Risk assessment model of expansive soil slope stability based on Fuzzy-AHP method and its engineering application

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ABSTRACT
To systematically evaluate the risk grade of stability of expansive soil slope during construction, the recognition model is established by using the fuzzy recognition theory and the analytic hierarchy process (AHP). Based on the characteristics of shallow slide of 22 expansive soil road sections constructed in China, the condition of slope body, swell–shrink grade, hydraulic and meteorological characteristics, supporting and improvement measures and other factors such as the construction technology and management level, vegetation coverage, soil layer, and drying–wetting cycles are selected as the evaluation indices, and the corresponding criteria of risk assessment system of expansive soil slope stability are also established. The weights of evaluation indices are obtained by using AHP method, and the measure functions are established to assess single index measurement and synthetic measurement. The maximum membership degree criterion is adopted to discern the risk grade of expansive soil slope stability. Based on this, the stability of expansive soil slope with different condition located in Nanyou highway and Beijing west 6th ring highway of China are analysed to verify the reliability of the present method. For practical purposes, the present risk assessment methodology can be used to systematically assess the slope stability in expansive soil zones.

1. Introduction
Expansive soil is a kind of geological body formed in the natural geological process which can have physical and chemical reactions with water, causing its own volume change. Expansive soil has the characteristics of fissure, consolidation, strong hydrophilicity, and repeated swell–shrink, which has negative influence on the environment and causes a hidden disaster in engineering. China is one of the countries with a wide distribution of expansive soils, and it has been estimated that the planned highways totalling 3300 km in length pass through expansive soils areas (Zheng and Yang 2004). The middle route of the South-North Water Transfer Project in China which is under construction will cross Nanyang, Shahe, Handan and some other areas with a total length of more than 300 km of expansive soils. The stability of the expansive slope seriously affects the safety of the projects, which will lead to great losses and damage to the environment (see Figure 1). Therefore, there is a need to analyse the risk assessment of expansive soil slope stability.
A large number of researches have been carried out to analyse the landslide mechanism and slope stability of expansive soil. However, due to the characteristics of expansiveness, fractures and over-consolidation, the slope stability of expansive soil appears as shallow slide and progressive damage which always happens during the rain or after rain (Liu et al. 1997; Zheng and Yang 2009). In recent years, a series of new results of risk assessment of slope stability have been made with the application of mathematical theory and artificial intelligence. Jibson et al. (2000) applied the probabilistic models for landslide risk and hazard analysis. Pistocchi et al. (2002) summarized many landslide hazard evaluation studies. Comprehensive lists of stability factors commonly employed in the factors mapping approach were given by Guzzetti et al. (1999). The analytical hierarchy process and its combinations, such as multi-criteria evaluation (MCE), multi-criteria decision analysis (MCDA) also have been used by different authors in landslide susceptibility mapping. Komac 2006; Yalcin 2008; Hasekiogullari and Ercanoglu 2012.

Based on the analysis of engineering geological survey of the unloading high slope of one hydroelectric power station, Wen et al. (2006) established the multistage fuzzy comprehensive evaluation model of slope stability to evaluate the stability of the slope. Based on the systematic study of red sandstone in China, Zhou (2010) carried out reliability analysis and risk assessment of red sandstone bedding slope in Xiangxi zone of China. Based on the results of the statistical analysis of a large amount of measured data in geotechnical engineering and civil engineering, Li (2015) established the fundamental fuzzy models of rock slope stability by using the theory of fuzzy measures. However, as one of the most widely distributed special soil, there is few research on risk assessment of expansive soil slope stability. Therefore, based on the fuzzy-analytic hierarchy process (FAHP) method, recognition model of risk assessment of expansive soil slope stability is established with comprehensive consideration of influence factors.

