Non-Fragile Passivity Synchronization Control for Complex Dynamical Networks With Dynamics Behavior Links

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ABSTRACT
Passivity synchronization of complex dynamical networks with time-varying dynamics behavior links is investigated in this paper. In many real-world complex dynamical networks have dynamic behavior which can cause synchronization losing in the networks. To make the systems synchronization, a non-fragile controller is given. By constructing a new Laypunov-Krasovskii functional and combining the reciprocal convex technique, sufficient conditions for complex dynamical networks to be synchronized are derived. The derived conditions can be solved by linear matrix inequalities (LMIs). In the end, two examples are presented to demonstrate the effectiveness of the proposed methods.

INDEX TERMS Non-fragile control, passive theory, dynamics behavior links, complex dynamical network.

I. INTRODUCTION
The dynamical behaviors of complex networks have been discussed for far and wide. In particular, lots of researchers have widely investigated the synchronization problem for complex networks [1]–[5]. It should be pointed out that since the dynamic of complex dynamic networks are coupled with topological structure, the qualitative analysis on their dynamic behaviors is more challenging and interesting issue compared to single general dynamic systems.

It has been shown that the notion of passivity plays an important role in the analysis and design of linear and nonlinear systems. And the passivity analysis approach has been used for a long time to deal with the control problems for some kinds systems, such as neural network systems [6]–[10], T-S fuzzy systems [11]–[13], Markovian jump system [14]–[17], singular system [18], [19], etc. very recently, the problem of passivity analysis has been discussed for complex networks in [20]–[26], in [20], [21], [22], the authors studied the synchronization of complex dynamical networks and general complex dynamical networks by using passivity theory. Wang et.al. [23] and Wang et.al. [24] investigated input passivity and output passivity for a generalized complex network with nonlinear, time-varying, non-symmetric and delayed coupling. In [25], the authors were devoted to the passivity problem for a class of Markovian jump switching complex dynamical networks with multiple time-varying delays and stochastic perturbations. The authors in [26] analyzed the passivity analysis and pinning control of multi-weighted complex dynamical networks. Various control schemes have been proposed to data to deal with the synchronization problem for complex dynamical networks. These include pinning control [27]–[29], adaptive control [30], [31], impulsive control [32]–[34], sampled data control [35], [36] and non-fragile control [37], [38]. Each control strategy has its own characteristics. For instance, the basic idea of pinning control strategy is to make the whole network behave expectantly by selectively controlling a few nodes in the network. The advantage of adaptive control is that it has certain adaptability. It can identify the parameters of the system continuously according to the input and output data of the system. When the parameters of the controller deviate very slightly, the stability of the closed-loop system will be destroyed or reduced, which will arouse widespread concern in the research of non-fragile robust control. In [37], the author studies the problem of non-fragile synchronization control for Markovian jumping complex dynamical networks with probabilistic time-varying coupling delays. The
problem of robust non-fragile synchronization is investigated in [38] for a class of complex dynamical networks subject to semi-Markov jumping outer coupling, time-varying coupling delay, randomly occurring gain variation, and stochastic noise over a desired finite-time interval. In [39] the author investigated exponential $H_{\infty}$ synchronization via non-fragile sampled-data.

In a practical point of view, there are usually several types of connection links between different nodes, and each type of link has its own special attributes, for example, the time delay. Recently, a number of networks are often referred to as complex multi-links dynamic networks [40]–[43]. In [40] the time delay has been introduced in to split the network, upon which a model of complex dynamical networks with multi-links has been constructed. The authors in [41] discussed the decay synchronization of complex multi-links time-varying dynamic network. In [42], multiple weighted complex networks were studied. Synchronization of complex dynamical networks with dynamical behavior links was studied [43]. As existence of links affect the synchronization. To the best of authors’ knowledge, dynamic model and multi-link time varying have not been considered for the link model. All the existing literatures of the links complicate network behavior, and make the complex dynamical networks model to be more general.

Motivated by the above attention, this paper concerns the non-fragile passivity synchronization for complex dynamical networks with dynamic links. The main contributions of this paper are given as follows:

1. The passivity synchronization problem of complex dynamical networks with dynamic behavior links is discussed. We give the state-space dynamic model used for the links.

2. By using a non-fragile control theory, passivity theory and Kronecker products, some new sufficient conditions are derived to guarantee the passivity synchronization of complex dynamical networks with behavior links.

3. By employing inequality techniques, non-fragile passivity synchronization criteria are established in the form of linear matrix inequalities, which can be efficiently solved by using MATLAB LMIs control toolbox.

4. Two examples to illustrate the obtained criteria is presented with simulation results.

Notation: The notation in this paper is standard. $R^n$ denotes the $n$ dimensional Euclidean space; $R^{m \times n}$ represents the set of all $m \times n$ real matrices. For a real asymmetric matrix $X$ and $Y$, the notation $X < Y$ (respectively, $X > Y$) means $X - Y$ is semi-positive definite (respectively, positive definite). The superscript $T$ denotes matrix transposition. Moreover, in symmetric block matrices, $*$ is used as an ellipsis for the terms that are introduced by asymmetry and $\text{diag} \{ \cdots \}$ denotes a block-diagonal matrix. The notation $A \otimes B$ stands for the Kronecker product of matrices $A$ and $B$. $\| \cdot \|$ stands for the Euclidean vector norm. $\mathcal{E}$ stands for the mathematical expectation. If not explicitly stated matrices are assumed to have compatible dimensions.

II. PRELIMINARIES AND PROBLEM STATEMENT

Consider the following the complex dynamical networks model with dynamic links:

$$\begin{cases} 
\dot{x}_i(t) = Ax_i(t) + B_1f(x_i(t)) + B_2f(x_i(t - \tau(t))) \\
+ \sigma_1 \sum_{j=1}^{N} g_{ij} \Gamma_1 z_j(t) + \sigma_2 \sum_{j=1}^{N} g_{ij} \Gamma_2 z_j(t - \tau(t)) \\
+ Ew_i(t) + U_i(t) 
\end{cases}$$

(1)

where $x_i(t) \in R^n$ and $y_i(t) \in R^h$ denote the state vectors of the $i$-th node and the link, respectively. $z_i(t) \in R^n$ is the output of the link and $U_i(t) \in R^m$ represents the control signal. $\sigma_1$ and $\sigma_2 > 0$ denote the non-delayed and delayed coupling strength. $w_i(t)$ is the disturbance; $G = (g_{ij})_{N \times N}$ is the out-coupling matrix representing the topological structure of the complex networks, in which $g_{ij} g_{ij}$ is defined as follows: if there exists a connection between node $i$ and node $j$ ($i \neq j$), then $g_{ij} = g_{ji} = 1$, otherwise, $g_{ij} = g_{ji} = 0$ ($i \neq j$). The row sums of $G$ are zero, that is, $\sum_{j=1}^{N} g_{ij} = -g_{ii}, i = 1, 2, \ldots, N$.

