The Roots of the Standard Model of Particle Physics

P.J. Mulders

Nikhef Theory Group and Department of Physics and Astronomy, VU University Amsterdam, De Boelelaan 1081, NL-1081 HV Amsterdam, the Netherlands

We conjecture how the particle content of the standard model can emerge starting with a supersymmetric Wess-Zumino model in 1+1 dimensions (d = 2) with three real boson and fermion fields. Considering SU(3) transformations, the lagrangian and its ground state are SO(3) invariant. The SO(3) symmetry extends the basic IO(1,1) Poincaré symmetry to IO(1,3) for the asymptotic fields of quarks and leptons. Striking connections between SU(3) invariant and the proton excess balances the electroweak and strong sectors are the vanishing of SU(2) ⊗ U(1) symmetry remains, broken down as in the standard model. The boson excitations in d = 4 are identified with electroweak gauge bosons and the Higgs boson. Fermion excitations come in three families of quarks living in E(1,3) Minkowski space or three families of quarks living in E(1,1). Many features of the standard model now emerge in a natural way. The supersymmetric starting point solves the naturalness problem. The underlying left-right symmetry leads to custodial symmetry in the electroweak sector. In the spectrum one has Dirac-type charged leptons and Majorana-type neutrinos. The electroweak behavior of the naturally confined quarks, leads to fractional electric charges and the doublet and singlet structure of left- and right-handed quarks, respectively. Most prominent feature is the link between the number of colors, families and space directions.

II. THE STARTING POINT

As starting point we take one space dimension (1D world) with a d = 2 Poincaré symmetry IO(1,1). We take a field theoretical route rather than a string theoretical one. The Poincaré symmetry is central in the Hilbert space, with Hamiltonian and momentum operator generating time and space translations and boosts transforming among momentum eigenstates. With only one time and one space direction, states live in an E(1,1) Minkowski space with coordinates xμ and metric x2. When appropriate, we will use light-cone components employing light-like vectors n = n+ = (1,1) and n− = (1,1), thus a·n = a+ and a·n = a−. The quantum states |k⟩ in the free theory are associated with the modes of field oscillations around the classical (minimum energy) solution, for free fields eigenstates of the momentum operator P. Together with the boost operator K, the operators H, P (combined into Pμ) and K generate the 2-dimensional Poincaré symmetry group IO(1,1), [H, P] = 0, [K, H] = iP and [K, P] = iH or [P+, P−] = 0 and [K, P±] = ±iP±, with Casimir operator P2 = PμPμ = H2 - P2 = P+P−. The IO(1,1) symmetry can be combined with an SO(N) symmetry to

I. INTRODUCTION

The standard model of particle physics is highly successful incorporating besides gravity all known particles and their interactions. The theoretical framework is a renormalizable gauge theory even if the gauge symmetry is a rather ad hoc combination of an unbroken SU(3) symmetry group with a spontaneously broken SU(2) ⊗ U(1) symmetry group for strong and electroweak interactions. It requires besides different coupling constants also a multitude of parameters governing the coupling with the Higgs sector, responsible for the electroweak symmetry breaking and the mixings and masses of quarks and leptons. Striking connections between the electroweak and strong sectors are the vanishing of baryon minus lepton number (the proton excess balances the electroweak and strong sectors are the vanishing of quarks and leptons. Striking connections between

*Electronic address: mulders@few.vu.nl
obtain the $IO(1,N)$ space-time symmetry with as generators $H$, $P^i$, $K^i$ and $J^{ij}$ (combined into $P^\mu$ and $J^{\mu
u}$, of course also including discrete space- and time-reversal symmetries.

