Spin-current Seebeck effect in quantum dot systems

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Abstract
We first bring up the concept of the spin-current Seebeck effect based on a recent experiment (Vera-Marun et al 2012 Nature Phys. 8 313), and investigate the spin-current Seebeck effect in quantum dot (QD) systems. Our results show that the spin-current Seebeck coefficient $S$ is sensitive to different polarization states of the QD, and therefore can be used to detect the polarization state of the QD and monitor the transitions between different polarization states of the QD. The intradot Coulomb interaction can greatly enhance $S$ due to the stronger polarization of the QD. By using the parameters for a typical QD whose intradot Coulomb interaction $U$ is one order of magnitude larger than the linewidth $\Gamma$, we demonstrate that the maximum value of $S$ can be enhanced by a factor of 80. On the other hand, for a QD whose Coulomb interaction is negligible, we show that one can still obtain a large $S$ by applying an external magnetic field.

Keywords: spin-current Seebeck effect, quantum dot, spin transport

(Some figures may appear in colour only in the online journal)

1. Introduction

The phenomenon where a temperature gradient across a conductor generates an electric voltage is called the Seebeck effect [1]. This effect has applications in thermal sensing devices such as thermocouples [1, 2]. The efficiency of the Seebeck effect is measured by the Seebeck coefficient $S$, which is defined as: $V = -S(T_2 - T_1)$. Here $V$ is the electric voltage, and $T_2$, $T_1$ are temperatures of the hot and cold regions, respectively. The Seebeck coefficient is a powerful tool to characterize materials because it provides information about the energetic difference between the relevant transport states and the Fermi level [3], while conductance only reflects the density of states near the Fermi level. The Seebeck coefficient $S$ is mainly determined by the energy dependence of the electron scattering in bulk materials. Much effort has been devoted to increasing $S$, for the purpose of enhancing thermoelectric properties [4–7].

Recently, Uchida et al [8] has observed a similar effect, where a temperature gradient in a metallic magnet can generate a spin voltage. This effect is known as the spin Seebeck effect, and it allows one to generate a pure spin current without electric currents, which is crucial for spintronic devices [9–12].

Here, we investigate another effect, which we call the spin-current Seebeck effect. In analogy to the Seebeck effect, where a heat current generates a charge voltage, here a spin current can also generate a charge voltage [13]. As shown in figure 1(a), under closed-circuit conditions, when a temperature gradient exists, a heat current is set up, with high-energy electrons moving from the left side to the right side and low-energy electrons moving in the opposite direction. When the transmission coefficient is energy dependent, this results in a non-zero net electric current. Under open-circuit conditions, an electric voltage is built up for the net electric current to be zero. Now consider spin transport (figure 1(b)). When a spin bias exists [14, 15], a spin current is set up, with spin-up electrons moving from the left side to the right side and spin-down electrons moving in the opposite direction. When the transmission coefficient is...
energy dependent, the net electric current $J_e = e(J_\uparrow + J_\downarrow)$ is typically non-zero. Under open-circuit conditions, an electric voltage is built up. Note that in both the Seebeck effect and the spin Seebeck effect, the driving force of the system is heat current, while in the spin-current Seebeck effect, the driving force is spin current. On the other hand, both the Seebeck effect and the spin-current Seebeck effect result in an electric voltage, while the spin Seebeck effect results in a spin voltage. As in the case of the Seebeck effect, we can also define a dimensionless spin-current Seebeck coefficient:

$$V = -\frac{S}{e} (\Delta \mu_L - \Delta \mu_R)$$  \hspace{1cm} (1)$$

where $\Delta \mu$ is the spin splitting of the chemical potential [13] and $V$ is the electric voltage. Here, it is important to emphasize that, for historical reasons, the name ‘Seebeck effect’ often implies the existence of a temperature gradient. However, in the spin-current Seebeck effect, there is NO temperature gradient in the system. Recently, the spin-current Seebeck effect has been observed in a graphene system [13], where large values of $\Delta \mu$ can be obtained. Using non-magnetic electrodes, they were able to measure the second-order component of signal: $V \propto I^2$, which they called ‘spin Seebeck voltage’. Experimental results and numerical modeling are in good agreement, indicating that the signal measured arises owing to the spin-current Seebeck effect [16].