$G_1$ and $G_2$ represent the inner coupling matrix of the complex networks. $f : R^n \rightarrow R^h$ is a nonlinear vector-valued function which will be defined later. The matrices $A, E \in R^{n \times n}$, $B_1, B_2 \in R^{n \times q}$, $A_l \in R^{k \times k}$, $B_l \in R^{k \times n}$, $C_l \in R^{l \times k}$, $D_l \in R^{l \times n}$. The time delay $\tau(t)$ is assumed to satisfy the condition as follows:

$$0 \leq \tau(t) \leq \tau, \quad 0 \leq \dot{\tau}(t) \leq \ddot{\tau} < 1$$

(2)

Assumption 1: For all $x, y \in R^n$, the nonlinear function $f(\cdot)$ is continuous and assumed to satisfy the following sector-bounded nonlinearity condition:

$$[f(x) - f(y) - F_1(x - y)]^T * [f(x) - f(y) - F_2(x - y)] \leq 0,$$

(3)

where $F_1$ and $F_2 \in R^{n \times n}$ are known constant matrices with $F_2 - F_1 > 0$. For presentation simplicity and without loss of generality, it is assumed that $f(0) = 0$.

The isolated node of network (1) is given by

$$\dot{x}_s(t) = Ax_s(t) + B_1f(x_s(t)) + B_2f(x_s(t - \tau(t)))$$

(4)

$x_s(t) \in R^n$ is a synchronization manifold, which can be either an equilibrium point, a periodic orbit or an orbit of a chaotic attractor.

In this way, the response of a link to $x_s(t)$, as its input, is described by $z_s(t)$ with the following equations:

$$\begin{cases} 
\dot{y}_s(t) = A_1y_s(t) + B_1x_s(t) \\
\dot{z}_s(t) = C_1y_s(t) + D_1x_s(t) 
\end{cases}$$

(5)

We set the error vectors as

$$e_i(t) = x_i(t) - x_s(t), \quad i = 1, 2, \ldots, N.$$

(6)
And introduction $N$ vectors
\[ m_i(t) = y_i(t) - y_\delta(t), \quad i = 1, 2, \ldots, N. \quad (7) \]
as link errors. The vectors $m_i(t)$ are the errors between the link responses and response of a virtual link connected to individual node. We can obtain the synchronization error dynamic equations of complex dynamical networks as follows:
\[
\begin{aligned}
\dot{e}_i(t) &= A e_i(t) + B_1 f(e_i(t)) + B_2 f(e_i(t) - (t - \tau(t))) \\
&+ \sigma_1 \sum_{j=1}^{N} g_{ij} \Gamma_j \left(C m_j(t) + D_i e_j(t)\right) \\
+ \sigma_2 \sum_{j=1}^{N} g_{ij} \Gamma_j \left(C m_j(t) - (t - \tau(t)) + D_i e_j(t) - (t - \tau(t))\right) \\
+ E w_j(t) + \mathcal{U}_i(t) \\
\dot{m}_i(t) &= A_i m_i(t) + B_i e_i(t)
\end{aligned}
\quad (8)
\]
where
\[
\begin{aligned}
f(e_i(t)) &= f(x_i(t)) - f(x_\delta(t)) \\
f(e_i(t) - (t - \tau(t))) &= f(x_i(t) - (t - \tau(t))) - f(x_\delta(t) - (t - \tau(t)))
\end{aligned}
\]

Design the following non-fragile controller
\[
\mathcal{U}_i(t) = (K_i + \Delta K_i) e_i(t), \quad i = 1, 2, \ldots, N.
\quad (9)
\]
where $K_i \in \mathbb{R}^{m_i \times m_i}$, $(i \in R)$ being the constant controller gain to be determined. $\Delta K_i$ are perturbed matrices. $\Delta K_i(t)$ can be given as
\[
\Delta K_i(t) = V_i H_i(t) M_i 
\]
where $V_i$ and $M_i$ are known constant matrices. $H_i(t) \in \mathbb{R}^{k \times l}$, $i = 1, 2, \ldots, N$ is an unknown time-varying matrix, satisfying
\[
H_i^T(t) H_i(t) \leq I
\quad (10)
\]

For convenience of later development, the error dynamics systems (8) can be written in the following Kronecker product forms:
\[
\begin{aligned}
\dot{e}(t) &= (\tilde{A} + \sigma_1 \tilde{D}_1 + K + V W M) e(t) + \sigma_1 \tilde{C}_j m(t) \\
&+ \sigma_2 \tilde{D}_2 e(t - (t - \tau(t))) + \sigma_2 \tilde{C}_2 m(t - (t - \tau(t))) \\
&+ \tilde{B}_1 F(e(t)) + \tilde{B}_2 f(e(t) - (t - \tau(t))) + \tilde{E} w(t) \\
\dot{m}(t) &= \tilde{A}_j m(t) + \tilde{D}_j e(t) \\
\end{aligned} 
\quad (11)
\]
where
\[
\begin{aligned}
\tilde{A} &= I_N \otimes A, \quad \tilde{D}_1 = G \otimes \Gamma_1 D_1, \quad \tilde{D}_2 = G \otimes \Gamma_2 D_1 \\
\tilde{C}_1 &= G \otimes \Gamma_1 C_1, \quad \tilde{C}_2 = G \otimes \Gamma_2 C_1, \quad \tilde{B}_1 = I_N \otimes B_1 \\
\tilde{B}_2 &= I_N \otimes B_2, \quad \tilde{E} = I_N \otimes E, \quad \tilde{A}_j = I_N \otimes A_i \\
\tilde{B}_i &= I_N \otimes B_i, \quad \tilde{C}_i = I_N \otimes C_i, \quad \tilde{D}_i = I_N \otimes D_i \\
K &= \Delta \text{diag} \{K_1, K_2, \ldots, K_N\}, \\
V &= \Delta \text{diag} \{V_1, V_2, \ldots, V_N\}, \\
H &= \Delta \text{diag} \{H_1, H_2, \ldots, H_N\}
\end{aligned}
\]

In this paper, we main aim is study the non-fragile passivity synchronization network (1) or (11). Before stating the main results, we introduce the following definitions.

**Definition 1** [44]: The complex dynamical networks (1) is said to be synchronized for any initial conditions $\phi_i(t)$, $i = 1, 2, \ldots, N$, if the following condition holds
\[
limit_{t \to \infty} \left\| e_i(t) \right\| = \lim_{t \to \infty} \left\| x_i(t) - x_\delta(t) \right\| = 0, \quad i = 1, 2, \ldots, N
\]

**Definition 2** [44]: Considered complex dynamical networks (1) are said to be passive, if there exists a scalar $\gamma > 0$ such that the following inequality holds under zero initial condition:
\[
2 \int_0^{\infty} z^T(t) w(t) dt \geq -\gamma \int_0^{\infty} w^T(t) w(t) dt
\]
To facilitate the presentation, the following lemmas are necessary to establish the main results.

