| $d = 4$ charged AdS BH | negative & positive | negative          | negative divergence | $2 - \frac{1}{8}$ [32] |
| $d \geq 5$ charged AdS BH | negative & positive | negative          | negative divergence | $2 - \frac{1}{8}$ [33] |
| $d = 5$ neutral GB-AdS BH | negative             | negative          | negative divergence | $2 - \frac{1}{8}$ |

Table I: Properties of the normalized scalar curvature $R_N$ along the saturated small black hole (liquid) and large black hole (gas) curves, and near the critical point for the VdW fluid, $d$-dimensional charged AdS black hole, and five-dimensional neutral GB-AdS black hole.

Then we constructed the Ruppeiner geometry and calculated the scalar curvature. Following Ref. [32], we adopted the normalized scalar curvature $R_N$ to test the microstructure of the five-dimensional neutral GB-AdS black hole. In order to give a clear comparison of the properties of $R_N$ for the VdW fluid, charged AdS black hole, and five-dimensional neutral GB-AdS black hole, we summarize the results in Table I. From the table, we find some results of $R_N$ for the five-dimensional neutral GB-AdS BH: i) $R_N$ is negative, which indicates attractive interaction of the microstructure of the thermodynamic systems. ii) At the critical point, $R_N$ goes to negative infinity. iii) $R_N$ has a critical exponent 2 and $R_N(1 - \tilde{T})^2 = -\frac{1}{8}$. One more intriguing property we observed for the neutral GB-AdS black hole is that, the normalized scalar curvature $R_N$ shares the same form for both the saturated small and large black holes. As we know, when the black hole system crosses the first order phase transition, its microstructures will experience a significant change. However, from the result of the normalized scalar curvature $R_N$, these two different black hole systems with different microstructures have the same interaction. This reveals a particular interesting nature for the AdS black hole in GB gravity.

In summary, we analytically investigated the property of the microstructure for the neutral AdS black hole in GB gravity, by combining with the thermodynamic phase transition and Ruppeiner geometry. The results show that the attractive interaction is dominant for the black hole system. Of particular interest is that during the small-large black hole phase transition, the scalar curvature implies that the interaction keeps unchanged while the microstructure of the black hole system does change. This is an interesting and important result for the black hole in the GB gravity. Our study is also worthwhile generalizing to charged and higher-dimensional AdS black hole in GB gravity. However, there may be no analytic result, and one needs to use the numerical calculation. Nevertheless, these will strengthen our knowledge on the black hole microstructures.

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[1] D. Kastor, S. Ray, and J. Traschen, *Enthalpy and the mechanics of AdS black holes*, Class. Quant. Grav. 26, 195011 (2009), [arXiv:0904.2765 [hep-th]].

[2] D. Kubiznak and R. B. Mann, *P-V criticality of charged AdS black holes*, J. High Energy Phys. 1207, 033 (2012), [arXiv:1205.0559 [hep-th]].
[3] N. Altamirano, D. Kubiznak, and R. B. Mann, *Reentrant phase transitions in rotating ads black holes*, Phys. Rev. D 88, 101502(R) (2013), [arXiv:1306.5756[hep-th]].

[4] N. Altamirano, D. Kubiznak, R. B. Mann, and Z. Sherkatghanad, *Kerr-AdS analogue of triple point and solid/liquid/gas phase transition*, Class. Quant. Grav. 31, 042001 (2014), [arXiv:1308.2672[hep-th]].

[5] N. Altamirano, D. Kubiznak, R. B. Mann, and Z. Sherkatghanad, *Thermodynamics of rotating black holes and black rings: phase transitions and thermodynamic volume*, Galaxies 2, 89 (2014), [arXiv:1401.2586[hep-th]].

[6] B. P. Dolan, A. Kostouki, D. Kubiznak, and R. B. Mann, *Isolated critical point from Lovelock gravity*, Class. Quant. Grav. 31, 242001 (2014), [arXiv:1407.4783[hep-th]].

[7] S.-W. Wei and Y.-X. Liu, *Critical phenomena and thermodynamic geometry of charged Gauss-Bonnet AdS black holes*, Phys. Rev. D 87, 044014 (2013), [arXiv:1209.1707[gr-qc]].

