The Drell-Hearn-Gerasimov Sum-Rule in QCD

S.D. Bass

Institut für Theoretische Kernphysik, Universität Bonn, Nussallee 14–16, D-53115 Bonn, Germany

Abstract

Photoproduction spin sum-rules offer a new window on the spin structure of the nucleon that complements the information we can learn from polarised deep inelastic scattering experiments. We review the theory and present status of the Drell-Hearn-Gerasimov sum-rule in QCD, emphasising the possible relation between the present “discrepancy” in this sum-rule and the nucleon’s strangeness magnetic moment. In the case of an elementary electron or photon target (say at the NLC) the Drell-Hearn-Gerasimov sum-rule provides a test for physics beyond the minimal Standard Model.

1sbass@pythia.itkp.uni-bonn.de
1 Introduction

The Drell-Hearn-Gerasimov sum-rule \([1]\) for spin dependent photoproduction relates the difference of the two cross-sections for the absorption of a real photon with spin anti-parallel \(\sigma_A\) and parallel \(\sigma_P\) to the target spin to the square of the anomalous magnetic moment of the target, viz.

\[
(DHG) \equiv -\frac{4\pi^2 \alpha S \kappa^2}{m^2} = \int_{\nu_{th}}^{\infty} \frac{d\nu}{\nu} (\sigma_A - \sigma_P)(\nu)
\]

Here \(S\) and \(m\) are the spin and mass of the target; \(\kappa\) is the anomalous magnetic moment in units of \((eS/m)\) where \(e\) is the electric charge. The sum-rule follows from the very general principles of causality, unitarity, Lorentz and electromagnetic gauge invariance and one assumption: that we can use an unsubtracted dispersion relation for the spin dependent part \(f_2(\nu)\) of the forward Compton amplitude \(f(\nu)\). Modulo the no-subtraction hypothesis, the Drell-Hearn-Gerasimov sum-rule is valid for a target of arbitrary spin \(S\), whether elementary or composite \([2]\).

In the Standard Model the anomalous magnetic moment \(\kappa = \frac{1}{2}(g - 2)\) of an elementary target starts at \(O(\alpha)\). It follows that the left hand side of the sum-rule Eq.(1) starts at \(O(\alpha^3)\). The Drell-Hearn-Gerasimov integral vanishes in the classical tree approximation \(\sigma \sim O(\alpha^2)\) for the \(2 \to 2\) processes \(\gamma a \to bc\) where \(a\) is either a real lepton, quark, photon, gluon or elementary Higgs target, viz.

\[
\int_{\nu_{th}}^{\infty} \frac{d\nu}{\nu} \Delta \sigma_{\text{Born}} = 0.
\]

This result was discovered for a charged lepton target by Altarelli, Cabibbo and Maiani \([3]\), and generalised to the other targets by Brodsky and Schmidt \([4]\). Any deviation from the sum-rule Eq.(2) in processes such as \(\gamma e \to W \nu\) and \(\gamma \gamma \to WW\) at the Next Linear Collider (NLC) would be evidence for physics beyond the minimal Standard Model such as virtual corrections from SUSY or composite structure of the quarks or vector bosons \([1, 2, 3]\).

In QCD the Drell-Hearn-Gerasimov sum-rule provides an important constraint on the spin structure of the composite nucleon which compliments the Bjorken and Ellis-Jaffe \([5]\) sum-rules of high \(Q^2\) polarised deep inelastic scattering. These high \(Q^2\) sum-rules relate the first moment of the nucleon’s first spin structure function \(g_1\) to the scale invariant axial charges of the target nucleon \([7-13]\). Polarised deep inelastic scattering experiments at CERN and SLAC have verified the Bjorken sum-rule for the iso-vector part of \(g_1\) to within 8\% \([14, 15]\). They have also revealed a four standard deviations violation of OZI in the flavour singlet axial charge \(g_A^{0|\text{inv}}\) \([14, 17]\). This discovery has inspired many would-be theoretical explanations involving the axial anomaly \([18-23]\) and chiral soliton models \([24]\) – for recent reviews see \([12, 25]\).

A longstanding puzzle in (pre-QCD) spin physics is the “discrepancy” between the inclusive Drell-Hearn-Gerasimov sum-rule for a nucleon target and the “measured” exclusive one and two pion photoproduction contributions to the Drell-Hearn-Gerasimov sum-rule. I. Karliner \([26]\), following earlier work by Fox and Freedman \([27]\), carried out a multipole analysis of (unpolarised) single pion photoproduction data to obtain the contribution of this exclusive channel, which seems to be well
saturated below $\nu_{\text{LAB}} \sim 1.2\text{GeV}$, to the inclusive sum-rule. When she combined this analysis with an estimate of the two pion background, Karliner found very close agreement between these exclusive “measurements” and the isoscalar part of the fully inclusive sum-rule, but not the isovector part, which differed both in sign and by a factor of 2. The theoretical predictions for the isoscalar and isovector parts of the DHG integral $(\text{DHG})^{\text{inclusive}}_{I=0,1}$ and Karliner’s results $(\text{DHG})^{\pi}_{I=0,1}$ are:

$$(\text{DHG})^{\text{inclusive}}_{I=0} = -219\mu b, \quad (\text{DHG})^{\pi}_{I=0} = -222\mu b$$

$$(\text{DHG})^{\text{inclusive}}_{I=1} = +15\mu b, \quad (\text{DHG})^{\pi}_{I=1} = -39\mu b.$$ 

No error was quoted for the “experimentally determined” integrals $(\text{DHG})^{\pi}_{I=0,1}$, which were evaluated up to an ultraviolet cutoff $\nu_{\text{LAB}} = 1.2\text{GeV}$. Whilst one should not worry that an exclusive channel does not saturate an inclusive sum-rule, this result is very curious in that we normally expect the isovector contribution to an inclusive sum-rule to saturate faster with increasing energy $\nu$ than the isoscalar contribution. (Pomeron-like exchanges which govern the high-energy behaviour of $f(\nu)$ are isoscalar.)

