Spatially Resolved Fast-Time Vibrometry Using Ultrawideband FMCW Radar Systems

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Abstract—Highly accurate vibrometry and ranging are important topics in the industrialized economy. Wherever optical measurement technology fails due to its high prices and vulnerability within harsh environments, millimeter-wave (mmWave) radar technology is well suited. This article introduces a signal processing chain for ultrawideband frequency-modulated continuous-wave (FMCW) radar. It uses fast-time measurement to evaluate the instantaneous phase, thus allowing for spatially resolved sensing of multiple simultaneously vibrating radar targets, faster than the chirp rate. In order to accomplish this, a sophisticated error model and a calibration scheme were derived. We used three FMCW radar systems covering a broad range of the mmWave spectrum to demonstrate the signal processing approach. In contrast to the commonly used slow-time measurement principle, the highest detectable frequency was improved from 55 Hz to at least 16 kHz, which is the upper limit of the audio range. Up to 10 kHz could be measured with an underlying large-scale motion of 0.4 m/s, while the vibration displacement was at a minimum of 30 nm.

Index Terms—Displacement measurement, distance measurement, estimation error, millimeter-wave (mmWave) radar, processing algorithms, radar measurements, radar theory, ultrawideband radar, vibration measurement, vibrometers.

I. INTRODUCTION

Within the last few years, millimeter-wave (mmWave) frequency-modulated continuous-wave (FMCW) radar systems have been widely used for short-range measurements. Due to their low peak power and their simple homodyne structure, they allow for low-cost chip integration with high bandwidth and, hence, with the highest range resolution. Current research deals with ranging, synthetic aperture radar (SAR) imaging, material characterization, and biomedical applications. Nowadays, well-known commercial applications for mmWave sensor technology are autonomous driving, ultrafast communication (5G), tank-level probing, or safety sensory. Here, mmWave sensory is outstanding due to its advantageous properties, which enables sensing even within the harshest environments.

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Fig. 1. Illustration of the slow- and fast-time measurement principle using FMCW radar systems.

Many applications require highly sensitive measuring of mechanical vibrations. A common example is wear indication, and fault detection of rotating machines [1]–[4] or structures [5]–[10], e.g., bridges or high-rise buildings. Furthermore, vibrometry deals with health monitoring to measure the heartbeat as well as the respiration rate [11]–[15] or extracting speech by vocal vibrations [16]–[19]. Typically, highly accurate instruments for measurement of vibrations and distances are contact-based [20] or use fragile laser devices.

In this article, we demonstrate a spatially resolved method for highly precise vibration measurement of radar targets in motion using ultrawideband FMCW radar systems. In contrast to common vibrometry [3]–[5], [8], [21]–[24] and ranging [11], [25]–[30], here, vibrometry is elaborated in the time domain of the intermediate frequency (IF) signal. This allows for fast-time measurement of vibration displacements with a significantly higher measurement rate compared with the ramp-repetition rate. Instead of measuring only in the region of a few Hz, it allows us to measure at least the entire audio range of 16 kHz. The proposed measurement principle does not require super-fast frequency chirps. Usually, these chirps are noisy and nonlinear, e.g., due to the large loop bandwidth of the phase-locked loop (PLL) [31], which is commonly used for frequency synthesis. For a better understanding, Fig. 1 shows the difference between slow- and fast-time measurements.

Compared with unmodulated continuous-wave (CW) radar, which is widely used for radio wave vibrometry [16], [19], [32]–[38], fast-time vibrometry using FMCW radar generally enables simultaneous measurements of multiple vibrating radar targets by separating them within the range domain, which is the radial distance from the radar sensor to the

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respective target. Generally, CW radar does not allow measuring distances unambiguously within the entire measurement range. Also, multiple reflecting radar targets cannot be separated. However, there are enhanced CW techniques that can partially overcome these restraints. For example, phase-comparison monopulse radar [15] or adaptive beam steering [37] could separate multiple targets based on their angular location, and multitone CW radar [38] could enhance the range unambiguity. FMCW radar does not have these restraints. Its measurement principle is unambiguous, and multiple targets can be separated according to the available bandwidth. Nevertheless, compared with CW radars, the hardware of FMCW radars is more complex and more expensive. However, as frequency increases up to mmWave and the lower THz range, these systems could be entirely on-chip, thus significantly reducing the price. Rodenbeck et al. [39] demonstrated a procedure for remote sensing of vibrating radar targets in motion using mmWave pulse-Doppler radar. Their approach also works in fast time, and they also use continuous motion compensation techniques. However, compared with pulse (Doppler) radar, an FMCW radar has a low peak power and can therefore be integrated into low-cost radar systems with ultrahigh bandwidth.

This article is an extension of [40], where we demonstrated a basic working principle of fast time, i.e., intrachirp vibrometry using the FMCW radar. This extended article consists of an in-depth investigation of the following features. We derived a sophisticated error model, which accurately describes the effects of the radar hardware on vibrometry and ranging accuracy. Furthermore, we present two signal processing chains for fast-time vibrometry and slow-time ranging, respectively. The slow-time measurement principle is based on a highly accurate phase evaluation [25]–[27], [41], [42], which we elaborated in [25]. Then, the methods of vibrometry and ranging are used together as a combined measuring procedure for measuring multiple vibrating radar targets in motion. As a consequence, a two-step technique for continuous self-calibration is necessary and will be introduced. After that, we propose an additional calibration technique, which is performed in the baseband and does not require any radio frequency (RF) metrology. Compared with conventional calibration methods, it provides the calibration of the radar, considering the propagation delay independent and dependent RF phase response, where the latter is primarily due to nonlinearities of the frequency ramp synthesizer. Fig. 2(a) and (b) shows a photograph of the experimental setup and the related results of our prior work, which is extended here. They clearly illustrate that we could measure the vibration frequencies of multiple nonmoving but simultaneously vibrating radar targets in fast time.

