The origin of the Meissner effect in new and old superconductors

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Abstract

It is generally believed that superconducting materials are divided into two classes: ‘conventional’ and ‘unconventional’. Conventional superconductors (the elements and thousands of compounds including MgB$_2$) are described by conventional London–BCS–Eliashberg electron–phonon theory. There is no general agreement as to what mechanism or mechanisms describe ‘unconventional’ superconductors such as the heavy fermions, organics, cuprate and pnictide families. However all superconductors, whether ‘conventional’ or ‘unconventional’, exhibit the Meissner effect. I argue that there is a single mechanism of superconductivity for all materials, that explains the Meissner effect and differs from the conventional mechanism in several fundamental aspects: it says that superconductivity is driven by lowering of kinetic rather than potential energy of the charge carriers, it requires conduction by holes rather than electrons in the normal state, and it predicts a non-homogeneous rigid charge distribution and an electric field in the interior of superconductors, and a spin current near the surface. Furthermore I argue that neither the conventional mechanism nor any of the other proposed unconventional mechanisms can explain the Meissner effect. Superconductivity in materials is discussed in the light of these concepts, some experimental predictions, connections to Dirac’s theory, and connections to the superfluidity of $^4$He.

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1. Introduction

BCS theory still reigns as the undisputed explanation of superconductivity in ‘conventional’ superconductors [1]. In a somewhat circular argument, a ‘conventional’ superconductor is defined to be a superconductor described by BCS theory. In addition there are by now at least ten different classes of materials that are generally believed to be ‘unconventional’, i.e. not described by BCS theory [2, 3]. Yet the belief that conventional BCS electron–phonon theory describes the simplest materials, elements and compounds, remains unwavering (reference [4] summarizes the alleged successes of BCS theory in the description of conventional superconductors), irrespective of the fact that in relative terms the phase space of ‘conventional’ superconductors is rapidly shrinking. It should also be stressed that the supposedly ‘unconventional’ superconductors are not necessarily characterized by having a large critical temperature, since they are found e.g. among heavy fermion materials with $T_c$’s of a few degrees, organic superconductors of the order of 10$^0$, electron-doped cuprates and many iron-pnictide materials with $T_c$ below 30$^0$, all lower than the supposedly conventional superconductor MgB$_2$ with $T_c = 39$ K. And certainly there is no single ‘unconventional mechanism’ proposed to describe all unconventional superconductors: new mechanisms are being proposed that apply specifically to one family only, e.g. the cuprates, or the iron pnictides, or the heavy fermion superconductors.

However all superconductors, whether conventional or not, exhibit the Meissner effect. I argue that BCS theory cannot explain the Meissner effect [5], so it cannot explain any superconductor. Furthermore, none of the unconventional mechanisms proposed to explain ‘unconventional’ superconductivity have addressed the question of how to explain the Meissner effect. I argue that none of these mechanisms describe any superconductor because they cannot explain the Meissner effect.

I propose that the Meissner effect can only be explained if (i) superconductivity is driven by lowering of the kinetic energy of the charge carriers [6], and (ii) superconductors...
expel negative charge from the interior to the surface in the transition to superconductivity [7]. This physics results in a macroscopically inhomogeneous charge distribution [8] and in the existence of macroscopic zero-point motion which manifests itself in the form of a spin current [9] in the ground state of superconductors. Neither BCS theory nor London electrodynamic theory describe this physics. Nevertheless, parts of both BCS theory and London theory are undoubtedly correct.

The points (i) and (ii) are intimately connected. Kinetic energy lowering means, e.g. via Heisenberg’s uncertainly principle, expansion of the electronic wave function which in turn implies outward motion of negative charge. That outward motion of negative charge explains the generation of the Meissner current is immediately seen from the action of the Lorentz force [10]. That the Meissner effect is impossible in the absence of outward motion of charge is immediately seen from the equations of motion [11] and from the fact that there is no other source of electromotive force [12]. That kinetic energy lowering drives superconductivity follows from the fact that the Meissner effect cannot occur unless there is outward motion of negative charge; outward motion of negative charge implies charge separation, hence increase in potential energy, so the ‘electromotive force’ driving it [12] has to be lowering of kinetic energy.

Furthermore our explanation of superconductivity is supported by the close relationship that is known to exist between superconductors and superfluids, e.g. 4He [13]. Both are macroscopic quantum phenomena [13]. Both exhibit frictionless flow with vanishing generalized vorticity. Kinetic energy lowering drives the superfluid transition in 4He [14], so it is natural to conclude that it also drives the superconducting transition in superconductors.

We quote from the preface of London’s book on superfluids, vol 2 [13]: ‘That something strange happens to liquid helium at about 2.2°K was noticed by Kammerlingh Onnes as early as 1911. He found that when the liquid is cooled below that temperature it starts expanding instead of continuing to contract, thus deviating from the behavior of most substances’. Indeed, the expansion of 4He below the critical temperature is clear indication that the transition is driven by kinetic energy lowering, and it parallels [14] the wavefunction expansion and charge expulsion that we propose exists in superconductors, also driven by kinetic energy lowering.

Instead, BCS theory and conventional London electrodynamic theory imply that superconductivity is driven by qualitatively different physics, that is non-existent in liquid 4He. Nobody has ever proposed that what drives conventional superconductivity within BCS theory, ‘a footing small interaction between electrons and lattice vibrations’ [15], has anything to do with the superfluidity of 4He. And if charge moves outward driven by kinetic energy lowering, conventional London electrodynamics will not apply because it requires absence of electric fields inside superconductors.

Remarkably, the only suggestion we could find in the scientific literature before the high Tc era that kinetic energy lowering has anything to do with superconductivity is in the preface of London’s book on superconductivity [13], where he writes: ‘It is not necessarily a configuration close to the minimum of the potential energy (lattice order) which is the most advantageous one for the energy balance, since by virtue of the uncertainty relation the kinetic energy also comes into play. If the resultant forces are sufficiently weak and act between sufficiently light particles, then the structure possessing the smallest total energy would be characterized by a good economy of the kinetic energy’. However, in the remainder of the book no mention whatsoever is made of how ‘a good economy of the kinetic energy’ would play any role in superconductivity.

The reason that London said this in his preface is presumably that he had superfluid He in mind together with the close relation in other properties of superconductors and superfluids. There was however no indication at that time either from experiment or from theory that kinetic energy lowering had anything to do with the transition to superconductivity.