**Lemma 1** [45]: Let $\otimes$ denotes the Kronecker product $A$, $B$, $C$ and $D$ are matrices with appropriate dimensions. The following properties are hold:
\[
\begin{aligned}
(1) \quad & (ca) \otimes b = a \otimes (cb), \quad \text{for any constant } c; \\
(2) \quad & (A + B) \otimes C = A \otimes C + B \otimes C \\
(3) \quad & (A \otimes B) (C \otimes D) = (AC) \otimes (BD) \\
(4) \quad & (A \otimes B)^T = A^T \otimes B^T \\
(5) \quad & (A \otimes B)^{-1} = (A^{-1} \otimes B^{-1})
\end{aligned}
\]

**Lemma 2** [36]: For a positive definition matrix $P$, and a differentiable function $x(a)$ [a \in [a, b]]$, 0 < a < b$, the following inequalities hold:
\[
\begin{aligned}
(1) \quad & \int_a^b \dot{x}(s) P(x(s)) ds \leq \frac{1}{b-a} \Xi_1 P \Xi_1 + \frac{1}{b-a} \Xi_2 P \Xi_2 \\
& (b-a) \int_a^b \dot{x}(s) P(x(s)) ds \\
(2) \quad & \geq \left( \int_a^b x(s) ds \right)^T P \left( \int_a^b x(s) ds \right) \\
& - \int_a^b \int_{t+\theta}^b \dot{x}(s) P(x(s)) ds d\theta \\
&(3) \quad & \leq \frac{1}{(b-a)^2} \int_a^b \dot{x}(s) P \Xi_2 P \Xi_1 x(s) ds d\theta
\end{aligned}
\]
where
\[
\Xi_1 = x(b) - x(a), \quad \Xi_2 = x(b) + x(a) - \frac{2}{b-a} \int_a^b x(s) ds
\]

**Lemma 3** [46]: Let $L^T = L$, $H$ and $E$ be real matrices of appropriate dimensions with $F(t)$ satisfying $F^T(t) F(t) \leq I$. Then, $L^H H^E + E^T F^H T^H < 0$, if and only if there exist a scalar $\varepsilon > 0$ such that $L + \varepsilon^{-1} H H^T + \varepsilon E^T E < 0$, or equivalently
\[
\begin{bmatrix}
L & H & \varepsilon E^T \\
* & -\varepsilon I & 0 \\
* & * & -\varepsilon I
\end{bmatrix} < 0
\]
III. MAIN RESULTS

The complex dynamical networks with dynamic links have been introduced in the previous sections. In this section, the synchronization will be analysed for error system (11) by Lyapunov functional method under the designed non-fragile controller based in passivity theory.

**Theorem 1:** Let Assumption 1 hold. For any given scalar, the error system (11) to be asymptotically stable with a passivity performance $\gamma > 0$, if there exist symmetric positive definite matrices $U_a$ ($a = 1, 2, \ldots, 10$), any appropriate dimensioned matrices $Q_1, Q_2$, and scalars $\varepsilon_1, \varepsilon_2, \beta_1, \beta_2, \beta_3, \beta_4, \delta > 0$ such that the following LMI holds:

$$\Pi = \begin{bmatrix} \Pi_{11} & \Sigma_1 & \theta \Sigma_2 \\ * & \delta P & 0 \\ * & * & \delta P \end{bmatrix} < 0 \quad (12)$$

where

$$\Pi_{11} = \begin{bmatrix} \Pi_1 & \Pi_2 \\ * & \Pi_3 \end{bmatrix}$$

$$\Sigma_2 = \begin{bmatrix} M, 0, 0, \ldots, 0, 0 \end{bmatrix}$$

$$\pi_{11} = \text{sym} ((I_N \otimes U_1) (A + \sigma_1 D_1 + K) + (I_N \otimes U_3)$$

$$+ (I_N \otimes U_4) - \frac{4}{\tau} (I_N \otimes U_7) - \frac{1}{\tau} (I_N \otimes U_8)$$

$$- 2 (I_N \otimes U_9) - (\varepsilon_1 + \varepsilon_2) \bar{F}_1$$

$$+ \text{sym} (\beta_1 (I_N \otimes Q_1) (A + \sigma_1 D_1 + K))$$

$$- \delta (I_N \otimes U_1)$$

$$\pi_{13} = \sigma_2 (I_N \otimes U_1) \bar{D}_2 + \sigma_2 \beta_1 (I_N \otimes Q_1) \bar{D}_2$$

$$+ \beta_2 (A + \sigma_1 D_1 + K)^T (I_N \otimes Q_1)^T$$

$$\pi_{14} = \sigma_1 (I_N \otimes U_1) \tilde{C}_1 + \bar{B}_1^T (I_N \otimes U_2)^T$$

$$+ \sigma_1 \beta_1 (I_N \otimes Q_1) \tilde{C}_1 + \beta_3 \bar{B}_1^T (I_N \otimes Q_2)^T$$

$$\pi_{16} = \sigma_2 (I_N \otimes U_1) \tilde{C}_2 + \sigma_1 \beta_1 (I_N \otimes Q_1) \tilde{C}_2$$

$$+ \beta_4 \bar{B}_1^T (I_N \otimes Q_2)^T$$

$$\pi_{12} = - (I_N \otimes U_3) - \frac{\tau}{4} (I_N \otimes U_7)$$

$$\pi_{13} = -(1 - \bar{\tau}) (I_N \otimes U_4) + \text{sym} (\sigma_2 \beta_2 (I_N \otimes Q_1) \bar{D}_2)$$

$$\pi_{14} = \beta_2 (I_N \otimes Q_1) \beta_1 \bar{C}_1, \pi_{15} = \frac{2}{\tau} (I_N \otimes U_3)$$

$$\pi_{14} = \text{sym} ((I_N \otimes U_2) \tilde{A}_1 - (I_N \otimes U_10))$$

$$+ \text{sym} (\beta_3 (I_N \otimes Q_2) \tilde{A}_1) + (I_N \otimes U_5)$$

$$+ (I_N \otimes U_6) - \frac{4}{\tau} (I_N \otimes U_8) - \delta (I_N \otimes U_2)$$

$$\pi_{15} = \frac{6}{\tau^2} (I_N \otimes U_7) + \frac{2}{\tau} (I_N \otimes U_9)$$

$$\pi_{16} = (A + \sigma_1 D_1 + K)^T (I_N \otimes Q_1)^T - \beta_1 (I_N \otimes Q_1)$$

$$\pi_{17} = \bar{B}_1^T (I_N \otimes Q_2)^T, \pi_{18} = (I_N \otimes U_1) \tilde{E} - \bar{D}_1^T,$$

$$\pi_{19} = \beta_2 (I_N \otimes Q_1) \tilde{E}$$

$$\pi_{10} = \text{sym} ((I_N \otimes U_1) V + \beta_1 (I_N \otimes Q_1) V)^T, 0, \beta_2 ((I_N \otimes U_1) V)^T, 0, \ldots, 0, ((I_N \otimes Q_1) V)^T, 0, 0$$

$$\Pi_1 = \begin{bmatrix} \pi_{11} & \pi_{12} & \pi_{13} & \pi_{14} & 0 & \pi_{16} \\ * & \pi_{22} & 0 & 0 & 0 & 0 \\ * & * & \pi_{33} & \pi_{34} & 0 & \beta_2 (I_N \otimes Q_1) \sigma_2 \hat{C}_2 \\ * & * & * & \pi_{44} & \pi_{45} & \beta_4 \hat{A}_1^T (I_N \otimes Q_2) \\ * & * & * & * & \pi_{55} & 0 \\ * & * & * & * & * & -(1 - \bar{\tau}) (I_N \otimes U_6) \end{bmatrix}$$

