**U(1) Gauge Field Localization on Bloch Brane with CHH Mechanism**

Zhen-Hua Zhao\(^a\), Yu-Xiao Liu\(^b\), and Yuan Zhong\(^{b,c}\)

\(^a\)Department of Applied Physics, Shandong University of Science and Technology, Qingdao, 266590 People’s Republic of China

\(^b\)Institute of Theoretical Physics, Lanzhou University, Lanzhou 730000, People’s Republic of China

\(^c\)IFAE, Universitat Autònoma de Barcelona, 08193 Bellaterra, Barcelona, Spain

E-mail: zhaozhh09@lzu.edu.cn, liuyx@lzu.edu.cn, yzhong2009@ifae.es

**ABSTRACT:** We follow the Chumbes-Holf da Silva-Hott (CHH) mechanism to study the (quasi-)localization of \(U(1)\) gauge field on the Bloch brane. The localization of \(U(1)\) gauge field is discussed for four kinds of Bloch brane solutions: the original Bloch brane solution, the generalized Bloch brane solution, and the degenerate Bloch brane solutions I and II. With the CHH mechanism, we find that the mass spectrum of the gauge field Kaluza-Klein modes is continuous and only the zero mode of the gauge field is localized on the brane. For the massive modes, there exist resonant modes only on the degenerate Bloch branes.

**KEYWORDS:** Extra dimensions, Braneworld, Gauge field, Localization
1 Introduction

In brane-world theory graviton can be localized on the Randall-Sundrum (RS) thin brane [1, 2] and on the thick brane [3], naturally. In addition to graviton localization, the localization of Standard Model particles is also an important issue for any brane-world model.

The localization of fermion on brane can be realized by introducing the usual Yukawa coupling between the background scalar field and the fermion field [4–10] when the background scalar field has a kink-like configuration interpolating between the two different vacua at the two sides of brane. However, when the scalar field is an even function of the extra dimension, one need to introduce the new localization mechanism presented in ref. [11]. Real scalar field can be localized on brane as long as graviton is localizable [5].

While for the U(1) gauge field, its localization is more complex than the fermion and scalar fields. In the RS thin brane scenario, U(1) gauge field with the following standard five-dimensional action

\[ S \sim \int d^5x \sqrt{-g} F_{MN} F^{MN} \]  

(1.1)
can not be localized [12]. Here, \( F_{MN} = \partial_M A_N - \partial_N A_M \) is the field strength of the U(1) gauge field. In order to localize U(1) gauge field on the RS brane, many ideas were proposed [13–21].

In the thick brane scenario, U(1) gauge field with the action (1.1) can be localized on some thick branes. For example, it can be localized on the thick de Sitter (dS) brane [22, 23], the Weyl thick brane [24], and the brane with finite extra-dimension [25]. Especially, in ref. [23], there the potentials in the corresponding Schrodinger equations for the Kaluza-Klein (KK) modes of the vector field are modified Poschl-Teller potentials, which lead to the localization of the vector zero mode on the brane as well as to mass gaps in the mass spectra. But it cannot be localized on thick brane models that are asymptotically RS.
In order to localize gauge field on thick brane, Kehagias and Tamvakis (KT) proposed a general mechanism, in which a coupling between the gauge field and an extra dilaton field is introduced [26]. Kehagias and Tamvakis mechanism has been applied in many different brane-world scenarios to localize the vector [27–30] and Kalb-Ramond fields [30–33]. Recently, Chumbes, Holf da Silva and Hott (CHH) propose a new mechanism to localize gauge and tensor fields on thick brane [34]. In their method, gauge and tensor fields are directly coupled to a function of the background scalar field.

