Some Implications of inverse-Compton Scattering of Hot Cocoon Radiation by relativistic jets in Gamma-Ray Bursts

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Abstract: Long Gamma-Ray Bursts (GRB) relativistic jets are surrounded by hot cocoons which confine jets during their punch out from the progenitor star. These cocoons are copious sources of X-ray photons that can be and are inverse-Compton (IC) scattered to MeV–GeV energies by electrons in the relativistic jet. We provide detailed estimates for IC flux resulting from various interactions between X-ray photons and the relativistic jet, and describe what we can learn about GRB jets and progenitor stars from the detection (or an upper limit) of these IC scattered photons.

1. Introduction

There is evidence that long duration gamma-ray bursts (GRBs) are produced when a massive star dies at the end of its nuclear burning life and its core collapses to a neutron star or a blackhole (e.g. Galama et al. 1998, Hjorth et al. 2003, Stanek et al. 2003, Modjaz et al. 2006, Campana et al. 2006, Starling et al. 2011, Sparre et al. 2011, Melandri et al. 2012). The newly formed compact object at the center of the progenitor star produces a pair of relativistic jets that make their way out of the star along polar regions. Punching their way to the stellar surface these jets shock heat the material they encounter and push it both sideways and along the jet’s direction. Therefore, the jet is surrounded by this shock heated plasma, or a hot cocoon, which provides collimation for it (see fig. 1). The total amount of thermal energy in the cocoon is equal to the work done by the jet on stellar material it encounters while inside the star and that is estimated to be of order $L_j t_* \sim 10^{51}$ erg — where $L_j$ is the jet luminosity and $t_*$ is the travel time for the jet inside the star at sub-relativistic speed (e.g. Meszaros & Rees, 2001; Ramirez-Ruiz et al. 2002; Matzner, 2003). We are here making the rough assumption that the bulk of the work goes to thermal energy though a significant amount goes into the cocoon forward (in jet direction) momentum.

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Fig. 1.— Schematic sketch of a hot cocoon surrounding the jet while the jet is still inside the star is shown on the left side of this figure. The sketch to the right shows half of the system (the hemisphere with right to the center jet) at a later time when the jet and the cocoon have punched through the surface of GRB progenitor star. Thermal radiation from the cocoon is inverse-Compton scattered by the same jet, and by any relativistic jet produced by the central engine at a later time when the \textit{late jet} emerges above the cocoon surface. The IC scattered photons form a halo peaked near the edge of the jet and roughly as wide as the jet.

The jet and the cocoon emerge from the stellar surface more or less at the same time. The central engine of long-GRBs remains active for $\sim 10^5 s$ after the jet emerges from the stellar surface (e.g. Burrows et al. 2005, Chincarini et al. 2007, 2010). The isotropic equivalent of luminosity from the cocoon at its peak, shortly after it emerges from the stellar surface, is of order $10^{49}$ erg s$^{-1}$ at a few keV (Ramirez-Ruiz et al. 2002). The cocoon luminosity decreases slowly with time for about $10^3 s$ and then drops to zero rapidly when the cocoon becomes transparent to Thomson scattering of photons. Any relativistic jet that is launched after the cocoon breaks through the stellar surface is expected to encounter this intense radiation field when it reaches the cocoon photosphere, and will produce a short-lived bright pulse of inverse-Compton scattered photons which form a halo peaked near the edge of the jet and roughly as wide as the jet (fig. 1). Furthermore, radiation from the cocoon is Thomson scattered by electrons in the wind of GRB progenitor star, and some of these wind-scattered photons collide with the relativistic jet at large angles to produce an intense pulse of high energy inverse-Compton radiation.

\footnote{The jet while inside the cocoon is shielded from the IC drag of its intense radiation by an optically thick layer of electron-positron plasma that is created at the interface of the jet and cocoon. These $e^\pm$ pairs are produced by the collision of thermal photons from the cocoon with IC scattered photons at this interface (Ceccobello & Kumar, 2014).}
We provide in this work an estimate of IC flux from these different interactions of cocoon radiation with the GRB relativistic jet, and suggest how the detection of this high energy radiation (or an upper limit) can be used to constrain GRB jet and progenitor star properties.

In section 2 we describe cocoon dynamics and radiation which follows closely the work of Ramirez-Ruiz et al. (2002) and Matzner (2003), and is included here for the sake of completeness and ease to follow the rest of the paper. Section 3 describes the IC scattering of cocoon radiation by a relativistic jet directly, and via an intermediate process of scattering first by electrons in the circum-burst medium (CBM). Application to GRBs is provided in §4.

2. Cocoon dynamics and radiation

We describe in this section the propagation of a relativistic jet through a star and how it shocks and pushes sideways stellar material it encounters on its way to the surface to evacuate a cavity in the polar region. This process creates a hot cocoon of plasma that encapsulates the jet. The properties of the hot cocoon and its radiation are discussed separately in two subsections. Much of the discussion in this section closely follows the work of Ramirez-Ruiz et al. (2002) and Matzner (2003).

2.1. Dynamics of cocoon

Let us consider a jet of luminosity $L_j$, speed $v_j$, and Lorentz factor $\Gamma_j$ produced by the central engine of a GRB. While the jet moves through the star it drives a shock wave through the stellar envelope and that in turn reacts back on the jet and slows down a section of it near the head via a reverse-shock traveling into the jet.

The speed of the jet head, $v_h$, can be calculated from ram pressure balance in the radial direction of the unshocked jet and stellar plasma as viewed from the rest frame of the jet-head:

$$\rho_j c^2 (\Gamma_j^2 / 4 \Gamma_h^2) \approx \rho_a \Gamma_h^2 v_h^2,$$

where $\rho_j$ and $\rho_a$ are densities of the unshocked jet and the stellar envelope respectively, and $\Gamma_j$ & $\Gamma_h$ are the Lorentz factors of the unshocked jet and the jet-head wrt the unshocked star (the Lorentz factor of the unshocked jet wrt jet-head is $\sim \Gamma_j / 2 \Gamma_h$). Considering that
the jet luminosity at the stellar surface can be written as

\[ L_j = \pi \theta_j^2 R_*^2 \rho_j \Gamma_j^2 c^3, \]  

(2)

and the total mass of the swept up gas by the jet is

\[ m_c \sim \pi \theta_j^2 \rho_a R_*^3, \]  

(3)

we obtain

\[ 2\Gamma_h^2 v_h \sim \left[ \frac{R_* L_j}{cm_c} \right]^{1/2}, \]  

(4)

where \( \theta_j \) is jet opening angle.

The work done by the jet on the stellar material can be obtained from momentum flux conservation and from the fact that the energy flux for relativistic outflows is \( c \) times the momentum flux whereas for the sub-relativistic swept-up gas it is smaller by a factor \( v_h/c \).

The difference in the energy flux for the jet-head and the unshocked jet gives the rate at which energy is deposited in the cocoon, and this way we obtain the total energy in the cocoon to be

\[ E_c \sim L_j \left[ 1 - v_h/c \right] R_* / v_h \quad \text{or} \quad E_c \sim L_j R_* / v_h \quad \text{for} \quad v_h/c \ll 1. \]  

(5)

With this expression for energy in the cocoon we can simplify equation (4) further:

\[ 4\Gamma_h^4 v_h \sim c \Gamma_c \]  

(6)

where

\[ \Gamma_c \equiv \frac{E_c}{mc c^2} \]  

(7)

is the terminal Lorentz factor of the cocoon plasma (provided that \( \Gamma_c \geq 1 \)) after it escapes through the stellar surface and its thermal energy is converted to bulk kinetic energy.

