High-Coherence Heralded Single-Photon Source based on Two-Photon Interference

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We present the first Heralded Single-Photon Source, to our knowledge, based only on linear optics and weak coherent states. By time-tuning a Hong-Ou-Mandel interferometer fed with frequency-displaced coherent states, the output photons can be synchronously heralded following sub-Poisson statistics, which is indicated by the second-order correlation function \(g^2(0) = 0.556\). When compared to state-of-the-art asynchronous spontaneous parametric down-conversion-based sources, the technique presents comparable yield and at least one order of magnitude higher coherence time.

Single-photon sources are a fundamental resource in quantum optics, with a broad range of applications [1–3]. Even though deterministic single photon generation is highly desirable, it is not yet available. Therefore, the photon number statistics of any pseudo-single-photon source is always a concern, and the compromise between vacuum, single- and multi-photon emission probabilities directly impacts the source’s performance.

A practical and low-cost way to probabilistically producing single photon pulses is to use a faint laser source (FLS), whose output pulses approach coherent states [4]. Each pulse is described as a superposition of Fock states with the number of photons following Poisson distribution according to the parameter \(\mu\), the average number of photons per pulse. A small value of \(\mu\) binds the proportion of multi-photon to non-vacuum pulses to \(\mu/2\) but leads to an increase in the probability of emitting vacuum pulses to about \(1 - \mu\). The weak coherent states (WCSs) lead to a trade-off, as multi- and vacuum pulses impose limitations to the security and performance of quantum key distribution (QKD) links. Safeguarded by strict security proofs [5, 6] and techniques [7–10], however, WCSs are widely used in QKD.

Alternatively to FLS, nonlinear optical processes can be explored to assemble a single-photon source with sub-Poisson photon-number distribution, which is the case of the Spontaneous Parametric Down Conversion (SPDC) [11, 13] and Four-Wave Mixing (FWM) [14, 15]. In such processes, pump photons can be converted into pairs of highly-correlated phase-matched photons. The detection of one photon can, thus, be used to announce the presence of the other photon. This so-called heralded single-photon source (HSPS) can be obtained from continuous-wave (CW) [16, 18] or pulsed [19, 20] optical pump even though the inherent low-coherence nature of the nonlinear processes yield broadband emission. The pulsed pump method creates low-coherence photons (with coherence time as low as 10 ps [17]), which renders the synchronization of QKD systems critical – specially if it involves the joint measure of two single-photon pulses from distinct remote sources (e.g. Bell-state measurements [21, 23]).

The CW pump method can improve the coherence time up to 285 ps if a narrow filtering scheme is employed [17] but asynchronous operation is required.

In this letter, we propose and experimentally demonstrate the first HSPS, to our knowledge, to be completely independent of nonlinear optical effects and fully based on WCSs. The High-Coherence HSPS (HC-HSPS) relies on the interference of two frequency-displaced WCSs in an HOM interferometer. The interference pattern exhibits anti-bunching peaks with increased correlation between the interferometer’s output modes. The local detection of a photon in one of these modes announces a high-coherence single-photon pulse at the output of the HC-HSPS.

It has been formerly shown that the incidence of two indistinguishable single photons in a beam splitter (BS) leads to the photon-bunching effect when the temporal modes are matched [24]. A quantum beat pattern is expected when the photons have different frequencies [25] and the effect can be observed in temporal modes if the coherence time of the single photons is high enough [26, 27]. The optical beat note corresponds to the frequency displacement between the photons even if independent WCSs are used [25].

In order to create the optical beat note and explore its anti-bunching peaks, we generated two frequency-displaced WCSs through an all-fibered self-heterodyne setup [28] as depicted in Fig.1. The self-heterodyne approach avoids stability issues regarding both optical power and relative frequency drift and makes use of only one laser source, even though two fully-independent continuous-wave (CW) laser sources could also be used.

The attenuated CW signal of an external cavity laser diode is frequency modulated with modulation depth \(\Delta\) and period \(T\) and sent into a balanced Mach-Zehnder Interferometer-like setup (MZI). An optical switch at one of the MZI’s arms selects the photons with constant frequency offset \(\Delta/(2\pi) = 2\Delta T/T = 40\ \text{MHz}\) relative to the other arm. This is verified by measuring the bright-light version of the optical beat note with a p-i-n photodiode and an electrical spectrum analyzer at the output of the MZI (Fig.1).

