The ‘classical tunnelling effect’—observations and theory

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Abstract. In ground-based laboratory experiments, in experiments under microgravity conditions and in parabolic flight experiments, it is often observed that individual microparticles appear sometimes to simply pass through a strongly coupled complex plasma without affecting the fabric (the structure) of the penetrated cloud. This is surprising at first sight, because the collisional mean free path in such ‘plasma crystals’ is of the order of the interparticle separation. We have termed this new effect ‘classical tunnelling’ in analogy to its quantum mechanical counterpart. To explain this anomalous transport, a ‘geometrical model for charge variation’ of moving and stationary ordered particles is proposed. Theoretical predictions are in good qualitative agreement with observations and numerical simulations, and show that the phenomenon can be considered as a consequence of the non-Hamiltonian character of complex plasmas.

\ldots It’s a mystery therefore I can think whatever I want \ldots
\ldots Scientists have a strange tendency to be insufficiently empirical sometimes \ldots

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1. Introduction

Complex plasmas consist of four components—electrons, ions, charged microparticles and neutral gas atoms (as a background). The complexity of this system is due to the very different space and timescales of the sub-component dynamics, which may vary by up to ten orders of magnitude. For instance, the microparticle sub-system of the complex plasma can be crystallized even if the entire system is far from thermal equilibrium [1]–[3].

In the past decade, it has been shown that complex plasmas can serve as a suitable prototype to model various transport phenomena at the kinetic level. These are, e.g., particle, momentum and energy transport in shocks (see, e.g. [4] and references therein), viscosity of liquid complex plasmas [5], heat transfer in strongly coupled (crystallized) complex plasma [6], diffusion [7] in liquid, gaseous complex plasmas, etc.

The other principal aspect of complex plasma dynamics, as it has been discussed in [8], is that the rate of momentum and energy exchange through binary (electrostatic) collisions between the microparticles can exceed the rate of dissipation by, e.g., (Epstein) neutral gas drag [9]. At lower neutral gas pressure (low friction), complex plasmas can be used successfully for wave excitation [10]–[19], and dynamical instability experiments, such as the two-stream particle flow instability [8], parametric and other instabilities [20]–[23]. In this paper, we investigate a new phenomenon—the anomalous transport of microparticles through strongly coupled complex plasmas and plasma crystals, where ‘anomalous’ transport means ‘unusual’ and ‘much enhanced’ as, e.g., in the tunnel effect.

In solid bodies, where free particle motion is inhibited, the anomalous transport refers to particular mechanisms allowing non-conductive or non-diffusive transport of particles or energy [24] such as momentum transfer along chains of interacting particles, so-called focusons, or focused collisions [25]–[28]. Focusons do not involve transfer of individual particles (the particles are displaced but normally restored to former positions after the excitation went over).
The transferred momentum, however, can concentrate at the edge of the chain and thus affect local damage relatively deep inside the body. The focused collisions are important for particle scattering by a solid surface [29], and for ion-induced sputtering [30, 31].

Channelling in solid bodies occurs when an ion enters a crystal with a small angle to the crystal planes. The electrostatic interaction between the incoming ion and the lattice then causes the ion to follow the crystal planes. (By mechanically bending of a crystal, it is possible to deflect ions.) It is well known that the ion will channel only if it is fast enough [32]–[36].

The tunnelling in strongly coupled complex plasmas, which is reported here, occurs at a typical microparticle velocity range of 1–10 mm s⁻¹, at least 5–10 times faster than the particle velocities in the surrounding strongly coupled environment (typical thermal speeds are in the sub-millimetre range). Sometimes it is observed that individual particles simply pass through a strongly coupled complex plasma (even in the crystalline state)—as if these particles can ‘walk through a wall’. We have studied this anomalous transport, a new phenomenon observed in ground-based laboratory experiments, in experiments under microgravity conditions and in parabolic flight experiments. To explain the observations, a model for a local density-dependent charge variation is proposed. The main assumption is that the ‘new’ particle shares the free electrons with the crystal particles in proportion to the overlap of the Debye spheres. Since the sum of all charges remains constant inside a cell, the charge of an individual particle should then decrease and the barrier for penetration will be lower. This phenomenon can be considered as a consequence of the non-Hamiltonian character of complex plasmas [37, 38].

The paper is organized as follows: in section 2 the observations obtained in parabolic flight experiments (section 2.1), in ground based experiments (section 2.2) and in experiments under microgravity conditions (section 2.3) on the International Space Station (ISS) are presented. In section 3 the main theoretical assumptions are formulated (section 3.1) and the geometrical model for charge variation is proposed (section 3.2). Using force balance, a simple penetration criterion is deduced (section 3.3). Theoretical predictions are compared with results of numerical simulations of ‘tunnelling’ in a two-dimensional (2D) double-chain geometry (section 3.4). Finally a possible generalization of the theory is discussed involving the in-cell and inter-cell pressure balance (sections 3.5 and 3.6). In section 4 the results are summarized and discussed.

2. Experimental evidence of anomalous transport in strongly coupled complex plasmas

2.1. Parabolic flight experiment

Abnormal inter-penetration of two particle types (figure 1) has been observed during the second parabolic flight campaign performed by the ‘Deutsches Zentrum für Luft- und Raumfahrt’ (DLR) (for parabolic flight experiments, see [39, 40]).