Therefore, the jet head speed is sub-relativistic when \( \Gamma_c < 4 \) and is given by

\[ v_h \sim c \Gamma_c / 4. \]  

(8)

For \( \Gamma_c > 4 \), the jet head speed is mildly relativistic and its Lorentz factor is given by

\[ \Gamma_h \sim (\Gamma_c / 4)^{1/4}. \]  

(9)

The expansion speed of the cocoon in the direction perpendicular to its surface, \( v_c \), is determined by equating the ram pressure with the thermal pressure inside the cocoon \( (p_c) \), i.e.

\[ v_c = \left( \frac{p_c}{\rho_a} \right)^{1/2}. \]  

(10)
The average thermal pressure inside the cocoon is approximately

$$p_c = \rho_a v_c^2 \sim \frac{E_c}{3V_c},$$

(11)

where

$$V_c \sim \pi t^3 v_h v_c^2 / 3 \sim R_s^3 (v_c / v_h)^2$$

(12)
is the volume of the cocoon. Using equations (11) & (12) we find

$$v_c^4 \sim \frac{E_c v_h^2}{3\rho_a R_s^3} \sim \pi \theta_j^2 \Gamma_c R_s^3 v_h^3.$$  

(13)

We made use of equations (3) and (7) to derive the last equality.

Substituting for $v_h$ from equations (8) and (9) we obtain

$$v_c \sim \left\{ \begin{array}{ll}
(\pi \theta_j^2 \Gamma_c^3 / 48)^{1/4} & \text{for } \Gamma_c < 4 \\
(\pi \theta_j^2 \Gamma_c / 3)^{1/4} & \text{for } 4 < \Gamma_c < \theta_j^{-2}
\end{array} \right.$$  

(14)
The thermal pressure of the cocoon can be obtained using equations (10) and (14) and is given by

$$p_c \sim \frac{L_j}{\theta_j R_s^2 c (3\pi \Gamma_c)^{1/2}},$$  

(15)
and its temperature is

$$k_B T_c = k_B (3p_c / \sigma_a)^{1/4} \sim (7 \text{keV}) \frac{[L_j^{(\text{iso})}]^{1/4}}{R_s^{1/2} \Gamma_c^{1/8}},$$  

(16)
where $\sigma_a$ is the radiation constant, and $L_j^{(\text{iso})} = 4L_j / \theta_j^2$ is isotropic equivalent of GRB jet average luminosity. We note that the cocoon temperature has a weak dependence on jet luminosity and its unknown angular size while inside the star.

Once the cocoon punches through the stellar surface its Lorentz factor increases linearly with radius, as is the case for any radiation dominated relativistic plasma, until it attains the terminal value of $\Gamma_c$. The temperature in the observer frame during this phase of acceleration is constant. The temperature decreases as $r^{-2/3}$ when the cocoon starts to coast at a constant speed of $v_c$ at $R_s \sim R_s \Gamma_c$ but before its radial width starts to increase linearly with distance at $r \sim R_s \Gamma_c^2$, the temperature declines as $r^{-1}$ for $r \gtrsim R_s \Gamma_c^2$. 
Fig. 2.— The left panel shows the isotropic equivalent of luminosity of cocoon in GRB host galaxy rest frame (divided by $10^{50}\text{erg/s}$) as a function of its distance from the center of explosion. The thermal energy in the cocoon is taken to be $10^{52}\text{erg}$ (isotropic equivalent), its terminal Lorentz factor ($\Gamma_c$) is 5, and the radius of GRB progenitor star is taken to be $10^{11}\text{cm}$. The cocoon radiation lasts for about $10^2\text{s}$ in observer frame, and is likely hidden under a much brighter GRB prompt emission and its X-ray tail. The right panel shows the cocoon temperature in eV as a function of its distance from the center.

2.2. Thermal radiation from cocoon

The average number density of electrons associated with baryons in the cocoon, in its rest frame, is

$$n'_e(R_*) \sim \frac{m_e}{m_p V_c} \sim \frac{3p_c}{m_p c^2 \Gamma_c} \sim (2 \times 10^{19}\text{cm}^{-3}) \frac{L_{j,52}^{(\text{iso})} \theta_{j,-1}}{R_{*,11}^2 \Gamma_{c,3/2}}.$$  \hspace{1cm} (17)

This density is much larger than the number density of thermal $e^\pm$ pairs at temperature $T_c$,

$$n'_\pm = \frac{2(2\pi k_B m_e T_c)^{3/2}}{h^3} \exp \left( -\frac{m_e c^2}{k_B T_c} \right),$$  \hspace{1cm} (18)

as long as $k_B T < 30\text{keV}$. 

The photon mean free path length in cocoon comoving frame is
\[
\lambda'(r) = \frac{1}{\sigma_T n'_p(r)} \approx 8 \times 10^4 \text{cm} \frac{R_{s,11}^2 \Gamma_{c,1}^{3/2}}{L_{j,52}^{(iso)} \theta_{j,-1}} \left[ \frac{r}{R_*} \right]^3 \min \{1, \max \left[ R_s / r, \Gamma_{c}^{-1} \right] \},
\]
where \( R_s = \Gamma_c R_* \) is the radius where Lorentz factor of the cocoon stops increasing, \( R_* \) is the GRB progenitor star radius, and \( \theta_j \) is its average opening angle during the time when it was making its way out of the star.

The cocoon thermal luminosity is governed by photon diffusion across the cocoon and is given by
\[
L_{c}^{iso}(r) = 4\pi R_*^2 \sigma_{sb} T_c^4(R_*) \left( r / R_* \right)^{2/3} (\lambda'(r) / ct')^{1/2}
\]
where \( \sigma_{sb} \) is the Stefan-Boltzmann constant, \( T_c(R_*) \) is cocoon temperature when it emerges from the stellar surface, and \( t' = r / (c \Gamma_c) \) is dynamical time in cocoon rest frame. This equation for cocoon luminosity is valid as long as the optical depth is much larger than unity, i.e. for \( \lambda'(r) \ll r / \Gamma_c \). The luminosity and temperature are shown in fig. 2. The radius where the cocoon becomes transparent to Thomson scatterings, i.e. \( \lambda'(r) \approx r / \Gamma_c \), is given by:
\[
R_{tr} = \left[ \frac{\sigma_T E_c}{4\pi m_p c^2 \Gamma_c} \right]^{1/2},
\]
where \( E_c \) is the total energy of the cocoon. The optical depth of the cocoon for \( r < R_{tr} \) scales as \( r^{-2} \).

The cocoon luminosity drops quickly for \( r > R_{tr} \) since the total rate of emission due to the bremsstrahlung process at radius \( R_{tr} \) can be shown to be rather small:
\[
L_{ff}(r = R_{tr}) \approx (1.3 \times 10^{40} \text{ergs}^{-1}) \left[ T_c(R_{tr}) / 10^7 \text{K} \right]^{1/2} R_{tr,14} \Gamma_c^{2.5}.
\]

3. IC scattering of cocoon radiation by jet

For simplicity we will consider the cocoon, after it emerges from the surface of the star and following a brief period of acceleration, to be a shell of plasma that moves away from the star at a constant speed \( v_c \) and Lorentz factor \( \Gamma_c \). After emerging from the star the cocoon undergoes adiabatic expansion and converts a part of its thermal energy to kinetic energy of bulk motion. During this phase its Lorentz factor increases linearly with radius and reaches the terminal value \( \Gamma_c \) at \( r \sim \Gamma_c R_* \). The distance of the cocoon from the center of the star is approximately given by
\[
R_c(t) \approx R_* + tv_c.
\]
Fig. 3.— The basic geometry of inverse-Compton scattering of thermal radiation from cocoon by electrons in a relativistic jet. A photon leaves the cocoon at \((r, \theta, t_1)\) — point marked “B” in the sketch, and arrives at “A” where the relativistic jet is at time \(t\). The angle between the photon momentum vector and jet axis is \(\theta_1\). After IC scattering the photon travels within an angle \(\Gamma_j\) of the radial direction because of relativistic beaming; \(\Gamma_j\) is the Lorentz factor of the jet.

