Probing the heavy Higgs mass by the measurement of the Higgs decay branching ratios in the minimal supersymmetric standard model

Jun-ichi Kamoshita\textsuperscript{a}, Yasuhiro Okada\textsuperscript{a} and Minoru Tanaka\textsuperscript{b}

\textit{Theory Group, KEK, Tsukuba, Ibaraki 305, Japan\textsuperscript{a}}

and

\textit{Department of Physics, Osaka University, Toyonaka, Osaka 560, Japan\textsuperscript{b}}

\textbf{ABSTRACT}

We examine whether the parameters in the Higgs sector of the minimal supersymmetric standard model can be determined by detailed study of production cross section and decay branching ratios of the Higgs particle. Assuming that the lightest CP-even Higgs boson ($h$) is observed at a future $e^+e^-$ linear collider with $\sqrt{s} = 300 \sim 500$GeV, we show that the value of CP-odd scalar mass is determined from the ratio of the two branching ratios, $Br(h \rightarrow b\bar{b})$ and $Br(h \rightarrow c\bar{c}) + Br(h \rightarrow gg)$, almost independently of the stop mass scale.
In the search for the theory beyond the standard model (SM), the supersymmetric (SUSY) extension is considered to be an attractive and promising candidate. It is, therefore, important to investigate how the idea of SUSY can be explored in future collider experiments such as LHC and $e^+e^-$ linear colliders. For this purpose, the Higgs sector of the SUSY standard models can play a unique role. Since the Higgs sector has distinct features, its close investigation can give important information on these models.

In the minimal supersymmetric standard model (MSSM) the Higgs sector consists of two Higgs doublets, therefore, there exist five physical states, i.e. two CP-even Higgs($h, H$), one CP-odd Higgs($A$), and one pair of charged Higgs($H^\pm$). Since the form of Higgs potential is very restricted in the MSSM compared to general two-Higgs models, it is possible to derive specific predictions for this Higgs sector. For example, the upper bound on the lightest CP-even neutral Higgs mass is given as about 130GeV[1]. As for the discovery of the Higgs bosons at the future linear collider, it is shown that at least one of the CP-even Higgs bosons is detectable at an $e^+e^-$ linear collider with $\sqrt{s} = 300 \sim 500$GeV [2, 3].

Since the discovery of at least one Higgs boson is guaranteed, here we would like to address the question of to what extent the values of the parameters in the MSSM Higgs sector will be determined from the detailed study of the Higgs properties. This is especially important in the case when only the lightest CP-even Higgs is discovered at a future $e^+e^-$ linear collider with $\sqrt{s} = 300 \sim 500$GeV, since in such a case the behavior of the Higgs may well be quite similar to that of the minimal SM Higgs. In order to obtain useful information on the Higgs sector it is therefore necessary to measure the production cross section and/or decay branching ratios precisely and detect possible deviations of the Higgs’s properties from those of the SM Higgs. In the following we consider the determination of the parameters in the MSSM Higgs sector assuming that only the lightest CP-even Higgs is observed at a future $e^+e^-$ linear collider with $\sqrt{s} = 300 \sim 500$GeV. We show that the determination of the Higgs decay branching ratios in the charm and gluonic modes are important to constrain the masses of the heavy Higgs sector which
may not be observed during the earlier stages of the linear collider experiment. Therefore precise measurements of branching ratios including these modes are useful to set the next beam energy of the $e^+e^-$ linear collider at which the heavy Higgs bosons can be directly produced.

