A Theoretical Determination of $N_{nn}/N_{np}$ in Hypernuclear Non–Mesonic Weak Decay

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Abstract

The ratio $N_{nn}/N_{np}$ between the number of neutron–neutron and neutron–proton pairs emitted in the non–mesonic weak decay of $\Lambda$–hypernuclei is calculated within a nuclear matter formalism extended to $^{12}\Lambda C$ via the local density approximation. The single–nucleon emission spectra, $N_p$ and $N_n$, are also evaluated. Our formalism takes care of both ground state correlations (gsc) and final state interactions (FSI). The evaluation of $N_{nn}/N_{np}$ — which, unlike $\Gamma_n/\Gamma_p \equiv \Gamma(\Lambda n \rightarrow nn)/\Gamma(\Lambda p \rightarrow np)$, is an actual observable quantity in non–mesonic decay — is performed within a fully microscopic model where a proper treatment of FSI, gsc and ground state normalization is considered. All the isospin channels contributing to one– and two–nucleon induced decays are included. Our final result for the coincidence number ratio, $N_{nn}/N_{np} = 0.374$, is in agreement with the KEK–E508 datum, $(N_{nn}/N_{np})^{exp} = 0.40 \pm 0.10$.

Keywords: $\Lambda$–Hypernuclei, Non–Mesonic Weak Decay, Two–Nucleon Induced Decay, FSI

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1. Introduction

Being one of the main sources of information on strangeness–changing baryon interactions, the non–mesonic weak decay of $\Lambda$–hypernuclei has attracted a great deal of attention for several years. In this decay, the final, observable product is the emission of two or more nucleons from the hypernucleus. The non–mesonic decay width, $\Gamma_{NM}$, is built up from the one– and two–nucleon induced decays, $\Gamma_1 \equiv \Gamma(\Lambda N \rightarrow nN)$ and $\Gamma_2 \equiv \Gamma(\Lambda NN \rightarrow nNN)$, respectively: $\Gamma_{NM} = \Gamma_1 + \Gamma_2$. All possible isospin components are: $\Gamma_n = \Gamma(\Lambda n \rightarrow nn)$, $\Gamma_p = \Gamma(\Lambda p \rightarrow np)$, $\Gamma_{nn} = \Gamma(\Lambda nn \rightarrow nnn)$, $\Gamma_{np} = \Gamma(\Lambda np \rightarrow nnp)$ and $\Gamma_{pp} = \Gamma(\Lambda pp \rightarrow npp)$.

One should note that the only observables in hypernuclear weak decay are the lifetime $\tau$, the mesonic rates $\Gamma_\pi = \Gamma(\Lambda \rightarrow \pi^\pm p)$ and $\Gamma_{\pi^0} = \Gamma(\Lambda \rightarrow \pi^0 n)$ and the spectra of the emitted particles (nucleons, pions and photons). None of the non–mesonic partial decay rates ($\Gamma_n$, $\Gamma_p$, $\Gamma_{nn}$, etc) is an observable from a quantum–mechanical point of view. Each one of the possible elementary non–mesonic decays occurs in the nuclear environment, thus subsequent final state interactions (FSI) modify the quantum numbers of the weak decay nucleons and new, secondary nucleons are emitted as well: this prevents the measurement of any of the non–mesonic partial decay rates.
Among the weak decay observables one has the spectra of emitted neutrons ($N_n$) and protons ($N_p$) as well as the $nn$ and $np$ nucleon coincidence spectra, $N_{nn}$ and $N_{np}$. In the present contribution we discuss a microscopic model to take care of the $N_{nn}/N_{np}$ ratio and of the single–nucleon spectra for the emission of protons and neutrons. We report results for $^{12}$C. The paper is organized as follows: in Section 2 we outline the formalism, results for $^{12}$C are discussed in Section 3 and finally, in Section 4 some conclusions are given.

2. Formalism

Using the formalism developed in [1], the number of particles, $N_N$, and of pair of particles, $N_{NN'}$, where $N (N') = n$ or $p$, emitted in non–mesonic decay can be written as follows:

\[
N_n = 2\Gamma_n + \Gamma_p + 3\Gamma_{nn} + 2\Gamma_{np} + \Gamma_{pp} + \sum_{i,i',j} N_{j(i)} \Gamma_{i,i'\rightarrow j},
\]

(1)

\[
N_p = \Gamma_p + \Gamma_{np} + 2\Gamma_{pp} + \sum_{i,i',j} N_{j(p)} \Gamma_{i,i'\rightarrow j},
\]

(2)

\[
N_{nn} = \Gamma_n + 3\Gamma_{nn} + \Gamma_{np} + \sum_{i,i',j} N_{j(nn)} \Gamma_{i,i'\rightarrow j},
\]

(3)

\[
N_{np} = \Gamma_p + 2\Gamma_{np} + 2\Gamma_{pp} + \sum_{i,i',j} N_{j(np)} \Gamma_{i,i'\rightarrow j},
\]

(4)

\[
N_{pp} = \Gamma_{pp} + \sum_{i,i',j} N_{j(pp)} \Gamma_{i,i'\rightarrow j}.
\]

(5)

All terms which contain the functions $\Gamma_{i,i'\rightarrow j}$ represent the action of FSI. We refer the reader to [1] for a complete explanation of the meaning of these functions. The factors $N_{j(N)}$ are the numbers of nucleons of the type $N$ contained in the multinucleon state $j$. In the same way, the $N_{j(NN')}$'s are the numbers of $NN'$ nucleon pairs in the state $j$. The summation over $i$, $i'$ and $j$ runs over all possible initial ($i$ and $i'$) and final ($j$) states. We have used a normalization for which $\Gamma \equiv \Gamma/\Gamma_{NM}$.

