QED Ward Identity for fermionic field in the light-front

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In a covariant gauge we implicitly assume that the Green’s function propagates information from one point of the space-time to another, so that the Green’s function is responsible for the dynamics of the relativistic particle. In the light front form, which in principle is a change of coordinates, one would expect that this feature would be preserved. In this manner, the fermionic field propagator can be split into a propagating piece and a non-propagating (“contact”) term. Since the latter (“contact”) one does not propagate information, and therefore, assumedly with no harm to the field dynamics we wanted to know what would be the impact of dropping it off. To do that, we investigated its role in the Ward identity in the light front.

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I. INTRODUCTION

One of the most important concepts in quantum field theories is the question of renormalizability. In QED (Quantum Electrodynamics) specifically, the electric charge renormalization is guaranteed solely by the renormalization of the photon propagator. This result is a consequence of the so-called Ward identity, demonstrated by J.C. Ward in 1950 [1, 2, 3]. The importance of this result can be seen and emphasized in the fact that without the validity of such an identity, there would be no guarantee that the renormalized charge of different fermions (electrons, muons, etc.) would be the same. In other words, without such identity, charges of different particles must have different renormalization constants, a feature not so gratifying nor elegant. Moreover, without the Ward identity, renormalizability would have to be laboriously checked order by order in perturbation theory.

What the Ward identity does is to relate the vertex function of the theory with the derivative of the self-energy function of the electron, and this important correlation is expressed in terms of equality between the renormalization constants, namely, $Z_1 = Z_2$, where $Z_1$ and $Z_2$ are the renormalization constants related to the vertex function and the fermionic propagator respectively. Since the renormalized electric charge is given in terms of the bare electric charge via the product $e_R = Z_1^{1/2} Z_2 Z_1^{-1} e_0$, it follows immediately that $e_R = Z_3^{1/2} e_0$, i.e., electric charge renormalization depends solely on the renormalization of the photon propagator.

We know that light-front dynamics is plagued with singularities of all sorts and because of this the connection between the covariant quantities and light-front quantities cannot be so easily established. If we want to describe our theory in terms of the light-front coordinates or variables, we must take care of the boundary conditions that fields must obey. Thus, a simple projection from the covariant quantities to light-front quantities via coordinate transformations is bound to be troublesome. This can be easily seen in our checking of the QED Ward identity in the light-front, where the fermionic propagator does bear an additional term proportional to $\gamma^+(p^+)^{-1}$ oftentimes called “contact term” in the literature, which, of course, is conspicuously absent in the covariant propagator. This term, as we will see, is crucial to the Ward identity in the light-front. The covariant propagating term solely projected onto the light-front coordinates therefore violates Ward identity, and therefore breaks gauge invariance. Such result is obviously wrong and unwarranted.

The outline of our paper is as follows: We begin by considering the standard derivation for the covariant case Ward identity and show explicitly that the fermionic propagator there cannot be analytically regularized, otherwise Ward identity cannot be achieved. Then we explicitly construct our fermionic propagator in terms of the light-front coordinates, with the proper contact term in it and in the following section we deal with the checking of the Ward identity proper. Finally, the next two sections are devoted to the concluding remarks and Appendix; in the latter we define our light-cone coordinates convention and notation and include explicit calculations showing that without the contact term in the fermionic propagator, Ward identity is not satisfied, and thus gauge invariance is violated.
II. THE WARD IDENTITY

There are several ways to write down the Ward identity for fermions, and one of them is inferred from manipulations of their propagator, namely, $S(p)$. Multiplying by its inverse, we get the identity

$$S(p)S^{-1}(p) = I,$$

Deriving both sides with respect to $p^\mu$ we get

$$\frac{\partial S(p)}{\partial p^\mu}S^{-1}(p) + S(p)\frac{\partial S^{-1}(p)}{\partial p^\mu} = 0$$

which leads to

$$\frac{\partial S(p)}{\partial p^\mu}S^{-1}(p) = -S(p)\frac{\partial S^{-1}(p)}{\partial p^\mu}$$