$$\Pi_2 = \begin{bmatrix} \beta_2 (I_N \otimes Q_1) \bar{B}_1 (I_N \otimes Q_1) \bar{B}_2 \\ 0 & 0 & \pi_{22}^2 & 0 & 0 & \pi_{33}^2 \\ 0 & 0 & 0 & \pi_{34}^2 & \pi_{35}^2 & \pi_{36}^2 - \bar{C}_1 \\ 0 & 0 & 0 & 0 & \pi_{34}^2 & \pi_{36}^2 - \bar{C}_1 \\ 0 & 0 & 0 & 0 & \pi_{34}^2 & \pi_{36}^2 - \bar{C}_1 \end{bmatrix}$$

$$\Pi_3 = \begin{bmatrix} -2 \varepsilon_1 P & 0 & 0 & 0 & \bar{B}_1^T (I_N \otimes Q_1)^T & 0 & 0 \\ * & -2 \varepsilon_2 P & 0 & 0 & \bar{B}_2^T (I_N \otimes Q_2)^T & 0 & 0 \\ * & * & \pi_{33} & 0 & 0 & 0 & 0 \\ * & * & \pi_{34} & 0 & 0 & 0 & 0 \\ * & * & * & \pi_{35} & 0 & (I_N \otimes Q_1) \tilde{E} \\ * & * & * & * & \pi_{66} & 0 & 0 \\ * & * & * & * & * & -\gamma P \end{bmatrix}$$
\[ \pi_{33}^2 = \sigma_2 D_2^T (I_N \otimes Q_1)^T - \beta_2 (I_N \otimes Q_1) \]
\[ \pi_{44}^2 = \frac{6}{\tau_3} (I_N \otimes U_8) + \frac{2}{\tau} (I_N \otimes U_{10}) \]
\[ \pi_{45}^2 = \sigma_1 C_1^T (I_N \otimes Q_1)^T \]
\[ \pi_{52}^2 = \bar{A}_1^T (I_N \otimes Q_2)^T - \beta_3 (I_N \otimes Q_2) \]
\[ \pi_{66}^2 = \frac{6}{\tau_4} (I_N \otimes U_8), \quad \pi_{65}^2 = \sigma_2 C_2^T (I_N \otimes Q_1)^T \]

Proof: Let us choose the following Lyapunov-Krasovskii functional candidate for the error system (11):

\[ V(t, e(t)) = V_1(t, e(t)) + V_2(t, e(t)) + V_3(t, e(t)) + V_4(t, e(t)) \]

where

\[ V_1(t, e(t)) = e^T(t) (I_N \otimes U_1) e(t) + m^T(t) (I_N \otimes U_2) m(t) \]

\[ V_2(t, e(t)) = \int_{t-\tau}^{t} e^T(s) (I_N \otimes U_3) e(s) ds \]
\[ + \int_{t-\tau(t)}^{t} e^T(s) (I_N \otimes U_4) e(s) ds \]
\[ + \int_{t-\tau}^{t} m^T(s) (I_N \otimes U_5) m(s) ds \]
\[ + \int_{t-\tau(t)}^{t} m^T(s) (I_N \otimes U_6) m(s) ds \]

\[ V_3(t, e(t)) = \int_{t-\tau}^{t} \int_{u}^{\tau(t)} e^T(s) (I_N \otimes U_7) \dot{e}(s) ds du \]
\[ + \int_{t-\tau}^{t} \int_{u}^{\tau(t)} \dot{m}^T(s) (I_N \otimes U_8) \dot{m}(s) ds du \]

\[ V_4(t, e(t)) = \int_{t-\tau}^{t} \int_{u}^{\tau(t)} \int_{v}^{t} e^T(s) (I_N \otimes U_9) \dot{e}(s) ds du dv \]
\[ + \int_{t-\tau}^{t} \int_{u}^{\tau(t)} \int_{v}^{t} \dot{m}^T(s) (I_N \otimes U_{10}) \dot{m}(s) ds du dv \]

Then, by calculating the first derivative of \( V(t, e(t)) \) along the trajectories of the error system (11), we can easily obtain

\[ \dot{V}_1(t, e(t)) = 2e^T(t) (I_N \otimes U_1) \dot{e}(t) \]
\[ + 2m^T(t) (I_N \otimes U_2) \dot{m}(t) \]
\[ = 2e^T(t) (I_N \otimes U_1) \left[ A + \sigma_1 D_1 + K + VHM \right] e(t) \]
\[ + \sigma_2 D_2 e(t - \tau(t)) + \sigma_1 C_1 m(t) + \sigma_2 C_2 m(t - \tau(t)) \]
\[ + \bar{B}_1 F(e(t)) + \bar{B}_2 F(e(t - \tau(t))) + \bar{E} \dot{v}(t) \]
\[ + 2m^T(t) (I_N \otimes U_2) \left[ \bar{A} m(t) + \bar{B} \dot{e}(t) \right] \]

\[ \dot{V}_2(t, e(t)) = \]
\[ e^T(t) (I_N \otimes U_3) e(t) + e^T(t) (I_N \otimes U_4) e(t) \]
\[ - e^T(t - \tau) (I_N \otimes U_3) e(t - \tau) \]
\[ + m^T(t) (I_N \otimes U_5) m(t) + m^T(t) (I_N \otimes U_6) m(t) \]
\[ - (1 - \bar{v}(t)) e^T(t - \tau(t))(I_N \otimes U_4) e(t - \tau(t)) \]
\[ - m^T(t - \tau)(I_N \otimes U_5) m(t - \tau) \]
\[ - (1 - \bar{v}(t)) m^T(t - \tau(t))(I_N \otimes U_6) m(t - \tau(t)) \]
\[ \leq e^T(t) (I_N \otimes U_3 + I_N \otimes U_4) e(t) \]
\[ - e^T(t - \tau) (I_N \otimes U_3) e(t - \tau) \]
\[ - m^T(t - \tau)(I_N \otimes U_5) m(t - \tau) \]
\[ - (1 - \bar{v}(t)) e^T(t - \tau(t))(I_N \otimes U_4) e(t - \tau(t)) \]
\[ + m^T(t)(I_N \otimes U_5 + I_N \otimes U_6)m(t) \]
\[ - (1 - \bar{v}(t)) m^T(t - \tau(t))(I_N \otimes U_6)m(t - \tau(t)) \]

\[ \dot{V}_3(t, e(t)) = \]
\[ \tau^2 e^T(t) (I_N \otimes U_7) \dot{e}(t) \]
\[ - \int_{t-\tau}^{t} \dot{e}^T(s)(I_N \otimes U_7) \dot{e}(s) ds \]
\[ + \tau m^T(t)(I_N \otimes U_8) \dot{m}(t) \]
\[ - \int_{t-\tau}^{t} \int_{u}^{t} \dot{e}^T(s)(I_N \otimes U_9) \dot{e}(s) ds du \]

\[ \dot{V}_4(t, e(t)) = \]
\[ \tau^2 \frac{2}{\tau} e^T(t) (I_N \otimes U_9) \dot{e}(t) \]
\[ + \frac{\tau^2}{\tau} m^T(t)(I_N \otimes U_{10}) \dot{m}(t) \]
\[ - \int_{t-\tau}^{t} \int_{u}^{t} \dot{e}^T(s)(I_N \otimes U_9) \dot{e}(s) ds du \]

