Pentaquark as a NK* bound state with $TJ^P=0^\frac{3}{2}^-$

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(Dated: June 5, 2018)

We have investigated negative-parity uudd\barsqbar pentaquarks by employing a quark model with the meson exchange and the effective gluon exchange as qq and q\bar{q} interactions. The system of five quarks is dynamically solved; the qq and q\bar{q} correlations are taken into account in the wave function. The masses of the pentaquarks are found to be reasonably low. It is found that the lowest-mass state is $TJ^P=0^\frac{3}{2}^-$ and the next lowest one is $0^\frac{1}{2}^-$. The former is reported to have a large width. We argue the observed narrow peak corresponds to the latter state. It is still necessary to introduce an extra attraction to reduce the mass further by 140 – 280 MeV to reproduce the observed Θ\barsqbar mass. Since their level splitting is less than 80 MeV, the lower level will not become a bound state below the NK threshold even after such an attraction is introduced. It is also found that the relative distance of two quarks with the attractive interaction is found to be by about 1.2 – 1.3 times closer than that of the repulsive one. The two-body correlation seems important in the pentaquark systems.

PACS numbers: 14.20.-c, 12.39.Mk, 12.39.Jh

Keywords: Pentaquark, quark model, quark correlation

I. INTRODUCTION

Since the experimental discovery of the baryon resonance with strangeness +1, $\Theta(1540)^+$, many attempts have been performed to describe the peak theoretically\[1\]. To describe this resonance by using a quark model, one needs at least five quarks, uudd\bar{q}, which is called a pentaquark. After one year of struggle, it gradually has become clear that a quark model has difficulties to explain some of the features of this peak. Namely, (1) the observed mass is rather low, (2) the observed width is very narrow, and (3) there is only one peak is found, especially no $T=1$ peak nearby. In order to reproduce the observed mass, about 100 MeV above the KN threshold, it is preferred to assign the $(0s)^5$ state with the most attractive channel, $TJ^P=0^\frac{3}{2}^-$. It has been pointed out, however, that width of this state would be about 1 GeV\[3\], which is far from the observed narrow width, 0.90 MeV\[4\]. The possibility that the pentaquark with $0^\frac{1}{2}^-$ as well as $0^\frac{3}{2}^-$ may be seen as a low-lying peak was pointed out by several works\[2, 3, 5\]. In this work, we would like to show that the pentaquark with $0^\frac{3}{2}^-$ is a promising candidate for the observed peak by performing a dynamical calculation of the five-quark system with the realistic qq and q\bar{q} interactions.

We employ two kinds of parameter sets for the hamiltonian: the one is with the one-boson exchange (OBE) as the qq interaction, the other is with the one-gluon exchange (OGE) as well as OBE. We find that the absolute value of the mass is low after a reasonable assumption for the zero-point energy is introduced, though it is still necessary to introduce extra attraction to reproduce the data. We also find that both of the two parameter sets predict that the mass of the $0^\frac{3}{2}^-$ state is lower than that of the $0^\frac{1}{2}^-$ state. Their difference, however, is less than 80 MeV. Thus, the $0^\frac{3}{2}^-$ state can be assigned to the observed peak without forming the $0^\frac{1}{2}^-$ bound state below NK threshold even after the extra attraction is introduced.

II. MODEL

We have employed a valence quark model. The hamiltonian is taken as:

$$H_q = \sum_i \sqrt{m_i^2 + p_i^2} + v_0$$

$$+ \sum_{i<j} (V_{OGE_{ij}} + V_{OBE_{ij}} + V_{conf_{ij}}).$$

(1)

The two-body potential term consists of the one-gluon-exchange potential, $V_{OGE}$, the one-boson-exchange potential, which consists of the PS and $\sigma$-meson exchange, $V_{OBE} = V_{PS} + V_{\sigma}$, and the confinement potential, $V_{conf}$, which are defined as:
In $V_{\text{OGE}}$, $\alpha_s$ is the strength of OGE, and $\Lambda_g$ is the form factor introduced because quarks cannot be considered as point-like particles in this picture. In $V_{\text{PS}}$, $g$ is the quark-meson coupling constant: $g = g_8$ for $\pi$, $K$, and $\eta$ and $g = g_9$ for $\eta'$ meson. From the asymptotic potential shape, $g_8$ can be obtained from the observed nucleon-pion coupling constant, $g_{\pi N N}$. $f$ and $\sigma$ are the flavor U(3) generators and Pauli spin operators, respectively. The term proportional to $(\Lambda_m/m_m)^2$ is originally the $\delta$-function term; the form factor for the meson exchange, $\Lambda_m$, is also introduced. $\Lambda_m$ is assumed to depend on the meson mass $m_m$ as: $\Lambda_m = \Lambda_0 + \kappa m_m$.

