Non-singular solutions of flux branes in M-theory and attractor solutions

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Abstract

Here we present a class of solutions of M-theory flux branes, which are non-singular at origin. These class of solutions help us to determine the field strength at origin together with the behavior of it near origin. Further we show a way to find the attractor solutions of such flux branes.

Keywords: Flux brane, M-theory, Attractor solution.

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I. INTRODUCTION

A magnetic flux tube is characterised by the integral of field strength over the transverse plane and the generalization of magnetic flux tubes to higher rank and dimensions are known as flux branes or F\(p\) branes. To be precise an F\(p\) brane in \(D\)-dimensions has \(p+1\) dimensional Poincare invariance in world volume together with \(SO(D-p-1)\) rotational invariance in \(D-p-1\) transverse directions and a non zero rank \(D-p-1\) field strength tangent to the transverse dimensions.

The example of a flux brane with gravity is the Melvin universe\([1]\), which is a F1 brane in 3+1 Einstein-Maxwell gravity and can be embedded in string theory (there are also dilatonic generalization of F1 brane in 3+1 dimensions\([2-4]\) in string theory). The embedding of the Melvin universe in M-theory and in string theory has created a lot of interest in particular in the type IIA F7 brane\([5-7]\), whose M-theory description suggests IIA-0A duality \([8,9]\) with a speculation that the end point of 0A (closed string) tachyon condensation is the IIA vacuum and F7-IIA cone duality with an alternate IIA description \([7,25-30]\).

Flux branes also play an important role in supergravity description of expansion of D\(p\) brane into D\((p+2)\) and in the stabilisation of tubular branes\([10-12]\). Further the study of magnetic background allowing a conformal theory description\([13-16]\) gives a direction for studying the decay of unstable background to stable supersymmetric ones\([17-22]\) and when several magnetic parameters are considered the supergravity background may preserve some supersymmetry\([23]\) and in the context of supersymmetric flux branes their construction and classification has been analysed extensively from supergravity perspective together with the search of dynamical objects in the theory\([24,42-46]\).

There is an interesting conjecture\([7]\) related to flux branes and holographic principle\([31,32]\), that the F\(p\) branes in string theory and M-theory are large N duals of field theories without gravity. This conjecture is in the spirit of Ads/CFT\([33]\) which is a realisation of holographic principle. (There also exists analogue of holographic principle at much larger length scale\([34-36]\)). Attempts are made to understand such non-supersymmetric holographic dualities studying solutions corresponding to non-supersymmetric brane configurations, but the
solutions turn out to be singular at origin[37-41].

Here we find a non-singular solutions of $F_p$ branes in M-theory. In M-theory there are two types of $F_p$ branes one is $F6$, a magnetic flux brane and the other is $F3$, an electric one. Their solutions can be obtained from the action in Einstein frame with graviton and $D - p - 1$ form field for $D = 11$ and for $p = 6$, $p = 3$(here both dilaton and dilaton coupling are set at zero). In section 2 we construct a class of non singular solutions near origin but for the sake of finite field strength we ultimately restrict ourselves to take solutions upto origin. Further these new solutions near origin help to determine the field strength at origin (in terms of the parameters of the solutions) together with the behavior of the field near origin.

In section 3 we extend the new solutions of section 2 with the help of the solutions in[7] which are singular at origin i.e we will prove that there exists a point at which the new solutions near origin and the corresponding old singular solutions intersect. Thus we get solutions of flux branes except at origin.

In sectin 4 we show that for non-singular solutions[7] which hold only near origin can be able to catch the corresponding solutions discussed in section 2. Thus we present non-singular solutions of M-theory flux branes with large field strength at origin. It is to be noted that the solutions in[7] i.e the non-singular solutions near origin and the singular solutions, do not intersect with each other.

In section 5 we propose a way to find the attractor solutions of M-theory flux branes without solving the metric explicitly but assuming some tentative form of the solutions of metric. Further this method is also applicable to determine the asymptotic solutions of II$q$ branes.(A II$q$ flux brane is obtained from an action in Einstein frame for $D = 10$ where dilaton is present with graviton and $q$ form field)

**II. SOLUTIONS NEAR ORIGIN BUT NOT AT ORIGIN**

As we have mentioned earlier that a $F_p$-brane will have $p + 1$ dimensional poincare invariance in the world-volume and $SO(q)$ rotational invariance in $q = D - p - 1$ transverse
directions. For notational convenience we will use the number of transverse dimensions $q$ to label the field strength of $F_p$-branes.

