Pair Creation of Black Holes by Domain Walls

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Abstract

In this paper we study the production of pairs of neutral and charged black holes by domain walls, finding classical solutions and calculating their classical actions. We find that neutral black holes whose creation is mediated by Euclidean instantons must be produced mutually at rest with respect to one another, but for charged black holes a new type of instanton is possible in which after formation the two black holes accelerate away from one another. These new types of instantons are not possible in Einstein-Maxwell theory with a cosmological constant. We also find that the creation of non-orientable black hole solutions can be mediated by Euclidean instantons and that in addition if one is prepared to consider entirely Lorentzian no-boundary type contributions to the path integral then mutually accelerating pairs may be created even in the neutral case. Finally we consider the production of Kaluza-Klein monopoles both by a standard cosmological term and in the presence of a domain wall. We find that compactification is accompanied by the production of pairs of Kaluza-Klein monopoles.

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I. INTRODUCTION

Motivated by the recent interest in black hole pair creation in Euclidean quantum gravity \footnote{1}, we consider additional such tunnelling processes in the presence of domain walls. The reason we expect tunnelling processes occur in a domain wall space-time is because the gravitational field of a domain wall is repulsive. An observer near a domain wall will experience a repulsive force, and will accelerate away from the wall. This is just the sort of ‘anti-gravitational’ background in which pair creation processes occur, as with de Sitter space-time \footnote{2}. In this paper we show that tunnelling geometries do indeed exist, and compute the probabilities for the pair creation of uncharged and charged black holes in a domain wall space-time. In section II, we discuss the novel gravitational properties of domain walls and give a cut-and-paste procedure for the construction of a domain wall space-time. In section III, we consider the tunnelling process by which vacuum domains are created from nothing. We construct the necessary instanton, and compute the tunnelling amplitude. This process is to be considered as the background process to the pair creation of black holes. In section V, we show that uncharged, static black holes may be pair created in the presence of a domain wall. This result is unusual, as few other processes for the pair creation of uncharged black holes are known. However, no instanton exists for the creation of accelerating, uncharged, static black holes. In section VI, we consider the pair creation of magnetically charged black holes. Here we find both static and accelerating solutions, with two unusual results. First, for a given domain wall surface energy there exists an infinite sequence of discrete values of the charge, describing the production of accelerating, charged black holes. Second, under the action of certain discrete involutive isometries, half of these configurations describe the creation of non-orientable black holes. In section VII we discuss the uniqueness and isoperimetry of instantons. In section VIII we discuss Lorentzian tunnelling geometries for the processes described in this paper. Finally, in section X, we discuss the implications of the creation of non-orientable black holes, the stability of domain walls against puncture.

Throughout this paper we use units in which \(\hbar = c = G = 1\).

II. DOMAIN WALLS

In this section we review the properties of domain walls. We discuss the novel gravitational properties of a domain wall space-time, features relevant for the calculations carried out in this paper. A procedure for the cut-and-paste construction of a domain wall space-time will be given.

A domain wall is a two-dimensional topological defect which forms at the boundary between two regions of space in which a field, such as a Higgs, has undergone the breaking of a discrete symmetry. If \(V_0\) is the submanifold of the field configuration space on which the field acquires a vacuum expectation value, then a necessary condition for the appearance of a domain wall is \(\pi_0(V_0) \neq 0\) which tells us that the vacuum manifold is not connected. The physics of domain walls is extensively reviewed in \footnote{3}.

We now summarize the gravitational features of a domain wall \footnote{4}. An idealized, thin domain wall located at \(x = 0\) has the stress-energy tensor

\[
T_{\mu\nu} = \sigma \delta(x) \text{diag}(1, 0, 1, 1) \tag{2.1}
\]
where the surface mass-density of the wall is given by $\sigma$. For this distributional source, the metric is $C^0$ but not $C^\alpha$, for any $\alpha \geq 1$. While it is not possible to find a static solution of the Einstein equations with this source term, a time-dependent solution exists, as shown by Vilenkin [4] and Ipser & Sikivie [5]. Their metric takes the form

$$ds^2 = \left(1 - K|x|\right)^2 dt^2 - dx^2 - \left(1 - K|x|\right)^2 e^{2Kt}(dy^2 + dz^2),$$

where $K = 2\pi\sigma$. As was pointed out in [4], the gravitational field of the vacuum domain wall described by (2.2) is repulsive, because the source (2.1) violates the strong energy condition; an inertial observer at $x = 0$ will see test bodies accelerated away from the wall with acceleration $a = K$. To understand this, note that the $t - x$ part of the metric (2.2) is the two-dimensional Rindler metric [5]. By analogy with the cases of electric fields and a positive cosmological constant, we would expect this repulsive force to provide a mechanism for a tunnelling process such as black hole pair production, as we find later in this paper.

Let us examine in more detail the global structure of the space-time described by the line-element (2.2). The hypersurfaces $x = \text{constant}$ are isometric to a portion of 2+1-dimensional de Sitter space-time:

$$ds^2 = dt^2 - e^{2Kt}(dy^2 + dz^2).$$

Since 2+1 de Sitter space-time has the topology $S^2 \times \mathbb{R}$ we expect the domain wall to have the topology $S^2 \times \mathbb{R}$, which means that at each instant of time, the domain wall is a sphere. To see this more clearly, recall that in [5] a transformation to the coordinates $(T, X, Y, Z)$ was found covering one side of the domain wall space-time, so that the metric becomes

$$ds^2 = dT^2 - dX^2 - dY^2 - dZ^2.$$

Thus the domain wall, which in the old coordinates is a plane located at $x = 0$, is in the new coordinates the hyperboloid

$$X^2 + Y^2 + Z^2 = \frac{1}{K^2} + T^2.$$

The metric induced on this hyperboloid by the ambient Minkowski metric is just the de Sitter metric. Therefore, the domain wall is a copy of 2+1 de Sitter. This provides a useful way of thinking of this domain wall. Consider the following topological construction: take two copies of Minkowski space, and in each copy consider the interior of the hyperboloid determined by equation (2.3), match these solid hyperboloids to each other across their respective boundaries; there will be a ridge of curvature (much like the edge of of a lens) along the matching surface, where the domain wall is located. This is illustrated in figure [6]. Thus, an inertial observer on one side of the wall will see the domain wall as a sphere which accelerates towards the observer for $T < 0$, stops at $T = 0$ at a radius $K^{-1}$, then accelerates away for $T > 0$. With this brief introduction to the properties of the domain wall space-time, we turn our attention to the problem of using this background to nucleate black holes.
III. CREATION OF DOMAINS

In order to study the pair creation of black holes in a domain wall space-time, we first consider the creation from nothing of a closed universe consisting of two vacua separated by a domain wall. This process is the background to the black hole pair production.