The output of the MZI is converted into an HOM in-
The theoretical prediction for the normalized coincidence counts \[ C_e = \frac{1}{2} \left( 2 - e^{-\frac{\pi^2}{\Delta^2}} \cos(\tau \Delta) \right) \] is fit to data. The operational point is obtained when the spatio-temporal modes are matched with \( \tau = \pm \pi/\Delta \), the anti-bunching peak.

The states produced at the HC-HSPS output are squeezed WCSs for which the photon-number distribution is narrower than the coherent state but with wider bandwidth, obeying Heisenberg’s Uncertainty Principle [5]. The frequency-broadening with respect to the original linewidth is due to the frequency modulation of the optical signal. The expected linewidth of the squeezed WCS heralded by the HC-HSPS is of 430 MHz – see the Appendix – which agrees well with the 447.5 MHz linewidth measured with a high resolution Optical Spectrum Analyzer connected to the source’s output (Fig. 1c) [29]. The correspondent coherence time of 2.2 ns is almost three orders of magnitude higher than synchronous and 7.8 times greater than highly-filtered asynchronous SPDC-HSPS [17].

The theoretical model for the output states is drawn based on the decomposition of the WCSs fed into the HOM interferometer as pairs of Fock states [28] with up to three photons with an error bounded above by 1% for \( \mu < 0.67 \) – see the Appendix. Two distinct values of the relative temporal mode delay, \( \tau \), are of importance in analyzing the source: \( \tau \) greater than the mutual coherence time of the WCSs (\( \tau_{coh} \)) and the operational point at the highest anti-bunching peak.

In the first case, the oscillatory pattern of the interference vanishes since the states are fully distinguishable, so each photon sent into the HOM interferometer takes a random output. Any attempt to herald such states will fall into the Poisson-like photon-number statistics of a FLS – see the Appendix. This condition is further used as reference for evaluating the second order correlation function of the HC-HSPS. In the anti-bunching peak case, coincident counts can reach up to 150% of the distinguishable case. The normalized heralding probabilities of vacuum, multi-photon, and single-photon pulses for the HC-HSPS (\( \tau = \pm \pi/\Delta \)) is presented below, as a function of \( \mu \) and the heralder SPD efficiency (\( \eta_H \)):

\[
P_v = \frac{8 + \mu (4 - 2\eta_H) + \mu^2 (1 - \eta_H + \eta_H^2/3)}{8 + \mu (16 - 2\eta_H) + \mu^2 (16 - 6\eta_H + \eta_H^2/3)}
\]

\[
P_{m} = \frac{5\mu^2}{8 + \mu (16 - 2\eta_H) + \mu^2 (16 - 6\eta_H + \eta_H^2/3)}
\]

\[
P_s = \frac{12\mu + \mu^2 (10 - 5\eta_H)}{8 + \mu (16 - 2\eta_H) + \mu^2 (16 - 6\eta_H + \eta_H^2/3)}
\]

In Table I we present the normalized heralding probabilities of vacuum, multi-photon and single-photon pulses.
for the HC-HSPS at the operational point considering \( \mu << 1 \) and \( \eta_H = 1 \), compared to the probability of finding similar pulses in an FLS. Even though the multi-photon emission probability is slightly higher for the HC-HSPS, the emission of vacuum is relatively suppressed, and the single-photon pulses occur with higher probability. This characterizes the narrowing of the photon-number distribution of the squeezed WCS at the output of the source.

The second-order correlation function parameter \( g^2(\tau) \) at zero time determines whether the source produces photons following sub- \(<1\) or super- \(>1\) ), or Poisson-like \(=1\) statistics \([16, 18]\). The experimental values are obtained with a Hanbury-Brown and Twiss Analyzer (HBT) set at the output of the HC-HSPS. The HBT setup is composed of two SPDs (F and G) triggered by the heralding signal and a coincidence station (CS) (Fig. 3b).