So taking the graviton, a $q$ form field strength $F_q$ and in the case of string theory a dilaton, the action in the Einstein frame is given by

$$S = \frac{1}{l_p^{D-2}} \int d^D x \sqrt{g} (R - \frac{1}{2} \partial \phi \partial \phi - \frac{1}{2q!} e^{a \phi} F_q^2)$$  \hspace{1cm} (2.1)$$

For M-theory we have $D = 11$ and $\phi = a = 0$, the field strength $F_q$ has either $q = 4$ for a magnetic flux branes or the dual $q = 7$ for an electric flux branes. Our ansatz for the metric is

$$ds^2 = e^{2A(r)} (-dt^2 + dx_1^2 + .... + dx_{D-q-1}^2) + dr^2 + e^{2C(r)} d\Omega_{q-1}^2$$  \hspace{1cm} (2.2)$$

The above metric has the asymptotic form

$$ds^2 \sim r^l (-dt^2 + dx_1^2 + .... + dx_{D-q-1}^2) + dr^2 + nr^2 d\Omega_{q-1}^2$$  \hspace{1cm} (2.3)$$

which is known as the attractor solution of the above metric, where $l$ and $n$ are positive constants. The equation of motion for the field strength $d \ast F = 0$ can be solved as

$$F_q = f M_p e^{-(D-q)A+(q-1)C} \epsilon_{r_1 \ldots r_{q-1}}$$  \hspace{1cm} (2.4)$$

where $f$ is dimensionless constant which measures the field strength at the origin. Again for large $r$ the field strength behaves as

$$F_q \sim sr^{q-2} \epsilon_q$$  \hspace{1cm} (2.5)$$

where $s$ is a positive constants and $\epsilon_q$ is the transverse volume.

So starting from above action we will seek for solutions of M-theory flux branes together with a zero energy constraint, coming from the $R_{rr}$ component of Einstein’s equation. For two fluxbranes here we first consider the F6-brane, and from above action we arrive at the following equations of motion

$$A'' + 7A' A' + 3A' C' - \frac{(f M_p)^2}{6} e^{-14A} = 0$$  \hspace{1cm} (2.6a)$$

$$C'' + 7A' C' + 3C' C' + \frac{(f M_p)^2}{3} e^{-14A} - 2e^{-2C} = 0$$  \hspace{1cm} (2.6b)$$
The zero energy constraint implies

\[ 7A'A' + 7A'C' + C'C' - \frac{(fM_p)^2}{12}e^{-14A} - e^{-2C} = 0 \quad (2.6c) \]

At the centre of the fluxbrane the metric is flat. So near origin we assume \( C = \ln \left( \frac{4m}{3k} \left( \frac{14}{27} \right)^{1/2} A \right) \). So for \( r \to 0, \ A \to 0 \) implies \( e^{2A} \to 1 \) and \( e^{2C} \to 0 \), which satisfy the boundary condition of F6 brane. Now defining

\[ A_2(r) = \frac{kr^{3/2}}{(r + m)^{3/2}} - \frac{3k}{2} \left( \frac{r}{m} \right)^{1/2} \left( 1 + \frac{r}{m} \right)^{1/2} + \frac{3k}{2} \left( \ln \left( \frac{r}{m} \right) \right)^{1/2} \left( 1 + \frac{r}{m} \right)^{1/2} \quad (2.7) \]

where \( 1 > m > 0 \) and \( k > 1 \) and \( A_2 \to 0 \) for \( r \to 0 \) and combining equations of (2.6) into a single equation

\[ A''(1 + 3A) + A'A'(2 + 7A) = 0 \quad (2.8) \]

it can be easily seen that for \( A = A_2 \) equation (2.8) holds near \( r = 0 \) and also it makes metric to be flat at origin. Here \( A_2 \) actually represents a class of solutions where for each \( m \) and \( k \) in their corresponding ranges we get each solution of the class and further for every \( m, A_2 \) remains a solution of (2.8) as long as \( \frac{r}{m} < 1 \)

