Cosmological constraints on interacting dark energy with redshift-space distortion after Planck data

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The interacting dark energy model could propose a effective way to avoid the coincidence problem. In this paper, dark energy is taken as a fluid with a constant equation of state parameter \( w_x \). In a general gauge, we could obtain two sets of different perturbation equations when the momentum transfer potential is vanished in the rest frame of dark matter or dark energy. There are many kinds of interacting forms from the phenomenological considerations, here, we choose \( Q = 3H\xi_x\beta\rho_x \) which owns the stable perturbations in most cases. Then, according to the Markov Chain Monte Carlo method, we constrain the model by currently available cosmic observations which include cosmic microwave background radiation, baryon acoustic oscillation, type Ia supernovae, and \( f\sigma_8(z) \) data points from redshift-space distortion. Joining the geometry tests with the large scale structure information, the results show a tighter constraint on the interacting model than the case without \( f\sigma_8(z) \) data. We find the interaction rate in 3\( \sigma \) regions: \( \xi_x = 0.00372^{+0.000768}_{-0.00072} \), \( f\sigma_8 = 0.0055 + 0.0102 \). It means that the recently cosmic observations favor small interaction rate between the dark sectors, at the same time, the measurement of redshift-space distortion could rule out large interaction rate in 1\( \sigma \) region.

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I. INTRODUCTION

In March 2013, the Planck Collaboration and European Space Agency (ESA) publicly released the new and precise measurements of the cosmic microwave background (CMB) radiation in a wide range of multiples (\( l < 2500 \)) [1–4]. There is no doubt that this data will improve the accuracy of constraining the cosmological models. After Planck data, the CMB data sets include two main parts: one is the low-l (up to a maximum multipole number of \( l = 49 \)) and high-l (from \( l = 50 \) to \( l = 2500 \)) temperature power spectrum likelihood from Planck; the other is the low-l (up to \( l = 32 \)) polarization power spectrum likelihood from nine-year Wilkinson Microwave Anisotropy Probe (WMAP9) [5]. The observational constraints on the standard model from the CMB data show us that the Universe is composed by 68% dark energy, 28% dark matter, and 4% baryons [3].

The Planck data are in good agreement with the \( \Lambda \)CDM model which is composed by the cosmological constant and cold dark matter (CDM), especially for the high multiples (\( l > 40 \)). However, the standard scenario itself is encountering the coincidence problem [4–8], which points out the fact that there is no reasonable explanation why the energy densities of vacuum energy and dark matter are of the same order today. In order to avoid this issue, one direct way is to describe dark energy as a fluid and consider its equation of state (EoS) \( w_x \) as a free parameter. This model is usually called as the \( w \)CDM model. Constraints on this extensional model from the CMB and baryon acoustic oscillation (BAO) data sets present that \( w_x = -1.13^{+0.24}_{-0.25} \) with 95% confidence levels (C.L.) [3].

Alternative powerful mechanism to alleviate the coincidence problem is to consider the interaction between dark matter and dark energy. Firstly, the standard model of particle physics thinks the interaction within the dark sectors could be a natural choice, uncoupled case would be an additional assumption on some model [4]. It is worth looking forward to obtain the concrete form of interaction from the first principles. However, this idea is scarcely possible because the physical nature of dark matter and dark energy are still unknown. In most cases, one could assume the form of interaction from the phenomenological considerations. A satisfactory interacting model at least requires that the interacting form \( Q \) should be expressed with respect to the energy densities of dark fluids and other covariant quantities, some possibilities of the interaction between the dark sectors have been widely discussed in Refs. [10–38]. Roughly, we divide these works into three main types. Interacting model (I) is \( Q = \beta\rho_\phi \) or \( Q = \beta(\phi)\rho_\phi \) [10–38] which might be motivated within the context of scalar-tensor theories. Although model (I) could have a significant physical motivation, but it meets with a challenge [28]: the accelerated scaling attractor is not connected to a matter era where structure grows in the standard way. Far from this defect, some other interacting models have been suggested and discussed. Interacting model (II) is \( Q = \Gamma_c\rho_c \), \( Q = \Gamma_x\rho_x \), or \( Q = \Gamma_c\rho_c + \Gamma_x\rho_x \) [34, 52] which is not...
TABLE I: The data points of $f\sigma_8(z)$ measured from RSD with the survey references. The former nine data points at $z \in [0.067, 0.78]$ were summarized in Ref. [103]. The data point at $z = 0.8$ was released by the VIPERS in Ref. [104]. Then, a lower growth rate from RSD than expected from Planck was also pointed out in Ref. [103].

| $z$   | $f\sigma_8(z)$ | Survey and Refs |
|-------|----------------|-----------------|
| 0.067 | 0.42 ± 0.06    | 6dFGRS (2012) [102] |
| 0.17  | 0.51 ± 0.06    | 2dFGRS (2004) [98] |
| 0.22  | 0.42 ± 0.07    | WiggleZ (2011) [99] |
| 0.25  | 0.39 ± 0.05    | SDSS LRG (2011) [100] |
| 0.37  | 0.43 ± 0.04    | SDSS LRG (2011) [100] |
| 0.41  | 0.45 ± 0.04    | WiggleZ (2011) [99] |
| 0.57  | 0.43 ± 0.03    | BOSS CMASS (2012) [101] |
| 0.60  | 0.43 ± 0.04    | WiggleZ (2011) [99] |
| 0.78  | 0.38 ± 0.04    | WiggleZ (2011) [99] |
| 0.80  | 0.47 ± 0.04    | VIPERS (2013) [104] |

in the light of physical interaction between the dark sectors but is assumed for mathematical simplicity. $\Gamma_c$ or $\Gamma_x$ is a constant interaction rate which is determined by local interactions. Furthermore, if one considers interaction could be influenced by the expansion rate $H$ of the Universe, interacting model (III) could be designed as $Q = 3H\xi_c\rho_c$, $Q = 3H\xi_x\rho_x$, or $Q = 3H(\xi_c\rho_c + \xi_x\rho_x)$ [58, 59]. This kind of model could produce an accelerated scaling attractor which might be connected to a standard matter era [58]. Apart from the three main types of interacting models, some other generalized interacting forms have been studied in Refs. [86–97].