For massless excitations in 1D, right-movers (depending on $x^+$) are independent from left-movers (depending on $x^-$). Right- and left-handed fields satisfy $[P^-,\phi_R] = i\partial_-\phi_R$, and $[P^+,\phi_L] = i\partial_+\phi_L = 0$. For massive fields left and right modes become coupled, while the other derivatives $[P^+,\phi_R] = i\partial_-\phi_R$ and $[P^-,\phi_L] = i\partial_+\phi_L$ acquire roles as (front form) canonical momenta [2]. For $M = 0$ the fermion fields in $d = 2$ satisfy $\gamma^{-}_L\xi_L = \gamma^{-}_L\xi_L = 0$ and $\xi_{R/L}$ are independent good fields [3]. Massive fermion fields satisfy the constraints $[P^-,\xi_R] = i\partial_-\xi_R = -iM\xi_L$ and $[P^+,\xi_L] = i\partial_+\xi_L = iM\xi_R$.

The $d = 2$ Poincaré algebra in $E(1,1)$ can be extended to a supersymmetric algebra (for a review see Ref. [4]) with anti-commuting fermionic operators $Q_{R/L}$,

\[
\{Q_R, Q_R^\dagger\} = 2P^+, \quad \{Q_L, Q_L^\dagger\} = 2P^-, \\
\{P^\pm, Q_R^\dagger\} = 0, \quad [K, Q_{R/L}] = \pm \frac{i}{2}Q_{R/L}.
\]

Supersymmetry connects the fields, $\{Q_{R/L}, \phi_{R/L}\} = \xi_{R/L}$ and $\{Q_{R/L}, \xi_{R/L}\} = [P^\pm, \phi_{R/L}]$. We will first consider one type of $R/L$ fields ($N = 1$) in a single space dimension and then extend this to a set of (three) real scalar and real fermionic (Majorana) fields $\phi$ and $\xi$. If the masses are zero, right-movers ($R$) and left-movers ($L$) are independent degrees of freedom: for bosons a simple doubling; for fermions coinciding with right- and left-handed fermions. Starting for $N = 1$ with the Wess-Zumino model [5] in two dimension,

\[
\mathcal{L} = \frac{1}{2}\partial_\mu\phi_R\partial^\mu\phi_R + \frac{1}{2}\partial_\mu\phi_L\partial^\mu\phi_L \\
+ \frac{1}{4}\xi_R\partial_\mu\xi_R + \frac{1}{4}\xi_L\partial^\mu\xi_L - V(\phi,\xi) \\
= \frac{1}{2}\partial^\mu\phi_R\partial_\mu\phi_R + \frac{1}{2}\partial^\mu\phi_L\partial_\mu\phi_L + i\overline{\psi}\gamma^\mu\psi - V(\phi,\xi),
\]

(real) right and left fields for bosons which can be combined into (real) scalar (CP-even) and pseudoscalar (CP-odd) fields $\phi_{R/L} = (\phi_R \pm \phi_L)/\sqrt{2}$. Real fermion fields can be combined in a (self-conjugate) spinor $\psi = (\xi_R, -i\xi_L)/\sqrt{2}$. Supersymmetry strongly restricts the interaction terms. The most compact expression is in terms of the scalar and pseudoscalar fields containing a mass term coupling left and right fields and a single Yukawa coupling that also governs the fermion-boson coupling,

\[
V(\phi,\xi) = \frac{1}{2}(M + g\phi_S)^2(\phi^2_S + \phi^2_P) + \frac{1}{2}g^2\phi^2_P(\phi^2_S + \phi^2_P) \\
+ \overline{\psi}(M + g\phi_S + g\phi_Pn^L)\psi + \lambda F
\]

using $\gamma_5 = \gamma^0\gamma^1$. The constraint is given by

\[
\lambda F = \frac{\lambda}{4g}\left((M + 2g\phi_R\sqrt{2})(M + 2g\phi_L\sqrt{2}) - M^2\right) \\
= \frac{\lambda}{g}\left((g\phi_S + M/2)^2 - g^2\phi_P^2 - M^2/4\right).
\]

