You Only Need End-to-End Training for Long-Tailed Recognition

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Abstract

The generalization gap on the long-tailed data sets is largely owing to most categories only occupying a few training samples. Decoupled training achieves better performance by training backbone and classifier separately. What causes the poorer performance of end-to-end model training (e.g., logits margin-based methods)? In this work, we identify a key factor that affects the learning of the classifier: the channel-correlated features with low entropy before inputting into the classifier. From the perspective of information theory, we analyze why cross-entropy loss tends to produce highly correlated features on the imbalanced data. In addition, we theoretically analyze and prove its impacts on the gradients of classifier weights, the condition number of Hessian, and logits margin-based approach. Therefore, we firstly propose to use Channel Whitening to decorrelate (“scatter”) the classifier’s inputs for decoupling the weight update and reshaping the skewed decision boundary, which achieves satisfactory results combined with logits margin-based method. However, when the number of minor classes are large, batch imbalance and more participation in training cause over-fitting of the major classes. We also propose two novel modules, Block-based Relatively Balanced Batch Sampler (B3RS) and Batch Embedded Training (BET) strategy, to increase the training iterations of tail classes in each epoch without affecting the representation learning of head classes.

The re-sampling methods include random over-sampling and random under-sampling [3, 8, 10, 24, 43]. The former augments the tail classes by duplicating samples, however, which results in over-fitting [8, 24, 43]. The latter randomly takes away some samples of head classes, leading to poorer generalization ability [3, 10, 41]. Therefore, in recent years, re-sampling methods have fallen out of favor, and the mainstream focused on how to combine the re-sampling approaches to learn better classifiers [17, 52, 56]. In this work, we propose two novel modules (Figure 1), Block-based Relatively Balanced Batch Sampler (B3RS) and Batch Embedded Training (BET) strategy, to increase the training iterations of tail classes in each epoch without affecting the representation learning of head classes.

1. Introduction

In the real-world recognition tasks, long-tailed label distributions (i.e., imbalanced datasets) are a common and natural problem, where a few categories (i.e., head classes) have more samples than others (i.e., tail classes). This challenging task has received increasing attention in recent years [2, 5, 7, 13, 16, 17, 19, 21, 22, 26, 28, 29, 34, 37–39, 44, 45, 48, 50, 52, 56]. In previous literature, the methods can be roughly categorized into three groups: re-sampling-based [3, 8, 10, 11, 13, 24, 41, 43, 47, 51], cost-sensitive re-weighting-based [2, 5, 16, 19, 20, 27, 29, 37–39, 44, 45, 54] and other methods [17, 22, 28, 50, 52, 53, 55, 56]. To improve the classification accuracy of tail classes, the re-sampling approaches change the sampling frequency to balance the label distribution, and the re-weighting approaches allocate large weights for tail classes via the loss function, thus an unbiased classifier is learned.

Recently, the simple and effective re-weighting methods received more attention on long-tailed recognition. Some works allocate large weights for tail classes in the loss function by increasing their logits margins [2, 29, 37, 45]. However, there is a large gap in the performance between the above logits margin-based approaches and two-stage decoupled training methods [17, 26, 29]. In this paper, we would like to explore the question: What causes the poor performance of end-to-end training on the long-tailed recognition?

We firstly demonstrate why the model tends to learn the representations with low entropy in the imbalanced data from the perspective of information theory. As we know, the lower entropy means the more compact features, which is consistent with the learning of cross-entropy loss. The principle of Maximum Entropy (ME) is widely used for regularization to increase the uncertainty of prediction, thereby improving the generalization of the model [30]. Different
from the above methods, we analyze the relationship between entropy and singular values of the features. We conclude that the curve of singular values is steeper when the entropy is lower, which means that the features are concentrated in certain directions, i.e. higher correlated features. Therefore, we observe that the correlation between channels of the classifier’s inputs (Figure 2), and theoretically analyze and prove its impacts on the gradients of classifier weights and the generalization of the model (i.e., the condition number of Hessian). The channel-wise features with high correlation cause the weights of each class to be updated in a nearly consistent direction and make the condition number larger, thus affecting the generalization of the model. In addition, under the above condition, the logits margin-based approaches, e.g., LDAM [2], exacerbate the imbalance of the gradient norm and weight norm of each category.

Therefore, we propose to use channel whitening to decorrelate the features before being fed into the classifier to decouple the weight update and reshape the skewed decision boundary, which achieves satisfactory results when combined with logits margin-based methods [2]. However, when the number of tail classes is large, the neural network is under-fitted to the tail classes. We propose two novel modules, Block-based Relatively Balanced Batch Sampler (B3RS) and Batch Embedded Training (BET) to solve the above problems. Our contributions are summarized as follow:

- We propose a channel whitening approach to decorrelate the features before being fed into the classifier, which not only decouples the weight update but also reshapes the skewed decision boundary.
- We also propose two novel modules, Block-based Relatively Balanced Batch Sampler (B3RS) and Batch Embedded Training (BET) to encourage tail-class samples to participate in more training. The above two modules can help end-to-end training to achieve state-of-the-art performance when the imbalance ratio is low.
- Experimental results on the long-tailed classification benchmarks, CIFAR-LT and ImageNet-LT, demonstrate the effectiveness of our method.

2. Related Works

In this section, we firstly introduce some important works on long-tailed recognition, including re-sampling methods, re-weighting methods, and decoupled training methods. Secondly, some applications of whitening in neural networks are introduced.

