Dark polariton-solitons in semiconductor microcavities

A.V. Yulin1, 2, O.A. Egorov1, 3, F. Lederer3, and D.V. Skryabin1
1 Centre for Photonics and Photonic Materials, Department of Physics, University of Bath, Bath BA2 7AY, United Kingdom
2 Department of Engineering Mathematics, University of Bristol, Bristol, BS8 1TR, United Kingdom
3 Institute of Condensed Matter Theory and Solid State Optics, Friedrich Schiller University Jena, Max-Wien-Platz 1, 07743 Jena, Germany

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We report the existence, symmetry breaking and other instabilities of dark polariton-solitons in semiconductor microcavities operating in the strong coupling regime. These half-light half-matter solitons are potential candidates for applications in all-optical signal processing. Their excitation time and required pump powers are a few orders of magnitude less than those of their weakly coupled light-only counterparts.

Polaritons are mixed states of photons and material excitations and are well-known to exist in many condensed matter, atomic and optical systems [1, 2, 3, 4, 5]. We are dealing below with a semiconductor microcavity, where polaritons exist due to mixing of quantum well excitons and resonant microcavity photons [2, 3, 4]. In the strong coupling regime photons, emitted as a result of electron transitions, excite the medium and are re-emitted in a cascaded manner, which gives rise to so-called Rabi oscillations [1, 3, 4]. This phenomenon results in the two peak structure of the microcavity absorption spectrum. The measured spectral width of the peaks corresponds to the picosecond polariton life time [3]. This is in contrast with the more usual weak-coupling regime (typical for operation of vertical cavity surface emitting lasers (VCSELs) [3]), where the slow (nanosecond) carrier dynamics does not catch up with the fast (picosecond) photon decay. Thereby most of the photons leave the cavity as soon as they are emitted. In this regime the response to a pulse, resonating with a cavity mode, results in a single spectral peak. Thus any potential application of microcavity polaritons in optical information processing leads to a 2-3 orders of magnitude response time reduction relative to the VCSEL-like operating regimes.

One of the topics of the recent research into the weakly coupled semiconductor microcavities has been the localised structures of light or cavity solitons [6, 7, 8, 9, 10, 11, 12], which have demonstrated rich physics and have been proposed for information processing applications [7, 8]. In the weak coupling regime formation of polaritons is irrelevant, since the dispersion of linear excitations is purely photonic. Slowness of the light-only cavity solitons is an outstanding problem, which can be rectified in the strong-coupling regime, where potentially much faster, but not yet reported, light-matter solitons are expected.

In the last few years extensive studies of the polaritons in strongly coupled microcavities have been strongly motivated by the smallness of the polariton mass leading to observation of the polariton Bose-Einstein condensation at few Kelvin temperatures [13, 14]. Polaritons have also been recently observed even at the room temperatures, see, e.g., [15], which has further boosted their potential for practical applications. Another very important feature of polaritons in semiconductor microcavities is their strong repulsive interaction (two-body scattering) resulting in a substantial defocusing nonlinearity [2, 3]. Amongst nonlinear effects predicted or observed with microcavity polaritons are optical bistability [16, 17, 18, 19, 20] and parametric conversion [18, 21, 22, 23, 24]. Observation of these effects with polaritons requires pump intensities of ~100W/cm² or below (see, e.g., Fig. 1 in [17]), which is less than the typical pump of 10kW/cm² required for semiconductor microcavities operating in the weak-coupling regime [6, 7, 8, 9, 10, 11] (see, e.g., Fig. 5 in [9]).

Solitonic effects with polaritons in bulk media have attracted a significant (mostly theoretical) attention since 70s till now, see, e.g., Ref. [2, 25]. In the latest wave of research on exciton-polaritons in strongly coupled microcavities the solitonic effects have not been much of a focus yet, with an important exception of a recent experimental paper [26]. In this work the authors claim observation of dark and bright localized structures or cavity solitons in a strongly coupled semiconductor microcavity. Some other papers have reported localisation of microcavity polaritons due to linear defects [14, 27], as a result of switching between two polarizations [28], or neglecting such important requisites of passive cavities as losses, external pump and hence bistability [29]. For studies of spatially dependent polariton dynamics, see, e.g., [30]. Our work is aimed at filling an existing gap in the theoretical knowledge about microcavity polariton-solitons. This is necessary not only for backing so far limited experimental observations [26], but also and mainly for guiding the future work in this direction.

