An automatic division method for multi-model control of a type of block-structured systems

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Abstract. To make full use of the special structure of a type of block-structured systems, an automatic division method is proposed. By setting up a definite criterion for division, a block-structured system can be automatically divided into linear subsystems with an initial threshold value and a step-sized. Thus, dependence on previous knowledge and repeatedly tuning of threshold value are largely reduced. Finally, a multi-model controller is designed based on the linear subsystems for the considered block-structured system. A Lab-tank system that can be approximated by a block-structured model is studied, and simulations show that the proposed method is effective and useful.

1. Introduction

One of the most popular block-structured models is the Hammerstein model, which has been successfully applied to modeling a lot of nonlinear systems, such as heat exchangers, CSTRs, distillation columns, and so on [1-3]. That is because the Hammerstein model has a special structure: It is a connection of a nonlinear element and a linear dynamic element [4, 5]. Although this structure is simple, it can capture the dynamical features of numerous practical systems. However, when to design controllers for this kind of block-structured systems, it becomes nontrivial since linear control techniques are not sufficient for it. Currently, the main solution is the nonlinear inversion control method [6, 7]. Unfortunately, the nonlinear inversion method is not applicable to Hammerstein systems with input multiplicity. Moreover, the method cannot cover the influence of the input nonlinearity of the static element, and may easily degrade the closed-loop performance [7].

To overcome the drawbacks of the nonlinearity inversion method, the multi-model control methods have been introduced into the block-structured systems in the past years. That is to divide a block-structured system into a set of linear subsystems to approximate it; then local linear controllers are designed based on the subsystems; and finally local controllers are integrated into a multi-model controller for the system. And some division methods have been proposed correspondingly [1, 6, 7]. An included angle based division method was initiated in [1] to partition the operating range of a Hammerstein-like system according to its special structure. Later, an included angle based dichotomy grid method was used in the division process to reduce the number of grid points [7]. And a min-max principle based on included angle was employed to choose the operating points [6] to reduce dependency on previous knowledge.
In this work, an automatic division method is proposed for one type of the block-structured models to reduce the dependency on a prior knowledge and decrease computational load. A typical Hammerstein system is studied, and simulations demonstrate that the proposed automatic division method is effective.

The paper is organized as follows. In Section 2, some basic background concepts are briefly introduced. In Section 3, the proposed automatic multi-model division method is detailed. In Section 4, closed-loop simulations are presented to demonstrate the effectiveness of the proposed method. And Section 5 concludes the paper.

2. Multi-model control of block-structured systems

2.1. Included angle
Consider a block-structured model described by Eq.(1), which is also called Hammerstein model.

\[
\begin{align*}
\dot{w}(k) &= f(u(k)) \\
y(k) &= \sum_{i=1}^{p} a_i y(k-i) + \sum_{i=1}^{q} b_i w(k-i)
\end{align*}
\]

(1)

where \( w \) is an intermediate variable; \( f(\cdot) \) is an nonlinear function; \( u \) is the input and \( y \) is the output of the system; and \( a_i \) and \( b_i \) are the coefficients.

Grid system (1) with \( n_g \) static points and linearize the system at every static point \((u_{0i}, y_{0i})\) to get the linearized models in the discrete transfer function. The \( i \)-th linearized is in Eq.(2):

\[
G_i(z) = \frac{\Delta Y(z)}{\Delta U(z)} = f'(u_{0i}) \times \frac{b_1 z^{-1} + b_2 z^{-2} + \cdots + b_q z^{-q}}{1 - a_1 z^{-1} - a_2 z^{-2} - \cdots - a_p z^{-p}}, i = 1, 2, \ldots, n_g
\]

(2)

where \( \Delta Y(z) \) and \( \Delta U(z) \) are the z-transfer function of \( \Delta y(k) = y(k) - y(k-1) \); and \( \Delta u(k) = u(k) - u(k-1) \).

Without loss of generality, suppose \( \frac{b_1 + b_2 + \cdots + b_q}{1 - a_1 - a_2 - a_p} = 1 \) in Eq. (1), then the included angle is defined as follows [3]:

\[
\theta(G_i,G_j) := \theta_i - \theta_j, \quad i, j = 1, 2, \ldots, n_g
\]

(3)

where \( \theta_i \) and \( \theta_j \) are the slope angles of the system (1) at steady state points \((u_{0i}, y_{0i})\) and \((u_{0j}, y_{0j})\) on its static I/O map; \( \theta_i = \arctan(f'(u_{0i})) \) and \( \theta_j = \arctan(f'(u_{0j})) \).

