Robust Intensity Mapping Analysis against Foregrounds for the Epoch of Reionization

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Intensity mapping of the HI 21 cm line and the CO 2.61 mm line from the epoch of reionization has emerged as powerful, complementary, probes of the high-redshift Universe. However, both maps and their cross-correlation are dominated by foregrounds. We propose a new analysis by which the signal is unbiased by foregrounds, i.e. it can be measured without foreground mitigation. We construct the antisymmetric part of the HI-CO cross-correlation, arising because the statistical fluctuations of two fields have different evolution in time. We show that the sign of this new signal can distinguish model-independently whether inside-out reionization happens during some interval of time.

Intensity mapping of the 21 cm hyperfine transition line of atomic hydrogen is currently considered to be one of the most promising probes of the epoch of reionization (EOR) [1, 2]. Upcoming large radio interferometer arrays promise to detect the 21 cm power spectrum from the EOR for the first time, and will attempt to obtain the tomographic 21 cm imaging (see, e.g. [3]). As a complementary probe, intensity mapping of the 2.61 mm (J = 1 → 0) spectral line of the 12CO (carbon monoxide) [4–8] can probe the gas mass of star-forming regions during the formation of first galaxies.

However, intensity maps of both 21 cm line and CO(1-0) line are dominated by foregrounds which are typically orders of magnitude brighter than the signals from the EOR. At low redshifts, two-point cross-correlation of two fields, e.g. the HI-galaxy survey cross-correlation [9,10] or the CO-galaxy survey cross-correlation [11], can be robust against foregrounds because the foreground contamination of each field, if any, is usually caused by different sources. (In this case, see also a one-point cross-correlation technique unbiased by foregrounds [12] unfortunately, that is not the case at high redshifts for the EOR: the same set of Galactic and extragalactic radio sources contribute to both 21 cm and CO foregrounds at the EOR, so both foregrounds depend on the observed frequency with exactly the same power law, even though their strengths are very different. Cross-correlation between the 21 cm and CO(1-0) line intensity maps [6, 7,14] consequently, will be affected significantly by foregrounds. Thus sophisticated foreground removal or avoidance techniques (see [16] and references therein) must be implemented in order to measure the auto-correlation as well as cross-correlation signals.

In a statistically isotropic universe, the two-point cross-correlation function between any pair of fields δ_i and δ_j, defined as ξ_ij(x) = ⟨δ_i(x')δ_j(x' + x)⟩, is symmetric under the exchange of the line-of-sight (LOS) coordinates or equivalently the order of two fields. However, evolution effect [17] can intrinsically break the parity symmetry along the LOS and cause the asymmetry of the cross-correlation in the large-scale structure. This effect is easy to understand: if the field δ_i evolves more rapidly than the field δ_j, then whether δ_i is in front of, or behind, δ_j would result in different cross-correlation for the same physical separation. In principle, larger asymmetry in cross-correlation is expected between two fields with more distinctive evolution in cosmic time.

Cross-correlation between the HI 21 cm line and CO(1-0) line intensity mapping from the EOR is indeed asymmetric due to evolution effect. During the EOR, ultraviolet and X-ray photons emitted from the first luminous objects ionize hydrogen atoms first in the surrounding intergalactic medium and form bubbles of ionized hydrogen regions, and eventually these ionized bubbles fill the whole Universe by z ≃ 6 [18]. We illustrate the evolution of the 21 cm and CO line intensity maps with a slice of lightcone box at three representative stages of reionization in Fig. 1 While the CO(1-0) line intensity maps mostly trace the cosmic density distributions that gradually form the filamentary structures, the HI 21 cm line intensity maps reflect the distributions of neutral hydrogen regions, showing the patchy patterns that rapidly percolate towards the end of reionization. This comparison clearly shows that the progressing of cosmic reionization is faster than the evolution of density fluctuations during the EOR. As such, we expect that the HI-CO cross-correlation is strongly asymmetric. The antisymmetric component of the cross-correlation, ξ^A_{ij}(x) = 1/2[ξ_{ij}(x) - ξ_{ji}(x)], or equivalently ξ^A_{ij}(x) = 1/2[ξ_{ij}(x) - ξ_{ij}(-x)], contains independent statistical information [19, 20] complementary to the symmetric component that the term “cross-correlation” was usually implicitly referred to in the literature.

