Decays of Higgs to $b\bar{b}$, $\tau\bar{\tau}$ and $c\bar{c}$ as Signatures of Supersymmetry and CP Phases

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Abstract

The branching ratio of the lightest Higgs decay into $b\bar{b}$, $\tau\bar{\tau}$ and $c\bar{c}$ is sensitive to supersymmetric effects. We include in this work the effects of CP phases on the Higgs decays. Specifically we compute the deviation of the CP phase dependent branching ratio from the standard model result. The analysis includes the full one loop corrections of fermion masses including CP phases involving the gluino, the chargino and the neutralino exchanges. The analysis shows that the supersymmetric effects with CP phases can change the branching ratios by as much as 100% for the lightest Higgs decay into $b\bar{b}$ and $\tau\bar{\tau}$ with similar results holding for the heavier Higgs decays. A detailed analysis is also given of the effects of CP phases on the Higgs decays into $c\bar{c}$. The deviations of $R_{b/\tau}$ and $R_{b/c}$ from the standard model result are investigated as a possible signature of supersymmetry and CP effects. Thus a measurement of the decays of the higgs into $b\bar{b}$, $\tau\bar{\tau}$ and $c\bar{c}$ may provide important clues regarding the existence of supersymmetry and CP phases.

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1 Introduction

It is known that supersymmetric contributions can significantly affect the Higgs decays into $b\bar{b}$, $\tau\bar{\tau}$ and $cc[1]$. We compute here the effects of CP phases on these arising from soft breaking parameters. Thus, for example, the simplest supergravity unified model mSUGRA[2], whose soft breaking sector is defined by the parameters $m_0, m_1^2, A_0, \tan\beta$ (where $m_0$ is the universal scalar mass, $m_1^2$ is the universal gaugino mass, $A_0$ is the universal trilinear coupling and $\tan\beta = \frac{<H_2>}{<H_1>}$ where $H_2$ gives mass to the up quark and $H_1$ gives mass to the down quark and the lepton) can accommodate two CP violating phases which can be chosen to be the phase of the Higgs mixing parameter $\mu$ and the phase of $A_0$. Extended versions of mSUGRA including nonuniversalities can accommodate more phases. In this analysis we will consider the more general case of the supersymmetric standard model (MSSM) which allows for several CP phases. Thus, for example, we will allow the gaugino masses to be in general nonuniversal so that $\tilde{m}_i = |\tilde{m}_i|e^{i\xi_i} (i = 1, 2, 3)$. In the analysis we use the results recently obtained regarding the effects of CP phases on the third generation quark and lepton masses[3]. The analysis of Ref.[3] extends the analyses of Refs. [4, 5, 6] where large corrections to third generation masses and specifically to the b quark mass, but without inclusion of CP phases, were found. The corrections to the quark and lepton masses are of considerable importance in the analysis of Yukawa unification[7, 8, 9]. In this paper we utilize these corrections to study their effects on the Higgs decays into $b\bar{b}$ and $\tau\bar{\tau}$.

The analyses which include CP phases must be constrained by the experimental upper limits on the electric dipole moment for the electron and for the neutron which are very stringent. Thus for the electron the current experimental limit for the magnitude of electron edm is $d_e < 4.3 \times 10^{-27}ecm[10]$ while for the neutron it is $d_n < 6.5 \times 10^{-26}ecm[11]$. There is a similar stringent limit on the edm of the $H_g^{199}$ atom, i.e., $d_{H_g} < 9 \times 10^{-28}ecm[12]$. A variety of techniques have been discussed in the literature to achieve consistency with experiment[13, 14, 15, 16, 17, 18, 19, 20, 21]. Specifically one finds that it is possible to accommodate large CP phases and still have consistency with the edm constraints either via the cancellation mechanism[16, 18, 20, 21] or via large phases in the third generation sector[19]. We note in passing the analyses of Refs.[20, 21] also include $H_g^{199}$ edm constraint and show that this constraint along with the electron and the neutron edm constraint can be satisfied in the presence of large phases. Of course, if the phases are large they will affect a variety of low energy phenomena. Such effects
have been investigated on a variety of processes. These include effects on the Higgs sector\cite{22, 23}, on $g_\mu - 2$\cite{24}, on collider physics\cite{25, 26}, in $B$ physics and in flavor violation\cite{27, 21, 28, 29} and a variety of other low energy phenomena. The number of phenomena investigated is rather larger and a more complete list can be found in Ref.\cite{30}.

The outline of the rest of the paper is as follows: In Sec.\ref{sec:basis} we give a brief description of the basic formalism needed for the evaluation of the full one loop effects on the decays of the Higgs into quarks and leptons including the effects of CP phases. In Sec.\ref{sec:loopcorr} we give the effective low energy interaction of the $b$ quark with the lightest Higgs including loop corrections. The branching ratios $BR(H_2 \rightarrow b\bar{b})$ and $BR(H_2 \rightarrow \tau\bar{\tau})$ are also computed. In Sec.\ref{sec:numanal} we take the limit of vanishing phases and compare our results to previous analyses. In Sec.\ref{sec:results} we give a numerical analysis of the deviations of the branching ratios from the standard model predictions because of supersymmetric effects and discuss the sensitivity of these deviations to CP phases. Conclusions are given in Sec.\ref{sec:concl}. In Appendix A we extend the results of Sec.\ref{sec:numanal} to include $H_2 \rightarrow c\bar{c}$ decay and compute the supersymmetry and CP effects on $\Delta R_{b/c}$. Extension of the results to decays of the Higgs bosons $H_1$ and $H_3$ is given in Appendix B.

\section{The basic formalism} \label{sec:basis}

At the tree level the $b$ quark couples to the neutral component of $H_1$ Higgs boson while the coupling to the $H_2$ higgs bosons is absent where

\[
(H_1) = \begin{pmatrix} H_1^0 \\ H_1^- \end{pmatrix}, \quad (H_2) = \begin{pmatrix} H_2^{++} \\ H_2^0 \end{pmatrix}
\]

Loop corrections produce a shift in the $H_1^0$ couplings and generate a non vanishing effective coupling with $H_2^0$. Thus the effective Lagrangian would be written as

\[
-\mathcal{L}_{\text{eff}} = (h_b + \delta h_b)\bar{b}_R b_L H_1^0 + \Delta h_b \bar{b}_R b_L H_2^{0*} + \text{H.c.}
\]

where the star on $H_2^{0*}$ is necessary in order to have a gauge invariant $\mathcal{L}_{\text{eff}}$. The same analysis holds for the tau-lepton sector where $h_\tau$, $\delta h_\tau$ and $\Delta h_\tau$ are used in the Lagrangian involving $\bar{\tau}_R \tau_L$.