That has changed now. Since the early days of the theory of hole superconductivity discussed in this paper it was clear that electron-hole asymmetry was intimately related to kinetic energy: the pairing interaction (correlated hopping) was termed Δt [16], indicating its relation with the hopping amplitude t and kinetic energy (in contrast to other works [17–19]). The relation between kinetic energy lowering, wavefunction expansion and charge asymmetry became clearer when it was realized that superconductors expel negative charge from their interior to the surface [7, 8]. Experimentally, evidence for kinetic energy lowering was found in optical properties of cuprates and pnictides [20] as predicted theoretically several years before the experiments [21].

2. Electromotive force

An electromotive force is a non-electric force that moves electric charges against the direction dictated by electric fields. In a voltaic cell, an electromotive force moves positive charges from the negative to the positive electrode (raising their electric potential energy). Similarly in the Meissner effect an electromotive force is needed to accelerate the electric charges near the surface carrying the developing Meissner current in direction opposite to that dictated by the electric force generated by Faraday’s law as the magnetic field lines are moving out [22].

Neither conventional BCS–London theory nor any of the unconventional theories of superconductivity proposed in recent years (except for the one discussed here) offer an explanation of what this electromotive force is in superconductors. In other words, while they describe the initial and final state, they cannot describe the process by which the system evolves from the initial to the final state. In the absence of such an electromotive force, metals in the presence of a magnetic field would never become superconducting. But they do.

There is a limited number of possibilities offered by the known laws of physics. We know of four fundamental forces: gravitation, strong, weak and electromagnetic. It is almost obvious that neither gravitational, nor strong (nuclear) nor weak interactions can play a role. We are
the electronic orbit expands from radius $r$ of the Meissner effect and hence of superconductivity. As diamagnetism increases, and negative charge moves $L$ fixed angular momentum

**Figure 1.** The essence of superconductivity. An electron in an expanding orbit with fixed angular momentum lowers its kinetic energy ($K_s < K_n$) increases its diamagnetic susceptibility and causes expulsion of negative charge. The top orbit represents the normal state, with $r_n = k_F^{-1}$, the bottom one the superconducting state, with $r_s = 2\lambda_L$.

left with electromagnetic, however we are seeking a non-electromagnetic electromotive force.

There is in fact a known fifth force in nature: the 'quantum force'. A quantum particle confined to a finite volume exerts 'quantum pressure' against the confining walls. This quantum pressure, times the area over which it acts, gives us a force. We have argued in reference [12] that this is the electromotive force that explains both the physics of voltaic cells and the Meissner effect.

This then leaves us with just two possibilities: either there is another force in physics, as yet unknown, that explains the electromotive force manifest in the Meissner effect. If so, the proponents of the conventional theory of superconductivity or of other unconventional theories should explain what that force is. Or, the quantum force mentioned above explains the Meissner effect. The theory described in this paper proposes the latter, and explores the consequences of this for the physics of superconductivity. We find that many resulting properties of superconductors are qualitatively different from the predicted properties of superconductors within conventional BCS–London theory. If proponents of the conventional theory were to argue that this quantum force also explains the Meissner effect in the conventional framework, they would have to explain how one would avoid the other consequences of this physics that our theory predicts to be unavoidable.

**3. Kinetic energy lowering, radial expansion and the Meissner effect**

Figure 1 shows what we propose is the essential physics of the Meissner effect and hence of superconductivity. As the electronic orbit expands from radius $r_n$ to $r_s$, with fixed angular momentum $L$, the kinetic energy decreases, diamagnetism increases, and negative charge moves outward. From the Larmor expression for the diamagnetic susceptibility

$$\chi_{\text{Larmor}} = -\frac{ne^2}{4\pi m_e c^2} < r^2 >,$$

one obtains Landau diamagnetism for $r = k_F^{-1}$ ($k_F =$ Fermi momentum) and perfect diamagnetism for $r = 2\lambda_L$, with $\lambda_L$ the London penetration depth [23]. This is easily seen using the expressions for the electronic density of states $g(\epsilon_F) = 3\pi/2\epsilon_F$ and the standard expression for the London penetration depth

$$\frac{1}{\lambda_L^2} = \frac{4\pi n e^2}{mc^2}.$$

Thus, the transition to superconductivity involves expansion of electronic orbits from $r_n = k_F^{-1} \sim $ Å to $r_s = 2\lambda_L \sim 1000$ Å. The correct theory of superconductivity should contain the physics depicted in figure 1. BCS theory does not.

The Hamiltonian for the conduction electrons in a metal is

$$H = K + U_{\text{pot}}.$$

The first term is the electronic kinetic energy, given by

$$K = \sum_i (-\frac{\hbar^2}{2m_e}) \nabla_i^2$$

and the second term is the potential energy, given by the sum of electron–ion and electron–electron interactions

$$U_{\text{pot}} = U_{\text{el–ion}} + U_{\text{el–el}}.$$

Conventional BCS theory attributes the energy lowering in going from the normal to the superconducting state to $U_{\text{el–ion}}$ together with the ion dynamics. Unconventional mechanisms proposed to describe new superconductors propose that the energy lowering originates in $U_{\text{el–el}}$, usually involving magnetic mechanisms. Instead, we propose that superconductivity in all materials originates in the fact that the average electronic kinetic energy is lower in the superconducting state than in the normal state:

$$\langle \Psi_{\text{super}} | K | \Psi_{\text{super}} \rangle < \langle \Psi_{\text{normal}} | K | \Psi_{\text{normal}} \rangle,$$

while the average potential energy is higher

$$\langle \Psi_{\text{super}} | U_{\text{pot}} | \Psi_{\text{super}} \rangle > \langle \Psi_{\text{normal}} | U_{\text{pot}} | \Psi_{\text{normal}} \rangle$$

albeit by a lesser amount, so that the difference yields the condensation energy of the superconductor.

