Is the physical vacuum a preferred frame?

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Abstract
It is generally assumed that the physical vacuum of particle physics should be characterized by an energy momentum tensor in such a way to preserve exact Lorentz invariance. On the other hand, if the ground state were characterized by its energy-momentum vector, with zero spatial momentum and a non-zero energy, the vacuum would represent a preferred frame. Since both theoretical approaches have their own good motivations, we propose an experimental test to decide between the two scenarios.

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1 Introduction
The possible existence of a preferred reference frame $\Sigma$ is an old and important issue that dates back to the origin of the theory of relativity and to the basic differences between Einstein’s special relativity and the Lorentzian point of view. No doubt, today, the former interpretation is widely accepted. However, in spite of the deep conceptual differences, it is not so obvious how to distinguish experimentally the two interpretations.

For a modern presentation of the Lorentzian approach one can, for instance, follow Bell [1]. Differently from the usual derivations within special relativity, one starts from physical modifications of matter (namely Larmor’s time dilation and Lorentz-Fitzgerald length contraction) to deduce the basic Lorentz transformation between $\Sigma$ and any moving frame.
S'. Due to the crucial underlying group property, two observers S' and S", individually connected to Σ by a Lorentz transformation, are then also mutually connected by a Lorentz transformation with relative velocity parameter fixed by the velocity composition rule. As a consequence, one deduces a substantial quantitative equivalence of the two formulations of relativity for most standard experimental tests.

Thus, one is naturally driven back to the old question: if there were a preferred frame Σ, could one observe the motion with respect to it? In Sect.2, after reviewing the general problem of vacuum condensation in present particle physics, we shall argue that this might indeed be possible. In fact, by accepting the idea that the physical vacuum might be defined by its energy-momentum vector, with zero spatial momentum and non-zero energy, one deduces that such a vacuum represent a preferred frame since any moving observer should feel an energy-momentum flow along the direction of motion.

After this first part, we shall consider in Sect.3 an alternative point of view where the vacuum is only characterized by a suitable expectation value of the energy-momentum tensor and the previous conclusion is not true.

Since the two theoretical approaches have their own good motivations, we shall compare phenomenologically the two scenarios. This other part will be discussed in Sects.4 and 5 where a possible experimental test will be proposed. Finally, Sect.6 will contain a summary and our conclusions.

2 Vacuum energy and Lorentz invariance

The phenomenon of vacuum condensation, that has changed substantially the old view of the vacuum in axiomatic quantum field theory [2], in the physically relevant case of the Standard Model of electroweak interactions can be summarized saying that “What we experience as empty space is nothing but the configuration of the Higgs field that has the lowest possible energy. If we move from field jargon to particle jargon, this means that empty space is actually filled with Higgs particles. They have Bose condensed” [3]. In the simplified case of a pure Φ⁴ theory, this condensation phenomenon can be explicitly checked by constructing [4] a variational approximation to the spontaneously broken phase as a coherent state built up with the creation and annihilation operators of an empty reference vacuum state |0⟩. Thus, it becomes natural to ask [5] if the macroscopic occupation of the same quantum state, i.e.
\( k = 0 \) in some reference frame \( \Sigma \), could represent the operative construction of a “quantum ether”. This would characterize the \textit{physically realized} form of relativity and could play the role of preferred frame in a modern Lorentzian approach.

Usually this possibility is not considered with the motivation, perhaps, that the average properties of the condensed phase are summarized into a single quantity that transforms as a world scalar under the Lorentz group (in the Standard Model, the vacuum expectation value \( \langle \Phi \rangle \) of the Higgs field). However, this does not imply that the physical vacuum state has to be Lorentz invariant. Namely, Lorentz transformation operators \( U', U'' \) might transform non trivially the basic vacuum state \( \Psi^{(0)} \rangle \) (appropriate to an observer at rest in \( \Sigma \)) into new vacuum states \( \Psi', \Psi'' \rangle \) (appropriate to moving observers \( S', S'' \)...) and still, for any Lorentz-invariant operator \( G \), one would find

\[
\langle G \rangle_{\Psi^{(0)}} = \langle G \rangle_{\Psi'} = \langle G \rangle_{\Psi''} = \ldots
\]  

