Improvement of photovoltaic efficiency by Fano coherence

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Abstract

We show that Fano resonance in the decay channels of a three-level system can lead to considerably absorption enhancement and emission suppression. We found that a coherence built up in the ground doublet states, with strength depending on a coupling parameter that arises from the Fano interference, can in principle lead to breaking of the detail balance between the absorption and emission processes in atomic systems.

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I. INTRODUCTION

Quantum coherence effects, such as Coherent Population Trapping (CPT) [1] and Electromagnetically Induced Transparency (EIT) [2–5], have been the focus of broad research activities for the last two decades, as they drastically change the optical properties of media. For example, for EIT in CW and pulsed regimes [3–7], absorption practically vanishes. Media with excited coherence may display high index of refraction without absorption [8]. It is possible to achieve manipulation of a coherent medium [9] and enhance nonlinear effects at a few photon level [10, 11].

The study of quantum interference and coherence effects in atomic and molecular systems [12] has found numerous fascinating phenomena, e.g., Fano interference [13], vacuum induced coherence [14], lasing without inversion [15, 16], quantum Carnot engine [17] and long lived coherences in biochemical molecules [18]. The application of coherence in solar energy physics in fact can change the balance processes that are limiting the operation of quantum systems [19]. For example, for a quantum photocell, the fundamental limit to the efficiency is accepted to be in the balance of radiative absorption and recombination. The coherence effects can in fact break this balance and significantly suppress the emission process, resulting in enhancement of the power generated by photocell. One of the possible ways to break the balance between recombination and absorption is via the coherent drive similar to the LWI process [20, 21], where the coherence between two levels induced by external source [22] can cancel the emission processes. It is also possible to generate the coherence without using external fields. This approach is based on the Fano effect [13] that manifests itself as an interference between the eigenstates of the system. For example, Fano interference was built up among states of two coupled quantum wells via tunneling [23]. The direct application in an optical system by means of lasing without inversion was analyzed in [21] and has the name ”Fano-Harris lasing without inversion” to distinguish it from the standard (externally driven) LWI. The latest results showed that the quantum coherence that arises from the Fano coupling can significantly enhance the power delivered to the load [24], as well as control over enhancement and suppression of the emission and absorption profiles [25].

In this work we have studied the effect of Fano interference on the probabilities of emission and absorption. This interference is induced by two spontaneous decays from discrete ground state doublet to an identical continuum.
FIG. 1: Scheme of the three-level system with continuums. The Fano interference generates among the ground state doublet and reservoir state $R_v$ (enclosed by dashed ellipse). $\mathcal{E}$ is the external weak electric field. $2\Gamma$ and $2\gamma_{1,2}$ are spontaneous decay rates from eigenstates to continuums.

The organization of this paper is as follows. In section II we discuss the theoretical model of the three-level system, the dynamical evolution of the system, and the probability of absorption and emission. In section III we simulate the effect of the Fano interference between the decay channels on the probability of absorption and emission. In section IV we show some analytical calculations of the Fano interference in both the probability amplitude and the density matrix approaches. Finally in section V we summarize our results.

II. THEORETICAL MODEL

An interesting example of Fano-like coupling is the three-level system as shown in Fig. 1, where the effects of coherence play a major role. This scheme is developed from the previous intersubband double quantum well structure [26]. Consider a ground state doublet $|v_{1,2}\rangle$ and an excited state $|c\rangle$ coherently driven by a weak electric field. This external field plays the role of the sunlight. We choose the central frequency $\nu$ such that the energies of the state $|v_{1,2}\rangle$ are related to $|c\rangle$ as $\hbar(\nu \pm \Delta)$, where $\Delta$ is now the frequency detuning, not the
tunneling in [26]. The ground state doublet decay to an identical continuum (we consider this continuum as a reservoir state $R_v$) with rate $2\gamma_{1,2}$, and the excited state $|c\rangle$ decays to a continuum (reservoir state $R_c$) with rate $2\Gamma$.

