Torsion contraints from the recent precision measurement of the muon anomaly

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Abstract

In this paper we consider non-minimal couplings of the Standard Model fermions to the vector (trace) and axial vector (pseudo-trace) components of the torsion tensor. We then evaluate the contributions of these vector and axial vector components to the muon anomaly and use the recent precision measurement of the muon anomaly to derive constraints on the torsion parameters.

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The Standard Model (SM) has been extremely successful in explaining the results of all experiments both at low, medium and high energies. In spite of this phenomenal success a majority of high energy physicists agree that the SM is at best a renormalizable effective field theory valid in a restricted energy range since it does not include quantum gravity. Many theorists further believe that the ultimate and fundamental theory of Nature will be provided some day by string theory since the low-energy effective Lagrangian of closed strings reproduces quantum gravity.

The low-energy effective action of closed strings predicts, along with the a massless graviton, a massless antisymmetric second rank tensor field \( B_{\alpha\beta} \) (Kalb-Ramond tensor) which enters the action via its antisymmetrised derivative \( H_{\alpha\beta\gamma} = \partial_{\alpha} B_{\beta\gamma} \). The third rank tensor \( H_{\alpha\beta\gamma} \) is referred to as the torsion field strength. Since torsion is always predicted by string theory it is highly interesting to investigate what low-energy observables and phenomenological effects torsion can produce. Some effects of both heavy (nonpropagating) and dynamical (propagating) torsion fields have already been considered [2] and bounds on the free parameters of the torsion action (torsion mass and couplings) have been derived using experimental data. In this paper we shall study the effects of the trace and pseudo-trace components of the torsion tensor on the anomalous magnetic moment of the muon.

Precision measurements of the muon anomaly \( a_{\mu} = (g_{\mu} - 2)/2 \) currently constitutes one of the most stringent tests for the SM and for new physics, particularly in the light of the recent and future promised results from the E821 experiment at BNL[4]. Recently the E821 collaboration has reported a new improved measurement of \( a_{\mu} \). The experimental value of the muon anomaly according to this latest measurement[5] is

\[
\mathcal{a}_{\mu}^{\text{exp}} = (11\ 659\ 204(7)(4)) \times 10^{-10}.
\]

The SM prediction, according to the latest and improved calculation[6] of \( a_{\mu}^{\text{hadron}} \), is

\[
\mathcal{a}_{\mu}^{\text{SM}} = (11\ 659\ 176 \pm 6.7) \times 10^{-10}.
\]

The experimental value of the muon anomaly differs, therefore, from the SM prediction by \( \delta a_{\mu} = (28 \pm 10.5) \times 10^{-10} \) i.e. by about 2.66\( \sigma \). The BNL collaboration is expected to
reach an eventual precision of $4 \times 10^{-10}$ during its final stage of operation\textsuperscript{[4]}. If the central value does not change, this could eventually differ from the SM by about $7 \sigma$ and provide, perhaps, the earliest proof of new physics beyond the SM.

It is clear, therefore, that it is extremely important to see if (a) the presence of torsion fields can explain why the measured value of the muon anomaly is different from the SM prediction, and/or (b) the highly accurate measurements from the E821 experiment can be used to constrain the parameter space of the torsion action. Such an analysis forms the subject of the present article.

§ Torsion couplings to SM fermions: In dealing with quantum theory on curved space times with torsion the metric $g_{\mu \nu}$ and the rank-3 torsion tensor $T_{\beta \gamma}^{\alpha}$ should be considered as independent dynamical variables. In this paper, we shall consider the observable consequences of torsion only and therefore we shall set the metric to be flat Minkowskian everywhere i.e. $g_{\mu \nu} = \eta_{\mu \nu}$.

The torsion field $T_{\beta \gamma}^{\alpha}$ is defined\textsuperscript{[3]} in terms of the non-symmetric connection $\tilde{\Gamma}_{\beta \gamma}^{\alpha}$

$$T_{\beta \gamma}^{\alpha} = \tilde{\Gamma}_{\beta \gamma}^{\alpha} - \tilde{\Gamma}_{\gamma \beta}^{\alpha} \, .$$

(1)

For convenience the torsion tensor $T_{\beta \gamma}^{\alpha}$ can be divided\textsuperscript{[2]} into three irreducible components. These are

- trace: $T_{\beta} = T_{\beta \alpha}^{\alpha}$;

- pseudo-trace: $S_{\nu} = \epsilon^{\alpha \beta \mu \nu} T_{\alpha \beta \mu}$;

- the third rank tensor $q_{\alpha \beta \gamma}$, which satisfies the conditions $q_{\beta \gamma}^{\alpha} = 0$ and $\epsilon^{\alpha \beta \mu \nu} q_{\alpha \beta \mu} = 0$.