This paper presents a recognition model by using the fuzzy recognition theory and the analytic hierarchy process (AHP) to systematically evaluate the risk grade of stability of expansive soil slope during construction. Based on the characteristics of shallow slide of 22 expansive soil road sections during construction, the condition of slope body, swell–shrink grade of expansive soil, hydraulic and meteorological characteristics, supporting and improvement measures and other factors such as the construction technology and management level, vegetation coverage, soil layer, and drying–wetting cycles during construction are selected as the evaluation indices, and the corresponding criteria of risk assessment are established. The weights of evaluation indices are determined by using the AHP method, and the measure functions are established to assess single index measurement and synthetic measurement. The maximum membership degree criterion is adopted...
to discern the risk grade of expansive soil slope stability. Moreover, the stability of expansive soil slope located in Nanyou highway and Beijing west 6th ring highway of China were analysed to verify the reliability of the present method.

2. FAHP evaluation model

First proposed by T.L. Saaty (1979, 1990), the AHP is suitable for dealing with complex systems related to making a choice from several alternatives and provides a comparison of the considered options. In this paper, this method (Chang 1996) is utilized to establish recognition model of risk assessment of expansive soil slope stability.

Let $U_t$ be a factor set and $V$ be a target set. According to some common characteristics of evaluation factors, the factor set $U_t$ can be divided into several subset: $U_t = \{\mu_{t1}, \mu_{t2}, \ldots, \mu_{tm}\}$ and evaluation target is classified using the fuzzy language to establish the evaluation set: $V = \{v_1, v_2, \ldots, v_m\}$. After establishing the factor set and evaluation set, the basic steps of FAHP can be given as follows:

Step 1. Choose the linguistic ratings for criteria and alternatives with respect to criteria. In this step, the important weights of evaluation criteria and the ratings of alternatives are considered as linguistic terms to assess alternatives under fuzzy environment.

Step 2. Determine the degree of membership and establishment of the fuzzy evaluation matrix for single factor $\mu_{ti} (i = 1, 2, \ldots, n)$.

$$R_t = (R_{t1}, R_{t2}, \ldots, R_{tm}) = (r_{ij})_{n \times m} \quad (t = 1, 2, \ldots, k) \quad (1)$$

where $R_{ti}$ is the performance rating of the $i$th alternative, $r_{ij}$ with respect to the $t$th criterion, $C_t$.

Step 3. Determine the index weight. Subjective weights can be derived from AHP. A judgment matrix can be obtained by using the 1–9 scale method suggested by T.L. Saaty (1990).

Step 4. Comprehensive evaluation. If we assume that the number of criteria is $n$ and the count of alternatives is $m$, fuzzy decision matrix of single factor will be obtained with $m$ rows and $n$ columns. After constructing fuzzy decision matrix, the first level comprehensive evaluation vectors can be obtained with the corresponding weights.

$$b_{ti} = \sum_{i=1}^{m} \omega_i r_{ij} \quad (2)$$

$$B_{t} = \omega_t \cdot R_t = (b_{t1}, b_{t2}, \ldots, b_{tm}) \quad (3)$$

Combined with the weights of secondary level evaluation indices, the secondary level evaluation vector can be calculated by using matrix composed of first level evaluation vectors $B_t$ shown as follows:

$$Q = w_t \sum_{i=1}^{m} B_i = w_t \cdot \begin{bmatrix} B_1 \\ B_2 \\ \vdots \\ B_m \end{bmatrix} = \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_m \end{bmatrix} \cdot \begin{bmatrix} b_{11} & b_{21} & \cdots & b_{k1} \\ b_{12} & b_{22} & \cdots & b_{k2} \\ \vdots & \vdots & \ddots & \vdots \\ b_{1m} & b_{2m} & \cdots & b_{km} \end{bmatrix} \quad (4)$$

Based on the weighted normalized fuzzy decision matrix, the risk grade of expansive soil slope stability can be recognized according to the maximum membership degree principle.
2.1. Evaluation indices

The evaluation indices have very important influence on the accuracy of evaluation result (Bolt 1956; Williams 1958; Peck et al. 1974; Mitchell 1976). During the process of survey and treatment of landslide in expansive soil area, the influence factors of expansive soil slope stability should be clarified. Yin et al. (2010) proposed that the landslide characteristics of shallow, traction, seasonality, directivity and chronicity resulted from the cracks on expansive soil slope. Yin et al. (2009) presented that the swelling deformation which reduces the safety factor obviously should be considered during the analysis of expansive soil slope stability. Cheng (2006) proposed that the swelling deformation and expansive force were the main reasons of shallow damage. To evaluate the swell–shrink grade more scientifically, free expansion rate, moisture content of standard moisture absorption and plasticity index were taken as the evaluation indices of swell–shrink grade (Yao et al. 2004; Yao et al. 2005).

