BARYOGENESIS AND DAMPING IN NONMINIMAL ELECTROWEAK MODELS

A. Kundu and S. Mallik
Saha Institute of Nuclear Physics
1/AF, Bidhannagar, Calcutta 700064, India

Abstract

We study the effect of damping on the generation of baryon asymmetry of the Universe in the standard model of the electroweak theory with simple extensions of the Higgs sector. The propagation of quarks of masses up to about 5 GeV are considered, taking into account their markedly different dispersion relations due to interaction with the hot electroweak plasma. It is argued that the contribution of the \( b \) quark can be comparable to that of the \( t \) quark calculated earlier.

PACS number(s): 98.80 Cq, 12.15 Ji
1 Introduction

The discovery of baryon number violation in the standard electroweak theory [1] has led to the possibility of constructing a scenario for baryogenesis at the electroweak scale in the early universe. This violation, although exceedingly small at the present epoch, can be unsuppressed at the then prevailing high temperature [3]. The other two Sakharov conditions [3] could also be met: C and CP violation originate from the interaction of quarks with the Higgs fields. Also, if the electroweak phase transition is of first order, the motion of the walls of bubbles of broken phase within the unbroken medium would produce the required departure from thermal equilibrium.

A natural and generic mechanism for electroweak baryogenesis was proposed by Cohen, Kaplan and Nelson (CKN) [4, 5]. Here a CP-odd charge (like the lepton number) is separated by the reflection and transmission of fermions by the bubble wall. It is then converted into an asymmetry in baryon number by the baryon number violating processes occurring outside the bubble. As a straightforward calculation of the observed baryon asymmetry requires too big a CP violation to be available in the minimal standard model (MSM), these authors chose to work with simple extensions of the minimal version. The required CP violation is achieved by the complex space dependent fermionic mass function within the bubble wall, which arise quite generally in such models [6].

Shaposhnikov [7] studied a similar mechanism of direct separation of baryonic number by the bubble wall, taking into account the effect of the hot ambient plasma on the quark propagation. As a result of interaction with the quanta of the medium, the quarks acquire temperature dependent effective masses and satisfy altered dispersion relations. The corresponding fermionic modes or quasiparticles have very different reflection and transmission coefficients in different regions of momenta. The details are worked out in Farrar and Shaposhnikov (FS) [8]. It is found that quark momenta relevant for baryogenesis are much lower than the temperature of phase transition. As a consequence, the CP violation of the MSM model suffices to generate the observed baryon asymmetry of the Universe.

Recently Gavela et al [9] and Huet and Sather [10] object to the FS analysis, pointing out that they do not include the imaginary part of the effective quark mass giving rise to damping of amplitudes. Including the damping they find the reflection coefficients and hence the baryon asymmetry to reduce to a negligible value.

This objection is not agreed upon by FS [11], however. According to them, although the leading quark interaction with the medium can be simulated by the real effective mass with an altered dispersion relation, higher order interaction giving rise to the imaginary part in the mass and hence damping cannot be treated in this way, as it gives non-unitary description. Instead, such dissipative processes must be treated in a many body context.

Pending a proper discussion of the many body problem, it is useful to study the dissipative quantum mechanical problem itself in connection with the baryon asymmetry. Here we consider simple extensions of MSM following the work of CKN. However, while they considered the t quark propagation, we consider the propagation of lighter quarks, for which the finite temperature corrections are important. The reflection and transmission
amplitudes are obtained in an iterative series in powers of the mass parameters. The results for reflection and transmission currents confirm the loss of unitarity for finite wall thickness. However, as the wall thickness tends to zero, unitarity is restored, but the fermion-antifermion asymmetry is lost in the model. We reintroduce the asymmetry by assuming a very large imaginary part of the mass function within the wall and carry out a representative calculation of the baryon asymmetry.

In sec. 2 we review the propagation properties of quark excitations in the electroweak plasma taking damping into account. In sec. 3 we solve the Dirac equation within the bubble wall where the mass function is space dependent. It provides the matching condition to be used in sec. 4 to find the reflection and transmission amplitudes. In sec. 5 we calculate the baryon asymmetry in the thin wall approximation but retaining the imaginary part of the mass function. Finally in sec. 6 we conclude with a discussion of the results obtained.