The first direct test of the Drell-Hearn-Gerasimov sum-rule will be made in experiments which are planned or underway at the ELSA, GRAAL, LEGS and MAMI facilities. Whilst we wait for the results of these experiments it is worthwhile to review what is known and what is not known about the spin structure of the nucleon at photoproduction. This Brief Review is about the Drell-Hearn-Gerasimov sum-rule in QCD. We refer to Drechsel \[28\] for an excellent review of nuclear physics aspects.

The structure of this article is as follows. In Section 2 we review the derivation of photoproduction sum-rules with emphasis on the validity of the no-subtraction hypothesis. In Section 3 we discuss the consequences of the Drell-Hearn-Gerasimov sum-rule for our understanding of nucleon structure. We then give an overview of the present status of the Karliner “discrepancy” (Section 4) and possible resolutions of this “discrepancy” based on strange quark dynamics (Section 5) and vector meson dominance arguments (Section 6). Section 6 also reviews the $Q^2$ dependence of $(\sigma_A - \sigma_P)$ from photoproduction to polarised deep inelastic scattering. Finally, we summarise the physics that can be learnt in future polarised photoproduction experiments.

### 2 The Drell-Hearn-Gerasimov Sum-Rule

To understand the derivation of the Drell-Hearn-Gerasimov Sum-Rule we start with the forward Compton scattering amplitude

$$f(\nu) = \chi_f^* \left[ f_1(\nu)\vec{\epsilon}_a^*\vec{\epsilon}_i + if_2(\nu)\vec{\sigma}.(\vec{\epsilon}_a^*\vec{\epsilon}_i) \right] \chi_i.$$  

If $|f_2(\nu)| \to 0$ for complex $\nu \to \infty$, then causality allows us to write an unsubtracted dispersion relation for the spin dependent part of $f(\nu)$. Using the Optical Theorem (unitarity) to rewrite $\text{Im} f_2$ in terms of the photoproduction cross-sections and
crossing symmetry

$$f_2(\nu) = -f_2^*(-\nu)$$

we obtain

$$\text{Re} f_2(\nu) = \frac{\nu}{4\pi^2} \int_{\nu_{th}}^{\infty} \frac{d\nu'}{\nu'^2} \frac{\nu'}{\nu^2} (\sigma_A - \sigma_P)(\nu').$$

(6)

Photoproduction spin sum-rules are obtained by taking the odd derivatives (with respect to \(\nu\)) of \(\text{Re} f_2(\nu)\) in Eq.(6), viz.

$$\left(\frac{\partial}{\partial \nu}\right)_{n+1} \text{Re} f_2(\nu),$$

and then evaluating the resulting expression at \(\nu = 0\). For small photon energy \(\nu \to 0\), we use the low energy theorems [29]:

$$f_1(\nu) \to -\frac{\alpha}{m} + (\bar{\sigma}_N + \bar{\beta}_N)\nu^2 + O(\nu^4)$$

and

$$f_2(\nu) \to -\frac{\alpha \kappa N^2}{2m^2} \nu + \gamma N\nu^3 + O(\nu^5)$$

(8)

which follow from Lorentz and electromagnetic gauge invariance. Here \(\gamma_N, \bar{\sigma}_N\) and \(\bar{\beta}_N\) are the spin, and electric and magnetic Rayleigh polarisabilities of the nucleon respectively.

The Drell-Hearn-Gerasimov sum-rule is derived when we evaluate the first derivative \((n = 0)\) of \(\text{Re} f_2(\nu)\) in Eq.(6) at \(\nu = 0\), viz.

$$\int_{0}^{\infty} \frac{d\nu'}{\nu'} (\sigma_A - \sigma_P)(\nu') = -\frac{2\pi^2\alpha}{m^2} \kappa_N^2.$$ 

(9)

The third derivative of \(\text{Re} f_2(\nu)\) yields a second sum-rule for the spin polarisibility \(\gamma_N\)

$$\int_{0}^{\infty} \frac{d\nu'}{\nu'^3} (\sigma_A - \sigma_P)(\nu') = \frac{1}{4\pi^2} \gamma_N.$$ 

(10)

These photoproduction sum-rules have a parallel in high \(Q^2\) deep inelastic scattering where the odd moments of the spin dependent structure function \(g_1\) project out the nucleon matrix elements of gauge-invariant local operators, each of which is multiplied by a radiative Wilson coefficient.

The no-subtraction hypothesis is the only “QCD input” to the derivation of the sum-rule. How good is it ?

Discounting for the moment any fixed pole contribution, \(\text{Im} f_2\) and \(f_2\) have the same \(\nu \to \infty\) behaviour in the Regge analysis of the high-energy behaviour of spin dependent hadronic scattering processes [30]. Regge arguments tell us that the Drell-Hearn-Gerasimov integral converges.

