For how long are particles accelerated in shells of recurrent Novae?

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ABSTRACT
Galactic Novae is at present well established class of γ-ray sources. We wonder for how long the mechanism of acceleration of electrons operates in shells of Novae. In order to put constraints on the time scale of the electron acceleration, we consider a specific model for the injection and propagation of electrons within the shell of the recurrent Nova RS Ophiuchi. We calculate the equilibrium spectra of electrons within the Nova shell and the γ-ray fluxes produced by these electrons in the comptonization of the soft radiation from the Red Giant within the Nova binary system and also radiation from the Nova photosphere. We investigate two component, time dependent model in which a spherically ejected Nova shell propagates freely in the polar region of the Nova binary system. But, the shell is significantly decelerated in the dense equatorial region of the binary system. We discuss the conditions for which electrons can produce γ-rays which might be detectable by the present and/or future γ-ray observatories. It is concluded that freely expending shells of Novae in the optimal case (strongly magnetised shell and efficiency of acceleration of electrons of the order of 10%) can produce TeV γ-rays within the sensitivity of the Cherenkov Telescope Array only within 1-2 years after explosion. On the other hand, decelerated shells of Novae have a chance to be detected during the whole recurrence period of RS Ophiuchi, i.e. ~15 years.

Key words: stars: novae — acceleration of particles — radiation mechanisms: non-thermal — gamma-rays: stars — cosmic rays

1 INTRODUCTION
Nova is a thermonuclear explosion in a layer of matter on the surface of a White Dwarf (WD). The matter appears as a result of the accretion process from a companion star in the WD binary system (Gallagher & Starrfield 1978, Bode & Evans 2008). Depending on the type of the companion star, a Red Giant (RG) or a Main Sequence star, the Nova is called a Symbiotic or a Classical Nova. Explosions occur on different time scales in different binary systems, depending on the mass of the WD and the accretion rate. Novae, observed many times in a human life, are classified as recurrent Novae.

During the last decade, several Novae (but only two symbiotic) have been detected by the Fermi-LAT (Large Area Telescope) at GeV γ-ray energies (Abdo et al. 2010, Ackermann et al. 2014, Franckowiak et al. 2018 and see also Table S1 in Chomiuk et al. 2021). The spectra of these sources are usually well described by a simple power law function, with the differential spectral index close to -2, and the exponential cut-off at a few to several GeV. γ-ray emission appears with some delay after the initial optical flash. It lasts up to several to a few tens of days after explosion. In the case of the recurrent Nova RS Ophiuchi (RS Oph), exploded for the last time in August 2021, γ-ray emission is observed within the GeV (Cheung et al. 2021) up to sub-TeV energy range (Acciari et al. 2022, Aharonian et al. 2022). The spectrum is well described by a broken power law function with the spectral index close to -2 in the GeV energy range, the spectrum steepens to about -4 at sub-TeV energies. On the other hand, γ-ray spectrum hardens with time after explosion. These results indicate that particles are accelerated in Novae at least up to TeV energies. They likely contribute to the cosmic ray background in the Galaxy locally around the source (Acciari et al. 2022).

γ-ray emission from Novae is expected to be appear either in the decay process of neutral pions produced by hadrons which collide with the matter of the expanding material in the nova shell, or by electrons which Inverse Compton up-scatter soft radiation from the Nova photosphere. This radiation dominates at the early time after explosion (see e.g. Abdo et al. 2010, Sitarek & Bednarek 2012, Mar-
tin & Dubus 2013, Metzger et al. 2015, Ahnen et al. 2015, Vurm & Metzger 2018, Martin et al. 2018). The observation of the GeV \( \gamma \)-ray emission, at the early stage of the Nova explosion, suggests that this emission is likely produced by hadrons, when the shell is still dense. But the transfer of energy from hadrons to radiation has to be rather efficient. On the other hand, the radiation field for relativistic electrons is very dense, allowing efficient transfer of energy from electrons to radiation. Other processes, e.g. the synchrotron energy losses, can additionally extract energy from electrons, especially at the largest energies. This effect can be responsible for the change of the spectral index of the \( \gamma \)-ray emission from RS Oph in the energy range between GeV emission and sub-TeV range. The leptonic model is consistent with the striking correlations between the optical and the \( \gamma \)-ray light curves observed by Aydi et al. (2020a) and Li et al. (2017).

The Nova RS Oph belongs to the special class of recurrent symbiotic Novae. Already six explosions of this Nova have been observed with the recurrence period of the order of several years. The binary system RS Oph contains a massive WD (with the mass estimated on 1.35 \( M_\odot \), i.e. close to the Chandrasekar limit, see Dobrzycka & Kenyon 1994, Shore et al. 1996, Hachisu et al. 2009), and MO-2 III donor Red Giant (RG) with the mass of 0.68-0.8 \( M_\odot \) (Ampaśa & Mikolajewska 1999) and the luminosity \( \sim 10^{58} \) times larger than the luminosity of the Sun. The main part of the RG surface has the temperature of 3600 K (Pavlenko et al. 2021).

Its radius is estimated on 67\( ^{+15}_{-16} \) \( R_\odot \), where \( R_\odot \) is the radius of the Sun (Dumm & Schild 1998). The axis of the binary system is inclined at the angle 39\( ^{+1}_{-1} \) degrees to the observer’s line of sight (Ribeiro et al. 2009). The mass transfer to the WD is estimated in the range 3.7 \( \times 10^{-8} \) \( M_\odot \) yr\(^{-1} \) (Schaefer 2009) and 10\(^{-6}\) \( M_\odot \) yr\(^{-1} \) (Iijina 2006). Then, the upper limit on the mass expelled in the Nova shell is scaled by the recurrence time of Nova RS Oph, i.e it is in the range 5.6 \( \times 10^{-7} \) \( M_\odot \) to 1.5 \( \times 10^{-5} \) \( M_\odot \). The RG is expected to produce the equatorial wind with the mass loss rate \( \sim 10^{-7} \) \( M_\odot \) yr\(^{-1} \) and the velocity 40 km s\(^{-1} \) (Wallerstein 1958). The kinetic and geometrical structure of the Nova ejecta looks very complicated. The velocity of ejecta, at about one day after explosion in 2006, was measured on \( \sim 4000 \) – 7500 km s\(^{-1} \) (Buil et al. 2006). The composite structure of the ejecta from the RS Oph Nova, at a few hundred days after explosion in 2006 as observed by the Hubble Space Telescope, shows two component symmetric flow, with the low velocity high density equatorial region and the high velocity polar region. A true expansion velocity for the material at the poles was measured on 5600 \( \pm 1100 \) km s\(^{-1} \) (Bode et al. 2007).

Based on these observations, it is concluded that the ejected shell of material moves with different velocities in the equatorial and the polar regions of the binary system. In the simplest two-component scenario, the Nova shell is composed of the equatorial slow and dense region and the fast, freely expanding polar region (see Shore et al. 2011, 2013 and Mason et al. 2018). Moreover, there are evidences of multiple ejections. Initial slow and dense material is reached by the fast wind from the WD surface formed at some time after initial explosion (Aydi et al. 2020b).

