Influence of Surface Tension on Nuclear Collective Properties

N. G. Goncharova

M.V. Lomonosov Moscow State University, Faculty of Physics, Moscow, 119991 Russia
e-mail: n.g.goncharova@gmail.com

Abstract—Rigidities of even-even nuclei were estimated and compared with nuclear charge radii. Correlation of maximal nuclear rigidities with minimal values of \( r_0 \) parameters was revealed. Influence of effective surface tension on nuclear properties was discussed.

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Shell structure in the nucleonic motions has a profound effect on collective properties of the individual nuclei. Competition between the nucleons in the closed shells which promote spherical shape of nucleus and nucleons in the unfilled shells which tend to polarize the nucleus and bring about nonspherical equilibrium shape is one of the origins in great diversities in nuclear characteristics. This source of differences in nuclear forms and shapes was indicated already at the dawn of nuclear physics [1–3]. These contradictory tendencies reveal themselves as well in the considerable diversities of nuclear responses to excitations, e.g. in great differences in the fragmentations of multipole resonances’ strengths. One of nuclear characteristics which reflect the competition between tends to minimize the nuclear surface and to minimize the Coulomb repulsion is nuclear rigidity. This quantity reveals in the Hamiltonian of collective vibrations, where it determines their potential energy [4]:

\[
\tilde{H}_{\text{coll.vib.}} = \frac{1}{2D} \sum_m |\tilde{p}_m|^2 + \frac{C}{2} \sum_m |\tilde{a}_m|^2,
\]  

(Only the most important quadrupole vibrations would be considered). In (1) \( D \) represents the mass transport in the vibrations; \( C \)—nuclear rigidity.

The energy of vibration connects both characteristics

\[
\hbar \omega = \sqrt{C/D}.
\]  

(2)

As were shown in [1], \( C \) represents the effective surface tension of nucleus:

\[
C = 4R^2\sigma - \frac{3}{10\pi} \frac{e^2Z^2}{R},
\]  

(3)

(\( \sigma \)—coefficient of surface tension, \( R \)—nuclear radius).

\( C \) values are connected with mean squared deformations of nucleus \( \beta^2 \). In the nuclear state with \( N \) phonons (e.g. see [4])

\[
\beta^2_N = \langle N, J, M | \sum_m \tilde{a}_m^2 | N, J, M \rangle = \frac{\hbar \omega}{2C} (2N + 5).
\]  

(4)

For the ground state of nucleus (\( N = 0 \) state) the rigidity could be estimated as

\[
C = \frac{5\hbar \omega}{4\beta^2_0}.
\]  

(5)

In the review [5] are listed the mean squared deformations \( \beta^2_0 \) for even-even nuclei from \(^{10}\)Be up to \(^{252}\)Cf. These values were extracted from measured transitions probabilities \( 0^+ \rightarrow 2^+ \). This procedure is not completely model independent but dispersion is not larger than 10—15%. It means that shown below results of estimations for \( C \) values based on (5) and \( \beta^2_0 \) from [5] have approximately the same reliability.

Since the rigidities represent the effective surface tensions of nuclei the systems of nucleons with maximal values of \( C \) should have as well the maximal pressure on nuclear matter. In classical physics of liquids the surface tension of liquid drop, pressure on it, and radii of ellipsoid are connected by Laplace’ formula:

\[
p = \sigma (1/R_1 + 1/R_2).
\]  

(6)

Below would be shown results for comparison the nuclear rigidities for even-even isotopes of the same element and their parameters \( r_0 \) [6] for charge radii

\[
r_0 = R_{\text{charge}} \cdot A^{-1/3}
\]  

(7)

As is seen from the Fig. 1 maximal values of nuclear rigidities and consequently of surface tension correlate with minimal values of \( r_0 \). This effect would be demonstrated almost for all investigated even-even nuclei. Since \( r_0 \) correspond to charge radii of nuclei

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under consideration, diminution of $r_0$ marks as well decrease the size of proton's well and increase of charge distributions densities. (The values of used in the fig’s nuclear radii are from [6]).

The comparison of rigidities for some light nuclei shows that for the even-even isotopes rigidity of the nucleus with closed neutron shell is several times larger than for other nuclei of the same element. This effect is especially striking for even-even calcium isotopes (Fig. 2).

The $^{48}\text{Ca}$ nucleus has the maximal rigidity among light nuclei and simultaneously the minimal $r_0$ parameter [7]. (The systematic investigation of radii for sd-shell nuclei [8] as well reveals the minima of $R_{\text{charge}}$ for nuclei with closed neutron shell).

The influence of proton shell occupation is demonstrated in the Fig. 3 where the rigidities for even-even nuclei with $Z$ from 18 up to $Z = 28$ with neutron numbers $N = 28$ are represented.