From the modeled single and multi-photon probabilities, we find that \( g^2(0) \approx 0.56 \) for the anti-bunching and \( 1 \) for the distinguishable case. Figure 3b presents the predicted values for \( g^2(0) \) and the experimental values. The inset exhibits the predicted values for \( g^2(\tau) \) considering \( \mu = 0.1 \) and the detectors efficiency \( \eta_{H,F,G} = 0.15 \). The agreement between experimental and theoretical results validates the model.

The performance of a QKD system is directly dependent on the photon statistics of the optical source. The secret key generation probability and the maximum achievable link distance are analyzed below for our Linear-HSPS, for a FLS, and for an SPDC-HSPS, following the GLLP [6] and the decoy states analysis [30].

The parameters \( P_0, P_1 \) and \( P_2 \) are the probability of Alice sending vacuum, single- or multi-photon pulses for each kind of source considered.

The yield is the conditional probability of detection for Bob, given that Alice has sent an \( i \)-photon pulse. This is given by \( Y_i \approx P_d + \eta_i \), where \( P_d \) is the detector dark count probability and \( \eta_i = 1 - (1 - \eta)^i \). The overall transmittance \( \eta \) includes the detection efficiency \( \eta_{SPD} \), the transmittance of Bob’s devices \( \eta_{Bob} \) and is proportional to the link loss \( (\eta \approx 10^{-\alpha L/10}) \) – where \( \alpha \) is the attenuation coefficient (dB/km) of the fiber with length \( L \) (km) at the operational wavelength. The gain of the \( i \)-photon state is given by \( Q_i = P_i Y_i \), and the overall gain is obtained by summing over the contribution of all states \( Q_{\mu} = \sum_{i=0}^{\infty} Q_i \).

The error probability of the \( i \)-photon state is given by \( e_i = (e_0 Y_0 + e_{opt} \eta_i) / Y_i \), where \( e_0=1/2 \) is the probability of a dark count occurrence at the wrong detector and \( e_{opt} \) corresponds to the optical misalignment of Bob’s apparatus. The overall error is given by \( E_\mu = \frac{1}{Q_{\mu}} \sum_{i=0}^{\infty} e_i Q_i \).

The secret key generation probability is computed as

\[
R \geq q \left\{ Q_1 \left[ 1 - H_2(e_1) \right] - Q_{\mu} H_2(E_\mu) f(E_\mu) \right\}
\]

where \( H_2(x) \) is the Shannon binary entropy, \( f(x) \) is the inefficiency of the error correction (1.16 here), and \( q=1/2 \) appears due to the bases matching probability.

Considering the GLLP security analysis [6], the gain of single-photon pulses is lower bounded by

\[
Q_1 = Q_\mu - \sum_{i=2}^{\infty} P_i \approx Q_\mu - P_m(\mu)
\]
This assumption is the pessimistic one where all multi-photon pulses sent by Alice are eavesdropped. In this analysis, the single-photon error is given by

\[ e_1 = E_\mu Q_\mu / Q_1 \]  

When using the decoy states method, the values of \( Q_1 \) and \( e_1 \) can be estimated without the pessimistic assumption. This is possible due to the random choice of intensities sent by Alice. The \( i \)-photon yield does not depend on Alice’s choice, so Eve cannot predict nor fake the output of Alice’s source. Furthermore, due to these tighter bounds, Alice can usually use higher intensity values without jeopardizing the system security.

The simulation parameters were extracted from [31]: \( \alpha = 0.21 \text{ dB/km}; \eta_{SPD} \times \eta_{in,Bob} = 0.045; P_{\text{dark}} = 0.85 \times 10^{-6}; e_{\text{opt}} = 0.033. \) The value of \( \mu \) was optimized at each link distance for the FLS and HC-HSPS. The efficiency of the heralder detector of the HC-HSPS is 15%. The probability for the SPDC-HSPS were obtained from [19]: \( P_1 = 0.42 \) and \( g^2(0) = 0.018, \) with \( P_2 = g^2(0) P_1^2 / 2 \) [16].