Here we investigate the consequences of the hypothesis that electrons are continuously accelerated in the expanding Nova shell applying a simple two-component geometrical model. It is assumed that the Nova exploded initially spherically symmetric. But due to the axially symmetric wind of the Red Giant, the shell is significantly decelerated in the equatorial region of the binary system but propagates almost freely in the polar regions. The presence of non-thermal particles, and the general two-component structure of the outflow, is supported by the observations of the non-thermal radio emission on a time scale of a month after explosion (e.g. Eyres et al. 2009). Such radio emission can last for years, as observed in the case of Nova V445 Pup (Nyamai et. al. 2021), indicating that particles can be accelerated in the Nova shells for a similar time scale.

We calculate the \( \gamma \)-ray spectra produced in the Inverse Compton (IC) process by electrons accelerated continuously during the propagation of the Nova shell. The equilibrium spectra of electrons are calculated at an arbitrary moment after the initial explosion taking into account different energy loss processes of electrons. Due to weaker energy losses late after explosion, electrons can reach multi-TeV energies, producing TeV \( \gamma \)-rays mainly in the comptonization process of the RG radiation. In order to test the hypothesis on the extended acceleration of electrons in the expanding shell of Nova RS Oph, we compare predicted \( \gamma \)-ray fluxes with the sensitivities of the present and future Cherenkov telescopes.

### 2 STRUCTURES OF NEBULAE AROUND RECURRENT NOVAE

Recurrent Novae are a sub-class of Novae which show regular explosions with the recurrence period shorter than about hundred years. Up to now, the largest number of explosions (known 6) have been observed in the case of the Nova RS Oph. RS Oph is a specially interesting Nova since it belongs to a class of recurrent symbiotic Novae in which the White Dwarf forms a binary system with a RG star. In such Novae, the geometry of explosion is particularly complex. The shell of expelled matter (even if ejected isotropically) can be modified due to the interaction with the RG wind which is concentrated in the orbital plane of the binary system. Therefore, a part of the Nova shell can be significantly decelerated in the orbital plane of the binary system. On the other hand, in polar regions, the shell expends with a constant velocity. In fact, such two-component expansion of the Nova shell has been observed in the case of the previous explosion of RS Oph in 2006 (see e.g. Ribeiro et al. 2009). Such geometrical two component model (originally proposed by Shore et al. 2011, 2013 and Mason et al. 2018) is considered in our work. Another complication might be due to multiple ejection of material by the Nova with different velocities (Aydi et al. 2020b). The material of initial ejection is expected to move with the lower velocity than the latter formed wind from the WD. As a result of their collisions, the shell is additionally energized.

In the case of a recurrent Nova RS Oph, we consider a simple two component geometrical structure for the Nova shell. We accept that the material is ejected isotropically from the White Dwarf surface with the mass of the order of \( M_{sh} = 10^{-6}M_\odot \) \( M_\odot \) and with the velocity of the order of \( v_{sh} = 3000v_8 \) km s\(^{-1} \). The total kinetic energy of the expanding shell of Nova is then \( L_{kin} = 0.5M_{sh}v_{sh}^2 = 9 \times 10^{43}M_\odot v_8^2 \) erg. During the recurrence time scale, this
shell is expected to move with the constant velocity in the polar region of the binary system but it can be significantly decelerated in the equatorial region of the RS Oph binary system on a time scale of months. The deceleration is due to the entrainment of the Red Giant wind by the shell. The Red Giant wind is assumed to be expelled within the equatorial region of the binary system, i.e. within a part of the whole sphere equal to $2\Omega_{RS}$. The Red Giant wind is expelled with the rate $M_{RG} = 10^{-7}M_{\odot}$ yr$^{-1}$, and the velocity $v_{RG} = 10^6v_{6}$ cm s$^{-1}$.

After ejection of the material by the WD, the nuclear energy is still generated in the remnant layer of matter on the WD surface. This energy is irradiated from the optically thick photosphere with the initial luminosity over the Eddington luminosity of the Solar mass object. During the time of several to a few tens of days, the photosphere becomes transparent to the soft thermal emission from the WD surface. This emission appears in the soft X-rays since the typical temperature of the surface of the WD is of the order of a few $10^5$ K. During the initial optically thick stage of the layer of matter on the WD surface, also a strong wind is expelled from the WD. This wind collides with the Nova shell, additionally energizing the material in the shell, already at some time after initial Nova explosion. We are interested in the processes occurring within the shell from the moment of this additional energization by the fast wind from the WD.

Recent observations of the sub-TeV $\gamma$-ray emission from RS Oph (Acciari et al. 2022, Aharonian et al. 2022) indicate that particles (electrons, hadrons) have to be accelerated during the Nova explosion. The site and mechanism of their acceleration is not presently known. Particles might be accelerated either in the wind or in the collision region between the Nova wind with the ejecta or in the expending Nova shell. We consider a scenario in which electrons are accelerated continuously in the Nova shell. At early phase after ejection, lasting from several days to a few weeks, accelerated electrons cool mainly on the IC process by scattering radiation from the nearby photosphere and from the RG. At latter phase, electrons lose a part of the energy mainly of their synchrotron process. Electrons are captured in the shell by the magnetic field being advected from the central part of the Nova binary system with the expending shell. If the shell moves with the constant velocity, relativistic electrons are transported to the distance from the WD equal to $R_{sh} = \tau_{rec}v_{sh} \approx 1.4 \times 10^{37}\tau_{sh}$ cm, where the recurrence time of the Nova is $\tau_{rec} = 15 \tau_{15}$ years. Electrons are still immersed in the radiation of the RG star producing $\gamma$-rays in the Inverse Compton process. We expect that this IC $\gamma$-ray component should appear at larger energies than sub-TeV $\gamma$-ray emission observed a few days after explosion, since electrons are expected to reach larger energies due to longer time scale for their energy losses on the synchrotron process and also due to the larger dynamical time scale of the shell.

3 ACCELERATION OF ELECTRONS IN THE NOVA SHELL

We follow the standard scenario for the explosion of Nova. Ejected material forms a shell with some thickness $\Delta R = v_{sh}t/R$, where $R$ is the shell radius and the scaling factor is $\beta = 0.1\beta_{-1}$. Particles are expected to be accelerated in the region of the shell. How this process occurs in the case of Nova explosions is at present an open question. In principle, two sites can be investigated, i.e. regions between the ejecta and the surrounding medium or between the shell material and the Nova wind. We concentrate on the first possibility since it provides eventual mechanism for acceleration of particles on a time scale of the propagation of the shell. The theory of shock acceleration argues that relativistic particles can take as much as $\sim 10 - 20\%$ of the kinetic energy of the shock. The acceleration process of particles can be limited either by their energy losses or the dynamical time scale of the shock. Therefore, in order to estimate the maximum energies to which electrons are accelerated, we consider a simple model for the magnetic field structure within the expending shell. This magnetic field determines the main processes for the energy gain and energy losses of particles in the shell.

