MIXING OF PSEUDOSCALAR MESONS
AND
ISOSPIN SYMMETRY BREAKING

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Mixing of the pseudoscalar mesons is discussed in the quark-flavor basis with the hypothesis that the basis decay constants follow the pattern of particle state mixing. The divergences of the axial vector currents which embody the axial vector anomaly, combined with this hypothesis, provide a calculational scheme for the parameters describing the mixing of the $0^-$, $0^0$, and $0^+$ mesons. Phenomenological applications of this mixing scheme are presented with particular interest focused on isospin symmetry breaking in QCD estimated as and $0^+$ admixtures to the pion. In contrast to previous work a possible difference in the basis decay constants $f_u$ and $f_d$ is considered and consequences of this potentially large effect on the strength of isospin symmetry breaking is discussed.

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1. Introduction

The mixing of the pseudoscalar mesons is a subject of considerable interest that has been examined in many theoretical and experimental investigations, for earlier references see [1]. New aspects of mixing which mainly concern the proper definition of meson decay constants and the consistent extraction of mixing parameters from experimental data have recently been discussed [2,3,4] and will be reviewed here. A prominent topic in this context is the interplay between the $U_A(1)$ anomaly and isospin symmetry breaking.

In the analysis of mixing presented in Ref. 3, the quark-flavor basis is used and assumed that the decay constants ($f_q, q = u, d, s$) in that basis follow the pattern of particle state mixing, i.e., there is a common mixing angle $	heta$, in contrast to the frequently used octet-singlet basis. Here different mixing angles for the decay constants and for the states are needed. With the help of the divergences of the axial vector currents, the basic parameters of that mixing scheme can be determined for given masses of the physical mesons. It has been found in [5] that this approach leads to consistent results and explains many empirical features of mixing. In Ref. 5, that mixing scheme has been generalized in order to estimate the and $0^+$ admixtures to the pion which is a source of isospin symmetry breaking (ISB). Due to a number of recent experiments the interest in ISB has been renewed. It therefore seems opportune to reexamine the mixing of $0^+$ and $0^-$ mixing. Thus,
for instance, a crucial examination of the value of the \( u \) \( d \) quark mass difference is necessary. This mass difference is an important ingredient in understanding and estimating ISB within QCD. Another point is the role of possible differences between the decay constants \( f_u \) and \( f_d \) which has not been explored so far. Such a contribution to ISB would by no means ruin our understanding of ISB as being generated by the \( u \) \( d \) quark-mass difference. This is so because decay constants which represent wavefunctions at zero spatial quark-antiquark separation, are functions of the quark masses. Since we are not able to calculate the decay constants to a sufficient accuracy within QCD at present one may consider them as independent soft parameters in the analysis of meson mixing and ISB. As I am going to explain below, a possible difference between these decay constants is an important source of theoretical uncertainty in the analysis of ISB.

The plan of the paper is as follows: In Sect. 2 the quark-avorm mixing scheme will be presented briefly. In Sect. 3 some theoretical and phenomenological results for \(- \) m mixing will be reviewed. M mixing with the \( 0 \) and consequences for ISB will be discussed in Sect. 4 and compared to experiment in Sect. 5. The summary is given in Sect. 6.

2. The quark-avorm mixing scheme

The quark-avorm basis is constructed by the states \( a \in \{ u; d; s \} \) which are understood to possess the parton component positions

\[
j_a i = \psi_a i + \psi_{a+} \tag{1}\]

in a Fock expansion where \( \psi \) denotes a (light-cone) wave function. The higher Fock states also include \( jgj \) components. The decay constants in that basis, defined as vacuum-particle matrix elements of axial-vector currents, \( J^a_5 = a \cdot 5 \cdot a \), are assumed to possess the property

\[
h^0 J^a_5 j^b \langle p | i = ab f_{a \cdot p} \quad a; b = u; d; s. \tag{2}\]

The motivation for choosing this specific basis comes from the fact that breaking of SU(3)\(_F\) by the quark masses induces the three states differently, and from the observation that vector and tensor mesons—where the axial-vector anomaly plays no role—have similar mixing angles very close to the ideal mixing one \( \alpha_m = \arctan \frac{1}{2} \). The quark-avorm mixing scheme holds to the extent that OZI rule violation except of those induced by the anomaly, are negligible small. Flavor symmetry breaking, on the other hand, is rather large and is to be taken into account in any analysis of pseudoscalar meson mixing.