To get the dynamical evolution of this system, we can use both the probability amplitude method and the density matrix method. In the probability amplitude method, we can write the state vector as

$$|\Psi\rangle = v_1|v_1\rangle + v_2|v_2\rangle + c|c\rangle.$$  
(1)

The dynamical equations of amplitude can be derived with the Weisskopf-Wigner approximation, which are also given in [26] for intersubband double quantum well structure by simply adding the decay term of $2\Gamma$ from the excited state $|c\rangle$ to the reservoir $|R_c\rangle$,

$$\dot{v}_2 = -(\gamma_2 + i\Delta)v_2 - p\sqrt{\gamma_1\gamma_2}v_1 - i\Omega_2 c,$$  
(2)

$$\dot{v}_1 = -(\gamma_1 - i\Delta)v_1 - p\sqrt{\gamma_1\gamma_2}v_2 - i\Omega_1 c,$$  
(3)

$$\dot{c} = -i\Omega_2 v_2 - i\Omega_1 v_1 - \Gamma c,$$  
(4)

where $\Omega_{1,2} = \varphi_{cv_1,2}\epsilon/\hbar$ are the Rabi frequencies of the applied field. $\varphi_{cv_1}(\varphi_{cv_2})$ is the dipole moment of the transition $|c\rangle \leftrightarrow |v_1\rangle$ ($|c\rangle \leftrightarrow |v_2\rangle$), and $\mathcal{E}_0$ is the applied electric field. The terms containing the product of the decay rates appear due to the interference introduced by the decay of the two optical transitions to the same state. This is the so-called Fano interference, which couples the doublet states. In order to measure the strength of the interference, we introduce the $p$ factor, which is the normalized scalar product of the corresponding dipole moments:

$$p = \frac{\varphi_{R_v v_1} \cdot \varphi_{R_v v_2}}{|\varphi_{R_v v_1} \varphi_{R_v v_2}|}.$$  
(5)

According to its definition, the alignment factor takes value 1 for parallel dipole moments and $-1$ for antiparallel dipole moments, both corresponding to the maximal coherences. $P$ takes value 0 for the orthogonal situation, which gives no interference. Intermediate $p$ values on the $[-1, 1]$ segment are also possible. The extremes of maximal and minimal coherences deserve special attention.

Similarly, we can derive the dynamical equations in the format of density matrix. In the rotating-wave approximation, the semi-classical time-dependent interaction Hamiltonian that describes the atom-laser coupling for this $\Lambda$ system is given by

$$H_{int} = -\hbar \left( \Omega_1 e^{-i\Delta t}|c\rangle\langle v_1| + \Omega_2 e^{i\Delta t}|c\rangle\langle v_2| + H.C. \right).$$  
(6)
The time evolution of the density matrix is given by the master equation
\[
\dot{\rho} = -\frac{i}{\hbar} [H_{\text{int}}, \rho] + L\rho,
\] (7)
where \( L\rho = L_1\rho + L_2\rho \) describes spontaneous emission terms. The spontaneous decay rate between two levels \(|1\rangle, |2\rangle\) is given by
\[
\gamma = \frac{1}{4\pi \epsilon_0} \frac{4\omega^3 \rho_{12}^2}{3\hbar c^3} = \frac{\omega^3}{3\pi \hbar c^3 \epsilon_0} \rho_{12}^2.
\] (8)
Let \( \gamma' = \frac{\omega^3}{3\pi \hbar c^3 \epsilon_0} \), then \( \gamma = \gamma' \rho_{12}^2 \), thus the relaxation terms become
\[
L_1\rho = -\gamma' [(\sigma_{cv_1}^\dagger \sigma_1^+ + \sigma_{cv_2}^\dagger \sigma_2^+) (\sigma_{cv_1} \sigma_1 + \sigma_{cv_2} \sigma_2) \rho
+ \rho (\sigma_{cv_1}^\dagger \sigma_1^+ + \sigma_{cv_2}^\dagger \sigma_2^+) (\sigma_{cv_1} \sigma_1 + \sigma_{cv_2} \sigma_2)]
- 2(\sigma_{cv_1} \sigma_1 + \sigma_{cv_2} \sigma_2) \rho (\sigma_{cv_1}^\dagger \sigma_1^+ + \sigma_{cv_2}^\dagger \sigma_2^+)\];
\[
L_2\rho = -\Gamma [\sigma_3^+ \sigma_3\rho + \rho \sigma_3^+ \sigma_3 - 2\sigma_3^0 \sigma_3^+]\).
\] (9)
Here \( \sigma_3^+ = |v_1\rangle \langle R_v|, \sigma_3^0 = |v_2\rangle \langle R_v|, \) and \( \sigma_3^+ = |c\rangle \langle R_e| \) are the atomic transition operators. We have taken into consideration the interference introduced by the two decays from the ground state doublet to the same continuum.