To demonstrate the proposed calibration and measurement methods, we utilized three ultrawideband FMCW radar systems, which cover the mmWave spectrum. The RF front ends [23], [43], [44] of these radar systems are still milestones regarding relative bandwidth and phase noise. Then, we utilized an experimental setup, where an adjustable vibrating radar target was moved along the 5-m linear rail.

Our article is a proof of concept for a new measurement technique, whose potential application is real-time and in-process monitoring of industrial production processes, e.g., in machine tools. A fusion of micrometer accurate ranging and highly precise vibrometry is applicable for accurate adjustment of the machine parameters and condition monitoring for indicating wear. Machines produce transient and continuous acoustic emissions during operation, whose characteristic frequency signature can be identified using machine learning and pattern recognition to indicate the tool lifetime or the machine state. Due to an increasing lack of resources and growing environmental awareness, defective machines, or worn-out tools, which cause faulty workpieces, are of particular importance. In addition, machine downtimes can be reduced, thus increasing production efficiency.

The organization of this article is as follows. Section II addresses the necessary fundamentals and introduces an error model for highly accurate ranging and vibrometry using the FMCW radar. Section III describes the proposed signal processing methods for slow-time ranging and fast-time vibrometry. Section IV demonstrates the experimental results using three radar systems covering the mmWave spectrum. Finally, Section V concludes this article.

The following notation is used for variables. Complex-valued variables, such as analytic signals, are written with an underscore (e.g., $s(t)$). Matrices and column vectors are written as boldface upper (e.g., $E$) and lower cases (e.g., $e_0$), respectively.

II. DISTANCE ESTIMATION USING FMCW RADAR

A. FMCW RADAR FUNDAMENTALS

FMCW radar systems use electromagnetic waves to determine the range, velocity, and angle of reflecting objects. Fig. 3 shows a generalized schematic of the monostatic\(^2\) linear

\(^2\)Transmitter and receiver are colocated (transceiver).
FMCW (LFMCW) principle, as used here. A frequency ramp synthesizer generates a linear frequency-modulated signal, which is also called frequency ramp or frequency chirp

\[ s_{tx}(t) = A_{tx} \cdot \cos \left( \omega_0 t + \int \pm \dot{\omega} dt \right), \quad -\frac{T}{2} \leq t \leq \frac{T}{2}. \] (1)

The center angular frequency is given by \( \omega_0 \), and the ramp slope is given by \( \dot{\omega} = B/T \), of which \( B \) is the ramp bandwidth in radians per second and \( T \) is the ramp sweep time. The parameter \( A_{tx} \) is the amplitude of the transmit signal. For the sake of convenience, “±” or “−” in all following equations applies to upchirp or downchirp, respectively. The signal in (1) is redefined as

\[ s_{tx}(t) = A_{tx} \cdot \cos \left( \omega_0 t + \frac{1}{2} \dot{\omega} t^2 \right) \] (2)

and radiated via a coupler or circulator into free space using an antenna. Here, the electromagnetic wave impinges among reflecting objects, the so-called radar targets. The receive signal for a single idealized point scattering radar target is damped to \( A_{rx} \) and delayed by \( \tau \) copy of the transmit signal \( s_{tx}(t) \)

\[ s_{rx}(t) = A_{rx} \cdot \cos \left( \omega_0 (t - \tau) \pm \frac{1}{2} \dot{\omega} (t - \tau)^2 \right). \] (3)

After the reception, a homodyne receiver compares \( s_{tx}(t) \) and \( s_{rx}(t) \). Assuming that \( K \) radar targets result in a superposition of \( K \) quasi sinusoidal beat signals within the baseband

\[ s_d(t) = \sum_{k=0}^{K-1} A_k \cdot \cos \left( \dot{\omega}_k \tau_k t \pm \omega_0 \tau_k - \frac{1}{2} \dot{\omega}_k \tau_k^2 \right) \] (4)

the so-called IF signal. The IF signal is characterized by the parameters amplitude \( A_k \) and propagation delay \( \tau_k \). Both frequency and phase of the particular signal component are proportional to the propagation delay \( \tau \). The distance from the radar to the respective target is indicated by \( \tau c/2 \), where \( c \) is the propagation speed of electromagnetic waves.

**B. Proposed Error Model**

However, each highly linear and low-noise radar system is impaired by perturbations of the hardware and dispersive free-space propagation. Consequently, this results in inaccurate and imprecise\(^3\) range measurements as well as false detections due to indistinguishable ghost targets. To overcome this problem, we derive a sophisticated device error model, which is

\(^3\)Precision: random errors and accuracy: systematic errors.

