Accounting of spontaneous magnetic fields in Focus 3D code

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Abstract. To study the interaction of laser radiation with matter, the Focus 3D code is being developed. The Focus 3D code uses the finite volume method. Within the framework of the program, the solution of the equations of ideal magnetic gas dynamics in conservation-law form is realized. To remove the divergence of the magnetic field, the projection scheme is used. Accounting of spontaneous magnetic fields is realized. Test problems are proposed with an analytical solution for verifying the correctness of numerical realization of accounting for spontaneous magnetic fields. A second order of approximation is obtained for calculating the gradient of smooth functions by the Gauss gradient theorem. The convergence of the solution of the problem to an analytic solution on refined grids is shown.

1. Introduction

Magnetic fields of considerable magnitude were discovered in various experiments on interaction of laser radiation with matter in plasma formed [1, 2]. Spontaneous magnetic fields in matter appear under condition of noncollinearity of pressure and density gradients. The presence of perturbations at the boundary of target and their further development only increase the growth of magnetic fields, which in turn affect the development of instabilities. To study the described processes, the Focus 3D code is being developed [3].

The Focus 3D code uses the finite volume method. Within the framework of the program, the solution of the equations of ideal magnetic gas dynamics in conservation-law form is realized. To approximate the flows in the centers of the cell faces, the HLL scheme is used [4]. To obtain the expression for the numerical flow on the faces of cells, we consider the one-dimensional Riemann problem. The time approximation is carried out according to the two-stage Runge-Kutta scheme. To remove the divergence of the magnetic field, the projection scheme is used [5, 6].

The code was tested by solving the system of equations of the ideal magnetic gas dynamics on one-, two- and three-dimensional problems. The comparison was carried out with the solutions obtained under the FLASH code [7], and the results were shown to be in a good agreement. It is shown that removing the divergence of the magnetic field gives more accurate result.

Test problems having analytic solutions are proposed to verify the correctness of the numerical realization of spontaneous magnetic fields accounting. The second order of approximation is obtained for calculation the gradient of smooth functions by the Gauss gradient theorem. The convergence of the solution on refined grids is shown.

2. Formulation of the problem

Magnetic gas dynamics (often named as MHD) is described by the equations of continuity, motion, energy and magnetic induction. The last equation describes the change in the magnetic field, the first
three equations express the laws of conservation of mass, momentum and energy with allowance for the magnetic component. To write the equations, we use the operator \( \nabla \) (nabla) [8], namely \( \nabla f \) is the gradient of the scalar function \( f \), \( \nabla \cdot \vec{F} \) and \( \nabla \times \vec{F} \) are the divergence and the rotor of the vector function \( \vec{F} \), respectively.

The system of MHD equations with allowance for spontaneous magnetic fields in a symmetric Gaussian system is written as follows:

\[
\begin{align*}
\frac{\partial \rho}{\partial t} &= -\nabla \cdot (\rho \vec{v}), \\
\frac{\partial (\rho v_i)}{\partial t} &= -\nabla \cdot (\rho v_i \vec{v}) - \frac{\partial p^*}{\partial x_i} + \nabla \cdot \left( \frac{\vec{B} \cdot \vec{B}}{4\pi} \right), \quad i = 1, 2, 3, \\
\frac{\partial (\rho E)}{\partial t} &= -\nabla \cdot \left( (\rho E + p^*) \vec{v} \right) + \nabla \cdot \left( \frac{(\vec{v} \cdot \vec{B}) \cdot \vec{B}}{4\pi} \right) + \frac{c \mathbf{A} m_p}{4\pi e \langle z \rangle} \frac{\vec{B} \cdot (\nabla p \times \nabla \rho)}{\rho^2}, \\
\frac{\partial \vec{B}}{\partial t} &= -\nabla \cdot (\vec{v} \cdot \vec{B} - \vec{B} \cdot \vec{v}) + \frac{c \mathbf{A} m_p}{e \langle z \rangle} \frac{(\nabla p \times \nabla \rho)}{\rho^2}, \quad i = x, y, z,
\end{align*}
\]

where \( \rho \) – density, \( \vec{v} \) – velocity, \( \vec{B} \) – magnetic induction, the total pressure is \( p^* = p + p_B \), \( p \) – gas dynamic pressure, \( p_B = \frac{B^2 + B^2 + B^2}{8\pi} \) – magnetic pressure, the total energy of the medium is \( \rho E = \rho e + \rho \frac{v_x^2 + v_y^2 + v_z^2}{2} + \frac{B^2 + B^2 + B^2}{8\pi} \), \( e \) – specific internal energy, \( c \) – velocity of light, \( A \) and \( \langle z \rangle \) – atomic weight and average ion charge of this substance, \( m_p \) – rest mass of the proton, \( e \) – electron charge.

