Developing Non-Linear Relationship Among Factors Affecting the Rutting Susceptibility of Asphalt Mixtures Using Two Parameter Weibull Distribution

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Abstract. This paper illustrates a novel approach to determine the probabilistic rutting in asphalt mixtures based on Weibull distribution statistical approach. In this paper, thirty wearing course asphalt mixtures prepared using different binder grades and aggregate sources, are analysed. Cooper Wheel tracker Test (CWTT) and Asphalt pavement analyser (APA) have been used to ascertain the rutting potential of asphalt mixtures with varying binder grade and aggregate source. Mixtures are then ranked based on their rutting susceptibility. The obtained data is processed using two parametric Weibull approach. Scale and shape parameters are figured out by multiple regression technique. Test results are analysed using goodness of fit test. Results indicate that rut depth predicted from two parameter Weibull distribution technique correlates well with the actual rut depth with $R^2$ value of 0.88 for both cooper wheel tracker and asphalt pavement analyser test. It has been found that Weibull distribution bests fit the NHA-A, NHA-B and SP-B graded mixtures with NRL 60/70 at 50°C. A good correlation has been observed between Cooper Wheel tracker and Asphalt pavement analyser as translated by $R^2$ of 0.86. This research is important in a sense that several agencies have adopted rutting as a failure criteria for flexible pavements. By mathematically modelling the rutting response, predicting the rutting behaviour of mastics becomes simpler and more efficient in approach.

1. Introduction:
Rutting is defined as an accumulation of unrecoverable strain resulting from applied load which is visible as depressed channel in wheel paths of asphalt pavement. Major and minor changes accumulated during the design life of pavement that eventually depreciates the pavement performance[1]. There are number of factors that affect the functional as well as structural rutting performance of asphalt mixtures. Deceleration and repeated loads from heavy traffic can create pronounce ruts in wheel path of asphalt concrete pavement [2]. Asphalt mixture properties i.e. binder type and aggregate gradations in addition to compaction efforts significantly affect the rutting potential[3]. Sabahat et. al.[4] investigated the factors effecting permanent deformation of asphalt mixtures using Asphalt pavement analyzer (APA) and concluded that temperature variable most significantly affects the rut depth[5]. Timely prediction of pavement response is critical to improve the design of new and existing pavements. Moreover, laboratory rutting tests show scatter results even by adopting high quality control and testing practices[6]. Therefore, it is required to move towards mathematical based pavement performance models that will reduce the testing effort and predict permanent deformation with more accuracy. As this distress has been associated with majority of flexible pavements, so there is a need to develop simple mechanistic empirical techniques in-parallel to ascertain the rut potential of asphalt mixtures and to reduce the time and efforts required in performing such tests in laboratory. A handful of studies are available that present different techniques of simulating laboratory rutting to actual in-field rutting. The models were developed to timely predict the pavement performance.

Haydari[7] modeled the rutting of asphalt pavements obtained from uniaxial creep test using Artificial Neural Network (ANN) and Adaptive Neuro-Fuzzy (ANFIS) techniques. XU and
Muhammad[8] developed a Mechanistic Empirical (ME) approach to model asphalt pavement rutting where the rutting is empirically correlated to the cyclic deflection which is evaluated using finite element modelling. Monismith[9] used a regression-based approach to describe the rutting behavior of asphalt mixtures on Wes track test sections. Gao et al.[10] employed the solution of Burger’s model to predict rutting and compared the final model with Finite Element Method (FEM), which validated the model pretty well. Collop et al.[11] studied the rutting behavior based on its viscoelastic behavior using VESYS software. Wong and Suo[12] established a nonlinear regression model by performing dynamic cyclic compression test. The development process and models of repeated load axial permanent deformation test was presented by Boateng and Maina[13] in the form of non-linear regression equation using a standard South African hot-mix asphalt mixture. Zhou et al.[14] proposed a three-stage model to accurately characterize three stages of permanent deformation. Gandomi,[15] concluded that flow number is an explanatory index to predict rutting potential of asphalt mixtures by utilizing a gene expression programming (GEP).

