Minimal SU(5) Resuscitated by Long-Lived Quarks and Leptons

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Abstract

The issue of gauge unification in the (non-supersymmetric) Standard Model is reinvestigated. It is found that with just an additional fourth generation of non-sequential and long-lived quarks and leptons, $SU(3) \otimes SU(2) \otimes U(1)$ gauge couplings converge to a common point $\sim 3.5 \times 10^{15}$ GeV ($\tau_p \sim 10^{34 \pm 1}$ years). This result is due to the non-negligible- but still perturbative-contributions of the top and fourth generation Yukawa couplings to the gauge two-loop $\beta$ functions, in contrast with the three generation case where such a contribution is too small to play an important role in unification.
It is a standard lore that present measurements of the gauge couplings at the $Z$ mass appear to indicate that all three couplings actually do not converge at the same point if they were to evolve according to the minimal Standard Model (SM) with three generations. This is somewhat problematic for the simple idea of Grand Unification, and in particular the minimal $SU(5)$ model. It is also a standard lore that with low-energy supersymmetry (generically referred to as MSSM from here on) broken at around 1 TeV or less, such a unification is possible and occurs at an energy scale $\sim 10^{16}$ GeV corresponding to a proton lifetime of $\sim 10^{36}$ years (roughly four orders of magnitude above the current limit). As such, the idea of Grand Unification with a desert (beyond 1 TeV) fits snugly with low-energy supersymmetry. There is, however, a catch. The lightest scalar in MSSM cannot be heavier than $\sim 150$ GeV. What would happen if no scalar is found below, say $2m_Z$, thus ruling out low-energy supersymmetry (or at least the simplest version of it)? Should one then simply give up the idea of simple unification with a desert and entertain more complex versions with many intermediate scales, or just give up the whole idea of Grand Unification itself? It is perhaps worthwhile to reexamine this whole issue within the context of the (non-supersymmetric) SM itself.

It is not entirely clear that one has exhausted all possibilities concerning the SM. One may ask, for example, what role the mass of the Higgs boson, $m_H$, has, especially when it is larger than $\sim 174$ GeV. (For a lighter Higgs boson in the presence of a heavy fermion, several constraints, especially from vacuum stability, have been discussed.) It is known from previous studies that when $m_H \geq 174$ GeV, the Higgs quartic coupling develops Landau poles below the Planck scale $\sim 10^{19}$ GeV. For example, the Higgs boson with a mass $\sim 208$ GeV would develop a Landau pole at $\sim 10^{10}$ GeV. It is therefore not at all clear how these Landau poles might influence the evolution of the gauge couplings.

One may also ask whether or not the addition of a fourth generation might change the evolution of the gauge couplings in such a way as to unify them again. It is well-known that the addition of an extra family does not change the result at one-loop. However, the two-loop $\beta$ functions for the gauge couplings contain contributions from Yukawa couplings.
With just three generations, the dominant Yukawa contribution comes from the top quark. However, it can be seen that the top Yukawa coupling actually decreases with energy. As a result, it practically does not help the convergence of the three gauge couplings. With four generations and with the fourth generation being sufficiently heavy (an issue explored below), it turns out that all Yukawa couplings grow with energy. It is found that this growth can significantly affect the evolution of the gauge couplings. In fact, when the fourth-generation quarks and leptons are sufficiently heavy, all Yukawa couplings (including that of the top quark) develop Landau poles below the Planck scale. In order to make sensible statements based strictly on the validity of perturbation theory, we shall restrict ourselves to the range of mass where these Landau poles lie above a few times $10^{15}$ GeV. There are two reasons for doing so. The first one is the fact that, in order to satisfy the current lower bound on the proton lifetime, the unification scale (if there is one) has to be larger than $10^{15}$ GeV. The second reason is the fact that, if the three gauge couplings were to converge at the same point— with that point being $\sim 10^{15}$ GeV— due to the effects of the Yukawa couplings which show up at two loops, we would like it to happen when these Yukawa couplings are still in the perturbative domain. That does not mean, however, that, if the fourth generation is sufficiently massive so that the Landau poles are below $10^{15}$ GeV, one could not have unification. It simply means that non-perturbative methods (such as a Higgs-Yukawa model on the lattice for example) should be used to examine this case. Unfortunately, such a study is not available at the present time (e.g. the difficulty with chiral fermions on the lattice). It is for these reasons that we shall restrict ourselves to the mass range where the associated Landau poles would lie above $10^{15}$ GeV.

