Black holes, Planckian granularity, and the changing cosmological ‘constant’

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Abstract
In a recent work we have argued that noisy energy momentum diffusion due to space-time discreteness at the Planck scale (naturally expected to arise from quantum gravity) can be responsible for the generation of a cosmological constant at the electro-weak phase transition era of the cosmic evolution. Simple dimensional analysis and an effectively Brownian description of the propagation of fundamental particles on a granular background yields a cosmological constant of the order of magnitude of the observed value, without fine tuning. While the energy diffusion is negligible for matter in standard astrophysical configurations (from ordinary stars to neutron stars), here we argue that a similar diffusion mechanism could, nonetheless, be important for black holes. If such effects are taken into account, two observational puzzles might be solved by a single mechanism: the ‘$H_0$ tension’ and the relatively low rotational spin of the black holes detected via gravitational wave astronomy.

Keywords Black holes · Dark energy · Unimodular gravity · Quantum gravity

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1 Introduction

Soon after A. Einstein proposed in 1915 the general theory of relativity, it became clear that its basic equations did not allow for a space-time describing a static universe, which is what was taken as “evident” at the time. He, therefore, introduced a term that changed that aspect of the theory, and which came to be known as the cosmological constant (CC). The discovery in 1929 of Hubble’s cosmic expansion seemed to make such term unnecessary (although possible, as long as its value was small enough). The development of the quantum theory of fields soon led researchers to argue that there must be a vacuum energy associated with the zero point fluctuations of all modes of all matter fields, just as the $\frac{1}{2} \hbar \omega$ ground state energy of a harmonic oscillator. Estimates indicated that the value of the resulting contribution to the CC should be of order $m_p^2$ (where $m_p$ denotes the Planck mass). It was clear, very early on, that the total value of the CC must be at least 120 orders of magnitude smaller than that. A severe fine tuning seemed to be required. Actually, for a long time most people were convinced that there was some deep physical principle that would explain why its value was, in fact, vanishing (see for instance [1]). Things took a sharp turn with the discovery in 1998 [2,3] of the fact that the universe’s expansion was accelerating, and the realization that the simplest explanation was a rather un-naturally small, but non-vanishing value of the cosmological constant. That has become part of the standard model of cosmology, also known as ΛCDM (involving, besides the well known components representing ordinary matter and radiation, a cosmological constant and cold dark matter). The last couple of years the situation has become more puzzling when, as a result of a sharp increase in the precision of cosmological observations, some inconsistencies have appeared in the value of the parameters characterizing the universe’s expansion extracted from observations focusing on different epochs. One of the simplest accounts for such observations would be a small change in the cosmological constant from the era of emission of the cosmic microwave radiation, about 13.7 billion years ago, to recent times (corresponding to the last few billion years). In the following we will discuss a proposal that seems to account in a rather natural way (i.e. without fine tuning), for both the actual value of the cosmological constant, its recent change, and an apparently disconnected issue: the unexpectedly low value of the angular momentum of the black holes that have been observed to collide generating gravity waves.

2 The present tension and our approach

There is, presently, a $4.4\sigma$ [4] tension between the present value of the Hubble expansion parameter $H_0$ inferred from the CMB measurements of Planck 2018 [5], using
the so-called concordance ΛCDM model [6], and the local supernova determination of \( H_0 \) [7]. If this tension were to be confirmed as a failure of the ΛCDM model, then a natural mechanism for its resolution would consist of having a dark energy component that deviates from an exact cosmological constant, and which has grown from the CMB recombination time until today. Independent analyses seems to suggest such interpretation [8].

In a recent series of papers [9–11] we have proposed a mechanism by which the usual (ΛCDM model) dark energy component of the late universe can be produced from the noisy diffusion of energy from matter degrees of freedom to the underlying granular structure of spacetime at the Planck scale, taking place during the cosmological radiation dominated epoch. One such proposal tied the effect to an hypothetical spacetime granularity which is expected from physical properties of black holes in the semiclassical regime\(^1\) [12], as well as from various approaches to quantum gravity.

On the other hand, the result in [13,14], together with constraints from the validity of Lorentz symmetry strongly suggest that any kind of Planckian granularity must be of a relational nature, i.e. only apparent when the spacetime geometry is curved and is probed with the suitable degrees of freedom, and, thus, its phenomenological manifestation is severely restricted (see for instance [15,16]). Those considerations also suggest that the degrees of freedom that might be sensitive to Planckian discreteness should be massive (ΛCDM model) dark energy component of the late universe can be produced from the noisy diffusion of energy from matter degrees of freedom to the underlying granular structure of spacetime at the Planck scale, taking place during the cosmological radiation dominated epoch. One such proposal tied the effect to an hypothetical spacetime granularity which is expected from physical properties of black holes in the semiclassical regime\(^1\) [12], as well as from various approaches to quantum gravity.

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\[
    u^\mu \nabla_\mu u^\nu = \alpha \frac{m}{m_\text{P}^2} \text{sign}(s \cdot \xi) R s^\nu, \tag{1}
\]

where \( \alpha > 0 \) is a dimensionless coupling, \( m \) is the mass of the particle, \( R \) is the scalar curvature, \( s^\mu \) is the spin of the particle, and \( \xi^\mu \) is a preferred time-like vector field characterizing the mean state of motion of the overall matter configuration that is

---

\(^1\) That is, under conditions where the use of classical general relativity together quantum field theory on curved spacetimes offers a reliable approximation

\(^2\) Scalar massive fields, like the inflaton or the Higgs, are one-dimensional probes: they only carry a spacetime arrow given by their four-momenta. Massless (scale invariant) fields are transverse and hence three-dimensional probes: null four-momenta and a spin state in the two-dimensional orthogonal space. Only spinning and massive particles can be seen as genuine four-dimensional probes of the spacetime geometry: their four momenta plus their spin state that can span the full 4-dimensional ‘tangent’ spacetime.
responsible for the spacetime curvature (in cosmology this is well approximated by the four velocity of comoving observers). Note that the RHS of 1 must be orthogonal to $u^\nu$ (from the simple kinematical requirement that $u_\nu u^\mu \nabla_\mu u^\nu = 0$) and this can easily be assured by requiring it to be proportional to the spin of the particle. This reinforces our heuristic argument (see Footnote 2) that it should be only the spin carrying particles that might be sensitive to the diffusion mechanism, because the spin provides the only proper (or canonical) direction orthogonal to the four-velocity of the probe.

That Eq. (1) embodies ‘diffusion of energy’ (or an anomalous violation of energy conservation) is clear from the behavior of the mechanical energy $E \equiv -m u^\nu \xi_\nu$ (defined in the frame $\xi^\mu$) along the particles world-line, namely

$$\dot{E} \equiv -m u^\mu \nabla_\mu (u^\nu \xi_\nu)$$

$$= -\alpha \frac{m^2}{m_p^2} |s \cdot \xi| \mathbf{R} - m u^\mu u^\nu \nabla_{(\mu} \xi_{\nu)}.$$ (2)

The last term on the r.h.s. of (2) encodes the standard change of $E$ associated with the non-Killing character of $\xi^\mu$ (e.g., it represents the effect of red-shift in cosmology), while the first term encodes the friction that damps out any motion with respect to $\xi^\mu$. Energy is lost due to the fundamental granularity until $u^\mu = \xi^\mu$ and the particle is at rest with the cosmological fluid, and thus the anomalous effect would cease.\(^3\) Note however that, in principle, one might have $\xi \cdot s = 0$ even when $u$ is not parallel to $\xi$, but as far as a gas of particles in thermal equilibrium is concerned (i.e. the situation considered in the previous works) that condition applies only to a set of measure zero and the inter-particle collisions will rapidly take any particle away from that set.

At this point, we should note that self-consistency of the proposal requires that, together with Eq. 1 above, one introduces a modification for the evolution equation for the spin, simply because by construction we ought to have $u \cdot s = 0$ at all times. In fact, the simplest expression at the same order of analysis as that of Eq. 1 is given by:

$$u^\mu \nabla_\mu s^\nu = \alpha \frac{m}{m_p^2} \text{sign} (s \cdot \xi) \mathbf{R} (s \cdot s) u^\nu.$$ (3)

This is just a minimalistic solution, other terms can be added if one makes further use of the direction $\xi^\alpha$.

