A Preliminary Lattice Study of the Glue in the Nucleon

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About half the mass of a hadron is given from gluonic contributions. In this talk we calculate the chromo-electric and chromo-magnetic components of the nucleon mass. These computations are numerically difficult due to gluon field ultra-violet fluctuations. Nevertheless a high statistics feasibility run using quenched Wilson fermions seems to show reasonable signals.

1. INTRODUCTION

One of the earliest experimental indications that the nucleon consists not only of three quarks, but also has a gluonic contribution came from the measurement of the fraction of the nucleon momentum carried by the quarks. That this did not sum up to 1 as is required from the energy–momentum sum rule gave evidence for the existence of the gluon. Denoting $\langle x \rangle (f)$ as the fraction of the nucleon momentum carried by parton $f$ we have

$$\sum_{q} \langle x \rangle^{(q)} + \langle x \rangle^{(g)} = 1.$$  \hspace{1cm} (1)

Experimentally $\langle x \rangle^{(u+d)} \approx 0.4$ so the missing component is large, at least 50% of the total nucleon momentum.

We have previously estimated the quark contribution from a lattice calculation (at least for the valence part in the quenched approximation) \cite{1}. In this contribution we shall consider $\langle x \rangle^{(g)}$.\textsuperscript{3}

Analogously to $\langle x \rangle^{(q)}$ we have, with $\mathcal{M}$ denoting Minkowski space and averaging over the polarisations,

$$\langle N, p | O^{\mathcal{M}(g)_{\mu_1 \mu_2}} - \frac{1}{2} \eta^{\mu_1 \mu_2} O^{\mathcal{M}(g)_{\alpha \alpha}} | N, p \rangle = 2 \langle x \rangle^{(g)} \left[ p^{\mu_1} p^{\mu_2} - \frac{1}{4} \eta^{\mu_1 \mu_2} m_N^2 \right].$$  \hspace{1cm} (2)

with $O^{\mathcal{M}(g)_{\mu_1 \mu_2}} = -\text{Tr} F^{\mathcal{M}i}_{\mu_1 \alpha} F^{\mathcal{M}j}_{\mu_2 \alpha}$. (Higher moments can also be considered, by inserting covariant derivatives between the $F$’s, \cite{3}.)

2. THE LATTICE METHOD

We now turn to the lattice. The gluon operators are euclideanised\textsuperscript{3} and discretised in the usual way. For the field strength we choose the usual clover leaf form\textsuperscript{3} \cite{4}, which belongs to an irreducible representation of the 4-dimensional hyper-

\hspace{1cm} $2 E^M_{\mu_1} = F^{\mathcal{M}i}_{\mu_1} \rightarrow i F_{i4} \equiv i E_i$ and $B^M_{\mu_1} = -\frac{1}{4} e_{ijk} F^M_{jk} \rightarrow \frac{1}{4} e_{ijk} F_{jk} \equiv B_i$.\textsuperscript{3}

\textsuperscript{3}Note that we, like most workers in the field, do not subtract the trace of the clover term, to make $F^{\text{lat}}$ traceless in the colour fields. This is an $O(a^2)$ operator and so does not contribute to the continuum identification of the clover term with $F$.
pergroup $H(4)$. Defining $O_{\mu\nu} = -\text{Tr} F^{\mu\alpha}_{\phantom{\mu\alpha}F^{\nu\alpha}}$ this then gives with the two obvious operator choices

$$O_a = O_{i4} = \text{Tr}(\vec{E} \times \vec{B})_i,$$

$$O_b = O_{44} - 4O_{jj} = \frac{2}{4} \text{Tr}(-\vec{E}^2 + \vec{B}^2),$$

(3)

($O_a^M \rightarrow iO_a$ and $O_b^M \rightarrow O_b$). Both choices have their problems: Operator (a) needs a non-zero momentum $p_i$, while operator (b) requires a delicate subtraction between two terms similar in magnitude. The relation to $\langle x \rangle(g)$ is given by

$$\langle N, \vec{p} | O_{Ra} | N, \vec{p} \rangle = -2iE_N p_i \langle x \rangle(g),$$

$$\langle N, \vec{0} | O_{Rb} | N, \vec{0} \rangle = 2m_N^2 \langle x \rangle(g),$$

(4)

with $O_R$ denoting the renormalised operator.