2 Quark propagation in hot plasma

In this section we collect the properties of light quark excitations as they propagate through the electroweak plasma at about the phase transition temperature. The most important effect of the medium on the quark is obtained by calculating the quark propagator at finite temperature \[8, 12\]. There arises a temperature dependent, chirally invariant complex mass with a modified dispersion relation. Neglecting the electroweak contributions compared to that due to strong interaction, the leading contribution to the real part \(E_0\) and imaginary part \(\gamma\) of the effective mass are the same for both the left\((L)\)- and right\((R)\)-handed quasiparticles,

\[E_0 = (2\pi\alpha_s/3)^{1/2}T \simeq .5T\]

and

\[\gamma = .15\alpha_s T \simeq .2T\]

with \(\alpha_s = .12\) at the Z boson mass. For excitations close to \(E_0\), the effective Lagrangian incorporating the altered dispersion relation is \[8, 9, 10\]

\[\mathcal{L} = iR^\dagger(\partial_0 + \frac{1}{3}\sigma \cdot \nabla + iE_0 + \gamma)R + iL^\dagger(\partial_0 - \frac{1}{3}\sigma \cdot \nabla + iE_0 + \gamma) + mL^\dagger R + m^* R^\dagger L, \ (1)\]

where we have also included the quark mass acquired through Higgs mechanism.

Having incorporated the effects of field degrees of freedom in (1), the problem reduces to quantum mechanics of left- and right-handed quasiparticles, having the structure of a resonance of width \(\gamma\). We prefer to make the momentum variable complex rather than the energy, so that the spatial propagation will be damped.
In the following we consider the one dimensional problem where the quasiparticles propagate along the \( z \)-axis, normal to the bubble wall. Writing 

\[
L = \left( \begin{array}{c} \psi_1 \\ \psi_2 \end{array} \right), \quad R = \left( \begin{array}{c} \psi_3 \\ \psi_4 \end{array} \right)
\]

the equations of motion derived from the Lagrangian (1) split into two independent sets. Defining 

\[
\Phi = \left( \begin{array}{c} \psi_1 \\ \psi_3 \end{array} \right), \quad \Phi' = \left( \begin{array}{c} \psi_4 \\ \psi_2 \end{array} \right),
\]

and considering solutions of positive energy \( E \), they are 

\[
\frac{d\Phi}{dz} = iQ(z)\Phi
\]

(2)

where 

\[
Q(z) = 3 \begin{pmatrix} E - E_0 + i\gamma & m^*(z) \\ -m(z) & -(E - E_0 + i\gamma) \end{pmatrix}
\]

(3)

and a similar one for \( \Phi' \) with \( m \) replaced by its complex conjugate. It suffices for us to work with \( \Phi \) only. Note that the current along the \( z \)-axis carried by the components \( \psi_1 \) and \( \psi_3 \) of \( \Phi \) is 

\[
j_z = \Phi^\dagger \sigma_3 \Phi,
\]

(4)

\( \sigma_3 \) being the third Pauli matrix.

The planar bubble wall has a finite thickness, extending from \( z = 0 \) to \( z = z_0 \). It separates the broken phase (\( z > z_0 \)) from the unbroken phase (\( z < 0 \)). The real part of the Higgs induced mass \( m(z) \) rises from zero in the unbroken phase through the bubble wall to the (almost) zero temperature mass \( m_0 \) in the broken phase. The imaginary part is non-zero only within the bubble wall. Their actual shapes will be conveniently chosen later in sec. 3 below.

In the unbroken phase (\( m = 0 \)), the components \( \psi_1 \) and \( \psi_3 \) (also \( \psi_2 \) and \( \psi_4 \)) decouple. Consider (damped) plane waves along \( z \) direction, \( \psi_{1,3} \sim e^{iKz}, \quad K = k + i\Gamma_u, k > 0. \)

They satisfy the dispersion relations [14], 

\[
E_\pm = E_0 \pm \frac{k}{3}, \Gamma_u = \pm 3\gamma,
\]

(5)

the (+) and(-) relations holding for \( \psi_1 \) and \( \psi_3 \) respectively. In contrast to the situation for a free massless particle at zero temperature satisfying \( E_\pm = \pm k \), here a part of the (-) branch \( (0 < E_- < E_0) \) in (5) is also available for quasiparticle propagation with positive energy. The (+) branches are called the normal and abnormal ones respectively. Unlike the energy \( E_\pm \), the variable \( k \), however, does not represent the true momentum \( \vec{k}_\pm \) of the excitation, the latter being given by 

\[
\vec{k}_\pm = \pm \frac{1}{3} E_\pm
\]

(6)
The reversal of sign of $\bar{k}_-$ in the abnormal branch is in conformity with the same for the
group velocity,

$$v_\pm = \frac{\partial E_\pm}{\partial k} = \pm \frac{1}{3} \tag{7}$$

Thus the above solution for $\psi_1 (\psi_3)$, belonging to the normal (abnormal) branch, advances
in the positive (negative) $z$-direction. For solutions $\psi_{1,3} \sim e^{-iKz}$, $\psi_1$ and $\psi_3$ belong to
opposite branches. Note that the propagation is always damped.