Finally, multiplying both sides from the left by the propagator itself

$$\frac{\partial S(p)}{\partial p^\mu} = -S(p)\frac{\partial S^{-1}(p)}{\partial p^\mu}S(p) \tag{1}$$

Now, using $S(p) = \frac{i}{\not p - m}$ it follows that its inverse is $S^{-1}(p) = -i(\not p - m)$. Deriving this last expression with respect to $p^\mu$ we get

$$\frac{\partial S^{-1}(p)}{\partial p^\mu} = -\gamma_\mu$$

which inserted into (1) leads to the differential form of the Ward Identity, namely,

$$\frac{\partial S(p)}{\partial p^\mu} = iS(p)\gamma_\mu S(p). \tag{2}$$

Here it is important to stress that if the propagator were raised to a power as in the analytic regularization scheme, i.e. if it had the form $S(p) = \frac{i}{(\not p - m)^\sigma}$, with $\sigma \neq 1$, the identity (2) would not be fulfilled.

III. FERMION PROPAGATOR IN THE LIGHT-FRONT

With the light-front coordinate transformations given in Appendix A, we can find the corresponding fermionic propagator, beginning with the term $\not p$, as in (11): 

$$\not p = p_\mu \gamma^\mu = (\gamma^+ p^- + \gamma^- p^+) - (\vec{\gamma} \cdot \vec{p} \perp),$$

then

$$S(p) = \frac{i}{[(\gamma^+ p^- + \gamma^- p^+) - (\vec{\gamma} \cdot \vec{p} \perp) - m]},$$

or, in another way, using $S(p) = \frac{i(\not p + m)}{p^2 - m^2}$,

$$S(p) = \frac{i[(\gamma^+ p^- + \gamma^- p^+) - (\vec{\gamma} \cdot \vec{p} \perp) + m]}{p^+(p^- - p_{on})},$$

$$S(p) = \frac{i(\not p_{on} + m)}{2p^+(p^- - p_{on})} + \frac{i\gamma^+}{2p^+}, \tag{3}$$

where 

$$\not p_{on} = (\gamma^+ p_{on} + \gamma^- p^+) - (\vec{\gamma} \cdot \vec{p} \perp),$$

$$p_{on} = \frac{p_{on}^2 + m^2}{2p^+}.$$
IV. THE WARD IDENTITY ON THE LIGHT-FRONT

There are two manners to test if the propagator \( S(p) \) on the light-front satisfy the Ward identity (2). The simplest and most direct one is to do the derivatives \( \frac{\partial S^{-1}(p)}{\partial p^\mu} \) for each component and put them in (1):

\[
\frac{\partial S^{-1}(p)}{\partial p^+} = -i\gamma^-
\]

\[
\frac{\partial S^{-1}(p)}{\partial p^-} = -i\gamma^+
\]

\[
\frac{\partial S^{-1}(p)}{\partial p_{1,2}} = -i\gamma^{1,2}.
\]

(4)

\[
\frac{\partial S(p)}{\partial p^+} = iS(p)\gamma^- S(p)
\]

\[
\frac{\partial S(p)}{\partial p^-} = iS(p)\gamma^+ S(p)
\]

\[
\frac{\partial S(p)}{\partial p_{1,2}} = iS(p)\gamma^{1,2} S(p),
\]

(5)

where \( p_{1,2} = p_\perp \) and \( \gamma^{1,2} = \gamma^\perp \) are the transversal or perpendicular components.

Comparing (5) and (2), one verifies that the Ward identity is satisfied if one includes the necessary factors due to the change of coordinate system, or, in other words, considering the Jacobian determinant of this transformation.