On the other hand, the thick brane is usually generated by a background scalar field. In ref. [35], Bazeia and Gomes introduced the Bloch brane generated by two real scalar fields. This brane model was further generalized in ref. [36], and investigated in refs. [11, 29, 33, 37–39]. It is known that $U(1)$ gauge field with the action (1.1) can not be localized on the Bloch brane [29]. In ref. [29] the localization of gauge field on the Bloch brane was discussed with the KT mechanism. In order to localize the zero mode on the Bloch brane, an extra dilaton scalar field is introduced in ref. [29].

In this paper, we will investigate the localization of $U(1)$ gauge field with the CHH mechanism. In this mechanism the third dilaton scalar field, which appears in ref. [29], is not needed. The localization of gauge field for four kinds of Bloch brane solutions are discussed and the localized zero mode is found.

This paper is constructed as follows. In section 2, the Bloch brane scenario and its four kinds of solutions are reviewed briefly. The localization of $U(1)$ gauge field is discussed in section 3. Finally, we give our conclusions in section 4.

2 Review of Bloch brane

The action for the Bloch brane model reads [35]

$$S = \int d^4x dy \sqrt{-g} \left[ \frac{1}{4} R - \frac{1}{2} \partial_M \phi \partial^M \phi - \frac{1}{2} \partial_M \chi \partial^M \chi - V(\phi, \chi) \right],$$

(2.1)

where $g = \det(g_{MN})$, $R$ is the scalar curvature of the five-dimensional space-time, $M, N = 0, 1, 2, 3, 4$, and $\phi, \chi$ are two real scalar fields depending only on the extra-dimensional coordinate $y$ for the static flat brane model.

The line element for the five-dimensional space-time is assumed as

$$ds^2 = g_{MN} dx^M dx^N = e^{2\alpha(y)} \eta_{\mu\nu} dx^\mu dx^\nu + dy^2,$$

(2.2)

where $e^{2\alpha(y)}$ is the warp factor, $\alpha(y)$ is only the function of extra-dimensional coordinate $y$, and $\eta_{\mu\nu} = \text{diag}(-1,1,1,1)$. From the above action (2.1), one can get the equations of motion of $\phi, \chi$, and the Einstein equations [35]:

$$\phi'' = -4\alpha' \phi' + \frac{\partial V(\chi, \phi)}{\partial \phi},$$

(2.3)

$$\chi'' = -4\alpha' \phi' + \frac{\partial V(\chi, \phi)}{\partial \chi},$$

(2.4)

$$\alpha'' = \frac{2}{3}(\phi'^2 + \chi'^2),$$

(2.5)

$$\alpha'^2 = \frac{1}{6}(\phi'^2 + \chi'^2) - \frac{1}{3} V(\phi, \chi),$$

(2.6)
where the prime stands for the derivative with respect to $y$. By introducing a superpotential $W(\phi, \chi)$, the above equations can be reduced to the following first-order form:

$$
\phi' = \frac{\partial W(\chi, \phi)}{\partial \phi},
$$

(2.7)

$$
\chi' = \frac{\partial W(\chi, \phi)}{\partial \chi},
$$

(2.8)

$$
\alpha' = -\frac{2}{3} W(\chi, \phi),
$$

(2.9)

and the scalar potential is determined in terms of the superpotential by

$$
V = \frac{1}{2} \left[ \left( \frac{\partial W(\chi, \phi)}{\partial \phi} \right)^2 + \left( \frac{\partial W(\chi, \phi)}{\partial \chi} \right)^2 \right] - \frac{4}{3} W^2(\chi, \phi).
$$

(2.10)

Then for the superpotential

$$
W(\phi, \chi) = \phi \left[ \left( 1 - \frac{1}{3} \phi^3 \right) - b \chi^2 \right],
$$

(2.11)

where $b$ is a real parameter, the solution of Eqs. (2.7)-(2.9) is given by [35]

$$
\phi(y) = \tanh(2by),
$$

(2.12a)

$$
\chi(y) = \sqrt{\frac{1}{b} - 2 \text{sech}(2by)},
$$

(2.12b)

$$
\alpha(y) = \frac{1}{9b} \left[ (1 - 3b) \tanh^2(2by) - 2 \ln \cosh(2by) \right],
$$

(2.12c)

where the parameter $b$ satisfies the constrain $0 < b < 1/2$. The above two-field solution represents a Bloch wall. When $b \to 1/2$, we will get the Ising wall of the one-field solution [35].

