Vacuum Energy, EoS, and the Gluon Condensate at Finite Baryon Density in QCD

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Abstract. The Equation of States (EoS) plays the crucial role in all studies of neutron star properties. Still, a microscopical understanding of EoS remains largely an unresolved problem. We use 2-color QCD as a model to study the dependence of vacuum energy (gluon condensate in QCD) as function of chemical potential $\mu \ll \Lambda_{\text{QCD}}$ where we find very strong and unexpected dependence on $\mu$. We present the arguments suggesting that similar behavior may occur in 3-color QCD in the color superconducting phases. Such a study may be of importance for analysis of EoS when phenomenologically relevant parameters (within such models as MIT Bag model or NJL model) are fixed at zero density while the region of study lies at much higher densities not available for terrestrial tests.

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INTRODUCTION

This talk is based on few recent publications with Max Metlitski [1]. Neutron stars represent one of the densest concentrations of matter in our universe. The properties of super dense matter are fundamental to our understanding of nature of nuclear forces as well as the underlying theory of strong interactions, QCD. Unfortunately, at present time, we are not in a position to answer many important questions starting from fundamental QCD lagrangian. Instead, this problem is usually attacked by using some phenomenological models such as MIT Bag model or NJL model. Dimensional parameters (e.g. the vacuum energy) for these models are typically fixed by using available experimental data at zero baryon density. Once the parameters are fixed, the analysis of EoS or other quantities is typically performed by assuming that the parameters of the models (e.g. bag constant) at nonzero $\mu$ are the same as at $\mu = 0$.

The main lesson to be learned from the calculations presented below can be formulated as follows: the standard assumption (fixing the parameters of a model at $\mu = 0$ while calculating the observables at nonzero $\mu$) may be badly violated in QCD.

The problem of density dependence of the chiral and gluon condensates in QCD has been addressed long ago in [2]. The main motivation of ref. [2] was the application of the QCD sum rules technique to study some hadronic properties in the nuclear matter environment. The main result of that studies is– the effect is small. More precisely, at nuclear matter saturation density the change of the gluon condensate is only about 5%. Indeed, in the chiral limit the variation of the gluon condensate with density can be expressed as follows [2],

$$\frac{b g^2}{32 \pi^2} \langle G_{\mu \nu}^a G_{\mu \nu}^{a \ast} \rangle_0 = -m_N \rho_B + 0(\rho_B^2), \quad b = \frac{11N_c - 2N_f}{3},$$

(1)
where the standard expression for the conformal anomaly is used, $\Theta^\mu_\mu = -\frac{b g^2}{32\pi^2} G^a_{\mu\nu} G^{a\mu\nu}$. We should note here that the variation of the gluon condensate is well defined observable (in contrast with the gluon condensate itself) because the perturbative (divergent) contribution cancels in eq. (1). The most important consequences of this formula: a) the variation of the gluon condensate is small numerically, and b) the absolute value of the condensate decreases when the baryon density increases. Such a behavior can be interpreted as due to the suppression of the non-perturbative QCD fluctuations with increase of the baryon density.

Our ultimate goal here is to understand the behavior of the vacuum energy (gluon condensate ) as a function of $\mu$ for color superconducting (CS) phases[3], [4]. It is clear that the problem in this case is drastically different from nuclear matter analysis [2] because the system becomes relativistic and binding energy ($\sim \Delta$) per baryon charge is order of $\Lambda_{QCD}$ in contrast with $\leq \frac{2}{3}$% of the nucleon mass at nuclear saturation density. The quark-quark interaction also becomes essential in CS phases such that the small density expansion (valid for dilute noninteracting nuclear matter) used to derive (1) can not be justified any more.

Unfortunately, we can not answer the questions on $\mu$ dependence of the vacuum energy in real $QCD(N_c = 3)$. However, these questions can be formulated and can be answered in more simple model $QCD(N_c = 2)$ due to the extended symmetry of this model. Some lessons for the real life with $N_c = 3$ can be learned from our analysis, see below.

**GLUON CONDENSATE FOR $QCD(N_c = 2)$**

We start from the equation for the conformal anomaly,

$$\Theta^\mu_\mu = -\frac{b g^2}{32\pi^2} G^a_{\mu\nu} G^{a\mu\nu} + \bar{\psi}M\psi, \quad b = \frac{11}{3}N_c - \frac{2}{3}N_f = 6$$

(2)

For massless quarks and in the absence of chemical potential, eq. (2) implies that the QCD vacuum carries a negative non-perturbative vacuum energy due to the gluon condensate.