In the Focus 3D code, the solution of the system of equations (1) is organized through splitting by processes. First, a system of equations for ideal MHD is solved. Then the procedure for cleaning the magnetic charge is performed with the help of the introduced artificial scalar potential. After this, the equations for changing the magnetic induction and energy are solved taking into account spontaneous magnetic fields. The convenience of this method of solving the system of equations (1) is that each of these processes can be taken into account or not taken into account for the user's calming. That allows you to explore each of these processes separately and in combination.

In the Focus 3D code, the three-dimensional system of equations of multicomponent magnetic gas dynamics in the form of conservation laws in Euler variables in the Cartesian coordinate system has the form

\[
\frac{\partial \vec{u}}{\partial t} + \nabla \cdot \vec{f} (\vec{u}) = 0,
\]

where

\[
\vec{u} = \left( \rho, \rho v_x, \rho v_y, \rho v_z, \rho E, B_x, B_y, B_z, \rho Y_1, \ldots, \rho Y_N \right)^T
\]

is the vector of conservative variables, \( Y_1, \ldots, Y_N \) – mass fractions of the components of the mixture of substances, subjected to the normalization condition \( \sum_{n=1}^{N} Y_n = 1 \), \( \vec{f} \) – tensor whose matrix consists of the vectors of physical flows in each direction \( \left( \vec{f}_x, \vec{f}_y, \vec{f}_z \right) \).
\[
\vec{f}(\vec{u}) = \begin{pmatrix}
\rho_v \\
\rho_v^2 + p^* \frac{B_z^2}{4\pi} \\
\rho_v v_x - \frac{B_x B_z}{4\pi} \\
\rho_v v_y - \frac{B_y B_z}{4\pi} \\
\rho_v v_z - \frac{B_z B_z}{4\pi} \\
\rho v v_x + p^* \frac{B_x^2}{4\pi} \\
\rho v v_y + p^* \frac{B_y^2}{4\pi} \\
\rho v v_z + p^* \frac{B_z^2}{4\pi} \\
(\rho E + p^*) v_x - K B_z \\
(\rho E + p^*) v_y - K B_x \\
(\rho E + p^*) v_z - K B_x \\
\end{pmatrix}, \tag{4}
\]

where \( K = \frac{\vec{v} \cdot \vec{B}}{4\pi} = \frac{v_x B_y - v_y B_x + v_B B_z}{4\pi} \), \( Y_N \) is determined from the normalization condition. Dependences of the specific internal energy and pressure on density and temperature are determined by the equation of state (EOS).

The discretization of special and temporal parts of the system of the MHD equations is carried out separately in accordance with the method of lines [9]. For spatial discretization, the finite volume method is used, which consists in integrating the equations into the computational grid cells [10] and the subsequent discretization of the integral terms. For time sampling, the Runge-Kutta method from the second-order approximation class SSP [11] is used. For spatial discretization the HLL scheme [4] of the second-order approximation is used. To approximate the fluxes \( \vec{f}_n(\vec{u}_j) \) from (4) along the normals in the centers of the cell faces, the HLL scheme adapted for unstructured grids is used:

\[
\vec{f}_n(\vec{u}_j) = \begin{pmatrix}
\rho v \\
\rho v v_x + n_j p^* - \frac{B_x B_n}{4\pi} \\
\rho v v_y + n_j p^* - \frac{B_y B_n}{4\pi} \\
\rho v v_z + n_j p^* - \frac{B_z B_n}{4\pi} \\
(\rho E + p^*) v_x - K B_n \\
(\rho E + p^*) v_y - K B_n \\
(\rho E + p^*) v_z - K B_n \\
\end{pmatrix},
\]

\[
a_j^+ = \max \left\{ \lambda_k \left( \frac{\partial f_j}{\partial u_j} u_j^p \right), \lambda_k \left( \frac{\partial f_j}{\partial u_j} (u_j^N) \right) \right\}, \tag{5}
a_j^- = \min \left\{ \lambda_k \left( \frac{\partial f_j}{\partial u_j} u_j^p \right), \lambda_k \left( \frac{\partial f_j}{\partial u_j} (u_j^N) \right) \right\},
\]

