Numerical study of shear band formation in triaxial compression tests

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ABSTRACT

Numerical simulations of triaxial compression tests are performed in order to investigate the applicability of the material point method (MPM) to large-deformation geotechnical problems. A Mohr-Coulomb constitutive model is adopted as a geotechnical nonlinearity, for which parameters are obtained by experimental results. The stress-strain relationships obtained in the simulations show good agreement with the experiments at high levels of strain, thus demonstrating the effectiveness of MPM. Parametric studies are also performed in order to investigate factors that influence the relationship between the initial and final shear bands observed during simulations of triaxial compression tests. The main findings of the study are followings: (1) initial shear bands are generated near the constrained edges of a specimen; (2) when both top and bottom edges of a specimen are constrained, initial shear bands near the edges at which the external forces are applied develop finally into clear shear bands; (3) specimen geometry has a major influence on the formation of shear bands in the case of rectangular specimens.

Keywords: material point method, triaxial compression test, shear band

1 INTRODUCTION

Failures of geomaterials occur upon deformation localization. In order to simulate the behavior of geomaterials after such failures, it is desirable to employ a numerical simulation method capable of high resolution and that handles large deformations while allowing the use of constitutive models. Natural ground forms with a complicated structure and without plane surfaces. The plane strain condition is applicable only to limited types of geomaterial structures (river banks, road or rail embankments, etc.). Most other geomaterial structures are structured three dimensionally. Therefore, any numerical simulation that targets naturally deposited ground must consider its three dimensional geometry.

Typical mesh-based methods, such as the Finite Element Method (FEM), are widely applied to engineering problems and are used in many commercial codes. However, mesh methods present a number of difficulties when applied to geotechnical problems such as the numerical modeling of complex strata or large deformations: 1) considerable time is required to generate the mesh; 2) mesh tangling occurs in strain localized zones; and 3) remeshing is computationally intensive. On the other hand, mesh-free methods are able to avoid mesh-tangling and provide high resolution analysis using constitutive models just like FEM. However, until now, triaxial compression tests using mesh-free methods have typically been two dimensional approaches (Yoshida 2011; Kiriyama 2013). There have been few three dimensional simulations using mesh-free methods, especially focusing on shear band formation. In this paper, the material point method (MPM) is employed to simulate a triaxial compression test. Based on this simulation, the applicability of MPM to geomaterials is investigated, while the formation of the shear band is discussed as well.

2 MATERIAL POINT METHOD

The material point method was first proposed by Sulsky et al. (1994). MPM is a particle based approach that applies the Particle In Cell (PIC) developed for fluid mechanics to solid mechanics. Areas of MPM application include collision and contact analyses, and effective applications have been reported (Bardenhagen et al. 2004; Coetzee et al. 2006). However, in the case of large deformation problems, numerical noise is often generated by the original MPM algorithm when particles move across numerical grids, making it difficult to apply the method to the numerical simulation of geomaterials, which exhibit significant nonlinear behavior. To reduce this numerical noise, Bardenhagen and Kober (2004) proposed the generalized interpolation material point (GIMP) method. GIMP minimizes numerical noise, thereby enhancing MPM and making it suitable for simulations of continua. The author has implemented a three dimensional GIMP method with a Mohr-Coulomb constitutive model and used it to perform the simulations described in the sections that follow.
3 NUMERICAL SIMULATION OF TRIAXIAL COMPRESSION TEST

In order to investigate the applicability of MPM to geomaterials, numerical simulations of a series of triaxial compression tests were performed. The experiments were triaxial compression tests under unconsolidated undrained (UU) conditions performed by the author (Kiriyama 2013). Specimens were created from unsaturated DL Clay (with a water content of 4.5%). Silicon oil, which is non-volatile in air, was used as pore liquid for the convenience of maintenance and reproducibility of the soil material. Specimens were monotonically compressed until the axial strain reached 15% under strain controlled loading (1%/min). The stress-strain relationship and material constants obtained experimentally from these tests are shown in Fig. 2 and Table 1, respectively. Post-test sketches of the specimens are shown in Fig. 2.