Due to the variation of the transmission power, the amplitude of the radiated wave \( A_{tx}(t) \) cannot be assumed constant. It depends on the frequency in the RF path and, hence, on the sweep time \( t \). The instantaneous phase is modulated by a perturbing component \( \epsilon(t) \) due to nonlinearities of the frequency ramp, leading to systematic frequency deviations of the frequency chirp. In addition, the frequency ramp is affected by phase noise \( \phi_{pn}(t) \) of the frequency synthesizer. The perturbed receive signal for \( K \) radar targets is

\[ s_{rx}(t) = s_{tx}(t) \ast \left( \sum_{k=0}^{K-1} h_{tx,\tau_k}(t - \tau_k) * h_{tx,\epsilon}(t) \right) * h_{rf}(t). \] (6)

as shown in Fig. 4. The receive signal is a convolution of the transmit signal \( s_{tx}(t) \) with a propagation delay-dependent impulse response of the RF path \( h_{tx,\epsilon}(t) \), where \( \epsilon(t) \) is the propagation delay of the respective target. In particular, with ultrawideband radar systems in the presence of adjacent and sharp absorption lines of gas molecules, \( h_{tx,\epsilon}(t) \) represents these perturbation propagation effects. Oxygen and water vapor components in the atmosphere could affect radio waves when measuring within the mmWave region.\(^4\) In addition, the signal is convolved with \( h_{rf}(t) \), which is the impulse response of the respective radar target. It expresses the near-field effects of nonpoint-shaped targets or radar targets with multiple inseparable scattering points due to insufficient radar bandwidth.

As investigated in [25], \( h_{tf,\epsilon}(t) \) refers to the variation of the target phase center. Especially with highly accurate ranging over a broad range, variation of \( h_{tf,\epsilon}(t) \) becomes crucial. The last component within the RF path impulse response is \( h_{rf}(t) \), which defines radar hardware-related perturbations between the mixer and the antenna. It represents antenna ringing, crosstalk within the radar hardware, and dispersion

\(^4\)Important molecular absorption lines in air within the mmWave spectrum: O\(_2\): 60 and 119 GHz, and H\(_2\)O: 22, 183, and 325 GHz [45].
in waveguides or striplines. The static nonlinear device error model for the IF signal is
\[ s_{if}(t) = (s_{tx}(t) \cdot s_{tx}(t) + n(t)) \ast h_{if}(t) \tag{7} \]
where the homodyne receiver generates the beat signal. In contrast to (4), the IF signal is perturbed by additive white Gaussian noise \( n(t) \). It unifies the hardware immanent receiver noise of the RF components, such as the mixer, amplifier or coupler, thermal noise, and noise within the baseband signal processing like quantization noise of the analog-to-digital converter. The signal is convolved with the impulse response of the IF path \( h_{if}(t) \) due to the IF filter. Consequently, this leads to a frequency and, thus, range-dependent variation of the IF signal parameters amplitude and phase.

The representation of the device model in the RF domain is difficult to characterize. Therefore, we propose a generalized device model in baseband representation, which is equivalent to the model, as described before. This kind of representation is well suited for simulation of a discrete quantity of radar targets. Fig. 5 shows the error model using complex-valued analytic signals. We redefine (7) for \( K \) radar targets by
\[ s_{if}(t) = \left( \sum_{k=0}^{K-1} A_k A_e(t) \cdot \cos(\phi_k(t) + \phi_{pm, \tau_k}(t)) + n(t) \right) \ast h_{if}(t). \tag{8} \]

The signal consists of a superposition of a sinusoid for each scatterer. Unchanged to (7), it is affected by additive white Gaussian noise and the IF path impulse response. The amplitude is composed of two components \( A_k \) and \( A_e(t) \), where \( A_e \) represents the free-space path lost and the radar cross section (RCS). It is given by the radar range equation for far-field conditions, as considered in \( h_{if, \tau_k}(t) \) and \( h_{if, \tau_k}(t) \). The amplitude modulation component \( A_e(t) \) covers \( A_{tx}(t) \) and the attenuation in \( h_{if}(t) \), e.g., due to dispersive components or even amplitude modulation by dispersive wave propagation in free space. In addition, phase noise \( \phi_{pm, \tau_k}(t) \) perturbs the instantaneous phase, which is \( \phi_{pm}(t) - \phi_{pm}(t - \tau) \). Due to correlation effects in the homodyne transceiver, the phase noise of the phase noise source partially cancels out depending on the propagation delay of the radiated wave \( \tau \). According to [46], the single-sided phase noise spectrum in the IF signal is
\[ L_{if, \tau}(\omega) = L(\omega) + 20 \log_{10} \left( 2 \sin(\omega \tau) \right) \tag{9} \]
where \( L(\omega) \) is the single-sided power spectral density (PSD) of the phase noise generated by the frequency ramp source.

The intermediate phase
\[ \phi_k(t) = \dot{\omega}_0 \tau + \epsilon_0 \tau_k - \frac{1}{2} \dot{\omega}_0^2 \tau_k^2 + \epsilon_1(t) \tau_k \tag{10} \]
which carries the frequency and zero-phase information, is disturbed by two parameters \( \epsilon_0(t) \) and \( \epsilon_1(t) \). The first parameter
\[ \epsilon_0(t) = \arg \left( H_{if}(j\omega) \right) + \arg \left( H_{if, \dot{\omega}}(j\omega) \right), \quad \text{with} \quad \omega = \omega_0 + \dot{\omega} t \tag{11} \]
defines the entire phase response of the RF path, which is independent of the propagation delay \( \tau \). Since
\[ \lim_{\tau \to 0} \frac{\epsilon(t) - \epsilon(t - \tau)}{\tau} = \frac{d}{dt} \epsilon(t). \tag{12} \]
is applicable, the second parameter \( \epsilon_1(t) \) is
\[ \epsilon_1(t) = \frac{\arg \left( H_{if, \dot{\omega}}(j\omega) \right) - \frac{d}{dt} \epsilon(t)}{\tau}, \quad \text{with} \quad \omega = \omega_0 + \dot{\omega} t \tag{13} \]
which summarizes nonlinearities of the frequency ramp and the distance-dependent RF path phase responses, due to molecular absorption lines. Hence, \( \epsilon_1(t) \tau \) is the deviation of the instantaneous phase and \( \epsilon_1(t) \) is the deviation of the instantaneous frequency in radians per second.