We estimate the strength of the magnetic field in the expending shell of Nova assuming some level of equipartition between the kinetic energy density of the ejected shell and the energy density of the magnetic field. The radius of the WD in the Nova RS Oph is $R_{WD} = 2.3 \times 10^8$ cm (see formula in Nauenberg 1972, for the WD mass $\sim 1.35$ M$_{\odot}$, see Dobrzycka & Kenyon 1994, Shore et al. 1996, Hachisu, Kato, Luna 2009). The distance from the WD is expressed in units of the stellar radius according to $R = rR_{WD}$. The density of the matter in the ejected shell is

$$n_{sh} = \frac{M_{sh}}{4\pi R^2 m_p} \approx \frac{8.2 \times 10^{25} M_{-6}}{\beta_{-1} v_{4}^3} \approx \frac{5.8 \times 10^{10} M_{-6}}{\beta_{-1} t_{3}^3 v_{4}^3} \text{ cm}^{-3}, \tag{1}$$

where $r = v_{sh}t/R_{WD} \approx 1.12 \times 10^3 t_{4} t_{3}$, the time $t = 8.6 \times 10^4 t_4$ s, and $m_p$ is the proton mass. The magnetic field strength, at some level of equipartition with the kinetic energy density of the shell, is obtained from $a n_{sh} m_p v_{sh}^2/2 = \cdots$
$B^2/(8\pi)$, where $\alpha = 10^{-2}\alpha_{-2}$ is so called equipartition co-
icient. The magnetic field is

$$B = (4\pi c n_{sh} m_p)^{1/2} v_{sh} \approx 32 v_{sh} \frac{\alpha_{-2} M_{-6}}{\beta_{-1} t_d^{3/4}}^{1/2} \text{ G.}$$

(2)

For the scaling value of the parameter $\alpha \sim 0.01$, this
prescription is generally consistent with the estimates of
the magnetic field in the shell during previous explosion
of RS Oph obtained at 20 days after explosion from the
equipartition arguments based on the radio observations,
i.e. $B \sim (0.08 - 0.11) \text{ G (Rupen et al. 2008)}$.

On the other hand, electrons are accelerated at the en-
ergy gain rate which can be parametrised by

$$\dot{E}_{\text{acc}} = \frac{\xi c E}{R_t} \approx 0.1 \xi - 5 B \approx 3.2 \xi - 5 v_8 \left( \frac{\alpha_{-2} M_{-6}}{\beta_{-1} t_d^{3/4}} \right)^{1/2} \text{ GeV/s}.$$  

(3)

where $\xi = 10^{-5} \xi_{-5}$ is the acceleration coefficient which is esti-
mented on $\xi \sim (v_{turb}/c)^2 \sim 10^{-5} v_8^2$, where $v_{turb} \sim 0.3 v_{sh}$ is the
turbulent velocity of scattering centres above the shock of
the order of the fluid flow for the strong shock. We use
this prescription for the slow acceleration of electrons which
is characteristic for the second order Fermi acceleration pro-
cess. However, in Sect. 6, we discuss the results for the pre-
scription for the fast acceleration of electrons, characteris-
tic for the first order Fermi acceleration process. In this case,
$\gamma$-ray spectra only slightly extend to larger energies since at
2 months after explosion the dynamical time scale of the shell
becomes shorter than the synchrotron cooling time scale of
electrons.

The energy loss rate of electrons on the synchrotron
process depends on the energy density of the magnetic field,

$$\rho_{\text{syn}} = \frac{B^2}{8\pi} \approx 2.6 \times 10^{-3} v_8^2 (\alpha_{-2} M_{-6}/\beta_{-1} t_d^{3/4}) \text{ GeV/cm}^3.$$  

(4)

It is

$$\dot{E}_{\text{syn}} = \frac{4}{3} \rho_{\text{syn}} \sigma_T \gamma^2 \approx 1.4 \times 10^{-3} v_8^2 (\alpha_{-2} M_{-6}/\beta_{-1} t_d^{3/4}) E_{\text{GeV}}^2 \text{ GeV/s}.$$  

(5)

By comparing the energy gain rate with the synchrotron en-
ergy loss rate, we obtain the maximum energies of electrons

$$E_{\text{max}}^{\text{syn}} = 34 (\xi_{-5}/v_8)^{1/2} (\frac{\beta_{-1} t_{d3}}{\alpha_{-2} M_{-6}})^{1/4} \text{ GeV.}$$  

(6)

Close to the central engine, the above process determines
the maximum energies of electrons. By comparing the max-
imum energies of accelerated electrons with the maximum en-
ergy of the $\gamma$-ray photons equal to $\approx 500 \text{ GeV}$ (Aharonian
et al. 2022), i.e. $E_{\gamma \text{max}}^{\text{syn}} > E_{\gamma \text{max}}^{\text{acc}} \approx 500 \text{ GeV}$, we constrain
the equipartition coefficient $\alpha < 2 \times 10^{-2} (\xi_{-5}/v_8)^2 t_d^{3/4} E_{\text{GeV}}^2/\beta_{-1} t_d^{3/4}$. For larger magnetiz-
tion, the synchrotron energy losses do not allow acceleration of electrons to the observed $\gamma$-ray ener-
gies. Therefore, shells cannot be strongly magnetized al-
ready a few days after formation. However, note that this
constraint strongly depends on the time after explosion.

At large distances from the WD, the energy losses of electrons becomes too low (due to the strong drop of
the magnetic field during the expansion of the Nova shell) to balance the energy gains from the acceleration mecha-
nism. Then, the maximum energies of electrons are limited
by the dynamical time scale of the Nova shell equal to

$$\tau_{\text{dyn}} = \frac{R_{sh}}{v_{sh}} = 8.6 \times 10^7 t_d \text{ s.}$$

Then, the electron energies are obtained from the comparison of the acceleration time
scale, $\tau_{\text{acc}} = E_0/\dot{E}_{\text{acc}}$, with $\tau_{\text{dyn}}$,

$$E_{\text{max}}^{\text{dyn}} \approx 2.75 \times 10^5 \xi_{-5} v_8 (\frac{\alpha_{-2} M_{-6}}{\beta_{-1} t_d^{3/4}})^{1/2} \text{ GeV.}$$  

(7)

The time scales for different energy loss processes of elec-
trons in the Nova shell, the dynamical time scale of the shell and the acceleration time scale of electrons in the shell, are
shown in Fig. 2, for specific time after the Nova explosion.

For the considered range of the magnetic field strengths in
the shell, the synchrotron energy loss process dominates for
the most energetic electrons accelerated in the Nova shell.
However, at early time after explosion, the IC scattering of
soft radiation from the Nova photosphere becomes dominant
mechanism for the electron energy losses. At the later time
after explosion, the energy losses on the Inverse Compton
scattering of soft radiation from the RG start to dominate,
accept the moment of the next explosion at 15 years after
original explosion. At the late stage of the Nova, the elec-
trons with energies below $\approx \text{TeV}$, lose energy mainly on the

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{timescales.png}
\caption{The time scales, $\tau$, are shown for different energy gains and loss processes of relativistic electrons in the shell of the re-
current Nova RS Oph at different moments after explosion (equal
to 3 days, 30 days, 300 days, 1000 days, 3000 days, and 15 years).
The following time scales are marked by: the acceleration time
scale for the equipartition coefficient $\alpha = 0.01$ (thin solid curve)
and $10^{-4}$ (thick solid), the time scale for the synchrotron energy
losses of electrons are marked by dashed lines, for $\alpha = 0.01$ (thin)
and for $10^{-4}$ (thick), the dynamical time scale (dot-dashed), the IC
time scale in the Red Giant radiation field (dotted) and in the radia-
tion field from the Nova photosphere as observed at 3 days and 15 years
after explosion of the Nova (dot-dot-dot-dashed). The parameters of
the Nova photosphere and the Red Giant are reported in the main
text.}
\end{figure}
The γ-ray production in the IC process. As we noted above, the maximum energies of electrons are determined by the balance between the energy gains from the acceleration mechanism and energy losses on the synchrotron process or on the dynamical time scale. In Fig. 3, we show how those maximum energies of electrons evolve with time after initial explosion of the Nova, for fixed parameters of the Nova shell and different values of the magnetic field strength which is described by the equipartition coefficient $\beta$. Electrons reach the maximum energies at the intermediate time after explosion. The maximum energies of electrons at the early time are constrained by the synchrotron energy losses and, at late time, by the dynamical time of the Nova shell.

