

The Two-Photon-Exchange and \( \gamma Z \)-Exchange Corrections to Parity-Violating Elastic Electron-Proton Scattering

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Leading electroweak corrections play an important role in precision measurements of the strangeness form factors. We calculate the two-photon-exchange (TPE) and \( \gamma Z \)-exchange corrections to the parity-violating asymmetry of the elastic electron-proton scattering in a simple hadronic model including the finite size of the proton. We find both can reach a few percent and are comparable in size with the current experimental measurements of strange-quark effects in the proton neutral weak current. The effect of \( \gamma Z \)-exchange is in general larger than that of TPE, especially at low momentum transfer \( Q^2 \lesssim 1 GeV^2 \). Their combined effects on the values of \( G_E^s + \beta G_M^s \) extracted in recent experiments can be as large as \(-40\%\) in certain kinematics.

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Strangeness content in the proton remains one of the most intriguing questions in hadron structure. Early indications on the contribution of strange quarks to the nucleon properties came from neutrino and electron deep inelastic scatterings and pion-nucleon sigma term, which suggested that strange quarks might give non-negligible contributions to the spin and mass of the proton \([1]\). Many other observables were later suggested, including excess \( \phi \) production in pp annihilation \([2]\), double polarizations in photo- and electroproduction of \( \phi \) meson \([3]\), and asymmetry in scattering of longitudinally polarized electrons from polarized targets, to probe the strangeness in the nucleon.

Parity-violating asymmetry \( A_{PV} = (\sigma_R - \sigma_L)/(\sigma_R + \sigma_L) \) in polarized electron elastic scattering arises from the interference of weak and electromagnetic amplitudes. Weak neutral current elastic scattering is mediated by the \( Z \)-exchange and measures form factors which are sensitive to a different linear combination of the three light quark distributions. When combined with proton and neutron electromagnetic form factors and with the use of charge symmetry, the strange electric and magnetic form factors, \( G_E^s \) and \( G_M^s \), can then be determined \([3]\). Since this is a rather clean technique to access the charge and magnetization distributions of the strange quark within nucleons, four experimental programs SAMPLE \([5]\), HAPPEX \([6]\), A4 \([7]\), and G0 \([8]\) have been designed to measure this important quantity, which is small and ranges from 0.1 to 100 ppm. This calls for greater efforts to reduce theoretical uncertainty in order to arrive at a more reliable interpretation of experiments.

At tree level, parity violation in electron scattering \( e(p_1) + p(p_2) \rightarrow e(p_3) + p(p_4) \) comes from the interference of diagrams with one-photon-exchange (OPE) and \( Z \)-boson exchange shown, respectively, in Figs. 1(a) and 1(b). Leading order radiative corrections include the box diagrams shown in Figs. 1(c) and 1(d) and other diagrams. The radiative corrections to \( A_{PV} \) have been discussed in \([9, 10]\). However, theoretical uncertainties remain.

Recently, the contribution of the interference of the two-photon-exchange (TPE) process of Fig. 1(c) with diagram of Figs. 1(a) and 1(b) to \( A_{PV} \), has been evaluated in \([11]\) in a parton model using GPDs. It was prompted by the fact that such a model calculation of the TPE effect was found \([12]\) to be arguably able to explain the discrepancy between the measurement of the proton electric to magnetic form factor ratio \( R = \mu_p G_E/G_M \), where \( \mu_p = 2.79 \), from Rosenbluth technique and polarization transfer technique at high momentum-transfer-squared \( Q^2 \) \([13]\). In \([11]\), it was found that indeed the TPE correction to \( A_{PV} \) can reach several percent in certain kinematics, becoming comparable in size with existing experimental measurements of strange-quark effects.

FIG. 1: (a) one-photon-exchange, (b) \( Z \)-boson-exchange, (c) TPE, and (d) \( \gamma Z \)-exchange diagrams for elastic \( ep \) scattering. Corresponding cross-box diagrams are implied.
in the proton neutral weak current. However, the partonic calculations of $11,12$ are reliable only for $Q^2$ large comparable to a typical hadronic scale, while all current experiments $3, 6, 7, 8$ have been performed at lower $Q^2$ values. In addition, the $\gamma Z$-exchange diagram in Fig. 1(d), expected to be of the same order as the TPE correction, was not considered in $14$.

In this paper, we report on calculations of the TPE and $\gamma Z$-exchange corrections to $A_{PV}$, where both are treated in the same hadronic model developed in $14$ to estimate the TPE contribution to $R$. The advantage of the calculation of $14$ is that it is also applicable to low $Q^2$ region and their results for $R$ are in agreement with the partonic calculation of $12$. We will follow $14$ and consider only the elastic intermediate states in the blobs of Figs. 1(c) and 1(d). We find both can reach a few percent and are comparable in size with the current experimental constraints of strange-quark effects in the proton neutral weak current.

