Abstract

The Higgs model is generalized so that in addition to the radial Higgs field there are fields which correspond to the themasy and entropy. The model is further generalized to include state and sign parameters. A reduction to the standard Higgs model is given and how to break symmetry using a non-zero VEV (vacuum expectation value) is shown. A 'fluid rotation' can be performed on the standard Higgs model to give a model dependent on the entropy and themasy and with a constant mass.

1 Introduction

1.1 The principles involved.

One of the outstanding problems of particle physics is whether the Higgs’ field exists; these hypothetical fields are conjectured to make non-Abelian gauge fields \(A_i^a\), describe massive particles as needed by the standard particle physics models. There are \(\exists\) alternatives to Higgs’ scalar fields which use fluids instead. Apart from it being a good idea to have alternative models, the reason for producing these particular fluid alternatives can be addressed after outlining the principles below:

The principle that: ”All fundamental fields are gauge fields”. The success of non-Abelian gauge vector fields in describing fundamental interactions suggests the principle that “ALL FUNDAMENTAL FIELD ARE GAUGE FIELDS.” Taken at face value this implies that the Higgs fields used in symmetry breaking should also be a gauge field: as it is a scalar field at first sight this seems impossible. A way around this is to have a collection of scalar fields with the properties that: firstly they encompass the standard Higgs mechanism, secondly they form a gauge system. This can be realized by using the scalar field description of a perfect fluid, and then “charging” it via \(\partial_a \rightarrow D_a = \partial_a + ieA_a\), as described below.
The: "Extreme principle of equivalence." The extreme principle of equivalence is that "There is only one concept of mass in physics." [2], and this extended principle requires the equivalence of these masses to all other masses, including those generated by symmetry breaking mechanisms.

The: "Redundancy of ad hoc procedures." This principle is that "Any principled explanation (perhaps not only scientific) of anything is preferable to an ad hoc calculational procedure which produces an equivalent or inferior result." It leaves open what to choose if the ad hoc calculational procedure produces a superior result, or how to address the quality of the result. In the present case Higgs’ scalar fields are ad hoc whereas fluids might arise from the statistical properties of the non-Abelian gauge fields.

1.2 Application of these principle to symmetry breaking.

Re-writing procedure. There are well known techniques by which stresses involving scalar fields can be rewritten as fluids, so that it is possible to re-write the standard model with fluids instead of Higgs scalars. My first attempt at using fluids for symmetry breaking [2] was essentially to deploy these rewriting procedures to convert the scalar fields to fluids; the drawback of this approach is the fluids that result are somewhat unphysical, however an advantage is that symmetry breaking occurs with a change of state of the fluid.

Velocity potential method. My second attempt [3] used the decomposition of a perfect fluid vector into several scalar parts called vector potentials and identifying one of these with the radial Higgs scalar; thus the theory is a generalization of the standard theory, so that any experimental verification of the standard theory does not negate it. The gauge fields are then introduced by using the usual covariant substitutions for the partial derivatives of the scalars, for example φ = ∂φ → ∇φ = ∂φ + ieA, and calling the resulting fluid the COVARIANTLY INTERACTING FLUID. This results in an elegant extension of standard Higgs symmetry breaking, but with some additional parameters present. Previous work on the vector potentials shows that some of these have a thermodynamic interpretation, it is hoped that this is inherited in the fluid symmetry breaking models. The additional parameters can be partially studied with the help of an explicit Lagrangian, see below. Both of my approaches have been so far restricted to abelian gauge fields.