4 EQUILIBRIUM SPECTRUM OF ELECTRONS WITHIN THE SHELL

We assume that electrons are injected into the Nova shell from some acceleration mechanism. The acceleration process starts just after the Nova explosion and continue, at a constant rate in time, up to the observation time $t_{\text{obs}}$. Electrons are injected with the power law spectrum to characteristic maximum energy $E_{\text{max}}$ (estimated by Eq. 6 or Eq. 7). It is assumed that electrons are confined in the shell by the magnetic field. The equilibrium spectrum of electrons within the shell at the time $t_{\text{obs}}$ is obtained by solving the continuity equation for electrons of the type discussed by Blumenthal & Gould (1970, see Eq. 5.2). However, since the spectrum of electrons evolve within the shell in which physical conditions change in a complicated way in time we solve this transport equation for electrons numerically. We create a grid in the energy of injected electrons (in fact in $\log E_0$) and the time. We do not consider any additional acceleration of electrons after initial injection at the time $t_{\text{obs}}$. The contribution to the electron spectrum at the time $t_{\text{obs}}$ from freshly injected electrons at the time $t < t_{\text{obs}}$ is calculated numerically by subtracting their energy losses on specific radiation processes (synchrotron, bremsstrahlung and inverse Compton). These different energy losses are described in Sect. 4 and below. In the case of the average energy losses of electrons on the inverse Compton process, we use the approximate formula which is also valid in the Klein-Nishina regime (Moderski et al. 2005). The numerical method is used due to complicated dependence of the energy losses of electrons on time. These losses are determined by varying physical parameters during propagation of the shell (radiation field, magnetic field, density of matter). However, we neglect the energy losses of electrons on the adiabatic expansion of the shell since this process is difficult to consider on the present stage of knowledge. In fact, the shell might expand in time but also contract under the external pressure of the cosmic space or the wind from the White Dwarf. Therefore, it is considered that only radiation processes will determine the equilibrium spectrum of electrons at a specific propagation time of the shell. We determine the effect of radiation processes on the electron’s spectrum at a specific distance $R_{\text{sh}}$ by calculating the energy loss time scales, $t_{\text{syn,IC,br}} = E_{\text{sh}}/E_{\text{syn,IC,br}}$. Those time scales are dependent on the energy density of the magnetic field, different types of soft radiation and the matter in the shell.

The energy density of the soft radiation field from the RG is

$$\rho_{\text{RG}} = aT_{\text{RG}}^4 \approx \frac{7.2 \times 10^{11} T_{\text{sh}}^4 R_{\text{sh}}^3}{r^2} \approx \frac{57 T_{\text{sh}}^4 R_{\text{sh}}^3}{(v_0 t_d)^2} \text{GeV cm}^{-3},$$

valid for $r > R_{\text{RG}}/R_{\text{WD}}$. The radius of the RG is scaled by $R_{\text{RG}} = 10^{13} R_{\text{sh}}$ cm, its surface temperature is $T_{\text{RG}} = 3 \times 10^9 T_d$ K, and $a$ is the radiation constant.

On the other hand, the soft radiation from the Nova photosphere dominates at the early time after the Nova explosion, but also at the recurrence period of the Nova explosion (see Fig. 2). The energy density of radiation from the Nova photosphere is given by

$$\rho_{\text{ph}} = aT_{\text{ph}}^4 \approx \frac{8.9 \times 10^{13} T_{\text{sh}}^4 R_{\text{sh}}^3}{r^2} \approx \frac{7.1 \times 10^7 T_{\text{sh}}^4 R_{\text{sh}}^3}{(v_0 t_d)^2} \text{GeV cm}^{-3},$$

where the temperature of the photosphere is $T_{\text{ph}} = 10^4 T_d$ K, and its radius is $R_{\text{ph}} = 10^{13} R_{\text{sh}}$ cm. The radiation field from the Nova photosphere dominates at several days after the Nova explosion and, also at the recurrence period of the Nova explosion (see Fig. 2). The energy density of radiation from the Nova photosphere is given by

$$\rho_{\text{ph}} = aT_{\text{ph}}^4 \approx \frac{8.9 \times 10^{13} T_{\text{sh}}^4 R_{\text{sh}}^3}{r^2} \approx \frac{7.1 \times 10^7 T_{\text{sh}}^4 R_{\text{sh}}^3}{(v_0 t_d)^2} \text{GeV cm}^{-3},$$

The cooling time scales of electrons on the IC process (in the general case) in those different radiation fields are calculated from the approximate formula given in Sect. 2.1 in Moderski et al. (2005). We also calculate the energy losses of electrons on the bremsstrahlung process in a completely ionized hydrogen using the formula,

$$E_{\text{brem}} \approx \frac{c n_{\text{sh}} m_p E}{X_0} F \approx \frac{4.5 \times 10^{-5} M_{\odot} E_{\text{GeV}} F}{\beta^{-1} v_0^2 t_d^2} \text{GeV cm}^{-3}$$
where $X_0 = 62.8 \text{ g cm}^{-2}$ is the radiation length in a neutral Hydrogen. The correction factor, $F = \ln \left(\frac{2E}{m_e c^2}\right)/\ln 183$, is introduced for the bremsstrahlung energy losses in the case of ionized hydrogen. The detailed formula for this process can be found in Haug (2004). Note that the bremsstrahlung energy losses are usually clearly lower in respect to other energy loss processes (see Fig. 2). Then, the cooling time scale on the bremsstrahlung process can be approximated by

$$\tau_{\text{brem}} = E / \dot{E}_{\text{brem}} \approx 2.2 \times 10^4 \beta_{-1} v_{sh}^3 t_\Delta^3 / (M_6 F) \text{ s.}$$  \hspace{1cm} (11)

The energy loss time scales of electrons on different processes, their energy gains from the acceleration mechanism and the dynamical time scale at different distance of the shell from the WD are shown in Fig. 2.

Electrons, confined within the shell, lose energy on different energy loss processes. Since the conditions within the shell change significantly in time (and propagation distance), we develop a numerical code in order to calculate the equilibrium spectrum of electrons at a specific distance (and propagation time) from the WD. At first, we consider the simple scenario in which the shell is expanding freely, without any deceleration by the medium surrounding the jet or any energy gain from the wind of the White Dwarf.

