Elementary students’ representations in solving word problems

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Abstract. The mathematical representation is essential for students as a means of communicating their ideas so it will be easier for them to solve the problems that they may face. In solving word problems, students learn how to develop their own models because, in principle, every individual has their way of understanding things. The purpose of this study was to describe the representation of third-grade elementary school students in solving mixed arithmetic operations word problems. Students’ representation data were obtained from the results of the test related to word problems, and task-based interviews with two students. The results showed that the students used images, mathematical expressions, and word representations. This research implies that the teachers need to develop tasks for the students along with the answer alternatives by taking into consideration various representations that might be provided by the students.

1. Introduction
The mathematical representation of this research is the mathematical expression of the idea in the form of images, symbols, mathematical expressions, words or written text that are used to show the students' thinking in solving problems in a certain way [1,2,3]. The students' ability in solving mathematical problems is closely related to the way how the students understand math knowledge. According to Bruner [4,5], to understand the knowledge, a person will go through three stages of representation, namely: enactive, iconic and symbolic stage. In the enactive stage, it will be easier for the students to use concrete objects that are related to the concept being studied, so students learn to use or to manipulate objects directly. This is called concrete model stage. In the iconic stage, students attempt to comprehend knowledge related to mental images, which are the description of concrete objects that have been manipulated. Furthermore, students do not directly manipulate concrete objects in this stage but only using visual images, drawings or diagrams outlining concrete activities or situations enacted in the enactive stage. In the symbolic stage, students change the shape of the images into symbolic systems, so the students directly manipulate symbols and no longer associated with the objects. Bruner’s stages then refined by Lesh [6]. Lesh [6] split the iconic stage into manipulative and pictorial models, the symbolic stage into the spoken language and written symbols. On the other hand, Ishida [7] and Manoy [8] classified representation into: (a) the realistic model, which is the representation with realistic or real-world situation, (b) manipulative model, the representation with manipulative aids, (c) Pictorial model, the representation with static figures or pictures, (d) language model, the representation with written everyday language, and (e) symbolic model, that is the
representation with mathematical symbols. In solving word problems related to mixed arithmetic operations, students learn to represent their ideas in different ways.

The question in this research is how the representation of third-grade elementary school students in solving word problem related to mixed arithmetic operations, while the purpose of the research is to describe the representation of third-grade elementary school students in solving word problems related to arithmetic operations.

2. Methods
This study includes qualitative research because it examines the phenomenon in its real-life context and tries to describe the phenomenon. Subjects of this study were third-grade primary school students, with the following criteria: (a) They have studied mixed arithmetic operation lesson consisting of addition, subtraction, multiplication, and division; and (b) They can communicate ideas/ opinions or thoughts orally and in written simply and clearly. Two chosen subjects who meet the criteria were interviewed by employing task-based interviews. Data were collected using the test and interview, while the instruments were word problems and interview guidelines. The word problems provided presented two mathematical problems. Results of interviews were analyzed using the following five steps:

- Selecting and identifying data related to ways of solving the problems by the subjects, and then generating code in many ways presented.
- Writing data set related to the ways of solving the problems by the subjects.
- Checking the validity of the data, this was done by comparing and examining written answers and clarified them with interview responses. If there is a match between the written answers and the response data, so it can be concluded that the data obtained are valid. In case of the data obtained are not valid, then interviews need to be re-conducted until valid data were obtained.
- Establishing the data that is writing the data set related to the ways the subjects solved the problems so conclusions can be drawn from the data. The data summarized were the subjects’ responses regarding the ways they solve the problems.
- Drawing conclusions from the collected data.

3. Result
The research subjects were given two word problems to describe their representations related to mixed arithmetic operations which were the word problem 1 (W1) and the word problem 2 (W2). Next, it was explored in more detail the mathematical representations of the two subjects –the subject 1 (S1) and the subject 2 (S2) Below are the word problems used in this research.

W1: Mrs. Janet bought a box of eggs containing 20 eggs. The eggs were placed into several plastic bags. Each plastic bag contained three eggs. How many plastic bags should Mrs. Janet prepare for all eggs? Provide as many possible ways to answer this problem.

W2: Today Dinda had a birthday party, and she invited 25 friends. Dinda set up some tables, and four people sit at each table. How many tables were needed to accommodate all her friends? Provide as many possible ways to answer this problem.

**Figure 1.** The problem given to students

The representations by S1 and S2 in solving the word problems involved mathematical expressions. This is in line with Hwang et al. [9] that state that representation means the process to make abstract models, symbols or concepts to describe inter-sentence relations. S1 and S2 presented their representations because every individual has different representations [10,11,12]. S1 and S2 also
used another representation called image representation. In solving the problem, drawing an image might appear as one of the representations [13,14,15].

3.1 Representations by S1
Data or information provided by S1 of W1 and W2 show the consistency of information (valid) so that the data displayed to answer the research question can use the W1 data. Here some representations of solving W1 by S1 are presented in Figure 2.

