The quark-hadron thermodynamics in magnetic field

V.D. Orlovsky and Yu.A. Simonov
Institute of Theoretical and Experimental Physics
117218, Moscow, B.Cheremushkinskaya 25, Russia
November 19, 2013

Abstract

Nonperturbative treatment of quark-hadron transition at nonzero temperature $T$ and chemical potential $\mu$ in the framework of Field Correlator Method is generalized to the case of nonzero magnetic field $B$. A compact form of the quark pressure for arbitrary $B, \mu, T$ is derived. As a result the transition temperature is found as a function of $B$ and $\mu$, which depends on only parameters: vacuum gluonic condensate $G_2$ and the field correlator $D^E(x)$, which defines the Polyakov loops and it is known both analytically and on the lattice. A moderate ($25\%$) decrease of $T_c(\mu = 0)$ for $eB$ changing from zero to 1 GeV$^2$ is found. A sequence of transition curves in the $(\mu, T)$ plane is obtained for $B$ in the same interval, monotonically decreasing in scale for growing $B$.

1 Introduction

Strong magnetic fields (m.f.) are now a subject of numerous studies [1, 2, 3, 4, 5, 6, 7, 8], since they can be present in different physical systems. Namely, in cosmology m.f. of the order of $10^{18}$ Gauss or higher can occur during strong and electroweak phase transition [1, 2]. In noncentral heavy ion collisions one can expect m.f. $O(10^{18} - 10^{21})$ Gauss [3, 4, 5, 6], while in some classes of neutron stars m.f. can reach the magnitude of $10^{14}$ Gauss, or even more in the central regions [7]. All this makes it necessary to study the effects of
strong m.f. in all possible physical situations and using different methods, for a recent review see [8].

One of most interesting aspects of strong m.f. is its influence on the QCD hadron-quark phase transition, which can occur both in astrophysics (neutron stars) and heavy ion experiments. On the theoretical side many model QCD calculations have predicted the increase of the critical temperature $T_c$ with growing $B$ [9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35], and only few obtained an opposite result [36, 37], see [38, 39] for reviews and additional references. Recently the lattice data of [40] with physical pion mass and extrapolated to continuum have demonstrated the decreasing critical temperature as a function of $B$. This phenomenon was called the inverse magnetic catalysis and the further study of the m.f. dependence of the quark condensate and of its magnetic susceptibility was done in [41] and [42] respectively. It is our purpose in this paper to exploit the formalism of Field Correlators (FC) developed earlier for the QCD phase transition at zero m.f. [43, 44, 45, 46, 47, 48, 49] to study the same problem in the case of arbitrary m.f.

The advantage of the FC method is that it is based only on the fundamental QCD input: gluonic condensate $\langle G^2 \rangle$, string tension $\sigma$, $\alpha_s(q)$ and current quark masses. In contrast to [36], where the same basic principle [41] was used, but pions were elementary in CPT, it treats all hadrons, including pions, as $q\bar{q}$ or $3q$ systems, which allows to consider high m.f. with $eB \gg m_\pi^2, \sigma$. As a result the critical temperature $T_c(B)$ decreases with the growing $B$ as in lattice data of [40].

Recently the FC method was successfully applied to the study of phase transition in neutron stars [50, 51] without m.f. and in the case of strange quarks and strange matter in [52]. It is interesting to investigate the role of m.f. in these transitions and our results below may be a reasonable starting point for this analysis.

The paper is organized as follows. In section 2 the general FC formalism as applied to the hadron-quark transition is given, and in section 3 the contribution of magnetic field is explicitly taken into account. In section 4 the quark and hadron thermodynamic potentials are estimated at large m.f. and the corresponding transition temperature $T_c(B)$ is found. In section 5 the case of nonzero chemical potential is treated and in section 6 a discussion of results and prospectives is presented.
2 General formalism

We shall follow the ideas of [43, 44, 45, 46, 47, 48, 49] (see [53] for a review) and consider the low-temperature hadron phase as the hadron gas in the confining background vacuum field, and the total free energy can be represented as

\[ F = \varepsilon_{\text{vac}} V_3 + F_h, \]  

where

\[ \varepsilon_{\text{vac}} = \frac{\beta(\alpha_s)}{16\alpha_s} \langle G^{a}_{\mu\nu} G^{a}_{\mu\nu} \rangle + \sum_q m_q \langle \bar{q}q \rangle, \]  

and \( F_h \) is the hadron free energy, which in absence of magnetic field and treating hadrons as elementary can be written as [54, 55] (\( \beta = 1/T \))

\[- F_{h/V_3} = \sum_i P_h^{(i)} = \sum_i g_i T^4 \int_0^\infty dpp^2 \eta \ln \left( 1 + \eta e^{-\beta E_i} \right), \]  

where \( \eta = -1 \) or +1 for bosons or fermions respectively and in the relativistic case \( E_i = \sqrt{\mathbf{p}^2 + m_i^2} \), while \( g_i \) is the spin-isospin multiplicity of hadron \( i \).

Taking integral in (3), one can write \( P_h^{(i)} \) as

\[ P_h^{(i)} = \frac{g_i T^4}{2\pi^2} \sum_{n=1}^\infty (-\eta)^{n+1} \frac{(\beta m_i)^2}{n^2} K_2(n\beta m_i), \]  

where \( K_2 \) is the McDonald function. As an example of another starting point we present below the derivation of the quark pressure from the statistical sum in the form of the generating function with the proper time integration [45, 47, 48, 49].

\[ \frac{1}{T} F_q = \frac{1}{2} \ln \det (m^2_q - \hat{D}^2) = -\frac{1}{2} tr \int_0^\infty \xi(s) \frac{ds}{s} e^{-sm^2_q + s\hat{D}^2} \]  

The latter expression can be written as a path integral with the background field containing both electromagnetic \( A^{(e)}(x) \) and color potential \( A_{\mu}(x) \), [53, 56, 57, 58]

\[ \frac{1}{T} F_q(A, A^{(e)}) = -\frac{1}{2} tr \int_0^\infty \xi(s) \frac{ds}{s} d^4x (\hat{D}^2)^w_{xx} e^{-K - s m_i^2} \langle W_o(C_n) \rangle, \]  

where \( K = \frac{1}{4} \int_0^\infty \left( \frac{d\eta}{d\tau} \right)^2 d\tau \),

3
\[ W_\sigma(C_n) = P_F P_A \exp(i g \int_{C_n} A_\mu dz_\mu + i e \int_{C_n} A_\mu^{(e)} dz_\mu) \exp \int_0^s (g \sigma_{\mu\nu} F_{\mu\nu} + e \sigma_{\mu\nu} F_{\mu\nu}^{(e)}) d\tau, \]

and

\[ \overline{(Dz)}_{xy}^\mu = \prod_{m=1}^{n} \frac{d^4 \Delta z_k(m)}{(4\pi \varepsilon)^2} \sum_{n=0,\pm 1,\pm 2} (-)^n \frac{d^4 p}{(2\pi)^4} e^{ip \cdot \sum \Delta z_k(m) - (x-y) - n \beta \delta_{\mu 4}). \]

It was shown in [48, 49], that \( P_q^{(i)} \equiv P_q \) can be written as

\[ P_q = 2N_c \int_0^\infty ds \frac{e^{-m_q^2 s}}{s} \sum_{n=1}^{\infty} (-)^{n+1} [S^{(n)}(s) + S^{(-n)}(s)] \]

and

\[ S^{(n)}(s) = \int (\overline{(Dz)}_{wm} e^{-K} \frac{1}{N_c} tr W_\sigma(C_n) \]

and in the case, when only one-particle contribution is retained,

\[ S^{(n)}(s) = \frac{1}{16\pi^2 s^2} e^{-\frac{n^2 \beta^2}{4s} - J_n^E}, \]

and \( J_n^E \) defines the Polyakov loop configuration expressed via the field correlator \( D_1(x) \) [48, 49, 59]

\[ J_n^E = \frac{n\beta}{2} \int_0^{n\beta} d\nu \left( 1 - \frac{\nu}{n\beta} \right) \int_0^\infty d\xi \xi D_1^E(\sqrt{\nu^2 + \xi^2}), \]

where \( D_1^E(x) \) is the colorelectric correlator, which stays nonzero above the deconfinement temperature.