The microparticles (monodisperse melamine-formaldehyde with radii of 1.69±0.05 μm and of 3.45±0.1 μm, mass density of 1.51 g cm⁻³) were injected into an argon plasma (at a gas pressure of 31.5 Pa) of an inductively coupled discharge. The discharge was ‘ignited’ and driven in a spherical chamber by two coils that were fed by an rf current at 13.6 MHz and a power 0.4 W. The microparticles were injected as a mixture by the same dispenser, and were illuminated from the side by a laser beam that was expanded by cylindrical lenses to support a thin (∼150 μm thickness) laser sheet, oriented perpendicular with respect to the observation direction. Plasma and particle cloud parameters are listed in tables 1 and 2. The electron density and temperature were obtained by extrapolating results of probe measurements [40]. From that we estimated the

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Figure 1. Different stages of anomalous particle transport observed in a parabolic flight experiment. Chains of smaller particles (a) passing through a cloud of larger particles (b). The ‘drop’ of smaller particles moving inside the cloud of larger particles (c, d). Final phase of inter-cloud interaction: penetration of larger particles through the coupled cloud of smaller particles, starting (e) and final (f) stages.

Ion density and particle charge using charging currents and quasi-neutrality:

\[
\begin{align*}
J_i &= J_e, \\
n_i &= n_e + Zn_d, \\
J_i &= (8\pi)^{1/2} R^2 n_i v_{Ti} (1 + \xi Ze^2 / RT_i), \\
J_e &= (8\pi)^{1/2} R^2 n_e v_{Te} \exp(-Ze^2 / RT_e), \\
v_{Ti,j} &= (T_{e,i} / m_{e,i})^{1/2}.
\end{align*}
\]
Table 1. Plasma parameters. \( n_\text{e}, T_\text{e,i}, \lambda_\text{e} \) and \( \lambda_\text{i} \) are the electron density, electron and ion temperatures, electron screening length, and the ion mean free path correspondingly.

| Experimental campaign | Gas  | \( P \) (Pa) | \( U_{pp} \) (V) | Power (W) | \( n_\text{e} \) (m\(^{-3}\)) | \( T_\text{e} \) (eV) | \( T_\text{i} \) (eV) | \( \lambda_\text{e} \) (\( \mu \text{m} \)) | \( \lambda_\text{i} \) (\( \mu \text{m} \)) |
|-----------------------|------|-------------|-----------------|-----------|----------------|----------------|----------------|----------------|----------------|
| Parabolic flight      | Ar   | 31.5        | 0.4             |           | \( 1.8 \times 10^{13} \)\textsuperscript{a} | 2.4             | 0.025           | 2700           | 85             |
| Ground-based          | Ar   | 95          | 71              |           | \( 8.2 \times 10^{15} \)\textsuperscript{b} | 6\textsuperscript{c} | 0.025           | 240\textsuperscript{c} | 28             |
| Experiment on ISS     | Ar   | 50          | 84              | 0.25      | \( 4.6 \times 10^{15} \)\textsuperscript{c} | 6\textsuperscript{c} | 0.025           | 320\textsuperscript{c} | 54             |

\( ^{a} \)\( n_\text{e} \) was estimated from probe measurements [40].

\( ^{b} \)Extrapolation from [56].

\( ^{c} \)Results of numerical simulation.

Table 2. Dust cloud parameters. \( R \) is the radius, \( Z \) is the charge (number of bounded electrons per particle), \( M \) is the mass, \( a \) is the interparticle separation (in separated clouds), \( \tau = \nu^{-1} \), \( \nu \) is the Epstein drag coefficient, \( n_\text{t}, \lambda_\text{i} \) are the ion density, and the ion screening length, \( H = Z/N_\text{e} \) is the Havnes parameter, \( N_\text{e} = \frac{4}{3} \pi a^3 n_\text{e} \) is the number of free electrons per particle.

| Experimental campaign | \( R \) (\( \mu \text{m} \)) | \( M \) (\( 10^{-11} \) g) | \( a \) (\( \mu \text{m} \)) | \( \tau \) (ms) | \( N_\text{e} \) (\( 10^3 \)) | \( Z \) (\( 10^3 \)) | \( n_\text{t} \) (m\(^{-3}\)) | \( \lambda_\text{i} \) (\( \mu \text{m} \)) | \( H \) |
|-----------------------|-----------------|------------------|-----------------|----------------|----------------|----------------|----------------|----------------|------|
| Parabolic flight      | 1.69            | 3.04             | 210             | 2.1            | 0.7            | OML            | 3.6             | 11 \( \times 10^{13} \) | 110  |
|                       | 3.45            | 26.0             | 330             | 17.8           | 2.7            | DML            | 1.5             | 5.7 \( \times 10^{13} \) | 160  |
|                       |                 |                  |                 |                |                | OML            | 8.6             | 8 \( \times 10^{13} \)  | 140  |
|                       |                 |                  |                 |                |                | DML            | 4               | 4.4 \( \times 10^{13} \) | 180  |
| Ground-based          | 0.6             | 0.14             | 150             | 0.23           | 85             | OML            | 5.2             | 6.4 \( \times 10^{15} \) | 15   |
|                       |                 |                  |                 |                |                | DML\textsuperscript{b} | 1.4             | 6.4 \( \times 10^{15} \) | 15   |
|                       |                 |                  |                 |                |                | DML\textsuperscript{c} | 3.8             | 6.3 \( \times 10^{15} \) | 15   |
| Experiment on ISS     | 1.7             | 3.13             | 120\textsuperscript{a} | 1.3            | 24             | OML            | 12.9            | 5 \( \times 10^{15} \)  | 16   |
|                       |                 |                  |                 |                |                | DML\textsuperscript{b} | 4.3             | 4 \( \times 10^{15} \)  | 19   |
|                       |                 |                  |                 |                |                | DML\textsuperscript{c} | 10.2            | 4.7 \( \times 10^{15} \) | 17   |

\( ^{a} \)Dense part of the cloud.

\( ^{b} \)Estimated at \( \lambda = \lambda_\text{e} \).

\( ^{c} \)Estimated at \( \lambda = \lambda_\text{i} \).

