A new method to determine Globular Cluster ages

Raul Jimenez
Royal Observatory Edinburgh, Blackford Hill EH9 3HJ, Edinburgh, UK

Paolo Padoan
Theoretical Astrophysics Center, Blegdamsvej 21, DK-2100 Copenhagen, Denmark
Dept. of Astronomy, University of Padova, Vicolo dell’ Osservatorio 5, I-35122, Padova, Italy

Received __________________; accepted __________________
ABSTRACT

We present a new method to compute stellar ages in Globular Clusters (GC) that is ten times more precise than the traditional isochrone fitting procedure. The method relies on accurate stellar evolutionary tracks and on photometry for GCs complete down to the main sequence, and it is based on counting number of stars in two different regions of the CMD: the red giant branch and the main-sequence. We have applied this method to the globular cluster M68 and found an age of 16.4±0.2 Gyr for \((m - M)_V = 15.3\). This new method reduces the error associated to the uncertainty in the distance modulus by a factor of two, the error due to the choice of the value for the mixing length parameter to almost zero and the error due to the colour-\(T_{\text{eff}}\) transformation to zero.

Subject headings: globular clusters: ages – globular clusters: M68
1. Introduction

Globular clusters are among the oldest objects in the Universe and also the tracers of the collapse of the Galaxy, and they are very important cosmological probes for the age of the Universe. The determination of the age of GCs is still an open problem. GCs ages have been recently reviewed by Chaboyer (1995). When all possible random and systematic errors are taken into account, an error bar of 5 Gyr is associated to any age determination using the main sequence turnoff isochrone fitting method (MSTO). An alternative method has been proposed in order to cure some of the problems of the MSTO procedure (Jimenez et al. 1996), giving lower ages than the MSTO.

The main problem in the MSTO comes from the fact that the isochrone has to fit the position of the main sequence turnoff that is not a point on the CMD, but rather an extended region. The same is true for the alternative method developed by Iben & Renzini (1984).

In this paper we present a new method to determine ages of GCs that does not rely on fitting any particular morphological feature in the CMD diagram and that allows us to reach a precision of 0.2 Gyr for a given distance modulus. The method is based on a careful computation of stellar evolutionary tracks and on counting stars in two different regions of the CMD: the red giant branch (RGB) and the main sequence (MS), down to a magnitude where the sample is complete.

The paper is organized as follows. In section 2 we describe the theoretical stellar evolution models used in this work. In section 3 we present the method and compute the age for M68. Section 4 contains a comparison with other methods. We finish with the summary and conclusions.

2. Theoretical stellar evolutionary models

The set of stellar evolutionary models has been computed with the latest version of James MacDonald’s code (JMSTAR9). The code incorporates the latest advances in opacities and updated physics, it also uses the elegant technique developed by Peter Eggleton to follow evolution up to the RGB tip. Tracks were computed for a range of masses from $0.50 M_\odot$ to $1.00 M_\odot$ with a mass interval of $0.001 M_\odot$. This is achieved using an adaptive mesh grid with 3000 points. A complete description of the code, as well as a detailed list of the physics used on it, is given in Jimenez & MacDonald (1996).

The tracks were started from a contracting initial gas cloud in the Hayashi track and follow up to the RGB tip, where the Helium core flash occurs. All evolutionary tracks were stopped at the Helium core
flash, that takes place under degenerate conditions. The mixing length parameter was chosen from the fit to the RGB position (Jimenez et al. 1996); for M68 we adopted a value of 1.38. The metallicity for all the tracks was $Z = 0.0002$ and $Y = 0.24$. The whole grid (that covers a range of masses from 120–0.01 $M_\odot$) with several metallicities and values of $Y$ will be made available shortly (Jimenez, Padoan & MacDonald 1996, in preparation). Some of the tracks are plotted in Fig. 1.

3. The method

Stars of different masses evolve at a different speed along the CMD – the more massive the faster – with the effect that the number of stars inside a fixed luminosity bin in the main sequence decreases as time increases. It seems natural to use this effect as a clock to measure the age of the GCs, since it is as simple as counting stars in the CMD.