Defining $M/2g = \nu$, we introduce fields $\phi_S + \nu = \nu\Phi_S$ and $\nu\Phi_P = \phi_P$ which can be re-defined as $\Phi_S = \cosh\eta$ and $\Phi_P = \sin\eta$ or if one likes one can use an imaginary representation for $\Phi_P$ by writing $\eta = i\theta$. The bosonic part of the potential including constraint becomes

\[
V(\Phi) = \frac{\nu^2M^2}{2}\Phi^2_S\Phi^2_P = \frac{\nu^2M^2}{2}\Phi^2_S(\Phi^2_S - 1)
\]

or $V(\Phi) = \frac{1}{2}\mu^2v^2\sinh^2(2\eta)$. Defining $|\Phi|^2 = (\Phi^2_S + \Phi^2_P)/2 = \Phi^2_R + \Phi^2_L$, we have $\Phi^2_R = \cosh^2(2\eta)$ and we have $\Phi^2_P = |\Phi|^2 + 1/2$ and $\Phi^2_P = |\Phi|^2 - 1/2$. Looking at the minimum of the potential $(\eta = 0 \text{ or } \theta = 0)$ we see that the boson field acquires a vacuum expectation value which is right-left symmetric, $\Phi_R = \Phi_L = |\Phi| = 1/\sqrt{2}$ (or $\Phi_S = 1$ and $\Phi_P = 0$). The real excitations around the vacuum are Majorana modes $\Psi = \Psi^c = (\xi, -i\xi)/\sqrt{2}$ and real scalar bosonic modes $\phi_S/\sqrt{2} = \Phi = \Phi^c = (1 + H)/\sqrt{2}$. Note that $\phi_S = vH$. The 1D pseudoscalar field $\phi_P$ can be identified as a vector field writing $i\partial_\mu\Phi_{R/L} = (i\partial_\mu \pm gA_\mu)/(\sqrt{2})$. In the ground state $A_\mu = 0$ and around the vacuum one has $A^\mu = \phi_P(n^\mu - i\bar{n}^\mu)$ or $A^\mu = -A^\mu = \phi_P$. This suggests working with a complex field $\Phi$ rather than left and right fields that are CP symmetric, $\Phi_R = \Phi^*_L$. For a single field a global $U(1)$ symmetry is not relevant and local symmetries don’t lead to dynamics either, but taking multiple scalar fields the symmetry pattern becomes much richer.

### III. EXTENSION TO THREE FIELDS

The symmetric extension to $N$ real boson and fermion fields (we take $N = 3$), $\phi = (\phi_1, \phi_2, \phi_3)$, has interesting consequences for the dynamics, which is studied by looking at the possible fluctuations around the vacuum, in the symmetric basis $\langle\Phi\rangle^T = (1, 1, 1)/\sqrt{3}$. Including complex phases we consider $SU(3)$ fluctuations, although the lagrangian is only invariant under $SO(3)$ transformations of the fields, which is also the symmetry of the groundstate. We propose to use the $SO(3)$ symmetry in combination with inversion and time reversal symmetry, to extend the $d = 2$ Poincaré symmetry to a $d = 4$ Poincaré symmetry. Implemented in Weyl mode, the asymptotic fields become real representations of $IO(1,3)$ living in $E(1,3)$.

At this stage, part of the freedom in fluctuations around the vacuum has been incorporated. The already accounted for real $SO(3)$ rotations are identified with the subalgebra generated by the $SU(3)$ generators $\lambda_2$, $-\lambda_5$ and $\lambda_7$, constituting the algebra of the factor group of the subgroup $G' = SU(2) \times U(1)$ with generators $\lambda_2/2$, $\lambda_3/2$, $\lambda_3/2$ and $\lambda_3/2$. This subgroup contains the $SU(3)$ Cartan subalgebra consisting of $I_3 = \lambda_3/2$ and $Y = \lambda_8\sqrt{3}$ that will serve as electroweak charge labels for weak isospin and hypercharge. Labelling the (massless) bosonic states using this Cartan subalgebra, gives fields $\phi^{I_3, Y}_{R/L}$ ($x,t$) living in $E(1,3)$. For two fields this would have been just a $U(1)$ charge assignment. The basic bosonic starting point for the three fields and their
electroweak quantum numbers is illustrated in Fig. 1.

