Measurement of Branching Fractions and Charge Asymmetries in $B^\pm \to \rho^\pm \pi^0$ and $B^\pm \to \rho^0 \pi^\pm$ Decays, and Search for $B^0 \to \rho^0 \pi^0$

B. Aubert,1 R. Barate,1 D. Boutigny,1 F. Couderc,1 J.-M. Gaillard,1 A. Hicheur,1 Y. Karyotakis,1 J. P. Lees,1 P. Robbe,1 V. Tisserand,1 A. Zghiche,1 A. Palano,2 A. Pompili,2 J. C. Chen,3 N. D. Qi,3 G. Rong,3 P. Wang,3 Y. S. Zhu,4 G. Eigen,4 I. Ofte,4 B. Stugu,4 G. S. Abrams,5 A. W. Borgland,5 A. B. Breon,5 D. N. Brown,5 J. Button-Shafer,5 R. N. Cahn,5 E. Charles,5 C. T. Day,5 M. S. Gill,5 A. V. Gritsan,5 Y. Groyzman,5 R. G. Jacobsen,5 R. W. Kadel,5 J. Kadyk,5 L. T. Kerth,5 Yu. G. Kolomensky,5 G. Kukartsev,5 C. LeClere,5 M. E. Levi,5 G. Lynch,5 L. M. Mir,5 P. J. Oddone,5 T. J. Orimoto,5 M. Pripstein,5 N. A. Roe,5 A. Rosmosan,5 M. T. Ronan,5 V. G. Shelkov,5 A. V. Telnov,5 W. A. Wenzel,5 K. Ford,6 T. J. Harrison,6 C. M. Hawkes,6 D. J. Knowles,6 S. E. Morgan,6 R. C. Penny,6 A. T. Watson,6 K. Goetzem,7 T. Held,7 H. Koch,7 B. Lewandowski,7 M. Pelizaueas,7 K. Peters,7 H. Schmuckel,7 M. Steinke,7 J. T. Boyd,8 N. Chevalier,8 W. N. Cottingham,8 M. P. Kelly,8 T. E. Latham,8 C. Mackay,8 F. F. Wilson,8 K. Abe,1 T. Cuhadar-Donszelmann,9 C. Hearty,9 T. S. Mattison,9 J. A. McKenna,9 D. Thiessen,9 P. Kyberd,10 A. K. McKemey,10 L. Teodorescu,10 V. E. Blinov,11 A. D. Bukan,11 V. B. Golubev,11 V. N. Ivanchenko,11 E. A. Kravchenko,11 A. P. Onuchin,11 S. I. Serednyakov,11 Yu. I. Skovpen,11 E. P. Solodov,11 A. N. Yushkov,11 D. Best,12 M. Brunschma,12 M. Chao,12 D. Kirkby,12 A. J. Lankford,15 M. Mandelkern,12 R. K. Mommsen,12 W. Roethel,12 D. P. Stoker,12 C. Buchanan,13 B. L. Hartfiel,13 J. W. Gary,14 J. L. Layter,14 B. C. Shen,14 K. Wang,14 D. del Re,15 H. K. Hadvabwand,15 E. J. Hill,15 D. B. MacFarlane,15 H. P. Paar,15 Sh. Rahatlou,15 V. Sharma,15 J. W. Berryhill,16 C. Campagnar,16 B. Dahmes,16 S. L. Levy,16 O. Long,16 A. M. Mazur,16 J. D. Richelm,16 W. Verkerke,16 T. W. Beck,17 J. Beringer,17 A. M. Eisen,17 C. A. Heusch,17 W. S. Lockman,17 T. Schalk,17 R. E. Schmirtz,17 B. A. Schumm,17 A. Seiden,17 P. Spradlin,17 M. Turri,17 W. Walkowiak,17 D. C. Williams,17 M. G. Wilson,17 J. Albert,18 E. Chen,18 G. P. Dubois-Felsmann,18 A. Dvoretzki,18 R. J. Erwin,18 D. G. Hitlin,18 I. Narsky,18 T. Piatenko,18 F. C. Porter,18 A. Ryd,18 A. Samuel,18 S. Yang,18 S. Jayatilleke,19 G. Mancinelli,19 B. T. Meadows,19 M. D. Sokoloff,19 T. Abe,20 F. Blanc,20 P. Bloom,20 S. Chen,20 P. J. Clark,20 W. T. Ford,20 U. Nauenberg,20 A. Olivas,20 P. Rankin,20 J. Roy,20 J. G. Smith,20 W. C. van Hoek,20 L. Zhang,20 J. L. Harton,21 T. Hu,21 A. Soffer,21 W. H. Toki,21 R. J. Wilson,21 J. Zhang,21 D. Altenburg,22 T. Brandt,22 J. Brose,22 T. Colberg,22 M. Dickopp,22 R. S. Dubitzky,22 A. Hauke,22 H. M. Lackner,22 E. Maly,22 R. Müller-Pfefferkorn,22 R. Rogowski,22 S. Otto,22 J. Schubert,22 K. R. Schubert,22 R. Schwierz,22 B. Spaan,22 L. Wilden,22 D. Bernard,23 G. R. Bonneauad,23 F. Brochard,23 J. Cohen-Tanugi,23 P. Grenier,23 Ch. Thiebaut,23 G. Vigen,23 M. Verderi,23 A. Khan,24 D. Lavin,24 F. Muheim,24 S. Playfer,24 J. E. Swain,24 M. Andreotti,25 V. Azzolini,25 D. Bettoni,25 C. Bozzi,25 R. Calabrese,25 G. Cibinetto,25 E. Luppi,25 M. Negrini,25 L. Piemontese,25 A. Sarti,25 E. Treadwell,26 R. Baldini-Ferroli27 K. Alcacer,27 R. de Sangro,27 F. Dallacasa,27 G. Finocchiari,27 P. Patteri,27 M. Piccolo,27 A. Zallo,27 A. Buzzo,28 R. Capra,28 R. Conti,28 G. Crosetti,28 M. Lo Vetere,28 M. Macri,28 M. R. Monge,28 S. Passaggio,28 C. Patrignani,28 E. Robutti,28 A. Santon,28 S. Tosi,28 S. Bailey,29 M. Morii,29 E. Von,29 W. Bhimji,30 D. A. Bowmerman,30 P. D. Dauncey,30 U. Egede,30 I. Eschrich,30 J. R. Gaillard,30 G. W. Morton,30 J. A. Nash,30 G. P. Taylor,30 G. J. Grenier,31 S.