Bogoliubov space of a Bose–Einstein condensate and quantum spacetime fluctuations

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Abstract

In this work, we consider the role that metric fluctuations could have upon the properties of a Bose–Einstein condensate. In particular, we consider the Bogoliubov space associated with it and show that there are, at least, two independent ways in which the average size of these metric fluctuations could be, experimentally, determined. Indeed, we prove that the pressure and the speed of sound of the ground state define an expression allowing us to determine the average size of these fluctuations. Afterward, an interferometric experiment involving Bogoliubov excitations of the condensate and the pressure (or the speed of sound of the ground state) provides a second and independent way in which this average size could be determined, experimentally.

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(Some figures may appear in colour only in the online journal)

1. Introduction

The mathematical and physical difficulties plaguing all theoretical models behind a quantum theory of gravity [1, 2] have spurred the so-called quantum gravity phenomenology, a topic that can be defined as the observational and experimental search for deviations from Einstein’s general relativity, or from quantum field theory. It embodies deformed versions of the dispersion relation [3, 4], or deviations from the 1/r-potential and violations of the equivalence principle [5, 6]. Of course, these aforementioned cases do not exhaust the extant possibilities. The use of cold atoms, either bosonic or fermionic, in this context is a point already considered [7, 8]. In particular, the possibility of constraining the energy–momentum relation resorting to cold atoms has already shown us that these kinds of systems could open up new landscapes in the context of gravitational physics [9].

In the present work, we consider the role that metric fluctuations could have upon the properties of a Bose–Einstein condensate. In particular, we address the issue of conformal metric fluctuations. Let us explain, briefly, the meaning of this last sentence. In this context,
the main idea corresponds to a Minkowskian background and in addition, small spacetime fluctuations are also present, and they are a consequence of some quantum gravity scenario. One of the assumptions in this approach is related to the fact that these spacetime fluctuations emerge as classical fluctuations of the background metric. There are several possibilities around the type of fluctuations that can be considered \[10\]. Among the huge spectrum of possibilities, we may find the so-called conformal fluctuations \[12, 13\]. They can be considered, mathematically, the simplest case and entail a redefinition of the inertial mass. In other words, the average size of these kinds of fluctuations, say \(\gamma\), appears in the corresponding motion equations in the form \(m/(1 + \gamma)\), where \(m\) denotes the usual inertial mass (when no fluctuations are present). Clearly, an experimental outcome detects \(m/(1 + \gamma)\), and not \(m\) or \(\gamma\) separately.

Of course, a possible objection to the analysis of conformal fluctuations could be related to its simplicity, which in physical terms implies a redefinition of the inertial mass. In this work, we will show that even the simplest case, i.e. conformal fluctuations, is endowed with a richness that leads us to detectable effects.

We must underline the fact that in this work, we do not analyze the model from which these fluctuations could emerge; this aspect can be consulted in the corresponding references \[12, 13\]; we use them only as initial physical information. It is also noteworthy to comment that determining the place that these fluctuations have in the general scheme for Lorentz violating scenarios \[14\] is a task that has not been done yet, namely it cannot be asserted that they are a special subset of the already existing data.

The idea in this work is related to the possibility of, by means of two or more experimental proposals, deducing separately, \(m\) and \(\gamma\). It will be proved that the Bogoliubov space of a Bose–Einstein condensate offers this option. Indeed, we consider the Bogoliubov space associated with a Bose–Einstein condensate and show that there are at least two independent ways in which the average size of these metric fluctuations can be experimentally determined. First, we consider the many-particle Hamiltonian of a bosonic gas immersed in a homogeneous gravitational field, and, in addition, the effects of the metric fluctuations upon the inertial mass will be introduced. It will be shown that the pressure and speed of sound of the ground state of the Bogoliubov space of the condensate allow us to put forward an experiment which, in principle, determines the average size of these fluctuations. As a by-product, the value of \(m\), i.e. the bare mass, can also be obtained. Secondly, we analyze an interferometric experiment, resorting to Bogoliubov excitations, and deduce the phase shift induced by the gravitational field and the metric fluctuations. It will be proved that this gravity-induced phase shift together with the pressure (or the speed of sound) of the ground state of the Bogoliubov space implies two expressions which determine the size of these metric fluctuations, and also of \(m\). In order to better understand this argument, let us denote by \(v\) the speed of sound of the ground state of the Bogoliubov space, and by \(\Delta\Phi\) the aforementioned gravity-induced phase shift of two Bogoliubov excitations. It will be shown that, roughly said, \(m/(1 + \gamma) = f_1(v, a, N, V, g)\), whereas \(m/(\sqrt{1 + \gamma}) = f_2(\Delta\Phi, a, N, V, g)\). In the last two expressions, \(f_1\) and \(f_2\) are two functions (deduced in this work) whose values can be measured independently, and \(N, V, a\) and \(g\) denote the number of particles, volume of the container of the bosonic gas, scattering length and, finally, acceleration of gravity, respectively. Clearly, \(f_2/f_1 = \sqrt{1 + \gamma}\). In other words, the experimental deduction of \(f_1\) and \(f_2\) leads us to the determination of a characteristic of these metric fluctuations, namely \(\gamma = (f_2/f_1)^2 - 1\). In a similar way, we may find an expression for \(m\).

