Optical properties of Chern-Simons systems

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Abstract. Chern-Simons (CS) systems interacting with electromagnetic radiation are described by a term $\int F \wedge F$ added to the Maxwell action. In (3+1)D, this CS term is a boundary term affecting the system behaviour in its borders. We study the consequences of the above in the properties of electromagnetic radiation, in particular, by considering the interplay between magneto-electric properties and topology. Apart from a modified Kerr polarization rotation, compared to that found for the particular case of topological insulators, we also found two Brewster angles, for s and p polarization of reflected radiation, respectively. Energy distribution between reflected and transmitted radiation is also studied in terms of the magneto-electric properties and topological condition of the system.

Keywords: Chern-Simons theories, $\theta$-vacuum, topological insulators, optical properties of surfaces.

1. Introduction

Chern-Simons (CS) theories arised in mathematics, in the study of topological invariants [1, 2]. Since then a variety of applications have been found from gauge field theory to cosmology [3]. On the other hand, a $\theta$-term (a CS form in 3+1 dimensions), was introduced in QCD by Roberto Peccei and Helen Quinn in 1977 [4]. A decade after, the idea was generalized by Franck Wilczek to other contexts, particularly to condensed matter physics [5]. As he observed, the CS form in the action could have applications in describing phenomena where topology plays a role, such as those exhibiting Quantum Hall phenomena [6]. In recent years, the novel topological insulators have become a new example of the above [7, 8].

In (3+1)D, CS contribution is a boundary term, not affecting the behaviour of the system in the bulk. But this $\theta$-term is important when the system is bounded [9], and nontrivial...
contributions arise from the boundary. An example of this is the modification in the Casimir energy inside a spherical \( \theta \)-region [10]. Effects in gravity are currently under study [11].

In \( \theta \)-systems, radiation experiences changes in its properties. Even the pure vacuum affects the radiation, if it is provided with nontrivial Chern-Simons properties -a \( \theta \)-vacuum. In this latter case, an unexpected reflection makes the energy diminishes after crossing the interface, with the possibility of considering this for cosmological issues [12]. In the context of topological insulators, an equivalent phenomenon to Kerr-Faraday polarization rotation occurs in reflected and refracted waves at the surface of such materials [7, 13].

In this brief report, we study the polarization and energy distribution of the reflected and transmitted radiation at the surface of a \( \theta \)-system. The existence of a standard s-Brewster angle (\( s \)-polarized reflected radiation) and a nonstandard Brewster angle (\( p \)-polarized reflected wave) is discussed. Finally, we consider the reflectance and transmittance of the \( \theta \)-system surface.

2. The basic approach

For electromagnetic radiation in a \( \theta \)-system, a CS term of the form

\[
\frac{\theta}{2} \int F \wedge F
\]  

(2.1)

is added to the Maxwell action. In noncovariant notation, this amounts to a \( \theta \mathbf{E} \cdot \mathbf{B} \) term. The field equations become [12]

\[
\varepsilon \nabla \cdot \mathbf{E} = \theta \delta(\Sigma) \mathbf{B} \cdot \mathbf{n}
\]  

(2.2)

\[
\frac{1}{\mu} \nabla \times \mathbf{B} - \partial_t \mathbf{E} = \theta \delta(\Sigma) \mathbf{E} \times \mathbf{n},
\]  

(2.3)

together with the two homogeneous Maxwell equations (we use units with the light speed \( c = 1 \); consequently, the electric permittivity, \( \varepsilon \), and magnetic permeability, \( \mu \), become adimensional). The delta function stands for evaluation at the interface, \( \mathbf{n} \) being the inwards unit normal to the interface surface \( \Sigma \).

