Comments on QCD Corrections to $b \to s\gamma$ Decay

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Abstract

We find some errors in previous calculation of leading log QCD corrections to $b \to s\gamma$ decay, which include corrections from $m_{top}$ to $M_W$ in addition to corrections from $M_W$ to $m_b$. The inclusive decay rate is found to be enhanced more than previous calculations. At $m_t = 170$GeV, the running from $m_{top}$ to $M_W$ results in 13% enhancement, and for $m_t = 250$GeV, 16% is found.

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It is well known that the process $b \to s\gamma$ is extremely sensitive to new physics beyond the Standard Model. In 1993, the CLEO collaboration placed an upper limit on the inclusive $b \to s\gamma$ decay $B(b \to s\gamma) < 5.4 \times 10^{-4}$ at 95% C.L. [1]. This has inspired a large number of papers [2]. It has been argued that this experiment provides more information about restrictions on the Standard Model, 2-Higgs doublet model, Supersymmetry, Technicolor and etc. Recently CLEO has measured the inclusive branching ratio to be [3]

$$Br(b \to s\gamma) = (2.32 \pm 0.51 \pm 0.29 \pm 0.32) \times 10^{-4}.$$  

(1)

Corresponding to 95% confidence level, the range is $1 \times 10^{-4} < Br(b \to s\gamma) < 4 \times 10^{-4}$. More stringent constraints are obtained by experiments, so more accurate theoretical calculation of this decay rate is needed.

The radiative $b$ quark decay has already been calculated in many papers [4, 5, 7]. It is found to be strongly QCD-enhanced. In other words, the strong interaction plays an important role in this decay. However, there are still some uncertainties in these papers. Most papers [4, 5] do not include the QCD running from $m_{t_{op}}$ to $M_{W}$. Since the top quark is found to be heavier than W boson ($m_{t_{op}} = 174 \pm 10^{\pm 13}_{12} \text{ GeV}$ [6]), it needs a detailed calculation of this effect. Ref. [7] does include this running, however there are some errors in calculation of anomalous dimensions which may lead to some changes in final result.

In our present paper, by using effective field theory formalism [8], we recalculate the $b \to s\gamma$ decay in Minimal Standard Model. We first integrate out the top quark, generating an effective five-quark theory. By using the renormalization group equation, we run the effective field theory down to the W-scale, including QCD corrections from $m_{t_{op}}$ to $M_{W}$, and correct some errors in ref. [7]. Then the weak bosons are removed. Untruncated anomalous dimensions of QCD running
from $M_W$ to $m_b$ are used. Finally we calculate the rate of radiative $b$ decay.

The effective Hamiltonian is written as

$$\mathcal{H}_{\text{eff}} = 2\sqrt{2}G_FV_{tb}V_{ts}^* \sum_i C_i(\mu)O_i(\mu).$$

(2)

The operators $O_i$ make a complete basis of dimension-6 operators:

$$O_{1 LR} = -\frac{1}{16\pi^2} m_b \overline{s}_L D^2 b_R,$$

$$O_{2 LR} = \mu^{\epsilon/2} \frac{g_3}{16\pi^2} m_b \overline{s}_L \sigma^{\mu\nu} X^a a_R b_R^a G_{\mu\nu},$$

$$O_{3 LR} = \mu^{\epsilon/2} \frac{e Q_b}{16\pi^2} m_b \overline{s}_L \sigma^{\mu\nu} b_R F_{\mu\nu},$$

$$Q_{LR} = \mu^{\epsilon} \frac{g_3}{2} m_b s_L b_R,$$

$$P_{1 L}^{A} = -\frac{i}{16\pi^2} \overline{s}_L T^{A}_{\mu\nu} D^\mu D^\nu b_L,$$

$$P_{2 L} = \mu^{\epsilon/2} \frac{e Q_b}{16\pi^2} \overline{s}_L \gamma^\mu b_L \partial^\nu F_{\mu\nu},$$

$$P_{3 L} = \mu^{\epsilon/2} \frac{e Q_b}{16\pi^2} F_{\mu\nu} \overline{s}_L \gamma^\mu b_L,$$

$$P_{4 L} = i \mu^{\epsilon/2} \frac{e Q_b}{16\pi^2} \tilde{F}_{\mu\nu} \overline{s}_L \gamma^\mu \gamma^5 D^\nu b_L,$$

$$R_{1 L} = i \mu^{\epsilon} \frac{2 g_3}{2} \phi_+ b_L,$$

$$R_{2 L} = i \mu^{\epsilon} \frac{g_3}{2} (D^\sigma \phi_+) b_L,$$

$$R_{3 L} = i \mu^{\epsilon} \frac{g_3}{2} (D^\sigma \phi_-) \overline{s}_L \gamma_\sigma b_L.$$

(3)

The coefficients $C_i(\mu = m_t)$ are calculated from matching diagrams, and agree with ref. [7].