Clearly $T$ behaves as a vector field and $S$ as an axial vector field. The simultaneous presence of both $S$ and $T$ as light dynamical fields could, therefore, be a likely signature of torsion. However, the simultaneous presence of both $S$ and $T$ as light dynamical fields in the low-energy effective field theory would lead to serious problems related to renormalizability. The predictions of the model about loop-induced corrections like the muon anomaly would then become cut-off dependent and somewhat uncertain. In this
paper we shall, therefore, work within the simplifying assumption that only one of them appears as a light, propagating degree of freedom, whereas the other is very heavy and non-propagating. Hence, we are led to consider the following cases:

1. $T$ is very heavy and only $S$ appears as the propagating degree of freedom;

2. $S$ is very heavy and only $T$ appears as the propagating degree of freedom.

We also assume that the third rank tensor $q_{\alpha\beta\gamma}$ does not couple to SM fermions.

We now consider the action for the torsion tensor with non-minimal couplings to fermions. This assumes the general form

$$S_{\text{torsion}} = \int d^4x \left[ -\frac{1}{4} S_{\mu\nu} S^{\mu\nu} + \frac{1}{2} M_S^2 S^\mu S_\mu + \eta S \bar{\psi} \gamma_\mu \gamma_5 \psi S^\mu \right] + \int d^4x \left[ -\frac{1}{4} T_{\mu\nu} T^{\mu\nu} + \frac{1}{2} M_T^2 T^\mu T_\mu + \eta_T \bar{\psi} \gamma_\mu \psi T^\mu \right],$$

(2)

where $S_{\mu\nu} = \partial_\mu S_\nu - \partial_\nu S_\mu$ and $T_{\mu\nu} = \partial_\mu T_\nu - \partial_\nu T_\mu$. Although in the above Lagrangian we have included the kinetic energies for both $S$ and $T$, it should be kept in mind that for the underlying theory to be renormalizable, only one of them should be considered as propagating. For phenomenological purposes only the first or the second line of Eqn.(2) is relevant, never both.

We note that in the minimal coupling scheme $\eta_T = 0$. A non-zero $\eta_T$, therefore, represents purely non-minimal effects. Moreover, since the vector current is exactly conserved (CVC), the mass $M_T$ of $T_\mu$ can be either zero or non-zero without affecting the renormalizability of the model. On the other hand, partial conservation of the axial vector current (PCAC) implies that $M_S$ must be non-zero for the above action to be renormalizable.

§ Torsion contributions to the muon anomaly: In the following we shall estimate the muon anomaly contribution due to the torsion tensor separately for the two cases mentioned above.

(a) In this case, only the axial vector field ($S_\mu$) appears as the propagating degree of freedom and the muon anomaly contribution due to torsion arises from the one-loop diagram in Fig. 1(a). Using the Gordon identity to express the vertex correction as a sum
of a vector current and spin current, we finally get

\[ a^{(s)}_\mu = \frac{m^2_\mu \eta^2_S}{4\pi^2} \int_0^1 dx \frac{x(1 - x)(x - 4) - 2x^3m^2_\mu M^2_S}{m^2_\mu x^2 + M^2_S(1 - x)}. \] (3)

Two particular limits of \( M_S \) are particularly interesting, namely \( M_S \gg m_\mu \) and \( M_S \approx m_\mu \). For \( M_S \gg m_\mu \), the above results simplifies to

\[ a^{(s)}_\mu \approx -\frac{5}{3} \left( \frac{m_\mu \eta_S}{2\pi M_S} \right)^2, \] (4)

while, for \( M_S \approx m_\mu \) we get

\[ a^{(s)}_\mu \approx -\frac{\eta^2_S}{4\pi^2} \int_0^1 dx \frac{4x - 5x^2 + 3x^3}{1 - x + x^2} = -1.3138 \left( \frac{\eta_S}{2\pi} \right)^2. \] (5)

Note that the contribution of \( S \) to the muon anomaly is always negative.

\[ V = (a) \ S \quad \text{or} \quad (b) \ T \]

\[ \begin{array}{c}
\mu \\
\downarrow p \\
\mu \\
\mu \\
\mu \\
\downarrow l+p \\
\mu \\
\downarrow l+p' \\
\mu \\
\downarrow p' \\
\eta \\
\gamma \\
\end{array} \]

**Figure 1.** One-loop Feynman diagrams that contribute to the muon anomaly through exchange of 
(a) \( S \) and (b) \( T \) fields.

