A Performance Analysis Method of High Speed and Small Diameter Propeller

Yuchuan Zhang\textsuperscript{1,2,a}, Zhiqiang Hu\textsuperscript{1}, Yi Yang\textsuperscript{1} Lingbo Geng\textsuperscript{1}, Chao Wang\textsuperscript{1}

\textsuperscript{1} Shenyang Institute of Automation, Chinese Academy of Sciences, Shenyang 110006, China;\textsuperscript{2} School of Mechanical, Northeastern University, Shenyang 110004, China.

\textsuperscript{a} 921985141@qq.com

Abstract. Recently, water-air drone has shown widespread applications in various fields, such as military and live entertainment. As a direct source of power, it is necessary to study and calculate the performances of water-air propeller. The underwater propeller has the maximum diameter limit and the air propeller has the minimum thrust limit. So, high speed and small diameter are the characteristics of water-air propeller. In this paper, the performance in air of a water-air propeller has been calculated using programs, which were based on the strip theory. The effect of rotate speed on thrust, torque and power-to-thrust ratio has been carefully investigated. Meanwhile, these results have been verified by physical experiments.

Keywords: Performance calculation, Propeller, Strip theory, MATLAB.

1. Introduction
The analysis method of air propeller, a theory developing from finite wings, usually applied in calculating performances according to lifting line theory. So the section of propeller can be deemed two-dimensional wing. The fluid of wing section has similar speed and direction to real blade. This way to design the shape of blade section is used in current studies. Propeller performance calculations are the most important part of its design and application. Therefore, many methods have been studied and developed to calculate and analyze the performances of propeller [1]. These mechanical models which are always developing enable the calculated propeller performances closing to the experimental results.

In this paper, the propeller performance analysis programs were compiled based on the strip theory. Through the program calculates the properties of a certain propeller were systematically calculated and studied. Meanwhile, some experimental experiments were researched to verify the accuracy of the calculated results [2].

2. Calculated theory
Strip theory is developed on the basis of Joukowsky’s vortex theory to analyze the propeller model. It does not consider the radial flow of the propeller and assume that the induced velocity is average velocity regardless of its periodicity. The induced velocity comes from eddy current and it is related to the number of propeller blades, the pitch of the blades and the aerodynamic data of the airfoil. So strip theory approaches the reality, which considers these effects.
2.1. Aerodynamic analysis of blade profile

In this paper, the propeller is mostly used for the situation that advance ratio (J) equals to 0. Therefore, this paper is mainly focus on this advance ratio to calculate [3,4,5]. At the radial direction (r), the length of the micro-segment (dr) is taken and the corresponding leaf length is c. And the fluid velocity and force of blade section can be found in Fig. 1.

![Fig. 1 Velocity and force of the blade section](image)

The trajectory of the leaf element is a spiral, so the synthesis rate of blade profile can be expressed as:

$$W = \sqrt{(2\pi nr - v_x)^2 + v_r^2}$$  \hspace{1cm} (1)

Where and the tangential velocity in the paddle is $2\pi nr$, $v_x$ represents the axial induced velocity and $v_r$ is the circumferential induced velocity. The angle between the geometric synthesis speed and the rotation plan is $\phi_0$. And this angle can be calculated by:

$$\tan \phi_0 = \frac{v_r}{2\pi nr - v_x}$$  \hspace{1cm} (2)

Assuming the installation angle of blade profile is $\theta$. Since the feed speed is zero, the interference angle generated by the blade wake vortex is $\phi_0$. The angle of attacking the airflow relative to the blade profile is:

$$\alpha = \theta - \phi_0$$  \hspace{1cm} (3)

And according to strip theory, the lift force L and resistant force D at the blade profile can be expressed as:

$$L = \frac{1}{2} \rho CW^2 C_l$$  \hspace{1cm} (4)

$$D = \frac{1}{2} \rho CW^2 C_d$$  \hspace{1cm} (5)
2.2. Aerodynamic analysis of propeller

The propeller pull is given by:

\[ T = N_B \int_{r_0}^{R} (L \cos \beta - D \sin \beta) dr \]  

(6)

where the number of propeller blades is NB, r0 represents the hub radius and R is the propeller radius. And the torque of propeller is:

\[ Q = N_B \int_{r_0}^{R} (L \cos \beta - D \sin \beta) r dr \]  

(7)

The power absorbed by the propeller can be expressed in following equation:

\[ P = Q \omega \]  

(8)

where \( \omega \) stands for the rotate speed of propeller. So the thrust-to-power ratio is:

\[ \tau = \frac{P}{T} \]  

(9)

When the aerodynamic analysis of propeller is calculated, \( N_B, D, c, C_l \) and \( C_d \) are known quantities. However, \( \beta, \nu_x, \nu_y, \tau, T \) and \( Q \) are computational quantities. In order to calculate the interference angle, the tension obtained by the momentum equation can be equal to the tensile force received by the blade profile, thereby obtaining:

\[ N_B c \frac{(C_l \cos \beta - C_d \sin \beta)}{\sin^2 \beta} = 8 \pi r \]  