The quantities $\delta h_f$ and $\Delta h_f$ receive SUSY QCD and SUSY electroweak contributions. They are calculated in Ref.\cite{3} on mass corrections to lepton and quark masses. In the analysis carried out there one finds that the couplings are generally
complex due to CP phases in the soft breaking terms. Electroweak symmetry is
broken spontaneously by giving vacuum expectation value to $H_0^1$ and $H_0^2$. Thus
one finds

$$(H_1) = \frac{1}{\sqrt{2}} \left( v_1 + \phi_1 + i \psi_1 \right), \quad (H_2) = \frac{e^{i\theta_H}}{\sqrt{2}} \left( v_2 + \phi_2 + i \psi_2 \right)$$

(3)

Inserting in $H_0^1$ and $H_0^2$ one finds

$$-\mathcal{L}_m = M_b \bar{b}_R b_L + H.c. \quad (4)$$

where

$$M_b = \frac{h_b v_1}{\sqrt{2}} \left\{ 1 + \frac{\delta h_b}{h_b} + \frac{\Delta h_b}{h_b} \tan \beta \right\}$$

(5)

Here $M_b$ is complex because $\delta h_b$ and $\Delta h_b$ are both complex. We carry out a
redefinition of the b quark field

$$b = e^{i\frac{1}{2} \gamma_5 \chi b} b', \quad \tan \chi_b = \frac{\text{Im} M_b}{\text{Re} M_b}$$

(6)

After the redefinition of Eq.(6) the mass term reads

$$-\mathcal{L}_m = m_b \bar{b}_R' b_L' + H.c. \quad (7)$$

where $m_b$ is real and positive and $b'$ is the physical field and $m_b$ and $h_b$ are related by

$$h_b = \frac{\sqrt{2} m_b}{v_1} \left[ (1 + \text{Re} \frac{\delta h_b}{h_b} + \text{Re} \frac{\Delta h_b}{h_b} \tan \beta)^2 + (\text{Im} \frac{\delta h_b}{h_b} + \text{Im} \frac{\Delta h_b}{h_b} \tan \beta)^2 \right]^{-\frac{1}{2}}$$

(8)

The above can be approximated by

$$h_b \simeq \frac{\sqrt{2} m_b}{v_1} \frac{1}{(1 + \Delta_b)}, \quad \Delta_b = \text{Re} \frac{\delta h_b}{h_b} + \text{Re} \frac{\Delta h_b}{h_b} \tan \beta$$

(9)

From now on we will drop the prime on b and we will assume that we are already
in the basis where the b field is the physical field for the b quark. The other terms
in Eq. (2) which have $\phi_1, \psi_1, \phi_2$ and $\psi_2$ dependence will produce the interaction
between the b-quark and the mass eigen states of the Higgs fields $H_i$. Next we
introduce the basis $\phi_1, \phi_2, \psi_{1D}, \psi_{2D}$ where

$$\psi_{1D} = \sin \beta \psi_1 + \cos \beta \psi_2, \quad \psi_{2D} = -\cos \beta \psi_1 + \sin \beta \psi_2$$

(10)
In this basis the field $\psi_{2D}$ is the would be Goldstone field. By considering one loop contributions to the Higgs boson masses and mixings from top-stop, bottom-sbottom, chargino and neutralino exchanges with CP violating phases, the mass eigen states of the Higgs fields would be mixed states of CP even and CP odd states. Thus the mass eigen states $H_i$ (i=1,2,3) are related to the $\phi_1, \phi_2, \psi_{1D}$ as follows\cite{22, 23, 31, 32}

$$
\begin{pmatrix}
H_1 \\
H_2 \\
H_3
\end{pmatrix}
= R
\begin{pmatrix}
\phi_1 \\
\phi_2 \\
\psi_{1D}
\end{pmatrix}
$$

(11)

In the absence of CP phases there is no mixing between the CP even and the CP odd Higgs and the Higgs mass matrix consists of a $2 \times 2$ matrix for the CP even fields and a one element for the CP odd Higgs. Solving for the eigen values in this case one may choose the first eigen value to be the heavier mass and the second eigen value to be the lighter mass. In previous analyses where we considered the effects of CP phases on the Higgs sector\cite{23}, we used the convention that in the limit of vanishing CP phases one has $H_1 \rightarrow H, H_2 \rightarrow h, H_3 \rightarrow A$\cite{23}. In the present analysis we continue to use the same convention. Thus the lightest Higgs boson field corresponds to $H_2$ in our notation.

3 The Decays $H_2 \rightarrow b\bar{b}$ and $H_2 \rightarrow \tau\bar{\tau}$ including effects of CP phases

In this section we study the decay of $H_2$ Higgs into $b\bar{b}$ and $\tau\bar{\tau}$. Consider the b quark first. The effective interaction of the b quark with the Higgs mass eigen states $H_2$ is given by

$$
-\mathcal{L}_{\text{int}}^b = \bar{b}[C_b^S + i\gamma_5 C_b^P]bH_2
$$

$$
C_b^S = \tilde{C}_b^S \cos \chi_b - \tilde{C}_b^P \sin \chi_b
$$

$$
C_b^P = \tilde{C}_b^S \sin \chi_b + \tilde{C}_b^P \cos \chi_b
$$

$$
\sqrt{2}\tilde{C}_b^S = Re(h_b + \delta h_b)R_{21} + [-Im(h_b + \delta h_b) \sin \beta
$$

$$
+ Im(\Delta h_b) \cos \beta]R_{23} + Re(\Delta h_b)R_{22}
$$

$$
\sqrt{2}\tilde{C}_b^P = -Im(h_b + \delta h_b)R_{21} + [-Re(h_b + \delta h_b) \sin \beta
$$

$$
+ Re(\Delta h_b) \cos \beta]R_{23} - Im(\Delta h_b)R_{22}
$$

(12)

In deriving $\mathcal{L}_{\text{int}}^b$ above, we redefined the quark field $b$ such that $h_b$ is real and is given by Eq.(8) while the quark mass $M_b$ is complex before field redefinition. In
Figure 1: One loop contribution to the bottom quark mass involving exchange of gluino, charginos and neutralinos in the loop.

Ref. [31] the authors redefined the fields such that $h_b$ is complex and is given by their Eq.(4.5) while the quark mass is real and positive. To compare now our Eq.(12) with Eq.(4.10) of Ref. [31] and Eq. (77) of Ref. [6], we have the phase $\chi_b$ of the quark mass apparent while $h_b$ in $C^S_b$ and $C^P_b$ is real. In Eqs.(4.11, 4.12) of Ref. [31] and Eq.(78, 79) of Ref. [6] the phase $\chi_b$ is absent because of their choice of phases which makes $h_b + \delta h_b + \Delta h_b \tan \beta$ a real and positive quantity and our Eq.(6) would lead then to a vanishing $\chi_b$. However $h_b$ in these expressions is complex and this will compensate for the absence of $\chi_b$. So both methods, the one given here and that of Refs.[31] and [6], lead to the same phenomenological results.

The $\tau$ lepton has similar interactions. Thus

$$-\mathcal{L}_{int}^\tau = \bar{\tau}[C^S_\tau + i\gamma_5 C^P_\tau]bH_2$$

(13)

where $C^S_\tau$ and $C^P_\tau$ are given by Eq. (12) with the transposition $b \rightarrow \tau$. We are interested in the ratio of the branching ratios for $H_2 \rightarrow \bar{b}b$ and $H_2 \rightarrow \bar{\tau}\tau$, i.e.,

$$R_{b/\tau} = \frac{BR(H_2 \rightarrow \bar{b}b)}{BR(H_2 \rightarrow \bar{\tau}\tau)}$$

(14)

Using the interactions given in Eqs. (12) and (13) we can estimate this to be

$$R_{b/\tau} = 3 \frac{m_b^2}{m_\tau^2} \frac{S_\tau}{T_b} \frac{T_\tau}{(1 + \omega)}$$

(15)

Here

$$T_f = \left( \frac{M_{H_2}^2 - 4m_f^2}{M_{H_2}^2} \right)^2 (A_f^S)^2 + \left( \frac{M_{H_2}^2 - 4m_f^2}{M_{H_2}^2} \right)^2 (A_f^P)^2$$