An increase factor of 1.15 is observed in the maximum achievable distance using the HC-HSPS when compared to both the FLS and SPDC-HSPS under GLLP analysis. The secret key generation probability outperforms the FLS with GLLP analysis and also with the decoy states method. The HC-HSPS is highly competitive with state-of-the-art SPDC-HSPS [19] [20] concerning both parameters, with the advantage of achieving higher heralding rates. Considering the operation of the HC-HSPS with two independent frequency-mismatched CW laser sources emitting 0.1 photons per time interval, the heralding rate at a 15% SPD operating in gated-Geiger mode at 1 GHz would be close to 30 MHz, more than 6 times greater than the SPDC-HSPS reported in [19].

We have presented for the first time, to our knowledge, a heralded single photon source that relies on linear optics and weak coherent states. The interference between the frequency-displaced weak coherent states in an Hong-Ou-Mandel interferometer results in a structured interference pattern that exhibits anti-bunching peaks. The system is time-tuned for heralding the photon pulses that match the highest of those peaks, enhancing the correlation between the locally-detected spatio-temporal mode and the heralded mode. Sub-Poissonian photon statistics can be synchronously achieved with high coherence time of 2.2 ns, one order of magnitude greater than highly filtered asynchronous spontaneous parametric down-conversion heralded single-photon sources. The photon statistics of our high coherence linear heralded single-photon source was modeled and an Hanbury-Brown and Twiss analysis assessed its sub-Poisson character, with a second-order correlation parameter down to 0.556. The performance of the HC-HSPS was compared to a faint laser source and to an SPDC-HSPS under both the GLLP security analysis and using the decoy states method, in a simulation of a BB84-like QKD section. The results reveal that the HC-HSPS outperforms the FLS and is highly competitive with the SPDC-HSPS.

Presently, the HC-HSPS is limited to 2 kHz heralding events, due to the low rate of heralded single-photons in the self-heterodyne scheme, since the interference at the beam splitter is only enabled for a short duration in order to assure the right displacement of frequencies between photons. This can, however, be overcome by employing two lasers with fixed frequencies. We estimate that the heralding rate can be as high as 30 MHz, surpassing the state-of-the-art SPDC-HSPS. The all-fibered telecom-compatible design and the high coherence of the HC-HSPS source makes it a proper choice for long-distance quantum communication.

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APPENDIX

In this section we give details of the theoretical model for the photon number statistics of the proposed High-Coherence Heralded Single-Photon Source (HC-HSPS). In order to do so, we decompose the frequency-mismatched weak coherent states (WCSs) fed into the input symmetrical beam splitter (BS) of the Hong-Ou-Mandel (HOM) interferometer into pairs of Fock states, up to 3 photons. The error associated to this simplification is estimated from the Poisson cumulative distribution function \( \sum_{M=0}^{3} \sum_{N=0}^{3} \mu^M N^N e^{-2\mu}/(M!N!) \), with \( M + N \leq 3 \) to be bounded above by 1% for \( \mu < 0.67 \).

The model considers that the input states to the HOM interferometer are generated by two independent continuous-wave (CW) optical sources emitting spatial-mode matched parallel-polarized photons with identical average number of photons per time interval, \( \mu \). The case in which any number of photons enter the interferometer through one of the input modes, with vacuum at the other, has a straightforward solution: a random distribution of photons between the two output spatial modes.

We focus, therefore, on the case of two frequency-displaced single photons entering the interferometer, one in each of the input modes. This main result is then used to compose the other non-trivial cases of one- and two-photon states entering each input mode. The possible outcomes of the interferometer are then combined and weighted by the probability of occurrence of the correspondent input states. This information consists on the source’s output statistics once the probability of a heralding event is considered. In possession of the latter, we present the expected result for the Hanbury-Brown and Twiss (HBT) analysis and the calculation of the second-order correlation function of the source, \( g^2(0) \). Finally, we present some discussion regarding the coherence of the heralded photons.

Theoretical Model for the HC-HSPS

Consider \( BS_1 \), a symmetric beam splitter with input spatial modes \( A \) and \( B \) and output spatial modes \( C \) and \( D \) which is depicted in Fig. 5.