The quantum dot (QD) is the simplest and fundamental structure in low-dimensional and mesoscopic transport devices. The energy levels in a QD are well separated due to the confinement in all three dimensions [17]. A QD is sometimes referred to as a zero-dimensional system and behaves in many ways as an artificial atom [18, 19]. The transport properties of a QD can be measured by coupling it to the leads and passing current through it, with the strength of couplings, the number of electrons in the dot, and energy levels in the dot under experimental control [18]. Since the 1990s, many novel and interesting transport phenomena in QD systems have been observed in experiments, such as Coulomb blockade [20–23] and the Kondo effect [24–27]. QD systems can be described by an Anderson model of a site weakly coupled to ideal leads with an on-site Coulomb interaction $U$ [22]. The intradot e–e interaction plays an important role in all the above-mentioned transport phenomena of the QD, such as the conductance oscillation at low temperatures in the Coulomb blockade effect [22, 23, 28].

In this paper, we study the spin-current Seebeck effect of a lead–QD–lead system. The schematic configuration of our system is shown in figure 2. We assume that the spin splitting of the chemical potential in the left lead is slightly larger than the right lead, thus setting up a spin bias across the QD. The energy levels of the QD can be tuned by changing the gate voltage $V_g$, thereby changing the energy dependence of the transmission coefficient of the dot. We do not discuss the detailed physical mechanisms of various ways to generate spin voltage, i.e. the spin splitting of chemical potentials in the leads, which can be found in the literature [10, 14, 15], but we use the generic chemical potential setup of figure 2 to drive a spin current through a QD. By using the non-equilibrium Green’s functions [17, 29], we have obtained an analytical expression of spin-current Seebeck coefficient $S$.
in the linear response regime. Applying this expression, we numerically calculated $S$ as a function of $V_g$ under different conditions. The $S - V_g$ curve is always antisymmetric due to the particle–hole symmetry, i.e. the symmetry between electron and hole with opposite spins. Our results demonstrate that the spin-current Seebeck coefficient is very sensitive to the particle–hole symmetry, i.e. the symmetry between the left and right leads $\Delta V_g \equiv \Delta \mu_{L} - \Delta \mu_{R}$. In the linear regime ($\Delta V_g \to 0$), the spin bias $V_s = \mu_{L\uparrow} - \mu_{R\uparrow} = -\mu_{L\downarrow} = -\mu_{R\downarrow}$.

\begin{equation}
\begin{aligned}
H &= H_L + H_c + H_i \\
H_L &= \sum_{k,\sigma \in L, R, \sigma} \epsilon_{ka} c_{k\sigma}^\dagger c_{k\sigma} \\
H_c &= \sum_{\sigma} \epsilon_{d\sigma} d_{\sigma}^\dagger d_{\sigma} + U n_{\uparrow} n_{\downarrow} \\
H_i &= \sum_{k,\sigma \in L, R, \sigma} \left[ V_{ka} c_{k\sigma}^\dagger d_{\sigma} + \text{h.c.} \right].
\end{aligned}
\end{equation}

Here $H_L$ describes the non-interacting leads; $\alpha = L, R$ represents the left and right leads respectively. $c_{k\sigma}^\dagger (c_{k\sigma})$ creates (annihilates) an electron with spin $\sigma$ in the $\alpha$ lead. $H_c$ describes the QD with one single-particle energy level $\epsilon_{d\sigma}$ and intradot Coulomb interaction $U$. The single-particle energy level $\epsilon_{d\sigma}$ is spin-degenerate in the absence of an external magnetic field $B$. When applying an external magnetic field $B$, $\epsilon_{d\sigma} = \epsilon_{d\sigma} - \sigma B$ due to the Zeeman splitting. $H_i$ is the Hamiltonian for tunneling between the leads and the QD, where $V_{ka}$ is the tunneling matrix element. Note that the notation we use here means we have assumed there is no spin-flipping mechanism in our system, so the spin of an electron does not change when tunneling between the leads and the QD.

By using the non-equilibrium Green’s function, the electric current with spin $\sigma$ polarization can be written as [29, 30]:

\begin{equation}
J_{\sigma} = \frac{i e}{\hbar} \int \frac{d\epsilon}{2\pi} \left[ f_{L\sigma}(\epsilon) - f_{R\sigma}(\epsilon) \right] T_{\sigma}(\epsilon)
\end{equation}