The instanton for domain creation is the analogue of the \( S^4 \) instanton which mediates the creation from nothing of a de Sitter space-time. Since the Lorentzian section of the instanton is almost everywhere flat, and consists of the interiors of two hyperboloids in Minkowski space-time glued back-to-back, the obvious choice for the Riemannian section is to take two flat 4−balls and glue them back-to-back. This gives a manifold topologically equivalent to \( S^4 \). Thus we obtain a ‘lens’, owing to the ridge of curvature running along the hemisphere at the location of the domain wall, which is topologically like \( S^4 \). This construction is illustrated in figure 1. This Riemannian section may be matched to the Lorentzian space-time across a nucleation surface \( \Sigma \), to describe the creation of a closed universe.

The Euclidean action for the domain creation is

\[
S_E = \int_M \sqrt{g} d^4x \left[ -\frac{R}{16\pi} + \frac{1}{2} (\partial \phi)^2 + V(\phi) \right]. \tag{3.1}
\]

Substituting the on shell condition

\[
\frac{R}{8\pi} = (\partial \phi)^2 + 4V \tag{3.2}
\]

we obtain

\[
S_E = -\int \sqrt{g} d^4x V(\phi). \tag{3.3}
\]

There are no boundary terms because the instantons we consider are compact, having no boundary. Also, provided the potential energy function \( V(\phi) \) is positive, the Euclidean action \( S_E \) will always be negative. For a domain wall in flat space, as a consequence of the equation of motion,

\[
\frac{1}{2} \left( \frac{d\phi}{dz} \right)^2 - V(\phi) = 0, \tag{3.4}
\]

where \( z \) is the proper distance in the direction perpendicular to the domain wall. The energy-per-unit-area \( \sigma \) is given by

\[
\sigma = \int_{-\infty}^{\infty} dz \left[ \frac{1}{2} \left( \frac{d\phi}{dz} \right)^2 + V(\phi) \right] = 2 \int_{-\infty}^{\infty} dz V(\phi[z]) \tag{3.5}
\]

Thus, for domain walls in the thin wall approximation,

\[
S_E = -\frac{1}{2} \sigma \int \sqrt{h} d^3x \tag{3.6}
\]
where the integration of $\sqrt{hd^3x}$ gives the volume of the $S^3$ ridge on the instanton. The ridge is located at radius $r = 1/(2\pi\sigma)$ from the center of either four-ball. Hence, the volume is

$$\text{vol}(S^3) = 2\pi^2 r^3 = \frac{1}{4\pi\sigma^3}$$  \hspace{1cm} (3.7)

Therefore, the action for nucleating a domain wall is $S_E = -1/(8\pi\sigma^2)$. The amplitude for this process, ignoring the pre-factor is, remembering that only half the Riemannian instanton is included in the complex path, $\exp(-S_E/2)$. This amplitude increases for increasing domain wall size, or decreasing $\sigma$. Such behaviour is analogous to the case of a universe containing a positive cosmological constant created from nothing. There, the tunnelling process amplitude grows for an increasingly large universe, or for decreasing cosmological constant.

**IV. Uniqueness and Isoperimetry**

In deciding which instantons are the most important to a given physical process one usually uses two criteria: (1) uniqueness, and (2) least action. In fact the latter criterion is more frequently used. Consider for example the decay of the false vacuum in 4-dimensional Euclidean space via the formation of a bubble of true vacuum. According to Coleman et al [6], the $O(4)$ invariant bubble has the least action among solutions. In the thin wall approximation this reduces to the isoperimetric inequality: the 3-sphere has the least area of any 3-surface enclosing the given 4-volume.

In fact Coleman’s assumption that we should restrict attention to the least action solution is redundant in this case. A celebrated theorem of Gidas, Ni and Nirenberg [7] implies that the only solutions of $\nabla^2 \phi = V'(\phi)$ on $\mathbb{R}^4$ tending to the false vacuum at infinity are $O(4)$ invariant. Thus independently of action considerations if we are to use a saddle point of the classical action at all we can only use an $O(4)$ invariant one.

In the thin wall approximation the result of Gidas, Ni and Nirenberg [7] reduces to an equally celebrated result of Aleksandrov [8] on the uniqueness of embedded closed hypersurfaces of constant mean curvature $K = g^{ij} K_{ij}$ in flat space: They must all be spherical. Another closely related result is that of Ros [1]: A sphere is the only compact hypersurface with constant Ricci scalar embedded in Euclidean space. The thin shell approximation is $K_{ij} = cg_{ij}$. The Gauss-Codazzi equation, $\nabla_i K^{ij} - \nabla^j K = 0$, then implies that $c$ is a constant. In a flat Riemannian 4-manifold the Ricci scalar of a hypersurface is given by

$$(3) R = -K_{ij} K^{ij} + K^2$$  \hspace{1cm} (4.1)

whence $(3) R$ is a constant.

Note that these global results may fail if we loosen our assumptions that we have embedded hypersurfaces to allow immersions. In our case the assumption that we have an embedding is essential because our domain wall must separate space-time into two regions: self-intersections are not allowed physically.

It follows directly from the results of Aleksandrov [8] or of Ros [9] quoted above that our construction of the instanton mediating the birth of two domains is unique. In other words, our almost everywhere flat $S^4$ is the only such almost everywhere flat $S^4$. It is interesting
that the corresponding uniqueness result for the usual round Einstein metric on $S^4$ remains elusive. Furthermore, although we have not investigated in detail the uniqueness of our $O(3)$ invariant thin shells in the Euclidean Schwarzschild geometry we conjecture that they are also unique.

V. CREATION OF UNCHARGED BLACK HOLES

We now analyse the problem of creating uncharged black holes in the presence of a domain wall. First, we construct the instanton for the nucleation of a pair of static black holes. Second, we demonstrate that no such instanton exists for accelerating black holes in this background space-time. Third, we compute the probability for the nucleation of a pair of static black holes.

Let us address whether it is possible to create a pair of black holes which are in static equilibrium relative to the domain wall. Naïvely, we would expect to be able to fine tune the black hole mass $m$ and domain wall surface energy $\sigma$ so that the repulsive force of the wall exactly balances the attractive force on the black holes. Indeed, this is always possible and there exists a Euclidean instanton which we can use to estimate the probability that such a pair of ‘finely tuned’ black holes would be created in the presence of the wall.

To obtain the instanton for static black hole nucleation, we first construct the Lorentzian section. In order to obtain a non-zero probability, we require a spatially closed universe whose 3-volume is finite. This guarantees that the total energy at the instant of nucleation vanishes. To obtain such a space-time, take two copies of the full vacuum Kruskal manifold, each of which has two asymptotically flat regions. Cut each along two static timelike hypersurfaces of the same radius, one in each asymptotically flat region outside each hole, discarding the exteriors. This procedure is illustrated in figure 2. We obtain a space-time with closed spatial sections having topology $\mathbb{R} \times S^1 \times S^2$ by identifying across the two static timelike hypersurfaces. The result is a space-time containing two domain walls and two domains containing a black hole in each. Moreover this space-time, with its identifications, has a Riemannian section.

