Inhomogeneous broadening of
a Mott insulator spectrum

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Abstract. We report an analysis of the inhomogeneous broadening of the excitation spectrum of a three-dimensional (3D) Mott insulator (MI) due to the presence of weak disorder. We present a theoretical model predicting the modification in the spectrum and compare the results to the experimental data obtained for a system of ultracold $^{87}$Rb atoms in a 3D optical lattice, where quasi-disorder is added by means of an auxiliary 1D optical lattice with non-commensurate wavelength. The model shows good agreement with the measured points, confirming a significant broadening of the MI spectrum resonances even for a very small height of the disordering potential.

Contents

1. Introduction 2
2. Inhomogeneous broadening of a MI spectrum 2
3. The model 4
4. Experimental procedure and results 9
5. Conclusions 12
Acknowledgments 12
References 12

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1. Introduction

Bose–Einstein condensates in optical lattices have proved to be a very interesting frame in which to study a range of various quantum phenomena. Recent research has in particular followed a natural connection with solid state physics, as laser standing waves provide ideal periodic potentials for the atoms, with no defects and with remarkable experimental control on the main parameters. Furthermore atomic Bose–Einstein condensates are made of weakly interacting particles. Interactions enrich the physical scenario with fascinating phenomena, such as sound propagation and superfluidity, and are responsible for the self-nonlinear behaviour of matter waves. By tuning the intensity of the standing waves creating the lattice, it is also possible to change from the weakly to the strongly interacting regime. In the latter case, a BEC in a three-dimensional (3D) optical lattice experiences the quantum phase transition from a superfluid to a Mott insulator (MI) state, first observed by Greiner et al \cite{1} in a pioneering experiment. The transition was predicted in the context of ultracold atoms in optical potentials as a clean experimental realization of the Bose–Hubbard (BH) model \cite{2}. However all possibilities have not yet been exhausted. In particular, recent works have enriched the system by introducing optical disorder, i.e. an additional energy scale in the BH model (for a theoretical review see \cite{3, 4}), by the use of laser speckles or two-colour lattices \cite{5–9}.

In this paper, we present an extended analysis of the theoretical model which has also been applied in \cite{9} to predict the observed broadening of the MI excitation resonances in the presence of a small amount of disorder. The main idea is that of considering the MI resonances inhomogeneously broadened as a consequence of the random distribution of the site energies due to the disordering potential, analogously to the inhomogeneous broadening of spectral lines. We consider in detail the case of quasi-disorder induced by a two-colour lattice. We then compare the predictions of the model to the experimental data obtained for a system of $^{87}\text{Rb}$ atoms in a 3D optical lattice (notably an experimental geometry different from that of \cite{9} in which 1D systems have been studied, also for values of disorder out of the perturbative regime). Starting from a MI state we experimentally add a small amount of disorder by superimposing along one direction a second weaker lattice onto the main one, with a non-commensurate wavelength. The model shows nice agreement with the measured data. The energy spectrum of the system is significantly modified and the resonances appear noticeably broadened even for very small disorder.

2. Inhomogeneous broadening of a MI spectrum

We start by considering the system of ultracold bosons in a regular optical lattice. The associated Hamiltonian is well defined in the context of the BH model \cite{2}:

$$\hat{H} = -J \sum_{\langle i, j \rangle} \hat{a}_i^\dagger \hat{a}_j + \frac{U}{2} \sum_j \hat{n}_j(\hat{n}_j - 1), \tag{1}$$

where $\hat{a}_j$ and $\hat{a}_j^\dagger$ are the annihilation and creation operators of one boson in the $j$th site of the lattice, $\hat{n}_j$ is the number operator and $\langle i, j \rangle$ indicates the sum on nearest neighbours. The two terms on the right-hand side of (1) refer respectively to the tunnelling energy ($J$) and to the on-site atomic interaction ($U$) and define the two significant energy scales. The competition between them determines the nature of the ground state of the system. In particular, assuming integer filling of the sites, in the limit in which interactions dominate ($U \gg J$), the system is in a MI state and the ground state is a product of Fock states localized at each lattice site. As number
fluctuations are significantly frozen out, the long-range phase coherence is lost. A characteristic feature of a system in the MI phase is to have a gapped excitation spectrum which is responsible for its insulating nature and vanishing compressibility.