Expanding Eq. (7) on the basis of \(|c\rangle, |v_1\rangle, |v_2\rangle, |R_e\rangle, |R_v\rangle\), and using the relaxation Eqs. (9)-(10), we obtain the dynamical evolution of the density matrix elements as,
\[
\dot{\rho}_{11} = i\Omega_1^* \rho_{c1} - i\Omega_1 \rho_{1c} - 2\gamma_1 \rho_{11} - p\sqrt{\gamma_1 \gamma_2} (\rho_{12} + \rho_{21}),
\] (11)
\[
\dot{\rho}_{22} = i\Omega_2^* \rho_{c2} - i\Omega_2 \rho_{2c} - 2\gamma_2 \rho_{22} - p\sqrt{\gamma_1 \gamma_2} (\rho_{12} + \rho_{21}),
\] (12)
\[
\dot{\rho}_{Re} = 2\Gamma \rho_{cc},
\] (13)
\[
\dot{\rho}_{Rv} = 2\gamma_1 \rho_{11} + 2\gamma_2 \rho_{22} + 2p\sqrt{\gamma_1 \gamma_2} (\rho_{12} + \rho_{21}),
\] (14)
and the non-diagonal terms
\[
\dot{\rho}_{12} = i\Omega_1^* \rho_{c2} - i\Omega_2 \rho_{1c} - p\sqrt{\gamma_1 \gamma_2} (\rho_{11} + \rho_{22}) - \Gamma_2 \rho_{12},
\] (15)
\[
\dot{\rho}_{1c} = i\Omega_1^* (\rho_{cc} - \rho_{11}) - i\Omega_2^* \rho_{12} - p\sqrt{\gamma_1 \gamma_2} \rho_{2c} - \Gamma_{1c} \rho_{1c},
\] (16)
\[
\dot{\rho}_{2c} = i\Omega_2^* (\rho_{cc} - \rho_{22}) - i\Omega_1^* \rho_{21} - p\sqrt{\gamma_1 \gamma_2} \rho_{1c} - \Gamma_{2c} \rho_{2c},
\] (17)
where
\[
\Gamma_{12} = \gamma_1 + \gamma_2 + 2i\Delta,
\] (18)
\[
\Gamma_{1c} = \Gamma + \gamma_1 + i\Delta,
\]
\[
\Gamma_{2c} = \Gamma + \gamma_2 - i\Delta,
\]
are complex dephasing. Here we already exploit the norm preserving condition (\(\|\rho\| = 1\)) of a density matrix for a closed system, so the time derivation of \(\rho_{cc}\) is not needed.

Defining the probability of emission as a sum of the population in the levels \(v_1, v_2\) and \(R_c\) for the system in Fig.1

\[
P_{emiss}(t) = 1 - \rho_{cc}(t) - \rho_{Rc,Rc}(t),
\]

with initial conditions \(v_{1,2}(0) = 0, \ c(0) = 1\), and taking into account the evolution of level \(R_c\): \(\dot{\rho}_{Rc,Rc}(t) = 2\Gamma \rho_{cc}(t)\), the probability of emission is:

\[
P_{emiss}(t) = 1 - \rho_{cc}(t) - 2\Gamma \int_0^t \rho_{cc}(t')dt'.
\]

Similarly, the probability of absorption is given by

\[
P_{abs}(t) = \rho_{cc}(t) + \rho_{Rc,Rc}(t) = \rho_{cc}(t) + 2\Gamma \int_0^t \rho_{cc}(t')dt'.
\]

Eqs. (11)-(17) and Eqs. (20)-(21) are the formulas we will exploit to investigate the photovoltaics system. In the following section III, \textit{ode45} function of MATLAB is used to solve these ordinary differential equations.

\section{NUMERICAL SIMULATIONS}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig2.png}
\caption{Time dependence of probability of emission and absorption. \(p = 1\) (solid red), \(p = 0\) (dashed blue) with \(\Omega_1 = \Omega_2 = 0.001, \Delta = 0.6, \Gamma = 1\), and \(\gamma = 1\).}
\end{figure}

Fig.2 shows the dependence of probability of emission and absorption on time with and without interference. The red solid curves correspond to the maximal coherence with \(p = 1\)
and the blue dashed curves correspond to no coherence with $p = 0$. The parameters we are using are Rabi frequency $\Omega_1 = \Omega_2 = 0.001$, detuning $\Delta = 0.6$, and the spontaneous decay rates $\Gamma = \gamma = 1$.