III. PROPOSED SIGNAL PROCESSING METHODS

A. Slow-Time Ranging

A basic method commonly used for slow-time range estimation using FMCW radar systems is calculating discrete Fourier transform (DFT) of the IF signal \( s_{if}[n] \), followed by a maximum peak detection within the magnitude spectrum. Nevertheless, this approach is very inaccurate and imprecise. Zero padding or parabolic interpolation is commonly used to improve the spectral estimation. These simple treatments are just as imprecise and do not consider radar hardware impairments or effects through near-field wave propagation, as elaborated in Section II-B. In the following, we introduce an enhanced procedure for sinusoidal parameter and range estimation, which is then used together with the signal processing approach for fast-time vibrometry, as shown in Section III-B. The slow-time procedure is similar to the enhanced phase evaluation, which we have introduced in [25]. We implemented changes in the RF path compensation and absolute phase estimation, which reduces the computational load. The signal processing method combines a time-domain impulse measurement principle (IF evaluation) with an interferometric measurement principle (IF signal zero-phase evaluation). It allows a very precise and unambiguous range estimation with the highest accuracy.

Fig. 6 roughly shows the algorithm for slow-time sinusoidal parameter and range estimation. First, the time-centered fast Fourier transform (FFT), a calculation-optimized version of the DFT, is
\[ s_{if}[n] = \left\{ \begin{array}{ll} \text{FFT}\{s_{if}[n + N/2] \cdot w[n]\}, & n = 0, 1, \ldots, N/2 - 1 \\ \text{FFT}\{s_{if}[n - N/2] \cdot w[n]\}, & n = N/2, \ldots, N - 1 \end{array} \right. \tag{14} \]

5The magnitude spectrum of the FMCW IF signal is approximately the time-domain radar impulse response.
These frequency estimates are combined upchirp and downchirp, assuming triangular frequency chirps. Evaluation steps are repeated twice for each adjacent pair of the slow-time range estimation becomes

\[ R = \frac{\phi_{un}}{\omega_0} \cdot \frac{c}{2}. \]  

The Cramér–Rao bound (CRB) expresses a lower bound for the efficiency of unbiased estimators \( \text{CRB}(\cdot) \leq \text{Var}(\cdot) \). Assuming that \( s_{ij}[n] \) is embedded in white Gaussian noise, then the CRB for phase evaluation is (see [25])

\[ \text{CRB} = \frac{c^2}{4\eta N \omega_0^2} \]  

where \( \eta \) is the linear signal-to-noise ratio (SNR). Considering [25], the CRB is \( 12/\eta^2 \) times smaller than the generally used frequency evaluation with maximum peak detection in the spectrum. The parameter \( B_{rel} = B/\omega_0 \) is the relative bandwidth of the radar sensor.

B. Fast-Time Vibrometry

Vibrometry using FMCW radar with spectral methods in slow-time is fundamentally limited by the ramp-repetition rate \( 1/T \). This means that the maximum measurable frequency using slow-time measurement is \( 1/(2T) \) and thus very low. This article introduces a signal processing approach for fast-time vibrometry using the FMCW radar to overcome the paradigm of signal processing within the slow-time range profile. Assuming a radar target in the distance \( R_0 \), which is superimposed by an oscillation with a frequency \( \omega_{\text{vib}} \) and an amplitude \( \Delta R \)

\[ R(t) = R_0 + \Delta R \sin(\omega_{\text{vib}}t). \]  

The target vibration is assumed to be radially oriented toward the radar. According to (4), the simplified IF signal is

\[ s_i(t) = \tilde{s}_i(t) \cdot \exp \left( jhB_{\text{rel}} \sin(\omega_{\text{vib}}t) \frac{t}{T} \pm jh \sin(\omega_{\text{vib}}t) \right) \]  

with

\[ \tilde{s}_i(t) = \exp \left( j \frac{2\phi R_0}{c} \pm j \frac{2\phi_0 R_0}{c} \right) \]  

and

\[ h = \frac{2\omega_0 \Delta R}{c}. \]  

The IF signal \( s_i(t) \) is frequency modulated, where \( \tilde{s}_i(t) \) is the carrier signal and \( h \) is the modulation index known from the communication theory. The first term \( \exp(jhB_{\text{rel}} \sin(\omega_{\text{vib}}t)/T) \) in (23) is negligible. Hence, the equation is reduced to

\[ s_i(t) = \tilde{s}_i(t) \cdot \exp \left( \pm jh \sin(\omega_{\text{vib}}t) \right). \]  

Fig. 7 shows the signal processing chain for fast-time vibrometry, which consists of a two-step self-calibration procedure, which significantly improves the quality of measurement.

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\( \pi \)

\( \omega_0 \)

\( \omega \)

\( s_{ij}[n] \)

\( \phi_{un} \)

\( \phi \)

\( \text{Var}(\cdot) \)

\( \text{CRB}(\cdot) \)

\( B_{rel} = B/\omega_0 \)

\( \omega_{\text{vib}} \)

\( \Delta R \)

\( R(t) \)

\( R_0 \)

\( \tilde{s}_i(t) \)

\( h \)

\( B_{\text{rel}} \)

\( \phi R_0 \)

\( \phi_0 R_0 \)
Instead of evaluating the spectrum within the entire sweep time, each fast-time sampling point of the instantaneous phase 
\[ \phi[n] = \arg \left\{ \frac{s[n] - s_{\text{cal}}[n]}{2 \cdot s_{\text{cal}}[n]} \right\} \]  
\[ \text{(27)} \]
is processed individually. Equation (27) denotes the entire procedure for estimating the instantaneous phase \( \phi[n] \), of which the details are as follows.