To obtain this Riemannian section, we start with the usual Riemannian section of Schwarzschild with mass $m$. There the topology is $S^2 \times \mathbb{R}^2$, where the $\mathbb{R}^2$ factor looks like a cigar. We snip each cigar along the radius $r = 3m$, where $m = (6\sqrt{3}\pi\sigma)^{-1}$, corresponding to the location of the domain wall. Next, we graft the two manifolds together along the surface where we made the cut. The resulting surface, which is rather like a ‘baguette’, having topology $S^2 \times S^2$ with a ‘ridge’ at the domain wall, is illustrated in figure 2. One may check that for a radius $r = 3m$ and no other,

1. the hypersurface $r$ is totally umbilic, that is the second fundamental form $K_{ij}$ is proportional to the induced metric $g_{ij}$ on the domain wall world sheet

2. the discontinuity in the second fundamental form on the hypersurface $r = 3m$ is $[K_{ij}] = 4\pi \sigma g_{ij}$.

Thus, if $r = 3m$, the metric satisfies the Israel junction conditions and is therefore a bona fide domain wall space-time.
The Riemannian section now has topology $S^2 \times S^2$, containing a single domain wall and two ‘bolts’, that is two 2–spheres on which the Killing field $\frac{\partial}{\partial \tau}$ vanishes. The Riemannian and Lorentzian sections are joined together across the nucleation surface $\Sigma$ which has topology $S^1 \times S^2$. The nucleation circle is located on the ‘baguette’ along $\tau = 0$ and $\tau = 4\pi m$. We now turn to the equations of motion of the instanton.

The above construction for the nucleation of a pair of static black holes in the presence of a domain wall is a special case of the work of Hiscock [10] and Berezin et al [11] on $O(3)$–invariant thin wall bubble nucleation. Applying the Israel junction conditions to the domain wall interface joining the two Euclidean black hole space-times, we obtain the equation of motion [10]

$$\sqrt{f - \dot{r}^2} = 2\pi \sigma r, \quad (5.1)$$

for a wall located at radius $r$. Here $f = 1 - 2m/r$, the $g_{00}$ Euclidean Schwarzschild metric coefficient in $(\tau, r)$ coordinates, and $\cdot \equiv f^{\frac{1}{2}} \frac{\partial}{\partial \tau}$. This equation may have the interpretation as describing a fictitious particle moving under the influence of a potential

$$V(r) = f^2 - (2\pi \sigma r)^2. \quad (5.2)$$

In the present case, the equation describes the motion of the domain wall relative to the Euclidean black holes. The general solution of (5.1) is periodic, but there is a solution for which $r =$ constant and the energy is zero, corresponding to $\partial V/\partial r = 0$. This has an infinite period, and occurs at $r = 3m$; it is the static domain wall.

We may also look for accelerating solutions describing the creation of accelerating black holes. The solution of the Euclidean equations of motion is periodic in $\tau$. The period $\beta_w$ is obtained by evaluating the line integral $d\tau$ over the closed path between the extrema $[r_{\text{min}}, r_{\text{max}}]$, defined by the radii at which $\dot{r} = 0$. Thus,

$$\beta_w \equiv \oint_{r_{\text{min}}}^{r_{\text{max}}} d\tau = \oint_{r_{\text{min}}}^{r_{\text{max}}} \frac{dr}{\sqrt{f - (2\pi \sigma r)^2}}. \quad (5.3)$$

The reader is cautioned that this expression for the period is inapplicable to the static, $m \to (6\sqrt{3}\pi \sigma)^{-1}$ case in that as $r_{\text{min}} \to r_{\text{max}}$, we find $\beta_w \to 1/\sigma$. This solution does not correspond to the static, $r = 3m$ solution for which the period in $r$ is infinite.

In order for solutions of the Euclidean equations of motion to define $\textit{bona fide}$ domain walls, they should not intersect themselves. Therefore the period $\beta_w$ must equal, or be an integer submultiple, of the period of Schwarzschild:

$$\beta_w = \frac{\beta_S}{n} = \frac{8\pi m}{n}, \quad n \in \mathbb{Z}_+. \quad (5.4)$$

However, the above condition cannot be satisfied for an accelerating black hole. There is no value of the black hole mass in the interval $m \in [0, (6\sqrt{3}\pi \sigma)^{-1}]$ for which the periods match, as shown in figure 3. It follows that the only non-self-intersecting domain wall trajectory is the static one, in which case $r = 3m$ and $m = 1/(6\sqrt{3}\pi \sigma)$. Hence, for a given domain wall surface energy density, there is a unique mass for the Schwarzschild black hole which may be created.
We now calculate the action for the pair creation of static black holes. From our earlier work, equation (3.6) in section III, the Euclidean action is

\[ S_E = -\frac{1}{2}\sigma \int_W \sqrt{h} d^3x = -\frac{1}{2}\sigma \text{vol}(W), \tag{5.5} \]

where \( W \) denotes the hypersurface supporting the domain wall. Provided the surface energy density of the domain wall is positive, the Euclidean action is negative. The volume of the domain wall \( W \) is given by

\[ \text{vol}(W) = \sqrt{1 - \frac{2m}{r}} \int_0^{\pi} d\tau \int_0^{\pi} d\theta \int_0^{2\pi} d\phi r^2 \sin \theta \]
\[ = \sqrt{1 - \frac{2m}{r}} \cdot 8\pi m \cdot 4\pi r^2 \]
\[ = 96\sqrt{3}\pi^2 m^3. \tag{5.6} \]

Thus, the action for the creation of a (static) Schwarzschild-domain wall universe is given by

\[ S_E = -48\sqrt{3}\pi^2 m^3\sigma. \tag{5.7} \]

Of course, we do not really want to create a whole new universe containing a pair of static black holes and a domain wall. Rather, we wish to calculate the probability for black holes to form given the presence of a domain wall. We want to divide out by the probability to nucleate a domain wall, as calculated in section III.

The probability \( P \) for the tunnelling process is the square of the amplitude, \( P = e^{-S_E} \). Dividing the probability for creating black holes with domain walls by the probability for domain walls only, we obtain the relative probability for static, uncharged black hole pair production in the presence of a domain wall:

\[ P = \exp \left( -\frac{11}{216\pi\sigma^2} \right). \tag{5.8} \]

As expected, the probability is heavily suppressed for small \( \sigma \).

This conclusion seems to be at variance with that of Hiscock [10], who has claimed that the Euclidean action for some \( O(3) \)-symmetric bubble solutions is smaller than those of the \( O(4) \)-symmetric solutions, and hence that black holes may act as effective nucleation centers rendering the nucleation of domains more likely. It seems that Hiscock is not considering precisely the same situation that we have in mind, since we are considering a situation in which the black holes appear at the same time as does the domain rather than being present beforehand. Moreover, Hiscock considers the general case when the energy densities \( \rho_1, \rho_2 \) outside the domain wall are non-vanishing. It seems that one cannot obtain in a simple way our solutions as a limit of his results. Thus, for example, his equation (27) [10] is not equivalent to our equation (5.7) in the limit that \( \rho_1 = \rho_2 = 0 \). This is for the same reason that (5.3) cannot be used in the limit \( r_{\text{min}} \to r_{\text{max}} \) to obtain the static domain wall solution.