Electrons are injected locally (at a specific distance $R$) with the power law spectrum and the spectral index equal to 2, with an exponential cut-off at characteristic energy $E_{\text{max}}$. The spectrum of electrons, $dN/dE/dt = A E^{-2} \exp(-E/E_{\text{max}})$, is normalized to their energy gain rate from the acceleration mechanism in the way that the total energy in relativistic electrons, injected from the beginning of the explosion up to the moment described by the time $t_{\text{max}}$, is equal to 10% of the kinetic energy of the shell. The kinetic energy of the shell is determined by $E_{\text{sh}} = 0.5 M_{\text{sh}} v_{\text{sh}}^2$.

Then, the injection rate of electrons, as a function of distance from the central engine, is obtained from

$$dN / dEdR = \frac{1}{v_{\text{sh}}} dN / dt$$  \hspace{1cm} (12)

Electrons, injected at a specific distance, suffer energy losses on different radiation processes which importance depends on physical conditions in the shell at specific distance from the Nova progenitor. Those physical conditions are defined above. We use a step space (and also time) method, in steps of $\Delta R$ (and corresponding step time $\Delta t = \Delta R / v_{\text{sh}}$), in order to determine the modification of the electron’s spectrum during propagation of the shell. The energies of electrons, at the $n$-th step and the propagation time $t_n$ of the shell, are obtained from $E_n = E_{n(-1)} - \Delta E$, where $\Delta E = \Delta t \cdot E_{\text{tot}}$, where $E_{\text{tot}}$ is the sum of the energy losses of electrons at the time $t_n$ on important energy loss processes, e.g. the IC on the RG radiation field and on the photosphere radiation field, on the bremsstrahlung process, and on the synchrotron process. The number of relativistic electrons, inside the shell, slowly builds up during the propagation of the Nova shell. However, energies of electrons drop due to their energy losses. At the every time step, also freshly accelerated electrons are added to the equilibrium spectrum of electrons calculated in the previous step.

Based on the above procedure, we calculate the equilibrium spectrum of electrons at an arbitrary moment after Nova explosion, corresponding to the specific distance of the shell from the WD. The example equilibrium spectra of electrons, at a specific time after explosion, equal to 3 days, 1 yr, 2 yrs and 15 yrs, are shown in Fig. 4. The spectra are shown for two values of the initial mass of the shell, $M_{\text{sh}} = 10^{-6} M_\odot$ (thick curves) and $10^{-7} M_\odot$ (thin), and the shell velocity $v_{\text{sh}} = 6 \times 10^8 \text{ cm s}^{-1}$. We investigate the dependence of the equilibrium spectrum of electrons on the magnetic field strength, which determines the acceleration process of electrons. The magnetic field strength is also responsible for the main energy loss mechanism. It is described by the equipartition coefficient $\alpha$. We show that, within a few days after explosion, the equilibrium spectra of electrons are steep since electrons are able to cool efficiently. This is due to the fact that soon after explosion, the radiative energy loss time scales of electrons are shorter than the dynamical time scale of the shell. Note that, these spectra can still extend up to $\sim$TeV energies. Note that, at latter stages of the shell propagation, the cut-off in the electron’s spectrum does not depend strongly on the value of the parameter $\alpha$. This is due to a relatively weak dependence of the maximum energies of injected electrons on $\alpha$ (see Eqs. 6 and 7, and also Fig. 3). We observe interesting behaviour of the electron spectra on the mass of the shell. Since the magnetic field is assumed to be at some level of the equipartition with the kinetic energy of the shell, the larger mass means also stronger magnetic field and more efficient cooling of injected electrons. Therefore, the equilibrium spectra of electrons are steeper, showing the cut-off at lower energies for larger masses of the Nova shell. However, a year after explosion the synchrotron cooling of electrons becomes limited by the dynamical time scale of the shell. Then, the spectra of electrons are well described by the power law function with the maximum energy extending up to a few tens of TeV. Therefore, we expect that the acceleration process of electrons in the Nova shell at the late time after explosion can be tested at TeV energies by the Cherenkov telescopes.

5 GAMMA-RAYS FROM THE FREELY EXPANDING SHELL

In the previous section we have calculated the equilibrium spectrum of electrons at an arbitrary time after the Nova explosion assuming that those electrons are confined within the freely expanding shell of the Nova. As we have noted above, the medium, in which the Nova shell propagates, is expected to be strongly inhomogeneous. In general, we distinguished two regions with different proprieties, in the equatorial region of the binary system and in its polar region. We apply the freely expanding shell model to describe propagation of the shell in the polar region.

We calculate $\gamma$-ray fluxes from electrons which comptonize the soft radiation produced by the Red Giant star in the binary system with the parameters mentioned in the Introduction. The distance to the Nova RS Oph is not well known. We assume the value 2.45 kpc (Rupen et al. 2008, and see discussion in Acciari et al. 2022). At the early stage of the Nova explosion, this soft radiation field is in fact dominated by the soft radiation from the Nova photosphere (defined in Sect. 4). The photosphere emission forms also dominant radiation field for electrons in the shell at the moment.
Electrons are continuously ejected with the power law spectrum with the spectral index $2$ up to a kinetic energy density of the shell with the equipartition coefficient assumed to be $\alpha$. The magnetic field in the shell is estimated from the equipartition of the magnetic field energy density with the energy of the shell. The magnetic field strength at the shell is calculated assuming the equipartition of the magnetic field energy density with the kinetic energy density of the shell. The results, for different values of the equipartition coefficient $\alpha = 0.1$ (solid curves), $10^{-2}$ (dotted), $10^{-3}$ (dashed), and $10^{-4}$ (dot-dashed) are reported. The energy loss processes of electrons are taken into account as considered in Sect. 4.

As an example, we show the $\gamma$-ray spectra from the IC scattering of the soft radiation from the Nova photosphere and the Red Giant by the equilibrium spectrum of electrons calculated at different moments after the initial Nova explosion: $t_{\text{obs}} = 3$ days (figure on the left), 1 yr (centre-left), 2 yrs (right-centre), and 15 yrs (right). In the case of 3 days and 15 yrs, the soft radiation from the Nova photosphere dominates. Electrons are continuously ejected with the power law spectrum with the spectral index $2$ up to $E_{\text{max}}$, collecting $10\%$ of the kinetic energy of the shell. The magnetic field in the shell is estimated from the equipartition of the magnetic field energy density with the kinetic energy density of the shell with the equipartition coefficient assumed to be $\alpha = 0.1$ (solid curve), $10^{-2}$ (dotted), $10^{-3}$ (dashed), and $10^{-4}$ (dot-dashed). The other parameters of the model are the same as in Fig. 4. For comparison we show the sensitivities of the Fermi-LAT (100 days, Funk et al. 2013), the MAGIC Cherenkov telescopes (50 hr sensitivity, marked by the dotted curve, Alexiš et al. 2012), the HESS (25 hrs sensitivity, marked by the dotted curve, see HESS Collaboration), and the Cherenkov Telescope Array (50hr, Maier et al. 2017).