-J. Lee,31 U. Mallick,31 J. Cochran,32 H. B. Crawley,32 J. Lamsa,32 W. T. Meyer,32 S. Prell,32 E. I. Rosenberg,32 J. Yi,32 M. Davier,33 G. Grosdidier,33 A. Hörk,33 S. Laplace,33 F. Le Diberder,33 V. Lefeltier,33 A. M. Lutz,33 T. C. Petersen,33 S. Plaszczynski,33 M. H. Schune,33 L. Tantot,33 G. Wormser,33 V. Briljevic,33 C. H. Cheng,34 D. J. Lange,34 M. C. Simani,34 D. M. Wright,34 A. J. Bevan,35 J. P. Coleman,35 J. R. Fry,35 E. Gabathuler,35 R. Gamet,35 M. Kay,35 R. J. Parry,35 D. J. Payne,35 R. J. Sloane,35 C. Touramanis,35 J. J. Back,36 P. F. Harrison,36 H. W. Shorthouse,36 P. B. Vidal,36 C. L. Brown,37 G. Cowan,37 R. L. Flack,37 H. U. Flachaer,37 S. George,37 M. G. Green,37 A. Kurup,37 C. E. Marker,37 T. R. McMahon,37 S. Ricciardi,37 F. Salvatore,37 G. Vaitasas,37 M. A. Winter,37 D. Brown,38 C. L. Davis,38 J. Allison,39 N. R. Barlow,39 R. J. Barlow,39 P. A. Hart,39 M. C. Hodgkinson,39 F. Jackson,39 G. D. Lafferty,39 A. J. Lyon,39 J. H. Weatherall,39 J. C. Williams,39 A. Farbin,40 D. Kovalskyi,40 C. K. Lae,40 V. Lillard,40 D. A. Roberts,40 G. Blaylock,41 C. Dallapiccola,41 K. T. Flood,41 S. S. Hertzbach,41 R. Kofer,41 V. B. Koptchev,41 T. B. Moore,41
L. Gladney, E. O. Olaiya, E. Grauges-Pous, R. Bartoldus, M. Rotondo, J. L. Ritchie, A. J. R. Weinstein, R. Prepost, G. Vasseur, P. David, M. L. Kocian, P. Poropat, H. Marsiske, D. Dujmic, D. Cote-Ahern, W. J. Wisniewski, N. Danielson, R. Aleksan, L. Vitale, W. Dunwoodie, J. H. von Wimmersperg-Toeller, J. Stelzer, M. Zito, M. Haire, D. H. Wright, C. C. Young, D. W. G. S. Leith, F. Di Lodovico, D. A. Sanders, S. Willocq, F. E. Elsen, O. L. Buchmueller, M. J. J. John, J. A. Ernst, K. Paick, M. K. Sullivan, H. Kim, M. Weaver, S. Ahmed, F. Taylor, M. Bona, G. S. Pal, M. J. J. John, R. S. Panvini, A. J. S. Smith, T. A. Gabriel, F. Simonetto, R. Swick, M. Haire, J. Walsh, J. Swanson, J. A. Ernst, M. Allton, G. Zeller.
16 University of California at Santa Barbara, Santa Barbara, CA 93106, USA
17 University of California at Santa Cruz, Institute for Particle Physics, Santa Cruz, CA 95064, USA
18 California Institute of Technology, Pasadena, CA 91125, USA
19 University of Cincinnati, Cincinnati, OH 45221, USA
20 University of Colorado, Boulder, CO 80309, USA
21 Colorado State University, Fort Collins, CO 80523, USA
22 Technische Universität Dresden, Institut für Kern- und Teilchenphysik, D-01062 Dresden, Germany
23 Ecole Polytechnique, LLR, F-91128 Palaiseau, France
24 University of Edinburgh, Edinburgh EH9 3JZ, United Kingdom
25 Università di Ferrara, Dipartimento di Fisica and INFN, I-44100 Ferrara, Italy
26 Florida A&M University, Tallahassee, FL 32307, USA
27 Laboratori Nazionali di Frascati dell’INFN, I-00044 Frascati, Italy
28 Università di Genova, Dipartimento di Fisica e INFN, I-16146 Genova, Italy