**Dipole.**— We use the dipole of the HI-CO cross-power spectrum as the antisymmetry estimator. The antisymmetric component of the cross-correlation between two fields δ_i and δ_j is Fourier dual to the imaginary part of the cross-power spectrum, 1/4π [δ^A_{ij}(k)] = (2π)^3δ^{(3)}_{ij}P^A_{ij}(k). Here “c.c.” stands for the complex conjugate of the first term, and δ^A(k) is the Fourier dual to the field δ(x). Since P^A_{ij}(k) = -P^A_{ji}(k),
FIG. 1. Evolution of brightness temperature maps. We show the 21 cm (top) and CO(1-0) maps (bottom) in a slice of simulated lightcone box perpendicular to the LOS in a region of 384 comoving Mpc on each side inside the simulation volume, (from left to right) at redshift \(z = 7.77, 8.48, \) and 9.96 (corresponding to global neutral fraction \(\bar{x}_{\text{HI}} = 0.25, 0.50, \) and 0.75 in our fiducial model, i.e. with time flow from right to left), respectively.

\(P_{ij}^1(k)\) is called the “antisymmetric cross-power spectrum”. Note that \(P_{ij}^1(-k) = -P_{ij}^1(k)\), i.e. \(P_{ij}^1(k)\) is also antisymmetric in \(k\)-space, so its averaging over a spherical \(k\)-shell (i.e. monopole) is zero, and only odd moments are nonzero. The leading order in terms of the expansion in spherical harmonics is the dipole. Thus we neglect higher order terms and assume a simple template for extracting the dipole \(P_{ij}^1(k) = P_{ij}^\Lambda(k) Y_{10}(\theta)\). At a given spherical \(k\)-shell, we first average \(P_{ij}^1(k)\) over polar angles in a ring with the same azimuthal angle \(\theta\) with respect to the LOS (\(z\)-axis), and obtain the average, \(P_{ij}^1(k,\theta)\). It is straightforward to derive the variance of \(P_{ij}^1(k,\theta)\):

\[
\sigma_{P_{ij}^1}^2(k,\theta) = \frac{1}{N(k,\theta)} \left[ P_i(k) P_j(k) - (P_{ij}^S(k))^2 \right] \left( P_{ij}^1(k,\theta) \right)^2 \right] ,
\]  

where \(P_i(k)\) and \(P_j(k)\) are the auto-power spectrum of the fields \(\delta_i\) and \(\delta_j\), respectively, and \(P_{ij}^S(k)\) is their symmetric (i.e. real part of) cross-power spectrum. The auto-power and the symmetric cross-power spectrum only depend on the magnitude of wavenumber \(k\) because the lightcone effect for each separate statistics is not important \(\cite{22}\). Here \(N(k,\theta)\) is the number of cells in the ring of \((k,\theta)\) in the upper hemisphere. We test that \(\sigma_{P_{ij}^1}^2(k,\theta)\) is always real and positive. Next, the dipole \(P_{ij}^\Lambda(k)\) and its variance can be estimated with \(\chi^2\)-fitting over all measures at different \(\theta\) in the spherical \(k\)-shell. The estimator of the dipole is

\[
P_{ij}^\Lambda(k) = \frac{\sigma_{P_{ij}^1}^2(k)}{\sigma_{P_{ij}^1}^2(k,\theta)} \sum_{\theta} \frac{P_{ij}(k,\theta) Y_{10}(\theta)}{\sigma_{P_{ij}^1}^2(k,\theta)} ,
\]

where the variance of the dipole estimation is

\[
\sigma_{P_{ij}^\Lambda}^2(k) = \left[ \sum_{\theta} \frac{Y_{10}^2(\theta)}{\sigma_{P_{ij}^1}^2(k,\theta)} \right]^{-1} .
\]

