Density Spectrums from Kinetic Inflations

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The pole-like accelerated expansion stages purely driven by the coupling between the gravity and the dilaton field without referring to the potential term. We consider three such scenarios based on the scalar-tensor gravity, the induced gravity and the string theory. Quantum fluctuations during the expansion stages (including more general situations) can be derived in exact analytic forms. Assuming that the pole-like acceleration stage provides a viable inflation scenario in the early universe we derive the generated classical density spectrums. The generated classical density field shows a generic tilted spectrum with $n \simeq 4$ which differs from the observed spectrum supporting $n \simeq 1$.

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I. INTRODUCTION

A pole-like accelerated expansion stage can be realized by a nontrivial coupling between the dilaton field and the gravity without referring to the potential term. These types of accelerated expansion can be considered as the potential candidates for inflation models [1]. One scenario based on the induced gravity is proposed by the authors of [2]. Recently, the string theory has motivated an action which is also known to allow a similar acceleration stage [3]. The similar inflation scenario in the context of the scalar-tensor gravity is proposed in [4] and is termed as the kinetic inflation. We adopt the name “kinetic inflation” for the pole-like acceleration stages realized in various generalized gravity without the potential term. We know that all these theories are related with each other by simple field redefinitions or conformal transformations. Although the acceleration stages can be realized, the specific realization of inflation model which resolves the conventional cosmological flatness and horizon problems with a successful graceful exit is yet to be made, and is currently under active investigation [5,6].

During the expansion stage the quantum fluctuations in the dilaton field and the metric may naturally arise from the vacuum expectation values of the fluctuating field and the metric. In this paper we will present the quantum fluctuations generated during the expansion stages in analytic forms for the scalar type perturbations. We will show that the generated power spectrums have the same spectral index independently of the specifics of considered gravity or the expansion stage.

Assuming that the acceleration stage provides a viable inflation stage in the early universe we can derive the generated spectrums of the large scale density field in the second horizon crossing epoch in the conventional era of cosmological evolution. In this part we will consider a scenario where a generalized gravity theory has the roles of the gravity theory in the early universe and Einstein’s theory takes over the role of gravity at some point. That is, the kinetic inflation is accepted as the acceleration stage while the observationally relevant scales leave the horizon scales. In such a case, the classical fluctuations in the large scale arise from quantum fluctuations of the metric and the scalar field during the kinetic polar inflation stage. Assuming that the fluctuations become superhorizon scale and are classicalized, it is known that the fluctuations freeze independently of the changes in the background equation of state and also changes in the underlying gravity sector. Such a freezing is conveniently characterized by a (temporally) conserved quantity which is usually represented by a curvature fluctuation in a certain gauge choice (spatial characters of the fluctuations are always conserved during the linear evolution stage independently of the horizon scale; in this sense, no structure formation [like self-organization] arises in the linear theory). The observationally relevant spectrums we will derive in the following will be valid as long as the transition of the gravity sector occurs while the observationally relevant scales stay in the superhorizon scale.

The generalized gravity theories to be considered in the following are simple subsets of the generalized gravity theories generally studied in [6]. Thus, we will present the structure generation and evolution processes in the generalized gravity theories by reducing the general results in [7]. Our study presented below will be based on the original frame of the generalized gravity theory without referring to the conformal transformation. Parallel analyses in the pre-big bang scenario based on the low energy effective action of the string theory are presented in [8].

In Sec. [9] we summarize the general and unified formulation for handling the quantum generation and clas-
sical evolution processes of the scalar type perturbations in the generalized gravity theories. In Sec. II applications are made to the expansion stages driven by the nontrivial coupling between the gravity and the dilaton field without potential terms. Cases include the scalar-tensor theory, the induced gravity, the string theory, and Einstein gravity with a minimally coupled scalar field. We present the power spectrums of the quantum fluctuations in analytic forms. Useful equations are deposited in the Appendix for a convenient reference. We show that the spectrums are valid for various types of expansion stages allowed by the potential-less assumption. We discuss the relations among results in different gravity theories. In Sec. III, for comparison, we briefly summarize the results for the ordinary inflation based on the field potential in a minimally coupled scalar field. In Sec. IV we derive the generated density spectrums in the second horizon crossing epoch. We assumed that the underlying gravity has swiched from the generalized gravity to the Einstein one while the relevant perturbation scales were in the superhorizon size. Sec. V is a brief discussion. We set $c \equiv 1 \equiv 8\pi G$.

II. GENERAL FORMULATION

In Sec. II we have considered gravity theories represented by the following action

$$S = \int d^4x \sqrt{-g} \left[ \frac{1}{2} f(\phi, R) - \frac{1}{2} \omega(\phi) \phi^\alpha \phi_\alpha - V(\phi) \right].$$

(1)

The gravitational field equation and the equation of motion are:

$$R_{ab} = \frac{1}{F} \left[ \omega \phi, a \phi, b + g_{ab} \left( V + \frac{RF - f}{2} \right) \right],$$

$$\phi + \frac{1}{2\omega} (\omega, \phi, \phi_\alpha + f, -2V, \phi) = 0,$$

(2)

where $F \equiv \partial f / \partial R$.