Further one can find the field strength \( fM_p \) at \( r = 0 \) by using \( A = A_2 \) and \( C = C_2 = \ln \left( \frac{4m}{3k} \left( \frac{14}{27} \right)^{1/2} A_2 \right) \), but it turns out to be infinite for the cost of non-singularenness of the solution at origin. So for the sake of finite \( fM_p \) we have to restrict ourselves to go upto origin with this solution i.e we can allow a very large value of \( fM_p \) and without any loss of generality using (2.6(a)) we can write

\[ fM_p = \left( \frac{27k}{2m^2\epsilon} \right)^{1/2} \quad (2.9) \]

where \( \epsilon = \frac{r}{m} \neq 0 \) for some \( r = \bar{r} \) together with the field strength near origin by

\[ F = \left( \frac{27k}{2m^2\epsilon} \right)^{1/2} e^{-7A_2 + 3C_2} \quad (2.10) \]

Clearly our choices of \( m \) and \( k \) make field strength very large, but we fail to take (2.7) as a solution upto origin, we can go as close as possible but we have to stop somewhere. In other words the field strength at origin stops the solution at some \( r \neq 0 \) and for larger \( fM_p \) we can get closer to origin with this non-singular solution. We can extend this solution upto origin and we will discuss it in section 4.
Now we consider the F3 brane and so from (2.1) the equations of motion are

\[ A'' + 4A'A' + 6A'C' - \frac{(f M_p)^2}{3} e^{-8A} = 0 \]  \hspace{1cm} (2.11a)

\[ C'' + 4A'C' + 6C'C' + \frac{(f M_p)^2}{6} e^{-8A} - 5e^{-2C} = 0 \]  \hspace{1cm} (2.11b)

and the zero energy constraint becomes

\[ 24A'A' + 60C'C' + 96A'C' - \frac{(f M_p)^2}{24} e^{-8A} - 60e^{-2C} = 0 \]  \hspace{1cm} (2.11c)

Here once again we want to find non-singular solution of \(A\) together with \(C\). So assuming \(C = \ln \left(\frac{4m}{3k} \left(\frac{20}{27}\right)^{1/2} A\right)\) and combining equations of (2.11) we get

\[ A''(4 + 3A) + 2A'A'(2 + 4A) = 0 \]  \hspace{1cm} (2.12)

Again for \(A = A_2\) of (2.7) we find such \(A\) satisfy (2.12) near origin, where \(C = C_2 = \ln \left(\frac{4m}{3k} \left(\frac{20}{27}\right)^{1/2} A_2\right)\). Further we can find the field strength at origin to be infinite so we restrict our solution not to reach origin as we have done in F6 case, and also arguing previously we can find the field strength at origin using (2.11(a))

\[ f M_p = \left(\frac{27k}{2m^2\epsilon}\right)^{1/2} \]  \hspace{1cm} (2.13)

where \(\epsilon = \frac{r}{m} \neq 0\) for some \(r = \bar{r}\) together with the behavior of \(F\) near origin

\[ F = \left(\frac{27k}{2m^2\epsilon}\right)^{1/2} e^{-4A_2 + 6C_2} \]  \hspace{1cm} (2.14)

We also extend this solution up to origin in section 4.

So for F3 case field strength at origin has the same expression of that of F6 but this does not necessarily imply that at origin both have the same value of field strengths because \(m\) and \(k\) of (2.7) can take any value out of their corresponding ranges. We will discuss it later.

III. ATTEMPT FOR SOLUTIONS EXCEPT AT ORIGIN

In previous section we have got solutions of \(A\) and \(C\) of (2.2) for F6 and F3 branes near origin. This solutions are not valid for any \(r\) specially for large \(r\). This can be verified by
substituting $A$ and $C$ (as obtained in previous section) in their equations of motions or by comparing to the attractor solution, as every solutions asymptotically have to match with it.

We have discussed earlier that there exist solutions of metric of F6 and F3 branes i.e the solutions of $A$ which is singular at origin. So here we can make an attempt to paste the solutions of $A$ given here with the corresponding solutions in [7] such that we can get a solution that can be extended to any larger $r$ for both F6 and F3 branes. First we will do it for F6 brane.