Here, \( J_{\text{e}(i)} \), \( n_{\text{e}(i)} \), \( v_{\text{Te}(i)} \), \( T_{\text{e}(i)} \) represent the electron (ion) current, the electron (ion) number density, the electron (ion) thermal velocity and the electron (ion) kinetic temperature, and \( Z \) is the particle charge. These relationships generalize those usually used in the so-called Orbital Motion Limited (OML) approximation [41]–[43]. The generalization parameter there is the \( \xi \)-factor, a measure of collisionality. In the collisionless case \( \xi = 1 \) (it corresponds to the ‘pure’ OML approximation). In case of frequent collisions, if the motion of ions collected by a particle is limited by the ion...
mobility, $\xi$ is given by equation (2) (similar results were found in [44, 45]):

$$\xi = (2\pi)^{1/2}(m_i/e)(v_{Ti}\mu_i/R) \equiv (2\pi/3)^{1/2}\lambda_c/R.$$  

(2)

Here, $\mu_i$ is the ion mobility, $\lambda_c$ is the ion mean free path and $R$ is the particle radius. Practically, equation (2) works well if the collision mean free path is less than one third of the screening length, $\lambda_c < \lambda/3$ [44]. For an arbitrary collision frequency, the correction parameter can be fitted successfully by using simulation results found in [44]. Namely, for the range $\lambda_c > 0.1\lambda$, where the deviation from OML is already well pronounced, the following simple fit is suitable (it fits simulation results found in [35] with an accuracy better that 5%):

$$\xi = (2\pi/3)^{1/2}\lambda_c/(R + R_c), \quad R_c \equiv (2\pi/3)^{1/2}\lambda_c^2/(3\lambda + \lambda_c).$$  

(3)

It is convenient (and we will do it below) to call this case as a DML (Drift Motion Limited) approximation, in analogy to the abbreviator ‘OML’ approximation. Results obtained by using both OML and DML approaches are collected in table 2. Note that the Havnes parameter and the particle charge are overestimated in OML. The Havnes parameter is of the order of a few, $H \sim 3–5$, in OML, whilst $H \sim 1–2$ in DML. The difference arises because the ion mean free path (estimated by using experimental data [46]) is 2–4 times less than the ion screening length and hence the correction equations (2) and (3) are important. The OML estimate for the particle charge is also approximately twice that obtained by using DML. Note, nevertheless, that the deviation for the estimated ion screening length is not so huge. Both models predict similar results: the ion screening length is less than the interparticle separation, $\lambda_i < a$.

Just after microgravity was reached during the parabolic flight manoeuvre, the discharge was ignited and shortly after the first particle injection started. The chamber was filled up by a dense stratified cloud of particles, in which the larger particles concentrated at the edges (close to the chamber walls). The events observed after the second injection (which was initiated a few seconds after the first one) are demonstrated in figure 1. Here, the field of view is $29.1 \times 21.8$ mm$^2$. The frame rate was 25 frame s$^{-1}$. Three different stages were observed. Just after the injection (figures 1(a) and (b)), the smaller particles were displaced from their equilibrium position in the centre of the chamber and became embedded deep in the cloud of larger particles. The small particles then moved toward the chamber centre where the big particles had left a ‘void’. At this stage they moved very fast and formed narrow separated streamers (or chains) allowing an easier penetration through the strongly coupled cloud of larger particles. Each streamer consisted of 5–10 aligned particles; transport velocity was of the order of $2\pm 0.5$ mm s$^{-1}$. After the void was filled up, the cloud of smaller particles self-organized forming a ‘macro-droplet’ (like a droplet of liquid which, being self-compressed by surface tension, forms a spheroid in microgravity; see figure 1). This self-sustained droplet of microparticles was freely ‘floating’ inside the cloud of larger particles, responding to the slightest changes in gravity during the remaining parabolic flight time. In the process its form changed in response to the force fields and the pressure of the surrounding (large) particle cloud. Firstly, it ‘fell’ slowly down as a whole (figures 1(c) and (d)), reorienting a bit. At this stage the mean velocity was about 1.5$\pm 0.5$ mm s$^{-1}$. Then the droplet stopped suddenly, reoriented, and then quickly moved upward and to the right. At this stage the droplet accelerated up to $4.1\pm 0.8$ mm s$^{-1}$. Finally, it collided with the cloud of larger particles. During the collision, the larger particles accelerated downwards (toward the droplet), acquiring a velocity of $2.8\pm 0.3$ mm s$^{-1}$, and penetrated through the droplet (figure 1(e)). The relative velocities of counter-streaming particles were about 7 mm s$^{-1}$ at this stage. The large
particles created deep channels inside the droplet that were visible quite some time after the
interpenetration was over (figure 1(f)). The mean kinetic energy of random particle motion
was difficult to resolve but estimated to be of the order of room temperature (∼0.025 eV).
The ‘thermal’ velocities are of the order of 0.2 mm s⁻¹ (0.6 mm s⁻¹) for the 1.69 µm (3.45 µm)
particles.

2.2. Laboratory observations: ground-based experiment

The experiments under gravity conditions (see table 1) were carried out in a radio-frequency (rf)
plasma chamber. Additionally, thermophoresis was used to compensate for gravity (experimental
conditions are similar to that in [8, 47]). The microparticles (diameter 1.2 µm), which were
injected into the discharge chamber, normally self-organize in a non-uniform cloud around the
void (particle-free region). They are visualized with reflected light from a laser sheet, which
illuminates a vertical plane of ∼100 µm thickness through the chamber axis. The particle cloud
was recorded with a charge-coupled device camera at a rate of 15 Hz. Size, form and position
of the void changed as the discharge parameters were changed (for details, see [8, 47, 48]).
At higher pressures (∼90–100 Pa), the upper part of the cloud became unstable, and it lost its
particles which then fell down. The particles formed a weak beam (one or two particles of width)
impinging on the lower part of the cloud and—sliding through it! Image analysis allowed us to
study the interaction of these particle beams with the strongly coupled cloud.