We shall not dwell here on details of the numerical calculations, but just mention that the method we use to extract matrix elements from the lattice is standard, namely evaluating the ratio of 3-point to 2-point nucleon correlation functions. The 3-point function is easy to calculate: we simply multiply the 2-point function with the appropriate gluon operator for every configuration. This sits at time $\tau$ between the baryon–anti-baryon (placed at $t$ and 0 respectively). $\tau$ and $t - \tau$ are (hopefully) large enough, $\geq d_m$ say, so that all the excited nucleon states have died out. We found it expedient to sum over all allowed values of $\tau$. Thus we consider

$$R(t) = \frac{\langle B(t) \frac{1}{t - 2d_m + 1} \sum_{\tau = d_m}^{t - d_m} O(\tau) \overline{B}(0) \rangle}{\langle B(t) \overline{B}(0) \rangle},$$

$$= \frac{2}{2E_N} \langle N, \vec{p} | O | N, \vec{p} \rangle. \quad (5)$$

We work with quenched Wilson fermions at $\beta = 6.0$ and $\kappa = 0.1515, 0.1530, 0.1550$ on a $16^3 \times 32$ lattice with antiperiodic time boundary conditions for the fermion. We have generated O(5000) sources (on 3000–3500 configurations with Jacobi smeared source/sink).

3. THE RAW DATA

We shall first consider $O_{44} = -\text{Tr}\vec{E}^2$. In Fig. 1 we show $R(t)$ for this operator. We fixed $d_m = 4$ and hope to see a signal after 7 (before this there is not enough time to insert the operator) and about 17 (after this the nucleon mixes with its

Figure 1. $R(t)$ for the vacuum subtracted operator $-\text{Tr}\vec{E}^2$. The vertical dotted lines denote the fit interval, the fit value being given by the horizontal dotted line.

Figure 2. $R(t)$ for the vacuum subtracted operator $\text{Tr}\vec{B}^2$. As expected there is a large cancellation between the chromo-electric and -magnetic pieces. While the error bars are uncomfortably large, there does seem to be a signal. This is worse using $O_a$; indeed the best we can say is that it is not inconsistent with $O_b$. 

parity partner). Indeed a signal is seen. A similar picture holds for $\frac{1}{2}(O_{44} - O_{jj}) = \text{Tr}\vec{B}^2$ as shown in Fig. 2. Considering $O_b$ directly is given
4. RENORMALISATION

As gluon operators are singlets, they can mix with the quark singlet. But there exists a combination of singlet operators with vanishing anomalous dimension. (This is due to the conservation of the energy-momentum tensor.) We may estimate the renormalisation factor $Z_b$:

- Using first order perturbation theory. We find $Z_b = 1 + a_1 g^2 + \ldots$, $a_1 = 0.220$. We shall use this in Fig. 4.

- Non-perturbatively. In [7] a proposal was made to determine $Z$ from MC simulations. We have looked at $\langle AOA \rangle$ for about 100 gauge fixed configurations. Huge noise is present but $Z_b$ is consistent with 1.

5. RESULTS AND DISCUSSION

Extracting $\langle x \rangle^{(g)}_b$ from $R$, we may now attempt a chiral extrapolation. This is shown in Fig. 4. While the quality of the fit is not so good, the result is at least encouraging $\approx 0.53 \pm 0.23$. This is at least not inconsistent with the expectations previously discussed. Of course our ultimate aim is to attempt a mass splitting of the nucleon, in the same spirit as [3]. This seems a difficult goal with our present method: probably a two order of magnitude improvement in statistics is required.

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