CP Violation in Hyperon Decays\textsuperscript{1}

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Abstract

The theory and phenomenology of CP violation in hyperon decays is summarized.

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1 Introduction

The CPT theorem was proved in 1955\cite{1} and soon thereafter Lüders and Zumino\cite{2} deduced from it the equality of masses and lifetimes between particles and anti-particles. In 1958 Okubo\cite{3} observed that CP violation allows hyperons and antihyperons to have different branching ratios into conjugate channels even though their total rates must be equal by CPT. Somewhat later, this paper inspired Sakharov\cite{4} to his famous work on cosmological baryon-antibaryon asymmetry. Pais\cite{5} extended Okubo’s proposal to asymmetry parameters in Λ and ¯Λ decays. The subject was revived in the ’80s and a number of calculations were made\cite{6,7}. Only now, over 40 years after Okubo’s paper, are these proposals about to be tested in the laboratory.

The reason for the current interest is the need to find CP violation in places other than just $K_L - K_S$ complex. Only a number of different observations of CP violation in different channels will help us pin down the source and nature of CP violation in or beyond the standard model (SM). From this point of view hyperon decay is one more weapon in our arsenal in addition to the K system, the B system, the D system, etc.

2 Phenomenology of Hyperon Decays

I summarize here the salient features of the phenomenology of non-leptonic hyperon decays \cite{8}. Leaving out $\Omega^-$ decays, there are seven decay modes $\Lambda \to N\pi$, $\Sigma^\pm \to N\pi$ and $\Xi \to \Lambda\pi$. The effective matrix element can be written as

$$i \bar{u}_\rho (a + b\gamma_5)u_\Lambda \phi$$

(1)
for the mode \( \Lambda \rightarrow p + \pi^- \), where \( a \) and \( b \) are complex in general. The corresponding element for \( \bar{\Lambda} \rightarrow \bar{p} + \pi^+ \) is then:

\[
i \bar{v}_p (-a^* + b^* \gamma_5) v_\Lambda \phi^+ \tag{2}\]

It is convenient to express the observables in terms of \( S \) and \( P \) and write the matrix element as

\[
S + P \sigma \hat{q} \tag{3}\]

where \( q \) is the proton momentum in the \( \Lambda \) rest frame and \( S \) and \( P \) are:

\[
S = a \sqrt{\frac{(m_\Lambda + m_p)^2 - m_\pi^2}{16\pi m_\Lambda^2}} \]

\[
P = b \sqrt{\frac{(m_\Lambda - m_p)^2 - m_\pi^2}{16\pi m_\Lambda^2}} \tag{4}\]

In the \( \Lambda \) rest-frame, the decay distribution is given by:

\[
\frac{d\Gamma}{d\Omega} = \frac{\Gamma}{8\pi} \left\{ 1 + \alpha < \sigma_\Lambda > \cdot \hat{\sigma} \right\} + < \sigma_p > \cdot [(\alpha + < \sigma_\Lambda > \cdot \hat{\sigma}) \hat{q} + \beta < \sigma_\Lambda > \times \hat{q} + \gamma (\hat{q} \times (\sigma_\Lambda > \times \hat{q}))] \}
\]

where \( \Gamma \) is the decay rate and is given by:

\[
\Gamma = 2 |q| \left\{ |S|^2 + |P|^2 \right\} \tag{6}\]

\( \alpha, \beta \) and \( \gamma \) are given by

\[
\alpha = \frac{2 Re (S^* P)}{\{ |S|^2 + |P|^2 \}},
\]

\[
\beta = \frac{2 Im (SP^*)}{\{ |S|^2 + |P|^2 \}}.
\]

\[
\gamma = \frac{\{ |S|^2 - |P|^2 \}}{\{ |S|^2 + |P|^2 \}} \tag{7}\]
For a polarized \( \Lambda \), the up-down asymmetry of the final proton is given by
\[
\alpha \quad (\alpha \text{ is also the longitudinal polarization of the proton for an unpolarized } \Lambda).
\]
\( \beta \) and \( \gamma \) are components of the transverse polarization of proton \[^9\].