In order to predict accurately the number of stars in a theoretical CMD it is necessary to have a large number of evolutionary tracks in the range of masses observed in GCs (0.80 $M_\odot$ to 0.70 $M_\odot$) for M68. To achieve a precision of 0.2 Gyr it is necessary to have one track every 0.001 $M_\odot$ (see section IV) and therefore a high number of grid points (2000 or more) in the adaptive mesh of the stellar evolution code.

In addition it is necessary to have complete photometry for the GC to a certain magnitude along the main sequence, and very accurate photometry along the RGB to be able to distinguish AGB stars from RGB stars.

To test our method we have used photometric data obtained by ourselves for the GC M68 (Jimenez et al. 1996). Very accurate photometry was obtained in several bands (UBVRIJHK). This allowed us to clearly distinguish the AGB from the RGB. M68 has a very low metallicity ($Z = 0.0002$) and therefore is representative of the oldest GCs in our galaxy and its age is a constraint on the age of the Universe.

The first step of the method consists in comparing the theoretical and observational luminosity functions for the main sequence stars in order to determine to which magnitude the observational data are complete. The second step consists in sampling the luminosity function using only two luminosity bins, one for the RGB and the other for the main sequence, down to the luminosity where the data are complete. Note however that in our application of the method to M68 we hardly include in the second bin the top of the main sequence since our data are complete only to $V = 19.0$. 
3.1. The theoretical luminosity function

We first draw a set of evolutionary tracks (luminosity vs. time) for a given value of the metallicity and helium content (the mixing length parameter has been fixed to 1.38 (see Jimenez et al. 1996)). We then choose an age, that is represented by a vertical line that intersects the tracks. Finally, we fix luminosity values that are horizontal lines in the same time-luminosity diagram.

The track that goes through the intersection between a given luminosity and the time gives the mass that corresponds to that luminosity at that time. The whole procedure is illustrated in Fig. 1.

In this way we can use stellar evolutionary tracks to determine the mass-luminosity ($M-L$) relation for stars of any mass and metallicity (Padoan & Jimenez, in preparation).

The luminosity function is determined by using this $M-L$ relation and by assuming a stellar initial mass function (Padoan 1995).

An example of a theoretical luminosity function is shown in Fig. 2 for the range of masses 0.71–0.77$M_\odot$ and for metallicity $Z = 0.0002$ with $Y = 0.24$. The observed luminosity function for M68, obtained excluding the AGB and HB stars, is plotted for comparison.

In the case of our data for M68, the theoretical luminosity function is remarkably well fitted by the theory down to a magnitude of $V = 19$ (notice the linear scale in the plot). The observational luminosity function deviates from the theoretical one only for magnitudes larger than $V = 19$. Therefore we consider our data complete down to a magnitude of $V = 19$.

3.2. The age of the GC

The second step of the method consists in sampling the luminosity function using only two luminosity bins, one for the RGB and the other for the main sequence down to the value where the data are complete.

The number of stars that populate the luminosity bin in the main sequence is decreasing, as time increases, more rapidly than the number of stars in the RGB. Therefore the ratio of these two numbers is a function of the age of the GC.

In Fig. 4 we show the two-bin luminosity function for different ages and compare it with the observational value.
For a distance modulus \((m - M)_V = 15.3\) the observations are best fitted by an age of \(16.4 \pm 0.2\) Gyr.

The age determination depends on the assumed distance modulus. In Table 1 ages are given for different distance moduli. As expected the cluster appears older when it is assumed to be closer (smaller distance modulus). Jimenez et al (1996) have determined the distance modulus with high precision by fitting the luminosity function of the RGB with theoretical luminosity functions (from stellar evolutionary tracks). Their result is \((m - M)_V = 15.3 \pm 0.1\). Therefore our best estimate of the age of the globular cluster M68 is \(16.4 \pm 0.2\) Gyr for the assumed distance modulus. If the uncertainty in the distance modulus is considered, the uncertainty in the age is \(\pm 1.5\) Gyr.