Of many techniques adopted, Weibull distribution is a widely used statistical technique in reliability and life data analysis due to its versatility and simplicity. The family of Weibull distribution was introduced in 1939 by a Swedish physicist Waloddi Weibull. It is one of the best known life time distributions which can adequately model various types of observed failures[16]. Two parameter Weibull Distribution requires shape parameter (β) and scale parameter (η) for reliability analysis. In order to evaluate whether data could be modelled using Weibull distribution, the best way is to use statistical software or by manually plotting data on Weibull probability plot. A straight line on Weibull probability plot is an indication that data follows Weibull distribution. There are limited number of studies conducted to date to model the laboratory rutting performance using Weibull Distribution technique as shown in Table 1.

| References | Year | Materials Used | Experimental Technique | Results |
|------------|------|----------------|------------------------|---------|
| Bala and Napiah[17] | 2018 | Modified asphalt mixture with polypropylene and polyethylene in addition to non-silica particles | Wessex Wheel Tracking Device | A good correlation was found between measured and laboratory rut depth. |
| Singh and Krishna[6] | 2018 | Asphalt mixture | Flat Rubber Wheeled Loaded Wheel Tester | Weibull distribution resulted in conservative rut results while lognormal presented overestimated rut curves |
| A.S.M Asif ur Rehman, Matias M. Mendez Larraín and Rafiqul A. Tarefde[18] | 2017 | Hot Mix Asphalt (HMA) | Hamburg Wheel tracking Device | The developed regression based predictive model using Weibull Distribution is found to be a good predictor of in-situ rutting. |
| Matias M. Mendez Larraín and Rafiqul A. Tarefde[19] | 2016 | Warm Mix Asphalt (WMA) | Hamburg Wheel Tracking Device | Weibull Function is a good estimator of lower rutting depths and where stripping phase has not been analyzed. |
In past literature, asphalt mixtures were prepared using single batch of binder grade and aggregate samples. These mixtures were then used to test the rutting performance of asphalt mixtures. Results obtained from single wheel tracking device were then simulated using Weibull distribution technique. In this study, extensive data has been collected to model the rut behavior. Three binder grades and aggregate samples were used to prepare asphalt mixture using Marshal Mix design. Performance testing was then conducted on Cooper Wheel Tracking Device and Asphalt Pavement Analyzer (APA). This wide results in terms of mixture variability and testing techniques were then simulated with modelled Weibull Distribution technique. One more modification in this research study is the correlation of rut results obtained from CWT and APA using non-linear regression equation. The main objective of this research study is to develop a probabilistic approach to predict the true rutting curve that will provide enough flexibility to finalize the pavement design based on type of materials being used and testing conditions being adopted at the field.

The contributions of this research study are threefold:
1. To investigate the rutting behaviour of asphalt mixtures composed of variety of bituminous and aggregates mixtures.
2. To develop relationship among various parameters effecting the rutting of asphalt mixtures based on CWT and APA using two parameter Weibull Distribution.
3. Comparing CWT and APA test techniques with two parametric Weibull distribution to verify the simulation results.

2. Methodological Approach:
A total of thirty wearing coarse mixtures with varying aggregate and bitumen sources have been considered for this research study. Each mixture is unique in its own characteristics based on type of bitumen and aggregate used. Figure 1 gives a summary of methodological approach adopted for modelling the rutting behavior of asphalt concrete mixtures.

| Author          | Year | Mixtures            | Test Methodology                                      | Results                                                                 |
|-----------------|------|---------------------|-------------------------------------------------------|----------------------------------------------------------------------|
| Coleri et al    | 2008 | Asphalt Concrete (AC) mixtures | Repeated simple shear test at repeated height (RSST-CH). | Integrated Weibull approach is a good simulator of stage I and II of in-situ rutting of asphalt mixtures. |
For each asphalt mixture, properties of individual materials were evaluated first and then a full material characterization was performed.