In the broken phase, the propagation properties are similar except that the Higgs $m_0$
couples the two components in $\Phi$ (and in $\Phi'$). Again consider damped plane wave along
$z$-direction, $\Phi \sim \chi e^{iPz}$, $P = p + i\Gamma_b, p > 0$. The spinor $\chi$ satisfies

$$\begin{pmatrix} 3(E - E_0 + i\gamma) - P & 3m_0 \\ -3m_0 & -3(E - E_0 + i\gamma) - P \end{pmatrix} \chi = 0 \tag{8}$$

We get the dispersion relations by setting the determinant of the $2 \times 2$ matrix equal to zero. Separating the real and imaginary parts, we get

$$(E - E_0)^2 - (p/3)^2 = \frac{m_0(p/3)^2}{\gamma^2 + (p/3)^2}, \quad \Gamma_b = \frac{3\gamma(E - E_0)}{p}. \tag{9}$$

The normal (+) and abnormal (-) branches arise on taking the square root,

$$E = E_0 \pm \frac{p}{3}g(p), \quad \Gamma = \pm \gamma g(p),$$

where $g(p) = \sqrt{1 + \frac{m_0^2}{\gamma^2 + (p/3)^2}}$. Note that the presence of damping ($\gamma \neq 0$) removes any gap
between the normal and the abnormal branches, which exists for $\gamma = 0$, the dispersion
relation then reducing to $E = E_0 \pm \sqrt{p^2/9 + m_0^2}$. The spinor $\chi$ is obtained by solving (8). Normalizing to unit current, we get

$$\chi = \begin{pmatrix} c \\ -s \end{pmatrix}, \tag{10}$$

where the components $c$ and $s$ stand for

$$c = \cosh \theta.e^{i\phi}, \quad s = \sinh \theta.e^{i\phi} \tag{11}$$

where

$$\cosh \theta = \sqrt{\frac{E - E_0 + p/3}{p/3}}, \quad e^{4i\phi} = \frac{p/3 + i\gamma}{p/3 - i\gamma}$$

The presence of non-zero damping also brings in the phase $\phi$.

We note here for later use the Lorentz invariant expression for the density of fermionic excitation,

$$n = (exp \beta p \cdot v + 1)^{-1}$$
where $\beta$ is the inverse temperature of the fluid in the frame where it is at rest, $p^\mu$ is the energy-momentum 4-vector of the excitation and $v^\mu$, the 4-velocity of the medium. In the wall rest frame, $p^\mu = (E, \bar{p}), v^\mu = \gamma(1, v)$ where $\gamma = 1/\sqrt{1 - v^2}$ and $\bar{p}$ is the true momentum also given by (6). For $p$ along the positive $z$-direction, we thus have in this frame, $p \cdot v = E_\pm(1 \mp v/3)$, up to linear term in $v$. In the following we require the densities of particles with true momenta towards the wall. In the unbroken phase these are given by

$$n^u_\pm = \frac{1}{e^{\beta E_\pm(1-v/3)} + 1}$$

for the $(\pm)$ modes respectively. In the broken phase the corresponding quantities $n^b_\pm$ are given by the same expressions with the reversal of sign of $v$.

