EFFECT OF MEASUREMENT ERRORS ON PREDICTED COSMOLOGICAL CONSTRAINTS FROM SHEAR PEAK STATISTICS WITH LARGE SYNOPTIC SURVEY TELESCOPE

D. BARD1, J. M. KRATCHEVIL2, C. CHANG1, M. MAY3, S. M. KAHN1, Y. ALSAYYAD4, Z. AHMAD5, J. BANKERT5, A. CONNOLLY4, R. R. GIBSON4, K. GILMORE1, E. GRACE6, Z. HAIMAN6, M. HANDEL5, K. M. HUFFENBERGER2, J. G. JERNIGAN6,7, L. JONES4, S. KRUGHOFF4, S. LORENZ5, S. MARSHALL1, A. MEERT5, S. NAGARAIAN6, E. PENGO5, J. PETERSON5, A. P. RAMSUSED1, M. SHMAKOVA1, N. SYLVESTRE5, N. TODD5, AND M. YOUNG5

1 KIPAC, Stanford University, 452 Lomita Mall, Stanford, CA 94309, USA; djbard@slac.stanford.edu
2 Department of Physics, University of Miami, Coral Gables, FL 33124, USA
3 Physics Department, Brookhaven National Laboratory, Upton, NY 11973, USA
4 Department of Astronomy, University of Washington, Seattle, WA 98195, USA
5 Department of Physics, Purdue University, West Lafayette, IN 47907, USA
6 Department of Astronomy and Astrophysics, Columbia University, New York, NY 10027, USA
7 Space Sciences Laboratory, University of California, Berkeley, CA 94720, USA

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Abstract

We study the effect of galaxy shape measurement errors on predicted cosmological constraints from the statistics of shear peak counts with the Large Synoptic Survey Telescope (LSST). We use the LSST Image Simulator in combination with cosmological N-body simulations to model realistic shear maps for different cosmological models. We include both galaxy shape noise and, for the first time, measurement errors on galaxy shapes. We find that the measurement errors considered have relatively little impact on the constraining power of shear peak counts for LSST.

Key words: cosmological parameters – dark energy – gravitational lensing: weak – methods: statistical

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1. INTRODUCTION

Weak gravitational lensing (WL) by large-scale cosmic structures has emerged as one of the most promising methods to constrain the parameters of both dark energy and dark matter (e.g., Albrecht et al. 2006; see also Hoekstra & Jain 2008; Munshi et al. 2008 for reviews). The COSMOS survey has provided independent evidence of the accelerated expansion of the universe from cosmological WL measurements (Schrabback et al. 2010; Semboloni et al. 2011). Over the next decade, the Large Synoptic Survey Telescope (LSST) and other large surveys covering several thousand square degrees will produce galaxy catalogs of unprecedented quality. These surveys will provide WL datasets with an enormous wealth of information about structure formation, enabling not just the study of traditional two-point statistics like the power spectrum, but also the extraction of additional non-Gaussianity information. The combination of these measurements will help substantially tighten the constraints on cosmological parameters.

Galaxy clusters are collapsed objects that provide a complementary probe of cosmology to the power spectrum. Indeed, the cluster mass function has long been considered a useful probe of cosmological models. It can be approximated analytically, and has a strong dependence on the cosmological parameters $\Omega_m$, $\sigma_8$, and $w$, where $\Omega_m$ is the fractional matter density of the universe, $\sigma_8$ is the normalization of the matter power spectrum at the length scale $8h^{-1}$ Mpc and $w$ is the evolution of the equation of state of dark energy. Clusters can be identified and their masses measured through several different techniques, including X-ray observations, the Sunyaev–Zeldovich effect, galaxy counts and WL (see Allen et al. 2011, for an overview). The measurement of cluster mass using WL has the advantage that it is independent of the luminous and dynamic properties of the galaxies, and is sensitive to both the baryonic and dark matter components. However, using WL to detect clusters, by searching for peaks in reconstructed lensing maps, is more problematic. Shear peaks detected in WL surveys are often not due to single galaxy clusters, but to chance alignments of structure along the line of sight (Hamana et al. 2004). In addition, genuine clusters that are aligned with matter underdensities along the line of sight can be missed. Cluster counts using WL alone therefore tend to have low completeness and low purity (Hamana et al. 2004).

Peaks in WL maps are a direct observable in WL surveys and can constrain cosmology, regardless of whether they originate from a single galaxy cluster or a random superposition of matter overdensities. While harder to predict theoretically than the cluster mass function, they are observationally cleaner with fewer opportunities for systematic errors to complicate the interpretation of the measurement. Contributions from filaments and other chance alignments encode additional information about the structure of matter beyond the cluster mass function, making peak counts a probe of cosmological parameters complementary to measurements from cluster counts.

In the past several years, there has been a significant increase in interest in lensing peaks and other closely related statistics. Most work has concentrated on peak counts in maps of convergence, which are easy to simulate but observationally harder

8 Forthcoming and planned surveys include those by LSST (www.lsst.org), by Hyper Suprime-Cam (www.naoj.org/Projects/HSC/HSCProject.html), the Dark Energy Survey (www.darkenergysurvey.org), the Kilo-Degree Survey (http://kids.strw.leidenuniv.nl), Pan-STARRS (http://pan-starrs.ifa.hawaii.edu/public), and Euclid (http://sci.esa.int/euclid).

9 To our knowledge, lensing peaks were first considered as a probe of cosmology in the early ray-tracing simulations by Jain & van Waerbeke (2000), who studied the $\Omega_m$-dependence of the peak counts.
to reconstruct than maps of reduced shear (see Section 2 for
definitions of convergence and reduced shear). Jain et al. (2000)
studied the probability distribution function of the convergence
and Wang et al. (2009) investigated its cumulative version, the
fractional area of “hot spots” on convergence maps. Both statis-
tics are similar to peak counts in the high-convergence limit
and have been shown to have useful cosmology sensitivity. The
fractional area statistic is also known as $V_0$, one of the three
Minkowski functionals for two-dimensional (2D) thresholded
fields. Minkowski functionals are related to peaks and had been
proposed as a weak-lensing statistic by Sato et al. (2001) and
Guimarães (2002). More recently, Maturi et al. (2010) con-
structed an analytical approximation to the $V_2$ Minkowski
functional, which is the genus statistic and also corresponds to peaks
in the high-threshold limit. The full set of Minkowski functionals in the context of WL has been studied extensively
both theoretically (Munshi et al. 2011) and in ray-tracing sim-
ulations (Kratochvil et al. 2012). In a different approach, peak
counts have also been studied in wavelet space (Pires et al.
2009), and found to break the degeneracy in $(\sigma_8, \Omega_m)$ cosmo-
lological models found in measurements of the power spectrum
alone.

Preliminary studies (Marian et al. 2009, 2010) that defined
peaks as local density maxima were based on 2D projections
of the 3D mass distribution in low-resolution N-body simula-
tions. Weak-lensing peak counts using ray-traced simulations
were subsequently studied by Dietrich & Hartlap (2010) and
Kratochvil et al. (2010) and more recently in Li (2011) and
Marian et al. (2012). Based on simulations with better mass
resolution, these references revealed that low-amplitude peaks
(which typically do not correspond to single collapsed dark
matter halos) contain more cosmological information than high-
amplitude peaks. It should be noted that the range of peak
heights qualifying peaks as “low” varies greatly between these
references.