We consider a homogeneous, isotropic and flat cosmological model with the general scalar type perturbations

$$ds^2 = -(1 + 2\alpha) dt^2 - a_\chi(1 + 2\varphi) dx^\alpha dx^\beta,$$

where $a(x, t), \chi(x, t)$ and $\varphi(x, t)$ are the scalar type metric perturbations. Without losing the generality, we have taken a spatial gauge choice which (in combination with the temporal gauge condition left for our freedom to choose) will fix the spatial gauge mode completely. We consider perturbations in the scalar (or dilaton) field as

$$\phi(x, t) = \bar{\phi}(t) + \delta\phi(x, t),$$

(4)

where a background quantity is indicated by an overbar which will be neglected unless necessary. Equations for the background are:

$$H^2 = \frac{1}{3F} \left( \frac{\omega}{2} \ddot{\phi}^2 + V + \frac{RF - f}{2} - 3H \dot{F} \right),$$

$$\ddot{\phi} + 3H \dot{\phi} + \frac{1}{2\omega} \left( \omega, \phi, \dot{\phi}^2 - f, \phi + 2V, \phi \right) = 0,$$

(5)

where $H \equiv \dot{a}/a$. The general formulation for handling the structure formation processes is presented in unified way in Sec. IV. In the following we briefly summarize the formulation which will be used in later sections.

Using a gauge invariant combination

$$\delta\phi_\varphi \equiv \delta\phi - \frac{\dot{\phi}}{H} \varphi_\delta \phi \equiv -\frac{\dot{\phi}}{H} \varphi_\delta \phi,$$

(6)

the action valid to the second order in the scalar type perturbation becomes (for derivation, see [9])

$$\delta S = \frac{1}{2} \int a^3Z \left[ \delta\phi_\varphi^2 - \frac{1}{a^2} \delta\phi_\varphi | \alpha \delta\phi_\varphi, \alpha \right]$$

$$+ \frac{1}{a^3Z} \frac{H}{\phi} a^3Z \left[ \frac{\dot{\phi}}{H} \right] \delta\phi_\varphi^2 dtd^3x.$$  

(7)

$$\delta\phi_\varphi$$ is a gauge invariant combination which is $\delta\phi$ in the uniform-curvature gauge ($\varphi \equiv 0$). The non-Einstein nature of the theory is present in a parameter $Z$ which is defined as

$$Z(t) \equiv \frac{\omega + \frac{3\dot{H}^2}{2\rho_\varphi}}{\left( 1 + \frac{3\dot{H}}{2\rho_\varphi} \right)^2}.$$  

(8)

The equation of motion of $\delta\phi_\varphi$ becomes

$$\delta\phi_\varphi + \left( \frac{a^3Z}{a^3Z} \right) \delta\phi_\varphi$$

$$- \left\{ \frac{1}{a^2} \nabla^2 + \frac{1}{a^3Z} \frac{H}{\phi} \left[ a^3Z \left( \frac{\dot{\phi}}{H} \right) \right] \right\} \delta\phi_\varphi = 0.$$  

(9)

In terms of $\varphi_{\delta\phi}$ we have

$$\frac{1}{a^3Z} \frac{H^2}{\dot{\phi}^2} \left( a^3Z \frac{\dot{\phi}^2}{H^2} \varphi_{\delta\phi} \right) - \frac{1}{a^2} \nabla^2 \varphi_{\delta\phi} = 0.$$  

(10)

The large and small scale asymptotic solutions are:

$$\delta\phi_\varphi(x, t) = -\frac{\phi}{H} \left[ C(x) - D(x) \int_0^t \frac{H^2}{a^3Z} dt \right],$$

(11)

$$\delta\phi_\varphi(k, \eta) = \frac{1}{a\sqrt{2k}} \left[ c_1(k) e^{i\kappa \eta} + c_2(k) e^{-i\kappa \eta} \right] \frac{1}{\sqrt{Z}},$$

(12)

where $C(x)$ and $D(x)$ are integration constants of the growing and decaying mode, respectively. $D(x)$ term is
higher order in the large scale expansion compared with the solutions in the other gauges; see \cite{[17]. At this point $c_1(k)$ and $c_2(k)$ are arbitrary integration constants.