The second manner to test the identity on the light-front is working explicitly with all the figures of (2). The details are presented in Appendix B, and below we put the principal results:

\[
\frac{\partial S(p)}{\partial p^+} = iS(p)\gamma^- S(p) = \frac{-ip^- (\hat{p} + m)}{2 [p^+ (p^- - p_{on})]^2} + \frac{i\gamma^-}{2p^+ (p^- - p_{on})},
\]

(6)

\[
\frac{\partial S(p)}{\partial p^-} = iS(p)\gamma^+ S(p) = \frac{-i(\hat{p}_{on} + m)}{2p^+ (p^- - p_{on})^2},
\]

(7)

\[
\frac{\partial S(p)}{\partial p_{1,2}} = iS(p)\gamma^{1,2} S(p) = \frac{ip_{1,2} (\hat{p} + m)}{2 [p^+ (p^- - p_{on})]^2} + \frac{i\gamma^{1,2}}{2p^+ (p^- - p_{on})},
\]

(8)

that is, again one corroborates the relations (5). An important point here is that, using the simplified propagator \( S(p) = \frac{i(\hat{p}_{on} + m)}{p^+ (p^- - p_{on})} \) as some authors do, the Ward Identity is not fulfilled, as shown in Appendix C.

V. CONCLUSIONS

We have shown here that the Ward identity for the fermionic field in the light-front is preserved to guarantee that the charge renomalization constant depends solely on the photon renormalization constant, as it is expected. However, one important point emerges in our computation, and that is that the Ward identity in the light-front is valid provided the fermionic field propagator bears the relevant “contact” term piece, which is absent in the covariant propagator and its straightforward projection into light-front variables.

Our computation has demonstrated once again the significance of the light-front zero-mode contribution that the so-called “contact” term bears in it, without which Ward identity would be violated. Although the zero-mode term does not carry physical information, its non-vanishing contribution nonetheless is crucial to the validity of the Ward identity in the light-front formalism. In other words, “contact” term may not carry information from one space-time point to another in the light front, but contains relevant physical information needed to ensure the Ward identity, and therefore, for the correct charge renormalization.
VI. APPENDIX

A. Light-front Coordinates

The Light-front is characterized by the null-plane $x^+ = t+z = 0$, which is its time coordinate. All of the coordinates are set regarding this plane, and one has new definitions of the scalar product, for example. The basic relations on the light-front are

$$x^+ = \frac{1}{\sqrt{2}} (x^0 + x^3)$$
$$x^- = \frac{1}{\sqrt{2}} (x^0 - x^3)$$
$$\vec{P}_\perp = x^1 \vec{t} + x^2 \vec{y},$$

so, the scalar product is given by

$$a^\mu b_\mu = (a^+ b^- + a^- b^+) - \vec{a}_\perp \cdot \vec{b}_\perp.\quad (10)$$

Using (10), one can write the product $\hat{p}$ on the light-front:

$$\hat{p} = p_\mu \gamma^\mu = (\gamma^+ p^- + \gamma^- p^+) - (\vec{\gamma}_\perp \cdot \vec{P}_\perp).\quad (11)$$

B. Checking the Ward Identity

In this Appendix, we show the details of the algebra necessary to arrive at (6-8). In the first place, we list the numerous properties that Dirac gamma matrices in the light-front obey and should be used:

$$\gamma^+ \gamma^+ = \gamma^- \gamma^- = 0$$
$$\gamma^+ \gamma^- \gamma^+ = 2 \gamma^+$$
$$\gamma^- \gamma^+ \gamma^- = 2 \gamma^-$$
$$\gamma^1 \gamma^\pm \gamma^\mp = \gamma^\pm$$
$$\gamma^2 \gamma^\pm \gamma^\mp = \gamma^\pm$$
$$\gamma^1 \gamma^1 \gamma^2 \gamma^2 = -I$$
$$\gamma^1 \gamma^2 \gamma^+ \gamma^- \gamma^+ \gamma^- = 0$$
$$\{\gamma^1, \gamma^2\} = 0$$

