Near Horizon Geometry and Black Holes in Four Dimensions

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Abstract

A large class of extremal and near-extremal four dimensional black holes in M-theory feature near horizon geometries that contain three dimensional asymptotically anti-de Sitter spaces. Globally, these geometries are derived from AdS\(_3\) by discrete identifications. The microstates of such black holes can be counted by exploiting the conformal symmetry induced on the anti-de Sitter boundary, and the result agrees with the Bekenstein-Hawking area law. This approach, pioneered by Strominger, clarifies the physical nature of the black hole microstates. It also suggests that recent analyses of the relationship between boundary conformal field theory and supergravity can be extended to orbifolds of AdS spaces.

1 Introduction

The recent success of string theory in describing black holes\(^1\) has left many unanswered questions. In particular, string theory does not yet offer a crisp analysis of the apparent problem with information loss. Such shortcomings arise because many geometrical concepts that are familiar in general relativity have no obvious counterparts in weak coupling constructions using intersecting D-branes. For example, we are presently unable to specify the spacetime loci of the black hole microstates, and we cannot consider nontrivial causal structure in the framework of string theory.

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\(^1\)For review see e.g. [1].
Recently, Strominger presented an alternative counting of black hole microstates which may provide a suitable framework where spacetime issues can be addressed \[2\]. Strominger’s main focus was on the BTZ black hole solutions to three dimensional gravity with a negative cosmological constant\[3\]. His arguments also work for certain five dimensional black holes whose near horizon geometries contain an anti-de Sitter space. The purpose of the present paper is to apply an analogous strategy to four dimensional black holes in M-theory, both in the extremal and near-extremal cases.

We focus attention on the near-horizon region by truncating the black hole metric to its leading terms close to the horizon. This can be justified by taking a suitable limit of parameters that decouples the near-horizon region from the asymptotic Minkowski space, as in the recent works \[8, 9, 10\]. The near horizon geometry contains a factor which is precisely a three dimensional BTZ black hole. This geometry is asymptotically anti-de Sitter space (AdS\(_3\)) and the diffeomorphisms at the outer boundary generate a 1+1 dimensional conformal field theory. This CFT has a classical central charge that depends only on the cosmological constant and its precise value was found some time ago by Brown and Henneaux \[11\]. We interpret the black hole as a state in the CFT and, since we know the central charge, the degeneracy of states for a given mass and angular momentum can be easily determined\[4\]. We find that the near-horizon BTZ geometry associated with our four dimensional black holes has an effective central charge, mass and angular momentum that yield a precise match between the degeneracy of the boundary conformal field theory and the four dimensional Bekenstein-Hawking formula.

Although the near-horizon geometries of our black holes are asymptotic to AdS\(_3\), they arise globally as orbifolds of anti-de Sitter space. The interpretation of the black hole spectrum as excitations of an associated boundary CFT suggests that recent analyses of the relationship between boundary conformal field theory and supergravity can be extended to orbifolds of AdS spaces \[8, 9, 11\].

### 2 Entropy From The Near Horizon Geometry

A simple construction of four dimensional black holes begins with three M5-branes that intersect orthogonally over a common string that carries momentum \[12, 13\]. Some energy is added to lift the system beyond the BPS limit. Wrapping this configuration on \(T^6 \times S^1\) gives the metric:

\[
\begin{align*}
\left. ds_{11}^2 \right| & = (H_1 H_2 H_3)^{-\frac{4}{3}} \left[ \left( -dt^2 + dx_{11}^2 + \frac{T_0}{r}(\cosh \delta_0 dt + \sinh \delta_0 dx_{11})^2 \right) + \right. \\
& + \left. (H_1 H_2 H_3)^{\frac{2}{3}} \left( \frac{1}{1 - \frac{r^2}{R^2}} dr^2 + r^2 d\Omega_2^2 \right) + \right. \\
& + \left. (H_1 H_2 H_3)^{-\frac{4}{3}} \left[ H_1 (dx_3^2 + dx_5^2) + H_2 (dx_6^2 + dx_7^2) + H_3 (dx_8^2 + dx_9^2) \right] \right].
\end{align*}
\]

\[2\] See also the early contribution by Carlip \[3\] and the recent works \[4, 5, 6\].

\[3\] For a review see \[6\].