### 3.3. Accuracy of the method

To estimate the error due to counting stars we proceeded in the following way. Several frames of the same clusters taken during a period of 15 nights and different seeing conditions were analysed to count the total number of star in the frame and the number of stars per bin of luminosity. This allowed us to estimate the error that comes from crowding and choice of the point spread function. For this purpose we used 20 frames. The difference in the total number of stars from frame to frame was not bigger than 2\%, and the same applies when the stars were counted in the bins used in the method. This corresponds to an error of 0.15 Gyr.

We have also checked the effect of the IMF on the age determination. In this work we use a power law IMF with exponent 2.0 (where Salpeter is 2.35). A steeper IMF with exponent 3.0 affects the age only slightly in the sense of making the GC older by only 0.1 Gyr. Therefore we conclude that for reasonable IMF slopes the error related to the IMF is about 0.1 Gyr.

It is interesting to point out how stable the stellar evolution code is. Stellar tracks spaced by 0.001\(M_\odot\) are clearly defined in the time-luminosity diagram (see fig. 1). We tried to understand how much the position of the tracks in such a diagram could change due to different initial conditions in the starting protostellar cloud, and to round-off errors in different computers. The result of computations with different initial conditions and with different machines has shown that the computed tracks are very stable, in the sense that they occupy the same position in the time-luminosity diagram with a precision such that two tracks spaced by only 0.0001\(M_\odot\) can be distinguished, when the stellar evolution code is run with 3000 mesh points. Therefore the uncertainty in the theoretical determination of stellar masses does not affect the
As stated by Chaboyer (1995) the main uncertainty in the MSTO method is the choice of the value of the mixing length parameter ($\alpha$). This gives uncertainties in the age as large as 10%. In our method the value of $\alpha$ does not affect the age determination, since we use only tracks in the time-luminosity diagram that look almost identical even for very different values of $\alpha$ (Jimenez et al 1996). Therefore the mixing length parameter is not a source of error for us as in the MSTO method.

Another source of error in the MSTO is the transformation between colour and $T_{\text{eff}}$. Chaboyer (1995) gives an estimate of 5%. In our method the error due to this is zero since no colour transformation is necessary to compute the luminosity function.

Finally we comment briefly about how the uncertainty in the value of the distance modulus affects our age determination. Again, Chaboyer (1995) computes an error of 25% on the age due to uncertainties in the distance modulus value for the MSTO. It transpires from Table 1 that in our method that uncertainty has been reduced to 15%.

The total error in our age determination is estimated to be 0.2 Gyr that is the sum of the uncertainty in the number of stars per bin and in the IMF slope. In addition, an error of 1.5 Gyr should be added when the distance modulus is not known better than 0.25$^m$. It should be stressed that the same uncertainty (0.25$^m$) gives in our method an uncertainty in the age of only 15%, but 25% in the MSTO.

In the case of MSTO if the distance modulus, $\alpha$, colour-$T_{\text{eff}}$ transformation and chemical composition are fixed to a certain value, the uncertainty in the age is 10%, while it is only 2% in the present method.

4. Discussion

The investigation of GCs ages requires the discussion of two basic problems:

- The determination of the stellar absolute luminosity from the observed stellar magnitudes, that is the problem of measuring distances accurately.

- The uncertainties in stellar evolution theory that translate into uncertainties in the prediction of stellar ages.
A third problem arises in the age determination method based on isochrone fitting. Namely this method presents the problem of fitting the position of the MSTO that is not a point on the CMD, but rather an extended region. In fact the position of the MSTO is very sensitive to the assume mixing length parameter and colour calibration. The same is true for the alternative method developed by Iben & Renzini (1984).

The method developed in the present work is also affected by the uncertainties in the estimated distance of the globular cluster and in the stellar evolution theory. Nevertheless it improves considerably on the previous ones (by a factor 10!) because it does not rely on fitting any particular morphological feature in the CMD, and does not depend at all on mixing length parameter and colour calibration. In fact it has been shown in the paper that an uncertainty of only 0.2 Gyr is achieved, for a given distance modulus, just by counting stars on the CMD, as long as the stellar counts are stopped at a magnitude where the data are known to be complete.

As far as the stellar evolution theory is concerned, two are the most important uncertainties:

- The enhancement of $\alpha$ elements in GCs (Pagel & Tautvaisiene 1995). How to handle them in stellar evolution theory is still an open problem (Vandenberg 1992, Salaris, Chieffi & Straniero 1993).
- The helium settling in the radiative core that can reduce the amount of H and therefore shorten stellar ages.

