Analytical approach in determination of the surface layer parameters of machine parts hardened by a moving elastic indenter

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Abstract. The reducing of machine time during surface hardening by smoothening is possible by increasing in the contact length between the hardening element and part surface. We study the case: the indenter-working surface is a fragment of a cylindrical surface, which the symmetry axis is parallel to the axis of symmetry of the part hardened surface. We suggest that such a process scheme is corresponds to a 2D contact problem in which a cylindrical stamp moves in an inertial coordinate system along an elastic half-plane (circle). We are tasking the boundary conditions in the contact zone, using the complex potentials developed in the works of L. Galin.

1. Relevance

The choice of the physical model of the movement of the stamp along the elastic half-plane depends on many factors: the elastic characteristics of the stamp and half-plane, the radius of the stamp in plan, the nature of the interaction of the contacting bodies, etc. In our opinion, the models in figure 1 a, b of previous work [1-3] are most acceptable for treatment.

The choice of models figure 1 a, b we explain as follows:

- known methods for measuring the contact length (length \(A_1A_4\)) [4-6];
- known methods of measuring forces acting through the stamp on the half-plane [7-9];
- known methods for measuring the speed of movement of the stamp on the elastic half-plane [10-12];
- known methods for measuring the average friction force and friction coefficient [13-16].

Next, we consider the physical model of the interaction of the stamp on the elastic half-plane shown in figure 1a [1]. This choice is due to the fact that, within the framework of the elastic (linear) model, we can determine the deformation of the boundaries of the elastic half-plane in the areas free of loads. From a practical point of view, this circumstance means: it is possible to determine (within the framework of the linear model) the bulge shape formed before and after the moving indenter [14].
Figure 1. Possible physical models of diamond treatment: a – not considering the bulge of deformed metal formed in front of the moving stamp; b - considering the bulge formed in front of the moving stamp.

2. Setting boundary conditions

Physical boundary conditions are as follows:

\[
\begin{align*}
[A_1A_5A_4]: & \quad \sigma_y = 0, \quad \tau_{xy} = 0 \quad - \text{the area is free from external loads} \\
[A_1A_2]: & \quad v_y = \Phi_x' z_{z_1}', \quad \tau_{xy} - f_1 \cdot \sigma_y = 0 \quad - \text{friction section} \\
[A_2A_3]: & \quad u_x(x) = 0, \quad v_y = \Phi_x z_{z_1}' \quad - \text{adhesive bond area} \\
[A_3A_4]: & \quad v_y = \Phi_x' z_{z_1}', \quad \tau_{xy} + f_2 \cdot \sigma_y = 0 \quad - \text{friction section}
\end{align*}
\]  

(1)

Here \(f\) – friction coefficient, \(\Phi(x)\) - function that describes the contour of the stamp. For computational convenience, we assume that

\[
\Phi(x) = \frac{x^2}{2R}
\]

(2)

Using the conformal mapping of the lower half-plane without an indent \(A_1A_2A_3A_4\) onto the half-plane \(Z_1\)

\[
z_1 = z + \frac{\sigma \cdot e^{z\alpha}}{8\pi} \cdot \frac{(i + z \cdot e^{-z\alpha})^3}{1 - z \cdot e^{-z\alpha}}
\]

(3)
Figure 2 shows the converted contact of the stamp with the hardened surface.

![Diagram](image)

**Figure 2.** Scheme of loading the contact zone of the stamp and the surface of the half-plane $Z_1$.

Note that the coordinates of the points along the 0X axis of the points $A_i$ and $a_i$ in figures 1 and 2 of present work do not coincide.