Several other aspects of WL peak counts have also been
explored. WL peaks were used by Marian et al. (2011) and
Maturi et al. (2011) to predict constraints on the primordial
non-Gaussianity parameter $f_{NL}$. Yang et al. (2011) studied the
origin of the cosmologically important low peaks, and found
that they are typically caused by a combination of 4–8 low-
mass halos. Kratochvil et al. (2012) and Marian et al. (2012)
demonstrated that cosmological constraints from peaks can be
tightened by combining several angular smoothing scales.
Pires et al. (2012) compared WL peak counts directly to two
other commonly used non-Gaussian statistics, skewness and
kurtosis, and found the peak counts to be superior in information
extraction from WL maps. Finally, VanderPlas et al. (2012)
studied the effect of masks on shear peak counts and showed
that using Karhunen–Loève analysis can mitigate biases on peak
count distributions caused by masked regions, and can also
reduce the number of noise peaks. A comprehensive study of
the uncertainty that the presence of masks introduces into peak
counts in real observational situations has yet to be performed.

Previous work on this subject has therefore determined the
value of peak counts in constraining cosmology, both alone
and in combination with other lensing measurements, where
peak counts can break degeneracies in cosmological parameter
estimation. Attention has also been paid to optimizing the
extraction of cosmological information. Work by Maturi et al.
(2010) and Marian et al. (2011) has concentrated on determining
the optimal filter size and shape, and how filters of different sizes
can be combined to increase the information extracted from
shear maps. Dietrich & Hartlap (2010) demonstrated the value
in redshift-dependent measurement of shear peaks. However, to
date there has been no effort to include measurement errors in
the predictions made from the above simulations.

This paper introduces a framework to produce realistic
galaxies that can be used to trace the shear maps produced in
cosmological simulations, with sizes, magnitudes, redshifts and
signal-to-noise (S/N) properties matching observed distributions,
and measurement errors matching expected uncertainties
from a 10 yr LSST survey. We take a forward-modeling ap-
proach where we compare a dataset produced with a particular
cosmology with datasets produced for other cosmological mod-
els. In this way, we can compare the expected results from the
different cosmologies and, eventually, determine the best fit to
the data.

This paper is organized as follows. The WL formalism
and aperture mass calculation are introduced in Section 2. To
calculate these shear peak statistics, we start with a large suite
of N-body simulations described in Kratochvil et al. (2012) and
Yang et al. (2011), which are ray-traced in order to obtain maps
of the shear and convergence parameters for seven different
cosmological models, covering the cosmological parameter
space in $\Omega_m, \sigma_8$ and $w$. These are described in Section 3. We
seed these shear maps with source galaxies in order to obtain
a mock dataset for each cosmological model. Realistic galaxies
are essential to predicting realistic constraints on cosmology, so
we must include all effects that will impact the quality of the
measurements. Measurement uncertainties depend largely on
the S/N of the flux of the galaxy, with fainter galaxies having
lower S/N. We therefore use the LSST Image Simulator (ImSim;
Connolly et al. 2010) input catalogs to identify the intrinsic
properties to be used for the source galaxies, such as size,
magnitude and redshift. Uncertainties in shape measurement
are determined from a large suite of LSST simulations, used to
model the expected errors due to atmospheric and instrumental
effects and the residual contributions to galaxy shape distortion
after the ellipticity of the point-spread function (PSF) has been
interpolated to the galaxy position and deconvoluted from the
galaxy shape. This process is described in Section 4, and
provides a mock catalog of galaxy shapes representative of that
which would be obtained after 10 yr of LSST data, for each of the
seven cosmological models. We then calculate the aperture mass
over each of these mock catalogs, and look for peaks in the maps
of S/N of the aperture mass statistic. The resulting peak counts
are described in Section 5, which allow us to make the first
realistic predictions of constraints on cosmological parameters
from shear peak statistics for LSST. We describe the process by
which we calculate the constraints on cosmology by comparing
the different mock datasets in Section 6, and discuss our results
in Section 7. Finally, we summarize our work in Section 8.

2. FORMALISM

Photons from distant galaxies are deflected by the tidal
gravitational field of matter along the line of sight. If the lensed
image of a galaxy is smaller than the characteristic scale of the
lensing potential, the distortion of the galaxy shape can be
described by a linearized lens mapping, given by the Jacobian

$$A = (1 - \kappa) \begin{pmatrix} 1 - g_1 & -g_2 \\ -g_2 & 1 + g_1 \end{pmatrix},$$

where $g$ is the reduced shear $g = \gamma/(1 - \kappa)$. The complex shear
$\gamma \equiv \gamma_1 + i\gamma_2$ describes the distortion of the galaxy shape, and the
convergence, $\kappa$, describes the magnification of the galaxy image relative to its source. For a full derivation of these parameters, see for example Bartelmann & Schneider (2001).

Of course, we cannot measure the shear parameters directly, but must estimate them from the resulting small distortions in observed galaxy shapes. We parameterize galaxy shapes by the complex ellipticity $\epsilon = \epsilon_1 + i \epsilon_2$, where the components of ellipticity are normalized moments of the light intensity of the object $I_{i,j}$ weighted by a Gaussian function $W(x_1, x_2)$:

$$
\epsilon_1 = \frac{I_{11} - I_{22}}{I_{11} + I_{22}}, \quad \epsilon_2 = \frac{2I_{12}}{I_{11} + I_{22}},
$$

$$
I_{ij} = \iint W(x_1, x_2) f(x_1, x_2) x_i x_j dx_1 dx_2, \quad i, j = 1, 2.
$$

In the weak limit, the observed galaxy ellipticity is a combination of the intrinsic galaxy ellipticity $\epsilon_{\text{int}}$ and reduced shear $g$:

$$
\epsilon_{\text{obs}} = \epsilon_{\text{int}} + g, \quad \text{Shape noise from the intrinsic ellipticity of galaxies } \sigma_{\epsilon_{\text{int}}} = (\epsilon_{\text{int}}^2) \text{ is much larger than } g^2, \text{ so to obtain } g \text{ we can average over large numbers of galaxies (assuming that galaxy shapes and orientations are random over a large enough area of the sky). In this case, the observed ellipticity (} \epsilon_{\text{obs}} = (g). \text{ The uncertainty in a measurement of } g, \sigma_g, \text{ is therefore a combination of the galaxy shape noise and measurement uncertainty } \sigma_{g}^2 = \sigma_{\epsilon_{\text{int}}}^2 + \sigma_{\text{meas}}^2. \text{ Previous work (e.g., Dietrich & Hartlap 2010; Maturi et al. 2010; Marian et al. 2011) has considered the impact of the shape noise } \sigma_{\text{int}} \text{ but not the measurement uncertainty } \sigma_{\text{meas}}. \text{ Matter overdensities along the line of sight will cause the shear field, and therefore the observed shapes of galaxies, to be tangentially aligned around the projected peak of the overdensity. We can use this property of shear fields to reconstruct the aperture mass, which is a weighted sum over the tangential components of galaxy shapes around a point. We define the aperture mass at position $\theta_0$ as in Schneider (2005),}

$$
M_{ap}(\theta_0) = \int d^2 \theta Q(\theta) g_0(\theta, \theta_0),
$$

where $g_0$ is the tangential component of reduced shear relative to $\theta_0$ defined as

$$
g_0(\theta, \theta_0) = -(g_1 \cos(2\phi) + g_2 \sin(2\phi)).
$$

$\phi$ is the angle with respect to the horizontal axis between positions $\theta_0$ and $\theta$ in the map. Note the minus sign, and the factor of two in the angles (necessary because shear is spin-2, not a vector). $Q(\theta)$ is the weighting function, and determines the statistical properties of $M_{ap}$. In practice, the shear field is sampled by galaxies and we measure the reduced shear of these galaxies. We therefore estimate the aperture mass by summing over the tangential components of galaxy shapes using

$$
M_{ap}(\theta_0) = \frac{1}{N_g} \sum_{i=1}^{N_g} Q(\theta) g_0(\theta, \theta_0).
$$

If the weight function $Q(\theta)$ follows the expected shear profile of a mass peak then the aperture mass is a matched filter for detecting mass peaks. We use the spherically symmetric function introduced by Schirmer et al. (2007), which follows an NFW (Navarro et al. 1996) profile with exponential cutoffs as $x \to 0$ and $x \to \infty$:

$$
Q_{\text{NFW}}(x, x_c) = \frac{1}{1 + e^{-(x-x_c)/\theta_c}}, \quad \text{tanh}(x/x_c).
$$