These ideas were used in a previous work [10,11] to provide an account for the nature and magnitude of the dark energy component that became dominant in the latter epochs of the cosmic evolution. The point is that, although standard general relativity is inconsistent with any violation of the conservation of energy momentum ($\nabla_\mu T^{\mu\nu} = 0$), a relatively mild modification known as unimodular relativity can

\(^3\) It is unclear if one should think of this energy as being ‘transferred’ to quantum gravity degrees of freedom, or rather lost in entities without any permanence (in time) as would be the case for spacetime `defects’. On the other hand, it is worth noting that general considerations discussed in [17] indicate that energy conservation is a notion which, strictly speaking (i.e. as an exact statement), we would probably be forced to forgo. However, if those entities correspond to degrees of freedom in the usual sense of the word (with permanence in time) they could be relevant in dealing with the information loss puzzle in black hole evaporation [18,19].
readily accommodate it (provided that a certain integrability condition holds). The equations of uni-modular gravity are:

\[ R_{ab} - \frac{1}{4} g_{ab} R = 8\pi G \left( T_{ab} - \frac{1}{4} g_{ab} T \right) \] (4)

which, as we noted, admits \( J_a \equiv 8\pi G \nabla^b T_{ab} \neq 0 \). After some simple manipulations (and making use of the standard Bianchi identity), and assuming that \( d J = 0 \), one finds,

\[ R_{ab} - \frac{1}{2} g_{ab} R + g_{ab} \left( \lambda_0 + \int J \right) = 8\pi G T_{ab} \] (5)

with \( \lambda_0 \) an integration constant. In the homogeneous and isotropic cosmological setting the integrability conditions hold automatically. The point is of course that \( \lambda_0 + \int J \) will act, at late times, when \( J \) becomes negligible, as an effective cosmological constant. A simple use of kinetic theory allows for the evaluation of \( J \) resulting from our postulated friction effect during the relevant cosmological epochs where the effect is strongest. This procedure, together with the assumption that a protective symmetry operating at the Planck epoch fixes \( \lambda_0 = 0 \), leads to an estimate of the effective cosmological constant whose order of magnitude naturally (i.e. with \( \alpha \sim O(1) \) and no fine-tuning) matches the observations. The detailed study reveals that the effect is overwhelmingly dominated by the contributions coming from the epoch when particles first acquire mass, namely the electro-weak transition.

We ought to point out that the term sign \((s \cdot \xi)\) in Eq. (22) has a non-differentiable behavior in the limit where \( u \) becomes colinear with \( \xi \) and \( s \cdot \xi \to 0 \). This corresponds to the (non-relativistic) limit where the kinetic energy of the particle, as measured by co-moving observers, goes to zero. The form of this factor in this limit was not important in the analysis of \([10,11]\) because in that case the contributions to the formula were always dominated by effects coming from the relativistic regimes (temperatures higher than, or of the order of the mass of the particles involved). In effect, it is natural to expect that quantum field theoretical modeling of the effect would lead to modified versions with a smooth low energy behavior, which would however result in similar effects in settings involving relativistic particles. An example is provided by the replacement of such term by \((s \cdot \xi/u \cdot \xi)\) which behaves as sign \((s \cdot \xi)\) for \( u^\mu \) corresponding to ultra-relativistic motion as seen in the frame defined by \( \xi^\mu \). Other natural regularizations are provided by expressions such as \((s \cdot \xi/u \cdot \xi)^{2n+1}\) for a natural number \( n \). None of those variants would modify the results as far as the generation of an effective cosmological constant during the electro-weak transition era in cosmology. As we will see, however, those modifications could have important impact in the low energy regimes, and, in particular, in the manifestation of the type of effect in question during the latter stages in the universe’s evolution.

2.1 Macroscopic objects and the Planckian diffusion force

We now consider possible effects that the force implied by Eq. (1) could produce on macroscopic objects. The first issue is what is the appropriate manner in which
such question must be addressed. The point is that, in the context of GR, the motion of extended objects is, in principle, a highly complex matter requiring for instance a suitable notion of macroscopic localization (e.g. center of mass) and the study of the manner in which it evolves [20]. The point is that even in highly idealized situations the motion of something like a center of mass in GR does not correspond to a geodesic world-line, but is instead described (to the lowest order in curvature corrections) by effective equations like, for example, the Papapetrou Eq. [21]. The general treatment of these issues, is thus highly non-trivial, as follows from the fact that no simple covariant notion of ‘addition of forces’ to compute the total force, nor ‘addition of positions weighted by mass’ to compute a suitable ‘center of mass’ position.4 In the regimes where the weak field approximations hold, the question simplifies dramatically, and that is in fact what we encounter in most practical situations when dealing with GR.

Let us for instance consider two point particles undergoing the effect described by Eq. (1) in a situation where the spacetime geometry is well approximated by Minkowski and the relative velocities of the particles are small so that a Newtonian analysis is justified. The force on the center of mass is controlled by the sum of the forces, but that is very different from the force one would estimate by considering the whole system as a point particle placed at the center of mass undergoing the effect of Eq. (1). This follows, among other reasons, just from the fact that the angular momentum of the composite object about its center of mass is not simply the sum of the spins of the constituent particles. It is therefore clear that, even if Eq. (1) applies to the single elementary particles, it cannot be taken as a general recipe for the deviation from geodesic motion for an extended object. In that case the best manageable estimate of the effect is obtained by applying Eq. (1) to each individual constituent and then combining the effects in the appropriate manner, be it in a Newtonian or in a fully relativistic regime (where an analysis analogue to that of Papapetrou’s [21] would have to be used).

Let us consider next the magnitude of such effects for some simple astrophysical objects. As we noted, in the ‘Newtonian regime’ we should simply add the forces on all the constituents of the macroscopic body. As the force is proportional to the scalar curvature (that is dominated by the density for non-relativistic matter) it seems that the ideal candidate for a macroscopic astrophysical object maximizing our force would be neutron stars (or quark stars). Let us estimate the force in that case. We will also assume that the neutrons are highly polarized along a common direction. This is the configuration where the individual particle contributions would contribute coherently. It is certainly not representative of the typical astrophysical situation but it gives an upper bound for our force. It follows (using $|s| = 1/2$) that

$$|F| \leq \alpha \sum_i \frac{m_i^2}{m_p^2} R = \alpha \int n \frac{m^2}{m_p^2} R dV,$$

4 Considerations motivated by the fact that in the real world, the characterization of a particle as truly a point-like object (with infinitely precise localization at all times), cannot be taken as anything more than a very good approximation, lead to rather interesting conclusion as about the nature of geometry as an emergent entity [22].
where in the first equation the sum is over constituent particles (say neutrons) with masses $m_i$, and in the second line we introduced the number density $n$ of particles and assumed that $m_i = m$ for all particles. The inequality follows, in part, from the statistical fluctuations in the behavior of what we wrote as $\text{sign}(s \cdot \xi)$ in (1), and the fact that, in realistic situations, not all the individual forces will be aligned. As noted above, in the low energy regime, such a simple characterization of the anomalous force requires some regularization near $s \cdot \xi = 0$ which would naturally make the total force even smaller. Ignoring for the moment all the above caveats, using the fact that $\rho = mn$ and assuming the $R$ might be taken as approximately constant over the region occupied by the matter, we can write the following estimate:

$$|F| \leq \alpha \frac{mM}{m_p} R = \alpha \frac{mM}{m_p} \frac{8\pi \rho}{m_p^2} = \alpha 8\pi \frac{mM}{m_p} \frac{m^4}{m_p^2} m_p^2,$$

where $M = \int \rho dV$ is the total mass of the star. In the second line we used Einstein’s equations, and in the last line we used that $\rho \approx (1 \text{GeV})^4 \approx m^4$ as would be appropriate for a neutron star. In order to characterize the relevance of the effect through a dimensionless quantity, we can compare the new force with the usual ‘gravitational force’ binding the neutron star to a galaxy like ours. Such comparison provides an indication of whether the Planckian friction can generate substantial deviations from the motion of the neutron star away from its expected Newtonian trajectory. For a galaxy with mass $M_g$, and using the virial theorem, we know that (in average) we have,

$$M v^2 \approx \frac{G M M_g}{R},$$

where $v$ is the (mean) magnitude of the velocity of the neutron star. From the previous equation we find that $R \approx \frac{G M g}{v^2}$. The gravitational force is then

