Modelling inflation in transportation, communication and financial services using B-Spline time series model

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Abstract. Inflation is an increase in the price of goods and services in general where the goods and services are the basic needs of society or the decline of the selling power of a country's currency. Significant inflationary increases occurred in 2013. This increase was contributed by a significant increase in some inflation sectors / groups i.e transportation, communication and financial services; the foodstuff sector, and the housing, water, electricity, gas and fuel sectors. However, significant contributions occurred in the transportation, communications and financial services sectors. In the model of IFIs in the transportation, communication and financial services sector use the B-Spline time series approach, where the predictor variable is $Y_t$, whereas the predictor is a significant lag (in this case $Y_{t-1}$). In modeling B-spline time series determined the order and the optimum knot point. Optimum knot determination using Generalized Cross Validation (GCV). In inflation modeling for transportation sector, communication and financial services obtained model of B-spline order 2 with 2 points knots produce MAPE less than 50%.

Keywords: B-spline, Inflation in Transportation Communication and Financial Services, GCV

1. Introduction

Inflation is a trend (trend) or movement of general price level rises continuously from one period to the next. Controlled and low’s of inflation can support the maintenance of people's purchasing power. While unstable inflation will complicate the business world in the planning of business activities, both in production and investment activities and in determining the price of goods and services it produces [1]. Inflation in Indonesia consists of seven sectors, including food stuffs sector; Fast Food, beverage, cigarette and tobacco sectors; Sectors of Housing, Water, Electricity, Gas and Fuel; Sector of Clothing; Health Sector, Education, Recreation and Sports sectors; Transportation, Communications and Financial Services Sectors. The following presents the descriptive statistics of each sector of inflation in Indonesia from January 2007 until August 2017. Based on Table 1 it is seen that the average inflation rate for the foodstuff sector is also greater than in other sectors. While inflation in the transportation, communication and financial services sectors is lowest value, but has the highest standard deviation among other sectors. This suggests that the variance or diversity in the transport sector, communications and financial services is highest. In addition, the transportation, communication and financial services sectors provide the highest value after the foodstuff sector. The following presents the scatter plot of inflation in the sectors of transportation, communications and financial services.
Table 1. Statistic Descriptive of seven sectors in Inflation from January 2007 until August 2017.

| Sector of Inflation                                      | Mean | Standard Deviation | Min  | Max  |
|----------------------------------------------------------|------|--------------------|------|------|
| Food Stuffs                                              | 8.81 | 4.317              | 1.452| 20.019|
| Fast Food, beverage, cigarette and tobacco               | 7.071| 1.862              | 4.25 | 12.931|
| Housing, Water, Electricity, and Gas                     | 4.933| 2.474              | 1.183| 12.398|
| Clothing                                                 | 5.18 | 2.858              | -0.413| 12.267|
| Health                                                   | 4.59 | 1.707              | 2.187| 9.691 |
| Education, Recreation and Sport                          | 4.87 | 1.799              | 2.747| 10.408|
| Transportation, Communication and Financial Services     | 3.715| 5.215              | -6.852| 16.333|

Figure 1. Scatter plot of Inflation in Transportation, Communication and Financial Services Sectors from January 2007 until August 2017

Based on Figure 1 the change in the value of inflation in the transportation, Communication and Financial Services Sectors sector occurs fluctuatively, especially in years of 2008, 2009, 2013 and 2015. The possibility of this fluctuating change can occur, because there is a certain event that causes fluctuating changes. Figure 1 shows that the data pattern is not linear so if it wants to be modeled with parametric will be biased.

One of the statistical methods used to model time series data for prediction is the Autoregressive Integrated Moving Average (ARIMA) method [2]. The ARIMA model is included in the linear model and parametric model, meaning that it requires assumptions, among others, the data must be stationary in the mean and variance, and the residual must be white noise and normally distributed. So with the strictness of these assumptions, many researchers were developing nonparametric models, meaning it does not require certain assumptions in modeling.