(16)
where

\[ A_f^S = \tilde{A}_f^S \cos \chi_f - \tilde{A}_f^P \sin \chi_f, \quad A_f^P = \tilde{A}_f^S \sin \chi_f + \tilde{A}_f^P \cos \chi_f \]

\[ \tilde{A}_f^S = [(1 + Re \delta h_f R_{21} - Im \delta h_f R_{23} \sin \beta + Re \Delta h_f R_{22} + Im \Delta h_f R_{23} \cos \beta] \]

\[ \tilde{A}_f^P = [-Im \delta h_f R_{21} - (1 + Re \delta h_f R_{23} \sin \beta - Im \Delta h_f R_{22} + Re \Delta h_f R_{23} \cos \beta] \tag{17} \]

\[ S_f = [(1 + Re \delta h_f + Re \Delta h_f \tan \beta)^2 + (Im \delta h_f + Im \Delta h_f \tan \beta)^2] \tag{18} \]

and \((1 + \omega)\) is the QCD enhancement factor and is given by[33]

\[ (1 + \omega) = 1 + 5.67 \frac{\alpha_s}{\pi} + 29.14 \frac{\alpha_s^2}{\pi^2} \tag{19} \]

so that \((1 + \omega) \simeq 1.25\) for \(\alpha_s \simeq 0.12\). CP phases enter in Eq. (15) through \(T_f\) and \(S_f\) since these depend on the phases through the couplings \(\Delta h_f, \delta h_f\) and through the matrix element \(R_{21}\). Now the CP dependence of \(\Delta h_b\) and \(\delta h_b\) is significantly different from the CP dependence of \(\Delta h_\tau\) and \(\delta h_\tau\) on phases since, for example, corrections to the \(b\) quark mass involve gluino contributions and are thus sensitive to the phase \(\xi_3\) while the corrections to the \(\tau\) lepton mass do not depend on this phase. Thus one can expect a sensitive dependence of the ratio \(R_{b/\tau}\) on the phases. We may compare the above result to the result from the Standard Model. Here one has

\[ (R_{b/\tau})_{SM} = 3(\frac{m_b^2}{m_\tau^2})(\frac{m_h^2 - 4m_b^2}{m_h^2 - 4m_\tau^2})^3(1 + \omega) \tag{20} \]

By identifying \(m_h\) with \(m_{H_2}\) we define the shift due to supersymmetric effects including the effects due to CP phases as follows

\[ \Delta R_{b/\tau} = \frac{R_{b/\tau} - (R_{b/\tau})_{SM}}{R_{b/\tau}} \tag{21} \]

As pointed out in Ref.[1] this ratio can be used to distinguish the SM Higgs from the lightest SUSY Higgs. The same holds here except that we also take into account the effects of CP phases. We will discuss the effects of the CP phases on \(\Delta R_{b/\tau}\) numerically in Sec.5. It is also of interest to analyze the ratio of the branching ratios \(H_2 \to b\bar{b}\) and \(H_2 \to c\bar{c}\). Thus define

\[ R_{b/c} = \frac{BR(H_2 \to b\bar{b})}{BR(H_2 \to c\bar{c})} \tag{22} \]
By repeating the analysis as for the previous case we get

\[ R_{b/c} = \frac{m_b^2}{m_c^2} \tan^2 \beta \frac{S'_c T_b}{S_b T'_c} \tag{23} \]

where

\[ T'_c = \left( \frac{M_{H_2}^2 - 4m_f^2}{M_{H_2}^2} \right)^2 (A_e^S)^2 + \left( \frac{M_{H_2}^2 - 4m_f^2}{M_{H_2}^2} \right) \frac{\Delta h_c R_{23} \sin \beta}{h_c} (A_e^P)^2 \tag{24} \]

where

\[ A_e^S = \tilde{A}_e^S \cos \chi_c - \tilde{A}_e^P \sin \chi_c, \quad A_e^P = \tilde{A}_e^S \sin \chi_c + \tilde{A}_e^P \cos \chi_c \]

\[ \tilde{A}_c^S = [(1 + Re \delta h_c/h_c) R_{22} - Im \delta h_c/h_c R_{23} \cos \beta + Re \Delta h_c/h_c R_{21} + Im \Delta h_c/h_c R_{23} \sin \beta] \]

\[ \tilde{A}_c^P = [-Im \delta h_c/h_c R_{22} - (1 + Re \delta h_c/h_c) R_{23} \cos \beta - Im \Delta h_c/h_c R_{21} + Re \Delta h_c/h_c R_{23} \sin \beta] \tag{25} \]

\[ S'_c = [(1 + Re \delta h_c/h_c) + Re \Delta h_c/h_c \cot \beta)^2 + (Im \delta h_c/h_c + Im \Delta h_c/h_c \cot \beta)^2] \tag{26} \]

We note that the prime is used on \( T'_c \) and \( S'_c \) for the c quark case since it has different couplings. The \( \Delta h_c \) and \( \delta h_c \) can be deduced similar to the analysis of \( \Delta h_t \) and \( \delta h_t \) given in Ref.[3]. Explicit results are given in Appendix A. An analysis similar to the above but for the decay of the heavier Higgs bosons \( H_1 \) and \( H_3 \) are given in Appendix B.

### 4 Limit of Vanishing Phases

We compare our results now with previous analysis of Ref.[1] by taking the limit of vanishing phases. In this limit we set

\[ \chi_f = 0, \quad Im(\delta h_f) = 0 = Im(\Delta h_f) \]

\[ R_{11} = \cos \alpha, \quad R_{12} = \sin \alpha \]

\[ R_{21} = -\sin \alpha, \quad R_{22} = \cos \alpha \]

\[ R_{13} = R_{23} = 0 \tag{27} \]

Neglecting the masses of \( b, c \) and \( \tau \) relative to the masses of the Higgs bosons, one obtains from Eqs. (15)-(18) the result

\[ R_{b/\tau} = 3 \frac{m_b^2}{m_\tau^2} \left( 1 + \epsilon'_b - \epsilon_b/\tan \alpha \right)^2 \left( 1 + \epsilon'_\tau + \epsilon_\tau \tan \beta \right)^2 \left( 1 + \epsilon'_\tau - \epsilon_\tau/\tan \alpha \right)^2 (1 + \omega) \tag{28} \]
where $\epsilon_{b,\tau}$ and $\epsilon'_{b,\tau}$ are defined so that

$$\epsilon_{b,\tau} = \Delta h_{b,\tau} / h_{b,\tau} \quad \epsilon'_{b,\tau} = \delta h_{b,\tau} / h_{b,\tau}$$

(29)

On neglecting the $\tau$ correction and $\epsilon'_b$ correction this result agrees with Eq.(11) of the first paper of Ref.[1]. Similarly in the limit of vanishing phases one finds for $R_{b/c}$ the result

$$R_{b/c} = \frac{m_b^2}{m_c^2} (\tan \alpha \tan \beta) \frac{(1 + \epsilon'_b - \epsilon_b \cot \alpha)^2 (1 + \epsilon'_c + \epsilon_c \cot \beta)^2}{(1 + \epsilon'_b + \epsilon_b \tan \beta)^2 (1 + \epsilon'_c - \epsilon_c \tan \alpha)^2}$$

(30)

Again neglecting the $\epsilon'_b$ and $\epsilon'_c$ terms the above result agrees with Eq.(19) of the first paper of Ref.[1]. We note, however, that in general the $\epsilon'$ term is not necessarily negligible.