We write boundary conditions (1) using two complex Galin’s potentials

$$w_1(z) = u_1 - iv_1 = \int_{-\infty}^{t} (\sigma_y)_{y=0} \frac{dt}{t-z}$$

$$w_2(z) = u_2 - iv_2 = \int_{-\infty}^{t} (\tau_{xy})_{y=0} \frac{dt}{t-z}$$

(4)

In this case, the boundary conditions (1) take the form:

on $[a_1 a_5]$:

$$\text{Im} \omega_1 = 0, \text{Im} \omega_2 = 0$$

(5)

on $[a_1 a_2]$:

$$v_2 + f_1 v_1 = \text{Im}(\omega_2 + f_1 \omega_1) = 0$$

$$u_1 - \beta v_1 = \text{Im}(\omega_1 + \beta \omega_2) = \frac{GN - MH}{CN - DM} \Phi_{x'z_4'} = 0$$

(6)

on $[a_3 a_4]$:

$$u_2 - f_2 v_1 = \text{Im}(\omega_2 - f_2 \omega_1) = 0$$

$$u_1 - \beta v_1 = \text{Im}(\omega_1 + \beta \omega_2) = \frac{GN - MH}{CN - DM} \Phi_{x'z_4'} = 0$$

(7)

The different signs of the first equations (6) and (7) are due to different directions of friction on the segments $[a_1a_2]$ and $[a_3a_4]$. For computational convenience, we introduce the following notation

$$p = \frac{CN - MH}{CN - DM}, \quad q = \frac{DG - CH}{CN - DM}$$

(8)

The coefficients C, N, D, M, H, G depend on the speed of movement of the stamp and the elastic characteristics of the deformable half-plane. To determine them, we introduce new parameters that have the form:
\[ \begin{align*}
    a k_1 &= A; & a k_2 &= B; \\
    b - k_1^2 &= C; & b - k_2^2 &= D; \\
    c k_1 - d k_1^3 &= K; & c k_2 - d k_2^3 &= F; \\
    f k_1 - g k_1^3 &= G; & f k_2 - g k_2^3 &= H; \\
    -h k_1^2 + l &= M; & -h k_2^2 + l &= N.
\end{align*} \] (9)

where
\[ \begin{align*}
    a &= -\frac{\lambda + \mu}{\mu}; & b &= \frac{\lambda + 2\mu}{\mu} - \frac{w^2}{c_2^2}; \\
    c &= -(\lambda + 2\mu) - \frac{\lambda w^2}{c_2^2}; & d &= \lambda; \\
    f &= (3\lambda + 4\mu) - (\lambda + 2\mu) \frac{w^2}{c_2^2}; & g &= \lambda + 2\mu; \\
    l &= \lambda + 2\mu - \mu \frac{w^2}{c_2^2}; & h &= -\lambda.
\end{align*} \] (10)

Here \( \lambda, \mu \) – Lame constants
\[ \lambda = \frac{vE}{(1 + v)(1 - 2v)}; \quad \mu = \frac{E}{2(1 + v)} \] (11)

where \( E \) is the elastic modulus of the half-plane, \( v \) is the Poisson's ratio.

\( c_1 \) – expansion waves velocity; \( c_2 \) – distortion wave velocity:
\[ \begin{align*}
    c_1 &= \sqrt{\frac{\lambda + 2\mu}{\delta}}, \\
    c_2 &= \sqrt{\frac{\mu}{\delta}},
\end{align*} \] (12)

\( \delta \) – density of the elastic half-plane, \( w \) – stamp speed.

The constants \( k_1 \) and \( k_2 \) have the following form:
\[ k_1^2 = 1 - \frac{w^2}{c_1^2}; \quad k_2^2 = 1 - \frac{w^2}{c_2^2} \] (13)