**Re-sampling.** Re-sampling methods as one of the classic approaches include over-sampling [3, 10, 41, 47] for the tail classes, under-sampling [8, 24, 43] for the head classes, and some heuristic re-sampling [36]. Pouyanfar et al. [36] introduce a new dynamic sampling method that adjusts sampling rates according to class-wise performance. Over-sampling methods duplicate tail samples might lead to over-fitting, while under-sampling may discard precious data thus impairing the generalization ability. The current mainstream focused on combining re-sampling approaches with other training strategies (e.g., decoupled training) to achieve state-of-the-art performance [17, 52, 56].

**Re-weighting.** Re-weighting methods usually allocate large weights for training samples of tail classes in the loss functions to learn an unbiased classifier. Cui et al. [5] proposed to adopt the effective number of samples instead of proportional frequency. Thereafter, Cao et al. [2] explored
the relationship between the margins of tail classes and the
genralization error and designed a label-distribution-aware
loss to encourage a larger margin for tail classes. Balance
Meta-Softmax (BALMS) [37] proposed an extended
margin-aware learning method. Aditya et al. [29] pro-
posed a “logits adjustment” approach by reducing the logits
margin. However, the above-
posed a “logits adjustment” approach by reducing the logits
value based on the label frequencies. 

Decoupled training. Kang et al. [17] proposed a decou-
pled training strategy to disentangle representation learning
from classifier learning and achieved surprising results. Zhou et al. [56] pro-
posed a unified Bilateral-Branch Network (BBN) and cumulative learning strategy to gradually
change the training from feature representation to the clas-
sifier. Similar work is that Zhang et al. [52] proposed to com-
bine two sampling approaches (class balanced sampling and
random sampling) with a feature extraction module and three classifier modules to balance the feature learning and
classifier learning. The decoupled training achieves better
performance than the re-weighting methods by end-to-end training.

Whitening. Whitening is a linear transformation that
transforms data and makes its covariance matrix is an identity
matrix, meaning that they are uncorrelated and each has
the same variance. Whitening is always used as a prepro-
cessing method [18]. In recent years, whitening has had
many applications on neural networks, including normaliza-
tion [14, 15, 33], generative adversarial networks [40], and
self-supervised learning [9]. In this work, we firstly explore
its application on long-tailed recognition.

3. Method

In this section, we firstly demonstrate why the learned
features with low entropy in the imbalanced data from the
respective of information theory, and analyze the relationship
of entropy and singular values. We then visualize the
correlation coefficients between the channel-wised features
before inputting them into the classifier and demonstrate a
key factor, correlated features, that causes the poor performance
of end-to-end training on the long-tailed data. We
also provide some theoretical proof on the impacts of corre-
lated features. Secondly, we propose channel whitening to
decorrelate the features before being fed into the classifier.
Finally, we present two novel modules, a new sampler, and
a training strategy.

3.1. Analysis based on Information Theory

In information theory, Mutual Information (MI) is de-
signed to measure the “amount of information” shared by
two random variables. In this paper, we are interested in
$I(X; Y)$ which represents the MI between learned features
$X$ and labels $Y$. The MI can be written as the difference of
two entropy terms:

$$I(X; Y) = H(Y) - H(Y|X) = H(X) - H(X|Y)$$

where $I(X; Y)$ is the reduction of uncertainty in $X$ when
$Y$ is observed. In classification task, the minimization
of cross-entropy loss means maximizing $I(X; Y)$. Com-
pared with balanced data, $H(Y)$ is fixed in long-tailed data.
However, as shown in Figure 3, the positive predictions in
most classes is very low meaning that the uncertainty of
$Y$ is higher and $H(Y|X)$ has larger value, thereby resulting
in smaller $I(X; Y)$ and reducing the generalization. The
above result leads to lower entropy $H(X)$ when $H(X|Y)$
is fixed. In the following, we will discuss the relationship
between entropy and covariance matrix and its singular val-
ues.

Given a $d$-dimensional features $X = [x_1, x_2, ..., x_B] \in \mathbb{R}^{B \times d}$, with probability density function $p_X(x)$, the entropy
of $X$ is defined by [4]:

$$H(X) = \int_{\mathbb{R}^d} p_X(x) \log p_X(x) dx,$$  

From the results on [32], suppose $X$ obeys Gaussian distribu-
tion, i.e., $X_i \sim D(\mu_i, \Sigma_i)$, then the $H(X)$ can be written as:

$$H(X) = \frac{d}{2} \log 2\pi e + \frac{d}{2} \log |\Sigma|,$$

where $\mu$ denotes the mean vector, $\Sigma$ stands for the covari-
ance matrix, and $|\Sigma|$ denotes the determinant of $|\Sigma|$. The
covariance matrix $|\Sigma|$ can be expressed as:

$$|\Sigma| = \prod_{i=1}^{d} \lambda_i,$$
where \( \lambda_i \) is the singular value of the covariance matrix \( \Sigma \).

From the above analysis, we can conclude that the entropy \( H(X) \) depends on the distribution of singular values. A low \( H(X) \) means more small singular values, which indicates that the features are concentrated in the direction of the eigenvector with larger singular values. The visualization results in the appendix demonstrate our conclusion (Figure 8, 7 11). Therefore, in order to improve the performance of the model in the long-tail data, we need to increase the entropy of features. As mentioned in Section 1, the re-sampling methods are proposed to balance the features, thereby getting larger \( H(X) \) and improving the generalization [3, 8, 10, 24, 43].

