Application of the zero thickness interface element approach to the upper bound limit analysis of shallow tunnel excavation

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Abstract. The aim of this study is to apply the zero thickness element approach to investigate the upper bound value of limit analysis with the different failure mechanism of a shallow tunnel excavation in soft clay. This interface is modelled as a contact element with zero thickness and with the Coulomb's friction behaviour. The procedure of numerical analysis adopted the zero thickness element particularly developed and installed into the finite element program in the laboratory. A series of finite element analysis is used to investigate the solutions of stability and simulate the soil progressive failure mechanism due to a shallow tunnel excavation. Four geometry of failure mechanism are taken into account the effects of overburden depth and critical collapse ratio. According to the results obtained, the upper bound values between the limit equilibrium analysis and the numerical calculation are theoretically coincidence, and also the progressive failure mechanism of cohesive soil can be practically predicted. The numerical analyses show that the predicted stability ratio is in reasonable agreement with the analytical solutions in the limit analysis.

1. Introduction
A basic engineering decision to be made in designing a tunnel in soft clay is whether or not the tunnel can be excavated without internal support. The safety of constructing a shallow heading in soft clay can be assessed in terms of the fluid support pressure which may be required to maintain stability. This can be estimated by means of the lower and upper bound theorems of plasticity. In this paper, only upper bound stability solutions are derived for collapse under undrained conditions. The results obtained of limit analysis are compared by finite element analysis that using the zero thickness interface elements is particularly investigated.

Engineering literature contains a variety of interface element formulations; however, they can generally be classified as either a stiffness approach (e.g. directionally stiff elements) or a constraint approach (e.g. Lagrange multipliers). A hybrid of these two approaches has also been developed. Several types of finite elements have been proposed for the modeling of joints and interfaces. They can be described and classified in the following categories.

A common interface element is proposed by Goodman et al [1] with relative displacements being the nodal unknowns. The thickness is assumed to be zero and to avoid overlapping and penetrating a very high normal stiffness of the interface. An element considering relative motions within the interface element as independent degree of freedom is developed [2]. A similar element, based on an isoparametric formulation has been proposed for two- or three-dimensional applications [3,4]. The linkage elements in which only the connections between opposite nodes which are connected by discrete springs is proposed by Frank [5]. Although the desired effect may be obtained, data preparation, mesh
generation and interpretation of results turn out to be very involved. Desai et al [6] proposed a thin-layer element based on an isoparametric formulation. The main features are the introduction of an independent shear modulus in the elastic range and the possibility of taking into account stick, slip, debonding and rebonding modes. Although some applications have been reported, the performance of this element is not fully investigated. The application of zero thickness elements to the analyses of a retaining wall and the numerical instability caused by ill-conditioning of the stiffness matrix and high stress gradients was also investigated by Day and Potts [7]. Recently, the developments on zero-thickness interface elements for coupled hydro-mechanics problems are described [8].

The approach used in this paper used the idea of the zero thickness elements described by Carol and Alonso [3] and Beer [4] for the analysis of rock joints, the main advantage being an easier implementation into existing finite element codes. The influence of interface stiffness, stability ratio, the failure mechanism and the deformed soil surface profile is studied. The numerically calculated stability ratio is compared with approximate analytically derived values used in limit analysis. Solutions to the stability problem of a shallow tunnel in cohesive material applying the proposed elements are briefly presented.

2. Interface element formulation

2.1. Geometry, stiffness and constitutive law

For the definition of the interface elements, an isoparametric formulation is chosen that it can be easily used in conjunction with the continuum elements. The interface element with four or six nodes is fully compatible with four and eight-node isoparametric 2D soil element. The use of an element based on an isoparametric formulation has the advantage of being easily implemented into existing finite element codes. Furthermore, the data preparation and the mesh generation can be done in a routine manner.

To obtain the basic element stiffness matrices, the interface stress is characterized by the normal and shear stresses is assumed. The normal stress $\sigma$ and the shear stress $\tau$ are related by the constitutive law to the normal and tangential interface element strain, $\varepsilon$ and $\gamma$ as shown the following:

$$
\begin{bmatrix}
\sigma \\
\tau
\end{bmatrix} = [D]egin{bmatrix}
\varepsilon \\
\gamma
\end{bmatrix}
$$

(1)

where $[D]$ is the material stiffness matrix, and represented by

$$
[D] = \begin{bmatrix}
K_n & 0 \\
0 & K_s
\end{bmatrix}
$$

(2)

where $K_n$ and $K_s$ are the shear and normal stiffness of elasticity, respectively (in units of F/L3).

The linear elastic-perfectly-plastic model using a Mohr-Coulomb failure criterion as the yield surface is investigated in the analyses. The formulation of the constitutive behavior was based on plasticity theory. The Mohr-Coulomb failure criterion is defined by the yield function $f$, and represented as

$$
f(\sigma, \tau) = \tau - \sigma \tan(\phi) - c = 0
$$

(3)

where $\phi$ is the internal friction angle, $c$ is the cohesion.