Here, $x = \theta/\theta_{\text{max}}$, where $\theta_{\text{max}}$ gives the radius to which the filter is tuned. We use a value of 5.6 arcmin. $x_c$ is a constant, set to 0.15, which has been empirically determined to be a good value for shear peak counting (Hettescheidt et al. 2005). The rms dispersion of $M_{ap}$ in the case of no lensing is determined from the dispersion of the intrinsic shape noise of galaxies (Bartelmann & Schneider 2001),

$$
\sigma(M_{ap}) = \frac{\sigma_\epsilon}{\sqrt{2n}} \sqrt{\sum_i Q^2(\theta_i)}.
$$

Providing the lensing is weak within the radius of the aperture, $\sigma(M_{ap})$ will be close to the rms dispersion in the presence of lensing. It can therefore be used as an estimate of the uncertainty of the aperture mass. We can calculate the noise directly from the data, and look for peaks in the map of $S/N$, where $S/N(\theta_0) = \sqrt{N \sum Q(\theta) g_0^2}$.

$$
S/N(\theta_0) = \frac{\sqrt{N \sum Q(\theta) g_0^2}}{\sqrt{\sum Q^2(\theta) g_0^2}}.
$$

We use a pixel size of $12.2''$ for this map. We define a peak as a group of pixels which are all above a threshold, that touch along the edges, or corners, and that are surrounded by pixels that are below that threshold, thereby being isolated from other such groups (sometimes called 8-connectivity). We count the number of peaks in our maps for a range of $S/N$ thresholds. The counts go to unity as the $S/N$ reaches the minimal (negative) value. The definition of peaks used in this paper is similar to that of Dietrich & Hartlap (2010). We note that there are other definitions of peaks in the literature (e.g., Kratochvil et al. 2010; Marian et al. 2011; Yang et al. 2011).

We are working with thousands of 12 deg$^2$ simulated shear and convergence maps, with each map containing ~1.5 million galaxies. Calculating the aperture mass for all maps is a significant computational problem, which we address by taking advantage of the properties of graphics processing units (GPUs). The implementation of the aperture mass calculation on the GPU is described in detail in Bard et al. (2013). By using the GPU we can reduce the computation time per map from several hours to a few minutes.

Previous work (Yang et al. 2011) has determined that peak counts in convergence maps contain additional information not provided by the power spectrum alone. In order to make a similar determination about the information in peak counts in aperture mass maps, we must also calculate the aperture mass power spectrum. To do this, we calculate the Fourier transform of the $S/N$ map, multiply by the complex conjugate, normalize the entries, and thus obtain a power spectrum as a function of angular wave vector $\ell$. The power spectrum is then binned radially into 1000 $l$-bins and summed angularly to a power spectrum depending only on the magnitude of $\ell$.

11 However, it is not yet clear whether this value or this filter shape in general is the best choice for the low shear peaks, which have been discovered to be cosmologically important (Dietrich & Hartlap 2010; Kratochvil et al. 2010) and been shown to be due to projections of multiple clusters (Yang et al. 2011) since the publication of Hettescheidt et al. (2005).
We will also use this information to constrain cosmological parameters, alone and in combination with the peak counts, as described in Section 7. We note that the filter we use in calculating the aperture mass suppresses long-wavelength modes. This will reduce the cosmological information contained in the aperture mass power spectrum compared to the power spectrum calculated from the two-point correlation function. However, the comparison is fair since the peak counts will reflect the same suppression.

3. SIMULATIONS

In order to predict peak counts from different cosmological models we must use a large suite of N-body simulations representing these models, ray-traced to produce shear maps. The large-scale structure simulations and shear maps we use in this analysis were created with the Inspector Gadget lensing simulation pipeline (Kratochvil, in preparation; Kratochvil et al., in preparation) on the New York Blue supercomputer, which is part of the New York Center for Computational Sciences at Brookhaven National Laboratory/Stony Brook University. In this section we describe the simulations and the cosmological models we chose to study.

3.1. N-body Simulations

The N-body simulations are the same as those used in Yang et al. (2011, 2013) and Kratochvil et al. (2012), and consist of a series of 80 cold dark matter N-body simulations with 512$^3$ particles each and a box size of 240 $h^{-1}$ Mpc. They were run with a modified version of the public N-body code Gadget-2 (Springer 2005). The linear matter power spectrum, which serves as input for the initial conditions generator N-GenIC associated with Gadget-2, was created with CAMB (Lewis et al. 2000) for $z = 0$, and scaled to the starting redshift of our simulations at $z = 100$ according to the linear growth factor.

The N-body simulations cover different cosmological models produced in multiple runs with different random initial conditions. A total of 50 of the runs is available in the fiducial cosmology, with parameters chosen to be \{$\Omega_m = 0.26$, $\Omega_\Lambda = 0.74$, $w = -1.0$, $n_s = 0.96$, $\sigma_8 = 0.798$, $H_0 = 0.72$\}. These 50 runs all used the same input power spectrum, but each one is a different and strictly independent realization. This yields a statistically robust set of simulations. In each of the other six cosmological models one parameter was varied at a time, keeping the others fixed, with the following values: $\Omega_m = \{0.23, 0.29\}$ (while $\Omega_\Lambda = \{0.77, 0.71\}$ such that the universe stays spatially flat), $w = \{-0.8, -1.2\}$, and $\sigma_8 = \{0.75, 0.85\}$. For each of these six non-fiducial cosmological models five simulations are available, where each simulation used a different realization of the initial conditions. Table 1 lists all the cosmological models with their parameters and number of N-body simulations.

| WL Map Set Identifier | $\sigma_8$ | $w$ | $\Omega_m$ | $\Omega_\Lambda$ | No. of Sims |
|-----------------------|-----------|-----|------------|-----------------|-------------|
| Primary               | 0.798     | $-1.0$ | 0.26 | 0.74 | 45 |
| Auxiliary             | 0.798     | $-1.0$ | 0.26 | 0.74 | 5  |
| Om23                  | 0.798     | $-1.0$ | 0.23 | 0.77 | 5  |
| Om29                  | 0.798     | $-1.0$ | 0.29 | 0.71 | 5  |
| w12                   | 0.798     | $-1.2$ | 0.26 | 0.74 | 5  |
| w08                   | 0.798     | $0.8$  | 0.26 | 0.74 | 5  |
| s175                  | 0.750     | $-1.0$ | 0.26 | 0.74 | 5  |
| s185                  | 0.850     | $-1.0$ | 0.26 | 0.74 | 5  |

The shear and convergence maps, described in more detail in the next subsection, were generated by mixing simulations with different random initial conditions, and by randomly rotating and shifting the simulation data cubes. For the maps in each non-fiducial cosmology a mixture of all five independent N-body runs was used. In the fiducial cosmology, two completely independent sets of maps are available. One of these sets, called hereafter the “auxiliary” map set, was created from the five independent N-body runs with the same five quasi-identical initial conditions as in the non-fiducial cosmologies. The second map set was created by mixing lens planes from the remaining larger ensemble of 45 independent N-body runs, and will be referred to as the “primary” map set. This is also reflected in Table 1.

3.2. Weak Lensing Maps

Our pipeline uses a standard 2D ray-tracing algorithm, as described in Hamana & Mellier (2001), to create the weak lensing maps from the N-body simulations. Earlier work using similar algorithms includes Schneider et al. (1992), Wambsganss et al. (1998), and Jain et al. (2000). We refer the reader to Kratochvil et al. (2010, 2012) and Yang et al. (2011) for the full description of our methodology and verification of the accuracy of the simulations used.