$$|F_g| = \frac{G M M_g}{R^2} = \frac{M v^4}{G M_g} = \frac{M v^4}{M_g m_p^2}.$$  

We can now compute the quantity of interest, namely

$$\left| \frac{F}{F_g} \right| \leq \alpha 8\pi \frac{mM}{m_p^2} \frac{m^4}{m_p} M_g M v^4 = \alpha 8\pi v^{-4} m^5 M_g m_p^4 M v^4 \approx \alpha (2 \times 10)(10^{12})(10^{-5 \times 19})(10^{12}10^{38}),$$

where we used $v \approx 10^{-3}$, $M_g \approx 10^{12} M_\odot$ and $M_\odot \approx 10^{38} m_p$. The result is then

$$\left| \frac{F}{F_g} \right| \leq 2 \alpha \times 10^{-32}$$

Thus for $\alpha \sim O(1)$ the force is negligibly small in comparison with the gravitational binding force for a neutron star in a galaxy. It seems clear that the previous bound will apply to any other star or macroscopic object for which the density is bounded by that of the neutron star, and for which spins will tend to average out to zero, leading to a
decrease in the value of the total cumulative force resulting from the ones described by Eq. (1) acting on each elementary particle constituent.

3 Granularity and diffusive effects in black holes

The successful prediction of the value of the Dark Energy component described in [10,11], corresponding to the generation of an effective cosmological constant during the electro-weak epoch, and which remains essentially constant afterwards, naturally suggests the search for a similar account for the recent observations that indicate that the dark energy content might have changed from recombination time until today. However, as the universe has cooled down about 17 orders of magnitude from the EW transition epoch the effects of the diffusion (1) applied to fundamental particles has become completely negligible. Thus that mechanism cannot produce the type of late time growth of $\Lambda$ that is indicated by the observations that lead to the $H_0$ tension. Nonetheless, a new (possibly unprecedented) situation of high density has become relevant in the later evolution of the universe with the appearance of galaxies and stars: this corresponds to the occurrence of a gravitational collapse leading to BH formation.\footnote{Although speculative ideas about the possibility of primordial black holes forming in much earlier times have certainly been contemplated.}

3.1 Black holes and translational energy diffusion from relational space-time granularity

Black holes share many features with fundamental particles in their relation with the external environment: when isolated, they are completely characterized by their mass, charge and spin with a gyromagnetic factor $g$ with the same value as the one predicted by the Dirac Eq. [23].\footnote{On the other hand, the considerations that allow the introduction of CoM in general relativistic contexts dealing with extended matter distributions [20,21] are grossly violated in the case of Black Holes, as clear-cut evidence of that we can point to the fact that such objects can exist and be produced as vacuum solutions.} This suggests that black holes might be the only kind of macroscopic objects for which the effects of the space-time granularity explored here could be described by an equation like (1) or something very close to it. Thus we will again postulate that, as a result of the space-time granularity of quantum gravitational origin, black holes would be affected by a friction-like term with the form:

$$u^\mu \nabla_\mu u^\nu = \bar{\alpha}_{bh} \frac{M}{m_p^2} \text{sign}(s \cdot \xi) \tilde{R} s^\nu,$$

(12)

where the new phenomenological dimensionless coupling $\bar{\alpha}_{bh}$ has been introduced. The point is that as a result of a possible Wilsonian ‘running’, or averaging procedure due to the macroscopic character of the black hole—keeping in mind the emergent nature of space-time geometry advocated here—that parameter needs not be the same as the $\alpha$ for fundamental particles in Eq. (1). That the new coupling $\bar{\alpha}_{bh}$ needs not be the same seems clear from the fact that its derivation would not only involve a
fundamental quantum gravity theory describing the relational discreteness, but also the detailed structure of macroscopic black holes in that theory.\(^7\) The factor \(\tilde{R}\) in the present case must be regarded as some average measure of the space-time curvature associated with the matter specifying the locally-preferential frame in the surroundings of the black hole;\(^8\) that is, it must be intimately tied to \(\xi^\mu\).

We will next consider the effects of such proposal for various processes occurring in the universe as a result of the late time formation of structure, and eventually, on the value of the effective cosmological constant. That is, we will consider black holes as the analogues of fundamental objects undergoing a similar effect at late times, as the elementary particles did in the electro-weak transition epoch, in the proposal contemplated in \([10,11]\). However, before any further analysis of that idea, we should obtain a rough estimate of the astrophysical importance of the effect of Eq. (12), by comparing its magnitude with that resulting from the relevant “gravitational (Newtonian) forces” in two interesting scenarios:

First, we compare its order of magnitude with that of the gravitational force at the moment of closest approach in black hole coalescence, \(F_{\text{BHC}}\). We estimate \(F_{\text{BHC}}\) for two equal mass BHs with the Newtonian expression when the separation is of the order of the Schwarzschild radius and the black holes are extremal so \(s = (M/m_p)^2\). This gives \(|F_{\text{BHC}}| \approx GM^2/(2GM) = m_p^2/4\) from which it follows that

\[
\left| \frac{F}{F_{\text{BHC}}} \right| \leq 4\alpha_{\text{bh}} \left( \frac{M}{m_p} \right)^4 \frac{\tilde{R}}{m_p^2} = 4\alpha_{\text{bh}} 10^{32} \left( \frac{M}{M_\odot} \right)^4,
\]

(13)

where we have used \(\tilde{R} \approx 10^{-120}m_p^2\), i.e., we have assumed that the mean scalar curvature around the black hole comes entirely from the contribution of the cosmological constant; we are assuming for concreteness that the environment of the black hole is not contributing to the definition of the curvature entering (12).\(^9\) The point is that

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\(^7\) It is clear that any attempt to derive these effective parameters would strongly depend on the fundamental theory embodying the granular structure of space-time along the relational lines described in the Appendix. Doing so is of course not feasible when such theory is lacking. One might try to get some idea of the possibilities by considering proposals made in quantum theoretical language and leading to similar effects at the level of fundamental particles—we note for instance the model describing a characterization of WKB trajectories of the Dirac theory on a pseudo-Riemannian geometry \([24]\)—and then, based on such theory attempt to incorporate radiative corrections and obtain a renormalization group analysis to estimate the running of the effective coupling with the mass scale of the object in question. That might provide an idea of what is possible. Such study is clearly outside the scope of this paper and in any event it is not completely clear that the result would be really representative of the generic case.

\(^8\) This is a delicate and complex conceptual issue, and we do not claim to have a clear control of all its ramifications as of yet. The point is that while it is true that GR is a purely local theory, we can expect quantum gravity to involve both, some of the local aspects of GR, but also some of the non-locality inherent to quantum theory, as demonstrated by the violation of Bell’s inequalities. Thus the relational notions involved when describing the quantum gravity granular structure of space-time might be expected to involve aspects that reflect a certain degree on non-locality. Of course, just as in the case of EPR and the situations considered in Bell’s results, we would expect those nonlocal aspects to coexist peacefully with relativity in the sense of not allowing faster than light communication. We will not pursue that interesting discussion further here but can point the interested reader to works such as \([25–28]\) for more analysis.