The comparison of calculated rigidities for even-even nuclei shows very large diversities in their values. If for Calcium isotopes with $A = 40$ and $48$ $C > 600$ MeV, for $^{28}\text{Si}$ it is less than 20 MeV. The low values of surface tension for $^{28}\text{Si}$ apparently lead to deviation of this nucleus from spherical shape. In the [9] the effect of this deviation on the structure of E1 resonance was traced.

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**Fig. 1.** Rigidities and parameters $r_0$ for Carbon (a), Oxygen (b), Silicon (c) and Sulfur (d) isotopes. Right axes—$r_0$ in fm (points connected with thin lines); left axes—rigidities $C$ in MeV (points connected with thick lines).

**Fig. 2.** Rigidities (points connected with solid line) and parameters $r_0$ (points connected with dashed line) for Calcium isotopes.
(It should be mentioned that in the work [10] was performed the calculation of rigidities for \( {}^{40}\text{Ca} \) and \( {}^{48}\text{Ca} \) based on a nuclear level scheme. Obtained values for rigidities are not far from shown in the Fig. 2 but with \( C({}^{40}\text{Ca}) > C({}^{48}\text{Ca}) \)).

Very striking correlation between rigidities and charge radii reveals the comparison of these values for Zirconium isotopes (Fig. 4). There clearly seen not only correlation of \( C \) and minimum of \( r_0 \) for \( {}^{90}\text{Zr} \) but as well the similar effect for \( {}^{96}\text{Zr} \). Moreover the value of \( r_0 \) \( \left( {}^{96}\text{Zr} \right) < r_0 \left( {}^{90}\text{Zr} \right) \). (The \( {}^{96}\text{Zr} \) nucleus has near the nuclear surface two filled neutron shells \( \left| \frac{1}{2}\left| \begin{array}{c} 9/2 \\ 5/2 \end{array} \right| \right. \).

The investigation of \( {}^{96}\text{Zr} \) as “new magic” nucleus with \( N = 56 \) as magic number was recently performed on the basis of spectroscopic data (see [11, 12] and references there).

The firmness of “magic” peculiarities’ in nuclei with neutron numbers \( N = 50 \) and \( N = 56 \) are very contrast. The rigidities of nuclei with \( N = 50 \) are about 3 times higher than for nuclei of the same isotope with number of neutrons \( N - 2 \) or \( N + 2 \). The addition or extracting a pair of neutrons changes effective surface tension drastically, exactly as it take place for \( N = 50 \) in Zr. The comparison \( C \) and \( r_0 \) for nuclei with \( Z = 38 \) (Strontium isotopes) is shown in the Fig. 5. The “magic” number \( N = 50 \) reveals itself as a prominent peak in rigidities distribution. Similar effect was observed for \( Z = 42 \) (Molybdenum isotopes) [7].

In both cases (\( Z = 38 \) and \( Z = 42 \)) the influence of “new magic” number \( N = 56 \) on the rigidity is visible but much weaker than for Zr isotopes. However the effect of high surface tension on charge radii could be clearly seen.

Correlation between rigidities and nuclear charge radii could be demonstrated for all nuclei with number of neutrons near \( N = 82 \). In the Figs. 6 and 7 it is shown for Ba and Ce even-even isotopes.

Effect of growing surface tension on nuclear size could be seen as well on heavy non-magic nuclei. Shown in the Fig. 8 plot of \( C \) and \( r_0 \) parameters for Mercury reflects the rise of pressure on nuclear matter with adding neutron pairs and manifestation of caused compression on nuclear charge distribution.

The distribution of rigidities and \( r_0 \) parameters for Lead even-even isotopes shown in the Fig. 9 demonstrates the maximal values of nuclear rigidities. This is a unique case of a slightly smaller \( C \) for a nucleus with magic \( N = 126 \) neutron numbers than for \( N = 128 \). In the performed method of the \( C \) estimations it is a consequence of very small parameters of quadrupole deformation \( \beta = 0.0224 \) for \( {}^{210}\text{Pb} \) in comparison with \( {}^{208}\text{Pb} \) \( (\beta = 0.0553) \) according to [5].
INFLUENCE OF SURFACE TENSION ON NUCLEAR COLLECTIVE PROPERTIES

CONCLUSIONS

Detected correlations of nuclear rigidities and the nuclear sizes manifest the surface tension' impact on collective properties of nuclei. The addition of neutron pairs to unclosed neutron shell leads to increase of surface tension and growing pressing on the nuclear matter.

Since only charge radii of nuclei are systematically explored, the influence of rigidities on nuclear sizes could be properly traced only for proton’s distributions in nuclei.

The distributions of neutron matter in nuclei with high rigidities are under investigation.

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