1) **Reference Signal and Frequency Conversion**: Fast-time measurement requires a reference signal
\[ s_{\text{ef}}[n] = A \cdot A_v[n] \cdot \exp\left\{ j(\omega t_n + \phi - \epsilon_0[n] \pm \epsilon_1[n] \tau) \right\} \]  
\[ \text{(28)} \]
used for frequency conversion similar to the carrier signal within frequency modulation in communication technology. The sinusoidal parameters \( A, \omega, \phi, \) and the propagation delay \( \tau \) are estimated in a prior slow-time measurement, and \( A_v[n], \epsilon_0[n], \) and \( \epsilon_1[n] \) are determined within a calibration process described later in Section III-C. The variable \( t_n \) subsumes
\[ t_n = \frac{n}{f_s} - \frac{T}{2}, \quad n = 0, 1, \ldots, N - 1. \]  
\[ \text{(29)} \]

2) **Single-Sided Range Gate**: A range gate is applied to restrain unwanted interfering targets and to increase the SNR by suppressing unneeded frequency components in the signal. Range gating is realized by multiplying the signal with the single-sided range gate function
\[ r_g[n] \rightarrow R:\!G[n] = \begin{cases} 1, & \forall |\omega - 2\pi f_n| \leq \frac{\omega_{\text{rg}}}{2} \\ 0, & \text{otherwise} \end{cases} \]  
\[ \text{(30)} \]
in the frequency domain, where the variable \( f_n \) is
\[ f_n = f_s \cdot \frac{n}{N}, \quad n = 0, 1, \ldots, N - 1. \]  
\[ \text{(31)} \]
The single-sided range gate is a rectangular function shifted to \( \omega \) with an IF bandwidth of \( \omega_{\text{rg}} \), which only passes positive frequency components. According to the Euler identity, leaving out the negative frequencies results in a complex-valued analytic signal, which is necessary to derive the instantaneous phase of the time-domain signal by applying the argument function \( \arg\{\cdot\} \). The IF signal is frequency modulated, resulting in sidebands. For sinusoidal modulating signals, the spectrum of these sidebands can be expressed as a Fourier series
\[ \exp\left( jh \sin(\omega_{\text{vib}}t) \right) = \sum_{n=-\infty}^{\infty} J_n(h) \cdot \exp(jn\omega_{\text{vib}}t) \]  
\[ \text{(32)} \]
where \( J_n(\cdot) \) is the Bessel function first kind of order \( n \). With Carson’s bandwidth rule [48], a lower bound for the minimum required IF bandwidth of the range gate \( \omega_{\text{rg}} \) to detect a certain vibration frequency \( \omega_{\text{vib}} \)
\[ \omega_{\text{rg}} \geq 2\omega_{\text{vib}} \cdot (h + 1) \]  
\[ \text{(33)} \]
is derived. The inequality is then reduced to
\[ \omega_{\text{rg}} \geq 2\omega_{\text{vib}} \]  
\[ \text{(34)} \]
for a small modulation index \( h \). Thus, the width of the range gate in meters is
\[ R_{\text{rg}} \geq \omega_{\text{vib}} \cdot \frac{c}{\omega_{\text{rg}}} \]  
\[ \text{(35)} \]
which decreases by increasing the bandwidth \( B \) or reducing the sweeping time \( T \) of the frequency ramp. Range gating, i.e., limitation of the frequency components, leads to a significant error in the time domain. Edge effects or settling effects on the instantaneous phase are the consequence. Fig. 8 shows the maximum error of the instantaneous phase due to edge effects, considering the IF bandwidth of the range gate in Hz \( f_{\text{rg}} \) and the modulation index \( h \). For each frequency ramp, at least the first and the last 0.5 \( f_s/f_{\text{rg}} \) samples of the instantaneous phase data should be discarded.

3) **Continuous Empty-Room Self-Calibration**: A continuous self-calibration method for the empty room is necessary to separate the wanted signal of the main target from surrounding interfering targets within the range gate. It must continuously adapt to the changing environment without suppressing the wanted signal. Fig. 9(a) shows the flow diagram for the continuous calculation of the empty-room calibration signal
\[ s_{\text{cal},m}[n] = s_{\text{cal},m-1}[n] \cdot a + (s_{m-1}[n] - s_{\text{ref},m-1}[n]) \cdot (1 - a) \]  
\[ \text{(36)} \]
C. Estimation of the Calibration Parameters

In this section, we demonstrate an approach to estimate the parameters \( \epsilon_0[n] \), \( \epsilon_1[n] \), and \( A_t[n] \), as introduced in Section II-B. These variations from the ideal sinusoidal IF signal cause impairments of the peaks in the range profile. The parameters are crucial because they lead to inaccurate estimates in slow-time ranging or variations within the instantaneous phase in the fast-time vibrometry procedure. Since RF linearity measurements of ultrawideband radar systems are hard to carry out, our approach can also be used for estimating nonlinearities of the frequency ramp.