The insertion of (11) into (9) yields

\[ \frac{1}{T^4} P_q = \frac{N_c n_f}{4\pi^2} \sum_{n=1}^{\infty} \frac{(-)^{n+1}}{n^4} \int_0^\infty ds \frac{e^{-m_q^2 s - \frac{n^2 \beta^2}{4s}} - J_n^E} = \]

\[ = \frac{4N_c n_f}{\pi^2} \sum_{n=1}^{\infty} \frac{(-)^{n+1}}{n^4} \varphi_q^{(n)} L^n, \quad \varphi_q^{(n)} = \frac{n^2 m_q^2}{2T^2} K_2 \left( \frac{m_q n}{T} \right), \]

where \( L^n = e^{-J_n^E} \).
At this point it is convenient to give one more representation of $P_q$, namely, using \[58\] one can extract in $z_4(\tau)$ the fluctuating part $\tilde{z}_4(\tau)$

$$z_4(\tau) = \tilde{z}_4(\tau) + z_4(\tau), \quad \tilde{z}_4(\tau) = 2\omega \tau = t_E, \quad s = \frac{T_4}{2\omega}, \quad T_4 = n\beta$$ (14)

$$P_q = 2N_c n_f \int_0^{\pi} \frac{d\omega}{\omega} \sqrt{\frac{\omega}{2\pi}} \sum_{n=1}^{\infty} \frac{(-)^{n+1}}{\sqrt{n\beta}} \int D^3 z e^{-K(\omega) - J^E_n}$$ (15)

$$K(\omega) = \int_0^{n\beta} dt_E \left( \frac{\omega}{2} + \frac{m^2}{2\omega} + \frac{\omega}{2} \left( \frac{dz}{dt_E} \right)^2 \right), \quad m_q \equiv m.$$ (16)

In this way we obtain the form, equivalent to (13)

$$P_q = \frac{N_c n_f}{\pi^2 \beta^2} \sum_{n=1}^{\infty} \frac{(-)^{n+1}}{n^2} \int_0^{\infty} \omega d\omega e^{-\frac{m^2}{2\omega} + \frac{\omega}{2}} n\beta - J^E_n.$$ (17)

Neglecting $J^E_n$ and for $m_q = 0$ one obtains the standard result

$$P_q = \frac{4N_c n_f T_4}{\pi^2} \sum_{n=1}^{\infty} \frac{(-)^{n+1}}{n^4} = \frac{7N_c n_f \pi^2}{180} T^4,$$ (18)

where we have used

$$\int_0^{\infty} \omega d\omega e^{-\frac{m^2}{2\omega} + \frac{\omega}{2}} n\beta = 2m^2 K_2(mn\beta).$$ (19)

For the following it will be useful to keep in (13) the $d^3 p$ integration, contained in $D^3 z$, which yields

$$P_q = \frac{n_f N_c}{\sqrt{\pi}} \int \frac{d^3 p}{(2\pi)^3} \sum_{n=1}^{\infty} \frac{(-)^{n+1}}{\sqrt{n\beta}} \int_0^{\infty} \frac{d\omega}{\sqrt{\omega}} e^{-\frac{m^2 + P^2}{2\omega} + \frac{\omega}{2}} n\beta.$$ (20)

One can see, that (20) coincides with (4), when $g_4 = 4N_c n_f$. Finally, one obtains from (4) or (18) the pressure for gluons

$$P_g = (N_c^2 - 1) \frac{2T_4}{\pi^2} \sum_{n=1}^{\infty} \frac{\langle \Omega^n \rangle + \langle \Omega^* n \rangle}{2n^4},$$ (21)

where $\Omega^n = \exp(i g \int_0^{n\beta} A_4 dz_4)$, and $\Omega$ is the adjoint Polyakov loop.
3 Quark and hadron thermodynamics in magnetic field

We discuss here the one-particle thermodynamics in constant homogeneous m.f. $B$ along $z$ axis, in which case one should replace $E_i$ in (3) by the well-known expression [55], which in the relativistic case has the form

$$E_{n_{\perp}}^\sigma (B) = \sqrt{p_z^2 + (2n_{\perp} + 1 - \bar{\sigma})|e_q B + m_q^2|}, \quad \bar{\sigma} \equiv \frac{e_q}{|e_q|}\sigma_z, \sigma_z = \pm 1. \quad (22)$$

We also take into account, that the phase space of an isolated quark in m.f. is changed as follows [54]

$$V_3d^3p \rightarrow \frac{dp_z |e_q B|}{2\pi} V_3, \quad \left(\frac{2\pi}{3}\right)^3 \rightarrow \frac{2\pi}{2\pi} \left|\frac{e_q B}{2\pi}\right|^2 V_3, \quad (23)$$

and hence (3) can be rewritten as $\bar{P}_q(B) = \sum_q P_q(B)$,

$$P_q(B) = \sum_{n_{\perp}, \sigma} 2N_c T |e_q B| \frac{1}{2\pi} (\chi(\mu) + \chi(-\mu)), \quad (24)$$

where

$$\chi(\mu) \equiv \int \frac{dp_z}{2\pi} \ln \left(1 + \exp\left(\frac{\bar{\mu} - E_{n_{\perp}}^\sigma (B)}{T}\right)\right). \quad (25)$$

We have introduced in (24) the chemical potential $\mu$ with the averaged Polyakov loop factor $\bar{L} = \exp(-\bar{J}/T)$, (see [53] for a corresponding treatment without m.f.)

$$\bar{\mu} = \mu - \bar{J}, \quad \bar{L}_\mu = \exp\left(\frac{\mu - \bar{J}}{T}\right). \quad (26)$$

Eq.(24) can be integrated over $dp_z$ with the result

$$P_q(B) = \frac{N_c |e_q B| T}{\pi^2} \sum_{n_{\perp}, \sigma} \sum_{n=1}^{\infty} \frac{(-)^{n+1}}{n} \left(\frac{1}{2}(\bar{L}_\mu^n + \bar{L}_{-\mu}^n)\varepsilon_{n_{\perp}}^\sigma K_1\left(\frac{n\varepsilon_{n_{\perp}}^\sigma}{T}\right)\right), \quad (27)$$

where

$$\varepsilon_{n_{\perp}}^\sigma = \sqrt{|e_q B|(2n_{\perp} + 1 - \bar{\sigma}) + m_q^2}. \quad (28)$$

The same result for $\mu = 0$ can be obtained, extending (20) to the case of nonzero $B$, using (23) and replacing the exponent in (20) as

$$\left(\frac{m_q^2 + p_z^2}{2\omega} + \frac{\omega}{2}\right)n\beta \rightarrow \left(\frac{m_q^2 + p_z^2 + (2n_{\perp} + 1 - \bar{\sigma})|e_q B|}{2\omega} + \frac{\omega}{2}\right)n\beta, \quad (29)$$
\[ P_q = 2N_c \left( \frac{|e_q B|}{(2\pi)^2} \right) \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n\beta} \int_{0}^{\infty} d\omega e^{-\left(\frac{\omega^2}{\omega_B^2} + \frac{\omega}{2}\right) n\beta}, \]

(30)

and using the equation

\[ \int_{0}^{\infty} d\omega e^{-\left(\frac{\omega^2}{\omega_B^2} + \frac{\omega}{2}\right) \tau} = 2\lambda K_1(\lambda \tau), \]

(31)

we come to the Eq. (27).