We can understand the energetics involved and its connection with orbit expansion by looking at a two-electron atom, i.e. $\text{H}^-$ or He, with atomic number $Z = 1$, $Z = 2$ respectively. Assuming the variational wavefunction

$$\Psi_{\alpha}(r_1, r_2) = \phi_{\alpha}(r_1)\phi_{\alpha}(r_2)$$

with

$$\phi_{\alpha}(r) = \left(\frac{1}{r_0 \pi}\right)^{1/4} e^{-r/r_0},$$

we have

$$\langle \Psi_{\alpha} | K | \Psi_{\alpha} \rangle = 2\frac{\hbar^2}{2m_e r_0^2}.$$

$$L = m_e v r_n$$

$$K_n = \frac{1}{2} m_e v^2 = -\frac{L^2}{2m_e r_n^2}$$

$$\chi_{\text{Larmor}} = -\frac{ne^2}{4\pi m_e c^2} < r^2 >$$

$$K_s = \frac{L^2}{2m_e r_s^2} < K_n$$
\[ \langle \Psi_n | U_{\text{el-ion}} | \Psi_n \rangle = -2 \frac{Z e^2}{r_0}, \quad (10b) \]
\[ \langle \Psi_n | U_{\text{el-\ion}} | \Psi_n \rangle = \frac{5 e^2}{8 r_0}. \quad (10c) \]

Therefore, if in going from the normal to the superconducting state \( r_0 \) changes from \( r_n \) to \( r_n > r_n \) we have from equations \((10a)\)–\((10c)\)

\[ \langle \Psi_n | K | \Psi_n \rangle < \langle \Psi_n | K | \Psi_n \rangle, \quad (11a) \]
\[ \langle \Psi_n | U_{\text{pol}} | \Psi_n \rangle > \langle \Psi_n | U_{\text{pol}} | \Psi_n \rangle. \quad (11b) \]

the latter valid as long as \( Z > 5/16 \) so that the kinetic energy decreases and the potential energy increases. More specifically,

\[ \langle \Psi_n | U_{\text{el-ion}} | \Psi_n \rangle > \langle \Psi_n | U_{\text{el-ion}} | \Psi_n \rangle, \quad (12a) \]
\[ \langle \Psi_n | U_{\text{el-\ion}} | \Psi_n \rangle < \langle \Psi_n | U_{\text{el-\ion}} | \Psi_n \rangle, \quad (12b) \]

so the potential energy increase results from increased electron–ion energy partially compensated by a decrease in electron–electron energy.

The minimum in the total energy occurs for

\[ \bar{r}_0 = \frac{a_0}{Z} \quad \text{(13)} \]

with \( a_0 = \hbar^2/(m_{e}e^2) \) the Bohr radius and

\[ \bar{Z} = Z - \frac{5}{16}. \quad \text{(14)} \]

Therefore, in order for the minimum value of the energy to correspond to the mesoscopic scale \( 2\lambda_L \), the ‘effective’ nuclear charge \( \bar{Z} \) has to be very small:

\[ \bar{Z} \sim \frac{a_0}{2 \lambda_L}. \quad \text{(15)} \]

This illustrates that orbit expansion and the Meissner effect will be associated with situations where carriers propagate through negatively charged anions, where the ‘effective’ nuclear charge \( \bar{Z} \) is close to zero, such as O\(^{--}\), As\(^{-}\), S\(^{--}\), Se\(^--\) and B\(^-\). It is not surprising therefore that high \( T_c \) superconductivity is found in cuprates, iron pnictides, iron chalcogenides and magnesium diboride.

When the orbits expand, they become highly overlapping, so they have to become phase coherent to avoid collisions that would raise the potential energy even further. In contrast, the small orbits in the normal state are non-overlapping and hence can have arbitrary phases. Therefore, the entropy is larger in the normal state (small orbits) than in the superconducting state (large orbits) and for this reason the normal state is favored at high temperatures and the superconducting state at low temperatures.

Finally, to achieve a low value of the ‘effective’ nuclear charge \( \bar{Z} \) requires the electronic conduction band to be almost full so that the negative electrons cancel the positive charge of the ions. Thus superconductivity will occur if there are almost full bands, giving rise to dominance of positive Hall coefficient (hole conduction) in the normal state.

4. Theory of hole superconductivity

The theory of hole superconductivity (see [24] and references therein) is consistent with the physics of the Meissner effect discussed in section 2 and depicted in figure 1. More specifically, it predicts the physics shown schematically in figure 2. Superconductors expel negative charge from the interior to the surface, due to the expanding orbits, giving rise to an excess negative charge density [25]

\[ \rho_- = e n_e \frac{\hbar}{4 m e \lambda_L}, \quad \text{(16)} \]

within a London penetration depth of the surface. An electric field exists in the interior pointing outward, with maximum value given by

\[ E_m = -\frac{\hbar c}{4 e \lambda_L^2}. \quad \text{(17)} \]

Electrons within a London penetration depth of the surface carry a spin current, with velocity given by [26]

\[ \bar{v}_s^0 = -\frac{\hbar}{4 m_e \lambda_L} \vec{s} \times \hat{n}. \quad \text{(18)} \]

with \( \hat{n} \) the outward-pointing normal to the surface. It originates in the superposition of rotational zero-point motion of electrons in the \( 2\lambda_L \) orbits throughout the bulk of the system, that cancels out in the interior but not near the surface, just like Amperian surface currents originate in the sum of local magnetic dipole currents in the bulk of a magnetized material. The electrodynamic equations governing the behavior of electric and magnetic fields, charge and spin currents, are given in [25].

The proposal that superconductors eject electrons from their interior to the surface depicted in figure 2(a), whether or not magnetic fields are present, has not yet been experimentally verified. However we argue that it is clearly illustrated in the situation shown in figure 3. When current flows from a normal conductor into a superconductor, flow lines go to the surface. Since the current is carried by charge carriers, charge carriers entering the superconductor in the interior region have to flow to the surface. The ‘London moment’ experiments [27] and the ‘gyromagnetic effect’ experiments [28] show that charge carriers in the superconducting state are always electrons [29], hence it is electrons (with their negative charge) that flow to the surface.

Figure 2. Illustration of three key aspects of the physics of superconductors proposed here. (a) Superconductors expel negative charge from their interior to the region near the surface; (b) carriers reside in mesoscopic overlapping orbits of radius \( 2\lambda_L \) (\( \lambda_L \) = London penetration depth); (c) a spin current flows near the surface of superconductors (the arrow perpendicular to the orbit denotes the direction of the electron magnetic moment).
as they enter the superconducting region. At the same time, magnetic field lines, which are circles throughout the interior in the normal region, are pushed out to the surface. It is only a small additional step to conclude that electrons in superconductors move to the surface whether or not a charge current is flowing, carrying any existing interior magnetic field lines with them.

The orbital angular momentum of electrons in orbits of radius \(2\lambda_L\) and speed \(v_0^2\) is \(L = m_e \frac{v_0^2}{2} (2\lambda_L) = \hbar/2\). Thus, it reflects the intrinsic spin angular momentum of the electron, which itself can be thought of as an orbital motion at speed \(c\) [30, 31] in an orbit of radius \(r_q = \hbar/2m_e c\), the ‘quantum electron radius’ [32] (as opposed to the classical electron radius \(r_e = e^2/2mc^2\)).