Now, by applying Lemma 2, we can obtain

\[ - \int_{t-\tau}^{t} \dot{e}^T(s)(I_N \otimes U_7) \dot{e}(s) ds \]
\[ \leq - \frac{1}{\tau} \Xi_1^T(I_N \otimes U_7) \Xi_1 - \frac{3}{\tau^2} \Xi_2^T(I_N \otimes U_7) \Xi_2 \]

where

\[ \Xi_1 = e(t) - e(t - \tau) \]
\[ \Xi_2 = e(t) + e(t - \tau) - \frac{2}{\tau} \int_{t-\tau}^{t} e(s) ds \]

\[ - \int_{t-\tau}^{t} \int_{u}^{t} \dot{e}^T(s)(I_N \otimes U_9) \dot{e}(s) ds du \]
\[ \leq - \frac{2}{\tau^2} \left( \tau e(t) - \int_{t-\tau}^{t} e(s) ds \right)^T(I_N \otimes U_9) \left( \tau e(t) - \int_{t-\tau}^{t} e(s) ds \right) \]
Similarly, we can obtain
\[
-\int_{t-\tau}^{t} \dot{m}^T(s)(I_N \otimes U_8) \dot{m}(s)ds \\
\leq -\frac{1}{\tau} \tilde{z}_1^T(I_N \otimes U_8) \tilde{z}_1 - \frac{3}{\tau} \tilde{z}_2^T(I_N \otimes U_8) \tilde{z}_2 \quad (23)
\]

\[
-\int_{t-\tau}^{t} \int_{\mu}^{t} \dot{m}^T(t)(I_N \otimes U_{10}) \dot{m}(t)d\mu \leq -\frac{2}{\tau^2} \left( \tau m(t) - \int_{t-\tau}^{t} m(s)ds \right)^T(I_N \otimes U_{10}) \tau m(t) - \int_{t-\tau}^{t} m(s)ds \right)^T \quad (24)
\]

According to Assumption 1, there exist positive scalars \( \varepsilon_1, \varepsilon_2 \geq 0 \) such that
\[
-\varepsilon_1 \left[ \begin{array}{c} e(t) \\ F(e(t)) \end{array} \right]^T \left[ \begin{array}{c} \bar{F}_1 - \bar{F}_2 \\ 2I_N \end{array} \right] \left[ \begin{array}{c} e(t) \\ F(e(t)) \end{array} \right] \geq 0 
\]
\[
-\varepsilon_2 \left[ \begin{array}{c} e(t) \\ F(e(t-\tau(t))) \end{array} \right]^T \left[ \begin{array}{c} \bar{F}_1 - \bar{F}_2 \\ 2I_N \end{array} \right] \left[ \begin{array}{c} e(t) \\ F(e(t-\tau(t))) \end{array} \right] \geq 0
\]

where
\[
\bar{F}_1 = I_N \otimes \left( F_1^2 F_2 + F_2^2 F_1 \right), \quad \bar{F}_2 = I_N \otimes \left( F_1^2 + F_2^2 \right)
\]

Moreover, for any matrix \( Q_1, Q_2 \) with appropriate dimension and scalars \( \beta_1, \beta_2, \beta_3, \beta_4 > 0 \), the following equation holds
\[
0 = 2 \left[ \dot{e}^T(t) + \beta_1 e^T(t) + \beta_2 e^T(t-\tau(t)) \right] (I_N \otimes Q_1) * \left[ \begin{array}{c} \varepsilon_1(t) \end{array} \right] + \sigma_2 \bar{D}_e(t-\tau(t)) + \sigma_1 \bar{C}_m(t) + \sigma_2 \bar{C}_2 m(t-\tau(t)) + \bar{B}_1 F(e(t)) + \bar{B}_2 F(e(t-\tau(t))) + \bar{E} \dot{w}(t) \quad (27)
\]

\[
0 = 2 \left[ \dot{m}^T(t) + \beta_1 m^T(t) + \beta_2 m^T(t-\tau(t)) \right] * \left[ \begin{array}{c} (I_N \otimes Q_1) [-\dot{m}(t) + \bar{A}_m(t) + \bar{B}_1 e(t) \end{array} \right) \right]
\]

By considering (17)-(28) together, we eventually have
\[
\hat{V}(t, e(t)) - \delta V(t, e(t)) - 2z_2^T(t) w(t) - \delta w^T(t) w(t) \leq \Omega^T(t) \left[ \begin{array}{c} \Pi_{11} + \Sigma_1 H \Sigma_2 + \Sigma_2^T H \Sigma_1 \end{array} \right] \Omega(t) \quad (29)
\]

where
\[
\Omega(t) = \left[ \begin{array}{c} e(t) \\ e(t-\tau) \\ m(t) \\ m(t-\tau) \\ T \end{array} \right] \left[ \begin{array}{c} \Pi_{11} + \Sigma_1 H \Sigma_2 + \Sigma_2^T H \Sigma_1 \end{array} \right] \left[ \begin{array}{c} e(t) \\ e(t-\tau) \\ m(t) \\ m(t-\tau) \end{array} \right] + \int_{t-\tau}^{t} e(s)ds \right]^T
\]

By Schur complement in Eq.(30), we can rewrite (29) as
\[
\hat{V}(t, e(t)) - \delta V(t, e(t)) - 2z_2^T(t) w(t) - \delta w^T(t) w(t) \leq \Omega^T(t) \Pi \Omega(t) \quad (31)
\]

where
\[
\Pi = \Pi_{11} + \theta^{-1} \Sigma_1 \Sigma_1^T + \theta \Sigma_2^T \Sigma_2
\]

Based on (12), we can know
\[
\hat{V}(t, e(t)) - \delta V(t, e(t)) - 2z_2^T(t) w(t) - \delta w^T(t) w(t) \leq \Omega^T(t) \Pi \Omega(t) < 0 \quad (32)
\]

Thus
\[
\hat{V}(t, e(t)) - \delta V(t, e(t)) < 2z_2^T(t) w(t) + \delta w^T(t) w(t) \quad (33)
\]

Pre-and post-multiplying (33) by \( e^{-\delta t} \) and integrating it from 0 to \( t_p \), under the zero initial condition, we can obtain
\[
0 < e^{-\delta t} V(t, e(t)) < V(0, e(0)) + \int_{0}^{t_p} e^{-\delta t} \left( \delta w^T(t) w(t) - \delta w^T(t) w(t) \right) dt \quad (34)
\]

That is to say
\[
\int_{0}^{t_p} \delta w^T(t) w(t) + \delta w^T(t) w(t) dt > 0, \quad \delta = \gamma \quad (35)
\]

And then the condition in Definition 2 is satisfied, which is shown the error dynamics (11) systems passive.