This is illustrated in figure 2. We superimposed and colour-coded five consecutive images
to demonstrate the particle motion, but each of the shown, seven subsequent sub-image
compositions start one frame after the one before. Penetrating particles (falling from top to
bottom) and coupled particles (bottom) are of the same size. The source of penetrating particles
(not visible in figure 2) is on the top, at a height of ∼10–15 mm above the boundary of
the cloud.

The mean separation between the particles is 150 µm, the penetration depth is about 10
lattice distances, corresponding to ∼1.5 mm. The maximum velocity of the beam particles is
15 mm s⁻¹ (shortly before they penetrate) whilst the vibration (thermal) velocity of cloud particles
∼2–3 mm s⁻¹ is much less (approximately five to ten times smaller). Evidently, the beam particles
and the coupled particles shown in figure 2 move almost at one plane (at least no more than half
of the average interparticle distance displaced) because the illuminating laser sheet is narrower
than the interparticle distance.

Our estimates for the cloud parameters are listed in table 2. They were obtained by applying
the balance equations (1)–(3) in the same manner as above. It should be noted, however, that now
the situation is more complicated. In the parabolic flight experiments, all measurements were
performed in the bulk plasma and we could certainly suppose that all the screening is due to
the ions. In this case the particle cloud is located in the region between the void and the sheath,
then pre-sheath region, and hence ion flow effects may be more pronounced, possibly leading
to systematic errors in our determination. (However, this does not affect the observations of the
penetrating particles, which ‘sampled’ the same region as the cloud.) We determined the cloud
parameters in the following way: in/near the sheath, ions are accelerated toward the electrode,
their screening length effectively grows, and the plasma screening length has a trend to be defined
by electrons [49]–[53]. In other words, one can expect that the screening length varies with depth
but certainly its value lies in the range between the ion and electron Debye lengths. A simple
Figure 2. Anomalous transport of particles under gravity conditions. Shown are seven panels, each of them (size 1.20 × 4.54 mm²) consists of five superimposed consecutive images, which are colour-coded (colour varies from dark blue, light blue, green, yellow to red). Only particles within the ~100 µm laser sheet are visible. The beam particles are shown as long multicoloured ‘streaks’, whereas the strongly coupled particles in the cloud appear as round, mostly red dots. This shows that these particles are practically immobile during the total observation period of one-third of a second. Note especially that this applies also to cloud particles in the immediate vicinity of the penetrating particle ‘projectiles’. The mean interparticle separation is ~150 µm.

expression

\[ \lambda = [\lambda_i^{-2}(1 + M_T^2)^{-1} + \lambda_e^{-2}]^{-1/2}, \quad M_T = V/v_{Ti}, \]  

where \( M_T \) is the thermal Mach number, \( v_{Ti} \) the ion thermal velocity and \( V \) the ion flow velocity, was used in [53]. The flow velocity (deep inside the sheath), in principle, is limited only by the ion acoustic speed, \( c_{ia} = (T_e/m_i)^{1/2}. \) So that it might be expected that \( M_T < M < c_{ia}/v_{Ti} = (T_e/T_i)^{1/2}. \) For the parameters used (see table 1), we estimate \( M \sim 15 \) as the upper limit for the thermal Mach number. We see that even substantially below the ion acoustic limit, the \( M_T \)-factor might be important. For example, \( \lambda \sim a/2 = 75 \mu m \) if \( M_T \sim 5(\equiv \frac{1}{3} M). \) The latter is three times lower than the ion acoustic limit.

For comparison, both limiting cases (\( \lambda = \lambda_e \) or \( \lambda = \lambda_i \)) are listed in table 2. Both limits correspond to \( H \ll 1. \) This is because the electron concentration is extremely large while the particle size is extremely small in this experiment and consequently the number of free electrons is large (see table 2).

2.3. Recent experimental observations: microgravity condition

The penetration of charged microparticles into a dense coupled complex plasma cloud has recently been observed on board the ISS with the PKE–Nefedov laboratory, which contains
Figure 3. PKE–Nefedov: formation of a dense cloud of coupled particles at the boundary of the sheath region. The field of view is 28.16 × 21.45 mm².

a symmetrically driven rf plasma discharge specially designed for the investigation of complex plasmas (more details can be found in [54]).

Monodisperse particles of different sizes—3.4 and 6.8 µm in diameter, as well as a mixture of both sizes—can be injected into the plasma chamber, between the two electrodes. The microparticles are illuminated by a thin (∼150 µm) sheet of laser light perpendicular to the electrode system (produced by a laser diode and cylindrical optics).

Two CCD cameras (768 × 576 pixels, 25 Hz, 8 bit) provide different magnifications of the complex plasmas. The overview camera shows about a quarter of the field between the electrodes, 28.16 × 21.45 mm², while the high-resolution camera covers 8.53 × 6.50 mm².

It is possible to change the number of particles, the plasma conditions and the neutral gas pressure during one experiment. The experiment in question was performed in an argon plasma at a frequency of 13.56 MHz. Discharge parameters are listed in Table 1. The particles injected into the plasma chamber charge up quickly and begin to form a strongly coupled cloud (a plasma crystal) starting at the boundary of the sheath region (figure 3). Estimates of the cloud parameters are listed in Table 2.

Using an image processing software, the positions of particles were identified and traced from one frame to the next, and the particle velocities were calculated.