There exists a remarkable parallel in the physics at the three different length scales \(r_q, a_0 = \hbar^2/m_e c^2\) (Bohr radius) and \(2\lambda_L\), corresponding to the length scale of the electron, the atom and the superconductor. Slater has already remarked [33] that for superconductors ‘the orbits must be of order of magnitude of 137 atomic diameters’. If we assume that the carrier density is given by

\[
n_s = \frac{1}{4\pi a_0^2}
\]

it yields a London penetration depth (using equation (2))

\[
\lambda_L = \frac{2a_0}{\alpha} = 137a_0
\]

with \(\alpha = e^2/\hbar c\) the fine structure constant. The atomic or band length scale \(a_0\) is then precisely the geometric mean of the electron length scale \(r_q\) and the superconducting length scale \(2\lambda_L\). It is as if an expansion takes place \(r_q \rightarrow a_0 \rightarrow 2\lambda_L\) with scaling factor \(2/\alpha\), i.e.

\[
2\lambda_L = \frac{2}{\alpha} a_0 = \left(\frac{2}{\alpha}\right) \left(\frac{2}{\alpha}\right) r_q.
\]

Similarly the corresponding energy scales are obtained by multiplying the Dirac energy scale \(E_{\text{Dirac}} = 2mc^2\) by powers of the fine structure constant

\[
E_{\text{sc}} = \left(\frac{\alpha}{2}\right)^2 E_{\text{band}} = \left(\frac{\alpha}{2}\right)^2 \left(\frac{\alpha}{2}\right)^2 E_{\text{Dirac}}.
\]

where the energies of quantum confinement over these length scales are given by

\[
E_{\text{Dirac}} = \frac{\hbar^2}{2m_e r_q^2} = 2mc^2 = 1.022 \text{ MeV},
\]

\[
E_{\text{band}} = \frac{\hbar^2}{2m_e a_0^2} = \frac{e^2}{2a_0} = 13.6 \text{ eV},
\]

\[
E_{\text{sc}} = \frac{\hbar^2}{2m_e (2\lambda_L)^2} = 2v = 181 \mu\text{eV},
\]

where

\[
v = \frac{\hbar^2 q_0^2}{4m_e}
\]

(with \(q_0 = (2\lambda_L)^{-1}\)) is the kinetic energy lowering per electron obtained within our theory in the transition to the superconducting state (the kinetic energy of electrons in the spin current is \(1/2m_e (v_0^2) = v/2\) [34]). Thus, spatial expansion with factor \((2/\alpha)^2\) and corresponding energy reduction by \((\alpha/2)^2\) is seen to connect these three very different realms.

The expelled charge density \(\rho_-\) is related to the total superfluid charge density \(\rho_s\) by the same factor [25]

\[
\rho_- = \frac{v_0^2}{c} \rho_s = \frac{r_q}{2\lambda_L} \rho_s = \left(\frac{\alpha}{2}\right)^2 \rho_s
\]

as is the spin current speed to the speed of light

\[
v_0 = \frac{r_q}{2\lambda_L} c = \left(\frac{\alpha}{2}\right)^2 c.
\]

Within Dirac theory, the ratio of the small to the large component of the electron wave function is \(\sim v/c\) in the non-relativistic limit. Thus equations (25) and (26) indicate that the expelled charge density \(\rho_-\) reflects the small component of the electron wave function. Denoting by \(|\psi\rangle\) and \(|\chi\rangle\) the large and small components of the electron wavefunction, \(\rho_- \sim \langle\chi |\psi\rangle\).

### 5. Electronic zitterbewegung

Within Dirac’s theory of the electron, the ‘instantaneous’ velocity of the electron is always \(c\), the speed of light [35, 36]: the time derivative of the position operator in the Heisenberg representation is \(d\hat{x}_s/dt = c\hat{a}_s\), with \(\hat{a}_s\) the Dirac \(\alpha\)-matrices, and \(a_s^2 = 1\). The motion of the electron with average speed \(v\) thus has superposed a rapidly oscillating component at speed \(c\) (termed ‘zitterbewegung’ by Schrödinger [35]), that has been interpreted as a circular motion of radius \(r_q = \hbar/2m_ec\) giving rise to the spin angular momentum \(\hbar/2\) and the electron magnetic moment [30, 31].

The rotational zero-point motion in \(2\lambda_L\) orbits with orbital angular momentum \(\hbar/2\) predicted by our theory can be seen as an amplified version of this microscopic zitterbewegung. It is remarkable that for each spin component the spin current in the absence of applied magnetic field is

\[
J_{\sigma} = \frac{n_s}{2} v_0^2 = \frac{\rho_- c}{2}
\]

using equation (25), hence it can be interpreted as originating in the excess electrons of each spin propagating at speed \(c\) in opposite directions. Moreover, when a magnetic field is applied the currents change to

\[
J_{\sigma} = \frac{n_s}{2} \left( v_0 - \frac{e\lambda_L}{m_0 c} \vec{a} \cdot \vec{B} \right)
\]

\[
E_{\text{Dirac}} = \frac{\hbar^2}{2m_e r_q^2} = 2mc^2 = 1.022 \text{ MeV},
\]

\[
E_{\text{band}} = \frac{\hbar^2}{2m_e a_0^2} = \frac{e^2}{2a_0} = 13.6 \text{ eV},
\]

\[
E_{\text{sc}} = \frac{\hbar^2}{2m_e (2\lambda_L)^2} = 2v = 181 \mu\text{eV},
\]

\[
\rho_- = \frac{v_0^2}{c} \rho_s = \frac{r_q}{2\lambda_L} \rho_s = \left(\frac{\alpha}{2}\right)^2 \rho_s
\]

\[
v_0 = \frac{r_q}{2\lambda_L} c = \left(\frac{\alpha}{2}\right)^2 c.
\]

Within Dirac theory, the ratio of the small to the large component of the electron wave function is \(\sim v/c\) in the non-relativistic limit. Thus equations (25) and (26) indicate that the expelled charge density \(\rho_-\) reflects the small component of the electron wave function. Denoting by \(|\psi\rangle\) and \(|\chi\rangle\) the large and small components of the electron wavefunction, \(\rho_- \sim \langle\chi |\psi\rangle\).
as the spin parallel (antiparallel) to \( \vec{B} \) slows down (speeds up). At the same time, it is found [25] that the excess charge density changes by the same factor:

\[
\rho_0 = \frac{n_0}{2e} \left( v_0^0 - \frac{e\lambda_L}{m_e c} \vec{B} \right)
\]

(29)
i.e. the excess charge density of spin parallel (antiparallel) to \( \vec{B} \) that slows down (speeds up) decreases (increases) in magnitude. Thus,

\[
j_0 = \rho_0 c
\]

(30)
holds for any applied magnetic field lower than the critical field. In other words, the predicted ground state currents both with and without applied magnetic field can be seen as resulting from the electrons giving rise to the excess charge density moving at the speed of light, just as in Schrödinger’s zitterbewegung.