For F6 brane the singular $A$ and $C$ of (2.2) (call them $A_3$ and $C_3$) are given by

$$A_3 = \frac{1}{7} \ln (r f M_p) - \frac{1}{14} \ln(18/7)$$  \hspace{1cm} (3.1a)

$$C_3 = \ln r - \frac{1}{2} \ln(27/14)$$  \hspace{1cm} (3.1b)

We will show that $A_3$ and $C_3$ intersect with $A_2$ and $C_2$ respectively. So we define

$$f(r) = A_2(r) - A_3(r)$$

Clearly $f$ is continuous in $(0, \infty)$. Now for $r \to 0$ as $A_3(r) \to -\infty$ and $A_2(r) \to 0$, $f(r) \to \infty$. So there exists $r_1$ very near to origin such that $f(r_1) > 0$.

Now if we can show that for some non zero $r_2$, $f(r_2) < 0$ then by Bolzano’s theorem of continuous function[47] there exists some $r_0$ in $(r_1, r_2)$ where $f(r_0) = 0$ which implies $A_3(r_0) = A_2(r_0)$ and to do so we have to carefull such that $r_0$ must not be much away from origin, and also $\frac{r}{m} < 1$ otherwise $A_2(r_0)$ will not be a solution there of (2.8).

So precisely we will show $f(r) < 0$, for $1 > \epsilon_2 > \frac{r}{m} > \epsilon_1 > 0$. We choose $k = 2n$ where $n$ is positive integer such that $\frac{3k}{2}(\epsilon_2)^{1/2} < 1$. Under such condition considering full expression of $A_2(r)$ where for all $\frac{r}{m} < 1$ we have $k(\frac{r}{m})^{3/2} < \frac{3k}{2}(\frac{r}{m})^{1/2}(1 + \frac{r}{m})^{3/2}$ and further there exists a constant $N$ such that $\ln (N(\frac{3k}{2})^2(\frac{r}{m})^{1/2}) > \ln ((\frac{r}{m})^{1/2} + (1 + \frac{r}{m})^{1/2})^{3k/2}$ So for

$$g(r) = E + k \frac{(\frac{r}{m})^{3/2}}{1 + (\frac{r}{m})^{3/2} - \frac{3k}{2} \frac{r}{m}^{1/2}(1 + \frac{r}{m})^{1/2}} + \frac{5}{14} \ln r$$

where

$$E = \ln (N(3k/2)^2(1/m)^{1/2}(18/7)^{1/14}(1/f M_p)^{1/7})$$
we can choose large $f M_p$ or small $\epsilon$ such that $E \leq 0$ then $g(r_2) < 0$ for some $\epsilon_1 < \frac{r_2 m}{m} < \epsilon_2$ i.e

$$f(r_2) = k\frac{(\frac{r_2}{m})^{3/2}}{(1 + \frac{r_2}{m})^{3/2}} - \frac{3k}{2} \frac{(\frac{r_2}{m})^{1/2}}{(1 + \frac{r_2}{m})^{1/2}}$$

$$+ \ln \left( \frac{((\frac{r_2}{m})^{1/2} + (1 + \frac{r_2}{m})^{1/2})^{3k/2}}{(rfM_p)^{1/4}} \right) < g(r_2) < 0$$

Further we can always choose $r_1 < r_2$ by continuity of $f$ and as $f(r) \to \infty$ for $r \to 0$. So for very small $r_0$ in $(r_1,r_2)$, $A_3(r_0) = A_2(r_0)$.

Now $C_3(r)$ and $C_2(r)$ both matches near origin and we can choose $r_0$ to be their point of pasting. So we define

$$A(r) = A_2(r) \quad \text{for } r \leq r_0$$

$$= A_3(r) \quad \text{for } r \geq r_0$$

$$C(r) = C_2(r) \quad \text{for } r \leq r_0$$

$$= C_3(r) \quad \text{for } r \geq r_0$$

(3.2)

Clearly these $A(r)$ and $C(r)$ are the solutions of F6 brane metric except origin.