Next, we will indicate the error system is asymptotic synchronization. For this paper, considering \( w(t) = 0 \), then we still have \( \hat{V}(t, e(t)) < 0 \). Therefore from Lyapunov stability theory and Definition 1, the error system (11) is asymptotically stable which means that the complex dynamical networks are synchronized asymptotically. Thus, the proof of this theorem is completed.

Remark 1: This paper introduced a modified Lyapunov-Krasovskii functional and the integral inequalities to reduce conservative. Add \( V_i(t, e(t)) \) \( i = 1, 2, 3, 4 \) conclude the form of Kronecker, in order to reduce conservative.

Remark 2: By proving the theorem, the sufficient conditions for synchronization of complex dynamic network systems with dynamic links are obtained. It can be seen from Eq.(12) that the sufficient condition of synchronization is very complex. Firstly, This is determined by the complexity of complex network system. Secondly, when selecting the appropriate Lyapunov-Krasovskii functional, the complexity of Lyapunov-Krasovskii function also increases the number of final linear matrix inequalities. Theorem 2 gives solvable linear matrix inequalities in the follow. It is also verified by the examples in the end of the paper.

Theorem 2: Suppose that Assumption 1 holds. If there exist any symmetric positive definite matrices \( \bar{U}_a \) \( a = 2, 3, \ldots, 10 \), any appropriate dimensioned matrices \( \bar{Q}_1, \bar{Q}_2 \), and
scalars $\varepsilon_1, \varepsilon_2, \beta_1, \beta_2, \beta_3, \beta_4, \theta > 0, \gamma > 0$ such that the following LMIs holds:

$$\tilde{\Pi} = \begin{bmatrix}
\tilde{\Pi}_{11 \times 11} & \tilde{\Sigma}_1 & \tilde{\Sigma}_2 \\
* & -\delta I_P & 0 \\
* & * & -\delta I_P
\end{bmatrix} < 0 \quad (36)$$

where

$$\tilde{\Pi}_{11 \times 11} = \begin{bmatrix}
\tilde{\Pi}_1 & \tilde{\Pi}_2 \\
* & \tilde{\Pi}_3
\end{bmatrix}$$

$$\tilde{\Sigma}_2 = \begin{bmatrix}
M(I_N \otimes P), 0, 0, \ldots, 0, 0 \\
\vdots
\end{bmatrix}
$$

$$\pi_{11} = \text{sym} \left( (\tilde{A} + \sigma_1 \tilde{D}_1)(I_N \otimes P) + Y \right) + (I_N \otimes \tilde{U}_3)$$

$$\pi_{13} = \sigma_2 \tilde{D}_2 (I_N \otimes P) + \beta_2 Y (I_N \otimes \tilde{Q}_1)^T$$

$$\pi_{14} = \sigma_1 \tilde{C}_1 (I_N \otimes P) + \beta_2 \bar{B}_I (I_N \otimes \tilde{U}_2)^T$$

$$\pi_{16} = \sigma_2 \tilde{C}_2 (I_N \otimes P) + \sigma_1 \beta_1 (I_N \otimes \tilde{Q}_1) \tilde{C}_2 (I_N \otimes P) + \beta_4 \bar{B}_I^T (I_N \otimes P)(I_N \otimes \tilde{Q}_2)^T$$
\[
\begin{align*}
\pi_{17}^1 &= E - (I_N \otimes P) \tilde{D}_1^T \\
\pi_{32}^1 &= (I_N \otimes \tilde{Q}_1) \tilde{B}_2 \\
\pi_{35}^2 &= \sigma_2 (I_N \otimes P) \tilde{D}_1^T (I_N \otimes \tilde{Q}_1)^T \\
&\quad - \beta_2 (I_N \otimes P) (I_N \otimes \tilde{Q}_1) \\
\pi_{44}^2 &= \frac{6}{\tau^2} (I_N \otimes \tilde{U}_8) + \frac{2}{\tau} (I_N \otimes \tilde{U}_{10}) \\
\pi_{45}^2 &= \sigma_1 (I_N \otimes P) \tilde{C}_1^T (I_N \otimes \tilde{Q}_1)^T \\
\pi_{46}^2 &= \tilde{A}_1^T (I_N \otimes P) (I_N \otimes \tilde{Q}_2)^T \\
&\quad - \beta_3 (I_N \otimes P) (I_N \otimes \tilde{Q}_2) \\
\pi_{54}^2 &= \frac{6}{\tau^2} (I_N \otimes \tilde{U}_8) \\
\pi_{65}^2 &= \sigma_2 (I_N \otimes P) \tilde{C}_1^T (I_N \otimes \tilde{Q}_1)^T \\
\pi_{66}^2 &= -\beta_4 (I_N \otimes P) (I_N \otimes \tilde{Q}_2) \\
\pi_{33}^3 &= -\frac{12}{\tau^2} (I_N \otimes \tilde{U}_7) - \frac{2}{\tau^2} (I_N \otimes \tilde{U}_9) \\
\pi_{44}^3 &= -\frac{12}{\tau^2} (I_N \otimes \tilde{U}_8) - \frac{2}{\tau^2} (I_N \otimes \tilde{U}_{10}) \\
\pi_{35}^3 &= \tau (I_N \otimes \tilde{U}_7) + \frac{\tau^2}{2} (I_N \otimes \tilde{U}_9) \\
&\quad - 2 (I_N \otimes \tilde{Q}_1) (I_N \otimes P) \\
\pi_{46}^3 &= \tau (I_N \otimes \tilde{U}_8) + \frac{\tau^2}{2} (I_N \otimes \tilde{U}_{10}) \\
&\quad - 2 (I_N \otimes \tilde{Q}_2) (I_N \otimes P)
\end{align*}
\]

Then, the complex dynamical networks with dynamical behavior links (1) is asymptotically and the non-fragile gain matrix \(K = Y(I_N \otimes P)^{-1}\).

**Proof:** By Lemma 1, pre-and post-multiply (12) by

\[
\text{diag} \left\{ (I_N \otimes U_1)^{-1}, \ldots, (I_N \otimes U_1)^{-1}, I, I, (I_N \otimes U_1)^{-1}, (I_N \otimes U_1)^{-1}, I, I, \ldots, I \right\}_6
\]

and define \((I_N \otimes U_1)^{-1} = (I_N \otimes P)\), based on the properties of Kronecker product we can obtain \(U_1^{-1} = P\), and setting

\[
\begin{align*}
(I_N \otimes \tilde{U}_k) &= (I_N \otimes U_1)^{-1} (I_N \otimes U_k) (I_N \otimes U_1)^{-1} \\
&\quad (k = 2, 3, \ldots, 10) \\
(I_N \otimes \tilde{Q}_1) &= (I_N \otimes U_1)^{-1} (I_N \otimes \tilde{Q}_1) \\
(I_N \otimes \tilde{Q}_2) &= (I_N \otimes U_1)^{-1} (I_N \otimes \tilde{Q}_2) \\
(I_N \otimes \tilde{F}_1) &= (I_N \otimes U_1)^{-1} (I_N \otimes \tilde{F}_1) \\
(I_N \otimes \tilde{F}_2) &= (I_N \otimes U_1)^{-1} (I_N \otimes \tilde{F}_2)
\end{align*}
\]

It is easy found that matrix inequality (12) can be transformed into linear matrix inequality (36). This proof is completed.