We turn now to thermodynamics of hadrons in m.f. The difficulty here is that hadrons are not elementary objects, unlike quarks, and we cannot use for them the energy expressions like (22). Hadrons in m.f. were studied analytically in [60, 61, 62, 63] and on the lattice in [64, 65, 41]. We can use for them an expression of the type of Eq. (24) or Eq. (27), however we should write it in a more general way for the charged hadrons

\[ P_{H_i}^{(i)}(B) = g_i \left( \frac{|e_B|}{2\pi} \right) \int dP_z \frac{1}{2\pi^2} \left( n_i - \chi(-\mu_i) \right), \]

(32)

where

\[ \chi(\mu_i) \equiv \sum_{n_{\perp},s_i} \ln \left( 1 + \exp \left( \frac{\mu_i - E_{N_{\perp}}^{s_i}(B)}{T} \right) \right), \]

(33)

and we take into account, that the total hadron energy \( E_{N_{\perp}}^{s_i}(B) \) depends on the set of 2d oscillator numbers \( N_{\perp} = \{ n_{\perp}(1), n_{\perp}(2), \ldots n_{\perp}(\nu) \} \) for each of \( \nu \) constituents and on the set \( s_i = \{ \sigma_1, \ldots \sigma_\nu \} \) of spin projections of all constituents. For very large m.f. \( eB \gg \sigma(\sigma \text{ is the string tension}) \) one can approximate \( E_{N_{\perp}}^{s_i}(B) \) as an average of the sum of constituents (2 for mesons and 3 for baryons),

\[ E_{N_{\perp}}^{s_i}(B) = \left\langle \sum_{k=1}^{\nu} \sqrt{m_{q_k}^2(k) + |e_k|B(2n_{\perp}(k) + 1 - \frac{e_k \sigma_k}{|e_k|}) + p_z^2(k)} \right\rangle_{P_z} \]

(34)

where the average is taken with the functions \( \chi(p_z(1), \ldots p_z(\nu)) \), satisfying \( \sum_{k=1}^{\nu} p_z(k) = P_z \), and taking into account confining dynamics along z axis (see explicit expressions in the Appendix). In the approximation used in [60, 61, 62, 63], when confinement is quadratic, the functions \( \chi \) are the oscillator eigenfunctions. For neutral hadrons one should use instead of (32) the form (1), where m.f. acts on the multiplicity \( g_i \) and the mass \( m_i \), which can strongly depend on m.f., as it is in the case of \( \rho_0 \) and \( \pi^0 \) mesons, see [61, 62].
At this point it is useful to compare the systematics of hadrons without m.f. with that of strong m.f. In the first case one classifies a hadron, using e.g. spin \( S \), partly \( P \) isospin \( I \), orbital momentum \( L \) and radial quantum number \( n_r \), or else total angular momentum \( J \). For strong m.f. both spin \( S \) (or \( J \)) and isospin are not conserved and one has e.g. instead of 2 states \( \rho^0(S_z = 0), \pi^0 \) linear combinations \( \langle u +, \bar{u} - | \equiv \langle + + | u, \langle - + | u, \langle + - | d, \langle - + | d \) and similarly \( \rho^0(S_z = +1) \) splits into 2 states: \( \langle + + | u, \langle + + | d \).

A similar situation occurs in baryons: neutron, \( S_z = -\frac{1}{2}, (ddu) \) splits into \( (−−+), (−+−), (−+−) \).

A specific role is here played by the so-called “zero states”: those are states for which all constituents have factors in (34) equal to zero:

\[
2n_\perp(k) + 1 - \frac{e_k}{|e_k|} \bar{\sigma}_k = 0, \quad k = 1, 2, \ldots \nu. \tag{35}
\]

Masses of zero states decrease fast with m.f. and for \( eB \approx \sigma \) can be \((30 \div 40)\%\) lower than in absence of m.f. \[61, 62\]. Therefore the role of these states in the forming of \( P_h(B) \) exponentially grows, while energies \( E_{N_i}^{\pm}(B) \) of all other states according to (34) grow proportionally to \( \sqrt{eB} \). Thus in \( \rho^0(S_z = 0), \pi^0 \) only the states \( \langle + - | u, \langle - + | d \) are zero states, while for the neutron with \( S_z = -\frac{1}{2} \) the only zero state is \( (−−+) \). In this way the most part of all excited hadron states have energies growing with m.f. and their contribution is strongly suppressed for \( eB > \sigma \). However, the same situation occurs for the system of free quarks at large m.f., which can be clearly seen comparing (34) with energies of free quarks, therefore the main difference occurs for not large m.f., when \( eB < \sigma \) and hadron energies change less rapidly than those of free quarks, and hence \( P_q \) may grow faster with \( eB \) than \( P_h \), which finally results in the decreasing \( T_c(B) \), as we show below.

Indeed, for each quark the zero states constitute one half of \( n_\perp = 0 \) states, namely the states with \( \bar{\sigma}_k = 1 \), and the corresponding pressure is proportional to \( \frac{|e_q| B \mu_q T}{n} K_1 \left(\frac{\mu_q n}{T}\right) \), tending to \( \frac{|e_q| B T^2}{n} \) at large m.f., thus growing linearly with \( B \).

For hadrons in the same limit the charged zero states contribute in (32) the amount \( \frac{|e_h| B T^2}{\nu} \), while neutral zero states, like \( \pi^0 \), contribute \( \frac{T^4}{\nu} \). Therefore for \( B \geq T^2 \) the growth of quark pressure with \( B \) is faster than that of hadrons, and one can assert, that for \( B \geq T^2, \sigma \) the inequality holds

\[
\Delta P_q(B, T) \equiv P_q(B, T) - P_q(0, T) > \Delta P_h(B, T) = P_h(B, T) - P_h(0, T). \tag{36}
\]
In the next section we shall show, that (36) leads to the decreasing of the deconfinement temperature \( T_c(B) \) with growing \( B \) independently of the character of this transition. In particular, the above arguments were based on the single-line approximation for quarks \([48, 49]\), when quarks are treated as independent and vacuum fields create only Polyakov line contributions (\( \bar{L}^n \) in (27)). A more accurate treatment, taking into account the \( q\bar{q} \) interaction due to the \( D_1 \) correlator (cf Eq. (12)), shows, that the \( q\bar{q} \) pairs can form bound states in this interaction \([59]\), and with increasing m.f. the binding energy grows, which lowers the \( q\bar{q} \) mass, thus leading to the growth of the quark pressure. Moreover, the introduction of this “intermediate state of deconfinement”, existing in the narrow region near \( T_c \), consisting of bound and decaying \( q\bar{q} \) pairs strongly affects the nature of the deconfining transition, making it softer. In addition, the colorelectric string tension, which disappears at \( T_c \), decreases gradually in the same region, lifting in this way the hadron pressure and making the transition continuous. However this remark does not change qualitatively the considerations of the present paper and will be treated in detail elsewhere.