One of the methods developed in nonparametric modeling is nonparametric regression. The concept of nonparametric regression modeling is the estimation curve looking for its own form of function against the distribution of data, so the modeling is not dependent on the parameters, but the parameters are generated from the approximate data pattern[3]. A nonparametric regression approach has been widely used, among others Spline [4], Local Polynomial, Wavelet, Fourier, Neural Network. Regression method is usually used for cross section data, but regression method can be developed for time series data modeling. The concept of time series regression is the determination of predictor variables. The number of predictor variables is determined by the number of insignificant lags. The insignificant lag determination uses the Partial Autocorrelation Function (PACF) plot [2].

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One of the nonparametric regression approach is Spline. Spline is a segmented piece of polynomial, so it has a high flexibility in modeling [5]. The spline approach can use the truncated function or the B-Spline function. It is adequate to model random effects in mixed model analysis and is also efficient in the estimation of covariance function [6]. B-Spline is a segmented approach that is able to reach distant data points. According to Misztal (2006) [7], spline function tend to be highly affected by the distribution of data, the degree of function, and especially by the choice of number and position of knots. Ruppert et. al (2003) [3] recommend knots to be placed at equidistant intervals. According Meyer (2005b) [9] the use of a B-spline function, which is a kind of penalized spline, attenuates the influence of knot position on the estimate. Some studies using B-splines include Duan et.al (2016) [12] performing B-spline modeling on steel strength modeling observed over time and Qiau et al. (2015) [10] conducted a B-spline modeling study on the durability of changes in the frequency signal over time. Therefore, this paper examines inflation modeling in the transportation, communication and financial services sector using B-Spline.

2. Nonparametric Regression Using B-Spline

Nonparametric regression modeling is one of nonlinear modeling. The shape is quite complex but does not require assumptions. Nonparametric approach is a model estimation method based on unconstrained approach of assumption of certain regression curve form where the regression curve is assumed only smooth, meaning it is contained in a certain function room so that nonparametric regression has high flexibility because the regression curve estimation form adjusts its data without being influenced by the research subjectivity factor [4]. The basic assumption of nonparametric regression is the presence of a smoothing that links the y response variable to one or more predictor variables x. Regression nonparametric model with observation \((x_i,y_i); \ i=1, 2,..., n\) as follow as:

\[
y_j = \mu(x_j) + \varepsilon_j, \quad i=1, 2, ..., n
\]

with \(\varepsilon_i\) is residual of model which random variable with mean 0 and constant variance \(\sigma^2\), while \(\mu(x_i)\) is value of function \(\mu\) which unknown in the value of \(x_1,..,x_n\), assumed that \(a_0 \leq x_1 \leq ... \leq x_n \leq a_1\) where \(a_0\) taken from the minimum value of x and \(a_1\) taken from the maximum value of x.

The Curve of function \(\mu\) in Eq. (1) while approach B-spline function with orde \(m\) with \(k\) knot, can be written as:

\[
\mu(x_i) = \sum_{j=1}^{m+k} b_j N_{j-m,m}(x_i), \quad i=1, 2, ..., n
\]

With \(N_{j-m,m}(x)\) is basis of B-Spline and \(b_j\) is parameter of B-Spline regression.

To make B-Spline function with orde \(m\) and \(k\) knot points \(\xi_1, \ldots, \xi_k\) which \(a_0 < \xi_1 < ... < \xi_k < a_1\), first defined additional knots as much as \(2m\), which \(\xi_{-(m-1)}, \ldots, \xi_{-1}, \xi_0, \xi_k+1, \ldots, \xi_{k+m}\), with \(\xi_{-(m-1)} = ... = \xi_0 = a_0\) and \(\xi_{k+1} = ... = \xi_{k+m} = a_1\).