Our results on the spatially-inhomogeneous domain wall space-time seem to be in general accord with those of Bousso & Hawking [12] who consider spatially-homogeneous, complex
solutions with a massive scalar field. They find that the probability of creating a Nariai-type solution with spatial topology \( S^1 \times S^2 \) is suppressed relative to the probability of creating a de Sitter-type solution with spatial topology \( S^3 \).

It is interesting to compare our result for the probability (5.8) with the rate for the pair creation of black holes from a cosmic string. In Eardley et al [13], the probability that a cosmic string will snap, producing two black holes on the bare string terminals, was estimated. For a string of mass-per-unit-length \( \mu \) producing black holes of mass \( m \), the rate is approximately \( \exp(-\pi m^2/\mu) \). To compare this result with (5.8), we look for black holes of mass \( m = (6\sqrt{3}\pi\sigma)^{-1} \). The ratio of the Euclidean actions for these processes is \( S_{E,\text{wall}}/S_{E,\text{string}} \sim 11\mu/2 \), so that for Planck scale defects only do the two processes have comparable probabilities.

VI. CREATION OF MAGNETICALLY CHARGED BLACK HOLES

In this section, we consider the creation of magnetically charged black holes. The case of electrically charged black holes is entirely analogous with the subtlety that the electromagnetic field in the electrically charged case must be pure imaginary on the Riemannian section [14]. In the case of the charged black hole, the equation of motion of the bubble wall is

\[
\sqrt{\tilde{f}} - \dot{r}^2 = 2\pi\sigma r
\]

where \( \tilde{f} = 1 - 2m/r + q^2/r^2 \), the \( g_{00} \) Euclidean Reissner-Nordström metric coefficient in \((\tau, r)\) coordinates, and \( \dot{\cdot} = \tilde{f}^{-1/2} \partial / \partial \tau \). As with the uncharged case, a static solution exists for the motion of the domain wall relative to the black hole, now located at a radius

\[
r_{\text{static}} = \frac{3}{2} m \left[ 1 + \sqrt{1 - \frac{8q^2}{9m^2}} \right].
\]

The mass of the created black holes, given by

\[
m = \frac{1}{6\sqrt{6}\pi\sigma} \left[ 1 + 36(2\pi\sigma q)^2 + \left(1 - 12(2\pi\sigma q)^2\right)^{3/2}/2 \right]^{1/2},
\]

as a function of the black hole charge, \( q \), and surface energy density, \( \sigma \), of the domain wall. Here we see that the mass runs between \((6\sqrt{3}\pi\sigma)^{-1} \leq m \leq (8\pi\sigma)^{-1}\) for \( 0 \leq q \leq m \).

The Euclidean action for the instanton now includes an electromagnetic contribution. Using the Einstein-Maxwell field equations, we obtain

\[
S_E = -\frac{1}{2} \sigma \int_W \sqrt{h} d^3x + \int_M \sqrt{g} d^4x \frac{F^2}{16\pi}.
\]

The first term, due to the presence of the domain wall, gives

\[
-\frac{1}{2} \sigma \int_W \sqrt{h} d^3x = -2\pi\sigma r^2 \beta_{RN} \tilde{f}^{1/2} |_{r_{\text{static}}}
\]

evaluated at \( r_{\text{static}} \). Here, \( \beta_{RN} \) is the instanton period for the Reissner-Nordström black hole,
\[ \beta_{RN} = 2\pi \left( \frac{m + \sqrt{m^2 - q^2}}{\sqrt{m^2 - q^2}} \right). \]  

(6.6)

For the second term, the integration over \( M \) covers both sides of the domain wall space-time, to give

\[ \int_M \sqrt{g}d^4x \frac{F^2}{16\pi} = q^2 \beta_{RN} \left( \frac{1}{r_+} - \frac{1}{r_{static}} \right). \]  

(6.7)

where \( r_+ = m + \sqrt{m^2 - q^2} \) is the outer black hole horizon radius. We may divide the amplitude for this process by the amplitude for domain creation to obtain the probability for the pair creation of static, charged black holes in the presence of a domain wall:

\[ P = \exp \left[ -\frac{1}{8\pi \sigma^2} + 2\pi \sigma r_{static}^2 \beta_{RN} \left( \frac{1}{r_+} - \frac{1}{r_{static}} \right) \right] \]  

(6.8)

In the limit \( q \to m \), the probability becomes \( P = \exp[-3/(32\pi \sigma^2)] \). Hence, we see that the probability for the pair creation of static, charged black holes is only slightly suppressed relative to the production of uncharged black holes.

For the case of a charged, accelerating black hole we must match the domain wall instanton to the Lorentzian space-time. The period of the instanton is now given by

\[ \beta_w \equiv \oint_{r_{max}}^{r_{max}} d\tau = \oint_{r_{min}}^{r_{min}} \frac{dr}{\sqrt{\left( \frac{1}{r} - \frac{2m}{r^2} + \frac{q^2}{r^4} \right) \left( 1 - \frac{2m}{r} + \frac{q^2}{r^2} - (2\pi \sigma r)^2 \right)}}. \]  

(6.9)

Matching the Riemannian to Lorentzian sections, we require that the period \( \beta_w \) must equal, or be an integer submultiple of, the period of Reissner-Nordström:

\[ \beta_w = \frac{\beta_{RN}}{n} = \frac{2\pi}{n} \left( \frac{(m + \sqrt{m^2 - q^2})^2}{\sqrt{m^2 - q^2}} \right), \quad n \in \mathbb{Z}_+. \]  

(6.10)

The behavior of \( \beta_w \) and \( \beta_{RN} \) as functions of \( q \) are shown in figure 4. As \( q \to m \), \( \beta_{RN} \) diverges, whereas \( \beta_w \) approaches a finite value. Examining figure 4, we see that for a certain values of the mass, there are values of the charge such that \( \beta_{RN} \geq \beta_w \). Alternatively, for certain values of the charge, there are black hole masses such that \( \beta_{RN} \geq \beta_w \). Hence, we find the interesting result that for a given domain wall surface energy density \( \sigma \), there exists a family of instanton solutions describing the pair creation of accelerating black holes with mass \( m \) and charge \( q_n \) for \( n \in \mathbb{Z}_+ \). These instantons are not \( SO(2) \)-invariant, merely \( D_n \)-invariant, where \( D_n \) is the dihedral group; the symmetries of a polygon with \( n \) sides. If \( n \) is odd, the maximum and minimum values of \( r \) are diametrically opposite each other in the \( r - \tau \) factor. A radial line passing through both lies on a reflection symmetric axis, a possible nucleation surface for the instanton. If \( n \) is even, there are two types of reflection symmetric axes, one passing through two opposite radial maxima, \( r_{max} \), and one passing through two opposite radial minima of \( r_{min} \). Thus, there appear to be two possible nucleation surfaces for the instanton in this case.

The two nucleation surfaces for \( n \)-even have interesting physical consequences. The Lorentzian data for the black holes formed at \( r_{min} \) describe decelerating black holes. These
black holes form, then collapse in on the domain wall. The data for the black holes formed at $r_{\text{max}}$ describe accelerating black holes. These black holes form, then accelerate away from the domain wall.