It is well known [31, 32] that the isotriplet contribution to \((\sigma_A - \sigma_P)\) behaves as

$$\left(\sigma_A - \sigma_P\right)_3(\nu) \sim \nu^{\alpha_{a_1}-1}, \quad \nu \to \infty$$

(11)

where \(\alpha_{a_1}\) is the intercept of the \(a_1\) Regge trajectory \((-0.5 \leq \alpha_{a_1} \leq 0\) [33]). The isosinglet contribution is still somewhat controversial and depends on the Lorentz structure of the short range part of the exchange potential [34]. The soft pomeron
dominates the large energy ($\sqrt{s} > 10\text{GeV}$) behaviour of the spin averaged cross section ($\sigma_A + \sigma_P$) but does not contribute to ($\sigma_A - \sigma_P$). However, the physics which generates the soft pomeron can contribute to ($\sigma_A - \sigma_P$). If the pomeron were a scalar, then we would find

$$\left(\sigma_A - \sigma_P\right)_0^{[1]} \sim 0, \quad \nu \to \infty. \quad (12)$$

In the Landshoff-Nachtmann approach, the soft pomeron is modelled by the exchange of two non-perturbative gluons and transforms as a $C = +1$ vector potential with a correlation length of about 0.1fm. This two non-perturbative gluon exchange gives a contribution

$$\left(\sigma_A - \sigma_P\right)_0^{[2]} (\nu) \sim \frac{\ln \nu}{\nu}, \quad \nu \to \infty. \quad (13)$$

It has also been suggested that there may be a negative-signature, two-pomeron cut contribution to ($\sigma_A - \sigma_P$)

$$\left(\sigma_A - \sigma_P\right)_0^{[3]} (\nu) \sim \frac{1}{\ln^2 \nu}, \quad \nu \to \infty. \quad (14)$$

It is an important challenge for future high-energy photoproduction experiments to determine the strength of the three possible large $\nu$ contributions in Eqs.(12-14). One could then make contact with the small $x$ behaviour of the nucleon’s first spin structure function $g_1$ which is measured in polarised deep inelastic scattering. The soft Regge contributions form a baseline for investigations of DGLAP and BFKL small $x$ behaviour in $g_1$. Each of the possible contributions in Eqs.(11-14) give a convergent integral in Eqs.(6,9). If the short range exchange potential transformed as an axial-vector, then the Drell-Hearn-Gerasimov integral would not have converged: the sea would be generated all with the same polarisation and ($\sigma_A - \sigma_P$) would grow to reach the Froissart bound for the spin averaged cross section

$$\left(\sigma_A + \sigma_P\right) \sim \ln^2 \nu, \quad \nu \to \infty. \quad (15)$$

However, this is not QCD. (In QCD the $qqg$ vertex is a vector coupling – see eg. )

Given that the Drell-Hearn-Gerasimov integral converges, the only way that the sum-rule can fail is if there exists a $J = 1$ Regge fixed pole so that $\text{Im}f_2 \to 0$ does vanish and $\text{Re}f_2$ does not vanish at asymptotic photon energy $\nu \to \infty$. In this case the condition $|f_2(\nu)| \to 0$ as $\nu \to \infty$ would not be fulfilled and we would find a constant spin flip (subtraction constant) correction to the sum-rule. There is both theoretical and phenomenological evidence against such a fixed pole correction to (DHG), Eq.(1).

Fixed pole contributions to current-hadron scattering amplitudes like $f(\nu)$ are, in principle, allowed by current algebra. They are not allowed in hadron-hadron amplitudes. Consider a photon scattering from a (composite) target, the
structure of which is described by a field theory with coupling constant $g$. Where they exist, arguments in favour of fixed poles in $f_1$ yield contributions which start at $\mathcal{O}(1)$ in the transverse cross-section [13] and $\mathcal{O}(g^2)$ in the longitudinal cross-section [14]. Explicit calculation [3, 4, 45] shows that both sides of the Drell-Hearn-Gerasimov sum-rule vanish at $\mathcal{O}(\alpha_s \sim g^2)$ in perturbative QCD, in agreement with the sum-rule Eq.(2). There is no constant spin flip correction to the sum-rule at either $\mathcal{O}(1)$ or $\mathcal{O}(\alpha_s)$ in perturbative QCD. This result suggests that any fixed pole contribution is most likely a non-perturbative effect.

It was suggested in [46] that an isovector $J = 1$ fixed pole might resolve the “discrepancy” between $(\text{DHG})_{I=1}^{\text{inclusive}}$ and $(\text{DHG})_{I=1}^{\pi}$. No suggestion was offered why the isoscalar partner of such a would-be fixed pole should be suppressed. As emphasised by Heimann [32], an isovector $J = 1$ fixed pole would also induce a violation of Bjorken’s sum-rule in polarised deep inelastic scattering. The Bjorken sum-rule is presently verified to within 8% [14, 15]. Henceforth, we accept the Drell-Hearn-Gerasimov sum-rule as valid.

### 3 Properties of the DHG sum-rule

The forward Compton amplitude $f(\nu)$ in photon-nucleon scattering has charge parity $C = +1$. The cross-sections for spin dependent photoproduction (on the right hand side of the Drell-Hearn-Gerasimov sum-rule) receive contributions from OZI violating processes where the photon couples to the target nucleon via a $C = +1$, colour neutral, gluonic intermediate state in the $t$ channel, for example two gluon exchange.

The anomalous magnetic moment $\kappa_N$ which appears on the left hand side of the Drell-Hearn-Gerasimov sum-rule has charge parity $C = -1$. It is measured in the nucleon matrix element of the conserved vector current operator which is not renormalised. (It is renormalisation group invariant and does not mix with any gluonic operators under renormalisation.) Furry’s theorem tells us that $\kappa_N$ receives no contribution from processes where the photon couples to a quark loop which, in turn, couples to an even number of gluons. The axial anomaly [17, 18] does not contribute to $\kappa_N$. OZI violation can arise in $\kappa_N$ via the photon’s coupling to the $\phi(1020)$, which, in turn, can couple to the three quark “valence nucleon” via a $C = -1$, colour neutral, three gluon intermediate state.