The widely accepted dimensionless mean-field model for excitons strongly coupled to the circularly polarized cavity photons is [2, 3, 21]

\[ \partial_t E - i(\partial_x^2 + \partial_y^2)E + (\gamma_e - i\Delta)E = E_p + i\Psi, \]
\[ \partial_t \Psi + (\gamma_0 - i\Delta + |\Psi|^2)\Psi = iE. \] (1)

Here \( E \) and \( \Psi \) are the averages of the photon and exciton creation or annihilation operators. Normalization is such that \( \langle \Omega_R/g \rangle |E|^2 \) and \( \langle \Omega_R/g \rangle |\Psi|^2 \) are the photon...
The sum of states |Ψ(∆)| = 1 physically corresponds to the zero homogeneous solution is absent. Therefore we proceed with a brief consideration of spatially homogeneous solutions (HS) and their stability. Then we report the existence of various cavity polariton solitons (CPSs) and study their stability and instability scenarios. HS having bistable dependence from $E_p$ is an important prerequisite for the soliton existence. $E(E_p)$ is multivalued provided that $f(\Delta) > 0$, where

$$f(\Delta) \equiv \Delta(\Delta^2 + \gamma_c^2 - 1) - \sqrt{3}\gamma_0(\Delta^2 + \gamma_c^2 + \gamma_c^2 / \gamma_0).$$

The cumbersome expressions for the roots of $f(\Delta) = 0$ simplify for $\gamma_c = \gamma_0 = 0$ and give two bistability intervals $\Delta > 1$ and $-1 < \Delta < 0$. These two intervals overlap with the $\Delta$ intervals allowed by the dispersion relation, see Eq. (2) and Fig. 1. The bistability in the interval $-1 < \Delta < 0$ appears because of the nonlinear resonance of the pump with the L-polaritons whereas the bistability in the semi-infinite interval is associated with the nonlinear resonance of the pump with the U-polaritons. Weaker coupled cavities with defocusing nonlinearities exhibit bistability only for $\Delta > 0$, see, e.g. 12. Below we focus our attention on the solitons linked to L-polaritons, therefore our studies are unique to the strong coupling regime. Stability analysis of the HS L-polaritons ($\Delta < 0$) has been previously reported for example in Refs. 21. The lower state of the L-polariton bistability loop can be modulationaly unstable within some interval of $E_p$, while the upper state is generally stable, see Figs. 2(a) and 3(a). Here, modulational instability (MI) we understand as the growth of linear perturbations in the form $e^{ik_x x + ik_y y + \alpha t}$ ($Re \kappa$ is the growth rate). As $\Delta$ is changing from the bottom of the L-polariton branch towards the linear exciton resonance, $\Delta = 0$, the point of MI is moving towards the left edge of the bistability loop and finally goes beyond the latter, cf. Figs. 2(a) and 3(a).

Restricting ourselves to the structures independent on the polar angle ($\theta = arg(x + iy)$) we find that the time-independent CPSs obey

$$-i \left( \frac{d^2 E}{dr^2} + \frac{1}{r} \frac{dE}{dr} \right) + (\gamma_c - i\Delta)E = E_p + i\Psi$$

where $r = \sqrt{x^2 + y^2}$, $\Psi = iE/[\gamma_0 - i\Delta + iz]$ and $z \equiv |\Psi|^2$ is found solving the real cubic equation $(\gamma_c^2 + (z - \Delta)^2)z = |E|^2$. $z$ turns out to be a single valued function of $|E|^2$ throughout the range of parameters corresponding to the bistability of L-polaritons. Thus the potential problem of ambiguity in choosing a root for $z$ is avoided.

We start our analysis of cavity polariton solitons from the case, when the MI point of low state L-polaritons is within the bistability interval. In many previously studied models bifurcation points of the homogeneous solutions have been the sites where localized structures branch off 12. Applying the Newton iterative method to Eq. (3) we have found a family of small amplitude
coupling regime, see, e.g. [9, 33, 34]. Unlike fiber solitons, mentally for semiconductor microcavities in the weak-
been previously studied both theoretically and experi-
be naturally selected by our system and the instability of
nonlinearity dark CPSs, see, e.g., [12], are expected to
detailed investigation and it will be analyzed elsewhere.

FIG. 2: (a) Amplitude of the homogeneous state (HS) (black line), max $|\Psi(x, y)|^2$ for bright solitons (blue line) and
min $|\Psi(x, y)|^2$ for dark solitons (red line) shown as functions of $E_p$; $\Delta = -0.7$, $\gamma_{0,c} = 0.1$. (b) is the zoom of the rectan-
gular area from (a) showing bifurcations of the dark solitons.
(c,d) Exciton density distribution $|\Psi(x, y = 0)|^2$ across the
bright (c) and dark (d) solitons for the points marked by 1,
2, 3 and 4. Full and dashed lines in (a)-(d) mark stable and
unstable solutions, respectively.

bright CPSs emerging from the MI point, see the dashed red line in Fig. 2(a). Going towards smaller values of
$E_p$, the CPSs become more intense, see Fig. 2(c). The $E_p$ value, at which the lower and upper homogeneous
states can be connected by a standing 1D front, is called
Maxwell point and this is the point where the branch of
the bright CPSs terminates ($E_p = 0.1748$). When the
pump approaches the Maxwell point the soliton broad-
ens and its peak intensity tends towards the intensity
of the upper homogeneous state. We also perform a
full 2D linear stability analysis of the found structures.
The linear perturbations are assumed in the general form
$\epsilon^J(x)e^{iJ\theta+\kappa t} + \epsilon^{*}_J(x)e^{-iJ\theta+\kappa_{c}t}$, where $J = 0, 1, 2, \ldots$. [32].
The resulting Jacobian operator is analysed using finite
differences in $r$. The linear stability analysis shows that
the bright CPSs are unstable with respect to the pertur-
bation with the azimuthal index $J = 0$ and that the
development of the instability splits the CPS into 2D
moving fronts. When $E_p$ is close to the Maxwell point
this instability is relatively weak and bright CPSs can
be easily stabilized by the spatial inhomogeneities of
the pump or cavity detuning. This problem deserves more
detailed investigation and it will be analyzed elsewhere.