Basis of B-Spline function in orde \(m\) with knot points in \(\xi_i\) where \(i = -(m-1), ..., k\) defined which recursive as follow as [11]:

\[
N_{i,m}(x) = \frac{x-\xi_i}{\xi_{i+m-1}-\xi_i} N_{i,m-1}(x) + \frac{\xi_{i+m}-x}{\xi_{i+m}-\xi_{i+1}} N_{i+1,m-1}(x)
\]

For \(i = -(m-1), ..., k\), and

\[
N_{i,1}(x) = \begin{cases} 1, & x \in [\xi_i, \xi_{i+1}] \\ 0, & \text{with else} \end{cases}
\]

The kinds of Basis B-Spline Function
Based on Eq. (3), the kinds of basis B-Spline function in $m$ are:

a. Orde $m = 2$ given basis B-Spline function linear, the function as follow as:

$$N_{i,2}(x) = \frac{x - \zeta_i}{\xi_{i+1} - \zeta_i} N_{i,1}(x) + \frac{\zeta_{i+2} - x}{\xi_{i+2} - \xi_{i+1}} N_{i+1,1}(x) \text{ with } i = -2, \ldots, k$$

b. Orde $m = 3$ given basis B-Spline function quadratic, the function as follow as:

$$N_{i,3}(x) = \frac{x - \zeta_i}{\zeta_{i+1} - \zeta_i} N_{i,2}(x) + \frac{\zeta_{i+3} - x}{\zeta_{i+3} - \zeta_{i+1}} N_{i+1,2}(x) \text{ with } i = -2, \ldots, k$$

c. Orde $m = 4$ given basis B-Spline function cubic, the function as follow as:

$$N_{i,4}(x) = \frac{x - \zeta_i}{\zeta_{i+1} - \zeta_i} N_{i,3}(x) + \frac{\zeta_{i+4} - x}{\zeta_{i+4} - \zeta_{i+1}} N_{i+1,3}(x) \text{ with } i = -2, \ldots, k$$

Model of B-spline nonparametric regression with $m$ orde and knot points $k$ in Eq. (2) can be written as:

$$y_i = b_1 N_{1-m,m}(x_i) + b_2 N_{2-m,m}(x_i) + \ldots + b_{(m+k)} N_{k,m}(x_i) + \varepsilon_i$$

As matrix can written as:

$$
\begin{bmatrix}
  y_1 \\
  y_2 \\
  \vdots \\
  y_n \\
\end{bmatrix} =
\begin{bmatrix}
  N_{1-m,m}(x_1) & N_{2-m,m}(x_1) & \ldots & N_{k,m}(x_1) \\
  N_{1-m,m}(x_2) & N_{2-m,m}(x_2) & \ldots & N_{k,m}(x_2) \\
  \vdots & \vdots & \ddots & \vdots \\
  N_{1-m,m}(x_n) & N_{2-m,m}(x_n) & \ldots & N_{k,m}(x_n) \\
\end{bmatrix}
\begin{bmatrix}
  b_1 \\
  b_2 \\
  \vdots \\
  b_{(m+k)} \\
\end{bmatrix} +
\begin{bmatrix}
  \varepsilon_1 \\
  \varepsilon_2 \\
  \vdots \\
  \varepsilon_n \\
\end{bmatrix}
$$

or:

$$\mathbf{y} = \mathbf{N} \mathbf{b} + \mathbf{\varepsilon} \quad (4)$$

The curve of regression $\mu$ can be approached by B-spline function with order $m$ and knot points $k$ for $\lambda = \{\xi_1, \ldots, \xi_k\}$ as follow as:

$$\mu_\lambda(x_i) = \sum_{j=1}^{m+k} b_{\lambda,j} N_{j-m,m}(x_i)$$

So that the model of B-spline for $\lambda = \{\xi_1, \ldots, \xi_k\}$ is:

$$y_i = \sum_{j=1}^{m+k} b_{\lambda,j} N_{j-m,m}(x_i) + \varepsilon_i$$

It can be developed as:

$$y_i = b_{\lambda_1} N_{1-m,m}(x_i) + b_{\lambda_2} N_{2-m,m}(x_i) + \ldots + b_{\lambda_{(m+k)}} N_{k,m}(x_i) + \varepsilon_i \quad (5)$$