To account for the fluctuations around the vacuum, we look at the covariant derivatives,

\[ E(1, 1): \quad iD_\mu \Phi^i = i\partial_\mu \Phi^i + g \sum_{a \in G} A^a_\mu (T_a)_{ij} \Phi^j, \quad (7) \]
\[ E(1, 3): \quad iD_\mu \Phi^i = i\partial_\mu \Phi^i + g \sum_{a \in G} A^a_\mu (T_a)_{ij} \Phi^j. \quad (8) \]

The first expression applies to fields in \( E(1, 1) \) and accounts for local \( SU(3) \) gauge invariance. It involves eight (color) gauge fields also living in \( E(1, 1) \). The second expression is relevant for (asymptotic) fields in \( E(1, 3) \). Coupling for real continuous \( SO(3) \) transformations the field and space rotations, there are no gauge fields for that part leaving only the complex transformations involving four (electroweak) gauge fields living in \( E(1, 3) \).

The embedding of \( SO(3) \) directions into \( SU(3) \) is not unique. The discrete symmetry group \( A_4 \) governs the possible oriented embeddings. For singlet representations of this embedding group one can consider \( SU(3) \supset SO(3) \times A_4 \times [SU(2) \otimes U(1)] \rightarrow SO(3) \otimes [SU(2) \otimes U(1)] \), decoupling space-time and internal symmetries [9]. The unitary transformation matrix for these singlet states [7-10] is the matrix \( W \) that rotates the symmetric embedding of the vacuum into an electroweak embedding,

\[
W = \begin{pmatrix}
\sqrt{1/3} \\
\sqrt{1/3} \\
\sqrt{1/3}
\end{pmatrix}
\begin{pmatrix}
0 \\
1 \\
0
\end{pmatrix}
\]

where \( \omega = \exp(i\frac{2\pi}{3}) \). Since the starting point only had \( SO(3) \) as a symmetry group, the vacuum indeed is not invariant under \( SU(2) \otimes U(1) \) transformations, but it is neutral for \( Q = I_3 + Y/2 \). The symmetry pattern and its breaking thus is summarized as

\[
\text{IO}(1, 1) \otimes SU(3) \supset \text{IO}(1, 1) \times SO(3) \otimes SU(2)_Y \otimes U(1)_Y.
\]

All bosons and fermions, however, still do originate as (finite dimensional) representations of the basic \( SU(3) \) symmetry group, which will become important later. There are three families of particles corresponding to the singlets of \( A_4 \). Going to three space dimensions the interaction changes from a confining potential to a \( 1/r \) (or Yukawa) potential between the (electroweak) charges, which thus can be free, in contrast to the (color) \( SU(3) \) charges in one space dimension.

**IV. ELECTROWEAK SECTOR**

After the introduction of the covariant derivatives, part of the potential is included in the term

\[
D^\mu \Phi^* D_\mu \Phi = \partial^\mu \Phi^* \partial_\mu \Phi + \frac{N C_A}{2} g^2 A^\mu A_\mu \Phi^* \Phi (10)
\]

With \( C_A(G = SU(3)) = 4/3 \), the second term in Eq. (10) is precisely \(-V(\Phi)\) and we are left with the 1 + 1 dimensional QCD lagrangian (without a Higgs mass-term),

\[
\mathcal{L} = \frac{1}{2} \partial^\mu \Phi \partial_\mu \Phi - \frac{1}{4} F^{\mu \nu} F_{\mu \nu} + \bar{\Psi}(i \partial - M - g v H) \Psi, \quad (11)
\]

but with a scalar field, which does not seem harmful [11]. We will first consider the electroweak structure of the lepton sector before returning to that of the colored fermions (quarks).