Let us begin by listing the parameters in the MSSM Higgs sector and the observables available in the experiment at the future $e^+e^-$ linear collider. Although at the tree level the masses and the mixings of the Higgs sector in this model are parametrized by two parameters, i.e. CP-odd scalar mass($m_A$) and the ratio of the vacuum expectation values($\tan \beta \equiv \frac{\langle H_2 \rangle}{\langle H_1 \rangle} = \frac{v_2}{v_1}$), the radiative correction to the Higgs potential brings new parameters into the discussion\[1\]. In the calculation of the Higgs effective potential at one loop level the most important contribution comes from top and stop loops, and therefore the relevant parameters are the two stop masses($m_{\tilde{t}_1}, m_{\tilde{t}_2}$), the higgsino mass parameter($\mu$) and the trilinear soft-breaking mass parameter($A_t$). For the moment, we assume that no significant effect is induced by the left-right mixing of the stop sector. Then, the Higgs sector is determined by three parameters, which we take to be $m_A$, $\tan \beta$ and the stop mass scale ($m_{\text{susy}} \equiv \sqrt{m_{\tilde{t}_1} m_{\tilde{t}_2}}$). Note that only this combination of the stop masses enters in the Higgs mass formulas through the radiative correction as long as the left-right mixing in the stop sector is neglected.

As for the observables, we assume that the lightest CP-even Higgs is produced through the Higgs bremsstrahlung process, $e^+e^- \rightarrow Z h$, and that the Higgs decay modes to the SUSY particles are not dominant. Then the main decay mode of the lightest Higgs is $h \rightarrow b\bar{b}$. With reasonable luminosity of the $e^+e^-$ linear collider($\sim 50 \text{fb}^{-1}\text{/year}$) we should be able to determine the mass of the Higgs to within a few percent\[2, 3\]. We can also expect to measure production cross section multiplied by the $h \rightarrow b\bar{b}$ branching ratio, $\sigma(e^+e^- \rightarrow Z h) \cdot Br(h \rightarrow b\bar{b})$, to within a few percent \[2, 3\]. Other measurable quantities are

\[1\]We can also measure the production cross section $\sigma(e^+e^- \rightarrow Z h)$ by recoil mass distribution independently of the Higgs decay modes. This may give additional information, but the following discussion does not depend on the availability of this quantity.
branching ratios\cite{4,5}. The lightest CP-even Higgs has sizable decay branching ratios in the modes $h \to b\bar{b}$, $\tau\bar{\tau}$, $c\bar{c}$ and $gg$\cite{6,7,8}. Since the $hb\bar{b}$ and $h\tau\bar{\tau}$ couplings originate from the Yukawa couplings with the same Higgs doublet, the ratio $Br(h \to \tau\bar{\tau})/Br(h \to b\bar{b})$ is the same as in the SM, and therefore no information on the parameters, $m_A$, $\tan\beta$ and $m_{\text{susy}}$ is obtained.\footnote{As stressed in Ref.\cite{4}, the $h \to WW^{(*)}$ mode is important to distinguish the MSSM Higgs from SM Higgs for $m_h \gtrsim 120\text{GeV}$. Since the branching ratio depends crucially on the Higgs mass, we will not consider this mode here.} On the other hand, the ratio $Br(h \to c\bar{c})/Br(h \to b\bar{b})$ depends on these parameters since the charm and bottom couplings to the Higgs boson come from Yukawa couplings with different Higgs doublets. The dependence of $Br(h \to gg)/Br(h \to b\bar{b})$ on the Higgs parameters is the same as that of $Br(h \to c\bar{c})/Br(h \to b\bar{b})$ in a good approximation since the dominant contribution to $h \to gg$ is almost always induced by the top quark loop. In the following we will consider the quantity

$$R_{br} \equiv \frac{Br(h \to c\bar{c}) + Br(h \to gg)}{Br(h \to b\bar{b})}. \quad (1)$$

It turns out to be possible to determine the sum of the charm and gluonic branching ratios to a reasonable precision($\pm20 \sim 25\%$) by the future experiment although it is very difficult to measure two branching ratios separately with enough precision\footnote{This ratio is important to determine the bottom mass as we discuss later.}.\footnote{In general, there could be two solutions for $\tan\beta$. Such a multiple solution, however, occurs only when $m_h \lesssim 80\text{GeV}$ and $m_A \lesssim 150\text{GeV}$.}

