On the Bekenstein-Hawking Entropy, Non-Commutative Branes and Logarithmic Corrections

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Abstract

We extend earlier work on the origin of the Bekenstein-Hawking entropy to higher-dimensional spacetimes. The mechanism of counting states is shown to work for all spacetimes associated with a Euclidean doublet \((E_1, M_1) + (E_2, M_2)\) of electric-magnetic dual brane pairs of type II string-theory or M-theory wrapping the spacetime’s event horizon plus the complete internal compactification space. Non-Commutativity on the brane worldvolume enters the derivation of the Bekenstein-Hawking entropy in a natural way. Moreover, a logarithmic entropy correction with prefactor \(1/2\) is derived.

Keywords: Bekenstein-Hawking Entropy, D-Branes

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1 Introduction

In the past few years M/string-theory made considerable progress towards the stabilization of its multitude of moduli arising from compactification and therefore towards predictivity [1]-[9]. Particularly interesting for addressing the real world is heterotic M-theory [10]-[12] with its flux compactifications [13]-[15]. The latter offer an intriguing rationale for why and how grand unified theories have to be combined with gravity. Due to its peculiar blend of classical and quantum physics [11], it is clear that eventually knowledge of the full quantum heterotic M-theory becomes intimately tied to its promising phenomenology. It is therefore also from a very pragmatic phenomenological point of view of direct interest to obtain the elusive non-perturbative formulation of M-theory which would require first of all to identify its fundamental microscopic states.

Unfortunately, experimentally, we are still far away from testing M-theory directly. In order to unravel the mysteries of M-theory we therefore have to look for other, necessarily theoretical, clues. Probably the best guidance in this respect comes from the Bekenstein-Hawking entropy (BH-entropy) [16]-[22]. Because of its universal applicability it seems to capture a generic feature of the underlying microscopic theory of quantum gravity. In this paper we would like to propose, following earlier work, [23]-[26], a possible set of microscopic chain-like states for the non-perturbative regime where the string-coupling constant $g_s$ becomes of order one.

We will base our argumentation on a microscopic mechanism to count states leading to the BH-entropy and its leading logarithmic correction. Along the line of reasoning which will involve electric-magnetic dual Euclidean brane pairs, we will see that non-commutativity on the brane worldvolume fits in naturally, suggesting a non-commutative event horizon. Let us mention that, also within the “membrane paradigm” approach (see e.g. [27],[28]), it was recently pointed out in [29] that the stretched horizon of a black hole should be thought of as a non-commutative membrane, suggesting as well a non-commutative event horizon. The results which we will present here will generalize the analysis of [23] in that they apply also to the BH-entropy of higher-dimensional $d > 4$ spacetimes while [23] was devoted to the study of the BH-entropy of $d = 4$ spacetimes. For some earlier other interesting ideas trying to understand the BH-entropy from string- or M-theory see [30]-[38].
2 BH-Entropy and Electric-Magnetic Dual Pairs

2.1 Type II String-Theory Case

Let us start with type II string-theory on a D=10 geometric background \( M^{1,d-1} \times M^{10-d} \) described by a metric \( (2 \leq d \leq 10) \)

\[
ds^2 = g^{(1,d-1)}_{\mu\nu}(x)dx^\mu dx^\nu + g^{(10-d)}_{mn}(y)dy^m dy^n
\]  

with Lorentzian signature. The \( d \)-dimensional external non-compact space \( M^{1,d-1} \) is the one whose BH-entropy we are interested in while the internal space \( M^{10-d} \) is taken to be compact with certain factorization properties as we will explain below. More specifically, because we are interested in an external spacetime with non-zero BH-entropy, let \( M^{1,d-1} \) possess a \((d-1)\)-dimensional future event horizon \( \mathcal{H}^+ \). Consider a \((d-1)\)-dimensional spacelike hypersurface \( \Sigma \) in \( M^{1,d-1} \) with one boundary at spatial infinity \( i_0 \) and another boundary, denoted \( \mathcal{H}^{d-2} \), on \( \mathcal{H}^+ \). For spacetimes \( M^{1,d-1} \) describing black holes, \( \mathcal{H}^{d-2} \) is commonly referred to as the “boundary” of the black hole and its area as the “area of the horizon”.