$$\gamma^1 \gamma^\pm \gamma^\mp = \gamma^\pm$$
$$\gamma^1 \gamma^1 \gamma^2 \gamma^2 = 2 \gamma^1 \gamma^2$$
$$\{\gamma^1, \gamma^2\} = 0$$

\begin{align*}
\{\gamma^1, \gamma^+\} &= 2I \\
\{\gamma^1, \gamma^-\} &= 2I
\end{align*}

\begin{align*}
\{\gamma^1, \gamma^+\} &= 2 \gamma^1 \gamma^+ \\
\{\gamma^1, \gamma^\mp\} &= 2 \gamma^1 \gamma^\mp
\end{align*}

Next, some useful relations:

$$\frac{\partial p_{on}}{\partial p^+} = \frac{p_+^2 + m^2}{2(p^+)^2} = \frac{p_{on}}{p^+}\quad (13)$$

$$\frac{\partial p_{on}}{\partial p^+} = -\gamma^+ p_{on} p^+ + \gamma^-\quad (14)$$

$$\frac{\partial p_{on}}{\partial p_1} = \frac{p_1}{p^+}\quad (15)$$

Remembering that the fermion propagator is $S(p) = \frac{i(\hat{p}_{on} + m)}{p^+ (p^- - p_{on})} + \frac{i\gamma^+}{2p^+}$, the plus component derivative is
\[ \frac{\partial S(p)}{\partial p^+} = \frac{i (\phi_{on} + m)}{2p^+ (p^- - p_{on})} - \frac{i (\phi_{on} + m)}{2 (p^+)^2 (p^- - p_{on})^2} - \frac{i (\phi_{on} + m)}{2 (p^+)^2 (p^- - p_{on})^2} \left( \frac{\partial p_{on}}{\partial p^+} \right) - \frac{i \gamma^+}{2 (p^+)^2} \]

\[ \frac{\partial S(p)}{\partial p^-} = \frac{-i \gamma^+ \phi_{on}}{2p^+ (p^- - p_{on})} + \frac{i \gamma^-}{2p^+ (p^- - p_{on})} - \frac{i (\phi_{on} + m)}{2 (p^+)^2 (p^- - p_{on})^2} - \frac{i \gamma^+}{2 (p^+)^2} \]

\[ \frac{\partial S(p)}{\partial p^+} = -\frac{i \gamma^+ (p^-) - i \gamma^- p^+ p_{on} + i (\gamma \cdot p^1) p^- - i m p^-}{2 [p^+ (p^- - p_{on})]^2} \]

\[ \frac{\partial S(p)}{\partial p^-} = -\frac{-i p^- (\phi + m) + \frac{2}{4} p^+ \gamma^- (p^- - p_{on})}{2 [p^+ (p^- - p_{on})]^2} + \frac{i \gamma^-}{2p^+ (p^- - p_{on})} \cdot (16) \]

Now, calculating the term \( i S(p) \gamma^- S(p) \) and exploiting the properties of the gamma functions, we have

\[ = -i \left\{ \frac{(\phi_{on} + m) \gamma^- (\phi_{on} + m)}{4 [p^+ (p^- - p_{on})]^2} + \frac{(\phi_{on} + m) \gamma^- \gamma^+}{4 (p^+)^2 (p^- - p_{on})} + \frac{\gamma^+ \gamma^- (\phi_{on} + m)}{4 (p^+)^2 (p^- - p_{on})} + \frac{\gamma^+ \gamma^- \gamma^+}{4 (p^+)^2} \right\} \]

\[ = -i \left\{ \frac{\phi_{on} \gamma^- \phi_{on} + m \{\phi_{on}, \gamma^-\} + m^2 \gamma^- + \gamma^+ \gamma^- \gamma^+ \phi_{on} + m \{\gamma^+, \gamma^-, \gamma^+\} + \frac{\gamma^+}{4 (p^+)^2} \} \right\} \]