(b) In this case only the trace \((T^\mu)\) of the torsion tensor appears as the propagating dynamical field and the muon anomaly contribution due to torsion arises from Fig. 1(b). Following the usual procedure we then find

\[ a^{(T)}_\mu = \frac{m^2_\mu \eta^2_T}{4\pi^2} \int_0^1 dx \frac{x(1 - x)}{x^2m^2_\mu + (1 - x)M^2_T}. \] (6)

For \( M_T \gg m_\mu \) we get

\[ a^{(T)}_\mu \approx \frac{1}{3} \left( \frac{m_\mu \eta_T}{2\pi M_T} \right)^2. \] (7)
while for $M_T \simeq m_\mu$,

$$a_\mu^{(T)} \simeq \frac{\eta_T^2}{4\pi^2} \int_0^1 dx \frac{x(1-x)}{1-x+x^2} = 0.2092 \left(\frac{\eta_T}{2\pi}\right)^2 . \quad (8)$$

The muon anomaly contribution due to $T$ is, therefore, always positive.

§ Determination of constraints: The above formulae can be now used in conjunction with the experimental results to obtain constraints on the parameter space of torsion fields. We first consider the constraints that arise when the torsion mass is much greater than the muon mass. Under this condition the muon anomaly due to $S$ can be expressed as

$$a_\mu^{(S)} = -\frac{5}{3\pi} \left(\frac{m_\mu}{\Lambda_S}\right)^2 \quad (9)$$

where $\Lambda_S$ is an effective scale given by $\Lambda_S^2 = 4\pi M_S^2/\eta_S^2$. Since the experimental result already shows a positive deviation ($2.66\sigma$) from the SM value, the effects of $S$ would cause an even further deviation. We can parameterize this excess deviation by

$$\xi_S = \frac{|\delta a_\mu^{\text{exp}}(\text{C.V.}) - a_\mu^{(S)}|}{\Delta(\delta a_\mu^{\text{exp}})} \quad (10)$$

where

$$\delta a_\mu^{\text{exp}} = \delta a_\mu^{\text{exp}}(\text{C.V.}) \pm \Delta(\delta a_\mu^{\text{exp}}) ,$$

C.V. standing for the experimental central value. In the decoupling limit, $\Lambda_S \rightarrow \infty$, $a_\mu^{(S)} \rightarrow 0$ and $\xi_S \rightarrow 2.66$ which is the SM result. The presence of $S$ cannot, therefore, account for the present experimental value of the muon anomaly for $\xi_S < 2.66$. The variation of $\xi_S$ with $\Lambda_S$ is shown in Fig. 2(a), together with the $3\sigma$ bound which corresponds to

$$M_S > \eta_S \ (1.13 \text{ TeV}) \quad (11)$$

A similar analysis can be done for the vector component $T$. For $M_T \gg m_\mu$ the muon anomaly contribution due to $T$ can be written as

$$a_\mu^{(T)} = \frac{1}{3\pi} \left(\frac{m_\mu}{\Lambda_T}\right)^2 \quad (12)$$

where $\Lambda_T$ is another effective scale given by $\Lambda_T^2 = 4\pi M_T^2/\eta_T^2$. Since the contribution of $T$ to the muon anomaly is positive, its presence can constitute an explanation for the muon
anomaly both for positive and negative deviations from $\delta a_\mu^{\text{exp}}(C.V.)$. As before, we can parametrize the contribution to the muon anomaly by

$$\xi_T = \frac{|\delta a_\mu^{\text{exp}}(C.V.) - a_\mu^{(T)}|}{\Delta (\delta a_\mu^{\text{exp}})}$$

