Modelling the reflection of radio waves from cold glaciers

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Abstract. Radio echo-sounding is a powerful technique for investigating glacier subsurface. Various remote sensing techniques use different wavelengths (e.g., 13.575 GHz for CryoSat and 20-25/200-600 MHz for ground-penetrating radar), and the results gained using different wavelengths may not coincide but rather complement each other. Here we describe the electrophysical model of a cold glacier based on glaciological input data and radiophysical laws; calculate the depth profile of the dielectric constant, and track variations in the effective part of the dielectric constant and the loss tangent depending on the surface temperature and frequency of the radio signals; calculate the frequency dependence of the reflectivity coefficient of the modelled glacier; reveal the frequency range of radio-waves most suitable for investigation of glacier characteristics.

1. Introduction
When glaciers are investigated by radiophysical methods, the measured value is the reflected radio signal: if the spectrum of the incident radio signal was $S_f$, the spectrum of the reflected would be equal to $R(f)S(f)$, where the complex function $R(f)$ is called the reflection coefficient of radio waves, it carries information about the properties of the reflecting medium [1]. The presence of water as a component of glacier ice and snow dramatically changes the physical properties of glaciers and divides them into “warm” and “cold”. In this paper, we consider the electrophysical model of a “cold” glacier as the most stable from the point of view of radiophysics, and discuss the features of its dielectric characteristics due to changes in surface temperature and frequency of the interacting electromagnetic field.

2. Methodology: theoretical basis for calculating the reflection coefficient of radio waves from a glacier
Consider the problem of determining the reflection coefficient of radio-waves from the ice surface $z = 0$ based on the following wave equation:

$$Y'' + k^2\varepsilon(z)Y = 0. \tag{1}$$

In this equation, $k$ is the wavenumber. The dielectric constant of a reflecting half-space, consisting of a glacier which is $D$ m thick and a dielectric-homogeneous bed, is given by the
complex function $\varepsilon(z)$:

$$\varepsilon(z) = \begin{cases} 1, & z < 0 \\ \varepsilon_{\text{ice}}(z), & 0 \leq z \leq D. \\ \varepsilon_p, & z > D \end{cases}$$  \hspace{1cm} (2)$$

Here $\varepsilon_{\text{ice}}(z)$ and $\varepsilon_p$ are dielectric constants of ice and bed, respectively. The solution of the equation (1) in the half-space $z < 0$ is:

$$Y_{z<0} = \exp(ikz) + R(f)\exp(-ikz),$$  \hspace{1cm} (3)$$

where the first term describes the incident radio-wave, the second term describes the wave reflected from the surface of the glacier with a reflection coefficient $R(f)$.

Snow, firn and ice are a mixture of three phases of water: solid (ice crystals, snowflakes), liquid (water), and gaseous (water vapor and air).

**Snow and firn** Since we consider snow and firn, which consist of crystals of fresh dry ice and air, we use the Looyenga formula modified to calculate the permittivity of the snow cover $\varepsilon = \varepsilon_{\text{sn}}$, air $\varepsilon_1 = \varepsilon_{\text{atm}} = 1$ and ice $-\varepsilon_2 = \varepsilon_{\text{ice}}$ [2,3,4]:

$$\varepsilon_{\text{sn}} = \left(\frac{\rho}{\rho_{\text{ice}}} \left(\frac{\varepsilon_{\text{ice}}^{1/3}}{3} - 1\right) + 1\right)^3$$  \hspace{1cm} (4)$$

Here $\rho_{\text{ice}}$ is the ice density, $\rho$ is the snow density.

**Dry ice** To model the complex dielectric constant of ice, the Debye formula is traditionally used [2,5]:

$$\varepsilon_{\text{ice}} = \varepsilon_\infty + \frac{\varepsilon_0 - \varepsilon_\infty}{1 + \tau^2w^2} + \frac{\varepsilon_0 - \varepsilon_\infty}{1 + \tau_0^2w^2}.$$  \hspace{1cm} (5)$$

In this formula $\varepsilon_0$ is the dielectric permittivity of ice in the permanent field, $\varepsilon_\infty$ is the optical permittivity of ice, $w = 2\pi f$, where $f$ is the electromagnetic field frequency in Hz. The relaxation time of ice $\tau$ depends on temperature according to the law $\log \tau = \frac{2900}{T + 273.15} - 15$, $T$ is in degrees Celsius. Figure 1 shows frequency dependence of the real part of the function (5), and figure 2 shows frequency dependence of the loss tangent of ice $\tan\delta_{\text{ice}} = \varepsilon''_{\text{ice}}/\varepsilon'_{\text{ice}}$. In both figures, line 1 is plotted for temperature $0^\circ C$, line 2 – for $-10^\circ C$ and line 3 – for $-20^\circ C$.

![Figure 1. Frequency dependence of $Re(\varepsilon_{\text{ice}})$.](image1.png)

![Figure 2. Frequency dependence of $tg(\delta_{\text{ice}})$.](image2.png)
Glacier bed The dielectric constant of the dry bed $\varepsilon_p$ is estimated between 6 and 12 [6].