Equation (\ref{eq:6}) can be written as

$$v'' = \left(\frac{z''}{z} + \nabla^2\right)v = 0,$$

$$v = \sqrt{Za\delta\phi_c}, \quad z = \sqrt{Za\delta\phi_c}/H.$$  \hspace{1cm} (13)

The similar form in Einstein gravity and some types of generalized gravity can be found in \cite{[16]. When we have $z''/z = n/\eta^2$ with $n = \text{constant}$, Eq. (\ref{eq:6}) leads to a solution

$$\delta\phi_c(\eta) = \frac{\sqrt{\pi|\eta|}}{2a} \left[c_1(k)H^{(1)}_\nu (k|\eta|) + c_2(k)H^{(2)}_\nu (k|\eta|)\right] \frac{1}{\sqrt{Z}}, \quad \nu \equiv \sqrt{n + \frac{1}{4}}.$$  \hspace{1cm} (14)

Considering $\delta\phi_c(\eta)$ as a mode function of $\delta\phi(x, t)$ which is regarded as a quantum Heisenberg operator, the canonical quantization condition leads to the following normalization condition

$$|c_2(k)|^2 - |c_1(k)|^2 = 1.$$  \hspace{1cm} (15)

The coefficients $c_i(k)$'s in Eqs. (\ref{eq:14} \ref{eq:15} \ref{eq:16}) have phase differences.] The power spectrum based on the vacuum expectation value is

$$P^{1/2}_{\delta\phi}(k, \eta) = \sqrt{k^3/2\pi^2} |\delta\phi_c(k)|.$$  \hspace{1cm} (16)

The corresponding power spectrum of $\delta\phi$ follows from Eq. (\ref{eq:6}) as

$$P^{1/2}_{\delta\phi}(k, t) = \frac{H}{\phi} P^{1/2}_{\delta\phi}(k, t).$$  \hspace{1cm} (17)

Using parameters

$$\epsilon_1 = \frac{H}{H^2}, \quad \epsilon_2 = \frac{\dot{\phi}}{H\phi}, \quad \epsilon_3 = \frac{1}{2} \frac{\dot{\phi}}{HF}, \quad \epsilon_4 = \frac{1}{2} \frac{\dot{E}}{HE},

E \equiv F \left(\omega + \frac{3\dot{F}^2}{2\phi^2 F}\right),$$  \hspace{1cm} (18)

for $\dot{\epsilon}_i = 0$ we have [for general expressions see Eq. (88) of \cite{[15]}]

$$n = \frac{(1 - \epsilon_1 + \epsilon_2 - \epsilon_3 + \epsilon_4)(2 + \epsilon_2 - \epsilon_3 + \epsilon_4)}{(1 + \epsilon_1)^2}.$$  \hspace{1cm} (19)

For $\dot{\epsilon}_1 = 0$ we have

$$\eta = -\frac{1}{aH} \frac{1}{1 + \epsilon_1}.$$  \hspace{1cm} (20)

III. APPLICATIONS TO KINETIC INFLATIONS

A. Scalar-tensor Theory

A scalar-tensor theory is given by an action

$$S = \int d^4x\sqrt{-g} \left[\phi R - \omega(\phi)\phi^a\phi_a - V(\phi)\right],$$  \hspace{1cm} (21)

which is a case of Eq. (\ref{eq:6}) with

$$f = 2\phi R, \quad F \equiv \frac{df}{dR} = 2\phi, \quad \omega \rightarrow 2\omega(\phi).$$  \hspace{1cm} (22)

Equation (\ref{eq:6}) becomes:

$$R_{ab} = -H \frac{\phi}{\phi} + \omega \phi^2 \frac{\phi_{ab}}{\phi} + \frac{1}{2} (V + \Box \phi) g_{ab}.$$  \hspace{1cm} (23)

Equation (\ref{eq:6}) becomes:

$$H^2 = -\frac{\phi}{\phi} + \frac{\omega}{6} \phi^2 + \frac{V}{6\phi}, \quad \frac{\phi}{2\phi} + 3H \frac{\phi}{\phi} + \frac{1}{2\omega + 3} (\omega, \phi^2 + \phi V, \phi - 2V) = 0.$$  \hspace{1cm} (24)

Ignoring the potential term, Eq. (\ref{eq:6}) leads to the following solution (\ref{eq:6}):

$$\dot{\phi} = C_1 \frac{1}{a^3 \sqrt{1 + \frac{2}{3} \omega}}, \quad \phi = \pm C_1 \frac{1}{a^3} \int \frac{dt}{a}, \quad H = \frac{\dot{\phi}}{2\phi} \left(-1 \pm \sqrt{1 + \frac{2}{3} \omega}\right).$$  \hspace{1cm} (25)

If we additionally have $\omega = \text{constant}$ we can show:

$$a \propto |t_0 - t|^{-q}, \quad \phi \propto |t_0 - t|^{1 + 3q}, \quad q = -\frac{1 + \omega \mp \sqrt{1 + \frac{2}{3} \omega}}{4 + 3\omega}.$$  \hspace{1cm} (26)

A pole-like acceleration stage can be realized when $q > 0$ which corresponds to the upper sign and $t_0 > t$. In the following analyses for generality we will consider both signs.