The effect of the Fano interference are apparent. We will take the steady states as an example. Without coherence, the probability of absorption is $1.47 \times 10^{-6}$, and the probability of emission is $2.94 \times 10^{-6}$. With $p = 1$, the probability of absorption is $1.8 \times 10^{-6}$ and the probability of emission is $1.8 \times 10^{-6}$. Due to the fano interference, we obtain a nearly 22% increase in absorption and a 38% decrease in emission. The result meets our expectation that the balance between absorption and emission has been broken by the coherence induced by the spontaneous decays. The output power of photovoltaic is proportional to the ratio of the population density on the excited state to that on the ground state\cite{24}. Our result provides a possible method to enhance the power output of photovoltaic.

We should also expect that the detuning plays a role in changing the probability of emission and absorption. Fig.3 shows the same plots as shown in Fig.2 but with a smaller detuning $\Delta = 0.1$. Under the same conditions, the probabilities of both emission and absorption increase with this smaller detuning. As with larger detuning, the interference plays a similar role, giving an increase of 30% in absorption and a decrease of 34% in emission.

FIG. 3: Time dependence of probability of emission (a) and absorption (b). $p = 1$ (solid red), $p = 0$ (dashed blue) with $\Omega_1 = \Omega_2 = 0.001, \Delta = 0.1, \Gamma = 1$, and $\gamma = 1$. 
IV. THEORETICAL ANALYSIS

To simplify the calculation, we assume a special case where $\gamma_1 = \gamma_2 = \gamma$. We will solve this problem analytically using both the probability amplitude and population density approaches.

A. Probability amplitude approach

We borrowed the method Scully exploited and modified it to fit our model. The two extremes of maximal and minimal coherences have special properties. We will only consider these two situations in our analysis. For the maximal coherence, we have $p = 1$, the dynamical Eqs. (2)-(4) become

$$\dot{v}_2 = -(\gamma + i\Delta)v_2 - \gamma v_1 - i\Omega_2 c,$$  \hspace{1cm} (22)

$$\dot{v}_1 = -(\gamma - i\Delta)v_1 - \gamma v_2 - i\Omega_1 c,$$  \hspace{1cm} (23)

$$\dot{c} = -i\Omega_2 v_2 - i\Omega_1 v_1 - \Gamma c.$$  \hspace{1cm} (24)

Writing Eqs. (22)-(24) in matrix form, we obtain

$$\frac{d}{d\tau} \begin{pmatrix} v_2 \\ v_1 \\ c \end{pmatrix} = -V_f \begin{pmatrix} v_2 \\ v_1 \\ c \end{pmatrix} - iV_p \begin{pmatrix} v_2 \\ v_1 \\ c \end{pmatrix},$$  \hspace{1cm} (25)

where $\tau = \gamma t$, and the Fano decay matrix is defined by

$$V_f = \begin{pmatrix} 1 + i\tilde{\Delta} & 1 & 0 \\ 1 & 1 - i\tilde{\Delta} & 0 \\ 0 & 0 & \tilde{\Gamma} \end{pmatrix},$$  \hspace{1cm} (26)

and the probe-field interaction is given by

$$V_p = \begin{pmatrix} 0 & 0 & \tilde{\Omega}_2 \\ 0 & 0 & \tilde{\Omega}_1 \\ \tilde{\Omega}_2 & \tilde{\Omega}_1 & 0 \end{pmatrix},$$  \hspace{1cm} (27)

with $\tilde{\Delta} = \frac{\Delta}{\gamma}$ and $\tilde{\Omega}_{1,2} = \frac{\Omega_{1,2}}{\gamma}$, $\tilde{\Gamma} = \frac{\Gamma}{\gamma}$. 

