The mode competition in a two-mode semiconductor laser

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The mode competition in a two-mode semiconductor laser

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Abstract. In this paper the dynamics of a two-mode semiconductor laser model is considered. The dynamic evolution of a two-mode semiconductor laser can be described by nonlinear rate equations, which take into account the gain saturation. Changes in parameters of the system can lead to changes in the character of the solutions. Qualitative analysis of the system is used for investigating the typical bifurcations and allowed conditions for the existence of different types of mode oscillations in the system. The phase portraits of steady-state solutions of the system are plotted using numerical methods. The competition phenomenon between modes is observed.

1. Introduction
Over the past decade instabilities in laser have become a widely discussed topic in quantum optics. Semiconductor laser is an excellent example of nonlinear dynamic systems. In current paper we consider a model of a two-mode semiconductor laser. The study of multimode lasers is by far one of the most actual problems of nonlinear dynamics and laser physics. Notable in this system is the phenomenon of mode competition, arising from connectivity modes in the laser. The phenomenon of mode competition can also be found in the systems with entirely different kind of nature, such as in environmental, biological, chemical, economy systems. For example, in the economy system the competition between producers under capitalism plays a major role in the development of the proportionality between the different areas of production [1]. The competition phenomenon is also observed in a well-known biological model of Voltaire, which describes the coexistence of several populations [2]. In recent years, a considerable amount of theoretical and experimental studies have been devoted to the practical application of these mechanisms in the creation of new types of lasers and other optical quantum devices.

Most laser generators tend to work in the multimode regime. Such as ring [3, 4], semiconductor lasers and also lasers with feedback.

We will analyze the competition events occurring in the two-mode semiconductor laser. To determine the conditions of stable and unstable states of lasing modes, we will use a qualitative and numerical analysis of the system. Qualitative analysis of a two-mode laser model, gives a clear explanation of the mode competition in the laser.

2. The Model
In this section we will discuss a two-mode laser model. The dynamical behaviour of such laser can be described by a set of coupled first order differential equations. The model is applicable to all solid-state lasers, including semiconductor lasers as well.

Rate equations for the laser photon density ($S$) in the two-mode approximation, that take into account the saturation of the medium gain has the following dimensionless form[5, 6]:
\[ \dot{S}_i = (\alpha_i - \theta_{i1}S_i - \theta_{i2}S_2)S_i, \]
\[ \dot{S}_2 = (\alpha_2 - \theta_{21}S_1 - \theta_{21}S_2)S_2, \]

(1)

where, the net amplification rate \( \alpha_i, i = 1, 2 \) equals to the difference of gain \( G_i = a(N - N_{th}) \) and loss \( 1/\tau_{ph} \) in resonator \( \alpha_i = G_i - 1/\tau_{ph} \).

\( N \) - carrier concentration,

\( N_{th} \) - carrier density at threshold,

\( \tau_{ph} \) - the photon lifetime of \( i \text{-th} \) mode,

\( \theta_{ij} \) - self-saturation coefficient,

\( \theta_{ij} \) - cross-saturation coefficient, \((i=1, 2, j=2, 1)\).

Differential equations for the concentration of carriers charge are omitted, since we are interested in time scales of the order of \( \tau_{ph} \approx 1 \text{ ps} \), and typical carrier lifetime of \( \tau_e \approx 1 \text{ ns} \). These parameters correspond to semiconductor lasers.

Thus, the concentration of charge carriers can be considered constant and equal to the initial value \( N_0 \).

In this case \( G_i = a(N_0 - N_{th}) = aN_0 \).

3. Discussions and Results

Let us now examine the stability of equilibrium points of the system using qualitative analysis.

Solutions of nonlinear differential equations depend on their parameters. There may be critical parameter values, at which the character of the solution changes completely. For this reason, it is important to know the qualitative behavior of the solutions and examine stability boundaries of equilibria.

Differential equations for the concentration of carriers charge are omitted, since we are interested in time scales of the order of \( \tau_{ph} \approx 1 \text{ ps} \), and typical carrier lifetime of \( \tau_e \approx 1 \text{ ns} \).