The observed properties of the hyperon decays can be summarised as:
(i) the \( \Delta I = 1/2 \) dominance i.e. the \( \Delta I = 3/2 \) amplitudes are about 5% of the \( \Delta I = 1/2 \) amplitudes; (ii) the asymmetry parameter \( \alpha \) is large for \( \Lambda \) decays, \( \Xi \) decays and \( \Sigma^+ \rightarrow p\pi^0 \) and is near zero for \( \Sigma^\pm \rightarrow n\pi^\pm \); and (iii) the Sugawara-Lee triangle sum rule
\[
\sqrt{3} A(\Sigma^+ \rightarrow p\pi^0) - A(\Lambda \rightarrow p\pi^-) = 2A(\Xi \rightarrow \Lambda\pi^-) \text{ is satisfied to a level of 5% in both } s \text{ and } p \text{ wave amplitudes.}
\]

### 3 CP Violating Observables

Let a particle \( P \) decay into several final states \( f_1, f_2 \) etc. The amplitude for
\( P \rightarrow f_1 \) is in general:
\[
A = A_1 e^{i\delta_1} + A_2 e^{i\delta_2} \tag{8}
\]

where 1 and 2 are strong interaction eigenstates and \( \delta_i \) are corresponding
final state phases. Then the amplitude for \( \bar{P} \rightarrow \bar{f}_1 \) is
\[
\bar{A} = A_1^* e^{i\delta_1} + A_2^* e^{i\delta_2} \tag{9}
\]

If \( |A_1| >> |A_2| \), then the rate asymmetry \( \Delta = (\Gamma - \bar{\Gamma})/(\Gamma + \bar{\Gamma}) \) is given
by:
\[
\Delta \approx -2 \frac{|A_2/A_1| \sin(\phi_1 - \phi_2)\sin(\delta_1 - \delta_2)}{\sin(\phi_1 - \phi_2)\sin(\delta_1 - \delta_2)} \tag{10}
\]

where \( A_i = |A_i| e^{i\phi_i} \). Hence, to get a non-zero rate asymmetry, one must
have (i) at least two channels in the final state, (ii) CPV weak phases must
be different in the two channels, and (iii) unequal final state scattering
phase shifts in the two channels\[^6\]. A similar calculation of the asymmetry
of \( \alpha \) shows that for a single isospin channel dominance,

\[
A = (\alpha + \bar{\alpha})/(\alpha - \bar{\alpha}) = 2\tan(\delta_s - \delta_p) \sin(\phi_s - \phi_p)
\]  

(11)

In this case the two channels are orbital angular momentum 0 and 1; and even a single isospin mode such as \( \Xi^- \to \Lambda \pi^- \) can exhibit a non-zero \( A \). In \( B \) decays an example of a single isospin mode exhibiting CP violating rate asymmetry is \( B \to \pi\pi \), i.e. In this case the two eigen-channels with different weak CP phases and different final state phases are \( B \to D\bar{D}\pi \to \pi\pi \) and \( B \to \pi\pi \to \pi\pi \).

To define the complete set of CP violating observables, consider the example of the decay modes \( \Lambda \to p\pi^- \) and \( \bar{\Lambda} \to \bar{p}\pi^+ \). The amplitudes are:

\[
S = -\sqrt{\frac{2}{3}} S_1 e^{i(\delta_1 + \phi_s^i)} + \frac{1}{\sqrt{3}} S_3 e^{i(\delta_3 + \phi_s^i)}
\]

\[
P = -\sqrt{\frac{2}{3}} P_1 e^{i(\delta_{11} + \phi_p^i)} + \frac{1}{\sqrt{3}} P_3 e^{i(\delta_3 + \phi_s^i)}
\]  

(12)

where \( S_i, P_i \) are real, \( i \) refers to the final state isospin (\( i=2I \)) and \( \phi_i \) are the CPV phases. With the knowledge that \( S_3/S_1, P_3/P_1 << 1 ; \) one can write\[12, 13\]

\[
\Delta_\Lambda = (\Gamma - \bar{\Gamma})/(\Gamma + \bar{\Gamma}) \approx \sqrt{2}(S_3/S_1)\sin(\delta_3 - \delta_1)\sin(\phi_3^s - \phi_1^s)
\]

\[
A_\Lambda = (\alpha + \bar{\alpha})/(\alpha - \bar{\alpha}) \approx -\tan(\delta_{11} - \delta_1)\sin(\phi_p^p - \phi_{1}^s)
\]

\[
B_\Lambda = (\beta + \bar{\beta})/(\beta - \bar{\beta}) \approx \cot(\delta_{11} - \delta_1)\sin(\phi_p^p - \phi_1^s)
\]  

(13)

For \( \pi N \) final states, the phase shifts at \( E_{c.m.} = m_\Lambda \) are known and are:

\( \delta_1 = 6^0, \ \delta_3 = -3.8^0, \ \delta_{11} = 1.1^0 \) and \( \delta_{31} = -0.7^0 \). The CPV phases \( \phi_i \) have to be provided by theory.

