Accurate Identification of Low-power Repulsive Double Coils’ Leakage and Mutual Inductance

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Abstract. The commonly used frequency domain method cannot accurately obtain the leakage and mutual inductance of the low-frequency and low-power repulsive double coils. Therefore, it is our top priority to accurately identify the leakage and mutual inductance of the repulsive double coils. In this paper, the leakage inductance of the static and dynamic coils, the mutual inductance of the coils are obtained by the method of adding a step voltage to the repulsive double coils. The simulation is based on the finite element software with the connection lead resistance and inductance added to the repulsive double coils’ circuit. By fitting the simulated current waveform of the static and dynamic coils with the experimental current waveform, it can be confirmed that the parameter identification is accurate.

1. Introduction

The electromagnetic repulsive mechanism needs to use finite element software to design and research, and its structural parameters are very important to finite element simulation. However, many parameters are not found on the nameplate, even many repulsive mechanism manufacturers do not test and calculate these parameters deliberately. Therefore, the accurate identification of the structural parameters of the electromagnetic repulsive mechanism has always been a relatively important topic in the field of repulsive mechanisms. Most scholars in the past have studied the analytical calculation of the leakage inductance of high-frequency transformers [1-9]. Less scholars have studied the leakage inductance of low-frequency, low-power transformers. Nowadays, the structural parameters of the electromagnetic repulsive mechanism are measured by the frequency domain method. Most of them use the Agilent 3442OA micro-ohmmeter and the Agilent 4294A impedance analyser to identify the connection lead resistance and inductance [10-12]. However, for the low-frequency, low-power electromagnetic repulsive mechanism, which value is at a micro-Heng level. Using the frequency domain method to calculate the leakage and mutual inductance will inevitably bring a big error. Therefore, this paper starts from the repulsive double coils and analyses the leakage and mutual inductance of the static and dynamic coils by adding a step voltage to the repulsive coils. Finally, the accurate identification of the repulsive double coils’ leakage and mutual inductance is verified by fitting the simulational current waveform to the experimental current waveform.

2. Traditional frequency domain method to measure repulsive coils’ structure parameters

A pair of repulsive double coils are fabricated in the laboratory. The coil wire diameter is 2mm, the coil ring diameter is 100mm, and the repulsive double coils are tightly fitted with a spacing of 3mm. The
coil’s loop has a connection lead resistance because of the need to measure the current. The lead inductance is negligible. The repulsive double coils model is shown in Figure 1. The leakage and mutual inductance of the repulsive double coils are measured by a constant current source, an AC power source, and a multimeter which are used in the laboratory commonly.

![Figure 1. Model of repulsive double coils.](image)

2.1. **Constant current source to measure the resistance of static and dynamic coils**
The static and dynamic coils are taken out to be opened, a DC source is used to supply static and dynamic coils with a current of 1A respectively, and a multimeter is used to measure the voltage. According to Ohm’s law, the resistance of static and dynamic coils can be obtained: $R_1 = 2.6 \Omega$, $R_2 = 2.9 \Omega$.

2.2. **Static coil to be powered by alternating current and RMS to be measured**
The static and dynamic coils are fitted tightly and the dynamic coil is opened, alternating current is added to the static coil. The multimeter is used to measure the voltage of the static and dynamic coils: $U_1 = 26 mV$, $U_2 = 2.8 mV$, static coil’s current: $I_1 = 4.0 A$.

2.3. **Static coil’s self-inductance and mutual inductance**

$$L_1 = \frac{1}{\omega} \sqrt{\frac{U_1^2}{I_1}} - R_1^2 = 19 \mu H, M_{21} = \frac{1}{\omega} \frac{U_2}{I_1} = 2.2 mH$$

2.4. **Dynamic coil to be powered by alternating current and RMS to be measured**
The static coil is opened, the alternating current is added to the dynamic coil. A multimeter is used to measure the voltage of the static and dynamic coils: $U_1 = 2 mV$, $U_2 = 34 mV$, dynamic coil current: $I_2 = 4 A$.