Next, let us take a pair of mutually orthogonal (with respect to the metric given in (1)) Euclidean electric-magnetic dual type II branes

\[
(E_1, M_1) \in \{(Dp, D(6-p)), (F1, NS5)\}.
\]  

Without exhibiting them explicitly, we understand that this set of dual type II branes includes as well all possible pairs in which any brane (including for short also the fundamental string F1) is replaced by its charge-reversed anti-brane. As the orthogonal Euclidean \( E_1 \) and \( M_1 \) together can cover an 8-dimensional submanifold, they possess the right dimensionality so that we can wrap the pair \((E_1, M_1)\) around the complete \( \mathcal{H}^{d-2} \times M^{10-d} \).

In case the (Euclidean) dimensions of \( E_1 \) and \( M_1 \) do not happen to coincide with \( d-2 \) resp. \( 10-d \) (in either order) we would require \( M^{10-d} \) resp. \( \mathcal{H}^{d-2} \) to factorize appropriately. For instance a torus compactification \( M^{10-d} = T^{10-d} \) would satisfy this requirement. In the special case of a Schwarzschild black hole in uncompactified 10-dimensional spacetime one would have to include the dual Euclidean pair \((D7, D(-1))\) as well.

For reasons which will soon become clear, we will wrap a second pair of mutually orthogonal Euclidean electric-magnetic dual branes

\[
(E_2, M_2) \in \{(Dq, D(6-q)), (F1, NS5)\} \tag{3}
\]
around $\mathcal{H}^{d-2} \times \mathcal{M}^{10-d}$. Here $(E_1, M_1)$ and $(E_2, M_2)$ can be chosen independently as far as the mechanism of counting states will be concerned. However, the choice of $(E_1, M_1)$ and $(E_2, M_2)$ has to be compatible with the metric background $^{[11]}$. For instance, a Schwarzschild black hole background geometry which is charge neutral, will require a second antibrane pair $(E_2, M_2) \equiv (\bar{E}_1, \bar{M}_1)$ to neutralize the charges of the first pair.

In passing let us mention that wrapping branes around the horizon is very reminiscent of the “membrane paradigm” in which the event horizon of a $d = 4$ black hole (or rather its “stretched horizon”) is conceived as an effective membrane (however not a fundamental one like in M/string-theory) which enjoys some intriguing non-relativistic properties $^{[?]}$. One might therefore view part of the current approach also as an embedding of this idea into M/string-theory where the role of the effective membrane is played by fundamental M/string-theory branes. Note, however, that while the effective membrane is Lorentzian and has a built in time direction, this is not the case for the Euclidean branes. The inherently small lifetime of a Euclidean brane, which is of order $\Delta t \sim \sqrt{\alpha'}/c$, gets however infinitely dilated by the time dilatation between the event horizon and any exterior observer, as we will discuss below.

The reason for introducing the dual pairs is that it will allow a useful rewriting of the BH-entropy, associated with the $(d-2)$-dimensional area of $\mathcal{H}^{d-2}$, as we will see next. Our goal is to express this BH-entropy exclusively in terms of string-theory entities. Since each pair $(E_i, M_i); i = 1, 2$ covers the space $\mathcal{H}^{d-2} \times \mathcal{M}^{10-d}$, we can write for the compactification volume in both cases

$$\text{vol}(\mathcal{M}^{10-d}) = \text{vol}(E_i)\text{vol}(M_i)\text{vol}(\mathcal{H}^{d-2}), \quad i = 1, 2.$$  

(4)

The effective $d$-dimensional Newton Constant can therefore be expressed as

$$G_d = \frac{G_{10}}{	ext{vol}(\mathcal{M}^{10-d})} = \frac{(2\pi)^6 \alpha'^4 g_s^2}{8} \times \frac{\text{vol}(\mathcal{H}^{d-2})}{\text{vol}(E_i)\text{vol}(M_i)}, \quad i = 1, 2$$  

(5)

where $\alpha'$ denotes the Regge slope.