After evaluating the loop diagrams, we find that the weak mixing of operators agrees with
ref.\[7\]. While the QCD anomalous dimensions for each of the operators in our basis are

\[
\begin{align*}
O_{1} & \quad O_{2}^{LR} & \quad O_{3}^{LR} & \quad P_{L}^{1,1} & \quad P_{L}^{1,2} & \quad P_{L}^{1,3} & \quad P_{L}^{1,4} & \quad P_{2}^{2} & \quad P_{3}^{3} & \quad P_{4}^{4} \\
\end{align*}
\]

\[
\gamma = \begin{pmatrix}
\frac{20}{3} & 1 & -2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-8 & \frac{2}{3} & \frac{4}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{16}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
6 & 2 & -1 & \frac{2}{3} & 2 & -2 & -2 & 0 & 0 & 0 \\
4 & \frac{3}{2} & 0 & -\frac{113}{36} & \frac{137}{18} & -\frac{113}{36} & -\frac{4}{3} & \frac{9}{4} & 0 & 0 \\
2 & 1 & 1 & -2 & 2 & \frac{2}{3} & -2 & 0 & 0 & 0 \\
0 & \frac{1}{2} & 2 & -\frac{113}{36} & \frac{89}{18} & -\frac{113}{36} & \frac{4}{3} & \frac{9}{4} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -\frac{4}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -\frac{4}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{pmatrix}
\]

\[
\gamma = \begin{pmatrix}
\frac{23}{3} & 0 & 0 & 0 \\
0 & \frac{23}{3} & 0 & 0 \\
0 & 0 & \frac{23}{3} & 0 \\
0 & 0 & 0 & \frac{23}{3} \\
\end{pmatrix}
\]

Comparing with ref.\[7\], there are some differences in the anomalous dimension matrix, which may lie in omiting a symmetric factor of 1/2 in ref.\[7\] in calculating Feynman diagram like Fig.1. And some changes may due to miscalculation.

After these changes, the whole matrix can be easily diagonalized, and gives all real eigenvalues while that in ref.\[7\] can not. Inserting anomalous dimension (4)(5) to the renormalization group
equation satisfied by the coefficient functions $C_i(\mu)$

$$\mu \frac{d}{d\mu} C_i(\mu) = \sum_j (\gamma^\tau)_{ij} C_j(\mu).$$

(6)

we can have the coefficients of operators at $\mu = M_W$. And some of these operators change a lot from ref.[7] due to our improvements.

In order to continue running the basis operator coefficients down to lower scales, one must integrate out the weak gauge bosons and would-be Goldstone bosons at $\mu = M_W$ scale. This leads to the well-known six four-quark operators[4, 5]. The other remaining two-quark operators can be reduced by using equations of motion(EOM) to the gluon and photon magnetic moment operators $O_{LR}^2$ and $O_{LR}^3$.

To be comparable with previous results, we rewrite our operators $O_{LR}^3$, $O_{LR}^2$ as $O_7$, $O_8$ like ref.[5],

$$O_7 = (e/16\pi^2)m_b \bar{s}_L \sigma^\mu\nu b_R F_{\mu\nu},$$

$$O_8 = (g/16\pi^2)m_b \bar{s}_L \sigma^\mu\nu T^a b_R G^a_{\mu\nu}.$$

(7)

For completeness, we give the explicit expressions of the coefficient of operator $O_8$ and $O_7$ at $\mu = M_W$,

$$C_{O_8}(M_W^-) = \left( \frac{\alpha_s(m_t)}{\alpha_s(M_W)} \right)^{\frac{23}{14}} \left\{ \frac{1}{2} C_{O_{LR}^1}(m_t) - C_{O_{LR}^2}(m_t) + \frac{1}{2} C_{P_L^{1,1}}(m_t) \\
+ \frac{1}{4} C_{P_L^{1,2}}(m_t) - \frac{1}{4} C_{P_L^{1,4}}(m_t) \right\} - \frac{1}{3},$$

(8)

$$C_{O_7}(M_W^-) = \frac{1}{3} \left( \frac{\alpha_s(m_t)}{\alpha_s(M_W)} \right)^{\frac{23}{16}} \left\{ C_{O_{LR}^1}(m_t) + 8 C_{O_{LR}^2}(m_t) \left[ 1 - \left( \frac{\alpha_s(M_W)}{\alpha_s(m_t)} \right)^{\frac{23}{25}} \right] \\
+ \left[ -\frac{9}{2} C_{O_{LR}^1}(m_t) - \frac{9}{2} C_{P_L^{1,1}}(m_t) - \frac{9}{4} C_{P_L^{1,2}}(m_t) + \frac{9}{4} C_{P_L^{1,4}}(m_t) \right] \left[ 1 - \frac{8}{9} \left( \frac{\alpha_s(M_W)}{\alpha_s(m_t)} \right)^{\frac{23}{25}} \right] \\
- \frac{1}{4} C_{P_L}(m_t) + \frac{9}{23} 16\pi^2 C_{W_L^1}(m_t) \left[ 1 - \frac{\alpha_s(m_t)}{\alpha_s(M_W)} \right] \right\} - \frac{23}{36}.$$
They are expressed by coefficients of operators at $\mu = m_t$ and QCD coupling $\alpha_s$. So it is convenient to utilize these formula.