Similar expressions can be written for other hyperon decays. For example, for \( \Lambda \to n\pi^0 \), \( \Delta \) is \(-2\Delta_\Lambda \) and \( A \) and \( B \) are identical to \( A_\Lambda \) and \( B_\Lambda \).
For $\Xi^- \to \Lambda\pi^-$ (and $\Xi^0 \to \Lambda\pi^0$) the asymmetries are\[13\]:

\[
\Delta_{\Xi} = 0
\]

\[
A_{\Xi} = -\tan(\delta_{21} - \delta_2) \sin(\phi^p - \phi^s)
\]

\[
B_{\Xi} = \cot(\delta_{21} - \delta_2) \sin(\phi^p - \phi^s)
\]

(14)

where $\delta_{21}$ and $\delta_2$ are the $p$ and $s$-wave $\Lambda\pi$ phase shifts at $m_{\Xi}$ respectively.

Somewhat more complicated expressions can be and have been written for $\Sigma$ decays\[13\]. There are no experimental proposals at the moment for measuring these.

4 Calculating CP Phases

In standard model description of the non-leptonic hyperon decays, the effective $\Delta S = 1$ Hamiltonian is

\[
H_{\text{eff}} = \frac{G_F}{\sqrt{2}} U_{ud}^* U_{us} \sum_{i=1}^{12} c_i(\mu) O_i(\mu)
\]

(15)

after the short distance QCD corrections (LLO + NLLO) where $c_i = z_i + y_i\tau (\tau = -U_{td} U_{ts}^*/U_{ud} U_{us})$, and $\mu \sim 0(1 \text{ GeV})$. For CP violation, the most important operator is:

\[
O_6 = \bar{d} \lambda_i \gamma_\mu (1 + \gamma_5) s q \lambda_i \gamma_\mu (1 - \gamma_5) q
\]

(16)

and $y_6 \approx -0.1$ at $\mu \sim 1 \text{ GeV}$. To estimate the CP phases in Eq. (12), one adopts the following procedure. The real parts(in the approximation that the imaginary parts are very small) are known from the data on rates and asymmetries. The real parts of the amplitudes have also been evaluated in SM with reasonable success with some use of chiral perturbation theory(current algebra and soft pion theorems) and a variety of choices for the
baryonic wave functions. The MIT bag model wave function is one such choice which gives conservative results. The same procedure is adopted for calculating the imaginary parts using $O_6$. The major uncertainty is in the hadronic matrix elements and the fact that the simultaneous fit of S and P waves leaves a factor of 2 ambiguity [14]. In the SM, with the Kobayashi-Maskawa phase convention there is no CPV in $\Delta I = 3/2$ amplitudes; and for $\Lambda$ decays $\phi_3 = 0$. There is a small electroweak penguin contribution to $\phi_3$ which is neglected. The rate asymmetry is dominated by the s wave amplitudes and the asymmetry $A_\Lambda$ is dominated by the $\Delta I = 1/2$ amplitudes. Evaluating the matrix elements in the standard way and with the current knowledge of the K-M matrix elements one finds for the decays [13, 15]

$\Lambda \to p\pi^-$ and $\Xi^- \to \Lambda\pi^-$:

$$\phi_s^\Lambda - \phi_p^\Lambda \simeq 3.5 \times 10^{-4}$$

$$\phi_s^\Xi - \phi_p^\Xi \simeq -1.4 \times 10^{-4}$$

(17)

With the $N\pi$ phase shifts known to be

$$\delta_s - \delta_p \simeq 7^0$$

(18)

one finds for the asymmetry $A_\Lambda$ in the standard model a value of about $-4.10^{-5}$. For the $\Xi \to \Lambda\pi^-$ decay mode the phase shifts are not known experimentally and have to be determined theoretically. There are calculations from 1965 [16] which gave large values for $\delta_s - \delta_p \sim -20^0$; however, all recent calculations based on chiral perturbation theory, heavy baryon approximation etc. agree that $\delta_s - \delta_p$ lies between $1^0$ and $3^0$ [17]. In this case the asymmetry $A_\Xi$ is expected to be $\sim -(0.2$ to $0.7)10^{-5}$. In the Table 1, the SM results for the expected asymmetries in SM are given. Using very crude back of the envelope estimates, similar results are obtained.
What is needed is some attention to these matrix elements from the Lattice community.