8
It is intuitive to introduce a dressed basis in which the Fano coupling is transformed away. We proceed from the bare basis via the $U$, $U^{-1}$ matrices of diagonalization.

$$
U^{-1} = \frac{1}{\sqrt{2}} \begin{pmatrix}
1 & 1 & 0 \\
x - i\Delta & -x - i\Delta & 0 \\
0 & 0 & \sqrt{2}
\end{pmatrix},
$$

(28)

$$
U = \frac{1}{\sqrt{2}x} \begin{pmatrix}
x + i\Delta & 1 & 0 \\
x - i\Delta & -1 & 0 \\
0 & 0 & \sqrt{2}x
\end{pmatrix},
$$

(29)

where $x = \sqrt{1 - \Delta^2}$.

The transformed state vector is defined by

$$
U \begin{pmatrix} v_2 \\ v_1 \\ c \end{pmatrix} = \begin{pmatrix} A_+ \\ A_- \\ B \end{pmatrix},
$$

(30)

which implies

$$
\begin{pmatrix}
\dot{A}_+ \\
\dot{A}_- \\
\dot{B}
\end{pmatrix} = -W_p \begin{pmatrix} A_+ \\ A_- \\ B \end{pmatrix} - iW_f \begin{pmatrix} A_+ \\ A_- \\ B \end{pmatrix},
$$

(31)

where the diagonal operator is

$$
W_f = UV_f U^{-1} = \begin{pmatrix}
1 + x & 0 & 0 \\
0 & 1 - x & 0 \\
0 & 0 & \tilde{\Gamma}
\end{pmatrix},
$$

(32)

and the transformed interaction potential is

$$
W_p = UV_p U^{-1} = \frac{1}{\sqrt{2}} \begin{pmatrix}
0 & 0 & \tilde{\Omega}_+ \\
0 & 0 & \tilde{\Omega}_- \\
\tilde{\Omega}'_+ & \tilde{\Omega}'_- & 0
\end{pmatrix},
$$

(33)

where

$$
\tilde{\Omega}_\pm = [\tilde{\Omega}_2(x \pm i\Delta) \pm \tilde{\Omega}_1]/x,
$$

$$
\tilde{\Omega}'_\pm = [\tilde{\Omega}_2 \pm \tilde{\Omega}_1(x \mp i\Delta)]/x.
$$

(34)
The equations of motion in terms of $A_\pm$ and $B$ are then

\[
\frac{dA_+}{d\tau} = -(1 + x)A_+ - \frac{i}{\sqrt{2}}\tilde{\Omega}_+ B, \quad (35)
\]

\[
\frac{dA_-}{d\tau} = -(1 - x)A_- - \frac{i}{\sqrt{2}}\tilde{\Omega}_- B, \quad (36)
\]

\[
\frac{dB}{d\tau} = -\frac{i}{\sqrt{2}}[\tilde{\Omega}_+ A_+ + \tilde{\Omega}_- A_-] - \tilde{\Gamma} B. \quad (37)
\]

Note that the transformed interaction matrix in Eq. (31) is not symmetric, i.e., it is non-Hermitian, thus the absorption-emission balance is broken.

From Eqs. (35)-(37), we can derive the analytical solutions of probability amplitude in the dressed states. To find the probability amplitude for emission, we take the initial condition as $B(0) = 1$ and $A_\pm(0) = 0$, and we assume the Rabi frequency of the driving fields $\Omega_{1,2}$ to be weak enough to apply the perturbation method. The first order approximation of $B$ is a pure exponential function $B(0) \approx e^{-\tilde{\Gamma} \tau}$. According to our numerical simulations, this equation only works well for a short time in the beginning. A higher order approximation is needed. Replacing $B$ in both Eqs. (35)-(36) with $B(0)$, we obtain

\[
\frac{dA_+^{(1)}}{d\tau} = -(1 + x)A_+^{(1)} - \frac{i}{\sqrt{2}}\tilde{\Omega}_+ e^{-\tilde{\Gamma} \tau}, \quad (38)
\]

\[
\frac{dA_-^{(1)}}{d\tau} = -(1 - x)A_-^{(1)} - \frac{i}{\sqrt{2}}\tilde{\Omega}_- e^{-\tilde{\Gamma} \tau},
\]

giving the $A_\pm$

\[
A_\pm^{(1)} \approx -\frac{i}{\sqrt{2}}\tilde{\Omega}_\pm \int_0^t e^{-(1 \pm x - \tilde{\Gamma} \tau)(t - t')} dt' = -\frac{i}{\sqrt{2}(1 \pm x - \tilde{\Gamma} \tau)}\tilde{\Omega}_\pm. \quad (39)
\]