We have employed two kinds of parameter sets: the set with $V_{\text{OBE}}$ but not $V_{\text{OGE}}$ (chiral model, $R\pi$ in the following) and the set with $V_{\text{OGE}}$ and $V_{\text{OBE}}$ (Rg$\pi$) as shown in Table 1. For reference, we also employ the parameter set given by the Graz group 2.

As for the confinement potential for pentaquarks, we replace the factor $(\lambda_i \cdot \lambda_j)$ by its average value as shown in eq. (3). This modified potential gives the same value as that given by the original confinement for the orbital $(0s)^5$ state. This replacement enables us to remove all the scattering states and to investigate only tightly bound states, which will appear as narrow peaks. It is based on the idea of the flux tube model; there the configurations where gluonic flux tubes bind all the five quarks can be distinguished from those of a baryon with a meson. After the coupling of the scattering states with an original confinement potential, some of the states we find will melt away into the continuum 11. Later we discuss which states disappear by comparing the masses of the pentaquarks and the baryon-meson states.

We take the zero-point energy, $v_0$, as:

$$v_0 = \begin{cases} V'_6 & \text{q$\pi$ systems} \\ 3V_0 & \text{q$^3$ systems} \\ 6V_0 & \text{q$^4$$\pi$ systems} \end{cases}$$

The zero-point energy of the pentaquark is taken to be twice as large as that of the q$^3$ systems. It is motivated by the result of lattice QCD calculation, which indicates that the q$\pi$-$\pi$qq type gluon configuration is favored for a pentaquark 12,13; namely, two Y-shapes which are connected by the $\pi$ quark gives the lowest energy. The value of the zero-point energy itself, however, is not uniquely determined in this kind of empirical models. Our main concern here is the level splitting of the states, though we believe the above assumption is not very far from reality.

The wave function we employ is written as:

$$\psi_{TSL}(\xi_A, \xi_B, \eta, R) = \sum_{i,j,n,m,\alpha,\alpha'} \phi_{q^4}(\alpha, \xi_A, u_i) \phi_{q^2}(\alpha', \xi_B, u_j) \psi(\lambda, \eta; v_n)|_{TSL} \chi_{\pi}(R; w_m)$$

where $\phi_{q^4}$ is the antisymmetrization operator over the four ud-quarks, and $\xi_A, \xi_B, \eta$ and $R$ are the coordinates defined as:

$$\xi_A = r_1 - r_2 \quad \text{and} \quad \xi_B = r_3 - r_4$$
$$\eta = (r_1 + r_2 - r_3 - r_4)/2$$
$$R = (r_1 + r_2 + r_3 + r_4)/4 - r_{\pi}$$

$\phi_{q^2}(\alpha, \xi; u)$ is the wave function for a qq pair with the size parameter $u$:

$$\phi_{q^2}(\alpha, \xi; u) = \varphi_\alpha \exp \left[-\frac{\xi^2}{4u^2}\right]$$
where the quantum number $\alpha$ stands for one of the four relative $S$-wave quark pairs: $(TS)C = (00), (01), (10), (11)$. The relative wave function between two quark pairs, $\psi(\lambda, \eta; v)$, and the wave function between the four-quark cluster and the $\pmb{\pi}$ quark, $\chi_\pi(R; w)$, are taken as:

$$\psi(\lambda, \eta; v) = \exp \left[-\frac{\eta^2}{2v^2}\right]$$

and

$$\chi_\pi(R; w) = \exp \left[-\frac{2R^2}{5w^2}\right].$$

The gaussian expansions are taken as geometrical series: $u_{i+1}/u_i = v_{n+1}/v_n = 2$ and $w_{m+1}/w_m = 1.87$. We take 6 points for $u (0.035 - 1.12)$, 4 points for $v (0.1 - 0.8)$, and 3 points for $w (0.2 - 0.7)$. Since we use a variational method, the obtained masses are the upper-limit. They, however, converge rapidly; the mass may reduce more, but probably only by several MeV.