The charge parity properties of $\kappa_N$ and $f_2$ on the left and right hand sides of the sum-rule Eq.(1) imply that processes which contribute to $C = +1$ observables but not to matrix elements of the conserved $C = -1$ vector current cancel in the logarithmic Drell-Hearn-Gerasimov integral for the difference of the two spin dependent photoabsorption cross-sections [49]. In the absence of a $J = 1$ fixed pole, the very general principles of causality, unitarity, Lorentz and electromagnetic gauge invariance imply that the contribution to the Drell-Hearn-Gerasimov integral from OZI violating exchanges which do not contribute to $\kappa_N$ is either zero or infinite (so that the integral would not converge and one would need a subtraction). In the infinite scenario the sea and glue would be generated all with one polarisation, which is not QCD.

In contrast to the Drell-Hearn-Gerasimov sum-rule, the Spin Polarisability su-
rule involves only $C = +1$ observables and is sensitive to all $C = +1$ OZI violating exchanges in the $t$ channel.

It is interesting to compare the physics of Drell-Hearn-Gerasimov with the spin averaged cross section. The negative sign of the Thomson term $-\frac{\alpha}{m}$ in the low energy theorem for $f_1(n)$, Eq.(7), means that we cannot write an unsubtracted dispersion relation for the spin averaged cross-section [50]. (Cross-sections are non-negative!) Furthermore, the Thomson term involves the square of the forward matrix element of the $C = -1$ vector current (the Dirac form-factor evaluated at zero momentum transfer), which measures the number of valence (current or constituent) quarks in the nucleon. Even without the sign, $C = +1$ OZI violating processes involving two gluon exchange (like Pomeron exchange) will contribute to the total spin-averaged cross-section so that any integral sum-rule for this cross-section must involve at least one $C = +1$ observable.

The Drell-Hearn-Gerasimov sum-rule also tells us that the contribution to the anomalous magnetic moment of a massive fermion due to internal structure vanishes in the limit $m_f r \to 0$, where $m_f$ is the mass and $r$ is the radius of the fermion [5]. Let $m^* \gg m_f$ be a scale that represents the internal bound-state structure of the fermion ($m^* \to \infty$ in the limit that $r \to 0$). In addition to pure QED contributions, the Drell-Hearn-Gerasimov integral will receive new resonance or continuum contributions beyond the threshold $s_{th} = m^*^2$ associated with the internal structure of the fermion. Such contributions yield

$$\left(\sigma_A - \sigma_P\right) \sim \alpha \frac{\pi}{m^*^2} f\left(\frac{m^*^2}{s^2}\right)$$

and a contribution to the anomalous magnetic moment

$$\delta\kappa^{(\text{non-QED})} \sim \frac{m_f}{m^*}$$

which vanishes in the zero radius limit. If we use the Drell-Hearn-Gerasimov sum-rule as a probe for physics beyond the minimal Standard Model in an NLC experiment, then $m^*$ represents the scale of “new physics”. In QCD Eq.(17) provides an important constraint on quark model expressions for the anomalous magnetic moment. In the MIT and Cloudy Bag Models the nucleon’s anomalous magnetic moment is proportional to the confinement radius $R$ [51] (see also [52]). On the other hand, the non-relativistic quark model expression

$$\kappa_N = \sum_q e_q \sigma_q \frac{m}{m_q} - 1,$$

is inconsistent with the Drell-Hearn-Gerasimov sum-rule because it contains no information about the radius of the nucleon. (In Eq.(18) $e_q$ is the quark charge, $m_q$ is the constituent quark mass and $m$ is the nucleon mass. The $C = -1$ quantity $\sigma_q$ denotes the fraction of the nucleon’s spin that is carried by its quarks minus the fraction that is carried by its antiquarks.)

\section{4 The Experimental Situation}

To understand Karliner’s observation of a “discrepancy” between the inclusive Drell-Hearn-Gerasimov sum-rule and the contribution to the sum-rule from exclusive one
and two pion photoproduction, it is helpful to write the anomalous magnetic moment $\kappa_N$ as the sum of its isovector $\kappa_V$ and isoscalar $\kappa_S$ parts, viz.

$$\kappa_N = \kappa_S + \tau_3 \kappa_V. \quad (19)$$

We can then write separate isospin sum-rules:

$$(\text{DHG})_{I=0} = (\text{DHG})_{VV} + (\text{DHG})_{SS} = -\frac{2\pi^2 \alpha}{m^2} (\kappa_V^2 + \kappa_S^2) \quad (20)$$

$$(\text{DHG})_{I=1} = (\text{DHG})_{VS} = -\frac{2\pi^2 \alpha}{m^2} 2\kappa_V \kappa_S.$$  

The physical values of the proton and nucleon anomalous magnetic moments $\kappa_p = 1.79$ and $\kappa_n = -1.91$ correspond to

$$\kappa_S = -0.06 \quad \kappa_V = +1.85 \quad (21)$$

Since $\left(\frac{\kappa_S}{\kappa_V}\right) \simeq -\frac{1}{30}$, it follows that $(\text{DHG})_{SS}$ is negligible compared to $(\text{DHG})_{VV}$. Indeed, it is safe to regard $(\text{DHG})_{SS}$ as negligible compared to any reasonable estimate of the present “experimental” error on $(\text{DHG})_{I=0}$. It follows that, first, the isoscalar sum-rule $(\text{DHG})_{I=0}$ measures the isovector anomalous magnetic moment $\kappa_V$. Given this isoscalar measurement, the isovector sum-rule $(\text{DHG})_{I=1}$ then measures the isoscalar anomalous magnetic moment $\kappa_S$. If we take the Drell-Hearn-Gerasimov sum-rule as an exact equation and just the Karliner data, then we can invert this equation to obtain

$$\kappa_S^{(\pi)} = +0.16 \quad \kappa_V^{(\pi)} = +1.86. \quad (22)$$

The “discrepancy” in the isovector sum-rule translates into a “discrepancy” in the isoscalar anomalous magnetic moment $\kappa_S$. (Note that $\left(\frac{\kappa_S}{\kappa_V}\right)^{(\pi)} \simeq -\frac{1}{12}$ so that $(\text{DHG})_{SS}^{(\pi)} \ll (\text{DHG})_{VV}^{(\pi)}$ with these values of $\kappa$.)