Model of B-spline (13) can be written as matrix as:

$$
\begin{bmatrix}
  y_1 \\
  y_2 \\
  \vdots \\
  y_n \\
\end{bmatrix} =
\begin{bmatrix}
  N_{1-m,m}(x_1) & N_{2-m,m}(x_1) & \ldots & N_{k,m}(x_1) \\
  N_{1-m,m}(x_2) & N_{2-m,m}(x_2) & \ldots & N_{k,m}(x_2) \\
  \vdots & \vdots & \ddots & \vdots \\
  N_{1-m,m}(x_n) & N_{2-m,m}(x_n) & \ldots & N_{k,m}(x_n) \\
\end{bmatrix}
\begin{bmatrix}
  b_{\lambda_1} \\
  b_{\lambda_2} \\
  \vdots \\
  b_{\lambda_{(m+k)}} \\
\end{bmatrix} +
\begin{bmatrix}
  \varepsilon_1 \\
  \varepsilon_2 \\
  \vdots \\
  \varepsilon_n \\
\end{bmatrix}
$$

It can be written as:

$$\mathbf{y} = \mathbf{N}_\lambda \mathbf{b}_k + \mathbf{\varepsilon}$$
The estimation parameter of $b_{\lambda} = (b_{\lambda 1}, b_{\lambda 2}, \ldots, b_{\lambda (m+k)})^T$ can be obtained by ordinary least square. Estimator of $\hat{b}_{\lambda}$ can be obtained by minimizing sum of squares residual (SSR). SSR be minimum if partial derivative SSR to $\hat{\lambda}$ was zero.

$$\text{RSS} = e^T e$$

$$= (y - N_\lambda b_{\lambda})^T (y - N_\lambda b_{\lambda})$$

$$= y^T y - 2 b_{\lambda} N_\lambda^T y + b_{\lambda} N_\lambda^T N_\lambda b_{\lambda}$$

Then derivatifed by $b_{\lambda}$ can be obtained:

$$\frac{\partial \text{RSS}}{\partial b_{\lambda}} = -2 N_\lambda^T y + 2 N_\lambda^T N_\lambda b_{\lambda} = 0$$

$$N_\lambda^T N_\lambda \hat{b}_{\lambda} = N_\lambda^T y$$

$$\hat{b}_{\lambda} = (N_\lambda^T N_\lambda)^{-1} N_\lambda^T y$$

with $\hat{b}_{\lambda} = (\hat{b}_{\lambda 1}, \hat{b}_{\lambda 2}, \ldots, \hat{b}_{\lambda (m+k)})^T$.

Estimator of curve regression $\hat{\mu}_{\lambda} = (\hat{\mu}_{\lambda 1}, \hat{\mu}_{\lambda 2}, \ldots, \hat{\mu}_{\lambda n})^T$ as follow as:

$$\hat{\mu}_{\lambda} = N_\lambda \hat{b}_{\lambda}$$

$$= N_\lambda (N_\lambda^T N_\lambda)^{-1} N_\lambda^T y$$

$$= S_\lambda y$$

with $S_\lambda = N_\lambda (N_\lambda^T N_\lambda)^{-1} N_\lambda^T$ simetris and definit positif (Eubank, 1999).

Estimator of curve regression can be written as:

$$\hat{\mu}_{\lambda} (t) = \sum_{j=1}^{m+k} \hat{b}_{\lambda j} N_{j-m,m} (x)$$

with $\hat{b}_{\lambda j}$ can be obtained from $\hat{b}_{\lambda} = (\hat{b}_{\lambda 1}, \hat{b}_{\lambda 2}, \ldots, \hat{b}_{\lambda (m+k)})^T$ so that the estimation of model for B-spline function in nonparametric regression is:

$$\hat{y} = \sum_{j=1}^{m+k} \hat{b}_{\lambda j} N_{j-m,m} (x)$$

(6)

In Eq. (6) can be written as:

$$\hat{y} = \hat{b}_{\lambda 1} N_{1-m,m} (x) + \hat{b}_{\lambda 2} N_{2-m,m} (x) + \ldots + \hat{b}_{\lambda (m+k)} N_{k,m} (x)$$
To model time series data using the B Spline model, the data must be modified into

where \( y \) is dependent variable in observation \( i \)-th of regression model and \( x_i \) is independent variables in observation \( i \)-th . The variable \( x_i \) represents the significant of lag value (p). To see what lags was significant will be used in modifying data the PACF plot views of the inflation data (Figure 2). Based on Figure 2, the significant value of PACF lag 1 has the greatest and most significant value. Therefore, variables \( (y_i, x_i) = (y_i, y_{i-1}) \) are used. Modeling uses data in the sample from January 2009 to March 2016. So \( (y_i, x_i) = (y_i, y_{i-1}) \) with \( i = 2, 3, ..., 111 \). The plot between x and y in the sample data is shown in Figure 3.