The $\gamma$-ray spectra from the IC scattering of the soft radiation from the Nova photosphere and the Red Giant by the equilibrium spectrum of electrons calculated at different moments after the initial Nova explosion: $t_{\text{obs}} = 3$ days (figure on the left), 1 yr (centre-left), 2 yrs (right-centre), and 15 yrs (right). The velocity of the shell is $v_{\text{sh}} = 6 \times 10^5 v_9 \text{ cm s}^{-1}$. In all calculations the thickness of the shell is $\beta = 0.1$. Electrons are injected with the power law spectrum with the spectral index $2$, with an exponential cut-off at $E_{\text{max}}$, where $E_{\text{max}}$ is calculated from the comparison of the acceleration time scale with different energy loss time scales of electrons or with the dynamical time scale of the shell. The magnetic field strength at the shell is calculated assuming the equipartition of the magnetic field energy density with the kinetic energy density of the shell. The results, for different values of the equipartition coefficient $\alpha = 0.1$ (solid curves), $10^{-2}$ (dotted), $10^{-3}$ (dashed), and $10^{-4}$ (dot-dashed) are reported. The energy loss processes of electrons are taken into account as considered in Sect. 4.

The $\gamma$-ray emission from the recurrent Nova RS Oph by the MAGIC Cherenkov Observatory (Acciari et al. 2022). The features of these spectra are investigated as a function of the magnetization of the shell described by the equipartition coefficient $\alpha$. We show that observed $\gamma$-ray emission from GeV to sub-TeV energies (Fermi, HESS and MAGIC telescopes, Acciari et al. 2022, Aharonian et al. 2022) is well described for reasonable parameters of the model (see dot-dashed and dashed curves in Fig. 5 for $\alpha \sim 10^{-4} - 10^{-3}$). We also calculate the $\gamma$-ray spectra at 1 yr and 2 yrs after explosion in order to confront them with the future observations with the Cherenkov telescopes. We predict that these $\gamma$-ray spectra extend up to the TeV energy range. They stay below the sensitivity limit of the present Cherenkov telescopes unless the kinetic energy of the Nova shell is clearly larger than considered in our calculations (the mass of the shell of the subsequent explosion of the recurrent Nova. Therefore, at the early stage of shell propagation, and also at the moment of the next explosion, additional radiation from the photosphere of the Nova is taken into account when calculating the $\gamma$-ray spectrum. At the early time, electrons in the shell lose efficiently energy on the radiation processes since their radiative cooling times are shorter than the dynamical time scale of the shell. However, at latter phase, electrons lose only a part of their ejected energy. Those electrons are advected with the shell to larger distances. They finally escape into the interstellar space when the Nova shell becomes decelerated as a result of the entrainment of the interstellar matter.
and from the surrounding cosmic space. (a) The parameters of the Nova shell depend on the parameters of the RG wind and the surrounding medium. The velocity profiles for the shell of the Nova which propa- gates in the equatorial region of the binary system, are considered. Depending on the parameters of the Nova shell, its mass, initial velocity, and the solid angle of the Red Giant wind, we observe moderate or very effective deceleration of the shell due to entrainment of the matter from the RG wind. Then, in the equatorial region of the binary system, closer than the radius of the Nova, the velocity of the shell evolves according to 

\[
v_{sh}(R) = \frac{\Omega M_{sh} v_{sh}^{\text{init}}}{(\Omega M_{sh} + M_{RG}(R))} \approx \frac{v_{sh}^{\text{init}}}{[1 + 4 \times 10^{-15} M_{\odot}/v_{sh}^{\text{init}}]}.
\]

where \( \Omega \) is the spin rate of the binary system. At distances, \( R > R_{RG} \), the RG wind is not present. Then, the Nova shell moves with the constant velocity equal to \( v_{sh}(R_{RG}) \). In fact, at very large distances, the Nova shell can become again decelerated due to the entrainment of the matter from the surrounding cosmic space. For the average density of matter in the cosmic space, of the order of \( n_{\text{cos}} = 0.1 \text{ particles cm}^{-3} \), this additional deceleration is expected to become important on a sub-parsec distance scale. We include this additional effect on the shell velocity model introducing additional deceleration factor, 

\[
v(R) = \frac{M_{sh} v_{sh}^{\text{init}}}{[M_{sh} + M_{\text{cos}}(R)]} \approx \frac{v_{sh}^{\text{init}}}{[1 + 0.3 R_{sh}^{3} n_{0.1}/M_{\odot}]}.
\]

where \( v_{sh}^{\text{init}} \) is the initial velocity of the shell but already after the process of the interaction with the RG wind, \( M_{\text{cos}} = (4/3) \pi R^{3} n_{\text{cos}} m_{p} \approx 3 \times 10^{-6} R_{17}^{3} n_{0.1} M_{\odot} \) is the amount of the mass entrained from the shell into the cosmic space, and the distance of the shell from the Nova progenitor is \( R = 10^{17} R_{17} \text{ cm} \). As a result of the above processes, the velocity profile of the shell, propagating in the equatorial region of the binary system, has complicated dependence on the distance from the WD. The example profiles of the shell velocity are shown in Fig. 6. The typical parameters of the RG wind, and the surrounding medium, are considered. Depending on the parameters of the Nova shell, its mass, initial velocity, and the solid angle of the Red Giant wind, we observe moderate or very effective deceleration of the shell (see velocity profiles in Fig. 6). We conclude that, a part of the Nova shell, propagating in the equatorial region of the binary system, stays much closer to the RG (the main source of soft radiation) then the freely expending part of the shell towards the polar region. It is expected that, due to the above mentioned effect, the \( \gamma \)-ray fluxes from the IC

\[
M_{sh} = 10^{-6} M_{\odot} \text{ and } v_{sh} = 6 \times 10^{8} \text{ cm s}^{-1}.
\]

However, we note that predicted TeV \( \gamma \)-ray emission at 1 yr after explosion has a chance to be detected by the future Cherenkov Telescope Array (CTA) Observatory. We also calculate the \( \gamma \)-ray spectra 15 yrs after initial explosion, i.e. at the moment of the next explosion of the recurrent Nova (RS Oph). Then, the dominant soft radiation field is provided by the Nova photosphere. In this case, we predict that the fluxes of \( \gamma \)-rays, produced in terms of this model, are below sensitivities of the present and future Cherenkov telescopes (see Fig. 5).
process of electrons in the equatorial part of the Nova shell reach clearly larger levels.

As in Sect. 5, we calculate the equilibrium spectra of electrons within the Nova shell applying those same assumptions on the acceleration model of electrons during the shell propagation. In these calculations, we take into account the dependence of the shell velocity on the distance from the central engine of the Nova. The example calculations of those spectra are shown on Fig. 7. The equilibrium spectra of electrons, in the decelerating shell model, extends to lower energies than observed in the case of freely expending shell model. This is due to stronger magnetic fields in the shell region at a specific moment of propagation of the Nova shell. However, the equilibrium spectra of electrons, in the decelerating shell model, still extend to a few TeV.

In Fig. 8, we also calculate the γ-ray spectra from the IC process for the equilibrium spectra of electrons. In contrast to the model of free expending Nova shell, the γ-ray fluxes, for these same parameters as considered in the case of freely expending shell, i.e. strongly magnetised shells with the mass equal to $M_{sh} = 10^{-6} M_\odot$ and the energy conversion efficiency from the shell to relativistic electrons equal to 10%, are expected to be within the sensitivity limit of the CTA. At 1 year after the Nova explosion and large masses of the Nova shell, the γ-ray fluxes are expected to be still close to the 50 hrs sensitivity limits of the present Cherenkov telescopes (such as HESS, MAGIC and VERITAS). This γ-ray emission have a chance to be also constrained by the future observations in the GeV energy range by the Fermi Observatory. We conclude that observations of the GeV-TeV γ-ray emission from late stages of Nova explosions should provide useful constraints on the acceleration of leptons in the Nova shells.