6. Energetics and the virial theorem

It has been argued that the virial theorem implies that the kinetic energy of electrons is necessarily increased in going from the normal to the superconducting state [37, 38], independent of what is the mechanism for superconductivity. According to the virial theorem [39], in a system where the only interactions are Coulomb interactions

\[
\langle K \rangle = -\frac{1}{2} \langle U_{\text{pot}} \rangle,
\]

(31)
where \( \langle \rangle \) denotes expectation value with a wavefunction that is an eigenstate of the Schrödinger equation. The total energy is then

\[
E = \langle K \rangle + \langle U_{\text{pot}} \rangle = -\langle K \rangle.
\]

(32)
In going from the normal state (or from an ensemble of normal states at finite temperature above \( T_c \)) to the superconducting state, the total energy necessarily has to decrease. Equation (32) then would seem to indicate that the kinetic energy increases, in contradiction with the discussion in our previous sections.

However, equation (31) only holds if the Schrödinger equation is assumed to apply without relativistic corrections. As discussed in the previous section, relativity plays a key role in the theory of hole superconductivity. We can estimate the magnitude of deviation in the energetics predicted by the non-relativistic virial theorem by the following argument. Electrons can be thought of as being ‘spread out’ over a distance \( r_q = \hbar/2m_e c \), so the Coulomb potential form \( e^2/r \) is not valid when \( r \) becomes comparable or smaller than \( r_q \). The average of the Coulomb potential over this region is

\[
\delta U = \frac{e^2}{r} \left( r/r_q \right) \sim \frac{e^2 r^2}{2 r_q} = 362 \mu eV
\]

(33)
for a wavefunction extending over a distance \( a_0 \) (Bohr radius). The magnitude of this term is certainly large enough to be relevant to superconducting condensation energies.

The term just discussed corresponds to the ‘Darwin term’ in the non-relativistic limit of the Dirac equation. An equally important correction comes from the spin–orbit interaction

\[
U_{s.o.} = -\frac{e\hbar}{4m_e^2 c^2} \vec{\sigma} \cdot (\vec{E} \times \vec{p}).
\]

(34)
and taking \( p = m_e v_0^0 \) and using equation (2), equation (34) yields

\[
U_{s.o.} = \frac{e^2 r^2}{4a_0^2}.
\]

(37)
In summary, we conclude that the argument that the virial theorem forbids kinetic energy driven superconductivity is invalid if relativity plays a key role in superconductivity. As discussed here and in earlier work, relativity plays a key role in the theory of hole superconductivity. This is also seen from the fact that within this theory the screening of electrostatic fields takes place over distance \( \lambda_4 \), that involves the speed of light, rather than over the Thomas Fermi length (that does not involve the speed of light) as in the conventional theory.

7. Superconductivity in materials

If indeed superconductivity is driven by kinetic energy lowering, it will occur when the kinetic energy of carriers in the normal state is high. That will be the case when bands are almost full, as shown schematically in figure 4 where the conduction in the normal state occurs through holes rather than electrons. We have discussed extensively in previous papers the vast empirical evidence indicating that it is always hole carriers that drive superconductivity, and that materials without hole carriers cannot be superconductors [40].

Another way to put it: if superconductivity involves an expansion of the electronic wavefunction as discussed in section 2, it should predominantly occur in systems where the electronic wavefunction is very compressed in the normal state, so that the drive to expand to lower the quantum kinetic energy is highest. It is interesting that precisely this criterion was formulated by W Meissner and G Schubert in 1943 [41]. They defined the quantity \( V_k \), volume per valence electron, as the difference between the atomic volume and the ionic volume, divided by the number of conduction electrons per atom, and noted that superconductivity is associated with low values of \( V_k \), with the smallest \( V_k \) values corresponding...
to the highest critical temperatures. Figure 5 shows the Meissner–Schubert diagram obtained from a paper by E Justi [42], where $V_{Ec}$ is plotted versus atomic number. Justi remarks [42] that this is a very marked correlation (‘ausgeprägte Regelmässigkeit’) that has ‘evident truth content’ (‘offenbare Wahrheitsgehalt’). It is also interesting to note that in the same paper Justi remarks that there is no correlation between superconductivity and Debye temperatures: ‘...finden wir weder eine spezielle Auszeichnung der $S$-Leiter durch besondere $\theta_D$ Beträge, die vielmehr über den gesamten vorkommenden Bereich von $\theta_D = 69^\circ$ bis $\theta_D = 400^\circ$ streuen, noch insbesondere einen Zusammenhang zwischen $\theta_D$ and $T_c$.1

It would be interesting to extend the Meissner–Schubert diagram to many more superconducting elements and compounds discovered since then.

B Matthias has emphasized that superconductivity is very frequently associated with lattice instabilities [43]. This observation follows naturally from the principles discussed here: firstly, carriers having high kinetic energy in the normal state gives rise to an unstable situation, and to lower the kinetic energy either the system will go superconducting or the lattice will distort. Secondly, when the Fermi level is near the top of the band the electronic wavefunction is antibonding, with small charge density between the ions, in contrast to the bonding wavefunction for electrons near the bottom of the band, as shown schematically in figure 4. The small electronic charge density between ions for antibonding electrons gives rise to repulsion between the ions and ‘antibinds’ the lattice, thus leading to lattice instabilities. Thus, the fact that many antibonding states are occupied by electrons (Fermi level near the top of the band) is associated with both lattice instabilities and superconductivity.

In contrast, as is well known BCS theory does not particularly care about superconducting materials having low values of $V_{Ec}$ nor predominantly hole carriers in the normal state. It ascribes the prevalence of lattice instabilities near superconductivity to a strong electron–phonon interaction. But it does not provide simple criteria to predict which materials will have ‘strong electron–phonon interaction’ and which will not; this is usually found out only after elaborate calculations that calculate $T_c$ after its value has been measured experimentally [2]. And BCS theory has nothing to say about the Meissner–Schubert correlation discussed above nor about the necessity of hole carriers for superconductivity to occur, which many workers have pointed out in the past [44].