3. Materials Used:

3.1 Aggregates:
The properties of aggregates play an important role in overall performance of asphalt mixtures as 90% of asphalt concrete mixture consists of aggregates. For this research study, aggregates selected from Uban shah quarry of Sindh province, Sargodha quarry and Margalla quarry located in Punjab Province were analyzed. Flakiness and Elongation index test, sand equivalent test, water absorption test, soundness test and los Angeles abrasion value tests were performed to assess the performance of aggregates in HMA mixture. Aggregates queried from Uban shah were found to have flakiness index of 12% and Elongation index of 17% which is more than the specified limit of 10% according to BS 812.108. While Sargodha and Margalla aggregates had less proportions of flaky and elongated particles. Uban Shah, Margalla and Sargodha aggregates were qualitatively tested by performing sand equivalent test (ASTM D2419), Los Angeles abrasion value test (ASTM C131), Water absorption (ASTM C127), soundness test (ASMT C88) and Uncompacted void test (ASTM C1252). It has been found that all aggregate samples fulfil the standard specified limit prescribed in their respective ASTM standards. These tests translated the quality performance of different aggregate samples used and NHA-A gradation aggregates were found to be more tough and stronger as compared to other ones. While aggregates queried from Uban Shah were found to be more prone to loading and other physical and environmental condition due to presence of large number of flat and elongated particles.

3.2 Bitumen:
The bitumen sources investigated in this research study i.e. NRL 60/70, NRL 40/50, ARL 60/70 were manufactured from National Refinery Limited, Karachi and Attock Refinery Limited. Softening point test (ASTM D36), Penetration test (ASTM D5), Ductility test (ASTM C88) and Flash and fire point test (ASTM C142) were carried out to determine the viscosity, penetration index and safe temperature range for mixing asphalt. NRL 40/50 was found to have softening point of 49°C, followed by ARL 60/70 having 48°C and NRL 60/70 with 46°C softening point. Penetration value of NRL 60/70 is found to be 70mm, followed by ARL 60/70 with 63mm and NRL 40/50 with 49mm. Highest penetration values along with lowest softening point value of NRL 60/70 indicates increased viscosity as compared to NRL 40/50 and ARL 60/70.

4. Asphalt Mixture Design:
Marshall Mix design has been adopted for preparation of asphalt concrete specimens according to standard ASTM D 1559 specifications. A total of fifteen samples including three trials of asphalt-aggregate binder blends with five samples for each blend were prepared for each mixture. Results of Marshall mix design are tabulated in Table 2.

| Aggregates | Gradation | OBC | Stability |
|------------|-----------|-----|-----------|
| NHA-A      | 3.93      | 1190|           |
| NHA-B      | 4.65      | 1198|           |
| SP-A       | 5.40      | 1160|           |
| SP-B       | 4.55      | 1182|           |
| MS-II      | 4.70      | 1375|           |
| NHA-A      | 3.78      | 1189|           |
| NHA-B      | 4.60      | 1006|           |
| SP-A       | 5.50      | 1176|           |
| SP-B       | 4.44      | 1073|           |
| MS-II      | 4.81      | 1255|           |
Marshall Mixture design summary has been tabulated in Table 1 which indicates the results for 30 wearing coarse mixtures using NRL 40/50, NRL 60/70 and ARL 60/70 bitumen samples and Uban Shah, Margalla and Sargodha aggregates.

It can be inferred from Table 1 that asphalt mixtures prepared from Uban Shah aggregates have low stability values as compared to asphalt mixtures prepared from other aggregates. As Margalla aggregates and NRL 40/50 binder, when qualitatively tested, performed better as compared to other samples. This is the reason that asphalt mixtures prepared with Margalla aggregates and NHA 40/50 binder showed highest stability values as referred to Table 1.

### Preparation of test specimens:
Research study has been conducted in two phases. In phase I of study, Asphalt pavement analyzer has been used to test the rutting of asphalt mixtures that are prepared from super pave gyratory compactor. APA is a laboratory method for an accelerated evaluation of permanent deformation. Rutting of mixes is assessed by placing a cylindrical sample under repetitive loading of 534N. APA features an automated data acquisition system that displays rut measurements in graphical format According to the standard protocol. Based on past literature, 5mm rut depth or 8000 loading cycles, whichever comes first, is considered as pass-fail criteria. A study was conducted at NCAT by Kandhal et al.[21] and in Florida by Choubane et al.[22] to evaluate the rutting potential of asphalt mixes using APA. Results indicated that APA has a fair potential of simulating the in-situ rutting of pavement.