(1)

As a matter of fact, this view of a non-Lorentz-invariant vacuum turns out to be unavoidable when combining the general idea of a non-zero vacuum energy with the algebra of the 10 generators \( P_\alpha, M_{\alpha,\beta} (\alpha, \beta=0, 1, 2, 3) \) of the Poincaré group. Here \( P_\alpha \) are the 4 generators of the space-time translations and \( M_{\alpha,\beta} = -M_{\beta,\alpha} \) are the 6 generators of the Lorentzian rotations with the following commutation relations

\[
[P_\alpha, P_\beta] = 0 \tag{2}
\]

\[
[M_{\alpha,\beta}, P_\gamma] = \eta_{\beta,\gamma} P_\alpha - \eta_{\alpha,\gamma} P_\beta \tag{3}
\]

\[
[M_{\alpha,\beta}, M_{\gamma,\delta}] = \eta_{\alpha,\gamma} M_{\beta,\delta} + \eta_{\beta,\delta} M_{\alpha,\gamma} - \eta_{\beta,\gamma} M_{\alpha,\delta} - \eta_{\alpha,\delta} M_{\beta,\gamma} \tag{4}
\]

where \( \eta_{\alpha,\beta} = \text{diag}(1,-1,-1,-1) \).

In the following we shall assume the existence of a suitable operatorial representation of the Poincaré algebra for the quantum theory (where \( \hat{M}_{32} = -i\hat{J}_1, \hat{M}_{01} = -i\hat{L}_1 \) and cyclic permutations). We shall also assume that the occurrence of vacuum condensation does not modify the structure of the basic commutation relations \([2]-[4]\). This hypothesis, that reflects the observed properties of the energy-momentum under Lorentz transformations and as such has a sound experimental basis, is also completely consistent with the general attitude toward the phenomenon of spontaneous symmetry breaking. This, while giving rise to a non-symmetric ground state, leaves unchanged the commutation relations among the generators of the underlying dynamical symmetry group.
Within this framework, one possible assumption behind a non-trivial vacuum is that the physical vacuum state $|\Psi^{(0)}\rangle$ maintains both zero momentum and zero angular momentum, i.e. (i,j=1,2,3)

$$\hat{P}_i |\Psi^{(0)}\rangle = M_{ij} |\Psi^{(0)}\rangle = 0$$

but, at the same time, is characterized by a non-vanishing energy

$$\hat{P}_0 |\Psi^{(0)}\rangle = E_0 |\Psi^{(0)}\rangle$$

This vacuum energy might have very different explanations. Here, we shall limit ourselves to explore the physical implications of its existence by just observing that, in interacting quantum field theories, there is no known way to ensure consistently the condition $E_0 = 0$ without imposing an unbroken supersymmetry (which is not phenomenologically acceptable).

To this end, let us consider the generator of a Lorentz-transformation along the 1-axis $\hat{M}_{01}$. For $E_0 \neq 0$, finite Lorentz transformations obtained by exponentiating $\hat{M}_{01}$ will produce new vacuum states $|\Psi'\rangle, |\Psi''\rangle$..that differ non trivially from $|\Psi^{(0)}\rangle$. They maintain zero eigenvalues of $\hat{P}_2$ and $\hat{P}_3$ (since these commute with $\hat{M}_{01}$) while the mean value of $\hat{P}_1$ and $\hat{P}_0$ can be computed by defining a boosted vacuum state $|\Psi'\rangle$ as

$$|\Psi'\rangle = e^{\lambda' \hat{M}_{01}} |\Psi^{(0)}\rangle$$

(recall that $\hat{M}_{01} = -i\hat{L}_i$ is an anti-hermitian operator) and using the relations

$$e^{-\lambda' \hat{M}_{01}} \hat{P}_1 e^{\lambda' \hat{M}_{01}} = \cosh \lambda' \hat{P}_1 + \sinh \lambda' \hat{P}_0$$

$$e^{-\lambda' \hat{M}_{01}} \hat{P}_0 e^{\lambda' \hat{M}_{01}} = \sinh \lambda' \hat{P}_1 + \cosh \lambda' \hat{P}_0$$