### III. MASS SPECTRUM

The masses of $q\bar{q}$, $q^3$, and $q^4\bar{q}$ systems are shown in Table II N, $\Sigma$, and $\Delta$ masses of $R\pi$ and Graz parameter sets were given in refs. [7, 10].

It is very difficult for a constituent quark model to describe the Goldstone bosons: they need a collective mode, which is constructed by the superposition of $Q(q\bar{q})$. Also, it is hard to justify the models with the kaon-exchange interaction between quarks to describe a kaon. We do not push the model to give the correct kaon mass. After fitting $\rho$-meson mass by adjusting $V_0'$ in the eq. (10), we use $K^*$ mass as a reference of the threshold.

Contrary to the $q\bar{q}$ systems, we have more satisfactory results for the $q^3$ baryons. The masses of the $S$-wave ground states are well reproduced. Each parameter set was taken so as to approximately reproduce $N$, $\Delta$, and $\Sigma$ masses. Though we do not recite other baryon masses, the octet baryon masses are predicted within less than 25 MeV error in the Graz parameter set, 41 MeV in $R\pi$, and 13 MeV in $Rg\pi$ parameter set. The decuplet baryon masses are predicted within less than 14 MeV in the Graz parameter set, 93 MeV in $R\pi$, and 5 MeV in $Rg\pi$ parameter set. The $R\pi$ parameter set tends to overestimate the strange baryons. The level splittings themselves are not very far from the observed values.

We have solved the system of the pentaquarks using the method described in the previous section. The masses of the pentaquark with $(TS) = (01)$ and (10) states are the lowest two among the $q^4\bar{q}$ systems with the flavor-spin interaction. Since there is no pion-exchange between $u$ or $d$ and $\pi$ quark, the three states, $(TS)J^P = (01)\frac{1}{2}^+, (01)\frac{3}{2}^-$, and $(10)\frac{1}{2}^-$, are essentially degenerated. In our case, $R\pi$ and Graz parameter sets are the chiral models. The mass difference of these three levels is 20–34 MeV in these parameter sets.

Both of the $V_{OGE}$ and $V_{OBE}$ are included in the $Rg\pi$ parameter set. Because of $V_{OGE}$ has non-vanishing spin-spin interaction between the $q^4$ cluster and $\pi$-quark, the

| Model | Kin | $m_a$ | $m_s$ | $\alpha_s$ | $\Lambda_q$ | $\frac{g_5}{\sqrt{2}} (m_0 g_5)^2$ | $\Lambda_0$ | $\kappa$ | $\alpha_{conf}$ | $V_0$ | $V_0' - 2V_0$ |
|-------|-----|-------|-------|----------|----------|-------------------------------|--------|--------|-------------|------|----------------|
| $Rg\pi$ | $\pi \eta$ | 313 | 530 | 0 | -0.69 | 0 | 1.81 | 0.92 | 767 | 170 | $-378.3$ | $-51.7$ |
| $Rg\pi_S$ | $\pi \eta$ | 340 | 560 | 0.35 | 3 | 0.69 | 1 | 1.81 | 0.92 | 767 | 172.4 | $-381.7$ | $-22.4$ |
| Graz | $\pi \eta$ | 340 | 560 | 0.35 | 3 | 0.69 | 1.34 | 2.87 | 0.81 | -172.4 | $-416$ | $-39.3$ |

$^1$ Ref. [4] $^2$ with new $V_0'$.
splitting between $(01)^{3/2}_-^-$ and $3^{3/2}_-$ is much larger in $\text{Rg}\pi$ than that of the chiral model: it is 71 MeV for the $\text{Rg}\pi$ parameter set whereas it is 32 MeV for $\text{R}\pi$, or 9 MeV for the Graz parameter set.

The absolute values of the pentaquark mass are from 1603 to 1835 MeV. Each of the states is below the NK\textsuperscript{*} threshold except for one exception, $1\frac{1}{2}^{+}\text{Graz}$ of Graz parameter set. Since the assumption we made for the zero-point energy has large ambiguity, we do not conclude that they are the pentaquark mass. More attraction is necessary to reproduce the observed pentaquark mass.