As we know, whitening has been widely used for decorrelating and scattering the features, thereby improving the generalization of the model [9, 25]. Therefore, as shown in Figure 2, we observe the correlations between the features are higher on the long-tailed CIFAR-10 when the model convergences.

### 3.2. The Impacts of Correlated Feature

We analyze the impacts of correlated features on the gradients of classifier weights, the convergence of model (i.e., the condition number of Hessian), and logits margin-based methods.

#### 3.2.1 Impact on Gradient

We present an abridged version of the exact gradient calculation here. The classifier uses the linear combination of the inputs as logits for prediction. We can formulate it as:

\[
\begin{align*}
    f_\theta &= W^T X + b \\
    &= [w_1, w_2, ..., w_N]^T [x_1, x_2, ..., x_B] + b,
\end{align*}
\]

(5)

where \( W \in \mathbb{R}^{C \times N} \) are the weights of the FC layer, and \( N \) is the number of classes. \( f_\theta \) are logits, and \( b \) are the biases of the linear function. We use \( Y = [y_1, y_2, ..., y_N]^T \in \mathbb{R}^{N \times B} \) to denote the logits, \( P, Y \in \mathbb{R}^{N \times B} \) to denote the class probabilities and ground-truth labels, respectively. For sample \( x_i \), its probabilities \( p_i \) and cross-entropy loss \( \ell(p_i, y_i) \), where \( i = 1, 2, ..., B, k = 1, 2, ..., N \). If there are \( B_k \) samples on the class \( k \), we have the following properties of the classifier weights:

**Property 1** The classifier weights are updated \( w_k = w_k^{t-1} - \eta \nabla w_k \), where \( \eta \) denotes the learning rate, and:

\[
\begin{align*}
    \nabla w_k^t &= w_k^{t-1} + \eta \left( \frac{1}{B_k} \sum_{i=1}^{B_k} (1 - p_{ik}) x_i^T \right) - \eta \left( \frac{1}{B_k} \sum_{j=1}^{B_k} p_{jk} x_j^T \right),
\end{align*}
\]

(6)

\[
egin{array}{ccc}
	ext{positive gradient} & \text{negative gradient} \\
\nabla w_k^t &= w_k^{t-1} + \eta \left( \frac{1}{B_k} \sum_{i=1}^{B_k} (1 - p_{ik}) x_i^T \right) & - \eta \left( \frac{1}{B_k} \sum_{j=1}^{B_k} p_{jk} x_j^T \right)
\end{array}
\]

Figure 3. The Visualization of true positive predictions, gradient norm and weight norm on sorted categories in descending order of frequency. We use ResNet-32 on CIFAR-10-LT with an imbalance ratio of 200.

where \( B = B_k + B_k, \nabla w_k \in \mathbb{R}^{1 \times C} \), positive gradient and negative gradient increases and decreases the magnitude of weights respectively.

From the above Property 1, we can obtain the following conclusion on the gradient of classifier weights: The input features \( x_i \in \mathbb{R}^{C \times 1} (i = 1, 2, ..., B) \) with higher correlation coefficients cause the directions of the gradients of each category more consistent, resulting in the descent directions of the weights of the tail classes are always not the best. As we know, head classes with a large number of samples participate in more training, which makes the directions of gradients more conducive to their weights converging to the minimum points. In addition, the model is trained by stochastic gradient descent (SGD) [42] with momentum resulting in the decent directions of gradients that are rarely the best for the weights of tail classes [25]. In other words, the loss drops quickly in some direction and slowly in another, which easily make the model converge to local minima.

Another conclusion is that: The gradient norm \( \| \nabla w_k \|_2 \) is proportional and inversely proportional to true positive prediction \( p_{ik} \) and false positive prediction \( p_{jk} \), respectively. As shown in Figure 3, when the model is trained with cross-entropy (CE) loss, tail classes tend to have lower true positive prediction \( p_{ik} \) and higher negative positive prediction \( p_{jk} \), which results in the norm values of positive gradient \( \nabla w_k^p \) and negative gradient \( \nabla w_k^n \) to be approximately equal \(( \| \nabla w_k^p \|_2 \approx \| \nabla w_k^n \|_2 \)). Therefore, the gradients of tail classes are always with smaller gradient norm \( \| \nabla w_k \|_2 \) and smaller weight norm \( \| w_k \|_2 \). In the following Section 3.2.3, we discuss how the logits margin-based method [2] affects true positive prediction \( p_{ik} \) and the gradient norm \( \| w_k \|_2 \) of each category.

#### 3.2.2 Impact on Generalization

Firstly, we present an abridged version of the Hessian calculation of classifier weight here and analyze the relationship between the covariance matrix of correlated features and the Hessian matrix. Secondly, we further analyze how the eigenvalue distribution of input features affects the condition number of Hessian, thereby affecting the generaliza-
tion of the model.