Table 1. Results of experiment. \( \sigma_{m0} \): initial confining pressure, \( G \): shear modulus, \( \rho \): density, \( c \): cohesion, \( \phi \): internal friction angle, \( \psi \): dilatancy angle.

| No. | \( \sigma_{m0} \) (kPa) | \( G \) (kPa) | \( \rho \) (g/cm\(^3\)) | \( c \) (kPa) | \( \phi \) (deg) |
|-----|-----------------|-------------|-----------------|-------------|-------------|
| 1   | 10              | 1311        | 1.53            | 8.5         | 30.5        |
| 2   | 50              | 6401        |                 |             |             |
| 3   | 100             | 8514        |                 |             |             |

During the simulation, the vertical stress values at the bottom of the specimen were totaled and the average vertical stress acting on the bottom of the specimen was calculated. A deviatoric stress corresponding to the experimental value was calculated by using this average vertical stress and the initial confining pressure, which was equal to the lateral stress. The effectiveness of the MPM can be determined by comparing the stress-strain relationships of the experiment and the simulation.

3.1 Analytical conditions

The numerical model is explained in Fig. 3. The specimen consisted of a set of particles representing a 5 cm x 10 cm (diameter x height) cylinder in a three dimensional calculation area. The cap and pedestal also consisted of a set of particles and are represented by cylinders 5 cm x 1 cm (diameter x height) in size. The constitutive property of the specimen was the Mohr-Coulomb failure criterion and the material constants were the values listed in Table 1. The particles arranged 2 x 2 x 2 in each cell (grid unit), so each cell contained 8 particles. A total of 212,112 particles comprised the specimen. A fixed boundary condition was assigned to the pedestal particles. As the initial confining pressure, homogeneous initial stresses of 10 kPa, 50 kPa and 100 kPa were applied to the particles. A strain controlled condition was achieved by applying to the cap particles a constant vertical velocity (0.02 m/sec) and a constant vertical acceleration (0.0 m/sec\(^2\)). Two types of horizontal boundary condition (fixed or free) were set to the cap particles in order to investigate the effects of this boundary condition.

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Fig. 1. Stress-strain relationship obtained by experiment.

Fig. 2. Post-test sketches of specimens.

Fig. 3. Numerical model of triaxial specimen.
### 3.2 Comparison with experimental results

The analytical stress-strain relationship and the distribution of maximum shear strain with $\sigma_{m0}=10$ kPa are shown in Fig. 4 and Fig. 5, respectively. The specimen with the fixed-cap condition shows no initial shear band at $\varepsilon_a=1\%$, multiple initial shear bands at $\varepsilon_a=2\%$, and two clear X-shaped shear bands at strains over $\varepsilon_a=5\%$. The stress-strain relationship with the fixed-cap condition shows good agreement with the experiment at large strains (over 5%) as seen in Fig. 4(a), thus demonstrating the effectiveness of the MPM. The specimen with the free-cap condition shows no initial shear band at $\varepsilon_a=1\%$, multiple initial shear bands in the lower part of the specimen, and one clear sash-shaped shear band at strains over $\varepsilon_a=5\%$. The stress-strain relationship with the free-cap condition rises to a peak deviatoric stress and then begins to decrease. Fig. 6 shows the distribution of vertical stress inside the specimens. With the fixed-cap condition, vertical stress is directly transferred from top to bottom, while with a free-cap the vertical stress is transferred only where the top and bottom surfaces overlap (when viewed from above). This apparent strain softening behavior arises because of the release of vertical stress due to the eccentricity of the cap position. It indicates that strain softening behavior occurs due to changes in the micro structure, friction, and cohesion as well as the macro geometric change of the specimen itself. A previous study using clay material reported that strain softening occurred due to buckling of the specimen (Kodaka et al. 2007). Although the mode of failure is different, the simulation results agree with the experimental results in that a geometrical change affects the stress-strain relationship.

The experimental results exhibit both a peak stress and a sash-shaped shear strain localization, indicating an intermediate status between the fixed-cap and free-cap conditions.

### 4 PARAMETRIC STUDY OF SHEAR BAND FORMATION

Deformation localization is highly dependent on shape, material, force, and boundary conditions. Simulation results at $\varepsilon_a=2\%$ show multiple initial shear bands (Fig. 5). It is interesting, investigating deformation localization, to consider which initial shear band develops to form the final shear band. In this section, factors influencing the formation of shear bands, especially their locations and shapes, are discussed.