The stellar evolution models used in this work do not include any of these effects. A simple solar-scaled composition has been used, and no He diffusion has been taken into account. Nevertheless, these uncertainties do not invalidate our procedure. If $\alpha$ elements and He diffusion affect significantly the stellar ages, our method would give an age estimate for the globular cluster shortened by 20–30%, that is an age in agreement with previous works (Chaboyer, Sarajedini & Demarque 1992, Jimenez et al. 1996).

5. Summary and conclusions

We have developed a new method to determine the ages of GCs. Using theoretical evolutionary tracks we have predicted the relative number of stars in the main sequence and in the RGB, as a function of age and distance modulus.
The dependence of the age on the distance modulus is twice smaller than what is found using the traditional isochrone fitting method, but the accuracy of the age determination for a given distance modulus is ten times higher, because the present method is based just on counting stars in different luminosity bins and therefore does not have troubles with fitting the morphology of the MSTO (mixing length parameter and colour calibration).

In Table 2 we show a comparison of the errors involved in computing GCs ages using the MSTO and our method.

We have applied this method to the old halo GC M68 and found an age of $16.4 \pm 0.2$ Gyr, if the distance modulus $(m - M)_V = 15.3$ determined by Jimenez et al. (1996), is used.

This value is in good agreement with previous age determinations found by Chaboyer, Sarajedini, Demarque (1992), using isochrone fitting, and Jimenez et al. (1996), using the HB morphology technique.

This work has been partly supported by the Danish National Research Foundation through its support for the establishment of the Theoretical Astrophysics Center. RJ thanks the Theoretical Astrophysics Center in Copenhagen where part of this work was carried out. PP enjoyed the hospitality of the Royal Observatory in Edinburgh were part of this work was carried out. We thank James Macdonald for kindly providing us the latest version of his stellar evolution code (JMSTAR9).
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This manuscript was prepared with the AAS \LaTeXX macros v4.0.
\( (m - M)_V \) & Age (Gyr) \\
15.5 & 14.5 ± 0.2 \\
15.4 & 15.0 ± 0.2 \\
15.3 & 16.4 ± 0.2 \\
15.2 & 17.5 ± 0.2 \\
15.1 & 18.8 ± 0.2 \\
15.0 & 20.0 ± 0.2 \\

Table 1: Age determination for different distance moduli

|                 | MSTO | This work |
|-----------------|------|-----------|
| distance modulus| 25%  | 15%       |
| mixing length   | 10%  | 0%        |
| colour-\( T_{\text{eff}} \) | 5%   | 0%        |
| He diffusion    | 7%   | 7%        |
| \( \alpha \)-elements | 10%  | 10%       |

Table 2: The values of the errors associated with different uncertainties when computing GCs ages. The MSTO and our method have been compared. Notice how the influence of this uncertainties in our method are smaller than in the MSTO. An accuracy in the age determination of 5% can be achieved with our method.
Fig. 1.— Evolutionary tracks for stars in the range of masses 0.7-0.8 $M_\odot$. The luminosity bins and the time are the ones used for computing the luminosity function shown in Fig. 2. The tracks are spaced by 0.001 $M_\odot$. 
Fig. 2.— The theoretical luminosity function (crosses and dotted line) for the estimated age of M68 is compared with the observational luminosity function (diamonds). The observations are fitted remarkably well by the theory down to the magnitude $V = 19.0$. This indicates that the data are complete down to $V = 19$. The largest error bars for the observations are about the size of the plotting symbols (diamonds).
Fig. 3.— This diagram shows the luminosity bins used to determine the age of M68. The vertical lines are the different ages considered.
Fig. 4.— The two-bin luminosity function. The left bin contains the RGB stars, the right bin the main sequence stars down to $V = 19.0$. Diamonds connected by dotted lines are the theoretical two-bin luminosity functions for different ages spaced by 0.5 Gyr. The continuous line is the observational value. We have plotted the 1% error bar due to the uncertainty in counting stars, which corresponds to an uncertainty in the age of 0.2 Gyr.