Then the included angle matrix \( \Theta = [\theta_{ij}]_{n_g \times n_g} \) composed of \( \theta_{ij} \) between any two of the \( n_g \) steady state points is computed. The biggest included angle in the matrix \( \Theta \) is denoted as \( \theta_{\text{max}} \), and the smallest is \( \theta_{\text{min}} \). The normalized included angle matrix is defined by \( \Theta^\circ = \Theta/\theta_{\text{max}} = [\theta_{ij}]_{n_g \times n_g}, \) which is bounded between 0 and 1. “0” means the system has similar static gains and similar dynamics at the two points. On the contrary, “1” means the system has quite different behaviours at the two points. In this work, we use the property to divide the considered block-structured system automatically into a multi-model framework.

2.2. Multi-model control of block-structured systems
Suppose \( m \) linearized models \( P_j \) \((j = 1, 2, \ldots, m)\) are chosen from the \( n_g \) models described by Eq.(2) to approximate system (1). Then the linear model bank is described by Eq.(4).

\[
P_j(z) = f'(u_{0j}) \times \frac{b_1 z^{-1} + b_2 z^{-2} + \cdots + b_q z^{-q}}{1 - a_1 z^{-1} - a_2 z^{-2} - \cdots - a_p z^{-p}}, j = 1, 2, \ldots, m
\]

(4)

For every local model described by Eq.(4), a MPC is designed based on it by considering the following objective function [8] as in Eq.(5):

\[
j_l = \sum_{j=1}^{N_f} \|r(k+j) - y(k+j|k)\|_{Q_l}^2 + \sum_{j=0}^{N_u-1} \|\Delta u_k(k+j)\|_{R_l}^2
\]

subject to
where
\[
\Delta u_i(k+j) = u_i(k+j) - u_i(k+j-1),
\]
and \(R_i\) is the weighting matrix; \(N_{yi}\) and \(N_{ui}\) are the prediction horizon and control horizon; and \(r_i\) is the reference signal, \(i = 1, 2, \ldots, m\).

Solve the Quadratic Programming Problem formed by Eqs(5)-(6) and get the solution \(\Delta u_i(k)\) according to the receding horizon philosophy. Therefore, the control input at time instant \(k\) is obtained:
\[
u_i(k) = \sum_{i=1}^{m} u_i(k) \varphi_i(k)
\]
where the weighting function \(\varphi_i\) is the included angle based weighting function [6].

3. Automatic multi-model division method for block-structured systems

3.1. Automatic division algorithm

Step 1. Choose an initial threshold value \(\gamma_0\) and a step-size \(\delta\) according to a prior knowledge. And set \(k = 1\).

Step 2. Set \(i = 1\) and \(m_k = 0\).

Step 3. If \(i \leq n_p\), set \(j = i\) and \(m_k = m_k + 1\). Otherwise jump to Step 11.

Step 4. Select a nominal model \(G^*\) from the \(i\)th to \(j\)th linearized models using Eq.(9).

\[
G^* : \min_{G_{i \leq j}} \{\theta^n(G_{i \leq j})\}
\]

Step 5. Calculate the biggest included angle of \(G^*\) using Eq.(10).

\[
\theta^n_{\max} : \max_{i \leq j} \{\theta^n(G_{i \leq j})\}
\]

Step 6. If \(\theta^n_{\max} \leq \gamma\) and \(j < n_p\), set \(j = j + 1\) and return to Step 4. Otherwise, go to Step 7.

Step 7. If \(\theta^n_{\max} > \gamma\), set \(j = j - 1\). Otherwise, go to Step 8.

Step 8. Choose a nominal model using

\[
(\text{min}) \{\theta^n(G_{i \leq j})\}
\]

Step 9. \(G^*\) is denoted as \(P_{mk}\), and its grid point is denoted as \(OP_{mk}\). Put \(P_{mk}\) into Queue \(\text{pQ}_k\), and \(OP_{mk}\) into Queue \(\text{opQ}_k\).

Step 10. Set \(i = j + 1\), and return to Step 3.