**Property 2** The Hessian of classifier weights can be written as:

$$H = \frac{\partial^2 \ell(p_i, \hat{y}_i)}{\partial w_k \partial w_{k'}} = \frac{\partial \nabla w_k}{\partial w_{k'}}$$  

(7)

where $H \in \mathbb{R}^{d \times d}$, $d = N \times C$, $k$ and $k'$ belong to different classes. Thus, the elements in $H$ can be computed as:

$$H_{kk',k'c'} = \begin{cases} \frac{1}{B} \sum_{i=1}^{B} p_{ik}(1-p_{ik})x_{ic}x_{ic'} & k = k' \\ -\frac{1}{B} \sum_{i=1}^{B} p_{ik}p_{ik'}x_{ic}x_{ic'} & k \neq k' \\ \end{cases}$$

(8)

$$= \frac{1}{B} \sum_{i=1}^{B} p_{ik}(\delta(k,k') - p_{ik})x_{ic}x_{ic'}$$

(9)

where $\delta$ denotes the Kronecker delta, and $c$ and $c'$ denote different channels of features $x_i$.

Previous works have studied the connection between eigenvalues of Hessian and generalization [1, 49]: The size of the positive eigenvalues is a measure of how well a minimum will generalize to unseen data. If a minimum is wide, and thus has small eigenvalues in many directions, the minimum is better resistant to noisy transformations of the weights, while a sharp minimum has a higher sensitivity to the noise in the weights.

The generalization is connected to the eigenvalues of the Hessian, in particular, to the ratio of the largest eigenvalue to the smallest one, e.g., the condition number of Hessian. Simon et al. [46] proved that: The condition number $\kappa$ of Hessian is determined by the ratio of largest eigenvalue $\lambda_{C}$ and smallest non-zero eigenvalue $\lambda_{i}(\lambda_{i} \neq 0)$ of covariance matrix $\Sigma = \sum_{i=1}^{B} x_{i}x_{i}^{T}$ of input features. $\kappa$ is:

$$\kappa(H) = \kappa(\Sigma) = \frac{\lambda_{C}(\Sigma)}{\min_{\lambda_{i}, \lambda_{i} \neq 0} \lambda_{i}(\Sigma)}$$

(10)

Simon et al. [46] also provided the proof that the condition number $\kappa(\Sigma)$ is bounded by the variance of channel-wised features $x_{c}$.

The strongly correlated features (larger $R_{c}$) make $\kappa(\Sigma)$ with larger upper bound, thereby impacting the generalization of the model. Therefore, as suggested by Simon et al. [46], we propose channel whitening to decorrelate the features.

### 3.2.3 Impact on Logits Margin

The logits margin-based approach, e.g., LDAM [2], allocates large loss for tail class by adding a larger margin into the softmax cross-entropy. For a sample $x_{i}$ belonging to class $k$, the logits margin can be written as:

$$p_{ik} = \frac{e^{y_{ik}-\delta_{ik}}}{\sum_{k' \neq k} e^{y_{ik'}-\delta_{ik}}}$$

(11)

$$\ell(p_{i}, y_{i}) = \log \left[ 1 + \sum_{k' \neq k} e^{(y_{ik'}-y_{ik})+\delta_{ik}} \right]$$

(12)

where $\delta_{ik} = \frac{1}{M_{k}}$ depends on the prior label distribution, $M_{k}$ denotes the number of samples on class $k$. As shown in Figure 3, the positive prediction confidences of the head classes are greatly reduced after logits margin, which results in more imbalanced gradient norms. Therefore, the directions of gradient tend to update the classifier weights of head classes causing larger weight norms of head classes. In addition, as shown in Figure 5 of the appendix, the loss curve of the LDAM method drops in the violent turbulence, which means the model tends to generalize to sharper minima compared with CE loss.

Based on the above analysis, we can obtain the following conclusions: 1) In order to decouple the classifier weights update, we must decorrelate the features in the channels. Therefore, we propose to use the following channel whitening approach to decorrelate features and help to speed up the convergence rate of model.

### 3.3. Channel Whitening

Let $X = [x_{1}, x_{2}, ..., x_{C}]^{T} \in \mathbb{R}^{C \times B}$ be a batch of channel-wised features before inputting into the classifier, where $C$ and $B$ are the numbers of channel and batchsize respectively. The whitening transformation $\phi$ is defined as:

$$\phi(X) = \Sigma^{-\frac{1}{2}}(X - \mu \cdot 1^{T}),$$

$$\mu_{c} = \frac{1}{B} \sum_{i=1}^{B} x_{ic},$$

$$\Sigma = \frac{1}{C}(X - \mu \cdot 1^{T})(X - \mu \cdot 1^{T})^{T} + \epsilon I,$$

where $\mu = [\mu_{1}, \mu_{2}, ..., \mu_{C}]^{T} \in \mathbb{R}^{C}$ is a column vector with dimension $C$, $1$ is a column vector where all entries are equal to one, $\Sigma$ is the covariance matrix of zero-mean $X$, and $\epsilon > 0$ is a small positive number for numerical stability (preventing a singular $\Sigma$), $\Sigma^{-\frac{1}{2}}$ is the inverse square root of the covariance matrix. The whitened $\phi(X)$ has identity covariance matrix $\Sigma$.

The ZCA whitening compute $\Sigma^{-\frac{1}{2}}$ through eigen decomposition: $\Sigma^{-\frac{1}{2}} = V \Lambda^{-\frac{1}{2}} V^{T}$, where $\Lambda = \text{diag}(\lambda_{1}, \lambda_{2}, ..., \lambda_{C})$ and $V = [v_{1}, v_{2}, ..., v_{C}]$ are the eigenvalues and eigenvectors of $\Sigma$, i.e., $\Sigma = V \Lambda V^{T}$. The above process means that the centered $X$ is rotated by $V^{T}$, scaled by $\Lambda^{-\frac{1}{2}}$, and then rotated by $V$ again.