8. Some experimental consequences

We have discussed a variety of experimental consequences in previous papers. Here we focus on three.

8.1. Plasmon dispersion relation

The dispersion relation for longitudinal bulk plasmons in a normal metal is

$$\omega_q^2 = \omega_p^2 + \frac{3}{5} \frac{\omega_c^2 q^2}{q^2}$$

with $w_p^2 = 4\pi ne^2 / n$, $n$ the electron density, and $v_F$ the Fermi velocity. Conventional BCS theory predicts that plasmons are essentially unchanged in the superconducting state [45, 46]. Instead, we predict [47] for longitudinal charge oscillations of the superfluid the dispersion relation

$$\omega_q^2 = \omega_p^2 + c^2 q^2,$$

which is a significant change since $v_F \ll c$ (we are assuming no change in the plasma frequency $\omega_p$ between normal and superconducting states for simplicity).

At finite temperatures the response should have a normal and a superfluid component. If $n_n$, $n_s$ are the normal and superfluid densities at temperature $T$ we have

$$\omega_q^2 = \omega_p^2 + \left[ \frac{n_n}{n} c^2 + \frac{n_s}{n} \frac{3}{5} \frac{\omega_c^2}{\omega_p^2} \right] q^2$$

$$= \omega_p^2 + \frac{v_{eff}^2}{2},$$

$$v_{eff} = \sqrt{\frac{n_n}{n} \frac{c^2}{2} + \frac{n_s}{n} \frac{3}{5} \frac{\omega_c}{\omega_p}},$$

so that the slope of the plasmon dispersion relation $\omega_p$ should increase from $\sqrt{3/5} v_F$ to $c$ as $T$ is lowered from $T_c$ to 0. The superfluid density at temperature $T$ is given by

$$n_s = n - \frac{\lambda_L^2}{\lambda_L^2(T)},$$

where $\lambda_L$ in the numerator is the zero-temperature London penetration depth. In a two-fluid model description one has approximately $n_s = n(1 - t^4)$, $n_n = nt^4$, with $t = T / T_c$.

To our knowledge the plasmon dispersion relation in superconductors has never been carefully studied experimentally, presumably because no change is expected within BCS theory since the energies involved are much larger than the superconducting energy gap. In [48], Nücker et al reported measurements of EELS (electron energy loss spectroscopy) spectra in Bi$_2$Sr$_2$CaCu$_2$O$_8$ at room temperature and at the end of the paper stated ‘we would like to mention that we have performed similar measurements on excitations

1 Translation: ‘...we find neither a special distinction of superconductors as having particular values of $\theta_D$ (Debye temperature), which in fact spread over the entire range of values from $\theta_D = 69^\circ$ to $\theta_D = 400^\circ$, nor in particular any relationship between $\theta_D$ and $T_c$ (superconducting critical temperature)’.
of valence and core electrons at 30 K which is well below the superconducting transitions temperature $T_c$, 83 K. Neither the loss function nor the plasmon dispersion show a significant difference between room temperature and 30 K." We believe the experiment should be repeated, since the theory discussed here predicts a significant change in the plasmon dispersion relation in that temperature range. Our theory also predicts a significant change in the dispersion relation of surface plasmons below $T_c$ [49].

8.2. Screening and compressibility

The dispersion relation equation (39) arises from the zeros of the longitudinal dielectric function of the superfluid which is very different from the Linhardt function of the normal metal according to our theory [47]. In the static limit this dielectric function is [47, 50]

$$\epsilon(q, \omega \to 0) = 1 + \frac{1}{\lambda^2 TF q^2}$$

(42)

in contrast to the Linhardt–Thomas–Fermi form valid in the normal state as well as in the superconducting state within BCS theory [45, 46]

$$\epsilon_{TF}(q) = 1 + \frac{1}{\lambda^2_{TF} q^2}$$

(43a)

with

$$\frac{1}{\lambda^2_{TF}} = 4\pi \epsilon^2 g(\epsilon_F)$$

(43b)

with $g(\epsilon_F)$ the density of states at the Fermi energy. These equations imply that static external electric fields are screened over distances $\lambda_{TF}$ and $\lambda_F$ for the superconductor and the normal metal respectively. For free electrons we have $g(\epsilon_F) = 3n/2\epsilon_F$ so that

$$\frac{1}{\lambda^2_{TF}} - \frac{6\pi n e^2}{\epsilon_F} = \frac{1}{\lambda^2 F} 2\epsilon_F$$

(44)

assuming the density of superconducting electrons $n_s$ is the same as that of normal electrons. From the compressibility sum rule

$$\epsilon(q \to 0, 0) = 1 + \frac{4\pi n e^2}{q^2} n^{-2} \kappa$$

(45)

with $\kappa^{-1} = -V \partial P/\partial V)_N$ it follows that the superconductor is much more rigid with respect to longitudinal charge distortions:

$$\kappa_s = \frac{1}{4\pi n^2 e^2 \lambda^2_{TF}} = \frac{1}{n \epsilon_F e^2}$$

(46a)

$$\kappa_n = \frac{1}{4\pi n^2 e^2 \lambda^2_{TF}} = \frac{g(\epsilon_F)}{n^2} = \frac{3}{2n\epsilon_F}$$

(46b)

(latter valid for a free electron gas). Furthermore the superfluid will propagate longitudinal charge oscillations at the speed of light rather than the Fermi velocity. Thus, as the temperature is lowered below $T_c$, the electronic bulk modulus (inverse compressibility) should increase rapidly as electrons condense into the superfluid phase. This is not predicted by conventional BCS theory and should provide an explanation for the anomalous behavior of electronic sound propagation found by Avramenko et al [51] in metals cooled into the superconducting state.

8.3. Nuclear quadrupole resonance

Nuclear quadrupole resonance (NQR) spectroscopy measures the interaction between the electric quadrupole moment of a nucleus and the electric field gradient at the site. Thus it may offer the possibility of detecting the internal electric fields in superconductors predicted by our theory.