Phase II of research considered the use of Cooper Wheel tracker (CWT) device to predict permanent deformation of samples compacted on roller compactor. Rutting measured by rolling a small loaded wheel device of 720N repeated over the specimen is then co-related to actual in-service pavement rutting.

### 5. Weibull Distribution:
The Weibull distribution in case of fitting CWTT and APA rut depth presented as the function of number of load cycles can be represented by hazard/failure rate function. The failure rate function of Weibull distribution function is expressed as follows:

\[
R(N) = \frac{\beta}{\eta} \left( \frac{N - \eta}{\eta} \right)^{\beta - 1}
\]  

Equation 1
Where R(N) is the deformation rate in “mm” at ‘N’ number of cycles and loading conditions, β is the shape factor also known as slope parameter and η is the scale factor. For two-parameter Weibull distribution, location parameter “y” is not used, and this value can be set to zero. Other than shape and scale parameters, Rut depth is also dependent on the number of loading cycles which is one the most important factor in modelling rut prediction models\cite{23}. Successful fitting of Weibull distribution to rut data would mean that thorough understanding of rutting behaviour can be ascertained without doing prior laboratory destructive testing. Shape parameter shows the effect of how deformation rate increases or decreases in a function and represented by the shape parameter bath curve as shown in Figure 2.

Weibull function with β <1 has a distorting rate that decreases with time, also known as early failure life whereas function with β >1 has a deformation rate that increases with time. Weibull function with β =1 translates constant rate of deformation. A change in the scale parameter “η”, has an effect of stretching out the function.

6. **CWTT and APA rutting test results:**
Total thirty wearing coarse mixtures with varying bitumen and aggregate sources were prepared using roller compactor and tested on APA and Cooper Wheel tracker to assess the rutting propensity of these asphalt mixtures. Figure 3 shows output results of CWTT test while Figure 4 shows the APA rutting results at temperature conditions of 40°C and 50°C.

![Figure 2: Weibull Shape parameter Bath curve](image-url)
Figure 3: CWTT rut curves for different HMA mixtures at (a) 50°C (b) 40°C
Test results indicate that mixtures are more susceptible to rutting at temperature condition of 50°C compared to 40°C for both CWTT and APA mixtures as shown in Figure 3 (a),(b) and Figure 4 (a),(b). It is also been observed that SP-A NRL 60/70 mixtures are more susceptible to rutting as compared to other mixtures because of the presence of flaky and elongated aggregate particles. Also, NRL 60/70 has high penetration index as compared to bitumen grade 40/50. No sign of stripping has been observed for AC mixtures up to the limiting value of 10,000 loading cycles for CWTT and 8000 cycles for APA mixtures.

7. Weibull Probability Plots:
Weibull probability plot is a graphical technique employed to ascertain whether data set logically fits 2-parameter Weibull distribution at first place. Probability plots show a graph with observed cumulative percentage on abscissa and expected cumulative percentage on ordinate. Scatter plots close to reference line translates that data set follows the Weibull distribution. There are number of ways to plot Weibull probability plot. For this research study, Weibull-DR reliability software has been used to plot probability graphs for all asphalt mixtures. Figure 5 (a), (b) shows the sample Weibull probability plots for CWTT and APA.
Figure 5: Weibull Probability Plot for (a) CWTT and (b) APA

Weibull plots has special scale designed so that if data follows Weibull distribution, points will be linear as can be shown on Figure 5. The least square fits of this line yield the scale and shape parameter of Weibull distribution. Mathematically, the shape and scale parameters can be figured out from probability plot as shape parameter is the reciprocal of slope of fitted line and scale parameter is exponent of intercept of fitted line. Table 3 shows the estimated values of shape and scale parameters for all asphalt mixtures.