Now, we can use the effective Lagrangian [5]

$$\mathcal{L} = \frac{F^2}{2} Tr \nabla_\sigma \Sigma \nabla^\sigma \Sigma', \quad \nabla_0 \Sigma = \partial_0 \Sigma - \mu B\Sigma + \Sigma B^T$$

(3)

to calculate the change in the trace of the energy-momentum tensor $\langle \Theta^\mu_\mu \rangle$ due to a finite chemical potential $\mu \ll \Lambda_{QCD}$. The energy density $\epsilon$ and pressure $p$ are obtained from the free energy density $\mathcal{F}$,

$$\epsilon = \mathcal{F} + \mu n_B, \quad p = -\mathcal{F}.$$  

(4)

Therefore, the conformal anomaly implies,

$$\langle \frac{bg^2}{32\pi^2} G^a_{\mu\nu} G^{a\mu\nu} \rangle_{\mu,m} - \langle \frac{bg^2}{32\pi^2} G^a_{\mu\nu} G^{a\mu\nu} \rangle_0 =$$

$$-4(\mathcal{F}(\mu,m) - \mathcal{F}_0) - \mu n_B(\mu,m) + \langle \bar{\psi}M\psi \rangle_{\mu,m}.$$  

(5)
where the subscript 0 on an expectation value means that it is evaluated at $\mu = m = 0$. Now we notice that all quantities on the right hand side are known from the previous calculations $^5$, therefore the variation of $\langle G^2_{\mu\nu} \rangle$ with $\mu$ can be explicitly calculated. As expected, $\langle G^2_{\mu\nu} \rangle$ does not depend on $\mu$ in the normal phase $\mu < m_\pi$ while in the superfluid phase $\mu > m_\pi$ this dependence can be represented as follows $^1$,

$$
\langle \frac{bg^2}{32\pi^2} G^a_{\mu\nu} G^{\mu\nu a} \rangle_{\mu,m} - \langle \frac{bg^2}{32\pi^2} G^a_{\mu\nu} G^{\mu\nu a} \rangle_{\mu=0,m} = 4F^2 (\mu^2 - m_\pi^2) \left( 1 - \frac{2m_\pi^2}{\mu^2} \right). \tag{6}
$$

The behavior of the condensate is quite interesting: it decreases with $\mu$ for $m_\pi < \mu < 2^{1/4}m_\pi$ and increases afterwards. The qualitative difference in the behaviour of the gluon condensate for $\mu \approx m_\pi$ and for $m_\pi \ll \mu \ll \Lambda_{QCD}$ can be explained as follows. Right after the normal to superfluid phase transition occurs, the baryon density $n_B$ is small and our system can be understood as a weakly interacting gas of diquarks. The pressure of such a gas is negligible compared to the energy density, which comes mostly from diquark rest mass. Thus, $\langle \Theta^\mu_\mu \rangle$ increases with $n_B$ in precise correspondence with the “dilute” nuclear matter case $^1$. On the other hand, for $\mu \gg m_\pi$, energy density is approximately equal to pressure, and both are mostly due to self-interactions of the diquark condensate. Luckily, the effective Chiral Lagrangian (3) gives us control over these self-interactions as long as $\mu \ll \Lambda_{QCD}$.

The main lesson to be learned for real $QCD(N_c = 3)$ from exact results discussed above is as follows. The transition to the CS phases is expected to occur $^1$ at $\mu_c \simeq 2.3\cdot\Lambda_{QCD}$ $^6, ^7$ in contrast with $\mu_c = m_\pi$ for transition to superfluid phase for $N_c = 2$ case. The binding energy, the gap, the quasi -particle masses are also expected to be the same order of magnitude $\sim \mu_c$. This is in drastic contrast with nuclear matter case when binding energy is very small. At the same time, $QCD(N_c = 2)$ represents a nice model where the binding energy, the gap, the masses of quasi -particles carrying the baryon charge are the same order of magnitude. This model explicitly shows that the gluon condensate can experience extremely nontrivial behavior as function of $\mu$. We expect a similar behavior for $QCD(N_c = 3)$ in CS phases when function of $m_\pi^2/\mu^2$ in (6) is replaced by some function of $\mu_c/\mu^2$ for $N_c = 3$. We should note in conclusion that the recent lattice calculations $^8, ^9$ are consistent with our prediction (6).

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$^1$ $\mu$ here for $QCD(N_c = 3)$ is normalized as the quark (rather than the baryon) chemical potential