Since mixing of the \( 0 \) with the \( 0 \) and \( 0 \) is weak while \( 0 \) m mixing is strong it is appropriate to use isoscalar and isovector combinations

\[
\frac{1}{2} \left[ \begin{array}{c} u \\ d \end{array} \right]; \tag{3}\]

\( ^{a} \)In Ref. 3 \( q \) is denoted by \( q \). Analogous changes occur for other quantities, e.g. \( f \).
as the starting point instead of \( u \) and \( d \). The unitary matrix \( U \) that transforms from the basis \( f \); \( \pi \); \( \rho \) to that one for the physical meson states \( f_{P_1} = 0; f_{P_2} = \pi; f_{P_3} = 0 \) can then be linearized in the \( 0 \) and \( 0 \) mixing angles. An appropriate parameterization of \( U \) reads

\[
U ( f ; \pi ) = \begin{pmatrix} 1 + \cos \theta & \cos \theta \sin \varphi \\ \sin \theta & \cos \theta \sin \varphi \end{pmatrix},
\]

which is also of advantage to introduce isovector and isoscalar axial vector currents

\[
J_5^a = \frac{1}{2} [ u \begin{array}{c} s \\ d \end{array} \begin{array}{c} d \\ s \end{array}],
\]

The matrix elements \( h^* j^b_j [ F_a ] i \) are then defined by \( f_i \) and \( f_a \) are the basic decay constants in the \( 0 - 0 \) sector while the parameter \( z = ( f_u + f_d ) / ( f_u - f_d ) \) being of order \( \alpha \), occurs in the \( 0 \) \( (5) \) sector. We note in passing that the basic decay constants are renormalization scale dependent. Ratios like \( y \) or mixing angle are on the other hand scale independent. Since the anomalous dimension controlling the scale dependence of the decay constants are of order \( \alpha^2 \), this effect is tiny and discarded here.

Taking vacuum-particle matrix elements of the current divergences, one ends with the help of (4) and (5) \( (a; a^0 = 1; 2; 3; b; b^0 = +; s) \)

\[
h^* j^b_j [ F_a ] i = M^2_{a} U_{a^b} [ F_{b^b} ];
\]

where \( M^2 = \text{diag} [ M_{\pi}, M_{\rho}, M_{\rho} ] \) is the particle mass matrix which appears necessarily quadratic here. Next, I recall the operator relation

\[
\theta J_5^a = 2 m_a i a + !;
\]

which holds as a consequence of the \( U \) \( (1) \) anomaly. The topological charge density is given by \( f = 4 \pi C_6 \), \( C_6 \) denotes the gluon field strength tensor and \( G \) its dual. Inserting \( f \) into \( \theta J_5^a \) and neglecting terms of order \( \alpha^2 \), one obtains after some algebraic manipulations a set of equations which can be solved for the mixing parameters

\[
\sin \varphi = \frac{\Phi \left( M^2_{\pi}, M^2_{\rho}, M^2_{\rho} \right)}{\Phi \left( M^2_{\pi}, M^2_{\rho}, M^2_{\rho} \right)}
\]

\[
h^* j^b_j [ F_a ] i = \frac{f_i \Phi \left( M^2_{\pi}, M^2_{\rho}, M^2_{\rho} \right)}{\Phi \left( M^2_{\pi}, M^2_{\rho}, M^2_{\rho} \right)} ;
\]
\[
M^2_0 = m_{ss}^2 = \frac{1}{2} (m_{uu}^2 + m_{dd}^2) ; \quad (9)
\]

\[
y = 2 \frac{M^2_{0} - m_{ss}^2}{M^2_{0} - m_{ss}^2} ; \quad (10)
\]

\[
y = \frac{1}{2} m_{ss}^2 \frac{m_{uu}^2}{M^2_{0}} + z ; \quad (11)
\]

In addition, the symmetry of the mass matrix forces relations between decay constants and the anomaly matrix elements

\[
y = \frac{f_{u}}{f_{d}} = \frac{\pi}{2} \frac{\hbar \gamma^{j} j_{a} i}{\hbar \gamma^{j} j_{a} i} ; \quad z = \frac{f_{u}}{f_{d}} + \frac{f_{d}}{f_{u}} = \frac{\hbar \gamma^{j} j_{a} i}{\hbar \gamma^{j} j_{a} i} ; \quad (12)
\]

Last of all the pion decay constant, \( f \), equals \( f_{s} \) up to corrections of order \( \epsilon \).