Let us discuss which of the above levels should be observed as a peak. It is known that for the $(TS)\equiv(01)$ and $(10)$ state, there is only one spin-flavor-color configuration which can be combined to the orbital $[4]$ symmetry. This means that a pentaquark which includes the above $q^4$ states can couple to the relative $S$-wave meson-baryon systems strongly.

Suppose that a peak is observed only when the level is below the 'S-wave threshold', by which we mean the mass of the meson-baryon system which can form the concerning $TJ^P$ state with relative $S$-wave. For example, the $S$-wave threshold of the $TJ^P=0\frac{1}{2}^-$ and $1\frac{1}{2}^-$ states is $m_N + m_K$ while that of $0\frac{3}{2}^-$ is $m_N + m_K$. Then, the levels of $0\frac{1}{2}^-$ and $1\frac{1}{2}^-$ disappear if they are higher than the NK threshold while the level of $0\frac{3}{2}^-$ may be seen if it is lower than the NK\textsuperscript{*} threshold. Also, $1\frac{3}{2}^-$ and $2\frac{3}{2}^-$ disappear if they are higher than the $\Delta K$ threshold.

As seen in Table III the $TJ^P=0\frac{1}{2}^-$ and $1\frac{1}{2}^-$ states are above the NK threshold in our present work. Thus these two are probably not observed. On the other hand, the mass of the $0\frac{3}{2}^-$ state is below the NK\textsuperscript{*} threshold. Because this level has to decay to the relative $D$-wave NK system by the tensor term of the interaction, this level may be seen as a peak. To investigate the situation quantitatively, one needs to perform, e.g., a resonating-group-method calculation for $q^4\pi$ systems, by which the width of the state can be obtained. This we will investigate elsewhere.

To assign the $0\frac{3}{2}^-$ state to the observed peak, it is still necessary to introduce extra attraction by 140 - 280 MeV. It is reported that there are other sources which contribute the absolute mass. For example, the instanton induced interaction, which should be taken into account to reproduce the $\eta-\eta'$ mass difference, gives a universal two-body attraction and a three-body repulsion.

The level splitting between the lowest two states, $TJ^P=0\frac{1}{2}^-$ and $0\frac{3}{2}^-$, is less than 80 MeV. So, the lowest state will not become a bound state as the extra attraction is introduced so that the $0\frac{3}{2}^-$ state becomes 100 MeV above the NK threshold. Other states which can be combined to the orbital $[4]$ symmetry are known to have a higher mass from the discussion based on the group theory. It is also pointed out that one of the positive-parity pentaquarks, $0\frac{1}{2}^+$ state, may be assigned to the observed single peak. Actually, this level can be as low as the negative-parity state. This $0\frac{3}{2}^-$ pentaquark seems more appropriate candidate of the observed single peak.

### IV. ROLES OF THE QQ CORRELATION

Except for the confinement force, all the interaction terms are short-ranged in the quark model. Thus, when the deformation by the quark-quark correlation is introduced in the model, quark pairs where the interaction is attractive become more tightly bound while those with repulsion tend to stay apart from each other. Then an
attractive pair may behave like a single particle; this is the qq correlation which motivates the diquark models \[19, 20\]. We have looked into how much the qq correlation is developed in our full calculation by checking the size of each quark pair.

In Table III we show the number of quark pairs with specific quantum numbers and the size of that pairs. The number of the pairs with the quantum number \(T_2S_2\), \(N_{T_2S_2}\), and the size of the pairs, \(r_{T_2S_2}\), are defined by using the projection operator \(P_{ij}^{(T_2S_2)}\) as:

\[
N_{T_2S_2} = \langle \sum_{i>j} P_{ij}^{(T_2S_2)} \rangle \\
\]

\[
r_{T_2S_2} = \sqrt{\langle \sum_{i>j} P_{ij}^{(T_2S_2)} \rangle \langle \sum_{i>j} P_{ij}^{(T_2S_2)} \rangle} / N_{T_2S_2} .
\]

The number of quark pairs, \(N_{T_2S_2}\), obtained by the full calculation is not very different from that of the group classification, as was also found in the nucleon case\[9\]. The contribution from each pair, however, can be different. The size of quark pairs is large when the interaction is repulsive while it becomes small for the attractive pairs. The ratio is about 1.2 – 1.3. We also find that the qq correlation in the pentaquarks have similar size to that in the nucleon.