Interacting dark energy could exert a non-gravitational ‘drag’ on dark matter, which will influences the evolution of matter density perturbations and the expansion history of the Universe. It means that some new features could be introduced into structure formation [12, 45, 49, 50, 55, 89]. So, in the process of exploring the interaction, it is necessary to consider the effects on the cosmological constraints from the large scale structure information. Moreover, comparing with the geometry information (CMB, BAO, and type Ia supernovae (SNIa)), the large scale structure information is a powerful tool to break the possible degeneracy of cosmological models, because the dynamical growth history of different models could be distinct even if they might undergo similar background evolution behavior. Based on the redshift-space distortion (RSD), the currently observed $f\sigma_8$ data could be closely associated with the evolution of matter density perturbations $\delta_m$ via the relation $f_m = d\ln\delta_m/d\ln a$, but it depends on the $\Lambda$CDM model. To keep away from this disadvantage, Song and Percival suggested to constrain the dark energy models by use of the model-independent $f\sigma_8(z)$ measurement [102], in which $\sigma_8$ is the root-mean-square mass fluctuation in spheres with radius $8h^{-1}\text{Mpc}$. Inspired by this paper, Xu combined the $f\sigma_8(z)$ data with the geometry measurements to constrain the holographic dark energy model in Ref. [107]. After Planck data, Xu compared the deviation of growth index $\gamma_L$ (the growth function is parameterized as $f = \Omega_L^\gamma$) in the Einstein’s gravity theory and modified gravity theory [108] and confronted Dvali-Gabadadze-Porrati braneworld gravity with the RSD measurement [109]. Besides, Yang and Xu explored the possible existence of warm dark matter from $f\sigma_8(z)$ test [110], and Yang et al. constrained a decomposed dark fluid with constant adiabatic sound speed by combining the RSD data with the geometry tests [111]. All the above constraints on the cosmological models from the RSD test [107, 111] obtained tighter constraints on the model parameter space than the case without the $f\sigma_8(z)$ data. Up to now, the ten observed data points of $f\sigma_8(z)$ are shown in Table I.

The interaction rate should be determined by the cosmic observations. After Planck data is released, several interacting dark energy models have been constrained by the recently cosmic observations [39, 61, 93, 95]. In our cosmological constraints, the CMB data is from Planck [3] and WMAP9 [3]. We use the measured ratio of $r_s/D_v$ as a ‘standard ruler’ to adopt the BAO data, according to the section 5.2 of Ref. [3], the concrete values at three different redshifts are, $r_s/D_v(z = 0.106) = 0.336 \pm 0.015$ [112], $r_s/D_v(z = 0.35) = 0.1126 \pm 0.0022$ [113], and $r_s/D_v(z = 0.57) = 0.0732 \pm 0.0012$ [114]. For the SNIa data, we use the SNLS3 data which is composed by 472 SN calibrated by SFTO and SALT2 [113, 115]. The geometry measurements slightly favor the interaction between dark matter and dark energy, meanwhile, the growth rate of dark matter perturbations possibly rules out large interaction rate which was pointed out in Ref. [53]. This would allow the use of the large scale structure information, which would significantly improve the constraints on the interacting models. So, in this paper, we will try to add the RSD measurement to constrain the interacting model. It is worthwhile to anticipate that the large scale structure measurement will give a tight constraint on the parameter space.

The outline of this paper is as follows. In section II, in the background evolution, the interaction between the
dark sectors could lead to the changes in the effective EoS of dark energy. Then, in a general gauge, via choosing the rest frame of dark matter or dark energy, two sets of different perturbation equations could be given by the vanishing momentum transfer potential. Furthermore, the stability of the perturbations determines the interacting form \( Q = 3H\xi_x\rho_x \) as our research emphasis, the model parameter \( \xi_x \) is also called as the interaction rate in this paper. In section III, when the interaction rate was varied, we presented the cosmological implications on the CMB temperature power spectra, matter power spectra, and evolution curves of \( f\sigma_8(z) \). Based on the Markov Chain Monte Carlo (MCMC) method, we performed the cosmological constraints on the IwCDM model (the wCDM model with interaction between the dark sectors). Section IV is the conclusion.