5 Numerical Analysis

In this section we discuss the size of the supersymmetric corrections including CP phases to $\Delta R_{b/\tau}$ and $\Delta R_{b/c}$ discussed in Sec.3. We carry out the analysis in MSSM. Since the general parameter space of MSSM is rather large we shall limit our selves to a more constrained set for the purpose of this numerical study. We shall use for our parameter space the set $m_A, m_0, m_{1/2}, |A_0|, \tan \beta, \theta_\mu, \alpha_{A_0}, \xi_1, \xi_2$ and $\xi_3$. The parameter $\mu$ is determined via radiative breaking of the electroweak symmetry. The other sparticle masses are obtained from this set using the renormalization group equations evolving the GUT parameters from the GUT scale down to the electroweak scale. We then use the sparticle spectrum generated by the above method to compute the supersymmetric corrections to the Higgs decays. As mentioned above the phases chosen here for the numerical analysis are $\xi_1, \xi_2, \xi_3, \alpha_{A_0}$ and $\theta_\mu$. This choice provides a convenient set of phases because of their appearance in the expressions for $R_{ij}$, $\Delta h_f$ and $\delta h_f$. By examining these expressions one finds linear combinations of the phases listed above that enter the analysis. Such linear combinations have been classified for EDMs in Ref.[17]. Thus not all the phases are independent and absorbing one of these phases by redefinition of the other phases in these linear combinations will not change the numerical analysis, i.e, the effect of CP phases on Higgs decay. However, for the numerical analysis here it is more convenient to work with the set listed above.

In Fig. 2 we give a plot of $\Delta R_{b/\tau}$ for the decay of Higgs $H_2$ as given by Eq.(21) as a function of the phase $\xi_3$ for values of $\tan \beta$ ranging from 5-50. The analysis shows
a very sharp dependence of $\Delta R_{b/\tau}$ on $\xi_3$. Thus $\xi_3$ affects not only the magnitude of $\Delta R_{b/\tau}$ but also its sign depending on the value of $\xi_3$. A similar analysis holds for $\Delta R^{H_1}_{b/\tau}$ where $R^{H_1}_{b/\tau}$ is given by Eq.(54). Fig. 3 gives a plot of $\Delta R^{H_1}_{b/\tau}$ as a function of $\xi_3$ for all the same inputs as for Fig. 2. In this case also one finds a very sharp dependence of $\Delta R^{H_1}_{b/\tau}$ on $\xi_3$. The analysis of $\Delta R^{H_3}_{b/\tau}$ follows a similar pattern as $\Delta R^{H_1}_{b/\tau}$ and is not exhibited. As discussed in Sec.4 the quantity $\Delta R_{b/c}$ is also of interest where $R_{b/c}$ is given by Eq.(30). In Fig. 4 we give a plot of $\Delta R_{b/c}$ as a function of $\xi_3$ for all the same inputs as in the analysis of Fig. 2. Here also we find a very substantial dependence of $\Delta R_{b/c}$ on $\xi_3$. Next we analyse the variations of $\Delta R_{b/\tau}$ and $\Delta R_{b/c}$ as a function of $\theta_\mu$. In Fig. 5 a plot of $\Delta R_{b/\tau}$ is given as function of $\theta_\mu$ for various values of $\xi_3$. Similar plots for $\Delta R^{H_1}_{b/\tau}$ and for $\Delta R_{b/c}$ are given in Fig. 6 and Fig. 7. Together Figs. 5, 6 and Fig. 7 show a rapid dependence of $\Delta R_{b/\tau}$, $\Delta R^{H_1}_{b/\tau}$ and of $\Delta R_{b/c}$ on $\theta_\mu$ in addition to their sharp dependence on $\xi_3$.

The analysis given in Fig. 2, Fig. 3 and Fig. 4, and also in Figs. 5, 6 and Fig. 7 include CP effects arising from both supersymmetric QCD and supersymmetric electroweak effects. Next we study the dependence of the branching ratios on the electroweak CP phase $\xi_2$. In Fig. 8 we give an analysis of $\Delta R_{b/\tau}$ for the decay of Higgs $H_2$ as a function of $\xi_2$ for values of $m_0, m_1$ ranging from 200 GeV -400 GeV. Here we see that the dependence of $\Delta R_{b/\tau}$ on $\xi_2$ is not as strong as it is on $\xi_3$. Nonetheless the $\xi_2$ dependence of $\Delta R_{b/\tau}$ is still quite substantial as the variations in $\Delta R_{b/\tau}$ can be as much as 40%. In Fig. 9 we exhibit the dependence of $\Delta R^{H_1}_{b/\tau}$ on $\xi_2$ and one finds that once again the dependence though not as strong as the $\xi_3$ dependence is still quite substantial and one can get variations of as much as 40% over the full range of $\xi_2$. The analysis of $\Delta R^{H_3}_{b/\tau}$ is very similar and is not displayed. In Fig. 10 the dependence of $\Delta R_{b/c}$ on $\xi_2$ for the $H_2$ decay is given. Again although the dependence of $\Delta R_{b/c}$ on $\xi_2$ is not as strong as on $\xi_3$ it is still quite substantial as one finds that $\Delta R_{b/c}$ can vary about 20% over the full range of $\xi_2$.

In Fig. 11 we give a plot of $\Delta R_{b/\tau}$ as a function of $\tan \beta$ for three different inputs which for the largest value of $\tan \beta$ satisfy the edm constraints including the $H^\text{199}_g$ edm constraint. (The $H^\text{199}_g$ edm constraint translates into $C_{Hg} = |d^C_d - d^C_u - 0.012d^C_s| < 3.0 \times 10^{-26}cm$[20, 21].) The lower set of curves are for the cases when phases are included while similar upper curves are without phases. First one finds the effect of phases is so large as to not only affect the magnitude of $\Delta R_{b/\tau}$ but also affect its sign. Further, one finds that $\Delta R_{b/\tau}$ is strongly dependent on
\( \tan \beta \) for cases with and without phases. An identical analysis for \( \Delta R_{b/\tau}^{H_1} \) is given in Fig. 12 where \( \Delta R_{b/\tau}^{H_1} \) is plotted as a function of \( \tan \beta \) for exactly the same set of inputs as in Fig. 11. One finds once again a very similar behavior in that both the magnitude and the sign of \( \Delta R_{b/\tau}^{H_1} \) are affected and also one finds a very strong dependence on \( \tan \beta \). The analysis of \( \Delta R_{b/\tau}^{H_3} \) is very similar to that of \( \Delta R_{b/\tau}^{H_1} \) and is not given. In Fig. 13 we give a plot of \( \Delta R_{b/c} \) as a function of \( \tan \beta \) again for the same identical inputs as in Fig. 11. In this case also the effects of phases though smaller than in the case of \( \Delta R_{b/\tau} \) are still substantial in that the CP phase effects can be up to 30%. Further, one finds about 20% variation due to variations in \( \tan \beta \).