The proposed channel whitening approach can decouple the classifier weight update by decorrelating the features before inputting them into the classifier, which not only alleviates the above-mentioned gradient and generalization problems but also “scatters” the features to reshape the skewed decision boundary toward the head classes. As shown in Figure 7 and 11 of the appendix, we also visualize
the channel-wised singular value distribution on different layers of ResNet-32. We observe that the singular value distributions on imbalanced CIFAR-10 become steeper as the network layers become deeper, especially on the layer $p$ compared with balanced CIFAR-10. Therefore, we propose to use channel whitening to decorrelate the features only on the layer $p$ that before inputting them into the classifier.

3.4. Block-Based Relatively Balanced Sampler & Batch Embedded Training

When the imbalance ratio is large, however, the widely used random sampler makes the tail classes difficult to participate in the batch training, resulting in them under-fitting. Therefore, we would like to propose a new sampler and a novel training strategy that can make the tail classes participate in more iterations without decreasing the learning of head classes. As shown in Figure 4, the proposed Block-based Relatively Balanced Sampler (B3RS) is divided into the following four steps:

- **All categories $N$ are sorted according to their number of samples, and the number of samples in each category is $Q = [Q_1, Q_2, ..., Q_N]$, the proportion of each category is $r = [r_1, r_2, ..., r_N]$.**

- **All sorted categories are equally divided into $G$ blocks, and each block has same number of classes $F = \frac{N}{G}$. In addition, the $F$ classes in each block $G_i$ are sampled from $\{N_i, N_i + G, N_i + 2G, ...\}^F$, where $i = 1, 2, ..., G$. The number of samples of each category in block $G_i$ can be written as $Q_i = [Q_i^1, Q_i^2, ..., Q_i^F]$, and their ratios are $r_i = [r_i^1, r_i^2, ..., r_i^F]$.**

- **We propose a Block-based Cyclic Reversed Sampling with Replacement method, which means that the samples in each batch are always collected from the same categories in the blocks with replacement. Because there are not enough samples of the tail classes. The meaning of “reversed” is: To get relatively balanced samples in each batch compared with random sampling, we need to manually specify a minimum sampling ratio $r_{min}$ for the tail classes in each block, which can be calculated by:**

$$r_{min} = \begin{cases} r_0 & S < 1 \\ S \times r_0 & 1 \leq S < 10 \\ \frac{1}{S} & S \geq 10 \end{cases}$$

(14)

where $r_0$ is a basic sampling ratio determined by the imbalance factor of a dataset. $S = \frac{QN}{F}\frac{N}{G}$ is a scale parameter and is inversely proportional to the imbalance factor. $Q_N$ is the number of samples in the minor category, $B$ is the batch size, $F$ is the number of classes in every block, $\alpha$ is an adjustable parameter. $\frac{QN}{\alpha}$ is used to ensure the same samples of tail classes must participate in training every $\alpha$ epoch to prevent over-fitting. $\frac{1}{\alpha}$ denotes the mean of samples per category in each batch. $\frac{1}{F}$ means that the number of samples in each batch category is the same, i.e., balanced samples of each category in the batch. The proposed B3RS has the following two properties to ensure categories in each batch have the same number of samples:

- **same number of samples in each batch tail classes, $B \times r_i^F$.**

- **same batch imbalance ratio.**

After minimum sampling ratio $r_{min}$ is determined, the new sampling ratio $r_i'$ of all categories in each block $G_i$ can be calculated by the following equation:

$$r_i' = \begin{cases} r_{min}' & r_i' \leq r_{min} \\ (1 - r_{min}) \times r_i^{F-F'} & r_i' > r_{min} \end{cases}$$

(15)

where $r_i' \in \mathbb{R}^F$, $F$ is the number of categories in each block, $F'$ is the number of categories whose class ratios are less than $r_{min}$, $r_i^{F-F'} = [r_i^1, r_i^2, ..., r_i^{F-F'}]$. The fixed $r_i'$ makes categories in each block have the same sampling ratio, i.e., the same batch imbalance ratio.

- **Shuffle the samples in each batch.**

As shown in Figure 1, the number of samples in the proposed B3RS still has a long-tailed distribution, but it is alleviated compared to a random sampler. If we only use the above proposed B3RS instead of the random sampler for training, there is not much difference from over-random sampling. Therefore, we further propose a novel Batch Embedded Training strategy. The batch samples with the same imbalance ratio in the B3RS will participate in the training intermittently (every $T$ iterations) in every epoch to promote the weight update of tail classes. The overall algorithm is detailed in Alg. 1.
Algorithm 1 An End-to-End Training Approach for Long-Tailed Recognition

Required Samplers: Random Sampler with iterations $T_1$, B3R. Sampler with iterations $T_2$

Required Models: Initialized Backbone $f_{01}$ and Classifier $f_{02}$

Required: Inputs $X$, Features $Z$, Iteration $T$, Channel Whitening $\phi$, Whitened features $\hat{Z}$

1: for $t_1=1$ to $T_1$ do
2: Extract features $Z = f_{01}(X)$
3: Channel whitening $\hat{Z} = \phi(Z)$
4: Output logits $\hat{Y} = f_{02}(\hat{Z})$
5: if $t_1/T = 0$ then
6: for $t_2=1$ to $T_2$ do
7: Extract features $Z = f_{01}(X)$
8: Channel whitening $\hat{Z} = \phi(Z)$
9: Output logits $\hat{Y} = f_{02}(\hat{Z})$
10: Compute cross-entropy loss
11: Update $f_{01}$ and $f_{02}$ by back-propagation
12: end for
13: end if
14: Compute cross-entropy loss
15: Update $f_{01}$ and $f_{02}$ by back-propagation
16: end for

4. Experiments

In this section, we firstly introduce the three long-tailed image classification datasets used for our experiments. Then we present some key implementation details of our methods. After that, the state-of-the-art methods are compared with our proposed method. Finally, some ablation studies are given to highlight some important properties of our method.