The quark mass term in the above relations are defined as matrix elements of the pseudoscalar currents

\[
m_{ss}^2 = \hbar \gamma^{j} j_{a} i \frac{m_{a}}{f_{s}} a_{i} j_{a} i ; \quad (13)
\]

The quark-avormixing scheme can readily be extended to the case of the \( \psi \). One then finds for instance that the charm admixture to the \( \psi \) and \( \psi' \) amounts to 0.6 and 1.6\%, respectively. Their corresponding charm decay constants are \( f_{c} = (2.4 \pm 0.2) \text{ MeV} \) and \( f_{c}' = (6.3 \pm 0.6) \text{ MeV} \). The radiative decays of the \( J = 1/2 \) into the \( \psi \) and \( \psi' \) provide a nice test of these results.

3. \( ^{0}m \) mixing

The three relations taken from Ref. 11 can be used to determine the \( ^{0}m \) mixing parameters for given masses of the physical mesons and adopting the well-known PCAC result

\[
m_{ss}^2 = 2M_{\psi}^2 - M_{\psi'}^2 ; \quad (14)
\]

a result that is also obtained from leading order chiral perturbation theory. This theoretical estimate of the \( ^{0}m \) mixing parameters provides

\[
f_{s} = 1.27 f ; \quad a^2 = \hbar \gamma^{j} j_{a} i = \frac{\pi}{2 f_{s}} = 0.276 \text{ GeV}^2 ; \quad (15)
\]

It implies a value of \( 134^\circ \) for the state mixing angle = \( \text{ideal} \) in the SU(3)\( f \) octet-singlet basis. Note that the theoretical estimate presented here differs slightly from the one presented in Ref. 11. A phenomenological determination of the mixing parameters has also been attempted in Ref. 11, using experimental data instead of the theoretical results. Thus, comparing processes involving \( \psi \) mesons with those
Involving $0^+_s$ one can determine the mixing angle provided the OZI rule holds. An example is set by the radiative decays of the $\pi^0$ meson into the $0^+_s$ or $0^+$. This decay proceeds through the emission of the photon from the strange quark and a subsequent $s\bar{s}$ transition into the $\pi^0$ component. Hence, the ratio of $0^+_s$ and $0^+$ decay widths reads

$$\frac{[\pi^0]}{[\pi^+]} = \frac{k_{\pi^0}}{k_{\pi^+}} \frac{g^3}{[\pi^+]} \frac{[\pi^0]}{[\pi^+]} \frac{[\pi^0]}{[\pi^+]},$$

where $k_{\pi^0}$ is the final state's state three momentum in the rest frame of the decaying $\pi^0$ meson. The quantity $k_{\pi^+}$ is a small correction due to the $\pi^+$ meson mixing.

Other reactions like the $\pi^+\pi^-\gamma$ or $\pi^+\pi^-\pi^0$ transitions are sensitive to the decay constants. An interesting piece of information comes from the radiative decays of the $J^P=0^+$ into the $s\bar{s}$ and $0^+$. According to Novikov et al., the photon is here emitted from the charm quarks which subsequently annihilate into lighter quark pairs through the effect of the anomaly. This mechanism leads to the following result for the ratio of decay widths

$$\frac{[\pi^0]}{[\pi^+]} = \frac{k_{\pi^0}}{k_{\pi^+}} \frac{g^3}{[\pi^+]} \frac{[\pi^0]}{[\pi^+]};$$