### 3 Solution inside the bubble wall

We solve equation (2) in a perturbation series in the mass function. Assume

$$\Phi(z) = e^{3i(E - E_0 + iv)z}\sigma_3 \Psi(z), \quad 0 \leq z \leq z_0$$

$\Psi(z)$ will then satisfy

$$\frac{d\Psi}{dz} = iR(z)\Psi.$$ 

$R(z)$ has only off-diagonal elements,

$$R(z) = \begin{pmatrix} 0 & M(z) \\ \tilde{M}(z) & 0 \end{pmatrix},$$

where $M(z) = 3m(z)e^{6i(E - E_0 + iv)z}$, and the tilde stands for complex conjugation and change in the sign of $\gamma$. We now convert it into an integral equation,

$$\Psi(z) = \Phi(0) + i \int_0^z R(z')\Psi(z')dz'$$

It has an iterative solution, $\Psi(z) = \Sigma(z)\Phi(0)$, where

$$\Sigma(z) = 1 + i \int_0^z dz' R(z') - \int_0^z dz' \int_0^{z'} dz'' R(z')R(z'') + \cdots$$

We shall actually need the solution for $\Phi(z)$ at $z = z_0$,

$$\Phi(z_0) = e^{3i(E - E_0 + iv)z_0}\sigma_3 \Sigma(z_0)\Phi(0) \equiv \Omega(z_0)\Phi(z_0).$$

Writing

$$\Omega(z_0) = \begin{pmatrix} \alpha & \beta \\ \bar{\beta} & \bar{\alpha} \end{pmatrix},$$
we get

$$\alpha = F(1 + \int_0^{z_0} dz' \int_0^{z'} dz'' \tilde{M}(z') M(z'') + \cdots)$$  \hspace{1cm} (14)$$

$$\beta = iF(\int_0^{z_0} dz' \tilde{M}(z') + \cdots)$$  \hspace{1cm} (15)$$

with $F = e^{-3z_0}e^{3(E-E_0)z_0}$. Note that in the absence of damping ($\gamma = 0$), the tilde operation reduces to complex conjugation.

As already mentioned, simple extensions of the Higgs sector of the standard model can provide an additional source of large CP violation for baryogenesis. In the standard model with a single Higgs doublet, the expectation value of the Higgs field is real everywhere during the phase transition. But in multi-Higgs models, some of the components acquire space dependent values within the bubble wall. This in turn leads to complex space dependent mass function for the quarks having Yukawa couplings to those multiplets. These are, in principle, calculable from the model considered but, in practice, will depend on the many Higgs self-couplings. Here we avoid this problem by assuming a simple but anticipated form the mass function,

$$m(z) = \frac{m_0}{z_0} z + i \frac{\delta}{z_0^2} z (z_0 - z),$$  \hspace{1cm} (16)$$

within the bubble wall. The parameter $\delta$ relates to the CP violation in the model.

It suffices to work out the perturbation series to second order to get the leading contribution to the asymmetry in the baryonic currents. With the parametrization (16), $\alpha$ and $\beta$ in (14-15) can be obtained explicitly to this order as

$$\alpha = F(1 + 9z_0^2 U + \cdots)$$

$$\beta = iF(3z_0 V + \cdots)$$  \hspace{1cm} (17)$$

$U$ is quadratic in $m_0$ and $\delta$ and $V$ is linear,

$$U = Am_0^2 + iBm_0\delta + C\delta^2$$

$$V = am_0 - ib\delta.$$  \hspace{1cm} (18)$$

Each of the coefficients $A, a$ etc. depend only on $\sigma = 6z_0\{\gamma - i(E - E_0)\}$,

$$A = e^{\sigma} \left( \frac{1}{\sigma^3} - \frac{1}{\sigma^4} \right) - \frac{1}{3\sigma} - \frac{1}{2\sigma^2} + \frac{1}{\sigma^4},$$

$$B = e^{\sigma} \left( \frac{1}{\sigma^3} - \frac{4}{\sigma^4} + \frac{4}{\sigma^5} \right) + \frac{1}{3\sigma^2} + \frac{1}{\sigma^3} - \frac{4}{\sigma^5},$$

$$C = e^{\sigma} \left( \frac{1}{\sigma^4} - \frac{4}{\sigma^5} + \frac{4}{\sigma^6} \right) - \frac{1}{30\sigma} + \frac{1}{3\sigma^2} + \frac{1}{\sigma^3} - \frac{4}{\sigma^6},$$

$$a = e^{\sigma} \left( \frac{1}{\sigma} - \frac{1}{\sigma^2} \right) + \frac{1}{\sigma^2},$$

$$b = e^{\sigma} \left( \frac{1}{\sigma^2} - \frac{2}{\sigma^3} \right) + \frac{1}{\sigma^2} + \frac{2}{\sigma^3}.$$  \hspace{1cm} (19)$$
4 Damping in reflection and transmission

The damping rate has its origin in the scattering of the fermion under consideration by other particles in the plasma. Although it does not alter the densities of particles in the unbroken or broken phase, a quantum mechanical amplitude gets attenuated as it traverses the plasma. In the present context only the attenuation within the bubble wall is relevant, where the baryon asymmetry is produced. Thus we prepare our states near the wall of the bubble \[15\].