![FIG. 5. Spatial modes of BS_1 and BS_2, the beam splitters used for modelling the HC-HSPS.](image)

Its transfer function can be described using creation operators \( \hat{a}^\dagger \).

\[
\hat{a}^\dagger_A = \left( j \hat{a}^\dagger_C + \hat{a}^\dagger_D \right) / \sqrt{2} \\
\hat{a}^\dagger_B = \left( \hat{a}^\dagger_C + j \hat{a}^\dagger_D \right) / \sqrt{2}
\]

Combining the description of the wave-function of each input photon \( \xi_{A,B} = \xi_{A,B} (\tau) e^{-j \Phi_{A,B}} \) to the creation operators, we write electric field operators that can be associated to the input modes of the BS [27].

\[
E^\dagger_{A,B} = \xi_{A,B} \hat{a}_{A,B} \\
E_{A,B} = \xi^*_{A,B} \hat{a}^\dagger_{A,B}
\]
The BS’s output spatio-temporal modes $C$ and $D$ can, therefore, be written as a function of the field operators $\beta$

\[
E_C^{\uparrow}(t) = \left[-jE_A^{\uparrow}(t) + E_B^{\uparrow}(t)\right]/\sqrt{2}
\]

\[
E_D^{\uparrow}(t) = \left[E_A^{\uparrow}(t) - jE_B^{\uparrow}(t)\right]/\sqrt{2}
\]  

(11)

The case of interest is that of two single photons entering the BS, one at each input mode, which can be written in terms of creation operators applied to vacuum as $|\text{in}\rangle_{A,B} = |1,1\rangle_{A,B} = a_A^\dagger a_B^\dagger |0\rangle_{A,B}$. We adopt the notation $P_{R,S}^{M,N}$ for the conditional probability of finding $M$ and $N$ photons at the output modes $C$ and $D$, respectively, given that we had $Q$ and $S$ photons at the input modes $A$ and $B$, respectively.

The probability of finding one photon in $C$ at $t = \tau_0$ and the other photon in $D$ at $t = \tau_0 + \tau$ can be computed as

\[
P_{1,1}^{\uparrow,\uparrow}(\tau_0, \tau) = \langle 0| a_A a_B E_C^{-\uparrow}(\tau_0) E_D^{-\uparrow}(\tau_0 + \tau) E_C^{\uparrow}(\tau_0 + \tau) E_D^{\uparrow}(\tau_0 + \tau) a_A^\dagger a_B^\dagger |0\rangle
\]

(12)

Eq. (12) can be solved considering gaussian-shaped wave-packets with identical half-width at 1/e, and well-defined angular frequency modes $\omega_A$ and $\omega_B$ ($\omega = \omega_A/2 + \omega_B/2$). The frequency difference $\Delta = \omega_A - \omega_B$ is fixed and the wave-functions can be expressed as

\[
\xi_A = \frac{1}{\sqrt{\pi}\sigma^2} e^{-\left((-\delta t/2)^2/(2\sigma^2)\right)} e^{-j(\omega - \Delta/2)t}
\]

\[
\xi_B = \frac{1}{\sqrt{\pi}\sigma^2} e^{-\left((-\delta t/2)^2/(2\sigma^2)\right)} e^{-j(\omega + \Delta/2)t}
\]  

(13)

Integration from $-\infty$ to $\infty$ over $\tau_0$ (as we are only interested in the effective time delay $\tau$ between the wave packets) and over $\delta t$ (to take into account the CW nature of the optical sources) yields

\[
P_{1,1}^{\uparrow,\uparrow} = \frac{1}{2} (1 - \beta)
\]  

(14)

with $\beta = e^{-\frac{\tau^2}{2\sigma^2}} \cos(\tau\Delta)$. Due to normalization and symmetry arguments, the probability of finding two photons in one output spatial mode and vacuum in the other is found to be

\[
P_{0,0}^{\uparrow,\downarrow} = P_{0,0}^{\downarrow,\uparrow} = \frac{1}{2} (1 + \beta)
\]  

(15)

The solution considering a single photon entering one of the input ports of the beam splitter with vacuum entering the other, $|1,0\rangle_{in}$ and $|0,1\rangle_{in}$, is straightforward:

\[
P_{1,0}^{\uparrow,\downarrow} = P_{0,1}^{\uparrow,\downarrow} = P_{0,1}^{\downarrow,\uparrow} = P_{0,1}^{\uparrow,\downarrow} = 1/2
\]  

(16)