\[ = -i \left\{ \frac{2 \gamma^+ (p_{on})^2 - 2 p_{on} (\gamma \cdot p_{on}) + (\gamma \cdot p_{on})^2 \gamma^- + 2 m p_{on} + m^2 \gamma^-}{4 [p^+ (p^- - p_{on})]^2} + \frac{4 \gamma^+ p_{on} - 2 (\gamma \cdot p_{on}) + 2 m}{4 (p^+)^2 (p^- - p_{on})} + \frac{\gamma^+}{2 (p^+)^2} \right\} \]

\[ = -i \left\{ \frac{2 p^- \phi - 2 \gamma^- p^+ (p^- - p_{on}) + 2 m p^-}{4 [p^+ (p^- - p_{on})]^2} \right\} \]

\[ = \frac{-i p^- (\phi + m)}{2 [p^+ (p^- - p_{on})]^2} + \frac{i \gamma^-}{2p^+ (p^- - p_{on})}, \quad (17) \]

One can see that, from (16) and (17), \( \frac{\partial S(p)}{\partial p^+} = i S(p) \gamma^- S(p) \).

For the minus component, the derivative is very simple,

\[ \frac{\partial S(p)}{\partial p^-} = -\frac{i (\phi_{on} + m)}{2 p^+ (p^- - p_{on})^2} \]

And the term \( i S(p) \gamma^+ S(p) \),

\[ = -i \left\{ \frac{(\phi_{on} + m) \gamma^+ (\phi_{on} + m)}{4 [p^+ (p^- - p_{on})]^2} \right\} \]
\[
\begin{align*}
&= -i \left\{ 2\gamma^- (p^+)^2 - 2p^+ (\gamma_\perp p_\perp) + 2p^+ p_{on}\gamma^+ + 2mp^+ \right\} \\
&= \frac{-i (p_{on} + m)}{2p^+ (p^- - p_{on})^2} \tag{19}
\end{align*}
\]
and again one has \( \frac{\partial S(p)}{\partial p} = iS(p)\gamma^+ S(p) \).

Finally, the derivative of the transversal components:
\[
\begin{align*}
\frac{\partial S(p)}{\partial p_1} &= \frac{i (\gamma_{on} p_1 \gamma^-)}{2p^+ (p^- - p_{on})} + \frac{i (p_{on} + m)}{2p^+ (p^- - p_{on})^2} \left( \frac{\partial p_{on}}{\partial p_1} \right) \\
\frac{\partial S(p)}{\partial p_1} &= \frac{i (\gamma_{on} p_1 \gamma^-)}{2p^+ (p^- - p_{on})} + \frac{ip_1 (p_{on} + m)}{(p^+)^2 (p^- - p_{on})^2} \\
\frac{\partial S(p)}{\partial p_1} &= \frac{ip_1 (\gamma^+ p^- + \gamma^- p^+ - (\gamma_\perp p_\perp) + m) + i\gamma^1 p^+ (p^- - p_{on})}{2 [p^+ (p^- - p_{on})]^2} \\
\frac{\partial S(p)}{\partial p_1} &= \frac{ip_1 (\gamma^+ p^- + p^+ - (\gamma_\perp p_\perp) + m) + i\gamma^1 p^+ (p^- - p_{on})}{2 [p^+ (p^- - p_{on})]^2} + \frac{i\gamma^1}{2p^+ (p^- - p_{on})}. \tag{20}
\end{align*}
\]
The term \( iS(p)\gamma^1 S(p) \) is very laborious and almost all of the gamma matrices properties must be used:
\[
\begin{align*}
&= -i \left\{ \frac{\gamma_{on} \gamma^- (p_{on}) + m \{ p_{on}, \gamma^- \} + m^2 \gamma^1}{4 [p^+ (p^- - p_{on})]^2} + \frac{\gamma_{on} \gamma^+ + \gamma^+ \gamma^1 p_{on} + m \{ \gamma^-, \gamma^- \} + \gamma^+ \gamma^1 \gamma^-}{4 (p^+)^2 (p^- - p_{on})} \right\} \\
&= -i \left\{ \frac{-2\gamma^1 p^+ p_{on} - 2\gamma^- p_1 p_{on} - 2\gamma^- p^+ p_1 - \gamma^1 (p_1)^2 + \gamma^1 (p_2)^2 - 2\gamma^2 p_1 p_2 + m^2 \gamma^- - 2mp^+}{4 [p^+ (p^- - p_{on})]^2} + \frac{2\gamma^1 p_1 - 2\gamma^- p^+}{4 (p^+)^2 (p^- - p_{on})} \right\}
\end{align*}
\]
\[
\begin{align*}
&= -i \left\{ \frac{-2p_1 [\gamma^+ p^- + \gamma^- p^+ - (\gamma_\perp p_\perp) + m] + \gamma^1 (p_1)^2 + \gamma^1 (p_2)^2 - 2\gamma^1 p^+ p^- + m^2 \gamma^-}{4 [p^+ (p^- - p_{on})]^2} \right\} \\
&= -i \left[ \frac{-2p_1 (\dot{p} + m) + \gamma^1 (p_1^2 + p_2^2 + m^2 - 2p^+ p^-)}{4 [p^+ (p^- - p_{on})]^2} \right] \\
&= -i \left[ \frac{-2p_1 (\dot{p} + m) - 2\gamma^1 (p^+ p^- - p^+ p_{on})}{4 [p^+ (p^- - p_{on})]^2} \right] \\
&= \frac{ip_1 (\dot{p} + m)}{2 [p^+ (p^- - p_{on})]^2} + \frac{i\gamma^1}{2p^+ (p^- - p_{on})}. \tag{21}
\end{align*}
\]
and from (20) and (21), one has \( \frac{\partial S(p)}{\partial p_{1,2}} = iS(p)\gamma^{1,2} S(p) \).
C. The Ward Identity for the propagator without contact term