In the vicinity of the surface \( \Sigma \), the field equations (2.2) and (2.3) imply that the normal component of \( \mathbf{E} \) and the tangential component of \( \mathbf{B} \) are discontinuous:

\[
[\varepsilon \mathbf{E}_n] = \theta \mathbf{B}_n|_\Sigma, \quad \left[ \frac{1}{\mu} \mathbf{B}_\tau \right] = -\theta \mathbf{E}_r|_\Sigma.
\]  

(2.4)

Subindices \( n \) and \( \tau \) stand for normal and tangent components of the fields, respectively, with respect to the interface \( \Sigma \). The symbol \([ \cdot ]\) must be interpreted as the difference between the fields evaluated immediately inside and immediately outside the \( \theta \)-medium. In addition to those boundary discontinuities, continuity holds for the tangential component of \( \mathbf{E} \) and the normal component of \( \mathbf{B} \), thus ensuring consistency of (2.4).
3. The radiation fields at the interface

Consider a small region in the boundary of the \( \theta \)-system where a linearly polarized planar electromagnetic wave incides from a non-\( \theta \) surrounding region with an angle \( \varphi \). The wave is reflected with the same angle and transmitted to the other medium with a different angle \( \psi \), these angles measured as usual. The standard laws of reflection and refraction hold for \( \theta \)-systems.

By applying the boundary conditions for the fields at \( \Sigma \), we derive the electric field amplitude for the refracted and reflected waves (the magnetic field is obtained from that by the standard relationship). As usual, we decompose the fields in parallel (\( \parallel \), \( p \)) and perpendicular (\( \perp \), \( s \)) components with respect to the incidence plane, and introduce the definitions

\[
e_i^\parallel \equiv \frac{E_i^\parallel}{E_i}, \quad e_r^\parallel \equiv \frac{E_r^\parallel}{E_i}, \quad e_t^\parallel \equiv \frac{E_t^\parallel}{E_i},
\]

with similar definitions for \( s \)-components of the fields. Then, we find for the transmitted wave,

\[
\left( \begin{array}{c} e_t^\parallel \\ e_t^\perp \end{array} \right) = \frac{2}{D} \left( \begin{array}{cc} \eta s + 1 & -\theta \\ \theta s & \eta + s \end{array} \right) \left( \begin{array}{c} e_i^\parallel \\ e_i^\perp \end{array} \right)
\]

(3.1)

and, for the reflected wave,

\[
\left( \begin{array}{c} e_r^\parallel \\ e_r^\perp \end{array} \right) = \frac{1}{D} \left( \begin{array}{cc} (\eta s + 1)(\eta - s) + \theta^2 s & 2\theta s \\ 2\theta s & - (\eta s - 1)(\eta + s) - \theta^2 s \end{array} \right) \left( \begin{array}{c} e_i^\parallel \\ e_i^\perp \end{array} \right)
\]

(3.2)

with \( D \equiv (\eta s + 1)(\eta + s) + \theta^2 s \). We have introduced the magneto-electric parameter

\[ \eta \equiv \frac{n_2 \mu_1}{n_1 \mu_2} \quad (n \text{ is the index of refraction}) \] and the so-called magnification factor

\[ s \equiv \cos \varphi / \cos \varphi. \]

To simplify notation, we have redefined \( \theta / \eta_1 \rightarrow \theta \) \( (\eta_1 = n_1 / \mu_1, \text{and if the surrounding medium is vacuum space, } \theta \text{ coincides with the parameter in the action}).

Figures 1 and 2 show the amplitude fields both for refracted and reflected waves compared to \( \theta \)-vacuum. It is interesting to note, in Fig. 2, the Brewster angle for \( \eta = 1.1 \) and \( \varphi = 60^\circ \), for a value of \( \theta = 0.504 \). This issue will be discussed in more detail below.

Figure 1: Field amplitude for the refracted wave. Comparison is made with \( \theta \)-vacuum.