The large-scale structure from the N-body simulations was output as particle positions in boxes at different redshifts, starting at redshift $z = 2$. Since the simulation boxes are much smaller than the distance from the observer to the source redshift plane, the line of sight must be tiled with multiple simulation boxes output at different redshifts. The boxes for such a tiling are randomly drawn from the five independent simulations available for each cosmology (45 simulations for the primary map set of the fiducial cosmology). The boxes are randomly rotated by 90° angles and shifted to avoid the repetition of structure in the same places. The particles were then projected perpendicularly onto planes spaced 80 $h^{-1}$ Mpc apart in a direction parallel to the central line of sight of the map. For each plane a separate box is used, only a third of a box thus gets used each time. We used the triangular shaped cloud scheme (Hockney & Eastwood 1998) to place the particles on a grid on these 2D density planes; the particle surface density was then converted into the gravitational potential via the Poisson equation. The algorithm then followed light rays from the observer back in cosmic time. The deflection angle as well as the weak lensing convergence and shear were calculated at each plane for each light ray. These depend on the first and second derivatives of the potential, respectively. Between the planes, the light rays traveled in straight lines.

Shear and convergence maps, spanning 12 deg$^2$, were created for 2048 $\times$ 2048 light rays. For simplicity, we created maps assuming the source galaxies to be at three fixed redshifts, $z_s = 1, 1.5, 2$. Each cosmological model is represented by 500 such 12 deg$^2$ maps for convergence and shear parameters for each of the three source galaxy redshifts.

For the fiducial model, we have 50 independent N-body simulations, which we split into a group of 5 and a group of...
45. From each of these two groups we generate 500 shear and convergence maps. We can use either of these two map sets to determine the best-fit points in the parameter space during our Monte Carlo procedure in Section 6.2 to determine the cosmological parameter error contours. We see no appreciable difference in the contours obtained from the two map sets, from which we conclude that for our purposes of parameter constraint from peak counts even five simulations produce enough variance. We use 500 maps created from five simulations for all other cosmological models, because creating many maps reduces the noise in our estimators. Low, ubiquitous structure in the maps is statistically well sampled by the simulations, and all different kinds of projections of this structure must also be well sampled. This can only be achieved by creating many maps, even if the underlying simulations are reused. We limit ourselves to 500 maps, because this is the quantity required to produce the stable parameter estimation contours described in Section 6.2.

4. SOURCE GALAXIES

In this section we describe how we characterize the source galaxies which we use to trace the shear field. We wish to make our prediction for shear peak counts as realistic as possible, and for that it is essential that we make our source galaxies as realistic as possible. The steps we take to create the ensemble of source galaxies can be summarized as follows:

1. Assign a spatial position for the galaxy.
2. Assign a redshift for the galaxy. Based on redshift, assign the galaxy a magnitude, size and intrinsic shape.
3. Add reduced shear to galaxy. Recalculate size and magnitude.
4. Add reduced shear error to galaxy.

4.1. Intrinsic Properties

As described in Section 3, we have 500 realizations of maps for each of seven different cosmological models. Each set consists of ray-traced maps of the lensing parameters $\gamma_1$, $\gamma_2$, $\kappa$ in three redshift bins, at $z = 1.0$, 1.5, and 2.0. We consider each of the 500 realizations of one cosmological model to be independent observations of the sky, and for each map we generate an independent ensemble of galaxies to use as tracers of the shear field in three dimensions. The same source galaxies are used for all seven cosmologies, so we are effectively observing the same "sky" with all cosmological models.

We scatter the galaxies randomly across the field, ensuring that we have an average galaxy density of 30 galaxies arcmin$^{-2}$, which is roughly the expected galaxy density usable for weak lensing analyses for an LSST 10 yr survey in $r$-band (Wittman et al. 2009). At this first step, we have already limited how realistic we can make this study: in randomly positioning the source galaxies, we do not take into account the shifts in their apparent positions due to lensing and that the source galaxy positions are in reality correlated with dark matter halos in the simulation. We decided to neglect these effects because it lets us shoot light rays backward in time through the $N$-body simulation indiscriminately, as opposed to having to determine which light ray hits a fixed galaxy position. Matching galaxy density with input shear maps is very difficult; see for example Behroozi et al. (2011).

One consequence of these simplifications is that we neglect the magnification bias present in lensing (Turner et al. 1984). The magnification bias arises from two competing effects:

1. high-shear regions magnify galaxies, thus making fainter galaxies visible in a flux-limited survey and adding source galaxies in those regions of the sky, (2) the magnification also spreads apart the apparent positions of the source galaxies, thus diluting the number of galaxies in these high-shear regions.

We anticipate that this variation in density will have a small impact on lensing peak counts, or at least on the cosmological constraints coming from lensing peak counts. This is because the constraints have been shown to be dominated by the numerous low peaks (Kratochvil et al. 2010; Dietrich & Hartlap 2010), which are to be found in low-shear regions, while the magnification bias is most noticeable in regions of high shear. For the high significance lensing peaks, the primary effect of magnification on galaxies will be a shift in the apparent position of galaxies which can also shift the position of a peak (particularly if it is not the central peak of a cluster). Since peak counts (measured using one smoothing scale as done in this paper) do not measure angular correlations, a shift in position will not affect the results. For the central peak of a cluster, however, the dilution of source galaxies associated with magnification will mostly cause an apparent broadening of the peak, which will make the peak appear larger than in our simplification. We do not expect this to be a significant effect for the cosmological constraints, because high central peaks are by far outnumbered by the others, but the importance of this effect should be studied in future work.

Next we assign each galaxy a redshift, size, and magnitude, taken from a distribution obtained from the input catalogs of the LSST ImSim. Galaxies in these catalogs have properties based on those produced by the Millennium dark matter simulations. The galaxy catalog is complete out to a magnitude of 28, which is approximately one magnitude deeper than the expected depth of the full LSST 10 yr survey. These quantities have been anchored to observations from a compilation of deep survey data,13 the DEEP2 survey (Coil et al. 2004), and data from the publicly available Hubble Deep Field catalogs.14 A redshift is assigned at random to the galaxies, shown in Figure 1, where the dashed line represents a simple model of the form $n(z) \propto z^2 e^{-z^2}$, as described in Wittman et al. (2000) and previously found to be a good fit to DEEP2 survey data (Coil et al. 2004). A redshift-dependent size and magnitude is assigned for each galaxy from

\[ n(z) \propto z^2 e^{-z^2}. \]

(A color version of this figure is available in the online journal.)

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13 http://astro.dur.ac.uk/~num/pubhtml/counts/counts.html
14 http://www.stsci.edu/ftp/science/hdf/archive/v2.html
the simulated input catalogs, where we define size as the product of the RMS of the semi-major and semi-minor axes of the galaxy.

An intrinsic ellipticity is then assigned to each galaxy. We base the assigned ellipticity on measurements made using COSMOS data (Joachimi et al. 2013; Leauthaud et al. 2007), where the intrinsic galaxy shape noise was found empirically to be 0.23 per reduced shear component. A small dependence on galaxy magnitude was identified in the COSMOS data, but no significant dependence on size or redshift was found. We next assign a reduced shear to the galaxy, taken from the input shear and convergence maps described in Section 3 and extrapolated to the galaxy position in R.A., decl. and redshift (where we linearly extrapolate the shear and convergence parameters between the three redshift planes at \(z = [1.0, 1.5, 2.0]\)). The galaxy size, magnitude, and ellipticity are recalculated to account for the effect of shearing and magnification.

4.2. Measurement Error

To assign a measurement error for the galaxy shape is somewhat complicated. Our aim is to obtain an error consistent with a measurement of galaxy shapes from a 10 yr stack of LSST observations, using only observations made with good seeing in the \(r\)-band. If we require the median seeing to be 0\(\prime\)066, which is an acceptable quality for weak lensing analyses (Wittman et al. 2009), we expect a 10 yr stack to consist of approx. 100 observations.