After the transformation [6, 7], we express the parameters \( C, D, G, Y, M, N \) through the elastic parameters of the half-plane \( (E, v) \):
\[ C = b - k_1^2 = \frac{1}{1-2v} - \frac{1}{2-2v} m^2; \]
\[ D = b - k_2^2 = \frac{1}{1-2v}; \]
\[ G = f k_1 - g k_1^3 = \frac{E}{(1+v)(1-2v)} \sqrt{1-\frac{1-2v}{2-2v} m^2} \left[ 1 + \left( \frac{3}{2} - 2v \right) m^2 \right]; \]
\[ H = f k_2 - g k_2^3 = \frac{E}{(1+v)(1-2v)} \sqrt{1-m^2} \left[ 1 + (2-2v)m^2 \right]; \]
\[ M = -h_1 k_1^2 + l = \frac{E}{(1-v)(1-2v)} \left[ 1 - \frac{1-2v}{2-2v} m^2 \right]; \]
\[ N = -h_2 k_2^2 + l = \frac{E}{(1-v)(1-2v)} \left[ 1 - \frac{1}{2} m^2 \right]. \]

Based on (14) we find the coefficients \( p, q \):

\[ p = \frac{GN - MH}{CN - DM} = \frac{E}{-\frac{1}{2} (1-v)m^2 \left[ 1 - \frac{1-2v}{2-2v} m^2 \right]} \times \]
\[ \left[ 1 + \left( \frac{3}{2} - 2v \right)m^2 \right] \left[ 1 - \frac{1}{2} m^2 \right] \sqrt{1 - \frac{1-2v}{2-2v} m^2} - \]
\[ \left[ 1 - \frac{1-2v}{2-2v} m^2 \right] \left[ 1 + (2-2v)m^2 \right] \sqrt{1 - m^2} \];
\[ q = \frac{DG - CM}{CN - DM} = \frac{1}{-\frac{1}{2} m^2 \left[ 1 - \frac{1-2v}{2-2v} m^2 \right]} \times \]
\[ \left[ 1 + \left( \frac{3}{2} - 2v \right)m^2 \right] \sqrt{1 - \frac{1-2v}{2-2v} m^2} - \]
\[ \left[ 1 - \frac{1-2v}{2-2v} m^2 \right] \left[ 1 + (2-2v)m^2 \right] \sqrt{1 - m^2} \].

The boundary condition on the segment \([a_2a_3]\), corresponding to the adhesive bond area, has the form:

\[ u_2 + \beta' v_1 = \text{Im}(i\omega_2 - \beta' \omega_1) = 0 \]
\[ u_1 - \beta v_1 = \text{Im}(i\omega_1 + \beta \omega_2) = \frac{GN - MH}{CN - DM} \Phi'_i z'_i = 0 \]  

n expressions (6) \( \beta \neq \beta' \) - this is due to the fact that the movement of the diamond sphere (stamp) causes some “anisotropy” of the deformation pattern. The coefficient \( \beta \) is determined by the formula [6, 7]:

\[ \beta = \frac{C}{D}; \]
\[ \beta' = \frac{G}{H}; \]
\[ \beta'' = \frac{M}{N}. \]
\[
\beta = \frac{x - 1}{x + 1} = \frac{1 - 2\nu}{2 - 2\nu}
\]  

(17)

We determine the coefficient \( \beta' \), characterizing the displacement along the \( x - u \) axis. For a moving die in accordance with [14, 17],

\[
u = i\left[ A\phi(z_1) - A\overline{\phi(z_1)} + B\phi(z_2) - B\overline{\phi(z_2)} \right]
\]

then

\[
\frac{du}{dx} = i \left[ A\phi'(z_1) - A\overline{\phi'(z_2)} + B\phi'(z_2) - B\overline{\phi'(z_2)} \right]
\]

or

\[
\frac{du}{dx} = 2 \text{Im}[A\phi'(z_1) - B\phi'(z_2)]
\]

Using the expression \( \psi'(z_1) \) and \( \psi'(z_2) \) from [4,5] we will change the last equation:

\[
\frac{du}{dx} = 2 \text{Im}\left[ \frac{AN + BM}{GN - MH} \int_{-\infty}^{+\infty} \left( \frac{\sigma_y}{\xi - x} \right)_{x=0} d\xi - \frac{BG + AH}{GN - MH} \int_{-\infty}^{+\infty} \left( \frac{\tau_{xy}}{\xi - x} \right)_{x=0} d\xi \right]
\]