Furthermore, of these instantons, the $n$–even solutions may describe the creation of non-orientable black holes [15] if points on the Riemannian section which differ by a shift in $\tau$ of half a period are identified together with reflection in the domain wall.

The Euclidean action for the instanton describing the pair production of charged, accelerating black holes with a domain wall is again given by equation (6.4) with the following changes. The first term, due to the presence of the domain wall, is

$$
-\frac{1}{2} \sigma \int_W \sqrt{h} d^3x = -2\pi \sigma \int_0^\beta d\tau r^2 \tilde{f}^{1/2}
$$

$$
= -4\pi \sigma \int_{r_{\text{min}}}^{r_{\text{max}}} dr \frac{r^2}{\sqrt{\tilde{f} - (2\pi \sigma r)^2}}
$$

(6.11)

The second term, due to the electromagnetic field, is

$$
\int_M \sqrt{g} d^4x \frac{F^2}{16\pi} = q^2 \int_0^\beta d\tau \int_{r_+}^{r} \frac{dr}{r^2}
$$

$$
= q^2 \frac{\beta}{r_+} - 2q^2 \int_{r_{\text{min}}}^{r_{\text{max}}} dr \frac{1}{r \sqrt{\tilde{f} - (2\pi \sigma r)^2}}
$$

(6.12)

Dividing the amplitude for this process by the amplitude for domain creation, we obtain the probability for the pair creation of accelerating, charged black holes in the presence of a domain wall. The argument of the exponent of the probability, as a function of $q = q_n$, for different values of $n$, are shown in figure 5. Here we see that the probability for the creation of accelerating, charged black holes decreases only slightly for increasing values of $n$.

It is interesting to compare our results with the recent work of Mann & Ross [16] who consider Reissner-Nordström - de Sitter solutions. They find three classes of instantons for the pair creation of charged black holes in a background de Sitter universe. The first class corresponds to black holes with zero surface gravity. These are analogous to our extreme, static solutions. The second class occurs if the surface gravities of the cosmological and event horizons are equal. As first pointed out by Mellor & Moss [17], this arises if $q = m$ in the usual notation. This class seems to be analogous to our non-extreme, static solutions. The third class is a generalized Nariai space-time. This has no obvious analogue in our work.

Because of the $O(3)$-invariance assumed by Mann & Ross, their Lorentzian solutions must, by Birkhoff’s theorem, be static and the Riemannian solutions invariant under $SO(2) \times SO(3)$. Thus, it is impossible to find analogues of our accelerating domain wall solutions which are invariant only under $O(3)$.

VII. LORENTZIAN SADDLE POINTS

In this section we would like to point out that there is a Lorentzian solution describing a pair of black holes accelerating in a domain wall background. As would be expected, this is obtained by taking two copies of Schwarzschild, ‘cutting’ each copy along the surface of
a hyperboloid, the three-surface of constant acceleration surrounding each hole, identifying these hyperboloids and then identifying the holes to compactify everything (see figure 2).

This solution, which we call the accelerating Schwarzschild-domain wall solution, can be used to construct a ‘Lorentzian path’ corresponding to the ‘birth’ of a pair of accelerating black holes in a domain wall background, even though no Euclidean instanton describing the process exists. To see how to do this, first notice that there exists a well-defined involutive isometry, which we shall denote as \( R_T \), on each side of the wall, which in Kruskal coordinates \((T, Z, \theta, \phi)\) can be defined simply by taking \( T \rightarrow -T \). This involution is obviously well defined on the entire Schwarzschild-domain wall solution. We remind the reader that the Kruskal coordinates \( T \) and \( Z \), which cover the maximal extension of Schwarzschild, are given in terms of the coordinates \( r \) and \( t \) by

\[
T = \sinh \left( \frac{t}{4m} \right) e^{\frac{r}{4m} \sqrt{r - 2m}}
\]

\[
Z = \cosh \left( \frac{t}{4m} \right) e^{\frac{r}{4m} \sqrt{r - 2m}}
\]  

Unfortunately, \( R_T \) has \( T = 0 \) as a fixed point set. In order to get a freely acting involution we therefore need to compose \( R_T \) with some other map which does not have any fixed points. Such a map is given, in terms of the natural Kruskal coordinates on each side of the domain wall, by simply constructing a map corresponding to ‘parity inversion’:

\[
P : (T, Z, \theta, \phi) \rightarrow (T, Z, \pi - \theta, \phi + \pi)
\]  

By forming the composition, \( PR_T \), of these two maps we therefore obtain a freely acting involutive isometry which acts on the the whole accelerating Schwarzschild-domain wall solution. On the domain wall itself, the map \( PR_T \) restricts to the usual antipodal map on de Sitter space \([18]\). If we identify this solution under the action of this involutive isometry, we obtain a non-time and non-space orientable Lorentzian manifold with a single boundary component homeomorphic to \( S^1 \times \mathbb{RP}^2 \). This is therefore an example of a solution where we can find a Lorentzian ‘history’ describing some process, but there exists no Euclidean instanton. More precisely, there exists a Lorentzian ‘saddle point’, which interpolates from ‘nothing’ (the empty set) to ‘something’ (the desired Schwarzschild-domain wall solution). One might object that this construction does not in fact yield the desired late-time solution since obviously the geometry is neither space nor time-orientable. However, this objection is not really relevant for two reasons. First of all, using \([15]\) we know that each side of the domain wall in the identified solution admits the pin structure corresponding to the superselection sector of fermions actually used in particle physics, and so we can consistently study solutions of the Dirac equation in this background. Second of all, as we have discussed in detail in \([15]\), whenever there exists some energy source which can contribute to decay processes such as black hole pair production (using instanton techniques), that same energy source can contribute to the birth of non-orientable black holes. Indeed, as we discuss in \([18]\) of this paper, many of the instantons which we have constructed above to describe the production of magnetically charged black holes can be identified to yield non-orientable instantons which mediate the decay of vacuum domain wall solutions into solutions containing a pair of non-orientable black holes. The purpose of this section is to point out that there are some decay processes which cannot be mediated even by non-orientable instantons, but which can be described using non-orientable Lorentzian manifolds.
Of course, we would expect this sort of thing to happen in a variety of other situations as well. For example, any description of gravitational kink-antikink pair production would require the use of Lorentzian histories since by definition, the ‘nucleation surface’ for a gravitational kink could not have positive definite signature.

An interesting point to make here is that very often the solutions which can be ‘created from nothing’ using instanton methods can also be identified under the action of some freely acting isometry to yield a Lorentzian saddle point which also describes the late time ‘creation’ event. This statement is formalized in the following

**Fact:** Let \( M^L = (M, g_L) \) be any Lorentzian manifold admitting a Riemannian section \( M^R \), and suppose that there exists some freely acting involutive isometry on \( M^L \). Then it is possible to find a Lorentzian saddle point which describes the ‘creation from nothing’ of \( M^L \).