For the second option of the covariant derivative (Eq. [8], we have \( C_A(G') = 2/3 \), the second term in Eq. (10) is only \(-V(\Phi) / 2 \). This leaves the scalar field massive with \( M_H = M / \sqrt{2} \). This is an experimentally interesting scenario for the standard model if the fermion mass is identified with the top quark mass \( M = M_t \).

For the bosons, we have (in principle arbitrarily) assigned right to the triplet and left to the anti-triplet. The fields can be rotated into a single scalar field with a nonzero vacuum expectation value as is done in the usual standard model treatment, even if they form triplets,

\[
\Phi_L = \frac{1}{\sqrt{2}} \exp(-i \sum_{a=1,2,3} \theta^a \lambda_a) \begin{pmatrix} 0 \\ 1 & H \\ 0 \end{pmatrix},
\]
\[
\Phi_R = \frac{1}{\sqrt{2}} \exp(i \sum_{a=1,2,3} \theta^a \lambda_a) \begin{pmatrix} 1 & H \\ 0 & 0 \end{pmatrix}.
\]

The electroweak charges and corresponding generators of gauge transformations are identified with the \( SU(2)_Y \otimes U(1)_Y \) transformations but with a single coupling constant within \( SU(3) \). The charged fields are neither \( I_3 = \lambda_3 / 2 \) or \( Y = \lambda_8 \sqrt{3} \) eigenstates but they are eigenstates
of $Q = I_3 + Y/2$. The breaking of the $SU(2)_I \times U(1)_Y$ symmetry to $U(1)_Q$ after the choice of ground state being neutral, produces three massive and one massless gauge boson. As discussed in a slightly different context [12], the $SU(3)$ embedding gives a weak mixing angle, $\sin \theta_W = 1/2$ after rewriting in

$$iD_\mu \Phi = i \partial_\mu \Phi + \frac{g}{2} \left( \sum_{i=1}^{3} W_\mu^i \lambda_i + B_\mu \lambda_8 \right) \Phi$$

(12)

the neutral combination $g W_\mu^0 I_3 + (g/2\sqrt{3}) B_\mu Y$ in terms of $Z^\nu$ and $A^\nu$. One obtains (using the dimensionful coupling in $d = 2$) $e = g/2$ and masses $M^2_W = M^2/4$, $M^2_Z = M^2_W / \cos^2 \theta_W = M^2/3$ and $M^2_T = 0$. In zeroth order, the weak mixing is fine and the Higgs mass and gauge boson masses are related and they are of the right order with $M = M_4$. Taking $v = M/2g = 1$, one even is tempted to compare $e/M = 1/4$ with $\sqrt{4\pi\alpha} \approx 0.3$. Besides providing a global zeroth order picture for electroweak bosons, we note that the left-right symmetric starting point also ensures custodial symmetry [13] [14].

For the fermionic excitations, the starting $SU(3)$ triplets $\xi_R$ and anti-triplets $\xi_L$ in 1D match those of the bosons, implying the underlying supersymmetry of the elementary fermionic and bosonic $d = 2$ excitations. Also in this case one fixes one direction for the $SU(3)$ representations (the $SO(3)$ embedding) and uses the (remaining) symmetry to fix the electroweak structure as an $SU(3)$ triplet or anti-triplet. The fermions then have electroweak charges corresponding to isospin doublets and singlets as shown in Fig. 2. We already mentioned the possible role of the $A_4$ embedding symmetry in the family structure of fermions, which allows three independent families. Besides the matrix $W$ that transforms between symmetric and electroweak basis,

$$Q_s = \frac{1}{\sqrt{3}} \begin{pmatrix} 0 & i & -i \\ -i & 0 & i \\ i & -i & 0 \end{pmatrix} = W \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix} W^\dagger.$$ 