\[
a_j = \left( \frac{1}{2} \left[ c_j^2 + \frac{B^2}{4\pi \rho} \pm \left( \left[ c_j^2 + \frac{B^2}{4\pi \rho} \right] - c_j^2 \frac{B^2}{4\pi \rho} \right)^{1/2} \right] \right)^{1/2},
\]
where $\vec{f}_n(\vec{u}) = f(\vec{u}) \cdot \vec{n} - \text{projection (4) per unit normal } \vec{n} = (n_x, n_y, n_z) \text{ to the cell edge, } \vec{u}_j^+ \text{ and } \vec{u}_j^- - \text{proprietary and neighboring cells, } a_j^+ \text{ and } a_j^- - \text{decision reconstruction in the centre of j-side, } \lambda_1 < \ldots < \lambda_K$ $-$ eigenvalues of the Jacobi matrix of the system (2), $\lambda_1 = v_n - a_j, \quad \lambda_K = v_n + a_j$, $v_n = n_x v_x + n_y v_y + n_z v_z$ $-$ normal component of the velocity, $a_j$ $-$ fast (superafven) magnetosonic velocity, $B_n = n_x B_x + n_y B_y + n_z B_z$ $-$ normal component of the magnetic induction, $c_0$ $-$ sound velocity determined by the EOS used, in the case of the EOS for an ideal gas $c_0 = (dp/d \rho)^{1/2}$.

The Focus code uses the reconstruction of density, temperature, velocity, magnetic induction and mass fractions on the cell faces using the TVD-slope limiter approach, for example, minmod and vanLeer.

The condition for the absence of a magnetic charge (or the solenoidal condition of the magnetic field) is automatically satisfied in the mathematical sense, if there is no magnetic charge in the initial conditions. This is not the case when carrying out multidimensional computations due to discretization errors of approximation, especially in cases where the derivatives of discontinuous functions are calculated.

There are several approaches for ensuring the solenoidal nature of the magnetic field [5]. There is no unequivocally correct choice - all approaches are aimed at obtaining the same results, but each of them has its own scope of applicability and limitations in implementation. A method was chosen which uses an artificial scalar potential to destroy a magnetic charge. The ideas of the method are based on the following assumptions. After a certain number of steps in time, a magnetic field is obtained such that $\text{div}(\vec{B}) \neq 0$. One can introduce a function $\varphi = \text{div}(\vec{B})$ satisfying the Poisson equation

$$\nabla^2 \varphi = \nabla \cdot (\vec{B}).$$

The new value of the magnetic field strength vector $\vec{B}'$ is calculated through the found value of the magnetic induction vector $\vec{B}$ taking into account the scalar potential, using the following formula

$$\vec{B}' = \vec{B} - \nabla \varphi.$$  

After correction of the magnetic field vector $\vec{B}'$, it is necessary to correct the specific total energy

$$E' = E + \frac{(\vec{B}')^2 - \vec{B}^2}{8\pi \rho},$$

where $E$ $-$ previously calculated specific total energy.

The comparison was conducted with the FLASH reference code [7, 12] recognized worldwide for solutions of magnetic gas dynamics. For a correct comparison with Focus, in FLASH a second-order scheme in space and time MUSCL-Hancock was selected, the Riemann solver is HLL, and the limiter is vanLeer. A numerical comparison was made for the three test problems [12]: one-dimensional Brio-Wu, two- and three-dimensional Orszag-Tang. Numerical calculations show that the approximation of ideal magnetic gas dynamics in the Focus code is implemented correctly.

Formulas for taking into account spontaneous magnetic fields in a laser plasma [13] are derived from the generalized Ohm’s law (9) and Maxwell’s equations (10) - (11):

$$\dot{E} = -\frac{\vec{E} \times \vec{B}}{c} + \frac{j \times \vec{B}}{\text{cen}_e} - \frac{\nabla P_e}{\text{en}_e} + \frac{\nabla \cdot \pi_{\text{eff}}}{\text{en}_e} - \frac{R_f}{\text{en}_e} + \frac{\sigma}{
abla \cdot \vec{E}},$$

$$j = \frac{c}{4\pi} \nabla \times \vec{B} - \frac{1}{4\pi} \frac{\vec{E}}{\dot{\vec{t}}},$$