At hadron level, the couplings of photon and Z-boson with proton are given as

$$\langle p'|J_\mu^Z|p \rangle = \frac{\tau(p')}{M^2} \left[ F_1^{Z,p} \gamma_\mu + F_2^{Z,p} i\sigma_{\mu\nu} q^\nu + G_{\gamma A}^Z \gamma_5 \right] u(p),$$

$$\langle p'|J_\mu^\gamma|p \rangle = \frac{\tau(p')}{M^2} \left[ F_1^{\gamma,p} \gamma_\mu + F_2^{\gamma,p} i\sigma_{\mu\nu} q^\nu \right] u(p),$$

(1)

where $M$ is the proton mass and $q = p' - p$. $F_{1,2}^{Z,p}$ and $G_{\gamma A}^Z$ are the proton electromagnetic/neutrual weak current and axial form factors, respectively.

Choosing the Feynman gauge and neglecting the electron mass $m_e$ in the numerators, the amplitudes of box diagrams Fig. 1(c) and Fig. 1(d) can be written as

$$M^{(d)} = -i \int \frac{dk}{(2\pi)^4} \tau(p_3) \left( -ie\gamma^\mu \right) \left[ \frac{i(\not{p}_1 + \not{p}_2 - \not{k})}{(p_1 + p_2 - k)^2 - m_e^2 + i\epsilon} \right] \times \left( -ig\gamma^\nu \right) \left( -1 + 4\sin^2 \theta W + \gamma_5 \right) u(p_1) \Gamma_\nu \left( \frac{p_4}{p_4 - k} - \frac{M^2}{2} + i\epsilon \right) \left( -i \right) \times \frac{i(k + M)}{k^2 - M^2 + i\epsilon} \tau_\nu u(p_2),$$

$$M^{(c)} = -i \int \frac{dk}{(2\pi)^4} \tau(p_3) \left( -ie\gamma^\mu \right) \left[ \frac{i(\not{p}_1 + \not{p}_2 - \not{k})}{(p_1 + p_2 - k)^2 - m_e^2 + i\epsilon} \right] \times \left( -ie\gamma^\nu \right) u(p_1) \Gamma_\nu \left( \frac{p_4}{p_4 - k} - \frac{M^2}{2} + i\epsilon \right) \left( -i \right) \times \frac{i(k + M)}{k^2 - M^2 + i\epsilon} \tau_\nu u(p_2),$$

(2)

where $\Gamma_\nu = ie(p'|J_\mu^Z|p)$ and $G_{\gamma A}^Z = -ie(p'|J_\mu^\gamma|p)$ with $4g^2/M_\gamma^2 = \sqrt{2}G_F, M_\gamma$ the Z-boson mass, and $G_F$ the Fermi constant. Amplitudes for cross-box diagrams can be written down similarly. The infinitesimal photon mass $\lambda$ has been introduced in the photon propagator to regulate the IR divergence. In the soft photon approximation, the sums of $M^{(c)}, M^{(d)}$ and their crossed diagrams can be factorized as

$$M_{\text{soft}}^{(c)} = \frac{1}{2} \delta_{MT} M^{(a)}, \quad M_{\text{soft}}^{(d)} = \frac{1}{2} \delta_{MT} M^{(b)},$$

(3)

where $\delta_{MT}$ denotes the correction from the box diagrams in the soft photon approximation given by the standard treatment of Mo and Tseytlin $12$. The IR divergence of the interference of $M_{\text{soft}}^{(c)}$ and $M_{\text{soft}}^{(d)}$ with Figs. 1(a) and 1(b) are exactly canceled by corresponding terms in the bremsstrahlung cross section involving the interference between real photon emission from the electron and from the proton. Under such an approximation, the box diagrams and their corresponding bremsstrahlung cross section give no correction to $A_{PV}$ since $\delta_{MT}$ is independent of the initial electron helicity. To go beyond the soft photon approximation to estimate the corrections to $A_{PV}$, we calculate the full amplitudes of $M^{(c)}$ and $M^{(d)}$ and subtract $M_{\text{soft}}^{(c)}$ and $M_{\text{soft}}^{(d)}$ from their respective full amplitudes. The interferences between the remaining box diagrams and the tree diagrams are thus IR safe.