In addition to the above original Bloch brane solution, the generalized Bloch brane solution was found in ref. [36] by using the following generalized super-potential

$$
W(\phi, \chi) = \phi \left[ a \left( v^2 - \frac{1}{3} \phi^3 \right) - b \chi^2 \right],
$$

(2.13)

It reads [36]

$$
\phi(y) = v \tanh(2by),
$$

(2.14a)

$$
\chi(y) = v \sqrt{\frac{a - 2b}{b}} \text{sech}(2by),
$$

(2.14b)

$$
\alpha(y) = \frac{v^2}{9b} \left[ (a - 3b) \tanh^2(2by) - 2a \ln \cosh(2by) \right],
$$

(2.14c)

where $a > 2b > 0$ or $a < 2b < 0$. 

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Other solutions of Bloch brane were also found in ref. [36] for the same super potential (2.13) with $a = b$ and $a = 4b$, namely, the degenerated Bloch brane solutions. They are

$$
\phi(y) = \frac{\sqrt{c_0^2 - 4} \, v \sinh(2bvy)}{\sqrt{c_0^2 - 4 \cosh(2bvy) - c_0}},
$$

(2.15a)

$$
\chi(y) = \frac{2v}{\sqrt{c_0^2 - 4 \cosh(2bvy) - c_0}},
$$

(2.15b)

$$
\alpha(y) = \frac{1}{2} \left[ \frac{4v^2}{9 \left( \sqrt{c_0^2 - 4 \cosh(2bvy) - c_0} \right)^2} - \frac{4 \left( c_0^2 - \sqrt{c_0^2 - 4c_0 - 4} \right) v^2}{9 \left( \sqrt{c_0^2 - 4 - c_0} \right)^2} \right] + \frac{1}{2} \log \left( \frac{\sqrt{c_0^2 - 4 - c_0}}{\sqrt{c_0^2 - 4 \cosh(2bvy) - c_0}} \right),
$$

(2.15c)

for $c_0 < -2$ and $a = b$, and

$$
\phi(y) = \frac{\sqrt{1 - 16c_0} \, v \sinh(4bvy)}{\sqrt{1 - 16c_0 \cosh(4bvy) + 1}},
$$

(2.16a)

$$
\chi(y) = \frac{2v}{\sqrt{1 - 16c_0 \cosh(4bvy) + 1}},
$$

(2.16b)

$$
\alpha(y) = \frac{1}{2} \left[ \frac{4 \left( 8c_0 + \sqrt{1 - 16c_0} + 1 \right) v^2}{9 \left( \sqrt{1 - 16c_0 + 1} \right)^2} - \frac{4v^2 \left( \sqrt{1 - 16c_0 \cosh(4bvy) + 8c_0 + 1} \right)}{9 \left( \sqrt{1 - 16c_0 \cosh(4bvy) + 1} \right)^2} \right] + \frac{1}{2} \log \left( \frac{\sqrt{1 - 16c_0 + 1}}{\sqrt{1 - 16c_0 \cosh(4bvy) + 1}} \right),
$$

(2.16c)

for $c_0 < 1/16$ and $a = 4b$.

In this paper we will call solutions (2.12), (2.14), (2.15), and (2.16) as the original, generalized, degenerate I, and degenerate II Bloch brane solutions, respectively.

From the above solutions one can find that the Bloch brane has a rich inner structure. The details of the above solutions can be found in refs. [35, 36].