In the same way we can also study F3. The singular solutions of it[7](here $A_3(r)$ and $C_3(r)$ are the solutions of $A(r)$ and $C(r)$ of (2.2) )are

$$A_3 = \frac{1}{4} \ln (rfM_p) - \frac{1}{8} \ln(9/2) \quad (3.3a)$$

$$C_3 = \ln r - \frac{1}{2} \ln(27/20) \quad (3.3b)$$

Again we can define a continuous function

$$f(r) = A_2(r) - A_3(r) \quad (3.8)$$

and arguing in the same we have a very small non zero $r_1$ such that $f(r_1) > 0$. Also under the same codition i.e for $1 > \epsilon_2 > \frac{r_1}{m} > \epsilon_1 > 0$ choosing $k$ to be an even positive integer such that for some, $\epsilon_1 < \frac{r_2 m}{m} < \epsilon_2$

$$f(r_2) = k\frac{(\frac{r_2}{m})^{3/2}}{(1 + \frac{r_2}{m})^{3/2}} - \frac{3k}{2} \frac{(\frac{r_2}{m})^{1/2}}{(1 + \frac{r_2}{m})^{1/2}}$$

$$+ \ln \left( \frac{((\frac{r_2}{m})^{1/2} + (1 + \frac{r_2}{m})^{1/2})^{3k/2}}{(rfM_p)^{1/4}} \right) < 0$$
and arguing in the previous way as we have done for F6 case that there exists $r_2 > r_1 > 0$ near origin, and $f(r_2)$ changes its sign. So by Bolzano’s theorem for some $r_0$ in $(r_1, r_2)$, $A_2(r_0) = A_3(r_0)$.

Here we can say the same thing for $C_2(r)$ and $C_3(r)$ so we can define

$$A(r) = A_2(r) \quad \text{for } r \leq r_0$$
$$= A_3(r) \quad \text{for } r \geq r_0$$

$$C(r) = C_2(r) \quad \text{for } r \leq r_0$$
$$= C_3(r) \quad \text{for } r \geq r_0$$

Thus this $A(r)$ and $C(r)$ are again the solution of $A(r)$ and $C(r)$ of (2.2) for F3 brane except at origin.

**IV. THE NON-SINGULAR SOLUTION**

In section 2 we have already mentioned that if we restrict $f$ from infinity we can not call (2.7) as a solution at origin or more precisely if we take $\epsilon = \frac{f}{m}$ then one can not carry (2.7) as a solution far from $r$ towards origin.

Let for $\bar{r}$, $fM_p$ is such, it manages $E \leq 0$, further $\bar{r}$ is small so $A_2(\bar{r}) = \frac{3kp}{4m}$. Now very close to origin

$$A_1(r) = \frac{1}{48}(rfM_p)^2$$

is a solution[7] of $A$ of (2.6) together with $C = C_1 = \ln\left((\frac{14}{27})^{1/2}r\right)$. So at $r = \bar{r}$, $A_1(\bar{r}) < A_2(\bar{r})$ and for some $r > \bar{r}$, $A_1(r) > A_2(r)$, if not then $A_1(r)$ must have an intersection with $A_3(r)$ of F6 but it is not so. So there exists $\bar{r}_0 < r_0$ such that $A_1(\bar{r}_0) = A_2(\bar{r}_0)$.

So using results of section 3 and with $A_1$, $A_2$, $A_3$ of F6 we have

$$A(r) = A_1(r) \quad \text{for } r \leq \bar{r}_0$$
$$= A_2(r) \quad \text{for } r \leq r_0$$
$$= A_3(r) \quad \text{for } r \geq r_0$$

(4.2)
Similar things can be done with $C$ and with $C_1$, $C_2$, $C_3$ of F6. This above $A$ is the non-singular solution of F6.

In the same way for F3 there also exists a solution of $A$

$$A_1(r) = \frac{1}{42} (r f M_p)^2 \quad (4.3)$$

near and upto origin with $C = C_1 = \ln \left(\frac{20}{27}\right)^{1/2} r$ and in the same way we can show that there exists $r_0 < r_0$ near origin where $A_1$ and $A_2$ intersect. So again with $A_1$, $A_2$, $A_3$ of F3 we have non-singular $A$ of F3 can be defined like F6 and same thing can be done for $C$ of F3 also.