In this way, one finds

$$\langle P_1 \rangle_{\Psi'} = E_0 \sinh \lambda'$$

$$\langle P_0 \rangle_{\Psi'} = E_0 \cosh \lambda'$$

so that a boost produces a vacuum energy-momentum flow along the direction of motion with respect to $\Sigma$. Therefore, in the spirit of both classical and quantum field theory, where global quantities are obtained by integrating local densities over 3-space, for a moving observer $S'$ the physical vacuum looks like some kind of ethereal medium for which, in general, one can introduce a momentum density $W_{0i}$ through the relation (i=1,2,3)

$$\langle P_i \rangle_{\Psi'} = \int d^3x W_{0i} \neq 0$$
3 The energy-momentum tensor of the vacuum

There is however another approach to the problem of the vacuum that leads to completely different conclusions. According to refs. [8, 9], the physical vacuum state $|\Psi(0)\rangle$ should not be considered an eigenstate of the energy-momentum operator but should rather be characterized by the expectation value of the local energy-momentum tensor $\hat{W}_{\mu \nu}$. Since the only Lorentz-invariant tensor is $\eta_{\mu \nu}$, this should have the form

$$\langle \hat{W}_{\mu \nu} \rangle_{\Psi(0)} = \rho \eta_{\mu \nu}$$

$\rho$ being a space-time independent constant. In this case, by introducing the Lorentz transformation matrices $\Lambda^\sigma_{\mu}$ to any moving frame $S'$, defining $\langle \hat{W}_{\mu \nu} \rangle_{\Psi'}$ through the relation

$$\langle \hat{W}_{\mu \nu} \rangle_{\Psi'} = \Lambda^\sigma_{\mu} \Lambda^\rho_{\nu} \langle \hat{W}_{\sigma \rho} \rangle_{\Psi(0)}$$

and using eq. (13), it follows that the expectation value of $\hat{W}_{0i}$ in any boosted vacuum state $|\Psi'\rangle$ vanishes, just as it vanishes in $|\Psi(0)\rangle$. Therefore, differently from eq. (12), one gets

$$\langle \hat{P}_i \rangle_{\Psi'} = \int d^3x \langle \hat{W}_{0i} \rangle_{\Psi'} = 0$$

To resolve the conflict, the author of ref. [8] advocates the point of view that the vacuum energy $E_0$ is likely infinite and represents a spurious concept. Thus one should definitely replace eqs. (5)-(6) with eq. (13) (“the question is not whether the vacuum has an energy-momentum vector but whether the vacuum has an energy-momentum tensor”).

The issue is non trivial and does not possess a simple solution. We can only observe that, by accepting this point of view, one might be faced with some consistency problems. For instance, in a second-quantized formalism, single-particle energies $E_1(p)$ are defined as the energies of the corresponding one-particle states $|p\rangle$ minus the energy of the zero-particle, vacuum state. If $E_0$ is considered a spurious concept, also $E_1(p)$ will become an ill-defined quantity.

At a deeper level, one should also realize that in an approach based only on eq. (13) the properties of $|\Psi(0)\rangle$ under a Lorentz transformation are not well defined. In fact, a transformed vacuum state $|\Psi'\rangle$ is obtained, for instance, by acting on $|\Psi(0)\rangle$ with the boost generator $\hat{M}_{01}$ as in eq. (11). Once $|\Psi(0)\rangle$ is considered an eigenstate of the energy-momentum operator as in Sect.2, one can definitely show that, for $E_0 \neq 0$, $|\Psi'\rangle$ and $|\Psi(0)\rangle$ differ non-trivially. On
the other hand, if \( E_0 = 0 \) there are only two alternatives: either \( \hat{M}_{01}|\Psi^{(0)}\rangle = 0 \), so that \( |\Psi'\rangle = |\Psi^{(0)}\rangle \), or \( \hat{M}_{01}|\Psi^{(0)}\rangle \) is a state vector proportional to \( |\Psi^{(0)}\rangle \), so that \( |\Psi'\rangle \) and \( |\Psi^{(0)}\rangle \) differ by a phase factor.