Here $f_{\sigma}(\epsilon) = 1/\exp[(\epsilon - \mu_{\sigma})/k_B T] + 1$ is the Fermi distribution function of the spin $\sigma$ electrons in the $\alpha$ lead. $T_{\sigma}(\epsilon)$ is the transmission coefficient for spin $\sigma$ electrons. $T_{\sigma}(\epsilon)$ can be written as:

\begin{equation}
T_{\sigma}(\epsilon) = \text{Tr} \left\{ \frac{\Gamma_{L\sigma}(\epsilon) \Gamma_{R\sigma}(\epsilon)}{\Gamma_{L\sigma}(\epsilon) + \Gamma_{R\sigma}(\epsilon)} \left[ G^R_{\sigma}(\epsilon) - G^L_{\sigma}(\epsilon) \right] \right\}.
\end{equation}

Here $G^R(\epsilon)$ is the standard retarded (advanced) Green’s function of the QD in the presence of coupling to the leads. $\Gamma^R(\epsilon) = 2\pi \sum_{\nu} |V_{\nu\alpha}|^2 \delta(\epsilon - \epsilon_{\nu\alpha})$ are the linewidth functions. The Green’s functions can be obtained from the Dyson equation and the Keldysh formalism [29, 31]:

\begin{equation}
G^r(\epsilon) = \begin{pmatrix} G^R_{\uparrow}(\epsilon) & 0 \\ 0 & G^R_{\downarrow}(\epsilon) \end{pmatrix} = g^r(\epsilon) + g^t(\epsilon) \Sigma^r g^r(\epsilon),
\end{equation}

\begin{equation}
G^a(\epsilon) = \begin{pmatrix} G^R_{\uparrow}(\epsilon) & 0 \\ 0 & G^R_{\downarrow}(\epsilon) \end{pmatrix} = g^a(\epsilon) \Sigma^a g^a(\epsilon).
\end{equation}

Here the boldface letters ($\mathbf{G}$, $\mathbf{g}$, and $\mathbf{\Sigma}$) represent the $2 \times 2$ matrices. $\mathbf{g}$ is the Green’s function of the QD without coupling to the leads. $\Sigma^r, \Sigma^a$ are self-energies (we have neglected the higher order of self-energy correction that originates from the combination of the e–e interaction and the tunneling terms) [31].

\begin{equation}
\Sigma^r = \begin{pmatrix} i\Gamma & 0 \\ 0 & i\Gamma \end{pmatrix},
\end{equation}

\begin{equation}
\Sigma^a = \begin{pmatrix} i\Gamma (f_{L\uparrow} + f_{R\uparrow}) & 0 \\ 0 & i\Gamma (f_{L\downarrow} + f_{R\downarrow}) \end{pmatrix},
\end{equation}

where we have neglected the energy dependence of linewidth functions $\Gamma^{L,R}$ and consider symmetric barriers: $\Gamma^{L,R} = \Gamma$. $\mathbf{g}$ can be calculated applying the equation-of-motion technique, and hence $\mathbf{G}$ can be obtained from equation (5):
Now we consider the spin-current Seebeck coefficient of our system in figure 2. $\mu_{\text{av}}$ can be conveniently chosen to be zero; thus we have $\Delta \mu_L = \mu_{L \uparrow} = -\mu_{L \downarrow}$, $\Delta \mu_R = \mu_{R \uparrow} = -\mu_{R \downarrow}$. We define spin bias as: $\Delta V_s = \Delta \mu_L - \Delta \mu_R = \mu_{L \uparrow} - \mu_{R \uparrow} = \mu_{R \downarrow} - \mu_{L \downarrow}$. As discussed in part I, $\Delta V_s$ will induce an electric voltage $V$ under open-circuit conditions. If we consider the linear response regime, i.e. $\Delta V_s \to 0$, we can keep only the first-order terms of equation (3), which yields the following expression for the spin-current Seebeck coefficient $S$ (see appendix):

$$
S = \lim_{\Delta V_s \to 0} - \frac{eV}{\Delta V_s} = \frac{\int \text{d}\omega [\frac{\partial}{\partial \omega}]_{\mu = \mu_R} \tau_{\uparrow}(\omega) - [\frac{\partial}{\partial \omega}]_{\mu = \mu_L} \tau_{\downarrow}(\omega)]}{\int \text{d}\omega [\frac{\partial}{\partial \omega}]_{\mu = \mu_R} \tau_{\uparrow}(\omega) + [\frac{\partial}{\partial \omega}]_{\mu = \mu_L} \tau_{\downarrow}(\omega)]}
$$

(11)

This formula plays the role of the starting point for the following numerical calculations.