V. A WAY TO FIND ATTRACTOR SOLUTION OF M-THEORY FLUX BRANES

In this section we attempt for a new way to find attractor solution of the metric of F6 and F3(except constant $n$ see (2.3)) without solving $A$ and $C$ of (2.2) explicitly. Looking at the the equations of motion of flux branes one can find $A$ and $C$ have to blow up for $r \to \infty$. So one of the choice out of the set of elementary functions may be both $A$ and $C$ are like $r^a$ for some $0 < a < 1$ but then $A''$ falls faster than $A'$ and $C'$. But if the choice of $A$ and $C$ are like $\ln r$ then $A''$, $A'A'$, $A'C'$ and $C''$ all fall like $\frac{1}{r^2}$ for large $r$. So for constants $\alpha$ and $\beta$ we choose $A = \alpha \ln r$ and $C = \beta \ln r$. Under such condition (2.6) implies at large $r$, both $(f M_p)^2 e^{-14A}$ and $e^{-2C}$ also behave as $\frac{1}{r^2}$ so

$$\alpha = \frac{1}{7} \quad \text{and} \quad \beta = 1 \quad (5.1)$$

and thus for large $r$ we have

$$ds^2 \sim r^{\frac{4}{7}} (-dt^2 + dx_1^2 + ... + dx_6^2) + dr^2 + r^2 d\Omega_{q-1}^2 \quad (5.2)$$

and this is the attractor solution of F6 flux brane(except constant $n$).

For F3 we can go through the same way, i.e taking $A = \alpha \ln r$ and $C = \beta \ln r$ and using (2.11) we get

$$\alpha = \frac{1}{4} \quad \text{and} \quad \beta = 1 \quad (5.3)$$
and asymptotically metric becomes

\[ ds^2 \sim r^{\frac{1}{2}}(-dt^2 + dx_1^2 + \ldots + dx_3^2) + dr^2 + r^2 d\Omega_{q-1}^2 \]  

(5.4)

which is the attractor solution of F3 flux brane (except \( n \)).

[Although here we only study the flux branes in M-theory but in this context it is very natural to discuss about the asymptotic solutions of type IIq flux branes, as the way we have described above the asymptotic solutions of M-theory flux branes also is a way for that of type IIq case.

In type II case there is dilaton in the action (2.1). For \( D = 10 \) and the dilaton coupling \( a = 1/2(5 - q) \), taking same ansatz (2.2) for the metric and using the integral of motion \( \phi = \frac{2}{5-q} A \) we have the equations of motion only for \( A \) and \( C \) as

\[ A'' + (10 - q)A'A' + (q - 1)A'C' - \frac{q - 1}{16}(f M_s)^2 e^{-2\frac{15+q}{4-q}A} = 0 \]  

(5.5a)

\[ C'' + (10 - q)C'A' + (q - 1)C'C' - \frac{9 - q}{16}(f M_s)^2 e^{-2\frac{15+q}{4-q}A} - (q - 2)e^{-2C} = 0 \]  

(5.5b)

together with the zero energy constraint

\[ [(10 - q)(9 - q) - 8\frac{(5 - q)^2}{(q - 1)^2}]A'A' + (q - 1)(q - 2)A'C' + 2(q - 1)(10 - q)A'C' \]

\[-(q - 1)(q - 2)e^{-2C} - \frac{1}{2}(f M_s)^2 e^{-2\frac{15+q}{4-q}A} = 0 \]  

(5.5c)

Again if one assume \( A = \alpha \ln r \) and \( C = \beta \ln r \) then in similar manner we can have

\[ \alpha = \frac{q - 1}{15 + q} \quad \text{and} \quad \beta = 1 \]  

(5.6)

and for large \( r \) we have

\[ ds^2 \sim r^{\frac{2}{15+q}}(-dt^2 + dx_1^2 + \ldots + dx_{9-q}^2) + dr^2 + r^2 d\Omega_{q-1}^2 \]  

(5.7)

. This is the asymptotic solution of metric of type IIq case except the multiplicative constants of \( r^{\frac{2}{15+q}} \) and \( r^2 \).]
VI. CONCLUSION

Although the solutions $A(r)$ of flux branes as given in section 4 is non-singular at origin but we have to show whether at $r_0$ and $\bar{r}_0$ derivatives of $A$ exist or not. However it can help to find the value of $fM_p$ (or atleast a range of values) for F6 and F3 cases.

So to get an estimate of $fM_p$ we have to fix $m$ and $k$ of the solutions or atleast to restrict their ranges further. To do so one can choose them in such a way that (if possible) the difference of one sided derivatives of $A(r)$ both at $r_0$ and $\bar{r}_0$ get minimised and otherwise we have to optimise. In this way we can compare the values of $fM_p$ for F3 and F6 cases.

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