Penetrating particles (figure 4) were observed at the final stage of injection when the cloud became dense enough. Figure 4 shows the superposition of 19 consecutive images, colour-coded to visualize the particle trajectories. Usually the length of the trajectories is a few particle separations at most, but some trajectories are longer (e.g., the length of the trajectory ‘1’ in figure 4 is about 3.4 mm, corresponding to 20–30 interparticle separations). The velocities (averaged) of the particles with longer trajectories (marked in figure 4) are \( V_1 = 4.8 \pm 0.5 \text{ mm s}^{-1} \), \( V_2 = 3.6 \pm 0.4 \text{ mm s}^{-1} \) and \( V_3 = 2.5 \pm 0.3 \text{ mm s}^{-1} \). The estimated mean kinetic energy of particles in the crystalline part of the cloud (bottom part in figure 3) is of the order of room temperature. Hence, thermal velocity \( \sim 0.6 \text{ mm s}^{-1} \).
Figure 4. PKE–Nefedov: example of anomalous transport observations. We marked different areas: (I) disordered cloud of condensing (crystallizing) particles; (II) ordered cloud of (quasi-crystallized) particles; (III) sheath region; (IV) lower electrode. 1–3 are the trajectories of penetrating particles.

3. Theoretical model for particle penetration

3.1. Main assumptions

As discussed in the previous sections, the following behaviour has been observed:

(i) particles that come to rest on the surface of the void (in microgravity experiments);
(ii) particles that penetrate several lattice layers and then come to rest (in microgravity experiments);
(iii) particles that penetrate completely through a plasma crystal (in laboratory experiments);
(iv) clouds of particles that penetrate through each other forming long ‘strings’ or ‘lanes’ (in parabolic flight experiments).

We will not discuss the inter-cloud penetration here, except for one point, since we believe that the observed behaviour is due to a different (collective) physical process that needs a different treatment. This will be discussed elsewhere. Instead, we concentrate in the following on observations (i)–(iii).

Although the experimental conditions (gas pressure, plasma and particle parameters, discharge conditions, etc) are very different in these experiments, there is one common point: particles may ‘walk’ through several interparticle layers practically freely, without visible deceleration. Such a behaviour seems impossible at first sight, since the particle cloud is strongly coupled (sometimes even crystalline) and (Coulomb) collisional losses would quickly decelerate any impacting particle. Hence there have to be some physical processes, operating in such a way that moving test particles may slide through a lattice of similar particles, without apparently disturbing the ‘fabric’ of the penetrated material. Such processes have so far only been encountered in ‘quantum-mechanical tunnelling’. Therefore we propose to name this new process ‘classical tunnelling’.

Our theoretical model starts from the premise that the penetrating particles ‘elastically deform’ the cloud or lattice as they slide through it. Since the interparticle forces are mainly
electrostatic, it is natural to investigate this coupling—particularly since the process operates in the bulk plasma as well as the sheath—with or without gravity. The theoretical description therefore requires two components: a large-scale ‘body’ force and local charge variations.

When a particle passes through the layers of a plasma crystal, its charge changes (following the overlap of the Debye shielding spheres) and the local equilibrium is temporarily disturbed. There are two aspects: (i) a particle which is leaving a layer will experience an increasing repulsion and acceleration and then; (ii) after passing through the inter-layer midpoint (where the electrostatic forces from the neighbouring particles balance), it will be decelerated as it approaches the next (lower) layer. This deceleration will be a little bit weaker if the proposed mechanism of local charge variation operates as envisaged. If the maximal ‘in-layer’ decelerating force is not sufficient to stop the particle against the body force, e.g., gravity or ion drag [55], then the particle ‘walks through a wall’. This explains qualitatively the situation in ground-based experiments as well as in microgravity.

3.2. Geometrical model

Let us consider a particle that penetrates through an ordered 3D lattice. We suppose that:

(i) there is a triangular multilayer structure consisting of three particles in an elementary cell (figure 5), and that initially all the charges are equal;
(ii) the screening length does not change, $\lambda = \text{const}$; particles are two Debye (screening) lengths apart $a = 2\lambda = \text{const}$;
(iii) the recharging process is fast compared to other processes and flight time;
(iv) a new particle (moving downward in figure 5) takes electrons away from the crystal particles in proportion to the overlap of the Debye spheres; the sum of all the charges remains constant, $n_i \approx \text{const}$, $n_e < \sim Zn_d$.

From these assumptions, we can investigate how the charge changes during the penetration. When the moving particle has reached the upper layer (see figure 5), four particles have to share the charge reservoir of three (formerly). We may assume $Q = Q_U$. Then the local charge balance yields:

$$Q + 3Q_U = 4Q_U = 3Q_0,$$

and $Q(z = h) = \frac{3}{4}Q_0 = Q_U$ while $Q_L$ is unaffected, $Q_L = Q_0$. Here, $Q_0$ is the initial charge of the unperturbed lattice, $Q_U$ the charge in the upper layer and $Q_L$ the charge in lower layer. When the particle is in the lower lattice plane, $Q_U = Q_0$, and we have

$$Q(z = 0) = \frac{3}{4}Q_0 = Q_L.$$  

(6)

Since the charge variation of particles in a given cell (from one plane to the next) follows the overlap of the Debye spheres, then for $h = 2\lambda$, this should be approximately a linear relationship:

$$Q_U = Q_0(1 - \frac{1}{4}z/h), \quad Q_L = Q_0[1 - \frac{1}{4}(h - z)/h].$$

(7)

In order to balance the charges of all the particles, we get

$$Q = 6Q_0 - 3Q_U - 3Q_L = \frac{3}{4}Q_0.$$  

(8)

The moving particle charge is independent of position, $z$. This is a consequence of the linearity assumption.
Figure 5. Geometrical model for charge variation of moving and crystal particles. Here, $a$ is the in-layer particle separation, $h$ is the inter-layer distance, $Q$ is the charge of the moving particle, $Q_U$ is the charge of the nearest-neighbour particles in the upper layer and $Q_L$ is the charge of the nearest-neighbour particles in the lower layer.