Karliner’s evaluation of $(\text{DHG})_{(I=0,1)}^{(\pi)}$, and hence $\kappa_{(V,S)}$, emerged as a delicate cancellation of individual multipole contributions, which we list in Table 1. (We refer to [26, 53] for the definitions of these resonances and multipoles.) A more recent multipole analysis has been performed by Workman and Arndt [54] using an updated data set for single pion photoproduction up to $\nu_{LAB} = 1.7\text{GeV}$. They obtained $(\text{DHG})_{I=1}^{(\pi,WA)} = -225\mu b$ and $(\text{DHG})_{I=0}^{(\pi,WA)} = -65\mu b$ [44] (see also [53]) by combining their new multipole analysis with Karliner’s estimate of the two pion background (up to $\nu_{LAB} = 1.2\text{GeV}$). The main difference between the two analyses is in the single pion photoproduction data between $400\text{MeV} < \nu_{LAB} < 600\text{MeV}$. If taken at face value, this new evaluation of the Drell-Hearn-Gerasimov integral would appear to increase the “discrepancy” in the sum-rule. However, it is important to bear in mind that different ultraviolet cut-offs were used to evaluate the one and two pion contributions to $(\text{DHG})_{(I=0,1)}^{(\pi,WA)}$. Furthermore, and most importantly, both the Karliner [26] and Workman and Arndt [54] determinations of $(\text{DHG})$ are indirect
Table 1: Multipole Structure of the DHG integral

| Resonance   | Multipole | Contribution to \((DHG)_{I=0}\) | Contribution to \((DHG)_{I=1}\) |
|-------------|-----------|----------------------------------|----------------------------------|
| \(P_{33, \frac{3}{2}^+}\) | \(M_{1+}(E_{1+})\) | \(-271 \mu b\) | \(+23 \mu b\) |
|             | \((\Delta(1232)\) only | \(-240 \mu b\) | \()\) |
| \(S_{11, \frac{1}{2}^-}\) | \(E_{0+}\) | \(+168 \mu b\) | \(-23 \mu b\) |
| \(P_{11, \frac{1}{2}^+}\) | \(M_{1-}\) | \(+10 \mu b\) | \(-3 \mu b\) |
| \(D_{13, \frac{1}{2}^-}\) | \(E_{2-}, M_{2-}\) | \(-67 \mu b\) | \(-13 \mu b\) |
| \(F_{15, \frac{3}{2}^+}\) | \(E_{3-}, M_{3-}\) | \(-10 \mu b\) | \(-8 \mu b\) |
| \(2\pi\) background | \()\) | \(-52 \mu b\) | \(-16 \mu b\) |

“measurements” extracted from unpolarised data. We eagerly await the results of the ELSA, GRAAL, LEGS and MAMI experiments, which will provide the first direct measurements of \(\sigma_A\) and \(\sigma_P\) up to \(\nu_{LAB} \approx 3\text{GeV}\).

To resolve Karliner’s “discrepancy” we are looking for non-resonant contributions below \(\nu_{LAB} = 1.2\text{GeV}\) as well as physics which contributes to \((\sigma_A - \sigma_P)\) at \(\nu_{LAB} > 1.2\text{GeV}\) which can shift \(\kappa_S\) by an amount \(\approx -0.22\) whilst keeping \(\kappa_V\) essentially constant. There is no elastic contribution to the DHG integral. In the Introduction we outlined how the DHG sum-rule can be used to look for physics beyond the minimal Standard Model in an NLC experiment. This argument\(^4\) can be extended to a nucleon target as follows. The size of an \textit{exclusive} channel contribution to the \textit{inclusive} Drell-Hearn-Gerasimov sum-rule measures the contribution of the physics involved in this channel to the anomalous magnetic moment with all other physics switched off.

Now assume that pion photoproduction can be described entirely by light quark dynamics. In this scenario, both \(\kappa_S^{(\pi)}\) and \(\kappa_V^{(\pi)}\) do not receive any contribution from the physics which generates the nucleon’s strangeness magnetic moment\(^2\). The physics of the strangeness magnetic moment (Section 5) is one possible candidate to fill the Karliner “discrepancy”.

\(^2\) For completeness, we note two places where dynamics involving the strange quark can contribute to pion photoproduction. First, in a full treatment, one should include virtual kaon corrections to the \(\gamma N \rightarrow N^*\) vertices. These corrections, which are in general much smaller than the corresponding virtual pion corrections, are absorbed into the electromagnetic form-factors for these transitions. Second, the prominent role of the \(S_{11}(1535)\) in eta photoproduction suggests that the wavefunction of this resonance contains a significant \(\Sigma K\) bound state component\(^5\). We assume here that the pure light-quark component of the \(S_{11}(1535)\) dominates its contribution to pion photoproduction.