Based on Figure 3, variables of x and y have a rather diffuse pattern. To get the optimal B-spline model is required the selection of optimal knot points as well. The method used to determine the location of the optimal knot point in this study is with Generalized Cross Validation (GCV). The knot points are between the minimum data and the maximum data from the sample data.

### 4. Results and Discussion

The data of this research is inflation data for years on years (yoy) in transportaion, communication, and financial services sectors from January 2009 until April 2017. The Data devided into two parts, are insample data and outsample data. Insample data used to train the model, while outsample data to test the model. Insample data taken from January 2009 until March 2016, while outsample data taken from April 2016 until April 2017. As bellow presents scatter plot of insample data and outsample data.

![Scatter Plot of Inflation in Transportation, Communication and Financial Services in Indonesia](image)

**Figure 2.** Scatter Plot of Inflation in Transportation, Communication and Financial Services in Indonesia (a) Insample Data From Jan 2009-March 2016 and (b) Outsample Data from Apr 2016-Apr 2017

Inflation data on the transportation sector, communications and financial services is the univariate time series data \( (y_i) \). To model time series data using the B Spline model, the data must be modified into cross section data with the form \( (y_i, x_i) \) whereas \( y_i \) is dependent variable in observation \( i \)-th of the model and \( x_i \) is independent variables in observation \( i \)-th . The variable \( x_i \) represents the significant of lag value (p). To see what lags was significant will be used in modifying data the PACF plot views of the inflation data (Figure 2). Based on Figure 2, the significant value of PACF lag 1 has the greatest and most significant value. Therefore, variables \( (y_i, x_i) = (y_i, y_{i-1}) \) are used. Modeling uses data in the sample from January 2009 to March 2016. So \( (y_i, x_i) = (y_i, y_{i-1}) \) with \( i = 2, 3, ..., 111 \). The plot between x and y in the sample data is shown in Figure 3.

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The best knot point is the knot point with minimum GCV, which will produce the best estimation. In this study, GCV calculations were performed using B-spline order 2 (linear), order 3 (quadratic) and order 4 (cubic) each of which had one to three knots. As below the values of knot points of some order based on Table 2.

| orde   | Knots | Knot points | GCV    |
|--------|-------|-------------|--------|
| linear | 1     | 13          | 5.245335 |
|        | 2     | 7; 12       | 4.750622 |
|        | 3     | -3; -2; -1  | 5.227852 |
|        | 4     | 9; 11; 13; 14 | 5.160862 |
| quadratic | 1   | 11          | 5.287010 |
|         | 2     | 6; 7        | 4.974516 |
|         | 3     | -6; -4; -3  | 5.347319 |
|         | 4     | 9; 10; 12; 13 | 5.091798 |
| cubic  | 1     | 16          | 5.392915 |
|        | 2     | 12; 13      | 4.783588 |
|        | 3     | -5; -4; -2  | 5.534105 |
|        | 4     | -4; -3; -2; 0 | 5.281709 |

The best model is the model with the smallest GCV value. Based on GCV modeling of B-spline order of 2, 3 and 4 with approach 1 to 4 knot point, the best model was obtained linear B-spline model with 2 knots, at point 7 and 12 with GCV value of 4.750622. Model of linear B-spline with 2 knot points, at point 7 and point 12 is
\[ \hat{y} = -6.549N_{-1,2}(x) + 7.297N_{0,2}(x) + 6.502N_{1,2}(x) + 16.958N_{2,2}(x) \]  \hspace{1cm} (17)

with 

\[ N_{i,2}(x) = \frac{x - \xi_{i+1}}{\xi_{i+2} - \xi_i} N_{i,1}(x) + \frac{\xi_{i+2} - x}{\xi_{i+2} - \xi_{i+1}} N_{i+1,1}(x), \quad i = -1, \ldots, 2 \]

with \( \xi_{-1} = 6.852, \quad \xi_1 = 7, \quad \xi_2 = 12, \quad \xi_3 = \xi_4 = 16.330 \)

\[ N_{i,1}(x) = \begin{cases} 1, & x \in [\xi_i, \xi_{i+1}] \\ 0, & \text{else} \end{cases} \]