Rainfall is the main cause of shallow sliding damage of expansive soil slope (Zheng and Yang 2009; Jubair Hossain 2012). Based on the Galerkin finite element method, Shimada et al. (1995) carried out the simulation of two-dimensional unsaturated seepage of cut slopes with different rainfall intensity and soil types. The simulation results show that the slope safety factor reduces with increasing rainfall intensity, and the permeability function has little impact on the slope safety factor. D.G.Fredlund (1993) discussed the influence of rainfall intensity, rainfall duration and different soil type on the rainfall infiltration to analyse the sensitivity of parameters in the process of seepage by using numerical simulation. Their numerical simulation results show that the permeability coefficient has little effect on the slope stability compared with rainfall intensity. However, the influence of fissure on permeability coefficient was not considered. Based on the calculation of slope stability, Yao et al. (2002) proposed that the influence of cracks on expansive soil slope stability should be considered. Liu et al. (1997) provided the crack depth of expansive soil slope in different areas and topography in China: 1.5 m for flat terrain, the maximum up to 2.0 m; 2.5~3.0 m for sloping field, and the maximum up to 4.0 m.

The natural slope ratio of expansive soil is only 1:4~1:6, all kinds of protection and reinforcement measures are adopted for vast majority of expansive soil slope, such as rigid supporting treatment (the rigid retaining wall, anti-slide pile et al.), flexible supporting treatment, modified method with mixed inorganic binder, the sandwich method, coupled with vegetation cover and drainage facilities (Liu et al. 1997; Zheng and Yang 2009). As one more effective treatment technology, geogrid flexible supporting technology has solved the expansive soil problem of Nanbai road in Guangxi, Yewu road in Henan, Beijing west 6th ring road and Nanning outer ring road in Guangxi province successfully (Zheng and Yang 2004). Structure style and damage degree of treatment measures can influence the stability of expansive soil slope directly. Construction technology and management level also have a great impact on slope stability during the construction process, such as atmospheric wetting–drying cycles, the control of moisture content and degree of compaction (Zheng and Yang 2009). To simplify the analysis, the construction technology and management level are taken as one influence factor of fuzzy evaluation.

Based on the influence factors of expansive soil slope stability comprehensively, 2 index layers with 5 types of evaluation factors consisting of 18 evaluation indices, which have important reference significance on expansive soil slope stability are selected as follows, and the risk assessment indices system of expansive soil slope stability is shown in Figure 2.

Target layer: risk grade of expansive soil slope stability $Q$. The secondary stage index layer was summarized as follows: condition of slope body $B_1$, swell–shrink grade of expansive soil $B_2$, hydraulic and meteorological characteristics $B_3$, supporting and improvement measures $B_4$, and other factors $B_5$. The first stage index layer was summarized as follows: slope height $R_1$, slope gradient $R_{12}$, natural water content $R_{13}$, standard moisture absorption water content $R_{21}$, free expansion rate $R_{22}$, plasticity index $R_{23}$, rainfall intensity $R_{31}$, rainfall duration $R_{32}$, crack depth $R_{33}$, hydrogeological condition $R_{34}$, drainage measures $R_{35}$, flexible supporting treatment $R_{41}$, rigid supporting treatment
$R_{42}$, soil improvement $R_{43}$, construction technology and management level $R_{51}$, vegetation coverage $R_{52}$, soil layer $R_{53}$, and times of dry–wetting cycles during construction $R_{54}$.

### 2.2. Criteria of risk assessment of expansive soil slope stability

Based on the above-mentioned analysis of influence factors of expansive soil slope stability, the FAHP (Saaty 1979, 1990; Komac 2006) and the analysis results of 22 expansive soil road sections in China (Zheng and Yang 2004, 2009) were used to establish the indices and criteria of risk assessment of expansive soil slope stability, as shown in Table 2.