1.3 The Lagrangian formulation of fluids.

A perfect fluid has a Lagrangian formulation in which the Lagrangian is the pressure $p$. Variation is achieved by using the first law of thermodynamics

$$dp = n dh - nT ds,$$

(1)

where $n$ is the particle number, $T$ is the temperature, $s$ is the entropy, and $h$ the enthalpy. In four dimensions a vector can be decomposed into four scalars, however the five scalar decomposition

$$hV = W = \sigma + \sum_i \theta_s^{(i)} s_i^\mu, \quad V^\rho = -1,$$

(2)

$(i) = 1, 2$ is often used, because for $i = 1$, $s$ and $\theta = \int T d\tau$ have interpretation, Roberts (1997) [3], as the entropy and the thermasy respectively. From now on the index $(i)$ is suppressed as it is straightforward to reinstate. Replacing the first law with $dp = -VdW - nT ds$, variation gives the familiar stress and scalar evolution equations. Previously, Roberts (1997) [3], all partial derivatives in the above have been replaced with vector covariant derivatives

$$D^\mu = \partial^\mu + ieA^\mu,$$

(3)

to obtain a generalization of scalar electrodynamics called fluid electrodynamics. For many interacting fluids the interaction terms can be disregarded, Anile (1990) [4], however here all interaction
terms are kept. Quantization of these models is not looked at here although fluids can be quantized, Roberts (1999) [5]. Here instead of using the first law for variation a specific explicit Lagrangian is assumed. This fixes the equation of state. Since previous work three things are approached differently. The first is the best conventions for the scalar decomposition are 2. This is because the spacetime index is put on $s$ as this will allow easier generalization to second order non-equilibrium thermodynamics, Israel and Stewart (1979) [6]. The + convention is used for $\theta$ and $s$ in 2 (rather than $-\theta, s$). The second is that the fluid remains isentropic after charging. The third is that two vector normalization conditions are required after charging. Equivalences between fluids and scalar fields were first studies by Tabensky and Taub (1973) [7]. Thermodynamical quantities can be introduced into the standard Higg’s model via the partition function Kapusta (1989) [8]. Clearly the entropy, temperature and so on cannot occur from both the vector 2 and the partition function, perhaps a suitable partition function for the present model might allow both to be identified. An approach which dispenses with Higgs fields is that of Nicholson and Kennedy (2000) [9], another approach which involves gravity is that of Kakushadze and Langfelder (2000) [10].

2 Explicit Lagrangians

Consider the Lagrangian

$$L(q, q_\rho) = \beta (-W_\rho W^\rho)^r - Q(q),$$

where $r$ is called the equation of state parameter (see equation 9), $\beta$ is called the sign parameter and $Q$ is the potential. Varying with respect to the metric and then assuming the stress is of the form of a perfect fluid implies

$$-2(-1)^r \beta r W_\rho^{2r-2} W_\mu W_\nu = (p + \mu) V_\mu V_\nu.$$  

Normalization $V_\rho^2 = 1$ and $hV_\mu = W_\mu$ implies

$$W_\rho^{2r} = (-1)^r h^{2r}.$$  

Recall that $L = p$, together with the above three equations this gives

$$p = \beta h - Q, \quad \mu = (2r - 1) \beta h^{2r} + Q, \quad n = 2\beta rh^{2r-1}.$$  

Varying $L$ with respect to $\sigma, \theta$ and $s$ respectively

$$(nV^\rho)_\rho - Q_\sigma = 0, \quad -n\dot{s} - Q_\theta = 0, \quad (n\theta V^\rho)_\rho - Q_\sigma = 0.$$  

This is a perfect fluid; when $Q = 0$ it has the $\gamma$-equation of state

$$1 < \gamma = \frac{2r}{2r - 1} < 2, \quad \frac{\gamma}{2\gamma - 2} = r < \frac{1}{2}.$$  

Particular cases are $r = 1$ which implies $\gamma = 2$ which is the equation of state for coherent radiation; and $r = 2$ which implies $\gamma = \frac{4}{3}$ which is the equation of state for incoherent radiation. The canonical momenta are

$$\Pi^\sigma = -n, \quad \Pi^\theta = 0, \quad \Pi^s = n\theta.$$  

The constrained Hamiltonian is

$$H_\lambda = \Pi^\sigma (\dot{\sigma} + \theta \dot{s}) + \lambda^1 (\Pi^\sigma - \theta \Pi^\theta) + \lambda^2 \Pi^\theta - L,$$  

where $\lambda^1$ and $\lambda^2$ are the Lagrange multipliers. The momenta and Hamiltonian are the same as in the general case. In the general case the Euler equations and the canonical stress vanish identically; however for explicit Lagrangians the Euler equations are the same as \[15\] and the canonical stress is the same as the stress.
3 Charged Explicit Lagrangian.