To study damping effects, consider, for example, the quasiparticle propagation in the normal mode. We send a right-handed fermion towards the domain wall from the unbroken phase. Noting the reversal of chirality after reflection at the wall, the incident wave (of unit current at \(z = 0\)) and a reflected wave of amplitude \(r\), say, is given by

\[
\Phi(z) = \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{iKz} + \begin{pmatrix} 0 \\ r \end{pmatrix} e^{-iKz}, \quad z \leq 0
\] (20)

On the right (broken phase), we have only the transmitted wave of amplitude \(t\), say. From (10) we get

\[
\Phi(z) = t \begin{pmatrix} c \\ -s \end{pmatrix} e^{iP(z-z_0)}, \quad z \geq z_0
\] (21)

Eq.(13) now serves as the matching condition needed to find the reflection and transmission amplitudes

\[
t \begin{pmatrix} c \\ -s \end{pmatrix} = \begin{pmatrix} \alpha \\ \beta \end{pmatrix} \begin{pmatrix} 1 \\ r \end{pmatrix},
\]
giving

\[
r = -(s\alpha + c\bar{\beta})/D, \quad t = 1/D, \quad D = c\bar{\alpha} + s\beta.
\] (22)

In the absence of damping (\(\gamma = 0\)), the tilde operation on \(\alpha\) and \(\beta\) reduces to complex conjugation and we immediately obtain from (22),

\[
|r|^2 + |t|^2 = 1 \quad (\gamma = 0),
\] (23)

expressing the equality of currents in the two phases. However, in the general case (\(\gamma \neq 0\)), this does not hold. The reflection and transmission coefficients can be found in the general case to the second order in the mass variables using the results for \(\alpha\) and \(\beta\) obtained in (17-19). The expression are rather clumsy and will not be presented here. But for large damping, i.e. large values of \((6\gamma z_0)\), the leading behaviour of these coefficients is simple to state,

\[
|r|^2 \rightarrow O \left( \frac{1}{(6\gamma z_0)^2} \right), \quad |t|^2 \rightarrow O \left( e^{-6\gamma z_0} \right)
\] (24)

badly violating the current conservation relation (23).

Thus the damping causes the calculational scheme to violate unitarity and results derived from such a scheme may be questioned. A satisfactory scheme can only be found in a many body formulation of the problem which incorporates the true mechanism responsible for damping.
5 Problem of baryogenesis

Unitarity can be restored in the present framework only if the thickness of the bubble wall tends to zero (thin wall approximation). In this limit the quasiparticles would not have to cross any intervening medium to go from one phase to the other, which is precisely the region where the loss in probability due to the damping matters in the present problem.

As \( z_0 \to 0 \), the functions \( A, B, C, a \) and \( b \) approach constants and we get

\[
\alpha = 1 + 9z_0^2\left(\frac{1}{6}m_0^2 + \frac{i}{30}m_0\delta + \frac{1}{72}\delta^2\right) + O(z_0^3) \tag{25}
\]

\[
\beta = 3z_0\left(-\delta + \frac{i}{2}m_0\right) + O(z_0^2) \tag{26}
\]

Accordingly the matrix \( \Omega(z_0) \to 1 \) and no asymmetry between fermion and antifermion currents would result.

Nevertheless, we plan to calculate the baryon asymmetry in this model for zero wall thickness to examine the effect of damping it. For this purpose we imagine that \( \delta \) is very large so that \( \delta z_0 \) goes to a non-zero constant, say, \( \Delta_0 < 1 \), even if \( z_0 \) is very small. Then

\[
\alpha = 1 + \frac{\Delta_0^2}{8} + O(\Delta_0^3) \tag{27}
\]

\[
\beta = \frac{1}{2}\Delta_0 + O(\Delta_0^2) \tag{28}
\]

Note that they are real and independent of \( \gamma \).

Since baryon non-conservation through sphaleron processes involves the left-handed fermions and antifermions, we are interested in calculating only the left-handed baryonic currents in the unbroken phase.

