Solution of nonlinear Gribov-Levin-Ryskin-Mueller-Qiu evolution equation for gluon distribution function

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Abstract. In this paper we have determined the behavior of gluon distribution function by solving the Gribov-Levin-Reskin-Mueller-Qiu (GLR-MQ) evolution equation, which is nonlinear in gluon density. The moderate $Q^2$ behavior of $G(x,t)$, where $t = \ln(Q^2/\Lambda^2)$, is obtained by employing the Regge like behaviour of gluon distribution function at small-$x$. Here $Q^2$ behavior of nonlinear gluon distribution function is investigated for small values $x = 10^{-2}, 10^{-3}, 10^{-4}$ and $10^{-5}$ respectively. Our predictions are compared with different parametrisations and are found in good agreement. It is observed from our results that with the nonlinear corrections incorporated, the strong growth of $G(x,t)$ that corresponds to the linear QCD evolution equation is slowed down. Moreover essential taming of gluon distribution function is observed for $R = 2$ GeV$^{-1}$ as expected.

1. Introduction
Studies of parton distribution function or more importantly the gluon distribution function can provide an exquisitely fine test of perturbative quantum chromodynamics (QCD) in the small-$x$ region and can disclose a systematic examination of new effects such as saturation of parton (quark and gluon) density in the nucleon. The determination of the gluon density at small values of the Bjorken scaling variable $x$ is particularly interesting, because in this region gluons are expected to dominate the proton structure function. The most precise determinations of the gluon momentum distribution in the proton can be obtained from a measurement of the deep inelastic scattering (DIS) proton structure function $F_2(x,t)$ and its scaling violation. In perturbative QCD, the high-$Q^2$ behavior of DIS has traditionally been determined by the linear Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) evolution equations [1]. Consequently, the approximate analytical solutions of DGLAP evolution equations have been reported in recent years with significant phenomenological success [2,3]. The sharp growth of the gluon distribution function as $x$ goes smaller observed at HERA [4] can ultimately violate unitarity and so it has to be controlled by considering the gluon recombination or shadowing effects at very small values of $x$. This leads to the modification of the evolution equation by a term which is non-linear in gluon density. Gribov, Levin, Ryskin, Mueller and Qiu first studied these effects and proposed a new nonlinear evolution equation known as the GLR-MQ equation [5,6]. The study of viable generalizations of the GLR-MQ equation has been performed with great interest in the last few years [7-10].
In the present work we provide a solution of the nonlinear GLR-MQ evolution equation for the calculation of $G(x, t)$ in leading order by employing Regge behavior of gluon distribution function. The moderate $Q^2$-behavior of nonlinear gluon distribution function is obtained and results are compared with the predictions of different parameterisations viz. GRV98 [11], MRST2001 [12] and EHKQS model [7] etc. Finally, we present our conclusions.

2. Solution of nonlinear GLR-MQ evolution equation

The nonlinear GLR-MQ equation is given as [13]

\[
\frac{\partial G(x, Q^2)}{\partial \ln Q^2} = \frac{\partial G(x, Q^2)}{\partial \ln Q^2} \bigg|_{\text{DGLAP}} - \frac{9}{2} \frac{\alpha_s^2(Q^2)}{R^2 Q^2} \int_x^1 \frac{d\omega}{\omega} \left[ G\left(\frac{x}{\omega}, Q^2\right)\right]^2. \tag{1}
\]

The size of the nonlinear term depends on the value of $R$ where, $R$ is the correlation radius between two interacting gluons. The negative sign in front of the nonlinear term is responsible for gluon recombination.