1.1. Metric fluctuations

Now we succinctly address the issue of metric fluctuations. In this context, a spacetime is present, the one that can be regarded as a classical background on which classical fluctuations
exist [12]. We suppose that they are a consequence of a quantum theory of gravity where the microscopic structure of spacetime exhibits quantum fluctuations.

At this point, let us explicitly state that the background geometry is a Minkowskian spacetime, and in this given spacetime our fluctuations shall be understood as deviations from the Minkowskian metric via

$$g_{\mu\nu}(x, t) = \eta_{\mu\nu} + h_{\mu\nu}(x, t), \quad |h_{\mu\nu}| \ll 1$$

and Greek indices run from 0 to 3. These spacetime fluctuations modify our current physics. Indeed, they entail a change of the motion equations. In addition, we restrict ourselves to the case of spacetime fluctuations characterized by short wavelengths and, in consequence, by large frequencies. The meaning of the words short and large emerges by the comparison of the spacetime resolution scale that each particle quantum mechanically defines [11, 12]. Under these conditions, the effects of these kinds of fluctuations upon a quantum particle can be represented by an averaging procedure which is correctly defined by the aforementioned resolution scale [12, 13].

In this general scheme, these fluctuations imply a modified Schrödinger equation, in which the effective Hamiltonian operator becomes [12, 13]

$$H = -\frac{\hbar^2}{2m} (\delta^{ij} + \gamma^{ij}) \partial_i \partial_j - mU(x) + \frac{m}{2} \sigma_0^2, \quad (1)$$

where $\gamma^{ij} > 0$ and $\sigma_0^2 > 0$ are the averaged quantities of the order $O(\hbar^2)$ in the fluctuations. Hence, $\gamma^{ij}$ and $\sigma_0^2$ are the particle-dependent constants which have to be taken into account in the general case. In this context, the last term can be understood as a redefinition of the Newtonian potential $\tilde{U}(x) = U(x) + \sigma_0^2/2$, which leads to a constant shift of the energy. Thus, the gravitational term does suffer a modification due to the presence of these fluctuations but it maintains its form and no modified gravitational mass appears. This behavior is in contrast to the inertial mass, which, as shown before, is different. In other words, in this case we expect a violation to the weak equivalence principle [15].

The simplest case involves the so-called conformal fluctuations defined by the following conditions: $\gamma^{ij} = 0$, if $i \neq j$, whereas $\gamma^{xx} = \gamma^{yy} = \gamma^{zz} = \gamma$ and is a constant. We may rephrase this last assertion stating that the idea of conformal fluctuations is depicted by matrices proportional to the unit matrix. In other words, at least concerning the spacetime resolution defined by our quantum particles, the average properties of our involved spacetime fluctuations are given by just one constant. Therefore, the Hamiltonian now reads

$$H = -\frac{\hbar^2}{2m} \gamma \partial_i \partial_j - mU(x). \quad (2)$$

Note that the last term of (1) is not present, since it is composed of a combination of off-diagonal elements of the perturbation tensor $h_{\mu\nu}$, which vanish in the case of conformal fluctuations. Clearly, the corresponding model of quantum fluctuations tells us that the term associated with a potential term will not be modified. This point will be relevant in the definition of the motion equation in which a coupling between atoms and a homogeneous gravitational field exists.