In the superconducting state we predict an internal electric field that grows linearly with the distance to the center as the surface is approached, reaching maximum value $E_m$ given by equation (17) at distance $\lambda_L$ from the surface and decreasing to zero approximately linearly as the surface is reached. Thus the electric field gradient created by this electric field within distance $\lambda_L$ of the surface is

$$\frac{\partial \vec{E}}{\partial n} \sim \frac{E_m}{\lambda_L}.$$  (47)

For example, for Nb, Pb and In we have respectively $E_m = 308 \times 10^4$ V cm$^{-1}$, $\lambda_L = 400 \AA$, $E_m = 240 \times 10^4$ V cm$^{-1}$, $\lambda_L = 390 \AA$ and $E_m = 87 \times 10^4$ V cm$^{-1}$, $\lambda_L = 640 \AA$.

The electric field gradient equation (47) is too small to be detected directly. For example, the electric field gradient in In metal measured by NQR is of order $10^3$ V cm$^{-2}$ [52], several orders of magnitude smaller than what equation (47) predicts. However the electric field itself will shift ionic positions and modify the electronic charge distribution around the nuclei which will change the electric field gradient at the nuclear site and this effect should be large enough to be observable. For example, application of an electric field of magnitude $17 \times 10^3$ V cm$^{-1}$ to KClO$_3$ was found to result in a substantial shift and change of lineshape of the Cl$^{+}$ quadrupole resonance [53].

In 1961 Simmons and Slichter [54] reported a marked shift in the nuclear quadrupole resonance frequency of In below the superconducting transition temperature, of approximately 2% downwards, as well as a change in lineshape from symmetric to asymmetric. They remarked that ‘volume changes associated with the superconducting phase transition are not, by several orders of magnitude, large enough to account for this large shift’ and they concluded that ‘the explanation of the large shift remains an open question at present.’ It is still an open question today, and the experiment has never been repeated.

There have been remarkably few other measurements of NQR resonance in zero magnetic field in the superconducting state. Hammond and Knight [55] reported a very small frequency shift in the superconducting state of Ga. In YBCO, a steep decrease in the frequency as $T_c$ was approached from above was measured, and an increase below $T_c$ [56]. That reference also gives a complete list of NQR measurements in the normal and superconducting states of various materials over the years. The list is remarkably short.

We suggest that a systematic study of NQR frequency shifts and lineshape changes between the normal and superconducting state of various materials, both those categorized as ‘conventional’ and ‘unconventional’, would be of great interest. No such effects are expected within the conventional theory of superconductivity nor have such effects been predicted within other unconventional theories. Measurement of such effects would provide direct evidence
Figure 6. Superfluid pressure causes (a) electrons in superconductors to be expelled from the interior to the surface and beyond, and (b) 4He to climb the lateral surfaces of a container and escape to the exterior (‘Onnes effect’).

Figure 7. Illustration of manifestations of superfluid pressure. In superconductors (left), it gives rise to asymmetric tunneling characteristics, frequently observed in high $T_c$ materials, reflecting (according to our theory) the pressure pushing out negative electrons from the interior of superconductors. In superfluid 4He (right) it gives rise to the fountain effect, reflecting the superfluid pressure that drives the superfluid flow into the hotter region that has lower superfluid density.

for the development of an electric field in the interior of superconductors as they enter the superconducting state, as predicted by our theory.

9. Superfluid pressure and relation with 4He

The essential common aspect of the physics of superconductors and superfluid 4He, unrecognized in the currently accepted theories of both systems, appears to be that superfluids exert quantum pressure that is larger than in the normal state. This is in contrast to the situation in conventional Bose condensation where the condensate exerts no pressure.

In other words, as the system goes superfluid or superconducting and long range coherence sets in, the wavefunction of the mobile carriers exerts additional outward pressure and expands its spatial extent, driven by lowering of quantum kinetic energy.

For superconductors, this quantum pressure manifests itself in the negative charge expulsion (not yet experimentally detected) that we predict exists in all superconductors (figure 6(a)) and associated with it in the ‘Meissner pressure’$^2$ that expels the magnetic field through orbit expansion, opposing the ‘Maxwell pressure’ that wants to keep the magnetic field inside. As discussed in section 2, Meissner pressure is proposed to originate in quantum pressure which in turn is proposed to originate in the fact that a rotating body with fixed angular momentum lowers its kinetic energy by expanding its orbit. In is interesting to note that in the early development of electromagnetism, Maxwell explained the ‘Maxwell pressure’ exerted by magnetic fields in direction perpendicular to the field also as originating in rotational motion (of ‘molecular vortices’) with axis along the magnetic field direction [57].

Another manifestation of the superfluid pressure for the case of superconductors is that tunneling currents in normal–insulator–superconductor (NIS) tunnel junctions are larger for negatively biased samples (figure 7 left panel) [58, 59], reflecting the tendency of superconductors to expel electrons. Yet another manifestation is the proximity effect,

$^2$ The term ‘Meissner pressure’ was coined by F London in [13] vol 1, p 111, 135, 138.

where the superconducting wavefunction extends from the superconducting into the neighboring normal region.

For superfluid 4He [13], the superfluid pressure manifests itself vividly in the ‘fountain effect’: when the superfluid concentration is depleted in one region of a container by heating, the superfluid from another region will spurt into the depleted region with great force, driven by this pressure (figure 7 right panel). It also manifests itself (according to our interpretation) [14] in the ‘Onnes effect’, the fact that the superfluid will climb up the walls of a container, defying gravity (figure 6(b)). It manifests itself in the anomalous negative thermal expansion of superfluid 4He below the $\lambda$ point. It manifests itself in the inverted $\lambda$ shape of the specific heat curve in 4He (that gives the name to the $\lambda$ transition) which is direct evidence for quantum kinetic energy lowering in the transition to superfluidity [60]. (The variation of the potential energy with temperature gives rise to non-inverted $\lambda$ behavior at the transition point. See [60]). All these phenomena we argue are manifestation of superfluid pressure originating in quantum pressure i.e. kinetic energy lowering. We have proposed that, just as in superconductors, in superfluid 4He this pressure originates in rotational zero point motion [14].

Furthermore, the enhanced charge rigidity that we predict in the superconducting state [47] is clearly associated with this quantum pressure and has its counterpart in 4He in the enhanced bulk modulus (inverse compressibility) observed below the $\lambda$-point [61].