\(^9\) It is of course possible to consider our kind of model in which it is the galactic matter that is providing the source of \(R\) in Eq. (13), but in that case, consistency with the relational view described in \([10,11]\), would

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this already gives an extremely large effect if $\alpha_{bh}$ would be order one, which strongly suggests that, if the effect is there at all, the dimensionless coupling $\alpha_{bh}$ must be very small. We note, however, that the above estimation concerns extremal black holes and it would be negligible for BHs with vanishing rotation (see also the discussion in the next section). Moreover we note that as discussed in the Footnote 9, it would not be surprising if the general relativistic ‘dragging of the frames’ was somehow also affecting the granular space-time structure and thus came to play a role in this situation. In the local frame of ‘quasilocal observers’ near the horizon the black hole is seen as non-rotating (this is apparent in the analysis of [29]). That could play a role in accounting for the suppression mechanism for the force as described in macroscopic terms. Again ignoring such caveats, we will explore two natural possibilities for suppression in what follows: the first one where we assume that

\[
\bar{\alpha}_{bh} = \alpha_{bh}^{01} \frac{m_p}{M} \implies \frac{F}{F_{BHC}} \leq 4\bar{\alpha}_{bh}^{01} 10^{-6} \left( \frac{M}{M_\odot} \right)^3,
\]

and a less suppressed possibility where

\[
\bar{\alpha}_{bh} = \alpha_{bh}^{02} \sqrt{\frac{m_p}{M}} \implies \frac{F}{F_{BHC}} \leq 4\bar{\alpha}_{bh}^{02} 10^{13} \left( \frac{M}{M_\odot} \right)^{\frac{7}{2}}.
\]

Note that this type of suppression could arise naturally from some fundamental description if the process is mediated by some stochastic process where the number of ‘area quanta’ $N_A \equiv M^2/m_p^2$ or the number of ‘energy-quanta’ $N_M \equiv M/m_p$ plays a role. In such case, the supression we are considering in $\bar{\alpha}_{bh}$ would be the usual stochastic factor $1/\sqrt{N}$. Such deviations from classical expressions arise naturally in some studies of BH entropy calculations in loop quantum gravity [30]. Commenting on the immediate apparent phenomenological implications, while the second possibility seems more susceptible to being ruled out empirically, it is not obviously that it is at this point in contradiction with observations if the mergers observed so far have corresponded to non spinning black holes. However, in the second case, and unless $a/M < 10^{-13}$, a strong suppression in $\bar{\alpha}_{bh}^{02}$ would be implied by the fact that data from LIGO has so far come out very well adjusted to the standard expectations from pure GR.

We can also compare the force (1) to the Newtonian gravitational force binding the BH to its galaxy (9). Then we get the previous estimate enhanced by the factor

Footnote 9 continued

require that the local preferential frame $\xi^\mu$ associated with the granular space-time structure, would also be determined by the ‘mean notion’ of the local matter distribution. In the case at hand that matter is that which is presumably differentially rotating with the galaxy, thus making the exploration of the issue a much more complex task. For instance, one would have to consider the various possibilities for what exactly one is supposed to take as the ‘mean motion’ mentioned above and/or the size of the region one is supposed to average over in determining that. The point is that, in such case, it is rather unclear what would it mean exactly, for a black hole to be ‘slowed down’ by the granularity induced friction, in its motion relative to the surrounding gravitating media. That kind of study, although in our view rather worthwhile, falls well outside the scope of what we intend to to do in the present manuscript.
\( M_g/(4Mv^4) \), and the non relativistic factor \( v^{2n+1} \) (recall discussion at the end of Sect. 2), namely

\[
\left| \frac{F}{F_g} \right| \leq \alpha_{bh} 10^{44} \left( \frac{M}{M_\odot} \right)^3 v^{2n-3} = \alpha_{bh} 10^{53-6n} \left( \frac{M}{M_\odot} \right)^3,
\]

(16)

where we used \( M_g \approx 10^{12} M_\odot \), \( M_\odot \approx 10^{38} m_p \), and we used \( v \approx 10^{-3} \). Thus, once more, unless \( \alpha_{bh} < 10^{-56+8n} \), the model would imply large deviations from the standard predictions of general relativity for spinning BHs of intermediate masses. In the two possible scenarios proposed in (14) and (15) we get

\[
\left| \frac{F}{F_g} \right| \leq \alpha_{bh}^{01} 10^{15-6n} \left( \frac{M}{M_\odot} \right)^2,
\]

(17)

and

\[
\left| \frac{F}{F_g} \right| \leq \alpha_{bh}^{02} 10^{34-6n} \left( \frac{M}{M_\odot} \right)^2.
\]

(18)

Note that the integer \( n \) controlling the low energy smoothing of the sign function in (1) may affect in an important manner the order of magnitude of the effects, and can, for low values—like \( n = 3 \) in the first case, and \( n = 6 \) in the second—completely dominate over the other large factors.

We note, however, that all the above bounds have been computed with the assumption that the BH is close to maximally rotating, i.e. the assumption that

\[
\sqrt{s_{BH}} \approx \left( \frac{M}{m_p} \right)^2 = 10^{76} \left( \frac{M}{M_\odot} \right)^2.
\]

(19)

The force is enhanced by this huge factor for highly rotating BHs but it could be substantially reduced for black holes that have negligible rotation. We will consider below a mechanism that would assure this for sufficiently old black holes allowing the exception of those that have just been formed from a gravitational collapse or a collision. This means that a very interesting path for the study of our proposal, potentially setting important constraints on the model’s parameters, is open by the detailed studies of the post collision part of the gravity wave signals, including the “ring down”, which are already leading to constraints on deviations from GR at the 10% level [31].

We will discuss the possible phenomenological implications of this effect on galaxies in Sect. 4. When it comes to the contribution of such forces to cosmological evolution what one needs to estimate is the total amount of energy that the mechanism can actually diffuse, and thus, according to the general ideas developed in [9–11], bring about a change in the effective value of the cosmological constant. It is clear that a precise analysis would necessitate a solid understanding of the cosmological distributions of black holes as a function of masses, spins and velocities and their evolution throughout cosmic history.
However, it is easy to see that the present effect would naturally be overshadowed by another one, namely the one associated with the rotational friction we will discuss next. The argument is rather simple, independently of the value of the parameter $\alpha_{bh}$, the effect on the cosmological constant is bound by the magnitude of the kinetic energy associated with the peculiar velocities of the black holes relative to the frame defined by their local gravitational environments (the mean motion of all matter in their environment). The black holes would be generically moving with non-relativistic velocities of the order of the mean velocities of stars in galaxies, i.e. $v \sim 10^2 \text{km s}^{-1} \sim 10^{-3} c$. Therefore, each each black hole could contribute at most with something of the order of $\frac{1}{2} M_{BH} v^2 \sim M_{BH} 10^{-6}$. That is, there would be at most $10^{-6}$ of $\rho_{BH}$, the energy density in black holes, as an available source to contribute to our effect. On the other hand, a rapidly rotating black hole can, in principle, lose a substantial fraction of its total energy if a friction-like effect of the type put forward here could completely stop its rotation in cosmological times.

### 3.2 Black hole rotational energy diffusion

Black holes are normally expected to be born with rather high values of spin [32]. In fact, the most natural scenario calls for black holes to be formed at or near extremality, simply because the single most important effect countering the gravitational collapse of gravitationally bound matter (for which usual physical pressures are not longer sufficiently relevant) is the conservation of angular momentum and the ‘centrifugal’ barrier it offers against gravitational collapse. Thus, the gravitational collapse can be expected to take place rather rapidly once the angular momentum barrier is finally overcome, and this will typically be associated with situations where the angular momentum is close to maximal for that condition. In other words, it seems rather natural to assume that a substantial proportion (and even the great majority) of black holes are born at or close to extremality. 10

The very fact that black holes are systems that carry huge values of entropy (as the combination of Einstein’s equations and quantum field theory imply) suggests that Planckian micro-states could play a relevant role in their dynamics. In this respect it is important to recall that black holes close to extremality do not seem to represent, from the entropic point of view, a most natural configurations. That is, for a given mass, an extremal black hole is very far away from a maximum of entropy which corresponds to the non-rotating Schwarzschild configuration.

It is thus natural to expect that nature would ‘find’ some physical mechanism which would tend to drive situations involving rotating black holes towards non-rotating ones. The Penrose process and Hawking radiation are two known effects that would contribute to this. Penrose’s process, however, requires the accretion of matter with appropriate distribution of angular momentum from the BH surroundings, and thus would not be active generically. Hawking radiation involving preferential emission of quanta carrying away angular momentum would be another process contributing to that, but it would be an extremely slow one. On the other hand, a friction force,

10 Although effective mechanisms of angular momentum transport within the BH progenitor might offer paths to alter such generic scenario [33].

\[\text{Springer}\]
as the one we are considering here would result in a natural diffusive effect acting in the appropriate direction and, as we will see, could potentially be a rather efficient one in cosmological time scales while producing only small deviations from general relativity for the natural timescales the black hole. This is exactly the type of effect that could have remained unnoticed given the limited observational data available so far.