3. Numerical results

In numerical investigation, we consider symmetric barriers: $\Gamma_L = \Gamma_R = \Gamma$ as mentioned above, and set $\Gamma = 1$ and $\mu_{\text{av}} = 0$ as the energy reference. Also we have chosen the atomic units for simplicity, where the electron charge, the electron mass and the Planck constant are set equal to one: $e = m_e = \hbar = 1$. Thus, the values of $U$, $V_s$ and $V_g$ are all given in units of $\Gamma$. In the linear regime, we can define $V_s = \mu_{L \uparrow} = \mu_{R \uparrow} = -\mu_{L \downarrow} = -\mu_{R \downarrow}$. Note that the linear regime means $\Delta V_s \to 0$, but $V_g$ does not necessarily tend to zero. Below, we also study a non-interacting QD, i.e. $U = 0$. In this case, the Green’s function of the QD in the presence of coupling can be exactly obtained as $G^0_s(e) = G^0_0(e) = 1/(e - \epsilon_{\text{d}} + i\Gamma)$; thus, the tunneling coefficient and spin-current Seebeck coefficient can be calculated straightforwardly. In an experimental setup, $\epsilon_{\text{d}}$ is tuned by gate voltage $V_g$, and $\epsilon_{\text{d}} = -V_g$. We first study the QD in the absence of external magnetic field; hence, the single-electron energy level is spin-degenerate.

We start our calculation from the case where $V_s = 0$. It is worth noting that in the traditional Seebeck effect, the temperature $T$ can never actually reach zero. Here, in the spin-current Seebeck effect, there is no problem with $V_s$ being zero. When $U = 0$, the spin-up and spin-down electrons are completely symmetric in the system; hence, $S = 0$. When $U$ is large enough, the QD will become spin-polarized and $S$ arises. For a real QD, $U$ is usually one order of magnitude larger than the linewidth $\Gamma$, which we have set as the energy scale in our system. Since the lead–QD–lead system can be modeled by an Anderson-type Hamiltonian, we learn that equations (5)–(10) can yield three sets of solutions for $n_{\uparrow}$, $n_{\downarrow}$: (i) $n_{\uparrow} > n_{\downarrow}$; (ii) $n_{\uparrow} > n_{\downarrow}$; (iii) $n_{\uparrow} < n_{\downarrow}$. Solution (i) represents a high-energy state of the system with the QD unpolarized, so $S = 0$. Solutions (ii) and (iii) are symmetric with opposite spin-polarization states of the QD, as shown in figure 3. It can be seen that the $S$–$V_g$ curves corresponding to the two different polarization states of the QD are opposite. This important feature indicates that: (1) $S$ is sensitive to the polarization state of the QD, and can be used to distinguish between state (i) and state (iii), while other quantities such as electric current and conductance cannot tell the difference; (2) when a transition from state (ii) to state (iii) occurs, $S$ will change sign; thus, one can monitor the transitions between the two states by measuring $S$. Although the value of the spin of a QD can be routinely identified from other methods such as Coulomb peak Zeeman shift [32], one is not able to monitor the transition between different polarization states of the QD as a function of time. However, it is possible to monitor the transition between polarization states of the QD as a function of time by measuring $S$, provided that the oscillation of tunneling current caused by spin flipping of the QD is not too fast [33, 34].

Next we consider the case where $V_s$ is small compared to the linewidth $\Gamma$. It can be readily learned from the Anderson model that the system has three sets of solutions when $V_s = 0$. We further show via a numerical approach that the system still has three sets of solutions when $V_s$ is non-zero: (i) $n_{\uparrow} \gtrsim n_{\downarrow}$; (ii) $n_{\uparrow} > n_{\downarrow}$; (iii) $n_{\uparrow} < n_{\downarrow}$. Hereafter we choose the solution $n_{\uparrow} > n_{\downarrow}$ for definiteness. Figure 4 shows the $S$ versus $V_g$ at different temperatures for $U = 0$ and $U = 30$, respectively. We find that $S$ is greatly enhanced when $U$ is large, by a maximum factor of 80 compared to the case when $U = 0$. Also, $S$ is more robust against
temperature increase at larger $U$. The reason is as follows. When $U = 0$, the QD is unpolarized, and the density of state (proportional to the transmission coefficient) is identical for spin-up and spin-down electrons in the QD. When $U \neq 0$, the transmission coefficient for electrons with different spins near the Fermi level will become rather different because of the spin polarization of the QD, and the difference will become more distinct as $U$ increases. To get a clearer view of the effect of $U$, we show the $S$ versus $V_g$ at different $U$ in figure 5, where we can see that $S$ is enhanced as $U$ increases. Moreover, the region with apparently non-zero $S$ corresponds to the Coulomb blockade regime, with occupation number of the QD $(n_\uparrow) + (n_\downarrow) \approx 1$; while the left and right regions with nearly zero $S$ correspond to an occupation number of 0 and 2 (the empty state and double-occupied state) respectively.