[8] S.-W. Wei and Y.-X. Liu, *Triple points and phase diagrams in the extended phase space of charged Gauss-Bonnet black holes in AdS space*, Phys. Rev. D 90, 044057 (2014), [arXiv:1402.2837[hep-th]].

[9] A. M. Frassino, D. Kubiznak, R. B. Mann, and F. Simovic, *Multiple reentrant phase transitions and triple points in Lovelock thermodynamics*, J. High Energy Phys. 1409, 080 (2014), [arXiv:1406.7015[hep-th]].

[10] R.-G. Cai, L.-M. Cao, L. Li, and R.-Q. Yang, *P-V criticality in the extended phase space of Gauss-Bonnet black holes in AdS space*, J. High Energy Phys. 1309, 005 (2013), [arXiv:1306.6233[gr-qc]].

[11] H. Xu, W. Xu, and L. Zhao, *Extended phase space thermodynamics for third order Lovelock black holes in diverse dimensions*, Eur. Phys. J. C 74, 3074 (2014), [arXiv:1405.4143[gr-qc]].

[12] B. P. Dolan, A. Kostouki, D. Kubiznak, and R. B. Mann, *Isolated critical point from Lovelock gravity*, Class. Quant. Grav. 31, 242001 (2014), [arXiv:1407.4783[hep-th]].

[13] R. A. Hennigar, W. G. Brenna, and R. B. Mann, *P-V criticality in quasitopological gravity*, J. High Energy Phys. 1507, 077 (2015), [arXiv:1505.05517[hep-th]].

[14] R. A. Hennigar and R. B. Mann, *Reentrant phase transitions and van der Waals behaviour for hairy black holes*, Entropy 17, 8056 (2015), [arXiv:1509.06798[hep-th]].

[15] R. A. Hennigar, R. B. Mann, and E. Tjoa, *Superfluid black holes*, Phys. Rev. Lett. 118, 021301 (2017), [arXiv:1609.02564[hep-th]].

[16] D.-C. Zou, R. Yue, and M. Zhang, *Reentrant phase transitions of higher-dimensional AdS black holes in dRGT massive gravity*, Eur. Phys. J. C 77, 256 (2017), [arXiv:1612.08056[gr-qc]]; M. Zhang, D.-C. Zou, and R.-H. Yue, *Reentrant phase transitions and triple points of topological AdS black holes in Born-Infeld-massive gravity*, Adv. High Energy Phys. 2017, 3819246 (2017), [arXiv:1707.04101[hep-th]].

[17] N. Altamirano, D. Kubiznak, R. B. Mann, and Z. Sherkatghanad, *Thermodynamics of rotating black holes and black rings: phase transitions and thermodynamic volume*, Galaxies 2, 89 (2014), [arXiv:1401.2586[hep-th]].

[18] D. Kubiznak, R. B. Mann, and M. Teo, *Black hole chemistry: thermodynamics with Lambda*, Class. Quantum Grav. 34, 063001 (2017), [arXiv:1608.06147[hep-th]].

[19] S.-W. Wei and Y.-X. Liu, *Insight into the microscopic structure of an AdS black hole from thermodynamical phase transition*, Phys. Rev. Lett. 115, 111302 (2015), [arXiv:1502.00386[gr-qc]]; Erratum: Phys. Rev. Lett. 116, 169903 (2016).

[20] A. Dehyadegari, A. Sheykhi, and A. Montakhab, *Critical behaviour and microscopic structure of charged AdS black holes via an alternative phase space*, Phys. Lett. B 768, 235 (2017), [arXiv:1607.05333[gr-qc]].

[21] M. Chabab, H. E. Moumni, S. Iraoui, K. Masmar, and S. Zhizeh, *More insight into microscopic properties of RN-AdS black hole surrounded by quintessence via an alternative extended phase space*, Int. J. Geom. Meth. Mod. Phys. 15, 1850171 (2018), [arXiv:1704.07720[gr-qc]].

[22] G.-M. Deng and Y.-C. Huang, *Q-Φ criticality and microstructure of charged AdS black holes in f(R) gravity*, Int. J. Mod. Phys. A 32, 1750204 (2017), [arXiv:1705.04923[gr-qc]].