### 2.3. Weights analysis of evaluation indices

The weights of evaluation indices have a direct impact on the evaluation results, which should be determined on basis of the significance of each index and its influence degree on the slope stability objectively. Weights can be derived from AHP in this paper. A judgment matrix can be obtained by using the 1~9 scale method suggested by Saaty (1979). The weight vector and the maximum eigenvalues of the matrix can be expressed in the following forms:

$$\omega_i = \left( \prod_{j=1}^{n} p_{ij} \right)^{1/n} \sum_{j=1}^{n} \left( \prod_{j=1}^{n} p_{ij} \right)^{1/n},$$

$$\lambda_{\text{max}} = \left( \frac{1}{n} \sum_{i=1}^{n} \left( (A \omega)_i / \omega_i \right) \right)^{1/n},$$

where $A$ is the judgment matrix; $\omega_i$ is the weight of evaluation index; $\omega = (\omega_1, \omega_2, ..., \omega_n)$ is the weight of $A$; $\lambda_{\text{max}}$ is the maximum eigenvalues of $A$; $n$ is the quantity of evaluation indices; $i = 1,2,..., n$; and $j = 1,2,...,n$.

Take the ratio of $CI$ and $RI$ as the discriminate of consistency check, and the ratio of $CI$ and $RI$ can be expressed in the following form:

$$CR = \frac{CI}{RI} = \frac{(\lambda_{\text{max}} - n) / (n - 1)}{RI}$$

where $CI$ is the coincidence index; $RI$ is the random coincidence index value; and $CR$ is the random coincidence coefficient, indicating the matrix satisfy the requirement of consistency check when the value of $CR$ is smaller than 0.1.
Take the weight of slope condition as an example (see Table 1), and the weights of slope height $R_{11}$, slope gradient $R_{12}$, natural water content $R_{13}$ – i.e. $v_1, v_2, v_3$ can be adopted as 0.540, 0.297, and 0.163, respectively. The weights of basic indices are listed in Table 2.

### 2.4. Calculation of measure functions

Single index measure function is constructed to compute the single index measure $R_{ij}$. Assume $a_1 < a_2 < \cdots < a_n$, the measure functions can be written as:

$$R_{i1} = \begin{cases} 
1 & x < a_1 \\
\frac{a_2 - x}{a_2 - a_1} & a_1 < x < a_2 \\
0 & x > a_2 
\end{cases} \quad (8)$$

$$R_{i2} = \begin{cases} 
1 - \frac{a_2 - x}{a_2 - a_1} & a_1 < x < a_2 \\
\frac{a_3 - x}{a_3 - a_2} & a_2 < x < a_3 \\
0 & x < a_1 \text{ or } x > a_3 
\end{cases} \quad (9)$$

$$R_{i3} = \begin{cases} 
1 - \frac{a_3 - x}{a_3 - a_2} & a_2 < x < a_3 \\
\frac{a_4 - x}{a_4 - a_3} & a_3 < x < a_4 \\
0 & x < a_2 \text{ or } x > a_4 
\end{cases} \quad (10)$$

$$R_{i4} = \begin{cases} 
0 & x < a_3 \\
1 - \frac{a_4 - x}{a_4 - a_3} & a_3 < x < a_4 \\
1 & x > a_4 
\end{cases} \quad (11)$$

where $i$ is the $i$th evaluation index.

Evaluation matrix $C_i$ is composed of membership degree of first level evaluation index $R_{ij}$, and the membership degree can be determined by using single index measure functions as shown in Table 3. To be simplified, related literatures (Saaty 1979, 1990; Zhou 2010; Li et al. 2013; Shi et al. 2014) can also be consulted for the specific steps.

Based on the quantitative criteria of the risk grade, the qualitative evaluation indices can be quantified as suggested by Cao and Zhang (2006), and the relative membership degree can be obtained by using the triangular membership function $R_i$ as shown in the last row of Table 3.