The time \(t\) is measured in the GRB host galaxy rest frame (as are all times unless specified otherwise), and its zero-point is taken to coincide with the emergence of the cocoon from stellar surface.

Let us assume that a relativistic jet is launched from GRB central engine at time \(t_j\) after the cocoon punches through the surface of the star. The jet carries a luminosity \(L_j^{(iso)}\) (isotropic equivalent), has Lorentz factor \(\Gamma_j\) and speed \(v_j\).

The time when the jet emerges above cocoon surface is given by

\[
t_{\text{emerge}} = \frac{R_* + t_j v_c}{v_j - v_c},
\]

when it is at a distance \(r_{\text{emerge}} = R_* + v_c t_{\text{emerge}}\) from the center.

We will not consider any inverse-Compton scattering of cocoon radiation by the jet until the jet punches through the cocoon photosphere or gets to a radius where the cocoon is transparent to photons (which ever comes first). This is to avoid the uncertainty regarding the escape of photons below the photosphere through the polar cavity which is likely to be partially filled by plasma flowing into it in between episodes of central engine activity.

Thermal photons from the cocoon can either travel directly to the relativistic jet and undergo inverse-Compton scattering there. Or photons from the cocoon could first be Thomson scattering by electrons in the circum-burst medium (CBM) toward the jet, and then undergo inverse-Compton scattering by electrons in the relativistic jet. We consider these two differ-
ent possibilities separately in subsections below. It might seem that the second process is less efficient than the first, and hence might not have any impact on observations. However, because of relativistic beaming of thermal photons from the cocoon in the forward radial direction, the angle between the jet axis and photons is of order $\Gamma^{-1}_c$ or less, and hence these IC scatterings boost photon energy by a factor $\sim (\Gamma_j/\Gamma_c)^2$ or less when electrons are cold in the jet comoving frame. On the other hand, thermal photons scattered first by electrons in the CBM can collide with the jet at larger angles and produce much higher energy photons.

3.1. Direct scattering of cocoon photons by the relativistic jet

The basic process considered in this section is sketched in figure 1. Photons from the cocoon travel directly to the relativistic jet and there they are IC scattered by electrons in the jet.

We consider all those photons from the cocoon that arrive at the jet at time $t$ when it is a distance $d$ from the center (fig. 3 – the point marked “A”), and determine the IC luminosity when some of these photons are scattered by the jet. Photons leave the cocoon photosphere at angle $\theta$, when it is at radius $r$, and meet the jet at radius $d$ (see fig. 3) at time $t$ when the following condition is satisfied:

$$t = (r - R_*)/v_c + r_1/c,$$

(25)

where

$$r_1^2 = d^2 + r^2 - 2rd \cos \theta.$$  

(26)

This pair of equations determine the locus of all points in the $(r, \theta)$ plane — the equal arrival time curve — from which photons arrive at “A” from the cocoon. We note that only a fraction of photons originating at the equal arrival time curve make it to the jet. Those that travel at an angle $(\theta + \theta_1) > \Gamma^{-1}_c$ are swept back up by the cocoon and scattered in a different direction depending on cocoon’s optical thickness at the time of this encounter. We keep track of this in our numerical calculations.

Photons IC scattered by the jet when it is at a distance $d$ arrive at a far away observer with a delay (wrt the arrival of first photons from the cocoon directly without suffering any scattering) of

$$t_{\text{obs}} \approx t_j + \frac{d}{2c\Gamma_j^2}.$$  

(27)

We are ignoring cosmological redshift factors in this and all other equations which can be easily included in the final expression for flux etc.
Fig. 4.— The left panel shows IC luminosity divided by the luminosity carried by the relativistic jet ($L_{ic}/L_j$) as a function of distance of the jet from the center of explosion. Electrons are taken to be cold in the jet comoving frame, i.e. $\gamma_e = 1$; $L_{ic} \propto \gamma_e^2$ as long as the energy of photons from the cocoon as seen in electron rest frame is less than $m_e c^2$. The thermal energy in the cocoon is $10^{52}$ erg (isotropic equivalent), its terminal Lorentz factor is 5, and the radius of GRB progenitor star is taken to be $10^{11}$ cm. The Lorentz factor of the relativistic jet ($\Gamma_j$) is 100 for all calculations shown in this figure. And its delay wrt to the time when cocoon punches through the stellar surface ($t_j$) is taken to be 10s (solid curve), 50s (dashed curve) and 250s (dot-dash curve). We note that the x-axis can be converted to observer frame time using the equation $t_{obs} = [t_j + d/(2c\Gamma_j^2)](1 + z)$. And to obtain the observed lightcurve we need to convolve the curve in the left panel with $L_j(t)$, and include contributions to IC scatterings from parts of the jet moving at an angle larger than $\Gamma_j^{-1}$ wrt observer line of sight; the latter effect prevents lightcurve from falling off faster than $[t_{obs}/(1 + z) - t_j]^{-3}$ (Kumar and Panaitescu, 2000). The right hand panel shows the photon energy at the peak of the IC spectrum as a function of jet distance from the center for the same three values of $t_j$ as in the left panel.

3.1.1. IC luminosity due to cocoon radiation scattering off of electrons in the jet

The specific intensity of thermal radiation from the cocoon in its rest frame when it is at radius $r$ is $I'_\nu(r)$. The specific intensity in the GRB host galaxy rest frame is related to
\( I'_{\nu'} \) as follows

\[
I_{\nu}(r) = I'_{\nu'}(r) \left[ \frac{\nu}{\nu'} \right]^3 = \frac{I'_{\nu'}(r)}{\Gamma^3_c [1 - \beta_c \cos(\theta + \theta_1)]^3},
\]

where

\[
\beta_c \equiv v_c/c, \quad \nu = \frac{\nu'}{\Gamma_c [1 - \beta_c \cos(\theta + \theta_1)]} \equiv \nu' \mathcal{D}_1
\]

is the relativistic Doppler shift formula, and angles \( \theta \) & \( \theta_1 \) are defined in fig. 3. The bolometric thermal luminosity of the cocoon for a far away observer is

\[
L_c^{\text{iso}}(r) = 4\pi d^2 \int d\Omega \int d\nu I_{\nu} = 2\pi^2 r^2 \nu'_{p} I'_{\nu'} \Gamma_c^2,
\]

where \( \nu'_{p} \) is frequency at the peak of cocoon’s thermal radiation spectrum.

The specific intensity in the jet comoving frame is given by

\[
I''_{\nu''} = I_{\nu''}/\nu'' = I_{\nu' \Gamma_j^3 (1 - \beta_j \cos \theta_1)^3} = I'_{\nu'} \left[ \frac{\Gamma_j (1 - \beta_j \cos \theta_1)}{\Gamma_c (1 - \beta_c \cos(\theta + \theta_1))} \right]^3,
\]

\[
\nu'' = \nu \Gamma_j (1 - \beta_j \cos \theta_1) \equiv \nu \mathcal{D}_2.
\]

Radiation from the cocoon is IC scattered by electrons in the jet. If the Lorentz factor of electrons associated with their random motion in the jet comoving frame is \( \gamma_e \), then the bolometric IC luminosity (isotropic equivalent) in observer frame when the jet is at a distance \( d \) from the center of explosion is

\[
L_{\text{ic}}^{\text{iso}} = 4\pi d^2 (\gamma_e \Gamma_j)^2 \int d\Omega''_1 \int d\nu'' I''_{\nu''} \min [1, \tau_T(\theta_1)],
\]

where

\[
\tau_T(\theta_1) = \frac{\sigma_T(I_{\nu''}^{\text{iso}}) \delta t_j}{4\pi d^2 m_p c^2 \Gamma_j \cos \theta'_1 \max \{1, c\delta t_j/(d/\Gamma_j^2)\}},
\]

is the Thomson scattering optical depth of the causally connected part of the jet for photons moving at angle \( \theta_1 \) wrt to the jet axis; \( \delta t_j \) is jet duration in GRB host galaxy rest frame, and

\[
\cos \theta'_1 = \frac{\cos \theta_1 - \beta_j}{1 - \beta_j \cos \theta_1}
\]

is cosine of the angle between jet axis and photon momentum in jet comoving frame.