Substituting Eq. (39) into Eq. (37), we obtain the next order of $B$,

\[
B^{(1)} = (a_0 \tau - a_+ - a_- + 1)e^{-\tilde{\Gamma} \tau} + a_+ e^{-(1 + x)\tau} + a_- e^{-(1 - x)\tau}, \quad (40)
\]

where

\[
a_0 = \frac{x^2 \tilde{\Omega}_+^\prime \tilde{\Omega}_+}{1 + x - \tilde{\Gamma}} + \frac{x^2 \tilde{\Omega}_-^\prime \tilde{\Omega}_-}{1 - x - \tilde{\Gamma}}, \quad (41)
\]

\[
a_\pm = \tilde{\Omega}_\pm^\prime \tilde{\Omega}_\pm \frac{x^2}{(1 + x - \tilde{\Gamma})^2}.
\]

Similarly, we can get the probability amplitude for absorption. There is no population in the ground state, and the initial condition is $c(0) = 0$, $\nu_1 = 1$ or $\nu_2 = 1$. Eq. (30) gives
\( B(0) = 0 \), and
\[
A_\pm = \frac{1}{\sqrt{2x}}[v_2(x \pm i\Delta) \pm v_1].
\] (42)

We need to consider \( v_1 = 1 \) and \( v_2 = 1 \) separately.

For \( v_1 = 1 \), \( A_\pm = \pm \frac{1}{\sqrt{2x}} \). In the situation of a weak field, we have the first order approximation of \( A_\pm \),
\[
A_\pm^{(0)} = \pm \frac{1}{\sqrt{2x}} e^{-(1\pm x)}. \] (43)

Substituting Eq. (43) into Eq. (37), the next order of \( B \) is
\[
B^{(1)} = (b_+ - b_-) e^{-\Gamma t} - b_+ e^{-(1+x)t} + b_- e^{-(1-x)t}, \] (44)
where
\[
b_\pm = -i \frac{\tilde{\Omega}_\pm}{2(1 \pm x - \Gamma)}. \] (45)

For \( v_2 = 1 \), we have \( A_\pm = \pm \frac{x \pm i\Delta}{\sqrt{2x}} \). Compared with the situation \( v_1 = 1 \), there is only a time-independent coefficient difference. We can directly obtain
\[
B^{(1)} = (b_+(x + i\Delta) - b_-(x - i\Delta)) e^{-\Gamma t}
+ b_+ e^{-(1+x)t}(x + i\Delta) + b_- e^{-(1-x)t}(x - i\Delta), \] (46)
where \( b_\pm \) are the same as defined for \( v_1 = 1 \).

For the situation without coherence, or \( p = 0 \), the calculation is straightforward. No dressed states are needed. Eqs. (2)-(4) become
\[
\dot{v}_2 = - (\gamma + i\Delta) v_2 - i\Omega_2 c, \] (47)
\[
\dot{v}_1 = - (\gamma - i\Delta) v_1 - i\Omega_1 c, \] (48)
\[
\dot{c} = - i\Omega_2 v_2 - i\Omega_1 v_1 - \Gamma c. \] (49)

For emission, we have the same initial condition of \( c(0) = 1 \), \( v_{1,2}(0) = 0 \). The first order approximations are
\[
c^{(0)} = e^{-\Gamma t}, \] 
\[
v_1^{(0)} = \frac{i\Omega_1}{\Gamma - \Gamma_1}(e^{-\Gamma t} - e^{-\Gamma_1 t}), \] (50)
\[
v_2^{(0)} = \frac{i\Omega_2}{\Gamma - \Gamma_2}(e^{-\Gamma t} - e^{-\Gamma_2 t}), \]
where
\[ \tilde{\Gamma}_1 = 1 - i \tilde{\Delta}, \]
\[ \tilde{\Gamma}_2 = 1 + i \tilde{\Delta}. \] (51)