An experimental measurement of the phase shifts $\delta_s - \delta_p$ in $\Lambda\pi$ system will put the predictions for $A_\Xi$ on a firmer basis. There is an old proposal due to Pais and Treiman [18] to measure $\Lambda\pi$ phase shifts in $\Xi \to \Lambda\pi\nu\bar{\nu}$, but this does not seem practical in the near future. Another technique, more feasible, is to measure $\beta$ and $\alpha$ to high precision in $\Xi$ and $\Xi$ decays. Then the combination

$$\frac{(\beta - \bar{\beta})}{(\alpha - \bar{\alpha})} = \tan(\delta_s - \delta_p)$$

(19)

can be used to extract $\delta_s - \delta_p$. To the extend CP phases are negligible one can also use the approximate relation:

$$\frac{\beta}{\alpha} \approx \tan(\delta_s - \delta_p)$$

(20)

5 Beyond Standard Model

Can new physics scenarios in which the source of CP violation is not K-M matrix yield large enhancements of these asymmetries? We consider some classes of models where these asymmetries can be estimated more or less reliably [12, 13].

First there is the class of models which are effectively super-weak [19]. Examples are models in which the K-M matrix is real and the observed CP violation is due to exchange of heavier particles; heavy scalars with FCNC, heavy quarks etc. In all such models direct CP violation is negligible and unobservable and so all asymmetries in hyperon decays are essentially zero. Furthermore, they need to be modified to accommodate the fact that direct CP violation has now been seen in the kaon decays( the fact that $\epsilon'/\epsilon$ is not
zero). In the three Higgs doublet model with flavor conservation imposed, the charged Higgs exchange tend to give large effects in direct CP violation as well as large neutron electric dipole moment \[20\].

There are two generic classes of left-right symmetric models: (i) Manifest Left-Right symmetric model without $W_L - W_R$ mixing \[21\] and (ii) with $W_L - W_R$ mixing \[22\]. In (i) $U^{L}_{KM} = \text{real}$ and $U^{R}_{KM}$ complex with arbitrary phases but angles given by $U^{L}_{KM}$. Then one gets the “isoconjugate” version in which

$$H_{eff} = \frac{G_F}{\sqrt{2}} U_{us} \left[ J^{\mu L} J^{\mu L} + \eta e^{i\beta} J^{\mu R} J^{\mu R} \right]$$ \hspace{1cm} (21)$$

where $\eta = m^2_{W_L}/m^2_{W_R}$ and $\beta$ is the relevant CPV phase. Then $H_{p.c.}$ and $H_{p.v.}$ have overall phases $(1 + i\eta\beta)$ and $(1 - i\eta\beta)$ respectively. To account for the observed CPV in K-decay, $\eta\beta$ has to be of order $4.5 \times 10^{-5}$. In this model, $\epsilon'/\epsilon = 0$ and there are no rate asymmetries in hyperon decays but the asymmetries A and B are not zero and e.g. A goes as $2\eta\beta \sin(\delta_s - \delta_p)$.

In the class of models where $W_L - W_R$ mixing is allowed, the asymmetries can be enhanced, and also $\epsilon'/\epsilon$ is not zero in general.

Models where the gluon dipole operator is enhanced beyond the SM value are especially interesting:

$$\bar{d}\lambda^\alpha (a + b\gamma_5)s \ G^\alpha_{\mu\nu}$$ \hspace{1cm} (22)$$

In SM, the coefficient of this operator is too small to be interesting. The parameter $a$(actually its imaginary part) is constrained by the known value of $\epsilon$ and contributes only to hyperon decays. The parameter $b$ can contribute to both $\epsilon'$ as well as the CP violating asymmetries in hyperon decays. The current range of $\epsilon'$ as given by the experimental values we heard here\[23\] allows $b$ to contribute to $A_\Lambda$ at a level of $5 \times 10^{-4}$\[24\]. Such enhancement
Table 1: Expectations for Hyperon CPV Asymetries.