Take drainage facility as an example. If the drainage facility for a certain slope is partially damaged, which is unable to drain adequately under the condition of heavy rain, there is saturated zone existing which is not enough to form hydrostatic pressure in the shallow layer yet. Therefore, the membership degree of each evaluation grade can be determined by using the membership function, $C_i = (0, 0.25, 0.75, 0)$.
Table 2. Indices and criteria of risk assessment of expansive soil slope stability.

| Secondary evaluation indices and the weights | First level evaluation indices | Weights of first level evaluation indices | Criteria for risk assessment of expansive soil slope stability |
|---------------------------------------------|---------------------------------|------------------------------------------|---------------------------------------------------------------|
|                                             |                                 |                                          | Level I (100~80)      | Level II (80~60)      | Level III (60~40)     | Level IV < 40         |
| Condition of slope body (0.239)             | Slope height (m)                | 0.540                                    | 0~4                 | 4~6                  | 6~15                 | >15                  |
|                                             | Slope ratio (°)                 | 0.297                                    | 0~15                | 15~25                | 25~35                | 35~45                |
|                                             | Natural water content (%)       | 0.163                                    | <25                 | 25~35                | 35~55                | >55                  |
| Swell–shrink grade of expansive soil (0.149)| Moisture content of standard moisture absorption (%) | 0.367                                    | <2.5                | 2.5~4.8              | 4.8~6.8              | ≥6.8                |
|                                             | Free expansive ratio (%)        | 0.301                                    | <40                 | 40~60                | 60~90                | >90                  |
|                                             | Plasticity index               | 0.332                                    | <15                 | 15~28                | 28~40                | >40                  |
| Hydraulic and meteorological conditions (0.342)| Rainfall Intensity (mm/d)  | 0.378                                    | <30                 | 30~60                | 60~120               | >120                 |
|                                             | Rainfall duration (h)          | 0.196                                    | <6                  | 6~24                 | 24~48                | >48                  |
|                                             | Depth of crack (m)             | 0.256                                    | <1                  | 1~2                  | 2~3                  | >3                   |
|                                             | Hydrogeology condition         | 0.068                                    | Without hydrops and groundwater activity | With little hydrops and groundwater activity | With hydrops and groundwater activity | Serious hydrops and frequent groundwater activity |
|                                            | Drainage measures              | 0.058                                    | Well-equipped facilties | Relatively intact | Partly damaged; inadequate drainage | Without drainage facilties |
| Support and improvement measures (0.181)   | The flexible supporting treatment | 0.582                  | Well-equipped facilties | Protective structure partly damaged | Without protective structure or partly broken | Badly broken |
|                                            | The rigid supporting treatment | 0.309                                   | Well-equipped facilties | Protective structure partly damaged | Without protective structure or partly broken | Badly broken |
|                                             | Soil improvement               | 0.110                                   | Mostly replaced      | Partly replaced and improved | Partly improved | Without improvement and replacement |
| Other factors (0.089)                       | Construction management and technology level | 0.454                  | Excellent reputation; rich construction experience and technical force | Favourable reputation; relatively rich construction experience and technical force | Common reputation; construction experience and technical force | Poor reputation; insufficient construction experience and technical force |
|                                             | Vegetation coverage (%)        | 0.321                                   | 75~100              | 75~50                | 50~25                | <25                  |
|                                             | Soil layer                     | 0.167                                   | Single layer; well-distributed | Multilayer; relatively well-distributed | Multilayer; without weak intercalated layer | Multilayer; with weak intercalated layer |
|                                             | Times of dry–wet cycling during construction | 0.099                  | 0                    | 1~3                  | 3~8                  | More than 8          |
### Table 3. Measure functions of single index.