From the point of view of the relational space-time granularity ideas that underlay our approach to these issues, it seems quite natural to expect such a friction-like effect. Indeed, if continuity is replaced by something discrete at the Planck scale, then it is natural to expect that (as in concrete approaches to quantum gravity) deviations from exact rotational invariance would open a channel for the non-conservation of angular momentum. This would be a genuine quantum gravitational effect that would drive black holes to their maximum entropy non-rotating state. In other words, as the granular aspects of the gravitational environment of the rotating black hole will not possess the exact axial symmetry expected from the purely classical perspective, it is natural to expect that an associated violation of conservation of angular momentum would ensue.

Now, we consider what is the amount of energy associated with the rotation of a black hole that can become available for dissipation via such process. The answer to this question is far from evident given that our effect is supposed to have a quantum gravity origin, and we, of course, do not have anything like a workable full quantum gravity theory to investigate the issue (or to derive our hypothetical effect). There is, however, a rather simple estimate that can be obtained by considering that, during the spin down process the entropy of the black hole itself should not decrease.\footnote{Note, however, that this is not evident in the present context because we are dealing with effects of a fundamentally quantum gravitational nature, and, as is well known, even QFT effects might lead to violations of the $dA/dt > 0$ classical expectations.} For concreteness we will assume from now on that the process is ‘adiabatic’, i.e., $A =$ constant. That is, we will consider for simplicity the case where all energy associated with rotation that was free to dissipate quasi-statically under the constraint that the area of the BH did not decrease.

In that case, the analysis is simple. The area of the event horizon for a Kerr black hole with mass $M$ and angular momentum $J$ (and thus $a = J/M$) is given by $A(M, a) = 8\pi M (M + \sqrt{M^2 - a^2})$. This means that the mass as function of the area is just given by

$$M = \frac{A}{8\pi} \frac{1}{\sqrt{\frac{A}{4\pi} - a^2}}.$$  \hspace{1cm} (20)

From the previous formula it is easy to see that, in the transition from an extremal ($a^2 = A/(8\pi)$) to a non-rotating black hole ($a = 0$) of the same area $A$, the amount of energy made available for dissipation is

$$\Delta M = \frac{2 - \sqrt{2}}{2} \sqrt{\frac{A}{8\pi}} \approx 0.3 \sqrt{\frac{A}{8\pi}} = 0.3 M_{\text{ext}},$$  \hspace{1cm} (21)
where $M_{\text{ext}}$ is the initial mass of the (close to) extremal black hole.\footnote{This is a well known result. See for instance \cite{34}.} In other words, there would be up to 30\% of the energy density in black holes available for dissipation under the above discussed conditions. As we see, it is indeed the case that (in ordinary conditions) the translational energy available in the peculiar motions of BH in galaxies is simply negligible compared with that available in their rotation.

We assumed that the process by which friction would slow down the spin of the BH is adiabatic. This assumption leads to the simple estimate of the energy loss (\ref{21}). There is another extreme situation (the most entropy producing one) which corresponds to the case where under an infinitesimal change $\delta M = 0$ and the diffusion process converts all of the work $\delta W = \Omega \delta J$ into heat (from the first law of BH mechanics $\kappa \delta A/(8\pi) + \Omega \delta J = 0$). Only a precise understanding of the microscopic physics can decide where between these two extreme situations physics lies. The previous analysis is therefore to be taken as an order of magnitude phenomenological estimate. However, note that even though the extreme situation where $M$ remains constant would make the contributions to the evolution of dark energy negligible, this would still have important phenomenological implications at the astrophysical level.

It should be noted that, although the above estimated efficiency bound seems rather solid when considering a single black hole subject to the friction torque we are proposing, this might not be the total efficiency of each single black hole in its contribution to energy dissipation on cosmological time scales. The point is that it is entirely possible that after two such black holes have lost their individual spins, they might collide with a high value of total angular momentum, leading, after the emission of gravity waves (now possibly modified by the kind of effect we have in question), to a standard highly rotating black hole. Such black hole would then undergo the same process as described above, contributing further to the energy dissipation. Thus, the net combined efficiency of our process could be higher than the 30\% estimated above. The detailed analysis of this possibility depends again on the evolution and distribution of black holes in galaxies and their resulting collision probabilities.

### 3.3 The explicit form of the black hole spin friction

We have argued generically that it is natural to expect the kind of friction inducing relational granular structure of space-time to affect the spin of the rotating black hole. Moreover, as noted in the discussion around Eq. (1), the consideration of a deviation from geodesic motion, such as that in Eq. (\ref{12}), cannot be done without, at the same time, including a modification of the evolution law for the spin of the object in question.

However, in contrast with the case of a fundamental particle, for which the spin is a basic quantum that cannot be changed, the black hole spin is a macroscopic quantity that can be lowered via the friction mechanism with the fundamental granularity. This allows for a new term in the spin evolution equation. Concretely, in the case of black holes we must postulate:

$$u^\mu \nabla_\mu s^\nu = \bar{\alpha}_{\text{bh}} \frac{M}{m_p^2} \text{sign}(s \cdot \xi) \tilde{R}(s \cdot s) u^\nu - \bar{\beta}_{\text{bh}} \frac{M}{m_p^2} \tilde{R}_{\text{BH}} s^\nu, \quad (22)$$
where $\bar{\beta}_{\text{bh}}$ is a new dimensionless coupling controlling the “intrinsic” spin diffusion term. The quantity $\tilde{R}_{\text{BH}}$ is a measure of the local curvature around the black hole. Such quantity could be associated to some averaged value of the local Kretschman scalar $\sqrt{R_{abcd}R^{abcd}}$, in the region occupied by the black hole, or something along those lines. We will not try to be very precise on that at this point because it is really not necessary. Whatever the correct quantity would be, it will have to be of the order of the inverse square of the characteristic size of the black hole which can be defined in terms of the horizon radius. Thus we take

$$\tilde{R}_{\text{BH}} \approx \frac{1}{r_{\text{bh}}^2}$$ (23)

The new term is responsible for an exponential decay of the black hole spin due to ‘rest frame torque’ on the rotating black hole. Namely, the previous equation implies

$$u^\mu \nabla_\mu (s \cdot s) = -2\bar{\beta}_{\text{bh}} \frac{M}{m_p^2} \tilde{R}_{\text{BH}} (s \cdot s), \quad (24)$$

In other words, the evolution of the black hole’s spin in its rest frame is simply $s^2(t) = s^2(0)e^{-2t/\tau_s}$ or $||s(t)|| = ||s(0)||e^{-t/\tau_s}$ with $\tau_s^{-1} = \bar{\beta}_{\text{bh}} M R_{\text{BH}}$. Then the life-time of the black hole’s spin can be estimated to be

$$\tau_s = \frac{2 \bar{\beta}_{\text{bh}}}{M M_\odot} 10^{-5}s \quad (25)$$

That is, for $\bar{\beta}_{\text{bh}}$ order unity, the lifetime would be extremely small, and just given by the classical BH characteristic time scale. This could have rather large effects potentially observable in LIGO, which is a rather exciting possibility. On the other

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13 The reader would understandably be concerned by the fact that there are two distinct measures of curvature entering the above formula referring to a single black hole. The point is that the first term is intimately connected with the one occurring in Eq. (12) which characterizes the translational friction experienced by the black hole as a result of its translational motion with respect to the matter that generates the mean curvature of the space-time. The black hole is, of course, translationally at rest with respect to its “own reference frame” (i.e. the one defined by its own four momentum as seen from a sufficiently large distance). Thus, the curvature entering in that term should not involve that associated with the black hole itself. On the other hand, it is well known that the rotational motion generically is connected with differential effects such as “dragging of the frames” and thus, from our point of view, it can be tied to frictional type effects between the “grains of space-time” that are, correspondingly, associated with different rotational velocities. All these considerations are, of course highly, speculative and tied to the heuristic picture outlined here and elsewhere [10,11,15,16]. The formulas must therefore be looked upon only as “educated guesses” to be applied at the phenomenological level. A fundamental theory of quantum gravity sufficiently developed to work out all the issues in detail, would be a prerequisite for a more solid justification of these expressions.

14 In this analysis, we are ignoring the time dependence of the mass implied by the effect as it would be a correction that does not affect the order of magnitude estimate.