What has first been done in order to experimentally individuate this quantum phase with ultracold atoms in a 3D optical lattice is: to look at the density profile after time-of-flight and to measure the excitation spectrum. Particularly, the latter gives a clear indication of entering the MI phase, as it appears with a dramatic modification with respect to the case of a superfluid system. Two different ways to measure the energy spectrum of a strongly interacting system are reported in literature namely adding a magnetic field gradient \[1\] or performing amplitude modulation of the height of the lattices with varying frequency \(\nu\) \[10\]. In both cases one measures the width of the central peak in the atomic density distribution after time-of-flight as an indicator of the energy transferred to the system. In the following, we will consider only the modulation technique. For a detailed description of the experimental procedure and results see \[10\]. In a typical experimental MI spectrum, it is usually possible to detect at least two excitation peaks. They correspond respectively to a modulation frequency close to the calculated interaction energy \(U\) and to \(2U\) and are consequences of the gapped structure of the energy spectrum. Naively speaking, for the simplest homogeneous case in the limit \(J \to 0\), the lowest lying energy excitation corresponds to removing an atom from one lattice site and adding it to a neighbouring one. For a MI this excitation has an energy cost \(\Delta E = U\), due to the on-site atom–atom repulsion. The second peak is commonly attributed to higher order processes and to the jumping of one atom to a neighbouring site with greater occupation number in the experimentally realized inhomogeneous systems.

If one now adds a weak disorder, the Hamiltonian has to include one more term describing the single site energies:

\[
\hat{H} = -J \sum_{\langle i,j \rangle} \hat{a}_i^\dagger \hat{a}_j + \frac{U}{2} \sum_j \hat{n}_j(\hat{n}_j - 1) + \sum_j \epsilon_j \hat{n}_j,
\]

where the bounded distribution \(\epsilon_j \in [-\Delta/2; \Delta/2]\) is due to the disordered potential. The presence of weak disorder introduces random site-to-site energy differences with the consequence that an elementary excitation costs now \(\Delta E = U + \delta_j\), where \(\delta_j = (\epsilon_{j+1} - \epsilon_j)\) is the energy difference between neighbouring sites, see figure 1. The excitation energy is not the same for all the particles, differently from the homogeneous MI case, and one expects to observe a broadening of the resonances at energies \(U\) and \(2U\). An experimentally accessible way to create site-to-site quasi-disorder in a lattice (main lattice) is to add a second weaker lattice (disordering lattice) with incommensurate wavelength. One expects that in a finite system this non-periodic modulation can
produce the same qualitative effects as the ones induced by a truly random potential, as suggested in recent theoretical works \cite{3, 4}. Theoretical simulations of the response of the system to a time modulation of the lattice height have been performed in \cite{11, 12} in the case of both a single and a two-colour lattice. Here we do not give an \textit{a priori} theoretical estimate of the energy transferred to the system by the modulation, but we predict a broadening of the experimentally observed resonances on the basis of a simple model derived from analogies with spectroscopy.

3. The model

The bichromatic potential resulting from the superposition of the two lattices (main lattice + disordered lattice) can be written as

\[ V(x) = V_1 \sin^2 k_1 x + V_2 \sin^2 k_2 x. \]  

We consider the limit \( V_2 \ll V_1 \) in which the lattice with wavenumber \( k_2 \) just produces a small perturbation of the main lattice with wavenumber \( k_1 \). In this case the minima of the potential \( V(x) \) can be approximated with the minima \( x_j = j\pi/k_1 \) of the unperturbed potential \( V_1 \sin^2 k_1 x \). We can calculate the deviation of the actual minima positions by expanding (3) in series around the points \( x_j \):

\[ V(x_j + \xi) \approx V_2 \sin^2 \tilde{k} j + \xi k_2 V_2 \sin 2\tilde{k} j + \xi^2 (k_1^2 V_1 + k_2^2 V_2 \cos 2\tilde{k} j), \]

where \( \tilde{k} = \pi k_2 / k_1 \). By requiring the stationarity of \( V(x_j + \xi) \) with respect to \( \xi \) one finds that the first-order correction to the minima position is

\[ \xi_j = -\frac{1}{2k_2 \cos 2\tilde{k} j + (k_1^2 V_1 / k_2^2 V_2)} \cos 2\tilde{k} j \approx -\frac{1}{2k_2 V_2} \cos 2\tilde{k} j. \]