(13)

one needs to transform Majorana fermions ($\xi_1, \xi_2, \xi_3$) into charged fermions ($\xi^+, \xi^0, \xi^-$), which we do by mixing $\xi_1$ and $\xi_3$ in the symmetric basis, such that

$$Q_{ew} = \begin{pmatrix} 0 & 0 & i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix} = V_Q \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix} V_Q^\dagger.$$ 

(14)

This shows that $Q_s = U_{HPS} Q_{ew} U_{HPS}^\dagger$ in which the tri-bimaximal mixing matrix [15] appears, $U_{HPS} = W V_Q^\dagger$.

$$U_{HPS} = \begin{pmatrix} \sqrt{2/3} & \sqrt{1/3} & 0 \\ -\sqrt{1/6} & \sqrt{1/3} & -\sqrt{1/2} \\ -\sqrt{1/6} & \sqrt{1/3} & \sqrt{1/2} \end{pmatrix}.$$ 

(15)

This looks like a promising zeroth order description for leptons providing arguments for the role of the discrete symmetry group $A_4$, which is a subgroups of both SO(3) and SU(3), in the structuring of families and the mixing matrices. The details of this, however, need to be worked out.

Finally note that (without looking at the role of the masses) the extension of 1D fermion fields leads to 3D 'good' light-front fields $\Psi = (\xi_R, -\xi_L)$ with two-component spinors $\xi_R/L$ and $\epsilon = i\sigma^2$. The rotations are represented by $J = \sigma/2$, boosts by $K = \pm i\sigma$ for right and left fields, respectively (thus $n^\nu \rightarrow \sigma^\nu$ and $\bar{n}^\nu \rightarrow \bar{\sigma}^\nu$). The coupling of fermions to the pseudoscalar fields, combined into a 3D vector field, becomes the $\overline{\Psi} A \Psi$ coupling.

V. STRONG SECTOR

The fermionic modes $\xi$ can also just live in $E(1, 1)$ and be arranged in three families of $SU(3)_C$ color triplets, which are identified as colored quarks but living in $E(1, 1)$ where color is confined via the instantaneous confining linear potential of the gauged $SU(3)$ symmetry. In order to study the electroweak structure of quarks (their valence nature) one has to study their interactions with the electroweak gauge bosons. We propose to do this by mapping the structure of the excitations into three spatial directions in a frozen color scheme in which we just consider fermions of one particular color (say $r$). Take the case of all $\xi_R$ states with color $r$ and all $\xi_L$ being $\bar{r}$. Taking a step back and looking at what was done in order to find leptons where the frozen colors were in essence space dimensions. The one-dimensional state would be labeled by a single momentum component, which is extended to states labeled by a 3-dimensional momentum vector in $E(1, 3)$. For two space dimensions, the fermions could be labeled by their helicity in $E(1, 2)$.
VI. CONCLUSIONS

Concluding, instead of extending the standard model of particle physics, I have described an attempt to start a minimal scenario is created to obtain the standard model of particle physics with also in 3D elementary fields, while confinement of color is implicit. Most prominent is that it links the number of colors, families and space directions. The Higgs or top quark mass are the natural basic scales for wave-lengths of the one-dimensional excitations producing the right orders of magnitude for masses of top quark, Higgs particle and gauge bosons. There are many details that need to be investigated to see if the proposed scheme can be made consistent, the embedding mechanism for the family structure, the origin of mixing matrices, the emergence of the scale of QCD, etc. The conjectures as put forward here will likely not invalidate the existing highly successful field theoretical framework for the standard model. Hopefully a more explicit treatment could provide ways to calculate its parameters. The 1D starting point for the strong sector also may provide insights why and to what extent descriptions like the AdS/QCD correspondence (see e.g. Ref. [17]), collinear effective theories (see e.g. review in Ref. [18]) or the many effective theories for QCD at low energies work. The link with the family structure might provide handles on universality breaking effects such as the 'proton radius puzzle'. The reason