4.1. Experimental Setup

Datasets. We perform experiments on three long-tailed datasets, including CIFAR-10-LT [23], CIFAR-100-LT [23], and ImageNet-LT [6]. Following prior work [2], the long-tailed versions of CIFAR datasets are sampled from the balanced CIFAR by controlling the number of samples for each category. An imbalance factor $\gamma$ is used to present the ratio of training samples for the most frequent class and the least frequent class, i.e., $\gamma = \frac{N_{max}}{N_{min}}$. In our experiments, we set the imbalance factors as 10, 50, 100, 200, 500, 1000 for the CIFAR-10-LT, and 10, 50, 100, 200, 500 for the CIFAR-100-LT. In the above settings, when the imbalance factor $\beta$ takes the maximum value, the least frequent classes only have $1$ or $5$ samples, which is similar to few-shot learning. The large-scale ImageNet-LT consists of 115.8K training images from 1000 classes and the number of images per class is decreased from 1280 to 5.

Table 1. Top 1 accuracy for CIFAR-10-LT.

| Imbalance Factor | 500  200  100  50  10 |
|------------------|-----------------------|
| End-to-end training |
| CE | 30.1 | 34.4 | 38.6 | 43.8 | 56.7 |
| CB-CE | - | 35.6 | 38.8 | 44.8 | 57.6 |
| Focal | - | 35.6 | 38.4 | 44.3 | 55.8 |
| MW-Net | - | 36.6 | 41.6 | 45.7 | 58.9 |
| LDAM | 33.2 | 38.8 | 42.8 | 47.3 | 57.5 |
| Mixup | - | - | 39.5 | 44.9 | 59.1 |
| Decoupled training |
| BBN | - | - | 42.6 | 47.0 | 59.1 |
| Ours (w / CW) | 30.7 | 36.4 | 40.0 | 43.2 | 57.7 |
| Ours (w / CW & LDAM) | 32.6 | 37.8 | 43.2 | 45.7 | 58.5 |
| Ours (w / BET) | 32.4 | 36.2 | 40.6 | 45.2 | 58.3 |
| Ours (w / BET & CW) | 33.7 | 38.2 | 41.8 | 46.5 | 59.1 |
| Ours (w / BET & LDAM) | 33.4 | 38.3 | 42.5 | 47.5 | 59.3 |
| Ours (w / BET & LDAM & CW) | 34.7 | 39.7 | 43.4 | 47.8 | 60.0 |

Table 2. Top 1 accuracy for CIFAR-100-LT.

4.2. Implementation Details

We implement our framework in PyTorch [35] and adopt TITAN-X GPUs for training. The details of the experimental setting are illustrated in the appendix.

4.3. Experimental Results

In this section, we present results to demonstrate the effectiveness of our method, including of the Channel Whitening (CW) and BET.

Effectiveness of Channel Whitening. As shown in Table 1, 2 and 3, the performance of models trained with channel whitening can be improved 0.6% ~ 4.4%, 0.6% ~ 2%
### Table 3. Top 1 accuracy on ImageNet-LT using ResNet-10.

| Method               | Many   | Medium | Few   | All  |
|----------------------|--------|--------|-------|------|
| End-to-end training  |        |        |       | 32.3 |
| CE                   | 55.1   | 22.4   | 2.2   | 32.3 |
| Focal                | 36.4   | 29.9   | 16    | 30.5 |
| Lifted Loss          | 35.8   | 30.4   | 17.9  | 30.8 |
| Range Loss           | 35.8   | 30.3   | 17.6  | 30.7 |
| OLTR                 | 43.2   | 35.1   | 18.5  | 35.6 |
| LDAM                 | 51.0   | 25.2   | 4.9   | 32.4 |
| Equalization Loss    | -      | -      | -     | 36.4 |
| Instance Balance     | 55.4   | 22.7   | 2.7   | 32.5 |
| Class Balance        | 45.5   | 34.4   | 14.8  | 36.0 |
| Decoupled training   |        |        |       |      |
| cRT                  | 48.8   | 36.2   | 22.8  | 39.2 |
| LWS                  | 49.9   | 36.8   | 13.1  | 38.7 |
| NCM                  | 41.1   | 31.8   | 19.7  | 33.7 |
| Ours (w/CW)          | 55.6   | 28.1   | 6.3   | 35.7 |
| Ours (w/CW & LDAM)   | 49.3   | 36.7   | 21.7  | 39.5 |
| Ours (w/BET)         | 52.5   | 13.0   | 0.7   | 26.5 |
| Ours (w/BET & CW)    | 56.8   | 30.2   | 8.8   | 37.5 |
| Ours (w/BET & LDAM)  | 50.8   | 15.7   | 2.4   | 27.4 |
| Ours (w/BET & LDAM & CW) | 55.6 | 31.7 | 12.4 | 38.3 |

Table 3. Top 1 accuracy on ImageNet-LT using ResNet-10.

on the CIFAR-10-LT and CIFAR-100-LT compared with baselines. The overall performance on ImageNet-LT is 35.7%, which is a competitive result compared with other end-to-end training methods [2, 28, 31, 45, 54]. Especially, channel whitening improves the performance of medium shot 5.7% and few shot 4.1% without compromising the performance of many shot. We also analyze how the channel whitening can “scatter” the features to generate impacts on the decision boundary in the appendix (8).