Of particular importance to the whole idea of Grand Unification is the feasibility of the search for proton decay, our only direct evidence of such a theory. The current prediction of supersymmetric GUT for the proton lifetime is approximately $10^{37}$ years, which puts it way beyond any foreseeable future search. (It may be a moot point if there is no light Higgs below 150 GeV.) For the minimal SU(5), the well-known prediction for the proton lifetime is roughly two orders of magnitudes lower than the current experimental lower bound.
of $5.5 \times 10^{32}$ years [7]. Is it possible that, if the proton does decay, its lifetime might be within reach of, say, SuperKamiokande which presumably could extend its search up to $10^{34}$ years? We would like to point out in this note that this might be possible.

In what follows, we shall distinguish two cases: the minimal SM with three generations and one Higgs doublet (case I), and the “non-minimal” case with four generations and one Higgs doublet (case II). We shall use two-loop RG equations throughout this paper. For case I, they are well-known and the explicit expressions can be found in the literature [8]. For the second case with four generations, we shall write down explicitly the two-loop RG equations below [9]. To set the notations straight, our definition of the quartic coupling in terms of the Higgs mass is 

$$m_H^2 = \frac{\lambda v^2}{3} \quad \text{(corresponding to } \frac{\lambda (\phi^\dagger \phi)^2}{6} \text{ in the Lagrangian)}$$

while the more common definition is 

$$m_H^2 = 2\lambda v^2.$$ 

Therefore our $\lambda$ is six times the usual $\lambda$. The RG equations given below reflect our convention on the quartic coupling.

We begin with the minimal SM with three generations. As mentioned earlier, we are particularly interested in the Higgs mass range, $m_H \geq 174$ GeV. In particular, if we restrict ourselves to the values of $m_H$ where the associated Landau poles would lie above $10^{15}$ GeV, we are then looking at the range $174\text{GeV} \leq m_H \leq 180\text{GeV}$. Fig. 1 shows the evolution of the three gauge couplings $g_1, g_2, g_3$ for $m_H = 174$ GeV. (The results are practically the same for $m_H = 180$ GeV.) One clearly sees that they do not converge to the same point, a result similar to the already well-known one. Here, we actually include the indirect effect of the Higgs mass, namely its effect on the top Yukawa coupling which feeds into the RG equations for the gauge couplings. As one might have suspected, we have found no effect: the three gauge couplings still do not meet. For heavier Higgs i.e. $m_H > 180$ GeV, the Landau poles will appear below $10^{15}$ GeV. We cannot say for sure, at least within the context of the perturbation theory, what influence such a “heavy” Higgs might have on the evolution of the gauge couplings.

We now turn to the second scenario with four generations and one Higgs doublet. These four generations fit snugly into $\bar{5} + 10$ representations of SU(5), except for the right-handed neutrinos which we should need if we were to give a mass to the neutrinos. For example,
this could be incorporated into a 16-dimensional representation of SO(10) which splits into 
5 + 10 + 1 under SU(5). We could then have a pattern of symmetry breaking like \( SO(10) \rightarrow SU(5) \) for example. All four neutrinos can acquire a mass via the see-saw mechanism for example. Furthermore there is no reason why the fourth one cannot be much heavier than the other three, namely its mass could be at least half the Z mass. These details are however beyond the scope of this paper.

The appropriate two-loop RG equations are given by:

\[
16\pi^2 \frac{d\lambda}{dt} = 4\lambda^2 + 4\lambda(3g_t^2 + 6g_q^2 + 2g_l^2 - 2.25g_2^2 - 0.45g_1^2)
-12(3g_t^4 + 6g_q^4 + 2g_l^4) + (16\pi^2)^{-1}[180g_t^6
+288g_q^6 + 96g_l^6 - (3g_t^4 + 6g_q^4 + 2g_l^4 - 80g_3^2(g_t^2
+2g_q^2))\lambda - \lambda^2(24g_t^2 + 48g_q^2 + 16g_l^2) - (52/6)\lambda^3
-192g_3^2(g_t^4 + 2g_q^4)]
\] (1a)