The IC luminosity equation (33) can be rewritten in a more convenient form by replacing jet comoving frame variable \( \Omega'' \) with host galaxy rest frame variable \( \Omega \), and \( \nu'' \) with \( \nu' \):

\[
L_{\text{ic}}^{\text{iso}} = 4\pi d^2 (\gamma_e \Gamma_j)^2 \int d\Omega_1 \int d\nu' I'_{\nu'} \min [1, \tau_T(\theta_1)] \frac{\Gamma_j^2 (1 - \beta_j \cos \theta_1)^2}{\Gamma_c^4 (1 - \beta_c \cos(\theta + \theta_1))^4},
\]
where we made use of $d\Omega'_i/d\Omega_1 = D_i^{-2}$, and equations (29), (31) & (32).

The peak of the IC spectrum in observer frame is at

$$\nu_p^{\text{(ic)}} = \frac{\Gamma_j \gamma_i^2}{\int d\Omega'_{\nu'} d\nu' \nu' I_{\nu'}^{\text{(ic)}} \min [1, \tau_T(\theta_1)]} = \frac{\Gamma_j \gamma_i^2}{\int d\Omega_1 \nu I_{\nu'} \min [1, \tau_T(\theta_1)]} \frac{D_i^2 D_j^2}{\int \nu I_{\nu'} \min [1, \tau_T(\theta_1)]} D_i^2 D_j^2. \tag{37}$$

Numerical results for IC luminosity and peak photon energy are shown in figure 4 and order of magnitude estimates are provided in the sub-section below. We point out that the IC luminosity peaks roughly at the time when the jet emerges above cocoon photosphere, i.e. IC scatterings of cocoon radiation is very efficient in extracting energy from the jet at least for a brief period of time. The phase of high luminosity lasts for a time of order the curvature time $(d/(2c\Gamma_j^2))$ or jet duration (in GRB host galaxy rest frame) whichever is larger. The IC luminosity decreases rapidly with jet distance from the center, as $\sim d^{-5}$, after the jet moves away from the cocoon photosphere. This decline is reduced to $d^{-2}$ for $d \gtrsim R_{tr}(\Gamma_j/c)\max\{1, |t_j/c/R_{tr}|^{1/2}\}$ when thermal photons are moving within an angle $\Gamma_j^{-1}$ of the jet axis and consequently IC scatterings do not boost the energy of photons; $R_{tr}$ is the radius where the cocoon becomes transparent to Thomson scatterings. The peak of the IC spectrum is roughly at $T_c(\Gamma_j/2c)^2 \sim 10^5$ eV at the time when the jet emerges above the cocoon-photosphere (fig. 4).

### 3.1.2. Order of magnitude estimate for IC luminosity

A relativistic jet launched with a delay of time $t_j$, and moving outward at speed $v_j$, is at a distance $d$ from the center of explosion at time $t_d = t_j + d/v_j \approx t_j + d/c + d/(2c\Gamma_j^2)$. The jet is met with photons from the cocoon which were emitted in a certain region of $(r, \theta)$ plane at different times. The smallest cocoon radius ($R_{\text{min}}$) from which photons could arrive at the jet at time $t_d$ corresponds to $\theta = \pi/2$ and the maximum radius ($R_{\text{max}}$) is when $\theta = 0$ (see fig. 3). Therefore, equations for the minimum and maximum cocoon radii are obtained by equating jet and photon travel times to $d$, and are given by

$$t_d = \frac{R_{\text{min}} - R_*}{v_c} + \frac{(R_{\text{min}}^2 + d^2)^{1/2}}{c} \approx \frac{R_{\text{min}} - R_*}{v_c} + \frac{d}{c} \Rightarrow R_{\text{min}} \approx R_* + v_c \left[t_j + \frac{d}{2c\Gamma_j^2}\right], \tag{38}$$

and

$$t_d = \frac{R_{\text{max}} - R_*}{v_c} + \frac{d - R_{\text{max}}}{c} \Rightarrow R_{\text{max}} \approx 2(ct_j + R_*)\Gamma_j c + d(\Gamma_j/c)^2 = 2\Gamma_j^2(cR_* + ct_{\text{obs}}). \tag{39}$$

Photons emitted at $\theta > 1/\Gamma_c$ in the direction of the jet could get swept back up by the cocoon and scattered in a different direction before reaching the jet. This is certainly true
when $R_{\text{min}}$ given by the above equation is much smaller than $R_{tr}$. In that case a better lower limit for cocoon radius from which photons can arrive at the jet is obtained by taking $\theta \approx 1/\Gamma_c$ and that gives $R_{\text{min}} \approx t_{\text{obs}}v_c\Gamma_c^2$. The IC luminosity vanishes when $R_{\text{min}} > R_{tr}$ or $t_{\text{obs}} \gtrsim R_{tr}/v_c$ since the cocoon luminosity drops off steeply for $r > R_{tr}$.

The equation for the curve in $(r, \theta)$ plane from which photons arrive at the jet at the same time is obtained by equating the time for a photon from the cocoon to arrive at the jet and the time it takes for the jet to arrive at $r = d$ (including the launch delay of $t_j$), i.e.

$$t_j = \frac{r - R_\ast}{v_c} + \frac{r_1}{c} - \frac{d}{v_j} \approx \frac{r - R_\ast + r_1 - d}{c} + \frac{r}{2c\Gamma_c^2} - \frac{d}{2c\Gamma_j^2}. \quad (40)$$

This equation can be rewritten as

$$t_j + R_\ast/c \approx \frac{r}{2c} \left( \theta^2 + \Gamma_c^{-2} \right) + \frac{r_1}{2c} \left( \theta_1^2 - \Gamma_j^{-2} \right), \quad (41)$$

where $r_1$, $\theta$ and $\theta_1$ are as defined in figure 3. Since, $r\theta \approx r_1\theta_1$, that means that the time for the cocoon to travel to radius $r$ is a factor $\sim r_1/r$ larger than the time it takes for a photon to travel the distance $r_1$ to the jet. We consider here the case where $d \ll 2ct_j\Gamma_j^2$, and for that the above time delay equation simplifies to

$$t_j + R_\ast/c \approx \frac{r}{2c\Gamma_c^2} + \frac{r\theta^2d}{2c(d - r)} \approx \frac{r}{2c\Gamma_c^2} + \frac{(d - r)\theta^2d}{2cr}. \quad (42)$$

Considering that $^3\theta \lesssim \Gamma_c^{-1}$ we see from equation (42) that photons reaching the jet were emitted from the cocoon when it was at a radius $r \approx \min\{2(ct_j + R_\ast)\Gamma_c^2, R_{tr}\}$ and at a latitude

$$\theta \approx \begin{cases} 
\Gamma_c^{-1} & t_j \ll R_{tr}/(2c\Gamma_c^2) \\
(2ct_j/R_{tr})^{1/2} & t_j \gg R_{tr}/(2c\Gamma_c^2)
\end{cases} \quad (43)$$

The expression for IC luminosity (eq. 36) can be simplified using small angle and large Lorentz factor expansion, and is given by

$$L^{(\text{iso})}_{\text{ic}} \approx 32\pi^2\gamma_c^2\Gamma_c^4 \int d\theta \theta I'(r)r^2\tau_T \frac{[1 + (\theta_1\Gamma_j)^2]^2}{[1 + (\theta\Gamma_c)^2]^4}, \quad (44)$$

where $I'(r) = L^{\text{iso}}_{\text{c}}(r)/(4\pi^2\gamma_c^2\Gamma_c^2)$ is the frequency-integrated specific intensity in cocoon comoving frame, $L^{\text{iso}}_{\text{c}}(r)$ is cocoon luminosity (in host galaxy rest frame) when at radius $r$ which