Let us now discuss how these three parameters, $m_A$, $\tan\beta$ and $m_{\text{susy}}$, will be determined from the above observables. Since we assume that one CP-even Higgs is observed at the $e^+e^-$ linear collider, the mass of the Higgs is supposed to be known precisely. We then can solve for one of the three parameters in terms of the other two using the formula for the lightest CP-even Higgs mass. We here solve $\tan\beta$ as a function of $m_A$ and $m_{\text{susy}}$ and try to determine these two parameters from the measurements of the production cross section and the branching ratios.\footnote{In general, there could be two solutions for $\tan\beta$. Such a multiple solution, however, occurs only when $m_h \lesssim 80\text{GeV}$ and $m_A \lesssim 150\text{GeV}$.}

The formulas for the partial decay width of the MSSM Higgs is found in Ref.\cite{6}. QCD corrections are important for the $h \to q\bar{q}$ mode\cite{9,10} as well as the $h \to gg$ mode\cite{11,12}. For the $h \to q\bar{q}$ partial width we use the formula given
in Ref.[10] where $O(\alpha_s^2)$ and $\frac{m_t^2}{m_h}$ corrections are taken into account. As for the $h \to gg$ mode we use the next to leading QCD formula for top loop diagrams[11],

$$\Gamma(h \to gg) = \Gamma_{LO}[\alpha_s(m_h)] \left(1 + \left(\frac{95}{4} - \frac{7}{6}n_F\right)\frac{\alpha_s(m_h)}{\pi}\right),$$  \hspace{1cm} (2)$$

where the leading order formula $\Gamma_{LO}$ is found in Ref.[13] and $n_F = 5$. QCD correction in this formula corresponds to the case where Higgs mass is far below the $t\bar{t}$ threshold. For the Higgs mass region considered here this is a good approximation. For the other quark’s loop we use the leading order result. Although the $b$ quark loop can have a sizable contribution for large $\tan\beta$, the branching ratio of $h \to gg$ is suppressed in such a case.

In the parameter region considered in this paper contribution from the stop loop is very small and therefore is neglected. Figure 1 (a) shows the contour plot of $R_{br}$ for $m_h = 120$ GeV. Here and in the following discussion we take the top mass ($m_t$) as 170 GeV, the MS running quark masses for charm and bottom as $\tilde{m}_c(m_c) = 1.2$ GeV, $\tilde{m}_b(m_b) = 4.2$ GeV and the strong coupling constant as $\alpha_s(m_Z) = 0.12$. Ambiguities associated with these inputs are discussed later. A striking feature of this plot is that $R_{br}$ is almost independent of $m_{susy}$. This property is useful for constraining the value of $m_A$. In figure 1 (b) the contour plot of $\sigma(e^+e^- \to Zh) \cdot Br(h \to b\bar{b})$ are shown for $\sqrt{s} = 300$GeV and $m_h = 120$ GeV. Contrary to the case of $R_{br}$ the constraint obtained from $\sigma(e^+e^- \to Zh) \cdot Br(h \to b\bar{b})$ depends on both $m_A$ and $m_{susy}$. It may be possible to detect the deviation from the SM Higgs by measuring this quantity to within a few percent[2, 3, 4], however, it is difficult to obtain the constraint on $m_A$ independently of $m_{susy}$.

We can explain the independence of $R_{br}$ from $m_{susy}$ in the following way. The CP-even Higgs mass matrix is given by

$$M^{2}_{ns} = \begin{pmatrix} m_A^2 \sin^2 \beta + m_Z^2 \cos^2 \beta & -(m_A^2 + m_Z^2) \cos \beta \sin \beta \\ -(m_A^2 + m_Z^2) \cos \beta \sin \beta & m_A^2 \cos^2 \beta + m_Z^2 \sin^2 \beta + \frac{\delta}{\sin^2 \beta} \end{pmatrix},$$  \hspace{1cm} (3)$$

where

$$\delta = \frac{3}{4\pi^2} \frac{m_t^4}{v^2} \ln \left(\frac{m_{susy}^2}{m_t^2}\right),$$  \hspace{1cm} (4)$$