**Channel Whitening with LDAM.** The results on Table 1 and 3 can well demonstrate that the performance of LDAM [2] can be greatly improved after decoupling weight update by channel whitening. In this way, the end-to-end training has achieved better performance than decoupled training method [17, 56] on the CIFAR-10-LT and ImageNet-LT datasets. Even more surprising is that channel whitening with the LDAM method achieves the best results 39.5% on the large scale ImageNet-LT. However, when the imbalance ratio is large on the CIFAR-100-LT, their combinations achieve worse performance than LDAM.

**Effectiveness of BET.** As shown in Table 1, when the imbalance ratios are 10 and 50, the model is trained only with BET can achieve the best performance by end-to-end training. In Table 2 and 3, the performances of BET are all lower than LDAM [2], even worse than the baseline, which means that the BET approach is suitable for a dataset with a low imbalanced ratio. When the BET combines with LDAM and CW methods can achieve the best performance on the CIFAR-100-LT. More importantly, the performance on ImageNet-LT (Table 3) can achieve 37.5% and 38.3% when CW and CW & LDAM are combined with BET respectively.

### 4.4. Ablation Study

**More Training Epochs** As shown in Figure 9 of the appendix, the evaluation accuracy of the model trained with “LDAM-CW” fluctuates relatively large, that is, the model has not converged after being trained with 90 epochs. Therefore, we train the model with 120 epochs, which enables the model to achieve better accuracy (39.8% on Table 4) and convergence (“LDAM-CW-120”).

**New Schedule of Learning Rate** The results in Figure 10 of the appendix demonstrate that BET enables the model to reach the local best performance in the first epoch (epoch 30, epoch 60) when the learning rate is reduced, and the performance at a fixed learning rate decreases. Therefore, we take a new schedule of learning rate to stabilize the training of the model. As demonstrated in Table 5, the new schedule of learning rate makes the “LDAM-BET-CW-new” method achieve the same accuracy 39.2% as decoupled training methods [17].

### 5. Conclusion

In this paper, we explore and identify a key factor, correlated features before inputting the classifier, that affects the learning of the classifier with end-to-end training. Firstly, we theoretically analyze the impact of correlated features and propose channel whitening to alleviate it. The channel whitening combined with logits margin-based method can achieve competitive results compared with decoupled training methods. Secondly, the proposed Batch Embedded Training approach can achieve the best performance when the imbalance ratio is small. Our method has shown considerable improvements on the benchmarks and has great potential for end-to-end training, which needs to be explored.
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Appendix

In this appendix, we present the calculation of the correlation coefficients, analyze the distribution of singular values, compare the loss curves and feature distribution after channel whitening. We also provide the detailed experimental settings, and some ablation studies on the hyper-parameters.

I. Calculation of Feature Correlation

We analyze the correlation of features between each channel by computing their correlation coefficients. Let the features before inputting the classifier as $X = [x_1, x_2, \ldots, x_C]^T \in \mathbb{R}^{C \times B}$, where $C$ is the number of channel, $B$ is the batchsize. In order to analyze the linear correlation of channels, we compute their Pearson product-moment correlation coefficient (PPMCC) by:

$$\rho(x_c, x_{c'}) = \frac{\sum_{i=1}^{B} (x_{ic} - \bar{x}_c)(x_{ic'} - \bar{x}_{c'})}{\sqrt{\sum_{i=1}^{B} (x_{ic} - \bar{x}_c)^2} \sqrt{\sum_{i=1}^{B} (x_{ic'} - \bar{x}_{c'})^2}}$$

where $c, c' = 1, 2, \ldots, C$, $\rho$ has value between $-1$ and $1$, and $\rho = 1$ means that the channels $x_c$ and $x_{c'}$ are linearly correlated.

II. Results of Channel Whitening

In the paper, we analyze the relationship between singular values and feature entropy $H(X)$, condition number of Hessian $\kappa(H)$. In this section, we will present some results to demonstrate the effectiveness of channel whitening.

Distribution of Singular Values and Features As shown in Figure 7 and Figure 11, we visualize the channel-wise singular value distributions on different layers of ResNet-32. From Figure 7 (a) and (b), we observe that the curves of singular values on imbalanced CIFAT-10 become steeper as the network layers become deeper, especially on the layer-$p$ compared with balanced CIFAR-10. After channel whitening, as shown in Figure 7 (c), the singular values on layer-$p$ have more larger values, i.e., the curve of singular value is smoothed. Therefore, the more larger singular values mean that the entropy $H(X)$ with higher value, thereby balancing the feature space to reshape the classification boundary. As illustrated in Figure 8, after channel whitening, the features are scattered and the samples are no longer concentrated in a certain direction. More samples of tail classes are mixed with head classes, so that the classification boundary is biased toward the head classes, thereby improving the performance of tail classes.