TABLE I: Fermionic excitations with their assignments in $SO(1,1) \times SU(3)_C$ and $SO(1,3) \times SU(2)_L \times U(1)_Y$ symmetry schemes. The column labeled space $L$ indicates one of the basic (left/right) modes of the $d = 2$ theory (the colored fermionic modes). The columns labeled $(T_1, T_2)$ contain charge eigenstates of two basic modes that can be combined with the $L$-mode, giving allowed $I_3$ states within $SU(3)$.

| $L$ | space | $T_1$ | $T_2$ | $I_1$ | $I_2$ | $Y$ | $Q$ | $R$ |
|-----|-------|-------|-------|-------|-------|-----|-----|-----|
| $\nu_L$ | $\xi^0_L$ | $\xi^0_L$ | $\xi^0_L$ | 1/2 | 1/2 | -1 | 0 | 1/2 |
| $\nu_R$ | $\xi^0_R$ | $\xi^0_R$ | $\xi^0_R$ | 1/2 | -1/2 | -1 | -1 | 1/2 |
| $\epsilon^+_L$ | $\xi^+_L$ | $\xi^+_L$ | $\xi^+_L$ | 0 | 0 | +2 | +1 | 1/2 |
| $\epsilon^+_R$ | $\xi^+_R$ | $\xi^+_R$ | $\xi^+_R$ | 1/2 | 1/2 | +1 | +1 | 1/2 |
| $\epsilon^-_R$ | $\xi^-_R$ | $\xi^-_R$ | $\xi^-_R$ | 0 | 0 | -2 | -1 | 1/2 |
| $u_L$ | $\xi^0_L$ | $\xi^0_L$ | $\xi^0_L$ | 1/2 | +1/2 | +1/3 | +2/3 | 3 |
| $d_L$ | $\xi^-_L$ | $\xi^-_L$ | $\xi^-_L$ | 1/2 | -1/2 | +1/3 | -1/3 | 3 |
| $\bar{u}_L$ | $\xi^+_L$ | $\xi^+_L$ | $\xi^+_L$ | 0 | 0 | -4/3 | -2/3 | 3 |
| $\bar{d}_L$ | $\xi^0_L$ | $\xi^0_L$ | $\xi^0_L$ | 0 | 0 | +2/3 | +1/3 | 3 |
| $u_R$ | $\xi^0_R$ | $\xi^0_R$ | $\xi^0_R$ | 1/2 | -1/2 | -1/3 | -2/3 | 3 |
| $d_R$ | $\xi^+_R$ | $\xi^+_R$ | $\xi^+_R$ | 1/2 | +1/2 | +1/3 | +1/3 | 3 |