The obvious differences from QCD correction to $C_7(M_W)$ and $C_8(M_W)$ can easily be seen from Fig.2. In comparison to ref.[7], the enhancement of coefficient of operator $O_7$ is almost the same size, but the values for $O_8$ are quite different. Here the effect to $O_8$ is an enhancement rather than a suppression as in ref.[7]. These changes come from the corrections of anomalous dimensions described earlier. Since $C_7(M_W)$ and $C_8(M_W)$ are both the input of the following QCD running from $M_W$ to $m_b$, it is expected to change the final result.

The running of the coefficients of operators from $\mu = M_W$ to $\mu = m_b$ was well described in ref.[5]. After this running we have the coefficients of operators at $\mu = m_b$ scale. Here we use $M_W = 80.22\text{GeV}$, $m_b = 4.9\text{GeV}$. Both $C_7(m_b)$ and $C_8(m_b)$ are enhanced in comparison to values obtained by Misiak where the QCD corrections from $M_W$ to $m_t$ are neglected[3].

The leading order $b \to s\gamma$ matrix element of $H_{\text{eff}}$ is given by the sum of operators $O_5$, $O_6$ and $O_7$, this disagrees with ref.[7], but agrees with Misiak[3]. The sought amplitude will be proportional to the squared modulus of

$$C_7^{\text{eff}}(m_b) = C_7(m_b) + Q_d [C_5(m_b) + 3C_6(m_b)]$$

instead of $|C_7(M_b)|^2$ itself.

Following ref.[3], applying eqs. (10), one finds

$$\frac{BR(\bar{B} \to X_s \gamma)}{BR(\bar{B} \to X_s e\bar{\nu})} \simeq \frac{6\alpha_{\text{QED}}}{\pi g(m_c/m_b)} |C_7^{\text{eff}}(m_b)|^2 \left(1 - \frac{2\alpha_s(m_b)}{3\pi} f(m_c/m_b)\right)^{-1},$$

where $g(m_c/m_b) \simeq 0.45$ and $f(m_c/m_b) \simeq 2.4$ corresponding to the phase space factor and the one-loop QCD correction to the semileptonic decay, respectively[3]. The electromagnetic fine
structure constant evaluated at the $b$ quark scale takes value as $\alpha_{QED}(m_b) = 1/132.7$. The results are summarized in Fig.3 as functions of the top quark mass. The QCD-uncorrected values are also shown. In this figure, one can easily see that, at $m_t = 170\text{GeV}$, it results in 13% enhancement from Misiak’s result[5], and for $m_t = 250\text{GeV}$, 16% is found.

As a conclusion, we have given the full leading log QCD corrections (include QCD runnings from $m_{top}$ to $M_W$), with whole anomalous dimension matrix untruncated. Comparison to the previous calculation[4], two points are improved:

(1) Correct errors of anomalous dimensions in ref.[7].

(2) Use untruncated anomalous dimensions in QCD running from $M_W$ to $m_b$ instead of truncated ones.

In fact, point(1) makes an enhancement while point(2) leads to a suppression. The total result does not change a lot, e.g. a suppression of 3% at $m_{top} = 170\text{GeV}$ comparing ref.[4].

The whole QCD-enhancement of the $BR(\overline{B} \to X_s\gamma)$ makes a factor of 3.9 at $m_t = 170\text{GeV}$, when $\Lambda_{QCD} = 175\text{MeV}$. Using the experimental branching ratio $BR(\overline{B} \to X_c\ell\nu) = 10.8\%$, one can find that $BR(\overline{B} \to X_s\gamma) \simeq 4 \times 10^{-4}$ at $m_t = 174\text{GeV}$. It just reaches the upper limit of present experiment of CLEO. That shows there is very little space for new physics.

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Figure Captions

Fig.1 One of the Feynman diagram in calculating Anomalous dimensions, with the heavy dot denoting high dimension operator.
Fig. 2 The photon and gluon magnetic moment operator’s coefficient $C_7(M_W)$ (upper) and $C_8(M_W)$ (lower) for different top quark mass. The ones with and without QCD corrections are indicated by solid and dashed lines respectively. ($\Lambda_{QCD} = 300\text{MeV}$ is used)

Fig. 3 $\text{BR}(\overline{B} \to x_s\gamma)$ normalized to $\text{BR}(\overline{B} \to x_c e\nu)$, as function of top quark mass. The upper solid lines indicated our results for a full QCD correction. Dashed lines correspond to Misiak’s results without QCD running from $m_{top}$ to $M_W$. 