**Table 2. Analysis cases and conditions.**

| Case   | Shape    | Material | Cap Horizontally | Pedestal Horizontally |
|--------|----------|----------|-------------------|-----------------------|
| C-fc-1 | Cylinder | ✔️       | ✔️                | ✔️                    |
| C-fc-2 | Cylinder | ✔️       | ✔️                | ✔️                    |
| C-fc-3 | Cylinder | ✔️       | ✔️                | ✔️                    |
| C-c-1  | Cylinder | ✔️       | ✔️                | ✔️                    |
| C-c-2  | Cylinder | ✔️       | ✔️                | ✔️                    |
| R-fc-1 | Cylinder | ✔️       | ✔️                | ✔️                    |
| R-fc-2 | Cylinder | ✔️       | ✔️                | ✔️                    |
| R-c-1  | Cylinder | ✔️       | ✔️                | ✔️                    |
| R-c-2  | Cylinder | ✔️       | ✔️                | ✔️                    |

Fig. 7. Specimen shapes.
4.1 Parameter selections

The process of shear band formation was studied through parametric studies focusing on specimen shapes (cylinder, rectangular), material (frictional, cohesive), and cap and pedestal boundary condition (fixed, free). The numerical model specimen with $\sigma_{\text{ml}}=10$ kPa was used. The combinations of parameters are listed in Table 2. Cases C-fc-1 and C-fs-2 are the same simulations as Fig. 5(a) and Fig. 5(b). The cylindrical and rectangular models are shown in Fig. 7. The same analytical conditions, such as the number of particles per cell and boundary conditions, are assigned to the rectangular specimen as used in the cylindrical model.

4.2 Factors influencing shear band formation

a) Force/boundary condition

Fig. 8 shows the maximum shear strain just after specimens C-fc-1 to 3 reached plastic status. According to Fig. 8(a), four initial shear bands (1 and 2 in the upper part and 3 and 4 in the lower part of the specimen) were observed in view 1. Similarly, another four initial shear bands (5, 6, 7, and 8) were observed in view 2. A total of eight initial shear bands were observed in C-fc-1. Specimen C-fc-1 shows X-shaped shear localization (Fig. 5(a)), consisting of two clear shear bands, in the upper part of the specimen. It is guessed that the initial shear band (1, 2) or (5, 6) developed to complete the formation of the final shear band. According to Fig. 8(b), which is the case with the free-cap condition, initial shear bands were not observed clearly in the upper part of the specimen, while two initial shear bands per section, or a total of four initial shear bands, were observed in the lower part of the specimen. Specimen C-fc-2 shows sash-shaped shear localization (Fig. 5(b)). The one initial shear band shown in Fig. 8(b) developed into the final shear band. According to Fig. 8(c), which is the case with the free-pedestal condition, initial shear bands were not observed in the lower part of the specimen, while two initial shear bands per section, or a total of four initial shear bands, were observed in the upper part of the specimen. Fig. 5(a) and 8(a) reveal that initial shear bands forming near the force condition outweighed the others and developed to form the final shear bands. Fig. 8(b) and (c) reveal that initial shear bands started near the fixed boundary conditions and no clear initial shear band formed around the free boundaries.

b) Material

Fig. 9 shows the maximum shear strain of a cohesive cylinder specimen. In Fig. 9(a)(C-c-1), the Barrel-shaped deformation typically observed when clay materials are used is visible. Specimen C-c-1 has eight initial shear bands at $\varepsilon_{\text{a}}=5\%$, which is the same as C-fc-1. With increasing axial strain, the initial shear bands, which formed independent X-shaped shear bands in the upper and lower parts of the specimen, merged near the centre to form the one clear shear band from top to bottom. Fig. 9(b)(C-c-2) shows the eight initial shear bands, which are the same as in both the cap and pedestal fixed conditions (C-c-1). Initial shear bands were also observed near the free boundary condition when cohesive material was used. This reveals that the boundary condition has little influence on the formation of initial shear bands when material is cohesive. An initial shear band near the force condition developed to complete forming the final shear band ($\varepsilon_{\text{a}}=20\%$).
in Fig. 9(b)).