Consider the propagation of quark excitation in the normal mode. We have already calculated for finite wall thickness the reflection and transmission amplitude when right-handed fermions are incident on the wall from the unbroken phase. We rewrite them for thin wall approximation,

\[
r = -(s\alpha + c\beta)/D, \quad t = \frac{1}{D}, \quad D = c\alpha + s\beta.
\]

With (25) and (26) the reflection coefficients become,

\[
T_+ = |t|^2 = 1/|D|^2, \quad R_+ = 1 - |t|^2.
\]

where

\[
|D|^2 = 1 + hm_0^2 + \frac{\Delta_0^2}{4} + \frac{2}{3}hpm_0\Delta_0, \quad h^{-1} = 4\left(\gamma^2 + (p/3)^2\right)
\]

The incident flux is the same for particles and antiparticles, viz. \( \frac{1}{3}n_+^u \), where \( n_+^u \) is given by (12). Considering both the particles and antiparticles, the net contribution to the reflected left handed baryonic current is

\[
\int \frac{dk}{2\pi} \frac{1}{3}n_+^u(R_+ - \bar{R}_+)
\]

\[\tag{29}\]
Here and in the following a bar on a reflection or transmission coefficient denotes the corresponding quantity for the antiparticle. It is obtained by solving the same eqn.(2) with \( m \) replaced by \( m^* \).

Next consider transmission in the unbroken phase due to incidence on the wall from the broken phase. On the left there is only a transmitted wave of amplitude \( t' \), say

\[
\Phi(z) = \begin{pmatrix} 0 \\ t' \end{pmatrix} e^{-iKz}, \quad z < 0
\]

On the right we have both the incident wave and the reflected wave of amplitude \( r' \), say.

\[
\Phi(z) = \begin{pmatrix} s \\ -c \end{pmatrix} e^{-iPz} + r' \begin{pmatrix} c \\ -s \end{pmatrix} e^{iPz}, \quad z > 0
\]

Again the matching condition (13) gives

\[
\begin{pmatrix} s + r'c \\ -(c + r's) \end{pmatrix} = \begin{pmatrix} \alpha \\ \beta \end{pmatrix} \begin{pmatrix} 0 \\ t' \end{pmatrix}
\]

giving

\[
r' = -(s\alpha + c\beta)/D, \quad t' = (s^2 - c^2)/D
\]

Note that the coefficients fail to satisfy the current conservation across the wall, \( |r'|^2 + |t'|^2 \neq 1 \). The problem here is that the current in the broken phase as calculated from the wave function using (4) is not \(-1 + |r'|^2\) but contains cross terms,

\[
j = -1 + |r'|^2 + \{r'(s^*c - sc^*) + c.c.\}
\]

\[
= -|1 - r'(s^*c - sc^*)|^2 + |r'|^2\{1 + |s^*c - sc^*|^2\}
\]

\[
= -|c^2 - s^2|^2 + |r'|^2|c^2 - s^2|^2,
\]

on using \(|c|^2 - |s|^2 = 1\). Thus with correct normalization, these reflection and transmission coefficients coincide, as usual, with those calculated for incidence from the unbroken phase.

The transmitted left handed baryonic current in the unbroken phase is given by

\[
\int \frac{dp}{2\pi} \frac{\partial E^b_+}{\partial p} n^b_+(T_+ - \bar{T}_+)
\]

where \( \partial E_+ / \partial p \) is the group velocity in the broken phase. Adding (29) and (30) we get the total baryonic current in the normal mode in the unbroken phase due to reflection and transmission as

\[
J_+ = \int \frac{dk}{2\pi} \frac{1}{3} (n^b_+ - n^u_+)(T_+ - \bar{T}_+)
\]

In a similar way we can work out the baryonic current in the abnormal mode in unbroken phase to be

\[
J_- = \int \frac{dk}{2\pi} \frac{1}{3} (n^b_- - n^u_-)(T_- - \bar{T}_-)
\]
where

\[ T_- = 1/|\alpha c' - \beta s'|^2 \]

and \( c', s' \) are given by same expressions as for \( c \) and \( s \) with changes of sign of \((E - E_0)\) and of \( \phi \).

It is now easy to make an order of magnitude estimate of these currents. Formally the \( k \)-integrals in (31) and (32) extend over \( 0 \) to \( \infty \) and \( 0 \) to \( 3E_0 \) respectively. But the integrands are highly damped at higher values of \( k \), not just because of the presence of the density functions; the transmission coefficients for a realistic (i.e. smooth and finite width) bubble wall would have fallen exponentially for even lower \( k \) values. We set \( E_0 \) as a reasonable upper limit for both the integrals. Hopefully the low momentum approximation, on which the effective Lagrangian (1) is based, would admit this upper limit.