Mock signal.— We generate realizations of the 21 cm and CO brightness temperature fields using the density and ionized fraction data from semi-numerical simulations with the code 21cmFAST \(\cite{23,24}\). Readers are referred to the companion Letter for details of the simulations and our modeling of the 21 cm and CO signals. We then interpolate the snapshots at different time to construct the lightcone data cube along the LOS. To reduce the interpolation error caused by insufficient sampling of snapshots, we output the simulation results at 100 different redshifts from \(z = 12.22\) to 6.56 (corresponding to \(\bar{x}_{\text{HI}} = 0.95\) to 0.01 in our fiducial model) in such a manner that these redshifts correspond to the comoving LOS distances with equal separation of 13.7 comoving Mpc. To avoid the impact of periodic boundary condition, we only use the inner cubic region of 384 comoving
FIG. 2. The dipole of the HI-CO cross-power spectrum, $P^A$, vs wavenumber $k$ in our fiducial reionization model, when the center of lightcone box is at $z = 9.96$ (green solid dots), 8.48 (blue solid dots), and 7.77 (black solid dots), respectively. We fit the dipole to a modified power law (dashed lines). For illustrative purpose, we also show the results in the hypothetical no-reionization scenario (open dots) and fit the dipole to a power law (solid lines) at each central redshift. The error bars are 1σ standard deviation for cosmic variance corresponding to the simulation volume of 100 realizations.

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Mpc on each side away from the boundary of the simulation box.[25] To mimic the observations from radio interferometers, we subtract from the lightcone field the mean of the 2D slice for each 2D slice perpendicular to the LOS, because radio interferometers cannot measure the mode with $k_\perp = 0$. This forms the mock lightcone fields of $\delta T_{21}(x)$ and $\delta T_{\rm CO}(x)$ in a cubic box with its location marked by its central redshift $z$. The Fourier transforms of the two fields are used to compute the cross-power spectrum and then its dipole, $P^A_{\rm HI-CO}(k)$, using the aforementioned prescription. Note that the antisymmetric cross-power spectrum flips its sign if the order of cross-correlation is swapped, so we fix our convention that the order of cross-correlation is HI-CO throughout the Letter, and thus the subscript “HI-CO” in the notation of the dipole $P^A_{\rm HI-CO}(k)$ is dropped for the rest of this Letter. Finally, we vary the initial condition and generate 100 different realizations, in order to lower the cosmic variance of dipole from simulations.

**Generic feature.**—In Fig. 2 we plot the dipole of the HI-CO cross-power spectrum $P^A(k)$. If, hypothetically, the universe was not reionized, i.e. $x_{\rm HI} = 1$ everywhere all the time, we find that the HI-CO dipole is generically positive, and it fits to a power law, $P^A(k) = A_{\rm NR}(k/k_*)^{-n_{\rm NR}}$ at each redshift of the box center (hereafter “central redshift”), where we choose $k_* = 1\, h\, \text{Mpc}^{-1}$. On the other hand, in the fiducial reionization simulation, while the signal is initially too small to be distinguished from its cosmic variance at high redshifts, we find that the dipole is generically negative, and it fits to a power law at large scales with suppression at small scales, $P^A(k) = -A_R(k/k_*)^{-n_R} \exp\left[-\beta_R(k/k_*)^{n_R}\right]$ at each central redshift.