5
represents the top-stop loop effect in the calculation of the Higgs effective potential. From this matrix the Higgs mixing angle, $\alpha$, is given by

$$\tan \alpha = \frac{(m_Z^2 + m_A^2) \sin \beta \cos \beta}{m_h^2 - (m_Z^2 \cos^2 \beta + m_A^2 \sin^2 \beta)}.$$  \hspace{1cm} (5)

Since the dependences of the branching ratios on the angles $\alpha$ and $\beta$ are given by

$$\text{Br}(h \rightarrow b\bar{b}) \propto \frac{\sin^2 \alpha}{\cos^2 \beta}, \quad \text{Br}(h \rightarrow c\bar{c}) \propto \frac{\cos^2 \alpha}{\sin^2 \beta},$$  \hspace{1cm} (6)

the ratio of $\text{Br}(h \rightarrow b\bar{b})$ and $\text{Br}(h \rightarrow c\bar{c})$ is proportional to

$$\frac{\text{Br}(h \rightarrow c\bar{c})}{\text{Br}(h \rightarrow b\bar{b})} \propto \left( \frac{1}{\tan \beta \tan \alpha} \right)^2.$$  \hspace{1cm} (7)

By using Eq.(8), $\frac{1}{\tan \beta \tan \alpha}$ is rewritten as

$$\frac{1}{\tan \beta \tan \alpha} = \frac{m_h^2 - m_A^2}{m_Z^2 + m_A^2} \left(1 + \frac{m_h^2 - m_Z^2}{m_h^2 - m_A^2} \frac{1}{\tan^2 \beta}\right),$$  \hspace{1cm} (8)

where the $m_{\text{susy}}$ dependence is implicit in $\tan \beta$. Since the second term in the parenthesis in Eq.(8) is negligible for $m_A^2 \gg m_h^2 \sim m_Z^2$, Eq.(8) is approximately given by

$$\frac{1}{\tan \beta \tan \alpha} \approx \frac{m_h^2 - m_A^2}{m_Z^2 + m_A^2}.$$  \hspace{1cm} (9)

This explains the independence from $m_{\text{susy}}$ for the ratio of the $h \rightarrow b\bar{b}$ and $h \rightarrow c\bar{c}$ branching ratios. Under the present assumption that the only lightest CP-even Higgs boson is discovered at an $e^+e^-$ linear collider with $\sqrt{s} = 300 \sim 500\text{GeV}$, the above mass relations among $m_A, m_h$, and $m_Z$ are naturally satisfied. For the $h \rightarrow gg$ mode, the situation is similar to $\text{Br}(h \rightarrow c\bar{c})/\text{Br}(h \rightarrow b\bar{b})$ because the dominant contribution to the $h \rightarrow gg$ mode almost always comes from the top-loop diagram and $\text{Br}(h \rightarrow gg)$ has the same $\alpha$ and $\beta$ dependence as $\text{Br}(h \rightarrow c\bar{c})$. This is the reason why $R_{br}$ is almost independent of $m_{\text{susy}}$.

Let us next consider how well $m_A$ will be constrained from the measurement of $R_{br}$. In figure 2 (a) we show the $R_{br}$ as a function of $m_A$ for $m_{\text{susy}} = 0.75, 1, 5, 10\text{ TeV}$ and
$m_h = 120$ GeV. Figure 2 (b) corresponds to the case for $m_{susy} = 0.5, 1, 2, 10$ TeV and $m_h = 100$ GeV. In these figures some of the lines terminate because we cannot obtain a solution beyond the end point for the assumed Higgs mass. Although $R_{br}$ becomes the SM value in the limit $m_A \to \infty$, we can see that the ratio is about 20% smaller than the SM value even at $m_A = 400$ GeV. Note that the direct search for the CP-odd Higgs can exclude only the mass region approximately half of the $\sqrt{s}$ (i.e. $m_A \lesssim 250$ GeV for $\sqrt{s} = 500$ GeV) since the associated production of $A$ and the heavy CP-even Higgs $H$ is the only practical production mechanism in this region. Therefore $R_{br}$ can be a good probe into the heavy Higgs bosons in the mass region larger than $\frac{\sqrt{s}}{2}$. For example, if the experimental result is given by $R_{br} = 0.10 \pm 0.02$, we will be able to constrain the value of $m_A$ to $260$ GeV $\lesssim m_A \lesssim 400$ GeV. On the other hand, in the case that the branching ratio is consistent with the SM value, the lower bound on $m_A$ may be obtained.