The same analysis applies to the case in which either two (Eq [17]) or three photons (Eq [18]) enter the same input port and vacuum enters the other

\[
P_{2,0}^{\uparrow,\downarrow} = P_{0,2}^{\uparrow,\downarrow} = P_{0,2}^{\downarrow,\uparrow} = P_{0,2}^{\uparrow,\downarrow} = 1/4
\]

\[
P_{2,1}^{\uparrow,\downarrow} = P_{1,2}^{\uparrow,\downarrow} = 1/2
\]  

(17)

\[
P_{3,0}^{\uparrow,\downarrow} = P_{0,3}^{\uparrow,\downarrow} = P_{0,3}^{\uparrow,\downarrow} = P_{0,3}^{\uparrow,\downarrow} = 1/8
\]

\[
P_{2,1}^{\uparrow,\downarrow} = P_{1,2}^{\uparrow,\downarrow} = P_{1,2}^{\uparrow,\downarrow} = P_{1,2}^{\uparrow,\downarrow} = 3/8
\]  

(18)

The cases $|2,1\rangle_{in}$ and $|1,2\rangle_{in}$ are modeled as a superposition of a pair of single photons, one in each input port, and an independent photon in either one of the input ports.

\[
P_{3,1}^{\uparrow,\downarrow} = P_{1,3}^{\uparrow,\downarrow} = P_{1,3}^{\uparrow,\downarrow} = P_{1,3}^{\uparrow,\downarrow} = 1/8
\]

\[
P_{2,2}^{\uparrow,\downarrow} = P_{2,2}^{\uparrow,\downarrow} = P_{2,2}^{\uparrow,\downarrow} = P_{2,2}^{\uparrow,\downarrow} = 3/8
\]  

(19)

The final probabilities for each output state in modes $C$ and $D$ are obtained by grouping the conditional probabilities of each output pair weighted by the probabilities of occurrence of each pair of input Fock states. The latter are given, for WCSs, by the Poisson distribution $P_{R,S}^{M,N}|\mu\rangle_{A,B} = \mu^{M+N} e^{-2\mu}$. The general form of the probability of occurrence of the output state $|R,S\rangle_{C,D}$ when the BS is fed by frequency-displaced WCSs is, thus, given by Eq [20],

\[
P_{R,S} = \sum_{m=0}^{3} \sum_{n=0}^{3} P_{R,S}^{M,N} P_{M,N} \begin{cases} \begin{align*} R + S &= 3 \\ m + n &\leq 3 \end{align*} \end{cases}
\]  

(20)

**Statistics of the HC-HSPS**

The output states of the HC-HSPS are characterized by a heralding event to which, disregarding dark counts, are associated to the presence of a photon – or more than one – at spatial mode $C$, the heralder arm. As we are only interested in those such cases for computing the statistics of the source’s output, the total probability,
used for normalization, is

$$P_T = \sum_{R=1}^{3} \sum_{S=0}^{2} \eta_P P_{R,S} \begin{cases} R, S \in \mathbb{N} \\ R + S \leq 3 \end{cases}$$ \hspace{1cm} (21)$$

Therefore, the probabilities of the three distinct events of heralding vacuum $P_v$, single- $P_s$ or multi-photon $P_m$ pulses at the spatial mode $D$ (disregarding the dark count probability) are given by Eqs. \[22\] \[24\].

$$P_v = (\eta_1 P_{1,0} + \eta_2 P_{2,0} + \eta_3 P_{3,0}) / P_T$$ \hspace{1cm} (22)

$$P_m = \eta_1 P_{1,2} / P_T$$ \hspace{1cm} (23)

$$P_s = (\eta_1 P_{1,1} + \eta_2 P_{2,1}) / P_T$$ \hspace{1cm} (24)