The sign of the antisymmetric cross-correlation can be explained by the evolution effect as illustrated in Fig. 3. In the hypothetical no-reionization scenario, the 21 cm fluctuations are simply equal to the matter density fluctuations which evolve relatively slowly during the EOR. Since the CO bias decreases with time[6], the CO brightness temperature fluctuations decrease with time statistically, too. Therefore, the HI-CO cross-correlation is stronger in the back end than in the front end, i.e. $\langle \delta T_{21}(x')\delta_{\rm CO}(x' + x) \rangle > \langle \delta T_{\rm CO}(x')\delta_{\rm CO}(x' - x) \rangle$. Thus the HI-CO antisymmetric cross-correlation is positive, or $\xi^A_{\rm HI-CO}(x) > 0$. On the other hand, in the reionization simulations, the universe is reionized with “inside-out”, i.e. overdense regions are reionized earlier on average, so the 21 cm and CO fluctuations are anti-correlated (i.e. their cross-correlation is negative) at large scales. Since reionization proceeds much faster than the CO bias evolution, the evolution effect due to reionization dominates over that due to the change of CO bias. Since the 21 cm fluctuations grow with time, the CO-HI cross-correlation is weaker in the back end than in the front end, i.e. $|\langle \delta_{\rm CO}(x')\delta_{21\text{cm}}(x' + x) \rangle| < |\langle \delta_{\rm CO}(x')\delta_{21\text{cm}}(x' - x) \rangle|$, so $\langle \delta_{\rm CO}(x')\delta_{21\text{cm}}(x' + x) \rangle > \langle \delta_{\rm CO}(x')\delta_{21\text{cm}}(x' - x) \rangle$, thus $\xi^A_{\rm CO-HI}(x) > 0$. So the HI-CO antisymmetric cross-correlation is negative, or $\xi^A_{\rm CO-HI}(x) < 0$. This picture is applied for a general class of inside-out reionization models, so the sign of the HI-CO antisymmetric cross-correlation can tell whether or not inside-out reionization happens during some interval of time, regardless of the detail of reionization models.

**Foreground.**—To generate the mock foregrounds for...
Cora redshifted 21 cm and CO line, we employ the code Cora[26], which uses the technique described in [27]. This code considers four sources of foregrounds — Galactic synchrotron, Galactic free-free, extragalactic diffuse free-free and extragalactic point sources. At a given observed frequency, it generates random numbers with Gaussian distribution as real and imaginary parts of the foreground frequency, it generates random numbers with Gaussian distribution as real and imaginary parts of the foreground frequency, and neglects the contribution of other molecular line emissions to the CO foreground[30]. We also assume that the foregrounds are completely uncorrelated with the 21 cm and CO signals from the EOR.

In Fig. 4 we compare the dipoles with foreground-contaminated maps and with foreground-free maps. For both inside-out reionization and the hypothetical no-reionization scenario, we find that the fractional difference between the dipoles with and without foreground is less than one part in 100,000. In other words, the HI-CO dipole is robust against foregrounds, and thus can be measured directly from the observed, foreground-contaminated, data. This advantage makes the dipole analysis a clean method to extract the information from the 21 cm and CO maps during the EOR.

This result can be explained by the fact that both 21 cm and CO foregrounds are caused by the same set of radio sources and therefore depend on the observed frequency with exactly the same power law, even though their strengths are orders of magnitude different. The foreground on a 3D data cube is generated by mapping the observed frequency of foreground to the corresponding comoving distance and cosmic time on the lightcone. Consequently, the 21 cm and CO foregrounds appear to “co-evolve” at the same pace on the lightcone. The evolution effect of antisymmetric cross-correlation for such two “co-evolving” foreground fields is negligible, and thus has no impact on the HI-CO dipole.

**Discussions.**— Other effects may contribute to the asymmetry as well, but they are suppressed at high redshifts. First, the field of view for radio interferometric observations corresponds to only a few hundred comoving Mpc at the EOR, at which scale the relativistic distortions[17, 29–32] are not important. Secondly, the wide-angle effect[33] is negligible for this field of view, and the distant-observer assumption holds well at high redshifts. Also, the large-angle effect[33] is irrelevant because different choices of angle used to measure the asymmetry do not make difference under this assumption. Lastly, gravitational lensing[17, 34] of the 21 cm field is a secondary, higher-order, effect. Since the HI-CO antisymmetric cross-correlation is dominated by the evolution effect due to reionization, it contains the information regarding the progressing of reionization, so we propose to extract this new signal, which is unbiased by foregrounds, as a clean probe of cosmic reionization. In the companion Letter, we will explore the astrophysical application of the HI-CO dipole.