[23] M. K. Zangeneh, A. Dehyadegari, A. Sheykhi, and R. B. Mann, *Microscopic origin of black hole reentrant phase transitions*, Phys. Rev. D 97, 084054 (2018), [arXiv:1709.04432[hep-th]].

[24] Y.-G. Miao and Z.-M. Xu, *Microscopic structures and thermal stability of black holes conformally coupled to scalar fields in five dimensions*, Nucl. Phys. B 942, 205 (2019), [arXiv:1711.01557[hep-th]].

[25] Y.-G. Miao and Z.-M. Xu, *Thermal Molecular Potential among Micromolecules in Charged AdS Black Holes*, Phys. Rev. D 98, 044001 (2018), [arXiv:1712.00545[hep-th]].
[26] Y.-G. Miao and Z.-M. Xu, Interaction Potential and Thermo-correction to the Equation of State for Thermally Stable Schwarzschild AdS Black Holes, Sci. China-Phys. Mech. Astron. 62, 010412 (2019), [arXiv: 1804.01743 [hep-th]].
[27] D. Li, S. Li, L.-. Mi, and Z.-H. Li, Insight into black hole phase transition from parametric solutions, Phys. Rev. D 96, 124015 (2017).
[28] Y. Chen, H. Li, and S.-J. Zhang, Microscopic explanation for black hole phase transitions via Ruppeiner geometry: two competing mechanisms, arXiv:1812.11765 [hep-th].
[29] X.-Y. Guo, H.-F. Li, L.-C. Zhang, and R. Zhao, Microstructure and continuous phase transition of RN-AdS black hole, Phys. Rev. D 96, 124015 (2017).
[30] Y.-Z. Du, R. Zhao, and L.-C. Zhang, Microstructure and continuous phase transition of the gauss-bonnet ads black hole, arXiv:1901.07932 [hep-th].
[31] Z.-M. Xu, B. Wu, and W.-L. Yang, The fine micro-thermal structures for the Reissner-Nordstrom black hole, arXiv:1910.03378 [gr-qc].
[32] S.-W. Wei, Y.-X. Liu, and R. B. Mann, Repulsive Interactions and Universal Properties of Charged AdS Black Hole Microstructures, Phys. Rev. Lett. 123, 071103 (2019). arXiv:1906.10840 [gr-qc].
[33] S.-W. Wei, Y.-X. Liu, and R. B. Mann, Ruppeiner Geometry, Phase Transitions, and the Microstructure of Charged AdS Black Holes, arXiv:1909.03887 [gr-qc].
[34] J.-X. Mo and G.-Q. Li, Coexistence curves and molecule number densities of AdS black holes in the reduced parameter space, Phys. Rev. D 92, 024055 (2015). arXiv:1604.07931 [gr-qc].
[35] G. Ruppeiner, Riemannian geometry in thermodynamic fluctuation theory, Rev. Mod. Phys. 67, 605 (1995); Erratum: Rev. Mod. Phys. 68, 313 (1996).
[36] F. Weinhold, Metric geometry of equilibrium thermodynamics, J. Chem Phys. 63, 2479 (1975); Erratum: J. Chem Phys. 63, 2484 (1975).
[37] D. G. Boulware and S. Deser, String Generated Gravity Models, Phys. Rev. Lett. 55, 2656 (1985).
[38] R. G. Cai, Gauss-Bonnet Black Holes in AdS Spaces, Phys. Rev. D 65, 084014 (2002), arXiv:hep-th/0109133.
[39] D. L. Wiltshire, Spherically Symmetric Solutions Of Einstein-maxwell Theory With A Gauss-bonnet Term, Phys. Lett. B 169, 36 (1986).
[40] M. Cvetic, S. Nojiri, and S. D. Odintsov, Black Hole Thermodynamics and Negative Entropy in deSitter and Anti-deSitter Einstein-Gauss-Bonnet gravity, Nucl. Phys. B 628, 295 (2002), arXiv:hep-th/0112045.
[41] S.-W. Wei, P. Cheng, and Y.-X. Liu, Analytical and exact critical phenomena of d-dimensional singly spinning Kerr-AdS black holes, Phys. Rev. D 93, 084015 (2016). arXiv:1510.00085 [gr-qc].