The significance of why we have chosen to use dual branes lies in the fact that the product of their tensions satisfies the generalized Dirac quantization condition

$$\tau_{E_i}\tau_{M_i} = \frac{1}{(2\pi)^6 \alpha'^4 g_s^2}.$$  

(6)

which allows us to express the inverse of the Newton Constant as

$$\frac{1}{G_d} = 8 \left(\tau_{E_i}\text{vol}(E_i)\right)\left(\tau_{M_i}\text{vol}(M_i)\right)\frac{\text{vol}(\mathcal{H}^{d-2})}{\text{vol}(E_i)\text{vol}(M_i)}, \quad i = 1, 2.$$  

(7)
Hence, the BH-entropy of the spacetime $M^{1,d-1}$, associated with the area of $H^{d-2}$, can be written purely in terms of the respective Euclidean Nambu-Goto actions $S_{E_i}, S_{M_i}$ for $E_i$ and $M_i$ as

$$S_{BH} = \frac{\text{vol}(H^{d-2})}{4G_d} = 2S_{E_i}S_{M_i}, \quad i = 1, 2,$$

(8)

where $S_{E_i} = \tau_{E_i}\text{vol}(E_i)$ and similarly for the magnetic branes.

The fact that we considered two dual pairs instead of just one becomes important now. Namely, it allows us to get rid of the prefactor 2 and write the BH-entropy instead as a sum over both dual pairs

$$S_{BH} = \sum_{i=1,2} S_{E_i}S_{M_i}. \quad (9)$$

For the mechanism to microscopically derive $S_{BH}$ by counting states, which we will present later, it will be important that we can replace the prefactor 2 by the sum at the expense of introducing two dual pairs instead of just one. If we had to use only one dual pair then the microscopic derivation of $S_{BH}$ would be off by precisely this factor 2. Moreover, taking two dual pairs instead of just one also makes sense from the point of view that a charge-neutral Schwarzschild black hole wouldn’t be compatible with just one dual pair. At least one other anti-brane pair is needed to dispose of the long-range U(1) RR- or NS-fields of the branes.

Let us finally comment on the stability of the Euclidean brane system. A Euclidean brane would exist for just a very short time-interval, given essentially by the string-time $\Delta t \sim \sqrt{\alpha'/c}$, if put into a flat spacetime background. In contrast, the Euclidean branes considered here, are located on an event horizon. The infinite redshift with which an exterior observer sees the horizon, implies, at the classical level, an infinite time dilatation, rendering the Euclidean branes classically stable to any such observer. It is worthwhile emphasizing that the entities which characterize an event horizon’s thermodynamics – and in particular its entropy – such as the mass, the angular momentum and charge of the interior region of spacetime enclosed by it, are also measured away from the horizon, in the exterior asymptotic regime. We should therefore analyze the system of Euclidean branes not from an observer’s point of view who is located on these branes but from an exterior observer’s point of view for whom they appear stable due to the infinite time dilatation \cite{24}.
2.2 M-Theory Case

Let us now try to obtain an analogous expression for the BH-entropy in M-theory. In M-theory we have a unique dual brane pair \((M2, M5)\) and essentially the same reasoning goes through as in the type II case. We will start with a \(D=11\) spacetime \(\mathcal{M}^{1,d-1} \times \mathcal{M}^{11-d}\) whose geometry describes a compactification from 11 to \(d\) dimensions \((2 \leq d \leq 11)\)

\[
ds^2 = g^{(1,d-1)}_{\mu\nu}(x)dx^\mu dx^\nu + g^{(11-d)}_{mn}(y)dy^m dy^n . \tag{10}
\]