In the above discussion we have assumed that the left-right mixing in the stop sector is negligible. This is especially used in deriving Eq.8. To see how the above results depend on the mixing effects we have calculated $R_{br}$ using a one loop potential including the trilinear soft breaking term for the stop ($A_t$ term) as well as the supersymmetric higgsino mass term ($\mu$ term) as in Ref.[14]. Examples are shown in figure 3. We can see that for reasonable values of $A_t$ and $\mu$, $R_{br}$ does not strongly depend on the parameters in the stop sector. Sbottom loops can also affect the Higgs mass formulas for large values of $\tan \beta$. As long as the left-right mixing in the sbottom sector is neglected, this effect is negligible for $R_{br}$. In such a case a term $\frac{\delta_b}{\cos^2 \beta}$ is added in the (1,1) component of the CP-even Higgs matrix in Eq.3 where $\delta_b = \frac{3}{4 \pi^2} m_b^2 \ln \frac{m_{stop}}{m_b}$, and Eq.8 is modified as

$$\frac{1}{\tan \beta \tan \alpha} = \frac{m_h^2 - m_A^2}{m_Z^2 + m_A^2} \left( 1 + \frac{m_h^2 - m_A^2}{m_Z^2} \frac{1}{\tan^2 \beta} - \frac{\delta_b}{m_h^2 - m_A^2} \frac{(1 + \tan^2 \beta)^2}{\tan^2 \beta} \right)$$

(10)

Even for $\tan \beta \approx \frac{m_t}{m_b}$ the last term in the parentheses is suppressed by $\frac{3}{4 \pi^2} \frac{m_t^2}{m_h^2 - m_A^2}$ and $R_{br}$.

\footnote{As pointed out in Ref.[15], it is possible to suppress the $b\bar{b}b\bar{b}$ coupling using the left-right mixing effect. But this only occurs for special choices of parameters for which the effect of $A_t$ and $\mu$ is large enough to cancel the off-diagonal term in Eq.3.}
is almost independent of the SUSY breaking scale.

So far we have neglected uncertainties in the quark masses and the strong coupling constant in the calculation of the branching ratios. The precise determination of the $\overline{\text{MS}}$ running quark mass ratio $\frac{\bar{m}_c^2(m_h)}{\bar{m}_b^2(m_h)}$ would be especially important in the calculation of $R_{br}$. For this purpose we need input parameters for charm and bottom quark masses at some renormalization scale. We may use results from several non-perturbative methods such as lattice QCD, QCD sum rule [16] and the heavy quark effective theory (HQET). As an illustration we estimate the uncertainties in $\frac{\bar{m}_c^2(m_h)}{\bar{m}_b^2(m_h)}$ using the results of lattice QCD and HQET. In Ref. [17] the $\overline{\text{MS}}$ bottom quark mass at the bottom scale is given as $\bar{m}_b(m_b) = (4.17 \pm 0.05 \pm 0.03)\text{GeV}$ by lattice QCD. On the other hand HQET gives the difference of the charm and bottom quark mass, which should be understood as pole masses, as $\Delta M_{bc} = M_b - M_c = (3.40 \pm 0.03 \pm 0.03)\text{GeV}$ [18]. Note that although the pole mass itself has no physical meaning in QCD the difference does have at least in the leading order of the $1/m$ expansion [19]. From these two inputs and the two-loop relation between the perturbative pole mass and the $\overline{\text{MS}}$ mass [20, 21], $M_Q = \overline{m}_Q(M_Q) \left(1 + \frac{3}{\pi} \frac{\alpha_s(M_Q)}{\pi}\right)$, we can calculate the $\overline{\text{MS}}$ running quark masses. Using $\alpha_s(m_Z) = 0.117 \pm 0.006$ [23] in addition to the above two inputs we obtain