II. THE BACKGROUND EQUATIONS AND PERTURBATION EQUATIONS

When the interaction \( Q \) between the dark sectors is considered, one can write the evolution equations for the energy densities of dark matter and dark energy as,

\[
\rho'_c + 3H\rho_c = aQ_c = -aQ, \tag{1}
\]

\[
\rho'_x + 3H(1 + w)x = aQ_x = aQ, \tag{2}
\]

where the prime denotes the derivative with respect to conformal time \( \tau \) and the subscript \( c \) and \( x \) respectively stand for dark matter and dark energy. \( w_x = p_x/\rho_x \) and \( H = d\ln a/d\tau \). \( Q \) represents the rate of energy density transfer, so \( Q > 0 \) means that the direction of energy transfer is from dark matter to dark energy, \( Q < 0 \) implies the opposite situation. Based on the above two equations, we could define the effective EoS of dark matter and dark energy,

\[
w_{c,\text{eff}} = \frac{aQ}{3H\rho_c}, \quad w_{x,\text{eff}} = w - \frac{aQ}{3H\rho_x}, \tag{3}
\]

when we consider the dark energy as quintessence case \((w \geq -1)\) and \( Q > 0 \), the effective EoS \( w_{x,\text{eff}} \) could cross the phantom divide \((w = -1)\), this interacting quintessence behaves like a uncoupled 'phantom' model, moreover, does not have any negative kinetic energies. At the same time, the possible existence of this case might be influenced by the instability of the perturbations.

In a general gauge, the perturbed Friedmann-Robertson-Walker (FRW) metric is \( 51, 53, 55 \)

\[
ds^2 = a^2(\tau)\{-(1 + 2\phi)d\tau^2 + 2\partial_iBd\tau dx^i + [(1 - 2\psi)\delta_{ij} + 2\partial_i\partial_jE]dx^idx^j\}, \tag{4}
\]

where \( \phi, B, \psi \) and \( E \) are the gauge-dependent scalar perturbations quantities.

The four-velocity of \( A \) fluid is given by \( 51, 53, 55 \)

\[
u_A^\mu = a\sum_{\nu}Q_A^\nu = 0, \tag{6}
\]

where \( T_A^{\mu\nu} \) represents the \( A \)-fluid energy-momentum tensor. When \( Q_A \) and \( E_A^\nu \), respectively, represent the energy and momentum transfer rate, relative to the four-velocity \( u_A^\mu \), one has \( 51, 53, 55 \)

\[
Q_A^\mu = \tilde{Q}_A^\mu + E_A^\nu, \tag{7}
\]

where \( \tilde{Q}_A = Q_A + \delta Q_A \) and \( E_A^\nu = a^{-1}(0, \partial^i f_A) \). \( Q_A \) is the background term of the general interaction, and \( f_A \) is a momentum transfer potential. The perturbed energy-momentum transfer four-vector can be split as \( 51, 53, 55 \)

\[
Q_A^0 = -a[Q_A(1 + \phi) + \delta Q_A], \quad Q_A^i = a\partial_i[Q_A(v + B) + f_A], \tag{8}
\]

The perturbed energy and momentum balance equations are \( 51, 53 \)

\[
\delta\rho_A' + 3H(\delta\rho_A + \delta p_A) - 3(\rho_A + p_A)\psi' - k^2(\rho_A + p_A)(v + E') = aQ_A\phi + a\delta Q_A, \tag{9}
\]
\[ \delta p_A + [(\rho_A + p_A)(v_A + B)]' + 4\mathcal{H}(\rho_A + p_A)(v_A + B) + (\rho_A + p_A)\phi - \frac{2}{3}k^2\rho_A\pi_A = aQ_A(v + B) + af_A, \] (10)

Defining the density contrast \( \delta_A = \delta\rho_A/\rho_A \) and considering \( \pi_A = 0 \), one has the general evolution equations for density perturbations (continuity) and velocity perturbations (Euler) equations for a fluid [51, 53, 55].

\[ \delta'_A + 3\mathcal{H}(c^2_{sA} - w_A)\delta_A + 9\mathcal{H}^2(1 + w_A)(c^2_{sA} - c^2_A)\frac{\theta_A}{k^2} + (1 + w_A)\theta_A - 3(1 + w_A)\psi' + (1 + w_A)k^2(B - E') = \frac{a}{\rho_A}(-Q_A\delta_A + \delta Q_A) + \frac{aQ_A}{\rho_A}\left[\phi + 3\mathcal{H}(c^2_{sA} - c^2_A)\frac{\theta_A}{k^2}\right], \]

(11)

\[ \theta'_A + \mathcal{H}(1 - 3c^2_A)\theta_A - \frac{c^2_A}{(1 + w_A)}k^2\delta_A - k^2\phi = \frac{a}{(1 + w_A)\rho_A}[(Q_A\theta - k^2f_A) - (1 + c^2_A)Q_A\theta_A], \]

(12)

where \( c^2_A \) is the adiabatic sound speed whose definition is \( c^2_A = \rho'_A/\rho_A = w_x + w_x/(\rho'_A/\rho_A) \), and \( c^2_A \) is the A-fluid physical sound speed in the rest frame, its definition is \( c^2_{sA} = (\delta\rho_A/\delta\rho_A)_\text{rest frame} \) [53, 118, 120]. In order to avoid the unphysical instability, \( c^2_{sA} \) should be taken as a non-negative parameter [53].