All partial derivatives are replaced by vector covariant derivative. Capital letters are used for the new quantities and small letters for the uncharged quantities. The Lagrangian becomes

\[ L \rightarrow L = \beta(-W\ast\rho W\ast\rho) - Q(q) - \frac{1}{4}F^2, \]  

(12)

where "\ast" denotes complex conjugate. The vector field becomes

\[ W_\mu = \sigma_\mu + \theta s_\mu + i(\sigma + \theta s)eA_\mu = w_\mu + i\bar{e}A_\mu, \]  

(13)

where \( \bar{e} \equiv (\sigma + \theta s)e \). Under the global transformations

\[ \sigma \rightarrow \exp(-ie\lambda)\sigma, \quad s \rightarrow \exp(-ie\lambda)s, \quad \theta \rightarrow \theta, \quad A_\mu \rightarrow A_\mu + \lambda_\mu, \]  

(14)

the vector field changes to \( W_\mu \rightarrow \exp(-ie\lambda)W_\mu \) so that \( W_\mu W^\ast_\mu \) is invariant, implying that the Lagrangian is invariant. One can choose that \( A_\mu = 0 \) and then these transformations are local.

The Noether current is

\[ J^\mu = \beta r(-W_\rho W^\ast_\rho)^{-1} \{ -i(\bar{e}^* w^\mu - \bar{e}w^* \mu) + 2\bar{e}\bar{e} A^\mu \}. \]  

(15)

One can introduce two normalization conditions

\[ V_\mu V^* \mu = -1, \quad v_\mu v^\mu = -1, \]  

(16)

which imply

\[ H^2 = h^2 - e^2 A^2_\rho. \]  

(17)

Proceeding as before

\[ P = \beta H^{2r} - Q - \frac{1}{4}F^2, \quad \bar{\mu} = \beta(2r - 1)H^{2r} + Q + \frac{1}{4}F^2, \quad N = 2\beta r H^{2r-1}, \]  

(18)

and the canonical momenta are

\[ \Pi^\sigma = -N, \quad \Pi^\theta = 0, \quad \Pi^s = -N\theta, \quad \Pi^{A_\mu} = 0, \]  

(19)

the last of which is surprising; because not all the components of \( \Pi^{A_\mu} \) vanish in usual gauged scalar electrodynamics. The stress is

\[ T_{\mu\nu} = NH(V_\nu V^*_\nu + V^*_\mu V_\nu) + Pg_{\mu\nu}, \]  

\[ = \frac{N}{H}(w_\mu w_\nu + e^2 A_\mu A_\nu) + Pg_{\mu\nu}, \]  

\[ T = 3P - \bar{\mu}, \]  

\[ v^\alpha v^\beta T_{\alpha\beta} = \mu + e \frac{N}{H}(A^2_\alpha + (v^\alpha A_\alpha)^2). \]  

(20)

The conservation law is

\[ T^\beta_{\mu,\beta} = w_\mu Q_\sigma + \frac{N}{H}w_\mu + e^2 \left( \frac{N}{H} A_\mu A^\beta \right) + P_\mu. \]  

(21)

The constrained Hamiltonian is

\[ H_\Lambda = \Pi^\sigma (\dot{\sigma} + \theta \dot{s}) + \lambda^1(\Pi^s - \theta \Pi^\sigma) + \lambda^2 \Pi^\theta + \lambda^3 \Pi^{A_\mu} - L, \]  