Fig. 10 shows the entire procedure for estimating the calibration parameters. A calibration parameter measurement series consists of \( M \) measurements that need to be executed consecutively with a single radar target at \( M \) arbitrary varying distances. The calibration signal is

\[
\mathbb{g}[n] = \text{sg}[n] \ast \text{rg}[n] \exp(j(\omega_{0n} + \phi))
\]

(39)

where the particular IF signals are range-gated and frequency-converted, as described in Section II-B. The system of linear equations is

\[
\epsilon_0 + \epsilon_1 \cdot \tau^T = E
\]

(40)

where the regressand vectors \( \epsilon_0 \) and \( \epsilon_1 \) are

\[
\epsilon_0 = [\epsilon_0[0], \ldots, \epsilon_0[N - 1]]^T
\]

(41)

and

\[
\epsilon_1 = [\epsilon_1[0], \ldots, \epsilon_1[N - 1]]^T
\]

(42)

with the parameter vector

\[
\tau = [\tau_0, \ldots, \tau_{M-1}]^T
\]

(43)

The parameter \( \tau_m \) is the propagation delay at the \( m \)th measurement point. The regressor matrix \( E \) containing the observations is

\[
E = \begin{bmatrix}
\phi_0[0] & \ldots & \phi_{M-1}[0] \\
\vdots & \ddots & \vdots \\
\phi_0[N - 1] & \ldots & \phi_{M-1}[N - 1]
\end{bmatrix}
\]

(44)

where the input variables \( \phi_m[n] \) denote the unwrapped instantaneous phase \( \text{arg}(\mathbb{g}_m[n]) \) of the \( m \)th measurement within the data set. To ensure the coherency of the unwrapping processes between the instantaneous phases over the entire data set, we recommend unwrapping by starting from the centered bin. This procedure ensures higher robustness against edge effects, as explained in Section III-B. The regressand variables \( \epsilon_0[n] \) and \( \epsilon_1[n] \) are then estimated, e.g., by linear regression.

Besides phase modulation, amplitude modulation is another primary source of radar imperfections. Especially for ultrawideband radar, this impairment becomes critical, e.g., due to a significant variation of the impedance by a mismatched antenna. The amplitude modulation is calculated as

\[
A_t[n] = \frac{1}{M} \sum_{m=0}^{M-1} \frac{|\mathbb{g}_m[n]|}{A_m}
\]

(45)

where the envelopes of the IF signals \( |\mathbb{g}_m[n]| \) are divided by the respective amplitudes \( A_m \), which are measured in slow-time before.

IV. EXPERIMENTAL RESULTS

A. Utilized Hardware and Calibration

As a test platform, we utilized three different radar systems, within a span from 68 to 250 GHz. These miniaturized ultrawideband FMCW radar systems are different in many ways. Thus, they have different impairments, which makes them
perfectly suitable for demonstrating the calibration and measurement procedures. Table I shows the parameters of the radar sensors, whose RF hardware was developed by our group [23], [43], [44] using custom monolithic microwave integrated circuits (MMICs) in silicon–germanium (SiGe) technology. All sensors utilize an offset PLL technology for frequency synthesis. This technique provides the lowest phase noise and an excellent linearization of the PLL, resulting in especially linear frequency ramps. A low-noise oven-controlled crystal oscillator (OCXO) was used as a reference oscillator, which is highly frequency stable. Sensor #3 uses two bistatic on-chip antennas. Due to their close proximity and their shared lens, the system is considered as quasi-monostatic.

The chirp rate, which is the measurement rate in slow time, was 110 Hz, whereas the sampling rate was 1 MHz, which is the measurement rate in fast time. Considering the Nyquist–Shannon sampling theorem, the upper frequency limit for fast-time vibrometry is 0.5 MHz. However, it will be shown that the available SNR reduces this boundary. In contrast to fast-time vibrometry, the settling behavior of the PLLs limits the chirp rate and, hence, the frequency limit for slow-time vibrometry. An increase in the chirp rate would cause that the PLLs are no longer in lock.

Fig. 11 shows a photograph of the experimental setup. We used an aluminum disk with a diameter of 80 mm, attached to a vibration actuator, as a radar target. Moreover, the assembly is situated on a tilted bar, which is mounted onto a linear track. The linear track has a working range of at least 5 m. The vibration actuator is a 5-W voice coil, which can produce sounds at a moderate volume. The linear track has a positioning repeatability of about 10 μm. The uncalibrated and calibrated echoes of the sensors are illustrated for demonstration. As shown in Fig. 12(n) and (o), the Y-band radar echo shows false pulses at half a distance due to parasitic radiation of the half transmit frequency [44].

The amplitude, hence the SNR of sensor #3, varies significantly with respect to the transmit frequency, as shown in Fig. 12(k). Fig. 12(b) shows the propagation delay-independent RF phase response $0[n]$ of the first sensor, which is mainly because of the highly dispersive waveguide used. In contrast to sensor #1, ringing due to internal reflections is the majority of $0[n]$ from sensor #2 and #3 [see Fig. 12(g), and (l)]. According to [45], the worst case variation of the dispersive delay due to absorption lines by gas molecules for the frequency range of the third sensor is less than 10 ps/km. This would cause a worst case error $1[n]$ not more than 0.75 MHz, fewer than shown in Fig. 12(m). The variation of the dispersive delay in the frequency range of sensor #1 and #2 is negligible. Therefore, we suppose that $1[n]$ [see Fig. 12(c), (h), and (m)] is an estimation for the nonlinearities of the frequency synthesizer.

Fig. 13 shows the measured noise floor in the vibration spectrum without any averaging to demonstrate the lower detection limit of each radar system at the given configuration. As known from the communication technology, there is a threshold effect for frequency modulation, which describes a significantly decreasing SNR if the carrier-to-noise ratio falls in front of the sensors to ensure a point scatterer. At each calibration point, 50 upchirp and downchirp measurements were coherently averaged. The range gate was 10 kHz, which provides sufficient IF bandwidth to demonstrate the sensor impairments. This thorough procedure serves to illustrate the limits of the radar systems and can be significantly reduced in practical use by a factory preset. The figure is subdivided into calibration plots for each radar sensor, which shows $A[n]$, $0[n]$, and $1[n]$. The uncalibrated and calibrated echoes of the sensors are illustrated for demonstration. As shown in Fig. 12(n) and (o), the Y-band radar echo shows false pulses at half a distance due to parasitic radiation of the half transmit frequency [44].