For the typical parameters of the experiments described in the following section this correction is less than 0.2\% of the main lattice spacing. This means that the bichromatic lattice essentially introduces a compositional disorder, i.e. an inhomogeneity in the site properties, rather than a topological disorder, i.e. an inhomogeneity in the lattice topology. The energies in the potential minima \( \epsilon_j \) can be calculated as

\[ \epsilon_j = V(x_j + \xi) \approx V_2 \sin^2 \tilde{k} j \left( 1 - 2\frac{k_2^2 V_2}{k_1^2 V_1} \cos^2 \tilde{k} j \right) \approx V_2 \sin^2 \tilde{k} j, \]

where we have neglected the correction due to the finite \( \xi_j \) since \( V_2 \ll V_1 \). The energy differences between adjacent sites can be analytically calculated from (6) in the following way:

\[ \delta_j = \epsilon_{j+1} - \epsilon_j = V_2 [ \sin^2 \tilde{k} (j + 1) - \sin^2 \tilde{k} j ] = \frac{V_2}{2} [ \cos 2\tilde{k} j - \cos 2\tilde{k}(j + 1) ] \]

\[ = \frac{V_2}{2} [ \cos 2\tilde{k} j - \cos 2\tilde{k} j \cos 2\tilde{k} + \sin 2\tilde{k} j \sin 2\tilde{k} j \sin 2\tilde{k} ] \]

\[ = \frac{V_2}{2} [ \cos 2\tilde{k} j (1 - \cos 2\tilde{k}) + \sin 2\tilde{k} j \sin 2\tilde{k} ] \]

\[ = V_2 \sin \tilde{k} [ \cos 2\tilde{k} j \sin \tilde{k} + \sin 2\tilde{k} j \cos \tilde{k} ] = \Delta \sin \tilde{k} (2j + 1), \]

where the maximum energy difference \( \Delta = V_2 \sin \tilde{k} \leq V_2 \) depends on the height of the disordered lattice and on the ratio between the two wavelengths.

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In order to evaluate the lineshape of the inhomogeneously broadened resonances, we need an explicit form for the distribution of the energy shifts across the lattice. We first note that, when \( k_1 \) and \( k_2 \) are not commensurate, the collection \( \{ \delta_j \} \) of energy differences corresponds to a non-periodic sampling of the periodic function

\[
\delta(\theta) = \Delta \sin \theta
\]  

at angles \( \theta_j = \tilde{k}(2j + 1) \) for \( j \in [1, j_{\text{max}}] \), where \( j_{\text{max}} \) is the number of occupied lattice sites, as schematically sketched in figure 2(a). All these points can be translated into the same first period of the sine function, as shown in figure 2(b). This representation suggests that in the limit \( j_{\text{max}} \gg 1 \) one can consider a uniform sampling of the \( \theta \)-axis, provided that the sampling frequency is sufficiently different from integer multiples of the oscillation frequency, i.e. that the ratio between the two wavenumbers \( k_1 \) and \( k_2 \) is sufficiently far from a simple ratio of integer numbers. In this case the density probability of the energy differences can be explicitly evaluated with a simple graphical argument. The probability \( p(\delta) \, d\delta \) of finding an energy difference in an interval \( d\delta \) centred around \( \delta \) is given by

\[
p(\delta) \, d\delta = \frac{d\theta}{\pi},
\]

as illustrated by figure 2(c), in which we have restricted our consideration to the interval between \( -\pi/2 \) and \( \pi/2 \) in which \( \delta(\theta) \) is monotonic. From this it follows that the density of probability \( p(\delta) \) can be written as

\[
p(\delta) = \frac{1}{\pi} \frac{d\theta}{d\delta}
\]
and by definition it automatically satisfies the norm condition \[ \int p(\delta) \, d\delta = 1, \quad \text{since} \quad \int_{-\pi/2}^{\pi/2} d\theta = \pi. \]

By inverting (8) one obtains the following analytical expression:

\[
p(\delta) = \frac{1}{\pi \Delta} \frac{1}{\sqrt{1 - (\delta^2/\Delta^2)}}.
\]

(11)

This function is plotted in figure 3 (solid line). One sees that this distribution diverges at the extremes \(-\Delta\) and \(\Delta\), owing to the sinusoidal dependence in (6). In the pure random case, in which one has a flat distribution of \(\epsilon_j\) between 0 and \(\Delta\), \(p(\delta)\) would be a triangular function, that is plotted as dashed line in the same figure.