From Eq. (\ref{eq:6}) we have

$$\epsilon_1 = \frac{1}{q}, \quad \epsilon_2 = -3, \quad \epsilon_3 = -\frac{1 + 3q}{2q}, \quad \epsilon_4 = 0.$$  \hspace{1cm} (27)

Thus, from Eqs. (\ref{eq:14} \ref{eq:19}) we have $n = -\frac{1}{4}$ and

$$\varphi_{\delta\phi}(\eta) \equiv \frac{1}{4} \sqrt{\frac{\pi}{3}} \left(\sqrt{\frac{|\eta|}{a^2 \phi}}\right)^{1/4} \times \left[c_1(k)H^{(1)}(k|\eta|) + c_2(k)H^{(2)}(k|\eta|)\right].$$  \hspace{1cm} (28)
The power-spectrum of fluctuating quantum field based on the vacuum expectation value is presented in Eq. (16) for the metric fluctuations (coupled with the dilaton field) \( \phi_{3\phi} \). The corresponding power spectra valid in general scales can be derived from Eqs. (16,28). In the large scale limit the power spectrum becomes:

\[
P_{\phi_{3\phi}}(k, \eta) = \frac{1}{\sqrt[3]{\sqrt{3}}} \left( \frac{|\eta|}{a^2\phi} \right)^{3/2} \ln (k|\eta|) \times |c_2(k) - c_1(k)|.
\]  

(29)

**B. Induced Gravity Theory**

The induced gravity theory is given by

\[
S = \int d^4x \sqrt{-g} \left[ \frac{1}{2} \xi \phi^2 R - \frac{1}{2} \phi^a \phi_a - V(\phi) \right],
\]

which is a case of Eq. (1) with

\[
f = \xi \phi^2, \quad F = \xi \phi^2, \quad \omega = 1.
\]

(31)

Equation (5) becomes:

\[
R_{ab} = \frac{1}{\xi \phi^2} \left[ \phi_a \phi_b + g_{ab} V + \xi (\phi^2)_{a;b} + \frac{1}{2} \xi g_{ab} \Box \phi^2 \right],
\]

(32)

Equation (6) becomes:

\[
\begin{align*}
H^2 + 2H \dot{\phi} &- \frac{1}{6 \xi} \ddot{\phi}^2 = \frac{V}{3 \xi \phi^2}, \\
\ddot{\phi} + 3H \dot{\phi} + \frac{\ddot{\phi}^2}{\phi} &+ \frac{1}{1 + 6 \xi} \left( \frac{4}{V} - V, \phi \right).
\end{align*}
\]

(33)

Assuming \( V = 0 \) Eq. (33) leads to the following solution

\[
a \propto |t_0 - t|^{-q}, \quad \phi \propto |t_0 - t|^{-1 + \frac{3q}{2q} + \frac{1}{6q}},
\]

(34)

Cases with \( q > 0 \), thus the upper sign, and \( t_0 > t \) include the pole-like acceleration stage. For generality, we will take both signs.

From Eq. (13) we have:

\[
\begin{align*}
\epsilon_1 &= \frac{1}{q}, \quad \epsilon_2 = \frac{1 - 3q}{2q}, \quad \epsilon_3 = \epsilon_4 = -\frac{1 + 3q}{2q}.
\end{align*}
\]

(35)

Thus, from Eqs. (9,14) we have \( n = -\frac{3}{2} \), and

\[
\begin{align*}
\delta \phi(t) &\propto \sqrt{\frac{\sqrt{|\eta|}}{a \phi}} \left[ c_1(k) H^{1(1)}_0(k|\eta|) + c_2(k) H^{1(2)}_0(k|\eta|) \right].
\end{align*}
\]

(36)

The power spectrum in general scale is given by Eq. (16). In the large scale limit we have:

\[
P_{\phi_{3\phi}}(k, \eta) = \frac{1}{\sqrt[3]{\sqrt{3}}} \left( \frac{|\eta|}{a^2 \phi} \right)^{3/2} \ln (k|\eta|) \times |c_2(k) - c_1(k)|.
\]

(37)

By the following changes the results in the scalar-tensor theory can be translated into the ones in the induced gravity

\[
\dot{\phi} \rightarrow \frac{1}{2} \xi \phi^2, \quad \omega \rightarrow \frac{1}{4 \xi}, \quad \delta \phi \rightarrow \xi \phi \delta \phi.
\]

(38)

**C. String Theory**

The low-energy effective action of string theory is given by

\[
S = \int d^4x \sqrt{-g} \frac{1}{2} e^{-\phi} \left[ R + \phi^a \phi_a - 2V(\phi) \right],
\]

which is a case of Eq. (1) with

\[
f = e^{-\phi} R, \quad \omega = -e^{-\phi}, \quad V \rightarrow e^{-\phi} V.
\]

(40)

Equation (6) becomes:

\[
R_{ab} = -\phi_{a;b} - g_{ab} V_{,\phi},
\]

\[
\Box \phi = \phi_{a;b} + 2 (V + V_{,\phi}).
\]

(41)

Equation (7) becomes:

\[
H^2 = H \dot{\phi} - \frac{\phi^2}{6} + \frac{1}{3} V,
\]

\[
\ddot{\phi} + 3H \dot{\phi} - \phi^2 + 2 (V + V_{,\phi}) = 0.
\]

(42)

For \( V = 0 \) Eq. (12) leads to the following solution

\[
a \propto |t_0 - t|^{-1/\sqrt{3}}, \quad e^\phi \propto |t_0 - t|^{-1 + \sqrt{3}}.
\]

(43)

The upper sign with \( t < t_0 \) represents a pole-like inflation stage which is called as a pre-big bang stage [3]. The corresponding studies of Sec. (11) for the pre-big bang scenario are presented in [3]. In the following we summarize the results considering both signs for generality.