In principle, the HI 21 cm map can be cross-correlated with other tracers of cosmological density fluctuations from the EOR, e.g. [CH][35] or other molecular line intensity maps, or high-redshift galaxy surveys[36, 37], if possible. As the evolution effect due to cosmic reionization dominates over that of the density fluctuations, we expect that the dipole of the cross-power spectrum between the 21 cm and a generic probe of density fluctuations, or high-redshift galaxy surveys[36, 37], if possible. The evolution effect due to cosmic reionization dominates over that of the density fluctuations, we expect that the dipole of the cross-power spectrum between the 21 cm and a generic probe of density fluctuations, or high-redshift galaxy surveys[36, 37], if possible. As the evolution effect due to cosmic reionization dominates over that of the density fluctuations, we expect that the dipole of the cross-power spectrum between the 21 cm and a generic probe of density fluctuations, or high-redshift galaxy surveys[36, 37], if possible.

**Note.**— While[38] was posted on the arXiv earlier than this Letter, the proposal of using antisymmetric cross-correlation between the 21 cm and CO line intensity maps as a new probe of cosmic reionization was originally presented with main results by Y.M. in his talk at the 2019 LIM conference held at the CCA, NYC, as acknowledged by[38] itself. Our Letter has essentially least overlapping with[38] but this proposal. [38] focused on the antisymmetric part of angular cross-power spectrum and its application to a special reionization model, instead of...
generic features.