2.5. **Dynamic coil’s self-inductance and mutual inductance**:

$$L_2 = \frac{1}{\omega} \sqrt{\frac{U_2^2}{I_2}} - R_2^2 = 25.4 \mu H, M_{12} = \frac{1}{\omega} \frac{U_1}{I_2} = 1.59 mH$$

The value of self-inductance calculated by the frequency domain method is smaller than the value of mutual inductance, which is impossible. The reason is that the measuring instrument cannot meet the accuracy requirement of the small-power repulsive double coils’ inductance value. Conclusion is that the traditional frequency domain method is not able to measure the leakage and mutual inductance of low-frequency, low-power repulsive double coils. We need to design a new method to accurately identify the leakage and mutual inductance of the repulsive double coils.
3. Stepping voltage method to identify the repulsive double coils’ leakage and mutual inductance

3.1. Mathematical model of charged capacitor discharging to static coil

Figure 2 is an equivalent circuit diagram of the charged capacitor discharging to the static coil. The initial stage of the charged capacitor discharging to the repulsive double coils can be regarded as a step voltage. In the picture, the connecting lead inductance and resistance of the static coil can be represent by $L_L$ and $R_L$, the self-inductance of static coil can be represent by $L_1$, the voltage and current of the static coil can be represent by $u_1$ and $i_1$.

\[ u_1 = i_1 R_L + L_1 \frac{di_1}{dt} \]  

(1)

Since the current is small in the initial stage of current rise, the influence of the value of resistance can be omitted. Simplify (1):

\[ u_1 = L_1 \frac{di_1}{dt} \]  

(2)

3.2. Mathematical model of charged capacitor discharging to repulsive double coils

Figure 3 is an equivalent circuit diagram of the charged capacitor discharging to the repulsive double coils. The leakage inductance of static and dynamic coils can be represent by $L_\sigma$ and $L_\sigma$, the connecting lead resistance of dynamic coil can be represent by $R_\sigma$, the mutual inductance of repulsive double coils can be represent by $M$, the current of static and dynamic coils can be represent by $i_1$ and $i_2$.

\[ u_1 = L_\sigma \frac{di_1}{dt} + M \frac{di_2}{dt} \]  

(3)

$\theta = L_\sigma \frac{di_1}{dt} + M \frac{di_2}{dt}$

(4)

$\frac{di_1}{dt} = \frac{u_1 - L_\sigma \frac{di_1}{dt} - M \frac{di_2}{dt}}{L_\sigma}$

(5)

$\frac{di_2}{dt} = \frac{M \frac{di_1}{dt} - u_1 + L_\sigma \frac{di_1}{dt}}{L_\sigma}$

(6)
Similarly, the current is small in the initial stage of current rise, the influence of the value of resistance can be omitted, simplified the model:

\[
\begin{align*}
    u_1 &= i_1 R_1 + L_{1\sigma} \frac{di_1}{dt} + M \frac{d(i_1 - i_2)}{dt} \\
    0 &= i_2 R_{2\sigma} + i_2 R_{2l} + L_{2\sigma} \frac{di_2}{dt} + M \frac{d(i_2 - i_1)}{dt}
\end{align*}
\]  

(3)

Integrate equation (4):

\[
\begin{align*}
    u_1' &= L_{1\sigma} \frac{di_1'}{dt} + M \frac{d(i_1' - i_2')}{dt} \\
    0 &= L_{2\sigma} \frac{di_2'}{dt} + M \frac{d(i_2' - i_1')}{dt} \\
    i_1 &= L_{1\sigma} + M \\
    i_2 &= L_{2\sigma} + M
\end{align*}
\]  

(4)

3.3. Experimental measurement of repulsive double coils’ current and voltage

3.3.1. Description of the experimental circuit. In the experiment, the static coil is discharged by an electrolytic capacitor, 103μF, charged with 106V. Two sets of Rogowski coils and oscilloscopes are used to measure the current of the static and dynamic coils and the voltage of the static coil. Connecting lead inductance and resistance is existed in current circuit. The experimental circuit model is shown in Figure 4.
3.3.2. The charging capacitor discharges to the static coil separately. The electrolytic capacitor discharges to the static coil separately through the thyristor, and the current waveform of the static coil is recorded by the Rogowski coil and the oscilloscope. The experimental circuit is shown in Figure 5.