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Table 2: Present status of the Spin Polarisability sum-rule

| $\gamma_N$ | Multipoles | $\chi_{\text{PT}}$ (1 loop without $\Delta$) | $\chi_{\text{PT}}$ (1 loop with $\Delta$) |
|------------|------------|------------------------------------------|------------------------------------------|
| $\gamma_p$ | -1.34$\mu$b | +2.16$\mu$b                              | -1.50$\mu$b                              |
| $\gamma_n$ | -0.38$\mu$b | +3.20$\mu$b                              | -0.46$\mu$b                              |

It is interesting to compare the present status of the Drell-Hearn-Gerasimov and Spin Polarisability sum-rules \[55\]. In Table 2 we show the values of the proton and neutron spin polarisabilities that are extracted from the pion photoproduction data in comparison with the predictions \[57\] of relativistic, one loop chiral perturbation theory, $\chi_{\text{PT}}$, with and without the $\Delta$ resonance included. There is good agreement between the “data” and this $\chi_{\text{PT}}$ calculation with the $\Delta$ included. This result is quite natural. First, the Spin Polarisability sum-rule is expected to saturate faster with increasing energy than the Drell-Hearn-Gerasimov sum-rule because of the $\frac{1}{m^3}$ denominator in Eq.(10). The $\Delta$ as the lowest lying nucleon resonance is therefore expected to play a more prominent role in the sum-rule for $\gamma_N$ than the sum-rule for the anomalous magnetic moment. Second, both the “experimental” numbers and theoretical predictions for $\gamma_N$ in Table 2 do not include dynamics associated with the strange quark.

5 Relation to the strangeness magnetic moment

The nucleon’s anomalous magnetic moment is measured in the $q^2 \to 0$ limit of the nucleon’s Pauli form-factor, viz. $\kappa_N = F_2(0)$ where

$$< N(p')|j_{\mu}^m(0)|N(p) > = \bar{\pi}(p')(\gamma_{\mu}F_1(Q^2) + \frac{i\sigma_{\mu\nu}q^\nu}{2m}F_2(Q^2))u(p)$$

and $q = (p - p')$. We can express $\kappa_p$ as the sum over up, down and strange quark contributions

$$\kappa_p = \frac{2}{3}F_2^u(0) - \frac{1}{3}F_2^d(0) - \frac{1}{3}F_2^s(0).$$

Motivated in part by the role that intrinsic strangeness plays in some theoretical explanations of the EMC spin effect and the value of the pion-nucleon sigma term, there is presently much interest in measuring the size of $\kappa_s$ with experiments planned at Bates, MAMI and TJNAF.

Theoretical predictions \[58-69\] of $F_2^s(0)$ are very model dependent with results that differ both in sign and order of magnitude. There are two main types of approach. The first has its starting point H"ohler et al.’s minimal three pole approximation \[70\] to the spectral function for the iso-scalar Pauli form-factor

$$F_2^{(i=0)}(q^2) = \sum_{i=1}^{3} \frac{A_im_i^2}{m_i^2 - q^2}.$$
Höhler et al. [70] found the second pole at $m_2^2 = 0.97\text{GeV}^2$ which they identified with the $\phi(1020)$. The $\phi(1020)$ couples to the nucleon with a large OZI violation and, in this approach, leads to a large negative strangeness magnetic moment $F_2^s(0) \approx -0.26$ [58]. This result has to be taken with care. First, Meissner et al. [59] have recently emphasised the role of a correlated $\pi\rho$ exchange which is modelled by a single effective $\omega'$ pole with mass $m_{\omega'} = 1.1\text{GeV}$ in the Bonn potential for the nucleon-nucleon interaction [72]. This effective $\omega'$ is an alternative candidate for the second pole $m_2$ in Eq. (25). It has the potential to remove the OZI violation found by saturating the $m_2$ pole with the $\phi(1020)$. Secondly, in a full treatment, it is necessary to extend the fit to $F_2^{(I=0)}(q^2)$ beyond the three pole approximation to include continuum contributions and to match onto perturbative QCD asymptotics at large $q^2$, viz. $F_2 \sim \left(\frac{1}{q^4}\right)^3, q^2 \to \infty$ [73].

The second approach to $F_2^s(0)$ involves calculating $N \to K\Lambda$ and $N \to K\Sigma$ loops within a given model. The first authors to do this were Henley and collaborators [11] who found $F_2^s(0) \approx -0.03$ in the Cloudy Bag Model [11] with SU(3) couplings. If SU(3) couplings are used (as in all the present calculations), then $\frac{g_{K\Sigma N}}{g_{K\Lambda N}}_{SU(3)} \approx \frac{1}{25}$ and the $N \to K\Sigma$ loop contributions to $F_2^s(0)$ are neglected in a first approximation. ($K\Sigma$ loops will play a more important role if phenomenological, SU(3) violating, couplings (\frac{g_{K\Sigma N}}{g_{K\Lambda N}}/ph \approx \frac{1}{4}$ [74] are used.) Musolf and Burkardt [78] subsequently argued that seagull terms can induce a large negative value of $F_2^s(0) \approx -0.35$. Melnitchouk and Malheiro [88] have recently estimated $F_2^s(0)$ using light-cone form-factors which are consistent with mesonic effects in deep inelastic scattering [74]. Without seagulls, they find that these kaon loops contribute between -0.04 and -0.1 to $F_2^s(0)$. In a recent quark model calculation, Geiger and Isgur [84] have included the processes $(p \to \Lambda K)_{P(\frac{1}{2})}$, $\gamma(\Lambda K)_{P(\frac{1}{2})} \to (\Lambda K^*)_{P(\frac{1}{2})} \to p$ and $(p \to \Lambda^*(1405)K)_{S(\frac{1}{2})}$, $\gamma[\Lambda^*(1405)K]_{S(\frac{1}{2})} \to [\Lambda^*(1405)K^*]_{S(\frac{1}{2})} \to p$ inside the kaon loop. They obtain $F_2^s(0) \approx +0.04$ in their full calculation and $F_2^s(0) \approx -0.08$ when they keep only the $K\Lambda$ intermediate state.