---

3This is because of forward beaming of photons from the cocoon to within an angle $\Gamma_c^{-1}$ of the cocoon’s velocity vector, and also because photons traveling at a larger angle wrt the radial direction will be swept back up and scattered in a different direction by the cocoon (if $r < R_{tr}$) before they can reach the jet.
is given by equation (20), and \( \theta \) \& \( \theta_1 \) (fig. 3) are related by \( \theta_1 \approx r \theta / r_1 \approx r \theta / d \). Most of the contribution to the integral in the above equation comes from \( \theta \sim \Gamma_c^{-1} \) for \( t_j \ll R_{tr}/(2c\Gamma_c^2) \), and \( \theta_1 \sim (2ct_j/R_{tr})^{1/2} \) when \( t_j \gg R_{tr}/(2c\Gamma_c^2) \). The IC luminosity with this approximation is given by (as long as energies of thermal photons from the cocoon are less than \( m_e c^2 \) in the comoving frame of electrons in the jet)

\[
L^{(iso)}_{ic}(d) \approx \tau_T \gamma^2_c \times \begin{cases} 
L^{iso}_c(r_t)[1 + (\theta_1 \Gamma_j)^2]^2 & t_j \ll R_{tr}/(2c\Gamma_c^2) \\
4L^{iso}_c(r_t)(\theta_1 \Gamma_c)^{-6}[1 + (\theta_1 \Gamma_j)^2]^2 & t_j \gg R_{tr}/(2c\Gamma_c^2)
\end{cases}
\]  
(45)

where

\[
r_t = \min \{ R_{tr}, 2ct_j \Gamma_c^2 \}, \quad \theta_t \sim \max \{ \Gamma_c^{-1}, (2ct_j/R_{tr})^{1/2} \},
\]

and

\[
\theta_1 \approx \begin{cases} 
2ct_j \Gamma_c / d & t_j \ll R_{tr}/(2c\Gamma_c^2) \\
(2ct_j R_{tr})^{3/2} / d & t_j \gg R_{tr}/(2c\Gamma_c^2)
\end{cases}
\]  
(46)

According to eq. (34) the optical depth of a relativistic jet to Thomson scatterings scales as \( \tau_T \propto d^{-1} \) when the jet duration is longer than \( d/(c\Gamma_j^2) \). In this case we see from equation (49) that \( L^{(iso)}_{ic} \propto d^{-5} \) when \( \theta_1 \Gamma_j > 1 \), i.e. when the energy of IC scattered photons is larger than incident photon energy even for \( \gamma_c = 1 \). At larger jet distances, when photons from the cocoon are traveling almost parallel to the jet axis in order to catch up with it, so that \( \theta_1 \Gamma_j < 1 \), the IC luminosity declines as \( L^{(iso)}_{ic} \propto d^{-2} \). Numerical calculations confirm these rapid decline of IC luminosity with distance (fig. 4).

The IC luminosity peaks at the time when the jet emerges above the cocoon surface where \( d \sim r_t \) (eq. 46), and \( \theta_1 \sim \min(\theta_t, \Gamma_c^{-1}) \). The maximum IC luminosity can be obtained from equations (45) and (47) and is given by

\[
L^{(iso)}_{ic,max} \approx \gamma^2_c \times \begin{cases} 
L^{iso}_c(r_t) \Gamma_j / \Gamma_c & t_j \ll R_{tr}/(2c\Gamma_c^2) \\
4L^{iso}_c(R_{tr}) \Gamma_j / \Gamma_c [R_{tr}/(2ct_j \Gamma_c^2)] & t_j \gg R_{tr}/(2c\Gamma_c^2)
\end{cases}
\]  
(48)

The IC lightcurve peaks in the observer frame time at: \( t_{peak} \sim t_j + \min(r_t, R_{tr})/(2c\Gamma_j^2) \), and the temporal width of the peak is larger of the curvature time at \( r_t \) (\( \approx r_t/2c\Gamma_j^2 \)) and the timescale for decline of relativistic jet luminosity.

The peak of the IC spectrum can be shown to be at a frequency

\[
h\nu_{p}^{ic} \sim \gamma^2_c \Gamma_j \left[ \frac{k_B T_c}{\Gamma_c} \right] \left( \frac{R_e}{r_t} \right)^{2/3} \left[ \frac{\Gamma_j(1 - \beta_j \cos \theta_1)}{\Gamma_c \{1 - \beta_c \cos(\theta_t + \theta_1)\}} \right] \sim \frac{k_B T_c \gamma^2_c [1 + (\theta_1 \Gamma_j)^2]}{1 + [(\theta_t + \theta_1) \Gamma_c]^2} \left( \frac{R_e}{r_t} \right)^{2/3}
\]  
(49)
as long as scatterings are not in Klein-Nishina regime; here $T_c$, $r_t$, $\theta_t$ and $\theta_1$ are given by equations (16), (46) and (47). A more explicit expression for the IC peak frequency is obtained by substituting for these variables:

$$
\sqrt{\nu_p^{ic}} \sim \frac{\gamma_e^2 k_B T_c}{R_{tr}} \left[ \frac{R_{\ast}}{R_{tr}} \right]^{2/3} \times \left\{ 1 + \left( \frac{ct_j \Gamma_j}{d} \right)^2 \left( \frac{R_{tr}}{R_{\ast} + 2ct_j \Gamma_j} \right)^{2/3} \right\}^{2/3}
$$

$$
\left[ 1 + 2ct_j \Gamma_j^2 R_{tr}/d^2 \right] \left[ 1 + 2ct_j \Gamma_j^2 / R_{tr} \right]^{-1}
$$

$$
t_j \ll R_{tr}/(2c \Gamma^2)
$$

$$
t_j \gg R_{tr}/(2c \Gamma^2)
$$

(50)

Fig. 5.— Thermal photons from the cocoon are scattered by electrons in the wind that left the GRB progenitor star within the last few years of its life. Some of these scattered photons collide with the relativistic jet at a fairly large angle wrt the jet axis, and undergo strong inverse-Compton scattering by electrons in the jet.

3.2. IC scattering of wind-scattered cocoon radiation by relativistic jet

Let us consider a spherical stream of photons produced by the cocoon moving outward to larger radii. Some of these photons are scattered by electrons in the circum-stellar medium
— wind from the GRB progenitor star — and arrive at the relativistic jet where they could suffer a second scattering by electrons in the jet. We calculate the average photon energy and luminosity of these IC photons in observer frame.