For superconductors we have shown that electron–hole asymmetry gives rise to positive thermoelectric power for NIS tunnel junctions [62]. Namely, if the superconductor is at lower temperature than the normal electrode, electrons will flow from the superconductor to the normal electrode, exactly the opposite to what would happen in the normal state. This is entirely analogous in the case of 4He: above $T_c$, the normal fluid will flow from the hotter to the colder region in a container. Instead, below $T_c$, the dominant flow is from the colder region to the hotter region (as in the fountain effect). This is again clear illustration of the identical effect of quantum pressure for superconductors and superfluids that we propose is responsible for these phenomena.
In superfluid $^4$He, a backflow of normal fluid flow occurs whenever the constraints of the system allow it when there is superfluid flow from a colder to a hotter region of the container. Similarly we may expect such backflow to occur in superconductors: when electrons in the interior condense into the superfluid state and are expelled towards the surface in the transition to superconductivity, the outward charge flow should be partially compensated by an inflow of normal electrons. In the presence of a magnetic field, the inflowing electrons will be deflected by the Lorentz force in opposite direction to the outflowing electrons and will transmit that angular momentum to the ionic lattice by scattering. This should play an important role in explaining the puzzle of how angular momentum is conserved when a metal goes superconducting in the presence of a magnetic field and the Meissner current that carries angular momentum develops [5].

In summary, we have seen that several different phenomena in superfluid $^4$He and superconductors can be explained if the superfluids exert quantum pressure due to quantum zero point motion. In addition, the frictionless flow of electric current in superconductors and the non-viscous flow of superfluid $^4$He and their respective critical velocities can be explained in a unified way as originating in quantum zero point diffusion [14, 63]. Of course the fact that quantum zero point motion plays an important role in liquid He is generally recognized. For example, everybody agrees that this is the reason why He remains liquid under its own vapor pressure down to zero temperature. However the fact that superfluid $^4$He exerts quantum pressure that is larger than that of the normal fluid is not generally recognized, despite the clear experimental evidence for it. A notable exception can be found in the writings of K Mendelssohn who consistently emphasized this key aspect of superfluid $^4$He, for example when he writes [64]: ‘A question of particular interest is whether or not the superfluid phase contributes to the pressure. According to the Bose–Einstein model (F. London, 1939) this contribution is zero, but it appears to us from the experimental results that such a zero-point pressure must exist and that it is of fundamental importance for the explanation of the transport phenomena.’

10. Discussion

We argue that the Meissner effect is an unresolved puzzle within the conventional theory of superconductivity. How can superconductors governed by conventional BCS–London theory expel a magnetic field when cooled from the normal into the superconducting state? The magnetic Lorentz force

$$\vec{F} = \frac{e}{c} \vec{v} \times \vec{B} = \frac{e}{c} B(v_r \hat{\theta} - v_\theta \hat{r}) = F_\theta \hat{\theta} + F_\phi \hat{\phi} \quad (48)$$

will not give rise to an azimuthal force $F_\theta$ that will set the Meissner current into motion unless there is a radial velocity $v_r$, i.e. a net outflow of charge: in equation (48), $F_\phi = 0$ if $v_\phi = 0$. BCS–London theory does not describe radial flow of charge, hence $v_r = 0$. Unless and until proponents of BCS–London theory explain which force in nature will propel the charge near the surface to move in the azimuthal direction and overcome the Faraday counter-electromotive force to generate the Meissner current, the Meissner effect will remain unexplained within the conventional theory. Since all superconductors exhibit the Meissner effect, we must conclude that the conventional theory in its present form does not apply to any real superconductor.

Instead, we have proposed that the Meissner effect in all superconductors is explained by orbit expansion driven by quantum pressure, that gives rise to outflow of negative charge, an outward pointing electric field in the interior of superconductors, an excess negative charge near the surface, enhanced charge rigidity, rotational zero point motion in $2\pi L$ orbits reflecting electronic zitterbewegung, and a macroscopic spin current near the surface. This non-conventional physics should exist in all superconductors that exhibit the Meissner effect. That is, in all new and old superconductors, ‘conventional’ and ‘unconventional’.

Concerning materials, this physics indicates that superconductivity should be particularly favored in materials that have a lot of negative charge, namely almost filled bands (hole conduction in the normal state) and negatively charged anions, as well as highly compressed electronic wavefunctions giving rise to high kinetic energy and quantum pressure, since all of these factors will contribute to the tendency to expel negative charge. Materials evidence in favor of this [40] is the high $T_c$ observed in the cuprates, pnictides and MgB$_2$, all possessing negative anions, that superconductors overwhelmingly display dominant hole conduction in the normal state (positive Hall coefficient), that they show a particularly small volume per electron (Meissner–Schubert diagram) and that they are often close to lattice instabilities, indicating high occupation of antibonding states.

Furthermore, we have argued that this view of superconductivity has close connection to and sheds new light into the physics of superfluidity in $^4$He. Both superconductivity and superfluidity are proposed to be kinetic energy driven, due to the fact that superfluids exert quantum pressure. This suggests a common explanation for many observed phenomena in superfluids and superconductors that would be unrelated otherwise, such as the superfluid fountain effect, the tunneling asymmetry in superconductors, the thermomechanical effect in $^4$He, the predicted positive thermoelectric power in superconducting tunnel junctions, the Onnes effect in superfluid He films, the Meissner effect in superconductors, the transfer of optical spectral weight from high to low frequencies in superconductors, and the negative thermal expansion of superfluid He. The non-relativistic virial theorem, which says that kinetic energy should be raised when the total energy is lowered if the dominant interactions in the system are Coulomb, is profoundly misleading for both superconductors and superfluids.

The physics of superconductivity as driven by kinetic energy lowering and exhibiting rotational zero point motion has led us to conclude that the fundamental origin of quantum pressure in nature is rotational zero-point motion, and in particular is responsible for the stability of matter and the Pauli exclusion principle [23, 65]. In contrast, in the conventional understanding of quantum mechanics there is no rotational zero point motion, and the origin of quantum pressure and the stability of matter is attributed to Heisenberg’s uncertainty principle. However, an alternative
interpretation of quantum mechanics connecting Heisenberg’s uncertainty principle to an intrinsic rotational motion of the electron due to its spin was already proposed in pioneering work by D Hestenes in 1979 [66]. The idea that rotational motion gives rise to pressure is of course very old: it was at the core of the early description of magnetic fields by Maxwell (molecular vortices) [57] as well as in the theory of ‘vortex atoms’ by Lord Kelvin [67]. Our suggestion derived from both the physics of superconductors (fermions) and superfluid 4He (bosons), that rotational zero point motion is at the root of quantum pressure quite generally, suggests that rotational zero point motion may be a fundamental element of the fabric of space-time itself, and it is natural to speculate that it may also be at the root of other phenomena such as the enigmatic dark energy that pervades the universe [68].

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