29 Harvard University, Cambridge, MA 02138, USA
30 Imperial College London, London, SW7 2BW, United Kingdom
31 University of Iowa, Iowa City, IA 52242, USA
32 Iowa State University, Ames, IA 50011-3160, USA
33 Laboratoire de l’Accélérateur Linéaire, F-91898 Orsay, France
34 Lawrence Livermore National Laboratory, Livermore, CA 94550, USA
35 University of Michigan, Ann Arbor, MI 48109, USA
36 Queen Mary, University of London, E1 4NS, United Kingdom
37 University of London, Royal Holloway and Bedford New College, Egham, Surrey TW20 0EX, United Kingdom
38 University of Louisville, Louisville, KY 40292, USA
39 University of Manchester, Manchester M13 9PL, United Kingdom
40 University of Maryland, College Park, MD 20742, USA
41 University of Massachusetts, Amherst, MA 01003, USA
42 Massachusetts Institute of Technology, Laboratory for Nuclear Science, Cambridge, MA 02139, USA
43 McGill University, Montréal, QC, Canada H3A 2T8
44 Università di Milano, Dipartimento di Fisica and INFN, I-20133 Milano, Italy
45 University of Mississippi, University, MS 38677, USA
46 Université de Montréal, Laboratoire René J. A. Lévesque, Montréal, QC, Canada H3C 3J7
47 Mount Holyoke College, South Hadley, MA 01075, USA
48 Università di Napoli Federico II, Dipartimento di Scienze Fisiche e INFN, I-80126, Napoli, Italy
49 NIKHEF, National Institute for Nuclear Physics and High Energy Physics, NL-1009 DB Amsterdam, The Netherlands
50 University of Notre Dame, Notre Dame, IN 46556, USA
51 Oak Ridge National Laboratory, Oak Ridge, TN 37831, USA
52 Ohio State University, Columbus, OH 43210, USA
53 University of Oregon, Eugene, OR 97403, USA
54 Università di Padova, Dipartimento di Fisica and INFN, I-35131 Padova, Italy
55 Universités Paris VI et VII, Lab de Physique Nucléaire H. E., F-75252 Paris, France
56 Università di Pavia, Dipartimento di Elettronica e INFN, I-27100 Pavia, Italy
57 University of Pennsylvania, Philadelphia, PA 19104, USA
58 Università di Perugia and INFN, I-06100 Perugia, Italy
59 Università di Pisa, Dipartimento di Fisica, Scuola Normale Superiore and INFN, I-56127 Pisa, Italy
60 Prairie View A&M University, Prairie View, TX 77446, USA
61 Princeton University, Princeton, NJ 08544, USA
62 Università di Roma La Sapienza, Dipartimento di Fisica and INFN, I-00185 Roma, Italy
63 Universität Rostock, D-18051 Rostock, Germany
64 Rutherford Appleton Laboratory, Chilton, Didcot, Oxon, OX11 0QX, United Kingdom
65 DSM/Dapnia, CEA/Saclay, F-91191 Gif-sur-Yvette, France
66 University of South Carolina, Columbia, SC 29208, USA
67 Stanford Linear Accelerator Center, Stanford, CA 94309, USA
68 Stanford University, Stanford, CA 94305-4060, USA
69 State Univ. of New York, Albany, NY 12222, USA
70 University of Tennessee, Knoxville, TN 37996, USA
71 University of Texas at Austin, Austin, TX 78712, USA
72 University of Texas at Dallas, Richardson, TX 75083, USA
73 Università di Torino, Dipartimento di Fisica Sperimentale and INFN, I-10125 Torino, Italy
74 Università di Trieste, Dipartimento di Fisica e INFN, I-34127 Trieste, Italy
75 Vanderbilt University, Nashville, TN 37235, USA
76 University of Victoria, Victoria, BC, Canada V8W 3P6
77 University of Wisconsin, Madison, WI 53706, USA
78 Yale University, New Haven, CT 06511, USA