Setting the differential terms of the equations (1) to zero,

\[ \dot{S}_i = 0, i = 1, 2, \ldots, \]

we obtain four possible steady-state solutions for the photon densities:

\[ (S_1^*, S_2^*) = (0, 0), \]  

(2)

\[ (S_1^*, S_2^*) = \left( \frac{\alpha_1}{\theta_{11}}, 0 \right), \]  

(3)

\[ (S_1^*, S_2^*) = \left( 0, \frac{\alpha_2}{\theta_{22}} \right), \]  

(4)

\[ (S_1^*, S_2^*) = \left( \frac{\alpha_2 \theta_{22} - \alpha_1 \theta_{21} - \alpha_1 \theta_{12} - \alpha_2 \theta_{11}}{\theta_{11} \theta_{22} - \theta_{12} \theta_{21}}, \frac{\alpha_1 \theta_{12} - \alpha_2 \theta_{21} - \alpha_1 \theta_{12} - \alpha_2 \theta_{21}}{\theta_{11} \theta_{22} - \theta_{12} \theta_{21}} \right). \]  

(5)

Let us consider the stability of steady states solutions (2) - (5) using linear approximation. Jacobian matrix of the system (1) has the following form:
The first equilibrium point (2) is at the origin and in all parameters of the system is an unstable node, since
\[ \text{tr}(J) = \alpha_1 + \alpha_2 > 0 \] and
\[ \text{det}(J) = \alpha_1 \alpha_2 > 0. \]

The solutions (3) and (4) are stable nodes when determinant \( \text{det}(J) = -\alpha_1 (\alpha_j - \theta_j \frac{\alpha_i}{\theta_i}) \) has a positive and trace \( \text{tr}(J) = \alpha_j - \alpha_i - \theta_j \frac{\alpha_i}{\theta_i} \) has a negative values, that is, under appropriate conditions
\[ \alpha_j < \theta_j \frac{\alpha_i}{\theta_i}. \]

The solution of the system is transformed into the saddle, when a determinant has a negative value \( \text{det}(J) = -\alpha_1 (\alpha_j - \theta_j \frac{\alpha_i}{\theta_i}) \), which is reached at \( \alpha_i > \theta_j \frac{\alpha_i}{\theta_i} \), where \( i, j = 1,2; i, j = 2,1 \) for the solutions of (3) and (4) respectively.

The nontrivial solution (5) is a stable node in case of performance ratio
\[ \frac{\theta_{i2} \alpha_i}{\theta_{22}} < \alpha_i < \frac{\theta_{i1} \alpha_i}{\theta_{21}}, \]
that implies the inequality
\[ \theta_{i2} \theta_{21} < \theta_{i1} \theta_{22}. \]

If the solution doesn’t satisfy above condition the steady state solution is an unstable saddle. Thus, the stability of solution 5 depends on the relation between the products of the coefficients of saturations.

Let us now explain the physical meaning of the conditions of stability of the steady states solutions.

Absolute instability of trivial solutions (2) means that the laser oscillates always at least in one mode.

Solution (3) shows that the laser generates in the first mode, when \( \alpha_2 < \theta_2 \frac{\alpha_1}{\theta_{11}} \), that corresponds to \( G_2 < G_1 \). A similar pattern can be observed for the condition (4), in this case the laser generates in a second mode when \( \alpha_1 < \theta_1 \frac{\alpha_2}{\theta_{22}} \), that corresponds to the inequality \( G_2 > G_1 \).

Thus, we can conclude that two-mode laser prefer to oscillate in a mode that has the largest gain.

Using numerical methods, such as Runge-Kutta of fourth order, it is possible to integrate the nonlinear equations of the system in different parameters and get the phase portraits of system states. The phase portraits of steady states solutions (3) and (4) are illustrated in Figure 1,a,b, respectively.

Solution (3) (see Figure 1, a) shows that the laser generates in the first mode. For the solution 4 (see Figure 2, b), the laser generates in a second mode since. We can see that these numerical results confirm the results of qualitative analysis of the system presented in previous section.

For the solution 5, there are two modes that oscillate simultaneously, since all integral curves tend to one intersection point of these modes. The type of the phase portrait in this case is the stable node (see Figure 2, a).
According to expression (6) for the solution (5), we can formulate a rule of coexistence of oscillation of two modes simultaneously (see Figure 2, a) in the laser: the product of the coefficients of cross- saturation must be less than the product of the coefficients of its own saturation.

The outcome of the competition for solution (5) in unstable state, that has the form of a saddle, depends on the initial conditions. The mode 1 or mode 2 will be oscillated according to initial values of $S_1$ and $S_2$. These laser states are plotted in Figure 2, b.

![Figure 1](image1.png)

**Figure 1.** The phase portraits of steady states solutions (3) and (4)

![Figure 2](image2.png)

**Figure 2.** The phase portraits of steady states solutions (5)

**Conclusion**

Thus, the two-mode laser has 3 types of mode generation: in the first case - one of the modes will suppress the other, in the second case - two modes will oscillate simultaneously, in the third case – the laser works as a trigger, where the oscillation in one of two possible modes depends on its initial conditions.

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