Backward Tamm states in left-handed metamaterials

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We study the electromagnetic surface waves localized at an interface separating a one-dimensional photonic crystal and left-handed metamaterial, the so-called surface Tamm states. We demonstrate that the metamaterial allows for a flexible control of the dispersion properties of surface states, and can support the Tamm states with a backward energy flow and a vortex-like structure.

The study of unusual properties of left-handed metamaterials is a rapidly developing field of physics. Left-handed metamaterial (LHM) is characterized by simultaneously negative effective dielectric permittivity and negative effective magnetic permeability, that gives rise to a variety of unusual properties of electromagnetic waves. Such materials have first been suggested theoretically by Veselago [1] almost 40 years ago, but they have been realized experimentally only a few years ago [2]. Later, it was shown that they possess many extraordinary wave-guiding properties [3-6, 8-10]. In this Letter we demonstrate another example of unusual properties of LHM and study electromagnetic surface waves guided by an interface between LHM and a one-dimensional photonic crystal, the so-called surface Tamm states [7, 8]. We demonstrate that the presence of a metamaterial allows for a flexible control of the dispersion properties of surface states, and the interface can support the Tamm states with a backward energy flow and a vortex-like structure.

Surface modes are a special type of waves localized at an interface between two different media. In periodic systems, staggered modes localized at surfaces are known as Tamm states [3, 4], first found in solid-state physics as localized electronic states at the edge of a truncated periodic potential. Surface states have been studied in many different fields of physics, including optics [3, 10], where such waves are confined to an interface between periodic and homogeneous dielectric media, as well as nonlinear dynamics of discrete chains [11]. In optics, the periodic structures have to be manufactured artificially in order to manipulate dispersion properties of light in a similar way as the properties of electrons are controlled in crystals. Such periodic dielectric structures are known as photonic crystals. An analogy between solid-state physics and optics suggests that surface electromagnetic waves should exist at the interfaces of photonic crystals, and indeed they were predicted theoretically [4, 10] and observed experimentally [12]. Such Tamm states can be very important for applications of photonic crystals, as they allow for the enhanced coupling of the electromagnetic waves to and from the photonic crystal waveguides [13, 14].

In this Letter, we study surface electromagnetic waves, or surface Tamm states, guided by an interface separating homogeneous LHM and one-dimensional photonic crystal. We assume that the terminating layer (or a cap layer) of the periodic structure has the width different from the width of other layers of the structure. We study the effect of the width of this termination layer on surface states, and explore a possibility to control the dispersion properties of surface waves by adjusting termination layer thickness. We find novel types of surface Tamm states at the interface with the metamaterial which have a backward energy flow and a vortex-like structure. We also compare our results with the case when the LHM medium is replaced by a conventional dielectric, which we refer to as right-handed material (RHM). The surface states in these two cases we call left- and right-handed Tamm states, respectively.

Geometry of our problem is sketched in Fig. 1. We consider the propagation of TE-polarized waves described by one component of the electric field, $E = E_y$ [15], and governed by a scalar Helmholtz-type equation. We look for stationary solutions propagating along the interface with the characteristic dependence $\sim \exp[-i\omega(t - \beta x / c)]$, where $\omega$ is an angular frequency, $\beta$ is the normalized wavenumber component along the interface, and $c$ is the speed of light, and present this equation in the form,

$$\left[ \frac{d^2}{dz^2} + k_x^2 + \frac{\omega^2}{c^2} \varepsilon(z) \mu(z) - \frac{1}{\mu(z)} \frac{d\mu}{dz} \frac{d}{dz} \right] E = 0, \quad (1)$$

where $k_x^2 = \omega^2/\varepsilon_0\mu_0$, both $\varepsilon(z)$ and $\mu(z)$ characterize the transverse structure of the media. Surface modes correspond to localized solutions with the field $E$ decaying from the interface in both the directions. In a left-side homogeneous medium $|z| < -d_s$, see Fig. 1, the fields are decaying provided $\beta > \varepsilon_0\mu_0$. In the right-side periodic
where \( K_b \) is the Bloch wave number, and \( \Psi(z) \) is the Bloch function which is periodic with the period of the photonic structure (see details, e.g., in Ref. \[16\]). In the periodic structure the waves will be decaying provided \( K_b \) is complex; and this condition defines the spectral gaps of an infinite photonic crystal. For the calculation of the Bloch modes, we use the well-known transfer matrix method \[17\].