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We present measurements of branching fractions and charge asymmetries in $B$-meson decays to $\rho^+\pi^0$, $\rho^0\pi^+$ and $\rho^0\pi^0$. The data sample comprises $89 \times 10^6 T(4S) \rightarrow B\overline{B}$ decays collected with the BABAR detector at the PEP-II asymmetric-energy $B$ Factory at SLAC. We find the charge-averaged branching fractions $B(B^+ \rightarrow \rho^+\pi^0) = (10.9 \pm 1.9\,(\text{stat}) \pm 1.9\,(\text{syst})) \times 10^{-6}$ and $B(B^+ \rightarrow \rho^0\pi^+) = (9.5 \pm 1.1 \pm 0.8) \times 10^{-6}$, and we set a 90% confidence-level upper limit $B(B^0 \rightarrow \rho^0\pi^+) < 2.9 \times 10^{-6}$.

We measure the charge asymmetries $A_{CP}^{\rho^+\pi^0} = 0.24 \pm 0.16 \pm 0.06$ and $A_{CP}^{\rho^0\pi^+} = -0.19 \pm 0.11 \pm 0.02$.

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The study of $B$-meson decays into charmless hadronic final states plays an important role in the understanding of $CP$ violation in the $B$ system. Recently, the BABAR experiment performed a search for $CP$-violating asymmetries in neutral $B$ decays to $\rho^0\pi^\pm$ final states, where the mixing-induced $CP$ asymmetry is related to the angle $\alpha \equiv \arg [-V_{ub}V^*_{ub}/V_{ub}^*V_{ub}]$ of the Unitarity Triangle. The extraction of $\alpha$ from $\rho^0\pi^\pm$ is complicated by the interference of decay amplitudes with differing weak and strong phases. One strategy to overcome this problem is to perform an SU(2) analysis that uses all $\rho\pi$ final states. Assuming isospin symmetry, the angle $\alpha$ can be determined free of hadronic uncertainties from a pentagon relation formed in the complex plane by the five decay amplitudes $B^0 \rightarrow \rho^0\pi^\pm$, $B^0 \rightarrow \rho^0\pi^0$, $B^0 \rightarrow \rho^0\pi^0$, $B^+ \rightarrow \rho^0\pi^0$ and $B^+ \rightarrow \rho^0\pi^+$. These amplitudes can be determined from measurements of the corresponding decay rates and $CP$-asymmetries. The branching fractions have been measured for $B^0 \rightarrow \rho^0\pi^\pm$ and $B^0 \rightarrow \rho^0\pi^0$, and an upper limit has been set for $B^0 \rightarrow \rho^0\pi^0$.

In this letter we present measurements of the branching fractions of the decay modes $B^+ \rightarrow \rho^0\pi^0$ and $B^+ \rightarrow \rho^0\pi^+$, and a search for the decay $B^0 \rightarrow \rho^0\pi^0$. All three analyses follow a quasi-two-body approach. For the charged modes we also measure the charge asymmetry, defined as

$$A_{CP} \equiv \frac{\Gamma(B^- \rightarrow f) - \Gamma(B^+ \rightarrow \overline{f})}{\Gamma(B^- \rightarrow f) + \Gamma(B^+ \rightarrow \overline{f})},$$

where $f$ and $\overline{f}$ are the final state and its charge-conjugate, respectively.

The data used in this analysis were collected with the BABAR detector at the PEP-II asymmetric-energy $e^+e^-$ storage ring at SLAC. The sample consists of $(88.9 \pm 1.0) \times 10^6 B\overline{B}$ pairs collected at the $T(4S)$ resonance (“on-resonance”), and an integrated luminosity of 9.6 fb$^{-1}$ collected about 40 MeV below the $T(4S)$ (“off-resonance”).

Each signal $B$ candidate is reconstructed from three-pion final states that must be $\pi^+\pi^0\pi^0$, $\pi^+\pi^-\pi^+$, or $\pi^+\pi^-\pi^0$. Charged tracks must have ionization-energy loss and Cherenkov-angle signatures inconsistent with those expected for electrons, kaons, protons, or muons. The $\pi^0$ candidate must have a mass that satisfies $0.11 < m(\gamma\gamma) < 0.16\,\text{GeV}/c^2$, where each photon is required to have an energy greater than 50 MeV in the laboratory frame and to exhibit a lateral profile of energy deposition in the electromagnetic calorimeter consistent with an electromagnetic shower.