The front of the photon stream from the cocoon is at radius

\[ r_{\gamma t} = R_\ast + tc, \]  

at time \( t \) in the host galaxy rest frame, and its rear end is at

\[ r_{\gamma l} = \max \{ R_\ast + tv_c, R_{tr} + (t - t_{tr})c \}, \]  

where \( R_{tr} \) is the radius where the cocoon becomes transparent to Thomson scattering\(^4\) which is given by equation (21), and

\[ t_{tr} = (R_{tr} - R_\ast)/v_c. \]  

The thermal flux from the cocoon at time \( t \), and radius \( r \) between \( r_{\gamma t} \) and \( r_{\gamma l} \), is

\[ f_c(r, t) = \frac{L_{iso}^{\ast}(r_1)}{4\pi r^2}, \]  

where \( L_{iso}^{\ast} \) is given by equation (20), and

\[ r_1 = R_\ast + [r_{\gamma l}(t) - r] \beta_c/(1 - \beta_c). \]  

Let us consider electron density at radius \( r \) associated with GRB progenitor wind to be

\[ n_e(r) = n_0(R_\ast/r)^2. \]  

The IC luminosity calculation requires as input the specific intensity of wind-scattered thermal photons at the location of the jet. Photons scattered in the wind at \((r, \theta, \phi = 0, t_1)\) will arrive at the jet location \((d, 0, 0, t)\) provided that

\[ r = \left[ d^2 + (t - t_1)^2c^2 - 2c(t - t_1)d \cos \theta_1 \right]^{1/2}, \]  

Where \( \theta_1 \) is the angle between jet axis and the photon momentum vector (fig. 6), and is related to \( \theta \) via the following equation

\[ r_1 \sin \theta_1 = r \sin \theta, \quad r_1 \equiv c(t - t_1). \]  

\(^4\)The thermal luminosity of the cocoon drops rapidly beyond the transparency radius \((R_{tr})\).
Equation (57) can be solved to determine the time $t_1$ when a photon at $(r, \theta)$ should be scattered so that it arrives at the jet at time $t$. This in turn allows us to calculate the specific intensity of wind-scattered photons at the location of the jet:

$$\delta I_{\nu}^{(s)}(\theta_1) = \sigma_T f_c(r, t_1)n_e(r)\delta r_1/4\pi$$

With specific intensity in hand we can calculate the IC luminosity using the following equation

$$L_{ic}^{(iso)} = 4\pi d^2(\gamma_c \Gamma_j)^2 \int d\Omega' \int d\nu' \min[1, \tau_T(\theta_1)] I_{\nu'}^{(s)''},$$

where $I_{\nu'}^{(s)''}$ is the specific luminosity of wind scattered photons as measured in the rest frame of the jet. The equation for IC luminosity can be rewritten in a more convenient form using Lorentz transformations of specific intensity, angle and frequency:

$$L_{ic}^{(iso)} = 4\pi d^2(\gamma_c \Gamma_j)^2 \int d\Omega_1 \int d\nu \int dr_1 \min[1, \tau_T(\theta_1)] \frac{dI_{\nu}^{(s)}}{dr_1} D_2^2,$$

where the Doppler factor $D_2$ is defined in equation (32).

Numerical results for IC luminosity and peak photon energy are shown in figure 7, and order of magnitude estimates are provided in the sub-section below. The IC luminosity peaks roughly at the time when the jet emerges above the cocoon photosphere, and then decreases as $\sim d^{-4}$ after the jet travels past the photosphere.

3.2.1. Order of magnitude estimate for IC luminosity

Consider a photon that is emitted by the cocoon at radius $r_c(t_2)$ and scattered by an electron in the CBM at radius $r$ toward the jet which is at a distance $d$ from the center of explosion. The angle between the photon momentum and the jet axis is $\theta_1$ (fig. 6). The requirement that photons arrive at the jet when it is at distance $d$ can be expressed as

$$\frac{r\theta^2}{2} + \frac{r_1\theta^2}{2} + \frac{r_c}{2\Gamma_c^2} = ct_j + \frac{d}{2\Gamma_j^2},$$

where as before $t_j$ is the time when the jet is launched, and other symbols are defined in figure 6.

At time $t$ the distance of the leading edge of the thermal radiation front from the center of explosion is given by

$$r_{\gamma l}(t) = tc + R_\ast \approx tc,$$
and the trailing edge is at
\[ r_{\gamma t}(t) = r_{\gamma t}(t) \left[ 1 - 1/2\Gamma_c^2 \right]. \] (64)

Photons near the leading edge left the cocoon when it was at radius \( R_s \) whereas photons near the trailing edge were produced close to \( r_{\gamma t} \). Therefore, for photons near the leading edge, the term \( r_c/2\Gamma_c^2 \approx R_s/2\Gamma_c^2 \) in equation (62) can be neglected, and the equation simplifies to
\[ t_j \approx \frac{r\theta^2 d}{2cr_1} \approx \frac{r_1\theta_1^2 d}{2cr}, \] (65)

where we made use of a geometrical relation \( r\theta = r_1\theta_1 \) for the triangle OAB (fig. 6).

---

\(^5\)We are also considering \( d \ll 2\epsilon_j\Gamma_j^2 \) so that the second term on the right side of equation (62) can be neglected.
The intersection of a circle of radius $r_1$ centered at the jet head with the region containing cocoon radiation at time $t_1 = t_j + d/v_j - r_1/c$ provides the locus of all points from which photons scattered at time $t_1$ arrive at the jet at the same time (fig. 6). The width of the overlap region of a circle of radius $r_1$ (centered at the explosion site) and a second circle of radius $r_1$ (centered at the jet) is $\delta = r + r_1 - d \approx r\theta^2/2 + r_1\theta^2/2 \approx ct_j$ (fig. 6). Since the width of the radiation front is $r/2\Gamma^2_c$, all points on the circle of radius $r_1$ that lie inside of the other circle are also inside the radiation front at time $t_1$ as long as $r > 2ct_j\Gamma^2_c$. Therefore, the set of these points (shown in magenta color in fig. 6) constitute the entire 1-D hyper-surface from which photons scattered in the CBM at $t_1$ arrive at the jet at the same time. From this little geometrical construction we see that the angular size of the beam of CBM scattered radiation that arrives at the jet at radius $d$ is equal to $\theta_1$ which is given by equation (65).

The intensity of CBM scattered radiation is given by

$$\delta I^{(s)} = \frac{\sigma_T L_c^{iso}(r, t_1)n_e(r)\delta r_1}{16\pi^2 r^2},$$

and the IC luminosity due to scattering of these photons by the jet, in the Thomson regime, is obtained from equation (61) using small angle expansion

$$L_{ic}^{(iso)} \approx \frac{d^2c^2\sigma_T}{8} \int_0^{\theta_1} \int d\theta_a \frac{\tau_T L_c^{iso}(d - r_1, t_1)n_e(d - r_1)}{(d - r_1)^2} [1 + (\theta_a\Gamma_j)^2]^2 \theta_a,$$  \hspace{1cm} (67)

where $\theta_1$ — the upper limit of $\theta_a$ integration — is given by equation (65). It is easy to modify the above equation for $L_{ic}^{(iso)}$ to include Klein-Nishina cross-section when photon energy in electron rest frame is larger than $m_e c^2$. For $d \gtrsim 2ct_j\Gamma^2_c$ and $\theta_1\Gamma_j \gg 1$ equation (67) can be rewritten as:

$$L_{ic}^{(iso)} \approx \frac{d^2c^2\gamma_c^2 \Gamma^4_j \tau_T \sigma_T}{48} \int dr_1 \frac{L_c^{iso}(d - r_1, t_1)n_e(d - r_1)}{(d - r_1)^2} \left(\frac{2ct_j(d - r_1)}{r_1d}\right)^3. $$  \hspace{1cm} (68)

We note that the integrand for $L_{ic}^{(iso)}$ is a rapidly decreasing function of $r_1$ and therefore most of the contribution to the IC luminosity comes from the smallest possible value of $r_1$ which is $ct_j$ (this is to ensure that photons and jet arrive together at radius $d$ even though the jet was launched with a delay of $t_j$).

We consider $d \gtrsim 2ct_j\Gamma^2_c$ since at smaller distances the jet is below the cocoon’s photosphere, and photons scattered in the CBM cannot reach the jet (unless $d > R_v$). Moreover, equation (68) is valid only for $d \lesssim 2ct_j\Gamma^2_j$, and so for $2ct_j\Gamma^2_c \lesssim d \lesssim 2ct_j\Gamma^2_j$ the IC luminosity is given by

$$L_{ic}^{(iso)} \approx (\tau_T d^2)\frac{\gamma_c^2 \Gamma^4_j \sigma_T L_c^{iso}(d)n_e(d)ct_j}{d^2}. $$  \hspace{1cm} (69)
This expression for IC luminosity has a simple physical interpretation. Since the integrand for \( L_{ic}^{(iso)} \) increases rapidly with decreasing \( r_1 \) (eq. 68) most photons arriving at the jet were scattered by electrons in CBM within a distance \( ct_j \) of the jet. Hence the incident flux at the jet is \( f_s \sim \sigma_T L_c^{iso} n_e ct_j / d^2 \) which is IC scattered by electrons in the jet to produce \( L_{ic}^{(iso)} \sim f_s \tau_T \gamma_e^2 \Gamma_j^4 d^2 \) as long as Klein-Nishina corrections are unimportant.