$$\frac{\partial \vec{B}}{\partial t} = -c \cdot \nabla \times \vec{E},$$

$$\frac{\partial \vec{E}}{\partial t} = \frac{\vec{E} \times \vec{B}}{c} + \frac{j \times \vec{B}}{\text{cen}_e} - \frac{\nabla P_e}{\text{en}_e} - \frac{R_f}{\text{en}_e} + \frac{\sigma}{\nabla \cdot \vec{E}}.$$
where \( m_e, e, n_e, P_e \) — mass, charge, concentration and pressure of electrons; \( \vec{J} \) — current density; \( c \) — velocity of light; \( R_t, R_f \) — thermal force and frictional force; \( \vec{E} \) and \( \vec{B} \) — electric and magnetic field; \( \sigma \) — electrical conductivity as a function of plasma density and temperature; \( P_L \) — laser radiation pressure; \( \pi_{\alpha\beta} \) — stress tensor of viscosity; \( \vec{v} = \vec{v} - \frac{\vec{J}}{en_e} \) — velocity of the electrons; \( \vec{v} \) — flow velocity.

Suppose that the displacement current determined by the term \( \frac{\partial \vec{E}}{\partial t} \) can be neglected. The substitution \( \vec{J} \) from equation (10) into equation (9) and \( \vec{E} \) from equation (9) into equation (11) yields

\[
\frac{\partial \vec{B}}{\partial t} = \nabla \times (\vec{v} \times \vec{B}) - \frac{c^2}{4\pi} \nabla \times \left( \frac{1}{\sigma} \nabla \times \vec{B} \right) + \frac{c}{e} \nabla \times \left( \frac{R_t + R_f}{n_e} \right) + \frac{c}{e} \nabla \times \left( \frac{1}{n_e} \left[ \nabla P_e + \nabla P_L + \nabla \cdot \pi_{\alpha\beta} - \frac{1}{4\pi} (\nabla \times \vec{B}) \times \vec{B} \right] \right) \tag{12}
\]

Allowance for the frictional force is important only in a superdense and highly nonequilibrium plasma, so if we neglect the influence of the frictional force, the error will be insignificant. The same assumption will also be true for the thermal force \( R_t \). The pressure of the laser radiation \( P_L \) and the viscous stress tensor \( \pi_{\alpha\beta} \) are much smaller than the electron pressure \( P_e \) in the laser plasma under the same irradiation conditions, and therefore they can be neglected. It is known that the gas-dynamic pressure dominates the electron pressure, and the effect of the magnetic field on its source and on the motion of the plasma can be neglected. The diffusion of the magnetic field in this paper is not considered. As a result, the magnetic induction equation takes the following form:

\[
\frac{\partial \vec{B}}{\partial t} = \nabla \times (\vec{v} \times \vec{B}) + \frac{c}{e} \nabla \times \left( \frac{\nabla P_e}{n_e} \right). \tag{13}
\]

According to the rules of action with the operator \( \nabla \) [8]

\[
\nabla \times \left( \frac{\nabla P_e}{n_e} \right) = \nabla P_e \times \nabla n_e. \tag{14}
\]

From the quasi-neutrality condition of the medium it follows that \( n_e = \langle z \rangle \cdot n_\rho, \rho = A \cdot m_p \cdot n_\rho \), then

\[
n_e = A \frac{\langle z \rangle}{\rho} \cdot \rho. \tag{15}
\]

In the Focus code, the three-temperature approximation has not yet been realized, therefore \( P_e = P \). Accordingly, the magnetic induction equation takes the form:

\[
\frac{\partial \vec{B}}{\partial t} = \nabla \times (\vec{v} \times \vec{B}) + \frac{cA m_p}{e \langle z \rangle} \frac{(\nabla P \times \nabla \rho)}{\rho^2} \cdot \vec{B}. \tag{16}
\]

In order to take into account the effect of the magnetic field on the energy change, the equation of magnetic induction multiplied by \( \vec{B}/4\pi \) is added to the equation for the change in the total gas-dynamic energy. As a result, we obtain:

\[
\frac{\partial}{\partial t} (\rho E) = - \nabla \cdot \left( (\rho E + P^*) \cdot \vec{v} \right) + \nabla \cdot \left( \frac{\langle \vec{v} \cdot \vec{B} \rangle}{4\pi} \cdot \vec{B} \right) + \frac{cA m_p}{4\pi e \langle z \rangle} \frac{\vec{B} \cdot (\nabla P \times \nabla \rho)}{\rho^2}. \tag{17}
\]

In the Focus code, the gradient of the scalar quantity is calculated by the Gauss gradient theorem [8].
3. Testing of spontaneous magnetic fields