2.2. Numerical stability

In the sense of simple formulation of the failure criterion, when the shear stress reaches and satisfies the failure criterion, the value of shear stiffness is set to zero, and the normal stiffness remains unchanged. The tensile stress in the interface element is generally not permitted. If tensile stress occurs, both the normal and shear stiffness are set to zero, and the tensile stress is redistributed by the solution algorithm. If the interface element moves such that the maximum normal tensile strength is exceed $c/\tan(\phi)$, the interface is allowed to subsequently open and close, and the residual tensile stress is redistributed. When
the interface is open the normal stress remains equal to \( c/\tan \phi \) and shear stress remains equal to zero. This is essentially the same as setting both normal and shear stiffness to zero. The amount of opening of the interface is recorded. When the interface recluses and reforms contact, the constitutive model again defines the interface behaviors.

Although the zero thickness interface elements have been successfully used by many others without reporting particular problems, but it also necessary to be taken into account that the Ill-conditioning can be reduced by careful selection of the size of the 2D elements adjacent to the interface, and the problem of steep stress gradients is entirely one of inadequate mesh design. For the convergence of solutions, an accelerated modified Newton-Raphson scheme with a sub-stepping stress point algorithm was employed to solve the non-linear finite element equations in this study.

3. Limit analysis – upper bound theorem

![Figure 1](image-url)

**Figure 1.** Schematic illustration of four upper bound mechanisms A, B, C and D.

As the working face of tunnel is advanced, a means of supporting the ground close the face may be needed. In soft ground with immediate support provided by such means as shield, the use of compress air or clay slurry under pressure may be required; otherwise collapse may occur as a result of gross plastic deformation of the soil, possibly accompanied by flooding. Under these circumstances, it is possible to idealize the process of tunneling as shown in figure 1. in which a circular un-lining tunnel of diameter \( D \), which there acts a uniform fluid pressure \( \sigma_f \), is shown being constructed with a depth of cover \( C \). The ground has a unit weight \( \gamma \) and a uniform pressure \( \sigma_s \) acts on the soil surface. The tunnel pressure is necessary to maintain the stability of the heading for different values of the parameters that have been defined \( (C, D, \sigma_s) \) and the strength of ground is investigated. The collapse of the tunnel heading will usually be a sudden event and hence it is appropriate to characterize the strength of the ground by its undrained shear strength \( c_u \). In following analysis, it is assumed that \( c_u \) is constant with
depth.

In the point of view of stability of un-lining tunnel, Broms and Bennermark [9] defined a stability ratio \( N \) equal to the difference between the total overburden stress in the ground at the axis of the tunnel and the tunnel pressure divided by the undrained shear strength \( c_u \).

\[
N = \left[ \sigma_s - \sigma_T + \gamma (C + D / 2) \right] / c_u
\]  

(4)

They concluded that if \( N \) is less than 6 then the opening will be stable. The upper bound theorem states that if a work calculation is performed for a kinematically admissible collapse mechanism then the loads thus deduced will be higher than (or equal to) those for collapse.

For a complete solution the requirements of equilibrium, compatibility, material behavior and the boundary conditions, both load and displacement, must all be satisfied. All four of the above requirements are satisfied only for a limit analysis solution in which both upper and lower bound calculations lead to the same results. Stability solutions will be obtained using the limit theorems of plasticity.

4. Applications – The plane strain circular tunnel

An upper bound to the true collapse load can be found by selecting any kinematically admissible collapse mechanism and performing an appropriate work rate calculation. It may be noted that the solution will lead to an unsafe or low value for the tunnel pressure. The accuracy of an upper bound calculation will depend, in part, on the closeness of the assumed mechanism to the real one, and it is in the selection of a collapse mechanism that the models tests have proved to be of value. In practice, it is important to establish the minimum tunnel pressure necessary for stability.

Four upper bound mechanisms are shown in figure 1. The mechanism A and B are simple ‘roof’ and ‘roof and sides’ mechanisms each containing one variable dimension (or angle \( \theta \)). The mechanism C has four variable angles (\( \theta, 2\alpha, \beta, \delta \)) in its specification and includes mechanism A and B as the special case. The mechanism D is a ‘roof, sides and bottom’ mechanism with three variable angles (\( \theta, \alpha, 2\beta \)). The work described in this paper has been limited to plane strain circular tunnel. The soil is idealized as a linear elastic material. The interface is assumed as an elastic-perfectly-plastic material with a cohesion \( c_u \). In addition, the input data of calculation is represented in table 1. The boundary conditions are constrained by the rollers in both sides and bottom of meshes proposed. The procedure for determining the critical collapse load is to derive an expression for (\( \sigma_s - \sigma_T \)) (involving the variable dimension or angle) and then to minimize the value of (\( \sigma_s - \sigma_T \)) with respect to the variable dimension or angle.