We use the LSST Photon Simulator (PhoSim) to simulate a 10 yr stack of LSST observations of an area on the sky the size of an LSST chip (1/16 the area of an LSST CCD, with approx. 13.6 arcmin\(^2\) field of view), at different positions on the LSST focal plane in order to sample the PSF as it varies across the focal plane. PhoSim is a high-fidelity, ray-traced end-to-end simulator of the LSST system. A detailed description of the system can be found in Peterson et al. (2009), Connolly et al. (2010), and Chang et al. (2012). Recent work by Chang et al. (2013) has studied in depth the impact on galaxy shape measurement made by the distortions introduced by the atmosphere and the LSST telescope itself. We wish to isolate the impacts of measurement and algorithmic effects, and to evaluate the impact of these errors separately from the error due to galaxy intrinsic shape noise.

Since we have already accounted for shape noise in a previous step in the pipeline, we use in these simulations an input catalog consisting of circular galaxies with a Gaussian profile to remove any effects of shape noise. The magnitudes, redshifts, S/Ns, and spectral energy distributions of these Gaussian galaxies are the same as the fully realistic galaxy distribution of the ImSim input catalogs. The advantage to this approach is that we can easily evaluate the measurement error without attempting to remove galaxy shape noise. We are not performing a redshift-dependent measurement, and we ignore for this work the potentially substantial errors in redshift measurement.

We take 500 values of \([\gamma_1, \gamma_2, \kappa]\) at random from one of the simulated shear maps described in Section 3, and produce 500 different sheared ImSim input catalogs by applying a single reduced shear value \(\{\gamma_{1,\text{in}}, \gamma_{2,\text{in}}\}\) to all galaxies in an existing catalog. For each of these sheared input catalogs, we produce 100 simulated images of the same area of sky, each time with different atmospheric conditions specified by the LSST Operations Simulator (Pinto et al. 2009) selected such that the median seeing is 0\(\prime\)066. We process the resulting images using the SourceExtractor object-detection package (Bertin & Arnouts 1996). For each exposure, we use the stars in the field to reconstruct the PSF, which is interpolated to the galaxy locations using a third-order polynomial interpolation function. The measured galaxy shapes are corrected for distortions due to the PSF using the popular KSB (Kaiser et al. 1995) algorithm implemented in the IMCAT\(^{15}\) pipeline. We use the KSB algorithm because it is well known in the community, and its strengths and weaknesses are well understood. For example, it is known that the process that converts ellipticity to reduced shear should be calibrated using simulations. We apply a “perfect” calibration, by applying a calibration factor that shifts the mean measured reduced shear in each of our simulated exposures to the input reduced shear value. The measured, PSF-corrected, calibrated shape for each galaxy is then averaged over the 100 atmospheric realizations, giving us an estimate of the stacked galaxy shape measurement. More sophisticated algorithms are expected to give a smaller uncertainty on galaxy shape measurement (see Kitching et al. (2012) for a summary of the performance of many current shape measurement methods). Despite applying a perfect calibration, for this reason we consider the uncertainty obtained from our KSB implementation to be conservative for LSST.

We compare the measured galaxy shapes to the input reduced shear values, and the difference between input and output gives the uncertainty on the reduced shear measurement. There is a dependence of the measurement uncertainty on magnitude, with fainter galaxies having larger uncertainties, as shown in Figure 2. This is in accordance with the dependence of reduced shear measurement uncertainty on object magnitude found in data from the COSMOS survey (Leauthaud et al. 2007). We account for this dependence as we assign measurement errors drawn from this distribution, which are added to the galaxy shape noise. Since we assign the noise to galaxies randomly, we do not consider any spatial correlations the noise may have across the field. Chang et al. (2013) have shown that, for a 10 yr stack of LSST images, the spatial correlation of measurement error (including an imperfectly modeled PSF) is at a level comparable to the statistical error on the weak lensing correlation function, around \(10^{-7}\). We therefore consider that the spatial correlations of measurement error will be similarly small for aperture mass statistics, and neglect it in this work. However, future work is planned to specifically quantify the impact of correlated error on peak counts.

\(^{15}\) http://www.ifa.hawaii.edu/~kaiser/imcat/
Figure 3. Standard deviation of the errors for input reduced shear parameters \( g_{1,\text{in}} \) and \( g_{2,\text{in}} \), for elliptical galaxies of different measured reduced shear \( g_{1,\text{meas}} \) and \( g_{2,\text{meas}} \). The standard deviation is taken from fitting a Gaussian to the distribution of galaxy shape measurement errors for each measured \( g \) bin. Black squares represent \( g_1 \), blue triangles \( g_2 \).

(A color version of this figure is available in the online journal.)

Figure 4. Distribution of galaxy shapes \( |\mathbf{g}| \), for intrinsic ellipticity only (black dotted line) and including measurement error (red solid line).

(A color version of this figure is available in the online journal.)

We also wish to investigate the dependence of measurement error on galaxy shape. To do this, we therefore made a set of simulations identical to those described above, but using elliptical galaxies. However, we are unable to separate the intrinsic shape noise from the measurement error in these simulations, so we are limited to examining the dependence of the total error on the measured reduced shear with the measured galaxy shape. Figure 3 shows the uncertainty distribution for input reduced shear values \( |g_{1,\text{in}}, g_{2,\text{in}}| \) for different values of measured galaxy shape \( |g_{1,\text{meas}}, g_{2,\text{meas}}| \). There is no dependence on galaxy shape, and the distribution is remarkably flat. We also check for a dependence of the measurement error on input shear and find none. We do see a significant dependence of the uncertainty on \( |g_{1,\text{in}}, g_{2,\text{in}}| \) with the raw ellipticity measurement of the galaxy, but the process of PSF deconvolution using KSB, and the calibration procedure remove this dependence. We therefore do not apply a shear- or shape-dependent measurement error.

Figure 4 shows the distribution of the assigned values of the reduced shear \( |\mathbf{g}| \) for all galaxies in our sample, comparing the intrinsic ellipticity alone to the combination of intrinsic ellipticity and measurement error. The measurement error has a much smaller contribution to the total galaxy shape error than the intrinsic ellipticity, but is not negligible. We shall see in the next sections what impact this has on the peak counts and cosmological constraints.

5. PEAK COUNTS

5.1. Cosmological Dependence of Shear Peaks

We consider these mock galaxy shape measurements to be a representative sample of an LSST 10 yr survey. We use these simulated datasets to perform the aperture mass calculation given in Section 2 using the GPU implementation described in Bard et al. (2013), and obtain peak counts above S/N thresholds for the seven cosmological models described in Table 1. The aperture mass is calculated using source galaxies with shape noise alone, and using source galaxies with both shape noise and measurement error. Figure 5 shows the distributions of peak counts for the different cosmological models, where each model is sampled by the same galaxies (including shape noise and measurement errors) scaled to the full-sky LSST survey size. We see significant peak counts at S/N thresholds around and below zero S/N. Although the majority of the area above this low S/N threshold consists of a single connected region, there exist isolated peaks within areas of negative S/N that contribute to the peak count.

To evaluate the impact of measurement error, compared to shape noise alone, we calculate the difference between the peak counts for the two cases. This is shown in Figure 6, where we plot the fractional difference between the peak counts for intrinsic shape noise alone, compared to shape noise and measurement error. The difference is largest at very low and high peak significance, where it reaches up to 25%. As we might expect, the difference is identical for all cosmological simulations, showing that measurement error should not bias constraints on cosmological parameters in favor of one model over another.