\[4\] We follow \[2\] and assume that the CFT is unitary. This should be justified because the derivation of \[11\] only gives the central charge.
In this solution, the mutual intersection of the 5-branes lies along the circle \( S^1 \) with coordinate \( x_{11} \) and \( x_4, \ldots, x_9 \) lie along the 6-torus \( T^6 \). The three kinds of M5-branes are associated with the harmonic functions \( H_i = 1 + \frac{q_i}{r} = 1 + \frac{r_0 \sinh^2 \delta_i}{r} \) and the physical charges \( Q_i = \frac{1}{2} r_0 \sinh 2\delta_i, \ i = 1, 2, 3. \) In general, nonextremal solutions cannot be unambiguously interpreted in terms of constituent extremal branes. We therefore focus on the “dilute gas” regime \( \delta_i \gg 1, \ i = 1, 2, 3 \) where the 3 M5-branes form an inert background for momentum-carrying waves that travel along both directions of \( x_{11} \). In this limit the energy flowing along \( x_{11} \) is \( P = \frac{r_0}{8G} \cosh 2\delta_0 \) and the net momentum is \( P = \frac{r_0}{8G} \sinh 2\delta_0. \)

The six compact dimensions \( x_4, \ldots, x_9 \) will play no role in our argument. In fact, the \( T^6 \) could equally well have been replaced by a general Calabi-Yau manifold. An effective five dimensional geometry – comprising the four non-compact dimensions and \( x_{11} \) – can be found simply by omitting the last line of Eq. [1]. In the following we will analyze this five-dimensional metric from several points of view.

**Four dimensional interpretation:** Define the harmonic function \( H_0 = 1 + \frac{q_0}{r} = 1 + \frac{r_0 \sinh^2 \delta_0}{r} \) associated with the Kaluza-Klein momentum along \( x_{11} \). Then compactification along \( x_{11} \) yields a regular four dimensional black hole with metric:

\[
d s^2_4 = -(H_0 H_1 H_2 H_3)^{-\frac{1}{2}}(1 - \frac{r_0}{r}) d t^2 + (H_0 H_1 H_2 H_3)^{\frac{1}{2}}[\frac{1}{1 - \frac{r_0}{r}} d r^2 + r^2 d \Omega^2_2] \quad (2)
\]

The entropy implied by the horizon area of this black hole is \( S = A_{4G_4} = \frac{\pi^2}{G_4} \prod_{i=0}^{3} \cosh \delta_i \). In the dilute gas regime \( \delta_i \gg 1, \ i = 1, 2, 3 \) this gives:

\[
S = \frac{\pi}{G_4} \sqrt{Q_1 Q_2 Q_3 r_0} \cosh^2 \delta_0 \quad (3)
\]

**Three dimensional interpretation:** A useful perspective on the four dimensional black hole in Eq. [2] is achieved by reconsidering the five dimensional metric on \( t, x^1, x^2, x^3, x_{11} \). In the near-horizon region \( (q_i/r \gg 1) \) we can take \( H_i = 1 + \frac{q_i}{r} \to \frac{q_i}{r}, \ i = 1, 2, 3. \) The resulting metric takes a simple form in terms of the rescaled time \( \tau = tl/R_{11} \), the radial coordinate \( \rho^2 = 2R^2_{11}(r + r_0 \sinh^2 \delta_0)/l \) and the angular variable \( \phi = x_{11}/R_{11} \) with \( l = 2(Q_1 Q_2 Q_3)^{\frac{1}{2}} \) and \( R_{11} \) the radius of the compact \( x_{11} \) direction. The metric is:

\[
d s^2_5 = d s^2_3 + \frac{l^2}{4} d \Omega^2_2 \quad (4)
\]

\[
d s^2_3 = -N^2 d \tau^2 + N^{-2} d \rho^2 + \rho^2 (d \phi + N_\phi d \tau)^2 \quad (5)
\]

where:

\[
N^2 = \frac{\rho^2}{l^2} - \frac{2R^2_{11} r_0 \cosh 2\delta_0}{l^3} + \frac{R^4_{11} r_0^2 \sinh^2 2\delta_0}{l^4 \rho^2} \quad ; \quad N_\phi = \frac{r_0 R^2_{11} \sinh 2\delta_0}{l^2 \rho^2} \quad (6)
\]