Having derived the function \(p(\delta)\), one can calculate the line shape of the MI resonances in the presence of the inhomogeneous distribution of on-site energies in (6). We assume that a resonance of the MI spectrum in the uniform case can be described by a Gaussian line profile

\[
f(\nu) = A \exp \left[ -\frac{(\nu - \nu_0)^2}{2\sigma^2} \right].
\]

(12)

where \(\nu\) is the excitation frequency, \(\nu_0\) is the position of the excitation peak (\(U\) for the uniform system) and \(\sigma\) is the width of the resonance. This assumption is well justified by the results of the experimental works [1, 10] in which the excitation spectrum of the MI has been measured. In the limit of vanishing tunnelling \(J \to 0\) and for the uniform system \(\epsilon_j = 0\) the width of the resonance is expected to vanish, since excitations can be produced only at energy \(U\). In the real spectra measured in the experiments many different effects may contribute to the finite width. There could be physical contributions, like the effect of the finite tunnelling \(J\) and the influence of thermal fluctuations connected to the finite temperature \(T\) of the system. Moreover, we expect a broadening to be present due to the weakly varying harmonic potential always present in the experiments. Finally, instrumental broadenings intrinsic to the excitation technique (e.g. power broadenings and interaction-time broadenings) could contribute as well.
Figure 4. Inhomogeneous broadening of the MI. At $\Delta = 0$ the resonance is described by the Gaussian line profile in (12). The other lines for $\Delta > 0$ are calculated with the integral of (13) describing the convolution of the homogeneously broadened line profile in (12) with the distribution of energy shifts in (11).

We assume that in the presence of disorder the resonance at $\nu_0$ is locally shifted by the energy differences between adjacent sites $\delta_j$, thus becoming $\nu_0 + \delta_j$. Starting from this assumption, we calculate the inhomogeneously broadened line profile of the Mott resonances as

$$g(\nu) = \int_{-\Delta}^{\Delta} f(\nu - \delta) p(\delta) \, d\delta.$$  (13)

This convolution integral cannot be calculated analytically. The result of a numerical integration is shown in figure 4 for varying values of $\Delta$ from 0 to $4\sigma$. In the following section, devoted to the experimental results, we will quantitatively compare the measured excitation spectra with the broadened line profiles calculated with this model.

Before discussing the experiment, we note that in (1) and (2) $J$ and $U$ are assumed to be constant. Actually, for the bichromatic potential in (3) we expect also $J$ and $U$ to become functions of the position, since the height of the optical barriers is modulated in space. In the limit $V_2 \ll V_1$ the corrections to $J$ and $U$ are sufficiently small to neglect their role in the above discussion. Let us derive them explicitly. We can start by assuming that in the bichromatic potential the effective height of the optical barriers can be taken as

$$\tilde{V}_j = \epsilon_j + \frac{1}{2}(\epsilon_j + \epsilon_{j+1})$$  (14)

with reference to figure 5, in which $\epsilon_j$ and $\epsilon_{j+1}$ are the energies at the bottom of neighbouring sites and $\epsilon_{j+(1/2)}$ is defined as the energy at the peak of the potential barrier between sites $j$ and $j + 1$. By using (6) and applying some trigonometry, as in the derivation of (8), one obtains

$$\tilde{V}_j = V_1 + V_2 \sin^2 \frac{k}{2} \cos k(2j + 1).$$  (15)
The maximum correction to the barrier height $V_1$ of the main lattice is thus

$$
\delta \tilde{V} = V_2 \sin^2 \frac{\tilde{k}}{2}.
$$

(16)

The quantities $J$ and $U$ depend on the height of the optical lattice and can be explicitly evaluated using the formulae of [13]

$$
J = \frac{4}{\sqrt{\pi}} E_{R1} \left( \frac{V_1}{E_{R1}} \right)^{3/4} \exp \left[ -2 \left( \frac{V_1}{E_{R1}} \right)^{1/2} \right],
$$

(17)

$$
U = \sqrt{\frac{8}{\pi}} k_1 a_s E_{R1} \left( \frac{V_1}{E_{R1}} \right)^{3/4},
$$

(18)

holding for an isotropic 3D optical lattice with wavenumber $k_1$ and height $V_1$ in all directions, where $E_{R1} = \hbar^2 k_1^2 / 2m$ is the recoil energy, $m$ the atomic mass and $a_s$ the scattering length. If just one of the three lattices ($\hat{x}$-axis) is perturbed by the presence of the additional lattice with wavenumber $k_2$, one can write the following scaling relations

$$
J_x \sim \tilde{V}^{3/4} \exp \left[ -2 \left( \frac{\tilde{V}}{E_{R1}} \right)^{1/2} \right],
$$