To find the Tamm states, we take solutions of Eq. \[1\] in a homogeneous medium and the Bloch modes in the periodic structure and satisfy the conditions of continuity in a homogeneous medium and the Bloch modes in the periodic structure (slow decay of the field for \( z < -d_s \) and fast decay into the periodic structure), thus the total energy flow is backwards with respect to the propagation wavevector. Physically, this difference can be explained by looking at the transverse structure of these two modes. In Figs. \[3\] (a,b) we plot the profiles of the two modes having the same frequency \( k = \omega/c = 0.85\text{cm}^{-1} \), with different longitudinal wavenumber \( \beta \). Corresponding points are shown in the inset in Fig. \[2\]. For the mode (a), the energy flow in the metamaterial exceeds that in the periodic structure (slow decay of the field for \( z < -d_s \) and fast decay into the periodic structure); thus the total energy flow is backward. For the mode (b) we have the opposite case, and the mode is forward. In the right-hand Tamm state approaches the lower edge of the first (lower) band gap, while these right-handed Tamm states approach the top edge. Another important difference between the two cases, is that for the LHM surface modes the slope of the dispersion curve becomes negative for small \( \beta \), and it remains positive for larger longitudinal wavenumbers (see the inset in Fig. \[2\]), while for the conventional dielectric media, the dispersion curves are always with a positive slope. The slope of the dispersion curve determines the corresponding group velocity of the mode. The extended control over the group velocity in the case of LHM bandgap structure is possible due to the backward energy flow in metamaterials. Similar effects have been already predicted for other types of the wave guiding structures \[14\,\,15\]. Similarly, the left-handed Tamm surface wave has a vortex-like energy flow pattern.

As a result of different slope of the dispersion curve of the left-handed Tamm states, we observe the mode degeneracy, i.e., for the same frequency \( \omega \) (or wavenumber \( k \)), there exist two modes with different value of \( \beta \). The mode with lower \( \beta \) has a negative group velocity (with respect to the propagation wavevector), while the other mode has a positive group velocity. Such modes are termed as backward and forward, respectively. In the forward wave, the direction of the total energy flow coincides with the propagation direction, while in the backward wave the energy flow is backwards with respect to the wavevector. Physically, this difference can be explained by looking at the transverse structure of these two modes. In Figs. \[3\] (a,b) we plot the profiles of the two modes having the same frequency \( k = \omega/c = 0.85\text{cm}^{-1} \), with different longitudinal wavenumber \( \beta \). Corresponding points are shown in the inset in Fig. \[2\]. For the mode (a), the energy flow in the metamaterial exceeds that in the periodic structure (slow decay of the field for \( z < -d_s \) and fast decay into the periodic structure); thus the total energy flow is backward. For the mode (b) we have the opposite case, and the mode is forward. In the right-hand Tamm state approaches the lower edge of the first (lower) band gap, while these right-handed Tamm states approach the top edge. Another important difference between the two cases, is that for the LHM surface modes the slope of the dispersion curve becomes negative for small \( \beta \), and it remains positive for larger longitudinal wavenumbers (see the inset in Fig. \[2\]), while for the conventional dielectric media, the dispersion curves are always with a positive slope. The slope of the dispersion curve determines the corresponding group velocity of the mode. The extended control over the group velocity in the case of LHM bandgap structure is possible due to the backward energy flow in metamaterials. Similar effects have been already predicted for other types of the wave guiding structures \[14\,\,15\]. Similarly, the left-handed Tamm surface wave has a vortex-like energy flow pattern.

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1.05 1.1 1.15 1.2 1.25
1.5
0.5
0
-0.5
1 1.05 1.1 1.15 1.2 1.25
FIG. 4: Total energy flow in RH (dashed) and LH (solid) Tamm modes vs. β. Cap layer thickness is \( d_c = 0.01 \text{ cm} \).

handed Tamm state geometry, the energy flow in all parts of the wave are directed along the wavenumber and the localized surface waves are always forward. To demonstrate this, in Fig. 4 we plot the total energy flow in the modes as a function of the wavenumber \( \beta \). These results confirm our discussion based on the analysis of the dispersion characteristics.

Increasing the thickness of the cap layer \( d_c \) will push the dispersion of both right- and left-handed modes inside the gap (see the curves for \( d_c = 0.65 \text{ cm} \) in Fig. 2), thus providing better localization of the modes. In the second gap, for the chosen parameters the LHM mode exists deeper in the gap than the right-handed mode, allowing for a better wave localization. An example of the second-gap-mode profile is shown in Fig. 2(d). One can see that the second band modes are generally weaker localized at the interface then the modes from the first band gap.

Finally, in Fig. 5 we plot the existence regions for the surface Tamm modes on the parameter plane \((d_c, \beta)\). Shaded is the area where the surface modes do not exist. Increasing the cap layer thickness, \( d_c \), we effectively obtain a dielectric waveguide, one cladding of which is a homogeneous medium, while the other one is a photonic crystal. In such a case a typical mode is shown in Fig. 2(c). The contours of the corresponding non-existence regions for right-handed Tamm states are also shown in Fig. 2. The regions have qualitatively different shapes. While the lower non-existence region moves down with increase of \( \beta \) preserving its width, the corresponding right-handed region shifts upwards, decreasing significantly in width.

In conclusion, we have studied novel types of electromagnetic surface waves guided by an interface between a left-handed metamaterial and a one-dimensional photonic crystal. We have shown that in the presence of a left-handed material the surface Tamm waves can be either forward or backward while for conventional structures the Tamm states are always forward. We have compared the properties of the backward Tamm states with the case of conventional dielectric structures and analyzed the existence regions for both right- and left-handed Tamm states. We believe our results will be useful for a deeper understanding of the properties of surface waves in plasmonic and metamaterial systems.

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