A New Remote User Authentication Scheme
Using Smart Cards with Check Digits

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Abstract — In 2000, Hwang and Li proposed a new remote user authentication scheme using smart cards. Chan and Chang showed that the masquerade attack is successful on this scheme. Recently Shen, Lin and Hwang pointed out a different type of attack on this scheme and presented a modified scheme to remove these defects. Further in 2003, Leung et al showed that this modified scheme is still vulnerable to the attack proposed by Chan and Cheng. In addition they showed that the extended attack proposed by Chang and Hwang also works well. Recently Kumar have suggested the idea of check digits to overcome the above attacks. In this paper we present a new scheme by using check digits which also overcomes these attacks and is more efficient than Kumar’s scheme.¹

Index Terms — Authentication, cryptography, security, cryptanalysis, smart cards.

1. INTRODUCTION

In 1993, Chang and Wu [5] introduced Remote password authentication scheme with smart cards. A number of remote password authentication schemes with smart cards have been proposed since then. These schemes allow a valid user to login a remote server and access the services provided by the remote server.

In 2000, Hwang and Li [7] proposed a new remote user authentication scheme using smart cards, which was based on ElGamal’s cryptosystem. Chan and Cheng [2] made a cryptanalysis of Hwang and Li’s scheme and pointed out that a legitimate user can easily create a valid pair of \((ID_i, PW_i)\) without knowing the secret key of the system ‘\(x_i\).’

Later, Shen, Lin and Hwang [10] showed that a legitimate user can masquerade as another user. The user can compute some other user’s password. They also proposed a modified scheme to patch up both ‘Chen and Chang’s attack’ and their own attack. In the same year, Chang and Hwang [4] proposed an extended attack to solve the problem of Chan and Cheng’s attack on Hwang and Li’s authentication scheme.

Further Leung et al [9] showed that weakness still exists in Shen, Lin and Hwang’s scheme. By using similar attack of Cheng’s attack or Chang and Hwang’s attack a legitimate user can still impersonate other legal users to login a remote server.

Recently Kumar [8] suggested the idea of check digits to repair the Shen et al’s scheme, which successfully removes the threats devised by Leung et al. In the same year Awasthi and Lal [1] also proposed a scheme which also removes these threats successfully.

In this paper we propose a new remote user authentication scheme, which is more efficient than Kumar’s scheme in terms of computational cost.

II. REVIEW OF THE HWANG-LI SCHEME

In this section, we briefly review Hwang-Li scheme [5]. This scheme is composed of the initial phase, registration phase, the login phase and the authentication phase. Whenever a user registers itself through registration phase, the server identifies the user and issues a password and a smartcard, holding related information, through a secure channel. To access the remote server, user inserts its smart card in to the device and keys the password. The server authenticates the user in authentication phase. Different phases work as follows:

A. Initial Phase

The System Administrator (SA) generates the following parameters

\[ p : \text{a large prime number} \]
\[ h(\cdot) : \text{a one-way function} \]
\[ x_i : \text{a secret key of the system} \]

B. Registration Phase

A user \( U_i \) who wants to register itself for accessing server services, submits it’s \( ID_i \) to the SA. SA computes \( PW_i \) as

\[ PW_i = ID_i^{x_i} \mod p. \]

Registration center issues a password \( PW_i \) and a smart card, which contains the public parameters \((h(\cdot), p)\).

C. Login Phase

User \( U_i \) inserts its smart-card and provides \( ID_i \) and \( PW_i \). The smart card then does the following:

1. Generate a random number \( r \).
2. Compute \( C_1 = ID_i^r \mod p \).
3. Compute \( t = h(T \oplus PW_i) \mod p -1, \) where \( T \) is the current time stamp.
4. Compute \( M = ID_i^t \mod p \).
5. Compute \( C_2 = M(PW_i)^r \mod p. \)
6. Sends a message $C = (ID_1, C_1, C_2, T)$ to the remote system.

D. Authentication Phase

The system receives the message $C$ at time $T'$, where $T'$ is the current time of system. Then the system does the following steps:

1. Test the validity of $ID_i$.
2. Test the validity of time interval between $T$ and $T'$.
3. Check whether the following equation holds:
   \[ C_2 = C_1^{p^2} \mod p. \]

   It is difficult for user $U_i$ to compute the secret key $x_i$ of the system from the equation:
   \[ PW_i = ID_i^{x_i} \mod p. \]

III. ATTACKS ON HWANG-LI SCHEME

A. Chan-Cheng’s attack

According to this attack a legitimate user can easily create a valid pair of $(ID_i, PW_{ij})$ without knowing the secret key of the system $x_i$.