Visualization of Loss and Gradient Norm As shown in Figure 5, the loss values of “CE-CW” and “LDAM-CW” are larger than their corresponding losses without channel whitening, which means the gradients of tail classes participate in more updates, i.e., tail classes with higher loss values. As illustrated in Figure 6, channel whitening decouples the weights updating to make the gradients of tail classes have larger norm values, instead of gradients with almost zero values due to the high correlation of features.

III. Experiments

III.1. Experimental Details

Long-tailed CIFAR For both long-tailed CIFAR-10 and CIFAR-100, following most of the existing work, we use ResNet-32 [12] as backbone to extract image representation. SGD optimizer is adopted to optimize model with momentum of 0.9, weight decay of 0.0002. The initial learning rate is set to 0.1 and is decreased to 1/10 of its previous value on the 160 and 180 epochs in the total 200 epochs. The batchsize is set to 128.

Long-tailed ImageNet For ImageNet-LT, we use ResNet-10 [12] as backbone model. SGD optimizer with momentum of 0.9, weight decay of 0.0005. The initial learning rate is set to 0.2 and is decreased to 1/10 of its previous value every 30 epochs in the total 90 epochs. The batch size is set to 512.

| Hyper-parameter | $G$ | $r_{\text{min}}$ | $\alpha$ | $T$ |
|-----------------|-----|-----------------|----------|-----|
| CIFAT-10_LT     | 1   | 0.05            | 2        | 60  |
| CIFAT-100_LT    | 10  | 0.01            | 2        | 30  |
| ImageNet_LT     | 100 | 0.01            | 2        | 60  |

Table 6. The hyper-parameters of B3RS and BET on different datasets. $G$ is the block number, $r_{\text{min}}$ is the minimum sampling ratio, $\alpha$ is the epoch interval for B3RS, $T$ is the iteration interval for BET.
Figure 6. The gradient norm over epochs on CIFAR-10-LT with ResNet-32.

Figure 7. The singular value distributions on different layers of ResNet-32. Results are from CIFAR-10-LT dataset with an imbalance factor 200. The sub-figures of (a), (b) from top to bottom, from left to right are: Conv_1, Layer_1, Layer_2, Layer_3 and Layer_p, where “p” denotes pooling. The last sub-figure on (c) is Layer_w, where “w” denotes whitening.

III.2. Ablation Study

Hyper Parameters The hyper-parameters of B3RS and BET on different datasets are summarized in Table 6. In this part, we construct experiments on the ImageNet-LT with varying hyper-parameters including block number $G$, minimum sampling ratio $r_{min}$ and training iterations $T_2$ on the B3RS. All the results are obtained by training the models
Figure 8. The t-SNE 2D visualization of feature distribution before classifier layer. Results from CIFAR-10-LT dataset on the imbalance ratio 200.

Figure 9. The evaluation accuracy on ImageNet-LT with ResNet-10.

As shown in the Table 8, 7 and 9, the performances under different varying hyper-parameters are similar, that is, the models are not sensitive to the choice of hyper-parameters.
Figure 10. The evaluation accuracy on ImageNet-LT with ResNet-10.

Table 7. Performance on varying basic sampling ratio on ImageNet-LT.

| Sampling ratio \(r_{min}\) | Samples | Many  | Medium | Few  | All  |
|-----------------------------|---------|-------|--------|------|------|
| 0.002                       | 1       | 56.5  | 29.9   | 8.1  | 37.2 |
| 0.01                        | 5       | 56.8  | 30.2   | 8.8  | 37.5 |
| 0.02                        | 10      | 56.5  | 30.1   | 10.0 | 37.5 |
| 0.03                        | 15      | 56.1  | 30.1   | 11.0 | 37.5 |
| 0.04                        | 20      | 55.7  | 29.8   | 12.1 | 37.4 |

Table 8. Performance on varying block number on ImageNet-LT.

| Block number \#G | Many  | Medium | Few  | All  |
|------------------|-------|--------|------|------|
| 10               | 53.6  | 30.3   | 11.2 | 36.7 |
| 20               | 55.9  | 30.2   | 11.5 | 37.4 |
| 50               | 56.9  | 29.2   | 10.2 | 37.3 |
| 100              | 56.8  | 30.2   | 8.8  | 37.5 |
| 200              | 56.2  | 29.6   | 9.1  | 37.0 |

Table 9. Performance on varying training iterations on B3RS.

| Iterations \#T_2 | Many  | Medium | Few  | All  |
|------------------|-------|--------|------|------|
| 100              | 56.8  | 29.8   | 8.5  | 37.3 |
| 200              | 56.5  | 29.6   | 8.7  | 37.1 |
| 300              | 56.3  | 29.8   | 8.3  | 37.1 |
| 400              | 56.5  | 29.9   | 8.7  | 37.3 |
| 500              | 56.8  | 30.2   | 8.8  | 37.5 |
| 600              | 56.2  | 29.7   | 8.8  | 37.1 |

At the same time, we can control the performance on different shots by selecting different hyper-parameter, e.g., small block number \(G\) and large basic sampling ratio \(r_{min}\) will make the model have higher performance on the few shot data.
Figure 11. The singular value distributions on different layers of ResNet-32. Results from CIFAR-10-LT dataset on imbalance factor 200. The sub-figures of (a), (b) from top to bottom, from left to right are: Conv_1, Layer_1, Layer_2, Layer_3 and Layer_p, where “p” denotes pooling. The last sub-figure on (c) is Layer_w, where “w” denotes whitening.