In our model, the snow thickness is 8 meters, the thickness of the “cold” firn is 50 meters [7], the thickness of the entire glacier reaches 110 meters. Figure 3 shows the depth distribution of temperature in snow/firn/ice body $T(T_0, z)$ for the surface temperature $T_0 = 0^\circ C$ (line 1), $T_0 = -10^\circ C$ (line 2) and $T_0 = -20^\circ C$ (line 3).

3. Results and discussion

For given $T(T_0, z)$ and $\rho(z)$ the depth profiles of the real part of the dielectric constant and loss tangent were calculated using equations (8) and (7). The respective plots are shown on figures 4 and 5. From the calculations it follows that the lower the temperature and the higher the frequency, the less radio waves are absorbed in the glacier, and the absorption is almost the same in depth and does not depend on the type of snow-ice cover.

The permittivity of a glacier is a fairly abstract concept, since for practical application the reflection coefficient of radio waves depending on it is of interest. The function $R(f) = \ldots$
$|R(f)| \times e^{i\phi}$ is complex. The method used in this work to calculate $R(f)$ from an arbitrary depth profile is described in detail in [8]. Figure 6 shows the absolute value of the reflection coefficient $|R(f)|$ of radio-waves with frequencies from $10^4$ Hz to $10^8$ Hz. In the figure, the letter A marks the frequency range, the wavelength of which corresponds to the condition $|\lambda| > 10$. They are reflected only from the glacier bed with a reflection coefficient equal to $R(f) = \frac{1 - \sqrt{\varepsilon_{\text{bed}}}}{1 + \sqrt{\varepsilon_{\text{bed}}}}$. The glacier itself is transparent to them. As $\lambda$ decreases, the waves reflected from the bed begin to interfere with the waves reflected from the surface [9]. Therefore, the frequency dependence of the reflection coefficient takes an oscillating form, the oscillation period depends on the effective thickness of the glacier, and the amplitude depends on the dielectric constant of the bed and ice. For waves of this range, the value of $tg(\delta_{\text{ice}}(z))$ is approximately equal to $0.5 \div 0.05$, which leads to a decrease in the amplitude of oscillations: due to absorption, the waves reflected from the bed cease to reach the surface, but the ice-firn layer is still transparent for them (in Fig. 6 this range is indicated by the letter B), therefore, reflection occurs from the surface. The reflection coefficient becomes $R(f) = \frac{1 - \sqrt{\varepsilon_{\text{ice}}(0)}}{1 + \sqrt{\varepsilon_{\text{ice}}(0)}}$. Figure 7 shows a fragment of the function $|R(f)|$ in the vicinity of the frequency of the radio altimeter signal — $13.575$ GHz. It should be noted that a change in temperature on the surface of the glacier does not affect $|R(f)|$. Figure 8 shows a graph of the temperature dependence of the phase of the reflection coefficient of a radio wave with a frequency of $13.575$ GHz from a glacier.

![Figure 6](image6.png)  
**Figure 6.** Frequency dependence of $|R(f)|$ for radio waves with a frequency of $10^5$ Hz to $10^8$ Hz.

![Figure 7](image7.png)  
**Figure 7.** Frequency dependence of $|R(f)|$ for radio-waves with frequencies from $13.55$ GHz till $13.6$ GHz.

4. Conclusions

As a result of the analysis of the calculations, it turned out that at low temperatures for frequencies above $1$ MHz the real part of the dielectric constant of the glacier does not change with the frequency and temperature on the surface, but depends on the structure of the glacier, while the depth profile of the loss tangent is constant throughout glacier, with the exception of the upper layer with a thickness of up to $10$ m. The range of radio-waves from $0.1$ to $1$ MHz is not optimal for sounding cold glaciers: the absorption of radio-waves by ice is large for studying thick layers of the glacier, and the wavelength does not allow studying thin layers, so the reflection signal reflected from the surface prevails. The small absorption of short radio waves by ice leads to the fact that the frequency dependence of the reflection coefficient of short radio-waves is practically the sum of the partial reflections of radio-waves from the surface and
Figure 8. The temperature dependence of the phase of the reflection coefficient of a radio wave with a frequency of 13.575 GHz from a glacier. The horizontal axis is the frequency, the vertical axis is the phase of the reflection coefficient, its value changes by almost an order of magnitude when the temperature changes by 20 degrees.

internal snow/firn, firn/ice and ice/bed boundaries [10]. Period and amplitude of oscillations of the function $|R(f)|$ depend on the depth of the internal boundaries and the gradient of dielectric characteristics of ice, snow, firn and glacier bed. Changes in surface temperature, leading to a change in the loss tangent of the upper glacier layers, are manifested in the phase magnitude of the reflection coefficient of radio-waves: it grows with the temperature. From the analysis of the calculation results it follows that the signal reflected from the glacier at frequencies above 100 MHz contains information about the structure of the cold glacier and the depth distribution of the dielectric constant in its body, but to restore the electrophysical parameters of the glaciers it is necessary to use a broadband signal with a smooth spectrum.

Acknowledgments
The work was performed as part of the state assignment of the IRE named after V.A. Kotelnikov RAS and partially supported by Russian Foundation of Basic Research RFBR (grant No. 18-05-00420).

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