Next, we need to specialize the energy and momentum transfer rate between the dark sectors. In order to find the perturbation equations which apply to the interacting models (II) and (III), firstly, we specialize the momentum transfer potential as the simplest physical choice which is zero in the rest frame of either dark matter or dark energy [50, 53]. This leads to two cases of simple interacting model which include energy transfer four-vector parallel to the four-velocity of dark matter or dark energy. In the light of Refs. [50, 52], the momentum transfer potential \( f_A \) is

\[ k^2f_A = Q_A(\theta - \theta_c), \text{ for } Q^e_A \parallel u^e_c, \]

(13)

\[ k^2f_A = Q_A(\theta - \theta_x), \text{ for } Q^e_A \parallel u^e_x, \]

(14)

furthermore, introducing a simple parameter of ‘choosing the momentum transfer’ \( b \) [54]

\[ b = \begin{cases} 1, & \text{for } Q^e_A \parallel u^e_c, \\ 0, & \text{for } Q^e_A \parallel u^e_x, \end{cases} \]

in the rest frame of dark matter or dark energy, the momentum transfer potential \( f_A \) could be unified as

\[ k^2f_A = Q_A[b(\theta - \theta_c) + (1 - b)(\theta - \theta_x)] = Q_A[\theta - b\theta_c - (1 - b)\theta_x], \]

(15)

Substituting the above relation into Eqs. (11) and (12), the continuity and Euler equations of A fluid could be reduced to

\[ \delta'_x + 3\mathcal{H}(c^2_{sA} - w_x)\delta_x + 9\mathcal{H}^2(1 + w_x)(c^2_{sA} - c^2_A)\frac{\theta_x}{k^2} + (1 + w_x)\theta_x - 3(1 + w_x)\psi' + (1 + w_x)k^2(B - E') = \frac{a}{\rho_x}(-Q_x\delta_x + \delta Q_x) + \frac{aQ_x}{\rho_x}\left[\phi + 3\mathcal{H}(c^2_{sA} - c^2_A)\frac{\theta_x}{k^2}\right], \]

(16)

\[ \theta'_x + \mathcal{H}(1 - 3c^2_{sA})\theta_x - \frac{c^2_{sA}}{(1 + w_x)}k^2\delta_x - k^2\phi = \frac{a}{(1 + w_x)\rho_x}[(b\theta_c - (1 - b)\theta_x - (1 + c^2_A)\theta_x], \]

(17)

For the IwCDM model, \( c^2_{sc} = c^2_{ac} = w_c = 0 = w_x' \) and \( c^2_{ax} = w_x \), so the continuity and Euler equations become

\[ \delta'_x + 3\mathcal{H}(c^2_{sA} - w_x)\delta_x + 9\mathcal{H}^2(1 + w_x)(c^2_{sA} - w_x)\frac{\theta_x}{k^2} + (1 + w_x)\theta_x - 3(1 + w_x)\psi' + (1 + w_x)k^2(B - E') = \frac{a}{\rho_x}(-Q_x\delta_x + \delta Q_x) + \frac{aQ_x}{\rho_x}\left[\phi + 3\mathcal{H}(c^2_{sA} - w_x)\frac{\theta_x}{k^2}\right], \]

(18)

\[ \delta'_c + \theta_c - 3\psi' + k^2(B - E') = -\frac{a}{\rho_c} (Q_c\delta_c - \delta Q_c) + \frac{aQ_c}{\rho_c}\phi, \]

(19)
\[ \theta_x' + \mathcal{H}(1 - 3c_{sx}^2)\theta_x - \frac{c_{sx}^2}{(1 + w_x)} k^2\delta_x - k^2\phi = \frac{\alpha Q_x}{(1 + w_x)\rho_x} [b\theta_c + (1 - b)\theta_x - (1 + c_{sx}^2)\theta_x], \quad (20) \]

\[ \theta_c' + \mathcal{H}\theta_c - k^2\phi = -\frac{\alpha Q_c}{\rho_c} (1 - b)(\theta_c - \theta_x), \quad (21) \]

When the interaction is introduced, the instability of the perturbations becomes an important topic. In most cases, the energy transfer rate \( Q = \Gamma_x\rho_x \) or \( Q = 3H\xi_x\rho_x \) owns the stable perturbations. In this paper, we will choose the interacting model (III) as our research emphasis, so we take the interacting form as \( Q = 3H\xi_x\rho_x \). So, we have \( Q_x = -Q_c = 3H\xi_x\rho_x \) and \( \delta Q_x = -\delta Q_c = 3H\xi_x\rho_x\delta_x \). At the moment, the continuity and Euler equations could be recast into

\[ \delta_x' + 3\mathcal{H}(c_{sx}^2 - w_x)\delta_x + 9\mathcal{H}^2(1 + w_x)(c_{sx}^2 - w_x)\theta_x + (1 + w_x)\theta_x - 3(1 + w_x)\psi' + (1 + w_x)k^2(B - E') = 3\mathcal{H}\xi_x\phi + 9\mathcal{H}^2(c_{sx,eff}^2 - w_x)\xi_x\theta_x, \quad (22) \]

\[ \delta_c' + \theta_c - 3\psi' + k^2(B - E') = 3\mathcal{H}\xi_x\rho_x(\delta_c - \delta_x) - 3\mathcal{H}\xi_x\rho_x\phi, \quad (23) \]

\[ \theta_x' + \mathcal{H}(1 - 3c_{sx}^2)\theta_x - \frac{c_{sx}^2}{(1 + w_x)} k^2\delta_x - k^2\phi = \frac{3\mathcal{H}\xi_x}{1 + w_x} [b(\theta_c - \theta_x) - c_{sx,eff}\theta_x], \quad (24) \]

\[ \theta_c' + \mathcal{H}\theta_c - k^2\phi = 3\mathcal{H}\xi_x\rho_x(\theta_c - \theta_x), \quad (25) \]

Moreover, one could judge the stability of the perturbations via the doom factor. Here, we also define the doom factor for our IwCDM model

\[ d \equiv \frac{-Q}{3H\rho_x(1 + w_x)} = \frac{-\xi_x}{1 + w_x}, \quad (26) \]

according to the conclusion of Refs. \([53, 59]\): when \( d < 0 \), the stable perturbations could be acquired for the interacting form \( Q = 3H\xi_x\rho_x \). It means that the perturbation stability requires the conditions \( \xi_x > 0 \) and \((1 + w_x) > 0 \) or \( \xi_x < 0 \) and \((1 + w_x) < 0 \). Here, in order to avoid the phantom doomsday \([121]\), we would discuss the stable case of \( \xi_x > 0 \) and \((1 + w_x) > 0 \).