\[
16\pi^2 \frac{dg_t^2}{dt} = g_t^2(9g_t^2 + 12g_q^2 + 4g_l^2 - 16g_3^2 - 4.5g_2^2 - 1.7g_1^2
(8\pi^2)^{-1}[1.5g_t^4 - 2.25g_t^2(6g_q^2 + 3g_l^2 + 2g_t^2)
-12g_q^4 - (27/4)g_t^4 - 3g_l^4 + (1/6)\lambda^2 + g_t^2
(-2\lambda + 36g_3^2) - (892/9)g_3^4)}
\] (1b)

\[
16\pi^2 \frac{dg_q^2}{dt} = g_q^2(6g_t^2 + 12g_q^2 + 4g_l^2 - 16g_3^2 - 4.5g_2^2 - 1.7g_1^2
(8\pi^2)^{-1}[3g_q^4 - g_q^2(6g_q^2 + 3g_l^2 + 2g_t^2)
-12g_q^4 - (27/4)g_t^4 - 3g_l^4 + (1/6)\lambda^2 + g_q^2
(-8/3)\lambda + 40g_3^2) - (892/9)g_3^4)}
\] (1c)

\[
16\pi^2 \frac{dg_l^2}{dt} = g_l^2(6g_t^2 + 12g_q^2 + 4g_l^2 - 4.5(g_2^2 + g_1^2)
(8\pi^2)^{-1}[3g_q^4 - g_q^2(6g_q^2 + 3g_l^2 + 2g_t^2) - 12g_q^4
-(27/4)g_t^4 - 3g_l^4 + (1/6)\lambda^2 - (8/3)\lambda g_l^2})
\] (1d)
In the above equations, we have assumed for the fourth family, for simplicity, a Dirac neutrino mass and, in order to satisfy the constraints of electroweak precision measurements, that both quarks and leptons are degenerate $SU(2)_L$ doublets. The respective Yukawa couplings are denoted by $g_q$ and $g_l$. Also, in the evolution of $\lambda$ and the Yukawa couplings, we have neglected, in the two loop terms, contributions involving $\tau$ and bottom Yukawa couplings as well as the electroweak gauge couplings, $g_1$ and $g_2$. For the range of Higgs and heavy quark (including the top quark) masses considered in this paper, these two-loop contributions are not important to the evolution of $\lambda$ and the Yukawa couplings.

In what follows, we shall assume that, whatever mechanism (a right-handed neutrino in this case) that is responsible for giving a mass to at least the 4th neutrino will not affect the evolution of the three SM gauge couplings. Also there are reasons to believe that this 4th generation might be rather special, distinct from the other three and having very little mixing with them. The physics scenario behind the 4th neutrino mass might be quite unconventional.

What masses for the fourth generation are we allowed to use in our analysis? For the quarks, the mass can even be lower than the top quark mass. As of now, there is no strict limit on the mass of the 4th generation quarks if the 4th family is non-sequential, i.e. having very little mixing with the other three. As discussed in [10], the current accessible but unexplored decay length for a long-lived heavy quark to be detected is between $100 \mu m$ and
1 m. As long as a member of the 4th generation quark doublet (e.g. the down-type quark) decays in that range, its mass can even be lower than the top mass. The phenomenology of a near degenerate long-lived doublet of quarks and its detection is discussed in full length in Ref. [10]. As for the 4th generation leptons, we shall assume that the mass is greater than \(m_Z\).

As we have stated above, we shall restrict ourselves to the mass range of the fourth generation that will have Landau poles only above \(10^{15}\) GeV. We shall require that, if there is convergence of the three gauge couplings, it should occur when the Higgs quartic coupling and the Yukawa couplings are still perturbative in the sense that one can neglect contributions coming from three loop (and higher) terms to the \(\beta\) functions.

Fig. 2 shows the evolution of \(g_1^2\), \(g_2^2\), and \(g_3^2\) for one particular set of masses, namely \(m_Q = 151\) GeV, \(m_L = 95.3\) GeV and \(m_t = 175\) GeV, where \(m_Q\), \(m_L\) and \(m_t\) denote the fourth quark, lepton and top masses respectively. Vacuum stability (\(\lambda > 0\)) and the requirement that \(\lambda/4\pi \sim 1\) above \(10^{15}\) GeV, for the fermion masses listed above, give a prediction for the mass for the Higgs boson to be \(m_H = 188\) GeV which is larger than the fourth generation quark mass. Two remarks are in order here.