4 The quark-antiquark contribution to the pressure at nonzero m.f.

As it was discussed above, the nonperturbative \( D_1 \) contribution to a single quark is given by Eqs. (11), (12), where \( J_n^E \) can also be written as

\[
L_{\text{fund}} = \exp(-J_1^E) = \exp \left(- \frac{V_1(\infty)}{2T} \right), \quad (37)
\]

where \( V_1(r) \) is the \( q\bar{q} \) nonperturbative (np) colorelectric interaction generated by the field correlator \( D_1^E(x) \) \([59]\)

\[
V_1(r, T) = \int_0^\beta \int_0^\nu (1 - \nu T)d\nu \int_0^T \xi d\xi D_1^E(\sqrt{\nu^2 + \xi^2}). \quad (38)
\]

As it was argued in \([60]\), the asymptotics of \( D_1^E(x) \) is expressed via the gluelump mass \( M_0 \approx 1 \) GeV \([67]\) and can be written as

\[
D_1^{(np)}(x) = \frac{A_1}{|x|} e^{-M|x|} + O(\alpha_s^2), \quad A_1 = 2C_2\alpha_s\sigma_{adj}M_0, \quad (39)
\]
which leads to

\[ V_1(r, T) = V_1(\infty, T) - \frac{A_1}{M_0^2} K_1(M_0 r) M_0 r + O \left( \frac{T}{M_0} \right), \quad (40) \]

and

\[ V_1(\infty, T) = \frac{A_1}{M_0^2} \left[ 1 - \frac{T}{M_0} \left( 1 - e^{-M_0/T} \right) \right], \quad \frac{A_1}{M_0^2} \approx \frac{6 \alpha_s(M_0) \sigma_f}{M_0} \approx 0.5 \text{GeV}. \quad (41) \]

At \( r = 0 \), \( V_1(0, T) = 0 \).

Above \( T_c \) the value of \( V_1(\infty, T) \) is decreasing, as seen from (41), (40). This is in agreement with lattice data on Polyakov loops in [68]. Recently, the potential \( V_1(r, T) \) was studied on the lattice in [69], yielding a behavior similar for \( V_1(\infty, T) \) at \( T = 1.2 T_c \).

We now consider the hadron and quark-gluon pressure in the single-line (the independent particle) approximations with the purpose to define the deconfinement temperature as a function of m.f.

One starts with the total pressure in the confined phase, phase I, which can be written in the form, generalizing the results of [43, 44, 45] for the case of nonzero m.f.

\[ P_I = |\varepsilon_{\text{vac}}| + \sum_i P_i^{(g)}(B), \quad (42) \]

where \( \varepsilon_{\text{vac}} = \varepsilon_{\text{vac}}^{(g)} + \varepsilon_{\text{vac}}^{(q)} \) is given in (2), and we assume, that the gluonic condensate does not depend on m.f. in the first approximation, while the quark condensate \( |\langle \bar{q} q \rangle| \) grows with m.f., as shown analytically in [70] and on the lattice [41, 42], however we neglect this contribution in the first approximation and discuss its importance at large \( eB \) in the concluding section.

In the deconfined phase (phase II) the pressure can be written in the form (cf [43, 44, 45, 48])

\[ P_{II} = \frac{1}{2} |\varepsilon_{\text{vac}}^{(g)}| + \sum_q P_q(B) + P_g \quad (43) \]

where \( P_q(B) \) is given in (24)-(26), and we assume, that vacuum colormagnetic fields, retained in the deconfined phase at \( T \approx T_c \), create one-half of vacuum condensate \( G_2 \) as it happens for \( T = 0 \).

\[ \frac{1}{2} |\varepsilon_{\text{vac}}^{(g)}| \approx \frac{(11 - \frac{2}{3} n_f)}{32} \Delta G_2, \quad \Delta G_2 \approx \frac{1}{2} G_2. \quad (44) \]
Taking into account the chemical potential $\mu$, one can rewrite (27) as

$$P_q(B) = \sum_{q=u,d,\ldots} \frac{N_c |\epsilon_q B| T}{\pi^2} \sum_{n_\perp,\sigma=\pm 1}^{\infty} \frac{(-)^{n+1}}{n} \cosh \frac{\mu n}{T} L_{fund} \epsilon_\sigma n_\perp K_1 \left( \frac{n \epsilon_\sigma}{n_\perp} \right)$$

(45)

Finally, for gluon pressure we are neglecting the influence of m.f., which appears in higher $O(\alpha_s)$ orders, and write

$$P_{gl}(B) \approx P_{gl}^{(0)} = \frac{2(N_c^2 - 1)}{\pi^2} \sum_{n=1}^{\infty} \frac{T^4 n}{n^4} L_{adj}.$$  

(46)

As a result we define the deconfinement temperature from the equality

$$P_I(T = T_c) = P_{II}(T = T_c).$$  

(47)

The contribution of zero levels of light quarks clearly dominates in (45), when $eB > T^2$, so that keeping for simplicity only the $n = 1, \sigma = 1$ terms for small $\mu$, one has

$$\bar{P}_q(B) \approx P_q^{(0)}(B) = \frac{N_c n_f |\bar{\epsilon}_q B| T^2}{\pi^2} L_{fund} \cosh \frac{\mu}{T},$$

(48)

where $|\bar{\epsilon}_q| = \frac{e}{n_f \sum_{i=1}^{n_f} |\epsilon_i|}$, $\bar{\epsilon}_q = \frac{4}{9} e$ for $n_f = 3$.

Neglecting as a first approximation the hadron pressure and $\epsilon_{vac}^{(q)}$ in (42), one obtains an equation for $T_c$:

$$\frac{1}{2} |\epsilon^{(q)}_{vac}| = P_{gl}^{(0)} + P_q^{(0)}(B),$$

(49)

and finally, neglecting the term $\sum_q m_q |\langle \bar{q}q \rangle|$, and $P_{gl}^{(0)}$ for large $B > T^2$, one obtains the asymptotic expression

$$T_c^2 = \frac{(11 - \frac{2}{3} n_f) G_2 \pi^2}{64 N_c n_f |\bar{\epsilon}_q| B L_{fund} \cosh \frac{\mu}{T}}.$$  

(50)

For $\mu = 0$, we take $L_{fund} = \exp \left( -\frac{V_1}{T_c} \right)$, $V_1 \approx 0.5$ GeV [59], $n_f = 3$, $\bar{\epsilon}_q = \frac{4}{9} e$, $G_2 = 0.006$ GeV$^4$ [71], and we obtain

$$T_c(eB = 1 \text{GeV}^2) \approx 0.125 \text{ GeV}.$$  

(51)
For the same parameters and \( B = 0 \) in \[71\] one gets \( T_c(0) \approx 0.165 \text{ GeV} \). These values are in a reasonable agreement with the corresponding lattice data in \[42\], \( T_{c,\text{lat}}(1 \text{ GeV}^2) \approx 0.138 \text{ GeV} \), \( T_{c,\text{lat}}(0) \approx 0.16 \text{ GeV} \).

One can now take into account at large \( B \) also the contribution of gluons, \( P_{gl} = \frac{16}{\pi^2} \exp \left( -\frac{9V_{18}}{8T} \right) \) and \( \pi^0 \) mesons, \( P_{\pi^0} \approx \frac{\pi^2}{90} T^4 \), since the mass of \( \pi^0 \) tends to zero for \( eB > \sigma \), while that of \( \pi^+, \pi^- \) grows as \( \sqrt{eB} \) and does not contribute appreciably to the pressure. One can check that solving equation \( \frac{1}{2} |\varepsilon^{(g)}| + P_{\pi^0} = P_{gl}^{(0)} + P_q^{(0)}(B) \) (52)

one obtains \( T_c(eB = 1 \text{ GeV}^2) \), which is 2\% larger, than in (51).