In the synchronous gauge \((\phi = B = 0, \psi = \eta, \text{ and } k^2E = -h/2 - 3\eta)\), we rewrite the continuity and Euler equations as

\[ \delta_x' + (1 + w_x)\left(\theta_x + \frac{h'}{2}\right) + 3\mathcal{H}(c_{sx}^2 - w_x)\delta_x + 9\mathcal{H}^2(1 + w_x)(c_{sx}^2 - w_x)\theta_x = 9\mathcal{H}^2(c_{sx}^2 - w_x)\xi_x\theta_x, \quad (27) \]

\[ \delta_c' + \theta_c + \frac{h'}{2} = 3\mathcal{H}\xi_x\rho_x(\delta_c - \delta_x), \quad (28) \]

\[ \theta_x' + \mathcal{H}(1 - 3c_{sx}^2)\theta_x - \frac{c_{sx}^2}{1 + w_x} k^2\delta_x = \frac{3\mathcal{H}\xi_x}{1 + w_x} [b(\theta_c - \theta_x) - c_{sx}^2\theta_x], \quad (29) \]

\[ \theta_c' + \mathcal{H}\theta_c = 3\mathcal{H}\xi_x\rho_x(1 - b)(\theta_c - \theta_x), \quad (30) \]

As usual, one does not consider the interaction between baryons and dark energy, so the interaction will lead to a bias between the density perturbations of dark matter and baryons. According to \( \delta_m = (\rho_0\delta_c + \rho_b\delta_b)/(\rho_c + \rho_b) \), the growth function of total matter is \( f_m = d\ln\delta_m/d\ln a = \delta_{m}'/(\mathcal{H}\delta_m) \). In order to adopt the RSD measurement, we modify the CAMB code \([122]\) and CosmoMC package \([123]\), and add a new module to calculate the theoretical values of \( f\sigma_8(z) \). For some details, please see Refs. \([107, 111]\).
A. Theoretical predictions of CMB temperature, matter power spectra, and $f\sigma_8(z)$ evolution

When the interaction between the dark sectors is considered, some cosmological effects could take place, so we try to look for theoretical predictions of CMB temperature power spectra, matter power spectra, and the evolution curves of $f\sigma_8(z)$. Here, the cosmological implications have been discussed under the stability condition of the perturbations. When the interaction rate $\xi_x$ is varied, the influences on the CMB temperature power spectra are presented in Fig. 1. According to Eqs. (1) and (3), increasing the value of $\xi_x$, which corresponds to enlarge the positive $w_{c,eff}$, dark matter redshifts faster than the uncoupled evolution $a^{-3}$, which make the equality of matter and radiation earlier; hence, the sound horizon is decreased. As a result, the first peak of CMB temperature power spectra is depressed. At large scales $l < 100$, the integrated Sachs-Wolfe (ISW) effect is dominant, the changed parameter $\xi_x$ affects the CMB power spectra via ISW effect due to the evolution of gravitational potential. In Fig. 2, we plot the influence on the matter power spectrum $P(k)$ for the different values of interaction rate $\xi_x$. The evolution law is opposite to the CMB temperature power spectra. With increasing the values of $\xi_x$, the matter power spectra $P(k)$ is enhanced due to the earlier matter-radiation equality. The case of $\xi_x = 0.00372$ (corresponds the IwCDM model with mean value) and that of $\xi_x = 0$ (corresponds to the uncoupled wCDM model) are almost the same.

Then, we keep the interaction rate $\xi_x$ varied in the positive axis and fix the other model parameters according to Table II, we plot the evolution curves of $f\sigma_8(z)$ in Fig. 3. At both lower and higher redshifts, the curves of $f\sigma_8(z)$ are enhanced with the increasing the values of $\xi_x$. Particularly, it is easy to see that the case of $\xi_x = 0.00372$ (corresponds the IwCDM model with mean value) and that of $\xi_x = 0$ (corresponds to the uncoupled wCDM model) are significantly distinguishing from the evolution curves of $f\sigma_8(z)$, which is different from the evolutionary curves of CMB temperature and matter power spectra. It means that, to some extent, the RSD test could break the possible degeneracy between the IwCDM model and the uncoupled wCDM model.
FIG. 2: The effects on matter power spectra for the different values of interaction rate $\xi_x$. The black solid, red thick dashed, green dotted-dashed, and blue dotted lines are for $\xi_x = 0, 0.00372, 0.1$, and $0.2$, respectively; the other relevant parameters are fixed with the mean values as shown in the fifth column of Table I.