(22)
Variation with respect to the scalar and vector fields gives
\[ \delta \sigma : Q_\sigma = (N \frac{h}{H} v^\alpha)_\alpha, \]
\[ \delta \theta : -Q_\theta = N \frac{h}{H}, \]
\[ \delta s : Q_s = (N \frac{h}{H} v^\alpha)_\alpha, \]
\[ \delta A_\mu : F^\beta_{\mu \beta} = e^2 N \frac{h}{H} A_\mu. \]  
(23)

Note that if one starts with the electrodynamical Lagrangian \( L = -\frac{F^2}{4} \) and tries to implement a substitution scheme from this via \( A_\mu \rightarrow A_\mu + kW_\mu \) one cannot recover the above type of charged fluids because \( F^2 \rightarrow F^2 + \) second derivatives in the scalar fields, which do not occur in the above. Thus there is no mirror mechanism.

4 Comparison with the Higgs’ Model

To recover scalar electrodynamics first note that
\[ W_\alpha W^{* \alpha} = w_\alpha^2 + e^2 A_\alpha^2. \]  
(24)

Now set the state parameter \( r = 1 \), the sign parameter \( \beta = -1 \), and the potential \( Q(q) = V(\rho^2) \) so that
\[ L = w_\alpha^2 + e^2 A_\alpha^2 - V(\rho^2) - \frac{1}{4} F^2, \]  
(25)

which has no cross term \( w_\alpha A^\alpha \). Now set \( \sigma = \rho \), \( \theta = s = 0 \). Changing the gauge \( A'_\mu = \nu_\mu + A_\mu \) and dropping the prime gives scalar electrodynamics in radial form. This has lagrangian, stress and equations of motion
\[ L = \rho_\alpha^2 + (\vec{\nabla}_\alpha \nu)^2 - V(\rho^2) - \frac{1}{4} F^2, \]
\[ \vec{\nabla}_\mu \nu = \rho(\nu_\mu + eA_\mu), \]
\[ T_{\mu \nu} = 2\phi_\mu \phi_\nu + 2\vec{\nabla}_\mu \nu \vec{\nabla}_\nu \nu + F_{\mu \gamma} F^\gamma_{\nu} + g_{\mu \nu} L, \]
\[ F^\mu_{\alpha \beta} + 2 e \rho \vec{\nabla}^\mu \nu = 0, \]
\[ 2(\vec{\nabla}_\nu + (\nu_\beta + e A_\beta)^2 + V') \rho = 0, \]
\[ 2(\vec{\nabla}_\nu + e A_\beta) = 0, \]  
(26)

respectively. Giving the Higgs field a non-zero expectation value \( \langle 0 \mid \rho \mid 0 \rangle = a \). \( \rho \) transfers to
\[ \rho \rightarrow \rho' = \rho + a, \]  
(27)

and the lagrangian becomes
\[ L = \rho_\beta^2 + (\rho + a)^2 e^2 A_\beta^2 - V((\rho + a)^2) - \frac{1}{4} F^2, \]  
(28)

the term \( a^2 e^2 A_\beta^2 \) is a constant mass term.

For a fluid one can perform a ‘scalar fluid rotation’
\[ \sigma \rightarrow \sigma' = \sigma + \theta s. \]  
(29)
Starting with scalar electrodynamics and identifying $\rho$ with $\sigma$ does not give one of the above fluids as gradients in $\theta$ appear. Alternatively there is the ‘vector fluid rotation’

$$\sigma_\mu = \rho_\mu \rightarrow w_\mu. \quad (30)$$

Starting with scalar electrodynamics and performing this rotation and replacing $\rho$ with $\sigma$ gives a fluid of the above type. One can then assume a VEV $\langle 0|\sigma|0 \rangle$ to generate a constant mass term as above, the difference being that some account of thermodynamics has been achieved.

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