At large \( eB \gg T^2 \) and \( \mu = 0 \), \( G_2 = 0.006 \text{ GeV}^4 \), Eq. (50) yields \( T_c^2 = \frac{0.00205 \text{ GeV}^2}{eB} \exp \left( \frac{0.25 \text{ GeV}}{T_c} \right) \) (53)

and one obtains \( T_c(eB = 6 \text{ GeV}^2) \approx 0.08 \text{ GeV} \) and a slow decrease for larger \( eB \), \( T_c(eB) \sim \frac{1}{\ln(eB)} \).

To investigate the behavior of \( T_c(B) \) at all values of \( B \) and \( \mu = 0 \) we take into account all Landau levels, as it was done in the Appendix, and write resulting expression for \( P_q(B) \) in the case \( \mu = 0 \)(cf. Eq. (A.5)),

\[
P_q(B) = \frac{N_c e_q B T}{\pi^2} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} L^n \left\{ m_q K_1 \left( \frac{mm_q}{T} \right) + \frac{2T e_q B + m_q^2}{m_q} K_2 \left( \frac{n}{T} \sqrt{e_q B + m_q^2} \right) - \frac{n e_q B}{12T} K_0 \left( \frac{n}{T} \sqrt{m_q^2 + e_q B} \right) \right\} \] (54)

In the limiting case of small quark mass, \( m_q \ll \{ T, e_q B \} \) one can rewrite (54) as

\[
P_q(B) \approx \frac{N_c e_q B T}{\pi^2} L \left\{ T + 2TK_2 \left( \frac{\sqrt{e_q B}}{T} \right) - \frac{e_q B}{12T} K_0 \left( \frac{\sqrt{e_q B}}{T} \right) \right\}. \] (55)

Note, that Eq. (55) for small \( eB \) tends to the limiting \( B \)-independent form (13), (18). We shall use the forms (54), (55) at all values of \( eB \), and hence recalculate (49) with \( P_q^{(0)} \equiv P_q(B) = \sum_{q=u,d,s} P_q(B) \), and \( P_q(B) \) from (54), (55).

As a result from Eq. (49) one obtains the curve \( T_c(B) \) shown in Fig. 1 together with the points obtained on the lattice in [40].
Figure 1: QCD phase diagram in $B-T$ plane for $\mu = 0$ as it is given by (49) in comparison with lattice data [40].

5 The case of nonzero chemical potential

We first consider the case of very large $eB$, when one can retain only the lowest Landau levels of quarks.

For nonzero $\mu$ and $e_qB \gg 4T^2$ one can keep only zero Landau level and rewrite (24) in the form

$$P_q(B) = \frac{N_c}{2\pi^2}e_qB(\phi(\mu) + \phi(-\mu)), \quad (56)$$

where $\phi(\mu)$ is

$$\phi(\mu) = \int_0^\infty \frac{p_zdp_z}{1 + e^{p_z-\mu}}, \quad \bar{\mu} = \mu - \bar{J} \equiv \mu - \frac{V_1(\infty)}{2}. \quad (57)$$

At large $\frac{\bar{\mu}}{T} \gg 1$ one can use the expansion [53]

$$\phi(\mu) \approx \frac{(\mu - \bar{J})^2}{2} + \frac{\pi^2}{6}T^2; \quad (58)$$

and one obtains in the lowest approximation (neglecting $\pi^0$ and gluon contribution at large $eB$), which yields $(\frac{1}{2}e_{vac} = P_q(B))$ the critical value of the chemical potential $\mu_c$,
\[ \tilde{\mu}_c^2 = \frac{1.386G_2}{e_qB}; \quad \mu_c = \frac{V_1(\infty)}{2} + 1.18\sqrt{\frac{G_2}{e_B}}. \] (59)

For \( eB \to \infty \) one has \( \mu_c(eB \to \infty) = \frac{V_1(\infty)}{2} = 0.25 \text{ GeV} \), where we have assumed, that \( V_1(\infty) \) is independent of \( eB \).

Near the critical point the critical curve is easily obtained from (56), (58).

\[ \left( \mu_c - \frac{V_1(\infty)}{2} \right)^2 + \frac{\pi^2}{3}T_c^2 = \frac{1.386G_2}{eB}. \] (60)

For small \( \mu \) the lattice data of [72] reveal, that \( V_1(\infty) \) depends on \( eB \) and may become negative for large \( T \) and \( B \).

However, for small \( T \) the behavior of \( L \) in [72] is compatible with our assumption, that \( V_1(\infty) \) is weakly dependent on \( T \) and being around 0.5 GeV, which supports our form of the phase transition curve (60). Also the \( \mu \) dependence of the color screening potential \( V_1(\infty, \mu) \) was studied in [69], and was found to be rather moderate for \( (\mu/T)^2 \leq 1 \) and \( T/T_c = 1.20 \) and 1.35. Therefore we can assume, that the behavior (60) is qualitatively correct for large \( eB \), \( eB > \sigma = 0.18 \text{ GeV}^2 \), and it should go over for \( B \to 0 \) into the form found earlier in [71].

Now we turn to the case of arbitrary m.f. As shown in the appendix, one can sum up in \( P_q(B) \) over \( n_\perp, \sigma \) in the following way

\[ P_q(B) = \frac{N_c e_q B}{2\pi^2} \left\{ \phi(\mu) + \phi(-\mu) + \frac{2}{3} \left( \frac{\lambda(\mu) + \lambda(-\mu)}{e_q B} - \frac{e_q B}{24} (\tau(\mu) + \tau(-\mu)) \right) \right\}, \] (61)

where \( \phi(\mu) \) is given in (57), while \( \lambda(\mu), \tau(\mu) \) are

\[ \lambda(\mu) = \int_0^\infty \frac{p^4 dp}{\sqrt{p^2 + \tilde{m}_q^2 \exp \left( \frac{\sqrt{p^2 + \tilde{m}_q^2 - \bar{\mu}}}{T} \right)}} \left( \frac{1}{\sqrt{p^2 + \tilde{m}_q^2 - \bar{\mu}}} \right) + 1, \] (62)

\[ \tau(\mu) = \int_0^\infty \frac{dp_z}{\sqrt{p^2 + \tilde{m}_q^2 \exp \left( \frac{\sqrt{p^2 + \tilde{m}_q^2 - \bar{\mu}}}{T} \right)}} \left( \frac{1}{\sqrt{p^2 + \tilde{m}_q^2 - \bar{\mu}}} \right) + 1. \] (63)

Here \( \tilde{m}_q^2 = m_q^2 + e_q B \). One can see, that \( \lambda(\mu), \tau(\mu) \) decay exponentially for \( e_q B \to \infty \), and hence one returns to Eq. (56) in this limit. In the opposite case, when \( e_q B \to 0 \), one recovers the form (61) with only \( \lambda(\mu) + \lambda(-\mu) \) present, which exactly coincides with one, studied in [71].
One can now calculate the transition curve $T_c(\mu, B)$ in the $(T, \mu)$ plane for different values of $eB$, using (61) for $\bar{P}_q(B) = \sum_q P_q(B)$ in the equation $\frac{1}{2}|\varepsilon^{(g)}_{vac}| = P_g^{(0)} + \bar{P}_q(B)$. The resulting sequence of curves $T_c(\mu, eB)$, $0 \leq \mu \leq \mu_c$ and $eB = (0, 0.2, 0.6, 1) \text{ GeV}^2$ is shown in Fig. 2. One can see from (59), that asymptotically for large $eB$ the limiting curve is a cut of the straight line $T_c = 0$, $0 \leq \mu \leq \frac{\sqrt{3}(\infty)}{2}$. Of special importance is the region of small $eB, eB \ll 4T^2$, where the asymptotic (large $eB$) $P_q(B, \mu)$ from Eq. (56) turns over into the m.f. independent form of Eq. (13).