To test the numerical implementation of accounting for spontaneous magnetic fields (SMF), the authors formulated two test problems having an analytical solution. The basic quantities in the statement of the problem were taken as characteristic values for problems on the compression of a thermonuclear target by a laser. Namely, a substance was considered - a monoatomic fully ionized deuterium, i.e. A=1 and \( z \)=1, the specific heat at constant volume \( c_v=1.2471708 \times 10^8 \ erg \cdot g^{-1} \cdot K^{-1} \), the EOS of an ideal gas \( p=(\gamma-1)\rho e \), \( e=c_v T \), \( \gamma=5/3 \), background temperature \( T_0=300 \ K \), background density \( \rho_0=10^{-6} \ g \cdot cm^{-3} \), background pressure \( p_0=(\gamma-1)\cdot(\rho_0+c_0)\cdot c_0 T_0 \). Problems are considered in the absence of motion conditions – only changes in magnetic induction and total energy due to the appearance of spontaneous magnetic fields. The calculations were compared at time \( t=10^{-14} \ s \), as a characteristic step in time in the problem of compression. The initial data of the test tasks are given in the CGS:

1. Test SMF 2D – computational domain \( (x,y)\in[0;1]^2 \),
   \[
   \rho = \rho_0 + \rho_1 \cdot \exp \left( -K_\rho \cdot \left( -x - x_\rho \right)^2 + (y - y_\rho)^2 \right), \quad \rho_1 = 100 \ g \cdot cm^{-3}, \quad K_\rho = 250, \quad x_\rho = y_\rho = 0.5 \ cm,
   \]
   \[
   p = p_0 + p_1 \cdot \exp \left( -K_\rho \cdot \left( -x - x_\rho \right)^2 \right), \quad p_1 = 10^{13} \ g \cdot cm^{-1} \cdot s^{-2}, \quad K_\rho = 10^4, \quad x_\rho = 0.35 \ cm ;
   \]

2. Test SMF 3D – computational domain \( (x,y)\in[0;1]^2 \),
   \[
   \rho = \rho_0 + \rho_1 \cdot \exp \left( -K_\rho \cdot \left( -x - x_\rho \right)^2 + (y - y_\rho)^2 + (z - z_\rho)^2 \right), \quad \rho_1 = 100 \ g \cdot cm^{-3}, \quad K_\rho = 250,
   \]
   \[
   x_\rho = y_\rho = z_\rho = 0.5 \ cm, \quad p = p_0 + p_1 \cdot \exp \left( -K_\rho \cdot \left( -x - x_\rho \right)^2 + (y - y_\rho)^2 + (z - z_\rho)^2 \right), \quad p_1 = 10^{13} \ g \cdot cm^{-1} \cdot s^{-2}, \quad K_\rho = 5, \quad x_\rho = y_\rho = z_\rho = 0 \ cm.
   \]

In the two-dimensional problem \( \nabla \rho = (\rho_x', \rho_y'; 0) \), \( \nabla p = (p_x'; 0; 0) \), then \( \nabla p \times \nabla \rho = (0; 0; p_x' \cdot \rho_y') \), where, respectively, \( p_x' = (p - p_0) \cdot (-2K_\rho \cdot (x - x_\rho)) \) and \( \rho_y' = (\rho - \rho_0) \cdot (-2K_\rho \cdot (y - y_\rho)) \). In the three-dimensional problem, the density and pressure gradients will have all three components, respectively, the changes due to spontaneous magnetic fields in the magnetic induction vector will occur in all three components. In both problems, the maximum density \( \rho_{max} = \rho_0 + \rho_1 \) and maximum pressure \( p_{max} = p_0 + p_1 \) are obtained.

The calculations for the two-dimensional problem were carried out on a uniform grid of 50, 100, 200, 400, 800 and 1600 cells in each direction with a fixed Courant number of 0.125 and a vanLeer tilt limiter. Figure 1 shows the gas-dynamic and magnetic pressures on a grid of 800x800 cells. It can be seen that at time \( 10^{-14} \ s \) the magnetic pressure is 6 orders of magnitude smaller than the gas pressure, which indicates that after \( 10^6 \) such steps, the magnetic pressure will become comparable in size to the gas-dynamic pressure, and by the time \( 10^{-6} \) it will fully influence the flow. Figure 2 simultaneously shows the values of the density, gas-dynamic and magnetic pressures, respectively, you can see the place of initiation of the magnetic field.
Figure 1. Test SMF 2D. The gas-dynamic (a) and magnetic (b) pressures.