3 Localization of gauge field

As was analyzed in ref. [29], for a gauge field with the following standard five-dimensional action

$$
S \sim \int d^5x \sqrt{-g} F_{MN} F^{MN},
$$

(3.1)

the corresponding zero mode cannot be localized on the Bloch brane. In order to localize gauge field on the Bloch brane, the authors of ref. [29] extended the Bloch brane scenario to the so called dilatonic Bloch brane model, which is described by the following action

$$
S = \int d^5x \sqrt{-g} \left[ \frac{1}{4} R - \frac{1}{2} (\partial \phi)^2 - \frac{1}{2} (\partial \chi)^2 - \frac{1}{2} (\partial \pi)^2 - V(\phi, \chi, \pi) \right],
$$

(3.2)
where the scalar fields $\phi$ and $\chi$ generate the brane, and the dilaton scalar field $\pi$ is used to localize gauge field on the brane. The action of gauge field is assumed to be

$$S \sim \int \sqrt{-g} d^5 x e^{-2\lambda\pi} \sqrt{2/3} F_{MN} F^{MN},$$

(3.3)

where the coupling between the dilation field $\pi$ and the gauge field is introduced. With the action (3.3), the zero mass mode of gauge field is found to be localized on the brane and some massive resonant modes also are found [29].

The method used in ref. [29] was first proposed by Kehagias and Tamvakis (KT) in ref. [26]. In this paper we will follow another mechanism proposed by Chumbes, Hoff, and Hott (CHH) in ref. [34] and study the localization of gauge field. Compared with the KT mechanism, the dilation scalar field is not needed in the CHH mechanism. We will show that $U(1)$ gauge field can be localized on the standard Bloch brane by introducing a coupling between the gauge field and background scalar field $\chi$. The action of the five-dimensional $U(1)$ gauge field reads

$$S = -\frac{1}{4} \int d^5 x \sqrt{-g} \chi(y) F_{MN} F^{MN},$$

(3.4)

By means of the decomposition of $A_\mu = \sum_n a_\mu(x) \rho_n(y)$ and the gauge $\partial_\mu A^\mu = 0$ and $A_4 = 0$, the above action (3.4) can be reduced to

$$S = -\frac{1}{4} \int dy \chi(y) \rho_0(y)^2 \int d^4 x (f_{\mu\nu} f^{\mu\nu} - 2m_0^2 a_\mu a^\mu),$$

(3.5)

where $f_{\mu\nu} = \partial_\mu a_\nu - \partial_\nu a_\mu$ is the four-dimensional gauge field strength tensor, and $\rho_n(y)$ should satisfy the equation

$$\rho''_n + \left( \chi' \chi + 2\alpha' \right) \rho'_n = -m_0^2 \rho_n e^{-2\alpha}.$$  

(3.6)

The localization of gauge field requires

$$I \equiv \int_\infty^\infty dy \chi(y) \rho^2_0(y) < \infty.$$  

(3.7)

3.1 Zero mode

First we discuss the localization of the zero mode of gauge field. Let $m_0 = 0$, Eq. (3.6) reads

$$\rho''_0 + \left( \chi' \chi + 2\alpha' \right) \rho'_0 = 0.$$  

(3.8)

By introducing the filed transformation [34]

$$\rho_0 = e^{-\gamma(y)} \tilde{\rho}_0(y)$$  

(3.9)

with $\gamma(y)$ satisfies $2\gamma' = 2\alpha + \chi'/\chi$, Eq. (3.8) can be reduced to

$$-\rho''_0 + \left( \gamma'' + \gamma'^2 \right) \tilde{\rho}_0 = 0,$$  

(3.10)
or
\[
\left( \frac{d}{dy} + \gamma' \right) \left( - \frac{d}{dy} + \gamma' \right) \hat{\rho}_0 = 0. \tag{3.11}
\]
The solution of the above equation is \( \hat{\rho}_0 = C_1 e^{-\gamma} \), where \( C_1 \) is a constant. So the zero mode solution is \( \rho_0 = C_1 \). Now we will check if the integration (3.7) is convergent for the four kinds of Bloch brane solutions.