Here, we considered the $P$-photon detection efficiency at the heralder mode, $\eta_P = 1 - (1 - \eta_H)^P$, which depends on the detector efficiency $\eta_H$. With $\alpha = \exp(-2\mu)$, the probabilities can be expressed as

$$P_v = \alpha [\eta_H \mu + (2\eta_H - \eta_H^2) \mu^2 (1/2 + \beta/4) + (3\eta_H - 3\eta_H^2 + \eta_H^3) \mu^3 (1/6 + \beta/8)] / P_{total}$$

$$P_m = \alpha \eta_H \mu^3 (1/2 - \beta/8) / P_{total}$$

$$P_s = \alpha [\eta_H \mu^2 (1 - \beta/2) + (2\eta_H - \eta_H^2) \mu^3 (1/2 - \beta/8)] / P_{total}$$

which equates to

$$P_v = \frac{24 + \mu [24 - 12\eta_H + (12 - 6\eta_H) \beta] + \mu^2 [12 - 12\eta_H + 4\eta_H^2 + (9 - 9\eta_H + 3\eta_H^2) \beta]}{24 + \mu (48 - 12\eta_H - 6\eta_H \beta) + \mu^2 [48 - 24\eta_H + 4\eta_H^2 + (3\eta_H^2 - 6\eta_H) \beta]}$$

$$P_m = \frac{24 + \mu (48 - 12\eta_H - 6\eta_H \beta) + \mu^2 [48 - 24\eta_H + 4\eta_H^2 + (3\eta_H^2 - 6\eta_H) \beta]}{24 + \mu (24 - 12\beta) + \mu^2 [24 - 12\eta_H + (3\eta_H - 6) \beta]}$$

$$P_s = \frac{24 + \mu (48 - 12\eta_H - 6\eta_H \beta) + \mu^2 [48 - 24\eta_H + 4\eta_H^2 + (3\eta_H^2 - 6\eta_H) \beta]}{24 + \mu (24 - 12\beta) + \mu^2 [24 - 12\eta_H + (3\eta_H - 6) \beta]}$$

Second-order correlation function analysis

The zero-time second-order correlation function, $g^2(0)$, can be measured using an HBT setup – see the main text for setup details. This consists of adding a symmetrical BS to the heralded mode $D$ with one SPDs connected to each of its output modes (modes $F$ and $G$) and which are triggered by the heralding signal.

The parameter $g^2(0)$ is experimentally defined as \[10\]

$$g^2(0) = Q_{FG} / (Q_F Q_G)$$ \hspace{1cm} (27)

where, $Q_F$, $Q_G$, and $Q_{FG}$ are, respectively, the probabilities of an event being registered at SPD $F$, SPD $G$, and in coincidence between them. These values can be computed from probability of an $i$-photon state being heralded (the source statistics) and detected in the desired detector (the single-photon detection probability given by $\eta_F$ and $\eta_G$). For negligible dark count probability, a detection event can be triggered by a single- or multi-photon pulse, so we can write

$$Q_{F,G} = \frac{P_s \eta_{F,G}}{2} + \left( \eta_{F,G} - \frac{\eta^2_{F,G}}{4} \right) P_m$$ \hspace{1cm} (28)

Given our suppositions, coincident detections are triggered by half the multi-photon probability events, resulting in

$$Q_{FG} = \frac{\eta_F \eta_G}{P_m / 2}$$ \hspace{1cm} (29)

The $g^2(0)$ parameter is then re-written as a function of the photon statistics of the HC-HSPS and the detection efficiency values as

$$g^2(0) = \frac{P_m}{P_m^2 / 2 + (2 - \eta_F / 4 - \eta_G / 4 + \eta_F \eta_G / 8) P_m^2 + (2 - \eta_F / 4 - \eta_G / 4) P_m P_s}$$ \hspace{1cm} (30)

HC-HSPS Output Linewidth

The frequency-modulation input port of the TLS used in the self-heterodyne setup of the HC-HSPS is excited by a triangular wave with frequency 3.1 kHz corresponding
to a period of 322.6 microseconds. The optical switch positioned at one of the MZI’s arm selects photons with a fixed frequency displacement in order to guarantee . The optimized duration of the optical switch pulse was experimentally determined to be of 30 microseconds – 9.3 % of the triangular wave period. This value yields a classical optical beat note of 40 MHz measured with a 3 MHz linewidth in an Electrical Spectrum Analyzer.

The total frequency sweep that characterizes the HC-HSPS output linewidth is, therefore, given by $\Delta \nu = 40 \times (0.093)^{-1} = 430$ MHz. This result is in agreement with the experimental 447.5 MHz linewidth measured with a high resolution Optical Spectrum Analyzer connected to the source’s output.