**c) Shape**

Fig. 10 and 11 show the distribution of maximum shear strain in rectangular specimens made of both frictional (c, φ) and cohesive (c) material. In the case of frictional material, the final shear bands form in a plane, starting from an upper vertex and extending to the diagonally opposite lower vertex, irrespective of the boundary condition. On the other hand, they take an X shaped with the fixed-cap condition and a sash shaped with the free-cap condition, which are the same as the result when the cylinder specimens is used. A rectangular specimen with cohesive material, shown in Fig. 11, forms final shear bands that are parallel to the edge of the top or bottom surface. These results indicate that the locations of shear bands are predictable on the basis of material types (frictional or cohesive) in the case of a rectangular specimen.

**4.3 Comparison with previous studies**

In the literature, there are numerous reports relating to triaxial compression tests, as well as numerical simulations under various analytical conditions and using other numerical methods. In this section, the numerical results reported by the author are compared with these previous studies.

Yamakawa et al. (2002) performed triaxial compression tests with cylindrical specimens consisting of Toyoura sand. They reported that the aspect ratio and size of specimens affected the stress-strain relationship, dilatancy characteristics, and shear band formation. The bifurcation mode was also analyzed and the shear band patterns formed were categorized into six types. Among these, the ‘diamond’ pattern had a similar shear strain distribution to the numerical results obtained here for cylindrical specimens (e.g. C-fc-1 in Fig. 8(a)). Although certain conditions differ between the two cases, the experimental results reported by Yamakawa et al. are an example of shear band formation in the same patterns as demonstrated here.

Kodaka et al. (2007) performed triaxial compression tests using rectangular specimens of normal and overconsolidated clay. In their paper, patterns of shear band formation were experimentally and numerically investigated for different loading conditions (strain rates). They categorized the resulting shear band patterns into three types: X-mode, buckling-like mode, and complicated-mode. All shear bands were reported to have started from the vertices or edges of the specimens and the shear strain distribution was dependent on specimen shape and loading conditions (strain rates). Stress-strain curves were related to shear band formation, and it was reported that strain softening behavior accelerated after shear band formation. Moreover, in numerical simulations, imperfections in geometry and heterogeneity of material were considered. Higo et al. took the discussion further by implementing numerical simulations method that considered loading conditions (strain rates), shear band formation patterns, dilatancy characteristics, effective stress distribution inside the specimens, and migration of pore water. They reported that their simulation results were not dependent on the mesh sizes chosen. According to Kodaka et al.’s experimental result, rectangular specimens with an aspect ratio of 2:1 exhibited X-shaped shear localization in the front face and unclear X-shaped shear localization in the side face. This type of shear band formation was also observed in R-c-1. In this case, the specimen, which has an aspect ratio of 3:1, shows a shear localized zone in the plane that joins the top and bottom vertices. Although the aspect ratio is different in this case, the shear strain localization observed in R-fc-1 and R-fc-2, which crosses the top and bottom vertices is like that categorized as the complicated-mode by Kodaka et al. and is likely to occur experimentally.

The emphasis of this paper is the numerical response of specimens under triaxial compression conditions; highly enhanced constitutive models are not used. However, MPM is able to accommodate constitutive models just like other mesh-based methods. MPM can also be used to develop simulations that consider pore
water and pore air. Such simulations have been already reported (Higo et al. 2010). The introduction of highly enhanced constitutive models into the author’s MPM numerical simulations is therefore desired in the future. On the issue of the numerical approach to shear band formation, shear band thickness and angle, mesh size dependency, and bifurcation have been discussed. In these respects, though, discussion of the simulation results reported in this paper need to be deepened. MPM uses a numerical grid. It is surmised MPM will exhibit a similar mesh size dependency to mesh-based methods. This will be investigated in a future study.

5 CONCLUSIONS

Three dimensional simulations of triaxial compression tests were performed using the material point method. Stress-strain relationships and the formation of shear bands were discussed first, with the main findings as follows.