3.3. Force on moving particle

The cell particles will, of course, be disturbed by an additional incoming particle. However, the observations demonstrate that neighbouring particles are hardly perturbed at all. This means that we may treat the layer geometry as approximately fixed (imperturbable).

The force on a moving particle (in the nearest-neighbour approximation) is:

$$F = F_{\text{ext}} + F_{\text{int}},$$

$$F_{\text{int}} = F_L - F_U,$$

$$F_{\text{ext}} = QE - mg + F_{\text{id}} + F_{\text{th}},$$

$$F_L = Q \sum Q_L \frac{1 + r_L/\lambda}{r_L^2} \exp(-r_L/\lambda),$$

$$F_U = Q \sum Q_U \frac{1 + r_U/\lambda}{r_U^2} \exp(-r_U/\lambda).$$

(9)

Here we consider gravity, ion drag ($F_{\text{id}}$), thermophoresis ($F_{\text{th}}$) and the external (discharge) electric field as possible sources of external forces; indices L (lower) and U (upper) correspond to particles in the lower and upper layers; the sum extends over all the (3) nearest neighbours in each layer of the considered cell (see figure 5); $Q$, $Q_L$, and $Q_U$ are the particle charges defined by relationship equations (7) and (8). Note that equations (9) are valid only in the vicinity of the midpoint. Just above the lower layer (or just behind the upper one), the influence of the next-neighbour layer is not negligible and has to be taken into account (see later).
Let us assume that the penetrating particle moves along the symmetry line as shown in figure 5. We then get:

\[ F_{\text{int}} = \left( \frac{Q_0}{\lambda} \right)^2 [\psi(z) - \psi(h - z)], \]

\[ \psi(z) = \frac{3}{4} \left( 1 - \frac{1}{4} \frac{h - z}{z} \right) \frac{z}{r(z)} \left( \frac{1 + r(z)/\lambda}{r(z)/\lambda} \right)^2 \exp(-r(z)/\lambda), \]

\[ r(z) = \sqrt{\frac{1}{2}a^2 + z^2}. \]  

(10)

At the midpoint \((z = h/2)\), by symmetry, we have \(F_{\text{int}} \equiv 0\). Away from the midpoint (but still in the vicinity, where \(|z - h/2|\) is small), the force changes linearly:

\[ F_{\text{int}} \approx -\kappa \left( z - \frac{h}{2} \right), \quad \kappa = -\frac{3}{2} \frac{Q_0^2}{\lambda^2} \psi' \left( \frac{h}{2} \right). \]

Here, \(\kappa\) is the proportionality coefficient (string constant) and is equal to (remembering that the parameters are \(h/\lambda = a/\lambda = 2\)):

\[ \kappa \approx 0.31 h^{-1} \frac{Q_0^2}{\lambda^2}. \]

(12)

The string constant equation (12) is positive and, hence, the midpoint position of a particle is stable. At the midpoint between the lattice planes, the electrostatic forces from the six neighbouring particles in the cell balance. Then the driving force \(F\) in equation (9) becomes

\[ F \left( z = \frac{h}{2} \right) = F_{\text{ext}} = \frac{3}{4} Q_0 E - mg + F_{\text{id}} + F_{\text{th}}. \]

(13)

This is in general not sufficient to support the particle as we will show later. This means that a penetrating particle will not come to rest at an interstitial site, and ‘doping’ of plasma crystals is somewhat complicated. Below the midpoint \((z < h/2)\), the net force from the neighbours is directed upwards, above it is the force directed downwards. Also, \(F_{\text{int}}\) approaches zero at \(z = 0\) and \(h\) (in the lattice planes \(F_{\text{int}} = 0\) identically, if all the other layers are accurately accounted for). If we take into account two more layers (an additional two extra layers introduce a minor correction, see figure 6), we can estimate the maximal decelerating force,

\[ |F_{\text{int}}|_{\text{max}} \approx 0.046 \frac{Q_0^2}{\lambda^2}, \quad z_{\text{max}} \approx \frac{1}{4} h, \frac{3}{4} h. \]

(14)

If this maximum force is not sufficient to stop the particle against the combined effect of the body forces, then we have the case of ‘classical tunnelling’—the particle may ‘walk through a wall’. The corresponding criterion is e.g. for a gravity-dominated situation:

\[ \frac{1}{4} Q_0 E \approx \frac{1}{4} mg > 0.046 \frac{Q_0^2}{\lambda^2}. \]

(15)

In reality the situation is more complicated since other forces, e.g., thermophoretic and ion drag forces, may play a role as mentioned before. The thermophoretic force can compensate for (or enhance) gravity, depending on the sign of the temperature gradient. The ion drag force, on the other hand, can compensate for the external electrostatic force. Unfortunately, it is still an unsolved question [37], how to calculate the ion drag force in a collisional complex plasma.
Figure 6. Internal force distribution inside the cell shown in figure 5 calculated for two (dashed line), four (solid line) and six neighbouring layers (dotted line). $F_\ast = (Q_0/\lambda)^2$ is the force normalizing factor, where $h/\lambda = a/\lambda = 2$.