It is hard to distinguish between the different cosmological models by eye in Figure 5, but if we plot the difference of the peak counts from the fiducial cosmology, as in Figure 7, we obtain a clearer view of the characteristics of each cosmology. The red curve in these figures represents the peak counts obtained from the auxiliary WL map set of the fiducial model, and acts as a control test to be compared with the primary map set of the fiducial model belonging to the same cosmology. In Figure 7 it is clear that the differences between the primary and auxiliary map sets of the fiducial model are consistent within the statistical error (shown by the error bars on the auxiliary fiducial
Figure 6. Fractional difference between peak counts with shape noise alone, and peak counts with shape noise and measurement error, given as the fractional difference from shape noise only. Solid curves are the average over the 500 different maps; error bars are the standard deviation of the 500 maps, shown for the auxiliary fiducial model to indicate the level of statistical error. See Table 1 for details of cosmological model parameters.

Figure 8. Peak counts above S/N threshold for different cosmological models for the “perfect” case where the aperture mass is calculated directly from the reduced shear maps. See Table 1 for details of cosmological model parameters. (A color version of this figure is available in the online journal.)

Figure 7. Difference between peak counts in the primary map set of the fiducial cosmology, and peak counts in other cosmologies for aperture mass calculated using realistic galaxies with both shape noise and measurement error, given as the fractional difference from fiducial cosmology counts above S/N threshold. Solid curves show the mean difference for the 500 maps used in the measurement; error bars are the standard deviation, shown for the auxiliary fiducial model to indicate the level of statistical error. See Table 1 for details of cosmological model parameters.

Figure 9. Difference between peak counts in the primary map set of the fiducial cosmology, and peak counts in other cosmologies for the “perfect” case where the aperture mass is calculated directly from the reduced shear maps, given as a percentage difference from fiducial cosmology counts above S/N threshold. Solid curves show the mean difference for the 500 maps used in the measurement; error bars are the standard deviation, shown for the auxiliary fiducial model to indicate the level of statistical error. See Table 1 for details of cosmological model parameters. (A color version of this figure is available in the online journal.)

Several of the cosmological models have very similar peak count distributions. In accordance with expectations of the $(\sigma_8, \Omega_m)$-degeneracy, the models Om23 and si75, and Om29 and si85, have very similar peak count profiles, which will result in our predicted cosmological constraints exhibiting a degeneracy in the corresponding direction in the parameter space.

We can also compare these peak counts to the counts obtained from calculating the aperture mass directly from the maps of reduced shear, without sampling the maps with source galaxies. This is the “perfect” case where we have perfect knowledge of the shear, and no noise is introduced by galaxy shape noise or measurement errors. It is therefore an impossible ideal, but serves as a useful comparison to examine how real measurements are affected by error. Since there is no noise in this measurement, constructing an S/N map is meaningless, and instead we count peaks in the map of aperture mass. The two quantities can be related by

$$M_{ap}(\theta_0) = \frac{S}{N(\theta_0)} \hat{\sigma}_{M_{ap}}$$

where $M_{ap}$, S/N and $\hat{\sigma}_{M_{ap}}$ are defined in Equations (1), (3) and (2) respectively. Figures 8 and 9 show the peak counts above aperture mass thresholds for different cosmologies, and the difference of the seven cosmological models compared to the primary map set of the fiducial model, respectively.

The addition of shape noise and measurement error has a significant impact on the shape of the peak counts, visible in a comparison of Figures 7 and 9. For the realistic case with errors included, the overall shape of the peak counts will be valuable in constraining cosmological parameters, since the deviation from the fiducial cosmology is visible at all S/N levels. The addition of noise has impacted both the significance of the peaks, as one would expect, and also the shape of the peak counts of the different cosmologies.

5.2. The Significance of Low Peaks

In this section we discuss further the significance of low peaks. Both Dietrich & Hartlap (2010) and Kratochvil et al. (2010) discovered that low peaks contribute the most to the cosmological constraints, which was later confirmed and elaborated upon by others (Yang et al. 2011, 2013; Kratochvil et al. 2012; Marian et al. 2012; Maturi et al. 2011). What has gone mostly unnoticed is that the different papers have completely different definitions of the word “low” in this
context, and different papers actually refer to completely disjoint peak ranges. Kratochvil et al. (2010) and the group’s follow-up works (Yang et al. 2011, 2013), subsequently referred to as Group A, define low peaks as having an $S/N$ between $0\sigma$ and $2\sigma$ or $1\sigma$ and $3.5\sigma$, depending on the publication. Dietrich & Hartlap (2010) define low peaks as lying in the range $3.25\sigma$–$4.5\sigma$, such that their entire range is higher than most of the previous group’s papers. The hierarchical peak finding algorithm of Marian et al. (2012) is overwhelmed by the number of peaks below $S/N \sim 3\sigma$ and breaks down, so these authors also restrict themselves to a range above $S/N \sim 3\sigma$ in their peak detection, while Maturi et al. (2011) conclude that to constrain the parameter $f_{\text{NL}}$ of primordial non-Gaussianity, only peaks with $S/N > 3\sigma$ are useful. We refer to this second group of three independent collaborations as Group B.

What has made a direct comparison of the works of these papers impossible is that Group A used peak counts in maps of convergence, while Group B used peak counts in shear maps and a somewhat more realistic simulation of galaxy shape noise. For the first time in this paper, we use reduced shear maps and include realistic LSST measurement errors for the same set of simulations used by Group A, which allows a direct comparison of the works. However, it should be noted that our comparison is not complete, since we use only one smoothing scale and a different aperture mass filter compared to the work in Group A.

We plot the $\Delta x^2$ between different cosmological models coming from the different $S/N$ ranges in Figure 10, where $\Delta x^2$ is the square of the fractional difference between the peak counts for a cosmological model and the fiducial model in one bin of $S/N$, divided by the variance for that $S/N$ bin. Neglecting correlations between individual $S/N$ ranges and simply interpreting the area under the curves as the strength of distinction between the cosmological models, we conclude that peaks with $S/N \sim 0\sigma$–$2\sigma$ carry approximately $1/3$ of all the information in the peak counts, peaks with $S/N > 3\sigma$ approximately half, and peaks with $S/N > 3.5\sigma$ also about $1/3$. We can compare this result to Kratochvil et al. (2010), which found according to the third panel of their Figure 5 that low peaks (by our definition of “low”) were somewhat more important for cosmology with convergence maps. However, we use a filter for aperture mass that emphasizes an NFW profile and so may de-emphasize smaller peaks, which would explain the discrepancy in our results.16

In the literature, there have been claims that low ($0\sigma$–$3\sigma$) peaks do not carry any useful cosmological information both due to galaxy shape noise dominating the peak counts, and due to the unknown influence of systematic errors in this range. Yang et al. (2011) have shown that the first of these issues is a misconception, and that real cosmological structure contributes significantly to peak counts at low $S/N$. We have shown here that, even in the presence of systematic errors from a realistic analysis pipeline, we can still extract that information.

Interestingly, an important result from Yang et al. (2013) is that peaks with $S/N \sim 1\sigma$–$3.5\sigma$ are largely unbiased by baryon effects and therefore lend themselves particularly well for cosmological parameter estimations at the sub-percent accuracy level to which LSST aspires. Their result is valid within the restrictions of their study of varying the concentration parameter within NFW halos. To obtain certainty that this is universally the case for all possible contributions of baryonic physics, full hydrodynamic simulations with different baryon prescriptions need to be run. We are in the process of investigating this for a future publication.

We conclude that there is substantial information content in low WL peaks, with “low” defined as $S/N \sim 0\sigma$–$3\sigma$. Low peak counts should not be dismissed as purely due to shape noise, especially since these low peaks have been shown to be less sensitive to uncertain baryonic physics in simulations.

6. ANALYSIS

We present in this section the methodology for extracting cosmological information from the peak counts and power spectra we have obtained from aperture mass calculations.

6.1. Statistical Descriptors

We generically refer to the different statistics one can obtain from a 2D WL map—e.g., power spectrum, peak counts, etc.—as statistical “descriptors”, and denote them by $N$. We can also combine them into a single vector, $N_i$, where $i$ indexes the peak height or the multipole for the power spectrum. Combining the data from several source redshifts makes the descriptor vector longer, but it is treated in the same way. We divide the range of peak height into 30 threshold bins. Similarly, we divide the power spectrum into 30 scale bins.