FIG. 3: The fitting evolutionary curves of $f\sigma_8(z)$ about the redshift $z$ for varied interaction rate $\xi_x$. The black solid, red thick dashed, green dotted-dashed, and blue dotted lines are for $\xi_x = 0, 0.00372, 0.01$, and $0.02$, respectively; the gray error bars denote the observations of $f\sigma_8(z)$ at different redshifts are listed in Table II; the other relevant parameters are fixed with the mean values as shown in the fifth column in Table II.
B. Cosmological constraint results

In our numerical calculations, the total likelihood $\chi^2$ can be constructed as

$$\chi^2 = \chi^2_{CMB} + \chi^2_{BAO} + \chi^2_{SNIa} + \chi^2_{RSD}.$$  

For the IwCDM model, we consider the eight-dimensional parameter space which reads

$$P \equiv \{\Omega_b h^2, \Omega_c h^2, \Theta_s, \tau, w_x, \xi_x, n_s, \log[10^{10} A_s]\},$$

where $\Omega_b h^2$ and $\Omega_c h^2$, respectively, stand for the density of the baryons and dark matter, $\Theta_s = 100\theta_{MC}$ refers to the ratio of sound horizon and angular diameter distance, $\tau$ indicates the optical depth, $w_x$ is the EoS of dark energy, $\xi_x$ is the interaction rate between the dark sectors, $n_s$ is the scalar spectral index, and $A_s$ represents the amplitude of the initial power spectrum. The priors to the basic model parameters are listed in the second column of Table II. Here, the pivot scale of the initial scalar power spectrum $k_{s0} = 0.05 Mpc^{-1}$ is used. Then, based on the MCMC method, we modify the publicly available CAMB code [122] and CosmoMC package [123] based on the perturbation equations in the synchronous gauge, and perform a global fitting for the interacting model with $Q_A^\mu \parallel w_x^\mu$ when the model parameters satisfy $\xi_x > 0$ and $(1 + w_x) > 0$. Here, we choose $c_{sx}^2 = 1$ which could avoid the unphysical sound speed [51, 53, 55].

After running eight chains in parallel on the computer, the constraint results for the IwCDM model are, respectively, presented in the fifth and sixth columns of Table II. We show the one-dimensional (1D) marginalized distributions of parameters and two-dimensional (2D) contours with 68% C.L., 95% C.L., and 99% C.L. in Figs. 4. We anticipate that the large scale structure test will give a tighter constraint on the parameter space than before. In order to compare with the constraint without RSD data, we also constrain the IwCDM model without the $f\sigma_8(z)$ data set, the results are shown in the third and fourth columns of Table II.

Here, we pay attention to the constraint result of the interaction rate. In the third of Table II, we find the interaction rate $\xi_x = 0.209^{+0.0711}_{-0.0403}$ in 1σ region. Some similar constraint results have been presented in the previous papers. Before Planck data, $Q = \Gamma_x \rho_x$ (belongs to the interacting model (III)) was considered in Ref. [52], the interacting dark energy with a constant EoS has been constrained by CMB from WMAP7 [124], BAO [125], HST (Hubble Space Telescope) [126] and SNIa from SDSS [127], the results of $Q_A^\mu \parallel w_x^\mu$ showed that the best-fit value of interaction rate was $\Gamma_x / H_0 = 0.366$. After Planck data, in Ref. [39], the perturbed expansion rate of the Universe and the interacting form $Q = H \xi_x \rho_x$ was considered, this interacting model has been tested by CMB from Planck + WMAP9 [3, 5], BAO [112, 114] and HST [128]. The constraint results from CMB and BAO presented that the mean values of interaction rate was $\xi_x = -0.61^{+0.12}_{-0.25}$ from CMB and BAO measurements, and $\xi_x = -0.67^{+0.086}_{-0.17}$ from CMB and HST tests (the minus is from the background evolution equations of dark matter and dark energy).

In brief summary, the geometry tests which mainly includes CMB, BAO, SNIa, and HST slightly favor the interaction between dark matter and dark energy. Meanwhile, the growth rate of dark matter perturbations possibly rules out large interaction rate which was pointed out in Ref. [55]. Instead of the case without RSD data, the large scale structure information evidently influences the expansion history of the Universe and the evolution of matter density perturbations, the parameter space of the interacting model is greatly improved. As expected, from the fifth column of Table II, we find the recently cosmic observations indeed favor small interaction rate $\xi_x = 0.00372^{+0.000768}_{-0.00372}$ after the RSD measurement is added. To some extent, the $f\sigma_8(z)$ test could rule out large interaction rate.

Furthermore, based on the same observed data sets (CMB from Planck + WMAP9, BAO, SNIa and RSD), the IwCDM model has another two parameters $w_x$ and $\xi_x$ which give rise to the difference of the minimum $\chi^2$ with the $\Lambda$CDM model, $\Delta\chi^2_{min} = 2.819$.