Table 3: Estimated Values of shape and scale parameters
8. CWTT and APA rut prediction model:

The Weibull distribution is one of the most commonly used distributions in reliability analysis because of its ability to take on various forms by adjusting its parameters. Previous researches have resulted that temperature has a very pronounced effect in rutting of asphalt mixes[3]. The relationship between shape factor “β”, temperature “T” and asphalt mixtures’ Stability “S” was determined by performing multiple regression with shape parameter being considered as dependent variables and temperature and stability is considered as independent variable.

Equation 2 shows the results of fitting a multiple linear regression model to describe the relationship between dependent variable and 2 independent variables. The equation of the fitted model is;

$$
\beta = 2.0759 - 0.3123 \times \log(S) + 0.0034 \times T \quad \text{……………………2}
$$

As above equation shows that β is the function of temperature and stability of asphalt mixes. Figure 6 (a) and (b) shows the effect of stability and temperature variations on the β for CWTT and APA respectively. Results have been graphically presented for both CWTT and APA. Stability has been plotted as an abscissa while β is plotted as an ordinate.
Stability is an indicator of collective properties of aggregates and bitumen in an asphalt mixture. As it can be seen from Figure 5 that $\beta$ found for the mixtures are greater than 1.0. This indicates that mixtures are experiencing normal/increasing wear out failure rate with time. Figure 6 shows the effect of temperature variation on $\beta$ for CWTT and APA data. The trend shows that temperature has a direct relation with $\beta$. As temperature increase from 40 to 50°C, value of $\beta$ gradually increases.

Figure 7 shows the effect of temperature variation on the shape parameter for CWTT and APA data. The trend shows that temperature has a direct relation with the shape parameter. As temperature increase from 40 to 50°C, value of shape parameter gradually increases.
This correlation depicts that increase in temperature will increase the value of $\beta$ which would then affect the rut depth, as equation 1 shows a direct relation between rut depth and $\beta$. Moreover, stability has an indirect relationship with $\beta$. So, increase in stability will in turn slows down the deformation rate in asphalt mixtures.

Based on the statistical approaches adopted in the past literature, it is considered that scale factor “$\eta$” is always dependent on the shape ($\beta$) and location factor ($y$). As location factor is taken as zero, $\eta$ is entirely dependent on $\beta$. Equation 3 shows the relationship obtained as the result of multiple regression technique. The output shows the results of fitting a multiple regression model to describe the relationship between scale ($\eta$) and shape ($\beta$) factor where $\eta$ is taken as dependent variable and $\beta$ as an independent variable. The equation of the fitted model is

$$\eta = -1.1 + 3.9877 \times \beta \quad \ldots \ldots \ldots \ldots \ldots 3$$

Equation 3 is graphically presented in Figure 7 to exhibit the relationship between dependent and independent variables. Test results of CWTT and APA data both has been plotted in the Figure 7 (a) and (b), respectively.

![Graphs showing the relationship between temperature and shape factor for APA and CWTT](image-url)
It is inferred from the Figure 7 that $\beta$ for CWTT is a bit more scattered than APA. Few values of CWTT are out of bound range. The reason might be that specimens tested by CWT test device were compacted using roller compactor where control of air voids is not possible. While APA specimens were compacted using super pave gyratory compactor where volumetric control is possible. Otherwise, a linear correlation has been observed between shape parameter and scale parameter in case of both CWTT and APA.

9. Model Validation:
In order to discuss the correlation among dependent and independent variables, goodness of fit of CWT and APA rut depth model exhibited in Equation 2 and 3 is evaluated at this stage. The output of results for shape and scale factor has been shown in Table 4 and 5 respectively. As shape parameter is considered to be dependent on stability of asphalt mixtures and temperature conditions and scale parameter is considered to be dependent on shape parameter, it is important to validate the statistical significance of these variables.
Table 4: Parameter Estimate of multiple regression for Shape Parameter for CWTT and APA

| Parameter | Estimate | Standard Error | t- statistics | P-Value |
|-----------|----------|----------------|---------------|---------|
| S         | -0.312   | 0.03           | 10.4          | 0.015   |
| T         | 0.003    | 0.0001         | 34            | 0.001   |
| Constant  | 2.075    | 0.32           | 6.5           | 0.000   |