Therefore, if the structure in eq.(13) were really equivalent to the exact Lorentz invariance of the vacuum, it should be possible to show similar results, for instance that such a \( |\Psi^{(0)}\rangle \) state can remain invariant under a boost, i.e. be an eigenstate of

\[
\hat{M}_{0i} = -i \int d^3x \ (x_i \hat{W}_{00} - x_0 \hat{W}_{0i})
\]

(16)

with zero eigenvalue. As far as we can see, there is no way to obtain such a result by just starting from eq.(13) (that only amounts to the weaker condition \( \langle \hat{M}_{0i} \rangle_{\Psi^{(0)}} = 0 \)). Thus, independently of the finiteness of \( E_0 \), it should not come as a surprise that one can run into contradictory statements once \( |\Psi^{(0)}\rangle \) is instead characterized by means of eqs.(5)-(6).

For these reasons, it is not so obvious that the local relations (13) represent a more fundamental approach to the vacuum, as compared to our previous analysis in Sect.2. Rather, in our opinion, both approaches have their own good motivations and, to decide between eqs.(12) and (15), one should try to work out the possible observable consequences. In this way, the non-trivial interplay between local and global quantities in quantum field theory will be checked experimentally. If the analysis of Sect.2 is correct, the physically realized form of relativity contains a preferred frame and one might be able to detect the predicted non-zero density flow of energy-momentum in a moving frame.

### 4 The vacuum as a medium

As anticipated, by assuming eq.(12), for a moving observer the physical vacuum looks like an ethereal medium with a non-zero momentum density along the direction of motion. To estimate the possible observable consequences, we shall adopt Eckart’s thermodynamical treatment [10] of relativistic media where the relevant quantities are the energy-momentum tensor \( W^{\alpha \beta} \) and the 4-velocity vector \( u^\mu \) of the medium.

In this context, it is natural to start from a 4-velocity of the vacuum medium \( u^\mu(\Sigma) \equiv (1,0,0,0) \) for an observer at rest in \( \Sigma \). It is less obvious, however, to deduce its value for a moving frame \( S' \) since the simplest choice of defining \( u^\mu(S') = \Lambda^\mu_\nu \ u^\nu(\Sigma) \) as for an ordinary medium, in terms of the Lorentz transformation matrix \( \Lambda^\mu_\nu \) that connects \( S' \) to \( \Sigma \),
can hardly be accepted. In fact, if this were the correct transformation law, the motion with respect to $\Sigma$ could be detected on a pure kinematical basis regardless of the value of the vacuum energy $E_0$ that, in the quantum theory, represents the only relevant quantity that can possibly determine a non-Lorentz invariant vacuum state. For this reason, using this quantum input in the classical analysis, one deduces that the motion with respect to $\Sigma$ cannot induce any kinematical change in the description of the ethereal medium itself as if it were seen simultaneously at rest in all frames. This means to fix

$$u^\mu(S') \equiv (1, 0, 0, 0)$$

(whatever the $S'$ frame) so that the $u^\mu$ of the vacuum does not transform as the 4-velocity of ordinary media and eq.(17) should be interpreted as an external constraint on the structure of the vacuum[11]. In this sense, for any moving observer $S'$, the vacuum medium appears at rest according to Eckart’s definition $u^\mu(S') \equiv (1, 0, 0, 0)$ but not according to Landau’s definition since $W^{0i} \neq 0$[12]. The two criteria coincide only for the observer at rest in $\Sigma$.