Figure 2. Test SMF 2D. The density, gas-dynamic and magnetic pressures.

Calculations for the three-dimensional problem were carried out on a uniform grid of 25, 50, 100, 200 and 400 cells in each direction for a fixed Courant number of 0.125 and a vanLeer tilt limiter. Figure 3 shows the gas-dynamic and magnetic pressures on a grid of 800x800 cells along a slice $z = 0.5$. Considering that in a three-dimensional setting the pressure is set more smoothly than in a two-dimensional one, i.e. the pressure gradient will have values about 3 orders of magnitude smaller, respectively, the magnitude of the magnetic pressure in this formulation is much smaller. Figure 4 shows simultaneously the values of density, gas-dynamic and magnetic pressures, with an analytical solution on the grid of 800x800 cells along the slice $z = 0.5$ on the left and a Focus solution on the 200x200x200 grid on the right. In a two-dimensional section, the magnetic pressure looks like a crescent, and in a three-dimensional image it is seen that the magnetic pressure has the shape of a cup. Also in Figure 5, you can see that this three-dimensional cup has a hole on the bottom, which is explained by the collinearity of pressure gradients and density in this place. Also in Figure 5 on the right, one can see on the cut along $z = 0.5$ 3D image of the magnetic field a crescent is obtained, similar to the analytical one.
Figure 3. Test SMF 3D. The gas-dynamic (a) and magnetic (b) pressures.

Figure 4. Test SMF 3D. The density, gas-dynamic and magnetic pressures. (a) – analytical solution, slice $z = 0.5$; (b) – the Focus code on a grid of 200x200x200 cells.

Figure 5. Test SMF 3D. The magnetic pressure – the Focus code on a grid of 200x200x200 cells. (b) – slice $z = 0.5$. 
As a criterion of accuracy, the relative error of the numerical solution from the standard one in the norm $L_1$ is calculated by

$$
\|u_h - u_0\|_{L_1} = \sum_i |u_h - u_0| \cdot V_i / \sum_i |u_h| \cdot V_i,
$$

where $u_0$ – exact solution in the $i$ cell; $u_h$ – numerical solution in the $i$ cell; $i$ - cell number; $V_i$ - cell volume. For grids obtained by successively doubling the number of cells in each direction, the order of convergence was calculated from the following formula

$$
\alpha_k = \log \left( \frac{\|u_{k+h} - u_0\|}{\|u_{k+2h} - u_0\|} \right).
$$

For both problems, the norm $L_1$ of the difference between the solution obtained by the Focus code and the analytical solution was calculated. Convergence was investigated for the magnitude of the pressure and density gradients, as well as the magnitude of the magnetic pressure. For a two-dimensional test, calculations were carried out on grids from 50 to 1600 cells obtained by successive doubling, for a three-dimensional problem, from 25 to 400. The comparison was made with an analytical solution on an appropriate grid. Table 1 gives the order of convergence in magnitude of the magnetic pressure of the Focus solution to the analytical one in the norm $L_1$ for two-dimensional and three-dimensional problems. Figure 6 shows the convergence graphs of the solutions obtained in the norm $L_1$; on the left for the two-dimensional problem, on the right for the three-dimensional problem.

Based on the presented table and graphs, the second order of convergence for the presented problems is obtained.

**Table 1.** The order of convergence in magnitude of the magnetic pressure of the Focus solution to the analytical one in the norm $L_1$ in tests SMF 2D and 3D.

| Grids $h_i$ and $h_{i+1}$ | 2D  | 3D  |
|---------------------------|-----|-----|
| 25 and 50                 | –   | 4.6116 |
| 50 and 100                | 5.6968 | 2.7299 |
| 100 and 200               | 1.9324 | 2.1812 |
| 200 and 400               | 1.9539 | 2.0439 |
| 400 and 800               | 1.9579 | –   |
| 800 and 1600              | 1.9897 | –   |

**Figure 6.** The convergence in the norm $L_1$: (a) – test SMF 2D, (b) – test SMF 3D.
Test problems are proposed with an analytical solution for verifying the correctness of numerical realization of accounting for spontaneous magnetic fields. A second order of approximation is obtained for calculating the gradient of smooth functions by the Gauss gradient theorem. The convergence of the solution of the problem to an analytic solution on refined grids is shown. The Focus 3D code can be used to calculate the problems of interaction of laser radiation with matter in a plasma that is formed taking into account spontaneous magnetic fields.

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