Table 1: EDM constraints for the points of \( \tan \beta = 50 \) of Figs. 11, 12 and 13.

| Case | \( |d_e| \) e.cm | \( |d_n| \) e.cm | \( C_{Hg} \) cm |
|------|----------------|----------------|----------------|
| (i)  | \( 1.67 \times 10^{-27} \) | \( 1.59 \times 10^{-27} \) | \( 1.18 \times 10^{-27} \) |
| (ii) | \( 6.05 \times 10^{-28} \) | \( 3.47 \times 10^{-27} \) | \( 1.29 \times 10^{-26} \) |
| (iii)| \( 2.14 \times 10^{-27} \) | \( 8.90 \times 10^{-28} \) | \( 1.25 \times 10^{-26} \) |

6 Conclusions

In this paper we have computed the effects of CP phases on the Higgs boson decays. In supersymmetry after spontaneous breaking one is left with three neutral Higgs bosons which in the absence of CP phases consist of two CP even Higgs and one CP odd Higgs. With the inclusion of CP phases the Higgs mass eigen states are no longer CP eigen states but rather admixtures of CP even and CP odd states when loop corrections to the Higgs boson masses are included. Further, inclusion of loop corrections to the quarks and lepton masses are in general dependent on CP phases and these effects can be very significant for the case of the b quark mass. Additionally inclusion of CP dependent loop effects on the quark and lepton masses modify the vertices involving the quarks and leptons and the Higgs mass eigen states and these modifications also affect the Higgs boson decays. In this paper we have computed the effects of CP phases on the decays of the light and the heavy Higgs boson decays to \( \bar{b}b \) and to \( \tau \tilde{\tau} \). Specifically we computed the deviation of \( R_{b/\tau} \) from the standard model prediction in the presence of CP phases. We find that these effects can be rather large. Thus \( \Delta R_{b/\tau} \) can vary by as much as 100%.
or more due to the variation in the $\theta_\mu$ and $\xi_3$ while variations with $\xi_2$ are relatively small although still substantial. A similar analysis was also carried out for $\Delta R_{b/c}$ and one finds that the variations of $\Delta R_{b/c}$ with phases are also substantial. The analysis presented here points to the possible detection of CP phases via accurate measurement of the branching ratios provided we have sufficient constraints on the remaining SUSY input parameters from other experiment. Thus the decays of the Higgs if measured with sufficient accuracy may provide a signal for the presence of both supersymmetry and CP phases.

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**Appendix A: Analysis of $\Delta h_c$ and $\delta h_c$.**

Following the same technique as given in Ref.[3] we give below explicit expressions for $\Delta h_c$ and $\delta h_c$. Thus the c quark mass at the Z scale is given by

$$m_c(M_Z) = h_c(M_Z) \frac{v}{\sqrt{2}} \sin \beta(1 + \Delta_c)$$

(31)

where $\Delta_c$ gives the loop correction to the c quark mass $m_c$. and $\Delta_c$ is given by

$$\Delta_c = (Re\frac{\Delta h_c}{h_c} cot \beta + Re\frac{\delta h_c}{h_c})$$

(32)

An analysis of $\Delta h_c$ to one loop order gives

$$\Delta h_c = -\sum_{i=1}^{2} \sum_{j=1}^{2} \sum_{k=1}^{2} \frac{2\alpha_i}{3\pi} e^{-\kappa_j} m_j F_{cij} D_{cl1} D_{c2j} f(m_{g_i}^2, m_{\epsilon_i}, m_{\epsilon_j})$$

$$- \sum_{i=1}^{2} \sum_{j=1}^{2} \sum_{k=1}^{2} g^2 H_{cij} \{U_{i1}^{*} D_{s1k}^{*} - k_s U_{i2}^{*} D_{s2k}^{*}\} (k_c V_{k2}^{*} D_{s1j}) \frac{m_{\chi_i}^+}{16\pi^2} f(m_{\chi_i}^+, m_{\chi_i}, m_{\chi_i}^-)$$

$$- \sum_{i=1}^{2} \sum_{j=1}^{2} \sum_{k=1}^{2} g^2 K_{cij} \{U_{i1}^{*} D_{s1k}^{*} - k_s U_{i2}^{*} D_{s2k}^{*}\} (k_c V_{k2}^{*} D_{s1j}) \frac{m_{\chi_i}^+}{16\pi^2} f(m_{\chi_i}^-, m_{\chi_i}, m_{\chi_i}^+)$$

$$- \sum_{i=1}^{2} \sum_{j=1}^{2} \sum_{k=1}^{2} 2F_{cij} \{\alpha_{ck} D_{clj} - \gamma_{ck} D_{c2j}\} \{\beta_{ck} D_{cl1}^{*} + \alpha_{ck} D_{c2k}^{*}\} \frac{m_{\chi_k}^0}{16\pi^2} f(m_{\chi_k}^0, m_{\chi_k}, m_{\chi_k}^0)$$

$$- \sum_{i=1}^{2} \sum_{j=1}^{2} \sum_{k=1}^{2} 2\Delta_{cij} \{\alpha_{ck} D_{cl1k} - \gamma_{ck} D_{c2k}\} \{\beta_{ck} D_{cl1k}^{*} + \alpha_{ck} D_{c2k}^{*}\} \frac{m_{\chi_k}^0}{16\pi^2} f(m_{\chi_k}^0, m_{\chi_k}, m_{\chi_k}^0)$$

(33)

where

$$\alpha_{ck} = \frac{g_2 m_c X_{1k}}{2m_W \sin \beta}$$

$$\beta_{ck} = e Q_c X_{1k}' + \frac{g}{\cos \theta_W} X_{2k}' (T_{3c} - Q_c \sin^2 \theta_W)$$

$$\gamma_{ck} = e Q_c X_{1k}' - \frac{g Q_c \sin^2 \theta_W}{\cos \theta_W} X_{2k}'$$

(34)
\[ k_{c(s)} = \frac{m_{c(s)}}{\sqrt{2}m_W \sin \beta (\cos \beta)} \]  \tag{35}

and where \( Q_c = \frac{2}{3} \) and \( T_{3c} = \frac{1}{2} \). In the above \( D_{cij} \) is the matrix that diagonalizes the \( c \) squark mass\(^2 \) matrix and \( \tilde{c}_i \) are the \( c \) squark mass eigen states so that

\[ \tilde{c}_L = \sum_{i=1}^{2} D_{c1i} \tilde{c}_i, \quad \tilde{c}_R = \sum_{i=1}^{2} D_{c2i} \tilde{c}_i \]  \tag{36}

Similarly \( D_{sij} \) is the matrix that diagonalizes the \( s \) squark mass\(^2 \) matrix and \( \tilde{s}_i \) are the \( s \) squark mass eigen states so that

\[ \tilde{s}_L = \sum_{i=1}^{2} D_{s1i} \tilde{s}_i, \quad \tilde{s}_R = \sum_{i=1}^{2} D_{s2i} \tilde{s}_i \]  \tag{37}