The five quantities $N_N$ and $N_{NN'}$ of Eqs. (1–5) are observables. Five decay widths, $\Gamma_N$ and $\Gamma_{NN'}$, also enter into these equations. The fact that FSI cannot be neglected makes it impossible to invert the above relations in order to obtain expressions for the decay widths in terms of the observables $N_N$ and $N_{NN'}$. By starting from the experimental values of $N_N$ and $N_{NN'}$, it is possible to extract the so–called experimental values for the decay widths, once a model for the FSI is implemented. This is done in the hope that several models for FSI will lead to the same values for the extracted decay widths.

In this contribution we adopt a different working method. Instead of extracting experimental values for the decay widths, we work out a microscopic model to report results for $N_N$ and $N_{NN'}$ given by Eqs. (1–5). FSI are modeled by the set of Feynman diagrams of Fig. 1. It should be noted that by considering all the possible time–orderings of these diagrams, some of the corresponding Goldstone diagrams are pure gsc contributions (to $\Gamma_2$), others are pure FSI terms and others are interferences between gsc and FSI contributions. Pauli exchange terms have not been evaluated.

3. Results

Here we report results on the coincidence number ratio, $N_{nn}/N_{np}$, and on the single–proton and single–neutron emission spectra, $N_p$ and $N_n$, as a function of the nucleon kinetic energy.
The calculation is performed in nuclear matter; by the local density approximation we obtain results for $^{12}$C. The weak transition potential, $V^{\pi-\pi N\to NN}$, is represented by the exchange of $\pi$, $\eta$, $K$, $\rho$, $\omega$ and $K^*$ mesons, with the coupling constants and cut-off parameters deduced from the Nijmegen soft-core interaction NSC97f [2]. For the residual strong interaction, $V^{NN}$, we use a $\pi + \rho$ potential with the addition of a $g'$ Landau–Migdal parameter, with $g' = 0.7$.

In Table 1 we present our results for the $N_{in}/N_{np}$ ratio. The inclusion of FSI brings to a good agreement with data (without FSI, one would have $N_{in}/N_{np} = \Gamma_{pp}/\Gamma_p = 0.321$). Conversely, the effect of the quantum interference terms (which correspond to the diagrams $pp'$, $hh'$ and $pp'$ in Fig. 1) is not important in the determination of $N_{in}/N_{np}$. However, from Table 2 we see that the individual values of the interference terms are sizable in comparison with the dominant ones ($pp$) in both $N_{in}$ and $N_{np}$. From Table 2 we also see that the reduction due to the energy and angular cuts suffered by the FSI terms is much bigger than the reduction for the FSI–free, 1N–ind term. This is expected, as FSI give a dominant contribution in the low–energy region.

![Figure 1](image_url) The set of Feynman diagrams considered in this work and which contain the action of the strong interaction. The dashed and wavy lines stand for the weak and strong potentials, $V^{\pi-\pi N\to NN}$ and $V^{NN}$, respectively.

The single–nucleon spectra obtained with the microscopic model are shown in Fig. 2. It is clear that an improvement has been obtained with respect to the FSI–free predictions. We
Figure 2: Single-proton ($N_p$) and single-neutron ($N_n$) kinetic energy spectra for $^{12}_Λ\Lambda C$. With continuous (dashed) lines we give our results with (without) FSI. Data are from KEK–E508 [4].

observe that it is the action of the residual strong interaction (through FSI and gsc) the mechanism which could lead, once additional many–body terms are considered, to an agreement with the experimental spectra.

4. Conclusions

From the experimental spectra of nucleons produced in hypernuclear non–mesonic weak decay it is not straightforward to determine the decay widths $\Gamma_n, \Gamma_p, \Gamma_{nn}$, etc: a theoretical model for FSI is required. In this contribution, a quantum–mechanical microscopic model for nucleon emission has been discussed. It has been shown that with this approach it is possible to obtain a rather good agreement with experiment for the $N_{nn}/N_{np}$ ratio. However, we note that one should also reproduce the individual spectra $N_{nn}, N_{np}, N_n, N_p$, etc.

From a comparison with data, the predictions of our microscopic model for FSI shows a clear improvement over the FSI–free results. Our results suggest that the interference terms are not important. The present approach requires further improvements: i) a more realistic residual strong interaction, ii) the inclusion of Pauli exchange terms for the FSI diagrams and iii) the study of the effect of the $\Delta(1232)$. Finally, we note that the microscopic model allows us to set constraints not only on $V^{ΛN→NN}$ but also on $V^{NN}$.

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