To calculate the full amplitudes of $M^{(c)}$ and $M^{(d)}$, we need explicit forms of the form factors. For simplicity, we choose to parameterize the Sachs form factors of the proton $G_{E}^{p}/G_{M}^{p}$, which are linear functions of $F_1^{Z,p}$ and $F_2^{Z,p}$ in monopole forms: $G_{E}^{p} \equiv G_{E}^{p}/G_{M}^{p} \equiv (\mu_p G_{E}^{p}/G_{M}^{p}) = 1/((1 + Q^2/0.71)^2, G_{A}^{p} = G_{A}^{p}(0)/((1 + Q^2)^2, Q \equiv \sqrt{Q^2}$ in unit of $GeV$, i.e., $c = 1$, a conversion to be used hereafter. $x, y, z$ are determined from relations $16$, $G_{E}^{p} = \rho(1 - 4\lambda\sin^2 \theta W / G_{E}^{p} - \rho G_{E}^{p} - \rho G_{E}^{p})$, and $G_{A}^{p} = -(1 + R_{T}^{=}) G_{A}^{p} + 3R_{T}^{=}/Q^2 G_{M}^{p} + \Delta s$ at $Q^2 = 0$.

The quantities $G_{E}^{p}$ and $\Delta s$ refer to the SU(3) isoscalar octet form factor and the strange quark contribution to the nucleon spin, respectively. The $\rho, \kappa$ and $R_{T}^{=} = 0$ are due to radiative corrections. This results in $x = 0.076 \pm 0.00264, y = 2.08 \pm 0.00813 - G_{M}^{p}(0), z = -0.95 \pm 0.37\Delta s(0)$. We fix $x = 0.076$ and vary the values of $y, z, \Lambda_1, \Lambda_2$ to check the sensitivity of the results on the parameters and get almost the same results. As in $14$, we use package FeynCalc $17$ and LoopTools $18$ to do the analytical and numerical calculations, respectively. The IR divergence has been checked in our calculations.

In Fig. 2, we show the TPE and $\gamma Z$-exchange corrections to $A_{PV}$ by plotting $\delta$, defined by

$$A_{PV}(\gamma + Z + 2\gamma + Z) = A_{PV}(\gamma + Z)(1 + \delta),$$

(4)

at different values of $Q^2 = 0.1, 0.5, 1.0, 5.0 GeV^2$. $A_{PV}(\gamma + Z)$
We now turn to examine the effects of the TPE and $\gamma Z$-exchange on the values of strange form factors extracted from HAPPEX \([6]\) and A4 \([7]\) experiments. The parity asymmetry is conventionally expressed in the following form \([10]\),

$$A_{PV}(\rho, \kappa) = A_1 + A_2 + A_3,$$

$$A_1 = -\alpha \rho \left[ 1 - 4 \kappa \sin^2 \theta_W - \frac{\epsilon G_E^{\gamma p} G_E^{\gamma n} + \tau G_M^{\gamma p} G_M^{\gamma n}}{\epsilon (G_E^{\gamma p})^2 + \tau (G_M^{\gamma p})^2} \right],$$

$$A_2 = a \left[ \frac{\epsilon G_E^{\gamma p} G_E^{\gamma n} + \tau G_M^{\gamma p} G_M^{\gamma n}}{\epsilon (G_E^{\gamma p})^2 + \tau (G_M^{\gamma p})^2} \right],$$

$$A_3 = a (1 - 4 \sin^2 \theta_W) \left[ \frac{\epsilon G_E^{\gamma p} G_Z^{\gamma n}}{\epsilon (G_E^{\gamma p})^2 + \tau (G_M^{\gamma p})^2} \right], \quad (5)$$

where $a = G_F Q^2 / 4 \pi \alpha \sqrt{2}$, $\epsilon' = \sqrt{(1 + \tau)(1 - \epsilon^2)}$, and $\alpha$ the fine structure constant. When the parameters $\rho$ and $\kappa$ are set to equal one, Eq. \((5)\) reduces to the expression obtained in tree approximation. The linear combination of the strange form factors $G_E^{\gamma p} + \beta G_M^{\gamma p}$, with $\beta = \tau G_M^{\gamma p} / \epsilon G_E^{\gamma p}$ has been extracted from $A_2$ in Eq. \((5)\).

The latest PDG values \([19]\) for $\rho$ and $\kappa$ are $\rho = 0.9876$, $\kappa = 1.0026$. They deviate from one because higher-order contributions like vertex corrections, corrections to the propagators and $\gamma Z$-exchange are taken into account. The effect of the $\gamma Z$ box diagram was estimated in \([8]\) for the case of zero momentum transfer $Q = 0$ and gives a contribution of $\Delta \rho = -3.7 \times 10^{-3}$ and $\Delta \kappa = -5.3 \times 10^{-3}$ if the onset scale is set to be $1 GeV$. To avoid double counting one should then subtract $\Delta \rho$ and $\Delta \kappa$ from $\rho$ and $\kappa$ and use $\rho' = \rho - \Delta \rho$ and $\kappa' = \kappa - \Delta \kappa$ in Eq. \((5)\) instead. Consequently, we will set the experimental parity asymmetry $A_{PV}^{(Exp)}$