First, Fig. 2 shows the evolution of the gauge couplings without taking into account the effects of the heavy particle threshold near the unification point. For example, that threshold could come from the 24 and 5 Higgs scalars of SU(5). In fact, as one can see from Fig. 2, the three gauge couplings come close (to 4\% or less) to each other but do not actually meet at the same point if one does not include heavy threshold effects. By itself, within errors, it is already a good indication of possible unification. We would like nevertheless to discuss the issues of heavy threshold for completeness. One may ask the following question: if we choose a scale, \(M_G\), where the uncorrected couplings \(\alpha_i \equiv g_i^2/4\pi\) \((i = 1, 2, 3)\) are within say 4\% of each other, can one bring them together after the inclusion of heavy threshold effects? As an example, let us take the following point (last point in Fig. 2): \(\ln(E/175\text{ GeV}) = 30.62\) which corresponds to \(M_G = 3.48 \times 10^{15}\) GeV. At this point, one has: \(\alpha_3(M_G) = 0.0278\), \(\alpha_2(M_G) = 0.0273\) and \(\alpha_1(M_G) = 0.0285\). If one defines \(\Delta\alpha/\alpha \equiv (\alpha_{\text{larger}} - \alpha_{\text{smaller}})/\alpha_{\text{larger}},\)
one can immediately see that $\Delta \alpha / \alpha \approx 2\% - 4\%$.

Let us, e.g., assume the minimal SU(5) with the following heavy particles: $(X, Y) = (\bar{3}, 2, 5/6) + c.c.$ with mass $M_V$, real scalars $(8, 1, 0) + (1, 3, 1) + (1, 1, 0)$ (belonging to the 24-dimensional Higgs field) with mass $M_{24}$, and the complex scalars $(3, 1, -1/3)$ (belonging to the 5-dimensional Higgs field). (The quantum numbers are with respect to $SU(3) \otimes SU(2) \otimes U(1)$.) The heavy threshold corrections are then

\begin{align}
\Delta_1 &= \frac{35}{4\pi} \ln \left( \frac{M_G}{M_V} \right) - \frac{1}{30\pi} \ln \left( \frac{M_G}{M_5} \right) + \Delta_1^{NRO}, \\
\Delta_2 &= -\frac{1}{6\pi} + \frac{21}{4\pi} \ln \left( \frac{M_G}{M_V} \right) - \frac{1}{6\pi} \ln \left( \frac{M_G}{M_{24}} \right) + \Delta_2^{NRO}, \\
\Delta_3 &= -\frac{1}{4\pi} + \frac{7}{2\pi} \ln \left( \frac{M_G}{M_V} \right) - \frac{1}{12\pi} \ln \left( \frac{M_G}{M_5} \right) - \frac{1}{4\pi} \ln \left( \frac{M_G}{M_{24}} \right) + \Delta_3^{NRO},
\end{align}

where

\begin{equation}
\Delta_i^{NRO} = -\eta k_i \left( \frac{2}{25\pi \alpha_G^3} \right)^{1/2} \frac{M_G}{M_{Planck}},
\end{equation}

with $k_i = 1/2, 3/2, -1$ for $i = 1, 2, 3$, is the correction coming from possible dimension 5 operators present between $M_G$ and $M_{Planck}$. The magnitude of the coefficient $\eta$ is constrained to be less than or equal to 10. The corrected couplings can be written as

\begin{equation}
\frac{1}{\tilde{\alpha}_i(M_G)} = \frac{1}{\alpha_i(M_G)} + \Delta_i,
\end{equation}

There are of course many choices for the different mass scales. As an example, we shall choose: $M_5 = M_G$, $M_{24} = M_G$, $M_V = 0.5 M_G$, and $\eta = 10$. With this choice and taking as $\alpha_G$ the average of $\alpha_3(M_G) = 0.0278$, $\alpha_2(M_G) = 0.0273$ and $\alpha_1(M_G) = 0.0285$, we obtain

\begin{align}
\tilde{\alpha}_1(M_G) &= 0.02705, \\
\tilde{\alpha}_2(M_G) &= 0.02662, \\
\tilde{\alpha}_3(M_G) &= 0.02735.
\end{align}
From the above values, one can say that the couplings are practically the same with all three $\sim 0.027$ or $1/\tilde{\alpha}_G \sim 37$. This little exercise shows that heavy threshold effects can indeed bring about better unification.