Further, \( U \) and \( V \) are the matrices that diagonalize the chargino mass matrix and the matrix \( X \) diagonalizes the neutralino mass matrix and the elements of the \( X' \) matrix are defined by

\[ X'_{1k} = X_{1k} \cos \theta_W + X_{2k} \sin \theta_W, \quad X'_{2k} = -X_{1k} \sin \theta_W + X_{2k} \cos \theta_W \]  \tag{38}

where we are using a notation of Ref.[3]. \( F_{cij} \) and \( H_{cij} \) are defined by

\[ \frac{F_{cij}}{\sqrt{2}} = -\frac{gM_Z}{2 \cos \theta_W} \left\{ \left( \frac{1}{2} - \frac{2}{3} \sin^2 \theta_W \right) D_{c1i}^* D_{c1j} + \frac{2}{3} \sin^2 \theta_W D_{c2i}^* D_{c2j} \right\} \cos \beta \]

\[ + \frac{g m_{c\mu}}{2 M_W \sin \beta} D_{c1i}^* D_{c2j} \]  \tag{39}

and

\[ \frac{H_{cij}}{\sqrt{2}} = -\frac{gM_Z}{2 \cos \theta_W} \left\{ \left( -\frac{1}{2} + \frac{1}{3} \sin^2 \theta_W \right) D_{s1i}^* D_{s1j} - \frac{1}{3} \sin^2 \theta_W D_{s2i}^* D_{s2j} \right\} \cos \beta \]

\[ - \frac{g m_s^2}{2 M_W \cos \beta} [D_{s1i}^* D_{s1j} + D_{s2i}^* D_{s2j}] - \frac{g m_A A_s}{2 M_W \cos \beta} D_{s2i}^* D_{s1j} \]  \tag{40}

while \( K_{ij} \) and \( \Delta_{ij} \) are defined by

\[ \frac{K_{ij}}{\sqrt{2}} = -\frac{g}{2} Q_{jii}, \quad \frac{\Delta_{ij}}{\sqrt{2}} = -\frac{g}{2} Q_{ij}'' \]  \tag{41}

where

\[ Q_{ij} = \sqrt{\frac{1}{2} U_{i2} V_{j1}} \]

\[ g Q_{ij}'' = \frac{1}{2} [X_{3i}^*(g X_{2j}^* - g' X_{ij}^*) + (i \leftrightarrow j)] \]  \tag{42}
Finally the function \( f(m^2, m_i^2, m_j^2) \) is given by

\[
f(m^2, m_i^2, m_j^2) = \frac{1}{(m^2 - m_i^2)(m^2 - m_j^2)(m_j^2 - m_i^2)}(m_{13}^2 m_{13}^2 + m_{23}^2 m_{23}^2 + m_{33}^2 m_{33}^2)(43)
\]

for the case \( i \neq j \) and

\[
f(m^2, m_i^2, m_j^2) = \frac{1}{(m_i^2 - m_j^2)^2} \left( m_i^2 m_j^2 \ln \frac{m_i^2}{m_j^2} + (m^2 - m_i^2) \right) \quad (44)
\]

for the case \( i=j \). Similarly for \( \delta h_c \) we find the result

\[
\delta h_c = -\sum_{i=1}^{2} \sum_{j=1}^{2} \frac{2\alpha_s}{3\pi} e^{-i\xi_3 m_3} E_{cij} D^{*}_{c1i} D_{c2j} f(m_{25}^2, m_{25}^2, m_{25}^2) \nonumber
\]

\[
-\sum_{i=1}^{2} \sum_{j=1}^{2} \sum_{k=1}^{2} g^2 G_{cij} \left\{ U_{i1}^* D_{s1k} - k_s U_{i2}^* D_{s2k} \right\} (k_c V_{k2}^* D_{s1j}) \frac{m_{x_k^+}}{16\pi^2} f(m_{25}^2, m_{28}^2, m_{28}^2) \nonumber
\]

\[
-\sum_{i=1}^{2} \sum_{j=1}^{2} \sum_{k=1}^{2} g^2 C_{cij}^+ \left\{ U_{i1}^2 D_{s1k} - k_s U_{i2}^* D_{s2k} \right\} (k_c V_{k2}^* D_{s1j}) \frac{m_{x_k^+} m_{x_j^+}}{16\pi^2} f(m_{25}^2, m_{28}^2, m_{28}^2) \nonumber
\]

\[
-\sum_{i=1}^{2} \sum_{j=1}^{2} \sum_{k=1}^{4} 2E_{cij} \left\{ \alpha_{ck} D_{c1j} - \gamma_{ck} D_{c2j} \right\} \left\{ \beta_{ck}^* D_{c1i} + \alpha_{ck} D_{c2i} \right\} \frac{m_{x_k^+} m_{x_i^+}}{16\pi^2} f(m_{25}^2, m_{28}^2, m_{28}^2) \nonumber
\]

\[
-\sum_{i=1}^{2} \sum_{j=1}^{2} \sum_{k=1}^{4} 2\Gamma_{ij} \left\{ \alpha_{cij} D_{c1k} - \gamma_{cij} D_{c2k} \right\} \left\{ \beta_{cij} D_{c1k} + \alpha_{cij} D_{c2k} \right\} \frac{m_{x_k^+} m_{x_i^+}}{16\pi^2} f(m_{25}^2, m_{28}^2, m_{28}^2) \nonumber
\]

where \( E_{cij} \) and \( G_{cij} \) are given by

\[
E_{cij} = \frac{g M_Z}{2 \cos \theta_W} \left\{ \left( \frac{1}{2} - \frac{2}{3} \sin^2 \theta_W \right) D_{c1i}^* D_{c1j} + \frac{2}{3} \sin^2 \theta_W D_{c2i}^* D_{c2j} \right\} \sin \beta \nonumber
\]

\[
-\frac{g m_{c2}^2}{2 M_W \sin \beta} \left[ D_{c1i}^* D_{c1j} + D_{c2i}^* D_{c2j} \right] - \frac{g m_{c2}^2 A_{c2}}{2 M_W \sin \beta} D_{c2i}^* D_{c1j} \nonumber
\]

and

\[
G_{cij} = \frac{g M_Z}{2 \cos \theta_W} \left\{ \left( -\frac{1}{2} + \frac{1}{3} \sin^2 \theta_W \right) D_{s1i}^* D_{s1j} - \frac{1}{3} \sin^2 \theta_W D_{s2i}^* D_{s2j} \right\} \sin \beta \nonumber
\]

\[
+\frac{g m_{s2}^\mu}{2 M_W \cos \beta} D_{s1i}^* D_{s2j} \nonumber
\]

while \( C_{ij} \) and \( \Gamma_{ij} \) are defined by

\[
C_{ij} = -\frac{g}{2 \sin \beta} \left[ \frac{m_{x_i^+}}{2 M_W} \delta_{ij} - Q_{ij}^* \cos \beta - R_{ij}^* \right] \nonumber
\]

and

\[
\Gamma_{ij} = -\frac{g}{2 \sin \beta} \left[ \frac{m_{x_i^0}}{2 M_W} \delta_{ij} - Q_{ij}^{**} \cos \beta - R_{ij}^{**} \right] \nonumber
\]
where

\[
R_{ij} = \frac{1}{2M_W} [\tilde{m}^2 U_{i1} V_{j1} + \mu^* U_{i2} V_{j2}]
\]

\[
R''_{ij} = \frac{1}{2M_W} [\tilde{m}_1^* X_{1i} X_{1j} + \tilde{m}_2^* X_{2i} X_{2j} - \mu^* (X_{3i} X_{4j} + X_{4i} X_{3j})] \tag{50}
\]

**Appendix B: Decays of \(H_1\) and \(H_3\) Higgs bosons.**

We can repeat the same analysis for the heavy Higgs bosons \(H_1\) and \(H_3\). The effective interaction of the b quark with the Higgs mass eigen state \(H_1\) is given by

\[
-\mathcal{L}_{\text{int}}^b = \bar{b}[C_b^S + i\gamma_5 C_b^{P'}]bH_1
\]

\[C_b^S = C_b^{S'} \cos \chi_b - C_b^{P'} \sin \chi_b \]

\[C_b^{P'} = C_b^{S'} \sin \chi_b + C_b^{P'} \cos \chi_b \]

\[\sqrt{2} C_b^{S'} = Re(h_b + \delta h_b) R_{11} + [ - Im(h_b + \delta h_b) \sin \beta \]

\[+ Im(\Delta h_b) \cos \beta] R_{13} + Re(\Delta h_b) R_{12} \]

\[\sqrt{2} C_b^{P'} = - Im(h_b + \delta h_b) R_{11} + [ - Re(h_b + \delta h_b) \sin \beta \]

\[+ Re(\Delta h_b) \cos \beta] R_{13} - Im(\Delta h_b) R_{12} \tag{51}\]