With such representation of the vacuum medium, by introducing the heat flow 4-vector $q^\alpha \equiv - s_\beta^\alpha W^{\gamma\gamma} u_\gamma$, where $s_\beta^\alpha = \delta_\beta^\alpha + u^\alpha u_\beta$, and using eq.(17), one finds $q^i = W^{0i}$. Therefore, from the general relation between $q^\alpha$ and the temperature $T$ [10]

$$q^\alpha = -\kappa s_\beta^\alpha \left( \frac{\partial T}{\partial x^\beta} + Tu^\gamma \frac{\partial u_\beta}{\partial x^\gamma} \right)$$

($\kappa$ being the thermal conductivity of the medium), by using again eq.(17), one can define an effective temperature gradient through the relation

$$\frac{\partial T}{\partial x^i} \equiv -\frac{W^{0i}}{\kappa_0}$$

Here $\kappa_0$ is an unknown parameter, introduced for dimensional reasons, that plays the role of vacuum thermal conductivity. Since its value is unknown, the effective thermal gradient is left as an entirely free quantity whose magnitude can be constrained by experiments.

Formally, eq.(19) is the same type of relation that one finds in ref.[10]. Notice, however, the basic conceptual difference. There, one starts from a real, external temperature gradient $\frac{\partial T}{\partial x^i}$ to determine the heat flow $q^i = W^{0i}$ in an ordinary medium. Here, we are starting from the vacuum momentum density $W^{0i} = q^i$ in eq.(12) to define an effective $\frac{\partial T}{\partial x^i}$. This gradient emerges therefore as a consequence of the motion with respect to $\Sigma$ and could induce different effects on moving bodies. For instance, for a small temperature gradient, one expects a pure
thermal conduction in a strongly bound system, as a solid or a liquid, and the possibility of convective currents in a loosely bound system as a gas.

A possible objection to the previous picture is that the vacuum should not be represented as an ordinary medium but rather as a superfluid, see e.g. ref. [13]. As this would carry no entropy, moving bodies should feel no friction and thus there could be no vacuum momentum flow and no thermal gradient. From this perspective, the only possible condition consistent with a perfect superfluid behaviour would be to fix $E_0 = 0$. On the other hand, it is also known that in superfluid $^4$He, for any $T \neq 0$, in addition to the pure superfluid component, there must be a small fraction of ‘normal’ fluid to explain the tiny residual friction measured in the experiments. For this reason, a non-zero vacuum energy $E_0$, that in a moving frame gives rise to the momentum density $W^{\text{eq}}$ eq. (12) and to the effective thermal gradient eq. (19), is equivalent to assume a small non-superfluid component of the vacuum.

5 The experimental test

The effective thermal gradient eq. (19), if capable to generate convective currents in a loosely bound system as a gas, could in principle be detected by a slight anisotropy of the speed of light. This can be understood, in very simple terms, by introducing the refractive index $N$ of the gas and the time $t$ spent by light to cover some given distance $L$. By assuming isotropy, one finds $t = NL/c$. This can be expressed as the sum of $t_0 = L/c$ and $t_1 = (N - 1)L/c$ where $t_0$ is the same time as in the vacuum and $t_1$ represents the average time of which light is “slowed” by the presence of matter. If there are convective currents in the gas, so that $t_1$ is different in different directions, one expects an anisotropy of the speed of light proportional to $(N - 1)$.

For this reason, one should try to design an experiment where two orthogonal optical cavities are filled with a gas and study the frequency shift $\Delta \nu$ between the two resonators that gives a measure of the anisotropy of the two-way speed of light $\bar{c}(\theta)$. As anticipated, the presence of a small temperature gradient should give rise to two basically different effects:

a) a pure thermal conduction in the solid parts of the apparatus. This can affect differently the cavity length (and thus the cavity frequency) upon active rotations of the apparatus or under the Earth’s rotation and can be preliminarily evaluated and subtracted out by running the experiment in the vacuum mode, i.e. when no gas is present inside
the cavities. The precise experimental limits from ref. [14] show that any such effect can be reduced to the level $10^{-15} - 10^{-16}$.