The details of this transition are given in the Appendix.

Figure 2: QCD phase diagram in $\mu – T$ plane for different values of $eB$.

6 Discussion of results and conclusions

We have derived in the paper the compact forms of the quark pressure for zero chemical potential and arbitrary $T$ and $eB$ in (54) and for nonzero chemical potential in (61), which take into account higher Landau levels.

Using that, we have calculated $T_c(eB, \mu = 0)$ in Eq. (50) and Fig. 1 and $T_c(eB, \mu)$ in Fig. 2 in the lowest approximation, neglecting hadron contributions, except for $\pi^0$, and neglecting possible dependence of gluon vacuum energy $\varepsilon^{(g)}_{vac}$ and Polyakov loop $L$ on m.f. This approximation, which can
be however a crude one, is supported by available lattice data on the m.f. dependence of interquark potential $V_1(\infty)$ \[69, 72\].

We have shown, that $T_c(eB, \mu = 0)$ is a moderately decreasing function of $eB$, tending to zero asymptotically as $\frac{1}{\ln eB}$. This fact agrees reasonably well with the realistic lattice data \[40\]. Qualitatively the decreasing pattern was obtained in \[36\] and \[37\]. However, in both cases $T_c$ dropped much faster, and in \[36\] $T_c$ passes zero at $eB \approx 0.7$ GeV$^2$.

A common feature of all approaches, resulting in the decreasing $T_c(eB)$, is that they contain a constant piece of pressure in the confinement phase, which is destructed by transition to the quark phase: it is the vacuum condensate in the present paper and \[36\] and the MIT bag pressure in \[37\]. However the treatment of the hadronic pressure in all available papers, assumes that hadrons are elementary and their masses are not modified by m.f. In contrast to that, our approach suggests to use the real composite hadrons, with masses strongly dependent on m.f., as was found in \[61\]. In the present paper we have used this notion only marginally, taking $\pi^0$ into account, massless at large $eB$, and neglecting heavy in this limit $\pi^+, \pi^-$. However, in the next paper we plan to return to this problem and to calculate $T_c(eB, \mu)$ with the lowest mass hadrons.

In the present paper we have used the only parameters of our approach: $|\varepsilon^{(g)}_{\text{vac}}|$ and $V_1(\infty)$, the latter defining the Polyakov loop average: $\bar{L} = \exp \left(-\frac{V_1(\infty)}{2T}\right)$. For $|\varepsilon^{(g)}_{\text{vac}}|$ we have used $G_2 \equiv \frac{\alpha_s}{\pi} \langle F^a_{\mu\nu} F^a_{\mu\nu} \rangle = 0.006$ GeV$^4$, which was found to give the realistic $T_c(\mu = 0, B = 0) \approx 165$ MeV, in reasonable agreement with most lattice data for $n_f = 3$.

The range of values of $G_2$ including $G_2 = 0.006$ GeV$^4$ was studied in \[50, 51\] and found to give reasonable values of stable mass configuration for a hybrid star configuration. From the purely theoretical point of view, the values of $G_2$ are not uniquely defined, and the value 0.006 GeV$^4$ is within the boundaries of the analysis in \[73\].

We have neglected the dependence of vacuum parameter $|\varepsilon^{(g)}_{\text{vac}}|$ on $B, \mu$, since this dependence can occur only in higher orders of $\alpha_s$ expansion. However we disregarded the quark component $\varepsilon^{(g)}_{\text{vac}} \equiv \sum_q m_q \langle \bar{q}q \rangle$ of the vacuum energy, $|\varepsilon_{\text{vac}}| = |\varepsilon^{(g)}_{\text{vac}}| + |\varepsilon^{(q)}_{\text{vac}}|$, assuming the limit $m_q \to 0$. In reality the contribution of the strange quark with $m_s(2\text{GeV}) \approx 0.1$ GeV is significant and grows linearly with $eB$, which changes the asymptotics of $T_c(eB)$ at large $eB$ from $1/\ln(eB)$ to a constant one. Thus the results of the present paper refer to the case of the $n_f = 3$ massless quarks, and we plan to extend our results
to the realistic (2+1) case in the next publication.

Concerning the $\mu$-dependence of $T_c(\mu, B)$ one can see in Fig. 2 a smooth decreasing behavior for an increasing $B$, with a limiting piece of straight line $T_c(\mu, B \to \infty) \to 0, \quad \mu_c \to \frac{\nu_1(\infty)}{2}$.

We have not analyzed in the paper the character of the quark-hadron transition, since it needs a careful analysis of the hadronic phase, and possible change of parameters below $T_c$, which will be published elsewhere.

The authors are grateful to N.O. Agasian, M.A. Andreichikov, A.M. Badalian and B.O. Kerbikov for useful suggestions and discussions.

Appendix

The quark thermodynamics in a weak m.f.

We start with the case of $\mu = 0$. Eq. (27) can be rewritten as $P_q(B) = \sum_q P_q(B), \quad c_q \equiv |c_q|$

$$P_q(B) = \frac{N_c e_q BT}{\pi^2} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \bar{L}^n \sum_{n_\perp, \sigma} \varepsilon_{n_\perp}^\sigma K_1 \left( \frac{n\varepsilon_{n_\perp}^\sigma}{T} \right), \quad (A\,1)$$

where $\varepsilon_{n_\perp}^\sigma$ is given in (28). The sum over $n_\perp, \sigma$ can be transformed as follows

$$\sum_{n_\perp=0}^{\infty} \sum_{\sigma=\pm1} \varepsilon_{n_\perp}^\sigma K_1 \left( n\varepsilon_{n_\perp}^\sigma T \right) = m_q K_1 \left( \frac{nm_q}{T} \right) + 2 \sum_{n_\perp=0}^{\infty} \bar{\varepsilon} K_1 \left( \frac{n\bar{\varepsilon}}{T} \right). \quad (A\,2)$$

Here $\bar{\varepsilon} = \sqrt{m_q^2 + e_q B + 2e_q B(n_\perp + \frac{1}{2})}$.

At this point one can use the well-known approximation for the summation in the limit of weak m.f. (see §59 of [55])

$$\sum_{k=0}^{\infty} F(k + \frac{1}{2}) \approx \int_0^{\infty} F(x) dx + \frac{1}{24} F'(0). \quad (A\,3)$$

In the integral in (A3) one can use Eq. (19) and Eq. (31) to write

$$\int_0^{\infty} dx \sqrt{m_q^2 + e_q B + 2e_q Bx} \ K_1 \left( \frac{n\sqrt{m_q^2 + e_q B + 2e_q Bx}}{T} \right) =$$
\[
\frac{T}{2neqB} \int_0^\infty \omega d\omega e^{-\frac{T}{2} \left( \frac{eB - \omega^2}{eB + m_q^2} \right)} = \frac{T e_q B + m_q^2}{n \sqrt{e_q B + m_q^2}} K_2 \left( \frac{n}{T \sqrt{e_q B + m_q^2}} \right). \quad (A \, 4)
\]

As a result (A 1) can be rewritten as
\[
P_q(B) = \frac{N_c e_q B T}{\pi^2} \sum_{n=1}^{\infty} \frac{(-)^{n+1}}{n} \bar{L} n \left\{ m_q K_1 \left( \frac{nm_q}{T} \right) + \frac{2T e_q B + m_q^2}{e_q B} K_2 \left( \frac{n}{T \sqrt{e_q B + m_q^2}} \right) - \frac{ne_q B}{12T} K_0 \left( \frac{n}{T \sqrt{m_q^2 + e_q B}} \right) \right\}. \quad (A \, 5)
\]

One can see in (A 5), that the sum over \( n \) is well saturated by the first term with \( n = 1 \) (a typical situation for \( \mu = 0 \)) and for \( m_q \to 0 \) one can write
\[
P_q(B) \approx \frac{N_c e_q B T}{\pi^2} \bar{L} \left\{ T + 2TK_2 \left( \frac{\sqrt{e_q B}}{T} \right) - \frac{e_q B}{12T} K_0 \left( \frac{\sqrt{e_q B}}{T} \right) \right\}. \quad (A \, 6)
\]

For large \( e_q B, e_q B \gg T^2 \), the first term in the curly brackets dominates and one returns to Eq. (18) (with \( \mu = 0 \)).