Figure 5. Experimental circuit of static coil

The experimental waveform of the current and voltage of the static coil is shown in Figure 6. In the initial stage of capacitor discharging, it can be seen as a step voltage applied to the static coil. The initial current rise rate of the static coil current can be read from the experimental waveform.

\[
\begin{align*}
\frac{du}{dt} &= 48V \\
\frac{di}{dt} &= 160A/\mu s
\end{align*}
\]

(6)

Figure 6. Experimental current and voltage’s waveform of static coil

3.3.3. Mathematical model of charged capacitor discharging to static coil. Electrolytic capacitor discharges repulsion double coils through thyristor. The current waveform of static and dynamic coils is recorded by Rogowski coil and oscilloscope. The experimental circuit is shown in Figure 7.
The current and voltage waveform of the static and dynamic coils are shown in Figure 8. In the initial stage of capacitor discharging, it can be seen as a step voltage applied to the coil. Initial current rise rate of static coil current $i_1$, initial current rise rate of dynamic coil current $i_2$ and $u_1$ can be read from the experimental waveform.

\[
\begin{align*}
\frac{du_1}{dt} &= 36V \\
\frac{di_1}{dt} &= 180A/\mu s \\
\frac{di_2}{dt} &= 100A/\mu s
\end{align*}
\]  
\tag{7}

Figure 8. Experimental current and voltage’s waveform of double coils

3.4. Solution of self-inductance, mutual inductance and leakage inductance of static and dynamic coils

Bring (6) into (2) and bring (7) into (5):

\[
\begin{align*}
L_1 &= 0.3\mu H, L_2 = 0.32\mu H, M = 0.18\mu H, L_{1a} = 0.12\mu H, L_{2a} = 0.14\mu H
\end{align*}
\]

4. Simulation Research

In order to verify the correctness of the repulsion double coils’ leakage and mutual inductance obtained by the analysis. In this paper, the original proportional two-dimensional rotationally symmetric finite element model of repulsive double coils is established for verification.
A circular section is established with a diameter of φ2mm as a static coil, the center of which is 50mm from the axis of symmetry. A circular section of the same size is established as a dynamic coil which is 3 mm directly above the static coil. Finally, a cylindrical section is established as the air domain which covers the static and dynamic coils. The magnetic field and circuit physics are selected to be coupled with.

The initial capacitor voltage is 160V. If the connection lead resistance and inductance are not added in the circuit module, the comparison of static coil’s simulational and experimental current is shown in Table 1.

### Table 1. Comparison of static coil’s simulational and experimental current.

|                        | Peak current (A) | Peak time (μs) | Initial current rise rate (A/μs) |
|------------------------|------------------|----------------|-------------------------------|
| Simulational current of static coil | 3000             | 8              | 600                           |
| Experimental current of static coil     | 1300             | 14             | 160                           |

If the effect of resistance on current is ignored, peak current: \( I_n = \sqrt{\frac{C}{L}} U_0 \), Peak time: \( T = \frac{\pi}{2} \sqrt{LC} \). It is necessary to add a connecting lead inductance and resistance in the simulated double coils’ loop in order to reduce the current amplitude and increase the peak time, thereby reflecting the actual situation accurately. So the static coil connection lead inductance is 0.74μH, the static coil connection lead resistance is 33mΩ and the dynamic coil connection lead resistance is 3.5mΩ. The waveform of the simulational and experimental current can be fully fitted. Figure 9 and 10 are the fitting diagrams of the repulsion double coils’ experimental and the simulational current. It can be seen from the figure that the waveform has a very high degree of fitting, which can prove the step voltage method used to measure the leakage and the mutual inductance of repulsive double coils reliable.

![Figure 9. Experimental and simulational current waveform of single coil.](image-url)
5. Conclusion

This paper proposes a method for accurately distinguishing the leakage and mutual inductance of low-frequency and low-power repulsive double coils. The leakage and mutual inductance of the static and dynamic coils are accurately analysed by the method of adding a step voltage, $L_\text{le} = 0.12 \mu H$, $L_\text{mu} = 0.14 \mu H$, $M = 0.18 \mu H$. The effectiveness of the scheme and the accuracy of identifying leakage and mutual inductance are confirmed through fitting the finite element simulational current waveform and the experimental current waveform. The solution to identify leakage and mutual inductance of adding a sudden step voltage can be directly applied to identify structural parameters of the repulsive mechanism which has high use value.

Acknowledgements

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