Additionally, these fluctuations can be comprehended (at least partially) as redefinitions of the inertial mass [13]. The new inertial mass for an atom $(m_{\text{eff}})$ is now given by

$$m_{\text{eff}} = m(1 + \gamma)^{-1}. \quad (3)$$

The parameter $\gamma$ encodes the information concerning the metric fluctuations and its value depends on the particle species under consideration [12, 13]. Clearly, they have to be considered very small; thus, $|\gamma| \ll 1$ (otherwise its existence would be a proved experimental fact).

The expression for the redefined inertial mass contains an experimental hurdle. Indeed, note that a kinematical experiment detects the relation $m(1 + \gamma)^{-1}$, but not $m$ or $\gamma$, separately. At this point, we pose the main question that will be addressed in this work: could $m$ and $\gamma$ be measured separately?
1.2. Weakly interacting gas

As mentioned before, the main idea in this work involves an experimental proposal for the detection of the bare inertial mass, here denoted by \( m \), and the average size of our conformal fluctuations, the parameter \( \gamma \). Indeed, the Laplacian operator becomes, under this condition [12, 13],

\[
\Delta_\gamma = (1 + \gamma) \delta^{ij} \partial_i \partial_j.
\]  

(4)

This modification will be considered in the \( N \)-body Hamiltonian operator (assuming that the gas is so dilute that only the two-body interaction potential is required [16]). Our model will be a Bose–Einstein gas enclosed in a container of volume \( V \); the particles of the gas are atoms with passive gravitational mass \( m \) and located at a height \( l \) with respect to the surface of the Earth. We now hark back to our previous discussion concerning the modified motion equation for an atom coupled to a homogeneous gravitational field. Equation (36) in [12] tells us that the coupling term will suffer no modification at all. This point plays a very relevant role in the present proposal. Indeed, while the mass coefficient associated with the Laplacian operator will be modified, i.e. now we have an effective inertial mass (see (3)), the passive gravitational mass (which is the coupling parameter between an atom and a gravitational field) will suffer no change in the presence of our fluctuations.

The interaction between two particles will be assumed to be dominated by \( s \)-scattering, i.e. the temperature of the system is very low (\( ka \ll 1 \), where \( k \) and \( a \) are the wave vector and the scattering length, respectively) [17]. This entails the following Hamiltonian for the \( N \)-body system:

\[
\hat{H} = \sum_{\vec{k}=0}^{\infty} \frac{\hbar^2 k^2}{2m} (1 + \gamma) \hat{a}_{\vec{k}}^\dagger \hat{a}_{\vec{k}} + \frac{U_0}{2V} \sum_{\vec{p}, \vec{q} = 0}^{\infty} \sum_{j=0}^{\infty} \hat{a}_{\vec{p}}^\dagger \hat{a}_{\vec{q}}\hat{a}_{\vec{p}+\vec{q}+\vec{k}} \hat{a}_{\vec{p}+\vec{q}+\vec{k}} + \sum_{\vec{k}=0}^{\infty} mgl \hat{a}_{\vec{k}}^\dagger \hat{a}_{\vec{k}}.
\]  

(5)

\[
U_0 = \frac{4\pi \hbar^2}{m} (1 + \gamma).
\]  

(6)

Demanding \( \gamma = 0 \), we recover the usual result [18]. These operators (\( \hat{a}_{\vec{k}} \) and \( \hat{a}_{\vec{k}}^\dagger \)) are bosonic creation and annihilation operators and fulfill the usual Bose commutator relations. Very close to the temperature \( T = 0 \), the second term in this Hamiltonian becomes [18]

\[
\sum_{\vec{k}=0}^{\infty} \sum_{\vec{p}, \vec{q} = 0}^{\infty} \sum_{j=0}^{\infty} \hat{a}_{\vec{p}}^\dagger \hat{a}_{\vec{q}} \hat{a}_{\vec{p}+\vec{q}+\vec{k}} \hat{a}_{\vec{p}+\vec{q}+\vec{k}} = N^2 + 2N \sum_{\vec{k}=0}^{\infty} \hat{a}_{\vec{k}}^\dagger \hat{a}_{\vec{k}} + N \sum_{\vec{k}=0}^{\infty} (\hat{a}_{\vec{k}}^\dagger \hat{a}_{\vec{k}}^\dagger + \hat{a}_{\vec{k}} \hat{a}_{\vec{k}}). 
\]  