For the original Bloch brane solution (2.12), the integration (3.7) reads
\[
I = C_1^2 \int_{-\infty}^{+\infty} dy \chi(y)
= C_1^2 \int_{-\infty}^{+\infty} dy \sqrt{\frac{1}{b} - 2 \text{sech}(2by)}
= C_1^2 \frac{\pi}{2b} \sqrt{\frac{1}{b} - 2},
\tag{3.12}
\]
which is finite, so the zero mass mode can be localized on the original Bloch brane.

For the generalized Bloch brane solution (2.14), the integration (3.7) is
\[
I = C_1^2 \int_{-\infty}^{+\infty} dy \sqrt{\frac{a - 2b}{b} \text{sech}(2by)} = C_1^2 \frac{\pi}{2b} \sqrt{\frac{a - 2b}{b}}. \tag{3.13}
\]
Obviously, it is also convergent.

For the degenerate Bloch solution I (2.15), we have
\[
I = C_1^2 \int_{-\infty}^{+\infty} dy \sqrt{\frac{2v}{\sqrt{c_0^2 - 4 \cosh(2by)} - c_0}}
= -\frac{2}{b} C_1^2 \text{arctanh} \left( \frac{1}{2} \sqrt{\frac{c_0^2 - 4 + c_0}{c_0^2 - 4 + c_0}} \right). \tag{3.14}
\]
So the above integration is convergent for \( c_0 < -2 \).

For the degenerate Bloch solution II (2.16), the result is
\[
I = C_1^2 \frac{4 \text{EllipticK} \left( 1 - \frac{2}{\sqrt{1 - 16c_0}} \right)}{\sqrt{1 - 16c_0 + 1}}. \tag{3.15}
\]
where the function \( \text{EllipticK}(x) \)\(^1\) gives the complete elliptic integral of the first kind. With the condition \( c_0 < 1/16 \), the result (3.15) is finite.

To sum up, with the CHH mechanism, the \( U(1) \) gauge field can be localized on the Bloch brane.

### 3.2 Massive modes

Next we investigate the localization of the massive modes of gauge field. In this part, it is more convenient to rewrite the metric (2.2) in a conformal flat form, namely,
\[
ds^2 = e^{2A(z)} (\eta_{\mu\nu} dx^\mu dx^\nu + dz^2). \tag{3.16}
\]
\(^1\)The details of this function can be found in: http://mathworld.wolfram.com/CompleteEllipticIntegraloftheFirstKind.html
With the above metric (3.16) and the gauge choice \( A_4 = 0 \), the action of the five-dimensional gauge field (3.4) is reduced to

\[
S = -\frac{1}{4} \sum_n \int dz \tilde{\rho}_n^2(z) \int d^4x \left( f^{(n)}_{\mu\nu} f^{(n)}{}^{\mu\nu} - 2m_n^2 a^{(n)}_{\mu} a^{(n)}{}^{\mu} \right), \tag{3.17}
\]

where \( \tilde{\rho}_n = \rho_n \chi^{1/2} e^A \), and the KK modes \( \tilde{\rho}_n(z) \) satisfies the following Schrödinger-like equation

\[
-\tilde{\rho}_n'' + V(z) \tilde{\rho}_n = m_n^2 \tilde{\rho}_n, \tag{3.18}
\]

where the effective potential is given by

\[
V(z) = \frac{1}{2} A''(z) + \frac{1}{4} A'^2(z) + \frac{A'(z) \chi'(z)}{2 \chi(z)} + \frac{\chi''(z)}{2 \chi(z)} - \frac{\chi'^2(z)}{4 \chi^2(z)}, \tag{3.19}
\]

and the prime denotes the derivative with respect to \( z \). The above equation (3.18) can be recast to

\[
\mathcal{T}^\dagger \mathcal{T} \tilde{\rho}_n = m_n^2 \tilde{\rho}_n, \tag{3.20}
\]

where

\[
\mathcal{T}^\dagger = -\frac{d}{dz} + \Gamma, \quad \mathcal{T} = \frac{d}{dz} + \Gamma,
\]

and

\[
\Gamma = -\frac{1}{2} \left( \frac{\chi'}{\chi} + A' \right).
\]