3.4. Numerical simulation

We performed numerical simulations of particle penetration into a 2D double-chain geometry. Physically, there is no principal difference between the 2D and the 3D cases but the calculations are much simpler for two dimensions. The main difference is only in the magnitude of the charge variation. In the 3D case, the elementary cell contains three particles plus one incoming particle, and the total charge redistributes as $3:4$. In the 2D geometry, the elementary cell contains two particles plus one incoming particle, and charge distributes in proportion of $2:3$. Accordingly, in two dimensions the particle charge is $Q^{(2D)} = \frac{2}{3} Q_0$ whereas in three dimensions it is $Q^{(3D)} = \frac{3}{4} Q_0$. Since the difference is small,

$$Q^{(3D)} - Q^{(2D)} = (\frac{3}{4} - \frac{2}{3}) Q_0 = \frac{1}{12} Q_0 \ll Q_0,$$

the geometric factor is not important.

To simulate charge variation, each particle was assigned by a ‘specific area’ (in the 2D simulations a square of size $\lambda \times \lambda$). The charge variation was chosen to be proportional to the overlap of the specific areas exactly in the same manner as it was proposed above, without the linearity hypothesis, of course. All parameters (the interparticle separation, the number of chain particles, etc) could be varied and all particles interacted via Yukawa (screened Coulomb) forces.

Simulations were carried out for particles moving along the symmetry line and with a small displacement (see figure 7). It was found that the ‘penetration barrier’ always became lower if the charge exchange mechanism was ‘switched on’ and the charges were allowed to vary. As a test, in the examples shown, penetration was impossible if the charges were kept constant.

Generally, we observed that during penetration, the particle charge and energy varied irregularly, but the amplitude of the oscillations remained approximately constant (figures 7(b) and (c)) so that the mean value of the particle energy also remained constant. This is in good
Figure 7. Particle penetration through a 2D double-chain (numerical simulation). (a) Positions of double-chain particles (1) and the trajectories of three penetrating particles: along the symmetry line (2) and initially displaced at a distance $\pm 0.25\lambda$ (3, 4). Geometry parameters: $a/\lambda = 2, h/\lambda = 2.2$. Initial velocities: $V_x = 0, V_z = -1.2V_0, V_0 = \lambda/T_Q$: $T_Q = (m\lambda^3/Z^3e^2)^{1/2}$ is the ‘Coulomb’ timescale. (b) Kinetic energy variation, $K/K_0$. (c) Normalized charge variation.

agreement with the observations that the penetrating particles slide through the crystal almost as if it did not exist—with very little or no slowing down. On a faster timescale, the particle decelerates and accelerates due to non-potential forces [37, 38] as it crosses the lattice planes. Normally, the particle kinetic energy inside the double-chain was a factor 2–3 lower than the initial energy. Since the mean energy of the penetrating particles is approximately conserved, it matters little how long the double chain is, again in good agreement with our measurements.

3.5. Pressure balance

In section 3.2, it was implicitly assumed that not only the number of particles in a cell but also the number of ion and electrons in a cell are essentially fixed. This needs to be discussed more, and we do this here using pressure balance. The discharge volume is large compared to the volume of each individual cell. Therefore, it is natural to suppose that the rf power is quasi-uniformly distributed across each individual cell considered here. Hence, the number of ions in a cell,
being controlled by the applied external rf power, should remain roughly constant independent of the appearance of an additional grain. (In other words, the ion recombination dominates the volumetric process.) Kinetic temperatures of plasma species should then be constant, as well. The cell size is controlled by the averaged pressure balance:

\[ p_e + p_i + p_E = \text{const}, \]

where \( p_{e,i} = n_{e,i}T_{e,i} \) are the pressures of electrons and ions, and \( p_E \) is ‘the electric field pressure’. The latter is proportional to the mean density of the electrostatic energy:

\[ p_E = \alpha \frac{1}{V} \int_{\text{cell}} \frac{E^2}{8\pi} dV, \]

Here, \( V \) is the cell volume and \( \alpha \) is the proportionality coefficient. In one dimension, for instance, it is well known that \( \alpha = -1 \), and, hence, \( p_E \) is negative (strictly speaking, it is a tension, not a pressure). In the multi-dimensional case, this might not be the case. In general, the electrostatic field can be characterized by a pressure tensor defined usually as

\[ p_{i,k} = -\frac{E^2}{8\pi} (\delta_{i,k} - 2e_i e_k), \quad e_{i,k} = \frac{E_{i,k}}{E}, \]

where \( \delta_{i,k} \) is the unit matrix. For a simplifying consideration, we will suppose that \( \alpha \) has a roughly constant value, \( |\alpha| \approx \text{const} \sim 1 \).

The cell electrostatic energy approximately equals the energy of charged grains (we ignore other possible contributions):

\[ \left\langle \frac{E^2}{8\pi} \right\rangle \approx \sum_1^N \frac{Z^2 e^2}{2RV}, \]

where \( N \) is the number of grains, \( Z \) is the grain charge and \( R \) is the grain radius. Substituting relationship equations (18) and (19) into equation (17), and combining the result with the charge balance equation, yields:

\[ n_i T_i + n_e T_e + \frac{\alpha}{V} \sum_1^N \frac{Z^2 e^2}{2R} = \text{const}, \quad n_i V - n_e V - \sum_1^n Z = 0. \]

These equations control the balance of free and bounded electrons inside the cell. If all the particles are of the same size, equations (20) can be simplified:

\[ n_i T_i + n_e T_e + \frac{N \alpha Z^2 e^2}{2R} = \text{const}, \quad n_i V - n_e V - NZ = 0. \]

Let us now add one more grain (of the same type) to the cell containing normally \( N_0 = 3 \) identical grains. Since \( n_i = \text{const} \), \( T_e = \text{const} \), the balance equations (21) yield the relationships:

\[ (N_e - ZN)_0 = (N_e - ZN)_1, \quad \left( N_e + \frac{\alpha Z^2 e^2}{2RT_e} N \right)_0 = \left( N_e + \frac{\alpha Z^2 e^2}{2RT_e} N \right)_1. \]
Here, the indices 0 and 1 mark the situations before and after variation of the grain number; \( N_e = n_e V \) is the number of free electrons. It is convenient to introduce the following new parameters:

\[
\delta N_e = N_{e1} - N_{e0}, \quad x = \frac{\delta Z}{Z} = \frac{Z_1 - Z_0}{Z_0}, \quad \eta = \frac{Z_0 e^2}{2RT_e}.
\]