![Figure 2. Representations by S1](image_url)
Based on the interview data, the first that came into S1’s thinking was to do the multiplication of 3 that the result was close to 20. S1 mentioned: \(3 \times 5 = 15, 3 \times 6 = 18\) but if \(7 \times 3 = 21\), then what is used was \(3 \times 6\) because \(3 \times 6 = 18\) then the result will be 6.2, but the first way provided by S1 was by using the division. After completing the division, then S1 drew the image as the next step. First, S1 drew 20 circles in one box. According to S1, the image shows that there are 20 eggs in a box. In the next picture, every three circles were encircled into one except for the two last circles. According to S1, he gave mark for every 3 eggs that were put in the bags made in the next image, while the last two circles were still included in the bag (representation 1). In the next picture, S1 put every three circles in one box while the seventh box contained only two circles. According to S1, there was no relation between the first and the second way. The representation 2 provided by S1 was correct. In the next representation, S1 wrote \(6 \times 3 = 18\). if there are 20 eggs, then \(20-18 = 2\). So, if there is the remainder, it was 6.2. According to S1, this idea was obtained by applying the multiplication that has been learned previously. The third way used is related to the first and second way. It can be concluded that the representation 3 was correct, S1 confirmed that S1 did the multiplication and remainder was 2. So, 2 at 6.2 means the remainder (representation 4). The researcher suggested the way to make it easier for the readers (peers) to understand. The researcher gave the option of writing 6.2 or 6 with a remainder of 2 and S1 chose the second so that all previous writing used 6.2 crossed without being given the cue by the researcher. S1 changed it to 6 with a remainder of 2. Next, clarification on representation 4 was done. In doing this, S1 20 subtracted from 3 for several times and the result still show that there was a remainder of 2. Looking at the results in the representation 4, there was a mistake in the way of writing \(≠ 20-317-3\), and so on, which should have been written as \(20-3 = 17\) and comma was needed to be placed or in the next line S1 should write \(17-3 = 14\), and so on. Based on the explanation of the completion, representation 4 was then declared incorrect. The following clarification is regarding representation 5 which was done by way of summation. S1 summed 3 for six times and the result was 18. S1 wrote \(6 \times 3 = 18\). According to S1, all the summations made was the same as \(6 \times 3\). Based on the written result by S1 (representation 5), there was a way of writing error because \(3 + 3 ≠ 6 + 3\) and so on, which should have been written as \(3 + 3 = 6\) and place a comma or \(6 + 3 = 9\) and so on should be written in the next line. Based on the explanation above, representation 5 was then considered incorrect. In the written reply before the clarification, S1 only answered until representation 5. The next answers raised when clarification was in progress. Consider the following clarification. When the researcher tried asking clarification on a number line, it turned out that S1 understood what is going to implement. S1 made the number line and wrote the skip correctly (skip counting by three) as representation 6. Although the line made was not straight and the dots placement, was not symmetrical in the distance, but it can be concluded that S1 had a good understanding of what he was doing. Completion of the immediately represented with “porogapit” method and directly did the long division as the representation 7, and it was correct. The researcher then asked for other possible answers by joining the existing answers. To respond to this question, S1 provided an answer as representation 8 and answered any questions from the researcher correctly. As seen on the representation 8, there was a mistake in drawing it, for in the image there should be no operation symbol. Based on the explanation, completion of the representation 8 was wrong. As the attempt to have another representation, S1 used the same way as in representation 5 but with the different writing, as representation 9, and the result was correct. Researcher still asked about the possibility of other ways that can be made. S1 answered perhaps by combining addition and subtraction. S1 made 20 circles in one box and wrote \(6 \times 3 = 18\). By way of long reduction, S1 subtracted 20 from 18, and the remainder was 2 (representation 10). S1 the explained that the answer is still seven bags. Researcher still gave an opportunity for S1 to provide any other alternatives. However, S1 responded "no" which means that S1 had no more idea to answer that question. It turned out that S1 could do ten different representations.

Based on the analysis of written answers and clarifications (wrong answers excluded), the researcher concluded that in completing W1, S1 used seven correct ways which were ordinary division
(Representation 1) and the long division (Representation 7), drawing (Representation 2), mixed multiplication (Representation 3 and 10), the addition (Representation 9), and the number line (Representation 6). There were some images as well as long divisions that appear. Ideas generated came from the previous ideas, so the way of the completion made by S1 was interrelated.

3.2 Representations by S2

The data or information provided by S2 of W1 and W2 show the consistency of information (valid) so that the data displayed to answer the research question could use the W1 data. Several representations made by S2 were presented in Figure 3.