The external non-compact \(d\)-dimensional spacetime \(\mathcal{M}^{1,d-1}\) is the one whose BH-entropy we are interested in. We therefore assume that it has a non-trivial future event horizon \(H^+\). The \((d-2)\)-dimensional intersection of \(H^+\) with a spacelike hypersurface \(\Sigma\), coming in from spatial infinity \(i_0\), is again denoted \(H^{d-2}\). The volume of \(H^{d-2}\) is known as the “area of the horizon”.

The pair \((M2, M5)\) of a Euclidean M2 and M5 brane which are mutually orthogonal in the metric \((10)\), covers a 9-dimensional spacelike submanifold. We will let the pair wrap \(H^{d-2} \times \mathcal{M}^{11-d}\). Except for \(d = 5\) and \(d = 8\), we would have to assume that the metric on either \(\mathcal{M}^{11-d}\) or \(H^{d-2}\) factorizes appropriately into a direct product. For \(d = 9, 10, 11\) both M2 and M5 would have to wrap \(H^{d-2}\) whose metric must then exhibit a corresponding direct product structure. At first, this seems to exclude the Schwarzschild black holes in \(d = 9, 10, 11\) dimensions from consideration as in these cases \(H^{d-2}\) is spherical and spheres don’t factorize. We had, however, seen before that the \(d = 9, 10\) cases can be covered by the richer type II brane description which e.g. also allows for a dual \((D7, D(-1))\) Euclidean brane pair to cover the \(d = 10\) situation. So it is really the \(d = 11\) Schwarzschild case which cannot be adressed. It might be possible to include it as well by invoking the less understood M9-brane but will not pursued further here.

For M-theory compactifications the compactification volume can then be expressed as

\[
\text{vol}(\mathcal{M}^{11-d}) = \frac{\text{vol}(M2)\text{vol}(M5)}{\text{vol}(H^{d-2})} \tag{11}
\]

such that the effective \(d\)-dimensional Newton Constant becomes

\[
G_d = \frac{G_{11}}{\text{vol}(\mathcal{M}^{11-d})} = \frac{(2\pi)^7 l_{11}^9}{8} \times \frac{\text{vol}(H^{d-2})}{\text{vol}(M2)\text{vol}(M5)} \tag{12}
\]

where \(l_{11}\) denotes the 11-dimensional Planck-length. The important property of the dual brane pair is that the product of their tensions satisfies

\[
\tau_{M2}\tau_{M5} = \frac{1}{(2\pi)^7 l_{11}^9} . \tag{13}
\]
This allows to write the inverse of $G_d$ as

$$\frac{1}{G_d} = \frac{8 (\tau_{M2} \text{vol}(M2)) (\tau_{M5} \text{vol}(M5))}{\text{vol}(\mathcal{H}^{d-2})}.$$  \hspace{1cm} (14)

The $d$-dimensional BH-entropy associated with the spacetime $\mathcal{M}^{1,d-1}$ can therefore be expressed as

$$S_{BH} = \frac{\text{vol}(\mathcal{H}^{d-2})}{4G_d} = 2S_{M2}S_{M5}$$ \hspace{1cm} (15)

where $S_{M2}, S_{M5}$ are the respective Nambu-Goto actions of the Euclidean M2 and M5.

For a second dual brane pair $(M2, M5)$ wrapped around $\mathcal{H}^{d-2} \times \mathcal{M}^{11-d}$ independently of the first pair (for the following expression of the BH-entropy, the M2’s resp. M5’s of the two pairs don’t have to wrap necessarily the same submanifolds) one would arrive at the same result \[15\]. Hence one obtains, by employing two $(M2, M5)$ pairs, also in the M-theory case the result

$$S_{BH} = \sum_{i=1,2} S_{M2,i}S_{M5,i}$$ \hspace{1cm} (16)

which expresses the $d$-dimensional BH-entropy exclusively in terms of the branes’ Nambu-Goto actions. Again, since the Nambu-Goto action does not recognize the difference between a brane and an anti-brane we will understand subsequently that each M2 or M5 could also be replaced by its anti-brane partner.