Step 11. If \(k > 1\) and \(m_k = m_{k-1}\), go to Step 12. If \(k > 1\) and \(m_k > m_{k-1}\), go to Step 13.

Step 12. Let \(\gamma = \gamma_0 - \delta\) and \(k = k + 1\). Return to Step 2.

Step 13. The division is ended with the final \(\gamma = \gamma + \delta\). The Hammerstein system is divided into \(m_k\) subregions with \(m_k\) local models.

Step 14. Design a multi-model MPC based on the selected model bank according to Eqs (4)-(8).

3.2. Case studies

In this section, we apply the proposed division method to a Lab-tank system [9] described by Eqs(11)-(12) to illustrate the effectiveness of the proposed method.

\[
\begin{align*}
\dot{x}_1 &= -0.625x_1 + 0.625u \\
\dot{x}_2 &= S^{-1}(F_{in} - F_{out}) \\
y &= x_2
\end{align*}
\]

with

\[
\begin{align*}
F_{in} &= (ae^{ax_1} + c) \sqrt{2gh_{pump}} \\
F_{out} &= k_v \sqrt{2g(x_2 + h_0)}
\end{align*}
\]

where \(x_1\) is the inlet valve position; \(x_2\) is the liquid height, also the output of the systems; \(u\) is the pressure signal, the input of the system. The ranges of the state variables are \(20 \leq x_1 \leq 55\) and \(10 \leq x_2\).
≤ 63; and the range of the input is 20 ≤ u ≤ 63. The constants in Eqs. (11) and (12) are $S = 36\pi \text{ cm}^2$, $a = 0.0522 \text{ cm}^2$, $b = 0.0325$, $c = -0.0638 \text{ cm}^2$, $g = 981 \text{ cm/s}^2$, $h_{\text{pump}} = 1100 \text{ cm}$, $h_0 = 38.62 \text{ cm}$ and $k_v = 0.5299 \text{ cm}^2$, respectively.

First grid the Lab-tank system using the included-angle based dichotomy method [10] with $\gamma_0 = 0.001$, and get 62 grid points. Then divide the system with an initial threshold value $\zeta_0 = 0.45$ and step-size $\delta = 0.01$. Finally we get the final value of threshold is $\zeta = 0.253$ and the division result in Table 1.

| Subregion | Linearized models included | Operating point of the local linear model (x, u) | Subrange |
|-----------|----------------------------|-----------------------------------------------|---------|
| 1st       | 1→35                       | 18th: ([38.8125, 18.2493], 38.8125)            | 4 ≤ y ≤ 33.17 |
|           | 36→62                      | 50th: ([43.3125, 48.9460], 43.3125)            | 33.17 < C_A ≤ 63 |

“1→35” means the 1st to 35th linearized models are divided into the first subregion. The operating point of the first subregion is the 18th grid point. The subrange of subregion 1 is [4, 33, 17]. It is clearly that the division result is the same as in [6], which has been validated as a good division.

4. Closed-loop simulations

A multi-model MPC is designed for the Lab-tank system based on Table 1 for both set-point track and disturbance rejection control. The closed-loop input and output corresponding to MMPC1 of the Lab-tank system are $u_1$ and $y_1$ in Figures 1 and 2. In order to make a comparison, another multi-model MPC denoted as MMPC2 is designed using the method from [1]. The input and output are $u_2$ and $y_2$.

![Figure 1. Set-point tracking control of the Lab-tank system.](image)

The set-point tracking control responses of the two MMPCs are shown in Figure 1. The two MMPCs both perform well as the outputs track the set-point signal closely in the whole operating range. But when the reference signal changes from 10 to 24.7, $y_2$ has a bigger overshoos than $y_1$, making MMPC2 inferior to MMPC1. And the IAE values are $IAE_1 = 802.9$ and $IAE_2 = 968.3$. Therefore, MMPC1 is better than MMPC2 in terms of IAE criterion.

The disturbance rejection control responses are displayed in Figure 2. Obviously, both MMPCs are able to reject the disturbances timely and effectively. The IAE values are $IAE_1 = 552.7$ and $IAE_2 = 557.9$. Therefore, the multi-model controllers are almost the same for disturbance rejection control of
the Lab-tank system. And the performance of the proposed one is better. Besides, the proposed division method is much less dependent on previous knowledge and reduces computational load and tuning efforts greatly.

![Figure 2. Disturbance rejection control of the Lab-tank system](image)

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