(19)

$$
U \sim \tilde{V}^{1/4},
$$

(20)

in terms of the effective height $\tilde{V}$ of the optical barriers. The different exponent in $U$ is due to the fact that the $V_1^{3/4}$ dependence in (18) can be factorized into equal contributions $V_1^{1/4}$ of each of the three lattices to the strength of the confinement in the potential wells. When just one axis of the lattice is modified and an effective lattice depth $\tilde{V}$ is introduced along that axis,
the term $V_{1}^{3/4}$ becomes $V_{1}^{2/4} \tilde{V}_{1}^{1/4}$, giving the dependence in (20). By writing $\tilde{V} = V_{1} + \delta \tilde{V}$ and expanding (19) and (20) to the first order in $\tilde{V}$ one obtains the following corrections to $J_{x}$ and $U$:

$$\frac{\delta J_{x}}{J_{x}} = \left( \frac{3}{4} - \sqrt{\frac{V_{1}}{E_{R1}}} \right) \frac{V_{2}}{V_{1}} \sin^{2} \frac{\tilde{k}}{2},$$

$$\frac{\delta U}{U} = \frac{1}{4} \frac{V_{2}}{V_{1}} \sin^{3} \frac{\tilde{k}}{2},$$

where (16) has been used. For the parameters of the experiments described in this paper $\delta J_{x}/J_{x} < 7\%$ and $\delta U/U < 0.4\%$. These corrections are small and can be neglected: the leading effect of the secondary lattice is the modification of the on-site energies $\epsilon_{j}$, which completely accounts for the observed modification of the excitation spectra.

4. Experimental procedure and results

Our $^{87}$Rb BEC samples are produced in a Ioffe–Pritchard magnetic trap with axial frequency $2\pi \times 8.7(1)\text{Hz}$ and radial frequency $2\pi \times 90(1)\text{Hz}$. They typically consist of $\sim 10^{5}$ atoms in the hyperfine ground state $|F = 1, m_{F} = -1\rangle$. The 3D optical lattice is obtained by retroreflecting three orthogonal beams, each focused on the BEC with a waist of $\sim 100 \mu\text{m}$. The lattice beams are derived from a Ti:Sa laser operating at a wavelength $\lambda_{1} = 830.7(1)\text{nm}$. The frequency of each beam is shifted with respect to the others by about 10 MHz to cancel cross interference terms. The second lattice is derived from a diode laser operating at a wavelength $\lambda_{2} = 1076.8(1)\text{nm}$. It is superimposed to the main lattice only along one direction, and focused on the atoms with a waist of $\sim 300 \mu\text{m}$. See figure 6(A) for a scheme of our experimental configuration and figure 6(B) for a picture of our actual two-colour lattice potential. In order to adiabatically load the ground state of the resulting two-colour lattice we increase the intensity of all the beams to the final value with a 100 ms long exponential ramp with a time constant of 30 ms. The final height of
Figure 7. (A) Excitation spectrum for $s_1 = 25$ and $s_2 = 0$. We report the width of the central peak in the time-of-flight distribution after excitation, as a function of the modulation frequency, see the text for details. (B) Adding a small amount of disorder $s_2 = 0.2$ the resonance peaks start to broaden and (C) completely disappear for $s_2 = 0.5$. Comparison between experimental points and theoretical curves: the solid line at $s_2 = 0$ is a double-Gaussian fit to the experimental data, while the dashed curves at $s_2 > 0$ are obtained from the convolution with the expected distribution of energy differences in the quasiperiodic potential.

the main lattice is fixed at $s_1 = 25$, where the height of the potential is expressed in units of the recoil energy, i.e. $V_1 = s_1 E_{R1}$ and $E_{R1} = h^2/(2m\lambda_1^2) \simeq h \times (3.3 \text{ kHz})$. This guarantees that the system is deeply in the MI regime. We vary instead the height of the second lattice to different values $s_2$, in terms of the recoil energy $E_{R2} = h^2/(2m\lambda_2^2) \simeq h \times (1.8 \text{ kHz})$.