In the extreme scenario that the whole Karliner “discrepancy” were attributed to $F_2^s(0)$ one would predict $F_2^s \approx +0.66$, which is considerably bigger and has the opposite sign to most theoretical calculations of $F_2^s(0)$. Of course, the non-strange part of the kaon loop $N \to K\Lambda$ also renormalises $\kappa_N$. To the best of our knowledge, this kaon loop contribution has not yet been calculated. Further contributions will come from $\gamma\Lambda \to \Lambda^*$ excitations inside the kaon loop. A large positive value of $F_2^s(0)$ ($\approx +\frac{1}{2}$) would also follow if the SU(3) flavour structure of the anomalous magnetic moment $\kappa_N$ reflects the up quark dominance of many low-energy nucleon observables in QCD, viz. if $|F_2^u(0)| > |F_2^d(0)|$ [70]. (The physical values of $\kappa_N$, Eq. (21), together with $F_2^s(0) = +0.5$, would imply $F_2^u(0) = +2.17$ and $F_2^d(0) = -1.53$.)

Given the important role that the nucleon resonances play in Karliner’s evaluation of the sum-rule, it seems reasonable to consider the electromagnetic excitations of the $\Lambda$ in the processes $(N \to K\Lambda, \gamma\Lambda \to \Lambda^*)$. The interference between this process, which is associated with the isoscalar anomalous magnetic moment, and the process $(\gamma N \to N^*, N^* \to K\Lambda^*)$, which is, in part, associated with the isovector anomalous magnetic moment, is a candidate to fill the Karliner “discrepancy".
Table 3: Thresholds for hyperon (resonance) production in a DHG experiment

| Hyperon       | $J^P$ | Resonance | $\nu_{\text{LAB}}^{th}$ |
|---------------|-------|-----------|------------------------|
| $\Lambda(1115)$ | $1^+$ |           | 0.91 GeV               |
| $\Lambda^*(1405)$ | $\frac{1}{2}^-$ | $S_01$ | 1.45 GeV               |
| $\Lambda^*(1520)$ | $\frac{3}{2}^-$ | $D_{03}$ | 1.69 GeV               |
| $\Sigma^+(1189)$ | $1^+$ |           | 1.05 GeV               |
| $\Sigma^0(1193)$ | $1^+$ |           | 1.05 GeV               |
| $\Sigma^*(1385)$ | $\frac{3}{2}^-$ | $P_{13}$ | 1.42 GeV               |
| $\Sigma^*(1680)$ | $\frac{1}{2}^+$ | $P_{11}$ | 2.06 GeV               |

In Table 3 we list the threshold energies $\nu_{\text{LAB}}^{th}$ for the photoproduction of the $\Lambda$ and $\Sigma$ hyperons, together with their first two excited resonances \[77\]. One priority for future polarised photoproduction experiments should be to reconstruct these $\gamma N \rightarrow K \Lambda^*$ and $\gamma N \rightarrow K \Sigma^*$ processes from events involving one or two kaons in the final state.

Whilst this article was being prepared, Hammer et al.\[78\] independently suggested investigating exclusive channels involving strange mesons ($K$, $\eta$ and $\phi$) as a possible source of the Karliner “discrepancy”. Using positivity arguments, they argue that unpolarised single $K$, $\eta$ and $\phi$ photoproduction data puts an upper bound $|\kappa_{\text{HDM}}| \leq 0.2\kappa_p$ on the contribution $\kappa_{\text{HDM}}$ of these processes to the value of the anomalous magnetic moment that is extracted from the Drell-Hearn-Gerasimov sum-rule.

6 $Q^2$ dependence of $(\sigma_A - \sigma_P)$

The $Q^2$ and $\nu$ dependence of $(\sigma_A - \sigma_P)$ is contained in two spin dependent nucleon form factors $G_1(\nu, Q^2)$ and $G_2(\nu, Q^2)$, viz.

$$\sigma_A - \sigma_P = \frac{16m\pi^2\alpha}{2m\nu - Q^2} \left( m\nu G_1(\nu, Q^2) - Q^2 G_2(\nu, Q^2) \right) .$$  \hspace{1cm} (26)$$

Following Anselmino, Ioffe and Leader \[79\], the $Q^2$ dependent quantity

$$I(Q^2) = m^3 \int_{\frac{Q^2}{2m}}^\infty \frac{d\nu}{\nu} G_1(\nu, Q^2)$$  \hspace{1cm} (27)$$

can be used to interpolate between polarised photoproduction and deep inelastic scattering. The Drell-Hearn-Gerasimov sum-rule Eq.(1) is the statement:

$$I(0) = -\frac{1}{4} \kappa_N^2 .$$  \hspace{1cm} (28)$$
In high $Q^2$ deep inelastic scattering
\[ m^2 \nu G_1(\nu, Q^2) \rightarrow g_1(x, Q^2), \quad m \nu^2 G_2(\nu, Q^2) \rightarrow g_2(x, Q^2) \tag{29} \]
where $x = \frac{Q^2}{2m\nu}$ is the Bjorken variable. In the deep inelastic limit
\[ I(Q^2) = \frac{2m^2}{Q^2} \int_0^1 dx g_1(x, Q^2). \tag{30} \]
As $Q^2$ tends to $\infty$, the first moment of $g_1$ is given by \cite{9, 10, 11, 12}
\[ \int_0^1 dx g_1(x, Q^2) = \frac{1}{2} \sum_q e_q^2 \Delta q_{\text{inv}} \{ 1 + \sum_{\ell \geq 1} c_{\ell} g_{2\ell}(Q) \} + O\left(\frac{1}{Q^2}\right) \tag{31} \]
Here $\Delta q_{\text{inv}}$ is the q flavoured, gauge and scale invariant axial charge \cite{13}. The coefficient $c_{\ell}$ is calculable in $\ell$-loop perturbation theory \cite{11}. The current experimental value of the singlet axial charge \cite{16, 17}
\[ g_A^{0|\text{inv}} = \Delta u_{\text{inv}} + \Delta d_{\text{inv}} + \Delta s_{\text{inv}} \tag{32} \]
is \cite{16, 17}
\[ g_A^{0|\text{inv}} = 0.28 \pm 0.07. \tag{33} \]
This number is four standard deviations below the value 0.58 that it would have taken if $\Delta s_{\text{inv}}$ were zero \cite{8}.