From Eq. (13) we have

\[
\epsilon_1 = \epsilon_2 = \pm \sqrt{3}, \quad 2 \epsilon_3 = \epsilon_4 = -\left( 3 \pm \sqrt{3} \right).
\]

(44)

Thus, from Eqs. (9,14) we have \( n = -\frac{3}{4} \) and...
\[ \varphi_{\delta \phi} (\eta) = \frac{1}{2} \sqrt{\frac{\pi}{6}} \left( \sqrt{\frac{|\eta|}{a^2 e^{-\varphi}}} \right) \times \left[ c_1 (k) H^{(1)}_0 (k|\eta|) + c_2 (k) H^{(2)}_0 (k|\eta|) \right]. \] (45)

The power spectrum valid in general scales can be derived from Eq. (16). In the large scale limit we have

\[ P^{1/2}_{\delta \phi} (k, \eta) = \frac{\sqrt{2}}{3} \left( \sqrt{\frac{|\eta|}{a^2 e^{-\varphi}}} \right) \left( \frac{k}{2 \pi} \right)^{3/2} \ln (k|\eta|) \times c_2 (k) - c_1 (k). \] (46)

The power spectrum of the dilaton field in Eq. (46) is derived in \[3\]; authors in \[3\] derived a similar spectrum in the context of the conformally transformed Einstein frame.

By following changes the results in the scalar-tensor theory can be translated into the ones in the pre-big bang scenario

\[ \omega \to -1, \quad \phi \to \frac{1}{2} e^{-\varphi}, \quad V \to e^{-\varphi} V, \]
\[ \delta \phi \to -\frac{1}{2} e^{-\varphi} \delta \phi. \] (47)

D. Einstein Gravity with a Minimally Coupled Scalar Field

The minimally coupled scalar field is a case of Eq. (1) with \( f = R \) and \( \omega = 1 \). Equation (1) becomes:

\[ R_{ab} = 8 \pi G (\phi, a \phi, b + g_{ab} V), \quad \Box \phi - V, \phi = 0. \] (48)

Equation (1) becomes:

\[ H^2 = \frac{8 \pi G}{3} \left( \frac{\dot{\phi}^2}{2} + V \right), \quad \ddot{\phi} + 3H \dot{\phi} + V, \phi = 0. \] (49)

By identifying \( Z = F = \omega = 1 \) equations in Sec. I are valid for the minimally coupled scalar field.

For \( V = 0 \) Eq. (49) has the following solution

\[ a \propto |t_0 - t|^{1/3}, \quad \phi \propto \ln |t_0 - t|. \] (50)

This expansion law corresponds to the ultra-relativistic limit with an equation of state \( p = \mu \). Even in this case, driven by the pure kinetic term, from Eqs. (18,19) we have \( \epsilon_1 = \epsilon_2 = -3 \) and \( \epsilon_3 = \epsilon_4 = 0 \), thus \( n = \frac{3}{2} \). For the mode function, Eq. (14) remains valid with \( \nu = 0 \) and \( Z = 1 \). Thus, in the large scale limit, the power spectrum in Eq. (14) becomes

\[ P^{1/2}_{\delta \phi} (k, \eta) = 2 \left( \sqrt{\frac{|\eta|}{a}} \right) \left( \frac{k}{2 \pi} \right)^{3/2} \ln (k|\eta|) \times c_2 (k) - c_1 (k). \] (51)

E. Conventional Inflation with a Minimally Coupled Scalar Field

Studies in the case of the ordinary inflation based on a minimally coupled scalar field are thoroughly presented in \[13\]. compared with the previous works in \[14,15\] the results in \[13\] are based on the uniform-curvature gauge or the equivalent gauge invariant combinations, \( \delta \phi, \phi \). In the following, for comparison, we briefly rederive the results by reducing the general results in Sec. I. Equations (18,19) remain valid. The cases where the background scale factor follows an exponential or a power-law expansion in time correspond to \( \epsilon_i = 0 \). Thus, in both cases \( n \) becomes constant and the solution in Eq. (14) applies.