In the opposite case, \( e_q B \ll T^2 \) one can write, expanding \( K_n(z) \) at small \( z \),
\[
P_q(B) = \frac{N_c \bar{L}}{\pi^2} \left\{ 4T^4 + (e_q B)^2 \left[ \frac{1}{6} \ln \left( \frac{2T}{\sqrt{e_q B}} \right) + \frac{3}{16} - \frac{C}{6} \right] \right\}. \quad (A \, 7)
\]

Here \( C = 0.577 \) is the Euler constant.

At the point \( e_q B = T^2 \) one has \( K_2(1) = 1.625; K_0(1) = 0.421 \) and the coefficient of \( N_c e_q B T \bar{L} \) is 4.21 instead of 4 in the limiting form for \( e_q B \to 0 \), which implies that the weak field asymptotics \( 4T^4 \) has the 5% accuracy at \( e_q B = T^2 \), whereas the form (A 7) yields at this point the 0.2% accuracy. Therefore to treat the whole intermediate region \( e_q B \lesssim T^2 \) and \( e_q B > T^2 \) with few percent accuracy one can use Eq. (A 6).

For \( \mu > 0 \) one can rewrite \( \sum_{\mu, \sigma} \chi(\mu) \) in (24) in the same way, as it was done in (A 2),
\[
\sum_{\mu, \sigma} \chi(\mu) = \int \frac{dp_z}{2\pi} \left\{ \ln \left( 1 + \exp \left( \frac{\mu - \sqrt{p_z^2 + m_q^2}}{T} \right) \right) \right\} + \nu
\]

18
\[ + 2 \sum_{n_\perp = 0}^{\infty} \ln \left( 1 + \exp \left( \frac{\bar{\mu} - \sqrt{p_\perp^2 + m_q^2 + e_q B} + 2e_q B(n_\perp + \frac{1}{2})}{T} \right) \right) = I_1 + I_2. \]

(A 8)

The first term \( I_1 \) is easily (integrating by parts) transformed to the m.f. independent term \( \phi(\mu) \) in (A 3),

\[ I_1 = \frac{1}{\pi T} \phi(\mu). \]

(A 9)

The second term \( I_2 \) can be rewritten using (A 3) as follows

\[ I_2 = \frac{1}{\pi T} \int_0^\infty \frac{p_\perp^2 dp_z}{e_q B} \int_0^\infty \frac{d\lambda}{\sqrt{p_\perp^2 + m_q^2 + e_q B + \lambda}} \frac{1}{e^{\sqrt{p_\perp^2 + m_q^2 + e_q B + \lambda - \bar{\mu}}}} + 1 \]

\[ - \frac{e_q B}{24\pi T} \int_{-\infty}^\infty \frac{dp_z}{\sqrt{p_\perp^2 + m_q^2 + e_q B}} \frac{1}{\exp \left( \frac{\sqrt{p_\perp^2 + m_q^2 + e_q B - \bar{\mu}}}{T} \right)} \equiv I_2' + I_2''. \]

(A 10)

One can estimate the large \( eB \) asymptotics of \( I_2, I_2 \sim \exp \left( -\frac{\sqrt{e_q B + m_q^2}}{T} \right) \), which supports Eq. (56) in this limit. In the opposite limit, \( eB \to 0 \) the first term \( I_2' \) in (A 10) behaves as

\[ I_2' \approx \frac{2}{3\pi T e_q B} \int_0^\infty \frac{p^4 dp}{\sqrt{p^2 + \tilde{m}_q^2}} \frac{1}{\exp \left( \frac{\sqrt{p^2 + \tilde{m}_q^2 - \bar{\mu}}}{T} \right) + 1}, \]

(A 11)

where \( \tilde{m}_q^2 = m_q^2 + e_q B \). One can see, that \( I'_2 (e_q B \to 0) \) ensures in (24) the correct limiting form found in [71], namely

\[ P_q = \frac{N_c T^4}{3\pi^2} \left[ \phi_\nu \left( \frac{\mu - \frac{V_1}{2}}{T} \right) + \phi_\nu \left( -\frac{\mu + \frac{V_1}{2}}{T} \right) \right], \quad P = \sum_{q=1}^{n_f} P_q \]

(A 12)

with

\[ \phi_\nu(a) = \int_0^\infty \frac{z^4 dz}{\sqrt{z^2 + a^2}} \frac{1}{\exp \left( \sqrt{z^2 + a^2} - a \right) + 1}, \]

(A 13)

and \( \nu = \sqrt{\frac{m_q^2 + e_q B}{T}} \). At the same time the term \( I_2'' \) has the order \( O \left( \frac{(e_q B)^2}{T^4} \right) \), as compared to the leading term (A 12).
References

[1] T. Vachaspati, Phys. Lett. B 265, 258 (1991).

[2] K. Enqvist and P. Olesen, Phys. Lett. B 319, 178 (1993), hep-ph/9308270.

[3] D. E. Kharzeev, L. D. McLerran and H. J. Warringa, Nucl. Phys. A 803, 227 (2008);
V. Skokov, A. Y. Illarionov, and V. Toneev, Int. J. Mod. Phys. A 24, 5925 (2009), 0907.1395.

[4] V. Voronyuk, V. Toneev, W. Cassing, E. Bratkovskaya, V. Konchakovski, et al., Phys. Rev. C 83, 054911 (2011), 1103.4239.

[5] A. Bzdak and V. Skokov, Phys.Lett. B710, 171 (2012), 1111.1949.

[6] W.-T. Deng and X.-G. Huang, Phys.Rev. C85, 044907 (2012), 1201.5108.

[7] R. C. Duncan and C. Thompson, Astrophys. J. 392, L9 (1992).

[8] D. E. Kharzeev, K. Landsteiner, A. Schmitt, and H. -U. Yee, Lect. Notes Phys. 871, 1 (2013).

[9] V. P. Gusynin, V. A. Miransky, and I. A. Shovkovy, Nucl. Phys. B 462, 249 (1996), hep-ph/9509320.

[10] I. Shushpanov and A. V. Smilga, Phys. Lett. B 402, 351 (1997), hep-ph/9703201.

[11] N. O. Agasian and I. Shushpanov, Phys. Lett. B 472, 143 (2000), hep-ph/9911254.

[12] N. O. Agasian, Phys. Atom. Nucl. 64, 554 (2001), hep-ph/0112341.

[13] T. D. Cohen, D. A. McGady, and E. S. Werbos, Phys. Rev. C 76, 055201 (2007), 0706.3208.

[14] J. O. Andersen, Phys. Rev. D 86, 025020 (2012), arXiv:1202.2051.