![Figure 3. Representations by S2](image)

In completing W1, S2 directly put three eggs and insert them into each plastic bag. When doing so, S2 just counted and did not use props prepared by the researcher. After collecting the eggs, the first thing to do that S2 divided twenty by three and the result was six with a remainder of 2. Written result of Representation 1 shows that there was a mistake in writing because S2 did not write the remainder of 2. Based on the explanation, completion on representation one was declared incorrect. S2 finished second way by drawing. According to S2, there are seven images which were six plastic bags containing three eggs each and the image of 2 eggs which are not in the bag (Representation 2). Both representations made were correct. The researcher asked S2 to write the results of his thinking. S2 was silent and took some time to think, and later researcher intervened with the question of how many as containing three eggs and how many are the rest. Finally, S2 wrote $3 \times 6 = 18 + 2 = 20$ (S41096). S2 used a mix of multiplication and addition. Based on the results written by S2 in the representation 3, there was a mistake in writing $3 \times 6 \neq 18 + 2$, which should have been written as $3 \times 6 = 18$ and placed
a coma or on the next line should be written as $18 + 2 = 20$. Based on this, the representation 3 was declared incorrect. The researcher asked S2 about how S2 did $3 \times 7$. The answer was because the result was 21. Next, the researcher asked S2 again why $3 \times 7 = 21$. S2 thought that if it was added, but later S2 revised the answer that when subtracted from one the result is 20. This means that S2 used a mix of multiplication and subtraction in the representation 4. base on the written result for Representation 4, there was mistake in the way of writing $3 \times 7 \neq 21 - 1$, which should have been written as $3 \times 7 = 21$ and put a coma or on the next line, it should be written as $21 - 1 = 20$. Based on the explanation, completion in representation 4 was declared false.

Furthermore, the researcher tried to ask S2 other ways to resolve the problem by asking “how else it can be done?” S2 spontaneously answered by addition- $3 + 3 + 3 + 3 + 3 + 3 + 2 = 20$ (Representation 5). Completion for representation 5 was correct. Next answer provided by S2 was by subtraction. S2 wrote $20 - 3 - 3 - 3 - 3 - 3 = 20$ (Representation 6). Completion of the representation 6 was correct. The next question was focused on the ideas that have been raised by S2. According to S2, there is a relation between the first and second ideas concerning the way of thinking to get answers which were from the eggs that had been grouped.

Based on the analysis of the written answers and the clarifications (wrong answers excluded), the researcher concluded that in completing W1, S2 used three representations to solve the problem: drawing (Representation 2), addition (Representation 5) and subtraction (Representation 6). Ideas generated were from the previous ideas of eggs that had been grouped. S2 solve the problem in an interrelated way.

4. Conclusion

Students solved word problems used images, mathematical expressions, and word representations. Students used pictorial representations to illustrate the completion of word problems, mathematical expressions to write arithmetic operations, while the representation of words or written text used by students to write down the information that was known and being asked. There were several representations of the images that appear as well as long division. Ideas generated came from the previous ideas, so the ways to solve the problem provided by the subject were inter-related.

References
[1] Pape S J and Tchoshanov M A 2001 The Role of Representation(s) in Developing Mathematical Understanding *Theory into Practice* **40** 118
[2] NCTM 2000 *Principles and Standards for School Mathematics* (Reston, VA: NCTM)
[3] Rosengrant D 2005 *An Overview of Recent Research on Multiple Representations* (Rutgers: The State University of New Jersey)
[4] Hudojo H 1988 *Mengajar Belajar Matematika* (Jakarta: Depdikbud Dirjen Dikti PPLPTK)
[5] Tall D 1990 *A Versatile Theory of Visualisation and Symbolisation in Mathematics*
[6] Lesh R 1981 Applied Mathematical Problem Solving *Educational Studies in Mathematics* **12**
[7] Ishida T 1984 A Study on the Representational System in Mathematics Teaching *International Conference for Psychology of Mathematics education* (Australia: Sydney)
[8] Manoy J T *Variasi Model Pemecahan Masalah Matematika yang Terkait dengan Operasi Hitung Campuran oleh Siswa Sekolah Dasar* [Dissertation] Surabaya: Unesa 2011
[9] Hwang W Y, Chen N S, Dung J J and Yang Y L 2007 Multiple Representation Skills and Creativity Effects on Mathematical Problem Solving using a Multimedia Whiteboard System *Journal of Educational Technology & Society* **10** 191
[10] Rowe H L and Trickett E J 2017 *Student Diversity Representation and Reporting in Universal School-Based Social and Emotional Learning Programs: Implications for Generalizability* (USA: Springer)
[11] Krawec J L *Problem Representation and Mathematical Problem Solving of Students of Varying Math Ability* [Dissertation] Miami: University of Miami 2010
[12] Boonen A J H et al. 2014 *The Role of Visual Representation Type, Spatial Ability, and Reading*
Comprehension in Word Problem Solving: An Item-Level Analysis in Elementary School Children *International Journal of Educational Research* **68** 15

[13] Carreira S 2015 Mathematical Problem Solving Beyond School: Digital Tools and Students Mathematical Representations *Selected Regular Lectures from the 12th International Congress on Mathematical Education* 93

[14] Rubin A, Storeygard J and Koile K 2015 Supporting Special Needs Students in Drawing Mathematical Representations *The Impact of Pen and Touch Technology on Education* 57

[15] Earnest D 2015 From Number Lines to Graphs in the Coordinate Plane: Investigating Problem Solving Across Mathematical Representations *Cognition and Instruction* **33** 46