### 3 Cell Structure and Non-Commutativity

Now that we have found an expression for the BH-entropy in terms of the Nambu-Goto actions of two dual brane pairs, our aim will be to propose a suitable set of microstates capable of explaining the entropy by counting the states of a microcanonical ensemble. For this we will need one more ingredient to which we will turn now. When dealing with a Euclidean brane, it is more natural to treat its Euclidean “time” and space dimensions not differently in contrast to the case of a Lorentzian brane where the Lorentzian signature leads to such a distinction. Consequently, one is led to interpret the tension of for instance a $(p+1)$-dimensional Euclidean Dp-brane not as its “mass” per unit spatial $p$-volume but instead as the inverse of a $(p+1)$-dimensional volume unit $v_{Dp}$. On any of the Euclidean branes introduced so far we will therefore have a volume unit $v_E$ resp. $v_M$ given by the
inverse of the brane’s tension

\[ v_E = \frac{1}{\tau_E}, \quad v_M = \frac{1}{\tau_M}. \]  

(17)

Such an elementary volume unit on the brane’s worldvolume can be naturally understood if the brane’s worldvolume would be considered being non-commutative. Taking the simplest non-commutativity arising from string-theory \[39,40\]

\[ [X^i, X^j] = 2i\epsilon^{ij}l^2 \]  

(18)

for the worldvolume coordinates \( X^i \) of a Euclidean Dp-brane, one derives the uncertainty relation

\[ \Delta X^i \Delta X^j \geq l^2 \]  

(19)

for the coordinates \( X^i \). From this follows directly the “brane worldvolume uncertainty principle” \[41\]

\[ \Delta X^1 \ldots \Delta X^{p+1} \geq l^{p+1}. \]  

(20)

Indeed in \[41\] such an uncertainty principle was shown to arise in string field theory for all branes (including F1, NS5, Dp, M2, M5) by using S- and T-dualities. The result was that the smallest allowed volume \( l^{p+1} \) in \( \text{(20)} \) is always given (up to factors of \( \mathcal{O}(1) \)) through the tension of the respective brane

\[ l^{p+1} \simeq \frac{1}{\tau_{\text{Dp}}} \]  

(21)

for all Dp-branes and similarly for F1, NS5, M2, M5. From the perspective of a brane with non-commutative worldvolume (which arises in string field theory even in the absence of a background magnetic flux or Neveu-Schwarz B-field along the brane \[41\]), it is therefore clear that \( v_E, v_M \) in \( \text{(17)} \) represent the smallest volume unit which is allowed by the “worldvolume uncertainty principle”. For the special case of the fundamental string this just states that \( 2\pi \alpha' = 1/\tau_{\text{F1}} \) constitutes a smallest volume unit resp. that the string-length \( l_s \) constitutes a smallest length – a familiar result which has been argued for based on string scattering amplitudes, worldsheet conformal invariance and other arguments \[42,43\].

With this interpretation of the tension of a Euclidean brane, its Nambu-Goto action adopts a new meaning. Namely, quite analogous to the decomposition of phase space into
cells of size $h$ in quantum mechanics, we are led here to think of the brane worldvolume as a lattice composed out of a certain number of cells with volume $v_E$ resp. $v_M$. It is then precisely the Nambu-Goto action of the brane, $S_E$ resp. $S_M$, which counts how many such cells, $N_E$ resp. $N_M$, the brane contains

$$N_E = \frac{\text{vol}(E)}{v_E} = \tau_E \text{vol}(E) = S_E$$

(22)

and similarly for the magnetic dual component

$$N_M = S_M.$$  \hspace{1cm} (23)