1. Exponential expansion: For \( a \propto e^{n \eta} \) with \( H = \text{constant} \), Eq. (19) has a solution with \( V = \text{constant} \) and \( \phi = 0 \). We have \( \epsilon_1 = 0 \), thus \( n = 2 \) and \( \nu = \frac{3}{2} \). From Eqs. (14,16) we have the power spectrum valid in general scales and for the general vacuum state. In the large scale limit we have

\[ P^{1/2}_{\delta \phi} (k, \eta) = \frac{H}{2 \pi} |c_2 (k) - c_1 (k)|. \] (52)

A choice of the adiabatic vacuum (known as the Bunch-Davies vacuum in de Sitter space) corresponds to \( c_2 (k) = 1 \) and \( c_1 (k) = 0 \). The spectrum in Eq. (52) with the adiabatic vacuum was found in \[14\].

2. Power-law expansion: For \( a \propto t^p \) with \( p = \text{constant} (> 1) \) Eq. (19) has a solution with \( \phi = \sqrt{2p}/t \) and \( V = p(3p - 1)/t^2 \propto e^{-\sqrt{2}/\phi} \), \[14\]. In this case we have \( \epsilon_1 = \epsilon_2 = -1/p \), thus \( \nu = \nu_0 = \frac{3(p-1)}{2(p-1)} \). The general power spectrum follows from Eqs. (14,16). In the large scale limit we have

\[ P^{1/2}_{\delta \phi} (k, \eta) = \frac{H^2}{\pi^{3/2}} \left( \frac{\nu - 1}{p} \right) \left( \frac{2}{k|\eta|} \right)^{\nu - 3/2} \times |c_2 (k) - c_1 (k)|. \] (53)

In the limit of \( p \to \infty \) Eq. (53) reproduces Eq. (52). The spectrum in Eq. (53) with an adiabatic vacuum was first rigorously derived in \[14\].

IV. DENSITY SPECTRUMS

In the previous section we have derived quantum fluctuations based on the vacuum expectation value of the fluctuating quantum fields of the scalar type perturbations which include the fluctuating dilaton field and the fluctuating metric. The derived mode functions and the power spectrums in Sec. III are valid for the general expansion stages considered in Eqs. (14,16) respectively. Notice that the expansion stages included in Eqs. (26,34,43) are not necessarily acceleratory. The reason why the power spectrums are insensitive to the specifics of the expansion law remains to be explained.
The expansion stages include the pole-like accelerated expansion stages. Although constructing the specific models for a successful inflation remains to be seen, in the following we will assume a scenario that the pole-like acceleration stages provide the inflation era in the early universe.

We consider an evolution scenario of the presently observable patch of the universe and the structures in it as follows. The first assumption concerns quantum generation processes: The observationally relevant structures in the present universe are supposed to exit the local horizon, thus becoming the superhorizon scale later on, during the kinetic inflation stage supported by one of the generalized gravity theories. This first assumption allows us to consider the quantum fluctuations based on the vacuum expectation value as possible natural seeds for the later evolution into the large scale structures. The second assumption concerns the classical evolution processes: the underlying gravity governing the dynamics of our patch of the universe is supposed to transit from one of the generalized gravity to Einstein gravity while the relevant scales we are considering were in the superhorizon scale. Since the evolution of linear structures in superhorizon scales is kinematic in nature, we can conveniently handle the evolution using the conserved quantities. For the scalar type perturbations the perturbed three space curvature variable $\varphi$ in the uniform-field gauge (equivalently, the comoving gauge in Einstein’s gravity), i.e. $\varphi_{\delta \phi}$, is the one which is temporally conserved. From Eqs. (52,53) ignoring the decaying mode which is higher order in the large scale limit, we have

$$\varphi_{\delta \phi}(x, t) = C(x).$$

We emphasize that Eq. (54) is valid considering genetic changes in $V(\phi)$, $\omega(\phi)$ and $f(\phi, R)$ in the generalized gravity $\Box$, and in the equation of state $p = \rho(\mu)$ in the fluid era $\Box^{[20]}$. This reflects the kinematic nature of the evolution in the superhorizon scale. Thus, this second assumption makes the handling of the classical evolution processes easy. Equation (54) remains valid virtually in all scales (more precisely, larger than Jeans scale) while the universe is in the matter dominated era, $\Box^{[20]}$. From the classical power spectrum of $C(x)$, we can derive the power spectrums for the rest of the scalar type perturbations, like, fluctuations in the density, potential, and velocity fields and also the directional fluctuations in the cosmic microwave background photons.

In the large scale limit the classical power spectrum of $C(x)$ based on the spatial averaging becomes

$$P_C^{1/2}(k) = P_{\varphi_{\delta \phi}}^{1/2}(k, \eta)$$

$$= P_{\varphi_{\delta \phi}}^{1/2}(k, \eta) \times \sqrt{Q(k)},$$

where in the first step we used Eqs. (52,53) neglecting the decaying mode, and in the second step we adopted an ansatz on matching the classical fluctuations with the quantum fluctuations. $Q(k)$ is a classicalization factor which may take into account of possible effects from the classicalization process $\Box$. From $P_C$ we have the power spectrums of the classical fields like the density, velocity, gravitational potential, scalar contribution to the temperature anisotropy of the CMBR, etc. In Eq. (53) the right hand side should be evaluated while the scale is in the large scale limit during the inflation era. The power spectrums of the quantum fluctuations, $P^{1/2}_{\varphi_{\delta \phi}}(k, \eta)$ during various kinetic inflation stages are derived Sec. III.