Substituting Eq. (50) into Eq. (49), we get the next order of probability amplitude,
\[ c^{(1)} = (a_0 t - a_1 - a_2 + 1) e^{-\tilde{\Gamma} t} + a_1 e^{-\tilde{\Gamma}_1 t} + a_2 e^{-\tilde{\Gamma}_2 t}, \] (52)
where
\[ a_0 = \left( \frac{\tilde{\Omega}_1^2}{\tilde{\Gamma} - \tilde{\Gamma}_1} + \frac{\tilde{\Omega}_2^2}{\tilde{\Gamma} - \tilde{\Gamma}_2} \right), \]
\[ a_1 = \frac{\tilde{\Omega}_1^2}{(\tilde{\Gamma} - \tilde{\Gamma}_1)^2}, \]
\[ a_2 = \frac{\tilde{\Omega}_2^2}{(\tilde{\Gamma} - \tilde{\Gamma}_2)^2}. \] (53)

At large time \( \tau \gg 1/\tilde{\Gamma} \) and with weak field approximation, by applying Eq. (20), we can obtain the probability of emission
\[ P_{\text{emiss}} \approx \frac{(4\Delta^2(3\Gamma - 1) + (\Gamma^2 - 1)(3\Gamma + 1))(4\Omega_1^2 + 4\Omega_2^2)}{(4\Delta^2 + 1)^2 \Gamma + 2(4\Delta^2 - 1)\Gamma^3 + \Gamma^5}. \] (54)

For absorption, we only show the initial condition \( v_1(0) = 1, c(0) = 0 \) as an example. Following the same process for emission, we obtain
\[ v_1^{(0)} = e^{-\tilde{\Gamma}_1 t}, \]
\[ c^{(0)} = \frac{i\tilde{\Omega}_1}{\Gamma_1 - \tilde{\Gamma}} (e^{-\tilde{\Gamma}_1 t} - e^{-\Gamma t}), \]
\[ v_2^{(0)} = \frac{\tilde{\Omega}_1 \tilde{\Omega}_2}{\Gamma_1 - \tilde{\Gamma}} \left( \frac{e^{-\Gamma t} - e^{-\Gamma_2 t}}{\Gamma - \Gamma_2} + \frac{e^{-\tilde{\Gamma}_1 t} - e^{-\Gamma_2 t}}{\Gamma_1 - \Gamma_2} \right). \] (55)

Therefore
\[ c^{(1)} = -ib_0 e^{-\Gamma t} - \frac{ib_1}{\Gamma - \tilde{\Gamma}_1} (e^{-\tilde{\Gamma}_1 t} - e^{-\Gamma t}) \]
\[ - \frac{ib_2}{\Gamma - \Gamma_2} (e^{-\Gamma_2 t} - e^{-\Gamma t}), \] (56)
where
\[ b_0 = \Omega_1 |\Omega_2|^2 \left( \frac{1}{(\Gamma_1 - \Gamma)(\Gamma_2 - \Gamma)} \right), \]
\[ b_1 = \Omega_1 |\Omega_2|^2 \left( \frac{1}{(\Gamma_1 - \Gamma)(\Gamma_1 - \Gamma_2)} + \frac{1}{|\Omega_2|^2} \right), \]
\[ b_2 = \Omega_1 |\Omega_2|^2 \left( \frac{1}{(\Gamma_2 - \Gamma)(\Gamma_1 - \Gamma_2)} \right). \] (57)
By applying Eq. (21), the probability of absorption is given by

\[
P_{\text{abs}}(\tau|c) = \rho_{cc}(\tau) + 2\bar{\Gamma} \int_{0}^{\tau} \rho_{cc}(\tau')d\tau' \\
\approx \frac{4\Omega_{1}^{2}(1+\Gamma)}{4\Delta^{2}+(\Gamma+1)^{2}}.
\]

\hspace{1cm} (58)

**B. Density matrix approach**

It will be interesting to derive the analytical solution of the probability of emission and absorption in the density element form. However, we found this too complex with the existence of coherence. Therefore, we take the special situation of no interference \( p=0 \) as an example. The density matrix becomes

\[
\dot{\rho}_{11} = i\Omega_{1}^{*}\rho_{c1} - i\Omega_{1}\rho_{1c} - \gamma\rho_{11},
\]

\hspace{1cm} (59)

\[
\dot{\rho}_{22} = i\Omega_{2}^{*}\rho_{c2} - i\Omega_{2}\rho_{2c} - \gamma\rho_{22},
\]

\hspace{1cm} (60)

\[
\dot{\rho}_{cc} + \dot{\rho}_{Rc} = -i\Omega_{1}(\rho_{c1} - \rho_{1c}) - i\Omega_{2}(\rho_{c2} - \rho_{2c}),
\]