\( V. \) SUMMARY

We have investigated the negative-parity uudd\(\bar{s}\) pentaquarks by employing a quark model. The system for the five quarks is dynamically solved; the effects of qq or q\(\bar{q}\) correlations on the wave function are taken into account. The model has realistic qq and q\(\bar{q}\) interactions: the meson exchange for the chiral models, and both of the meson and the effective gluon exchange for the other parameter set, \(Rg\pi\).

It is found that the masses of the pentaquarks are reasonably low, though it is still necessary to introduce an extra attraction to reduce the mass further by 140 – 280 MeV to reproduce the observed \(\Theta^+\) mass. The pentaquark of the lowest mass is found to be \(TJ^P=0^+\). The next lowest is \(0^3^-\); we argue the observed peak corresponds to the latter state because the width can be narrow for this state. Since the level splitting of these two states is no more than 80 MeV, the lower level will not become a bound state below the NK threshold even if we introduce the extra attraction so that the mass of the upper state become as low as the observed one. The lower level will melt into the continuum after the coupling to the meson-baryon states is introduced.

It is also found that the size of quark pairs with the attractive interaction is found to be by about 1.2 – 1.3 times closer than that of the repulsive one. The two-body correlation seems important in the pentaquark systems.

\section*{Acknowledgments}

This work is supported in part by a Grant-in-Aid for Scientific Research from JSPS (No. 15540289).

\begin{thebibliography}{10}
\bibitem{1} T. Nakano et al. [LEPS Collaboration], Phys. Rev. Lett. 91, 012002 (2003)
\bibitem{2} M. Oka, Prog. Theor. Phys. 112, 1-19 (2004) and references therein.
\bibitem{3} A. Hosaka, M. Oka, and T. Shinozaki, [hep-ph/0409102]
\bibitem{4} S. Eidelman et al, Phys. Lett. B 592, 1 (2004); http://pdg.lbl.gov/
\bibitem{5} T. Inoue, et al, hep-ph/0407305
\bibitem{6} A. De Rujula, H. Georgi and S.L. Glashow, Phys.Rev. D12, 147 (1975).
\bibitem{7} L. Ya. Glozman et al, Phys. Rev. D58, 094030 (1998).
\bibitem{8} K. Shimizu and L.Ya. Glozman, Phys. Lett. 477B, 59 (2000).
\bibitem{9} M. Furuchi and K. Shimizu, Phys. Rev. C65, 025201 (2002)
\bibitem{10} M. Furuchi, K. Shimizu, and S. Takeuchi, Phys. Rev. C68, 034001 (2003)
\bibitem{11} E. Hiyama, et al, proceedings of PENTAQUARK04.
\bibitem{12} F. Okiharu, H. Suganuma, T. T. Takahashi, hep-lat/0407001
\bibitem{13} Y. Kanada-En'yo, O. Morinatsu and T. Nishikawa, proceedings of PENTAQUARK04, hep-ph/0410221
\bibitem{14} C. E. Carlson, et al., Phys. Lett. B 573, 101 (2003).
\bibitem{15} Fl. Stancu and D. O. Riska, Phys. Lett. B 575, 242 (2003); Fl. Stancu, Phys. Lett. B 595, 269 (2004), B 598, 295 (2004)(E).
\bibitem{16} K. Shimizu and S. Takeuchi, in preparation; S. Takeuchi and K. Shimizu, proceedings of PENTAQUARK04, hep-ph/0411016
\bibitem{17} S. Takeuchi, S. Nussinov, and K. Kubodera, Phys. Lett. B 318, 1 (1993).
\bibitem{18} T. Shinozaki, M. Oka, and S. Takeuchi, proceedings of PENTAQUARK04, hep-ph/0409103
\bibitem{19} R. L. Jaffe and F. Wilczek, Phys. Rev. Lett. 91, 232003 (2003)
\bibitem{20} M. Karliner and H. J. Lipkin, Phys. Lett. B 575, 249
\end{thebibliography}
(2003).