As the temperature of the phase transition is about 100 GeV, we may expand the density functions in \( v \) for small \( v \) and approximate the exponential by unity to get

\[ n_b^h - n_u^h \simeq -\frac{1}{6} \beta v E_0 \]

Thus the currents (31) and (32) become

\[ J_\pm = \pm \frac{\beta v m_0 \Delta_0}{2 + \Delta_0^2} \int_0^{E_0} \frac{dk}{2\pi} (E_0 \pm k \frac{p}{3}) \frac{p}{p^2 + 9\gamma^2} \]

Observe the large cancellation in the sum of \( J_+ \) and \( J_- \) giving the total CP-violating left-handed baryonic current in the unbroken phase. Evaluating the resulting integral approximately we get

\[ J_{CP}^L = J_+ + J_- \]

\[ \simeq \frac{\beta v m_0 \Delta_0}{2 + \Delta_0^2} \]

The final step is to obtain the baryonic density \( n_B \) in the broken phase from the steady state solution to the rate equations in the two phases. CKN\[5\] find numerical solution to the Boltzmann equation. We shall follow FS \[8\], who solve the diffusion equations for small bubble wall velocity to get

\[ n_B = J_{CP}^L f \]

where \( f \) is a given function of the diffusion coefficients for quarks and leptons, the wall velocity and the sphaleron induced baryon number violation rate. Their estimate for \( f \) is \( 10^{-3} \leq f \leq 1 \) in MSM, which should also be valid for its simple extensions.

Noting the one dimensional entropy density \( s = 73\pi/3\beta \), the baryon to entropy ratio is obtained as

\[ n_B/s \sim 1.3 \times 10^{-2} v \beta^2 m_0 \Delta_0 f/(2 + \Delta_0^2) \]

With \( m_0 = 5 GeV, \beta = 10^{-2} GeV^{-1}, v = 0.1 \) and \( \Delta_0 = 1 \), we get \( n_B/s \sim 3 \times 10^{-7} f \), to be compared with the observed value \( n_B/s \sim 5 \times 10^{-11} \).
6 Conclusion

We have studied the effect of damping on quasiparticle propagation in the electroweak plasma in connection with the problem of baryogenesis in non-minimal versions of the standard model, where the Higgs sector is extended to include more than one multiplet. In general, such an extension gives rise to a complex mass function for a quark within the bubble wall formed during the phase transition. This constitutes the CP violation needed for baryogenesis. Its real and imaginary parts are parametrized in a simple way making the integrals simple to evaluate. We follow closely the technique of CKN\cite{5}, but consider a direct separation of baryon number by the bubble wall rather than of some other CP-odd charge\cite{7}.

The inclusion of the temperature dependent effective mass gives rise to two modes of quasiparticle propagation in the plasma. Our calculation shows that both modes must be taken into account. In fact, the net baryon asymmetry current results after large cancellation between the baryonic currents carried separately by the two modes.

We also include the recently discussed damping suffered by the quasiparticles while propagating through the plasma\cite{3,10}. Like the effective mass, it also arises from interaction of a fermion with other particles in the plasma. This damping appears to ruin an otherwise successful calculation\cite{8} of baryon asymmetry in the MSM. Besides spatial attenuation, it also affects the spinor components of $\Phi$ and the dispersion relations are no longer separated by a mass gap. Also the two spinor components acquire equal and opposite phase.

The quasiparticle propagation is damped everywhere within the medium, in the broken and unbroken phases, as well as within the bubble wall. However, it is the damping associated with the propagation within the wall, which is of concern in the problem of baryogenesis. It causes the magnitudes of the reflection coefficients to be reduced, more so the larger the value of $\gamma z_0$. The problem one faces here is that the calculational scheme is non-unitary and the question of reliability of a calculation within such a scheme remains open.

Unitarity is restored in this formalism only in the thin wall approximation, when a calculation of baryon asymmetry would be free from the above objection. But $\Omega(z_0) \rightarrow 1$ as $z_0 \rightarrow 0$ and there is no asymmetry between the fermion and the antifermion as they interact with the wall. We are then led to consider a rather unrealistic situation where the coefficient $\delta$ in the imaginary part of the mass function is so large that $\delta z_0 = \Delta_0$, a non-zero constant, even in the thin wall approximation. It is now simple to calculate the baryon to entropy ratio of the Universe. As expected, it does not suffer any reduction due to damping rate when compared with a similar expression calculated without taking it into account\cite{17}.