This geometry is locally \( \text{AdS}_3 \times S_2 \) where the cosmological constant of the \( \text{AdS}_3 \) is \( \Lambda = -l^{-2} \) and the radius of the \( S_2 \) is \( l/2. \) The 3-geometry is recognized as the BTZ
black hole with mass and angular momentum:

\[ M_3 = \frac{2r_0 R_{11}^2}{l^3} \cosh 2\delta_0 \quad ; \quad 8G_3 J_3 = \frac{2r_0 R_{11}^2}{l^2} \sinh 2\delta_0 \]  

(7)

The BTZ black holes have a spectrum with \( M_3 \geq 0 \). The singularity in the causal structure is hidden behind a horizon when \( M_3 l \geq |8G_3 J_3| \). The AdS\(_3\) geometry is obtained by setting \( M_3 = -1 \), \( J_3 = 0 \) and is separated by a gap from the black hole spectrum. In the limit \( r_0 \to 0 \) at fixed \( \delta_0 \) and fixed \( Q_i \), we reach the \( M_3 = J_3 = 0 \) black hole. In terms of M theory this corresponds to three intersecting 5-branes without any momentum or non-extremality. The four dimensional extremal limit is achieved by taking \( r_0 \to 0 \) while sending \( \delta_0 \) and \( \delta_i \) to infinity to keep \( Q_i \) and \( Q_0 = \frac{1}{2} r_0 \sinh 2\delta_0 \) fixed. In three dimensions this is an extremal black hole satisfying \( M_3 = 8G_3 J_3 \). The horizons of the three dimensional effective BTZ black hole coincide with the four dimensional horizons, as does the singularity.

**Effective theory in 3d:** The coupling constant of the effective three-dimensional description can be identified as follows. We first write the five dimensional action in terms of the four dimensional coupling:

\[ \mathcal{L} = \frac{1}{16\pi G_4} \int \mathcal{R}^{(5)} d^4x \frac{d\phi}{2\pi} \]  

(8)

The Ricci-scalar on the five dimensional spacetime \( AdS_3 \times S_2 \) is simply the sum of the Ricci-scalar of each factor. We consider only spherically symmetric excitations and so the \( S_2 \) contributes an additive constant that can be omitted. The measure can be decomposed as:

\[ d^4x \frac{d\phi}{2\pi} = A_2 drdt d\phi = \frac{A_2}{2\pi R_{11}} \rho d\rho d\tau d\phi = \frac{A_2}{2\pi R_{11}} d^3x_{AdS} \]  

(9)

where \( A_2 = \pi l^2 \) is the area of the \( S_2 \). Combining the formulae we find the effective three dimensional action:

\[ \mathcal{L} = \frac{1}{16\pi G_3} \int \mathcal{R}^{(3)} d^3x_{AdS} \]  

(10)

where:

\[ \frac{1}{G_3} = \frac{1}{G_4} \frac{l^2}{2R_{11}} \]  

(11)

**Counting black hole states:** The BTZ black hole reduces to three dimensional anti-de Sitter space at large distances. It was shown in [11] that the algebra of diffeomorphisms of asymptotically \( AdS_3 \) spaces generates a conformal field theory on the cylinder at the boundary of spacetime. What is more, by explicit computation it was shown that the left and right moving Virasoro algebras both have a *classical* central charge:

\[ c = \frac{3l}{2G_3} \]  

(12)
It is tempting to identify the ground state of the CFT at infinity with \( A_{dS}^3 \) or a \( M_3^3 = -1 \), \( J_3^3 = 0 \) BTZ black hole. With this identification, the spectrum has a gap — the next higher level is the \( M_3^3 = J_3^3 = 0 \) black hole. It has been argued in [16] that in a supersymmetric theory the \( A_{dS}^3 \) spacetime would correspond to the Neveu-Schwarz vacuum while \( M_3^3 = J_3^3 = 0 \) is the Ramond vacuum.