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**Fig. 11.** Photograph of the experimental setup used to validate the proposed signal processing and calibration methods. We consecutively used three different mmWave radar sensors within the W-, D-, and Y-bands.

**Fig. 12.** Shows the achieved calibrations for all sensors. The radar target was placed with an increment of 2.5 mm along the axis. We decided to limit the travel range from 1.5 to 5 m in front of the sensors to ensure a point scatterer. At each calibration point, 50 upchirp and downchirp measurements were coherently averaged. The range gate was 10 kHz, which provides sufficient IF bandwidth to demonstrate the sensor impairments. This thorough procedure serves to illustrate the limits of the radar systems and can be significantly reduced in practical use by a factory preset. The figure is subdivided into calibration plots for each radar sensor, which shows $A[n]$, $0[n]$, and $1[n]$. The uncalibrated and calibrated echoes of the sensors are illustrated for demonstration. As shown in Fig. 12(n) and (o), the Y-band radar echo shows false pulses at half a distance due to parasitic radiation of the half transmit frequency [44].

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**TABLE I**

**Parameters of the Radar Sensors**

| Sensor | #1 | #2 | #3 |
|--------|----|----|----|
| Frequency range (GHz) | 68...92 | 124...169 | 200...250 |
| (W-band) | (D-band) | (Y-band) |
| Bandwidth (GHz) | 24 | 45 | 50 |
| Chirp rate (Hz) | 110 |
| Max. TX power (dBm) | -4 | -10 | 1.8 |
| Phase noise (dBc/Hz) | -85 | -80 | -75.6 |
| Antenna | 28 dBi lens | 25 dBi | 35 dBi lens |
| WR-10 WG | rect. horn | bistatic |
| Operation mode | LFM CW |
| Sampling rate | 1 MS/s |
| Reference oscillator | OCXO (-105 dBc/Hz @ 10 Hz) |
| Reference | [43], [51] | [23] | [44], [52] |

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*a* Sensor exceeds lower W-band specification by 7 GHz

*b* Measurement rate in slow-time

*c* Power at waveguide port / patch antenna EIRP

*d* at 10 kHz offset to the center frequency

*e* Measurement rate in fast-time
Fig. 12. Identified calibrations of the three radar sensors. The respective upper illustrations show $A_c$, $\epsilon_0$, and $\epsilon_1$, whereas the lower illustrations show the normalized IF power spectrum before and after applying the calibration data. (a) Measured $A_c$ calibration of sensor #1. (b) Measured $\epsilon_0$ calibration of sensor #1. (c) Measured $\epsilon_1$ calibration of sensor #1. (d) Uncalibrated IF spectrum of sensor #1. (e) Calibrated IF spectrum of sensor #1. (f) Measured $A_c$ calibration of sensor #2. (g) Measured $\epsilon_0$ calibration of sensor #2. (h) Measured $\epsilon_1$ calibration of sensor #2. (i) Uncalibrated IF spectrum of sensor #2. (j) Calibrated IF spectrum of sensor #2. (k) Measured $A_c$ calibration of sensor #3. (l) Measured $\epsilon_0$ calibration of sensor #3. (m) Measured $\epsilon_1$ calibration of sensor #3. (n) Uncalibrated IF spectrum of sensor #3. (o) Calibrated IF spectrum of sensor #3.
below a certain point. Due to the very low SNR, i.e., carrier-to-noise ratio, of the third radar sensor for the lower frequency range [see Fig. 12(k)], the respective noise floor is poor (see Sensor #3.1 in Fig. 13). Therefore, in the following, we reduced the sampling points of $R[n]$ to fit the RF range from 215 to 250 GHz. As a consequence, the noise floor could be improved significantly (see sensor #3.2 in Fig. 13). Since the shape of the radar target changes the RCS and, hence, the SNR, similar to a change in distance, it is unnecessary to investigate differently shaped radar targets in the subsequent experiments.

B. Vibrometry of a Fixed Radar Target

We did three vibrometry measurements without underlying large-scale motion. The radar targets were positioned at 1.3 m for sensor #1 and 0.5 m for sensor #2 and #3, thus providing sufficient SNR, according to Fig. 13. Reducing the range from the radar to the target would further limit the maximum IF bandwidth of the range gate and, hence, the upper limit of the detectable vibration frequency. A sequence of sinusoidal chirped vibrations was executed. The sequence took about 12 s and was within a frequency range of 300 Hz–16 kHz, covering the entire audio signal range. Sweeping through a single tone is sufficient to validate the proposed methods since a Fourier decomposition can represent complex vibrations as a sum of multitone.

Fig. 14 shows the experimental results. The entire frequency sweep is clearly visible. The noise floor is at 30 nm for the first and 50 nm for the second and third sensors. The figure shows the doubled-frequency components due to the harmonics of the audio power amplifier, which was used as a signal source. Smearing along the time axis, which appears with broadband events, e.g., the sudden shutdown of the vibration actuator, is due to the phase self-calibration. The mirrored ghost vibrations within Fig. 14(b) and (c) occurred because of multiple reflections, which are caused by ringing of the electromagnetic wave between the antenna and the target. Due to the short distance, the range gate cannot suppress these multiple reflections. These reflections are interfering targets, which are vibrating just as the main target. Consequently, they do not cancel out with the empty-room self-calibration.