In other words, given a decay process mediated by an instanton one can often find a Lorentzian path which corresponds to the same process. To see why this is true, recall that since there exists an instanton \( M^R \) for \( M^L \) we can match \( M^L \) to \( M^R \) across a spacelike three-surface \( \Sigma \) of vanishing extrinsic curvature. We can therefore cut \( M^L \) along \( \Sigma \) and form the ‘double’ \( 2M^L \) such that \( \Sigma \) is the \( t = 0 \) fixed point set of time reversal \( T : t \to -t \) on \( 2M^L \). Given the existence of another, freely acting involution ‘\( F \)’, we can form the freely acting isometry \( FT \) and identify \( 2M^L \) under the action of this involution. The resulting manifold will then be a non-time-orientable Lorentzian manifold with a single boundary component, such that the surface \( t = 0 \) will be the initial data ‘created’ by the instanton. Whether or not the identified Lorentzian path is space-orientable or not will depend upon whether or not the freely acting involution is space-orientation preserving or reversing. In the above example with the domain wall, we were forced to take \( P \) as our freely acting isometry, and \( P \) is space-orientation reversing. In other scenarios, it may be possible to choose space-orientation preserving maps. Such subtle differences will depend upon the detailed geometry of the solution in question.

### VIII. PRODUCTION OF KALUZA-KLEIN MONOPOLES AND THE DYNAMICS OF COMPACTIFICATION

In this section we shall give examples of the creation of Kaluza-Klein monopole - anti-monopole pairs. Our construction has something in common with the creation of pairs of monopoles by magnetic fields in Kaluza-Klein (KK) theory [13].

To begin with, consider the creation of a 5-dimensional universe by a positive cosmological constant. By the obvious analogy with the 4-dimensional case the instanton is \( S^5 \) with its standard round metric. We write this as

\[
ds_5^2 = d\tau^2 + \cos^2 \tau d\Omega_4^2
\]

where \( d\Omega_4^2 \) is the round metric on \( S^4 \). The entire 5-sphere is obtained by allowing \(-\frac{\pi}{2} \leq \tau \leq \frac{\pi}{2}\), but for the real tunnelling geometry we have \(-\frac{\pi}{2} \leq \tau \leq 0\). The Lorentzian section has \( \tau = it \) with \( t > 0 \). This gives a 5-dimensional expanding de Sitter universe whose spatial cross-sections are the round metric on \( S^4 \). To obtain the 3-dimensional space and
4-dimensional space-time we must find a Killing vector field $\partial/\partial x^5$ whose trajectories are generically circles and implement the KK reduction procedure.

Whatever Killing field we choose it is clear from (8.1) that both the internal dimensions and the 3 spatial dimensions will expand exponentially at the same rate in the Lorentzian portion of the geometry. Thus this example is not very satisfactory from a physical point of view but it does serve to illustrate the nature of the initial creation process. In a more elaborate model one might introduce some mechanism which would cause the scale of the internal dimension to settle down to some fixed, small value.

In choosing the $U(1)$ Killing field we recall that in the language of [20] $S^4$ may be regarded as containing a NUT and an anti-NUT. In KK theory these NUT’s correspond to monopoles. Explicitly we express the round metric on $S^4$ as

$$d\Omega_4^2 = d\rho^2 + \sin^2 \rho \left[ \frac{1}{4}(d\theta^2 + \sin^2 \theta d\phi^2) + \frac{1}{4}(dx^5 + \cos \theta d\phi)^2 \right]$$

(8.2)

where $0 \leq \rho \leq \pi$ and $\theta, \phi, x^5$ are Euler angles on $SU(2) \cong S^3$. The metric in the square brackets above is the standard round metric on $S^3$ with unit radius. Thus $0 \leq \theta \leq \pi$, $0 \leq \phi \leq 2\pi$, $0 \leq x^5 \leq 4\pi$. The vector field $\partial/\partial x^5$ generates the Hopf fibration on the $3$-sphere $\rho =$ constant. This acts freely on $S^3$ but it has fixed points at the north and south poles of $S^4$, $\rho = 0$ and $\rho = \pi$. To reduce to $4 + 1$ dimensions we write

$$ds_5^2 = e^{-\frac{4\sigma}{\sqrt{3}}}(dx^5 + 2A_\mu dx^\mu)^2 + e^{\frac{2\sigma}{\sqrt{3}}} g_{ab} dx^a dx^b$$

(8.3)

where $\sigma = \sigma(x^a)$ is the “modulus field” and $g_{ab}$ is the space-time metric. From equations (8.1),(8.2),(8.3) we have

$$e^{-\frac{4\sigma}{\sqrt{3}}} = \frac{1}{4} \sin^2 \rho \cos^2 \tau$$

$$g_{ab} dx^a dx^b = \frac{2}{\sin \rho \cos \tau} \left[ d\tau^2 + \cos^2 \tau \left( d\rho^2 + \frac{1}{4} \sin^2 \rho (d\theta^2 + \sin^2 \theta d\phi^2) \right) \right]$$

$$A = \frac{1}{2} \cos \theta d\phi$$

(8.4)

The last expression above shows clearly that magnetic monopoles are involved. The spatial metric is

$$ds_3^2 = \frac{2 \cos \tau}{\sin \rho} \left[ d\rho^2 + \frac{1}{4} \sin^2 \rho (d\theta^2 + \sin^2 \theta d\phi^2) \right].$$

(8.5)

Near $\rho = 0$ we have, with $r = 2\sqrt{\rho}$

$$ds_3^2 \sim 2 \cos \tau \left[ dr^2 + \frac{r^2}{16} (d\theta^2 + \sin^2 \theta d\phi^2) \right].$$

(8.6)

This metric is singular near the monopole at $r = 0$, because the area of a small 2-sphere of radius $r$ is $\pi r^2/4$. Thus the monopoles are rather like global monopoles in that they have a solid angle deficit (of $15\pi/4$).

To understand better the physics going on here, recall that substituting the ansatz (8.3) into the 5-dimensional Einstein action we get a 4-dimensional action of the form...
where $\Lambda_5$ is the 5-dimensional cosmological constant. Thus, one expects the modular field $\sigma$ will roll down the potential $\Lambda_5 e^{\frac{2\sigma}{\sqrt{3}}}$, which is precisely what equation (8.4) shows, because at late, real time $t = -i\tau$ we have as $t \to \infty$, $\sigma \sim -\sqrt{3}\tau/4$.