To constrain cosmology, we are interested in the true ensemble average17 (denoted henceforth by brackets ⟨⟩) and covariance of these descriptors as a function of cosmological parameters $p = \{\Omega_m, \sigma_8, \omega_b\}$. These of course are not available to us, but can be estimated from the simulations. Averaging over the pseudo-independent map realizations within a given cosmology, we can estimate the ensemble average by

$$\langle N_i(p) \rangle \approx \bar{N}_i(p) \equiv \frac{1}{R} \sum_{r=1}^{R} N_i(r, p),$$

where $N_i(p)$ is the descriptor vector for one set of cosmological parameters $p$, $N_i(r, p)$ is the descriptor vector for a single realization and $r$ runs over our $R = 500$ map realizations.

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16 Also see Pires et al. (2012) which claims that convergence can contain complementary information to shear if one manages to extract it observationally.

17 Averaged over all possible universes with the same cosmological parameter values.
We call this estimate the simulation mean. It differs from the true ensemble average both because of the limited number of realizations and also because of the limitations inherent in our simulations. In the absence of a fitting formula for the peak counts in the non-Gaussian case (analogous to the power spectrum formula from Smith et al. 2003) the simulation mean serves as our proxy for theoretically predicted peak counts.18

Because of the computational expense of producing cosmological simulations, we can only produce a limited number at the selected cosmologies given in Table 1. We have calculated the simulation mean at these points and must extrapolate to other cosmologies not explicitly simulated. Using finite differences between the simulated cosmologies, we construct a first-order Taylor expansion around our fiducial cosmology to estimate \( \bar{N}_i(p) \) for other cosmologies:

\[
\bar{N}_i(p) \approx \bar{N}_i(p_0) + \sum_a \frac{\bar{N}_i(p_0^{(a)}) - \bar{N}_i(p_0)}{p_{0a}} \cdot (p_a - p_{0a}).
\] (5)

Here, index \( a = 1, 2, 3 \) refers to an individual parameter, such as \( \Omega_m, \omega, \text{or } \sigma_8 \), \( p_0^{(a)} \) denotes the cosmological parameter vector of a simulated non-fiducial cosmology (where only the parameter \( p_a \) has been varied), and \( p_0 \) denotes the parameter vector for the fiducial cosmology.

The fraction in Equation (5) is the finite difference derivative. If the non-fiducial cosmology is chosen such that \( p_{0a} = p_0 \), we call it a “forward derivative,” if it is positive, we call it a “backward derivative.” We compare the parameter constraints calculated from each derivative to assess the robustness of our results.

Similarly to the simulation mean, we estimate the covariance of the statistical descriptors from the simulations, \( \text{Cov}(N_i, N_j) \approx C_{ij} \), where

\[
C_{ij}(p) \equiv \frac{1}{R - 1} \sum_{r=1}^{R} [N_i(r, p) - \bar{N}_i(p)][N_j(r, p) - \bar{N}_j(p)].
\] (6)

This covariance matrix contains contributions both from the sample variance of the true aperture mass signal and from the noise contributions. When the size of this covariance matrix is large, inaccuracies in its estimate can become challenging, as we explore further below.

6.2. Monte Carlo Probability Contours

To compute our parameter error contours, we choose a Monte Carlo based approach as described in and applied to weak lensing simulations in Kratochvil et al. (2012). In this approach, the best-fit cosmological parameter values are found for each map of the fiducial cosmology by minimizing a cost function like \( \chi^2 \), and then the contour is drawn based on the spread of these best-fit values in the parameter space. We have many individual maps, so finding the best-fit point in parameter space for each map and interpreting their spread as the uncertainty is a more natural approach for our analysis, compared to obtaining the Fisher matrix. In addition, we do not need to correct any bias in the calculation of the inverse of the covariance matrix (as described in Hartlap et al. 2007 and Anderson 2003), and we can simply numerically invert the matrix of the estimator obtained from (6). The \( \chi^2 \) we compute for each map would require this bias-correction, but it will change at every point in the parameter space by the same factor, and therefore the cosmological parameter values which minimize this \( \chi^2 \) for each individual WL map remain the same. We can therefore neglect this bias-correction factor and avoid the additional uncertainty which enters from its estimation.

Each of our WL maps spans a 12 deg\(^2\) field of view, yet we wish to obtain parameter contours for the full 20,000 deg\(^2\) LSST survey volume. We therefore employ bootstrapping to generate approximations to full-sky maps. In this procedure, we draw a map 20,000/12 \( \approx \) 1667 times from our 500 aperture mass maps, with replacement. The resulting 20,000 deg\(^2\) map is not a true composite: we do not place the drawn maps edge to edge, but rather compute the descriptor values for each patch individually and then average over them to get their values for the full-sky map. We create 10,000 such full-sky maps to obtain smooth parameter contours in our Monte Carlo procedure.

To estimate the cosmological parameter error contour from a set of WL maps from one cosmology, we use \( \chi^2 \)-minimization to fit for the best-fit cosmological parameters for each of the above bootstrapped full-sky maps. Thus our whole set of maps provides an ensemble of Monte Carlo realizations, and the distribution of those best-fit points can be used to draw probability contours at desired confidence levels.

For realizations drawn from the fiducial cosmology \( p_0 \), \( \chi^2 \) is

\[
\chi^2(r, p) \equiv \sum_{i,j} \Delta N_i(r, p) \left[ \text{Cov}^{-1}(p_0) \right]_{ij} \Delta N_j(r, p)
\]

where

\[
\Delta N_i(r, p) \equiv N_i(r, p_0) - \langle N_i(p) \rangle.
\]

For each Monte Carlo realization, we minimize \( \chi^2 \) with respect to \( p \) using a simulated annealing algorithm (other minimization techniques work just as well). The covariance matrix is computed from the auxiliary map set with Equation (6) and inverted with single vector decomposition, discarding any problematic eigenvectors. The simulation mean for the fiducial cosmology is computed from the primary map set, whereas the finite difference derivatives are computed from the auxiliary map set. The maps for which best-fit parameters are computed come from the primary map set. It is important that this map set is statistically completely independent from the map set used to compute the covariance matrix, to prevent correlations between the fitted data and the covariance matrix, which acts as a weighting function for the individual bins. It is also important to choose the map set with potentially the largest statistical scatter (i.e., based on the most simulations) to compute the best-fit parameters for, as the spread of the best-fit points will determine the size of the error contour (an underestimation of the scatter in the covariance matrix matters far less for this Monte Carlo technique).

7. COSMOLOGICAL CONSTRAINTS

In this section we present the results obtained by applying the methods described in Section 6 to the peak counts described in Section 5 in order to constrain the cosmological parameters \( \Omega_m \), \( \sigma_8 \), and \( \omega \).

To determine the sensitivity of peak counts to cosmological parameters, one can use the backward, forward, or symmetric derivative in the Taylor expansion in Equation (5). It is important to check that all derivatives give the same contours, and as expected we find very small shifts in the contour size and shape.
Figure 11. 68% error contours on the cosmological parameters $\Omega_m$, $\sigma_8$, and $w$ for peak counts, using the backward derivative in the Taylor expansion. The contours have been marginalized over the third parameter not shown in each of the panels. The dashed curves show the constraints for measurements including shape noise only, and the solid curves for both shape noise and measurement error.

(A color version of this figure is available in the online journal.)

Figure 12. 68% error contours on the cosmological parameters $\Omega_m$, $\sigma_8$, and $w$ for peak counts, using the forward derivative in the Taylor expansion. In each of the panels the third parameter not shown is marginalized over. The dashed curves show the constraints for measurements including shape noise only, and the solid curves for both shape noise and measurement error.