The mass of the reconstructed $\rho$ candidate must satisfy $0.4 < m(\pi^+\pi^0) < 1.3\,\text{GeV}/c^2$ for $\rho^+$ and $0.53 < m(\pi^+\pi^-) < 0.9\,\text{GeV}/c^2$ for $\rho^0$. The tight upper mass $m(\pi^+\pi^-)$ cut at 0.9 GeV/$c^2$ is to remove contributions from the scalar $f_0(980)$ resonance, and the tight lower cut is to reduce the contamination from $K_0^0$ decays. To reduce contributions from $B^0 \rightarrow \rho^+\pi^-$ decays, a $B^0 \rightarrow \rho^0\pi^0$ candidate is rejected if $0.4 < m(\pi^+\pi^0) < 1.3\,\text{GeV}/c^2$. For the $B^+ \rightarrow \rho^0\pi^0$ and $B^0 \rightarrow \rho^0\pi^0$ modes, the invariant mass of any charged track in the event and the $\pi^0$ must be less than 5.14 GeV/$c^2$ to reject $B^+ \rightarrow \pi^+\pi^0\pi^0$ background. For the $B^+ \rightarrow \rho^0\pi^+$ mode, we remove background from charmed decays $B \rightarrow D^0X$, $D^0 \rightarrow K^+\pi^-\pi^+$, and $D^0 \rightarrow K^+\pi^-\pi^-$, by requiring the masses $m(\pi^+\pi^-)$ and $m(K^+\pi^-)$ to be less than 1.844 GeV/$c^2$ or greater than 1.884 GeV/$c^2$. We take advantage of the helicity structure of $B \rightarrow \rho\pi$ decays by requiring that $|\cos\theta_p| > 0.25$, where $\theta_p$ is the angle between the $\pi^0$ ($\pi^+$) momentum from the $\rho^+$ ($\rho^0$) decay and the $B$ momentum in the $\rho$ rest frame.

Two kinematic variables, $\Delta E$ and $m_{ES}$, allow the discrimination of signal $B$ decays from random combinations of tracks and $\pi^0$ candidates. The energy difference, $\Delta E$, is the difference between the $e^+e^-$ center-of-mass (CM) energy of the $B$ candidate and $\sqrt{s}/2$, where $\sqrt{s}$ is the total CM energy. The beam-energy-substituted mass, $m_{ES}$, is defined by $\sqrt{(s/2 + p_B \cdot p_B)/E^2 - p_B^2}$, where $p_B$ is the momentum of the initial state $(E_i, \mathbf{p}_i)$ are measured in the laboratory frame. For $B^+ \rightarrow \rho^0\pi^+$ we require that $-0.05 < \Delta E < 0.05\,\text{GeV}$ while for both modes containing a $\pi^0$ we relax this requirement to $-0.15 < \Delta E < 0.10\,\text{GeV}$. For both $B^+ \rightarrow \rho^0\pi^+$ and $B^0 \rightarrow \rho^0\pi^0$ we require that $5.23 < m_{ES} < 5.29\,\text{GeV}/c^2$ while for $B^+ \rightarrow \rho^0\pi^0$ it is relaxed to $5.20 < m_{ES} < 5.29\,\text{GeV}/c^2$.

Continuum $e^+e^- \rightarrow q\overline{q}$ $(q = u, d, s, c)$ events are the dominant background. To enhance discrimination between signal and continuum, we use neural networks (NN) to combine six discriminating variables: the reconstructed $\rho$ mass, $|\cos\theta_p|$, the cosine of the angle between the $B$ momentum and the beam direction in the CM frame, the cosine of the angle between the $B$ thrust axis and the beam direction in the CM frame, and the two event-shape variables that are used in the Fisher discriminant of Ref. The event shape variables are sums
over all particles $i$ of $p_i \times |\cos \theta_i|^n$, where $n = 0$ or 2 and $\theta_i$ is the angle between momentum $i$ and the $B$ thrust axis. The NN for each analysis weighs the discriminating variables differently, according to training on off-resonance data and the relevant Monte Carlo (MC) simulated signal events. The final $\pi\tau$ candidate samples are selected with cuts on the corresponding NN outputs.

To further discriminate further between signal and continuum background, for the $B^{+} \rightarrow \rho^{0}\pi^{0}$ mode, we use the separation between the vertex of the reconstructed $B$ and the vertex reconstructed for the remaining tracks. This separation is related to $\Delta t$, the difference between the two decay times, by $\Delta z = \beta \gamma \Delta t$, where for PEP-II the boost is $\beta \gamma = 0.56$.