The second remark we wish to make is the question of the validity of perturbation theory. The usual requirement encountered in the literature is that $g_t^2/4\pi$, $g_q^2/4\pi$, $g_l^2/4\pi$, and $\lambda/4\pi$ have to be less than unity. First, notice that our definition of $\lambda$ is six times greater than the usual one which means that, instead of $\lambda/4\pi < 1$, we should require here $\lambda/24\pi < 1$. Let us then, for the moment, adhere to this common requirement. (We shall come back to this point below.) At the point where we refer to as the unification point, namely $M_G = 3.48 \times 10^{15}$ GeV, we found the following values for the Higgs quartic and Yukawa couplings: $g_t^2/4\pi = 0.4$, $g_q^2/4\pi = 0.16$, $g_l^2/4\pi = 0.48$, and $\lambda/24\pi = 0.19$. This clearly shows that, according to this common criterion, one is well inside the perturbative domain when unification occurs. Although it is not needed here, one might even relax this criterion and replace $4\pi$ or $24\pi$ by $16\pi^2$. In any case, this is perturbative unification. A side remark is in order here. For the three generation case, the value of the top Yukawa coupling at a similar scale is $g_t^2/4\pi \sim 0.016$, a factor of 20 smaller than in the four generation case. This explains why the Yukawa couplings are important enough in this case, but not in the three generation case, to modify the evolution of the gauge couplings.

An exhaustive study of different mass combinations for the fourth generation is beyond the scope of this paper. We wish to point out however a puzzling feature of the four generation case. As we increase the fourth generation mass, the Landau poles move lower in energy. Eventually, perturbation theory ceases to be valid and the question of gauge unification cannot be answered in this context. If one naively keeps the two-loop approximation, there seems to be an appearance of ultraviolet fixed points at rather large values of the Higgs quartic and Yukawa couplings. If one evolves the gauge couplings with the presence of these ultraviolet fixed points, there again is unification at roughly similar energy scales as the example discussed previously. Since it is not clear that one can trust this result using only the two-loop approximation, we are currently investigating the “Padé” approximation.
to the three loop $\beta$ functions but the “large” number of independent couplings makes this procedure non-trivial although we are optimistic that it can be carried through. However, for the purpose of this paper, as stated previously, we shall stay with our predictions coming from *perturbation theory*.

The proton partial mean lifetime as represented by $\tau_{p\to e^+\pi^0}$ is predicted to be $\tau_{p\to e^+\pi^0}(yr) \approx 10^{31\pm1}(M_G/4.6 \times 10^{14})^4$. In our case, we obtain the following prediction: $\tau_{p\to e^+\pi^0}(yr) \approx 3.28 \times 10^{34\pm1}$. This is comfortably larger than the current lower limit of $5.5 \times 10^{32}$ years. In addition, the prediction is not too much larger than the current limit which means that it might be experimentally accessible in the not-too-distant future, in contrast with the MSSM predictions.

This scenario made a number of predictions: 1) the proton decays at an accessible rate $\sim 10^{34\pm1}$ years; 2) there is a fourth generation of long-lived quarks and leptons with a quark mass $\sim 151$ GeV; 3) the Higgs mass is predicted to be $\sim 188$ GeV $> 2m_Z$, a value which is well suited for the “Gold Plated” signal $H \to l^+l^-l^+l^-$ at the LHC. All of these features can be tested in a not-too-distant future. For example, the fourth generation can be *non-sequential* and can have exceptionally long lifetimes. This could provide a distinct signature [10].

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FIGURES

FIG. 1. The evolution of the SM gauge couplings squared versus ln(E/175 GeV) for the three generation case. $g_3$, $g_2$, and $g_1$ are the couplings of SU(3), SU(2) and U(1) respectively. We have used $g_3^2(m_t) = 1.392$, $g_2^2(m_t) = 0.421$, and $g_1^2(m_t) = 0.2114$. The Higgs mass is $m_H = 174$ GeV.

FIG. 2. The evolution of the SM gauge couplings squared versus ln(E/175 GeV) for the four generation case. $g_3$, $g_2$, and $g_1$ are the couplings of SU(3), SU(2) and U(1) respectively. We have used $g_3^2(m_t) = 1.392$, $g_2^2(m_t) = 0.421$, and $g_1^2(m_t) = 0.2114$. Also we use $m_Q = 151$ GeV, $m_L = 95.3$ GeV and $m_t = 175$ GeV, where $m_Q$, $m_L$ and $m_t$ denote the fourth quark, lepton and top masses respectively. The heavy threshold effects are not taken into account here. They are discussed in the text and are shown to improve the unification point.