IV. SUMMARY

In this paper, we considered a type of interaction which was relative to the expansion rate $H$ of the Universe. When the interaction were introduced, the effective EoS of dark energy brought about the departure from the uncoupled case. In a general gauge, via introducing the parameter of ’choosing the momentum transfer’ $b$ for A fluid, we obtained two sets of different perturbation equations in the rest frame of dark matter or dark energy. Furthermore, in the synchronous gauge, based on the interaction form $Q = 3H\xi_x \rho_x$ whose perturbation equations were stable in most cases, the continuity and Euler equations were gained for the IwCDM model. According to the density perturbations of dark matter and baryons, we added a module to calculate the theoretical values of $f\sigma_8(z)$ which could be used to constrain...
the IwCDM model. In the aspect of theoretical predictions, we have plotted the effects of the varied interaction rate on CMB power spectra, matter power spectra and the evolution equations of $f\sigma_8(z)$. From $f\sigma_8(z)$ evolution curves, we could clearly distinguish from the IwCDM model with mean value to the uncoupled wCDM model, meanwhile, the CMB and matter power spectra could not make it. It meant that, to some extent, $f\sigma_8(z)$ could break the possible degeneracy of the cosmological models. Based on the MCMC method, we constrained the interacting model by CMB from Planck + WMAP9, BAO, SNia, with or without RSD data sets have been used.

| Parameters | Priors | IwCDM without RSD | Best fit | IwCDM with RSD | Best fit | ΛCDM with RSD | Best fit |
|------------|--------|-------------------|----------|----------------|----------|----------------|----------|
| $\Omega_b h^2$ | 0.0050, 0.1 | 0.00220$^{+0.00244}_{-0.00257}$ | 0.00221 | 0.00224$^{+0.00253}_{-0.00246}$ | 0.00223 | 0.00224$^{+0.00245}_{-0.00249}$ | 0.00225 |
| $\Omega_c h^2$ | 0.010, 0.99 | 0.0390$^{+0.0154}_{-0.0171}$ | 0.0464 | 0.0468$^{+0.0217}_{-0.0225}$ | 0.0474 | 0.0480$^{+0.0237}_{-0.0240}$ | 0.0498 |
| $\Omega_{DE}h^2$ | 0.0005, 0.0030 | 0.00035 | 0.00050 | 0.00055 | 0.00060 | 0.00065 | 0.00070 |
| $\tau$ | 0.010, 0.8 | 0.0882$^{+0.0548}_{-0.0599}$ | 0.0828 | 0.0862$^{+0.0655}_{-0.0695}$ | 0.0910 | 0.0950$^{+0.0784}_{-0.0835}$ | 0.0999 |
| $\xi$ | 0.1 | 0.2099$^{+0.0971}_{-0.0995}$ | 0.230 | 0.0932$^{+0.0950}_{-0.0955}$ | 0.0950 | 0.0950$^{+0.0952}_{-0.0960}$ | 0.0960 |
| $w_x$ | -1.0 | -0.9407$^{+0.0112}_{-0.0144}$ | -0.998 | -0.977$^{+0.0109}_{-0.0145}$ | -0.995 | -0.995$^{+0.0109}_{-0.0145}$ | -0.997 |
| $n_s$ | 0.5, 1.5 | 0.967$^{+0.0244}_{-0.0190}$ | 0.976 | 0.976$^{+0.0229}_{-0.0217}$ | 0.976 | 0.976$^{+0.0229}_{-0.0217}$ | 0.976 |
| $\ln(10^{10} A_s)$ | 2.24 | 3.0974$^{+0.0244}_{-0.0190}$ | 3.0910 | 3.0895$^{+0.0229}_{-0.0217}$ | 3.0784 | 3.0761$^{+0.0229}_{-0.0217}$ | 3.0559 |

**TABLE II**: The mean values with 1, 2, 3σ errors and the best fit values of the parameters for the IwCDM model, where CMB from Planck + WMAP9, BAO, SNia, with or without RSD data sets have been used.

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**Appendix A: Verifying the perturbation equations**

For an application example for Eqs. (18), (19), (20), if we follow Ref. 55 and take the interaction $Q_x = -Q_c = Q = \Gamma_\rho \rho_s$ so $\delta Q_x = -\delta Q_c = \Gamma_\rho \rho_s \delta_s$. Moreover, in order to avoid the unphysical sound speed, we choose $c_s^2 = 1$. Under these conditions, we could obtain the continuity and Euler equations which are compatible with Eqs. (32-37) in Ref. 55.

\[
\delta_x' + 3H(1 - w_x)\delta_x + \nabla^2 (1 - w_x)^2 \frac{\theta}{k^2} + (1 + w_x)\theta_s - 3(1 + w_x)\psi + (1 + w_x)k^2(B - E') = a\Gamma_x \left[ \phi + 3H(1 - w_x)\frac{\theta}{k^2} \right],
\]

(A1)
FIG. 4: The 1D marginalized distributions on individual parameters and 2D contours with 68% C.L., 95% C.L., and 99.7% C.L. between each other using the combination of the observed data points from the CMB from Planck + WMAP9, BAO, SNIa, and RSD data sets.

\[
\delta_c' + \theta_c - 3\psi' + k^2(B - E') = a\Gamma_x \frac{\rho_x}{\rho_c} (\delta_c - \delta_x - \phi), \tag{A2}
\]

\[
\theta_x' - 2H\theta_x - \frac{k^2\delta_x}{(1 + w_x)} - k^2\phi = \frac{a\Gamma_x}{(1 + w_x)} [b\theta_c - (1 + b)\theta_x], \tag{A3}
\]

\[
\theta_c' + H\theta_c - k^2\phi = a\Gamma_x \frac{\rho_x}{\rho_c} (1 - b)(\theta_c - \theta_x), \tag{A4}
\]

where

\[
b = \begin{cases} 
1, & \text{for } Q_A^\mu \parallel u_c^\mu, \\
0, & \text{for } Q_A^\mu \parallel u_x^\mu. 
\end{cases}
\]
Appendix B: Verifying the perturbation equations when the expansion rate of the Universe is perturbed