Approximately 33%, 7%, and 8% of the events have more than one candidate satisfying the selection in the $B^{+} \rightarrow \rho^{0}\pi^{0}$, $B^{+} \rightarrow \rho^{0}\pi^{+}$, and $B^{0} \rightarrow \rho^{0}\pi^{0}$ decay mode, respectively. In such cases we choose the candidate with the reconstructed $\rho$ mass closest to the nominal value of 0.77 GeV/$c^2$. Table I summarizes the numbers of events selected from the data sample and the signal efficiencies estimated from MC simulation. Some of the actual signal events are misreconstructed; this is primarily due to the presence of random combinations involving low momentum pions. For the charged $B$ modes we distinguish misreconstructed signal events with correct charge assignment from those with incorrect charge assignment. These numbers, estimated from MC, are also listed in Table I.

We use MC-simulated events to study the background from other $B$ decays, ($B$-background), which include both charmed ($b \rightarrow c$) and charmless decays. In the selected $\rho^{+}\pi^{-}$ ($\rho^{0}\pi^{0}$, $\rho^{0}\pi^{-}$) sample we expect $205 \pm 46$ (73 $\pm$ 19, 59 $\pm$ 18) $b \rightarrow c$ and $228 \pm 77$ (92 $\pm$ 11, 74 $\pm$ 22) charmless background events. All the three analyses share the major $B$-background modes: $B^{+} \rightarrow \rho^{0}\pi^{-}$, longitudinally polarized $B^{0} \rightarrow \rho^{+}\rho^{-}$, and $B^{0} \rightarrow \rho^{+}\rho^{0}$. Other important modes include $B^{+} \rightarrow \rho^{0}\pi^{0}$ (for $B^{0} \rightarrow \rho^{0}\pi^{0}$), $B^{+} \rightarrow (a_{1}\pi)^{+}$ (for $B^{+} \rightarrow (a_{1}\pi)^{+}$), $B^{+} \rightarrow K^{+}(892)^{0}\pi^{+}$ (for $B^{+} \rightarrow (a_{1}\pi)^{+}$), and background modes containing higher kaon resonances.

An unbinned maximum likelihood fit is used for each analysis to determine event yields and charge asymmetries. To enhance discrimination between signal and background events, we use the $B$-flavor-tagging algorithm developed for the BABAR measurement of the $CP$-violating amplitude $\sin 2\beta$ \cite{ref}, where events are separated into categories based on the topology of the event and the probability of misassigning the $B$-meson flavor. The likelihood for the $N_{k}$ candidates tagged in category $k$ is

$$L_{k} = e^{-N_{k}} \prod_{i=1}^{N_{k}} \left\{ N_{k}^{\rho\pi} \epsilon_{k} P_{k,i}^{\rho\pi} + N_{k}^{\rho\pi} P_{k,i}^{\rho\pi} + \sum_{j=1}^{N_{B}} L_{ij,k} \right\}, \quad (2)$$

where $N_{k}^{\rho\pi}$ is the number of signal events in the entire sample, $\epsilon_{k}$ is the fraction of signal events tagged in category $k$, $N_{k}^{\rho\pi}$ is the number of continuum background events that are tagged in category $k$, and $N_{B}$ is the number of $B$-background modes. $N_{k}^{\rho\pi}$ is the sum of the expected event yields for signal ($\epsilon_{k}N_{k}^{\rho\pi}$), continuum ($N_{k}^{\rho\pi}$) and fixed $B$ background. For the charged modes the asymmetries are introduced by multiplying the signal yields by $\frac{1}{2}(1 - Q_{i} A_{CP})$, where $Q_{i}$ is the charge of $B$-candidate $i$. The likelihood term $L_{ij,k}$ corresponds to the $j$th $B$-background contribution of the $N_{B}$ $B$-background classes. The total likelihood is the product of likelihoods for each tagging category.

The probability density functions (PDF) for signal and continuum, $P_{k,i}^{\rho\pi}$ and $P_{k}^{\rho\pi}$, are the products of the PDFs of the discriminating variables. The signal PDFs are given by $P_{k,i}^{\rho\pi} \equiv P_{k,i}^{\rho\pi}(m_{ES}) \cdot P_{k,i}^{\rho\pi}(\Delta E) \cdot P_{k,i}^{\rho\pi}(\Delta t)$ (NN) for the charged $B$ decay modes, and by $P_{k}^{\rho\pi} \equiv P_{k}^{\rho\pi}(m_{ES}) \cdot P_{k}^{\rho\pi}(\Delta E) \cdot P_{k}^{\rho\pi}(\Delta t)$ for $B^{0} \rightarrow \rho^{0}\pi^{0}$. Each signal PDF is decomposed into two parts with distinct distributions: signal events that are correctly reconstructed and signal events that are misreconstructed. For the charged $B$ modes, each PDF for the misreconstructed events is further divided into a right-charge and wrong-charge part. The $m_{ES}$, $\Delta E$, and $N$ PDFs for signal and for $B$ background are taken from MC simulation. For continuum, the yields and PDF parameters are determined simultaneously in the fit to on-resonance data.