The analysis of a large class of such reactions leads to the following set of phenomenological mixing parameters

$$f_s = (1.07, 0.02) \text{ GeV}^2; \quad f_1 = (1.04, 0.06) \text{ GeV}^2;$$

$$a^2 = (0.265, 0.01) \text{ GeV}^2.$$}

These values of absorb corrections from higher orders of flavor symmetry breaking and higher orders of $\pi^0\pi^+$. Transforming from the quark-flavor basis to the SU(3)$_F$ basis by an appropriate orthogonal matrix, one observes the need for two more angles, $\theta_1$ and $\theta_2$, besides the state mixing angle, in order to parametrize the constants for the weak decay of a physical $\pi^0$ meson through the action of a singlet or octet axial vector current

$$f_8 = f_8 \cos \theta_1; \quad f_1 = f_1 \sin \theta_1;$$

$$f_8^1 = f_8 \sin \theta_1; \quad f_1^1 = f_1 \cos \theta_1;$$

The singlet-octet mixing parameters are related to those for the quark-flavor basis by

$$f_8 = \sqrt{\frac{1}{2}f_8^2 + 2f_1^2}; \quad \theta_1 = \arctan \left( \frac{2f_8}{f_1} \right);$$

$$f_1 = \sqrt{3f_8^2 + 1}; \quad \theta_2 = \arctan \left( \frac{2f_8}{f_1} \right);$$

Only in the flavor symmetry limit, i.e., if $f_1 = f_8$, the three angles, $\theta_1$, $\theta_2$, and $\theta_3$, fall together. Evaluating the new parameters, say, from the theoretical set of mixing parameters one obtains

$$f_8 = 1.18 \text{ GeV}^2; \quad \theta_1 = 19.3;$$

$$f_1 = 1.09 \text{ GeV}^2; \quad \theta_2 = 6.9.$$
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The mixing angles differ from each other and from the state mixing angle (13½) substantially.

4. Isospin symmetry breaking

That the character of the approximate avor symmetry of QCD is determined by the pattern of the quark masses is a well-known fact that has extensively been discussed in the literature for decades. Isospin symmetry in particular which would be exact for identical u and d quark masses, holds to a rather high degree of accuracy empirically, although the ratio \( m_d/m_u \approx (m_d + m_u) \) is about 1/3, i.e. of order 1, and not, as one would expect for a true symmetry, much smaller than unity.

The violation of isospin symmetry for pseudoscalar mesons within QCD is usually estimated as an admixture of the avor-octet state to the pion. Exploiting the divergences of the axialvector currents, Gross, Treiman and Wilczek obtained

\[
0 = \frac{\alpha}{4} \frac{m_d}{m_u} \frac{m_u}{m_u + m_d} = \frac{2}{3};
\]

a result that also follows from lowest order chiral perturbation theory. We learn from this that due to the effect of the \( U(1) \) anomaly which is embodied in the divergences of the axialvector currents, ISB for the pseudoscalar mesons is of the order of \( m_d/m_u = \frac{2}{3} \) instead of the expected order \( m_d/m_u = \frac{1}{3} \). Isospin symmetry is thus partially restored and amounts to only a few percent. It is therefore to be interpreted rather as an accidental symmetry which comes about as a consequence of the dynamics. For hadrons other than the pseudoscalar mesons the strength of ISB is not necessarily set by the mass ratio, for comments on ISB in the vector meson sector see Ref. 17.

Defining the isospin-zero admixture to the \( ^0 \) by

\[
j^0|1 = j^1 + j + |0 \equiv 0^0 (2);
\]

one finds with the help of (10) and (11)

\[
0 = \sin \theta = \sin \frac{1}{2} \frac{m_d}{M} \frac{m_u}{M} + z;
\]

(24)

The \( f_0 = f_d \) limit of this result, termed \( ^0 (z = 0) \) and \( ^0 (z = 0) \) in the following, coincides with the result reported in Ref. 5. The quark mass term may be estimated from the \( K^0 \) \( K^+ \) mass difference corrected for mass contributions of electromagnetic origin

\[
m_{Kd}^2 - m_{Ku}^2 = 2 m_{K0}^2 \left( M_{K}^2 - M_{K+}^2 \right);
\]

(25)

A ccording to Dashen’s theorem, the electron magnetic correction is given by the corresponding difference of the pion masses

\[
M_{K+}^2 - M_{K}^2 = M_{K}^2 - M_{K0}^2 ;
\]

