Velocity continuation with Fourier neural operators for accelerated uncertainty quantification

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challenges of seismic imaging

noisy data and linearization errors
bandwidth and aperture limitations
presence of shadow zones

uncertainty quantification compliments imaging

assessing variability amongst feasible seismic images
translation of imaging uncertainty to downstream tasks, e.g., horizon tracking
challenges: high-dimensionality of seismic images and forward operator costs

Gilles Lambaré, Jean Virieux, Raul Madariaga, and Side Jin. “Iterative asymptotic inversion in the acoustic approximation”. In: Geophysics 57.9 (1992), pp. 1138–1154.

Gerard T Schuster. “Least-squares cross-well migration”. In: 63rd Annual International Meeting, SEG. Expanded Abstracts, 1993, pp. 110–113. DOI: 10.1190/1.1822308.
Conditional mean estimate

Normalized point-wise standard deviation

Pointwise histograms at (1.550 km, 1.175 km)

Pointwise histograms at (4.550 km, 1.738 km)

Mean and 99% confidence intervals of horizons

Ali Siahkoohi, Gabrio Rizzuti, and Felix J. Herrmann. “Deep Bayesian inference for seismic imaging with tasks”. In: Geophysics 87.5 (June 2022). DOI: 10.1190/geo2021-0666.1. URL: https://arxiv.org/abs/2110.04825.
Two main sources of uncertainty

errors in the data, e.g., measurement noise

forward modeling (background velocity model) errors
Focus of this talk

quantifying uncertainty w.r.t errors in the background model

main contributing factor to imaging uncertainty due to its effect on reflector positioning

requires solving numerous imaging problems, e.g., via reverser-time migration

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Sergey Fomel and Evgeny Landa. “Structural uncertainty of time-migrated seismic images”. In: Journal of Applied Geophysics 101 (2014), pp. 27–30.

Oleg V Poliannikov and Alison E Malcolm. “The effect of velocity uncertainty on migrated reflectors: Improvements from relative-depth imaging”. In: Geophysics 81.1 (2016), S21–S29.
Parihaka dataset

real prestack Kirchhoff time migrated dataset

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Veritas. *Parihaka 3D Marine Seismic Survey - Acquisition and Processing Report*. Tech. rep. New Zealand Petroleum Report 3460. New Zealand Petroleum & Minerals, Wellington, 2005.

WesternGeco. *Parihaka 3D PSTM Final Processing Report*. Tech. rep. New Zealand Petroleum Report 4582. New Zealand Petroleum & Minerals, Wellington, 2012.
Proposed method

a novel velocity continuation method

mapping images associated with one background model to another virtually for free

based on the recent advances in deep learning for operator learning
Linearized wave-equation based imaging

\[ d_i = \mathcal{F}(m, q_i) + \epsilon_i \]

\[ \approx \mathcal{F}(m_0, q_i) + \nabla_m \mathcal{F}(m, q_i) \bigg|_{m=m_0} \delta m + \mathcal{O}(\delta m^\top \delta m) + \epsilon_i \]

J(\cdot): Born modeling operator
linearization error

observed seismic data \( d = \{d_i\}_{i=1}^{n_s} \)
discretized acoustic wave equation \( \mathcal{F}(m, q_i) \)
squared-slowness model of the subsurface \( m \)
source signature \( q_i \)
measurement noise \( \epsilon_i \)
smooth squared-slowness model \( m_0 \)
Velocity continuation

\[ T_{(m_{\text{init}}, m_{\text{target}})} : \delta M \rightarrow \delta M \]

space of seismic images $\delta M$

initial and target background models $m_{\text{init}}, m_{\text{target}}$

maps images associated with one background model to another without explicitly imaging

Sergey Fomel. “Time-migration velocity analysis by velocity continuation”. In: *Geophysics* 68.5 (2003), pp. 1662–1672.

Mengmeng Yang, Marie Graff, Rajiv Kumar, and Felix J Herrmann. “Low-rank representation of omnidirectional subsurface extended image volumes”. In: *Geophysics* 86.3 (2021), S165–S183.
Fourier neural operator surrogate

\[ G_w^* (m_{\text{target}}, \delta m_{\text{init}}) \approx T_{(m_{\text{init}}, m_{\text{target}})}(\delta m_{\text{init}}) = \delta m_{\text{target}}, \]

Fourier neural operator \( G_w \)

initial and target seismic images \( \delta m_{\text{init}}, \delta m_{\text{target}} \)

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Zongyi Li et al. “Fourier Neural Operator for Parametric Partial Differential Equations”. In: 9th International Conference on Learning Representations. OpenReview.net, 2021. URL: https://openreview.net/forum?id=c8P9NQVtmn0.
Training

$$w^* = \arg \min_w \frac{1}{n_{\text{train}}} \sum_{i=1}^{n_{\text{train}}} \| G_w(m^{(i)}_0, \delta m_{\text{init}}) - \delta m^{(i)}_{\text{RTM}} \|_2^2$$

training tuples \( \left\{ (m^{(i)}_0, \delta m_{\text{init}}), \delta m^{(i)}_{\text{RTM}} \mid i = 1, \ldots, n_{\text{train}} \right\} \)

$$\delta m_{\text{RTM}} = \sum_{i=1}^{n_s} J(m_0, q_i) \top d_i.$$ 

trained with only 200 background and seismic image pairs

Maarten De Hoop, Daniel Zhengyu Huang, Elizabeth Qian, and Andrew M Stuart. “The Cost-Accuracy Trade-Off In Operator Learning With Neural Networks”. In: arXiv preprint arXiv:2203.13181 (2022).
Uncertainty in the tracked horizons

Xinming Wu and Sergey Fomel. “Least-squares horizons with local slopes and multi-grid correlations”. In: GEOPHYSICS 83 (4 2018), pp. IM29–IM40. DOI: 10.1190/geo2017-0830.1. URL: https://doi.org/10.1190/geo2017-0830.1.
Conclusions

Uncertainty quantification is rendered impractical when
the forward operators are expensive to evaluate
the problem is high dimensional

Fourier neural operator surrogate for velocity continuation
can lead to computational improvements
can be pretrained on existing data followed by a cheaper finetuning stage to the
pertaining survey
can act as “summary functions” for scalable amortized variational inference, i.e., full
posterior learning
Acknowledgment

This research was carried out with the support of Georgia Research Alliance and partners of the ML4Seismic Center
Contributions

- A survey-specific Fourier neural operator surrogate to velocity continuation
- Mapping images associated with one background model to another virtually for free
- Trained with only 200 background and seismic image pairs
- Enables accelerated uncertainty quantification w.r.t. background model

**Code:** [https://github.com/slimgroup/fno4vc](https://github.com/slimgroup/fno4vc)