|       | SM  | 2-Higgs | FCNC | L-R-S | L-R-S |
|-------|-----|---------|------|-------|-------|
|       |     |         |      |       |       |
|       |     |         |      | Superweak | (1) | (2) |
| $\Delta_A$ | $10^{-6}$ | $10^{-5}$ | 0   | 0     | 0     |
| $A_A$   | $-4.10^{-5}$ | $-2.10^{-5}$ | 0   | $-10^5$ | $6.10^{-4}$ |
| $B_A$   | $10^{-4}$ | $2.10^{-3}$ | 0   | $7.10^{-4}$ | -     |
| $\Delta_{\Xi}$ | 0   | 0       | 0   | 0     | 0     |
| $A_{\Xi}$ | $-4.10^{-6}$ | $-3.10^{-4}$ | 0   | $2.10^{-5}$ | $10^{-4}$ |
| $B_{\Xi}$ | $10^{-3}$ | $4.10^{-3}$ | 0   | $3.10^{-4}$ | -     |

of this operator takes place naturally in models where CP violation occurs due to the exchange of charged higgs scalars (such as the Weinberg model) and can also occur in several scenarios based on supersymmetry [25].

6 Experiments

There have been several proposals to detect hyperon decay asymmetries in $\bar{p}p \to \bar{\Lambda}\Lambda$, $\bar{p}p \to \bar{\Xi}\Xi$ and in $e^+e^- \to J/\psi \to \Lambda\bar{\Lambda}$ but none of these were approved [26]. The only approved and on-going experiment is E871 at Fermilab. In this experiment $\Xi^-$ and $\Xi^+$ are produced and the angular distribution of $\Xi^- \to \Lambda\pi^- \to p\pi^-\pi^-$ and $\Xi^+$ compared. This experiment effectively measures $A_{\Lambda} + A_{\Xi}$ and will be described in detail by Sharon White[27]. To summarize the implications for the measurement of $A_{\Lambda} + A_{\Xi}$ by E871: the SM expectation is about $-4.10^{-5}$ with a factor of two uncertainty; if new physics should contribute it could be as large as $7.10^{-4}$. A measurement by E871 at the $10^{-4}$ level, therefore, will already be a strong discriminant.
7 $\epsilon'/\epsilon$ and Hyperon Decay Asymmetries

It might seem that now that $\epsilon'/\epsilon$ has been measured and direct CP violation in $\Delta S = 1$ channel been observed, a study of CP violation in hyperon decays is unnecessary and no new information will be obtained. Why is it worthwhile measuring another $\Delta S = 1$ process like hyperon decay? The point is that there are important differences and the two are not at all identical. First, there are important differences in the matrix elements. Hyperon matrix elements do not have the kind of large cancellations that plague the kaon matrix elements. The hadronic uncertainties are present for both, but are different. Next, a very important difference is the fact that the $K \rightarrow \pi\pi$ decay (and hence $\epsilon'$) is only sensitive to CP violation in the parity violating amplitude and cannot yield any information on parity conserving amplitudes. Hyperon decays, by contrast, are sensitive to both. Thus, $\epsilon'/\epsilon$ and hyperon decay CP asymmetries are different and complimentary. The hyperon decay measurements are as important and significant as $\epsilon'/\epsilon$.

Conclusion

The searches for direct CPV are being pursued in many channels. $K \rightarrow 2\pi, \Lambda \rightarrow N\pi$, B decays and D decays. Any observation of a signal would be the first outside of $K^0 - \bar{K}^0$ system and would be complimentary to the measurement of $\epsilon'/\epsilon$. This will constitute one more step in our bid to confirm or demolish the Standard Kobayashi-Maskawa description of CP violation.

Hyperon decays offer a rich variety of CP violating observables, each with different sensitivity to various sources of CP violation. For example,
\( \Delta_A \) is mostly sensitive to parity violating amplitudes, \( \Delta_{\Sigma^+} \) is sensitive only to parity conserving amplitudes, \( A \) is sensitive to both etc. The number of experimental proposals is rather small so far. The one on-going experiment Fermilab E871 can probe \( A \) to a level of \( 10^{-4} \) which is already in an interesting range. In addition to more experiments, this subject sorely needs more attention devoted to calculating the matrix elements more reliably.

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