If the region around the polar axis of the star is evacuated by the passage of a relativistic jet at an earlier time then there might be a conical cavity of opening angle \( \theta_m \) in the CBM containing few electrons to scatter cocoon photons toward the jet. In this case the lower limit for the radial integral in equation (68) is restricted to \( r_1,\text{min} = \frac{\theta_m^2 d^2}{2ct_j + \theta_m^2 d} \) which follows from eq. 65, and therefore the IC luminosity is given by

\[
L_{ic}^{(iso)} \approx \frac{\gamma_e^2 \Gamma_j^4 \tau_T d^2 \sigma_T n_e(d) L_c^{iso}(R_{tr})(ct_j)^3}{r_{1,\text{min}}^2 d^2}. \tag{71}
\]

Equations (69) and (71) show that the IC luminosity decreases with jet distance as \( d^{-4} \) or faster which is consistent with numerical calculations shown in fig. 7, the luminosity peaks when the jet emerges just above the cocoon photosphere.

Another case we discuss is when the jet distance from the center of explosion is much larger than \( 2ct_j \Gamma_j^2 \). In this case the second term on the right side of equation (62) dominates and the various angles are given by

\[
\theta \approx \left( \frac{r_1}{\Gamma_j^2} \right)^{1/2}, \quad \text{and} \quad \theta_1 \approx \left( \frac{r}{r_1 \Gamma_j^2} \right)^{1/2}. \tag{72}
\]

Substituting this into equation (67) we find

\[
L_{ic}^{(iso)} \approx \frac{d^2 \gamma_e^2 \tau_T \sigma_T}{16} \int dr_1 L_c^{iso}(R_{tr}) n_e(d - r_1) \left( \frac{d - r_1}{r_1} \right) \left[ 1 + \frac{d - r_1}{r_1} + \frac{(d - r_1)^2}{3r_1^2} \right]. \tag{73}
\]

For \( \theta_1 \Gamma_j \gg 1 \) the integrand is proportional to \( 1/[(d - r_1)r_1^2] \), and therefore most of the contribution to IC luminosity comes from the smallest possible value of \( r_1 \). A lower limit to \( r_1 \) is provided by the consideration that the stellar wind within some angle, \( \theta_m \), of the polar axis might have been swept-up and evacuated by a relativistic jet that moved through the region before the current jet we are considering came along. Substituting \( \theta = \theta_m \) into equation (72) we obtain the following lower bound for \( r_1 \)

\[
r_{1,\text{min}} = \frac{\theta_m^2 r_j^2 d}{1 + \theta_m^2 r_j^2}. \tag{74}
\]
Finally, the IC luminosity when $\theta_1 \Gamma_j = (d/r_{1,\text{min}} - 1)^{1/2} \gg 1$ is given by

$$L_{\text{iso}}^{(\text{iso})} \approx \frac{\gamma_2^2 \tau_T d^2}{48} \frac{L_{\text{iso}}^{(\text{iso})}(R_{tr}) \sigma_T n_e(d) d}{2 \Gamma_j^2 r_{1,\text{min}}^2},$$

which declines with distance as $d^{-3}$.

### 4. What can we learn from IC scattering of cocoon photons?

Thermal radiation from cocoon can be IC scattered by GRB relativistic jets during the prompt $\gamma$-ray emission phase and also at later times by jets associated with X-ray flares. The IC luminosity depends on cocoon and jet properties, and also on the density of the circum-burst medium\(^6\). Therefore, IC photons could provide information regarding these different aspects of a GRB and its progenitor star.

A jet launched with a delay of $t_j$ emerges above the cocoon photosphere at a distance $r_t = \min\{2c t_j \Gamma_c^2, R_{tr}\}$ from the center, and there it is bombarded with X-ray photons moving at an angle $\gtrsim \Gamma_c^{-1}$ wrt jet axis. The IC scatterings of these photons by electrons in the jet produce high energy photons with a luminosity $L_{\text{iso}}^{(\text{iso})} \sim L_{\text{iso}}^{(\text{iso})} \tau_T \gamma_e^2 (\Gamma_j/\Gamma_c)^4$ (eq. 48). The value for $L_{\text{iso}}^{(\text{iso})}$ is of order the luminosity carried by GRB prompt relativistic jet even when electrons are cold in the jet frame (fig. 4), and the energy of IC scattered thermal photons is a few hundred keV for $\Gamma_j = 10^2$. Therefore, IC scatterings of cocoon photons by the GRB jet is an important process that should be included in any discussion of $\gamma$-ray radiation mechanism and jet energetics\(^7\).

The IC luminosity declines rapidly with distance as the jet moves above the cocoon photosphere ($L_{\text{iso}}^{(\text{iso})} \propto d^{-5}$) since fewer and fewer cocoon-photons are able to catch up and collide with the jet at larger distances ($\theta_1 \propto d^{-1}$; eq. 47). Thus, a jet with a delay of $t_j$ provides information regarding cocoon radiation at a radius $\min\{2c t_j \Gamma_c^2, R_{tr}\}$.

If GRB jet energy is dissipated and electrons accelerated to high $\gamma_e$ somewhere within a radius of $\sim 10 r_t$ (so that $\gamma_e \Gamma_j \gtrsim 10^4$) then the IC radiation would peak at $\sim \text{GeV}$ carrying a good fraction of the luminosity of the jet (fig. 4). This process, therefore, should be useful for

---

\(^6\)IC luminosity depends on circum-burst medium density for the case where photons from the cocoon are first scattered by electrons in the CBM before bouncing off of the jet.

\(^7\)The spectrum of IC scattered cocoon radiation is a Doppler broadened thermal spectrum and not the usual Band function shaped GRB spectrum unless electron distribution in jet comoving frame is a power-law function of its energy. Therefore, IC radiation is only a part of the observed GRB emission.
Fig. 7.— The left panel shows IC luminosity divided by the luminosity carried by the relativistic jet ($L_{ic}/L_j$) as a function of distance of the jet from the center of explosion. The difference between this and figure (4) is that here we consider thermal photons from the cocoon to have been first scattered by electrons in the circum-burst medium (CBM) and a fraction of those run into the jet and undergo IC scattering whereas in the other case photons from the cocoon traveled to the jet directly. The density of the CBM for these calculations is taken to be $n_e(r) = 50 r_{17}^{-2} \text{cm}^{-3}$ which corresponds to mass loss rate of $10^{-5} \text{M}_\odot \text{yr}^{-1}$, and wind speed of $10^3 \text{km/s}$, during the last 10 years of the star’s life; $r_{17} \equiv r/10^{17} \text{cm}$ is the distance from the center of the star. A conical region of angular size 0.1 rad along the jet axis is assumed to have been evacuated, i.e. $n_e = 0$, by the passage of an earlier relativistic jet, launched within a few seconds of the cocoon break-out, which passed through this region. Electrons are taken to be cold in the jet comoving frame, i.e. $\gamma_e = 1$; $L_{ic} \propto \gamma_e^2$ as long as the energy of photons from the cocoon as seen in electron rest frame is less than $m_e c^2$. The thermal energy in the cocoon is assumed to be $10^{52} \text{erg}$ (isotropic equivalent), its terminal Lorentz factor $\Gamma_c = 5$, and the radius of GRB progenitor star is taken to be $10^{11} \text{cm}$. The Lorentz factor of the relativistic jet ($\Gamma_j$) is 100 for all calculations shown in this figure. And its delay wrt to the time when cocoon punches through the stellar surface ($t_j$) is taken to be 10s (solid curve), 100s (dashed curve) and $10^3$s (dot-dash curve). The right hand panel shows the photon energy at the peak of the IC spectrum as a function of jet distance from the center for the same three values of $t_j$ as in the left panel.
investigating jet dissipation and the poorly understood $\gamma$-ray radiation mechanism provided that $\gamma$-rays are produced within a radius of $\sim 10r_t$.