We now come back to comment on the real problem with finite wall thickness. Though a satisfactory framework for calculation should avoid any non-unitarity due to damping by including the many body processes responsible for it, it, nevertheless, appears that the damping effect in baryon asymmetry will persist even in such a framework. Thus the result calculated without damping\cite{7} is likely to be reduced by suppression factor...
appearing in the reflection and transmission coefficients (24). Note that this suppression is much less than that encountered in MSM where one has to go to higher order in perturbation expansion in the mass matrix. Furthermore, the magnitude of the wall thickness in extended versions of the standard model is not known. In view of the many parameters in the potential of such a model, the indication is that it could be small in several regions of the parameters. For \( z_0 \leq \frac{1}{10} \text{GeV}^{-1} \), the damping factor is of order \( 10^{-4} \) at most. Within the uncertainties of the transport properties of the electroweak plasma, the baryon to entropy ratio would still be around the observed magnitude.

CKN \( [5] \) find that the t quark propagation through the bubble wall can produce the observed baryon to entropy ratio. We here argue that the contribution of the b quark, for which finite temperature correction in mass, in both the real and imaginary parts, is important, is also likely to be of similar magnitude.

References

[1] G. ’tHooft, Phys. Rev. Lett., 37, 8 (1976); Phys. Rev.D14, 3432 (1976)
[2] V. A. Kuzmin, V. A. Rubakov and M. E. Shaposhnikov, Phys. Lett. 155b, 36 (1985).
   See also
   V. Soni, Phys. Lett. 93B, 101 (1980)
   N. S. Manton, Phys. Rev.D28, 2019 (1983)
   F. R. Klinkhammer and N. S. Manton, Phys. Rev.D30, 2212 (1984)
   P. Arnold and L. McLerran, Phys. Rev.D36, 581 (1987)
[3] A. D. Sakharov, JETP Lett. 6, 24 (1967)
[4] A. G. Cohen, D. B. Kaplan and A. E. Nelson, Phys. Lett. 245B, 561 (1990), Nucl.
   Phys. B349, 727 (1991), Phys. Lett. 294B, 57 (1992)
[5] A. E. Nelson, D. B. Kaplan and A. G. Cohen, Nucl. Phys. B373, 453 (1992)
[6] J. F. Gunion, H. E. Haber, G. L. Kane and S. Dawson, The Higgs Hunter’s Guide
   (Addison-Wesley, Reading MA, (1990)
[7] M. E. Shaposhnikov, Phys. Lett. 277B, 324 (1992); ibid Phys. Lett. 282B, 483(E), (1992)
[8] G. R. Farrar and M. E. Shaposhnikov, Phys. Rev. Lett., 70, 2883 (1993); Phys.
   Rev.D50, 774 (1994)
[9] M. B. Gavela, P. Harnandez, J. Orloff and O. Pene, Mod. Phys. Lett. 9A, 795 (1994)
    M. B. Gavela, P. Harnandez, J. Orloff and O. Pene and C. Quimbay, preprint CERN-
    TH. 7263/94

[10] P. Huet and E. Sather, preprint SLAC-PUB-6479

[11] G. R. Farrar and M. E. Shaposhnikov, preprint RU-94-90

[12] H. A. Weldon Phys. Rev. D26, 2789 (1982)

[13] The wall thickness is known only in MSM, see e.g.
    M. Dine, R. Leigh, P. Huet, A. Linde and D. Linde Phys. Rev. D46, 550 (1992)
    N. Banerjee, S. Mallik and D. Rai Chaudhuri, Phys. Rev. D47, 4277 (1993)

[14] In Ref.[8] nonleading contributions to the self energy are included making the effective
    mass different for the left- and right-handed fermions. This leads to four branches in
    the dispersion relation instead of two considered here.

[15] The plane wave treatment below is expected to give basically the same result as with
    the more satisfactory wave packet approach. See Ref. [9]

[16] Note that we are working here in the wall rest frame. But the reflection coefficients
    obtained in the static situation can be used, as we keep only the linear term in the
    bubble wall velocity in our calculation of baryon asymmetry.

[17] A. Kundu and S. Mallik, preprint SINP/TNP/94-9, hep-ph 9411400.