In the second horizon crossing epoch where the matter is dominated we have

$$P^{1/2}_{\delta}(k, t_{HC}) = \frac{2}{5} P^{1/2}_{C}(k) = \frac{(aH)^2}{\sqrt{2\pi}} k^{-1/2} |\delta_k(t)|.$$  \hspace{1cm} (56)

Conventionally we take $|\delta_k(t)|^2 \equiv A(t)k^n$ where $n$ is a spectral index. Ignoring the classicalization factor ($Q \equiv 1$), the vacuum dependence (thus, taking $c_2 = 1$ and $c_1 \equiv 0$), and the logarithmic dependence on $k$ we have a generically tilted spectrum with $n = 4$ for the scenarios considered in $\Box$. The observation of the large scale cosmic structures and particularly the large scale anisotropy of the CMBR indicate $n \approx 1$. The spectrum with $n \approx 1$ is the Zeldovich spectrum which is a natural outcome of the near exponential inflation considered in Eqs. (54,55). The observationally relevant temperature fluctuations of the CMBR in different direction, $\delta T$, can be related with the fluctuating metric as $\Delta T = \frac{\delta \varphi}{\sqrt{2\pi}}$. Using $\Delta T(\theta, \phi) = \sum_{lm} a_{lm} Y_{lm}(\theta, \phi)$, we have

$$a_l^2 \equiv \langle |a_{lm}|^2 \rangle = \frac{4\pi}{25} \int_0^\infty \frac{1}{k} P_C(k) j_l^2(kx) dk,$$  \hspace{1cm} (57)

where $x = 2/H_0$.

Thus, the scenario we have considered in this section with one of the quantum fluctuations in $\Box$ as the seed fails to produce the observationally relevant structures. This implies that the pole-like inflation stages are not suitable for the seed generating mechanism for the observed large scale structures.

**V. Discussion**

In Sec. $\Box$ we have shown that the power spectrum $P_{\delta \phi_{\varphi_{\delta \phi}}} \propto k^4$ is a robust prediction from all the expansion stages driven by the kinetic (more precisely, potential-less) parts in the gravity. If one accepts this quantum fluctuations as the seed for the later evolution into the large scale classical structures through the inflation mechanism, as shown in Sec. $\Box$, it leads to a completely different spectrum for the large scale structures compared with the observed ones. We may call it a “structure problem” of the kinetic inflation scenarios. Together with the “graceful exit problem”, which generally appears in the kinetic type inflation scenarios, this can be accepted as another negative news for constructing inflation models.
based on a pole-like expansion stage in the generalized gravity theories.

We can think of some ways out of this generic structure problem in the kinetic inflation. Firstly, the power spectrums of the quantum fluctuations depend on the vacuum choice which is almost an arbitrary function of $k$ with a constraint in Eq. (15); the choice of the vacuum state needs physical consideration. Secondly, the classicalization process of the quantum fluctuations can possibly lead to a modification factor $Q(k)$ which in general may depend on the wave number $k$; however, the effects may arise from considering the nonlinear field effect which goes beyond the linear treatment considered in this paper [18]. Finally, probably the most reasonable approach would be to accept it as a problem for making the observed part of the large scale structures. In such a case, the observed part of the large scale structure can possibly arise from other seed generating mechanisms, like defects. This implies that kinetic inflation based on the above gravity theories is not appropriate for the seed generating mechanism for the currently observable patch of the universe. However, one remarkable feature which goes beyond the linear treatment considered in this paper is that the observed part of the large scale structures. In such a case, the classical actions used in Sec III should be modified by the quantum correction terms. Recently, it was suggested that the graceful exit problem in the kinetic inflation based on the string gravity can be resolved by the one loop quantum correction effect [21]. In such a modified scenario, the observationally relevant scales may exit the horizon during the quantum era. Deriving the generated quantum fluctuations in such modified gravity sectors may lead to completely different outcomes and require considering the action which is more general than the one in Eq. (6). This is currently an important open question especially in the context of the recently popular string theory.

The case of a decoupled gravitational wave can be treated in a similar manner. Each of two polarization states of the gravitational wave behaves similarly to the scalar type perturbation and we can analogously derive the exact forms of the power spectrums of the quantum fluctuations. The power spectrums show similar dependences on $k$ to the power spectrums for the scalar type perturbations presented in Sec. III. Results for the gravitational wave will be presented separately.

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**APPENDIX**

This appendix contains the equations and the general asymptotic solutions for the scalar type perturbations in three types of gravity theories considered in Sec. III A.