Suppose Bob is a legal user with a pair of $(ID_b, PW_b)$ and he wants to create a valid pair of $(ID_j, PW_{ij})$ such that it satisfies
\[ PW_i = ID_i^{x_j} \mod p. \]

He first computes $ID_j$ by
\[ ID_j = (ID_b * ID_i) \mod p. \]

Then, he can computes $PW_{ij}$ as follow:
\[ PW_{ij} = ID_j^{x_i} \mod p = (PW_b * PW_i) \mod p. \]

As a result, Bob can create a valid pair of $(ID_j, PW_{ij})$ without knowing the secret key $x_j$.

B. Shen-Lin-Hwang’s attack

According to this attack a legitimate user who wants to masquerade as another user $U_b$ may login a remote server and gain access.

Suppose Bob wants to masquerade as another user $U_b$. First computes $ID_j$ by
\[ ID_j = ID_i^r \mod p, \]
where $r$ is a random number such that $gcd(r, p) = 1$. Then, he submits his $ID_j$ to the remote server. SA will verify identity and provide a smart card and password
\[ PW_i = ID_i^{x_j} \mod p. \]

Now Bob can compute $PW_{ij}$ as follow:
\[ PW_{ij} = ID_j^{x_i} \mod p = PW_b \mod p. \]

As a result, Bob can successfully use user $U_i$’s password to masquerade as the user $U_i$ to login the remote server.

C. Chang and Hwang’s attack

Chang and Hwang pointed out that the Chan and Cheng’s attack is not always success. Chan and Cheng did not consider the validity checking of $ID_i$ in the authentication phase of Hwang and Li’s scheme and cannot guarantee that the specific format of the user identity always satisfies the square of a legitimate identity, that is, $ID_i \mod p$ [3]. Chan and Hwang also designed two attack mechanisms to obtain a valid identity which is based on Chan and Chang’s one.

I-Mechanism

Following Chan and Chang’s equation
\[ ID_i = (ID_b * ID_i) \mod p. \]

is modified to:
\[ ID_i = (ID_b)^r \mod p. \]

where $r$ is an arbitrary integer and the corresponding password can be obtained by:
\[ PW_{ij} = (PW_b)^r \mod p. \]

If $ID_b$ is primitive element of the modulus $p$, any valid identity $ID_i$ as well as its password can be computed. And now any intruder with identity $ID_i$ successfully passes the validity check for identity in Hwang and Li’s scheme.

II-Mechanism

Several intruders may cooperate to get a valid identity $ID_i$ as well as its corresponding password $PW_{ij}$ as follows
\[ ID_i = ID_b \mod p. \]

Suppose identity fulfills the required format, its corresponding password can be obtained by:
\[ PW_{ij} = ID_i^{x_i} \mod p = PW_b \mod p. \]

IV. SHEN-LIN-HWANG’S MODIFICATION

Shen, Lin and Hwang proposed a modified scheme in [7]. They suggested that instead of an identity a shadow identity will be given to the legal user. The scheme has the just hiding of identities to prevent from the forgery attack [section III-B, III-C]. The steps in login and authentication phase are not changed except $ID_i$ is replaced by $SID_i$. The modified registration phase is as follows:

User $U_i$ submits his $ID_i$ to the remote server for registration. The remote server computes the pair $(SID_i, PW_i)$ as follows:
\[ SID_i = Red(ID_i) \mod p. \]

Where $Red(.)$ is a shadow function, such that user cannot compute $ID_i$ having the knowledge of $SID_i$ and $Red(.)$. The shadow function is only maintained in the remote server. $SID_i$ is user’s shadow identity which is made public instead of $ID_i$. Registration center issues a shadow identity $SID_i$, a password $PW_i$ and a smart card, which contains the public parameters $(h, p)$. In login phase and authentication phase each $ID_i$ is replaced with $SID_i$. Login and authentication processes remain same. Now message $C$ coming to server through login page is $(SID_i, C_1, C_2, T)$ instead of $(ID_i, C_1, C_2, T)$.

According to Shen et al., Though shadow identity of $ID_i$ is known, shadow identity of $ID_b = ID_i^r \mod p$ (or $ID_b = ID_i \mod p$) cannot be computed. That is why this scheme is secure.

V. LEUNG-CHENG-FONG-CHAN’S OBJECTION

Leung, Cheng, Fong and Chan pointed out that although Shen et al. defend the attack of registration of a new $ID_i$ via $ID_i$ from a legal user $U_i$, the scheme can not withstand attack that similar to Chan and Cheng’s attack and Chang and Hwang’s attack in authentication phase.
According to Leung et al, shadow identity does not enhance
the security in authentication phase and hence intruder can still
log in to the remote system by having a pair (SID), PW).
Attack works as follows:
Replace IDj with SIDj in Chang and Hwang’s attack
(mechanism-I), we get
\[ SIDj = (SIDj)^r \mod p. \]
where \( r \) is an integer and the corresponding password
can be obtained by:
\[ PWj = (SIDj)^{\text{check digit}} \mod p. \]
\[ = [(SIDj)^{\text{check digit}}]^{\text{check digit}} \mod p. \]
\[ = (PWj)^{\text{check digit}} \mod p. \]
This shows that a legitimate user ui can impersonate other
legal user with valid pair (SID, PW) to login the remote
server. Note SIDj is primitive element of the modulus p, any
valid shadow identity SIDj as well as its password can be
computed. Similarly, chan-cheng’s attack works fine.