[15] J. O. Andersen, JHEP 1210, 005 (2012), arXiv:1205.6978.
[16] S. P. Klevansky and R. H. Lemmer, Phys.Rev. D 39, 3478 (1989).
[17] D. P. Menezes, M. Benghi Pinto, S. S. Avancini, and C. Providencia, Phys. Rev. C 80, 065805 (2009), [0907.2607].
[18] R. Gatto and M. Ruggieri, Phys. Rev. D 83, 034016 (2011), [1012.1291].
[19] R. Gatto and M. Ruggieri, Phys. Rev. D 82, 054027 (2010), [1007.0790].
[20] K. Kashiwa, Phys. Rev. D 83, 117901 (2011), [1104.5167].
[21] J. O. Andersen and R. Khan, Phys. Rev. D 85, 065026 (2012), [1105.1290].
[22] S. S. Avancini, D. P. Menezes, M. B. Pinto, and C. Providencia (2012), [1202.5641].
[23] K. Fukushima and J. M. Pawlowski (2012), [1203.4330].
[24] A. J. Mizher, M. N. Chernodub, and E. S. Fraga, Phys. Rev. D 82, 105016 (2010a), [1004.2712].
[25] J. O. Andersen and A. Tranberg, JHEP 08 (2012) 002; arXiv: 1204.3360.
[26] S. Kanemura, H.-T. Sato, and H. Tochimura, Nucl. Phys. B 517, 567 (1998), hep-ph/9707285.
[27] K. G. Klimenko, Theor. Math. Phys. 90, 1 (1992).
[28] J. Alexandre, K. Farakos, and G. Koutsoumbas, Phys. Rev. D 63, 065015 (2001), hep-th/0010211.
[29] D. D. Scherer and H. Gies, Phys. Rev. B 85, 195417 (2012), arXiv: 1201.3746.
[30] C. V. Johnson and A. Kundu, JHEP 12, 053 (2008), 0803.0038.
[31] F. Preis, A. Rebhan, and A. Schmitt, JHEP 1103, 033 (2011), 1012.4785.
[32] A. J. Mizher, E. S. Fraga, and M. Chernodub, PoS FACESQCD, 020 (2010b), 1103.0954.
[33] J. Gasser and H. Leutwyler, Phys. Lett. B 184, 83 (1987).
[34] J. Gasser and H. Leutwyler, Phys. Lett. B 188, 477 (1987b).
[35] P. Gerber and H. Leutwyler, Nucl. Phys. B 321, 387 (1989).
[36] N. Agasian and S. Fedorov, Phys. Lett. B 663, 445 (2008), arXiv: 0803.3156.
[37] E. S. Fraga and A. J. Mizher, Nucl. Phys. A 820, 1030 (2009); arXiv:0810.3693;
E. S. Fraga and L. F. Palhares; Phys. Rev. D 86, 016008 (2012); arXiv:1201.5881.
[38] E. S. Fraga, arXiv:1208.0917 in: Lect. Notes Phys. (Springer).
[39] R. Gatto and M. Rugieri, arXiv:1207.3190 in: Lect. Notes Phys. (Springer).
[40] G. Bali, F. Bruckmann, G. Endrodi et al., JHEP 1202, 044 (2012); arXiv:1111.4956.
[41] G. S. Bali, F. Bruckmann, G. Endr̆odi, Z. Fodor et al., Phys. Rev. D 86, 071502 (2012); arXiv:1206.4205.
[42] G. Bali, F. Bruckmann, M. Constantinou et al., arXiv:1301.5826.
[43] Yu. A. Simonov, JETP Lett. 54, 249 (1991).
[44] Yu. A. Simonov, JETP Lett. 55, 605 (1992).
[45] Yu. A. Simonov, Phys. At. Nucl. 58, 309 (1995).
[46] Yu. A. Simonov, Proc. Varenna 1995, Selected Topics in Nonperturbative QCD, p. 319.
[47] H. G. Dosch, H.-J. Pirner and Yu. A. Simonov, Phys. Lett. B 349, 335 (1993).
[48] Yu. A. Simonov, Ann. Phys. (NY) 323, 783 (2008).
[49] E. V. Komarov and Yu. A. Simonov, Ann. Phys. (NY) 323, 1230 (2008).
[50] D. Logoteta and I. Bombaci, Phys. Rev. D 88, 063001 (2013); arXiv:1309.0096 [nucl.th].

[51] M. Baldo, G. F. Burgio, P. Castorina, S. Plumari and D. Zappala, Phys. Rev. D 78, 063009 (2008).

[52] F. I. M. Pereira, Nucl. Phys. A 860, 102 (2011); Nucl. Phys. A 897, 151 (2013).

[53] A. V. Nefediev, Yu. A. Simonov and M. A. Trusov, Int. J. Mod. Phys. E. 8, 549 (2009).

[54] L. D. Landau and E. M. Lifshitz, Quantum Mechanics: Non-Relativistic Theory, Pergamon, New-York, 1991.

[55] L. D. Landau and E. M. Lifshitz, Statistical Mechaniscs, part I, vol. 5, Pergamon, New York, 1980.

[56] Yu. A. Simonov, Nucl. Phys. B 307, 512 (1988).

[57] Yu. A. Simonov and J. A. Tjon, Ann. Phys. (NY) 223, 1 (1993), ibid 300, 54 (2002).

[58] Yu. A. Simonov, Phys. Rev. D 88, 025028 (2013).

[59] Yu. A. Simonov, Phys. Lett. B 619, 293 (2005), A. Di Giacomo, E. Meggiolaro, Yu. A. Simonov and A. I. Veselov, Phys. At. Nucl. 70, 908 (2007), hep-ph/0512125.

[60] M. A. Andreichikov, V. D. Orlovsky and Yu. A. Simonov, Phys. Rev. Lett. 110, 162002 (2013).

[61] M. A. Andreichikov, B. O. Kerbikov, V. D. Orlovsky and Yu. A. Simonov, Phys. Rev. D 87, 094029 (2013).

[62] V. D. Orlovsky and Yu. A. Simonov, JHEP 1309, 136 (2013), arXiv:1306.2232.

[63] Yu. A. Simonov, Phys.Rev. D 88, 053004 (2013); arXiv:1304.0365.

[64] Y. Hidaka and A. Yamamoto, Phys. Rev. D 87, 094502 (2013); arXiv:1209.0007 [hep-lat].
[65] E. V. Luschevskaya and O. V. Larina, arXiv:1203.5699 [hep-lat].

[66] Yu. A. Simonov, Phys.Atom.Nucl. 69, 528 (2006), hep-ph/0501182.
Yu. A. Simonov and V. I. Shevchenko, Adv. High En. Physics, 2009, 873061 (2009).

[67] M. Foster and C. Michael Phys. Rev. D 59, 094509 (1999); Yu. A. Simonov, Nucl. Phys. B592, 350 (2001).

[68] S. Gupta, K. Hübner and O. Kaczmarek, Nucl. Phys. A 785, 278 (2007).

[69] J. Takahashi, K. Nagata, T. Saito et al., arXiv:1308.2489.

[70] Yu. A. Simonov, arXiv:1212.3118.

[71] Yu. A. Simonov and M. A. Trusov, Phys. Lett. B 650, 36 (2007); JETP Lett. 85, 730 (2007).

[72] F. Bruckmann, G. Endrődi and T. G. Kovacs, arXiv:1303.3972

[73] B.L.Ioffe and K.Zyablyuk, Eur. Phys. J. C 27, 229 (2003); hep-ph/0207183.