In the $B^{0} \rightarrow \rho^{0}\pi^{0}$ decay the $\Delta t$ distributions for signal and $B$ background are modeled from fully reconstructed $B^{0}$ decays from data control samples \cite{ref}. The continuum $\Delta t$ parameters are free in the fit to on-resonance data.

To validate the fit procedure, we perform fits on large MC samples that contain the measured number of signal and continuum events and the expected $B$-background. Biases observed in these tests are largely due to correlations between the discriminating variables, which are not accounted for in the PDFs. For $\rho^{+}\pi^{-}$ and $\rho^{0}\pi^{0}$ they are not negligible and are used to correct the fitted signal yields. In addition, the full fit biases are assigned as systematic uncertainties on all three signal yields.

Contributions to the systematic errors are summarized in Table III. Uncertainties in the signal MC simulation
are obtained from a topologically similar control sample of fully reconstructed \( B^0 \to D \rho^+ \rho^- \rho^0 \) decays. For the \( B^+ \to \rho^+ \pi^0 \) channel we also use \( B^+ \to K^+ \pi^0 \) decays to estimate the uncertainty in the \( \Delta E \) model. We vary the signal parameters, that are fixed in the fit, within their estimated errors and assign the effects on the signal yields and charge asymmetries as systematic errors. The expected yields from the \( B \)-background modes are varied according to the uncertainties in the measured or estimated branching fractions. Since \( B \)-background modes may exhibit direct \( CP \) violation, the corresponding charge asymmetries are varied within their physical ranges. For \( B^0 \to \rho^0 \pi^0 \), the systematic uncertainty due to interference with \( B^0 \to \rho^0 \pi^- \) is found to be 1.5 events. This is obtained by repeating the fit to data, after removing the cut on \( m(\pi^+ \pi^-) \). Systematic errors due to possible nonresonant \( B^0 \to \pi^+ \pi^- \pi^0 \) decays are derived from experimental limits [8]. Contributions from nonresonant \( B^+ \to \pi^+ \pi^0 \pi^0 \) for the \( \rho^0 \pi^0 \) mode and \( B^+ \to \pi^+ \pi^- \pi^+ \) for the \( \rho^+ \pi^- \) mode are estimated to be negligible. For the \( B^+ \to \rho^0 \pi^+ \) and \( B^0 \to \rho^0 \pi^0 \) decay modes, systematic uncertainties due to interference between \( \rho^0 \) and \( f_0(980) \) or a possible broad scalar \( \sigma (400 - 1200) \) were also studied and found to be negligible. Repeating the selection and fit for all three modes, without using the \( \rho \)-candidate mass and helicity angle, gives results that are compatible with those reported here. In the \( B^+ \to \rho^0 \pi^+ \) case, the analysis was repeated in the region \( |\cos \theta_\rho| < 0.25 \), and the resulting signal yield was consistent with zero.

After correcting for the fit biases we find from the maximum likelihood fits the event yields, \( N(\rho^+ \pi^0) = 169.0 \pm 28.7 \), \( N(\rho^0 \pi^+) = 237.9 \pm 26.5 \), and \( N(\rho^0 \pi^0) = 24.9 \pm 11.5 \), where the errors are statistical only. Figure 1 shows distributions of \( m_{ES} \) and \( \Delta E \), enhanced in signal content by cuts on the signal-to-continuum likelihood ratios of the other discriminating variables. The statistical significance of the previously unobserved \( B^+ \to \rho^+ \pi^0 \) signal amounts to 7.3\( \sigma \), computed as \( \sqrt{2 \Delta \log L} \), where \( \Delta \log L \) is the log-likelihood difference between a signal hypothesis corresponding to the bias-corrected yield and a signal hypothesis corresponding to a yield that equals one standard deviation of the systematic error. We find the

![Figure 1: Distributions of \( m_{ES} \) and \( \Delta E \) for samples enhanced in \( \rho^+ \pi^0 \) signal (a), \( \rho^0 \pi^+ \) signal (c,d) and \( \rho^0 \pi^0 \) signal (e,f). The solid curve represents a projection of the maximum likelihood fit result. The dashed curve represents the contribution from continuum events, and the dotted line indicates the combined contributions from continuum events and \( B \)-related backgrounds.](image)