VI. REVIEW OF THE KUMAR’S SCHEME

In this section, we briefly review Kumar’s scheme [8]. This
scheme is composed of the initial phase, registration phase, the
login phase and the authentication phase.

This scheme is the modified form of the Shen- Lin-
Hwang’s scheme and uses one more function \( C_K \) to generate
the check digit [6] for each registered identity. In this scheme,
only the AS can generate a valid identity and the corresponding
check digit.

A. Initialization Phase

In this phase, the AS generates the following parameters
\( \mathcal{P} \) : a large prime number.
\( f() \) : a one-way function.
xj : a secret key of the system, which is only possessed with
the AS.
\( \text{Red}() \) : a “shadowed” identity of the device which is only
possessed with the AS.
\( C_K() \) : a one way secure check digit function, which is used
to generate unique check digit for the registered identity,
and also is possessed with the AS Only.

B. Registration Phase

Assume that this phase is executed over a secure channel.
User \( U \) submits her/his identity string IDj to the AS. The string
IDj consists of the name of the user and a unique identification
number etc, which are unique for the user \( U \). Then, AS
computes the following:
\[ S_{ID} = \text{Red}(IDj), C_{ID} = C_K(S_{ID}) \text{ and } PW = (S_{ID})^{xj} \mod p. \]
Furthermore, the AS distributes the smart card and \( (S_{ID}, PW) \)
to the user \( U \) in a secure way (say physically). The smart
card contains the public parameters \( (f, p) \).

C. Login Phase

The user \( U \) attaches her/his smart card to the smart card
reader at any time \( T \) and keys her/his identity \( S_{ID} \) and
password \( PW \). The smart card conducts the following computations:
1. Generate a random number \( r \).
2. Computes \( C_1 = (S_{ID})^r \mod p. \)
3. Compute \( t = f(T \oplus PW) \mod p - 1. \)
4. Compute \( m = (S_{ID})^t \mod p. \)
5. Compute \( C_2 = m \mod p. \)
6. Sends a message \( L_k = (S_{ID} || C_{ID} || C_1, C_2, T) \) to the AS.

D. Authentication Phase

Assume that the AS receives the login request \( L_k \) at time \( T \).
Then, AS does the following computations to check the
validity of the login request \( L_k \).
1. Check the specific format of \( S_{ID} \). If the format of the \( S_{ID} \)
is incorrect, then AS rejects the login request \( L_k \).
2. Check, whether the condition \( C_{ID} = C_K(S_{ID}) \) holds, if
not, then AS rejects the login request \( L_k \).
3. Check, whether \( T_e - T \leq \Delta T \), where \( \Delta T \) is the legal time
interval due to transmission delay, if not, then AS rejects
the login request \( L_k \).
4. Check, if \( C_2 = (C_{1})^s(S_{ID})^f(T \oplus PW) \mod p, \) then the AS
accepts the login request. Otherwise, the login request
will be rejected by AS.

VII. A NEW REMOTE USER AUTHENTICATION SCHEME
USING SMART CARDS WITH CHECK DIGITS

In this section, we propose our new remote user
authentication scheme. This scheme is also composed of the
initial phase, registration phase, the login phase and the
authentication phase.

This scheme is the modified form of the Hwang-Li’s scheme
and uses one more function \( C_K \) to generate the check digit [8]
for each registered identity. In this scheme, only the AS can
generate a valid identity and the corresponding check digit.

A. Initialization Phase

The System Administrator (SA) generates the following parameters
\( p \) : a large prime number
\( f() \) : A one-way function
\( x_j \) : a secret key of the system
\( C_0() \) : a “function” to generate check digit for the
registered identity, which is only possessed with the AS.

B. Registration Phase

A user \( U \), who wants to register itself for accessing server
services, submits it’s \( ID \) to the SA. SA assigns a unique
registration number \( R \) and computes \( PW \) as
\[ PW = (ID_j \oplus R)^y \mod p. \]
\[ C_{ID} = C_K(ID_j \oplus R) \]
Registration center issues a password \( PW \) and a smart
card, which contains the public parameters \( (h, p, R, C_{ID}) \).