Here we work on the Ward Identity for the simplified propagator \( S(p) = \frac{i(p_{\text{on}} + m)}{2p^+(p^- - p_{\text{on}})} \).

For the minus component, the derivative is the same as the one obtained before,

\[
\frac{\partial S(p)}{\partial p^-} = \frac{-i(p_{\text{on}} + m)}{2p^+(p^- - p_{\text{on}})^2};
\]

(22)

and the term \( iS(p)\gamma^+ S(p) \) is equal too, because in the other case, the terms \( \frac{i\gamma^+}{2p^+} \) do not contribute due to the property \( \gamma^+\gamma^+ = 0 \):

\[
iS(p)\gamma^+ S(p) = \frac{-i(p_{\text{on}} + m)}{2p^+(p^- - p_{\text{on}})^2},
\]

(23)

so, for the negative component, the Ward identity is satisfied

\[
\frac{\partial S(p)}{\partial p^-} = iS(p)\gamma^+ S(p).
\]

For the plus component, one has

\[
\frac{\partial S(p)}{\partial p^+} = \frac{-i\gamma^+ p_{\text{on}}}{2p^+(p^- - p_{\text{on}})} + \frac{i\gamma^-}{2p^+(p^- - p_{\text{on}})} - \frac{i(p_{\text{on}} + m)}{2(p^+)^2(p^- - p_{\text{on}})^2};
\]

(24)

\[
\frac{\partial S(p)}{\partial p^+} = \frac{ip^+\gamma^- - p^+p_{\text{on}}\gamma^- - p^- p_{\text{on}} \gamma^+ + (p_{\text{on}})^2 \gamma^+ - p^- (p_{\text{on}} + m)}{2(p^+)^2(p^- - p_{\text{on}})^2};
\]

\[
\frac{\partial S(p)}{\partial p^+} = \frac{-ip^- (p_{\text{on}} + m)}{2[p^+(p^- - p_{\text{on}})]^2} + \frac{i\gamma^-}{2p^+(p^- - p_{\text{on}})} - \frac{i\gamma^+ p_{\text{on}}}{2(p^+)^2(p^- - p_{\text{on}})};
\]