We also investigate the dependence of the γ-ray emission for the broader range of the model parameters. In Fig. 9a, the γ-ray spectra are shown for different initial velocities of the Nova shell and other parameters fixed to constant values. The shell with larger velocity produce γ-ray spectra with larger fluxes. This effect is the result of a few counter-working effects. The increase of the fluxes is due to proportionally larger kinetic energy of the shell and less efficient acceleration of electrons in a weaker magnetic filed of the shell. However, due to larger shell velocity, the shell reaches larger distances from the central engine. Then, the soft radiation field, determining the IC process, is reduced. In Fig. 9b, the γ-ray spectra are shown as a function of the mass of the Nova shell. For the larger mass, the Nova shell propagates easily over larger distances since the deceleration process is less significant. Then, the magnetic field within the shell should drop to lower values. On the other hand, larger mass of the shell results in stronger magnetic field due to the equipartition arguments. Therefore, the interpretation of the obtained γ-ray spectra becomes complicated. The γ-ray spectra extend to larger energies with the mass of the shell. However, if the mass of the shell becomes very large, then it moves to large distances from the source of soft radiation (i.e. the Red Giant). Then, the efficiency of γ-ray production drops (see dotted and solid curves in Fig. 9b). In Fig. 9c, we show the dependence of the γ-ray spectra on the solid angle of the RG wind. For larger solid angle, a larger part of the shell (and so its kinetic energy) is able to interact with the RG wind. Therefore, the energy available for the acceleration of electrons becomes larger. From another site, a part of the shell within the equatorial region of the RG wind propagates over larger distances during this same time scale, since the amount of the matter entrained from the Red Giant wind is fixed. This results in a weaker magnetic field in the shell region and in consequence larger energies of accelerated electrons. Therefore, the γ-ray spectra extends to larger energies with larger solid angle in which the RG wind is confined. Finally, in Fig. 9d we investigate the dependence of the γ-ray spectra on the parameters which determine the acceleration process of electrons. We compare the γ-ray spectra for two different prescriptions for the acceleration parameter ξ, i.e. depending linearly $\xi = v_{sh}/c$, or quadratically on the shell velocity (see Eq. 3). It is found that for more efficient acceleration, the γ-ray spectra do not extend proportionally to the value of ξ. This effect is due to the fact that at a few years after explosion the bulk of electrons within the shell reach the maximum energies which are determined by the dynamical time scale of the shell but not by their synchrotron energy losses. However, in the case of both prescriptions for the acceleration efficiency, γ-rays in the TeV energy range are still expected provided that the parameter α is relatively large ($\alpha = 0.01$). From Eq. 7, it is clear that electrons are not able to produce TeV γ-rays for the case of weakly magnetized shells of Novae (i.e. $\alpha < 0.01$). We also show the γ-ray spectrum obtained in the case of electrons injected with the steeper power law spectrum then considered in the above calculations, i.e. spectral index equal to 2.5. As expected, the γ-ray spectra become also steeper in such a case. They might become two weak at TeV energies to be detected by the Cherenkov telescopes. We conclude that, at the late stage of the Nova expansion, γ-ray emission can be easier constrained by the γ-ray telescopes in the case of Novae which shells are moving with larger velocities, having larger injection masses, and which equatorial winds of their Red Giants are confined within larger solid angles. Moreover, the ejection process of electrons should operate for at least a few years and the spectra should be relatively flat, i.e. spectral indexes close to 2, as expected in the case of strong shocks.

7 DISCUSSION AND CONCLUSION

We investigate the problem of the time scale for the acceleration of electrons in shells of recurrent Novae. Recent observations of the γ-ray emission from several Novae in the GeV energy range, and also sub-TeV energies in the case of the Nova RS Oph (see references in the Introduction), indicate that such energetic emission is observed typically up to several (or in some cases to a few tens) of days after initial explosion of Nova. This is clearly different than observed in the case of supernovae, in which γ-ray emission becomes detectable at latter stages of expansion of the supernova shell. We wonder whether such persistent acceleration of particles is also present in the case of Nova shells. In order to put light on this question, we calculate the γ-ray IC emission expected from electrons accelerated in the expanding Nova shell at the late time after explosion. As an example, we consider in a more detail the case of the Recurrent Nova RS Oph, recently observed by the Cherenkov telescopes at a few days after explosion. A simple, geometrical two component
As in Fig. 4 but for the decelerating Nova shell, which is decelerated due to the interaction with the wind from the RG companion. The model for the shell deceleration is defined in Sect. 7. The electrons, with the equilibrium spectrum, up-scatter the soft radiation from the RG at 1 yr (left figure) and 2 years (centre) after explosion. The radiation from the Nova photosphere, produced during the subsequent flare, dominates over the radiation from the RG at 15 yrs after the Nova explosion. It is assumed that the RG wind is confined within the $\Omega_{RG} = 0.3$ of the solid angle in the equatorial region of the binary system. The mass loss rate of the RG is $M_{RG} = 10^{-7} M_\odot$ yr$^{-1}$ and the RG wind velocity is 40 km s$^{-1}$. The specific $\gamma$-ray spectra are calculated for the coefficient $\alpha = 0.1$ (solid curve), $10^{-2}$ (dotted), $10^{-3}$ (dashed), and $10^{-4}$ (dot-dashed).

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig7}
\caption{As in Fig. 4 but for decelerating Nova shell. The $\gamma$-ray spectra are calculated for the parameters of the equilibrium spectrum of electrons as shown in Fig. 7.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig8}
\caption{The $\gamma$-ray spectra produced in terms of the decelerated shell model, as considered in Figs. 7 and 8, at 2 yrs after initial explosion of Nova. (a) The dependence on the velocity of the shell $v_{sh} = 10^8$ cm s$^{-1}$ (solid curve), $3 \times 10^8$ cm s$^{-1}$ (dotted), $6 \times 10^8$ cm s$^{-1}$ (dashed), $9 \times 10^8$ cm s$^{-1}$ (dot-dashed). The mass of the shell is fixed on $M_{sh} = 10^{-6} M_\odot$ and the parameter $\alpha = 0.01$. (b) The dependence on the mass of the expanding shell of Nova $M_{sh} = 10^{-5} M_\odot$ (solid), $10^{-6} M_\odot$ (dotted), and $10^{-7} M_\odot$ (dashed). The other parameters are $v_{sh} = 6 \times 10^8$ cm s$^{-1}$, $\Omega_{RG} = 0.3$ and $\alpha = 0.01$. (c) the dependence on the solid angle of the Red Giant wind for $\Omega_{RG} = 0.1$ (solid), 0.5 (dotted), and 0.9 (dashed), for $M_{sh} = 10^{-6} M_\odot$, $v_{sh} = 3 \times 10^8$ cm s$^{-1}$ and $\alpha = 0.01$. The spectral index of injected electrons is fixed on 2. (d) Different prescriptions for the acceleration coefficient, $\xi = v_{sh}/c$ (solid) and $\xi = 0.1(v_{sh}/c)^2$ (dashed). The dotted curve shows the $\gamma$-ray spectra for the spectral index of ejected primary electrons equal to 2.5. The other parameters are $v_{sh} = 6 \times 10^8$ cm s$^{-1}$, $\Omega = 0.3$ and $\alpha = 0.01$.}
\end{figure}
model is elaborated for the expansion of the shell in the medium surrounding the Nova. In the equatorial region of the binary system, the shell efficiently decelerates due to the interaction with the Red Giant wind. At the polar regions, the shell expands with a constant velocity.