(10)

Let the blade solidity be \( \sigma = \frac{N_B C}{2 \pi r} \) and substitute it into the above formula. This can be obtained in below:

\[ \sigma \frac{(C_l \cos \beta - C_d \sin \beta)}{\sin^2 \beta} = 4 \]  

(11)

Circular and axial induced velocity can be acquired by momentum moment equation and moment equation and the equation is shown in below.

\[ \sigma \frac{(C_l \sin \beta + C_d \cos \beta)}{\sin^2 \beta} \nu_y = 4 \]  

(12)

The relationship between the circular and axial induced speed can be obtained by combining equation (11) and (12), and the relationship is:


\[
\frac{(C_y \sin \beta + C_d \cos \beta)}{(C_y \cos \beta - C_d \sin \beta)} = \frac{v_v}{v_i}
\]

(13)

Through solving the equation (13) to determine \( r \) and \( \beta \), and then know \( \alpha \). The aerodynamic characteristics of the actual angle of attack are calculated by the airfoil aerodynamic analysis software, and the aerodynamic characteristics of the propeller are finally obtained.

3. Results and discussion

The program bases on strip theory written by MATLAB. It can calculate the aerodynamic characteristics of each blade profile under the corresponding installation angle. According to the results, the aerodynamic characteristics, such as thrust, torque, power to thrust ratio of the propeller, are obtained. Table 1 shows the propeller geometric parameters including the installation angle and the chord length.

| Radiul (M) | Installation angle (degree) | Chord length (M) |
|------------|----------------------------|-----------------|
| 0.024      | 40.29                      | 0.0263          |
| 0.030      | 32.33                      | 0.2640          |
| 0.036      | 26.87                      | 0.2770          |
| 0.042      | 23.43                      | 0.2770          |
| 0.048      | 21.20                      | 0.2610          |
| 0.054      | 19.99                      | 0.2170          |
| 0.057      | 19.78                      | 0.1670          |
| 0.060      | 19.65                      | 0.0072          |

The program calculates the propeller performance of different rotating speed. Table 2, 3 and 4 show the calculation results of the thrust, torque and the thrust power to thrust ratio, respectively.

| Rotating speed | Thrust   | Rotating speed | Thrust |
|----------------|----------|----------------|--------|
| 8000           | 3.0633   | 12000          | 6.8925 |
| 9000           | 3.877    | 13000          | 8.0891 |
| 10000          | 4.7864   | 14000          | 9.3814 |
| 11000          | 5.7916   | 15000          | 10.7695|

| Rotating speed | Torque   | Rotating speed | Torque |
|----------------|----------|----------------|--------|
| 8000           | 0.0606   | 12000          | 0.1362 |
| 9000           | 0.0766   | 13000          | 0.1599 |
| 10000          | 0.0946   | 14000          | 0.1855 |
| 11000          | 0.1145   | 15000          | 0.2129 |

| Rotating speed | Power to thrust | Rotating speed | Power to thrust |
|----------------|-----------------|----------------|-----------------|
| 8000           | 16.5730         | 12000          | 24.8319         |
| 9000           | 18.6210         | 13000          | 26.9104         |
| 10000          | 20.6972         | 14000          | 28.9891         |
| 11000          | 22.7734         | 15000          | 31.0527         |
The propeller was obtained by 3D printing technology, which is shown in Fig. 2. The propeller was tested its dynamometers and speed properties to verify calculated results using existing experimental equipment. Fig. 3 gives the experimental equipment using in this paper. The experiment measured the change of propeller thrust, torque and thrust power to thrust ratio with the different rotating speed, when the advance ratio is zero by the dynamometer.

![Propeller](image1)

**Fig. 2** Propeller

![Experiment conditions](image2)

**Fig. 3** Experiment conditions

Table 5 and the Fig. 4 show the power to thrust power ratio of program calculation and the experimental test in different rotating speed. It can be concluded that calculated results have high accuracy.

| Rotating speed | \( \tau \) (experiment) | \( \tau \) (calculation) |
|----------------|-------------------------|-------------------------|
| 8000           | 17.5936                 | 16.5730                 |
| 9000           | 20.0978                 | 18.6210                 |
| 10000          | 21.8604                 | 20.6972                 |
| 11000          | 24.1195                 | 22.7734                 |
| 12000          | 26.6740                 | 24.8319                 |
| 13000          | 28.9258                 | 26.9104                 |
| 14000          | 30.8147                 | 28.9891                 |
| 15000          | 32.5156                 | 31.0527                 |

![Power ratio program](image3)

**Fig. 4** The power ratio program of experiment and calculation
4. Summary
The program-based strip theory calculates the propeller performance is good consistency with the result of the experience. The conclusion is following:

1. The power to thrust ratio increases as the rotating speed increases. It can guide the design of high efficiency water-air propeller.
2. The results of calculation and experience are good consistency. The program can be used to preliminary calculate the performance of propeller.
3. The power to thrust ratio of experiment is a little bit bigger than the result of program calculation. The reason of the error is the holder that must be used in the experiment.

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