We further note that $S$ is always antisymmetric under various values of all parameters. This is due to the particle–hole symmetry of our system. More specifically, one can show that the Hamiltonian (2) is invariant under the following particle–hole transform: $d_\alpha \rightarrow d_\beta^\dagger$, $d_\alpha^\dagger \rightarrow -d_\beta$, provided that $2\epsilon_d + U = 0$, which is exactly the symmetric center of the $S$–$V_g$ curves shown above. Also, the Fermi distribution function of the leads: $f(\epsilon - \mu_\alpha) \rightarrow f(-\epsilon + \mu_\beta) = 1 - f(\epsilon - \mu_\beta)$. Thus, the electrons and holes with opposite spins are symmetric in our system, which yields the antisymmetric behavior of $S$.

As we have emphasized at the beginning of this part, although we take the limit $\Delta V_s \rightarrow 0$ in the linear regime, $V_s$ does not have to be small. Figure 6 takes into account the case where $V_s$ is large compared to $\Gamma$. Figures 6(a) and (b) depict the $S$ versus $V_g$ at different $V_s$, with the temperature $T = 0.1$ and $T = 2$ respectively. We find that $S$ is greatly enhanced when $V_s$ is enlarged; in addition, when $T$ is lower, $S$ begins to increase at smaller $V_s$. The reasons can be explained as follows. When $V_s$ is enlarged, the Fermi levels for different spins are well separated, which leads to a distinct difference in transmission coefficient for spin-up and spin-down electrons; hence, $S$ is enhanced. The excited states of electrons above the Fermi energy due to the increase of temperature will offset the separation of Fermi levels, so $S$ begins to increase at smaller $V_s$ at lower temperatures. It is interesting to note that, by comparing figure 6(a) with 6(c), new peaks begin to emerge at large $V_s$, in the presence of $U$. These new peaks emerge when one of the energy levels in the QD coincides with either Fermi level in the lead, while the other energy level in the QD as well as the other Fermi level is far away, so new peaks emerge only when both $V_s$ and $U$ are large. Figure 6(d) indicates that thermal fluctuations caused by increase of temperature will smear out all structures of peaks.

So far, we have been studying the QD in the absence of an external magnetic field; therefore, the single-particle energy level in the dot is spin-degenerate. When applying an external magnetic field $B$, $\epsilon_{d0} = \epsilon_d - \sigma B$ due to the Zeeman splitting. For simplicity, we consider only a non-interacting QD and set $U = 0$. Figure 7 shows $S$ at different $B$, with $V_s = 0$. From the discussions above, we learn that in the case of a non-interacting QD, $S \equiv 0$ when $V_s = 0$ and $B = 0$. Here we find that $S$ is greatly enhanced upon applying a magnetic field, even for a non-interacting QD at $V_s = 0$. This indicates that we are still able to achieve large $S$ for a QD whose intradot e–e interactions are negligible by applying an external magnetic field. In addition, it is worth noting that, compared to figure 3, $S$ is obviously non-zero here even in the regions with an occupation number of 0 and 2. This can be understood as follows. In figure 3 the tunneling coefficient for each spin has two resonant peaks, located at $\epsilon_d$ and $\epsilon_d + U$ respectively;
while in the case here the tunneling coefficient for each spin has only one resonant peak. Therefore, when the Fermi energy level is close to one resonant peak, the transmission coefficient for the other spin is much less, resulting in a non-zero $S$ in these regions.

Since the result for small $V_s$ is similar to $V_s = 0$, we hereby study the case when $V_s$ is large, as shown in figure 8. We find that no new peak emerges at large $V_s$ compared with figure 6(c), because there is only one resonant peak for the transmission coefficient for each spin. Also, $S$ is more robust against temperature in the existence of magnetic field.

4. Conclusions

We investigate the spin-current Seebeck effect in quantum dot systems. We show that the spin-current Seebeck coefficient $S$ is always antisymmetric, originating from the particle–hole symmetry of our system. Our results demonstrate that $S$ is sensitive to different polarization states of the QD, and thus can be used to distinguish between different states and monitor the transitions between them. The intradot e–e Coulomb interaction $U$ can greatly enhance $S$ due to the stronger polarization of the QD induced by $U$. For a typical QD whose $U$ is an order of magnitude larger than $\Gamma$, the
maximum $S$ can be enhanced by a factor of 80. For a QD whose intradot Coulomb interaction is negligible, our work demonstrates that a large $S$ can still be obtained by applying an external magnetic field.