(7)

With this approximation, the \( N \)-body Hamiltonian has the following structure:

\[
\hat{H} = \frac{U_0 N^2}{2V} + mglN + \sum_{\vec{k}=0}^{\infty} \left[ \frac{\hbar^2 k^2}{2m} (1 + \gamma) + mgl + \frac{U_0 N}{V} \right] \hat{a}_{\vec{k}}^\dagger \hat{a}_{\vec{k}} + N \frac{U_0}{2V} \left[ \hat{a}_{\vec{k}}^\dagger \hat{a}_{\vec{k}}^\dagger + \hat{a}_{\vec{k}} \hat{a}_{\vec{k}} \right].
\]  

(8)

This Hamiltonian can be diagonalized introducing the Bogoliubov transformations [17]

\[
\hat{b}_{\vec{k}} = \frac{1}{\sqrt{1 - \alpha_{\vec{k}}^2}} \left[ \hat{a}_{\vec{k}} + \alpha_{\vec{k}} \hat{a}_{\vec{k}}^\dagger \right],
\]  

(9)

\[
\hat{b}_{\vec{k}}^\dagger = \frac{1}{\sqrt{1 - \alpha_{\vec{k}}^2}} \left[ \hat{a}_{\vec{k}}^\dagger + \alpha_{\vec{k}} \hat{a}_{\vec{k}} \right].
\]  

(10)

In this last expression, the following definitions have been introduced:

\[
\epsilon_{\vec{k}} = \frac{\hbar^2 k^2}{2m} (1 + \gamma) + mgl,
\]  

(11)
\[ \alpha_k = 1 + \frac{V\epsilon_k}{U_0 N} - \frac{\sqrt{V\epsilon_k}}{U_0 N} \sqrt{2 + \frac{V\epsilon_k}{U_0 N}}. \quad (12) \]

These two operators fulfill the same algebra related to \( \hat{a}_k \) and \( \hat{a}_k^\dagger \), i.e. they are also bosonic operators. The final form for our Hamiltonian is

\[ \hat{H} = \frac{U_0 N^2}{2V} + mglN + \sum_{\vec{k} \neq 0} \left\{ \sqrt{\epsilon_k \left( \epsilon_k + \frac{2U_0 N}{V} \right)} \hat{b}_k^\dagger \hat{b}_k - \frac{1}{2} \left( \frac{U_0 N}{V} + \epsilon_k - \sqrt{\epsilon_k \left( \epsilon_k + \frac{2U_0 N}{V} \right)} \right) \right\}. \quad (13) \]

The last summation diverges, a result already known \([19, 20]\), and this divergence disappears introducing the so-called pseudo-potential method, which implies that we must perform the following substitution \([20]\):

\[ \frac{1}{2} \left( \frac{U_0 N}{V} + \epsilon_k - \sqrt{\epsilon_k \left( \epsilon_k + \frac{2U_0 N}{V} \right)} \right) \rightarrow \frac{1}{2} \left( \frac{U_0 N}{V} + \epsilon_k - \sqrt{\epsilon_k \left( \epsilon_k + \frac{2U_0 N}{V} \right)} - \frac{1}{2\epsilon_k} \left( \frac{U_0 N}{V} \right)^2 \right). \quad (14) \]

Finally, this last summation will be approximated by an integral. It is noteworthy to mention that the original expression has as lower limit the condition \( k \neq 0 \), which implies that the integral has as lower limit the value \( mgl \). In other words,

\[ - \frac{1}{2} \sum_{\vec{k} \neq 0} \left[ \frac{U_0 N}{V} + \epsilon_k - \sqrt{\epsilon_k \left( \epsilon_k + \frac{2U_0 N}{V} \right)} - \frac{1}{2\epsilon_k} \left( \frac{U_0 N}{V} \right)^2 \right] \]

\[ = - \frac{\hbar^2 V}{8m^2\pi^2} \left( \frac{8\pi aN}{V} \right)^{5/2} \int_{mgl}^{\infty} f(x) dx. \quad (15) \]

In this last expression, we have that

\[ f(x) = x^2 \left[ 1 + x^2 - x\sqrt{2 + x^2} - \frac{1}{2x^2} \right], \quad (16) \]

where we have

\[ x = \sqrt{\frac{\epsilon_k V}{U_0 N}} \quad (17) \]