| Parameter | Value | Parameter | Value |
|-----------|-------|-----------|-------|
| Cohesive soil and interface | Failure mechanism | | |
| Elastic modulus, \( E \) (MPa) | 300.0 | Mechanism A (degree) | \( \theta = 30 \sim 60 \) |
| Poisson's ratio, \( v \) | 0.25 | Dimension (B, C, D) (m) | 20\( \sin \theta \), 20, 20 |
| Assumed unit weight, \( \gamma \) (MPa/m) | 0.0 | Mechanism B (degree) | \( \theta = 30 \sim 60 \) |
| Undrained cohesion, \( c_u \) (MPa) | 0.1 | Dimension (B, C, D) (m) | 40, 20, 20 |
| Friction angle, \( \phi \) (°) | 0.0 | Mechanism C (degree) | \( 2\alpha = \beta = \delta = \theta = 30 \sim 60 \) |
| Interface elements | | Dimension (B, C, D) (m) | 35, 20, 20 |
| Elastic modulus, \( E \) (MPa) | 300.0 | Mechanism D (degree) | \( \alpha = 2\beta = \theta = 30 \sim 60 \) |
| Poisson's ratio, \( v \) | 0.0 | Dimension (B, C, D) (m) | 45, 20, 20 |

After finite element numerical modeling, the deformed meshes of four upper bound mechanisms are shown in figure 2. The reason that \( \sigma_s \) and \( \sigma_T \) appear only in the form (\( \sigma_s - \sigma_T \)) in the upper bound calculations is that since a kinematically permissible mechanism for cohesive material involves no volume change then the decrease in area of the tunnel must equal the area of ground loss at the surface. Hence the work done by the pressures in the work calculation will be (\( \sigma_s - \sigma_T \)) multiplied by that area.
The sliding block mechanisms also suggest that a movement of soil inwards near the tunnel is accompanied by an (approximately) equal settlement of the ground surface.

According to the results obtained by the graphic expressions, Figure 3 shows that the failure mechanism is indicated by the vectors of incremental displacement in the last increment of the analysis. The failure mechanism for mechanism A to D in which the interface has been fully mobilized is also observed. All soil movement occurs within a different shape failure.
Figure 3. Displacement fields of four upper bound mechanisms by the finite element analysis.

Figure 4. Parametric studies of four upper bound mechanisms A, B, C and D.

The parametric studies of variable angles of each mechanism are particularly examined. Figure 4 show that in the neighborhood of the optimum upper bound, changes in the variable angles lead to small changes in the collapse load. For obtaining such optimum upper bound, the variable angle $\theta$ is approximately equal to 60 degrees for the mechanism A and B. On the other hand, for the mechanism C and D, the optimum value of $\theta$ is about 50 degrees. The results of a numerical optimization to discover the critical mechanisms are only for $\gamma D/c_u=0$ as a criterion for deciding the tunnel pressure should be a
safe procedure. This is because it always corresponds to a lower value of $N$, or a higher value of $\sigma_T$.

Examining the results obtained, the comparison between analytical solution and numerical calculation is investigated. Figure 5 clearly indicate the mechanism C is more critical for low values of $CID$ and is superseded by mechanism D for high values of $CID$. It also shows that the ability of the simple upper bound solutions to predict the stability of shallow tunnel in soft ground with satisfying accuracy. The numerical results presented in this paper can be used for the calculation of the undrained stability of tunnels when $CID<3$.

![Figure 5. Comparison of results obtained between analytical solution and numerical calculation for a circular tunnel ($\gamma D/c_u = 0$).](image)

There is no much difference between mechanisms B, C and D for practical purposes (i.e. in estimated collapse load) although the mechanisms of deformation very different. In general, it seems that a mechanism with one variable will yield an adequate upper bound (i.e. close to a good lower bound) providing an appropriate pattern of collapse is chosen. The stability solutions of analytical limit analysis methods are compared with numerical results which indicate the zero thickness interface elements provided a useful means to model cohesive material in finite element analysis of a shallow tunnel. The simple elastoplastic Mohr-Coulomb constitutive model appears to be satisfactory for the modeling of interface behavior in this type of analysis.

5. Conclusion
The numerical application of zero thickness interface elements considered in the finite element program with a simple elastoplastic Mohr-Coulomb failure criterion has predicted the upper bound of stability ratio for various types of a shallow tunnel failure mechanism in reasonable agreement with analytical solutions. These analyses have provided numerical confirmation of the analytical solutions such as Davis et al [10] that used the limit theorems of plasticity to estimate the tunnel pressures necessary for the support of shallow tunnels and underground openings in soft ground.

The results obtained in the analysis of the failure modes of a shallow tunnel modeled with zero thickness interface elements assuming associated plastic flow are presented. In addition, the upper bound of stability ratio is independent of the elastic stiffness.
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