The CFT at infinity has left and right moving Virasoro algebras whose zero modes are \( L_0^0 \) and \( \bar{L}_0^0 \). In terms of these generators, the mass and angular momentum are given by [17]:

\[
M_3^3 = \frac{8G_3}{l}(L_0 + \bar{L}_0) \quad (13)
\]
\[
J_3^3 = L_0 - \bar{L}_0 \quad (14)
\]

(Additive constants in the definition of \( L_0 \) and \( \bar{L}_0 \) have been omitted.) A state of given \( M_3^3 \) and \( J_3^3 \) is degenerate because there are many ways of exciting the oscillators of the CFT to yield the same macroscopic quantum numbers. Cardy’s formula for unitary CFTs [18] gives the entropy:

\[
S = 2\pi \left( \sqrt{\frac{cn_R}{6}} + \sqrt{\frac{cn_L}{6}} \right) \quad (15)
\]

where \( n_{R,L} \) are the oscillator levels of \( L_0 \) and \( \bar{L}_0 \). Using Eqs. (12)–(14) we find:

\[
S = \frac{\pi}{4G_3} \left[ \sqrt{l(lM + 8G_3J_3)} + \sqrt{l(lM_3 - 8G_3J_3)} \right] \quad (16)
\]

and so Eq. 7 and Eq. 11 gives:

\[
S = \frac{\pi}{G_4} \sqrt{Q_1Q_2Q_3r_0 \cosh^2 \delta_0} \quad (17)
\]

which agrees precisely with the macroscopic expression Eq. 3 that was deduced from the area of the black hole.

The entropy is a sum of two terms in Eq. 16 because it receives contributions from both sectors of the CFT. It has been suggested that the general geometric prescription for writing the entropy as a sum of two terms is [19]:

\[
S = S_R + S_L \quad ; \quad S_{R,L} = \frac{1}{2} \left( \frac{A_+}{4G} \pm \frac{A_-}{4G} \right) \quad (18)
\]

where \( A_\pm \) are the areas of the outer and inner horizons. This rule has previously been verified for extremal and near-extremal black holes in four and five dimensions, and it may hold in general. Interestingly, the two terms in Eq. 16 are also the sum and difference of the inner and outer horizon areas. The rule is therefore valid in three dimensions for general nonextremal black holes. Similar splits into inner and outer horizon contributions apply to the other thermodynamic variables.
3 The Relation to String Theory

So far we have employed units that are natural in general relativity, highlighting the fact that the calculation can be interpreted independently of conventional string theory. The connection to other ideas is clearest in standard string units. Then the quantization condition on the $M_5$-branes is $Q_i = n_i g l_s^3 / 4\pi V_2$ where $V_2$ is the volume of the part of the compact space transverse to the $M_5$, $g$ is the string coupling and $l_s = 2\pi \sqrt{\alpha'}$. The quantization condition on the Kaluza-Klein charge is similarly $Q_0 = n_0 g l_s^7 / 4\pi V_6$ and the gravitational coupling constant is $G_4 = \frac{1}{8} g^2 (2\pi)^6 (\alpha')^4 / V_6$.

In M-theory units $R_{11} = g \sqrt{\alpha'}$ and the Planck length is $l_p = (2\pi g)^{1/3} \sqrt{\alpha'}$. With these conventions we have:

$$l = \frac{2\pi l_p^3 (n_1 n_2 n_3)}{V_6^{1/3}} ; \quad G_3 = \frac{\pi l_p^3}{2 (n_1 n_2 n_3) \frac{1}{3} V_6^{1/3}}$$

and the effective central charge becomes:

$$c = 6 n_1 n_2 n_3$$

**The effective string:** Eq. (20) for the central charge normally emerges in string theory in a description of the effective dynamics of triply intersecting $M_5$-branes [12, 13, 20, 21]. Essentially, there are $n_1 n_2 n_3$ intersection strings of the $M_5$-branes and 6 momentum carrying degrees of freedom propagate on each of them. This picture has led to the effective string description of black holes, an efficient means of describing black hole scattering processes [22, 14]. Even though our description is reminiscent of the effective string in that there is a CFT with $c = 6 n_1 n_2 n_3$, there are differences in perspective. The effective string is usually introduced as a model parametrizing the collective excitations of the intersecting brane configuration and is therefore naturally localized close to the branes. In contrast, the present approach seems to associate the degrees of freedom with an auxiliary surface in the asymptotic $AdS_3$ space. The equivalence of the two interpretations requires a “holographic” principle in the sense advocated in [8, 10].