(13)

where $\xi_T$ denotes the number of standard deviations by which $a_\mu^{(T)}$ differs from $\delta a_\mu^{\text{exp}}(C.V.)$. The variation of $\xi_T$ with $\Lambda_T$ is shown in Fig. 2(b) along with the upper and lower bounds at $1\sigma$ and $2\sigma$ and the lower bound at $3\sigma$. The left (right) branch corresponds to positive (negative) deviations from $\delta a_\mu^{\text{exp}}(C.V.)$. For $\Lambda_T \approx 792$ GeV, $\xi_T$ vanishes — indicating this is the most favored value if torsion is the cause for the present muon anomaly. In fact, we obtain the following bounds

$$\Lambda_T = \begin{cases} 
792^{+209}_{-117} \text{ GeV at } 1\sigma \\
792^{+791}_{-194} \text{ GeV at } 2\sigma \\
> 543 \text{ GeV at } 3\sigma
\end{cases}$$

(14)

Figure 2. Illustrating constraints on the torsion parameter space from the muon anomaly with exchange of (a) $S$ and (b) $T$ fields in the limit $M_{S,T} \gg m_\mu$.

We now examine the situation when $M_{S,T} \approx m_\mu$. In this case the muon anomaly due to torsion cannot be parameterised in terms of a single parameter $\Lambda_{S,T}$ as we have done
in the above discussion. Here the integrals in Eqns. (3) and (6) have to be evaluated exactly. Noting that \( a_\mu^{(S)} \) is always negative and only a 3\( \sigma \) bound can be obtained, while the positive \( a_\mu^{(T)} \) allows for both upper and lower bounds at 1\( \sigma \) and 2\( \sigma \), we require

\[
|a_\mu^{(S)}| \leq 3\Delta(\delta a_\mu^{\text{exp}}) - \delta a_\mu^{\text{exp}}(\text{C.V.}) = 3.5 \times 10^{-10} \quad \text{at } 3\sigma
\]

\[
|a_\mu^{(T)} - \delta a_\mu^{\text{exp}}(\text{C.V.})| \leq n\Delta(\delta a_\mu^{\text{exp}}) = 10.5n \times 10^{-10} \quad \text{at } n\sigma \tag{15}
\]

Using these conditions we can now constrain the \( \eta_S^* - M_S \) and \( \eta_T^* - M_T \) planes. These are shown in Figs. 3(a) and (b) respectively. Broad hatching represents the allowed region at 3\( \sigma \), close hatching represents the allowed region at 2\( \sigma \) and cross hatching represents the allowed region at 1\( \sigma \). We have considered the range \( M_{S,T} = 50 \text{ MeV} - 1 \text{ GeV} \). Beyond a GeV it is reasonable to use the approximation \( M_{S,T} \gg m_\mu \), which has already been discussed. We note that for light \( S \) or \( T \) the muon anomaly result requires \( \eta_S \) and \( \eta_T \) to be very small. This constitutes an explanation as to why torsion effects have not been seen so far in low-energy precision experiments.

![Figure 3](image-url)

**Figure 3.** Illustrating constraints on the torsion parameter space from the muon anomaly with exchange of (a) \( S \) and (b) \( T \) fields in the limit \( M_{S,T} \approx m_\mu \).
Discussion of results: It is interesting to compare the bounds on torsion-fermion couplings and torsion masses obtained from the muon anomaly with the existing direct bounds obtained from collider data. Since collider bounds are available only for the $S$ field and that too for $M_S \gg m_\mu$, we must perforce confine the discussion to this case only. Using LEP-1.5 data on $A^{(e)}_{FB}$, Belyaev and Shapiro\cite{2} have found a lower bound $M_S > 2.1$ TeV for $\eta_S \simeq 1$. On the other hand, from the muon anomaly we obtain a lower bound $M_S \simeq 1.13$ TeV for $\eta_S \simeq 1$. The constraints obtained from the muon anomaly are therefore slightly weaker — but nevertheless comparable — with the direct collider bounds. The bounds obtained from the muon anomaly would however improve once the BNL collaboration enters the final stage of operation. It would also be interesting to update\cite{7} the collider bounds on torsion parameters in the light of more recent data from LEP-2 and from Run-I of the Tevatron.

Figure 4. Comparing direct constraints on the parameter space for the $S$ field with those obtained from the muon anomaly. Vertical hatching represents the region disallowed by $A^{(e)}_{FB}$ and oblique hatching represent the CDF constraint, both as obtained by Belyaev and Shapiro\cite{2}. The straight line corresponds to the $3\sigma$ bound from the muon anomaly.

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