At this point, in order to measure the excitation spectrum we modulate the intensity of the main lattice beam just along the direction of disorder with an amplitude of 30% for 30 ms. After the modulation we decrease the intensity of all the beams in 15 ms to $s_1 = 5$ and $s_2 = 0$, where the system has reverted back to the superfluid regime, then wait 5 ms for the system to thermalize and switch off all the confining potentials. We then image the atomic cloud density profile after 20 ms time-of-flight and take the width of central peak in the density distribution as a measure of the energy transferred to the system during the modulation. The connection between the experimental observable, i.e. the width of the central peak in the density distribution, and the energy transferred to the system has already been addressed in [1, 10, 11]. We note that at $s_1 = 25$ the tunnelling time ($\sim 300 \text{ ms}$) is much longer than the duration of the modulation (30 ms). Consequently the excitations are mostly produced along the $x$-direction, i.e. along the direction where we have introduced disorder, as tunnelling is enhanced by the resonant reduction of the barrier height.

In figures 7(A)–(C) we report the measured width of the central density peak as a function of the modulation frequency for different heights of the disordering lattice $s_2 = 0, 0.2$ and $0.5$, so that it always is $\Delta < U$. In figure 7(A), corresponding to $\Delta = 0$, one clearly detects the presence of a first peak at 1.7(1) kHz and of a second peak at twice the frequency 3.4(1) kHz, typical signatures of an ordered MI state. The points in figures 7(B) and (C), corresponding to the disordered case, show a significative broadening of the MI spectrum that loses the distinguishable resonance profile even for a small amount of disorder. The solid curve in figure 7(A) is a double-Gaussian fit of the experimental data. The dashed curves in figures 7(B) and (C) are obtained from the
Figure 8. Measurements of the width of the central density peak after modulation at 1.7 kHz (filled circles) and 2.5 kHz (empty circles) for $s_1 = 25$ and different heights of the disordering lattice. The lines are calculated from the model of inhomogeneous broadening of the MI resonances assuming zero tunnelling and a site-to-site energy difference distribution due to the presence of the disordering lattice.

convolution of this curve with the expected distribution of energy differences in the quasiperiodic case as illustrated above. To have a more quantitative comparison between the model and the experimental data we show in figure 8 the measured excitation corresponding to the MI peak at 1.7 kHz (filled circles) and to the minimum between the two peaks at 2.5 kHz (empty circles) as a function of the disorder height $s_2$. The peak excitation starts being suppressed even for very small disorder $s_2 \leqslant 0.5$, while the minimum starts to grow exceeding the peak excitation for $0.5 < s_2 < 0.8$. We recall that the experimental parameters introduced in the model are only the ones derived from the double-Gaussian fit of the actual MI spectrum in the absence of disorder and that the curves in figure 8 are not fitted to the experimental points as a function of $s_2$ but they are calculated according to the inhomogeneous broadening model presented above in the limit of zero tunnelling. We find the agreement between experiment and theory to be quite good, demonstrating the validity of the inhomogeneous broadening model described above. This mechanism has a spectroscopic analogy in the inhomogeneous broadening of spectral lines, occurring when each atom or molecule absorbs or emits light at different frequencies (as happens in the case of crystalline solids due to the presence of defects). We also notice that the accordance between calculated and measured points confirms that the broadening of the spectrum is not related to a possible heating of the system due to the loading of the disordering second lattice, which is also confirmed by the experimental observation of a pure Bose–Einstein condensate when the lattices are adiabatically switched back off. For the aim of completeness we point out that we have also applied the model to a different experimental geometry and in a wider range of disorder, also showing good agreement with experimental data until $\Delta < U$ [9].
5. Conclusions

We have analysed the modification of the MI state due to the presence of weak site-to-site disorder. We have modelled the system assuming an inhomogeneous broadening of the MI excitation spectrum resonances due to the site-to-site energy difference distribution. A comparison of the model with experimental data gives good agreement. We observe that the spectrum is significantly broadened even for a very small amount of disorder. The MI structure thus seems to be extremely fragile if subjected to this site-to-site inhomogeneity. In previous experiments [14], the MI phase has instead appeared quite resistant to modulation of the local chemical potential on a scale longer than the lattice site distance. For the inhomogeneity typically introduced by a confining magnetic potential, the system maintains at least locally the properties of the homogeneous system [15]. In the presence of this two-colour lattice disorder however, one expects a more intimate alteration of the structure, eventually leading to a new Bose glass phase when the disordering potential is strong enough.

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