15 It should be pointed out, however, that the strict analysis of the situation is a bit more delicate than what was presented here. The point is that there is a competing process which can be characterized as a type of precession driving the spin orientation into the plane that is perpendicular to both $\mathbf{u}$ and $\mathbf{\xi}$. In fact, the equation above can be used to compute the proper time evolution of $q \equiv \xi \cdot s$ which takes the form
hand, life-times comparable with cosmological scales would clearly be possible if the coupling constant is sufficiently weak. For instance if we take $\beta_{bh} = \sqrt{m_p/M} = \sqrt{M_\odot/M} 10^{-19}$ we would get:

$$\tau_s = 2 \sqrt{\frac{M}{M_\odot}} 10^{19-5} s \approx \frac{1}{158} \sqrt{\frac{M}{M_\odot}} \text{byrs.}$$

(26)

Note that for this simple estimate we have used $r_{BH} = 2M$ which is incorrect if the black hole has spin. However, this gives the right order of magnitude and simplifies the analysis. With the choice (23) there will be indeed an additional spin dependence in Eq. (24), which would have a more complicated solution, but with similar qualitative behavior so that the previous life-time estimate would still be reasonable. The previous expression is just an estimate from the natural ansatz $\beta_{bh} \approx \sqrt{m_p/M}$. The correct estimate of such effect necessitates a deeper understanding of the fundamental nature of black holes in quantum gravity. The key point here is that for a small enough coupling (say $\beta_{bh} \lesssim 10^{-19} \sqrt{M/M_\odot}$) the numbers become interesting: the previous ansatz for $\beta_{bh}$ gives a spin lifetime of the order of 6 million years for a solar mass black hole, while the life-time for the spin of a super'-massive black hole remains of the order of or larger than the age of the universe. But all this has to be discussed together with the effects on the translational degrees of freedom controlled by the magnitude of $\alpha_{bh}$.

4 Possible phenomenological consequences in galaxies

Before studying the cosmological effects of our proposal on the evolution of dark energy, we will briefly discuss possible (more local) consequences regarding galaxies. In order to organize the discussion, we will only consider the two models introduced in (14) and (15). It is clear that, from a phenomenological perspective, other possibilities should be explored. To give an idea of the type of effects that the new physics we have put forward can have, we will concentrate on these two natural examples.

4.1 The weaker translational coupling (14)

A possible consequence of this scenario is that black holes of masses of the order of solar masses should be closer than expected to galactic centers or they may even be

Footnote 15 continued

$$\frac{dq}{d\tau} = -B \text{sign}(q) - Cq + H s^0 u^0,$$

where $B = -\alpha_{bh} \frac{M}{m_p} u \cdot \xi \cdot s \cdot s, C = \alpha_{bh} \frac{M}{m_p}$ and $H$ is the Hubble parameter. We note that as $s^0 u^0$ has opposite sign as $q$ and vanishes when $q$ vanishes, all the terms tend to decrease the value of $q$. Thus, at the same time that the spin decreases it also tends to align along directions orthogonal to $\xi$ ( and $u$). However, all of the above ignores, of course, possible interactions with other celestial bodies and thus a phenomenologically precise study of the situation will be much more involved that what we intend to do in this first analysis.
falling into the central super-massive black hole. The reason is that the friction might be more important than the Newtonian force when they are spinning (recall estimate (17)). More precisely, let us assume in this section that $\alpha_{bh} = \alpha_{bh}^0 m_p/M$ and that $\beta_{bh} \approx \beta_{bh}^0 \sqrt{m_p/M}$ (and also take $n = 0$ in (17)). With these numbers, a solar mass BH would stop in 6 myrs, the force ratio (17) would be about 0.1 at its maximum. This is probably an effect that is too weak and lasts too short of a time to have important consequences. However, for a 100 solar mass BH the spin will last for about 60 myrs, and the force (17) would reach a maximum of about 100 times the Newtonian binding force. Before the spin goes down and the friction disappears the energy loss via our mechanism would induce such black holes to fall towards an orbit closer to the galactic center or even to merge with the central black hole. On the other hand, super-massive BH’s are thought to spin for times comparable to the age of the universe. In that case, our model indicates that those must be always in the center of galaxies, never alone (without an important accretion disk selecting their rest frame) orbiting around other isolated sources. As far as we know this is compatible with observations.

Therefore, if the order of magnitude of the couplings proposed in the previous paragraph are correct, the spin friction would be quite important for (highly spinning) black holes of the order of a few solar masses so that they would stop rotating quickly in galactic times (a few hundred million years myrs for the sun’s galactic orbit). However, the deviations from Newtonian translational motion would not be an important effect in those cases, because the force is small and lasts for too short of a time to produce appreciable deviations. This mechanism would imply that intermediate mass black holes found in galaxies would strongly populate the low spin part of their phase space, either because they were born with low spin or because they have lost it via the friction that we postulate here. This seems a rather attractive scenario, phenomenologically speaking, because of the data suggesting that progenitor BH’s observed via gravitational wave astronomy have unexpectedly low spins (see [35]).

On the other hand, larger mass black holes ($M \gtrsim 100 M_\odot$) have spins lasting longer, while the translational force becomes more important according to (17). Such black holes would have gone through a dramatic slow down in their motion around the galaxy and, as a result they might have fallen down into the galactic center coalescing with other black holes there. Such process could be relevant for the formation and/or the growth of super-massive black holes in the galactic centers. This might account for the origin of super-massive black holes, as well as for the generic correlation between the galaxy’s mass and the mass of the central BH [36]. We note that, if just a fraction of about $10^{-7}$ of the Milky way’s galactic halo mass (estimated at about $10^{13}$ solar masses), was made up of rapidly rotating solar mass black holes, then within a period of the order of a few hundred million years, they might have led to enough mass accretion into the central galactic black hole to account for most of its mass (estimated to be about 4 million solar masses).

A related effect would be a higher than expected rate of mergers between super-massive BHs in galactic centers and medium mass BHs. The gravitational signal of such events would be in a low frequency range, and hence would have remained undetected via gravitational wave detectors so far. However, they would produce secondary effects due to the disruption of the local accretion disks around super-massive BHs. A
detailed study is required to analyze the expected frequency range and establish their possible detectability and expected event rate by, say, advanced LIGO and LISA, as a function of the model’s parameter.

One place in which the effect might become relevant is in BH mergers. The estimate (14) implies (at least in the range of values of couplings explored in this section) that such effects could become comparable to the gravitational forces for spinning black holes with $M \gtrsim 100M_\odot$, and would already be at the 3% level for $M = 30M_\odot$. Note that these numbers would be suppressed by factors of the order of $a/M$, so high spin values is what would maximize signals. Such effects could become measurable in the near future if spinning black hole collisions are detected.

Finally, recall that in the estimates (17) as well as (13) we are assuming the existence of a background scalar curvature $R$ of the order of the present value of the cosmological constant, i.e., $R \approx 10^{-120}m_p^2$. This is important because the translational part of the friction force depends on having produced the cosmological constant by the time black holes form. This would be in agreement with the scenario of [10,11] as the cosmological constant is quickly generated during the early evolution of the universe around electroweak transition time. We will see in the next section that the energy lost via the friction effects postulated in the previous section can make this initial cosmological constant evolve into a higher value, and that that can alleviate (and possibly resolve) the so-called $H_0$ tension. These specific phenomenological aspects will be considered in some detail in a companion paper [37].

4.2 The stronger translational coupling (15)

For the situation where the suppression associated with the parameter $\alpha_{bh}$ is not as strong as in the previously considered case, the effects of our model would be rather dramatic. However, there will be a strong interplay between the rotational and translational aspects of the model. That is, either the rotational friction would be rather effective and would bring down all rotation of the black holes rapidly to essentially vanishing values (and in particular the progenitors black holes involved in LIGO would have $a/M < 10^{-13}$), or the translational friction would rapidly stop the rotation of the black holes around the galaxy, and they would therefore quickly fall towards the center, merging rapidly with the super-massive black hole of the corresponding galaxy. Once more, with a fraction of the order of $10^{-7}$ of the Milky way’s galactic halo mass made up of rapidly rotating solar mass black holes, the ensuing mass accretion into the central galactic black hole could account for most of its value, but in the present scenario that process would take a much shorter time. The case of the galaxy M87, taking its halo mass to be of the same order as that of the Milky Way, and with a central black hole of about $6 \times 10^9$ solar masses, would presumably indicate a situation in which a larger proportion of the halo (made up of rotating black holes) would have undergone accretion into the super-massive central black hole. Such situation could be more naturally accounted for within the stronger translational coupling scenario.