![Fig. 13. Measured noise floor within the vibrometry spectrum of the three radar sensors. Sensor #3.1 and #3.2 belong to sensor #3 with a full or reduced number of sampling points to overcome the threshold effect for frequency modulation.](image)

![Fig. 14. Experimental results of the vibrometry measurements of a stationary radar target. The range of the vibration frequency is 300 Hz–16 kHz. (a) Fast-time vibrometry using sensor #1 with a radar target in $r = 1.3$ m. (b) Fast-time vibrometry using sensor #2 with a radar target in $r = 0.5$ m. (c) Fast-time vibrometry using sensor #3 with a radar target in $r = 0.5$ m.](image)

![Fig. 15. Frequency response of the vibration actuator, which is obtained from the experimental result in Fig. 14. The resonance frequencies of the vibration actuator are evident.](image)

![Fig. 15. Frequency response of the vibration actuator. The frequency response was extracted from the linear chirp as shown in Fig. 14(a), where sensor #1 was used, which has the highest SNR. A highly nonlinear curve with resonance frequencies at 2.7 and 11.6 kHz is evident.](image)

C. Vibrometry of a Radar Target in Motion

This section demonstrates the ability of the measurement approach to simultaneously detect vibrations in fast time together with an underlying large-scale motion. The vibration sequence was a triangular, followed by sinusoidal chirp, which took about 26 s in total. Compared with the previous experimental series, the upper limit of the frequency range
Fig. 16. Experimental results for the vibrating radar target with an underlying large-scale motion. The upper plots show the slow time, whereas the lower plots show the fast-time measurement results. For slow-time ranging, the proposed phase evaluation approach was used. For velocity estimation, we used the Doppler frequency, which leads to a significant error for accelerated movements. (a) Slow-time ranging with sensor #1. (b) Fast-time vibrometry with sensor #1. (c) Slow-time ranging with sensor #2. (d) Fast-time vibrometry with sensor #2. (e) Slow-time ranging with sensor #3. (f) Fast-time vibrometry with sensor #3.
was reduced to 10 kHz. For sensor #1, the range of motion was within 4 m, whereas the velocity was up to 0.4 m/s. Again, to meet the SNR requirements, the range of motion for the second and third sensors was reduced to 1 m. Fig. 16 shows the measurement results. Fig. 16(a), (c), and (e) shows the results of slow-time ranging and velocity estimation, whereas Fig. 16(b), (d), and (f) shows the results of fast-time vibrometry. For slow-time ranging, phase evaluation was used, whereas the velocity was estimated using the Doppler estimation.

Accelerated movements highly disturb the Doppler estimation. However, using the proposed slow-time phase evaluation, the range measurements are not affected by this. Regarding vibrometry, the sequence of vibrations is clearly visible even if the radar target is within a fast movement. During motion, blurring is due to an insufficient continuous phase self-calibration and underlying vibrations by the moving linear actor. Fig. 16(b) shows a disturbing target within 20–25 s. This artifact is due to an imperfect empty-room self-calibration. Interfering targets occur as lines at frequencies, matching the IF signal frequency deviation between the main and the interfering target. Interfering targets occur as lines at frequencies, matching the IF signal frequency deviation between the main and the interfering target. Fig. 17 further shows the effect of the individual calibration mechanism. It shows a detailed view of the vibrometry plot of the first sensor with a moving radar target. The figure is divided into four subfigures, where specific calibration mechanisms are inactivated.

V. CONCLUSION

In this article, we proposed a sophisticated error model for highly accurate sensing using FMCW radar systems. We introduced a combined measurement approach for spatially resolved vibrometry and ranging of vibrating radar targets with underlying large-scale motions. Our novel signal processing chain in fast time allows sensing of significantly higher vibration frequencies than the chirp rate. Several simultaneously vibrating targets or interfering targets could be unambiguously separated in the range domain in contrast to traditional CW radar sensors. Multiple calibration techniques were introduced to overcome the modeled imperfections in order to achieve these features.

We demonstrated the proposed techniques using three ultra-wideband mmWave FMCW radar systems in the W-, D-, and Y-bands. The experimental results are shown in Table II. Vibrations up to 16 kHz with a displacement of a minimum of 30 nm could be measured, with a fixed radar target. Frequencies of at least 10 kHz were measured with a moving radar target of up to 0.4 m/s using the W-band sensor. We want to emphasize that with the commonly used slow-time measurements, only vibrations with frequencies up to 55 Hz could have been detected with the radar systems used. Consequently, we improved the upper detection limit for the vibration frequency by several orders of magnitude, which is outstanding in terms of spatially resolved vibrometry using FMCW radar systems.

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TABLE II

SUMMARY OF THE EXPERIMENTAL RESULTS

| Sensor | #1 | #2 | #3 |
|--------|----|----|----|
| Max. detectable vibration frequency (kHz) | 55 |
| Proposed Fast-Time Vibrometry<sup>a</sup> |
| Distance (m) | 1.3 | 0.5 | 0.5 |
| Min. detectable vibration displacement (nm) | 30 | 50 | 50 |
| Measured vibration frequency (kHz) | 16 |
| Proposed Fast-Time Vibrometry<sup>b</sup> |
| Distance (m) | 2–4 | 0.5–1 | 0.5–1 |
| Large-scale motion (m/s) | 0.4 | 0.1 | 0.1 |
| Measured vibration frequency (kHz) | 10 |

<sup>a</sup> Fixed radar target
<sup>b</sup> Radar target in motion
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