With these conditions, we deduce the final structure of the \( N \)-body Hamiltonian

\[ \hat{H} = E_0 + \sum_{\vec{k} \neq 0} E_k \hat{b}_k^\dagger \hat{b}_k. \quad (18) \]

In this last expression appear two energy terms, \( E_0 \) and \( E_k \), where \( E_0 \) denotes the energy of the ground state of the corresponding Bogoliubov space \([17]\),

\[ E_0 = \frac{2\pi a\hbar^2 N^2 (1 + \gamma)}{mV} \left[ 1 + \frac{128}{15} \sqrt{\frac{a^3 N}{V\pi}} \left( 1 - \frac{15}{16\sqrt{2}} \sqrt{\frac{m^2 gLV}{4\pi a\hbar^2 N(1 + \gamma)}} \right) \right] + Nmg l. \quad (19) \]
and we have the energy of the Bogoliubov excitations \( E_k \) given by [17]

\[
E_k = \sqrt{\epsilon_k \left( \epsilon_k + \frac{2U_0 N}{V} \right)}.
\]  

(20)

Concerning (18), if we impose the condition of the vanishing gravitational constant, i.e. \( g = 0 \), then we recover the usual Hamiltonian [20].

1.3. Speed of sound and pressure of the ground state

The pressure \( P_0 = -\frac{\partial E_0}{\partial V} \) and speed of sound \( v_s = \frac{\sqrt{\epsilon_s}}{Nm} \) associated with the ground state of the Bogoliubov space become, respectively,

\[
P_0 = \frac{2\pi a h^2 N^2 (1 + \gamma)}{m V^2} \left[ 1 + \frac{192}{15} \sqrt{\frac{a^2 N}{V \pi}} \left( 1 - \frac{5}{8\sqrt{2}} \sqrt{\frac{m^2 gV}{4\pi a h^2 N (1 + \gamma)}} \right) \right],
\]

(21)

\[
v_s^2 = \frac{4\pi a h^2 N (1 + \gamma)^2}{m V^2} \left[ 1 + 16 \sqrt{\frac{a^2 N}{V \pi}} \left( 1 - \frac{1}{2\sqrt{2}} \sqrt{\frac{m^2 gV}{4\pi a h^2 N (1 + \gamma)}} \right) \right].
\]

(22)

A fleeting glimpse at these last expressions tells us that they depend upon the bare inertial mass and the size of the fluctuations, i.e. upon \( m \) and \( \gamma \), respectively.

Note that they imply the following relation:

\[
\frac{m}{\sqrt{1 + \gamma}} = \frac{v_s^2 \pi a h^2 N^3 (1 + \alpha)^2 - P_0^2 V^3 (1 + \frac{5}{4} \alpha)}{2 \alpha v_s^2 \pi a h^2 (1 + \alpha) - P_0^2 V^3 \alpha \beta},
\]

(23)

\[
\alpha = \frac{192}{15} \sqrt{\frac{a^2 N}{V \pi}}; \quad \beta = \frac{5}{8\sqrt{2}} \sqrt{\frac{gV}{4\pi a h^2 N}}.
\]

(24)

Clearly, it does not contain the usual relation, namely \( m(1 + \gamma)^{-1} \).

Let us now explain how these two parameters \( m \) and \( \gamma \) could be measured separately. A careful look at expression (23) tells us that all the parameters appearing on the right-hand side can be detected experimentally. Indeed, the speed of sound \( v_s \), the number of particles \( N \), the pressure \( P_0 \), the volume of the container \( V \) and the scattering length associated with the s-wave case \( a \) can be determined in an experiment. This last parameter can be detected by resorting to spectroscopic methods [21]. In order to be able to use our results, we now show that \( U_0 \) can be measured experimentally. This can be achieved by recalling that the density profile of a condensate on the surface of the cloud is related to the force of the external potential and to \( U_0 \) (see p 158 of [18]). This last remark tells us that, concerning (6), we may deduce from an experiment the left-hand side of it.