Because of the orthogonality of $E$ and $M$, the cells of the dual pair $(E, M)$ will be of size

$$V_{\text{cell}} = v_E v_M.$$  \hspace{1cm} (24)

Hence, each dual pair $(E, M)$ contains

$$\frac{\text{vol}(E)\text{vol}(M)}{V_{\text{cell}}} = \frac{\text{vol}(E)\text{vol}(M)}{v_E v_M} = N_E N_M$$  \hspace{1cm} (25)

cells. Since the product of the tensions of two dual branes is independent of the specifically chosen dual branes, $V_{\text{cell}}$ is the same for all dual pairs. Two dual pairs $(E_1, M_1)$ and $(E_2, M_2)$, both covering the same volume, will therefore contain

$$\frac{\text{vol}(E_1)\text{vol}(M_1)}{V_{\text{cell}}} + \frac{\text{vol}(E_2)\text{vol}(M_2)}{V_{\text{cell}}} = \sum_{i=1,2} N_{E_i} N_{M_i} = N$$  \hspace{1cm} (26)

cells. By virtue of (16) resp. (10) plus (22), (23) together with (26), this implies that the $d$-dimensional BH-entropy associated with $\mathcal{M}^{1,d-1}$ simply becomes an integer

$$S_{BH} = \sum_{i=1,2} S_{E_i} S_{M_i} = \sum_{i=1,2} N_{E_i} N_{M_i} = N \in 2\mathbb{N},$$  \hspace{1cm} (27)

with $N$ the total number of cells contained in the combined worldvolume of $(E_1, M_1) + (E_2, M_2)$. This result is valid at sufficiently large $N$ where $N \gg \Delta$ and we can neglect expected but unknown microscopic quantum corrections $\Delta$ and set $N + \Delta \simeq N$. Tiny as these small quantum corrections may be, they will generically shift the value of the corrected expression $N + \Delta$ away from being an integer. We will in the following not consider $\Delta$ further and work in the $N \gg \Delta$ regime. Notice that $N$ has to be even because both $(E_1, M_1)$ and $(E_2, M_2)$ cover the same volume which implies the equality

$$S_{E_1} S_{M_1} = \frac{\text{vol}(E_1)\text{vol}(M_1)}{V_{\text{cell}}} = \frac{\text{vol}(E_2)\text{vol}(M_2)}{V_{\text{cell}}} = S_{E_2} S_{M_2}$$  \hspace{1cm} (28)

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and leads to $N_{E_1}N_{M_1} = N_{E_2}N_{M_2}$.

The scaling of the BH-entropy with an integer $N$ at large $N$ shouldn’t be a surprise because it is precisely what the holographic principle \cite{34} demands. Holography implies that the number of fundamental degrees of freedom $N_{dof}$ of a system does not scale with the volume but with its area in Planckian units. In our case the area is the area of the horizon

$$N_{dof} \sim \frac{vol(H^{d-2})}{G_d}.$$  \hspace{1cm} (29)

With the BH-entropy itself being proportional to the area of the horizon, one obtains, via the holographic principle, a scaling of the BH-entropy with an integer, namely the number of fundamental degrees of freedom

$$S_{BH} \sim N_{dof}.$$  \hspace{1cm} (30)

_Holography therefore demands that we identify $N_{dof} \sim N$ and consequently interpret the cells, or rather one degree of freedom per cell, as the $N$ fundamental degrees of freedom._

Let us note that in the limit where the string coupling constant $g_s \to 0$ goes to zero and we understand branes as smooth hypersurfaces, we won’t see the discrete cell structure. The reason is that the smallest allowed discrete volume $V_{cell} = 1/(\tau E \tau M) \propto g_s^2 \to 0$ goes to zero in this limit and the involved worldvolumes become quasi-smooth. However, in the non-perturbative regime where $g_s \simeq 1$, $V_{cell}$ is finite and the discrete brane worldvolumes become visible.
Figure 2: A closed chain possesses one more link than an open chain which is necessary to close the chain by connecting the last and the first link on the same cell. The closed chain gives the same number of different states as the open chain.