$$A_{PV}^{(Exp)} \equiv A_{PV}(1 + \gamma Z + 2 \gamma + \gamma Z),$$

$$= A_{PV}(\rho', \kappa')(1 + \delta). \quad (6)$$

With the value we obtain for $\delta$, we can then determine $A_{PV}(\rho', \kappa')$ and extract strange form factors from the resultant $A_2$. We introduce

$$\overline{G}_E + \beta \overline{G}_M = (G_E^{\gamma p} + \beta G_M^{\gamma p})(1 + \delta G), \quad (7)$$

to quantify the effect of the TPE and $\gamma Z$-exchange effect on the extracted values of strange form factors, where $G_E^{\gamma p} + \beta G_M^{\gamma p}$ and $\overline{G}_E + \beta \overline{G}_M$ denote those extracted from $A_{PV}(\rho, \kappa)$ and $A_{PV}(\rho', \kappa')$, respectively. From Eq. \((3)\) we then obtain

$$\delta_G = \frac{A_{PV}^{(Exp)} \left( \Delta \rho - \delta \right) + 4 a \rho \sin^2 \theta_W \Delta \kappa - A_3 \Delta \rho}{A_{PV}^{(Exp)} - A_0}, \quad (8)$$

where $A_0 = A_1 + A_3$. Note that in HAPPEX and A4 experiments, the values of $A_{PV}$ are negative while the values of $A_1$ and $A_3$ are both negative and $A_2$ is positive.
We present our results for $\delta_G$ in Table 1 for HAPPEX and A4 experiments. We see that at forward angles, even though $\delta$, the corrections to $A_{PV}$, are at most around 1%, the corrections to $G_E^s + \beta G_M^s$, $\delta_G$, are large and negative and can reach as negative as $\sim$40%. We find it is dominated by the second term of Eq. (5). This is because that though $\Delta \kappa$ is small, its coefficient is very large.

Our results of large $\delta_G$ can be understood by looking at the $Q^2$ evolution of $\delta$, depicted in Fig. 3. Previous estimate of the $\gamma Z$ box diagrams of Fig 1. (d) considered only the case of vanishing momentum transfer between initial and final electrons, corresponding to $Q^2 \equiv 0$ and $\epsilon = 1$. It is clear from Fig. 3 that the combined effects of the TPE and $\gamma Z$-exchange effect drops rapidly in the region of $0 < Q^2 < 0.1 GeV^2$. In addition, it drops faster at larger $\epsilon$ than at small $\epsilon$. Hence the use of the results of Table I for any finite $Q^2$ value would grossly overestimate the $\gamma Z$-exchange effect. However, $\delta (G_E^s + \beta G_M^s)$ is still smaller than the current experimental errors even $\delta_G$ is large. It is because the extracted values of $G_E^s + \beta G_M^s$ are very small.

In summary, we estimate both the TPE and $\gamma Z$-exchange corrections to the parity-violating asymmetry of the elastic polarized electron-proton scattering in a hadronic model. We find both can reach a few percent in a wide range of momentum transfer, and are comparable in size to the current experimental measurements of strange-quark contributions in the proton neutral weak current. The effect of $\gamma Z$-exchange is seen in general to be larger than that of TPE, especially in low $Q^2$ region. Their combined effects on the extracted values of $G_E^s + \beta G_M^s$ is surprisingly large, up to $\sim$40% in recent HAPPEX and A4 experiments. The reason is because previous estimate of $\gamma Z$-exchange effects, as used in current experimental analyses, was made at $Q^2 = 0$ and greatly overestimates them at nonvanishing $Q^2$ region.

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![Figure 3: Full TPE and $\gamma Z$-exchange corrections to parity-violating asymmetry as a function of $Q^2$ from 0.01 to 1 at $\epsilon = 0.1, 0.5, 0.9, 0.99$.](image)

| $Q^2 (GeV^2)$ | I | II | III | IV | V |
|---------------|---|----|-----|----|---|
| 0.477         | 0.1 | 0.109 | 0.23 | 0.108 |
| $\epsilon$   | 0.974 | 0.994 | 0.994 | 0.83 | 0.83 |
| $\delta (%)$  | 0.25 | 0.36 | 0.34 | 0.86 | 1.3 |
| $\delta_G (%)$ | -22.0 | -12.30 | -39.75 | -3.95 | -3.5 |

**TABLE I:** The corrections $\delta_G$ to $G_E^s + \beta G_M^s$ for HAPPEX and A4 experiments. I, III and IV refer to HAPPEX data in 2004, 2006, and 2007, respectively. II and IV correspond to A4 data in 2004 and 2005, respectively.

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