The \(\tau\) lepton has similar interactions. Thus

\[-\mathcal{L}_{\text{int}}^\tau = \bar{\tau}[C^{S'}_\tau + i\gamma_5 C^{P'}_\tau] \tau H_1 \tag{52}\]

and \(C^{S'}_\tau\) and \(C^{P'}_\tau\) can be obtained from Eq. \(51\) by the interchange \(b \to \tau\). For the \(H_1\) decays one finds

\[
R_{b/\tau}^{H_1} = \frac{BR(H_1 \to \bar{b}b)}{BR(H_1 \to \tau\tau)} \tag{53}\]

Using the interactions given by Eqs. \(51\) and \(52\) we determine this to be

\[
R_{b/\tau}^{H_1} = \frac{3m_b^2}{m_b^{2*}} \frac{S_\tau}{S_\tau} T_{b/\tau}^{H_1} (1 + \omega) \tag{54}\]

where

\[
T_{f}^{H_1} = \left( \frac{M_{H_1}^2 - 4m_f^2}{M_{H_1}^2} \right)^2 (A_{f}^{S'})^2 + \left( \frac{M_{H_1}^2 - 4m_f^2}{M_{H_1}^2} \right)^2 (A_{f}^{P'})^2 \tag{55}\]

where

\[
A_{f}^{S'} = \tilde{A}_{f}^{S'} \cos \chi_f - \tilde{A}_{f}^{P'} \sin \chi_f, \quad A_{f}^{P'} = \tilde{A}_{f}^{S'} \sin \chi_f + \tilde{A}_{f}^{P'} \cos \chi_f
\]

\[
\tilde{A}_{f}^{S'} = [(1 + Re h_f) R_{11} - Im h_f R_{13} \sin \beta + Re \frac{\Delta h_f}{h_f} R_{12} + Im \frac{\Delta h_f}{h_f} R_{13} \cos \beta]
\]

\[
\tilde{A}_{f}^{P'} = [ - Im h_f R_{11} - (1 + Re h_f) R_{13} \sin \beta - Im \frac{\Delta h_f}{h_f} R_{12} + Re \frac{\Delta h_f}{h_f} R_{13} \cos \beta] \tag{56}\]
The interaction that governs the $H_3$ decay is

$$-\mathcal{L}_{\text{int}}^b = \bar{b}[C_b^{S''} + i\gamma_5 C_b^{P''}]bH_3$$

$$C_b^{S''} = \tilde{C}_b^{S''}\cos\chi_b - \tilde{C}_b^{P''}\sin\chi_b$$

$$C_b^{P''} = \tilde{C}_b^{S''}\sin\chi_b + \tilde{C}_b^{P''}\cos\chi_b$$

$$\sqrt{2}\tilde{C}_b^{S''} = \text{Re}(h_b + \delta h_b)R_{31} + [-\text{Im}(h_b + \delta h_b)\sin\beta + \text{Im}(\Delta h_b)\cos\beta]R_{33} + \text{Re}(\Delta h_b)R_{32}$$

$$\sqrt{2}\tilde{C}_b^{P''} = -\text{Im}(h_b + \delta h_b)R_{31} + [-\text{Re}(h_b + \delta h_b)\sin\beta + \text{Re}(\Delta h_b)\cos\beta]R_{33} - \text{Im}(\Delta h_b)R_{32}$$

(57)

Similarly the $\tau$ lepton interaction with $H_3$ is given by

$$-\mathcal{L}_{\text{int}}^\tau = \bar{\tau}[C_\tau^{S''} + i\gamma_5 C_\tau^{P''}]\tau H_3$$

(58)

and $C_\tau^{S''}$ and $C_\tau^{P''}$ can be obtained from Eq. (57) by the interchange $b \to \tau$. For the $H_3$ decays one finds

$$R_{b/\tau}^{H_3} = \frac{BR(H_3 \to \bar{b}b)}{BR(H_3 \to \bar{\tau}\tau)}$$

(59)

Using the interactions given in Eqs. (57) and (58) we determine this to be

$$R_{b/\tau}^{H_3} = 3\frac{m_b^2 S_\tau T^{H_3}_b}{m_\tau^2 S_\tau T^{H_3}_\tau}(1 + \omega)$$

(60)

where

$$T^{H_3}_f = \left(\frac{M_{H_3}^2 - 4m_f^2}{M_{H_3}^2}\right)^2 (A^{S''}_f)^2 + \left(\frac{M_{H_3}^2 - 4m_f^2}{M_{H_3}^2}\right)^2 (A^{P''}_f)^2$$

(61)

and

$$A^{S''}_f = \tilde{A}^{S''}_f\cos\chi_f - \tilde{A}^{P''}_f\sin\chi_f, \quad A^{P''}_f = \tilde{A}^{S''}_f\sin\chi_f + \tilde{A}^{P''}_f\cos\chi_f$$

$$\tilde{A}^{S''}_f = [(1 + \text{Re}\frac{\delta h_f}{h_f})R_{31} - \text{Im}\frac{\delta h_f}{h_f}R_{33} \sin\beta + \text{Re}\frac{\Delta h_f}{h_f}R_{32} + \text{Im}\frac{\Delta h_f}{h_f}R_{33} \cos\beta]$$

$$\tilde{A}^{P''}_f = [-\text{Im}\frac{\delta h_f}{h_f}R_{31} - (1 + \text{Re}\frac{\delta h_f}{h_f})R_{33} \sin\beta - \text{Im}\frac{\Delta h_f}{h_f}R_{32} + \text{Re}\frac{\Delta h_f}{h_f}R_{33} \cos\beta]$$