Consider the kinematical relation associated with \( U_0 \) (see (6)). If \( F \) denotes the right-hand side of (23), then we obtain

\[
\sqrt{(1 + \gamma)} = \frac{FU_0}{4\pi a h^2},
\]

(25)

\[
m = \frac{F^2 U_0}{4\pi a h^2}.
\]

(26)

These last two expressions allow us to deduce \( m \) and \( \gamma \), separately. Their right-hand side involves parameters which can be measured in an experiment.
Figure 1. The interferometer with the arm lengths $l_1$ and $l_2$. The source for the Bose–Einstein condensates is located at point A, where it is coherently split into two sub-condensates which travel along different paths. Finally, due to the different times of flight, the phase shift $\Delta \phi$ can be detected at the detector at point D by means of the interference pattern.

1.4. Bogoliubov excitations and metric fluctuations

We now present a second manner in which $m$ and $\gamma$ could be detected separately. Here we will resort to the properties of Bogoliubov excitations of the condensate. It is already known [17] that, even at $T = 0$, the presence of two-body interactions entails the existence of excitations in the condensate [18, 20] whose energy is given by (20). At this point, we consider two Bogoliubov excitations located, initially, at point (A) (see figure 1), and whose wave vector fulfills the following condition:

$$\frac{\hbar^2 k^2}{2m} (1 + \gamma) + mgl > \frac{2U_0N}{V}.$$  (27)

Then, we have, approximately,

$$E_k = \frac{\hbar^2 k^2}{2m} (1 + \gamma) + mgl + \frac{4\pi a^2 N(1 + \gamma)}{mV}.$$  (28)

These conditions allow us to consider an interferometric proposal along the lines of a semi-classical approximation. In other words, we consider an experiment similar to the Colella–Overhauser–Werner (COW) idea [22–25]. This last experiment shows the effects of the gravitational field of the Earth upon the phase shift of two neutron beams. Here we consider the same kind of proposal but with a different intention. Indeed, we seek an extra expression whose dependence is not of the kind $[m(1 + \gamma)^{-1}]^s$, where $s$ is a real number. We now resort to the WKB approximation in order to deduce the corresponding gravity-induced phase shift [26].

The time of flight for the beam moving along the path (A)–(B)–(D) reads

$$(1) T = \frac{ml_1}{(1 + \gamma)\hbar k_0} + \frac{1}{g} \left[ \frac{(1 + \gamma)}{m} \frac{\hbar k_0}{m} - \sqrt{\left( \frac{(1 + \gamma)}{m} \frac{\hbar k_0}{m} \right)^2 - 2gl_1} \right].$$  (29)

Concerning (A)–(C)–(D), the time of flight is

$$(2) T = \frac{1}{g} \left[ \frac{(1 + \gamma)}{m} \frac{\hbar k_0}{m} - \sqrt{\left( \frac{(1 + \gamma)}{m} \frac{\hbar k_0}{m} \right)^2 - 2gl_2} \right] + \frac{\sqrt{l_2^2}}{\sqrt{\frac{(1 + \gamma)}{m} \frac{\hbar k_0}{m}^2 - 2gl_2}}.$$  (30)

In these last expressions, $k_0$ denotes the wave number at point (A). These expressions are valid for short times of flight or for large velocities for the motion of the center of mass of the BEC. Additionally, we have also assumed that the line passing through points (B) and (D) is parallel to the direction of the gravitational field. If this line forms an angle $\theta$ with...
the gravitational field, then our expression remains valid but now we have to replace \( g \) by an effective gravitational acceleration, namely \( g \cos(\theta) \). Clearly, with these two flight times, we may deduce, easily, the difference in the time of arrival [26] and, in consequence, the gravity-induced phase shift, here denoted by \( \Delta \phi \),

\[
\Delta \phi = \frac{m^2 g l_1 l_2}{(1 + \gamma) \hbar^2 k_0} \left[ 1 - \frac{2m^2 g l_2}{(1 + \gamma) \hbar^2 k_0^2} \right].
\]

(31)

Introducing the condition \( \gamma = 0 \), we recover the COW result [26]. We now note that this phase shift contains a dependence of the kind \( m^2/(1 + \gamma) \). Taking the dominant term in the expression for the gravity-induced phase shift and resorting to our previous results concerning the speed of sound of the ground state (see (22)), we may, separately, deduce \( m \) and \( \gamma \). Indeed (here \( \lambda_0 \) denotes the wavelength at (A) divided by \( 2\pi \)),

\[
m = \frac{v_s}{g l_1 l_2 \lambda_0} \sqrt{\frac{V}{4\pi aN}} \left[ 1 - 16 \sqrt{\frac{a^3 N}{\pi V}} \right].
\]

(32)

In a similar way, we find that

\[
1 + \gamma = \frac{v_s^2 \hbar^2}{g l_1 l_2 \lambda_0 \Delta \phi} \frac{V}{4\pi aN} \left[ 1 - 32 \sqrt{\frac{a^3 N}{\pi V}} \right].
\]

(33)

In the deduction of these last two expressions, we have resorted to the speed of sound of the ground state; nevertheless, the pressure could have been used, as well.