C. Login Phase

The user \( U \) attaches its smart card to the smart card reader at
any time \( T \) and keys the identity \( ID \) and password \( PW \). The
smart card computes as folloowing:
1. Generate a random number \( r \).
2. Computes \( C_i = (ID_i \oplus R)^i \mod p \).
3. Compute \( t = f(T \oplus PW_i) \mod p - 1 \).
4. Compute \( m = (ID_i)^i \mod p \).
5. Compute \( C = m (PW_i)^i \mod p \).
6. Sends a message \( C = (ID_i, R, C_{ID}, C_1, C_2, T) \) to the AS.

D. Authentication Phase

The system receives the message \( C \) at time \( T' \), where \( T' \) is the current time of system. Then the system does the following steps:
1. Test the validity of \( ID_i \).
2. Checks \( C_{ID} = C_k (ID_i \oplus R) \), if true then proceed else reject the request.
3. Test the validity of time interval between \( T \) and \( T' \), in case of invalid time interval give an appropriate message and stop the protocol.
4. Check whether the following equation holds:
   \[ C (C_i)^i = (ID_i)^i (r \oplus PW_i) \mod p. \]

VIII. SECURITY ANALYSIS OF THE SCHEME

In this section, we shall only discuss the enhanced security features. Rests are the same as original schemes in literature.

A. Chan-Cheng’s attack and Chang-Hwang’s Attack

The security explanation is very similar to Kumar’s discussion. The functionality of the \( C_k \) (.) blocks the attacks via authentication phase as in Chan-Cheng’s attack and Chang-Hwang’s Attack. Assume, Alice chooses a \( R' \) such that \( ID_b \oplus R' = (ID_i \oplus R)^i \mod p \), where \( k \) is a random number. Then, he can compute the corresponding password
\[ PW_b = PW_i^k \mod p. \]

But, this result is incomplete; still, it is essential to obtain the check digit corresponding to \( ID_b \). In the proposed scheme only the SA can generate the check digit corresponding to \( ID_b \) and \( R' \). As a result, a legal user \( ID_b \) cannot compute a valid pair of identity and password for the other user \( ID_b \) to login at the server. Thus, Chang-Hwang’s Attack will not work. As being the one of the case of this attack, Chan–Cheng’s attack also does not work.

A. Shen-Lin-Hwang’s attack

The functionality of the assignment of registration number blocks the masquerade attack via registration phase. Assume that an intruder intercepts the logon request \( C = (ID_i, C_{ID}, C_1, C_2, T_c) \) from a public network and computes
\[ ID_b = (ID_i)^i \mod p. \]

Now he submits this \( ID_b \) to the SA for the registration. Upon receiving the registration request from \( ID_b \) the SA will reject the request because the format of \( ID_b \) is incorrect. If in any case format holds, SA computes assigns a unique registration number \( r \) and computes a corresponding password \( PW_b = (ID_i \oplus r)^i \mod p. \) With this \( PW_b \) it is infeasible to compute \( PW_c \) because \( R \) (Registration number of user \( U_c \)) and \( r \) (Registration number of user \( U_r \)) corresponding to \( ID_b \) are predefined.

B. Leung-Cheng-Fong-Chan’s attack: is extension of Chan-Cheng’s attack and Chan and Hwang’s attack. In this scheme we are not using shadow function so there is no need to discuss this attack.

IX. COMPARISON

In this section we compare the computational cost of our scheme with other schemes. Let us denote \( E \) as computational cost of an exponential operation, \( H \) as cost of hashing, \( M \) cost of multiplication, \( R \) cost of redirected function (It is approximately hashing cost) and \( C \) as Check digit counting cost. Since \( R \) and \( C \) both functions are equivalent to hash function, we can not neglect their computational cost. Other costs are negligible or minor. Then following table shows the computational cost in various phases in terms of these costs.

| Schemes          | Computation Cost |
|------------------|------------------|
| Registration     | Login            | Authentication | Security |
| Phase            | Phase            | Phase          | in authentication and registration |
| Phase            | Phase            | Phase          | phases |
|                  |                  |                |        |
| Hwang-Li Scheme  | \( E \)          | \( 3E+H+M \)    | \( 3E+H+M \) | NIL |
| Shen et al’s     | \( R+E \)        | \( 3E+H+M \)    | \( 3E+H+M \) | NIL |
| Scheme           | \( R+E+C \)      | \( 3E+H+M \)    | \( 3E+H+M+C \) | Yes |
| Proposed Scheme  | \( E+C \)        | \( 3E+H+M \)    | \( 3E+H+M+C \) | Yes |

Above table shows that our proposed scheme bears a less redirection cost in registration phase than Kumar’s scheme and holds similar security level.

X. COMPARISON

This paper presents New Remote User Authentication Scheme Using Smart Cards with check digits. This scheme is more efficient than Kumar’s scheme. The proposed scheme is secure against the both types of attacks: attacks via registration phase and the attack via authentication phase. One enhanced feature of this scheme is that more than one user with same identity may register to the server.

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