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16 We note that M87, an elliptical galaxy, has about 10 times more stars than the Milky Way for about the same total mass. This feature suggests the possibility of a larger rate of black hole formation than that of our galaxy.
More generally, the formation of a super-massive black hole, as usually understood, requires a relatively high mass density located within a relatively small volume and with low angular momentum. The usual process of accretion is thought to be responsible for removing from the system the excess angular momentum that can be presumed to have been present at the start of the gravitational collapse. Thus the efficiency of that process would be the limiting factor in black hole growth. On the other hand, observations indicate that quasars were much more frequent in the early universe, favouring a scenario of early formation of super-massive black holes. Once more, the stronger translational coupling scenario, which clearly involves a strong diffusion of orbital angular momentum for the early black hole population, of say, 30 solar mass black holes, which, in turn, was presumably the result of the rapid gravitational collapse of first generation giant starts, might help in accounting for that, still unexplained, feature of our universe.

The detailed study of such scenario clearly deserves close attention, but that task is beyond the scope of the present paper. The point, however, is that the model is clearly susceptible to empirical examination, a rather welcome feature in a field where most models seem to be either very hard or sometimes even impossible to be explored observationally, beyond the specific feature for which they were designed to account for.

5 Possible resolution of the $H_0$-tension via diffusion from black holes

The considerations above illustrate how the basic equations we proposed in [10,11] to describe the noisy -energy-diffusion of fundamental particles due to Planckian granularity could be extended (and suitably modified) to describe, at the effective level, a similar friction-like effect on both the translational and rotational motions of black holes. In the case of fundamental particles, and as analyzed in detail in [10,11], the diffusion of energy can generate a dark energy component of the order of the present cosmological constant during the cosmological electro-weak transition epoch. Similarly, if the effects we have put forward here apply to black holes, then the associated energy diffusion might have a significant impact in the evolution of dark energy at late cosmological times. However, in order to make a detailed analysis of the effect we would need to have a rather clear and detailed understanding of the cosmological history of black hole formation, their relative abundances, the fraction of dark matter that might have been involved in their formation, their distribution

17 One might, at first sight, worry that in order to deal with the effect we have been considering in the context of black holes a complex and detailed study of the effect on each individual black hole must be undertaken. The point is that, just as we can proceed to do cosmology at late times considering all galaxies, the intergalactic gas, etc., as collectively representing a fluid that, at a certain level of approximation can be taken as homogeneous and isotropic, as far as cosmological studies are concerned, we should be able to proceed in the same way in dealing with the black hole component of such fluid and the dominant aspects that our effect would have at that level of analysis. It is clear, of course, that the more detailed study of these question at the local level, could be expected to become much more complex, just as the internal dynamics of galaxies is a much more complex problem than that of estimating their aggregate and averaged contribution to the cosmological mean density. We expect to undertake such studies of the effects of our hypothesis on the detailed dynamics of black holes in the near future.
inside galaxies, and their dynamical behavior during galaxy formation and growth. The point is that, although enormous amount of work has gone into modelling all these issues, the fact is that there seems to be at the present time no clearly unified consensual view regarding such questions. As far as we know, it is possible, although it is usually taken as very unlikely, that black holes might have formed directly from dark matter clustering in very early times (see however [38–41]). In such rather speculative scenario, the usually expected fragmentation of some gigantic dark matter clouds did not take place to a sufficient extent, and the gravitational collapse of large enough parts of the primordial cloud could have led directly to the formation of some super-massive black holes, even before galaxies formed and stars first lightened up. The point is, of course, that we still do not have a solid account of how such supermassive black holes formed. The above scenario would have supermassive black holes forming very early after decoupling, and, for rotational life times like the one contemplated in Eq. (26), the energy flux into the dark energy sector could continue until the present times. On the other hand such scenario might or might not coexist peacefully with recent surveys indicating rather large spin values for most super-masive black holes [42].

The observational constraints on primordial black hole abundances, in fact, allow for over 10% of $\Omega_{\text{DM}}$ (depending on the precise value of BH mass) to be in the form of black holes for BH masses up to 1 solar mass [43]), with sharp decreases for more massive black holes, coming from the CMB and for much less massive ones, from micro-lensing studies [43], as well as bounds on even smaller primordial BH’s from consideration of Hawking radiation in the $\gamma$-ray part of the spectrum. On the other hand, there are intriguing and yet unexplained correlations between the galactic structure and dynamics and the central super-massive black hole usually found at the galactic center [36]. All these questions would need to be considered with great care in a detailed analysis of the precise interplay between the effect we are proposing and the cosmological evolution of the black hole distribution functions.

What we seem to have a much more solid handle on is the amount of matter that is available for black hole formation starting at the CMB and the amount of matter that must be left at late times as indicated by galaxy and galactic cluster mass estimates. The data at this time allows for a gap. The idea is that a fraction of the matter both baryonic and dark that was present at the CMB went to form black holes, and that a good portion of that energy took the form of rotational and kinetic energy of those black holes. According to our ideas, a fraction of that energy was dissipated as a results of the friction-like forces and torques effectively generated by the space-time granularity of quantum gravitational origin. In that case, such energy dissipation would lead through the dynamics of uni-modular gravity to a change in the value of the cosmological constant, in conjunction with an anomalous decrease of the cosmological matter density component. That is, the “cosmological constant” would change in

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18 This would provide a physical mechanism for the effect described in Eq. (27) below and some of the models, specifically those involving a continuous energy transfer, which are described in more detail in the companion paper [37].

19 The detailed structure of the bounds as a function of black hole masses can be seen in figure 6 of [43]. We note however that the analysis carried out there seems to focus on a purely monocromatic case (i.e. where all black holes had the same mass) rather than on what seems more natural, namely a broad spectrum of black hole mass distribution.
tandem with an anomalous deviation from the dust $1/a^3$ behaviour of the matter density. Both effects would mimic a deviation from the expected equation of state making $w_{DE} \equiv (P/\rho)_{DE}$ (DE stands for Dark Energy) seem less than $-1$, while making $w_{\text{matter}}$ appear as anomalous (it would in fact become negative in the limit where we take matter as “powder” with its ordinary pressure strictly vanishing).

The correct interpretation comes from the modification of the continuity equation for the matter density component in the context of unimodular gravity, which takes the form:

$$\dot{\rho}_{\text{matter}} + 3\rho_{\text{matter}} \frac{\dot{a}}{a} = -\dot{\rho}_\Lambda$$

and can be written in the more convenient form

$$\frac{d(a^3 \rho_{\text{matter}})}{dt} = -\dot{\rho}_\Lambda a^3,$$

where $\rho_{\text{matter}}$ and $\rho_\Lambda$ denote the (dark matter plus baryon) matter density and the (time dependent) dark energy energy respectively. These equations follow directly from the Bianchi identity and Eq. (5) specialized to a perfect fluid and the FLRW cosmological case.

The exact analysis of the evolution of both $\rho_{\text{matter}}$ and $\rho_\Lambda$ requires detailed modeling of the fraction

$$f_{BH}(t, M, a) = \frac{\rho_{BH}(t, M, a)}{\rho_{\text{matter}}(t)}$$

of matter that is present in the form of black holes at each time $t$ during the cosmological evolution as a function of their mass $M$, angular momentum $a$, as well as the efficiency $\epsilon(a)$ of energy dissipation. Taking the simplifying assumption that all black holes are born as extremal ones, that $\epsilon(a = M) = 0.3$, and that the process of energy dissipation is rather fast on the relevant cosmological scales, one would be able to compute things explicitly.