Equation (3.20) means that there is no tachyonic mode with \( m^2 < 0 \) in the spectrum of the KK modes [40]. Note that, in order to get the effective action (3.17) of the four-dimensional gauge fields from the five-dimensional one (3.4), we have introduced the orthonormalization condition between different massive KK modes:

\[
\int dz \tilde{\rho}_m(z) \tilde{\rho}_n(z) = 0. \quad (m \neq n) \tag{3.21}
\]

So the localization condition for \( \tilde{\rho}_n(z) \) is

\[
\int dz \tilde{\rho}_n^2(z) < \infty. \tag{3.22}
\]

The property of \( \tilde{\rho}_n \) is determined by the effective potential (3.19). There are two methods to get the explicit expression of the effective potential \( V(z) \) (3.19). The first one is to resolve the Einstein equations and the equations of motion of the background scalar fields with the line element (3.16). From our knowledge, with this method, there is no analytic solution. The second one is to write the expression of \( V(z(y)) \) in the \( y \) coordinate by the use of the coordinate transformation \( dz = e^{-\alpha(y)}dy \), the result is

\[
V(z(y)) = \frac{1}{2} e^{2\alpha(y)} \left( \alpha''(y) + \frac{2\alpha'(y) \chi'(y)}{\chi(y)} + \frac{\chi''(y)}{\chi(y)} \right). \tag{3.23}
\]

Then, we can use the numerical relation between \( y \) and \( z \), \( y = y(z) \), to obtain \( V(z) \) from (3.23).
Figure 1. The shapes of the effective potential $V(z)$ for the original Bloch brane solution with different values of the parameter $b$.

Figure 2. The shapes of the effective potential $V(z)$ and the massive KK modes of the gauge field for the original Bloch brane solution. The parameters are set to for $b = 0.4$, $m^2 = 0.2$ (left), and $m^2 = 1$ (right).

3.2.1 Original and general Bloch branes

We first consider the original Bloch brane solution. The shapes of the effective potential $V(z)$ for different values of the parameter $b$ are shown in fig. 1. Figure 1 shows that $V(z)$ is a volcano-type potential and

$$V(z \to \pm \infty) \to 0.$$  \hfill (3.24)

So the mass spectrum of the gauge field is continued and it has any value satisfies $m^2 \geq 0$, and the massive KK mode for any $m^2 > 0$ is oscillated when far away from the brane along the extra dimension. When $m^2 \gg V_{\text{max}}$, where $V_{\text{max}}$ is the max value of the $V(z)$, the massive KK mode approaches the plane wave solution. The shapes of the massive KK mode are shown in fig. 2 for a typical potential.

Since the effective potential trends to vanish at the boundary of the extra dimension, the massive KK modes cannot be normalized. For such massive KK modes, we can use
the relative probability method [31, 41] to check whether there are the resonant modes or not in the spectrum of the gauge field. The relative probability function is defined as [41]

\[ P(m) = \frac{\int_{z_b}^{z_c} \tilde{\rho}^2(z) dz}{\int_{z_c}^{z_b} \tilde{\rho}^2(z) dz}, \]  

(3.25)

where \( z_c > z_b \) and \( 2z_b \) is about the thickness of the brane. Here and after, we set \( z_c = 10z_b \). If \( m^2 \gg V_{\text{max}} \) the solution of \( \tilde{\rho} \) will be a plane wave, so \( P(m) = z_b/z_c = 0.1 \). In order to get the solution of Eq. (3.18), we introduce two boundary conditions:

\[ \tilde{\rho}(0) = 0, \quad \tilde{\rho}'(0) = 1 \]  

(3.26)

for the odd parity solution, and

\[ \tilde{\rho}(0) = 1, \quad \tilde{\rho}'(0) = 0 \]  

(3.27)

for the even one.