We investigate a simple time dependent model for the acceleration of electrons and their energy losses. In terms of such a model, the equilibrium spectra of electrons are calculated at an arbitrary time after Nova explosion. We show that electrons can be accelerated to TeV energies in the late stage of the Nova. They produce TeV $\gamma$-rays in the inverse Compton scattering process of the RG soft radiation or the soft radiation from the Nova photosphere during the time of its domination, i.e. at several days after explosion or at the moment of the next explosion of the recurrent Nova (in the case of RS Oph it is $\sim 15$ yrs). The IC $\gamma$-ray emission, calculated at a few days after explosion, has the features similar to that observed by the Fermi at GeV energies and the Cherenkov telescopes at sub-TeV energies.

We show that $\gamma$-ray fluxes, produced in terms of such an extended in time acceleration model of electrons, could be detectable by the planned CTA Observatory for specific conditions in the Nova shell. In general, the $\gamma$-ray emission from a part of the shell, propagating within the solid angle of the RG wind (equatorial region of the binary system), is more likely to be observed. In fact, this part of the shell becomes significantly decelerated staying relatively close to the soft radiation from the RG. We predict that the CTA should be able to detect TeV $\gamma$-ray emission from the equatorial part of the shell of RS Oph even up to the moment of the next explosion (i.e. $\sim 15$ yrs), provided that the mass of the shell is of the order of $\sim 10^{-6} M_\odot$, the initial velocity of the shell is $v_{\text{sh}} = 6 \times 10^8$ cm s$^{-1}$, and the magnetic field strength in the shell is not very far from the equipartition with the kinetic energy of the shell. At the time of $\sim 1$ yr after explosion, the predicted $\gamma$-ray fluxes are even close to the sensitivity of the present Cherenkov telescopes. Note however, that in this case the acceleration efficiency of electrons can be also constrained at GeV energies by the future observation with the Fermi Observatory. We also show that $\gamma$-ray fluxes, expected in terms of such acceleration model of electrons in shells of Novae, are easier to detect in the case of a more geometrically extended RG wind, since in such a case larger part of initially spherical shell of the Nova is decelerated by the RG wind.

Almost freely expanding part of the Nova shell (in the polar regions of the binary system), is expected to produce $\gamma$-ray fluxes generally on a lower level than considered above decelerated part of the shell. This is due to the larger distance of the shell from the soft radiation field of the RG, or from the photosphere resulted in the next explosion of the recurrent Nova. In the case of a freely expanding shell, our model predicts $\gamma$-ray fluxes, which are above the sensitivity limit of the CTA, at the time shorter than $\sim 1$ yr (for optimum parameters of the shell).

The $\gamma$-ray fluxes, produced in the model of a freely expanding shell, can be additionally dissolved in time due to the difference in distances of specific parts of the shell to the observer (the propagation times of produced $\gamma$-rays may significantly differ). This time delay is of the order of $t_{\text{diff}} = R_{\text{sh}}/c = t_{\text{sh}} v_{\text{sh}}/c \approx 3.65 t_{\gamma} v_8$ days, where $t_{\gamma} = 3 \times 10^7 t_{\text{yr}}$ s is the propagation time of the shell. This effect is important in the case of the $\gamma$-ray emission from the shell at the moment of the recurrence time of the Nova, when the radiation from the photosphere of the next RS Oph Nova explosion dominate over the radiation from the Red Giant.

Although, we took into account energy losses of electrons on different radiation processes, we calculated the $\gamma$-ray spectra only from the Inverse Compton process. In fact, other radiation processes can even dominate as the main energy loss process of electrons with specific energies or at specific time after explosion. They are not expected to contribute significantly to the GeV-TeV $\gamma$-ray energy range. For example, the bremsstrahlung process is important only for electrons with energies below a few GeV, but only at a few days after explosion (see Fig. 2). On the other hand, the synchrotron process becomes the main energy loss process for electrons at the largest (multi-TeV) energies at early time after explosion, when the maximum energies of electrons are due to the synchrotron energy losses (see Figs. 2 and 3). At late time after Nova explosion, the maximum energies of electrons are due to the balance between the time scale for the electron acceleration and the dynamical time scale of the expending shell (Fig. 3). Then, IC energy losses of electrons dominate over the energy losses on the synchrotron process (see Fig. 2). For typical energies of electrons, $E = 1E_{\text{TeV}}$ TeV, and the strengths of the magnetic field, $B = 10^{-3}$ $B_{\text{mag}}$ G, at $2$ yrs after Nova explosion (see Eq. 2), the synchrotron emission falls into the UV to soft X-ray energy range, $\varepsilon_{\text{syn}} \approx m_e c^2(B/B_{\text{mag}})\gamma B^{3} \approx 46B_{\text{mag}}E_{\text{TeV}}^{3} eV$, $B_{\text{mag}} = 4.4 \times 10^{13}$ G is the critical magnetic field strength. This radiation can be over-come by either the hard thermal X-ray emission, with the peak luminosity of the order of $\sim 10^{33} - 10^{34}$ erg s$^{-1}$, produced by the hot gas heated by the shocks within the shell (Mukai et al. 2008, Gordon et al. 2021), or by the super-soft X-ray emission from the WD surface (Kahabka & van den Heuvel 1997).

In the considered here two-component geometrical model, we do not take into account the modification of the electron spectrum due to the adiabatic expansion (or contraction) of the shell. We neglected this effect since it is difficult to predict how the shell behaves during its propagation in the medium surrounding the recurrent Nova. The shell might naturally expend due to internal pressure, but it might also contract due to the external pressure of either the surrounding interstellar medium, or the wind from the White Dwarf. Therefore, we leave this complicated problem for the future more detailed modelling.

In our modelling, we use simple prescription for the magnetic field during the propagation of the Nova shell. In fact, if the surrounding of the shell is strongly inhomogeneous, then the downstream turbulent magnetic field would grow to milliGauss (see e.g. Giacalone & Jokipii 2007, Inoue et al. 2012; Fraschetti 2013), which can be above the values predicted by our simple prescription in the case of low value of the magnetization parameter $\alpha$ at a few years after the Nova explosion. Then, the faster synchrotron cooling might prevent acceleration of electrons to TeV energies (Vink & Laming 2003). As a result, the $\gamma$-ray spectra, predicted for low values of $\alpha$ (and years after explosion), might be tested only in the GeV energy range by the satellite observatories.
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DATA AVAILABILITY

The simulated data underlying this article will be shared on reasonable request to the corresponding author.

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