However, in the absence of a clear understanding of the cosmological evolution of black holes distribution, and of a clear answer to the question of how much dark matter can be converted into black holes, is it still possible to make an estimate of the contribution of our mechanism to the late cosmological evolution of dark energy? It seems that, despite the uncertainties, one can still produce an encouraging and rather safe estimate in terms of two basic ingredients: On the one hand, by the requirement that a sufficient amount of matter present at the CMB has not become black holes and has remained to account for what we see at present in galaxies, galactic clusters, inter-stellar gas, etc. On the other hand, the demand that at most 30% of all the matter that went into forming black holes (recall the efficiency estimate in Eq. (21)) would be dissipated into feeding the dark energy component according to Eq. (28). We should note that it is entirely conceivable that an important fraction of the dark matter known to be there in galaxies and galaxy clusters takes the form of black holes (most of which, by our arguments, would be non-spinning), and that a significant fraction of that was not directly attributable to baryonic matter which is known to represent about $0.04/0.3 = 0.13$ of the total matter density at the time of last scattering, which itself...
represented about 75% of the critical density at that time (the dark energy contribution at that time is negligible, but the photons and neutrinos, unlike what occurs in the present, do contribute significantly). In fact, it seems that the baryonic matter that we can directly observe as luminous components (or light absorbing gas and dust) does not really account for all baryonic matter that was present at the CMB and at nucleosynthesis [44]. And thus it seems attractive to consider a scenario in which an important fraction went to form black holes, with part of that energy dissipated via the kind of process contemplated in this work.

In a companion paper [37] we have considered some simple models of the situation discussed above, and have found that such simple scenarios can naturally account for the now famous “$H_0$ tension”. The basic outcome of the analysis presented indicate that having as little as a few % of the matter (including dark and baryonic components) originally appearing at the CMB being dissipated via the process we have outlined, taking place during the intervening period between the Last Scattering Surface (LSS) and today, it is possible, through the use of the equations of uni-modular gravity to account for the so called $H_0$ tension without negatively affecting the overall viability of the modified $\Lambda$-CDM model, as far as late time observations is concerned. Needless is to say that the modified model presented here does not imply any deviation from the standard $\Lambda$-CDM model regarding the conditions at the LSS or before.

### 6 Discussion

We have considered a scheme that would unify the successful estimate of the order of magnitude of the value of the cosmological constant presented in previous works with a related process having several potential observable consequences, and which, in addition, has the potential of accounting for certain behavior of the dynamics of the late universe that seems to indicate real tensions in the standard $\Lambda$CDM model.

The original model for a generation of a cosmological constant relied on the hypothesis of a ‘friction inducing’ granularity of space-time resulting from quantum gravity, together with the use of unimodular gravity, a theory, that, in principle, allows for departures from the exact conservation of energy momentum tensor. In the present paper, we extend these ideas arguing that, while they would be unsuitable for application to ordinary macroscopic bodies, they might apply with only small modifications to black holes. The outcome of such effects would be the change during the relative late cosmological times of the value of the cosmological constant, together with a change in the matter content of the universe. Additional consequences of the proposal include: an account for the fact that gravity wave detection of black hole collisions up to this time seem to involve progenitor black holes with very low or no spin. A natural account for the whereabouts of the invisible baryonic matter [44]. We have mentioned a scenario in which supermassive black holes would form very early on after decoupling and lose their rotational energy in cosmological times scales. That would offer a mechanism for a continuous change of the dark energy component, and

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20 Although recent studies indicate a significant portion of the missing baryons might have been found [45].
thus would fit with some of the detailed phenomenological models for resolution of the $H_0$ tension described in the companion paper [37]. We have also briefly considered scenarios in which the flux of energy from the matter to the dark matter sector would start later on in the cosmological history, once black holes have formed by the usual mechanisms (i.e. those involving gravitational collapse of stars of various sizes at the end of their normal cycle). Such scenarios would underly the physics behind other models for resolution of the “$H_0$ tension” treated in [37], as well as a natural path for the growth of the super-massive black holes at the center of galaxies, and other potential consequences discussed in Sect. 4.

Much more work will be needed to help establishing whether the present account is supported by further analysis and observations. In the meanwhile, we consider the proposal as a rather conservative one in comparison with those currently available, in the sense of connecting the account for the rough value of the cosmological constant obtained in previous works [10,11] with the details of its late time variation, into a relatively unified model, while requiring no exotic equation state for the dark energy sector. The fact that the latter is often postulated in a completely ad hoc manner with the only purpose of fitting the data, while offering no clear path for the independent study of the proposals, is compounded, in our view, with the fact, which we see as rather problematical (particularly when associated with a classical regime), that such equations of state, usually involve violation of the dominant energy condition.21

We end by acknowledging the highly speculative nature of the present proposal, while emphasizing one of its very attractive features: The possibility of its empirical exploration. We do hope that the present manuscript would stimulate other colleagues into contributing to the exploration of these and related ideas.

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Appendix A: On possible manifestations of space-time discreteness resulting from quantum gravity

This is a brief review containing the essence of the historical development of the considerations and ideas motivating the present work.

Over 20 years ago, there was a period of great excitement concerning searches for possible phenomenological signatures of granularity of space-time tied to quantum gravity (for a review see [46]). The proposals relied on the idea that such a granularity of space-time (presumably at the Planck scale) would be associated with a violation of local Lorentz invariance. Relatively concrete proposals in that direction soon appeared in connection with programs such as loop quantum gravity [47,48] and string theory [49–52]. The general scheme involved a modification of the dispersion relations for

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21 That condition is essentially the statement that energy flows are causal and its violation in any type of energetic component of the universe would imply faster than light energy transport.
particles of the form

\[ E^2 = p^2 + m^2 + \xi \frac{E^3}{m_p}. \]  

(A1)

The fact that such equation is not relativistically invariant implied there could be at most one frame in which it would hold (usually identified with the one defined by the CMB). This led to a focused effort to obtain bounds on the parameter \( \xi \) characterizing the violations of special relativity [53–56]. However, it was soon realized that any serious attempt to include the field theoretical radiative corrections in the context of a granularity associated with a preferential frame naturally led to violations of special relativity that would have been so large that they would have been observed long ago [13,14].

This observation has led to an intense debate in the community working on theoretical aspects of Quantum Gravity about the extent to which the results impinge on some of the most promising approaches towards the subject. On the other hand, it was soon noticed in some theoretical accounts [57,58] that a fundamental discreteness of space-time did not necessarily implied such special relativity violations. Moreover, the study of the renormalization flow of Lorentz violating contributions showed that in the presence of generic interactions these violations would be amplified by radiative corrections becoming large (i.e. not suppressed by the Planck mass as in A1) at low energies [13,14]. However, a key hypothesis in the no-go theorem of [14] was the assumption that the spacetime geometry is flat in the effective quantum field theory description, leaving the possibility that curvature modulated effects could survive. In view of this situation, we were driven to look for alternative manners in which a space-time granularity would manifest itself without involving the breakdown of Lorentz invariance. A first model of this sort was proposed and some steps were taken to study its most direct phenomenology [15,16]. The model involves an anomalous coupling of the Weyl curvature to fermions, with the fundamental object encoding such curvature, constructed out of the eigenvectors and eigenvalues of the Weyl tensor, considered as an operator in the space of two forms i.e. solutions of \( W_{cd}^{ab} X_{cd} = \lambda X_{ab} \), with the “eigenvalues” \( \lambda X \), characterizing the strength of the effect, and the corresponding two-forms \( X_{ab} \) representing the distinguished space-time planes. In fact, an experiment designed with the purpose of exploring these ideas has been carried out already by the EötWash experimental group [59,60]. This first proposal soon evolved into a more generic point of view, based on the idea that a space-time granularity that would only become manifest, or, furthermore, only emerge at all, as a result of the relative state of motion between the matter “responsible” for the space time curvature and that affected by it (the perspective here is motivated by the Dirac rule implying that true observables in gravity arise from relations between interacting degrees of freedom). More precisely, we adopt the view that some sub-spaces of the tangent space would be singled out by features of the underlying granular spacetime structure, and that those would affect, in anomalous ways, the matter probes in the corresponding space time regions. In order to take the next step and connect with phenomenolog-

\[ \text{An alternative route was considered seeking schemes that would allow modification of dispersion relations without the existence of preferred frames, a program known as Double Special Relativity, which unfortunately was shown to suffer from some serious problems.} \]
ical considerations, the point of view we take is that the same structure would be reflected, at least in part, at the macroscopic level, as “the curvature of space-time”. Thus, such features would characterize the anomalous behavior of the matter fields that underwent dynamical evolution in the corresponding regions of space-time (as distinct realizations of this general idea see [9–11,15,16]). The seemingly non-local aspects of such features are in turn taken not to be really problematic, if limited to the quantum regime, given the apparent inescapable conclusion from J. Bell’s results [61,62] (and the experimental confirmations [63]) indicating the presence, in nature, of, at least, some non-local features.

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