**IV. EXAMPLE**

In this section, we shall present two examples to demonstrate the effectiveness and applicability of the proposed method.

**Example 1:** In this example, a time-delayed Chua oscillator is chosen as the isolate node of the complex networks.

\[
\dot{x}(t) = Ax(t) + B_1f(x(t)) + B_2f(x(t - \tau(t)))
\]

where

\[
x(t) = (x_1(t), x_2(t), x_3(t))^T \\
f(x(t)) = (-0.5\alpha (m_1 - m_2) (|x_1(t)| + 1) - |x_1(t)| - 1), 0, 0)^T \\
f(x(t - \tau(t))) = (0, 0, -\eta w_0 \sin(vx_1(t - \tau(t))))^T \in \mathbb{R}^3
\]

\[
A = \begin{bmatrix}
-\alpha (1 + m_2) & \alpha & 0 \\
1 & -1 & 1 \\
0 & -\eta & -w
\end{bmatrix}
\]

\[
B_1 = B_2 = \begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}
\]

\(\alpha = 10, \eta = 19.53, w = 0.1636, m_1 = -1.4325, m_2 = -0.7831, v = 0.5, w_0 = 0.2. \) Let \(\tau(t) = 0.1 + 0.01 \sin(10t)\).

It obvious that \(\tau = 0.11, \tau = 0.1.\) The time-delayed Chua oscillator is chaotic, the dynamical chaotic is shown in Fig. 1. Let this result be the synchronization manifold, i.e. We consider four numbers of identical Chua’s circuits to create the complex dynamical networks as (1). The other parameters are given as follows. And it can be seen from Fig. 2 that the synchronization error is tending to zero, which implies that the synchronization of complex dynamical networks can be achieved by the designed non-fragile controller.

\[
\begin{align*}
A_l &= \begin{bmatrix}
2.5 & 0 & 0 \\
0 & 3 & 0 \\
0 & 0 & 1
\end{bmatrix}, \\
B_l &= D_l = \begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}
\end{align*}
\]

\[
\begin{align*}
C_l &= \begin{bmatrix}
2 & 0 & 0 \\
0 & 2 & 0 \\
0 & 0 & 5
\end{bmatrix}, \\
V &= \begin{bmatrix}
1 & 0 & 5 \\
0 & 1 & 0 \\
0 & 0 & 0
\end{bmatrix}
\end{align*}
\]

\[
\begin{align*}
M &= \begin{bmatrix}
-0.5 & 0 & 0 \\
0.2 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}, \\
\tilde{F}_1 &= \tilde{F}_2 = \begin{bmatrix}
1 & 0 & 0 \\
0 & 0 & 1
\end{bmatrix}
\end{align*}
\]

\[
\begin{align*}
G &= \begin{bmatrix}
-5 & 1 & 0 & 1 & 1 \\
1 & -3 & 1 & 0 & 0 \\
0 & 1 & -4 & 1 & 0 \\
1 & 0 & 0 & 1 & -3 \\
1 & 0 & 1 & 0 & 1 & -4
\end{bmatrix}
\end{align*}
\]

\(\sigma_1 = 0.2, \sigma_2 = 0.5, \beta_1 = 2, \beta_2 = 3, \beta_3 = 4, \beta_4 = 10, \varepsilon_1 = 8, \varepsilon_2 = 5, \delta = 5, \gamma = 15, \vartheta = 2.\)

Choose the initial conditions as \(x_1(0) = [-2 - 3 5]^T, \)

\(x_2(0) = [-1 2 4]^T, \)

\(x_3(0) = [5 - 1 2]^T, \)

\(x_4(0) = [-1 - 2 - 5]^T, \)

\(x_5(0) = [-1 - 2 6]^T, \)

\(x_6(0) = [1 - 6 8]^T \) and \(s(0) = [1 0 - 2]^T.\) Then by using the
Matlab toolbox, the LMIs in (36) can be solved with the feasible solutions as follows:

\[
P = \begin{bmatrix} 22.9867 & -1.5422 & 0.7879 \\ -1.5422 & 3.7985 & 0.1750 \\ 0.7879 & 0.1750 & 10.2182 \end{bmatrix}
\]

\[
U_3 = \begin{bmatrix} 37.5144 & -3.7379 & 1.5247 \\ -3.7379 & 5.2872 & 0.9547 \\ 1.5247 & 0.9547 & 37.5260 \end{bmatrix}
\]

\[
U_2 = 1.0e + 0.3 \times \begin{bmatrix} 0.3512 & -0.0245 & 0.0240 \\ -0.0245 & 0.0406 & 0.0710 \\ 0.0240 & 0.0710 & 2.1089 \end{bmatrix}
\]

\[
U_4 = 1.0e + 0.3 \times \begin{bmatrix} 0.8698 & 0.0869 & -0.0421 \\ 0.0869 & 1.1462 & 0.0060 \\ -0.0421 & 0.0060 & 0.6414 \end{bmatrix}
\]

\[
U_5 = 1.0e + 0.3 \times \begin{bmatrix} 0.8040 & -0.0935 & 0.0543 \\ -0.0935 & 0.0576 & 0.0272 \\ 0.0543 & 0.0272 & 1.0405 \end{bmatrix}
\]

\[
U_6 = 1.0e + 0.3 \times \begin{bmatrix} 0.0272 & 3.3963 & 0.1088 \\ -0.3963 & 1.1443 & 0.1055 \\ 0.1088 & 0.1055 & 4.1753 \end{bmatrix}
\]

\[
U_7 = \begin{bmatrix} 52.4257 & 7.8279 & -6.5331 \\ 7.8279 & 165.0222 & -0.5222 \\ -6.5331 & -0.5222 & 114.8096 \end{bmatrix}
\]

\[
U_8 = \begin{bmatrix} 49.7328 & -2.3872 & -0.9161 \\ -2.3872 & 23.8934 & 1.0626 \\ -0.9161 & 1.0626 & 54.8368 \end{bmatrix}
\]

\[
U_9 = 1.0e + 0.3 \times \begin{bmatrix} 0.9446 & -0.0760 & 0.0330 \\ -0.0760 & 0.3534 & 0.0046 \\ 0.0330 & 0.0046 & 1.0813 \end{bmatrix}
\]

\[
U_{10} = 1.0e + 0.3 \times \begin{bmatrix} 3.6520 & -0.2448 & 0.1347 \\ -0.2448 & 0.3929 & 0.0215 \\ 0.1347 & 0.0215 & 1.0743 \end{bmatrix}
\]

\[
Y = \text{diag}(Y_0, Y_0, Y_0, Y_0, Y_0)
\]

\[
Y_0 = \begin{bmatrix} 25.6978 & -5.6888 & -18.1239 \\ -5.6888 & -24.4703 & 18.6398 \\ -18.1239 & 18.6398 & 34.4446 \end{bmatrix}
\]

The corresponding state feedback control gain matrix is estimated as follows:

\[
K_1 = K_2 = K_3 = K_4 = K_5 = K_6 = \begin{bmatrix} 1.1168 & -0.9593 & -1.8434 \\ 0.7756 & -6.8492 & 2.0013 \\ -0.6004 & 4.5095 & 3.3400 \end{bmatrix}
\]

Remark 3: It can be seen from the Example 1 that we select the Chua oscillator system as the isolated node model. It can be seen from Figure 1 that Chua system is a chaotic system. By using non-fragile controller for complex dynamic network system, the feedback gain matrix is obtained through linear matrix inequality, and the complex dynamical network system is synchronization through data simulation.