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[1] D. Scott and M. J. Rees, The 21-cm line at high redshift: a diagnostic for the origin of large scale structure, Mon. Not. R. Astron. Soc. 247, 510 (1990).
[2] P. Madau, A. Meiksin, and M. J. Rees, 21 Centimeter Tomography of the Intergalactic Medium at High Redshift, Astrophys. J. 475, 429 (1997).
[3] S. R. Furlanetto, S. P. Oh, and F. H. Briggs, Cosmology at low frequencies: The 21cm transition and the high-redshift Universe, Phys. Rep. 433, 181 (2006).
[4] M. Righi, C. Hernández-Monteagudo, and R. A. Sunyaev, Carbon monoxide line emission as a CMB foreground: tomography of the star-forming universe with different spectral resolutions, Astron. Astrophys. 489, 489 (2008).
[5] C. L. Carilli, Intensity Mapping of Molecular Gas During Cosmic Reionization, Astrophys. J. 730, L30 (2011).
[6] Y. Gong, A. Cooray, M. B. Silva, M. G. Santos, and P. Lubin, Probing reionization with intensity mapping of molecular and fine-structure lines, Astrophys. J. 728, L46 (2011).
[7] A. Lidz, S. R. Furlanetto, S. P. Oh, J. Aguirre, T.-C. Chang, O. Doré, and J. R. Pritchard, Intensity mapping with carbon monoxide emission lines and the redshifted 21 cm line, Astrophys. J. 741, 70 (2011).
[8] L. Vallini, A. Pallottini, A. Ferrara, S. Gallerani, E. Sobacchi, and C. Behrens, CO line emission from galaxies in the Epoch of Reionization, Mon. Not. R. Astron. Soc. 473, 271 (2018).
[9] C. J. Anderson, N. J. Luciwi, C. Y. Li, C. Y. Kuo, J. Yadav, K. W. Masui, T.-C. Chang, X. Chen, N. Oppermann, Y.-W. Liao, U.-L. Pen, D. C. Price, L. Staveley-Smith, E. R. Switzer, P. T. Timbie, and L. Wolz, Low-amplitude clustering in low-redshift 21-cm intensity maps cross-correlated with 2dF galaxy densities, Monthly Notices of the Royal Astronomical Society 476, 3382 (2018), https://academic.oup.com/mnras/article-pdf/476/3/3382/25076525/sty434.pdf.
[10] M. Colless, G. Dalton, S. Maddox, W. Sutherland, P. Norberg, S. Cole, J. Bland-Hawthorn, T. Bridges, R. Cannon, C. Collins, W. Couch, N. Cross, K. Deeley, R. de Propris, S. P. Driver, G. Efstathiou, R. S. Ellis, C. S. Frenk, K. Glazebrook, C. Jackson, O. Lahav, I. Lewis, S. Lumsden, D. Madgwick, J. A. Peacock, B. A. Peterson, I. Price, M. Seaborne, and K. Taylor, The 2dF Galaxy Redshift Survey: spectra and redshifts, Monthly Notices of the Royal Astronomical Society 328, 1039 (2001), https://academic.oup.com/mnras/article-pdf/328/4/1039/3780131/328-4-1039.pdf.
[11] K. W. Masui, E. R. Switzer, N. Banavar, K. Bandura, C. Blake, L.-M. Calín, T.-C. Chang, X. Chen, Y.-C. Li, Y.-W. Liao, A. Natarajan, U.-L. Pen, J. B. Peterson, J. R. Shaw, and T. C. Voytek, MEASUREMENT OF 21 cm BRIGHTNESS FLUCTUATIONS AT z ∼ 0.8 IN CROSS-CORRELATION, The Astrophysical Journal 763, L20 (2013).
[12] H. Padmanabhan, A. Refregier, and A. Amara, Cross-correlating 21 cm and galaxy surveys: implications for cosmology and astrophysics, Mon. Not. R. Astron. Soc. 495, 3935 (2020) arXiv:1909.11104 [astro-ph.CO].
[13] D. T. Chung, M. P. Viero, S. E. Church, R. H. Wechsler, M. A. Alvarez, J. R. Bond, P. C. Breysse, K. A. Cleary, H. K. Eriksen, M. K. Foss, J. O. Gundersen, S. E. Harper, H. T. Ilhe, L. C. Keating, N. Murray, H. Padmanabhan, G. F. Stein, I. K. Wehus, and COMAP Collaboration, Cross-correlating Carbon Monoxide Line-intensity Maps with Spectroscopic and Photometric Galaxy Surveys, Astrophys. J. 872, 186 (2019) arXiv:1809.04550 [astro-ph.GA].
[14] P. C. Breysse, C. J. Anderson, and P. Berger, Canceling out intensity mapping foregrounds, Phys. Rev. Lett. 123, 231105 (2019).
[15] E. Visbal and A. Loeb, Measuring the 3D clustering of undetected galaxies through cross correlation of their cumulative flux fluctuations from multiple spectral lines, J. Cosmol. Astroparticle Phys. 2010, 016 (2010).
[16] A. Liu and J. R. Shaw, Data Analysis for Precision 21 cm Cosmology, Publ. Astron. Soc. Pac. 132, 062001 (2020).
[17] C. Bonvin, L. Hui, and E. Gaztañaga, Asymmetric galaxy correlation functions, Phys. Rev. D 89, 083535 (2014).
[18] X. Fan, C. L. Carilli, and B. Keating, Observational Constraints on Cosmic Reionization, Annu. Rev. Astron. Astr. 44, 415 (2006).
[19] L. Dai, M. Kamionkowski, E. D. Kovetz, A. Raccanelli, and M. Shiraishi, Antisymmetric galaxy cross-correlations as a cosmological probe, Phys. Rev. D 93, 023507 (2016).
[20] A. Hall and C. Bonvin, Measuring cosmic velocities with 21 cm intensity mapping and galaxy redshift survey cross-correlation dipoles, Phys. Rev. D 95, 043530 (2017).
[21] See Supplemental Material at [URL will be inserted by publisher] for the derivation.
[22] K. K. Datta, G. Mellema, Y. Mao, I. T. Iliev, P. R. Shapiro, and K. Ahn, Light-cone effect on the reionization 21-cm power spectrum, Mon. Not. R. Astron. Soc. 424, 1877 (2012).
[23] https://github.com/21cmfast/21cmFAST
[24] A. Mesinger, S. Furlanetto, and R. Cen, 21cmfast: a fast, seminumerical simulation of the high-redshift 21-cm signal, Mon. Not. R. Astron. Soc. 411, 955 (2011).
[25] See Supplemental Material at [URL will be inserted by publisher] for convergence tests and justification of choosing this size.
[26] https://github.com/radiocosmology/cora
[27] J. R. Shaw, K. Sigurdson, U. L. Pen, A. Stebbins, and M. Sitwell, All-Sky Interferometry with Spherical Harmonic Transit Telescopes, Astrophys. J. 781, 109 (2014).
[28] M. G. Santos, A. Cooray, and L. Knox, Multifrequency Analysis of 21 Centimeter Fluctuations from the Era of Reionization, Astrophys. J. 625, 575 (2005).
[29] P. McDonald, Gravitational redshift and other redshift-
space distortions of the imaginary part of the power spectrum, J. Cosmol. Astroparticle Phys. **2009**, 026 (2009).