So that the B-spline model with knot point at 7 and 12 as follow as:

\[ \hat{y} = -6.549N_{-1,2}(x) + 7.297N_{0,2}(x) + 6.502N_{1,2}(x) + 16.958N_{2,2}(x) \]

with 

\[ N_{-1,2}(x) = \frac{x - \xi_{-1}}{\xi_0 - \xi_{-1}} N_{-1,1}(x) + \frac{\xi_1 - x}{\xi_1 - \xi_0} N_{0,1}(x) \]

\[ N_{0,2}(x) = \frac{x - \xi_0}{\xi_1 - \xi_0} N_{0,1}(x) + \frac{\xi_2 - x}{\xi_2 - \xi_1} N_{1,1}(x) \]

\[ N_{1,2}(x) = \frac{x - \xi_1}{\xi_2 - \xi_1} N_{1,1}(x) + \frac{\xi_3 - x}{\xi_3 - \xi_2} N_{2,1}(x) \]

\[ N_{2,2}(x) = \frac{x - \xi_2}{\xi_3 - \xi_2} N_{2,1}(x) + \frac{\xi_4 - x}{\xi_4 - \xi_3} N_{3,1}(x) \]

After obtaining parameter estimation value using linear B-spline model with 2 knot point, data prediction is done. Comparison of actual inflation data and estimated or predicted inflation data can be illustrated in Figure 7 below:

**Figure 5.** Scatter plot of Actual data and prediction Data used linear B-Spline model

The estimation results in the form \((x, y)\) are subsequently returned in the form \((t, x)\) with the following syntax, and the results are presented in figure 6. Comparison of actual data out samples and sample predicted out data are presented in Tables 5 and 8.
Figure 6. The Actual data and prediction data based on time (t) using linear B-spline

Table 3. The Comparison between Actual and prediction of Outsample Data using linear B-Spline model

| Time      | Actual | Prediction |
|-----------|--------|------------|
| April 2016 | -1.51  | -1.20588   |
| Mei 2016   | -1.50  | -1.19606   |
| Juni 2016  | -0.99  | -0.68463   |
| Juli 2016  | -1.49  | -1.19048   |
| Agustus 2016| -1.93  | -1.62625   |
| September 2016| -1.35 | -1.04555   |
| Oktober 2016| -1.40  | -1.09485   |
| November 2016| -1.39  | -1.085     |
| Desember 2016| -0.73  | -0.42753   |
| Januari 2017| 2.75   | 3.044308   |
| Februari 2017| 3.05   | 3.352872   |
| Maret 2017  | 3.15   | 3.445784   |
| April 2017  | 5.11   | -1.20588   |

Figure 5 shows the scatter plot between the actual data and the predicted data. It is seen that inflation data modeling in transportation, communication, and financial services sector using B-spline model yields curve following actual data distribution. This shows that modeling using B-spline yields good modeling. Table 3 shows the comparison between predictive data and samples data. From sample data and estimation, MAPE value is 49.98% indicating that model has good performance because MAPE value is between 30% and 50%.

5. Conclusion

Time series data modeling using B-spline nonparametric regression approach, determination of predictor variable based on PACF plot. In inflation modeling for the transport sector, communications and financial services, the significant lag value is the 1st lag. Determination of optimum knot point using GCV, obtained optimum knot value 2 that is at point 7 and 12 with linear order. The data modeling of the insample data produces very good model accuracy, but the MAPE of outsample data reach 49.98%. In the future research we modelling linear, non-linear and combining linear and non-linear. For linear we will use intervention method, and for non linear we will use local polynomial.
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