To solve the GLR-MQ equation we consider a simple form of Regge like behavior of the gluon distribution function as

\[
G(x, t) = M(t)x^{-\lambda}, \tag{2}
\]

where $M(t)$ depends only on $t$, where $t = \ln(Q^2/\Lambda^2)$. Now using this behavior and substituting the expression for the usual DGLAP term [14], Eq. (1) can be expressed as

\[
\frac{\partial M(t)}{\partial t} = \alpha_s(t) M(t) \left[ \left( \frac{11}{12} - \frac{N_f}{18} + \ln(1 - x) \right) + \int_x^1 \frac{d\omega}{\omega} \left( \omega(1 - \omega) + \frac{1 - \omega}{\omega} \omega^{\lambda - 1} \right) \right] - \frac{9}{2} \frac{\alpha_s^2(t)}{R^2 \Lambda^2 e^t} M^2(t) \int_x^1 \omega^{2\lambda - 1} d\omega. \tag{3}
\]

Performing the integrations explicitly and rearranging the terms, equation (3) takes the form

\[
\frac{\partial M(t)}{\partial t} = P(x) \frac{M(t)}{t} - Q(x) \frac{M^2(t)}{t^2 e^t}, \tag{4}
\]

which can be solved as

\[
M(t) = \frac{t^{P(x)}}{C - Q(x) \Gamma[-1 + P(x), t]}. \tag{5}
\]

Here, $\Gamma$ is the incomplete gamma function and $C$ is a constant. $P(x)$ and $Q(x)$ are functions of $x$. However, the dependence of the functions $P(x)$ and $Q(x)$ on $x$ can be considered as mild which could be neglected in the region of small-$x$. Thus this solution is valid only for small-$x$.

Substituting Eq. (5) in Eq. (2) we get

\[
G(x, t) = \frac{t^{P(x)}}{C - Q(x) \Gamma[-1 + P(x), t]} x^{-\lambda}. \tag{6}
\]

Now, defining

\[
G(x, t_0) = \frac{t_0^{P(x)}}{C - Q(x) \Gamma[-1 + P(x), t_0]} x^{-\lambda}, \tag{7}
\]

at $t = t_0$ for some lower value of $Q = Q_0$, we obtain from Eq. (6)

\[
G(x, t) = \frac{t^{P(x)} G(x, t_0)}{t_0^{P(x)} x^{-\lambda} + Q(x) \left\{ \Gamma[-1 + P(x), t_0] - \Gamma[-1 + P(x), t] \right\} G(x, t_0)} x^{-\lambda}, \tag{8}
\]

which gives the $Q^2$ behavior of nonlinear gluon distribution function at fixed $x$ in leading order.
3. Result and discussion

Figures 1a,b; 2a,b and 3a,b respectively represents the plot of $G(x,Q^2)$ versus $Q^2$ for $R = 2 \text{ GeV}^{-1}$ and $R = 5 \text{ GeV}^{-1}$ obtained from Eq. (8) for small values of $x = 10^{-2}, \ 10^{-3}, \ 10^{-4}$ and $10^{-5}$. Our predictions of $G(x,Q^2)$ are compared with those obtained by the global fits to the parton distribution functions GRV98LO, MRST2001LO and EHKQS model. In order to find out the input distribution $G(x,t_0)$, we use MRST2001 LO parton distributions at $Q^2_0 = 1 \text{ GeV}^2$. In our analysis we consider the range $3 \text{ GeV}^2 < Q^2 < 40 \text{ GeV}^2$ for GRV98LO, $1.5 \text{ GeV}^2 < Q^2 < 20 \text{ GeV}^2$ for MRST2001LO and $1.4 \text{ GeV}^2 < Q^2 < 40 \text{ GeV}^2$ for EHKQS.
Figure 3. $G(x, Q^2)$ versus $Q^2$ for $R = 2 \text{ GeV}^{-1}$ and $R = 5 \text{ GeV}^{-1}$ using Eq. (8) for four small values of $x$ compared with EHKQS model (dashed lines).

4. Conclusion
In this paper we have obtained the $Q^2$ behavior of nonlinear gluon distribution function at fixed $x$ by solving GLR-MQ evolution equation and made an analysis of GRV98LO, MRST2001LO parametrisations and EHKQS model with it. We observe that for fixed-$x$ the gluon distribution function increases with increasing $Q^2$ as usual, but with the nonlinear corrections incorporated, the strong growth of gluon distribution function is eventually slowed down. Moreover essential taming of gluon distribution function is observed for $R = 2 \text{ GeV}^{-1}$ as expected.

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