$$\frac{\bar{m}_c^2(m_h)}{\bar{m}_b^2(m_h)} = 0.026 \pm 0.004 \pm 0.004 \pm 0.003,$$  \hspace{1cm} (11)

where the first (second, third) error comes from $\alpha_s(m_Z)$ ($m_b, \Delta M_{bc}$). Uncertainty in $Br(h \to gg)/Br(h \to b\bar{b})$ comes from top and bottom masses and $\alpha_s$ as well as the next-to-next-to-leading order QCD corrections in the gluonic partial width. If the top and bottom quark masses are known precisely, the strong coupling constant gives the largest uncertainty. Varing the strong coupling constant as $\alpha_s(m_Z) = 0.117 \pm 0.006$, the gluonic partial width changes by $\pm 12\%$.  \hspace{1cm} [6]

\footnote{Although a three-loop result has already appeared in the literature [22], we use the two-loop results throughout this illustration for the consistency with the remaining part of the calculation.}

\footnote{Recently, it is pointed out that the uncertainty in the strong coupling constant causes errors in}
Although the present uncertainty is relatively large, we can expect theoretical and experimental improvements in future. For example the $\alpha_s$ measurement at the $t\bar{t}$ threshold is expected to reduce the error in $\alpha_s(m_Z)$ by factor of 3 at the $e^+e^-$ linear collider experiment[25]. Also direct measurement of the MS running bottom quark mass at the scale of $m_h$ may be possible from the measurement of $\frac{Br(h \rightarrow \tau\bar{\tau})}{Br(h \rightarrow b\bar{b})}[2]$.

To summarize, we have examined whether the parameters in the MSSM Higgs sector can be determined by detailed study of the Higgs properties. We pointed out that the ratio of $Br(h \rightarrow c\bar{c}) + Br(h \rightarrow gg)$ and $Br(h \rightarrow b\bar{b})$ is sensitive to the heavy Higgs mass scale but its dependence on the stop mass scale is very weak. Therefore, if this ratio is measured with enough precision at the future $e^+e^-$ linear collider, we may be able to constrain the heavy Higgs mass scale even if the heavy Higgs bosons cannot be produced directly.

The authors would like to thank A. Djouadi, K. Hagiwara, K. Kawagoe, I. Nakamura, M. Peskin and P.M. Zerwas for useful discussions. They also wish to thank B. Bullock and K. Hikasa for reading the manuscript and useful comments. This work is supported in part by the Grant-in-aid for Scientific Research from the Ministry of Education, Science and Culture of Japan.

---

the theoretical predictions of the branching ratios of the modes $h \rightarrow c\bar{c}, gg$ as large as 20 % [24]. Our estimation is consistent with their result. As is stated in the text, however, this uncertainty in the strong coupling constant could reduce much in the linear collider era.
References

[1] Y. Okada, M. Yamaguchi and T. Yanagida, *Prog. Theor.Phys.* **85** (1991) 1; *Phys.Lett. B262* (1991) 54; J. Ellis, G. Ridolfi and F. Zwirner, *Phys.Lett. B257* (1991) 83; H.E. Haber and R. Hempfling, *Phys.Rev.Lett.* **66** (1991) 1815.

[2] JLC-I, KEK Report 92-16, December 1992.

[3] P. Janot in *Proceedings of the Workshop on Physics and Experiments with Linear e^+e^- Colliders*, Waikola, Hawaii, 1993 edited by F. Harris *et al.* (World Scientific, Singapore, 1993).

[4] M.D. Hildreth, T.B. Barklow and D.L. Burke, *Phys.Rev. D49* (1994) 3441.

[5] I. Nakamura and K. Kawagoe, to be published in *Proceedings of the Workshop on Physics and Experiments with Linear e^+e^- Colliders*, Morioka-Appi, Japan, September 8-12, 1995.

[6] V. Barger, M.S. Berger, A.L. Stange and P.J.N. Phillips, *Phys.Rev. D45* (1992) 4128.

[7] A. Yamada, *Mod.Phys.Lett. A7* (1992) 2877.