(26)
in the chiral limit. This correction amounts to \(12.6 \times 10^6 \text{ GeV}^2\) and leads to the estimate \(m_{ud}^2 \approx 0.0104 \text{ GeV}^2\). However, the quark masses increase \(M_{\pi}^2\) substantially. The exact size of this enhancement is subject to controversy. Different authors obtained rather different values for the electromagnetic mass splitting of the \(K^0\) mesons. For an estimate of the \(z = 0\) values of the and \(K^0\) mixtures to \(0\), I take the average of the results for \(M_{\pi}^2\) quoted in Ref. 19 and assign a generous error to the mixing angles in order to take into account the uncertainties in the electromagnetic contribution to the \(K^0\) mass. This way I obtain:

\[
^\wedge = (z = 0) = 0.017 \quad 0.003; \quad ^\wedge = 0(z = 0) = 0.004 \quad 0.001.
\]  

Due to the different value of the electromagnetic mass corrections, the value for \(^\wedge\) is now what larger than the one quoted by us previously.

Chao also investigated the mixing on the basis of the axial anomaly but, instead of diagonalizing the mass matrix, he exploited the PCAC hypothesis. He works in the conventional singlet-octet basis and assumes that the octet and singlet decay constants follow the pattern of state mixing, an assumption that has above been shown to be inadequate and theoretically inconsistent. Despite this his results on and \(0\) agree with our \(z = 0\) ones within the errors quoted in Ref. 19.

It is elucidating to express the mass terms simply by quark masses. With the help of the spontaneously broken SU(3)$_L$ \(\times\) SU(3)$_R$ quark model, one obtains:

\[
^\wedge = \frac{P}{3} \cos \theta; \quad ^\wedge = \frac{P}{3} \cos \theta.
\]  

with \(\theta\) given in Ref. 19. As we now see there is an additional factor of \(\frac{P}{3} \cos \theta = 1.34\) in comparison with GTW result. It would be unity if \(= \) ideal, i.e., if the physical and \(0\) mesons are pure flavor octet and singlet states, respectively. The small GTW value of \(\theta = 0.011\) has its source in the disregard of \(0\) mixing and the use of Dasher's result for the electromagnetic mass splitting. If the decay constants \(f_0\) and \(f_\pi\) differ from each other the mixing angles may deviate from the values quoted in Ref. 19. This potentially large effect is a source of considerable theoretical uncertainty of our understanding of ISB in the pseudoscalar meson sector.

It is emphasized that ISB as a consequence of \(0\) mixing is accompanied by a non-zero vacuum - \(0\) anomaly matrix element. From one obtains:

\[
h_{ij}^0 = 2 \left( \frac{M_{\pi}^2}{M_0^2} \right) \frac{M_{\pi}^2}{M_0^2} h_{ij}^0 = 2 \left( \frac{M_{\pi}^2}{M_0^2} \right) \frac{M_{\pi}^2}{M_0^2} h_{ij}^0.
\]  

The \(z\)-dependence cancels in this anomaly matrix element. Unavoidably the result goes along with a strange quark contamination of the pion.

5. Phenomenology of ISB

In this section I am going to compare experimental results on ISB in strong interactions with the theoretical expectation.
The decays \( (2S)! J^P = P \) are expected to be anomalous dominated as the radiative \( J^P = 0^+ \) decays, i.e., the \( 0^+ \) ratio is controlled by the ratio of the corresponding anomalous matrix elements:

\[
\frac{[ (2S)! J^P = 0^+ ]}{[ (2S)! J^P = ]} = \frac{\hbar \chi^{0 \downarrow} i^2}{\hbar \chi^{0 \downarrow} i} \frac{k_{(2S)! J^P = 0^+}}{k_{(2S)! J^P = }}, \quad (30)
\]