\hspace{1cm} (61)

\[
\dot{\rho}_{12} = i\Omega_{1}^{*}\rho_{c2} - i\Omega_{2}\rho_{1c} - \Gamma_{12}\rho_{12},
\]

\hspace{1cm} (62)

\[
\dot{\rho}_{1c} = i\Omega_{1}^{*}(\rho_{cc} - \rho_{11}) - i\Omega_{2}^{*}\rho_{12} - \Gamma_{1c}\rho_{1c},
\]

\hspace{1cm} (63)

\[
\dot{\rho}_{2c} = i\Omega_{2}^{*}(\rho_{cc} - \rho_{22}) - i\Omega_{1}^{*}\rho_{21} - \Gamma_{2c}\rho_{2c}.
\]

\hspace{1cm} (64)

Notice that Eq. (61) is already the time derivation of the core of the probability of emission and absorption formulas. It is the key to solve these equations. With the initial condition of \( \rho_{cc} = 1 \), Eq. (63), Eq. (64) will give us the same time evolution of \( \rho_{1c} \) and \( \rho_{2c} \), thus

\[
\rho_{1c} = \frac{i\Omega_{1}}{\Gamma_{13} - \gamma} (e^{-\gamma t} - e^{-\Gamma_{13}t}).
\]

\hspace{1cm} (65)

Integrating on both sides of Eq. (61) from zero to infinity, and substituting the results into Eq. (19), we get the probability of emission

\[
P_{\text{emiss}} = \frac{4(\Omega_{1}^{2} + \Omega_{2}^{2})(4\Delta^{2} + \Gamma^{2})\Gamma + \Gamma^{3} - 4\Delta^{2}}{4\Delta^{2} + \Gamma^{2}}.
\]

\hspace{1cm} (66)

For absorption, we have \( \rho_{11}(0) = 1 \); substituting it into Eq. (59), we get \( \rho_{11}(t) = e^{-t} \). From Eq. (63), we get

\[
\rho_{1c} = \frac{i\Omega_{1}}{\gamma - \Gamma_{13}} (e^{-t} - e^{-\Gamma_{13}t}).
\]

\hspace{1cm} (67)
According to our numerical simulations, $\rho_{2c}$ is always much smaller than $\rho_{1c}$, we can ignore the contribution by $\rho_{2c}$. Thus

$$
\dot{\rho}_{cc} + \dot{\rho}_{Rc} = -i\Omega_1(\rho_{c1} - \rho_{1c}) \\
= \frac{\Omega_1^2}{\Delta^2 + \Gamma^2} \{2\Gamma e^{-\gamma t} \\
- e^{-(\Gamma + \gamma)}t(2\Gamma \cos(\Delta t) + 2\Delta \sin(\Delta t))\}.
$$

Again integrating on both sides of Eq.(68) from zero to infinity, and substituting the result into Eq.(21), we get

$$
P_{abs} = \frac{4\Omega_1^2}{4\Delta^2 + \Gamma^2} \frac{(4\Delta^2 + \Gamma^2)\Gamma + \Gamma^2 - 4\Delta^2}{4\Delta^2 + (\Gamma + 1)^2}.
$$

To verify the analytical solutions, we can compare them with the numerical results. Let’s take the same parameters we have used in Fig.2 except that here we have $p = 0$. In the steady states, we have $P_{emiss} = 3.992 \times 10^{-6}$ for analytical solution and $P_{emiss} = 3.998 \times 10^{-6}$ for numerical solution, and $P_{abs} = 1.996 \times 10^{-6}$ for analytical solution and $P_{abs} = 1.999 \times 10^{-6}$ for numerical solution. Both sets match well, showing that these two analytical solutions are good enough to describe the emission and absorption dependence on the system parameters.

V. CONCLUSION

We have studied the effect of the Fano interference in a three-level system with reservoir. We found that the balance between emission and absorption for the original system has been broken, because the interference largely suppresses the emission process and enhances the absorption process. This property can possibly be applied to improve the efficiency of solar cell, as the interference can increase the probability of absorption and decrease the probability of emission. Under the weak field approximation, the analytical solutions of probability amplitude and density elements are derived. The results matched well with our numerical simulations.

VI. ACKNOWLEDGEMENT

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