Tend \( z_1 \) and \( z_2 \) when \( y = 0 \) to an arbitrary point on the \( x \)-axis and, using the property of limit values of Cauchy type integrals, we have

\[
\frac{du}{dx} = 2 \text{Im}\left[ \frac{AN + BM}{GN - MH} \left( \frac{1}{2\pi} \int_{-\infty}^{+\infty} \frac{\sigma_y}{\xi - x} d\xi - \frac{BG + AH}{GN - MH} \left( \frac{1}{2\pi} \int_{-\infty}^{+\infty} \frac{\tau_{xy}}{\xi - x} d\xi \right) \right) \right]
\]

or

\[
\frac{du}{dx} = 2 \text{Im}\left[ \frac{AN + BM}{GN - MH} \left( \frac{\sigma_y}{\xi - x} \right)_{x=0} - \frac{BG + AH}{GN - MH} \left( \frac{\tau_{xy}}{\xi - x} \right)_{x=0} \right]
\]

From which, using \( w_1(z) \) and \( w_2(z) \) we have:

\[
\frac{du}{dx} = \frac{AN + BM}{GN - MH} w_1 - \frac{BG + AH}{GN - MH} w_2
\]

Since on the segment \([a_1 a_3] \) \( u = 0 \) the last expression can be written as

\[
u_1 - \frac{BG + AH}{AN - BM} v_2 = 0
\]

i.e. \( \beta' = \frac{BG + AH}{AN - BM} \), where the analytical form of the coefficients G, H, N, M, A, B are given in [14, 17, 18].

Thus, the boundary conditions of the considered contact problem take the form:

\[
\begin{align*}
[a_1 a_3]: & \quad \text{Im} \omega_1 = 0, \quad \text{Im} \omega_2 = 0 \\
[a_4 a_2]: & \quad \text{Im}(\omega_2 + \beta \omega_1) = 0, \quad \text{Im}(i\omega_1 + \beta \omega_2) = \Phi \\
[a_2 a_3]: & \quad \text{Im}(i\omega_2 + \beta \omega_1) = 0, \quad \text{Im}(i\omega_1 + \beta \omega_2) = \Phi \\
[a_3 a_4]: & \quad \text{Im}(\omega_2 - \beta \omega_1) = 0, \quad \text{Im}(i\omega_1 + \beta \omega_2) = \Phi
\end{align*}
\]

(18)
Here for brevity we denote

\[ \phi = \frac{GN - MH}{CN - DM} \Phi'x'z' \]

Since we have previously suggested that the stamp is not an absolutely rigid body and has elastic characteristics. Additionally, we assume that the contact zone in the plan has a length that is much less than the radii of the contacting bodies.

For both bodies, there is the following relation of displacements on the contact area:

\[ y_1 = f_1(x); \]
\[ y_2 = -f_2(x) \]  \( (19) \)

Here \( f_1(x) = \frac{x^2}{2R_1} \) - function describing the contour of the stamp in the contact zone, \( R_1 \) – radius of the cylinder of the working surface of the indenter.

\( f_2(x) = \frac{x^2}{2R_2} \) – function describing the contour of the hardened surface in the contact zone. \( R_2 \) – is the radius of the part whose surface is hardened by the indenter. For computational convenience, we also assume that the origin is at the point of initial contact (see figure 3).

\[ \text{Figure 3. The scheme of interaction of a non-rigid indenter with a hardened part.} \]

**Conclusions**

Under the action of forces, the stamp (indenter) will receive the movement \( \delta_1 \), and the second body (the surface of the hardened part) will receive \( \delta_2 \) movement. Point A on the surface of the stamp (indenter), and point B, on the surface of the part, will be combined as a result of elastic deformation. In this case, the stamp and the second body will receive, respectively, displacements along the \( y \) axis: \( \vartheta_1 \) and \( \vartheta_2 \).

Note that the symmetric nature of the interaction of two bodies is not restrictive.

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