Toma, Wu and Meszaros (2009) suggested that this process could explain a delay of a few seconds for GeV emission detected by Fermi/LAT for a number of GRBs. However, that seems unlikely. Toma et al. considered radiation from the cocoon when it becomes transparent to Thomson scattering at a radius of $\sim 10^{14}$cm and that resulted in a delay of a few seconds for the IC emission. However, the cocoon starts radiating soon after it breaks through the stellar surface and its luminosity declines monotonically after it stops accelerating and attains the terminal Lorentz factor of $\Gamma_c$ at $r \sim R_\ast \Gamma_c \sim 10^{12}$cm, and so the delay for the arrival of cocoon photons IC-scattered by the jet should be smaller than a few seconds. We showed in §3 that IC luminosity peaks at the time when the relativistic jet emerges above the cocoon surface of optical depth 1 (and not the transparency radius) where it is bombarded with photons from the cocoon moving at an angle $\sim \Gamma_c^{-1}$ wrt the jet axis. Moreover, the IC luminosity declines rapidly with distance ($d^{-5}$) as photons from the cocoon have to move increasingly parallel to the jet in order to catch up to it. In this regime, i.e. when $d \gg 2t_j c \Gamma_c^2$ — which is what Toma et al. (2009) considered in their work — one needs $\gamma_e \gg 10^2$ in order to obtain a significant IC luminosity. However, in this case the radiation produced within the jet — which according to Toma et al. is observed as sub-MeV prompt emission — is much brighter than the radiation from the cocoon as measured in the jet comoving frame, and hence it is hard to see how the IC scattering of cocoon photons by the jet could be more important than scatterings of sub-MeV prompt $\gamma$-ray photons.

A detection of IC quasi-thermal component would provide information regarding cocoon luminosity and the ratio $\Gamma_j/\Gamma_c$. These quantities are related to the structure of the outer envelope of GRB progenitor star. IC photons would also provide information regarding electron thermal Lorentz factor in the region where prompt $\gamma$-ray photons are produced provided that that takes place within $\sim 10R_t$ as mentioned above. IC radiation is polarized, and its measurement would shed light on GRB jet structure.

Even an upper limit on IC emission provides useful information. For instance, the fact that $>100$ MeV emission from a typical GRB falls below Fermi/LAT detection threshold suggests that sub-MeV $\gamma$-ray prompt radiation is not produced between the distance of $\sim 10^{12}$cm and $10^{14}$cm from the center of explosion at least not involving a process that accelerates electrons to Lorentz factor larger than $\sim 10^2/(\Gamma_j/100)$; $\gamma$-ray source radius of less than $10^{15}$cm can be ruled out if $\gamma_e \gtrsim 10^2/(\Gamma_j/100)$ for those GRBs with high energy flux below the sensitivity of Fermi/LAT (fig. 4). This result is useful for constraining the mechanism by which $\gamma$-rays are produced in GRBs.

Late time jets ($t_j \gtrsim 10^2$s), such as those associated with X-ray flares, are useful for explor-
ing CBM density. Cocoon photons scattered first by electrons in the CBM and subsequently IC scattered by late jets produce a bright transient that peaks at a few $x \gamma^2_e$ MeV (fig. 7). The flux is directly proportional to the density of the CBM which is related to the mass loss rate of GRB progenitor star during the last $\sim 10$ years of its life.

GeV emission associated with late X-ray flares ($t_j \sim 500$ s) is reported for a long duration GRB 100728A at redshift 1.57 (Abdo et al. 2011). The X-ray and GeV emissions during the flare might be correlated, although the low photon statistics for Fermi/LAT precludes a definitive answer. The X-ray isotropic luminosity during the flare was $\sim 10^{49}$ erg s$^{-1}$ which in the jet comoving frame is significantly smaller than the estimated cocoon’s luminosity (fig. 2); X-ray flare data suggest jet Lorentz factor to be larger than 30 (Abdo et al. 2011). So from a theoretical point of view we expect cocoon photons to scatter off of the late X-ray flare jet, both directly as well as after bouncing off of CBM electrons, and produce high energy photons with luminosity comparable to that in the X-ray band and that is consistent with observations for this burst.

Another result of some interest is that the IC drag on a jet composed of electron-positron pairs, as opposed to electrons and protons, is so strong that the jet would lose most of its energy soon after emerging above cocoon photosphere. Since that is inconsistent with energy measured in GRB blastwave (from afterglow data) we conclude that GRB jets cannot be dominated by $e^\pm$.

5. Conclusion

Relativistic jets of long duration GRBs push aside stellar material, and evacuate a cavity, through the progenitor star on their way out to the surface. This process creates a hot cocoon of plasma surrounding the jet with energy of order $10^{52}$ erg (isotropic equivalent). A fraction of this energy is radiated away on time scale of a few hundred seconds (in observer frame) when the cocoon punches through the stellar surface. The interaction of this cocoon radiation with the relativistic jet has been investigated in this work and shown to be useful for exploring GRB jet and progenitor star properties. The basic idea is easy to explain. The radiative luminosity of the cocoon is of order $10^{48}$ erg/s (isotropic equivalent), and photons from the cocoon collide with the jet at an angle of order $\Gamma_c^{-1}$ wrt jet axis; $\Gamma_c$ is the Lorentz factor of the cocoon. The cocoon luminosity as viewed in the jet comoving frame is a factor $\Gamma_j^2/(3\Gamma_c^4)$ larger. The rate at which radiation is produced by the jet — which we see as prompt $\gamma$-ray or X-ray flare radiation — is of order $10^{51}\Gamma_j^{-2}$ erg s$^{-1}$ (isotropic equivalent) in its comoving frame. Therefore, the ratio of cocoon thermal-radiation and jet radiation energy densities is $\sim 3 \times 10^{-4}(\Gamma_j/\Gamma_c)^4$ in the jet comoving frame. This ratio is larger than
1 for $\Gamma_j/\Gamma_c > 10$, and in that case cocoon radiation is more important than the radiation produced within the relativistic jet for radiative cooling of electrons. Figures 4 and 7 show that the IC scattered cocoon radiation — which forms a halo peaked near the edge of the jet and roughly as wide as the jet — is of order the luminosity carried by the relativistic jet if electrons in the jet are heated to a thermal Lorentz factor larger than about $10^2$ within a distance from the central engine of $\sim 10^{15}$ cm.

The interaction of cocoon radiation with jet and predictions for high energy emission have been investigated in detail in this work. A lack of detection of IC scattered cocoon thermal-radiation suggests either that the jet energy is not dissipated and imparted to electrons out to a radius of at least $10^{15}$ cm — which would rule out a certain class of models for GRB prompt emission — or that the cocoon moves outward with a high Lorentz factor such that $\Gamma_j/\Gamma_c \lesssim 5$ (this possibility can be constrained by afterglow observations).

Photons from the cocoon scattered by electrons in the circum-burst medium (CBM) can collide with the jet at a larger angle than photons traveling from the cocoon to the jet directly. These collisions result is very high energy photons ($\sim$GeV) of considerable luminosity even for a modest thermal Lorentz factor for electrons (fig. 7). There is considerable uncertainty, however, in this estimate because the CBM could have been partially evacuated by an earlier passage of a relativistic jet through this region. Detection of this signal, or an upper limit, would provide a handle on the stellar mass loss rate during the last few years of the life of the GRB progenitor star.

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