Parity symmetry in the effective three dimensional theory takes $J \rightarrow -J$ and interchanges the right and left sectors of the CFT. This forces a symmetric appearance of the two sectors. In contrast, the standard effective string of four dimensional black holes is chiral, with $(0, 4)$ supersymmetry. We do not rely on string theory, or even on supersymmetry, and in the absence of such structure the two sectors appear symmetrically. More details appear when the CFT is realized in string theory and then the right and left sectors turn out to be very different, albeit with the same central charge.

**The Decoupling Limit:** In this paper we have truncated the black hole metric to the near horizon region and studied its physics. This procedure can be justified exactly in limits where the collection of branes creating the black hole is decoupled from the asymptotic space. A suitable limit, in the spirit of [8], is $l_p \rightarrow 0$ with:

$$r / l_p^3 ; \quad r_0 / l_p^3 ; \quad \delta_0 ; \quad n_i ; \quad V_6 / l_p^6 ; \quad R_{11} \quad \text{fixed}$$

(21)
This limit automatically implies the dilute gas conditions $\delta_i \gg 1$. The decoupling limit essentially isolates the throat region of the black hole from the asymptotic spacetime. We can ensure that supergravity remains valid as a description of the throat by taking the quantum numbers $n_i$ to be large \([23, 8, 24]\).

The focus on the near horizon region may leave the inaccurate impression that our description does not apply to black holes in asymptotically flat spacetime. Recent calculations of scattering from black holes shed some light on this issue\(^5\). In such problems it is often useful to solve the black hole perturbation equations approximately, by independently treating the near horizon region and the asymptotically flat Minkowski space. Combining the results into a solution that is valid everywhere requires a “matching” region that is well described by either truncation of spacetime. The matching region is exactly the asymptotic AdS\(^3\) of the present treatment. The approximations of the scattering calculation can be justified at long wavelength where the observer in the asymptotically flat space sees the entire near horizon region, including the asymptotic AdS\(^3\), as one entity. In view of these results it is reasonable to identify the near horizon region with the internal structure of the black hole independently of the nature of the surrounding space.

4 Discussion

The construction in this paper displays the nature of the black hole microstates clearly: the Virasoro algebra arises from the diffeomorphisms acting on the asymptotic boundary. So, the different degenerate states correspond to different microscopic ripples placed along $x_{11}$. The tension between the no hair theorem and the microscopic interpretation of black hole entropy is therefore resolved in the manner discussed in \([27, 28]\): the microstates are hidden in an extra dimension and are therefore fully consistent with the no hair theorem in four dimensions. It is intriguing that the BTZ black hole appears to violate the spirit of the no hair theorem, in that the microscopic states reside in the noncompact dimensions. However, we are not aware of a precise no hair theorem that applies in asymptotically AdS\(^3\) spaces.

The recent conjectures \([8, 9, 10]\) that CFTs and AdS spaces are dual to each other involve the realization of the isometry groups of AdS spaces as conformal symmetries of the boundary. The BTZ black hole at the center of our work is constructed from AdS\(^3\) by taking a quotient by some of the isometries \([17, 7]\). This orbifolding breaks the isometry group $SO(2, 2) \simeq SL(2, R) \times SL(2, R)$ to its Cartan subalgebra $U(1)^2$. Nevertheless, the boundary at infinity has a conformal symmetry which we have exploited to match the Bekenstein-Hawking formula. This suggests that the relationship between boundary conformal field theory and supergravity can be extended to orbifolds of AdS spaces.

In this context, it is natural to ask whether the interior of the black hole is also described by the CFT, giving a completely unitary description of the spacetime. Horowitz and Ooguri recently considered the related problem for an extremal 3-brane and found that the 3+1 dimensional CFT dual to AdS\(^5\) describes regions on both sides

\(^5\)The considerable literature includes \([14, 25, 26]\).
of the horizon. This follows because the \( SO(4, 2) \) conformal group in \( 3+1 \) dimensions is realized as isometries of \( \text{AdS}_5 \) which include translations across the horizon [29]. This argument could be carried over without modification to \( \text{AdS}_3 \). However, the isometries that remain in the BTZ black hole after the quotienting of \( \text{AdS}_3 \) do not relate regions inside and outside the horizon, so we are unable to generalize the argument of [29]. It remains a fascinating problem to determine whether and how the asymptotic CFT describes the region behind the horizon, thereby giving a unitary and non-singular description of black hole spacetimes.

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