[30] R. A. C. Croft, Gravitational redshifts from large-scale structure, Mon. Not. R. Astron. Soc. **434**, 3008 (2013).

[31] C. Bonvin, Isolating relativistic effects in large-scale structure, Classical Quant. Grav. **31**, 234002 (2014).

[32] C. Bonvin, L. Hui, and E. Gaztañaga, Optimising the measurement of relativistic distortions in large-scale structure, J. Cosmol. Astroparticle Phys. **2016**, 021 (2016).

[33] E. Gaztañaga, C. Bonvin, and L. Hui, Measurement of the dipole in the cross-correlation function of galaxies, J. Cosmol. Astroparticle Phys. **2017**, 032 (2017).

[34] M. Jalilvand, E. Majerotto, C. Bonvin, F. Lacasa, M. Kunz, W. Naidoo, and K. Moodley, New estimator for gravitational lensing using galaxy and intensity mapping surveys, Phys. Rev. Lett. **124**, 031101 (2020).

[35] Y. Gong, A. Cooray, M. Silva, M. G. Santos, J. Bock, C. M. Bradford, and M. Zemcov, Intensity Mapping of the [C II] Fine Structure Line during the Epoch of Reionization, Astrophys. J. **745**, 49 (2012).

[36] S. R. Furlanetto and A. Lidz, The Cross-Correlation of High-Redshift 21 cm and Galaxy Surveys, Astrophys. J. **660**, 1030 (2007).

[37] A. Lidz, O. Zahn, S. R. Furlanetto, M. McQuinn, L. Hernquist, and M. Zaldarriaga, Probing Reionization with the 21 cm Galaxy Cross-Power Spectrum, Astrophys. J. **690**, 252 (2009).

[38] G. Sato-Polito, J. L. Bernal, E. D. Kovetz, and M. Kamionkowski, Antisymmetric cross-correlation of line-intensity maps as a probe of reionization, arXiv e-prints, arXiv:2005.08977 (2020).
SUPPLEMENTAL MATERIAL