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Appendix

In this appendix, we present the derivation of equation (11) in detail. Under open-circuit conditions, the net electric current $J_c$ must be zero, i.e.:

$$J_c = J_\uparrow + J_\downarrow = 0.$$ 

Recall from equation (3) the expression for electric current with spin $\sigma$ polarization:

$$J_\sigma = \frac{ie}{\hbar} \int \frac{d\epsilon}{2\pi} \left[ f_{L\sigma}(\epsilon) - f_{R\sigma}(\epsilon) \right] \beta(\epsilon),$$

where the Fermi distribution functions for the left and right leads with spin $\sigma$ are:

$$f_{L\sigma}(\epsilon) = \frac{1}{e^{\beta(\epsilon - \mu_{L\sigma})} + 1},$$

$$f_{R\sigma}(\epsilon) = \frac{1}{e^{\beta(\epsilon - \mu_{R\sigma} + eV)} + 1},$$

where $V$ is the induced electric voltage for the net current to be zero. Since we have set the average chemical potential in the leads $\mu_{avr} = 0$, the spin splitting of the chemical potential in the left and right leads is: $\Delta \mu_L = \mu_{L\uparrow} - \mu_{L\downarrow}$, $\Delta \mu_R = \mu_{R\uparrow} - \mu_{R\downarrow}$; thus, the spin bias $\Delta V_s$ is given by: $\Delta V_s \equiv \Delta \mu_L - \Delta \mu_R = \mu_{L\uparrow} - \mu_{R\uparrow} = -(\mu_{L\downarrow} - \mu_{R\downarrow})$. We can expand $f_{L\uparrow}(\epsilon)$ near $\mu = \mu_{R\uparrow}$, and in the linear response regime, we only keep the first-order term in the expansion:

$$f_{L\uparrow}(\epsilon) = f(\epsilon)_{\mu = \mu_{R\uparrow}} + \left( \frac{\partial f}{\partial \mu} \right)_{\mu = \mu_{R\uparrow}} (\mu_{L\uparrow} - \mu_{R\uparrow})$$

$$= f(\epsilon)_{\mu = \mu_{R\uparrow}} - \left( \frac{\partial f}{\partial \mu} \right)_{\mu = \mu_{R\uparrow}} \Delta V_s$$

where we have used the definition for $\Delta V_s$ and the fact: $\frac{\partial f}{\partial \mu} = -\frac{f}{\partial \mu}$. Similarly, we can expand $f_{R\uparrow}(\epsilon)$ near $V = 0$, i.e. $\mu = \mu_{R\uparrow}$:

$$f_{R\uparrow}(\epsilon) = f(\epsilon)_{\mu = \mu_{R\uparrow}} + \left( \frac{\partial f}{\partial \mu} \right)_{\mu = \mu_{R\uparrow}} (-eV)$$

$$= f(\epsilon)_{\mu = \mu_{R\uparrow}} - \left( \frac{\partial f}{\partial \mu} \right)_{\mu = \mu_{R\uparrow}} (-eV).$$

Plugging the above two expansions into the expression for electric current, we get:

$$J_\uparrow = \frac{ie}{\hbar} \int d\omega \left[ - \left( \frac{\partial f}{\partial \omega} \right)_{\mu = \mu_{R\uparrow}} \Delta V_s \right.$$

$$+ \left. \left( \frac{\partial f}{\partial \omega} \right)_{\mu = \mu_{R\uparrow}} (-eV) \right] T_\uparrow(\omega).$$

Similarly, we can get:

$$J_\downarrow = \frac{ie}{\hbar} \int d\omega \left[ \left( \frac{\partial f}{\partial \omega} \right)_{\mu = \mu_{R\downarrow}} \Delta V_s \right.$$

$$+ \left. \left( \frac{\partial f}{\partial \omega} \right)_{\mu = \mu_{R\downarrow}} (-eV) \right] T_\downarrow(\omega).$$

Using the above two expressions for electric currents with spin up and down, then letting $J_c = J_\uparrow + J_\downarrow = 0$, we can get equation (11), which is the expression for the spin-current Seebeck coefficient in the linear response regime.

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