The shapes of \( P(m) \) for different values of the parameter \( b \) are shown in fig. 3. For a resonant mode, there will be a peak in the curve of \( P(m) \). From fig. 3, we cannot find any peak in the curves, so there is no resonant mode.
The potential $V(z)$ shown in fig. 1 has an attractive well, why there is not any resonant mode? In order to illustrate this question, one can make use of the following dual equation

$$
T T^\dagger \hat{\rho}_n = m_n^2 \hat{\rho}_n,
$$

(3.28)

with

$$
\hat{V}(z) = -\frac{1}{2} A''(z) + \frac{1}{4} A^2(z) + \frac{A'(z)\chi'(z)}{2\chi(z)} - \frac{\chi''(z)}{2\chi(z)} + \frac{3\chi'^2(z)}{4\chi^2(z)}.
$$

(3.29)

Equation (3.28) is similar to (3.20) except the reversed order of the operators. In supersymmetric quantum mechanics, $\hat{\rho}_n$ is called as the superpartner of $\tilde{\rho}_n$, and they share the same mass spectrum except the zero mode.

The shapes of the dual potential $\hat{V}(z)$ (3.29) are shown in fig. 4 with different values of the parameter $b$. From fig. 4, one can see that the attractive potential in fig. 1 now has changed to a repulsive one. In order to get resonant modes, we need a quasipotential with a deep enough “well”. From fig. 4, we know that there will be a “well” when the parameter $b$ is small, but this well is not deep enough. So, there is no resonant mode.

For the generalized Bloch brane solution, there are three parameters, which are $a$, $b$, and $v$. The shapes of the potentials $V(z)$ and $\hat{V}(z)$ will be affected by them. Their effects on the potentials have been checked in three cases by fixing two parameters and varying the third one. The shapes of the potentials $V(z)$ and $\hat{V}(z)$ are shown in figs. 5, 7, and 9.

In fig. 5, we fix the parameters $a$ and $b$ and tune the parameter $v$. From the shapes of $\hat{V}(z)$ (fig. 5(a)), we can see that there is no resonant mode. It can be verified form the shapes of the $P(m)$ in fig. 6: there is no peak in the curves. In fig. 7, we fix the parameters $a$ and $v$ and tune the parameter $b$. Form fig. 7(a), we can see that the height of the barriers and depth of the well increase with the parameter $b$, which means that there is a hope for finding the resonant modes with the increase of $b$. But fig. 7(b) shows that the well of the potential $\hat{V}(z)$ disappears as the increase of $b$. So, we will not to find the resonant mode with the the increase of parameter $b$. It can be also verified form the shapes of the $P(m)$.
Figure 5. The shapes of the potentials $V(z)$ (left) and $\hat{V}(z)$ (right) with different values of the parameter $v$ for the generalized Bloch brane solution, the values of other parameters are fixed as $a = 3$ and $b = 1$.

Figure 6. The shapes of $P(m)$ with different values of the parameters $v$ for the generalized Bloch brane solution, the values of other parameters are fixed as $a = 3$ and $b = 1$.

in fig. 8. In fig. 9, we fix the parameters $b$ and $v$ and tune $a$. Although the height of the barriers increase with $a$ for both $V(z)$ and $\hat{V}(z)$, resonant modes are also not found, see fig. 10.

The potential of $V(z)$ with a large range of values of the parameters are checked for both the original and generalized Bloch solutions, but no resonant mode is found.