| R_{11} | R_{12} | R_{13} | R_{14} |
|--------|--------|--------|--------|
| \begin{align*}
R_{11} &= \begin{cases}
1 - \frac{5}{1} x & x < 4 \\
1 - \frac{5}{1} & 4 < x < 5 \\
1 & x > 5
\end{cases} \\
R_{12} &= \begin{cases}
1 - \frac{5}{1} x & 4 < x < 5 \\
1 - \frac{5}{1} & x < 4 \text{ or } x > 11 \\
1 & x > 11
\end{cases} \\
R_{13} &= \begin{cases}
1 - \frac{5}{1} x & 4 < x < 5 \\
1 - \frac{5}{1} & x < 4 \text{ or } x > 11 \\
1 & x > 11
\end{cases} \\
R_{14} &= \begin{cases}
1 & x < 11 \\
1 - \frac{5}{1} x & 11 < x < 15 \\
1 & x > 15
\end{cases}
\end{align*} |
Combined with the corresponding weight, the first level comprehensive evaluation vector can be obtained after each membership degree of evaluation index is determined, and the secondary level evaluation vector can be calculated by the matrix composed of first level evaluation vectors. Risk grade of expansive soil slope stability is recognized according to maximum membership degree principle eventually.

3. Engineering application

To illustrate the feasibility and universality of the FAHP model proposed in this paper, two expansive soil slope cases, which have totally different hydrogeological condition and climates, are selected.

3.1. Engineering application I

Nanning–Youyiguan highway was one of the most important branch lines of the Hengyang–Nanning–Kunming highway (G275) with a total length of 220 km. This highway was located in the typical expansive soil area, and shallow sliding collapse happened to almost every cutting slope within half year. Take the K136+00~K136+370 section of Nanyou highway as an example; the slope body was composed of grey–white expansive soil with gravel and a small number of over-coarse-grained soil inside. The soil property parameters are shown in Table 4. The slope ratio was adopted as 1:1.5, and the height of slope was 10 m with a crack depth of 2 m. The basic structure characteristic of expansive soil slope of Nanyou highway is shown in Figure 3. Cutting slope was excavated down successfully on 28 November 2003, a large number of secondary cracks appeared on the slope and the whole slope was relatively stable with slight collapse. The small-type collapse appeared on the slope due to the influence of atmospheric wetting–drying cycles, and the whole slope was then destroyed. To improve the stability of the slope, geogrid flexible supporting treatment technology was used. The height of reinforced body was about 8 m, and upright interval of two adjacent reinforced layers was 0.5 m. The 3.5-m-wide adjacent geogrids were connected by connecting lever, with the U-shaped nails fixed.

Based on the above-mentioned information, the membership degree of each evaluation index can be calculated by using the grade criteria (see Table 2) and the single index measure functions (see Table 3), and the first level comprehensive evaluation vectors can be obtained with the corresponding weights. Take the calculation value of $B_i$ as an example, the value of $B_i$ can be expressed as

$$B_i = w_i C_i = w_i \sum_{j=1}^{3} R_{ij} = \begin{bmatrix} 0.540 \\ 0.297 \\ 0.163 \end{bmatrix} \cdot \begin{bmatrix} 0 & 0 & 0 \\ 1 \frac{1}{6} & 0 & 1 \\ 5 \frac{1}{6} & 5 \frac{1}{5} & 0 \\ 0 & 4 \frac{1}{5} & 0 \end{bmatrix} = (0, 0.253, 0.509, 0.238) \quad (12)$$

Table 4. Parameters of expansive soil for two cases.

| Case                | Dry density (g/m³) | Free expansive ratio (%) | Liquid limit (m³/g) | Plastic limit (%) | Plasticity index (%) | Grain composition (mm. %) | Content of montmorillonite (%) | Specific surface area (m²/g) | Relative contents of clay minerals (%) |
|---------------------|--------------------|--------------------------|--------------------|-------------------|----------------------|---------------------------|-------------------------------|-----------------------------------|------------------------------------------|
| Nanyou highway      | 1.74               | 62                       | 46.04              | 23.77             | 22.27                | 0.30                      | 56.14                         | 43.56                            | 42.29                                    | 58 11 20 11                              |
| Beijing west 6th ring highway | 1.91           | 42                       | 64.91              | 33.21             | 31.70                | 0.13                      | 46.31                         | 53.56                            | 48.00                                    | 45.44 331.55                     |
Combined with the weights of secondary level evaluation indices, the secondary level evaluation vector can be calculated by using matrix composed of first level evaluation vectors \((B_1, B_2, B_3, B_4, B_5)\) shown as follows:

\[
B_1 = (0, 0.253, 0.509, 0.238); \quad B_2 = (0, 0.211, 0.10, 0.689);
B_3 = (0.904, 0.042, 0.054, 0); \quad B_4 = (0, 0.1, 0.6, 0.3); \quad B_5 = (0.886, 0.114, 0, 0)
\]

\[
Q = w_0 \sum_{i=1}^{5} B_i = w_0 \cdot \begin{bmatrix} B_1 \\ B_2 \\ \vdots \\ B_5 \end{bmatrix} = 0.149 \\ 0.181 \\ 0.342 \\ 0.239 \\ 0.089 \end{bmatrix} \cdot \begin{bmatrix} 0 & 0 & 0.904 & 0 & 0.886 \\ 0.253 & 0.211 & 0.042 & 0.1 & 0.114 \\ 0.509 & 0.1 & 0.054 & 0.6 & 0 \\ 0.238 & 0.689 & 0 & 0.3 & 0 \end{bmatrix}
\]

\[
= (0.3880, 0.1243, 0.2558, 0.2319)
\]

(13)

Based on the maximum membership degree principle, the risk grade of expansive soil slope stability can be recognized. In this circumstance, the stability grade of the expansive soil slope is adopted as level I which agrees well with the engineering practice.

Based on the above-mentioned information, the stability grade of the expansive soil slope under the condition of rainfall and reinforcement treatment can be obtained.

After rainfall, the slope condition \(B_1\) and swell-shrink grade of expansive soil \(B_2, B_3,\) and \(B_5\) can be calculated in the following forms:

\[
B_1 = (0, 0.253, 0.509, 0.238); \quad B_2 = (0, 0.211, 0.10, 0.689);
B_3 = (0, 0.159, 0.267, 0.574); \quad B_4 = (0, 0.1, 0.6, 0.3); \quad B_5 = (0.486, 0.273, 0.241, 0)
\]

\[
Q = w_0 \sum_{i=1}^{5} B_i = w_0 \cdot \begin{bmatrix} B_1 \\ B_2 \\ \vdots \\ B_5 \end{bmatrix} = 0.0724 \\ 0.1785 \\ 0.3500 \\ 0.4281 \end{bmatrix}
\]

(14)

Based on the maximum membership degree principle, the stability grade of expansive soil slope is adopted as level IV. In practice, the whole slope was damaged with the gradual collapse of small-type slope, and the treatment measures were proposed.
With the treatment of geogrid flexible supporting, $B_3$, $B_4$, $B_5$ will change due to the improvement of seepage condition and drainage facilities. One obtains

$$Q = w_0 \sum_{i=1}^{5} B_i = w_0 \begin{bmatrix} B_1 \\ B_2 \\ \vdots \\ B_5 \end{bmatrix} = (0.4426, 0.1362, 0.1391, 0.2821)$$

(15)

After treatment of slope, the stability grade of expansive soil slope is adopted as level I suggesting that the support effect was satisfied and the results agree well with engineering practice. The comparison conditions of Nanyou highway before and after treatment are shown in Figure 4.

### 3.2. Engineering application II

Liangxiang–Zhaikou highway was the most difficult part of Beijing west 6th ring highway. Take the K10+260~K10+310 section of Nanyou highway as an example; the slope body was composed of deep blue and purple grey expansive soil, and the average height of these slopes was about 18 m with the width of deep crack less than 0.5 m. This project was constructed in June 2008. Based on the initial design scheme, the slopes were excavated with the ratio of 1:0.85. There was perched water infiltration on the cut slope surface during the construction process. Due to increased water loss, there were large number of arc tension cracks that occurred with average width of 20~50 cm at the trailing edge of slope, and the maximum visible depth was up to 3 m. The soil parameters are shown in Table 4.