b) small convective currents of the gas inside the cavities that can induce a slight anisotropy $\Delta \tilde{c}_\theta \equiv \tilde{c}(\pi/2) - \tilde{c}(0)$ in the two-way speed of light. On the base of the simple argument given above, the characteristic signature would be to measure a light anisotropy that, in two gaseous media of refractive index $N$ and $N'$, scales as

$$\frac{\Delta \tilde{c}_\theta(N)}{\Delta \tilde{c}_\theta(N')} \sim \frac{N - 1}{N' - 1}$$  \hspace{1cm} (20)

On the other hand, for strongly bound systems, such as when solid or liquid transparent media are filling the optical cavities, a small temperature gradient should mainly induce a heat conduction with no appreciable particle flow and thus with no light anisotropy in the rest frame of the apparatus, consistently with the classical experiments in glass and water.

This interpretation would be in agreement with the pattern observed in the classical and modern ether-drift experiments, as illustrated in refs. [15] [16]. This suggests (for gaseous media only) a relation of the type

$$\frac{\Delta \tilde{c}_\theta(N)}{c} \sim 3(N - 1) \frac{V^2}{c^2}$$  \hspace{1cm} (21)

where $V$ is the Earth’s velocity with respect to some preferred frame (projected in the plane of the various interferometers). In fact, in the classical experiments performed in air at atmospheric pressure, where $N \sim 1.000293$, the observed anisotropy was $\frac{\Delta \tilde{c}_\theta}{c} \lesssim 10^{-9}$ thus providing a typical value $V/c \sim 10^{-3}$, as that associated with most cosmic motions. Analogously, in the classical experiments performed in helium at atmospheric pressure, where $N \sim 1.000035$ (and in a modern experiment with He-Ne lasers where $N \sim 1.00004$), the observed effect was $\frac{\Delta \tilde{c}_\theta}{c} \lesssim 10^{-10}$ so that again $V/c \sim 10^{-3}$. This means that, if there were a preferred frame, by filling the optical resonators with gaseous media, the magnitude of the signal might increase by 5–6 orders of magnitude with respect to the limit $10^{-15} - 10^{-16}$ placed by the present ether-drift experiments in vacuum, namely from a typical $\Delta \nu \lesssim 1$ Hz up to a $\Delta \nu \sim 100$ kHz.

6 Summary and outlook

Summarizing: in this paper we have considered two basically different views of the vacuum. In a former approach, motivated by the observation that (with the exception of an unbroken
supersymmetry) there is no known way to produce consistently a vanishing vacuum energy, by using the Poincaré algebra of the boost and energy-momentum operators one deduces that the physical vacuum cannot be a Lorentz-invariant state and that, in any moving frame, there should be a vacuum energy-momentum flow along the direction of motion.

On the other hand, in an alternative picture where the vacuum is only characterized by a suitable form of the expectation value of the energy-momentum tensor (and the vacuum energy is considered a spurious concept), one is driven to completely different conclusions.

Since it is not so simple to decide between the two scenarios on a pure theoretical ground, we have tried to work out the possible phenomenological difference of the two approaches. To this end, we have argued that the non-zero density flow of energy-momentum, expected in a moving frame, should behave as an effective thermal gradient. This might induce small convective currents in a loosely bound system as a gas and produce a slight anisotropy in the speed of light proportional to $N - 1$, $N$ being the refractive index of the gas. This picture is consistent with the phenomenological pattern observed in the classical ether-drift experiments, the only ones performed so far in gaseous systems (air or helium at atmospheric pressure).

For this reason, we look forward to future, precise experimental tests where optical cavities can be filled with gaseous media (nitrogen, carbon dioxide, helium,...). In this way, one will be able to study the beat note of the two resonators, look for modulations of the signal that might be synchronous with the Earth’s rotation and check the trend in eq. (20). If a consistent non-zero signal will be found, besides providing evidence for the existence of a preferred frame, one will also give an experimental answer to the non-trivial questions, concerning the interplay of global and local quantities, mentioned in Sect.3.

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