Example 2: Considering the following 3-nodes complex dynamical network systems with the following parameters:

\[
A = \begin{bmatrix} 0.5 & 0.5 & 0.6 \\ -0.2 & 1.2 & 0.9 \\ 1.2 & 0.3 & 1.6 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2.5 & 0 \\ 0 & 0 & 1 \end{bmatrix}
\]

\[
\Gamma_1 = \Gamma_2 = E = D_l = B_l = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}
\]

\[
C_l = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 5 \end{bmatrix}, \quad A_l = \begin{bmatrix} 2.5 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 2 \end{bmatrix}
\]

\[
B_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad V = \begin{bmatrix} 0.3 & -0.5 & 0.6 \\ 0.2 & 0.3 & -0.7 \\ 0.1 & -0.9 & 1.5 \end{bmatrix}
\]

\[
M = \begin{bmatrix} -0.5 & 0.2 & 0.5 \\ -0.3 & 0.5 & -0.6 \\ 1.3 & 0.2 & 1.6 \end{bmatrix}, \quad G = \begin{bmatrix} -1 & 1 & 0 \\ 1 & -2 & 1 \\ 0 & 1 & -1 \end{bmatrix}
\]

\[
\sigma_1 = 0.2, \sigma_2 = 0.5, \beta_1 = 2, \beta_2 = 3, \beta_3 = 4, \beta_4 = 10, \epsilon_1 = 10, \epsilon_2 = 14, \delta = 5, \gamma = 10, \theta = 2. \text{ Let } \tau(t) = 0.02 + 0.01 \sin(10t). \text{ It is obvious that } \tau = 0.03, \tilde{\tau} = 0.1. \text{ Then by using the function } f(x(t)) = \frac{1}{2} (|x(t) + 1| + |x(t) - 1|). \text{ Then by using the function } f(x(t)) = \frac{1}{2} (|x(t) + 1| + |x(t) - 1|). \text{ Then by using the function } f(x(t)) = \frac{1}{2} (|x(t) + 1| + |x(t) - 1|). \text{ Then by using the function } f(x(t)) = \frac{1}{2} (|x(t) + 1| + |x(t) - 1|). \text{ Then by using the function } f(x(t)) = \frac{1}{2} (|x(t) + 1| + |x(t) - 1|). \text{ Then by using the function } f(x(t)) = \frac{1}{2} (|x(t) + 1| + |x(t) - 1|).
Matlab toolbox, the LMIs in (36) can be solved with the feasible solutions as follows:

\[
P = \begin{bmatrix} 6.3541 & 0.0716 & -0.9578 \\ 0.0716 & 8.8327 & -0.2113 \\ -0.9578 & -0.2113 & 4.6082 \end{bmatrix},
\]

\[
U_2 = \begin{bmatrix} 114.1120 & -0.2819 & 1.2332 \\ -0.2819 & 85.8525 & 2.3173 \\ 1.2332 & 2.3173 & 143.6112 \end{bmatrix},
\]

\[
U_3 = \begin{bmatrix} 0.0421 & -0.0125 & 5.2252 \\ -0.0125 & 6.9587 & 0.7548 \\ 5.2252 & 0.7548 & 161.1538 \end{bmatrix},
\]

\[
U_4 = \begin{bmatrix} 185.7171 & -1.2493 & -11.6004 \\ -1.2493 & 231.5431 & 0.7548 \\ -11.6004 & 0.7548 & 161.1538 \end{bmatrix},
\]

\[
U_5 = \begin{bmatrix} 568.4554 & -0.4777 & 15.0533 \\ 0.4777 & 550.3954 & 6.6598 \\ 15.0533 & 6.6598 & 552.1587 \end{bmatrix},
\]

\[
U_6 = \begin{bmatrix} 10.999 & 0.0071 & -0.1182 \\ 0.0071 & 1.1824 & -0.0264 \\ -0.1182 & -0.0264 & 1.0024 \end{bmatrix},
\]

\[
U_7 = \begin{bmatrix} 2.1412 & 0.0080 & 0.0189 \\ 0.0080 & 1.9857 & -0.0600 \\ 0.0189 & -0.0600 & 2.6322 \end{bmatrix},
\]

\[
U_8 = \begin{bmatrix} 2.1519 & 0.0099 & -0.0135 \\ 0.0099 & 2.0488 & -0.0697 \\ -0.0135 & -0.0697 & 2.6485 \end{bmatrix},
\]

\[
U_9 = \begin{bmatrix} 211.4718 & 0.3845 & -1.0305 \\ 0.3845 & 202.9218 & -3.7688 \\ -1.0305 & -3.7688 & 241.0608 \end{bmatrix},
\]

\[
U_{10} = \begin{bmatrix} 1.0013 & 0.0056 & -0.0869 \\ 0.0056 & 1.2501 & -0.0138 \\ -0.0869 & -0.0138 & 0.9830 \end{bmatrix},
\]

\[
Y = \begin{bmatrix} Y_0 & 0 & 0 \\ 0 & Y_0 & 0 \\ 0 & 0 & Y_0 \end{bmatrix},
\]

\[
Y_0 = \begin{bmatrix} 1.8179 & -1.1101 & -6.1874 \\ -1.1101 & 5.8168 & -3.0656 \\ -6.1874 & -3.0656 & 3.0246 \end{bmatrix},
\]

The corresponding state feedback control gain matrix is estimated as follows:

\[
K_1 = K_2 = K_3 = \begin{bmatrix} 0.0871 & -0.1582 & -1.3318 \\ -0.2808 & -0.6743 & -0.7545 \\ -1.1058 & -0.3597 & -0.9027 \end{bmatrix},
\]

After that, by selecting the initial values of the state node for error system (11) \( e_1(0) = [0.1 - 0.7 0.4]^T, e_2(0) = [0.2 - 0.1 0.2]^T, e_3(0) = [-0.1 0 -0.2]^T \) the state trajectories of the corresponding nodes are obtained. Fig.3 represents the synchronization error by applying the controller.

**V. CONCLUSION**

In this paper, the passivity synchronization of complex dynamical networks with dynamic behavior links has been investigated. With constructing a new Lyapunov-Krasovskii functional and combining the reciprocal convex technique, sufficient conditions for complex dynamical networks to be synchronized are derived. In the end, two examples are given to demonstrate the effectiveness of the proposed methods.

The dynamic behavior links which is introduced to the complex dynamical networks, shows its rich features in the complex dynamical. However, the fractional order complex dynamical networks about dynamic behavior links is not still fully utilized in the analysis of network synchronization. Therefore, our further research will turn on focus on the root of synchronization control, and try our best to play its important features.

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