(62)
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Figure 2: Plot of $\Delta R_{b/\tau}$ for the decay of $H_2$ Higgs as a function of the phase $\xi_3$. The other input parameters are: $m_A = 200$ GeV, $m_0 = m_1 = 200$ GeV, $\xi_1 = .5$, $\xi_2 = .7$, $\theta_\mu = .1$, $\alpha_{A_0} = 1.0$, and $|A_0| = 4$. The curves in descending order at the point $\xi_3 = \pi$ correspond to $\tan \beta = 5, 10, 20, 30, 50$. All angles here and in succeeding figure captions are in radians.
Figure 3: Plot of $\Delta R_{b/\tau}$ for the decay of $H_1$ Higgs as a function of the phase $\xi_3$. The other input parameters are: $m_A = 200$ GeV, $m_0 = m_{A^0} = 200$ GeV, $\xi_1 = .5$, $\xi_2 = .7$, $\theta_\mu = .1$, $\alpha_{A_0} = 1.0$, and $|A_0| = 4$. The curves in descending order at the point $\xi_3 = \pi$ correspond to tan $\beta = 5, 10, 20, 50, 30$. 
Figure 4: Plot of $\Delta R_{b/c}$ for the decay of $H_2$ Higgs as a function of the phase $\xi_3$. The other input parameters are: $m_A = 200$ GeV, $m_0 = m_{\tilde{\tau}} = 200$ GeV, $\xi_1 = .5$, $\xi_2 = .7$, $\theta_\mu = .1$, $\alpha_{A_0} = 1.0$, and $|A_0| = 4$. The curves in descending order at the point $\xi_3 = \pi$ correspond to $\tan \beta = 5, 10, 20, 30, 50$. 
Figure 5: Plot of $\Delta R_{b/\tau}$ for the decay of $H_2$ Higgs as a function of the phase $\theta_\mu$. The other input parameters are: $m_A = 200$ GeV, $m_0 = m_{\tilde{\chi}_0^0} = 300$, $\xi_1 = .5$, $\xi_2 = .7$, $\alpha_{A_0} = 1.0$, $|A_0| = 5$, and $\tan \beta = 20$. The curves in descending order at $\theta_\mu = 0$ correspond to $\xi_3 = 0, .75, 1.5, 2.25, 3$. 
Figure 6: Plot of $\Delta R_{b/\tau}$ for the decay of $H_1$ Higgs as a function of the phase $\theta_\mu$. The other input parameters are: $m_A = 200$ GeV, $m_0 = m_\frac{A}{2} = 300$, $\xi_1 = .5$, $\xi_2 = .7$, $\alpha_{A_0} = 1.0$, $|A_0| = 5$, and $\tan \beta = 20$. The curves in descending order at $\theta_\mu = 0$ correspond to $\xi_3 = 0, .75, 1.5, 2.25, 3$
Figure 7: Plot of $\Delta R_{b/c}$ for the decay of $H_2$ Higgs as a function of the phase $\theta_\mu$. The other input parameters are: $m_A = 200$ GeV, $m_0 = m_{\frac{1}{2}} = 300$, $\xi_1 = .5$, $\xi_2 = .7$, $\alpha_{A_0} = 1.0$, $|A_0| = 5$, and $\tan \beta = 20$. The curves in descending order at $\theta_\mu = 0$ correspond to $\xi_3 = 0, .75, 1.5, 2.25, 3$. 
Figure 8: Plot of $\Delta R_{b/\tau}$ for the decay of $H_2$ Higgs as a function of the phase $\xi_2$. The other input parameters are: $m_A = 200$ GeV, $\xi_1 = .5$, $\xi_3 = 0$, $\theta_\mu = .1$, $\alpha_{A_0} = 1.0$, $|A_0| = 5$, and $\tan \beta = 50$. The curves in ascending order correspond to $m_0 = m_{1/2} = 200, 250, 300, 350, 400$ GeV.
Figure 9: Plot of $\Delta R_{b/\tau}$ for the decay of $H_1$ Higgs as a function of the phase $\xi_2$. The other input parameters are: $m_A = 200$ GeV, $\xi_1 = .5$, $\xi_3 = 0$, $\theta_\mu = .1$, $\alpha_{A_0} = 1.0$, $|A_0| = 5$, and $\tan \beta = 50$. The curves in ascending order correspond to $m_0 = m_{1/2} = 200, 250, 300, 350, 400$ GeV.
Figure 10: Plot of $\Delta R_{b/c}$ for the decay of $H_2$ Higgs as a function of the phase $\xi_2$. The other input parameters are: $m_A = 200 \text{ GeV}$, $\xi_1 = .5$, $\xi_3 = 0$, $\theta_\mu = .1$, $\alpha_{A_0} = 1.0$, $|A_0| = 5$, and $\tan \beta = 50$. The curves in ascending order correspond to $m_0 = m_1 = 200, 250, 300, 350, 400 \text{ GeV}$. 
Figure 11: Plot $\Delta R_{b/\tau}$ for the decay of $H_2$ Higgs as a function of $\tan\beta$. The three lower curves are for the cases (i), (ii) and (iii) as follows: (i) $m_A = 200$ GeV, $m_0 = m_A = 300$ GeV, $A_0 = 4$, $\alpha_{A_0} = 1$, $\xi_1 = .5$, $\xi_2 = .659$, $\xi_3 = .633$, $\theta_\mu = 2.5$ (solid); (ii) $m_A = 200$ GeV, $m_0 = m_{1/2} = 555$ GeV, $A_0 = 4$, $\alpha_{A_0} = 2$, $\xi_1 = .6$, $\xi_2 = .653$, $\xi_3 = .672$, $\theta_\mu = 2.5$ (dot-dashed); (iii) $m_A = 200$ GeV, $m_0 = m_{1/2} = 480$ GeV, $A_0 = 3$, $\alpha_{A_0} = .8$, $\xi_1 = .4$, $\xi_2 = .668$, $\xi_3 = .6$, $\theta_\mu = 2.5$ (long-dashed). The edm constraints including the $H_g^{199}$ are satisfied for the above curves at $\tan\beta = 50$ as shown in Table 1. The three similar curves in the upper half plane are for the three cases above when the phases are all set to zero.
Figure 12: Plot $\Delta R_{b/\tau}$ for the decay of $H_1$ Higgs as a function of $\tan \beta$. The three lower curves are for the cases (i), (ii) and (iii) as follows: (i) $m_A = 200$ GeV, $m_0 = m_{\frac{1}{2}} = 300$ GeV, $A_0 = 4$, $\alpha_{A_0} = 1$, $\xi_1 = .5$, $\xi_2 = .659$, $\xi_3 = .633$, $\theta_\mu = 2.5$ (solid); (ii) $m_A = 200$ GeV, $m_0 = m_{\frac{1}{2}} = 555$ GeV, $A_0 = 4$, $\alpha_{A_0} = 2$, $\xi_1 = .6$, $\xi_2 = .653$, $\xi_3 = .672$, $\theta_\mu = 2.5$ (dot-dashed); (iii) $m_A = 200$ GeV, $m_0 = m_{\frac{1}{2}} = 480$ GeV, $A_0 = 3$, $\alpha_{A_0} = .8$, $\xi_1 = .4$, $\xi_2 = .668$, $\xi_3 = .6$, $\theta_\mu = 2.5$ (long-dashed). The edm constraints including the $H_{g}^{199}$ are satisfied for the above curves at $\tan \beta = 50$ as shown in Table 1. The three similar curves in the upper half plane are for the three cases above when the phases are all set to zero.
Figure 13: Plot $\Delta R_{b/c}$ for the decay of $H_2$ Higgs as a function of $\tan \beta$. The three lower curves are for the cases (i), (ii) and (iii) as follows: (i) $m_A = 200$ GeV, $m_0 = m_A = 300$ GeV, $A_0 = 4$, $\alpha_{A_0} = 1$, $\xi_1 = .5$, $\xi_2 = .659$, $\xi_3 = .633$, $\theta_\mu = 2.5$ (solid); (ii) $m_A = 200$ GeV, $m_0 = m_A = 555$ GeV, $A_0 = 4$, $\alpha_{A_0} = 2$, $\xi_1 = .6$, $\xi_2 = .653$, $\xi_3 = .672$, $\theta_\mu = 2.5$ (dot-dashed); (iii) $m_A = 200$ GeV, $m_0 = m_A = 480$ GeV, $A_0 = 3$, $\alpha_{A_0} = .8$, $\xi_1 = .4$, $\xi_2 = .668$, $\xi_3 = .6$, $\theta_\mu = 2.5$ (long-dashed). The edm constraints including the $H_{g}^{199}$ are satisfied for the above curves at $\tan \beta = 50$ as shown in Table 1. The three similar curves in the upper half plane are for the three cases above when the phases are all set to zero.