3.2.2 Degenerate Bloch branes

For the degenerate Bloch brane solutions I and II, the shapes of $V(z)$ are shown in fig. 11, which shows that the width of the potential well increases with the parameter $d$ and there are two potential wells and two barriers with vanishing potential between them when $d$ is large enough or $c_0 \to -1$ and $1/16$ for solutions I and II, respectively. Here, the parameter
Figure 7. The shapes of the potentials $V(z)$ and $\hat{V}(z)$ with different values of the parameter $b$ for the generalized Bloch brane solution, the values of other parameters are fixed as $a = 10$ and $v = 1$.

Figure 8. The shapes of $P(m)$ with different values of the parameters $b$ for the generalized Bloch brane solution, the values of other parameters are fixed as $a = 10$ and $v = 1$.

d is related to $c_0$ by $c_0 = -2 - 10^{-d}$ and $c_0 = 1/16 - 10^{-d}$ for for solutions I and II, respectively. We will see that this feature of $V(z)$ will lead to the appearance of resonant modes.

The curves of $P(m)$ are shown in figs. 12 and 13 for the degenerate Bloch brane solutions I and II, respectively. In the curves of $P(m)$, every peak corresponds to a resonant mode. For example, in fig. 12(a), the curves have three peaks, so there are three resonant modes, and the corresponding solutions are shown in fig. 14.

By the comparison between the two diagrams in fig. 12 or 13, we find that the number of the resonant modes increases with the width of the degenerate Bloch branes.
Figure 9. The shapes of the potentials $V(z)$ and $\hat{V}(z)$ with different values of the parameter $a$ for the generalized Bloch brane solution, the values of other parameters are fixed as $b = 1$ and $v = 1$.

Figure 10. The shapes of $P(m)$ with different values of the parameters $a$ for the generalized Bloch brane solution, the values of other parameters are fixed as $b = 1$ and $v = 1$.

4 Conclusions

We have studied the localization of U(1) gauge field on the Bloch brane with the CHH mechanism. In the Bloch brane scenario, there are two scalar fields, and one of them couples to U(1) gauge field, directly. So, compared to the KT mechanism, the CHH mechanism is simpler to study the localization of $U(1)$ gauge field in the Bloch brane scenario.

In this work, four kinds of Bloch brane solutions were discussed, they are the original, generalized, and degenerate I and II Bloch brane solutions, respectively. With the CHH mechanism, the equation of motion of the KK modes of the U(1) gauge field can be recast to a Schrödinger-like equation with the supersymmetric quantum mechanics form, so the tachyonic KK modes are excluded. We found that the zero mode of the U(1) gauge field can be localized on the brane and the mass spectrum is continuous with $m^2 \geq 0$. The resonant
(a) The degenerate Bloch brane solution I

(b) The degenerate Bloch brane solution II

Figure 11. The shapes of the effective potential $V(z)$ with different values of parameter $c_0$ for the degenerate Bloch brane solutions I (2.15) (left) and II (2.16) (right). The other parameters are set to $v = 1$ and $b = 1$.

Figure 12. The $P - m$ curves for the degenerate Bloch brane solution I (2.15). The parameters are set to $b = 1$, $v = 1$, $c_0 = -2 - 10^{-10}$ ($d = 10$) (Left), and $c_0 = -2 - 10^{-20}$ ($d = 20$) (Right).

modes in the KK spectrum were also discussed. For the original and generalized Bloch brane solutions, we did not find any resonant mode. While for the degenerate Bloch brane I and II solutions, we found some the resonant modes, and the number of the resonant modes is related with the inner structure of the Bloch brane and increases with the brane width.

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Figure 13. The $P - m$ curves for the degenerate Bloch brane solution II (2.16). The parameters are set to $b = 1$, $v = 1$, $c_0 = 1/16 - 10^{-10}$ ($d = 10$) (Left), and $c_0 = 1/16 - 10^{-20}$ ($d = 20$) (Right).

Figure 14. The shapes of the resonant modes corresponding to the three peaks in fig. 12(a).

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