The extension of this idea to the case of domain walls is now immediate. We replace the two solid 4-balls $B^4$ by two solid 5-balls $B^5$. Joining them together gives an almost everywhere flat $(AEF)$ $S^5$. The spatial sections are $AEF S^4$. These admit an obvious $SO(4) \subset SO(5)$ invariant action whose orbits are 3-spheres. We pick out the $U(1)$ subgroup corresponding to the Hopf fibration. Explicitly, the flat metric on $B^5$ can be written as:

$$ds^2 = d\rho^2 + \rho^2 (d\tau^2 + \cos^2 \tau d\Omega_3^2)$$

$$= d\rho^2 + \rho^2 \left[d\tau^2 + \cos^2 \tau \left(\frac{1}{4}(dx^5 + \cos \theta d\phi)^2 + \frac{1}{4} \sin^2 \theta d\phi^2 + \frac{1}{4} d\theta^2\right)\right]. \quad (8.8)$$

The Riemannian half of the geometry is given by $-\frac{\pi}{2} \leq \tau \leq 0$. The Lorentzian section is given by $\tau = it, \ t > 0$. Note that $t = \tau = 0$, the initial nucleation surface, corresponds to the flat 4-ball with metric

$$d\rho^2 + \rho^2 \left(\frac{1}{4}(dx^5 + \cos \theta d\phi)^2 + \frac{1}{4} \sin^2 \theta d\phi^2 + \frac{1}{4} d\theta^2\right). \quad (8.9)$$

As is well known, this is a special case of the self-dual multi-center metrics used to construct KK monopoles. In other words, it may be given by the form

$$ds^2 = V^{-1}(dx^5 + \vec{\omega}d\vec{x})^2 + Vd\vec{x}^2 \quad (8.10)$$

with $\vec{\nabla} \times \vec{\omega} = \nabla V \Rightarrow \nabla^2 V = 0$. The asymptotically locally flat (ALF) $k-$monopole metric corresponds to

$$V = 1 + \sum_{i=1}^{k} \frac{1}{|\vec{x} - \vec{x}_i|}. \quad (8.11)$$

The asymptotically locally Euclidean (ALE) metrics which tend to the flat metric are $\mathbb{R}^4/C_k$, $C_k$ being the cyclic group of order $k$, and have the same form of the metric as (8.10) with the replacement $V \to V - 1$ in (8.11). The flat metric (8.4) corresponds to $k = 1$.

The modular field $\sigma$, which has no potential in this case, is now given by

$$e^{-\frac{4\sigma}{\sqrt{3}}} = \frac{\rho^2}{4} \cosh^2 t \quad (8.12)$$

and so at late times it rolls: as $t \to \infty$, $\sigma \sim -\sqrt{3}t/2$. The three dimensional metric obtained by reduction is

$$\left(\frac{1}{\cosh^2 t} d\rho^2 + \frac{\rho^2}{4}(d\theta^2 + \sin^2 \theta d\phi^2)\right) \quad (8.13)$$

The area of a small sphere of proper radius $r$ is $\frac{\pi}{4} \cosh^2 t$. The solid angular deficit is thus $\pi(16 - \cosh^2 t)/4$ which starts off at $15\pi/4$ as in the previous case but decreases with time, becoming negative for $t > \cosh^{-1} 4$. 

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The generalization to higher-dimensional reductions on a circle is straightforward and follows closely the work in [19]. If the dimensionality of the space-time is odd, one takes $S^{2n+1}$ as the instanton. The spatial sections are now $S^{2n}$:

$$ds^2 = d\tau^2 + \cos^2 \tau d\Omega^2_{2n}.$$  

(8.14)

Now consider spherical polar coordinates on $S^{2n}$:

$$d\Omega^2_{2n} = d\rho^2 + \sin^2 \rho d\Omega^2_{2n-1}.$$  

(8.15)

The group $SO(2n)$ acts on $S^{2n}$, fixing the north and south poles, $\rho = 0, \pi$. Its orbits are the $S^{2n-1}$’s given by $\rho =$constant. Now, pick the Hopf $U(1) \subset SO(2n)$ acting freely on $S^{2n-1}$ as the KK circle. The reduced spatial manifold is now of the form

$$d\rho^2 + \sin^2 \rho (d\Omega^{FS}_{2(n-1)})^2.$$  

(8.16)

where the $d\Omega$ is the Fubini-Study metric on $\mathbb{CP}^{n-1}$. There are two singularities at the north and south poles which correspond to a Bais-Batenberg monopole - anti-monopole pair [21]. These cannot exist as isolated objects because the metric of a single Bais-Batenberg pole is not asymptotically flat; they exhibit a kind of confinement. However a monopole - anti-monopole pair is allowed.

In the dimensionality of the space-time is even and one takes $S^{2n+2}$ as the instanton, the discussion is similar but now in polar coordinates analogous to (8.13) the sphere $S^{2n}$ does not admit, for topological reasons, a non-vanishing vector field. As explained in [19] one could use a $U(1)$ subgroup of $SO(2n+1)$ acting on $S^{2n}$ whose fixed point set is a circle. As $\rho$ varies we would get a 2-brane of fixed points. Alternatively we could Hopf-fibre the entire spatial $S^{2n+1}$. Thus, the $(2n+2)$–dimensional metric would look like

$$ds^2 = d\tau^2 + \cos^2 \tau \left[ (d\Omega^{FS}_{2n})^2 + (dx^5 + A)^2 \right]$$  

(8.17)

for the Hopf 1-form $A$. Thus at $\tau = 0$ a closed inflating universe is born whose spatial sections have the geometry of $\mathbb{CP}^n$ with its Fubini-Study metric.

This model is especially interesting if $n = 2$ because $\mathbb{CP}^2$ is not the sole boundary of any compact 5-manifold and so it cannot be born from nothing in a non-singular way in a purely 5-dimensional theory, whether Lorentzian or Riemannian. Our example shows that it could be born in a 6-dimensional KK theory in a perfectly non-singular way. Thus KK theory may allow the evasion of no-go theorems derived using co-bordism theory.

Kaluza-Klein theory on a single $U(1)$ is not an especially attractive theory but it serves to point the way forward to more elaborate examples. Thus one could exploit the fact that $S^7$ is an $S^3$ bundle of $S^4$. Taking the round metric on $S^8$ as an instanton one has

$$ds^2 = d\tau^2 + \cos^2 \tau \left[ \frac{1}{4} d\Omega^2_{4} + (\sigma^a + A_\mu^a dx^\mu)^2 \right],$$  

(8.18)

where $d\Omega^2_4$ is the round metric on $S^4 \cong \mathbb{HP}^1$, $\sigma^a$, with $a = 1, 2, 3$, are left-invariant 1-forms on $S^3$, and $A_\mu^a$ in the Yang-Mills instanton field. Thus at $\tau = 0$ a universe with $S^4$ spatial cross-section is born together with an $SU(2)$ Yang-Mills instanton.
IX. NON-ORIENTABLE BLACK HOLES

As was mentioned briefly in \([\text{VI}]\), it is possible to define certain discrete involutive isometries on the instantons which mediate the production of charged black holes in the presence of a domain wall, such that if one identifies the instantons under the action of these isometries the resulting manifolds are non-orientable. It is easy to see that these involutions on the instantons extend to well defined maps on the Lorentzian sections and therefore that these ‘identified instantons’ mediate the nucleation of non-orientable black holes in the presence of a domain wall. We will now describe this construction in more detail.