4 Derivation of BH-Entropy and Logarithmic Correction

Next, we want to propose a set of microscopic states whose entropy should, in a microcanonical ensemble description, account for the BH-entropy and ideally also for its logarithmic correction. For this purpose we will consider on the combined \((E_1, M_1) + (E_2, M_2)\) worldvolume, taken as a lattice of \(N\) cells, open chains built out of \(N - 1\) successive links where each link connects two arbitrary cells (see fig.1). As each link will be allowed to start and end on any of the cells with same probability, the number of all such chain configurations is clearly \(N^N\). Besides the open chains there is a similar but topologically different class of chains which possesses the same number of configurations, \(N^N\). These are the closed chains made out of \(N\) links where the last link connects back to the first link (see fig.2). As far as the state counting is concerned they lead to the same results as the open chains and might therefore be considered as an alternative set of microstates until further selection criteria are found.

We could have called the counting of the different chain configurations so far “classical” because it considered all cells as being distinguishable. The cells, which we identified as the fundamental degrees of freedom via holography, should however at the quantum level rather be regarded as bosonic degrees of freedom and therefore considered being indistinguishable. We will see that this quantum feature leads to the correct counting of
states. As well-known from statistical mechanics, one can easily account for this quantum bosonic symmetry at large $N$ by dividing the classical number of configurations through the Gibbs-correction factor $N!$. Therefore, quantum-mechanically we obtain a number of

$$\Omega(N) = \frac{N^N}{N!}$$

(31)
different open or closed chain states.

By assuming that all chains with the same length $N - 1$ (open) resp. $N$ (closed) will possess the same energy, we can now determine in a microcanonical ensemble description the chain entropy. It is given by

$$S_{\text{chain}} = \ln \Omega(N)$$

(32)
and can be evaluated in the large $N$ limit (as appropriate for an area of $\mathcal{H}^{d-2}$ of macroscopic size) by using Stirling’s series

$$N! = \sqrt{2\pi N} N^N e^{-N} \left(1 + \frac{1}{12N} + \mathcal{O}\left(\frac{1}{N^2}\right)\right)$$

(33)
to approximate $\ln(N!)$. The result is

$$S_{\text{chain}} = N - \frac{1}{2} \ln N - \ln \sqrt{2\pi} - \frac{1}{12N} + \mathcal{O}\left(\frac{1}{N^2}\right)$$

(34)
which by virtue of the identification (27) becomes

$$S_{\text{chain}} = S_{\text{BH}} - \frac{1}{2} \ln S_{\text{BH}} - \ln \sqrt{2\pi} - \frac{1}{12S_{\text{BH}}} + \mathcal{O}\left(\frac{1}{S_{\text{BH}}^2}\right).$$

(35)

Thus indeed the chain entropy matches at leading order the BH-entropy and in addition at subleading order gives the expected logarithmic correction including the precise numerical prefactor (there has been a debate in the literature over whether the prefactor in front of the logarithm should be $1/2$ or $3/2$. While initially a factor $3/2$ had been universally favored [45], [46] arguments have been put forward more recently favoring $1/2$, see [47]-[50]. The discussion is still ongoing but it is certainly clear that the prefactor depends on the ensemble chosen and will change when one leaves the microcanonical ensemble description and uses the less fundamental canonical ensemble [51],[52]).