(23)

Then the equations (22) can be rewritten in a very simple form:

\[
\frac{\delta N_e}{Z_0} = 1 + 4x, \quad x + \frac{1}{4} = -\alpha \eta (x^2 + 2x + \frac{1}{4}).
\]

(24)

Equations (24) can be solved in general but the solution is extremely simple if the parameter \( \eta \) is small, \( \eta < 1 \), then,

\[
\delta N_e \approx 0, \quad x \approx -\frac{1}{4}, \quad Z_1 \approx \frac{3}{4} Z_0.
\]

(25)

This yields exactly what the geometrical model predicts (see equation (5)): the charge redistributes itself mainly between the particles and the number of free electrons in a cell does not change significantly. It is not difficult to explain: a small value of \( \eta \) corresponds, by definition, to a high electron temperature \( T_e \). In case of higher temperature, ‘the electron fluid’ tends to be incompressible: small deviation in \( N_e \) would affect a too large pressure response. This is exactly what equation (25) shows. Assuming \( \eta \) is small but finite, one can calculate the next order corrections to equation (25) by successive approximations, and verify that the geometrical model is applicable to well within an accuracy of 10–20% for the experiments described in section 2.

3.6. Particles of different sizes

The geometrical model is easy to generalize for particles which are not of the same size. It cannot be done directly in the frames of model equation (21) because particles of different sizes acquire different charge under similar conditions. It means that the adapted model has to contain some relationship accounting for the flux balance. Let us assume that the charges which are acquired by different particles are in some proportion \( Z = kz \), where \( k > 1 \) is the proportionality coefficient, \( Z \) (conditionally) is the charge of the larger grain and \( z \) is the charge of the smaller grain. Let us assume further that a new smaller grain is introduced into the cell. Then the new generalized balances equation (22) are:

\[
(N_e - 3Z)_0 = (N_e - 3Z)_1 - z_1, \quad \left( N_e + 3\frac{\alpha Z^2 e^2}{2RT_e} \right)_0 = \left( N_e + 3\frac{\alpha Z_1^2 e^2}{2RT_e} \right)_1 + \frac{\alpha z_1^2 e^2}{2RT_e}, \quad Z_1 = kz_1,
\]

(26)

where \( z_1 \) is the charge of the new little neighbour and \( Z_1 \) is the charge of each larger grain. (Note that equations (26) are identical to equations (22) if \( k = 1 \)). Assuming the parameter \( \eta \) in equation (23) is small, one can find:

\[
Z_1 = Z_0 \frac{3k}{3k + 1}, \quad z_1 = Z_0 \frac{3}{3k + 1}.
\]

(27)

Here, \( Z_0 \) is the unperturbed charge of the larger particles in the lattice.

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A two-size mixture of particles \((R_1/R_2 \approx 2)\) has been used in the parabolic flight experiments (see tables 1 and 2). For these experimental conditions, we estimate \(k^{(DML)} \approx 2.67\) (note: \(k^{(OML)} \approx 2.4\)), hence, \(Z_1/Z_0 \approx 0.9, z_1/Z_0 \approx 0.3\). This corresponds to \(\sim 10\%\) variation of charge (with respect to that for a single grain, either smaller or larger).

Formally equations (27) are still valid if a larger grain is introduced into a cell of smaller grains (the parameter \(k\) should then be replaced formally by \(1/k\)):

\[
Z_1 = z_0 \frac{3}{3 + k}, \quad Z_1 = z_0 \frac{3k}{3 + k}.
\]

(28)

The corresponding charge variation is then about \(50\%\): \(z_1/z_0 \approx 0.5\) for the smaller grains and \(Z_1/z_0 \approx 1.4\) for the larger grain. The huge difference of the acquired charges for the two cases (smaller grain slides through a cloud of larger particles or a larger grain through smaller particles) explains qualitatively some of the observations shown in figure 1. There is nearly no damage if smaller particles penetrate the cloud of larger particles (figures 1(a) and (b)), whereas a clearly visible structural damage occurs if larger particles penetrate through the cloud of smaller particles (figures 1(e) and (f)).

4. Discussion and conclusion

\ldots sometimes the shorter the walk the better the walk \ldots

Our observations have shown that strongly coupled complex plasmas can give rise to various anomalous transport properties, which are believed to be due to the non-Hamiltonian nature of the dust–dust interactions (in particular the charge variation). We have focused our attention on a new process, which we have termed ‘classical tunnelling’ (classical tunnel effect), and which has been observed at the kinetic level for the first time under microgravity conditions, in parabolic flight experiments and in laboratory experiments for a wide range of complex plasma parameters.

Typical ‘tunnelling velocities’ were \(5\text{–}15\, \text{mm s}^{-1}\), which is 10 times greater than the thermal velocity of particle vibrations. The penetration depth depends on the individual conditions but it was normally not less than \(5\text{–}10\) interparticle separations before the particles relaxed, in many cases the penetration continued through the whole cloud.

It has been proposed (and confirmed in numerical simulations) that a simple possible explanation for this ‘tunnel effect’ is a geometrical model for charge variation. The main assumption of the model is that the dust particles compete among each other for the charge in the plasma. We demonstrated that this model qualitatively is applicable both in cases of large \((H \sim 1)\) or small \((H \ll 1)\) values of the Havnes parameter if the effective particle charge is small.

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