Because of the defocusing nature of the polaritonic
nonlinearity dark CPSs, see, e.g., [12], are expected to
be naturally selected by our system and the instability of
bright CPSs is not surprising. Dark cavity solitons, have
been previously studied both theoretically and experi-
mentally for semiconductor microcavities in the weak-
coupling regime, see, e.g. [33, 34]. Unlike fiber solitons,
the dark cavity solitons have no conceptual disadvantage
over the bright ones as information carriers. The branch
of dark CPSs have been found to detach from the left
folding point of the bistability loop and tend towards the
Maxwell point, see Fig. 2(a) and the zoomed area in (b).
At the onset of their existence the dark solitons are seen
only as a very deviation from the homogeneous back-
ground. As $E_p$ tends towards the Maxwell point from
the right, they become much dipper. Near the Maxwell
point the dark solitons become very broad and can be
roughly considered as superpositions of infinitely separ-
ated 1D fronts ($1/r$ term in Eq. (4) can be disregarded
for large distances and the equation becomes effectively
1D). It is important to note that the relaxation of the
fronts towards the upper state happens without oscilla-
tions, however the relaxation towards the lower state is
oscillatory, see Fig. 2(c). Thus pinning of the two fronts

FIG. 3: a) Amplitude of the homogeneous state (HS) (black line) and min $|\Psi(x, y)|^2$ for dark solitons (red and blue lines)
shown as functions of $E_p$; $\Delta = -0.5$, $\gamma_{0,c} = 0.1$. B1 and
B2 mark two branches of dark CPSs. (b) is the zoom of the rectangular area from (a) showing bifurcations of the B2
dark CPSs. (c,d) Exciton density distribution $|\Psi(x, y = 0)|^2$ across B1 (c) and B2 (d) CPSs for the points marked by 1,
2, 3 and 4 in panels (a) and (b). Full and dashed lines in
(a)-(d) mark stable and unstable solutions, respectively. (e,f)
show development of the symmetry breaking instabilities of
the CPSs marked as 5 and 6 in (b).
and hence stabilization of CPSs is possible only for the dark structures (see the thick line in Fig. 3(c)). The stable branches of dark CPSs are shown by full lines in Fig. 3(b). The unstable ones correspond to the instabilities with $J = 0$.

In the case when the lower branch HS is unstable within the whole range of the bistability ($\Delta = -0.5$) we have not found bright solitons, see Fig. 3. This result is not surprising because the bright soliton branch is expected to detach from the point where the lower HS changes its stability. This point is now well out of the bistability range, which is another prerequisite for their existence. However, we have found two distinct branches of dark CPSs marked as B1 and B2 in Figs. 3(a),(b). The B1 branch bifurcates subcritically from the folding point of the upper homogeneous state. Initially unstable ($J = 0$) CPSs become stable after the turning point. Close to this turning point the B1 CPSs have a deep growing radius Fig. 3(c). Note, that close to the turning points additional destabilization of dark CPS happens due to linear eigenmodes with complex $\kappa$ and $J = 0$ (Hopf instability, see Fig. 3(a)) resulting in the formation of oscillating dark CPSs.

The branch B2 consists of ring shaped structures, see Fig. 3(d). The linear stability analysis shows that the B2 CPSs can be stable (see the interval marked by 4 in Fig. 3(b)). However, more often, they are unstable with respect to perturbations breaking the radial symmetry, i.e. with $J \neq 0$. An example of this instability development is shown in Figs. 3(e,f). The dark ring CPS shown in Fig. 3(f) is unstable against linear eigenmode with $J = 3$. The broader CPSs undergo azimuthal instabilities with larger azimuthal numbers $J$. For example, $J = 8$ for the concentric ring CPS shown in Fig. 3(f).

In summary: Following a series of recent experiments on observation of microcavity polaritons, we have studied the formation of spatially localised polariton-soliton structures in the strong coupling regime. In particular, our results can be used for the interpretation of the experimental measurements reported in [26], where the polariton Rabi splitting has been observed simultaneously with the formation of various bright and dark localised structures. Microcavity polariton solitons reported here exhibit a picosecond excitation time and can be observed at pump powers few orders of magnitude lower than those required in the weak coupling regime of the semiconduc-


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