### Table II: Summary of the systematic uncertainties.

| Error source     | \( \rho^+ \pi^0 \) | \( \rho^0 \pi^+ \) | \( \rho^0 \pi^0 \) | \( A_{CP}^{\rho^+} \) | \( A_{CP}^{\rho^0} \) |
|------------------|-------------------|-------------------|-------------------|-------------------|-------------------|
| Signal model     | 10.7              | 3.8               | 3.3               | 3.4               | 0.3               |
| Fit procedure bias| 14.4              | 8.2               | 2.0               | -                 | -                 |
| B background     | 11.2              | 2.3               | 3.3               | 5.0               | 2.2               |
| Detector charge bias | -                | -                 | -                 | 1.0               | 0.9               |
| Total fit error  | 21.1              | 9.3               | 5.1               | 6.1               | 2.4               |
| Relative efficiency error | 11.6% | 7.2% | 7.0% | - | - |

Branching fractions to be

\[
B(B^+ \to \rho^+ \pi^0) = (10.9 \pm 1.9 \pm 1.9) \times 10^{-6},
\]

\[
B(B^+ \to \rho^0 \pi^+) = (9.5 \pm 1.1 \pm 0.8) \times 10^{-6},
\]

\[
B(B^0 \to \rho^0 \pi^0) = (1.4 \pm 0.6 \pm 0.3) \times 10^{-6},
\]

where the first errors are statistical and the second systematic. The systematic errors include the uncertainties in the efficiencies, which are dominated by the uncertainty in the \( \pi^0 \) reconstruction efficiency and in the case of \( \rho^0 \pi^+ \), by the uncertainty due to particle identification. Here we define the \( B^0 \to \rho^0 \pi^0 \) branching ratio by including those events that pass our selection and are fitted as signal but excluding those events that can be interpreted as \( B^0 \to \rho^+ \pi^- \) with a \( \rho^- \), whose mass is closer to 0.77 GeV/c\(^2\) than the mass of the reconstructed \( \rho^0 \). The signal significance for \( \rho^0 \pi^0 \), including statistical and systematic errors, is 2.1\( \sigma \), and we use a limit setting procedure similar to Ref. 11 to obtain a 90% Confidence-Level upper limit on its branching fraction. Fits on MC samples are used to find the signal hypothesis for which the ratio of the probability that the fitted signal yield is less than that observed in data, and the probability that the fitted yield is less than that in data under the null signal hypothesis, is 0.1. This signal hypothesis is shifted up by one sigma of the systematic error and the efficiency.
is shifted down also by one sigma. This method gives an upper limit of \( B(B^0 \to \rho^0 \pi^0) < 2.9 \times 10^{-6} \).

Theoretical predictions of the ratio of branching fractions \( R \equiv B(B^0 \to \rho^+\pi^-)/B(B^+ \to \rho^0\pi^+) \), vary over a wide range. Tree level estimates suggest \( R \approx 6 \) \(^{12}\), while the inclusion of penguin contributions, off-shell \( B^* \) excited states and scalar \( \pi^+\pi^- \) resonances leads to lower values, \( R \approx 2 - 3 \) \(^{13}\). Using the measured \( B^+ \to \rho^0\pi^+ \) branching fraction and the \( B^0 \to \rho^+\pi^- \) branching fraction from Ref. \(^1\) we find \( R = 2.38^{+0.37}_{-0.31}(\text{stat})^{+0.24}_{-0.20}(\text{syst}) \), which is in agreement with previous experimental results \(^2\).

For the charged \( B \) decays we find the charge asymmetries, \( A_{CP}^+ = 0.24 \pm 0.16 \pm 0.06, A_{CP}^{0+} = -0.19 \pm 0.11 \pm 0.02 \), with contributions to the systematic errors listed in Table \(^{11}\).

In summary, we have presented measurements of branching fractions and CP-violating charge asymmetries in \( B^+ \to \rho^0\pi^+ \) and \( B^+ \to \rho^0\pi^- \) decays, and a search for the decay \( B^0 \to \rho^0\pi^0 \). We observe the decay \( B^+ \to \rho^0\pi^0 \) with a statistical significance of 7.3\( \sigma \). We also find a branching fraction for \( B^+ \to \rho^0\pi^+ \) that is consistent with previous measurements \(^3\), and set an upper limit for \( B^0 \to \rho^0\pi^0 \). We do not observe evidence for direct CP violation.

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\(^*\) Also with Università della Basilicata, Potenza, Italy
\(^\dagger\) Also with IFIC, Instituto de Física Corpuscular, CSIC-Universidad de Valencia, Valencia, Spain
\(^\ddagger\) Deceased

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