We can therefore conclude that the proposed microscopic chain states for the non-perturbative $g_s \simeq \mathcal{O}(1)$ regime allow for a general mechanism of counting states with the correct reproduction of the BH-entropy and its leading logarithmic correction not only for 4-dimensional [23] but also, as demonstrated in this paper, for $d$-dimensional spacetimes possessing event horizons.
5 Final Comments

Let us briefly address the situation where one treats the Euclidean branes not as probe branes, as we have done it here, but includes their backreaction on the spacetime geometry. First steps in this direction for the 4-dimensional Schwarzschild black hole have been undertaken in [24]. It is clear that the uncharged $d$-dimensional hyperspherically symmetric Schwarzschild-Tangherlini black hole has to be associated with branes where the second doublet, $(E_2, M_2) \equiv (\tilde{E}_1, \tilde{M}_1)$ consists of the anti-branes of the first doublet in order to be compatible with an uncharged configuration. This also fits because both the Schwarzschild-Tangherlini black hole and the brane anti-brane configuration break all supersymmetry. Moreover, because the Schwarzschild-Tangherlini black hole is a non-dilatonic black hole, one should employ in the type II case a self-dual $(D3, D3) + (\overline{D3}, \overline{D3})$ doublet, as the $D3$-brane is the only non-dilatonic type II brane.

Let us finally return to the cell-volume. As mentioned before, the cell-volume $V_{\text{cell}}$ on each of the two dual pairs $(E_i, M_i)$ is given, due to the orthogonality of $E_i$ and $M_i$, by the product of their minimal volumes

$$V_{\text{cell}} = v_{E_i} v_{M_i} = \frac{1}{\tau_{E_i} \tau_{M_i}} = \begin{cases} (2\pi)^6 \alpha'^4 g_s^2 & (D = 10) \\ (2\pi)^7 f_1^0 & (D = 11) \end{cases}.$$  \hspace{1cm} (36)

Restoring the constants $\hbar$ and $c$ and noticing that $G_d \hbar/c^3$ has length-dimension $L^{d-2}$, this result can be written as

$$V_{\text{cell}} = \begin{cases} 8G_{10} \frac{\hbar}{c^3} & (D = 10) \\ 8G_{11} \frac{\hbar}{c^3} & (D = 11) \end{cases}.$$  \hspace{1cm} (37)

It shows first that the $D=10/11$ Newton Constant acquires a geometric meaning in terms of the cell volume and second that a non-vanishing cell-volume is a quantum effect which vanishes in the classical limit where $\hbar \rightarrow 0$. This agrees also with the understanding of the cell-volume in terms of a non-commutative structure on the brane worldvolume which likewise results from a promotion of the classical coordinates $x^i$ to non-commuting quantum operators $X^i$. Moreover, we see that when gravity becomes weak, i.e. when $G_{10,11}$ becomes small, the cell volume shrinks and we end up with the ordinary smooth hypersurface description of branes which we expect from perturbative string-theory in this regime. On the other hand, it is clear that when gravity becomes strong, i.e. when $G_{10,11}$ becomes large, the discrete cell structure should show up prominently and hence signals a significant deviation from ordinary string-theory in the non-perturbative regime.
Given the relations (37) in D=10/11 dimensions, one readily obtains formulae for the effective d-dimensional Newton Constant $G_d$ (and effective Planck-length $l_d$ defined through $l_d^{-2} = G_d \frac{\hbar}{c^3}$) which are purely geometrical. These are

$$G_d \frac{\hbar}{c^3} = \frac{G_{10}}{\text{vol}(\mathcal{M}^{10-d})} \frac{\hbar}{c^3} = \frac{V_{\text{cell}}}{8 \text{vol}(\mathcal{M}^{10-d})} \quad (D = 10) \quad (38)$$

for the 10-dimensional type II case and

$$G_d \frac{\hbar}{c^3} = \frac{G_{11}}{\text{vol}(\mathcal{M}^{11-d})} \frac{\hbar}{c^3} = \frac{V_{\text{cell}}}{8 \text{vol}(\mathcal{M}^{11-d})} \quad (D = 11) \quad (39)$$

for the 11-dimensional M-theory case. The size of the effective d-dimensional Newton Constant appears as the ratio of the cell volume to the compactification volume.

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