(A color version of this figure is available in the online journal.)

between the different derivative methods consistent with the statistical limitations of the cosmological simulations used. The contours we present in this section are 68% error contours, corresponding to data obtained by a full LSST 10 yr survey using only good quality $r$-band data. Flat priors are assumed in the calculation.

Figure 11 shows the contours for the peak counts above S/N thresholds for the backward derivative. Each of the three panels depicts predicted constraints for two of the three cosmological parameters $\sigma_8$, $\Omega_m$, and $w$, where the contours have been marginalized over the third parameter not shown. The contours in dashed curves show the predicted constraints for measurements made with shape noise only, and solid curves show the constraints for both intrinsic shape noise and the realistic measurement errors described in previous sections. The predicted constraints with measurement error appear at first inspection to be smaller than with shape noise alone. In fact this is due to the accuracy with which we can derive the contours from the available set of cosmological simulations. A set of simulations covering a larger range of cosmological parameter space would yield smoother contours, but at present this is computationally unfeasible. If we look at the contours calculated for the forward derivative shown in Figure 12, we see that in this case the contours with measurement error appear at first inspection to be appreciably smaller than for shape noise alone. In fact this is due to the accuracy with which we can derive the contours from the available set of cosmological simulations. A set of simulations covering a larger range of cosmological parameter space would yield smoother contours, but at present this is computationally unfeasible.

We also calculate the power spectrum of the aperture mass maps, and use that to predict constraints on cosmological parameters. Previous work (Yang et al. 2011) has shown that peak counts in convergence maps contain additional information beyond the power spectrum, and can tighten constraints on cosmological parameters, as well as break degeneracies in constraints from the power spectrum alone. As discussed in Section 2, using the aperture mass statistic results in a suppression of long-wavelength modes, reducing the cosmological constraining power of the aperture mass power spectrum compared to the shear correlation power spectrum. However, the peak counts we obtain from the aperture mass maps will also contain the same suppression of information, and as such we can make a like-to-like comparison of the information contained in the aperture mass statistic. We compare here the constraints on cosmological parameters obtained from the power spectrum of the aperture mass maps, traced by galaxies with shape noise included, and the constraints from peak counts, traced by the same galaxies. We calculate the aperture mass power spectrum up to the Nyquist frequency of our maps, which is $l = 106,311$. We have verified that the results do not differ appreciably from taking the aperture mass power spectrum out to $l = 20,000$, since the shear filter we use in the aperture mass calculation cuts off the power spectrum above 20,000.

We note that we have neglected spatial correlations in measurement errors, which we expect to have a small impact on the number counts of peaks but are known to have a more significant impact on measurements of the power spectrum. In comparing constraints obtained with peak counts and our measurement of the aperture mass power spectrum, we therefore restrict the comparison to the case with shape noise only. This is shown in Figure 13. We see that the predicted constraints obtained with peak counts are better than using the aperture
mass power spectrum alone, and a small improvement is found when combining the two measurements. This has been shown to be true for peak counts in combination with the shear correlation power spectrum in previous work (Kratochvil et al. 2012; Yang et al. 2011).

It should be noted that our finding that measurement errors have a small impact on constraints from peak counts is only valid under the assumptions that we have made in our analysis framework. For example, there may be a larger error associated with the KSB algorithm than we find in this work, because our simulated galaxies are modeled with Sersic profiles rather than real galaxy shapes. Since the ImSim input catalogs are anchored to real data, this is a limitation from the current survey data, and will be improved with future observations. The accuracy of our constraints is also limited by the accuracy of our N-body simulations, which is discussed in detail in Section V.F of Kratochvil et al. (2012).

The predicted constraints in this work are comparable to those found in other analyses which use the same cosmological simulations, but different measurement techniques (Kratochvil et al. 2012; Yang et al. 2011). For example, Figure 12 in Yang et al. (2011) shows constraints on cosmological parameters from peak counts from convergence in a single redshift plane, combined with the power spectrum and scaled to an LSST-size survey, that are very similar to our constraints. The comparison is not direct, however, due to the differences in analysis methodology. In particular, most of the previous work has looked at peaks in maps of convergence, whereas we study aperture mass peak counts. The similarity of our constraints to those in Yang et al. (2011) implies that the difference between peak counts in convergence and aperture mass is small, but we are unaware of any work directly comparing the two methodologies.

It is even harder to make a comparison with other work that uses an entirely different set of cosmological simulations. In that case, the cosmological parameters varied in the simulations, as well as the details of the N-body algorithms, make it almost impossible to make meaningful comparisons between results.

It should be noted that this work does not use multiple smoothing scales. Kratochvil et al. (2012) have shown that combining smoothing scales is important to extract the maximum amount of information from weak lensing maps with a non-Gaussian descriptor, and Marian et al. (2012) have explicitly demonstrated this for peak counts. It has been shown in Hilbert et al. (2012) that the power spectrum contributes additional information to peak counts when a combination of smoothing scales is used for peaks, but it remains to be confirmed whether peaks can manage to extract all information when enough smoothing scales are used.

8. SUMMARY

We have produced the first framework for including realistic galaxies and measurement errors in predictions of shear peak counts, using information from the LSST ImSim. Galaxies are drawn from realistic distributions, based on observational data, in redshift, size, magnitude, and ellipticity. We use information from ImSim to assign uncertainties to the galaxy shape measurements based on these properties, using the KSB shape measurement algorithm. We use these realistic galaxies to trace the reduced shear maps produced from ray-traced cosmological N-body simulations, and distort the galaxy shapes appropriately according to the shear parameters interpolated in three dimensions. The aperture mass and S/N are calculated for the resulting simulated catalogs, using an implementation of the aperture mass statistic on the GPU.

We count peaks above S/N thresholds, and use the resulting peak count distributions to predict constraints on cosmological parameters with LSST. We also calculate the aperture mass for an idealized case where we know the reduced shear perfectly, with no uncertainty from galaxy shape noise or measurement error. Comparing the two cases, we find that the majority of the discriminating power for the ideal case is in the high S/N peaks, whereas for the realistic measurements the power comes from the full range of peak counts. This confirms for the case of peak counts in reduced shear maps what has already been seen in peak counts for convergence maps by Kratochvil et al. (2010) and Yang et al. (2011)—that low and medium significance peaks with S/N < 3.5σ in reduced shear maps contribute most of the cosmological constraining power.

We calculate the 68% confidence contours for the realistic and noiseless peak counts, and find that there is a significant degeneracy in Ω_m and σ_8, and smaller degeneracies in the other planes. Even in the presence of noise, there is substantial information in peak counts beyond that which can be extracted from the aperture mass power spectrum alone—the contours from peak counts are approximately 25% smaller compared to contours from the aperture mass power spectrum alone. Combining the two measurements (from peak counts and

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19 It should be noted that the analysis in Yang et al. (2011) used only 15 galaxies arcmin^-2 for one redshift plane, whereas we use 30 galaxies arcmin^-2 over the full redshift range.
the aperture mass power spectrum), we calculated expected constraints on $\Omega_m$, $\sigma_8$, and $w$ for a 10 yr LSST survey of $0.257 < \Omega_m < 0.263$, $0.792 < \sigma_8 < 0.804$, and $0.98 < w < 1.02$. Note that these constraints are given for illustration and are not intended as expected final constraints for LSST. LSST analyses will use multiple redshift bins for power spectrum and peaks and include measurements of baryon acoustic oscillations, supernovae etc. to achieve substantially better constraints.

We have shown that reduced shear peak counts are a useful probe of cosmology, and that the presence of realistic instrument noise and measurement uncertainties have very little impact on the power of the cosmological constraints. This work is the first step in a series of analyses needed to develop the analysis of shear peak counts to the level of sophistication currently enjoyed by the study of shear correlation functions. Future work should consider the impact of photometric errors, masked areas, and galaxy clustering around areas of high shear on peak counting, and should start to consider ways to potentially mitigate these effects.

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