The large violation of OZI in $g_A^{0|\text{inv}}$ is commonly associated with the $U_A(1)$ axial anomaly in QCD \cite{18-23}. The axial anomaly does not contribute to the anomalous magnetic moment for the reasons outlined in Section 4. This source of OZI violation gives a vanishing contribution to the Drell-Hearn-Gerasimov integral $I(0)$ at photoproduction. Strange resonance excitations $\gamma N \rightarrow K \Lambda^*$, and $\gamma N \rightarrow K \Sigma^*$ have the potential to make an important contribution to $I(0)$ at photoproduction. These resonance excitations contribute to polarised deep inelastic scattering only at non-leading twist. They are not present in $g_A^{0|\text{inv}}$.

Several models \cite{57, 79-86} have been proposed for the non-leading twist $Q^2$ variation of $I(Q^2)$, which changes sign between polarised photoproduction, Eq.(28), and deep inelastic scattering, Eqs.(31-33). Bag model \cite{79} and QCD sum-rule \cite{81, 82} calculations are available for the twist four $O(\frac{1}{Q^2})$ corrections to $g_1$ in Eq.(31). These calculations do not yet include the physics of the axial anomaly, which is potentially very important in the flavour singlet channel \cite{77}. Ioffe and collaborators \cite{79, 83} have proposed a simple phenomenological model for the $Q^2$ dependence of the inelastic part of $(\sigma_A - \sigma_P)$. They argue that the contribution from resonance production, denoted $I^{\text{res}}(Q^2)$, has a strong $Q^2$ dependence for small $Q^2$ and then drops rapidly with $Q^2$. The non-resonant part of $I(Q^2)$ is then parametrised by a smooth function which they took as the sum of a monopole and a dipole term, viz.
\[ I(Q^2) = I^{\text{res}}(Q^2) + 2m^2 \Gamma^{\text{as}} \left( \frac{1}{Q^2 + \mu^2} - \frac{C\mu^2}{(Q^2 + \mu^2)^2} \right). \tag{34} \]
Here
\[ \Gamma^{\text{as}} = \int_0^1 dx g_1(x, \infty) \tag{35} \]
and

\[ C = 1 + \frac{1}{2} \frac{\mu^2}{m^2} \frac{1}{\Gamma_{\text{as}}} \left( \frac{1}{4} k^2 + I_{\text{res}}(0) \right). \]  

(36)

Using vector meson dominance arguments, the mass parameter \( \mu \) was identified with rho meson mass, \( \mu^2 \simeq m_\rho^2 \). Karliner’s analysis [26] suggests that \( I_{\text{res}}(0) = -1.01 \). (There is no elastic contribution to (DHG).) In this approach, the “discrepancy” in the Drell-Hearn-Gerasimov sum-rule \((I - I_{\text{res}})_p(0) = +0.21\) is associated with the non-resonant vector meson dominance term.

The E143 Collaboration at SLAC have recently announced the first low-\( Q^2 \) measurements of \( I_p(Q^2) \) and \( I_d(Q^2) \) at \( Q^2 = 0.5\text{GeV}^2 \) and \( Q^2 = 1.2\text{GeV}^2 \) [88]. Both the proton and deuteron data points are consistent with the predictions of [83, 84, 88]. The \( Q^2 = 1.2\text{GeV}^2 \) data is consistent with the QCD sum-rule calculations [81, 82] of the twist four contribution to Eq.(31) with coefficients \( c_\ell \) which are evaluated to \( O(\alpha_s^3) \) [11, 89]. (Theoretical uncertainties due to higher order coefficients may be difficult to estimate [90].)

### 7 Conclusions and Opportunities

Photoproduction spin sum-rules offer a new window on the spin structure of the nucleon that complements the information that we can learn from polarised deep inelastic scattering experiments. The first direct measurements of the spin photoproduction cross-sections \( \sigma_A \) and \( \sigma_P \) will soon be available from the ELSA, GRAAL, LEGS and MAMI facilities up to \( \sqrt{s} \leq 2.5\text{GeV} \). These experiments will make a precise check of \( \gamma N \to N^* \) resonance contributions to the Drell-Hearn-Gerasimov and Spin Polarisability sum-rules. Further important information on the photoproduction spin structure of the nucleon will come from measuring:

- Non-resonant contributions associated, in part, with vector meson dominance (Section 6) and
- Strange resonances (\( \gamma N \to K\Lambda^* \)) and (\( \gamma N \to K\Sigma^* \)) as possible sources of the Karliner “discrepancy” (Sections 4 and 5);
- Tests of spin dependent Regge theory to distinguish the Lorentz structure of pomeron-like exchanges Eqs.(12-14). This requires varying \( \sqrt{s} \geq 10\text{GeV} \) to ensure that we are in the region where the pomeron dominates over other Regge contributions [35] (Section 2). Regge behaviour provides a good description of \( (\sigma_A + \sigma_P) \) starting at \( \sqrt{s} \approx 2.5\text{GeV} \).

When combined with measurements of the \( Q^2 \) dependence of \( (\sigma_A - \sigma_P) \) from SLAC and TJNAF, we will soon have a much more complete picture of the nucleon’s internal spin structure.
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