Table 5: Parameter Estimate of multiple regression for Scale Parameter for CWTT and APA

| Parameter | Estimate | Standard Error | t- statistics | P-Value |
|-----------|----------|----------------|---------------|---------|
| Shape     | 3.98     | 0.08           | 49.75         | 0.000   |
| Constant  | -1.10    | 0.12           | 9.16          | 0.034   |

The Estimate column in Table 3 and 4 represents the coefficient of the respective variables. Standard error column shows the accuracy of calculations being made. A t-statistic is calculated by dividing the estimated value by its standard error which is then compared with t-critical value which is determined from selected confidence level and degree of freedom. Confidence level of 95% has been selected. The P value in last column tells us whether a variable is statistically significant or not. Since the P-value in table is less than 0.05, there is a statistically significant relationship between the variables at the 95.0% confidence level.

Now as the predicted models for shape and scale parameters have been verified, these values are used to find rut depth based on Weibull distribution by putting values of shape and scale parameter obtained from Equation 2 and 3 respectively, in Equation 1. Rut depth is the function of temperature and stability only as stability is the indicator of both aggregates and bitumen properties, so it is convenient way of analysing different aggregates and bitumen sources. Figure 9 shows the correlation obtained between actual rut depth value and rut depth for CWTT as predicted form Equation 1.
Figure 9: Actual VS Predicted Rut depth for CWTT (a) Uban Shah NRL 60/70 (b) Margalla NRL 40/50 (c) Sargodha NRL 40/50
(a) Weibull ▲ APA

β = 1.23
η = 4.2
RMSE = 0.16
Se/Sy = 0.1
R² = 0.97

(b) APA ▲ Weibull

β = 1.33
η = 4.5
RMSE = 1.3
Se/Sy = 0.15
R² = 0.95

(c) APA ▲ Weibull

β = 1.33
η = 3.4
RMSE = 0.09
Se/Sy = 0.2
R² = 0.92
A good correlation has been observed between actual and predicted values as translated from Figure 9 and 10. It has been observed that same Weibull distribution model predicts both APA and CWTT data with precision. A correlation has been developed between CWTT and APA rut depth as shown in Figure 11. Quadratic equation correlates both values well with R2 value of 0.86 for 50 °C temperature and 0.84 for 40 °C temperature.

It can be seen from the above figures that rut values are higher at 50 °C temperature as compared to 40 °C for both CWTT and APA. It is also observed that rut values are comparatively lower for APA. This is because of the fact that loading frequency is less in case CWTT than APA, slower the frequency of load, greater is the permanent deformation of asphalt mix. The correlation between CWTT and APA recommends that instead of dealing with the costly and complicated rut simulation technique of APA, CWTT can be used as an alternative tool.
10. Conclusion:
In this research study, an effort has been made to develop a probabilistic model to ascertain the rutting resistance of asphalt concrete mixtures prepared from using varying gradations of aggregates and three distinct types of binder and aggregate samples. Results of CWT and APA has been modelled using 2 parameter Weibull distribution technique. Following conclusions are drawn based on the analysis and modelling of test results.

- It has been observed that rut depth variability is quite prominent in the case of Urban shah aggregates and 60/70 penetration grade bitumen because of the presence of flaky and elongated aggregates. While bitumen with penetration grade of 40/50 are more resistant towards rut phenomenon. None of the mixtures reached their failure state prior to the test conditions in both CWTT and APA.
- Weibull distribution technique can be used for better prediction of laboratory rutting. Asphalt mixture stability, temperature conditions and Number of loading cycles are dependent variables for modelling Weibull Rutting prediction. All samples have shown rut value up-to secondary phase. This is also one of the reasons that Weibull distribution modelled the rutting behaviour precisely.
- Model Validation shows that temperature condition plays a very significant role in the rutting of asphalt mixtures irrespective of the type of material being used. Smaller p value in case of temperature and stability factor shows that these variables have strong significance in affecting rutting potential of asphalt concrete mixtures.
- It is inferred that CWTT and APA rut data correlates well with R² value of 0.84 for 50°C and 0.84 for 40°C.

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