2. Conclusions

We have introduced into the \( N \)-body Hamiltonian the effects of conformal fluctuations of the metric, which are usually associated with a redefinition of the inertial mass of the involved particles, i.e. the mass appears as \( m(1 + \gamma)^{-1} \). Two different manners in which, experimentally, these two parameters could be detected have been put forward.

At this point, we must emphasize the fact that this work contains an experimental proposal aimed at the detection of some implications of quantum fluctuations. Firstly, the deduction of the pressure and speed of sound of the ground state of the Bogoliubov space has been calculated. These expressions allow us, when compared against \( U_0 \), to achieve our goal. Secondly, an interferometric proposal, resorting to Bogoliubov excitations, has been analyzed. It has been shown that the gravity-induced phase shift, along with the pressure (or speed of sound) of the ground state, renders a second way in which we may, in an experiment, deduce, separately, \( m \) and \( \gamma \).

Clearly, another issue has to be addressed: what are the implications of the present results if we compare them against the current experimental outcomes in the context of the weak equivalence principle? In order to do this, let us consider the dominant term in (31). The experimental precision associated with interferometric devices, in connection with gravitational physics, is very high [27], i.e. \( \sim 10^{-9} \). Taking this kind of precision, our result entails a limitation for our proposal. Indeed, our proposal, with the current technology, will be able to detect a violation to this principle if the following restriction is fulfilled:

\[
|\gamma| > 10^{-9}.
\]

(34)

Finally, we now analyze the relevance of additional effects, for instance, the presence of particles not included in the case of s-scattering. The present approach assumes that the dominant role is played by the condensed part of the system, and therefore additional terms are neglected. Clearly, a more precise analysis, which could include the presence of scattering processes not restricted to s-scattering, can be done. This possibility takes us outside the
validity realm of the Gross–Piatevskii equation and leads us to the Zaremba–Nikuni–Griffin equation. In this model, the aforementioned study can be carried out, at least in principle. Anomalous terms do include these kinds of processes [28].

It is noteworthy to mention that this approach can be used in the context of deformed dispersion relations. Let us explain this idea better. The breakdown of Lorentz symmetry appears as a consequence in some quantum gravity models [29]. The main point in this aspect is related to the fact that a modification in the relation dispersion impinges upon the thermodynamical properties of a bosonic gas [30, 31]. In particular, the pressure and speed of sound would be changed by the modified dispersion relation and the measurement of these two thermodynamical properties could, in principle, lead to the determination of some parameters associated with the new dispersion relation. In other words, the present proposal could be considered, in some restricted sense, as some kind of complementary study to [9]. These last comments allow us to address an additional issue, namely the possibility of using condensed matter systems, in a more regular basis, as precision tools in gravitational physics. Currently, gravitational physics there, at least in the context of its quantization, faces a dead end road. The usage of phenomenological information becomes an important source, due to the lack of more relevant results. No approach should be neglected, and it is in this realm that condensed matter systems could disclose new physics.

As an additional bonus related to this work, let us mention the connection with the Einstein equivalence principle (EEP) [32]. This principle tells us that locally the laws of physics are the special-relativistic laws. We may rephrase this statement asserting that locally the gravitational field can be gauged away. In other words, in a freely falling frame, the pressure and speed of sound, related to the ground state of the Bogoliubov space, must be given by our expression, if $g = 0$. Similarly, the gravity-induced phase shift shall vanish. These kinds of experiments are currently a hot topic in gravitational physics [33], since nowadays it is possible to create condensates, in a regular basis, under microgravity. The present idea can therefore be considered as a proposal for the use of Bogoliubov fluctuations as an additional tool for experiments in fundamental physics.

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