If one considers the expansion rate of the Universe is perturbed in the light of Ref. [60], \( \dot{H} = H + \delta H \). When the interacting form is taken as \( Q_x = -Q_c = \dot{H} \xi_c \rho_c \), one could obtain the continuity and Euler equations of dark energy and dark matter

\[
\delta Q_x = -\delta Q_c = 3H \xi_c \rho_c \left( \frac{\delta H}{H} + \frac{\delta \rho}{\rho_c} \right) = 3H \xi_c \rho_c (\mathcal{K} + \delta_x),
\]

where \( \mathcal{K} = \delta H/H \), according to Eqs. (18,19,20,21), the continuity and Euler equations for the IwCDM model read

\[
\begin{align*}
\delta' + 3H(c_{x}^{2} - w_{x})\delta_x + 9H^{2}(1 + w_{x})(c_{x}^{2} - w_{x}) \frac{\theta_{x}}{k^{2}} + (1 + w_{x})\theta_{x} - 3(1 + w_{x})\psi' + (1 + w_{x})k^{2}(B - E') &= \mathcal{H}\xi_c \theta_c + 3H^{2}(c_{x,eff}^{2} - w_{x})\xi_c \frac{\theta_{x}}{k^{2}} + \mathcal{H}\xi_c \mathcal{K}, \quad (B2) \\
\delta' + \theta_c - 3\psi' + k^{2}(B - E') &= \mathcal{H}\xi_c \frac{\theta_{x}}{\rho_c}(\delta_c - \delta_x) - \mathcal{H}\xi_c \frac{\rho_c}{\rho_c}(\theta_c - \theta_x) - \mathcal{H}\xi_c \frac{\rho_c}{\rho_c} \mathcal{K}, \quad (B3) \\
\theta' + \mathcal{H}(1 - 3c_{x,eff}^{2})\theta_x - \frac{c_{x,eff}^{2}}{1 + w_{x}}k^{2}\delta_x - k^{2}\phi &= \frac{\mathcal{H}\xi_c}{1 + w_{x}}[b(\theta_c - \theta_x) - c_{x,eff}\theta_x], \quad (B4) \\
\theta' + \mathcal{H}\theta_x - k^{2}\phi &= \mathcal{H}\xi_c \frac{\rho_c}{\rho_c}(1 - b)(\theta_c - \theta_x), \quad (B5)
\end{align*}
\]

Furthermore, in the synchronous gauge, \( \mathcal{K} = \delta H/H = (\theta + h'/2)/(3H) \) [60], so the continuity and Euler equations become

\[
\begin{align*}
\delta' + (1 + w_{x}) \left( \theta_x + \frac{h'}{2} \right) + 3H(c_{x,eff}^{2} - w_{x})\delta_x + 9H^{2}(c_{x,eff}^{2} - w_{x})(1 + w_{x}) \frac{\theta_{x}}{k^{2}} &= 3H^{2}(c_{x,eff}^{2} - w_{x})\xi_c \frac{\theta_{x}}{k^{2}} + \xi_c \frac{3}{3} \left( \theta + \frac{h'}{2} \right), \quad (B6) \\
\delta' + \theta_c + \frac{h'}{2} &= \mathcal{H}\xi_c \frac{\theta_{x}}{\rho_c}(\delta_c - \delta_x) - \frac{\xi_c}{3} \frac{\rho_c}{\rho_c} \left( \theta + \frac{h'}{2} \right), \quad (B7) \\
\theta' + \mathcal{H}(1 - 3c_{x,eff}^{2})\theta_x - \frac{c_{x,eff}^{2}}{1 + w_{x}}k^{2}\delta_x &= \frac{\mathcal{H}\xi_c}{1 + w_{x}}[b(\theta_c - \theta_x) - c_{x,eff}\theta_x], \quad (B8) \\
\theta' + \mathcal{H}\theta_x &= \mathcal{H}\xi_c \frac{\rho_c}{\rho_c}(1 - b)(\theta_c - \theta_x), \quad (B9)
\end{align*}
\]

where \( (\rho + p)v = \sum_{A}(\rho_{A} + p_{A})v_{A} \) [53, 60] and \( \theta_{A} = -k^{2}(v_{A} + B) \) [53, 117].

In the case of \( Q_{A}^{\mu} \parallel u_{A}^{\mu} \) (b=1), when \( c_{x,eff}^{2} = 1 \) [51, 53, 55], we could obtain the continuity and Euler equations which are compatible with Eqs. (4.1-4.4) in Ref. [60]. (Here, \( \xi_c = -\xi \) [60] because the background evolution equations of dark matter and dark energy are different between the two works.) Moreover, we generalize the continuity and Euler equations into the case of \( Q_{A}^{\mu} \parallel u_{A}^{\mu} \) (b=0).

[1] P.A.R. Ade, et al., Planck Collaboration, arXiv:1303.5062
[2] P.A.R. Ade, et al., Planck Collaboration, arXiv:1303.5075
[124] E. Komatsu et al., Astrophys. J. Suppl. Ser. 192, 18 (2011).
[125] W.J. Percival et al., Mon. Not. R. Astron. Soc. 381, 1053 (2007).
[126] A.G. Riess, Astrophys. J. 699, 539 (2009).
[127] R. Kessler et al., Astrophys. J. Suppl. Ser. 185, 32 (2009).
[128] A.G. Riess et al., Astrophys. J. 730, 119 (2011); 732, 129(E) (2011).