being $(\xi^- \xi^-)$, $(\xi^+ \xi^+)$ and $(\xi^0 \xi^0)$. For leptons in three space dimensions $\xi^0_L$ was combined with $(\xi^0_L \xi^0_L)$ to find an asymptotic charge eigenstate with $(Q, I_3) = (0, 1/2)$, which we already discussed as the left-handed Majorana neutrino $\nu_L$. For colored eigenstates we specify how states are 'viewed' in 3 dimensions by combining the (frozen) anti-red $\xi^0_L$ state with the (frozen) $rr$ combinations $(\xi^0_R \xi^0_R)$, $(\xi^0_L \xi^0_R)$ or $(\xi^0_L \xi^0_L)$. Then only the combination $(\xi^0_L \xi^0_R)$ leads to acceptable $SU(3)$ quantum numbers (roots), being an asymptotic acceptable $SU(2)_L$ weak eigenstate with $I_3 = 1/2$, which has $U(1)$ charge $Q = +2/3$, identified as the weak iso-doublet quark state $u_L$ with color $r$ belonging to a color triplet. Combining the (frozen) color $\bar{r}$ state $\xi^0_L$ with the (frozen) $rr$ combination $(\xi^0_R \xi^0_R)$, $(\xi^0_L \xi^0_R)$ or $(\xi^0_L \xi^0_L)$ gives only for $(\xi^0_L \xi^0_R)$ an acceptable (frozen) color $\bar{r}$ state with $(Q, I_3) = (-2/3, 0)$, the weak iso-singlet antiquark state $\bar{u}_L$. The full set of electroweak assignments of quarks as viewed in three space dimensions is shown in Table I. The resulting allowed $SU(3)$ quantum numbers are for each family a left-handed quark doublet and right-handed antiquark doublet and two singlets of opposite handedness. The way in which the electroweak structure emerges resembles the rishon model [16], but rather than having two fractionally charged preons ($V$ and $T$) in $d = 4$, our basic modes are charged or neutral preons living in $d = 2$. The family mixing would also for quarks originate from symmetries in fixing a direction, but in zeroth order there is only a single heavy quark, the top quark (with $M_t = M_t$), so the mixing would be trivial. But it is fair to say, that a complete mechanism for masses and mixing for quarks and leptons requires further study.
is that atomic Hydrogen involves all degrees of freedom of just one family while muonic Hydrogen is different in this respect. It could also be interesting to look at more (or maybe less) than three fields, which could be relevant in the context of the evolution of our universe into the world which above hadronic scales, i.e. the visible part at nuclear, atomic, molecular scales up to astronomical scales, is governed by three space dimensions.

Acknowledgements

I acknowledge useful discussions with colleagues at Nikhef, in particular with Tomas Kasemets. This research is part of the FP7 EU "Ideas" programme QWORK (Contract 320389).

[1] A first presentation on this work was given at the 6th International Conference on Physics Opportunities at Electron-Ion Collider (POETIC 6), September 7-11, 2015, Ecole Polytechnique, Palaiseau, France, for which a write-up will appear in the conference proceedings.
[2] P. A. M. Dirac, Rev. Mod. Phys. 21, 392 (1949).
[3] J. B. Kogut and D. E. Soper, Phys. Rev. D1, 2901 (1970).
[4] S. P. Martin, Adv. Ser. Direct. High Energy Phys. 21, 1 (2010), ArXiv: hep-ph/9709356
[5] J. Wess and B. Zumino, Phys. Lett. B49, 52 (1974).
[6] S. R. Coleman and J. Mandula, Phys. Rev. 159, 1251 (1967).
[7] N. Cabibbo, Phys. Lett. B72, 333 (1978).
[8] L. Wolfenstein, Phys. Rev. D18, 958 (1978).
[9] E. Ma and G. Rajasekaran, Phys. Rev. D64, 113012 (2001), ArXiv: hep-ph/0106291
[10] G. Altarelli and F. Feruglio, Nucl. Phys. B741, 215 (2006), ArXiv: hep-ph/0512103
[11] D. B. Kaplan (2013), ArXiv: 1306.5818 [nucl-th].
[12] S. Weinberg, Phys. Rev. D5, 1662 (1972).
[13] M. Veltman, Nucl. Phys. B123, 189 (1977).
[14] P. Sikivie, L. Susskind, M. B. Voloshin, and V. I. Zakharov, Nucl. Phys. B173, 189 (1980).
[15] P. F. Harrison, D. H. Perkins, and W. G. Scott, Phys. Lett. B530, 167 (2002), ArXiv: hep-ph/0202074
[16] H. Harari and N. Seiberg, Nucl. Phys. B204, 141 (1982).
[17] G. F. de Teramond and S. J. Brodsky, Phys. Rev. Lett. 102, 081601 (2009), 0809.4899.
[18] T. Becher, A. Broggio, and A. Ferroglia (2014), ArXiv: 1410.1892 [hep-ph].