First of all, recall that from eq. \(6.10\) and figure \(4\) we have the fundamental result that for a given domain wall surface energy density \(\sigma\), there exists a countable infinite set of instantons, where each instanton in the set mediates the production of a pair of accelerating black holes of mass \(m\) and charge \(q_n\). We shall denote each orientable instanton as \(M^n\). The value of \(n\) determines the behaviour of the domain wall on the instanton. It is very useful to visualize the motion of the domain wall on the instanton. To this end, imagine viewing the \((\tau, r)\) ‘cigar’ section of the instanton, illustrated in figure \(2\) ‘head on’. Viewed in this way, we see that when \(n\) is odd, the domain wall will sweep out an ‘odd-leafed clover’ on the surface of the instanton. When \(n\) is even, the clover will have an even number of leaves. Because the leaves of a regular even-leafed clover are always diametrically across from each other, we see that there exists an obvious isometry on the instanton: Namely, if \(\beta_{RN}\) is the period of the instanton then the map defined by

\[
I : (\tau, r) \rightarrow (\tau + \frac{\beta_{RN}}{2}, r)
\]  

is a discrete isometry. However, this map is not freely acting. If we attempt to identify the instanton under the action of \(I\) the resulting manifold will have singularities. We therefore need to find a freely acting involution and compose it with \(I\) to obtain a freely acting involutive isometry. Such a freely acting map is given by simply taking ‘parity inversion’, which was introduced above in section \([\text{VII}]\) and which is given as

\[
P : (\tau, r, \theta, \phi) \rightarrow (\tau, r, \pi - \theta, \phi + \pi)
\]  

We therefore obtain a freely acting isometric involution on each even \(n\) instanton by forming the map \(PI\). We can identify each even \(n\) instanton under the action of \(PI\). The resulting smooth manifolds will be non-orientable. In particular, the nucleation surface will now have the topology \(S^1 \times \mathbb{R}P^2\). We shall denote these identified instantons as \(M^n_{PI}\), where the notation is meant to read ‘the \(n\)-th instanton identified under the action of \(PI\)’.

Of course, \(P\) by itself is a perfectly respectable freely acting involutive isometry and so we are free to identify all instantons under the action of \(P\). In fact, we can even identify the odd \(n\) instantons under the action of \(P\) since we are no longer identifying the period. We shall denote these instantons as \(M^n_P\), where again the notation means ‘the \(n\)-th instanton identified under the action of \(P\).

One might now ask, what is the difference between \(M^n_{PI}\) and \(M^n_P\), for a given even value of \(n\)? Well, for one thing \(M^n_{PI}\) has exactly half the volume (and hence half the action) that \(M^n_P\) does. It follows that the decay process mediated by \(M^n_{PI}\) is four times less likely as the corresponding process mediated by \(M^n_P\). We therefore see that non-orientable black holes
will be produced with less frequency than their orientable counterparts. Furthermore, since the instanton periods are different the corresponding Lorentzian sections will have distinct thermodynamical properties. It would be interesting to study the properties of these black holes further.

X. CONCLUSION

In this paper we have studied the production of pairs of neutral and charged black holes by domain walls, finding classical solutions and calculating their classical actions. We have found that neutral black holes whose creation is mediated by Euclidean instantons must be produced mutually at rest with respect to one another, but for charged black holes a new type of instanton is possible in which after formation the two black holes accelerate away from one another. These new types of instantons are not possible in Einstein-Maxwell theory with a cosmological constant. Another unusual property is that there exist a countably infinite sequence of pair created charged, accelerating black holes with charge $q_n$ for a given mass $M$ and domain wall surface density $\sigma$. Surprisingly, the probability amplitude for the creation of these pairs asymptotes, rather than decays, for $q_n \to M$.

This process of black hole formation in the presence of a domain wall is similar in spirit to the snapping of a cosmic string to form a pair of black holes. However, true analog of the cosmic string process would seem to be the puncture of a domain wall with a black string at the boundary of the rupture. As this necessarily involves n-branes rather than simply black holes, we have not considered such a process in this paper.

We have also found that creation of non-orientable black hole solutions can be mediated by Euclidean instantons and that in addition if one is prepared to consider entirely Lorentzian no-boundary type contributions to the path integral then mutually accelerating pairs may be created even in the neutral case.

Finally we have considered the production of Kaluza-Klein monopoles both by a standard cosmological term and in the presence of a domain wall. We obtain in this way a better understanding of the putative process of "compactification" which plays a central role, albeit in a slightly different context, in almost all recent unification attempts. The main point is that the compactification process is accompanied by the production of pairs of topological defects – Kaluza-Klein monopoles in the simplest case. Thus we have obtained a unified picture in which three hitherto largely separate themes in quantum gravity:

1. the birth of the universe from nothing,
2. the production of primordial topological defects, and
3. the existence of extra dimensions

are brought together. It seems plausible that the basic ideas in this paper will extend to more complicated and hopefully more realistic examples.

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FIGURES

FIG. 1. The cut-and-paste construction of the domain wall space-time is presented. In the top row, two copies of Minkowski space-time, depicted by their conformal diagrams, are joined along an asymptotically-null surface in each region, describing the location of the domain wall. The time-symmetric surface $T = 0$ is shown on the diagrams. In the middle row, the procedure for the construction of the instanton for domain wall creation is depicted. Two flat 4-balls are glued back-to-back, yielding a ‘lens’ owing to the ridge of curvature running along the hemisphere at the location of the domain wall. At the bottom, the Riemannian space has been joined to the Lorentzian space-time on the $T = 0$ hypersurface, depicting the creation from nothing of Minkowski domains partitioned by a domain wall.

FIG. 2. The cut-and-paste construction of the domain wall and Schwarzschild black hole space-time is presented. In the first row, two copies of Schwarzschild space-time are joined along the $r = 3m$ timelike hypersurface at the location of the domain wall. In the second row, the identification of the two external regions of the Schwarzschild space-times is shown. This yields a space-time with topology $\mathbb{R} \times S^1 \times S^2$. In the third row, the construction of the domain wall and Schwarzschild instanton is shown. Two manifolds with topology $S^2 \times \mathbb{R}^2$ are cut at the radius $r = 3m$, and glued back-to-back, yielding a surface with topology $S^2 \times S^2$ with a ridge of curvature at the domain wall. In the fourth row, the Riemannian space has been joined to the Lorentzian space-time on the $T = 0$ surface, depicting the creation from nothing of a closed space-time containing two domain walls and two domains containing a black hole in each.

FIG. 3. The instanton period $\beta_w$ for uncharged, accelerating black holes (solid line) as a function of black hole mass. We see that there is no value of the mass for which $\beta_w$ matches the Schwarzschild period $\beta_S = 8\pi M$ (dashed line).

FIG. 4. The instanton period for charged, accelerating black holes, as a function of charge. The period $\beta_w$ is shown for $2\pi \sigma M = 27^{-1/2}, 1/7, 1/10$ (the long dashed, short dashed, and dotted lines respectively). The Reissner-Nordström period $\beta_{RN}$ is shown (solid line). Accelerating, charged black holes may be produced for discrete values of the charge $q_n$ such that $\beta_w = \beta_{RN}/n$.

FIG. 5. The probability $P$ for the creation of accelerating, charged black holes as a function of charge. The probability is shown for $2\pi \sigma M = 20^{-1/2}, 27^{-1/2}, 1/10$ (the solid triangle, open square, and solid square respectively) for discrete values of the charge $q_n$ for $n = 1, 2, 5, 10, 20, 50, 100$. The probability for the creation of static, charged black holes is also shown (solid line).
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