Figure 2: Field amplitude for the reflected wave. Note the Brewster angle, where p-component of the field vanishes, for \( \theta = 0.5 \).
4. Polarization rotation

We see from (3.1) and (3.2), and unlike the non-$\theta$ systems, the p- and s-components of fields mix each other for both outcoming waves. Therefore, the reflected and refracted waves experience a polarization plane rotation, similar to the Kerr-Faraday rotation (Kerr case corresponds to the reflection of light by a magnetic surface) [14]. Figures 3 and 4 show those polarization angles (measured with respect to the incidence plane) for a p-polarized incident wave.

![Refracted wave polarization](image1.png)  ![Reflected wave polarization](image2.png)

Figure 3: Refracted and reflected waves polarization for different incidence angles.

Figure 4: Refracted and reflected waves polarization for different incidence angles.

On the other hand, Figures 5 and 6 show the waves polarization rotation for different values of $\eta$ (for $\eta = 1$ we have the $\theta$-vacuum case).

![Refracted wave polarization](image3.png)  ![Reflected wave polarization](image4.png)

Figure 5: Refracted waves polarization for different system electric properties.

Figure 6: Reflected waves polarization for different system electric properties.
5. Brewster angle for $\theta$-systems

The radiation reflected by a nonconducting surface could eventually become $s$-polarized, for the well-known Brewster angle of incidence, which we call the $s$-Brewster angle. The novelty here is that a nonstandard $p$-Brewster angle appears, where reflected wave becomes polarized in the plane of incidence.

The Brewster angles in terms of $\theta$ are shown in Figures 7 and 8. From Fig. 7, we see that even for rather small values of $\theta$, the standard $s$-Brewster angle changes notoriously. For large values of $\theta$ compared to $\eta$, no $s$-Brewster angle is obtained (it goes to $90^\circ$) and, as an important difference with normal case, the standard $s$-Brewster angle depends on incident wave polarization, $\alpha_i$.

On the other hand, in order to obtain the $p$-Brewster angle the incident wave must not be $p$-polarized and $\eta$ has to be larger enough. Also, its existence is restricted to a definite domain of values of $\theta$, different for each value of the magneto-electric parameter $\eta$, with a strong variation of the angle for the values of $\theta$ at the ends of the interval. Moreover, a tendency is observed for the angle to come closer to $90^\circ$. Actually, as $\eta$ goes to 1 (vacuum case), $p$-Brewster angle will no longer exist.

![Image](image1.png)

**Figure 7:** Standard $s$-Brewster angle for different values of $\eta$ and when incident wave is $p$-polarized.

![Image](image2.png)

**Figure 8:** Nonstandard $p$-Brewster angle for different values of $\eta$.

6. $\theta$-system reflectance and transmittance

Energy distribution between reflected and refracted waves is modified with respect to normal systems. Figures 9 and 10 show the situation in terms of $\theta$.

For increasing values of $\theta$, the material becomes progressively more opaque, this being increased by higher values of the magneto-electric parameter $\eta$. 

![Image](image3.png)
7. Discussion

Results presented here show that electromagnetic radiation inciding in a bounded \( \theta \)-system exhibits interesting behaviour in its polarization properties and the energy distribution of the reflected and refracted waves at the interface. In particular, compared to a \( \theta \)-vacuum, where no matter is present (\( \eta = 1 \)) but topology is not trivial, we find that magneto-electric properties modify significantly the system behaviour. In particular, for a small \( \theta \), the reflected wave polarization can be very different according to the value of \( \eta \), and differing in both cases from the \( \theta \)-vacuum case, where reflection polarization plane rotates in 90° with respect to the incident wave polarization. We also have two Brewster angles, both significantly depending on the interplay between the magneto-electric properties and \( \theta \) parameter. A nonstandard p-Brewster angle arises, where reflected radiation becomes polarized in the plane of incidence. Finally, we find that, if \( \theta \) is large enough, the reflected wave could carry a larger part of the energy compared to normal non-\( \theta \) systems. A more detailed examination and the analytical expressions for some of the results presented here are discussed in Ref. [15].

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