1) Under the fixed-cap condition, the numerical stress-strain relationship indicated a peak stress in good agreement with the experimental results at large strain levels (over 5%), with the numerical simulation showing correctly that specimens form an X-shaped shear band.

2) Under the free-cap condition, the numerical stress-strain relationship showed an apparent strain softening behavior while the numerical specimen developed a sash-shaped shear band.

These two findings agree with a previous study involving two dimensional numerical simulations. The three dimensional numerical simulations reported here showed similar shear band formation regardless of cap condition.

Following the initial simulations above, a parametric study was performed focusing on force/boundary conditions, specimen material, and specimen shape. The formation of shear bands was investigated as these parameters were varied. The numerical results indicated that specific initial shear bands, which developed within the specimen according to certain factors, develop into the final shear bands. The findings of this parametric study are as follows.

3) Under the fixed-cap and fixed-pedestal condition, initial shear bands were observed crossing near the constrained edges. On the contrary, no clear initial shear band was observed near the free edges.

4) Among initial shear bands, specific ones that were near the loading edge developed to form the final shear bands.

5) Rectangular specimens exhibited the shear bands that were dependent on boundary conditions; there were X-shaped under the fixed-cap condition and sash-shaped under the free-cap condition, respectively.

6) The initial shear band of cohesive specimens was observed near an edge with a free boundary condition. This is because the shear strength of the cohesive material is independent of confining pressure.

7) Shear band formation in rectangular specimens could be predicted on the basis of material properties whether it is frictional or cohesive. That is, a frictional rectangular specimen developed a shear band that crossed from an upper vertex to the opposite vertex at the bottom, while the shear band of a cohesive rectangular specimen formed parallel to the edges of the top or bottom surface.

Shear band formation depends upon the conditions and topology of the specimen. At the analytical resolution used in this work, the maximum number of shear bands that appeared was eight, with specific bands indicating strain localization and developing into the final shear bands. For example, two shear bands formed in the upper part of a cylinder specimen with a fixed-cap, while different ones formed in the lower part of a cylinder specimen with a free-cap condition. It should be noted that increasing the resolution of the analysis, i.e., increasing the number of particles in the soil specimen, may reveal more initial shear bands.

REFERENCES

1) Bardenhagen, S.G., Guilkey, J.E., Roessig, K.M., Brackbill, J.U., Witzel, W.M. and Foster, J.C. (2001): An Improved Contact Algorithm for the Material Point Method and Application to Stress Propagation in Granular Material, CMES, 2(4), 509-522.

2) Bardenhagen, S.G. and Kobier, E.M. (2004): The generalized interpolation material point method, Computer Modeling in Engineering and Science, 5(6), 447-495.

3) Coetzee, C.J., Basson, A.H. and Vermeeren, P.A. (2006): Discrete and continuum modeling of silo discharge, R&D Journal incorporated into The S.A Mechanical Engineer, 22(2).

4) Higo, Y., Oka, F., Kimoto, S., Morinaka, Y., Goto, Y. and Zhen, C. (2010): A coupled npm-fdm analysis method for multi-phase elasto-plastic soils, Soils and Foundations, 50(4), 515-532.

5) Kiriyama, T. (2013): Numerical simulations of progressive failure of triaxial compression tests using generalized interpolation material point method, Journal of Japan Society of Civil Engineers, Ser.A2, 69(2), 1321-1332. (in Japanese)

6) Kodaka, T., Higo, Y., Kimoto, S. and Oka, F. (2007): Effect of sample shape on strain localization of water-saturated clay, Int. J. Numerical and Analytical Methods in Geomechanics, Vol.31, pp.483-521.

7) Sulsky, D., Chen, Z. and Schreyer, H.L. (1994): A particle method for history-dependent materials, Comput. Methods Appl. Mech. Engrg., 118, 179-196.

8) Yamakawa, Y., Ikeda, K., Sudo, Y., Terai, N., and Torii, K. (2002): Bifurcation deformation modes and size/shape effects of triaxial sand specimens, Journal of Geotechnical Engineering, JSCE, 70(III), 58, 357-371. (in Japanese)

9) Yoshida, I. (2011): Basic study on failure analysis with using MPM method, Journal of Japan Society of Civil Engineers, Ser.A2, 67(1), 45-48. (in Japanese)