### III C

1. **Scalar-tensor theory:** It is convenient to have:

\[
\frac{\dot{\phi}}{H \phi} = -\frac{1 + 3q}{q}, \quad \frac{1 + q}{1 + 3q} = \sqrt{1 + \frac{2}{3} \omega},
\]

\[
Z = 12 \left(\frac{q}{1 + 3q}\right)^2 \frac{1}{\phi},
\]

\[
\eta = -\frac{1}{1 + q} \frac{t_0 - t}{a} = -\frac{q}{1 + q} \frac{1}{aH}.
\]

For the scalar mode Eqs. (46-52) become:

\[
\delta \phi = \frac{1 + 3q}{q} \phi \phi_0\phi,
\]

\[
\delta S = \frac{1}{2} \int a^3 Z \left[ \left( \frac{1}{a^2} \phi_0 \phi + \phi_0 \phi \right) \right] \frac{1}{t_0 - t} \left( \frac{1 + 3q}{t_0 - t} \right)^2 \frac{1}{\phi_0} \delta \phi \delta \phi \alpha \delta \phi_0 \phi_0
\]

\[
\delta S = \frac{1}{2} \int a^3 Z \left[ \left( \frac{1}{a^2} \phi_0 \phi + \phi_0 \phi \right) \right] \frac{1}{t_0 - t} \left( \frac{1 + 3q}{t_0 - t} \right)^2 \frac{1}{\phi_0} \delta \phi \delta \phi \alpha \delta \phi_0 \phi_0
\]

2. **Induced gravity:** It is convenient to have:

\[
\frac{\dot{\phi}}{H \phi} = -\frac{1 + 3q}{2q} \frac{1 + q}{1 + 3q} = \sqrt{1 + \frac{1}{6\xi}},
\]

\[
Z = 6\xi \left( \frac{2q}{1 + 3q} \right)^2
\]

\[
\eta = -\frac{1}{1 + q} \frac{t_0 - t}{a} = -\frac{q}{1 + q} \frac{1}{aH}.
\]

For the scalar mode Eqs. (53-59) become:

\[
\delta \phi = \frac{1 + 3q}{2q} \phi \phi_0\phi,
\]

\[
\delta S = \frac{1}{2} \int a^3 Z \left[ \left( \frac{1}{a^2} \phi_0 \phi + \phi_0 \phi \right) \right] \frac{1}{t_0 - t} \left( \frac{1 + 3q}{t_0 - t} \right)^2 \frac{1}{\phi_0} \delta \phi \delta \phi \alpha \delta \phi_0 \phi_0
\]
\[
\frac{d^2\delta\phi}{dt^2} + \frac{3q}{t_0 - t} \frac{d\delta\phi}{dt} + \left[ \frac{1 + 3q}{2(t_0 - t)} \right]^2 - \frac{1}{a^2} \nabla^2 \right] \delta\phi = 0,
\]
\[
(68)
\]
\[
\ddot{\varphi}_\delta - \frac{1}{t_0 - t} \dot{\varphi}_\delta - \frac{1}{a^2} \nabla^2 \varphi_\delta = 0,
\]
\[
(69)
\]
\[
\varphi_\delta = C + \frac{D}{6\xi} (t_0 - t) \ln (1 - t/t_0),
\]
\[
(70)
\]
\[
\varphi_\delta = \frac{1}{2\sqrt{3\xi}} \left( \frac{\sqrt{\eta \xi}}{a^2} \right) \ln [c_1 e^{i\eta} + c_2 e^{-i\eta}].
\]
\[
(71)
\]
3. String theory: It is convenient to have
\[
\frac{\dot{H}}{H} = 3 \pm \sqrt{3}, \quad Z = \left(3 \mp \sqrt{3}\right) e^{-\phi},
\]
\[
\eta = -\frac{3 \mp \sqrt{3} t_0 - t}{2} = -\frac{3 \mp \sqrt{3} H}{2}.
\]
\[
(72)
\]
For the scalar mode Eqs. (14,23) become:
\[
\delta\phi = -\left(3 \mp \sqrt{3}\right) \varphi_\delta,
\]
\[
(73)
\]
\[
\delta S = \frac{1}{2} \int a^3 Z \left[ \frac{\delta^2 \varphi^2_\delta}{a^2} \frac{1}{a^2} \delta\phi_{\varphi,\alpha} \right] dt \, dx,
\]
\[
(74)
\]
\[
\ddot{\varphi}_\delta - \frac{1}{t_0 - t} \dot{\varphi}_\delta - \frac{1}{a^2} \nabla^2 \varphi_\delta = 0,
\]
\[
(75)
\]
\[
\dot{\varphi}_\delta = C + \frac{D}{6} \left( \frac{t_0 - t}{a^2 e^{-\phi}} \right) \ln (1 - t/t_0),
\]
\[
(76)
\]
\[
\varphi_\delta = -\frac{1}{2\sqrt{3}} \left( \frac{\sqrt{\eta \xi}}{a^2 e^{-\phi}} \right) \ln [c_1 e^{i\eta} + c_2 e^{-i\eta}].
\]
\[
(77)
\]

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