[8] S. Moretti and W.J. Stirling, *Phys.Lett. B347* (1995) 291.

[9] E. Braaten, J.P. Leveille, *Phys.Rev. D22* (1980) 715; N. Sakai, *Phys.Rev. D22* (1980) 2220; M. Drees, K. Hikasa, *Phys.Lett. B240* (1990) 455.

[10] S.G. Gorishny, A.L. Kataev, S.A. Larin and L.R. Surguladze, *Phys.Rev. D43* (1991) 1633; L.R. Surguladze, *Phys.Lett. B341* (1995) 60.

[11] T. Inami, T. Kubota and Y. Okada, *Z.Phys. C18* (1983) 69; A. Djouadi, M. Spira and P.M. Zerwas, *Phys.Lett. B264* (1991) 440.

[12] M. Spira, A. Djouadi, D. Graudenz and P.M. Zerwas, *Nucl.Phys. B453* (1995) 17.

[13] J.F. Gunion, H.E. Haber, G.L. Kane and S. Dawson, The Higgs hunter’s guide (Addison-Wesley, 1990).

[14] J. Ellis, G. Ridolfi and F. Zwirner, *Phys.Lett. B262* (1991) 477.

[15] G.L. Kane, G.D. Kribs, S.P. Martin and J.D. Wells, [hep-ph/9508265](https://arxiv.org/abs/hep-ph/9508265)

[16] S. Narison, *Phys. Lett. B341* (1994) 73.

[17] M. Crisafulli, V. Giénez, G. Martinelli and C.T. Sachrajda, preprint CERN-TH.7521/94, ROME prep. 94/1071, SHEP 94/95-14, [hep-ph/9506210](https://arxiv.org/abs/hep-ph/9506210).
[18] M. Neubert, preprint CERN-TH/95-107, [hep-ph/9505238].

[19] C.T. Sachrajda, Southampton Preprint SHEP-95/32, [hep-lat/9509085], and references therein.

[20] R. Tarrach, Nucl. Phys. B183 (1981) 384.

[21] J. Gasser and H. Leutwyler, Phys. Rep. 87 (1982) 77.

[22] N. Gray, D.J. Broadhurst, W. Grafe and K. Schilcher, Z. Phys. C48 (1990) 673.

[23] S. Bethke, preprint PITHA 95/14.

[24] A. Djouadi, M. Spira and P.M. Zerwas, preprint DESY 95-210, KA-TP-8-95, [hep-ph/9511344].

[25] K. Fujii, T. Matsui and Y. Sumino, Phys. Rev. D50 (1994) 4341.
Figure Captions

Fig.1: (a) $R_{br} \equiv \frac{Br(h \rightarrow c\bar{c}) + Br(h \rightarrow gg)}{Br(h \rightarrow b\bar{b})}$ in the parameter space of the CP-odd Higgs mass ($m_A$) and the stop mass scale ($m_{susy}$) for the lightest CP-even Higgs mass $m_h = 120$ GeV. We have used $m_t = 170$ GeV, $\tilde{m}_c(m_c) = 1.2$ GeV, $\tilde{m}_b(m_b) = 4.2$ GeV and $\alpha_s(m_Z) = 0.12$. (b) $\sigma(e^+e^- \rightarrow Zh) \cdot Br(h \rightarrow b\bar{b})$ for $\sqrt{s} = 300$ GeV. Other parameters are the same as in (a).

Fig.2: $R_{br}$ as a function of $m_A$ for several values of $m_{susy}$ for $m_h = 120$ GeV (a) and for $m_h = 100$ GeV (b). Other input parameters are the same as those for figure 1 (a).

Fig.3: $R_{br}$ as a function of $m_A$ including the left-right mixing effects of two stops. We take $m_h = 120$ GeV and two stop masses as $m_{\tilde{t}_1} = 1$ TeV and $m_{\tilde{t}_2} = 700$ GeV. The values shown in the parentheses represent ($A_t, \mu$) in GeV.
Fig. 1(a)

Fig. 1(b)
Fig. 2(a)

Fig. 2(b)
Fig. 3