And the term \( iS(p)\gamma^- S(p) \),

\[
i \left[ \frac{i(p_{\text{on}} + m)}{2p^+(p^- - p_{\text{on}})} \right] \gamma^- \left[ \frac{i(p_{\text{on}} + m)}{2p^+(p^- - p_{\text{on}})} \right] = -i\frac{(p_{\text{on}} + m)\gamma^- (p_{\text{on}} + m)}{4[p^+(p^- - p_{\text{on}})]^2}
\]

\[
= -i \left\{ \frac{2\gamma^+ (p_{\text{on}})^2 \gamma^- - 2p_{\text{on}}(\gamma_\perp p_\perp) + (p_\perp)^2 \gamma^- + 2mp_{\text{on}} + m^2 \gamma^-}{4[p^+(p^- - p_{\text{on}})]^2} \right\}
\]

\[
= -i \left\{ \frac{2p^- (p_{\text{on}} + m) + [-2\gamma^+ p_{\text{on}} - 2\gamma^- p^+ + 2(\gamma_\perp p_\perp) - 2m(p^- - p_{\text{on}})]}{4[p^+(p^- - p_{\text{on}})]^2} \right\}
\]

\[
= -i \left\{ \frac{-ip^- (p_{\text{on}} + m)}{2[p^+(p^- - p_{\text{on}})]^2} + \frac{i\gamma^-}{2p^+(p^- - p_{\text{on}})} + \frac{i\gamma^+ p_{\text{on}}}{2(p^+)^2(p^- - p_{\text{on}})} - \frac{[(\gamma_\perp p_\perp) - m]}{2(p^+)^2(p^- - p_{\text{on}})} \right\};
\]

(25)

and because of the presence of the last term and the wrong signal of the third, one has

\[
\frac{\partial S(p)}{\partial p^+} \neq iS(p)\gamma^- S(p).
\]

For the transversal components, the derivative is the same as obtained before,
\[ \frac{\partial S(p)}{\partial p_1} = \frac{ip_1 (p + m)}{2 [p^+ (p^- - p_{on})]^2} - \frac{i \gamma^1}{2 p^+ (p^- - p_{on})}. \]  

(26)

And the term \( i S(p) \gamma^1 S(p) \),

\[
i \left[ \frac{i (\slashed{p}_{on} + m)}{2 p^+ (p^- - p_{on})} \right] \gamma^1 \left[ \frac{i (\slashed{p}_{on} + m)}{2 p^+ (p^- - p_{on})} \right] = -i \frac{(\slashed{p}_{on} + m) \gamma^- (\slashed{p}_{on} + m)}{4 [p^+ (p^- - p_{on})]^2}
\]

\[
= -i \left\{ \frac{-2p_1 [\gamma^+ p^- + \gamma^- p^+ - (\gamma^1 p_{\perp}) + m] + 2\gamma^+ p^- p_1 + \gamma^1 (p_1)^2 + 2\gamma^+ p^+ p_{on} - 2\gamma^1 p_1 p_{on} + \gamma^1 (p_2)^2 + m^2 \gamma^1}{4 [p^+ (p^- - p_{on})]^2} \right\}
\]

\[
= \frac{ip_1 (p + m)}{2 [p^+ (p^- - p_{on})]^2} - \frac{\gamma^+ p_1}{2 (p^+)^2 (p^- - p_{on})}.
\]

(27)

then, comparing (26) and (27), one has \( \frac{\partial S(p)}{\partial p_{1,2}} \neq i S(p) \gamma^{1,2} S(p) \).

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[1] J.C. Ward, Physical Review, **77**, (1950) 293-293
[2] J.C. Ward, Physical Review, **78**, (1950) 182-182
[3] Y. Takahashi, Nuovo Cimento, **6**, ser. 10, (1957) 370