Variance.— We derive the variance of antisymmetric cross-power spectrum (Eq. 1) here. We start from the definition in the discretized form, \( V_{\text{tot}} P_{ij}^l (k) = \frac{1}{2l} \left[ \tilde{\delta}_l (k) \tilde{\delta}_j^* (k) - \tilde{\delta}_j^* (k) \tilde{\delta}_l (k) \right] \), where \( V_{\text{tot}} \delta_{k,k'} \to (2\pi)^3 \delta_{ij} (k-k') \) in the limit \( V_{\text{tot}} \to \infty \). This yields

\[
\langle P_{ij}^l (k) P_{ij}^l (k') \rangle = -\frac{1}{4 V_{\text{tot}}^2} \left[ \langle \delta_i (k) \tilde{\delta}_j^* (k) \tilde{\delta}_l (k') \rangle + \langle \tilde{\delta}_j (k') \delta_l (k) \tilde{\delta}_i (k) \rangle - \langle \delta_i (k) \tilde{\delta}_j^* (k') \tilde{\delta}_l (k) \rangle - \langle \tilde{\delta}_j (k') \delta_l (k) \tilde{\delta}_i (k) \rangle \right] + \text{c.c.}
\]

Here “c.c.” stands for the complex conjugate of the preceding terms. In the third line above, we used the Wick theorem. Since \( P_{ij}^l (-k) = -P_{ij}^l (k) \), the \(-k\) mode is not independent from the \(k\) mode, so we only consider the upper hemisphere in \(k\)-space, and the variance combines the contribution from both \(k\) and \(-k\) modes, i.e.

\[
\sigma_{P_{ij}^l}^2 = \left[ \langle (P_{ij}^l (k))^2 \rangle - \langle P_{ij}^l (k) \rangle^2 \right] - \left[ \langle P_{ij}^l (k) P_{ij}^l (-k) \rangle - \langle P_{ij}^l (k) \rangle \langle P_{ij}^l (-k) \rangle \right].
\]

Therefore

\[
\sigma_{P_{ij}^l}^2 = P_i (k) P_j (k) - (P_{ij}^S (k))^2 + (P_{ij}^l (k))^2.
\]

When average the signal over modes with the same \((k, \theta)\), the variance is reduced by a factor of \(N(k, \theta)\) (the number of these modes in the upper hemisphere), and thus we find Eq. 1.

Effect of redshift-bin size and finite simulation box.— Not only does the antisymmetric cross-power spectrum depend on the central redshift of a volume, but also it depends on the frequency bandwidth or the size of redshift-bin, corresponding to the physical size of the correlated fields. While large redshift-bin size enlarges the accessible scales of dipole, it may also smooth out small-scale asymmetry across different cosmic time, if any. Another issue is the effect of the periodic boundary condition on a finite simulation volume which may also cause systematic errors to the theoretical computation of the dipole from simulations.

To test these effects, we consider three scenarios for the cross-correlation — (i) using the full simulation volume (768 comoving Mpc on each side), (ii) using the inner cubic region of 384 comoving Mpc on each side away from the boundary of box, and (iii) further shrinking the cross-correlated volume to 192 comoving Mpc on each side. In Supplemental Material Fig. 1 the results of \((384 \text{Mpc})^3\) volume agree with those of \((192 \text{Mpc})^3\) volume, while the results of full volume significantly over-/under-estimate the magnitude of dipole at the earlier/later time. As such, the redshift-bin size corresponding to 384 comoving Mpc is a trade-off between accessing the large-scale modes and keeping small-scale asymmetry, which also avoids the finite box effect in our simulations. Our analysis in this Letter, therefore, used the data cube constructed from the inner cubic region of 384 comoving Mpc on each side away from the boundary of simulation box.
Supplemental Material FIG. 1. Effect of redshift-bin size and finite simulation box. We show the dipole of the HI-CO cross-power spectrum $P_A$ vs central redshift $z$ for our fiducial reionization model at the fixed wavenumber $k = 0.30 \, h \, \text{Mpc}^{-1}$ (left) and $k = 0.60 \, h \, \text{Mpc}^{-1}$ (right). The results are obtained by cross-correlating the fields in the full simulation volume of 768 comoving Mpc on each side (black), in the inner cubic region of 384 comoving Mpc on each side away from the boundary of box (green), or shrinking the fields to even smaller size of 192 comoving Mpc on each side (blue), respectively. The error bars are 1σ standard deviation for cosmic variance corresponding to the simulation volume of 100 realizations.