Based on the above-mentioned information, the membership degree of each evaluation index can be calculated by using the grade criteria (see Table 2) and the single index measure functions (see Table 3), and the first level comprehensive evaluation vectors can be obtained with the corresponding weights. The computed values of $B_1, B_2, B_3, B_4$, and $B_5$ are as follows:

$$B_1 = (0, 0.147, 0.016, 0.837); \quad B_2 = (0.211, 0.123, 0.299, 0.367);$$
$$B_3 = (0.528, 0.305, 0, 0.167); \quad B_4 = (0, 0, 0.891, 0.110); \quad B_5 = (0.454, 0, 0, 0.546)$$

$$Q = w_0 \sum_{i=1}^{5} B_i = w_0 \begin{bmatrix} B_1 \\ B_2 \\ \vdots \\ B_5 \end{bmatrix} = (0.2522, 0.1569, 0.2106, 0.3803)$$

(16)

The stability grade of expansive soil slope is adopted as level IV provided with rather high risk which agrees well with engineering practice.
Based on the analysis of geological conditions, climatic conditions and soil characteristics, the initial design scheme was modified, and the geogrid flexible supporting treatment technology was applied on expansive soil cut slopes. The whole slope was divided into two layers with the slope ratio of 1:1.5. Considering hydrops on the slope during the rainy season, a 0.8-m-thick gravel drainage layer was adopted at the back of the backfill body. The treatment construction process has been completed in August 2009 successfully.

After a series of treatment measures, the values of $B_1$, $B_2$, $B_3$, $B_4$, and $B_5$ varied with the change of slope height and ratio. The computed values of $B_1$, $B_2$, $B_3$, $B_4$, and $B_5$ are as follows:

$$B_1 = (0, 0.201, 0.562, 0.237); \quad B_2 = (0.211, 0.123, 0.299, 0.367);$$
$$B_3 = (0.599, 0.401, 0, 0); \quad B_4 = (0.614, 0.077, 0.309, 0); \quad B_5 = (0.821, 0.146, 0.033, 0)$$

$$Q = w_0 \sum_{i=1}^{5} B_i = w_0 \begin{bmatrix} B_1 \\ B_2 \\ \vdots \\ B_5 \end{bmatrix} = (0.4205, 0.2306, 0.2377, 0.1112)$$ (17)

After treatment of slope, the stability grade of expansive soil slope is adopted as level I suggesting that the support effect was satisfied, and the results agree well with engineering practice. The comparison conditions of Beijing west 6th ring highway before and after treatment are shown in Figure 5.

4. Conclusions

This paper presents a recognition model by using the fuzzy recognition theory and the AHP to systematically evaluate the risk grade of stability of expansive soil slope during construction. Based on the characteristics of shallow slide of 22 expansive soil road sections constructed in China, the condition of slope body, swell–shrink grade of expansive soil, hydraulic and meteorological characteristics, supporting and improvement measures and other factors such as the construction technology and management level, vegetation coverage, soil layer, and drying–wetting cycles during construction are selected as the evaluation indices. The weights of evaluation indices are obtained by using the analytic hierarchy process method, and the measure functions are established to assess single index measurement and synthetic measurement. The maximum membership degree criterion is adopted to discern the risk grade of expansive soil slope stability. Moreover, the stability of expansive soil slope located in Nanyou highway and Beijing west 6th ring highway in China are analysed.
to verify the reliability of the present method. For practical purposes, the present risk assessment methodology can be used to systematically assess the slope stability in expansive soil zones. It is worth noting that the FAHP method has its limitations. Since the values of some indices are derived from expert evaluation method with a certain subjective, the evaluation indices system should also be modified based on regional variations due to differences in climate and soil characteristics. In addition, comprehensive assignment method of weight can be adopted based on the analysis of volumes of integrated data. The subjective and objective weights can be derived from AHP and Frequency Statistical Method, respectively.

Acknowledgement

All the authors would like to thank the reviewers for their valuable advice and suggestions, which help to improve the level of paper.

Disclosure statement

No potential conflict of interest was reported by the authors.

Funding

The National Natural Science Foundation of China [grant number 51608053]; the State Key Development Program for Basic Research Program of China [grant number 2013CB036000].

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