The ratio of the anomalous matrix elements calculated in the described mixing approach (see (29)), reads

\[
\frac{\hbar \chi^{0 \downarrow} i^2}{\hbar \chi^{0 \downarrow} i} = \frac{\hat{c} 0^+}{\cos^2}; \quad (31)
\]

where, for simplicity, \( M_0^2 \) is neglected as compared to \( M^2 \) and \( M_2^2 \). It is important to realize that there is no \( z \)-dependence in (31). According to Donoghue and Wyle possible electromagnetic contributions to the transitions \( (2S)! J^P \) are expected to be strongly suppressed. The recent accurate measurement of the branching ratios for these decays can lead to

\[
\hat{^0}_{(2S)} = 0.031 \quad 0.002; \quad (32)
\]

which is rather large as compared with the theoretical estimates. This causes a problem since the decays \( (2S)! J^P = P \) are considered as a clean measurement of the anomalous mixing angle. We don’t know what the origin of this discrepancy is. ISB through mixing seems to be well understood, it is very difficult to obtain a mixing angle as large as this way. Thus, one may suspect that either the decays \( (2S)! J^P = P \) receive substantial contributions from other mechanisms than the anomalous angle (e.g., from higher Fock states) and/or there are other sources of ISB within QCD beyond mixing.

Recently clear signals for ISB and/or charge symmetry breaking have experimentally been observed in a number of hadronic reactions. However, the extraction of the mixing angles from these data is difficult and model-dependent. The experimental ratio of the \( ^+d! pp \) and \( ^+d! nn \) deviates from unity, the charge symmetry result on the basis of a rather simple model that includes state mixing but ignores mixing with the \( 0^+ \), and that takes into account a number of corrections such as di erences in the meson-nucleon coupling constants or the proton-neutron mass di erence, the authors of Ref. 6 extracted a mixing angle of

\[
\hat{d} = 0.026 \quad 0.007; \quad (33)
\]

from their experimental data. State mixing considered in the analysis performed in Ref. 6 involves and not just the \( z = 0 \) value, \( \hat{c} \), that occurs for instance in the anomalous matrix elements. One may assign the di erence between the results and (33) to the quantity \( z \). It then turns out that the small value of

\[
z_d = 0.012 \quad 0.010; \quad (34)
\]
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Mixing

i.e. a small difference in the individual decay constants

\[ f_d' f_i(1 - z); \quad f_i' f_i(1 + z); \]  

(35)
of about 2\%, successes to bring the theory into agreement with the experimental value.

However, this explanation would leave the result unexplained.

The non-zero forward-backward asymmetry in np ! d ! measured at TRIUM

Compatibility with charge symmetry. The phenomenological analysis of this data suffers from large ambiguities. A product of the not to well-known n -nucleon coupling constant and the 0 ! mixing angle controls the asymmetry. Moreover, there are additional charge symmetry violating contributions from the rescattering amplitude of the exchanged vector. The latter are practically unknown. Despite this the result presented in Ref. 7 is in agreement with the mixing ideas advocated here. For instance, taking the mixing angle together with the small -nucleon coupling constant that follows from dispersion theory and is compatible with the generalized Goldberger-Treiman relation, one obtains about the same value for the product of both as is quoted in Ref. 7.

The cross section for pd ! 4He ! 0 has not yet been analyzed theoretically while
the COSY measurement of the ratio of the pd ! 3He ! and pd ! 3He ! ! cross sections provides only a very weak signal for ISB.

Other processes in which ISB occurs are e.g. the (5) ! 3 decay or CP violations in K ! 0 ! 0. A somewhat larger value of the 0 ! mixing angle than

seems to be required by the data. With regard of the new experimental result

a revision of these analyses perhaps advisable. Recently a new scalar meson

D s (2317) has been observed that has an isospin symmetry violating decay into the D s ! 0 channel. The branching ratio for this decay channel, which has not yet been measured, would again probe the mixing picture of ISB.

6. Summary

A detailed theoretical and phenomenological analysis revealed that the quark-avor mixing scheme provides a clean and consistent description of the mixing of pseudoscalar mesons. On exploiting the divergencies of the axialvector currents all basis mixing parameters can be determined. It turned out that the mixing breaking manifests itself differently in the mixing properties of states and decay constants. The mixing of the \( ^0 \) with the \( ^0 \) induces ISB of a few percent. As the comparison with experiment reveals the exact magnitude of ISB is not well understood as yet. Here, more work is clearly needed.

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